

Junctions between two-dimensional plasmonic waveguides in the presence of retardation

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Abstract. Plasmons in two-dimensional waveguides are traditionally analysed within the electrostatic approximation, which assumes that the plasmon phase velocity is much smaller than the velocity of light. However, novel effects have recently been demonstrated for plasmons whose velocity is comparable to the velocity of light. In this retardation regime, electrostatic models are inaccurate. For a junction between two plasmonic waveguides, we present an analytical and a numerical model both valid in the retardation regime and compare them to an electrostatic model. We quantify the reflected and transmitted powers and the radiation loss in several scenarios. We found that power is radiated from a junction at the expense of the power of the reflected plasmon, but retardation has little effect on the phases of the reflected and transmitted plasmons. The radiation loss is typically below several percent when the plasmon velocities are five or more times below the light velocity. However, radiation still persists for slower plasmon velocities for a junction between a two-dimensional waveguide and a perfectly conducting sheet. As a result, retardation is expected to degrade the quality factors of plasmonic resonators without affecting their eigenfrequencies.

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1. Introduction

Two-dimensional (2D) electron systems can act as waveguides supporting guided electromagnetic waves known as plasmons. The current interest in 2D plasmons has been led by advances in material fabrication, such as graphene [1, 2], as well as by the search for novel terahertz devices [2, 3, 4, 5, 6]. Plasmon terahertz emitters and detectors often rely of a combination of 2D waveguides with different parameters (such as electron density and chemical potential) [7, 8, 9]. The frequency response of these devices will be determined by plasmon reflection from and transmission through the junctions formed by different 2D waveguides [10, 11, 12, 13].

In the simplest geometry, the junction is formed by two ungated 2D waveguides as shown in figure 1. In Ref. [14], Rejaei and Khavasi obtained elegant analytical expressions for the plasmon reflection and transmission coefficients. These expressions, however, were derived in the electrostatic approximation, assuming that the plasmon velocities in both waveguides are much smaller than the light velocity in the dielectric. Radiation is ignored, so that the power of the incident plasmon is split only between a reflected and a transmitted plasmon. Although the electrostatic approximation is valid in many practical scenarios, recent experimental and theoretical works [15, 16, 17] have also shown novel effects in the retardation regime, in which the plasmon velocity is comparable to the light velocity and the electrostatic approximation fails. It is, therefore, of interest to consider the junction shown in figure 1 in the presence of retardation and, in particular, to quantify the radiation loss.

Section 2 reviews the properties of the eigenmode spectrum of an isolated 2D waveguide. Section 3 formulates the problem of a plasmon incident on a junction and shows how the plasmon reflection and transmission coefficients as well as the radiated power can be found using a numerical and an approximate analytical techniques. Section 4 applies the formulation to several examples. Section 5 discusses the results and draws conclusions.

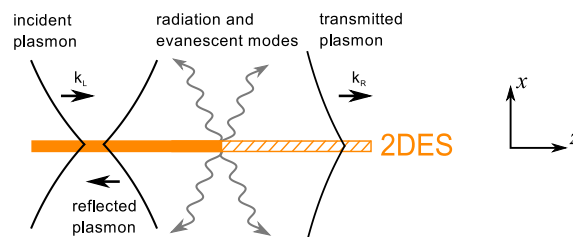


Figure 1. Schematic presentation of a junction between two-dimensional waveguides. Evanescent and radiation modes are excited at the junction in addition to transmitted and reflected plasmons.

2. Waveguide eigenmode spectrum

One method to analyse a waveguide junction, developed previously for metallic [18] and dielectric [19, 20, 21, 22] waveguides, is to expand the fields at the both sides of the junction into the eigenmodes of the left and right waveguides. The eigenmode spectra are derived under the condition that the field amplitudes do not grow away from the waveguide [23]. The spectrum of an ungated 2D plasmonic waveguide was discussed in Ref. [24], and we restate the results below. The waveguide occupies the plane $x = 0$, as shown in figure 1. A time variation of the form $\exp(j\omega t)$ is implied throughout.

The eigenmode spectrum can be separated into a discrete part and a continuous part. The discrete part contains a single mode that is the plasmon. It is a TM wave with the x - and z -components of the electric field and a y -component of the magnetic field. The magnetic field component for $x \geq 0$ can be written in the form

$$H(x, z) = Ae^{-jkz - \kappa x} \quad (1)$$

where A is a constant; k is the longitudinal wavenumber (in the z -direction) and κ is the decay rate. Both κ and k are positive and satisfy the dispersion relation in the dielectric in the form

$$k^2 - \kappa^2 = k_0^2, \quad (2)$$

where $k_0 = \omega/c$ and c is the light velocity in the dielectric.

We will assume that the waveguide has a lossless conductivity σ of the standard Drude form $\sigma(\omega) \sim 1/(j\omega)$. Applying the standard field boundary conditions at $x = 0$ leads then to the following dispersion relation

$$\Omega^2 \kappa = \omega^2, \quad (3)$$

where Ω is a constant. It is determined by the electron density and the effective mass in 2D semiconductor systems. In graphene, it is determined by the chemical potential. Figure 2 shows the normalised plasmon dispersion curve together with the light line $\omega = kc$. Also shown is the dispersion curve in the electrostatic approximation, $k = \kappa$. The full dispersion curve approaches the light line from the left at low frequency, whereas the electrostatic dispersion curve fails, expectedly, to reproduce this behaviour. The electrostatic dispersion curve approaches the full one as the frequency and the wavenumber increase.

