

The Structure of a Turbulent Boundary Layer at Low Reynolds Number..

by

John Murlis B.Sc

Department of Aeronautics

Imperial College of Science and Technology

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Summary.

The object of this work was to explain the observed Reynolds number dependence of the mean velocity defect profile of a turbulent boundary layer.

Mean flow measurements confirmed previous findings that the wake component of the "Universal Velocity Profile" grows rapidly in the range $800 < Re < 4750$ from about half its asymptotic value. This has been attributed to viscous effects at the fluctuating interface between the turbulence and the external irrotational stream, which were explored using "conditional sampling" analysis of the digitised turbulence data. It was found that traditional approaches to the analysis of interface statistics, particularly intermittency, failed to distinguish between "large eddy" structure and the fine scale structure seen, for example, in smoke pictures. A new definition of intermittency was devised to make this distinction and subsequent analysis gave the variation of burst length statistics and intermittency with Reynolds number. It was found that the shape of large-scale bursts was different at low Reynolds number and that the bursts themselves were larger by a factor of up to 10.

Turbulence measurements revealed that the ratio of longitudinal to normal intensity in the outer layer also varied at low Reynolds numbers. Analysis of the turbulent energy balances showed that production is considerably less at low Reynolds number and that advection is virtually non-existent.

A mechanism for the production of low Reynolds number effects, and modifications to represent these effects in boundary layer calculation methods, are suggested.

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List of Symbols.

a	constant in Bull's (1968) wake function
a_1	kinetic energy parameter
A	additive constant in hot-wire calibration law
B	slope of hot-wire calibration law
C	log-law additive constant
$G = \frac{\tau_w}{\frac{1}{2}\rho U_e^2}$	coefficient of skin friction
C_p	pressure coefficient
E	mean output from anemometer
$f(x)$	"a function of x "
$F_c(t)$	general criterion function
F_u, F_v	flatness factors of u and v components of fluctuating velocity
G	Clauser equilibrium parameter
G	diffusion function
H	shape factor
$ID(t)$	intermittency function
J	step number in digital processing
k	roughness element height
l	burst length
l	mixing length
$L(4/5)$	dissipation length parameter
L	dissipation length
n	velocity power in hot wire calibration law
n_p	number of points
n_b	number of bursts
p	static pressure

p	fluctuating pressure
P	probability
q	general fluctuating velocity
Q	general instantaneous velocity
Re	Reynolds number
Re_θ	Momentum thickness Reynolds number
$\bar{R} \equiv \frac{u_\tau \delta}{V_w}$	Green's (1971) wall co-ordinate parameter
S_u, S_v	Skewness of u and v components
$SC14, SC15$	threshold level factors
t	time
$TH1, TH2$	threshold levels
u, v, w	components of fluctuating velocity
U	mean streamwise velocity
V	mean normal velocity
U_e	mean free stream velocity
U_c	mean streamwise convection velocity
$u^+ \equiv U/u_\tau$	
$u_\tau \equiv \sqrt{(\tau_w/\rho)}$	friction velocity
V_τ, V_q	normal convection velocities
$w(y/\delta)$	wake function
x	streamwise distance (from L.E.)
y	normal distance (from wall)
z	lateral distance (from Q)
$y^+ \equiv \frac{u_\tau y}{\nu}$	
$z \equiv y/\delta$	
β	pressure gradient parameter
γ	intermittency factor
γ^*	wholesale intermittency factor

δ_D	"drawn" intermittency
δ_F	flatness factor intermittency
δ_K	Klebanoff's intermittency
δ	general boundary layer thickness
δ_c	Coles' boundary layer thickness
δ_s	sub-layer thickness
$\delta_{.995}$	boundary layer thickness at $U/U_e = 0.995$
δ^*	displacement thickness
δ	angle of yaw (of hot wire)
$\delta(x, y, t)$	intermittency function
ϵ	dissipation rate
η	Kolmogorov length scale
θ	momentum thickness
κ	von Karman's constant
λ	Taylor microscale
μ	viscosity
ν	Kinematic viscosity
ν_T	eddy viscosity
π	wake strength constant
ρ	density
τ	shear stress
τ_w	wall shear stress
φ	effective wire angle
$\bar{}$	overall mean value
$\bar{}_i$	irrotational zone mean value
$\bar{}_t$	turbulent zone mean value

Chapter One.

Introduction.

In 1904 Prandtl showed that if the viscosity of a fluid is small, its effects on the flow around a body are confined to a thin layer. This layer became known as the boundary layer (German - 'Grenzschicht') because it satisfied the no-slip surface boundary condition. The flow in the boundary layer could be laminar or, more frequently in cases of practical interest, turbulent. It was found to be responsible for most of the flow resistance of a streamlined body, an aircraft wing for example, and soon became a major subject in fluid mechanics research.

Studies of turbulent boundary layers at high Reynolds numbers have led to dimensionally-simple scaling laws for mean velocity profile. These laws rely on the independence from viscosity of the energy containing eddies, except in a very thin sublayer close to the surface. It has also been found that, in the absence of longitudinal pressure gradient, certain turbulent boundary layers may be specified uniquely by Reynolds number based upon a suitable boundary thickness. These are flows in which transition is having no further effect and they are known as 'fully developed turbulent boundary layers'.

On small-scale test rigs it has been found that although the flow is fully developed at low Reynolds numbers the scaling laws appear to break down. This can increase the surface shear stress by up to 15% above that predicted by the high-Reynolds-number scaling laws and is therefore of considerable significance.

In engineering practice low-Reynolds-number turbulent boundary layers are found in the flow over turbine blades, in the throats of rocket

nozzles, in combustion chamber mixing regions as well as over a considerable proportion of a high-Mach-number wing or lifting body.

The object of the research described in this thesis was to improve understanding of the mechanics behind the departure from high-Reynolds-number scaling laws in a fully-developed low-Reynolds-number turbulent boundary layer.

1.1 Mean Flow. The Symptoms of Reynolds Number Effects.

One of the most significant advances in the study of the turbulent boundary layer was the discovery of the universal law of the wall. This scaling law is based upon the assumption that near the wall the average shear stress, τ , is constant and equal to the wall shear stress, τ_w , and that the mean velocity distribution in this region depends entirely upon the wall shear stress, the distance from the wall, y , and the local fluid properties ρ and ν . Prandtl (1932) gives the law in the form:

$$U = u_\tau f\left(\frac{u_\tau y}{\nu}\right) \quad 1.1$$

where

$$u_\tau = \sqrt{\left(\frac{\tau_w}{\rho}\right)} \quad 1.2$$

is the friction velocity. It has been found that the function f is a universal function for any ^{smooth} solid boundary turbulent flow. Outside the viscous sublayer this becomes:

$$\frac{U}{u_\tau} = \frac{1}{K} \ln\left(\frac{u_\tau y}{\nu}\right) + C \quad 1.3$$

where

K is a constant, Von Karman's constant $\triangleq 0.41$

and

C is the additive constant ≈ 5.0

The expression (1.3) holds for a small part of the boundary layer, approximately $\delta_s < y/\delta < 0.2$, where δ_s is the sublayer thickness. The search for a unifying framework for the analysis of the entire boundary layer culminated in the formulation by Coles (1962) of a universal velocity profile law, adding to the established law of the wall a further function, the 'wake', to describe the outer layer profile. Then:

$$\frac{U}{u_\tau} = \frac{1}{K} \ln\left(\frac{u_\tau y}{\nu}\right) + C + \frac{\Pi}{K} W\left(\frac{y}{\delta}\right) \quad 1.4$$

where

$W\left(\frac{y}{\delta}\right)$ is the wake function, originally defined numerically but approximated by Hinze (1959) as

$$W\left(\frac{y}{\delta}\right) = \left(1 - \cos\left(\pi \frac{y}{\delta}\right)\right) \quad 1.5$$

where

$$\begin{aligned} W(1) &= 2 \\ \text{so that } \Pi &= \frac{K}{2} \Delta \frac{U}{u_\tau} \end{aligned} \quad \text{see Fig. (1.1.1)}$$

More recently Bull (1968) has suggested a modified form of the wake function

$$W\left(\frac{y}{\delta}\right) = \left(1 - \cos \frac{(\pi(\frac{y}{\delta} - a))}{1 - a}\right) \quad 1.6$$

where

$$a = \text{constant} \approx 0.08$$

Dean (1974) re-examined the 'law of the wake' and, using revised boundary conditions, obtained a semi-empirical formulation

$$\frac{\Pi}{K} W\left(\frac{y}{\delta}\right) = \frac{1}{K} \left[\left(\frac{y}{\delta}\right)^2 - \left(\frac{y}{\delta}\right)^3 + 2\Pi \left(3\left(\frac{y}{\delta}\right)^2 - 2\left(\frac{y}{\delta}\right)^3 \right) \right] \quad 1.7$$

Using his universal profile law Coles (1962) examined most of the available zero pressure gradient boundary layer data. He was able for the first time to make a consistent assessment of its quality and thus to distinguish between anomalies arising from the physical situation and those caused by the experimental method. For high Reynolds numbers he was able to show that Π was very nearly constant, so that equation (1.4) was consistent with the velocity defect law

$$\frac{U_e - U}{u_\tau} = f\left(\frac{y}{\delta}\right) \quad 1.8$$

but that for $R_\theta \equiv \frac{U_e \theta}{\nu} < 6000$, Π changed very rapidly, falling to zero at about $R_\theta = 500$. A law for the function $\Pi = f(R_\theta)$ in a zero-pressure-gradient incompressible boundary layer, based upon the best fit to all the experimenters' data, was derived and presented numerically. The data of Wieghardt (1944) agreed most closely with this law and can be taken as standard.

Hastings and Sawyer (1970) measured $\Pi(R_\theta)$ for their constant-pressure boundary layer at a free stream Mach number of 4.0. They found that the failure of the defect law persisted to higher R_θ , approaching 20,000.

Simpson (1970) suggested that the departure of the mean velocity profile from a universal form could equally well be explained by a variation in K , the Von Karman constant, Π remaining constant. He proposed the form

$$\frac{K}{0.41} = \left(\frac{R_\theta}{600}\right)^{-\frac{1}{8}} \quad 1.9$$

which gives $K \approx 0.33$ at $R_\theta = 1000$

Green (1971) discussed the available compressible low Reynolds number data, and found that Hastings and Sawyer's data agreed well with

Coles incompressible data plotted in the form

$$\pi = f(\bar{R}) \quad 1.10$$

where

$$\bar{R} = \frac{u_\tau \delta}{v_w} \quad 1.11$$

but added a note of caution regarding the pitot displacement corrections used. Green obtained π from a consideration of the outer profile shape without reference to the skin friction coefficient and suggests that the data of Hastings and Sawyer do not support Simpson's proposal because merely scaling u^+ and y^+ axes would not bring about the collapse of the high Reynolds number profiles onto those obtained at low Reynolds number.

Huffman and Bradshaw (1972) re-examined the low speed data and also considered that Simpson's suggestion was implausible. They did, however, find evidence for a dependence of C upon $\frac{\partial \tau}{\partial y}$, but it would be very weak in a zero pressure gradient boundary layer. Huffman and Bradshaw also note that the mean defect profiles in fully-developed pipe flow show no departure from the universal form at quite low Reynolds number, suggesting that $\pi(R_\theta)$ is due to the presence of the interface between the turbulent flow within the boundary layer and the irrotational flow in the free stream.

1.2 Intermittency in the Outer Layer.

Flow visualisation of turbulent shear flows with a free stream boundary shows that the edge of the turbulence is highly contorted. This may be seen in the U.S. Army B.R.L. shadowgraph pictures of the turbulent wake of a bullet reproduced in Corrsin and Kistler (1955), amongst others,

and the photographs of a smoke filled boundary^{layer} taken by Fiedler and Head (1966) Falco (1974) and Pongprudhanon (1974). The contortions of the boundary between turbulent and irrotational flow lead to the intermittency found in signals of turbulence properties, for example the output from hot wire anemometers taken at fixed positions in the layer. In fact it seems as if bursts of turbulent activity are driven past the measuring probe and separated by regions of quiescent flow. This intermittency should not be confused with the high wave number intermittency described by Batchelor (1953) and the bursts of turbulence are not equivalent to the bursts of turbulence production found in the regions near a solid wall and described by Kline, Reynolds, Schraub and Runstadler (1967), although some connection between the two latter phenomena may exist.

The proportion of time for which the flow at a certain boundary layer position may be considered 'turbulent' is known as the intermittency factor, $\gamma(x,y)$ which is the time mean of a quantity formally defined as having the value unity in turbulent fluid and zero in non-turbulent fluid and known as the intermittency function, $\delta(x,y,t)$.

The initial crude concept of intermittent turbulence as an on-off Gaussian process was refined by Townsend (1948) who showed from measurements taken in the wake of a circular cylinder that to a good approximation

$$\begin{aligned}\gamma_y &= \frac{\text{Flatness factor of fully turbulent fluid (at } y = 0)}{\text{Flatness factor of intermittently turbulent fluid at } y} \\ &= \frac{\frac{\overline{u^4}}{(\overline{u^2})^2}_{y \rightarrow 0}}{\frac{\overline{u^4}}{(\overline{u^2})^2}_y}\end{aligned}\quad 1.12$$

This method has more recently been extended by Sandborn (1959).

The intermittency factor may be calculated by the visual analysis of records of turbulence properties made by chart recorder or photographed oscilloscope traces. Effectively these methods reconstruct the intermittency function and have been used by Klebanoff (1954), Corrsin and Kistler (1955) and by Bradshaw (1966a). More recently turbulent flow properties have been calculated instantaneously using analogue (continuous) or digital (sampled) systems and the intermittency function has been constructed by comparing these values with some turbulent/irrotational threshold value. Analogue techniques were used by Corrsin and Kistler (1955) Bradbury (1964) and Kovasznay et al (1970) amongst others, whilst digital techniques have been used by Kaplan and Laufer (1968), Antonia and Bradshaw (1971) and Castro (1973). The turbulent flow property used in the detection procedure, known as the turbulence criterion parameter, and the intermittency system itself have received considerable attention and are discussed by Kibens and Kovasznay (1970), Wygnanski and Fiedler (1970), Antonia and Atkinson (1974) and Hedley and Keffer (1974). There are clearly many difficulties both in the choice of criterion parameter and in the design of suitable intermittency systems. There is no agreed definition of turbulence and it has been found that approximations used, for example one or other component of fluctuating vorticity, contain no well defined step in value between typical 'turbulence' and 'non-turbulence' and cover a whole range of wavelengths down to the smallest scales expected in turbulence (the Kolmogorov scale associated with dissipating eddies). These smallest wavelengths are the most populous, and have a considerable effect upon averages of such properties as burst length, yet in most reported intermittency measurement systems they are removed by low pass filtering or by 'smoothing'. The problems associated with small scale events in intermittency are discussed in detail in Chapter Four. Here we note that,

despite the difficulties, considerable advances in the understanding of shear flow turbulence have been made through the study of intermittency.

The statistics of the interface position, and the scales of turbulent bursts have been investigated by many experimentors. Corrsin and Kistler (1955) and Hedley and Keffer (1974) gave the probability distribution of burst length; Kovasznay, Kibens and Blackwelder (1970) measured the frequency of detection of the interface and Kaplan and Laufer (1968) derived a two-dimensional picture of the boundary layer using an array of hot wire probes, a method more recently used by Paizis and Schwartz (1974).

The dynamic properties of the fluid within a turbulent or non-turbulent region, or at a particular position within a region may be obtained by 'Conditional Sampling' techniques, described in general terms by Antonia and Bradshaw (1972). These techniques, used by Kaplan and Laufer (1968), Kovasznay et al (1970), Antonia (1972) and Hedley and Keffer (1974), have produced turbulent zone averages of mean velocities and certain turbulence parameters, also maps of space-time auto and cross correlations of u and v components of velocity and of intermittency function.

From these results a composite picture has been constructed by Laufer (1972); a series of bursts of turbulence occurs in random fashion well down towards the wall; these bursts move out into the layer with momentum defect, producing a turbulent wake downstream, and curl over, finally spreading out in a downstream direction. The irrotational flow rides over the bursts, following the interface closely. Entrainment takes place when the mean flow between bursts collides with the slower leading edges of the bursts.

The mechanism of entrainment was discussed by Corrsin and Kistler, who introduced the concept of a 'viscous superlayer' propagating

into the irrotational fluid and suggested that such a layer should have a thickness of order the Kolmogorov length scale (actually in the range 2η to 6η). Some recent range-conditioned measurements by La Rue and Libby (1974) in a heated wake tend to confirm this estimate of superlayer thickness.

Although the available data covers a wide range of Reynolds number a systematic comparison is not possible in view of the importance of variations in the intermittency scheme, and any comparison must be made where a Reynolds number range is covered in one experiment. The effects of Reynolds number have explicitly been examined by Fiedler and Head (1966) in a boundary layer, Sherman (1972) in a channel flow and Falco (1974) in a smoke filled boundary layer. Fiedler and Head do not reach definite conclusions on Reynolds number effect, but their smoke pictures agree well with those of Falco who infers that small scale structure is more significant at high Reynolds and that 'typical eddy size', a measure of burst length, decreases considerably with Reynolds number increase. Sherman's data were taken immediately after transition and show untypically large typical burst lengths. Many of the other data were obtained at low Reynolds number; Kovasznay et al at $R_\theta = 3000$; Kaplan and Laufer at $R_\theta \approx 3000$ and Antonia at $R_\theta = 2160$ for example, where the boundary layer may still be subject to significant Reynolds number effects in the outer layer.

At low Reynolds number it may be expected from an analogy with the viscous sublayer that the viscous superlayer would be proportionally thicker and, since the interface is highly contorted may, in fact, occupy a considerable volume of the flow. The outer layer structure, particularly burst length statistics and intermittency, is of great importance, therefore, in the present study of low Reynolds number turbulent flows.

1.3 Turbulence Structure.

The general Navier Stokes equations for a steady two dimensional flow field $U, V(x, y)$ and a turbulent field $u, v, w(x, y, z, t)$ are

$$\begin{aligned} \frac{\partial u}{\partial t} + (U+u)\frac{\partial}{\partial x}(U+u) + (V+v)\frac{\partial}{\partial y}(U+u) + w\frac{\partial u}{\partial z} \\ = -\frac{1}{\rho}\frac{\partial}{\partial x}(P+p) + \nu\left(\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} + \nabla^2 u\right) \end{aligned} \quad 1.13$$

$$\begin{aligned} \frac{\partial v}{\partial t} + (U+u)\frac{\partial}{\partial x}(V+v) + (V+v)\frac{\partial}{\partial y}(V+v) + w\frac{\partial v}{\partial z} \\ = -\frac{1}{\rho}\frac{\partial}{\partial y}(P+p) + \nu\left(\frac{\partial^2 V}{\partial x^2} + \frac{\partial^2 V}{\partial y^2} + \nabla^2 v\right) \end{aligned} \quad 1.14$$

$$\begin{aligned} \frac{\partial w}{\partial t} + (U+u)\frac{\partial w}{\partial x} + (V+v)\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z} \\ = -\frac{1}{\rho}\frac{\partial p}{\partial z} + \nu\nabla^2 w \end{aligned} \quad 1.15$$

Equations for the products of turbulent components of velocity may be found by multiplying the above equations by the appropriate fluctuating velocity component and taking the average. Making use of

$$\frac{\partial}{\partial x}(U+u) + \frac{\partial}{\partial y}(V+v) + \frac{\partial w}{\partial z} = 0$$

and re-arranging with the boundary layer simplifications gives

$$U\frac{1}{2}\frac{\partial \overline{u^2}}{\partial x} + V\frac{1}{2}\frac{\partial \overline{u^2}}{\partial y} + \overline{uw}\frac{\partial U}{\partial y} + \frac{1}{2}\frac{\partial}{\partial y}\overline{u^2v} = \frac{1}{\rho}\overline{p}\frac{\partial u}{\partial x} + \nu\overline{u\nabla^2 u} \quad 1.17$$

$$U\frac{1}{2}\frac{\partial \overline{v^2}}{\partial x} + V\frac{1}{2}\frac{\partial \overline{v^2}}{\partial y} + \frac{\partial}{\partial y}\overline{v\left(\frac{1}{2}v^2 + \frac{p}{\rho}\right)} = \frac{1}{\rho}\overline{p}\frac{\partial v}{\partial y} + \nu\overline{v\nabla^2 v} \quad 1.18$$

$$U \frac{1}{2} \frac{\partial \overline{w^2}}{\partial x} + V \frac{1}{2} \frac{\partial \overline{w^2}}{\partial y} + \frac{1}{2} \frac{\partial \overline{vw^2}}{\partial y} = \frac{1}{\rho} \overline{p \frac{\partial w}{\partial z}} + V \overline{w \nabla^2 w} \quad 1.19$$

which are the equations for Reynolds normal stresses, $\overline{u^2}$, $\overline{v^2}$, $\overline{w^2}$. The equation for Reynolds shear stress, $\overline{uv}$, may similarly be obtained by adding (1.13) multiplied by v to (1.14) multiplied by u

$$U \frac{\partial \overline{uv}}{\partial x} + V \frac{\partial \overline{uv}}{\partial y} + \overline{v^2} \frac{\partial U}{\partial y} - \frac{1}{\rho} \overline{p \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)} + \frac{\partial}{\partial y} \left(\overline{vw^2} + \frac{\overline{vp}}{\rho} \right) - V \left(\overline{v \nabla^2 u} + \overline{u \nabla^2 v} \right) = 0 \quad 1.20$$

To obtain the equation for turbulent kinetic energy $\overline{q^2} = (\overline{u^2} + \overline{v^2} + \overline{w^2})$ equations (1.17) (1.18) and (1.19) may be added. Then, using (1.16) again,

$$U \frac{1}{2} \frac{\partial \overline{q^2}}{\partial x} + V \frac{1}{2} \frac{\partial \overline{q^2}}{\partial y} + \overline{uv} \frac{\partial U}{\partial y} + \frac{\partial}{\partial y} \overline{v \left(\frac{1}{2} \overline{q^2} + \frac{p}{\rho} \right)} + \epsilon - V \frac{\partial^2}{\partial y^2} \left(\frac{1}{2} \overline{q^2} + \overline{v^2} \right) = 0 \quad 1.21$$

Outside the viscous sublayer, viscous diffusion (the last term) is negligible and (1.21) becomes

$$\left(U \frac{\partial}{\partial x} + V \frac{\partial}{\partial y} \right) \frac{1}{2} \overline{q^2} + \overline{uv} \frac{\partial U}{\partial y} + \frac{\partial}{\partial y} \left[\overline{q^2 v} + \frac{\overline{pv}}{\rho} \right] + \epsilon = 0 \quad 1.22$$

The first two terms in this equation give the advection of kinetic energy by the mean flow, the third gives the production of turbulent kinetic energy from the working of shear stress against the mean velocity gradient, the fourth term gives the diffusion, by normal component turbulence velocity and by fluctuating pressure, and the last term is the dissipation of energy to heat by viscosity.

It may be seen that the production term comes exclusively from the $\overline{u^2}$ equation, 1.17. The terms $\frac{1}{\rho} \overline{p \frac{\partial u}{\partial x}}$, $\frac{1}{\rho} \overline{p \frac{\partial v}{\partial y}}$ and $\frac{1}{\rho} \overline{p \frac{\partial w}{\partial z}}$ do not

appear in the equation 1.21 as their point average over all components is zero.

The shear stress equation outside the sublayer may be reduced to the form

$$U \frac{\partial \overline{uv}}{\partial x} + V \frac{\partial \overline{uv}}{\partial y} + \overline{v^2} \frac{\partial U}{\partial y} - \frac{1}{\rho} \overline{\rho \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)} + \frac{\partial}{\partial y} \left(\overline{uv^2} + \overline{pu} \right) = 0 \quad 1.22$$

The first two terms may be considered as a mean-flow-transport of shear stress, the third as a generation of turbulent shear stress in the presence of normal velocity fluctuations and mean flow gradient and the fifth as a lateral transport of shear stress. The fourth term is known as the pressure-strain term.

Equations 1.21 and 1.22 represent a balance of turbulent kinetic energy and shear stress respectively; the distribution between terms and the changes in distribution with position in a flow give the turbulent structure of the flow.

The structure of turbulence in a two dimensional boundary layer was first measured in detail by Townsend (1947) and Schubauer (1954). Measurements in a fully developed pipe flow were made by Laufer (1954). These results were discussed by Townsend (1956) who noted that over a large proportion of the flow the diffusion and advection of turbulent energy were negligible and proposed a two layer model: an inner layer in which most of the production took place and where the flow was approximately in energy equilibrium and an outer layer reliant upon diffusion from the inner layer for its turbulent energy. Rotta (1962) discusses the mechanism of turbulent energy production in equilibrium shear flows: in the presence of a turbulence field the normal component velocity fluctuations combine with the velocity gradient to produce shear stress which, working against the

mean velocity gradient, produces fluctuation initially in the streamwise direction; these are redistributed by the pressure fluctuation, such terms as $\frac{1}{\rho} \frac{\partial p}{\partial x}$, to contribute to $\overline{v^2}$ and $\overline{w^2}$. Thus the level of turbulence is maintained, the energy finally dissipating from the isotropic smallest eddies through viscosity.

Klebanoff (1954) made extensive turbulence measurements in a zero pressure gradient boundary layer. In particular, he emphasised the importance in energy considerations of the region very close to the wall ($\frac{u_\tau y}{\nu} < 30$) where most of the turbulence energy production and dissipation takes place and where there is a net energy inflow towards the wall because of pressure forces. He also demonstrated the inadequacy of the local-isotropy assumptions in obtaining turbulent-energy dissipation, particularly near the wall. Klebanoff had 'measured' dissipation and obtained diffusion by difference. Bradshaw (1966a) measured diffusion directly and demonstrated the enormous increase in production found in a strongly retarded boundary layer's outer region.

Bradshaw (1966b), following Townsend (1961) showed that the inner layer turbulence may be considered to be composed of two parts, an 'active' component producing shear-stress and scaling on y and u_τ and an 'inactive', effectively irrotational component generated partly by static pressure fluctuations and partly by the turbulence in the outer layer.

It has been known for some time that flow visualisation in the region very close to the wall shows a 'streaky' structure which intermittently breaks down. Using a novel system of flow visualisation based on hydrogen bubbles, Kline, Reynolds, Schraub and Runstadler (1967) were able to determine a cyclic pattern in this intermittent structure. Visual analysis was combined later with hot wire measurements which showed that a strong link existed between the pattern of flow breakdown and periods

of high uv activity. These results were discussed by, amongst others, Kim, Kline and Reynolds (1971) and Wallace, Eckelmann and Brodkey (1972) who describe the sequence of events: a low speed fluid lump separates from the surface; an unstable velocity profile is formed and the fluid 'bursts' outwards as newly formed turbulence. It is suggested that these events constitute the normal process of turbulence productions, but the link between this sub-layer bursting and the large-scale intermittency of the outer layer is not understood at the moment. From low Reynolds number turbulence measurements made in the pipe flow wall regions by Pennell, Sparrow, and Eckert (1972) it would seem that the inner layer is unaffected by Reynolds number effects and it might be expected, therefore, that it would be in the outer layer, the region of large scale intermittency, where low Reynolds number effects would be seen.

1.4 The Calculation of Low Reynolds Number Boundary Layers.

The Reynolds number dependence of the velocity profile presents a new challenge to the inventors of prediction methods, and several modifications to existing calculation schemes have been published.

Analytical forms of the velocity profile, with $\pi = f(R_\theta)$ scaled according to the Coles (1962) 'Law' have been used by Green (1971), McDonald (1970) and Cebeci (1973) to calculate shear stress, eddy viscosity and mixing length profiles from the integrated equations of motion. Baker and Launder (1974) deduced a mixing length in the outer layer

$$l = 0.075 \delta_{99} \left(1 - e^{-\frac{R_{\max}}{400}}\right) \quad 1.23$$

where

$$R_{\max} = \frac{u_z \delta_{99}}{V}$$

from their own measurements in a zero pressure gradient boundary layer with moderate injection.

In this thesis the results of a modification to the calculation method of Bradshaw, Ferriss and Atwell (1967) are described. Essentially Bradshaw's method solves the turbulent energy equation assuming a dissipation length profile which, in the modified version, is scaled on Reynolds number. Since this work was undertaken several more recent methods have been reported. Hanjalic and Launder (1972), for example, adopt a similar approach.

Chapter Two.

Test Equipment and Instrumentation.

In this chapter the apparatus and instrumentation are described, together with the procedures used in setting up and taking measurements.

2.1 Wind Tunnel.

The wind tunnel used was the Imperial College Aeronautics Department No. 1 boundary layer tunnel described in detail by Bradshaw (1972). The tunnel is a conventional "blower tunnel" driven by an Airscrew-Weyroc HEBA type 27" centrifugal fan and speed is controlled by an Electrical Construction Varispeed Mark 10 thyristor drive. The blower is coupled to a 30" x 5" working section via a two-dimensional contraction with an area ratio of 9:1, preceded by a rectangular section duct in which honeycombs and screens may be fitted. The working section carries a flexible roof and the floor has $3\frac{1}{2}$ " diameter ports pitched at 6" to take instrumentation discs. Two sections each 4'9" long were joined together to make one 9'6" section with 18 disc stations, the n 'th station being at $(6n + 1.5)$ inches from the leading edge. The nominal speed for all tests was 50 feet per second. Initially a 4", $\frac{3}{8}$ " cell high quality honeycomb (Bonded Structures Ltd. Aeroweb aluminium foil) and, downstream, screens of 16 mesh 28 s.w.g. woven wire, open ratio 0.58 (see Bradshaw 1965), were fitted. It was found, however, that spanwise variations of surface shear stress had the unacceptable magnitude of 10 % of the mean value at the tunnel exit. Fig. (2-1.1); the spanwise shear stress variation ^{measured by Preston tube} shows that the wavelength of this disturbance

was about 2", roughly the boundary layer thickness at this position. Similar results were obtained by Mojola (private communication) in the same tunnel and have been reported by Bradshaw (1965), de Bray (1969) and Patel (1964) elsewhere. Neither adding more screens nor reversing screens or honeycomb modified the pattern. Finally the honeycomb was placed downstream of the screens, following Patel (1964). Non-uniformity effects in the contraction boundary layer were eliminated by a $\frac{3}{8}$ " bleed, formed by raising the working section floor and fitting a sharp leading edge to it. The final variation in shear stress was 5 % of mean. Fig. (2-1.2). The repositioning of the honeycomb was expected to raise the free stream turbulence level in the working section. Measurements showed that with the honeycomb upstream the turbulence level, $\frac{\sqrt{u'^2}}{U_e}$, was 0.08 % rising with the honeycomb downstream to 0.1 % at 50 ft/sec and with frequencies below 10Hz (wave lengths about 5ft.) removed by high pass filtering. The final turbulence level and c_f variation were regarded as satisfactory.

To obtain stability of working speed over the long periods necessary the speed control designed by Castro (1973) was used in a slightly modified form.

The working section roof profile was adjusted using the screw jacks provided on each side. The pressure variation achieved down the length of the working section was less than 1 % of the dynamic pressure, an acceptable approximation to zero pressure gradient. The static pressure distribution is shown in Fig. (2-1.3); table (2.1) gives the screw jack co-ordinates required.

The importance of correct choice of tripping device has been pointed out by Coles (1962). A wire of 22 s.w.g., 0.028" diameter, was chosen which, when taped to the leading edge section $\frac{1}{2}$ " from the leading edge itself, gave a device height of 0.030". Stethoscope tests showed that

transition was complete across the working section within $\frac{1}{2}$ " downstream of the device. Graham (1969) gives a criterion for trip wires $R_k = \frac{U_\infty k}{\nu} > 1000$ where k = device height, and Gibbings (1959) suggests $R_k > 826$. The wire used gives $R_k = 800$ and Graham suggests that the wake settling length would then be $100 \times k = 3.0$ ", all measurements were made for $x \geq 6.0$ ".

2.2 Surface Tubes.

Preston (1954) describes the use of surface pitot tubes to determine skin friction. This method depends upon the existence of the similarity law known as the law of the wall. The tubes must be chosen so that their flow fields, roughly the height of the tube diameters, are contained within the inner layer, roughly $y < .1\delta$ in this case. Preston tubes of diameter 0.113", 0.080" and 0.058" were used giving $y/\delta_{995} = 0.377, 0.267, 0.193$, at station 1, $Re = 800$, and 0.068, 0.048, 0.035, at station 17, $Re = 4750$.

At high Reynolds number the tubes all fell well within the wall law limit. At low Reynolds numbers it seems that the region dominated by the log law may extend further into the layer (see for example the low Re profiles of Wieghardt given in Coles and Hirst (1967), although this may only be an impression because Π is small. As doubts exist about the validity of the universal log law at low Re , (Simpson 1970) for example, a further check on the skin friction values, independent of the log law (and of the Preston tubes) is, in any case, required. Small fence-type surface tubes designed to occupy the viscous sublayer were used as described by Bradshaw and Gregory (1961). These were first cleaned with acetone and then calibrated against the Preston tubes at high Reynolds number. Preston tube and Stanton tube readings were taken over a range of tunnel speeds and,

at each speed, Stanton tube pressure was plotted against τ_w calculated from Preston tube readings. The Preston tubes were calibrated using Patel's (1965) law in the graphical form given by Wong (1970). Measurements were taken using several Preston tubes of each size and it was found that the three tubes finally used agreed to within 1 % in pressure at $Re = 4750$ and 0.7 % at $Re = 800$, considerably better than the agreement within the whole range of tubes. Tube dimension errors were investigated and it was found that a 4 % tube diameter error produced less than 1 % change in c_f ; it is considered that dimensions are accurate to within this limit.

2.3 Mean Flow Measurement.

The mean velocity profiles were measured using a round section pitot tube of external diameter 1mm, mounted in a micrometer-screw traverse gear with 6" travel. The traverse gear is in turn mounted on a $3\frac{1}{2}$ " diameter brass instrumentation disc fitted with a static tapping. The traverse gear was insulated from the disc by a perspex sheet so that contact between disc and pitot tube mouth could be checked with an Avometer. The pitot tube position transducer was a 2" linear potentiometer (Pye Ether type PD 13) having a combined hysteresis and linearity error of 1.1 % of full scale (manufacturer's figures).

Pitot pressure measurements were made using a Combist micromanometer (Combustion Instruments Ltd.) fitted with a potentiometer geared to the central spindle to give electrical readout. The working fluid of this instrument is silicone oil, s.g. = 0.82. Scale sensitivity is 0.001mm, equivalent to 0.005 % of tunnel reference pressure, and the accuracy is better than 0.002mm (manufacturer's figures) which is 0.01 % of tunnel reference pressure in this case. Zero drift was found to be

negligible.

Tunnel reference dynamic pressure was taken from pitot and static tapings in the contraction and was registered on a Betz manometer in millimetres of water.

The use of pitot tube corrections is discussed by MacMillan (1956), Coles (1967) and Miller (1971). It was decided to include solely a displacement correction of $0.12d$ since doubt exists about the validity of turbulence corrections where the intensities are low, and the viscous corrections for circular section tubes would be of small order for the Reynolds numbers concerned (Re based on tube internal diameter ≈ 750). The pitot tube position signal, the micromanometer reading signal and a code signal were fed via a Solartron Data Transfer Unit and digital voltmeter (Solartron LM 1420) to an ASR 33 teletype, producing a punched paper tape and a printed record. The data tape was processed in the Aeronautics Department Honeywell H21 computer.

The arrangement of apparatus used for taking mean velocity profiles is shown in Fig. (2-3.1).

2.4 Hot Wire Apparatus.

A DISA type 55A38 X array hot wire probe was used with 5 micron diameter tungsten wire, welded to the prongs. The probe was clamped to the traverse gear stem, projecting forward 1.75" from the disc centre. The probe was connected to two DISA 55D01 anemometers by 5 metre shielded DISA Probe Cables. The anemometer output was recorded on an Ampex FR 1300 frequency modulated tape recorder after filtering and amplification in DISA auxiliary units (type 55 D25) and PHI system amplifiers (Servicon Dynamics Ltd.). Fluctuating voltages were summed in a signal.

conditioner DISA Type 55 D26, the output of which was measured by a DISA RMS meter. The hot wire electronics system is shown in Fig. (2-4.1).

The detailed operating instructions to be found in the DISA Instruction Manual for the 55D01 Anemometer Unit (1968)

were followed. The overheat ratio was 1.8, and it was possible to obtain an upper frequency limit of 40KHz in the wire response to a calibration square wave.

Wires were calibrated to the law:

$$E^2 = A + Bu^n \quad \text{where} \quad n = 0.45$$

and in yaw according to the law of Castro (1972)

$$\tan \psi = \frac{[\cos \delta - \left[\frac{E_{\delta}^2 - E_{01}^2}{E_{\delta=0}^2 - E_{01}^2} \right]^{\frac{1}{n}}]}{\sin \delta}$$

where ψ is the effective wire angle and δ is the angle of yaw. It was found that $\tan \psi$ was very close to 1.0 for all wires used, (typically in the range 1.05 - 0.95).

The anemometer signal was filtered at 1Hz high pass and 20KHz low pass, the frequency limit of the tape recorder at the tape speed used (60 inches per second). The amplifiers were used to amplify the anemometer output so that the signal recorded occupied the ± 1 volt recorder range. A Hatfield Instruments type 591/A decade resistor was attached in series with the amplifier feedback resistor giving a gain range of 1 - 10.

The recorder modules were set to unit gain and zero d.c. level, but because of difficulties in assessing exactly the gain of the whole recording signal a calibration signal, a sine wave of known R.M.S. voltage, was played through the system from auxiliary unit to tape recorder and recorded between each run.

2.5 Experimental Procedure.

At the start of each run conditions were allowed to settle for at least 30 minutes at the test speed of 50ft/sec. Preston tube measurements were made at every station using all three Preston tubes, all unused static tappings being sealed. Two similar Stanton tubes were calibrated at Station 17 ($Re = 4750$) and measurements were taken with each at Station 1-8. Four sets of repeat Preston tube measurements were made at Stations 1-8, 11, 14 and 17. These data were analysed using the computer program devised for the purpose by Young (1974).

The pitot tube in its traverse mechanism was aligned with the working section centreline where c_f was near spanwise mean value. The pitot tube surface pressure reading was taken and checked against Preston tube readings at the same position. Once good agreement was obtained in c_f a pitot traverse was taken; each traverse consisted of about 60 readings spaced across the boundary layer to give roughly equal increments on a logarithmic scale. At each position the Combist micromanometer was carefully adjusted until the meniscus movement was equidistant either side of the sight line, at which point readings were taken of tube position, manometer reading and code. After a spurious reading a corrected reading was taken using a different code for over writing. The end of each run was defined as the third successive position of equal pitot static pressure difference.

Traverses were made at all stations and repeated twice at stations 1-8, 11, 14 and 17. These data were analysed on the Honeywell H21 Computer using programs JMFO 1 and JMFO 2 (Appendix i).

For hot wire measurements probe cables were run into the working section either from the exit or through an empty disc position at least two stations downstream from the wire. The B.N.C. connectors

between probe and cable were insulated from, and firmly attached to, the floor with masking tape. Traverses were made at stations 1-8, 11, 14 and 17 each comprising 10 recordings made at positions $y/\delta_{995} = 1.1$ to 0.1 or as near to the wall as possible. Probes were calibrated at the beginning, the half-way point and the end of each traverse. Amplifier gains were adjusted at each position. The signal conditioner output was measured so that an analogue check on the data could be made.

The mean output of each anemometer was directly measured by digital volt meter. At the beginning of each tape recorder track a 10 second recording of the calibration sine wave was made, and between each 30 second data recording, 10 seconds of the calibration output from the system was recorded. The r.m.s. levels of these signals were also measured and noted.

The analogue recordings were replayed at 15 i.p.s. into an analogue-to-digital converter and a sample-and-hold unit interfaced by a PDP8 minicomputer to an Ampex TM16 digital tape transport as described in Weir and Bradshaw (1974).

The digital data were analysed on the Imperial College CDC 6400 computer using the program described by Weir and Bradshaw (1974).

Chapter Three.

The Mean Flow.

In this chapter the mean flow results derived from surface tube and pitot traverse data are discussed and compared with equivalent data obtained by other experimenters, in particular Wieghardt's (1944) as analysed by Coles and Hirst (1967).

3.1 The Physical Environment.

The properties of the physical environment affecting the mean flow are :

- (i) free-stream turbulence
- (ii) three dimensional effects arising from irregularities in the contraction boundary layer or the free stream, spanwise variation of transition position and streamwise vorticity induced by corners and small obstacles
- (iii) local pressure gradient variation

3.1.1 Effects of Free Stream Turbulence.

It is to be expected that the entrainment of free stream turbulence will affect the form of the mean velocity profile and the surface shear stress.

Coles (1962) suggested that the wake component could be expected to halve its value for an external turbulence level of 1.8 % and to decrease to zero for a level of 4.5 %.

The effects of free stream turbulence on c_f have been investigated by Robertson and Holt (1970), Tsuji and Iida (1972), Huffman, Zimmerman and Bennett (1972) and Charnay (1971). These data do not agree well together, presumably because of differences in experimental arrangement. Huffman et al, for example, used a monoplane grid for producing their free stream turbulence in contrast to the more usual biplane grid. Bradshaw (1974) in discussing these results suggest that a good correlation for the effects of turbulence on skin friction is given by:

$$\frac{C_f}{C_{f0}} = 1 + 3.2 \frac{\sqrt{u_e^2}}{U_e}$$

The free stream turbulence in the present experiment is of order 0.1 % at the working section entrance, so that c_f may be expected to be high by the negligible amount of 0.3 %.

3.1.2 Three Dimensional Effects.

The effect of irregularities in the contraction boundary layer was eliminated by the bleed at the beginning of the working section, which removed the boundary layer completely.

The transition device chosen was towards the small end of the acceptable range (section 2.1) but a stethoscopic survey showed that the transition was complete across the span of the working section within $\frac{1}{2}$ " downstream of the trip wire. It is expected that by the first measuring station 6" downstream the effects of irregular transition, principally spanwise variations in boundary layer thickness, will have diminished to negligible proportions.

The presence of spanwise variations of surface shear stress

has been noted above (section 2.1), and one possible explanation for this phenomenon is the existence of weak streamwise vortices in the free stream. It was suggested by Perkins (1970) that the formation of streamwise vorticity was a result of the secondary flow formation arising from inhomogeneity of the wall turbulence, such vorticity having a scale of order the boundary layer thickness. Perkins' analysis depends upon the spontaneous formation of shear stress gradients, which is not thought to be physically realistic.

Spangler and Wells (1970) artificially produced longitudinal vortices in a turbulent boundary layer and found that, with a vortex generating element of height 20 % of boundary layer thickness, they could produce variations of shear stress of lateral scale δ and magnitude + 12 % to -8 % of a 'smooth wall' shear stress. Averaged over the span, the increase in c_f was 3 %. The effects of the vortex generators were detectable for about 90 element heights, that is about 25 boundary layer thicknesses, downstream. Irregularities or imperfections of this order do not, however exist in the present experiment and the spanwise variation of skin friction remains at present unexplained.

3.1.3 Local Pressure Gradients.

The general effect of a favourable (negative) pressure gradient is to raise c_f and to decrease the wake strength, and conversely for adverse (positive) gradients. From the mildly retarded boundary layer of Bradshaw (1966) in which $U_e \propto x^{-0.15}$ and the constant pressure boundary layer of Wieghardt a value of 0.68 may be deduced for $\frac{d\pi}{d\beta}$

where

$$\beta = \frac{\delta^*}{\tau_w} \cdot \frac{dp}{dx} = \frac{\delta^*}{C_f} \cdot \frac{dC_p}{dx}$$

The largest pressure gradient occurs between $x = 3.5\text{ft.}$ and $x = 4.0\text{ft.}$, where $\frac{dP}{dx} = 0.02$ per foot. Hence $\beta = 0.06$ and the change in Π will be 0.06 or 10 % if $\Pi = 0.6$.

3.2 Surface Shear Stress.

Surface shear stress was measured using both the Preston tubes and Stanton tubes for which inner-layer similarity arguments give:

$$\Delta P \frac{d^2}{\rho v^2} = f\left(\frac{\tau_w d^2}{\rho v^2}\right)$$

where d is some typical tube length scale (the height or diameter) and the function depends upon geometry. Measurements are presented in non-dimensional form as skin friction coefficient c_f .

The Preston tube is designed to occupy the inner layer where

$$\frac{U}{u_\tau} = \frac{1}{K} \ln \frac{u_\tau y}{\nu} + C$$

and the validity of its readings at low Re depends on the validity of the constants K and C .

The Stanton tube, however, is ideally supposed to fit within the linear sublayer where:

$$\frac{U}{u_\tau} = \frac{u_\tau y}{\nu}$$

and its calibration should therefore, be independent of variations in the log law. In fact tubes cannot be made so small, but it may be assumed that departures from inner-layer similarity are more likely to be seen in the much larger Preston tubes.

The Stanton tubes were calibrated against the Preston tubes at high Reynolds number ($Re = 4750$) where inner-layer similarity is well established. If, now, agreement is obtained between the Preston and the

Stanton tube results at low Reynolds number, the universality of log law constants for this flow is demonstrated.

3.2.1 Standard Skin Friction Values.

In his constant-pressure boundary layer measurements, Wieghardt (1944) used a very careful experimental technique, and the free stream turbulence level in his tunnel was low. Coles (1962) found that these data lay close to a consensus of all the other experimenters' results which he examined. In particular the growth of the 'wake' component (Fig. 1-1.1) was found to be virtually monotonic from the lowest Reynolds number, an important indication of 'normal' development: excessively large trip wires or other initial disturbances lead to a fall in Π . These data were reanalysed by Coles and Hirst (1967) who calculated c_f using the 'Clauser chart' method of comparing the measured inner layer velocity profiles with the logarithmic law ($K = 0.41$, $C = 5.0$). The reanalysed data are used below as a basis for comparison.

3.2.2 Results.

In Fig. (3-2.1) the present results, $c_f = f(Re)$ are compared with Wieghardt's, given as his data points and the mean line through them. Preston tube and Stanton tube results are shown together with measurements obtained at the beginning of each pitot traverse, using the traverse pitot tube as a Preston tube: the results are displaced ahead of the others because the pitot mouth is $1\frac{1}{2}$ " in front of the disc centre.

It was found that, although individual Preston tubes consistently produced higher or lower c_f , no trend with Reynolds number was discernible

in differences between them. Differences were therefore attributed to geometric imperfections (0.003" uncertainty in diameter of the 0.058" tube gives a 1 % uncertainty in c_f). The three tubes in closest agreement were averaged over some 5 separate readings each so that each point in Fig. (3-2.1) represents the mean of 15 readings.

Re values in Fig. (3-2.1) were obtained taking a smooth curve through pitot traverse values of θ plotted against streamwise position and interpolating to allow for differences in position between Preston tube and pitot tube mouths.

The scatter of points in the present Preston tube data, about $\pm \frac{1}{2} \%$, is of the same order as the scatter in the results of Wiegardt, but there is a consistent difference between the two sets with the present data lying about 2 % below those of Wiegardt as interpreted by Coles and Hirst (1967), c_f from $\frac{1}{41} \ln y^+ + 5.0$. The Stanton tube results Fig. (3-2.2) show a larger scatter, + 1 % to - $1\frac{1}{2} \%$ about the mean Preston tube line. This may be attributed to the very poor sensitivity of the Stanton tube. (The largest Stanton tube pressure difference recorded, at $x = 0.5\text{ft}$, was only 0.05cms of combist fluid (density 0.82). The sensitivity of the combist is about + 0.001cms, so that the uncertainty in c_f would be $\pm 2 \%$). The agreement between Stanton tube and Preston tube results demonstrates that, assuming the log law is valid at high Reynolds numbers, it is also valid throughout the experimental Reynolds number range. Preston tube results may therefore be considered to be reliable.

Brederode and Bradshaw (1974a) re-examined the measurements of surface shear stress made by the present writer and other members of the Imperial College Aeronautics Department and found that c_f values taken directly from Preston tube measurements using Patel's calibration showed a consistent difference from the values obtained by Coles using an indirect

method based on the Clauser chart. Brederode and Bradshaw considered that the Preston tube results of Patel based on c_f deduced from the pressure drop in pipe flow, were intrinsically more reliable than the 'indirect' results of Coles, implying a change in the log law constant recommended by Coles. These revised log law constants, $K = 0.41$, $C = 5.2$, give c_f 2 % below that of Wieghardt, which would completely explain the differences obtained in this experiment.

3.3 The Growth of the Experimental Boundary Layer.

Despite the importance of boundary layer thickness, there seems to be no consensus as to which value should be used. Some experimenters use ' δ ' presumably the position at which $\frac{U}{U_e} = 1$, in which case $\delta \rightarrow \infty$, others use δ_{99} at which point $\frac{U}{U_e} = 0.99$ and Coles used the position at which $\frac{\partial w}{\partial y} = 0$, which is very difficult to measure accurately. These differences are not trivial, δ has been found to vary from definition to definition by up to 15 %.

For present purposes it has been decided to use δ_{995} , the position at which $\frac{U}{U_e} = 0.995$.

3.3.1 Boundary Layer Thickness.

δ_{995} was measured by plotting the outermost points of the pitot traverses on a large scale, drawing a smooth curve and picking off the point at which $\frac{U}{U_e} = 0.995$. This procedure is not entirely satisfactory because of the shallow slope of the velocity profile at this position, but it is considered the best available. The uncertainties are of order $\pm 0.01''$ in a layer about 1.7" thick, which is about twice the pitot position error

(0.005"). At highest Re ($= 4750$) the cumulative error in δ_{995} will be of order 1 % and at the lowest Re ($= 800$) about 2 %.

The Reynolds number most appropriate to the turbulent boundary layer would be

$$Re = \frac{u_z \delta}{\nu}$$

where u_z is a turbulence velocity scale and δ is a turbulence length scale, and a close approximation to this,

$$Re = \frac{u_z \delta_{995}}{\nu}$$

is shown in Fig. (3-3.1) plotted against Re in comparison with Wieghardt's data.

The scatter of the present data is well within the $\pm 1\%$ confidence limits of δ_{995} , but Coles' analysis gives Wieghardt's data a scatter about twice that, due probably to the difficulty of positioning $\frac{\partial w}{\partial y} = 0$.

3.3.2 Integral Thicknesses.

Following Coles (1968) standard integrals were used to represent the velocity profile close to the wall:

$$\int_0^{50} \left(\frac{U}{u_z} \right) d\left(\frac{u_z y}{\nu} \right) = 540.6$$

$$\int_0^{50} \left(\frac{U}{u_z} \right)^2 d\left(\frac{u_z y}{\nu} \right) = 6546.0$$

Then the momentum thickness, θ , and the displacement thickness, δ^* are given by:

$$\theta = \int_{\frac{50\nu}{u_z}}^{\delta} \frac{U}{U_e} \left(1 - \frac{U}{U_e} \right) dy + 540.6 \frac{\nu}{U_e} - 6546.0 \frac{u_z \nu}{U_e^2}$$

$$\delta^* = \int_{\frac{50v}{u_t}}^{\delta} \left(1 - \frac{U}{U_e}\right) dy + \frac{50v}{u_t} - \frac{540.6v}{U_e}$$

The scatter obtained in the calculation of θ and δ^* was about $\pm \frac{1}{2} \%$, Fig. (3-3.2).

3.3.3 Momentum Balance.

The momentum integral equation for a zero pressure gradient boundary layer is given by:

$$\frac{d\theta}{dx} = \frac{C_f}{2}$$

integrating both sides with respect to x from some initial position

$$x = x_0, \theta = \theta_0 \quad \text{gives:} \quad \frac{\theta - \theta_0}{\theta_0} = \frac{1}{\theta_0} \int_{x_0}^x \frac{C_f}{2} dx$$

The two sides of this equation, PL = $\frac{\theta - \theta_0}{\theta_0}$ and PR = $\frac{1}{\theta_0} \int_{x_0}^x \frac{C_f}{2} dx$, were separately calculated starting at station 1 ($Re = 800$) where $x = 0.5ft$ and $\theta = 0.0025ft$. The results, Fig. (3-3.3) show a slight divergence between the two sides of the momentum integral equation due to some lateral convergence or divergence, combined of course with any experimental errors.

The convergence/divergence term may be written

$$\frac{\theta}{x - x_0} = \frac{C_f}{2} - \frac{d\theta}{dx}$$

where $x - x_0$ gives the position of the local virtual origin (at x) of the convergence/divergence. $\frac{C_f}{2}$ and $\frac{d\theta}{dx}$ were separately plotted against x , Fig. (3-3.4) and assuming $\frac{d\theta}{dx}$ constant for $x > 3.5ft$, a good approximation, the virtual origin position, $\frac{1}{x - x_0} = f(x)$ was obtained, Fig. (3-3.5). The very small values of $\frac{1}{x - x_0}$ imply a virtual origin distant from x and therefore that divergence or convergence is very small indeed. The zero crossover, where x_0 changes from $-\infty$ to $+\infty$ represents a minimum or maximum point in the working section width. The virtual origin moves from about

24.5ft upstream at $\chi = 1.5\text{ft}$ to about 650ft downstream at $\chi = 8.5\text{ft}$.

Much of this imbalance may, however, be due to the spanwise variations in c_f .

3.4 The Shape Factor.

The streamwise development of shape factor, $H = \frac{\delta^*}{\theta}$, is shown in Fig. (3-4.1) together with the data of Wiegardt. It may at first appear that there is a considerable difference between the two sets of data, but the scale used is fine and the actual difference is of order - 2 %, a far smaller difference than the scatter in the smooth wall data of Hama (1954). In view of the scatter in Wiegardt's data it is difficult to attach significance to the difference between the two sets of measurements. The large difference between the 1962 and the 1968 values of the shape factor illustrates the effect on profile integral parameters of a comparatively small change in analytical technique.

