Can overpricing technology stocks be good for welfare? Positive spillovers vs. equity market losses*

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Abstract

This paper examines the real impact of "booms-and-busts" of equity prices of technology-intensive firms, such as the late 1990s episode. We emphasize that what makes such episodes different from "booms-and-busts" related to other assets is the presence of knowledge spillovers. Such spillovers imply underinvestment in R&D at the aggregate level. Therefore, when temporarily high equity prices create incentives to invest more in R&D there are permanent wage and productivity gains. Sufficient conditions for these gains to always offset the direct negative effects from losses of equity trading and firm-level overinvestment are that overpricing is small and lasts longer.

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"...in the 20th century, we all know, we had this dot-coms boom-bubble-and-bust. Most people, at the end, lost money on dot-coms, but it left us with this incredible internet highway, on which Google and Microsoft emerged and forged whole new industries."
Thomas L. Friedman (New York Times), interview at MSNBC, October 22nd, 2008

1 Introduction

The financial crises of the late 1990s and 2000s reignited a lively academic and policy discussion about the real economic impact of asset price fluctuations. Both crises were associated with not only outstanding "booms" in equity prices, but also "busts" involving losses in the equity market. However, these two crises had a very different follow-up effects on real outcomes, like productivity and output. In his review of the history of stock market crashes between 1800-2000, Bordo (2003) points out that the asset price busts related to new technologies have had relatively limited negative implications for the aggregate economy.[1] In sharp contrast, he shows that recessions associated with real estate market busts last longer and lead to twice as large output losses compared to the ones of any other assets.[2]

Such experiences have triggered a growing interest in incorporating frictions in the asset market in general equilibrium models.[3] While this literature invariably accounts for the important role of asset markets for aggregate outcomes, it often does not shed light on why and how "booms-and-busts" associated with new technologies, such as the Information and Communication Technologies (ICT) in the late 1990s, are different. This paper contributes to the literature of asset bubbles by offering an understanding of the implications of technology-related episodes. We ask: Can a boom in stocks of technology-intensive firms be welfare increasing in the

[1] In addition to the late 1990s episode, other well-known episodes regard the introduction of railways in the United Kingdom and the United States in the mid 19th century, electricity in the United States in the 1920s (see also Shiller 2000, Kidleberger 2005, Crafts 2000, Jovanovic and Rousseau 2006).

[2] A similar argument is found in Catão (2002).

[3] See, among others, Bernanke and Gertler (1989), S.Bernanke, Gertler, and Gilchrist (2000), Bernanke and Gertler (2001), Kiyotaki and Moore (1997), Charlstrom and Fuerst (1997), Holmstrom and Tirole (1997), Cooley, Marimon, and Quadrini (2004), Jermann and Quadrini (2010), Kyotaki and Gertler (2010).

short-run and long-run, even if the original boom was ex-post unjustified by firm-level fundamentals?

To investigate the welfare implications of technology stocks overpricing we develop an analytically tractable general equilibrium model that captures a number of empirical regularities. Our model accounts for the trade-off that is emphasized in our opening quote. First, equity overpricing results in short-term losses for investors who own the equity of these firms at the time of bust. Second, when equity prices affect R&D producers' incentives to invest in R&D, it also leads to negative net present value (NPV) investments at the firm level. Taken together, these two direct effects amount to a negative welfare impact. This impact is present also in a partial equilibrium analysis. At the same time, there is a positive effect that is realized indirectly and appears only at the general equilibrium level. Namely, by increasing aggregate investments in R&D, overpricing induces a permanent increase in aggregate productivity and workers' wages through the new products that become available via such investments.

The main finding is that such productivity gains can translate in permanent gains in the aggregate expected welfare. Importantly, we show also that there are potential gains in terms of the ex-post realized welfare. We find that the sufficient conditions for the realized gains, even in the short run, are that overpricing is not too high and that it lasts long enough. In this regard, our results are particularly strong as they describe the environment under which an agent would always be better off in the "bubble-like" economy, even at the date when equity prices fall. The predictions of our model for short-run capital losses and long-run productivity and wage gains from technology-related equity price fluctuations are confirmed empirically for the case of the 1990s' "dot-com bubble" (see discussion in Section 1.1).

It is important to note that our model delivers innovative insights and results because it considers two distinct features of the 1990s' episode in a unified framework. First, we consider an overpricing episode that is temporary. Second, we show that the presence of welfare gains relies on the type of assets that are overpriced (i.e., productive assets with positive knowledge externalities).

The first feature differentiates our framework and its implications from the real bubbles literature (e.g., Tirole (1985), Ventura (2006) and Ca-

ballero, Farhi, and Hammour (2006)), where "bubbles" are commonly tied to unproductive assets that are infinitely traded in a pyramid scheme, i.e. that never go bust. For the case of bubble-like episodes related to productive, technology-related assets like in our setting, Johnson (2007), Pastor and Veronesi (2005), Pastor and Veronesi (2006), and Pastor and Veronesi (2009), show how "bubbles" can arise as a consequence of rational learning about new technologies. Because these models do not involve overinvestment in equilibrium, there are no real costs when asset prices go "bust". Nevertheless, such costs are empirically relevant. For this reason, they are incorporated in our welfare analysis.

The same argument extends to Olivier (2000) who examines the welfare effects of an equity price bubble on productive assets that never go "bust" and thereby there are never any capital losses in the equity market. Similarly, to our paper, Olivier (2000) analyzes the case of R&D related assets and highlights that for the presence of positive welfare effect is crucial to have positive externatities. However, the only costs of overpricing that he considers come from an endoegenous increase of the real interest rate that is of a second-order importance in comparison with the direct costs through capital losses at the time when equity prices fall.[4] Moreover, his framework predicts predicts long term welfare gains though steady state growth rate of GDP (productivity and wages). We emphasize in the next section that the 1990s were rather characterized by long-term gains through only the level of GDP (productivity and wages), which is what our model predicts.

Kraay and Ventura (2007) relates to the 1990s episode and considers a bubble driven by investors sentiment that was temporarily diverging from actual fundamentals. While they consider a bubble that goes "bust", the main contrast to our paper is that they study the real implications of bubbles in unproductive assets as opposed to productive assets. Namely, they do not model the interaction between stock market dynamics and the distinct characteristics of R&D sector that drives the aggregate productivity, which is the main focus of our paper. Also, the main interest in their paper is on the dynamics of the US government debt and current account in the early 2000s'.

We highlight why there are distinct benefits associated with technology-

[4]We also consider endogenous interest rate in the extension of our basic model.

related assets' "booms", as opposed to other assets. In particular, we emphasize that knowledge spillovers in R&D production induce permanent productivity and wage benefits, despite the negative NPV investments at the R&D firm-level.[5] This is because such spillovers cause a well-understood appropriability problem for private sector R&D that induce underinvestment in R&D. Namely, a claim on the future flow of profits from selling the product, i.e., an equity contract, is not a claim on the full productivity benefits created by developing and manufacturing a new product. Therefore, in this context, an episode of equity overpricing temporarily creates the "missing" market incentives for R&D investment and delivers net welfare gains.

The prevalence and importance of knowledge spillovers in R&D production is supported by numerous empirical studies (for a review see Griliches 1992, Ishaq 1993, Hall 1995). There is also substantial support that there is not enough R&D in countries close to the technological frontier. For example, Jones and Williams (1998) suggest that optimal R&D in the United States is at least four times larger than actual R&D (see also Jones and Williams 2000, Comin and Gertler 2006).[6] Underinvestment in R&D has also been found to be more relevant in sectors with high R&D intensity and expenditures, like the ICT producing one that was at the centre of the late 1990s episode.[7]

The presence of knowledge externalities alone is nevertheless insufficient to generate the positive aggregate effects of the equity market boom. This is because in a frictionless world where the firms' incentives to invest in technology are not affected by stock prices and there are no credit constraints, the R&D investments do not increase during an equity price boom. In our setting, we allow for a direct feedback from equity price movements to incentives to innovate.

[5] Knowledge spillovers have a central role in R&D-based models of endogenous growth (starting from Romer 1990, Aghion and Howitt 1992, Grossman and Helpman 1991).

[6] These findings are despite the United States government's strong involvement in R&D, which suggests that this friction is not already corrected by R&D subsidies.

[7] The ICT-producing sector is highly intensive in R&D and patenting activity (Carlin and Mayer 2003), and receives below-average federal support for its R&D (NSF). A calculation in the spirit of Jones and Williams (1998) for the ICT-producing sector would imply a much higher scale of market underinvestment in R&D compared to the average industry.

Such a link is motivated in a wide set of economic environments. When entrepreneurs have no superior information compared to the market participants, they would react to prices as they learn information from their movements. However, incentives to invest can be affected by equity prices also when entrepreneurs have superior information. For example, Holmes and Schmitz Jr. (1990) show that equity markets enable efficient specialization, since innovators can specialize in creating new firms by selling them to agents who have a comparative advantage in management. Such exit opportunities are particularly important for R&D intensive sectors, where innovative talent is scarce and therefore gains from specialization are high. Moreover, the existence of exit opportunities is a crucial factor for venture capitalists and growth capital in their investment decisions, both of which are major sources of funding of innovative projects and are generally seen as not credit-constrained (see Jovanovic and Szentes 2007, Kortum and Lerner 2000).

In our baseline model, deviations of equity prices from fundamentals arise because short-horizoned, risk neutral investors rationally react on noisy public information.[8,9] In particular, we examine the case when economic agents receive unexpected public news that there is a possibility that the productivity level will increase permanently at some future date. This modelling choice of the source and nature of optimism is motivated by the fact that the late 1990s "boom" period was associated with the widespread euphoria that the new technologies paved the way for a "new economy". One example of that euphoria, is Alan Greenspan's speech at the Federal Reserve Bank of Chicago, March 6, 1999[10]:

"A perceptible quickening in the pace at which technological innovations are applied argues for the hypothesis that the recent acceleration in labor productivity is not just a cyclical phenomenon or a statistical aberration, but reflects, at least in part, a more deep-seated, still developing, shift in our economic landscape." [11]

[8] There is ample empirical evidence that equity prices can deviate from fundamentals (see extensive surveys by Shleifer 2000, Shiller 2000, Barberis and Thaler 2002).

[9] The role of "news" for future productivity in driving economic fluctuations has also been analyzed in the business cycles literature (starting from Pigou 1927); see also Beaudry and Portier (2006) for evidence.

[10] See also Gordon (1999), or Goldfarb, Kirsch, and Miller (2007).

[11] While harder to distinguish in the data, in our model there is a clear separation

It is however noteworthy that our assumption that optimism is exogenously driven by public information is made both because it allows for a simple analytical solution and permits flexible interpretation of the source of overpricing.[12] One could endogenize it though using both behavioral or rational mechanisms.[13] Concerning the latter, in a related paper, Tinn (2010) considers an environment where short-horizoned entrepreneurs have superior information about the value of their firm compared to the average equity market participant and care about the exit value of their firm. Tinn (2010) shows that in such a case investment itself becomes a positive public signal about the value of their firm. This creates further incentives to invest in technology and can lead to negative NPV at the firm level, as in the current paper.[14] Therefore, the afore mechanism argues for overpricing, rather than underpricing, in the context of new technologies.

Our baseline model is an overlapping generations one, which we consider the preferred approach in the context of episodes similar to the 1990s one. This is because it captures better the speculative nature of institutional equity market participants, while it makes our results directly comparable to the existing rational bubbles literature. Importantly, this framework also allows us to perform a more elaborate and conservative welfare analysis, since we trace the evolution of consumption, gains and losses over time.[15]

We also extend our analysis to consider an infinitely lived agent and endogenous interest rate. We show that our main results are similar to our baseline results when the representative agent is risk neutral as in the baseline model. They also remain valid more generally for a wide set of parameters in the case of a risk averse representative agent.

It is important to acknowledge at this point that equity prices can be

between the labor productivity related to the economy's ability to use the new technology and the one related to the creation of new technological products. The optimism is our setting is about the former.

[12] For example, in DeMarzo, Kaniel, and Kremer (2007) overpricing arises because of a "keeping up with Joneses" motive.

[13] While the source of this positive public signal is open to interpretation, it should also be noted that all agents in this model are rational given their information at any point in time.

[14] Angeletos, Lorenzoni, and Pavan (2010) analyze a setting where learning is both-sided, entrepreneurs learn from equity prices and market participants learn from investments. However their setting does not incorporate knowledge spillovers.

