

TURBULENT FLOW PAST ROUGH WALLS, AND
THE MOVEMENT OF GRAINS ON STREAM BEDS

By

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SYNOPSIS

Adequate information is already available on flow through pipes and on flat plates under constant pressure. In pipes, where the stress does not vary along the wall, the velocity distribution is logarithmic for most of the flow region. For flat plates, where the wall stress decreases systematically in the downstream direction, the velocity profile concaves around the logarithmic law. Between these two cases remains a gap to be closed.

This thesis concerns a theoretical and experimental investigation on a turbulent boundary layer where the pressure distribution is so arranged to give a constant stress along the wall as in pipes. Various sand roughnesses were used, and the distributions of mean velocity, pressure, and turbulence were measured at various sections.

Chapter II concerns the theoretical and experimental study of the flow in this particular type of boundary layer, whereas in Chapter III the mean velocity distributions are examined in

the light of earlier work on both rough and smooth walls, and the effect of roughness is investigated. Also a law for the velocity profile close to a smooth wall is derived, and an experiment on separation from a rough wall is reported.

Chapter IV deals with turbulence measurements near rough walls, whereas Chapter V examines the usual methods of calculation for turbulent boundary layers under pressure gradients.

In Chapter VI, loose sand was used for the boundary and the movement of grains was examined and compared with that in parallel flow.

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CHAPTER I

FULLY DEVELOPED TURBULENT FLOW AND CONCEPT OF THE BOUNDARY LAYER

1.1. Fully Developed Turbulent Flow

A fully developed turbulent flow may be defined as a flow where the velocity and the pressure at any point perform very irregular fluctuations of high frequency through the mixing of very small parcels of fluid (Dryden, 1939; Hinze, 1959).

The first mathematical attack on this motion is due to Osborne Reynolds (1895), who separated the instantaneous motion at any point in the turbulent flow into two parts; a mean flow whose components are u , v , w , and a superposed fluctuating flow whose components are u' , v' , w' , the mean values of which are zero. As a result of these fluctuations the fluid exhibits an apparent increase in the resistance to deformation through what is called "apparent shear stresses".

The methods available so far for the calculation of turbulent flows are based on hypotheses aimed at establishing a relationship between these apparent shear stresses and the mean values of the velocity components

1.1.1. Momentum Transfer Theory

In the Kinematic theory of gases as discussed by Roberts in (Heat and Thermodynamics, 1933), viscosity is explained as a result of momentum transfer from one layer to another, through the thermal action of molecules. In turbulent fluid motion there is a similar process of transfer, though on a larger scale; the main difference being that the Kinematic theory of gases concerns itself with the microscopic motion of molecules, whereas for turbulent flows the concept deals with the microscopic motion of lumps of the fluid particles which results in momentum transfer from one layer of the fluid to another.

Prandtl (1925) in his theory for the transfer of momentum begins by neglecting the viscosity effect on the turbulent process on the basis that the additional turbulent stresses are much greater than those due to viscosity. Consequently the eddying motion as Reynolds showed will be controlled by such apparent shear stresses τ .

$$\tau = -\rho \bar{u'v'} \quad (1.1)$$

where ρ is the fluid density.

In developing this concept, Prandtl considered an element of the fluid at a point A where the mean velocity is u as moving across the flow a distance l called "the mixing length" without any change of velocity and consequently without any change of momentum, to another point B where it mixes with the surrounding fluid. Such mixing process is supposed to take place in both directions, and during the process, particles from faster layers of the flow will enter the slower layers accelerating them and vice versa, and the mutual action of this process is such as if friction were present.

If l is small, the velocity at point B will be $(u + l \frac{du}{dy})$, where $\frac{du}{dy}$ is the velocity gradient in the direction y normal to the main flow, and consequently the displaced fluid element on arrival at B has a fluctuating velocity component u' , equal to $-l(\frac{du}{dy})$.

At this stage, to substitute for τ in equation (1.1) some assumption about v' has to be made, and hence Prandtl assumes that v' may arise from the fact that two elements of the fluid having different velocities and occupying different positions in front of one another may collide and move sideways from one another much in the manner of solid bodies. Consequently v'

may be assumed to be of the same order of magnitude of u' , thus $v' \propto l \left(\frac{du}{dy} \right)$. On the assumption that $\overline{uv} = u' v'$ it then follows that:

$$\tau = \rho l^2 \left| \frac{du}{dy} \right| \cdot \frac{du}{dy} \quad (1.2)$$

Here l is considered to contain any necessary numerical coefficients, which will actually mean somewhat different definition for l from that of the mixing length. The straight bars surrounding $\frac{du}{dy}$ in the above equation indicate that the numerical value is taken reckoned positive always, so allowing for a change of sign of τ with a change in sign of $\frac{du}{dy}$.

Equation (1.2) can be written in a form similar to that for laminar flow as follows:

$$\tau = \rho \overline{v} l \left(\frac{du}{dy} \right) \quad (1.3)$$

where the quantity $\rho \overline{v} l$ has the dimensions of viscosity, and has been described as an "apparent viscosity". Contrary to the Kinematic viscosity which is a physical constant of the fluid only, the apparent viscosity varies from point to point in the flow being a function also of the velocity.

In view of (1.2), the apparent viscosity in

equation (1.3) will vanish where $\frac{du}{dy}$ is equal to zero, i.e at points of maximum or minimum velocities, and this actually is not the case because the turbulent mixing does not vanish at points of maximum velocities (Fage, 1936; Laufer, 1953; Klebanoff, 1954). To overcome this difficulty, Prandtl (1942) introduced an alternative approximation for τ in the form:

$$\tau = \rho l^2 \left[\left(\frac{du}{dy} \right)^2 + l^2 \left(\frac{d^2u}{dy^2} \right)^2 \right]^{\frac{1}{2}} \cdot \left(\frac{du}{dy} \right)$$

where l is a new length to be determined experimentally. However this form is inconvenient for analytical purposes, and is of interest only for problems in which equation (1.2) would fail.

1.1.2. Vorticity Transfer Theory

In his theory for the vorticity transfer, Taylor (1932) assumes that each element of the fluid retains the vorticity of the layer from which it parted and not the momentum as supposed in Prandtl's theory. This assumption realizes the fact that, in two-dimensional motion, the local differences of pressure do not affect the vorticity of an element, whereas they may affect its momentum. Apart from that, Taylor's theory resembles Prandtl's theory in that an element is

supposed to move a distance 1 before mixing with the surrounding fluid.

For two-dimensional motion, the vorticity of the fluid $\mathcal{E}$ at any point can be expressed as:

$$\mathcal{E} = \frac{1}{2} \left(\frac{\partial u_1}{\partial y} - \frac{\partial v_1}{\partial x} \right)$$

where u_1 and v_1 are the instantaneous components of the velocity in the x and y-directions respectively, and if the flow is uniform in the x - direction $\mathcal{E}$ will equal $\frac{1}{2} \frac{du_1}{dy}$ which varies with time.

Neglecting the effect of viscosity, the equation of motion for a two-dimensional flow takes the form:

$$-\frac{1}{\rho} \frac{\partial p}{\partial x} = \frac{\partial u_1}{\partial t} + u_1 \frac{\partial u_1}{\partial x} + v_1 \frac{\partial u_1}{\partial y} \quad (1.4)$$

where $\frac{\partial p}{\partial x}$ is the pressure gradient causing the flow.

This equation can be written conveniently as:

$$-\frac{\partial}{\partial x} \left(\frac{p}{\rho} + \frac{u_1^2}{2} + \frac{v_1^2}{2} \right) = \frac{\partial u_1}{\partial t} + v_1 \left(\frac{\partial u_1}{\partial y} - \frac{\partial v_1}{\partial x} \right)$$

or

$$-\frac{\partial}{\partial x} \left(\frac{p}{\rho} + \frac{u_1^2}{2} + \frac{v_1^2}{2} \right) = \frac{\partial u_1}{\partial t} + 2 v_1 \mathcal{E} \quad (1.5)$$

When the turbulent motion is uniform in the flow direction, the time mean values of $\frac{\partial u_1}{\partial t}$ and $\frac{\partial}{\partial x} (v_1^2 + u_1^2)$ will equal to zero; on the other hand the temporal averages of $\frac{\partial p}{\partial x}$ and of $2v_1 \mathcal{E}$ will be different from

zero. Consequently equation (1.5) takes the form:

$$-\frac{1}{\rho} \frac{\partial \bar{P}}{\partial x} = 2 \bar{v} \bar{e} \quad (1.6)$$

where the bar denotes the time average values.

Now, an element of the fluid is transferred from one layer to another after travelling a distance l without any change in its vorticity until it mixes with the new layer. This element will possess a vorticity greater by an amount $-1 \frac{d}{dy} \left(\frac{1}{2} \frac{d\bar{u}}{dy} \right)$ than that of the layer with which it mixes. Taking v as the velocity fluctuation v' , then:

$$\frac{1}{\rho} \frac{\partial \bar{P}}{\partial x} = \bar{v} l \left(\frac{d^2 \bar{u}}{dy^2} \right) \quad (1.7)$$

For uniform flow $\frac{\partial \bar{P}}{\partial x} \frac{\partial \tau}{\partial y}$ thus:

$$\frac{\partial \tau}{\partial y} = \rho \bar{v} l \left(\frac{d^2 \bar{u}}{dy^2} \right) \quad (1.8)$$

which is identical with Prandtl's equation (1.3), only if $\bar{v} l$ is independent of y .

When the motion is not two-dimensional, the vorticity components are not conserved (Goldstien pp. 211) and hence they are not transferable, but will change continuously due to the action of the three-dimensional turbulence field. To account for this,

Taylor (1932) in his generalized vorticity transfer theory extended the original one to a three dimensional case of flow, but the new expression became intractable for practical application. To gain simplicity, Taylor (1935) modified the generalized theory by assuming that the vorticity remains conserved in a three dimensional flow, an assumption justified when the turbulence is isotropic.

1.1.3 Von Karman's Similarity Hypothesis (1930)

According to this hypothesis, the turbulent fluctuations are similar at all points in the flow field, and they differ from point to point only by a length scale and a velocity scale. For the velocity scale, Von Karman uses the shear stress velocity V_* and for the length scale he uses the mixing length l due to Prandtl.

To establish the dependence of the mixing length on space co-ordinates, Karman assumed that it is independent of the magnitude of the velocity, and is a function of the velocity distribution only. To define this characteristic length scale, the ratio between the first and second derivatives of the mean velocity was chosen, thus:

$$1 = \gamma \frac{du}{dy} / \frac{d^2 u}{dy^2} \quad (1.9)$$

where γ is a constant to be determined experimentally.

A mathematical argument to deduce equation (1.9) is found in Schlichting's (Boundary layer theory, 1955 pp. 392 - 393).

Substituting from equation (1.9) into (1.2), then:

$$\tau = \rho \gamma \left(\frac{du}{dy} \right)^4 / \left(\frac{d^2 u}{dy^2} \right)^2 \quad (1.10)$$

and this equation is clearly invalid in regions where

$\frac{d^2 u}{dy^2}$ is zero.

1.1.4. Comparison Between The Theories

In order to test which of the two theories, the momentum transfer theory and the vorticity transfer theory, is most nearly correct, Fage and Falkner (1932) conducted a series of velocity and temperature measurements in the wake behind a heated cylinder. These measurements gave support in this particular case of flow to the vorticity transfer theory, the predicted distributions of velocity and temperature agreeing closely with those obtained by direct measurements.

As indicated by Reynolds (1874), the transport of heat by turbulent mixing is similar to the transport of

momentum, and therefore the temperature and velocity distributions based on the momentum transfer theory will be similar. On the other hand, Fage (1932) has shown that the vorticity transfer theory predicts different distributions for the temperature and the velocity. Therefore in flows where the distributions of temperature and velocity are similar, Prandtl's theory will offer a better approximation such as in the case of the flow along plates (Elias, 1929), whereas Taylor's theory will offer a better representation where the distributions of temperature and velocity are not similar such as in the case of flow selected by Fage and Falkner (1932). A detailed comparison between the two theories is contained in Goldstien (pp. 646-673).

1.2 Concept Of The Boundary Layer

This concept was introduced by Prandtl (1904) who showed that the flow near a solid surface can be divided into two regions; a thin region near the wall where the friction forces are of appreciable magnitude (boundary layer), and an outside region where the friction forces are negligible and where the theory of perfect fluids offers an exact solution (Potential flow). While there is no sharp limit between these two regions, it is convenient for analytical purposes

to assume there is, and that friction effects are wholly restricted to the layer whose thickness is δ .

Due to the retarding effect of the friction forces in the boundary layer, there is an energy gradient across the flow, and this is supposed to vanish outside the layer. The region of the retarded flow becomes thicker with distance in the downstream direction as more and more fluid become affected by the friction at the wall.

The equations of motion for two-dimensional incompressible viscous flow can be written in the form:

$$\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \left(\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} \right) \quad (1.11)$$

and

$$\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial y} + \nu \left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} \right)$$

where u and v are the mean velocity components in the x and y -directions respectively, P is the static pressure, and ρ and ν are the density and the Kinematic viscosity of the fluid respectively. The accompanying equation of continuity is:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

Prandtl simplified these equations by dropping the small terms and finally he arrived at the form:

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{\partial P}{\partial x} + \nu \frac{\partial^2 u}{\partial y^2} \quad (1.12)$$

which together with the continuity equation:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (1.13)$$

and the boundary conditions:

at $y = 0$, $u = v = 0$ and at $y = \infty$ $u = U_x$

are sufficient to describe the motion.

If we substitute for $\mu \left(\frac{du}{dy} \right)$ by the shear stress τ , then equation (1.12) can be reduced to:

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{\partial P}{\partial x} + \frac{1}{\rho} \frac{\partial \tau}{\partial y} \quad (1.14)$$

which is valid for both laminar and turbulent flows.

1.2.1. Boundary Layer, Displacement, and Momentum Thicknesses.

The definition of δ , the boundary layer thickness is to a certain extent arbitrary, because the velocity of the flow in the boundary layer approaches that of the main stream asymptotically. However it is often an advantage to have some numerical limit and an arbitrary value such as $u = 0.99 U$ will serve.

To avoid such uncertainties in the definition of the boundary layer thickness, certain mathematical quantities which are more accurately defined are usually used, such as the displacement thickness δ_* , and the

momentum thickness θ .

The displacement thickness is defined as:

$$\delta_{\pi} = \int_{y=0}^{y=\infty} \left(1 - \frac{u}{U}\right) dy \quad (1.15)$$

and it indicates the distance by which the external stream lines are shifted due to the formation of the boundary layer.

The momentum thickness on the other hand is defined by the integral:

$$\theta = \int_{y=0}^{y=\infty} \frac{u}{U} \left(1 - \frac{u}{U}\right) dy \quad (1.16)$$

and it is proportional to the loss of momentum in the boundary layer, since $\rho U^2 \theta$ is the difference between the momentum of the mass inside the layer flowing with the potential velocity U , $\rho U \int_0^{\infty} u dy$ and between the actual momentum $\rho \int_0^{\infty} u^2 dy$.

The quantities δ_{π} and θ have the advantage that they are independent of the upper limit of δ , since a negligible increase is obtained from the integrals when extending the integration beyond the thickness δ into the main stream.

1.2.2. Von Karman's Momentum Equation

The solution of the boundary layer equations (1.12 and 1.13) for a given flow field is in many cases

very difficult and time consuming. Therefore it becomes convenient to possess at least approximate methods of solution where an exact solution of the boundary layer equations can not be obtained with a reasonable amount of work.

Most of these approximate methods are based on a momentum equation which can be derived from the equation of motion by integration. This momentum equation was introduced by Von Karman (1930) and is valid both for laminar and for turbulent flows with or without a pressure gradient.

For a steady incompressible two-dimensional flow, upon integrating the boundary layer equation (1.14) with respect to y to $y = h$ (greater than δ), then:

$$\int_0^h \left(u \frac{\partial u}{\partial x} - v \frac{\partial u}{\partial y} - U \frac{dU}{dx} \right) dy = - \frac{\tau_0}{\rho}$$

where τ_0 is the wall shear stress. Substituting for v the quantity $-\int_0^y \left(\frac{\partial u}{\partial x} \right) dy$ from the continuity equation, then:

$$\int_0^h \left(u \frac{\partial u}{\partial x} - \frac{\partial u}{\partial y} \int_0^y \frac{\partial u}{\partial x} dy - U \frac{dU}{dx} \right) dy = - \frac{\tau_0}{\rho}$$

Integrating by parts we get:

$$\int_0^h \frac{\partial}{\partial x} [u(U-u)] dy + \frac{dU}{dx} \int_0^h (U-u) dy = \frac{\tau_0}{\rho} \quad (1.17)$$

which is the original form of the momentum equation. Since the values of the integrals in equation (1.17)

do not depend on the limit h as long as it is greater than the boundary layer thickness, h can be given any value short of ∞ provided it exceeds δ .

Now introducing the definitions of the displacement thickness and the momentum thickness (equations 1.15 and 1.16), equation (1.17) can be reduced to:

$$\begin{aligned} \text{or } \frac{\tau_o}{\rho} &= \frac{d}{dx} (U^2 \theta) + \delta^* U \frac{dU}{dx} \\ \frac{\tau_o}{\rho U^2} &= \frac{d\theta}{dx} + \frac{\theta}{U} \frac{dU}{dx} (H+2) \end{aligned} \quad (1.18)$$

where $H = \frac{\delta^*}{\theta}$ = a "shape" factor for the velocity profile. This is the form of the momentum equation given by Gruschwitz (1931).

1.3. Universal Velocity Distribution Law

The velocity distribution in a pipe or a rectangular channel can be obtained from the mixing length hypotheses as follows:

According to Prandtl's momentum transfer theory, the turbulent shear stress τ is expressible as:

$$\tau = \rho l^2 \left(\frac{du}{dy} \right)^2$$

Assuming that τ remains constant with y in the immediate neighbourhood of the wall, then:

$$\tau = \tau_o = \rho l^2 \left(\frac{du}{dy} \right)^2 \quad (1.19)$$

At this stage, a statement about the mixing

length δ has to be made, and for that Prandtl introduces the experimental observation that δ is unaffected by viscosity in which case the only length we have at our disposal on which δ could depend, is the distance from the wall y . The only dimensionally correct formula for δ is therefore:

$$\delta = \gamma y$$

where γ is a constant.

Substituting into equation (1.19), we get:

$$\tau_o = \rho \gamma^2 y^2 \left(\frac{du}{dy} \right)^2 \quad (1.20)$$

or

$$\left(\frac{du}{dy} \right) = \frac{1}{\gamma} \sqrt{\frac{\tau_o}{\rho}} \cdot \frac{1}{y}$$

on integrating we get:

$$\frac{u}{V_*} = \frac{1}{\gamma} \ln y + \text{constant} \quad (1.21)$$

where $V_* = \sqrt{\frac{\tau_o}{\rho}}$ = The shear stress velocity.

To evaluate the constant in equation (1.21) the condition at the wall is used, namely $u = 0$ at a certain distance y_1 from the wall, thus:

$$\begin{aligned} \frac{u}{V_*} &= \frac{1}{\gamma} [\ln y - \ln y_1] \\ \text{or } \frac{u}{V_*} &= \frac{1}{\gamma} \ln \frac{y}{y_1} \end{aligned}$$

For smooth walls, the viscosity becomes important in the immediate neighbourhood of the wall and

therefore Prandtl suggests that:

$$y_1 = \beta \frac{v}{V_*}$$

which is dimensionally correct with β as a dimensionless constant. Accordingly equation (1.22) takes the form:

$$\frac{u}{V_*} = \frac{1}{\beta} \left[\ln \frac{y V_*}{v} - \ln \beta \right]$$

$$\text{or } \frac{u}{V_*} = A \log \frac{y V_*}{v} + B \quad (1.23)$$

which is called the universal velocity distribution law for smooth pipes with A and B as constants.

In the case of rough walls, to determine y_1 , a second length is available here, namely the roughness height k , and the question is now whether the length $\frac{v}{V_*}$ is involved or not. The criterion is the ratio of these two lengths $\frac{k V_*}{v}$ which may be regarded as a Reynolds number characteristic of the single bump. If $\frac{k V_*}{v}$ is sufficiently large, $\frac{v}{V_*}$ becomes small in comparison with k and y_1 is taken as a fraction of the roughness height, thus:

$$y_1 = \alpha k$$

where α is a constant depending on the type, shape and arrangement of the roughness elements. Substituting for y_1 into equation (1.22), then:

$$\begin{aligned}\frac{u}{V_{\infty}} &= \frac{1}{\gamma} \left[\ln \frac{y}{k} - \ln \alpha \right] \\ &= A \log y/k + B_1\end{aligned}\quad (1.24)$$

where A and B₁ are constants. A detailed study of the variation of y, with wall conditions is found in a paper by C.M. White and C.F. Colebrook* (1937).

Von Karman's formula for τ in the form:

$$\tau = \rho \gamma^2 \left(\frac{du}{dy} \right)^4 / \left(\frac{d^2 u}{dy^2} \right)^2 \quad (1.25)$$

will have an advantage over Prandtl's formula (equation 1.2) only if γ proves to be constant with y. To check this we have at our disposal the experimental velocity distributions from the present test. From figures (3.5 to 3.8), the velocity distribution is logarithmic for most of the boundary layer thickness and as will be shown in section (3.3) can be represented by the equation:

$$\frac{u}{V_{\infty}} = 5.75 \log \frac{y}{k} + \text{constant}$$

Differentiating and substituting into (1.25), then:

$$\begin{aligned}\tau &= \rho \gamma^2 \left(\frac{2.5 V_{\infty}}{y} \right)^4 / \left(\frac{2.5 V_{\infty}}{-y^2} \right)^2 \\ \text{or } \frac{\tau}{\tau_0} &= 6.25 \gamma^2 y^2\end{aligned}$$

here γ will remain constant only if $\frac{\tau}{\tau_0}$ does not vary with y, i.e if $\frac{\partial \tau}{\partial y} = 0$ which actually is not the case,

* "Experiments with fluid friction with roughened pipes", Proc. Roy. Soc. of London, Ser.A, 161.

since the equation of motion is reduced to $\frac{\partial \tau}{\partial y} = \frac{\partial p}{\partial x}$ for the boundary layer under test as will be shown in section (3.3) and $\frac{\partial p}{\partial x}$ is in fact considerably different from zero.

Had we assumed a linear relationship for τ in the form:

$\frac{\tau}{\tau_0} = \frac{(R-y)}{R}$ where R is the pipe radius, equation (1.25) can lead to a velocity distribution of the form:

$$\frac{U - u}{V_{\infty}} = -\frac{1}{Y} \left[\ln \left(1 - \sqrt{1 - \frac{Y}{R}} \right) + \sqrt{1 - \frac{Y}{R}} \right] \quad (1.26)$$

which seems less easier than Prandtl's equation (equation 1.22), and in the same time is in less agreement with the experimental results for pipes (Schlichting pp. 409), indeed a simple calculation of the value of $\frac{U - u}{V_{\infty}}$ at, for example $y = 0.5R$, gives:

$$\frac{U - u}{V_{\infty}} = 1.298 \quad \text{according to Von Karman's} \quad (1.26)$$

$$\text{and } \frac{U - u}{V_{\infty}} = 1.73 \quad \text{according to Prandtl's} \quad (1.22)$$

whereas experimental values for pipes range from 1.68 to 1.88. In the present experiments the velocities in the boundary layer gave the following values:

k_{mm}	x/k	$\frac{(U - u)}{V_*} \frac{y}{\delta} = 0.50$
1.56	320	1.68
1.96	252	1.62
3.16	220	1.70
4.75	123	1.68

So it appears that the simpler form due to Prandtl better represents the facts.

CHAPTER II

NOZZLE OF CONSTANT DRAG

2.1 Concept of the Nozzle of Constant Drag.

The concept of the nozzle of constant drag was originally introduced by C.M. White (1940) in his paper "The equilibrium of grains on the bed of a stream". * The nozzle was designed such as to insure a constant shear stress along its bed, through accelerating the main flow nearly exponentially in the downstream direction. The turbulence in this case was restricted to the boundary layer on the rough surface with the grains on the bed as the origin of turbulence, while the flow in the mainstream was steady and invicid.

In the theory, White assumes that the velocity distribution on the rough surface is like that near the wall of a rough pipe, where the longitudinal mean velocity component u may be described by the law:

$$u = 2.5 V_* \ln \frac{y}{y_1} \quad (2.1)$$

* Proceedings of the Royal Society of London series A No. 958, vol. 174 pp. 322-338, Feb. 1940.

where V_x is the shear velocity $= \sqrt{\frac{\tau_0}{\rho}}$ and y_1 is the shift of origin which accounts for the turbulent mixing ~~that~~ persists even where $u \rightarrow 0$.

Now equation (2.1) implies that, for a constant V_x and for values of y smaller than the thickness of the boundary layer δ , the stream function is independent of x , and hence the stream lines will be parallel to the wall. Consequently there exists a certain discontinuity at the edge of the boundary layer, outside which the stream lines are somewhat inclined and ~~on~~ ^{before} entering the boundary layer, they ~~will turn abruptly~~ through a small angle (Figure 2.1).

Since the pressure increase across the boundary layer is very small ^{or zero} ~~the~~ the pressure normal to the layer is practically constant, and the pressure inside the layer is determined by the outside flow. Thus from figure (2.1), for the equilibrium of an element of the boundary layer, we have according to White :

$$\tau_0 \cdot x = \int_{y_1}^{\delta} (P - P_2) dy$$

where τ_0 is the wall shear stress, and x is the distance from the leading edge of the wall, ~~though this expression as shown later neglects the stress due to singularities at the edge of the boundary layer and accordingly may underestimate the drag~~

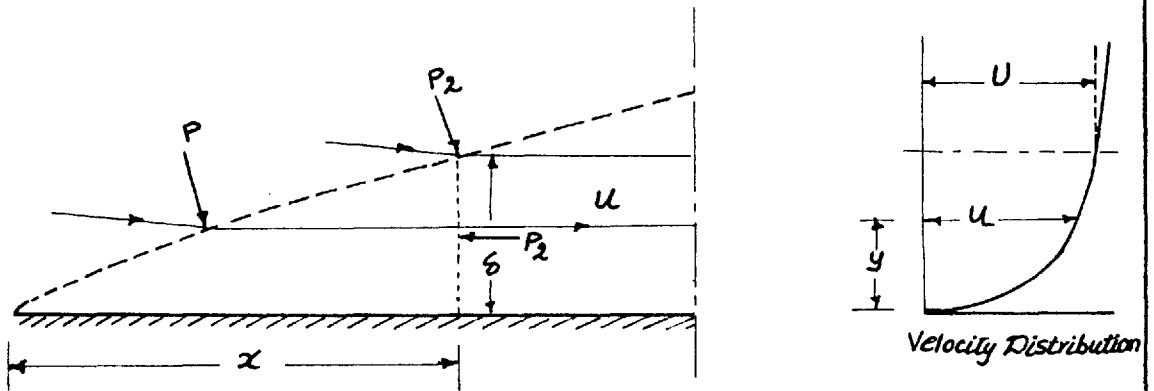


FIG. (2.1) Boundary Layer near Rough Wall With Constant Drag.
after C.M. White (vertical scale Exaggerated)

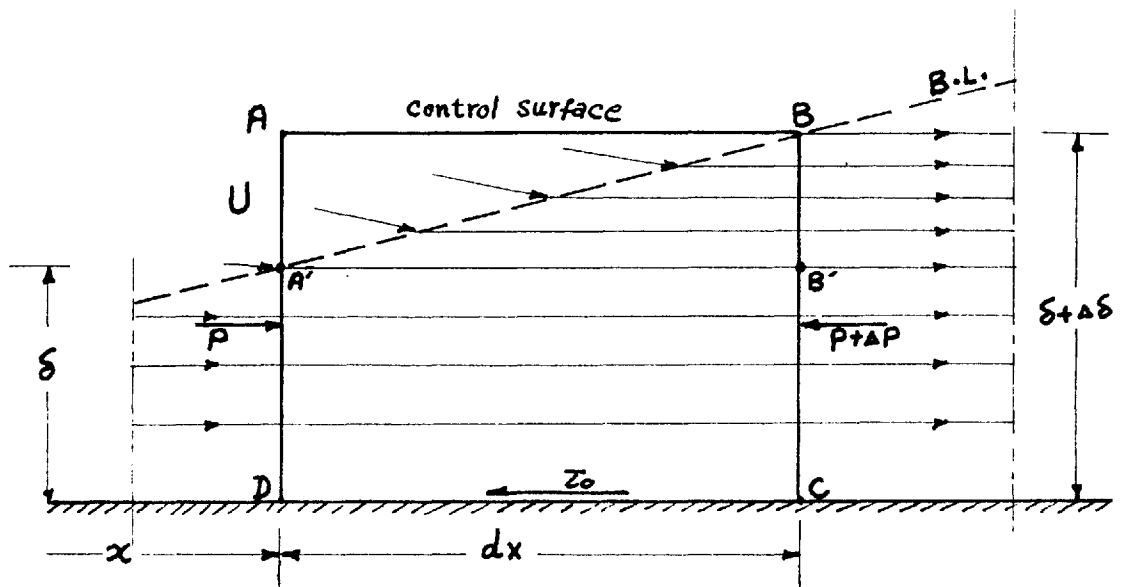


FIG. (2.2) Modified Theory for Nozzle with Constant Drag.

by from 20 to 30%.

Since P is determined by the irrotational motion outside the boundary layer, and since once it has entered the layer, the speed of a stream line remains unchanged, equation (2.2) may be written as:

$$V_{\infty}^2 \cdot x = \frac{1}{2} \int_{y_1}^b (U^2 - u^2) dy \quad (2.3)$$

where U is the velocity of the flow at the edge of the relevant boundary layer.

Substituting χ for $\frac{U}{2.5V_{\infty}}$ and z for $\frac{y}{y_1}$, the two limits $z = 1.0$ and $z = \frac{6}{y_1}$ correspond respectively to $\ln z = 0$ and $\ln z = \chi$, and equation (2.3) is thus reduced to:

$$\frac{x}{y_1} = \frac{6.25}{2} \int_1^{e^{\chi}} (\chi^2 - \ln^2 z) dz$$

or

$$\frac{x}{6.25y_1} = e^{\chi} (\chi - 1) - \frac{\chi^2}{2} + 1 \quad (2.4)$$

which is the general equation for the nozzle of constant drag.

To substitute for y_1 , White (1940) used the value $y_1 = \frac{k}{9}$ given by Schlichting (1936) for loosely packed spherical particles 1.50 diameters apart, since it more nearly represented the test conditions better than the value $y_1 = \frac{k}{33}$

corresponding to Nikuradse's closely packed grains cemented to the wall of a pipe. Therefore equation (2.4) takes the form:

$$1.5 x/k = e^{\chi} (\chi - 1) - \frac{\chi^2}{2} + 1 \quad (2.5)$$

where k represents the mean diameter of the grains.

Strictly speaking, equation (2.5) implies that each size of the sand requires a different nozzle, but the differences are small, and in erosion experiments when a nozzle is used with an inappropriate size of sand, the error is compensated by the bed adjusting itself in a slightly curved form.

To measure the average drag along the bed, White used a suspended metal strip mounted on a micrometer bar by two fine short cotton threads. The strip in this case was coated with sand grains similar in size and shape to those of the bed, and a travelling microscope was used to measure the displacement of the strip which has been related to the bed shear stress. On measuring the shear stress at $\frac{x}{k} = 45$, the hanging strip after allowing for the form drag, gave 19% greater drag than that predicted from theory. ~~This excess drag may be explained by the fact that, when the stream lines turn abruptly at the edge of the~~

~~boundary layer, they actually supply, as shown below, an additional force to the boundary layer, which is ultimately transmitted to the wall.~~

Further details of the

2.2. Modified Theory For The Nozzle of Constant Drag

Considering an element dx of the steady two-dimensional boundary layer, at distance x from the nose of the plate and bounded by the control surface ABCD (figure 2.2). For constant wall stress and according to White's concept, the stream function is independent of x , and the stream lines once they enter the boundary layer will continue to be parallel to the wall since there is no change in their speeds with distance. Thus there is no change in the momentum flux between $\acute{A}D$ and $\acute{E}C$. On the other hand the flux of fluid per unit time leaving across $\acute{B}B$ will be:

$$dx \cdot \frac{\partial}{\partial x} \int_{y_1}^{\delta} \rho u dy$$

and the corresponding outward flux of momentum will thus be:

$$-dx \cdot \frac{\partial}{\partial x} \int_{y_1}^{\delta} \rho u^2 dy$$

From continuity ~~and due to the inclination of the stream lines just outside the boundary layer,~~ fluid must enter through $A\acute{A}$ and AB , the sum of

which is equal to that leaving through BB'. The flux of fluid crossing AA' per unit time is $\rho U \cdot \Delta \delta$ and the corresponding inward flux of momentum is $\rho U^2 \cdot \Delta \delta$, whereas the fluid flux crossing AB per unit time will be $(dx \cdot \frac{\partial}{\partial x} \int_{y_1}^{\delta} u dy - \rho U \cdot \Delta \delta)$, and the corresponding inward flux of momentum will thus be:

$$(dx \cdot \rho U \frac{\partial}{\partial x} \int_{y_1}^{\delta} \rho u dy - \rho U^2 \cdot \Delta \delta)$$

Now considering the equilibrium of the forces acting on ABCD in the x-direction we get:

$$-\tau_0 \cdot dx - dx \cdot \frac{\partial}{\partial x} \int_{y_1}^{\delta} \rho u^2 dy + \rho U^2 \cdot \Delta \delta + dx \cdot \rho U \frac{\partial}{\partial x} \int_{y_1}^{\delta} u dy - \rho U^2 \cdot \Delta \delta - \frac{\partial P}{\partial x} \cdot dx \cdot \delta - \Delta P \cdot \Delta \delta = 0$$

the term $\Delta P \cdot \Delta \delta$ is of the second order and may be neglected, then:

$$\begin{aligned} \text{or} \quad & -\tau_0 - \frac{\partial}{\partial x} \int_{y_1}^{\delta} \rho u^2 dy + \rho U \frac{\partial}{\partial x} \int_{y_1}^{\delta} u dy - \frac{\partial P}{\partial x} \cdot \delta = 0 \\ & -\tau_0 - \frac{\partial}{\partial x} \int_{y_1}^{\delta} \rho u^2 dy + \rho U \frac{\partial}{\partial x} \int_{y_1}^{\delta} u dy + \rho U \delta \frac{dU}{dx} = 0 \quad (2.6) \end{aligned}$$

On the previous assumption that u is described by the velocity distribution law for pipes :

$$u = 2.5 V_{\pi} \ln \frac{y}{y_1}$$

equation (2.6) takes the form:

$$\begin{aligned} & \frac{-\tau_0}{\rho} - \frac{\partial}{\partial x} \int_{y_1}^{\delta} (2.5 V_{\pi})^2 \ln^2 \frac{y}{y_1} dy + U \frac{\partial}{\partial x} \int_{y_1}^{\delta} 2.5 V_{\pi} \ln \frac{y}{y_1} dy + \delta U \frac{dU}{dx} \\ & \text{Substituting } \chi \text{ for } \frac{U}{2.5 V_{\pi}} \text{ and } z \text{ for } \frac{y}{y_1}, \text{ then} \quad = 0 \quad (2.7) \end{aligned}$$

$\delta = y, e^{\chi}$ and the two limits $z = 1$ and $z = \frac{\delta}{y}$,
 correspond respectively to $\ln z = 0$ and $\ln z = \chi$,
 and thus equation (2.7) is transformed to:

$$- \frac{V_{\infty}^2}{6.25y} - 6.25y, \frac{\partial}{\partial x} \int_1^{e^{\chi}} \frac{V_{\infty}^2}{y} \ln^2 z \, dz + 6.25 V_{\infty} \chi y, \frac{\partial}{\partial x} \int_1^{e^{\chi}} V_{\infty} \ln z \, dz + 2.5 V_{\infty} \chi y, e^{\chi} \frac{dU}{dx} = 0$$

Since V_{∞} is independent of x , then:

$$- \frac{V_{\infty}^2}{6.25y}, - \frac{V_{\infty}^2}{\partial x} \int_1^{e^{\chi}} \ln^2 z \, dz + \chi \cdot \frac{V_{\infty}^2}{\partial x} \int_1^{e^{\chi}} \ln z \, dz + V_{\infty}^2 \chi e^{\chi} \frac{d\chi}{dx} = 0$$

$$\text{or } - \frac{1}{6.25y}, - \frac{\partial}{\partial x} \int_1^{e^{\chi}} \ln^2 z \, dz + \chi \frac{\partial}{\partial x} \int_1^{e^{\chi}} \ln z \, dz + \chi e^{\chi} \frac{d\chi}{dx} = 0$$

Integrating and substituting between the limits

$z = 1$ and $z = e^{\chi}$, then:

$$- \frac{1}{6.25y}, - \frac{\partial}{\partial x} [\chi^2 e^{\chi} \chi e^{\chi} + 2e^{\chi} - 2] + \chi \frac{\partial}{\partial x} [\chi e^{\chi} \chi e^{\chi} + 1] + \chi e^{\chi} \frac{d\chi}{dx} = 0$$

on differentiating we get:

$$- \frac{1}{6.25y}, - [\chi^2 e^{\chi} + 2\chi e^{\chi} - 2\chi e^{\chi} - 2e^{\chi} + 2e^{\chi}] \frac{d\chi}{dx} + \chi^2 e^{\chi} \frac{d\chi}{dx} + \chi e^{\chi} \frac{d\chi}{dx} = 0$$

or

$$\frac{1}{6.25y} \, dx = [\chi^2 e^{\chi} - \chi e^{\chi} - \chi] \, d\chi$$

By integrating this equation then:

$$\frac{x}{6.25y} = \frac{\chi^2 e^{\chi} - 3\chi e^{\chi} + 3e^{\chi} - \frac{\chi^2}{2} + c}{2}$$

at $x = 0$, $\chi = 0$ and hence $c = -3$, thus:

$$\frac{x}{6.25y} = \frac{\chi^2 e^{\chi} - 3\chi e^{\chi} + 3e^{\chi} - \frac{\chi^2}{2} - 3}{2}$$

(2.8) χ

which takes into account the ~~discontinuity~~ ^{on entering} inclination of the

stream lines just outside the boundary layer, and

χ . [An algebraic error has been duly corrected here. Assuming (2.8) uncorrected was used for the nozzle (p.40), the resulting differences are listed on p.41 and the effect of these can just be detected in Fig. 2.11, p.61.]

differs from the previous approximate formula (2.4) by the ^{small} amount $c \frac{\chi(\chi^2 - 4\chi + 4)}{\chi^2} \cdot \frac{\chi^2}{2}$

For rough walls ($\frac{v_* k}{\nu} > 60$), y_1 can be substituted by αk , where α is a fraction of the roughness height k , depending on the type, shape and arrangement of the roughness and consequently equation (2.8) can be reduced to:

$$c \frac{x}{k} = \frac{e^{\chi(\chi-1)} + 1}{e^{\chi(\chi^2 - 3\chi + 3)} - \frac{\chi^2}{2} - 3} \quad (2.9)$$

where c is a constant.

For smooth walls on the other hand, equation (2.8) is still valid, and according to White and Colebrook (1937), y_1 can be substituted by $0.10 \frac{\nu}{v_*}$ and equation (2.8), then takes the form:

$$\frac{x}{0.625 \frac{\nu}{v_*}} = \frac{e^{\chi(\chi-1)} + 1}{e^{\chi(\chi^2 - 3\chi + 3)} - \frac{\chi^2}{2} - 3} \quad (2.10)$$

2.3 Apparatus And Equipment

2.3.1. Construction of Apparatus

The general lay out of the apparatus is shown in figure (2.3). The main nozzle in this apparatus consisted of three parts, the smooth curved surface, the rough straight surface and the smooth sides.

The correct shape of the curve was important in this investigation, and special care was directed

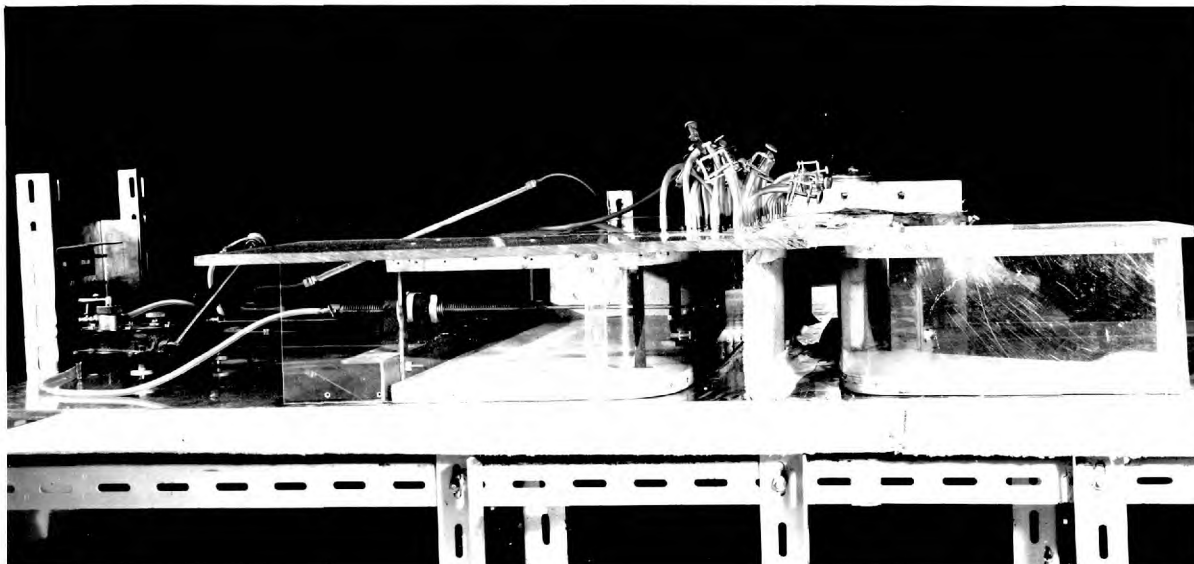
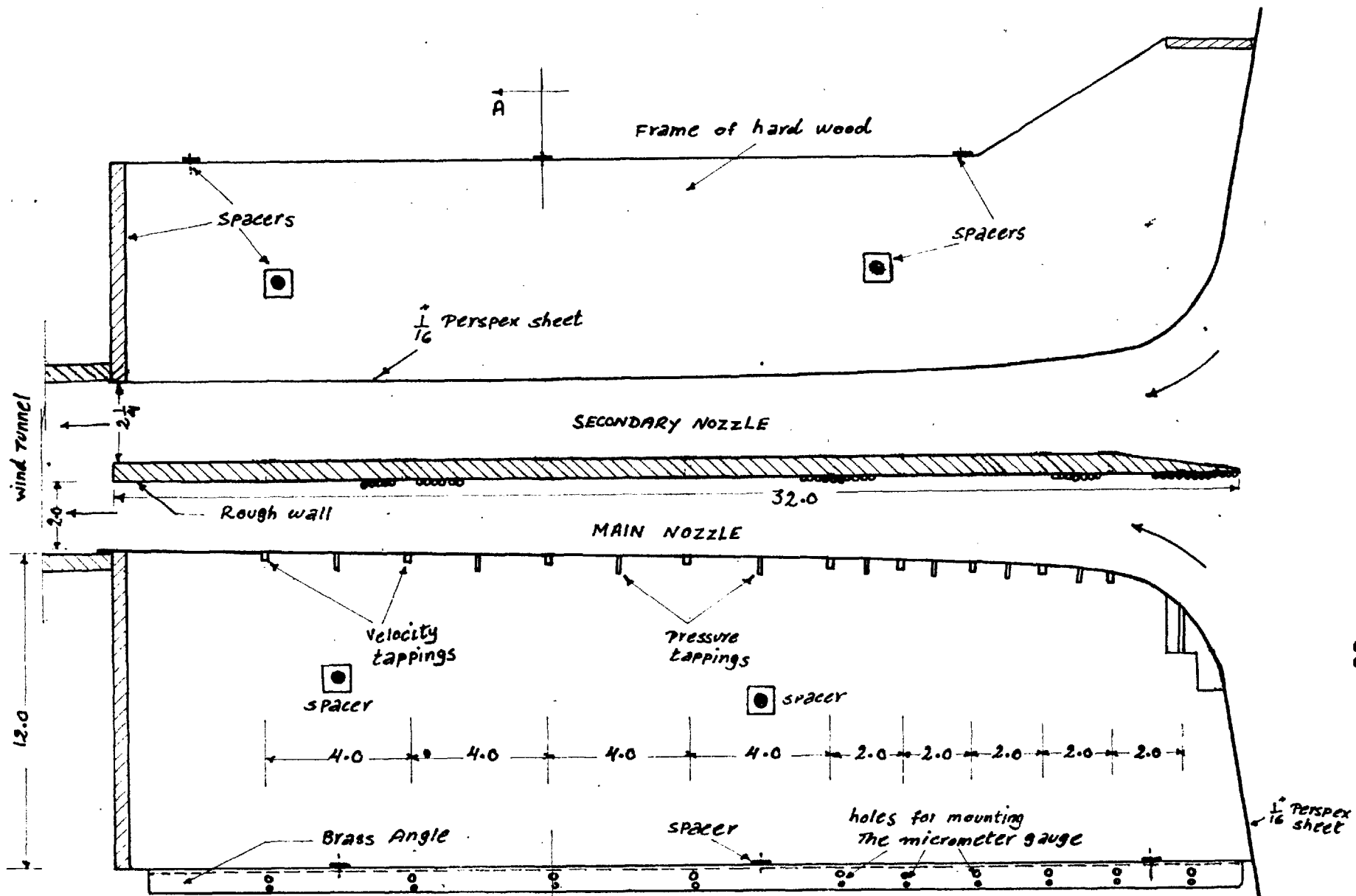
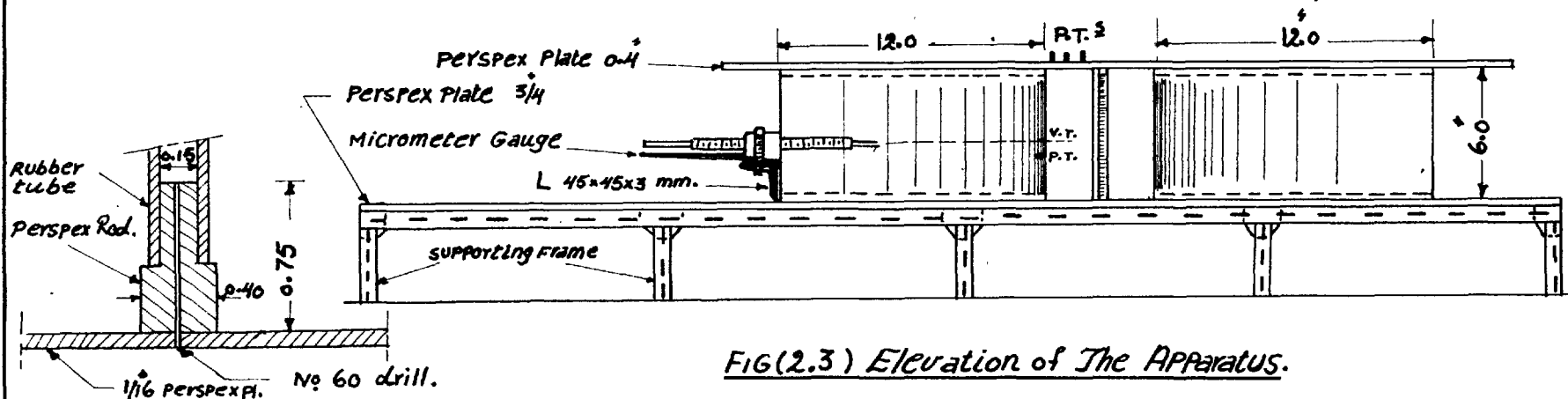


PLATE I. APPARATUS (Elevation)



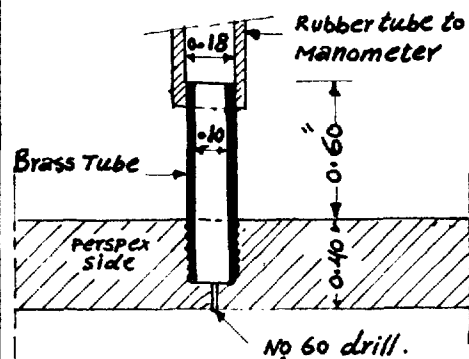
Dimensions in inches

FIG(2.3) Cross-sectional Plan for The Apparatus



FIG(2.3) Elevation of The Apparatus.

FIG(2.5) Details for P.T.s on the curve.



FIG(2.6) Details for P.T.s on the side.

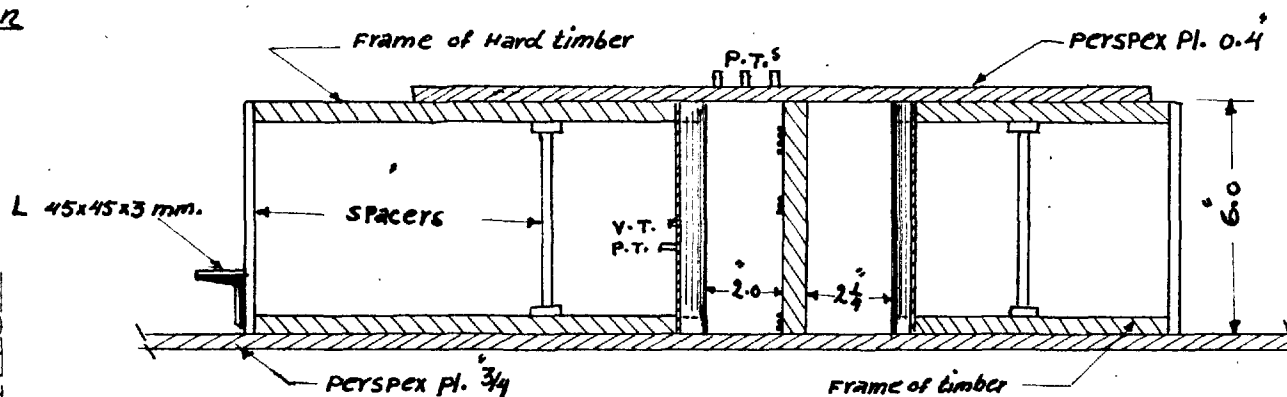


FIG. (2.3) SECTION A-A

to its construction. The theoretical curve was calculated using equation (2.8) for a sand roughness of 1.56 mm. diameter, with the help of a flow net, and the resulting dimensions for the curve are given in Table (2.1), where the nozzle of the plate is taken as the origin for measuring the co-ordinates x and y.

TABLE (2.1)

Co-ordinates of Nozzle according to Equation (2.8) in its original Form.

X inches	Y inches	X inches	Y inches	X inches	Y inches
0.94	4.94	2.50	3.20	9.50	2.41
1.00	4.70	2.75	3.10	10.0	2.38
1.10	4.40	3.00	3.02	11.00	2.34
1.20	4.22	3.25	2.97	12.00	2.30
1.30	4.03	3.50	2.93	13.00	2.28
1.40	3.92	4.00	2.86	14.00	2.26
1.50	3.83	4.50	2.79	16.00	2.23
1.60	3.74	5.00	2.74	18.00	2.20
1.70	3.65	5.50	2.70	20.00	2.165
1.80	3.58	6.00	2.66	22.00	2.130
1.90	3.51	6.50	2.62	24.00	2.10
2.0	3.44	7.00	2.58	26.00	2.07
2.10	3.38	7.50	2.54	28.00	2.04
2.20	3.33	8.0	2.50	30.00	2.02
2.30	3.28	8.50	2.47	32.00	2.00
2.40	3.24	9.00	2.44		

It is interesting to note that both equations (2.4) and (2.8), except for small values of $\frac{x}{y_1}$, give practically the same shape of curve, since the ratio $\frac{\chi(2.8)}{\chi(2.4)}$ is found to be substantially constant except near the nose of the plate. For example if we take $y_1 = k/9$ the values of χ basic to the two equations will be as follows:

TABLE (2.2)

x/k	χ (equation 2.4)	χ (eqn 2.8)	$\frac{\chi(2.8)}{\chi(2.4)} \frac{\chi(2.8)_{corrected}}{\chi(2.4)}$	
			$\chi(2.4)$	$\chi(2.4)$
50	3.45	3.12	0.905	0.988
100	3.95	3.46	0.875	0.990
200	4.48	3.85	0.860	0.992
300	4.76	4.09	0.860	
500	5.20	4.42	0.850	
800	5.58	4.70	0.840	
1000	5.70	4.82	0.840	
3000	6.62	5.56	0.840	
5000	7.12	5.95	0.835	
10000	7.67	6.40	0.835	
20000	8.32	6.93	0.835	
50000	9.10	7.62	0.835	
80000	9.54	7.97	0.835	0.999

~~At $x/k = 50$, equation (2.8) predicts 18.0% greater drag than that predicted from equation (2.4) and may well explain why the hanging strip in White's experiments gave~~

~~a 19.0% greater drag at $x/k = 45$. With large values of x/k the difference amounts to 30% of the drag.~~

After obtaining the co-ordinates of the curve, the curve was plotted full size on a template of card board which was used in cutting two symmetrical plates of hard timber (32" x 12" x $\frac{3}{4}$ ") which were fixed together by spacers at convenient distances to form the supporting frame for the curve. A thin sheet of perspex, 6.0 inches wide and $1/16$ " thick was fixed to the supporting frame, thus forming a smooth clean surface free from kinks and irregularities. This surface was extended at the upstream end tangent to the theoretical curve in order to avoid disturbing the flow at the entrance. The curve was provided with a set of pressure tappings, together with tappings for total head explorations.

The straight rough wall was made from a $\frac{3}{4}$ " thick plate of hard timber, 32 inches long and 6.0 inches wide, tapered from the back at its leading edge, and roughened by closely sieved sand grains. The sand was stuck to the plate by Durofix, and care was directed to form as nearly as possible one layer only of grains on the surface.

The sides of the nozzle were made from smooth clean plates of perspex, and were extended considerably upstream to insure two-dimensional flow at the entrance. Pressure tapings for measuring the static pressure along the boundary layer were provided on the upper side, together with another set of tapings for measuring the static pressure across the flow at the various stations.

The different parts of the nozzle were assembled together with the rough wall vertical, using two identical templates of card board having exactly the same shape of the space between the curve and the straight wall. After fixing the different parts together, adhesive tape and plasticine were used to make the nozzle air-tight, and the tapings were sealed by rubber seals when they were not in use.

Another smooth curved surface, similar in shape to the main curve, was used to form a symmetrical nozzle with the back of the rough surface and was given a slight divergence at its downstream end. The secondary nozzle served to make the flow symmetrical about the leading edge of the rough wall, thus helping to prevent any possible separation of the flow on entering the main nozzle. Smoke applied to

the flow showed that separation at the nose of the plate did not take place in the main nozzle. Also the slight divergence given to the secondary nozzle at its downstream end helped to establish steady conditions at the nose of the plate.

The whole apparatus was connected to the intake of an open circuit wind tunnel (Figure 2.4), having a 55.0 HP blower discharging into the surrounding space through a baffled diffuser. The discharge of the air through the nozzle was controlled either by changing the motor speed or by means of a flap valve in the wind tunnel.

2.3.2. Wall Roughnesses.

Four different sizes of closely sieved sands were used in the present investigation to form the roughness of the plate. In each case, the surface was as even as possible with one layer only of grains.

To determine the mean size of the grains, a few hundred grains were weighed, after determining their specific gravity, and on the assumption that they were perfect spheres, the mean diameter k was calculated. The values of k obtained by this

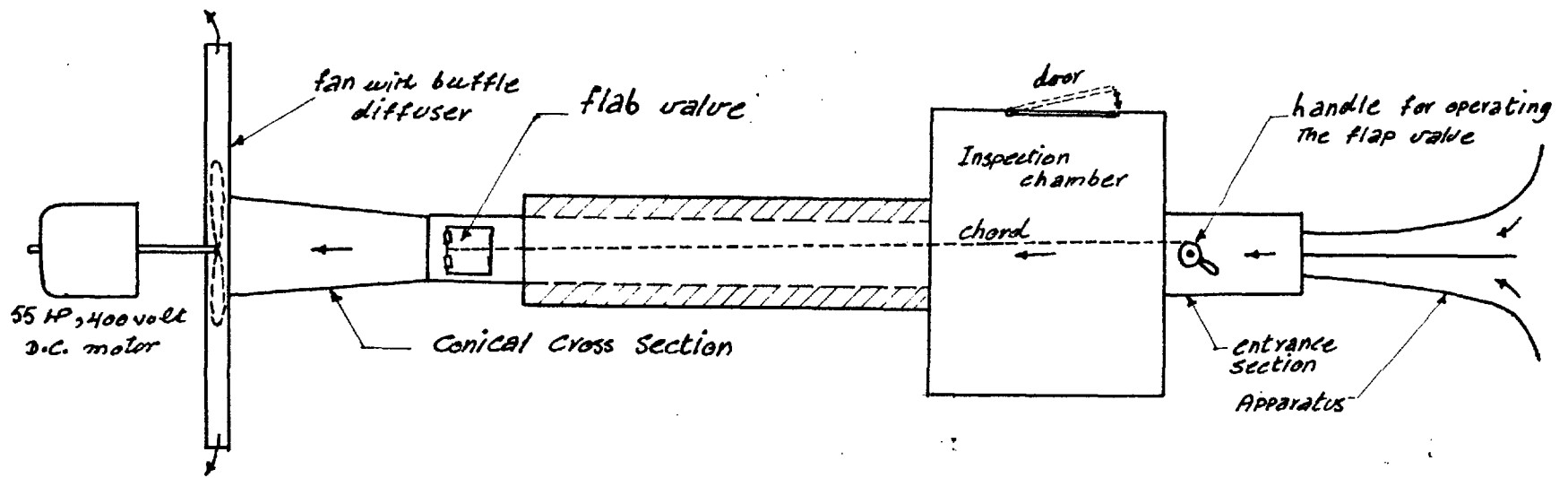


FIG.(2.4) A Diagramatic Sketch for The Wind Tunnel.

method proved to be an average between the values obtained by taking the mean value of 20 readings with a travelling microscope and those obtained using the sieve size. The following table gives the results of these measurements:

ROUGHNESS	Table 2.3				k mms. k mms.		k mms. by weight.
	PASSED BY SIEVE	RETAINED ON SIEVE			from sieve size.	travelling microscope.	
	mesh No.	opening mms.	mesh No.	opening mms.			
A	12	1.40	14	1.20	1.30	1.87	1.56
B	10	1.68	12	1.40	1.54	2.39	1.96
C	6	2.80	7	2.40	2.60	3.40	3.16
D	3 16	4.73	5	3.35	4.04	5.60	4.75

The travelling microscope will lead to values of k in agreement with those obtained by weight, provided that the measurements include the third dimension of the grain which usually is relatively small. For example taking three readings for each grain including the smallest dimension, the average size of 10 grains from sand "A" was found to be 1.54 mms, and the corresponding value for sand B was found to be 1.98 mms.

2.3.3. Static Pressure Tappings.

Static pressure tappings were provided for

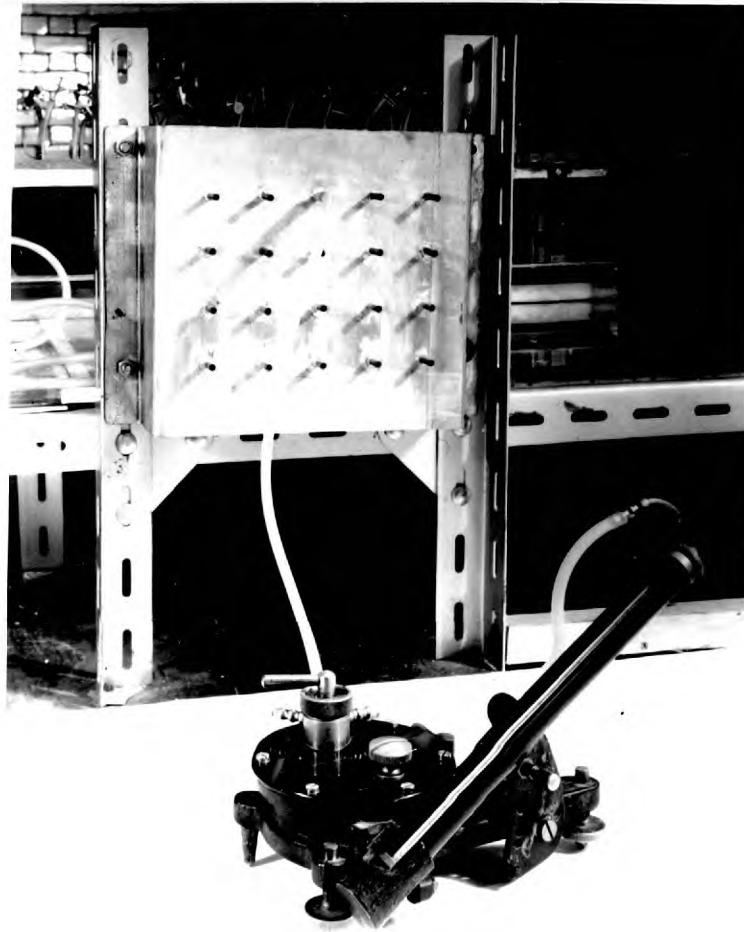


PLATE 2. SINGLE TUBE SWIVEL ARM MANOMETER.