The Clauser equilibrium parameter, given by

$$G = \frac{\int_0^\delta \left(\frac{U-U_e}{u_\tau} \right)^2 dy}{\int_0^\delta \left(\frac{U-U_e}{u_\tau} \right) dy} = \frac{H-1}{H} \sqrt{\frac{2}{C_f}}$$

was calculated and is given in Table (3.1). Values range from 6.96 at $\chi = 0.5\text{ft}$ to 6.74 at $\chi = 8.5\text{ft}$ which should be compared with the absolute value of 6.8 at high Reynolds number due to Coles and Hirst (1967). G is plotted against Π in Fig. (3-4.2) and it may be seen that the present data agree tolerably with Wiegardt's data.

The values of $G(\Pi)$ derived by Green (1972) from Huffman's data, and the correlation he obtained, are also shown to illustrate the effects expected in the presence of free stream turbulence, as an additional check on the absence of such effects in the present data.

3.5 Mean Velocity Profiles.

Mean velocity profiles were analysed in the form:

$$\frac{U}{u_\tau} = f\left(\frac{u_\tau y}{\nu}\right) + \frac{\pi}{K} w\left(\frac{y}{\delta}\right) \quad 3.5.1$$

where $f\left(\frac{u_\tau y}{\nu}\right) = \frac{1}{K} \ln\left(\frac{u_\tau y}{\nu}\right) + C \quad 3.5.2$

for $\frac{u_\tau y}{\nu} > 50$

The values of constants were taken from Brederode and Bradshaw (1974a):

$$C = 5.2 ; \quad K = 0.41$$

Semi-logarithmic plots of the results are shown in Fig. (3-5.1).

The solid line represents equation 3.5.2 and the dotted line gives the greatest extent of the wake. The distance between these two parallel lines gives the strength of the wake .

In general the present data fit the log law well: the slope, so far as can be seen, remains constant, but the low Reynolds number data seem to rise above the log law giving $C = 5.45$ at $R_\theta = 800$. The latter must be attributed to experimental error since the log law constants are implicitly universal in the Preston tube calibrations. Corrections have not been made, however, since the C variation is not consistent and disappears for $R_\theta > 2000$.

Experimental values of the wake strength, Fig. (3-5.2) are expressed with reference to the log law with $C = 5.2$ and to the best fit C' . The scatter in the latter case is less, tending to confirm that the scattered C values are due to experimental error.

Wieghardt's (1944) data is also shown, both in the original Coles (1962) form and in the revised Coles (1968) form (Coles and Hirst

(1967) do not give $\Pi(R_\theta)$). The earlier analysis was based on an inner-region fit, whilst the later values were derived by fitting the whole profile to the log law plus wake formation. The effect of this change was to give larger values for Π in the 1968 analysis, because the cosine wake function does not go to zero in the inner region and the effective log law slope to which Π is measured has a shallower slope, see Fig. (3-5.3):

$$\Delta \frac{U}{u_\tau} (1962) \rightarrow 2.7, \Delta \frac{U}{u_\tau} (1968) \rightarrow 3.0 \text{ as } R_\theta \rightarrow \infty$$

The scatter in the present data, ± 0.1 in $\Delta \frac{U}{u_\tau}$, is less than the scatter in Wieghardt's, ± 0.2 in $\Delta \frac{U}{u_\tau}$. Coles (1962) suggests that a reasonable accuracy would be 5% - 10% or ± 0.075 to ± 0.15 in $\Delta \frac{U}{u_\tau}$ at high Reynolds numbers.

The asymptotic value of $\Delta \frac{U}{u_\tau}$ obtained was 2.5, below both versions of Wieghardt's data, implying that the profile may not have been fully developed by $\chi = 8.5$, $R_\theta = 4750$.

3.6 Wake Function.

Bull (1968) shows that a confusion arises from the Coles (1968) form of the wake function:

$$\omega\left(\frac{y}{\delta_c}\right) = 1 - \cos\left(\frac{\pi y}{\delta_c}\right) \quad \text{where } \delta_c \equiv \text{Coles' } y \text{ at } \frac{\partial \omega}{\partial y} = 0$$

because of the definition of δ_c which in effect assumes that at $y = \delta_c$, $U = U_\theta$ and $\frac{\partial U}{\partial y}$ jumps to zero.

Bull suggests a modification of the form:

$$\omega\left(\frac{y}{\delta_c}\right) = 1 - \cos\left(\frac{\pi\left(\frac{y}{\delta_c} - a\right)}{1-a}\right)$$

where $Q = 0.08$, a value based upon Bull's own experimental data. See Fig. (3-6.2).

The present data were re-analysed to extract $w\left(\frac{y}{\delta_c}\right)$ based:

(i) upon $C = 5.2$

(ii) upon the best fit value, C'

These results are shown in Fig. (3-6.2) compared with the original (Coles) form and the later (Bull) form of the wake function. At high Reynolds numbers the fit to the Bull form is remarkably good, but at low Reynolds numbers some real discrepancies do occur. The main difficulty lies in the interpretation of the constant C in the log law equation, changes in the value of which make a considerable difference to the wake functions, see for example Fig. (3-6.2b) where the change from 5.2 to 5.4 makes such a large difference. The values of C' range from 5.2 at $R_\theta = 4750$ ($C' = C$) to 5.45 at $R_\theta = 800$, Bull used higher values, 5.6 and 5.8 and found that, using 5.2, wake values on average were 2.5 % higher, but that the fit near the wall was very poor. (This may also be seen at Station 2 $R_\theta = 1100$).

Thus it seems that there is a change in the profile shape with Reynolds number, effectively $Q = f(R_\theta)$ for $R_\theta < 6000$, values of Q at low R_θ being higher.

3.7 Conclusions.

The excellent agreement in skin coefficient between the present data and those of Wieghardt show that the experimental boundary layer may be considered fully developed and that no residual effects of transition should be expected. Agreement between Stanton tube and Preston tube data at low Reynolds number confirms the universality of the log law constants, so that Reynolds number dependence can be confined to the outer layer. The measured mean velocity profiles also show the constancy of the

log law at low Reynolds number. The wake component grows monotonically with

Reynolds number and is in good agreement with the original analysis of the profiles of Wieghardt and others by Coles (1962), suggesting that the later fitting procedure of Coles (1968) is less successful. A careful analysis of the shape of the wake components supports Bull (1968) in the re-evaluation of the Coles (1968) wake formula and, further, suggests that a change in the wake shape takes place at low Reynolds number.

Chapter Four

Conditional Sampling and the Measurement of Intermittency.

The object of conditional sampling as used in the present work is to produce separate averages of flow parameters for the turbulent and irrotational regions, called "zone averages", and the success of this technique is heavily dependent upon the ability of the intermittency scheme to accurately distinguish turbulent from irrotational zones.

A 'criterion parameter', some instantaneous property of the flow with effectively - zero value outside the turbulence, is chosen to make the distinction. In practice the criterion parameter is squared or rectified so that there are only positive values, which makes computing simpler. To make the contrast between 'turbulence' and 'non-turbulence' greater, some function of the criterion parameter, the 'criterion function', is used in the intermittency scheme. Any measurable fluid property has some non-zero free stream values and there will inevitably be a certain level of electronic noise: therefore, a non-zero 'threshold level' is chosen such that the criterion function must exceed the threshold level to register 'turbulence'. A unit on/off function, the 'intermittency function', may be constructed to show the regions of 'turbulence' and 'non-turbulence'. The time mean of the intermittency function gives the intermittency factor, δ , the fraction of sample time for which the flow past the measuring point is turbulent.

Even when the problems of spurious points occurring through non-zero values of criterion parameters in irrotational flow and the electronic noise have been solved there remain difficulties arising out of the physical nature of the turbulence:

(i) Any simple function of velocity used as criterion parameter will have occasional zeroes in the turbulence, as fluctuations change sign.

(ii) in addition, the turbulence itself contains 'high wave number intermittency' (see below)

(iii) physically the interface is highly re-entrant and fine-structured.

(i) and (ii) above, result in the prediction of spurious short irrotational troughs within a turbulent burst. Traditionally the practice has been to reject all short period events, turbulent or irrotational, by some form of smoothing, effectively low-pass filtering the criterion function, with the danger that the genuine occurrence of short bursts because of (iii) will be suppressed.

This chapter discusses the problems mentioned above, describes our attempts to circumvent them and gives the intermittency measurements finally taken.

4.1 The Intermittency Problem.

A major problem associated with the measurement of intermittency is the lack of a clear quantitative definition of turbulence on which to base the criterion parameter. The obvious function, fluctuating vorticity, was used by Corrsin and Kistler (1955) who based their intermittency scheme upon its longitudinal component, but such was the complexity of instrumentation (Corrsin and Kistler had a probe with four hot wires) that practice since then has been to use one component of fluctuating velocity or its spatial or temporal derivative.

Ideally the chosen function should have a zero value in irrotational zones and a clearly non-zero value everywhere else; zones

should be sharply distinct. In practice, however, it is found that the situation is not so simple.

It is comparatively easy to solve the problems of spurious indications of turbulence within irrotational regions (leading to the need for a non-zero threshold level) and of spurious indications of irrotational flow at zeroes of the criterion function within the turbulence ((i) above). The other and more serious problems (ii) and (iii) are associated with the physics of the very turbulence which it is endeavoured to understand.

Photographs taken in smoke-filled boundary layers, for example by Fiedler and Head (1966), Falco (1974) and Pongprudhanon (1974), the latter being reproduced in Fig. (4.1.1), show clearly that the interface between turbulent and non-turbulent fluid is very complex indeed. It appears highly re-entrant, wispy and finely structured. Kovasznay et al (1970) point out that, since the wavelengths of surface irregularities in the interface could extend over the full range of turbulent wavelengths (limited only by viscosity) the definition of the interface depends on the coarseness of the detection systems spatial resolution. The problem is analogous to that of definition of a continental coastline which, as Mandelbrot (1967) observes, entirely depends upon the sensitivity of measurement. In the present context, the attempt to understand the changes which occur with Reynolds number, the small scales are of great importance since at high Reynolds number it might be expected that a larger range of scales will be found. Falco (1974) also emphasises the importance of small scales in understanding Reynolds number effects on the intermittency. Clearly if 'smoothing' is used to dispose of difficulty (i), the occurrence of 'zeroes', then this fine structure may also be lost.

Difficulty (ii) arises from the use of a non-zero threshold level to deal with electronic noise, leading to the misinterpretation of

high wave number intermittency and short irrotational events within turbulent regions. If the intensity (spectral density) of the very smallest eddies is averaged over a distance corresponding to a large number of small eddy wavelengths, but small compared to the wavelength of the energy containing eddies, then, because of viscous dissipation, it is found that there is a variation in average intensity from point to point. Thus, contributions to the intensity of the smallest eddies are highly intermittent on a scale one order of magnitude larger than the smallest eddies. Batchelor (1953) described this as 'an uneven distribution in space of the energy associated with the large wave number components of turbulence - becoming more uneven with increase in wave number'. Areas of low high-wave-number intensity could be mistaken for 'irrotational' regions and Thomas (1973) attributes some irregularities in his intermittency results to this effect.

Thus the difficulties are that practical criterion functions have occasional spurious 'zeroes' within the turbulence, that even well inside the interface the turbulence contains its own high-wave-number 'intermittency' and that the interface is finely structured.

4.2 Some solutions to the Intermittency Problem.

The traditional approach to these problems, the use of smoothing, assumes that all short events are spurious. If the intermittency function obtained without smoothing has the form of a 'pulse train' then the smoothing of the criterion function is adjusted until a 'box car' intermittency function is obtained, see Fig. (4.2.1). It is found, for example by Antonia and Bradshaw (1971), that the average burst lengths and intermittency factors are not affected by a further increase in smoothing and the plateau values are taken as correct.

We have chosen, however, not to accept this assumption because of the possible importance of fine structure in Reynolds number effects. To distinguish between the different approaches results of the traditional method are defined as 'wholesale intermittency' and the results retaining fine structure as 'retail intermittency'. Thus the 'wholesale intermittency' will define the envelope of the turbulence containing the fine scale 'retail intermittency'. This is shown in Fig. (4.2.2) with some hypothetical criterion function, $F_c(t)$.

Wholesale intermittency as well as retail intermittency is of interest in the present study because it gives a picture of the behaviour of the largest eddies and Table (4.1) shows other experimenters' smoothing times, which give a valuable insight into the spatial resolution they obtained. The smoothing time values are expressed as the distance moved downstream by the free stream in the stated time, divided by a suitable length scale (in this case the boundary layer thickness). It may be seen that some of these 'smoothing lengths' are very large compared with the small scales apparent in smoke pictures.

Since wholesale intermittency concerns the larger energy containing eddies, a suitable length scale would be the boundary layer thickness, δ . The retail intermittency, however, is concerned with the smaller range of eddy lengths so that some other scale must be chosen.

It has been suggested that the Taylor microscale $\lambda \equiv \frac{u^2}{\sqrt{\frac{d^2 u^2}{dx^2}}}$ should be used as the normalising length scale for retail spatial resolution. However, the numerator is characteristic of energy containing eddies whereas the denominator is characteristic of dissipating eddies; λ , is roughly $5L^{1/3}\eta^{2/3}$ where η is the Kolmogorov length scale $\left(\frac{\nu^3}{\epsilon}\right)^{1/4}$ and L is the dissipation length parameter $\frac{(-\overline{uv})^{3/2}}{\epsilon}$ (which may be considered to be a scale associated with energy-containing eddies as ϵ equals the transfer

of energy from them). It is difficult, therefore, to ascribe a physical meaning to the Taylor microscale in this context.

Corrsin and Kistler (1955) put forward the hypothesis that a thin layer which they called the viscous super-layer separates the turbulent from the irrotational fluid. The viscous super-layer would then transport vorticity fluctuations to the initially 'irrotational' fluid through viscous forces. They suggested, plausibly, that this interfacial layer would have a thickness of the order of the Kolmogorov length scale. Supposing the smallest possible burst to consist of just a fold in the interface, the smallest burst length would be of order twice the Kolmogorov length. The Kolmogorov length is a typical scale of the dissipating or viscous eddies and it is suggested that η be used as the scale for retail intermittency. On this basis even the small smoothing lengths of, for example, Hedley and Keffer give large numbers of Kolmogorov length scales when re-analysed. (For their $R_\theta = 8500$ boundary layer at $y/\delta = 0.7$ their smoothing length is approximately 30η).

If both wholesale and retail intermittency are to be physically meaningful the spurious small scale structure must still be separated from the spurious, electronic, small scale events. A threshold level is thus required, which must be chosen very carefully. This is fully discussed in 4.4, below. The danger, then, is that spurious zeroes will appear in the nominally turbulent regions. It was suggested above that the rectified (or squared) criterion parameter could be made more sensitive to changes in regime by using its time derivative as criterion function. In a digital sampling system the time derivative of the rectified criterion parameter has a negligible probability of identical zeroes, but some low values will be found in turbulent regions so that it is dangerous to base intermittency on that one function. If, however, the second time derivative is used as

an alternative criterion function then, unless both functions are zero together, which is most rare, zeroes will be eliminated. This is known as the signal plus derivative approach.

4.3 Choice of Criterion Function.

A successful criterion function should:

- (i) contain a good contrast between turbulent and irrotational fluid
- (ii) not suffer from a high level of high-wave number intermittency
- (iii) preserve the small scale features of the flow
- (iv) be comparatively easy to measure.

The use of vorticity, apart from its technical difficulty, gives frequent spurious 'irrotational' zones within turbulence since the vorticity signal is seriously contaminated by high-wave number intermittency. (Vorticity is produced mainly in the small, but not smallest, eddy range.) Criterion functions chosen by various experimentors are summarised in Table (4.2); most are based on derivatives of one component of fluctuating velocity.

Antonia (1972) used the instantaneous product of u and v component fluctuating velocities, a quantity whose time mean, $\overline{uv}$, is the local turbulent shear stress. Castro (1973) also used uv ; he took the sensitised signal, $\frac{\partial uv}{\partial t}$, as criterion function and added as an alternative criterion function, $\frac{\partial^2 uv}{\partial t^2}$ to remove the effects of spurious zero values of $\frac{\partial uv}{\partial t}$. To produce a single sided criterion function Castro 'rectified', which is faster than 'squaring' in the digital computer.

The present study uses Castro's system, which has the

following advantages:

- (i) u and v are uncorrelated in the locally-isotropic range of wave numbers (by definition). Although the contribution to $\overline{uv}$ from the higher wave numbers is not instantaneously zero, it is smaller than their contributions to $\overline{u^2}$ or $\overline{v^2}$ and so problems with high-wave-number intermittency are reduced.
- (ii) Similarly w is found in practice to have a larger ratio of amplitude in the turbulent zone to amplitude in the irrotational zone than other conventionally used signals (i.e. u^2, v^2, w^2).
- (iii) The main interest here is $\overline{uv}$, so regions of small w have less importance.

4.4 Choice of Threshold Levels.

The choice of threshold level is critical to the correct assessment of intermittency factor γ and to the accurate identification of the interface.

Given that turbulent bursts cover the whole range of turbulent wave length and amplitudes, it might be expected that the measured intermittency factor, γ , would vary continuously with threshold level and such indeed is the present experience: see for example, Fig. (4.4.1) where γ is plotted against threshold level. Thus the traditional approach, adjusting threshold level until some standard $\gamma(\frac{y}{\delta})$ is achieved, is conspicuously inadequate where the spatial distribution of zones is required in detail.

Threshold levels may also be determined from comparison based on visual analysis of film or paper records of turbulence or the examination of oscilloscope traces. These methods, however, are highly

subjective, involving the observer's experience and predudice. Traces of instantaneous UV produced by Calcomp plotter were shown to several persons with experience in the field of intermittency measurement who were asked to indicate the regions of turbulent and irrotational flow. Almost without exception they drew the broad envelope of the turbulence, and on this basis we concluded that visual analysis in general gives wholesale intermittency.

Recently Antonia (1974) has used a square-wave-modulated random signal to calibrate an 'intermittency meter', but this also involves the experimenter's subjective ideas about the structure of the turbulence and the form of $\gamma(\frac{y}{\delta})$.

In the present work it was found in the early stages that varying the pair of threshold levels and comparing retail intermittency with a standard intermittency factor (taken as the Klebanoff (1954) values), at two boundary layer positions did not yield threshold levels which gave the 'correct' γ at the two positions. The results of this matching are shown in Fig. (4.4.2): any pair of threshold values chosen to give correct γ at low γ gives too low a γ at high γ and vice versa. These measurements were taken well away from the wall ($y/\delta > 0.6$) since there is a disagreement about γ close to the wall.

In any case we feel that attempts to choose threshold levels solely on the basis of the intermittency factor may well ignore the more important aspect, the distribution of the intermittency function in time and the accurate interpretation of the instantaneous flow state.

The final choice in the present work was based upon agreement between γ^* , the wholesale intermittency factor, with Klebanoff's data, and good agreement between the shape of the intermittency function and a visual analysis of the UV trace. This process was carried out at all

boundary layer positions at the highest experimental Reynolds number, $Re = 4750$, until a satisfactory compromise was reached.

The present experiments cover a wide range of Reynolds number and some scaling of threshold level is required. Although, in theory, it should be possible to make threshold level dimensionless with typical velocity and length scales of the flow, in practice a scaling based on the turbulence itself is probably more reliable. In the present work the threshold level is defined as a proportion of the mean value of the rectified criterion function within the turbulence, in fact within the Retail turbulence. An initial guess based upon an adjacent boundary layer position is used for a time equivalent to about 35 boundary layer thickness after which the threshold level is updated every seven boundary layer thicknesses. It was found that this process was rapidly convergent, usually within 350 boundary layer thicknesses.

4.5 Computer Analysis.

The calculation of intermittency is carried out within the digital analysis program (described by Weir and Bradshaw 1974) by the subroutine THRESH. The basic steps are as follows: (see Fig. (4.5.1))

- (i) Calculation of threshold levels (at the beginning of each record (group of 600 readings) only).
- (ii) Calculation of the rectified first and second time derivatives of the chosen criterion parameter for turbulence (u' in this case).
- (iii) Comparison of the criterion function formed in (ii) with threshold levels calculated in (i) and formation of the retail intermittency function (unity if either derivative exceeds its

threshold, else 0).

- (iv) Formation of the wholesale intermittency function according to the wholesale/retail scheme (v).

A noise clip (vi) may also be used, to remove electronic noise.

(i) Threshold Levels: At the start of the run these are set to values based upon estimates from previous runs. Using these input values a set number of records (usually 10: about 6000 pairs of u and v values) is processed to derive reasonably stable running average values of $\left| \frac{\partial uv}{\partial t} \right|$ and $\left| \frac{\partial^2 uv}{\partial t^2} \right|$ within the turbulent regions. These averages are then used to update the threshold levels as follows:

$$TH1 = SC14 \left| \overline{\frac{\partial uv}{\partial t}} \right|$$

$$TH2 = SC15 \left| \overline{\frac{\partial^2 uv}{\partial t^2}} \right|$$

where SC14 and SC15 are input constants (finally chosen as 0.30 and 0.35 respectively) known as 'threshold level factors'. The new TH1 and TH2 values are used to process further records, creating new running conditional averages, which are used to update TH1 and TH2 at the beginning of each record.

(ii) Criterion Function: Finite difference values of $\frac{\partial uv}{\partial t}$ and $\frac{\partial^2 uv}{\partial t^2}$ at each time step, neglecting time step magnitude which cancels out, are given by

$$\left| \frac{\partial uv}{\partial t} \right|_j \approx Abs(0.5(uv(j+1) - uv(j-1)))$$

$$\left| \frac{\partial^2 uv}{\partial t^2} \right|_j \approx Abs(uv(j+1) - 2uv(j) + uv(j-1)))$$

(iii) Intermittency Function: Both retail and wholesale intermittency are given within the intermittency function, $ID(j)$, a step function

containing four levels defined as follows: (see Fig. (4.5.2))

- ID(J) = 0 - irrotational point
- ID(J) = 1 - retail turbulence but wholesale irrotational
- ID(J) = 2 - retail irrotational but wholesale turbulent
- ID(J) = 3 - retail and wholesale turbulent

i.e. one mark is awarded for retail turbulence, two for wholesale turbulence and three for both.

At each time step, the calculated values of criterion function, $\left| \frac{\partial w}{\partial t} \right|$ and $\left| \frac{\partial^2 w}{\partial t^2} \right|$ are compared respectively with TH1 and TH2. Should either exceed the relevant threshold level, the intermittency function at that time step is set, initially, to 3. Otherwise, it is set to zero.

(iv) Wholesale Intermittency Function: The initial values within ID(J) give retail intermittency and the wholesale intermittency is subsequently formed partly at each time step and partly in a re-run of ID(J) carried out after each record. A re-run is necessary when points ahead of the current time step as well as before are required, for example, to find the complete length of a particular turbulent burst.

(v) Wholesale/Retail Scheme: Two forms of this algorithm exist; 'unbiased' and 'turbulence biased'.

Unbiased: (see Fig. (4.5.3)) At each change of flow state, from irrotational to turbulent or vice versa, a counter is set. The number of time steps to the next change is compared with an input minimum, called NMIN; excursions shorter than NMIN time steps are reset to retail values, 1 or 2. It was found, by comparing Calcomp traces of ID(J) with UV(J) that turbulent bursts were often inaccurately indicated, the fine structure at the beginning of bursts being missed by the wholesale intermittency. An alternative scheme was therefore devised.

Turbulence Biased: This algorithm passes through ID(J) twice:

on the first pass short 'irrotational troughs' are reset to value 2, (retail irrotational) and on the second pass short turbulent bursts separated from the principal neighbouring full turbulent burst by more than N_{MIN} time steps are reset to value 1 (retail turbulent) see Fig. (4.5.4). This scheme gave a more satisfactory representation of burst pattern, see section 4.7.

(vi) It was found that on one particular set of analogue recordings the noise present dominated the signal where the r.m.s. value of the latter was low (at the outer edge of the boundary layer). A 'noise clip' devised by Young (1975) and fully described in Weir and Bradshaw (1974) was used to see whether, in this case, it would be possible to analyse the recordings.

The noise clip effectively introduces a third threshold level:

$$TH3 = perc \times Abs(Dev_t - Dev_b)$$

where $perc$ is an input value (in the range 0.15 - 0.3). Dev_t is the standard deviation of the turbulence function (uv in this case) in turbulent zones and Dev_b is the standard deviation of the overall signal. Since noise and signal should be uncorrelated Young suggests that their variances may be separately calculated and would constitute a measure of the expected excursions from the mean. Near the surface Dev_t is very close to Dev_b , hence $TH3$ will be small and uv at virtually all points will exceed it. Near the outer edge, however, $TH3$ will take on larger values and, in a noisy signal, many points satisfying the gradient-type turbulence test will fall below $TH3$. These latter points will be rejected from the turbulence, on the assumption that they do not have the expected excursion value. The results of this system are discussed in section 4.7.

4.6 Spatial Resolution of the Intermittency Function.

The limit of the spatial resolution is set by the frequency response of the analogue tape recorder, the upper limit of which is 20 kHz at 60 ins./sec. recording speed. Assuming a convection velocity of 50 ft./sec. (the free stream speed) the smallest recorded wavelength will be 0.03 ins.

The typical retail intermittency length scale, the Kolmogorov length scale, $\eta = [V^3/\epsilon]^{1/4}$ where $\epsilon = \tau \frac{dU}{dy}$, gives, at $Re = 4750$ (Stn. 17), $y/\delta = 0.7$ and $\frac{\tau}{\rho} \frac{dU}{dy} = 104.0$, $\eta = 0.0053$ ins. According to Corrsin and Kistler the smallest possible burst length, 2η will then be of order 0.01 ins. The smallest wavelength recorded will be of order 6η , so that the very smallest bursts will, in any case, be missing from the analogue tape. Assuming, however, that the superlayer is of a similar scale as the sublayer, the usual thickness will be about 15η so that these smallest bursts will be rare.

In a digital recording the length scales are determined by the analogue tape replay speed, the digitisation rate and the low pass filtering.

It is suggested, by Gold and Raeder (1970) that for accurate representation of the 'shape' of a signal by a digital process, 4 sampling points would be required to define the smallest wavelengths. Offen and Kline (1973) recommended an 8 point representation. Digital recordings were made at 15 ins./sec. analogue replay speed and 10,000 samples/sec. digitisation rate. The maximum analogue frequencies for each channel would then be $(\frac{15}{60} \times 20,000 =)5$ kHz. Assuming the smallest recorded wavelength to be 0.03 ins., this smallest wavelength would be defined by only $(\frac{10,000}{5,000} =)2$ points. It was decided, therefore, to set the low pass filter to 2.5 kHz, 10 kHz real time, in which case the smallest scale would be 0.060 ins.

but defined by 4. samples.

To test the effects of digital resolution and the exclusions of the smallest wavelengths, tests were made as follows:

(i) data were digitised at $7\frac{1}{2}$ ins./sec. and 1250 H z low pass filter (equivalent to 2.5 KH z at 15 ins./sec.) to give an 8 point definition of the 0.060 ins minimum wavelength.

(ii) data were digitised at $3\frac{3}{4}$ ins./sec. and 1250 H z low pass to give an 8 point definition of the smallest analogue-recording-wavelength (0.030 ins.).

From these tests (fully described in 4.7) it seemed that the 4 point definition worked adequately and that the inclusion of wave lengths in the range 0.030 to 0.060 ins. made negligible difference to intermittency or to conditional averages.

The minimum burst length passing the wholesale criterion would be N_{MIN} times the 'sampling length', $U\Delta t$, where U is the convection velocity and Δt is the time step. Thus for $N_{MIN} = 10$, features of length less than 0.150 ins. would be considered to be retail features only. Tests were made at $R_\theta = 4750$ to find the best value of N_{MIN} on the basis of a visual comparison between the wholesale intermittency function and $uv(t)$ and a numerical comparison, with results derived by other experimenters (e.g. Klebanoff (1954)).

It was found with the biased scheme that far from the surface ($y/\delta_{995} \rightarrow 1.0$) the value of N_{MIN} did not make a large numerical difference to γ^* , the wholesale intermittency factor, but that near the surface ($y/\delta_{995} \rightarrow 0.1$) the effect of a large N_{MIN} ($N_{MIN} = 20$ minimum burst length = $0.300''$ or $0.19 \delta_{995}$ at this station) was to increase the value of γ^* by some 8 % compared with its value with $N_{MIN} = 5$, from 0.82 to 0.89 approximately.

The biasing of the intermittency scheme means that short turbulent bursts attach themselves to the fronts and backs of full turbulent wholesale bursts. As N_{MIN} is increased it might be expected that a significant increase in the number of such attached short bursts might be found. The fact that this is not so, the increase in γ^* is modest, implies that there are few extra short bursts added as N_{MIN} increases and that, at high y/δ_{995} turbulent bursts are well separated with few free short bursts appearing in otherwise irrotational regions.

Tests performed on the unbiased scheme showed it to be generally unsatisfactory: the numerical comparisons could be obtained but usually the intermittency functions left much to be desired.

The final deciding factor in the choice of N_{MIN} was the resolution, based on δ_{995} , at low R_θ . At $R_\theta = 790$, $N_{MIN} = 5$ gives a minimum wholesale wavelength of $0.258 \delta_{995}$. This was felt to represent the coarsest meaningful scale for wholesale intermittency (it is about twice Kovasznay et al's (1970) scale) which could be supported by visual comparison between $ID(J)$ and $UV(J)$. The same value of N_{MIN} was used throughout, giving a minimum wholesale wavelength of $0.053 \delta_{995}$ at $R_\theta = 4750$. Scaling N_{MIN} on δ would not be physically meaningful since the retail scale η , is proportional to $\delta^{1/4}$: the constant N_{MIN} option is the safest at present.

4.7 Visual Analysis of the Intermittency Scheme.

The next three sections describe the efforts made to ensure that the intermittency scheme performed satisfactorily and give the results finally obtained. Because so much importance was attached to visual comparisons, the visual analysis is presented separately from the numerical analysis; the distinction is to an extent artificial but

inevitable. The first of these sections, 4.7, describes the visual analysis used in the intermittency scheme design, in particular, the pattern found in the intermittency function and the changes in this pattern as certain parameters were changed. The next section, 4.8, gives the strictly numerical output, simply in terms of intermittency factor $\delta(y/\delta)$ as a function of the various systems examined. The last of these sections, 4.9, describes in more detail the patterns found in the criterion parameter, $uv(t)$, and their implications for the conditionally sampled quantities.

This section contains an evaluation of the intermittency scheme based upon a comparison between the computed intermittency function and a subjective visual analysis of the criterion parameter. Initially all the visual analysis was carried out from Calcomp plots, (ii) below, Fig. (4.7.1) for example. These showed, from top to bottom the intermittency trace $ID(t)$, then $u(t)$ followed by $v(t)$ and, at the bottom $uv(t)$. The magnitude of the velocity traces was scaled with $\sqrt{u^2}$ or $\sqrt{v^2}$ in the case of uv , and the time scale is in sampling period units scaled by $\frac{1}{10}$ i.e. 10.00 is equivalent to 100 sampling periods. The intermittency trace gives wholesale and retail intermittency based on the values given in section 4.5, (iii) above. The visual analysis was carried out by drawing on top of the trace of criterion parameter, usually $uv(t)$, a simple two valued step function to show irrotational and turbulent regions. For convenience of comparison this 'drawn intermittency function' has been repositioned in Figs. (4.7.1) etc., under the computed intermittency trace. The drawn intermittency trace was produced by carefully marking out clearly turbulent regions i.e. where uv activity was high, without reference to the computed intermittency. This process depends upon a subjective idea of what constitutes turbulence, but some subjectivity is considered to be inevitable in this work. As a check on drawn intermittency, its mean value,

$\delta_0(y/\delta)$ was calculated at high R_θ and compared with Klebanoff's (1954) $\delta(y/\delta)$.

Each Calcomp plot gave results for 10 records of data, 6000 sampling periods or about 0.2 seconds, equivalent to about 7δ at $R_\theta = 4750$. In this thesis about 3 records are shown, to give the typical details of each plot.

At a later stage in the work an interactive visual display system, Digigraphics, (1), became available through the work of Mahgoub (1974), and was used as an a posteriori justification of the choice of intermittency scheme. Because of data storage limits it was possible only to display one record of data (7δ at $R_\theta = 4750$) together with the two intermittency traces to be compared. This display length is too short for quantitative values of δ to be obtained, but does give a good illustration of the broad features.

(i) Digigraphic. Figs. (4.7.9 ; 4.7.10 ; 4.7.11) show a high Reynolds number ($R_\theta = 4750$) trace of $UV(t)$ and $ID(t)$ at a position where $\delta \approx 0.5$. Both biased and unbiased schemes are shown with minimum burst length of 5, 12 and 20 sampling periods, that is, about 0.05δ , 0.12δ and 0.2δ . The unbiased scheme takes into account only the history of the intermittency function; Figs. (4.7.10 ; 4.7.11) show clearly that it is not able to predict that short bursts at the beginning of a large burst might be attached to it, or that a group of short bursts might overall constitute a wholesale burst. It also seems that the larger values of $REYN$ are too coarse, used in conjunction with the biased scheme. Fig. (4.7.12), which should be compared with the biased trace in Fig. (4.7.9), illustrates the effect of threshold level. In general as threshold level factor decreases wholesale bursts are widened, but their number is not increased. The highest values of threshold level factor 0.3 and 0.35, give what seems

to be best positioning for the fronts and backs of bursts, based upon the $w(t)$ trace shown.

(ii) Calcomp Analysis. The digital sampling system was tested by making recordings at a range of replay speeds and low-pass filter settings (detailed in 4.6). Effectively these give a 4-point definition of 0.060ins. minimum wavelength, an 8-point definition of the same minimum wavelength and an 8-point definition of the smallest wavelength to be found on the analogue tape, 0.030ins. The results are shown in Fig. (4.7.6), Fig. (4.7.13) and Fig. (4.7.14) respectively. Were the 4-point definition inadequate, additional small scale features would appear in the intermittency function of Fig. (4.7.13). In fact the general absence of such additional features in Fig. (4.7.13) demonstrates that the 4-point definition, Fig. (4.7.6) works very well indeed. In fact Fig. (4.7.13) has an appearance similar to Fig. (4.7.6); but stretched in the time axis. The computed intermittency factors, both δ and δ^* , were also very similar in the two cases.

At low Reynolds number, where the dimension-less sampling length $\frac{U\Delta t}{\delta}$ is larger than at high Reynolds number a check was required to ensure that important small scale features were not clipped by the low pass filtering. Since filtering removed features of length of the order of η within the expected wavelength range, the smallest bursts would be lost, but if these make little difference to the intermittency or to the turbulence quantities the loss is acceptable. The data shown in Fig. (4.7.14) contains the minimum wavelengths recorded on the analogue tape. Comparison with Fig. (4.7.6) suggests that very few additional features appear outside turbulent bursts. It might be expected that, using a turbulence biased scheme, wholesale bursts would become wider with the increase in 'fuzziness' implicit in the extended wavelength range. In fact, the traces certainly contain more retail activity within wholesale bursts, but this does not

seem to significantly widen them, an impression confirmed by the closely similar computed intermittencies and turbulent quantities.

The sampling system used, then, gives an adequate reproduction of flow features, with a 4-point definition of smallest wavelengths, and the wavelength range used covers the most significant scales.

The effects of threshold level on the intermittency function were investigated by processing the same sets of records at various threshold levels. As suggested, in section 4.4, the choice of threshold level must depend as much on the accurate form of $ID(t)$ and its ability to determine the position of flow features as upon its time mean value, the intermittency factor. The physical matching between computed intermittency factor and the 'drawn' intermittency is described below, the numerical results are detailed in section 4.8. It is expected that a visual impression of the flow, such as the drawn intermittency, will give the large scale flow features only, so that comparison should be made with the wholesale intermittency. Results are shown for high Reynolds number ($R_\theta = 4750$) at a position near the outer edge ($y/\delta_{995} = 0.9$) in Figs. (4.7.15 ; 4.7.16 ; 4.7.17). These show the effects of increasing the pair of threshold level factors from 0.22 ; 0.28 to 0.28 ; 0.30 and should be compared with Fig. (4.7.1) which shows the same data with factors 0.30 ; 0.35. Figs. (4.7.18 ; 4.7.19) show the effects of a low threshold factor, 0.25 ; 0.32 at two positions, ($y/\delta_{995} = 0.7 ; 0.9$) at low Reynolds number ($R_\theta = 900$) and should be compared with Figs. (4.7.6 ; 4.7.7) which show the same data with factor 0.30 ; 0.35.

In general the result obtained by the digigraphics analysis is confirmed: bursts do not increase significantly in number as threshold level decreases, but become wider. This suggests that fronts and backs of bursts do not contain a sudden increase in criterion function value but

rather a gradual increase so that, as threshold level changes, turbulence is first detected, and is cut off, at different positions. The effect of this is to make the choice of threshold level factor from visual analysis very difficult, because it now becomes a question of deciding at which position on gently increasing slopes the turbulent bursts begin and end,

Comparison between Fig. (4.7.7) (high factor) and Fig. (4.7.19) (low factor) illustrate the difficulty of making a choice of factor at positions of relatively high intermittency: irrotational regions are so narrow that they are very difficult to detect visually. The detection of the turbulent bursts within a principally irrotational environment, Figs. (4.7.6 ; 4.7.18), is far more simple.

The results of a visual analysis must be qualified by the above. A comparison of Figs. (4.7.15 ; 4.7.16 ; 4.7.17) and Fig. (4.7.1) suggests that the burst lengths drawn are best reproduced by the latter, where the threshold factors are 0.30 ; 0.35. Similarly Fig. (4.7.6) looks better than Fig. (4.7.18). Thus from the visual analysis, threshold level factor should be 0.30 ; 0.35.

An alternative criterion parameter, v^2 , was suggested and Figs. (4.7.20 ; 4.7.21 ; 4.7.22) show the use of this parameter with the same intermittency scheme and threshold level factor as finally chosen for uv criterion parameter. These results should be compared respectively with Figs. (4.7.1 ; 4.7.2 ; 4.7.3). In the v^2 results the second trace from the bottom shows $v^2(t)$, which has a deceptively high contrast between regions of low and high activity. The v^2 criterion parameter gives wider bursts near the edge of the layer and narrower bursts close to the wall so that scaling threshold level factors would not produce an agreement between intermittency factor from $v^2(t)$ and from $uv(t)$. The numerical reasons for preference towards $uv(t)$ are given in

section 4.8.

The visual analysis of intermittency using the chosen scheme with threshold level factor 0.30 ; 0.35 is shown for several values of y/δ_{995} in Figs. (4.7.1 to 4.7.5) at high Reynolds number and Figs. (4.7.6 to 4.7.8) at low Reynolds number. The remarks made and values given below relate to the full 10-record trace produced by the Calcomp plotter, of which the 3 record extract reproduced is representative.

At high Reynolds number, agreement between drawn and computed wholesale intermittency is good throughout the layer. Very close to the wall, at $y/\delta_{995} = 0.1$ (Fig. (4.7.5)), the wholesale scheme seems to overestimate intermittency, probably because the small separation between bursts brings any retail, (small scale) turbulence comparatively closer to the bursts. At this position other experimenters from Carrsin and Kistler (1955) onwards, predict $\sigma = 1$. In the present results the criterion parameter $uv(t)$ is clearly intermittent at $y/\delta_{995} = 0.1$, and it is suggested that the closeness of bursts has caused other experimenters to overestimate intermittency factor close to the wall.

At low intermittency, e.g. $y/\delta_{995} = 0.9$ (Fig. (4.7.1)), the drawn intermittency factor, σ_D , the computed retail intermittency factor, σ , and Klebanoff's (1954) intermittency factor σ_K agree well together being respectively 0.350 ; 0.355 and 0.368, but σ^* , the computed wholesale intermittency is high, 0.41. Hence, although the wholesale intermittency function has the better visual comparison, the fine balance of small scales in the computed retail intermittency factor brings the latter closer to drawn intermittency in value. By $y/\delta_{995} = 0.7$ (Fig. (4.7.2)), however, σ^* , (= 0.81) and σ_K (=0.82) are in better agreement with the drawn intermittency σ_D (= 0.78) than the retail intermittency, σ (= 0.73).

At low Reynolds number the agreement between drawn and

computed intermittency is not quite as close, mainly because the increased activity of $uv(t)$ in nominally irrotational regions makes the production of an accurate drawn intermittency more difficult. At $y/\delta_{995} = 0.7$, Fig. (4.7.7) and $y/\delta_{995} = 0.5$, Fig. (4.7.8), the drawn intermittency function contains more bursts than the computed wholesale intermittency function, mainly as the computed large bursts are subdivided in the visual analysis. This implies that irrotational regions of very small size can be detected, in fact the visual analysis at high intermittency factor relies heavily upon recognition of some burst pattern, (detailed in 4.9 below) so that bursts can be separately identified even if very close together.

Fig. (4.7.23) and Fig. (4.7.24) show the effects of the noise clip at the outer edge of the layer $y/\delta_{995} = 1.1$. The conventional scheme overestimates intermittency Fig. (4.7.24), but the noise clip, with a low value of $\text{perc} = 0.2$ (see 4.5) enables the separation of noise and signal Fig. (4.7.23).

4.8 Computed Intermittency Results.

This section describes the behaviour of the computed intermittencies, the effects of changes in sampling system, criterion parameter and threshold level factor. The final scheme is detailed and the intermittency results for the range of experimental Reynolds number are given.

Klebanoff (1954) measured intermittency in a high Reynolds number, zero pressure gradient boundary layer, using visual analysis of vorticity and u-component fluctuating velocity traces together with the flatness factor intermittency of Townsend (1948):

$$\gamma_F = 3 / \frac{\overline{u^4}}{(\overline{u^2})^2}$$

Klebanoff's results were found to approximate closely to a Gaussian integral curve given by

$$\gamma = \frac{1}{2} (1 - \operatorname{erf} \zeta) \quad , \quad \zeta = 5 \left(\frac{y}{\delta} - 0.78 \right)$$

principally on the basis of $\gamma(y/\delta)$, Fig. (4.8.4). This curve is used as a standard for the present intermittency values because it has been very important historically: most other experimenters have fitted their intermittency schemes to it.

The sampling systems tested, detailed in section (4.6) and evaluated visually in section (4.7) give very similar values of intermittency factor, Fig. (4.8.1). Thus taking more samples or considering smaller scales gives very little advantage, and for economy in processing time the 4-point / 0.060" - scale system was chosen.

The results of the use of V^2 as criterion parameter are shown in Fig. (4.8.2). It can be seen that V^2 gives a higher γ at the edge and a lower γ near the surface and, in agreement with the visual analysis, scaling threshold level factor does not improve overall agreement with $\gamma_K(y/\delta)$. It must therefore, be concluded that $V^2(t)$ is less well behaved as a criterion parameter than $W(t)$. It is also interesting that values of γ from $V^2(t)$ should be so much lower than those obtained from $W(t)$ near the wall, since $W(t)$ is known to be intermittent in the sub-layer and has a high flatness factor in the log-law region.

The effects of threshold level on intermittency factor are shown in Fig. (4.8.3), from which it is evident that no clear 'best fit' threshold level factor can be selected. Although low factors give γ closer to unity near the wall, they greatly overestimate intermittency factor in the outer part of the boundary layer. The values of factor finally chosen, 0.30 and 0.35, give a good overall fit to $\gamma_K(y/\delta)$ with γ^* slightly high

and γ slightly low except at extreme values of intermittency factor where both γ and γ^* are low. This latter may be due at low γ_K to uncertainty in Klebanoff's boundary layer thickness. The values of γ, γ^* close to the wall confirm the findings of visual analysis, that intermittency reaches down to the wall: even γ^* tends towards a value less than unity, typically 0.95. There is, in fact, no direct experimental evidence that γ does reach unity close to the wall, except for flatness factor intermittencies which are forced to do so. Experimental values of intermittency factor close to the wall are rare: Kovaszny et al (1970) give $\gamma = 0.96$ at $y/\delta = 0.4$; Hedley and Keffer (1974), $\gamma = 0.93$ at $y/\delta = 0.4$; the closest to unity given by Corrsin and Kistler (1955) is 0.88 and Klebanoff (1954) finds $\gamma = 0.92$ based on film analysis. Analogue intermittency meters may produce intermittency factors greater than unity, but such data would in the normal course of events be disregarded as physically unacceptable and would not be reported.

The sensitivity of conditional averages to threshold level was investigated and the results are shown in Figs. (4.8.7 to 4.8.14). Since the effect of threshold level is seen directly in intermittency factor, the results are plotted against γ, γ^* . Both the turbulent zone average $\bar{Q}$ and the contribution of the turbulent zone to the overall average, $\gamma \bar{Q}$ are given. Over the range of intermittency factor considered, which is of order $\pm 10\%$ there is an almost linear relationship between turbulent zone average and intermittency which is continuous from retail to wholesale i.e. $\bar{Q}_R(\gamma)$ and $\bar{Q}_W(\gamma^*)$ lie nearly on the same straight line. Thus the wholesale scheme, despite its turbulence bias, makes little numerical difference to turbulence quantities beyond that expected from the change of intermittency.

Roughly, a 10% increase in intermittency effects a 7%

decrease in $\overline{u^2}$, Fig. (4.8.7), $\overline{v^2}$ (4.8.9) and $\overline{uv}$ (4.8.11) and a 10 % decrease in $\overline{uv^2}$, (4.8.13). The turbulent zone contribution to overall average, however, changes by a far smaller amount: 10 % increase in intermittency factor leads to a 3 % increase in $\overline{u^2}$, $\overline{v^2}$ and $\overline{uv}$ and barely a 1 % decrease in $\overline{uv^2}$. Now, if $\overline{Q}$ is roughly independent of δ , then it is implied that values of Q near the edges of turbulent zones are very small, so that changes in δ make little difference to total contribution to unconditional averages.

The flatness factor intermittencies, Fig. (4.8.4) show a good general agreement between the data points of Klebanoff and the present high-Reynolds-number results, except that the latter are better behaved near the wall. Close to the wall there is scatter in Klebanoff's results and values of intermittency factor greater than unity are found. Near to the wall F_u does not in fact tend towards 3, the Gaussian flatness factor, but to some slightly lower value. The near-wall value of F_u obtained in the present work was, therefore, taken as a basis and Fig. (4.8.5) shows that a better collapse onto $\delta_K(y/\delta)$ was obtained. It also proved possible to collapse high and low Reynolds number results closer together, so that $F_{u(0)}/F_{u(y)}$ varied with Reynolds number less than $3/F_{u(y)}$.

Fig. (4.8.6) shows computed intermittency factors, δ , δ^* and the flatness factor intermittency factor δ_F at high Reynolds number. The latter collapse better on to the curve $\delta_K(y/\delta)$.

Intermittency results at typical high and low Reynolds numbers are shown in Fig. (4.8.15). Close similarity of $\delta(y/\delta_{995})$, $\delta^*(y/\delta_{995})$ is found throughout the Reynolds number range. The position of $\delta^* = 0.5$, $0.84 \delta_{995}$ in agreement with Klebanoff $0.78 \delta \approx 0.82 \delta_{995}$, is fairly constant throughout the Reynolds number range. Kovasznay et al find $\delta = 0.5$ at 0.8δ ($R_\theta \approx 3300$) and Hedley and Keffer at 0.75δ ($R_\theta \approx 9000$). The position of

$\delta^* = 0.9$, however, moves closer to the wall with increasing Reynolds number, whilst $\delta^* = 0.2$ moves away from the wall. Thus $\delta^*(y/\delta_{995}^*)$ appears to pivot about $\delta^* = 0.5$ with $\frac{\partial \delta^*}{\partial y}$ decreasing with increasing Reynolds number. This is also seen, though less dramatically, in $\delta(y/\delta_{995}^*)$. Higher values of δ^* close to the wall are found at lower Reynolds numbers, $\delta_{MAX}^* = 0.97$ at $R_\theta = 800$ and $\delta_{MAX}^* = 0.935$ at $R_\theta = 4750$.

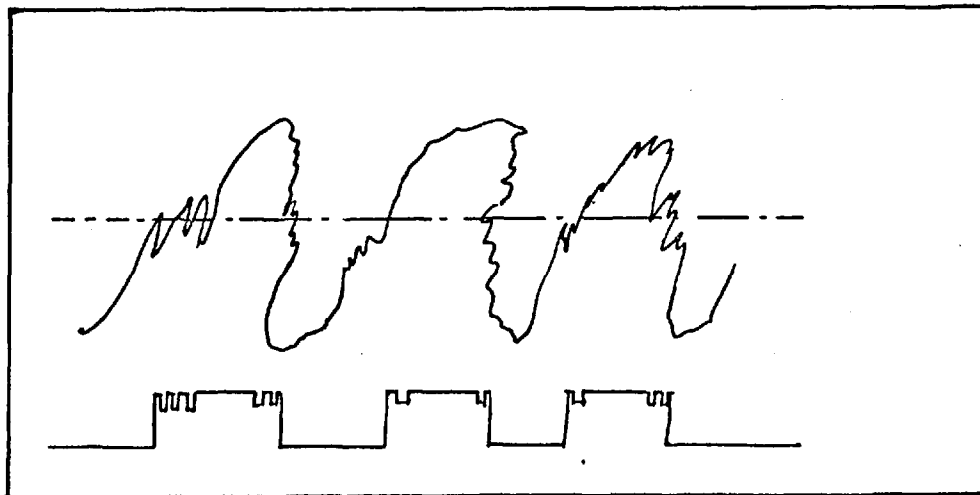
The intermittency results for all experimental stations are shown in Fig. (4.8.16).

4.9 Visual Analysis of Intermittency Results.

This section describes in further detail the physical appearance of the intermittency function $ID(t)$ and, comparing it with the criterion parameter, $W(t)$ suggests typical burst patterns. Unless specifically stated, bursts are wholesale turbulent regions, troughs are wholesale irrotational regions.

Firstly, it must be noted that, as expected, bursts coincide

with regions of high $w(t)$ activity. In addition it seems that troughs, defined as having low $w(t)$ slope with respect to time generally also coincide with regions of very small $w(t)$. This latter is particularly noticeable at high Reynolds number and low intermittency, Fig. (4.7.1) for example. At low Reynolds numbers there is more activity of $w(t)$ in troughs, but it seems even so that the mean, $\overline{w}$, is very close to zero. This is in marked contrast to the $u(t)$ and $v(t)$ traces which in most cases show a considerable slope within irrotational regions. This slope is usually of the duration of the trough and remains fairly steady throughout. Fig. (4.7.1) shows this particularly clearly for the $v(t)$ trace which in general has a negative slope in irrotational regions, crossing the zero line somewhere about the middle of the regions. Thus on the backs of bursts the flow is instantaneously away from the wall whilst near fronts it is towards the wall in agreement with the findings of Kovasznay et al (1970). Retail activity in troughs (small scale turbulent events) is comparatively rare, but there are considerable retail irrotational events at leading and trailing edges of bursts.



Thus although there are few 'free' small scale bursts there is a considerable amount of fuzziness around bursts and some small irrotational patches

within bursts, the so called 'marble cake' effect clearly seen in the smoke pictures of Pongprudhanon, Fig. (4.1.1).

As predicted in section 4.7, the leading and trailing edges of bursts are frequently regions where $uv(t)$ slope is gently increasing, again this is seen in Figs. (4.7.7) and (4.7.6), at low δ . These slopes are often of considerable length, up to 0.5δ , so that a considerable interface position error is possible. At higher values of intermittency, ≈ 0.5 , the slopes are shorter, but the apparent linear relationship between δ^* and threshold level factor is explained.

The smaller scale bursts seen near the outer edge, e.g. at $y/\delta_{995} = 0.9$, have a very similar ^{absolute} physical length at high and low Reynolds numbers, but at high Reynolds number a wider range of scales is seen, up to 1.6" or about 1.0δ . Typical smaller scales are physically about 0.2", about 0.15δ at $R_\theta = 4750$ and 0.3δ at $R_\theta = 900$.

Towards the wall bursts become longer and more closely spaced, but even at the nearest point to the wall ($y/\delta_{995} = 0.1$ at $R_\theta = 4750$) troughs of considerable length, $\approx 0.25 \delta$ are found.

The behaviour of $uv(t)$ itself within bursts is very interesting. It is clear that uv 'spikes' are larger at low Reynolds number; as a proportion of $\overline{u^2}$ instantaneous values of uv become larger at low Reynolds number. Spikes are also larger at the outer edge than near the wall. The wavelength of uv fluctuations is very similar at high and low Reynolds number, being of order 0.15λ . Near the wall $uv(t)$ is more one-sided (towards negative uv).

The typical burst pattern is as follows: an initially gentle negative uv excursion is followed by a larger positive excursion and a series of alternatively negative and positive spikes, occurring about once in a boundary layer thickness. Larger bursts contain on average larger

spikes and the smaller bursts contain only one spike each. From Fig. (4.7.6) it appears that, in one spike bursts, the spike is usually positive. The $u(t)$ and $v(t)$ traces do not show such a clear pattern, but, again a typical behaviour may be seen: $v(t)$ at the leading edge is sharply negative and $u(t)$ shows strong positive values; suggesting a pushing forward and downwards towards the wall; trailing edges are not so strongly marked, the general direction seems to be outwards and at a stream-wise velocity defect.

