

Understanding Near-Field Wireless Power Transfer Systems With Complex Mutual Inductance and Negative Mutual Resistance

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Abstract—Mutual inductance traditionally appears as a scalar quantity in electric circuit theory. However, in wireless power transfer (WPT) systems with dissipative media such as seawater and human tissues, the complex nature of mutual inductance has been observed. Another phenomenon of negative mutual resistance (NMR) also puzzles many engineers in the WPT community. While complex mutual inductance (CMI) and NMR have been studied separately in the literature, they cannot be separated in many WPT systems. This article reviews and examines CMI and NMR in a holistic manner for WPT. Based on electric circuit theory, this article presents an extended WPT theory with a generalized analysis and circuit model of WPT systems to explain the physical nature and coexistence of CMI and NMR. This extended WPT theory shows that these two elements not only co-exist in WPT systems with dissipative media but also in any WPT system with at least one extra induced dissipative current loop in intended relay resonators and unintended conducting paths such as unintended metallic objects and electromagnetic shielding materials. A methodology for optimizing the WPT systems using the CMI and NMR through recompensation of the resonator design to enhance mutual coupling is provided. The theory is illustrated with several WPT examples with practical measurements and simulations.

Index Terms—Complex mutual inductance (CMI), magnetic coupled resistance, wireless power transfer (WPT).

I. INTRODUCTION

MUTUAL inductance and magnetic coupling coefficient usually appear as a real number under ideal and lossless conditions. They are normally expressed as a scalar quantity in electric circuit theory [1]. For wireless power transfer (WPT) applications (with air as medium [2], [3], [4], [5], [6] and human tissues as medium [7]), mutual inductance and

magnetic coupling coefficient are commonly represented as a scalar. However, mutual inductance or its equivalent magnetic coupling coefficient has been observed and expressed as a complex number as early as 2008 in [8] and then in other biomedical [9], [10], [11] and underwater [12], [13], [14], [15] WPT applications. Published in 2008, Su et al. [8] use the mutual impedance comprising a real term and an imaginary term to account for the power loss in the electromagnetic shield and mutual inductance, respectively, in a WPT system. In 2011, Poon et al. [9] point out that early biomedical WPT studies were conducted with near-field magnetic coupling below 10 MHz because of the tissue absorption loss, but the optimal operating frequencies for WPT in human tissues fall into the low-GHz range. Therefore, while a magnetically coupled equivalent circuit is used to represent the biomedical WPT systems, wave equations are adopted to analyze the WPT process in high-frequency (low-GHz) studies. It is explained in [10] that the mutual inductance for near-field magnetic coupled implantable devices can be represented as a scalar quantity, but it could become a complex number for high-frequency operation through a combination of inductive and radiative modes.

In 2012, Lee et al. [11] show that the relay resonators can shift the optimal operating frequency of a wireless power domino system slightly below the nominal resonant frequency of the resonators (i.e., slightly capacitive mode), indicating that the presence of the relay resonators could cause a phase shift in the WPT process. It is interesting to note that a similar shift of optimal operating frequency is also observed in an underwater WPT application reported in 2015 [12]. In 2020, eddy current loss and detuning effect of seawater were examined for WPT in [13], which includes a phase-shift factor to the mutual inductance. Following [13], a study on WPT in attenuating media is reported in a physics journal in 2021 [14], in which the self-inductances and mutual inductances are generally described in complex forms. Chu et al. [14], Sengupta and Weisshaar [15], Chen et al. [16], Wang et al. [17], and Harris and Ricketts [18], published in subsequent years, point to the same complex nature of the mutual inductance highlighted in [8], [9], and [13] for WPT systems in the presence of conductive and magnetic slabs or metasurfaces.

While complex mutual inductance (CMI) has emerged as a research topic in WPT, the presence of negative mutual resistance (NMR) also puzzles the WPT community. NMR is not a new concept, and it was previously studied in

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coupled oscillator circuits and core-based coupled inductors and transformers. Instead of dissipating energy like a resistor, a negative resistor device can inject energy into a circuit. Negative resistance plays a crucial role in oscillator design. Active circuits, comprising bipolar junction transistors (BJTs) or metal–oxide–semiconductor field-effect transistors (MOSFETs), can provide negative resistance by supplying energy to the LC resonant tank to sustain continuous oscillation. In the absence of this energy contribution, the oscillations would gradually diminish due to the circuit's inherent internal resistance. Two NMR LC oscillator techniques are reported in [19] and [20], in which the oscillation is sustained by coupling two LC oscillators through a transistor circuit.

NMR is also noted in recent power electronics research. For closely coupled core-based inductors and transformers, the proximity effects could induce power losses in the closely coupled winding [21], [22]. These effects are not noticeable at low frequencies, but become obvious in high-frequency applications. While proximity effects between the transmitter resonator and receiver resonator in a WPT system are normally negligible, the circuit transformation of the coupled circuit model (including the mutual resistance in [21] and [22]) to the T-model is relevant to this article.

While WPT has been described in physics journals and engineering journals, this article explains WPT with electric circuit theory that is easily understood by researchers and engineers. This article has two major contributions. First, while the CMI and NMR were reported separately over the last two decades in various power electronics and WPT publications, these topics are addressed together under one roof. Second, a generalized reduced-order WPT model is presented to explain the CMI and NMR phenomena in an integrated manner. A general analysis of a WPT system is presented with this generalized circuit model that can cover a wide range of WPT applications including the presence of: 1) lossless or lossy medium; 2) relay resonators; 3) intended and unintended electromagnetic shielding materials; and 4) metallic objects. The analysis shows that CMI occurs whenever there is at least one induced dissipative current loop in the WPT system apart from the currents in the transmitter and receiver resonators. This induced dissipative current loop can exist not only in dissipative media [14] such as seawater [12], [13] and human tissues [9], [10], but also in relay resonators [11], intended metallic object such as a corona ring [22], and other conductive and magnetic materials [8], [15], [16], [17], [18].

II. TWO-COIL WPT SYSTEM WITH AIR AS POWER FLOW MEDIUM

Fig. 1 shows a typical two-coil WPT system with air as the power flow medium. The resonator capacitors for circuit compensation are not included here because they do not contribute directly to the magnetic coupling between the transmitter and receiver coils. The mutual inductance M_{TR} is a scalar in this model. Based on the equivalent circuit model in Fig. 1(b), the mathematical relationship governing this two-coil WPT system is obtained

$$\begin{bmatrix} V_T \\ V_R \end{bmatrix} = \begin{bmatrix} j\omega L_T + R_T & j\omega M_{TR} \\ j\omega M_{TR} & j\omega L_R + R_R \end{bmatrix} \begin{bmatrix} I_T \\ I_R \end{bmatrix} \quad (1)$$

where L_T and L_R are the self-inductance of transmitter and receiver coils, respectively, while R_T and R_R are the equivalent

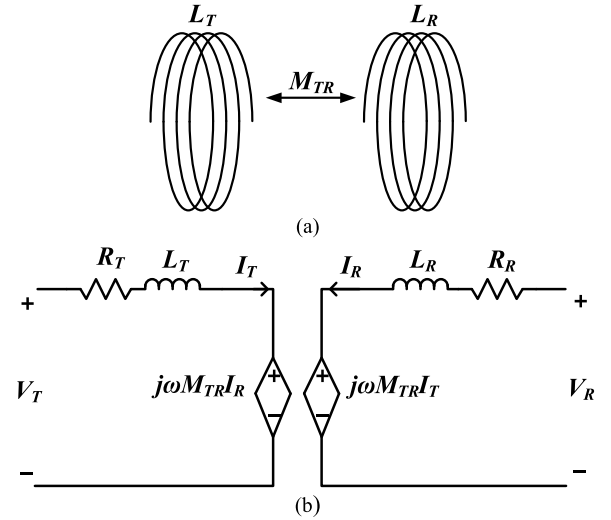


Fig. 1. Typical two-coil WPT system with air as medium. (a) Two magnetically coupled coils. (b) Equivalent magnetically coupled circuit model.

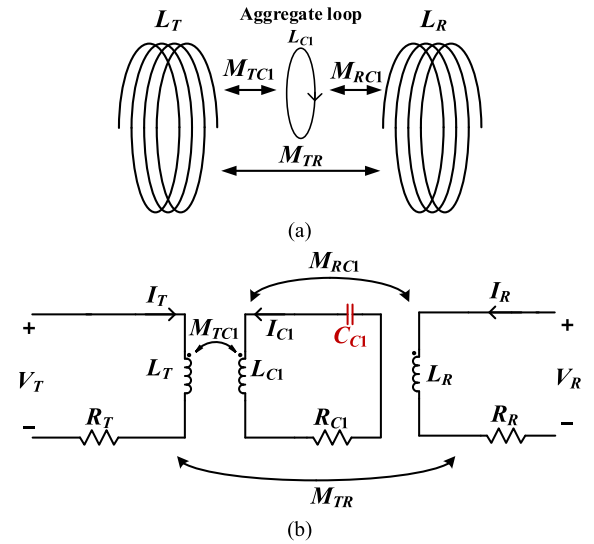


Fig. 2. Two-coil WPT system in lossy media. (a) Aggregate loop for modeling eddy current and power loss in the media. (b) Equivalent circuit model with an aggregate loop L_{C1} .

series resistance of the coils, respectively. This second-order system equation will be used as a reference in the remaining parts of this article.

III. GENERALIZED WPT SYSTEM MODEL WITH AN EXTRA AGGREGATE CURRENT LOOP

In this section, a generalized WPT equivalent circuit covering a wider range of applications is presented with a thorough analysis. To account for various WPT situations, an aggregate current loop represented by a closed loop of a resistor R_{C1} , capacitor C_{C1} , and inductor L_{C1} is introduced into the two-coil WPT system to form an arbitrary three-coil system, as shown in Fig. 2. It is noteworthy that using a three-coil system to model a WPT system is not new. In a study of a WPT system in the presence of a corona ring, which is an intended metallic object, the induced current loop in this ring is modeled as an inductive–resistive loop in the form of a third coil in

addition to the transmitter and receiver coils [22]. For WPT in seawater, Kim et al. [23] and Niu et al. [24] model the induced eddy current as if it is the loop current in a third inductive-resistive coil. After reducing the three-coil WPT system to an equivalent two-coil system, Kim et al. [23] have an equivalent circuit with CMI, but the corresponding mutual resistance is positive. In [24], the equivalent circuit consists of extra resistance arising from the eddy current power loss, but the mutual inductance is scalar.

In the generalized model with an aggregate induced current loop of this extended WPT theory, the mutual inductance is complex and co-exists with mutual resistance. These characteristics have physical meanings. In this model, M_{TC1} is the mutual inductance between the transmitter coil and the aggregate loop (C1), while M_{RC1} is the mutual inductance between the receiver coil and the aggregate loop. The characteristics of this aggregate current loop can be represented by proper choices of the parametric values. In principle, this induced aggregate current loop could represent the induced eddy current loop in a dissipative medium, the induced current in intended/unintended metallic, magnetic, and ferromagnetic objects, and the induced current in relay resonators.

A. Generalized Reduced-Order Circuit Model

For general analysis, a compensation capacitor C_{C1} is introduced to the aggregate current loop. Assuming the aggregate loop impedance to be

$$Z_{C1} = R_{C1} + j\left(\omega L_{C1} - \frac{1}{\omega C_{C1}}\right) \quad (2)$$

where the reactance of the aggregate current loop is $X_{C1} = \omega L_{C1} - 1/(\omega C_{C1})$.

The mathematical relationship governing this arbitrary three-coil system is derived as

$$\begin{bmatrix} V_T \\ V_R \\ 0 \end{bmatrix} = \begin{bmatrix} R_T + j\omega L_T & j\omega M_{TR} & j\omega M_{TC1} \\ j\omega M_{TR} & R_R + j\omega L_R & j\omega M_{RC1} \\ j\omega M_{TC1} & j\omega M_{RC1} & R_{C1} + Z_{C1} \end{bmatrix} \begin{bmatrix} I_T \\ I_R \\ I_{C1} \end{bmatrix} \quad (3)$$

where R_T , R_{C1} , and R_R are the equivalent resistances in the transmitter resonator, relay resonator, and the receiver resonator, respectively. Note that the equivalent load resistance R_L of the WPT system is not separately represented in (3). The effect of R_L will be addressed in Section III.

To apply the Kron reduction technique to eliminate the matrix elements related to the aggregate loop [25], the third-order matrix equation of (3) can be reduced to an equivalent second-order matrix equation [22] as follows:

$$\begin{bmatrix} V_T \\ V_R \end{bmatrix} = \begin{bmatrix} j\omega L'_T + R'_T & Z_{TCR} \\ Z_{TCR} & j\omega L'_R + R'_R \end{bmatrix} \begin{bmatrix} I_T \\ I_R \end{bmatrix} \quad (4)$$

where the off-diagonal element $Z_{TCR} = j\omega M_{TCR}$ represents the impedance of the equivalent mutual inductance (M_{TCR}) between the transmitter coil and receiver coil in the reduced-order mathematical model.

The equivalent mutual inductance (M_{TCR}) of the reduced-order model can be expressed as

$$M_{TCR} = M_{TR} - \frac{j\omega M_{TC1} M_{RC1}}{Z_{C1}}$$

$$= M_{TR} - \frac{j\omega M_{TC1} M_{RC1} (R_{C1} - jX_{C1})}{X_{C1}^2 + R_{C1}^2}$$

$$M_{TCR} = \left[M_{TR} - \frac{\omega M_{TC1} M_{RC1} \left(\omega L_{C1} - \frac{1}{\omega C_{C1}}\right)}{R_{C1}^2 + \left(\omega L_{C1} - \frac{1}{\omega C_{C1}}\right)^2} \right] - j \left[\frac{\omega M_{TC1} M_{RC1} R_{C1}}{R_{C1}^2 + \left(\omega L_{C1} - \frac{1}{\omega C_{C1}}\right)^2} \right] \quad (5a)$$

$$M_{TCR} = M_x - jM_y. \quad (5b)$$

It is important to note that the addition of an extra aggregate current loop to the two-coil model would lead to an effective mutual inductance expressed in complex number form. From (5a) and (5b), the real part of the M_{TCR} is

$$M_x = M_{TR} - \frac{\omega M_{TC1} M_{RC1} \left(\omega L_{C1} - \frac{1}{\omega C_{C1}}\right)}{R_{C1}^2 + \left(\omega L_{C1} - \frac{1}{\omega C_{C1}}\right)^2} \quad (6)$$

and the magnitude of the imaginary part is

$$M_y = \frac{\omega M_{TC1} M_{RC1} R_{C1}}{R_{C1}^2 + \left(\omega L_{C1} - \frac{1}{\omega C_{C1}}\right)^2}. \quad (7)$$

The equivalent mutual inductance in (5b) can also be expressed in its AC impedance form as

$$Z_{TCR} = j\omega M_{TCR} = j\omega M_x + \omega M_y. \quad (8)$$

In (8), the positive real part ωM_y is related to the dissipative (resistive) nature of this induced current loop. The imaginary part $j\omega M_x$, related to WPT through mutual magnetic coupling, affects the WPT and its efficiency.