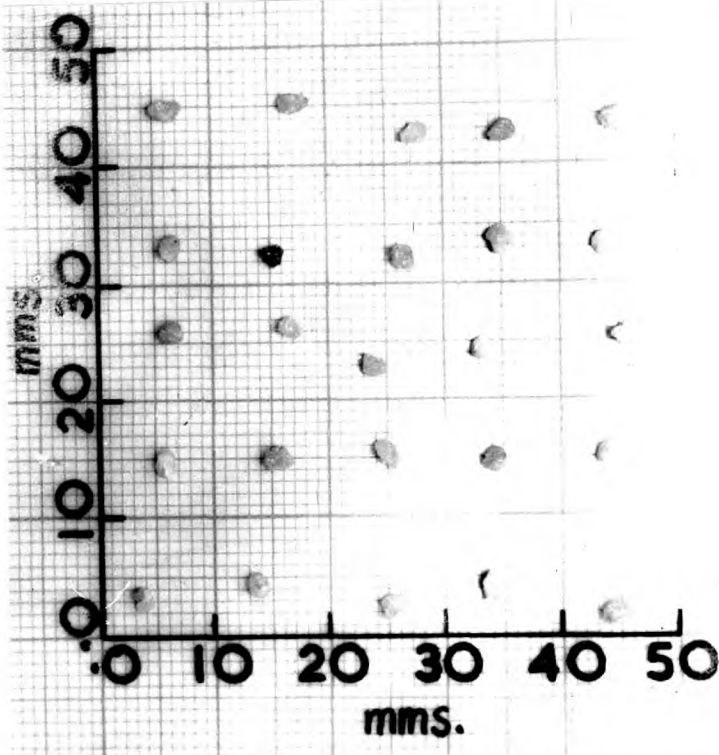


PLATE 3. TYPICAL GRAINS OF SAND 'A' $R = 1.56$ mms.

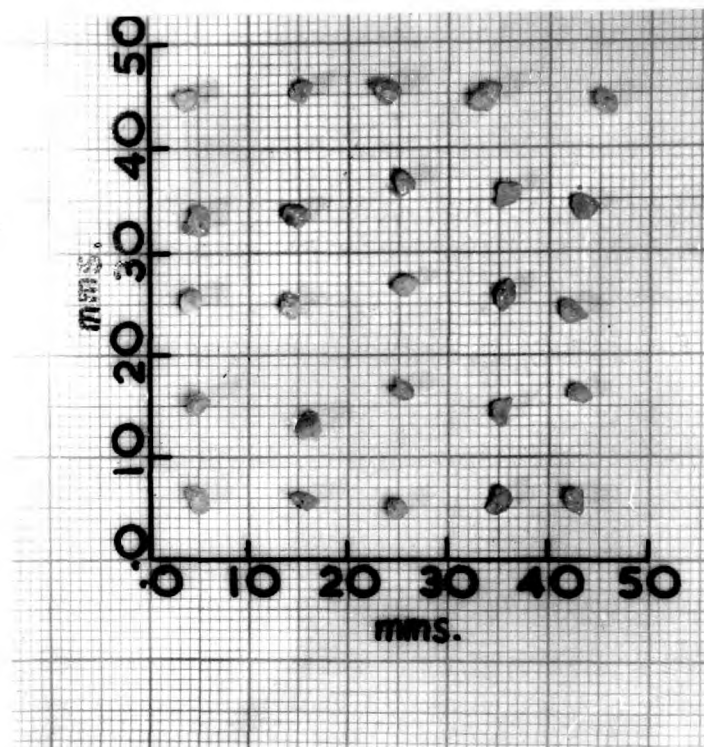


PLATE 4. TYPICAL GRAINS OF SAND 'B' $R = 1.96$ mms.

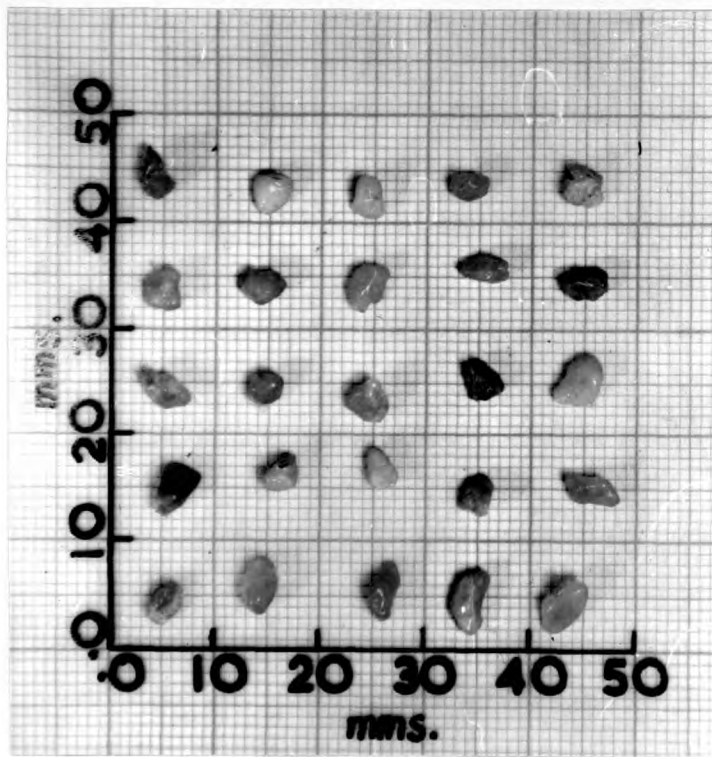


PLATE 5. TYPICAL GRAINS OF SAND 'C' $R = 3.16$ mm.

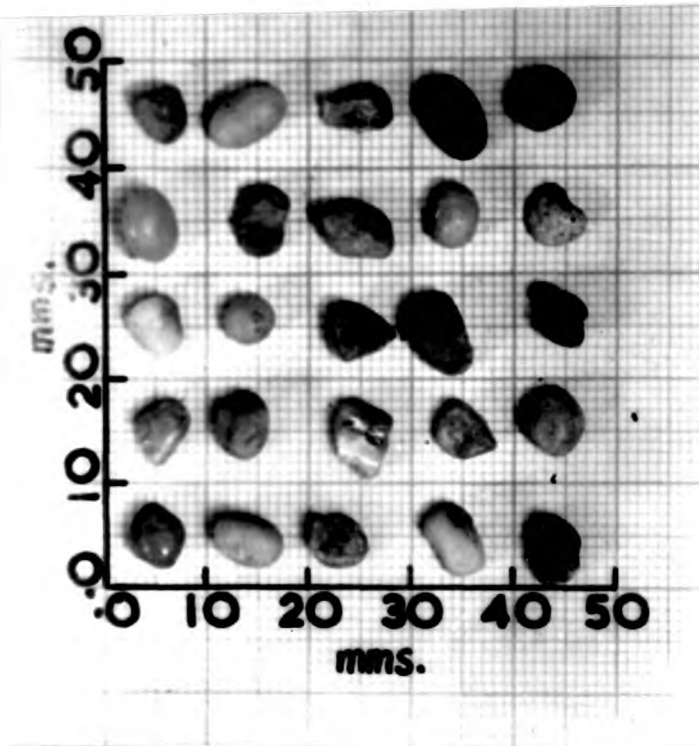


PLATE 6. TYPICAL GRAINS OF SAND 'D' $R = 4.75$ mm.

measuring the static pressure along the boundary layer of the rough wall, along the curve and across the flow at the various stations where total head explorations were made.

The tappings for the static pressure along the boundary layer were located on the upper side of the nozzle. The first 13 of these tappings were spaced at distances of $\frac{1}{2}$ " due to the steep pressure gradient in this region of the flow, whereas the next 10 tappings were spaced at 1.0" intervals and the rest of the tappings at 2.0" intervals as the variations in the pressure become smaller. The first 13 tappings were located at a height of 0.10 inches above the surface of the roughness elements whereas the other tappings were kept at a height of 0.20 inches as the boundary layer became thicker. These tappings were made from brass tubes which were screwed in the perspex side to a depth of 0.32 inches, then the rest of the perspex thickness was drilled by a No. 60 drill to form the tapping hole. The connections between these tappings and the perspex side were made air-tight by the use of white lead and hemp.

Pressure tappings were also provided on the upper side of the nozzle for measuring the variation of static pressure across the flow and were the same as mentioned above.

2.3.4. Total Head Measurements.

Total head explorations in the flow were made using a small total head pitot tube made from stainless steel tube 0.10 inches external diameter and 0.0415" internal diameter. The pitot tube was introduced in the flow through the total head tappings provided on the curved surface, and the mouth of the tube could travel normal to the rough wall by means of a micrometer gauge which served as a carrier for the tube. The micrometer gauge was subdivided to 0.002" and was supported on a brass angle provided with holes at the various stations to house the micrometer gauge (figure 2.3). The angle was fixed to the supporting frame at a known distance from the plane of the rough wall, thus helping to check the location of the pitot tube with respect to the reference wall.

The effect of viscosity on the reading of a total head pitot tube was studied by Miss Baker (1922), who showed that the velocity of the flow

can be found accurately from the difference between the pitot tube reading and the static pressure reading opposite to its mouth, provided that $\frac{ud}{\nu} > 60$ where d is the diameter of the pitot opening, and for smaller values of $\frac{ud}{\nu}$ the velocity would be overestimated due to viscosity effect. In the present investigation, however, ~~this~~ difficulty does not arise. Since the minimum velocity measured in the test was about 35.30 ft./sec., and the kinematic viscosity for air was 160×10^{-6} ft.²/sec., the corresponding value of $\frac{ud}{\nu}$ thus was 760 which is well above the danger zone given by Miss Baker.

The turbulent velocity fluctuations on the other hand as pointed out by Goldstien (1936) may have a falsifying effect on the readings of the pitot tube. As indicated by Tillmann (1946) the time averages of the squares of velocity, as measured by the pitot tube, can be approximated by:

$$u_p^2 = \bar{u}^2 + \overline{u'^2}$$

$$\text{or } \bar{u} = \sqrt{u_p^2 - \overline{u'^2}} = u_p \left[1 - \frac{1}{2} \left(\frac{\overline{u'^2}}{u_p^2} \right) \right] \quad (2.11)$$

where u_p is the velocity measured by the pitot tube,

$\bar{u}$ is the true mean velocity and u' is the longitudinal velocity fluctuation about the mean. Since $\bar{u}^2$

is a positive quantity, the pitot tube always tends to overestimate the velocity. In the present investigation, however, the maximum value of $\frac{\sqrt{u'^2}}{u}$ measured in the tests (section 4.3) was 0.165 and the corresponding error in u according to (2.11) will be of the order of 1.4%. However, the value of $\frac{\sqrt{u'^2}}{u}$ was usually much smaller than 0.165, and the effect of turbulence on the present pitot tube readings can be ignored.

2.3.5. Manometers

2.3.5.1. Single Arm Swivel Manometer.

Both the static pressure and the total head pressure were recorded on a single tube swivel arm manometer (made in Berlin by R. Fuess) with alcohol as metering fluid (plate 2). The arm of the manometer could be set in any one of five different inclinations having factors of 0.80, 0.40, 0.20, 0.10, or 0.05 to give the equivalent height of the vertical column of water in each case (based on 0.80 specific gravity for alcohol), the reading being the difference between the atmospheric pressure and the measured pressure in millimeters. The manometer was leveled repeatedly during the readings and the time average values for the pressures were recorded.

This manometer has an advantage over a multi-tube manometer in eliminating errors resulting from slightly different inclinations of the individual tubes in the multi-tube case.

2.3.5.2. Reference Manometer.

To have all measurements for a single roughness done under the same flow conditions, a separate manometer was used to check the flow during the measurements. The manometer used for this purpose was an inclined differential alcohol manometer which could be kept at any suitable inclination, and after being leveled, was connected to a pressure tapping on the side of the nozzle. The reading of this manometer was maintained constant during the measurements, either by slightly adjusting the motor speed or by adjusting the opening of a flap valve in the wind tunnel, thus ensuring the same discharge throughout the particular measurements.

2.4 Experimental Results

2.4.1. Velocity and pressure surveys.

Mean velocity distributions for the boundary layer were estimated from the total head explorations made with the pitot tube, together with the

measured static pressure variation across the flow. For each roughness, several velocity profiles were measured along lines normal to the rough wall, and in all cases the normal distance y was measured from the centre line of the grains. When changing the roughness, the plane of the rough wall was shifted parallel to itself so that the line passing through the centre of the grains occupied the same position in all cases.

The measured velocity distributions for the various roughnesses are seen in figures (2.7 to 2.10) plotted against the normal distance y , and their positions are shown relative to the noze of the plate ($x = 0$). The corresponding pressure distribution along the boundary layer of the rough wall, as obtained from the pressure tappings, are plotted in its correct position below the velocity distributions, and the pressures are given in cms of water below atmospheric pressure. The figures also show the approximate pressure gradient $\frac{dP}{dx}$ obtained by differentiating the pressure distribution, and again plotted against the same distance scale of x . As seen from the figures, the pressure gradient is maximum at the noze of the plate and drops systematic-

FIG. (2.7) Velocity and Pressure Distributions for Roughness "A" ($k=1.56\text{ mm}$)

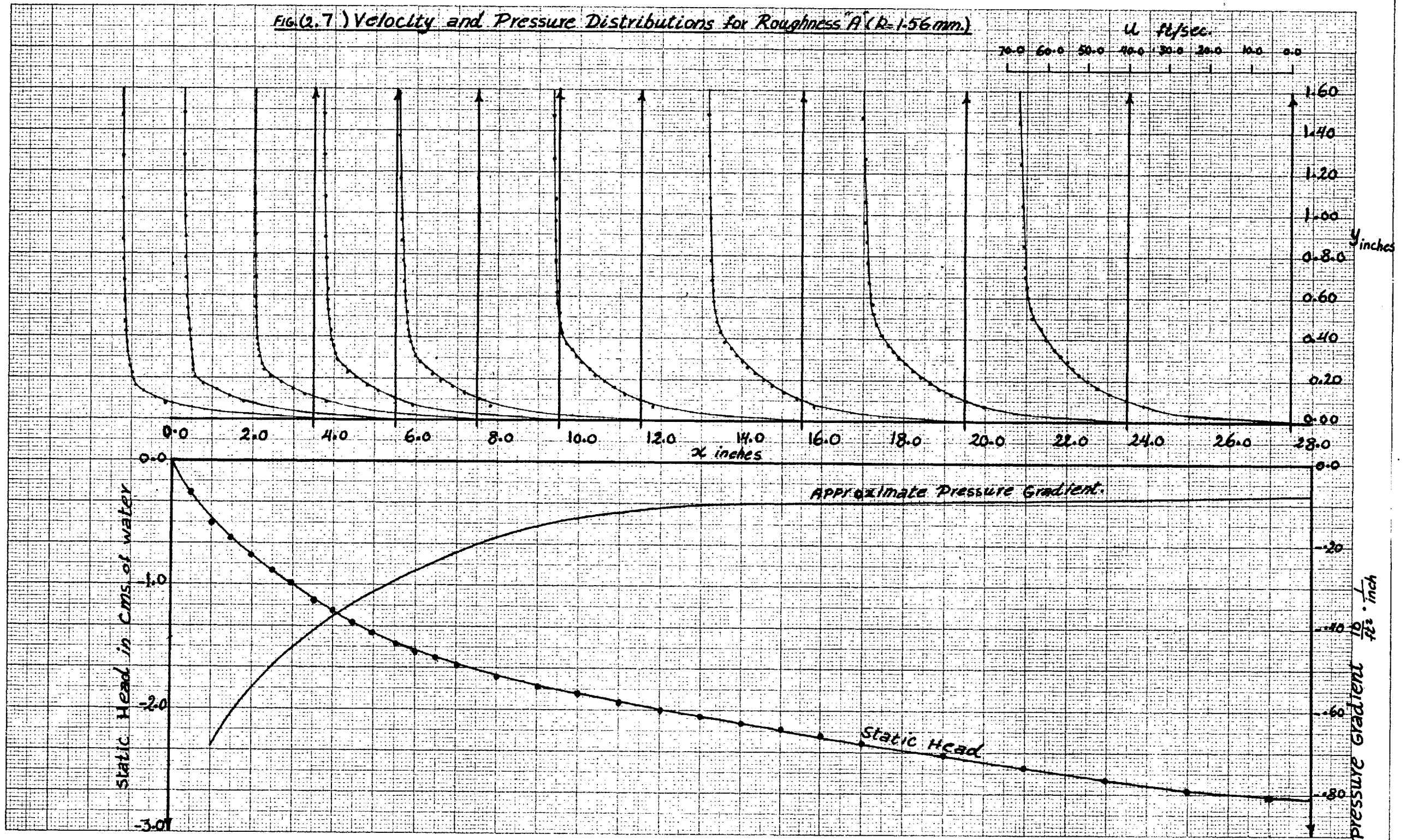


Fig. (2.8) Velocity and Pressure Distributions for Roughness B^* ($k=1.96 \text{ mm}$)

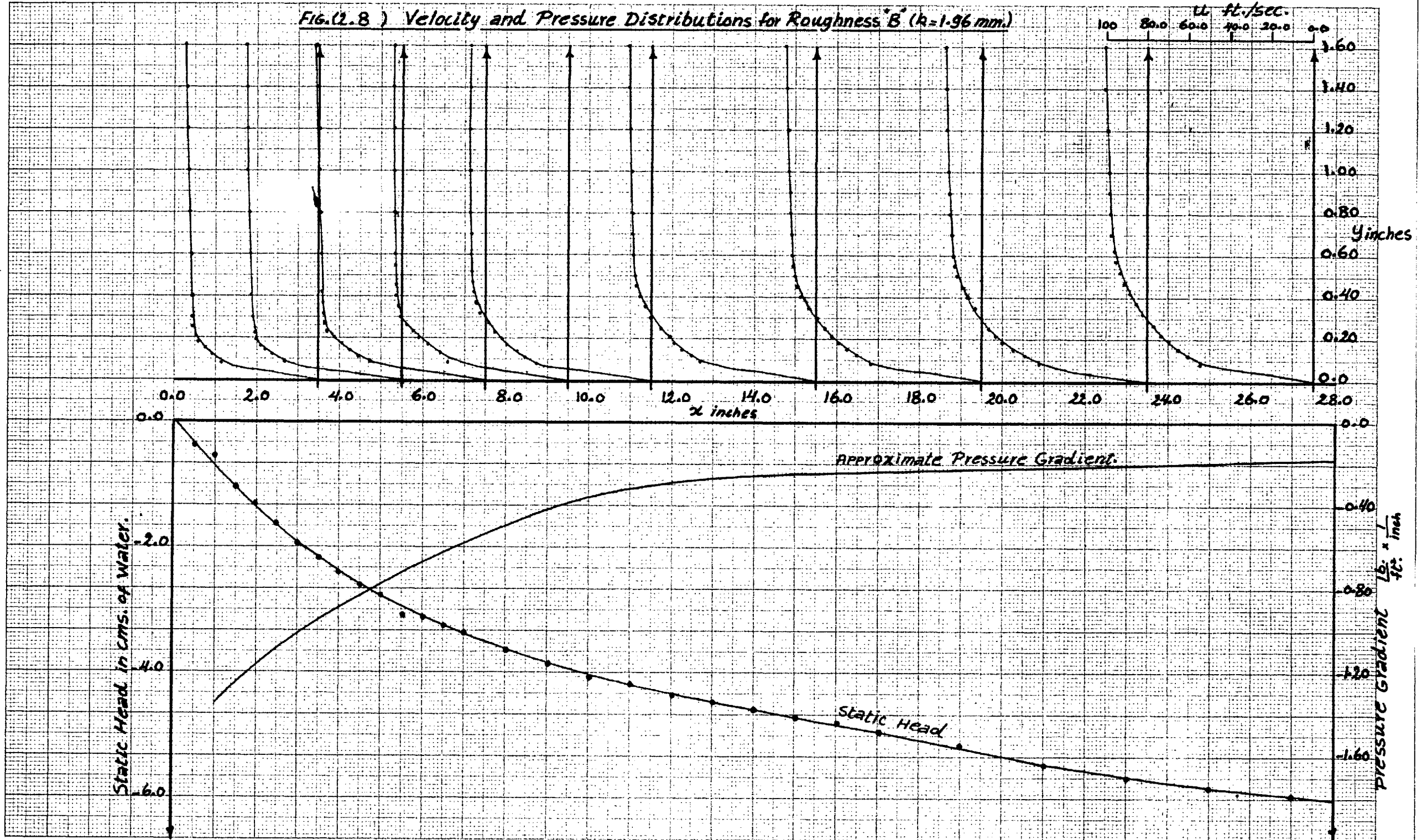


FIG. (2.9) Velocity and Pressure Distributions for Roughness C^* ($k=3.16 \text{ mm.}$)

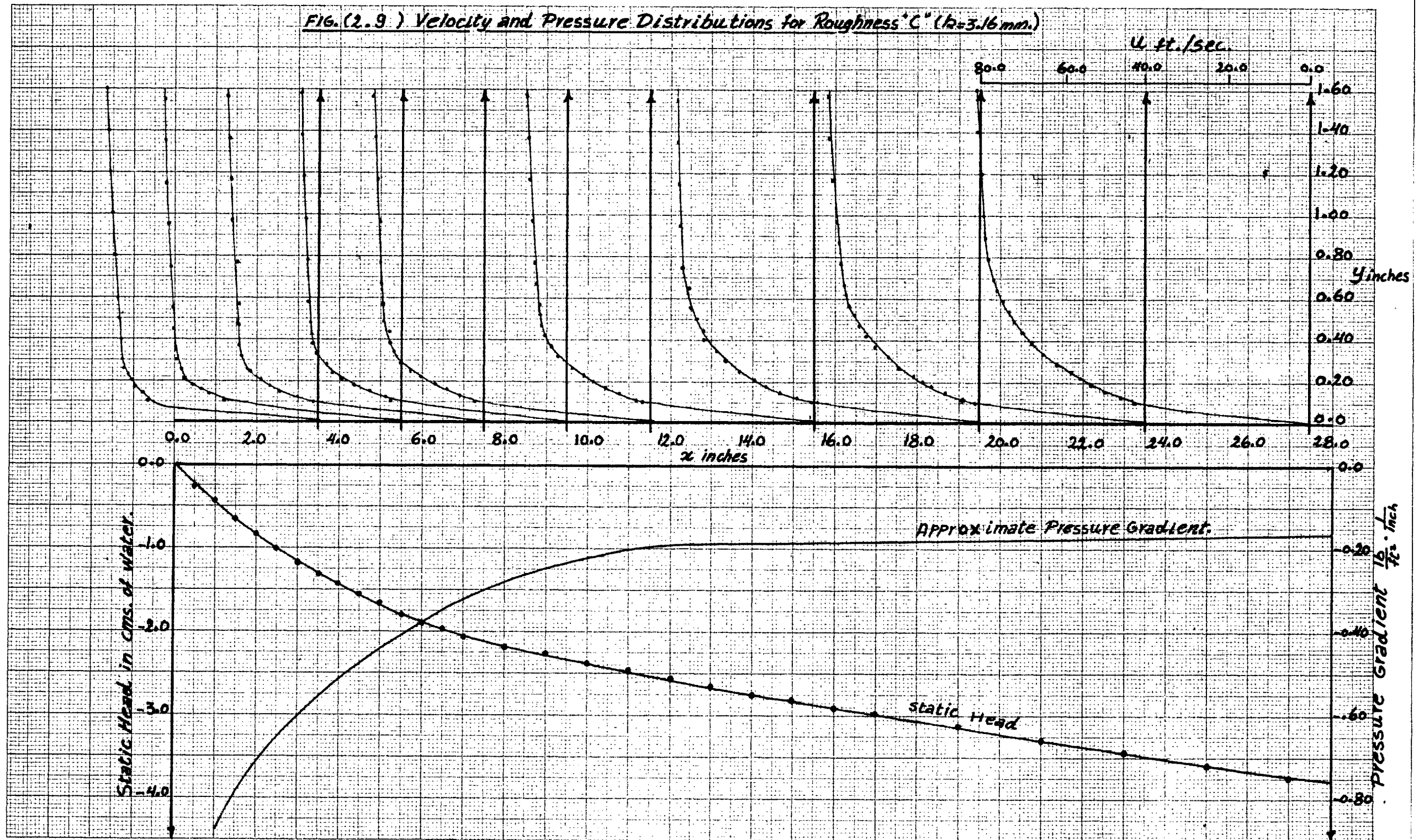
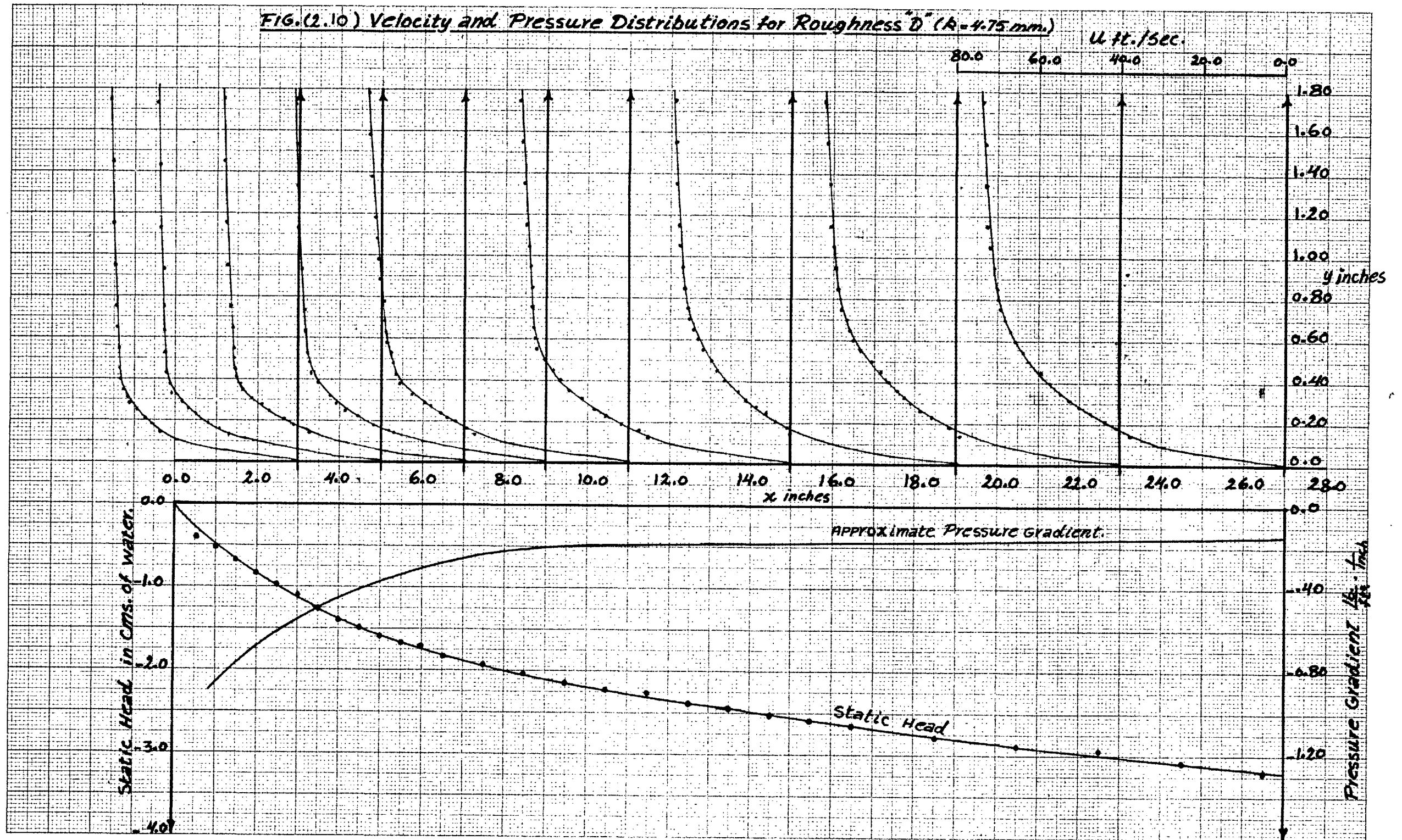


FIG. (2.10) Velocity and Pressure Distributions for Roughness D^* ($k = 4.75 \text{ mm.}$)



cally in the downstream direction in a near exponential manner, i.e. $\frac{dp}{dx}$ tends slowly to zero as x tends to ∞ . Figure (2.11) shows the variation, in the downstream direction of the velocity for various values of y in the case of the 1.56 mms sand, and as seen from this figure u is substantially constant for small values of y which is in line with the theory.

A plot of the type of figure (2.11) seems convenient for presenting the velocity distribution within the flow for any case of pressure distribution. For comparison, figure (2.11a) was plotted for the case of a constant pressure, though on a smooth wall, and the figure shows clearly the variation of u with x and y .

2.4.2. Evaluation of the Wall Shear Stress

To calculate the shear stress at the wall, τ_o , a method based on the momentum equation is introduced. This method involves the use of the experimental surveys of the total pressure, static pressure and mean velocities in the boundary layer as follows:

Considering a steady two-dimensional flow along a flat wall under the influence of a pressure gradient, and taking a rectangular control surface ABCD (Figure 2.12) placed in the flow such as to have

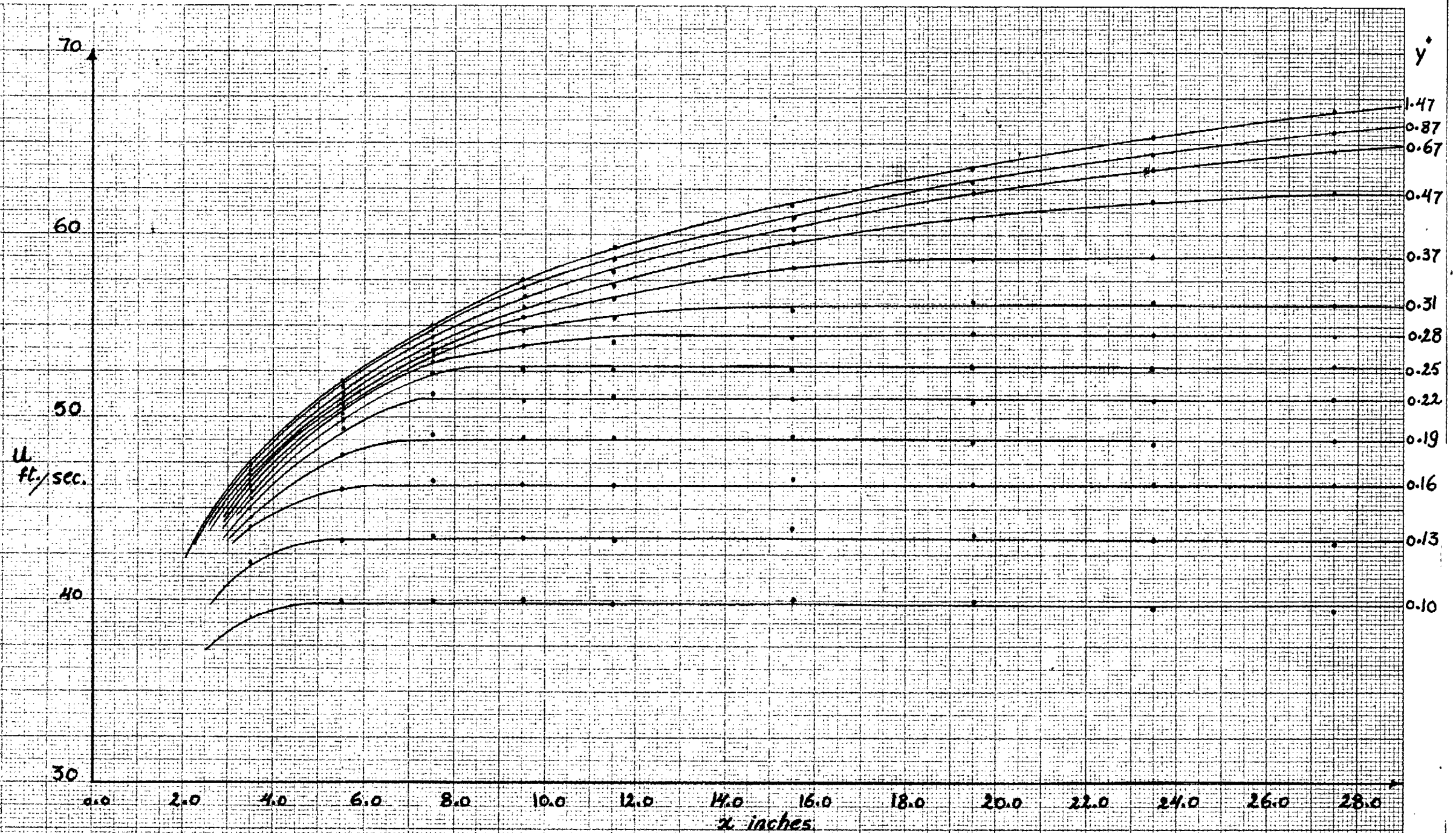


FIG. (2.11) Velocity Distribution for various values of y , $b = 1.56$ mm.

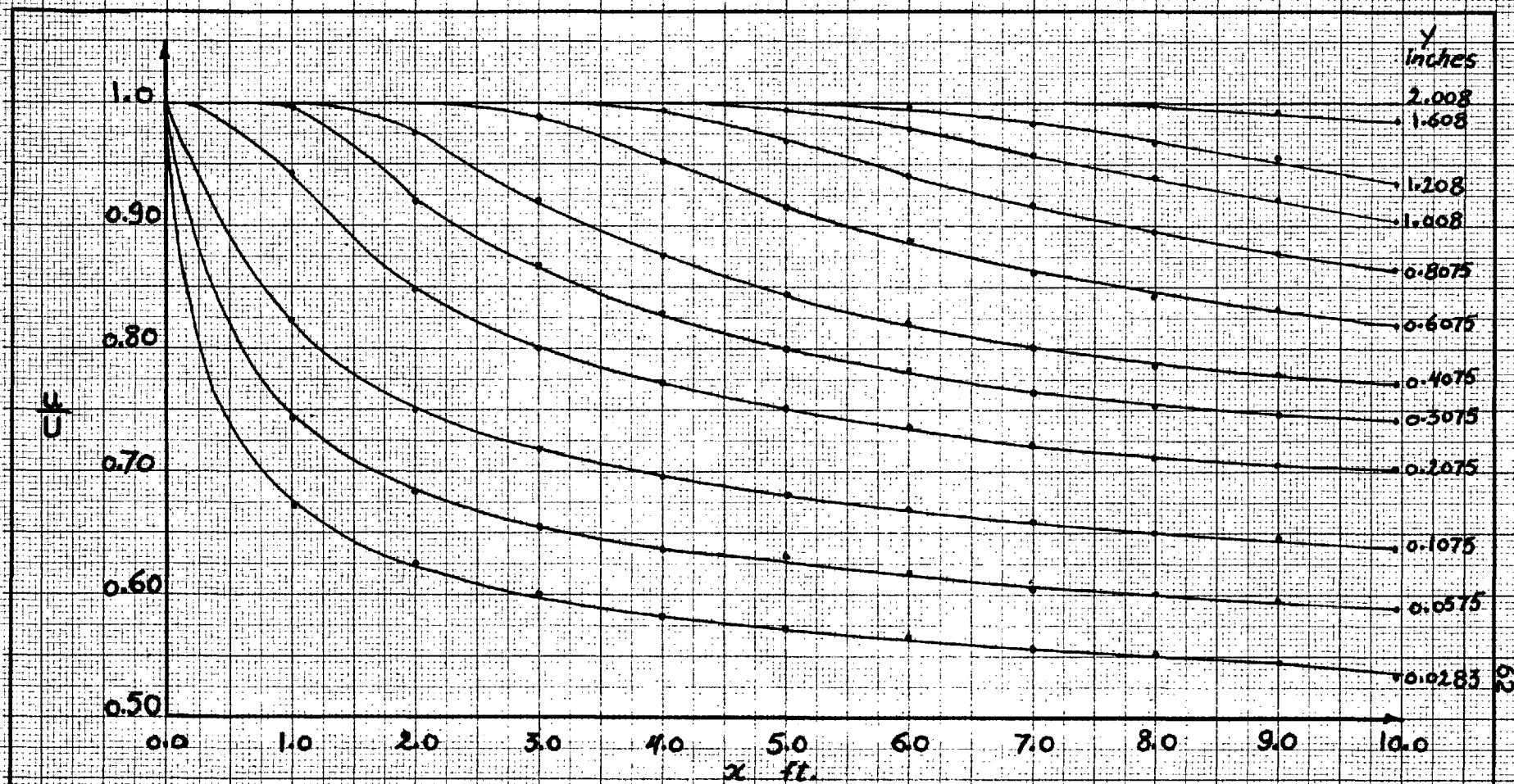
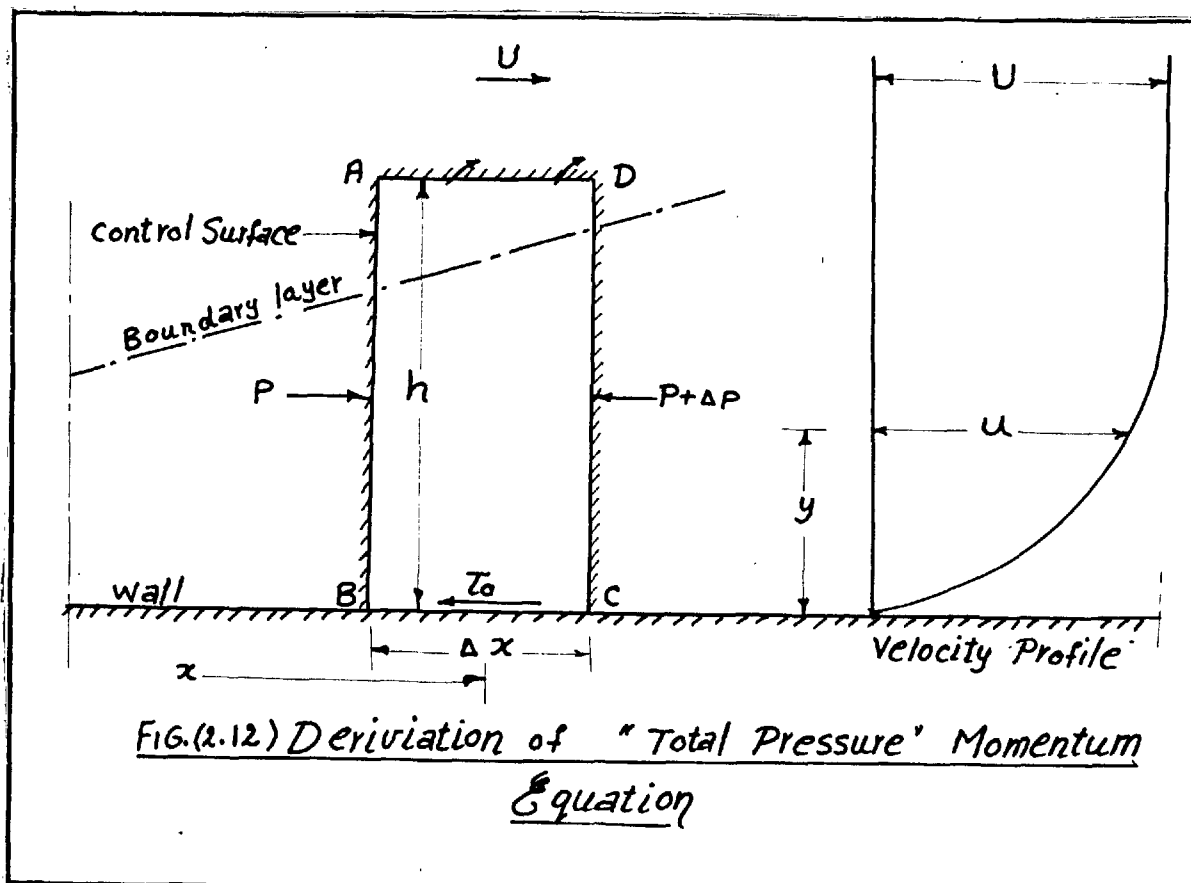


FIG (2.11 a) Velocity Distribution for various values of x and y for Constant Pressure Flow on a Smooth wall. (Plotted from data by the Iowa Institute, 1958)

the side AD parallel to the wall and at such a distance h from it greater than the thickness of the boundary layer. The forces acting on the control surface in the x -direction will thus be: that due to the friction at the wall $-\tau_0 dx$, that



due to the pressure gradient $-dx \cdot h \cdot \frac{dP}{dx}$; and those contributed by the change of momentum in the x -direction.

Due to continuity, fluid must leave through AD, the quantity leaving being equal to the difference between that entering through AB and that leaving through CD, i.e. equal to $dx \cdot \frac{\partial}{\partial x} \int_{y_1}^h \rho u dy$, and the corresponding momentum flux per unit time across AD will thus be $dx \cdot \rho U \frac{\partial}{\partial x} \int_{y_1}^h u dy$. Similarly the force due to the difference in the momentum flux between the inflowing masses through AB and the outgoing masses through CD will equal $-dx \cdot \rho \frac{\partial}{\partial x} \int_{y_1}^h u^2 dy$.

For the equilibrium of the forces acting on the control surface in the x-direction we have:

$$-h \frac{\partial P}{\partial x} - \frac{\partial}{\partial x} \int_{y_1}^h \rho u^2 dy + \rho U \frac{\partial}{\partial x} \int_{y_1}^h u dy - \tau_o = 0$$

$$\therefore \frac{\partial}{\partial x} \int_{y_1}^h 2(P + \frac{\rho u^2}{2}) dy - \frac{\partial}{\partial x} \int_{y_1}^h P dy - \rho U \frac{\partial}{\partial x} \int_{y_1}^h u dy + \tau_o = 0$$

or

$$\frac{\partial}{\partial x} \int_{y_1}^h 2P' dy - \frac{\partial}{\partial x} \int_{y_1}^h P dy - \rho U \frac{\partial}{\partial x} \int_{y_1}^h u dy + \tau_o = 0 \quad (2.12)$$

where $P' = \text{total pressure} = p + \frac{1}{2} \rho u^2$

From the definition of the displacement thickness δ_* (equation 1.15) we have:

$$\delta_* = \int_0^h (1 - \frac{u}{U}) dy$$

or

$$\int_{y_1}^h u dy = U (h - \delta_*) \quad (2.13)$$

Substituting from (2.13) into equation (2.12), then:

$$\frac{\partial}{\partial x} \int_{y_1}^h 2 P' dy - \frac{\partial}{\partial x} \int_{y_1}^h P dy - \rho U \frac{\partial}{\partial x} [U(h - \delta_*)] + \tau_0 = 0$$

$$\therefore \frac{\partial}{\partial x} \int_{y_1}^h 2 P' dy - \frac{\partial}{\partial x} \int_{y_1}^h P dy + \rho U^2 \frac{\partial}{\partial x} \delta_* - \rho U \frac{dU}{dx} (h - \delta_*) + \tau_0 = 0$$

$$\text{or } \frac{\partial}{\partial x} \int_{y_1}^h 2 P' dy - \frac{\partial}{\partial x} \int_{y_1}^h P dy + \rho U^2 \frac{\partial}{\partial x} \delta_* + \frac{\partial}{\partial x} P(h - \delta_*) + \tau_0 = 0 \quad (2.14)$$

Now writing $p = p_h + \Delta p$ where P_h is the static pressure at $y = h$, then:

$$\frac{\partial}{\partial x} \int_{y_1}^h P dy = h \frac{\partial P_h}{\partial x} + \frac{\partial}{\partial x} \int_{y_1}^h \Delta P dy \quad (2.15)$$

Substituting from (2.15) into equation (2.14) then:

$$\tau_0 = - \frac{\partial}{\partial x} \int_{y_1}^h 2 P' dy + \frac{\partial}{\partial x} \int_{y_1}^h \Delta P dy + \delta_* \frac{\partial P}{\partial x} - \rho U^2 \frac{\partial}{\partial x} \delta_* \quad (2.16)$$

which is the exact solution for the flow considered.

The term $\frac{\partial}{\partial x} \int_{y_1}^h \Delta P dy$ was usually very small, in which case equation (2.16) reduces to:

$$\tau_0 = - \frac{\partial}{\partial x} \int_{y_1}^h 2 P' dy - \rho U^2 \frac{\partial}{\partial x} \delta_* + \delta_* \frac{\partial P}{\partial x} \quad (2.17)$$

as compared to Gruschwitz's formula:

$$\tau_0 = U^2 \frac{d\theta}{dx} - \frac{\partial P}{\partial x} (2\theta + \delta_*)$$

Equation (2.17) seems easier due to the fact that it avoids the momentum thickness θ .

To avoid the differentiation of three different quantities in equation (2.17), the total shear force D can be used instead of τ_o , and by definition is equal to $\int_0^x \tau_o dx$. Thus integrating equation (2.17) with respect to x we get:

$$D = \int_0^x \tau_o dx = -2 \int_{y_1}^h P' dy - \int_0^x \rho U^2 \frac{d\delta_*}{dx} dx + \int_0^x \delta_* \frac{dP}{dx} dx$$

$$D = -2 \int_{y_1}^h P' dy - \int_0^x \rho U d\delta_*$$

(2.18)

To evaluate the first term of the above equation, the total pressure p' was plotted to base y and the area bounded by this curve was then integrated graphically up to a value of h greater than the thickness of the boundary layer everywhere, thus twice this area gave the term $2 \int_{y_1}^h P' dy$. The resulting values for $2 \int_{y_1}^h P' dy$ are shown plotted in figures (2.13 to 2.16) against the distance x from the nose of the plate. These diagrams also show the variation with x of the displacement thickness $\delta_* = \int_{y_1}^h (1 - \frac{u}{U}) dy$. For the purpose of evaluating the second term in equation (2.18), the velocity U at $y = h$, was

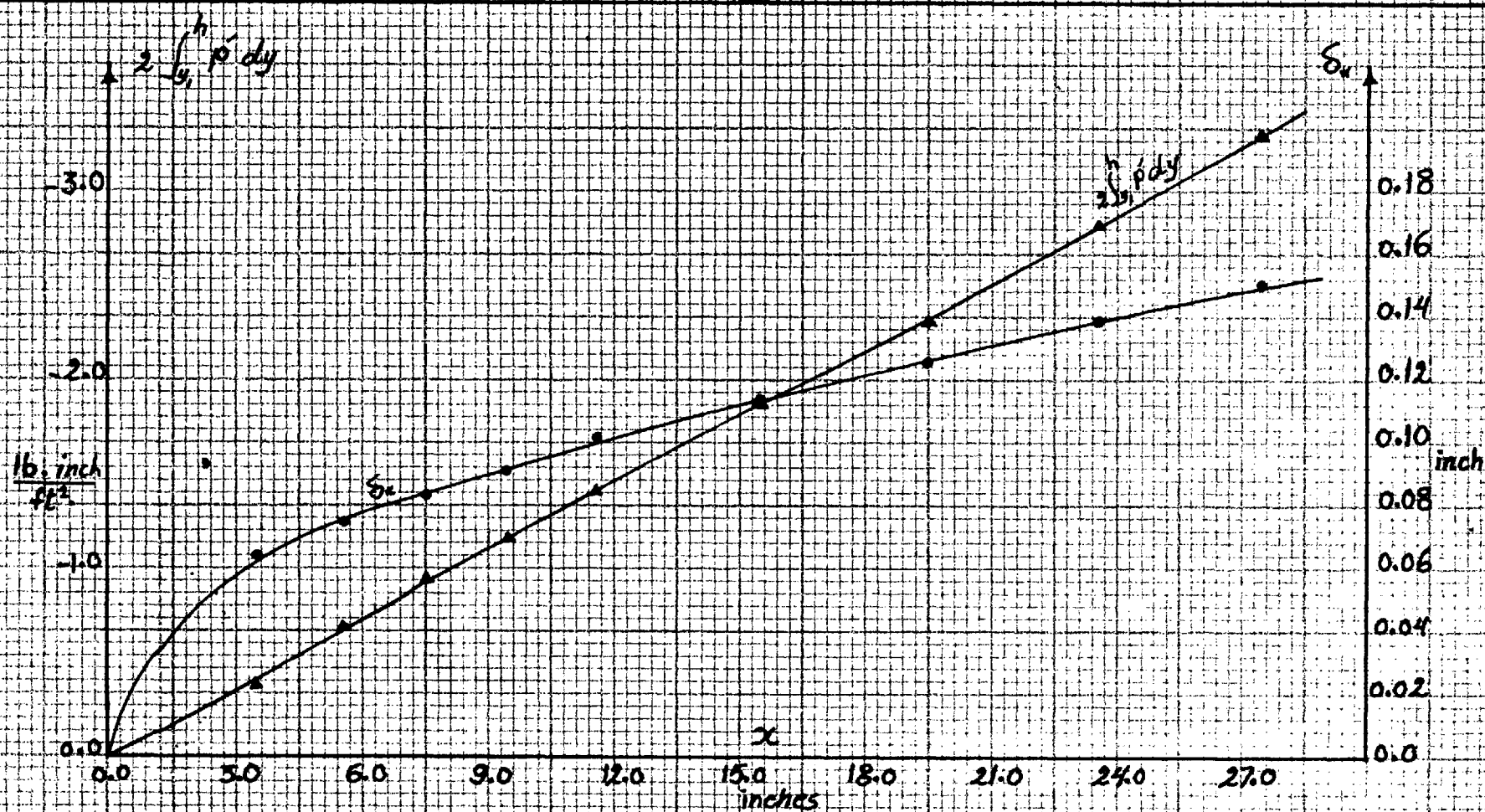


Fig. (2.13) Distribution of S_x and $2 \int_0^a p dy$ for Roughness A^* ($k=1.56 \text{ mm}$)
($h=1.40 \text{ inches}$)

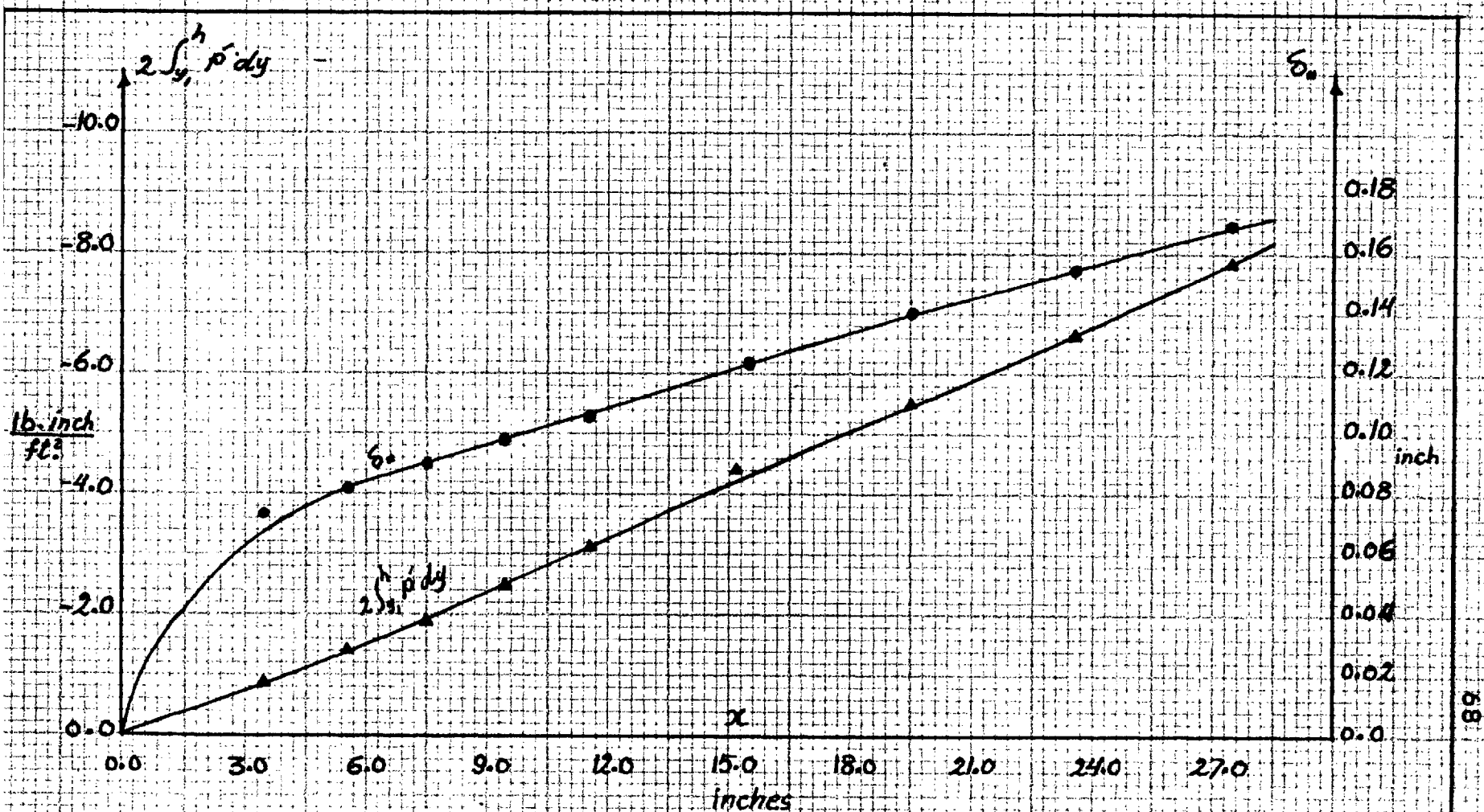


Fig. (2.14) Distribution of δ_s and $2 \int_{y_1}^h \rho dy$ for Roughness "B" ($k = 1.96 \text{ mm.}$)
($h = 1.40 \text{ inches}$)

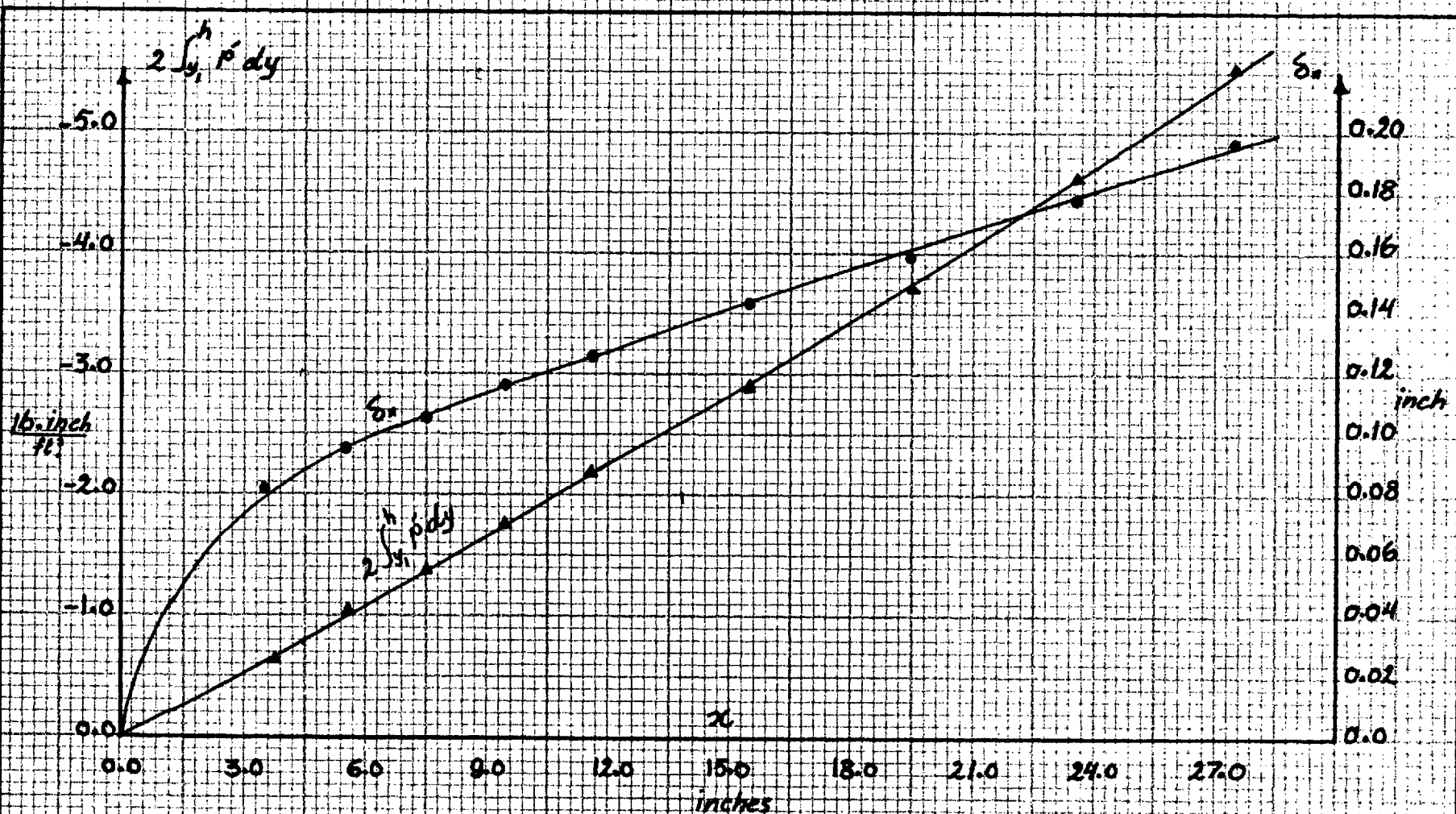


Fig. (2.15) Distribution of δ_r and $2 \int_{y_1}^h \rho dy$ for Roughness "C" ($k=3.16 \text{ mm}$)
($h=1.40 \text{ inches}$)

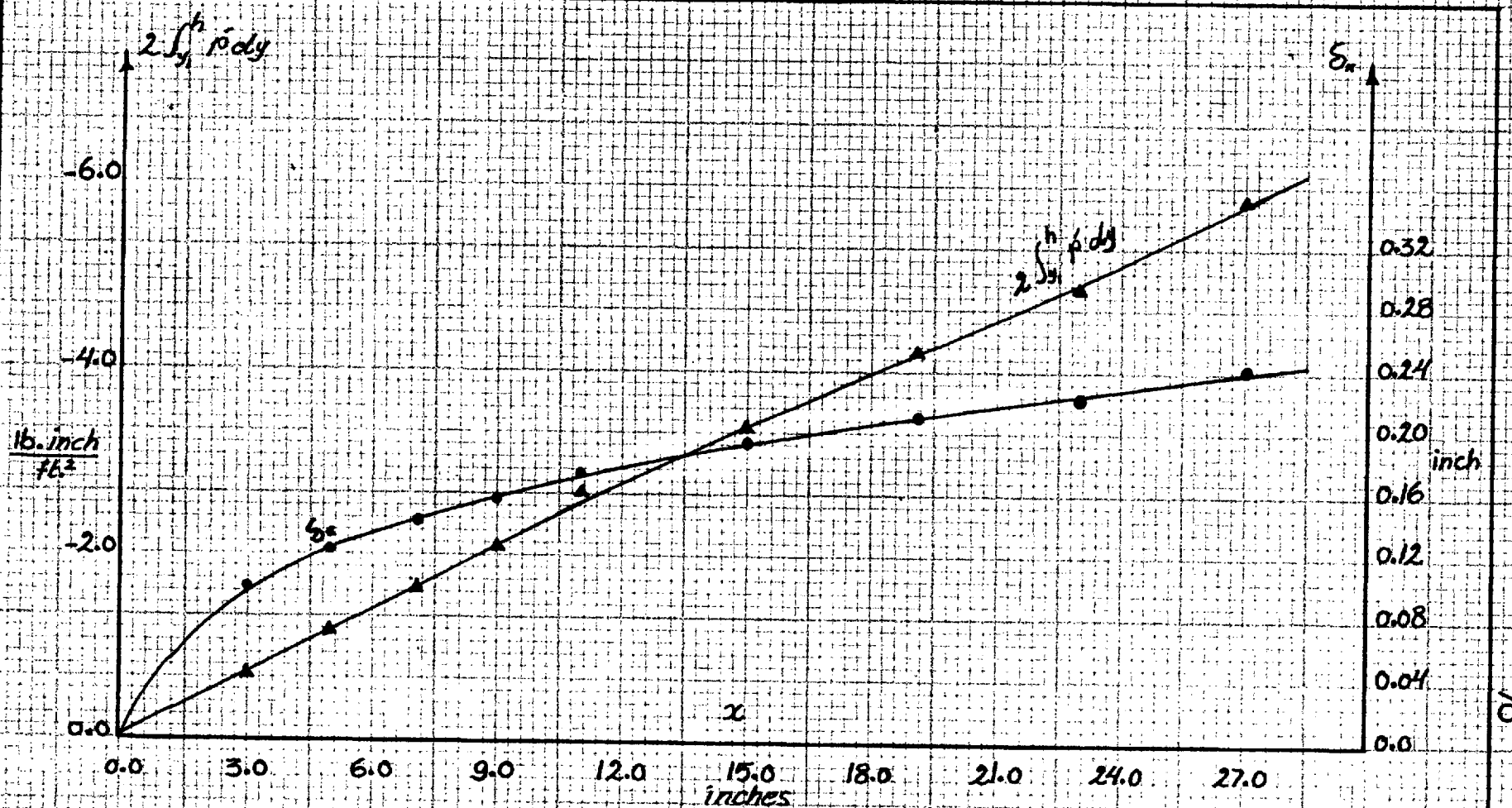


Fig. (2.16) Distribution of τ_x and $2 \int_{y_1}^h \tau dy$ for Roughness "D" ($k = 4.75 \text{ mm.}$)
($h = 1.40 \text{ inches}$)

plotted to base of $U\delta_*$, for various distances from the noze of the plate, and the area under this curve was then integrated up to the point under consideration. The final distribution for the total drag D is shown in Figures (2.17 to 2.20) for the various roughnesses. By inspecting these figures it appears that the variation of the drag with x is substantially linear and a straight line is seen satisfactorily in agreement with the points. Accordingly the local shear stress τ_o is substantially constant, since by definition is equal to $\frac{1}{\delta x} D$. However, to check if there is any systematic change in (D) , apart from the experimental scatter, specially for the bigger roughnesses, the local slope of the line D against x was calculated at the intermediate values of x on the assumption that a broken line joins between the points. Again the resulting values for τ_o showed no systematic change with x apart from a certain amount of scatter above and below the average value obtained from the slope of the lines in Figures (2.17 to 2.20), bearing in mind that an error of $\pm 2.0\%$ in D at some points may lead to

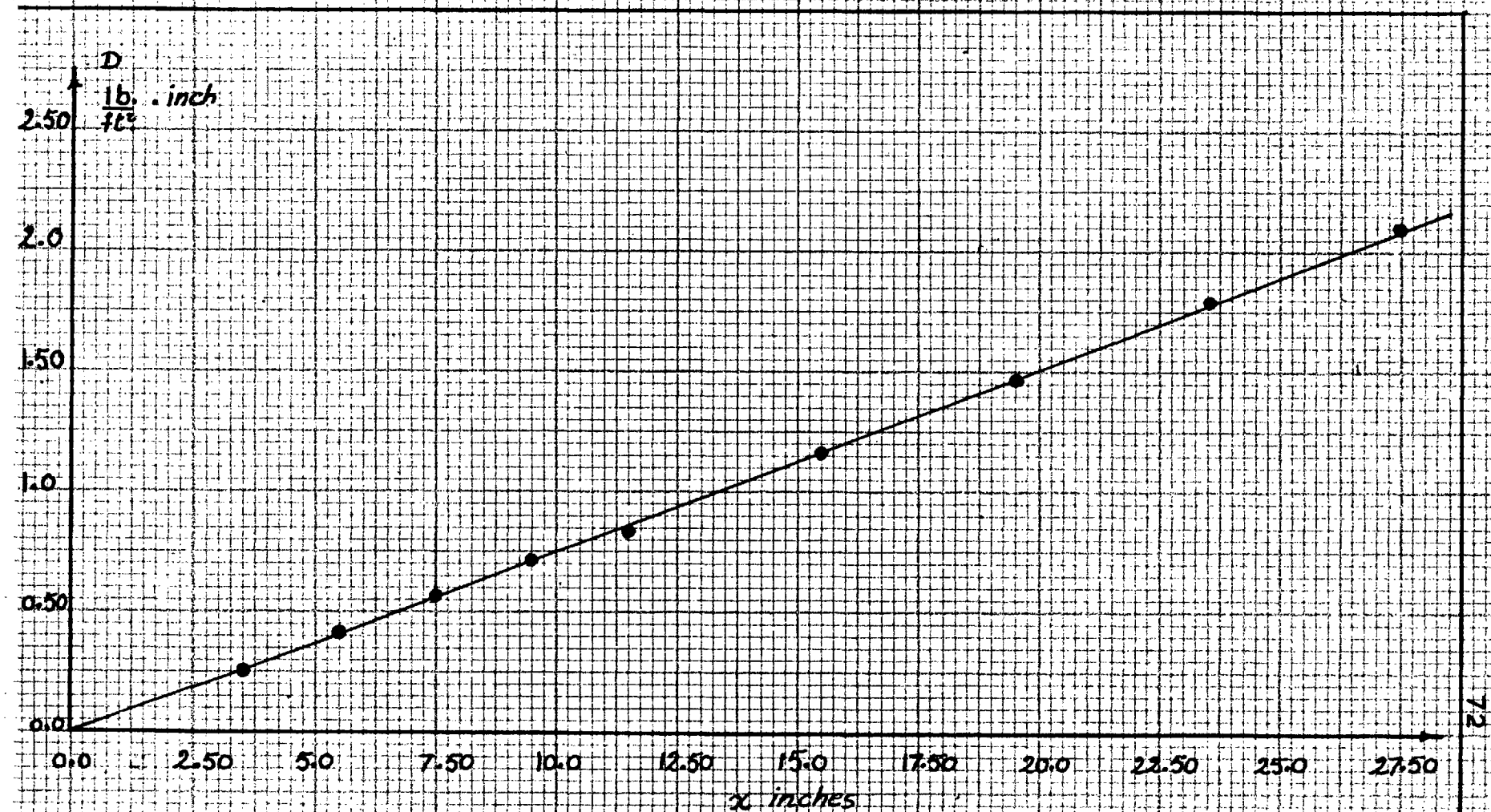


FIG. (2.17) Distribution of Total Drag D for Roughness 'A' ($k = 1.56$ mm.)