[15] Conclusions derived from an OLG framework, are also more informative about policy implications in an environment, where policy makers are likely to be short horizoned.

positively correlated to R&D investments also in the case that there are credit constraints. This is because, when there are credit constraints, higher equity prices reduce the cost of financing, so that (at least some) firms can undertake profitable investments they could not otherwise (see e.g., Stein 1996).[16] In the context of the 1990s episode, Jermann and Quadrini (2007) suggest that the productivity gains can be interpreted as the result of relaxed credit constraints.[17]

The positive real effects of equity overpricing due to relaxed credit constraints features also in the analysis of Farhi and Panageas (2007). They emphasize though that equity overpricing encourages just as well negative NPV investments due to asymmetric information and moral hazard problems. They find that the negative effect of such unproductive investments drove the sector-level outcomes for the period 1980-2005. Polk and Sapienza (2009) shed further light on the potential negative effects of overpricing on firm investment. Specifically, they support that equity overpricing leads to overinvestments at the firm level, even after controlling for the equity issuance channel. Moreover, their finding is stronger for R&D intensive sectors. Both aforementioned findings are consistent with our modelling approach.

As is clear from the above discussion, incorporating credit constraints in our framework would strengthen our baseline results. This is because increases in equity prices that relax credit constraints would imply unambiguous benefits from financing the positive NPV projects of credit-constrained firms, as in Jermann and Quadrini (2007). Therefore, we abstract from credit constraints in this paper purely for the clarity of our argument and not because this channel is not empirically relevant.

The remainder of the paper is organized as follows. Section 1.1 discusses the empirical facts about 1990s technology related episode. Section

[16] There is overwhelming support for the positive relation between equity prices and investment at the aggregate economy, or firm-level (e.g., Barro 1990, Morck, Shleifer, and Vishny 1990, Blanchard, Rhee, and Summers 1993). There is also evidence that for credit-constrained firms equity prices affect investment through the equity issuance channel (see Baker, Stein, and Wurgler 2003).

[17] Indeed, Hall and Lerner (2010) review theoretical arguments why credit constraints matter for the case of R&D intensive firms and cite ample evidence in their support. Also, Allen and Gale 2000 survey theoretical arguments why some activities, like R&D, may be dependent on equity finance.

2 sets up the baseline model with overlapping generations of risk-neutral consumers. Section 3 presents the results of the baseline model focusing on the impact of temporary equity overpricing on consumption. Section 4 considers the extension with infinitely lived consumers with CRRA utility. Section 5 concludes.

1.1 The 1990s technology-related episode

As already argued, our model offers the background to interpret the late 1990s episode of equity price "boom-and-bust" as a period of temporary overpricing of technology stocks driven by optimism. Indeed, during this period, the equity price movements were positively correlated with the temporary acceleration of R&D investment that was almost entirely concentrated in a few high-tech industries, like pharmaceuticals, medical equipment and ICT (see Brown, Fazzari, and Petersen 2009). In particular, from 1994 to 2004 there was a dramatic boom in R&D: the ratio of privately financed industrial R&D to GDP rose from 1.40% in 1994 to an all-time high of 1.89% in 2000.

The 1990s optimism regarding rising profitability that lead to the boom, collapsed with the disappointing announcements of sales at the end of 1999 (see the historical review in Goldfarb, Kirsch, and Miller (2007)). In line with our model, investors received losses in the aftermath of the equity market bust. Goldfarb, Kirsch, and Miller (2007) document that as of September 2002 and following an 18-month long decline, the NASDAQ index closed with a 4.4 trillion USD market value loss. About a quarter of this loss concerned the 150 largest Silicon Valley firms.[18] Along with the decline in the equity market, the R&D expenditures followed suit with a downward correction in the ratio of privately financed industrial R&D to GDP.

Nevertheless, the technology firms were overall productive, as their resulting survival rate was "on par with or higher than other emerging industries" (Goldfarb, Kirsch, and Miller 2007).[19] Further to this, another

[18] Also, part of these losses came from the "dot-com" firms that had an IPO during the boom period, but never proceeded further from the business planning stage (about 19% of all IPOs).

[19] In line with this, Helbling and Terrones (2003) report that the market-to-book ratios

well-documented fact during the NASDAQ boom period was the acceleration of new technologies, output and wages (see, e.g. Staiger, Stock, and Watson 2001): In the mid-1990s, the United States labour productivity accelerated by 1.05 pp. (see Table 8.3 in Jorgenson, Ho, and Stiroh 2005). Consistent with our model's predictions, the trends in these variables eventually regressed to their original levels after 2001,[20] but the event of their short-term acceleration brought important gains in their levels by the end of the 1990s. In fact, over the entire decade wage gains amounted to 6.9 percent, and employment gains to 15 percent (Ilg and Haugen 2000).[21]

2 The Model

2.1 Consumers

The economy is small and open, and is populated with overlapping generations of risk-neutral consumers, who work and invest in assets in the first period of their lives, and consume and retire in the second period of their lives. There is a representative consumer for each generation. In Section 4 we discuss the implications of the set-up of a closed economy with infinitely lived representative agent with more general preferences over risk.

A consumer born in period t is endowed with L units of labor that he supplies to the final goods producing sector for a wage w_t. There are two available assets: a risk-free bond and equity. The internationally traded

of dot-com firms remained broadly above their pre-bubble levels in the aftermath of the 2000 bust.

[20] Robert J. Gordon emphasized the short-lived nature of the gains in trend-productivity (e.g., see Gordon 2000), while that of wages is reported by Jared Bernstein and Lee Price in their Economic Policy Institute report of September 2nd, 2005. See also the retrospective study of Jorgenson, Ho, and Stiroh (2007).

[21] Jared Bernstein and Lawrence Mishel of the Economic Policy Institute report: "Income and poverty data for 1998 [..] show that American families are clearly benefiting from the tight labor market and the strong economy. Median family income grew by $1,475 in 1998, a 3.3% increase over 1997 [..] the largest one-year gain since 1986. Taken in tandem with last year's growth of 3.0%, median workers have not had two such strong consecutive years in well over two decades."

In 2007 Jared Bernstein reported in the New York Times that according to the United States Census, the "median annual family income" had increased by 11 percent between its late 1980s' peak and its peak in 2000, while by 2007 it fell only by 1 percent with the latter decline invariably attributed to within-cohort income distribution changes that are beyond the scope of our study.

risk-free bond offers a gross return $R > 1$ and its supply is infinitely elastic. The supply of equity in period t involves shares of monopolistic firms that engage in R&D and intermediate goods production. All firms pay out their profits as dividends, π_t, and their shares are traded in the equity market at the post-dividend price P_t. The symmetry across assets is a conjecture to be verified in equilibrium and is due to the lack of firm-specific risks.

The budget constraint of a consumer born in period t is

$$\begin{aligned} w_t L &= B_t + P_t H_t, \\ C_{t+1} &= (P_{t+1} + \pi_{t+1}) H_t + R B_t, \end{aligned} \tag{1}$$

where C_{t+1} is his consumption in period $t+1$, B_t is his demand of risk-free bond and H_t is his equity demand.

Consumers choose their bond and equity holdings to

$$\max_{b_t(i), h_t(i)} \left\{ E\left[C_{t+1}\right], \text{ s.t. } (1) \right\}. \tag{2}$$

2.2 Final good production

Competitive final good producers use labour, L, and all available intermediate goods to produce output Y_t. There is a continuum of distinct intermediate good varieties available in period t, denoted with $x_t(j)$, where $j \in [0, A_t]$ and $A_1 > 0$. The production function is

$$Y_t = (\phi_t L)^{1-\alpha} \textstyle\int_0^{A_t} x_t^{\alpha}(j) dj; \ \alpha \in (0,1), \tag{3}$$

where ϕ_t is the labour augmenting productivity shock that is the only source of uncertainty in this economy, whose process is presented in detail in Section 2.4.

The price of the final good is normalized to one. Final goods producers take the wage, w_t, and the price of intermediate goods, $p_{x_t}(j)$, as given and maximize profits:

$$\max_{L, \{x_t(j)\}_j} \left\{ Y_t - \textstyle\int_0^{A_t} p_{x_t}(j) x_t(j) dj - w_t L \text{ s.t. } (3) \right\}. \tag{4}$$

The intermediate goods depreciate fully within a period.

2.3 Intermediate good production and R&D

The final good producers buy each intermediate good, $x_t(j)$, from a monopolistic intermediate goods producing firm j.

The production of every unit of intermediate good requires the investment of η units of final good. In each period t, an intermediate goods producer j maximizes profits

$$\pi_t(j) = \max_{p_{x_t}(j), x_t(j)} \left\{ p_{x_t}(j) x_t(j) - \eta x_t(j),\ s.t.\ p_{x_t}(j) = \frac{\partial Y_t}{\partial x_t(j)} \right\}. \qquad (5)$$

In period t, there is a continuum of active intermediate goods producers indexed with $j \in [0, A_t]$.

An intermediates good firm is established by the development of a new variety. The development of a new variety in period t, $e \in (A_t, A_{t+1}]$, requires R&D investment one period before the new variety becomes available, i.e., period $t+1$. This provides the intermediate goods firm, e, with infinitely lasting monopolistic power (e.g., by the means of a patent) and its profits are $\pi_{t+k}(e)$, where $k \geq 1$.

The R&D production sector is fully competitive. R&D producers take the value of their firm, P_t, and aggregate productivity of R&D, $\bar{\lambda}_t$, as given and maximize their expected net gain from R&D investment, I_t,

$$\max_{I_t} \left\{ P_t \left(A_{t+1} - A_t \right) - I_t, \text{ s.t. } A_{t+1} - A_t = \bar{\lambda}_t I_t \right\}. \qquad (6)$$

To capture the direct effect of equity prices on the incentives to invest in R&D, it is assumed that the value of a new intermediate good firm for its owner is equal to firm's equity market value. All intermediate good firms already established in period t, i.e., those established before and already producing, $j \in [0, A_t]$, and those currently conducting R&D, $e \in (A_t, A_{t+1}]$, are listed in the equity market. Each of these firms corresponds to one divisible share that is held by the consumers. The value of all shares listed in period t is the same, which is a conjecture that is verified from the equilibrium results of Section 3.1.

While each individual R&D producer takes the R&D productivity $\bar{\lambda}_t$ as given, it is a function of aggregate decisions. In the spirit of Comin and

Gertler (2006) and Jones (1995), the basic analysis assumes that

$$\overline{\lambda}_t = \lambda I_t^{\rho-1} A_t^{1-\rho};\ \lambda > 0,\ \rho \in [\alpha^{\frac{1}{1-\alpha}}, 1), \tag{7}$$

which ensures a well-defined steady-state, bears an exogenous component, λ, and captures two spillover effects of the individual R&D entrepreneurs' decisions. First, an increase in the number of known varieties, A_t, increases permanently the R&D productivity (positive knowledge externality). Second, as the aggregate R&D investment increases, the marginal productivity of individual R&D investment decreases (negative congestion externality). The parameter ρ captures the overall productivity of R&D production, as it governs the relative strength of knowledge externalities in R&D production over the congestion ones.

It is worth noting that parameter ρ in (7) is crucial in building up a hypothesis for technology related assets. As already emphasized in our Introduction, a defining feature of R&D production is that it bears positive spillovers that are sufficiently strong to imply aggregate underinvestment in a competitive equilibrium without mispricing compared to the optimal allocations, aka the Social Planner's equilibrium. The condition that $\rho \in [\alpha^{\frac{1}{1-\alpha}}, 1)$ ensures that our model bares this R&D feature, which is why it is maintained throughout our basic analysis.

Yet, the specification in (7) is flexible enough to allow the discussion of the model's implication for values of parameter ρ out of this interval. In Section 3.2.3 we contrast our results to the ones of the limit case where $\rho = 0$. When $\rho = 0$, then the intermediate goods sector is a "non-R&D sector" because we effectively shut down the effect of positive knowledge spillovers on R&D production. It is also clear from (7) that the congestion externality is the strongest when ρ is close to zero. This allows us to highlight why the aggregate impact of technology related "booms-and-busts" is different from those related to other sectors featuring strong congestion externalities, like real estate.[22]

Figure 1 summarizes the interaction among all agents in this economy

[22] We acknowledge that we abstract from some other features of real estate, but our aim is to analyze the case of technology related assets. Therefore, we refer to real estate only for illustrative reasons as it is a common example of a sector with a strong congestion externality.

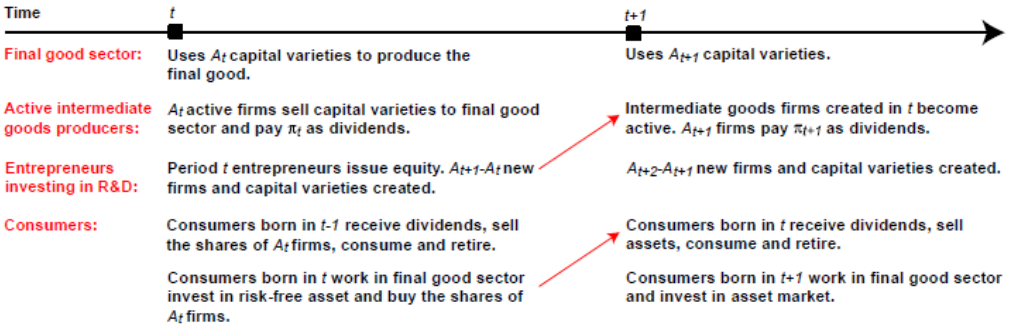


Figure 1: Timing of production and consumption decisions.

and the timing of their actions.