The mathematical expressions of other circuit parameters of the generalized reduced-order circuit model are listed as follows:

$$L'_T = L_T - \frac{\omega M_{TC1}^2 \left(\omega L_{C1} - \frac{1}{\omega C_{C1}}\right)}{R_{C1}^2 + \left(\omega L_{C1} - \frac{1}{\omega C_{C1}}\right)^2} \quad (9a)$$

$$L'_T = L_T - \Delta L_T \quad (9b)$$

$$R'_T = R_T + \frac{(\omega M_{TC1})^2 R_{C1}}{R_{C1}^2 + \left(\omega L_{C1} - \frac{1}{\omega C_{C1}}\right)^2} \quad (10a)$$

$$R'_T = R_T + \Delta R_T \quad (10b)$$

$$L'_R = L_R - \frac{\omega M_{RC1}^2 \left(\omega L_{C1} - \frac{1}{\omega C_{C1}}\right)}{R_{C1}^2 + \left(\omega L_{C1} - \frac{1}{\omega C_{C1}}\right)^2} \quad (11a)$$

$$L'_R = L_R - \Delta L_R \quad (11b)$$

$$R'_R = R_R + \frac{(\omega M_{RC1})^2 R_{C1}}{R_{C1}^2 + \left(\omega L_{C1} - \frac{1}{\omega C_{C1}}\right)^2} \quad (12a)$$

$$R'_R = R_R + \Delta R_R \quad (12b)$$

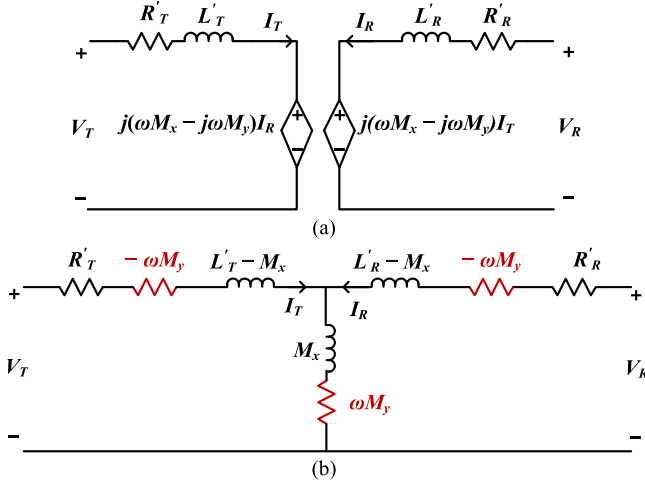


Fig. 3. (a) Coupled inductor model with CMI. (b) T-circuit equivalent of the coupled inductor model with CMI.

where $\Delta L_T, \Delta L_R, \Delta R_T$, and ΔR_R are the “modification factors” arising from the presence of the extra induced current in the WPT process.

Based on (6), (9), and (11)

$$M_x = M_{TR} - \sqrt{\Delta L_T \Delta L_R} \quad \forall \left(\omega L_{C1} > \frac{1}{\omega C_{C1}} \right) \quad (13a)$$

$$M_x = M_{TR} + \sqrt{\Delta L_T \Delta L_R} \quad \forall \left(\omega L_{C1} < \frac{1}{\omega C_{C1}} \right) \quad (13b)$$

$$M_x = M_{TR} \quad \forall \left(\omega L_{C1} = \frac{1}{\omega C_{C1}} \right). \quad (13c)$$

Based on (7), (10), and (12)

$$\omega M_y = \sqrt{\Delta R_T \Delta R_R}. \quad (14)$$

Based on these equivalent circuit parameters, the reduced-order coupled inductor model and the corresponding T-circuit model are shown in Fig. 3(a) and (b), respectively. As mainstream power electronics circuit simulation software, such as PLECS and LTSPICE, does not support CMI, the equivalent T-circuit model in Fig. 3(b) is particularly useful and compatible with circuit simulators. The mutual resistance ωM_y introduces a negative resistance at both the transmitter and receiver sides, with the potential of reducing the overall winding resistance and thus loss. The mathematical proof verifying the equivalence between the T-circuit model and the coupled inductor model with CMI is presented in the Appendix.

Important Remark: If the NMR (ωM_y) takes an arbitrary value, it could pose a risk of presenting a net negative resistance to the primary inverter, potentially causing oscillator-like instability. To maintain passivity in the model, the parameters are strictly physically bounded

$$\begin{cases} R'_T \geq 0 \\ R'_R \geq 0 \\ R'_T R'_R - (\omega M_y)^2 \geq 0. \end{cases} \quad (15)$$

The proof is given in the Appendix.

B. Model Parameter Extraction for the Generalized Reduced-Order Circuit Model

Similar to the traditional transformer model, open-circuit tests and short-circuit tests can be applied to the T-circuit model in Fig. 3(b) in order to extract the model parameters. These tests assume that practical tests can be conducted on the transmitter and receiver sides. In the case where practical tests cannot be conducted on the receiver side (e.g., medical implants), finite-element simulations can be used to perform the tests on the receiver side.

1) *Open-Circuit Tests:* Under open-circuit tests, the open-circuit voltage is

$$V_T^{oc} = (R'_T + j\omega L'_T) I_T^{oc} \quad (16)$$

where the subscript T refers to the transmitter side, and the superscript OC refers to open-circuit tests.

Let the open-circuit voltage vector be the reference, $V_T^{oc} = |V_T^{oc}| \angle 0^\circ$. The open-circuit current is

$$\begin{aligned} \angle I_T^{oc} &= |I_T^{oc}| \angle \left[-\tan^{-1} \left(\frac{\omega L'_T}{R'_T} \right) \right] \\ \angle &= \frac{|V_T^{oc}|}{\sqrt{(R'_T)^2 + (\omega L'_T)^2}} \angle \left[-\tan^{-1} \left(\frac{\omega L'_T}{R'_T} \right) \right]. \end{aligned} \quad (17)$$

By measuring the open-circuit voltage V_T^{oc} and open-circuit current I_T^{oc} , their phase angle θ_T^{oc} can be obtained

$$\theta_T^{oc} = -\tan^{-1} \left(\frac{\omega L'_T}{R'_T} \right). \quad (18)$$

The power loss in the open-circuit test on the transmitter side is

$$P_T^{oc} = (I_T^{oc})^2 R'_T.$$

With the measurements of P_T^{oc} and I_T^{oc} , R'_T can be obtained as

$$R'_T = \frac{P_T^{oc}}{(I_T^{oc})^2}. \quad (19)$$

From (18) and (19)

$$L'_T = \left(\frac{R'_T}{\omega} \right) \tan(\theta_T^{oc}). \quad (20)$$

If the open-circuit tests can be repeated (or a finite-element simulation can be performed) on the receiver side

$$I_R^{oc} = \frac{|V_R^{oc}|}{\sqrt{(R'_R)^2 + (\omega L'_R)^2}} \angle \left[-\tan^{-1} \left(\frac{\omega L'_R}{R'_R} \right) \right].$$

By measuring the open-circuit voltage V_R^{oc} and open-circuit current I_R^{oc} , their phase angle θ_R^{oc} can be obtained

$$\theta_R^{oc} = -\tan^{-1} \left(\frac{\omega L'_R}{R'_R} \right). \quad (21)$$

The power loss in the open-circuit test on the receiver side is

$$P_R^{oc} = (I_R^{oc})^2 R'_R.$$

With the measurements of P_R^{oc} and I_R^{oc} , R'_R can be obtained as

$$R'_R = \frac{P_R^{oc}}{(I_R^{oc})^2}. \quad (22)$$

From (21) and (22)

$$L'_R = \left(\frac{R'_R}{\omega} \right) \tan(\theta_R^{oc}). \quad (23)$$

2) *Short-Circuit Tests*: By shorting the output terminals of the receiver resonator, measurements for short-circuit tests can be conducted on the transmitter side. The input impedance as seen from the transmitter side Z_T^{sc} can be derived from the short-circuit voltage V_T^{sc} and I_T^{sc}

$$Z_T^{sc} = \frac{V_T^{sc}}{I_T^{sc}} = \left[R'_T + j\omega L'_T - \frac{(\omega M_y + j\omega M_x)^2}{R'_R + j\omega L'_R} \right]. \quad (24)$$

By measuring the short-circuit voltage V_T^{sc} , short-circuit current I_T^{sc} , and their phase angle θ_T^{sc} , the imaginary part of Z_T^{sc} can be obtained

$$\Im(Z_T^{sc}) = \left| \frac{V_T^{sc}}{I_T^{sc}} \right| \sin(\theta_T^{sc}). \quad (25)$$

By measuring the power loss in the short-circuit test on the transmitter side P_T^{sc} , the real part of Z_T^{sc} can be obtained

$$\Re(Z_T^{sc}) = \frac{P_T^{sc}}{(I_T^{sc})^2}. \quad (26)$$

With these measurement data and the parameters obtained from the open-circuit test, M_x and M_y can be obtained as

$$M_x = \frac{\Im \left(\sqrt{\left(R'_T - \frac{P_T^{sc}}{(I_T^{sc})^2} + j \left(\omega L'_T - \left| \frac{V_T^{sc}}{I_T^{sc}} \right| \sin(\theta_T^{sc}) \right) \right) (R'_R + j\omega L'_R)} \right)}{\omega} \quad (27)$$

$$M_y = \frac{\Re \left(\sqrt{\left(R'_T - \frac{P_T^{sc}}{(I_T^{sc})^2} + j \left(\omega L'_T - \left| \frac{V_T^{sc}}{I_T^{sc}} \right| \sin(\theta_T^{sc}) \right) \right) (R'_R + j\omega L'_R)} \right)}{\omega}. \quad (28)$$

It should be noted that the square-root operation results in two solutions. One solution is in the first quadrant and the other in the third quadrant. In practice, the first-quadrant solution is typically adopted.

3) *Finite-Element Simulation Method*: If the magnetically coupled system can be modeled in a simulation software, an alternative and convenient way to obtain the circuit parameters of the generalized reduced-order T-circuit model is to use finite-element simulation software. Modern electromagnetic field simulation software such as High Frequency Structure Simulator (HFSS) generates a 2×2 matrix equation in the form of (4) to represent a WPT system. The matrix equation is typically derived through full-wave electromagnetic simulation using the finite-element method (FEM). The software models the physical geometry of the transmitter and receiver coils, along with their surrounding environment, and solves Maxwell's equations to compute the system's S-parameters (scattering parameters). These S-parameters form a 2×2 system matrix that characterizes how electromagnetic energy is transmitted and reflected between the two ports—one for the transmitting coil and one for the receiving coil. Such 2×2 system matrix can be turned into the Z-parameters in the frequency domain like the following equation for electric circuit analysis. Each element in (4) comprises components that can be derived based on model equations (5)–(14)

$$\begin{bmatrix} V_T \\ V_R \end{bmatrix} = \begin{bmatrix} R'_T + j\omega L'_T & \omega M_y + j\omega M_x \\ \omega M_y + j\omega M_x & R'_R + j\omega L'_R \end{bmatrix} \begin{bmatrix} I_T \\ I_R \end{bmatrix}. \quad (29)$$

C. Physical Meanings and Insights Provided by the Generalized Reduced-Order Circuit Model

The theoretical analysis of this generalized reduced-order model is important because the extended WPT theory provides useful insights and physical meanings to explain the phenomena in a wide range of WPT applications in various types of power flow media and under different conditions.

It is important to note the following points.

- 1) Apart from the currents in the transmitter coil (I_T) and the receiver coil (I_R), the effective mutual inductance can be expressed in complex form whenever there is an extra induced current loop (represented as I_{C1} in the aggregate current loop of the generalized model) that has positive resistance (i.e., dissipative with $R_{C1} > 0$).
- 2) Equations (5)–(8) of this general analysis provide the mathematical details of this CMI. This feature explains why the CMI was observed in WPT applications in WPT system with magnetic shields [8], dissipative media such as seawater [13], [16], [17], [26], and human tissues [9], [10].
- 3) The presence of this extra induced current loop would create its magnetic field that interacts with the magnetic fields created by the transmitter current and the receiver current. Consequently, all equivalent circuit parameters in the reduced-order model have two terms. In (6) and (9)–(12), the first term on the right-hand side of the parameter equation (for M_x , L'_T , R'_T , L'_R , and R'_R) is the same as that in the basic two-coil WPT model with air or vacuum as the power flow medium in Fig. 1. The second term is the “modification factor” arising from the interactions caused by the extra current loop.
- 4) The magnitude of the imaginary part (M_y) of effective mutual inductance M_{TRC} in (5) and (7) is proportional to the resistance R_{C1} in the aggregate current loop. On the one hand, if the aggregate current loop is ideally lossless (i.e., $R_{C1} = 0$), $M_y = 0$ and $M_{TRC} = M_x$, which becomes a scalar. The magnetic parameters M_x , L'_T , and L'_R will retain their second modification factor, but the resistive parameters will be $R'_T = R_T$ and $R'_R = R_R$. On the other hand, if the resistance in this aggregate current loop approaches infinity (i.e., $R_{C1} \rightarrow \infty$), $M_y = 0$, $I_{C1} = 0$, and $M_{TRC} = M_{TR}$, which is also a scalar. In this case, the situation is like the wireless power flow medium is air or vacuum, and there are no nearby conductive or magnetic materials. The aggregate current loop does not exist, and the circuit model is reverted to the standard two-coil WPT model in (1). All modification factors of the reduced-order model in (6) and (9)–(12) become zero.
- 5) For the impedance form of the effective mutual inductance $Z_{TRC} = j\omega M_x + \omega M_y$, the real part (ωM_y) is related to the resistive or dissipative nature of the aggregate induced current loop. This real part is reflected as two NMR terms in the transmitter side and receiver side of the T-circuit model in Fig. 3(b). Therefore, the concepts of CMI and NMR are inseparable in a WPT system in general. The imaginary part ($j\omega M_x$) is related to the magnetic coupling of the transmitter and receiver coils in the presence of the aggregate induced current loop. The general mutual inductance form agrees with that in [8] and [9].