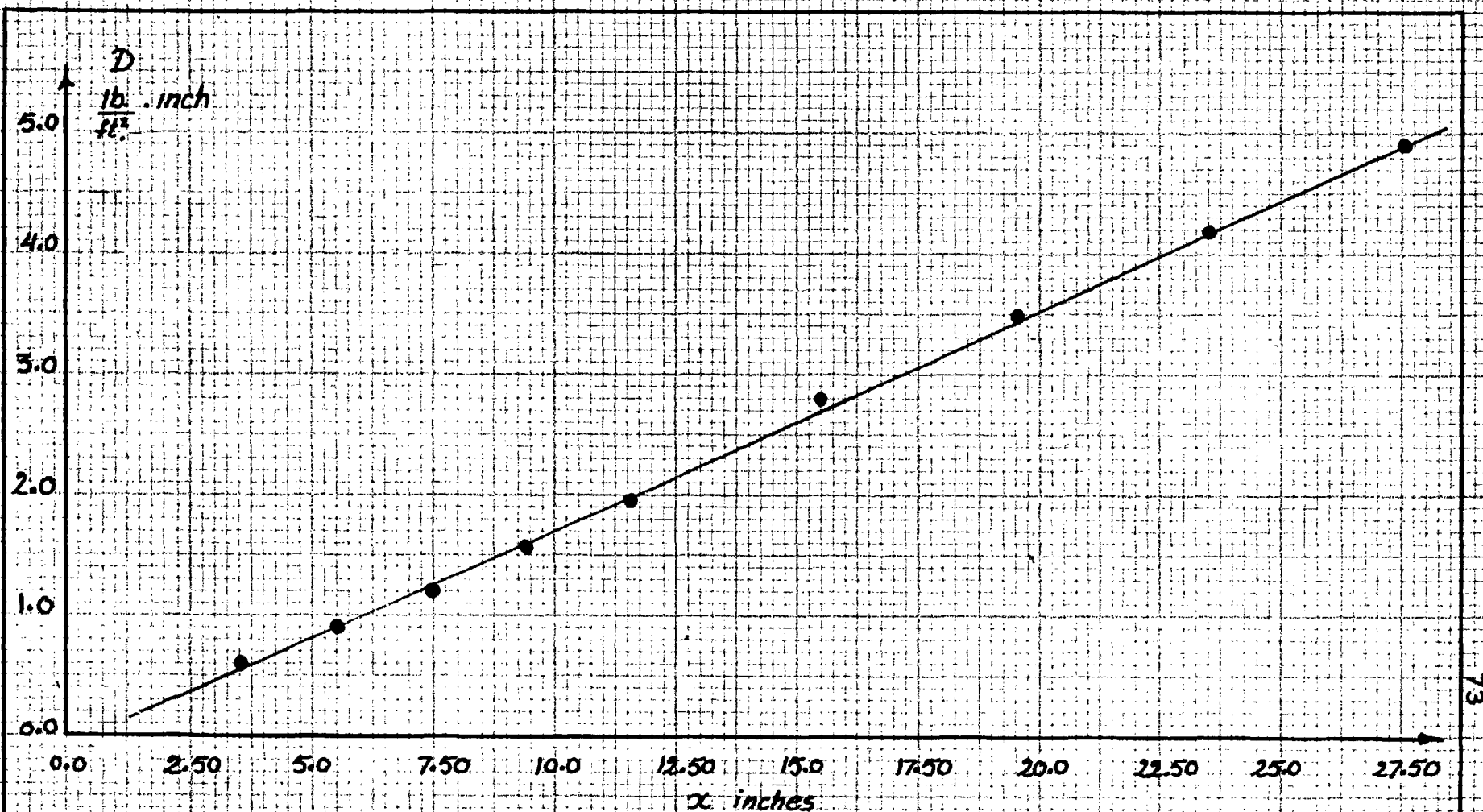


FIG (2.18) Distribution of Total Drag D for Roughness B ($k=1.96 \text{ mm.}$)

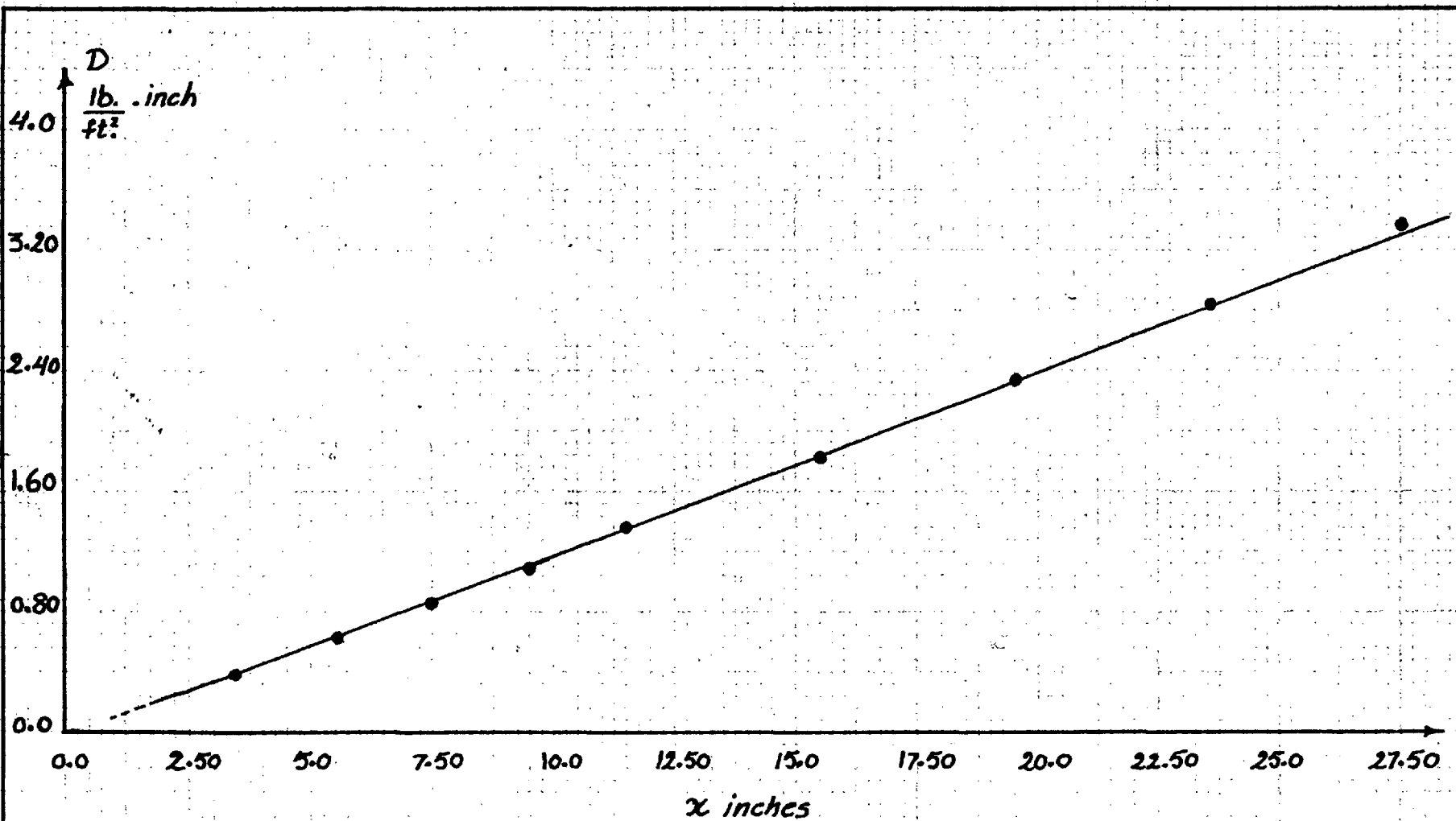


FIG.(2.19) Distribution of Total Drag D for Roughness " C " ($k = 3.16 \text{ mm.}$)

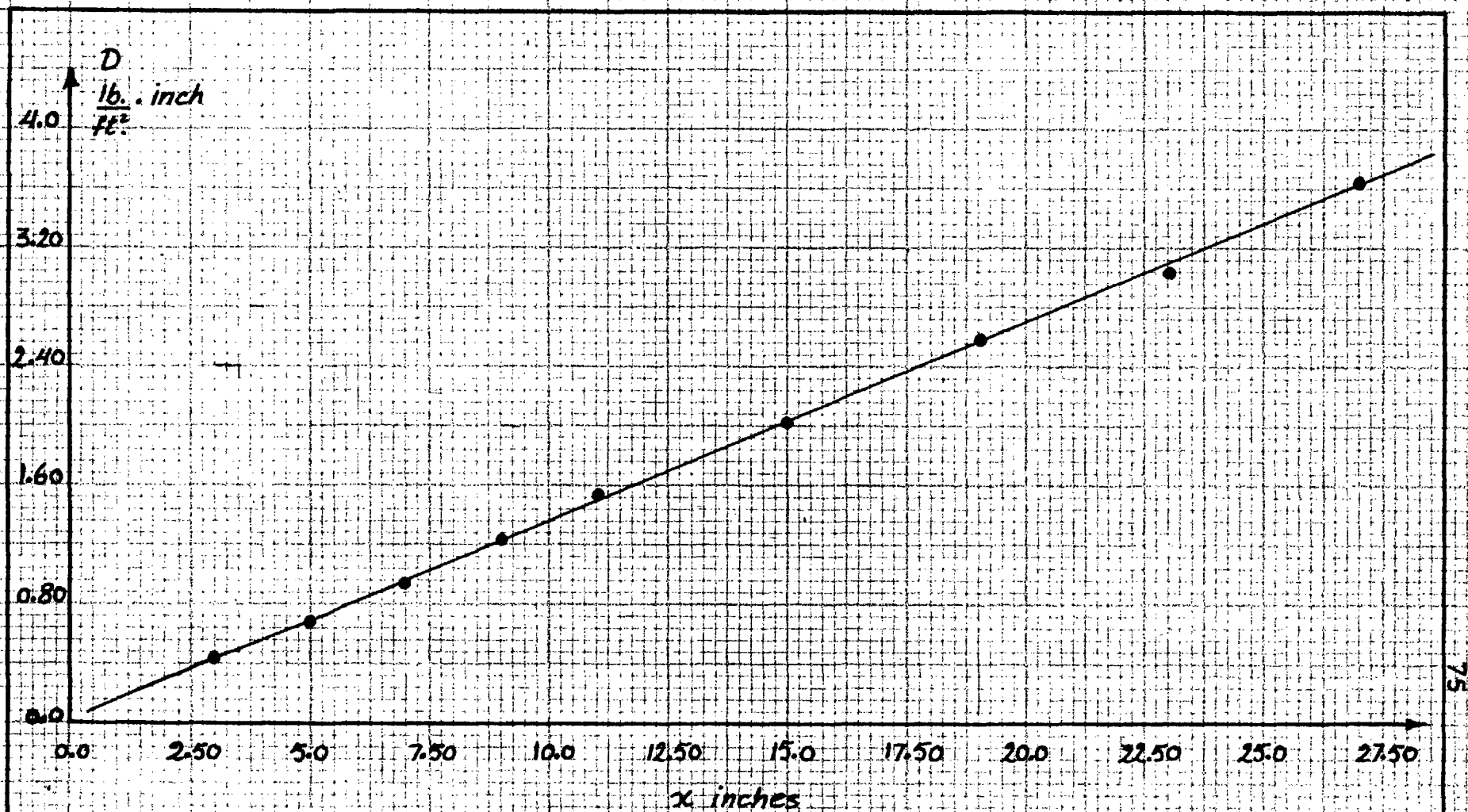


FIG.(2.20) Distribution of Total Drag D for Roughness D^* ($k = 4.75$ mm.)

an error of more than $\pm 25\%$ in τ_o in this method.

In equation (2.17) for τ_o , the term $\frac{\partial}{\partial x} \int_{y_1}^h \Delta P dy$ has been assumed negligible. However, in the equation for the total drag (equation 2.18) this term changes to $\int_{y_1}^h \Delta P dy$ which may not be negligible and the equation thus becomes:

$$D = - \int_{y_1}^h 2 P' dy + \int_{y_1}^h \Delta P dy - \int_0^x \rho U dU \delta_*$$

or

$$D = - \int_{y_1}^h 2 P' dy + \int_{y_1}^h (P - P_h) dy - \int_0^x \rho U dU \delta_* \quad (2.19)$$

The evaluation of the term $\int_{y_1}^h (P - P_h) dy$ showed, as seen from the values accompanying tables (III) in the appendix, that it had an appreciable value at small distances from the nose of the plate and decreases rapidly with increasing distance, the accuracy of the calculation was thus improved by adding this term. In general however, the scatter in the points of D against x is small and may be due partly to the use of graphical integration and partly to experimental errors.

2.4.3. Effect of Roughness on the Design of the Nozzle.

The experimental values of $X = \frac{U}{2.5V_*}$ are seen plotted against $\log \frac{x}{k}$ in figures (2.21 to 2.24).

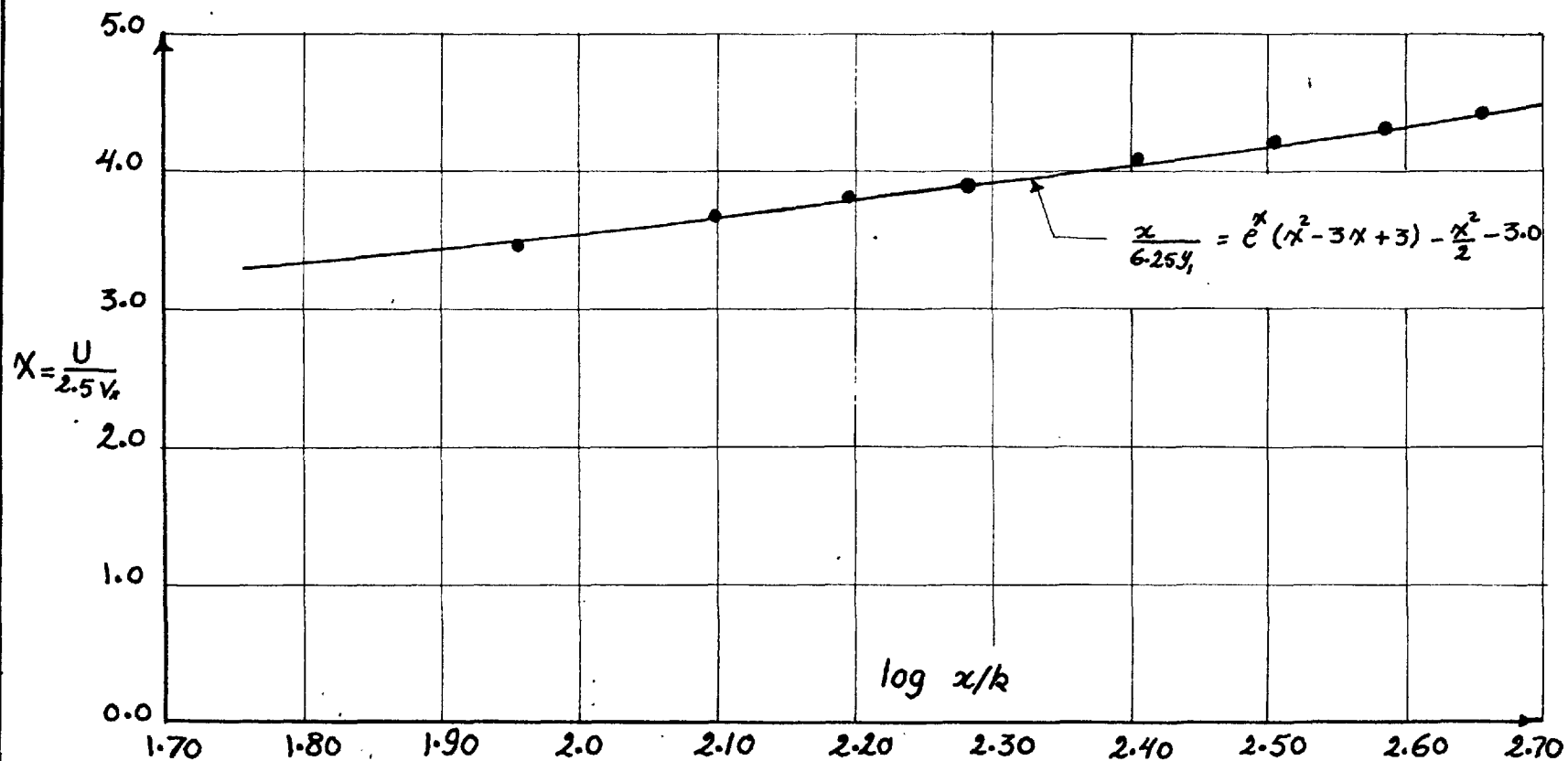


FIG.(2.21) Variation of χ with x/k for Roughness A ($y_1 = 0.00618$ inches)

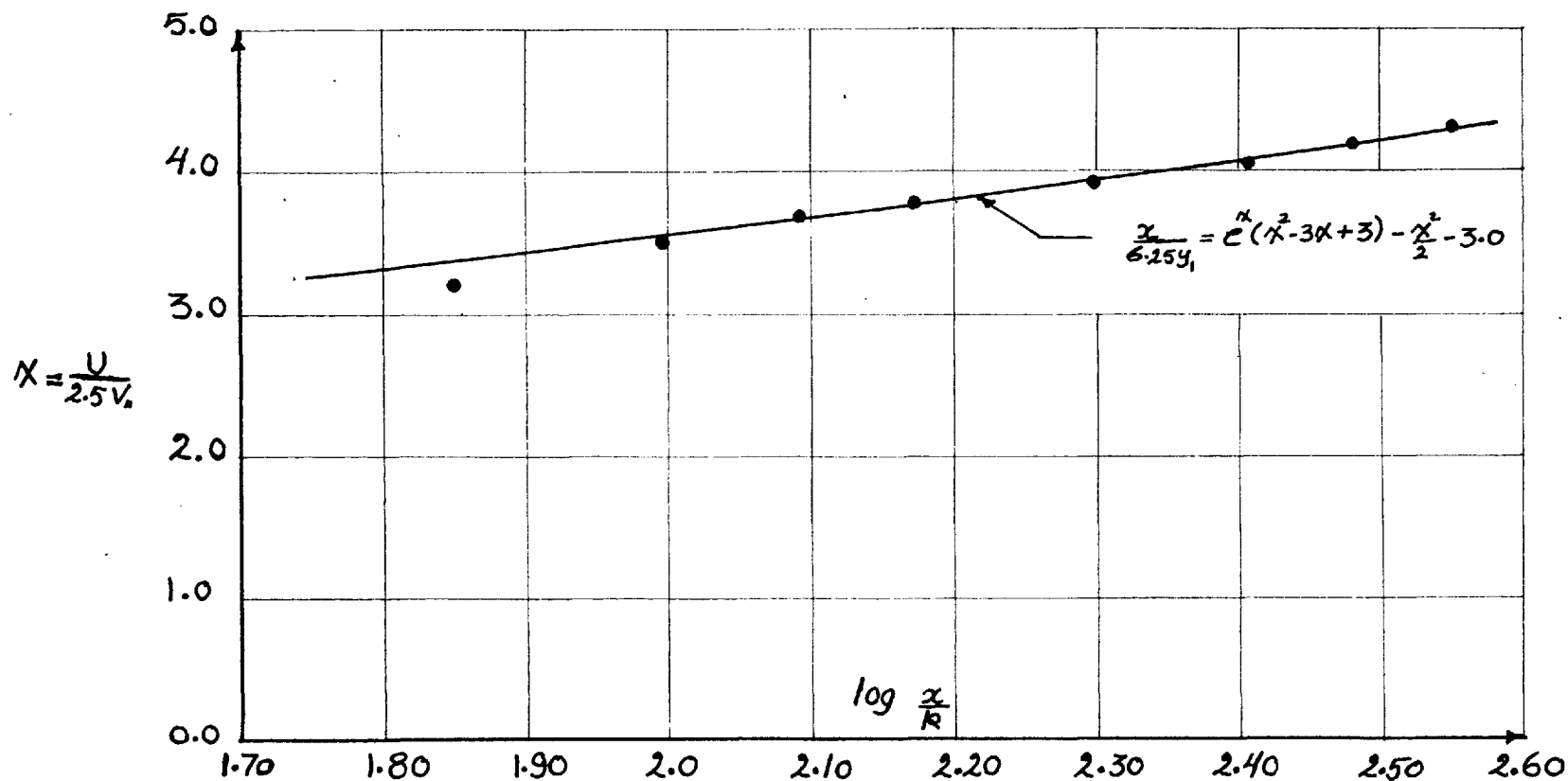


FIG. (2.22-) Variation of λ with x/k for Roughness "B" ($y_s = 0.0073''$)

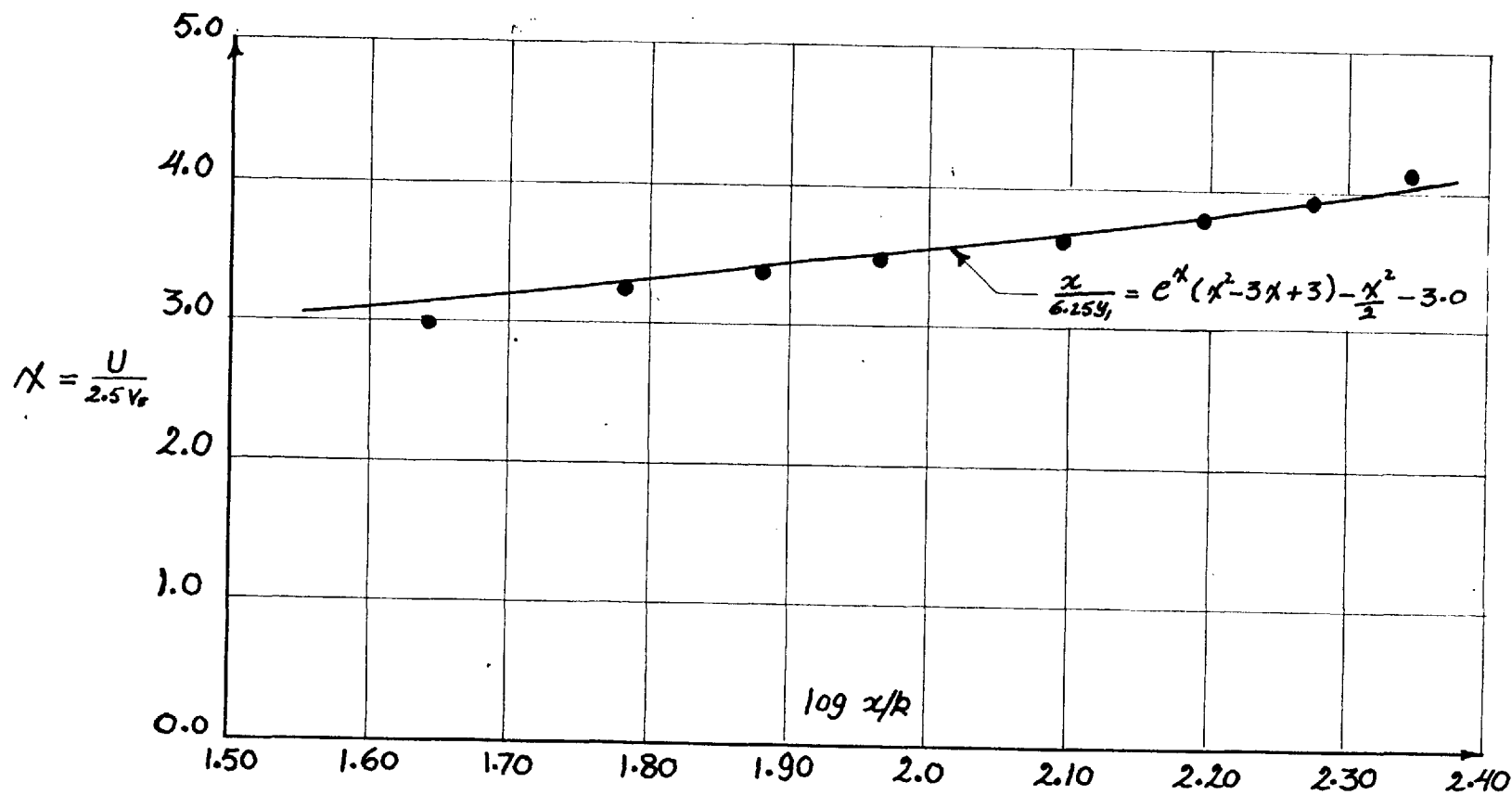
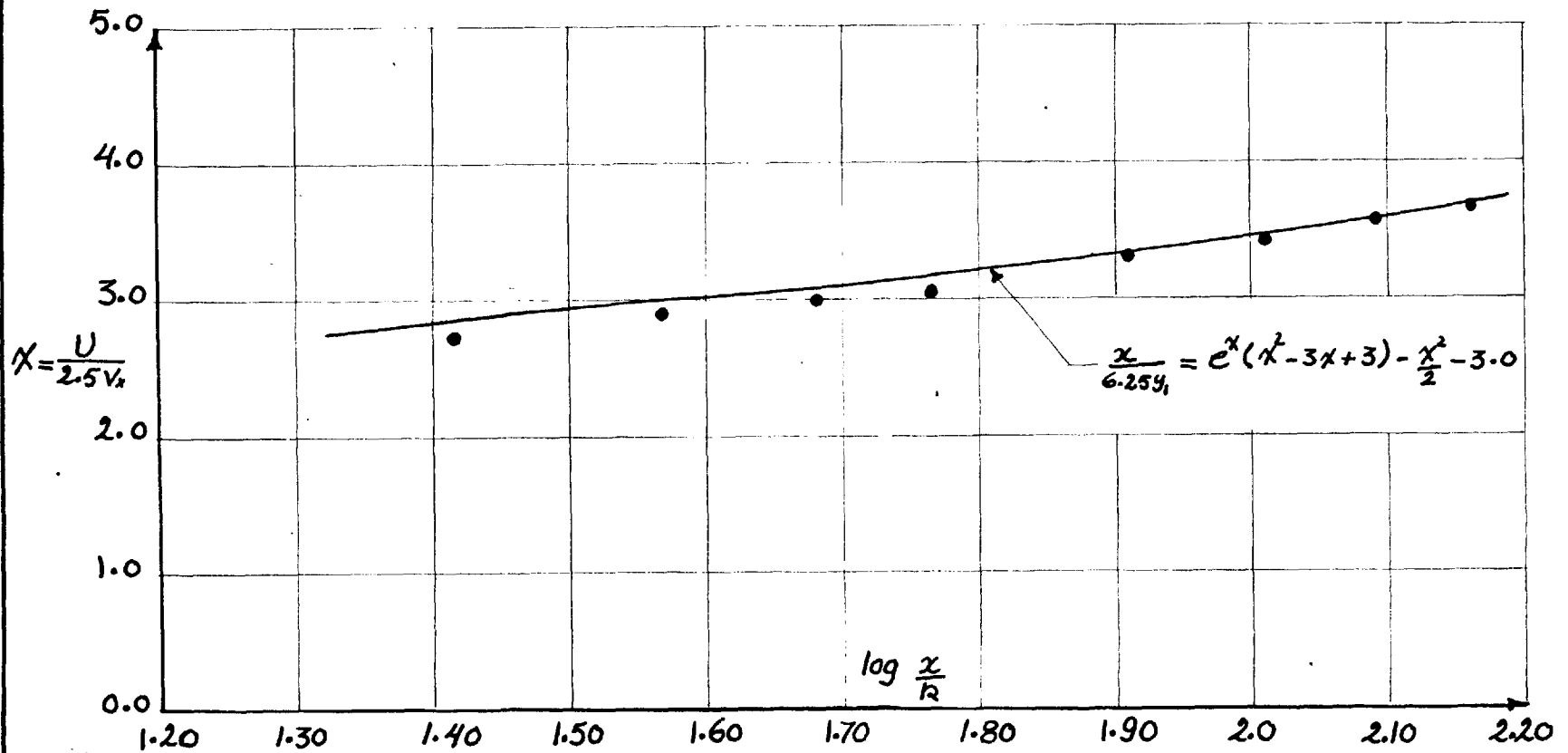


FIG. (2.23) Distribution of X with x/k for Roughness "c" ($y_s = 0.0112$ inches)



FIG(2.24) Variation of X with x/k for Roughness "D" ($y_1 = 0.0209$ inches)

for the various roughnesses used. In determining these values, the average value of V^* was used and the values of U were obtained from the intersection found by extrapolating the velocity profile of the boundary layer and that of the main stream as shown later in section (3.3). For comparison the corresponding curves from equation (2.8) are shown on the same figures. In calculating these curves from equation (2.8) the average value of y_i is required. To obtain (y_i) average, the velocity distributions of the boundary layer were plotted against $\log y$ and the intersections found with the axis $u = 0$ gave the local values of y_i at the different stations, thus (y_i) average was determined from:

$$(y_i)_{\text{average}} = \left[(y_i)_1 \times (y_i)_2 \times (y_i)_3 \times \text{-----} \times (y_i)_n \right]^{\frac{1}{n}}$$

This method for determining (y_i) average seems of more advantage than taking the arithmetic mean value, since errors in (y_i) local will have a small effect on (y_i) average.

By inspecting figures (2.21 to 2.24), it seems that the design of the nozzle is tolerant to changes in the wall roughness since the diagrams showed very small deviations from equation (2.8)

in the range of y_1 from 0.00618" to 0.0209", i.e about 3.4 times fold range of y_1 .

However, from continuity the amount of flow passing across any section is the same, and since V_x is substantially constant, the shape of the curve is in fact determined by the successive values of the ratio $\frac{x_r}{x}$ where x_r is the value of x at some arbitrary reference point, the ratio being independent of the absolute value of V_x . In other words, the ratio between the width of the nozzle at $x = x_1$ and $x = x_2$ is simply $\frac{x_2}{x_1}$.

Now to check theoretically the change in the curve due to the change in roughness, equation (2.8) was solved numerically for the various roughnesses used and the ratio $\frac{x_r}{x}$ was determined at various distances from the nozzle of the plate. The reference value was taken at $x = 27.5$ inches in all cases and the results of the calculations are given in the following table:

Table 2.4.

Effect of Roughness on the Shape of the Curve.

x inches	ROUGHNESS A y ₁ =0.00618"		ROUGHNESS B y ₁ =0.0073"		ROUGHNESS C y ₁ =0.01120"		ROUGHNESS D y ₁ = 0.02090"	
	χ	$\frac{\chi_r}{\chi}$	χ	$\frac{\chi_r}{\chi}$	χ	$\frac{\chi_r}{\chi}$	χ	$\frac{\chi_r}{\chi}$
3.5	3.257	1.3458	3.165	1.3552	2.965	1.3610	2.717	1.3618
5.5	3.488	1.2566	3.385	1.2673	3.174	1.2723	2.898	1.2787
7.5	3.642	1.2033	3.543	1.2106	3.329	1.2130	3.040	1.2190
9.5	3.766	1.1638	3.675	1.1673	3.451	1.1701	3.146	1.1779
11.5	3.878	1.1302	3.780	1.1348	3.542	1.1376	3.237	1.1438
15.5	4.040	1.085	3.951	1.0857	3.705	1.0898	3.39	1.093
19.5	4.174	1.0500	4.089	1.0505	3.838	1.0520	3.509	1.056
23.5	4.292	1.0214	4.200	1.0218	3.940	1.0247	3.609	1.0269
27.5	4.383	1.00	4.290	1.00	4.038	1.000	3.706	1.000

As seen from the above table, the differences are small and were compensated when the plate was shifted parallel to itself in the case of the larger roughnesses. As an example, if the width of the nozzle at $x = 27.5$ " was 1.0 inch, the width at $x = 3.5$

inches will be 1.3458 inches for roughness 'A' and 1.3618 inches for roughness 'D', thus a difference of 0.016 inches is obtained from the theory. In other words, the error in the width of the nozzle due to the use of roughness 'D' with a curve designed for roughness 'A' will be of the order of 1.20% which is very small particularly because 'D' is 3.4 times greater than roughness 'A'. Therefore it may be concluded that one design for the curve can be used, without much error, for a wide range of roughnesses.

CHAPTER III

MEAN VELOCITY DISTRIBUTION

3.1 Velocity Distribution In Pipes

In 1932, the experiments of Nikuradse on fully developed turbulent flow through pipes established that the velocity near the wall can be expressed by the functional relationship:

$$\frac{u}{V_*} = f\left(\frac{yV_*}{\nu}\right) \quad (3.1)$$

where u is the mean velocity at distance y from the wall, V_* is the shear velocity and ν is the kinematic viscosity of the fluid.

The experimentally observed velocity distribution, as given by Nikuradse (1932) for a number of smooth circular pipes, is plotted in figure (3.1) in the form $\frac{u}{V_*}$ against $\log \frac{yV_*}{\nu}$ and is represented by a heavily marked strip of experimental points.

For most of the flow region between the wall and the axis of the pipe, the experimental results are in good agreement with Prandtl's logarithmic velocity distribution law in the form:

$$\frac{u}{V_*} = A \log \frac{yV_*}{\nu} + B \quad (3.2)$$

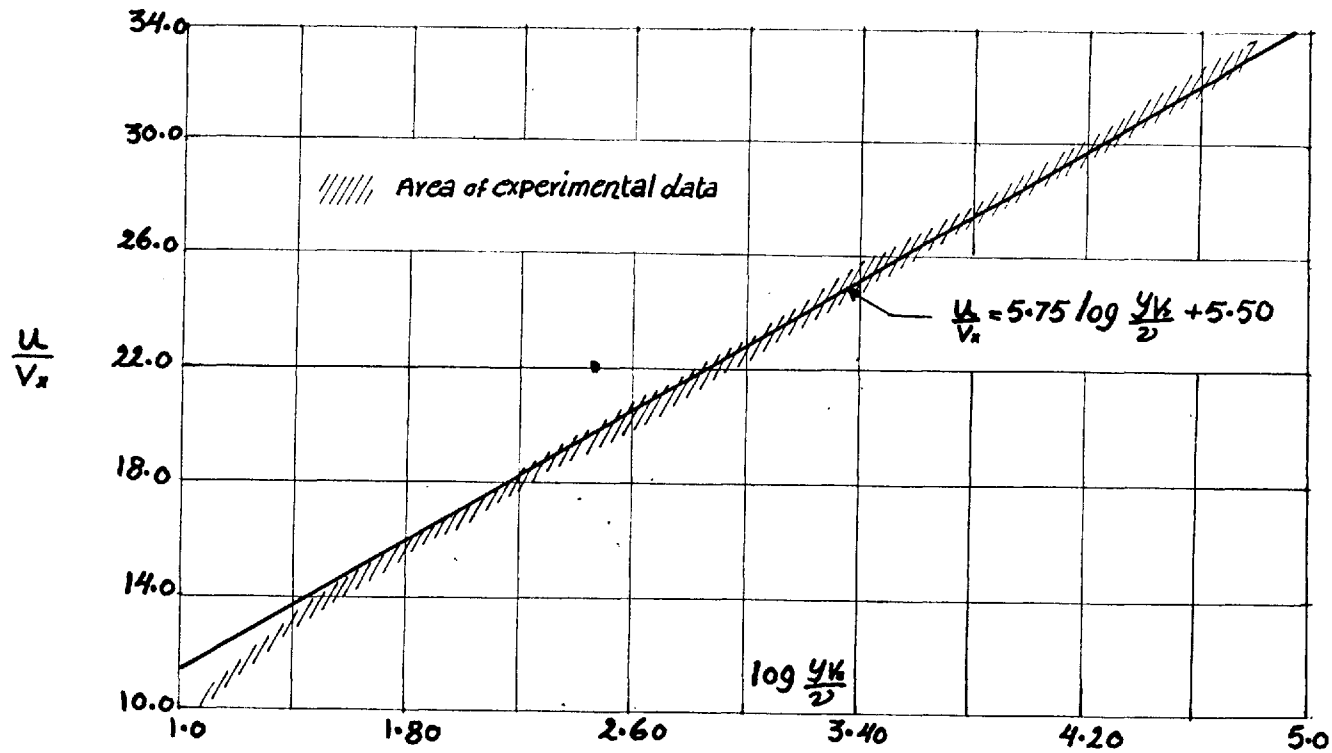


FIG. (3.1.) Velocity Distribution for Smooth Pipes.
(after J. Nikuradse)

with the constants $A = 5.75$ and $B = 5.5$.

Equation (3.2) being one for turbulent flow is valid only in regions where the viscous stresses can be neglected in comparison with the turbulent shearing stresses, and this is the case for nearly the whole flow region except very close to the wall. In the region very close to the wall, the turbulent velocity components and so the apparent turbulent stresses approach zero at the wall itself where the viscous stresses predominate. Consequently the velocity distribution in this region tends to deviate from equation (3.2) and this can be seen from Nikuradse's results (figure 3.1) where the recorded velocities become smaller than those indicated by equation (3.2) for small values of $\frac{yV_*}{\nu}$.

Regarding the behaviour of the velocity profile in this region, we have on record the experimental results of Laufer (1953) and Reichardt (1940). The measurements by Laufer were carried out with air flowing through a smooth pipe of 9.72 inches internal diameter at a velocity of 10 and 100 ft/sec. and a corresponding Reynolds number of 50,000 and 500,000 respectively based on the pipe diameter. His measurements in the region very close to the wall are shown

in figure (3.2) plotted in the form $\frac{u}{V_*}$ against $\log \frac{yV_*}{\nu}$. The relevant measurements by Reichardt on the flow through a rectangular channel are also shown on the same figure.

To compare the experimental evidence with theory, the logarithmic law (equation 3.2) and the law for laminar flow ($\frac{u}{V_*} = \frac{yV_*}{\nu}$) are shown on the diagram. It seems clear from this diagram that the relation $\left[\frac{u}{V_*} = f\left(\frac{yV_*}{\nu}\right) \right]$ deviates from its logarithmic form for values of $\frac{yV_*}{\nu}$ smaller than 30. For values of $\frac{yV_*}{\nu}$ of the order of 5.0 or less, although the experimental results in general fall below the curve for laminar flow, the discrepancies are not very large. With increasing distance from the wall, the contribution from the turbulent stresses becomes larger, and the errors introduced through using the laminar law to describe the velocity profile, become greater.

To arrive at the shape of the turbulent velocity profile in this region of the flow, the following analysis is used:

For a steady incompressible turbulent flow, the Reynolds equation in cylindrical co-ordinates in

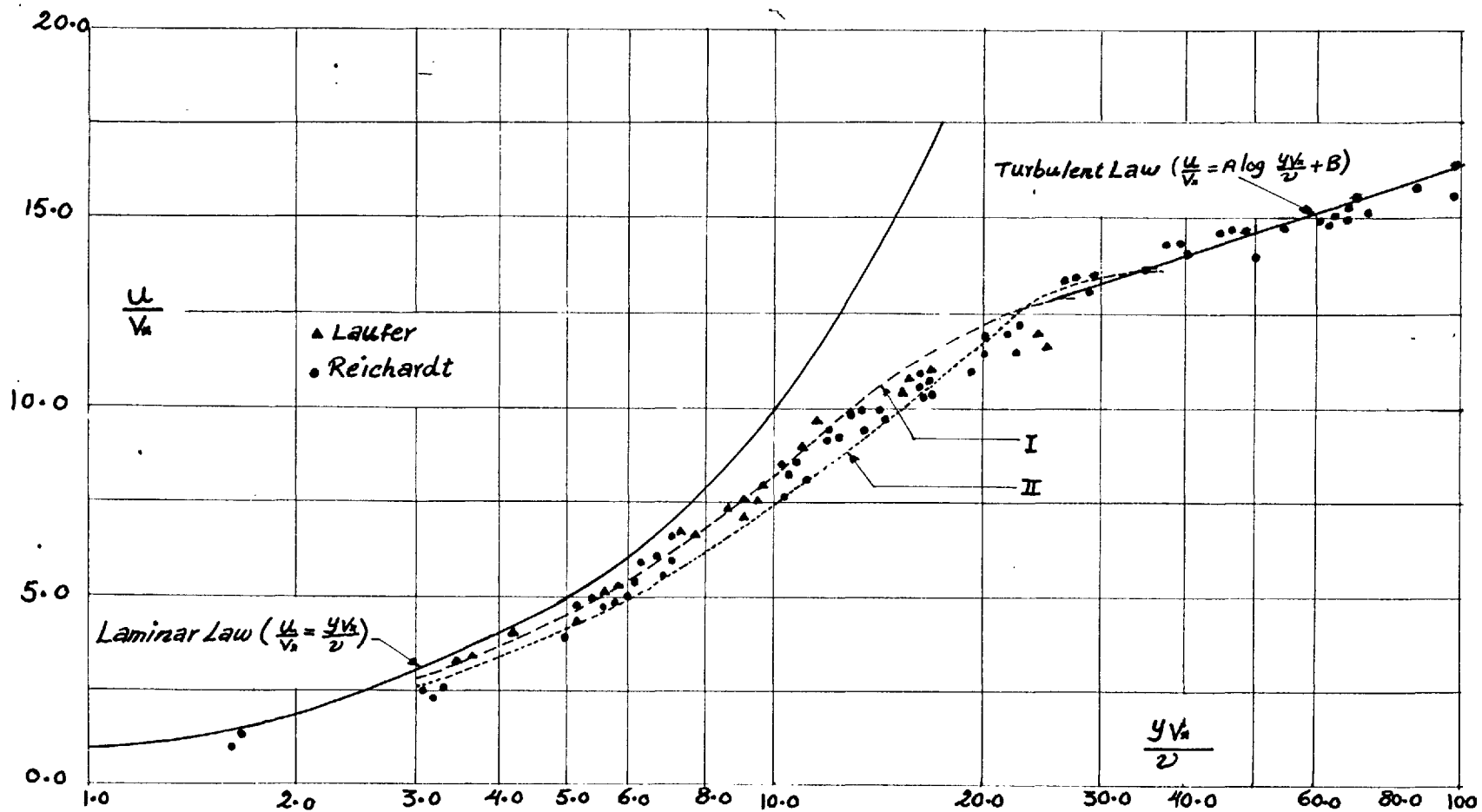


FIG. (3.2.) Velocity Distribution for Small values of "y" (Smooth walls)

the x-direction has the following form:

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial r} + \frac{w}{r} \frac{\partial u}{\partial \phi} = -\frac{1}{\rho} \frac{\partial P}{\partial x} - \left(\frac{\partial}{\partial x} \overline{u'^2} + \frac{1}{r} \frac{\partial}{\partial r} r \overline{u'v'} + \frac{1}{r} \frac{\partial}{\partial \phi} \overline{u'w'} \right) + \nu \nabla^2 u$$

where

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial r^2} + \frac{1}{r} \frac{\partial}{\partial r} + \frac{1}{r^2} \frac{\partial^2}{\partial \phi^2}$$

In this equation, u , v and w are the mean velocity components in the x , r and ϕ directions respectively, and u' , v' and w' are the turbulent velocity fluctuations in the same directions respectively.

For a fully developed pipe flow, the velocity field is independent of x . Also we have $v=w=0$ and

$$\frac{\partial}{\partial \phi} = 0, \text{ thus the equation is reduced to:}$$

$$\begin{aligned} \frac{1}{\rho} \frac{\partial P}{\partial x} &= -\frac{1}{r} \frac{d}{dr} r \overline{u'v'} + \nu \left(\frac{d^2 u}{dr^2} + \frac{1}{r} \frac{du}{dr} \right) \\ &= -\frac{1}{r} \frac{d}{dr} r (\overline{u'v'} - \nu \frac{du}{dr}) \end{aligned}$$

Integrating, then:

$$\frac{r^2}{2\rho} \frac{\partial P}{\partial x} = -r (\overline{u'v'} - \nu \frac{du}{dr}) + c \quad (3.3)$$

At the centre of the pipe, $r = 0$ and $\frac{du}{dr} = 0$, hence $c = 0$, and equation (3.3) can be reduced to:

$$\frac{r^2}{2\rho} \frac{\partial P}{\partial x} + \nu \frac{du}{dr} = \overline{u'v'} \quad (3.4)$$

where R is the pipe radius.

Replacing r by $(R-y)$, equation (3.4) takes the form:

$$\frac{du}{dy} = - \frac{\overline{u'v'}}{\nu} + \frac{V_{\pi}^2}{\nu} \frac{(R-y)}{R}$$

Integrating, we get;

$$u = - \int_0^y \frac{\overline{u'v'}}{\nu} dy + \frac{V_{\pi}^2 y}{\nu} \frac{(R - y/2)}{R} + c$$

At the wall where $y=0$, $u = 0$ and $\overline{u'v'} = 0$, hence $c = 0$.

For small values of y , the equation is reduced to:

$$\frac{u}{V_{\pi}} = \frac{V_{\pi} y}{\nu} - \int_0^y \frac{\overline{u'v'}}{\nu V_{\pi}} dy \quad (3.5)$$

Which gives the general shape of the velocity profile for the region of the flow close to the wall, and it contains the effect of both the viscous stresses and the Reynolds stresses on the velocity.

In order to be able to evaluate the second term on the R.H.S. of equation (3.5), the limit of integration as well as the variation of $\overline{u'v'}$ with y have to be known. At $y = 0$, $u' = v' = 0$ and so is $\overline{u'v'}$, where as for the upper limit of integration, the particular distance where the turbulent stresses

become maximum is a possible point at which the velocity distribution has to join the logarithmic law.

Assuming, as a first approximation, a linear distribution for $\overline{u'v'}$ with respect to y (figure 3.3), equation (3.5) will take the form:

$$\frac{u}{V_{\infty}} = \frac{yV_{\infty}}{\nu} - \frac{1}{2} \frac{yV_{\infty}}{\nu} = \frac{1}{2} \frac{yV_{\infty}}{\nu}$$

From the experimental velocity distribution

(figure 3.2), the point where $\frac{u}{V_{\infty}} = \frac{1}{2} \frac{yV_{\infty}}{\nu}$ lies

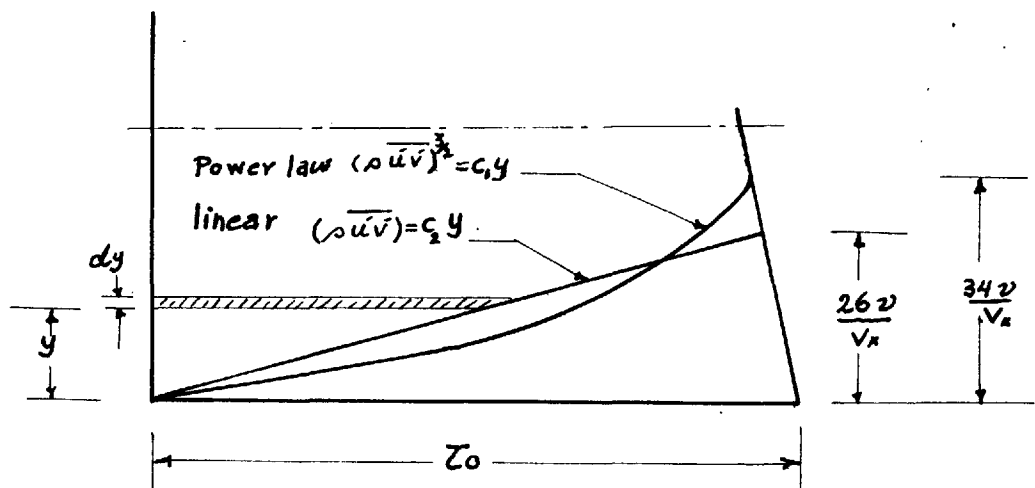


FIG. (3.3) Assumptions for the Turbulent Shearing stresses.

at $\frac{yV_{\pi}}{V} = 26$ and the resulting velocity profile takes the shape of curve I. The assumption of this linear distribution for $\overline{u'v'}$ involves a point of discontinuity at the upper limit of y . However, had we assumed a power law distribution for $\overline{u'v'}$ of the form: $y = \text{constant} (\rho \overline{u'v'})^{\frac{3}{2}}$, equation (3.5) would take the form:

$$\frac{u}{V_{\pi}} = \frac{yV_{\pi}}{V} - \frac{3}{5} \frac{yV_{\pi}}{V} = \frac{2}{5} \frac{yV_{\pi}}{V}$$

and the resulting velocity profile (curve II) is also included in figure (3.2). In calculating curve II, the value $\frac{yV_{\pi}}{V} = 34$ has been used as the upper limit of integration as this value gives best agreement.

From these two curves, it seems that equation (3.5) accurately agrees with the observed velocity distribution near the wall; any practical distribution of turbulent shear stress having a power law with the power between 1.0 and $\frac{3}{2}$ with the corresponding upper limit for y lying between $\frac{26V}{V_{\pi}}$ and $\frac{34V}{V_{\pi}}$, will provide the necessary assumptions for evaluating the integral.

3.2 Velocity Distribution For Constant Pressure Boundary Layers

Flow conditions for a flat plate are different from those for a pipe. First, a pressure gradient is essential to maintain the flow through a pipe; but not along a plate. Second, in a pipe, far enough from the entrance, the opposite boundary layers meet at the centre line and the velocity field becomes independent of the x-direction; whereas for a flat plate the flow field undergoes a process of gradual change with distance in the downstream direction, and strictly speaking, there is no guarantee that the flow will display similar velocity profiles at successive sections.

The difference between the two types of flow can be seen from the boundary layer equations which write as follows:

$$\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} = - \frac{\partial P}{\partial x} + \frac{\partial \tau}{\partial y}$$

$$\text{and } \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

thus:

$$\rho u \frac{\partial u}{\partial x} - \rho \frac{\partial u}{\partial y} \int_0^y \frac{\partial u}{\partial x} dy = - \frac{\partial P}{\partial x} + \frac{\partial \tau}{\partial y} \quad (3.6)$$

In a pipe with a developed flow, we have;

$\frac{\partial u}{\partial x} = 0$ and $v = 0$, thus τ_0 is wholly determined by the pressure gradient. On the other hand, for

a flat plate where $\frac{\partial P}{\partial x} = 0$, a change in u with x is necessary, according to equation (3.6), to overcome the wall friction and one must expect a different velocity profile for every different distance.

In general, the whole process of a boundary layer is one of gradual diffusion outwards of the wall condition. That is to say, the velocity profile near the wall is governed primarily by the stress at the wall, whereas well away from the plate the fluid has a memory of a quite different stress occurring nearer the nose of the plate. Thus normally the velocity profile tends to concave upwards when plotted against $\log y$. This curvature in the velocity profile can be seen in figure (3.4) which shows two velocity profiles on flat plates measured in the Göttingen Institute by Tillmann (1946).

The first profile is for the flow along a smooth plate at constant main stream velocity of 31.0 meters/sec., whereas the second profile is for the flow along a rough plate at a constant velocity of 17.8 meters/sec. For comparison, the pipe formula (equation 3.2) is plotted. The

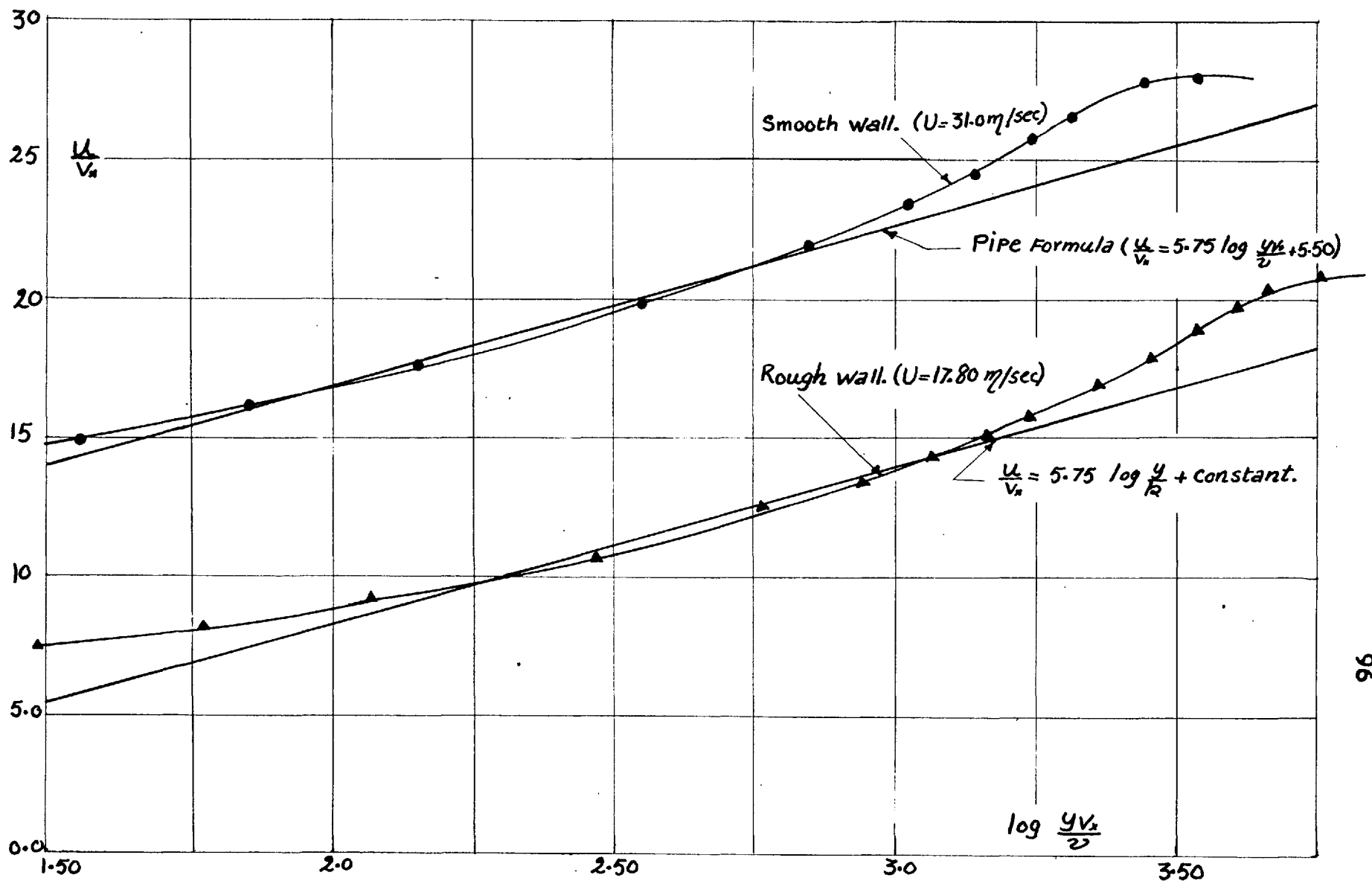


FIG.(3.4.) Velocity Distribution For Flat Plates (Göttingen Institute measurements)

figure indicates that the velocity profile for a flat plate, wheather smooth or rough, is different from that for a pipe in the sense that it sags around it. The figure also indicates that the velocity profile for a rough wall is shifted downwards parallel to that for a smooth wall, when plotted in the form $\frac{u}{V_x}$ against $\log \frac{yV_x}{\nu}$.

Regarding the region near to the wall where the velocity profile is governed primarily by the local wall stress, reference may be made to the experiments by Schultz-Grunow (1941), Ludweig and Tillman (1949); and Klebanoff and Diehl (1951) all of whom experimented with smooth flat plates at constant pressure. These experiments indicate that the velocity distribution is logarithmic for values of $\frac{yV_x}{\nu}$ between 30 and 500, and beyond that the data deviate from the logarithmic relation. This range corresponds to values of $\frac{y}{\delta} < 0.15$ where δ is the boundary layer thickness. The experiments however do give slightly different values for the constant A in equation (3.2). On the average they give $A = 5.6$ as compared with 5.75 found from velocity distribution measurements in pipes, or 5.65 found from resistance measurements in pipes.

3.3. A Particular Case of The Boundary Layer

The nozzle of constant drag constitutes a particular case of the boundary layer where the flow accelerates according to the equation:

$$\frac{x}{6.25y_1} = e^{\chi} (\chi^2 - 3\chi + 3) - \frac{\chi^2}{2} - 3$$

which was derived on the assumption that the stream lines inside the boundary layer are parallel to the wall. This was confirmed by the velocity surveys (figures 2.7 to 2.11) where u was found to be substantially constant, at different sections in the flow, for equal values of y smaller than the boundary layer thickness. Thus inside the boundary layer, $\frac{du}{dx} = 0$ and equation (3.6) reduces to:

$$\frac{\partial P}{\partial x} = \frac{\partial \tau}{\partial y}$$

which resembles that for a pipe. Therefore one might expect that the velocity distribution in the present boundary layer will possess some resemblance to that in a pipe.