Other eigenmodes of the waveguide have a continuous spectrum, for which the value of the transverse wavenumber k varies between zero and infinity. Since the plasmon is an odd mode (for which the magnetic field has opposite values at the opposite sides of the waveguide), we need consider only the continuous spectrum of TM modes having the same symmetry. Their magnetic field component can be written in the form

$$h(q, x, z) = A \left[\sin(qx) - \frac{q}{\kappa} \cos(qx) \right] e^{-jkz}, \quad (4)$$

where A is a constant. The wavenumbers k and q satisfy the dispersion relation in the dielectric of the form

$$k^2 + q^2 = k_0^2. \quad (5)$$

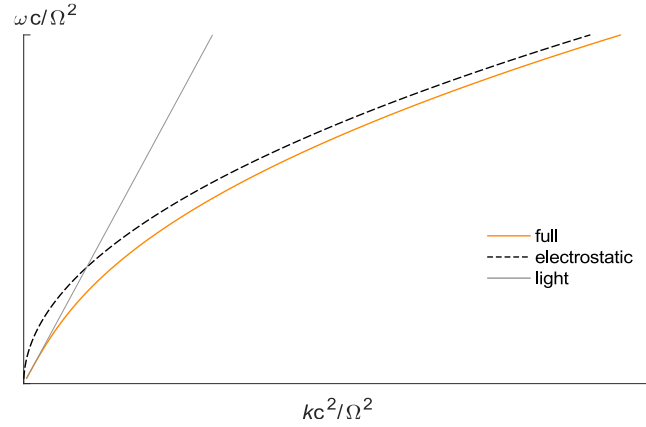


Figure 2. The full dispersion curve deviated from that obtained in the electrostatic approximation when the plasmon velocity approaches the light velocity.

When $0 \leq q \leq k_0$, the value of k is a real positive number and corresponds to a radiation mode. When $q > k_0$, the value of k is imaginary and corresponds to an evanescent mode.

All waveguide modes obey orthogonality relations in the form

$$\int_0^\infty H(x, z) h^*(q, x, z) dx = 0$$

$$\int_0^\infty h(\tilde{q}, x, z) h^*(\tilde{q}, x, z) dx = \frac{\pi}{2} \left(\frac{q^2}{\kappa^2} + 1 \right) \delta(q - \tilde{q})$$
(6)

where $*$ denotes complex conjugation, and $\delta(q)$ is the Dirac delta.

3. Theoretical formulations

We now consider a junction at $z = 0$ formed by two different waveguides as shown in figure 1. A plasmon is incident on the junction from the left waveguide, and we denote its wavenumbers by k_L and κ_L . This plasmon will partially reflect back. It will also partially transmit into the plasmon in the right waveguide with the wavenumbers k_R and κ_R . The continuous-spectrum modes will also be excited at the both sides of the junction.

The boundary conditions are the continuity of the y -component of the magnetic field and the x -component of the electric field leading to [25]

$$(1 + R)H_L(x, 0) + \int_0^\infty r(q)h_L(q, x, 0)dq$$

$$= TH_R(x, 0) + \int_0^\infty t(q)h_R(q, x, 0)dq$$
(7)

for the magnetic field, and to

$$\begin{aligned} k_L(1-R)H_L(x,0) - \int_0^\infty q r(q)h_L(q,x,0)dq \\ = k_RT H_R(x,0) + \int_0^\infty q t(q)h_R(q,x,0)dq \end{aligned} \quad (8)$$

for the electric field. Here, the amplitude of the incident plasmon is equal to unity; subscripts L and R refer to the left and the right waveguide; R is the plasmon reflection coefficient; T is the plasmon transmission coefficient; $r(q)$ and $t(q)$ characterise the scattering into the continuous modes of the left and right waveguides.

Equations (7) and (8) need to be solved for R , T , $r(q)$, $t(q)$. They have the same form as the equations for step discontinuities in dielectric waveguides, and can be solved using the same methods [19, 20]. Denoting the x -component of the electric field at the junction as $\mathcal{E}(x)$, we can rewrite (8) as

$$k_L(1-R)H_L(x,0) - \int_0^\infty q r(q)h_L(q,x,0)dq = \mathcal{E}(x) \quad (9)$$

and

$$k_RT H_R(x,0) + \int_0^\infty q t(q)h_R(q,x,0)dq = \mathcal{E}(x) \quad (10)$$

Multiplying (9) and (10) in turns by $H_L(x,0)$, $H_R(x,0)$, $h_L(q,x,0)$ and $h_R(q,x,0)$, integrating $\int_0^\infty dx$, and using the orthogonality relations (6) leads to

$$R = 1 - \frac{2\kappa_L}{k_L} \int_0^\infty \mathcal{E}(x)H_L(x,0)dx \quad (11)$$

$$T = \frac{2\kappa_R}{k_R} \int_0^\infty \mathcal{E}(x)H_R(x,0)dx \quad (12)$$

$$r(q) = -\frac{2\kappa_L^2}{\pi k(q^2 + \kappa_L^2)} \int_0^\infty \mathcal{E}(x)h_L(q,x,0)dx \quad (13)$$

$$t(q) = \frac{2\kappa_R^2}{\pi k(q^2 + \kappa_R^2)} \int_0^\infty \mathcal{E}(x)h_R(q,x,0)dx \quad (14)$$