2.4 Information structure

All agents have identical information and there is only a short episode of uncertainty. Initially the labour productivity is constant at the level ϕ and expected to stay constant for all periods $t < \tau$. To capture the idea of the emergence of a "new economy", like the 1990s, we assume that in some arbitrary period $t = \tau$ there is an unexpected public signal of strictly positive likelihood of that there is may be a permanent shift in long-run productivity from $t = \tau + T$ onwards. Namely, there are two possible states, $u \in \{H, L\}$, that can be realized in period $t = \tau$:

$$\phi_t = \begin{cases} \phi^H \text{ w.p. } Q \\ \phi^L \text{ w.p. } 1 - Q \end{cases} \quad \text{for any } t \geq \tau + T,$$

where $\phi^H > \phi^L \geq \phi$. Perfect information concerns the case when $Q = 0$, so as to capture the presence of erroneous optimism of a permanent improvement in productivity thereby profits (dividends) of intermediate goods producers (see Section 3.1.1 below).

For the sake of clarity, we present the results for the case where $\phi^L = \phi$. Note that in periods $\tau \leq t \leq \tau + T - 1$, it holds that period t expectations are

$$E_t \left[\phi_{\tau+T+k}\right] = Q\phi^H + (1 - Q)\,\phi = \phi\,(1 + \kappa)\,,$$

where $k = 0, 1, ...$ and

$$\kappa = Q\frac{\phi^H - \phi}{\phi} \tag{8}$$

measures the "degree of optimism". All the main results are valid for some permanent improvement, i.e., $\phi_L > \phi$.[23]

It is worth keeping in mind, and as is shown below, that uncertainty about labour productivity implies uncertainty about the demand and thereby profits (dividends) of intermediate goods producers (see Section 3.1.1).

2.5 Markets

In every period t, the final good production, Y_t, and the returns from the previous period's risk-free asset investment, RB_{t-1}, are used to finance aggregate expenditures on consumption, $C_t \equiv \int_0^1 c_t(i)di$, investment in intermediate goods production, $X_t \equiv \int_0^{A_t} \eta x_t(j)dj$, R&D investment, I_t, and purchases of the risk-free asset $B_t \equiv \int_0^1 b_t(i)di$. The goods market clearing condition is

$$RB_{t-1} + Y_t = C_t + X_t + I_t + B_t. \tag{9}$$

All consumers are employed by the final goods sector, and the labour market clears for the equilibrium wage w_t.

The supply of equity in period t consists of A_t shares of current intermediate goods producers and $A_{t+1} - A_t$ new shares issued by R&D producers, so that supply is A_{t+1}. The equity price ensures market clearing, $H_t = A_{t+1}$.

[23] An earlier draft examines the case of adding private infomartion in a setting which is very close to the one of the present paper. The main results are robust to this extension, and are available from the authors upon request.

3 Results

3.1 Equilibrium outcomes

3.1.1 Final and intermediate good production

From (3) and (4), the final good producers' optimal demand for variety j is independent of the demand for any other variety $j' \neq j$, i.e., $p_{x_t}(j) = \alpha L^{1-\alpha} x_t^{\alpha-1}(j)$ for any j. Using this in (5), the demand for intermediate goods is linear in labor productivity and is the same across intermediate goods producers,

$$x_t(j) = x_t \equiv \left(\frac{\alpha^2}{\eta}\right)^{\frac{1}{1-\alpha}} L\phi_t. \tag{10}$$

The price of each variety is $p_{x_t}(j) = \frac{\eta}{\alpha}$. From (5), this implies that all intermediate goods producers have the same profits,

$$\pi_t = \Gamma\phi_t \ ; \ \Gamma \equiv \alpha(1-\alpha)\left(\frac{\alpha^2}{\eta}\right)^{\frac{\alpha}{1-\alpha}} L. \tag{11}$$

This shows that uncertainty about future labor productivity implies uncertainty about the future profits of intermediate goods producers. Because profits are perfectly correlated across intermediate goods producers, then the equity price, P_t, needs to be the same across firms in any period t. The latter confirms the original conjecture for symmetry of equity prices across firms in Sections 2.1 and 2.3.

3.1.2 R&D production

An individual R&D producer's decision for R&D investment (see equation (6)) determines the relationship between the equity price (the value of a new variety) and individual R&D productivity, $P_t = \frac{1}{\bar{\lambda}_t}$. Using (7), the endogenous $\bar{\lambda}_t$ implies

$$I_t = A_t \lambda^{\frac{1}{1-\rho}} P_t^{\frac{1}{1-\rho}}. \tag{12}$$

Perfect competition and free entry to R&D production implies zero profits for R&D producers and from (6),

$$P_t(A_{t+1} - A_t) = I_t. \tag{13}$$

This condition shows that aggregate R&D investment in period t equals the total stock market value of the new firms producing intermediate goods that are established in the same period.

Using (12) and (13), the R&D growth rate is an increasing function of equity price,[24] i.e.,

$$g_{A,t} \equiv \frac{A_{t+1}-A_t}{A_t} = \lambda^{\frac{1}{1-\rho}} P_t^{\frac{\rho}{1-\rho}}. \tag{14}$$

This highlights that the equilibrium equity price determines the development of R&D over time and therefore real economic activity. Given the elasticity of R&D growth with respect to equity price, $\frac{\rho}{1-\rho} > 0$, the real impact of equity price is higher the higher the R&D productivity ρ is.

In sharp contrast, in a "non-R&D sector", where $\rho = 0$, it is clear from (14) that the growth rate becomes exogenous as $\lim_{\rho \to 0} g_{A,t} = \lambda$.

3.1.3 Asset prices

From consumer problem (2), it follows that in any period t, the following no arbitrage condition holds, i.e. the equilibrium equity price is

$$P_t = \frac{E[P_{t+1} + \pi_{t+1}]}{R}. \tag{15}$$

By (15) and the law of iterated expectations and the equilibrium equity price equals the present discounted value of the expectations of future profits, i.e.,

$$P_t = \sum_{k=1}^{\infty} \frac{E[\pi_{t+k}]}{R^k}. \tag{16}$$

There is certainty in periods $t < \tau$ and $t \geq \tau + T$ and using (11) and (16) it holds that $P_t = \frac{\Gamma\phi}{R-1}$ for $t < \tau$. Similarly $P_t = \frac{\Gamma\phi^u}{R-1}$ for $t \geq \tau + T$ and $u \in \{H, L\}$. To relate to the 1990s episode, we consider the case where $u = L$ realizes.[25] During the period of uncertainty, the future profits are uncertain and prices can be found from (8), (11) and (16). Overall, in the

[24] $\frac{\partial g_{A,t}}{\partial P_t} = \lambda^{\frac{1}{1-\rho}} \frac{\rho}{1-\rho} P_t^{\frac{\rho}{1-\rho}-1} > 0$.

[25] Our results would be straightforwardly reversed if state $u = H$ realizes instead.

model economy the prices are

$$P_{\tau+k} = \frac{\Gamma\phi}{R-1} + \kappa\frac{\Gamma\phi}{(R-1)\,R^{T-k-1}}, \text{ for } k \in \{0, ..., T-1\}, \tag{17}$$
$$P_t = \frac{\Gamma\phi}{R-1}, \text{ for } t < \tau \text{ and } t \geq \tau + T.$$

In order to account for the effect of equity market overpricing, a useful benchmark is an economy where consumers have perfect public information; referred as the "PI economy" from now onwards. The PI equilibrium equity price, P_t^{PI}, $Q = 0$, and by (8) it also implies that $\kappa = 0$, i.e.,

$$P_t^{PI} \equiv \lim_{\kappa \to 0} P_t = \frac{\Gamma\phi}{R-1} \text{ for any } t. \tag{18}$$

The wedge between the model's and the PI economy's equity price is due to the perceived positive probability of the high state.

Finally, from consumer's first period budget constraints (1), and using that equity supply is A_{t+1}, then the aggregate risk free asset holding is

$$B_t = w_t L - P_t A_{t+1}. \tag{19}$$

3.1.4 Consumption, output and goods market clearing

Using (10) in final goods production function (3), the equilibrium final good production is linear in ϕ_t, given the endogenous productivity level A_t, i.e.,

$$Y_t = \frac{\Gamma}{\alpha(1-\alpha)}\phi_t A_t, \tag{20}$$

where Γ is defined in (11).

The equilibrium wage is determined by solving the final goods' producers profit maximization problem and is also linear in ϕ_t given A_t, i.e., from (4) and (20) $w_t = (1-\alpha)\frac{Y_t}{L} = \frac{\Gamma}{\alpha L}\phi_t A_t$.

From the consumers' budget constraint (1), the equity market clearing, (19) and (20), the aggregate consumption is

$$C_t = (P_t + \pi_t - RP_{t-1})A_t + Rw_{t-1}L = (P_t + \pi_t - RP_{t-1})A_t + R\tfrac{\Gamma}{\alpha}\phi_t A_{t-1}. \tag{21}$$

This highlights the channels through which imperfect equity market

affects welfare. There is a direct channel from the excess capital gains, $P_t + \pi_t - RP_{t-1} > 0$, or losses, $P_t + \pi_t - RP_{t-1} < 0$, from investing in one unit of equity. There is also an indirect channel from the impact that equity price has on the R&D production through (14) and thereby the number of varieties in every period. This indirect channel works by determining both the level of wages, $\frac{\Gamma}{\alpha}\phi_t A_{t-1}$, and level of excess capital gains or losses, $(P_t + \pi_t - RP_{t-1})A_t$.

Finally, using (10) for X_t, and (11), (19), (20), (21) in the goods market clearing condition (9), gives the free-entry condition for R&D production (see (13)). This confirms that the goods market clears in every period, which completes the equilibrium outcomes of the model.

3.2 Impact of temporary equity overpricing

This section addresses the main question of the paper by examining the impact that short-term equity overpricing has on economic allocations. Sections 3.2.1 and 3.2.2 consider the effect of overpricing on the R&D sector, where aggregate productivity is given by (7). Section 3.2.3 contrasts these findings to effects of overpricing on non-R&D assets, where $\rho = 0$.

3.2.1 Consumption gains

Given the equilibrium path of prices, (17) and (18), equation (21) fully specifies the path for aggregate consumption. In particular, from the equity price in PI economy (18) and profits (11), it is straightforward that there are no excess capital gains or losses in the PI economy, i.e., $P_t + \pi_t - RP_{t-1} = 0$. Consumption in PI economy is determined by wages which grows at the constant R&D growth rate $g_A^{PI} = \lambda^{\frac{1}{1-\rho}} \left(P^{PI}\right)^{\frac{\rho}{1-\rho}}$.

Further, from (14) and (17), R&D growth in the model economy is greater than in the PI economy for the same T periods that its equity prices exceed these in the PI economy, i.e., $g_{A,t+k} > g_A^{PI}$ for $k \in \{0, ..., T-1\}$. In turn, inspection of the path of consumption (21) suggests a potential trade-off between higher wages and excess capital losses. The following proposition summarizes the outcome of the interplay of the direct and indirect effects that temporary equity overpricing has on the consumption (21) of each generation.

Proposition 1 *When $T > 1$, and there is overpricing, $\kappa > 0$, then aggregate consumption in the model economy is at least as high as in the PI economy*

$$C_t \geq C_t^{PI},$$

for all periods, except $t = \tau + T$. In period $t = \tau + T$, the comparison of consumption between the model and PI economy is ambiguous and depends on the parameters of the model.

Proof. See Appendix A. ■

This result is driven by the positive indirect effect that higher equity prices have on consumption through higher R&D growth. The level of R&D in the model economy is higher than in the PI economy from $t = \tau + 1$ onwards, and all generations of consumers consuming from period $t = \tau + 2$ onwards receive higher wages compared to consumers in the PI economy.

At the same time, equity prices have a direct effect on the path of excess capital returns. This effect is present only for two generations. It is positive for the generation consuming in period $t = \tau$, which sells its equity holdings in the first period that equity price increases because of the optimistic public signal. This generation has higher consumption than the PI one, solely due to receiving excess capital gains. It is negative for the generation consuming in period $t = \tau + T$, which is the only one receiving excess capital losses. This generation faces high equity prices when investing, but it consumes when the impact of the single optimistic public signal on expectations is over and prices return to their original level. How much consumption decreases depends on the indirect impact of the signal through the expansion of R&D products, as well as the size of equity market, i.e., $A_{\tau+T}$. Figure 2 considers the case of a single-release of an optimistic signal in period t for $T = 6$ and illustrates its impact on economic variables of interest. Note that equity prices, R&D growth, consumption and wages are ratios of the model economy variables to the corresponding variables in PI economy.[26]

[26] The variables are "Equity price" - P_t/P_t^{PI}; "R&D growth" - $g_{A,t}/g_{A,t}^{PI}$; "Consumption" - C_t/C_t^{PI}, "Excess capital gains" - $(P_t + \pi_t - RP_{t-1})\, A_t$; "Riskfree-asset holdings" - $B_t/\left(B_t + P_t A_{t+1}\right)$ and "Wages compared" - w_{t-1}/w_{t-1}^{PI}. Parameters are were chosen for purely illustrative purposes with $\rho = 0.8$ and $\alpha = 0.3$.