To test this resemblance, some of the measured velocity profiles were plotted in a semi-logarithmic form in figures (3.5 to 3.8)

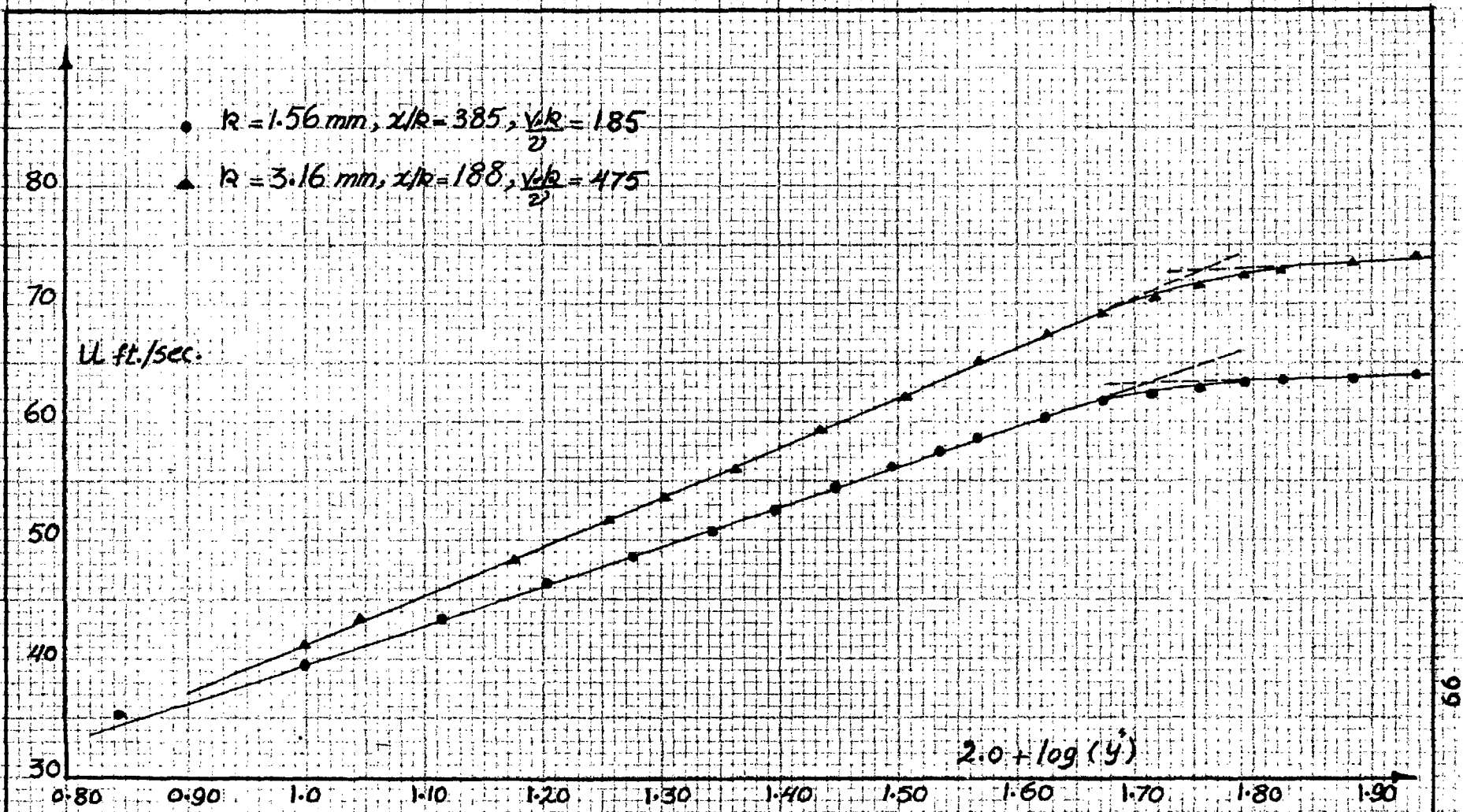


FIG.(3.5) MEASURED VELOCITY PROFILES IN THE PRESENT BOUNDARY LAYER.

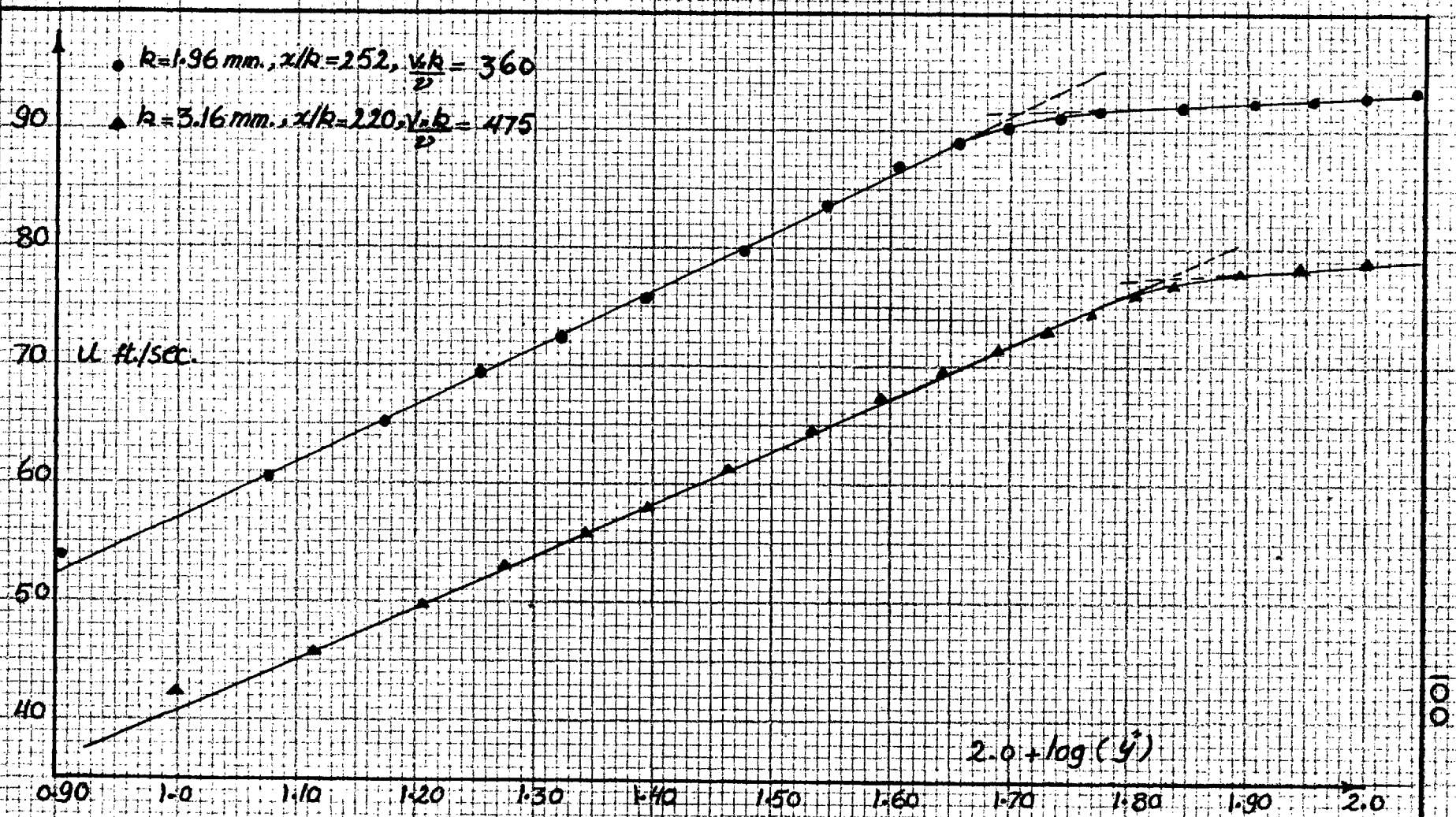


FIG.(3.6) MEASURED VELOCITY PROFILES IN THE PRESENT BOUNDARY LAYER.

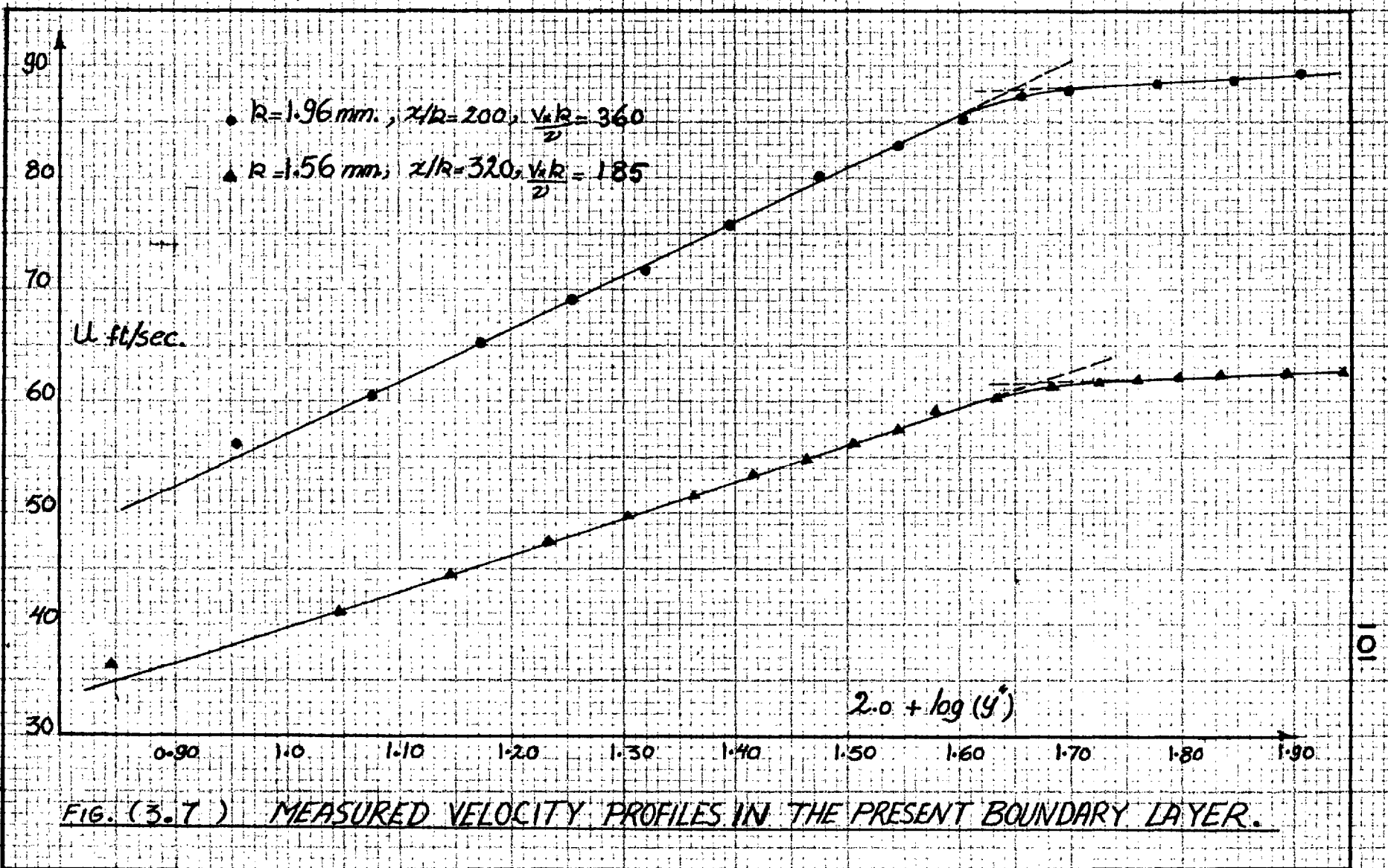


FIG. (3.7) MEASURED VELOCITY PROFILES IN THE PRESENT BOUNDARY LAYER.

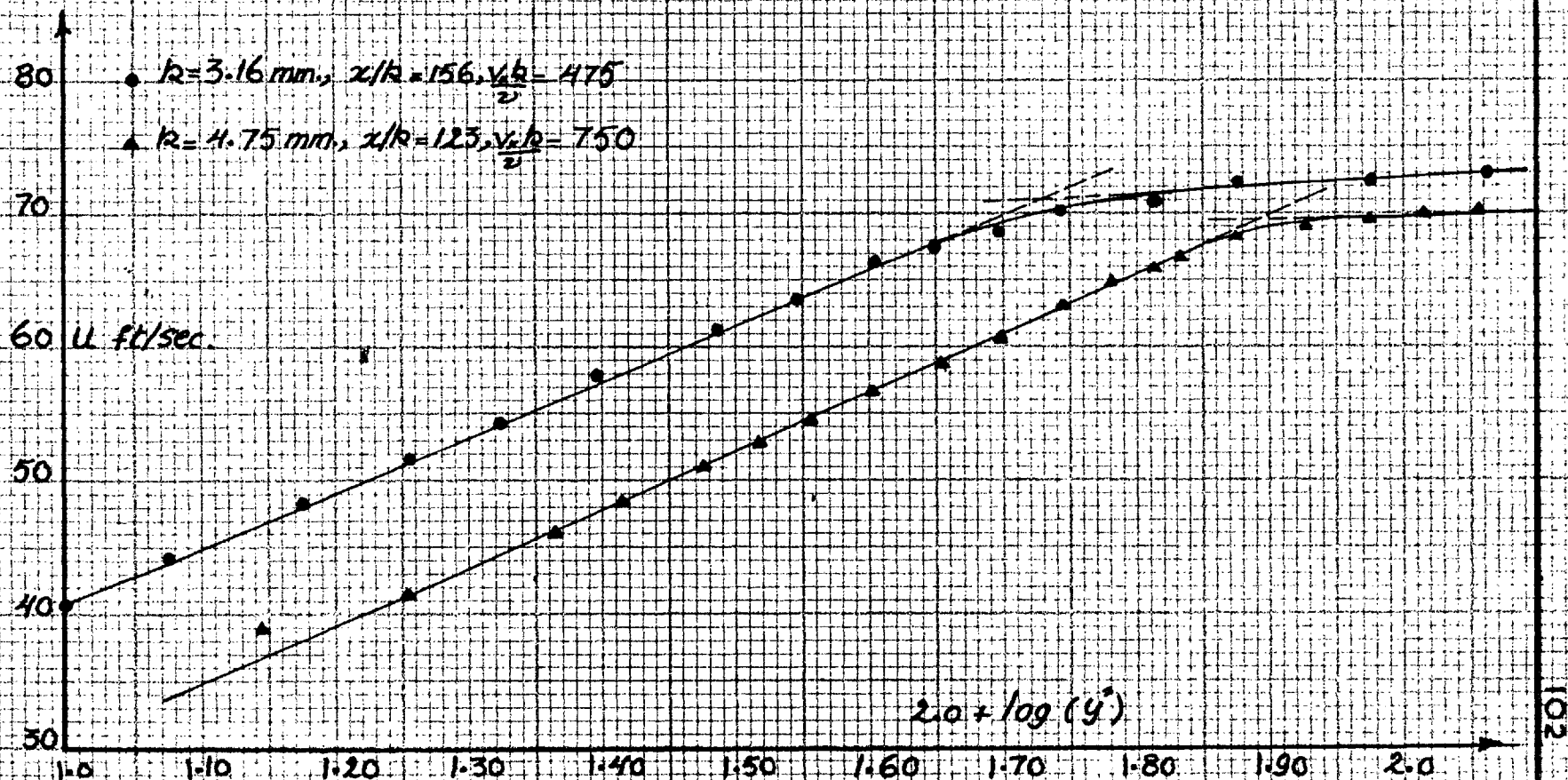


FIG. (3.8) MEASURED VELOCITY PROFILES IN THE PRESENT BOUNDARY LAYER.

for different wall roughnesses. In all figures, the experimental results fall nicely on straight lines for most of the boundary layer thickness. Due to the pressure gradient acting on the flow, the velocity in the main stream outside the boundary layer tends to increase, though very slightly, with distance from the wall. The figures also show that this part of the velocity profile connects smoothly with the logarithmic part in the boundary layer.

Regarding the definition of the boundary layer thickness δ , usually some arbitrary definition such as $u = 0.99 U$ is used for flat plates, and it involves a certain amount of uncertainties since u approaches U more or less asymptotically. When the velocity in the main stream is not constant, such definition becomes impractical. However, in figures (3.5 to 3.8), δ can be fairly well defined from the intersection found by extrapolating the velocity profile of the boundary layer and that of the main stream. Using this definition, the uncertainty in δ is reduced considerably since the two lines will meet at one point only.

Comparing the velocity profiles with those for flat plates, it is noticed that the present profiles follow a logarithmic relationship for values of $\frac{y}{\delta}$ up to 0.90 or more according to figures (3.5 to 3.8) compared to a value of $\frac{y}{\delta} = 0.15$ which was found by Schultz-Grunow (1941) and Klebanoff and Diehl (1951) for flat plates and beyond which the velocity distribution was found to deviate from the logarithmic law.

The difference in the behaviour of the two boundary layers is understandable since for a flat plate, the outside part of the velocity profile is determined as seen in section (3.2) by a wall stress occurring nearer the nose of the plate and which is different from the local stress; whereas in the present boundary layer, although the relevant part of the velocity profile is determined by an earlier shear stress, it still behaves as if it was determined by a local one since τ_0 is substantially constant along the wall. This makes the velocity profile in the present boundary layer similar to that in a pipe as will be shown in more detail later in this chapter.

This boundary layer also resembles that for a flat plate in the sense that the part of the velocity profile next to the wall is determined in both cases by the local wall stress, and this part, as indicated by Schultz-Grunow (1941), occupies only 0.15 of the boundary layer thickness. In other words, it can be said that a boundary layer of the present form offers a link between the pipe flow and the flat plate flow, and reduces the gap between them.

It is also noticed from the velocity profiles (figures 3.5 to 3.8) that the first point nearest the wall often deviates from the logarithmic law and gives a slightly greater velocity. This may partly be explained by the possibility that, when the pitot tube is in contact with the roughness elements which were loosely packed, its reading is falsified by the pressure rise in the wake behind the particle just upstream of its nose; and partly by the fact that in a velocity gradient, a pitot tube tends to read the pressure at a point slightly to one side of its centre always tending to read high.

To analyse in more detail the experimentally

obtained velocity distribution, we have Prandtl's logarithmic law for pipes as follows:

$$\frac{u}{V_*} = A \log \frac{y}{y_1} \quad (3.7)$$

where y_1 is the distance from the wall of the point where $u = 0$, and the law is valid for both smooth and rough pipes. In 1937, C.M. White and C.F. Colebrook from studies on fluid friction in roughened pipes, showed that for values of $\frac{V_* k}{\nu} > 60$ when the resistance becomes independent of viscosity, the value y_1 can in effect be replaced by a fraction of the grain size k so that $y_1 = \frac{k}{33}$ measured from the centre line of the grains inwards; whereas on the other hand, for values of $\frac{V_* k}{\nu} < 4$ the rough pipes become indistinguishable from perfectly smooth ones and y_1 becomes equal to $0.1 \frac{\nu}{V_*}$. Between these two extremes, there is a range ($4 < \frac{V_* k}{\nu} < 60$) in which both viscosity and grain size influence the resistance to the flow and y_1 will vary gradually between $0.1 \frac{\nu}{V_*}$ and $\frac{k}{33}$ and can be expressed in the form:

$$\frac{V_* y_1}{\nu} = \phi, \left(\frac{V_* k}{\nu} \right) \quad (3.8)$$

From Nikuradse's resistance measurements in pipes, the observed resistance law was satisfactorily expressed according to Karman-Prandtl theory in the general form:

$$\frac{\tau_0}{\rho \bar{U}^2} = \frac{1}{8} \left(2 \log \frac{0.225 R}{y_1} \right)^{-2} \quad (3.9)$$

For both smooth and rough pipes, where R is the pipe radius and $\bar{U}$ is the mean velocity of the flow as obtained by integrating the logarithmic law over the cross-sectional area of the pipe. For values of $\frac{V_* k}{\nu} > 60$, the equation then takes the particular form:

$$\frac{\tau_0}{\rho \bar{U}^2} = \frac{1}{8} \left(2 \log \frac{7.4R}{k} \right)^{-2} \quad (3.10)$$

Now combining equations (3.8) and (3.10), White and Colebrook obtained a general form of the law of the wall in the form:

$$2 \log \frac{7.4R}{k} - \frac{\bar{U}}{\sqrt{8} V_*} = \phi_2 \left(\frac{V_* k}{\nu} \right) \quad (3.11)$$

To trace the variation of the function ϕ_2 for uniform roughness, Nikuradse's results on rough pipes (1933) were plotted according to equation (3.11) in figure (3.9). This diagram is not just a

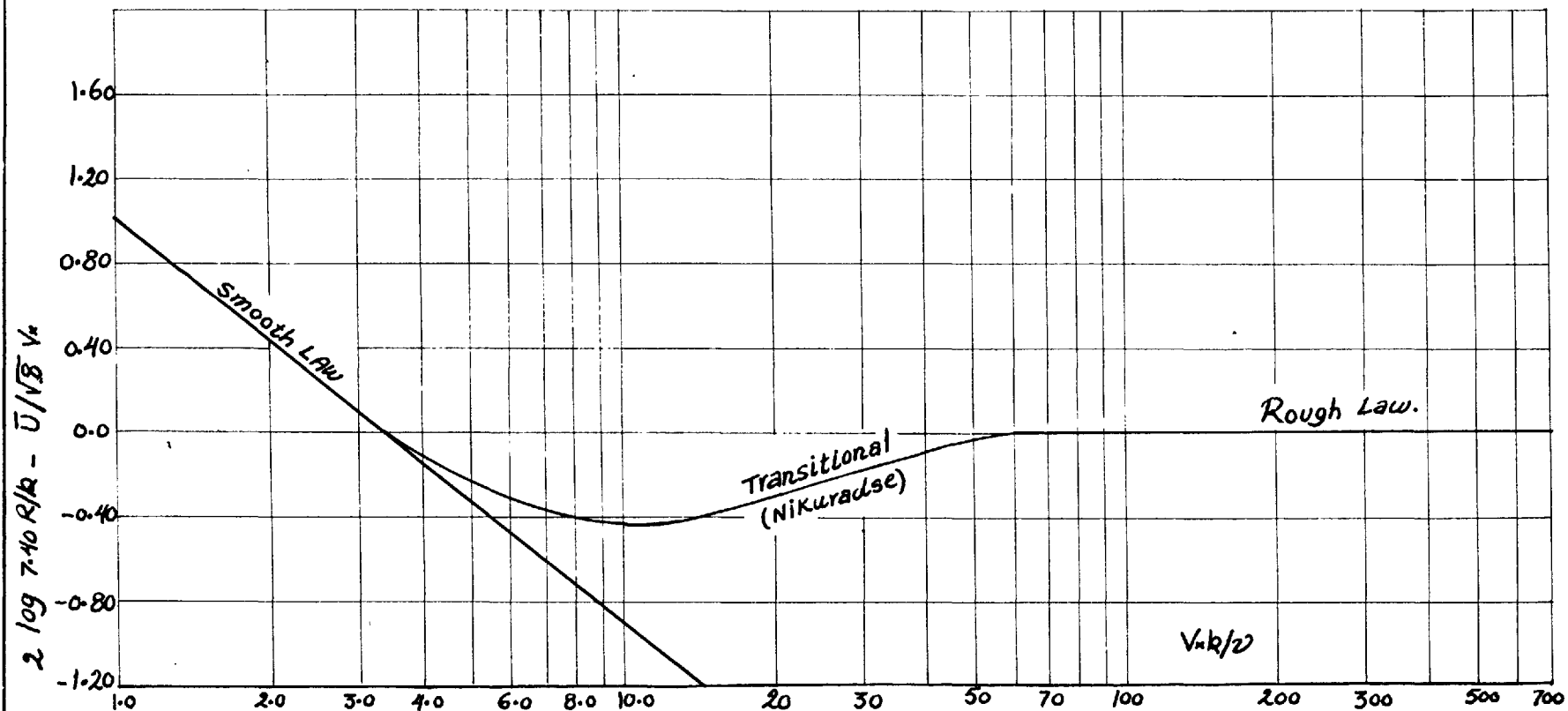


FIG.(3.9) Law of The Wall for smooth and Rough Surfaces.
(after C.M. White and C.F. Colebrook)

correlation of resistance coefficients, but actually describes the velocity distribution for the different wall conditions. The smooth law appears as the sloping straight line on the left and equation (3.11) will reduce in this case to:

$$\frac{\bar{U}}{V_*} = \sqrt{8} \left(2 \log 2.25 \frac{RV_*}{\nu} \right) = 5.65 \log 2.25 \frac{RV_*}{\nu}$$

or
$$\frac{u}{V_*} = 5.65 \log \frac{yV_*}{\nu} + 5.65$$

where $\bar{U}$ has been substituted by u at $y = 0.255 R$ as obtained by integrating the velocity profile across the pipe. The horizontal line to the right, on the other hand, indicates that the completely rough conditions have been obtained, and in this case equation (3.11) takes the form:

$$\frac{\bar{U}}{V_*} = \sqrt{8} \left(2 \log 7.4 \frac{R}{k} \right) = 5.65 \log 7.4 \frac{R}{k}$$

or
$$\frac{u}{V_*} = 5.65 \log \frac{y}{k} + 9.60 \quad (3.12)$$

where $\bar{U}$ has been substituted by u at $y = 0.255 R$.

The values of the constants in equation (3.12) differ slightly from the values 5.75 and 8.5 given by Nikuradse (1933), the differences being 1.75% and 1.10% respectively are less than the uncertainty in the measurements of the velocity distributions.

Plotting the velocity distribution for the present boundary layer in the form $\frac{u}{V_*}$ against $\log \frac{y}{k}$ for the roughnesses used, figures (3.10 to 3.13) were obtained. The resulting curves follow the rough pipes formula, except for the value of the constant 8.6 which is seen here to vary between 5.47 and 6.12. However, in Nikuradse's experiments on rough pipes, the roughness elements were glued to the wall as closely as possible, i.e. with maximum packing density, whereas in most practical cases the roughness density is considerably smaller than that of Nikuradse and consequently y_0 would be different from $k/33$.

Writing equation (3.12) in the general form:

$$\frac{u}{V_*} = A \log \frac{y}{k} + A \log \frac{k}{y_0}$$

the values of $\frac{k}{y_0}$ were calculated from figures (3.10 to 3.13) and were found to vary between 9.0 and 11.2, which are within the range of the values originally given by Schlichting (1936) for loosely packed spherical particles.

Now it may be of interest to proceed further by analysing the effect of the roughness parameter $\frac{V_* k}{\nu}$ on the velocity profile for the fully rough conditions ($\frac{V_* k}{\nu} > 60$). In 1935 Prandtl

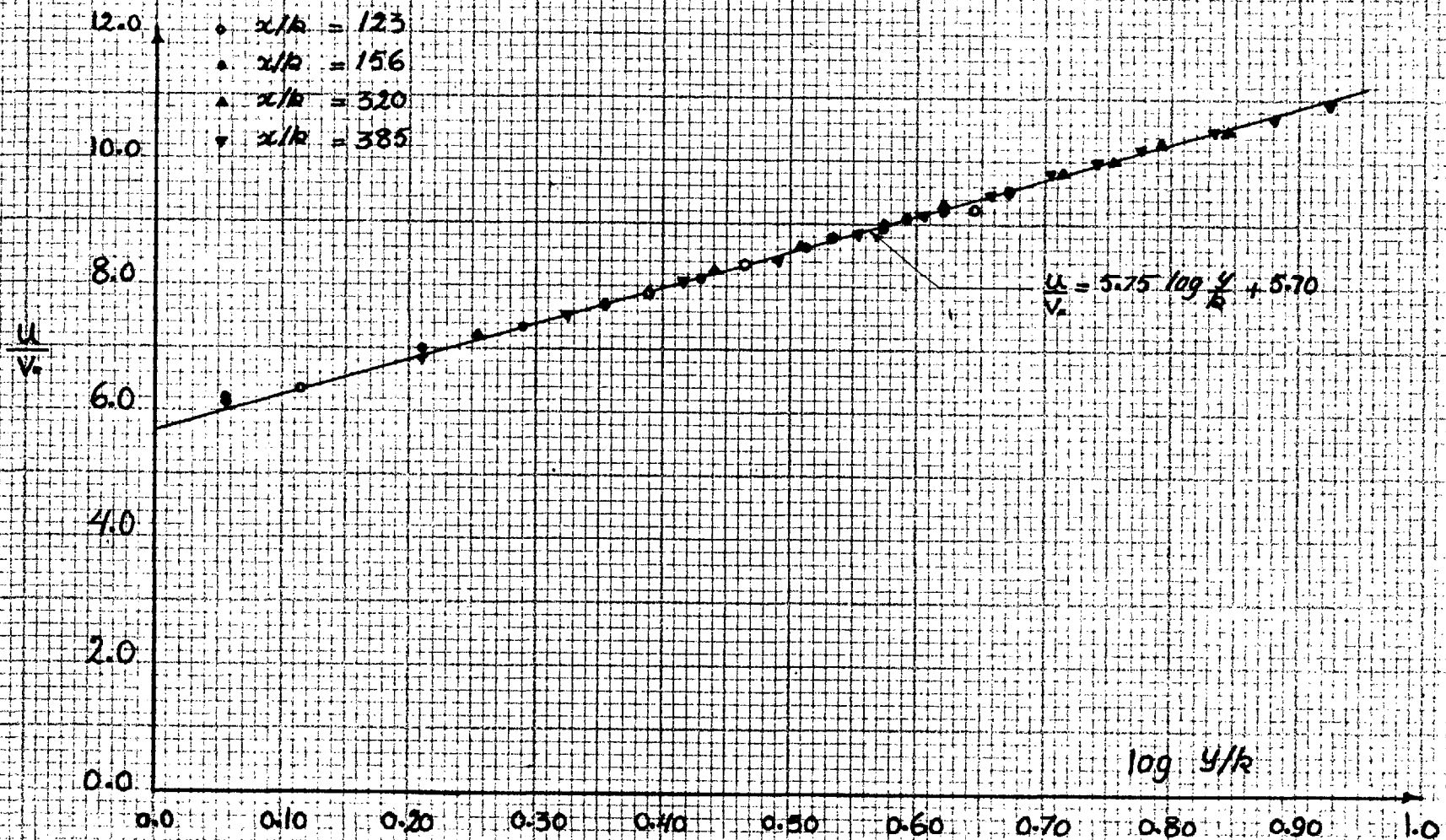


FIG. (3.10) Non-dimensional Velocity Distribution (Roughness "A", $k=1.56$ mm.)

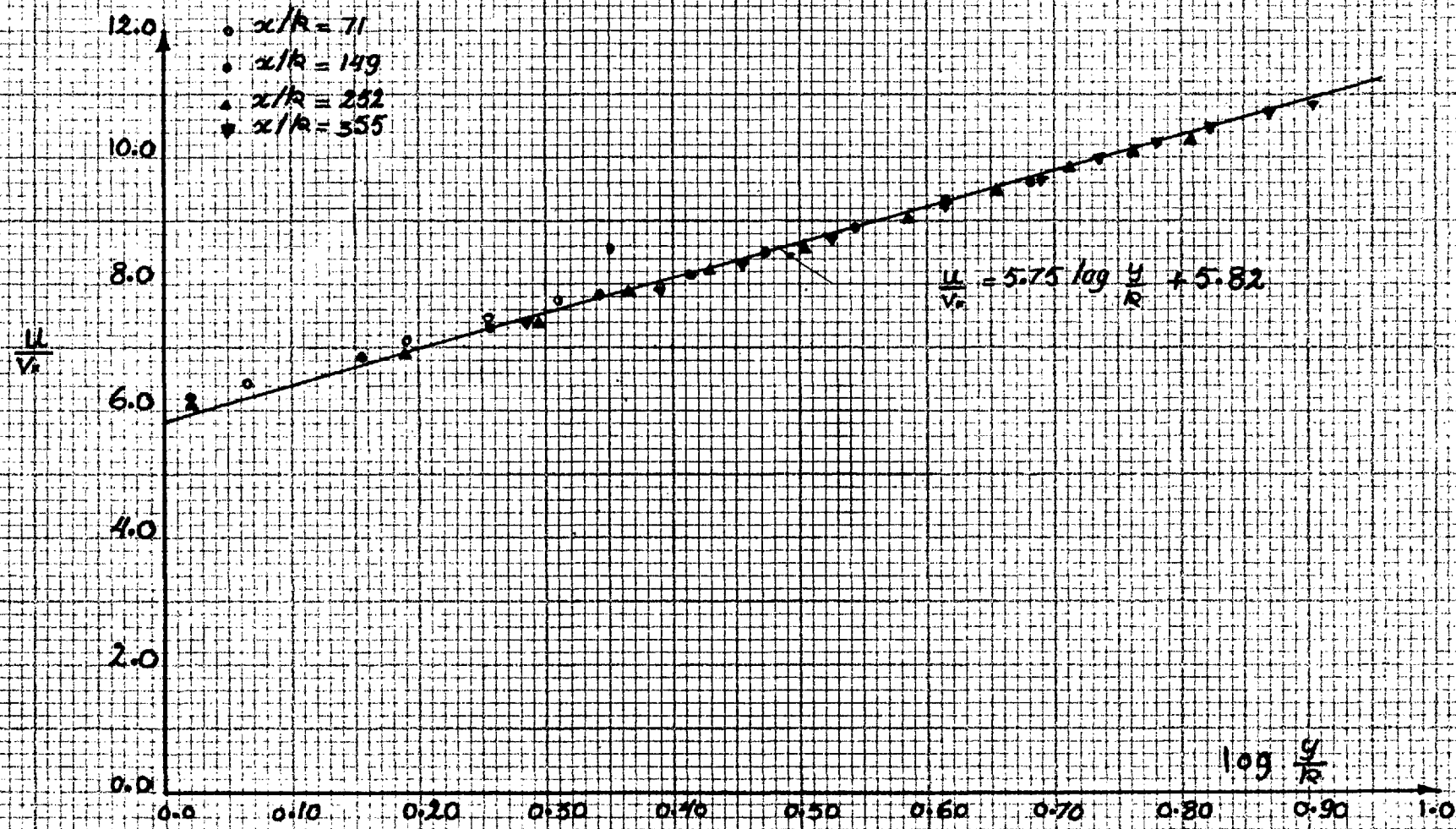


FIG. (3.11) Non-dimensional Velocity Distribution. (Roughness "B", $k = 1.96 \text{ mm}$)

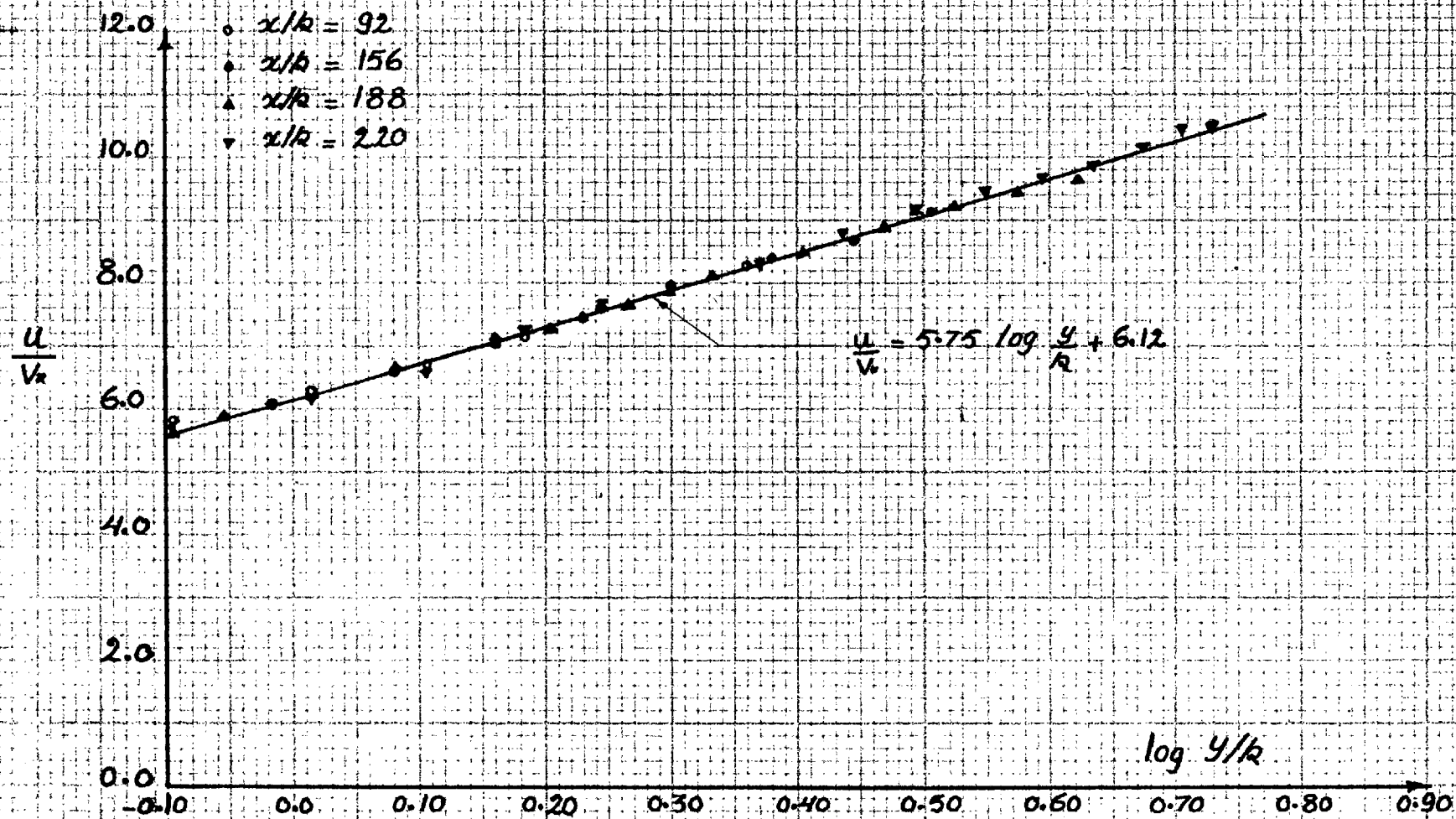


FIG. (3.12) Non-dimensional Velocity Distribution (Roughness "C", $k = 3.16$ mm.)

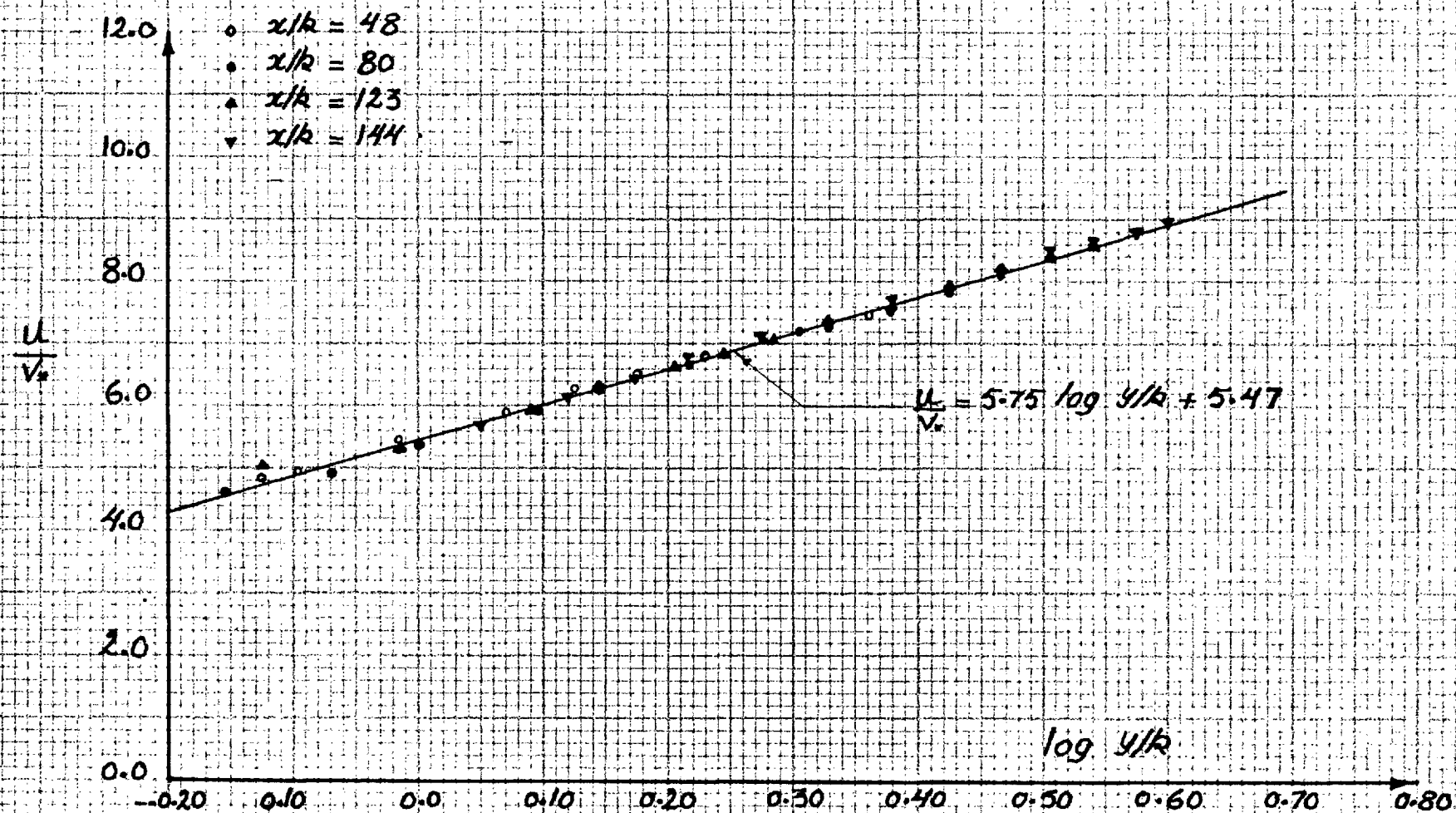


FIG. (3.13) Non-dimensional Velocity Distribution (Roughness "D", $k = 4.75$ mm)

pointed out that, when plotted in the form $\frac{u}{V_{\pi}}$ Versus $\log \frac{yV_{\pi}}{v}$, the logarithmic part of the velocity profile for rough walls is displaced downwards parallel to that for smooth walls by an amount

$$\frac{\Delta u}{V_{\pi}} \quad \text{depending on the roughness parameter} \quad \frac{V_{\pi} k}{v}.$$

This displacement in the velocity profile is seen from the Gottingen Institute measurements (figure 3.4), where the profile has undergone a shift $\frac{\Delta u}{V_{\pi}}$ of approximately 8.7 for a roughness parameter $\frac{V_{\pi} k}{v} = 112$, the effect of the roughness being to reduce the velocity by the amount Δu . This shift in the velocity profile is seen to be self evident from the velocity distribution law for rough pipes which may be written as follows:

$$\begin{aligned} \frac{u}{V_{\pi}} &= 5.75 \log \frac{y}{k_s} + 8.5 \\ &= 5.75 \log \frac{yV_{\pi}}{v} + 5.5 - (5.75 \log \frac{k_s V_{\pi}}{v} - 3.0) \\ &= 5.75 \log \frac{yV_{\pi}}{v} + 5.5 - \frac{\Delta u}{V_{\pi}} \end{aligned} \quad (3.13)$$

hence:

$$\frac{\Delta u}{V_{\pi}} = 5.75 \log \frac{k_s V_{\pi}}{v} - 3.0 \quad (3.14)$$

where k_s is the equivalent sand diameter, i.e. corresponding to Nikuradse's conditions.

To demonstrate the physical significance of

the quantity $\frac{\Delta u}{V_{\pi}}$ we use the following analysis:

For smooth pipes we have:

$$\frac{U}{V_{\pi}} = 5.75 \log \frac{RV_{\pi}}{\nu} + 5.5 = \sqrt{\frac{\rho U^2}{\tau_0}} \quad \text{smooth} \quad (3.15)$$

where $\frac{\tau_0}{\rho U^2}$ is the coefficient of skin friction. For rough pipes on the other hand we have according to equation (3.13):

$$\frac{U}{V_{\pi}} = 5.75 \log \frac{RV_{\pi}}{\nu} + 5.5 - \frac{\Delta u}{V_{\pi}} = \sqrt{\frac{\rho U^2}{\tau_0}} \quad \text{rough} \quad (3.16)$$

Subtracting equation (3.16) from equation (3.15) then:

$$\frac{\Delta u}{V_{\pi}} = \sqrt{\frac{\rho U^2}{\tau_0}} \quad \text{smooth} - \sqrt{\frac{\rho U^2}{\tau_0}} \quad \text{rough} = \frac{U}{V_{\pi}} \quad \text{smooth} - \frac{U}{V_{\pi}} \quad \text{rough}$$

thus the quantity $\frac{\Delta u}{V_{\pi}}$ indicates the loss of velocity due to the increase in the pipe resistance for the same pressure drop.

To trace the quantity $\frac{\Delta u}{V_{\pi}}$ for the profiles measured in the present boundary layer, figures (3.14 and 3.15) were plotted as $\frac{u}{V_{\pi}}$ versus $\log \frac{yV_{\pi}}{\nu}$. The law for smooth pipes was also included in the figures for comparison. By measuring the shift $\frac{\Delta u}{V_{\pi}}$ from the figures, the parameter $\frac{V_{\pi} k_s}{\nu}$ was calculated from equation (3.14) and is shown on the relevant profiles.

Based on this method, the nozzle of constant drag offers a convenient and accurate way for determining the equivalent sand roughness of

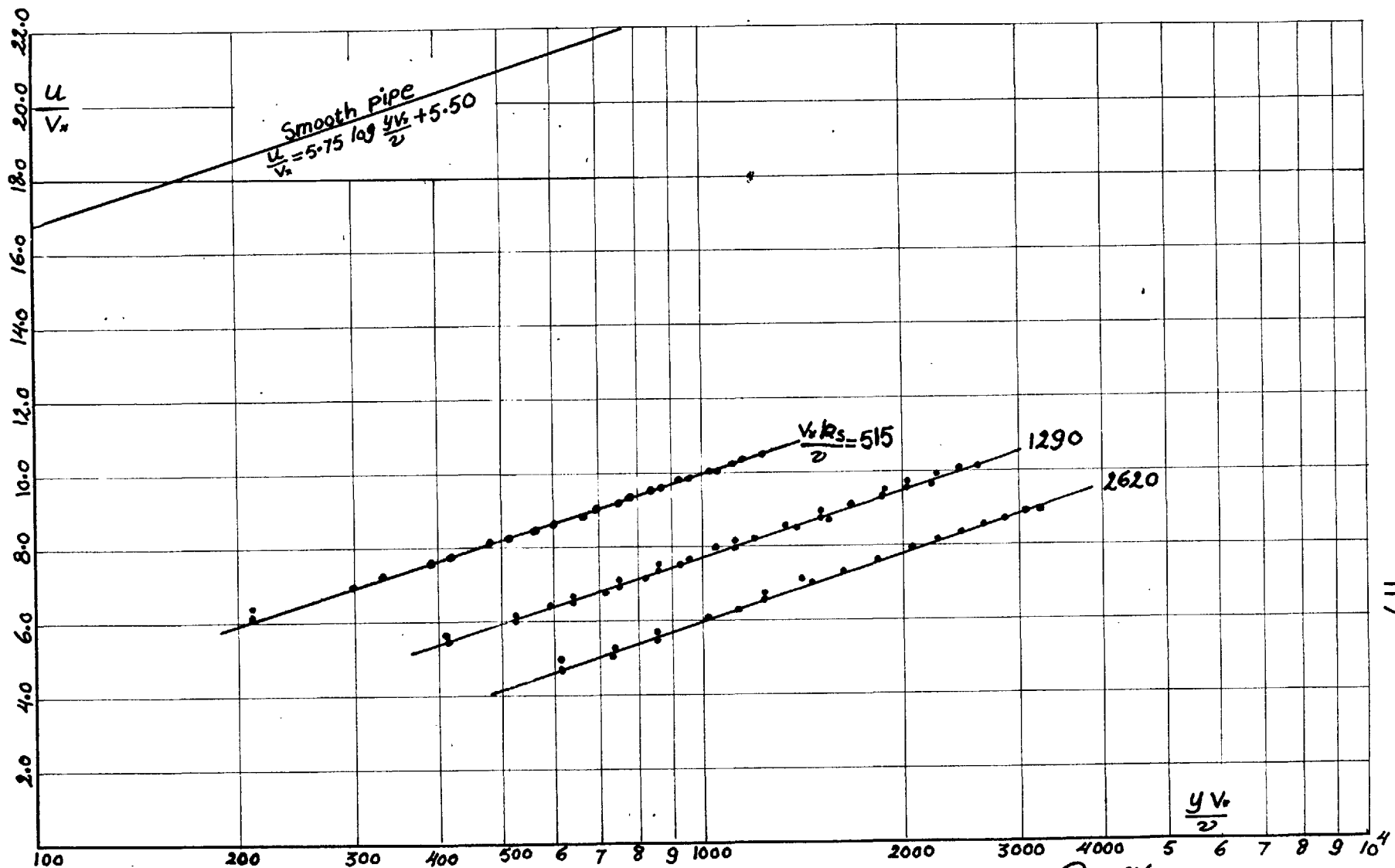


FIG.(3.14) Effect of The wall Roughness on The velocity Profile.

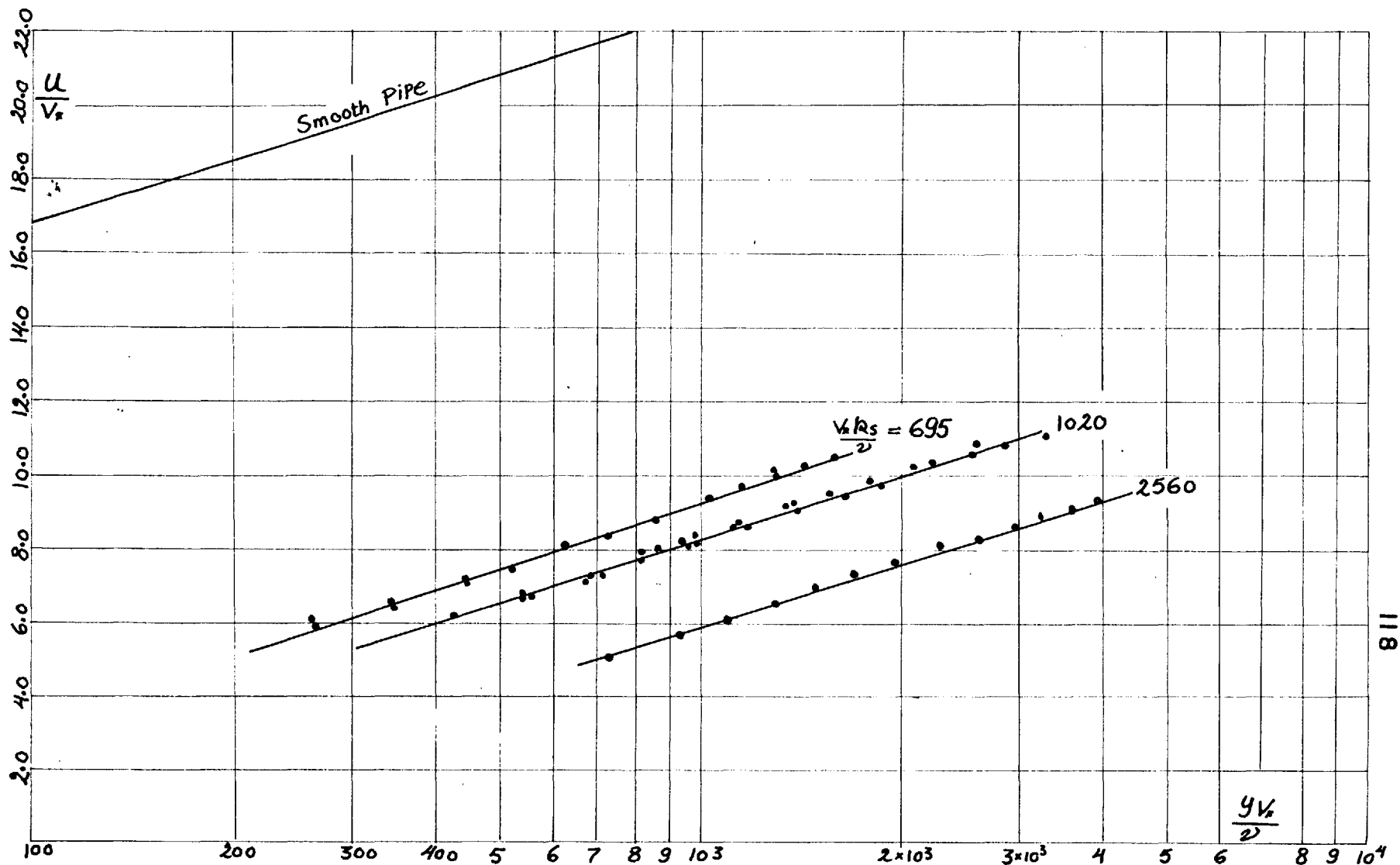


FIG.(3.15) Effect of The wall Roughness on The velocity Profile.

artificial roughnesses, ship and aircraft paints, etc. Since it was found in section (2.4.3.), that the curve required for the nozzle wall is insensitive to the particular value of the wall roughness used in calculating the curve, one design can be used without much error for measuring a wide range of roughnesses. Indeed it seems the logical development of the method originally described by Schlichting (1936).

Instead of using the equivalent sand diameter in deriving equation (3.14) from equation (3.7), the actual diameter may be used and equation (3.7) will then be:

$$\frac{u}{V_x} = 5.75 \log \frac{y}{\alpha k} \quad (3.17)$$

where $\alpha = \frac{y}{k}$, α being a function of the type, shape, and spacing of the roughness elements. On combining (3.17) with (3.13), α is obtained in the form:

$$5.75 \log \alpha = \frac{\Delta u}{V_x} - 5.75 \log \frac{V_x k}{2\nu} - 5.5 \quad (3.18)$$

By measuring $\frac{\Delta u}{V_x}$ from figures (3.14 and 3.15), the value of α was found from equation (3.18) to vary between 0.089 and 0.11 for the four different

roughnesses as compared to an average value of 0.03 for the roughnesses used by Nikuradse (1933), the difference being mainly due to the spacing of the roughnesses. In Nikuradse's experiments on pipes (1933), the grains were packed together as closely as possible; whereas plates (7 to 10) for the present roughnesses show a relatively wide spacing between the grains. These plates are arranged for stereoscopic viewing and were taken by a camera at a distance of 12 inches from the plates, the shift of camera being 3.0 inches and the angle of overlap 28° .

It is interesting to note that the original experiments by Schlichting (1936) on artificially roughened plates have shown clearly that the effective size of the roughness depends greatly on their spacing; and figure (3.16) calculated from Schlichting's experiments with spheres shows the variation of $\frac{k}{y_1}$, i.e. of $\frac{1}{\alpha}$ with the spacing ratio $\frac{L}{k}$. Beginning from high values of $\frac{L}{k}$, the figure shows that at first $\frac{k}{y_1}$ decreases in a power law with a decrease in the spacing ratio, and attains a minimum before beginning to increase again with still closer spacing. Thus from this

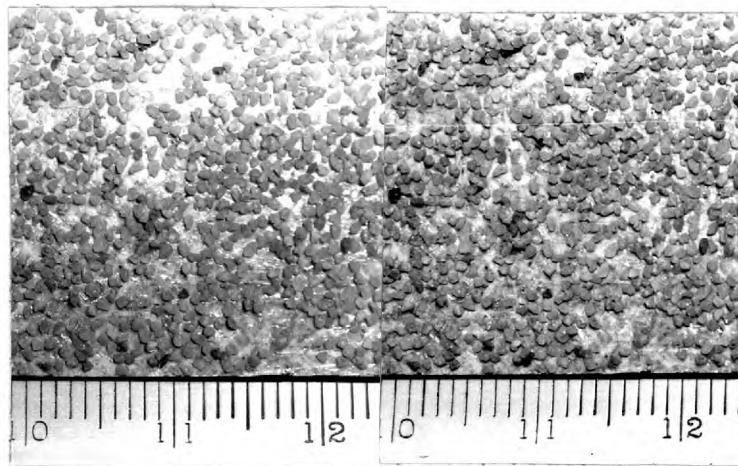


PLATE 7. ROUGHNESS "A" ARRANGED FOR STEREOSCOPIC VIEWING., $R = 1.56 \text{ mms.}$

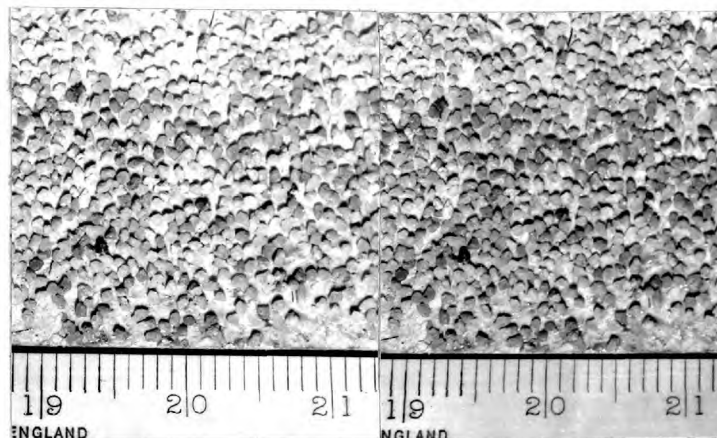


PLATE 8. ROUGHNESS "B" ARRANGED FOR STEREOSCOPIC VIEWING. $R = 1.96 \text{ mms.}$

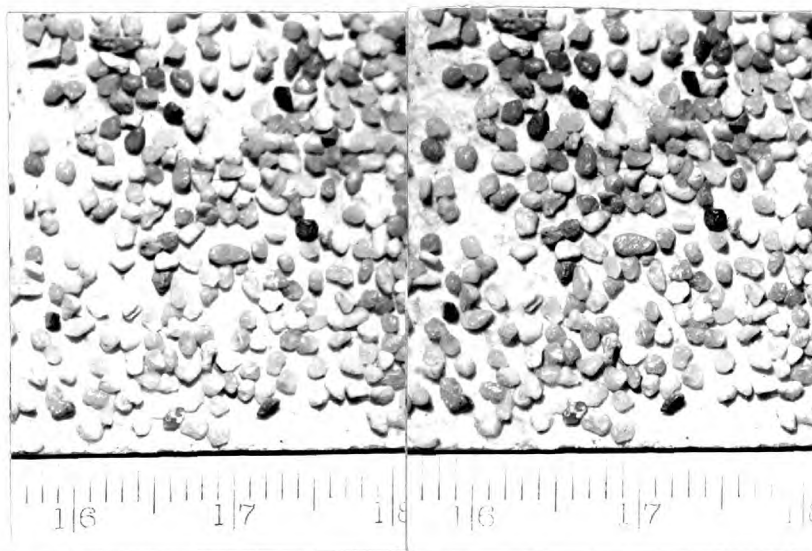


PLATE 9. ROUGHNESS 'C' ARRANGED FOR STEREOSCOPIC VIEWING . $R = 3.16$ mms.



PLATE 10. ROUGHNESS 'D' ARRANGED FOR STEREOSCOPIC VIEWING. $R = 4.75$ mms.

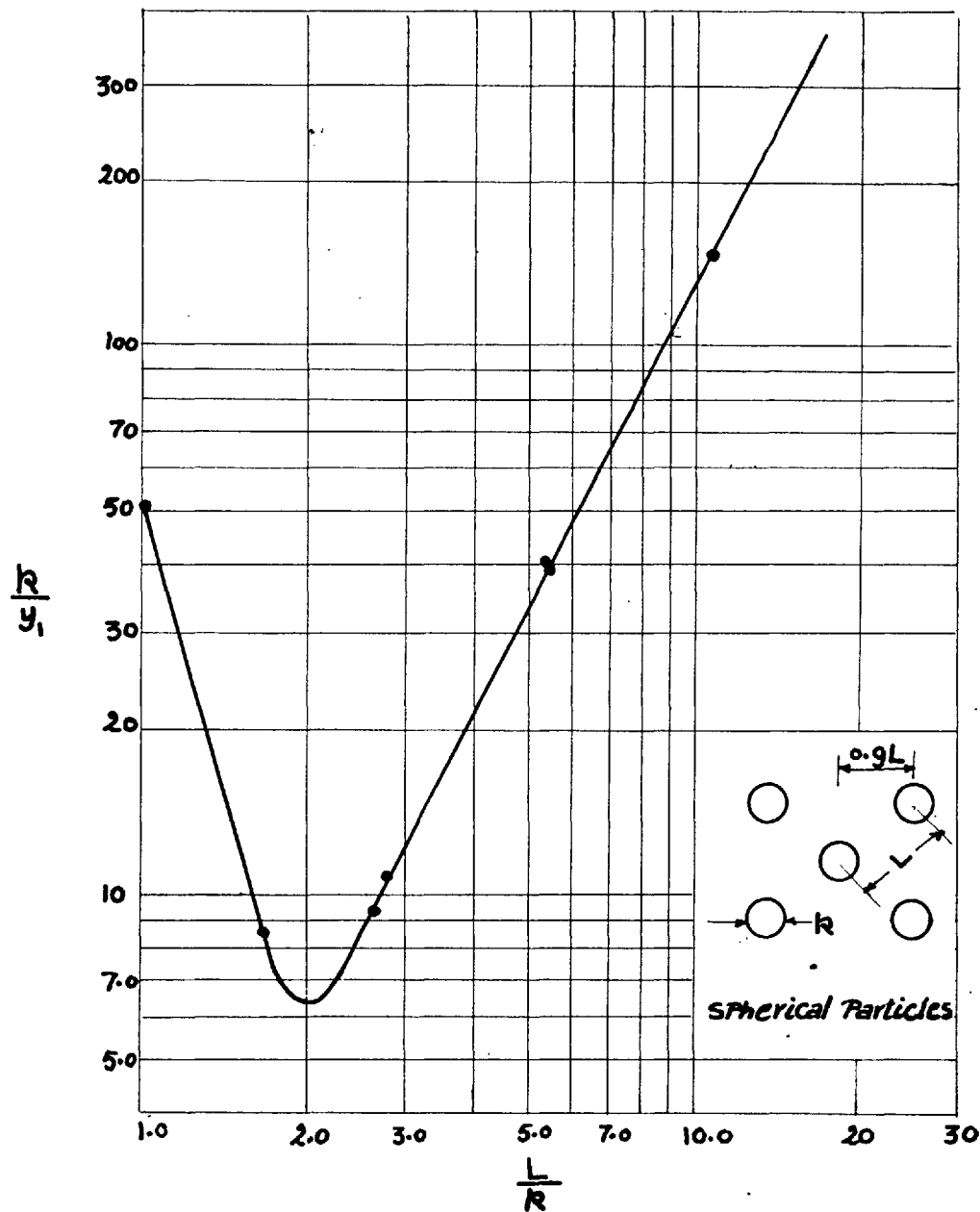


Fig. (3.16) Variation of $\frac{h}{y_1}$ with Spacing of Spherical Particles (calculated from data by Schlichting, 1936)

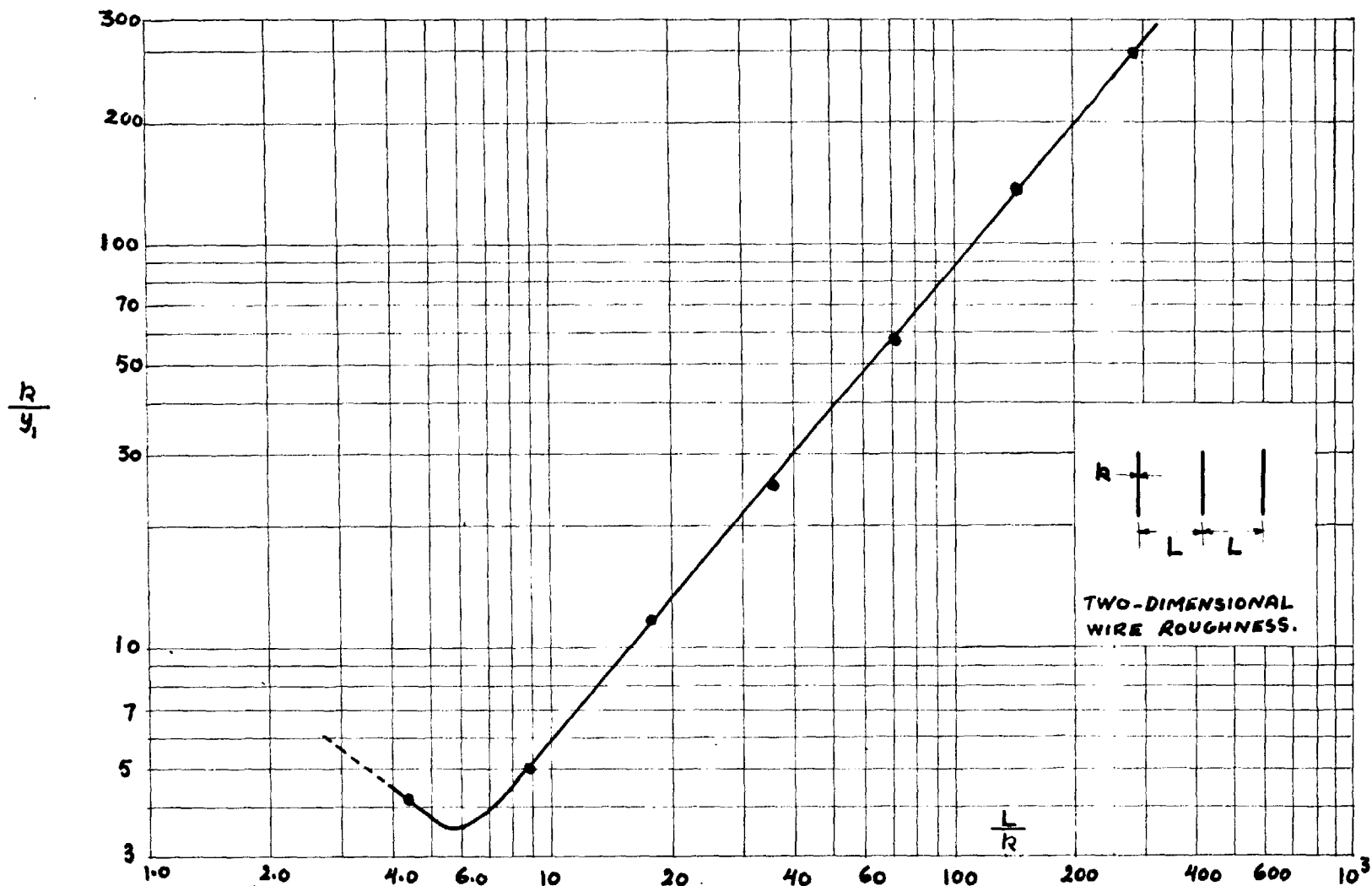


Fig. (3.17) Variation of $\frac{k}{y_1}$ with Spacing of Wire Roughness (calculated from data by Stevenson, 1960)

figure for perfect spheres, α can be five times greater than that found by Nikuradse, Figure (3.17) calculated from data by Stevenson (1960), shows again the same trend for two-dimensional wire roughnesses.