4.10 Burst Length Statistics.

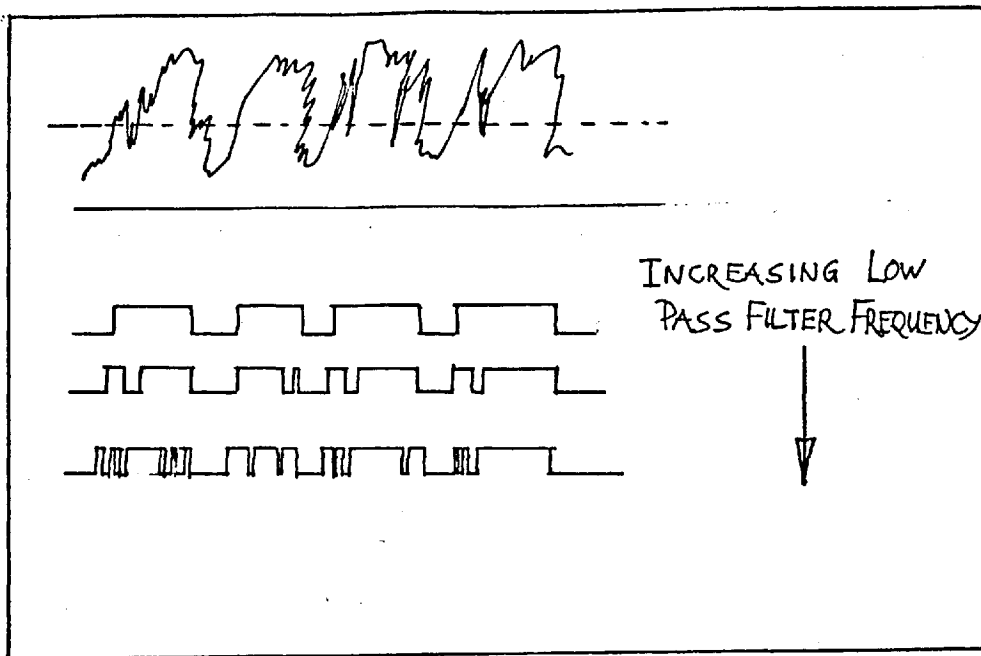
Burst lengths have been qualitatively described in the previous section, (4.9), and it is now required to quantify their distribution so that typical "average burst lengths" can be found at various Reynolds numbers and boundary layer positions.

The descriptions in meaningful terms of burst length statistics presents a difficult problem to the experimenter. The measure of disagreement between average burst lengths presented by different experimenters, see for example Table 5 in Hedley and Keffer (1974), makes it hard to believe that the values given represent the same property, even given the slightly different flow conditions.

The present author believes that the burst length problem arises from a conceptual mistake concerning intermittency. Corrsin and Kistler (1955) showed the intermittent nature of the turbulent shear flow free stream boundary and suggested that the bursts were well formed discrete lumps of fluid covered with an interfacial layer, the viscous superlayer, like the skin on an orange. Since then research workers have carried such a model with them, assumed that the interface has a simple

shape and that the turbulent bursts cover a finite range of lengths, thus giving a simple and meaningful average length. If now the smoke pictures of Fiedler and Head (1966), Falco (1974) or Pongprudhanon (1974), reproduced in Fig. (4.1.7), are examined it may clearly be seen that the interface, assuming that it exists as a layer at all, is highly re-entrant, finely structured and wispy. Even given that smoke pictures do slightly exaggerate the small scale structure (momentum or vorticity is diffused by viscosity faster than smoke by brownian motion) the interface appears rather more like the surface of a cauliflower than the peel of an orange.

Traces of such properties as vorticity (Corrsin and Kistler 1955) or $uv(t)$ e.g. Fig. (4.7.1) do seem to present a more ordered pattern, but the time scale is highly magnified, smallest scales are removed by filtering and the picture is essentially one-dimensional. Certainly the present computed intermittency function shows a considerable amount of fine retail structure around wholesale bursts, which is effectively fuzziness. Given that in general the population of a given burst length increases with diminishing length, the exclusion of smallest scales would have a considerable effect upon simple average-burst-length, which would then become a strong function of the low pass filtering of the turbulence signal. Assuming the existence of a continuous superlayer the statistical problem may be seen as analogous to that of the definition of a coastline. Mandelbrot (1967) has indeed used the coastline analogy to explain the principle of statistical self-similarity. In a process which obeys this principle, any small section has the same statistical properties as the whole and thus the coastline, for example, has a length dependent upon the segment length used.



Further the length of coast, $L(G)$ where G is the segment length is given in Richardson's Formula (quoted by Mandelbrot) as $L(G) = MG^{1-D}$, where M is some constant and D is a fractional dimension. This relationship can operate as a definition of D which then becomes an indicator of the amount of irregularity. For a straight line D is obviously 1; for the most irregular known coastline, that of the West of England, $D \approx 1.25$. The same arguments may be used for a surface, where D takes some value between 2 and 3 and, clearly, as G becomes small the area of the surface would become extremely large. In the present context a whole range of properties: intermittency; average burst length; crossing frequency; will all depend upon the sampling rate, the low pass filter setting and, in the cases where they are used, smoothing and hold time. Thus the real features of the interfacial properties may be lost by simply summing together all the turbulent bursts and irrotational troughs and calculating properties as if the process were simply continuous.

In an attempt to give more meaningful statistics, the burst length probability density has been calculated and is presented as a function of burst length from one sampling period to the largest observed burst. A difficulty with this system is the length of sample required to

obtain sufficient occurrences of large bursts, a problem also encountered by Corrsin and Kistler. These results, shown in Figures (4.11.1 ; 4.11.2 ; and 4.11.3), demonstrate clearly the power of the smallest bursts, which in all cases have the largest probability density. If now the weighted probability density, pl , where p is the probability density and l is the burst length in sampling points, is calculated and given as a function of l Fig. (4.11.5) it is found that the highest values $pl(l)$ are found above the minimum burst length, demonstrating that the principal contribution to intermittency comes from bursts well above the minimum burst length.

The moments of these probability distributions would give more plausible typical burst lengths and are discussed in the following section.

4.11 Burst Length Statistics : Numerical Results.

This section describes in detail the burst length probability distributions for wholesale and retail bursts at high, medium, and low Reynolds numbers and at various boundary layer positions. The first and second moments of these distributions are given as a function of boundary layer position at the chosen Reynolds numbers. The general length unit is the sampling length, in number of samples. The scaling length used for both wholesale and retail bursts is δ_{995} . Average burst lengths obtained by other workers are included, not so much as a basis for comparison as in an attempt to interpret them and to define a suitable average burst length for further work. In the free stream one sampling length is equivalent to 0.015 ins.

The probability distributions of retail and wholesale burst length as a function of burst length are shown in Fig. (4.11.1) (Station 2 $R_\theta \approx 1100$), Fig. (4.11.2) (Station 6, $R_\theta \approx 2200$) and Fig. (4.11.3) (Station 17, $R_\theta = 4750$) for different boundary layer positions of high

and low intermittency. As intermittency increases, towards the wall, y/δ decreasing, a wider range of both retail and wholesale burst lengths is found, and the probability distribution becomes flatter towards the smaller lengths.

The wholesale burst length probability distribution is generally the flatter and, as may be expected, extends to a larger burst length. Although the retail distribution remains very similar at all Reynolds numbers, the wholesale distribution becomes flatter with increasing Reynolds number. The wholesale distributions at large burst length contain considerable scatter because of the infrequency of their occurrence in the lengths of data processed.

The largest retail bursts found at $y/\delta_{995} = 0.4$, where intermittency is at its asymptotic highest value ($\gamma \approx 0.93$) are of order 60 sampling lengths, equivalent to about $30 \delta_{995}$ at the lowest Reynolds number ($Re \approx 1100$) and about $0.6 \delta_{995}$ at high Reynolds number ($Re = 4750$). The largest wholesale bursts are of order 200 sampling lengths, well below the digital record length.

So that the effects of Reynolds number may be seen more clearly these distributions have been replotted in Fig. (4.11.4) on a linear scale, with burst length scaled with δ_{995} and the probability scaled with ℓ the burst length. The main features to emerge from these plots are their pronounced peaks at lowest burst lengths. For $Re = 4750$, at $y/\delta_{995} = 1.0$, where $\gamma = 0.15$, the probability of retail burst length 1 approaches very close to unity, and even close to the wall, $y/\delta_{995} = 0.1$, $\gamma = 0.89$, the probability distribution has a noticeable peak at $\ell = 1$.

The low Reynolds number burst lengths are certainly proportionally larger (for a given probability, bursts are longer) but this is less marked towards the wall as the range of burst lengths becomes larger.

The wholesale distributions show this most clearly, where bursts are typically twice as large at $R_\theta = 1100$ and $y/\delta_{995} = 1.0$ as at $R_\theta = 4750$ at the same position. Thus in general small bursts are more significant in terms of population at high Reynolds numbers and therefore the 'fuzziness' around bursts would be greater, as seen in the smoke pictures of Falco (1974).

The values of peak probability of retail bursts are higher than published results, almost certainly because the bursts considered in the present experiment extend to smaller lengths. The peak probability values for wholesale bursts, however, compare somewhat better at low R_θ and low intermittency with Antonia (1972) and at high Reynolds numbers with Hedley and Keffer (1974) or Corrsin and Kistler (1955). This seems to confirm that wholesale intermittency defines the large scale structure which alone was considered by other experimenters.

The length weighted retail probability distributions are shown in Fig. (4.11.5) at Reynolds numbers $R_\theta = 1100, 2200$ and 4750 , in the semi log form $l \cdot p(l)$ with l scaled logarithmically. These plots give the contribution to intermittency of the various burst lengths. The peak contributions to intermittency $l \cdot p(l)$ maximum, come from longer bursts nearer the wall, as might be expected. It also seems that peaks occur at slightly longer burst length with increasing Reynolds number. Typically, at $y/\delta_{995} = 1.0$ the peak is around 4 sampling lengths and at $y/\delta_{995} = 0.4$ around 15 sampling lengths. At low Reynolds number near the wall, the major contribution to intermittency comes from fairly large bursts, of order $0.5 \delta_{995}$. At high Reynolds number from bursts of $0.14 \delta_{995}$. Again it seems that, proportionally, smaller bursts are responsible for intermittency at higher Reynolds numbers.

The moments of the burst length probability distributions were obtained by two methods.

(i) From the computed probability distributions:

$$\bar{l}_1 = \frac{\sum l \cdot p(l)}{\sum p(l)} \quad 4.11.1$$

$$\bar{l}_2 = \frac{\sum l^2 \cdot p(l)}{\sum l \cdot p(l)} \quad 4.11.2$$

(ii) it was feared that (i) would be sensitive to the large scatter of $p(l)$ at high burst length values, the distributions were therefore graphically smoothed and integrated so that:

$$\bar{l}'_1 = \frac{\int l^2 p(l) d \log l}{\int l p(l) d \log l} \quad 4.11.3$$

$$\bar{l}'_2 = \frac{\int l^3 p(l) d \log l}{\int l^2 p(l) d \log l} \quad 4.11.4$$

Additionally the average burst length was obtained from the simple relationship:

$$\bar{l}'' = \frac{\bar{n}p}{nb} \quad 4.11.5$$

These methods are compared for retail bursts in Fig. (4.11.7) the average burst length $\bar{l}''(y/8)$ agrees well with $\bar{l}_1(y/8)$ and the values obtained for $\bar{l}'_2(y/8)$, although slightly low, suggest that the simpler method, given by $\bar{l}_2$ (expression 4.11.2) is adequate.

The interpretation of these moments is open to some doubt, but roughly $\bar{l}_1$ is an average or typical burst length, whilst $\bar{l}_2$ is the

typical burst length contributing most significantly to intermittency.

Results are shown in Figures (4.11.8 ; 4.11.9 ; 4.11.10), retail and wholesale in the general form $\bar{l}_1(y/\delta), \bar{l}_2(y/\delta)$. The equivalent δ_{995} values have been given for reference.

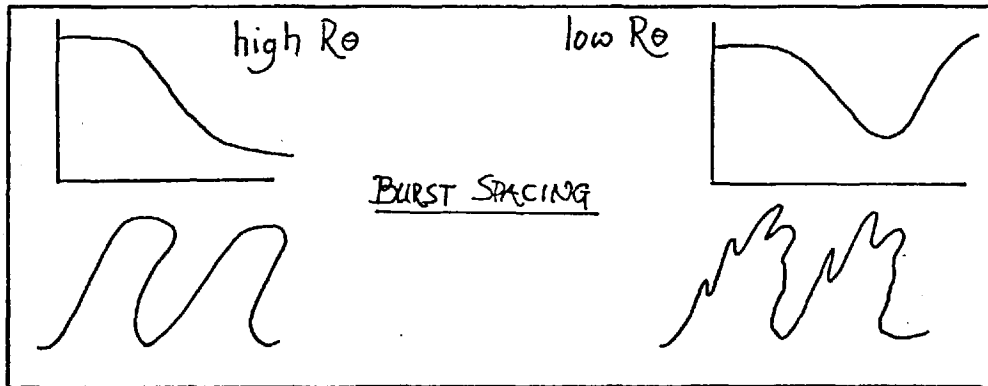
Close to the wall $y/\delta_{995} < 0.4$ the typical retail moments $\bar{l}_{1r}$ and $\bar{l}_{2r}$ have very similar values throughout the Reynolds number range (i.e. $\bar{l}_{1r}, \bar{l}_{2r} \propto R_\theta$) $\bar{l}_{1r} \approx 15$ points, about 0.225 ins. The values of $\bar{l}_{1r}(y/\delta), \bar{l}_{2r}(y/\delta)$ fall off uniformly in the range $0.4 < y/\delta < 0.95$. At the very outer edge of the boundary layer $\bar{l}_{1r}, \bar{l}_{2r}$ again achieve stable values, but in this case $\bar{l}_{1r}$ and $\bar{l}_{2r}$ increase with Reynolds number, $\bar{l}_{1r}$ from about 4 points at $R_\theta = 1100$ (the smallest possible scale) to about 6 points at $R_\theta = 4750$.

The wholesale values, Fig. (4.11.9) show, surprisingly, that near the wall the typical burst length actually decreases with increasing Reynolds number, from about $7\delta_{995}$ to $1\delta_{995}$ in $1100 < R_\theta < 4750$. At the outer edge the absolute lengths increase with Reynolds number, from 0.225 ins. to 0.6 ins. in the same range. This is a decrease from about $0.7\delta_{995}$ to about $0.4\delta_{995}$.

Fig. (4.11.10) compares $\bar{l}_{1w}(y/\delta)$ with scaled intermittency function $\gamma^*(y/\delta)$ at high Reynolds number $R_\theta = 4750$. Although the curves are similar in shape, average burst length falls off with y/δ (at about $y/\delta_{995} = 0.4$) before intermittency (at about $y/\delta_{995} = 0.7$). In fact, quite generally, the average burst length begins to decrease closer to the wall than intermittency factor. To maintain intermittency factor in a region where burst length is decreasing it is necessary to pack the bursts closer together, and Fig. (4.11.13) and Fig. (4.11.14) show burst spacing calculated simply by:

$$\frac{\text{total number of points in record}}{\text{number of turbulent bursts}}$$

In the outer regions of the layer, $y/\delta_{995} > 0.8$, there is a considerable general increase in spacing, which, taken together with the fairly constant average burst length in this region suggests that intermittency decreases by successively thinning out the number of bursts found at a particular y/δ



In the region $0.4 < y/\delta < 0.7$ the spacing decreases, implying that δ is, in fact, maintained by a closing up of the bursts, or that on average bursts sub-divide. The comparatively large change in spacing at low Reynolds numbers would indicate that a sub-dividing process would be stronger there. This suggests a burst pattern as shown in Fig. (4.11.15).

Average burst lengths are shown, normalised with δ_{995} , as a function of Re in Fig. (4.11.11) for a series of boundary layer positions. These plots show the relatively larger size of typical bursts at low Reynolds number, particularly near the wall. Wholesale averages can be seen to be considerably larger, by an order of magnitude.

Other workers who quote average burst lengths include:

Corrsin and Kistler (1955) who obtained $\bar{\ell} = 2\delta$ at $y/\delta = 0.72$ and 0.89 at $y/\delta = 0.98$ in a boundary layer at $Re = 8000$; Kovaszny et al (1970) found $\bar{\ell} = 0.16\delta$ at $y/\delta = 1.0$ and 0.44δ at $y/\delta = 0.7$ at $Re = 3000$; Sherman (1972) measured $\bar{\ell}$ for the range $700 < Re < 1000$ in channel flow

just down stream from natural transition, resulting in $\bar{L} = 12\delta - 100\delta +$; Falco quotes 'typical eddy sizes' from analysis of his smoke pictures, $= \delta$ at $R_\theta = 600$, 0.4δ at $R_\theta = 2000$ and 0.1δ at $R_\theta = 6000$. The results are difficult to interpret: Sherman's must be too large and probably show the effects of a low high-pass-filter setting; Falco finds an increase in eddy size at low Reynolds numbers, and, in fact, his typical eddies have scales similar to the present retail averages $\bar{L}_{IR}$ with the latter close to the wall; the values of Corrsin and Kistler also seem large, particularly considering the comparatively high Reynolds number. Fig. (4.11.12) summarises these data. It may be seen that the results of Kovaszny et al closely compare with the present $R_\theta = 4750$ data.

The overall conclusions about intermittency are that, although the intermittency functions $\delta(y/\delta)$, $\delta^*(y/\delta)$ show little change with Reynolds number, bursts are larger at low Reynolds number, and more small scale activity (fuzziness) is found at high Reynolds number. Bursts at high Reynolds number tend, in agreement with Kovaszny et al, to be solitary, but at low Reynolds number successively sub-divide in their progress from wall to outer edge. The present intermittency scheme reproduces both the expected form of the intermittency function obtained by visual analysis of criterion parameter $uv(t)$, and the intermittency factor, $\delta(y/\delta)$ obtained by other workers. The wholesale intermittency gives the large scale flow features and agrees well with other data, particularly impressive is the close agreement between average wholesale burst length $\bar{L}_{1w}(y/\delta)$ and the data of Kovaszny et al. The retail intermittency, however, contains the small scale features which give rise to fuzziness around bursts.

It is found that irrotational regions of considerable length, up to $\frac{1}{2}\delta$ occur close to the wall, so that intermittency exists closer to the wall than hitherto envisaged.

Chapter Five.Properties of the Turbulence.

In this chapter the results of the digital analysis of hot-wire data are discussed and, where possible, comparison is made with other workers' data. The presentation of these results is designed to highlight Reynolds-number-dependent changes in the structure of the turbulence and particularly in the mechanisms governing shear stress. In order to do this it has occasionally been necessary to calculate quantities, the physical interpretation of which is uncertain, (for example, conditional averages of moments of fluctuating velocity probability distributions) but which, it is argued, illustrate these changes at least qualitatively.

The fluctuating velocities, both conditionally and conventionally arranged, are calculated with respect to a 'total' baseline - the conventional mean velocity - and not to regional baselines as, for example, in Hedley and Keffer (1974), see Fig. (5.1)

Thus, using the notation detailed in Fig. (5.1),

$$q = Q - \bar{Q} ; \quad \tilde{q} = \overline{Q - \bar{Q}} ; \quad \tilde{\tilde{q}} = \overline{\overline{Q - \bar{Q}}} \quad 5.1$$

Hence:

$$\bar{q} = \gamma \tilde{\tilde{q}} + (1 - \gamma) \tilde{q} \quad 5.2$$

where $\gamma \tilde{\tilde{q}}$, is the contribution from the 'turbulent' regions, and $(1 - \gamma) \tilde{q}$ that from the 'irrotational' regions, to the overall average. In this analysis, only the turbulent zone averages have been directly calculated; irrotational zone averages, where given, have been obtained from (5.2).

The advantages of using a conventional average baseline is the simplicity of the relationship (5.2) and of other relations between

conditional and conventional averages which engineers wish to predict. Fundamentally, perhaps the most meaningful baseline for turbulent zone averages is the local mean velocity in the irrotational regions into which the eddies are spreading. Evaluation of turbulent zone averages with respect to the turbulent zone mean velocity ignores the fundamental component of the eddy motion.

To separate Reynolds-number effects in the turbulence from the changes which occur in surface shear stress, the velocity scale u_τ the local 'friction velocity' has been used in producing non-dimensional velocity parameters, except where changes in the shear stress profile shape are expected to contribute, in which case a velocity scale is defined based upon the local turbulent shear stress, $\overline{uv}$.

The conditional averaging schemes used have been discussed in Chapter Four, where it was concluded that, although 'wholesale' intermittency schemes give the envelope of the turbulence they miss the fine structure which, it is conjectured, plays an important part in Reynolds number dependence. Consequently retail conditional averages are used in this chapter to represent the properties within turbulent or irrotational zones.

5.1 Conditionally Sampled Mean Velocities.

Conventional averages of the streamwise component of mean velocity obtained from hot wire data are shown in Fig. (5.1.1) compared with pitot tube results for the highest experimental Reynolds number, $R_\theta = 4750$ and Hedley and Keffer's (1974) data. A close agreement exists between the hot wire and pitot tube results at Station 17 ($R_\theta = 4750$) except close to the wall, $y/\delta_{995} < 0.3$, where hot wire readings are low, by about 2% at $y/\delta_{995} = 0.2$, probably due to shear displacement effects

on the hot wire probe. At low Reynolds number the profile is fuller, i.e. there is a higher shape factor. Hedley and Keffer took their measurements in an artificially thickened boundary layer with a wake component much below its asymptotic value. Thus their profile is fuller than either the Reynolds number, $R_\theta = 9700$ or the shape factor, $H = 1.3$, would suggest and the associated Clauser parameter, $G = 6.23$, lies at the low Π end of $G(\Pi)$ calculated from Wieghard's data and shown in Fig. (3.4.2).

Kovaszny et al (1970) and Hedley and Keffer (1974) measured zone averages of streamwise and normal components of mean velocity. They note that in the outer layer ($y/\delta_{995} > 0.4$) the irrotational regions have higher and the turbulent regions lower, streamwise mean velocity than the conventional average and that in turbulent regions the mean normal velocity is directed away from the wall whereas in irrotational regions it is directed towards the wall. The results of Kovaszny et al and Hedley and Keffer are in good agreement despite the Reynolds number difference between them, Kovaszny et al $R_\theta = 3000$, Hedley and Keffer $R_\theta = 9700$, although it appears that the former's streamwise velocity results give higher values for $(\bar{U} - \tilde{U}) / U_e$ by about 50 % over the range of y/δ_{995} measured.

The present results for streamwise mean velocity, Fig. (5.1.2), and normal mean velocity, Fig. (5.1.3), show marked differences from Hedley and Keffer's data. These results are shown in terms of differences between zone and conventional averages and it appears that Hedley and Keffer's irrotational zone averages do not tend to the conventional averages towards the wall (as δ tends towards unity) as do the present data.

In general the pattern of zone averages agrees with that of Kovaszny et al and Hedley and Keffer, but with the additional interesting result that whereas the turbulent-zone-velocity-difference is a maximum at the outer edge of the boundary layer, $y/\delta_{995} \approx 1.0$, $\delta \approx 0.12$, the

irrotational zone finds its maximum difference at $y/\delta_{995} \approx 0.7$, close to the $\delta = 0.5$ position. This implies that irrotational fluid accelerates towards the wall only as far as $y/\delta_{995} = 0.7$ after which point it slows down to reach the bottom of the 'trough' at a small relative velocity. Turbulent fluid, however, thrusts out energetically to the edge of the boundary layer before slowing down.

The values of velocity-difference found in the present results are considerably less than those found by Kovasznay et al or Hedley and Keffer, particularly for positions close to the wall.

Changes occurring with Reynolds number were investigated by plotting zone velocity difference against streamwise position and it may be seen that both the streamwise differences, Fig. (5.1.4), and the normal differences, Fig. (5.1.6), increase as R_θ increases, although the irrotational zone-differences at the extremes, close to the wall and at the edge of the layer, are somewhat ambiguous in their behaviour. When these results are replotted, normalised by u_z instead of U_θ this trend is emphasised Fig. (5.1.5).

Bradshaw (1966b) suggested that an 'energy flux velocity' given by:

$$\frac{\frac{1}{2} \overline{q^2} \overline{U} + \frac{1}{2} \overline{q^2} \overline{u} + \overline{p} \overline{u} \cdot \frac{1}{\rho}}{\frac{1}{2} \overline{q^2}} \quad 5.3$$

could be a physically plausible definition of streamwise component convection velocity, and that a good approximation to (5.3) is:

$$U_c = \overline{U} + \frac{\overline{u^3}}{\overline{u^2}} \quad 5.4$$

Hedley and Keffer (1974) define U_c by:

$$U_c = \bar{U} + \frac{\bar{u}^3}{\bar{u}^2} \quad 5.5$$

$$U_c = \bar{U} + \frac{\bar{u} \bar{q}^2}{\bar{q}^2} \quad \text{or} \quad 5.6$$

These parameters were estimated using Hedley and Keffer's assumptions that:

$$\bar{q}^2 = \frac{3}{2} (\bar{u}^2 + \bar{v}^2) \quad 5.7$$

and also, based on Lawn (1970),

$$\bar{q}^2 \bar{u} = \frac{5}{4} (\bar{u}^2 \bar{v} + \bar{u}^3) \quad 5.8$$

The values of U_c calculated from 5.4, 5.5 and 5.6 are compared in Fig. (5.1.7) at Station 17, ($R_\theta = 4750$) which also shows the mean velocity profile.

Kovaszny et al (1970) calculated an overall convection velocity for turbulent 'bulges' of order $0.93 U_e$, and Hedley and Keffer found that U_c calculated from the relations 5.5 and 5.6 agreed well together and lay about 0.02 below $\bar{U}/U_e$.

The present data also show convection velocities below the turbulent-zone-mean-velocity and by a similar quantity. Agreement between U_c calculated according to 5.4, 5.5 and 5.6 is good for $y/\delta_{995} < 0.5$, but above this value U_c calculated from zone average intensities falls below 5.4 and, at high y/δ_{995} both 5.5 and 5.6 give values well below Bradshaw's mean-convection-velocity, 5.4, actually decreasing with increase in y/δ_{995} . A decrease of convection velocity with increase in y/δ_{995} at the outer edge of the layer seems implausible, unless bulges of turbulent fluid are bending back on themselves which smoke pictures tend to deny, and Bradshaw's relationship 5.4 therefore proves the most well behaved at

large y/δ_{995} .

Convection velocity in the form:

$$\bar{U} - \left(\bar{U} + \frac{\bar{u^3}}{\bar{u^2}} \right) \quad 5.9$$

is shown in Fig. (5.1.8) plotted for both low and high Reynolds number stations: the data of Hedley and Keffer are also shown, and give much lower values in the middle of the profiles, about $y/\delta_{995} = 0.8$. A direct comparison of these and later results is not possible because of baseline differences.

These results are shown plotted against streamwise distance in Fig. (5.1.9). The convection-velocity-difference (5.9) decreases with Reynolds number in general, even normalised with respect to U_τ (5.1.9b).

The concept of a lateral bulk convection velocity, representing the transport of turbulent energy by the large eddies, introduced by Bradshaw (1966b) has been extended by Hedley and Keffer in the form:

$$\bar{V}_q = \frac{\overline{q^2 v}}{\overline{q^2}} \quad 5.10$$

Alternatively the lateral bulk convection velocity can be defined by shear stress:

$$\bar{V}_\tau = \frac{\overline{uv^2}}{\overline{uv}} \quad 5.11$$

these, together with equivalent mean convection velocities:

$$\bar{V}_q = \frac{\overline{q^2 v}}{\overline{q^2}} \quad 5.12$$

and

$$\bar{V}_\tau = \frac{\overline{uv^2}}{\overline{uv}} \quad 5.13$$

are shown in Figures (5.1.10 ; 5.1.11).

The agreement between conventionally averaged relationships and their equivalent conditionally averaged parameters is closer in the

case of $\overline{V}_z$ and $\overline{V}_r$ than $\overline{V}_q$ and $\overline{V}_\theta$, probably because of inaccuracies in the derivation of $\overline{q^2 v}$ and $\overline{q^2 v}_q$.

In general $\overline{V}_z$ is larger than $\overline{V}_q$ by a factor of about 3, but approaches zero more rapidly near the wall. $\overline{V}_q$ at $y/\delta = 1.0$ is to a good approximation the same value as $\frac{d(\delta - \delta^*)}{dx}$ when $\delta = \delta_{995}$. Hedley and Keffer's line for $\overline{V}_q$ is shown in Fig. (5.1.10), but has a much shallower slope than the present data.

The streamwise variation of lateral convection velocity, represented by $\overline{V}_z/u_z$, Fig. (5.1.12), shows a very similar trend to streamwise convection velocity difference, a slight decrease with increasing Reynolds number, of order 10% over the range of experimental Reynolds numbers ($R_\theta = 800 - 4750$).

5.2 Turbulence Intensities.

The turbulence intensities in the three coordinate directions u , v and w , were measured by Klebanoff (1954) and presented as the root mean squares of fluctuating velocity normalised with the free stream velocity. Klebanoff's data, for which $R_\theta = 8000$, are taken as the asymptotic standard for the zero pressure gradient boundary layer and form the basis for comparison with present data.

The comparison between present data and Klebanoff's results re-calculated in the form $\frac{\overline{u^2}}{\overline{u_z^2}}$ and $\frac{\overline{v^2}}{\overline{u_z^2}}$ is made in Fig. (5.2.1). The present results are taken from the highest experimental Reynolds number $R_\theta = 4750$, and in general agreement with Klebanoff is excellent, except near the outer edge of the layer where our intensities are higher, both for u and v . At the outer edge where intensities are of small magnitude, a high degree of accuracy cannot be expected.

Figures (5.2.2 to 5.2.4) show the root mean square, second and fourth moments of U and V for typical low and high Reynolds numbers, $Re = 800$ and $Re = 4750$. Conventional and conditional averages, both retail and wholesale, are given: the scatter of these results is well within 5 % on $\overline{U^2}$, even at the lower Reynolds number. The highest mean square values recorded were of order $0.01 \frac{\overline{U^2}}{U_e^2}$, but even within turbulent zones an intensity greater than $0.07 \frac{\overline{U^2}}{U_e^2}$ was not found, so that these points should all lie within the limits of a conventional hot wire system without the use of analogue linearisers.

Turbulent zone conditional averages are shown as their contribution to the conventional average, that is, as $\delta(\overline{U^2})_{\text{Retail}}/U_e^2$ $\delta^*(\overline{U^2})_{\text{Wholesale}}/U_e^2$ etc. The difference between the conventional and conditional averages, then, will be the contribution from the irrotational regions. In fact, the conditional averages lie very close to the conventional averages (closer in the case of wholesale averages, probably due to the biasing of the wholesale intermittency scheme) so that there is only a very small contribution from irrotational zones, of order 5 % of $\overline{U^2}$ or $\overline{V^2}$. The fourth moments of U component intensity do, however, show a considerable excess of $\overline{U^4}$ over $\overline{U^4}$ at the outermost edge $y/\delta_{995} \approx 1.0$ which occurs later in this chapter as a very large turbulent-zone U -component flatness factor.

Turbulence intensity profiles for U and V components, Fig. (5.2.5), show that the profile shape is roughly preserved through the Reynolds range but that at low Reynolds numbers intensities are higher by about 17 % of U or V r.m.s. intensity when normalised with free stream velocity.

The intensity ratio, $\overline{U^2}/\overline{V^2}$ has been calculated for the data of Klebanoff (1954) and the present data. The results, Fig. (5.2.6), show

that considerable changes take place with Reynolds number. At high Reynolds numbers, Stations 17 and 11, for example, good agreement with Klebanoff is obtained, but at low Reynolds numbers $\overline{u^2}/\overline{v^2}$ is much lower near the wall by a factor as small as $\frac{1}{2}$ at Station 1, $y/\delta_{995} = 0.4$. In the outer part of the layer $y/\delta_{995} > 0.8$ agreement between stations becomes close.

Intensities normalised with u_z and $(\overline{uw})$ are compared with values computed from Klebanoff's data in Figures (5.2.7 ; 5.2.8). It may be seen that the shapes of $\overline{u^2}/\overline{uw}(y/\delta)$ and $\overline{v^2}/\overline{uw}(y/\delta)$ are quite different from the profiles normalised with u_z showing that $\overline{uw}$ changes more rapidly than $\overline{u^2}$ or $\overline{v^2}$ with y/δ . Agreement between Klebanoff and the present in Fig. (5.2.8). Station 17 data is again remarkably close, except at the outer edge where differences in $\overline{u^2}$ and $\overline{v^2}$ have been noted. The data are cross-plotted against x in Figures (5.2.9 ; 5.2.10 ; 5.2.11), which show considerable scatter but nevertheless clearly indicate the trends with x (or R_θ). Both $\overline{u^2}/u_z^2$ and $\overline{v^2}/\overline{uw}$ increase with R_θ , particularly noticeably near the wall, at $y/\delta_{995} \approx 0.35$. $\overline{u^2}/u_z^2, \overline{v^2}/\overline{uw}$ increase by about 30 % through the Reynolds number range. The picture for $\overline{v^2}/u_z^2$ and $\overline{v^2}/\overline{uw}$ is not so clear, but apparently $\overline{v^2}/u_z^2$ does not show a remarkable change, except at the outer edge, where an increase with R_θ may be seen; $\overline{v^2}/\overline{uw}$ seems to decrease from $R_\theta = 800 - 1500$ and there after to increase to an asymptotic level similar to its value at Station 1.

These overall averages, for which the major contributions come from the turbulent zones, show that the normal velocity fluctuations are proportionally more significant at low than high Reynolds numbers and that, for a given shear stress, the lateral turbulent intensity remains roughly constant with R_θ whilst lower streamwise component intensity is seen at low R_θ than at high R_θ .

5.3 Turbulent Shear Stress.

Turbulent shear stress controls the development of the boundary layer and its prediction has been the focus of considerable attention. These experiments have been carried out largely in order to improve understanding of the mechanisms governing shear stress, particularly at low Reynolds number.

The results of the present experiments, normalised by the wall shear stress, are compared at high Reynolds number ($R_\theta = 4750$) with the measurements of Klebanoff, ($R_\theta = 8000$) and the profiles calculated by Green (1971) at $R_\theta \approx 500$ and $R_\theta \approx 10,000$ in Fig. (5.3.1). Green's calculations predict a considerable change in profile shape with Reynolds number and the present results lie roughly between the computed profiles. Klebanoff's results, however, agree quite closely with Green's high Reynolds number profile, except towards the boundary layer edge where disagreement is probably due to uncertainty in Klebanoff's δ . Neglecting an obviously erroneous point at $y/\delta_{99} = 0.3$ the scatter in the present results is within 10 %. Fig. (5.3.2) shows conventional and conditional averages of shear stress $\frac{\overline{uv}}{U_e^2}$ at typical low ($R_\theta = 800$) and high ($R_\theta = 4750$) Reynolds numbers. The small difference between conventional and turbulent zone conditional averages is an indication of the very low level of shear stress outside the turbulence and, since turbulent-zone conditional averages are less than conventional averages, it may be assumed that the 'irrotational regions' shear stress has the same 'direction' as that within turbulent zones.

In Fig. (5.3.3) shear stress profiles are shown in the form $\frac{\overline{uv}}{U_e^2} \left(\frac{y}{\delta} \right)$ for each measurement station with the point $\tau_w = \frac{1}{2} c_f$ added;

the growth of $\frac{\overline{uv}}{U_e^2} = f\left(\frac{y}{\delta_{995}}\right)$ with R_θ may be seen clearly.

τ/τ_w profiles are shown in Fig. (5.3.4). The data lie between Green's calculated profiles in roughly the expected station order, with low R_θ profiles, Stations 1 - 2 for example, close to Green's $R_\theta \approx 500$ profile.

The correlation coefficient $\frac{\overline{uv}}{\sqrt{\overline{u^2}}\sqrt{\overline{v^2}}} \left(\frac{y}{\delta}\right)$ was calculated for each station and is shown in Fig. (5.3.5). The shear stress correlation was given the value 0.44 by Corrsin (1949), but Klebanoff found that 0.495 proved to be a better fit to his data. The present data are in closer agreement with the former estimate and in Fig. (5.3.6) have been plotted against streamwise distance with 0.44 as a basis. Close to the wall

$y/\delta_{995} < 0.6$ shear correlation coefficient remains fairly close to the value 0.44 throughout the Reynolds number range, but a slight decrease in value with R_θ increase is detected, particularly towards the outer edge

$y/\delta_{995} > 0.9$.

5.4 Triple Products of Fluctuating Velocity.

Triple product measurements have a great practical importance in themselves, since the development of 'second order closure' methods of boundary layer prediction require a good knowledge of their behaviour: they dominate diffusion terms in the turbulent energy equation and normal transport terms in the shear stress equation.

Physically the triple product terms involving u may be interpreted as normal diffusion, in which case $\overline{u^2v}$, $\overline{v^3}$ and $\overline{uv^2}$ would be the normal transport, respectively of $\overline{u^2}$, $\overline{v^2}$ and $\overline{uv}$ within turbulent zones. This is helpful because it is not possible to produce zone-averaged diffusion in the turbulent energy transport equation.

Despite their importance, satisfactory measurements of triple products in a turbulent boundary layer are rare: Bradshaw (1966) measured $\frac{\overline{u^2 v}}{(\overline{uv})^2}$ and $\frac{\overline{v^3}}{(\overline{uv})^2}$ directly and Hedley and Keffer (1974) made absolute measurements of all four triple products of u and v , both conditionally and conventionally averaged. These measurements are compared with the present data for Station 17 in Figures (5.4.1 to 5.4.5). Fair agreement is obtained between the present data and Bradshaw (1966), Fig. (5.4.1), except near the outer edge where the present data are higher, probably because of numerical errors in the manipulation of the very small values of $\overline{u^2 v}$, $\overline{v^3}$ and $\overline{uv}$. A better agreement in $\overline{v^3}$ than in $\overline{u^2 v}$ is found. Townsend (1961) suggested that, for considerations of inner law similarity, the transport of turbulent energy, $\left(\frac{\overline{pv}}{\rho} + \frac{1}{2} \overline{q^2 v}\right) \frac{1}{(\overline{c}/\rho)^2}$ must be a constant near the wall; Bradshaw (1966) supported this suggestion and in addition proposed that its value should be small, reflected in the values of $\frac{\overline{u^2 v}}{(\overline{uv})^2}$ and $\frac{\overline{v^3}}{(\overline{uv})^2}$. Certainly the present data show a leveling out near the wall and the values are indeed small.

Fig. (5.4.4) and Fig. (5.4.2) show $\frac{\overline{v^3}}{U_e^3}$ and $\frac{\overline{u^2 v}}{U_e}$ compared with Hedley and Keffer's (1974) data. It could be argued that $\frac{\overline{u^2 v}}{U_e}$ shows good agreement, except for $y/\delta_{995} < 0.6$, but $\frac{\overline{v^3}}{U_e}$ shows a poor agreement throughout. The c_f of Hedley and Keffer's experiment was 0.00274 compared with 0.00298 for the present measurement so that, to achieve agreement in $\frac{\overline{u^2 v}}{(\overline{uv})^2}$ and $\frac{\overline{v^3}}{(\overline{uv})^2}$ Hedley and Keffer's should be below the present data; certainly they would not agree with the direct measurements of $\frac{\overline{u^2 v}}{(\overline{uv})^2}$ and $\frac{\overline{v^3}}{(\overline{uv})^2}$ of Bradshaw (1966). At $y/\delta_{995} = 0.8$, $\frac{\overline{u^2 v}}{\overline{v^3}}$ values for the present experiment, Hedley and Keffer and Bradshaw are, respectively 1.43, 1.30, 1.13. These differences could well be due to the different Reynolds numbers involved, but the artificial thickening used by Hedley and Keffer has a considerable effect on the velocity profiles in the outer

layer and consequently upon shear stress, so that systematic comparison is not possible. A similarly poor agreement near the wall is found for $\frac{\overline{u^3}}{\overline{u_e^3}}$ and $\frac{\overline{uv^2}}{\overline{u_e^3}}$, Fig. (5.4.3) and Fig. (5.4.5). In view of the requirement for a constant and small value of diffusion near the wall it is very difficult to believe that Hedley and Keffer's data would prove a reliable basis for comparison with the present experiment, and therefore evaluation rests upon agreement with Bradshaw's (1966a) data.

The conventional and turbulent conditional averages of $\overline{u^3}$ and $\overline{v^3}$ are shown in Fig. (5.4.6) and Fig. (5.4.7) for stations 1 to 7, 11, 14 and 17. The turbulent zone averages lie very close to, but slightly above, the conventional averages. The position of maximum $\overline{u^3}$ seems to move inwards towards the wall with R_θ , from 0.6 at Station 1 ($R_\theta = 900$) to 0.4 at Station 17 ($R_\theta = 4750$).

The triple cross products, $\overline{u^2v}$ and $\overline{uv^2}$, Fig. (5.4.8) and (5.4.9) show, similarly, a very small contribution to conventional average from the irrotational zones. It can be seen that the mean lateral transport of $-\overline{uv}$, $\overline{u^2}$, $\overline{v^2}$ is always towards the edge of the layer and that this takes place almost exclusively, as expected, in the turbulent bursts.

To examine the effects of Reynolds numbers upon the turbulent zone products the non-dimensional quantities $\frac{\overline{q^3}}{(\overline{\sigma(-\overline{uv}))^2}$ and $\frac{\overline{q^3}}{\tau_w^2}$ were calculated for each triple product and each station, and crossplotted against α . The signs of the triple products were all considered positive for convenience of scalar comparison.

Non-dimensionalised with τ_w^2 , Fig. (5.4.10), the triple products are, of course, the same general shape as in dimensional form, but Fig. (5.4.12) shows the changes which occur with downstream distance effectively (R_θ).

In general, $\overline{u^3}$, $\overline{v^3}$, $\overline{u^2v}$ increase, and $\overline{uv^2}$ decreases, with increasing Reynolds number throughout the layer. These effects are more marked close to the wall (taking into account the differences in scale with y/δ) than near the boundary layer edge. A degree of overshoot; particularly in $\overline{u^2v}$, may be seen, with maximum (or minimum) at about $R_\theta = 2500$. These longitudinal plots are characterised by a large amount of scatter (up to $\pm 12\frac{1}{2}\%$), but the shape and general trend may easily be seen. Longitudinal plots of triple products non-dimensionalised with $(\sigma(\overline{uv}))^{\frac{3}{2}}$ however, are more ambiguous, Fig. (5.4.13). The scatter is larger because both the triple products and shear stress contribute to it, but once again a general trend does emerge: $\frac{\overline{u^3}}{(\sigma(\overline{uv}))^{\frac{3}{2}}}$ increases with R_θ throughout the layer; $\frac{\overline{v^3}}{(\sigma(\overline{uv}))^{\frac{3}{2}}}$ is roughly constant; $\frac{\overline{u^2v}}{(\sigma(\overline{uv}))^{\frac{3}{2}}}$ is somewhat ambiguous but seems to remain roughly constant or slightly increases and $\frac{\overline{uv^2}}{(\sigma(\overline{uv}))^{\frac{3}{2}}}$ decreases with R_θ , except that near the wall $\frac{\overline{u^2v}}{(\sigma(\overline{uv}))^{\frac{3}{2}}}$ shows a definite increase.

Thus the change in shape of $\frac{\tau}{\tau_w} = f(y/\delta)$ with Reynolds number makes a significant difference to the observed changes in non-dimensional triple products with Reynolds number. Considered on the basis of local shear stress, it seems that lateral transport of $\overline{uv}$ is stronger at low Reynolds number, diffusion of $\overline{u^2}$ is weaker and that of $\overline{v^2}$ remains roughly constant through the Reynolds number range. This may be compared to the behaviour of the turbulence intensities ($\overline{u^2}$ is less at low Reynolds numbers whilst $\overline{v^2}$ is roughly constant throughout the Reynolds number range) and is in accordance with the guideline that diffusion of a property is proportional to the value of the property. To obtain the relative importance of these processes the ratios of $\frac{\overline{u^2v}}{\overline{v^3}}$ Fig. (5.4.14) and $\frac{\overline{u^2v} + \overline{v^3}}{\overline{uv^2}}$ Fig. (5.4.15) were calculated and are shown as a function

of streamwise position; the former giving the turbulent zone comparative strength of diffusion of $\overline{u^2}$ compared with $\overline{v^2}$ and the latter the relative strength of kinetic energy and shear stress normal transport. The results are quite striking: at low Reynolds numbers $\frac{\overline{u^2 v}}{\overline{v^3}}$ increases rapidly, by about 60 % as R_θ rises from 900 to 2200, at positions in the outer layer, $y/\delta_{995} > 0.4$; $\frac{\overline{u^2 v} + \overline{v^3}}{\overline{uv^2}}$ increases similarly but by a more modest amount about 15 %. It is particularly interesting that these rapid changes take place in a range similar to that in which wake strength $\pi(R_\theta)$ grows most rapidly to its asymptotic value. The above results imply that at low Reynolds number $\overline{v^2}$ is comparatively more efficiently diffused and that the lateral transport of $\overline{uv}$ is comparatively more effective than that of turbulent kinetic energy.

5.5 Statistical Description of Turbulence.

The third and fourth moments of fluctuating velocity, suitably non-dimensionalised, constitute the properties known as skewness, which describes the asymmetry of distribution about the mean, and flatness or kurtosis, which gives a measure of the width of the distribution or the influence of the extremes. In a Gaussian process the values would be zero for skewness and 3 for flatness; turbulence is not quite Gaussian because different scales (eddies) are not entirely independent, but values close to the Gaussian, particularly near the wall where the intermittency is almost unity may be expected. Townsend (1947) found that the u -component of fluctuating velocity in homogeneous turbulence gave $F_u = 2.9 - 3.0$ $S_u(x) = 0.0$. High values of flatness factor imply that the probability density function has a higher peak and broader skirts than a Gaussian function of the same standard deviation, effectively that large deviations from the

mean are more significant. A non-zero value of skewness implies that deviations occur with greater probability on one side of the mean: negative skewness shows values below the mean occurring with better than random probability and positive skewness that fluctuations above the mean are more probable.

In the derivation of these properties in turbulent shear flow, Klebanoff (1954) suggests that inaccuracies must be expected particularly in skewness measurements, because skewness is very sensitive to the value of the mean of the probability distribution.

The flatness and skewness factors for u and v fluctuating velocities:

$$\bar{F}_u = \frac{\overline{u^4}}{(\overline{u^2})^2}$$

$$\bar{F}_v = \frac{\overline{v^4}}{(\overline{v^2})^2}$$

$$\bar{S}_u = \frac{\overline{u^3}}{(\overline{u^2})^{3/2}}$$

$$\bar{S}_v = \frac{\overline{v^3}}{(\overline{v^2})^{3/2}}$$

are shown in Figures (5.5.1 ; 5.5.11 ; 5.5.12) Klebanoff (1954) calculated

$\bar{F}_u$ for his constant pressure boundary layer, and the above range of

moments were also calculated by Hedley and Keffer (1974). The skewness of

u and v , Fig. (5.5.1) show that near the wall both $\bar{S}_u$ and $\bar{S}_v$ approach zero, that $\bar{S}_u$ is everywhere negative and $\bar{S}_v$ everywhere positive and that $\bar{S}_u$ increases more rapidly than $\bar{S}_v$ as the outer edge is approached, by a factor of approximately 2. The maximum values are of order $\bar{S}_u = -2.3$,

$\bar{S}_v = +1.5$. The flatness factors of u and v , Figures (5.5.11 ; 5.5.12) show a value of about 3 close to the wall, but both rise steeply for $y/\delta_{995} > 0.7$, $\bar{F}_u$ rising the more sharply, to very high values in the range 20 - 30.

It may be seen Fig. (5.5.11) that Klebanoff's results agree well with the present data at the highest experimental Reynolds number,

$Re = 4750$ (Station 17), except at high flatness factors, $y/\delta_{995} > 0.9$, where, some difference might be expected. The flatness factor values given by Hedley and Keffer (1974) are higher throughout the layer, both for u and v components. Fig. (5.5.1) shows that the skewness factor values given by them are also slightly high, probably because of small variations in mean value. In general a good agreement with both Klebanoff and Hedley and Keffer can be claimed.

The near zero values of $\bar{S}_u$ and $\bar{S}_v$ close to the wall are a measure of the small intensity gradients there. The strong negative values of $\bar{S}_u$, according to Klebanoff, reflect the contribution from the irrotational regions of low u and the comparatively weaker positive $\bar{S}_v$ probably shows the authority of the fairly large fluctuating v found in the turbulent zones (mean v is very close to zero so that the fluctuations, found for example, in strong movements away from the wall, affect $\bar{S}_v$ considerably even if they are relatively infrequent).

Close to the wall $\bar{F}_u$ and $\bar{F}_v$ show values very close to the Gaussian value of 3.0. As the flow becomes more intermittent, however, the flatness rises.

Some difficulty is associated with the analysis of conditionally sampled data so far as the calculation of meaningful zone average probability distribution moments is concerned. There is the problem of choice of mean level and the problem of the formation of an ensemble from intermittent signals.

Hedley and Keffer (1974) used the zone averages as the basis for probability distributions, but Dean (1974) used the overall average so that the difference in the turbulence zone with respect to the mean moment values could be seen. In the present work the system of Dean (1974) has been used, the turbulent zone moments then become:

$$\begin{aligned}\overline{\overline{F}}_u &= \frac{\overline{\overline{u^4}}}{(\overline{\overline{u^2}})^2} & \overline{\overline{F}}_v &= \frac{\overline{\overline{v^4}}}{(\overline{\overline{v^2}})^2} \\ \overline{\overline{S}}_u &= \frac{\overline{\overline{u^3}}}{(\overline{\overline{u^2}})^{\frac{3}{2}}} & \overline{\overline{S}}_v &= \frac{\overline{\overline{v^3}}}{(\overline{\overline{v^2}})^{\frac{3}{2}}}\end{aligned}$$

which are the discrete moments within turbulent zones, but they may be affected by the intermittency scheme which will determine the fluctuation intensity of the leading and trailing edges of each burst and hence may create an extra source of below-mean fluctuations.

Dean (1974) suggests alternative zone mean moments; the contribution of turbulent zones to the overall moments.

$$\begin{aligned}\overline{\overline{F}}'_u &= \frac{\delta \overline{\overline{u^4}}}{(\delta \overline{\overline{u^2}})^2} = \frac{\overline{\overline{F}}_u}{\delta} & ; & \quad \overline{\overline{F}}'_v = \frac{\overline{\overline{F}}_v}{\delta} \\ \overline{\overline{S}}'_u &= \frac{\delta \overline{\overline{u^3}}}{(\delta \overline{\overline{u^2}})^{\frac{3}{2}}} = \frac{\overline{\overline{S}}_u}{\sqrt{\delta}} & ; & \quad \overline{\overline{S}}'_v = \frac{\overline{\overline{S}}_v}{\sqrt{\delta}}\end{aligned}$$

These do not have a simple statistical meaning, but they do give at least qualitatively the changes in turbulent zones with Reynolds number. Both sets of moments were calculated and are shown in Figures (5.5.2 ; 5.5.3 ; 5.5.4 ; 5.5.13 ; 5.5.14 ; 5.5.15 ; 5.5.16). $\overline{\overline{S}}_u$ and $\overline{\overline{S}}_v$, Figures (5.5.2 and 5.5.3) are very close indeed to $\overline{\overline{S}}_u$ and $\overline{\overline{S}}_v$, $\overline{\overline{S}}_u$ becomes slightly more negative, and $\overline{\overline{S}}_v$ becomes slightly less negative, as the flow becomes more intermittent, probably due to the influence of leading and trailing edges. It is not understood why, if the negative value of $\overline{\overline{S}}_u$ is due to the presence of irrotational regions, $\overline{\overline{S}}'_u$ has an almost identical value.

$\overline{\overline{F}}_u$ and $\overline{\overline{F}}_v$, Figures (5.5.13 and 5.5.14) show very different behaviour; $\overline{\overline{F}}_u$ exceeds $\overline{\overline{F}}_v$ by a factor of up to 2 in the highly intermittent region, whereas $\overline{\overline{F}}_v$ remains virtually constant

throughout the layer, hardly varying from its near-wall value, 3. Thus u fluctuations based upon an overall mean are distributed over a wide range of values in the turbulent zones but v fluctuations are less widely distributed. The flatness factors based upon regional averages obtained by Hedley and Keffer (1974) are much more uniform and credible, both $\overline{F}_u$ and $\overline{F}_v$ being close to 3 throughout the layer.

$\overline{S}_u$ and $\overline{S}_v$, Fig. (5.5.4) show close agreement with $\overline{S}_u$ and $\overline{S}_v$ near the wall, as may be expected, but in the outer part of the layer, $y/\delta_{995} > 0.07$, both $\overline{S}_u$ and $\overline{S}_v$ take higher absolute values. This latter is particularly noticable in $\overline{S}_u$ where the maximum value of - 9 is found.

Similarly both $\overline{F}_u'$ and $\overline{F}_v'$ Figures (5.5.15 ; 5.5.16) are much larger than $\overline{F}_u$ and $\overline{F}_v$ in the outer part of the layer.

These differences must all be attributed to the non zero ensemble mean used in the calculations, and make the clear interpretation of the results difficult. These results were cross plotted against streamwise distance, x , to show the effects of Reynolds number. Figures (5.5.5 ; 5.5.6) and (5.5.17 ; 5.5.18) show the overall mean skewness and flatness for u and v , Fig. (5.5.18a) shows $\frac{3}{\delta}$, so that intermittency effects may be seen. Both $\overline{S}_u$ and $\overline{F}_u$ show a strong dependence upon Reynolds number certainly larger than $\frac{3}{\delta}$, both reducing by about 25% - 30% in the outer regions ($y/\delta_{995} = 0.9$), from $Re_\theta = 800$ to $Re_\theta = 4750$, close to the wall $\overline{F}_u$, as may be expected, remains virtually constant but $\overline{S}_u$ does seem to decrease by about 10% over the Reynolds number range. The results for the v component show a much reduced Reynolds number dependence: $\overline{S}_v$ increases slightly, $Re_\theta = 800 \sim 1500$, and then decreases to close to its original value, at $Re_\theta = 4750$; $\overline{F}_v$ shows a similar pattern except that close to the wall a gentle decrease is seen first followed by an increase to its

value at Station 17.