Substituting (11)–(14) into (7) and rearranging leads to the following integral equation for $\mathcal{E}(x)$

$$2H_L(x,0) = \int_0^\infty Y(x,x')\mathcal{E}(x')dx' \quad (15)$$

with

$$\begin{aligned} Y(x,x') = & \frac{2\kappa_L}{k_L} H_L(x,0)H_L(x',0) + \frac{2\kappa_R}{k_R} H_R(x,0)H_R(x',0) + \\ & \frac{2}{\pi} \int_0^\infty \frac{\kappa_L^2}{k(q^2 + \kappa_L^2)} h_L(q,x,0)h_L(q,x',0)dq + \\ & \frac{2}{\pi} \int_0^\infty \frac{\kappa_R^2}{k(q^2 + \kappa_R^2)} h_R(q,x,0)h_R(q,x',0)dq \end{aligned} \quad (16)$$

To derive (15), we used the standard field boundary conditions at the junction ($z = 0$) and at the waveguides ($x = 0$). In our electromagnetic approach no other boundary conditions are needed, and the ac current and electron densities can be calculated once (15) has been solved and the modal coefficients (11)–(14) have been found. As suggested in Ref. [10], the field boundary conditions are consistent with continuity of the ac current density and the ac electron density at the junction. The boundary conditions on the current and electron density satisfy the continuity equation, and results obtained from the electromagnetic approach conserve power. While the field boundary conditions alone are sufficient for our model, boundary conditions on the electron density and current may need to be invoked when using mode orthogonality in analysis of plasmons in the presence of dc currents [10, 26]. A further attractive feature of our electromagnetic approach is that it can automatically incorporate electron diffusion [27], whereas other approaches may need to postulate additional boundary conditions [28]. Alternative approaches to the boundary conditions at a junction include the ballistic current condition and continuity of the power flux, of the potential, and of the carrier energy [11, 13]. Having compared several boundary conditions to each other, Petrov *et al.* [13] found (albeit for a different geometry and ignoring retardation) that they gave similar results for plasmon reflection coefficients.

We solved (15) by two methods, a numerical and an analytical one, as described below.

3.1. Numerical solution

Equation (15) can be solved numerically using the Ritz-Galerkin procedure [19, 20]. Choosing a complete set of basis orthogonal functions $\psi_n(x)$, the unknown field $\mathcal{E}(x)$ is expanded into a series of the form

$$\mathcal{E}(x) = \sum_n \lambda_n \psi_n(x), \quad (17)$$

where λ_n is a constant. Substituting (17) into (15), multiplying by $\psi_m(x)$ and integrating over x transforms the integral equation into an infinite matrix equation of the form

$$2S_m(\kappa_L) = \sum_n Y_{mn} \lambda_n, \quad (18)$$

where

$$\begin{aligned} Y_{mn} = & \frac{2\kappa_L}{k_L} S_n(\kappa_L) S_m(\kappa_L) + \frac{2\kappa_R}{k_R} S_n(\kappa_R) S_m(\kappa_R) + \\ & \frac{2}{\pi} \int_0^\infty \frac{\kappa_L^2}{k(q^2 + \kappa_L^2)} s_n(q, \kappa_L) s_m(q, \kappa_L) dq + \\ & \frac{2}{\pi} \int_0^\infty \frac{\kappa_R^2}{k(q^2 + \kappa_R^2)} s_n(q, \kappa_R) s_m(q, \kappa_R) dq \end{aligned} \quad (19)$$

and $S_n(\kappa)$ and $s_n(q, \kappa)$ are the overlap integrals with the waveguide eigenmodes and the basis functions, defined as $S_n(\kappa) = \int_0^\infty \psi_n(x) H(x, 0) dx$ and $s_n(q, \kappa) =$

$\int_0^\infty \psi_n(x) h(q, x, 0) dx$. Provided the basis functions $\psi_n(x)$ are chosen well, the series in (17) will converge quickly and can be truncated. Equation (18) can then be solved numerically. We choose the basis functions as the Laguerre polynomials $L_n(x)$ weighted by an exponential, so that

$$\psi_n(x) = L_n\left(\frac{x}{x_0}\right) \exp\left(-\frac{x}{2x_0}\right). \quad (20)$$

Here x_0 is a constant. These basis functions were originally employed by Rozzi [19] to represent fields outside open dielectric waveguides. When a non-radiative surface wave is incident on a step discontinuity in a waveguide, some of its power is converted into radiation. Rozzi found that less than ten Laguerre polynomials were sufficient to ensure convergence of the results. This method was found to be in excellent agreement with others (see e.g. [29, 20]), including those based on discretising the continuous mode spectrum. We present convergence results for our geometry in Sec. 4.

The constant x_0 is chosen to represent well the plasmons in the left and right waveguide, and we found that, for the examples discussed in Sec. 4, the value $x_0 = 4/(\kappa_L + \kappa_R)$ satisfied the formal criterion defined in Ref. [19]. The overlap integrals $S_n(\kappa)$ and $s_n(q, \kappa)$ can be found analytically, but the matrix coefficients Y_{mn} need to be evaluated numerically.