- 6) In circuit theory, NMR can be related to a power source. In the generalized WPT model, the aggregate induced current loop stores energy in its associated magnetic field. The presence of this extra magnetic field interacts with the magnetic fields of the transmitter coil and receiver coil, resulting in an extra phase shift between the transmitter circuit and receiver current. Stored energy transfer from the transmitter coil to the receiver coil momentarily leads to the presence of the two NMR terms in the T-circuit model, as shown in Fig. 3(b) and practically demonstrated later in Section IV-C.
- 7) The off-diagonal element of (4) is the equivalent mutual impedance that is related to the wireless power flow and energy efficiency. Equation (6) for M_x provides an important tool to optimize the power flow and energy efficiency. It is important to note that there is a negative sign before the second term on the right-hand side of (6). This negative second term is proportional to $(\omega L_{C1} - 1/(\omega C_{C1}))$. If the aggregate induced current loop represents one or more relay resonators, by operating the frequency slightly below the natural resonant frequency of the relay resonator (i.e., slightly capacitive mode), the term $(\omega L_{C1} - 1/(\omega C_{C1}))$ becomes negative, and the negative second term of (6) becomes positive. Therefore, the effective mutual inductance can be enhanced to improve the power flow and energy efficiency through frequency compensation. This feature agrees with the independent observations in two WPT systems with relay resonators that the optimal frequency occurs at a frequency slightly below the resonant frequency of the relay resonators in [11] and [12].
- 8) The two NMR terms occur in the horizontal circuit branch of the T-circuit model. This horizontal branch represents the WPT path. The two NMRs have the effect of reducing the overall resistance in this power flow path. By using compensating capacitors to eliminate the leakage inductance in this horizontal branch, the impedance can be minimized for WPT.
- 9) If the aggregate induced current loop represents the induced currents in the relay resonator and ferrite shielding material as shown in Fig. 4, Zhong et al. [27] show that a three-coil WPT system can have energy efficiency higher than a two-coil counterpart for the same transmission distance. In addition, the extra phase shift between the transmitter resonator current and the receiver resonant current caused by the extra induced currents in the relay resonator and ferrite shield is illustrated in [27].

The generalized reduced-order WPT model can be used to explain the phase shift between the transmitter and receiver currents caused by the magnetic field created by the extra aggregate current loop. The practical example in Fig. 4 is used here as an illustration. If series compensating capacitors are included to cancel the leakage inductance in the transmitter and receiver circuits in Fig. 3(b) in WPT applications, two simplified equivalent electric circuits for WPT systems with a medium of air and with a relay resonator with ferrite electromagnetic shields are shown in Fig. 5(a) and (b), respectively. For Fig. 5(b), the aggregate current loop combines the effects of the induced current in the relay resonator

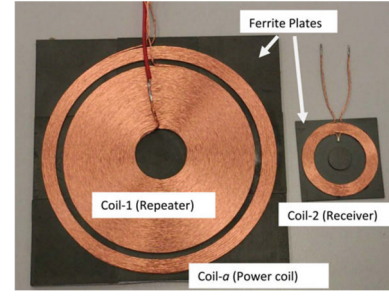


Fig. 4. Three-coil WPT system with a relay resonator and ferrite shields demonstrated in [27].

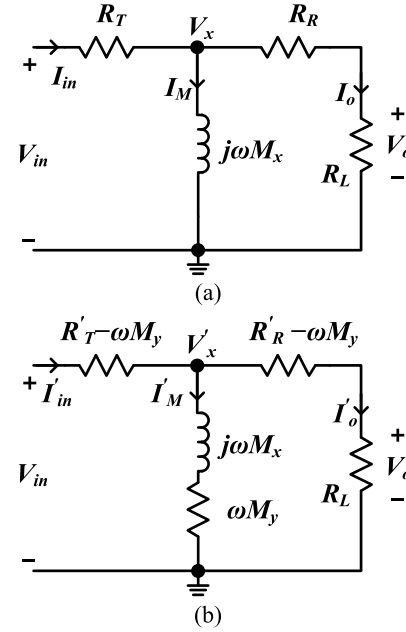


Fig. 5. (a) Equivalent circuit model of the scaler mutual inductance WPT system with canceled leakage inductance. (b) Equivalent circuit model of the CMI WPT system with canceled leakage inductance.

and the eddy current in the ferrite electromagnetic shields. Typical terminologies for transformer circuits are used for explanation. In these two examples, the input current, magnetizing current, and output current are labeled as I_{in} , I_M , and I_o in Fig. 5(a), and I'_{in} , I'_M , and I'_o in Fig. 5(b), respectively.

If the WPT systems in Fig. 5(a) and (b) are driven by a sinusoidal voltage source V_{in} , loaded by a resistor R_L , and the leakage inductance is canceled by the compensation capacitors. The current vectors for the circuit in Fig. 5(a) can be expressed as

$$\vec{I}_o = \left(\frac{V_x}{R_R + R_L} \right) \angle 0^\circ \quad (30)$$

$$\vec{I}_M = \left(\frac{V_x}{\omega M_x} \right) \angle (-90^\circ) \quad (31)$$

$$\vec{I}_{in} = \vec{I}_o + \vec{I}_M = \left(\frac{V_x}{\omega M_x} \right) \angle (-90^\circ) + \left(\frac{V_x}{R_R + R_L} \right) \angle 0^\circ. \quad (32)$$

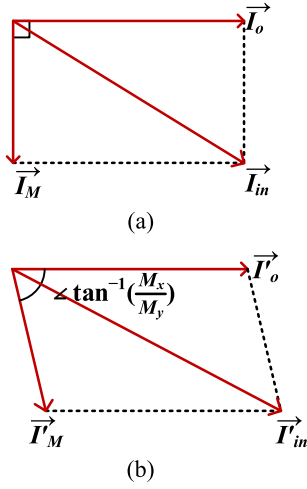


Fig. 6. Current vectors of (a) scalar mutual inductance WPT system and (b) CMI WPT system.

Similarly, the current vectors for the circuit with nonzero mutual resistance in Fig. 5(b) are

$$\vec{I}'_o = \left(\frac{V'_x}{R'_R + R_L - \omega M_y} \right) \angle 0^\circ \quad (33)$$

$$\vec{I}'_M = \left(\frac{V'_x}{\sqrt{(\omega M_x)^2 + (\omega M_y)^2}} \right) \angle \left(-\tan^{-1} \frac{M_y}{M_x} \right) \quad (34)$$

$$\begin{aligned} \vec{I}'_{in} = \vec{I}'_o + \vec{I}'_M &= \left(\frac{V'_x}{\sqrt{(\omega M_x)^2 + (\omega M_y)^2}} \right) \angle \left(-\tan^{-1} \frac{M_y}{M_x} \right) \\ &+ \left(\frac{V'_x}{R'_R + R_L - \omega M_y} \right) \angle 0^\circ. \end{aligned} \quad (35)$$

Comparing the set of (30)–(32) and the set of (33)–(35), both the current magnitudes and angles are altered by the presence of the extra aggregate current loop. The current vector diagrams for the two cases are illustrated in Fig. 6(a) and (b). The effects of the interactions of the magnetic field created by the extra induced currents in the relay resonator and ferrite material result in the phase shift of current vectors.

D. Circuit Parameters of the Aggregate Current Loop

The effects of any extra induced current or currents in the WPT system other than the transmitter and receiver currents are combined into the aggregate current loop in the generalized WPT model. While the generalized loop consists of a closed loop of an inductor L_{C1} , a capacitor C_{C1} , and a resistor R_{C1} in series connection, the values of the circuit parameters depend on the practical situations. For example, a relay resonator comprises these three passive circuit elements. For the induced eddy current in seawater, or a ferrite-based electromagnetic shield, or an intended corona ring, it can be represented by the induced current in an aggregate loop comprising an inductor and a resistor only. The generalized model allows users to evaluate the performance of the WPT systems based on theoretical or simulation studies.

The reactance of the aggregate current loop in (2) is $X_{C1} = (\omega L_{C1} - 1/(\omega C_{C1}))$, which is frequency-dependent.

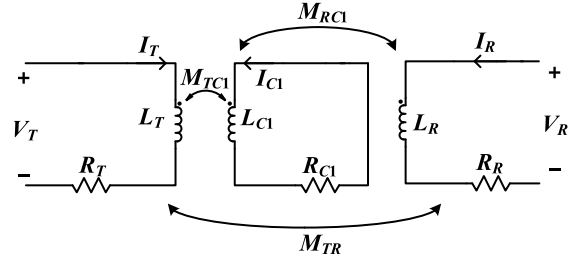


Fig. 7. Equivalent three-coil WPT system with an inductive-resistive aggregate loop.

By controlling the operating frequency, X_{C1} can be positive, negative, or zero. This feature gives the flexibility of the aggregate current loop to represent different types of induced currents in the WPT process.

1) *Inductive-Resistive Aggregate Loop* $X_{C1} > 0$: WPT systems in the presence of a dissipative medium, conductive or magnetic material, can normally be represented by the generalized reduced-order WPT model in which the aggregate current loop comprises an equivalent inductor and equivalent resistor. Fig. 7 shows the circuit model of the three-coil WPT system with an inductive aggregate loop. In this case, the compensation capacitor C_{C1} can be excluded. The parameter equations in (5)–(14) can be converted to the following:

$$\begin{cases} L'_T = L_T - \frac{(\omega M_{TC1})^2}{(\omega L_{C1})^2 + R_{C1}^2} L_{C1} \\ R'_T = R_T + \frac{(\omega M_{TC1})^2}{(\omega L_{C1})^2 + R_{C1}^2} R_{C1} \\ L'_R = L_R - \frac{(\omega M_{RC1})^2}{(\omega L_{C1})^2 + R_{C1}^2} L_{C1} \\ R'_R = R_R + \frac{(\omega M_{RC1})^2}{(\omega L_{C1})^2 + R_{C1}^2} R_{C1} \\ M_x = M_{TR} - \frac{\omega^2 M_{TC1} M_{RC1}}{(\omega L_{C1})^2 + R_{C1}^2} L_{C1} \\ M_y = \frac{\omega M_{TC1} M_{RC1} R_{C1}}{(\omega L_{C1})^2 + R_{C1}^2}. \end{cases} \quad (36)$$

It can be seen that these parameters have the modification factors that are functions of the ω , R_{C1} , and L_{C1} .

2) *Resistive Aggregate Loop* $X_{C1} = 0$: By selecting the compensation capacitor C_{C1} and the angular frequency ω such that $X_{C1} = 0$, the generalized aggregate loop becomes purely resistive. Under this situation, the conversion relationships are expressed as

$$\begin{cases} L'_T = L_T \\ R'_T = R_T + \frac{(\omega M_{TC1})^2}{R_{C1}} \\ L'_R = L_R \\ R'_R = R_R + \frac{(\omega M_{RC1})^2}{R_{C1}} \\ M_x = M_{TR} \\ M_y = \frac{\omega M_{TC1} M_{RC1}}{R_{C1}}. \end{cases} \quad (37)$$

With $X_{C1} = 0$, the self-inductance and mutual inductance are not affected (i.e., without the modification factors). But the equivalent resistances on the transmitter and receiver sides

are increased. However, the imaginary part of the complex coupling (M_y) becomes inversely proportional to the eddy current loss resistor R_{C1} . The form of compensation has the potential to improve the transmission efficiency of WPT systems.

3) *Capacitive Aggregate Loop* $X_{C1} < 0$: By selecting an angular frequency ω and a compensation capacitor to make X_{C1} less than zero, all the modification factors in the parameter equations become positive as shown in the following:

$$\begin{cases} L'_T = L_T + \frac{\omega M_{TC1}^2 |X_{C1}|}{X_{C1}^2 + R_{C1}^2} \\ R'_T = R_T + \frac{(\omega M_{TC1})^2 R_{C1}}{X_{C1}^2 + R_{C1}^2} \\ L'_R = L_R + \frac{\omega M_{RC1}^2 |X_{C1}|}{X_{C1}^2 + R_{C1}^2} \\ R'_R = R_R + \frac{(\omega M_{RC1})^2 R_{C1}}{X_{C1}^2 + R_{C1}^2} \\ M_x = M_{TR} + \frac{\omega M_{TC1} M_{RC1} |X_{C1}|}{X_{C1}^2 + R_{C1}^2} \\ M_y = \frac{\omega M_{TC1} M_{RC1} R_{C1}}{X_{C1}^2 + R_{C1}^2} \end{cases} \quad (38)$$

in which $|\cdot|$ represents the absolute value.

This is an important capacitive-compensation operating mode with $X_{C1} < 0$. The effective mutual inductance M_x is now greater than M_{TR} . This enhanced mutual inductance allows for further improvements in transmission efficiency, potentially exceeding that achieved with resistive compensation with $X_{C1} = 0$. Without the physical explanations, some previous studies using numerical optimization also observed this advantage. In [11], the optimal efficiency frequency is found to be slightly less than the resonant frequency of the relay resonator. After setting the operating frequency at the resonant frequency of the relay resonator, Liu et al. [28] report that maximum efficiency occurs when they slightly reduce the resonant capacitance value. This observation can be explained by the theory of the generalized reduced-order model. For the initial $X_{C1} = (\omega L_{C1} - 1/(\omega C_{C1})) = 0$, if C_{C1} is slightly reduced, the modified $X_{C1} = (\omega L_{C1} - 1/(\omega C_{C1})) < 0$. According to (13b) and (38), the effective mutual inductance is increased to:

$$M_x = M_{TR} + \frac{\omega M_{TC1} M_{RC1} |X_{C1}|}{X_{C1}^2 + R_{C1}^2}.$$

The generalized reduced-order model gives the explanation that under this capacitive compensation, the effective mutual inductance is now enhanced. This extended WPT theory explains why (13b) and (38) provide the important conditions for optimizing the energy efficiency performance of the WPT systems.

E. Energy Efficiency Optimization Based on the Generalized Reduced-Order Model and Compensation Circuit

To further investigate how complex coupling improves transmission efficiency, this section details the underlying loss

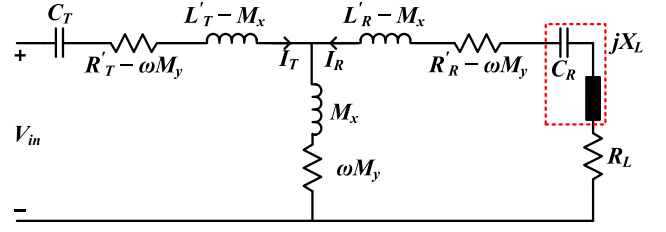


Fig. 8. CMI model with compensation capacitors and loads.

mechanisms. Fig. 8 shows the complex coupling model that includes compensation capacitors C_T and C_R , driven by a sinusoidal source V_{in} and loaded with R_L and X_L . R'_T and R'_R are the parasitic resistances in the transmitter resonator and receiver resonator in the reduced-order model. Note that R'_R does not include the external load R_L here. Also, X_L denotes the sum of the reactance of C_R and the load reactance, if present. The efficiency of the system can be derived as

$$\eta(X_L, R_L) = \frac{|I_R|^2 R_L}{|I_T|^2 (R'_T - \omega M_y) + |I_R|^2 (R'_R - \omega M_y + R_L) + |I_R + I_T|^2 (\omega M_y)}. \quad (39)$$

This efficiency expression indicates a possibility that the NMR ωM_y could lower the conduction power loss on both transmitter and receiver sides, i.e., $|I_T|^2 (R'_T - \omega M_y)$ and $|I_R|^2 (R'_R - \omega M_y)$.