The shape of the roughness, on the other hand, as indicated by Schlichtings experiments on cones and spherical segments (1936), is also of importance in determining their equivalent size, and some of the sands of the present experiments may have been more angular than those of Nikuradse, so partly accounting for the larger values of α in the present experiments.

3.4 Turbulent Velocity Profile at Separation

3.4.1. Mechanism and Definition of Separation

In a flow with a pressure rise along a rigid wall, the boundary layer increases its thickness considerably in the downstream direction until it reaches a place where the pressure is so high that the fluid no longer has sufficient energy to advance further downstream, and some of the flow then becomes reversed.

According to Schubauer and Klebanoff (1950),

since the kinetic energy of the flow is vanishingly small at the wall, and since the pressure is least at a point of minimum pressure, the potential energy must be minimum at that point. After this point of pressure minimum, the flow at the wall could never advance downstream against the rising pressure, unless it received energy from the flow farther out. This transfer of energy takes place through the turbulent shearing stresses and consequently a turbulent boundary layer will have a greater resistance for separation than a laminar boundary layer due to the fact that the former exhibits greater stresses.

The direct measurements by Schubauer and Klebanoff (1950) on the turbulent shearing stresses in boundary layers in the region of separation indicated that these stresses have their maximum values away from the wall, and the point of maximum value is shifted outwards as the flow passes downstream. Thus a stress gradient is towards the wall and they argued that the region between the wall and the point of maximum stress is receiving energy from the region beyond through the action of turbulent mixing.

These measurements by Schubauer and Klebanoff confirmed the older ones by Dryden (1947) which showed that the maximum values of the turbulent stresses are found at greater distances from the wall as the flow approaches the point of separation. This also seems clear from Prandtl's boundary layer equation. At the wall $\frac{\partial p}{\partial x} = \frac{\partial \tau}{\partial y}$ and for a positive pressure gradient, $\frac{\partial \tau}{\partial y}$ should be positive too, i.e. the point of maximum shearing stress should be away from the wall.

The point of separation itself is determined, and indeed defined, by the condition that the velocity gradient at the wall should vanish, i.e.

$$\left(\frac{du}{dy}\right)_{y=0} = 0 \quad \text{at separation.}$$

3.4.2. Prediction of Separation Point

Certain characteristics of the main flow have been used to predict separation point, such as the dimensionless number $\delta^2 \frac{dU}{dx} = -12$, given by Pohlhausen (1921) for laminar separation, where δ is the boundary layer thickness and $\frac{dU}{dx}$ is a measure of the pressure gradient. However, besides the boundary layer thickness and the local

pressure gradient, the evidence seems to point also to a dependence of the flow separation upon the previous history of the flow before it reaches separation point. This was shown by Von Karman and Millikan (1934), in a more complete mathematical treatment for laminar separation. For cases in which the potential flow undergoes a pressure drop to some point then undergoes a pressure rise, they showed that when the distance along the wall from the upstream stagnation point to the point of maximum velocity is increased, the potential velocity at which separation occurs is found to approach more and more closely to the point of maximum velocity.

For turbulent separation on the other hand, the "shape" parameters H or η have been used to predict the separation point. According to Gruschwitz (1931), imminent separation occurs at $\eta = 0.8$ or $H = 1.80$, while the value $H = 2.8$ was found by both Schubauer and Klebanoff (1950) and Sandborn (1953) for separation on smooth straight walls, and the value $H = 2.7$ was given by Dryden (1947) for flows on aerofoil-like sections.

In general however from various published observations, it seems that lower values for H occur at separation on boundaries having a short length of boundary layer growth and large pressure gradients; whereas separation occurs with higher values of H after a greater length of boundary layer growth and smaller pressure gradients. This shows that the value of H at separation is closely linked with the previous history of the flow.

3.4.3. Separation Test.

In separation tests, in order to restrict separation to one only of the walls, a boundary layer control system is desirable at the other sides to remove any displacement effect on the flow at the other sides, so insuring that separation occurs on the measurements side only. However, in the present test a simple but effective alternative method was used which avoids the use of such control systems. The boundary layer on the rough wall of the nozzle was forced to separate using a flap plate at the downstream end of the measurements side as indicated

in figure (3.18). The plate was pointing in an upstream direction and having the same width as the nozzle. Its length as well as its angle with the wall were varied until separation was finally obtained at a station 24.5 inches from the leading edge of the wall. The separation profile was measured at that station which was half an inch upstream of the plate. The plate's final length was 7.0 inches, with its upstream end 1.25 inches from the wall.

The presence of the plate in the flow induced a region of pressure rise that extended approximately 3.0 inches upstream. Figure (3.18) gives the pressure distribution in relation to the position of the plate and to the distance along the wall.

The hot wire anemometer of section (4.2) was used for measuring the velocity profile at separation particularly because the pitot tube which normally is more reliable and simple would here have been subject to errors due to the large velocity fluctuations associated with separation, and also the local variation of static pressure presented difficulties.

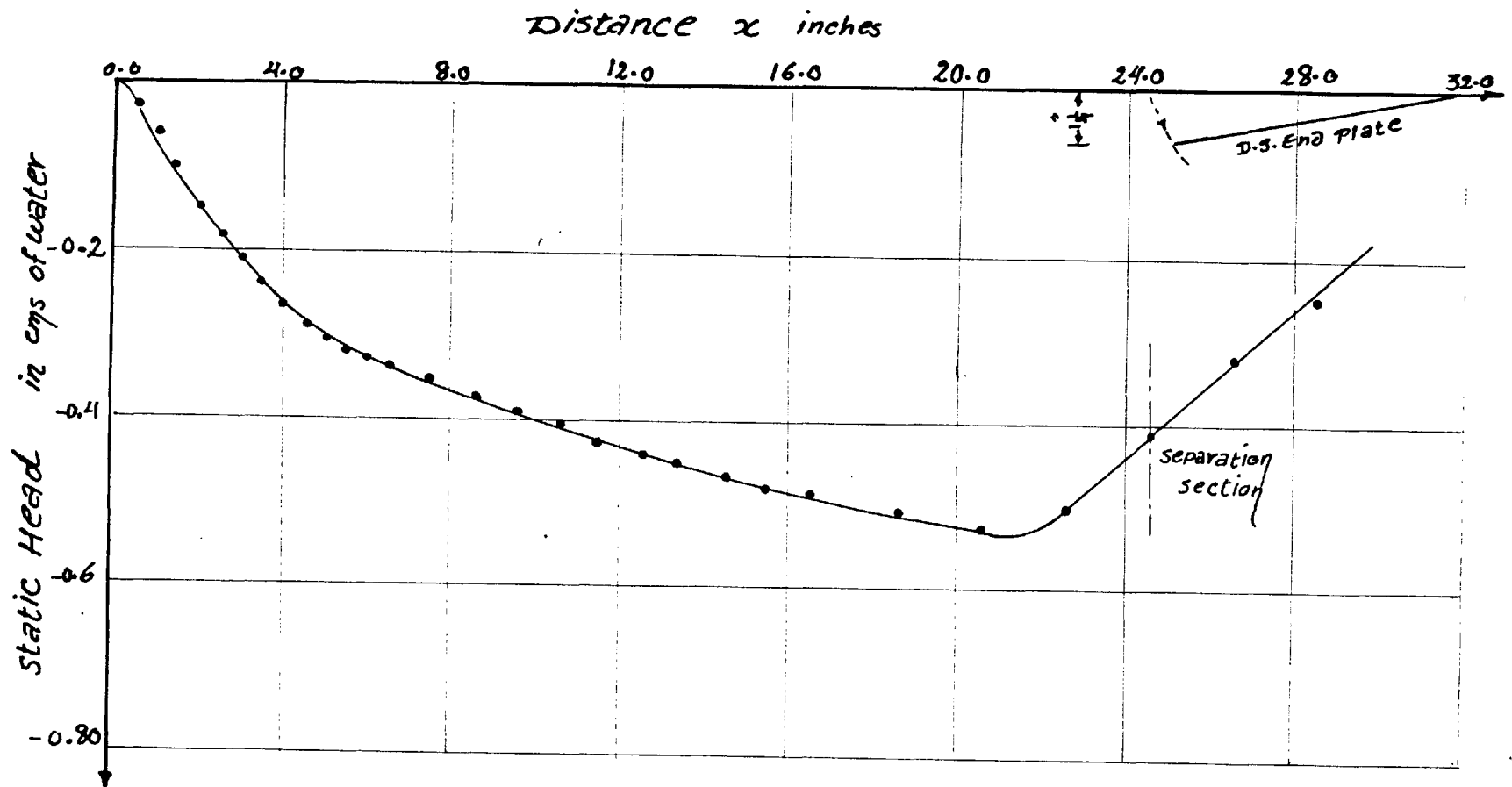


FIG.(3.18) Pressure Distribution for Separation Gas

The approximate thickness of the boundary layer at separation was 1.8 inches and the displacement thickness was 0.69 inches, while the shape factor H was found to be equal to 2.8.

It is interesting to compare the velocity profile obtained for the smoothest of the rough walls with a profile for a smooth wall, one profile for which has been given by Schubauer and Kelbanoff (1950) who use a hot wire as in the present test for measuring the velocities, and whose final shape factor H had the same value 2.8 as in the present test although the boundary layer and displacement thickness were 9.0 and 3.81 inches respectively. The pressure gradients for the rough wall and the smooth wall were respectively 0.018 and 0.021 inches of water head/inch length; and the two profiles are plotted non-dimensionally in figure (3.19).

Due to the unsteady nature of the flow at separation, the position of separation fluctuates along an appreciable length of the wall and in this case the hot wire readings in the neighbourhood of the wall will represent an average for the velocity along this zone of

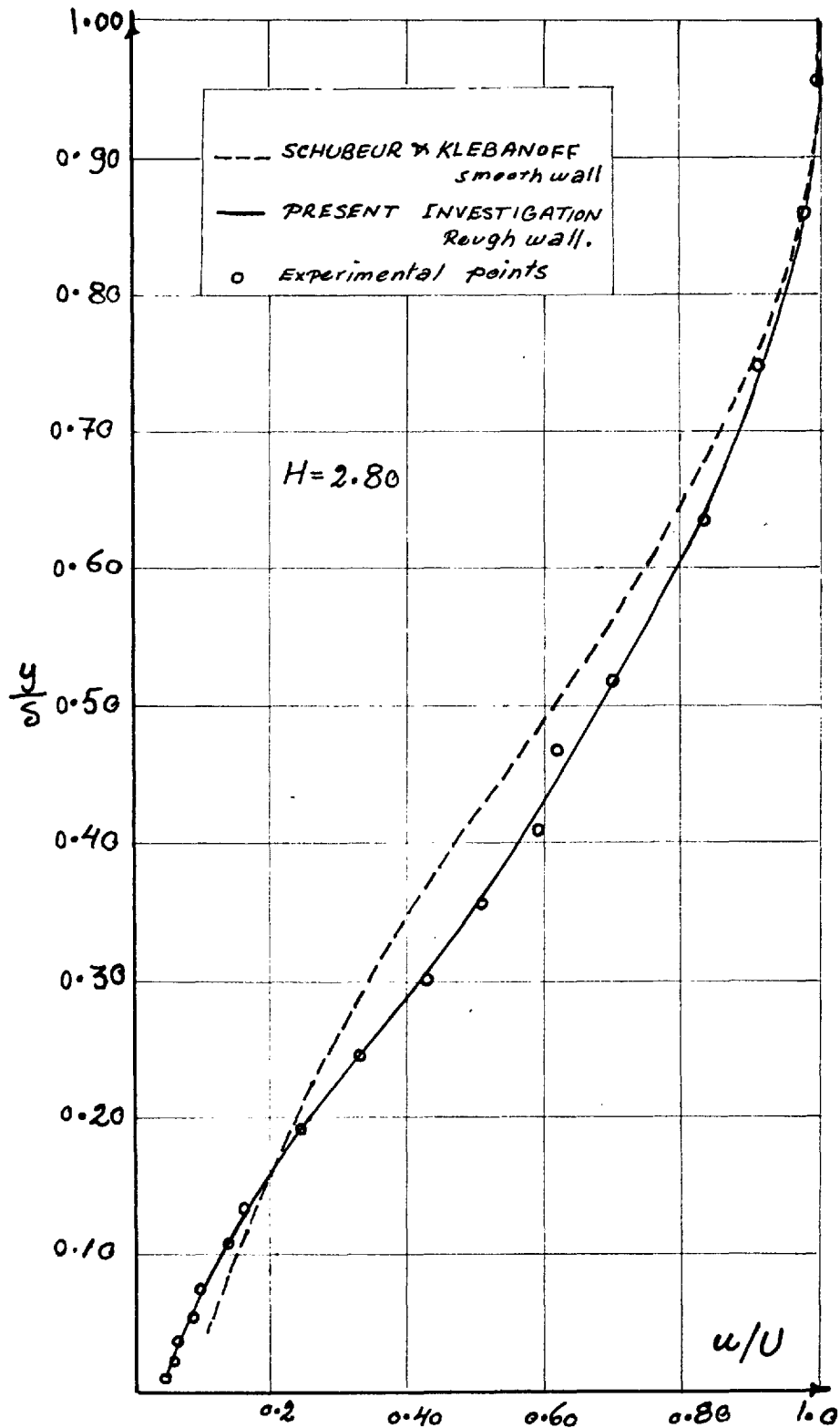


FIG.(3.19) Comparison of The velocity Profiles near Separation

separation. For the smooth wall, Schubauer and Kelbanoff (1950) reported that the position of separation was observed to fluctuate along a distance of approximately 5.0 inches; whereas in the present test this distance was considerably smaller due to the presence of the plate in the close neighbourhood of separation. Also in the present case, the pressure drop farther upstream had a stabilizing effect on the flow before it reached separation.

The Pohlhausen parameter $\delta^2 \frac{\frac{du}{dx}}{V}$ in the present test was -1360 at separation, i.e. the present boundary layer is about 115 times as stable as the laminar boundary layer for which Pohlhausen predicted a value of -12 as the limit.

CHAPTER IV

TURBULENCE INTENSITY MEASUREMENTS IN THE FLOW PAST ROUGH WALLS

4.1 Turbulence Characteristics

Turbulence is introduced into a stream, through some small random fluctuations of the velocity with time and space co-ordinates at relatively high frequencies (Dryden, 1939). According to Taylor and Von Karman (1937), turbulence can be generated by friction forces at fixed walls as in the case of pipe, channel, and boundary layer flows (wall turbulence); or by the flow of layers of the fluid with different velocities past one another as in jets and wakes behind bodies (free turbulence).

The characteristics of turbulence in a stream can be specified by distinct statistical average values of the intensity, scale, and microscale of turbulence (Hinze, 1959). The turbulence intensity is usually defined as the statistical root-mean-square of the velocity fluctuations about the mean, which is a convenient measure of the magnitude of the fluctuations in the velocity; If u is the mean velocity along the flow at any point, and u' is the

velocity fluctuation about the mean at any time, then the longitudinal component of intensity will be $\sqrt{F_u^2}$. Similarly the other two components of intensity normal to the flow direction will be $\sqrt{F_v^2}$ and $\sqrt{F_w^2}$. The scale of turbulence is a length measure of the average size of eddies present in the flow in any direction, while the micro-scale of turbulence is a measure of the average size of the small eddies which are responsible for the viscous dissipation of turbulent energy.

In the past, Fage (1936), Laufer (1953), and others have made systematic progress in measuring the characteristics of turbulence, though almost all the early work concerned boundaries where the surfaces were smooth. However regarding the effect of roughness, very few experiments are on record. Apart from the turbulence intensity profiles given by Fage (1934) for rough square pipes, by Tillmann (1946) for a sand roughened plate with zero pressure gradient, and by Corrsin and Kistler (1954) for constant pressure flow along a corrugated wall, there seems to be no other available information about the turbulence characteristics for the flow past rough walls.

These measurements by Fage, Tillman, and Corrsin and Kistler, indicate that the intensity of turbulence and the rate of dissipation of turbulent energy are greater for rough walls than for smooth walls; but they leave unanswered, the question as how the fluctuations develop as the flow passes along the wall in the downstream direction.

In the present investigation, to explore the development of these turbulent fluctuations along the boundary layer, a series of experiments were undertaken to measure the longitudinal component of turbulence intensity in flows past rough walls with three different roughness arrangements.

4.2. Hot Wire Anemometer

4.2.1. Principle of the Hot Wire Anemometer

The principle of the hot wire anemometer is based on King's semi-empirical law for the thermal equilibrium of electrically heated wires. When a wire is heated or cooled, its resistance changes with temperature, and this change of resistance can be related to the air speed causing the change.

For thermal equilibrium of the wire, the heat generated per unit time by the current heating the

wire, should equal the heat transferred to the fluid from the wire per unit time; thus according to King:

$$I^2 R = (A + B \sqrt{u}) (\theta_w - \theta_f) \quad (4.1)$$

where R is the wire's total resistance, I is the wire's current, A and B are constants to be determined experimentally, and θ_w and θ_f are the temperatures of the wire and the fluid respectively which are related directly to the corresponding resistances. As seen from equation (4.1), the relation between I^2 and $\sqrt{u}$ is substantially linear.

For measuring the fluctuations in the speed, the temperature of the wire and consequently its resistance vary rapidly with the speed fluctuations and two different methods are in use for measuring these fluctuations.

In the first method, the temperature of the wire and so its electric resistance is kept constant, and the fluctuations in the current necessary to do this are measured and correlated with the fluctuations in the speed; whereas in the second method, the heating current is kept constant and therefore the temperature and the resistance fluctuate with

the speed. The constant temperature method, although it has the advantage of giving higher accuracy, requires a feed-back system to keep the wire's temperature constant, and such systems are often unstable.

For the constant current method which was used in the present investigation, a very fine wire was placed in the stream and connected to one arm of a Wheatstone bridge, which was then adjusted to bring the galvanometer to zero, the balance thus obtained concerns the time average. Actually the temperature of the wire and consequently its resistance are fluctuating with the fluctuating velocity in the stream, though the resulting potential fluctuations are too small to be measured directly with an A.C. instrument. Accordingly the bridge is coupled to an amplifier which receives the out-of-balance signal, and the amplifier's output is measured by a thermo-couple type ammeter. The relation between the voltage fluctuations and the speed fluctuations can be calculated, since the electrical constants of the Wheatstone bridge and the amplifier together with the calibration parameters of the hot wire are known.

4.2.2. Circuit for measuring the $\bar{u}$ component of Turbulent Intensity

A circuit for measuring $\sqrt{\bar{u}^2}$ was constructed as shown in figure (4.1). The hot wire was inserted in one arm of a Wheatstone bridge with 500 ohms ratio-arm resistances, and the fourth arm of the bridge was formed by a variable constantan resistance R_3 . Another variable resistance R_2 connected in series with the circuit provided a convenient control for balancing the bridge, and this resistance consisted of two variable resistances, one of 1000 ohms and the other of 50 ohms for fine adjustment.

In order to avoid heating the wire when measuring its cold resistance, a very large resistance R_1 of 100,000 ohms was temporarily inserted in the circuit. The power supply to the circuit was a 24 volts D.C. battery. An A.C. spot galvanometer (230 volts, 50 ohms, 50 C/S) was adjusted for zero reading to indicate the balance of the bridge, and the milliammeter reading then gives double the wire's current.

After determining the cold resistance of the wire, R_1 was disconnected thus allowing the

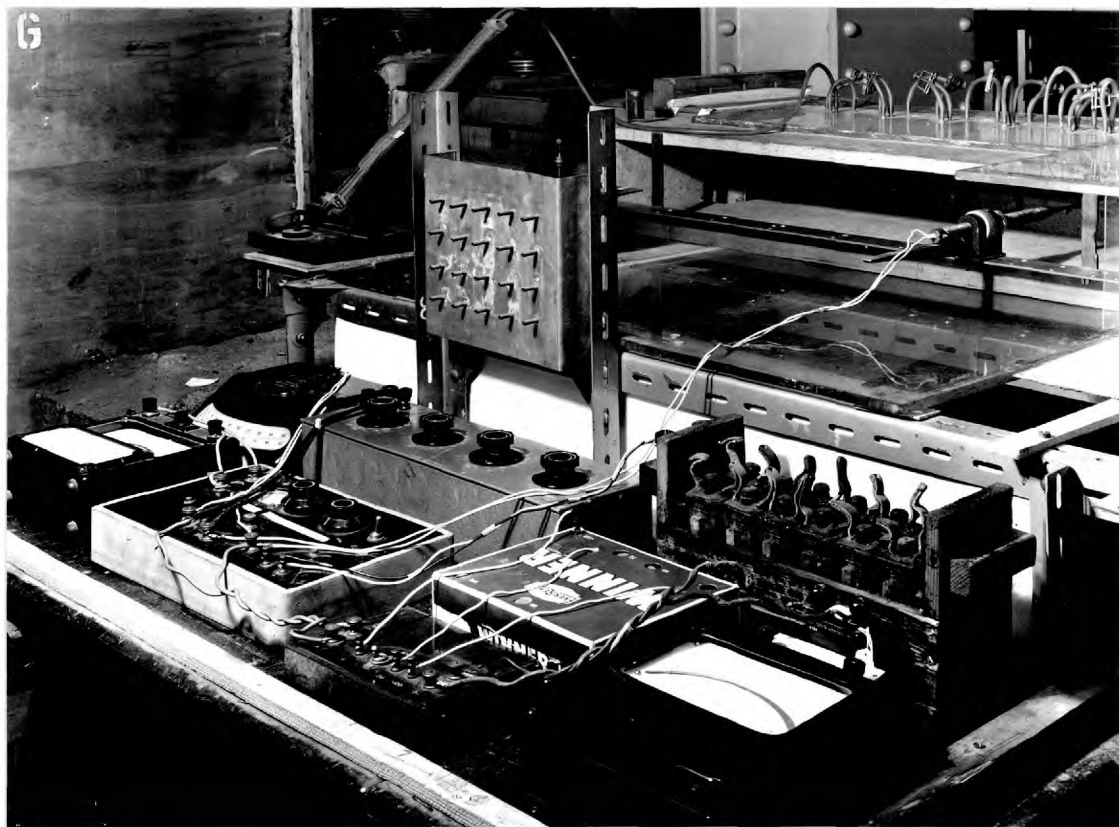


PLATE II. CIRCUIT FOR MEASURING THE $\dot{u}$ FLUCTUATION

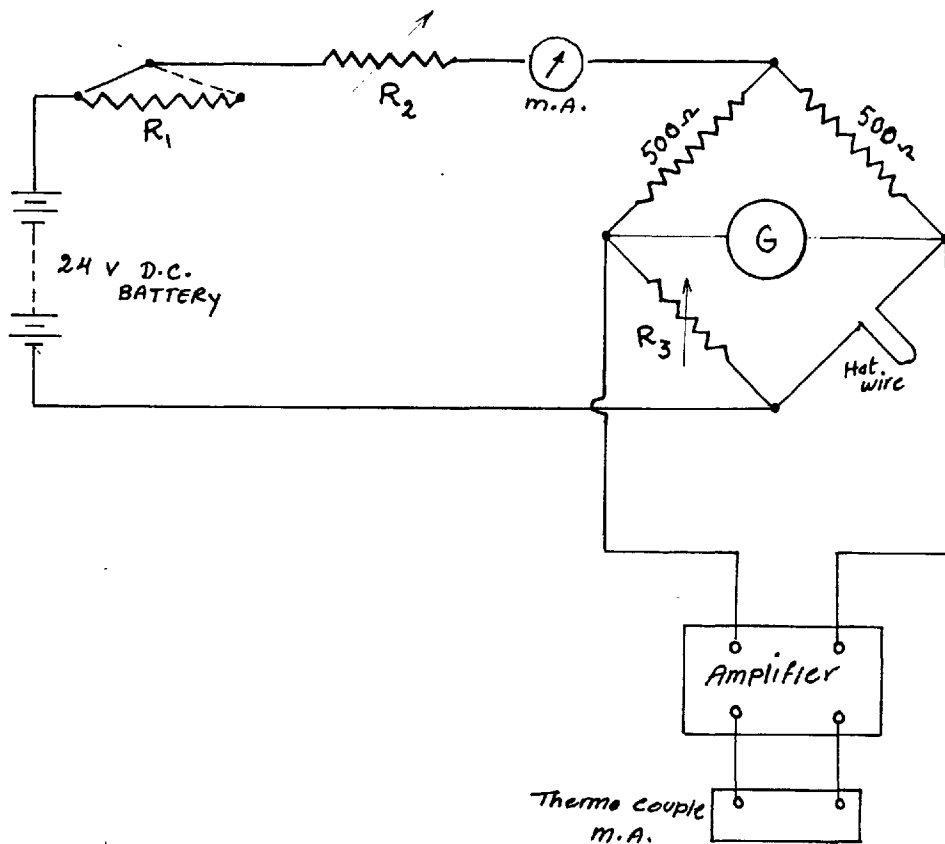


FIG. (4.1) *Circuit for Measuring $\sqrt{u^2}$*

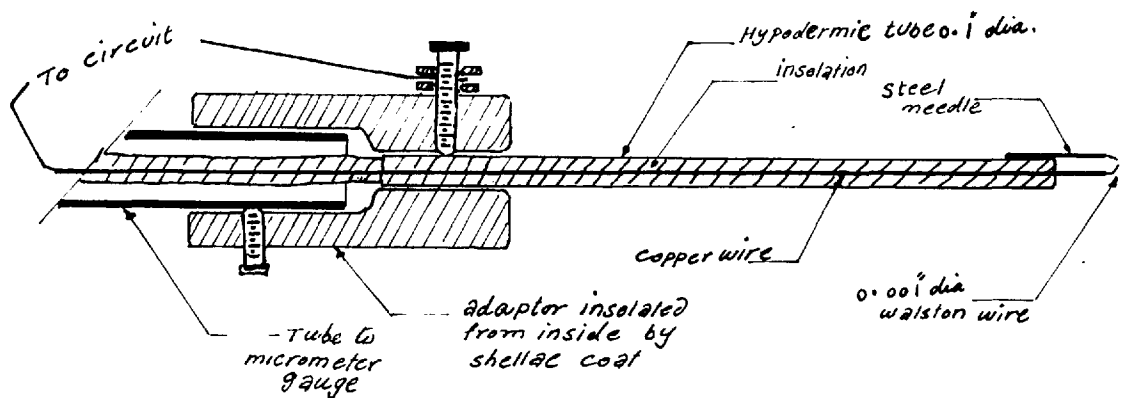


FIG. (4.2) *Hot Wire Holder*

bridge current to increase and heat the wire to about 150°C above the atmospheric temperature. R_3 was set to a chosen suitable value (usually 1.20 to 1.50 times the wire's cold resistance), and with the hot wire in the air stream, the bridge was then balanced by means of R_2 . Due to the velocity fluctuations causing the wire's resistance to fluctuate, the bridge was temporally unbalanced and this out-of-balance was fed to an amplifier to be magnified, and the output was then read on the thermo-couple-type milliammeter.

The thermo-couple measures directly the mean square value of the current corresponding to the voltage signal picked by the amplifier, or in other words, the reading of the milliammeter is proportional to the R.M.S. of the fluctuating voltage.

4.2.3. Material and Dimensions of the Hot Wire

Platinum, Nickel, or Tungsten, pure metals with accurately known electrical constants are used for hot wires; but Platinum is used most, chiefly because of its chemical inertness at high

temperatures.

The dimensions of the hot wire should be small, not only to cause a minimum disturbance to the flow, but also to have sensitivity for recording fluctuations of small size. Also from the strength point of view, the length-diameter ratio of the wire should not exceed 200.

In order to record rapid fluctuations of velocity accurately, the wire should be heated and cooled instantaneously, but due to the finite thermal inertia of the wire, there is a delay and a consequent reduction in reading. Thinner wires will have smaller thermal inertia, thus helping to minimize the compensation required to cover for that delay. The effect of the delay becomes more pronounced at velocity fluctuations with high frequencies and the turbulence intensity would be underestimated.

The raw material for the wire used in the present experiments was a silver Walston wire, 0.001 inches diameter with a core of Platinum 0.0001 inches in diameter. A piece of this wire about 2.0 mm. long was soft-soldered to the wire holder (figure 4.2), and a length of about 0.50 mm.

of the Platinum core was then exposed by removing the silver cover at the middle by a process of etching and electrolysis. The wire centre was immersed in a bubble of dilute nitric acid, and an electrolysing potential of 1.50 volts was applied across the electrolyte for about five seconds.

After soldering the hot wire to its holder, the holder was mounted on a microgauge and was inserted into the air stream. By careful adjustment of the gauge, the wire was brought very close to the roughness elements without touching them. The plane of the wall for the velocity fluctuations profiles was assumed to be at the mid-height of the roughness elements.

For measuring $\sqrt{u'^2}$, the wire was always kept normal to the flow direction, and in this position it does not record the v' component whose effect is negligible compared with that of the u' component.

4.2.4. Calibration of the Hot Wire

The formula used for the calculation of $\sqrt{u'^2}$ was derived by Betchov and Kuyper (1947), from

King's law and based on the assumption that the velocity fluctuation u follows a simple periodic function with angular frequency ω , and that the change in the average current due to rapid changes in the wire's resistance can be neglected. This formula can be written as follows:

$$\frac{\sqrt{u}^2}{u} = \frac{4 \left[\frac{1}{R R_a} \cdot \left(\frac{dI}{d(1/R)} \right)_u + \frac{I}{2R} \right] [R + S] \cdot e_i}{(I^2 - I_0^2) \cdot S \cdot A_1} \quad (4.2)$$

and a suitable compensation circuit is used to compensate for the finite thermal inertia of the wire. In equation (4.2), u is the local mean velocity, I is the wire's current, R and R_a are the actual and cold resistances of the wire respectively, I_0 is the wire's current corresponding to $u = 0$, S is the ratio arm resistance of the Wheatstone bridge, and A_1 and e_i are the amplifier's gain and out put respectively.

The values of the different constants in equation (4.2), were determined by placing the hot wire on the centre-line of an air jet whose speed was varied and known from pitot tube readings, and the working range of the wire's resistance values was then explored. The current required to heat the wire in each case was

recorded, together with the resistance and the corresponding wind speed. By extrapolating the curves of I^2 to zero velocity in figure (4.3), the I^2_0 values were determined for each value of the wire's resistance. To determine the values of $\frac{dI}{d(\sqrt{R})}$ in the equation, the current I was plotted against $\frac{1}{R}$ for different wind speeds (figure 4.4), and the slope of the resulting lines was measured at the corresponding wind speeds and then plotted versus u to give a curve convenient in using equation (4.2).

A change in the atmospheric pressure, temperature or degree of humidity of the air usually affected the performance of the hot wire. Also the accumulation of dust changed the thermal conditions at the surface, so usually one or two calibrations were needed every day.

4.2.5. Calibration of the Amplifier

The amplifier's gain A was determined for a range of frequencies varying from 50 C/S to 20 K.C/S, by applying a signal of known frequency from a generator and measuring the amplifier's input by a millivoltmeter. The output was measured by a thermo-couple-type milliammeter. The

FIG. (4.3) A Typical Calibration Curve for The Hot Wire.

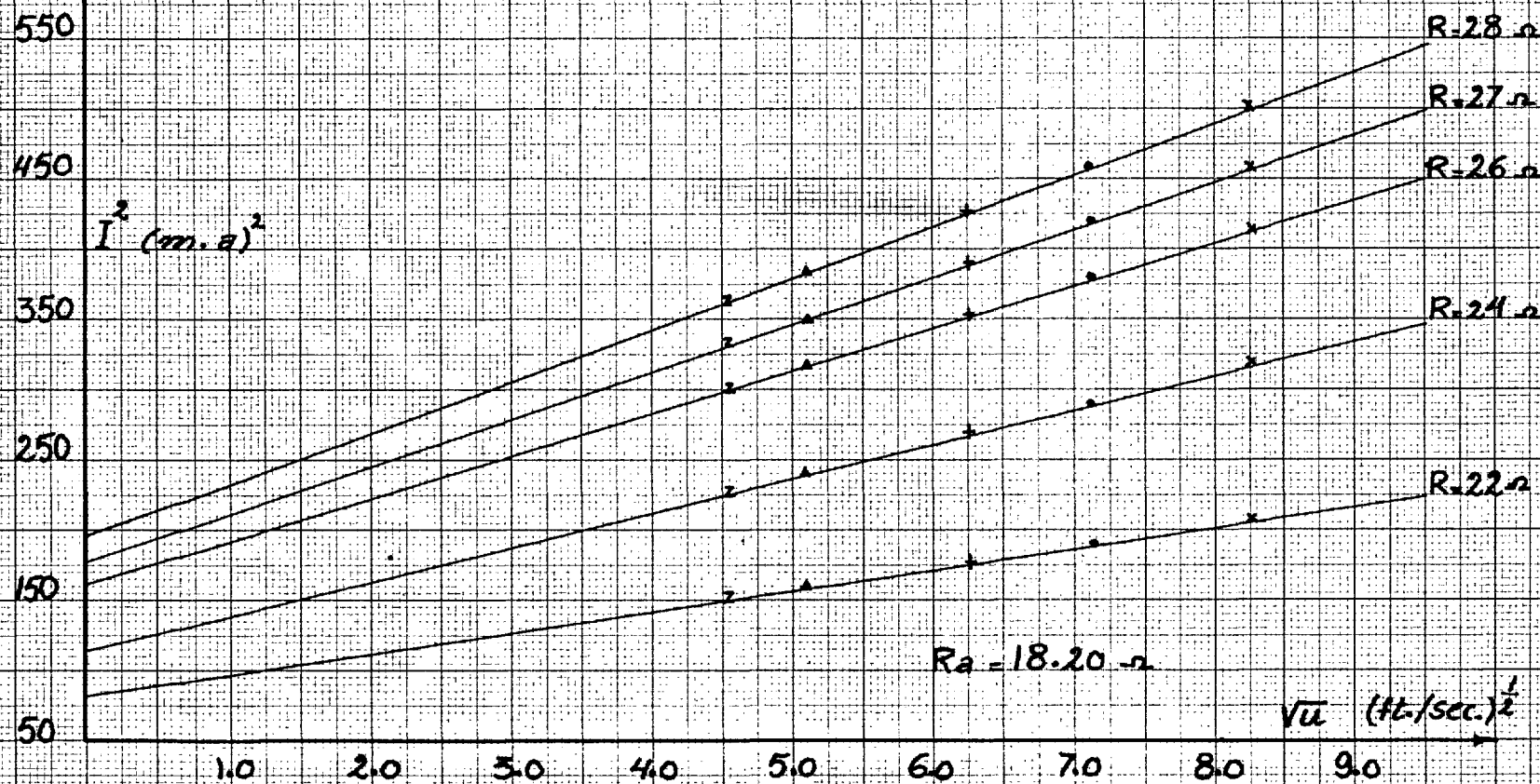


FIG. (4.4) A Typical Calibration Curve
for The Hot Wire.

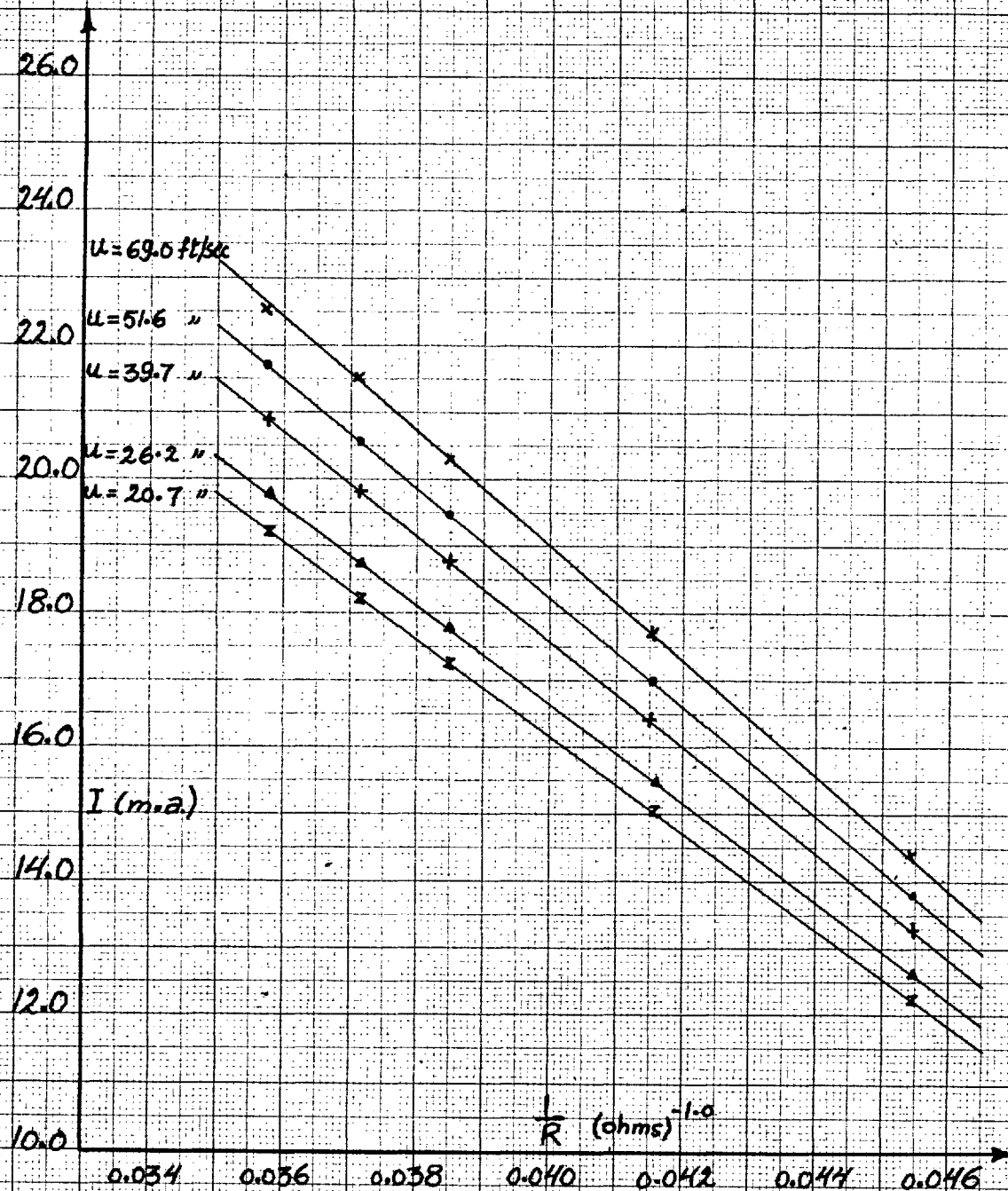
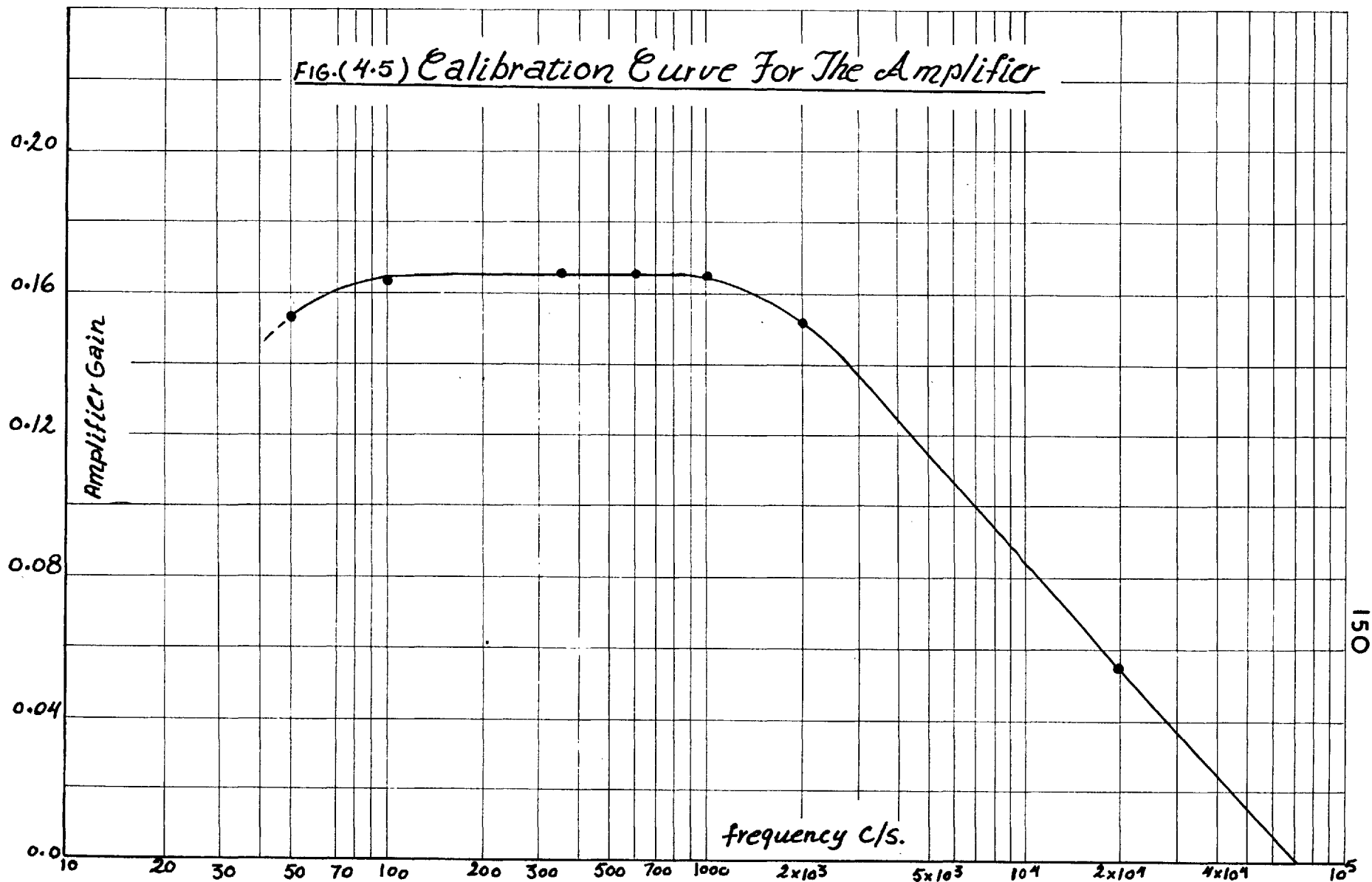


FIG.(4.5) Calibration Curve For The Amplifier



variation of the amplification with frequency is shown in figure (4.5), a curve which has a flat slope in the range of working frequencies between 100 and 1000 C/S.

When desired, the amplifier's output was connected to a cathode-ray oscillograph giving a visual representation of the velocity fluctuations. Plates (12 and 13) show typical oscillograms for the velocity fluctuations, both at the wall and outside the boundary layer of the 4.75 mm. sand-roughned plate. These oscillograms were taken at $x = 20.5$ inches from the leading edge of the plate, and the corresponding values of $\frac{\sqrt{u^2}}{U}$ were 0.066 and 0.0135 respectively. A calibration frequency of 50 C/S is shown on both plates for comparison, and the velocity fluctuations in plate (13) are shown to a scale three times larger than that in plate (12).

4.3. Experimental Results

Measurement of the $\sqrt{u^2}$ component in the present investigation were carried out for the flow in the nozzle of constant drag under the

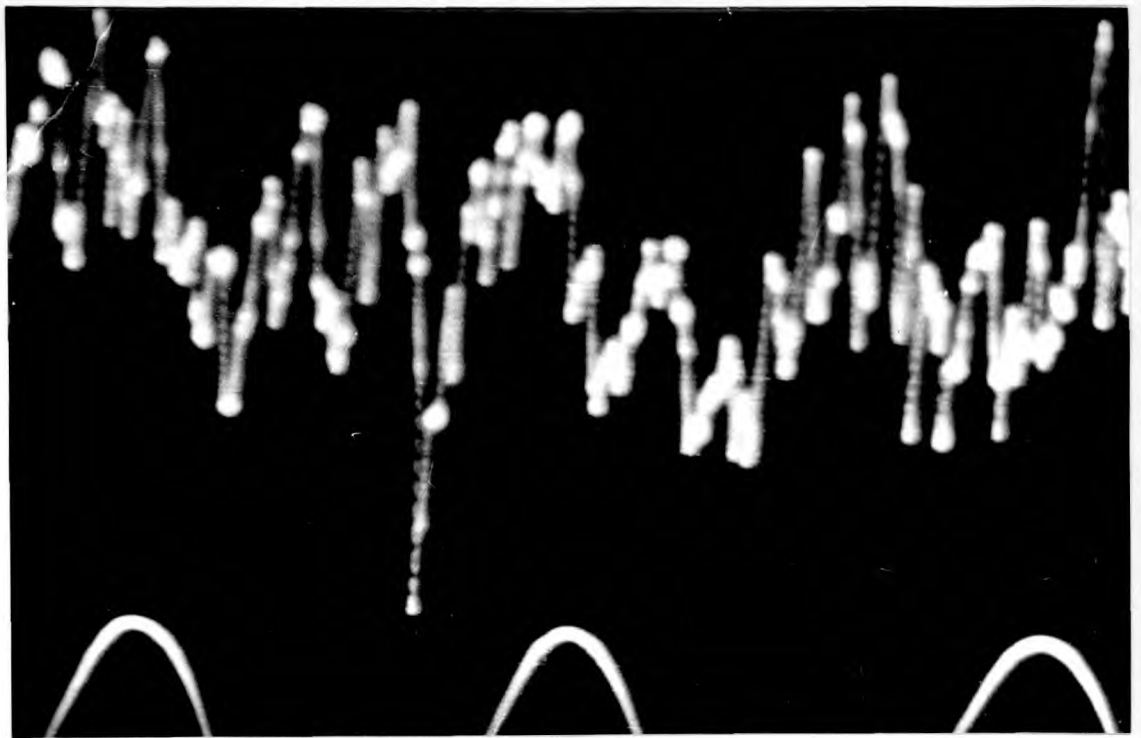


PLATE 12. OSCILLOGRAM FOR u' NEAR THE WALL ($r=4.75$ mm)
 $\frac{\sqrt{u'^2}}{U} = 0.066$, Calibration frequency 50 cycles per second

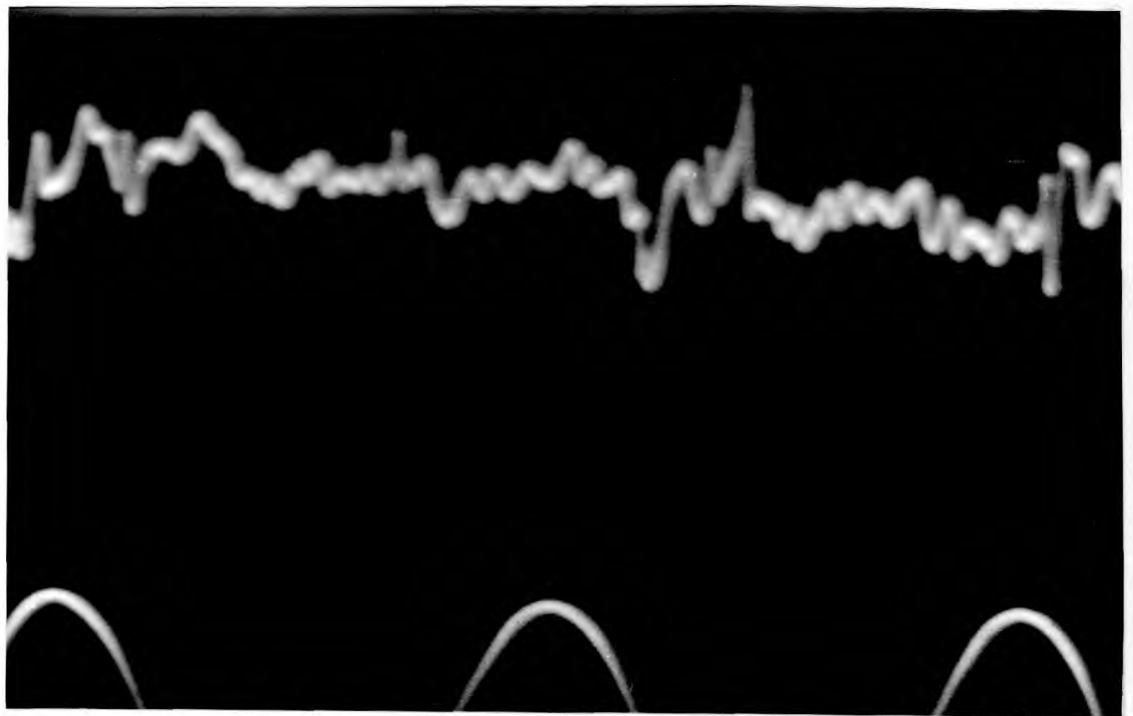


PLATE 13. OSCILLOGRAM FOR u' OUTSIDE THE BOUNDARY LAYER
 $r = 4.75$ mm. , $\frac{\sqrt{u'^2}}{U} = 0.0135$, Calibration frequency 50 C/S .

influence of a negative pressure gradient. For roughening the wall, sands A, C and D with grain diameters of 1.56, 3.16 and 4.75 mms respectively were used.

In each case the $\sqrt{u^2}$ values were measured at different stations along the boundary layer, for heights above the rough wall extending into the main stream. Figures (4.6a to 4.8a) show the $\sqrt{u^2}$ values in terms of the local mean velocities, while figures (4.6b to 4.8b) give the $\sqrt{u^2}$ values in terms of the velocity at the edge of the relevant boundary layer. The distribution of $\sqrt{u^2}$ in figures (4.6 to 4.8) shows the same general shape as that obtained by Corrsin and Kistler (1954) and Tillmann (1946) who dealt with rough walls under constant pressure. They also resemble the curves of Page (1934 and 1936) and Laufer (1953) for flow in pipes.

By inspection of figures (4.6b to 4.8b), the $\sqrt{u^2}$ values reach a maximum in a region close to the wall, and since the diagrams disclose a clear dependence on the roughness height, it seems clear the the velocity fluctuations due to the presence of the wall extend far outside

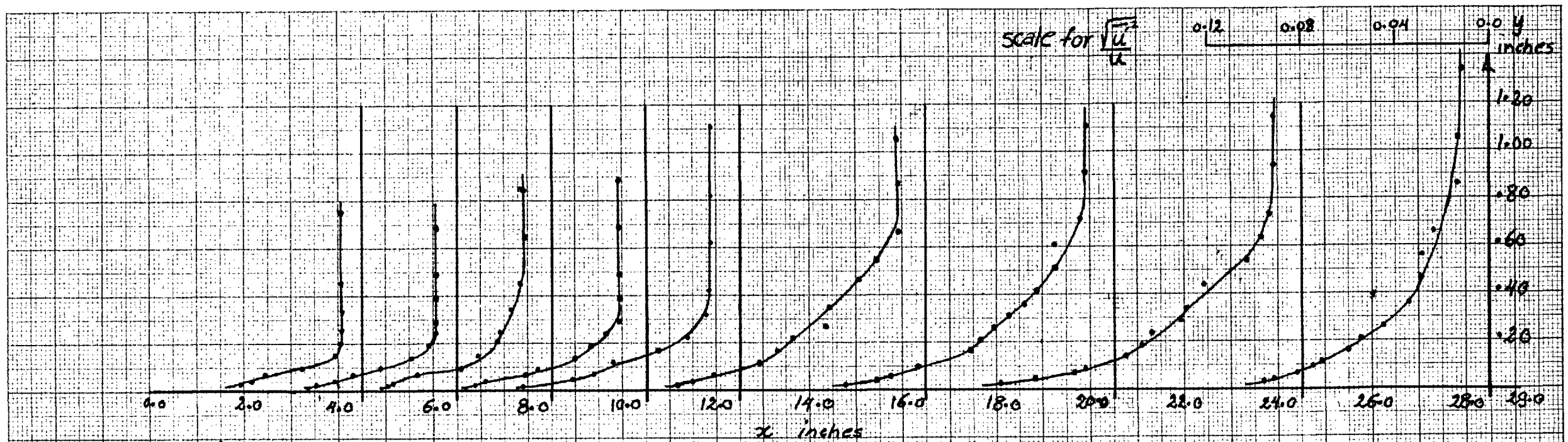


FIG. (4.65) Distribution of The Longitudinal Component of Turbulence Intensity as a ratio of The Local Velocities (Roughness "A", $k=1.56$ mm)

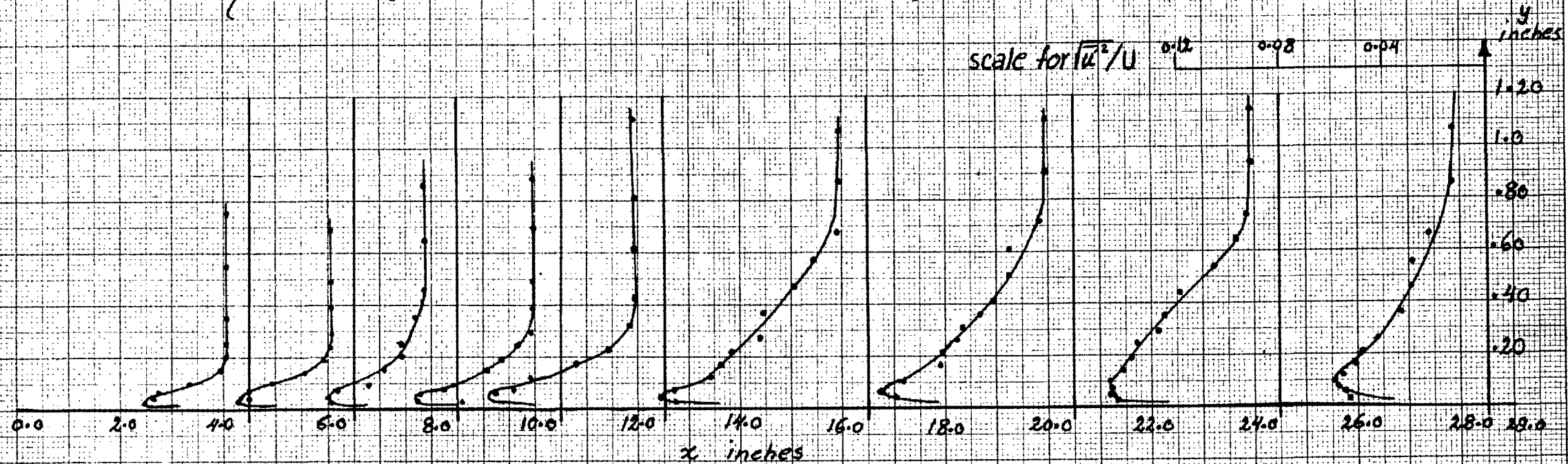


FIG. (4.66) Distribution of The Longitudinal Component of Turbulence Intensity as a ratio of Main Stream Velocities (Roughness "A")

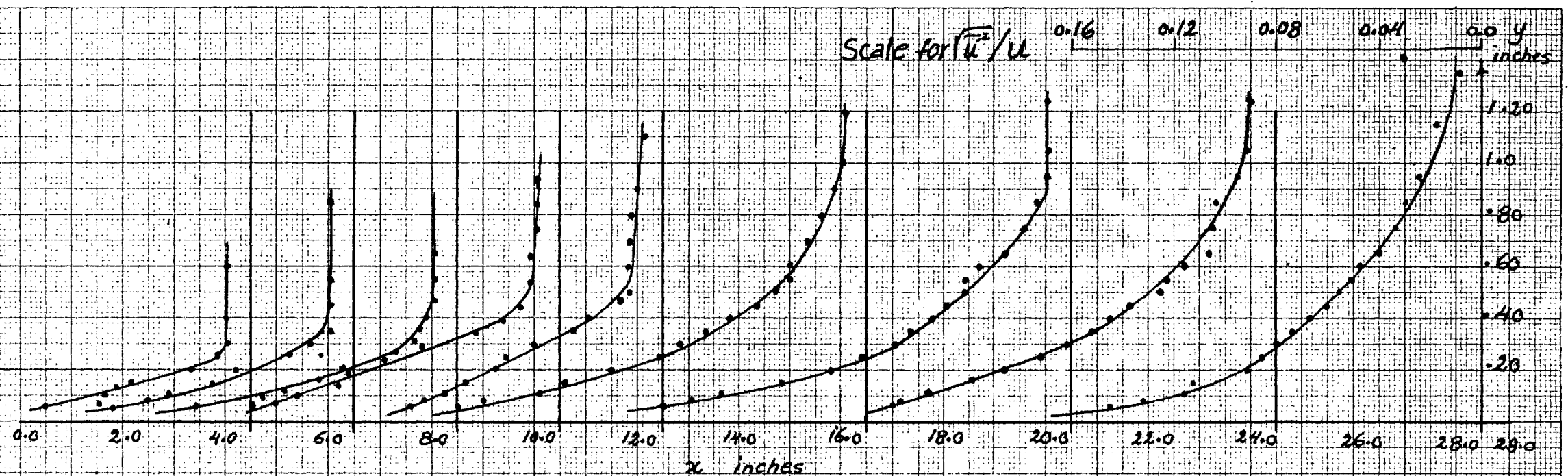


FIG. (4.7a) Distribution of The Longitudinal Component of Turbulence Intensity as a ratio of The Local velocities (Roughness " ϵ ", $R=3-16$ mm)

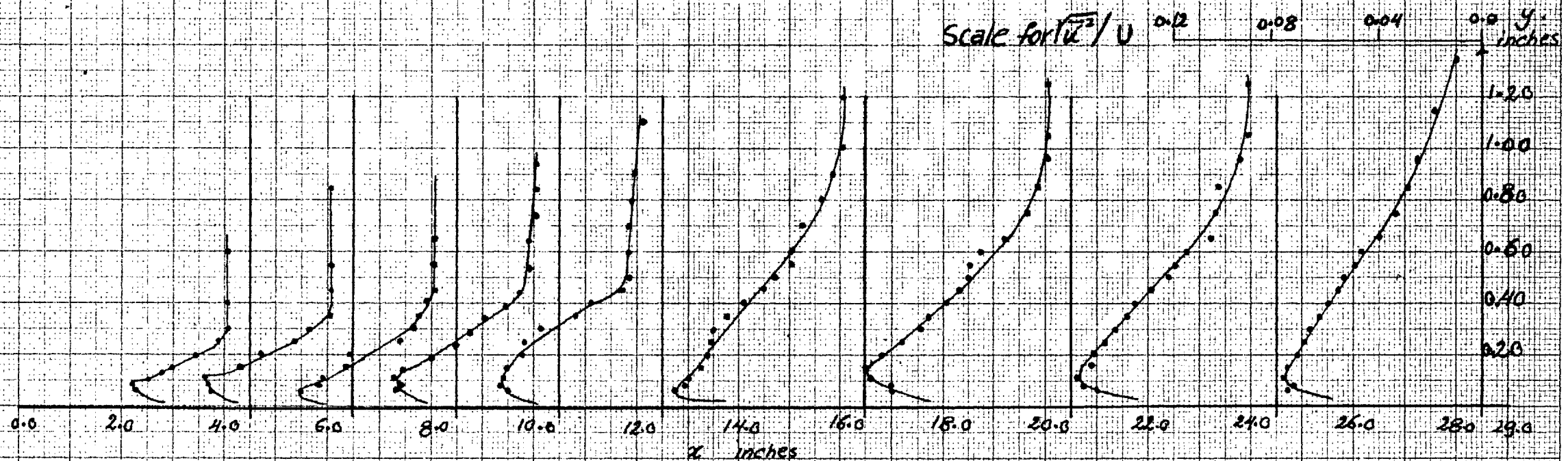
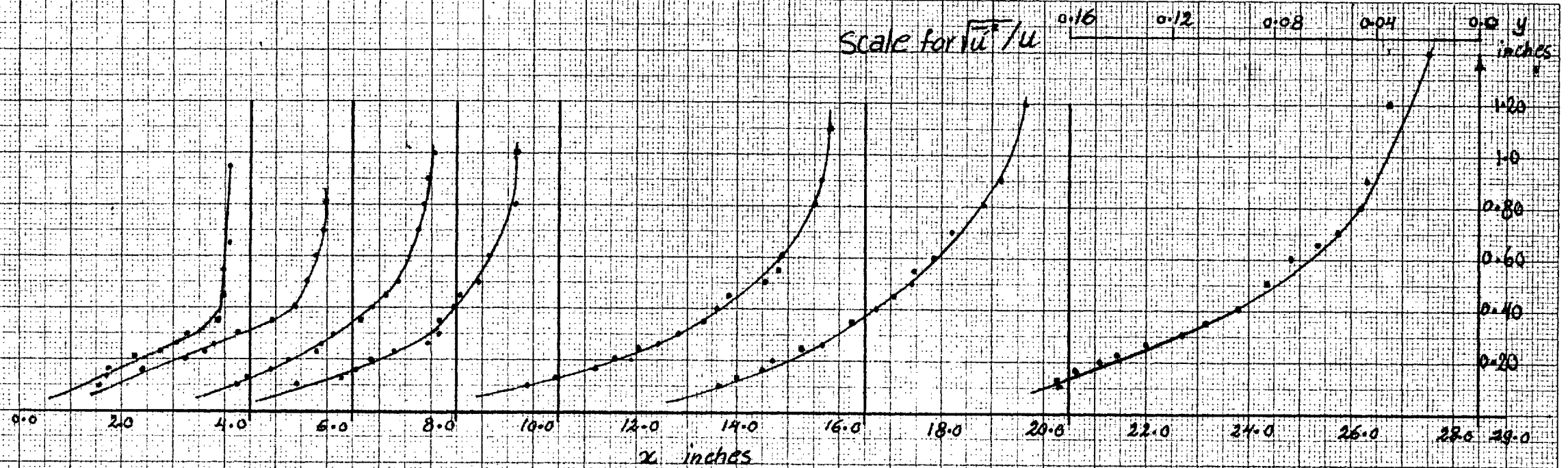
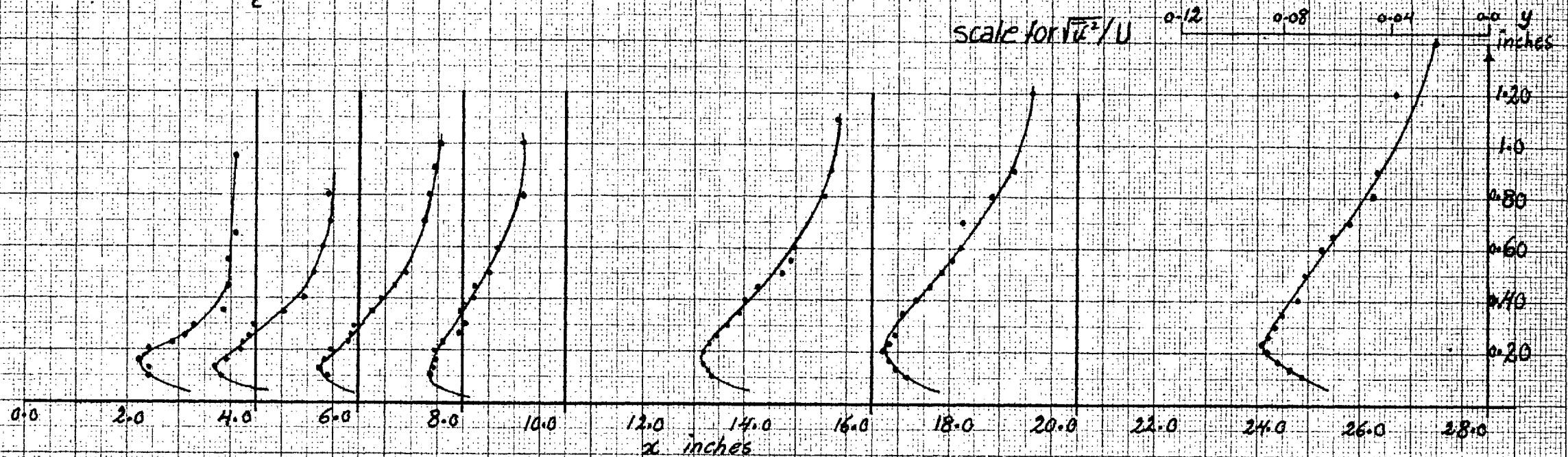


FIG. (4.7b) Distribution of The Longitudinal Component of Turbulence Intensity as a ratio of Main stream velocities (Roughness " ϵ ")



FIG(4.8a) Distribution of The Longitudinal Component of Turbulent Intensity as a ratio of The Local velocities (Roughness " D_s " 4.75 mm)



FIG(4.8b) Distribution of The Longitudinal Component of Turbulence Intensity as a ratio of Main stream Velocities (Roughness " D_s ")

the region of the boundary layer into the main stream itself. In the profiles of $\frac{\sqrt{u'^2}}{u}$, the values of $\frac{\sqrt{u'^2}}{u}$ tend to infinity as the distance from the wall y tends to zero, though strictly speaking these values are indeterminate, because both $\sqrt{u'^2}$ and u tend to zero at the wall. The $\frac{\sqrt{u'^2}}{u}$ values (figures 4.6b to 4.8b) however after passing through the maximum, tend to zero as the wall is approached where the absolute values of the fluctuations are zero. From the figures it is also clear that, except close to the wall, $\sqrt{u'^2}$ is greatest where the gradient of mean velocity is greatest and vice versa.