3.2. Analytical solution

In approximate models (especially when the difference between the left and right waveguides is small), the field at the boundary is often presented either in the form of an incident surface wave or as a combination of an incident and a transmitted surface waves [19, 20]. Accordingly, we write $\mathcal{E}(x)$ as a combination of the incident and the reflected plasmons

$$\mathcal{E}(x) = C_L H_L(x, 0) + C_R H_R(x, 0), \quad (21)$$

where C_L and C_R are two unknown constants. Substituting (21) into (15) and (16), multiplying in turns by $H_L(x, 0)$ and $H_R(x, 0)$ and integrating over x leads to two equations for C_L and C_R in the form

$$\begin{aligned} D_{11}C_L + D_{12}C_R &= 1 \\ D_{21}C_L + D_{22}C_R &= 2 \end{aligned} \quad (22)$$

where

$$D_{11} = \frac{2\kappa_R\kappa_L}{k_R(\kappa_R + \kappa_L)^2} + \frac{1}{2k_L} + \frac{2(\kappa_R - \kappa_L)^2}{\pi\kappa_L^4} \int_0^\infty \frac{1}{\sqrt{\frac{k_0^2}{\kappa_L^2} - \xi^2}} \frac{\xi^2 d\xi}{(\xi^2 + 1)^2 \left(\xi^2 + \frac{\kappa_R^2}{\kappa_L^2}\right)} \quad (23)$$

$$D_{12} = \frac{\kappa_L}{\kappa_L + \kappa_R} \left(\frac{1}{k_L} + \frac{1}{k_R} \right) \quad (24)$$

$$D_{21} = \frac{k_L + k_R}{k_L k_R} \quad (25)$$

$$D_{22} = \frac{2\kappa_L}{k_L(\kappa_L + \kappa_R)} + \frac{\kappa_L + \kappa_R}{2\kappa_R k_R} + \frac{2(\kappa_R - \kappa_L)^2}{\pi \kappa_R^4} \int_0^\infty \frac{1}{\sqrt{\frac{k_0^2}{\kappa_R^2} - \xi^2}} \frac{\xi^2 d\xi}{(\xi^2 + 1)^2 \left(\xi^2 + \frac{\kappa_L^2}{\kappa_R^2} \right)} \quad (26)$$

Once C_L and C_R are known, the reflection and transmission coefficients can be found as

$$R = 1 - \frac{1}{k_L} \left(C_L + \frac{2\kappa_L C_R}{\kappa_L + \kappa_R} \right) \quad (27)$$

$$T = \frac{1}{k_R} \left(C_R + \frac{2\kappa_R C_L}{\kappa_R + \kappa_L} \right) \quad (28)$$

$$r(q) = -\frac{2q\kappa_L(\kappa_L - \kappa_R)}{\pi k(q^2 + \kappa_L^2)(q^2 + \kappa_R^2)} C_R \quad (29)$$

$$t(q) = \frac{2q\kappa_R(\kappa_R - \kappa_L)}{\pi k(q^2 + \kappa_L^2)(q^2 + \kappa_R^2)} C_L \quad (30)$$

In a recent study [30], we used a similar presentation of the fields at a junction to find plasmon transmission and reflection coefficients, albeit for a different geometry of a junction between gated and ungated 2D waveguides. We found excellent agreement between this method and numerical calculations based on discretised modal equations and full-wave simulations.

3.3. Analytical solutions in the electrostatic approximation

Analytical expressions for plasmon transmission and reflection coefficients were presented in Ref. [14]. In our notation, they are

$$R = \frac{k_L - k_R}{k_L + k_L} e^{-j\theta} \quad (31)$$

$$T = \frac{2\sqrt{k_L k_R}}{k_L + k_R} \quad (32)$$

where

$$\theta = \frac{\pi}{4} - \frac{2}{\pi} \int_0^\infty \frac{\text{atan} \frac{k_L x}{k_R}}{x^2 + 1} dx \quad (33)$$

4. Examples

In the electrostatic regime, the only parameter determining plasmon reflection and transmission coefficients is the ratio between the plasmon phase velocities in the left and right waveguides, k_R/k_L [14]. In the retardation regime, however, the relationship between the plasmon velocities and the light velocity also plays a role. The following examples cover different possibilities.

The calculations were done as follows. First, the frequency ω and the parameter Ω_L were fixed. The plasmon wavenumber k_L for the left waveguide was found from the dispersion relations (2) and (3). Then Ω_R was varied and the plasmon wavenumber k_R found, again, from the dispersion relation. However, because the equations are scalable, the values of the transmission and reflection coefficients found for a particular set of ω , Ω_L , and Ω_R will remain the same for a different set provided the ratios Ω_R/Ω_L and $k_L(k_R)/k_0$ are also unchanged. We will therefore present results in a normalised form. In the following three examples, the plasmon wavenumber in the left waveguide will be fixed, and the plasmon wavenumber in the right waveguide will vary (by varying the ratio Ω_R/Ω_L) in the range $k_0 < k_R \leq 100k_0$.

In the first example, we set $k_L = 10k_0$, and the corresponding results are shown in figure 3. The dispersion relation for the plasmon in the left waveguide is shown in figure 3(a) together with the light line. Figures 3(b)–(e) show the results for the plasmon amplitude reflection and transmission coefficients. Figures 3(f) and (g) show the results for the relative reflected, transmitted, and radiated powers. These are the analytical and numerical solutions described in Sec. 3 and the electrostatic solutions using the model of [14]. As can be expected, for $k_R > k_L$ (both plasmons in the electrostatic regime) the numerical solution matches almost perfectly the electrostatic one. The only difference is a non-zero, but very small, phase of the plasmon transmission coefficient predicted by the numerical model, as shown in figure 3(e). The relative radiated power does not exceed 0.01 even for the extreme value of k_R . The analytical solutions for the plasmon reflection and transmission coefficients and the powers deviate slightly from the numerical ones as the difference between k_R and k_L increases. The analytical model also predict a higher (but still small) value of the phase of the transmission coefficient.

