

A STUDY OF FLOW CHARACTERISTICS IN A CIRCULAR SEDIMENTATION
TANK UNDER THE INFLUENCE OF A FORCED VORTEX

by

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NOTE

When, as is stated on p.84, the flow is sufficiently turbulent for minor changes in Reynolds Number to have little effect, this number can be dropped from the dimensional analysis. In this case the kinematic viscosity must be removed from other terms and equation (7), p.84 must become

$$\frac{D}{Vr_2} = f\left(\frac{V^2}{r_2 g}, \frac{\omega_1^2 r_2}{g}, \frac{\omega_1}{\omega_2}, \frac{r_1}{r_2}, \frac{r_1}{d}\right)$$

with consequent changes elsewhere. That is to say $\frac{\omega^2 r}{g}$ is a more appropriate secondary parameter than $\frac{\omega v}{rg}$. Alternatively $\frac{v}{r\omega}$ can be used as in the case of rotodynamic machinery.

If the Reynolds Number effects are important the Reynolds Number must be retained. This may be of practical significance in scaling up to full-size tanks.

ABSTRACT

This thesis presents an analytical and experimental study of the effects of a superimposed forced vortex on the flow characteristics of a circular sedimentation tank.

In the analytical section, a theoretical flow through curve for a circular sedimentation tank was derived as a result of considering the flow through the tank on a diffusion basis. The theoretically derived equation was then compared with the N-formula. A new parameter for measuring the tank hydraulic efficiency was introduced, and the manner the diffusion coefficient, D , of the tank affects its short-circuiting behaviour studied. With the help of Reynolds stresses and turbulent energy equations, the effects of a superimposed forced vortex on the tank diffusion coefficient was investigated. An analysis of the problem dimensionally revealed that the tank characteristic number N was a function of Froude number and a dimensionless term $\frac{\omega \nu}{rg}$ where ω , ν , r and g are the angular speed, kinematic viscosity, radius of the paddles and the acceleration due to gravity respectively.

The experimental investigation was conducted with a model circular sedimentation tank, covering a suitable range of speeds of the paddles,

and flow rates through the tank. N was found to have a relationship with Froude number and $\frac{\omega \nu}{rg}$ as predicted by the dimensional analysis. The effects of the forced vortex on N and D were found to be in accordance

with the analytical considerations. As a result of the experimental study, the existence of critical flow rates, the optimum flow rates and the critical speeds of the paddles were revealed. The setting up of secondary currents by the forced vortex which has both beneficial and harmful effects was also studied.

The present investigation has established analytically and experimentally that the effects of a forced vortex on a circular sedimentation tank are, up to a limit, to inhibit the short-circuiting of flow through the tank and to increase the tank hydraulic efficiency. Experimental results obtained with three types of inlets showed that the uniform flow inlet with baffles which was specially designed for this investigation was definitely superior to other inlets having slotted cylinder or cylindrical submerged weir diffuser.

Some tracing techniques were modified and improved, especially those concerning the injection of dye and the dye concentration measurement.

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CHAPTER 1

INTRODUCTION

Sedimentation as a means of separating solids from liquids has been long practised, and is now widely employed in many diversified fields. It is one of the most important processes in the treatment of water, sewage and industrial wastes, because it enables the settleable solids to be removed economically. Figures given by Townend (1960) indicate that the capital cost of sedimentation equipment may account for about 7 to 10 % of the overall cost of sewage treatment plant, depending upon its size.

The cost of sedimentation treatment is affected by tank shape and by other features of the design. Experience, together with knowledge gained from research has led to various designs of sedimentation tanks, differing in shape, in the nature of flow, and in the methods of operation. Tanks may be rectangular, circular or square in plan, and the flow in them is usually either horizontal or vertical. Horizontal flow rectangular tanks and radial flow circular tanks are the two main types commonly found in the treatment of sewage and industrial wastes, although vertical flow tanks with hopper bottom, or the so called Dortmund tanks, are also widely used. Vertical flow tanks are more common in water treatment where chemicals are added to promote flocculation. These tanks have considerable advantages where the influent is passed through a curtain of settling sludge; flocculation is thus improved.

As far as shape is concerned, much can be said about the advantages and disadvantages of rectangular and circular sedimentation tanks. From the hydraulic point of view, the radial flow tank is characteristically unstable, owing to the deceleration of flow as the peripheral weir is approached, although satisfactory performance can be achieved with a proper design (Hubbell, 1938). In a rectangular tank where the length is several times the width, the influent turbulence dies down long before the effluent weir is reached. A uniform flow can thus be obtained through a greater volume of the tank than in the case of a circular tank.

In spite of theoretical considerations, the extensive investigations carried out by the National Research Council into the sewage treatment at military establishments during World War II showed little difference between the performances of circular and rectangular sedimentation tanks (National Research Council, 1946 and 1948; cited by Richard Hazen 1959). From the point of view of economics, circular tanks proved to be a little cheaper than rectangular tanks for plants ranging from 5 to 20 million U.S. gallons per day (mgd.) (Richard Hazen, 1959).

As with other types of tanks, the design of circular sedimentation tanks is usually based upon experience and empirical rules rather than on fundamental considerations. Moreover, less research has been carried out on circular tanks than on rectangular tanks. Although the primary function of a sedimentation tank is the separation of suspended solids from liquids, designs based upon the settling properties of the solids alone are inadequate. The hydraulic characteristics of the tank are also vitally

important, and, in this respect, it appears that little use has been made of fundamental knowledge in the field of Fluid Mechanics in improving the hydraulic performance of sedimentation tanks.

As mentioned above, radial flow circular sedimentation tanks are inherently unstable because of the reduction of radial velocity towards the periphery. Short-circuiting between the inlet and the outlet weir is possible, especially when the incoming flow is at a different temperature from the general contents of the tank. The purpose of this research is to study the effects of superimposed forced vortex on the hydraulic behaviour of a circular tank. Such a vortex would cause the liquid to follow a spiral path from the central inlet to the peripheral overflow weir and would be expected to reduce the risk of short-circuiting. Besides, deposited matters would be moved towards the centre of the tank by the secondary currents induced by the forced vortex.

The investigation is divided into two parts: analytical and experimental. In the analytical section a theoretical flow-through curve is derived, and the parameters characterizing the hydraulic performance of the tank are studied. The effects of a forced vortex on these parameters are then considered. Dimensional analysis of the problem is also carried out to serve as a guide to the experimental study, tests are carried out with a model tank in which a forced vortex is induced by rotating paddles at the tank periphery. The tests are performed to cover the range of flow rates occurring in practice and the speed of rotation of the paddles is varied over a suitable range.

Before the improvement in the hydraulic performances of a circular sedimentation tank by a superimposed forced vortex is considered theoretically, it is relevant to review the literature and previous research in the field of sedimentation with particular reference to circular sedimentation tank, and related research in Fluid Mechanics.

1.1. Review of Literature

Sedimentation is believed to have been practised from time immemorial. The first application of sedimentation in water treatment for the public was probably carried out in or soon after 260 B.C. when Antiochus Theos built an aqueduct from the River Caprus to Laodicea, at the terminus of which were two settling reservoirs. It was also reported that a number of settling reservoirs, believed to be built by Emperor Hadrian, were excavated from the ruins of Carthage. Similar structures were built by the Romans in Rome, Alexandria, and at Lugdunum (now Lyons, France) (Baker, 1949).

Little has been recorded of the use of settling reservoirs for water treatment until the early nineteenth century when sedimentation tanks were built in Scotland at Paisley and Glasgow, in France at Paris and in London. Later in the century, sedimentation was more widely used to effect partial clarification of water before slow sand filtration.

Early sedimentation tanks were different from those commonly in use nowadays. They were invariably of the fill-and-draw type in which water is admitted at the bottom of the tank and, after a pre-determined period of settling, to withdraw the clarified water by means of a swing pipe so arranged that the free end of the pipe will be slightly submerged below the water surface. Although some of them are still in existence, their number is decreasing. Modern sedimentation tanks are of the continuous flow type which have advantages over the former in efficiency and head loss. With

the growth of the population and industry, the demands for water increase, and so does the volume of liquid wastes to be treated. It has become necessary to understand the settling process of particles and to apply this knowledge in sedimentation tank design to improve its efficiency and thus render the treatment by this method economical.

1.1.1. Sedimentation tanks: Research in general

The natural laws governing the settling of particles in fluid were studied by Newton (¹⁶⁸⁷~~1910~~) who discovered that a particle with a greater density than a fluid would settle in it with an increasing velocity until a velocity known as the 'terminal velocity' or the 'settling velocity' was reached. It would then continue to settle at this velocity, because the drag the fluid exerted upon the particle was equal to the gravitational force pulling the particle down. Stokes (1845 & 1933) studied the settling of sphere in fluid and derived a formula by which the settling velocity of a small sphere in an incompressible fluid of infinite extent could be calculated once its diameter and the difference in density between the sphere and the fluid were known. Despite this knowledge, the settling velocity of particles to be removed in water and sewage treatments had not been taken into consideration in the design of sedimentation tanks until Seddon (1889 & 1890) attempted to design a new system of settling basins for St. Louis, Mo., on a rational basis, making use of the knowledge obtained from the studies of Newton and Stokes. Later in 1904 A. Hazen developed a theory for the design of continuous flow sedimentation tanks. The theory was

based upon the unhindered settling of uniform discrete particles in a rectangular continuous flow tank and led to the conclusion that the efficiency of removal of the particles depended upon the overflow rate or surface loading of the tank (i.e. the rate of flow through the tank divided by the surface area) and was independent of the depth and detention period.

Camp (1936) further developed the theory to cover suspensions of particles of varying sizes. Slade (1937)^{alb} extended Hazen's theory also to account for suspensions containing particles with a range of settling velocities and varying degrees of turbulence. However, the results obtained were too empirical and theoretical to be of any practical value. Pöpel (1958) presented a new approach by equating two kinds of time, namely the detention period and the time required by the smallest particle to settle. He also assumed that the ratio $\frac{V \cdot d}{V_s}$, where V is the forward velocity in the tank, d the particle diameter and V_s its settling velocity, was constant. There appeared to be no reason why this should be so, and Pöpel himself did not offer any experimental verification.

In practice the tanks are far from being ideal. Turbulence, short-circuiting and scouring are significant factors in rendering the theory inapplicable in predicting the efficiency of a sedimentation tank. Moreover, the suspended solids in water and sewage are not entirely discrete. Some are flocculent and for these both the liquid overflow rate and the detention period are now known to be significant (Fitch, 1958). The concentration of the suspension and the shape of the particles also affect

their settling rates. Thus there are two main problems to be considered in designing a sedimentation tank, namely the settling properties of suspensions and the flow conditions in the tank.

1.1.2. Settling properties of suspensions

The settling properties of suspensions have attracted the interest of research workers in various fields. Physicists and chemists are more concerned with the effects of shape, size, orientation, concentration and flocculation of the suspended solids on their settling rates, and their analytical and experimental investigations are mostly confined to the suspensions whose particles are of regular size and shape (Taylor, 1923 a; von Smoluchowski, 1917; Jeffrey, 1922; Pettyjohn & Christiansen, 1948; Hawksley, 1951; Heiss & Coull, 1952 and Dallavalle, 1956). Because of the complexity of the suspensions encountered in the field of water and sewage treatments it is difficult for the sedimentation tank designer to apply the results obtained by the scientists directly to his problems. He is obliged, therefore, to carry out experiments with the actual suspensions under treatment. Dobbins (1944) and Camp (1946) did some tests on the hindered settling of discrete particles and applied the results in calculating solids removal in sedimentation tanks. Later Fitch (1957) and McLaughlin (1959) extended the experiments to cover the cases of hindered settling and settling by compression which was also investigated by Cse & Clevenger (1917) Robinson (1926), Steinour (1944), Behn (1957) and many others.

It must be pointed out here that the experiments cited above were mostly carried out under quiescent conditions, and would therefore be strictly applicable for calculation of removal in an ideal tank only. Density currents, turbulence and scouring are among those factors influencing the removal efficiency of a sedimentation tank. Realizing these limitations, attempts were made by some investigators to include some of those factors in their experiments. Camp (1942) and Dobbins (1944) introduced turbulence factor, but some doubts were expressed regarding the relationship between the degree of turbulence created in the tests and that actually occurring in a full scale sedimentation tank (Einstein, 1944). Bond (1950) also investigated the behaviour of alum and lime hydroxide suspensions both in still water and in upflow, and discussed the application of his theory to the precipitates formed in sedimentation tanks.

So far, it can be safely said, no single experiment on the settling of suspension has included all the conditions in a sedimentation tank to enable the results to be applied directly to a practical problem. Several investigators were therefore obliged to resort to establishing empirical formulae showing the relationship between the removal, the solid concentration, the detention time and the flow rate from the existing experimental data (Holmes & Gyatt, 1929; Babbitt & Schlenz, 1929; Schröpfer, 1933; Carpenter & Speiden, 1935; Pöpel, 1945; Escritt, 1956). However, these formulae had no rational basis and are applicable only to the tanks from which experimental data were collected and to the particular suspensions used. In order to predict the removal of particles in a sedimentation

tank it is therefore better in general to study the behaviour of suspensions and the tank characteristics separately.

1.1.3. Previous research on the hydraulic characteristics of sedimentation tanks

While there are possibilities for improving the settling properties of suspensions, for example, by addition of chemicals, the hydraulic conditions play an equally important role in determining the removal efficiency of a tank, and these also require investigation.

In an ideal tank all portions of the flow are retained for the same length of time which is the theoretical detention period (i.e. the volume of the tank divided by the flow rate). In practice, however, it is found that some parts of the flow remain in the tank for less than the theoretical detention period while others remain longer. In order to give each portion of flow an equal opportunity of sedimentation it is desirable to ensure uniform velocity distribution so as to avoid short-circuiting of excessive period of detention. Much of the research on sedimentation tanks has been devoted to tracing flow through curves to measure the extent to which short-circuiting or extensive retention occurs. These curves are usually obtained by adding a concentrated dose of a suitable tracer to the incoming flow and measuring the concentration of this tracer in the effluent from the tank over a period of time. The tracer used and dosing techniques are discussed in Section 3.2.

The shape of the tank and the inlet and outlet arrangements are major factors influencing the flow patterns in a sedimentation tank. Those flow patterns may be affected also by the difference in density between

different portions of the liquid in the tank.

Many of the basic principles apply equally to rectangular and circular tanks, and both shapes may be discussed together. It will be necessary, however, to give more detailed consideration to the special nature of flow in circular tanks.

2.1.4. The shape of the tank

There appears to be two schools of thought regarding the proper shape of modern mechanically cleaned sedimentation tanks. Some designers prefer the rectangular tank in which the length is several times the width, while others prefer the circular or the square tank, either fed at the centre of the tank with a peripheral overflow weir, or fed at one side and overflowing at the opposite side. The question is still debateable and various opinions can be cited.

An inherent drawback of a circular tank with centre feed is the influent flows radially or radially and vertically outwards through the tank at a constantly decelerated rate. This diverged flow is susceptible to disturbances and results in instability of the flow (Hubbell, 1938; Bergman, 1958).

From the theoretical and practical points of view, a long rectangular tank is to be preferred, because uniform flow with less turbulence can be obtained throughout much of the tank (Camp, 1946; R. Hazen, 1959). Sawyer (1956) studied density currents in rectangular and circular

tanks and concluded that the effects of such currents were more serious in circular tanks.

The floor of rectangular tank is usually slightly inclined towards the inlet ^e and where a hopper for sludge is provided. A circular tank usually have^s a floor sloping towards the centre. Dallas (1958) reported that a 'uniflow tank' which is rectangular in plan but triangular in longitudinal section produced by the use of a radically sloped bottom was efficient and economical. The effects of density currents were minimized and even though its volume was about half of that of a rectangular tank with uniform depth, the 'uniflow tank' exhibited some 40% longer delay in showing a tracer in its effluent. Townend & Wilkinson (1955) put forward an idea of making the depth of a circular tank inversely proportional to the radius in such a way as to keep the radial velocity constant throughout the tank. An improvement was expected from having no change in kinetic energy, hence no eddy turbulence. Because of the increase in Froude number with the radius, greater tank stability was also expected. Sludge removal should be improved due to the sloping floor. It was also economical in capacity. Some tanks of this design have been constructed at the West Middlesex Works, but at the time of writing of this thesis they have not yet been put into operation.

An interesting design which departed from all conventional sedimentation tanks was made by Hinderks (1929). The tank had the shape of an open spiral. The curvature of the spiral induces secondary currents counterbalancing the inflow turbulence. Solid matters were deposited on the inner side (convex), and by making the channel floor slope towards this side, they

could be easily removed by gravity. Unfortunately, there appears to be no further study of this kind of tank. This experiment showed the application of a hydraulic principle in the design of the tank which is an important step forward.

1.1.5. Inlet and outlet arrangements

The function of an inlet is to introduce the flow uniformly into the tank with minimum turbulence. An inlet structure is usually composed of a conduit leading the flow into the tank, and a diffuser which distributes it. For a rectangular tank, the flow usually comes into the tank from a pipe or a channel. It is common to bend the pipe upwards in such a way that the surface of water in the tank serves as a wide baffle destroying the kinetic energy of the flow as well as distributing it. Clifford (1935) designed a very successful inlet which bears his name. With this inlet, water from the pipe discharges into the open end of an 'eddy bucket' which is designed to destroy the kinetic energy and turbulence of the flow. Some tanks have their inlets in the forms of submerged weirs (Hinderks, 1929; Burgess & Green, 1960). Abdalla (1955) designed a 'manifold flow' inlet and reported that on the model scale it functioned satisfactorily. Hinderks (1929) also did some experiments with several types of inlets.

For a circular tank, the inlet is usually located at the centre with the riser opening at a distance below the liquid surface. Hubbell (1938) reviewed several types of riser and concluded that the main objection to most of them was the unequal velocity distribution caused by the

bends in the risers themselves and the consequent production of turbulence. To overcome this, Hubbell used guide vanes as equalizers. Instead of having a bell mouth at the opening of the riser, he arranged a series of circular plates, having holes of decreasing sizes in the centre, above it. The lowest plate had a hole of the same diameter as the riser and the top plate was solid. These plates distributed the flow and diffused it uniformly into the tank. However, this type of inlet had⁵ been little used. In most circular tanks, diffusers are in the form of slotted cylinders or circular submerged weirs. In some installations, the influent enters along small arc of the periphery wall of the tank and leaves it at the opposite side (Walton & Key, 1939). Fitch & Lutz (1960) designed a new type of inlet within which the influent was divided into two halves flowing in the opposite directions by means of submerged races. When met, the two streams were made to shear each other completely along their lengths. In the zone of shear the kinetic energy of the influent is instantaneously dissipated as turbulence. Many other arrangements have been used with a view, not only to destroying the kinetic energy of the influent and to distributing the flow uniformly, but also to create particular flow patterns. These types of arrangements will be more appropriately reviewed in Section 1.1.7.

Much research has been done on the design and use of baffles for the control of flow (~~Metcalf & Eddy, 1930~~; Hubbell, 1938; Walton & Key, 1939; Clifford and Windridge, 1935; ~~Tatcher, 1940~~; ~~Howitt & Roberts, 1945~~; Fair & ⁶Feyer, 1954; Dallas, 1958; Fitch & Lutz, 1960). It appears to be generally accepted that baffles required⁶ very carefully designed⁶ if they were

to function very satisfactorily. Badly designed baffles instead of destroying the kinetic energy of the influent they may induce cross currents which lead to severe short-circuiting.

Perforated baffle plates placed a short distance in front of the inlet and extending over the entire width and depth are used in several tanks for water treatment (American Society of Civil Engineering^s, 1940). Rohlic (1951) reported that the performance of oil separators was greatly improved by having a single vertical-slotted baffle, while Wiegmann & Müller-Neuhaus (1950) found that two vertical-slotted baffles in series were better than one. Both studies led to recommendation on the solidity ratio and other factors in both single and double baffles. Mau (1959) also studied the vertical-slotted baffles and found that they ^eaffected a greater improvement in the efficiency of both rectangular and circular conventional sedimentation tanks than any other inlets. Abdalla (1955) concluded from his experiment with perforated baffles of the solidity ratio of 60% that the upstream conditions of the baffle appeared to be important. It must be such as to promote favourable pre-distribution across the tank since losses through the holes were not big enough to smoothen the non-uniformity in flow distribution. Besides, he found that the perforated plates did not eliminate excessive turbulence for an appreciable distance on the downstream side.

Outlet arrangement is relatively simple although it is equally important. The usual design of an outlet is in the form of a weir. The weir loading and its siting are two major factors to be considered in the design of an outlet. The usual loading was recommended at less than 5000 U.S.

gallons per day per foot. If the loading is too high, the updraft currents will be too strong for good sedimentation, and the weir will influence the flow pattern over a large proportion of the tank. Kalinske (1948) used this as the criterion in designing the outlet for his circular tank. The problem was also studied by Abdalla (1955) for a rectangular tank. If the weir loading is too small, on the other hand, surface tension may break up the flow into streams. This takes place when the depth of the flow over the weir is less than $1/10$ th of an inch (Escritt, 1956). When this occurs it may be very difficult to secure uniform discharge over the entire length unless a saw-toothed edge is used to break up the weir into triangular notches

For a rectangular tank the weir is usually sited at the opposite end to the inlet, and extended across the full width of the tank. If this is found to be inadequate, a series of effluent launders in the form of channels providing a double length of weir may be constructed. Dallas (1958) reported the use of multiple weirs spaced along the surface of a tank to encourage the use of low weir loadings in order to reduce weir drag. Anderson (1945) demonstrated that for circular sedimentation tank a circular weir placed at two-third or three-quarters of the radial distance from the centre was in optimum position to draw off effluent. However, for simplicity in construction, they are usually formed on the periphery wall of the tank, often with triangular notches.

Sometimes the positions of the inlet and the outlet are interchanged. When this is done, the feed is now either from the whole of the

periphery wall or from a port formed at the periphery (Kiersted, 1924; Walton & Key, 1939; Voyses & Jenkins, 1943; Katz & Geinopolos, 1958; Bergman, 1958).

1.1.6. The effects of density currents on the hydraulic characteristics of sedimentation tanks

Density currents may be produced as a result of either temperature variation or difference in suspended solids. When there is a temperature difference between the incoming liquid and the liquid already in the tank, short-circuiting and stratification of flow are often observed. When the influent is colder than the tank contents, it tends to subside and flow along the floor towards the weir, leaving the upper layer of the liquid in the tank undisturbed. On the other hand, if the incoming liquid is warmer, it will flow in the upper portion of the tank. The liquid in the lower portion is therefore stagnant. Both phenomena reduce the effective volume of the tank and are undesirable.

The effects of the density currents are now well known and have been reported by many authors (Schröpfer, 1933; Walton & Key, 1939; Anderson, 1945; Sawyer, 1956). Bergman (1958) studied the effects of temperature differences in a circular sedimentation tank of the Dortmund type and found that the temperature difference between the liquid already in the tank and the incoming liquid had a far reaching effect on the tank efficiency. It was found that when the liquid in the tank was warmer than the influent the liquid appeared to be detained for a longer period in the tank

than when it was colder. His experiments cover a range of temperature difference between 0.5° to 7.2° F.

Density currents are also set up by the difference in specific gravity between the incoming liquid and the liquid in the tank because the former has a higher solid concentration. This phenomenon is more pronounced in final rather than in primary sedimentation tanks. Anderson (1945) discovered this fact while attempting to improve the efficiency of the circular sedimentation tanks at the Southwest Plant in Chicago. Sawyer (1956) suggested that the effects of density currents should be used as a limiting factor in the design of a sedimentation tank. Fitch & Lutz (1960) studied the problem again on a more rational basis, and attempts were made to utilize the tendency of the new feed to plunge and to stratify at the bottom of the tank to minimize short-circuiting. As a result, they designed an inlet which was previously described in Section 1.1.5.

1.1.7. Previous research on circular sedimentation tanks: flow path through a tank

Control of the flow path through a sedimentation tank is yet another field of investigation. It is quite a common practice when rectangular tanks are used to make the flow going over and under a series of transverse baffles. Other forms of control of the flow path have been provided by the use of horizontal trays (Camp, 1946) and inclined planes (Ingersoll, 1955; Abdalla, 1955; Ballance, 1959). Camp (1935) also attempted to create uniform flow in a rectangular tank by having the floor of the tank construct-

ed with a movable rubble^{er} belt.

Similar types of baffles as mentioned above can be used also in circular sedimentation tanks while there are other possibilities of controlling the flow in a spiral path or in rotational flow. Aitken (1945) constructed a circular tank having baffle walls built concentrically with the tank itself with the upward flow passages being larger than the downward ones. The maximum velocity throughout the tank was maintained lower than the critical velocity at which the floc would be destroyed. He claimed that the performance of his tank was equal to that of upflow tanks with the advantages of being easier to construct and more economical.

Kiersted (1924) reported the performance of a circular tank in which the flow was caused to take an inward spiral path with the speed of rotation increased as the centre of the tank was approached. This was achieved by arranging the liquid to come into the tank tangentially from three inlet ports at 120° apart, and withdrawing it at the centre through a vertical shaft. A study of the movement of the floats carefully placed in the tank at various depths showed that water completed several circuits before reaching the outflow vortex. One disadvantage of this type of arrangement was that the updraft current near the outlet was too strong.

In 1938 Hubbell obtained a spiral outward flow tank by installing a series of baffles around the central inlet. The baffles caused the flow to take spiral paths in travelling through the tank. Again water was observed to take several circuits before the outlet weir was reached. The baffles used were flat and were found to be as effective as curved or angular ones.

They need not be extended deeper than the bottom of the diffuser. This arrangement gave an improvement in the hydraulic characteristics of the tank compared with the standard arrangement, but the tank still showed short-circuiting of part of the flow and excessive detention of the other.

A year later, in 1939, Walton & Key put a plant into operation for the treatment of water for the supply of Alexandria. This had a tangential inlet at the periphery. The outlet was also at the periphery of the tank opposite to the inlet port. Before the final arrangement described above was adopted several experiments with outlet and inlet arrangements were carried out. Walton & Key also experienced strong updraft currents when the outlet was at the centre of the tank, as described by Kiersted. In one of their experiments they arranged such that the flow comes into the tank radially at the periphery and was withdrawn at the centre. Theoretically this arrangement should improve on the standard arrangement since the flow was now converging towards the centre. However, this proved to be deceptive as a consequence of the production of an induced uplift around the central draw off. Experiments with baffles and a larger diameter of the draw off shaft proved to be futile. It was after this test that Walton & Key decided to discard the idea of a circular tank operated on the radial flow principle.

Vokes and Jenkins (1942) altered one of their three upward flow tanks in Birmingham in such a way that the flow was introduced at the periphery wall tangentially in order to cause the tank content to rotate about the vertical axis. By building a wall on the existing periphery, the flow

was caused to pass two thirds of the periphery before coming to the submerged weir which occupied the rest of the periphery. From the results obtained over a considerable period and covering a range of different flow rates, they came to the conclusion that for separating activated sludge under these conditions the performance was inferior to the conventional tanks. They did not offer any explanation, but it was believed that either the rotation was not strong enough to suppress the density currents or that the uplift near the weir was too strong. It was possible also that the density difference between the liquid in the tank and the incoming liquid was large.

The idea of spiral flow had not been taken up again until Bergman (1958) postulated that by introducing the flow tangentially into an annular compartment adjacent to the periphery of the tank and drawing off the water at the centre, the combination of a vortex and radial flow introduced would cause the flow to take longest possible path between the inlet and the outlet. In fact his experiment was carried out with a conical bottom tank where the flow was originally combined upward and radial. Bergman's experiments indicated that this new arrangement was an improvement on the conventional one, especially when dealing with flow with a temperature different from that of the liquid in the tank.

Katz & Geinopolos (1958) made a comparative study of two types of circular tank, one with the conventional centre feed and the other with a peripheral feed. They found the peripheral feed tank more stable with less short-circuiting than in the centre feed tank. Tests with wax spheres of

known physical properties also indicated that the peripheral feed tank gave a higher solid removal efficiency than the conventional tank.

Having reviewed the researches in sedimentation thus far, it would be evident that, with a few exceptions notably the works of Hinderks (1930) and Camp (1936) little attempt has been made to apply the findings obtained in the field of Fluid Mechanics to the design of sedimentation tanks. It is well known in Fluid Mechanics that when the streamlines of a flow are curved, other properties of the flow itself change. It will be of interest to review researches in Fluid Mechanics which have bearing on the hydraulic conditions in a circular sedimentation tank.

1.1.8. Related researches in Fluid Mechanics.

1.1.8.1. Curvilinear flow: secondary currents

When a stream takes a curved path, there exist pressure gradients normal to the curved streamlines and parallel to the surface. Under the action of these pressure gradients the more slowly moving fluid in the boundary layer will follow paths of smaller radius of curvature. The transverse curvature of the streamlines increases as the wall is approached and a spiral motion of fluid may result.

The spiral motion is particularly noticeable if the aspect ratio is small, i.e. small depth to width ratio. Thomson (1876) used this fact, which he confirmed experimentally, to explain the natural phenomenon in which the secondary flow at a river bend removes material from the outer bank and piles it up on the inner bank. The beds of rivers are thus

continually scoured near the outer bank and silted near the inner bank.

If the aspect ratio is large, on the other hand, exactly the opposite result is found. The secondary flow along the longer vertical face is insufficient to carry water from the slow moving water at the concave surface to the convex. This results in a thick layer of dead water being accumulated on the concave side of the channel. Curved tubes with rectangular cross sections of high aspect ratios were used by Wilcken (Betz, 1930) to confirm the above phenomenon.

Eustice (1910, 1911, 1924, and 1925) and Hinderks (1927) also carried out some experiments with curved channels of rectangular cross sections of comparatively low aspect ratios, when spiral vortices were observed except at very low speeds. By constructing long radius elbows in split sections and using paint to coat the inside surfaces, Hinderks was able to demonstrate the existence of such motion.

The setting up of these secondary motions means that some energy is taken from the primary flow in addition to the energy lost in normal boundary resistance. With these facts in mind, Hinderks later applied this discovery to the design of a spiral flow sedimentation tank (see also Section 1.1.4.).

1.1.8.2. The effects of curvature on turbulence

A series of experiments on two dimensional flow in a curved channel was carried out by Wilcken, as reported by Betz in his paper in 1930, to study the development of the boundary layers along curved surfaces. He

found that the thickness of the boundary layers increased much more rapidly on the outer surface than on the inner surface. This was explained by the supposition that the turbulent interchange between fluid layers is promoted near the outer wall and diminished near the inner wall as a result of the centrifugal forces. According to this explanation the rapid increase of the boundary layer thickness is due to the increased interchange. These experiments are regarded as the confirmation of Prandtl's idea of the effect of centrifugal forces on turbulence (Prandtl, 1930; Dryden, Francis & Bateman, 1956).

Finding it difficult to draw any conclusion on the effects of curvature on turbulence from these experiments because the fluid paths are not parallel to the wall, Maccoll & Wattendorf (Wattendorf, 1935) employed a modified form of Wilcken apparatus to study the curvature effect on fully developed turbulent flow in which the main flow is parallel to the walls. Later Wattendorf carried the experiment further, trying to isolate the effect of constant curvature. He found slight increase in channel resistance probably influenced greatly by the induced secondary flows. Calculation of exchange factor and mixing length also lend support to Rayleigh's stability criterion which ^{predicts} the instability and increased mixing at the outer wall of the curved channel, and stability and decreased mixing at the inner wall (Rayleigh, 1916). From momentum considerations Prandtl concluded that there will be a stabilizing effect in the region where $\theta = \frac{u/r}{(du/dr) - u^2/r^2} > 0$ and a stabilizing effect if $\theta < 0$, u and r being the velocity and the radius respectively.

White (1929) experimented with curved tubes and pointed out the existence of a definite critical velocity for flow in helical pipes which was higher than the corresponding critical velocity for flow in straight pipes. Taylor's experiments (1929) showed clearly that, below a certain critical velocity, the effect of the tube curvature was to eliminate initial turbulence. The study of Eskinazi & Yeh (1956) also confirmed the curvature effect on turbulent quantities. For example, it was found that the radial fluctuation was suppressed near the inner wall and promoted near the outer wall respectively. It is clear from this study that the curvature of the streamlines alone is not sufficient to determine uniquely the characteristics of turbulence. The velocity distribution must also be considered.

1.1.8.3. Fluid in Rotation

Rayleigh's instability criterion mentioned previously was stated in 1916 in connection with the stability of revolving fluids. Applying Kelvin's circulation theorem and considering a disturbance with rotational symmetry, he came to the conclusion that for the system to be stable the square of the circulation $k = 2\pi r v$, v being the tangential velocity at any radius r , must increase outwards. Later Taylor (1923 b) applied this principle to his work on the stability of laminar flow between two rotating cylinders and brought in the viscosity effect. He found also (1922 & 1924) that if any small motion was communicated to a fluid which was rotating steadily like a solid body the resulting flow must be two dimensional though small oscillations about this state of motion were

possible. He demonstrated this by dropping colouring matter into rotating liquid. The colouring matter was not found to diffuse uniformly throughout the liquid. Instead, the slow movement of the liquid drew the coloured portion of the liquid out into sheets which remained always parallel to the axis of rotation. Fultz (1951) in his experiments combining convection and rotation did this test and came to the conclusion that the relative motion between the coloured portion and the rotating fluid was quasi-geostrophic, similar to meteorological motion on a large scale. Under rotation, the fluid appears to possess strongly anisotropic coefficients of eddy transfer between direction parallel to or across the axis of rotation.

An analytical study of the motion of a perturbed fluid element was discussed by Yeh (Traugott, 1956) who concluded that no matter what the initial disturbance was the subsequent relative motion of a fluid particle was oscillatory with a frequency of 2ω , where ω was the angular velocity of the main liquid body. Morgan (1951) considered the case of a small forced oscillation and found that the ratio of the amplitudes of the radial and tangential velocities was $\delta/2\omega$ where δ was the frequency of oscillation. It was clear from this study that when ω was very much greater than δ , the radial component of the oscillation was suppressed and the motion appeared to be two dimensional which agrees with Taylor's analysis. Traugott (1956) experimented with screen produced turbulence in a flow between concentric rotating cylinders. It was found that the radial fluctuating velocity was suppressed, the peripheral fluctuating velocity increased and the fluctuating velocity in the direction of the axis was not largely affected by the rotation.

An analytical and experimental study of rotating fluid was also conducted by Squire (1956). The flow of liquid from a point source on the axis of rotation was also considered. It was found that there was a region of irrotational flow around the axis created by the liquid from the source. Outside this region the liquid was rotational. The position of the surface separating the two regions depends upon the pressure head producing the source flow and the speed of rotation of the liquid. However, he only dealt with cases where pressure heads were low, and where the rotating fluid was unconfined in the direction parallel to the axis of rotation.

The study of secondary currents set up due to the contact of a fixed ground with the fluid in rotation and the movement of sedimentary particles was carried out as far back as in 1881 (Möller, 1881). This phenomenon causes small bodies at the bottom of a vessel to be carried to the centre and deposited there when the liquid in the vessel is put into rotation (e.g. undissolved sugar or tea leaves in a tea cup). The analytical study of the system was first conducted by Bödewadt (1940) in a very laborious way. Browning, in an unpublished paper (Schlichting, 1955), re-examined the problem and corrected Bödewadt's numerical solutions.

1.2. Introduction to the Problems under Investigation

It appears from the study of the works reviewed that by making use of the properties of liquid in curvilinear flow and in rotation, the hydraulic efficiency of a sedimentation tank may be improved. The advantages to be derived in this manner arise largely from the suppression of turbulence and also from the lengthening of the path the liquid has to take in flowing through the tank. We may now consider the advantages and disadvantages of various methods which have been suggested for modifying flow patterns in sedimentation tanks.

If the flow is introduced tangentially into the tank at the centre and drawn off at the periphery (Hubbell, 1938) the flow decelerates and is therefore susceptible to disturbances. Only a portion of the tank contents rotates because the main flow, having left the inlet tangentially tends to travel in a straight line towards the outlet weir. The angular speed is greater at the centre than at the periphery, and this also leads to instability of the system.

If the flow enters the tank tangentially at the periphery and is withdrawn at the centre, (Kierstead, 1924; Bergman, 1958) the outlet weir is short and heavily loaded. If on the other hand, water is drawn off also at the periphery (Walton & Key, 1939; Volkas & Jenkins, 1942), short-circuiting of water following the periphery is certain while water trapped at the centre of the tank would tend to remain there. The effective volume of the tank is thus reduced.

It appears that the disadvantages of these methods could be overcome by making the contents of the tank rotate as a solid body so that there is no relative movement between fluid. The advantages to be derived in this manner are as follows:-

(1) Settling solids in such a body of fluid would be similar to that in a quiescent liquid, but for the fact that there exists a field of centrifugal forces.

(2) If the influent comes into the tank at the centre it will be made to rotate by the surrounding liquid as it travels through the tank. The velocity of the flow in this case will increase with the radius of the tank and the system will be stable.

(3) Short-circuiting of the flow is now impossible because any fluid moving too fast radially will be pushed back as soon as it moves out into a region which rotates at a higher speed than its own.

(4) Turbulence at the inlet will be suppressed by the centrifugal forces.

(5) Dead water cannot exist at the centre since the incoming flow will push it outwards and the whole volume of the tank can be made effective.

(6) Density currents will be minimized or even eliminated.

(7) Secondary currents will help in moving the sedimentary particles towards the centre. The removal of deposited solids should therefore be easier and the scraping mechanisms may be dispensed with.

1.3. Introduction to the Investigations

(1) The first problem to be considered in the study of the performance of a sedimentation tank is to find a parameter or parameters that can be used to characterize the tank. In the past, investigators in this field employed parameters which are not rational. They are derived from the geometry of flow through curves without establishing their relationships, if any, with the tank hydraulic properties. The 'coefficient of quiescence' N in

$$\frac{c}{c_0} = \frac{N^N}{(N-1)!} \left(\frac{t}{T}\right)^{N-1} e^{-N\frac{t}{T}} \quad (\text{see Section 2.1.4.})$$

is virtually the same as 'n' in Hazen's theory, where 'n' denotes the number of imaginary cells constituting a sedimentation tank. Within each cell the mixing condition is assumed to be perfect and instantaneous. The assumption of perfect mixing is reasonable in the case of an aeration tank, because of the presence of baffle induced turbulence, but does not necessarily apply in the case of a sedimentation tank. This equation fails to explain why the coefficient of quiescence N or the number of the imaginary cells in a sedimentation tank should be different for different flow rates and inlets. The fact that the above equation describes very well the flow through curve of a sedimentation tank is significant and indicates that there possibly exists a relationship between N and the tank hydraulic conditions.

In Chapter 2, a theoretical flow through curve is derived as a result of the study of the flow in a sedimentation tank on a turbulent diffusion model. A parameter involving the turbulent diffusion coefficient is found and its relationship with N established.

(2) Having found such a parameter, it is now possible to conduct investigations into the effects of a forced vortex flow on the tank hydraulic characteristics.

This is done in 2 stages:-

(a) The nature of the newly introduced variable D , the diffusion coefficient, is discussed in terms of statistical analysis. This discussion, incidentally, also leads to the conclusion that the variance of the flow through curve is in fact dependent upon the diffusion coefficient.

(b) With the help of Reynolds stresses and turbulent energy equations, the effects of a forced vortex on the turbulent characteristics, which in turn affect D , are studied.

(3) Before designing an apparatus and performing experiments to verify the analytical results, dimensional analysis is carried out to serve as a guide in the experimental study.

(4) Experimental techniques are reviewed and improved where necessary, notably the methods of dye injection and concentration measurement. These are discussed in Chapter 3.

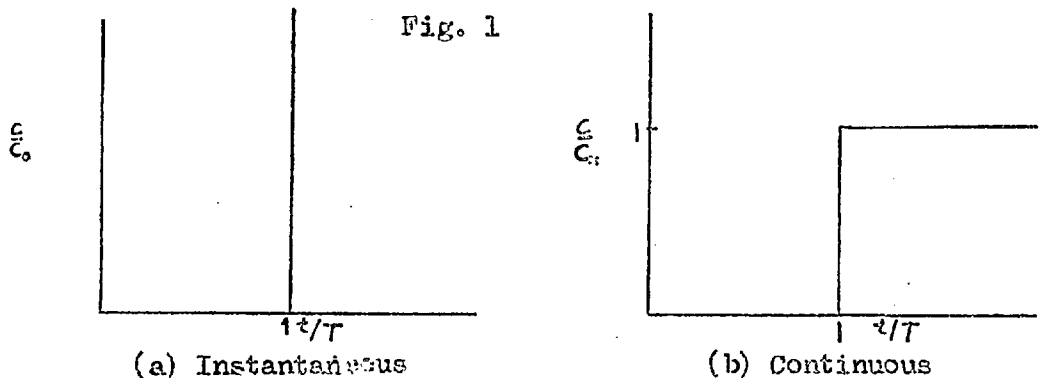
(5) In Chapter 4 the experimental investigation is described and the experimental phenomena encountered discussed. The conclusions drawn from the analytical and experimental study are given in Chapter 5. There will be found also some suggestions for further study.

CHAPTER 2

ANALYTICAL STUDY

2.1. Short-circuiting in a Circular Tank

In an ideal tank, each unit volume of the fluid is detained for a time equal to the theoretical detention time which is equal to the tank volume divided by the flow rate. The output of tracer introduced at the inlet will be shown in fig. 1 (a and b) depending on the method of introduction.



t/T is the ratio of time of concentration measurement to the theoretical detention time. c_x is the concentration of the incoming fluid. c/c_0 is the normalized concentration of tracer in fluid.

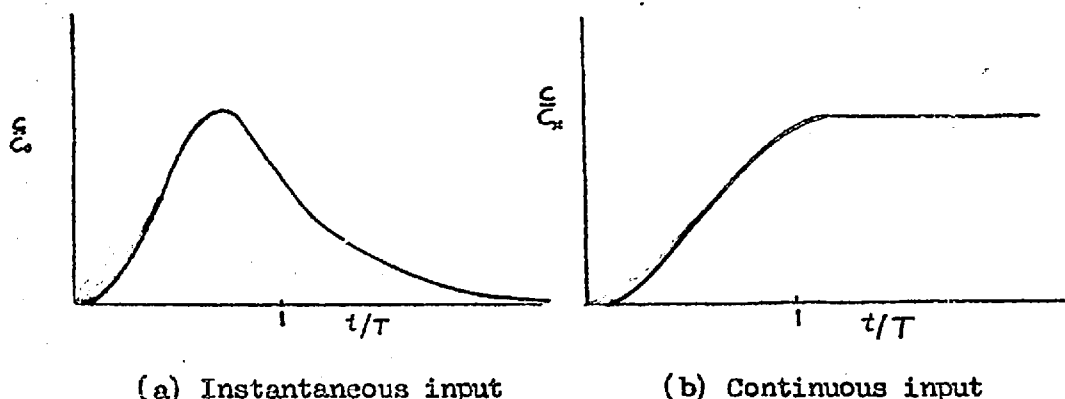
In actual operation, however, some of the inflow reaches the outlet in less than the theoretical detention time while some takes longer time. This phenomenon is usually termed 'short-circuiting'. This is the result of three main causes: -

- (a) The turbulent diffusion in the tank.
- (b) The non-uniform velocity distribution caused by the inlet.
- (c) Density currents, caused by difference in temperatures or

solids concentration.

Only short-circuiting due to turbulent diffusion will be treated mathematically here. The latter two causes of short-circuiting are best discussed qualitatively, and this is carried out in section 4.6.2. There are also other causes such as wind and desludging mechanisms which will not be considered here. All the causes mentioned above help in spreading the cloud of tracer and the actual flow through curves may look something like the following:-

Fig. 2



2.1.1. Short-circuiting due to turbulent diffusion

If the motion of a body of fluid is turbulent, the turbulent fluctuations tend to distribute the local characteristic which the fluid may possess. This may be momentum, kinetic energy, thermal, optical or chemical properties. A cloud of dye, for example, will be dispersed throughout an ever increasing volume. Turbulent diffusion is in addition to and can be considered as distinct from the ordinary diffusion due to molecular

agitation. The actual rate of increase or decrease in dye concentration evidently depends on two independent variables, namely, the carrying capacity of the cross flow, and the difference of the concentration. These can be written as a coefficient of lateral diffusion, D , and the lateral rate of change of concentration respectively (Rouse, 1950). The diffusion equation then takes the form: -

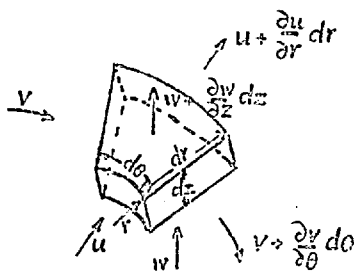
$$F = -D \frac{dC}{dy} \quad (1)$$

where F represents the rate of lateral transport of dye, D the coefficient of diffusion and C the dye concentration. (For a complete list of symbols see Appendix I. p. 193)

The nature of D and its relation to the kinematic eddy coefficient will be discussed in detail later when the effects of a forced vortex on it is considered (Section 2.3.)

The Differential equation

Fig. 3



Consider the fluid element as shown in fig. 3. Let F_{xx} be the rate of transport of tracer in x coordinate

$$\begin{aligned} F_{xx} &= -\left[D_{xx} \frac{\partial C}{\partial x} + D_{xy} \frac{\partial C}{\partial y} + D_{xz} \frac{\partial C}{\partial z} \right] \\ \text{and } F_{yy} &= -\left[D_{yx} \frac{\partial C}{\partial x} + D_{yy} \frac{\partial C}{\partial y} + D_{yz} \frac{\partial C}{\partial z} \right] \\ \text{and also } F_{zz} &= -\left[D_{zx} \frac{\partial C}{\partial x} + D_{zy} \frac{\partial C}{\partial y} + D_{zz} \frac{\partial C}{\partial z} \right] \end{aligned} \quad (2)$$

(Crank, 1956)

where $D_{xx} \frac{\partial c}{\partial x}$ is the rate of transport in the x direction due to the coefficient of diffusion in the x direction, $D_{yx} \frac{\partial c}{\partial y}$ is the rate of transport in the y direction due to the coefficient of diffusion in the x direction.

For a circular sedimentation tank in which the flow is axially symmetrical and two dimensional, the differential equation is

$$\frac{\partial c}{\partial t} = D \frac{\partial^2 c}{\partial r^2} + \frac{D}{r} \frac{\partial c}{\partial r} - u \frac{\partial c}{\partial r} \quad (3)$$

For the derivation of this equation see Appendix III.

The boundary conditions

Following the fact that the rate at which tracer (in the experiments to be described, a dye was used as the tracer) is introduced into the tank is equal to the rate at which it crosses the circle $r = a$, a being the radius of the inlet, by combined flow and diffusion, the boundary condition at this point can be expressed mathematically as:-

$$u c_x = u c - D \frac{\partial c}{\partial r}$$

where c_x is the dye concentration of the incoming flow.

At $r = b$, the radius of the overflow weir of the sedimentation tank

$$u c_e = u c - D \frac{\partial c}{\partial r}$$

where c_e is the dye concentration of the exit liquid. Looking at the equation more closely it will be found that if $\frac{\partial c}{\partial r}$ is negative the concentration of dye in the exit stream is greater than that before reaching the outlet weir. This is impossible. Again, if $\frac{\partial c}{\partial r}$ is positive, it means that the concentration would pass through a minimum somewhere in the tank and rise

towards the weir, which is also impossible. The boundary condition at $r = b$ must therefore be:-

$$D \frac{\partial c}{\partial r} = 0$$

So the boundary conditions of the present problem are

$$uc_r = uc - D \frac{\partial c}{\partial r} \quad r = a \quad (4)$$

$$D \frac{\partial c}{\partial r} = 0 \quad r = b \quad (5)$$

These boundary conditions are also used by Krammers & Aberda (1957) in their frequency response analysis of continuous flow systems.

Writing K for $ur = Q/2\pi$ where Q is the tank flow rate per unit depth and u the radial velocity at any radius r , Equation (3) now becomes

$$\frac{\partial c}{\partial t} = D \frac{\partial^2 c}{\partial r^2} + \frac{D}{r} \frac{\partial c}{\partial r} - \frac{K}{r} \frac{\partial c}{\partial r} \quad (6)$$

This can be conveniently solved by the method of Laplace transform (Appendix II). The Laplace transform of (5) is

$$\frac{\partial^2 \bar{c}}{\partial r^2} + \frac{1}{r} (1 - 2\nu) \frac{\partial \bar{c}}{\partial r} - q^2 \bar{c} = 0 \quad (7)$$

where $\nu = \frac{Q}{4\pi D}$, $q^2 = p/D$

Solving (6) with the general boundary conditions

$$r = a \quad k_1 \frac{\partial \bar{c}}{\partial r} - k_2 \bar{c} = k_3 \quad (8)$$

$$r = b \quad k_1' \frac{\partial \bar{c}}{\partial r} - k_2' \bar{c} = k_3' \quad (9)$$

$$\bar{c} = \frac{r^\nu}{p} \left[\frac{\phi I_\nu(qr) + \psi K_\nu(qr)}{(\eta\beta - \alpha\zeta)} \right] \quad (10)$$

$$\alpha = k_1 a^{(\nu-1)}(qa) I_{\nu-1}(qa) - k_2 a^\nu I_\nu(qa)$$

$$\beta = k_1' a^{(\nu-1)}(qa) K_{\nu-1}(qa) - k_2' a^\nu K_\nu(qa)$$

$$\eta = k_1' b^{(\nu-1)}(qb) I_{\nu-1}(qb) + k_2' b^\nu I_\nu(qb)$$

$$\zeta = k_1' b^{(\nu-1)}(qb) K_{\nu-1}(qb) - k_2' b^\nu K_\nu(qb)$$

$$\phi = k_2\beta - k_3\zeta$$

$$\psi = k_3\alpha - k_2\eta$$

This is the solution of the case in which the input is continuous. By theorem IV of Laplace transform (Appendix II) it can be shown that the solution for an instantaneous injection of dye is

$$\bar{C} = r^y \left[\frac{\zeta I_y(qr) + \xi K_y(qr)}{(\eta\beta - \alpha\zeta)} \right] \quad (11)$$

Inversion of the transform

In order to obtain a real solution from the transformed solution, the Inversion Theorem shown below is used:-

$$C = \frac{r^y}{2\pi i} \int_{\gamma-i\infty}^{\gamma+i\infty} e^{\lambda t} \left[\frac{\zeta I_y(\mu r) + \xi K_y(\mu r)}{(\eta\beta - \alpha\zeta)} \right] d\lambda \quad (12)$$

λ is now written for p and μ for q because $\bar{C}$ is now regarded as a function of complex variable, while p need not be complex at all. γ is a quantity so large that all the singularities of $\bar{C}$ lie to the left of the line $(\gamma-i\infty, \gamma+i\infty)$. The integrand has simple poles at $\lambda = -D\alpha_n^2$ where α_n is the n^{th} root of

$$\begin{aligned} & [k_1' i\alpha I_{y-1}(i\alpha b) + k_2' I_y(i\alpha b)] [k_1 i\alpha K_{y-1}(i\alpha a) + k_2 K_y(i\alpha a)] - \\ & - [k_1 i\alpha I_{y-1}(i\alpha a) + k_2 I_y(i\alpha a)] [k_1' i\alpha K_{y-1}(i\alpha b) + k_2' K_y(i\alpha b)] = 0 \end{aligned} \quad (13)$$

To evaluate the integrand the contour shown below is employed. The value of the integral is equal to the sum of the residues at poles inside this contour (fig. 4^(a)).

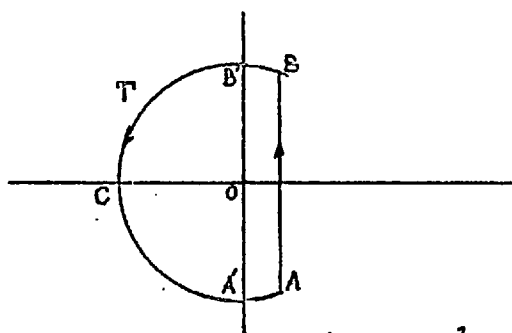


Fig. 4(a)

In order to find the residues at poles $\lambda = -D\alpha_n^2$; $\left[\frac{d(\eta\beta - \alpha\xi)}{d\lambda} \right]$ at $\lambda = -D\alpha_n^2$ has to be evaluated.

$$\begin{aligned} \left[\frac{d(\eta\beta - \alpha\xi)}{d\lambda} \right]_{\lambda = -D\alpha_n^2} &= \left[\frac{1}{2} \frac{d(\eta\beta - \alpha\xi)}{D\mu^2 d\mu} \right]_{\mu = i\alpha_n} \\ &= \left(\frac{1}{2D\mu^2} \right) [a'b'] \left[a \left\{ k_1' \mu I_{\nu-1}(\mu b) + k_2' I_{\nu}(\mu b) \right\} \left\{ k_1 \mu K_{\nu-1}(\mu a) + k_2 K_{\nu}(\mu a) \right\} + \right. \\ &\quad + b \left\{ k_1' \mu I_{\nu-1}(\mu b) + k_2' I_{\nu}(\mu b) \right\} \left\{ k_1' \mu K_{\nu-1}(\mu a) + k_2 K_{\nu}(\mu a) \right\} - \\ &\quad - b \left\{ k_1 \mu I_{\nu-1}(\mu a) - k_2 I_{\nu}(\mu a) \right\} \left\{ k_1' \mu K_{\nu-1}(\mu b) - k_2' K_{\nu}(\mu b) \right\} - \\ &\quad \left. - a \left\{ k_1 \mu I_{\nu-1}(\mu a) - k_2 I_{\nu}(\mu a) \right\} \left\{ k_1' \mu K_{\nu-1}(\mu b) - k_2' K_{\nu}(\mu b) \right\} \right]_{\mu = i\alpha_n} \end{aligned}$$

But at $\mu = i\alpha_n$ from (13)

$$\frac{k_1' \mu I_{\nu-1}(\mu b) + k_2' I_{\nu}(\mu b)}{k_1 \mu I_{\nu-1}(\mu a) - k_2 I_{\nu}(\mu a)} = \frac{k_1' \mu K_{\nu-1}(\mu b) - k_2' K_{\nu}(\mu b)}{k_1 \mu K_{\nu-1}(\mu a) + k_2 K_{\nu}(\mu a)} = \chi \text{ (say)}$$

and using Wronskian relation

$$\begin{aligned} &\left[\left(\frac{1}{2D\mu^2} \right) \frac{d(\eta\beta - \alpha\xi)}{d\mu} \right]_{\mu = i\alpha_n} \\ &= \left[\left(\frac{1}{2D\mu^2} \right) a'b' \chi \left\{ -k_1^2 \mu^2 a + k_2^2 a - 2k_1 k_2 \nu \right\} + \frac{1}{2\chi} a'b'^{-1} \left\{ -k_1^2 \mu^2 b + k_2^2 b + 2\nu k_1' k_2' \right\} \right]_{\mu = i\alpha_n} \\ &= -\left(\frac{1}{2D\alpha_n^2} \right) a'b' \chi \left[k_1^2 \alpha_n^2 a + k_2^2 a - 2k_1 k_2 \nu \right] + \left(\frac{1}{2\nu D\alpha_n^2} \right) a'b'^{-1} \left[k_1'^2 b + 2\nu k_1' k_2' + k_2'^2 b \right] \end{aligned} \quad (14)$$

Therefore

$$C = \sum_{n=1}^{\infty} \frac{r e^{-\alpha_n^2 t} [1][2] [6e^{\frac{1}{2}\nu\pi i} J_{\nu}(\alpha_n r) + \xi(-\frac{1}{2}\pi e^{\nu\pi i} H_{\nu}^{(2)}(\alpha_n r))]}{\frac{1}{2} a'b' D\alpha_n^2 [a \{ k_1'^2 b + k_2^2 b + 2\nu k_1' k_2' \} [2] - b \{ k_1^2 \alpha_n^2 a + k_2^2 a - 2k_1 k_2 \nu \} [1]]} \quad (15)$$

where

$$\begin{aligned}
 \phi &= [k_3' a^p \{k_1(i\alpha_n) (-\frac{1}{2}\pi e^{\frac{-\frac{1}{2}\pi(\nu-1)i}{H_{p-1}^{(2)}(\alpha na)} + k_2(-\frac{1}{2}\pi e^{\frac{-\frac{1}{2}\pi(\nu-1)i}{H_p^{(2)}(\alpha na)}\} \\
 &\quad - [k_3 b^p \{k_1'(i\alpha_n) (-\frac{1}{2}\pi e^{\frac{-\frac{1}{2}\pi(\nu-1)i}{H_{p-1}^{(2)}(\alpha nb)} + k_2'(-\frac{1}{2}\pi e^{\frac{-\frac{1}{2}\pi(\nu-1)i}{H_p^{(2)}(\alpha nb)}\} \\
 \xi &= [k_3' a^p \{k_1(i\alpha_n) e^{\frac{\frac{1}{2}\pi(\nu-1)i}{J_{p-1}(\alpha na)}} - k_2 e^{\frac{\frac{1}{2}\pi\nu i}{J_p(\alpha na)}}\} - k_3 b^p \{k_1'(i\alpha_n) e^{\frac{\frac{1}{2}\pi(\nu-1)i}{J_{p-1}(\alpha nb)}} \\
 &\quad - k_2' e^{\frac{\frac{1}{2}\pi\nu i}{J_p(\alpha nb)}}\} \\
 [1] &= k_1'(i\alpha_n) e^{\frac{\frac{1}{2}\pi(\nu-1)i}{J_{p-1}(\alpha nb)}} + k_2' e^{\frac{\frac{1}{2}\pi\nu i}{J_p(\alpha nb)}}; [2] = k_1(i\alpha_n) e^{\frac{\frac{1}{2}\pi(\nu-1)i}{J_{p-1}(\alpha na)}} + k_2 e^{\frac{\frac{1}{2}\pi\nu i}{J_p(\alpha na)}} \\
 [1]/[2] &= \chi
 \end{aligned}$$

Substitute

$$k_1 = k_1' = D; \quad k_2 = U; \quad k_3 = UC_x \quad \text{and} \quad k_3' = k_2' = 0$$

as the result of comparing (4) with (7). With these new constants (15)

becomes:-

$$\begin{aligned}
 C &= \sum_{n=1}^{\infty} r^p e^{-D\alpha_n^2 t} D^{\frac{1}{2}} i^{2p} [\alpha_n J_{p-1}(\alpha nb)] \left[k_1 J_{p-1}(\alpha na) - \frac{U}{D} J_p(\alpha na) \right] \\
 &= \frac{UC_x b^p D\alpha_n^{\frac{1}{2}} \pi (-1)^{p-1} i^{2p} [J_p(\alpha na) J_{p-1}(\alpha nb) - J_{p-1}(\alpha nb) J_p(\alpha na)]}{(\frac{1}{2} a^p b^p / D\alpha_n^2) [a \{ D\alpha_n^2 b \} \{ i^2 \alpha_n J_{p-1}(\alpha na) + \frac{U}{D} J_p(\alpha na) \}^2 D^2 \\
 &\quad - b \{ D\alpha_n^2 a + U^2 a - 2DU \} \{ D \alpha_n J_{p-1}(\alpha nb) \}^2]} \quad (16)
 \end{aligned}$$

At $r = b$, write

$$\begin{aligned}
 UC_x &= \frac{Q}{2\pi a} C_x = \frac{Q \cdot C_x}{2\pi a} \cdot \frac{KT}{KT} = \frac{Q_d}{2\pi KT} \cdot \frac{K}{a} \cdot T \\
 &= \frac{Q_d}{V} \cdot UT = C_0 UT
 \end{aligned}$$

where Q_d = volume of dye injected instantaneously into the tank

$2KT\pi$ = volume of the tank since $2KT = (b^2 - a^2)$.

equation (16) becomes:-

$$\frac{C}{C_0} = \sum_{n=1}^{\infty} \frac{2UT b^{p-1}}{a^p} e^{-D\alpha_n^2 t} \left[\frac{\{J_{p-1}(\alpha nb)\} \{J_{p-1}(\alpha na) - \frac{U}{D\alpha_n} J_p(\alpha na)\}}{\{J_{p-1}(\alpha na) - \frac{U}{D\alpha_n} J_p(\alpha na)\}^2 - J_p^2(\alpha na)} \right] \quad (17)$$

and at any radius r

$$\frac{C}{C_0} = \sum_{n=1}^{\infty} 2UT \left(\frac{r}{a}\right)^{\gamma} \left(-\frac{\gamma \alpha_n}{2}\right) e^{-D \alpha_n^2 t} \times \left[J_{\gamma-1}(\alpha_n b) Y_{\gamma}(\alpha_n r) - J_{\gamma}(\alpha_n r) Y_{\gamma-1}(\alpha_n b) \right] \left[\frac{J_{\gamma-1}(\alpha_n b) \left\{ J_{\gamma-1}(\alpha_n a) - \frac{U}{D \alpha_n} J_{\gamma}(\alpha_n a) \right\}}{\left\{ J_{\gamma-1}(\alpha_n a) - \frac{U}{D \alpha_n} J_{\gamma}(\alpha_n a) \right\}^2 - J_{\gamma-1}^2(\alpha_n b)} \right] \quad (18)$$

$$\text{and } \chi = \frac{D \alpha_n J_{\gamma-1}(\alpha_n b)}{D \alpha_n J_{\gamma-1}(\alpha_n a) - U J_{\gamma}(\alpha_n a)} = \frac{D \alpha_n Y_{\gamma-1}(\alpha_n b)}{D \alpha_n Y_{\gamma-1}(\alpha_n a) - U Y_{\gamma}(\alpha_n a)} \quad (19)$$

from which α_n can be determined.

2.1.2. Simplified Solution

To compare the equation (17) with the experimental curves and to obtain any other information from it, (19) must first be solved. The process is very tedious although it could be easily done by an automatic computer. It has been suggested that (19) might be simplified by letting a tend to zero (Jones, Talbot & Gordon). Again (19) involves the Bessel function of the second kind of the order γ and $\gamma-1$ which are difficult to deal with. It appears advisable therefore to reconsider the problem with $a = 0$.

As in many problems in the conduction of heat and diffusion where the solutions are easily found by obtaining first what is known as the Green's function of that particular problem. We shall use the same procedure here.

Green's function

The function is widely used in the theory of potential. When it

is known, the solution of a particular problem for the given region and boundary conditions, in which the initial condition is taken as an arbitrary function of position, can be written down immediately.

Green's function may be defined for a closed surface S as the potential which vanishes over the surface, and is infinite when r is zero, at the point $P(x', y', z')$ inside the surface. (Carslaw & Jaeger, 1959).

It can be shown that the concentration at time t in the tank with zero initial concentration is that obtained from a distribution of instantaneous sources at $t = 0$ over its volume. The solution of our problem is thus reduced to the determination of the Green's function for the liquid in which the concentration is required.

The solution for the instantaneous injection of dye of an amount Q_0 in an infinite medium is

$$\bar{C} = B(rq)^{\frac{1}{2}} K_0(qr) \quad (6) \text{ (Appendix III)}$$

where $B = Q_0 / (2^{\frac{1}{2}} \pi D v^{\frac{1}{2}})$ (Appendix III).