Through solving the circuit equation at the receiver side, the mathematical relationship between I_T and I_R is derived

$$\frac{I_T}{I_R} = -\frac{j(\omega L'_R + X_L) + R'_R + R_L}{j\omega M_x + \omega M_y}. \quad (40)$$

By substituting (40) into (39), I_T and I_R can be eliminated, and the efficiency of the system is simplified as in (41), shown at the bottom of the page.

The optimal load conditions for achieving maximum transmission efficiency are

$$X_{L,opt} = \frac{(\omega M_x)(\omega M_y)}{R'_T} - \omega L'_R \quad (42)$$

$$R_{L,opt} = \sqrt{R_R'^2 + R'_R \frac{(\omega M_x)^2 - (\omega M_y)^2}{R'_T} - \frac{(\omega M_x)^2 (\omega M_y)^2}{R_T'^2}}. \quad (43)$$

Under these optimal load conditions, substituting (42) and (43) into (41) yields the maximum transmission efficiency

$$\eta_{opt}(X_{L,opt}, R_{L,opt}) = \frac{(kQ)^2}{(1 + \sqrt{1 + (kQ)^2})^2} \quad (44)$$

where

$$kQ = \sqrt{\frac{(\omega M_x)^2 + (\omega M_y)^2}{R'_T R'_R - (\omega M_y)^2}} \quad (45)$$

$$\eta(X_L, R_L) = \frac{R_L}{(R'_R + R_L) + \frac{(\omega L'_R + X_L)^2 + (R'_R + R_L)^2}{(\omega M_x)^2 + (\omega M_y)^2} R'_T - 2 \left[\frac{(\omega M_y)(R'_R + R_L) + (\omega M_x)(\omega L'_R + X_L)}{(\omega M_x)^2 + (\omega M_y)^2} \right] (\omega M_y)}. \quad (41)$$

TABLE I
KEY ELECTROMAGNETIC PROPERTIES BETWEEN AIR AND SEAWATER

	Air	Seawater
Relative Permittivity ϵ_r	1.0006	81
Relative Permeability μ_r	1.0000004	0.999991
Electrical Conductivity σ	~ 0	4 S/m

Several important points about the efficiency optimization should be listed as follows.

- 1) This generalized kQ factor involves both the CMI and NMR.
- 2) It agrees with the kQ factor derived in [29] and [30]. It has a similar meaning to the conventional kQ factor used in the scalar mutual inductance model. That is, a higher kQ factor leads to a higher maximum efficiency [31], [32], [33], [34].
- 3) If the mutual resistance ωM_y is zero, the optimal efficiency equation is consistent with that derived from the scalar mutual inductance model [35].
- 4) The transmission efficiency is independent of the transmitter-side compensation capacitor C_T . The transmission power does depend on C_T . One possible design was proposed in [22], in which C_T can be selected such that $\omega L'_T - 1/(\omega C_T) = \omega L'_R - 1/(\omega C_R)$. This design can improve the transmission power.
- 5) The generalized kQ factor in (45) suggests that both mutual inductance (ωM_x) and resistance (ωM_y) contribute positively to the optimal efficiency, whereas the resistors (R'_T and R'_R) at the transmitter and receiver sides compromise the optimal efficiency.

IV. SOME APPLICATIONS OF THE GENERALIZED WPT SYSTEM MODEL

This section includes several examples of the generalized reduced-order model for WPT applications, covering the use of air and seawater as WPT medium, the presence of ferromagnetic electromagnetic shields, and relay resonators.

A. Similarities and Differences of a WPT System in Air and Seawater

The generalized reduced-order model can be used to study the similarities and differences of a WPT system working in air as medium and seawater as medium. The 3-D HFSS finite-element simulation package is used to simulate the two-coil WPT system in air and seawater. The same set of transmitter resonator and receiver resonator is used in this example, as shown in Fig. 9(a) and (b). The radii of the transmitter and receiver coils are identical and are 7 cm. The transmission distance is 3 cm. The operating frequency is 100 kHz. Fig. 9(c) shows the induced eddy current in seawater. The main electromagnetic properties of air and seawater are shown in Table I.

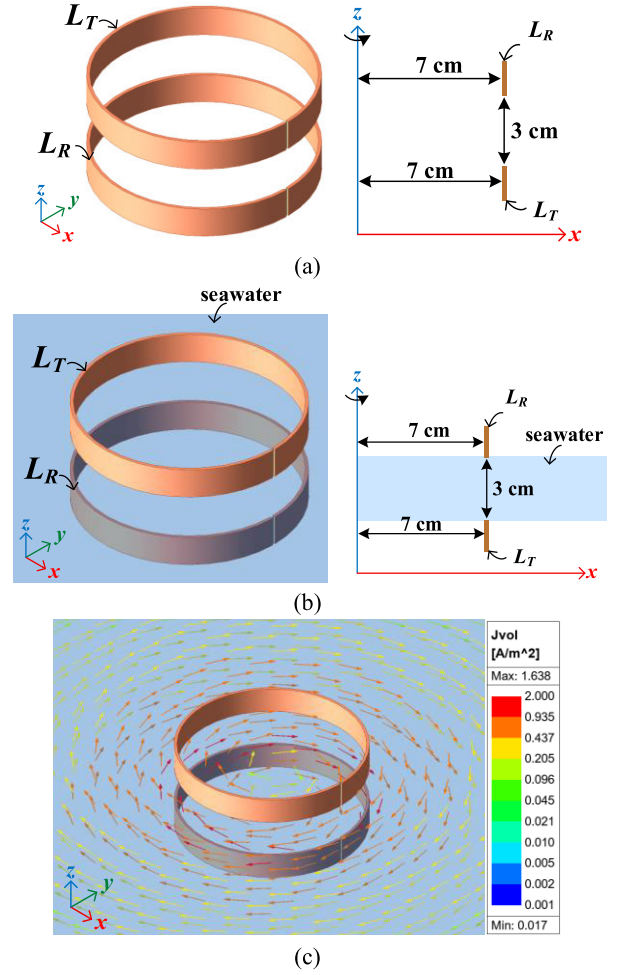


Fig. 9. (a) Simulation setup for the WPT system in air. (b) Simulation setup for the WPT system in seawater. (c) Simulation results of the WPT system in seawater with induced eddy current.

The system matrix for the two-coil WPT system in air generated by the HFSS software is shown in the following equation:

$$\begin{bmatrix} V_{T \text{ air}} \\ V_{R \text{ air}} \end{bmatrix} = \begin{bmatrix} 0.001517 + j0.139354 \, \Omega & j0.035649 \, \Omega \\ j0.035649 \, \Omega & 0.001463 + j0.140330 \, \Omega \end{bmatrix} \begin{bmatrix} I_{T \text{ air}} \\ I_{R \text{ air}} \end{bmatrix}. \quad (46)$$

The corresponding system matrix equation for the WPT system in seawater is shown in the following equation:

$$\begin{bmatrix} V_{T \text{ sea}} \\ V_{R \text{ sea}} \end{bmatrix} = \begin{bmatrix} 0.001684 + j0.136306 \, \Omega & 0.000103 + j0.035208 \, \Omega \\ 0.000103 + j0.035208 \, \Omega & 0.001629 \, \Omega + j0.137170 \, \Omega \end{bmatrix} \begin{bmatrix} I_{T \text{ sea}} \\ I_{R \text{ sea}} \end{bmatrix}. \quad (47)$$

The equivalent T-circuit models of the two-coil WPT system in air and seawater are shown in Fig. 10(a) and (b), respectively.

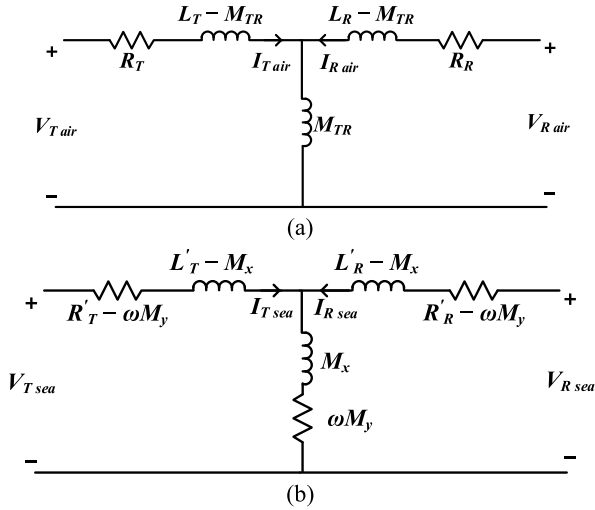


Fig. 10. (a) Equivalent circuit model for the WPT system in air. (b) Equivalent circuit model for the WPT system in seawater.

TABLE II

CIRCUIT PARAMETERS FOR THE WPT SYSTEMS IN AIR AND SEAWATER

WPT systems in air	WPT systems in seawater	Parametric differences
$L_T = 221.7888 \text{ nH}$	$L'_T = 216.9377 \text{ nH}$	$\Delta L_T = L'_T - L_T = -4.8511 \text{ nH} \text{ } (-2.19\%)$
$R_T = 1.517 \text{ m}\Omega$	$R'_T = 1.684 \text{ m}\Omega$	$\Delta R_T = R'_T - R_T = 0.1670 \text{ m}\Omega \text{ } (+11.01\%)$
$L_R = 223.3421 \text{ nH}$	$L'_R = 218.3128 \text{ nH}$	$\Delta L_R = L'_R - L_R = -5.0293 \text{ nH} \text{ } (-2.25\%)$
$R_R = 1.463 \text{ m}\Omega$	$R'_R = 1.629 \text{ m}\Omega$	$\Delta R_R = R'_R - R_R = 0.1660 \text{ m}\Omega \text{ } (+11.35\%)$
$M_{TR} = 56.7371 \text{ nH}$	$M_x = 56.0353 \text{ nH}$	$\Delta M_{TR} = M_x - M_{TR} = -0.7018 \text{ nH} \text{ } (-1.24\%)$
	$\omega M_y = 0.103 \text{ m}\Omega$	
Equivalent resistance in the WPT path $R_T + R_R = 2.980 \text{ m}\Omega$	Resistance in the WPT path $R'_T + R'_R - 2\omega M_y = 3.107 \text{ m}\Omega$	Resistance increase $0.127 \text{ m}\Omega \text{ } (+4.26\%)$

Now, (28) is rewritten as follows for comparison with (46) and (47) to extract the model parameters for the WPT system in air and seawater, as shown in Table II:

$$\begin{bmatrix} V_T \\ V_R \end{bmatrix} = \begin{bmatrix} (R_T + \Delta R_T) + j\omega(L_T - \Delta L_T) & \omega M_y + j\omega M_x \\ \omega M_y + j\omega M_x & (R_R + \Delta R_R) + j\omega(L_R - \Delta L_R) \end{bmatrix} \begin{bmatrix} I_T \\ I_R \end{bmatrix}. \quad (48)$$

Based on the information listed in Table II, two important points can be observed. First, the change of effective mutual inductance between the transmitter coil and the receiver coil is small ($\Delta M_{TR} = -1.24\%$). The reason is due to the almost identical values of permeability of air and seawater (close to 1.0). Second, the total horizontal resistance in the T-circuit model represents the total resistance of the WPT path. There is an increase in value in all resistance terms in the equivalent T-circuit model when the WPT medium is seawater. The overall increase of resistance in the WPT path is 4.26%,

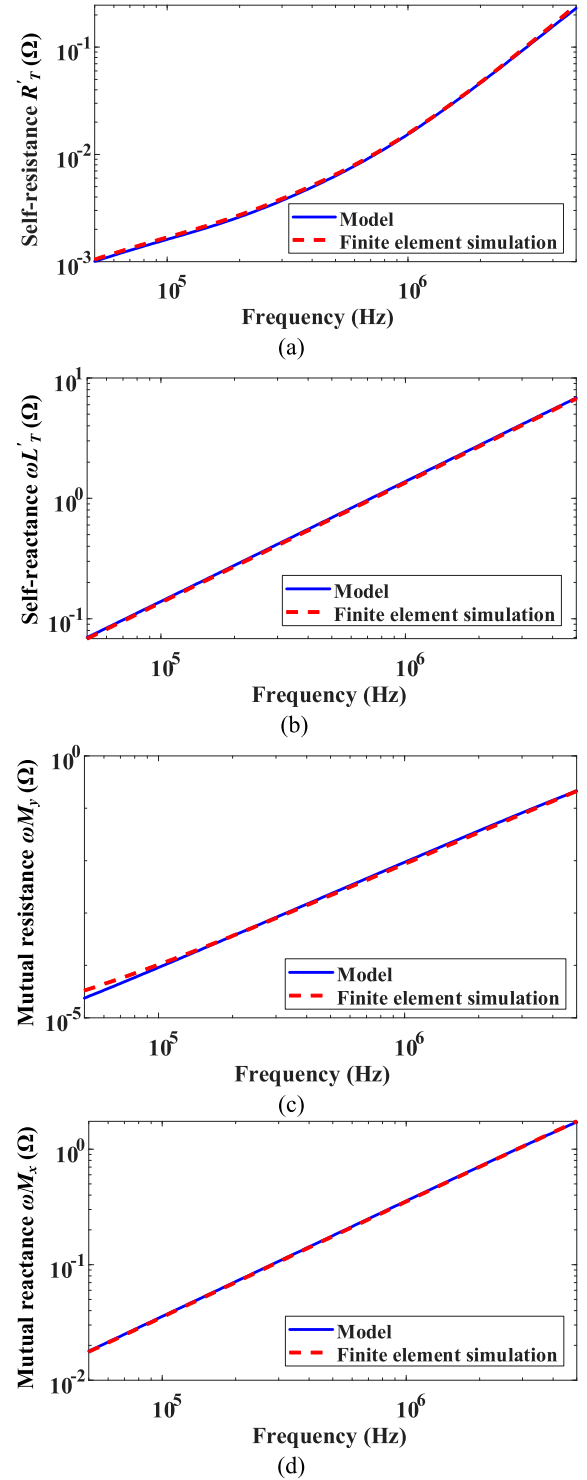


Fig. 11. Comparison between a three-coil representation and finite-element simulation for a WPT system in seawater. (a) Results for self-resistance R'_T . (b) Results for the self-reactance $\omega L'_T$. (c) Results for the mutual resistance ωM_y . (d) Results for the mutual reactance ωM_x .

indicating the dissipative nature of seawater arising from the induced eddy current in seawater.