On the other hand, when the value of k_R decreases below the value of k_L and becomes closer to the value of k_0 , the numerical and the analytical solutions deviate from the electrostatic one. It can be seen in the behaviour of the absolute values of the transmission and the reflection coefficients as well as of the phase of the transmission coefficient. The phase of the reflection coefficient, however, agrees well with the electrostatic solution, especially for the numerical model. The power radiated from the junction increases, reaching the relative value of 0.1 for $k_R = 1.14k_0$, as obtained from the numerical model and shown in figure 3(f). Compared to this value, the analytical model overstates the amount of radiation, as seen in figure 3(g). Both models show that power is radiated into the free space at the expense of the power in the reflected plasmon, as the transmitted plasmon power is identical to that predicted

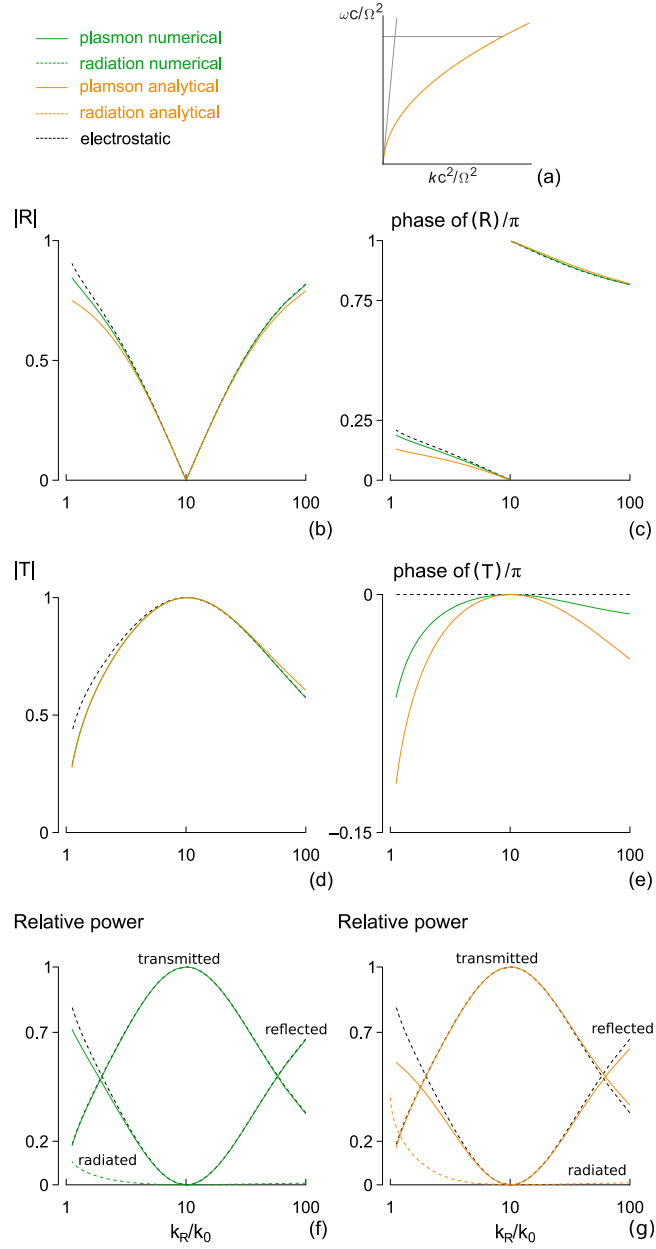


Figure 3. Comparison between numerical, analytical and electrostatic models. The plasmon wavenumber in the left waveguide is $k_L = 10k_0$. The radiated power is small except when the plasmon wavenumber in the right waveguide approaches the free-space wavenumber.