The differential equation is

$$\frac{\partial^2 \bar{C}}{\partial r^2} + \frac{1}{r} (1-2\nu) \frac{\partial \bar{C}}{\partial r} - q^2 \bar{C} = 0 \quad (3) \text{ (Section 2.1.1.)}$$

and the solution of this equation which is finite at $r = 0$ is

$$A r^{\frac{1}{2}} I_0(qr)$$

Therefore the general solution is

$$A r^{\frac{1}{2}} I_0(qr) + B r q^{\frac{1}{2}} K_0(qr)$$

The boundary condition at $r = b$ is

$$\frac{\partial \bar{c}}{\partial r} = 0$$

Therefore

$$A = \frac{B q^p K_{p-1}(qb)}{I_{p-1}(qb)}$$

The solution then becomes:-

$$\begin{aligned} \bar{c} &= B b^p \left[\frac{K_{p-1}(qb) I_p(qb) + I_{p-1}(qb) K_p(qb)}{I_{p-1}(qb)} \right] \\ &= B (bq)^p / I_{p-1}(qb) \end{aligned} \quad (20)$$

To obtain C from (20) we use

$$C = \frac{B b^{p-1}}{2\pi i} \int_{\gamma-i\infty}^{\gamma+i\infty} \frac{\mu^{p-1} e^{\lambda t}}{I_{p-1}(\mu b)} d\lambda \quad (21)$$

The integrand of (21) is a single-value function of λ . The zeros of $I_{p-1}(\mu b)$ are at $\lambda = -D\alpha_n^2$ where α_n , $n = 1, 2, \dots$ are roots of

$$J_{p-1}(\alpha_n b) = 0 \quad (22)$$

To evaluate the integral we use

$$\left[\frac{d}{d\lambda} I_{p-1}(\mu b) \right]_{\lambda = -D\alpha_n^2} = -\frac{1}{2D\alpha_n^2} \left[(\mu b) I_p(\mu b) \right]_{\mu = i\alpha_n}$$

(21) then becomes

$$\begin{aligned} C &= \sum_{n=1}^{\infty} \left[e^{-D\alpha_n^2 t} \cdot \frac{2\pi i}{\alpha_n b} b^{p-1} \alpha_n D \right] / (\alpha_n b) I_p(\alpha_n b) 2\pi i \gamma! \\ \frac{C}{C_0} &= \sum_{n=1}^{\infty} e^{-D\alpha_n^2 t} \frac{(b\alpha_n)^p}{2^p p! J_p(\alpha_n b)} \end{aligned} \quad (23)$$

where $C_0 = Q_d / \pi b^2$.

(23) can also be written as

$$\frac{C}{C_0} = \sum_{n=1}^{\infty} \frac{e^{-\left(\frac{\alpha_n b}{\lambda D}\right) \frac{t}{\gamma}} (\alpha_n b)^p}{2^p p! J_p(\alpha_n b)} \quad (24)$$

Curves of c/c_0 v. t/T for (24) for various values of γ are plotted in fig. 5.

2.1.5. Some properties of the simplified solution

(a) The time at which c/c_0 is maximum, $(t/T)_{\hat{c}/c_0}$

The following table has been prepared from fig. 5.

TABLE 1

γ	$(t/T)_{\hat{c}/c_0}$	$(\hat{c}/c_0)$
0.0	0.0	1.0
0.5	0.33	0.97
1.0	0.50	0.95
2.0	0.66	1.03
5.0	0.83	2.40

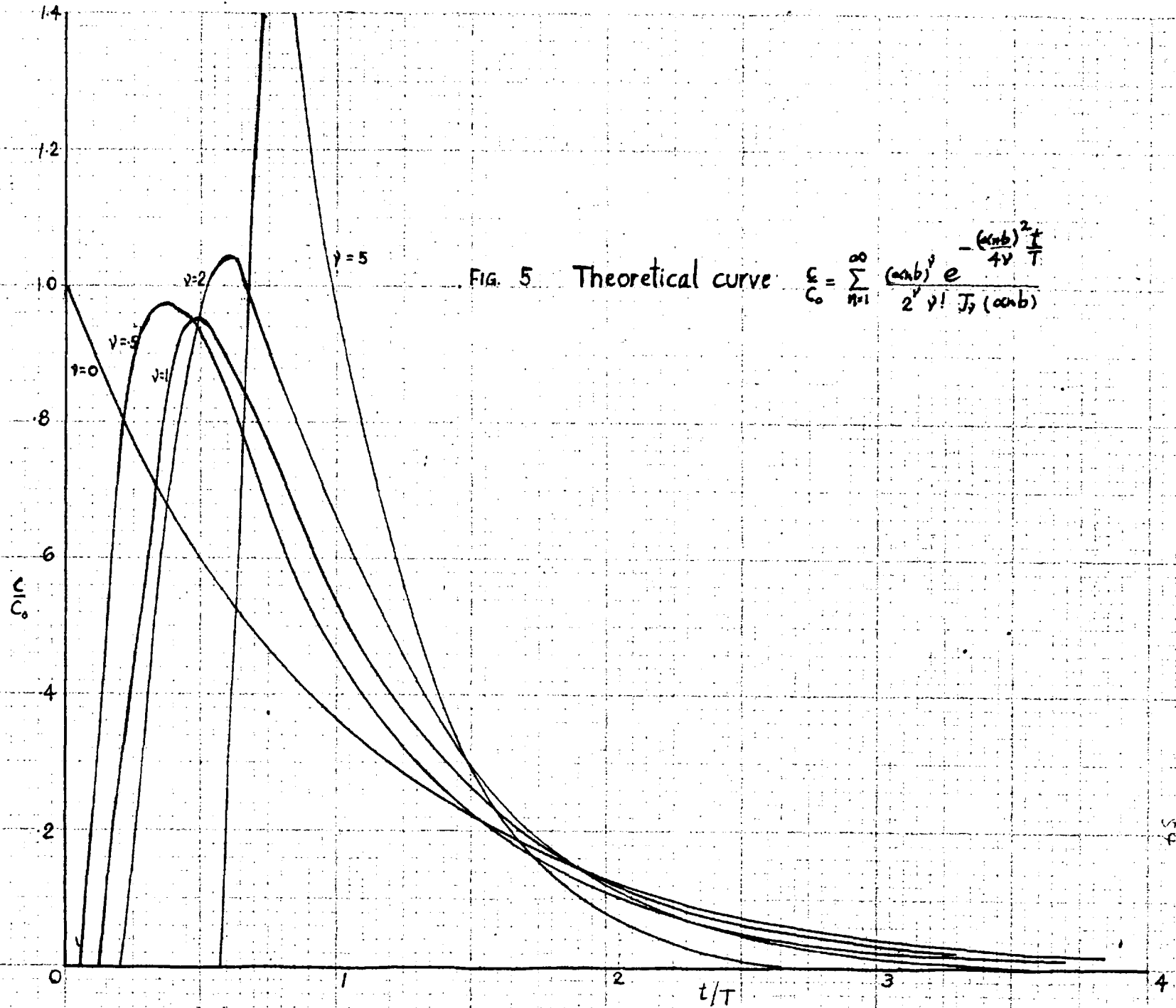
Examination of the table reveals that

$$\left(\frac{t}{T}\right)_{\hat{c}/c_0} = \frac{\gamma}{\gamma+1} \quad (25)$$

(b) The variance

The variance of (24) is

$$\begin{aligned} \sigma^2 &= \int_0^{\infty} \left(\frac{c}{c_0}\right) \left(\frac{t}{T}\right)^2 d\left(\frac{t}{T}\right) - \int_0^{\infty} \left(\frac{c}{c_0}\right) \left(\frac{t}{T}\right) d\left(\frac{t}{T}\right) \\ &= \int_0^{\infty} \left(\frac{c}{c_0}\right) \left(\frac{t}{T}\right)^2 d\left(\frac{t}{T}\right) - 1 \end{aligned}$$



Example of Calculation I

1. Theoretical Flow Through Curve

$$\frac{C}{C_0} = \sum_{n=1}^{\infty} \frac{e^{-\left(\frac{\alpha n b}{4\gamma}\right)^2 \frac{t}{T}}}{2^n \gamma! J_\gamma(\alpha n b)}$$

where $J_{\gamma-1}(\alpha n b) = 0$

$\gamma = 1$ $2^\gamma = 2$ and $\gamma! = 1$

n	$(\alpha n b)$		$C/C_0 = \sum_{n=1}^{\infty} [e^{-\left(\frac{\alpha n b}{4\gamma}\right)^2 \frac{t}{T}}] / [2^\gamma \gamma! J_\gamma(\alpha n b)]$ when t/T									
	$(\alpha n b)$	$J_1(\alpha n b)$	.2	.4	.6	.8	1.0	1.5	2.0	2.5	3.0	4.0
1	2.4048	+0.5191	+1.740	+1.300	+0.980	+0.737	+0.550	+0.267	+0.130	+0.063	+0.032	+0.007
2	5.5201	-0.3403	-1.768	-1.389	-0.090	-0.002	(b)	(b)	(b)	(b)	(b)	(b)
3	8.6537	+0.2715	+0.383	(b)	(b)	(b)						
4	11.7915	-0.2325	-0.022									
5	14.9309	+0.2065	(b)									
			Σ 0.332	0.911	0.890	0.735	0.550	0.267	0.130	0.063	0.032	0.007

(a) Jahnke, E. & Emde, F.

Tables of functions p. 166, Dover Publication, New York, 1945.

(b) The values are negligible.

2. The Variance

$$\sigma^2 = \sum_{n=1}^{\infty} \frac{2 \cdot (\alpha n b)^2}{2^n \gamma! J_\gamma(\alpha n b)} \left[\frac{4\gamma}{(\alpha n b)^2} \right]^3 - 1$$

where $J_{\gamma-1}(\alpha n b) = 0$

n	$(\alpha n b)$	$J_1(\alpha n b)$	$(\alpha n b)/J_1(\alpha n b)$	$[(\alpha n b)^2/4]^3$	$\sigma^2 + 1$	$\frac{1}{\sigma^2}$
1	2.4048	+0.5191	+4.650	+3.000	+1.550	$\frac{1}{.513} \pm 2.0$
2	5.5201	-0.3403	-16.200	+443.000	-0.037	
3	8.6537	+0.2715	+31.900	+6630.000	(b)	
					$\Sigma = 1.513$	

since the second term on the right is equal to unity.

$$\begin{aligned}
 G^2 &= \sum_{h=1}^{\infty} \int_0^{\infty} \frac{(\alpha n b)^y}{2^y \gamma! J_y(\alpha n b)} e^{-\frac{(\alpha n b)^2}{2\gamma} \left(\frac{t}{T}\right)^2} \left(\frac{t}{T}\right)^2 d\left(\frac{t}{T}\right) - 1 \\
 &= \sum_{h=1}^{\infty} \frac{(\alpha n b)^y}{2^y \gamma! J_y(\alpha n b)} \int_0^{\infty} e^{-\frac{(\alpha n b)^2}{2\gamma} \left(\frac{t}{T}\right)^2} \left(\frac{t}{T}\right)^2 d\left(\frac{t}{T}\right) - 1 \\
 &= \sum_{h=1}^{\infty} \frac{2 \cdot (\alpha n b)^y}{2^y \gamma! J_y(\alpha n b)} \left[\frac{4\gamma}{(\alpha n b)^2} \right]^{\frac{3}{2}} - 1
 \end{aligned} \tag{26}$$

Evaluation of (26) is shown in Example of Calculation I, and the results shown in Table 2.

TABLE 2

γ	G^2
0.0	1.00
0.5	1.53
1.0	1.95
2.0	2.90
5.0	6.05

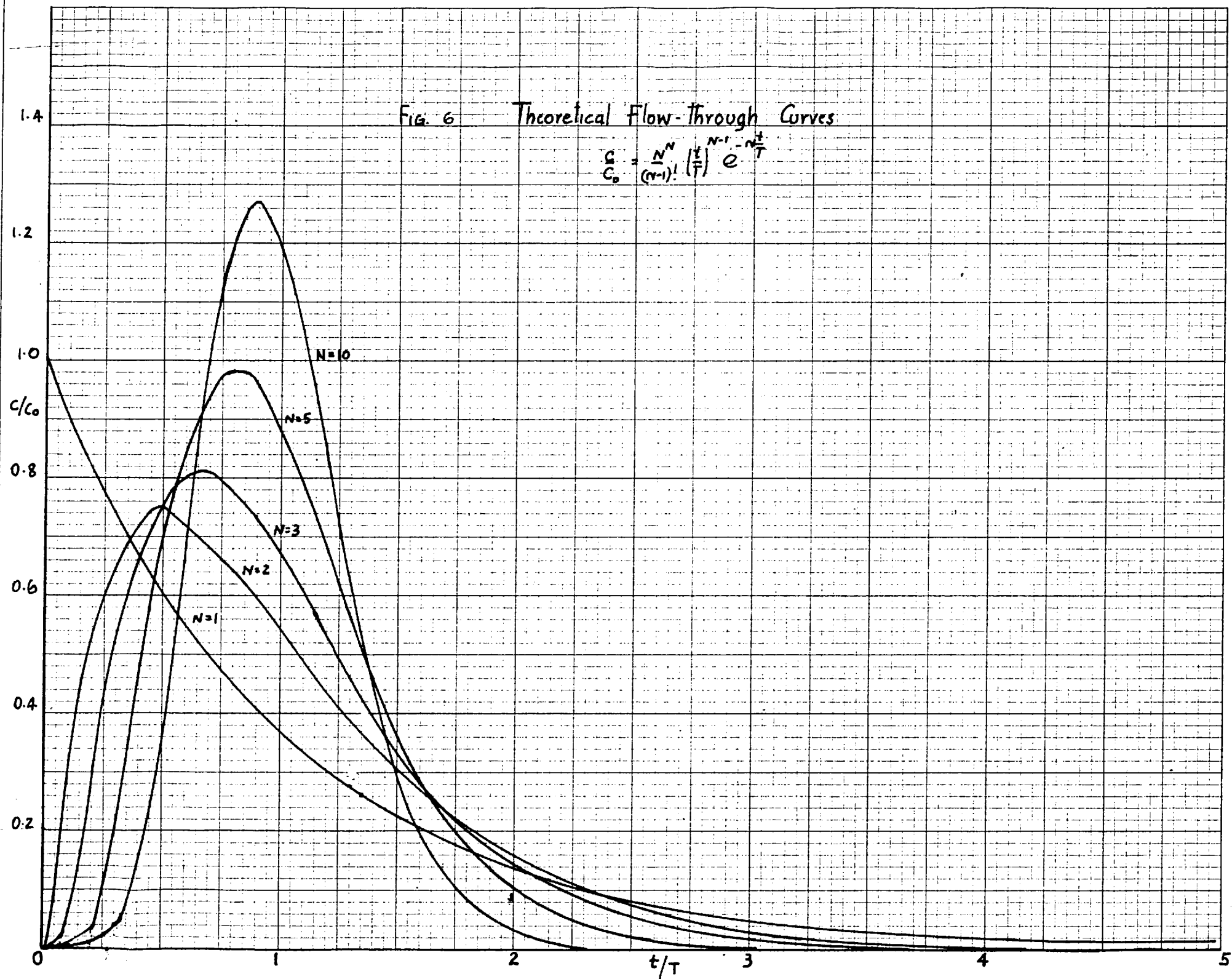
$$\text{or } G^2 = \frac{1}{\gamma+1}$$

2.1.4. The N- formula

Imagine a sedimentation tank divided into N compartments in each of which perfect mixing occurs instantaneously. It can be shown that:-

Fig. 6 Theoretical Flow-Through Curves

$$\frac{C}{C_0} = \frac{N^N}{(N-1)!} \left(\frac{t}{T}\right)^{N-1} e^{-\frac{Nt}{T}}$$



When $N = 1$ (27) becomes
$$\frac{c}{c_0} = \frac{N^N}{(N-1)!} \left(\frac{t}{T}\right)^{N-1} e^{-N\frac{t}{T}} \quad (27)$$

$$\frac{c}{c_0} = e^{-\frac{t}{T}} \quad (27a)$$

which agrees with (24) when $\gamma = 0$. It can be shown further that (see Section 2.1.5.)

$$\left(\frac{t}{T}\right) \frac{1}{c_0} = \frac{N-1}{N} \quad (29)$$

$$\sigma^2_{\text{with (24)}} = \frac{1}{N} \quad (30)$$

Comparing (27), (29) and (30), (25) and (26), it is evident that

$$\gamma = N-1 \quad (28)$$

Incidentally, 2γ is also known as Peclet number and the result shown in (30) agrees with those obtained by other investigators with different flow systems (Kramers & Aberda, 1957; Van der Laan, 1957). Following this analysis it seems justifiable to use the N -formula as an approximate formula governing the relationship between c/c_0 and t/T in an experimental analysis.

A similar equation

$$p dt = \frac{1}{T} \frac{N^N}{(N-1)!} \left(\frac{t}{T}\right)^{N-1} e^{-N\frac{t}{T}} dt$$

where $p dt$ represents the proportion of the flow, and p the concentration, has been found to fit many time-concentration distributions (Thomas & Archibald, 1952). Thomas & McKee (1944) also derived an equation

$$\frac{c}{c_0} = \frac{N^N}{(N-1)!} x^{N-1} e^{-N\frac{t}{T}}$$

However, this equation is dimensionally homogeneous only if $x = t/T$. This had not been clearly stated by them. Camp (1955) reported that Stein had developed (27) for the instantaneous dispersion curve for N -tanks of equal size in series.

2.1.5. Some properties of the N-formula

(a) Time at which c/c_0 is maximum, i.e. $(\frac{t}{T})_{\xi}^{\wedge}$

By differentiating c/c_0 with respect to t/T and equating it to zero

$$\frac{N^N}{(N-1)!} \left(\frac{t}{T}\right)^{N-1} (-N) e^{-N\frac{t}{T}} + \frac{N^N}{(N-1)!} e^{-N\frac{t}{T}} (N-1) \left(\frac{t}{T}\right)^{N-2} = 0$$

we have

$$\left(\frac{t}{T}\right)_{\xi}^{\wedge} = \frac{N-1}{N} \quad (29)$$

(b) Maximum concentration, i.e. $\hat{\frac{c}{c_0}}$

Putting (29) into (27) we have

$$\begin{aligned} \frac{c}{c_0} &= \frac{N^N}{(N-1)!} \frac{(N-1)^{N-1}}{N^{N-1}} e^{-N-1} \\ &= \frac{N(N-1)^{N-1}}{(N-1)!} e^{-(N-1)} \end{aligned} \quad (29a)$$

The curve representing (29a) is plotted in fig. 7.

(c) Variance

The variance of a curve is given by

$$\sigma^2 = \int_a^b x^2 F(x) dx - \left[\int_a^b x F(x) dx \right]^2$$

In this case x corresponds to t/T and $F(x)$ to c/c_0 . The interval a to b is the domain of variable x namely $0 \rightarrow \infty$

Therefore

$$\begin{aligned} \sigma^2 &= \int_0^{\infty} \frac{N^N}{(N-1)!} \left(\frac{t}{T}\right)^{N+1} e^{-N\frac{t}{T}} d\left(\frac{t}{T}\right) - \left[\int_0^{\infty} \frac{N^N}{(N-1)!} \left(\frac{t}{T}\right)^N e^{-N\frac{t}{T}} d\left(\frac{t}{T}\right) \right]^2 \\ &= \frac{N+1}{N} - 1 = \frac{1}{N} \end{aligned} \quad (30)$$

(30) can be approximated as follows:-

$$\sigma^2 \doteq \frac{\sum \left(\frac{\xi}{c_0}\right) \left(\frac{t}{T}\right)^2}{\sum \left(\frac{\xi}{c_0}\right)} - \left[\frac{\sum \left(\frac{\xi}{c_0}\right) \left(\frac{t}{T}\right)}{\sum \left(\frac{\xi}{c_0}\right)} \right]^2 \quad (30a)$$

(d) Time to the centre of gravity of the flow through curve,

i.e. $\bar{t}_c$ can be calculated from

$$\frac{\bar{t}_c}{\bar{t}} = \frac{\int_0^{\infty} \left(\frac{N''}{(N-1)!} \left(\frac{t}{\bar{t}} \right)^{N-1} e^{-N \frac{t}{\bar{t}}} d\left(\frac{t}{\bar{t}} \right) \right)}{\int_0^{\infty} \left(\frac{N''}{(N-1)!} \left(\frac{t}{\bar{t}} \right)^{N-1} e^{-N \frac{t}{\bar{t}}} d\left(\frac{t}{\bar{t}} \right) \right)} \quad (31)$$

This shows that the ratio of the time to the centre of gravity of the curve to the theoretical detention time is always unity, and it is not influenced by the mixing conditions in the tank - unless part of the tank is not functioning.

(e) Median time, i.e. $\bar{t}_m$

This is the time at which 50% of the dye has passed through the tank, i.e.

$$\begin{aligned} \int_0^{\bar{t}_m} \left(\frac{N''}{(N-1)!} \left(\frac{t}{\bar{t}} \right)^{N-1} e^{-N \frac{t}{\bar{t}}} d\left(\frac{t}{\bar{t}} \right) \right) &= \frac{1}{2} \\ \frac{1}{2} &= \frac{N''}{(N-1)!} \left[\frac{1}{N} \left(\frac{\bar{t}_m}{\bar{t}} \right)^{N-1} e^{-N \frac{\bar{t}_m}{\bar{t}}} + \frac{(N-1)}{N^2} \left(\frac{\bar{t}_m}{\bar{t}} \right)^{N-2} e^{-N \frac{\bar{t}_m}{\bar{t}}} + \frac{(N-1)(N-2)}{N^3} \left(\frac{\bar{t}_m}{\bar{t}} \right)^{N-3} e^{-N \frac{\bar{t}_m}{\bar{t}}} \right. \\ &\quad \left. + \dots + \frac{1}{N''} \{ (N-1)(N-2) \dots 1 \} e^{-N \frac{\bar{t}_m}{\bar{t}}} \frac{1}{N''} (N-1)! \right] \end{aligned} \quad (32)$$

(32) is plotted in fig. 8.

Fig. 7 Maximum Concentration versus N

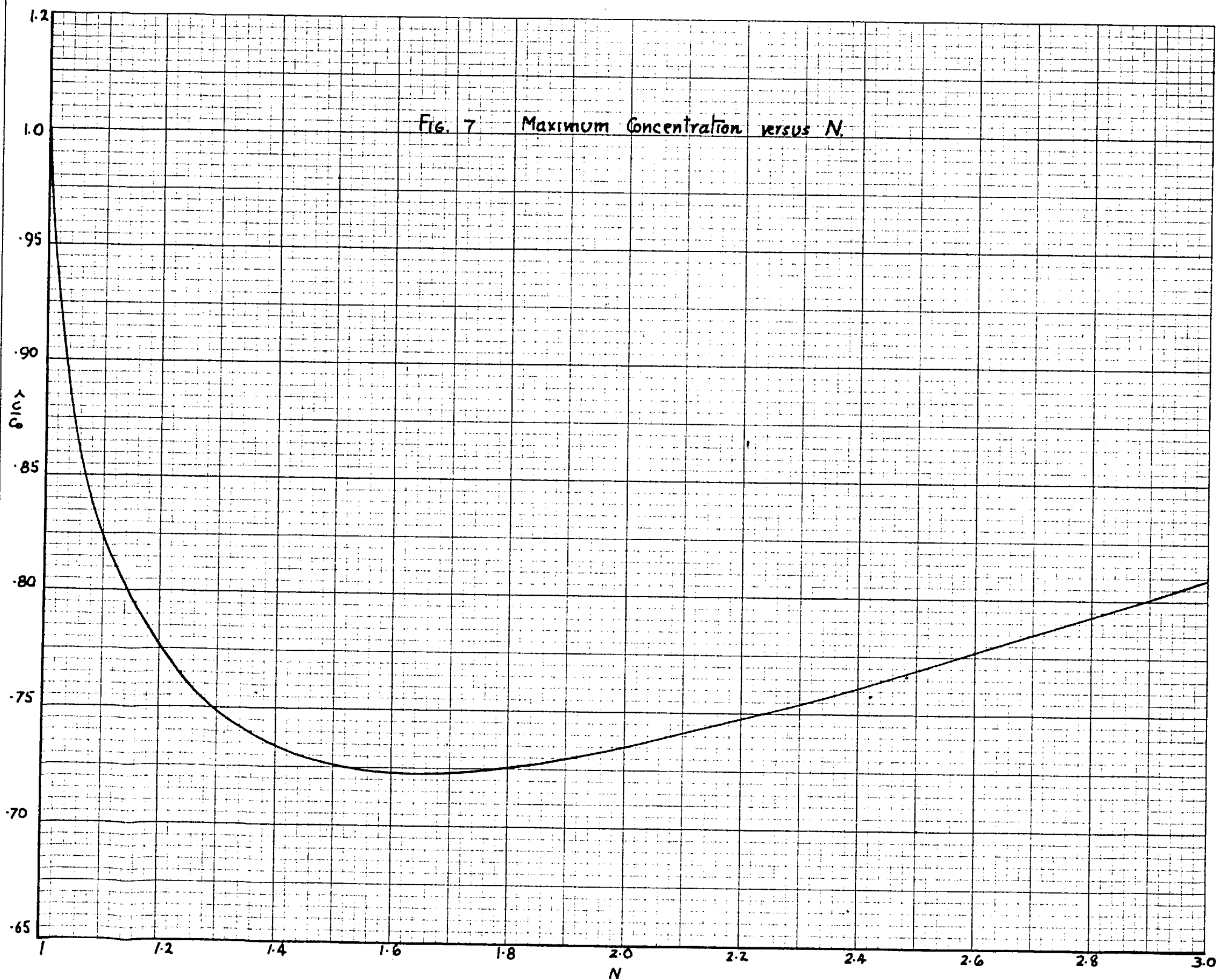
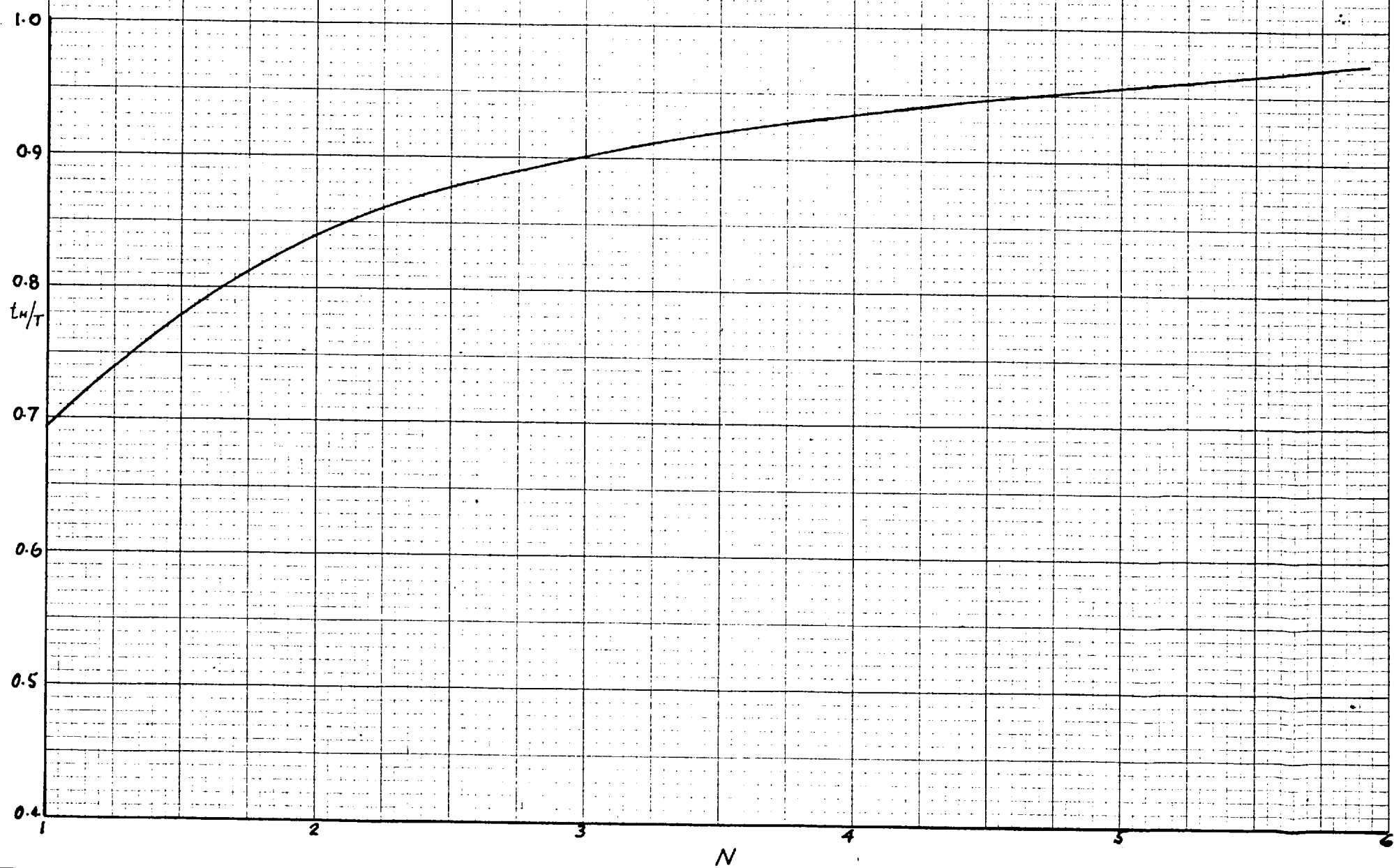


FIG. 8 t_w/T versus N



2.2. The Effects of a Forced Vortex on the Flow Characteristics of a Circular Sedimentation Tank

2.2.1. Theoretical Flow Pattern

When there is no forced vortex superimposed on it, the flow in the tank is essentially two dimensional except in the neighbourhood of the outlet weir. Therefore for a greater portion of the tank, the continuity equation takes the form

$$\frac{\partial(ur)}{\partial r} = 0 \quad (1)$$

The flow pattern changes completely, however, once a forced vortex is introduced. If the turbulence created by the paddles is neglected and the tank floor is assumed frictionless, the problem can be treated as that of the flow between two rotating cylinders with the inner and the outer cylinders acting as a cylindrical source and the outlet respectively.

The assumption of a frictionless floor is necessary since it is desirable to neglect the effect of secondary currents at this stage so as to obtain a general idea of what is happening in the tank when the contents are put into rotation.

(a) Differential equations

The equations governing the viscous flow of an incompressible fluid are those given by Navier-Stokes (Rouse, 1959a). They can be written in cylindrical polar coordinate form as follows:-

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial r} + \frac{v}{r} \frac{\partial u}{\partial \theta} + w \frac{\partial u}{\partial z} - \frac{v^2}{r} = -\frac{1}{\rho} \frac{\partial p}{\partial r} + \nu \left(\nabla^2 u - \frac{u}{r^2} - \frac{2}{r^2} \frac{\partial v}{\partial \theta} \right) \quad (2)$$

$$\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial r} + \frac{v}{r} \frac{\partial v}{\partial \theta} + w \frac{\partial v}{\partial z} + \frac{uv}{r} = -\frac{1}{\rho} \frac{\partial p}{r \partial \theta} + \nu \left(\nabla^2 v + \frac{2}{r^2} \frac{\partial u}{\partial \theta} - \frac{v}{r^2} \right) \quad (3)$$

$$\frac{\partial w}{\partial t} + u \frac{\partial w}{\partial r} + \frac{v}{r} \frac{\partial w}{\partial \theta} + w \frac{\partial w}{\partial z} = -\frac{1}{\rho} \frac{\partial p}{\partial z} + \nu \nabla^2 w \quad (4)$$

where

$$\nabla^2 = \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{1}{r^2} \frac{\partial^2}{\partial \theta^2} + \frac{\partial^2}{\partial z^2}$$

and the equation of continuity is

$$\frac{\partial(ur)}{r \partial r} + \frac{\partial v}{r \partial \theta} + \frac{\partial w}{\partial z} = 0 \quad (5)$$

For axially symmetrical and two dimensional flow (2) & (3) can be written as

$$u \frac{\partial u}{\partial r} - \frac{v^2}{r} = -\frac{1}{\rho} \frac{\partial p}{\partial r} + \nu \left(\frac{\partial^2 u}{\partial r^2} + \frac{\partial u}{r \partial r} - \frac{u}{r^2} \right) \quad (6)$$

$$u \frac{\partial v}{\partial r} + \frac{uv}{r} = \nu \left(\frac{\partial^2 v}{\partial r^2} + \frac{\partial v}{r \partial r} - \frac{v}{r^2} \right) \quad (7)$$

(4) disappears.

(6) & (7) can be written also as

$$u \frac{\partial u}{\partial r} - \frac{v^2}{r} = -\frac{1}{\rho} \frac{\partial p}{\partial r} + \nu \left(\frac{\partial}{\partial r} \frac{1}{r} \frac{\partial(ur)}{\partial r} \right) \quad (8)$$

and

$$\frac{u \partial(ur)}{r \partial r} = \nu \left(\frac{\partial}{\partial r} \frac{\partial(ur)}{\partial r} \right) \quad (9)$$

The equation of continuity becomes

$$\frac{\partial(ur)}{\partial r} = 0 \quad (10)$$

which is essentially the same as (1).

Integrating (1) gives $ur = \text{constant} = Ua$ where U is the velocity at $r = a$. (11)

Substitute (10) & (11) in (8)

$$\frac{1}{\rho} \frac{\partial p}{\partial r} = \frac{v^2}{r} + \left(\frac{Ua}{r} \right) \left(-\frac{Ua}{r^2} \right) = \frac{v^2}{r} - \frac{(Ua)^2}{r^3} \quad (12)$$

(12) shows that the pressure at any radius r is the difference between the pressure due to the centrifugal and the potential forces.

Put (12) in (9) we have

$$Ua \left(\frac{1}{r} \frac{\partial v}{\partial r} + \frac{v}{r^2} \right) = v \left(\frac{\partial^2 v}{\partial r^2} + \frac{1}{r} \frac{\partial v}{\partial r} - \frac{v}{r^2} \right)$$

which can be written also as

$$r^2 \frac{\partial^2 v}{\partial r^2} + (1-R)r \frac{\partial v}{\partial r} - (1+R)v = 0 \quad (13)$$

where $R = Ua/\gamma$

General Solution

(13) is a second order homogeneous differential equation and has the solution of the type

$$v = r^m \quad (14)$$

(Put (14) into (12) we have

$$m(m-1) + (1-R)m - (1+R) = 0$$

from which

$$m_1 = -1, m_2 = (1+R)$$

The general solution of (13) is therefore

$$v = Ar^{-1} + Br^{1+R} \quad (15)$$

Boundary conditions

Three cases will be considered here:

(a) With the inner and the outer cylinders rotating

$$v = \omega_1 r_1 \quad \text{at } r = r_1$$

$$v = \omega_2 r_2 \quad \text{at } r = r_2$$

(15) becomes

$$v = \frac{1}{r_1^{2+R} - r_2^{2+R}} \left[(\omega_1 r_1^{2+R} - \omega_2 r_2^{2+R}) r - r_1^{\frac{1+R}{2}} r_2^{\frac{1+R}{2}} (\omega_1 r_2^{\frac{1+R}{2}} - \omega_2 r_1^{\frac{1+R}{2}}) / r \right] \quad (16)$$

(b) With the inner cylinder stationary and the outer rotating

$$v = 0 \quad \text{at } r = r_1$$

$$v = \omega_2 r_2 \quad \text{at } r = r_2$$

(15) is now
$$v = \frac{\omega_1 r_1^2}{r_1^{2+R} - r_2^{2+R}} \left[r^{1+R} - \frac{r_2^{2+R}}{r} \right] \quad (17)$$

(c) With the inner cylinder rotating and the outer stationary

$$v = 0 \text{ at } r = r_1$$

$$v = \omega_1 r_1^2 \text{ at } r = r_2$$

we have

$$v = \frac{\omega_1 r_1^2}{r_1^{2+R} - r_2^{2+R}} \left[\frac{r_1^{2+R}}{r} - r^{1+R} \right] \quad (18)$$

2.2.2. Stream function

At any radius r

$$u = \frac{Q}{2\pi r}$$

and for case (a)

$$v = \frac{1}{r_1^{2+R} - r_2^{2+R}} \left[(\omega_1 r_1^2 - \omega_2 r_2^2) r^{1+R} - r_1^2 r_2^2 (\omega_1 r_2^R - \omega_2 r_1^R) / r \right]$$

the angle of the direction of flow to the radius is

$$\theta = \tan^{-1} \frac{\text{Eq. (16)}}{Q} \cdot 2\pi r \quad (19)$$

If ψ denotes the stream function

$$-\frac{\partial \psi}{r \partial \theta} = u \quad (20)$$

and

$$\frac{\partial \psi}{\partial r} = v \quad (21)$$

As the system is axially symmetrical

$$\begin{aligned} \psi &= - \int u r d\theta + \int v dr \\ &= - U a \theta + \frac{1}{r_1^{2+R} - r_2^{2+R}} \left[(\omega_1 r_1^2 - \omega_2 r_2^2) \frac{r^{2+R}}{2+R} - r_1^2 r_2^2 (\omega_1 r_2^R - \omega_2 r_1^R) \times \ln r \right] \\ &= K_1 \theta + K_2 r^{2+R} - K_3 \ln r \quad (22) \end{aligned}$$

If $\omega_1 r_1^2 = \omega_2 r_2^2$ $K_2 = 0$, the flow pattern is then logarithmic spiral.

As r approaches zero (22) tends to $K_1 \theta - K_3$ which represents a radial flow, and as r very large (22) tends to $K_2 r^{2+R}$ which represents rotating flow, which, if R is very small or zero,

$$v = 2K_2 r$$

This is when we have solid body rotation.

2.3. Statistical Analysis of Diffusion

2.3.1. The diffusion coefficient

The lagrangian technique of following the motion of individual fluid particles in the analysis of turbulent diffusion will be used. By this means, the spread of marked fluid particles can be predicted statistically from purely kinematic relationship.

Let us consider first the case when there is no general flow of the fluid and assume that the system is statistically steady and homogeneous.

The Lagrangian correlation coefficients

$$\begin{aligned} R_{Lx} &= \frac{\overline{u(0)} \overline{u(t)}}{\sqrt{\overline{u(0)^2} \overline{u(t)^2}}} & R_{Ly} &= \frac{\overline{v(0)} \overline{v(t)}}{\sqrt{\overline{v(0)^2} \overline{v(t)^2}}} \\ R_{Lz} &= \frac{\overline{w(0)} \overline{w(t)}}{\sqrt{\overline{w(0)^2} \overline{w(t)^2}}} \end{aligned} \quad (1)$$

therefore depend only on space and the interval t_1 . Bars over the quantities indicate that mean - ensemble mean - has been taken. For each of these correlations, a Lagrangian time scale is defined as the area under the appropriate curve of R_L versus t_1 :

$$T_{Lx} = \int_0^{\infty} R_{Lx} dt_1 \quad \text{etc.} \quad (2)$$

Imagine at time $t = 0$, a large number of particles are placed at $r = 0$. The spreading of this cloud of particles can be described statistically in terms of variance $\overline{r^2}$ of the radial distances r of its composite

particles. If the system is isotropic, i.e. $\overline{u^2} = \overline{v^2} = \overline{w^2}$ and $\overline{u} = \overline{v} = \overline{w} = 0$, a single velocity term is adequate to describe the system, and the mean displacement of the particles which pass any given point is zero. It can then be shown that

$$\overline{r^2} = 2\overline{u^2} \int_0^T \int_0^\xi R_L dt d\xi \quad (\text{Rouse, 1959b}). \quad (3)$$

In the present investigation, the spread of the cloud of dye in the radial direction is of interest.

Differentiate (3) with respect to t we have

$$\frac{d \overline{r^2}}{d t} = 2 \overline{u^2} \int_0^T R_L^u dt \quad (4)$$

where R_L^u is the coefficient of correlation in the radial direction.

The spread of particles which diffuse in random manner can also be described as:

$$\frac{\partial c}{\partial t} = D \frac{\partial^2 c}{\partial r^2} \quad (5)$$

$$\text{or} \quad c = A t^{-\frac{1}{2}} e^{-r^2/4Dt} \quad (\text{Taylor, 1954}) \quad (6)$$

A is a constant depending on the total amount of the marked particles.

$$\begin{aligned} \text{From (6)} \quad \overline{r^2} &= \int_{-\infty}^{+\infty} r^2 \frac{\partial c}{\partial r} dr / \int_{-\infty}^{+\infty} \frac{\partial c}{\partial r} dr \\ &= 4Dt \end{aligned} \quad (7)$$

$$\text{or} \quad \frac{d \overline{r^2}}{d t} = 4D \quad (8)$$

Equating (8) to (4) we have

$$\begin{aligned} 4D &= 2 \overline{u^2} \int_0^T R_L^u dt \\ D &= \frac{1}{2} \overline{u^2} \int_0^T R_L^u dt \end{aligned} \quad (9)$$

The diffusion coefficient is thus shown to be dependent upon the fluctuating velocity in the radial direction and the area of the Lagrangian correlation curve coefficient_A for the radial direction. The time T here is the time when $R_L^u = 0$. The above analysis shows that the tracing of the flow through curve of a sedimentation tank by injection of dye at the inlet and measuring its concentration at the outlet is in fact the measuring of the turbulent diffusion of the tank.

Certain important characteristics can be derived from the foregoing results:

(a) If the diffusivity is regarded as the product of the intensity and scale of the turbulent mixing process, then the scale is:

$$L_M = \frac{D}{\sqrt{u'^2}} \quad (\text{Rouse, 1959c}) \quad (10)$$

(b) When T is large, i.e. at any time which is larger than the Lagrangian time scale T_L , the integral in (9) becomes constant and is equal to T_L . D becomes constant. This may be regarded as the basis for the assumption of constant D which is used in Section 2.1. It may be of interest to note that when time is small $D \propto t$.

Even if D is not constant throughout the tank, equation (3) in 2.1. is still valid. The only difference is that D in that equation will be the average of $D_1, D_2, D_3, \dots, D_n$ of n imaginary cells composing the tank in the following manner:

$$\begin{array}{rcl}
r_0 \rightarrow r_1 & \frac{\partial \bar{C}}{\partial r_1} + \frac{1}{r} (1-2\gamma_1) \frac{\partial \bar{C}}{\partial r} - q_1^2 \bar{C} & = 0 \\
r_1 \rightarrow r_2 & \frac{\partial \bar{C}}{\partial r_2} + \frac{1}{r} (1-2\gamma_2) \frac{\partial \bar{C}}{\partial r} - q_2^2 \bar{C} & = 0 \\
\vdots & \vdots & \vdots \\
r_{n-1} \rightarrow r_n & \frac{\partial \bar{C}}{\partial r_n} + \frac{1}{r} (1-2\gamma_n) \frac{\partial \bar{C}}{\partial r} - q_n^2 \bar{C} & = 0
\end{array}$$

Adding the above equations together, we have, for r_0 to r_n

$$\frac{\partial \bar{C}}{\partial r} + \frac{1}{r} (1-2\gamma) \frac{\partial \bar{C}}{\partial r} - q^2 \bar{C} = 0$$

where

$$\gamma = \frac{\gamma_1 + \gamma_2 + \gamma_3 + \dots + \gamma_n}{n}, \quad q^2 = \frac{q_1^2 + q_2^2 + q_3^2 + \dots + q_n^2}{n}$$

2.3.2. Diffusion coefficient versus Turbulent mixing

The basic equation of diffusion is

$$F = -D \frac{dc}{dy} \quad (\text{Sec. 2.1. eq. 1.})$$

A closer examination of this equation reveals that the same rate of diffusion could result either from low velocity cross flow which carries over a considerable distance or from a high velocity cross flow which carries only a short distance (Rouse, 1950). In fact, as has been shown in 2.3.1., D can be regarded as the product of the velocity and the length scale, and is therefore analogous to ϵ , the eddy viscosity.

Boussinesq (Prandtl, 1935) introduced ϵ into the theory of turbulence by writing

$$\overline{u'v'} = -\epsilon \frac{d\bar{u}}{dy}$$

for a simple mean flow where $\bar{u} = \bar{u}(y)$, $\bar{v} = \bar{w} = 0$.

since a new ϵ is required for every new problem in fluid dynamics, the hypothesis is a failure (Pai, 1957). Prandtl pointed out the similarity

of the turbulent theory with the kinetic theory of gases, and put forward the mixing length theory in which

$$\epsilon = l^2 \left| \frac{d\bar{u}}{dy} \right|$$

The advantage of Prandtl's approach lies in the fact that the variable l is simply a length, so that a plausible assumption can be made more easily than for ϵ of Boussinesq. However, it is now well known that this concept has serious limitations and inconsistencies (Batchelor, 1950). One of the defects is that the mixing length deduced from the measurements of the mean flow is not very small, but of the same order of magnitude as the dimension of the mean flow.

Taylor (1920) was the first person to develop a statistical theory of turbulence which is applicable to continuous movements, and which satisfies the equation of motion. A series of papers by him on the same subject followed in 1935, and the application of statistical method is much more satisfactory. In very much the same way, the methods of statistics must be applied to turbulent diffusion so that the flow condition effects can be added to the effects of molecular agitation.

2.4. The effects of a Forced Vortex on the Diffusion Coefficient

2.4.1. Shear Stresses

Let θ , r , and z be the cylindrical coordinates in tangential, radial and axial directions, v , u and w the velocities and v' , u' , and w' the turbulent fluctuating velocities along θ , r and z respectively.

The Reynolds equations for two dimensional and axially symmetrical flow are (Goldstein, 1952):

$$u \frac{\partial u}{\partial r} - \frac{v^2}{r} = -\frac{1}{\rho} \frac{\partial p}{\partial r} - \frac{1}{r} \frac{\partial}{\partial r} (r \overline{u'^2}) + \frac{v'^2}{r} + \nu (\nabla^2 u - \frac{u}{r^2}) \quad (1)$$

$$u \frac{\partial v}{\partial r} + \frac{uv}{r} = -\frac{1}{r} \frac{\partial}{\partial r} (r^2 \overline{u'v'}) + \nu (\nabla^2 v - \frac{v}{r^2}) \quad (2)$$

$$0 = -\frac{1}{r} \frac{\partial}{\partial r} (r \overline{u'w'}) \quad (3)$$

and the continuity equation is $\frac{\partial(uv)}{r \partial r} = 0 \quad (4)$

From (4) $u = \frac{K}{r}$ $K = \frac{Q}{2\pi} \quad (5)$

(3) shows that $\overline{u'w'}$ is constant throughout the system.

From (2)

$$\begin{aligned} \rho \left(u \frac{\partial v}{\partial r} + \frac{uv}{r} \right) &= -\frac{1}{r} \rho \left(2r \overline{u'v'} + \frac{r^2}{\partial r} \overline{u'v'} \right) + \mu \left(\frac{\partial^2 v}{\partial r^2} + \frac{\partial v}{r \partial r} - \frac{v}{r^2} \right) \\ &= -\frac{1}{r^2} \frac{\partial}{\partial r} \left[-r^2 (\overline{T}_2 + \overline{T}_4) \right] + \mu \frac{\partial^2 v}{\partial r^2} + \mu \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial v}{\partial r} \right) \end{aligned} \quad (6)$$

But, from equation (15) Section 2.2.1.

$$v = Ar^{-1} + Br^{1/2} \quad (7)$$

Therefore
$$\frac{\partial V}{\partial r} = -Ar^{-2} + B(1+R)r^R \quad (8)$$

and
$$\frac{\partial^2 V}{\partial r^2} = 2Ar^{-3} + BR(1+R)r^{R-1} \quad (9)$$

Substitute (7), (8) and (9) for V , $\frac{\partial V}{\partial r}$ and $\frac{\partial^2 V}{\partial r^2}$ in (6) we obtain:-

$$\tau_t + \tau_l = \tau = - \left[\frac{2\mu A \ln r}{r^2} - \rho K B r^R + \mu \frac{R(1+R)}{(2+R)} B r^R \right] + \frac{C}{r^2} \quad (10)$$

C = constant of integration

Consider the following cases:

(1) When $R \neq 0$.

Let $\tau = \tau_{r_2}$ at $r = r_2$

Therefore

$$\begin{aligned} \tau &= - \left[\frac{2\mu A \ln r}{r^2} - \rho K B r^R + \mu \frac{R(1+R)}{2+R} B r^R \right] \\ &+ \left[\tau_{r_2} + \left\{ \frac{2\mu A \ln r_2}{r_2^2} - \rho K B r_2^R + \mu \frac{R(1+R)}{2+R} B r_2^R \right\} \right] \\ &= \frac{\mu}{r^2} \left[2A \left\{ \ln r_2 - \ln r \right\} + B R \left\{ r^{2+R} - r_2^{2+R} \right\} \left\{ 1 - \frac{1+R}{2+R} \right\} \right] + \tau_{r_2} \end{aligned} \quad (11)$$

and
$$\tau - \tau_{r_2} = \tau' = \frac{\mu}{r^2} \left[2A (\ln r_2 - \ln r) + B R (r^{2+R} - r_2^{2+R}) \left(1 - \frac{1+R}{2+R} \right) \right] \quad (12)$$

$$= A \text{ (negative)} + B \text{ (positive)} \quad (13)$$

because r is always greater than r_2 and $(1+R)/(2+R) < 1$

From eq. (16) Section 2.2.1.

$$A = - \frac{\omega_1^2 r_1^2 - \omega_2^2 r_2^2}{r_1^2 - r_2^2} \left(\omega_1 r_1^R - \omega_2 r_2^R \right) / \left(r_1^{2+R} - r_2^{2+R} \right)$$

$$B = \left(\omega_1 r_1^2 - \omega_2 r_2^2 \right) / \left(r_1^{2+R} - r_2^{2+R} \right)$$

Therefore:

(a) If $\omega_1 r_2^R > \omega_2 r_1^R$ A will be negative and B positive.

τ' is always positive, and $\overline{uv'} < 0$.

(b) if $\omega_1 r_2^R < \omega_2 r_1^R$ A will be positive and B negative.

τ' will always be negative, and $\overline{uv'} > 0$ provided that $\omega_1 r_1^2 < \omega_2 r_2^2$. On the other hand if $\omega_1 r_1^2 > \omega_2 r_2^2$ both A and B will be positive. Examination of (12) shows that under these conditions τ' will be negative near the inner wall and positive near the outer wall.

(ii) When $Rm = 0$, i.e. when there is no flow, (12) becomes

$$\tau' = \frac{\mu}{r^2} [2A (\ln r_2 - \ln r_1)] \quad (14)$$

A is now the only influencing factor = $-r_1^2 r_2^2 (\omega_1 - \omega_2) / (r_1^2 - r_2^2)$

τ' will be positive and $\overline{uv'}$ negative if $\omega_1 > \omega_2$ and negative ($\overline{uv'}$ positive) if $\omega_1 < \omega_2$.

(iii) If $\omega_1 r_1^2 = \omega_2 r_2^2$ i.e. $\frac{\omega_1}{\omega_2} = \frac{r_2^2}{r_1^2}$, B will again be equal to zero. (14) is the equation for the shear stress in this case, but τ' will always be negative.

2.4.2. Turbulent Energy Equation

The turbulent energy equation for two dimensional and axially symmetrical flow in polar coordinate can be written as follows (Wislicenus & Yeh, 1952):

$$\frac{1}{2} u \frac{\partial}{\partial r} \overline{u'^2} + \frac{1}{2} \frac{\partial}{\partial r} \overline{ru'^2} + \left[\overline{u'^2} \frac{\partial u}{\partial r} - 2 \overline{uv'} \frac{v}{r} \right] = - \frac{1}{\rho} \overline{u' \frac{\partial p}{\partial r}} + \nu \left[\overline{u' \nabla^2 u'} - \frac{\overline{u'^2}}{r} \right] \quad (15)$$

$$\frac{1}{2} u \frac{\partial}{\partial r} \overline{v'^2} + u \frac{\overline{v'^2}}{r} + \frac{1}{2r} \frac{\partial}{\partial r} \overline{ru'v'^2} + \frac{\overline{uv'^2}}{r} + \overline{uv'} \frac{\partial v}{r \partial r} = \nu \left(\overline{v' \nabla^2 v'} - \frac{\overline{v'^2}}{r} \right) \quad (16)$$

$$\frac{1}{2} u \frac{\partial}{\partial r} \overline{w'^2} + \frac{1}{2r} \frac{\partial}{\partial r} \overline{ru'w'^2} = \nu \left(\overline{w' \nabla^2 w'} \right) \quad (17)$$

$$\overline{q'^2} = \overline{u'^2 + v'^2 + w'^2}$$

The usual interpretation of groups of terms from left to right in (15),

(16) and (17) is as follows:

- (1) $\frac{1}{2} u \frac{\partial}{\partial r} \overline{u'^2}$ the rate of convection of turbulent energy by mean flow.
- (2) $\frac{1}{2} \frac{\partial}{\partial r} \overline{ru'^2} - \frac{\overline{uv'^2}}{r}$ the rate of turbulent diffusion by fluctuating velocities.
- (3) $\left[\overline{u'^2} \frac{\partial u}{\partial r} - 2 \overline{uv'} \frac{v}{r} \right]$ the rate of production of turbulence by the mean flow and the shear stress.
- (4) $-\frac{1}{\rho} \overline{u' \frac{\partial p}{\partial r}}$ the rate of transfer of turbulent energy by the work of fluctuating pressure gradients.
- (5) $\nu \left[\overline{u' \nabla^2 u'} - \frac{\overline{u'^2}}{r} \right]$ the rate of the viscous dissipation of the fluctuating velocities.

2.4.3. Influence of rotation on the fluctuating velocities

(a) Flow without rotation

In this case all terms containing V vanish, and the turbulent energy equations reduce to

$$\frac{1}{2} u \frac{\partial}{\partial r} \overline{u'^2} + \frac{1}{2} \frac{\partial \overline{ru'^3}}{r \partial r} + \frac{\overline{u'^2}}{r} \frac{\partial u}{\partial r} = - \frac{1}{\rho} \frac{\partial \overline{p' u'}}{\partial r} + \nu \left(\overline{u' \nabla^2 u'} - \frac{\overline{u'^2}}{r} \right) \quad (15a)$$

$$\frac{1}{2} u \frac{\partial}{\partial r} \overline{v'^2} + u \frac{\overline{v'^2}}{r} + \frac{1}{2} \frac{\partial \overline{ru'v'^2}}{r \partial r} + \frac{\overline{u'v'^2}}{r} = \nu \left(\overline{v' \nabla^2 v'} - \frac{\overline{v'^2}}{r} \right) \quad (16a)$$

$$\frac{1}{2} u \frac{\partial}{\partial r} \overline{w'^2} + \frac{1}{2} \frac{\partial \overline{ru'w'^2}}{r \partial r} = \nu \left(\overline{w' \nabla^2 w'} \right) \quad (17a)$$

Consider the equation (15a) for example, the third term, $\overline{u'^2} \frac{\partial u}{\partial r}$, represents the production of turbulent energy by u' and $\frac{\partial u}{\partial r}$. As $\frac{\partial u}{\partial r}$ is always negative, this term will always be negative, and the negative sign of this term means production of $\overline{u'^2}$. The following conclusions can be drawn from the above equations:

- (1) The radial fluctuating velocity, $\overline{u'}$ and $\overline{v'}$, receives the entire turbulent energy production directly.
- (2) $\overline{v'}$ and $\overline{w'}$ will be dissipated by the action of viscosity since they receive no turbulent energy.
- (3) There is turbulent energy production throughout the tank in the radial direction because $\overline{u'^2} \frac{\partial u}{\partial r}$ is always negative.

The above conclusions explain indirectly why a radial flow tank is characteristically unstable.

(b) Flow with rotation, i.e. with superimposed forced vortex.

From the turbulent energy equations (15) to (17) the following conclusions have been drawn:-

- (1) The production of turbulent energy affects both $\overline{u'^2}$ and $\overline{v'^2}$.
- (2) The superimposed forced vortex affects the production term in which $\frac{V}{r}$ occurs. (This term disappears when there is radial flow only).

Consider equation (15). If $\overline{u'v'}$ is positive, the production term is negative which means an increase of $\overline{u'^2}$ and vice versa. From this, together with the results obtained in section 2.4.1., the following conclusions can be drawn:

- (a) When $R \neq 0$ and (i) $\omega_1 r_1^R > \omega_2 r_1^R$ the production term will be positive which means a suppression of $\overline{u'^2}$
- (ii) $\omega_1 r_1^R < \omega_2 r_1^R$ and $\omega_1 r_1^2 < \omega_2 r_2^2$, $\overline{u'^2}$ is increased by the turbulent fluctuating velocity.
- (iii) $\omega_1 r_1^R < \omega_2 r_1^R$ and $\omega_1 r_1^2 > \omega_2 r_2^2$, $\overline{u'^2}$ is increased near the inner wall and suppressed near the outer wall.
- (b) When $R = 0$ and (i) $\omega_1 > \omega_2$, $\overline{u'^2}$ is suppressed.
- (ii) $\omega_1 < \omega_2$, $\overline{u'^2}$ is increased.
- (iii) $\omega_1 r_1^2 = \omega_2 r_2^2$, $\overline{u'^2}$ is increased since $\overline{u'v'}$ is positive.

The above conclusions check with Raleigh's criterion for the stability of a revolving fluid and also agree with Taylor's criterion for stability of fluid flow between two rotating cylinders. The works of Eskinasi & Yeh (1956) and Traugott (1956) also support the conclusions shown above.

- (3) The suppression of $\overline{u'^2}$ always means the increase of $\overline{v'^2}$ since the signs of the production terms in (15) and (16) are opposite. Similarly, an increase in $\overline{u'^2}$ means the suppression of $\overline{v'^2}$. In other words, each is increased or suppressed at the expense of the other.
- (4) Suppression of $\overline{u'^2}$ means also a small value of the integral scale in the radial direction (Eskinasi & Yeh, 1956).

2.5. Dimensional Analysis

The studies in the previous sections indicate how the characteristic number N of a sedimentation tank is affected by a superimposed forced vortex. However, as has been seen, the problem is so complicated that it is impracticable, if not altogether impossible, to obtain a single equation relating all variables. In order to conduct an experiment with a minimum of wasted effort, and to distribute the experimental results so as to define graphically a specific form of function describing the occurrence under study over the entire field of interest, full advantage is now to be taken of dimensional analysis.

Since the fundamental equation of physics are dimensionally homogeneous, and since the relationships that are deducible from these equations are consequently dimensionally homogeneous, it is justifiable to apply dimensional analysis to a practical problem (Langhaar, 1951). The first step in this analysis is to decide what variables enter the problem.

2.5.1. Variables

As a result of the analytical considerations, all the variables involved in the present problem have been revealed. They can be grouped in three classes as follows:

(a) Variables describing boundary geometry

Here we have r_1 , r_2 , and d as lengths that characterize the system. It may seem at first to be convenient to combine the three

independent variables into one, namely R , the hydraulic mean depth.

$$R = (r_1 - r_2)d / (2d + r_1 - r_2)$$

By doing this, the characteristics of the system are at once lost because for any given R , an infinite number of values of r_1 , r_2 , and d can be found. Since our problem is thought to be very much dependent upon the ratios of r 's and d , especially when the problem of secondary flows is to be considered, it appears advisable to keep them separate.

(b) Variables describing the flow

They are ω_1 , ω_2 , V , D and g . ω_1 and ω_2 are the angular speeds of the paddles at r_1 and r_2 respectively. V is the flow velocity, D the coefficient of diffusion and g the acceleration due to gravity.

(c) Variables describing the fluid

They are ρ , μ and D_v . D_v is the molecular diffusivity of the dye used. ρ and μ are the density and viscosity of the fluid respectively.

2.5.2. Grouping of the variables into the dimensionless terms

A function involving the variables discussed above can be written as

$$f(D, V, \omega_1, \omega_2, d, r_1, r_2, \rho, D_v, \mu, g) = 0$$

In order to facilitate the use of the algebraic approach to the problem, it is convenient to display the dimensions of the variables by a tabular arrangement as follows:

	1	2	3	4	5	6	7	8	9	10	11
	D	V	ω_1	ω_2	d	r_1	r_2	ρ	D_v	μ	ϵ
M	0	0	0	0	0	0	0	1	0	1	0
L	2	1	0	0	1	1	1	-3	2	-1	1
T	-1	-1	-1	-1	0	0	0	0	-1	-1	-2

This tabulated array of the dimensions of the variables is known as a dimensional matrix.

By crossing out the columns that are dependent on the others, the original matrix can be reduced to a non zero determinant of order 3, which is the rank of the matrix. Three homogeneous linear algebraic equations whose coefficients are the numbers in the rows of the dimensional matrix can be written as the following:-

$$\begin{aligned}
 k_8 + k_{10} &= 0 \\
 2k_1 + k_2 + k_5 + k_6 + k_7 - 3k_8 + 2k_9 - k_{10} + k_{11} &= 0 \\
 -k_1 - k_2 - k_3 - k_4 - k_9 - k_{10} - 2k_{11} &= 0
 \end{aligned} \quad (1)$$

From the theory of linear algebra (1) possesses exactly $(11-3) = 8$ linearly independent solutions (Langhaar, 1951), disregarding the trivial solution $k_1 = 0$. Consequently any values may be assigned to $k_1 : k_2 : k_3 : \dots : k_8$ and the corresponding values of k_9 , k_{10} and k_{11} can then be found by solving (1) by the elementary elimination procedure.

$$\begin{aligned}
 k_{10} &= -k_8 \\
 k_{11} &= -(1/3)(k_2 + 2k_3 + 2k_4 - k_5 - k_6 - k_7) \\
 k_9 &= -k_1 - (1/3)k_2 + (1/3)k_3 + (1/3)k_4 - (2/3)k_5 - (2/3)k_6 \\
 &\quad - (2/3)k_7 + k_8
 \end{aligned} \quad (2)$$

Values $k_1 = 1, k_2 = k_3 = k_4 = \dots k_8 = 0$ are now assigned for the first solution $\mathcal{A}_1$; $k_2 = 1, k_1 = k_3 = \dots k_8 = 0$ for the second solution and so on.

The solution can then be written in a matrix form as follows:

k_1	k_2	k_3	k_4	k_5	k_6	k_7	k_8	k_9	k_{10}	k_{11}
1	0	0	0	0	0	0	0	-1	0	0
0	1	0	0	0	0	0	0	-1/3	0	-1/3
0	0	1	0	0	0	0	0	+1/3	0	-2/3
0	0	0	1	0	0	0	0	+1/3	0	-2/3
0	0	0	0	1	0	0	0	-2/3	0	+1/3
0	0	0	0	0	1	0	0	-2/3	0	+1/3
0	0	0	0	0	0	1	0	-2/3	0	+1/3
0	0	0	0	0	0	0	1	1	-1	0

The 9th, 10th and 11th in the matrix of the solutions are merely the coefficients in the equations for k_9, k_{10} and k_{11} of (2). The first 8 columns of the solution matrix consist of zeros, except for the ones on the principal diagonal. Consequently the matrix can be written down immediately by inspection of (2).

By considering the determinant at the left hand side of the solution matrix, it is evident at once that the rank of the matrix is invariably equal to the number of rows in the matrix of solutions. These rows are therefore linearly independent. Since it contains 8 rows, the matrix of solutions constitutes a fundamental system of solutions.