Figures (4.9 and 4.10) show the variation of $\frac{\sqrt{u'^2}}{V_*}$ with $\frac{y}{\delta}$ for two of the roughnesses tested. In all, nine curves are included each for a different distance from the nose of the plate, and as can be seen in these figures the turbulence varies considerably from place to place along the boundary becoming steadily greater in the downstream direction. For example in the distance from $\frac{x}{K} = 85$ to $\frac{x}{K} = 230$ for the 3.16 mms. sand, the u' component increases more than 40%. Again in figure (4.10) for the 4.75 mms sand, the u' component

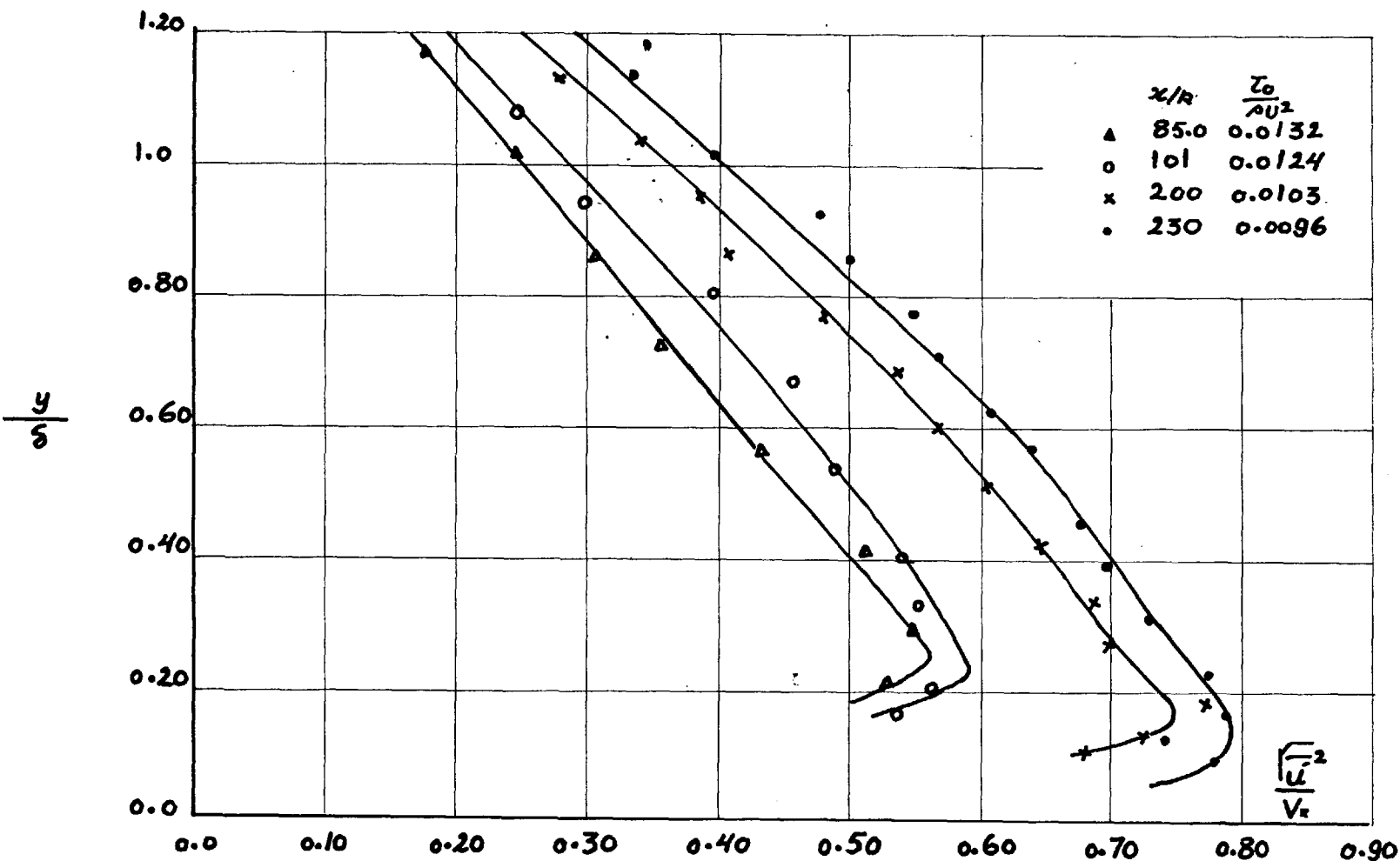


FIG.(4.9). Variation of $\sqrt{\bar{u}^2}$ with Distance from Nozzle of Plate, $h = 3.16$ mms.

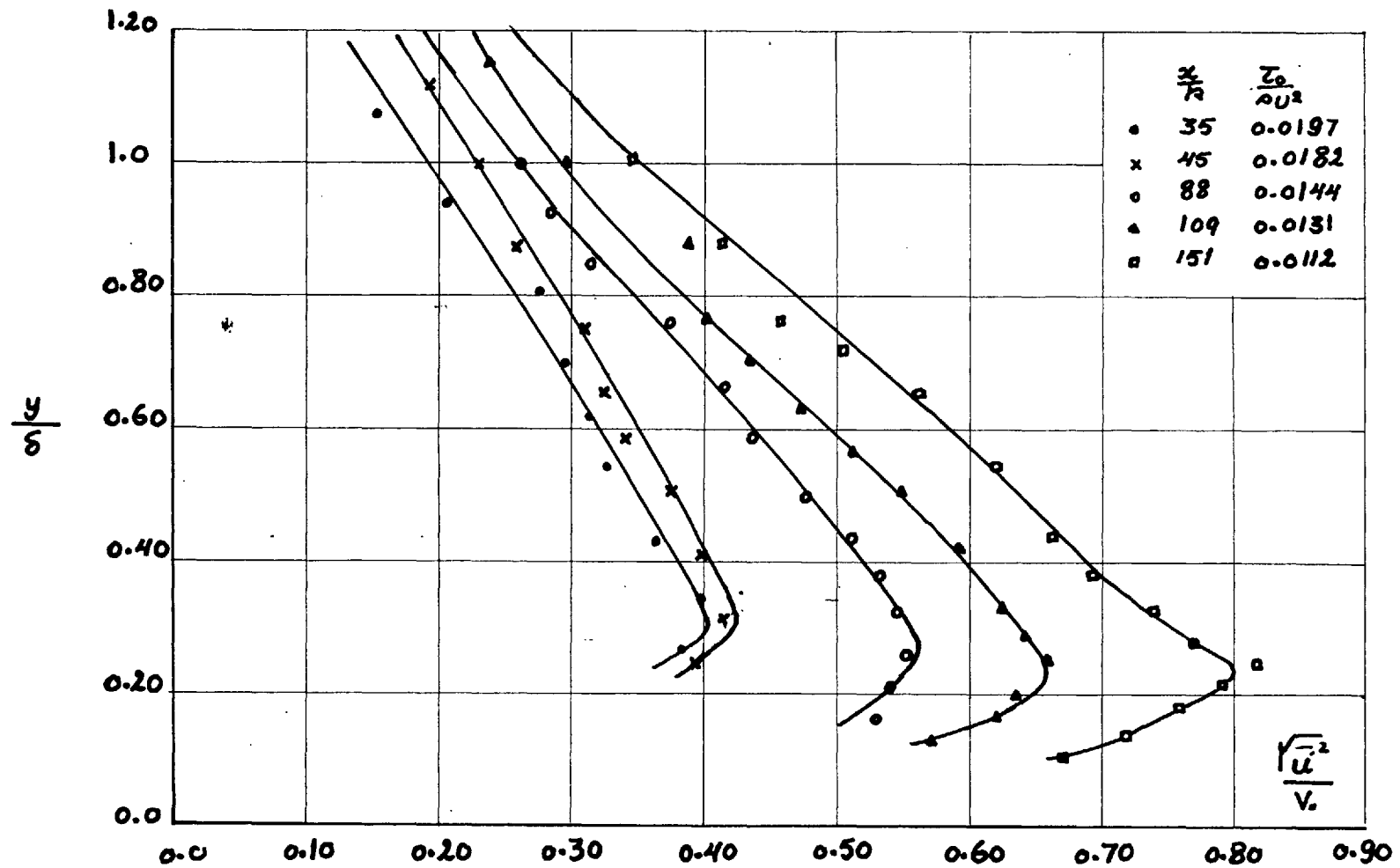
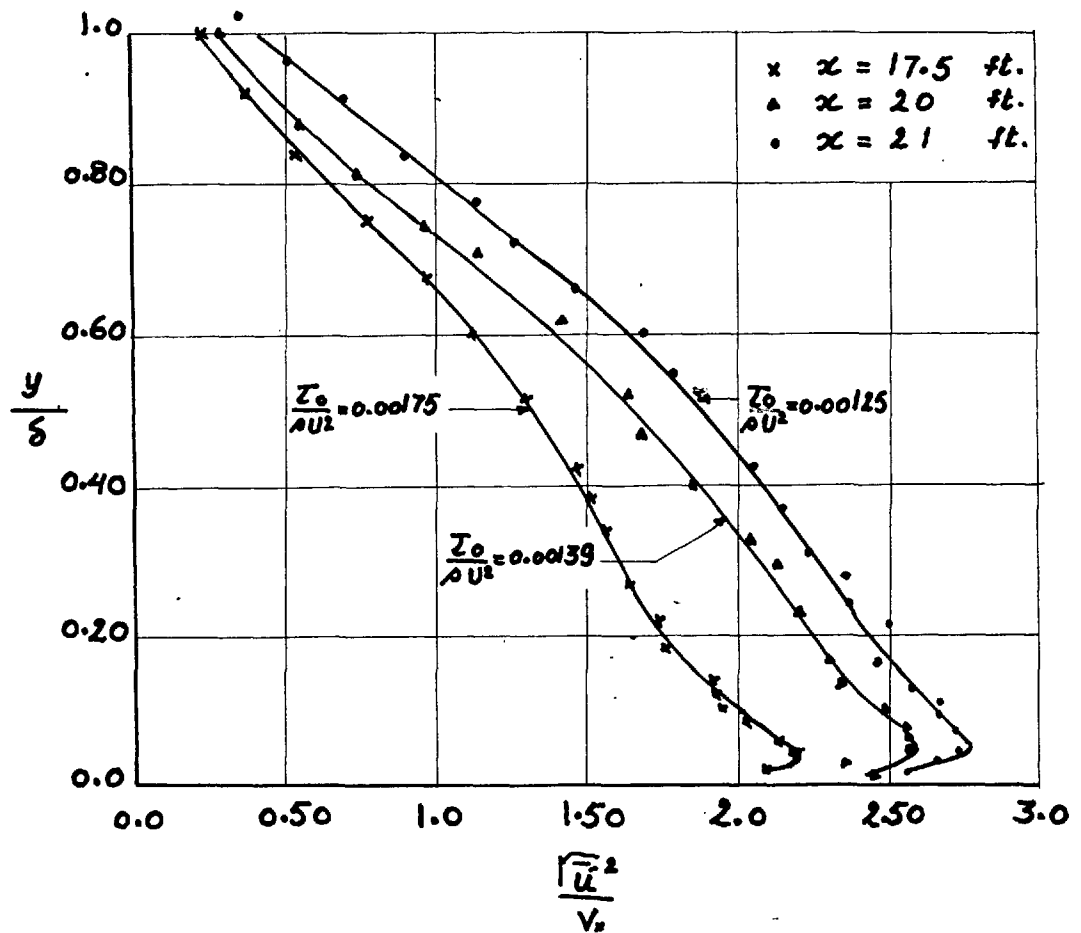


FIG.(4.10) Variation of $\bar{u}^2$ with Distance from Noze of Plate, $R = 4.75$ mms.

increases about 80% in the distance from $\frac{x}{K} = 35$ to $\frac{x}{K} = 151$. Since V_{*} was substantially constant throughout the length of the boundary, the growth of the turbulence can be attributed only to the greater speeds downstream, or more precisely to the greater values of the dimensionless ratio $\frac{U}{V_{*}}$.

To compare rough walls with smooth, figure (4.11) was plotted from the results of Schubauer and Klebanoff (1950) who experimented on a smooth wall, though in this case with a rising pressure. As can be seen from this figure, the values of $\frac{\sqrt{\bar{u}^2}}{V_{*}}$ varies along the flow again increasing steadily with $\frac{U}{V_{*}}$ in a way similar to rough walls. Also the results of smooth pipes by Fage (1936) who experimented on the maximum fluctuations using an ultramicroscope, and by Laufer (1953) who experimented on the R.M.S. values using a hot wire, showed a tendency for $\frac{\bar{u}_{max}}{V_{*}}$ and $\frac{\sqrt{\bar{u}^2}}{V_{*}}$ to increase with $\frac{U}{V_{*}}$. Thus for a constant V_{*} , $\frac{\sqrt{\bar{u}^2}}{V_{*}}$ is expected to increase with U , and when U is constant $\frac{\sqrt{\bar{u}^2}}{V_{*}}$ has a tendency to decrease with V_{*} as found in the experiments by Klebanoff and Diehl (1951) in boundary layers



FIG(4.11) Profiles of $\sqrt{u^2}$ for smooth walls.

(Schubauer and Klebanoff, 1950)

where U was constant.

This tendency of $\frac{\sqrt{u'^2}}{V_*}$ to increase with $\frac{U}{V_*}$ also seems consistent with the turbulent energy equation as seen from the following argument:

As G.I. Taylor (1935) pointed out, the rate of production of turbulent energy per unit volume, which is mainly due to the work done by the shearing stresses on the mean flow, can be represented by the term $\tau \frac{du}{dy}$. Taylor (1935) showed that the rate of dissipation of this energy in a pipe, on the assumption that $\overline{u'^2} = \overline{v'^2} = \overline{w'^2}$, can be expressed by the term $15 \rho \nu \cdot \frac{\overline{u'^2}}{\lambda^2}$ where λ is a length scale depending on the size of eddies. In order to represent the energy balance across the pipe, another term is needed, namely to represent the rate of energy diffusion from places where the rate of production of energy exceeds the rate of dissipation to places where the opposite occurs. For this term, Laufer (1951) deduced the expression $\frac{d}{dy} (\rho \nu' \frac{\overline{u'^2} + \overline{v'^2} + \overline{w'^2}}{2} + \nu' P)$ in the case of a channel or a pipe based on the Reynold's equation. Therefore the energy balance across the pipe can satisfactorily be represented

by:

$$\tau \frac{du}{dy} = 15\rho v \frac{\bar{u}'^2}{\lambda^2} + \frac{d}{dy} \left(\rho v' \cdot \frac{\bar{u}'^2 + \bar{v}'^2 + \bar{w}'^2}{2} + v'P \right)$$

dividing by τ_0 , then:

$$\frac{\tau}{\tau_0} \frac{du}{dy} = \frac{15v}{\lambda^2} \frac{\bar{u}'^2}{v_*^2} + \frac{d}{dy} \left(\frac{1}{2} v' \frac{\bar{u}'^2 + \bar{v}'^2 + \bar{w}'^2}{v_*^2} + \frac{v'P}{\tau_0} \right) \quad (4.3)$$

There seems no doubt that, in a pipe, $\frac{du}{dy}$ tends to increase with U , and therefore $\frac{\sqrt{\bar{u}'^2}}{v_*}$ according to equation (4.3) will have a tendency to increase with U , which seems in line with observation. Accordingly we may conclude here that $\frac{\sqrt{\bar{u}'^2}}{v_*}$ has a tendency, both theoretically and experimentally, to increase with $\frac{U}{v_*}$.

Unexpectedly, the individual curves of figures (4.9 and 4.10) for rough walls converge to single curves when plotted in the form $\frac{\sqrt{\bar{u}'^2}}{v_*} \frac{v_*^2}{U^2}$ as seen in figures (4.12 and 4.13).

Also the data on smooth walls fall on another single curve, though with $\frac{\sqrt{\bar{u}'^2}}{v_*} \cdot \frac{v_*^2}{U^2}$ values considerably smaller than those for rough walls. That the curves of $\frac{\sqrt{\bar{u}'^2}}{v_*}$ should converge when multiplied by $\frac{v_*^2}{U^2}$ is a paradox explained partly by the following:

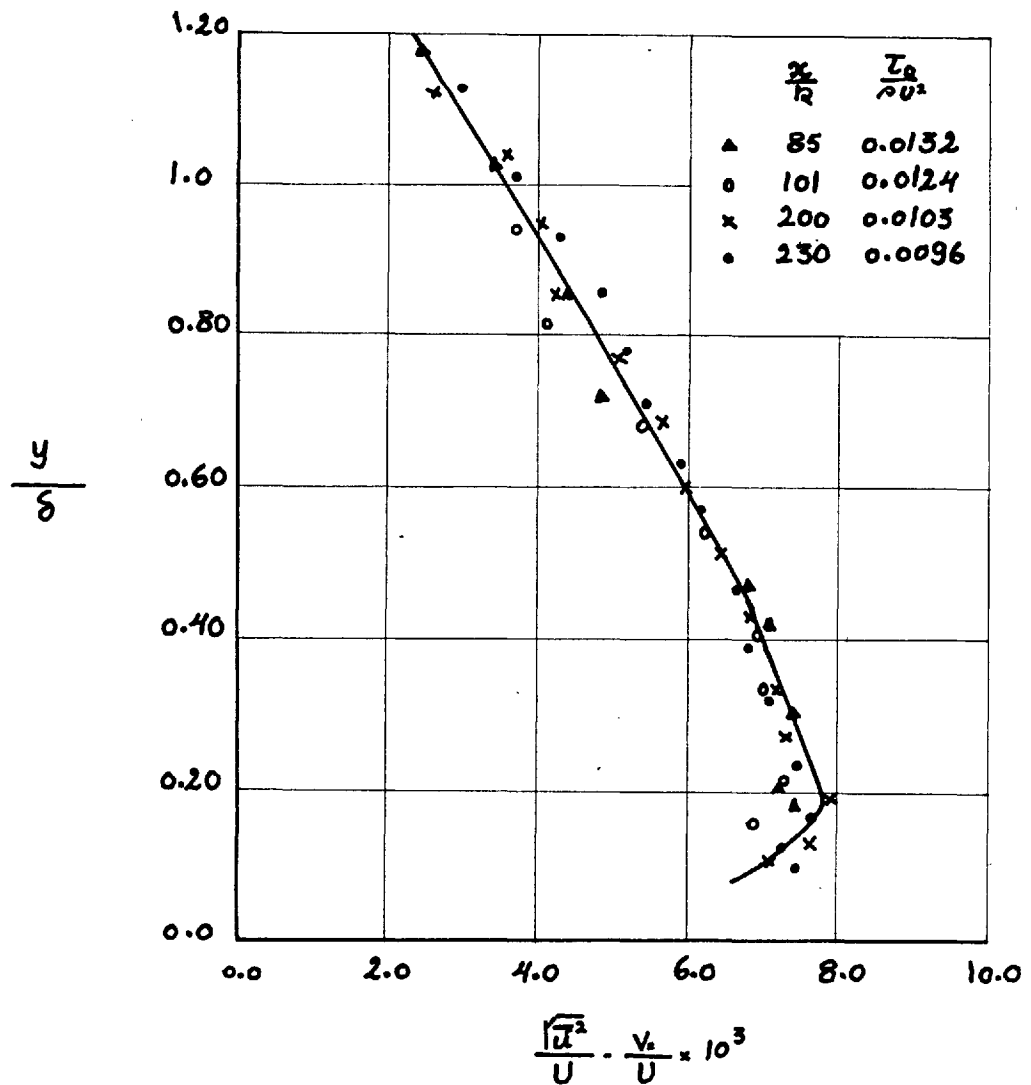


FIG. (4.12) A Dimensionless Representation for the $\overline{u^2}$ Profiles of Roughness 'c', $k=3.16$ mms.

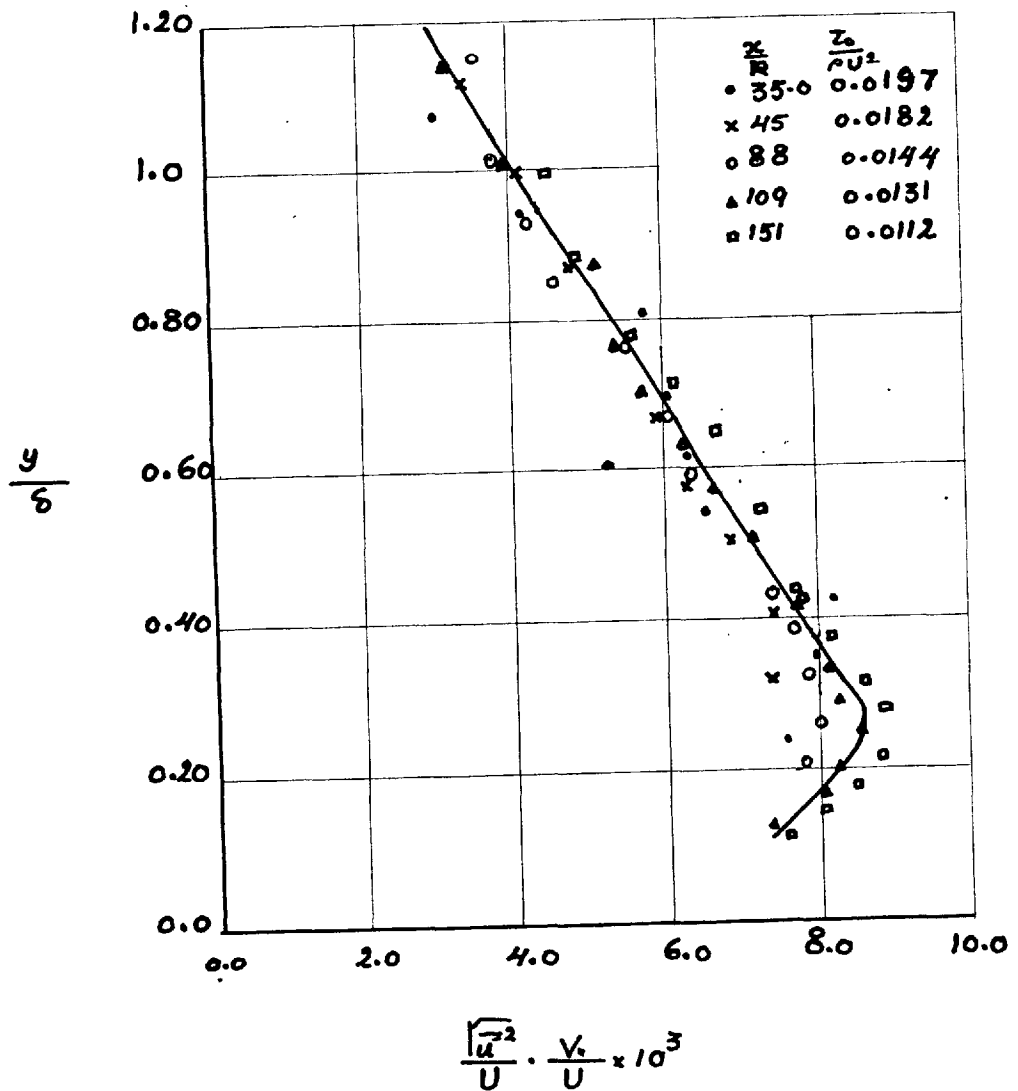
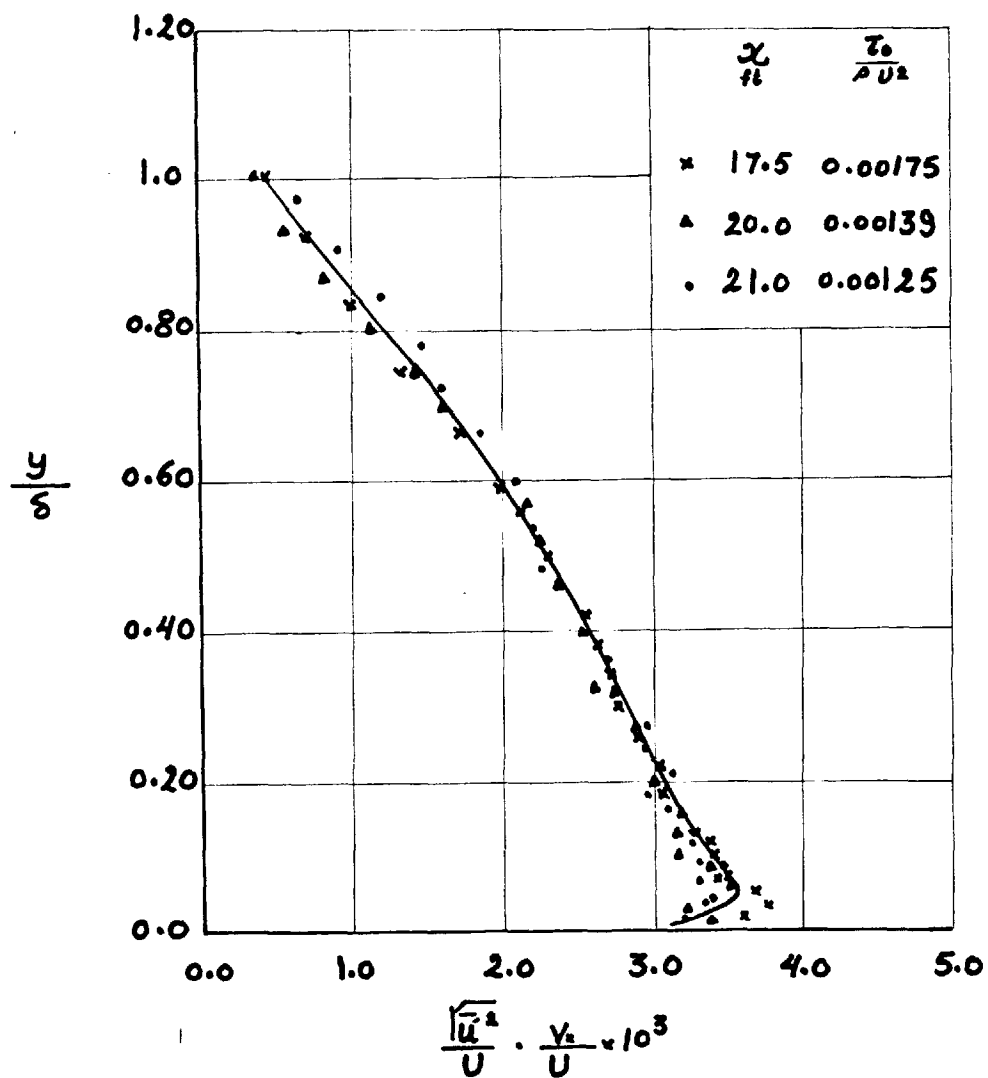


FIG. (4.13) A Dimensionless Representation for the $\sqrt{u'^2}$ Profiles of Roughness "D", $k = 4.75$ mms.



FIG(4.14) A Dimensionless Representation for the $\sqrt{u^2}$ Profiles in figure (4.11).

By definition we have:

$$\left. \begin{aligned} \tau &= \overline{\rho u' v'} = -\rho \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T u' v' dt \\ \sqrt{\overline{u'^2}} &= \left[\lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T u'^2 dt \right]^{\frac{1}{2}} \\ \text{and} \quad \sqrt{\overline{v'^2}} &= \left[\lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T v'^2 dt \right]^{\frac{1}{2}} \end{aligned} \right\} \quad (4.4)$$

where t is time.

As seen from these expressions, τ differs from $\rho \sqrt{\overline{u'^2}} \cdot \sqrt{\overline{v'^2}}$ by including two independent variables and to correlate between the two a coefficient K has been introduced (Goldstien, p 194) such that:

$$\rho \frac{\sqrt{\overline{u'^2}} \cdot \sqrt{\overline{v'^2}}}{\tau} = \frac{\sqrt{\overline{u'^2}} \cdot \sqrt{\overline{v'^2}}}{\overline{u' v'}} = \frac{1}{K} \quad (4.5)$$

According to (4.5), the values of $\frac{\sqrt{\overline{u'^2}}}{V_*}$ depend on the value of K . From the experiments on smooth walls by Klebanoff (1954) K was found to be substantially constant across the flow between $\frac{y}{\delta} = 0.05$ and $\frac{y}{\delta} = 0.8$, whereas on the other hand K was found to vary considerably from section to section depending on $\frac{\tau_0}{\rho U^2}$ as seen in figure (4.15) which contains the results of various investigators. Thus according to figure

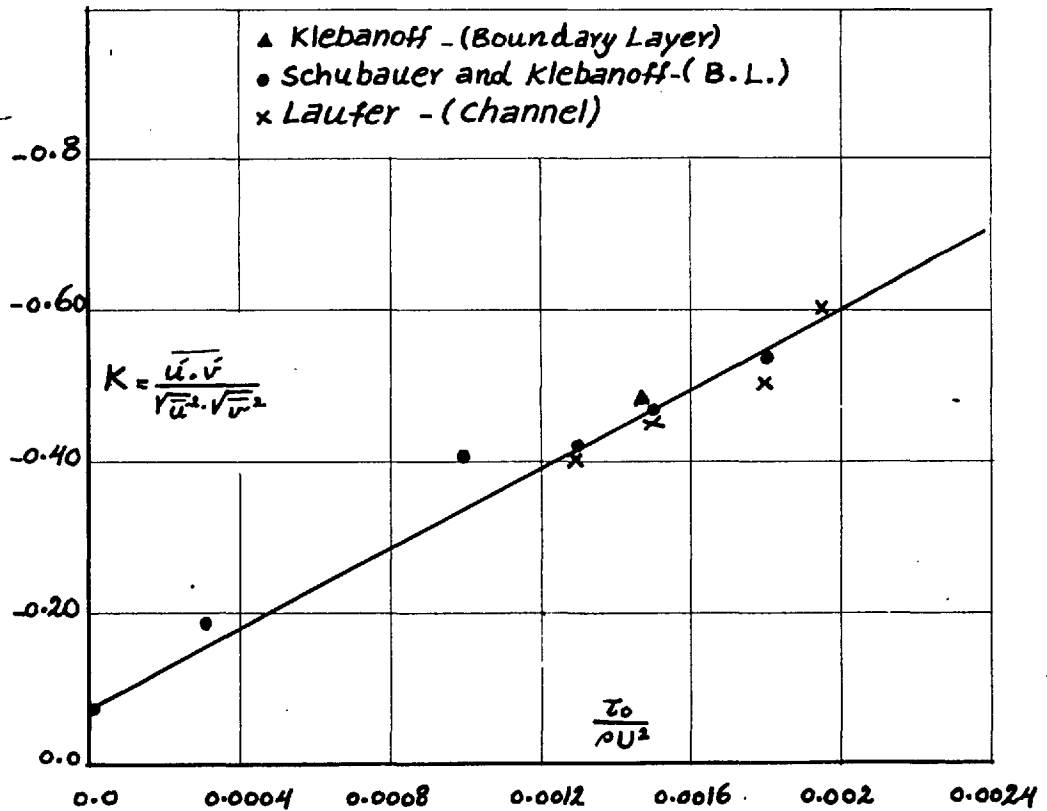


FIG. (4.15) Variation of Correlation Coefficient K with coefficient of Skin Friction

(4.15) we have:

$$K \propto \frac{v_*^2}{U^2}$$

which together with (4.5) seem to tie with the behaviour of $\frac{\sqrt{u'^2}}{v_*}$ in figures (4.12 to 4.14).

A physical explanation for the variation of K however seems, according to expressions (4.4), to be connected with the manner in which both u' and v' vary with time, i.e. it is expected to depend on the frequency, the amplitude and the phase between the two oscillations, an aspect of the problem which lies beyond the scope of this work.

CHAPTER V

CALCULATION OF TURBULENT BOUNDARY LAYERS UNDER A PRESSURE GRADIENT

5.1 Laminar Boundary Layers

The growth of the boundary layer in a pressure gradient depends on the geometrical conditions and the pressure field within the flow, and the purpose of the investigation described below is to predict the friction at the wall and to determine the shape of the velocity profile at different distances along the boundary and thus to predict beforehand wheather separation is going to occur.

For the calculation of laminar boundary layers, Pohlhausen (1921) introduced an original method in which he makes use of a differential equation of Karman's type. To characterize the shape of the velocity profile, Pohlhausen assumed a polynomial of the fourth degree for the velocity function in terms of the dimensionless distance from the wall $z = \frac{y}{\delta}$, where δ is the boundary layer thickness, thus:

$$\frac{u}{U} = Az + Bz^2 + Cz^3 + Dz^4 \quad (5.1)$$

In order to determine the constants in this equation, the boundary conditions are:

At $z = 0$, $v \frac{\partial^2 u}{\partial y^2} = -U \frac{dU}{dx}$ and $\frac{u}{U} = 0$
 and at $z = 1$, $\frac{u}{U} = 1$, $\frac{\partial u}{\partial y} = 0$ and $\frac{\partial^2 u}{\partial y^2} = 0$
 The first of the above conditions gives $B = -\frac{\delta^2}{2v} \frac{dU}{dx}$,
 whereas the other constants A, C and D are
 obtained in terms of B, thus:

$A = 2 - \frac{B}{3}$, $C = -2 - B$ and $D = 1 + \frac{B}{3}$, and
 equation (5.1) reduces to:

$$\frac{u}{U} = (2z - 2z^3 + z^4) + \frac{\Lambda}{6} z (1-z)^3 \quad (5.2)$$

where $\Lambda = -2B = \frac{\delta^2}{v} \frac{dU}{dx}$

At separation where $\left(\frac{du}{dy}\right)_{y=0}$ equals zero,
 equation (5.1) gives $A = 0$ or $\frac{\delta^2}{v} \frac{dU}{dx} = -12$.
 Also the relation between the displacement thick-
 ness δ_x , the momentum thickness θ and the bound-
 ary layer thickness δ can be obtained by
 substituting $\frac{u}{U}$ from equation (5.2) into the
 expressions for δ_x and θ (equations 1.15 and
 1.16), thus:

$$\frac{\delta_x}{\delta} = \frac{3}{10} - \frac{\Lambda}{120} \quad \text{and} \quad \frac{\theta}{\delta} = \frac{37}{315} - \frac{\Lambda}{945} - \frac{\Lambda^2}{9072}$$

and also $\tau_o = \mu \left(\frac{\partial u}{\partial y}\right)_{y=0} = \frac{\mu}{\delta} U \left(2 + \frac{\Lambda}{6}\right)$

In order to determine the parameter $\frac{\delta^2}{v} \frac{dU}{dx}$

at different locations along the flow, Pohlhausen substitutes $\frac{\delta^2}{\nu}$ in his differential equation which is then solved in steps. A detailed account of this method and further suggestions are contained in Schlichting, 1955, p 206.

As seen above, the presence of the second power of z in equation (5.1) is basic for including the pressure gradient in Pohlhausen's method, and also the presence of the first power of z is essential for determining separation point.

In this method, the parameter $\frac{\delta^2}{\nu} \frac{du}{dx}$ is in fact a parameter for representing the local value of the pressure gradient, and hence it can hardly represent the shape of the velocity profile whose development depends as well on the previous history farther upstream. The importance of the upstream history in determining the shape of the velocity profile is seen from the boundary layer equation which when integrated with respect to y between $y = 0$ and $y = h > \delta$, can lead to:

$$\tau_0 + h \frac{\partial p}{\partial x} - \frac{\rho}{2} \frac{\partial}{\partial x} \int_0^h u^2 dy + \rho U \frac{\partial}{\partial x} \int_0^h u dy$$

Integrating with respect to x , then:

$$\int_0^x \tau_0 \, dx + \int_0^x \delta x \frac{\partial P}{\partial x} \, dx = \rho U^2 \delta \int_0^1 \frac{u}{U} (1 - \frac{u}{U}) d(\frac{y}{\delta}) \quad (5.3)$$

from which it is seen that the shape of the velocity profile as represented by the integral on the R.H.S., depends upon the accumulated effects of both the pressure gradient and the friction at the wall, as well as upon the local values of δ and U . Consequently the measured value of $\frac{\delta^2}{\nu} \frac{dU}{dx}$ is expected to differ from those predicted, particularly near separation, depending on the different memories of the flow.

For high positive values of $\frac{\delta^2}{\nu} \frac{dU}{dx}$, the velocity distribution predicted from equation (5.2) becomes erroneous. For example, when $\frac{\delta^2}{\nu} \frac{dU}{dx} = +18$, equation (5.2) gives $\frac{u}{U} = 1$ both at $\frac{y}{\delta} = 0.50$ and $\frac{y}{\delta} = 1$. Again when $\frac{\delta^2}{\nu} \frac{dU}{dx} = +30$, it gives $\frac{u}{U} = 1$ both at $\frac{y}{\delta} = 0.25$ and $\frac{y}{\delta} = 1$.

However, in spite of its limitations and approximations, Pohlhausen's method has been shown to agree with many cases of laminar flow.

5.2 Turbulent Boundary Layers

The usual methods for calculating the growth

of turbulent boundary layers under a pressure gradient assume that turbulent velocity profiles can be described by a single parameter. Different methods use different parameters for describing the velocity profile, as for example, the shape factors $H = \frac{\delta^*}{\theta}$ used by Von Dochoff and Tetervin (1943) or $\eta = \left[1 - \left(\frac{u_0}{U} \right)^2 \right]$ used by Gruschwitz (1931), and unlike the laminar case, they emerge from experimental observations only.

Instead of the boundary layer thickness the momentum thickness θ is often preferred as the length scale for plotting the velocity profile (Gruschwitz, 1931; Wieghardt and Tillmann, 1951). The displacement thickness δ^* seems, however, easier than θ and thus offers a more convenient length scale for the velocity profile. To test this, the data by the Iowa Institute for Hydraulic Research (Landweber and Siao, 1958) on smooth walls under a constant pressure are shown in figure (5.1) plotted in the form $\frac{u}{U}$ against $\frac{y}{\delta^*}$. As seen from this figure, the points fall closely on a single curve, and bearing in mind the wide range involved, viz. $5.40 \times 10^5 < \frac{Ux}{\nu} < 5.40 \times 10^6$, it seems that the velocity profile

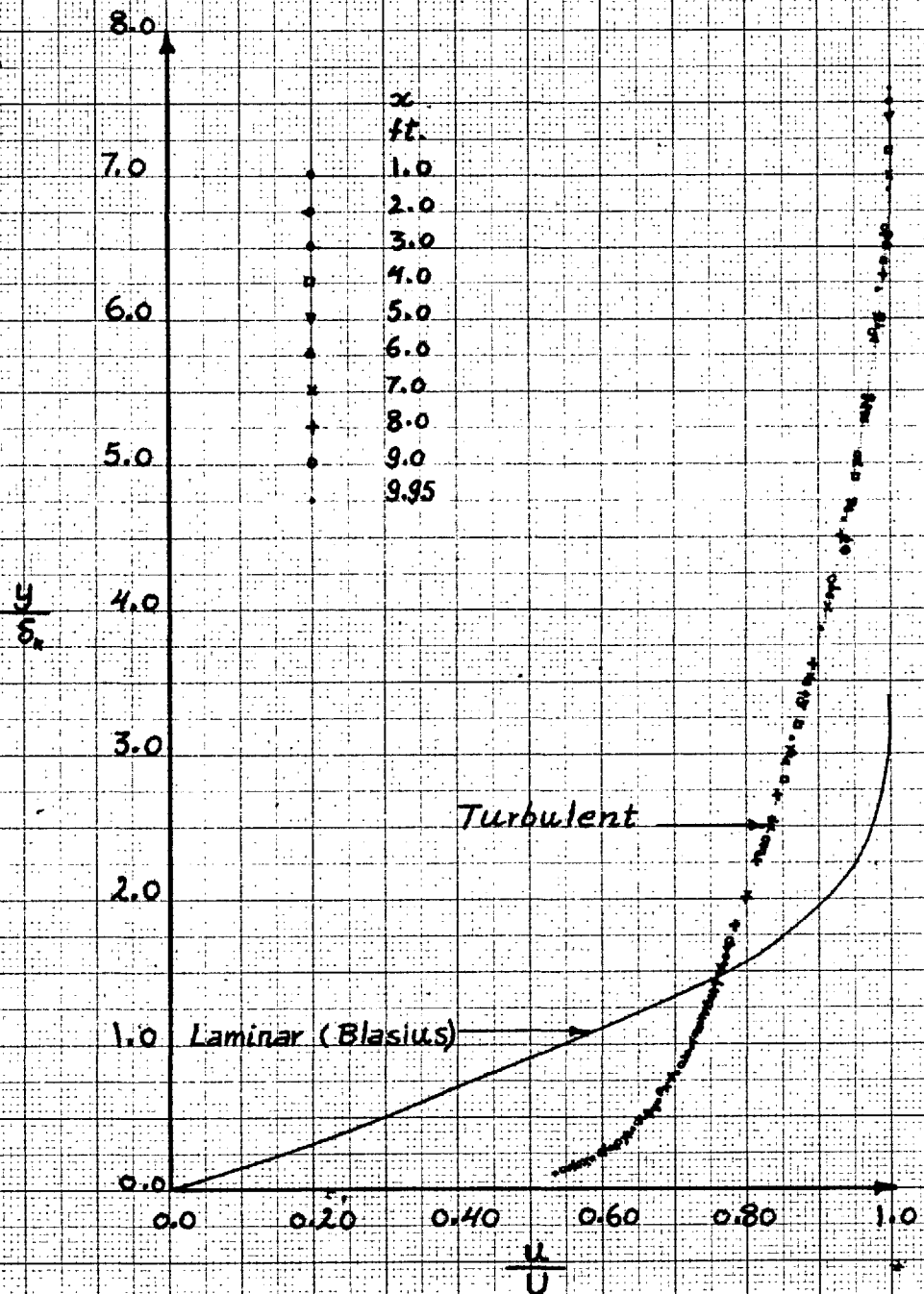


FIG. (5.1) Velocity Profiles for Constant Pressure Boundary Layers. Smooth Wall. (data by Iowa Institute) 1958

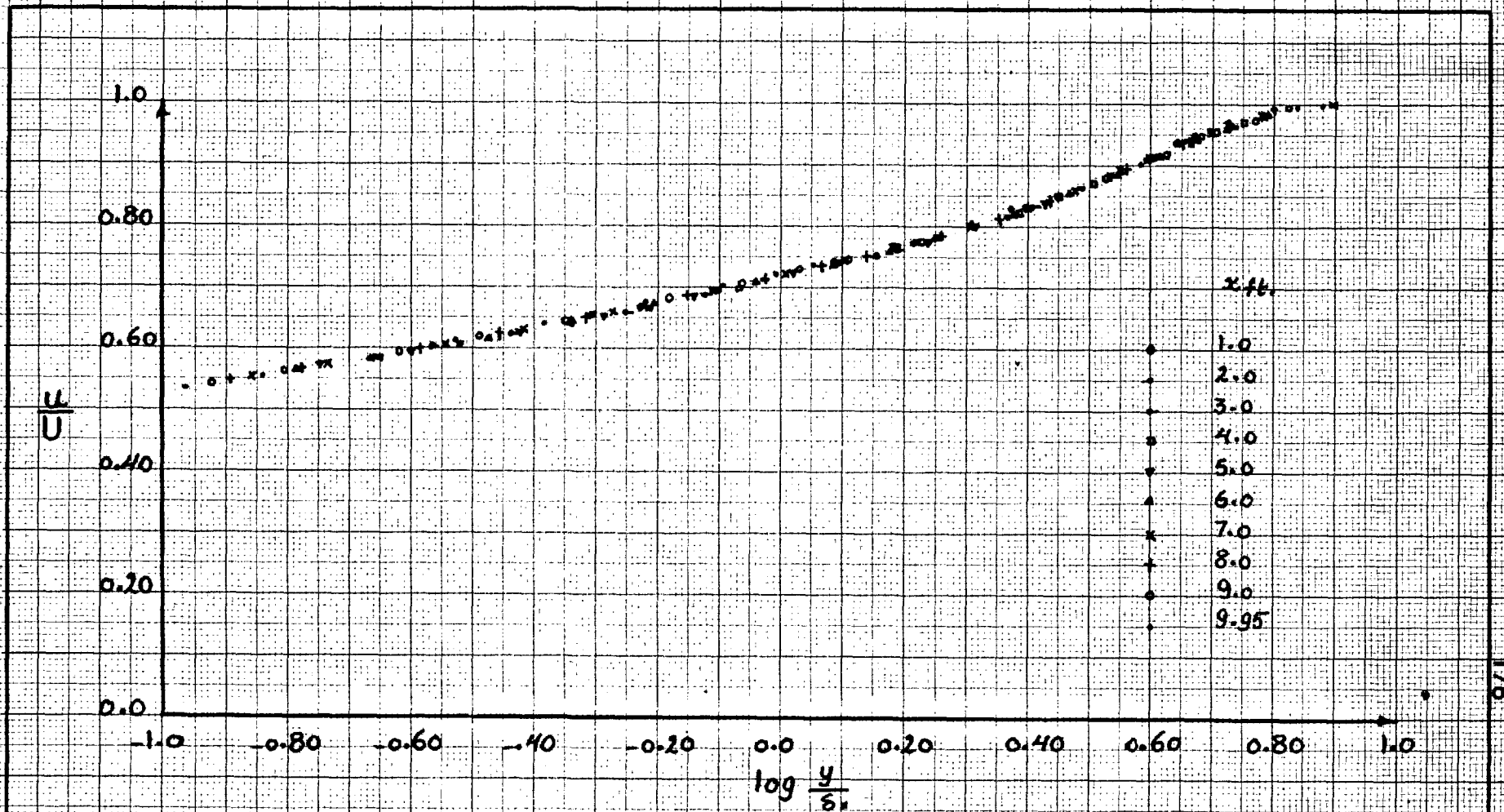


FIG. (5.2) Semi-logarithmic Representation for the Velocity Profiles of Figure (5.1).

for the constant pressure case can accurately be represented by a single function involving $\frac{u}{U}$ and $\frac{y}{\delta^*}$. The same data are also plotted semi-logarithmically in figure (5.2) which clearly shows that the velocity profile deviates from its logarithmic form beyond $\frac{y}{\delta} = 0.15$, a value again in agreement with those previously discussed in sections (3.2 and 3.3).

Similarly for the case where the boundary layer is under a pressure drop, a group of 25 different velocity profiles from data by Schubauer and Klebanoff (1950) were plotted in the form $\frac{u}{U}$ against $\frac{y}{\delta^*}$, and although $\frac{\delta^2}{\nu} \frac{dU}{dx}$ reached at times the value +240, i.e. $\frac{\delta^{*2}}{\nu} \frac{dU}{dx} = 5.70$, the resulting profiles were found to deviate only slightly from that for constant pressure as shown by the dotted curve in figure (5.3)

For pressure rise on the other hand, the velocity profile in the form $\frac{u}{U}$ against $\frac{y}{\delta^*}$ undergoes a systematic change with $\frac{\delta^2}{\nu} \frac{dU}{dx}$ as shown in figure (5.3). The two profiles included in this figure for the pressure rise case were plotted from the data by Schubauer and Klebanoff

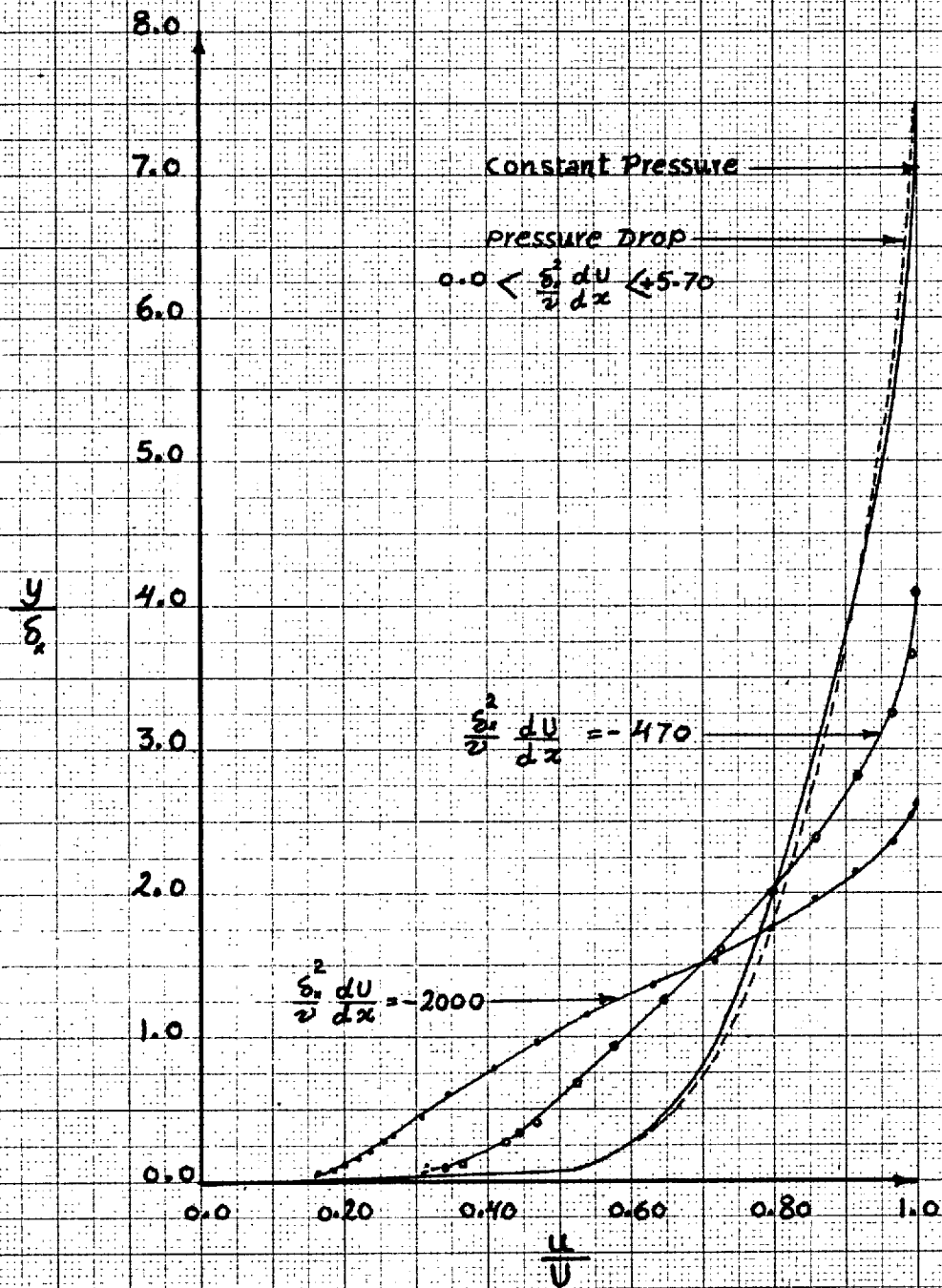


FIG. (5.3) Velocity Profiles for Various Pressure Distributions

(1950) for a smooth wall. However, since the curves of figure (5.3) do not intersect at a single point and since they have their maximum value of $\frac{u}{U}$ at different values of $\frac{y}{\delta_H}$, it is difficult to represent all profiles by a single function

5.2.1 Buri's Method

The first method for calculating turbulent boundary layers with pressure gradients was introduced by Buri (1931) who used **his** own experiments on accelerated flows as well as those of Nikuradise (1929) on both accelerated and retarded flows.

To describe the shape of the velocity profile, Buri used the following analysis:

For a steady two-dimensional incompressible flow, Prandtl's boundary layer equation writes as:

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = -\frac{1}{\rho} \frac{\partial p}{\partial x} + \frac{1}{\rho} \frac{\partial \tau}{\partial y}$$

At the wall, both u and v must vanish and the equation reduces to:

$$\frac{dp}{dx} = \left[\frac{\partial \tau}{\partial y} \right]_{y=0} = -\rho U \frac{dU}{dx}$$

Since the shape of the velocity profile depends

on the pressure gradient, Buri therefore uses $\left[\frac{\partial \tau}{\partial y}\right]_{y=0}$. though in a non-dimensional form, to describe the shape of the velocity profile, hence:

$$\Gamma_a = \left[\frac{\frac{d(\frac{\tau}{\tau_0})}{d(\frac{y}{\theta})}} \right]_{y=0} = - \rho U \frac{dU}{dx} \cdot \frac{\theta}{\tau_0} \quad (5.4)$$

The shape factor Γ_a in the above equation involves the wall stress τ_0 which can not be directly measured, and to eliminate it from the equation Buri uses Blasius law of resistance in the form:

$$\frac{\tau_0}{\rho U^2} = \epsilon \left(\frac{U\theta}{\nu} \right)^{\frac{1}{4}} \quad (5.5)$$

In the case where U is constant, ϵ has the constant value 0.01256, whereas when U is varying Buri assumes that ϵ would vary too, depending on the pressure gradient, i.e. on Γ_a , and is expected to increase in the case of a pressure drop, and to diminish in the case of a pressure rise. Substituting from (5.5) into (5.4), then:

$$\Gamma_a = - \frac{1}{\epsilon} \frac{\theta}{U} \frac{dU}{dx} \left(\frac{U\theta}{\nu} \right)^{\frac{1}{4}}$$

Introducing the new shape factor $\Gamma = -\Gamma_a$, then:

$$\Gamma = \frac{\theta}{U} \frac{dU}{dx} \left(\frac{U\theta}{\nu} \right)^{\frac{1}{4}}$$

It is interesting to note that this parameter resembles Pohlhausen's parameter $\frac{\delta^2}{\nu} \frac{dU}{dx}$ which can be written as $\frac{\delta}{U} \frac{dU}{dx} \cdot \left(\frac{U\delta}{\nu}\right)$, the difference between the two parameters being the power $\frac{1}{4}$ only, since θ is related to δ in Pohlhausen's method.

Now combining the momentum equation

$$\frac{\tau_o}{\rho U^2} = \frac{d\theta}{dx} + \frac{\theta}{U} (2+H) \frac{dU}{dx}$$

together with Blasius equation (5.5) the following equation is obtained:

$$\frac{d}{dx} \left[\theta \left(\frac{U\theta}{\nu} \right)^{\frac{1}{4}} \right] = \frac{5}{4} \left[\epsilon - \frac{\theta}{U} \frac{dU}{dx} \left(\frac{U\theta}{\nu} \right)^{\frac{1}{4}} \left(\frac{9}{5} + H \right) \right] \quad (5.6)$$

This equation can be readily checked as follows:

Differentiating the L.H.S. we get:

$$\frac{d}{dx} \left[\theta \left(\frac{U\theta}{\nu} \right)^{\frac{1}{4}} \right] = \frac{5}{4} \frac{d\theta}{dx} \left(\frac{U\theta}{\nu} \right)^{\frac{1}{4}} + \frac{1}{4} \frac{\theta}{U} \frac{dU}{dx} \left(\frac{U\theta}{\nu} \right)^{\frac{1}{4}} \quad (5.7)$$

Substituting into equation (5.6), then

$$\frac{d\theta}{dx} + \frac{\theta}{U} (2+H) \frac{dU}{dx} = \epsilon \left(\frac{U\theta}{\nu} \right)^{-\frac{1}{4}}$$

which is the momentum equation with the term

$\epsilon \left(\frac{U\theta}{\nu} \right)^{-\frac{1}{4}}$ substituted for $\frac{\tau_o}{\rho U^2}$ from Blasius equation.