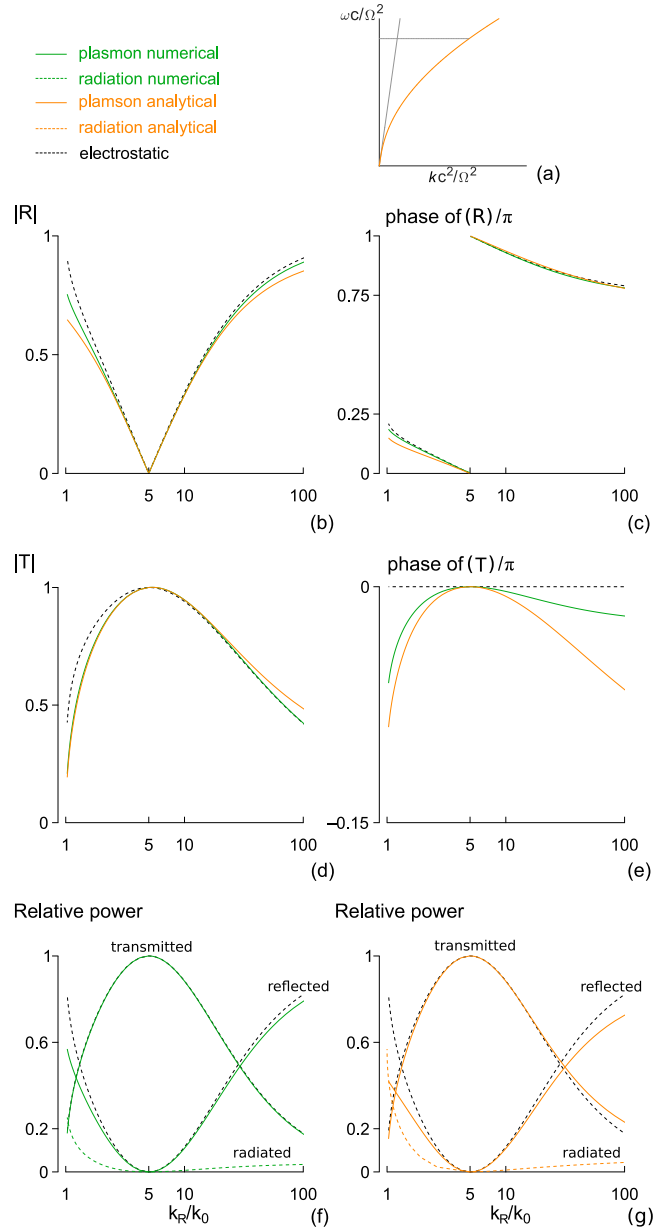


Figure 4. Comparison between numerical, analytical and electrostatic models. The plasmon wavenumber in the left waveguide is $k_L = 5k_0$. Radiation plays a role at high and low values of k_R .

by the electrostatic model.

In our next example, $k_L = 5k_0$, so that the plasmon velocity in the left waveguide is closer to the light velocity. The results, shown in figure 4, reveal an increased role of radiation both at high and at low values of k_R compared to the previous example. As the value of k_R increases above that of k_L , the plasmons in the left and right waveguides initially have similar field distributions, so that plasmon transmission is high and both plasmon reflection and free-space radiation are low, as seen in figure 4(f). For higher values of k_R , however, the incident plasmon also starts to couple into free-

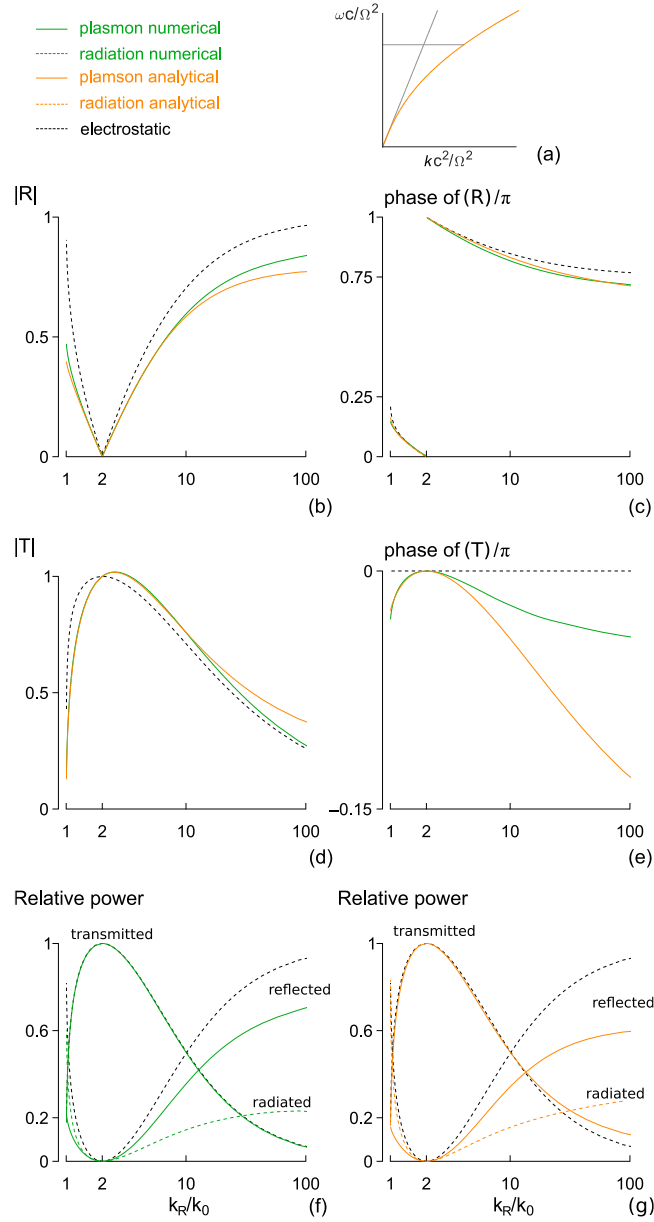


Figure 5. Comparison between numerical, analytical and electrostatic models. The plasmon wavenumber in the left waveguide is $k_L = 2k_0$. Radiation is significant for a wide range of the values of k_R .

space radiation, although the total relative radiated power does not exceed 0.05 at the maximum value of k_R . The radiation plays, however, a more prominent role for the range $k_R < k_L$. The analytical solution predicts, as before, higher relative radiated powers, as seen figure 4(g). Both the analytical and the numerical solutions coincide with the electrostatic one only in the range $k_R \approx k_L$. However, the electrostatic solution is still able to predict almost the same values of the phase of the reflection coefficient as the other two models. Also, the transmitted plasmon power calculated from the electrostatic and the numerical models coincide, which again shows that an increase of