Each row in the matrix of solutions is a set of exponents in a dimensionless product, which can be written as the following:

$$\begin{array}{ll}
 \pi_1 = D/D_v & \pi_5 = d \cdot g^{1/3} / D_v^{2/3} \\
 \pi_2 = V / (D_v \cdot g^{1/3}) & \pi_6 = r_1 g^{1/3} / D_v^{2/3} \\
 \pi_3 = \omega_1 D_v^{1/3} / g^{2/3} & \pi_7 = r_2 g^{1/3} / D_v^{2/3} \\
 \pi_4 = \omega_2 D_v^{1/3} / g^{2/3} & \pi_8 = \rho D_v / \mu
 \end{array}$$

We can now write $f_1(\pi_1, \pi_2, \pi_3, \dots, \pi_8) = 0$ (3)

or $\pi_1 = f_2(\pi_2, \pi_3, \dots, \pi_8)$ (4)

2.5.3. Transformation of dimensionless products

Townsend (1954, 1951) showed that molecular diffusion can be ignored if the ratio of the molecular diffusivity of the tracer to the kinematic viscosity of the fluid is much less than unity. The tracer used here is methylene blue dye. A visual observation of a drop of it in still water showed that it has a negligible molecular diffusivity compared with the turbulent diffusion coefficient. It is noticed that D_v occurs in every term. If it were to be neglected, it would mean that there will be no dimensionless term left. These 8 terms will have to be transformed, therefore, in such a way that D_v does occur in only one term which can be discarded. It will be seen from the transformation that, by carefully carrying out the operation, familiar dimensionless products will be obtained, and, as a result, a greater experimental control of variables is achieved.

The justification for replacing $f_1(\pi_1, \pi_2, \dots, \pi_8)$ by $f'_1(\pi_1, \pi_2, \dots, \pi_8)$

lies in the fact that relationships among $\pi_1, \pi_2, \pi_3; \dots \pi_8$ terms is not known; all that is known is that the relationship exists. A relationship among $\pi_1, \pi_2, \pi_3, \dots \pi_8$ implies a relationship among $T_1, T_2, \dots T_8$ since the variables $\pi_1, \pi_2, \pi_3, \dots \pi_8$ are determined by $T_1, T_2, \dots T_8$ and vice versa. This follows from the fact that either set of dimensionless variables is complete. (Buckingham, 1921).

By transformation:

T_1	$= (\pi_1 / \pi_2 \cdot \pi_7)^{-1} = (D/vr_2)^{-1}$	Peclet number
T_2	$= \pi_2 \cdot \pi_7 \cdot \pi_8 = vr_2 \rho / \mu$	Reynolds number
T_3	$= \pi_2^2 / \pi_7 = v^2 / r_2 g$	Froude number
T_4	$= \pi_3 / \pi_6 \cdot \pi_8 = \omega_1 v / r_1 g$	Nameless
T_5	$= \pi_8 = \rho D_v / \mu$	Schmidt number
T_6	$= \pi_3 / \pi_4 = \frac{\omega_1}{\omega_2}$	
T_7	$= \pi_6 / \pi_7 = \frac{r_1}{r_2}$	
T_8	$= \pi_6 / \pi_5 = \frac{r_1}{d}$	

Therefore
$$\frac{D}{vr_2} = f \left(\frac{vr_2 \rho}{\mu}, \frac{v^2}{r_2 g}, \frac{\omega_1 v}{r_1 g}, \frac{\rho D_v}{\mu}, \frac{\omega_1}{\omega_2}, \frac{r_1}{r_2}, \frac{r_1}{d} \right) \quad (5)$$

The Schmidt number is the only term now containing D_v . Neglecting Schmidt number (5) becomes

$$\frac{D}{vr_2} = f' \left(\frac{vr_2 \rho}{\mu}, \frac{v^2}{r_2 g}, \frac{\omega_1 v}{r_1 g}, \frac{\omega_1}{\omega_2}, \frac{r_1}{r_2}, \frac{r_1}{d} \right) \quad (6)$$

Since the range of the flow in the present investigation is well within the turbulent region, Reynolds number can also be neglected. (6) is therefore

$$\frac{D}{Vr_1} = f \left(\frac{V^2}{r_1 g}, \frac{\omega_1 V}{r_1 g}, \frac{\omega_1}{\omega_2}, \frac{r_1}{r_2}, \frac{r_1}{d} \right) \quad (7)$$

For any sedimentation tank of known r_1/r_2 and r_1/d (7) can be modified to

$$\frac{D}{Vr_2} = f \left(\frac{V^2}{r_2 g}, \frac{\omega_1 V}{r_1 g}, \frac{\omega_1}{\omega_2} \right) \quad (8)$$

Equation (8) indicates that for any given ratio of $\frac{\omega_1}{\omega_2}$, the plotting of N , i.e. $1 + Vr_2/2D$ (See Section 2.1. taking now V as the velocity at r_2) versus Froude number will result in a family of curves having $\frac{\omega_1 V}{r_1 g}$ as the parameter. (By having paddles at the outer periphery only as in the experiments to be described later, we are dealing with a case of $\frac{\omega_1}{\omega_2} = \infty$).

It is interesting to note that if r_1 denotes instead the mean radius and r_2 the difference in the outer and the inner radii of the tank, and the transform for T_L is done in the following manner:

$$T_L = \frac{\pi_3 \cdot \pi_6}{(\pi_6)^{1/2} (\pi_7)^{3/2}} = \frac{\omega_1 V_1 r_2^{1/2}}{\gamma}$$

is Taylor's number

(8) can be written in the form

$$\frac{D}{Vr_2} = f \left(\frac{V^2}{r_2 g}, \frac{\omega_1 r_1 r_2^{1/2}}{\gamma}, \frac{\omega_1}{\omega_2} \right) \quad (8a)$$

Compare (8) and (8a), it is evident that (8) should be preferred because the flow in the present problem has a free surface which is not horizontal, especially when rotation is introduced. Taylor's number does not include g which is one of the influencing variables.

* See Note opposite p. 3.

2.6. Summary

From the analytical study made above the following conclusions may be drawn:-

(1) Short-circuiting in a circular sedimentation tank depends upon the parameter $N = \gamma + 1$ where γ is $K/2D$.

(2) The diffusion coefficient D depends upon the fluctuating velocity and the integral scale of turbulence in the radial direction and for over a long period it can be taken as a constant.

(3) The effects of a superimposed forced vortex such that $\omega_1 > \omega_2$ will always result in a suppression of $\overline{u^2}$ and $\int_0^\infty R_L^u dt$, hence a decrease in D . Therefore, the superimposed forced vortex has been shown to minimize short-circuiting due to turbulent mixing. Without the forced vortex, the tank has also been shown to be susceptible to disturbances.

(4) Dimensional analysis indicates that a relationship between N , Froude number and $\frac{\omega_1 \gamma}{r_1 g}$ exists. The plotting of N v F_r should result in a family of curves having $\frac{\omega_1 \gamma}{r_1 g}$ as the parameter.

CHAPTER 3

EXPERIMENTAL TECHNIQUES

3.1. Apparatus

Experimental equipments are shown photographically and diagrammatically in Figs. 9-17. They consist of a circular tank, a constant head tank and a valve for the controlling of flow, a dosing device to inject dye into the tank and an absorptiometer for measuring the dye concentration.

3.1.1. Model sedimentation tank (Fig. 13)

A circular tank 3 feet in diameter and 6 inches deep was constructed with perspex for this investigation. It was made up in two sections: the cylindrical section and the floor. The cylindrical section consisted of two concentric cylinders 3 feet and 3 feet 6 inches in diameters respectively. The length of the inner cylinder was 6 inches, but that of the outer was $6\frac{1}{2}$ inches. They were joined together by a perspex ring in such a way that the cylindrical walls and the ring formed a channel. The top of the inner cylindrical wall was machined so that it also acted as an overflow weir.

The floor of the tank was also constructed with perspex $\frac{1}{2}$ inch thick and 3 feet 6 inches in diameter. It was attached to the cylindrical section by a series of bolts and nuts uniformly spaced along the outlet channel floor. In this way the floor of the tank was detachable for trans-

port purpose and the depth of the tank could be altered by insertion of another cylindrical section between the floor and the cylindrical section described above. A hole 3 inches in diameter was made at the centre of the floor for insertion of inlet risers of different sizes.

Water leaving the overflow weir flowed into the effluent channel the floor of which was made of Keon's cement and was coated with Glean. The floor sloped towards 4 outlets, each 1 inch in diameter. Water flowed from these outlets into an effluent chamber $3\frac{1}{2}$ inches in diameter (fig. 16). Part of the effluent was passed through the measuring cell of an Eel absorptiometer by which its dye concentration could be measured continuously. The rest of the effluent was run to waste.

3.1.2. Inlets

Three types of inlet were used in the experiments:

(1) Submerged weir inlet (2 "Ø P)

This inlet had a 2 inch diameter riser, the opening of which was located $\frac{1}{2}$ inch below the tank water surface. The diffuser was in the form of a cylindrical submerged weir suspended concentrically with the riser to a depth of 3 inches. (Fig. 14b).

(2) Slotted cylinder inlet (2 "Ø S)

This inlet had also a 2 inch diameter riser, but the diffuser was in the form of a slotted cylinder whose external and internal diameters were 6 and $5\frac{1}{2}$ inches respectively. (Fig. 14a).

(3) Uniform flow inlet with baffles (UFBI)

The riser of this inlet had a series of holes drilled along its

wall. The dimensions and spacing of these holes were calculated according to the Manifold Flow Theory (Appendix IV). A series of 4 circular plates were placed above each ring of holes to deflect the flow horizontally because water tended to leave the holes at an angle to the riser. Six baffles made of perspex, $5'' \times 2'' \times \frac{1}{8}''$, were attached tangentially to the circular plates in front of the holes. (Fig. 15 a & b).

3.1.3. Flow control unit

This consisted of a constant head tank (fig. 17) a valve and an orifice meter. The constant head tank was situated 6 feet above the sedimentation tank. It was made of perspex, $2'6'' \times 1' \times 9''$. The inflow turbulence was damped by gravels deposited in front of the inlet.

A water manometer was used to measure the difference in hydrostatic head upstream and downstream of the orifice respectively. The orifice meter was previously calibrated and the calibration curve was plotted in fig. 18. A screw valve was used to control the rate of flow through the sedimentation tank at any predetermined rate.

3.1.4. Rotating device

To rotate the water in the tank, eight paddles made of perspex of the size $6'' \times 3'' \times \frac{1}{4}''$ were attached to a perspex ring 3 feet in diameter and 3 inches wide, in such a way that the edges of the paddles had a very small clearance from the tank wall and floor. The ring was supported on ball bearings above the tank and was driven by a variable speed motor through

a friction drive. The speed of rotation of the ring, hence the paddles, could be varied between 0 and 25 rpm, by means of a 'variac'. (Fig. 11).

The dosing device and the measurement of effluent dye concentration are described in detail under Sections 3.2.2. and 3.2.4.

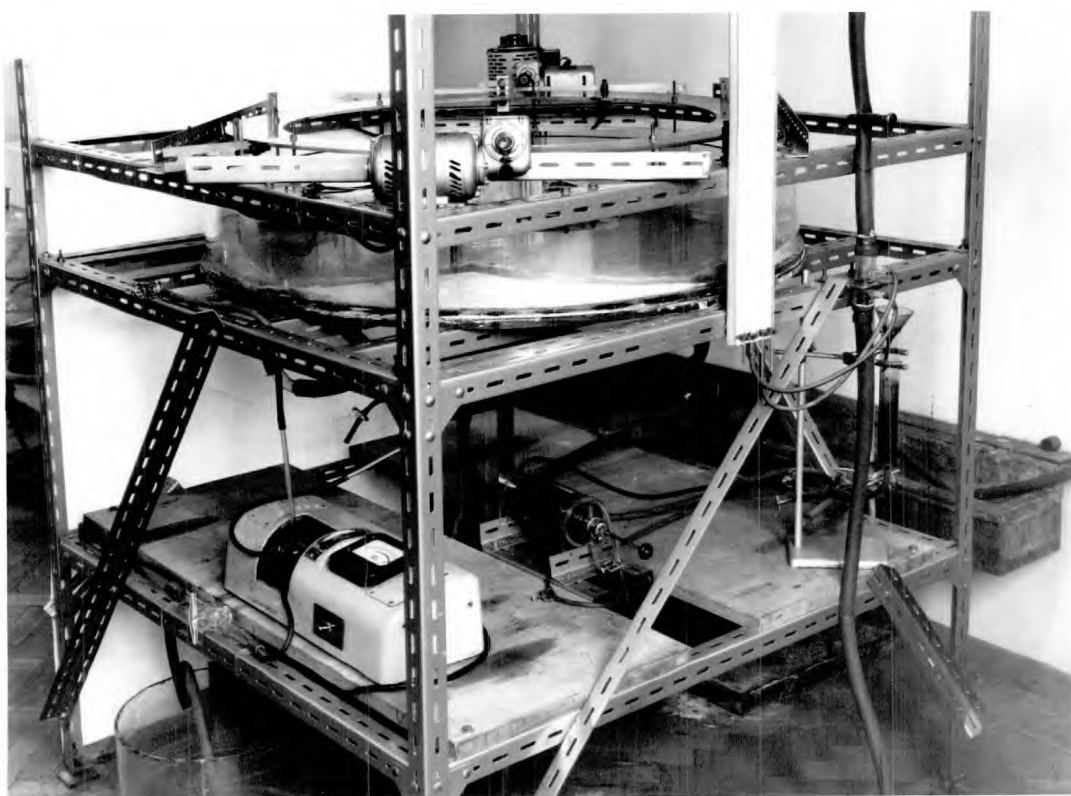


Fig. 9 Experimental apparatus: general view

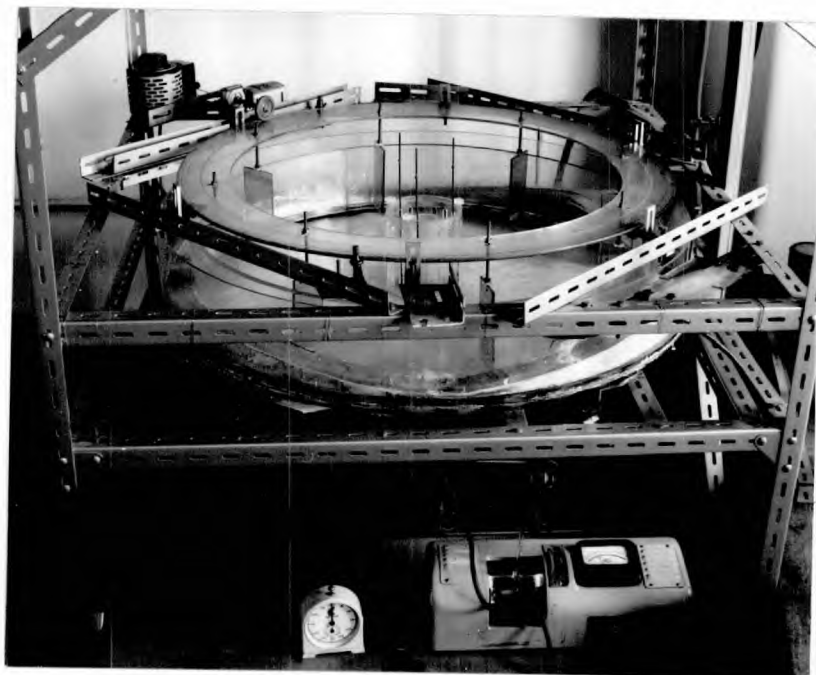


Fig. 10 The sedimentation tank

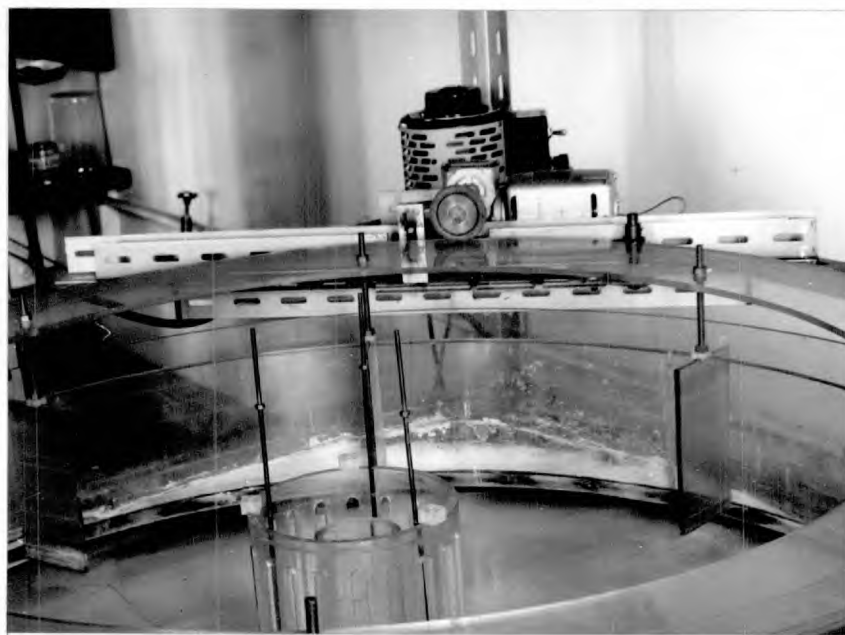


Fig. 11 Closed up view showing inlet, paddles and motor

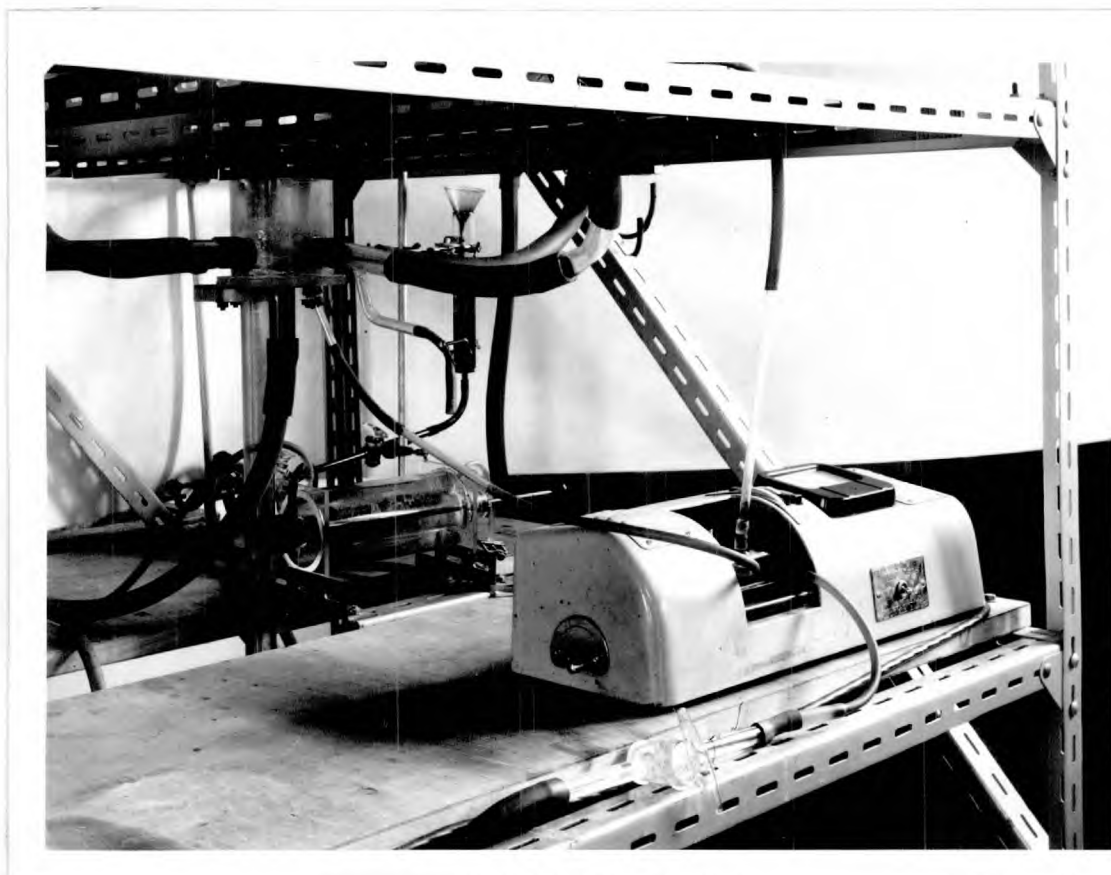
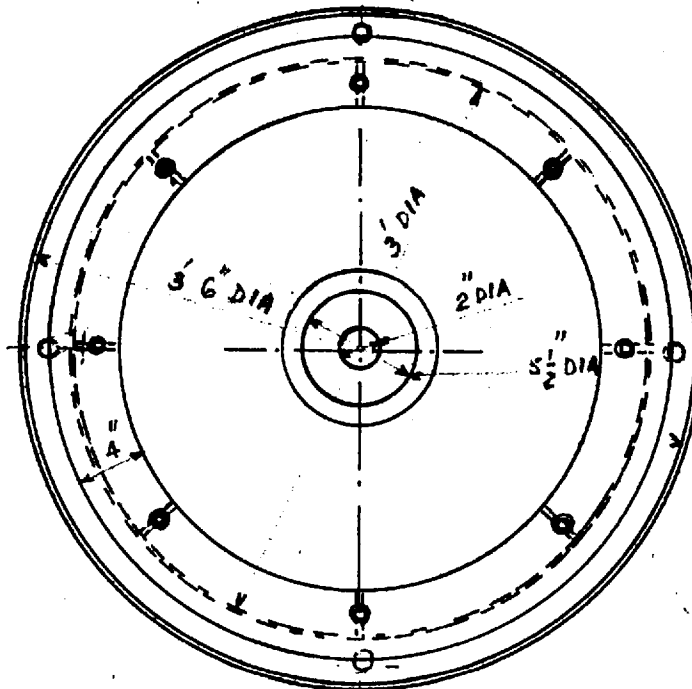
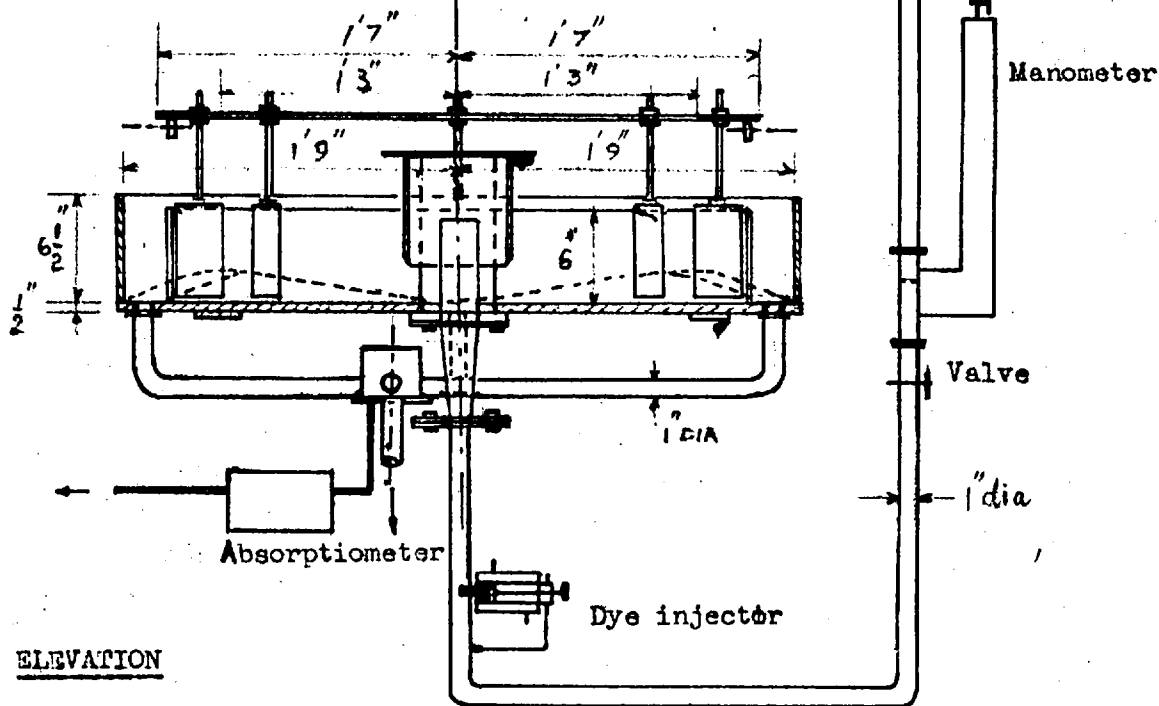


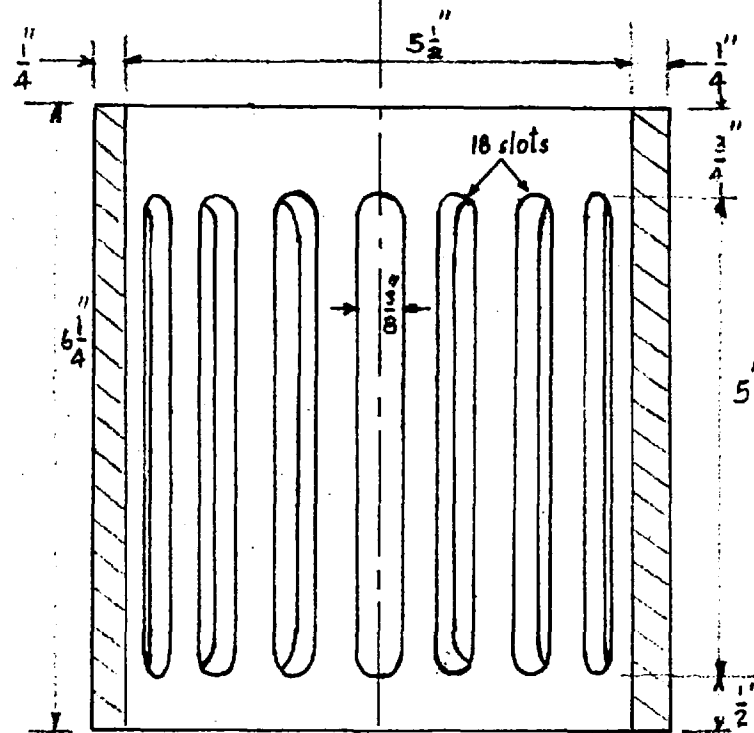
Fig. 12 Closed up view showing the absorptiometer and the dye injector

FIG. 13 THE SEDIMENTATION TANK.



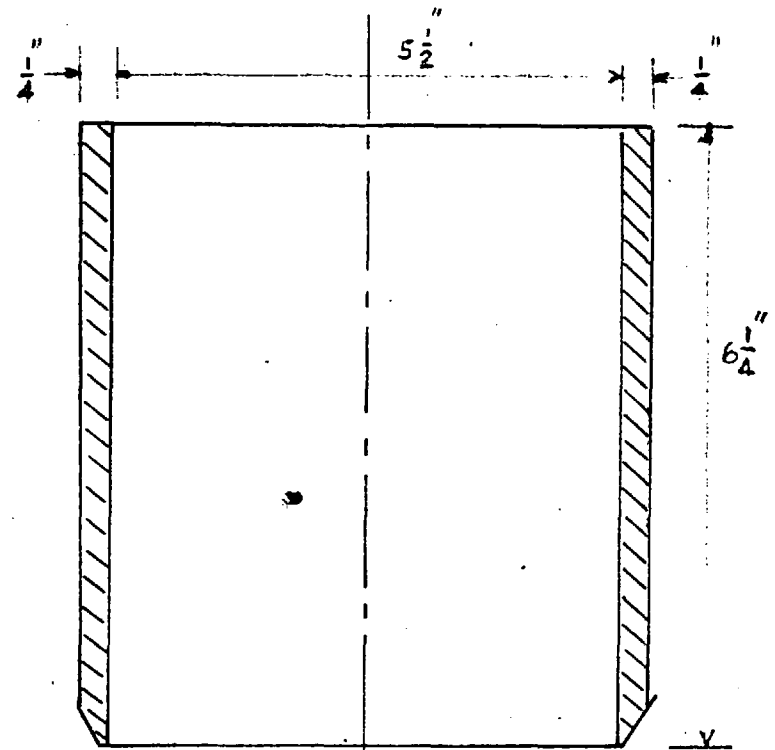
PLAN (not showing details under the tank)

FIG. 14 THE INLET DIFFUSERS.



THE SLOTTED CYLINDER

(a)



THE CYLINDRICAL SUBMERGED WEIR

(b)

FIG. 15 THE UNIFORM FLOW INLET WITH BAFFLES.

(a)

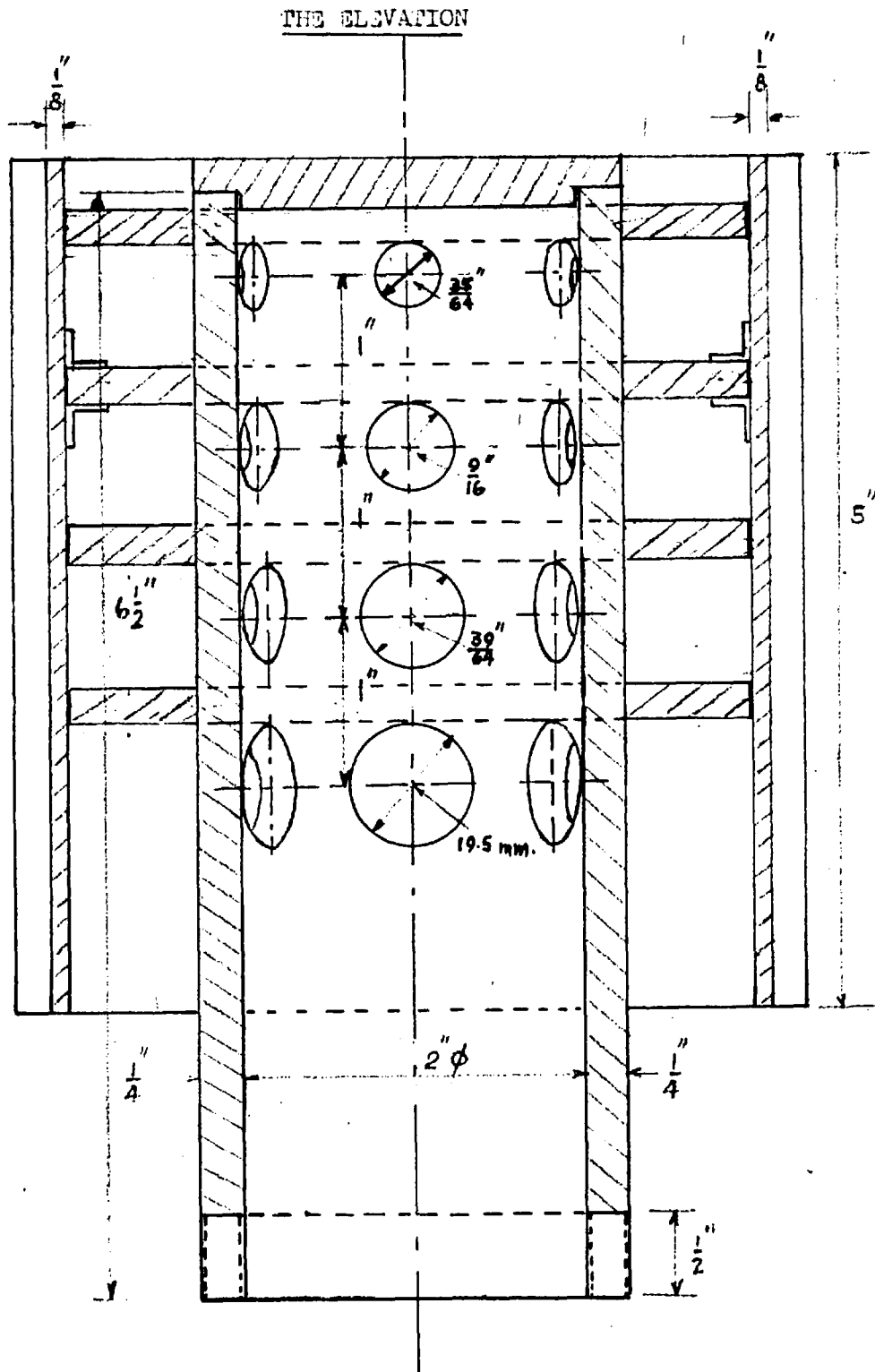


FIG. 15 THE UNIFORM FLOW INLET WITH BAFFLES.

(b)

PLAN

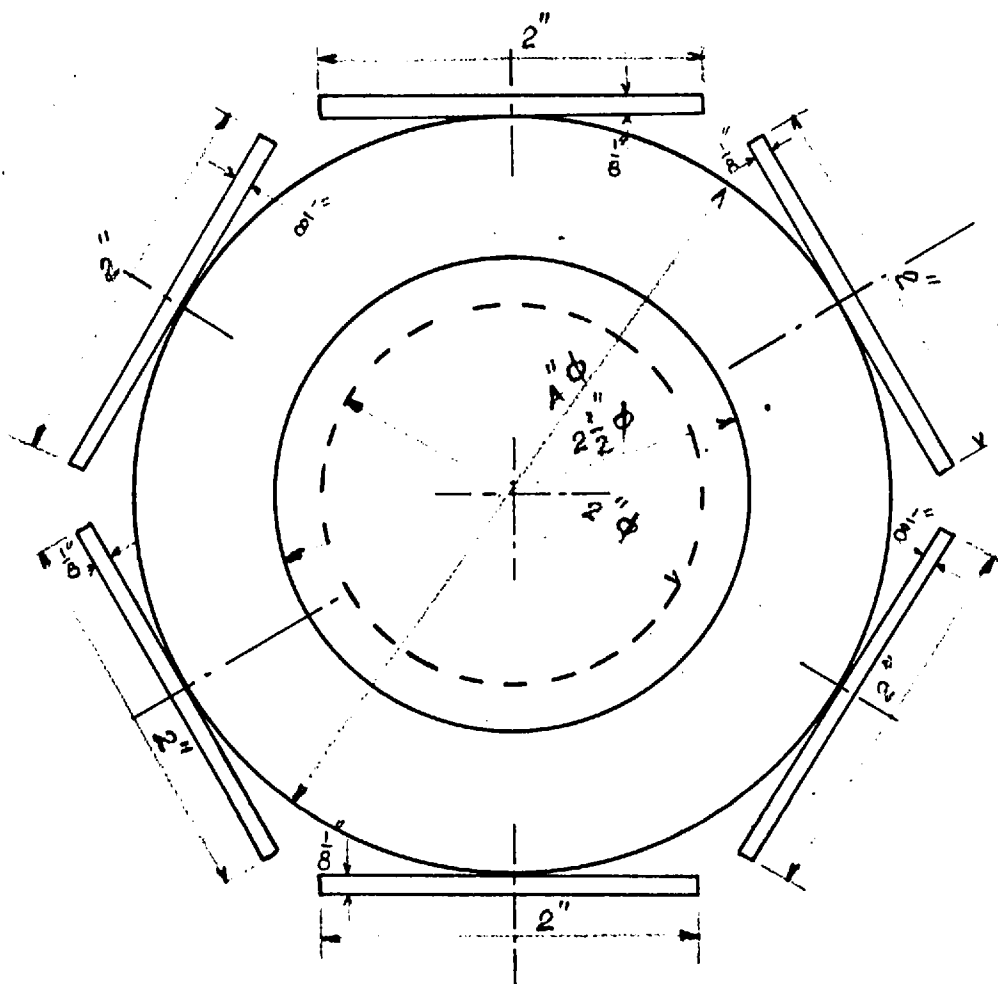


FIG. 16 THE EFFLUENT CHAMBER.

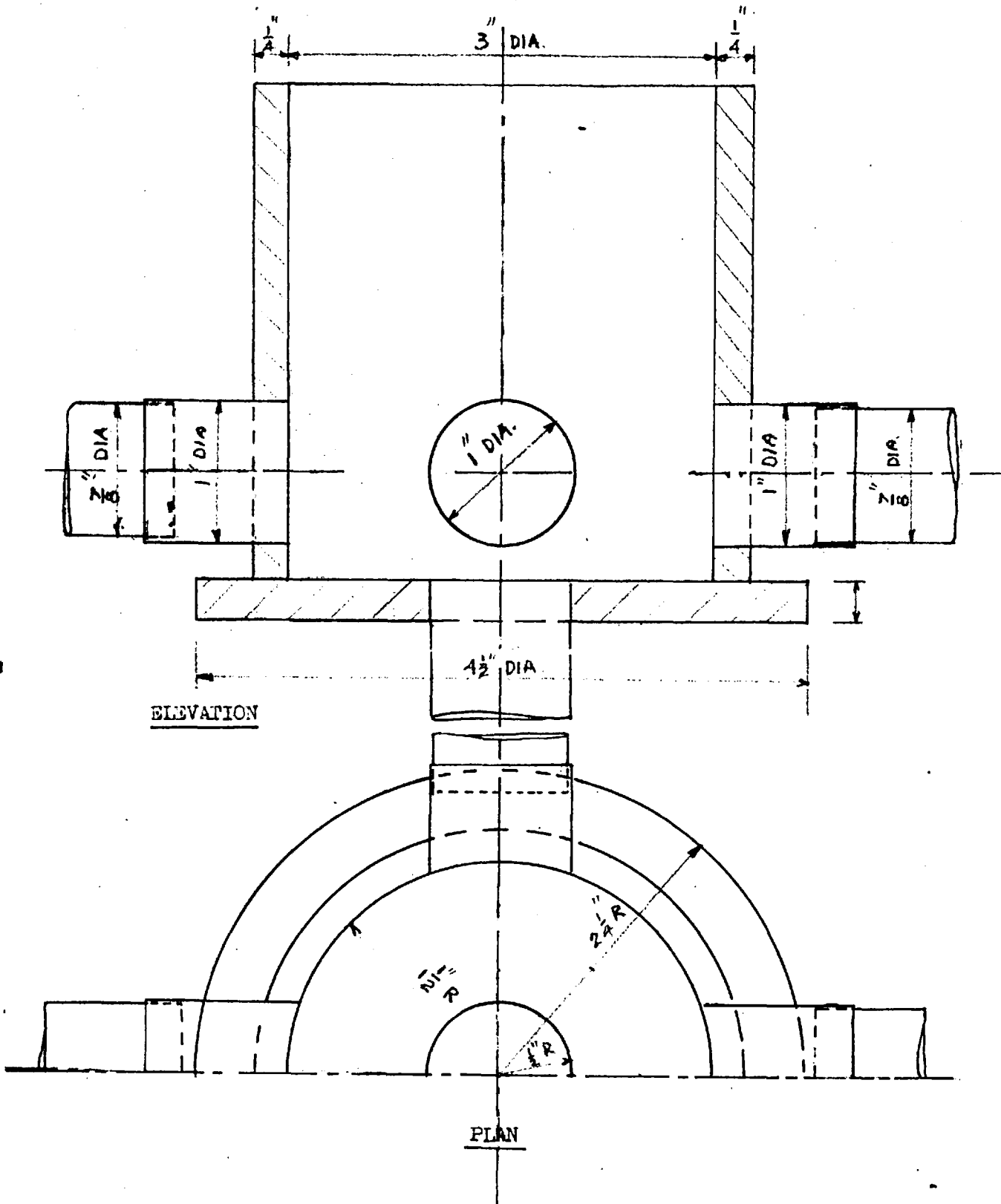


FIG. 17. THE CONSTANT HEAD TANK.

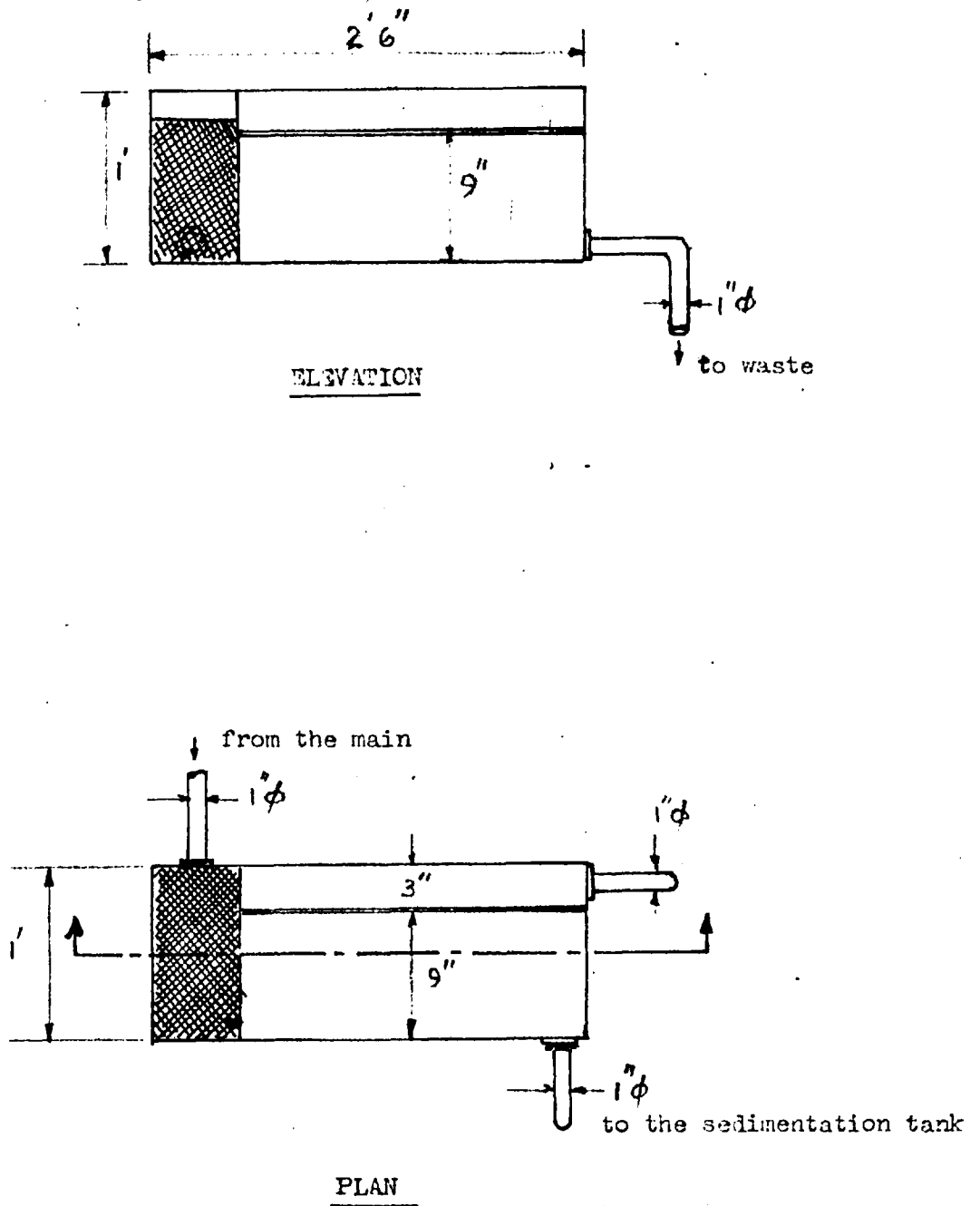
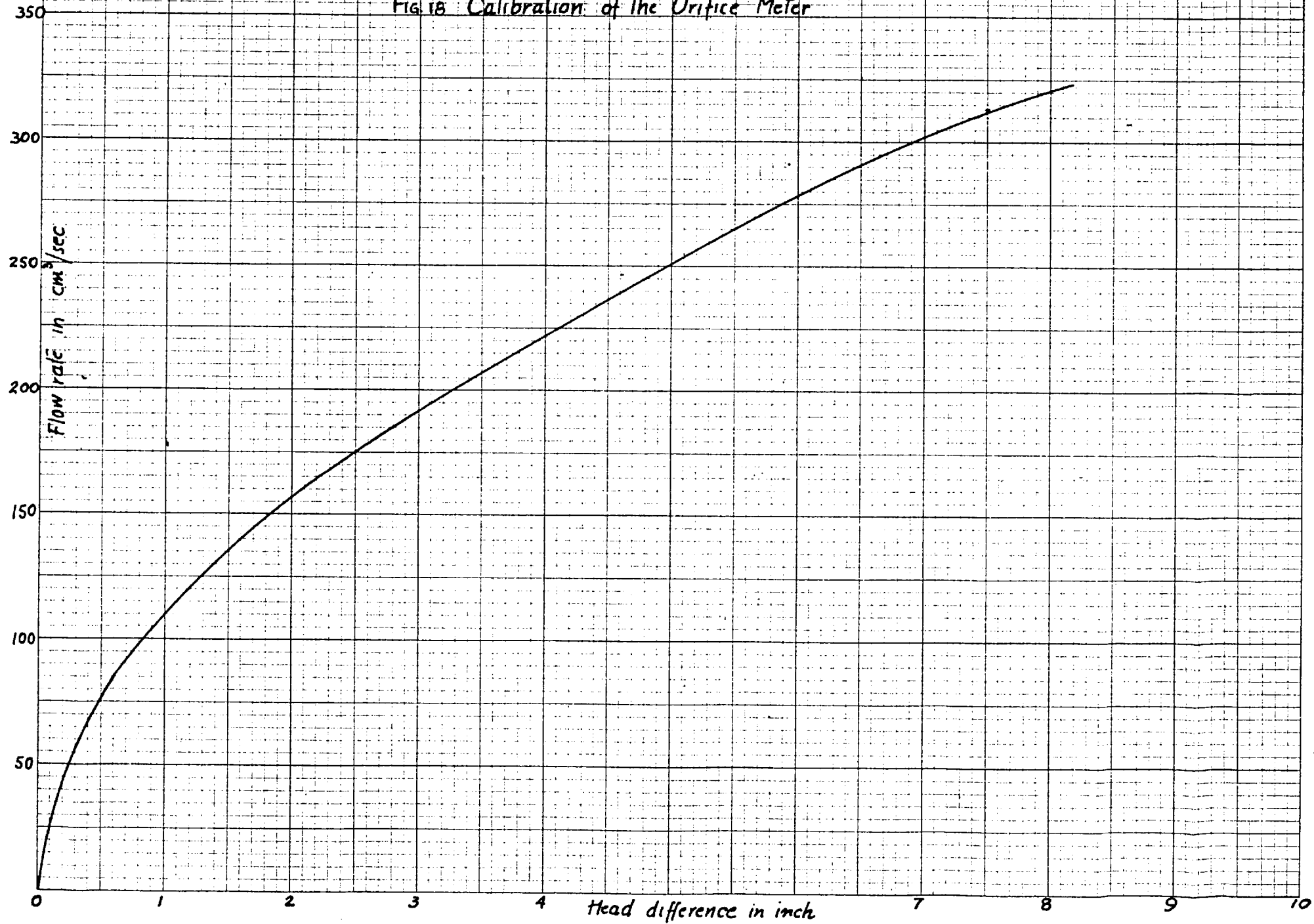


FIG 18 Calibration of the Orifice Meter



3.2. Tracing Techniques

Tracers may be introduced continuously or instantaneously into the tank. Sometimes it is convenient to use continuous injecting of tracers subject to a sinusoidal or other variation in concentration (Klinkenberg & Sjenitzer, 1956; Krammers & Aberda, 1957), especially when the system under investigation is not steady. For steady flow, as in the system under the present investigation, the first two methods are generally used.

3.2.1. Continuous versus instantaneous injection

The continuous dose method was originated about 1890 at the Lawrence Experimental Station of the Massachusetts State Board of Health. It depended upon the continuous displacement of a solution of known chloride concentration by a solution with a different chloride concentration. The flow through time was assumed to be the overall interval required for the effluent to attain the concentration of the second chloride solution. This method has not been widely used among sedimentation tank investigators, partly because of the large amount of tracer required, and partly because of the difficulty of controlling the dosing rate and the low degree of accuracy of the results. Taras (1956) found that the sampling interval in this method materially affected the overall results compared with the instantaneous dose method. The merit of the continuous method appears to lie in the fact that, in practice, the normal treatment chemicals can be used in determining the flow through time of the plant; also the maintenance of a constant

chemical feed reproduces routine plant operation. In sedimentation tank investigation, Muszkally & Vágás (1958) employed the continuous dose method and argued that the characteristic numbers derived from the flow through curve by instantaneous injection have no satisfactory physical basis. They maintained that the shape of the flow through curve varied with the dose and dosing time. In their experiments, the dosing time employed in injecting the tracer 'instantaneously' varied from 1.5 to 15 minutes compared with the tank theoretical^{detention} time of 6.9 minutes. It is believed that if the resultant curves are properly interpreted by taking the dosing time into account, they should reduce to the same curve. In experiments using the instantaneous method the dosing time must be kept short. Besides, the results obtained in the present investigation (see Sections 4.1 to 4.5) and those of other research workers (Ambrose, Baumann & Fowler, 1957) give no indication that the amount of the tracer used affects the shape of the flow through curve.

With the dye injector recently developed for laboratory use (see Section 3.2.2.) tracer could be introduced into the tank nearer to delta - function than any other dye injectors commonly used. The dosing time is about 1 to $1\frac{1}{2}$ seconds. Besides, the flow rate into the tank remains the same during the period of injecting, a distinct advantage over the continuous method where the flow rate through the tank is increased because of the added tracer. By injecting the tracer instantaneously, turbulence created at the point of injection help in mixing the tracer with the incoming liquid. This is very important or the tracer will not be representative of the flow.

The instantaneous method was introduced in 1907 by Clifford to the study of the time passage of liquid through percolating filters for sewage treatment. Capen (1927) adapted Clifford's idea to the determination of the flow through time of a number of sewage sedimentation tanks, and to this day, this Capen-Clifford approach remains the method of choice for estimating the hydraulic characteristics of sedimentation tanks.

3.2.2. Instantaneous injection of dye

(a) The static head method

A 40 cc of methylene blue dye slug is placed in a double wall glass vessel A (fig. 19), and its temperature is brought to that of the liquid in the tank by means of water circulating in the outer wall B. One end of the vessel is connected to the tank supply pipe at a point 1 foot upstream of the tank inlet. The other end is connected to the flow control unit. These connections are interrupted by two stop cocks (a) and (b). Before the injection of the dye (b) is turned on in order to bring the dye under the same static head as that which supplies the tank. Now, when (a) is turned on, the dye will be pushed by this pressure into the tank. The time taken for this operation is about 5 seconds, during which 40 cc. of dye and about 40 cc flushing water will have been delivered. Air valve (d) is also provided for emptying the dye injector before filling it with dye.

(b) The syringe method

This injector has been developed recently in Public Health

Engineering Section of the Department of Civil Engineering, Imperial College of Science and Technology, at the time of the present study. Figs. 12 and 20 illustrate the instrument.

With tap (c) open, a measured amount of dye is drawn into the cylinder (b) by withdrawing piston (a). The circulating water in the water jacket (b') helps ⁱⁿ bringing the temperature of the dye to that of the water in the tank. Tap (c) is now closed. Water in the tank cannot leak into the cylinder during the withdrawal of the piston because of the rubber tube (d) surrounding the injecting point of the injector. However, it is found that water can still seep into the dye reservoir if the tap (c) is not closed immediately after the withdrawal of the piston because the whole system of the injector is situated under the tank, and is thus subjected to a pressure head of about 2'6" of water. Water cannot at any rate leak into the dye in the cylinder (b) because this pressure is not great enough to push the piston backwards.

After the dye has attained the temperature of the water in the tank, the piston is pushed forwards to inject the dye into the tank. The rubber tube (d) will expand during the operation and allow the dye to leave the injector because of the built up pressure in (b). The injecting point is located 2' below the tank floor. This is found to give ample time for the dye to mix with the incoming flow.

While the piston moves forwards and delivers dye to the water an equal amount of water is sucked into the cylinder (b'') from a point 1' upstream of the injecting point. In this manner, there is neither an increase

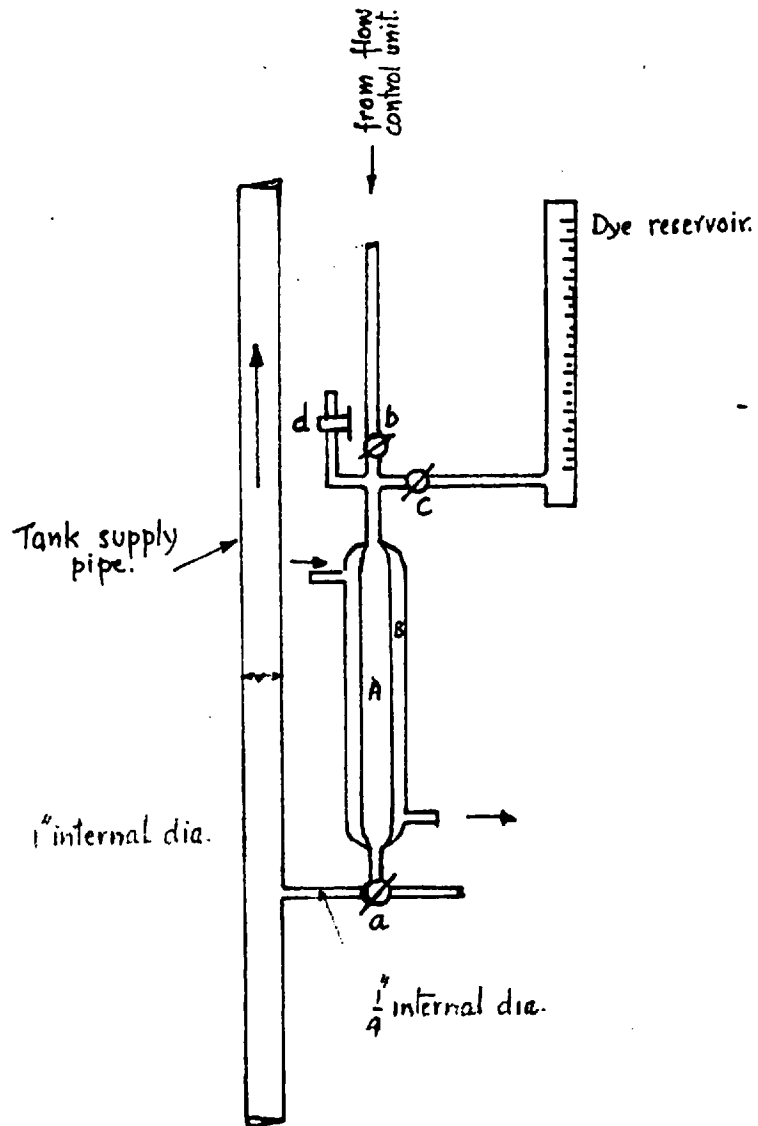


FIG. 19 THE STATIC HEAD DYE INJECTOR.

in tank flow rate nor the kinetic energy of the flow during the operation. No air bubbles can interfere with the injection as in the case of the static head method.

3.2.3. Tracers

One of the most important considerations in tracing the flow through a sedimentation tank is the choice of a suitable tracer. It should be inexpensive, small in quantity needed, harmless and must be representative of the flow. The last requirement means that the tracer must have the same density as the liquid and its chemical and physical properties must not change as it travels through the tank.

(a) Salt

Sodium chloride is the first substance used in tracing the flow through curve of a sedimentation tank. Several investigators in this field often reported very low detention efficiency or the efficiency of displacement, as Hyde (1950) called it. It must be admitted here that in these tests the percentage tracer recoveries were also low. Manson (1940) and Hooper (1940) reported that large errors are introduced when salt is used as tracer below a certain critical velocity. It is now known that salt tracer is not representative of the flow because its density is invariably greater than that of water (Ambrose et al, 1957),

(b) Other chemicals

Occasionally, Soda (Walton & Key, 1939), chlorine, fluorides, sodium silicate (Taras, 1956), and phosphate (Mau, 1959) are employed.

The methods of detecting them are generally more complicated than those for dye, and their properties often change when added to water. Mau reported some difficulties when using sodium phosphate in his experiments because of the presence of chlorine in water.

(c) Dyes

A dye tracer has the advantage of visualisation of the flow. At a suitable concentration it can have a density near to that of water. Many research workers (Clifford & Windridge, 1935; Camp, 1942) preferred KMnO_4 , but it has been known to stain and often results in a poor recovery of tracer since its quantity is reduced through deposition. Fluorescein dye is also used by many investigators (Ambrose et al, 1957; Davis, 1961, Ettinger, Witherow & Coulter, 1958). It has a weak colour at low concentration which necessitates the use of large dose. Geiger (1949), Thomas & Archibald (1951) and Abdalla (1955) used methylene blue dye and reported consistency in results and good percentage of dye recovery.

With regard to the facility in its concentration measurement, it was decided to use methylene blue dye made up with distilled water to a concentration of 2500 ppm. as the tracer in this study. At this concentration and at 20°C its density is very near to that of the water from the supply main (Abdalla, 1955).

(d) Radiotracers

They have advantages in that only a small amount is needed and are valuable in experiments carried out in full scale sewage tanks where salt and dye may be affected by chemical reaction or absorption on solids.

However, they are not so easily handled as dyes, and the method of detecting them remains to be perfected. One of the disadvantages of using radiotracers lies in the hazard to personnel when over exposed to radiation. Thomas and Archibald (1951) reported that the results obtained from using I-131 compared favourably with dye. Truesdale (1953), Seaman (1956) and Bergman (1958) also reported using radiotracers with satisfactory results. However, Ambrose et al. (1957) in their comparative study of dye, salt and Rubidium-86 as tracers experienced inconsistencies in results obtained from Ru - 86 as compared with fluorescein dye and suggested that, owing to the insufficient mixing of Ru-86 with the incoming flow, it might not represent the flow. Burgess & Green (1960) used Br-82, as ammonium bromide, in investigating flow through several sewage tanks and had done much to improve the techniques of using radiotracers.

3.2.4. Dye concentration measurement

This was carried out with an Eel absorptiometer which is a photo-electric instrument employing the principle of light absorption of a coloured liquid. A solution is measured by using light of complementary colour. A colour filter wheel is provided so that light of peak wave lengths between 6900 - 4700 Å can be selected. It has been found that the filter no. 606,500 Å is most suitable for methylene blue solution. Two graduated scales are provided; one relates directly to the solution concentration and the other reads the percentage transmission.

The instrument is found to 'drift', the phenomenon which gives rise to change in zero reading of the instrument. To overcome this it is also provided with a zero adjustment wheel.

The instrument is equipped with a pair of acid-proof fused glass cells 10 cm long and each has a capacity of 50 cc. One is to be used as a reference cell while the other as a sample cell. This involves filling and emptying the sample cell at intervals during a test. The method is therefore rather tedious. Besides, with this method it was impossible to obtain an accurate $\left(\frac{t}{T}\right)_{\infty}^{\wedge}$ and $\left(\frac{t}{T}\right)_i$. A continuous reading of the effluent concentration is therefore preferred.

The manufacturer does make a 'flow through cell' through which the effluent can be made to flow. Its dye concentration can then be measured continuously. Unfortunately, the cell is only 1 cm long. With the range of the concentration encountered in the tests, this cell is unsuitable since only a portion of the graduated scale would be used. Besides, no reference cell is provided. This is serious in view of the 'drift' of the instrument. Two glass cells 10 cm long with optical flat glasses at the end have been made. Each cell has a capacity of 50 cc. (fig. 21). One of them is used as the reference cell, being filled with distilled water and the other as the sample cell. The latter has a tangential inlet and outlet at the ends in addition to a filling tube at the centre of the cell.

3.2.5. Calibration of the absorptiometer

A series of methylene blue dye solutions of concentration varying from 0 to 2.5 ppm. were first prepared. Using distilled water as the reference solution the meter readings of these solutions were taken against the known dye concentration. Fig. 22 shows the calibration curve.

Because the two cells are not identical, it is found that the curve obtained above does not pass through the origin, but through a point (0,98) instead. Another straight line is then drawn parallel to this line passing through the origin, and is used as the calibration curve. This is necessary because the positions of the cells affect the zero too, owing to their being unidentical. At the beginning of an experiment, with distilled water in the reference cell and water from the tank in the sample cell, the meter reading is taken, and is found to be, say, 97, which is equivalent to 0.03 ppm. Any subsequent concentration obtained from the meter reading and the calibration curve will have to be reduced by 0.03 ppm. to arrive at the true concentration. If the zero reading is found to be 98 (which is equivalent to 0.02 ppm.) 0.02 ppm. will have to be subtracted from any measured concentration, and so on.

FIG. 21 THE CELLS FOR MEASURING DYE CONCENTRATION.

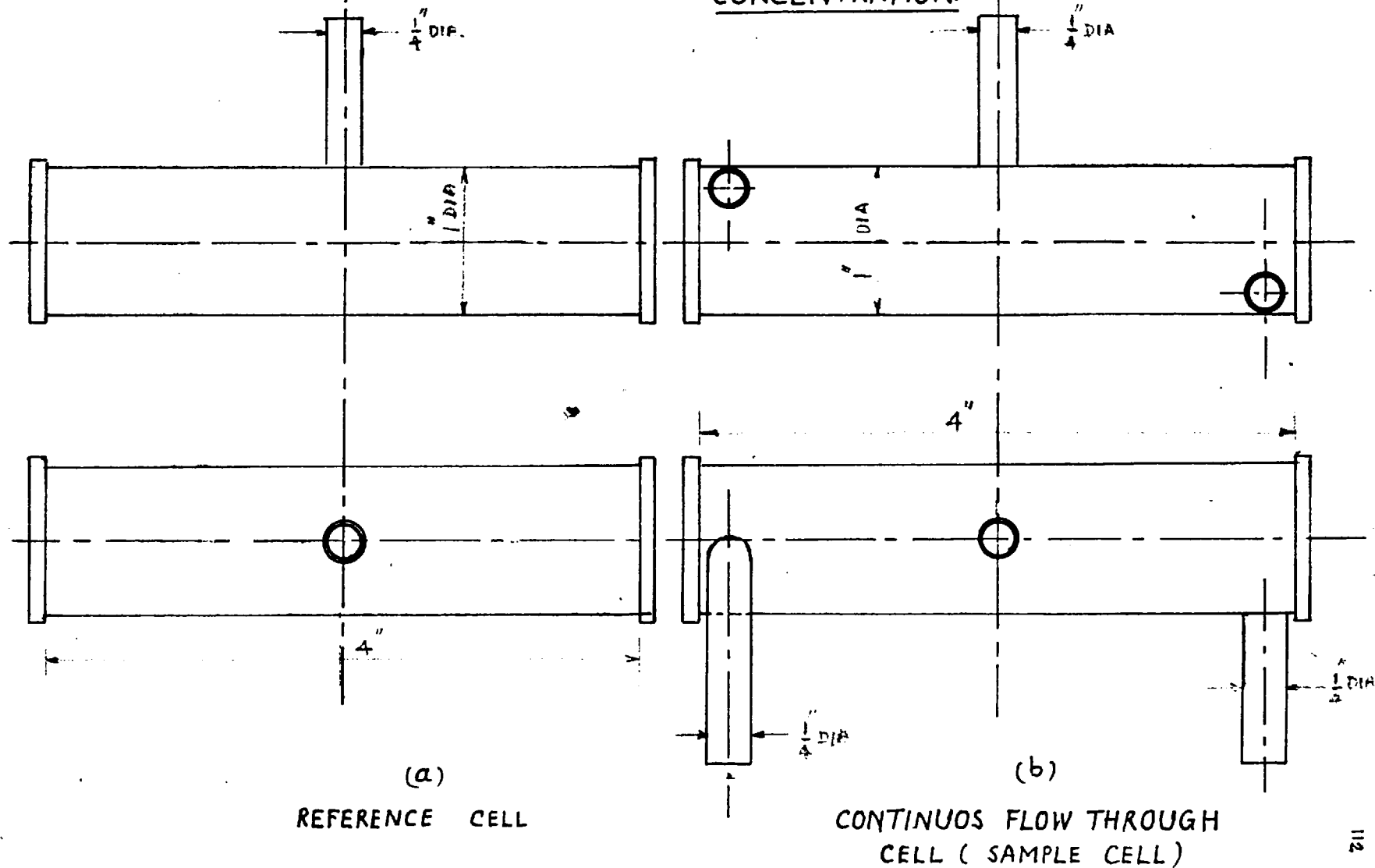
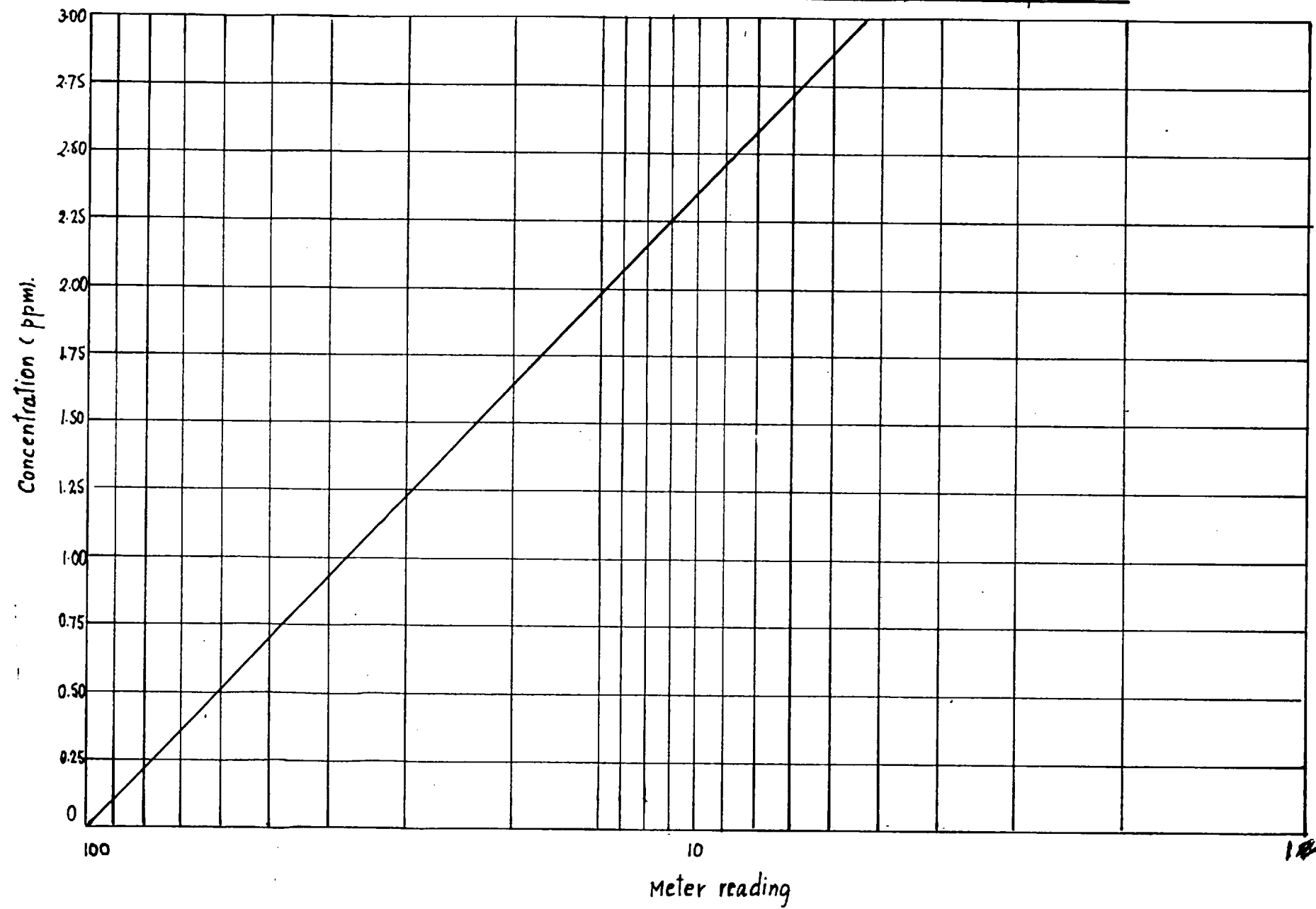


Fig. 22 Calibration curve of the absorptiometer.