The conditionally sampled values $\bar{S}_u, \bar{F}_u, \bar{S}_v$ and $\bar{F}_v$ Figures (5.5.7 ; 5.5.8) and (5.5.19 ; 5.5.20) show the same pattern as their equivalent conventional moments, the dependence is slightly less, except that $\bar{F}_v$ remains virtually independent of Reynolds number throughout the Reynolds number range.

Again the same pattern exists for $\bar{S}'_u, \bar{F}'_u, \bar{S}'_v$ and $\bar{F}'_v$ except that in this case outer layer values will be distorted by changes in δ with R_θ , Figures (5.5.9 ; 5.5.10), (5.5.21 ; 5.5.22).

In general, then, the Reynolds number dependence of the probability distribution moments of u is more pronounced than that of U , which changes little from low to high Reynolds number. Dependence is stronger in the outer part of the flow where intermittency becomes significant. The effect of intermittency is to increase flatness factor and to produce more negative skewness.

5.6 Turbulent Energy Balance.

The equation for the turbulent kinetic energy in a two-dimensional thin shear layer is given by

$$\underbrace{\left(U \frac{\partial}{\partial x} + V \frac{\partial}{\partial y} \right) \frac{1}{2} \overline{q^2}}_1 = \underbrace{-\overline{uv} \frac{\partial U}{\partial y}}_2 - \underbrace{\frac{\partial}{\partial y} \left(\underbrace{\overline{p'v}}_{\textcircled{a}} + \frac{1}{2} \overline{q^2 v} \right)}_3 - \underbrace{V \overline{u_i \frac{\partial^2 u_i}{\partial x_j^2}}}_4$$

which is the rate of change of turbulent energy along a mean streamline in terms of the local quantities on the right hand side of the equation.

These terms have the following meanings: term 1 gives the rate of transport of turbulent kinetic energy along a mean streamline and is

called the advection ; term 2 represents the local production of energy ; term 3 gives the loss of energy by diffusion, which occurs through pressure fluctuations (3a) and the normal transport of energy (3b) and term 4 gives the dissipation of energy through viscosity together with spatial transport by viscous forces, the latter being usually negligible.

The present measurements are confined to the $u-v$ plane and no attempt has been made to measure the w (spanwise) component of velocity. Consequently some assumption about the value of w is required in the estimates of $\overline{q^2} (= \overline{u^2} + \overline{v^2} + \overline{w^2})$ which are necessary for calculation of advection. The data of Klebanoff (1954) suggest that $\sqrt{\overline{w^2}}(y/\delta)$ lies about half way between $\sqrt{\overline{u^2}}(y/\delta)$ and $\sqrt{\overline{v^2}}(y/\delta)$ and when the average position of $\sqrt{\overline{w^2}}(y/\delta)$ is measured from these data it is found that $\sqrt{\overline{w^2}} = \frac{1}{2}(\sqrt{\overline{u^2}} + \sqrt{\overline{v^2}})$ is in fact a very close approximation. This would give

$$\overline{w^2} = \frac{1}{4} (\overline{u^2} + \overline{v^2} + 2\sqrt{\overline{u^2}}\sqrt{\overline{v^2}})$$

which was used to determine $\overline{q^2}$ in the present calculations. This method does beg the question of variations in $\overline{w^2} = f(u, v)$ with Reynolds number, particularly since it has been shown above that $\frac{\overline{u^2}}{\overline{v^2}}(y/\delta)$ varies with Re . It is probable, however, that these changes will be a less serious source of error than the graphical methods used to calculate advection. In any case, the calculation of advection where $\frac{\overline{u^2}}{\overline{v^2}}(y/\delta)$ changes most significantly, that is near the wall, is difficult and inaccurate for other reasons.

The experimental results for V , the mean normal velocity, were found to be unreliable, so this property was re-evaluated using the relation

$$-V_y = \int_0^y \frac{\partial U}{\partial x} dy$$

obtained from the integrated equation of continuity,

$$\frac{\partial V}{\partial y} + \frac{\partial U}{\partial x} = 0$$

The streamwise gradient of mean streamwise velocity, $\frac{\partial U}{\partial x}$ was obtained graphically at several y - positions and then integrated graphically with respect to y . This process proved inherently difficult because of the low values of $\frac{\partial U}{\partial x}$, so $V(y)$ was determined by this method for three streamwise positions corresponding to Stations 1, 7 and 17 and $V(y)$ for intermediate stations was obtained by scaling on $V_e = U_e \frac{d\delta^*}{dx}$. Thus the parts of the advection term could be obtained separately by graphical differentiation of profiles $q^2(x, y)$. Unfortunately the less reliable x derivatives dominate the term and a high degree of scatter was found from station to station. The calculated advection profiles for stations 1, 2, 4, 7 and 17 are shown in Fig. (5.6.4) where the streamwise scatter can be seen clearly. At each station, however, the profiles are fairly smooth. More points are shown where the boundary layer is thicker (station 17, for example) because calculations were based upon equal increments of y not y/δ . These data are shown plotted in a non-dimensional form normalised by δ_{995}^2, τ_w^2 and whilst a good collapse cannot be claimed, it is remarkable that the results for the middle range of stations 2, 3, 4, and 7, lie so close together. This may point to errors in the estimation of $V(y/\delta)$ and $q^2(y/\delta)$ at the extreme ends, stations 1 and 17.

The production term presents little difficulty: the local Reynolds shear stress was obtained by digital analysis and the mean-streamwise-velocity gradient was found graphically. The results, shown in Fig. (5.6.2) give smooth profiles and, again, remarkable degree of collapse when normalised with τ_w^2 .

Klebanoff (1954) obtained dissipation by direct measurement

and diffusion by difference, but Bradshaw (1966) has shown that more reliable results may be obtained by direct measurement of diffusion. The term $\overline{q^2 v}$, expanded, gives $\overline{u^2 v} + \overline{v^3} + \overline{w^2 v}$: the last part of this expansion and the pressure diffusion term $\frac{\overline{p v}}{\rho}$ are unobtainable from the present measurements. Following Lawn (1970), it was assumed that

$$\overline{w^2 v} \approx \frac{1}{2} \overline{v^3} \approx \frac{1}{2} \overline{u^2 v} \quad \text{and that} \quad \overline{p v} \approx \frac{1}{4} \overline{q^2 v}$$

so

$$\left(\overline{q^2 v} + \frac{\overline{p v}}{\rho} \right) \approx \frac{3}{4} (\overline{u^2 v} + \overline{v^3})$$

Both parts of this expression were graphically differentiated across the layer to obtain the diffusion term values, which are shown in Fig. (5.6.3). The closest to the wall that values could be obtained with any confidence was $y/\delta_{995} = 0.3$. The integral of diffusion over the whole width of the layer must be zero, and in fact a condition of close antisymmetry about $y/\delta_{995} \approx 0.55$ may easily be imagined except for the values at Station 2 which are clearly numerically overestimated.

The dissipation term was obtained by difference.

The energy balance is shown for the highest experimental Reynolds number, $R_\theta = 4750$, in Fig. (5.6.1f) in which the present results are compared with the values given by Bradshaw (1966) with diffusion calculated directly by Lawn's method. Bradshaw's values are based upon Klebanoff (1954), and the experimental Reynolds number is approximately $R_\theta = 8000$. The balances show a very similar form: 'dissipation $\approx$ production' near the wall and, at the outer edge 'advection $\approx$ diffusion'. The values are also similar, except that production is higher in the present results partly, though not wholly, due to Reynolds number effects.

It is interesting to consider these balances in the light of Townsend's (1956) two layer model. These results show that production and dissipation are the dominant processes over most of the layer, certainly

out to $y/\delta_{995} = 0.8$ where diffusion exceeds production for the first time. In addition it is not until $y/\delta_{995} = 0.55$ that energy is gained in control volume by diffusion. The advection shows a net loss of energy throughout. Thus it is difficult to believe that the outer layer is principally dependent upon diffusion for its energy; it is only the very edge of the layer where intermittency factor has a value $\gamma < 0.5$ which receives its energy more from diffusion than from production.

The turbulent energy balances for Stations 1, 2, 3, 4, 7 and 17 Fig. (5.6.1) shows that the general form of the energy distribution is retained throughout the Reynolds number range. It seems however, that at low Reynolds number the production and dissipation near the wall dominate even more, but that away from the wall, $y/\delta_{995} > 0.7$, diffusion becomes more significant than at high Reynolds number. The point at which diffusion = production moves towards the wall as Reynolds number decreases, from $y/\delta_{995} = 0.8$ at $R_\theta = 4750$ to $y/\delta_{995} = 0.72$ at $R_\theta = 800$. The magnitude of diffusion in this 'outer layer' is also proportionally larger at low Reynolds number, whereas advection is weaker leading to a balance diffusion $\approx$ dissipation rather than the high Reynolds number balance diffusion $\approx$ advection. The weakness of the advection term, if the values given can be believed, implies that negligible amounts of turbulent kinetic energy are advected downstream at low Reynolds number except at the outer edge of the layer.

Production, diffusion and dissipation are shown as functions of streamwise distance in Figures (5.6.5), (5.6.8) and (5.6.13) respectively.

At a position relatively close to the wall, $y/\delta_{995} = 0.4$, outside the inner logarithmic layer but where intermittency has reached its asymptotic value ($\gamma < 1.0$), all three terms decrease markedly with increasing Reynolds number. Production and diffusion have about twice their $R_\theta = 4750$ value at $R_\theta = 800$. Dissipation decreases by a smaller

amount, about 30 % from $R_\theta = 800$ to $R_\theta = 4750$. Far from the wall at the outer edge of the layer, $y/\delta_{995} \rightarrow 1.0$, production remains virtually constant with Reynolds number, but diffusion and dissipation behave as they do at $y/\delta_{995} = 0.4$, except of course that diffusion is negative. From Fig. (5.6.1) it would seem that advection increases with Reynolds number, throughout the layer.

The convergence of these profiles when normalised with $\tau_w^{3/2}$ has been noted, but when streamwise cross plots are made, some tendencies do emerge. Non dimensional production Fig. (5.6.6) increases throughout the Reynolds number range at all boundary layer positions from $y/\delta_{995} = 0.4$ to $y/\delta_{995} = 0.8$: at $y/\delta_{995} = 0.4$, by 50 %, from $R_\theta = 800$ to $R_\theta = 4750$; at $y/\delta_{995} = 0.8$ by a slightly greater amount. Non-dimensional diffusion Fig. (5.6.9) again falls with increasing Reynolds number but by a lesser amount than in its dimensional form ranging from 10 % at $y/\delta_{995} = 0.4$ to 40 % at $y/\delta_{995} = 0.8$ from $R_\theta = 800$ to $R_\theta = 4750$. The advection term grows very rapidly with Reynolds number, but high scatter makes it difficult to attach a value to the increase. High scatter is also found in the dissipation results which in general increase near the wall and decrease near the outer edge with increasing Reynolds number. An interesting feature of the dimensionless diffusion, advection and dissipation is that most of the streamwise change takes place in the early stages, between $R_\theta = 800$ and , typically, $R_\theta = 2500$, a similar range to that in which $\Pi(R_\theta)$ the wake strength parameter grows most rapidly.

The same general tendencies may be seen when the terms are normalised with $(\overline{uv})^2$, Figures (5.6.7), (5.6.10), (5.6.12) and (5.6.14).

If the non-dimensional diffusion term Fig. (5.6.10) is compared with the terms $\frac{\partial \overline{v^3}}{(\partial \overline{uv})^2}$ and $\frac{\partial \overline{u^2 v}}{(\partial \overline{uv})^2}$ shown in Fig. (5.4.13b) and (5.4.13c) respectively it may be seen that the changes in the diffusion term

with Reynolds number are much larger than the changes in the turbulent zone normal transport of $\overline{v^2}$ or $\overline{u^2}$. There is no obvious physical explanation for this; perhaps Lawm's hypothesis upon which the present diffusion values are based (and which assumes a constant proportional relationship between turbulent and pressure diffusion) does not apply at low Reynolds number, This implies that the ratio of pressure to turbulent diffusion would have to be lower at low Reynolds numbers to bring the diffusion term more in line with its turbulent zone 'components'. The other possibility is that the value of the spanwise fluctuating velocity does not hold a constant relationship with $\overline{u^2}$ and $\overline{v^2}$ at low Reynolds numbers.

A discussion of the turbulent kinetic energy balance and of its relationship to the shear stress balance is contained in the next section after the presentation of the shear stress balance results.

5.7 Shear Stress Balance.

The equation for the (x, y) plane Reynolds shear stress, $-\rho \overline{uv}$, transport is given by Bradshaw (1972), in the two dimensional thin shear layer approximation

$$\underbrace{\left(U \frac{\partial}{\partial x} + V \frac{\partial}{\partial y} \right) (-\overline{uv})}_{1} = \overbrace{\overline{v^2} \frac{\partial U}{\partial y}}^2 - \underbrace{\frac{p_1}{\rho} \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)}^3 + \underbrace{\frac{\partial \overline{uv^2}}{\partial y}}^4 + \underbrace{\frac{1}{\rho} \left(\frac{\partial \overline{p u}}{\partial y} \right)}_5 - \underbrace{V \left(\overline{u \nabla^2 v} + \overline{v \nabla^2 u} \right)}_6$$

where term 1 represents the transport of Reynolds shear stress along a mean streamline (mean-flow-transport), term 2 gives the generation of Reynolds shear stress, term 3 the redistribution by pressure fluctuations, term 4 and term 5 give the transport by, respectively, fluctuating

normal velocity and pressure fluctuations (turbulent - transport) and term 6 shows the destruction of shear stress by viscous forces. Bradshaw suggests that term 6 is negligible and that 5, which is much smaller than 4 may also be neglected.

The pressure redistribution term 3, also called the pressure-strain term remains and must be determined by difference. The balance then reduces to

$$\left(U \frac{\partial}{\partial x} + V \frac{\partial}{\partial y} \right) (\overline{uv}) = \overline{v^2} \frac{\partial U}{\partial y} - \frac{1}{\rho} \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) + \frac{\partial \overline{uv^2}}{\partial y}$$

Each of these terms, except the pressure-strain term, was determined using graphical procedures similar to those outlined in the previous section, (5.6).

The estimated balances are shown in Fig. (5.7.1) and the most obvious feature is the dominance of the generation term, far more so than the production term in the turbulent energy balances. For this reason the generation is shown both to the same scale as other terms and to a highly reduced scale so that the whole profile can be seen. The pressure-strain term is not included because it will behave as the generation term, but with opposite sign. Castro (1974) also noted the very large values of the generation term in a free shear layer. A suitable basis for precise comparison with these data is not known to the present author, and so their accuracy is difficult to assess. In any case high accuracy cannot be expected in the mean-flow-transport term, which depends upon streamwise gradients nor in the turbulent-transport term where graphical differentiation has been used on fairly scattered $\overline{uv^2} (y/\delta)$ data. It is believed, however, that the values given here preserve the broad features of the processes concerned.

It is only at high Reynolds numbers $Re > 3000$ that the

mean-flow-transport achieves the same significance as the turbulent-transport and at the lowest Reynolds number ($R_\theta = 800$), Fig. (5.7.1a) the former is negligible throughout the layer. At this lowest Reynolds number at the outer edge, $y/\delta_{995} > 0.8$ the turbulent transport is of roughly the same value as the generation term, and of the same sign, so that the pressure strain term will have twice the value of the the generation term making it the most significant term. At high Reynolds numbers, however, the mean-flow-transport in this outer part of the layer is more nearly balanced by the turbulent-transport term so that pressure-strain will be somewhat less, although still larger than generation.

Non-dimensional profiles of generation, turbulent transport and mean-flow transport are shown in Fig. (5.7.2), Fig. (5.7.3) and Fig. (5.7.4) respectively. Some collapse may be seen in the generation and turbulent transport terms, but the mean-flow transport clearly increases with Reynolds number. When these results are plotted against streamwise position, the trends emerge as follows: generation, Fig. (5.7.5), increases throughout the layer with Reynolds number by an average of some 20 %, larger increases occurring in the outer part of the layer, $y/\delta_{995} = 0.8$; as predicted, mean-flow transport, Fig. (5.7.9), increases, by up to 5 times, in the Reynolds number range $800 < R_\theta < 4750$; the turbulent transport term, Fig. (5.7.7), has a rather more complex behaviour. Non-dimensional turbulent transport (based on $\tau_w^{\frac{3}{2}}$) near the wall, $y/\delta_{995} = 0.4$ seems to increase initially with Reynolds number up to $R_\theta \approx 2500$ and there after to fall, ending up at $R_\theta = 4750$ at a value about twice its value at $R_\theta = 800$: at the outer edge, $y/\delta_{995} = 0.8$ a steady decrease with increasing Reynolds number is found. These results were considered highly ambiguous and they were re-analysed using $(\overline{uv})^{\frac{3}{2}}$ as the normalising function, Fig. (5.7.8) It may be seen that the outer edge decrease of turbulent transport with

Reynolds number is amplified, but that the behaviour closer to the wall remains ambiguous. It must be concluded therefore that there may be non-monotonic changes of this term with Reynolds number as has already been noted for some triple-product terms (Section (5.5)).

The generation and mean-flow-transport terms non-dimensionalised with $\overline{uv^2}$ are shown in Fig. (5.7.6) and Fig. (5.7.10) respectively and it may be seen that they behave as when normalised with $\tau_w^{\frac{3}{2}}$.

Again, comparing the turbulent transport term, Fig. (5.7.8) with its turbulent zone 'component', $\frac{\overline{uv^2}}{(\overline{\partial uv})^{\frac{1}{2}}}$, shown in Fig. (5.4.13d) it can be seen that $\frac{\partial \overline{uv^2}}{\partial y} \cdot \frac{1}{\overline{uv^2}}$ (the turbulent transport term) changes the more rapidly with Reynolds number. Again, it is not easy to offer a physical explanation, particularly since shear stress seems to be proportionally lower than turbulent kinetic energy components in irrotational zones. Perhaps the only conclusion that can be drawn on this question is that such terms as $\overline{v^3}$, $\overline{u^2v}$ and $\overline{uv^2}$ do not after all give such a close approximation to turbulent transport terms.

In general the equivalent terms in the two transport equations for turbulent kinetic energy and Reynolds shear stress behave similarly, except that the low values of transport terms found through most of the boundary layer in the latter cause the destruction terms to behave differently: dissipation of kinetic energy is roughly constant whilst pressure-strain increases with Reynolds number (both in the non-dimensional form with $\tau_w^{\frac{3}{2}}$). Fig. (5.7.11) shows the ratio of generation of shear stress to production of turbulent kinetic energy. Over the first six measuring stations, $800 < R_\theta < 2500$ the ratio decreases by about 20 % (or more, depending upon how the scatter of points is interpreted): beyond, $R_\theta > 2500$ there may be a slight increase with Reynolds number, but the data is too unreliable to confirm this. The behaviour below $R_\theta \approx 2500$

implies that, at low Reynolds number, the generation of $\overline{uv}$ is proportionally larger than the production of turbulent kinetic energy, or that it is more affected by the changes in turbulence at low Reynolds number.

The ratio of turbulent transport of Reynolds shear stress to diffusion of turbulent kinetic energy, shown in Fig. (5.7.12), contains a high degree of scatter and two conflicting behaviours: at positions where the transport is moderately positive or negative ($\frac{y}{\delta_{995}} = 0.4, \frac{y}{\delta_{995}} = 0.8$) the ratio increases with Reynolds number, but where the values are low near the crossover region between positive and negative values ($\frac{y}{\delta_{995}} = 0.6$) the ratio falls rapidly at low Reynolds numbers. Surprisingly only this latter behaviour is in agreement with Fig. (5.4.15), which shows the turbulent zone kinetic energy to shear stress 'transport' ratio, and it must be concluded that, well away from the wall at least, shear stress is less powerfully transported by turbulence than kinetic energy is diffused, particularly at low Re .

5.8 Conclusions.

In so far as it was possible to make comparisons, the present results at high Reynolds number are in good agreement with other data. A particular difficulty in assessing conditionally averaged turbulence concerns the baseline used in forming averages: in most reported results regional baselines have been used, but for physical reasons total baselines were used in the present work making comparison of zone averages virtually impossible.

The conditionally sampled mean flow results are in broad agreement with the data of Kovaszny et al (1970) and Hedley and Keffer (1974). Turbulent zones have a lower streamwise mean velocity than the irrotational zones, the normal mean velocity is directed away from the wall

in turbulent zones, towards the wall in irrotational zones. In addition the present results show that the turbulent zones move with a velocity closer to that of the irrotational zones at low Reynolds number, but at high Reynolds number have a higher normal mean velocity.

It was found that the most well behaved form of streamwise bulk convection velocity was Bradshaw's (1966) definition based upon overall average fluctuating velocity parameters which tended in value towards the mean streamwise velocity at the outer edge. Other definitions based upon turbulent-zone-averages gave values closer to $0.9 U_e$ predicted for irrotational regions, but dropped off sharply at $y/\delta_{995} > 0.9$. The difference between turbulent zone streamwise mean velocity and convection velocity was found to be significantly larger at low Reynolds numbers.

Unfortunately it was not found possible to obtain good agreement between normal convection velocities formed according to various definitions and although it was possible to show that this property has higher values at low Reynolds numbers more consideration of the parameter is required.

The u and v component turbulence intensities show an excellent agreement with Klebanoff's (1954) data, even at the outer edge, when both sets of results are non-dimensionalised on a basis of U_e . Perhaps the most striking result of these measurements is that the anisotropy of the turbulence changes radically with Reynolds number: at low Reynolds number and roughly mid layer positions the ratio $\overline{u^2}/\overline{v^2}$ is smaller than at high Reynolds numbers by about 25 %. It is shown that this is a consequence of change in $\overline{u^2}$, which increases with Reynolds number, whilst $\overline{v^2}$ remains roughly constant in the range $800 < R_e < 4750$.

As a proportion of wall shear stress τ_w , the Reynolds shear stress $\overline{uv}$ was found to be higher at high Reynolds numbers in

agreement with Green's (1971) and McDonald's (1970) predictions.

A fair agreement was obtained between the triple products $\frac{\overline{u^2v}}{(\overline{uv})^{\frac{3}{2}}}$ and $\frac{\overline{v^3}}{(\overline{uv})^{\frac{3}{2}}}$ and Bradshaw's (1966) data except at the outer edge of the layer where small inaccuracies in $\overline{uv}$ can become significant. It was found that the major contribution to overall averages of turbulence properties came from the turbulent zone averages. Suitably non-dimensionalised, using $(\gamma(\overline{uv}))^{\frac{3}{2}}$, the turbulent zone triple products behaved as follows, as Reynolds number increases: $\overline{u^3}$ increases; $\overline{v^3}$ remains roughly constant and both $\overline{u^2v}$ and $\overline{uv^2}$ decrease.

In the statistical analysis of fluctuating velocities good agreement was found between the present skewness values at moderate Reynolds and those of Hedley and Keffer (1974), particularly u component skewness, and between the present u component flatness values and the data of Klebanoff (1954). An attempt was made to produce some meaningful indication of probability distribution moments for turbulent zones. In general it was found that both skewness and flatness fell with increasing Reynolds number, but that the v component moments fell by a much smaller amount.

Turbulent energy balances were carefully constructed, and a good general agreement was found between the present high Reynolds number data and Bradshaw's (1966) results based on Klebanoff (1954). The balances of terms in the Reynolds shear stress transport equation were also determined for various Reynolds numbers, but unfortunately a suitable basis for comparison was not discovered. A very noticeable feature of the balances is that mean flow transport terms, advection in the case of the turbulent energy balances, increase very rapidly with Reynolds number from a very low initial value. In general the generation and production terms increase and the diffusion and turbulent transport terms decrease with increasing Reynolds number.

The largest part of all these changes takes place in the Reynolds number range $800 < R_\theta < 2500$, in a manner analogous to the change in $\Pi(R_\theta)$. At the outer edge of the boundary layer the balance changes from diffusion $\rightleftharpoons$ advection to diffusion $\rightleftharpoons$ dissipation as Reynolds number falls from $R_\theta = 4500$ to $R_\theta = 800$.

The behaviour of the diffusion terms suggests that Lawn's (1971) hypothesis of constant proportionality between pressure and turbulence diffusion breaks down at low Reynolds number. Bradshaw (1966a), however, obtained the same numerical result as Lawn by assuming that $\overline{w^2 v} = \frac{1}{2}(\overline{u^2 v} + \overline{v^3})$ and that $\overline{p v} = 0$, leading again to

$$\left(\frac{\overline{p v}}{\rho} + \frac{\overline{q^2 v}}{2} \right) \rightleftharpoons \frac{3}{4} (\overline{u^2 v} + \overline{v^3})$$

In both these hypotheses the real effect of pressure diffusion is concealed, but no reliable measurements of the term exist at present and it is difficult, therefore to see how they could be improved.

Chapter Six.Calculation of Low Reynolds Number Boundary Layers.

This chapter describes modifications made to the boundary layer calculation method of Bradshaw, Ferriss and Atwell (1967) to account for low Reynolds number effects. The modifications were designed to give agreement with the present mean flow data. Profiles of mixing length and eddy viscosity obtained by analysis of the present data are compared with the results of Green (1971), McDonald (1970) and Klebanoff (1954).

6.1 Program Modifications.

The boundary layer calculation method of Bradshaw et al converts the turbulent energy equation into an equation for turbulent shear stress using empirical functions relating energy dissipation, turbulent intensity and diffusion to the shear stress profile. An hyperbolic system is then formed with the mean momentum equation and the continuity equation and the set is solved numerically using the method of characteristics. Three empirical functions are used to close the equations; A_1 relating the shear stress profile to the turbulent kinetic energy profile; G , the diffusion function and L , the dissipation-length parameter. The shear stress/turbulent energy profile function is considered to be a constant, $A_1 \approx 0.15$, and the calculations have been found to be insensitive to the behaviour of G . The dissipation length parameter

$$L \equiv \left(\frac{\tau}{\rho} \right)^{\frac{3}{2}} \cdot \frac{1}{\epsilon}$$

may be considered as a generalised mixing length, equal to the mixing length

$$l = \left(\frac{\tau}{\rho}\right)^{\frac{1}{2}} / \frac{\partial U}{\partial y} \quad 6.2$$

in the special case of local energy equilibrium, Townsend (1961), or where advection and diffusion balance. L may be expected therefore to affect the calculations in much the same way as l . Over most of the layer dissipation is the dominant process: it was decided to base program modifications upon $L(\frac{y}{\delta})$. The values of the dissipation length parameter, $L(\frac{y}{\delta})$, and of the other functions were originally chosen so that the calculation method would reproduce Klebanoff's (1954) experimental shear stress profile when used with Coles' (1962) asymptotic velocity profile.

The program modification introduces another degree of freedom for L , in Re , arranged so that the velocity profiles computed agree with Coles $\pi(Re)$ derived from Wiegardt's data.

Originally, $L(\frac{y}{\delta})$ was given by

$$\left. \begin{aligned} L/\delta &= 0.4(y/\delta) & 0 < y/\delta < 0.1 & \quad 0.18 \\ L/\delta &= 0.095 - 0.055(2z - 1)^2 & 0.18 < y/\delta < 1.1 & \\ L/\delta &= 0.016 e^{10(y/\delta - 1.1)} & 1.1 < y/\delta & \end{aligned} \right\} 6.3$$

The function modification shifts the profile along the line $L = 0.4y$, so as to produce larger L in the outer layer at low values of Re . The positions of transition from linear to parabolic and from parabolic to exponential, Z_{01} and Z_{02} respectively, are then given by:

$$\left. \begin{aligned} Z_{01} &= 0.18 + Z_0 e^{-\left(\frac{Re}{3630}\right)^2} \\ Z_{02} &= Z_{01} + 0.92 \end{aligned} \right\} 6.4$$

where Z_0 is an input scaling factor. Then L is given by

$$\left. \begin{aligned}
 L/\delta &= 0.4 (y/\delta) & 0 < y/\delta < Z_{01} \\
 L/\delta &= 0.4 (Z_{01} - 0.18) + 0.095 - 0.055 (2(y/\delta - Z_{01}) + 0.18 - 1)^2 & Z_{01} < y/\delta < Z_{02} \\
 L/\delta &= (0.16 + 0.4 (0.01 + 0.18)) e^{-(10(Z - Z_{02}))} & Z_{02} < y/\delta
 \end{aligned} \right\} 6.5$$

The original form of $L(y/\delta)$ is obtained by setting $Z_0 = 0$. Initially computer calculations were made using synthetic starting profiles, generated by the program from Coles' profile, but it was found that a very sharp rise in wake strength, $\Delta \frac{U}{u_r}(Re)$, was obtained, reaching well above the asymptotic value before settling down, Fig. (6.1.1). Redesigned starting profiles based on a pure log law velocity distribution and an empirically chosen shear stress distribution were input numerically and the results were found to be better behaved, Fig. (6.1.2). From these calculations performed for a range of values of Z_0 the optimum value, $Z_0 = 0.05$ was obtained.

An alternative form for $L(y/\delta)$ given by

$$L = L_0 (1 + 10 \mu \frac{\partial U}{\partial y} / -\bar{u}\bar{v}) \quad 6.6$$

was also investigated. The results for both methods and the experimental points from the present tests together with the data of Wieghardt's are shown in Fig. (6.1.3) in the form $\Delta \frac{U}{u_r}(Re)$. It may be seen that a good agreement exists between the results of the first method, (6.5), the present data and Wieghardt's data but that the second method is unsatisfactory because $\Delta \frac{U}{u_r}(Re)$ rises too rapidly at low values of Re . The scatter in the computed results at low Reynolds number is due to residual effects of the starting conditions, but a fair degree of scatter is found even at high Reynolds number, reflecting the difficulty in estimating wake strength.

Brederode and Bradshaw (1974) made use of the modified

boundary layer calculation program to predict the boundary layer development in a square duct and found excellent agreement, within 1 % on $\overline{T}(Re)$, with their experimental data. They also used the method of Baker and Launder (1972), but with half the recommended correction which they found to agree less well with their experimental $\overline{T}(Re)$, within 5 %, principally because the initial rise in $\overline{T}(Re)$ was too rapid.

6.2 Results : Boundary Layer Development.

The modified program gave generally good agreement with experimental data in the range $800 < Re < 4000$ using the input starting profiles, but at Reynolds numbers $Re < 800$ the results from the calculations show a large scatter (experimental results in this range also tend to be unreliable).

The development of skin friction coefficient with Reynolds number is shown in Fig. (6.2.1), and it may be seen that the experimental results have a larger scatter than the individual points obtained from the calculations. The uncertainty in c_f at low Reynolds number, about $Re = 500$, is also apparent but is of fairly small order, about 2 %.

Shape factor development $H(Re)$ is shown in Fig. (6.2.2) and again an excellent agreement between calculation and experiment exists, except that at low Reynolds numbers the curve shows a tendency to flatten and to asymptote to $H \approx 1.5$. This behaviour was found by Brederode and Bradshaw (1974) both in the present calculation method and Baker and Launder's (1972).

The Clauser equilibrium parameter, Fig. (6.2.3) does not, however, exhibit such a simple agreement. There is an initial sharp rise in $G(Re)$ up to $Re \approx 800$ and subsequently a gentle decrease with increasing Re .

Although the general agreement for $R_\theta > 800$ is fairly good with the experimental data, there is a steeper slope in $G(R_\theta)$ in the calculated results. Since Π increases with R_θ , the calculations predict that G decreases as Π increases, which is unexpected.

6.3 Results : Boundary Layer Profiles.

In this section the computed profiles of boundary layer parameters are compared with the data from the present experiments and with the results obtained by Green (1971) and McDonald (1970) through their respective calculation methods. Where necessary the value of δ_{995} , required in scaling for length, has been extracted from the calculated velocity profiles by graphical methods. Since in most cases exactly comparable profiles do not exist the closest available in terms of R_θ are compared for typical high and low Reynolds numbers. Green's results are given for specified values of $\bar{R}$, the wall co-ordinate Reynolds number, $= \frac{u_\tau \delta}{\nu_w}$, which was determined for each of the present computed profiles.

Mean velocity profiles are shown in Fig. (6.3.1). The agreement between calculated and experimental profiles is good at high Reynolds number, but at low Reynolds number the computed profiles at the outer edge, $y/\delta > 0.6$, shows an irregular behaviour due to residual effects of the starting conditions. The calculated low Reynolds number profile ($R_\theta \approx 800$) appears too full at the outer edge, which might be expected to lead to an excessive value of $\Delta \frac{U}{u_\tau}$ and in fact $\Delta \frac{U}{u_\tau}(R_\theta)$, Fig. (6.1.3), is slightly high at $R_\theta \approx 800$.

The shear stress profiles, Fig. (6.3.2), show a good agreement between the present experimental data and the calculations throughout the

layer despite a slight tendency for calculated values to be low near the outer edge. The agreement with Green's calculated profiles is, in general, poor although this may in part be because the profiles are not at exactly equivalent Re . It was found that the agreement between Green's profiles and the experimental data, presented in Chapter Five, was better for his low Reynolds number case and near the wall (see Fig. (5.3.)).

Mixing length values were derived by graphical differentiation of mean velocity profiles, and the results for the calculations and the present experimental profiles together with those of Klebanoff (1954) given by Bradshaw (1966) are shown in Fig. (6.3.3). Fair agreement exists between the present experimental data at high Reynolds number and those of Klebanoff. The calculation profiles are different in shape from the experimental profiles whose trends they appear to exaggerate, particularly the sudden rise in l at $y/\delta > 0.7$. At low Reynolds numbers $l(y/\delta)$ is larger, but by a modest amount.

Calculation mixing length profiles $l(y/\delta)$ are shown in Fig. (6.3.4) together with the profiles of the empirical dissipation length parameter, $L(y/\delta)$. As expected the inner layer values for $l(y/\delta)$ and $L(y/\delta)$ are very similar, but for $y/\delta > 0.2$ the behaviour of the two functions is quite different, $l(y/\delta)$ increasing rapidly towards the outer edge where l may be expected to be of order δ , $L(y/\delta)$ tending towards zero at high y/δ . The difference between the values of $L(y/\delta)$ at low and at high Reynolds number is not large, the former being higher by some 20 % beyond the inner layer. It is noteworthy that such a modest change in $L(y/\delta)$ can reproduce the large changes in mean velocity profile found at low Reynolds number.

Comparison is made with the mixing length profiles of Green and of McDonald in Fig. (6.3.5) and (6.3.6) respectively. Outside the inner layer agreement with the present results from calculated profiles is,

in both cases, poor. Both Green and McDonald require large changes in $\frac{l}{\delta}(\frac{y}{\delta})$ in the outer layer to allow for Reynolds number effects, their high Reynolds number profiles reach a maximum at about $\frac{y}{\delta} = 0.4 - 0.6$ and then drop off, but in the low Reynolds number profiles, most noticeably Green's, $\frac{l}{\delta}$ rises throughout the layer. These figures suggest that the present calculation method compares more favourably with experimental data than with other calculation methods.

In Fig. (6.3.8) profiles of eddy viscosity, $V_T = \frac{\tau}{\rho \frac{\partial U}{\partial y}}$, are shown for the present calculation method, Klebanoff's data and the present experimental data. The high Reynolds number experimental results agree fairly well with Klebanoff's except that the latter are about 10 % higher throughout the layer. The low Reynolds number results suggest that eddy viscosity is more affected by Reynolds number than mixing length, since the former is some 30 % larger throughout the layer at low Reynolds number. The values obtained from the calculated profiles show a larger scatter than the experimental values, probably because the mean flow profiles obtained by calculation contain less datum points, making estimation of $\frac{\partial U}{\partial y}$ more difficult.

The calculation eddy viscosity profiles are compared with those obtained by Green and by McDonald in Fig. (6.3.9) and Fig. (6.3.10) respectively. Again the profile changes with Reynolds number given by Green and McDonald are much larger than those obtained in the present calculations, which in these figures appear very small indeed (and certainly smaller than the changes in the present experimental profiles).

All the eddy viscosity profiles have, unlike the mixing length profiles, the same basic shape, $\frac{V_T}{\delta^* U_e}$ tending towards zero at the outer edge of the boundary layer.

6.4 Conclusions.

The low Reynolds number program modification, effectively increasing $L(\frac{y}{\delta})$ at low Reynolds number until a fit between calculated and experimental $\Pi(R_\theta)$ is obtained, gives results which compare excellently with the present experimental data both for boundary layer growth, $c_f(R_\theta)$, $H(R_\theta)$, and $G(R_\theta)$ and for profiles of mean velocity and shear stress.

The calculation method is shown, however, to be very sensitive to the starting profiles. Under normal (high Reynolds number) circumstances the effects of starting profiles will have largely disappeared, but low Reynolds number calculations require accuracy even in the first few steps so that a careful choice of initial velocity and shear stress profiles is essential.

Mixing length profiles obtained from the calculated mean velocity profiles are closer in appearance to experimentally obtained results than to the profiles used in typical 'mixing length' calculation methods, particularly at the outer edge of the boundary layer.

In general, low Reynolds numbers have a slighter effect on profiles of mixing length and eddy viscosity in the present calculations and experiments than in the calculations of Green or McDonald.

In the outer layer both the Green/McDonald data analysis and the present calculations predict an increase in ℓ as R_θ decreases. The agreement of the detailed distributions is not very good, but this is true throughout the Reynolds number range. It may be said that the present modifications reproduce the experimental results for c_f , Π and H satisfactorily while the predictions of profile shape are within the general accuracy of the calculation method.

Chapter Seven.

General Discussion of Results and Conclusions.

The starting point for the investigations described in this thesis was the remarkable change in mean velocity profile of a turbulent boundary layer at low Reynolds number: the present work was designed to describe and if possible to explain the responsible turbulence phenomena. Inevitably in pursuit of this main objective many new questions have been raised, and a whole new range of measurements is required before a full explanation of the low Reynolds number effect can be advanced. Previous chapters have presented and discussed results in detail; certain immediate conclusions were drawn. It is the object of this chapter to produce a more connected account of the main structural features of the boundary layer and of their response to low Reynolds numbers and to pose some of the new problems encountered.

The skin-friction law derived from the overlap of the law of the wall and the velocity-defect law predicts that c_f increases as Reynolds number decreases, but in fact the increase is larger than predicted, for example by Schoenherr (1932) or Spalding and Chi (1958). This is because in the same range of low Reynolds numbers the mean velocity profile displays a sharply reducing wake strength as Reynolds number decreases until at some finite value of R_θ , about 600, the wake disappears altogether. Effectively this latter phenomena means that over much of the boundary layer outside the wall region there is a lower mean velocity gradient, $\frac{\partial U}{\partial y}$, at low Reynolds numbers.

The present measurements of shear stress confirm the low Reynolds number skin friction values obtained by Coles (1962) from Wieghardt's

results, but also lend support to the findings of Brederode and Bradshaw (1974) that the log law additive constant $C = 5.2$ fits available data better than Coles (1962) value $C = 5.0$, if Patel's (1965) Preston tube calibration is believed. The mean velocity profiles measured show the expected changes in wake strength, although it was not possible to obtain profiles at the lowest Reynolds numbers where $\Pi \rightarrow 0$: the lowest experimental Reynolds number, $R_\theta = 800$, gives a wake strength of about half the asymptotic value. Wake strength values obtained agreed far better with Coles analysis of (1962) than with the results obtained in Coles (1968) using a modified fitting procedure. Actual wake shapes support Bull's (1968) form of the cosine wake function, but there is apparently a change in shape with Reynolds number, though of small order.

Measurements of turbulence at low Reynolds number revealed a remarkable change in the structure of the turbulent field. This is of great significance since it is the turbulence which is responsible for the shear stress which itself determines the mean velocity distribution. Turbulence measurements in these experiments concern only the region outside the log law region, the hot wire probes used could not be taken closer to the wall. It was found that, at low Reynolds numbers and in an intermediate region

$y/\delta < 0.8$, $\bar{u}^2/\bar{v}^2$ was some 25 % less at the lowest Reynolds number and $y/\delta = 0.4$ than its asymptotic value of about 2.5 Fig. (5.2.6). Further analysis showed that this was in effect due to a decrease in $\bar{u}^2$, $\bar{u}^2/u_\tau^2$ Fig. (5.2.9) decreased by about 30 % whilst $\bar{v}^2/u_\tau^2$ Fig. (5.2.10) decreased by about 8 % only. This change took place principally in the same Reynolds number range as the increase in wake strength $\Pi(R_\theta)$. That is up to $R_\theta \approx 2500$, or station 7: this raised the possibility of a direct link between the two phenomena. The result of this change upon the measured shear stress profiles was remarkable: in the intermediate region shear stress was

found to be less for a given wall value by some 20 % Fig. (4.3.4a) at low Reynolds numbers ($R_\theta = 800$ compared with $R_\theta = 4750$). The profiles of the ratio of "turbulent kinetic energy" (actually a function of $\overline{u^2}$ and $\overline{v^2}$ since $\overline{w^2}$ was not measured) to shear stress, Fig. (5.2.8), were also found to be Reynolds number dependent. Klebanoff (1954) noted that this ratio seemed fairly constant for $0.2 < y/\delta < 0.8$ and the present high Reynolds number data also shows this constancy. At low Reynolds number, however, a steady increase from $y/\delta \approx 0.2$ to 0.8 was noted, a further result of the asymmetry of changes in the turbulence field at low Reynolds number. Also $\frac{\overline{uv}}{q^2}$ was seen to go to zero as $y \rightarrow \delta$ more rapidly at low Reynolds number, probably an indication of a stronger irrotational field.

The reduction in shear stress and the smaller values of mean velocity gradient at low Reynolds numbers both contribute to a lower production of turbulent energy. At $y/\delta \approx 0.6$, production normalised by $(\tau_w U_e)^{3/2} \cdot \frac{1}{\delta}$ is estimated to be 50 % of its $R_\theta = 4750$ value at $R_\theta = 800$, Fig. (5.6.6). The turbulent field generates shear stress principally through normal fluctuations, $\overline{v^2}$, and the generation of shear stress was found to suffer less from low Reynolds number effects; proportionally more shear stress than turbulent energy is generated at low Reynolds numbers, Fig. (5.7.11).

The redistribution of turbulent energy, produced as $\overline{u^2}$, to $\overline{v^2}$ and $\overline{w^2}$ is of great importance in the understanding of low Reynolds number effects. The redistribution is carried out by fluctuating pressure in such terms as $\frac{1}{\rho} \overline{p \frac{\partial u}{\partial x}}$ in equations 1.17, 1.18 and 1.19. At the moment there are no reliable measurements of pressure fluctuations, even at high Reynolds number so that further work is needed before progress can be made in this respect.

The production and generation terms in the Reynolds stress equations give the quantities involved, but do not improve our understanding

of the mechanisms which are responsible for the injection of new energy and shear stress into the flow. Considerable attention has been focused upon the "bursting" phenomenon as the mechanism of turbulence production, for example by Kim, Kline and Reynolds (1968). The large-scale turbulent bursts found in the outer layer have frequently been described in terms of lumps of turbulence, thrust from the point of their origin far down in the boundary layer towards the outer edge, and some comparison between the two phenomena seems inevitable, although a connection has yet to be established. Lin et al (1966) found that surface roughness eliminated "bursting"; but Antonia and Luxton (1973) noted that the intermittent structure in the outer layer was little affected by surface roughness, so that the link must be fairly obscure. Supposing, however, that intermittent production affects the outer layer intermittency, some inferences about the former may be drawn from the latter. Present measurements confirmed that, in agreement, with Klebanoff (1954) and Kovaszny et al (1970) turbulent bursts moved relatively slower than irrotational regions Fig. (5.1.2), at low Reynolds number it was found that this difference was less Fig. (5.1.4a) because turbulent bursts moved faster. It was also found that, unlike high Reynolds number bursts which are solitary Fig. (4.11.15) low Reynolds number bursts are more split, like a series of hands, fingers outstretched. Burst statistics revealed that low Reynolds number bursts were longer, up to the order of the boundary layer thickness Fig. (4.11.11) but irrotational regions did not reach so far into the layer. This suggests a far less organised structure at low Reynolds number with bursts of a wider variety of intensities originating further from the wall (having a higher "mean velocity"). These results would seem to agree with Falco's (1974) findings from his smoke picture analysis that whilst "typical eddies" were of order the boundary layer thickness in length at low Reynolds number ($Re_\theta = 600$) they were an order of magnitude smaller at high Reynolds

number ($R_\theta = 6000$). Falco's pictures also show a well defined, but complex structure at low Reynolds number: at high Reynolds number a more consistent "turbulent mush" is to be seen. Kovasznay (1970) describes a "reverse cascade" process of small bursts rolling together to create larger ones; present measurements of the ragged bursts at low Reynolds number, would lend support to this, the process being developed with Reynolds number. It would be interesting, supposing some connection to exist between "bursting" and outer layer turbulent bursts, to find out whether "bursting" was less regular at low Reynolds number and less evenly spaced.

The production of Reynolds stresses is balanced by transport and destruction as described in section 1.3 and Klebanoff has defined three regions of the boundary layer in which these processes have different characteristics: a viscous region from the viscous sublayer to $y^+ \approx 30$; the log law region from $y^+ \approx 30$ to $y/\delta \approx 0.2$ and the outer layer including the intermittent region $y/\delta > 0.4$. For the present purposes it will be convenient to subdivide this latter region into two: the intermittent region in which generation and destruction predominate, $0.2 < y/\delta < 0.8$, and the outer layer beyond $y/\delta \approx 0.8$ where transport terms become equally significant, at high Reynolds number at least.

In the intermediate region, which was found to be narrower at low Reynolds number, the ratio of production to diffusion was estimated to be lower at low Reynolds number, = 8 at $R_\theta = 800$, = 11 at $R = 4750$ ($y/\delta = 0.35$) Fig. (5.6.1). As Lawn's hypothesis was used this is effectively diffusion by turbulence, and if, as conjectured, pressure fluctuations are more powerful at low Reynolds numbers the whole diffusion term would be larger. The estimated diffusion thus showed larger values in both negative and positive lobes at low Reynolds number Fig. (5.6.3), with peak values perhaps twice at $R_\theta = 800$ what they were at $R_\theta = 4750$. There are two consequences of this:

less local dissipation takes place in the intermediate regions at low Reynolds number (proportionally more energy is transferred to the outer region); secondly, since transfer towards the wall of turbulent energy is by means of pressure diffusion, Klebanoff (1954), proportionally more energy will be transferred to the wall to be dissipated through viscosity.

The second transport mechanism is mean flow transport or, for turbulent energy, advection. This term was carefully estimated and showed the largest Reynolds number dependence Fig. (5.6.11), particularly in the outer region where its value increased threefold in the range $800 < R_\theta < 4750$, again principally in $R_\theta < 2500$. At low Reynolds number this has a considerable effect on the outer layer: energy diffused there is locally dissipated rather than advected downstream as at high Reynolds number. The physical interpretation of this is not easily seen: if diffusion may be considered as the movement of large scale bursts carrying contained energy to the outer edge, then advection is analogous, perhaps equivalent, to the process by which bursts bend over downstream and spread their energy along the flow. Kijayasu (1970) found that, although fronts of bursts were well defined their downstream edges merged into the free stream, suggesting a rapid transfer of energy downstream within bursts. The present findings may show that this process is less able to operate at low Reynolds number where the contrast between mean velocities in turbulent and irrotational regions is less marked.

Transport terms in the shear stress balance contribute far less than the equivalent terms in the turbulent energy balance, for example, the ratio of generation to turbulent transport is about $25 \left(\frac{y}{\delta} = 3.5 \right)$ at $R_\theta = 4750$ Fig. (5.7.1). The streamwise transport of $\overline{uv}$ is, however, so small at low Reynolds number Fig. (5.7.9) that the shear stress transported by turbulence to the outer region is chiefly destroyed locally by the "pressure-strain" term which, again, must therefore be more powerful at

low Reynolds number.

The mechanism by which the low Reynolds number boundary layer is able to destroy both ^{more} turbulent energy through viscosity and ^{more} turbulent shear stress through "pressure-strain" is not clear. The former, ^{larger} dissipation, could however be evidence for a more extensive viscous superlayer region, at low Reynolds number.

Statistical analysis of the distributions of fluctuating velocities showed that U -component fluctuations are strongly negatively skewed and V -component fluctuations positively skewed, Fig. (5.5.1). An interesting conflict appears in the Reynolds number dependence of these results. The U -components are more negatively skewed at low Reynolds number, Fig. (5.5.5), suggesting that below-mean values were larger, yet the mean velocity in turbulent regions was proportionally higher. V -components had slightly more positive skewness at low Reynolds number, Fig. (5.5.6), but initially there was an increase with Reynolds number in keeping with the relatively lower above-mean fluctuations. These tendencies were seen particularly in the region of high intermittency, $y/\delta > 0.8$, and suggest that the irrotational regions are far from passive and that considerable stream-wise and normal velocity fluctuations take place within them. Kovasznay et al (1970) suggest that the irrotational flow in effect "rides-out" the turbulent bursts; perhaps the turbulent bursts also shake the irrotational flow to and fro. The interpretation of these movements of distribution is a hazardous matter, but it may be conjectured that in respect of Reynolds number effects, there is more "shaking" and less "riding" at low Reynolds numbers.

An important part of this work has been the development of suitable modifications for boundary layer calculation methods, both as a practical application of the low Reynolds number effect and as a guide to the effect of low Reynolds numbers on the calculation method.

Bradshaw's turbulent-energy-equation method is described in section 1.4 and the modifications in chapter 6. Effectively dissipation length was increased with decreasing Reynolds number, $Re < 6000$, and it was found that a comparatively small increase in dissipation length, of order 18 %, was sufficient to enable the calculation of accurate velocity profiles down to below experimental Reynolds numbers ($5000 > Re > 500$). This is an excellent illustration of the sensitivity of this method to the dissipation length parameter. It cannot be claimed that, except in the wall region, agreement between mixing length and dissipation length was good, indicating the general inadequacy of local equilibrium assumptions despite the small (relative) values of transport terms in the intermediate region.

In summary, then, at low Reynolds number the turbulence is weaker as a proportion of surface shear stress but more evenly redistributed. The result is a lower shear stress in intermediate regions whilst generation of shear stress is less but turbulent energy production is proportionally still lower. Although production is smaller diffusion is a more efficient process carrying turbulent energy to the outer layer where it is dissipated locally and to the wall where it may be instrumental in maintaining a proportionally higher skin friction coefficient. Pressure fluctuations play a major role in this process, but measurements are not available at present.

These are the main conclusions, but a large volume of data has been presented and there must be considerably more information concealed in it. To facilitate the use of the results, mean velocity profiles are tabulated in Appendix (ii).

Two outstanding possibilities exist for extension of this work which would help to answer some of the unsolved questions. The first is an analysis of the three dimensional structure of the intermittent region, in particular to establish whether turbulent bursts are jet-like with a

spanwise scale of order of their streamwise scale, as the "mushroom" shapes seen for example by Falco (1974) would suggest, or if some spanwise co-ordination exists to make bursts more roll-like. Findings that bursts have well defined leading edges even if backs are somewhat diffuse encourages the hope that good point averages might be obtained once intermittency schemes have been improved, recognising the role of small scale events. Further, the three dimensional surface shear stress structure discussed in Chapter 2 and suggesting a regular spanwise waviness in boundary layer thickness (assuming $c_f = f(R_\theta)$) may have some role in controlling burst dimensions which would then become larger in the spanwise direction at low Reynolds number as a proportion of δ .

Secondly a spectral analysis of the data would be of great value, and could reveal the true aspects of the apparently poor large scale organisation at low Reynolds number as well as providing a further insight into low Reynolds number dissipation.

References.

Antonia R.A. (1972) Conditionally sampled measurements near the outer edge of a turbulent boundary layer. J. Fluid Mech. 56.

Antonia R.A. and Atkinson J.D. (1974) Use of a pseudo turbulent signal to calibrate an intermittency measuring circuit. J. Fluid Mech. 64.

Antonia R.A. and Bradshaw P. (1971) Conditional sampling of turbulent shear flows. Imperial College Aero. Dept. 71-04.

Antonia R.A. and Luxton R.E. (1973) Some statistical properties of turbulence in smooth and rough wall boundary layers. Univ. Sydney TNF 34

Baker R.J. and Launder B.E. (1974) The turbulent boundary layer with foreign gas injection. Int. J. Heat and Mass Transfer 17.

Batchelor G.K. (1953) The theory of homogeneous turbulence. Cambridge University Press.

Bradbury L.J.S. (1964) A simple circuit for the measurement of intermittency factor in turbulent flow. Aero. Quart. 15.

Bradshaw P. (1965) The effect of wind tunnel screens on nominally two-dimensional boundary layers. J. Fluid Mech. 22.

Bradshaw P. (1966a) The turbulence structure of equilibrium boundary layers. N.P.L. Aero. Rept. 1184 ARC 27675.

Bradshaw P. (1966b) "Inactive" motion and pressure fluctuations in turbulent boundary layers. N.P.L. Aero. Rept. 1172 ARC 27338.

Bradshaw P. (1972) The understanding and prediction of turbulent flow. Aero. J. 76.

Bradshaw P. (1972) Two more wind tunnels driven by aerofoil-type centrifugal blower. I.C. Aero Rept. 72-10

Bradshaw P. (1974) Effect of free stream turbulence on turbulent shear layers. Imperial College Aero. Rept. 74-10.

Bradshaw P. Ferriss D.H. and Atwell N.P. (1967) Calculation of boundary layer development using the turbulent energy equation. J. Fluid Mech. 28.

Bradshaw P. and Gregory N. (1961) The determination of local turbulent skin friction from observation in the viscous sublayer. ARC R & M 3202.

de Bray B.G. (1967) Some investigations into the spanwise non-uniformity of nominally two dimensional incompressible boundary layers downstream of gauze screens. ARC R & M 3578.