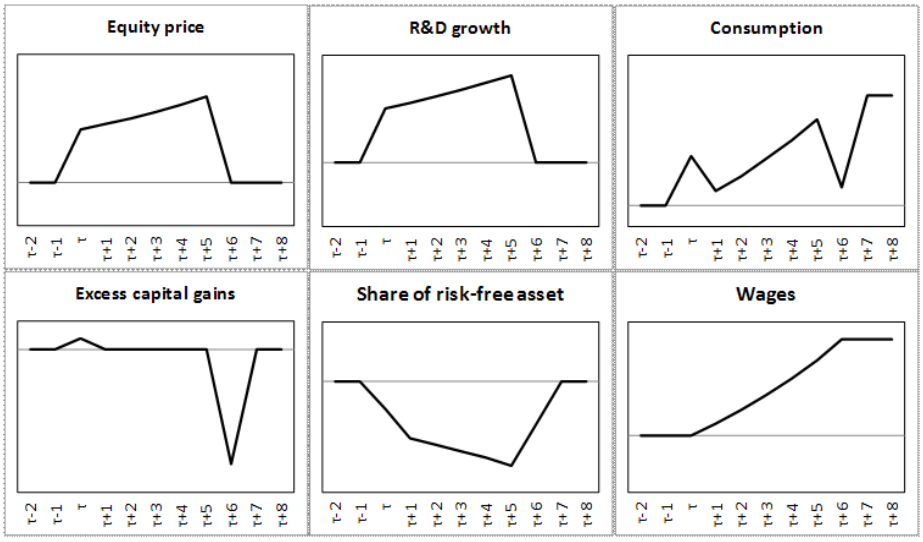


Figure 2: The impact of an optimistic signal on main economic variables.

3.2.2 Welfare gains

Proposition 1 above reveals that temporary overpricing in the equity market involves important trade-offs. First, there is an intergenerational consumption trade-off. Second, there is an intratemporal trade-off concerning the generation consuming in period $\tau + T$. This generation assumes the entire cost of raising R&D investment for T periods through excess capital losses, but gains though higher wages. To answer the question whether there is scope for welfare improvement in the presence of temporary equity overpricing, the criterion is Pareto improvement over the PI economy in the welfare of each generation.

Concerning ex-ante welfare, we look into the expected consumption level in period $t+1$ for the generation born in period t and ask whether the consumer cohort is ex-ante better off being in the Model economy, as opposed to the PI one.

Corollary 1.1 *Assume $\rho > 0$ and any degree of equity overpricing $\kappa > 0$. Then, no generation of consumers is ex-ante worse off compared to the PI economy, $U_t = E_t C_{t+1} \geq U_t^{PI} = E_t C_{t+1}^{PI}$, for any t.*

Proof. Because of the no arbitrage condition (15), there are no expected capital gains or losses for the risk-neutral individuals. Therefore, using (21) to calculate expected utility in equilibrium, it follows that $U_t = E_t C_{t+1} =$

$R\frac{\Gamma}{\alpha}\bar{\phi}A_t$, and $U_t^{PI} = E_t C_{t+1}^{PI} = R\frac{\Gamma}{\alpha}\bar{\phi}A_t^{PI}$, so that unless $\rho = 0$, then $U_t > U_t^{PI}$ $\forall t \geq \tau + 1$, while $U_t = U_t^{PI}$ $\forall t \leq \tau$. ■

We strengthen our conclusions by investigating further whether there is scope for Pareto improvement even in terms of ex-post realized welfare, i.e., realized consumption level. In other words, if consumers in period t were offered the choice between retaining the overpriced equity of the Model economy until $t+1$ and assuming any related capital gains, or shifting to the state of no history of equity overpricing, they would prefer to be in the Model economy.[27] A sufficient condition for this is the one ensuring no net losses for the realized consumption of generation $\tau + T$.

Corollary 1.2 Assu*me that* $T > 1$ *and* $\rho > \bar{\rho} > 0$.[28] *Then, there exists some* degree of equity overpricing, $\kappa > 0$, *such that no generation of consumers is worse off compared to the PI economy,* $C_t \geq C_t^{PI}$, *for any* t. *Furthermore, the higher is* T, *the higher is* κ.

Proof. See Appendix B. ■

Corollary 1.2 suggests that the net effect of temporary equity overpricing on $C_{\tau+T}$ depends on the degree of equity overpricing κ, and duration, i.e., the length of horizon that the optimistic public signal affects expectations T. The net effect of temporary equity overpricing also depends on all model's parameters relating to the productivity of R&D that are summarized in terms of ρ. In the welfare analysis that follows we continue to restrict attention to welfare calculations based on ex-post consumption outcomes.

The necessary condition for Pareto improvement of the model economy is that a small degree of equity overpricing has a big R&D productivity outcome. In such environment, the increase of R&D investment bears a very low cost in terms of (unit) excess capital losses, while it generates high wage returns. The length of the period during which the single optimistic signal affects equity market participants' expectations is also highly important.

[27] Of course, this is not to say that a consumer born in t in the Model economy would prefer to keep the ex-ante known overpriced equity, over the the alternative of being in the Model economy with no overpriced equity holdings.

[28] Namely $\bar{\rho}$ solves $\frac{R-1}{R}\frac{\bar{\rho}}{1-\bar{\rho}} = \alpha\frac{\left(1+\lambda^{\frac{1}{1-\bar{\rho}}}\left(\frac{\Gamma\phi}{R-1}\right)^{\frac{\bar{\rho}}{1-\bar{\rho}}}\right)^2}{\lambda^{\frac{1}{1-\bar{\rho}}}\left(\frac{\Gamma\phi}{R-1}\right)^{\frac{\bar{\rho}}{1-\bar{\rho}}}}$. See Appendix B for details.

The larger is T, the higher is the likelihood of Pareto improvement due to a single optimistic public signal. This is because the productivity gains from equity overpricing are accumulated over a longer time period and therefore suffice to cover higher excess capital losses.[29]

In order to understand what allows equity overpricing to be potentially Pareto improving, it is useful to compare the PI economy with the social planner's solution. The social planner maximizes consumption of all generations at the beginning of time,

$$W_1 = \sum_{t=1}^{\infty} \frac{1}{R^{t-1}} C_t,$$

on a steady-state path, where the initial period $t \equiv 1$, M_0, $A_1 > 0$ are given and technology evolves according to $A_{t+1} - A_t = \lambda I_t^{\rho} A_t^{1-\rho}$ (see equations (6) and (7)).[30]

Comparing the valuation of firms listed in the PI economy's equity market to their social value leads to the following Corollary.

Corollary 1.3 *The social value of an additional unit of R&D, P^{SP}, is always higher than the private one, P^{PI}, i.e.,*

$$\frac{P^{SP}}{P^{PI}} = \frac{1}{\alpha^{\frac{1}{1-\alpha}}} \frac{R-1}{R-1-(1-\rho)g_A^{SP}} > 1 \tag{22}$$

and the socially optimal growth rate solves

$$R - 1 = \rho \lambda^{1/\rho} \left(g_A^{SP}\right)^{-\frac{1-\rho}{\rho}} \frac{\Gamma \phi}{\alpha^{\frac{1}{1-\alpha}}} + (1-\rho) g_A^{SP}. \tag{23}$$

Proof. See Appendix C. ■

Corollary 1.3 shows that equity is "underpriced" in the PI economy. In particular, the return from equity of any individual intermediate-good firm is the future profit flow of that firm. However this does not compensate for increasing productivity for all future R&D producers. Furthermore, this

[29] As an illustration, when $T = 1$, then the generation consuming in $\tau + 1$ finances the higher R&D, but cannot yet benefit in terms of higher wages. As a result, its consumption is strictly lower than in the PI economy.

[30] The social planner is assumed to discount the consumption of future generations with the risk-free interest rate. This is because assuming a social discount factor of β, then the case when $R\beta = 1$, is the only one that admits a well-defined interior solution for endogenous growth, where the transversality conditions are not violated.

does not compensate for the positive effect of a new variety on the final good productivity. Therefore, both the monopolistic distortion and the positive spillover effects imply that the private return to R&D investment is lower than its social return.

More specifically, the first term, $\alpha^{-\frac{1}{1-\alpha}}$, in (22) captures the effect of monopolistic distortion, which implies that the production of intermediate goods in the PI economy is too low.[31] The second term, $\frac{R-1}{R-1-(1-\rho)g_A^{SP}}$, highlights that the social discount rate is lower than the private one. The effective discount rate accounts for the knowledge externality's growth effect, given by the product of optimal growth rate, g_A^{SP}, with the elasticity of new varieties production to the existent knowledge stock, $1-\rho$.

However, the low private return to R&D in the PI economy does not necessarily imply that there is also underinvestment in R&D. This is because of the negative congestion externality of any individual R&D producer on others' productivity. Therefore, increasing R&D investment and thereby R&D growth (see (14)) is not socially optimal.

Corollary 1.4 *A sufficient condition that there is underinvestment in R&D in PI economy compared to the socially optimal one is* $\rho \geq \alpha^{\frac{1}{1-\alpha}}$. *Under this condition*

$$\iota^{SP} > \iota^{PI} \Leftrightarrow g_A^{SP} > g_A^{PI},$$

where $\iota^{SP} \equiv \frac{I_t^{SP}}{A_t^{SP}}$ and $\iota^{PI} \equiv \frac{I_t^{PI}}{A_t^{PI}}$.

Proof. See Appendix D. ■

Given (7), Corollary 1.4 shows that it is always beneficial to increase R&D investment. While incentives to perform more R&D are absent in the market, the analysis of Section 3.2.1 showed that these incentives are created by the temporary equity overpricing. This brings R&D investment in the model economy temporarily closer to the socially desirable level, which provides scope for a Pareto improvement.[32]

[31] Appendix C shows that $x^{SP} = \left(\frac{\alpha}{\eta}\right)^{\frac{1}{1-\alpha}} \phi L$. From (10) it holds that $\frac{x^{SP}}{x^{PI}} = \frac{1}{\alpha^{\frac{1}{1-\alpha}}}$.

[32] It is worth highlighting though, that had we asked the social planner to make a choice from the viewpoint of a short-lived consumer born in t, and retiring in $t+1$, then to the extent that he is not altruistic, he would pick to spend *no* resources on I_t. In other words, for Pareto efficiency it is essential to have both the long-horizons and internalization of any spillovers.

3.2.3 Non-R&D sector

This Section concerns the case where $\rho = 0$, while maintaining all other assumptions of Section 2. As discussed in Section 2.3, this concerns a non-R&D sector that features strong congestion externalities but no knowledge spillovers. It is clear that ρ does not affect the equilibrium in the equity market and therefore the equity price in the Model economy and PI economy are still given by (17) and (18) of Section 3.1.3. However, the effect of temporary overpricing on consumption is dramatically different from Proposition 1 and Corollary 1.2.

Corollary 1.5 *If* $\rho = 0$, *then consumption in the Model economy and PI economy* are equal (i.e., $C_t = C_t^{PI}$) *for any* $t \neq \{\tau, \tau + T\}$ and it always holds that

$$C_\tau > C_\tau^{PI} \text{ and } C_{\tau+T} < C_{\tau+T}^{PI}, \text{ where } \left|C_\tau - C_\tau^{PI}\right| < \frac{\left|C_\tau - C_\tau^{PI}\right|}{R^T}.$$

Proof. See Appendix E. ■

Corollary 1.5 highlights that in a setting with no positive knowledge spillovers there cannot be ex-post Pareto improvement due to temporary equity overpricing. The reason is the following. As shown at the end of Section 3.1.2, there is no endogenous growth when $\rho = 0$. Therefore, developments in equity market have no impact on aggregate output and wages, and the positive indirect effect of temporary overpricing on consumption is absent. This implies that the only effect of temporary overpricing on aggregate consumption arises through excess capital gains and losses. When overpricing starts there are always realized capital gains, and when it finishes there are always realized losses for investors. Corollary 1.5 also shows that the eventual capital losses are bigger in discounted absolute value than the initial gains. The reason for this is that the economy is growing and there are new (non-R&D) firms entering the equity market. Therefore the "bust" has a relatively larger negative effect on aggregate consumption.

The negative effect of overpricing of non-R&D assets can be further seen by comparing investments in this setting with the socially optimal investments.