3.3. Experimental Errors

3.3.1. Error due to dye concentration measurement

During the early part of the experimental programme, it was found that the areas of the flow through curves were consistently greater than unity. As the random error in any measurement should be both positive and negative, it was believed that the error might arise from the technique employed in measuring dye concentration.

(a) Mixing in the flow through cell

If the volume of the flow through cell is denoted by V_c and the rate of flow through it Q_c , at any time t , the concentration in the cell

$$C_c = \frac{1}{2} (C_t + C_{t-\Delta t})$$

where C_t is the dye concentration at the time t when it enters the measuring cell and is assumed to be the same as when it leaves the tank (see later) and $C_{t-\Delta t}$ the concentration at the time $t-\Delta t$, Δt being the flow through cell theoretical detention time = $\frac{V_c}{Q_c}$

$$\text{But } \frac{C_t}{C_o} = \frac{N^N}{(N-1)!} \left(\frac{t}{T}\right)^{N-1} e^{-N\frac{t}{T}} \quad (\text{Eq. 27 Sec. 2.1.4.}) \quad (1)$$

Therefore $\frac{C_c}{C_o}$ can be shown to be approximately equal to

$$\begin{aligned} \frac{C_c}{C_o} &= \frac{1}{2} \frac{C_t}{C_o} + \frac{1}{2} \frac{N^N}{(N-1)!} \left(\frac{t}{T}\right)^{N-1} \left(1 - \frac{\Delta t}{T}\right)^{N-1} e^{-N\frac{(t-\Delta t)}{T}} \\ &= \frac{C_t}{C_o} \left(\frac{1+\frac{N}{2} \frac{\Delta t}{T}}{e^{\frac{N}{2} \frac{\Delta t}{T}}} \right) + \frac{1}{2} \frac{N^N}{(N-1)!} \left(\frac{t}{T}\right)^{N-1} \left\{ - (N-1) \left(\frac{\Delta t}{T}\right) + \frac{(N-2)(N-1)}{2!} \left(\frac{\Delta t}{T}\right)^2 - \dots \right\} e^{-N\frac{t}{T}} \\ &\approx \left(\frac{1+\frac{N}{2} \frac{\Delta t}{T}}{e^{\frac{N}{2} \frac{\Delta t}{T}}} \right) \frac{C_t}{C_o} \quad (2) \end{aligned}$$

and the area of the flow through curve

$$A_c = \left(\frac{1+\frac{N}{2} \frac{\Delta t}{T}}{e^{\frac{N}{2} \frac{\Delta t}{T}}} \right) \quad (3)$$

The detention and mixing in the cell thus cause the area of the flow through curve to be increased.

(b) Tank characteristic number N

This number is not affected by the rate of flow through the measuring cell because the multiplying factor $(\frac{1+e^{N\frac{4t}{T}}}{2})$ will cancel itself out in the process of the calculation of the variance.

(c) Mixing effect during the travel of dye from the overflow weir to the measuring cell

The dye concentration of the liquid on leaving the tank is related to time by $\frac{C_t}{C_0} = \frac{N''}{(N''-1)!} \left(\frac{t}{T}\right)^{N''-1} e^{-N''\frac{t}{T}}$ and the variance of the curve is $\frac{1}{N}$. If we now consider the system as including the effluent channel, the connecting tubes, the effluent chamber and the measuring cell, and let n be its characteristic number, we have:-

$$\frac{C_t}{C_0} = \frac{n}{(n-1)!} \left(\frac{t}{T}\right)^{n-1} e^{-n\frac{t}{T}} \quad (4)$$

The variance of this equation is $\sigma_c^2 = \frac{1}{n}$ where n now representing the number of imaginary cells constituting the whole system.

It can be shown that

$$\frac{1}{N} - \frac{1}{n} = \frac{1}{n_{co}} \quad (\text{Aris, 1959}) \quad (5)$$

where n_{co} is the number of imaginary cells constituting the system excluding the sedimentation tank itself.

However, observation of the mixing taken place during the time the water takes to reach the flow through cell after having left the tank is negligible due to the fact that the spread of a cloud of dye added to the water as it left the overflow weir took only about 5 seconds to flow through

the measuring cell. This is much shorter than the interval between each reading of the absorptiometer (see Section 4.2.). Therefore there was no marked additional error due to mixing in the channel outside the sedimentation tank. We can write therefore

$$\frac{1}{N} = \frac{1}{N} \quad (6)$$

i.e.

$$N = N \quad (7)$$

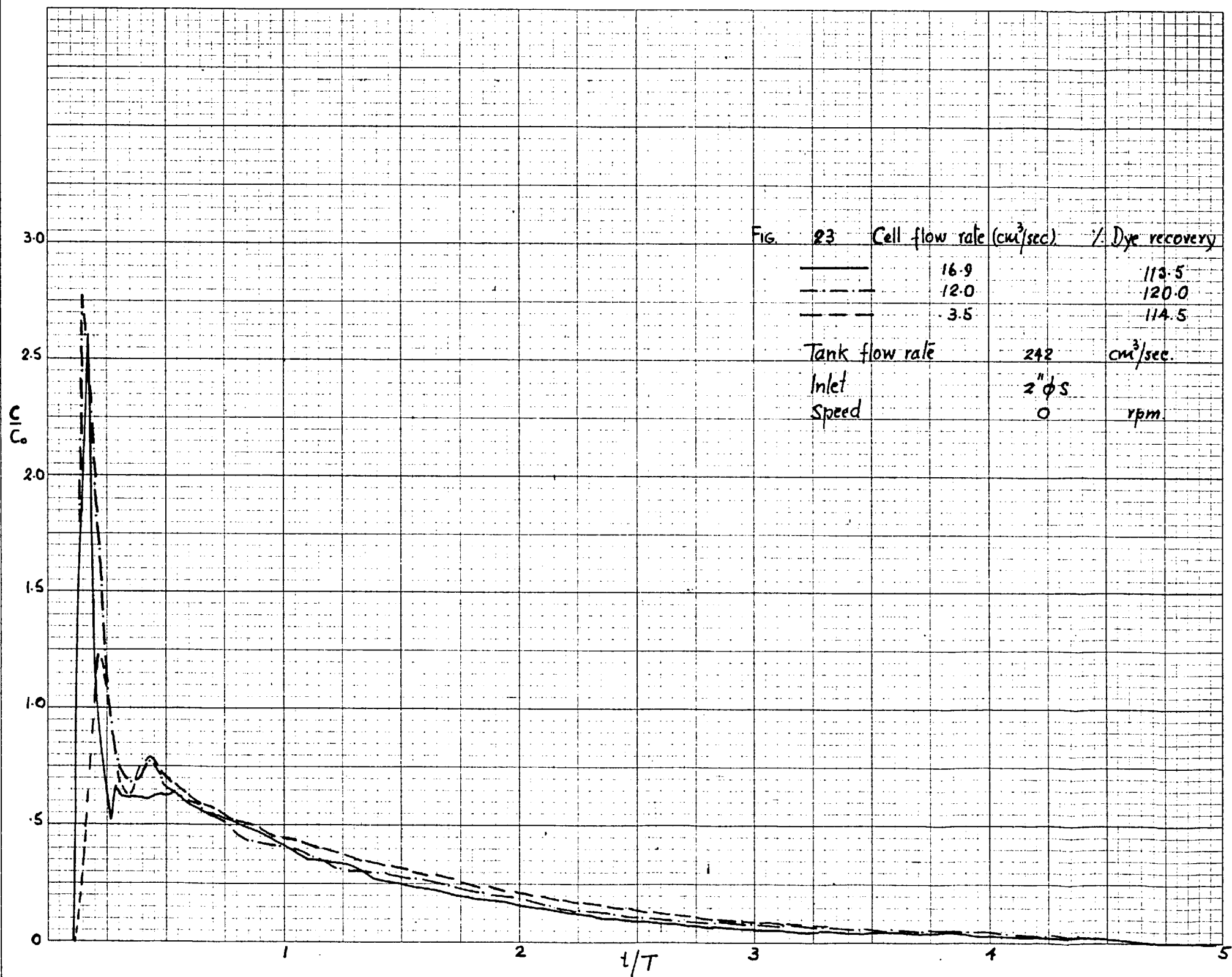
3.3.2. Experimental investigation of 3.3.1.

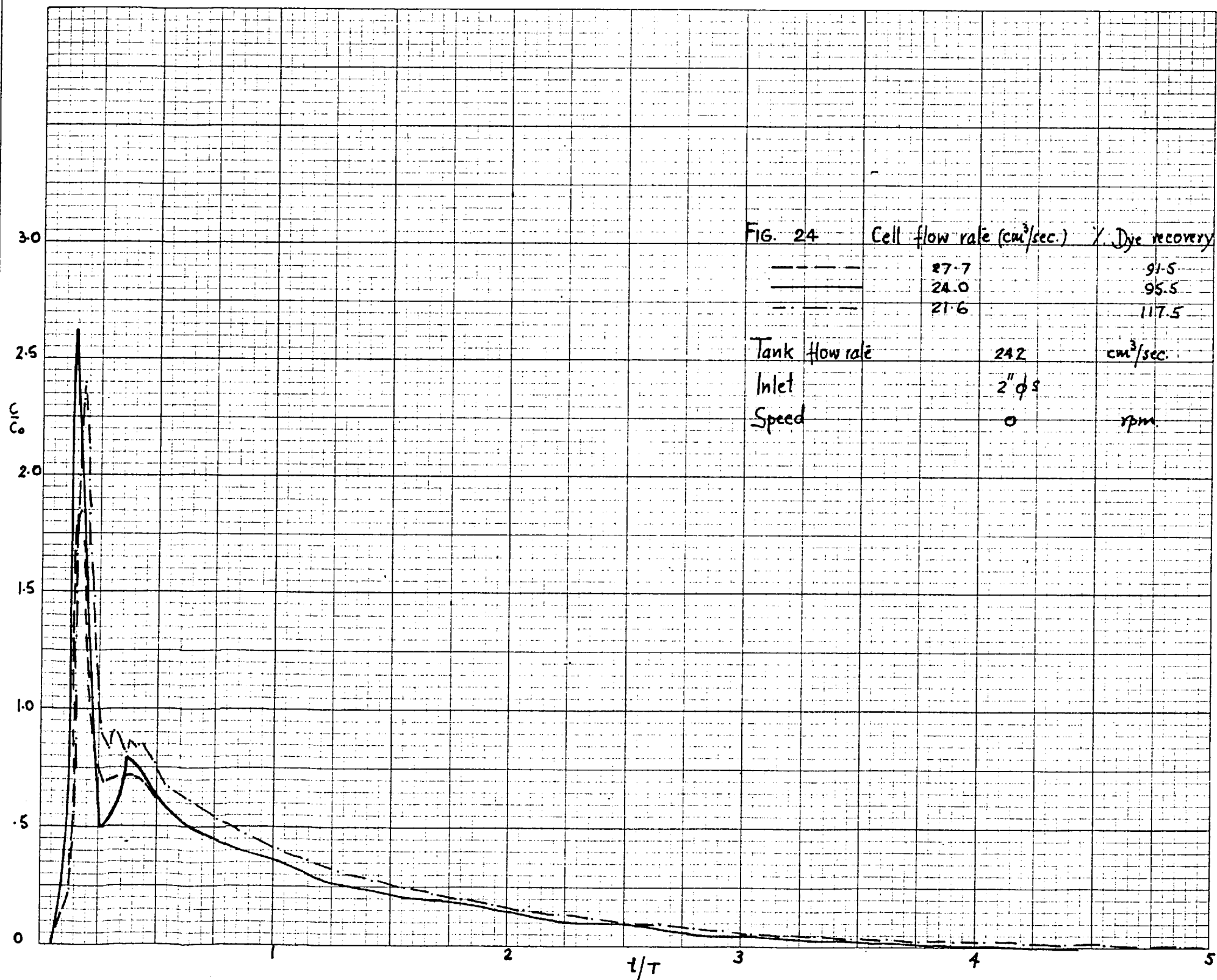
(1) Experiments were carried out with 2"Ø S at 242 cm³/sec and speeds of rotation of 0 and 5^{10 & 15} rpm. ^{Different} Two speeds were employed in order to create different N numbers. Figs. 23 to 28 show the effect of the cell flow rate on the shape of the flow through curve. In all cases the flow through curves obtained with low cell flow rates lagged behind those obtained with a high cell flow rate. It was evident also that a low cell flow rate gave rise to high percentage of tracer recovery.

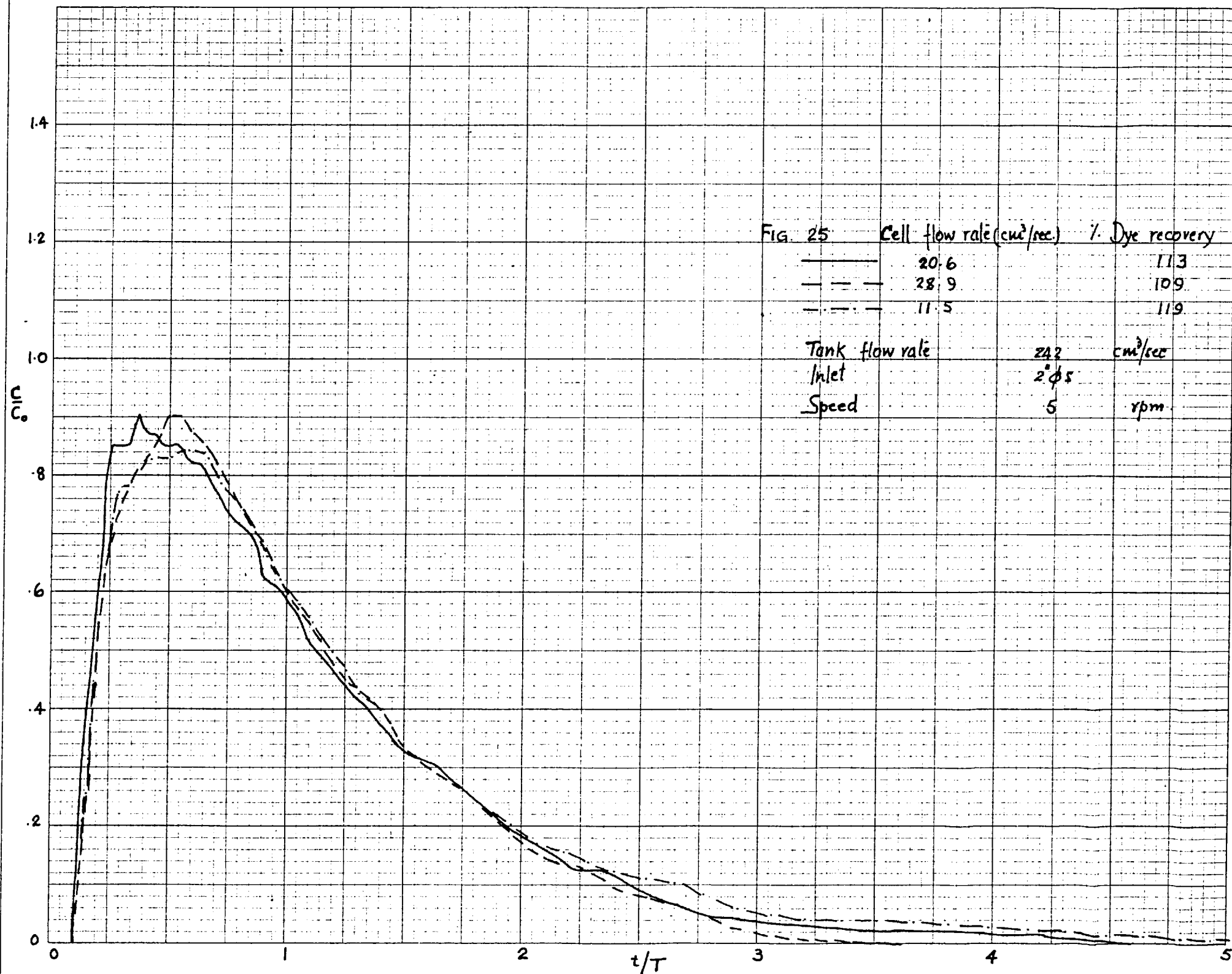
(2) In order to check on the above reasoning, experiments were also carried out with different flow rates and speeds of rotation. Sampling of the effluent for concentration measurement was also done by stopping the flow through the cell and emptying it at intervals. The results were tabulated in Table 3.

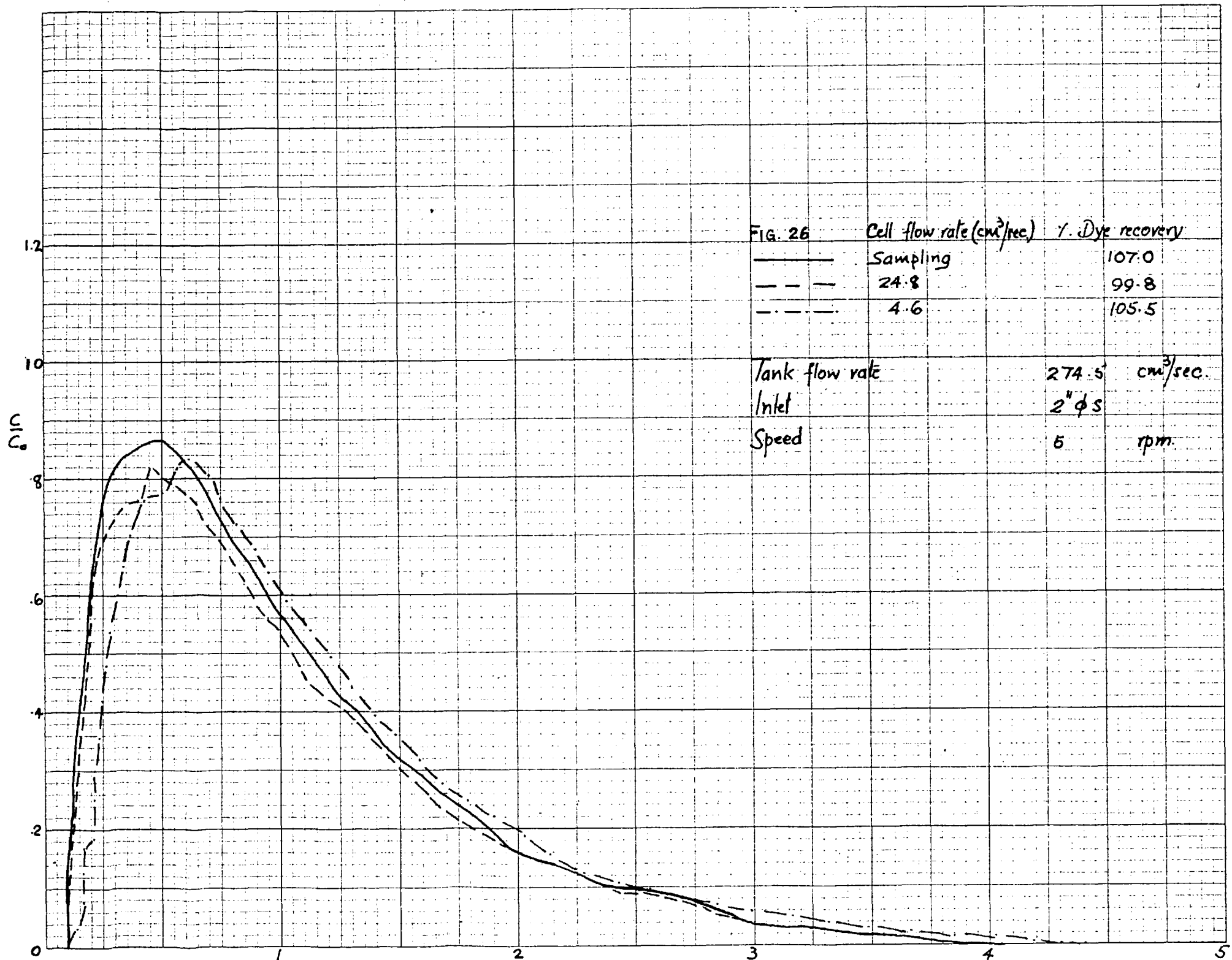
(3) The following conclusions had been drawn from the above experiments:-

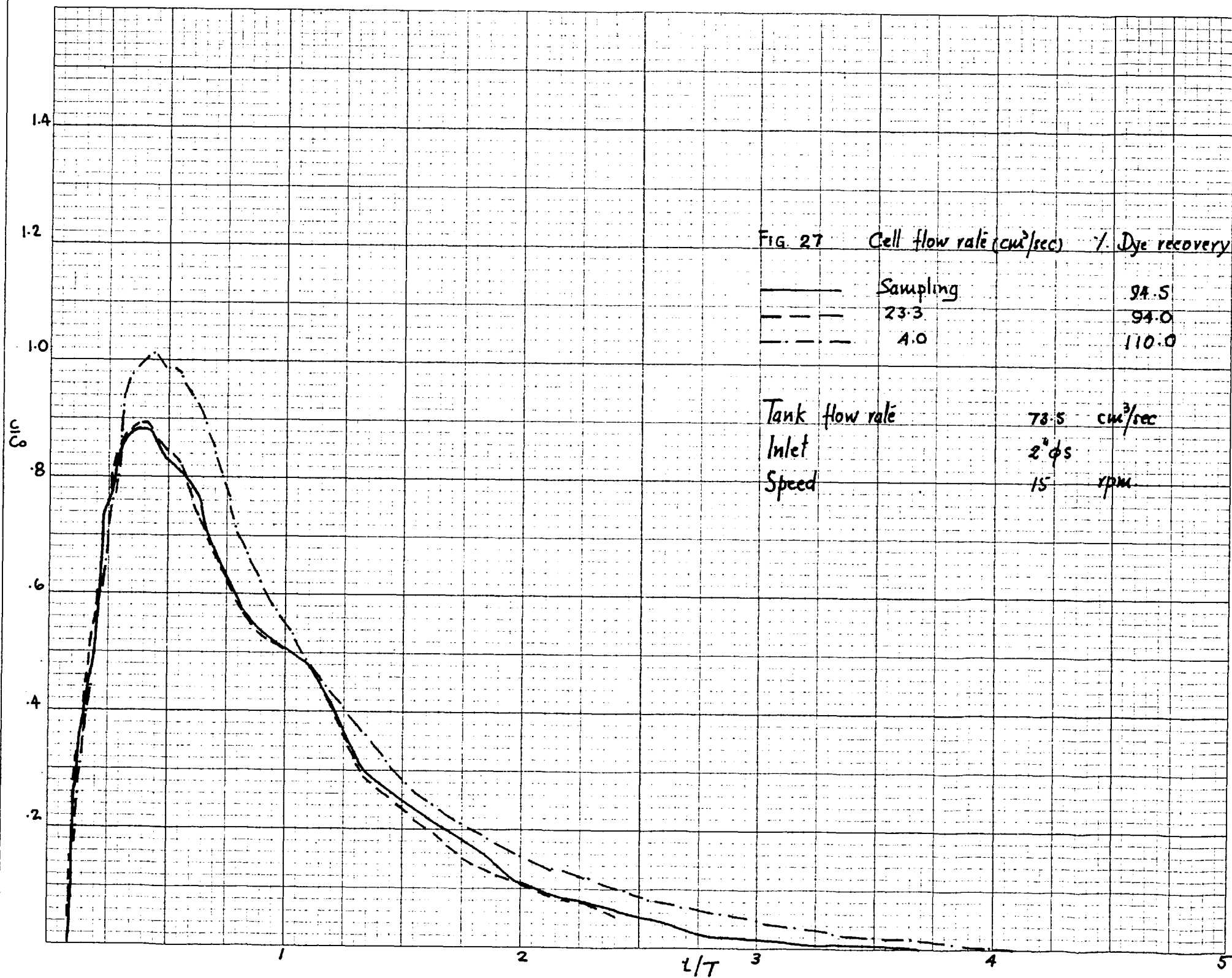
(a) The ratio of the cell theoretical detention time to that of the tank was found to affect the concentration measurement. The larger the











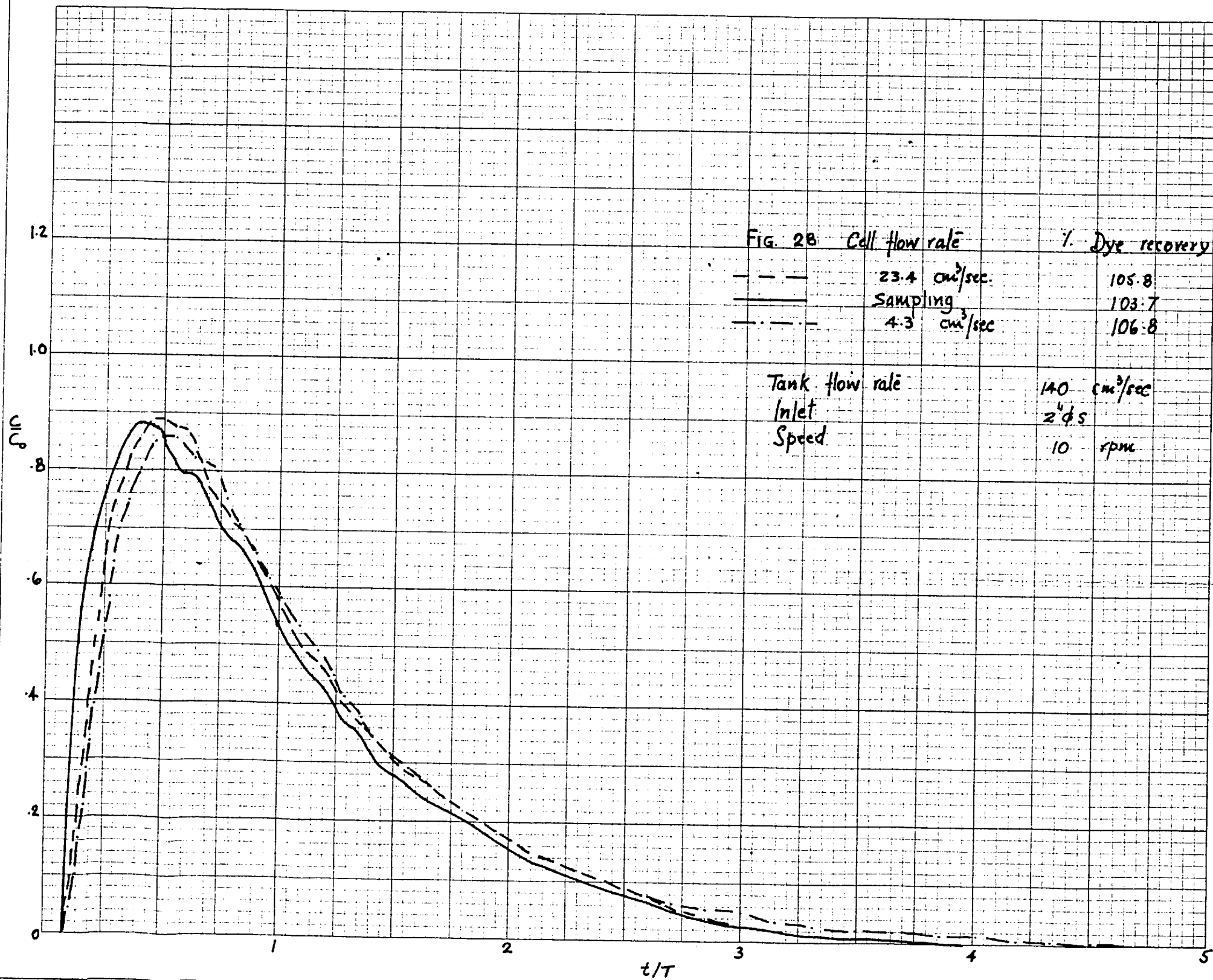


TABLE 3

Percentage dye recovery at various cell flow rates.

Tank flow rate cm/sec	Percentage dye recovery			
	Slow	Medium	Fast ³	Sampling
73.5 (5) ¹	107.5 (4) ²	99.2 (20)	-	103.5
73.5 (10)	104.5 (4)	91.5 (26)	-	101.0
73.5 (15)	110.0 (4)	94.0 (23)	-	94.5
140 (5)	107.8 (4.5)	100.9 (23.8)	-	99.8
140 (10)	106.8 (4)	105.8 (23.4)	-	103.7
199 (5)	107.6 (7)	104.5 (30.5)	101.0 (45.5)	101.5
199 (15)	116.0 (4)	-	101.0 (50)	103.5
274.5 (5)	105.5 (4.6)	99.8 (24.8)	-	107.0
274.5 (10)	-	-	-	104.5

- (1) Figures in brackets after tank flow rate are the speeds of rotation of the paddles.
- (2) Figures in brackets after percentage dye recovery are cell flow rates.
- (3) Fast flow rate introduces air bubbles which interfere with the meter readings. Only two experiments are carried out.

ratio the greater the error. However, at a cell flow rate of about 30 cm^3/sec the error involved was found to be comparable to those due to other sources (see later), and the percentage tracer recovery was generally satisfactory.

(b) Calculation shows that with N number within the range of the experiment the error should be less than 1%. It was found, however, that errors obtained in the experiments were much greater than those predicted by the theory. This was believed to be due to the presence of air bubbles and dust which were found collected in the measuring cell, especially at a low and a very high cell flow rate. Their presence always gave rise to an error on the positive side.

(c) By stopping the flow through the cell and emptying it at intervals it was possible to get rid of these bubbles, and in general, a good tracer recovery was obtained.

3.3.3. Correction of results for true volume of liquid in the tank

As the calculated volume of the tank was 100,000 cc., it was decided to use 40 cc of dye at 2500 ppm. as tracer in order to make c_0 equal to 1 ppm. The computation of c/c_0 was thus simplified. To facilitate experimental calculations further, the tank was taken as having a capacity of 100,000 cc and all the flow through curves had been plotted on this basis.

The actual volume of the liquid in the tank was in fact different in each because of the following:-

(a) The additional depth required by the head above the overflow weir which varied according to the rate of the flow through the tank.

(b) When the tank content was rotated the surface of the liquid took a parabolic shape which reduced the volume of the tank according to the speed of rotation.

The actual volume of the tank up to the level of the outlet weir, with allowances for the inlet structures and the paddles, was measured volumetrically and was found to be 102,500 cc.

The volume increase at each flow rate was calculated from the Francis formula

$$Q = 3.33 h^{(3/2)} b$$

Q is the flow rate, h the depth over the weir and b the weir length.

The calculated results were shown in Table 4.

In rotation the shape of the surface was parabolic with the maximum depression at the centre of the tank, and the decrease in volume of the

tank is $\frac{1}{8} \pi r^2 \frac{1}{60} \cdot \frac{4\pi^2 R^2}{2g} = 0.0096 \Omega^2$ litres (Ω in rpm.)

The calculated results were shown in Table 5, and the correction factors for (t/T) and $(t/T)^2$ in Tables 6 and 7 respectively.

TABLE 4Volume increased due to head above weir.

Flow rate $\text{cm}^3/\text{sec.}$	Volume increased Litre.
0	0
50	0.096
100	0.152
150	0.193
200	0.242
250	0.283
300	0.318
350	0.354

TABLE 5Volume reduced due to rotation

Speed of rotation rpm.	Volume reduced Litre.
0	0
1	0.010
2	0.038
3	0.086
4	0.133
5	0.240
7	0.470
10	0.960
15	2.160

TABLE 6

Correction factors for t/T .

Flow cm/sec	Speed of rotation Ω rpm.								
	0	1	2	3	4	5	7	10	15
0	1.025	1.025	1.025	1.024	1.023	1.023	1.020	1.015	1.003
50	1.026	1.026	1.026	1.025	1.024	1.023	1.021	1.016	1.004
100	1.026	1.026	1.026	1.026	1.025	1.024	1.022	1.017	1.005
150	1.027	1.027	1.026	1.026	1.025	1.024	1.022	1.017	1.005
200	1.027	1.027	1.027	1.026	1.026	1.025	1.023	1.018	1.006
250	1.028	1.028	1.027	1.027	1.026	1.025	1.023	1.018	1.006
300	1.028	1.028	1.028	1.027	1.027	1.026	1.023	1.018	1.006
350	1.028	1.028	1.028	1.028	1.027	1.027	1.024	1.019	1.007

TABLE 7

Correction factors for $(t/T)^2$

Flow cm/sec	Speed of rotation Ω rpm								
	0	1	2	3	4	5	7	10	15
0	1.051	1.050	1.050	1.049	1.047	1.046	1.041	1.031	1.007
50	1.052	1.052	1.052	1.051	1.049	1.048	1.043	1.033	1.009
100	1.054	1.053	1.053	1.052	1.051	1.049	1.044	1.034	1.010
150	1.054	1.054	1.054	1.053	1.051	1.050	1.045	1.035	1.011
200	1.055	1.055	1.055	1.054	1.052	1.051	1.046	1.036	1.012
250	1.056	1.056	1.055	1.054	1.053	1.051	1.047	1.037	1.012
300	1.057	1.057	1.056	1.055	1.054	1.052	1.047	1.037	1.013
350	1.058	1.058	1.057	1.056	1.055	1.055	1.048	1.038	1.014

3.3.4. Compounding of experimental errors: accuracy of the experimental results

If the final result y is related to the component x_i by the relation

$$y = F(x_1, x_2, x_3, \dots, x_n) \quad (1)$$

it can be shown that

$$G^2 = \sum_{i=1}^n \left(\frac{\partial F}{\partial x_i} \right)^2 G_i^2 \quad (2)$$

(Wilson, 1952)

where G^2 is the variance of y and G_i^2 measures the variance of the i th component.

Now the area of the flow through curve A is $\int_0^{\left(\frac{t}{T}\right)} \xi d\left(\frac{t}{T}\right) \quad (3)$

Therefore variation in A , i.e. G_A^2 is, using (2) and assume that differentiation under the integral sign is permissible:

$$G_A^2 = \left(\frac{t}{T}\right)^2 G_{\xi}^2 + \left(\frac{\xi}{T}\right)^2 G_{\left(\frac{t}{T}\right)}^2 \quad (4)$$

Again, with the help of (2) the variation in ξ denoted by G_{ξ}^2 and in $\left(\frac{t}{T}\right)$ denoted by $G_{\left(\frac{t}{T}\right)}^2$ are respectively,

$$G_{\xi}^2 = \left(\frac{t}{T}\right)^2 G_c^2 + \left(\frac{\xi}{T}\right)^2 G_{c_0}^2 \quad (5)$$

$$G_{\left(\frac{t}{T}\right)}^2 = \left(\frac{1}{T}\right)^2 G_t^2 + \left(\frac{t}{T}\right)^2 G_T^2 \quad (6)$$

If we let the deviation G_c be 0.005 ppm, and T be of the order 10 min, and $G_t = 0.01$ min, then the first terms on the right hand side of (5) and (6) become negligible compared with the second terms. Let $G_T = 0.05$ min. and $G_{c_0} = 0.01$ ppm. also t/T is of the order 4. (5) and (6) becomes respectively

$$G_{\xi_0}^2 = (0.005)^2 + (0.02)^2 \doteq 0.0004$$

$$G_{\left(\frac{\xi}{\tau}\right)}^2 = \frac{1}{100} (0.01)^2 + \frac{16}{100} (0.05)^2 \doteq 0.0004$$

Put (5) & (6) in (4) we have

$$G_A^2 = 16 \times 0.0004 + 0.004 = 0.0068$$

$$G_A = \pm 0.087$$

Error in the variance of the flow through curve, hence N_0

$$G_{\text{variance}}^2 = \int_0^{t/\tau} \left(\xi \right) \left(\frac{t}{\tau} \right)^2 d\left(\frac{t}{\tau} \right) - \left[\int_0^{t/\tau} \left(\xi \right) \left(\frac{t}{\tau} \right) d\left(\frac{t}{\tau} \right) \right]^2$$

$$\frac{\partial(G_{\text{variance}})}{\partial\left(\frac{t}{\tau}\right)} = \left(\frac{\xi}{\xi_0} \right) \left(\frac{t}{\tau} \right)^2 - \left(\frac{\xi}{\xi_0} \right) \left(\frac{t}{\tau} \right) \cdot 2 \cdot \int_0^{t/\tau} \left(\xi \right) \left(\frac{t}{\tau} \right) d\left(\frac{t}{\tau} \right)$$

$$\frac{\partial(G_{\text{variance}})}{\partial\left(\frac{\xi}{\xi_0}\right)} = \frac{1}{3} \left(\frac{t}{\tau} \right)^3 - 2 \left[\int_0^{t/\tau} \left(\xi \right) \left(\frac{t}{\tau} \right) d\left(\frac{t}{\tau} \right) \right] \cdot \frac{1}{2} \cdot \left(\frac{t}{\tau} \right)^2$$

To simplify the calculation let

$$\int_0^{t/\tau} \left(\frac{\xi}{\xi_0} \right) \left(\frac{t}{\tau} \right) d\left(\frac{t}{\tau} \right) = 1$$

Therefore

$$G_{\text{variance}}^2 = \left(\frac{\xi}{\xi_0} \right)^2 \left(\frac{t}{\tau} \right)^2 \left[\left(\frac{t}{\tau} \right)^2 - 2 \right] G_{\xi_0}^2 + \left[\left\{ \left(\frac{t}{\tau} \right)^2 \left[\frac{1}{3} \left(\frac{t}{\tau} \right) - 1 \right] \right\} G_{\frac{t}{\tau}}^2 \right]$$

$$= (4 \times 2) \times 0.0004 + \frac{16}{9} \times 0.0004$$

$$= 0.0256 + 0.0114 = 0.037$$

$$G_{\text{variance}} = \pm 0.192$$

Note that these are absolute errors. The percentage errors compound differently since the percentage errors of each component will be great when the value of the component is small, i.e. at a small concentration the percentage error of measuring the concentration will be very much greater than when the concentration is high even though the deviation in concentration measurement is the same in both cases.

CHAPTER 4

EXPERIMENTAL STUDY

4.1. Preliminary Experiments

Some trial runs with 1" $\varnothing$ and 2" $\varnothing$ risers were carried out during the development of the syringe dye injector. In these runs the static head method of dye injection as described in Section 3.2.1. was employed. One drawback of this method was the production of turbulence and air bubbles during each injection. Besides, there was always an inevitable increase in the tank flow rate during the operation because of the additional dye and flushing water. At a low tank flow rate this meant a considerable percentage increase in the flow. It was estimated that during the 5 seconds of injection 40 cc. of dye and 40 cc. of flushing water were delivered. This was equivalent to an increase of 16 cc./sec. in the tank flow rate.

Experience gained from these experiments led to the decision to design an inlet which would introduce the flow into the tank uniformly in the vertical direction. Theoretical calculation showed that 1" diameter tube was unsuitable (see Appendix IV). A 2" diameter tube was therefore used, and later adopted as the size of the risers of all the inlets to be tested in the actual experiments. At the conclusion of the experimental programme, however, it was found that 1" diameter riser with the slotted and submerged weir cylindrical diffusers behaved quite as well as 2" $\varnothing$ S and 2" $\varnothing$ P.

The results of these tests are shown in Table 8. The experimental

procedure employed was the same as that described below except for the use of the static head dye injector.

4.2. Experimental Procedure

Before the commencement of each test, water was allowed to run through the model tank at a predetermined rate for twice the theoretical detention period in order to ensure that the water in the tank and the incoming flow had the same temperature. This was in line with the generally accepted procedure which was used by many investigators in this field (Clifford and Windridge, 1935; Mau, 1959; Abdalla, 1955). During this time 40 cc. of methylene blue dye at 2,500 ppm. were introduced into the dye injector where its temperature was brought to that of the incoming water. When experimenting with paddles in rotation their angular speed was checked at intervals to make sure that it was constant before dye was injected. This step was necessary only when the motor was started from cold. The zero reading of the absorptiometer was then taken.

The timing was started as soon as the dye was injected into the tank. Readings of the absorptiometer were taken every 10 secs. for the first three minutes and every 20 secs. for the next two minutes. The reading interval was then increased to half a minute until the first ten minutes had passed, when it was again increased to one minute for the next ten minutes. After this a reading was taken every 5 minutes until there was no trace of dye left in the effluent. The average number of the readings taken

for a run was 39, and the total length of each run was from 4 to 5 times the theoretical detention time.

The flow rate was varied between 80 to 300 cc./sec. at intervals of about 25 cc./sec., and for each rate of flow two complete runs were made with each speed of rotation of 0, 1, 2, 3, 4, 5, 7, 10, and 15 rpm. The average of two runs was found to be adequate since they appeared to be in good agreement, especially after the maximum concentration had passed. For each run the flow through curve was obtained by plotting $\frac{C}{C_0}$ v. $\frac{t}{T}$. The area of each flow through curve was measured by a planimeter.

4.3. Critical Examination of Experimental Results: Tests for Validity of Results.

Two parameters are useful in indicating whether the results from a run may be considered as acceptable.

(a) The area of the flow through curve represents the total quantity of the tracer used in the experiment and from the method of plotting the curve this area should be unity, indicating that all the tracer injected into the tank has been recovered. However, some of the tracer may be retained on the surface of the tank, and there may be some errors introduced by the absorptiometer. The area generally varies above or below unity. (See also Section 3.3.4.).

(b) $\frac{t_c}{T}$, the ratio of the mean time of the flow through curve to the theoretical detention time. This ratio should be equal to unity unless a portion of the tank is completely stagnant. However $\frac{t_c}{T} > 1$ is also possible. Ingersoll (1955), Abdalla (1955) and Mau (1959) experienced this. Ingersoll believed that this is due to some of the dye being swept upstream of the injecting point. Observations made during the present experiments showed that this was not the case, and yet occasionally $\frac{t_c}{T}$ tended to be greater than unity. It appears that the conditions at the tail end of the flow through curve influence this ratio markedly. When there is a long detention of small concentration, the moments of these concentrations about the ordinate are often great enough to pull the ratio to the right. It is therefore affected by the area of the flow through curve.

Examination of the data obtained showed that for the tracer recovery between 90 - 110%, $\frac{I_s}{I}$ was not affected appreciably. It was therefore decided to accept the results whose tracer recoveries lie within this range. This is considerably narrower than the range used by some investigators (Mau, 1959; Rohlic, 1951, used the range between 84 - 116%).

4.4. Experimental Results: General Observation

4.4.1. With no flow through the tank.

(a) Effects of rotation on the spread of dye.

The tank was first filled with water which was later set into rotation by the action of the paddles rotating at 3 rpm. An interval of 10 minutes was allowed to elapse to ensure that the system was steady, when 10 cc. of methylene blue dye at 2,500 ppm. was dropped into the tank half-way between the centre and the paddles, and the timing started. It was observed that the dye did not diffuse in all directions due to turbulence, as it would have done if there were no rotation. Instead, it was drawn out gradually by the rotating liquid into fine sheets which always remained parallel to the axis of rotation. These sheets were seen to mix with the tank content gradually as time passed. Eventually, at about 30 minutes after its introduction, dye was observed to have mixed almost completely with the liquid in the tank. This latter phenomenon was believed to be due to the action of the secondary currents which will be discussed in the following sections. A series of photographs was taken, showing the behaviour of the dye cloud at different times. (See Fig. 29).

(b) The existence of secondary currents and the variation of their magnitudes with the speed of rotation.

The tank was again filled with water and was allowed to stand until the turbulence had died down when sand was carefully strewn throughout the tank. The paddles were then set in rotation at a slow speed (1 rpm.)

at first. At this speed no general movement of the sand was observed. On gradually increasing the speed up to about 7 rpm., movements of sand grains were noticed. Eventually, when the paddles were left rotating at 15 rpm., sand was found to have been carried inwards and now formed a ring surrounding the inlet (Fig. 30).

4.4.2. With flow through the tank

(a) No rotation

(i) The flow through curves. When there was no rotation, the flow through curves were observed to have a high early peak with a long tail of small concentration. The agreement between experimental and theoretical curves was generally worse than with rotation, especially between the rising parts of the curves. (See Section 4.5.3. and Figs. 45 and 46). Often two peaks were observed, the second peak being smaller in magnitude. Sometimes there were several peaks close together, of about the same magnitude, which might be due to non-uniform velocity distribution. This, however, happened very rarely. (See Figs. 31 - 34).

(ii) Flow distribution. The distribution of the flow horizontally was generally good for all inlets. The distribution of the flow vertically was, however, not so good. With the exception of the UFBI, the 2" $\varnothing$ P and the 2" $\varnothing$ S inlets showed a strong tendency for the flow to follow the tank floor regardless of the flow rate, although at a high flow rate the high degree of turbulence tended to distribute it more uniformly at a

distance away from the inlet in both cases.

The UFBI inlet distributed the flow best of all. Even though the inlet was designed to operate at about 200 cc./sec., it was found that the flow was also satisfactorily distributed at flow rates much different from this value. (See Fig. 34). With this inlet, the positioning of the baffles surrounding it was very delicate because if they were slightly out of adjustment the flow distribution would not be uniform horizontally. (See Fig. 35).

(b) With rotation

(i) Secondary currents. When rotation was introduced, it was observed that the radial movement of the flow was checked momentarily at a certain radius and a circular motion was transferred to it. The region between this radius and the inlet may be termed the inlet zone. The size of this zone depended on the speed of rotation and the flow rate, being small for a low flow rate or a high speed of rotation (Figs. 33 - 34). However, at 15 rpm., the front boundary of the dye surrounding the inlet tended to oscillate in the radial direction. Dye was seen to behave as though it was diffusing outwards as well as oscillating.

Observations made from the side of the tank revealed that at the moment when the dye appeared to stop its radial motion when viewed from above, it was actually rising. With the circular motion now imparted to it, the dye appeared to rise spirally outwards towards the outlet weir. This was believed to be due to the existence of the secondary currents.

On reaching the weir, some of the dye passed over it while the rest was carried downwards towards the centre in a spiral manner. Near the boundary of the inlet zone it started to rise spirally towards the outlet weir again. It must be remembered that this process was not abrupt, as in the case with no rotation.

Secondary currents were therefore assisting in mixing the content of the tank while the circular motion of the liquid suppressing the turbulence. This phenomenon was also observed and studied by Möller (1881), Hédewadt (1940) and Browning (Schlichting, 1955).

The magnitudes of these currents depended upon the speed of rotation, the flow distribution and the flow rate. If the incoming flow was directed against the tank floor, the radial velocities of the flow along the floor were opposite to those of the secondary currents, which were therefore minimized.

If, on the other hand, the flow was more towards the upper portion of the tank, it helped in promoting the secondary currents. This was the reason why the influent tended to flow in the upper half of the tank near the inlet at high speed of rotation, especially in the case of the 2" ϕ S inlet (see Fig. 34) and, in a lesser degree, the UFBI.

The existence of these currents was also evident in the movement of deposited particles at the tank floor. Before the commencement of rotation, they were scattered everywhere on the floor, but were seen to form a ring near the centre after the liquid was set in rotation.

(ii) The effect of rotation on the flow distribution in a hori-

zontal plane. The analytical study indicates that the effect of rotation is to increase $\sqrt{v^2}$. The mixing coefficient in the tangential direction was therefore increased. This was useful, because it meant

(a) a suppression of $\overline{u'^2}$

(b) a more uniform distribution of flow horizontally.

Fig. 33 shows the flow leaving the UFBI inlet, the baffles of which had been deliberately adjusted to create a non-uniformity of flow. It is evident from Fig. 33 that rotation helps in distributing the flow in this direction and makes the assumption of axially symmetrical flow more justifiable.

(iii) Water surface in the tank. With rotation, the surface of the water in the tank took a parabolic shape. At 10 rpm. the curvature of the surface was very noticeable, and at 15 rpm. water seemed to be thrown out of the tank by the centrifugal force, rather than flowing curvilinearly over the weir (see also Section 3.3.3.).

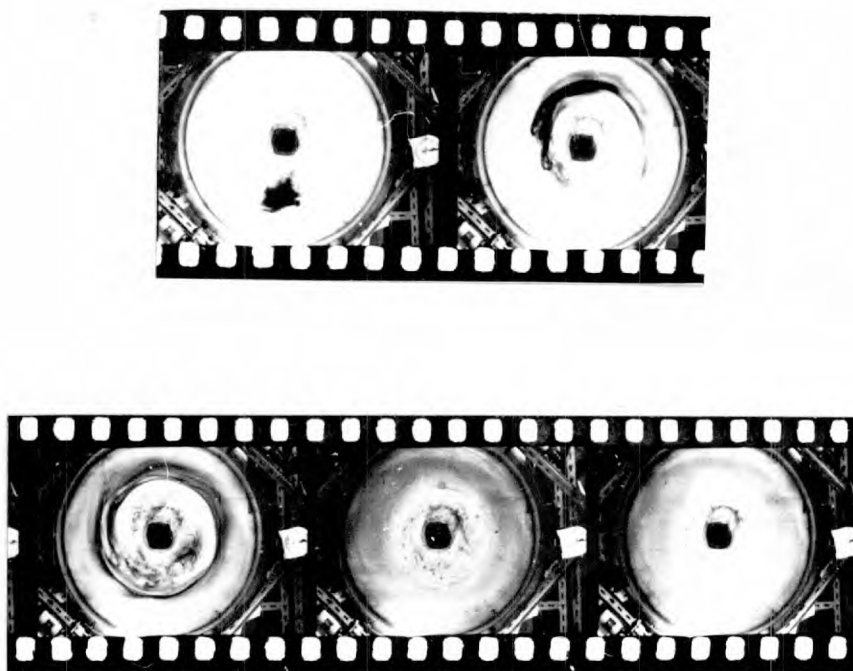
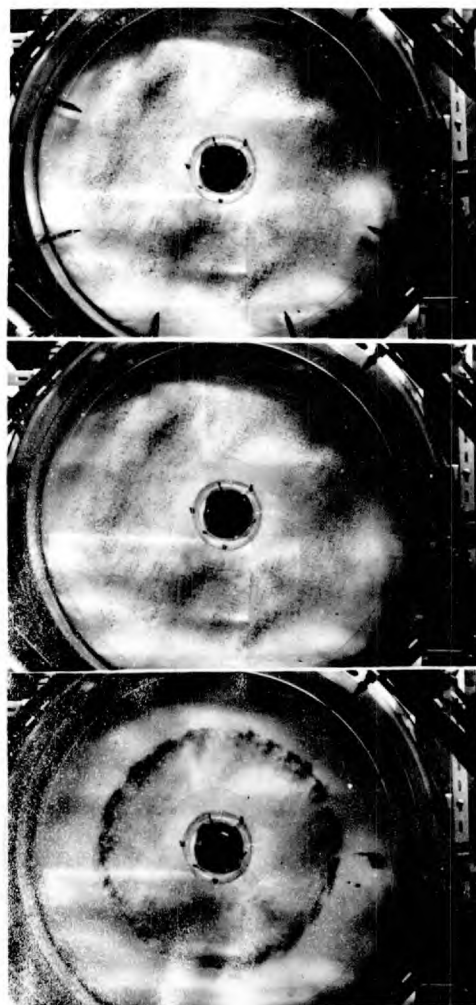


Fig. 29 The effect of rotation on the spread of a cloud of dye at 0, 1, 5, 15 and 30 min. after its introduction (1-5). No flow, 5 rpm.



MOVEMENT OF ^{252}Cf DEPOSITED PARTICLES DUE TO THE BOUNDARY CURRENTS.

Fig. 30 Movement of the deposited particles due to the secondary currents (p. 135).

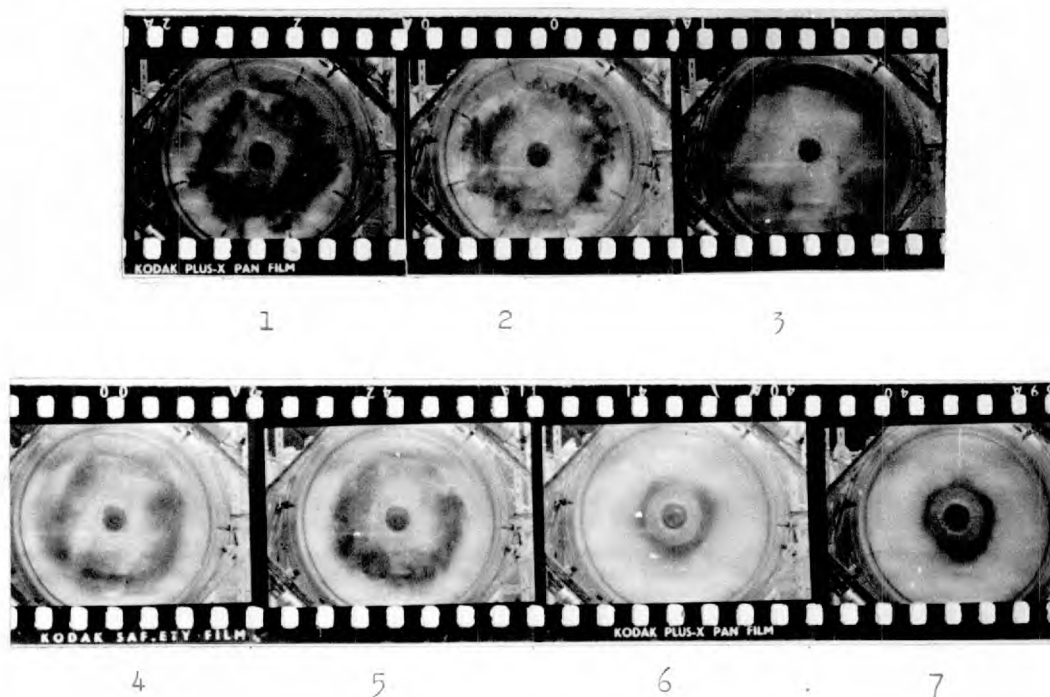


Fig. 31 The effect of rotation on short-circuiting (2"Ø P)

Photographs were taken immediately after the injection of dye

(1) Flow rate $150 \text{ cm}^3/\text{sec}$. speed 0 rpm.

(2) " 250 " 0

(3) Same as (2) but the photograph was taken about 10 sec. after the injection

(4) Flow rate $150 \text{ cm}^3/\text{sec}$. speed 3 rpm

(5) " 250 " 3

(6) " 150 " 15

(7) " 250 " 15

See pp. 136-137.

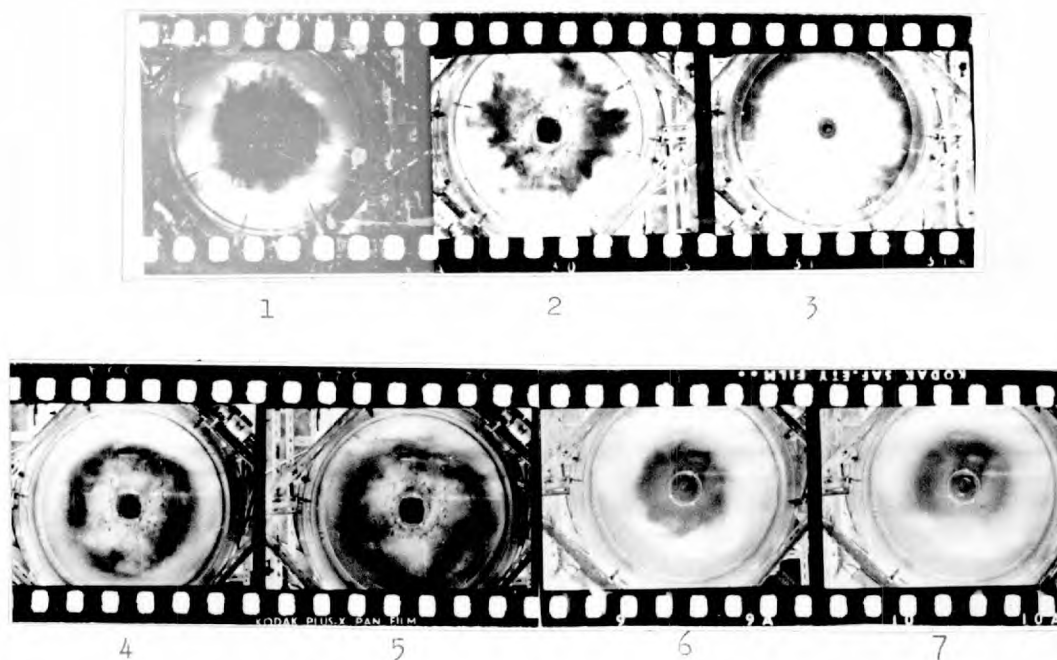


Fig. 32 The effect of rotation on short-circuiting ($2''\varnothing$ S)

Photographs were taken immediately after the injection of dye

- (1) Flow rate $150 \text{ cm}^3/\text{sec.}$ speed 0 rpm.
- (2) " 250 " 0
- (3) Same as (2) but the photograph was taken about 10 sec. after the injection
- (4) Flow rate $150 \text{ cm}^3/\text{sec.}$ speed 3 rpm
- (5) " 250 " 3
- (6) " 150 " 15
- (7) " 250 " 15

See pp. 136-137.

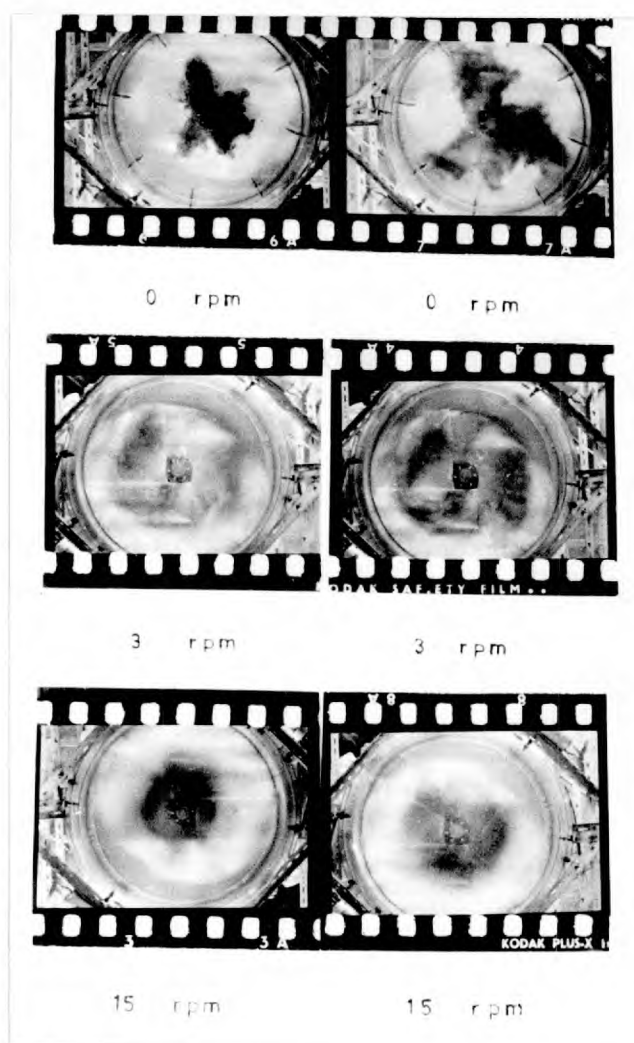


Fig. 33 The effect of rotation on short-circuiting (UFBI)

The baffles of the inlet had been deliberately adjusted to distribute the flow non-uniformly in a horizontal plane (see pp. 136-137). The flows in the first and the second columns are 150 and 250 $\text{cm}^3/\text{sec.}$ respectively

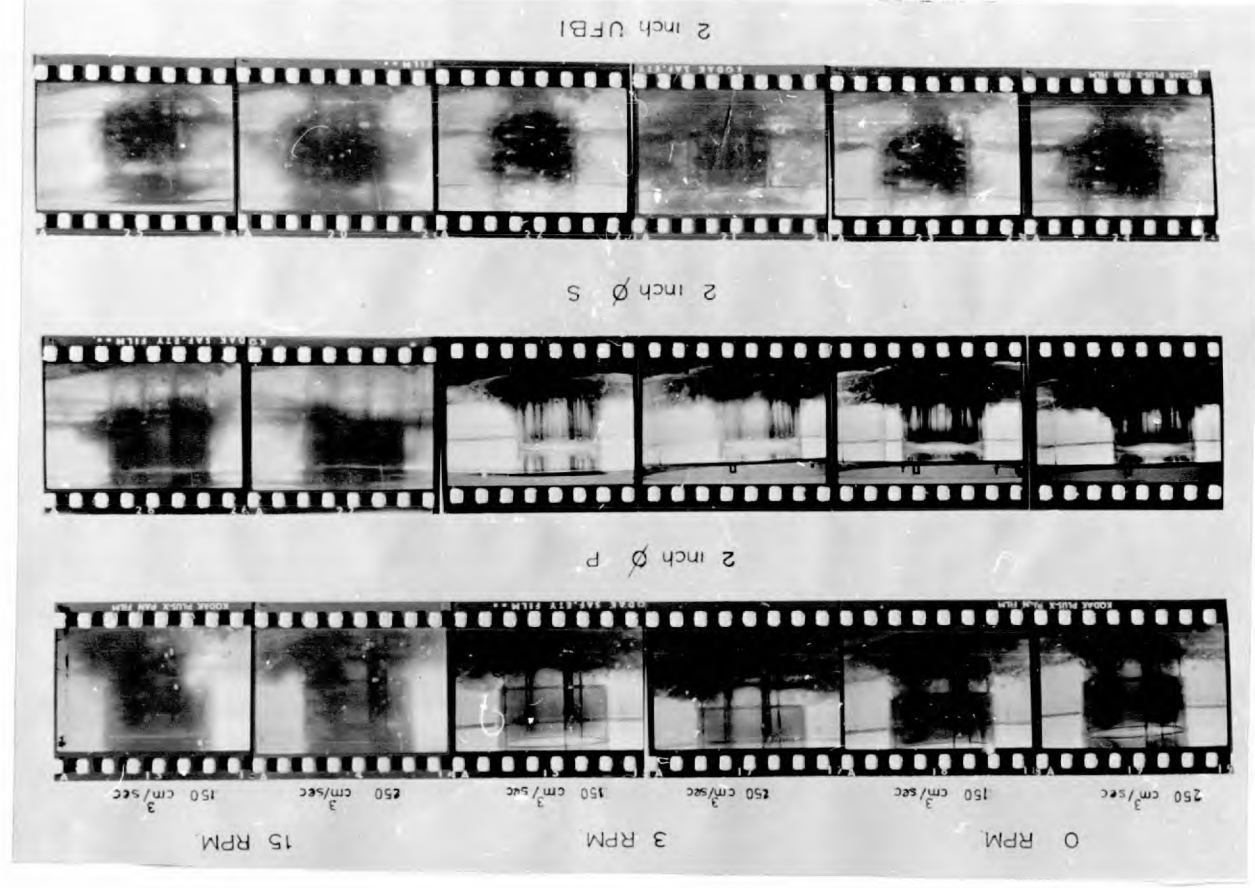


Fig. 34 Flow distribution in the verticle direction
(pp. 137-139).