Bull M.K. (1969) Velocity profiles of turbulent boundary layers. Aero. J. 72.

de Brederode V. and Bradshaw P. (1974) A note on the empirical constants appearing in the logarithmic law for turbulent wall flows. Imperial College Aero. Rept. 74-03.

de Brederode V. and Bradshaw P. (1974) Influence of the side walls on the turbulent centre-plane boundary layer in a square duct. Imperial College Aero. Rept. 74-06.

Castro I.P. (1973) A highly distorted turbulent free shear layer. Ph.D Thesis Imperial College.

Cebeci T. (1973) Kinematic eddy viscosity at low Reynolds numbers. AIAA. J. 11 pp. 102-104.

Charnay G. (1971) Caracteristiques d'une couche limite turbulente evoluant en presence d'un ecoulement exterieur turbulent. These Dres Sc. Universite Claude-Bernard, Lyon.

Coles D. (1956) The law of the wake in the turbulent boundary layer. J. Fluid Mech. 1.

Coles D.E. (1962) The turbulent boundary layer in a compressible fluid. Rand Rept. R 403-PR, ARC 24478. Appendix A. A manual of experimental boundary layer practice for low speed flow.

Coles D.E. (1967) A young persons guide to the Data. Proc. AFOSR-IFP. Stanford Conference on computation of turbulent boundary layers.

Coles D.E. (1968). Data Tabulation in Proc. AFOSR - IFP Stanford Conference on computation of turbulent boundary layers.

Coles D.E. and Hirst E.A. (1967) Memorandum on data selection. Proc. AFOSR-IFP. Stanford Conference on computation of turbulent boundary layers.

Corrsin S. and Kistler A.L. (1955) Free stream boundaries of turbulent flows. NASA Rept. 1244.

Corrsin S. (1949) Shear stress correlation in turbulent shear flow. (letter) J. Aero. Sci. 16 757.

Dean R.B. (1974) Reynolds number dependence of skin friction in a 2D rectangular duct flow and a discussion of the law of the wake. Imperial College Aero. Rept. 74-11

Dean R.B. (1974) Turbulent shear layer interaction in ducts and diffusers. Ph.D Thesis Imperial College.

Dean R.B. (1974) Personal Communication.

Falco R.E. (1974) Some comments on turbulent boundary layer structure inferred from the movements of a passive contaminant. AIAA paper 74-99 12th Aerospace Sciences meeting.

Fiedler H. and Head H.R. (1966) Intermittency measurements in the turbulent boundary layer. J. Fluid Mech. 25.

Gibbins J.C. (1959) On boundary layer transition wires. ARC C.P. No. 462.

Gold and Raeder (1970?) Analysis of random data.

Graham J.M.R. (1969) The development of the turbulent boundary layer behind a transition strip. ARC 31 492 FM 4090.

Green J.E. (1971) A note on the turbulent boundary layer at low Reynolds number in compressible flow at constant pressure. Personal Communication.

Green J.E. (1972) On the influence of free stream turbulence on a turbulent boundary layer, as it relates to wind tunnel testing at subsonic speeds. RAE TR 7220.

Hama F.R. (1954) Boundary layer characteristics for smooth and rough surfaces. Trans. Soc. Naval Arch. Nav. Engrs. 62.

Hanjalic K. and Launder B.E. (1972) A Reynolds stress model of turbulence and its application to thin shear flows. J. Fluid Mech. 52.

Hastings R.C. and Sawyer W.G. (1970) Boundary layers on a large flat plate at $H = 4$. Roy. Aircraft Est. Tech. Rept. 70040.

Hedley T.B. and Keffer J.F. (1974) Turbulent / non turbulent decisions in an intermittent flow. J. Fluid Mech. 64, 4.

Hedley T.B. and Keffer J.F. (1974) Some turbulent / non turbulent properties of an intermittent flow. J. Fluid Mech. 64, -4.

Hinze J.O. (1959) Turbulence. McGraw Hill.

Huffman G.D. and Bradshaw P. (1972) A note on von Karman's constant in low Reynolds number turbulent flow. J. Fluid Mech. 53, 1.

Huffman G.D., Zimmerman D.R. and Bennett W.A. (1972) The effects of free stream turbulence on turbulent boundary layer behaviour.
Detroit Diesel Allison.

Kaplan R.E. and Laufer J. (1968) The intermittently turbulent layer of the boundary layer. Proc. 12th Int. Congress of App. Mech. Stanford.

Kibens V. and Kovasznay L.S.G. (1970) Detection of the turbulent / non turbulent interface. John Hopkins Univ. Dept. of Mechanics Rept. 1-1970.

Kim H.T., Kline S.J. and Reynolds W.C. (1968) An experimental study of turbulence production near a smooth wall in a turbulent boundary layer with zero pressure gradient. AFOSR 68-6383.

Kim H.T., Kline S.J. and Reynolds W.C. (1971) The production of turbulence near a smooth wall in a turbulent boundary layer. J. Fluid Mech. 48.

Kijayasu I. (1970) Structure of super layer in a turbulent boundary layer. Tokio Univ. Int. of Sp. and Aero. Sci. Bull. 6.

Klebanoff P.S. (1954) Characteristics of turbulence in a boundary layer with zero pressure gradient. NACA T.N. 3178. also in NACA Rept. R47 (1955).

Kline S.J., Reynolds W.C., Schraub and Runstadler (1967).

Kovasznay L.S.G. (1970) The turbulent boundary layer. Ann. Rev. Fluid Mech.

Kovasznay L.S.G., Kibens V. and Blackwelder R.F. (1970) Large-scale motions in the intermittent region of a turbulent boundary layer. 41.

la Rue J.C. and Libby P.A. (1974) Temperature fluctuations in the plane turbulent wake. Phys. Fluids 17.

Laufer J. (1954) The structure of turbulence in fully developed pipe flow. NACA Rept. 1174.

Laufer J. (1972) Recent developments in turbulent boundary layer research. Inst. Nazionale Di Ata Mat. Milan Symposia Mathematica IX.

Lawn C.J. (1970) Diffusion of turbulence energy in flow through axisymmetric ducts. Imperial College Dept. of Mech. Eng. Rept.

McDonald H. (1970) Mixing length and eddy viscosity in a low Reynolds number boundary layer. UAR J. 214153-1.

MacMillan F.A. (1956) Experiments on Pitot-tubes in shear flow. Aero. Res. Counc. R & M No. 3028.

Mahgoub H. (1974) Application of interactive computer graphics to the solution of problems in turbulent shear flow. Imperial College M.Sc. Thesis.

Mandelbrot B.B. (1967) "How long is the coastline of Britain". Sci. 156.

Miller B.L. (1971) Shear stress measurement in an adverse pressure gradient turbulent boundary layer over a smooth flat plate. Univ. of Leicester Eng. Dept. 71 / 16.

Patel R.P. (1964) The effects of wind tunnel screens and honeycombs on the span wise variation of skin friction in two dimensional turbulent boundary layers. McGill Univ. TN 64-7

Patel V.C. (1965) Calibration of the Preston tube and limitations on its use in pressure gradients. J. Fluid Mech. 23, 1.

Paizis and Schwartz (1974) An investigation of the topography of motions of the turbulent interface. J. Fluid Mech. 63.

Pennel W.T., Sparrow E.H. and Eckert E.R.G. (1972) Turbulence intensity and time mean velocity distributions in low Reynolds number turbulent pipe flows. Int. J. Heat Mass Transfer. 15.

Perkins H.J. (1970) The formation of streamwise vorticity in a turbulent flow. J. Fluid Mech. 44.

Pongprudhanon P. (1974) Personal Communication.

Preston J.H. (1954) The determination of turbulent skin friction by means of pitot tubes. J. Roy. Aero. Soc. 58.

Prandtl L. (1932) Zur turbulenten strouiling in rohren und tangs platten. Ergeton Aerodyn. Versuchsant Gottingen 4, 18-29.

Robertson J.H. and Holt C.F. (1970) Stream turbulence effects on the turbulent boundary layer. Proc. ASCE. J. Hydr. Div. 98.

Rotta J.C. (1962) Turbulent boundary layers in incompressible flow. Progress in Aero. Sci. 2. Ed. Ferri A. Muchemmann D. and Sterne L.H.G. Pergamon.

Sandborn V.A. (1959) Intermittency in the turbulent boundary layer. J. Fluid Mech. 6.

Schoenherr K.F. (1932) Resistance of flat surfaces moving through a fluid. Trans. Soc. Nav. Arch. Nav. Eng. 40.

Schubauer (1954) Turbulent processes as observed in boundary layer and pipe. J. Apl. Phys. 25.

Sherman D. (1972) Some measurements of the intermittency function in an open channel flow in the region immediately downstream of a natural transition. S. and M. Note. 374.

Simpson R.L. (1970) Characteristics of turbulent boundary layers at low Reynolds numbers with and without transpiration. J. Fluid Mech. 42.

Spalding D.B. and Chi S.W. (1958) Drag of a compressible turbulent boundary layer on a smooth flat plate with and without heat transfer. J. Fluid Mech. 18.

Spangler J.G. and Wells C.S. (1970) Effects of spiral longitudinal vortices on turbulent boundary layer skin friction. NASA CR 145.

Sunyach M. (1971) Contribution a l'etude de frontieres d'ecoulements turbulent re. D.Sc. Thesis Lyons.

Townsend A.A. (1947) Measurements in the turbulent wake of a cylinder. Proc. Roy. Soc. A. 190.

Thomas R.M. (1973) Conditional sampling and other measurements in a plane turbulent wake. J. Fluid Mech. 59.

Townsend A.A. (1948) Local isotropy in the turbulent wake of a cylinder. Australian J. Sci. Res. Ser. A 1, 2.

Townsend A.A. (1956) The structure of turbulent shear flow. Cambridge.

Townsend A.A. (1961) Equilibrium layers and wall turbulence. J. Fluid Mech. 11.

Tsuji and Iida (1972) The influence of free stream turbulence on a zero pressure gradient turbulent boundary layer. Trans. Japan Soc. for Aero. and Space Sci. 15.

Wallace J.M., Eckelmann H. and Brodley R.S. (1972) The wall region in turbulent shear flow. J. Fluid Mech. 54, 1.

Weir A.D. and Bradshaw P. (1974) Apparatus and programs for digital analysis of fluctuating quantities in turbulent flow. Imperial College Aero. Rept. 74-09.

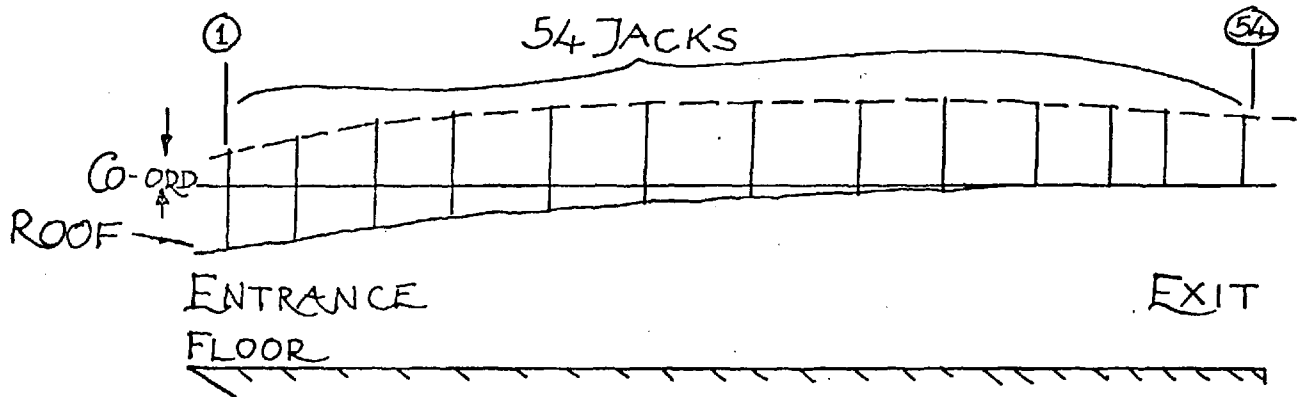
Wieghardt K. (1944) Zum reibungswiderstand ranther platten. Kaiser-Wilhelm-Institute fur Stromingsforschung Gotingen. UM 6612.

Wong F.Y.F. (1970) Shear flow over a rearward facing step. Imperial College M.Sc. Thesis.

Wynanski I. and Fiedler (1970) The two dimensional mixing region. J. Fluid Mech. 41.

Young S.T.B. (1974) Personal Communication.

Table 2.1



Jack No.	cms Co-ord	Jack No.	cms Co-ord	Jack No.	cms Co-ord	Jack No.	cms Co-ord
1	3.5	17	4.0	33	4.55	49	5.16
2	3.5	18	4.05	34	4.60	50	5.10
3	3.6	19	4.05	35	4.65	51	5.15
4	3.65	20	4.05	36	4.70	52	5.20
5	3.65	21	4.05	37	4.70	53	5.20
6	3.70	22	4.05	38	4.75	54	5.20
7	3.70	23	4.10	39	4.75		
8	3.70	24	4.15	40	4.80		
9	3.75	25	4.25	41	4.85		
10	3.75	26	4.25	42	4.85		
11	3.75	27	4.30	43	4.90		
12	3.80	28	4.40	44	4.95		
13	3.85	29	4.40	45	5.00		
14	3.90	30	4.45	46	5.05		
15	3.95	31	4.50	47	5.05		
16	3.95	32	4.55	48	5.10		

Table 3.1

x ft.	H	G	π	$\Delta U/u_2$	Re
0.5	1.490	6.96	0.257	1.25	791
1.0	1.458	6.97	0.358	1.75	1112
1.5	1.433	6.89	0.384	1.88	1368
2.0	1.419	6.85	0.425	2.08	1640
2.5	1.407	6.83	0.443	2.16	1900
3.0	1.401	6.83	0.462	2.25	2192
3.5	1.397	6.89	0.467	2.28	2387
4.0	1.393	6.95	0.472	2.30	2620
5.5	1.374	6.84	0.507	2.47	3362
7.0	1.356	6.72	0.492	2.40	4108
8.5	1.353	6.74	0.510	2.50	4750

Table 4.1

Rough values of Smoothing Time expressed as

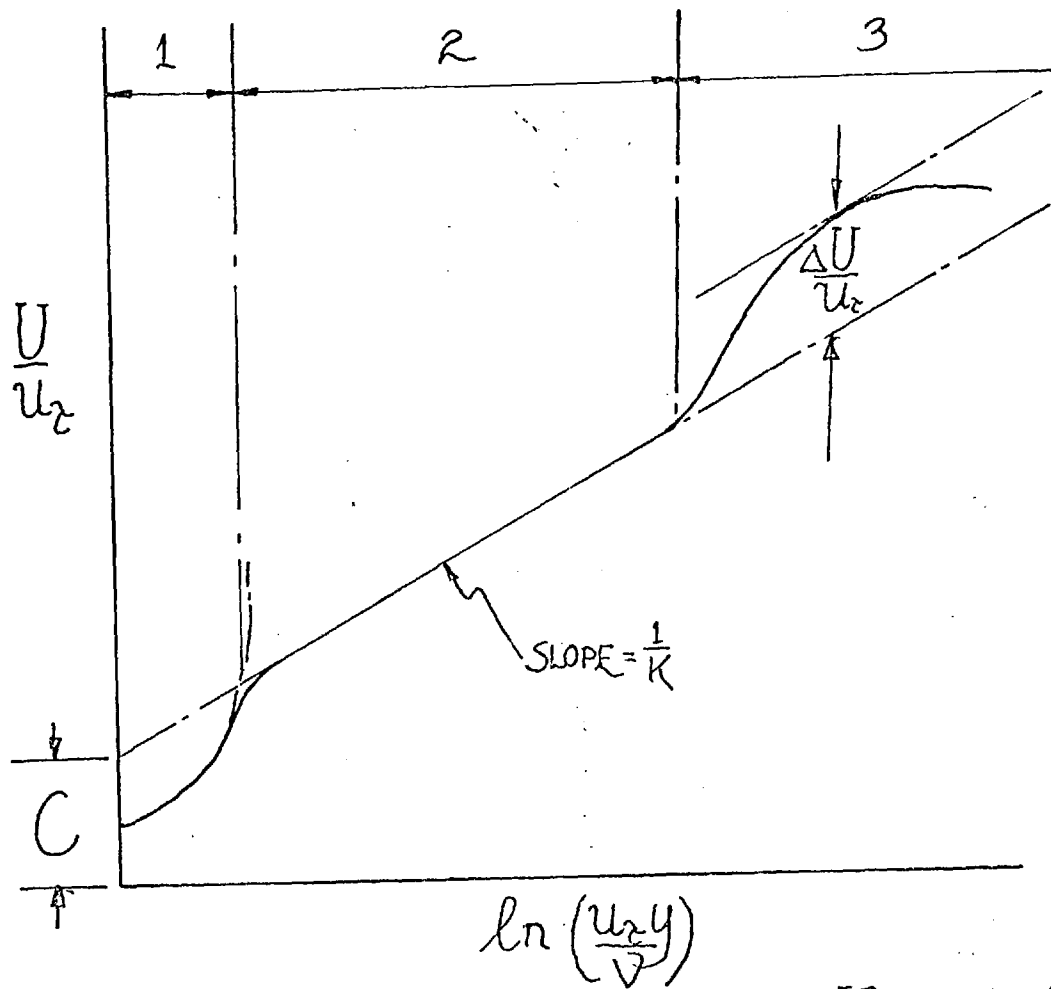
Author (s)	Distance travelled by Fluid in Hold time	
	(Shear Layer thickness)	
		Smoothing(Equivalent distance)
Kovaszny et al	1970	0.17
Sumyach	1971	0.03
Antonia	1972	0.06
Hedley and Keffer	1974	0.05
Thomas *	1973	0.3

* Based on half wake width

TABLE 2 Choices of criterion functions

Corrsin and Kistler (1955)	$ \omega_x $, smoothed
Bradbury (1964)	$ \partial u / \partial t $
Fiedler and Head (1966)	$ \partial u / \partial t $ + (the same, phase-shifted); smoothed?
Kaplan and Laufer (1968)	$(\partial u / \partial t)^2$, smoothed
Kovaszny et al. (1970)	$\partial^2 u / \partial y \partial t$, smoothed
Wyganski and Fiedler (1970)	$(\partial u / \partial t)^2 + (\partial^2 u / \partial t^2)^2$; smoothed?
Antonia and Bradshaw (1971)	$(\partial u / \partial t)^2$, smoothed
Sunyach (1971)	$(\partial u / \partial t)^2$, bandpass filtered + (the same, time-delayed)
Antonia (1972)	$(\partial uv / \partial t)^2$, smoothed
Thomas (1973)	$ u $ or $ \partial u / \partial t $, both bandpass filtered and smoothed
Castro (1973)	$ \partial uv / \partial t $ or $ \partial^2 uv / \partial t^2 $

FIGURE [1-1.1] THE UNIVERSAL VELOCITY PROFILE LAW



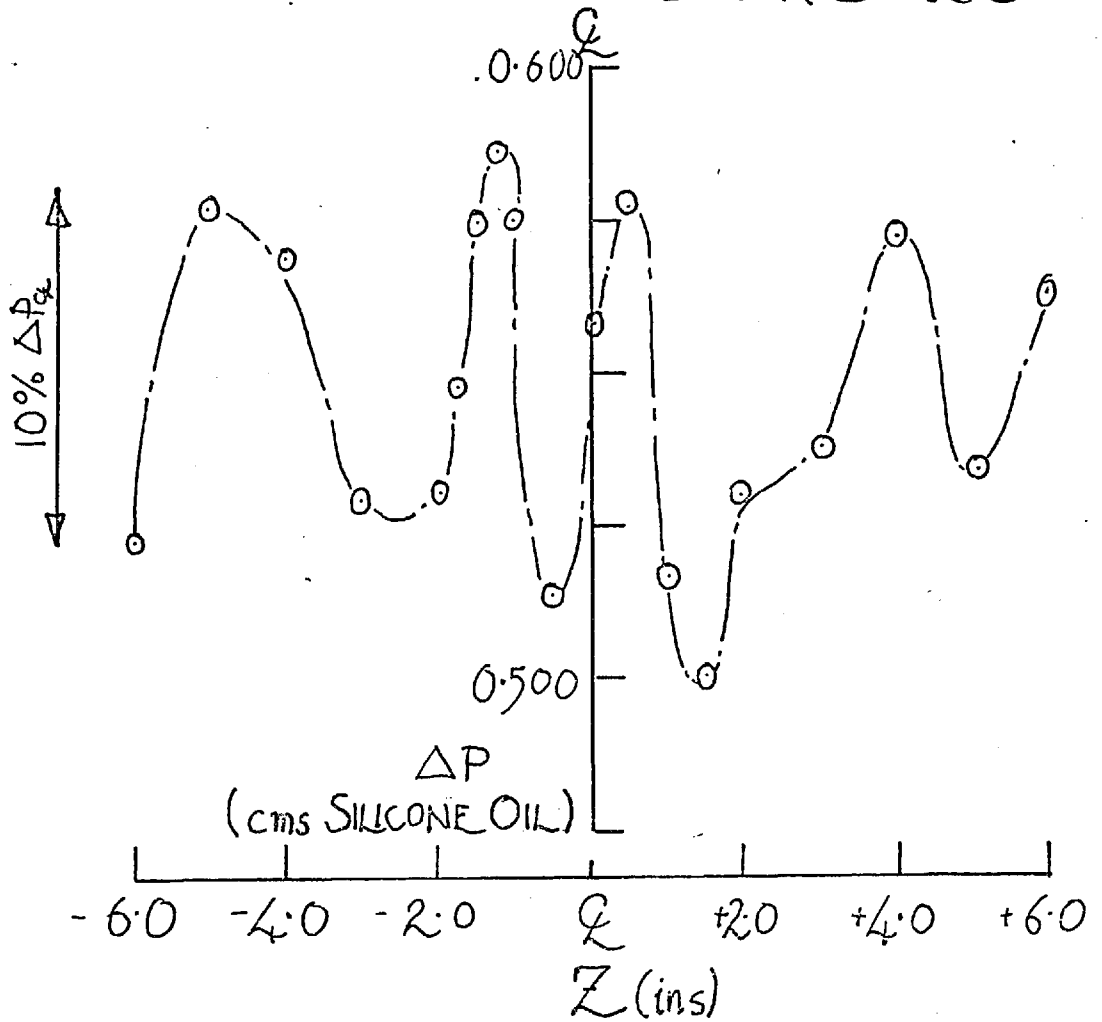
REGION 1 - VISCOUS SUBLAYER $\frac{U}{u_2} = \frac{u_2 y}{\nu}$

REGION 2 - INNER LAYER $\frac{U}{u_2} = \frac{1}{K} \ln \frac{u_2 y}{\nu} + C$

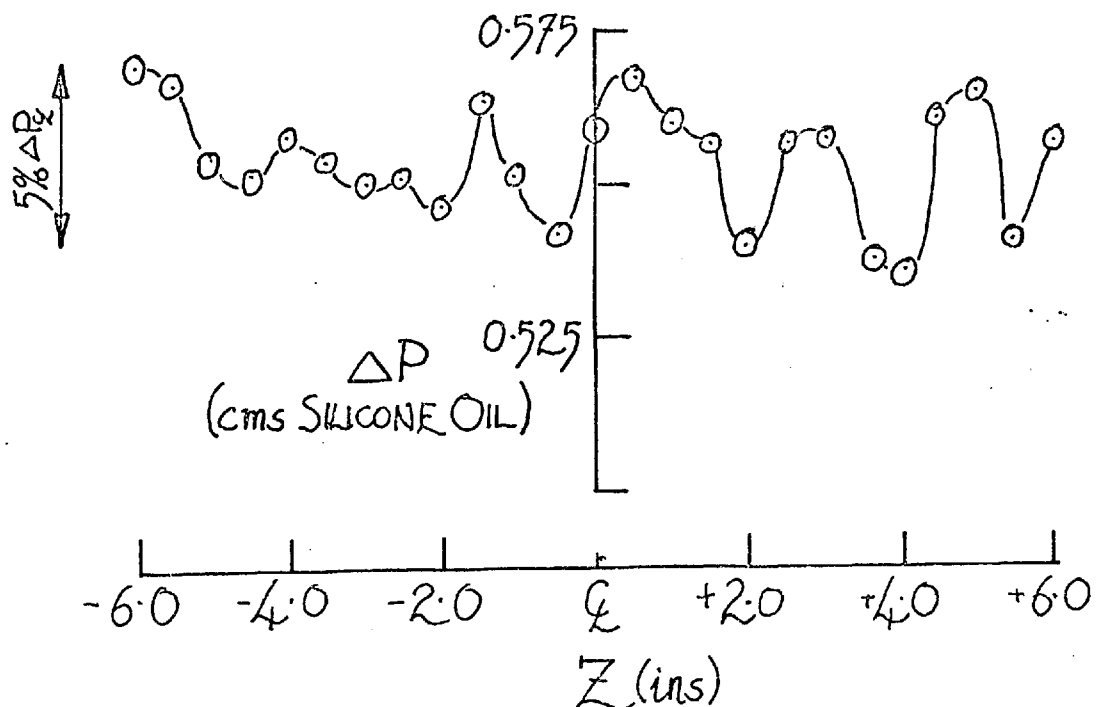
REGION 3 - OUTER LAYER $\frac{U}{u_2} = \frac{1}{K} \ln \frac{u_2 y}{\nu} + C + \frac{\pi w(y)}{K}$

WAKE STRENGTH $\equiv \frac{\Delta U}{u_2} = \frac{2\pi}{K}$

FIG[2-1.1] INITIAL SPANWISE DISTRIBUTION OF SURFACE SHEAR STRESS



FIG[2-1.2] FINAL SPANWISE DISTRIBUTION OF SURFACE SHEAR STRESS



FIG[2-1.3] DISTRIBUTION OF STATIC PRESSURE
ALONG THE WORKING SECTION

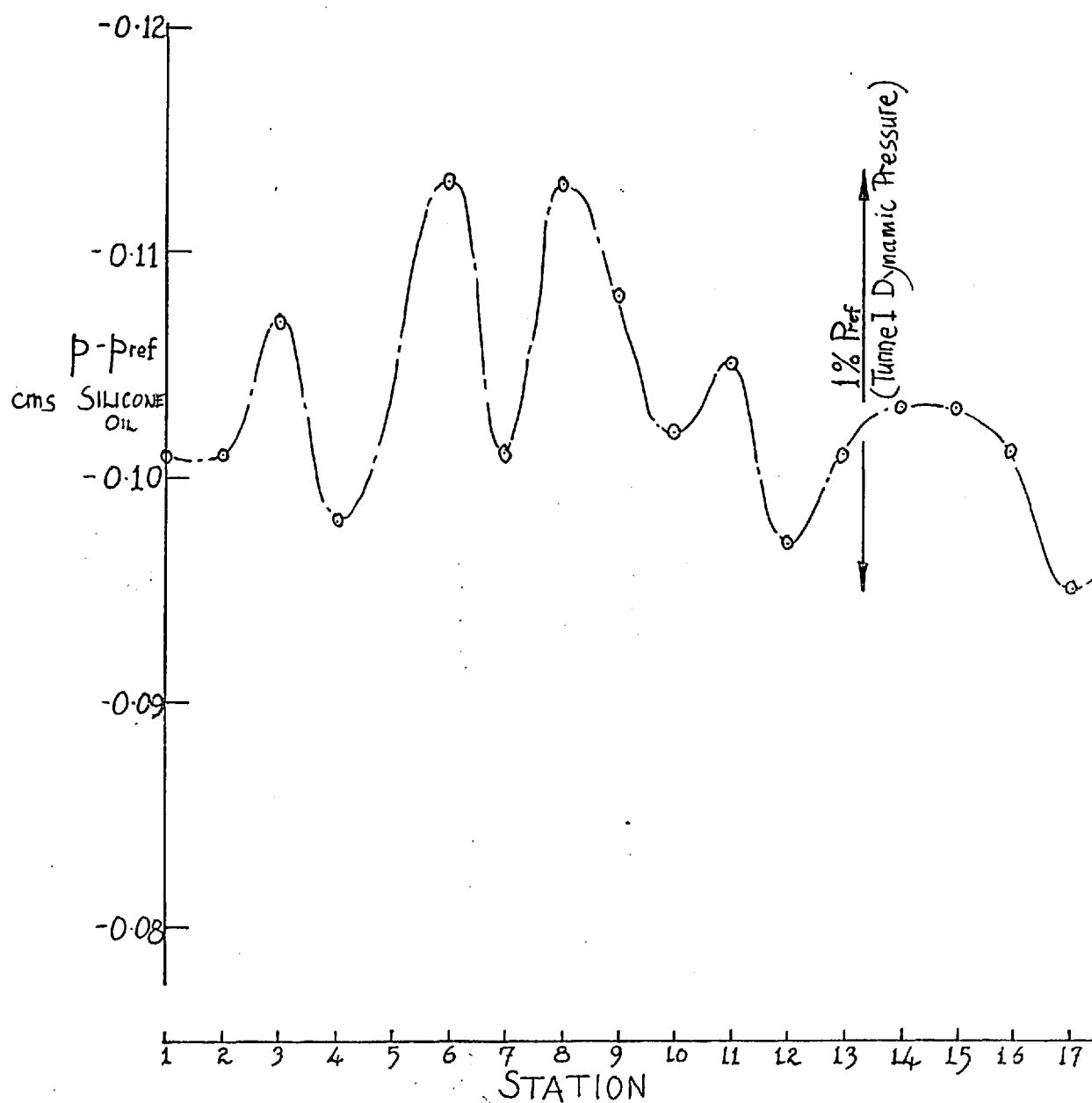
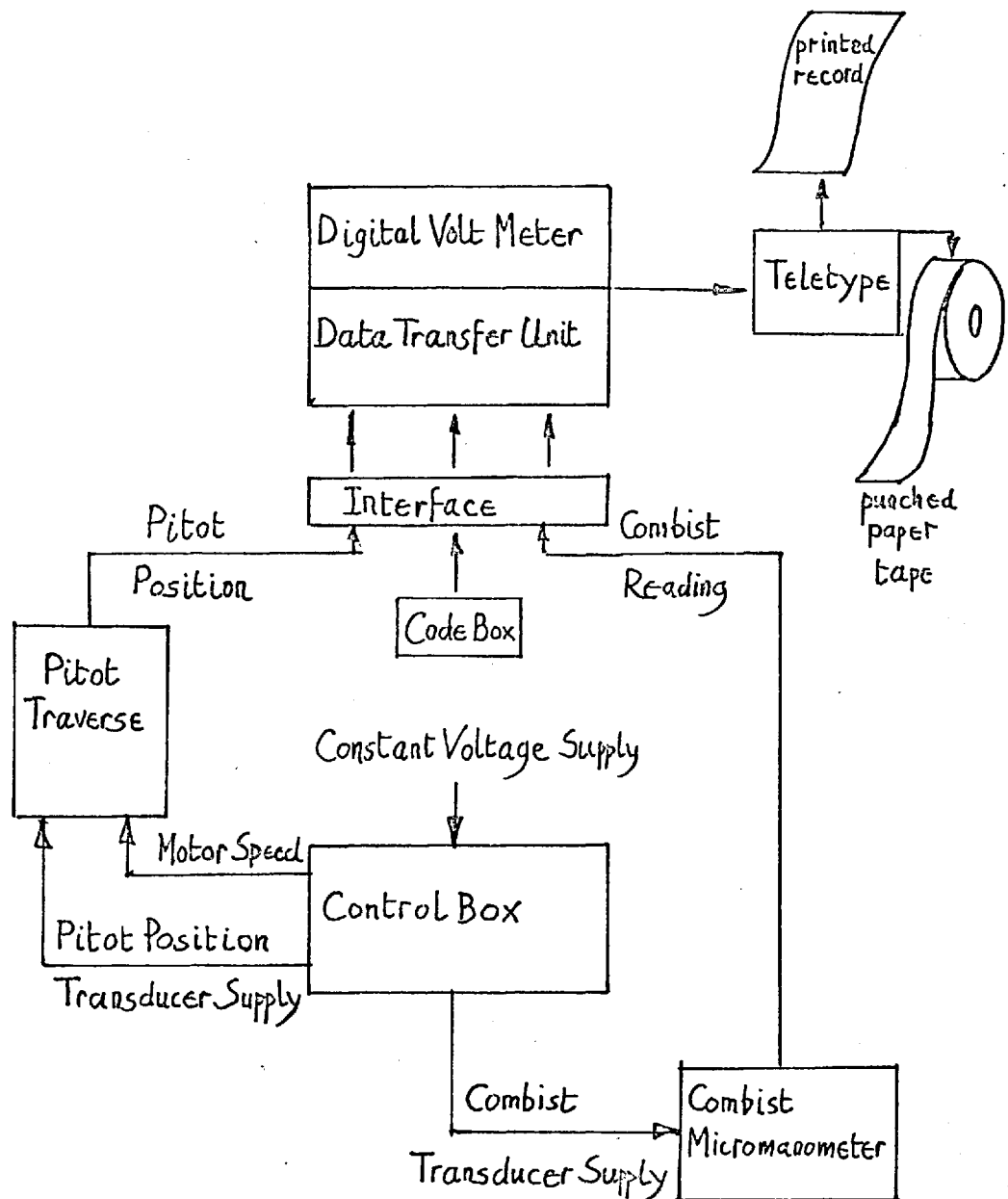


FIGURE [2-3.1] MEAN FLOW ELECTRONICS



FIGURE[2-4.1] HOT WIRE ELECTRONICS

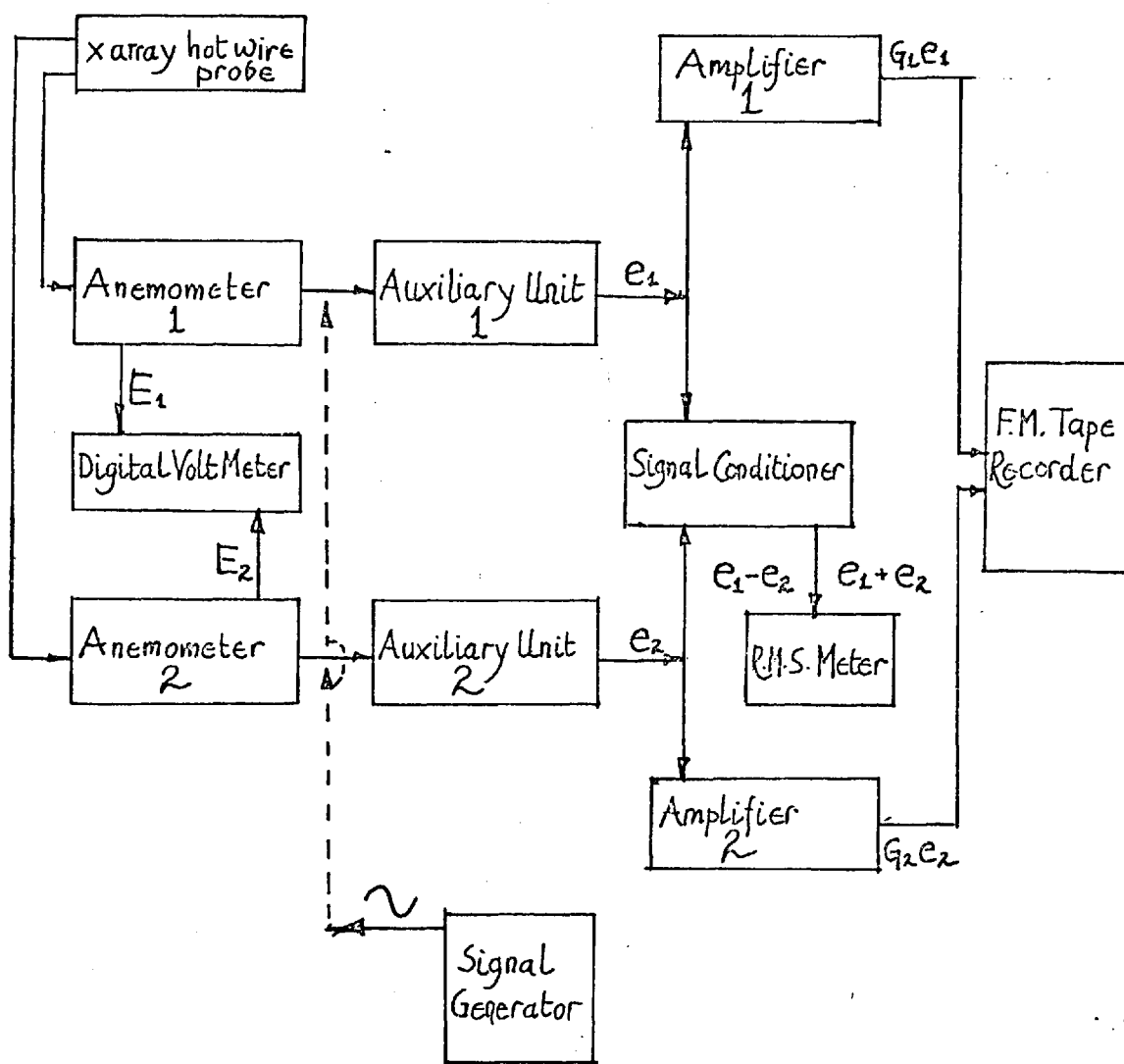


FIGURE [3-2.1] DEVELOPMENT OF SKIN FRICTION WITH REYNOLDS NUMBER

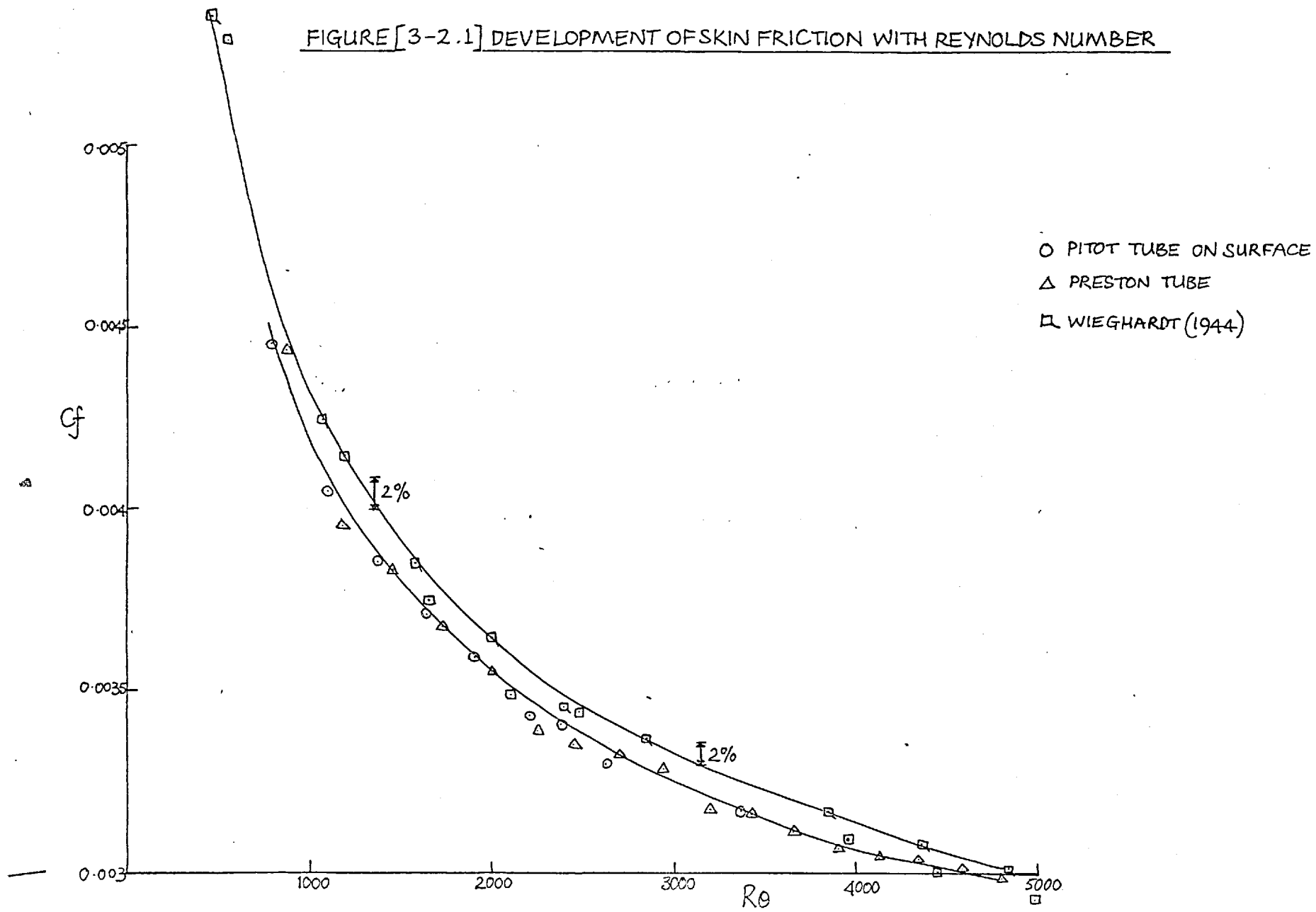
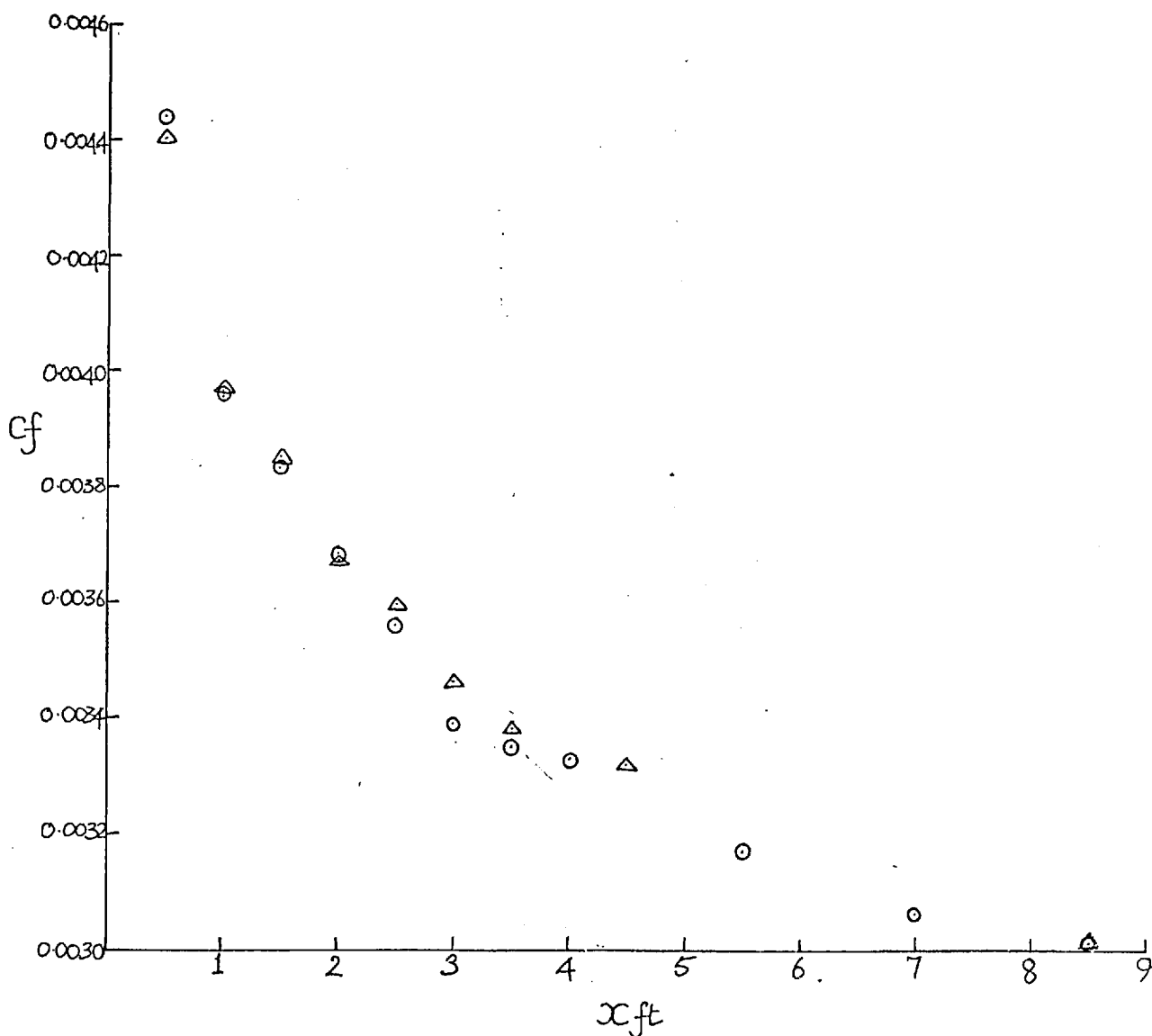
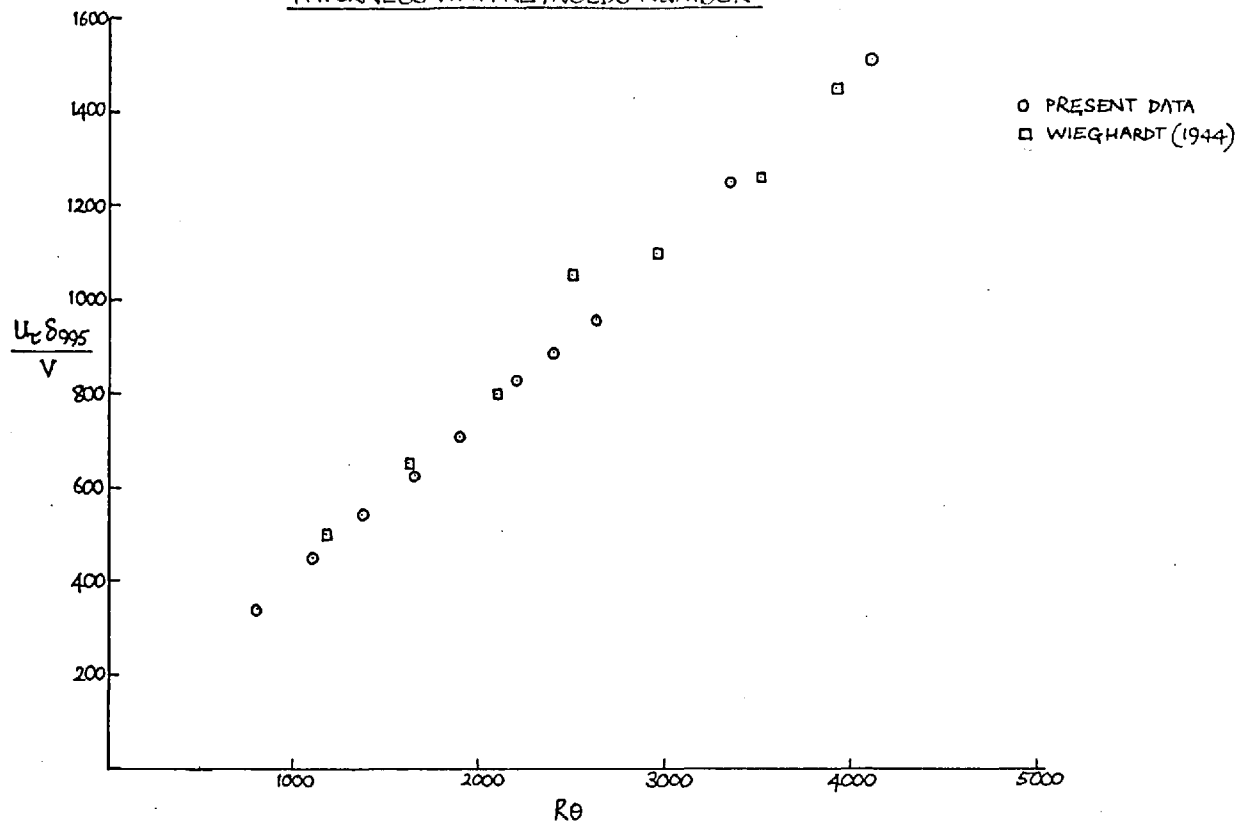


FIGURE [3-2.2] STANTON TUBE AND PRESTON TUBE VALUES
FOR SKIN FRICTION COEFFICIENT

○ PRESTON TUBE 0.113" DIA.
△ STANTON TUBE No. 2.