The terms on the R.H.S. of equation (5.6) involve Γ , H and ϵ all functions of the shape factor Γ , therefore Buri writes equation (5.6) conveniently as:

$$\frac{d}{dx} \left[\theta \left(\frac{U\theta}{\nu} \right)^{\frac{1}{4}} \right] = f(\Gamma) \quad (5.8)$$

Plotting the experimental values of $\frac{d}{dx} \left[\theta \left(\frac{U\theta}{\nu} \right)^{\frac{1}{4}} \right]$ against $\sqrt{x}$, Buri obtained a linear relation for equation (5.8) in the form:

$$\frac{d}{dx} \left[\theta \left(\frac{U\theta}{\nu} \right)^{\frac{1}{4}} \right] = a - b \sqrt{x}$$

where $a = 0.01475$ and $b = 3.945$ for accelerated flow, and $a = 0.0175$ and $b = 4.15$ for retarded flow. Separation in this method will occur at a value of $\sqrt{x} = 0.06$. Buri found also that Blasius resistance formula describes the friction at the wall in varied flow provided the local values of U and θ are used together with a constant mean value for $\epsilon = 0.0116$. This constant value for ϵ may be a good approximation for Buri's range of Reynolds number ($2 \times 10^2 < Re < 2 \times 10^3$), though strictly speaking ϵ is expected to vary with the pressure gradient and the Reynolds number.

5.2.2. Gruschwitz Method

At about the same time as Buri's Gruschwitz (1931) introduced another method for the calculation of turbulent boundary layers. To describe the shape of the velocity profile, Gruschwitz uses the momentum thickness θ as the length scale, and he chooses the value of $\frac{u}{U}$ at the particular

distance from the wall $\frac{y}{\theta} = 1$, to form his "shape" parameter η , thus:

$$\eta = 1 - \frac{(u\theta)^2}{U^2}$$

where u_θ is the velocity at $y = \theta$. Since η and H each independently can describe the shape of the velocity profile, it is expected that the two would be closely related. Indeed, Pretsch (1942) by using a power theorem for the velocity profile of the form $\frac{u}{U} = \left(\frac{y}{\delta}\right)^{\frac{1}{n}}$, arrived at the following relation between η and H :

$$\eta = 1 - \left[\frac{H-1}{H(H+1)} \right]^{(H-1)} \quad (5.9)$$

which is in good agreement with Gruschwitz experiments (figure 5.4). According to Gruschwitz,

η is smaller than 0.45 in the case of a pressure drop and is greater than 0.45 in the case of a pressure rise where it reaches 0.80 at separation.

To trace the variation of η in the downstream direction, Gruschwitz uses a dimensional argument and deduces a relation between η and the pressure gradient similar to Buri's equation (5.8), thus:

$$\frac{\theta}{\frac{1}{2}\rho U^2} \frac{d}{dx} \left(\frac{1}{2}\rho U^2 \cdot \eta \right) = f(\eta) \quad (5.10)$$

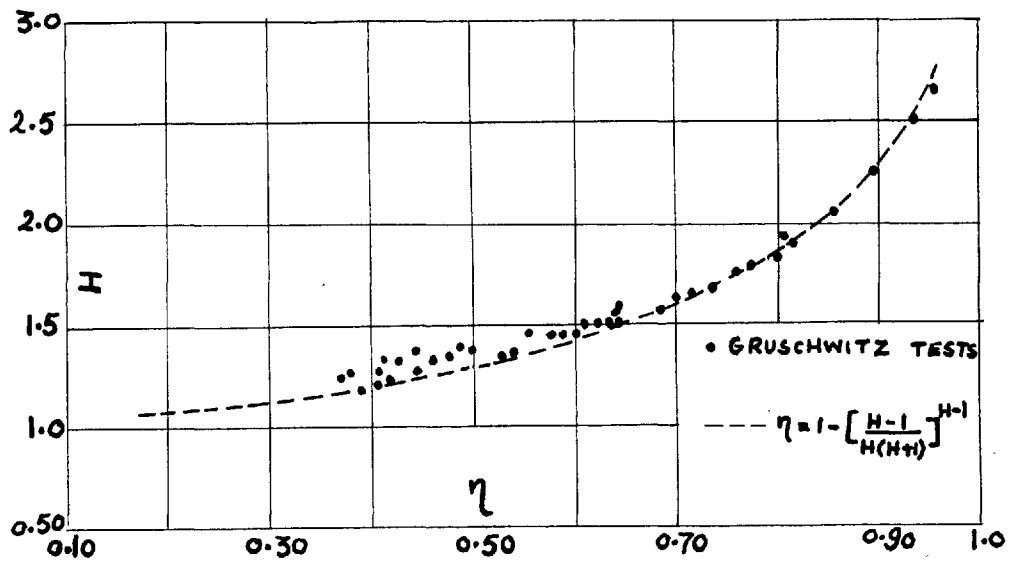


FIG. (5.4) Relationship between η and H .

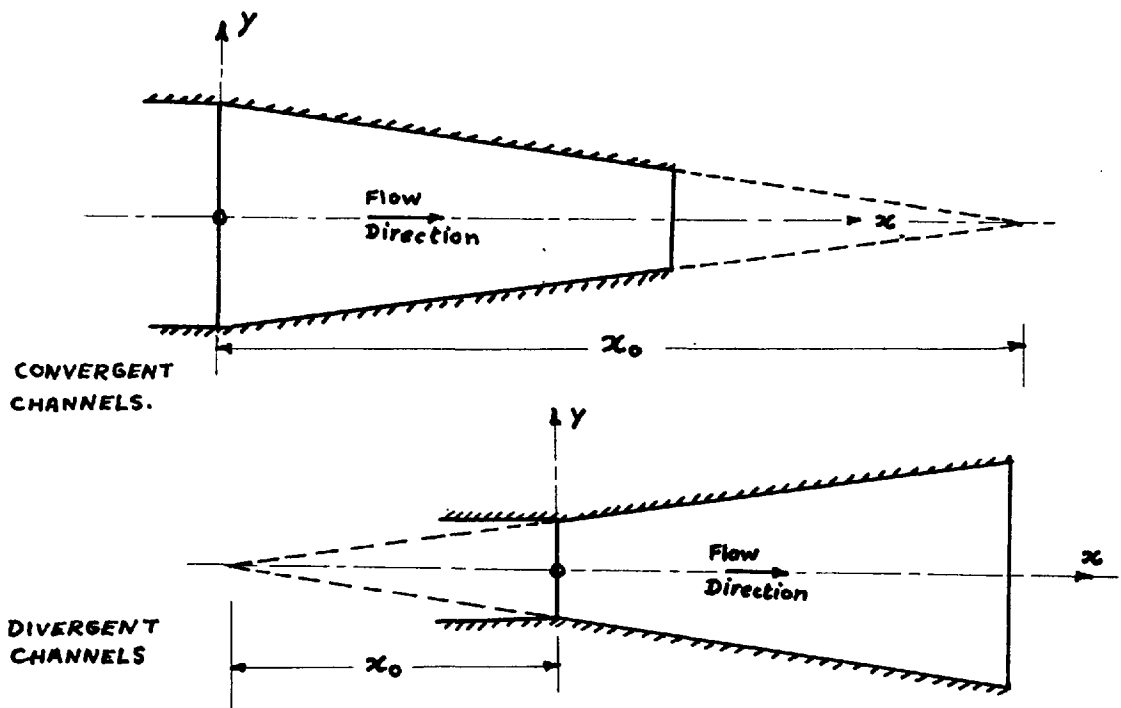


FIG.(5.5) Kehl's Notations for Convergent and Divergent Channels.

Plotting the experimental results, Gruschwitz obtained, as in Buri's case, a linear relation for the function in equation (5.10), namely:

$$\frac{\theta}{\frac{1}{2}\rho U^2} \frac{d}{dx} \left(\frac{1}{2}\rho U^2 \cdot \eta \right) = a\eta - b \quad (5.11)$$

where $a = 0.00894$ and $b = 0.00461$

For the friction at the wall, Gruschwitz uses Blasius law according to which U is constant and the $\frac{1}{7}$ power law as a basis, thus:

$$\frac{\tau_o}{\rho U^2} = 0.01256 \left(\frac{U\theta}{\nu} \right)^{\frac{1}{4}} \quad (5.12)$$

though strictly speaking this law is valid only for a limited range of Reynolds number and in the particular case where there is no pressure gradient.

Once the necessary relations are established (equations 5.9, 5.11 and 5.12) the calculation procedure in this method will be as follows:

1 - Using the momentum equation $\left[\frac{\tau_o}{\rho U^2} = \frac{d\theta}{dx} + \frac{\theta}{U} (H+2) \frac{dU}{dx} \right]$ as a differential equation in θ , constant mean values for $\frac{\tau_o}{\rho U^2}$ and H are assumed as a first approximation, and knowing an initial value for θ the equation is differentiated to obtain the successive values of θ . It is clear that the errors introduced in the momentum equation due to

the possible error in the assumed value of H would be small due to the fact that H always occur in the group $(H + 2)$. On the other hand, the possible errors in the assumed value of $\frac{\tau_0}{\rho U^2}$ will have a correspondingly large effect on the θ values obtained from this step.

2 - The approximate values of θ from step (1) are then used in equation (5.11) to obtain the corresponding values of η provided an initial value for η is known. Equation (5.9) is then used to obtain the corresponding values of H .

3 - Gruschwitz now returns to the momentum equation with the quantity H improved and the quantity $\frac{\tau_0}{\rho U^2}$ determined more precisely from Blasius formula. He works out a second approximation for θ , and hence the different parameters can be improved furthermore.

5.2.3. Kehl's Method

In an attempt to improve Gruschwitz method, Kehl (1943) conducted a series of experiments on converging and diverging flows which covered a range of Reynolds number seven times larger than that of Gruschwitz, i.e. up to $\frac{U\theta}{\nu} = 3.5 \times 10^4$.

To take into account the effect of the inclination of the channel walls, Kehl corrected the momentum equation by adding the term $\frac{\theta}{x+x_0}$ where x_0 is the distance from the entrance section of the point where the two walls meet when extended, x_0 being positive for diverging channels and negative for convergent channels. The new form of the momentum equation will thus write:

$$\frac{d\theta}{dx} + \frac{\theta}{U} (H+2) \frac{dU}{dx} + \frac{\theta}{x+x_0} = \frac{\tau_0}{\rho U^2}$$

and the additional term will tend to zero as x_0 increases.

To investigate the variation along the flow of Gruschwitz shape parameter η for the now wider range of Reynolds number, Kehl plotted his experimental values of η against $\frac{\theta}{\rho U^2} \frac{d}{dx} (\rho U^2 \eta)$ in a similar way to Gruschwitz. Instead of a single line as in Gruschwitz's case, Kehl obtained a set of parallel lines, the shift of each line being dependent on the Reynolds number whose effect was not clear in Gruschwitz's limited range. Consequently the constant b in Gruschwitz's equation:

$$\frac{\theta}{\rho U^2} \frac{d}{dx} (\rho U^2 \eta) = a \eta - b$$

was replaced by a function of R_θ in the form:

$$b = \frac{0.0146}{\log R_\theta} - \frac{0.85}{R_\theta - 300}$$

where the second term becomes insignificant for high Reynolds numbers and b can thus be substituted by the variable $\frac{0.0146}{\log R_\theta}$ alone.

Apart from these modifications, the calculation procedure in this method is exactly the same as in Gruschwitz's method. The various methods of calculation are summerized in table (5.1).

As seen from the table, the methods are similar in the sense that they all use the momentum equation and an auxiliary equation for the shape parameter, together with a suitable law for the friction along the wall. Since the experimental data from which the auxiliary equation is obtained in each method cover only a limited range of flow conditions, the results likewise are limited to a particular range. Also in these methods, no account is taken of how the velocity gradient $\frac{dU}{dx}$ is related to the

TABLE 5.1

Methods of Calculation for Turbulent Boundary Layers

Investigator	Shape Factor	Criterion for separation	Auxiliary Equation	Formula for Skin Friction
Buri (1931)	$\Gamma = \frac{\theta}{U} \frac{dU}{dx} \left(\frac{U\theta}{\nu} \right)^{\frac{1}{4}}$	$\Gamma = -0.06$	$\frac{d}{dx} \left[\theta \cdot \left(\frac{U\theta}{\nu} \right)^{\frac{1}{4}} \right] = a - b\Gamma$	Blasius $\frac{\tau_0}{\rho U^2} = \left(\frac{U\theta}{\nu} \right)^{-\frac{1}{4}}$
Gruschwitz (1931)	$\eta = 1 - \left(\frac{U\theta}{U} \right)^2$	$\eta = 0.80$	$-\frac{\theta}{\rho U^2} \frac{d}{dx} (\rho U^2 \eta) = 0.00894 \eta - 0.00461$	Blasius $\frac{\tau_0}{\rho U^2} = 0.01256 \left(\frac{U\theta}{\nu} \right)^{-\frac{1}{4}}$
Kehl (1943)	$\eta = 1 - \left(\frac{U\theta}{U} \right)^2$		$-\frac{\theta}{\rho U^2} \frac{d}{dx} (\rho U^2 \eta) = 0.00894 \eta - \frac{0.0164}{\log R\theta} + \frac{0.85}{R\theta - 300}$	Blasius $\frac{\tau_0}{\rho U^2} = 0.01256 \left(\frac{U\theta}{\nu} \right)^{-\frac{1}{4}}$
Von Doenhoff & Tetervin (1943)	$H = \frac{\delta^*}{\theta}$	$1.8 < H < 2.6$	$\theta \frac{dH}{dx} = \left[-2.035(H-1.268) - \frac{2\rho U^2 \theta}{\tau_0} \frac{dU}{dx} \right] \times e^{4.68(H-2.975)}$	Squire-Young $\frac{\tau_0}{\rho U^2} = \left[\frac{1}{5.89 \log 4.075 \frac{U\theta}{\nu}} \right]^2$
Garner (1944)	$H = \frac{\delta^*}{\theta}$	$H = 2.6$	$\theta \frac{dH}{dx} = \left[\frac{0.0135(H-1.4)}{R\theta^{\frac{1}{2}}} - \frac{\theta}{U} \frac{dU}{dx} \right] \times e^{5(H-1.4)}$	Falkner $\frac{\tau_0}{\rho U^2} = 0.0131 \left(\frac{Ux}{\nu} \right)^{-\frac{1}{7}}$

skin friction. Indeed, Page and Raymer (1946) used both Von Doenhoff-Tetervin's and Garner's methods, both based on the same experimental data, and found for an identical case of flow with $H = \text{constant}$, two quite different results. Further, it has yet to be proved that the velocity profile of a turbulent boundary layer can be specified implicitly by a single parameter only, since as seen from equation (5.3), the shape of the profile is affected by the previous history farther upstream.

CHAPTER VI

THE MOVEMENT OF GRAINS ON STREAM BEDS

6.1 Approaches to the Problem

The movement of grains, on a stream bed has been explained in several ways; by the impact or momentum of water against the grains, by the drag exerted on the bed by the flow, and by the hydraulic lift resulting from the difference in pressure at its bottom and top.

6.1.1. "Impact" or Momentum Approach

The simplest and perhaps the oldest approach to the problem dates back to Brahms who in 1753 explained the movement of grains as a result of the fluid pressures on the grain, which are determined by the velocity of the stream. Brahms and indeed many others using this approach arrived at the sixth-power law for the velocities necessary to initiate the grains movement, and the derivation can be summarized as follows:

Movement of an individual grain on the bed starts when the force component tending to push the grain up a slope angle ϕ , becomes equal to

the oppositely directed component of its weight (figure 6.1).

The dynamic force on the grain due to momentum, is $\alpha (\rho_f \cdot u^2) (\pi k^2 / 4)$, where ρ_f is the fluid density, u is the velocity, k is the grain diameter, and α is a coefficient depending partly on the proportion of the force that actually is exerted on the grain and partly on the proportion of the grain exposed to the flow.

The component of this force acting up the slope angle ϕ is thus:

$$\alpha (\rho_f u^2) \cdot (\pi k^2 / 4) \cdot \cos \phi$$

which must balance the component of the weight before the movement starts, hence:

$$\alpha (\rho_f u^2) \cdot (\pi k^2 / 4) \cos \phi = \rho' g \cdot \frac{\pi k^3}{6} \sin \phi$$

$$\text{or } \frac{2}{3\alpha} \rho' g k \tan \phi = \rho_f u^2 \quad (6.1)$$

where ρ' is the effective density of the grains material in water.

Equation (6.1) becomes the sixth-power law, when both sides of the equation are cubed, thus

$$k^3 \propto u^6$$

$$\text{or } u \propto k^{1/6}$$

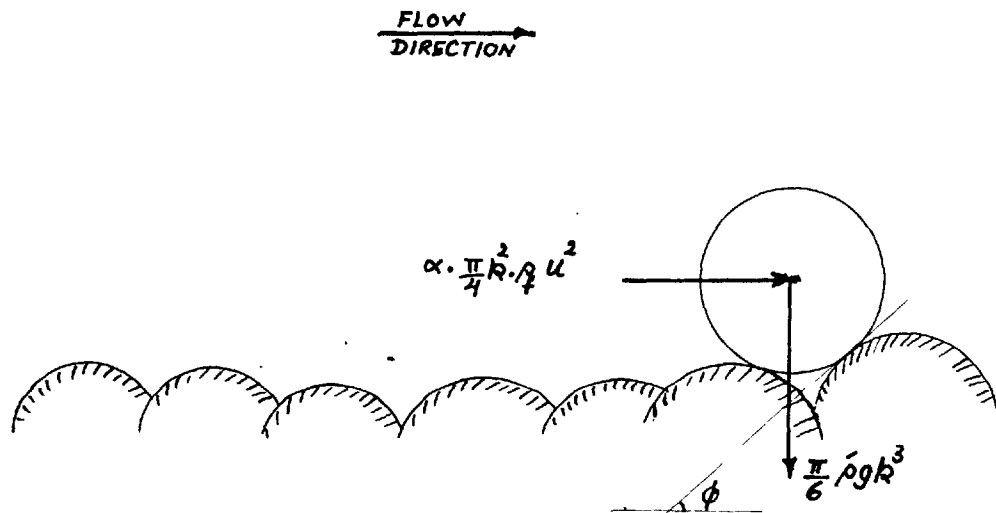
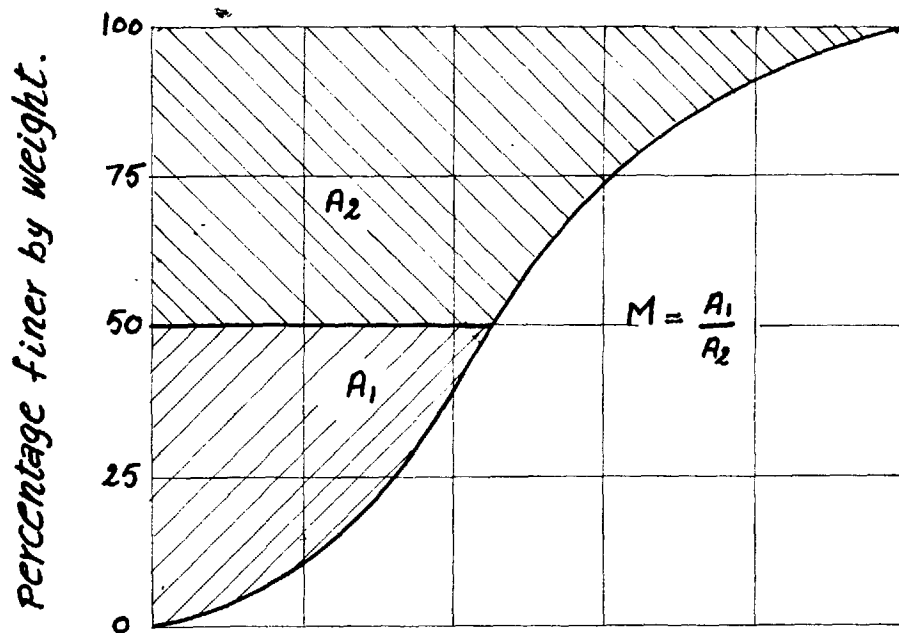


Fig. (6.1) Equilibrium of an Individual Grain using the "Momentum" Approach.



Grain diameter k in millimeters.

Fig. (6.2) Definition of Uniformity Modulus 'M'
(After H. Kramer, 1935)

where W is the submerged weight of the grain, and the law has been formulated independently by Brahms, 1753; Hopkins, 1844; Airy, 1855; Gilbert, 1914; and many others.

This law however, is based upon a velocity u which is not precisely defined. In fact, u varies greatly with height, and it is difficult to relate it to the measurable mean velocity of the stream.

6.1.2 "Drag or Shearing Force" Approach

The "critical drag" approach is of more practical significance to engineers, since it relates the sizes of the moving grains to certain readily measurable quantities, namely the depth of water; and slope of stream.

Based on the theories of Du Buat (1786), and du Boys (1879) many investigators have approached the problem of grains movement, using the tractive force exerted by the flow on a unit area of the bed, as a criterion for the movement. This tractive force τ can be satisfactorily approximated in the case of open channels by:

$$\tau = \rho_f g i d \quad (6.2)$$

where d is the water depth, i is the slope of total energy gradient, and g is the acceleration due to gravity. There seems no doubt that when the grain is about to move, the force τ_c exerted by the flow is equal and opposite to the resisting force offered by the grain. The former is easily obtained from (6.2), whereas on the other hand, the latter is more complex and depends upon the density, size, and shape of the grain as well as upon the arrangement of the grains on the bed.

Following this approach, many formulae for τ_c have emerged, though most are empirical being based on experimental findings. Most can be represented by a functional relationship of the form:

$$\tau_c = \rho' \cdot g \cdot k \cdot f \quad \begin{array}{l} \text{(shape and arrangement of} \\ \text{grains)} \end{array} \quad (6.3)$$

in which f is represented by a different parameter in each case. For example, Kramer (1935) used the uniformity modulus M defined as the ratio between the areas below and above the (50% finer by weight) line in the grading curve for the sand mixture (figure 6.2), and his formula

writes as follows:

$$\tau_c \text{ in gms/cm}^2 = \frac{100}{6} \cdot (\rho' \cdot g \cdot k_{\text{mms}}) \cdot \frac{1}{M} \quad (6.4)$$

which was modified by the U.S. Waterways experiment station (1935), later experiments having shown that the square root of the right hand side with a suitable new constant better follow the facts.

Instead of M , Chang (1939) used the parameter O , defined as the ratio between the longest and shortest side of a sand grain, and his formula is written as:

$$\tau_c = \text{constant} \cdot \left(\frac{\rho' k}{\rho_f} \right) \cdot O^{1/3} \text{ for } k > 0.50 \text{ mms.}$$

$$\text{and } \tau_c = \text{constant} \cdot \left(\frac{\rho' k}{\rho_f} \right)^{1/2} \cdot O^{1/6} \text{ for } k < 0.50 \text{ mms.}$$

Chang's experiments include a wide range of grain sizes varying from 0.134 mms to 8.09 mms and will be discussed later with reference to the present experiments.

In a more refined treatment, Shields (1936), using the modern concept of flow near a boundary, considered the shear stress term in non-dimensional form when divided by $\rho' g k$, i.e. in the form $\frac{\tau_c}{\rho' g k}$ and related this to $\frac{V_* k}{\nu}$ in a similar manner to the variation of the coefficient of friction with $\frac{V_* k}{\nu}$

for rough pipes. Thus for high values of $\frac{V_* k}{\nu}$ the term $\frac{\tau_c}{\rho' g k}$ was shown to be constant and equal to 0.060. However White's subsequent experiments (1940) and also the present tests show that this term varies considerably between 0.04 and 0.151 in the range of $\frac{V_* k}{\nu}$ between 27.0 and 1280. Also the experiments by the U.S. Waterways experiment station (1935) show that $\frac{\tau_c}{\rho' g k}$ depends to a great extent on the shape of the grains, which therefore requires an additional term in the formula.

Instead of $\frac{\tau_c}{\rho' g k}$, White and Quraishy (1942) introduced the term $\frac{V_*}{V_s}$, where V_s is the settling velocity of the grain in the fluid thus considering the dynamic and not the static equilibrium of the grains as in Shield's case. On the assumption that the moving layer of the grains on the top of the bed is separated on the whole from the stationary bed by a thin fluid film, White and Quraishy through a mathematical analysis, arrived at the following functional relationship for the critical tractive force:

$$\frac{V_*}{V_s} = F \left(\frac{V_* k}{\nu} \right) \quad \text{or} \quad (6.5)$$

$$\frac{V_*}{V_s} = f \left(\frac{V_* k}{\nu} \right)$$

The use of the settling velocity V_s as an independent variable in equation (6.5) simplifies the analysis to a great extent since it combines the effect of gravity and the grain size, shape, surface character and density. For the same size, grains of light material for which V_s is relatively small will be set in motion at relatively less shear stress than grains of a heavy material. Also grains of complex surface texture for which V_s is relatively greater, will be set in motion at much higher shear stresses than grains approaching spherical shape as required by the particular combination of V_* and V_s in equations (6.5).

On analysing the experimental data on τ_c from various sources, Quraishy came to the conclusion that; for values of $\frac{V_* k}{\nu}$ between 2.1 and 2.4, the inception of grain movement is largely controlled by viscosity and τ_c can be represented by an equation in the form:

$$\tau_c = \rho_f v_s^2 f \left(\log \frac{V_* k}{\nu} \right)$$

On the other hand for high values of $\frac{V_* k}{\nu}$ greater than 100, the fluid's viscosity plays

no part and the inertia of the fluid is the controlling factor, and consequently $\frac{V_*}{V_s}$ is independent of $\frac{V_* k}{2}$ and assumes a constant value. Between these two extremes there is a transitional range where both viscosity and inertia play equally important roles.

In 1940, a new approach to the problem of grains movement on stream beds was introduced by C.M. White who used the following analysis:

The flowing fluid on a loose granular bed tends to move the grains forward, and they in turn will exert a drag on the fluid depending upon the speed and nature of the stream. The shearing stresses at the bed will be transmitted wholly by the grains in the uppermost layer, and each such grain will transmit a small force depending on its diameter and its location relative to the adjacent grains, as well as on the type of motion of the fluid, though it is now the local motion which is concerned rather than that of the main stream.

White (1940), considers two extreme cases for the equilibrium of an individual grain:

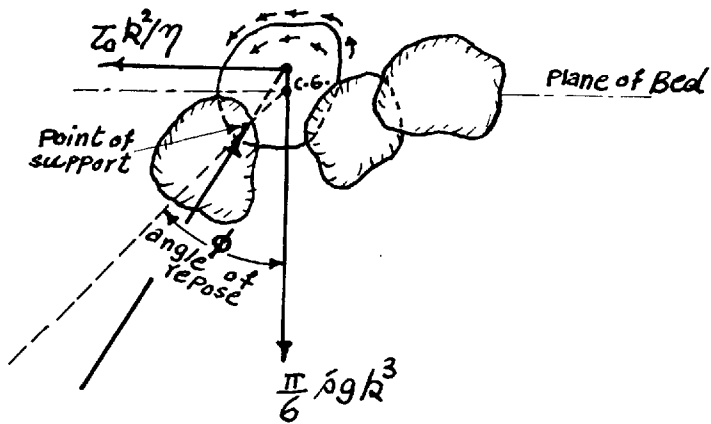
a) At slow speeds and with small grains ($\frac{V_* k}{\nu} < 3.5$), the force exerted by the viscous tangential stresses becomes important, and these stresses are dominant in determining the resultant force applied to the grain. Since these tangential stresses act on the exposed upper half of the grain surface, the resultant force will not pass through the centre of gravity, but lies at a point somewhat above it and can be transmitted to the centre by an equal force and a couple. Consequently the grain in this case has a tendency to rolling than to sliding motion (figure 6.3a). When the grain is about to move we have:

$$\tau_c = \alpha \eta \frac{\pi}{6} \rho' g k \tan \phi \quad (6.6)$$

where η is a packing coefficient defined as k^2 times the number of grains per unit area and consequently expresses the closeness of the grains, ϕ is the wetted angle of repose of the surface layer of grains and α is an experimental coefficient taking into account the effect of the shift of the resulting drag.

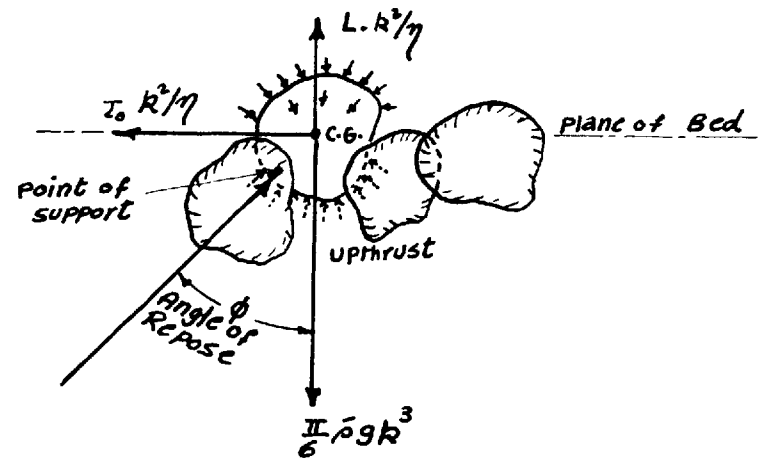
b) At higher speeds and with large grains ($\frac{V_* k}{\nu} >$

(a) $v_* k / \nu < 3.5$



For Equilibrium $\tau_0 = \alpha \eta \frac{\pi}{6} \rho g k \tan \phi$

(b) $v_* k / \nu > 3.5$



For Equilibrium $\tau_0 = \eta \frac{\pi}{6} \rho g k \tan \phi$

FIG.(6.3) Model for Equilibrium of an Individual Sand Grain
(after C.M. WHITE (1940))

3.5), the viscous shear stresses become relatively negligible compared with the forces due to pressure difference. In this case, the pressure distribution on the two sides of the grain will not be symmetrical, but less over the downstream half. The resultant force which is the integral of the pressures on the grain surface area will pass through the centre of gravity of the grain (figure 6.3b), and the particle will have more tendency to slide than to roll. When the grain is about to move, taking into account the upthrust neglected by White, we have:

$$\tau_c = \eta \cdot \left(\frac{\pi}{6} \rho' g k - u_p / k^2 \right) \tan \phi \quad (6.7)$$

In the range of $\frac{v_* k}{\nu}$ between 33.0 and 1280, the observed drag in White's experiments, which were made in a nozzle with a non-turbulent main flow, was in general half that predicted from theory, the difference being attributed by White to local turbulence, though clearly in the light of later work, the difference was mainly due to neglect of the uplift pressure. That this must have been the case is seen from the present experiments on the turbulent fluctuations in air flow (section 4.2.5.) which show that the u'

fluctuation has a frequency in the range of 500 C/S for the 4.75 mm. sand. Assuming that the dimensionless number $\frac{n d^2}{\nu}$ has the same value for both air and water flow in the nozzle, where n is the frequency of the u' fluctuation, d is a length scale describing the size of the apparatus, and ν is the kinematic viscosity of the fluid, then:

$$\frac{n_a d_a^2}{\nu_a} = \frac{n_w d_w^2}{\nu_w}$$

where a and w refer to air and water respectively.

Since $\frac{d_w}{d_a}$ was 1.5 in the present experiments, and $\frac{\nu_w}{\nu_a}$ was approximately 0.080, then n_w is expected to be in the range of 18 C/S. Consequently the duration of a single pulsation on a 4.75 mm. grain will be about $\frac{1}{36}$ of a second, whereas the duration of the shear stress velocity V_{τ} on the same grain is approximately $\frac{1}{17}$ of a second according to table (6.2). Therefore the turbulent pulsations are relatively rapid and while they may have accounted for some of the excess drag, particularly of the finer grains, they can hardly have accounted for the two fold difference measured by White.

White indeed pointed out that later experiments on lift gave considerable values for the lift, which when included in equation (6.7) explains the discrepancy.

6.1.3. Lift Magnitude

The lifting effect on solid bodies immersed in moving fluid has been the subject of investigations by Jeffreys (1929), Einstien and El-Samni (1949), White and Feir (1954) and many others. This lifting effect on a grain resting on a stream bed has been attributed by Rubey (1937), and Chang (1939) to the velocity gradient, that is to say, they regard it as a result of the difference in velocities at the top and bottom of the grain, though theoretically speaking this requires the assumption that Bernolli's equation is valid near the bed which is not the case because of the rotation due to the friction.

Prandtl (1952) on the other hand, in a discussion on the movement of solid bodies in moving water (Essentials of Fluid Dynamics, p 335) stated that, when a grain begins to move there

seems to be a lifting effect due to eddies, similar to that in a waterspout, which first ^a rises the grain a little so that its resistance to horizontal movement is decreased. The existence of eddies near a rough bed has indeed been observed by Gilbert who in 1914 during experiments on flume transportation observed the formation of vortex rings with transverse axes near the bed. However, there seems no doubt that vortices are formed in the wake behind obstacles immersed in the flow, and for two-dimensional obstacles such as cylinders or the roughness pattern photographed by White (plate 14) using alluminium dust, the vortices are established with their axes normal to the flow direction. When the obstacles are three-dimensional such as spheres or sand grains, the vortices are expected to have different inclinations for their axes from those for two-dimensional obstacles. The spaces between the grains in the top layer of a deposit of perfect spheres can in fact be considered as consisting of a series of hollows of curved shape, and a Potassium Permanganate dye applied to a series of hollows of similar shape

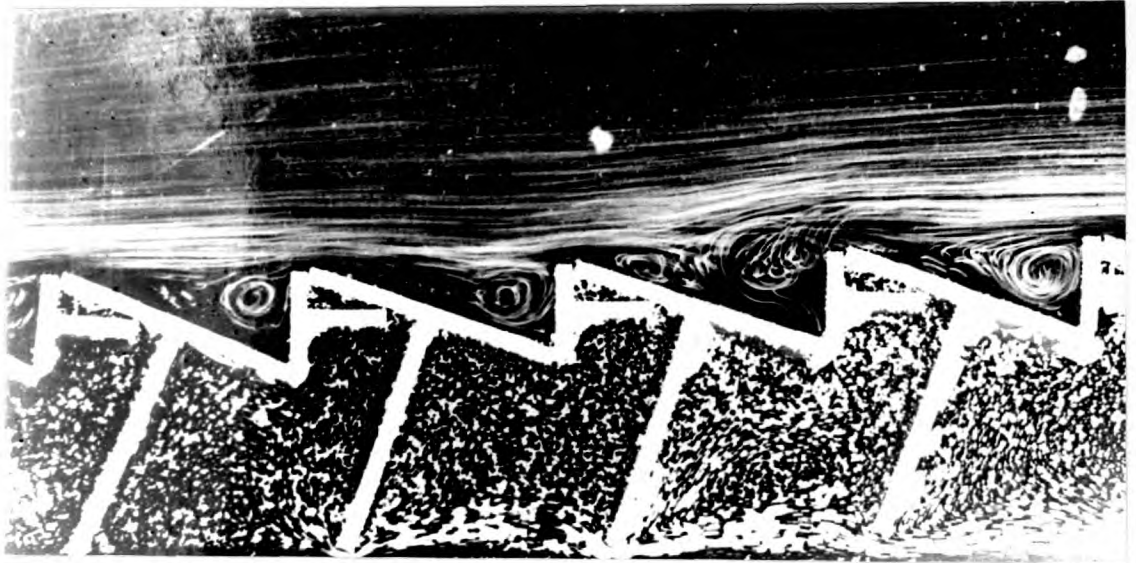


PLATE 14. TRANSVERSE VORTICES IN A TWO-DIMENSIONAL ROUGHNESS PATTERN (after C.M. WHITE.)

flow direction normal to paper

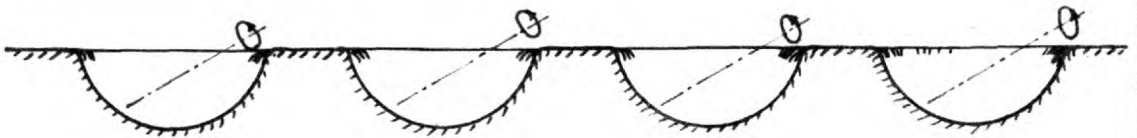


FIG. 6.4 INCLINED VORTICES IN A SERIES OF HOLLOWES AS REVEALED BY POTASIUUM PERMINGNATE DYE

in the bottom of a flume revealed that the vortices formed in these hollows had their axes inclined to the normals to the flow direction (figure 6.4). Thus a suction exists tending to lift the grains upwards. This lifting effect relieves the bed below the grains of part of their weight which is transferred to the fluid in the pores, thus resulting in a higher pressure for the fluid in these pores. Accordingly the lifting forces on the grains could be estimated by measuring the pressure increase in the pores as was suggested by Einstien in 1939 and verified by him and El-Samni (1949) and also by White and Feir (1954).

It is interesting to note here that an experiment made in the present tests with a deposit of four layers of glass spheres 2.93 mms. in diameter showed that, the spheres in the top layer vibrated vertically for some time before being pushed along the bed by the flow, thus confirming Prandtl's statement that the lifting forces raise the grains a little before they move, which results in their resistance to movement being decreased.

The lifting phenomenon has also been interpreted in a mathematical way by circulation.* For example, on the assumption that Bernolli's equation is still valid in the region of the flow near the bed, then on replacing the grain by an imaginary cylinder, circulation leads to a lift on the body equal to $\rho_f u \Gamma$, where ρ_f is the fluid density, u is the velocity of the flow and Γ is the circulation which tends to be proportional to u . In other words this kind of theory leads to the lift being proportional to $\rho_f u^2$. Although the assumption of Bernolli's validity near the bed is certainly not true, the proportionality between the lift and $\rho_f u^2$ is tacitly assumed in most investigations concerned with this problem. For example, Jeffreys (1929) treated mathematically a two-dimensional potential flow over a cylindrical strut resting on the bed of a deep stream and showed that the cylinder tends to lift when:

$$1.4299 \rho_f u^2 = \rho' \cdot g \cdot r$$

*"The Elements of the Prandtl-Lanchester Theory of Aeroplane Lift and Drag", Engineering, Vol. 67, 1924.

where u is the stream velocity and r is the radius of the cylinder. Comparing velocities predicted by this equation with experimental observations, Jeffreys found that the later are about four to five times greater. However, when an individual grain on the bed is surrounded by other grains and in contact with them at many points, the flow pattern around the body will be a complicated three-dimensional one whose lifting forces would be different from those in the two-dimensional case.

Regarding the three-dimensional case we have on record two different groups of experimental results. The first group is due to Einstien and El-Samni (1949) who undertook a series of experiments on the lift acting on an idealized sediment deposit consisting of hemi-spheres 0.225 ft. in diameter and arranged in a hexagonal pattern on the bottom of a flume. Since lift exerted by the flow on the hemispheres would relief the bed of that part of their weight which can be transfered to the fluid between the particles, then the lift can be measured as an increase of the

general pressure existing in the pores of the bed. These measurements led Einstien and El-Samni to an empirical formula for the lift in the form:

$$L = 0.178 \left(\frac{\rho_f}{2} u_1^2 \right) \quad (6.8)$$

where L is the average lift per unit area of the bed, and u_1 is the velocity at $y = 0.35 k$ from the reference line for the velocity profile. Substituting from the universal velocity distribution law, the equation can be written as:

$$\frac{L}{\tau_0} = 2.95 \log^2 \left(\frac{0.35k}{y_1} \right) \quad (6.9)$$

Taking the limits of y_1 to vary between $\frac{k}{9}$ for loose arrangement of the grains and between $\frac{k}{33}$ for very close arrangement, the lift to drag ratio according to (6.9) will vary between 0.75 and 3.3, i.e. the lift can be as high as three times the drag.

The second group of experimental results is due to White and Feir (1954) who experimented with air flow in a circular pipe containing a loose grain bed, on three sizes of grains. The lift in this case was measured as a difference in pressure between the bottom and top of the

grains. This difference in pressure was expressed in terms of a lift velocity u_L in the form:

$$L = \frac{\rho_f}{2} \cdot u_L^2 \quad (6.10)$$

From the measurements, White and Feir found that the lift velocity is linearly related to the bed shear velocity and on the average varies between $1.78 V_*$ and $2.20 V_*$ where V_* is the shear velocity at the bed. This means that according to equation (6.10), the lift to drag ratio roughly varies in the range between 1.59 and 2.42 which confirms with the value 2.0 quoted by C.M. White in 1948 (proceedings of U.G.G.I., Oslo Meeting, p. 263).

Another simple relation between the lift and drag on a grain starting to move was given by Chang (1939). According to Chang, the grain will tend to lift just before the upthrust balances its effective weight in water, thus the balance can be expressed by:

$$\frac{\rho_f}{2} u_L^2 \cdot \frac{\pi k^2}{4} = \frac{\pi}{6} \rho' g k^3 \quad (6.11)$$

On the other hand, the shear velocity V_* , necessary to overcome the frictional resistance of the bed can be expressed by:

$$\rho_f V_*^2 \cdot \frac{\pi k^2}{4} = \frac{\pi}{6} \rho' g k^3 \tan \phi \quad (6.12)$$

Dividing equation (6.11) by (6.12), then:

$$\frac{u_L}{\sqrt{2} V_*} = \sqrt{\frac{1}{\tan \phi}} \quad (6.13)$$

$$\text{or } \frac{L}{\tau_0} = \frac{1}{\tan \phi} \quad (6.14)$$

The measured value of $\tan \phi$ for the sands used in the present experiments actually varied between 0.55 and 0.58 and accordingly the lift to drag ratio deduced from (6.14) lies between 1.73 and 1.82 which seems in fair agreement with the value 2.0 predicted by C.M. White.

It is also interesting to note that (6.14) when inserted in White's expression for the critical drag explains completely the difference between theory and observation as follows:

The forces acting on the individual grain are:

- a) own weight in fluid = $\frac{\pi}{6} \rho' g k^3$
- b) Drag force exerted by the flow $\tau_c k^2 / \eta$
- c) Lifting force exerted by the flow $L k^2 / \eta$
- d) Reaction forces from neighbouring grains

Thus for equilibrium we have:

$$\left(\frac{\pi}{6} \rho' g k^3 - \frac{L k^2}{\eta} \right) \tan \phi = \frac{\tau_c k^2}{\eta}$$

Substituting for L from equation (6.14), then:

$$\left(\frac{\pi}{6} \rho' g k^3 - \frac{\tau_c k^2}{\eta} \cdot \frac{1}{\tan \phi} \right) \tan \phi = \frac{\tau_c k^2}{\eta}$$

$$\text{or } \frac{\pi}{6} \rho' g k^3 \tan \phi = 2 \tau_c \frac{k^2}{\eta}$$

Thus:

$$\tau_c = \eta \cdot \frac{\pi}{12} \rho' g k \tan \phi \quad (6.15)$$

which accounts for the lift and is in excellent agreement with White's original measurements.

6.2. Experimental

6.2.1. Apparatus

To insure constant shear stress throughout the length of the bed for the critical tractive force experiments, the nozzle of section (2.2) designed for constant drag was used. The general layout of the apparatus is shown in figures (6.5). The nozzle consisted of three main parts, a smooth curved surface, a straight sand bed, and two smooth sides. The curved surface was made from a smooth thin sheet of aluminium designed according to equation (2.8) and was supported on a frame of hard timber with stiffeners at the back of the curve to prevent it from buckling under the water pressure. The bed of the nozzle was formed by closely sieved sand laid to a depth of 0.40 inches on a straight plate of perspex fixed to the sides of the nozzle, and the bed was

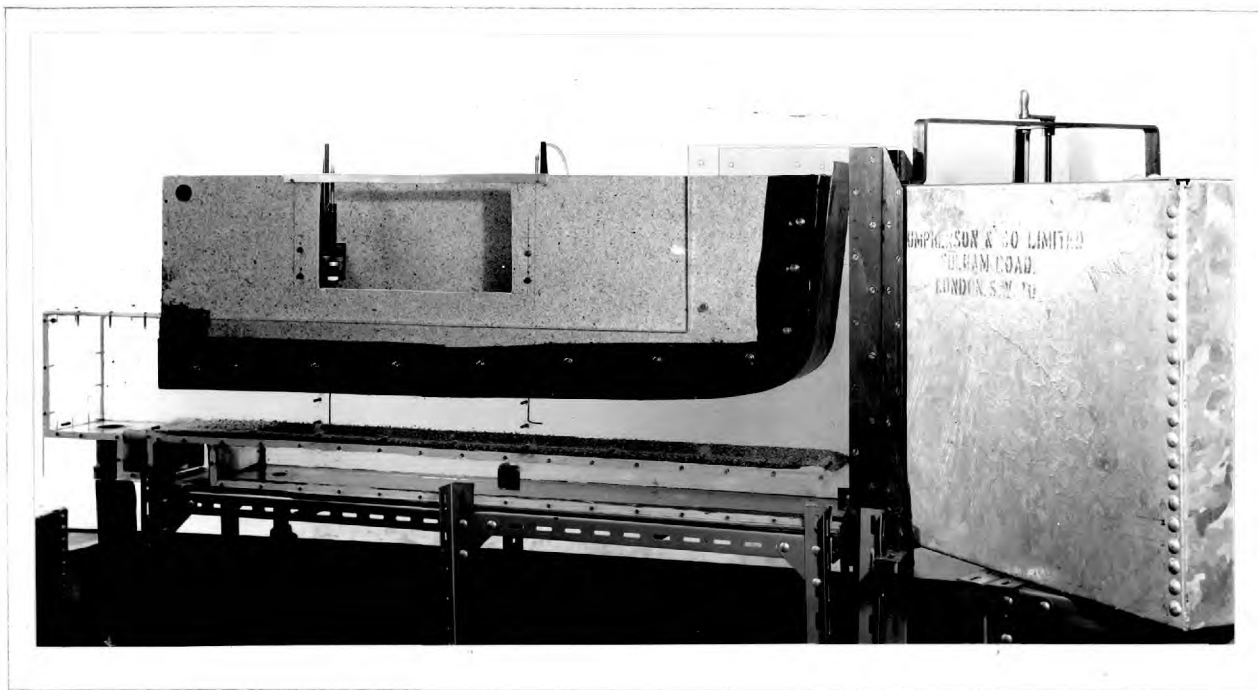


PLATE 15. APPARATUS FOR EROSION EXPERIMENTS.

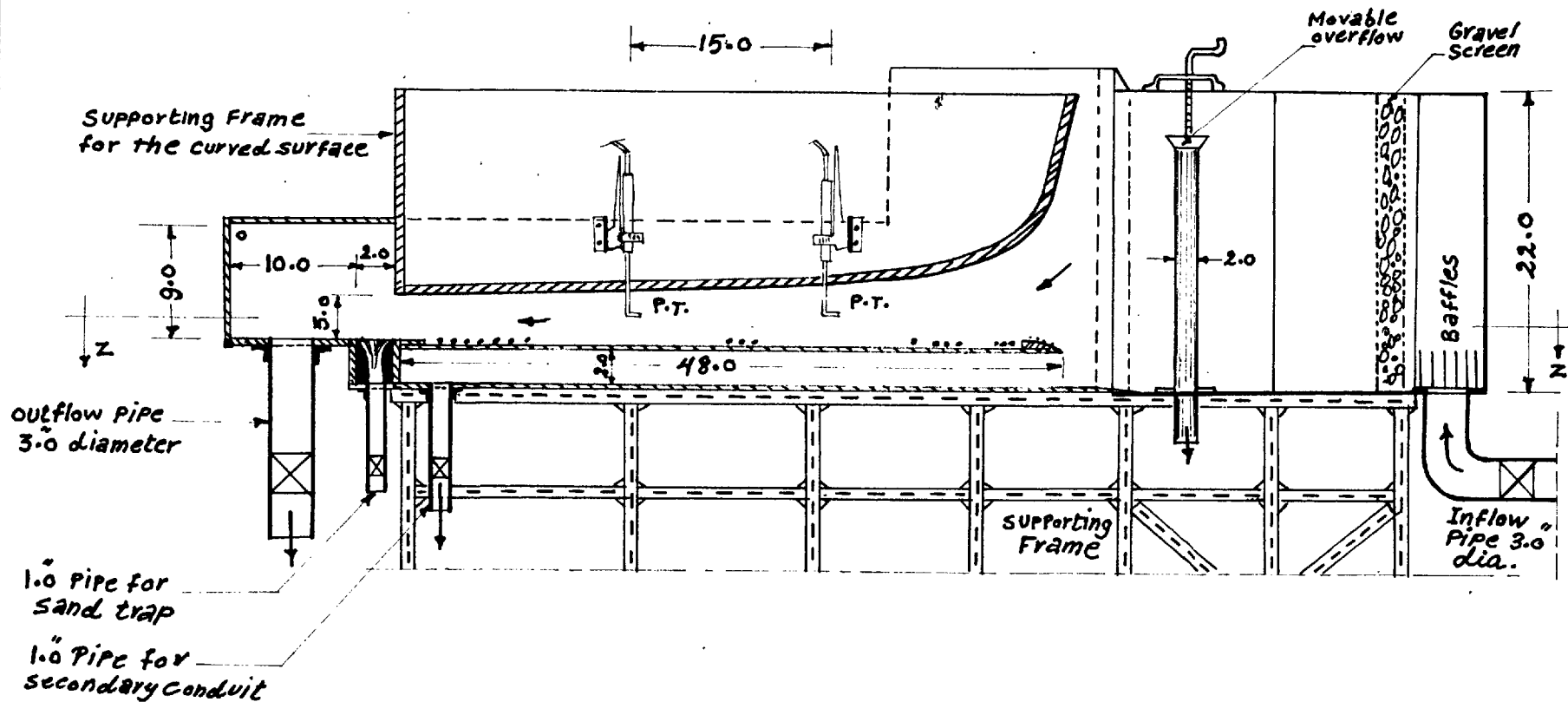


FIG.(6.5) Sectional Elevation for The Apparatus.
Dimensions in inches.

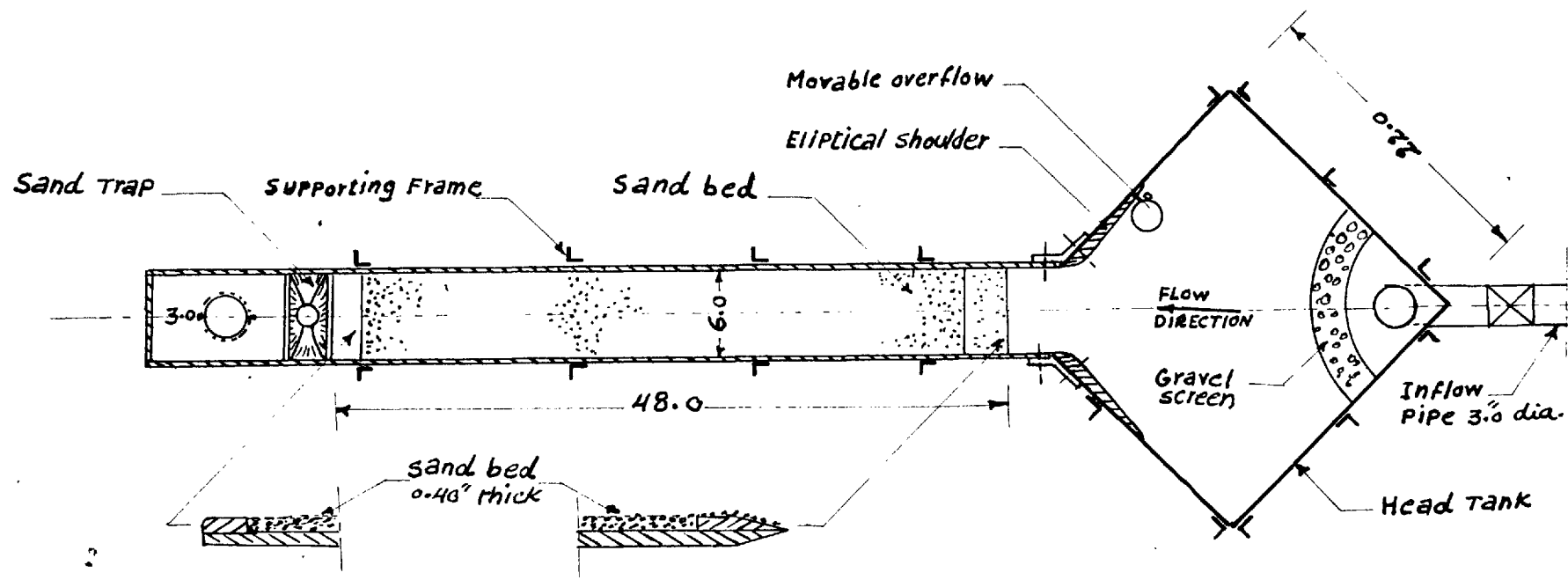


FIG (6.5) Sectional Plan z-z For The Apparatus.

Dimensions in inches.

6.0 inches wide and 48.0 inches long. To prevent the sand near the leading edge being washed away by the flow, a piece of perspex 0.40 inches thick tapered from the front and carrying grains cemented to it, was used to replace the sand of the first two inches of the bed. The sides of the nozzle were made from smooth perspex allowing visual observation of the grains movement.

The water was supplied to the nozzle through a 3.0 inches diameter pipe discharging into a 22 x 22 x 22 inches tank which was provided by a series of baffles and a gravel screen to distribute the flow. The tank was connected diagonally to the nozzle (figure 6.5) and the junction between the two was provided by elliptical shoulders to avoid separation at the entrance. The water level in the tank was controlled by a movable overflow pipe 2.0 inches diameter enlarged to 3.0 inches at the top. The flow out of the nozzle passed through a chamber 12.0 inches long before discharging into the atmosphere through a 3.0 inches pipe provided with a valve to control the discharge.

The sand grains eroded from the bed were trapped in a sand trap made from wax stream lined in shape. To insure a satisfactory flow at the leading edge of the bed, a secondary conduit 2.0 inches deep was constructed under the bed, and the flow through it was regulated by a valve on the exit pipe. The shearing stresses at the bed were determined from the pressure gradient in the nozzle in conjunction with the velocity profiles located at distances of 20 and 35 inches respectively from the leading edge of the bed. The velocity at any point on the profile was determined by measuring the total head at that point by a pitot tube, together with the static pressure measured through pressure tappings on the side of the nozzle. The pitot tubes were supported in the flow by means of a micrometer mounting, and the pressures were recorded on an inclined manometer.

6.2.2. Geometrical Properties of the Sands.

Eight different closely sieved sands were used in the present tests, and the sizes of the grains were determined, as discussed before in

section (2.3.2), using three different methods, the travelling microscope, the sieve size and by weighing. The sizes obtained by weighing a few hundred sand grains on the assumption that they were perfect spheres proved to be a good average between the other two methods, and were therefore used in the analysis.

The density and the angle of internal friction in water for each sand were determined and are given in table (6.1) together with the other properties of the sands. The coefficient of packing η in this table was determined for each sand by counting the top grains on the bed. All sands except sands 4 and 8 were from the same pit.

TABLE 6.1

Geometrical Properties of the Grains

Sand No.	Passed by Sieve No.	Retained on Sieve	k, mm by weighing	Average of	ρ gm/cm ³	$\tan \phi$	η
1	18	20	1.04	800 grains	2.61	0.58	0.34
2	14	16	1.42	500 "	2.61	0.58	0.40
3	12	14	1.56	500 "	2.58	0.55	0.40
4	12	14	1.57	500 "	2.61	0.58	0.40
5	10	12	1.96	500 "	2.61	0.58	0.40
6	8	10	2.55	500 "	2.608	0.55	0.60
7	6	7	3.16	300 "	2.60	0.55	0.40
8	3" 16	5	4.75	200 "	2.61	0.58	0.65

The sieve numbers in this table relate to B.S. sieves and indicate the number of wires per inch.

6.2.3. Experimental Results

In beginning a test run, the sand bed was leveled and the coefficient of packing was determined by counting the top grains. Water was then introduced by gradually opening the valve in the feed pipe, and the water level in the head tank was kept constant at a height of 15.5 inches above the bed by means of the movable overflow pipe. The pressure drop along the nozzle was then increased gradually using the valve on the exit pipe until the grains began moving; usually grains were scoured simultaneously from all parts of the bed, so confirming that the drag was substantially uniform.

To estimate the drag along the bed, the pressure distribution along the bed was measured through the pressure tappings on the side, and the total head distribution across the flow was measured at sections 20 inches and 35 inches from the nozzle of the bed. Using the measured pressure gradient and total head for the various tests, the bed shear stress τ_0 was determined from equation

(2.17). The pressure distributions at which grains started moving are shown in figure (6.6) for the various sands and the experimental results are summarized in table (6.2) which also contains two other groups of experimental results on converging streams. The first of these groups is due to White (1940) who conducted a series of experiments on the equilibrium of grains in a nozzle of constant drag, in which the main flow was accelerating and inviscid, and the turbulence was restricted only to the boundary layer on the bed. The calculated drag on the bed in his experiments was also checked by measuring the displacement of a hanging strip of metal coated with sand grains similar in size and shape to those of the bed.

The second group of experimental results is due to Chang (1939) who experimented in open channels with horizontal beds. His observations on the grains movement were made in a part of the channel 6 feet long and converging from a width of 12 inches to a width of 6 inches which caused the flow to accelerate considerably in the downstream direction. In order to determine the bed shear

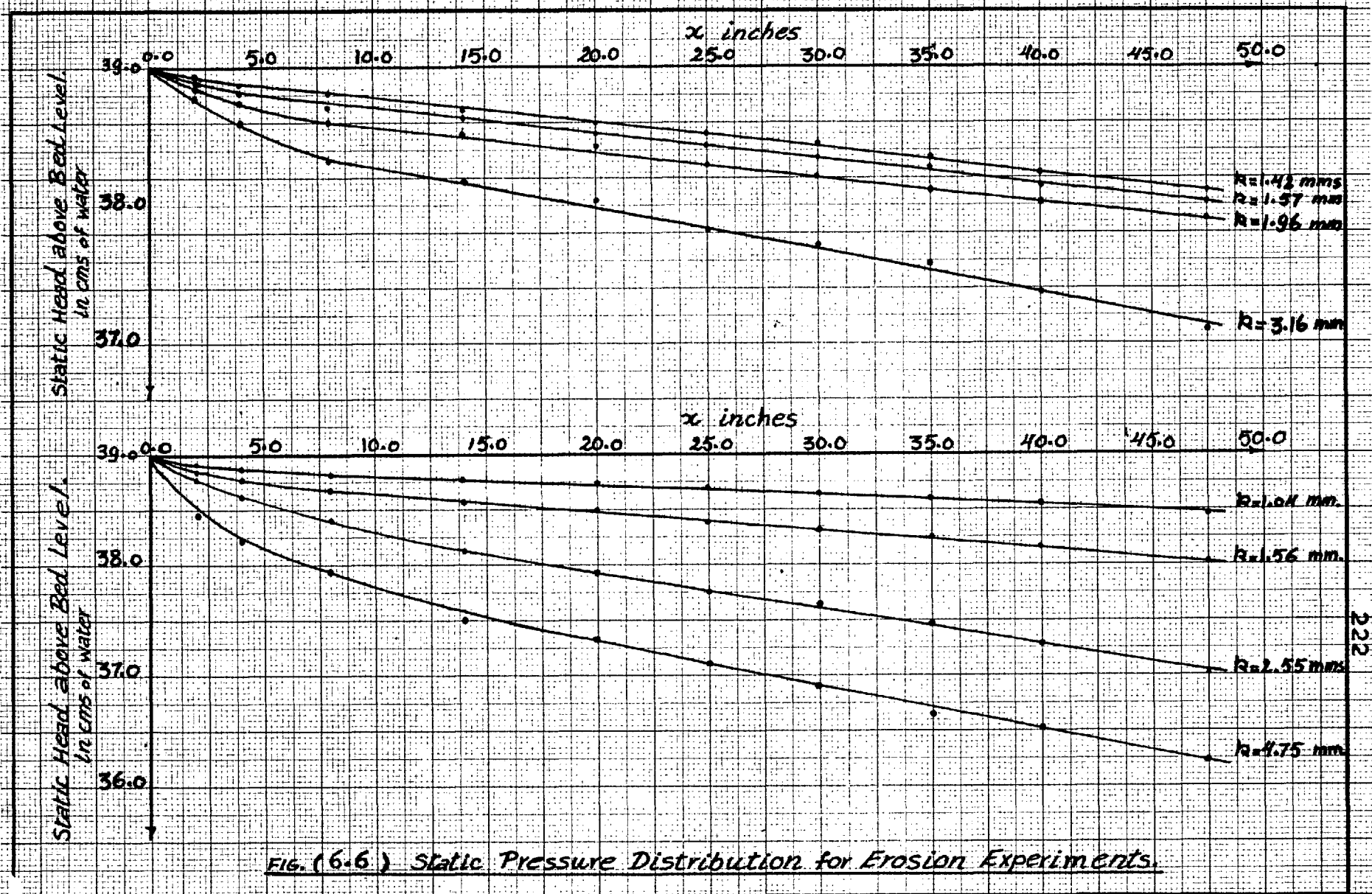


FIG. (6.6) Static Pressure Distribution for Erosion Experiments.