4.5. Analyses of Results

In order to compare the performances of various types of sedimentation tanks under different conditions, it is a common practice to obtain a flow through curve by plotting $\frac{C}{C_0}$ against $\frac{t}{T}$. There are many ways of interpreting this curve.

4.5.1. Parameters measuring the short-circuiting.

(a) N. This is the number of the imaginary cells which constitute the sedimentation tank. It is rational and has been shown to be equal to $\frac{K}{2D} + 1$ (see Section 2.1). Its relationship with the Froude number of the tank and the speed of rotation of the paddles has also been proved. Three families of curves of N v. Q^2 were plotted in Figs. 35 - 37. The values of N were calculated from the variance of the flow through curves obtained from the experiments.

Thomas and Archibald (1951) suggested calculating N from the ratio $\frac{\frac{t_0}{T}}{\frac{t_0}{T} - \left(\frac{t_0}{T}\right)^2}$. The comparison between the experimental and theoretical curves calculated from N obtained from both methods were shown in Figs. 38 - 40. It is evident that N calculated from the variance of the curve agrees more closely with the experiment. It has therefore been accepted in this analysis.

(b) The Dispersion index, i.e. $\frac{t_{10}}{t_{90}}$.

This was first put forward by Morrill (1932). It is the ratio of the times taken for 10% and 90% of dye to pass through the tank. It thus measures the spread of the flow through curve and is analogous to the variance of the curve. However, it has not been derived on a rational basis.

This ratio has not been used in the experimental analysis because the value of the t_{90} cannot be accurately determined due to the low concentration in this region. The accuracy of the computation of this ratio was also very much affected by the number of strips into which a flow through curve is divided for the calculation. This is clearly shown by the Example of Calculation II. The difference between the values of N calculated by the 20-strip and the 40-strip methods was only 1.5%, while that of the dispersion index was 5.5%. The values of $\frac{t_s}{\bar{t}}$ and $\frac{t_m}{\bar{t}}$ (see later) were also little affected.

(c) $\frac{t_s}{\bar{t}}$ This is the ratio of the arithmetic mean time to the theoretical detention time. It has also been known as the efficiency of displacement. Its value will be less than unity if there exists a stagnant portion of the liquid in the tank. The shape of the flow through curve has no influence upon this ratio. If the whole tank is used, i.e. there is no completely stagnant portion, $\frac{t_s}{\bar{t}}$ will be equal to unity regardless of the turbulent mixing in the tank. Therefore a tank which has a mass of fluid rotating upon itself, as well as exchanging with the fluid in the vicinity will have its $\frac{t_s}{\bar{t}} = 1$, even though its N number is low.

This ratio is very much affected also by the tail end of the curve, and inaccurate calculation may lead to values of $\frac{t_s}{\bar{t}}$ greater than unity. It is therefore not suitable as a parameter measuring the hydraulic performance of a sedimentation tank, but it is useful in testing whether an experimental result is to be accepted. ($\frac{t_s}{\bar{t}}$ is tabulated against Q^2 in

Tables 9 (a), (b) and (c) for the 2" ϕ P, 2" ϕ S and the UFBI respectively).

(d) $\frac{t_m}{T}$ This is the ratio of the time at which 50% of the dye has passed through the tank to the theoretical detention time. It is related to N (equation 32, Section 2.1.5.). Its sensitivity to the change in the tank conditions is, at any rate, very poor compared with N . While N can vary between 0 to ∞ , this ratio only varies between 0.669 to 1.0. Therefore, with the range of N in the present investigation, i.e. between 1 and 3, the variation in the value of $\frac{t_m}{T}$ is too small to be of any use. (See Tables 9 (a), (b) and (c)).

4.5.2. The Coefficient of Diffusion D.

This is the newly introduced parameter as a result of the analysis of the problem on a statistical basis. Its magnitude has been shown to be dependent upon the turbulence condition in the tank, and can therefore be regarded as a measure of the tank hydraulic efficiency. Its relationship with N and v has also been shown in Section 2.1. A small value of D corresponds to a high value of N , and a small degree of turbulence. This fact substantiates the reasoning that a high efficiency of sedimentation of suspended matters should be obtained with a high N number.

The curves of D v. Q^2 are plotted in Figs. 41 - 44.

It should be evident now that all the parameters cited above indicate only the turbulent mixing of a tank and therefore give a measure of the total short-circuiting only. It is possible, however, that a tank

exhibits severe short-circuiting caused by stagnant spaces and non-uniform velocity distribution when its turbulent mixing is comparatively small in degree. The resulting flow through curve in this case has a high, narrow peak with a high N value. Here $\frac{t_0}{T}$ will reveal this short-circuiting by giving a low value. On the other hand, if the tank has no completely stagnant spaces, but has a great deal of turbulent mixing, its N value will be low, and $\frac{t_0}{T}$ will be high. This is the situation which is common in most sedimentation tanks, and in circular radial flow tanks in particular.

Therefore, using the parameters characterizing short-circuiting due to turbulent mixing alone are inadequate in studying the hydraulic performance of a sedimentation tank. Some sort of indication of the existence of stagnant space and non-uniform velocity distribution in the tank is necessary. The limitation of using $\frac{t_0}{T}$ as the parameter has already been mentioned. We shall now discuss the possibility of revealing short-circuiting due to stagnant spaces and non-uniform velocity distribution by some other means.

4.5.3. Parameters measuring the Short-circuiting due to Stagnant Spaces and Non-uniform Distribution.

Two parameters have been frequently used to characterize severe short circuiting in a sedimentation tank, namely $\left(\frac{C}{C_0}\right)_t$, the ratio of the initial appearance of dye to the theoretical detention time, and $\left(\frac{C}{C_0}\right)_{\frac{t}{2}}$. The disadvantages of these parameters lies in the fact that they are very difficult to determine accurately. Examination of the flow through

curves obtained in the experiments together with visual observation during the test does not show any marked difference in the values of the ratios obtained with widely differing flow patterns, i.e. with and without the rotation of the paddles.

On the other hand, if the actual curves obtained in the experiment are compared with the theoretical ones, having the same N number, the effect of short-circuiting due to the above causes is evident. As severe short-circuiting is caused by non-uniform velocity distribution, the flow through curve of a sedimentation tank where this occurs will depart from the theoretical curve. This may be explained qualitatively as follows:-

When the dye streams rapidly across the tank due to non-uniform velocity distribution, the assumption of uniform and axially symmetrical flow breaks down. The tank effective volume is less than the assumed volume used in calculating C_0 and T . Thus, when the flow through curve is plotted, an early and high peak results, since the values of $\frac{C}{C_0}$ are larger and of $\frac{t}{T}$ smaller than they should be if the new value of the tank effective volume were used. The experimental curve is therefore different from the theoretical one. The greater the difference, the more severe the short-circuiting due to stagnant spaces and non-uniformity in velocity distribution.

Consequently, the comparison between the experimental and the theoretical curves is a better indication of this kind of short-circuiting. In Figs. 45 and 46, the representatives of the experimental curves are compared with the theoretical curves from both the v - and the N -formulae. Note also the better agreement obtained with a superimposed forced vortex.

Fig. 35 Curves of N versus Q^2 for the $2''\phi P$

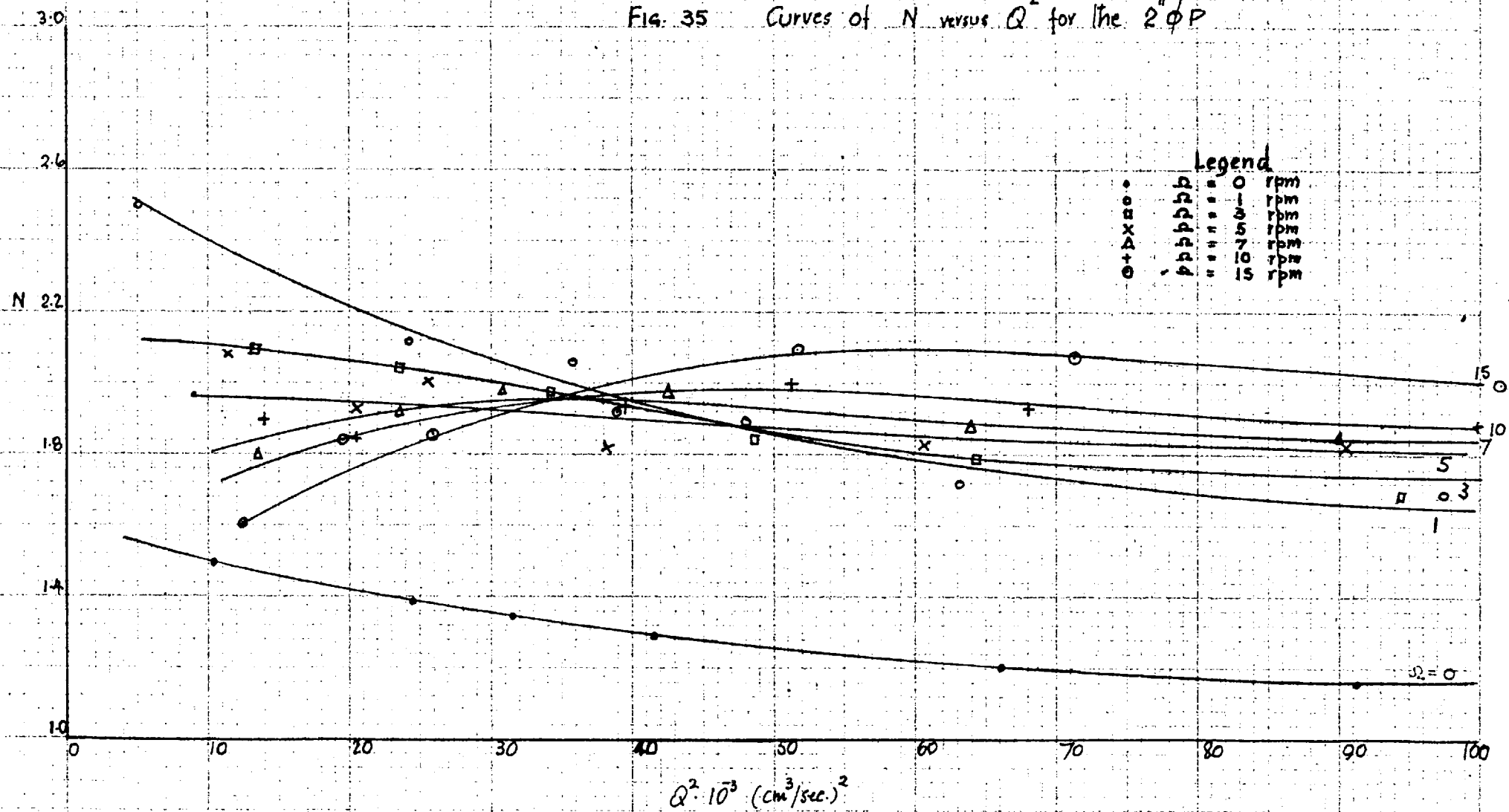


FIG. 36 Curves of N versus Q^2 for $2\phi S$.

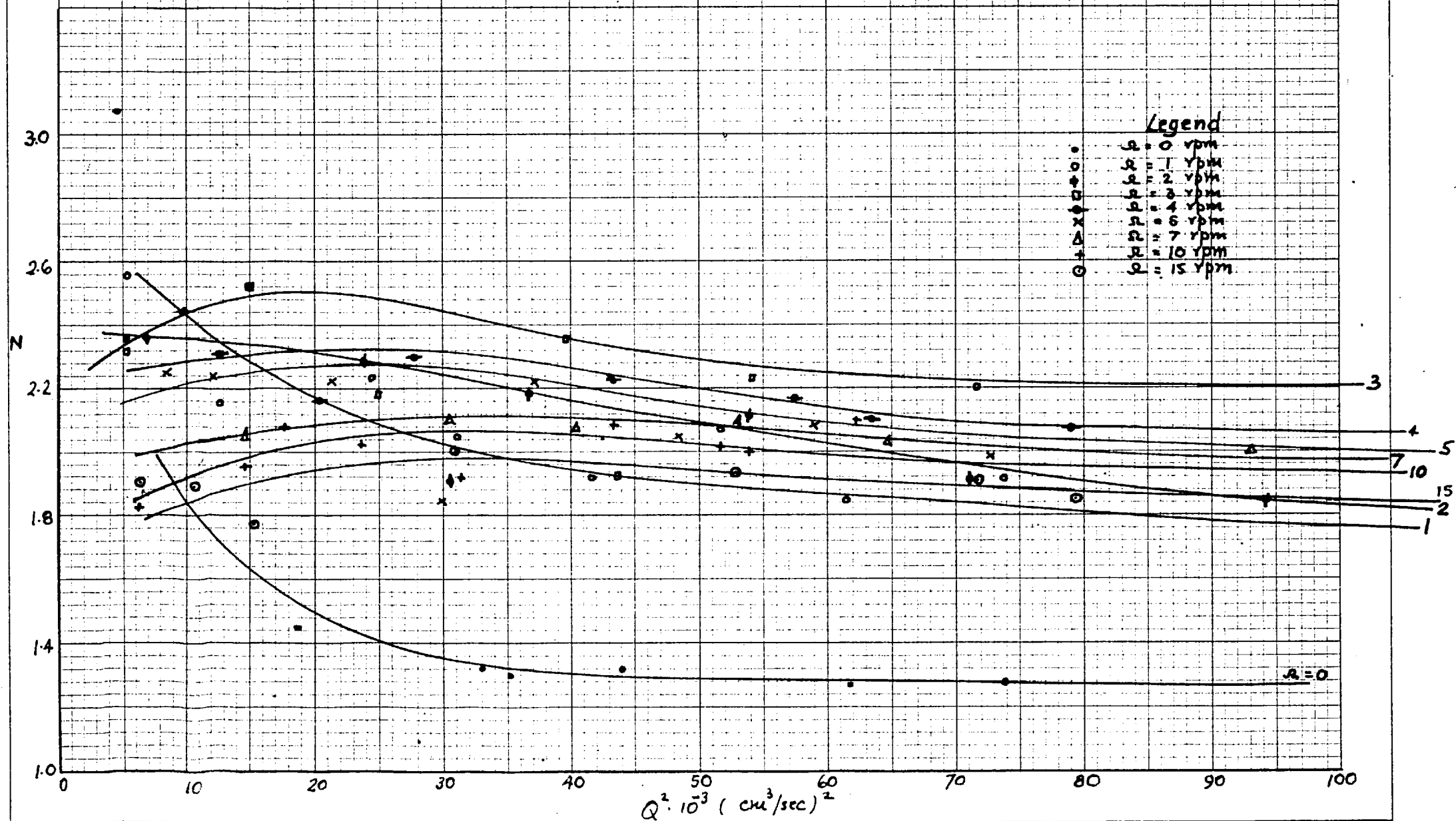


FIG. 37 Curves of N versus Q^2 for the UFBI Inlet.

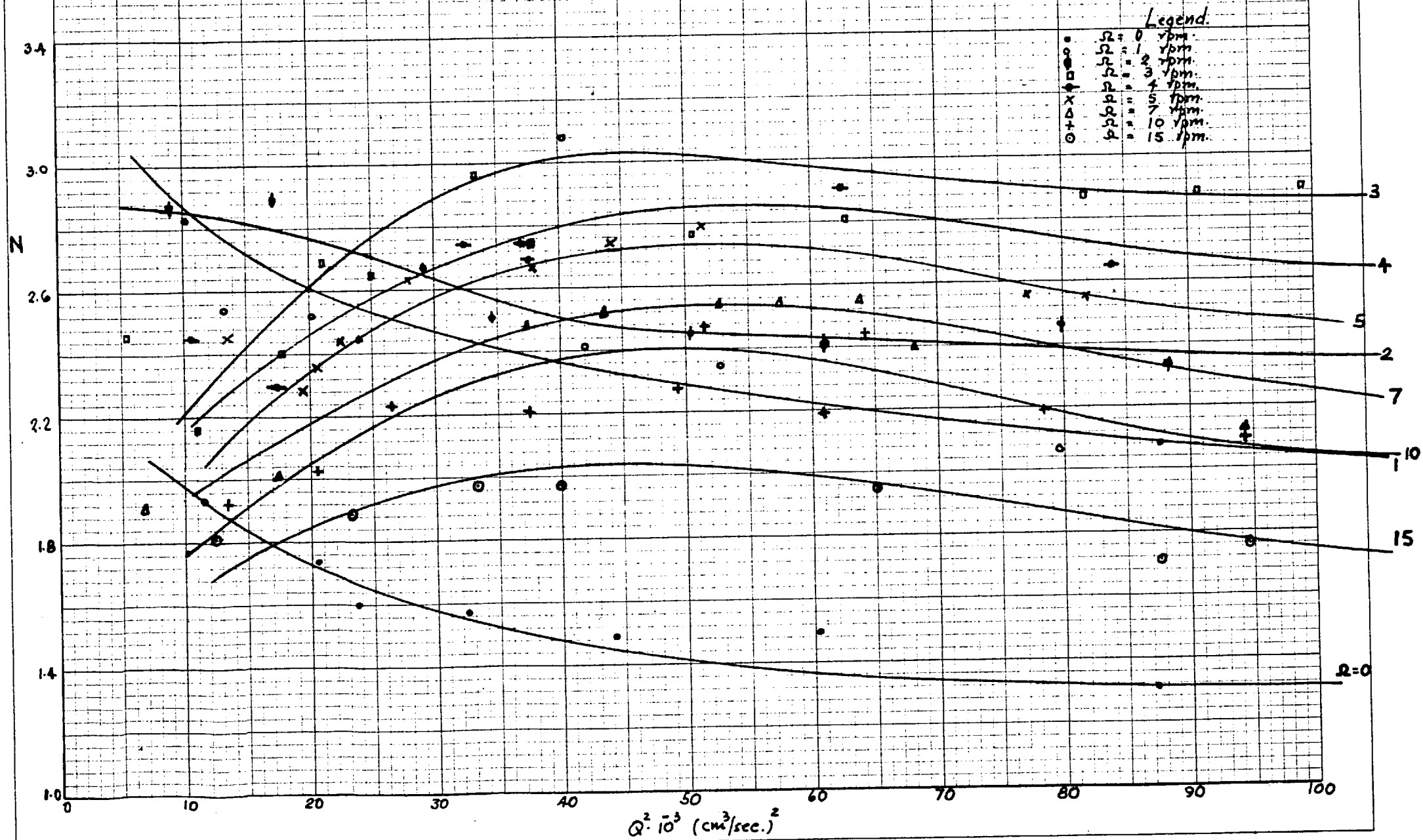
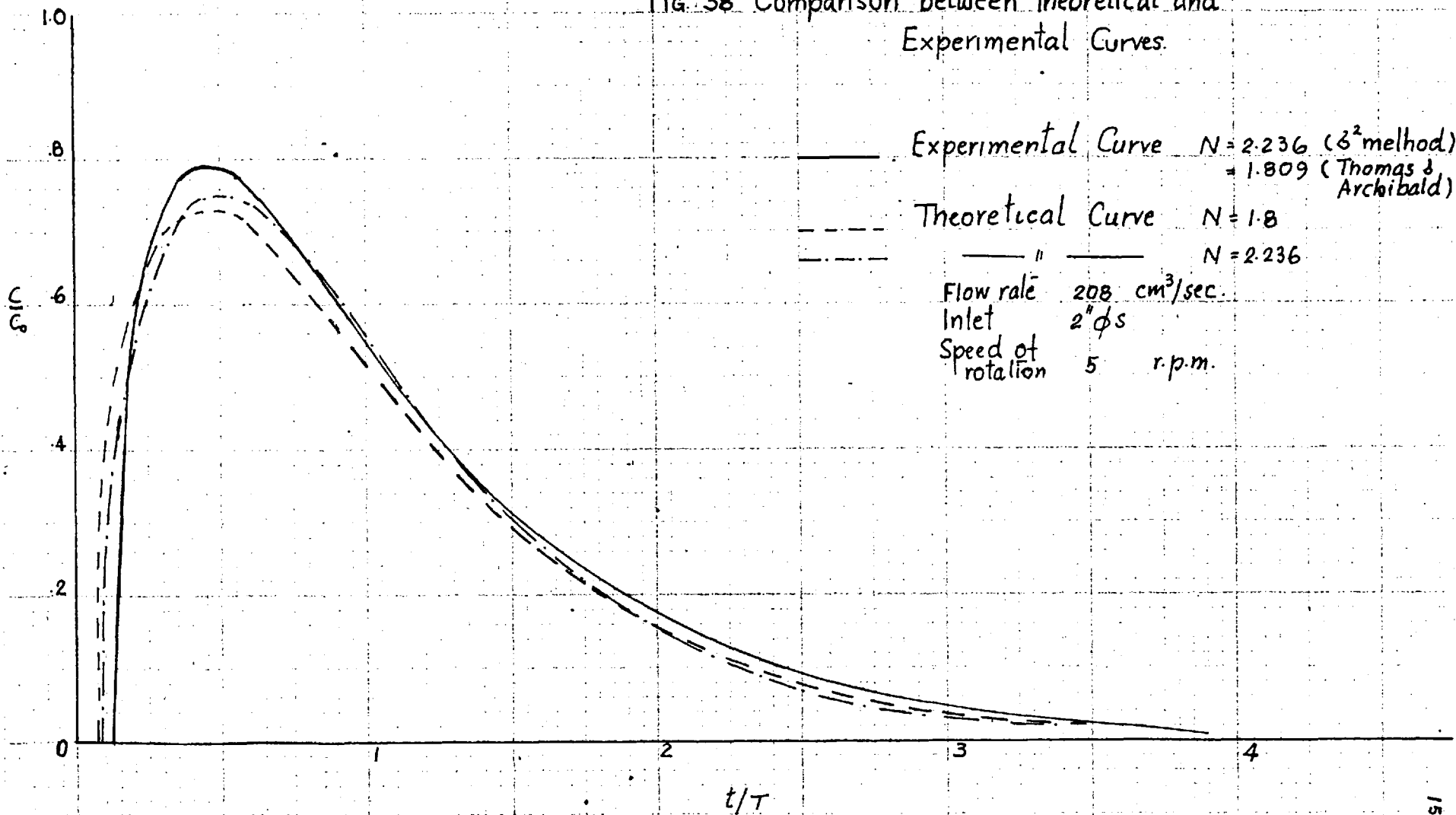
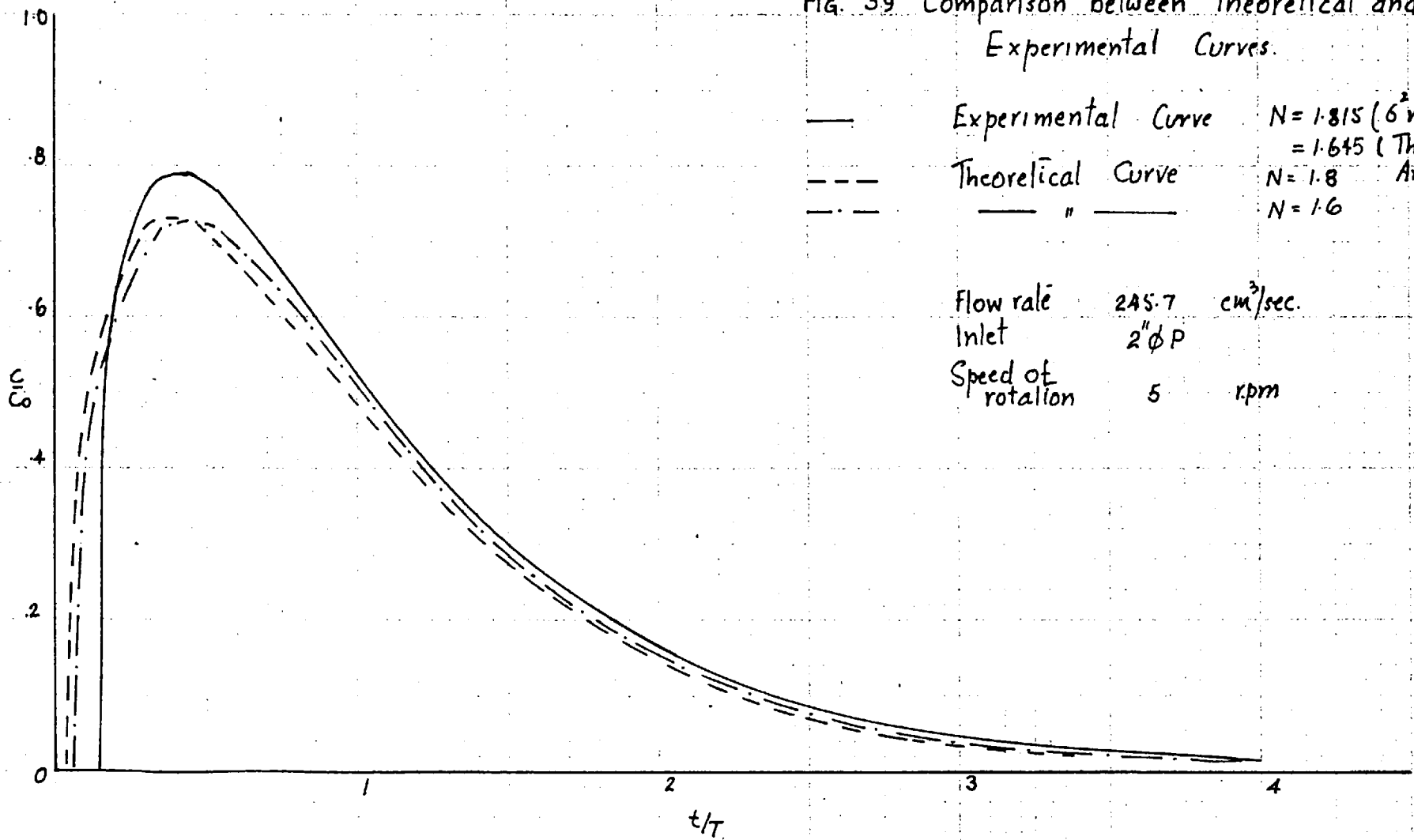


FIG. 38 Comparison between Theoretical and Experimental Curves.





—	Experimental Curve	$N = 1.815$ (6 ² method)
		$= 1.645$ (Thomas & Archibald)
---	Theoretical Curve	$N = 1.8$
-.-.-	— " —	$N = 1.6$

Flow rate	245.7	cm ³ /sec.
Inlet	2" ϕ P	
Speed of rotation	5	rpm

Fig. 40 Comparison between Theoretical and Experimental Curves.

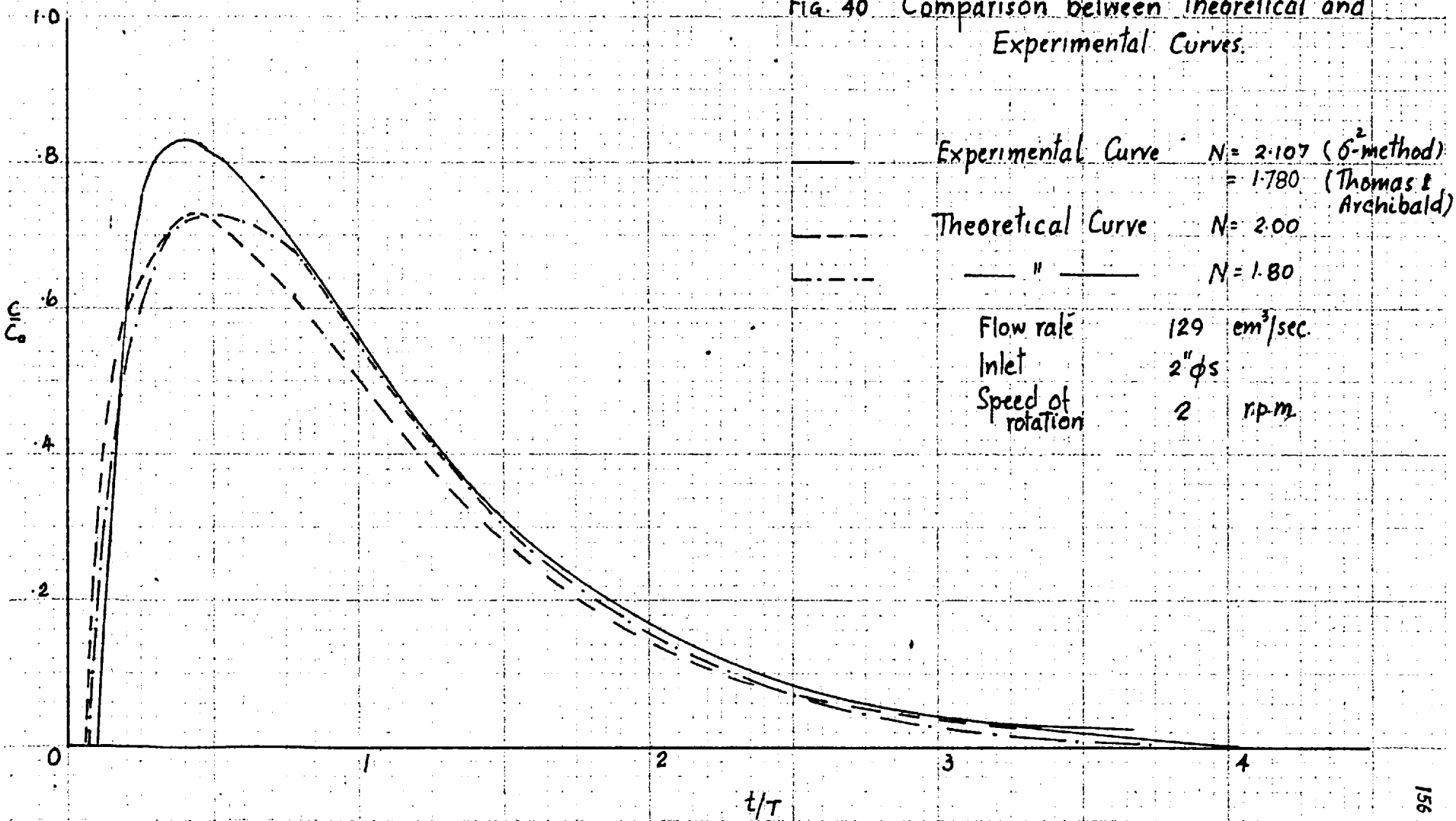


FIG. 41 Curves of D versus Q^2 for the three inlets.

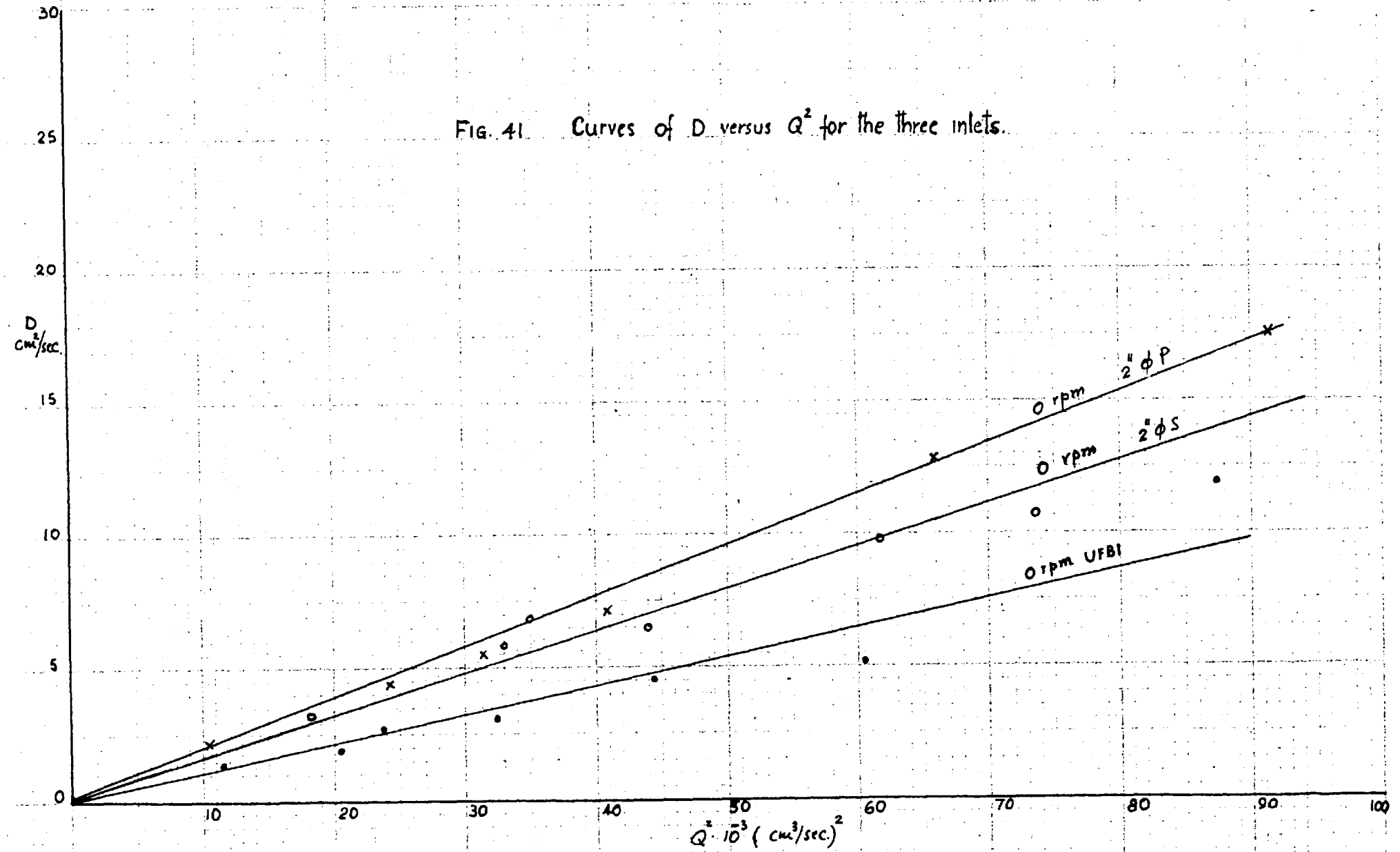


FIG. 42 Curves of D versus Q^2 for the $2''\phi P$.

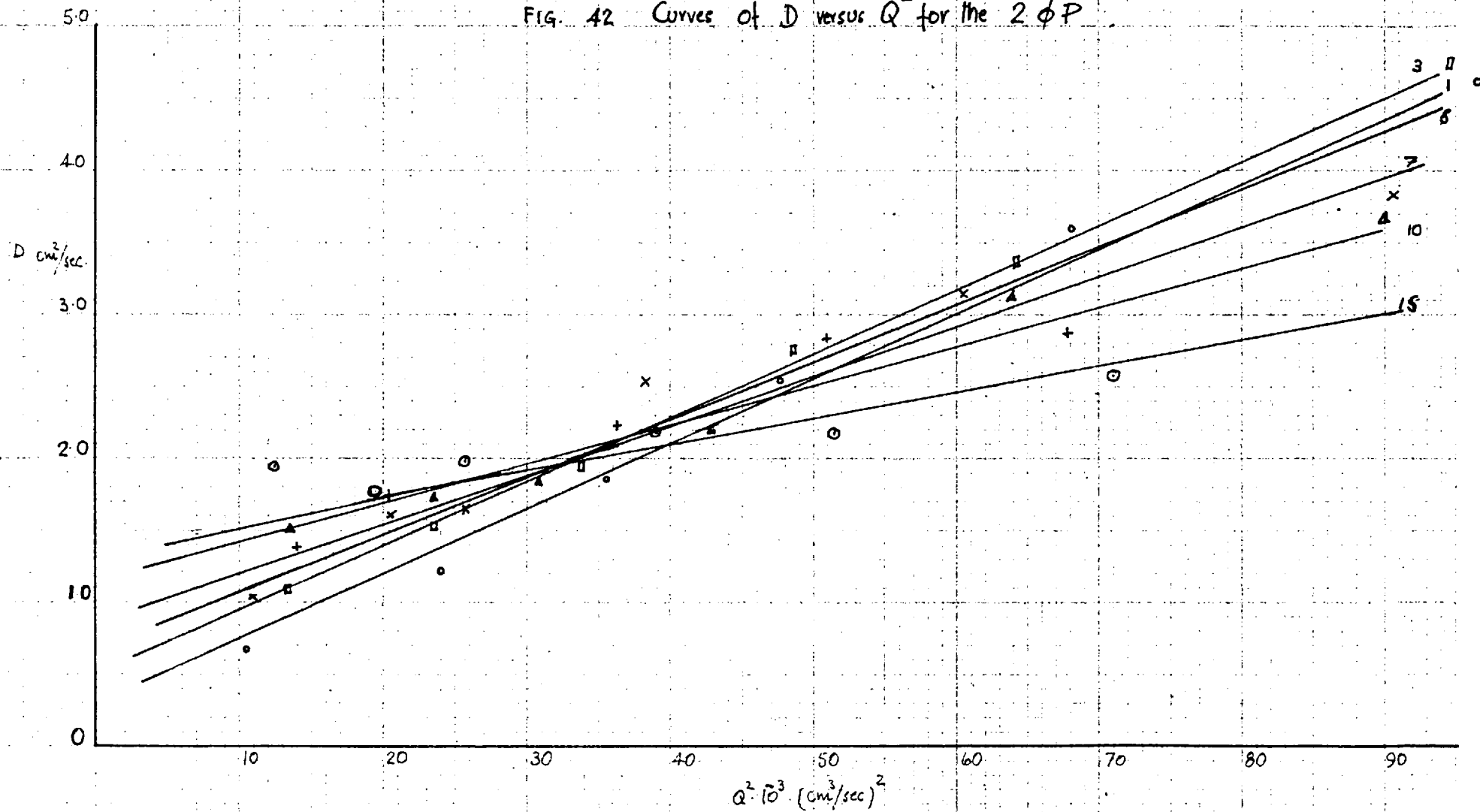


FIG. 43 Curves of D versus Q^2 for the 2" ϕ S Inlet

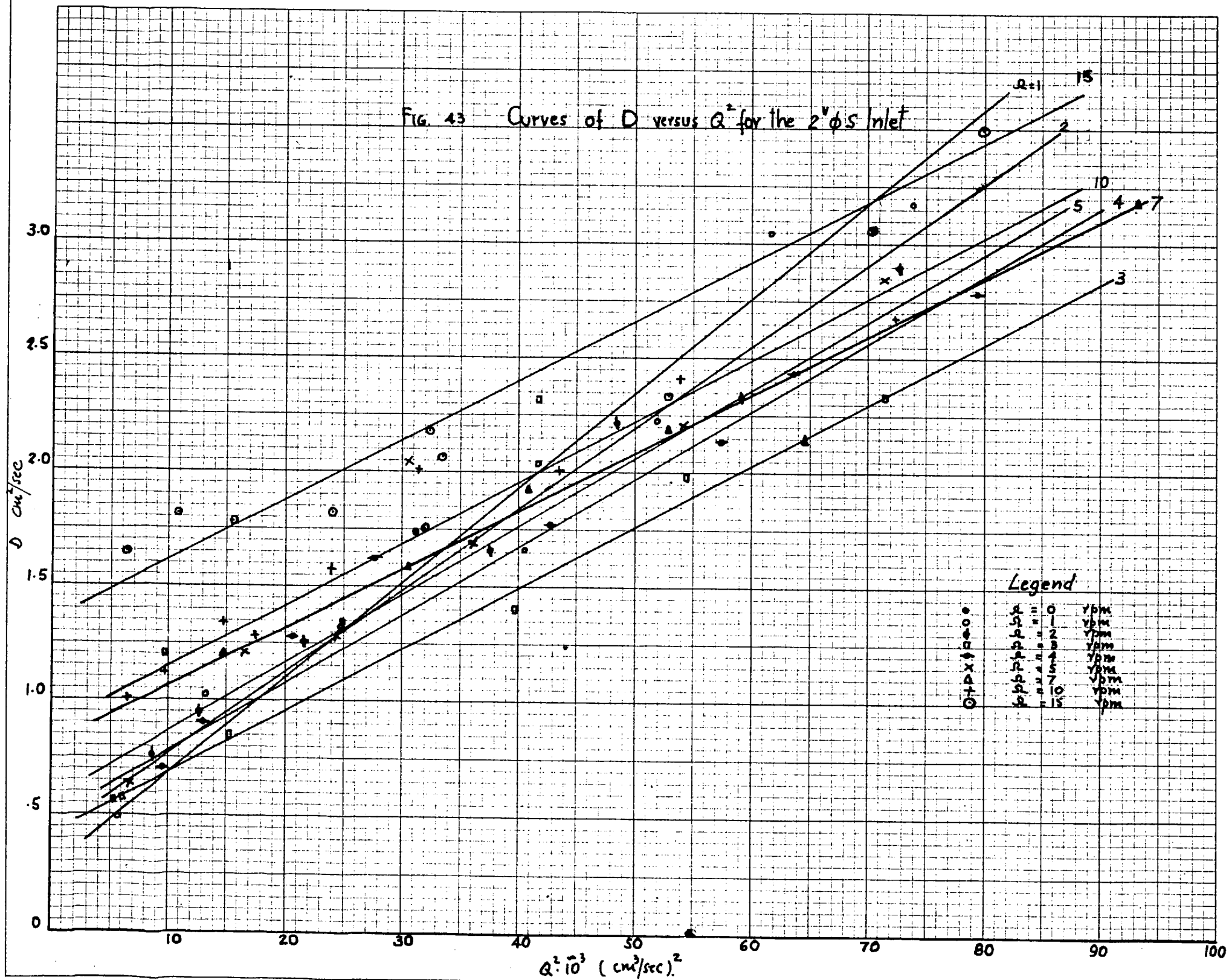
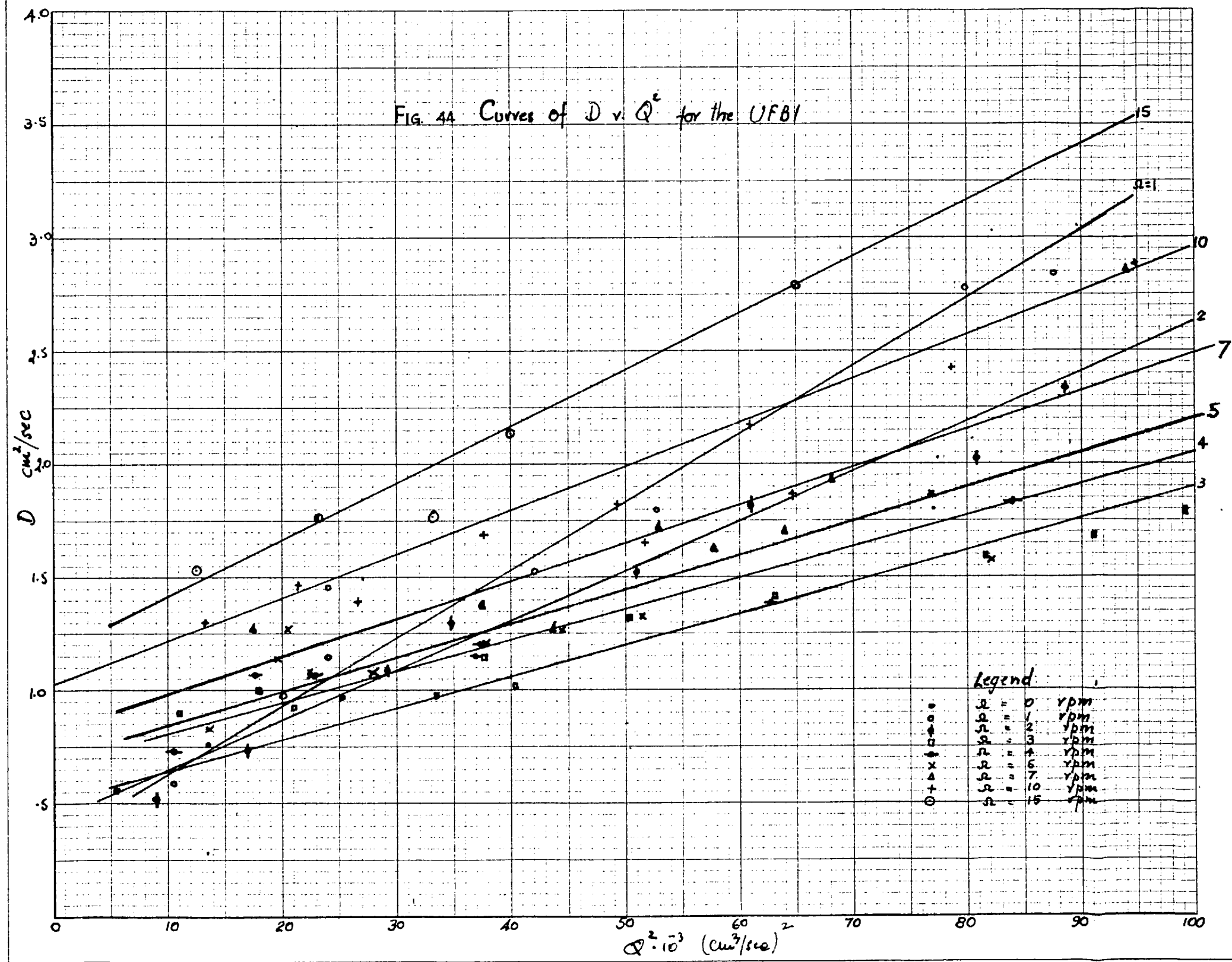


FIG. 44 Curves of D v. Q^2 for the UFB1



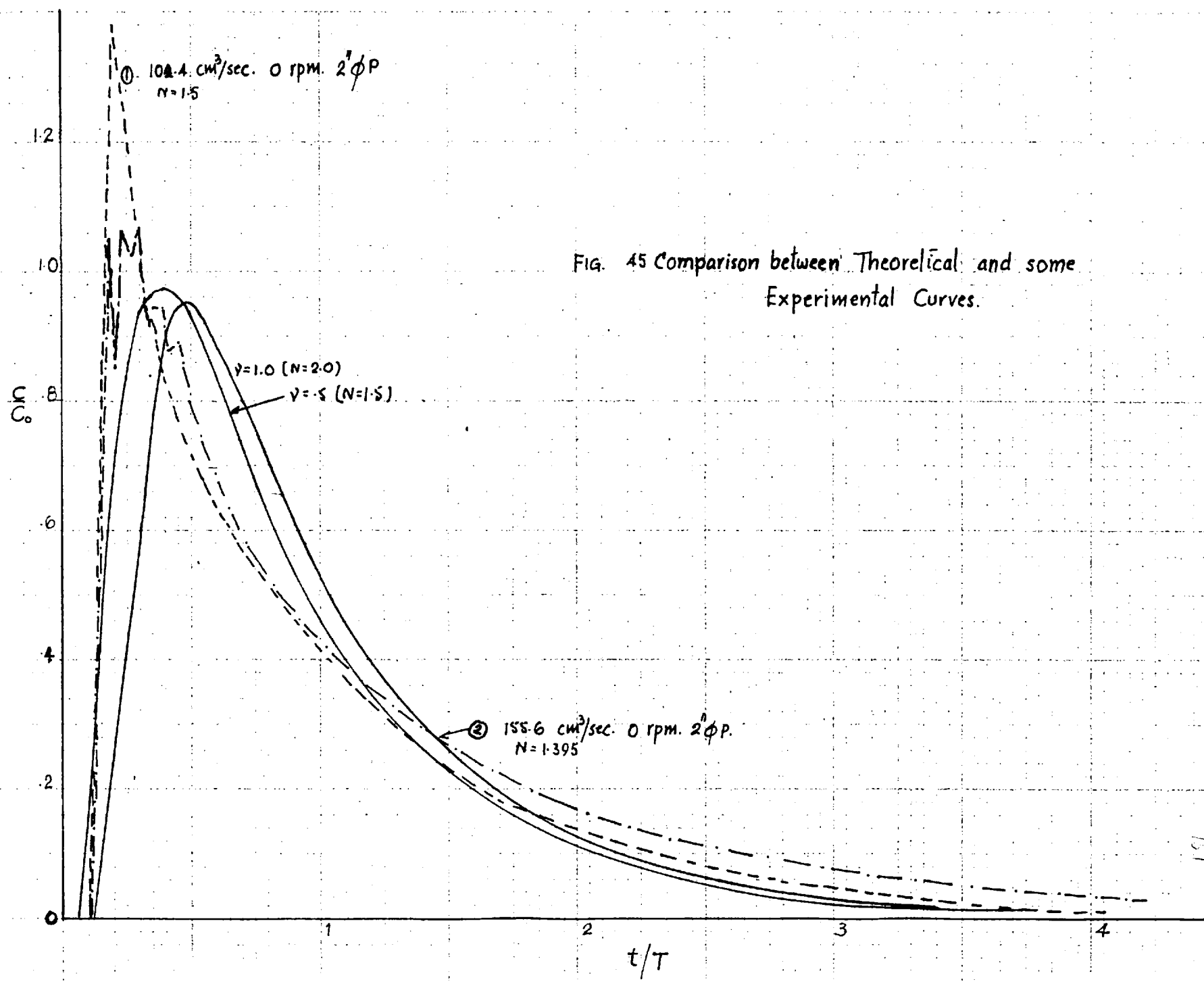


FIG. 45 Comparison between Theoretical and some Experimental Curves.

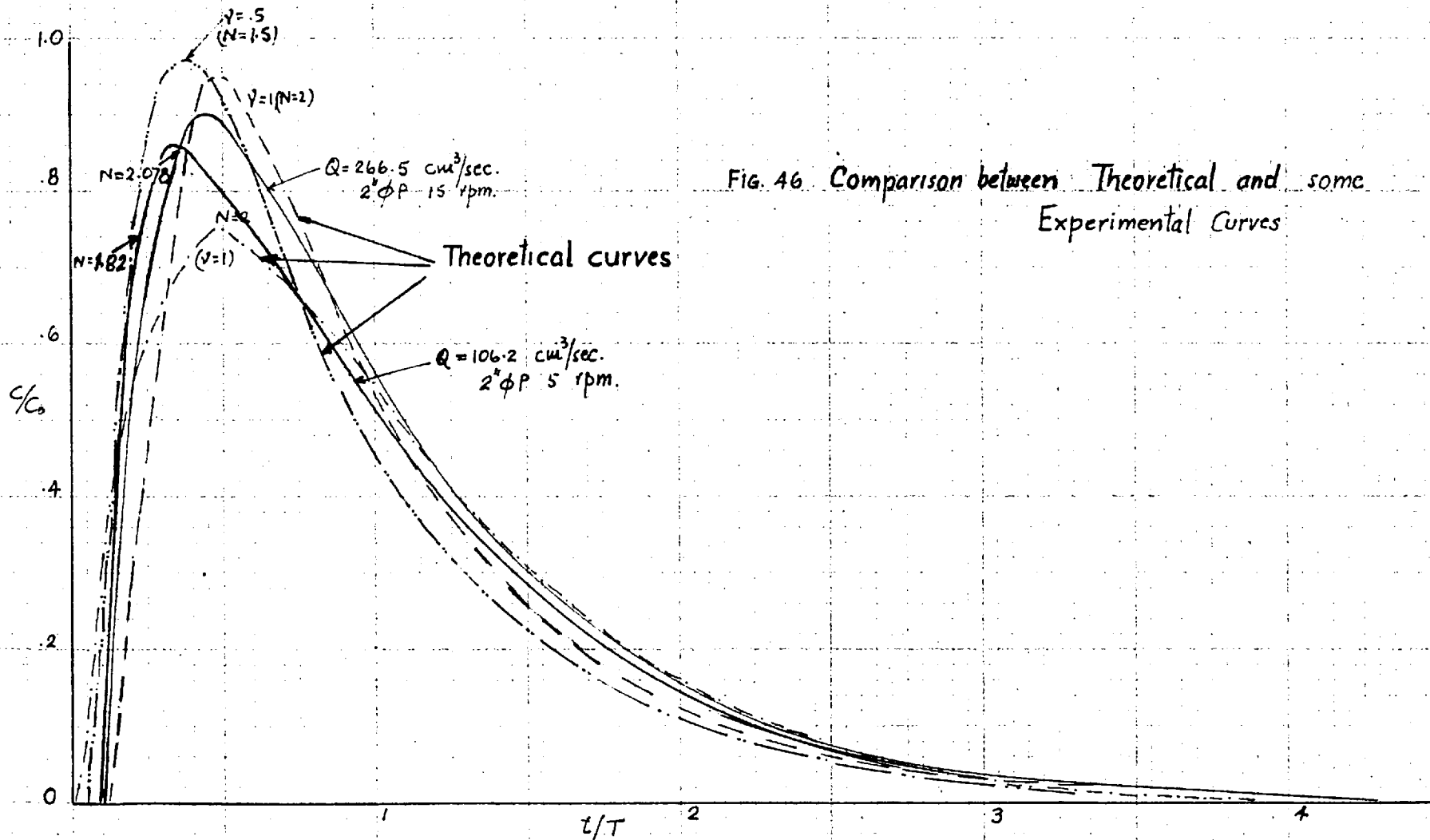


FIG. 46 Comparison between Theoretical and some Experimental Curves

Example of Calculations II

Comparison between 20- and 40- Strip Methods

40-strip			
t/T	c/c_0	$(t/T)(\frac{c}{c_0})$	$(\frac{t}{T})^2(\frac{c}{c_0})$
.05	0	0	0
.15	.230	.034	.005
.25	.580	.145	.036
.35	.700	.245	.086
.45	.760	.342	.154
.55	.770	.423	.233
.65	.750	.487	.316
.75	.700	.525	.394
.85	.640	.544	.462
.95	.600	.570	.541
1.05	.540	.567	.595
1.15	.490	.568	.659
1.25	.440	.550	.687
1.35	.388	.523	.706
1.45	.350	.507	.735
1.55	.310	.480	.744
1.65	.280	.462	.762
1.75	.245	.429	.751
1.85	.210	.388	.718
1.95	.185	.361	.704
2.05	.160	.328	.672
2.15	.140	.301	.647
2.25	.120	.270	.607
2.35	.110	.258	.606
2.45	.095	.233	.571
2.55	.088	.224	.571
2.65	.075	.199	.527
2.75	.070	.192	.528
2.85	.060	.171	.487
2.95	.052	.153	.451
3.05	.050	.152	.464
3.15	.040	.126	.397
3.25	.035	.114	.370
3.35	.030	.100	.335
3.45	.028	.097	.335
3.55	.023	.082	.291
3.65	.019	.069	.252
3.75	.014	.052	.195
3.85	.010	.038	.146
3.95	.008	.032	.126
4.05	.006	.024	.097
4.15	.004	.017	.070
4.25	.001	.004	.017
4.35	0	0	0
Σ	10.406	11.386	18.050

20-strip			
t/T	c/c_0	$(t/T)(\frac{c}{c_0})$	$(\frac{t}{T})^2(\frac{c}{c_0})$
.125	.200	.025	.003
.375	.720	.270	.101
.625	.750	.469	.293
.875	.640	.560	.490
1.125	.500	.562	.632
1.375	.380	.522	.718
1.625	.285	.463	.752
1.875	.205	.384	.720
2.125	.145	.308	.654
2.375	.108	.256	.608
2.625	.080	.210	.551
2.875	.058	.167	.480
3.125	.043	.134	.419
3.375	.030	.101	.341
3.625	.020	.072	.261
3.875	.010	.039	.151
4.125	.005	.021	.087
4.375	0	0	0
Σ	4.179	4.563	7.261

	40-STRIP	20-STRIP	% DIFFERENCE
N	1.862	1.835	1.50
t_g/T	1.092	1.092	0.18
t_w/T	0.912	0.914	0.20
t_{g0}/t_{w0}	6.94	6.56	5.50

$$N = \frac{1}{G^2}; \quad G^2 = \frac{\sum (t/T)(\frac{c}{c_0})}{\sum (\frac{c}{c_0})} - \left[\frac{\sum (t/T)(\frac{c}{c_0})}{\sum (\frac{c}{c_0})} \right]^2$$

Table 8
Preliminary Experiments.

No.	Q cu ³ /sec	Q^{2-3} $\times 10$	N	Remark
1	58.0	3.364	1.664	1" ϕ Riser: Cylindrical Submerged Weir Diffuser. 0 rpm.
2	92.5	8.556	1.580	
3	132.5	18.881	1.484	
4	157.5	24.806	1.442	
5	190.0	36.100	1.504	
6	203.0	41.209	1.445	
7	209.0	43.681	1.365	
8	212.0	44.904	1.596	
9	235.0	55.325	1.267	
10	67.5	4.556	1.845	1" ϕ Riser: Slotted Cylindrical Diffuser. 0 rpm.
11	130.5	17.030	1.731	
12	148.0	21.904	1.886	
13	149.23	22.269	1.780	
14	173.75	31.951	1.597	
15	187.0	34.969	1.443	
16	212.5	45.156	1.506	
17	227.0	51.529	1.248	
18	250.0	62.500	1.480	
19	275.0	75.625	1.455	2" ϕ Riser: Slotted Cylindrical Diffuser 0 rpm.
20	100.8	10.160	1.967	
21	143.0	20.449	1.799	
22	173.0	29.929	1.513	
23	196.0	38.416	1.436	
24	227.0	51.529	1.510	
25	254.0	64.516	1.627	
26	275.0	75.615	1.383	1" ϕ Riser: Slotted Cylindrical Diffuser. 1 rpm.
27	44.35	19.67	2.630	
28	81.20	6.593	2.250	
29	105.0	11.025	2.035	
30	123.5	15.25	2.205	
31	139.0	19.321	2.125	
32	140.0	19.60	2.015	
33	149.23	22.269	1.842	
34	164.4	27.027	2.032	
35	179.0	32.041	1.857	
36	193.55	37.461	1.884	
37	207.0	42.849	1.935	
38	227.0	51.529	1.634	
39	250.0	63.50	1.702	1" ϕ Riser: Slotted Cylindrical Diffuser 5 rpm.
40	59.5	3.540	3.040	
41	92.5	8.556	2.045	
42	155.9	24.305	2.331	
43	201.4	40.561	2.211	
44	234.9	55.084	2.041	
45	254.0	64.561	1.860	
46	273.8	74.966	2.045	

Table 9(a)

Experiments with the 2" ϕ P Inlet

No.	Q cm ³ /sec.	$Q^2 \rightarrow 3$ $\times 10$	Percent Dye Recovery	t_0/T	t_H/T	$(t/T)_{\infty}$	N	D	Remark
1	102.4	10.486	105.4	1.074	.808	.0195	1.780	1.370	0 rpm.
2	155.6	24.211	98.5	0.894	.669	.195	1.560	2.900	
3	178.0	34.684	102.7	1.036	.746	.156	1.480	3.880	
4	203.0	41.209	93.5	0.912	.681	.170	1.401	5.300	
5	257.0	66.049	102.5	1.060	.797	.146	1.300	8.940	
6	302.0	91.204	101.9	1.070	.772	.121	1.260	12.100	
7	102.4	10.486	95.4	0.884	.822	.341	2.500	.670	1 rpm.
8	155.0	24.024	100.0	1.041	.864	.389	2.120	1.215	
9	168.9	35.633	100.0	0.974	.801	.268	2.056	1.866	
10	218.5	47.702	100.4	1.014	.828	.341	1.894	2.550	
11	261.0	68.121	90.0	1.023	.826	.389	1.757	3.600	
12	312.0	97.344	98.8	1.050	.861	.389	1.700	4.600	
13	115.0	13.215	100.6	1.017	.856	.390	2.095	1.09	3 rpm.
14	153.5	23.592	98.0	1.026	.859	.435	2.048	1.536	
15	184.0	33.856	103.3	1.005	.836	.341	1.987	1.945	
16	220.0	48.400	105.6	1.033	.854	.317	1.830	2.771	
17	253.3	64.160	101.4	1.069	.893	.341	1.790	3.342	
18	307.1	94.310	110.0	1.120	.907	.379	1.680	4.720	
19	106.2	11.278	94.6	.935	.771	.391	2.080	1.030	5 rpm.
20	143.5	20.592	100.5	1.036	.892	.488	1.927	1.617	
21	160.1	25.760	102.5	1.014	.847	.449	2.003	1.670	
22	195.9	38.377	103.5	1.048	.870	.435	1.802	2.540	
23	245.7	60.386	100.5	1.024	.840	.390	1.815	3.166	
24	300.75	90.420	100.0	.993	.807	.341	1.904	3.830	
25	117	13.689	101.0	1.078	.913	.411	1.800	1.535	7 rpm.
26	153.5	23.562	104.0	1.052	.872	.440	1.920	1.740	
27	176.0	30.976	101.0	1.021	.852	.440	1.983	1.870	
28	207.0	42.849	103.0	1.036	.868	.391	1.980	2.210	
29	253.0	64.009	104.5	1.068	.892	.440	1.836	3.157	
30	300.0	90.00	100.8	1.054	.874	.440	1.847	3.695	
31	117.9	13.90	105.2	1.035	.856	.442	1.893	1.377	10 rpm.
32	142.4	20.278	106.9	1.038	.854	.442	1.841	1.766	
33	190.2	36.176	106.9	.986	.803	.344	1.891	2.228	
34	226.0	51.076	103.9	1.018	.845	.442	2.008	2.341	
35	260.8	68.016	101.5	.994	.819	.363	1.942	2.887	
36	315.6	99.063	106.9	1.011	.828	.403	1.880	3.740	
37	112.0	12.549	98.2	.926	.817	.398	1.616	1.950	15 rpm.
38	145.0	19.60	100.8	.981	.787	.423	1.820	1.780	
39	160.9	25.889	105.5	.987	.809	.398	1.848	1.979	
40	197.7	39.085	101.5	.989	.813	.417	1.924	2.192	
41	227.1	51.574	99.7	.975	.798	.417	2.087	2.180	
42	266.5	71.022	104.5	1.007	.826	.467	2.078	2.579	
43	322.2	103.813	101.0	.993	.807	.422	1.999	3.368	

TABLE 9(b)
Experiments with the 2" ϕ S.