FIGURE[3-3.1] GROWTH OF NON-DIMENSIONAL BOUNDARY LAYER THICKNESS WITH REYNOLDS NUMBER



FIGURE[3-3.2] GROWTH OF MOMENTUM AND DISPLACEMENT THICKNESSES WITH DISTANCE DOWNSTREAM

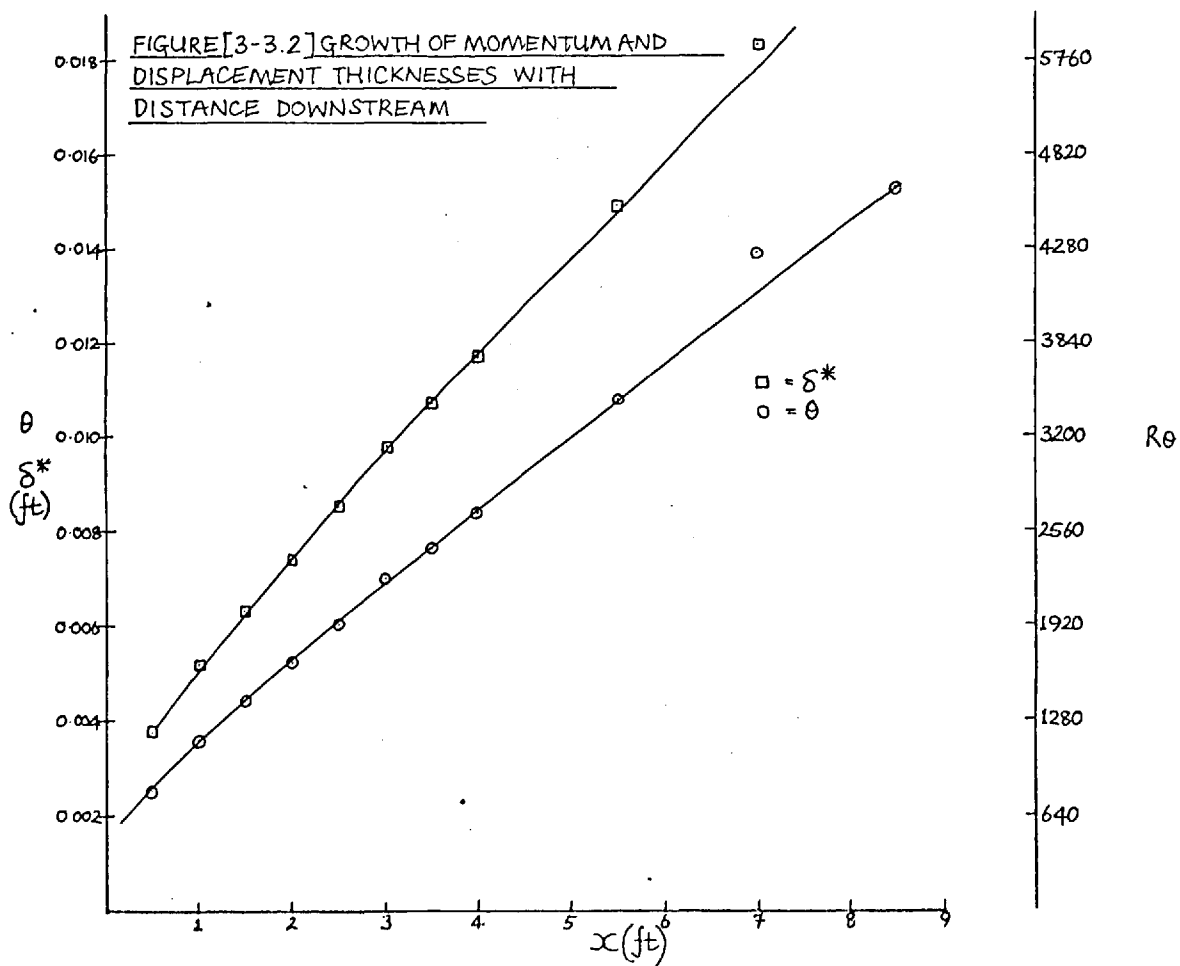


FIGURE [3-3.3] MOMENTUM BALANCE

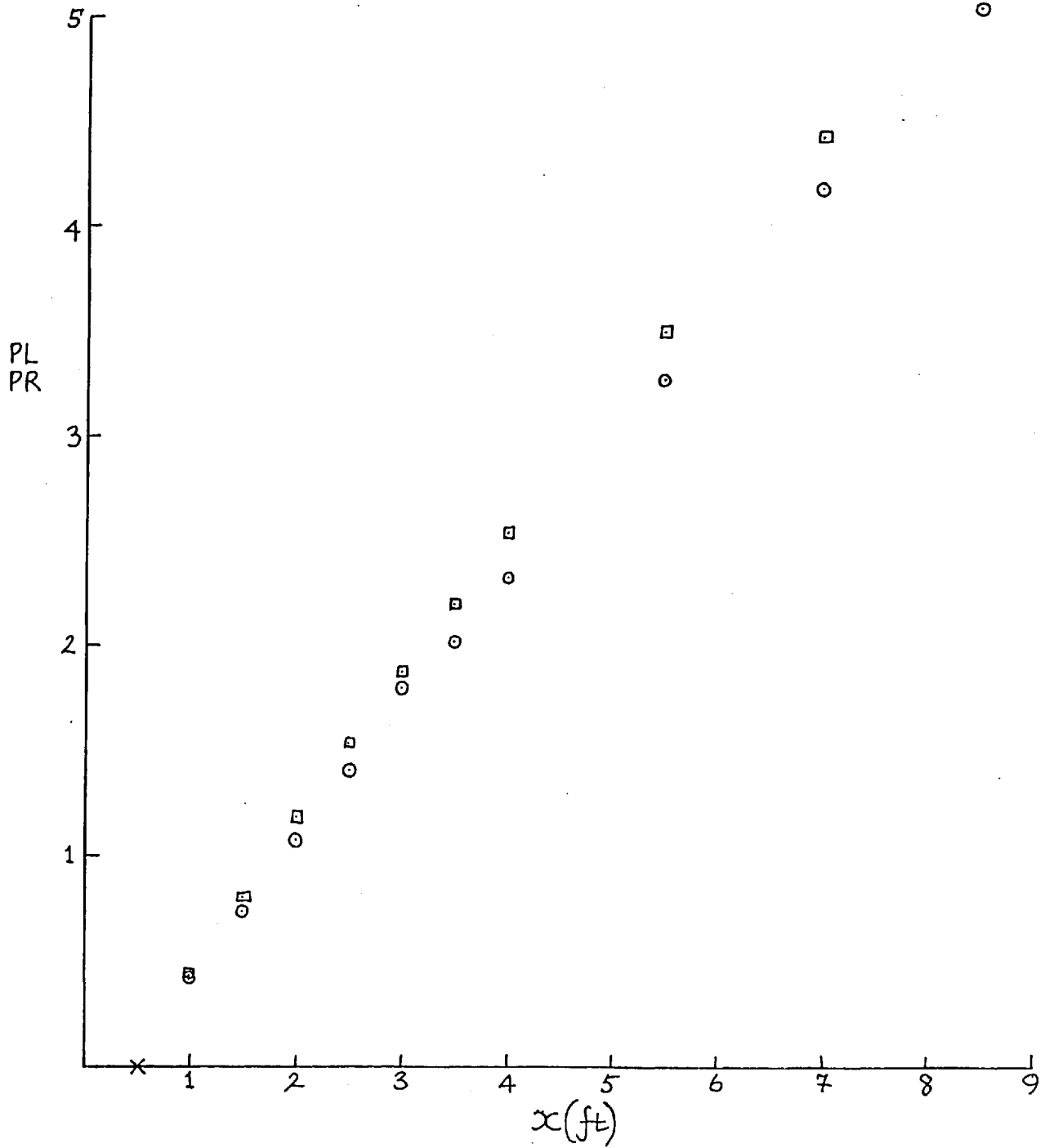
$$PL = \frac{\theta - \theta_1}{\theta_1}$$

$$PR = \frac{1}{\theta_1} \int \frac{C_f}{2} dx$$

x - ORIGIN

o - PL

□ - PR



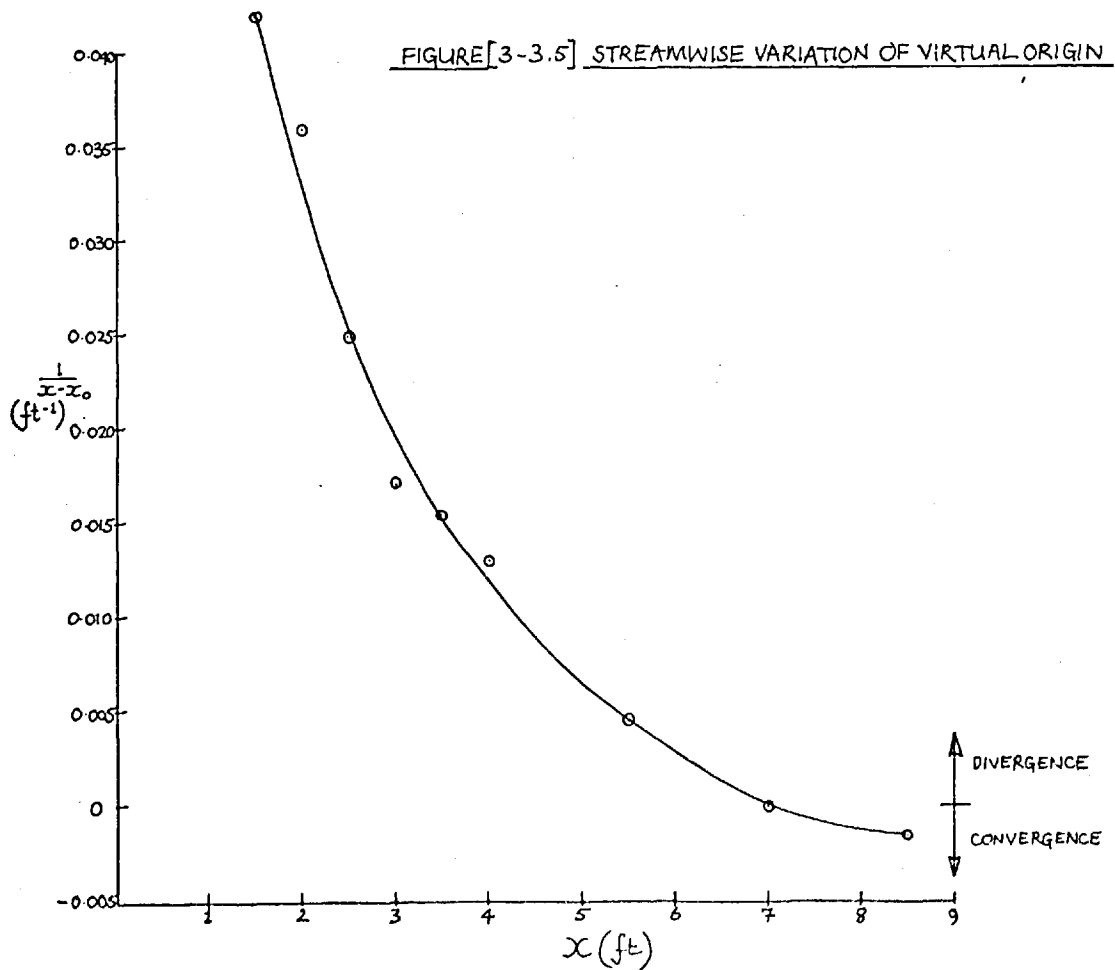
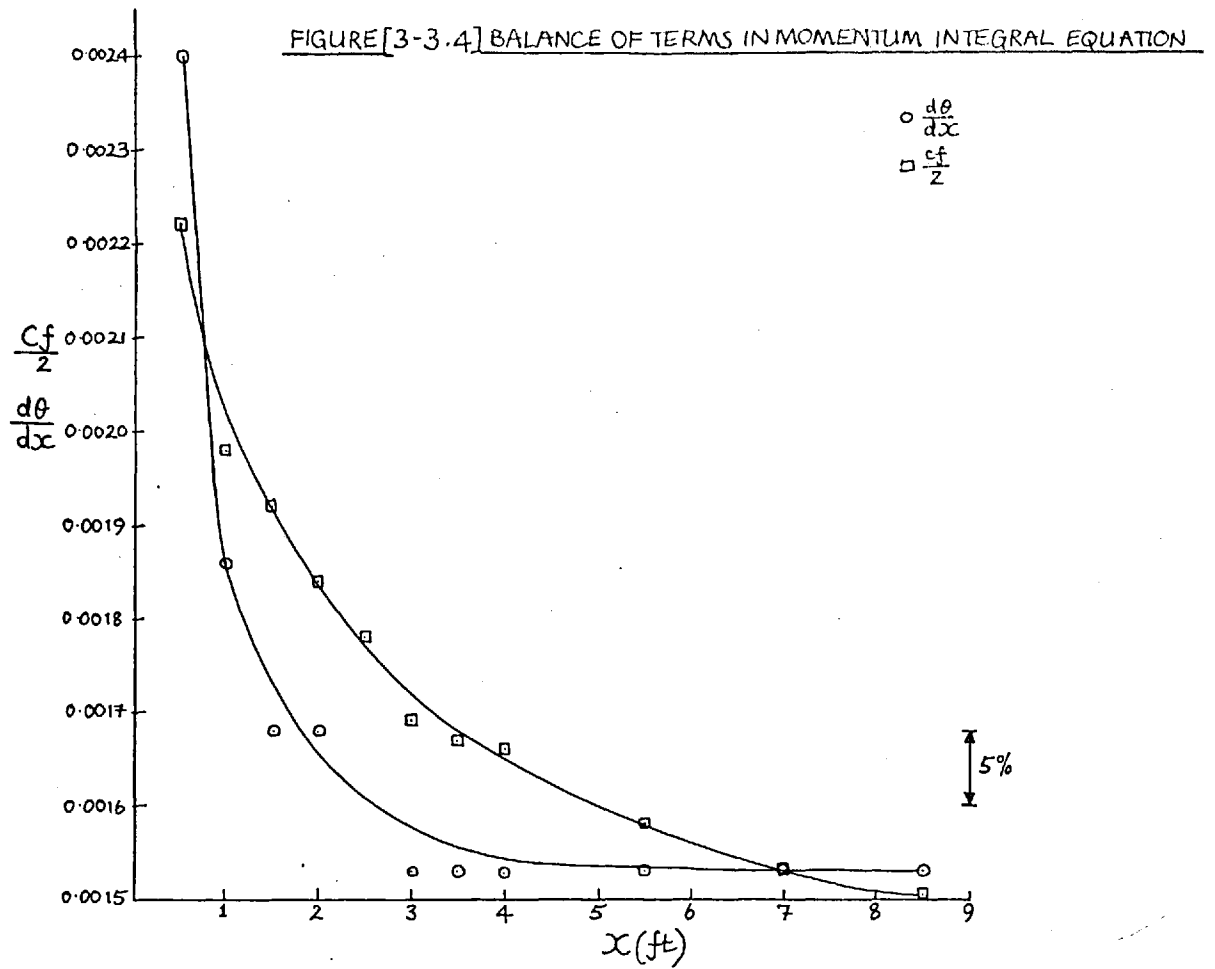


FIGURE [3-4.1] DEVELOPMENT OF SHAPE FACTOR, H , WITH REYNOLDS NUMBER

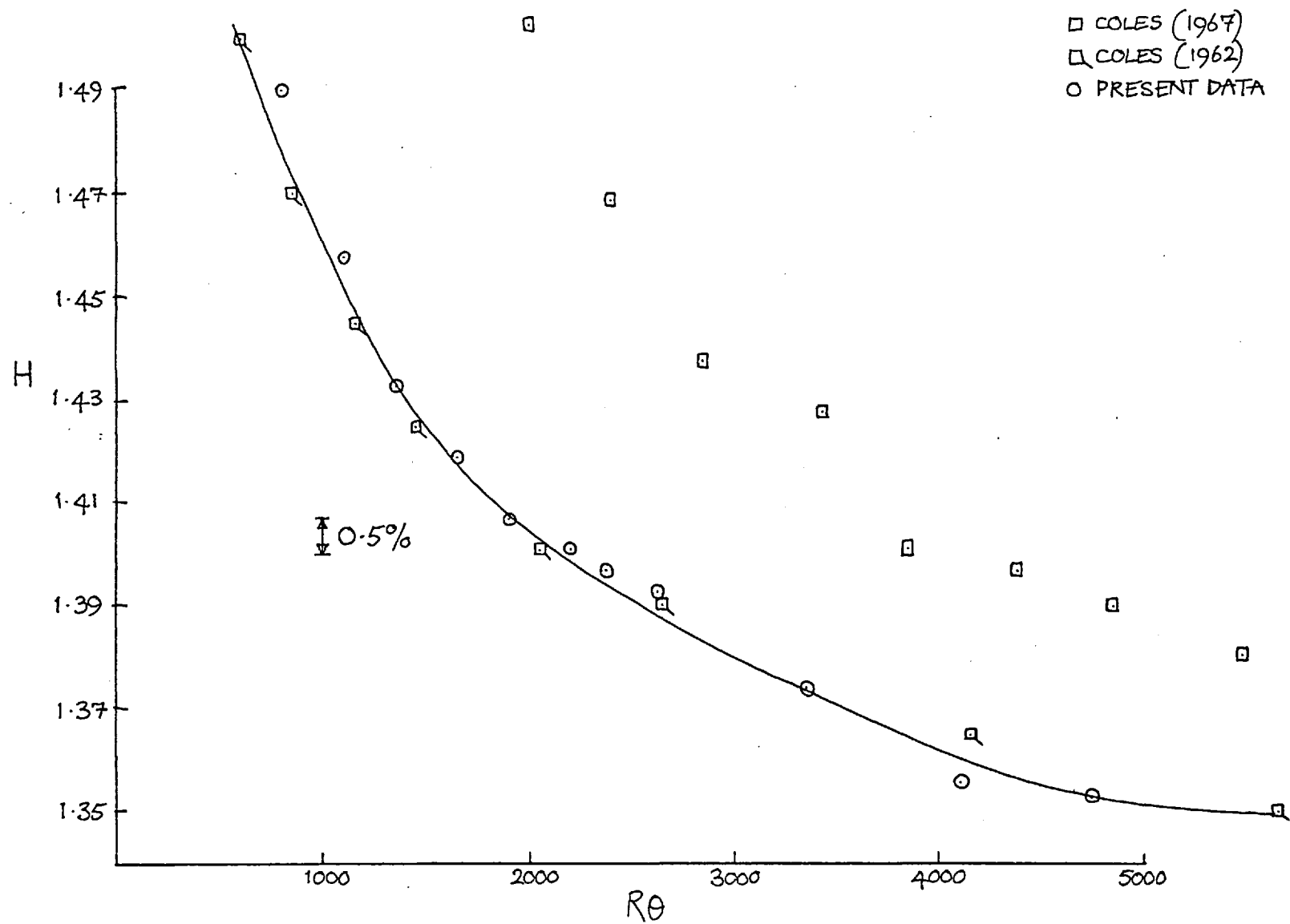
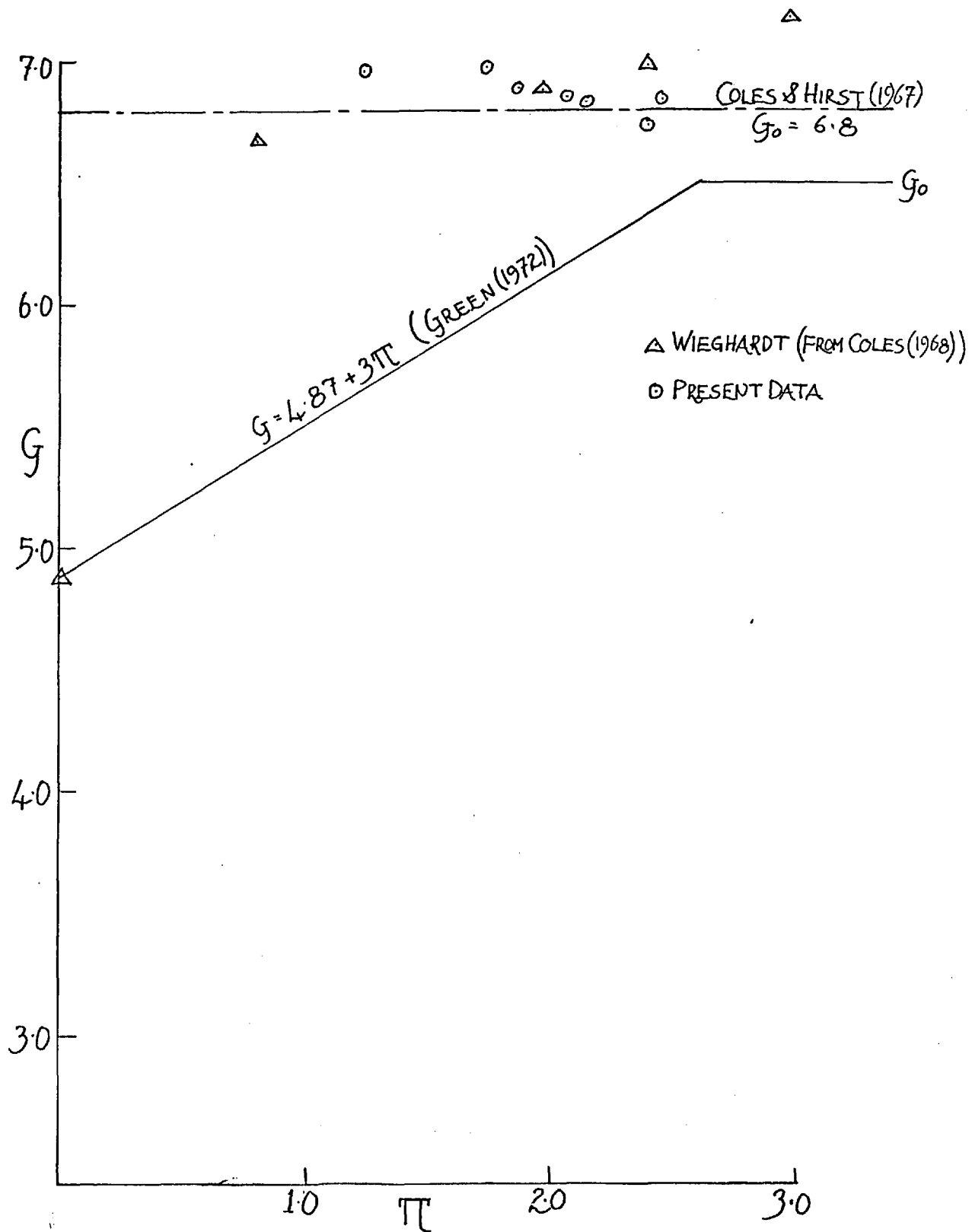
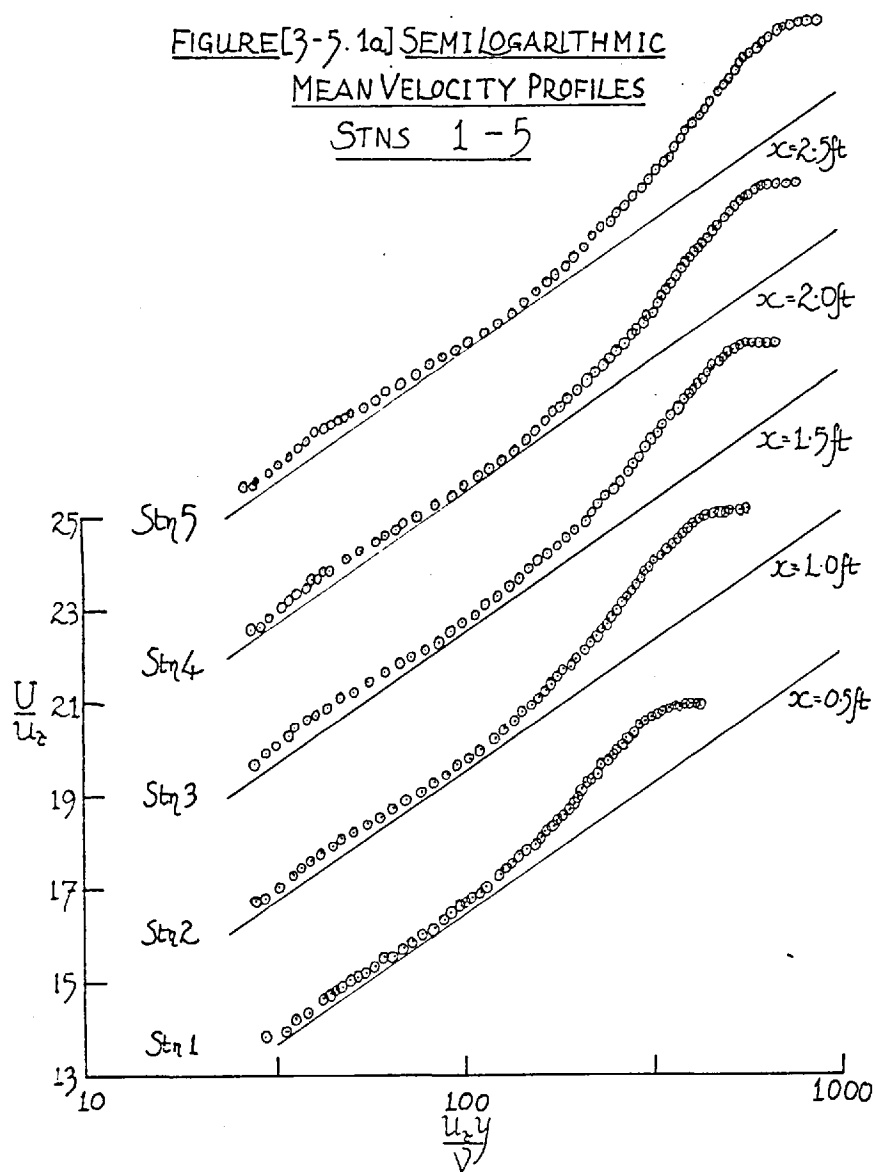


FIGURE [3-4.2] CLAUSER EQUILIBRIUM PARAMETER.



FIGURE[3-5.1a] SEMI LOGARITHMIC
MEAN VELOCITY PROFILES
STNS 1-5



FIGURE[3-5.1b] SEMI LOGARITHMIC
MEAN VELOCITY PROFILES
STNS. 6-17

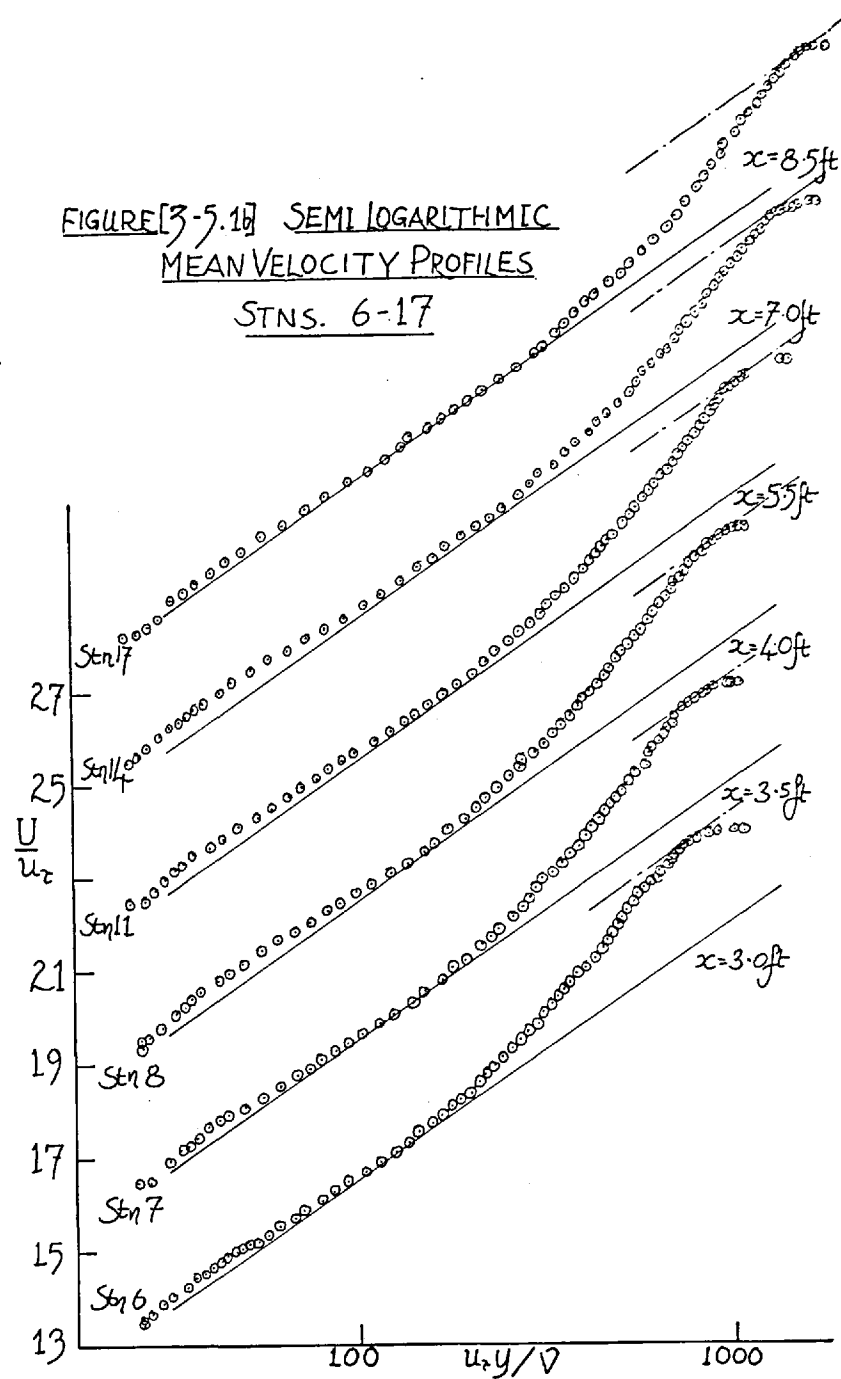


FIGURE [3-5.2] GROWTH OF STRENGTH OF WAKE COMPONENT, $\frac{\Delta u}{u_\tau}$ WITH REYNOLDS NUMBER

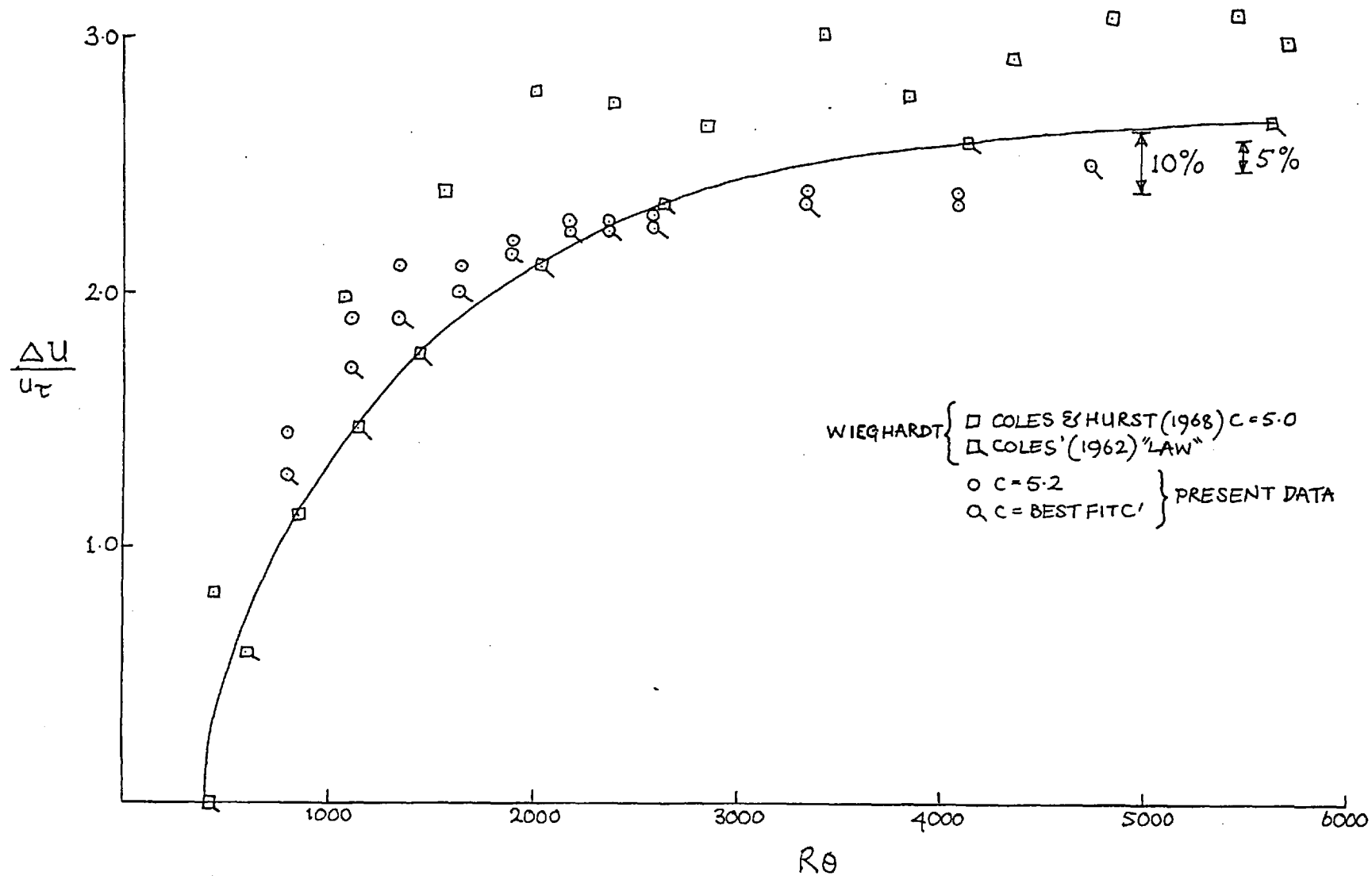
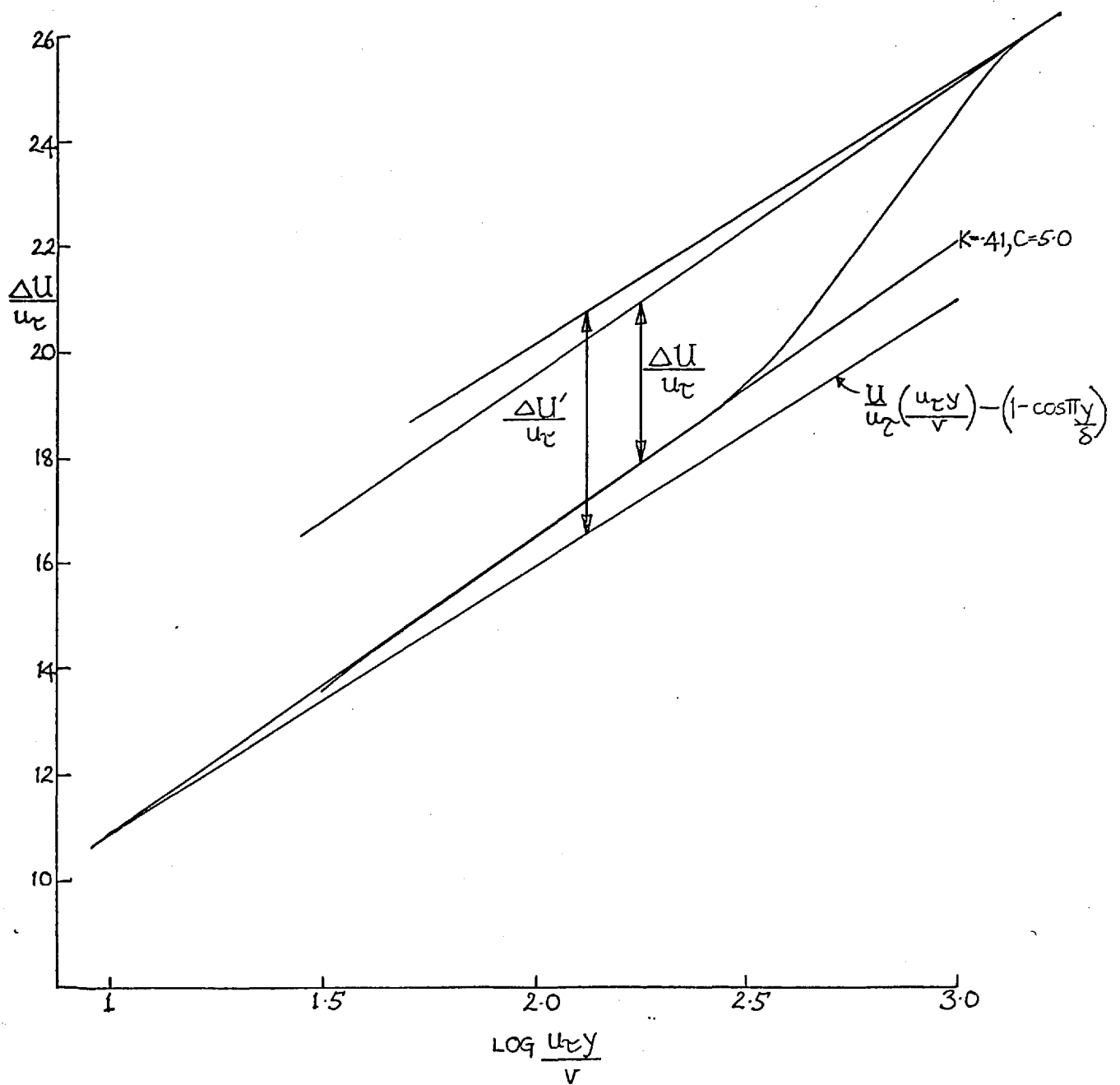
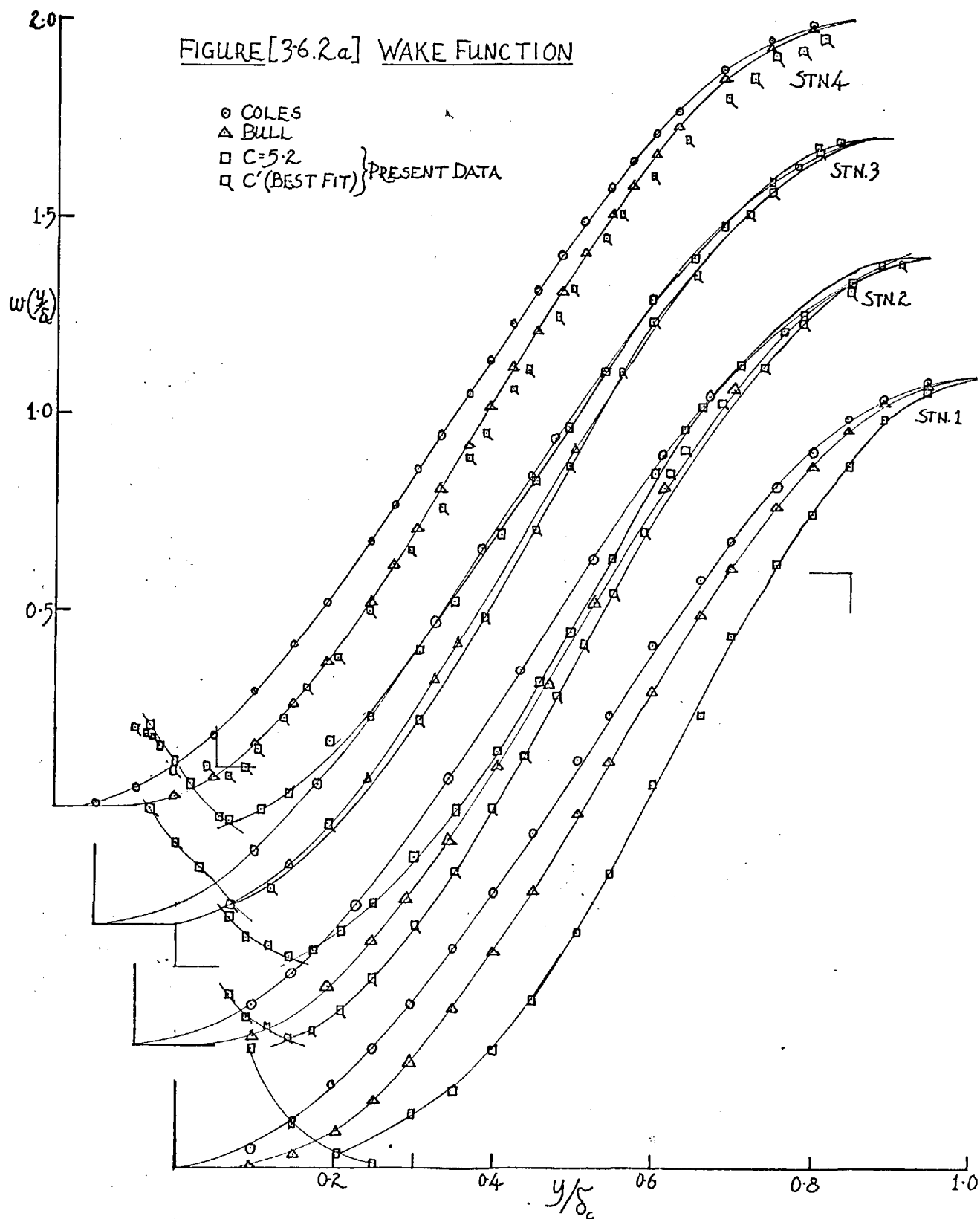
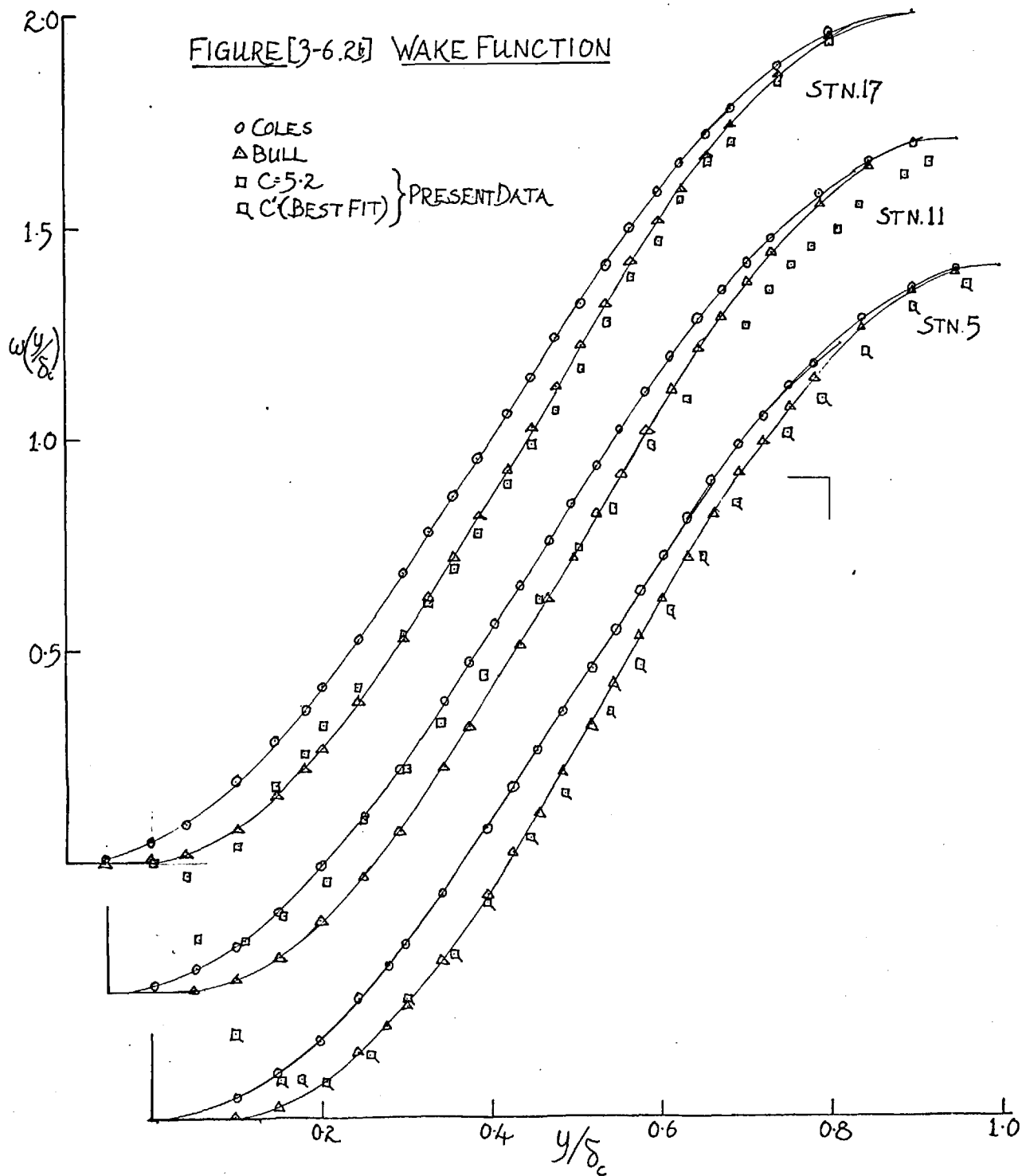


FIGURE [3-5.3]







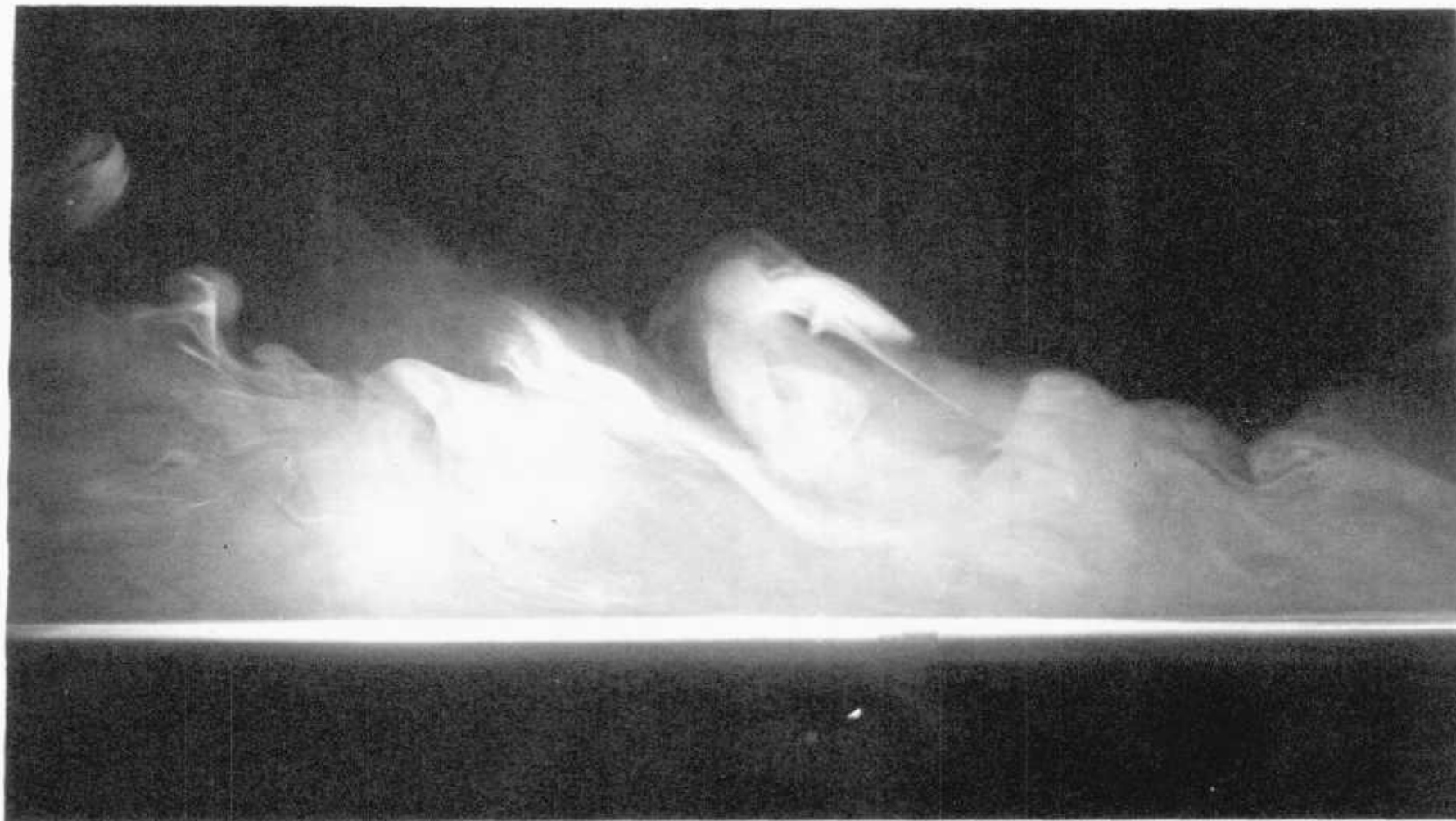


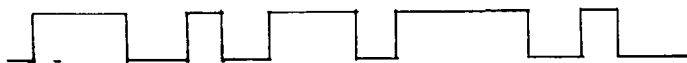
FIG 4-11 SMOKE - FILLED BOUNDARY LAYER, FLOW RIGHT TO LEFT. THE REGULARITY IS UNTYPICAL - OTHER PHOTOGRAPHS SHOW MORE HIGHLY CONTORTED BOUNDARIES.

FIGURE[4-2.1] TYPES OF INTERMITTENCY FUNCTION

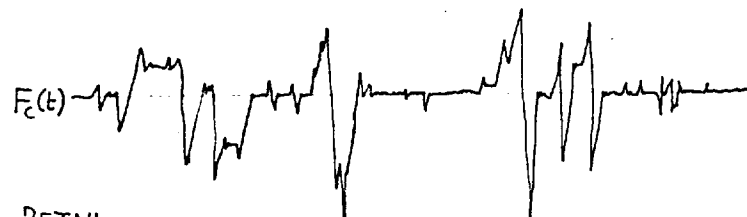
1 - PULSE TRAIN



2 - BOX CAR (SMOOTHED)



FIGURE[4-2.2] CRITERION PARAMETER
AND
INTERMITTENCY FUNCTIONS



RETAIL



WHOLESALE



FIGURE [4-4.1] CURVE OF INTERMITTENCY, γ , AGAINST
THRESHOLD, SC 14. AT STN. 17 POSTN. 5.
X COMPUTED γ (SC 14).

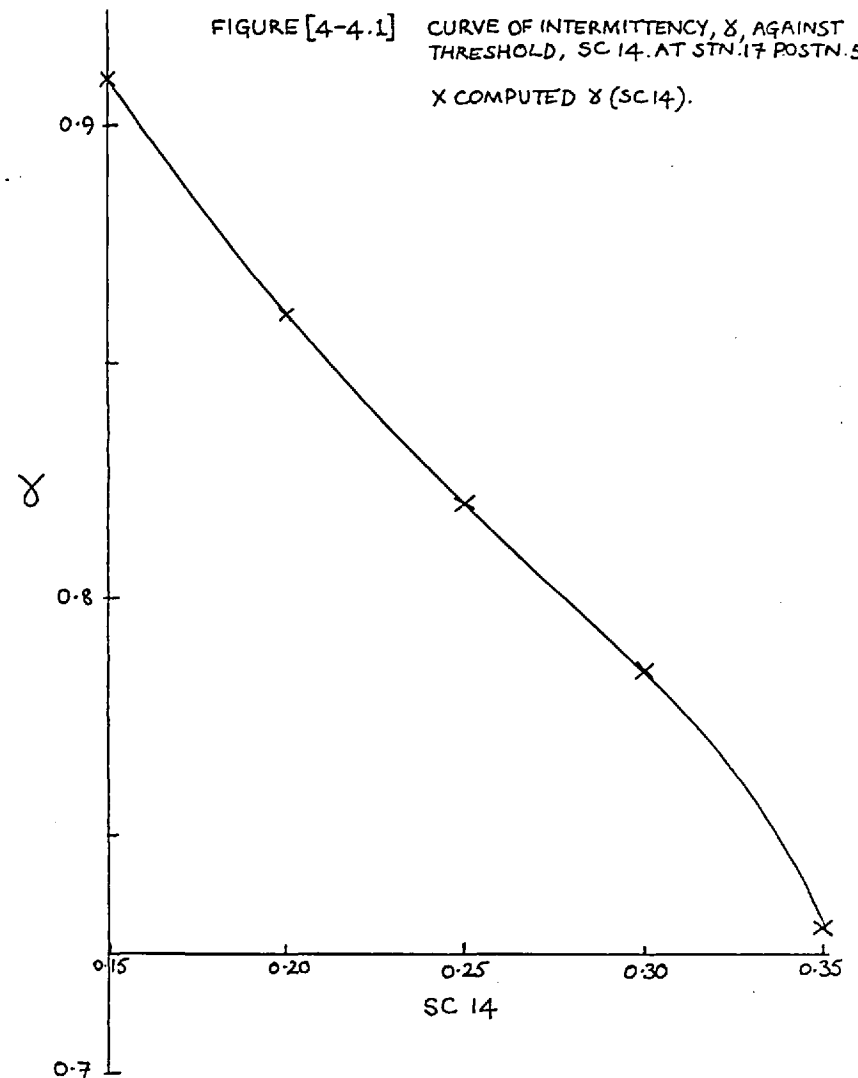


FIGURE [4-4.2a] INTERMITTENCY FACTOR FOR THRESHOLD LEVELS

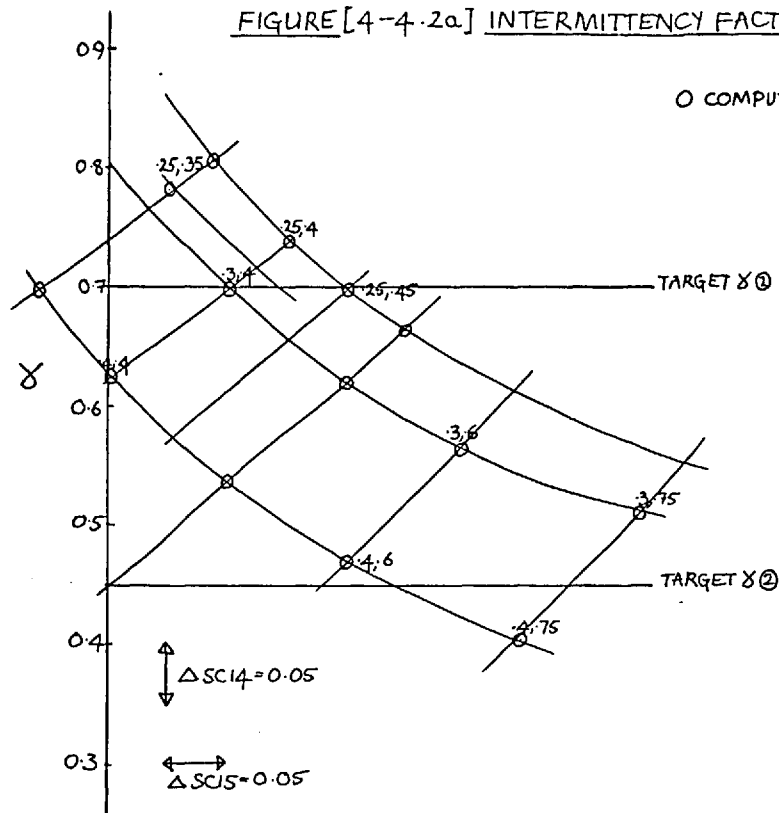
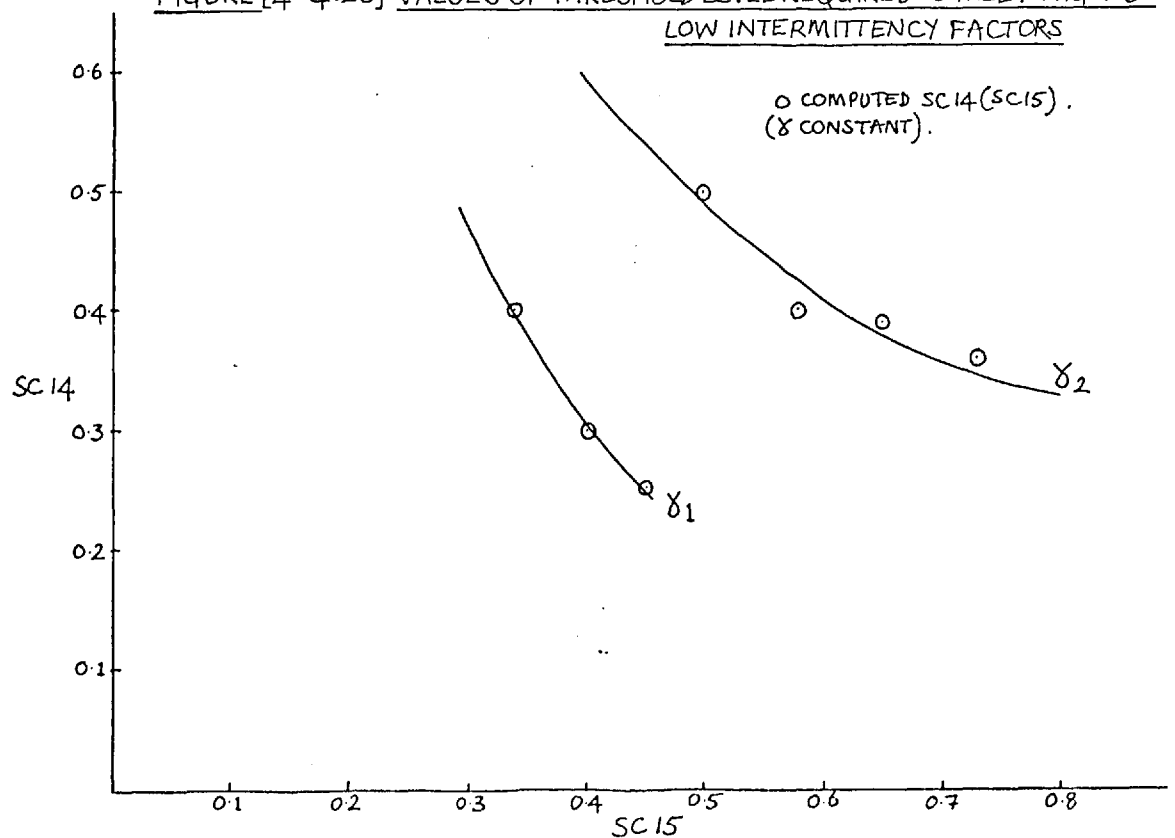
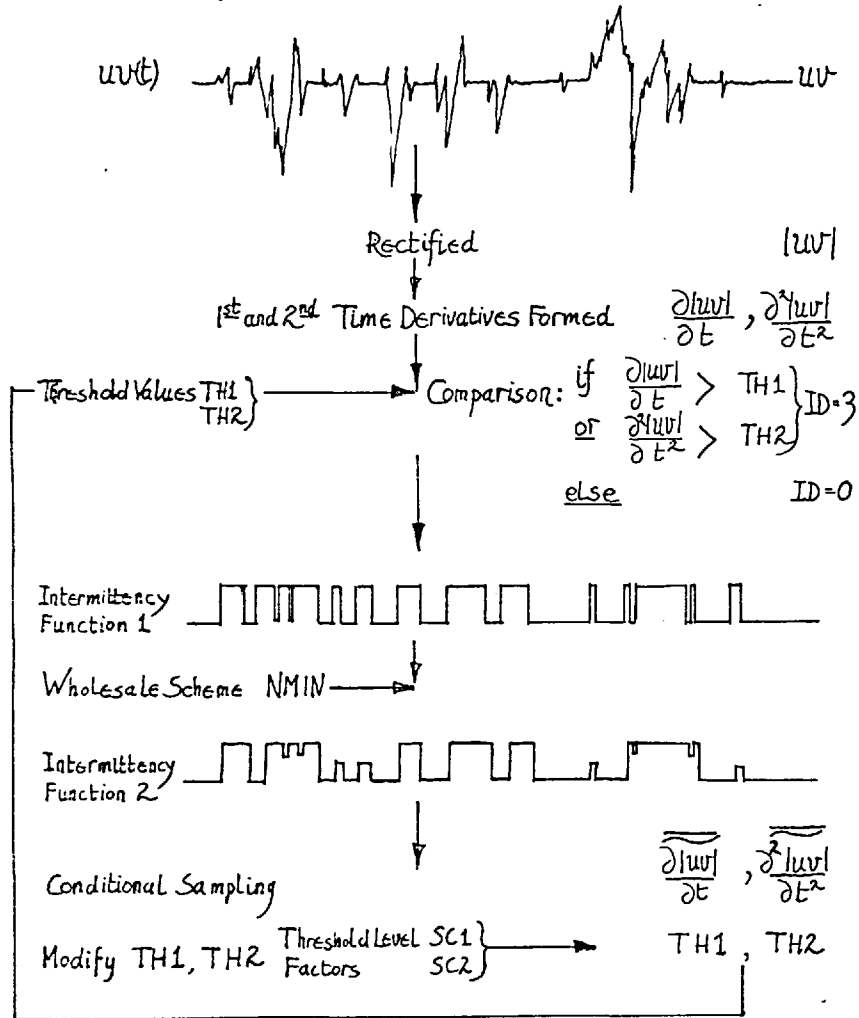