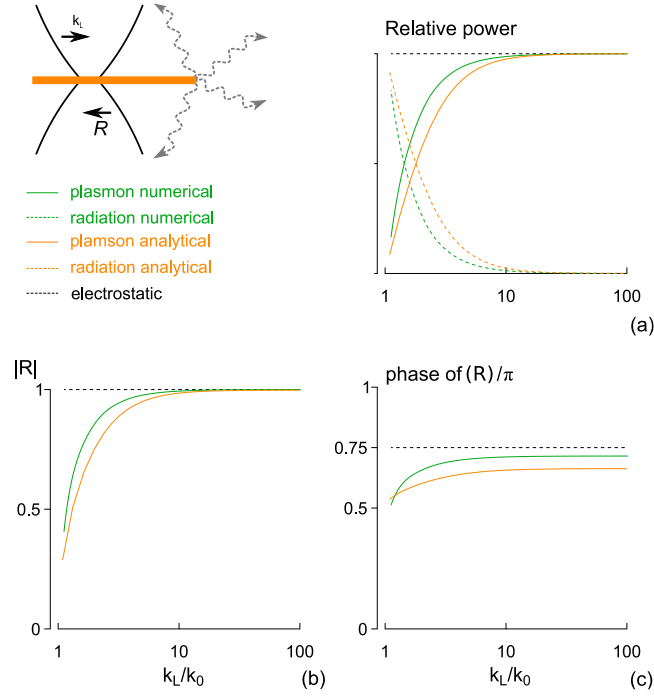


Figure 6. Comparison between numerical, analytical and electrostatic models for an abruptly terminated 2D waveguide. Power is radiated from the junction at low values of k_L but decreases rapidly as k_L grows.

the radiated power is accompanied by a decrease only in the reflected plasmon power.

In the next example, $k_L = 2k_0$, see figure 5. The trends seen in the previous example are now more pronounced. Radiation plays a prominent role, and the numerical model predicts the relative radiated power in excess of 0.1 when $k_R > 6.5k_0$ and $k_R < 1.2k_0$, as shown in figure 5(f). The quantitative disagreement between the electrostatic model on the one hand and the numerical and analytical model of the other is larger than in the previous two examples, although the numerical and the electrostatic models predict the same amount of the transmitted plasmon power.

Two further configurations emerge as special cases. The first one is an abruptly terminated two-dimensional waveguide ($\Omega_R \rightarrow 0$). The other is a junction between a two-dimensional waveguide and a thin perfectly conducting sheet ($\Omega_R \rightarrow \infty$). Figures 6(b) and (c) show the absolute value and the phase of the reflection coefficient for the abruptly terminated waveguide calculated from our numerical and analytical models. The corresponding values obtained from the electrostatic model are unity and $3\pi/4$ for any value of k_L . Figure 6(a) shows the relative reflected and radiated powers. Most power goes into free-space radiation when $k_L \sim k_0$. The radiated power decreases rapidly, however, as k_L increases; already for $k_L > 5k_0$ it is below 0.05 according to the numerical model. The analytical model shows the same qualitative behaviour but predicts a slower decrease of the radiated power with increase of k_L . The phase of the plasmon reflection coefficient is initially lower than the value $3\pi/4$ predicted by the

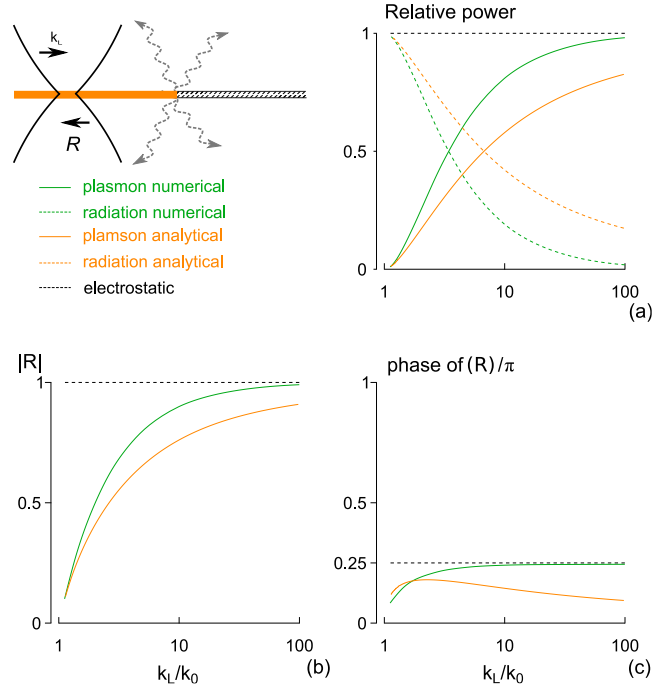


Figure 7. Comparison between numerical, analytical and electrostatic models for junction between a 2D waveguide and a perfectly conducting sheet. Radiation persists to high values of k_L .

electrostatic model, but it tends to this value as k_L increases.

Figure 7 show the results for the junction between a plasmonic waveguide and a conducting sheet. Qualitatively, the curves follow the same trend as for the abruptly terminated waveguide. The radiated power decreases with increasing k_L ; the absolute value of the plasmon reflection coefficient tends to unity, and its phase to the value of $\pi/4$ predicted from the electrostatic model. The radiation, however, persists to higher values of k_L than it did in figure 6. Even for k_L as large as $10k_0$ the relative radiated power is around 0.2, and it decreases to below 0.05 only for $k_L > 40k_0$.

Figure 8 shows how results for the reflected, transmitted and radiated power depend on the number of the Laguerre polynomials assumed in (17). These results were calculated for the parameters of Fig. 5, which shows the strongest effects of retardation. Figure 8(a) shows results for $k_R = 1.5k_0$, and Fig. 8(b) is for $k_R = 10k_0$. According to both figures, the solutions converge as the number of polynomials increases. The convergence is slower for the case of Fig. 8(b), presumably because of the larger difference between the plasmon wavenumbers and the larger radiated power. The number of polynomials used in the calculations of Figs. 3–7 was 10.