No.	Q cm ² /sec.	$\frac{Q^2}{10}$	Percent Dye Recovery	t_g/T	t_n/T	$(t/T)_{\Sigma}^2$	N	D cm ² /sec.	Remark.
44	67.0	4.489	105.0	1.067	.661	.711	3.076	.337	0 rpm.
45	137.0	18.769	104.5	.966	.716	.253	1.444	3.25	
46	182.5	33.306	104.5	.992	.748	.243	1.320	5.953	
47	188.0	35.344	103.5	.921	.679	.146	1.301	6.515	
48	210.0	44.100	107.0	1.097	.794	.292	1.325	6.74	
49	248.5	61.752	104.5	.972	.676	.146	1.266	9.750	
50	279.0	73.984	103.5	.967	.728	.46	1.268	10.597	
51	73.5	5.402	100.5	.992	.844	.555	2.554	.493	1 rpm.
52	114.0	12.496	106.0	1.033	.858	.390	2.153	1.032	
53	157.0	24.649	104.0	.974	.813	.219	2.234	1.328	
54	176.5	31.152	106.0	.989	.804	.325	2.055	1.745	
55	204.0	41.616	101.0	.989	.875	.341	1.939	2.267	
56	228.0	51.984	103.0	.945	.754	.195	2.070	2.264	
57	248.5	61.572	103.0	1.006	.822	.316	1.850	3.047	
58	272.0	73.984	109.0	.979	.786	.292	1.914	3.107	3 rpm.
59	73.5	5.402	108.5	1.023	.864	.439	2.336	.574	
60	74.0	5.520	100.5	.972	.811	.390	2.352	.573	
61	97.7	9.545	104.5	1.056	.853	.400	1.948	1.203	
62	122.6	15.031	107.0	.962	.821	.400	2.524	.840	
63	158.0	24.964	106.0	1.04	.851	.487	2.185	1.392	
64	181.5	39.942	105.5	.990	.835	.390	2.351	1.402	
65	204	41.616	105.5	1.058	.868	.350	1.937	2.308	5 rpm.
66	233.0	54.289	102.5	1.024	.835	.400	2.229	1.978	
67	268.0	71.826	104.5	1.022	.833	.400	2.202	2.325	
68	92.0	8.464	102.5	.991	.822	.449	2.255	.765	
69	112.3	10.544	102.5	.975	.806	.439	2.232	.950	
70	146.5	21.462	103.0	1.008	.855	.439	2.220	1.250	
71	173.2	29.998	100.5	1.066	.876	.410	1.842	2.150	
72	193.5	37.423	100.02	.993	.840	.439	2.222	1.653	7 rpm.
73	208.0	43.264	100.15	1.011	.854	.468	2.236	1.756	
74	220.0	48.40	102.0	1.017	.854	.439	2.041	2.209	
75	243.0	59.042	104.8	1.022	.833	.390	2.093	2.334	
76	270	72.90	104.0	1.042	.865	.419	1.980	2.837	
77	122.0	14.884	99.1	.966	.796	.400	2.059	1.205	
78	175.0	30.635	102.5	1.00	.838	.425	2.142	1.595	
79	202.0	40.804	106.9	1.089	.906	.500	2.080	1.920	10 rpm.
80	230.0	52.900	98.8	1.051	.878	.470	2.100	2.180	
81	252.0	64.764	101.1	1.125	.891	.500	2.228	2.145	
82	306.0	93.636	102.4	1.070	.874	.500	2.000	3.190	
83	79.7	6.352	106.0	1.040	.849	.541	1.826	1.007	
84	95.0	9.025	95.7	1.025	.841	.393	1.905	1.129	
85	121.75	14.823	103.2	1.021	.842	.453	1.948	1.340	
86	131.4	17.266	101.5	1.011	.841	.492	2.081	1.268	15 rpm.
87	154.3	23.808	104.0	1.032	.865	.492	2.021	1.577	
88	177.0	31.329	104.0	1.044	.867	.442	1.918	2.013	
89	209.0	43.681	101.5	1.002	.838	.450	2.084	2.011	
90	232.5	54.056	99.2	1.050	.862	.480	2.009	2.403	
91	250.0	62.500	100.5	1.055	.873	.480	2.121	2.328	
92	269.4	72.576	101.9	1.010	.835	.400	2.074	2.616	
93	80.5	6.483	107.0	.990	.770	.388	1.901	.933	4 rpm.
94	104.3	10.878	105.5	1.003	.801	.450	1.995	1.230	
95	124.6	15.529	106.0	1.023	.820	.450	1.795	1.635	
96	155.2	24.087	103.5	.954	.880	.400	2.249	1.297	
97	176.7	31.223	98.0	.987	.799	.400	2.060	1.740	
98	179.87	32.353	103.0	.904	.736	.490	2.385	1.354	
99	182.5	33.306	96.50	.946	.762	.492	2.258	1.514	
100	196.6	38.651	107.0	1.007	.825	.450	2.016	2.108	2 rpm.
101	230.4	53.084	100.8	.961	.817	.380	1.931	2.320	
102	268.2	71.931	104.0	1.024	.830	.477	1.917	3.050	
103	292.23	79.936	107.5	1.054	.853	.447	1.841	3.510	
104	98.86	9.773	101.0	.976	.823	.480	2.455	.708	
105	113.93	12.980	105.8	1.001	.845	.500	2.305	.910	
106	122.0	14.884	93.0	1.00	.801	.360	1.825	1.390	
107	142.43	20.288	106.2	1.003	.870	.450	2.153	1.286	4 rpm.
108	152.0	23.104	105.2	1.00	.807	.400	2.137	1.270	
109	172.47	27.746	109.0	1.041	.852	.450	2.100	1.635	
110	208.25	43.368	102.3	1.016	.862	.440	2.233	1.760	
111	239.50	57.36	106.0	1.055	.858	.450	2.175	2.130	
112	251.25	63.63	103.0	1.010	.860	.350	2.085	2.430	
113	282.0	79.524	103.2	.983	.857	.380	2.057	2.785	
114	92.5	6.806	101.0	.965	.774	.439	2.361	.632	2 rpm.
115	129.0	16.693	104.5	1.004	.836	.390	2.107	1.218	
116	155.0	24.190	106.0	1.045	.793	.361	2.271	1.274	
117	171.5	30.625	104.8	1.025	.845	.429	1.893	2.045	
118	192.0	36.864	103.0	.960	.792	.331	2.169	1.685	
119	233.0	54.289	101.5	.988	.812	.370	2.110	2.190	
120	267.0	71.289	108.5	1.025	.838	.381	1.894	2.840	
121	307.5	94.536	106.0	1.041	.849	.360	1.843	3.800	

TABLE 9(C)
Experiments with The UFBI Inlet

No.	Q cm ³ /sec	Q^2 10 ⁻³	Percent Dye Recovery	t_g/T	t_m/T	$(\frac{t}{T})^2$ $\frac{t_g}{t_m}$	N	D cm ² /sec	Remark
122	108.0	11.664	95.0	1.086	.692	.270	1.872	1.295	0 rpm.
123	143.5	20.592	101.2	.948	.720	.250	1.762	1.970	
124	154.0	23.716	107.0	.976	.708	.270	1.595	2.700	
125	180.2	32.472	103.0	.946	.660	.280	1.583	3.090	
126	210.5	44.310	102.5	1.124	.868	.325	1.483	4.550	
127	246.0	60.516	109.5	1.018	.750	.325	1.495	5.190	
128	295.5	87.320	106.0	1.068	.791	.290	1.258	11.951	
129	102.0	10.404	92.0	.976	.856	.487	2.823	.590	1 rpm.
130	116.0	13.456	101.5	1.015	.856	.487	2.535	.728	
131	142.0	20.164	100.0	1.011	.962	.584	2.516	.978	
132	155.0	24.025	105.9	1.070	.910	.608	2.455	1.155	
133	205.0	42.025	100.3	1.095	.903	.535	2.403	1.525	
134	230.0	52.900	106.2	1.091	.921	.584	2.340	1.790	
135	282.5	79.806	101.0	1.050	.987	.486	2.063	2.795	
136	296.0	87.616	92.3	.993	.849	.584	2.089	2.840	
137	73.5	5.402	106.0	1.030	.877	.536	2.456	.527	3 rpm.
138	105.1	11.025	108.5	1.096	.944	.555	2.155	.905	
139	134.0	17.956	103.5	1.026	.866	.500	2.397	1.005	
140	145.2	21.083	97.2	1.015	.841	.570	2.645	.920	
141	158.5	25.132	102.5	1.017	.879	.530	2.659	.996	
142	183.0	33.489	104.5	1.006	.882	.536	2.951	.979	
143	194.0	37.636	99.9	1.007	.973	.570	2.759	1.150	
144	400.5	40.200	104.0	1.001	.878	.536	3.072	1.010	5 rpm.
145	224.0	50.176	102.5	1.029	.901	.600	2.752	1.335	
146	251.0	63.001	101.0	1.035	.900	.560	2.802	1.452	
147	296.0	81.796	103.0	1.002	.839	.570	2.875	1.595	
148	301.75	91.053	100.0	1.023	.890	.570	2.875	1.683	
149	315.3	99.325	103.5	1.026	.898	.600	2.882	1.749	
150	117.0	13.689	101.0	.993	.879	.537	2.457	.838	
151	140.0	19.600	102.3	1.043	.987	.439	2.280	1.141	7 rpm.
152	143.5	20.592	102.5	1.048	.987	.614	2.361	1.285	
153	150.0	22.500	98.4	1.020	.892	.537	2.436	1.090	
154	167.0	28.056	105.0	1.040	.876	.557	2.634	1.070	
155	194.75	37.927	102.0	1.019	.883	.536	2.674	1.212	
156	210.5	44.310	105.5	1.026	.890	.585	2.751	1.253	
157	227.0	51.529	99.6	1.008	.872	.557	2.796	1.320	
158	278.0	77.284	102.0	1.041	.870	.556	2.558	1.867	10 rpm.
159	286.5	80.082	101.1	1.059	.932	.584	2.544	1.942	
160	83.3	6.939	105.5	1.033	.837	.452	1.914	.952	
161	132.0	17.424	97.6	.995	.813	.438	2.070	1.288	
162	193.8	37.558	105.0	1.013	.863	.535	2.482	1.362	
163	204.16	43.748	106.0	1.036	.898	.556	2.538	1.251	
164	230.0	52.900	110.0	.984	.901	.555	2.539	1.780	
165	240.5	57.840	107.5	1.039	.900	.556	2.538	1.630	15 rpm.
166	253.0	64.009	102.5	1.062	.896	.584	2.554	1.700	
167	261.0	68.121	107.5	1.039	.889	.537	2.400	1.945	
168	308.0	94.864	95.5	1.049	.940	.585	2.125	2.860	
169	115.0	13.225	105.0	.973	.773	.438	1.924	1.300	
170	143.95	20.722	107.0	1.037	.873	.482	2.025	1.463	
171	163.50	26.739	103.0	1.017	.847	.492	2.230	1.388	
172	194.0	37.636	103.0	1.017	.834	.510	2.309	1.675	4 rpm.
173	222.0	49.284	100.9	1.00	.833	.491	2.273	1.819	
174	227.5	51.756	105.0	.997	.813	.491	2.468	1.650	
175	247.0	61.009	105.0	102.5	.859	.540	2.192	2.165	
176	254.25	64.643	102.5	1.021	.863	.511	2.442	1.865	
177	280.0	78.40	106.0	1.050	.889	.491	2.197	2.420	
178	308.0	94.864	106.5	1.054	.860	.540	2.090	2.870	
179	111.8	12.499	110.0	1.095	.899	.511	1.805	1.580	2 rpm.
180	158.0	23.104	98.5	1.000	.781	.428	1.897	1.769	
181	182.6	33.343	99.3	1.045	.862	.492	1.973	1.761	
182	200.0	40.000	97.1	.991	.827	.447	1.978	2.135	
183	255.0	65.025	96.5	1.030	.841	.497	1.953	2.790	
184	296.0	87.616	104.5	1.075	.866	.457	1.717	4.31	
185	308.0	94.860	98.4	1.045	.845	.477	1.762	4.22	
186	101.85	10.373	103.5	.985	.876	.536	2.454	.730	2 rpm.
187	133.0	17.689	107.5	1.041	.882	.537	2.287	1.080	
188	180.0	32.400	104.5	1.00	.865	.557	2.743	1.079	
189	192.5	37.056	99.0	1.039	.887	.614	2.746	1.150	
190	194.0	37.636	99.2	1.013	.832	.537	2.687	1.200	
191	250.5	62.75	101.5	1.013	.882	.556	2.903	1.375	
192	290.0	84.10	100.0	1.009	.875	.487	2.650	1.835	
193	95.15	9.07	1.006	.957	.864	.536	2.869	.517	2 rpm.
194	131.05	17.174	104.1	1.018	.876	.536	2.871	.730	
195	170	29.241	103.5	1.011	.874	.536	2.630	1.095	
196	187.0	34.969	98.0	1.014	.834	.545	2.499	1.303	
197	226.0	51.076	108.0	1.048	.902	.555	2.453	1.625	
198	247.0	61.009	104.5	1.043	.832	.556	2.410	1.830	
199	283.0	80.089	106.0	1.035	.890	.584	2.460	2.020	
200	297.5	88.506	105.2	1.038	.875	.632	2.826	2.340	

TABLE 10THE VALUE OF A IN THE EQUATION $D = A\omega^2 + B$

Speed of rotation (rpm)	THE VALUE OF A (cm. sec. ⁻⁴) for		
	2" Ø P	2" Ø S	UFBI
0	19.4	16.0	10.8
1	4.90	4.14	3.02
2	-	3.55	2.20
3	4.44	2.68	1.40
4	-	3.04	1.38
5	4.07	2.98	1.50
7	3.36	2.32	1.66
10	2.70	2.64	1.92
15	1.86	2.64	2.50

TABLE 11THE VALUE OF B IN THE EQUATION $D = A\omega^2 + B$

Speed of rotation (rpm)	THE VALUE OF B (cm. sec. ⁻¹) for			
	2" Ø P	2" Ø S	UFBI	REMARK
0	0.00	0.00	0.00	
1	0.30	0.28	0.32	
2	-	0.42	0.42	
3	0.50	0.42	0.50	
4	-	0.45	0.67	
5	0.65	0.57	0.70	
7	0.85	0.82	0.82	
10	1.15	0.90	1.02	
15	1.35	1.34	1.17	

4.6. Discussions.

4.6.1. The Effects of the Forced Vortex on the Tank Hydraulic Characteristics.

The effects of rotation on the flow conditions in the tank were evident in the difference between the shapes of the flow through curves with and without rotation. When there was no rotation, the curve had a high peak, $\hat{\xi}_c$ being of the order of 1.4 at t/T of about 0.25, which was often followed by a smaller peak. With rotation, the curve was smooth, and there was only one peak, $\hat{\xi}_c$ of the order of 0.8 at t/T of about 0.45.

The occurrence of a high peak at comparatively small t/T followed by a smaller one shortly after was attributed to severe short-circuiting due to stagnant spaces and non-uniform velocity distribution. Dye was carried through the tank by fast stray currents. Some of it was deflected by the periphery wall and travelled towards the centre to be mixed further with the incoming flow.

When a forced vortex was superimposed, these stray currents could only carry the dye outwards until they came to a certain radius where the centrifugal force balanced the pressure head causing the stray currents. If a fluid particle was pushed further outwards, it would come into a region of a higher speed of rotation than its own. The centripetal force of this region would be greater than the centrifugal force the fluid particle possessed. This unbalanced force would push the particle inwards. On the other hand, if a fluid particle was carried inwards by disturbances, its centrifugal force was greater than the centripetal force of the fluid in

this region. It would therefore be similarly forced back to its original position. A fluid particle could only travel through the tank by the building up of pressure head at the centre of the tank due to the incoming liquid. This build-up in pressure would push the fluid outwards into the new region where the increase in the centripetal force balanced the increased pressure head. Further addition of the incoming flow meant further building up of pressure, which in turn pushed the fluid further outwards. This process would go on and on and represented the way the fluid travelled through the tank. The superimposed forced vortex rotated the fluid as it flowed outwards and resulted in a spiral flow.

It would be interesting at this point to refer to the works of Walton and Key at Alexandria, 1939. During one of their tests, a circular tank was adapted so that the flow came into the tank tangentially at the periphery and left the tank over a circular weir at the centre. The liquid had a higher speed of rotation at the centre of the tank than at the periphery. This was the opposite of the condition of flow in the present investigation. Under such rotation - free vortex - any fluid particle which was pushed outwards by disturbances would come into a region of slower speed of rotation. The centrifugal force of the fluid particles was greater than the centripetal force of this region. As a result, the fluid particles would travel further outwards. In the same manner, any fluid particle which travelled inwards due to disturbances would be carried further inwards. The system was therefore unstable, and there was no suppression of turbulence. The above discussion explains why, under such an arrangement, Walton and Key

obtained a result inferior to those obtained in another design of tank in which the fluid was drawn off not at the centre, but at the periphery. Under this arrangement, the speed of rotation was greater at the periphery and decreased inwards, as in the present investigation.

4.6.2. Comparison between the theoretical and experimental flow through curves.

The comparison between the theoretical and experimental curves showed that the flow through curves agreed more closely with theory with rotation than without. This was attributed to the inhibition of short-circuiting due to stagnant spaces and non-uniform velocity distribution (see also Section 4.3.) by the forced vortex. Further examination of the curves showed that the peak concentration of a theoretical curve always lagged behind that of an experimental curve, and the agreement between the rising parts of the two curves was not so good as that between the falling parts of the curves (Figs. 45 and 46). In general, the relation $\frac{C}{C_0} = \left(\frac{N}{N_0}\right)^{\frac{N_0}{N}} e^{\frac{N_0 - N}{T}}$ was found to describe the curve well, even though the experiments were carried out in the region of low N value, where it is only an approximate equation of the flow through curve. It was evident in Figs. 38 - 40 that N calculated from the variance was more accurate than N calculated according to Thomas and Archibald's suggestion (see also Section 4.5.1.).

The theoretically derived flow through curve (the y -formula, eq. 24, Section 2.1.) agrees better with the experimental curves than the N curve, both in the rising and the falling parts of the curve. The agreement was best at $\Omega = 15$ rpm.

4.6.3. The Effects of the Forced Vortex on the Diffusion Coefficient.

The analytical study made earlier (Chapter 2) indicates that the diffusion coefficient, D , will be decreased by the superimposition of a forced vortex, and a decrease in D means an increase in N . Furthermore, dimensional analysis shows that there is a relationship between N and the Froude number of the tank.

The curves of N v. Q^2 (Figs. 35 - 37) verified the above analysis.

Three families of curves, having Ω as the parameter, were obtained for the three inlets employed in this investigation and were found to have similar shape. When there was no rotation, N increased as Q^2 decreased. This is in accordance with the analytical and physical considerations. At the limit when Q^2 was equal to zero, that is no flow in the tank, the state of quiescence was reached because there was no turbulence, and N was seen to approach infinity. N seemed to decrease rapidly at first, but decreased more slowly at a flow rate greater than about 200 cc./sec. This limiting flow rate varied with the type of inlet.

With the superimposition of a forced vortex, curves of N v. Q^2 of similar shape to the ones without it were obtained when the speed of rotation of the paddles was low. There appeared to be a critical flow rate for every speed of rotation. This was the flow rate below which N was found to be smaller with a forced vortex than without. The value of the critical flow rate, $Q_{crit.}$, depended upon the speed of rotation, being small with a small speed. With the angular speed of 1 rpm. there appeared

to be no critical flow rate. This was probably because at low speed of rotation the critical flow rate might occur at a very low flow rate, at which the tank was unstable, and it was difficult to obtain repeatable results. It should be mentioned here that at a flow rate lower than about 80 cc./sec., the whole of the periphery weir was not used. Water flowing over the weir was observed to break into patches because of the surface tension. The tank was therefore unstable and the flow non-uniform.

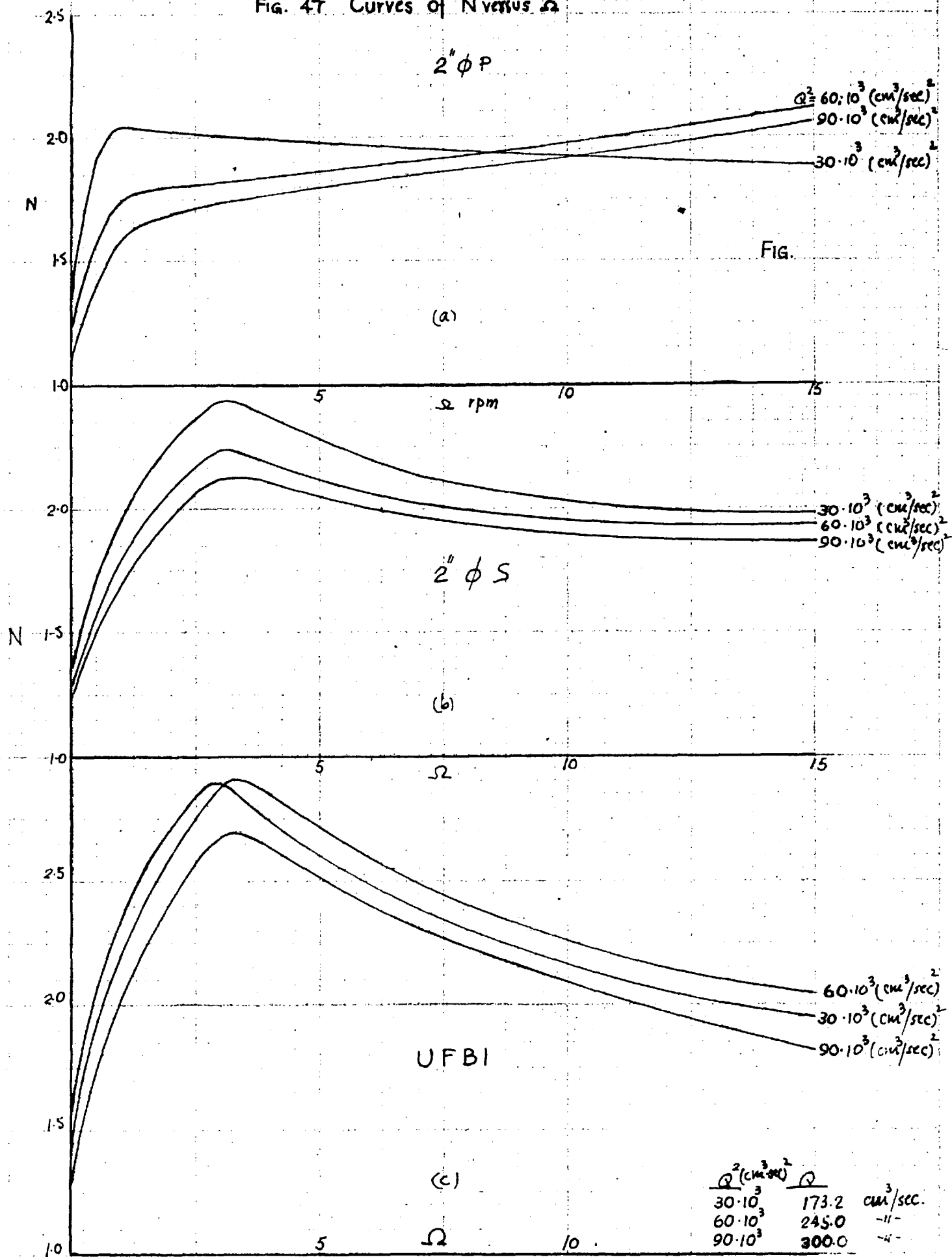
The existence of critical flow rate could be explained by referring to the setting up of secondary currents and the production of turbulence by the paddles. At a flow rate smaller than the critical flow rate, the mixing effects of the secondary currents and the paddles turbulence were greater than the suppression of inlet turbulence and non-uniformity in flow distribution by the forced vortex, as described in 4.6.1. The net result was that N was less than when there was no forced vortex. The fact that the critical flow rate was greater with higher speed of rotation substantiates the argument. The mixing effects of the secondary currents and the paddle turbulence will be later proved to be dependent upon the speed of rotation (see Section 4.6.6.).

For any speed of rotation there was also an optimum flow rate at which the value of N was highest. (See also Section 4.6.4.).

4.6.4. The Variation of the Tank Characteristic Number N with the Tank Flow Rate.

The curves of N v. Q^2 shows that N was not increased indefinitely

Fig. 47. Curves of N versus Ω



with the increase in the speed of rotation. This is a departure from the analytical study. It must be remembered, however, that in this study the effects of paddle turbulence and the mixing due to secondary currents had been neglected to render the problem mathematically soluble. It appeared that 3 rpm. was the speed of rotation at which the highest value of N could be obtained with the 2" $\varnothing$ S and the UFBI inlets. For the 2" $\varnothing$ P inlet 15 rpm. was found to give the highest value of N . The reason for this will be explained in 4.6.5. The greatest increase in the value of N was with the UFBI inlet where N was raised from 1.44 at 0 rpm. to 3.03 at 3 rpm. at the flow rate of 212 cc./sec. (The designed flow rate for the inlet was 216.5 cc./sec.).

The curves of N v. Q^2 were then reconstructed with N now plotted against Ω for the flow rates of 174, 245, 300 cc./sec., as shown in Figs. 47 (a), (b) and (c), in order to show the effects of rotation on the tank N number.

It is evident that N was decreased for Ω greater than 3.5 rpm. in the case of the 2" $\varnothing$ S and the UFBI inlets. For the 2" $\varnothing$ P the effect of Ω was similar to the other two inlets for a low flow rate through the tank. However, at a high flow rate N appears to increase with the increase in Ω . The reasons for this difference in behaviour are given in Section 4.6.5.

4.6.5. Behaviour of the Three Inlets.

The reason for using these three different inlets was as follows:

The first two inlets, that is the 2" ϕ P and the 2" ϕ S, did not introduce the flow uniformly throughout the depth of the tank, and, in the case of the first inlet, the flow was directed against the bottom of the tank and tended to flow along the floor. The flow would therefore counter-balance the secondary flow along the floor which was towards the centre. As a result, the mixing capacity of the secondary flow was hindered. This inlet should give high N number at a higher speed of rotation and flow rate. In the case of the 2" ϕ S, there was a tendency for the flow to follow the tank floor as in the first case, but, as it was not directed against the floor, the secondary currents could inhibit the outward flow and make the liquid flow more towards the upper portion of the tank. This action of the secondary currents was very noticeable at a high speed of rotation (Fig. 34, and see also Section 4.4.).

The UFBI inlet introduced the flow uniformly throughout the depth of the tank. The flow of water in the upper half of the tank would tend to promote the secondary currents, being in the same direction, while that in the lower half would tend to inhibit them. There was therefore no change in the nature of the secondary currents, and at a high speed of rotation water could be seen being forced to flow more towards the top. This inlet, like the 2" ϕ S, should also give a low N value at a high speed of rotation.

The study of the shapes of the N v. Q^2 curves made in 4.6.4. indicated that the above reasonings were correct.

4.6.6. The Variation of the Diffusion Coefficient with the Tank Flow Rate.

When D was plotted against Q^2 , three families of straight lines were obtained for the three inlets (Figs. 41 - 44). They could be represented by an equation of the type:

$$D = AQ^2 + B \quad (1)$$

where A and B are constants depending upon the inlet and the speed of rotation.

Fig. 41 showed that when there was no rotation B was equal to zero and (1) becomes

$$D = AQ^2 \quad (2)$$

This equation agrees with the physical conditions in the tank, because when Q is zero, D should be zero also; that is the quiescent condition. On the other hand, when Q approaches infinity D also approaches infinity, which is also physically correct. Comparison made among the three inlets showed that A was least with the UFBI and greatest with the 2" ϕ P. As Q and D appear to be dependent upon the kinetic energy of the flow before and after the inlet, A can be regarded as a measure of the effectiveness of an inlet, being small with an efficient inlet.

When a forced vortex was superimposed, the constants A and B in (1) appeared to be dependent upon the speed of rotation and the type of the inlet. For any inlet, the value of A measures the effectiveness of the superimposed forced vortex; the smaller the value of A the more

effective the forced vortex. 3 rpm. was found to be the optimum speed for the suppression of D for the 2" Ø S and the UFBI inlets, while 15 rpm. was the best speed for the 2" Ø P. The values of A and B were tabulated in Tables 11 and 12, p. 168.

The values of B's are of great interest because they are the measure of the turbulent mixing caused by the rotation, that is due to the paddles as well as the secondary currents. B is the value of D at the point where Q^2 is zero, that is, the value of D where a straight line in Figs. 41 - 44 cuts the D-axis and is therefore independent of the inlet. The values of B obtained from the experiments with the three inlets at different speeds of rotation were shown in Table 11. They were found to agree well.

These facts help substantiate the analytical study and the reasonings with regard to the effect of the paddle turbulence and the secondary currents (Section 4.6.3.), and the existence of critical and optimum flow rates.

4.6.7. The Ratio of the Arithmetic Mean Time to the Theoretical Detention Time.

The examination of the experimental results obtained from the three inlets indicates that $\frac{\bar{t}_e}{T}$ does not vary markedly from unity. This is in accordance with the analytical study which indicates that $\frac{\bar{t}_e}{T}$ will be equal to unity regardless of the turbulent condition in the tank. The results also showed that there was no appreciable stagnant portion of liquid

in the tank.

It appears that $\frac{t_g}{T}$ has a connection with the percentage of tracer recovery. With the help of the flow through cell, continuous measurement of dye concentration until no trace of it ^{was left} in the effluent was possible. It must be admitted, however, that at the tail end of the curve, where the concentration was always low, the error in the concentration measurement was large, and air bubbles and dust in water always help in making this error on the positive side. Although the concentration was small at the tail end, its moments about the origin were large because of the large order of the abscissa. As a result, the positive error in concentration measurement tended to pull the ratio to the right.

4.6.8. The Ratio of the Median to the Theoretical Detention Times.

The experimental results indicate that the ratio of $\frac{t_M}{T}$ is raised by the superimposed forced vortex, and there appears to be a tendency for it to be small at a very low or a very high flow rate. As $\frac{t_M}{T}$ is a function of N , it is expected to have a relationship with Q^2 similar to that between N and Q^2 . A closer examination of the results reveals that for the 2" Ø S and the 2" Ø P inlets $\frac{t_M}{T}$ may be regarded as independent of Q^2 for any speed of rotation of the paddles. $\frac{t_M}{T}$ is raised from about 0.7 to about 0.84 by the superimposed forced vortex, the speed of which does not seem to influence it.

For the UFBI inlet, the tendency for the ratio to be small at very low and very high flow rate is more marked. Again the ratio is

definitely raised by the superimposed forced vortex. At $Q = 245$ cc./sec.

$\frac{t_H}{T}$ is raised from 0.785 to 0.92 at $\Omega = 1$ and 15 rpm. respectively.

CHAPTER 5

CONCLUSIONS AND SUGGESTIONS FOR FURTHER STUDY

5.1. Conclusions Drawn from the Analytical Study.

5.1.1. The Theoretical Flow Through Curve.

The use of the diffusion type approach to study the mixing of the liquid in a continuous flow circular sedimentation tank results in an equation describing the flow through curve of the tank as follows:

$$\frac{C}{C_0} = \sum_{n=1}^{\infty} \frac{2UT\Gamma^{\gamma}}{a^{\gamma}} \left(-\frac{J_0(\alpha_n)}{2}\right) e^{-D\alpha_n^2 t} \left[J_{\gamma-1}(\alpha_n b) y_{\gamma}(\alpha_n r) - J_{\gamma}(\alpha_n r) y_{\gamma-1}(\alpha_n b) \right] \times \left[\frac{J_{\gamma-1}(\alpha_n b) \left\{ J_{\gamma-1}(\alpha_n a) - \frac{U}{D\alpha_n} J_{\gamma}(\alpha_n a) \right\}}{\left\{ J_{\gamma-1}(\alpha_n a) - \frac{U}{D\alpha_n} J_{\gamma}(\alpha_n a) \right\}^2 - J_{\gamma-1}^2(\alpha_n b)} \right] \quad (1)$$

and for $r = b$

$$\frac{C}{C_0} = \sum_{n=1}^{\infty} \frac{2UTb^{\gamma-1}}{a^{\gamma}} e^{-D\alpha_n^2 t} \left[\frac{J_{\gamma-1}(\alpha_n b) \left\{ J_{\gamma-1}(\alpha_n a) - \frac{U}{D\alpha_n} J_{\gamma}(\alpha_n a) \right\}}{\left\{ J_{\gamma-1}(\alpha_n a) - \frac{U}{D\alpha_n} J_{\gamma}(\alpha_n a) \right\}^2 - J_{\gamma-1}^2(\alpha_n b)} \right] \quad (2)$$

where α_n is the n th root of

$$\frac{D\alpha_n J_{\gamma-1}(\alpha_n b)}{D\alpha_n J_{\gamma-1}(\alpha_n a) - U J_{\gamma}(\alpha_n a)} = \frac{D\alpha_n y_{\gamma-1}(\alpha_n b)}{D\alpha_n y_{\gamma-1}(\alpha_n a) - U y_{\gamma}(\alpha_n a)} \quad (3)$$

To evaluate (1) the values of α_n , $n = 1, 2, 3, \dots$ were needed, and equation (3) must first be solved. (3) involved Bessel functions of the first and second kinds of the order γ and $\gamma-1$. It was therefore rather tedious to obtain α_n from it. The problem was then reconsidered with $a = 0$ and a new solution was obtained with the help of a Green function, i.e. by first obtaining the solution for instantaneous injection of dye

in an infinite medium and combining this solution with the one that is valid for the boundary condition at $r = b$. In this manner, the required solution was found to be:

$$\frac{c}{c_0} = \sum_{n=1}^{\infty} \frac{(\alpha_n b)^3 e^{-\frac{(\alpha_n b)^2 t}{T}}}{2^3 \gamma! J_\gamma(\alpha_n b)} \quad (5)$$

where α_n is now the n th root of

$$J_{\gamma+1}(\alpha_n b) = 0 \quad (6)$$

(4) reduces to $\frac{c}{c_0} = e^{-\frac{t}{T}}$ when $\gamma = 0$ (i.e. $D = \infty$). Comparing the time of the maximum concentration and the variance of (4) with those of the equation $\frac{c}{c_0} = \frac{N^N}{(N-1)!} \left(\frac{t}{T}\right)^{N-1} e^{-N\frac{t}{T}}$ it was found that in general

$$N = \gamma + 1 \quad (7)$$

N , the number of the imaginary cells constituting a sedimentation tank, which was introduced by Thomas and Archibald as a parameter measuring the hydraulic efficiency of a tank had thus been shown to be dependent upon the flow rate through the tank and the tank diffusion coefficient.

The ratio $\frac{t_R}{T}$ had been found to be independent of N while the relationships between $\left(\frac{t}{T}\right)_{\frac{c}{c_0}}$ and $\frac{t_R}{T}$ and N are as follows:

$$\begin{aligned} \left(\frac{t}{T}\right)_{\frac{c}{c_0}} &= \frac{N-1}{N} \\ \frac{1}{2} &= \frac{N^N}{(N-1)!} \left[\frac{1}{N} \left(\frac{t_R}{T}\right)^{N-1} e^{-N\frac{t_R}{T}} + \frac{N-1}{N^2} \left(\frac{t_R}{T}\right)^{N-2} e^{-N\frac{t_R}{T}} + \frac{(N-1)(N-2)}{N^3} \left(\frac{t_R}{T}\right)^{N-3} e^{-N\frac{t_R}{T}} + \dots \right. \\ &\quad \left. + \frac{1}{N^N} \{(N-1)(N-2) \dots 1\} e^{-N\frac{t_R}{T}} + \frac{1}{N^N (N-1)!} \right] \quad (8) \end{aligned}$$

5.1.2. The Diffusion Coefficient.

Lagrangian analysis of diffusion was used to study D which was found to be equal to

$$\frac{1}{2} \overline{u'^2} \int_0^T R_L^u dt \quad (9)$$

when the time was large

$$D = \frac{1}{2} \overline{u'^2} T_L = \text{Constant} \quad (10)$$

D can also be regarded as the product of the intensity and the scale of the turbulent mixing process and can be written as

$$D = L_M \sqrt{\overline{u'^2}} \quad (11)$$

where L_M is called the diffusion length. D is therefore analogous to the turbulent mixing coefficient which is equal to $\ell \sqrt{\overline{u'^2}}$. This analysis provides two informations:

(1) By tracing the flow through curve of dye in a sedimentation tank, a number of parameters measuring the hydraulic performance of the tank can be derived. These parameters depend upon D which is the turbulent diffusion coefficient of dye in water.

(2) The diffusion coefficient depends upon the radial fluctuation and the Lagrangian integral scale.

5.1.3. The Effects of the Forced Vortex on the Diffusion Coefficient.

From the Reynolds turbulent stresses and turbulent energy equations it has been shown that:

(1) When only the outer paddles rotate, there will be a suppression of $\overline{u'^2}$ resulting in a decrease in D . N is therefore increased

because D is inversely proportional to it.

(2) When only the inner paddles rotate, $\overline{u}^2$ is increased, resulting in an increase in D and a decrease in N .

(3) When both sets of paddles rotate in the same direction, there will be an increase in D near the inner paddles, while the opposite takes place near the outer. The net effect will depend therefore upon the relative speeds of the two.

5.1.4. The Dimensional Analysis.

The problem has also been analysed dimensionally. It is found that

$$\frac{D}{V_2} = N^{-1} = f\left(\frac{V_2 \rho}{\mu}, \frac{V^2}{r_2 g}, \frac{\omega_1 V}{r_1 g}, \frac{\rho D_1}{\mu}, \frac{\omega_1}{\omega_2}, \frac{r_1}{r_2}, \frac{r_1}{d}\right) \quad (12)$$

which on simplification becomes

$$\frac{D}{V_2} = N^{-1} = f\left(\frac{V^2}{r_2 g}, \frac{\omega_1 V}{r_1 g}\right)^* \quad (12a)$$

The above equation indicates therefore that N has a relationship with the Froude number and the dimensionless term $\frac{\omega_1 V}{r_1 g}$. In other words, if N is plotted against Q^2 , a family of curves will be obtained having $\frac{\omega_1 V}{r_1 g}$ as the parameter.

* See Note opposite p. 3.

5.2. Conclusions Drawn from the Experimental Study.

5.2.1. The Effects of the Superimposed Forced Vortex on the Diffusion Coefficient.

The present investigation^{was} confined experimentally to the case of $\frac{\Omega_i}{\Omega_c}$ and $\frac{r_i}{d} = \infty$ and $\frac{3}{4}$ respectively. Three types of inlets were used, and the curves of N v. Q^2 obtained from them are found to be similar in shape. They are considered as the verification of the dimensional analysis.

The curves of D v. Q^2 , D being obtained from N by the relation

$$N = \gamma + 1 = \frac{K}{2D} + 1 \quad (1)$$

have also Q as the parameter. They are considered as the verification of the dimensional analysis as well as the justification of the diffusion type approach to the problem. These curves can be represented by

$$D = A Q^2 + B \quad (2)$$

where A and B are constants depending on the inlet, the speed of rotation and the turbulent diffusion caused by the paddles and the secondary currents. A can be further regarded as the measure of the effectiveness of the inlet and the superimposed forced vortex.

These curves show clearly that the forced vortex lowers the value of D and increases the value of N .

5.2.2. The Effects of the Superimposed Forced Vortex on the Tank

Characteristic Number N .

When there is no forced vortex superimposed, the hydraulic performances of the three inlets are practically the same, having a high N value at a low flow rate. At a high flow rate the curves tend to be parallel to the Q^2 axis.

When the paddles are in rotation, the resultant flow through curves change in their characteristics. The occurrence of high peak concentration at a relatively short time compared with the theoretical detention time and long persistence of low concentration are minimized. The peak concentration takes place at a higher $\frac{t}{T}$ and is smaller in magnitude. The increase in the N value is greatest in the case of the uniform flow inlet, and least in the case of the submerged weir inlet. In general, the N values are low at low flow rate and very high flow rate, and they are not increased indefinitely with the increase in the speed of rotation. There appears to be a certain speed beyond which they are decreased instead of increased. This is called the optimum speed of rotation of the paddles.

5.2.3. Optimum Speed of Paddles Rotation.

3 rpm. seems to be the optimum speed of rotation of the paddles. At this speed highest N values are obtained with the UFBI and the 2" $\emptyset$ S inlets. With the 2" $\emptyset$ P, N is increased to the greatest extent at 15 rpm.

5.2.4. The Critical Flow Rates.

Because of the existence of the secondary currents associated with the rotating flow above a fixed ground, and the production of turbulence by the paddles, there is a critical flow rate below which N values are less with rotation than without. The value of the critical flow rate depends upon the speed of rotation and the type of the inlet. An optimum flow rate also exists, and its value too depends upon the speed of rotation and the type of inlet. They increase with the speed of rotation.

5.2.5. The Advantages and Disadvantages of Using Various Parameters Characterizing Short-Circuiting.

From the experimental and analytical study in the present investigation, the following conclusions may be drawn regarding the use of various parameters by investigators in the past:

(a) The use of $\frac{t_c}{T}$ as a parameter measuring short-circuiting is inadequate because it is not influenced by the mixing condition in the tank. Its determination is also very much affected by the condition at the tail end of the flow through curve where the accuracy in concentration measurement is poor. It is at any rate useful in the examination of experimental results. The results obtained in the present investigation do not show that the ratio varies markedly from unity, and there appears to be no effect of the forced vortex upon it.

(b) $\frac{t_M}{T}$ is better than $\frac{t_c}{T}$ in measuring short-circuiting, since it varies with the mixing condition in the tank as well as revealing the

presence of any stagnant portion of the tank. However, it is not so sensitive as γ and N , which should be obtained from the variance of the flow through curve. In the present investigation, it is found that $\frac{t_H}{T}$ is increased by a superimposed forced vortex. This indicates the lengthening of the average detention time of a fluid particle.

(c) The use of $\left(\frac{t}{T}\right)_{\frac{t}{T_0}}$ and $\left(\frac{t}{T}\right)_L$ as the parameters indicating serious short-circuiting due to non-uniform velocity distribution and secondary currents are not suitable because they cannot be accurately determined. Besides, on examining the experimental flow through curves, it appears that they do not vary markedly between cases where the degrees of short-circuiting are different. The existence of short-circuiting due to non-uniform velocity distribution and secondary currents is best revealed by comparing the experiment and theoretical curves. It is evident that the superimposed forced vortex minimizes this kind of short-circuiting.

5.2.6. Secondary Currents.

The deposited particles at the tank floor are observed to move towards the centre and gather in a ring surrounding the inlet. This phenomenon established further the existence of secondary currents and agrees with the observations and experiments of previous investigators in the field of secondary flow. It is possible that, in practice, the scraping mechanisms may be found unnecessary.

5.3. Suggestions for Further Study

The superimposition of a forced vortex on the flow of a circular sedimentation tank as a means of improving its flow characteristics is novel, and it is believed that the present investigation has opened up a new field of research in sedimentation tank design. It is obvious that not all the problems arising as a result of this attempt can be dealt with in a single study. From the experience and the result of experimental and analytical study of this kind of sedimentation tank, it is felt that the following may represent some of the problems worth studying.

5.3.1. Inlet Design and Inlet Number

The introduction of circular motion into the flow in the tank requires a new kind of inlet which should not only introduce the flow uniformly into the tank, but also tangentially in such a way that the incoming flow is caused to join the main flow in the tank with a minimum degree of turbulence. The application of the manifold flow theory in the design of an inlet appears to be successful as far as uniform distribution of flow is concerned. The problems now lie in the positioning and designing of baffles which will destroy the kinetic energy of the incoming flow as well as directing it tangentially into the tank. It may be possible also that an entirely new approach to the design problem is called for.

After the problem of designing an inlet comes naturally the question of finding a parameter to characterize it. To evaluate the performance

of an inlet it is a usual practice to obtain a flow through curve of a sedimentation tank with that inlet, and by obtaining several such curves for different types of inlets, their performances can be compared. The dimensional analysis shows that N has a relationship with the tank Froude number, and the experimental results reveal that each inlet has its own family of curves. It seems possible therefore to include an 'inlet number' into the dimensional analysis to complete the picture. The modified dimensionless equation now becomes:

$$N = F \left(Fr, \frac{\omega^2 r}{g}, I \right)$$

where I is the 'inlet number'. But what is an inlet number?

The kinetic energy of the influent seems to be very important to the performance of a sedimentation tank. Bloodgood, Boegly and Smith (1956) experimented with a rectangular sedimentation tank and found that there is a relationship between the kinetic energy of the influent and the removal efficiency of the tank. This, together with the fact that

$$D = A Q^2 + B$$

obtained in this research make it reasonable to propose that the inlet number be defined as the ratio of the kinetic energy of the incoming liquid and the liquid leaving the inlet zone. It is therefore a measure of the efficiency of an inlet in destroying the kinetic energy of the influent and appears to be related with A .

The use of the suggested inlet number would permit the evaluation of the performance of the inlet independently of the performance of the tank.

5.3.2. The Design of an Effective Means of Rotating the Tank Content

One inherent drawback of using paddles as the means of rotating the liquid in the tank is the production of turbulence. Number, shape and positioning of the paddles are among those factors which have to be studied. The analytical study shows the possibility of having two sets of paddles with different effects on the value of the diffusion coefficient.

5.3.3. Experiments with Particulate and Flocculent Particles

There is no reason to believe that a tank with a high hydraulic efficiency will not give also a high removal efficiency. Experiments with flocculent particles should prove very interesting because of the existence of shear forces due to the superimposed forced vortex. It is well known that shear forces up to a certain magnitude help ⁱⁿ forming flocs (Camp, 1946) and that the settling of flocculent particles depends upon the detention time.

5.3.4. Experiments with Various Ratios of the Radius to the Depth of the Tank.

The Nature of secondary currents is known to be dependent upon the aspect ratio of the system in which the main body of the liquid flows. The harmful and the beneficial effects of the secondary currents in a sedimentation tank under the influence of a forced vortex have also been revealed in the present research. The control of the currents, and also indirectly the control of B of equation (1) Section 4.6.6. are therefore very important,

and further study into their behaviour in a sedimentation tank is worthwhile. A suggestion to be made here is the study of the $N v. q^2$ curves for different ratios of r/d . By accumulating experimental data of this kind, the optimum ratio of r/d for a sedimentation tank can be derived and used in the design of a tank.

5.3.5. Short-circuiting due to Density Currents

This research has shown the effects of the superimposed forced vortex in inhibiting short-circuiting due to turbulent mixing and non-uniform velocity distribution. It would be of great interest if the effect of rotation on the short-circuiting by density currents could be studied. The performance of full scale tank is very much influenced by these currents, and they present a serious problem in the design of sewage settling tanks, because the incoming liquid is normally denser than the liquid in the tank. The difference in temperatures between the influent and the tank contents also gives rise to density currents. Experiments of Bergman (1958) along similar lines showed that rotation does improve the performance of his tank in comparison with the conventional tank.

APPENDIX ILIST OF SYMBOLS

The following list includes all algebraic symbols appearing in the thesis, except for standard mathematical notation and special local use. The listing under each item ranges from the general use to the specific.

A	a constant cross sectional area (L^2)
B	a constant width (L)
C	a constant concentration (M/L^3)
C_d	coefficient of discharge (dimensionless)
C_{de}	coefficient of discharge at the end of the slit (dimensionless)
C_{dx}	coefficient of discharge at a distance x from the closed end of the conduit (dimensionless)
C_ϕ	ratio of the fraction of recovered momentum to the ratio of the flow leaving the main conduit to the flow upstream of the opening
C-curve	flow through curve obtained with instantaneous injection of tracer
D	diffusion coefficient (L^2/T)
D_v	molecular diffusivity (L^2/T)
F	rate of lateral transport of tracer (M/L^2T) function of
F-curve	flow through curve obtained with continuous injection of tracer
H	total energy head (L)
H_f	head loss (L)
K	ratio of tank flow rate per unit depth to 2π
K'	friction coefficient (L/T)
L	length length dimension

L_M	mixing length (L)
M	mass mass dimension
N	number of imaginary cells constituting a sedimentation tank
P_d	ratio of the flow leaving the main conduit to the flow upstream of the opening (dimensionless)
Q	flow rate per unit depth (L^2/T) flow rate (L^3/T)
Q_{crit}	critical flow rate (L^3/T)
Q_o	optimum flow rate (L^3/T)
Q_d	amount of dye injected into the tank (M)
R	hydraulic mean radius (L)
R_e	Reynolds number (dimensionless)
R_L	Lagrangian correlation coefficient (dimensionless)
R_{Lx}	Lagrangian correlation coefficient in the x direction (dimensionless)
R_{Ly}	" " y direction
R_{Lz}	" " z direction
R_L^R, R_L^T, R_L^A	Lagrangian correlation coefficients in the radial, tangential and axial directions respectively (dimensionless)
T	time dimension tank theoretical detention time (T)
U	radial velocity at the inner radius of the tank (L/T)
V	forwarding velocity (L/T) tangential velocity (L/T)
V_s	settling velocity of a particle (L/T)
W	velocity in the axial direction (L/T)

a	inner radius of the tank (L)
b	outer radius of the tank (L)
c	tracer concentration (M/L^3)
c_o	tracer concentration in the flow through cell (M/L^3)
c_e	tracer concentration at the exit of a system (M/L^3)
c_o	amount of tracer used divided by the volume of the tank (M/L^3)
c/c_o	normalized concentration of tracer in fluid (dimensionless)
c_t	tracer concentration at the tank overflow weir at any time (M/L^3)
c_x	tracer concentration at the entrance of a system (M/L^3)
d	depth (L)
	diameter of a tube (L)
	diameter of a sphere (L)
e_n	width of the slit at the closed end of a conduit (L)
e_x	width of the slit at a distance x from the closed end of a conduit (L)
g	acceleration due to gravity (L/T^2)
h	hydrostatic head (L)
$k, k_1, k_2 \dots$	constants
k_d	fraction of recovered momentum (dimensionless)
l	length
	mixing length (L)
n	number of imaginary cells constituting the system, i.e. sedimentation tank, connecting tubes, effluent chamber, effluent channel and the measuring cell (dimensionless)
p	pressure intensity (M/LT^2)
	a positive number in $g(p) = \int_0^{\infty} \exp(-pt)u \, dt$ - Laplace transform of u
	$1/2$
$\frac{q}{q^2}$	$\frac{(p/D)}{u^2 + v^2 + w^2}$
r	radius (L)
	polar co-ordinate (radial distance)(L)
r_1	inner radius (L)
r_2	outer radius (L)

r_m	mean radius (L)
t	time
t_G	arithmetic mean time: time to the centre of gravity of the flow through curve
t_M	median time: time at which 50% of the tracer has passed through the tank
t_i	time of the initial appearance of the tracer
$t_{\frac{c}{c_0}}$	time at which c/c_0 is maximum
u	radial velocity (L/T), velocity
u'	fluctuating velocity in radial direction (L/T)
v	velocity (L/T) tangential velocity (L/T)
v'	fluctuating velocity in tangential direction (L/T)
w	velocity (L/T) velocity in the axial direction (L/T)
w'	fluctuating ^{velocity} in axial direction (L/T)
x	Cartesian co-ordinate distance in the direction of flow (L)
y	Cartesian co-ordinate distance normal to the direction of flow (L)
z	Cartesian co-ordinate distance in the axial direction (L)
Γ	circulation = $2\pi r v$
Ω	angular speed of the paddles in rpm. (T^{-1})
γ	specific gravity ($M/L^2 T^2$)
ν	kinematic eddy viscosity (L^2/T)
η	efficiency (%)
θ	polar co-ordinate (angular displacement)
μ	dynamic viscosity (MT/L^2)
ν	kinematic viscosity (L^2/T)
ρ	mass density (M/L^3)

σ^2	variance
σ_i^2	variance of the <i>i</i> th component
τ	shear intensity (M/LT^2)
τ_l	laminar flow
τ_t	turbulent flow
ϕ	diameter (L)
	function of
ψ	stream function
	function of
ω	angular velocity of the paddles (T^{-1})

APPENDIX II
NOTES ON LAPLACE TRANSFORMS AND BESSEL FUNCTIONS

II.1. Laplace Transforms

The use of the Laplace transform enables a partial differential equation to be transformed into an ordinary linear differential equation. The advantages of this method increase with the complexity of the problem.

If u denotes a function of x, y, z and t , then the function $g(p)$ is given by

$$g(p) = \int_0^{\infty} e^{-pt} u \, dt \quad p > 0 \quad (1)$$

$g(p)$ is called the Laplace transform of u assuming that the integral exists. It is also assumed that

$$\int_0^{\infty} e^{-pt} \nabla^2 u \, dt = \nabla^2 \int_0^{\infty} e^{-pt} u \, dt \quad (2)$$

With these assumptions, the 'subsidiary equation' of a differential equation can be found. Let the equation be, for example,

$$\nabla^2 u + A_2(x, y, z) \frac{\partial^2 u}{\partial t^2} + A_1(x, y, z) \frac{\partial u}{\partial t} + A_0(x, y, z) u = B(x, y, z, t) \quad (3)$$

with a boundary condition to be satisfied in the form

$$G(x, y, z) u + H(x, y, z) \frac{\partial u}{\partial n} = K(x, y, z, t) \quad (4)$$

and also conditions for $t = 0$ within the region e.g.

$$\left. \begin{aligned} \lim_{t \rightarrow 0} u(x, y, z, t) &= u_0(x, y, z) \\ \lim_{t \rightarrow 0} \frac{\partial u(x, y, z, t)}{\partial t} &= u_1(x, y, z) \end{aligned} \right\} \quad (5)$$

The subsidiary equation of (3) is

$$\nabla^2 \bar{u} + [A_2(x, y, z) p^2 + A_1(x, y, z) p + A_0(x, y, z)] \bar{u} = A_2(x, y, z) [p u_0 + u_1] + A_1(x, y, z) u_0 + \int_0^\infty e^{-pt} B(x, y, z, t) dt \quad (6)$$

and the boundary equation becomes

$$G(x, y, z) \bar{u} + H(x, y, z) \frac{\partial \bar{u}}{\partial n} = \int_0^\infty e^{-pt} u(x, y, z, t) dt \quad (7)$$

If $\bar{u}$ can be found from (6) and (7), u can be found from

$$\bar{u}(x, y, z, p) = \int_0^\infty e^{-pt} u(x, y, z, t) dt \quad (8)$$

If $\bar{u}$ appears in the Table of Transforms, u can be written down directly.

If this is not the case, u can be obtained from $\bar{u}$ by the Inversion

Theorem, i.e.

$$u(x, y, z, t) = \frac{1}{2\pi i} \int_{\gamma-i\infty}^{\gamma+i\infty} e^{+t\lambda} \bar{u}(x, y, z, \lambda) d\lambda \quad (9)$$

The line integral in (9) can be evaluated by transformation to a suitable closed contour and the application of the calculus of residues. Generally two types of contours are used as shown in Fig. 4 (a) and (b) according to the nature of $\bar{u}(x, y, z, \lambda)$. If it is a single valued function of λ with an infinite number of poles (a) is used. When $\bar{u}(\lambda)$ has a branch point at the origin but otherwise only a finite number of poles, (b) is used.

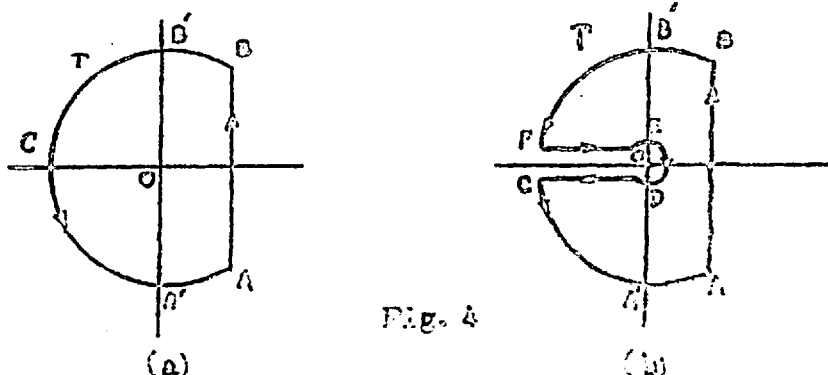


Fig. 4

Some Fundamental Properties of Laplace Transforms

$$\text{Theorem I} \quad L(v_1 + v_2) = L(v_1) + L(v_2)$$

$$\text{Theorem II} \quad L\left(\frac{\partial v}{\partial t}\right) = pL(v) - v_0$$

where v_0 is the value of $Lt v$. In general v_0 will be a
 $t \rightarrow 0$

function of the space variables x , y and z .

$$\text{Theorem III} \quad L\left(\frac{\partial^2 v}{\partial x^2}\right) = \frac{\partial^2 \bar{v}}{\partial x^2}$$

$$\text{Theorem IV} \quad L\left(\int_0^t v(t') dt'\right) = (1/p)L(v)$$

II.2. Bessel Functions

$J_\nu(z)$ is the Bessel function of the first kind of the order ν and is defined by

$$J_\nu(z) = \sum_{r=0}^{\infty} \frac{(-1)^r \left(\frac{1}{2}z\right)^{\nu+2r}}{r! \Gamma(\nu+r+1)} \quad (1)$$

where ν is real and z may be complex, its argument being given its principal values. (1) satisfies Bessel equation of the order ν

$$\frac{d^2 y}{dz^2} + \frac{1}{z} \frac{dy}{dz} + \left(1 - \frac{\nu^2}{z^2}\right) y = 0 \quad (2)$$

Particular cases of (1) occurred in the solution of differential equations of physical problems for over a century. As far back as in 1732, equation of the type

$$\frac{d^2 y}{dz^2} + \frac{1}{z} \frac{dy}{dz} + k^2 \frac{y}{z} = 0 \quad (2a)$$

was obtained by Bernoulli as a result of an investigation into the problem of the oscillation of a heavy flexible chain suspended with its

lower end free, but it was not until 1824 when F. W. Bessel, a German astronomer began to deal with these functions in a systematic way (McLachlan, 1955).

The property of $J_\nu(z)$ is such that

$$J_n(z) = (-1)^n J_{-n}(z) \quad (3)$$

where n is an integer. Therefore there are no independent solutions of (2). In order to obtain a second solution of (2) which is available for all values of ν , the following function is defined;

$$Y_\nu(z) = \frac{J_\nu(z) \cos \nu\pi - J_{-\nu}(z)}{\sin \nu\pi} \quad (4)$$

$$\text{and } Y_n(z) \text{ is } \lim_{\nu \rightarrow n} Y_\nu(z) \quad (5)$$

$$y_1, y_2 = \alpha J_\nu(z) + \beta Y_\nu(z) \quad (6)$$

where α and β are non zero constants.

The Modified Bessel Functions

$$\frac{d^2 y}{dz^2} + \frac{1}{z} \frac{dy}{dz} - \left(1 + \frac{\nu^2}{z^2}\right) y = 0 \quad (7)$$

is satisfied by

$$I_\nu(z) = \sum_{r=0}^{\infty} \frac{\left(\frac{1}{2}z\right)^{\nu+2r}}{r! \Gamma(\nu+r+1)} \quad (8)$$

Similarly, $I_{-\nu}(z)$ is not independent of $I_\nu(z)$ if ν is an integer. In this case a second solution is needed and is defined as

$$K_{\nu}(z) = \frac{1}{2} \pi \frac{I_{1-\nu}(z) - I_{\nu}(z)}{\sin \nu \pi} \quad (9)$$

$$\text{and} \quad K_n(z) = \lim_{\nu \rightarrow n} K_{\nu}(z) \quad (10)$$

The complete solution of (7) is therefore

$$y = A I_{\nu}(z) + B K_{\nu}(z) \quad (11)$$

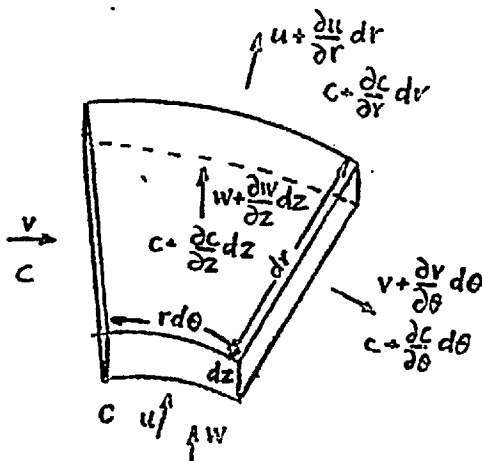
APPENDIX III

DIFFERENTIAL EQUATION OF THE DIFFUSION IN A CIRCULAR SEDIMENTATION TANK AND THE SOLUTION FOR AN INSTANTANEOUS INJECTION OF DYE IN AN INFINITE MEDIUM.

III.1. Differential Equations.

Consider a fluid element as shown below (Fig. 3, Section 2.2.1.)