To capture the eddy current effects in seawater, the two-coil WPT system can be represented by a three-coil model through the introduction of an aggregate L - R loop. Specifically, the impedance variation ΔZ caused by the seawater is modeled

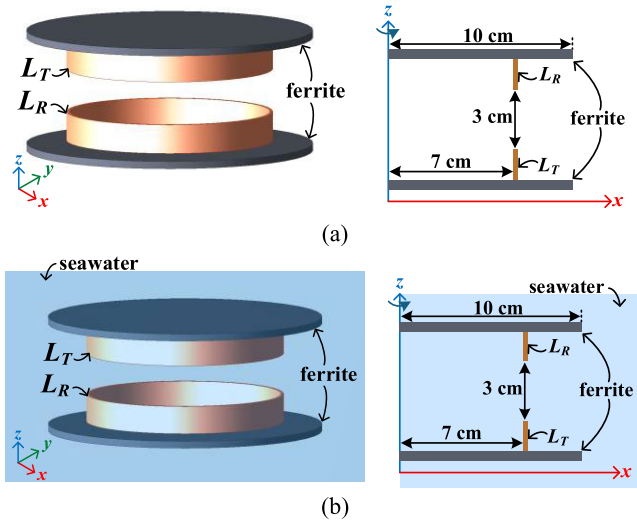


Fig. 12. (a) Simulation setup for the WPT system with ferrite in air. (b) Simulation setup for the WPT system with ferrite in seawater.

using this aggregate loop to account for modification factors such as ΔL_T , ΔR_T , and ωM_y . For the coreless WPT system, the fit parameters of the aggregate loop are $L_{C1} = 0.5905$ nH, $R_{C1} = 0.0581$ Ω , and $M_{TC1} = M_{RC1} = 3.7325$ nH. Fig. 11 shows the comparison of the impedance of the proposed three-coil model ($Z_{air} + \Delta Z$) with the finite-element analysis results (Z_{sea}) over a frequency range of 50 kHz–5 MHz, demonstrating a good agreement. Furthermore, this modeling approach remains valid for WPT systems equipped with ferrite shields. For the WPT system with ferrite as Fig. 12, the seawater effect in the presence of ferrite can also be accurately represented by a single aggregate loop, as shown in Fig. 13, with the parameters being $L_{C1} = 1.226$ nH, $R_{C1} = 0.0381$ Ω , and $M_{TC1} = M_{RC1} = 7.522$ nH. A more detailed discussion on the modeling of WPT systems with ferrite will be presented in Section IV-B.

B. Modeling a WPT System With Ferrite Electromagnetic Shields Over a Wide Frequency Range

Ferrite materials are often employed in WPT systems as magnetic shields to suppress leakage flux and enhance magnetic coupling between coils. The losses on the ferrite also result in a CMI. Similar to the CMI, this phenomenon is characterized by the complex permeability μ

$$\mu = \mu' - j\mu'' \quad (49)$$

where μ' represents the energy-storage capability of ferrite, and μ'' represents the loss. Fig. 14 shows the curve of the real and imaginary parts of the complex permeability of PC40 Mn–Zn ferrite [36]. At low frequencies (< 200 kHz), losses are negligible, and the scalar mutual inductance model can be used. However, at higher frequencies (> 500 kHz), losses become significant, necessitating the use of the CMI model. For example, the PC40 Mn–Zn ferrite exhibits a real permeability μ' of 1422 and an imaginary permeability μ'' of 2343 at 2 MHz.

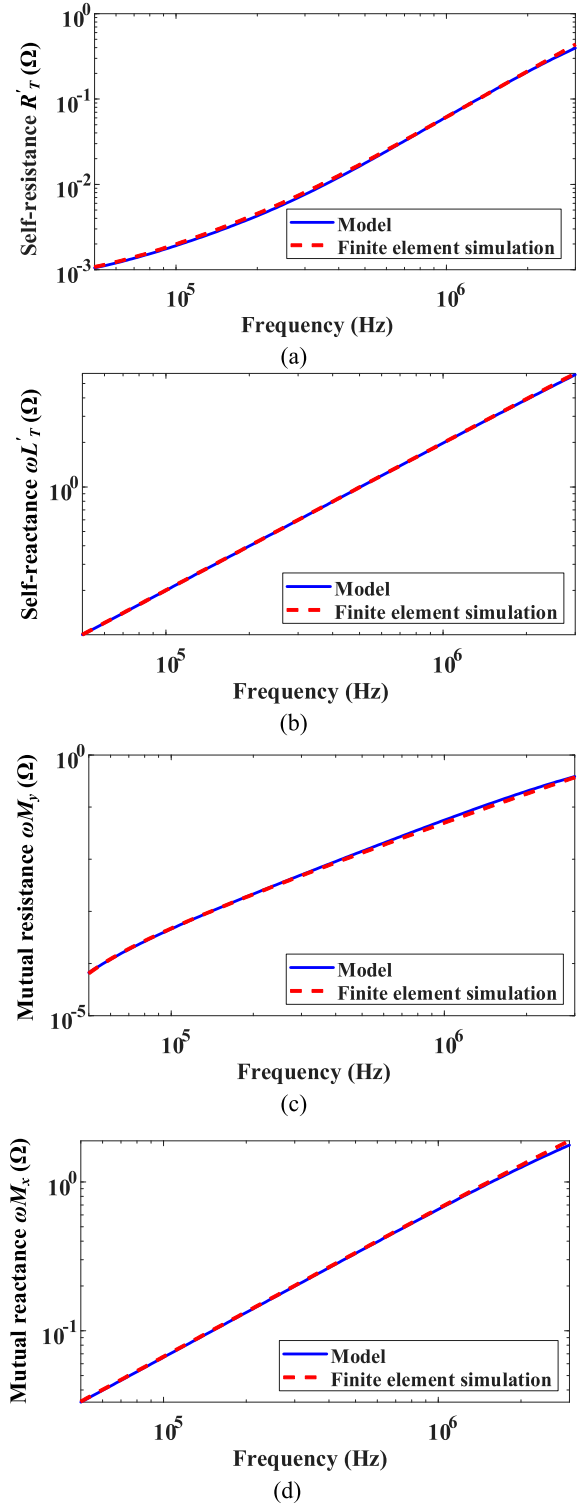


Fig. 13. Comparison between a three-coil representation and finite-element simulation for a WPT system with ferrite in seawater. (a) Results for self-resistance R'_T . (b) Results for the self-reactance $\omega L'_T$. (c) Results for the mutual resistance ωM_y . (d) Results for the mutual reactance ωM_x .

1) *FEA-Based Modeling for WPT Coils With Ferrite Electromagnetic Shields*: Following the guidelines of HFSS [37], two PC40 ferrite plates are used as the electromagnetic shields for a magnetically coupled system, as shown in Fig. 15(a), which displays half of the symmetric cross section of the setup

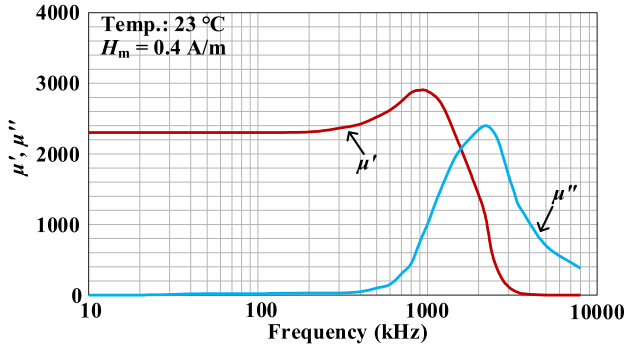


Fig. 14. Magnetic permeability of PC40 Mn-Zn ferrite as a function of frequency [36].

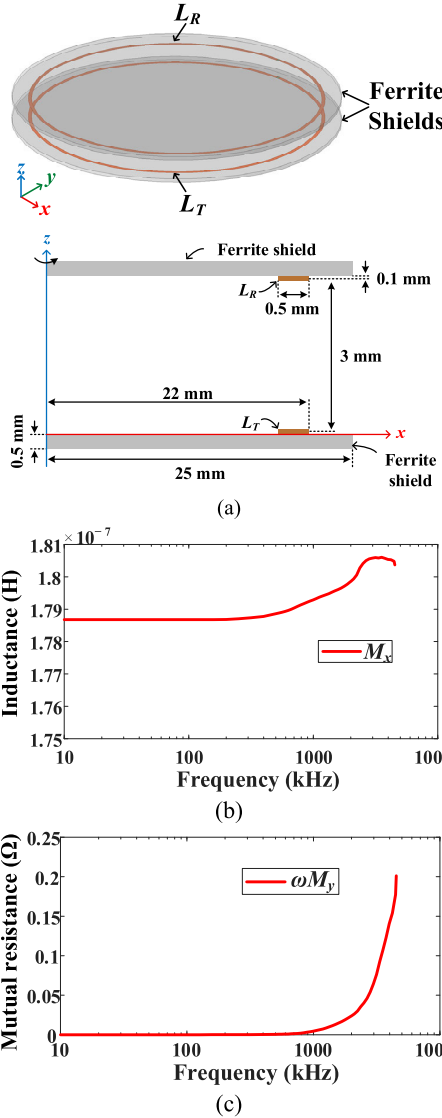


Fig. 15. (a) Two-coil WPT system with ferrite magnetic shields of 25mm radius. The distance between two coils is 3 mm. (b) Mutual inductance M_x as a function of frequency. (c) Mutual resistance ωM_y as a function of frequency.

for finite-element analysis. Fig. 15(b) shows the relationship between the mutual inductance (M_x) and the operating frequency obtained from finite-element analysis. It is noteworthy that while M_x remains almost constant up to 200 kHz, it starts to rise slightly from 200 kHz to 2 MHz. This slight

TABLE III
CIRCUIT PARAMETERS OF THE WPT SYSTEM WITH FERRITE SHIELD
WITH 3 MM DISTANCE

Operating frequency	200 kHz	2 MHz	Parametric differences
L'_T	304.2 nH	281.0 nH	-23.2 nH (-7.63%)
R'_T	84.64 mΩ	142.8 mΩ	58.16 mΩ (+68.71%)
L'_R	303.2 nH	280.7 nH	-22.5 nH (-7.42%)
R'_R	85.18 mΩ	139.9 mΩ	54.72 mΩ (+64.24%)
M_x	180.5 nH	182.8 nH	2.3 nH (+1.27%)
ωM_y	0.9567 mΩ	10.98 mΩ	10.02 mΩ (+1,047.7%)
Equivalent resistance in the WPT path	167.91 mΩ	260.74 mΩ	92.83 mΩ (+55.29%)

mutual inductance rise corresponds to the profile of real permeability μ' over the same frequency range. Fig. 15(c) shows the mutual resistance (ωM_y) as a function of the operating frequency. It can be observed that the profile of ωM_y matches the profile of imaginary permeability μ'' . In the generalized reduced-order model, ωM_y , corresponds to resistance in the aggregate current loop. In the case of a ferrite magnetic shield, it represents the equivalent resistance of the induced eddy and hysteresis power losses in the ferrite shielding material. Therefore, the aggregate induced current loop of the generalized reduced-order model can account for the presence of conductive and magnetic materials in the WPT environment.

Two specific operating frequencies, namely, 200 kHz and 2 MHz, are selected for comparative study based on the same generalized model. The corresponding parameters of the same WPT system at these two operating frequencies are listed in Table III. It is noted that the effective mutual inductance (M_x) remains largely constant and has only 1.27% variation between 200 kHz and 2 MHz. However, the equivalent resistances R'_T and R'_R due to the presence of the eddy and hysteresis power loss in the ferrite shielding material increase by over 64% from 200 kHz to 2 MHz. In particular, the NMR increases over ten times. Nevertheless, the two NMRs ($-2\omega M_y$) in the horizontal circuit branch of the T-circuit model counteract this resistance increase slightly, resulting in a net increase of 55.29% (less than 64%) in the total equivalent resistance in the WPT path. This equivalent resistance arises from the eddy current and hysteresis-related power loss in the ferrite shielding material. It increases with the operating frequency.

Two important points can be derived from the results in Figs. 14 and (c) 15(b) and and Table III.

- 1) The same generalized reduced-order model can incorporate the parametric variations of the WPT system over the whole frequency range from kilohertz to tens of megahertz. In the generalized reduced-order model, the circuit parameters are functions of operating frequency.
- 2) The generalized model can account for the nonlinear and complex characteristics of the permeability of the

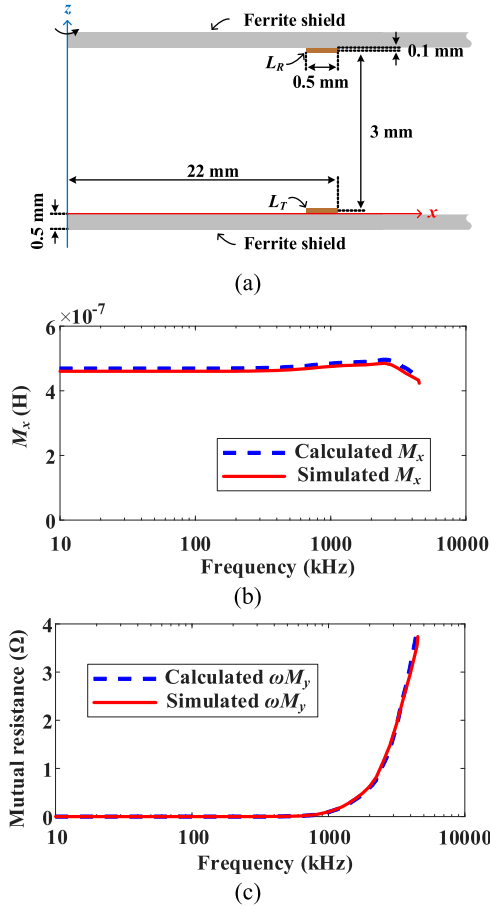


Fig. 16. (a) Two-coil WPT system with ferrite magnetic shields of 100mm radius. The distance between two coils is 3 mm. (b) Mutual inductance M_x as a function of frequency. (c) Mutual resistance ωM_y as a function of frequency.

ferromagnetic shielding materials such as ferrite, in WPT applications.

2) *Analytical Modeling for WPT Coils With Ferrite Electromagnetic Shields*: Based on the methodologies presented in [8], [38], and [39], the CMI can be derived analytically. As shown in Fig. 16, for a sufficiently large ferrite shield, the simulation results are consistent with the theoretical calculations. However, it should be noted that these analytical equations are established upon the assumption of an infinitely large ferrite plane. Consequently, in practical implementations where the ferrite shields are of finite dimensions, the actual values of M_x and M_y will be smaller than those predicted by the analytical expressions.