For a typical value of the dc electron density of 10^{12} cm^{-2} in a GaAs 2D electron system, the plasmon wavelength is $3 \mu\text{m}$ at 1 THz, corresponding to $k/k_0 = 28$. The plasmon wavelength can be increased (so that the plasmon wavenumber becomes closer to k_0) by, for example, increasing the concentration or decreasing the frequency. At

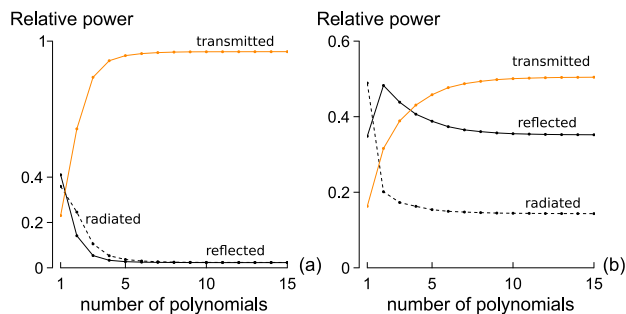


Figure 8. As the number of the Laguerre polynomials taken in the field expansion increases, calculations converge. Figure (a) corresponds to $k_R = 1.5k_0$, figure (b) corresponds to $k_R = 10k_0$; the other parameters are the same as in Fig. 5.

1 THz, $k/k_0 = 10$ for the electron density of $2.8 \times 10^{12} \text{ cm}^{-2}$; and $k/k_0 = 2$ for the density of $1.6 \times 10^{13} \text{ cm}^{-2}$. Decreasing the operating frequency may be a more practical way, and indeed most experimental demonstrations of retardation in plasmon have been so far at sub-THz frequencies, see e.g. [15, 17]. At 0.1 THz, for example, $k/k_0 = 3$ for the electron density of 10^{12} cm^{-2} .

As is it common for studies of 2D junctions [7, 8, 11, 14, 13], we assumed that the dc electron density has a step discontinuity at a junction. One may expect the assumption to hold well for the junction between a 2D waveguide and a perfectly conducting sheet and for an abruptly terminated waveguide. In real junctions between 2D waveguides, the dc electron density will vary smoothly. However, the concentration profile could still be approximated by a step function if the junction width is small compared to the plasmon wavelength. For a given geometry, this condition is easier to fulfil in the retardation regime (when the wavelength is larger) than in the electrostatic regime. Estimates assuming electron diffusion to be the main mechanism affecting the junction width suggest diffusion lengths of the order of 100 nm for two-dimensional semiconductor systems at the 77 K. On the other hand, at 1 THz, the condition $k = 10k_0$ corresponds to the plasmon wavelength of $8.5 \mu\text{m}$ in GaAs.

5. Discussion and conclusions

Our numerical and analytical models allowed us to quantify the effects of retardation on junctions between two-dimensional waveguides. We could also compare these two models to a previously developed electrostatic formulation [14].

When a plasmon is incident upon a junction, its power is split between a reflected plasmon, a transmitted plasmon, and free-space radiation. The amount of power radiated depends both on how close the phase velocities of the plasmons in the two waveguides are to the velocity of light and on how close they are to each other. If the plasmon velocities are close to each other, most of the power on the incident plasmon goes into the transmitted one. However, if the plasmon velocities are different and the

velocity at least in one waveguide is comparable to the light velocity ($k_{\text{plasmon}} < 5k_0$ in our examples), radiation loss becomes noticeable and may exceed 5% of the incident power (see figure 4(f)). For higher plasmon velocities, the radiated power increases dramatically; in the example of figure 5, the maximum radiated loss exceeded 50% as found from the numerical model. Although the electrostatic model was inaccurate in this situation, it still predicted the same plasmon transmission coefficient and a similar phase of the plasmon reflection coefficient as our numerical model. It follows, first, that radiation depletes power of the reflected but not of the transmitted plasmon. Second, because the retardation does not affect the phases considerably, the electrostatic model would be able to predict the eigenfrequencies of plasmonic resonators made of several sections of two-dimensional waveguides but will be unable to predict their quality factors.

Our analytical model agreed qualitatively with the numerical one across the whole range of parameters. The two models also agreed quantitatively when the plasmon wavenumbers in both waveguides, k_L and k_R , were comparable to each other, but the analytical model over-predicted the amount of the radiated power at the extremes of the k_R range. Such behaviour is expected, as the analytical model assumes that the field at a junction is dominated by the plasmons and will be more accurate when the amplitudes of other modes are small.

We also considered plasmons incident upon an abrupt termination followed by a dielectric and upon a junction between a waveguide and a conducting sheet. Both situations resulted in radiation when the plasmon velocity was high. However, the relative radiated power decreased rapidly with increasing the plasmon velocity for the abruptly terminated waveguide, and for $k_L < 5k_0$, the radiated power was below 5% of the incident one (see figure 6). By contrast, the radiated power decreased much more slowly with increase of the plasmon velocity for reflection from a conductor (see figure 7). According to the numerical model, the radiated power was around 35% of the incident power for $k_L = 5k_0$, around 20% for $k_L = 10k_0$, and it reduced to less than 5% only for $k_L > 40k_0$. These results agree with an earlier analysis of a slot diode [31], and may have implications for the design of graphene plasmonic devices that use thin metallic contacts. Reflection of plasmons from the contacts will lead to free-space radiation, which will degrade the resonance quality factors compared to those predicted from the electrostatic approximation.

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