TABLE 6.2.
CONVERGING STREAMS

experiment	bed material	ρ gms/cm ³	h mms.	$\tan \phi$	η	$\tan \theta$	Actual Drag dynes/cm ²	T_c (Eqn. 6.15) dynes/cm ²	$\frac{T_c \text{ Actual}}{T_c \text{ Calc'd.}}$	$\frac{V_c h}{\nu}$	$\frac{T_c}{\rho g h}$
present	1 sand in water	2.61	1.04	0.58	0.34	0.0	6.50	8.40	0.78	27	0.04
experiments	2 "	2.61	1.42	0.58	0.40	0.0	13.0	13.50	0.96	51	0.058
	3 "	2.58	1.56	0.55	0.40	0.0	13.50	14.0	0.96	56	0.055
	4 "	2.61	1.57	0.58	0.40	0.0	13.20	15.10	0.88	57	0.0535
	5 "	2.61	1.96	0.58	0.40	0.0	16.20	18.70	0.87	78	0.053
	6 "	2.608	2.55	0.55	0.60	0.0	29.50	34.80	0.86	137	0.074
	7 "	2.60	3.16	0.55	0.40	0.0	30.80	28.60	1.07	175	0.062
	8 "	2.61	4.75	0.58	0.65	0.0	67.0	74.0	0.91	390	0.09
White's	4 "	2.60	0.90	1.0	0.40	0.0	16.90	14.80	1.14	33	0.119
(1940)	5 steel shots "	7.90	0.71	0.685	0.40	0.0	30.80	32.70	0.94	35	0.064
	6 sand in air	2.60	0.90	1.0	0.40	0.0	22.50	24.0	0.94	80	0.098
	7 sand in water	2.60	5.60	1.0	0.40	0.45	50.0	46.20	1.08	360	0.057
	8 "	2.60	5.60	1.0	0.40	0.0	89.0	93.0	0.96	480	0.101
	9 "	2.60	5.60	1.0	0.40	0.50	133.0	123.0	1.08	590	0.151
	10 sand in air	2.60	5.60	1.0	0.40	0.0	146.0	149.0	0.98	1280	0.102
Chang's	6205 sand in water	2.547	8.09	(1.0)	(0.40)	0.0	162.0	130.0	1.24	810	0.132
(1939)	6204 "	2.558	6.54	(1.0)	(0.40)	0.0	120.0	106.0	1.13	560	0.12
	6203 "	2.567	5.29	(1.0)	(0.40)	0.0	95.0	86.0	1.10	410	0.117
	6202 "	2.590	3.77	(1.0)	(0.40)	0.0	78.0	61.50	1.27	242	0.132
	6201 "	2.633	2.54	(1.0)	(0.40)	0.0	39.60	42.20	0.94	125	0.098
	6200 "	2.634	1.76	(1.0)	(0.40)	0.0	31.20	29.50	1.05	76	0.112
	6199 "	2.640	0.886	(1.0)	(0.40)	0.0	14.60	14.80	0.99	27.5	0.105
	6198 "	2.643	0.396	(1.0)	(0.40)	0.0	6.60	6.60	1.0	7.9	0.105

Mean = 1.005

θ is angle of bed slope

stress for this type of flow, L. Straub (1939), H. Rouse (1939); and others,* each independently showed that by using the energy equation for gradually varied flow, and by considering the gain in momentum due to the acceleration of the flow, the drag at the horizontal bed of the flume can be satisfactorily approximated by the expression:

$$\tau_o = \rho_f g i_s d \left(1 - \frac{\bar{U}^2}{gd} \right) \quad (6.16)$$

where τ_o is the average drag at the bed, ρ_f is the fluid density, g is the acceleration due to gravity, d and $\bar{U}$ are the depth and mean velocity respectively, and i_s is the slope of the water surface which can be determined from Manning formula in the form:

$$i_s = \frac{n^2 \bar{U}^2}{2.21 m^{4/3} \left(1 - \frac{\bar{U}^2}{gd} \right)}$$

m and n being hydraulic mean radius and the Manning coefficient respectively, and the term $\left(1 - \frac{\bar{U}^2}{gd} \right)$ corrects for the effect of acceleration. Generally speaking, for high depth and with comparatively small velocity, the term $\frac{\bar{U}^2}{gd}$ in equation (6.16)

* Discussions on (Laboratory Investigations of Flume Traction and Transportation), Trans. Am. Soc. Civil Eng., Vol. 104, 1939. p. 1297.

has only a very small effect on τ_0 , whereas for Chang's conditions, its value at times reached 0.11.

By inspecting table (6.2) with its wide range of $\frac{V_{*k}}{\nu}$ between 7.9 and 1280, and of grain size between 0.396 mms and 8.09 mms, i.e. about 20 times fold range of k ; the actual drag in most cases is seen to be in good agreement with equation (6.15) which takes lift into account. The overall mean value of $\frac{\tau_c \text{ actual}}{\tau_c \text{ calculated}}$ for the 23 tests contained in the table is almost unity, though in general, this ratio is slightly smaller than unity for values of $\frac{V_{*k}}{\nu}$ below 70 (figure 6.7). The discrepancy may well be explained by the turbulent fluctuations in the boundary layer. As seen in section (6.1.2), for high speeds and large grains, these fluctuations are too rapid to affect the results, whereas for slower speeds, the duration of these fluctuations becomes larger, and thus may add slightly to the effective stress responsible for dislodging the grains.

In parallel flow on the other hand, the

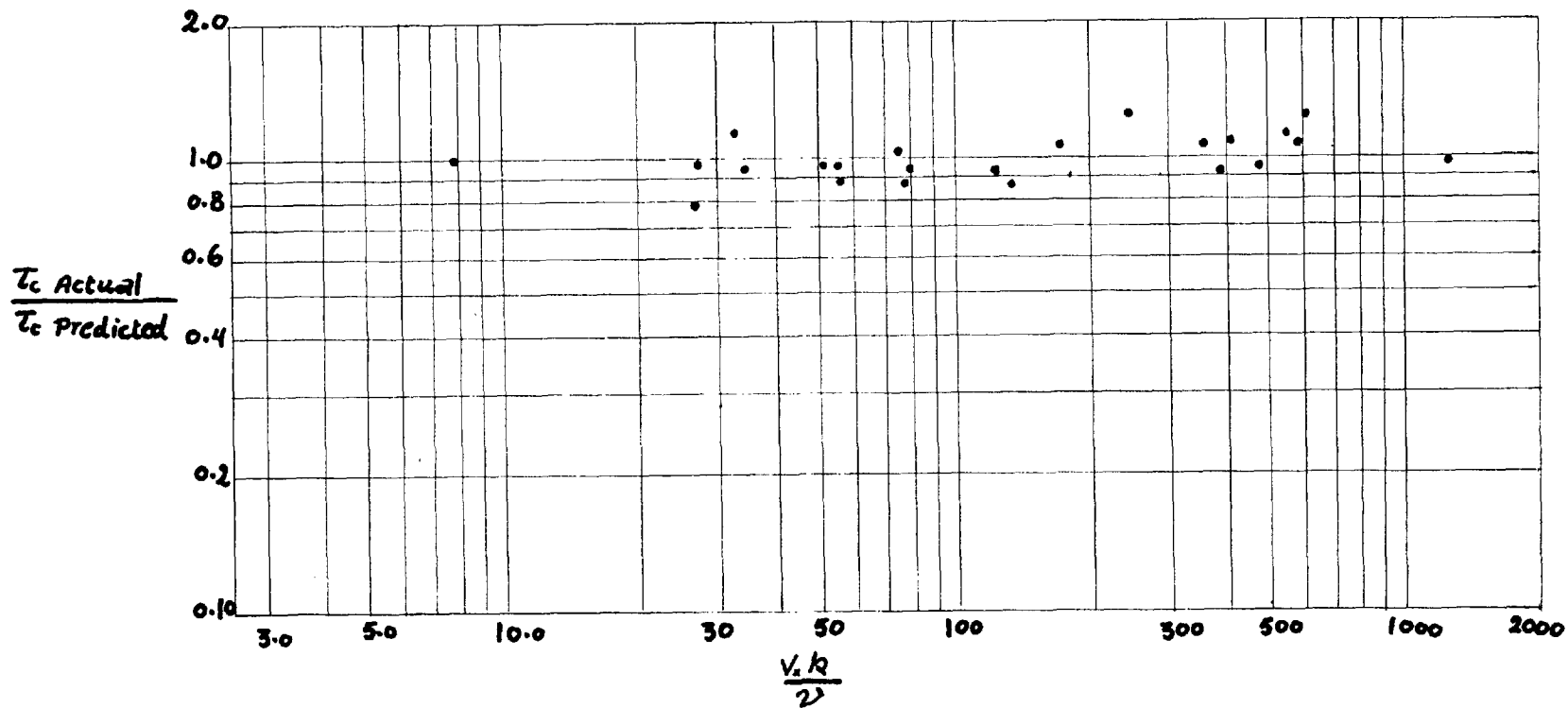


FIG.(6.7) Comparison between Theory and Experiment for
converging Flow.

turbulence which was restricted to the boundary layer in the case of converging flow, extends now throughout the stream. Although this large scale turbulence may not add materially to the mean stress at the bed, it can substantially add to the maximum stress fluctuations there. Consequently the grains on the bed in this case would start moving at much smaller mean stress, the difference between the effective stress and the measurable mean stress being contributed by large scale disturbances far from the bed. Indeed this was examined and confirmed by an experiment by White (1940) with a divergent nozzle, where large scale turbulence was strong enough to raise clouds of grains, though at the bed itself the time average of the tangential drag was zero. White also pointed out in the Oslo Meeting of the U.G.G.I (1948) that most experiments on parallel flow had confirmed this concept of large scale turbulence. Indeed table (6.3) containing a collection of experimental data on parallel flow from various sources, show that on average, the mean bed stress necessary to start movement of grains is nearly half that for

converging flow. For example sand (3) in the present experiments would, according to the experiments of the Prussian Institute, start moving at 7.0 dynes/cm² instead of the present 13.5 dynes/cm². Again sand (8) would start moving at 42.0 dynes/cm² instead of the present 67.0 dynes/cm².

The results for converging flow are compared with those for parallel flow in figure (6.8) in which the grain Reynolds number $\frac{V_{xk}}{\nu}$ is abscissa and the term $\frac{\tau_c}{\rho' g k}$ is ordinate. The ordinate to some extent represents the movability of the bed material, since τ_c is proportional to the force necessary to start movement, and $\rho' g k$ is proportional to the force resisting movement. As seen from the figure, the experimental results for converging flow lie above those for parallel flow thus possibly indicating that the movability of the later is greater than of the former, and on the average the $\frac{\tau_c}{\rho' g k}$ values are nearly half those for converging flow at the same value of $\frac{V_{xk}}{\nu}$.

For the case when the flow is parallel, although the lower limit of $\frac{\tau_c}{\rho' g k}$ can be

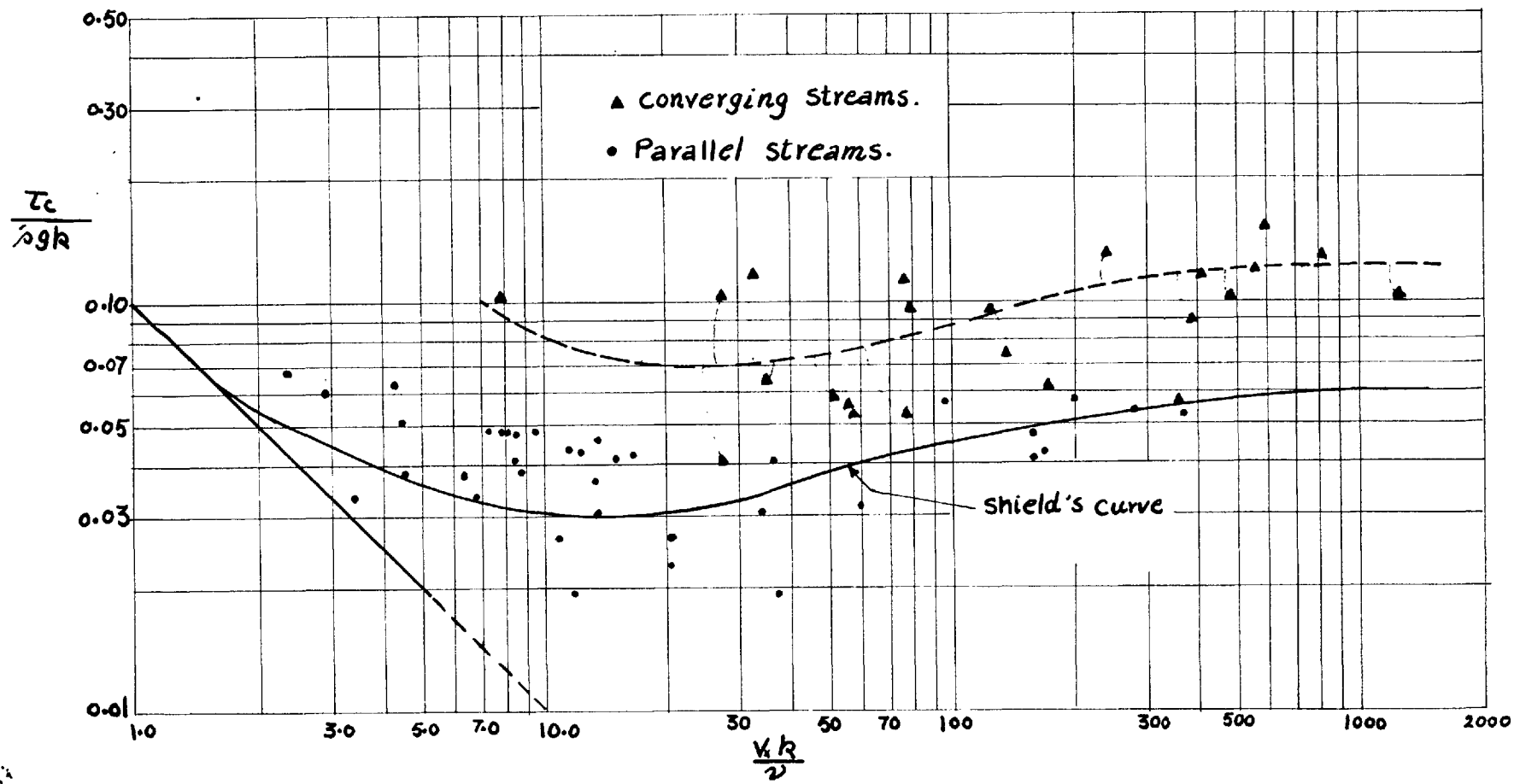


FIG. (6.8) Drag to Start Movement in Converging Flow and in Parallel Flow.

represented by Shield's curve (1936), the scatter in the points is too large to be due to experimental errors alone. However, in order to represent correctly the movability of the bed material, the term $\frac{\tau_c}{\rho' g k}$ should also contain a parameter representing the closeness of the grains and the composition of grain sizes. With the absence of this parameter, $\frac{\tau_c}{\rho' g k}$ would vary with the variations in the packing of the top grains and with the composition of the sand mixtures.

Regarding the effect of the closeness on $\frac{\tau_c}{\rho' g k}$, as White pointed out in 1940, the top layer of the granular bed exerts a drag on the moving fluid, each grain in this layer transmitting a small force depending on how many grains are there per unit area. Increasing the number of grains, i.e. increasing their closeness will result in a decrease in the force transmitted by each grain, and consequently in an increase in the value of $\frac{\tau_c}{\rho' g k}$. Again decreasing the number of grains per unit area will have the reverse effect.

Variation in the uniformity of size of the various sands could be expected to have an effect on $\frac{\tau_c}{\rho' g k}$. For example, a finer component in a mixture would be expected to move first before grains of the mean size began to move, and so the presence of the finer component would tend to increase the movability of the mixture. Secondly the finer grains would fill the voids and tend to increase the closeness, so tending to counterbalance the first effect. Again a coarser component in the mixture will result in an increase in the mean size of the grains and a decrease in their closeness. Unexpectedly, the experiments of the U.S. Waterways Station (1935) do not show any systematic variation in $\frac{\tau_c}{\rho' g k}$ with the composition of the grains as represented by the Modulus Factor M defined in section (6.1.2.). On the other hand, the results of the Prussian Institute do show a clear systematic decrease in $\frac{\tau_c}{\rho' g k}$ with a decrease in M, i.e. with an increase in the finer components in the mixture. This may be due to the fact that the effect of the reduction in the grain size in this case, has

outbalanced the effect of the closeness of the grains; whereas in the experiments by the U.S. Waterways Station, the two effects had more or less counterbalancing effects. In general however, the data contained in table (6.3) do not establish a relation between $\frac{\tau_c}{\rho' g k}$ and M .

A third factor which could affect the values of $\frac{\tau_c}{\rho' g k}$, is the scale of turbulence or the size of eddies present in the flow. Naturally, the sizes of the channels and the arrangement of experiments differ greatly from one set of experiments to another, and consequently the size of eddies would be different. Accordingly, the contribution of these eddies to the effective drag responsible for dislodging the grains would vary and may have contributed differently to the measured mean drags in the different laboratories.

TABLE (6.3)
PARALLEL STREAMS *

Experiment		k mms	ρ gms/cm ³	M	τ_c dynes/cm ²	$\frac{V_{mk}}{v}$	$\frac{\tau_c}{\rho g k}$
H. Kramer	I	0.706	1.70	0.358	5.15	12.5	0.0435
(sand							
mixtures)	II	0.558	1.70	0.461	3.80	8.40	0.0415
	III	0.800	1.70	0.414	4.80	13.50	0.036
F. Schaffernak	A	1.536	1.65	0.564	7.40	34.0	0.030
A. Schoklitsch	B	6.52	1.60	1.0	53.5	375	0.053
(uniform sand)	C	4.00	1.60	1.0	25.80	164	0.0415
	D	2.256	1.60	1.0	11.00	60	0.0312
	E	1.24	1.60	1.0	4.30	20.5	0.0223
	F	0.916	1.60	1.0	2.75	12.0	0.0195
H. Krey	G	0.376	1.68	0.825	2.35	4.50	0.038
(sand							
mixtures)	H	0.526	1.61	0.869	2.75	6.80	0.0335
	J	0.800	1.57	0.778	3.15	11.00	0.026
Prussian							
Experiment							
Institute	K	1.154	1.65	0.265	5.00	20.50	0.0265
	L	0.846	1.65	0.278	4.10	13.40	0.030
(sand							
mixtures)	M	0.836	1.65	0.366	5.70	16.50	0.0425

* Calculated from data contained in a paper by Chang,
Transactions Am. Soc. Civil Eng. Vol. 104, 1939, p. 1259.

TABLE (6.3) cont.

Experiment		k mms	ρ gms/cm ³	M	τ_c dynes/cm ²	$\frac{V_{\infty}k}{\nu}$	$\frac{\tau_c}{\rho g k}$
	N	0.744	1.65	0.399	5.45	13.60	0.0455
	O	0.244	1.65	0.435	2.35	2.90	0.060
	P	0.806	1.65	0.357	5.45	15.00	0.042
	Q	0.686	1.65	0.424	4.90	11.80	0.044
H. Engels	R	1.484	1.65	0.229	9.80	36.50	0.041
G.K. Gilbert	S	0.576	1.69	0.896	3.80	8.70	0.0395
(sand							
mixtures)	T	1.906	1.69	0.714	6.00	37.0	0.019
	U	3.71	1.69	0.867	29.0	160.0	0.047
	V	5.296	1.69	0.820	48.0	280	0.054
E. Indri	a	0.324	1.63	0.69	1.75	3.40	0.0335
(sand							
mixtures)	b	0.474	1.70	0.481	3.05	6.30	0.0385
	c	2.50	1.70	0.818	23.70	96.0	0.056
	d	4.00	1.70	0.777	40.00	200	0.059
U.S. Waterways	e	0.586	1.65	0.28	4.65	9.50	0.0495
Experiment							
Station	f	0.541	1.65	0.439	4.20	8.60	0.0485
(paper 17)	g	0.525	1.65	0.539	4.20	8.40	0.0495
	h	0.506	1.65	0.406	4.00	8.00	0.0495
	i	0.483	1.65	0.438	3.85	7.40	0.049

TABLE (6.3) cont.

j	0.347	1.65	0.643	2.85	4.50	0.051
k	0.310	1.65	0.525	3.15	4.30	0.063
l	0.205	1.65	0.560	2.25	2.35	0.068
m	4.077	1.65	0.566	27.50	170	0.0415

SUMMARY AND CONCLUSIONS

The turbulent flow past rough walls is investigated theoretically and experimentally with various sand roughnesses for a boundary layer under a particular pressure distribution; and the results are examined in the light of earlier investigations on both rough and smooth walls. Another set of tests were made with loose roughness on the bed to investigate the movement of grains.

From the theoretical and experimental results, the following conclusions are drawn:

1 - From theoretical considerations, in order to ensure a constant stress throughout the length of the wall, the main flow has to be accelerated according to the equation:

$$\frac{x}{6.25y_1} = \frac{e^{\chi}(\chi-1)+1}{\chi(\chi^2-3\chi+3)} - \frac{\chi^2}{2} - 3 \quad (1)$$

where $\chi = \frac{U}{2.5v_*}$ and y_1 is the shift in origin for the velocity profile, the equation being valid for both rough and smooth walls.

2 - In comparison with White's approximate formula:

$$\frac{x}{6.25y_1} = e^{\chi}(\chi-1) - \frac{\chi^2}{2} + 1 \quad (2)$$

~~equation (1) predicts up to 30% higher drag, the~~

~~excess drag being due to the inclination of stream lines just outside the boundary layer which was neglected in equation (2).~~

3 - ~~Although~~ equation (1) predicts higher drag than equation (2), the two equations, except for small values of $\frac{x}{y}$, give practically the same shape of curve since the ratio $\frac{x_1}{x_2}$ is found to be substantially constant at high values of $\frac{x}{y}$.

4 - The velocity was measured and found to be substantially constant at equal values of y smaller than the thickness of the boundary layer.

5 - The total drag D was evaluated and found to be substantially linear with x , and subsequently the local wall stress can not have varied appreciably since $\tau_0 = \frac{\partial}{\partial x} D$.

6 - In the range of k from 1.56 to 4.75 mms., the experimental values of χ are in good agreement with those predicted from equation (1). Solving equation (1) numerically for the various roughnesses used, the differences in the dimensions of the curve of the nozzle obtained from theory were very small being in the order of 1.2% or less. Consequently a single nozzle can be used, without much error,

for a wide range of roughnesses.

7 - Comparing the velocity profiles of the nozzle of constant drag with those for pipes and for flat plates, the following conclusions emerged:

(a) For pipes, the velocity profile is logarithmic for most of the flow region, except close to the wall; whereas for flat plates, whether rough or smooth, the velocity profile concaves upwards, particularly away from the wall, when u is plotted against $\log y$. This is due to the fact that the part of the flow near the wall is governed primarily by the local wall stress; whereas well away from the wall the fluid has a memory of a higher stress occurring nearer the nose of the plate. In the region near the wall where $\frac{y}{\delta} < 0.15$, the velocity profile for the flat plates can be approximated by the logarithmic law, though with slightly different values for the constants.

(b) The velocity profiles of the nozzle of constant drag are logarithmic up to values of $\frac{y}{\delta} = 0.90$ or more. This is due to the fact that the part of the flow away from the wall has a memory of an earlier wall stress which is identical with the local stress since τ_0 is substantially constant as for pipes. Therefore a boundary layer of the present type offers

a link between pipe flow and plate flow and reduces the gap between them.

(c) Regarding the shape of the velocity profile close to the wall of a smooth pipe in the region where both the turbulent and viscous stresses influence the velocity, the Reynolds equation was solved to obtain the following expression for the velocity profile:

$$\frac{u}{V_*} = \frac{yV_*}{\nu} - \int_0^y \frac{u'v'}{\nu V_*} dy \quad \text{in which the}$$

integral may be calculated by assuming a practical distribution of turbulent shear stress having a power law with the power between 1.0 and 3/2 with the corresponding upper limit for y between $\frac{26\nu}{V_*}$ and $\frac{34\nu}{V_*}$. The velocity profiles predicted on this basis were found to be in close agreement with the observations of two different investigators.

(d) When plotted in the form $\frac{u}{V_*}$ against $\log \frac{yV_*}{\nu}$ the velocity profile for rough walls is displaced downwards parallel to that for smooth pipes by an amount depending on $\frac{KV_*}{\nu}$. Based on this, the nozzle of constant drag offers a convenient and accurate method for determining the equivalent sand size of artificial roughnesses. This seems the logical development of the method originally used

by Schlichting (1936).

(e) The values of $\frac{y_1}{k}$ for the present roughnesses varied between 0.089 and 0.11 as compared to an average value of 0.03 for the closely packed roughnesses of Nikuradse (1933), the difference being mainly due to the difference in spacing the roughness elements. An examination of the experiments on spheres by Schlichting (1936) showed that $\frac{y_1}{k}$ can be five times greater than Nikurads'es values, depending on the spacing between the spheres.

(f) Separation of the flow from the smoothest of the rough walls was obtained using a flap plate at the downstream end of the rough wall. The velocity profile at separation had a shape factor $H = 2.80$, and the parameter $\frac{\delta^2}{\nu} \frac{dU}{dx}$ was -1360, i.e. the present boundary layer is about 115 times as stable as the laminar boundary layer for which Pohlhausen (1921) gave the value of -12 as the limit.

(g) An examination of the similarity between the velocity profiles from various published investigations show that, all profiles for constant pressure flow on smooth walls are similar when plotted in the form $\frac{u}{U}$ against $\frac{y}{\delta_*}$. For pressure drop, the profiles of $\frac{u}{U}$ against $\frac{y}{\delta_*}$ deviate only

slightly from that for constant pressure, whereas for pressure rise on the other hand, the profiles undergo a clear and systematic change with $\frac{\delta^2}{2} \frac{dU}{dx}$

8 - Measurements of the turbulence intensity

on various roughnesses showed the following:

(a) The profiles of $\sqrt{u'^2}$ in the tests show the same general shape as those for pipes. They also have some resemblance to those for rough walls under constant pressure.

(b) The profiles of $\frac{\sqrt{u'^2}}{U}$ after passing through the maximum tend to zero as the wall is approached, the maximum values disclosing a clear dependence on the roughness height.

(c) The curves of $\frac{\sqrt{u'^2}}{V_*}$ against $\frac{y}{\delta}$ show a clear tendency to increase with $\frac{U}{V_*}$ for both rough and smooth walls. This is seen to be consistent with the turbulent energy equation.

(d) Unexpectedly, the curves of $\frac{\sqrt{u'^2}}{V_*}$ tend to converge to single curves when multiplied by $\frac{V_*^2}{U^2}$ both for rough and for smooth walls, though with values of $\frac{\sqrt{u'^2}}{V_*} \frac{V_*^2}{U^2}$ considerably higher for the rough walls.

This is due to the variation of the correlation factor $K = \frac{\overline{u'v'}}{\sqrt{u'^2}\sqrt{v'^2}}$ which is seen to vary with $\frac{V_*^2}{U^2}$ in a substantially linear manner.

9 - The usual methods of calculation for turbulent boundary layers under a pressure gradient, are similar in that all use an auxiliary equation obtained for a limited range of flow conditions, and the results likewise are limited to a particular range. These methods however do not take into account how the pressure gradient is related to the skin friction. Further, it is shown that a single parameter is not adequate to describe the exact form of the velocity profile since this depends on the previous history farther upstream.

10 - Using a loose sand bed for the nozzle, the following points regarding the movement of grains have emerged:

(a) The velocity fluctuations near the bed are rapid, and therefore they can contribute only slightly, particularly for the finer grains, to the effective drag responsible for the grains movement.

(b) The existence of lift/^{was}evident during an experiment with a deposit of glass spheres in which the top layer vibrated vertically for some time before starting to move horizontally. The existence of inclined vortices in the spaces between the top

grains was confirmed from visual observations on the flow in a series of hollows of similar shape in the bottom of a flume.

(c) An estimation for the lift magnitude showed it to be between 1.73 and 1.82 times the drag, values favourably agreeing with the value 2.0 predicted by White.

(d) Introducing lift in White's expression for the equilibrium of an individual grain leads to:

$$\tau_c = \frac{\pi}{12} \cdot \eta \cdot \rho' g \cdot k \cdot \tan \phi$$

which is in good agreement with observations on converging flow from the present tests as well as from the tests by White (1940) and by Chang (1939).

(e) For parallel streams, the drag necessary to start the grains moving is on the average half that for converging streams, the difference is explained as a result of large scale disturbances having their origin away from the bed.

(f) For parallel flow, the lower limit of $\frac{\tau_c}{\rho' g k}$ can be represented by Shields curve, though the scatter of the points is large which may be explained partly by the variation in the uniformity of size of the various sands and partly by the difference in the size of eddies present in the

different flumes.

This covers the last point raised in the synopsis.

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APPENDIX I

POSSIBLE ERRORS IN THE MEASUREMENTS OF $\sqrt{u^2}$

1. Error Due To The Lack Of compensation

The velocity fluctuations are rapid and the hot wire has to be heated and cooled with the same frequency, but due to the thermal inertia of the wire, there is a time lag between the velocity fluctuations and the corresponding fluctuations in the wire's resistance. Accordingly the voltage input to the amplifier follows the velocity fluctuations, but does so with a reduced amplitude and a retarded phase, both depending on the time scale of the wire. The time scale itself depends on the flow conditions as well as on the properties of the hot wire.

With higher frequency fluctuations, the errors involved become larger, and an electronic compensator would have been necessary to compensate properly for the wire's thermal inertia. However, the effect of the lack of such a compensator was reduced by using very small wires having a correspondingly small thermal inertia.

Due to the thermal lag, the R.M.S. values of the fluctuations is underestimated by the factor $\sqrt{1 + M^2 \omega^2}$, where M is the time scale of the hot wire and ω is the angular frequency of the velocity fluctuations. For 0.0001 inches diameter wires, Klebanoff and Diehl (1951) gave

for M the value 0.25×10^{-3} seconds, and from the oscillograms (plates 12 and 13), ω is in the range of 500 c/s, consequently the error due to the lack of compensation may have amounted to 8.0% of the R.M.S. values.

2. Error due to the Length of the Wire

As a result of the non uniform velocity distribution along the length of the wire, specially if the wire's length is comparable to the microscale of turbulence, the wire tends to underestimate the turbulence intensity.

Corrections for the wire length were given by Dryden (1941), who suggested that in order to get the correct readings, the measured intensities should be multiplied by the factor $(1 + 0.166 \frac{L_1}{L_m})$, where L_1 is the wire's length and L_m is the microscale of turbulence. The wire length used never exceeded 1.0 mm., while the microscale L_m can be obtained by the method suggested by Laufer and Liepman (1947). They simply suggest that:

$$\frac{1}{L_m} = \frac{\pi N_0}{U}$$

where N_0 = number of zero $\dot{u}$ fluctuations per second as obtained from the $\dot{u}$ oscillogram, and U is the main stream velocity.

Taking a typical case, where $L_1 = 1.0$ mm., $N_0 = 1000$ and $U = 50$ ft./sec., L_m will be 4.8 mm., and the corresponding possible error in $\sqrt{\overline{u^2}}$

will be of the order of 3.5%. This compares with the errors of 1.25% and 2.5% found by Tatchell (1951) during experiments in wind tunnel contractions using a wire length of 0.75 mm.

3. Error due to the Variations in the Wire Current

The error introduced in equation (4.2) by assuming a constant heating current is negligibly small, because the small changes in the wire's resistance accompanying the velocity fluctuations do not appreciably alter the value of the heating current, noticing that the resistance of the ratio-arm of the bridge (500 ohms) is very large compared to that of the wire (15-25 ohms).

Supposing that R is the mean value of the wire's resistance, and that the change in R due to the instantaneous velocity fluctuation is ΔR , then the corresponding change in the wire's current will be proportional to $\frac{\Delta R}{500 + R}$. Taking a typical value of $R = 15$ ohms, and assuming that the variation in R is as high as 20%, then the change in the wire's current will still be less than 0.6%. However, variations in the wire's resistance are usually smaller than this amount and eventually the error due to the assumption of constant current is very small.

In general, the possible overall error in the present measurements of turbulence intensity is expected to be of the order of 10%. Rouse, Siao and Nagaratnam (1958), suggested that the hot wire, like any other means of turbulence

measurements, must be expected to have a probable error of ± 5 to 10%, which is a reasonable estimation for the errors in the present measurements.

APPENDIX II
TABULATED RESULTS

TABLE I.1.

Static Pressure Distribution Along the Boundary
 Layer Roughness A, $k = 1.56$ mms.

x inches	Manometer's reading cms of Alcohol		Pressure cms of water
	$\Delta \sin \alpha = 0.10$	$\Delta \sin \alpha = 0.20$	
0.50	- 2.50		- 0.25
1.0	5.00		0.50
1.50	6.20		0.62
2.0	7.60		0.76
2.5	8.70		0.87
3.0	9.80		0.98
3.5	11.50		1.15
4.0	12.0		1.20
4.5	13.0		1.30
5.0	13.80		1.38
5.5		- 7.40	1.48
6.0		7.70	1.54
6.5		7.90	1.58
7.0		8.20	1.64
8.0		8.80	1.76
9.0		9.10	1.82
10.0		9.30	1.86
11.0		9.70	1.94
12.0		10.0	2.00
13.0		10.20	2.04
14.0		10.50	2.10
15.0		10.70	2.14
16.0		11.0	2.20
17.0		11.30	2.26
19.0		11.80	2.36
21.0		12.30	2.46
23.0		12.70	2.54
25.0		13.20	2.64
27.0		13.50	2.70
29.0		14.00	2.80

TABLE I.2.

Static Pressure Distribution Along the Boundary
Layer. Roughness B, $k = 1.96$ mms.

x inches	Manometer's reading cms of Alcohol		Pressure cms of water
	$\rho \sin \alpha = 0.20$	$\rho \sin \alpha = 0.40$	
0.50	- 2.0		- 0.40
1.0	2.60		0.52
1.5	5.10		1.02
2.0	6.50		1.30
2.5	8.0		1.60
3.0	9.80		1.96
3.5	10.90		2.18
4.0	12.0		2.40
4.5	13.0		2.60
5.0	13.80		2.76
5.5	15.60		3.12
6.0	15.80		3.16
6.5		- 8.10	3.24
7.0		8.30	3.32
8.0		9.10	3.64
9.0		9.80	3.92
10.0		10.30	4.12
11.0		10.50	4.20
12.0		11.0	4.40
13.0		11.20	4.48
14.0		11.50	4.60
15.0		11.80	4.72
16.0		12.0	4.80
17.0		12.40	4.96
19.0		12.80	5.12
21.0		13.70	5.48
23.0		14.20	5.68
25.0		14.50	5.80
27.0		14.90	5.96
29.0		15.70	6.28

TABLE I.3.

Static Pressure Distribution Along the Boundary

Layer. Roughness C, $k = 3.16$ mms.

x inches	Manometer's reading cms of Alcohol		Pressure cms of water
	$\Delta \sin \alpha = 0.2$	$\Delta \sin \alpha = 0.40$	
0.50	- 1.30		- 0.26
1.0	2.20		0.44
1.50	3.20		0.64
2.0	4.20		0.84
2.50	5.0		1.00
3.0	6.0		1.20
3.5	6.60		1.32
4.0	7.20		1.44
4.5	7.80		1.56
5.0	8.30		1.66
5.5	9.0		1.80
6.0	9.40		1.80
6.5	9.70		1.94
7.0	10.30		2.06
8.0	11.0		2.20
9.0	11.20		2.24
10.0	11.90		2.38
11.0	12.30		2.46
12.0	12.80		2.56
13.0	13.20		2.64
14.0	13.90		2.78
15.0	14.0		2.80
16.0		- 7.30	2.92
17.0		7.40	2.96
19.0		7.80	3.12
21.0		8.30	3.32
23.0		8.60	3.44
25.0		9.0	3.60
27.0		9.40	3.76
29.0		10.10	4.04

TABLE I.4

Static Pressure Distribution Along the Boundary
 Layer. Roughness D, $k = 4.75$ mms.

x inches	Manometer's reading cms of Alcohol $\rho \sin \alpha =$			Pressure cms of water
	0.05	0.10	0.20	
0.50	-8.30			-0.415
1.0	10.50			0.525
1.5	13.90			0.695
2.0	16.50			0.825
2.5		-9.50		0.950
3.0		10.90		1.09
3.5		12.70		1.27
4.0		14.20		1.42
4.5		14.90		1.49
5.0		16.10		1.61
5.5		16.60		1.66
6.0		16.80		1.68
6.5		18.50		1.85
7.5			-9.60	1.92
8.5			10.10	2.02
9.5			10.70	2.14
10.5			11.0	2.20
11.5			11.20	2.24
12.5			12.0	2.40
13.5			12.20	2.44
14.5			12.70	2.54
15.5			12.90	2.58
16.5			13.20	2.64
18.5			14.10	2.82
20.5			14.40	2.88
22.5			14.70	2.94
24.5			15.40	3.08
26.5			16.20	3.24
28.5			16.50	3.30

TABLE II.1

VELOCITY DISTRIBUTION

Roughness A, $k = 1.56$ mms.

x = 3.50 inches x = 5.5 inches x = 7.5 inches

y inches	u ft/sec.	y inches	u ft/sec.	y inches	u ft/sec.
0.07	36.50	0.08	37.30	0.08	36.80
0.10	39.00	0.11	40.80	0.12	42.20
0.13	42.10	0.14	43.90	0.15	45.20
0.16	44.10	0.17	47.00	0.18	48.00
0.19	44.40	0.20	49.30	0.21	50.60
0.22	44.80	0.23	49.70	0.24	52.20
0.25	45.70	0.26	49.90	0.27	52.80
0.28	45.90	0.29	50.00	0.30	53.40
0.32	46.20	0.33	50.10	0.33	53.70
0.37	46.30	0.38	50.40	0.36	53.90
0.42	46.50	0.43	50.70	0.39	54.00
0.47	46.80	0.48	50.90	0.44	54.10
0.57	46.90	0.53	51.00	0.49	54.20
0.67	47.00	0.58	51.20	0.54	54.30
0.77	47.10	0.68	51.40	0.59	54.30
0.87	47.20	0.78	51.60	0.69	54.40
1.07	47.30	0.88	51.70	0.79	54.50
1.27	47.40	0.98	51.80	0.89	54.50
1.47	47.40	1.08	51.80	0.99	54.60
1.67	47.40	1.28	51.90	1.09	54.70
		1.48	51.90	1.29	54.80
		1.68	51.90	1.49	54.80
				1.69	54.80

TABLE II.1. (cont.)VELOCITY DISTRIBUTIONRoughness A, $k = 1.56$ mms.

x = 9.5 inches		x = 11.5 inches		x = 15.5 inches	
y inches	u ft./sec.	y inches	u ft./sec.	y inches	u ft./sec.
0.07	36.00	0.07	36.70	0.07	37.00
0.10	40.10	0.10	39.80	0.10	40.10
0.14	44.10	0.13	43.20	0.13	44.00
0.17	46.90	0.16	46.10	0.16	46.50
0.20	49.50	0.19	49.00	0.19	48.80
0.23	51.40	0.22	51.10	0.22	51.10
0.26	52.90	0.25	52.40	0.25	52.40
0.29	54.20	0.28	54.20	0.28	54.30
0.32	54.70	0.31	55.30	0.31	55.70
0.35	55.10	0.34	55.90	0.34	56.70
0.38	55.50	0.37	56.40	0.37	58.20
0.43	55.80	0.40	56.70	0.42	59.40
0.48	56.20	0.43	56.90	0.47	59.60
0.53	56.40	0.47	57.10	0.52	59.90
0.58	56.60	0.52	57.30	0.57	60.10
0.63	56.70	0.57	57.60	0.62	60.20
0.68	56.90	0.62	57.80	0.67	60.30
0.78	57.00	0.67	58.00	0.77	60.50
0.88	57.10	0.77	58.20	0.87	60.70
1.08	57.20	0.87	58.40	0.97	60.80
1.28	57.30	0.97	58.70	1.07	60.90
1.48	57.30	1.17	59.00	1.27	61.10
1.68	57.30	1.37	59.20	1.47	61.20
		1.57	59.30	1.67	61.20

TABLE II.1 (Cont.)
 VELOCITY DISTRIBUTION
 Roughness A, $k = 1.56$ mms.

x = 19.5 inches		x = 23.5 inches		x = 27.5 inches	
y inches	u ft./sec.	y inches	u ft./sec.	y inches	u ft./sec.
0.07	36.80	0.07	35.30	0.08	36.00
0.11	41.10	0.10	39.50	0.11	40.10
0.14	44.50	0.13	43.30	0.14	44.40
0.17	47.40	0.16	46.40	0.17	47.20
0.20	49.90	0.19	48.30	0.20	49.80
0.23	51.40	0.22	50.80	0.23	52.00
0.26	53.60	0.25	52.70	0.26	53.50
0.29	54.90	0.28	54.60	0.29	54.90
0.32	56.10	0.31	56.30	0.32	56.60
0.35	57.40	0.34	57.30	0.35	58.00
0.38	59.30	0.37	58.80	0.40	60.10
0.43	60.30	0.42	60.30	0.45	61.40
0.48	61.10	0.47	61.70	0.50	63.40
0.53	61.60	0.52	62.20	0.55	64.00
0.58	62.00	0.57	62.80	0.60	64.30
0.63	62.20	0.62	63.20	0.65	64.70
0.68	62.40	0.67	63.50	0.70	65.00
0.78	62.60	0.77	63.90	0.75	65.20
0.88	62.70	0.87	64.20	0.85	65.60
0.98	62.90	0.97	64.40	0.95	65.90
1.08	63.00	1.07	64.60	1.05	66.10
1.28	63.10	1.27	64.90	1.25	66.50
1.48	63.20	1.47	65.30	1.45	66.90
1.68	63.30	1.67	65.50	1.65	67.30

TABLE II.2

Velocity Distribution

Roughness B, $k = 1.96$ mms.

$x = 3.50$ inches		$x = 5.5$ inches		$x = 7.5$ inches	
y inches	u ft./sec.	y inches	u ft./sec.	y inches	u ft./sec.
0.08	47.80	0.09	57.00	0.09	55.80
0.12	51.90	0.12	62.70	0.11	60.90
0.15	55.40	0.14	66.10	0.14	64.90
0.18	58.60	0.16	68.60	0.17	69.20
0.21	59.70	0.19	70.10	0.20	72.70
0.25	61.20	0.22	70.80	0.23	76.50
0.30	61.30	0.25	71.00	0.27	77.70
0.40	61.40	0.30	72.20	0.32	77.90
0.60	61.90	0.40	72.60	0.37	78.00
0.80	62.40	0.60	73.10	0.42	78.40
1.00	62.90	0.80	73.70	0.60	78.70
1.20	63.30	1.00	74.10	0.80	79.20
1.40	63.90	1.20	74.50	1.00	79.50
1.60	64.20	1.40	74.60	1.20	79.90
1.80	64.50	1.60	74.80	1.40	80.20
		1.80	74.90	1.60	80.50
				1.80	80.70

TABLE II.2 (cont.)

VELOCITY DISTRIBUTION

Roughness B, $k = 1.96$ mm.

x = 9.5 inches		x = 11.5 inches		x = 15.5 inches	
y inches	u ft./sec.	y inches	u ft./sec.	y inches	u ft./sec.
0.08	58.20	0.08	55.50	0.09	56.00
0.13	62.00	0.11	60.90	0.12	60.30
0.17	67.90	0.14	64.70	0.15	65.10
0.20	72.10	0.17	69.50	0.18	69.10
0.23	75.20	0.20	72.00	0.21	71.30
0.26	78.00	0.23	75.50	0.25	75.60
0.30	80.90	0.27	78.80	0.30	80.50
0.35	82.00	0.32	82.70	0.35	82.70
0.40	83.10	0.37	84.30	0.40	85.00
0.45	83.30	0.42	85.80	0.45	87.70
0.55	83.30	0.47	86.00	0.50	87.90
0.60	83.70	0.52	86.30	0.60	88.10
0.80	83.80	0.60	87.00	0.70	88.70
1.00	83.90	0.70	87.10	0.80	89.10
1.20	83.90	0.80	87.20	1.00	89.80
1.40	84.00	1.00	87.20	1.20	90.10
1.60	84.00	1.20	87.30	1.40	90.40
1.80	84.10	1.40	87.30	1.60	90.50
		1.60	87.30	1.80	90.60
		1.80	87.40		

+
TABLE II.2 (cont.)

VELOCITY DISTRIBUTION

Roughness B, $k = 1.96$ mms.

x = 19.5 inches		x = 23.5 inches		x = 27.5 inches	
y inches	u ft./sec.	y inches	u ft./sec.	y inches	u ft./sec.
0.08	54.00	0.08	52.40	0.08	54.80
0.12	60.60	0.12	59.30	0.12	60.30
0.15	65.00	0.15	64.60	0.15	65.20
0.18	69.40	0.18	70.10	0.19	69.90
0.21	72.20	0.21	73.30	0.22	73.80
0.25	75.80	0.25	76.70	0.26	77.00
0.30	79.90	0.30	80.20	0.32	82.10
0.35	83.80	0.35	83.20	0.37	85.40
0.40	87.20	0.40	87.00	0.42	88.30
0.45	89.00	0.45	88.80	0.47	91.00
0.50	90.10	0.50	92.10	0.52	93.00
0.55	91.40	0.55	93.10	0.57	95.10
0.60	91.60	0.60	93.80	0.62	95.80
0.70	91.80	0.70	94.40	0.70	97.20
0.80	92.10	0.80	95.00	0.80	97.70
0.90	92.50	0.90	95.60	0.90	98.20
1.00	92.80	1.00	95.90	1.00	98.70
1.20	93.60	1.20	96.30	1.20	99.60
1.40	94.00	1.40	96.70	1.40	100.10
1.60	94.20	1.60	97.50	1.60	100.30
1.80	94.30	1.80	97.60		

TABLE II.3

VELOCITY DISTRIBUTION

ROUGHNESS C, $k = 3.16$ mms.

x = 3.5 inches		x = 5.5. inches		x = 7.5 inches	
y inches	u ft./sec.	y inches	u ft./sec.	y inches	u ft./sec.
0.10	41.50	0.11	43.00	0.10	41.10
0.14	42.30	0.14	46.40	0.12	45.20
0.17	45.00	0.16	48.40	0.15	49.30
0.20	45.30	0.18	50.80	0.18	51.80
0.23	46.20	0.21	52.70	0.21	54.10
0.26	47.30	0.25	53.10	0.25	56.60
0.30	47.50	0.30	54.20	0.32	59.00
0.40	47.90	0.35	54.70	0.37	59.10
0.50	48.50	0.45	55.10	0.47	59.40
0.60	49.00	0.55	55.50	0.57	59.50
0.80	49.40	0.75	56.00	0.77	59.90
1.00	50.10	0.95	56.90	0.97	61.10
1.20	50.50	1.15	57.10	1.17	61.20
1.40	51.10	1.35	57.40	1.37	61.90
1.60	51.60	1.55	57.70	1.57	62.30
1.80	51.90	1.75	58.10	1.77	62.50

TABLE II.3 (Cont.)
 VELOCITY DISTRIBUTION
 ROUGHNESS C, $k = 3.16$ mms.

x = 9.5 inches		x = 11.5 inches		x = 15.5 inches	
y inches	u ft./sec.	y inches	u ft./sec.	y inches	u ft./sec.
0.10	42.80	0.10	42.50	0.10	41.90
0.12	43.80	0.13	46.00	0.11	43.00
0.15	47.60	0.16	49.00	0.14	47.30
0.18	51.30	0.19	52.00	0.17	50.80
0.21	54.30	0.22	55.20	0.20	53.80
0.24	56.50	0.25	58.00	0.23	56.00
0.28	58.00	0.29	60.10	0.27	58.90
0.33	60.90	0.34	61.90	0.32	62.30
0.38	61.40	0.39	62.90	0.37	63.90
0.43	61.80	0.44	62.85	0.42	65.30
0.48	62.00	0.49	64.40	0.47	66.20
0.58	62.30	0.57	64.70	0.52	66.30
0.78	62.90	0.67	65.00	0.57	66.70
0.98	63.15	0.77	65.10	0.67	67.30
1.18	63.80	0.97	65.20	0.77	68.00
1.38	64.30	1.17	65.80	0.97	68.70
1.58	64.40	1.37	66.40	1.17	69.20
1.78	64.60	1.57	67.00	1.37	69.50
		1.77	67.50	1.57	69.80
				1.77	69.90

TABLE II.3 (cont.)

VELOCITY DISTRIBUTION

ROUGHNESS C, $k = 3.16$ mms.

x = 19.5 inches		x = 23.5 inches		x = 27.5 inches	
y inches	u ft./sec.	y inches	u ft./sec.	y inches	u ft./sec.
0.10	40.60	0.10	41.00	0.10	42.50
0.12	44.30	0.11	43.50	0.13	45.80
0.15	48.50	0.15	48.20	0.16	49.70
0.18	51.70	0.18	51.60	0.19	53.00
0.21	54.20	0.20	53.20	0.22	55.50
0.25	58.00	0.23	55.90	0.25	57.70
0.30	61.50	0.27	59.50	0.29	61.10
0.35	63.40	0.32	62.00	0.34	64.80
0.40	66.90	0.37	65.30	0.39	67.40
0.45	67.00	0.42	67.70	0.44	69.70
0.50	68.80	0.47	69.10	0.49	71.50
0.55	70.10	0.52	70.20	0.54	72.90
0.65	70.20	0.57	71.50	0.59	74.30
0.75	72.10	0.62	72.30	0.64	76.00
0.95	72.40	0.67	72.90	0.69	76.70
1.15	73.00	0.77	73.70	0.79	78.00
1.35	73.20	0.87	74.10	0.89	78.80
1.55	73.40	0.97	74.90	1.00	79.00
1.75	73.80	1.17	76.00	1.20	79.70
		1.37	76.50	1.40	80.50
		1.57	76.80	1.60	81.00
		1.77	76.80	1.80	81.50

TABLE II.4

VELOCITY DISTRIBUTION

ROUGHNESS D, $k = 4.75$ mms.

x = 3.0 inches		x = 5.0 inches		x = 7.0 inches	
y inches	u ft./sec.	y inches	u ft./sec.	y inches	u ft./sec.
0.14	33.70	0.13	37.00	0.14	37.30
0.18	35.50	0.16	39.10	0.18	41.20
0.21	37.60	0.19	42.00	0.21	43.60
0.25	39.70	0.23	45.00	0.25	47.50
0.28	41.30	0.26	46.80	0.28	49.10
0.31	41.80	0.29	48.40	0.31	51.80
0.35	42.80	0.33	51.00	0.35	54.00
0.40	43.40	0.38	51.40	0.38	54.30
0.45	43.60	0.43	52.30	0.41	55.30
0.55	43.90	0.53	52.50	0.45	55.40
0.65	44.10	0.63	52.60	0.50	55.80
0.75	44.40	0.73	52.70	0.55	56.00
0.95	44.80	0.93	53.00	0.65	56.10
1.15	45.20	1.13	53.70	0.75	56.70
1.45	45.50	1.43	54.00	0.95	57.70
1.75	46.00	1.73	54.50	1.15	57.90
				1.45	58.10
				1.75	58.30

TABLE II. 4 (cont.)

VELOCITY DISTRIBUTION

ROUGHNESS D, $k = 4.75$ mms.

x = 9.0 inches		x = 11.0 inches		x = 15.0 inches	
y inches	u ft./sec.	y inches	u ft./sec.	y inches	u ft./sec.
0.14	37.10	0.14	37.40	0.13	35.30
0.15	38.10	0.17	39.50	0.16	37.50
0.18	42.00	0.20	42.30	0.19	41.60
0.22	45.60	0.22	44.40	0.23	45.50
0.25	48.80	0.24	46.00	0.26	48.20
0.28	50.70	0.27	48.20	0.31	51.50
0.32	52.20	0.29	49.60	0.35	54.80
0.38	55.30	0.33	52.70	0.40	56.50
0.43	57.20	0.38	55.70	0.45	58.30
0.48	57.30	0.43	56.50	0.50	60.40
0.53	58.00	0.48	57.30	0.55	62.40
0.63	58.50	0.53	57.70	0.60	62.80
0.73	58.60	0.58	58.70	0.65	63.10
0.93	59.30	0.63	59.00	0.75	63.20
1.13	60.00	0.68	59.20	0.85	63.40
1.33	60.20	0.78	59.60	0.95	63.90
1.53	60.40	0.88	60.30	1.05	64.20
1.73	61.00	0.98	60.90	1.15	65.00
1.93	61.10	1.08	61.30	1.35	65.50
		1.18	61.60	1.55	66.00
		1.38	62.40	1.75	66.50
		1.58	63.00		
		1.78	63.10		

TABLE II.4 (cont)

VELOCITY DISTRIBUTION

ROUGHNESS D, $k = 4.75$ mms.

x = 19.0 inches		x = 23.0 inches		x = 27.0 inches	
y inches	u ft./sec.	y inches	u ft./sec.	y inches	u ft./sec.
0.15	40.30	0.14	39.00	0.14	37.50
0.18	40.80	0.18	41.20	0.18	40.80
0.21	44.00	0.23	45.90	0.21	43.80
0.25	46.20	0.26	48.20	0.25	47.30
0.28	48.80	0.30	51.00	0.28	50.00
0.31	51.30	0.33	52.70	0.31	52.40
0.35	53.20	0.36	54.20	0.35	54.90
0.40	56.60	0.40	56.50	0.40	57.20
0.45	58.70	0.45	58.40	0.45	59.70
0.50	59.90	0.50	60.60	0.50	62.30
0.55	62.00	0.55	63.60	0.55	63.50
0.60	63.30	0.60	65.30	0.60	65.70
0.65	64.30	0.65	66.00	0.65	66.80
0.70	65.20	0.70	66.70	0.70	68.00
0.75	65.40	0.75	68.20	0.75	69.50
0.85	66.30	0.85	68.90	0.80	69.70
0.95	66.90	0.95	69.50	0.85	70.00
1.05	67.30	1.05	70.00	0.95	70.70
1.15	67.70	1.15	70.40	1.05	72.00
1.35	68.10	1.35	70.80	1.15	72.60
1.55	68.50	1.55	71.30	1.35	72.70
1.75	68.80	1.75	72.00	1.55	73.0
				1.75	73.40

TABLE III .1.

TOTAL DRAG ALONG THE WALL

ROUGHNESS "A", $k = 1.56$ mms.,
 $h = 1.40$ inches

x inches	U ft./sec.	δ_x inch	$U\delta_v$ $\frac{\text{ft.}}{\text{sec.}} \cdot \text{inch}$	$\int_0^x \rho U dU \delta_x$ $\frac{\text{lb.}}{\text{ft}^2} \cdot \text{inch}$	$2 \int_0^h \rho dy$ $\frac{\text{lb.}}{\text{ft}^2} \cdot \text{inch}$	$\int_0^h \Delta p dy$ $\frac{\text{lb.}}{\text{ft}^2} \cdot \text{inch}$	D $\frac{\text{lb.}}{\text{ft}^2} \cdot \text{inch}$
3.50	47.40	0.064	3.03	0.200	-0.385	-0.082	0.267
5.50	51.90	0.0735	3.82	0.290	-0.685	-0.020	0.415
7.50	54.80	0.0815	4.47	0.375	-0.940	-0.010	0.575
9.50	57.40	0.091	5.22	0.470	-1.170	0.00	0.700
11.50	59.20	0.102	6.04	0.570	-1.40	0.00	0.83
15.50	61.20	0.112	6.85	0.700	-1.85	0.00	1.15
19.50	63.15	0.125	7.89	0.850	-2.30	0.00	1.45
23.50	65.20	0.139	9.06	1.020	-2.82	0.00	1.80
27.50	66.85	0.151	10.09	1.180	-3.31	0.00	2.13

TABLE III.2.

TOTAL DRAG ALONG THE WALL

ROUGHNESS "B ", $k = 1.96$ mms.,
 $h = 1.40$ inches

3.50	63.90	0.074	4.73	0.380	-0.91	-0.125	0.655
5.50	74.60	0.082	6.12	0.605	-1.45	-0.07	0.915
7.50	80.20	0.089	7.14	0.785	-1.905	-0.042	1.162
9.50	84.00	0.097	8.15	0.990	-2.53	-0.01	1.55
11.50	87.30	0.105	9.17	1.180	-3.10	0.00	1.92
15.50	90.40	0.124	11.21	1.610	-4.43	0.00	2.82
19.50	94.00	0.140	13.16	2.030	-5.54	0.00	3.51
23.50	96.70	0.154	14.89	2.420	-6.62	0.00	4.20
27.50	100.10	0.170	17.02	2.90	-7.80	0.00	4.90

TABLE III.3

TOTAL DRAG ALONG THE WALL
ROUGHNESS C, $k = 3.16$ mms,

$h = 1.40$ inches

x in.	U ft./sec.	δ_x in.	$U\delta_x$ $\frac{ft.}{sec.} \cdot inch$	$\int_0^x \rho U \delta_x dx$ $\frac{lb.}{ft^2} \cdot inch$	$2 \int_0^x p' dy$ $\frac{lb.}{ft^2} \cdot inch$	$\int_0^x \Delta p dy$ $\frac{lb.}{ft^2} \cdot inch$	D $\frac{lb.}{ft^2} \cdot inch$
3.50	51.10	0.082	4.19	0.275	-0.61	-0.04	0.375
5.50	57.45	0.095	5.46	0.44	-1.05	-0.01	0.62
7.50	61.90	0.105	6.49	0.56	-1.39	-0.01	0.84
9.50	64.30	0.117	7.52	0.70	-1.78	0.0	1.08
11.50	66.40	0.127	8.43	0.84	-2.20	0.0	1.36
15.50	69.50	0.143	9.93	1.10	-2.90	0.0	1.80
19.50	73.25	0.158	11.57	1.38	-3.70	0.0	2.32
23.50	76.50	0.178	13.60	1.78	-4.62	0.0	2.84
27.50	80.50	0.197	15.86	2.15	-5.55	0.0	3.40

TABLE III.4

TOTAL DRAG ALONG THE WALL
ROUGHNESS D, $k = 4.75$ mms.,

$h = 1.40$ inches

x in.	U ft./sec.	δ_x in.	$U\delta_x$ $\frac{ft.}{sec.} \cdot inch$	$\int_0^x \rho U \delta_x dx$ $\frac{lb.}{ft^2} \cdot inch$	$2 \int_0^x p' dy$ $\frac{lb.}{ft^2} \cdot inch$	$\int_0^x \Delta p dy$ $\frac{lb.}{ft^2} \cdot inch$	D $\frac{lb.}{ft^2} \cdot inch$
3.0	45.45	0.109	4.95	0.315	-0.67	-0.10	0.455
5.0	53.95	0.121	6.53	0.54	-1.20	-0.04	0.70
7.0	58.00	0.143	8.29	0.76	-1.68	-0.01	0.93
9.0	60.30	0.156	9.41	0.92	-2.15	0.0	1.23
11.0	62.40	0.175	10.92	1.14	-2.70	0.0	1.56
15.0	65.60	0.192	12.59	1.39	-3.39	0.0	2.00
19.0	68.10	0.208	14.16	1.63	-4.23	0.0	2.60
23.0	70.90	0.222	15.74	1.89	-4.90	0.0	3.01
27.0	72.80	0.242	17.62	2.20	-5.87	0.0	3.67

TABLE IV

VELOCITY PROFILE AT SEPARATION

k = 1.56 mms.

y inches	u ft./sec.	$\frac{\sqrt{u'^2}}{u}$	$\frac{\sqrt{u'^2}}{u}$
0.02	2.25	0.67	0.032
0.04	2.40	0.65	0.0335
0.07	2.45	0.70	0.0365
0.10	3.80	0.63	0.051
0.14	4.50	0.61	0.0585
0.19	6.25	0.54	0.072
0.24	7.60	0.52	0.084
0.34	12.20	0.52	0.1350
0.44	15.50	0.35	0.115
0.54	20.20	0.27	0.115
0.64	23.50	0.205	0.102
0.74	27.50	0.128	0.075
0.84	29.00	0.100	0.0615
0.94	32.50	0.062	0.043
1.14	39.00	0.035	0.029
1.34	43.00	0.016	0.0146
1.54	46.00	0.010	0.0098
1.74	47.00	0.005	0.005
1.84	47.00	0.005	0.005

TABLE V

VELOCITY DISTRIBUTION IN THE REGION VERY CLOSE TO

SMOOTH WALLS.

$$\frac{u}{V_*} = \frac{yV_*}{\nu} - \int_0^y \frac{\overline{u'v'}}{\nu V_*} dy$$

Assumption: $(\overline{\rho u'v'}) = c_1 y$			Assumption: $(\overline{\rho u'v'})^2 = c_2 y$		
$\frac{yV_*}{\nu}$	$\int_0^y \frac{\overline{u'v'}}{\nu V_*} dy$	$\frac{u}{V_*}$	$\frac{yV_*}{\nu}$	$\int_0^y \frac{\overline{u'v'}}{\nu V_*} dy$	$\frac{u}{V_*}$
1.0	0.01923	0.98077	1.0	0.057168	0.942832
2.0	0.07692	1.92308	2.0	0.181480	1.81852
3.0	0.17307	2.82693	3.0	0.35674	2.64326
4.0	0.30768	3.69232	4.0	0.57624	3.42376
5.0	0.48077	4.51923	5.0	0.83579	4.16421
6.0	0.69232	5.30768	6.0	1.1327	4.8673
7.0	0.94232	6.05768	7.0	1.4644	5.5356
8.0	1.23078	6.76922	8.0	1.8294	6.1706
9.0	1.55771	7.44229	9.0	2.2261	6.7739
10.0	1.92310	8.07690	10.0	2.6536	7.3464
12.0	2.76926	9.23074	12.0	3.5958	8.4042
14.0	3.76928	10.23072	14.0	4.6492	9.3508
16.0	4.92314	11.07686	16.0	5.8080	10.1920
18.0	6.23084	11.76916	18.0	7.0677	10.9323
20.0	7.69240	12.30760	20.0	8.4240	11.5760
22.0	9.30780	12.69220	22.0	9.8634	12.1366
24.0	11.07692	12.92308	24.0	11.415	12.5850
26.0	13.00	13.00	26.0	13.0450	12.9550
			28.0	14.7600	13.2400
			30.0	16.559	13.441
			32.0	18.440	13.560
			34.0	20.40	13.60