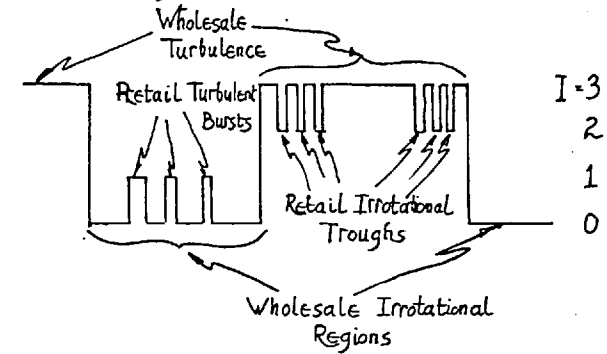
FIGURE [4-4.26] VALUES OF THRESHOLD LEVEL REQUIRED TO MEET HIGH & LOW INTERMITTENCY FACTORS



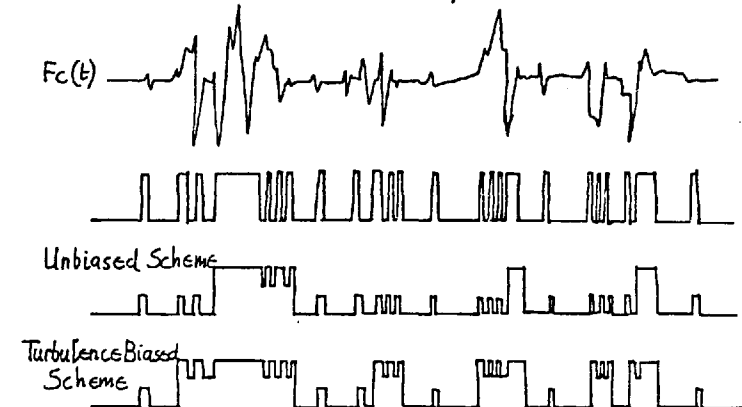
FIGURE[4-5.1] SUBROUTINE "THRESH"



FIGURE[4-5.2] INTERMITTENCY NOTATION

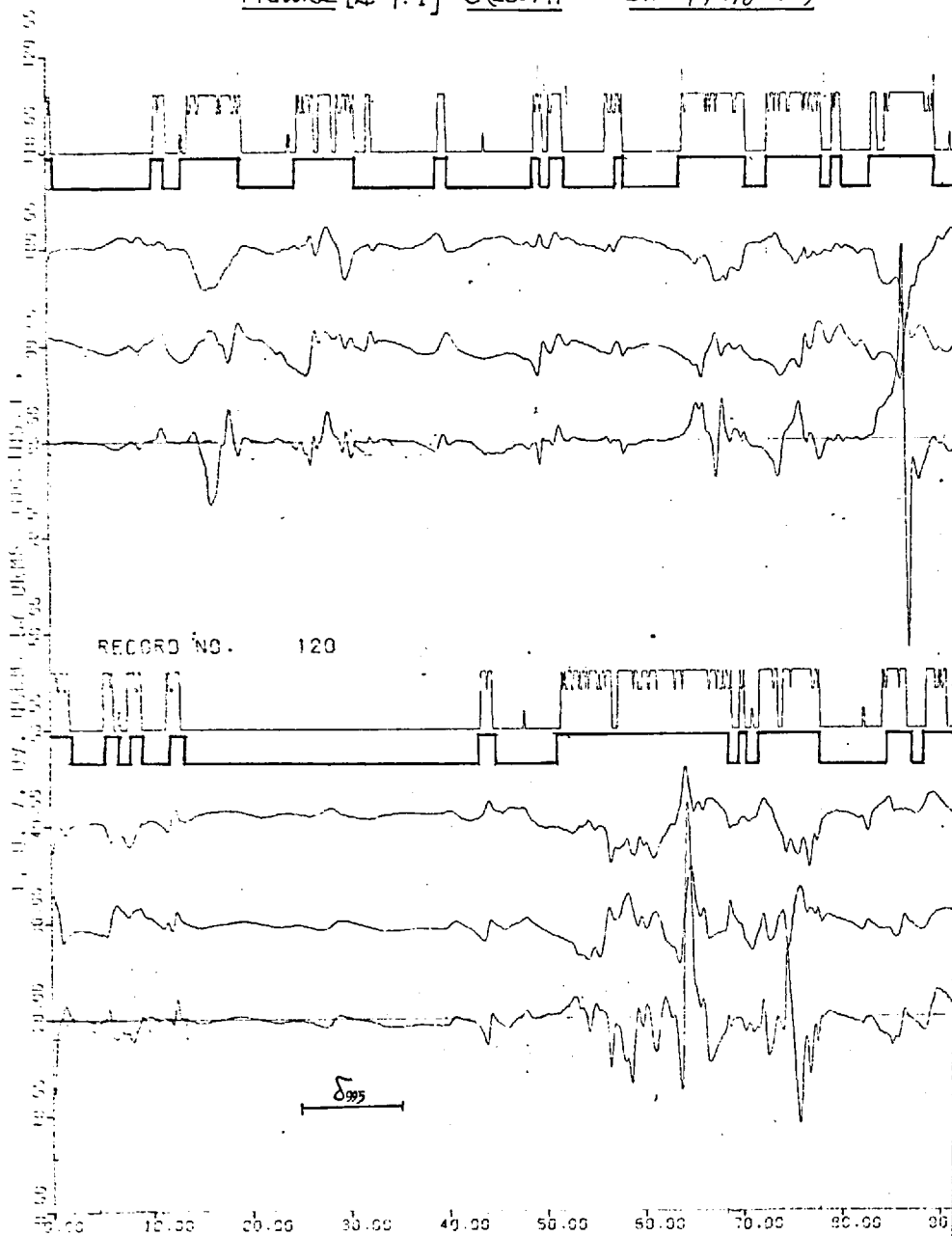


FIGURE[4-5.3] WHOLESALE/RETAIL SCHEMES



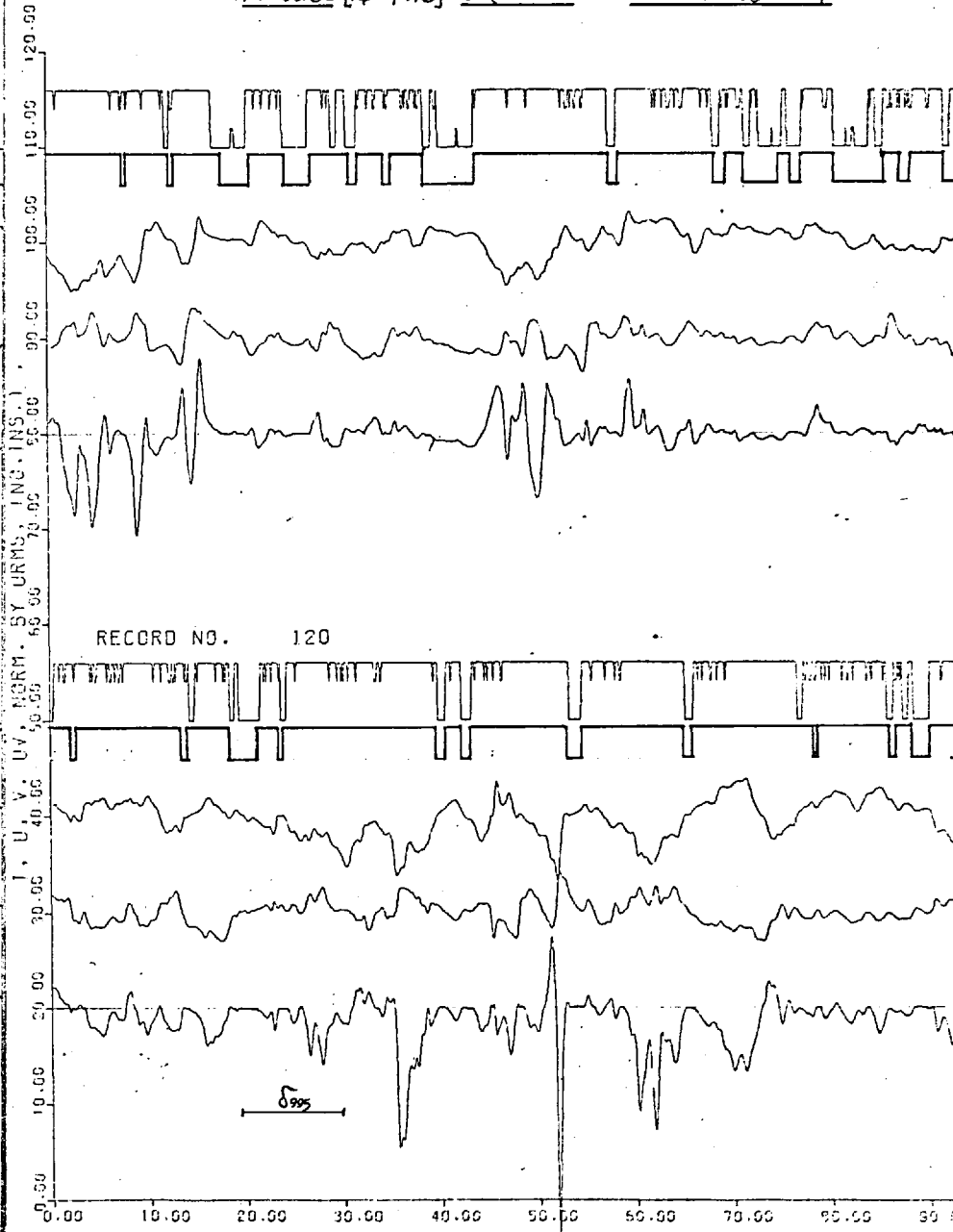
-RESULTS- IDAC RMSD CODE- 2359 0.0295 17090

FIGURE [4-7.1] CALCOMP STN17, $4/8 = 0.9$



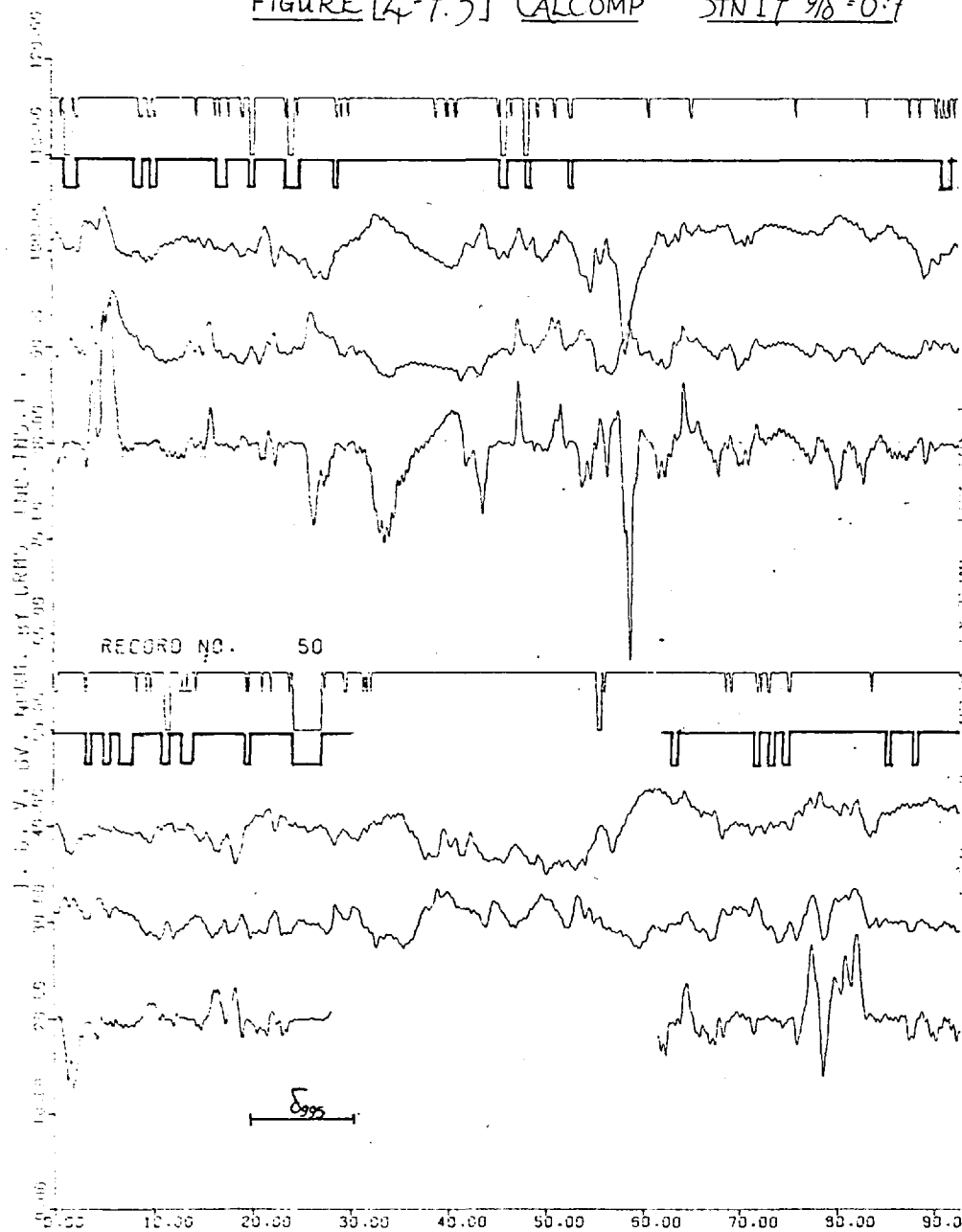
-RESULTS- IDAC RMSU CODE- 2359 0.0462 17070

FIGURE [4-7.2] CALCOMP STN. 17 $\frac{y}{8} = 0.7$



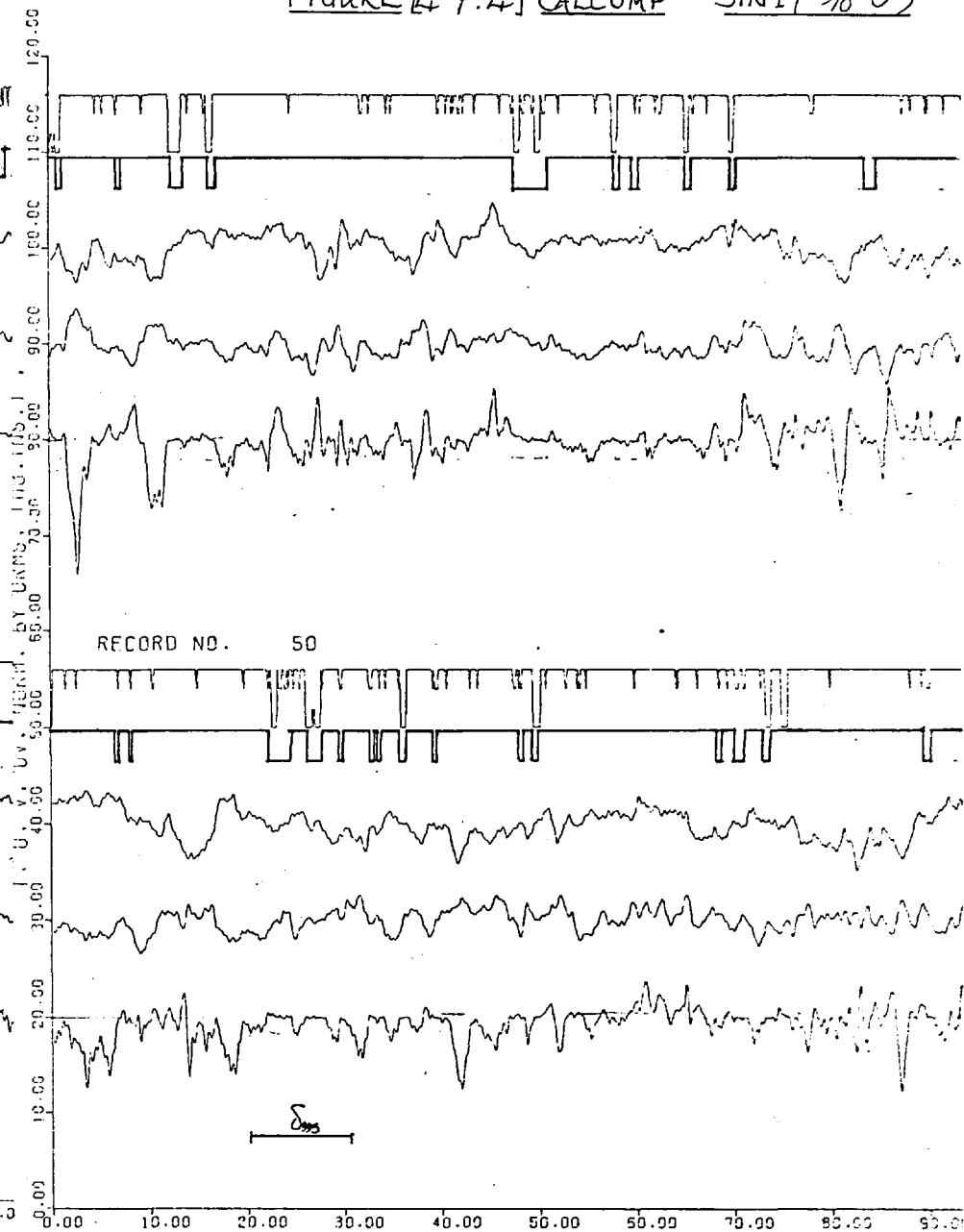
-RESULTS- IDAC RMSU CODE- 2359 0.0438 17070

FIGURE [4-7.3] CALCOMP STN17 $\frac{y}{\delta} = 0.7$



-RESULTS- IDAC RMSU CODE- 2359 0.0543 17050

FIGURE [4-7.4] CALCOMP STN17 $\frac{y}{\delta} = 0.5$



-RESULTS- IDFC RMSU CODE- 2359 0.0912 17010

FIGURE [4-7.5] CALCOMP STN17 $\frac{1}{8}$ -0.1

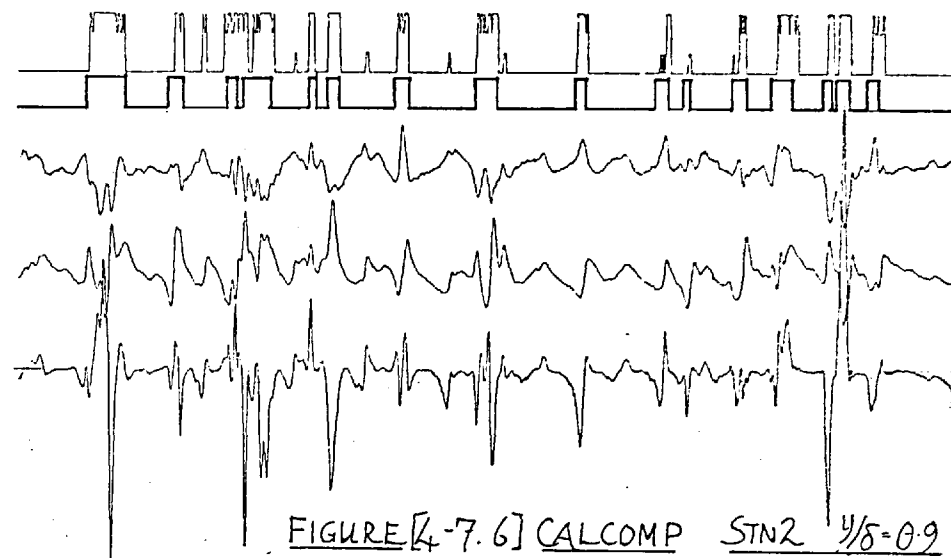
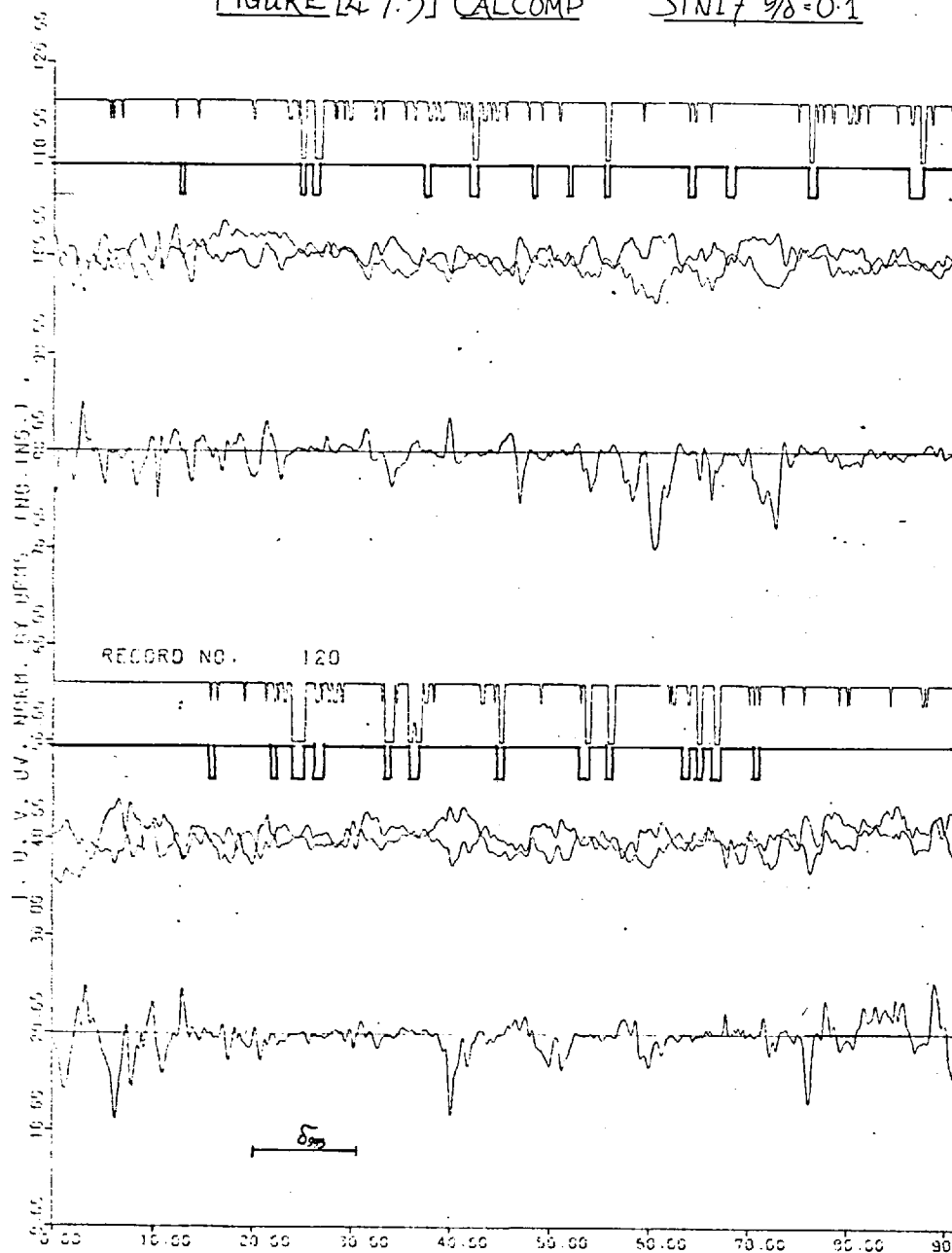
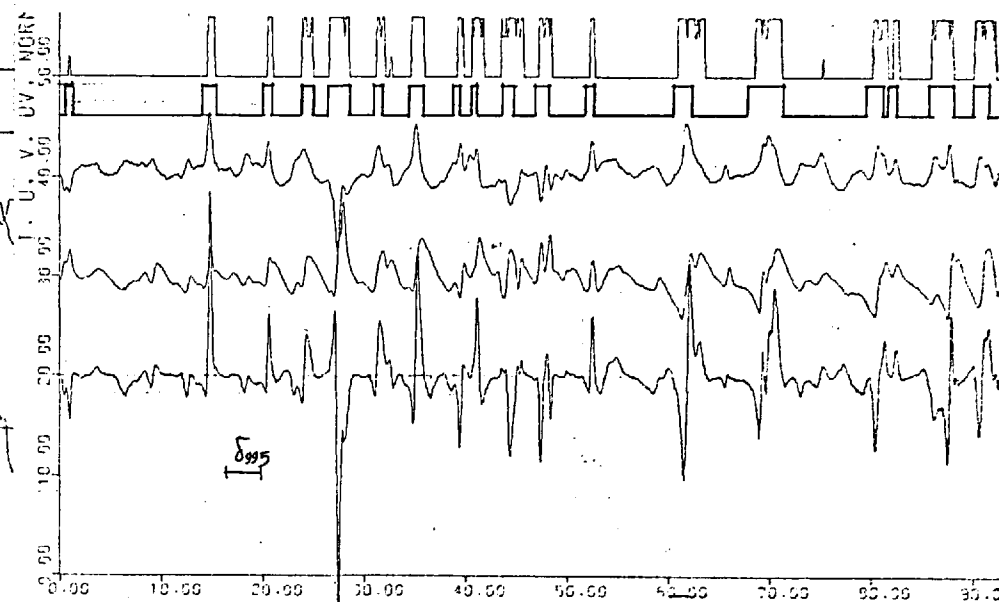


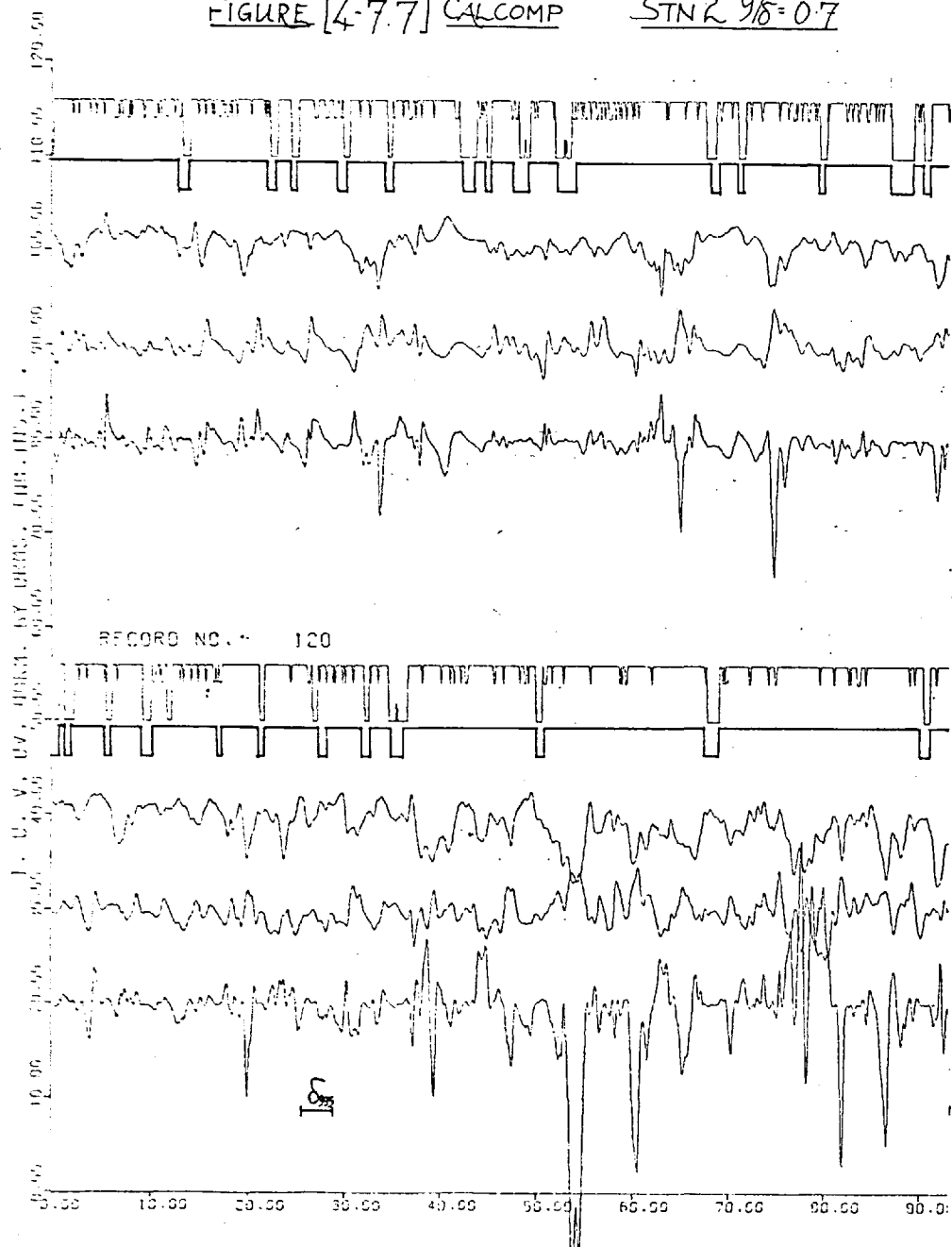
FIGURE [4-7.6] CALCOMP STN2 $\frac{1}{8}$ -0.9

-RESULTS- IDFC RMSU CODE- 2715 0.0198 2030



-RESULTS- IDAC RMSU CODE- 2716 0.0420 2070

FIGURE [4-7.7] CALCOMP STN.2 $y/\delta = 0.7$



-RESULTS- IDAC RMSU CODE- 2716 0.0567 2050

FIGURE [4-7.8] CALCOMP STN.2 $y/\delta = 0.5$

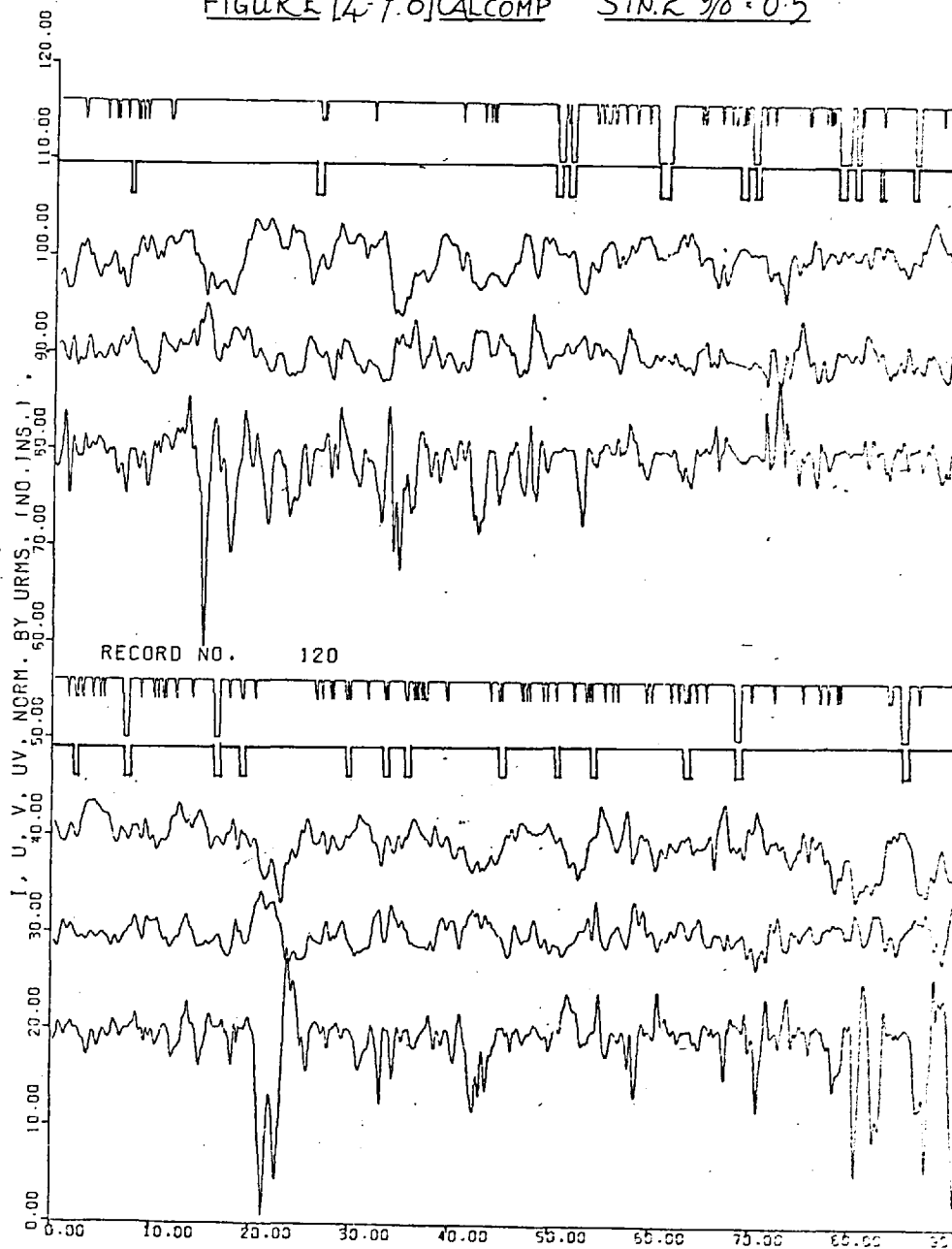


FIGURE [4-7.9] DIGIGRAPHIC

RECORD NO. 8
 $N_{min} = 5$
 $SC1 = 0.5$
 $SC2 = 0.35$

1 = $UV(t)$
 2 = $ID(t)$ BIASED
 3 = $ID(t)$ UNBIASED

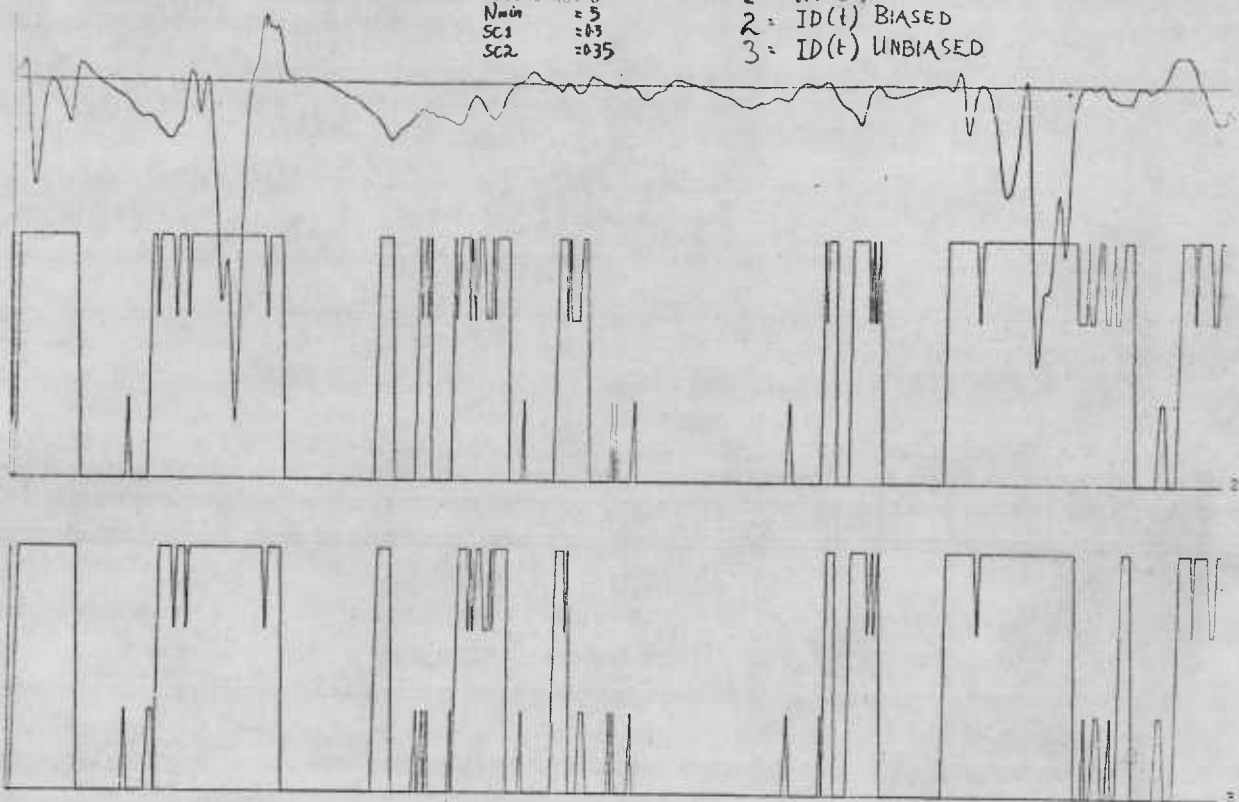


FIGURE [4-7.10] DIGIGRAPHIC

RECORD NO. 8
 $N_{min} = 12$
 $SC1 = 0.5$
 $SC2 = 0.35$

1 = $UV(t)$
 2 = $ID(t)$ BIASED
 3 = $ID(t)$ UNBIASED

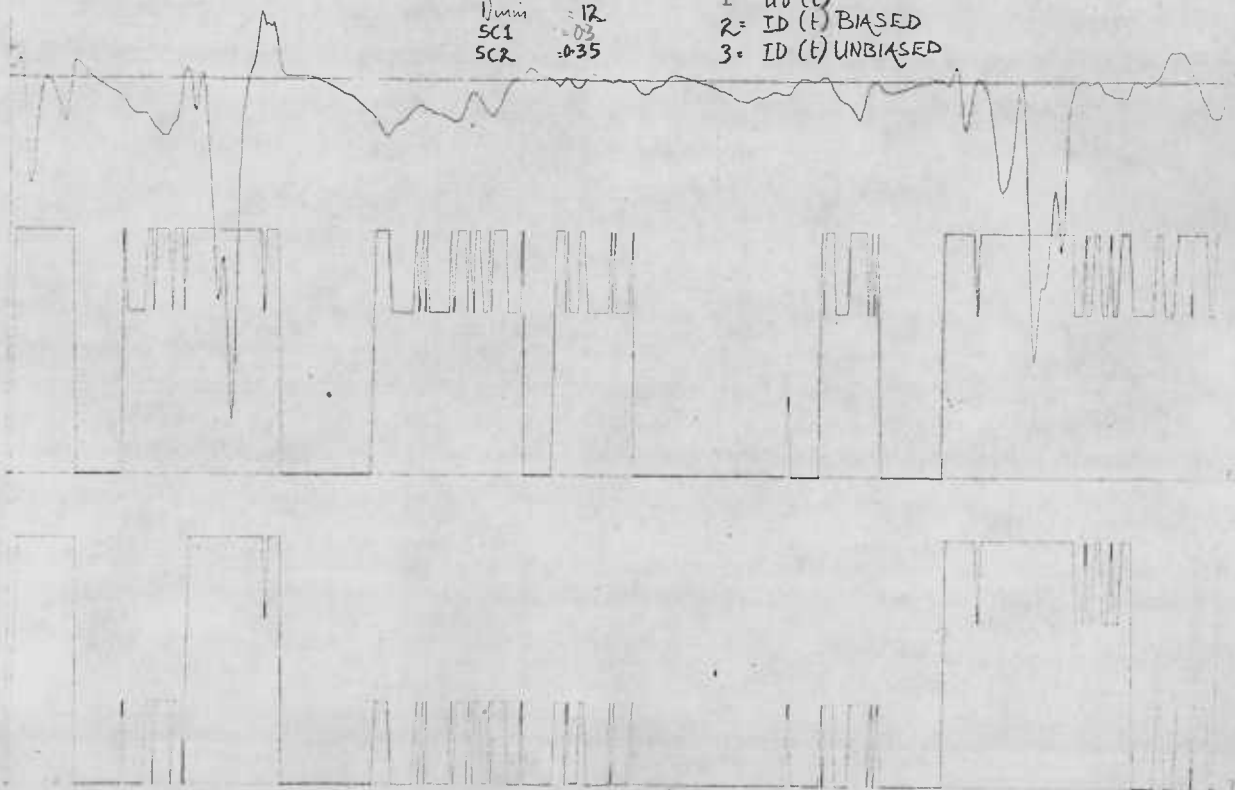


FIGURE [4-7.11] DIGIGRAPHIC

(2)

RECORD NO. 8
 $N_{mth} = 20$
 $SC1 = 0.30$
 $SC2 = 0.35$

1 = $uv(t)$
 2 = $ID(t)$ BIASED
 3 = $ID(t)$ UNBIASED

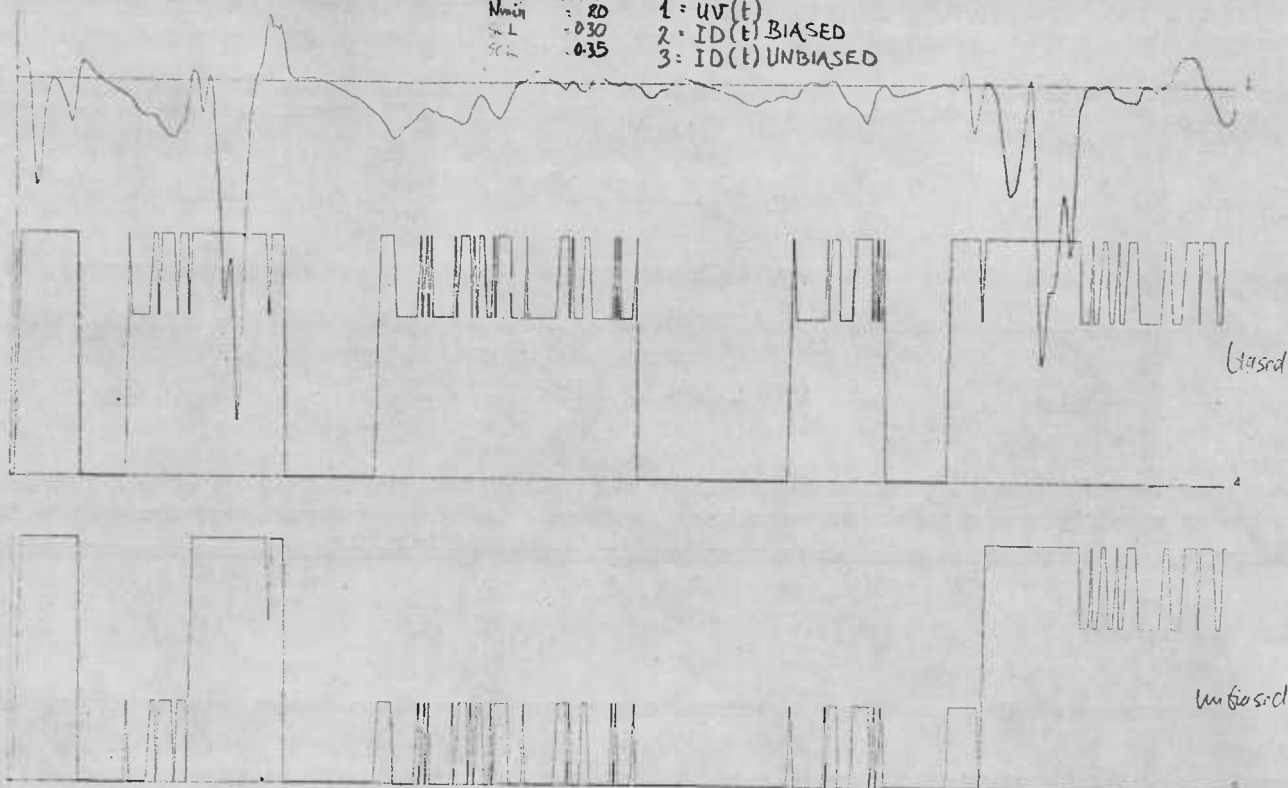


FIGURE [4-7.12] DIGIGRAPHIC

(4)

RECORD NO. 8
 $N_{mth} = 5$
 BIASED

1 = $uv(t)$
 8 = $ID(t)$ $SC1 = 0.25$, $SC2 = 0.32$
 9 = $ID(t)$ $SC1 = 0.20$, $SC2 = 0.28$

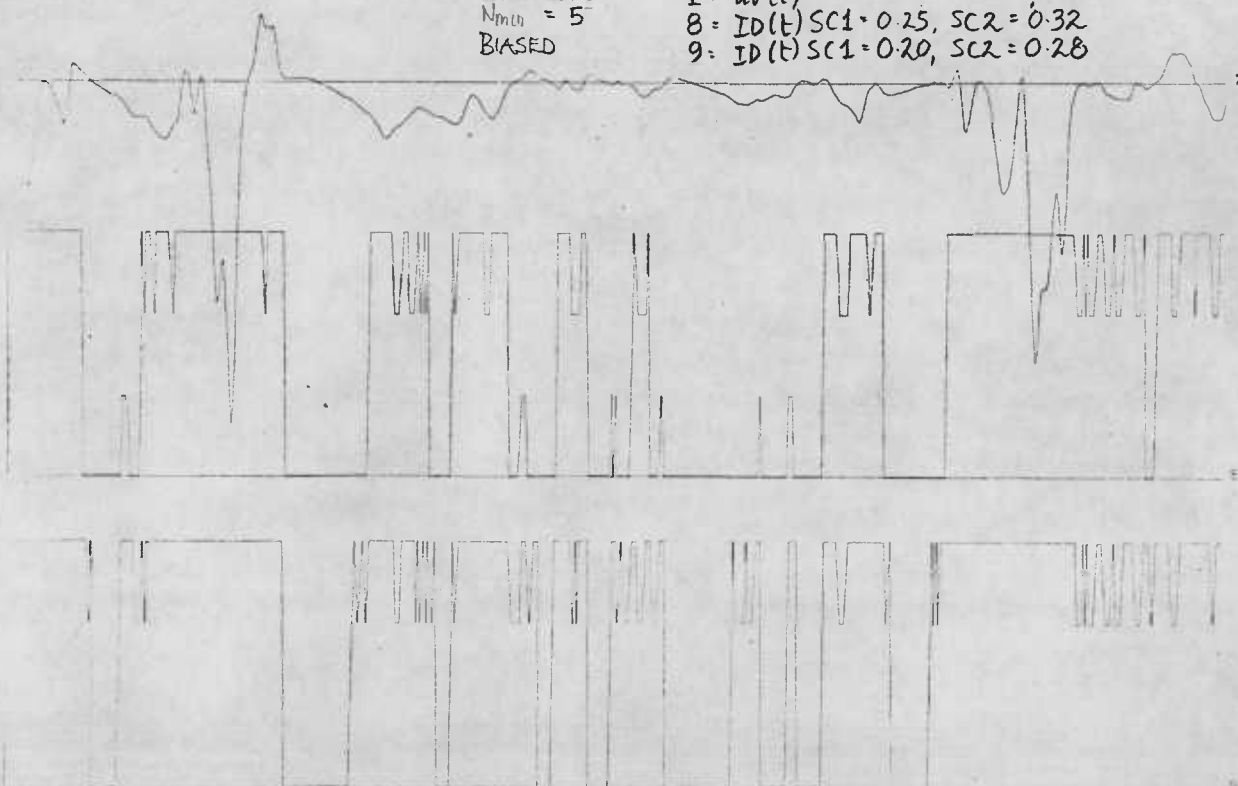


FIGURE [4-7.13] CALCOMP STN 2 4/8=0.9

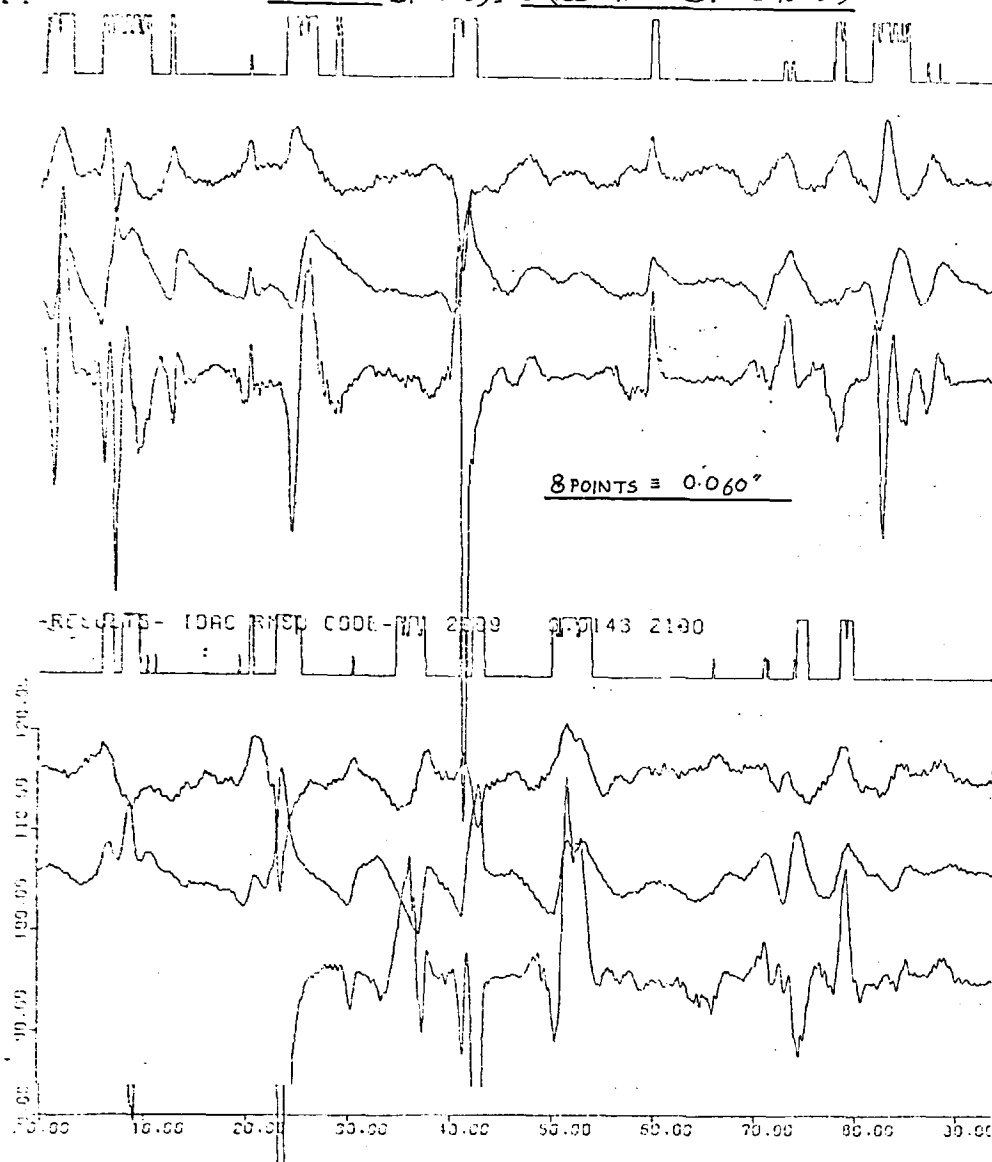
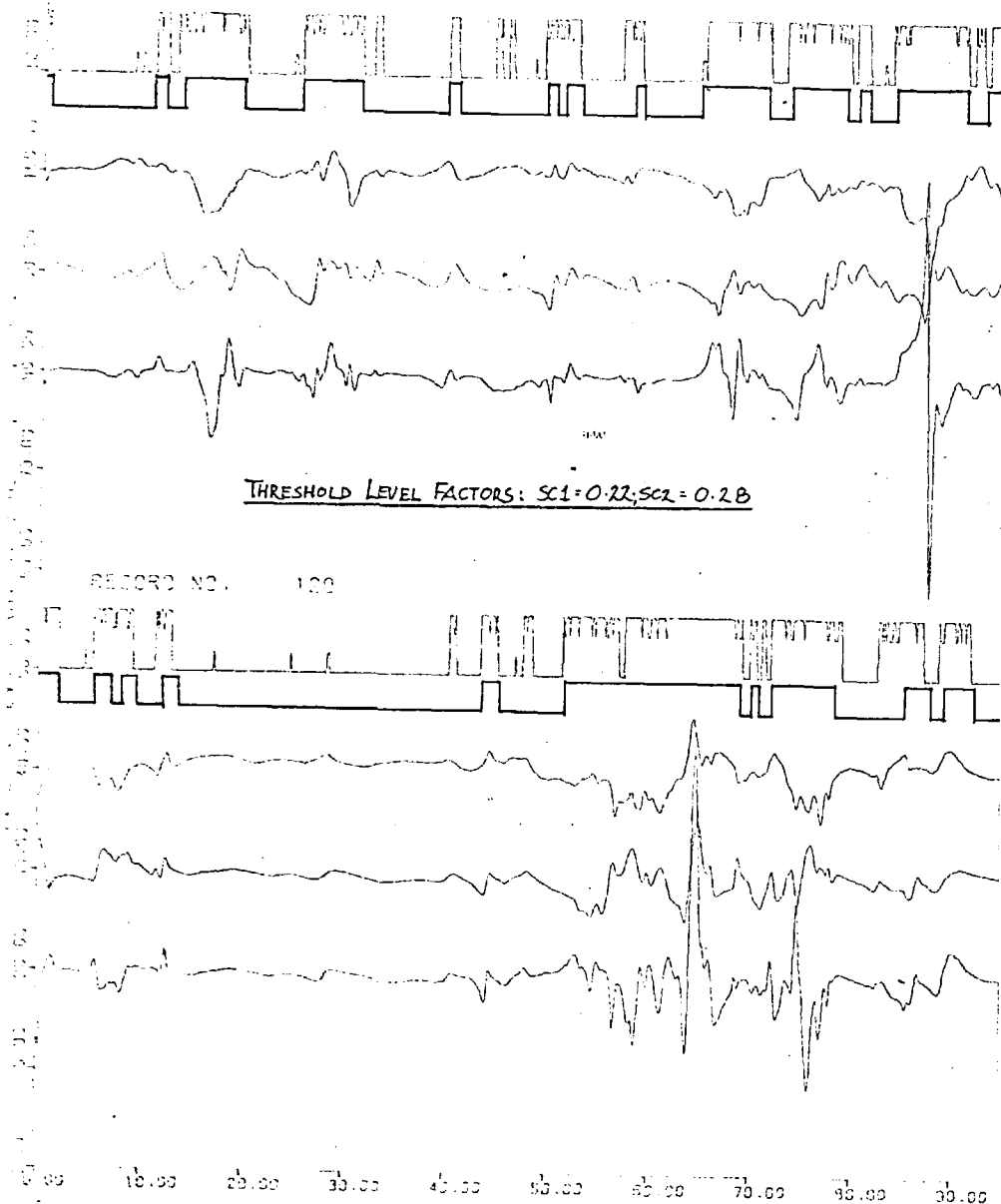