C. Mutual Resistance, Phase Shift, and Energy Efficiency Caused by Extra Current Loop in a WPT System

This section uses the generalized model and its associated theory to examine the CMI, NMR, and phase shift caused by an extra induced current loop in a WPT system based on the generalized reduced-order model. The practical study is based on a two-coil WPT system and a three-coil WPT system, as shown in Fig. 17(a) and (b), respectively. The coils have a uniform radius of 7 cm. The transmission distance between the Tx and Rx coils is 14 cm, with the relay coil positioned at the midpoint (7 cm from each). Their circuit parameters

TABLE IV
MEASURED PARAMETERS OF TWO- AND THREE-COIL WPT SYSTEMS

Symbol	Value	Symbol	Value
L_T (μH)	25.044	R_T (m Ω)	94.53
L_R (μH)	25.077	R_R (m Ω)	96.95
L_{C1} (μH)	24.828	R_{C1} (m Ω)	85.98
M_{TR} (μH)	1.066	M_{RC1} (μH)	3.595
M_{TC1} (μH)	3.442	ω	$2\pi \times 100$ kHz

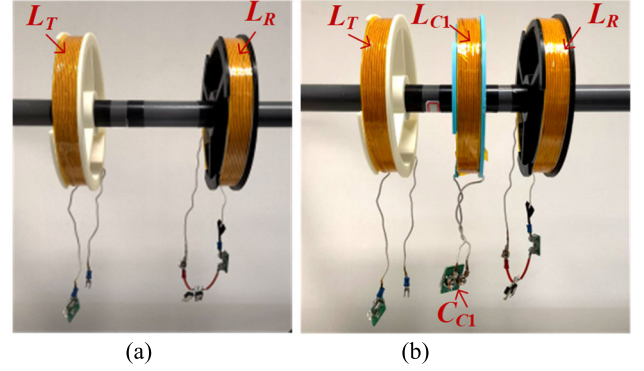


Fig. 17. (a) Two-coil WPT system and (b) three-coil WPT systems with various compensation capacitors for comparative study.

measured by the HIOKI IM3536 LCR meter are displayed in Table IV. The air-gap distance between the transmitter and receiver is 14 cm.

To illustrate the mechanism regarding NMR and the phase shift caused by the extra induced current loop, the two-coil WPT system is used as a reference for comparison with the three-coil WPT systems with three X_{C1} conditions. Recall from (2) that the reactance of the aggregate loop is $X_{C1} = (\omega LC_1 - 1/(\omega C_{C1}))$. The characteristics of X_{C1} are set as inductive, resistive, and capacitive, respectively (i.e., $X_{C1} = 15.600 \Omega$, 0Ω , and -1.206Ω) in order to examine its effects on the phase shift between the transmitter current I_T and the receiver current I_R .

The current vectors of the four cases are shown in Fig. 18(a)–(d). The current vectors $\vec{I}_T$ and $\vec{I}_{C1}$ are normalized with the output current $\vec{I}_R$, which is set as 1.0 per unit. Fig. 18(a) shows the $\vec{I}_T$ and $\vec{I}_R$ for the two-coil system. It can be seen that $\vec{I}_R$ lags behind $\vec{I}_T$ by 90° . Regardless of X_{C1} being inductive, resistive or capacitive, Fig. 18(b)–(d) indicates that the presence of the induced current $\vec{I}_{C1}$ will cause a phase shift between $\vec{I}_T$ and $\vec{I}_R$. The physical reason is that the magnetic field created by the extra induced current loop will interact with the magnetic fields created by $\vec{I}_T$ and $\vec{I}_R$.

Another important observation is that the mutual resistance (ωM_y) is maximum when $X_{C1} = 0$. In Fig. 18(c), this can be explained by (7) in which the square of X_{C1} exists in the denominator of the modification factor. Therefore, M_y is maximum when $X_{C1} = 0$. M_y will decrease as X_{C1} becomes more inductive or capacitive.

The instantaneous power flow and current phasors can be linked up and provide a more intuitive understanding. Taking the power transferred from the transmitter coil to the receiver coil as an example, the average transferred power can

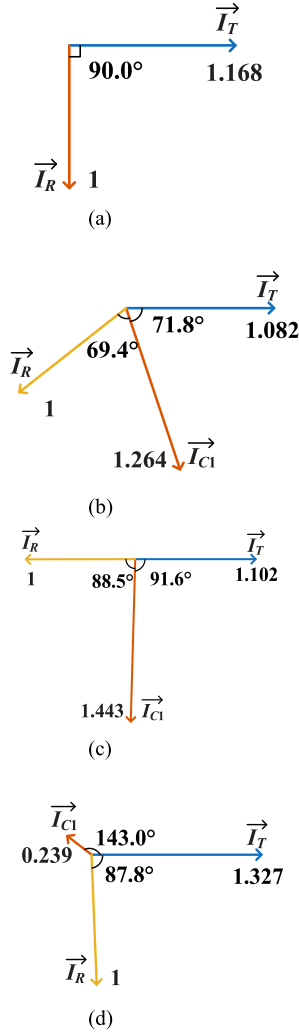


Fig. 18. Per-unit current vectors of these WPT systems. $\omega M_x = 0.357 \Omega$ and $\omega M_y = 0.002 \Omega$. These current vectors are normalized to the output current. (a) Two-coil WPT system. (b) Three-coil WPT system with $X_{C1} = -1.206 \Omega$. (c) Three-coil WPT system with $X_{C1} = 0$. (d) Three-coil WPT system with $X_{C1} = 15.600 \Omega$.

be calculated as

$$P_{T \rightarrow R} = \omega M_{TR} |I_T| |I_R| \sin(\angle I_T - \angle I_R) = \omega M_{TR} (\vec{I}_R \times \vec{I}_T) \cdot \vec{n} \quad (50)$$

where the operator $\times$ represents the vector (cross) product between the current vectors $\vec{I}_R$ and $\vec{I}_T$, and the operator $\cdot$ represents the dot product with the outward unit normal vector $\vec{n}$.

Based on the properties of the cross product in (50), the active power is proportional to the area of the parallelogram formed by the current vectors, and the power flow direction can be determined using the right-hand rule. This provides an intuitive relationship between the current phasors and the power flow. Fig. 19 shows the geometry explanation using current vectors in Fig. 18(a) as an example.

The existence of the mutual resistance (ωM_y) can also be seen from the direct and indirect negative energy transfer from the transmitter resonator back to the receiver resonator.

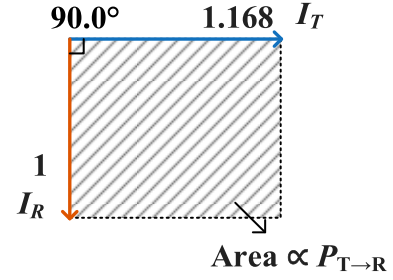


Fig. 19. Geometry explanation between current vectors and power flow using the current vectors in Fig. 18(a) as an example.

Power transfer can occur directly between the transmitter resonator and the receiver resonator (i.e., $P_{T \rightarrow R}$), or indirectly through the relay resonator (i.e., $P_{T \rightarrow C1}$ and $P_{C1 \rightarrow R}$). The power transfer waveforms of the four cases mentioned in Fig. 18(a)–(d) are shown in Fig. 20(a)–(d), respectively. Per-unit power values are adopted, with the output power used as the reference.

Fig. 20(a) shows the time-domain waveforms of the input power (P_{in}), output power (P_{out}), and the power flow from the transmitter resonator to the receiver resonator ($P_{T \rightarrow R}$). It is important to note that $P_{T \rightarrow R}$ is always positive. Because there is no energy transfer from the receiver resonator back to the transmitter resonator, the mutual resistance (ωM_y) is zero.

In Fig. 20(b) and (c), the shaded part of the power flow waveform of $P_{T \rightarrow R}$ indicates the direct energy transfer from the receiver resonator back to the transmitter resonator. This reverse energy transfer leads to nonzero mutual resistance. The reverse energy flow from the receiver resonator to the transmitter resonator can also take place indirectly through the relay resonator, as shown in Fig. 20(d).

Several important observations from Fig. 20(a)–(d) are as follows.

- 1) Mutual resistance exists whenever there is reverse power flow from the receiver resonator to the transmitter resonator in a WPT system.
- 2) When the relay resonator is designed at the capacitive mode ($X_{C1} < 0$), wireless power flows through both the direct path ($P_{T \rightarrow R}$) and indirect paths ($P_{T \rightarrow C1}$ and $P_{C1 \rightarrow R}$).
- 3) When the relay resonator is designed at the resistive mode ($X_{C1} = 0$), wireless power flows through the direct path ($P_{T \rightarrow R}$) that is almost zero. WPT occurs mainly through the relay resonator ($P_{T \rightarrow C1}$ and $P_{C1 \rightarrow R}$).
- 4) When the relay resonator is designed at the inductive mode ($X_{C1} > 0$), large power flows through the direct path ($P_{T \rightarrow R}$) but a substantial amount of it flows back through the relay resonator to the transmitter resonator, resulting in low efficiency.
- 5) The relay resonator can be designed to make the energy efficiency of the three-coil WPT system higher than that of the two-coil counterpart by being in the capacitive mode. According to (6) and (13b), the effective mutual inductance (which is proportional to the energy efficiency) is enhanced with $X_{C1} < 0$.
- 6) The energy efficiency plots of the four cases for a wide load range are displayed in Fig. 21. The results

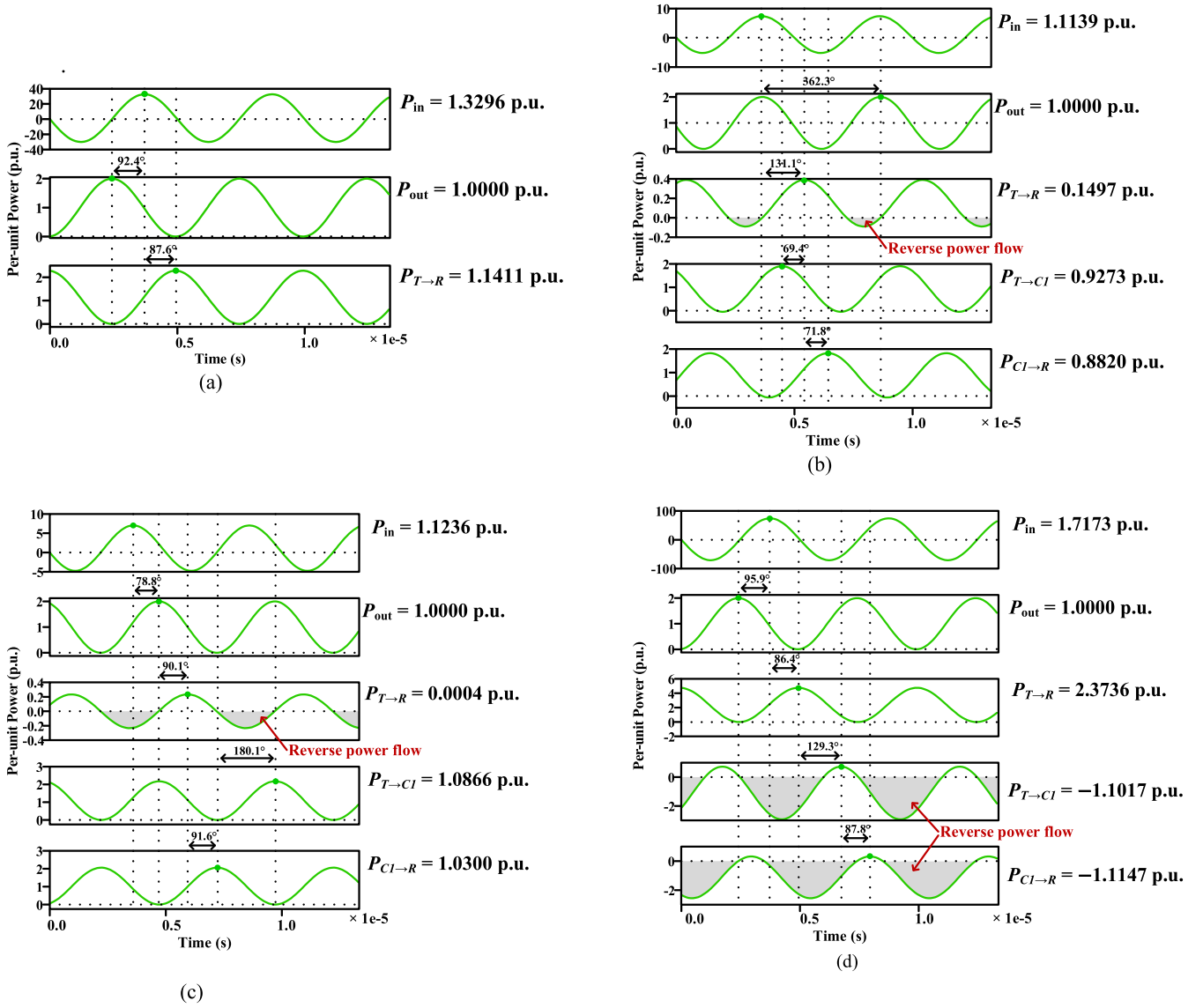


Fig. 20. Normalized transmission power of WPT systems, with the average transmission power labeled. (a) Two-coil WPT system. (b) Three-coil WPT system with $X_{C1} = -1.206 \Omega$. (c) Three-coil WPT system with $X_{C1} = 0$. (d) Three-coil WPT system with $X_{C1} = 15.600 \Omega$.

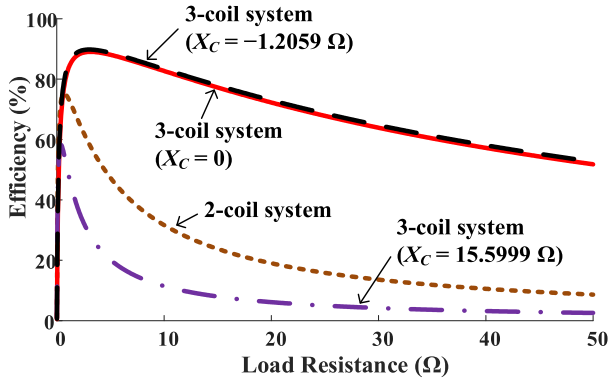


Fig. 21. Efficiency comparison of the WPT systems.

confirm that a three-coil WPT system with the relay resonator operating in the capacitive mode has an

energy efficiency much higher than that of the two-coil counterpart.

Four short videos are included as supplementary materials of this article on the time-varying magnetic field vectors of: 1) a two-coil system; 2) a three-coil system under the capacitive mode; 3) a three-coil system under the resistive mode; and 4) a three-coil system under the inductive mode. The visual displays indicate that the three-coil system under the capacitive mode transfers energy most effectively.

D. Design Criteria for a Three-Coil System to Be More Efficient Than a Two-Coil System

This section provides an example demonstrating how to apply the proposed optimization method to a three-coil WPT system (see Fig. 22). For a comparative study, the two-coil WPT system shown in Fig. 17(a) is used as a benchmark. In this comparative study, the radii of the transmitter and

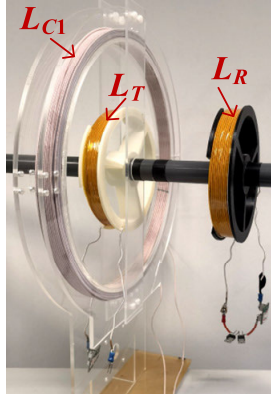


Fig. 22. Three-coil WPT system achieves higher efficiency than the two-coil system.