Fig. 3



The rate of change of concentration of the fluid element is, by considering the conservation of dye:-

$$\begin{aligned} \frac{\partial c}{\partial t} r \cdot d\theta \cdot dr \cdot dz &= - \left(D_{rr} \frac{\partial c}{\partial r} + \frac{D_{r\theta}}{r} \frac{\partial c}{\partial \theta} + D_{rz} \frac{\partial c}{\partial z} \right) r d\theta \cdot dz - \left\{ - \left(D_{rr} \frac{\partial c}{\partial r} + \frac{D_{r\theta}}{r} \frac{\partial c}{\partial \theta} + D_{rz} \frac{\partial c}{\partial z} \right) + \frac{D_{r\theta}}{r^2} \frac{\partial c}{\partial \theta} \right. \\ &\quad \left. - D_{rr} \frac{\partial^2 c}{\partial r^2} \right\} (r+dr) d\theta \cdot dz + u \cdot c \cdot r d\theta \cdot dz - \left(u + \frac{\partial u}{\partial r} dr \right) \left(c + \frac{\partial c}{\partial r} dr \right) r d\theta \cdot dz \\ &\quad - \left(D_{\theta r} \frac{\partial c}{\partial r} + \frac{D_{\theta\theta}}{r} \frac{\partial c}{\partial \theta} + D_{\theta z} \frac{\partial c}{\partial z} \right) dr \cdot dz - \left\{ - \left(D_{\theta r} \frac{\partial c}{\partial r} + \frac{D_{\theta\theta}}{r} \frac{\partial c}{\partial \theta} + D_{\theta z} \frac{\partial c}{\partial z} \right) - \right. \\ &\quad \left. - \frac{D_{\theta\theta}}{r} \frac{\partial^2 c}{\partial \theta^2} \right\} dr \cdot dz + v \cdot c \cdot dr \cdot dz - \left(v + \frac{\partial v}{\partial \theta} d\theta \right) \left(c + \frac{\partial c}{\partial \theta} d\theta \right) dr \cdot dz \\ &\quad - \left(D_{zr} \frac{\partial c}{\partial r} + \frac{D_{z\theta}}{r} \frac{\partial c}{\partial \theta} + D_{zz} \frac{\partial c}{\partial z} \right) dr \cdot r \cdot d\theta - \left\{ - \left(D_{zr} \frac{\partial c}{\partial r} + \frac{D_{z\theta}}{r} \frac{\partial c}{\partial \theta} + D_{zz} \frac{\partial c}{\partial z} \right) \right. \\ &\quad \left. - D_{zz} \frac{\partial^2 c}{\partial z^2} \right\} dr \cdot r \cdot d\theta + w \cdot c \cdot r \cdot d\theta \cdot dr - \left(w + \frac{\partial w}{\partial z} dz \right) \left(c + \frac{\partial c}{\partial z} dz \right) r \cdot d\theta \cdot dr \end{aligned}$$

(1)

$$\begin{aligned}
&= D_{rr} \frac{\partial^2 c}{\partial r^2} dr dz \cdot r d\theta - u r d\theta dz \frac{\partial c}{\partial r} dr + \frac{D_{\theta\theta}}{r} \frac{\partial^2 c}{\partial \theta^2} d\theta dr dz \\
&\quad - r dr dz \frac{\partial c}{\partial \theta} d\theta + D_{zz} \frac{\partial^2 c}{\partial z^2} dz dr r d\theta - v r d\theta dr \frac{\partial c}{\partial z} dz \\
&\quad + (D_{rr} \frac{\partial c}{\partial r} + \frac{D_{r\theta}}{r} \frac{\partial c}{\partial \theta} + D_{rz} \frac{\partial c}{\partial z}) dz \cdot dr \cdot d\theta
\end{aligned} \quad (2)$$

If the medium is orthotropic, i.e. D_r , D_θ and D_z are mutually perpendicular and $D_{r\theta} = D_{rz} = 0$, (2) becomes

$$\frac{\partial c}{\partial t} = D_{rr} \frac{\partial^2 c}{\partial r^2} - u \frac{\partial c}{\partial r} + \frac{1}{r} D_{rr} \frac{\partial c}{\partial r} + \frac{D_{\theta\theta}}{r^2} \frac{\partial^2 c}{\partial \theta^2} - \frac{v}{r} \frac{\partial c}{\partial \theta} + D_{zz} \frac{\partial^2 c}{\partial z^2} - w \frac{\partial c}{\partial z} \quad (3)$$

Further, if the medium is also isotropic, i.e. $D_{rr} = D_{\theta\theta} = D_{zz} = D$, we can write (2) as

$$\frac{\partial c}{\partial t} = D \frac{\partial^2 c}{\partial r^2} + \frac{D}{r} \frac{\partial c}{\partial r} - u \frac{\partial c}{\partial r} + \frac{D}{r^2} \frac{\partial^2 c}{\partial \theta^2} - \frac{v}{r} \frac{\partial c}{\partial \theta} + D \frac{\partial^2 c}{\partial z^2} - w \frac{\partial c}{\partial z} \quad (4)$$

For a two-dimensional flow (4) becomes:

$$\frac{\partial c}{\partial t} = D \frac{\partial^2 c}{\partial r^2} + \frac{D}{r} \frac{\partial c}{\partial r} - u \frac{\partial c}{\partial r} \quad (5)$$

However, if the medium is not isotropic, D in (5) will be D_{rr} . If it is anisotropic D will be $l^2 D_{rr} + m^2 D_{\theta\theta} + n^2 D_{zz}$ where l , m and n are the direction cosines in the r , θ and z direction respectively.

III.2. The Solution for an Instantaneous Injection of Dye in an Infinite Medium

The Laplace transform of (5) is:

$$\frac{\partial^2 \bar{c}}{\partial r^2} + \frac{1}{r} (1-2p) \frac{\partial \bar{c}}{\partial r} - q^2 \bar{c} = 0 \quad (6)$$

which is another form of the modified Bessel equation.

The complete solution of (6) is

$$\bar{c} = r^{\nu} A I_{\nu}(qr) + B K_{\nu}(qr) \quad (7)$$

As $I_{\nu}(qr)$ is infinite when r approaches infinity, the solution of (6) which is finite at $r = \infty$ is therefore

$$\bar{c} = r^{\nu} B K_{\nu}(qr) \quad (8)$$

If the amount of the dye injected into the tank is Q_d then

$$\frac{Q_d}{p} = \int_0^{\infty} 2\pi r^{\nu+1} B K_{\nu}(qr) dr \quad (9)$$

(9) implies that the total amount of dye in the medium is always Q_d .

Put $qr = z$ and $p = q^2/D$

$$\begin{aligned} Q_d &= B D \int_0^{\infty} 2\pi z (z/q)^{\nu} K_{\nu}(z) dz \\ &= (BD 2^{\nu+1} \pi^{\nu+1}) / q^{\nu} (q)^{\nu} \quad (\text{McLachlan, 1946}) \end{aligned}$$

$$\text{Therefore} \quad B = (Q_d q^{\nu}) / D 2^{\nu+1} \pi^{\nu+1} \quad (10)$$

(8) then becomes

$$\bar{c} = (Q_d (qr)^{\nu} K_{\nu}(qr) / (2^{\nu+1} D \pi^{\nu+1})) \quad (11)$$

The equation whose Laplace transform is (11) is

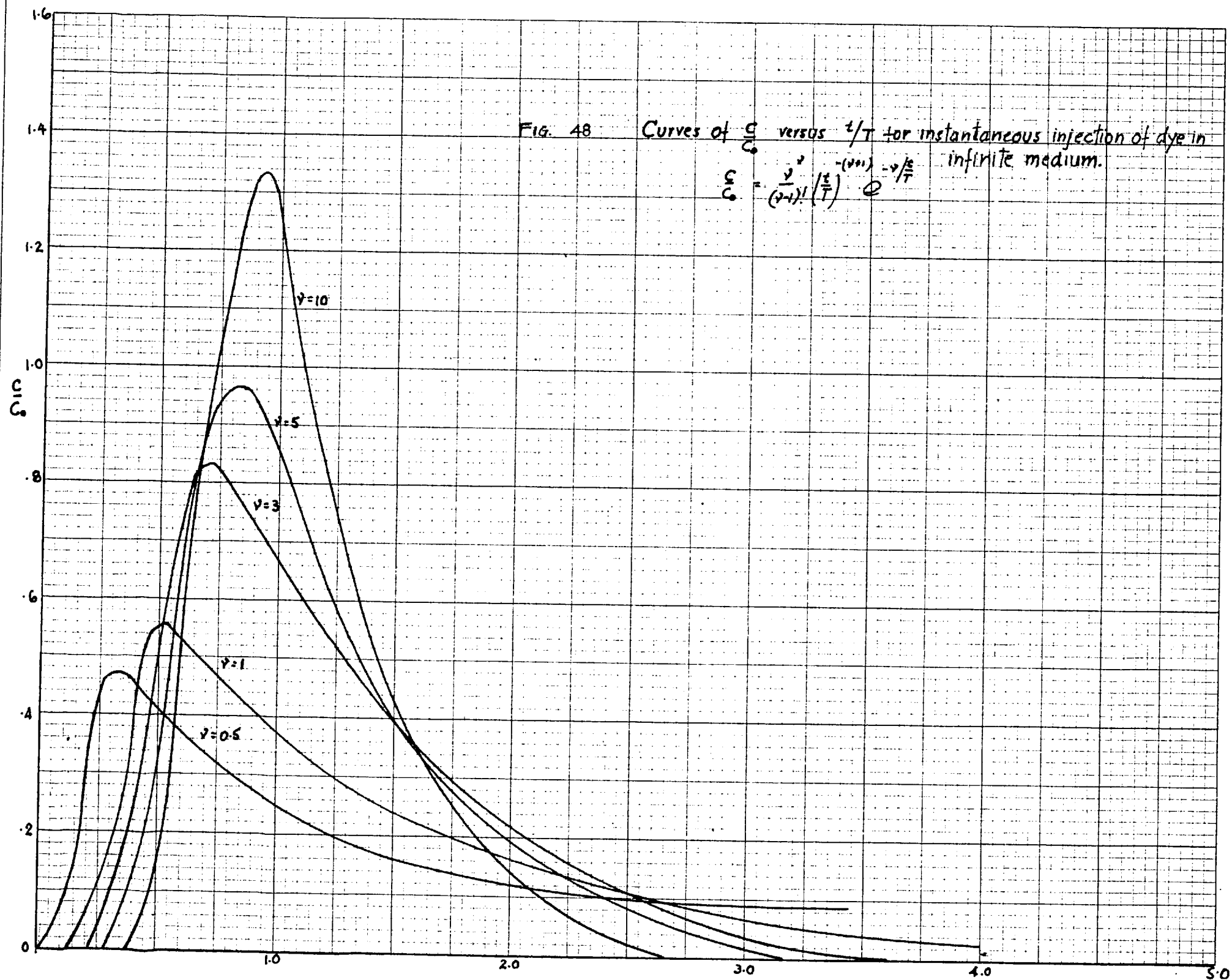
$$c = (Q_d / 4\pi D t^{\nu+1}) (r^2/4Dt)^{\nu} \exp(-r^2/4Dt) \quad (12)$$

But $Q_d/2\pi KT = c_0$, $K/2D = \nu$, and $r^2 = 2KT$ where $K = Q/2\pi$. (12)

can therefore be written as

$$\frac{c}{c_0} = \frac{\nu^{\nu}}{(\nu-1)!} \left(\frac{t}{T}\right)^{-(\nu+1)} e^{-\nu/(t/T)} \quad (12a)$$

(12a) is thus the solution for the instantaneous injection of dye of an amount Q_d in an infinite medium. It is plotted in Fig. 48.



APPENDIX IV

THE DESIGN OF UNIFORM FLOW INLET

A great deal of experimental and analytical work has been contributed to the problem of distributing the flow to a series of pipes along the main conduit. The approaches to this problem can be grouped into two different methods: the use of the modified energy equation and the momentum principle on the one hand, and the application of the classical hydrodynamic principles on the other.

Analysis

As water flows along the conduit it loses some of its quantity through the opening, with a resultant rise in the total head. If the discharges are to be uniform along the conduit, the variation in the total head must correspond to the variation in the coefficients of discharge of the openings. Three factors will therefore have to be considered.

- (1) The pressure changes due to discharging through the openings.
- (2) The friction losses.
- (3) The variation of the coefficient of discharge with the velocity head and the boundary geometry.

Pressure Changes

At any point x along the conduit, the change in the pressure head Δh due to a discharge ΔQ in a distance Δx is given by

$$\Delta h = -\frac{1}{ag} (Q - \Delta Q) \Delta v + K' V_o \frac{x}{L} \Delta u - k_d \frac{\Delta Q \cdot V}{ag} \quad (1)$$

where K' is the friction coefficient $= 32\nu/d^2$, ν being the kinematic viscosity and d the diameter of the conduit. k_d is the fraction of recovered momentum.

Published data indicate that it is either a function of Pd (the ratio of the flow leaving the conduit to the flow upstream of the opening) or a constant (McNown, 1945). It is noticed further that

(a) If the head loss is zero, $k_d = 0.5 Pd$.

(b) If the head loss is maximum, $k_d = 0$.

It appears logical therefore to write $C_\phi = k_d/Pd$. C_ϕ had been found to vary between 0 and 0.5.

The term on the left hand side of (1) represents the rise in pressure head, and those on the right hand side are respectively from left to right the change in momentum of the continuing flow, friction head loss and the recovered momentum of the flow leaving the conduit. Equation (1) agrees with Soucek and Zelnick's (1945) equation if the friction loss term is neglected, and with Enger and Levy's (1929) if it is further assumed that there is full recovery of the momentum of the flow leaving the conduit, i.e. neglecting head loss term altogether. The last assumption will be true only when the flow leaves the conduit at right angles to it. A conduit with a slit or hole in its wall produces a free jet at a certain natural angle to the main flow (Teakonas, 1957).

Equation (1) represents the change in pressure head along a conduit which distributes flow uniformly along its length, and can be modified in the following way to facilitate computation:-

$$dh = \frac{-1 \cdot V_o^2}{gL^2} [x + (1 - C) dx] dx + \frac{K^o V_o x}{g L} dx \quad (2)$$

where $H = h$, $V = 0$ at $x = 0$.

$H_o = h$, $V = V_o$ at $x = L$.

If $(1-C)(dx)^2$ is neglected, (2) is essentially the same as the equation used by Abdalla (1955) in his design of a manifold flow inlet for his rectangular sedimentation tank. He varied the cross sectional area of his conduit so that there was no change in pressure head along its length. Here, the cross sectional area of the conduit (the riser) will be kept constant to facilitate construction. Uniform flow will be obtained by varying the sizes of the holes along the riser wall.

Coefficient of Discharge

Experiments show that the rate of discharge can be calculated satisfactorily from

$$Q = C_d A \sqrt{2gh} \quad (3)$$

where C_d is the coefficient of discharge, A the area of the opening and h the static head at that point. The pressure head has been defined in the same way as it has been done by previous workers. It appears logical and convenient not to add velocity head to the discharge, since it has been shown that the discharge is less at a high conduit velocity (Sowack

and Zelnick, 1945).

If C_d were constant, it would have been a simple matter to calculate the area of the opening once h is known. Unfortunately, this is not the case. It has been found that C_d depends upon two factors (McNown, 1954), namely:-

- (1) The geometry of the opening and the main conduit.
- (2) The ratio of the opening velocity and the velocity in the main conduit beyond the opening.

As the ratio of the area of the hole to that of the conduit will not vary greatly in the present study, it can be assumed that C_d depends only upon the piezometric head at the opening and the velocity in the main conduit (Enger and Levy, 1929):-

$$C_{dx} = \frac{h - V^2/2g}{h} C_{de} \quad (4)$$

where C_{de} = the coefficient of discharge at the closed end of the conduit,

C_{dx} = the coefficient of discharge at any distance x from the closed end,

h = the piezometric head at the point x .

Inlet Design

For simplicity and convenience in construction it was decided to keep the riser cross sectional area constant. It was necessary therefore to vary the areas of the openings so that the discharge was uniform.

In the calculation the opening was assumed to be in the form of a slit. In construction the slit would be represented by a series of holes having a total area equal to that of the slit. This is permissible and had been confirmed experimentally by Eger and Levy (1929).

Equation (3) can also be written as:

$$dQ = C_{di} e_x dx \sqrt{2gh} \quad (5)$$

Let the width of the slit at the closed end be e_n . Put (4) into (5) and we have

$$\frac{e_x}{e_n} = \frac{\sqrt{Hh}}{h - \sqrt[3]{2g}} \quad (6)$$

h is given by equation (1).

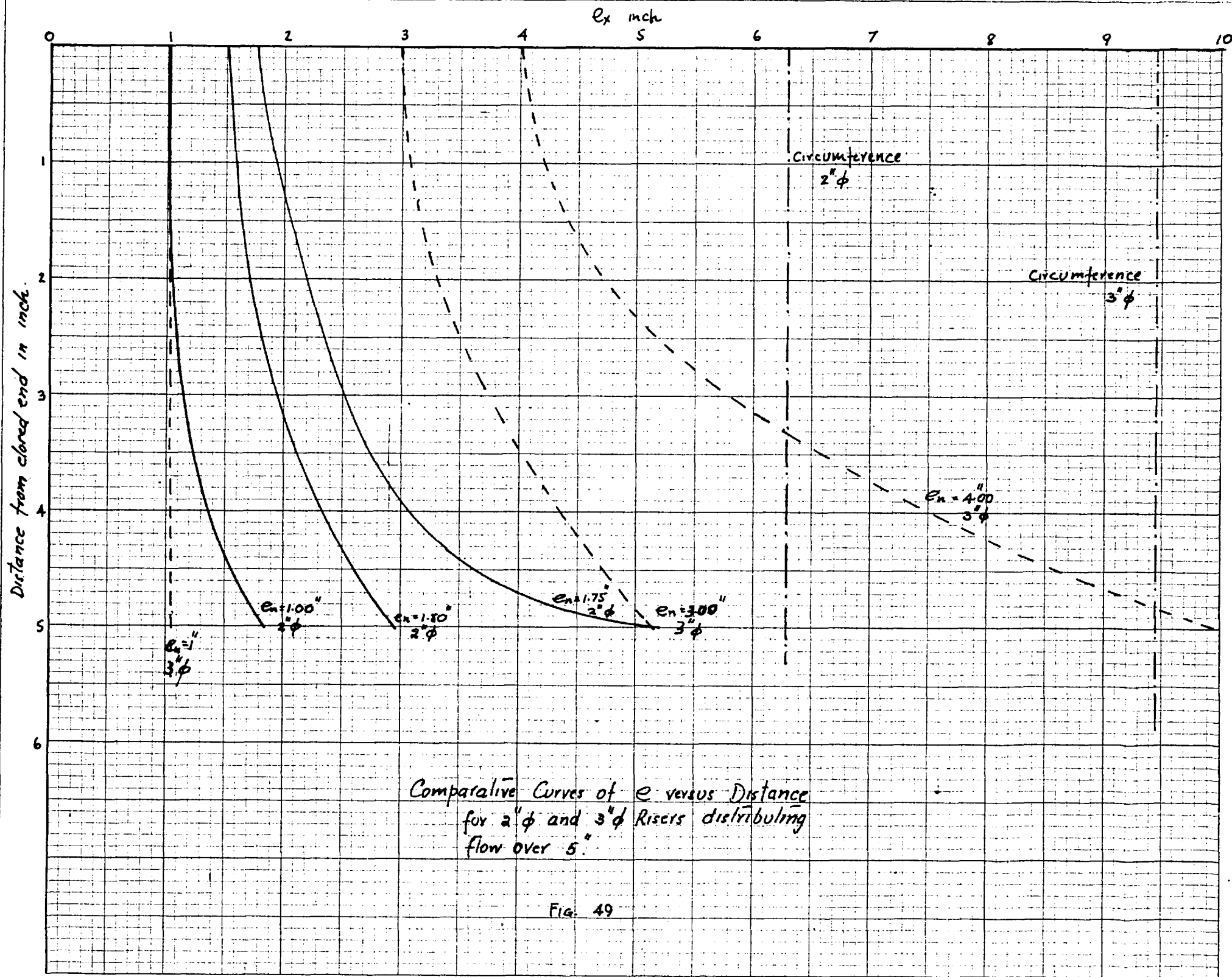
Computation of the Sizes of the Holes

It was decided to design an inlet for an average flow of about 200 $\frac{3}{\text{cm}^3/\text{sec}}$. (actually 216.5 $\frac{3}{\text{cm}^3/\text{sec}}$) to be distributed over a 4-inch length of riser.

The calculation is a trial and error one. A certain width e_n of the slit at the closed end was first assumed. Taking C_d as being 0.745, the static head H was calculated, from which h at any section x can be computed with the help of equation (1). It was found that if the assumed e_n was too large, h at a certain distance will be zero. This would mean mathematically that the width of the slit at that point must be infinite. This was impracticable. The physical meaning of this

was that beyond this point the pressure was negative. Therefore, instead of water flowing into the tank from the riser, we would have water flowing from the tank into the riser at the section where the pressure was negative. Water would therefore be sucked into the riser at sections far away from the closed end (near the tank floor) and delivered at the sections near the closed end (in the vicinity of the tank surface).

A smaller e_n would have to be assumed. If it was too small, the resulted slit would be too small in area, giving rise to a high velocity jet leaving the slit, which was also undesirable. The calculation would be performed with several values of e_n and L , the depth over which the flow was desired to be distributed, and the results plotted and compared. From this a suitable pair of e_n and L was selected. An example of the calculation was presented in Table 12, and the comparative plottings of the results were shown in Figs. 49-51.



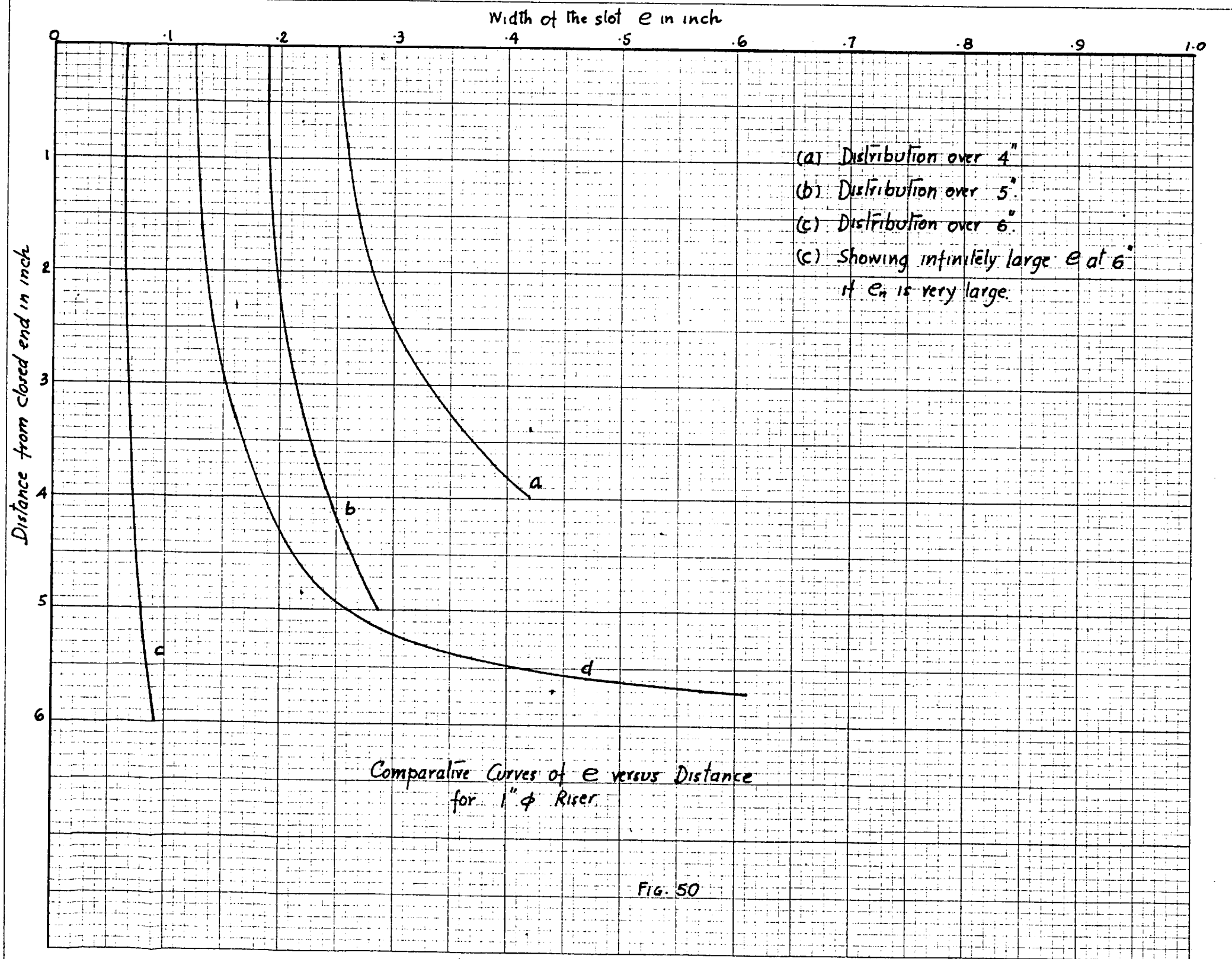
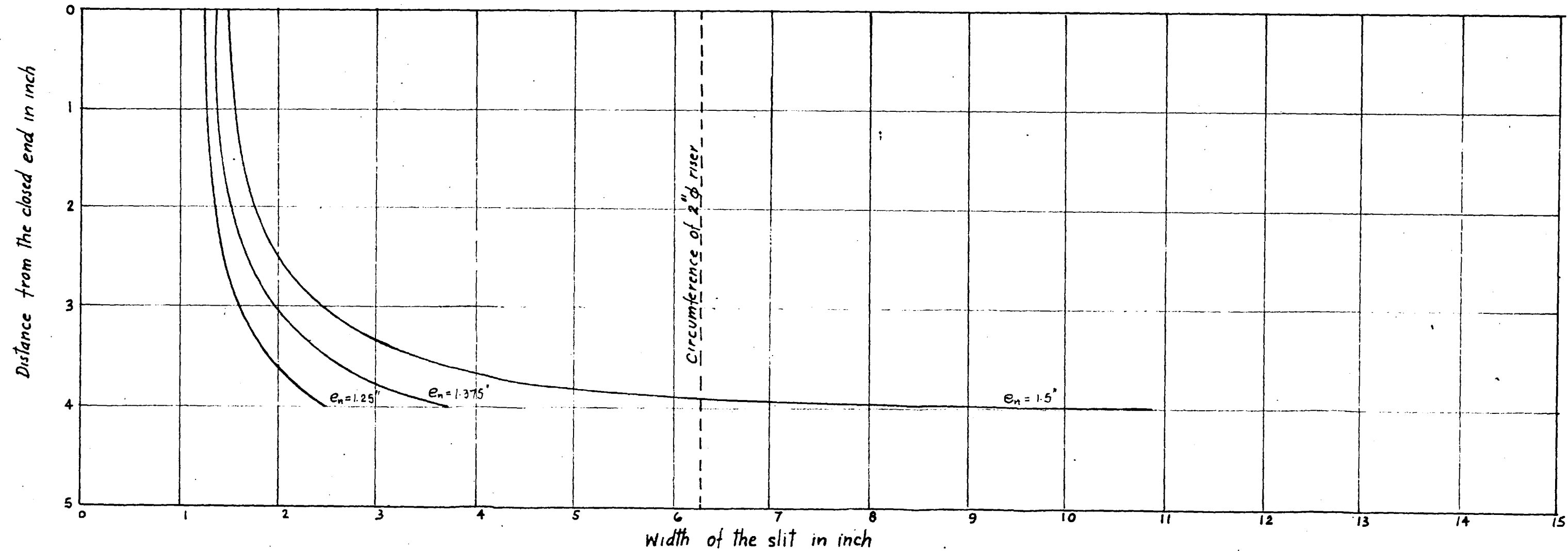


TABLE 12

CALCULATION OF SIZES OF THE HOLES FOR 2ϕ RISER DISTRIBUTING OVER 4" $e_n = 1.25$ "

(1) x (inch)	(2) x (feet)	(3) (1-c)dx (ft)	(4) (1-c) dx · x × 10 ⁴ (ft) ²	(5) ($\frac{x^2}{2}$) × 10 ⁴ (ft) ²	(6) (4) + (5) (ft) ²	(7) (b) × ($\frac{V_0}{Z}$) ² × 10 ⁴	(8) ($\frac{K \cdot V_0}{9L}$) · $\frac{x^2}{2}$ · 10 ⁴	(9) (8) - (7)	(10) ($\frac{V_0}{Z}$) ² · $\frac{1}{2}$ × (2) · 10 ⁴ (ft)	(11) h · 10 ⁴ (ft)	(12) h' = (11) - (10)	(13) e (in.)	Area of slit = ($e_{n1} + e_{n2}$)	Diameter of each of 6 holes (inch)	Nearest size of Drills
0	0	0.0625	0	0	0	0	0	0	0	54.7	54.7	1.25	1.40	.545	($\frac{35}{64}$) (.5469)
1	0.0834		52.1	34.8	86.9	2.95x	0.655	-2.295	1.97	52.43	50.43	1.30	1.49	.562	($\frac{9}{16}$) (.5625)
2	0.1668		104.0	139.0	243.0	8.35	2.615	-5.735	4.78	48.96	44.18	1.38	1.73	.605	($\frac{39}{64}$) (.6094)
3	0.2500		156.2	312.0	468.0	16.10	5.88	-10.220	10.70	44.48	33.78	1.64	2.83	.774	(195) (.7677)
4	0.4170		208.5	557.0	765.5	26.30	10.50	-15.800	19.15	38.90	19.75	2.52			

FIG.51 COMPARATIVE CURVES OF e VERSUS DISTANCE FOR 2ϕ RISER.

APPENDIX V

EXPERIMENTAL CURVES

The following are the flow through curves obtained in the experiments. In order to reduce the number of curves presented here to the minimum, so that each set of experiments for any speed of rotation and inlet can be shown comparatively on the same page, some curves have been left out. They are the curves whose flow rates come in between those already plotted. Their shapes can be roughly estimated from the shapes of the curves with flow rates immediately above and below it. Each curve shown here is the average of two tests which were found to agree closely with each other.

1.4

1.2

1.0

.8

.6

.4

.2

0

 t/T

FIG. 52

2" ϕ P

Flow rate

Speed of rotation

1

102.4 cm³/sec

0 rpm

2

155.6 "

0 "

1

2

FIG. 53

2" ϕ P

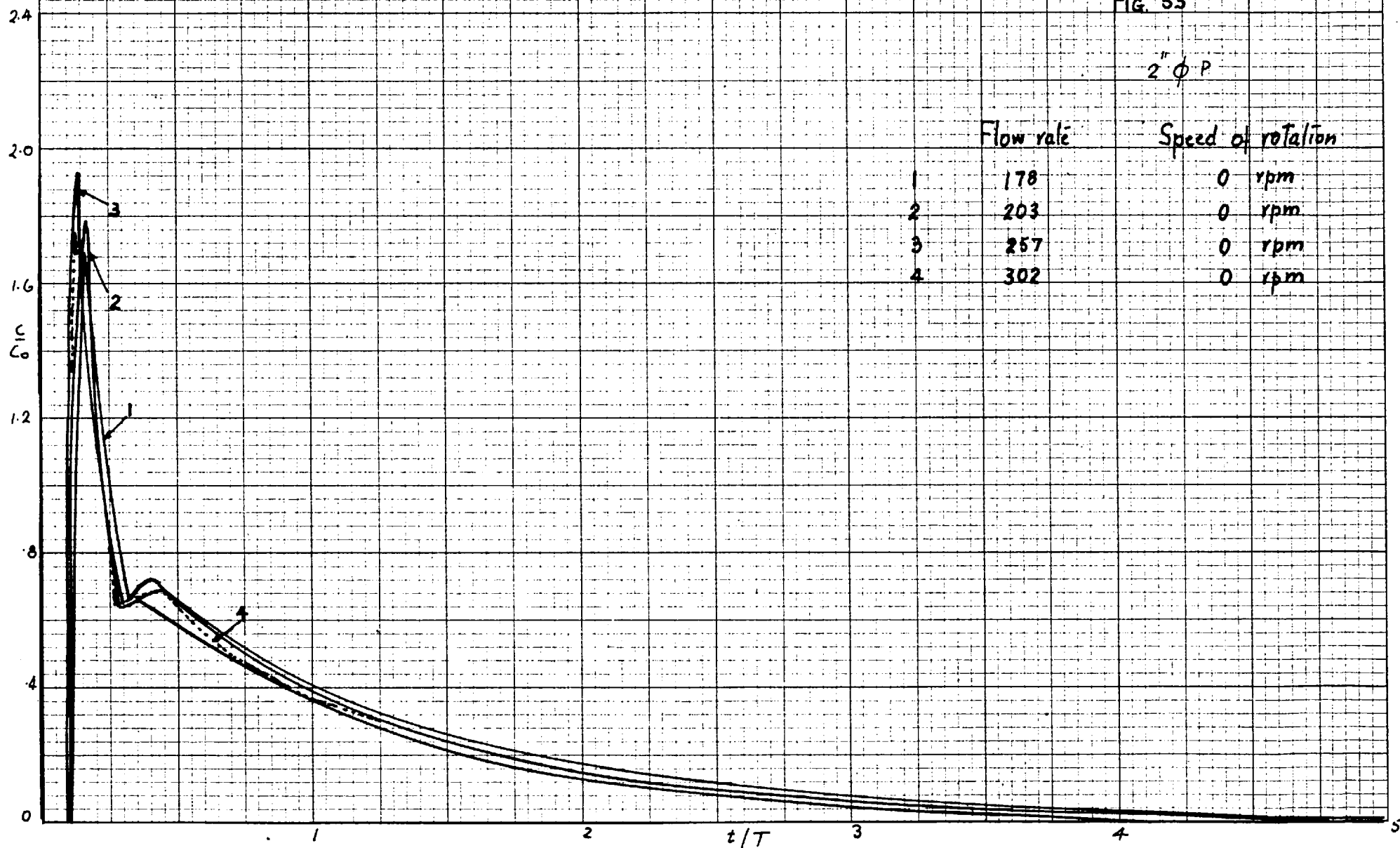


FIG. 54

$2 \phi P$

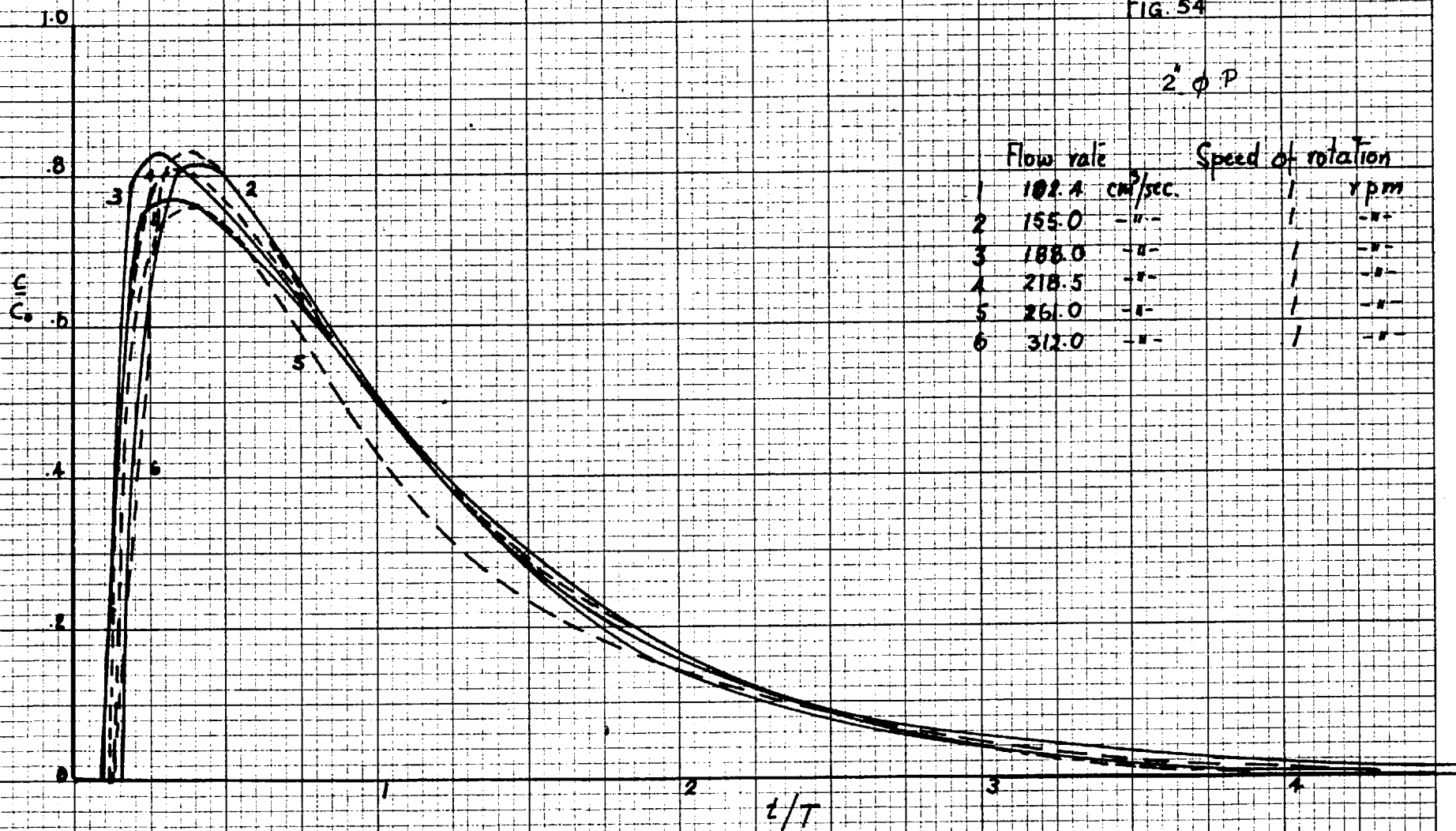


FIG. 55

$2 \phi p$

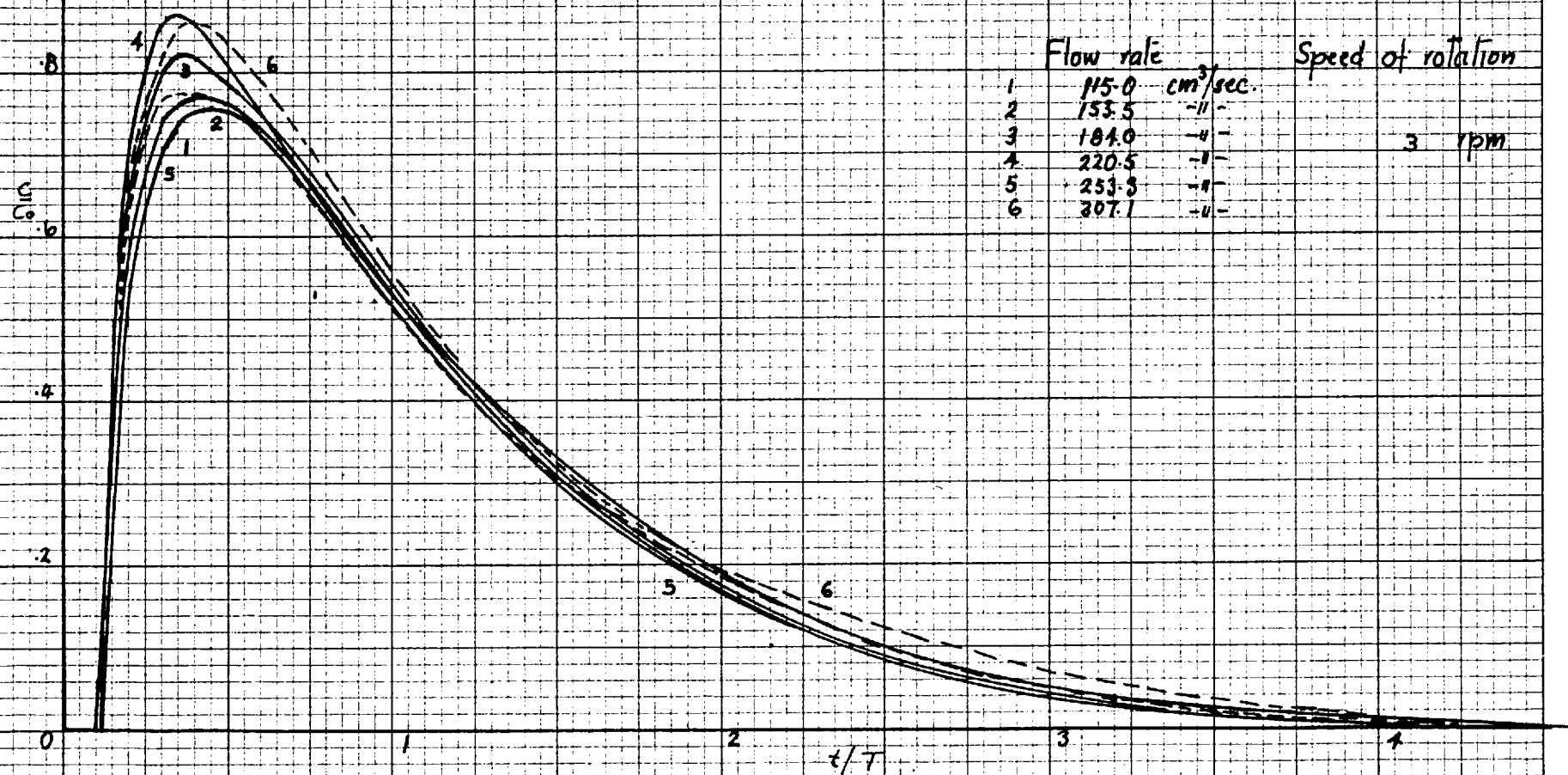


FIG. 56

$2\phi P$

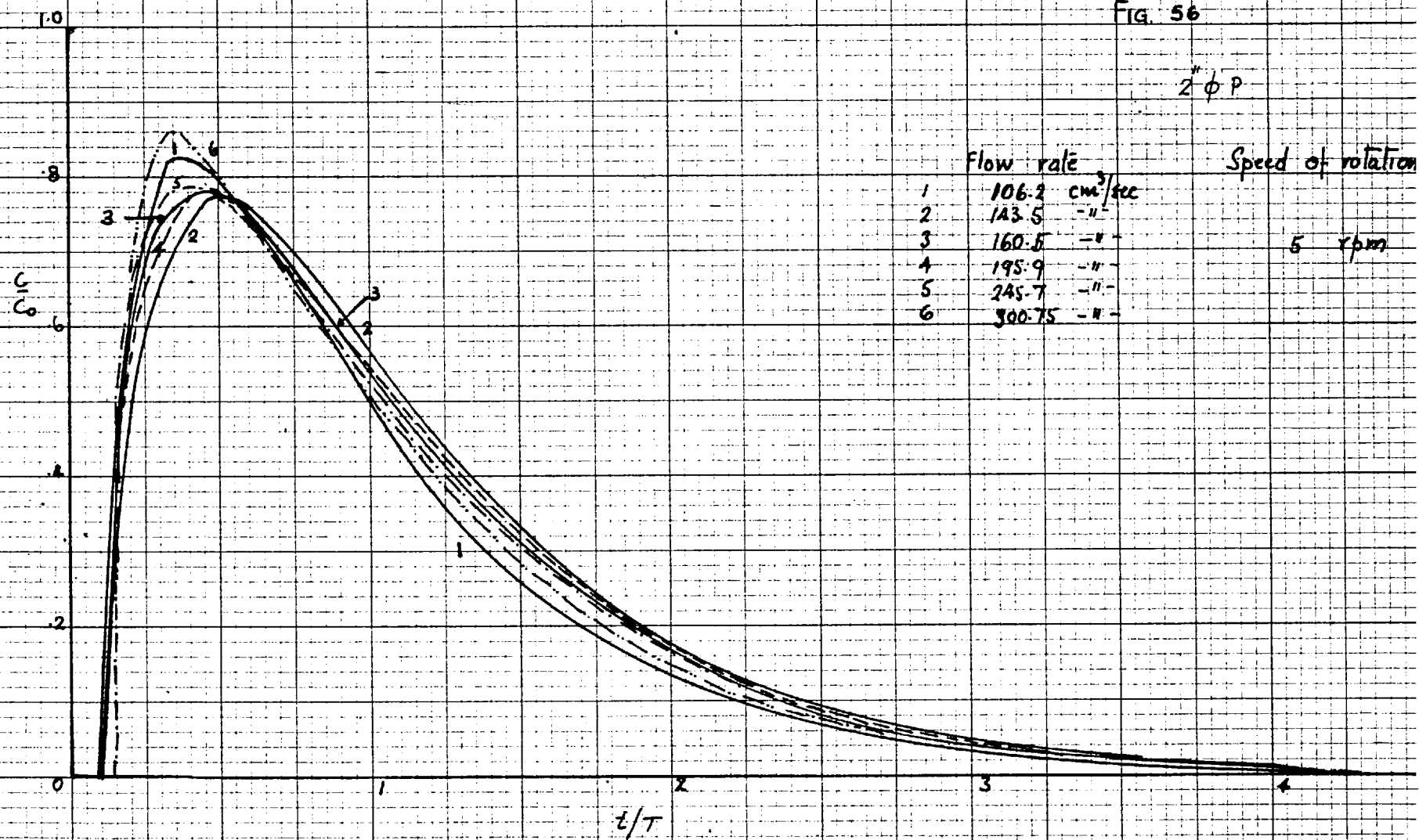


FIG. 57

$2' \phi P$

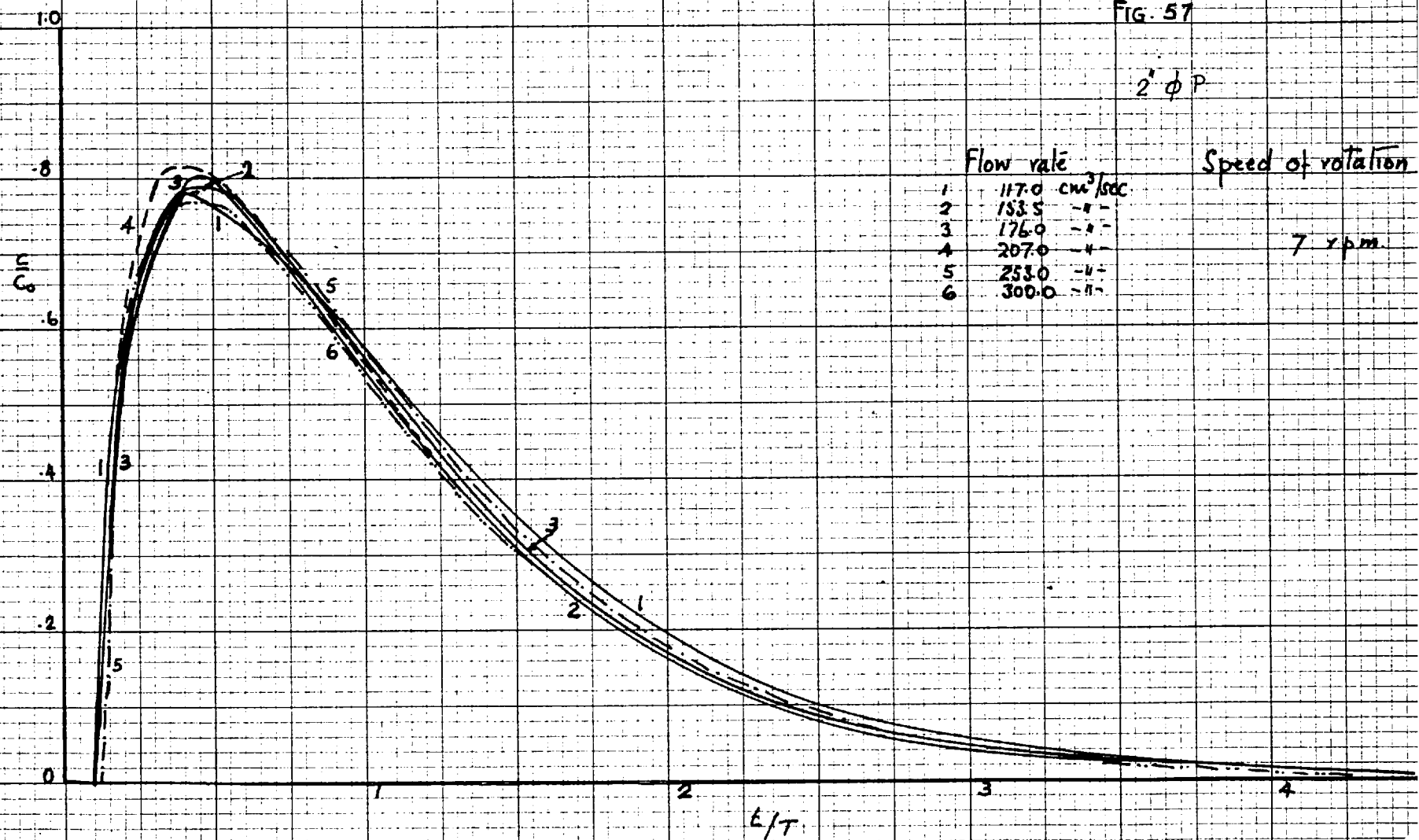


FIG. 58

2" ϕ P

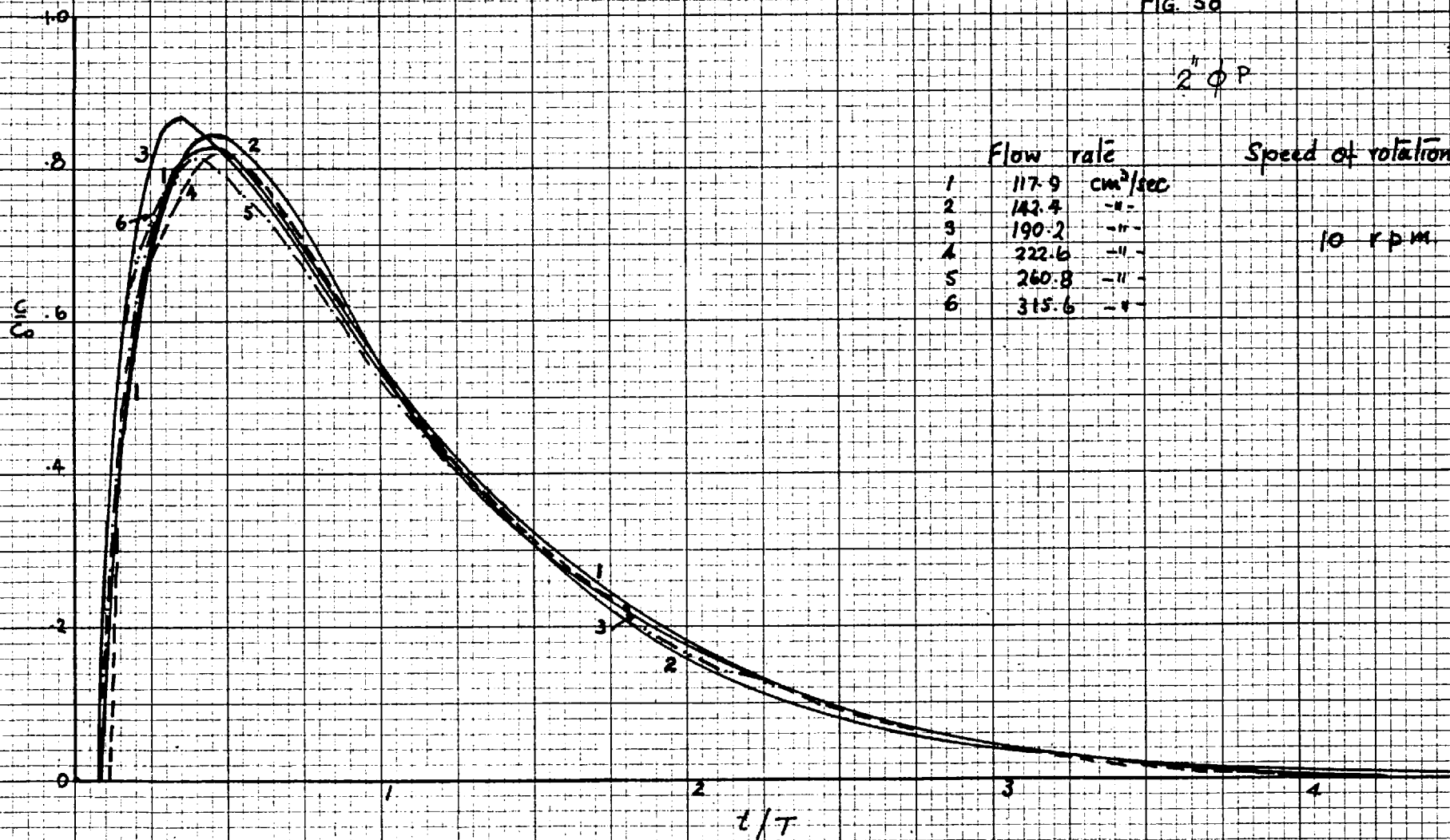


FIG. 59

$2^{\circ} \phi P$

	Flow rate	Speed of rotation
1	227.1 cm^3/sec	15 rpm
2	266.5 " "	
3	322.2 " "	

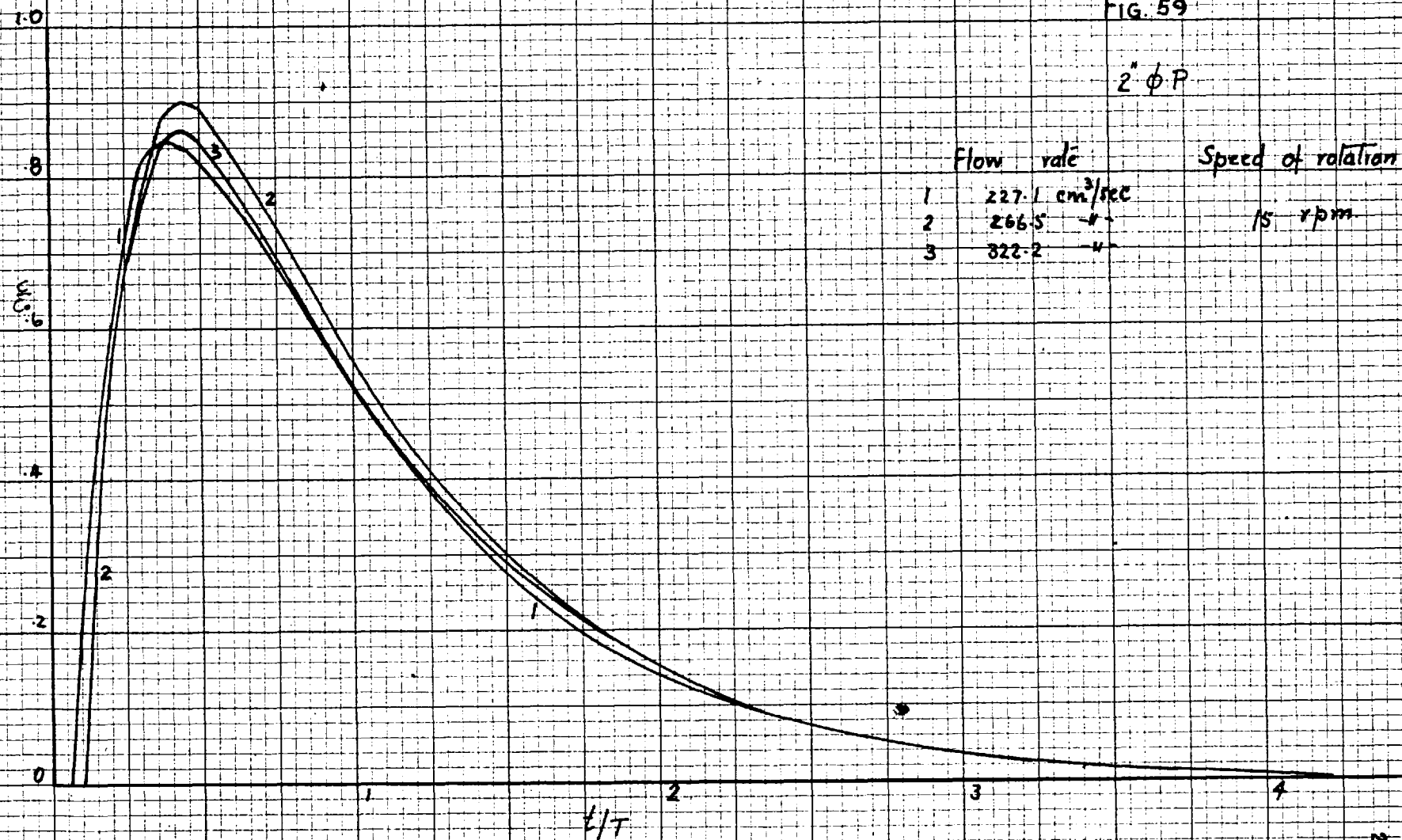
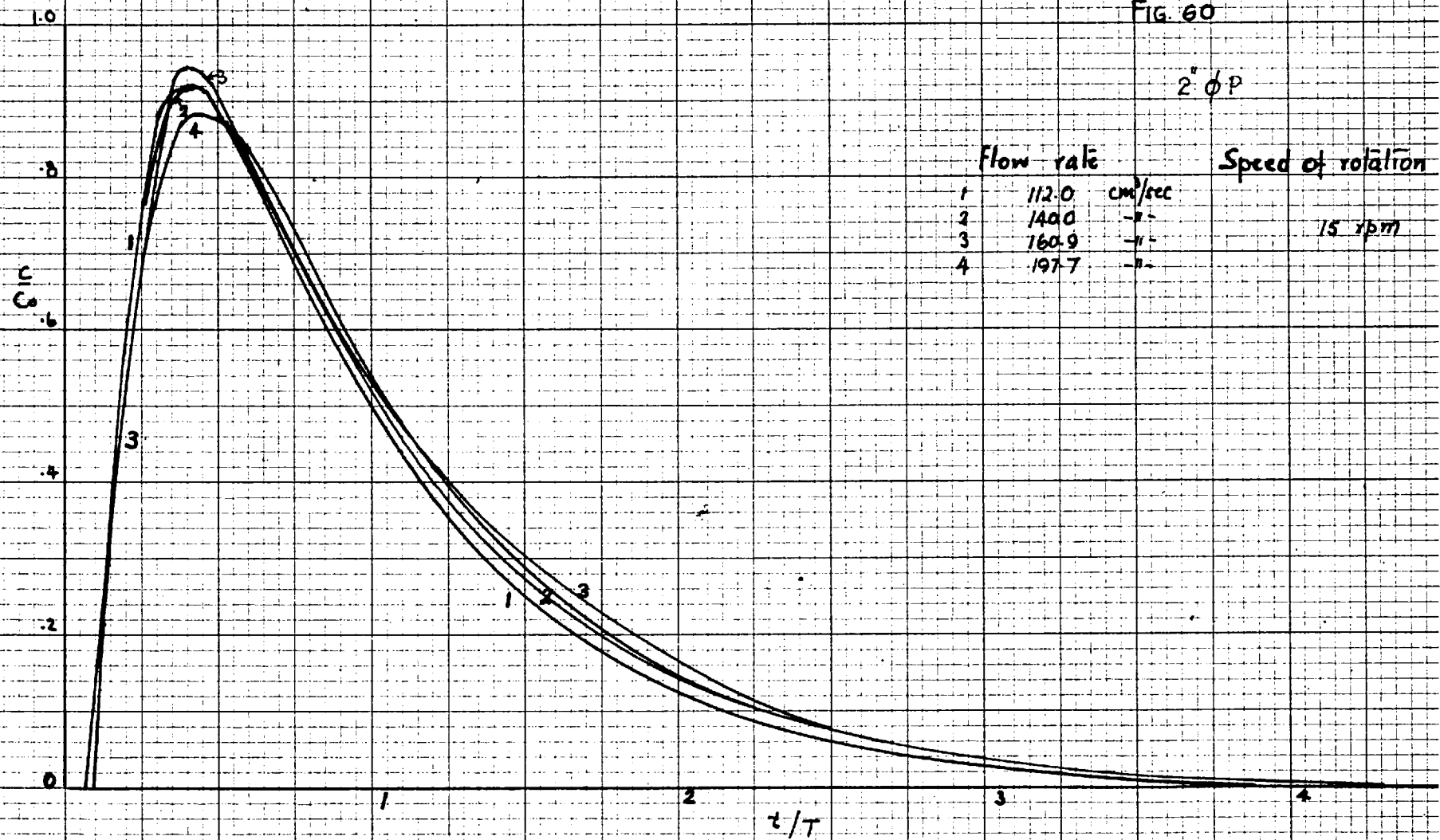
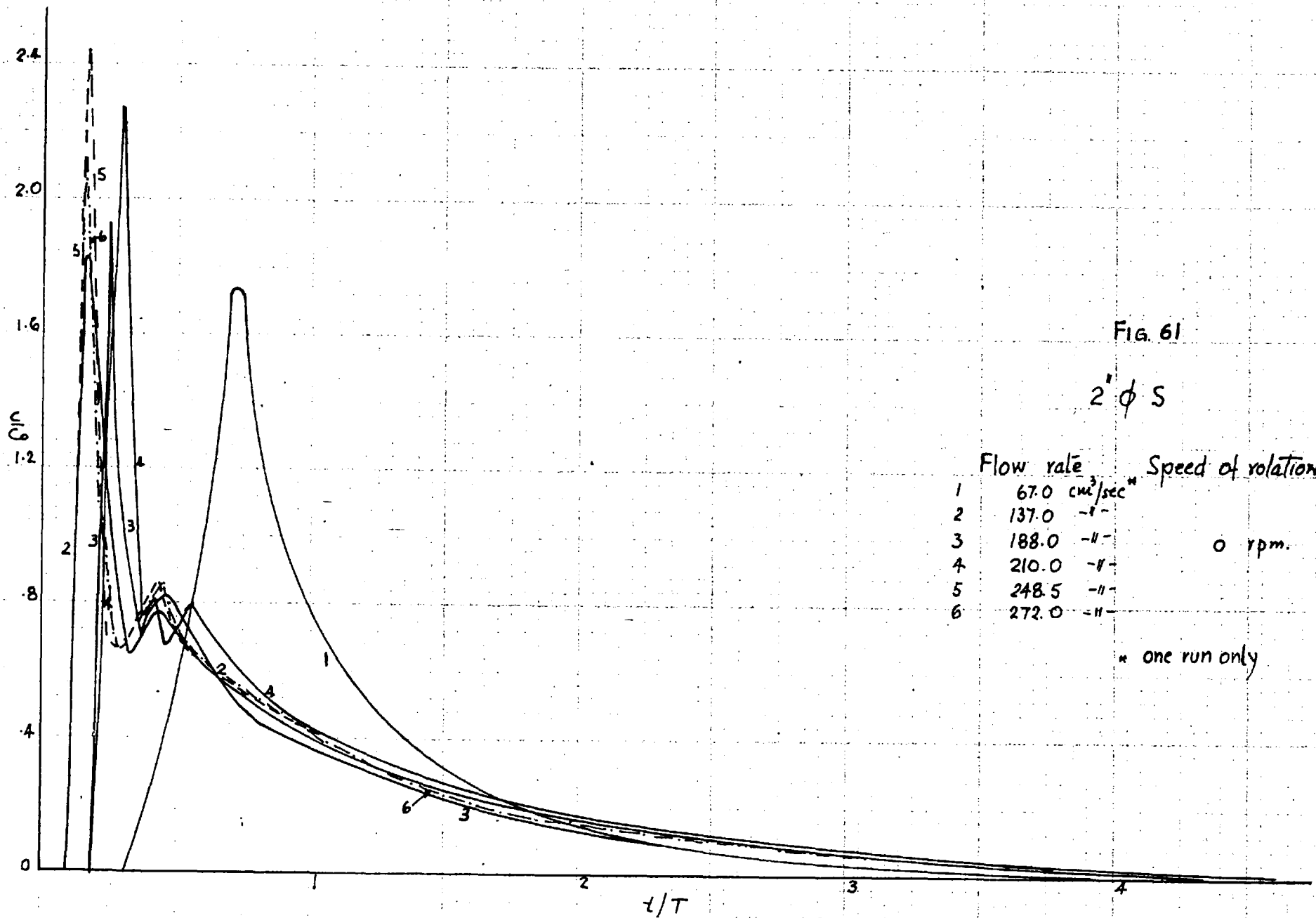
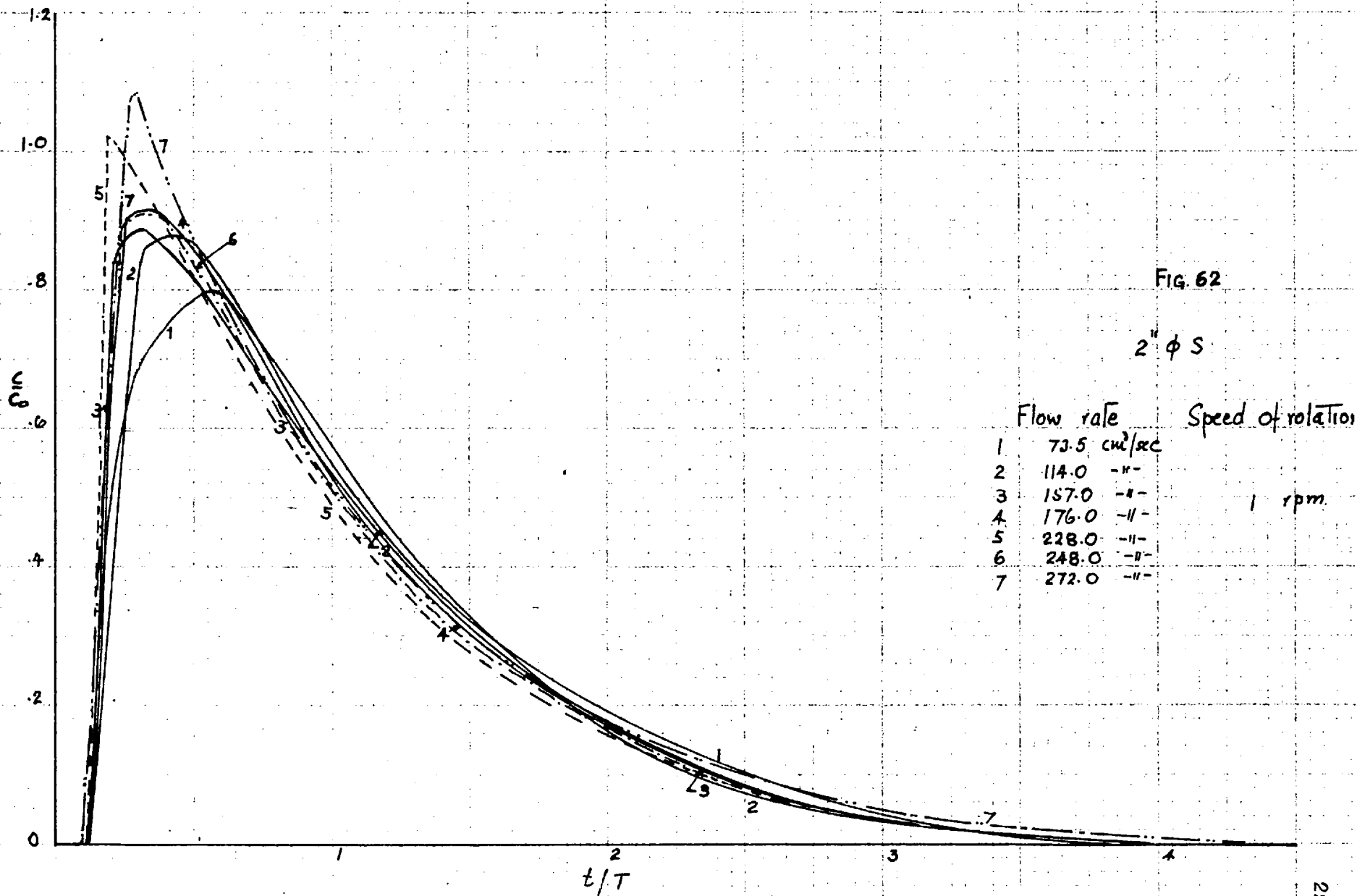