Corollary 1.6 *If* $\rho = 0$, *then there is always positive investment the Model*

and PI economy as $\iota_t = \frac{I_t}{A_t} = \lambda P_t > 0$ and $\iota_t = \lambda P_t^{PI}$, while the Social Planner would set $\iota^{SP} = 0$.

Proof. See Appendix E. ■

Corollary 1.6 shows that in contrast to the case of R&D investment, there is always overinvestment in an economy where the congestion externality is strong. Therefore, higher equity prices that encourage higher investments in this setting always have a harmful effect on aggregate consumption. The positive investment in the market equilibrium is driven by the fact that these investments are essentially entry costs that give future profits and market power to firms. This creates distortions at the aggregate level that are not offset by any positive externalities and unlike the technology sector market power and investments are unnecessary for growth.[33]

The results above highlight that considering knowledge spillovers is crucial for understanding the aggregate effect of events such as technology stocks boom and why the benefits of similar booms related to firms in other sectors do not incorporate similar indirect benefits at the aggregate level.

4 Extension: infinitely lived consumer in closed economy.

In Appendix F we solve the general case of the model with infinitely lived consumer with CRRA utility in a closed economy, where the interest rate is endogenous. As our preferred setting is the overlapping generation model above, we only discuss some of the main findings to see the degree to which our main conclusions extend to this setting. This setting maintains all the assumptions of the main model regarding production side and uncertainty (i.e., the news about potential productivity improvement is unexpected). The representative consumer problem is

$$\max E\left[U_t\right] = E\left[\sum_{s=t}^{\infty} \beta^{s-t} \frac{C_s^{1-\theta}-1}{1-\theta}\right]. \tag{24}$$

[33] It is clear that if we were to consider a small ρ instead of the extreme case of $\rho = 0$, some positive investments would be optimal, however from Corollary 1.4 and 1.6 we would expect the same qualitative effects as there would always be a tendency towards overinvestment at the aggregate level.

subject to the budget constraint for every period s and state $u \in \{H, L\}$[34]

$$B_s^u + H_s^u P_s^u + C_s^u = w_s^u L + R_{s-1}^u B_{s-1}^u + H_{s-1}^u \left(P_s^u + \Gamma \phi^u\right). \tag{25}$$

Namely there is a budget constraint for either states $u \in \{H, L\}$ for all variables in $s > \tau + T$ and subscript u is irrelevant (or equivalently $u = L$) for all variables in $s < \tau + T$ as the state has not realized yet.

The goods market clearing condition in this setting becomes

$$Y_t = C_t + X_t + I_t. \tag{26}$$

Lemma 2 *During the period of uncertainty $\tau \leq t \leq \tau + T - 1$, the growth rate of R&D $g_{A,t} > g_{A,t}^{PI}$, if and only if there is equity overpricing during the period of uncertainty. If consumers are risk neutral, then there is always overpricing and higher growth in the model economy. When consumers are risk averse then the necessary and sufficient condition for equity overpricing is satisfied more easily if the elasticity of the long term growth rate with respect to labour productivity is high. Consumption in any period is given by $C_t = A_t \left(\frac{1+\alpha}{\alpha}\Gamma\phi - \left(\frac{g_{A,t}}{\lambda}\right)^{\frac{1}{\rho}}\right)$ where $g_{A,t} = g_{A,t}^{PI} = g_A^{PI}$ for $t < \tau$ and $t > \tau + T - 1$.*[35]

Proof. The first statement is an immediate consequence of (14). The rest is in Appendix F. ■

Lemma 2 shows that for various parameter values the main features of the model remain valid, i.e., there is overpricing and temporary acceleration of R&D growth. If this is the case, then it is straightforward that after the resolution of uncertainty consumption is permanently higher and utility from $t > \tau + T - 1$ onwards is also always higher than in the PI economy. However, there must be an initial fall in consumption at least in period $t = \tau$. The main reason for this is that unlike the main model where agents had an opportunity to borrow from abroad, there is no such opportunity in a closed economy setting. This always holds at the limit case of $\theta \to 0$ (high

[34] Similarly to the main model, in every period, the available assets is equity of R&D and intermediate goods producers and a one period bond. Differently from the main model the bond supply is fixed at B_0 due to closed economy assumption. We would obtain the same solution for all real variables with other types of assets.

[35] We maintain the implicit assumuption that the parameters are such that consumption is always non-negative.

intertemporal elasticity) that is most directly comparable to our baseline setting with risk-neutral agents.[36]

We can further characterize the equilibrium growth rate in the risk-neutral case as follows.

Corollary 2.1 *When $\theta \to 0$ then the interest rate is always constant $R = \frac{1}{\beta}$ and the R&D growth rate in PI economy is*

$$g_A^{PI} = \left(\frac{\beta}{1-\beta}\lambda^{\frac{1}{\rho}}\Gamma\phi\right)^{\frac{\rho}{1-\rho}} \tag{27}$$

and during the period of uncertainty the growth rate in the model economy is

$$g_{A,\tau+k} = \left(\frac{\beta}{1-\beta}\lambda^{\frac{1}{\rho}}\Gamma\phi + \frac{\beta^{T-k}}{1-\beta}\lambda^{\frac{1}{\rho}}\Gamma\kappa\right)^{\frac{\rho}{1-\rho}} \text{ for } k = 0, ..., T-1. \tag{28}$$

Proof. Taking the limit of $\theta \to 0$ of R&D growth rate in the more general solution in Appendix F and using (8) proves the corollary. ■

Corollary 2.1 highlights again the result stated in Lemma 2 that during the period of uncertainty the growth rate of R&D is (and equivalently equity prices are) higher in the model economy than in the PI economy. In fact, the prices are the same as in the baseline model in Section 3.1.3.[37] Also notice that during the transition period, the growth rate is at its lowest level in period τ when the news about potentially higher long-term labour productivity arrives and at its highest level just in period $\tau + T - 1$, just before the uncertainty resolves ($\frac{\partial g_{A,\tau+k}}{\partial k} > 0$). This observation is useful for the following result.

Lemma 3 *When $\theta \to 0$ the growth rate in PI economy is always lower than the socially optimal one. Furthermore, there always exists a degree of*

[36] The reason why results are likely to change for high values of θ is the following. High θ makes consumers less willing to substitute lower consumption today for higher consumption in the future. Therefore, a positive news about future productivity makes them to prefer to increase their consumption immediately and unwilling to invest in technology during the period of uncertainty. The latter leads to slower growth and lower equity prices than in the PI economy. It is worth emphasising that this result is again largely driven by closed economy assumption. It is worth noting that in order in the case of 1990s and other similar episodes both inability to borrow from abroad and high risk aversion are arguable unrealistic.

[37] Recall that from (14) $P_{t+k} = (g_{A,t+k})^{\frac{1-\rho}{\rho}} \lambda^{-\frac{1}{\rho}}$ and therefore $P_{t+k} = \frac{\beta}{1-\beta}\Gamma\phi + \frac{\beta^{T-k}}{1-\beta}\Gamma\kappa$, which is the same as (17) given that $R = \frac{1}{\beta}$.

optimism, $\kappa > 0$, such that consumer's utility is higher in any period $t \geq \tau$ in the model economy than in the PI economy.

Proof. Because $\theta \to 0$ implies that utility is linear in consumption and $R = 1/\beta$, the Social Planner's problem in this case is exactly as the one already discussed in Section 3.2.2. Therefore, welfare maximizing growth rate is given by (23). Given this and (27) we find that $\left(g_A^{PI}/g_A^{SP}\right)^{\frac{1-\rho}{\rho}} = \left(1 - \frac{\beta}{1-\beta}(1-\rho)g_A^{SP}\right) \Big/ \rho\alpha^{-\frac{1}{1-\alpha}}$. Because $1-\frac{\beta}{1-\beta}(1-\rho)g_A^{SP} < 1$ and $\rho\alpha^{-\frac{1}{1-\alpha}} > 1$, it is clear that $g_A^{PI} < g_A^{SP}$. For the second part of the lemma notice that by Lemma 2 consumer's utility in any period $t > \tau + T - 1$ and the same holds for expected welfare for any $t > \tau + T - 1$ as utility in the high state productivity state is necessarily higher. For the period of uncertainty it is sufficient for strictly higher welfare that that the R&D growth is always $g_A^{PI} < g_{A,\tau+k} \leq g_A^{SP}$. As $g_A^{PI} < g_{A,\tau+k}$ holds for any $\kappa > 0$ and $g_{A,\tau+T-1} > g_{A,\tau+k}$ for any $k = 0, ..., T-2$, we can show that there is exists $\kappa = \hat{\kappa} > 0$ such that $g_{A,\tau+T-1} = g_A^{SP}$. From (23) and (28) we find that

$$1 = \frac{\left(g_{A,\tau+T-1}\right)^{\frac{1-\rho}{\rho}} - \frac{\beta}{1-\beta}\lambda^{\frac{1}{\rho}}\Gamma\kappa}{\left(g_A^{SP}\right)^{\frac{1-\rho}{\rho}}}\rho\alpha^{-\frac{1}{1-\alpha}} + \frac{\beta}{1-\beta}(1-\rho)g_A^{SP}.$$

Setting $g_{A,\tau+T-1} = g_A^{SP}$ and $\kappa = \hat{\kappa}$, we find that

$$\hat{\kappa} = \frac{\left(\rho\alpha^{-\frac{1}{1-\alpha}} - 1\right)\left(g_A^{SP}\right)^{\frac{1-\rho}{\rho}}\frac{1-\beta}{\beta} + (1-\rho)\left(g_A^{SP}\right)^{\frac{1}{\rho}}}{\lambda^{\frac{1}{\rho}}\Gamma},$$

which is indeed always positive as $\rho\alpha^{-\frac{1}{1-\alpha}} > 1$. ■

Lemma 3 confirms the main result of the paper for the case of infinitely lived risk-neutral consumers in a closed economy. Provided that there are positive knowledge externalities, some degree of equity overpricing (even if it is temporary) leads to welfare improvement. While this result is most stark in the case of risk neutrality, it holds more generally, at least for the case where risk aversion is not too high.

5 Conclusion

This paper builds a model to analyze the effect of equity overpricing on aggregate welfare, given the trade-off between excess capital losses from the equity holdings of R&D intensive firms and productivity gains from the new R&D products. We emphasize that for a full account of the costs and benefits of equity overpricing of technology-related assets, it is necessary to use a general equilibrium framework that captures a number of empirical regularities associated with technology-related assets. Our main result is that equity overpricing can increase (ex-post) welfare at all times. In an overlapping generations setting this implies higher consumption than in a frictionless economy even at the period when the maximum losses from equity trading are realized. This result highlights that even when an equity price increase is ex-post unjustified by realized profits, it induces temporarily higher productivity growth and thereby higher long-run level of consumption.

The benefits come from the wedge between the private and social returns to R&D that is due to the presence of knowledge spillovers in R&D production. While overpricing leads to overinvestment in R&D at the firm level, at the aggregate level, R&D investment rises temporarily closer to its socially optimal level. This has a permanent effect on the level of productivity of the economy and through this brings wage gains for all generations. These benefits would be left out from the welfare accounting of firm or sector-level analyses.

The sufficient conditions for welfare gains is that equity overpricing is lasting and is not too high. This means that there are small deviations of equity prices that are spread over more generations. In such a case, the consumption gains of overpricing can cover its costs and bring aggregate welfare gains at all times. The welfare gains are easier to achieve the higher the productivity of R&D is (i.e., strong knowledge and/or small congestion effect). In view of this, the benefits of equity overpricing are more likely related to firms engaging in investments in industries producing new technologies, rather than in "old industries".

In this spirit, the model sheds light on why the aggregate impact on technology-related stocks overpricing is relatively limited compared to other

sectors. As an illustration, we may contrast the characteristics of new technologies, like ICT, to those of real estate. In the real estate sector it is more likely that congestion externalities dominate any positive spillovers. Under these conditions, a real estate equity price boom that induces firm-level overinvestment, would always create suboptimally high investment at the aggregate level and an unambiguously negative effect on aggregate consumption from losses in the equity market. Therefore, it is more likely to observe a much stronger aggregate negative effect at the time of "bust". Such an asymmetry between movements of technology and real estate asset prices is in line with numerous historical examples.

Finally, our model provides the background to review policy in the event of equity overpricing. This is the case to the extent that public information about future returns is indeed the driver of equity mispricing, as is built in our model, as well as to the extent that policy statements can act as public signals. Indeed, there has been a debate regarding the role of policy statements in the late 1990s episode. The Economist, September 5th, 2002 featured the following view: *"...A less forgivable mistake was that Mr. Greenspan acted as something of a cheerleader for the 'new economy'. Even if some increase of productivity was real, his enthusiasm contributed to investors' euphoria. They seized on all of his comments to justify their bullishness about future profits..."*. Our results suggest that when policy makers can influence the public signal by issuing statements about equity market developments during a new technologies boom, they have a clear incentive to always encourage some overpricing. It is clear, however, that always encouraging optimism cannot be credible. While a more elaborate analysis of policy makers' incentives is left for future research, a more plausible way to interpret our current insight is that policy makers may lack incentives to "burst a technology bubble", if it has already emerged. More broadly, it suggests that policy makers' ability to issue credible statements about asset market developments may be limited. This is due to the fact that financial market participants' returns from asset trading are not necessarily aligned with the policy makers' objectives who care for aggregate outcomes.