TABLE V
MEASURED PARAMETERS OF THE THREE-COIL WPT SYSTEM

Symbol	Value	Symbol	Value
L_T (μH)	24.908	R_T (m Ω)	81.62
L_R (μH)	25.128	R_R (m Ω)	87.55
L_C (μH)	851.12	R_C (m Ω)	1069
M_{TR} (μH)	1.021	M_{TC} (μH)	25.593
M_{TC} (μH)	8.938	ω	$2\pi \times 100$ kHz

receiver coils are 7 cm, and the relay coil C has a radius of 15 cm. The transmission distances between the transmitter and receiver coils are 14 cm for both the two- and three-coil systems. The system parameters are listed in Table V. The unknown variables that need to be designed include load resistance R_L , the transmitter compensation capacitor C_T , the receiver compensation capacitor C_R , and the relay coil compensation capacitance C_{C1} .

Zhong et al. [27] introduce an inequality that makes the design of a three-coil WPT system to be more efficient than a two-coil system. As an alternative, by applying the complex coupling model, the inequality can be derived. The general kQ factor for a three-coil WPT system can be expressed as

$$kQ_{3\text{ coil}} = kQ_{TR} \sqrt{\frac{\left(\frac{X_{C1}}{R_{C1}} - \frac{kQ_{TC1}kQ_{RC1}}{kQ_{TR}}\right)^2 + 1}{\frac{X_{C1}^2}{R_{C1}^2} + kQ_{TC1}^2 + kQ_{RC1}^2 + 1}} \quad (51)$$

where $kQ_{TR} = \frac{\omega M_{TR}}{\sqrt{R_T R_R}}$, $kQ_{TC1} = \frac{\omega M_{TC1}}{\sqrt{R_T R_{C1}}}$, and $kQ_{RC1} = \frac{\omega M_{RC1}}{\sqrt{R_R R_{C1}}}$. The coefficient kQ_{TR} represents the kQ factor of the two-coil WPT system.

The inequality that makes the design of a three-coil WPT system to be more efficient than a two-coil system is derived based on the kQ factor. The necessary and sufficient condition for $kQ_{3\text{ coil}} \geq kQ_{TR}$ is

$$\frac{kQ_{RC1}^2 kQ_{TC1}^2 - kQ_{TR}^2 kQ_{TC1}^2 - kQ_{TR}^2 kQ_{RC1}^2}{2kQ_{TR} kQ_{TC1} kQ_{RC1}} \geq \frac{X_{C1}}{R_{C1}}. \quad (52)$$

The results in Fig. 23 demonstrate that the proper design of X_{C1} can make the three-coil WPT system more efficient than the two-coil WPT system.

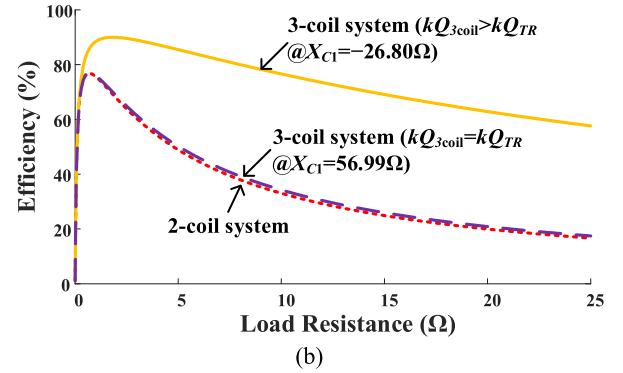
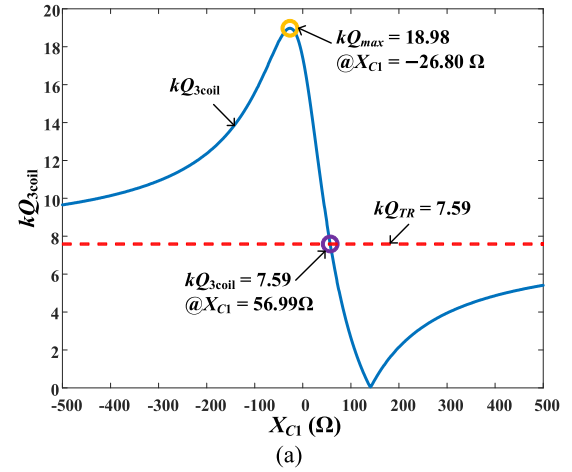


Fig. 23. (a) General kQ factor for making a three-coil WPT system as efficient as the three-coil system. (b) Transmission efficiency of the WPT systems.

When equality holds (i.e., $kQ_{3\text{ coil}} = kQ_{TR}$), the peak efficiency of the three-coil WPT system becomes identical to the two-coil WPT system. In this case, the relay coil becomes functionally neutral. Fig. 23(a) shows the general kQ factor as a function of X_{C1} . When $X_{C1} = \pm\infty$, the $kQ_{3\text{ coil}}$ also converges to kQ_{TR} . In these cases, the relay coil behaves as an open circuit and thus becomes functionally neutral. In addition, it is observed that $kQ_{3\text{ coil}} = kQ_{TR}$ is achieved when $X_{C1} = 56.99 \Omega$. Correspondingly, Fig. 23(b) shows the efficiency curves as a function of load resistance. The efficiency curve with $X_{C1} = 56.99 \Omega$ closely matches that of the two-coil system. This result verifies the effectiveness of the design.

These WPT systems are verified using experimental data. The operating frequency is set at 100 kHz. Fig. 24 shows the experimental waveforms of WPT systems operating at their optimal efficiency point. Fig. 25 shows the efficiency of the WPT systems. The measurement results are in good agreement with the model. Moreover, when X_{C1} is optimally designed ($C_{C1} = 2.834$ nF), the transmission efficiency of the three-coil system is higher than that of the two-coil system. When X_{C1} is designed ($C_{C1} = 3.311$ nF) such that $kQ_{3\text{ coil}} = kQ_{TR}$, both systems achieve identical transmission efficiency with optimal load resistance. Moreover, the difference between these two curves is very small. These results verify the inequality for making the three-coil system to be more efficient than a two-coil system.

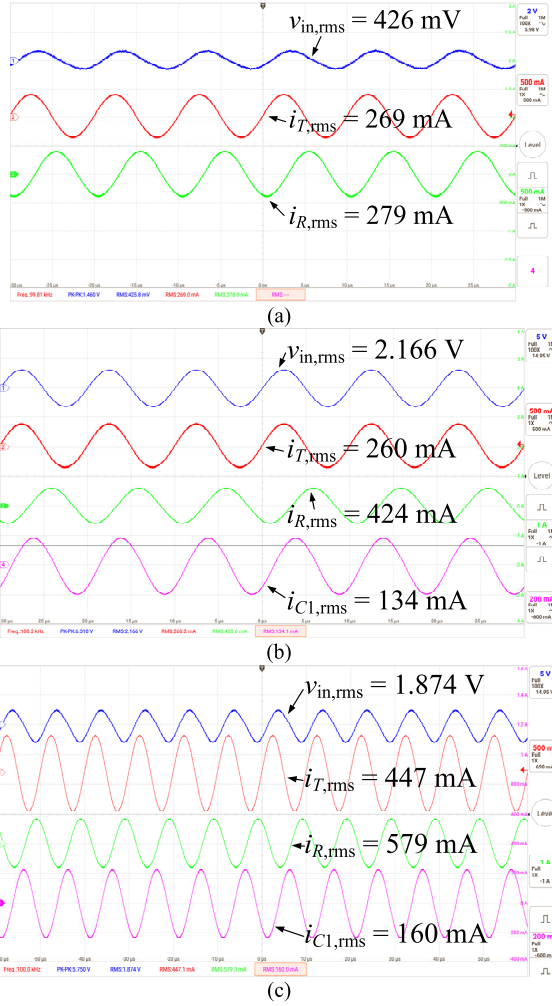


Fig. 24. Experimental waveforms at the peak efficiency for (a) two-coil WPT system, (b) three-coil WPT system with optimal kQ_{3-coil} , and (c) three-coil WPT system with $kQ_{3-coil} = kQ_{TR}$.

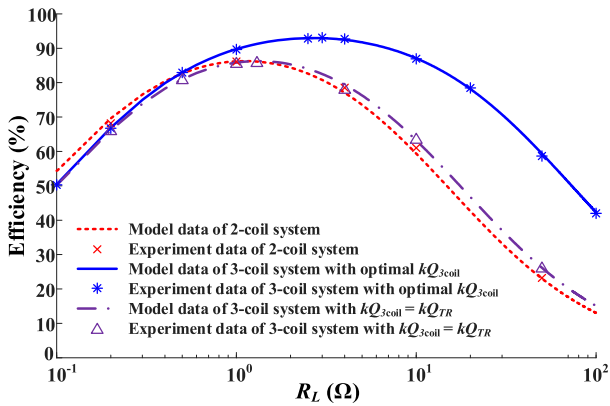


Fig. 25. Theoretical and measured efficiency curves for a two-coil WPT system, a three-coil WPT system with optimal kQ_{3coil} , and a three-coil WPT system with $kQ_{3coil} = kQ_{TR}$.

E. Comparison Between Generalized and Traditional WPT Models

In the traditional WPT model, mutual resistance is typically assumed to be negligible. However, when lossy media are

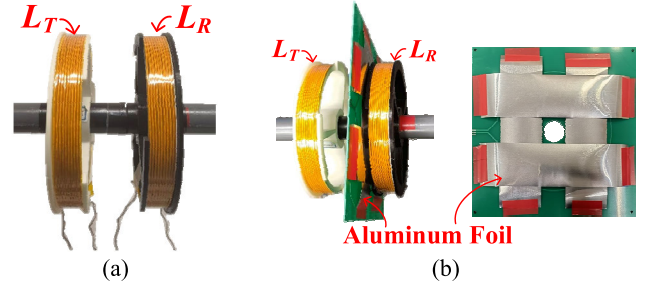


Fig. 26. Experimental setups for comparison. (a) Two-coil WPT system in air. (b) Two-coil WPT system with aluminum foil.

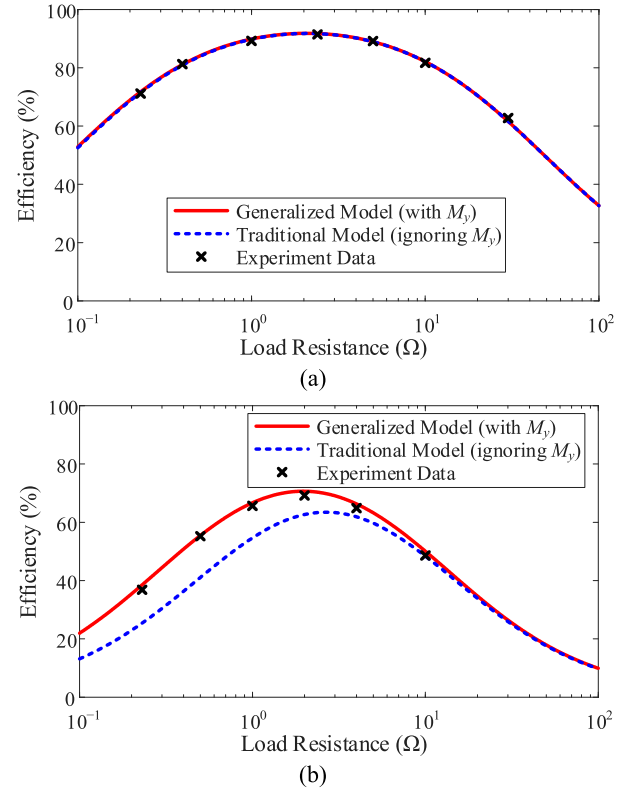


Fig. 27. Efficiency of WPT systems. (a) Efficiency of the WPT system in air. (b) Efficiency of the WPT system with aluminum foil.

introduced into the system, the mutual resistance becomes highly pronounced. Consequently, the traditional model leads to significant discrepancy in efficiency prediction.

Fig. 26 shows two 2-coil WPT experimental setups for comparison. Specifically, Fig. 26(a) shows a two-coil WPT system operating in air, while Fig. 26(b) shows the same system configuration with a 0.1-mm-thick layer of aluminum foil. The circuit parameters are summarized in Table VI.

As shown in Fig. 27(a), both models accurately predict the efficiency of the WPT system in air. However, Fig. 27(b) illustrates that the traditional model underestimates the efficiency in the presence of aluminum foil, whereas the generalized model maintains close agreement with the experimental data.

In addition, designing with the traditional model results in suboptimal $C_{R,opt}$ and $R_{L,opt}$ (see Table VI), failing to achieve the system's maximum efficiency potential.

TABLE VI
CIRCUIT PARAMETERS OF THE TWO-COIL WPT SYSTEM WITH AND WITHOUT ALUMINUM FOILS

Parameters	2-coil WPT system in air		2-coil WPT system with aluminium foil	
L'_T (μH)	24.782		23.982	
R'_T (m Ω)	103.5		158.4	
L'_R (μH)	24.847		19.624	
R'_R (m Ω)	89.0		483.4	
M_x (μH)	3.813		2.096	
ωM_y (m Ω)	8.3		151.8	
	Generalized Model (with M_y)	Traditional Model (ignoring M_y)	Generalized Model (with M_y)	Traditional Model (ignoring M_y)
Optimum compensation capacitor $C_{R,opt}$ (nF)	103.2	101.9	143.8	129.1
Optimum load resistance $R_{L,opt}$ (Ω)	2.21	2.23	1.97	2.67

V. CONCLUSION

The complex nature of mutual inductance and NMR has been noted in WPT and self-oscillator literature over the past two decades. This article reviews related works and introduces a generalized reduced-order circuit model for magnetically coupled WPT systems. The model integrates inseparable CMI and resistance via an aggregate current loop, representing extra induced currents and power losses in practical cases such as dissipative media (e.g., seawater and human tissue), conductive and magnetic materials (e.g., corona rings and electromagnetic shields), and relay resonators. It applies across frequencies from kilohertz to tens of megahertz. By reducing the model to a second-order matrix equation, a comprehensive analysis of generalized WPT systems is presented. This extended WPT theory enables the derivation of new parametric equations with clear physical significance and practical design implications. The modification factors in these equations reveal the influence of the aggregate current loop, and their interpretations indicate how system performance can be optimized through appropriate control. The model is validated through simulations and experimental results across multiple WPT scenarios. Furthermore, its underlying principles are applicable to multicoil and domino WPT systems, as demonstrated in Appendix C. This extended WPT theory provides the theoretical basis for emerging technologies such as new methods for enhancing effective mutual coupling and WPT efficiency [41] and foreign metal detection [42].