FIG. 60

$2^\circ \phi P$







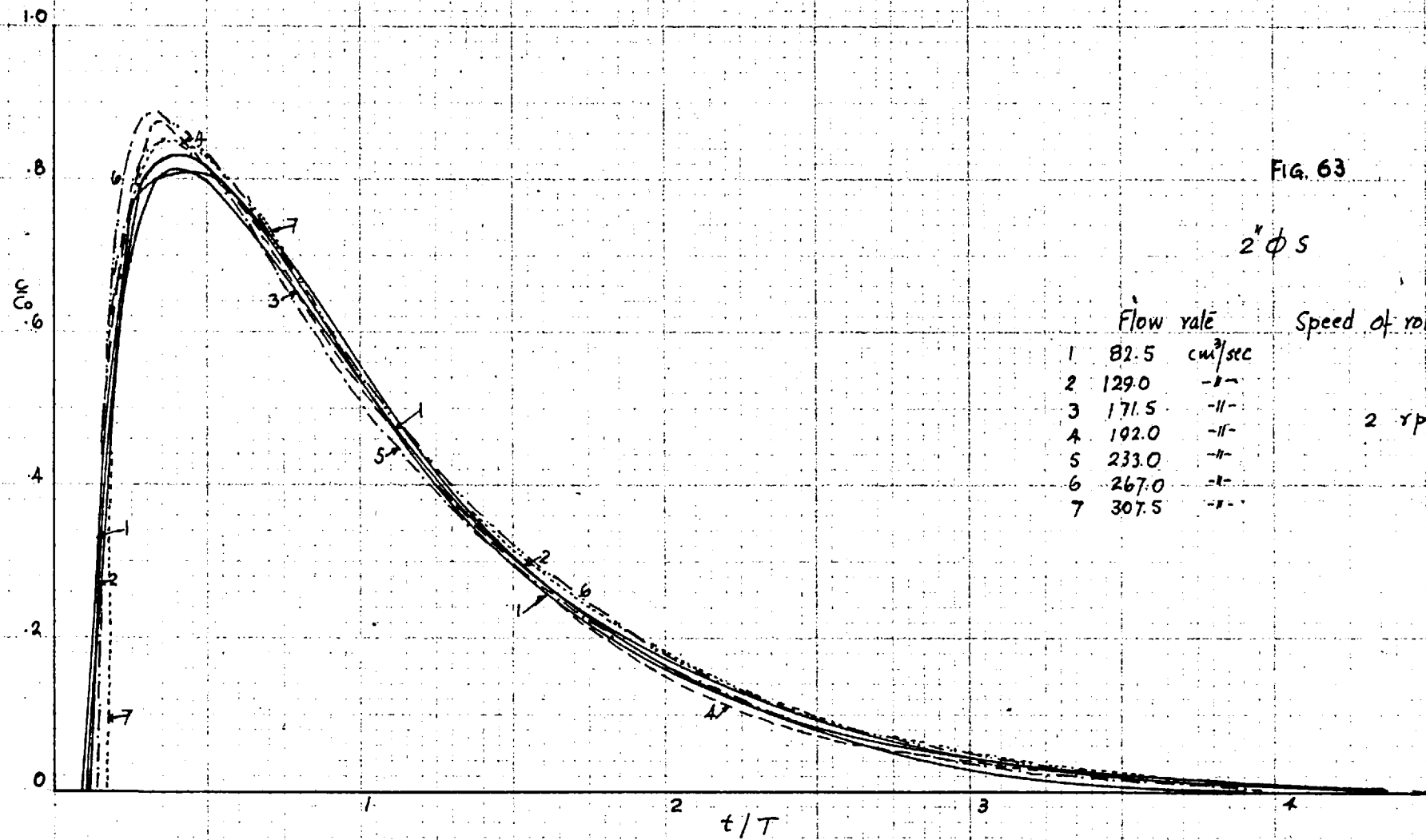


FIG. 63

$2\phi_s$

	Flow rate	Speed of rotation
1	82.5 cm ³ /sec	2 rpm.
2	129.0 —	
3	171.5 —	
4	192.0 —	
5	233.0 —	
6	267.0 —	
7	307.5 —	

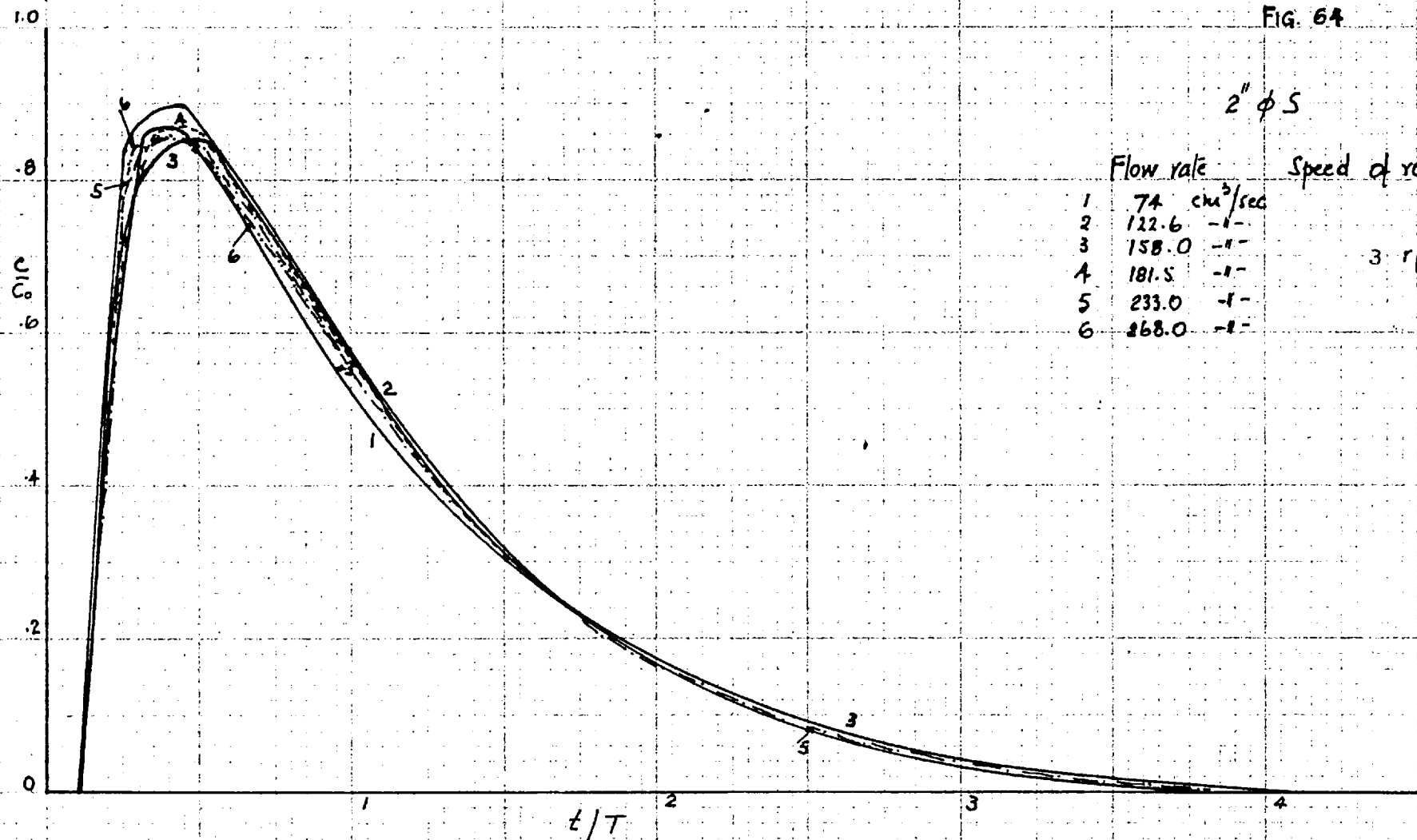
FIG. 64

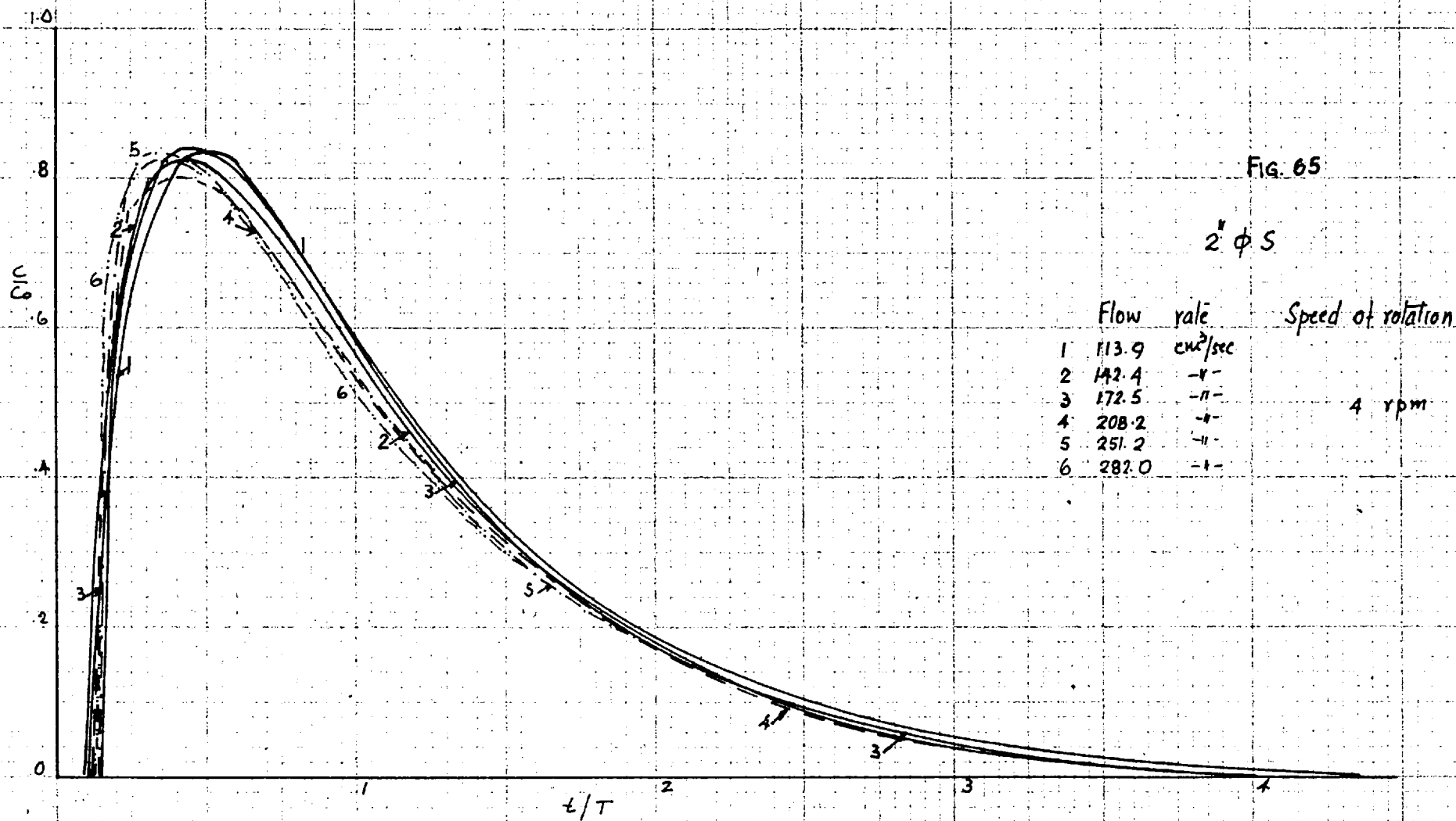
2" ϕ S

Flow rate Speed of rotation

1	74	cm ³ /sec
2	122.6	"
3	158.0	"
4	181.5	"
5	233.0	"
6	268.0	"

3 rpm.





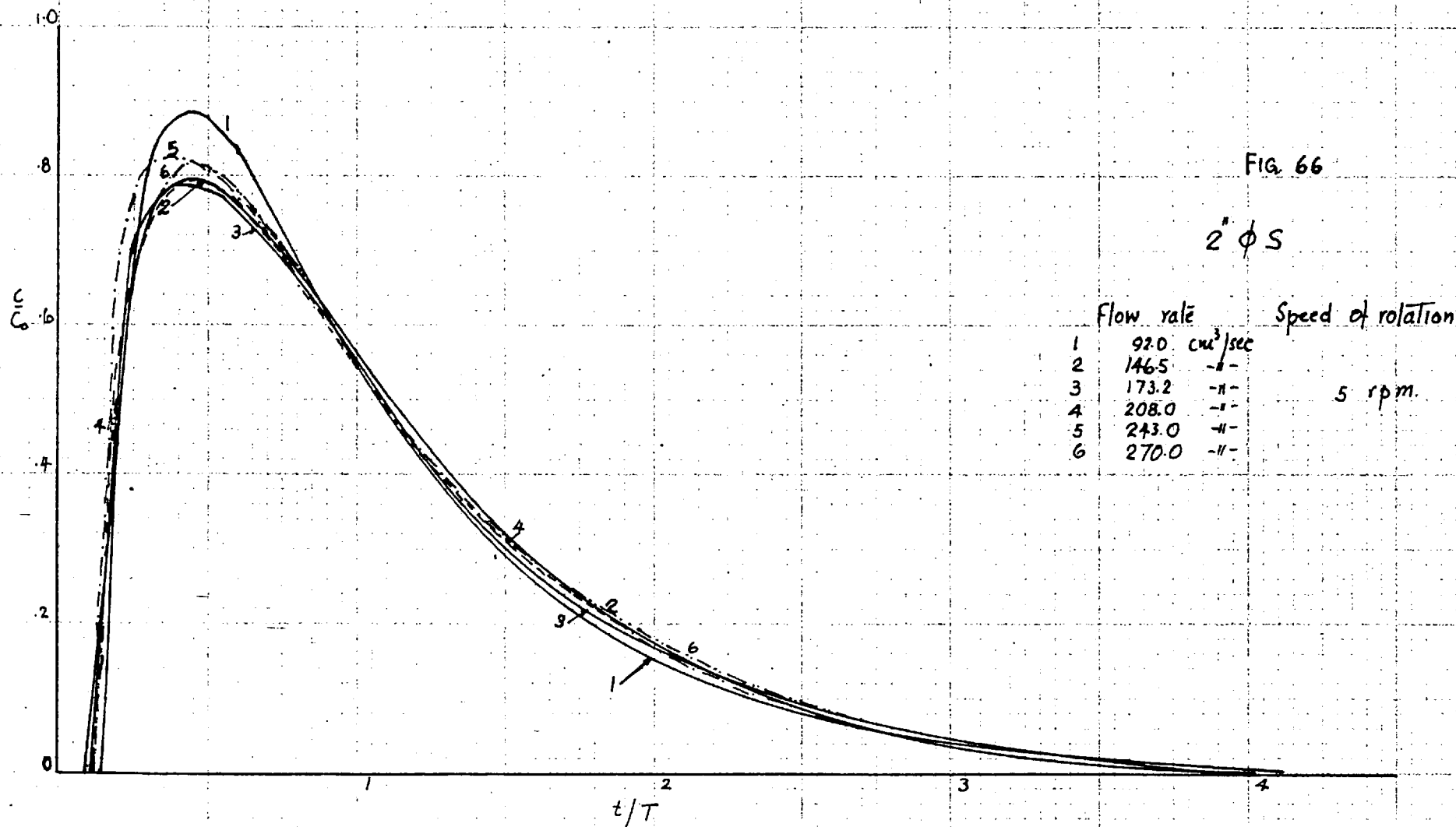


FIG. 67

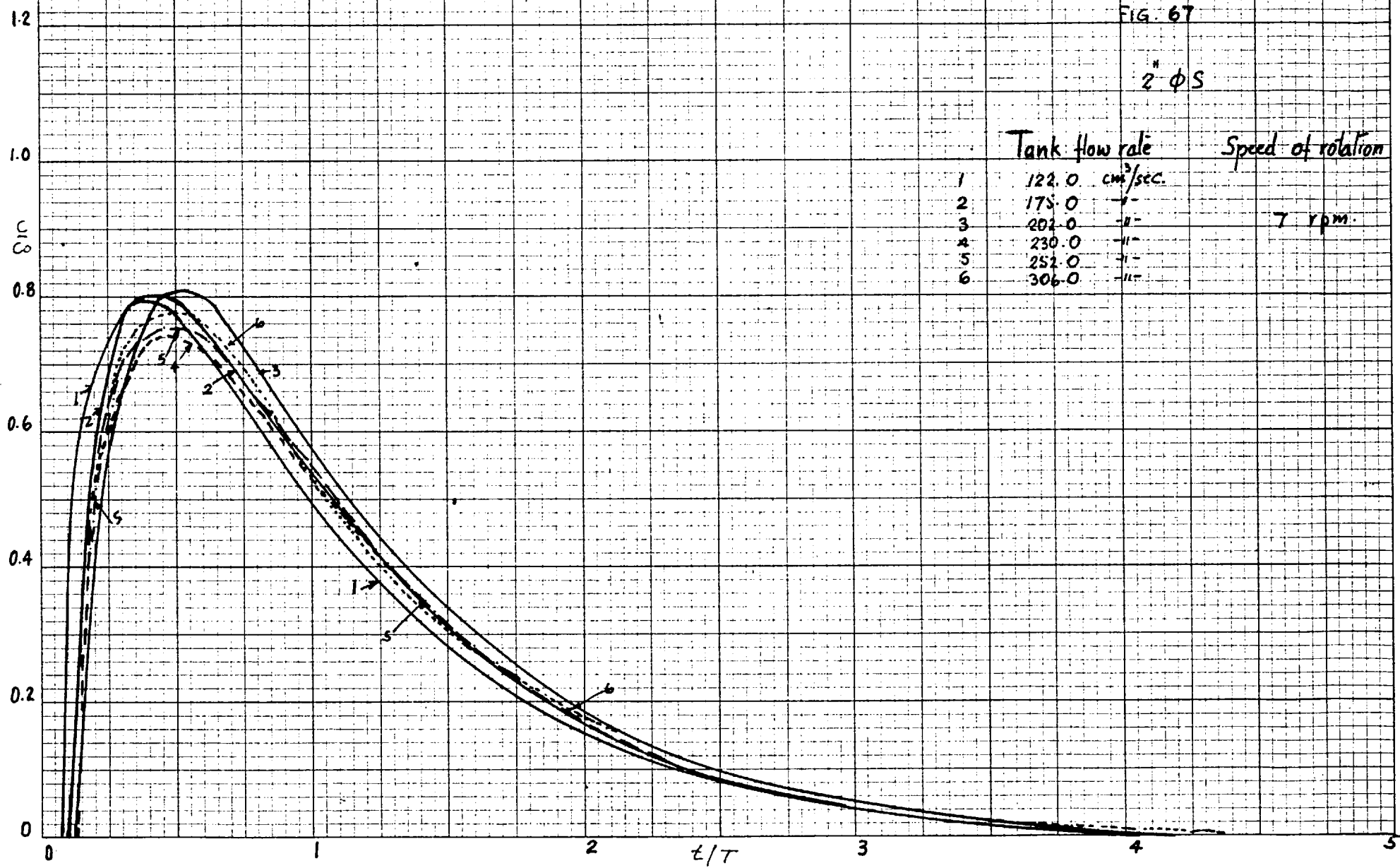
$2^* \phi S$

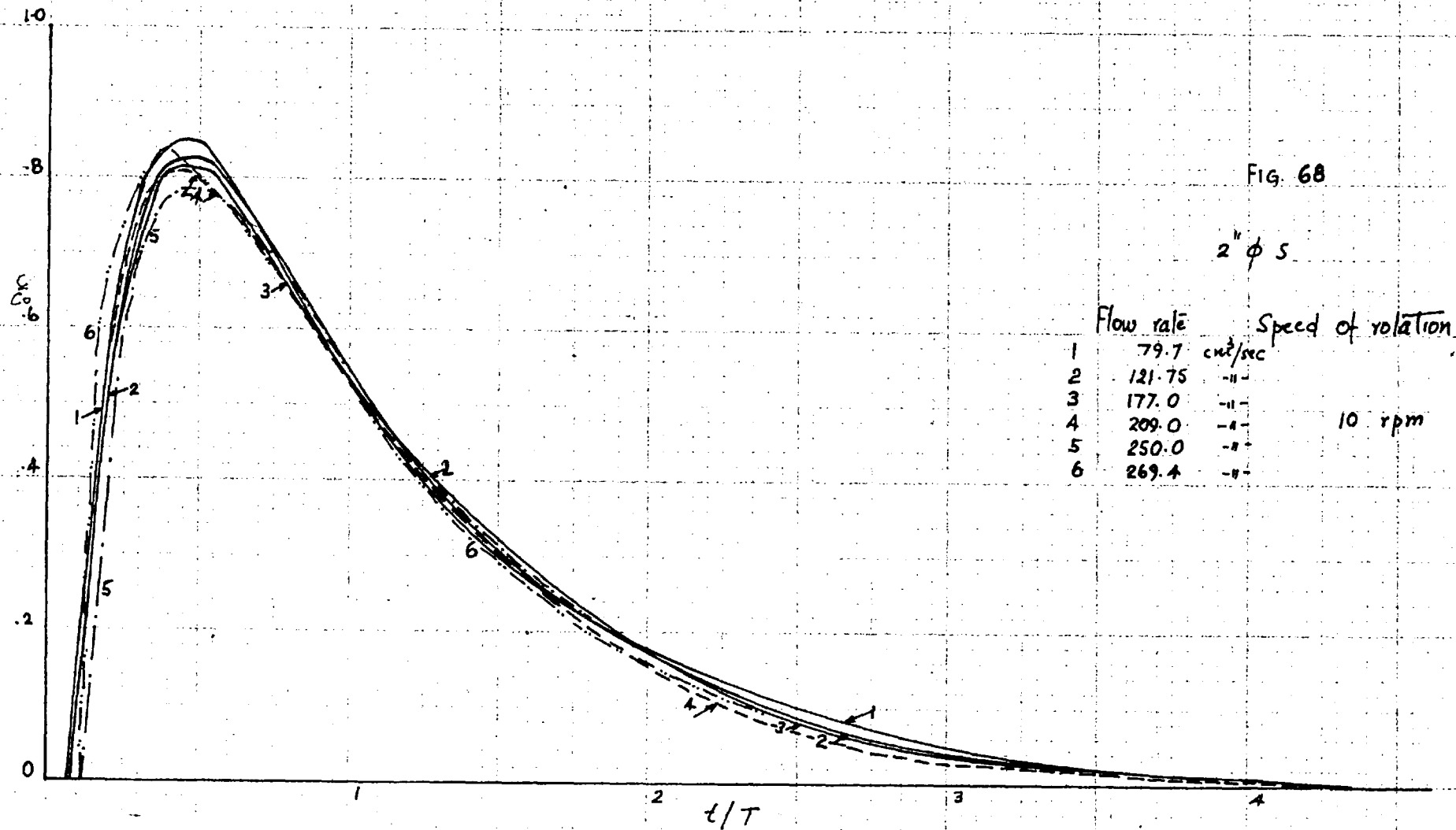
Tank flow rate

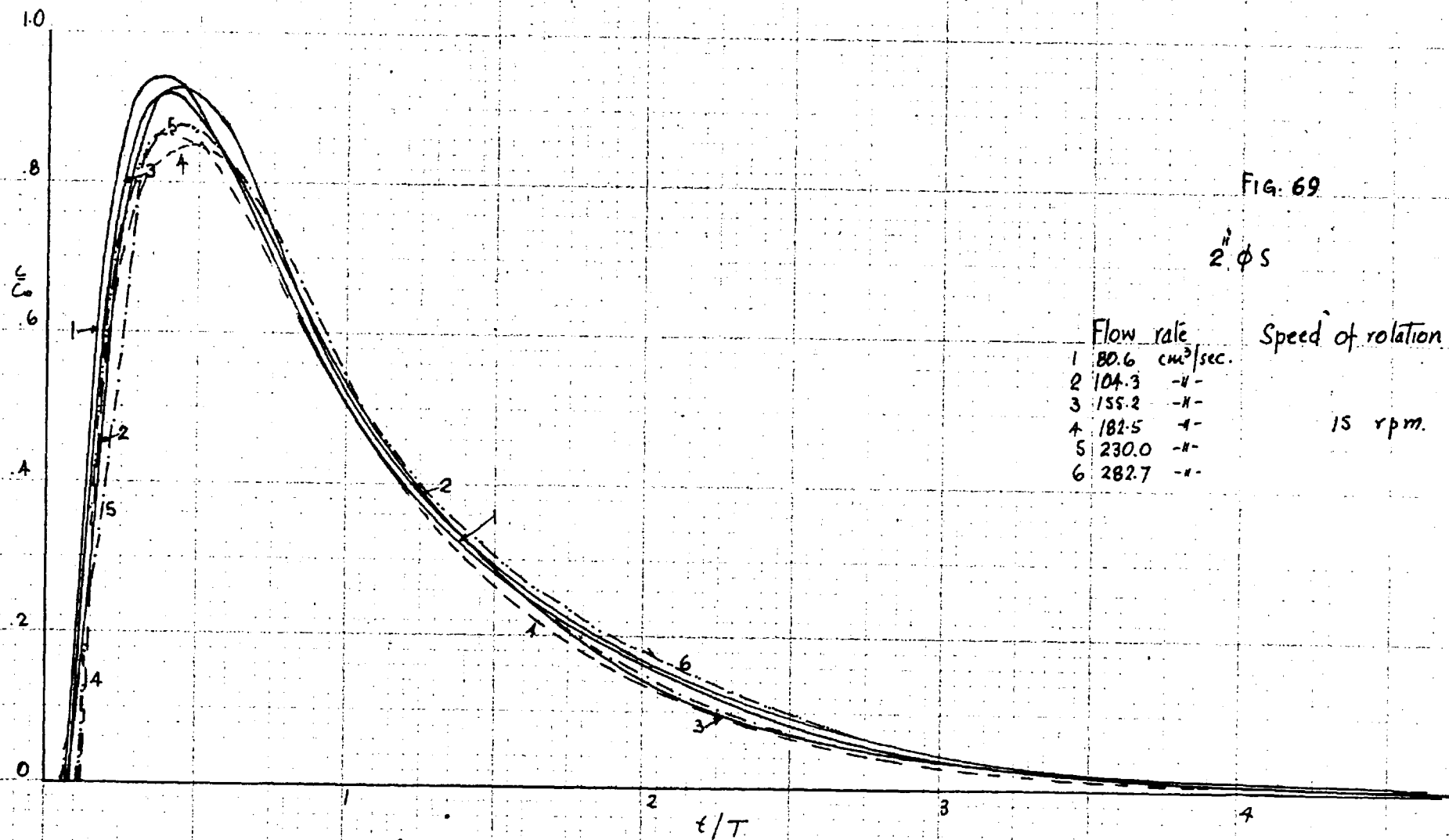
Speed of rotation

1	122.0	cm ³ /sec.
2	175.0	"
3	202.0	"
4	230.0	"
5	252.0	"
6	306.0	"

7 rpm.







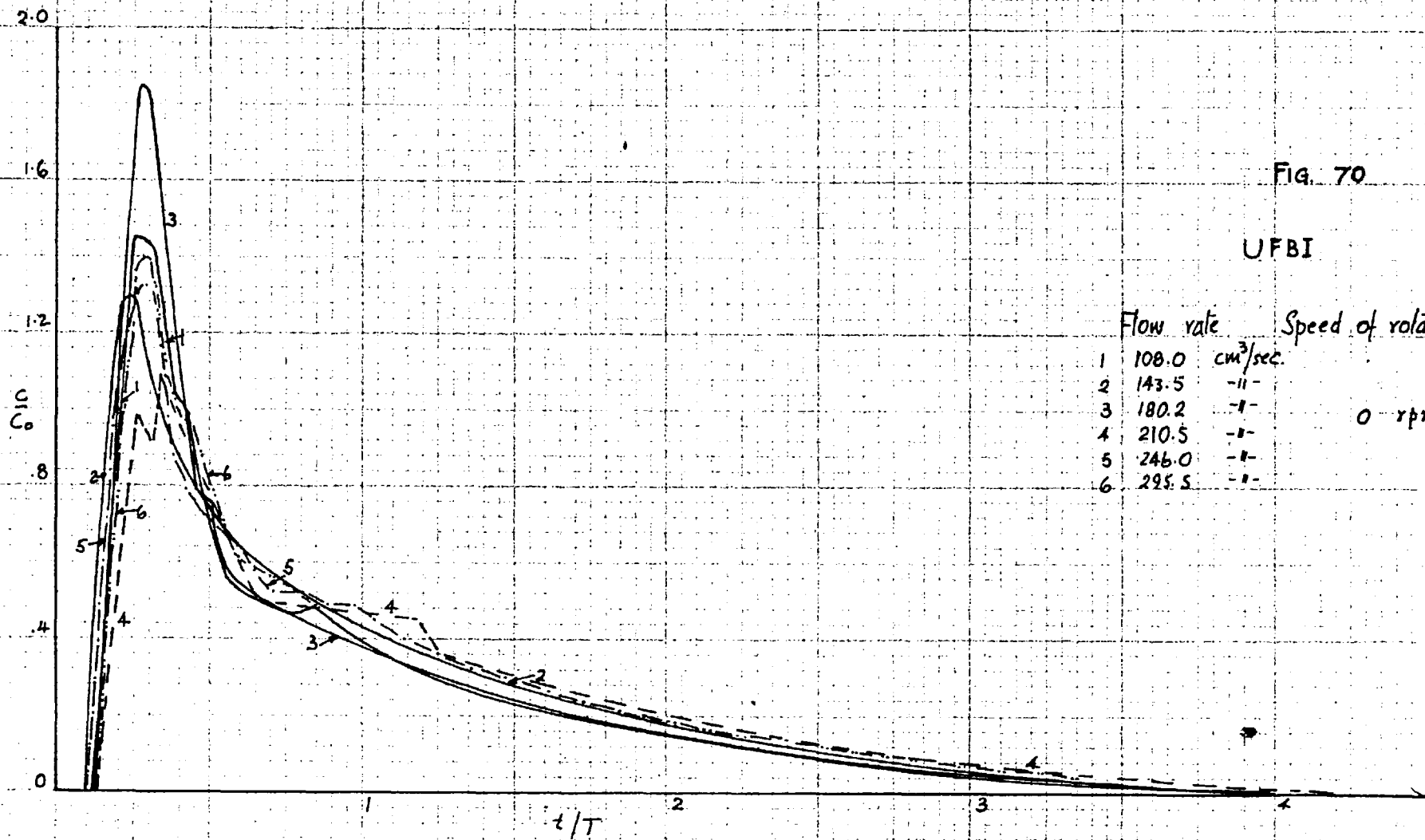


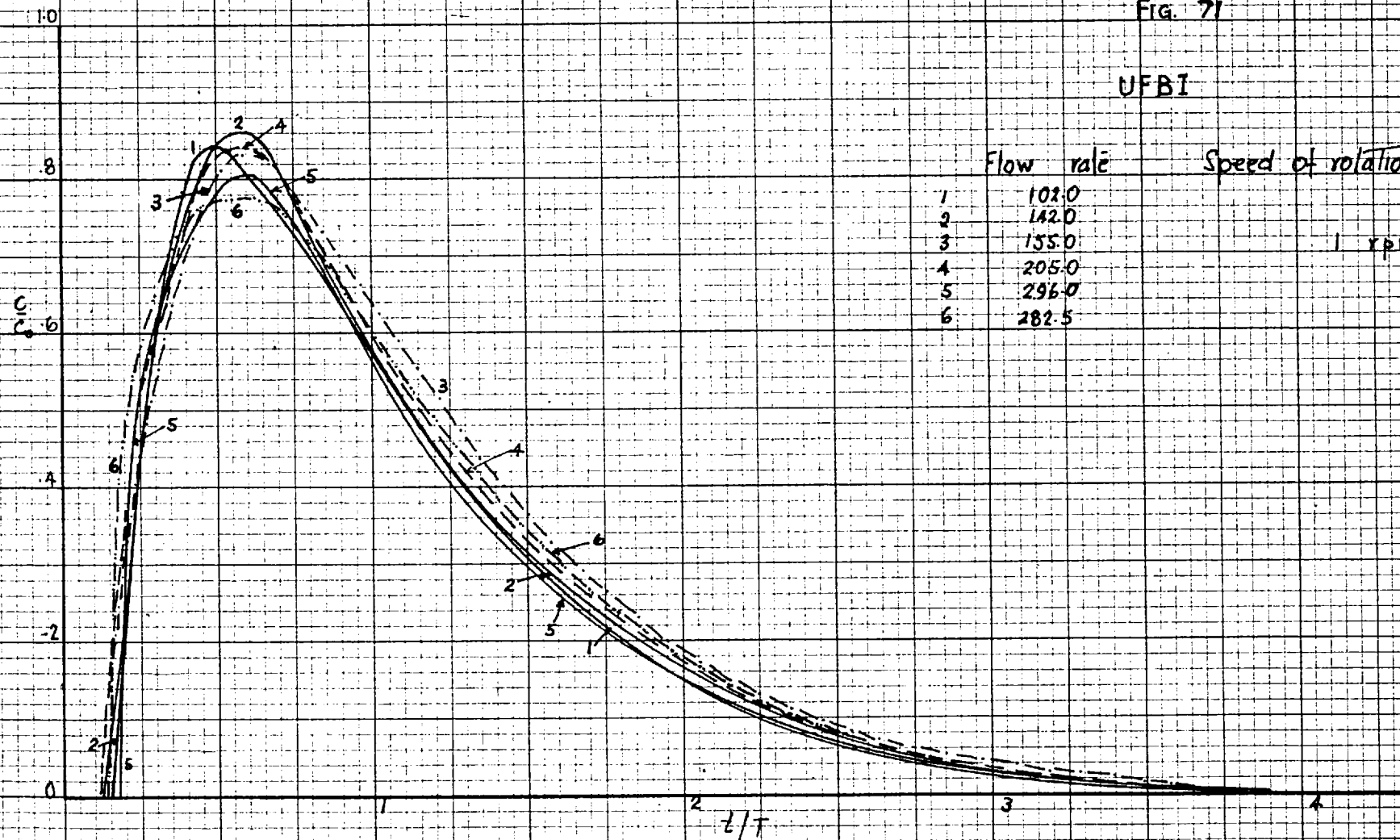
Fig. 71

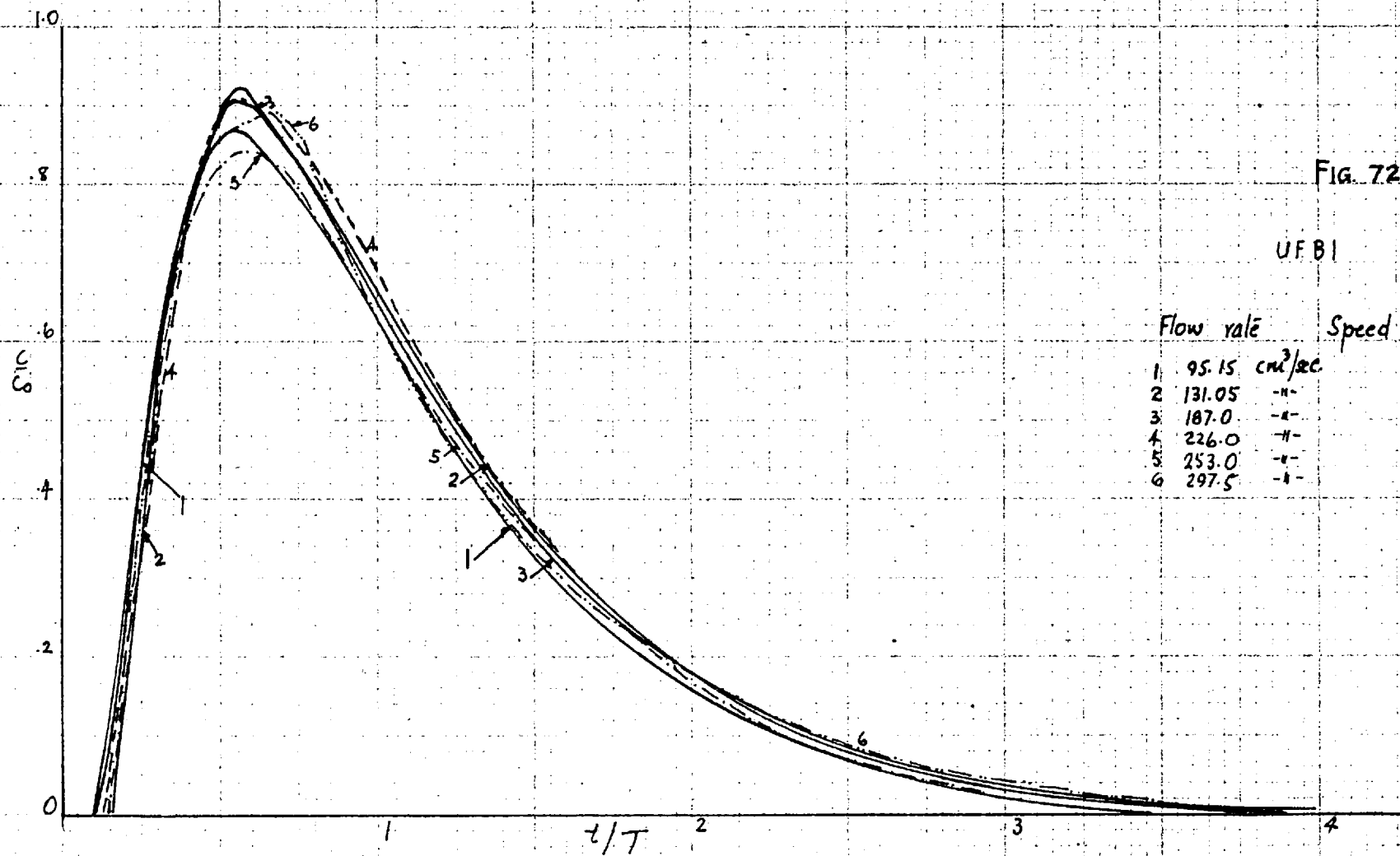
UFBI

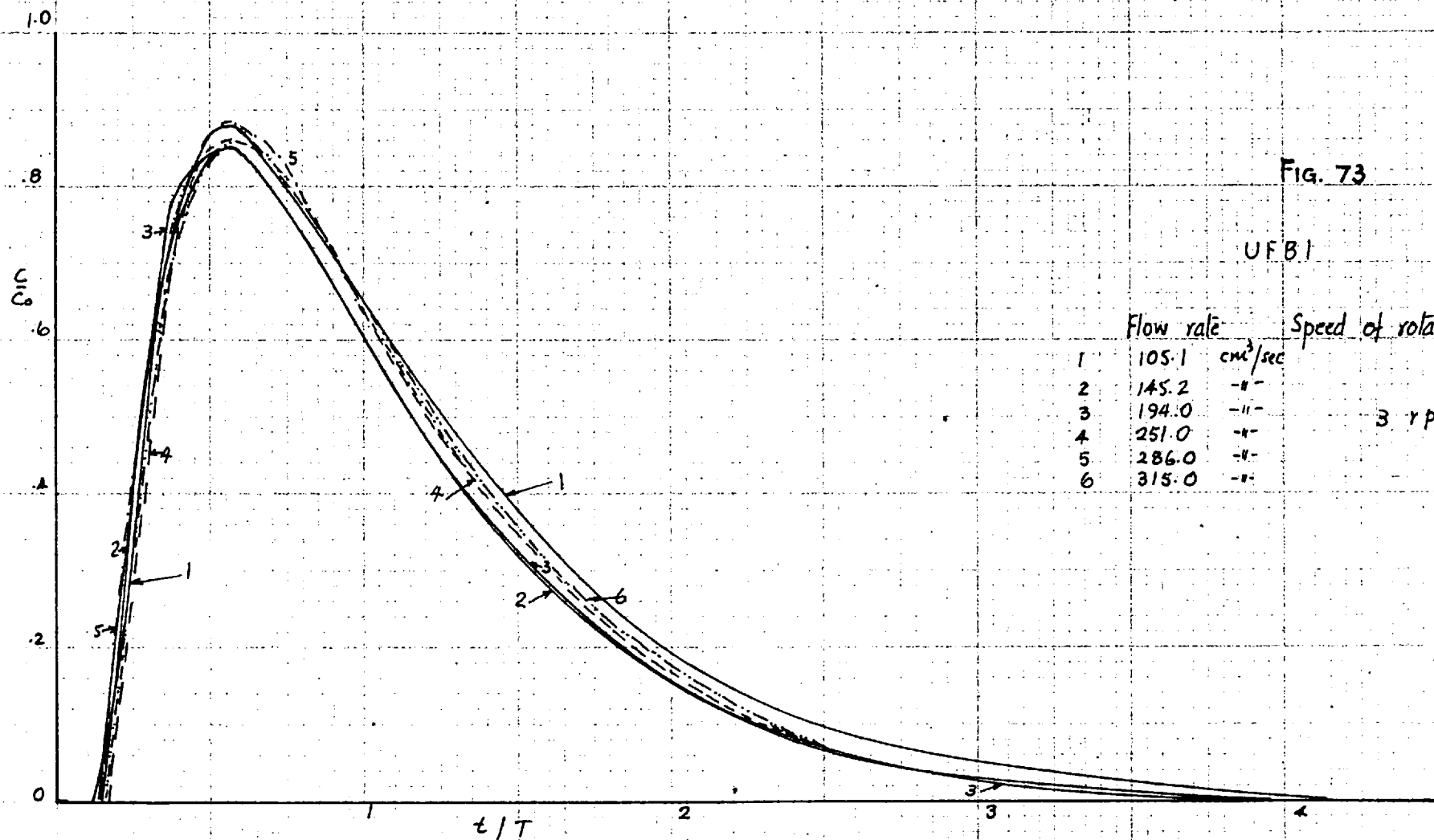
Flow rate Speed of rotation

1	102.0
2	142.0
3	155.0
4	205.0
5	296.0
6	282.5

1 rpm







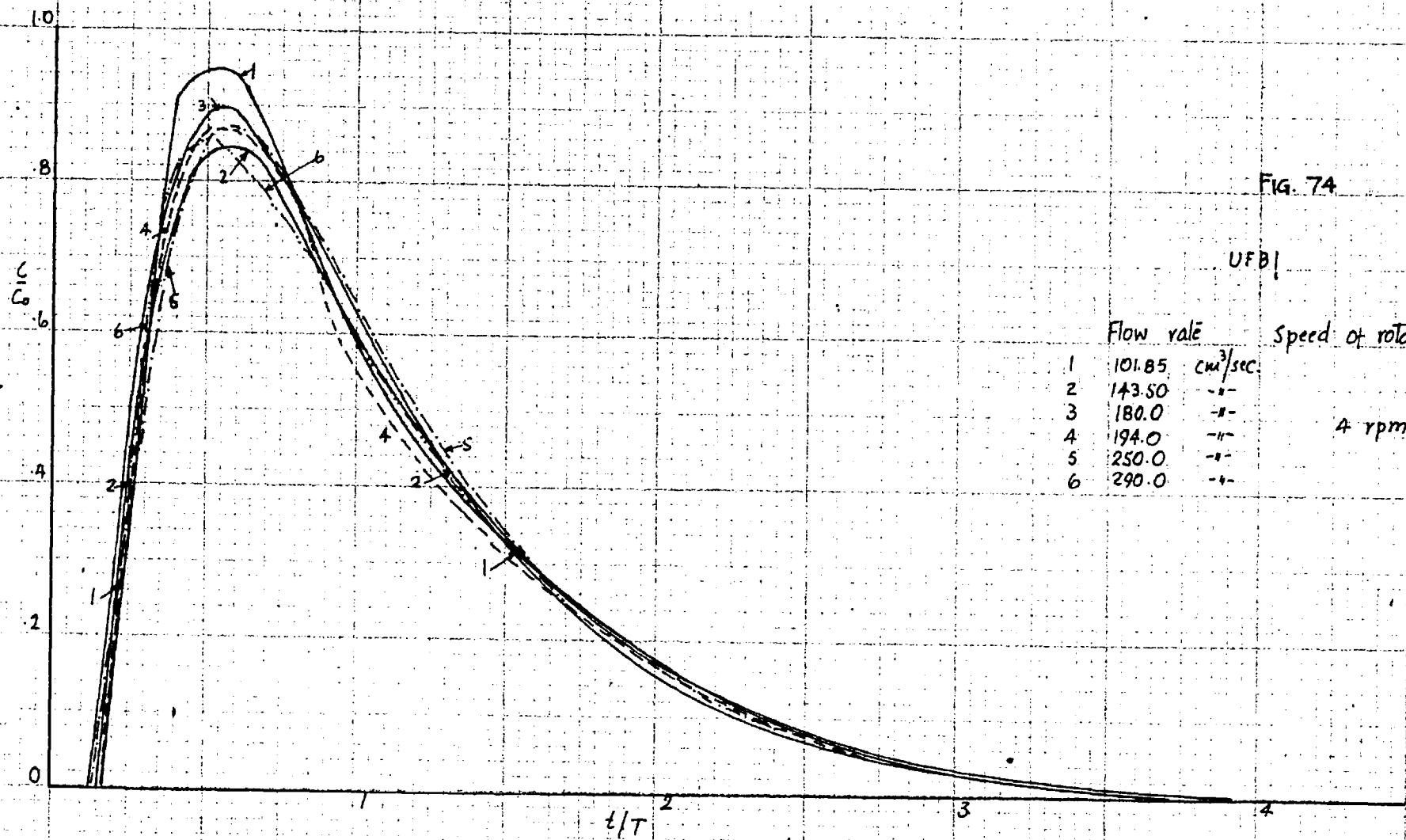


FIG. 74

UFB/

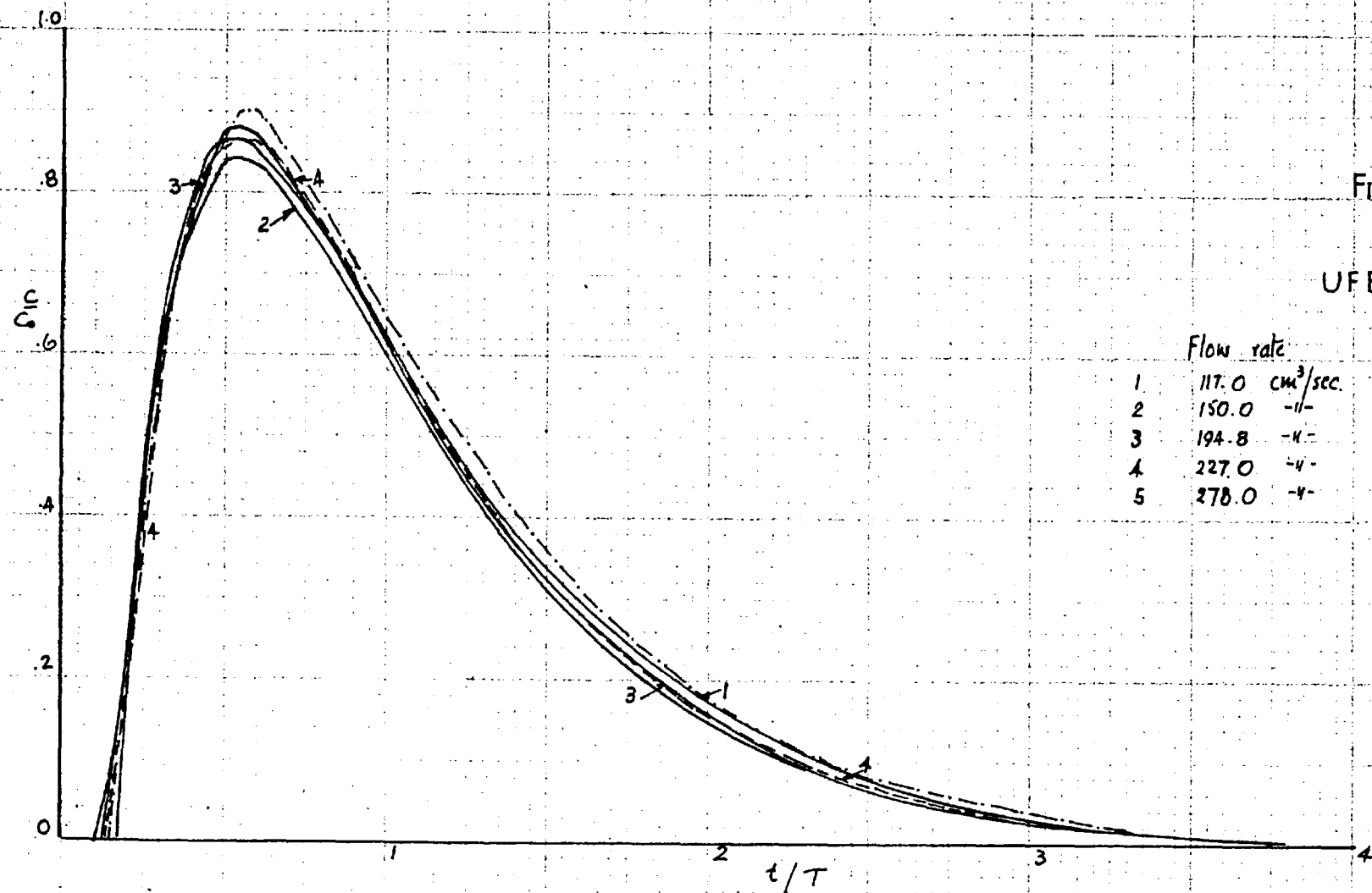
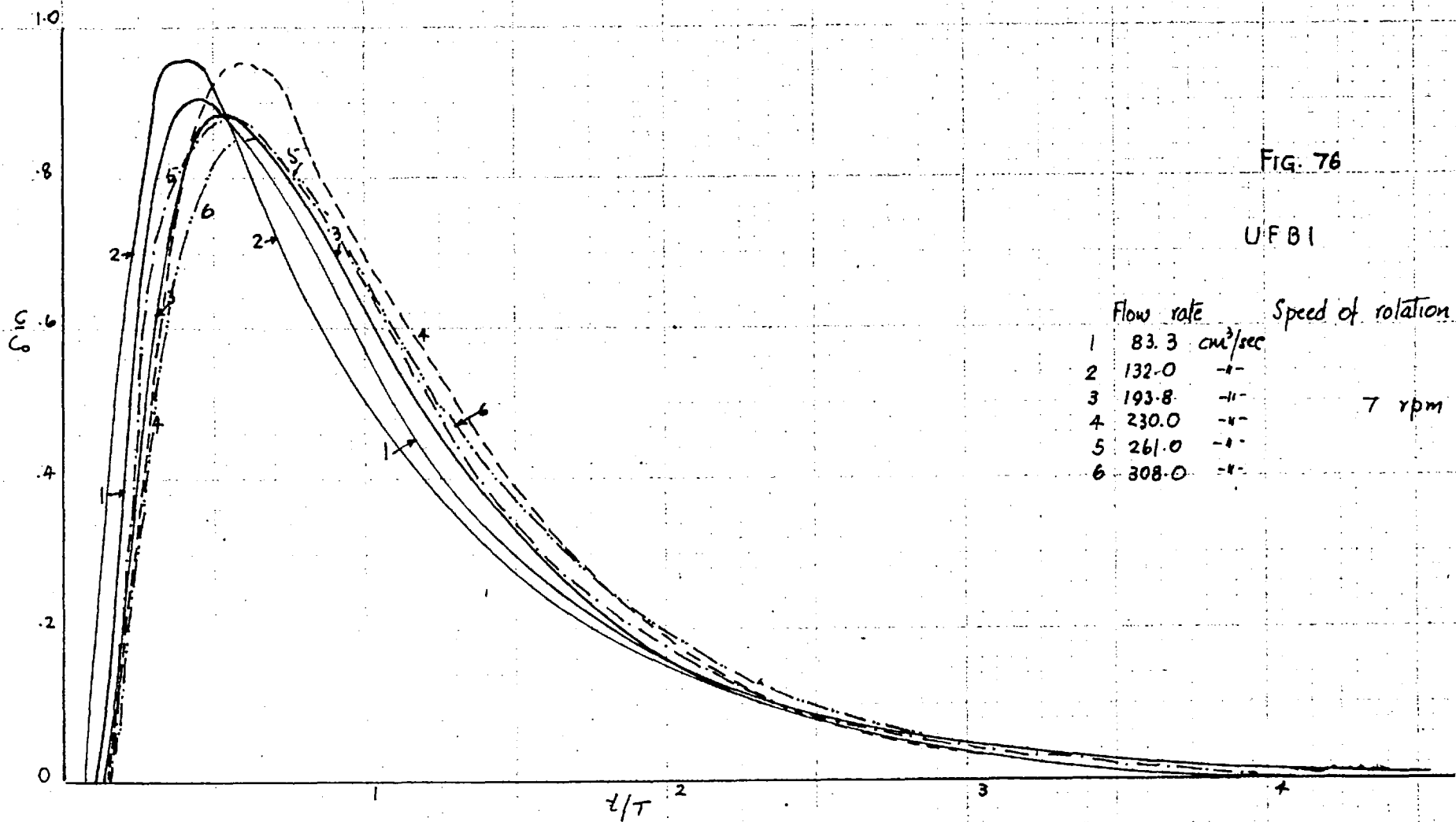
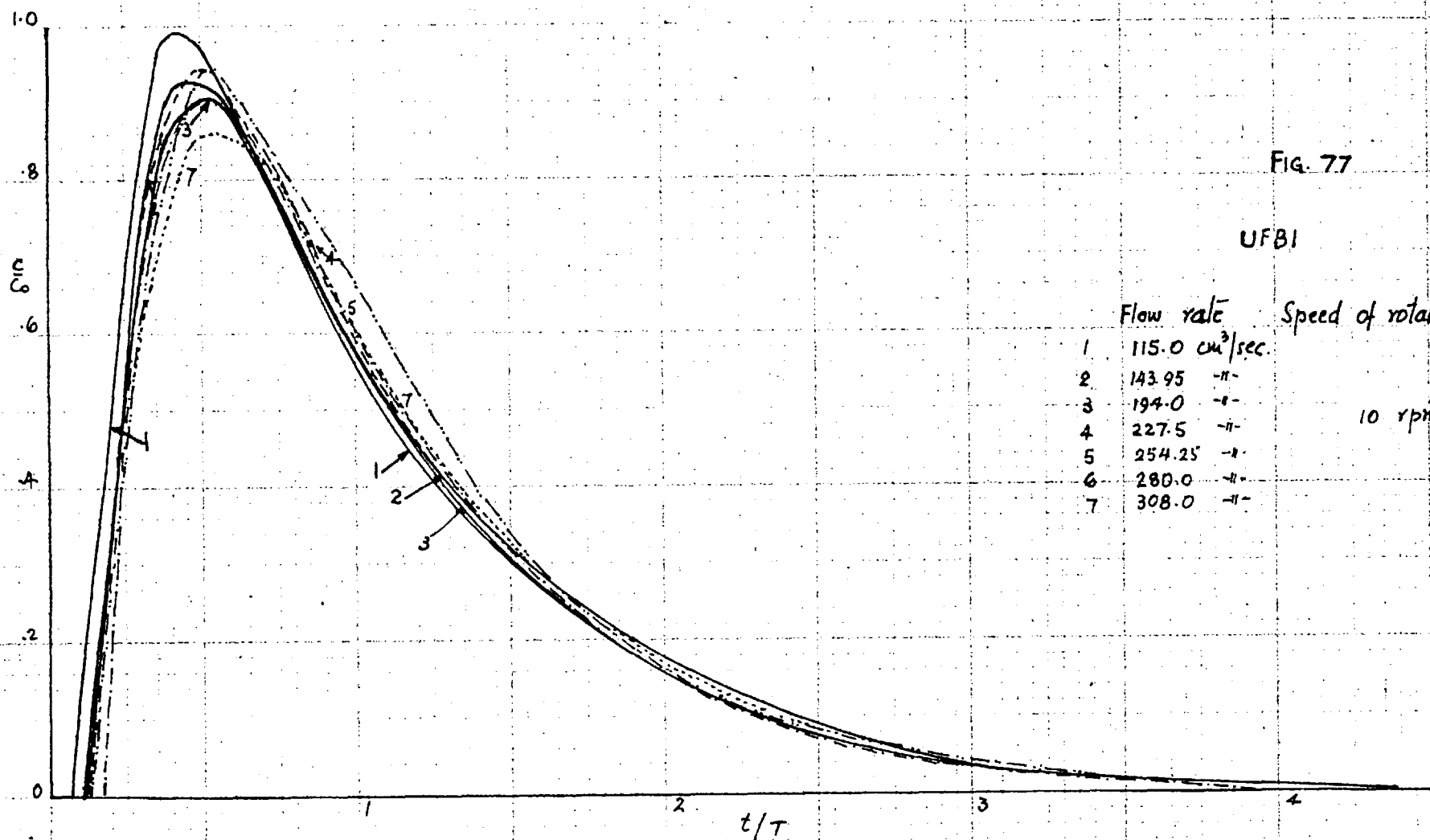


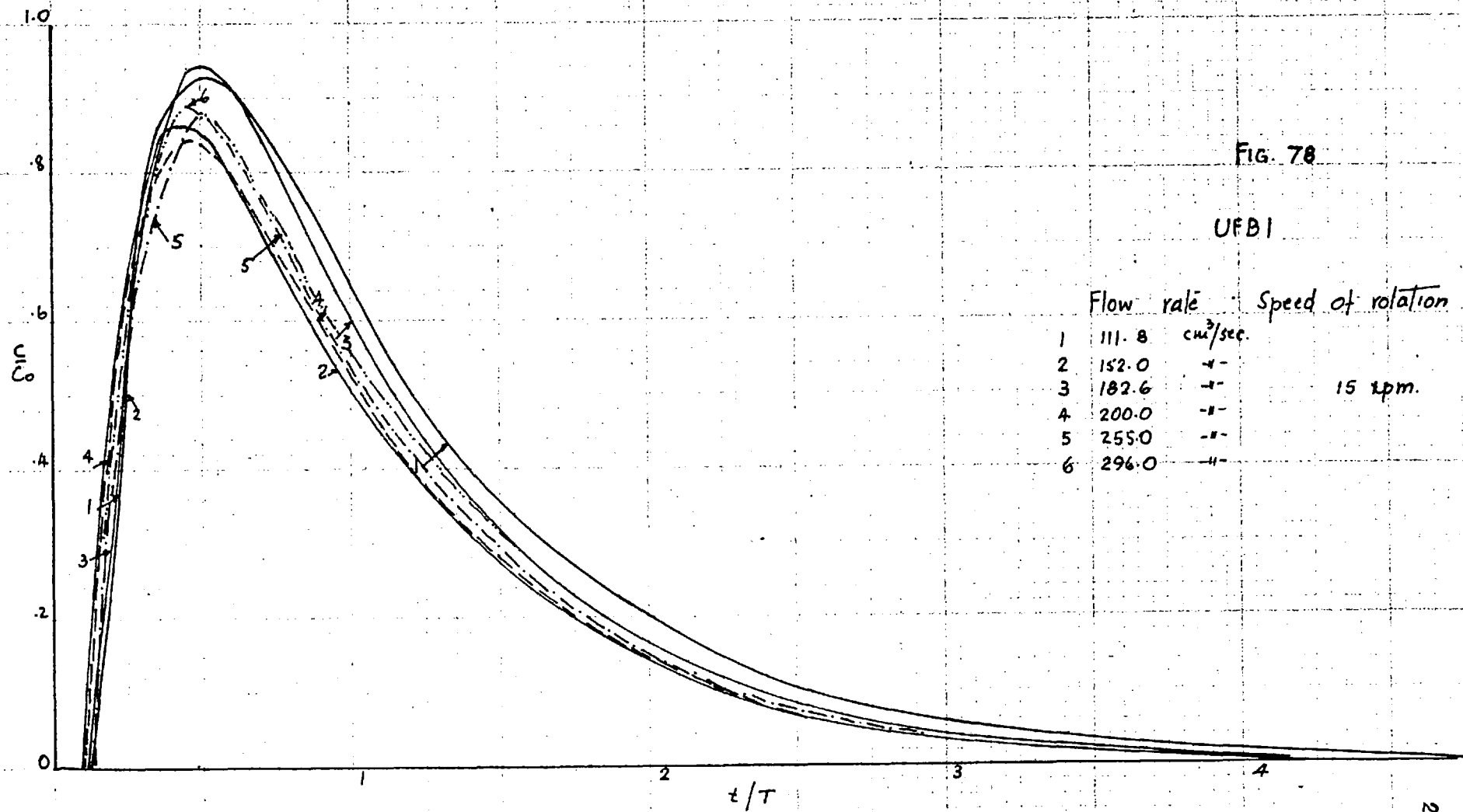
FIG. 75

UFBI

	Flow rate	Speed of rotation
1	117.0 $\text{cm}^3/\text{sec.}$	5 rpm.
2	150.0 -"-	
3	194.8 -"-	
4	227.0 -"-	
5	270.0 -"-	







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