A Proof of Proposition 1

The proof starts with specifying the equilibrium R&D growth rate and consumption in PI economy. From (6) and (18), the growth rate in PI economy is constant at

$$g_A^{PI} \equiv g_{A,t}^{PI} = \lambda^{\frac{1}{1-\rho}} \left(\frac{\Gamma\phi}{R-1}\right)^{\frac{\rho}{1-\rho}}.$$

Given that from (11) and (18), $P^{PI} + \pi_t = \frac{\Gamma\phi}{R-1} + \Gamma\phi = R\frac{\Gamma\phi}{R-1} = RP^{PI}$ and there are no excess capital gains (or losses) in PI economy. Therefore, aggregate consumption (21) simplifies to $C_t^{PI} = Rw_{t-1}^{PI}L$. Using, $w_{t-1}^{PI} = (1-\alpha)\, Y_{t-1}^{PI}$ and (20) this can be expressed as

$$C_t^{PI} = \tfrac{R}{\alpha}\Gamma\phi A_{t-1}^{PI}. \tag{29}$$

In the model economy, the equilibrium prices are given by (17). Given that $P_{\tau+k} > P^{PI}$ for $\forall k \in \{0, ..., T-1\}$ and $P_t = P^{PI}$ for any $t < \tau$ and $t \geq \tau + T$, the R&D growth rate (14) is $g_{A,t} = g_A^{PI}$, $t < \tau$ and $t \geq \tau + T$, and $g_{A,\tau+k} > g_A^{PI}$, $\forall k \in \{0, ..., T-1\}$. As a result, $A_t > A_t^{PI}$, $\forall\ t > \tau$, so that $Rw_{t-1}L = \frac{R}{\alpha}\Gamma\phi A_{t-1} > \frac{R}{\alpha}\Gamma\phi A_{t-1}^{PI}$, $\forall\ t \geq \tau + 2$.

Given the path of prices, the corresponding path of excess capital gains or losses is:

$$\begin{aligned}
&(P_t + \pi_t - RP_{t-1})A_t = 0,\ \forall t \in \mathbb{Z} :\ t \neq \tau+1, \tau+T,\\
&(P_\tau + \pi_\tau - RP_{\tau-1})A_\tau = \kappa\tfrac{\Gamma\phi}{(R-1)R^{T-1}}A_\tau,\\
&(P_{\tau+T} + \pi_{\tau+T} - RP_{\tau+T-1})A_{\tau+T} = -\kappa\tfrac{R\Gamma\phi}{R-1}A_{\tau+T}.
\end{aligned}$$

Consolidating the above information,

$$\begin{aligned}
&C_t = C_t^{PI},\ \forall t < \tau,\\
&C_\tau = \kappa\tfrac{\Gamma\phi}{(R-1)R^{T-1}} + \tfrac{R}{\alpha}\Gamma\phi A_{\tau-1} = \kappa\tfrac{\Gamma\phi}{(R-1)R^{T-1}} + C_\tau^{PI} > C_\tau^{PI},\\
&C_{\tau+k} = \tfrac{R}{\alpha}\Gamma\phi A_{\tau+k-1} > \tfrac{R}{\alpha}\Gamma\phi A_{\tau+k-1}^{PI} = C_{\tau+k}^{PI}, \forall k \in \{2, ..., T-1\},
\end{aligned}$$

In period $t = \tau + 1$, there are no excess gains or losses and there are no

realized wage benefits, so that

$$C_{\tau+1} = \frac{R}{\alpha}\Gamma\phi A_\tau = C^{PI}_{\tau+1},$$

while in period $t = \tau + T$, consumers on the one hand receive excess capital losses and on the other hand wage gains. They consume

$$C_{\tau+T} = \tfrac{R}{\alpha}\Gamma\phi A_{\tau+T-1} - \kappa\tfrac{R\Gamma\phi}{R-1}A_{\tau+T}. \tag{30}$$

Whether this is higher or lower than in PI economy depends on the parameters of the model.

B Proof of Corollary 1.2

Given the results in Proposition 1, all generations of consumers gain from a single optimistic public signal iff $\frac{C_{\tau+T}}{C^{PI}_{\tau+T}} \geq 1$. For period $t = \tau + T - 1$, the equilibrium equity price is $P_{\tau+T-1} = \frac{\Gamma\phi}{R-1} + \kappa\frac{\Gamma\phi}{R-1}$, such that by (14) the R&D growth rate is $g_{A,\tau+T-1} = \lambda^{\frac{1}{1-\rho}}\left(\frac{\Gamma\phi}{R-1}\right)^{\frac{\rho}{1-\rho}}(1+\kappa)^{\frac{\rho}{1-\rho}} = g_A^{PI}(1+\kappa)^{\frac{\rho}{1-\rho}}$. From (29) and (30), the condition $\frac{C_{\tau+T}}{C^{PI}_{\tau+T}} \geq 1$ can be expressed as

$$\begin{aligned} &G_1(\kappa,T)G_2(\kappa) \geq 1, \text{ where} \\ &G_1(\kappa,T) \equiv \tfrac{A_{\tau+T-1}}{A^{PI}_{\tau+T-1}} \text{ and } G_2(\kappa) \equiv 1 - \kappa\tfrac{\alpha}{R-1}\left[1 + g_A^{PI}(1+\kappa)^{\frac{\rho}{1-\rho}}\right] \end{aligned} \tag{31}$$

Since $A_{\tau+T} = (1+g_{A,\tau+T-1})A_{\tau+T-1}$ and $A_t = A_t^{PI}$, $G_1(\kappa,T) = \frac{\prod_{k=\tau}^{\tau+T-2}(1+g_{A,k})}{(1+g_A^{PI})^{T-1}}$. Using, (14) and (17), $g_{A,k} = g_A^{PI}\left(1 + \frac{R}{R-1}\kappa\frac{R-1}{R^{T-k+\tau}}\right)^{\frac{\rho}{1-\rho}}$ for any $k \in \{\tau, .., \tau + T - 2\}$. From here $g_{A,\tau+T-2} > g_{A,\tau+T-3} > ... > g_A^{PI} > 0$, it is clear $G_1(\kappa,T) > 1$ and $G_1(\kappa,2) = \frac{1+g_{A,\tau+T-2}}{1+g_A^{PI}} \leq G_1(\kappa,T)$ for any $T \geq 2$. Therefore, a sufficient condition for (31) to hold is that

$$F(\kappa) \equiv \left[1 + g_A^{PI}\left(1 + \tfrac{\kappa}{R}\right)^{\frac{\rho}{1-\rho}}\right]G_2(\kappa) - \left(1 + g_A^{PI}\right) \geq 0.$$

One needs to investigate the conditions under which there exists $\kappa > 0$, such that the above condition holds true. Note that $F(0) = 0$ and

$\lim_{\kappa\to\infty} F(\kappa) = -\infty$.[38] Furthermore note that $F(\kappa)$ is continuous in κ. For the purpose of this proof, it is sufficient to exclude that $F(\kappa) < 0$ for any $\kappa > 0$. Therefore, it becomes sufficient that $F(\kappa) > 0$ when making a small step away from $\kappa = 0$, i.e., $F'(0) > 0$. Given that

$$\begin{aligned} F'(\kappa) = & \tfrac{1}{R}\tfrac{\rho}{1-\rho} g_A^{PI} \left(1+\kappa\right)^{\frac{\rho}{1-\rho}-1} G_2(\kappa) + \\ & - \frac{\alpha}{R-1}\left(1+g_A^{PI}\left(1+\tfrac{\kappa}{R}\right)^{\frac{\rho}{1-\rho}}\right)\left(1+g_A^{PI}\left(1+\kappa\right)^{\frac{\rho}{1-\rho}}\right) \\ & - \frac{\kappa\alpha}{(R-1)} g_A^{PI} \tfrac{\rho}{1-\rho}\left(1+g_A^{PI}\left(1+\tfrac{\kappa}{R}\right)^{\frac{\rho}{1-\rho}}\right)\left(1+\kappa\right)^{\frac{\rho}{1-\rho}-1}, \end{aligned}$$

then the result is

$$F'(0) = \tfrac{1}{R}\tfrac{\rho}{1-\rho} g_A^{PI} - \tfrac{\alpha}{R-1}\left(1+g_A^{PI}\right)^2.$$

Therefore, $F'(0) > 0$ iff

$$\tfrac{R-1}{R}\tfrac{\rho}{1-\rho} > \alpha \tfrac{\left(1+g_A^{PI}\right)^2}{g_A^{PI}} = \alpha \frac{\left(1+\lambda^{\frac{1}{1-\rho}}\left(\frac{\Gamma\phi}{R-1}\right)^{\frac{\rho}{1-\rho}}\right)^2}{\lambda^{\frac{1}{1-\rho}}\left(\frac{\Gamma\phi}{R-1}\right)^{\frac{\rho}{1-\rho}}}. \tag{32}$$

Ceteris paribus, for $\lambda\Gamma\phi > R-1$, which ensures that $\frac{\partial g_A^{PI}}{\partial\rho} > 0$, the above condition is more likely to hold for higher values of ρ. Under the assumption $\lambda\Gamma\phi > R-1$, the LHS of the above inequality becomes an increasing function of ρ, while the RHS a decreasing one. Therefore, there exists $\bar{\rho} = \bar{\rho}(\alpha, \eta, L, \lambda, \phi, R)$ such that the condition (32) holds true, $\forall\rho > \bar{\rho}$, where $\bar{\rho}$ is a solution of (32) with equality. This proves the first part of Corollary 1.2. Note that since $F(\kappa)$ is a continuous function in κ, then ceteris paribus, $\forall\rho > \bar{\rho}$, Corollary 1.2 is true at least for a range of $\kappa \in (0, \bar{\kappa}]$, where $F(\bar{\kappa}) = 0$.

Finally, in order to show that $\frac{d\kappa}{dT} > 0$, apply the implicit function theorem for

$$G(\kappa, T) \equiv G_1(\kappa, T) G_2(\kappa) - 1 = c > 0.$$

[38] This is because $F(\kappa) = \kappa^{\frac{\rho+1}{1-\rho}} \cdot$
$\left\{\left[\kappa^{-\frac{\rho}{1-\rho}} + g_A^{PI}(\kappa^{-\frac{\rho}{1-\rho}} + z\tfrac{R-1}{R^2})^{\frac{\rho}{1-\rho}}\right]\left\{\kappa^{-\frac{1}{1-\rho}} - z\tfrac{\alpha}{R}\left[\kappa^{-\frac{\rho}{1-\rho}} + g_A^{PI}(1+z\tfrac{R-1}{R})^{\frac{\rho}{1-\rho}}\right]\right\} - \frac{1+g_A^{PI}}{\kappa^{\frac{\rho+1}{1-\rho}}}\right\}$
and since $\lim_{\kappa\to\infty}\kappa^{\tilde{\rho}} = \infty$ for any $\tilde{\rho} > 0$, then because the term in parentheses is finite and negative, it follows that $\lim_{\kappa\to\infty} F(\kappa) = -\infty$.

Then, $\frac{d\kappa}{dT} = -\frac{G_T}{G_\kappa} = -\frac{\frac{\partial G_1(\kappa,T)}{\partial T}G_2(\kappa)}{\frac{\partial G_1(\kappa,T)}{\partial \kappa}G_2(\kappa)+G_1(\kappa,T)G_2'(\kappa)}$, where given the analysis above it is straightforward that $\frac{\partial G_1(\kappa,T)}{\partial \kappa} > 0$ and $G_2'(\kappa) > 0$.
Therefore, $\frac{\partial G_1(\kappa,T)}{\partial \kappa}G_2(\kappa)+G_1(\kappa,T)G_2'(\kappa) > 0$, and for $\frac{d\kappa}{dT} > 0$, it is sufficient that $\frac{\partial G_1(\kappa,T)}{\partial T} < 0$.