APPENDIX A

DERIVATION OF T-CIRCUIT EQUIVALENT

Fig. 28(b) shows the T-circuit model. The mathematical relationship is written as

$$\begin{aligned}
 V_T &= (j\omega L'_T - j\omega M_x + R'_T - \omega M_y) I_T \\
 &\quad + (j\omega M_x + \omega M_y) (I_T + I_R) \\
 0 &= (j\omega L'_R - j\omega M_x + R'_R - \omega M_y + R_L) I_R \\
 &\quad + (j\omega M_x + \omega M_y) (I_T + I_R). \tag{A1}
 \end{aligned}$$

Through simplifying (A1), we have

$$V_T = (j\omega L'_T - j\omega M_x + R'_T - \omega M_y) I_T$$

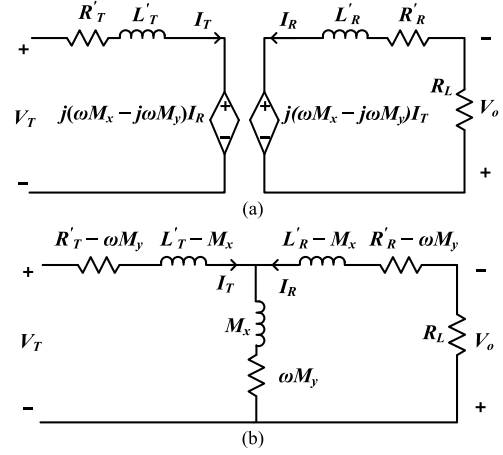


Fig. 28. (a) Complex coupling model. (b) T-circuit equivalent.

$$\begin{aligned}
 &+ (j\omega M_x + \omega M_y) (I_T + I_R) \\
 \Rightarrow V_T &= (j\omega L'_T + R'_T) I_T \\
 &+ (j\omega M_x + \omega M_y) I_R \\
 0 &= (j\omega L'_R - j\omega M_x + R'_R - \omega M_y + R_L) I_R \\
 &+ (j\omega M_x + \omega M_y) (I_T + I_R) \\
 \Rightarrow 0 &= (j\omega L'_R + R'_R + R_L) I_R \\
 &+ (j\omega M_x + \omega M_y) I_T. \tag{A2}
 \end{aligned}$$

The simplified expressions presented in (A2) are the mathematical relationship of the complex coupling model [see Fig. 28(a)]. This derivation demonstrates the equivalence between the complex coupling model and its T-circuit equivalent.

APPENDIX B

PASSIVITY OF COMPLEX COUPLING MODEL

The complex power $\tilde{S}$ injected into the WPT system with CMI is given by

$$\begin{aligned}
 \tilde{S} &= \begin{pmatrix} V_T \\ V_R \end{pmatrix}^H \begin{bmatrix} I_T \\ I_R \end{bmatrix} \\
 &= \begin{pmatrix} R'_T + j\omega L'_T & \omega M_y + j\omega M_x \\ \omega M_y + j\omega M_x & R'_R + j\omega L'_R \end{pmatrix} \begin{bmatrix} I_T \\ I_R \end{bmatrix}^H \begin{bmatrix} I_T \\ I_R \end{bmatrix} \\
 &= \begin{pmatrix} I_T \\ I_R \end{pmatrix}^H \begin{pmatrix} R'_T + j\omega L'_T & \omega M_y + j\omega M_x \\ \omega M_y + j\omega M_x & R'_R + j\omega L'_R \end{pmatrix} \begin{bmatrix} I_T \\ I_R \end{bmatrix} \tag{A3}
 \end{aligned}$$

where superscript H denotes the conjugate transpose.

The real part of the complex power $\tilde{S}$ represents the power consumption of the WPT system with CMI. To maintain passivity, this power consumption must be greater than zero

$$\Re(\tilde{S}) = \begin{pmatrix} I_T \\ I_R \end{pmatrix}^H \begin{bmatrix} R'_T & \omega M_y \\ \omega M_y & R'_R \end{bmatrix} \begin{bmatrix} I_T \\ I_R \end{bmatrix} \geq 0. \tag{A4}$$

Applying Sylvester's criterion for a positive semidefinite matrix, the passivity conditions are given

$$\begin{cases} R'_T \geq 0 \\ R'_R \geq 0 \\ R'_T R'_R - (\omega M_y)^2 \geq 0. \end{cases} \tag{A5}$$

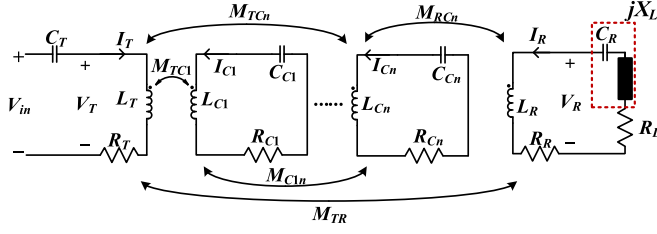


Fig. 29. Circuit diagram of the multicoin or domino WPT system.

This fundamental passivity constraint ensures that the maximum possible value of the NMR is strictly bounded by the resistances of the coils.

Furthermore, this physical boundary imposes a limit on the generalized kQ factor ($kQ = \sqrt{\frac{(\omega M_x)^2 + (\omega M_y)^2}{R'_T R'_R - (\omega M_y)^2}}$) that the denominator is always nonnegative.

APPENDIX C

OPTIMAL DESIGN FOR THE MULTICOIL AND DOMINO WPT SYSTEM

The CMI model gives new insights to derive the optimal efficiency conditions for a multicoin WPT.

Fig. 29 shows the circuit diagram of the multicoin WPT system. In this system, the coils L_T and L_R are used as the transmitter and receiver coils, respectively, while the relay coils are labeled from L_{C1} to L_{Cn} . The compensation capacitors are labeled according to their respective coils. For instance, the compensation capacitor for the relay coil L_{C1} is labeled as C_{C1} . The overall reactance is denoted in a similar way; for example, the overall reactance of relay coil L_{C1} is given by $X_{C1} = \omega L_{C1} - 1/(\omega C_{C1})$. Mutual inductances are denoted based on pairs of coupled coils. For example, the mutual inductance between the transmitter coil L_T and the relay coil L_{C1} is labeled as M_{TC1} . The load is denoted as R_L .

A. Methodology

Step 1: Kron Reduction of n -Coil WPT Systems:

According to the circuit diagram shown in Fig. 29, the multicoin WPT system without including compensation capacitors and load can be represented by a matrix expression. The transmitter and receiver coils are assigned as the first and second rows in the matrix

$$\begin{bmatrix} V_T \\ V_R \\ 0 \\ \vdots \\ 0 \end{bmatrix} = \begin{bmatrix} R_T + j\omega L_T & j\omega M_{TR} & j\omega M_{TC1} & \cdots & j\omega M_{TCn} \\ j\omega M_{TR} & R_R + j\omega L_R & j\omega M_{RC1} & \cdots & j\omega M_{RCn} \\ j\omega M_{TC1} & j\omega M_{RC1} & R_{C1} + jX_{C1} & \vdots & j\omega M_{C1n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ j\omega M_{TCn} & j\omega M_{RCn} & j\omega M_{C1n} & \cdots & R_{Cn} + jX_{Cn} \end{bmatrix}$$

$$\begin{bmatrix} I_T \\ I_R \\ I_{C1} \\ \vdots \\ I_{Cn} \end{bmatrix}. \quad (A6)$$

To simplify its expression, the matrix is partitioned into block matrices

$$\begin{bmatrix} V_\alpha \\ \mathbf{0} \end{bmatrix} = \begin{bmatrix} \mathbf{Z}_{\alpha\alpha} & \mathbf{Z}_{\alpha\beta} \\ \mathbf{Z}_{\beta\alpha} & \mathbf{Z}_{\beta\beta} \end{bmatrix} \begin{bmatrix} \mathbf{I}_\alpha \\ \mathbf{I}_\beta \end{bmatrix} \quad (A7)$$

where $V_\alpha = [V_T \ V_R]^T$, $\mathbf{I}_\alpha = [I_T \ I_R]^T$, $\mathbf{I}_\beta = [I_{C1} \ \cdots \ I_{Cn}]^T$, $\mathbf{Z}_{\alpha\alpha} = \begin{bmatrix} R_T + j\omega L_T & j\omega M_{TR} \\ j\omega M_{TR} & R_R + j\omega L_R \end{bmatrix}$, $\mathbf{Z}_{\alpha\beta} = \begin{bmatrix} j\omega M_{TC1} & \cdots & j\omega M_{TCn} \\ j\omega M_{RC1} & \cdots & j\omega M_{RCn} \end{bmatrix}$, $\mathbf{Z}_{\beta\alpha} = \mathbf{Z}_{\alpha\beta}^T$, and $\mathbf{Z}_{\beta\beta} = \begin{bmatrix} R_{C1} + jX_{C1} & \cdots & j\omega M_{C1n} \\ \vdots & \ddots & \vdots \\ j\omega M_{C1n} & \cdots & R_{Cn} + jX_{Cn} \end{bmatrix}$.

We reduced the system model to an equivalent two-port model via Kron reduction [25], [40]. The reduced-order matrix equation can be expressed using a 2×2 matrix $\mathbf{Z}_{Kron}$

$$\mathbf{V}_\alpha = (\mathbf{Z}_{\alpha\alpha} - \mathbf{Z}_{\alpha\beta} \mathbf{Z}_{\beta\beta}^{-1} \mathbf{Z}_{\beta\alpha}) \mathbf{I}_\alpha = \mathbf{Z}_{Kron} \mathbf{I}_\alpha \quad (A8)$$

where $\mathbf{Z}_{Kron} = \mathbf{Z}_{\alpha\alpha} - \mathbf{Z}_{\alpha\beta} \mathbf{Z}_{\beta\beta}^{-1} \mathbf{Z}_{\beta\alpha}$. From the reduced-order matrix $\mathbf{Z}_{Kron}$, the parameters of the equivalent circuit element of the complex coupling model (see Fig. 3) are extracted

$$\begin{aligned} L'_T &= \frac{\Im(\mathbf{Z}_{Kron})_{11}}{\omega} = L_T - \frac{\Im(\mathbf{Z}_{\alpha\beta} \mathbf{Z}_{\beta\beta}^{-1} \mathbf{Z}_{\beta\alpha})_{11}}{\omega} \\ R'_T &= \Re(\mathbf{Z}_{Kron})_{11} = R_T - \Re(\mathbf{Z}_{\alpha\beta} \mathbf{Z}_{\beta\beta}^{-1} \mathbf{Z}_{\beta\alpha})_{11} \\ L'_R &= \frac{\Im(\mathbf{Z}_{Kron})_{22}}{\omega} = L_R - \frac{\Im(\mathbf{Z}_{\alpha\beta} \mathbf{Z}_{\beta\beta}^{-1} \mathbf{Z}_{\beta\alpha})_{22}}{\omega} \\ R'_R &= \Re(\mathbf{Z}_{Kron})_{22} = R_R - \Re(\mathbf{Z}_{\alpha\beta} \mathbf{Z}_{\beta\beta}^{-1} \mathbf{Z}_{\beta\alpha})_{22} \\ M_x &= \frac{\Im(\mathbf{Z}_{Kron})_{21}}{\omega} = M_{TR} - \frac{\Im(\mathbf{Z}_{\alpha\beta} \mathbf{Z}_{\beta\beta}^{-1} \mathbf{Z}_{\beta\alpha})_{21}}{\omega} \\ M_y &= \frac{\Re(\mathbf{Z}_{Kron})_{21}}{\omega} = -\frac{\Re(\mathbf{Z}_{\alpha\beta} \mathbf{Z}_{\beta\beta}^{-1} \mathbf{Z}_{\beta\alpha})_{21}}{\omega} \end{aligned} \quad (A9)$$

where $\Re$ and $\Im$ are the real and imaginary parts of a complex value, respectively. The subscript indicates the position of an element in the matrix. For example, $\Im(\mathbf{Z}_{Kron})_{21}$ denotes the element in the second row and first column of the imaginary part of $\mathbf{Z}_{Kron}$.

Step 2: Optimization Using the CMI Model:

The general kQ factor is derived under optimal load conditions. It reflects the maximum achievable efficiency of the WPT system. In the expression for the general kQ factor in a multicoin WPT system, only the relay coil compensation capacitances C_{C1} , C_{C2} , ..., C_{Cn} are included. This suggests that optimizing these relay coil capacitances facilitates achieving the maximum achievable efficiency in a multicoin WPT system.

To summarize, the design procedure is summarized as follows.

$$R_L = R_{L,opt} = \sqrt{(\Re(Z_{Kron})_{22})^2 + \Re(Z_{Kron})_{22} \frac{(\Im(Z_{Kron})_{21})^2 - (\Re(Z_{Kron})_{21})^2}{\Re(Z_{Kron})_{11}} - \left(\frac{\Re(Z_{Kron})_{21} \cdot \Im(Z_{Kron})_{21}}{\Re(Z_{Kron})_{11}} \right)^2}$$

$$C_R = \frac{1}{\omega X_{L,opt}} = \frac{\Re(Z_{Kron})_{11}}{\omega [\Re(Z_{Kron})_{21} \Im(Z_{Kron})_{21} - \Re(Z_{Kron})_{11} \Im(Z_{Kron})_{22}]} \quad (A11)$$

Design Procedure 1: Substitute the parameters in (A6) into (A7) to obtain the general kQ factor. Determine the compensation capacitance of the relay coils (i.e., $C_{C1}, C_{C2}, \dots, C_{Cn}$) to maximize the general kQ factor, i.e.,

$$C_{C1}, C_{C2}, \dots, C_{Cn} = \sqrt{\frac{(\Im(Z_{Kron})_{21})^2 + (\Re(Z_{Kron})_{21})^2}{\Re(Z_{Kron})_{11} \Re(Z_{Kron})_{22} - (\Re(Z_{Kron})_{21})^2}} \quad (A10)$$

Design Procedure 2: Calculate the optimal load resistance $R_{L,opt}$ and select the receiver compensation capacitor C_R based on (A7) and (A8). It is worth noting that C_R compensates both the receiver inductance and the load reactance to achieve the optimal $X_{L,opt}$ as in (A11), shown at the top of the page.

Design Procedure 3: Select the transmitter compensation capacitor C_T such that the input impedance of the multi-coil system becomes purely resistive under the desired load condition

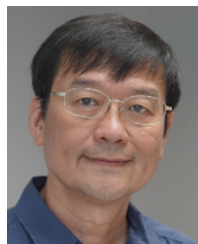
$$C_T = \frac{1}{\omega \Im \left((Z_{Kron})_{11} - \frac{(Z_{Kron})_{21}^2}{(Z_{Kron})_{22} + R_L + jX_L} \right)} \quad (A12)$$

The selection of C_T does not affect the transmission efficiency of the system but influences the transmission power.

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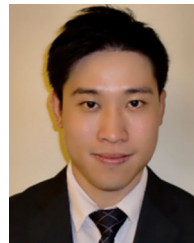
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