Given that T is a discrete variable, $\frac{\partial G_1(\kappa,T)}{\partial T} < 0$ holds if $\frac{G_1(\kappa,2)}{G_1(\kappa,3)} > \frac{G_1(\kappa,3)}{G_1(\kappa,4)} > \frac{G_1(\kappa,4)}{G_1(\kappa,5)} > ...$, etc. From the results at the beginning of this appendix $G_1(\kappa,2) = \frac{1+g_A^{PI}\left(1+\frac{\kappa}{R}\right)^{\frac{\rho}{1-\rho}}}{1+g_A^{PI}}$, $G_1(\kappa,3) = G_1(\kappa,2)\frac{\left(1+g_A^{PI}\left(1+\frac{\kappa}{R^2}\right)^{\frac{\rho}{1-\rho}}\right)}{\left(1+g_A^{PI}\right)^2}$, $G_1(\kappa,4) = G_1(\kappa,3)\frac{\left(1+g_A^{PI}\left(1+\frac{\kappa}{R^3}\right)^{\frac{\rho}{1-\rho}}\right)}{\left(1+g_A^{PI}\right)^2}$, etc. As $\left(1+\frac{\kappa}{R^{T-1}}\right)^{\frac{\rho}{1-\rho}}$ is decreasing in T, it is clear that $\frac{G_1(\kappa,T-2)}{G_1(\kappa,T-1)} > \frac{G_1(\kappa,T-1)}{G_1(\kappa,T)}$ and $\frac{\partial G_1(\kappa,T)}{\partial T} < 0$. Therefore given that $\rho > \bar{\rho}$, the degree of temporary equity overpricing, κ, that increases consumption of the generation consuming in period $\tau + T$ is higher.

C Proof of Corollary 1.3

The first-best allocations maximize the PDV of aggregate consumption in the economy (or equivalently the PDV of wealth).[39] The solution focuses on a steady-state growth path.

From (3), $X_t = \int_0^{A_t}\eta x_t(j)dj$, and (9), consumption in period t is $C_t = (\phi L)^{1-\alpha}\int_0^{A_t}x_t^\alpha(j)dj - \int_0^{A_t}\eta x_t(j)dj - I_t + RM_{t-1} - M_t$. Replacing this in aggregate welfare $W_1 = \sum_{t=1}^{\infty}\frac{1}{R^{t-1}}C_t$, and using the law of motion for R&D ($A_{t+1} = \lambda I_t^\rho A_t^{1-\rho} + A_t$) from (6) and (7), the social planner chooses I_t, $x_t(j)$ and A_{t+1} to maximize

$$\sum_{t=1}^{\infty}\frac{1}{R^{t-1}}\left[(\phi L)^{1-\alpha}\int_0^{A_t}x_t^\alpha(j)dj - \int_0^{A_t}\eta x_t(j)dj - I_t + q_{A,t}\left(A_t + \lambda I_t^\rho A_t^{1-\rho} - A_{t+1}\right)\right],$$

where $q_{A,t}$ is the social value of a variety in terms of final output. Given the welfare criterion, the decision regarding M_t drops out of the aggregate welfare given the discount rate of $\frac{1}{R}$, M_0 is given and $\lim_{t\to\infty}\frac{1}{R^{t-1}}M_t \to 0$.

The FOC with respect to investment in intermediate-good varieties $\frac{1}{R^{t-1}}\left[\alpha\left(\bar{\phi}L\right)^{1-\alpha}x_t^{\alpha-1}(j) - \eta\right] = 0, \forall j$, implies that its optimal level is con-

[39] It is assumed that there is perfect information regarding ϕ_t $\forall t$. The solution to the deterministic equilibrium provides with a useful benchmark for the PI and the model economy.

stant over time and the same across varieties

$$x^{SP} = \left(\frac{\alpha}{\eta}\right)^{\frac{1}{1-\alpha}} \phi L, \forall j.$$

The condition for optimal R&D investment $\frac{1}{R^{t-1}}\left[-1 + q_{A,t}\rho\lambda I_t^{\rho-1} A_t^{1-\rho}\right] = 0$, implies that opportunity cost of R&D investment within each period equals its marginal product

$$\frac{1}{q_{A,t}} = \rho\lambda \left(\iota_t^{SP}\right)^{\rho-1}, \tag{33}$$

where $\iota_t^{SP} \equiv \frac{I_t^{SP}}{A_t^{SP}}$. In a steady state R&D growth rate is constant $g_{A,t}^{SP} = g_A^{SP}$. Given $A_{t+1} - A_t = \lambda I_t^{\rho} A_t^{1-\rho}$, then $\iota_t^{SP} = \left(\frac{g_A^{SP}}{\lambda}\right)^{\frac{1}{\rho}} = \iota^{SP}$ is constant as well and (33) implies constant value of a variety $q_{A,t} = q_A$. The optimal decision on A_{t+1} implies that

$$Rq_A = \left[(\phi L)^{1-\alpha}\left(x^{SP}\right)^{\alpha} - \eta x^{SP}\right] + q_A\left[1 + (1-\rho)\lambda\left(\iota^{SP}\right)^{\rho}\right],$$

where the path of A_t ensures that the TVC condition, $\lim_{T\to\infty} \frac{q_A}{R^T} A_T = 0$, is satisfied. For the latter it is sufficient that $g_A^{SP} < R - 1$. The last two marginal conditions jointly require that the returns on the two assets are equal,

$$R - 1 = \rho\lambda^{1/\rho}\left(g_A^{SP}\right)^{\frac{\rho-1}{\rho}} \frac{\Gamma\phi}{\alpha^{\frac{1}{1-\alpha}}} + (1-\rho)g_A^{SP}. \tag{34}$$

Using the returns' equation result back into $\frac{1}{q_A} = \rho\lambda^{1/\rho} g_A^{SP\frac{\rho-1}{\rho}}$, it follows that the value of a new variety is

$$P^{SP} \equiv q_A = \frac{\Gamma\phi}{\alpha^{\frac{1}{1-\alpha}}} \frac{1}{R-(1-\rho)g_A^{SP}-1}, \tag{35}$$

where $R - 1 > (1-\rho)g^{SP}$ holds true from the TVC condition. The latter directly contrasts with the market value of a new variety in the PI economy, which is given by its equity market valuation (18). Comparing (18) with (35),

$$\frac{P^{SP}}{P^{PI}} = \frac{1}{\alpha^{\frac{1}{1-\alpha}}} \frac{R-1}{R-(1-\rho)g_A^{SP}-1}.$$

Given that $\alpha < 1$, $P^{SP} > P^{PI}$ for any parameter values.

D Proof of Corollary 1.4

From (33), the socially optimal investment is $\iota^{SP} = (q_A \rho \lambda)^{\frac{1}{1-\rho}}$. Using $A_{t+1} - A_t = \lambda I_t^{\rho} A_t^{1-\rho}$, the corresponding growth rate of R&D is $g_A^{SP} = \lambda \left(\iota^{SP}\right)^{\rho} = \lambda^{\frac{1}{1-\rho}} (q_A \rho)^{\frac{\rho}{1-\rho}}$. In the PI economy, the growth rate of R&D is given by (14) when $P_t = P^{PI}$ as $g_A^{PI} = \lambda^{\frac{1}{1-\rho}} \left(P^{PI}\right)^{\frac{\rho}{1-\rho}}$ and the investment $\iota^{PI} = \left(P^{PI}\lambda\right)^{\frac{1}{1-\rho}}$.

This implies that there is underinvestment in R&D compared to the socially optimal ($\iota^{PI} < \iota^{SP}$) and lower R&D growth ($g_A^{PI} < g_A^{SP}$) if $P^{PI} < q_A \rho$. From (18) and (35), this holds if

$$\frac{\rho q_A}{P^{PI}} = \frac{\rho}{\alpha^{\frac{1}{1-\alpha}}} \frac{R-1}{R-(1-\rho)g_A^{SP}-1} > 1.$$

A sufficient condition for this is $\rho \geq \alpha^{\frac{1}{1-\alpha}}$, which is assumed to be the case in Section 2.

If $\rho < \alpha^{\frac{1}{1-\alpha}}$, there would be a tendency toward "overinvestment" in R&D in PI economy because of congestion externality and the optimal growth rate could be lower than in PI economy.[40]

E Proofs of Corollaries 1.5 and 1.6

Proof of Corollary 1.5. From (14) and $\rho = 0$, we know that the growth rate $g_{A,t} = \frac{A_{t+1}-A_t}{A_t} = \lambda$ and does not depend on investment and equity price. It is therefore immediate that $A_t = A_t^{PI}$ for any t and output and wages and realized profits are always the same in the Model and PI economy (see Section 3). From (21) we then know that any difference between consumption in the Model and PI economy is driven by capital gains and losses. Given that there are no capital gains and losses in the PI economy, it holds for any t that

$$C_t - C_t^{PI} = (P_t + \pi_t - RP_{t-1})A_t = (P_t + \pi_t^{PI} - RP_{t-1})A_t^{PI}.$$

[40] Notice that in the extreme case where $\rho \to 0$, there is always overinvestment in the PI economy, i.e., $\frac{\rho q_A}{P^{PI}} = 0 < 1$ and two high R&D growth, i.e., $\lim_{\rho \to 0} g_A^{SP} = 0$ and $\lim_{\rho \to 0} g_A^{PI} = \lambda$.

Furthermore, the equilibrium in the asset market remains unchanged, so that equity prices in the Model economy are given by (17), $\pi_t = \pi_t^{PI} = \Gamma\phi$. The only two periods where there are capital gains or losses in the model economy are τ and $\tau + T$ (see Appendix A). Therefore,

$$C_\tau - C_\tau^{PI} = \kappa \frac{\Gamma\phi}{(R-1)R^{T-1}} A_\tau = \kappa \frac{\Gamma\phi}{(R-1)R^{T-1}} A_\tau^{PI} > 0$$
$$C_{\tau+T} - C_{\tau+T}^{PI} = -\kappa \frac{R}{R-1}\Gamma\phi A_{\tau+T} = -\kappa \frac{R}{R-1}\Gamma\phi A_{\tau+T}^{PI} = -\kappa \frac{R}{R-1}\Gamma\phi A_\tau^{PI}\left(1+\lambda\right)^T < 0$$

Finally, comparing the discounted absolute values of gains and losses, it holds that $\left|C_\tau - C_\tau^{PI}\right| < \left|C_\tau - C_\tau^{PI}\right| / R^T$, because $\kappa \frac{\Gamma\phi A_\tau^{PI}}{(R-1)R^{T-1}} < \kappa \frac{\Gamma\phi A_\tau^{PI}(1+\lambda)^T}{(R-1)R^{T-1}}$ $\iff 1 < (1+\lambda)^T$ and $\lambda > 0$.

Proof of Corollary 1.6. From the first order condition of (6), we know that $P_t \bar{\lambda}_t = 1$. Using that $\bar{\lambda}_t = \lambda I_t^{\rho-1} A_t^{1-\rho}$ and $\rho = 0$ implies that $\frac{I_t}{A_t} = P_t\lambda > 0$. Similarly $\frac{I_t^{PI}}{A_t^{PI}} = P^{PI}\lambda > 0$. At the same time, the Social Planner fully acknowledges that A_t grows exogenously, so that he chooses I_t, $x_t(j)$ and A_{t+1} to maximize

$$\sum_{t=1}^{\infty} \frac{1}{R^{t-1}} \left[(\phi L)^{1-\alpha} \int_0^{A_t} x_t^\alpha(j) dj - \int_0^{A_t} \eta x_t(j) dj - I_t + q_{A,t}\left(A_t(1+\lambda) - A_{t+1}\right) \right],$$

where it is immediate that the first order condition with respect to I_t gives $I_t^{SP} = 0$.

F Solution of the model with infinitely lived consumer with CRRA utility

Consumer problem

Consumer perceive the world as certain in all periods until τ (recall that the arrival of uncertainty is unexpected) and uncertainty resolves in $\tau + T$. Therefore for all $t \leq \tau - 1$ and $t \geq \tau + T$ the consumer problem ((24) and (25)) is standard give the following Euler equations and no arbitrage

conditions

$$\left(\frac{C_{t+1}}{C_t}\right)^{\theta} = \beta R_t \text{ and } P_{t+1} + \Gamma\phi = R_t P_t. \text{ for } t \leq \tau - 1 \tag{36}$$
$$\left(\frac{C^u_{t+1}}{C^u_t}\right)^{\theta} = \beta R^u_t \text{ and } P^u_t = \frac{P^u_{t+1}+\Gamma\phi^u}{R^u_t} \text{ for } t \geq \tau + T \text{ and } u \in \{H, L\}$$

For period $t = \tau$ until $t = \tau + T - 1$ the consumer's expected utility can be expressed as

$$U_{\tau+k} = \sum_{s=\tau+k}^{\tau+T-1} \beta^{s-\tau-k} \frac{(C_s)^{1-\theta}-1}{1-\theta} +$$
$$+ Q\sum_{s=\tau+T}^{\infty} \beta^{s-\tau-k} \frac{\left(C^H_s\right)^{1-\theta}-1}{1-\theta} + (1-Q)\sum_{s=\tau+T}^{\infty} \beta^{s-\tau-k} \frac{\left(C^H_s\right)^{1-\theta}-1}{1-\theta},$$

for $k = 0, ..., T-1$. Using this and (25) we find that that the only date where uncertainty affect the consumer directly is $t = \tau+T-1$ and therefore Euler in that period is

$$\left(C_{\tau+T-1}\right)^{-\theta} = \beta R_{\tau+T-1}\left(Q\left(C^H_{\tau+T}\right)^{-\theta} + (1-Q)\left(C^L_{\tau+T}\right)^{-\theta}\right) \tag{37}$$

and no arbitrage condition is

$$P_{\tau+T-1} = \frac{Q\left(P^H_{\tau+T}+\Gamma\phi^H\right)\left(C^H_{\tau+T}\right)^{-\theta}+(1-Q)\left(P^L_{\tau+T}+\Gamma\phi^L\right)\left(C^L_{\tau+T}\right)^{-\theta}}{R_{\tau+T-1}\left(Q\left(C^H_{\tau+T}\right)^{-\theta}+(1-Q)\left(C^L_{\tau+T}\right)^{-\theta}\right)}. \tag{38}$$

while due to the lack of new information in $t = \tau$ until $t = \tau + T - 2$, we have the same FOCs as in before, i.e.,

$$\left(\frac{C_{t+1}}{C_t}\right)^{\theta} = \beta R_t \text{ and } P_{t+1} + \Gamma\phi = R_t P_t. \text{ for } \tau \leq t \leq \tau + T - 2 \tag{39}$$

Steady state

Let as start by looking at the periods after resolution of uncertainty. We guess and verify a constant interest rate, from $\tau + T$ onwards. Given that for $t \geq \tau + T$ the equity price is

$$P^u_t = \frac{\Gamma\phi^u}{R^u - 1} \tag{40}$$

and from (14)

$$R^u - 1 = \Gamma \phi^u \lambda^{\frac{1}{\rho}} \left(g^u_{A,t}\right)^{\frac{\rho-1}{\rho}} \tag{41}$$

Note that there is a standard negative relationship between growth and interest rate

We know from the solution of the production side that $\frac{Y^u_{t+1}-Y^u_t}{Y^u_t} = \frac{X^u_{t+1}-X^u_t}{X^u_t} = g_A$. From free entry condition (13) that $P^u_t \frac{\left(A^u_{t+1}-A^u_t\right)}{A^u_t} = P^u_t g^u_A = \frac{I^u_t}{A^u_t}$ and therefore the ratio of investments to number of varieties $\frac{I^u_t}{A^u_t}$ must be constant as well in steady state . Therefore $\frac{I^u_{t+1}}{A^u_{t+1}} = \frac{I^u_t}{A^u_t} \Longrightarrow \frac{I^u_{t+1}}{I^u_t} = \frac{A^u_{t+1}}{A^u_t} = 1 + g^u_A$. From market clearing (26), we obtain that

$$\begin{aligned} C^u_{t+1} - C^u_t &= Y^u_{t+1} - Y^u_t - \left(X^u_{t+1} - X^u_t\right) - \left(I^u_{t+1} - I^u_t\right) \\ C^u_{t+1} - C^u_t &= g^u_A \left(Y^u_t - X^u_t - I^u_t\right) = g^u_A C^u_t \end{aligned}$$

Therefore consumption grows at the same rate as technology. So we need to solve for $g_A = g_C = g$

$$\begin{cases} R^u - 1 = \Gamma \phi^u \lambda^{\frac{1}{\rho}} \left(g^u\right)^{\frac{\rho-1}{\rho}} \\ \left(1 + g^u\right)^\theta = \beta R^u \end{cases} \tag{42}$$

which gives the following expression for the steady state growth rate

$$\Gamma \phi^u \lambda^{\frac{1}{\rho}} \left(g^u\right)^{\frac{\rho-1}{\rho}} + 1 = \frac{\left(1+g^u\right)^\theta}{\beta} \tag{43}$$

It holds that $\frac{dg^u}{d\phi^u} = \frac{\Gamma \lambda^{\frac{1}{\rho}} \left(g^u\right)^{\frac{\rho-1}{\rho}}}{\left(\frac{\theta\left(1+g^u\right)^{\theta-1}}{\beta} + \frac{1-\rho}{\rho} \Gamma \phi^u \lambda^{\frac{1}{\rho}} \left(g^u\right)^{-\frac{1}{\rho}}\right)} > 0$. The growth rate is higher if the state is realized to be high and due to the negative relationship between growth and interest rate, it also holds that the interest rate is lower if the high state realizes.

Using (26), (3), (10) $X_t \equiv A_t \eta x_t$, (13) and (42) we find the consumption on date $\tau + T$ as

$$C^u_{\tau+T} = \frac{1+\alpha}{\alpha} \Gamma \phi^u A_{\tau+T} - \frac{\Gamma \phi^u g^u A_{\tau+T}}{R^u - 1} = \left(\frac{1+\alpha}{\alpha} \Gamma \phi^u - \left(\frac{g^u}{\lambda}\right)^{\frac{1}{\rho}}\right) A_{\tau+T} \tag{44}$$

Notice that $C^u_{\tau+T} > C^{u,PI}_{\tau+T}$ if and only if $A_{\tau+T} > A^{PI}_{\tau+T}$, which implies that consumption in $\tau + T$ and in all the following period is higher if only

if the period of uncertainty is associated with higher R&D growth rate. Because (14) implies that growth rate in the model economy is higher that in PI economy if and only if prices are higher in the model economy than in PI economy.

Because the PI economy is always in steady state and $\phi^L = \phi$, it is straightforward that the steady state in PI economy has follows

$$\Gamma\phi\lambda^{\frac{1}{\rho}}\left(g^{PI}\right)^{\frac{\rho-1}{\rho}}+1=\frac{\left(1+g^{PI}\right)^{\theta}}{\beta}$$
$$C^{PI}_{\tau+T}=\left(\tfrac{1+\alpha}{\alpha}\Gamma\phi-\left(\tfrac{g^{PI}}{\lambda}\right)^{\frac{1}{\rho}}\right)A_{\tau+T}$$

for any t and the model and PI economy are identical for any $t \leq \tau - 1$.

Period of uncertainty

Given (37), (38), (40), ,(42), (44), (14), (26), (3), (10) $X_t \equiv A_t\eta x_t$, and (13), the growth rate of technology in $\tau + T - 1$ solves

$$\frac{\left(g_{A,\tau+T-1}\right)^{\frac{1-\rho}{\rho}}}{\beta\lambda^{\frac{1}{\rho}}}\left(\frac{1+g_{A,\tau+T-1}}{\frac{1+\alpha}{\alpha}\Gamma\phi-\left(\frac{g_{A,\tau+T-1}}{\lambda}\right)^{\frac{1}{\rho}}}\right)^{\theta}= \tag{45}$$
$$Q\frac{\Gamma\phi^H+\lambda^{-\frac{1}{\rho}}\left(g^H\right)^{\frac{1-\rho}{\rho}}}{\left(\frac{1+\alpha}{\alpha}\Gamma\phi^H-\left(\frac{g^H}{\lambda}\right)^{\frac{1}{\rho}}\right)^{\theta}}+(1-Q)\frac{\left(\Gamma\phi^L+\lambda^{-\frac{1}{\rho}}\left(g^L\right)^{\frac{1-\rho}{\rho}}\right)}{\left(\frac{1+\alpha}{\alpha}\Gamma\phi^L-\left(\frac{g^L}{\lambda}\right)^{\frac{1}{\rho}}\right)^{\theta}}$$

and given this we can find all the other relevant variables of the model as (14) gives $P_{\tau+T-1} = \left(g_{A,\tau+T-1}\right)^{\frac{1-\rho}{\rho}}\lambda^{-\frac{1}{\rho}}$, (26), (3), (10) $X_t \equiv A_t\eta x_t$, and (13)) give

$$C_{\tau+T-1}=\tfrac{1+\alpha}{\alpha}\Gamma\phi A_{\tau+T-1}-\left(\tfrac{g_{A,\tau+T-1}}{\lambda}\right)^{\frac{1}{\rho}}.$$

Similarly from above, (39), (14), (26), (3), (10) $X_t \equiv A_t\eta x_t$, and (13)) in periods $t = \tau$ until $t = \tau + T - 1$ the growth rate of technology solves

$$\frac{\left(g_{A,t}\right)^{\frac{1-\rho}{\rho}}}{\beta\lambda^{\frac{1}{\rho}}}\left(\frac{1+g_{A,t}}{\frac{1+\alpha}{\alpha}\Gamma\phi-\left(\frac{g_{A,t}}{\lambda}\right)^{\frac{1}{\rho}}}\right)^{\theta}=\frac{\left(g_{A,t+1}\right)^{\frac{1-\rho}{\rho}}\lambda^{-\frac{1}{\rho}}+\Gamma\phi}{\left(\frac{1+\alpha}{\alpha}\Gamma\phi-\left(\frac{g_{A,t+1}}{\lambda}\right)^{\frac{1}{\rho}}\right)^{\theta}}, \tag{46}$$

$P_t = \left(g_{A,t}\right)^{\frac{1-\rho}{\rho}}\lambda^{-\frac{1}{\rho}}$ and

$$C_t = A_t\left(\tfrac{1+\alpha}{\alpha}\Gamma\phi-\left(\tfrac{g_{A,t}}{\lambda}\right)^{\frac{1}{\rho}}\right).$$

As we have also found the initial steady state and model and PI economy are initially identical, this characterizes the equilibrium given the initial conditions.

Necessary and sufficient condition for overpricing and higher long-term technology level

Note that the left hand side of (45) is monotonically increasing in $g_{A,\tau+T-1}$. If $g_{A,\tau+T} = g_L$ with probability one (as in PI economy), then we would go back to the initial steady-state condition, as the right hand side would give $RHS^{PI} = \left((g_L)^{\frac{1-\rho}{\rho}} \lambda^{-\frac{1}{\rho}} + \Gamma\phi\right) \Big/ \left(\frac{1+\alpha}{\alpha}\Gamma\phi - \left(\frac{g_{A_L}}{\lambda}\right)^{\frac{1}{\rho}}\right)^{\theta}$ and then it should have been also necessarily that $g_{A,\tau+T-1} = g_L$. We care for sequences of growth rates during the boom that result in $\{g_{A,\tau+k}\}_{k=0}^{T-1}$ that are strictly bigger than g_L. Given the above, it suffices that the right hand side of (45) is strictly above RHS^{PI}. We can express it general condition as requiring that the following function of $\tilde{\phi}$

$$\frac{\Gamma\tilde{\phi} + g\left(\tilde{\phi}\right)^{\frac{1-\rho}{\rho}} \lambda^{-\frac{1}{\rho}}}{\left(\frac{1+\alpha}{\alpha}\Gamma\tilde{\phi} - \left(\frac{g(\tilde{\phi})}{\lambda}\right)^{\frac{1}{\rho}}\right)^{\theta}}$$

is increasing in $\tilde{\phi}$. Differentiating and simplifying, we obtain that the necessary and sufficient condition is

$$\frac{\left[\Gamma + \left(\frac{1-\rho}{\rho}\right)\lambda^{-\frac{1}{\rho}} g\left(\tilde{\phi}\right)^{\frac{1-\rho}{\rho}} \frac{g\prime(\tilde{\phi})}{g(\tilde{\phi})}\right]\tilde{\phi}}{\left[\Gamma\tilde{\phi} + g\left(\tilde{\phi}\right)^{\frac{1-\rho}{\rho}} \lambda^{-\frac{1}{\rho}}\right]} > \theta \frac{\left[\frac{1+\alpha}{\alpha}\Gamma - \frac{1}{\rho}\lambda^{-\frac{1}{\rho}} g\left(\tilde{\phi}\right)^{\frac{1}{\rho}} \frac{g\prime(\tilde{\phi})}{g(\tilde{\phi})}\right]\tilde{\phi}}{\left[\frac{1+\alpha}{\alpha}\Gamma\tilde{\phi} - \lambda^{-\frac{1}{\rho}} g\left(\tilde{\phi}\right)^{\frac{1}{\rho}}\right]}$$

Effectively, the left hand side of this gives on the elasticity of $P_{\tau+T} + \pi_{\tau+T}$ with respect to the change in the value of $\tilde{\phi}$. The right hand side is the elasticity of consumption-technology ratio $\left(\frac{C_{\tau+T}}{A\tau+T}\right)$ with respect to the change in ϕ, that is multiplied with the risk aversion coefficient. This is capturing the response of the cost in utility terms of an increase in $\tilde{\phi}$. Note that as $\theta \to 0$, then the right hand side of the expression goes to zero and the above condition is clearly satisfied. In contrast, when $\theta \to \infty$, then the utility cost is so high of an additional unit of savings today, that the afore expression is clearly violated (note that also in this case $g, g' \to 0$).

In the way that it is written, also see that $\frac{g\prime(\phi)}{g(\phi)} = \frac{\varepsilon_{g_\phi}}{\phi}$, where ε_{g_ϕ} is the elasticity of the growth rate with respect to ϕ. When this ratio is sufficiently

high, then the afore expression would be more easily be satisfied since the left hand side is increasing in $\varepsilon_{g_\phi}/\phi$ whereas the right hands side decreases in it, which proves the second statement on the necessary and sufficient condition in Lemma 2.

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