FIGURE [4-7.14] CALCOMP STN 2 4/8=0.9



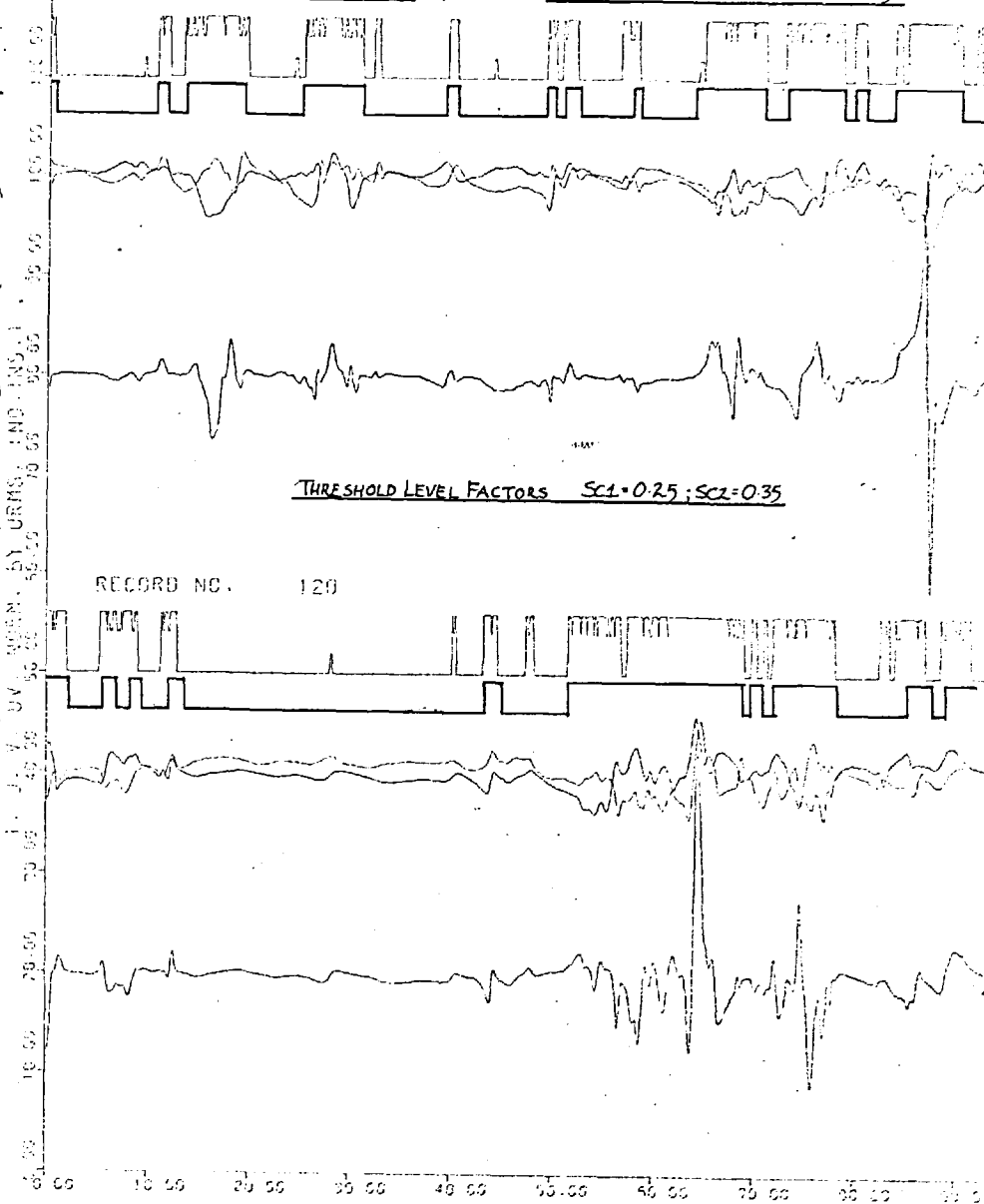
007 0115-1440 8MSD 1001- 1300 010206 17000
M1. 2. 1000-03

FIGURE [4-7.15] CALCOMP STN 17 $\frac{y}{x} = 0.9$



007 0115-1440 8MSD 1001- 1300 010206 17000
M1. 2. 1000-03

FIGURE [4-7.16] CALCOMP STN 17 $\frac{y}{x} = 0.9$



RESULTS- 1040 RMSU CODE- 2050 0 0100 2050
 10 00 20 00 30 00

FIGURE [4-7.17] CALCOMP STN 17 $\frac{y}{b}=0.9$

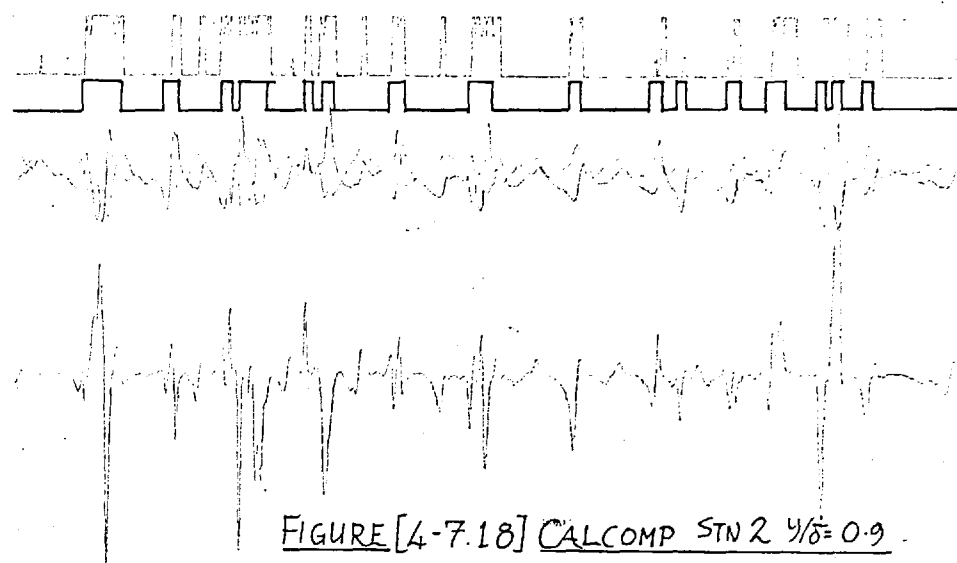
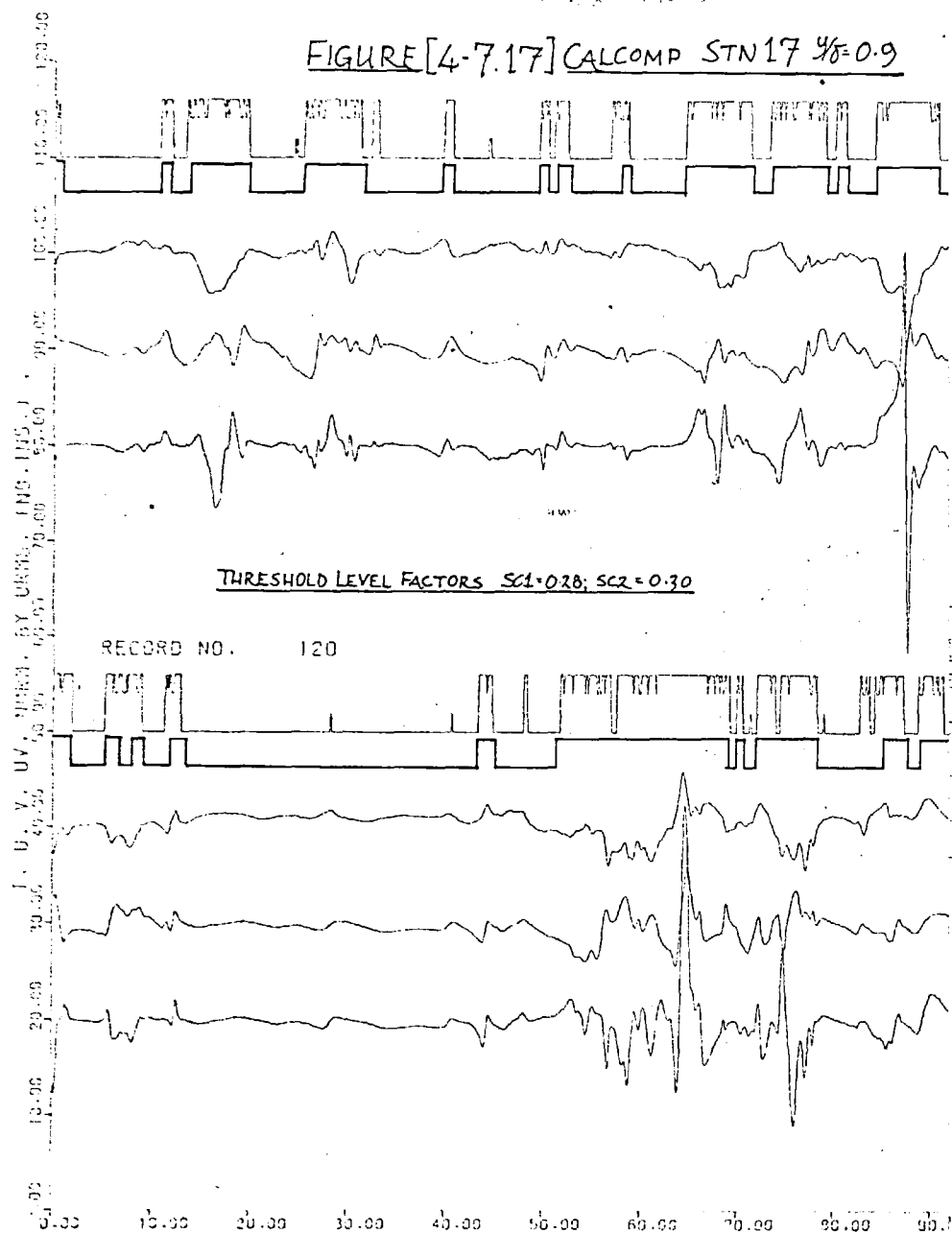


FIGURE [4-7.18] CALCOMP STN 2 $\frac{y}{b}=0.9$

RESULTS- 1040 RMSU CODE- 2716 0 0100 2050 SC1=0.25 SC2=0.32
 RECORD NO 120

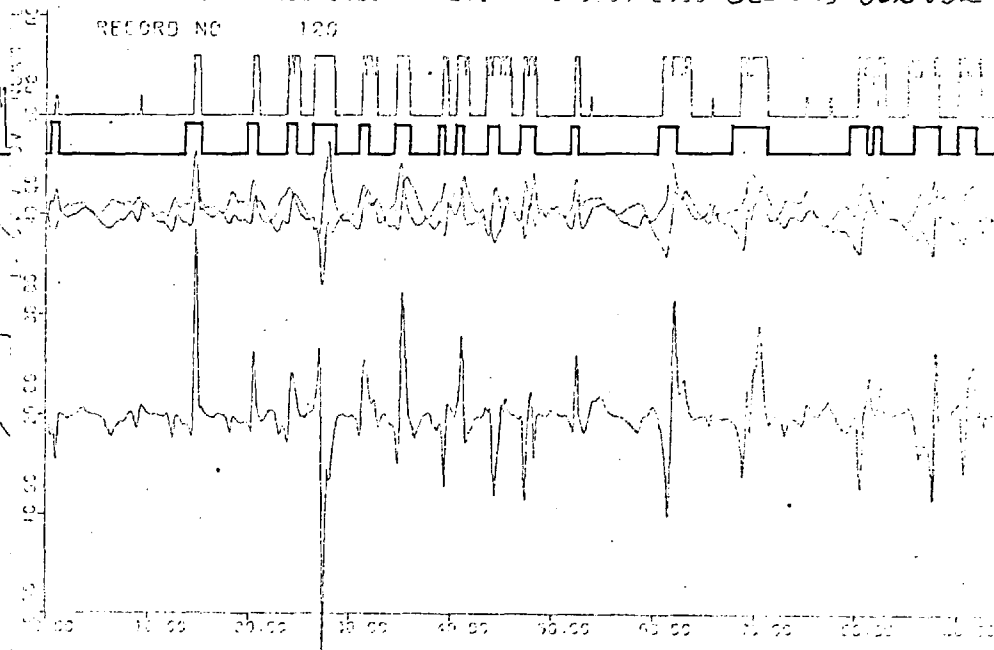


FIGURE [4-7.19] CALCOMP STN2 1/8-0.7

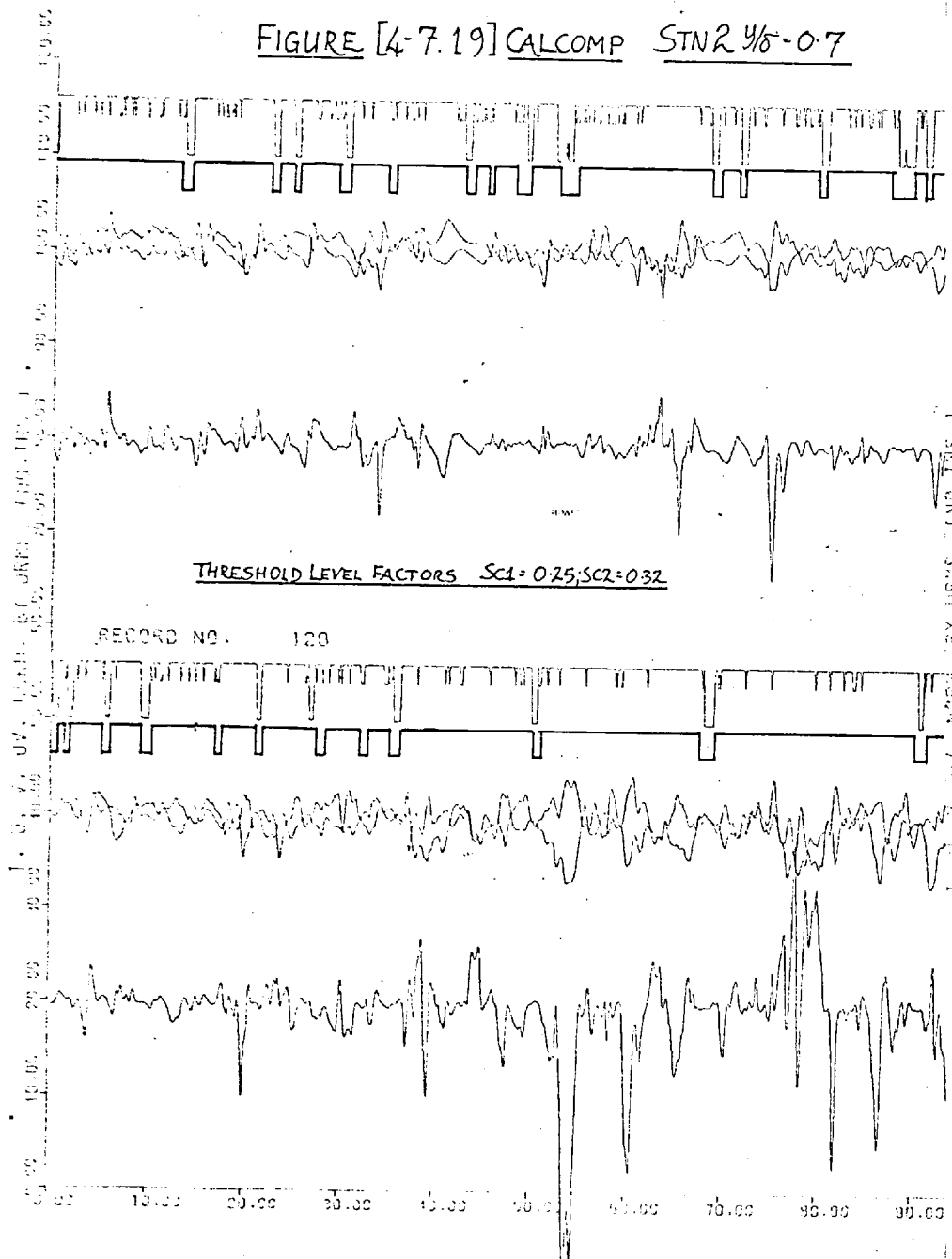
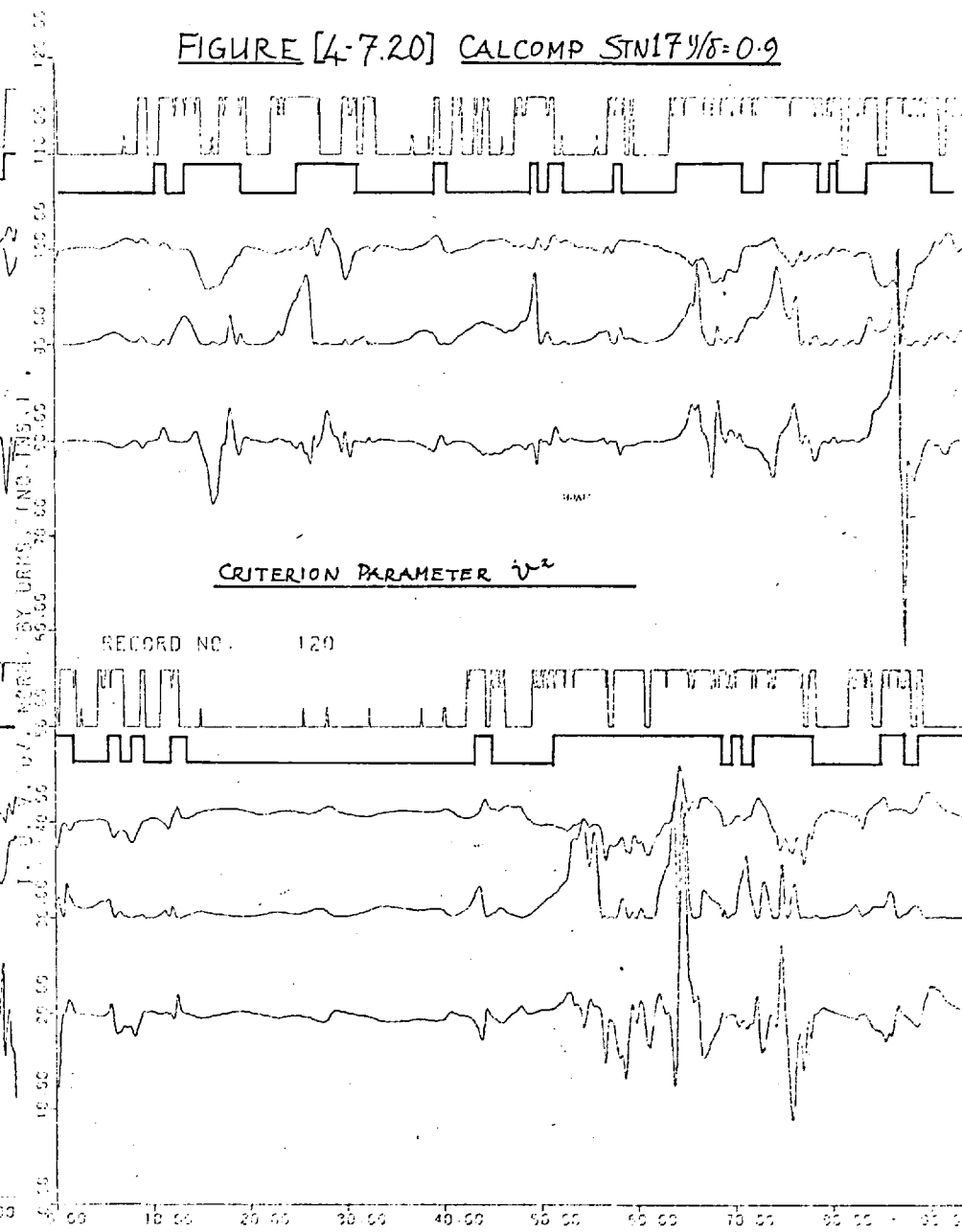
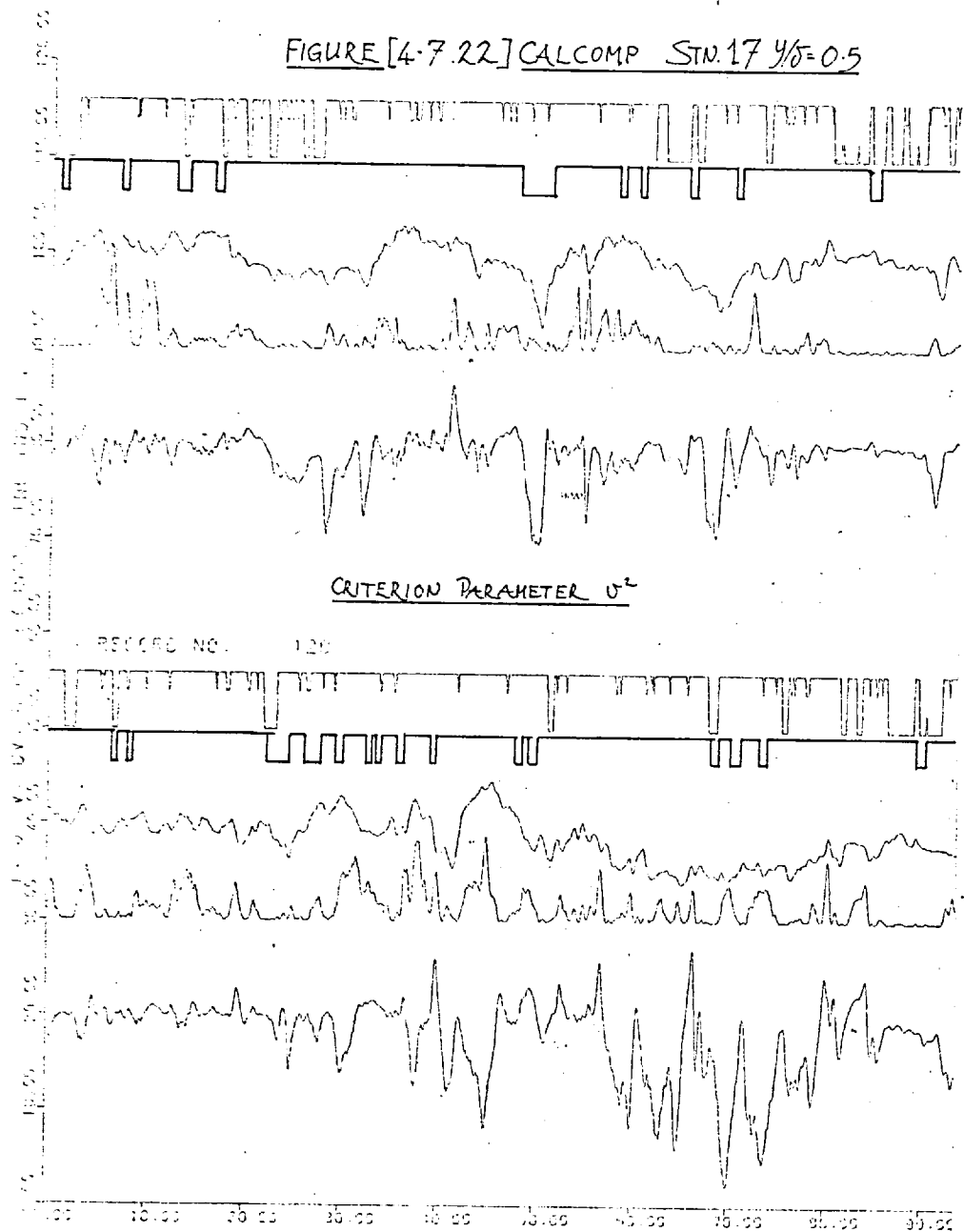


FIGURE [4-7.20] CALCOMP STN17 1/8-0.9



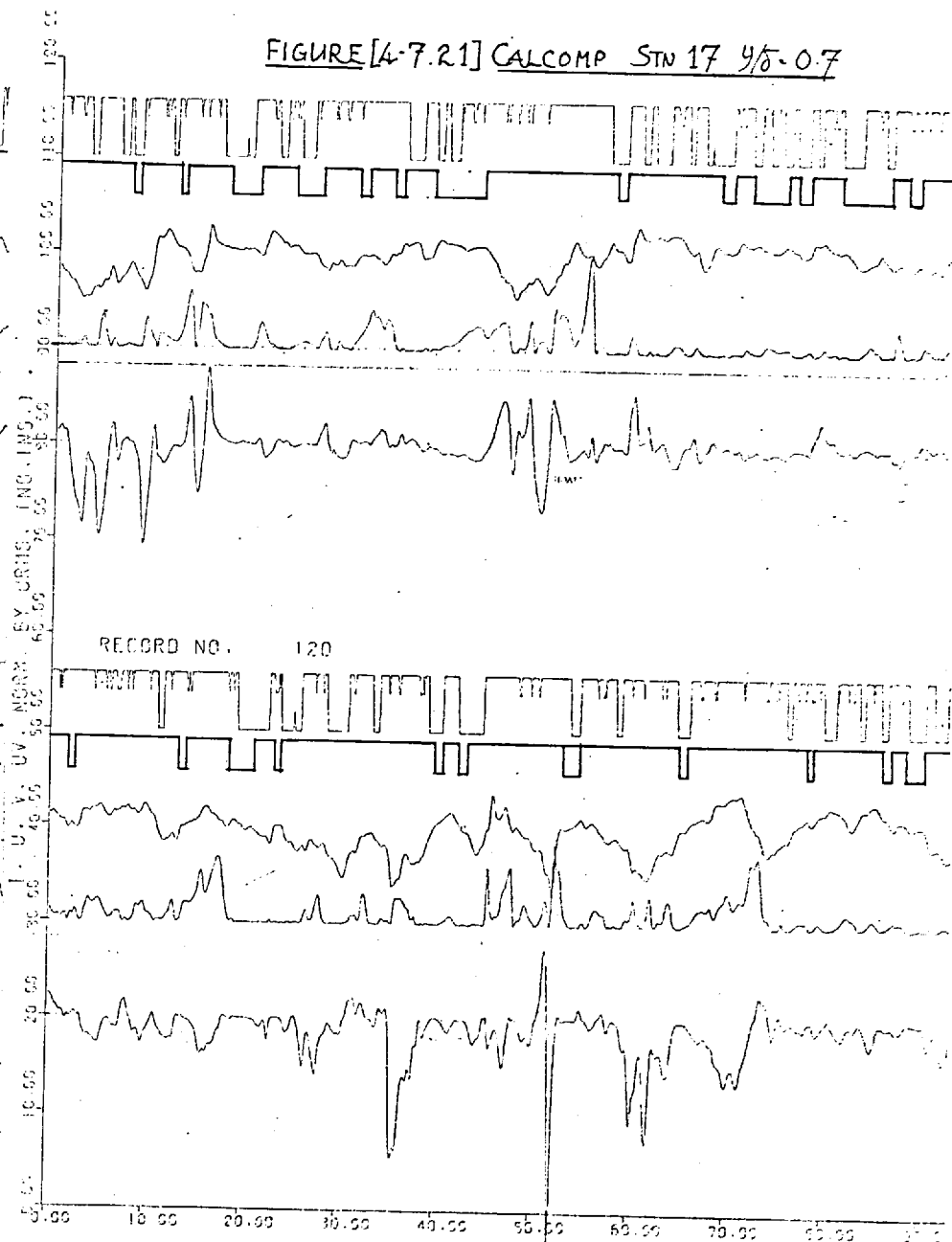
RESULTS- 1340 RMS CODE - 2357 0.0452 17070 Vcrit. 4-7-22

FIGURE [4-7.22] CALCOMP STN.17 $\gamma/\delta=0.5$



RESULTS- 1340 RMS CODE - 2357 0.0452 17070 Vcrit.

FIGURE [4-7.21] CALCOMP STN 17 $\gamma/\delta=0.7$



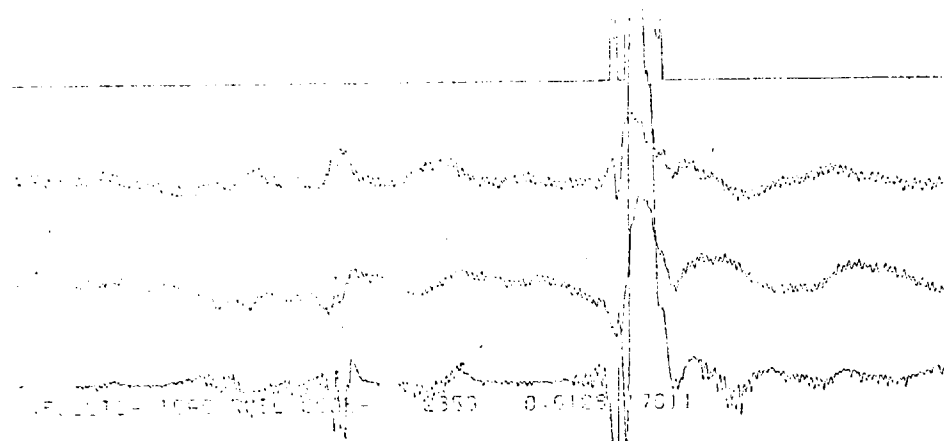


FIGURE [4-7.23] CALCOMP
STN 17 $y/\delta = 1.1$ WITH NOISE CLIP

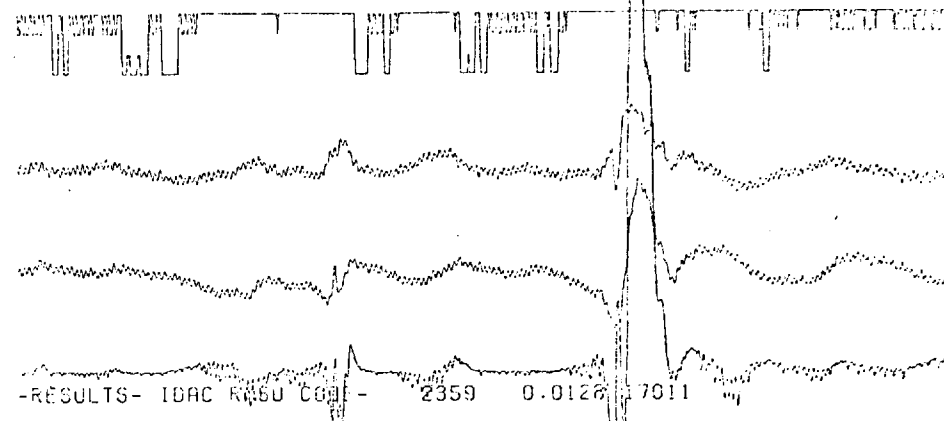
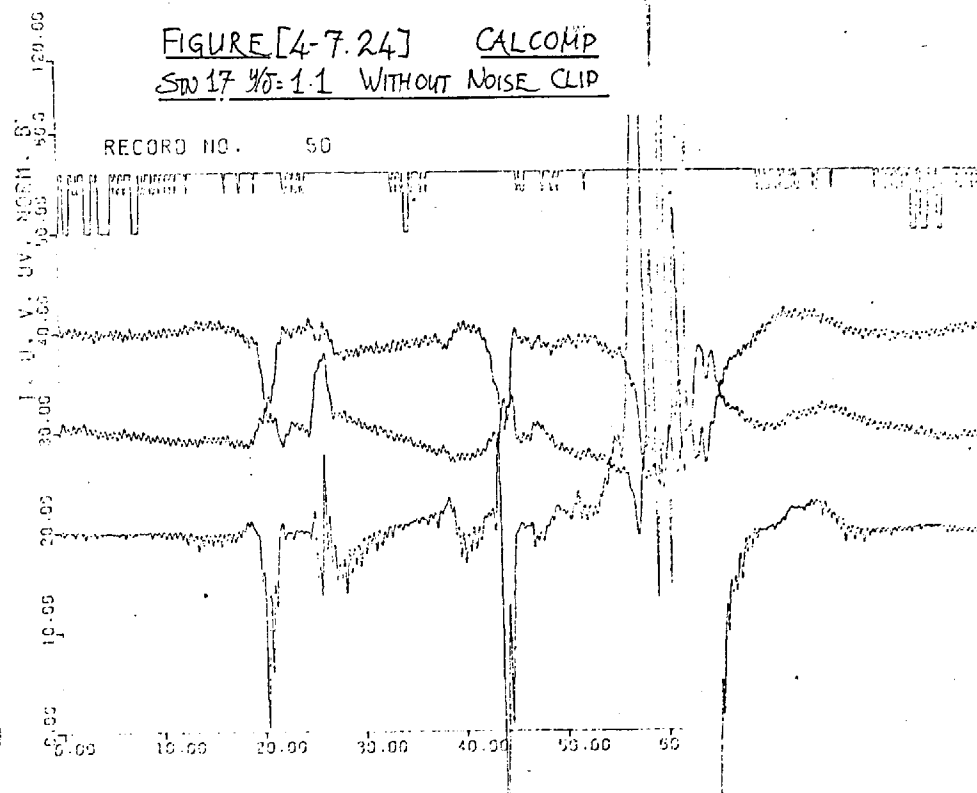


FIGURE [4-7.24] CALCOMP
STN 17 $y/\delta = 1.1$ WITHOUT NOISE CLIP



RECORD NO. 50

FIGURE [4-8.1] INTERMITTENCY FACTOR AT STATION 2
EFFECTS OF DIGITISATION RATE

SC 14 = .3 SC 15 = .35

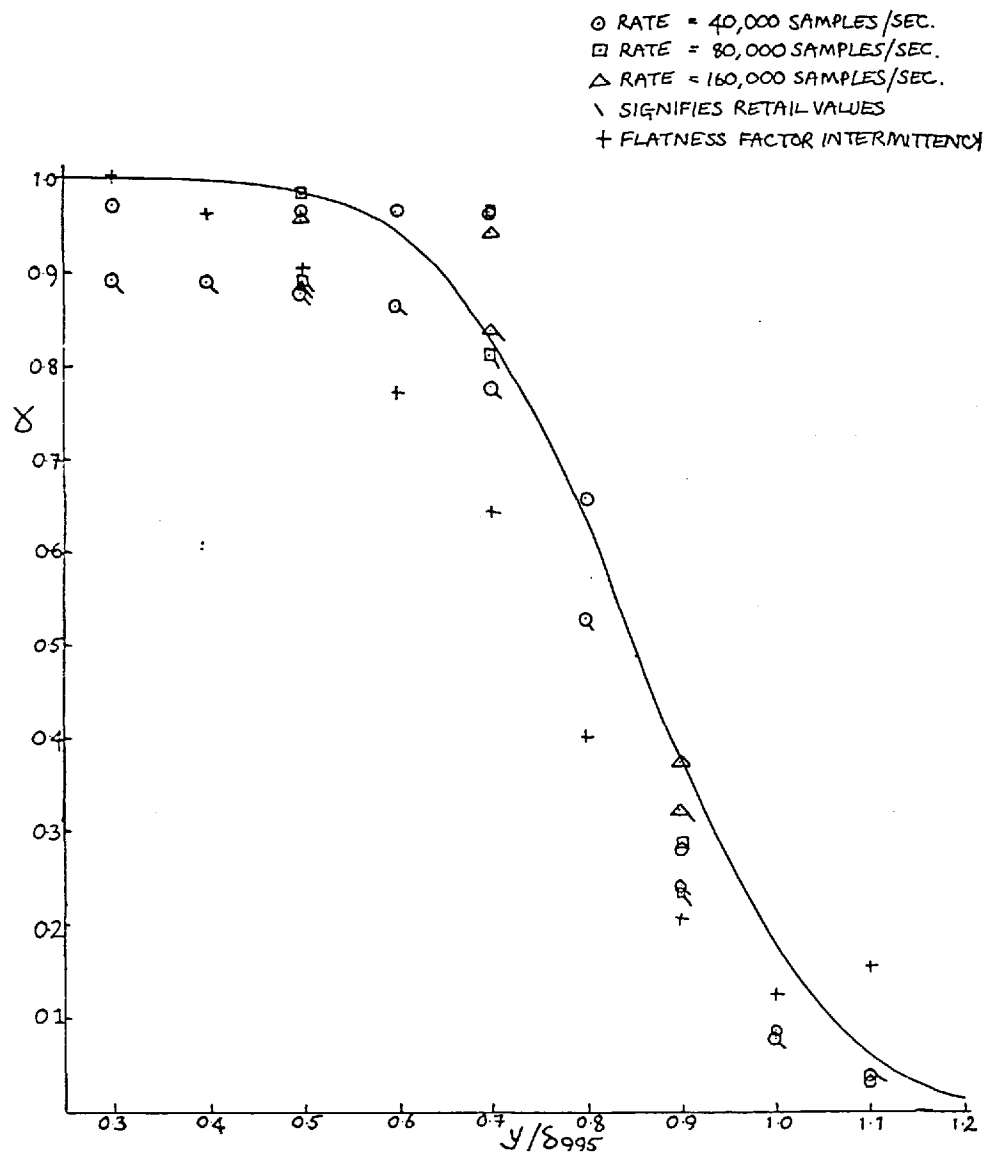
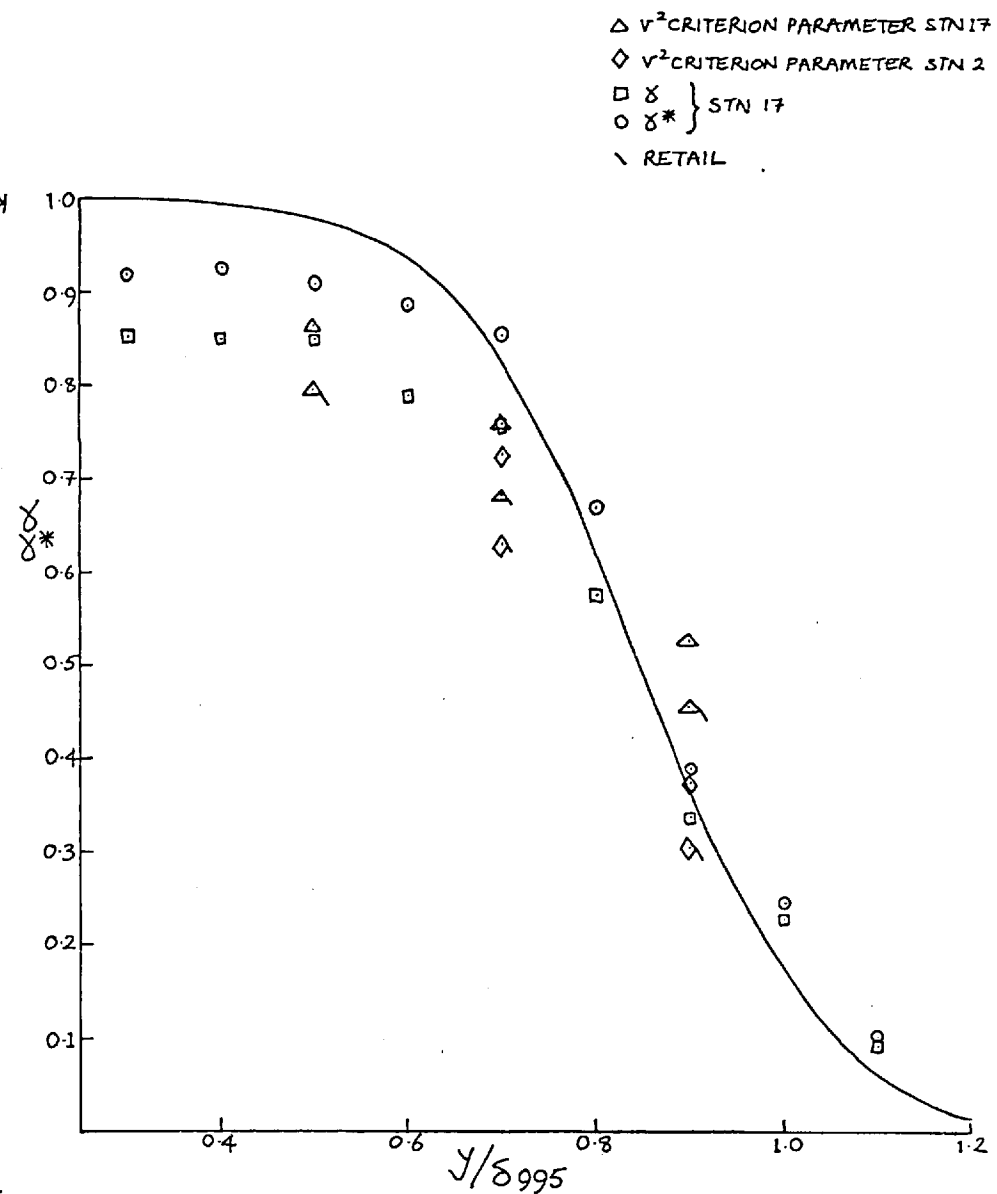


FIGURE [4-8.2] INTERMITTENCY FACTOR WITH V AS
CRITERION PARAMETER



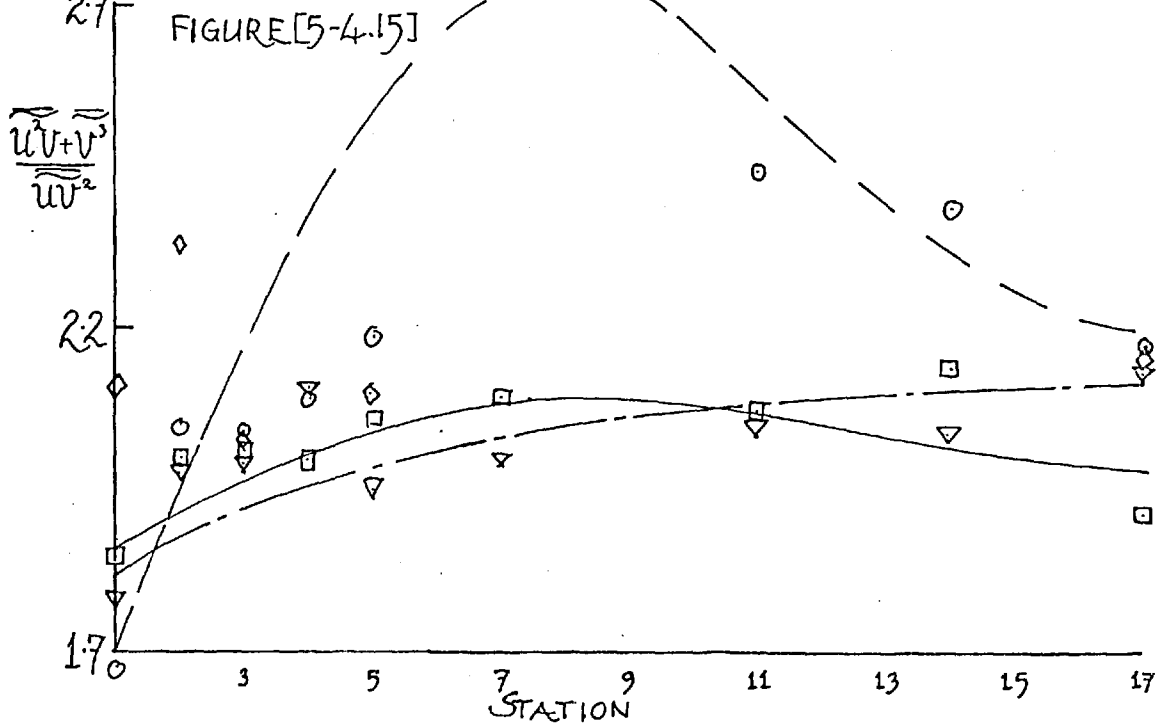
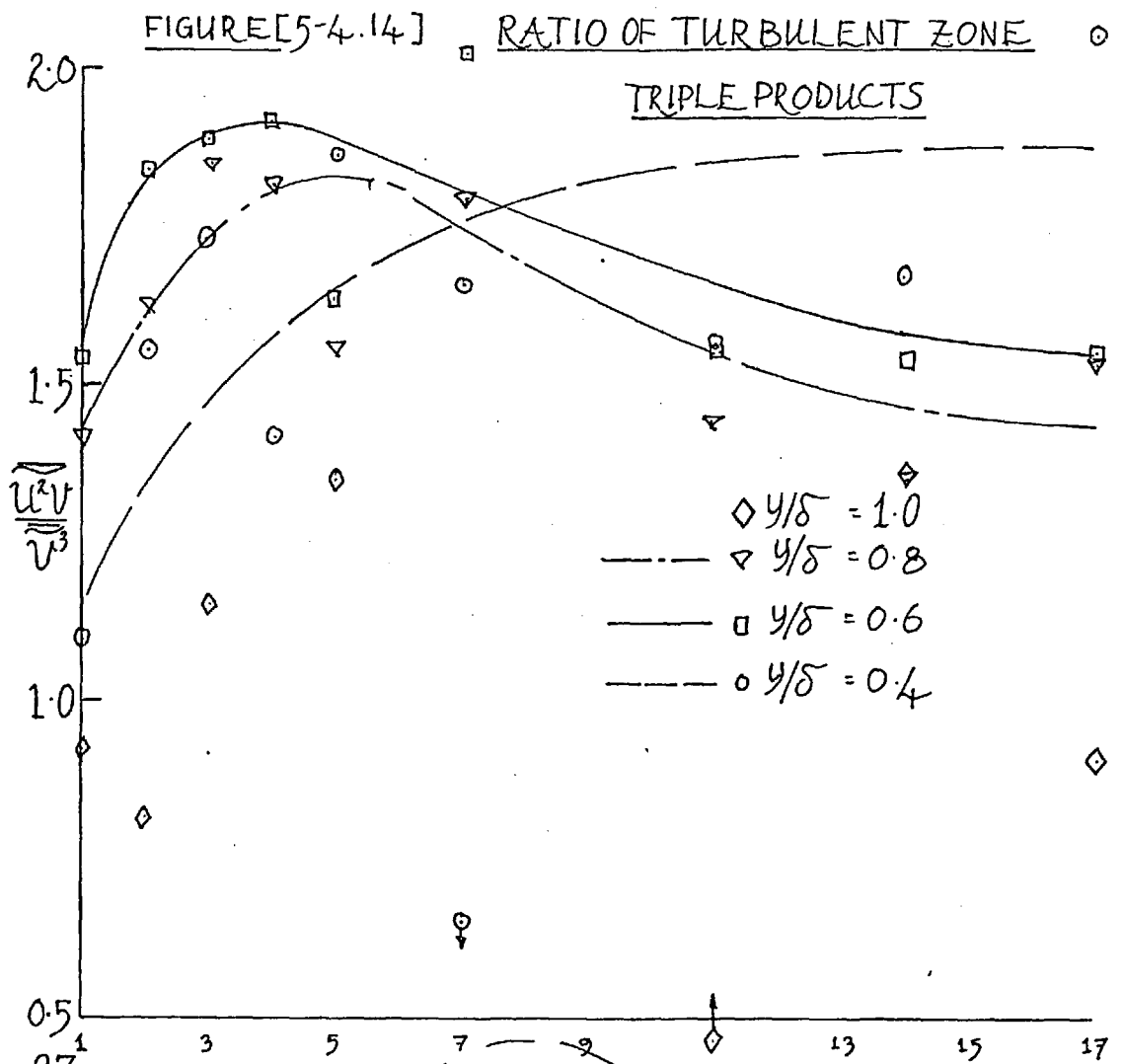


FIGURE [5-5.1] SKEWNESS OF u & v COMPONENT
FLUCTUATING VELOCITIES

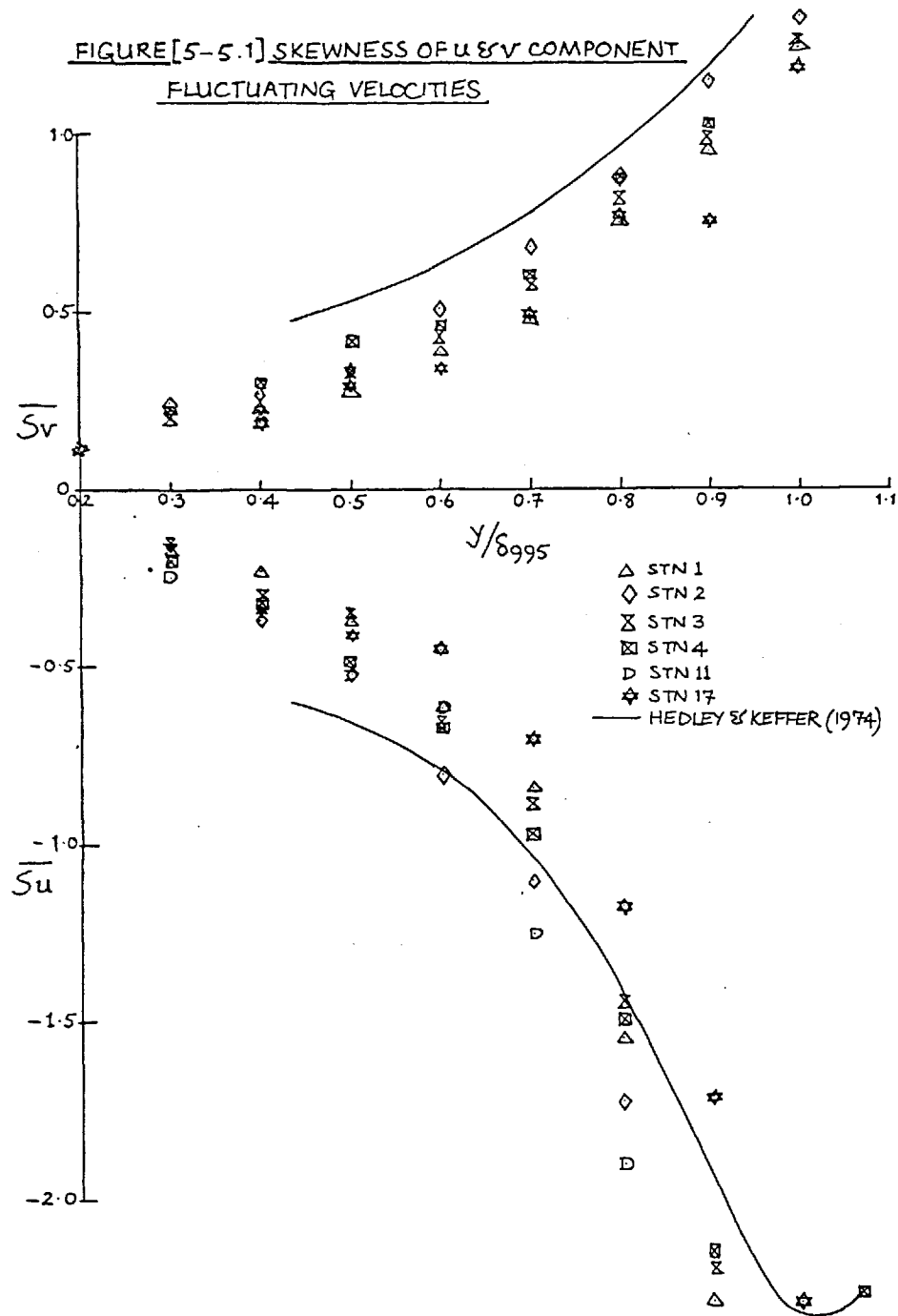


FIGURE [5-5.2] SKEWNESS OF CONDITIONALLY SAMPLED
 u & v COMPONENT FLUCTUATING VELOCITY
(TURBULENT ZONE) AT LOW Re .

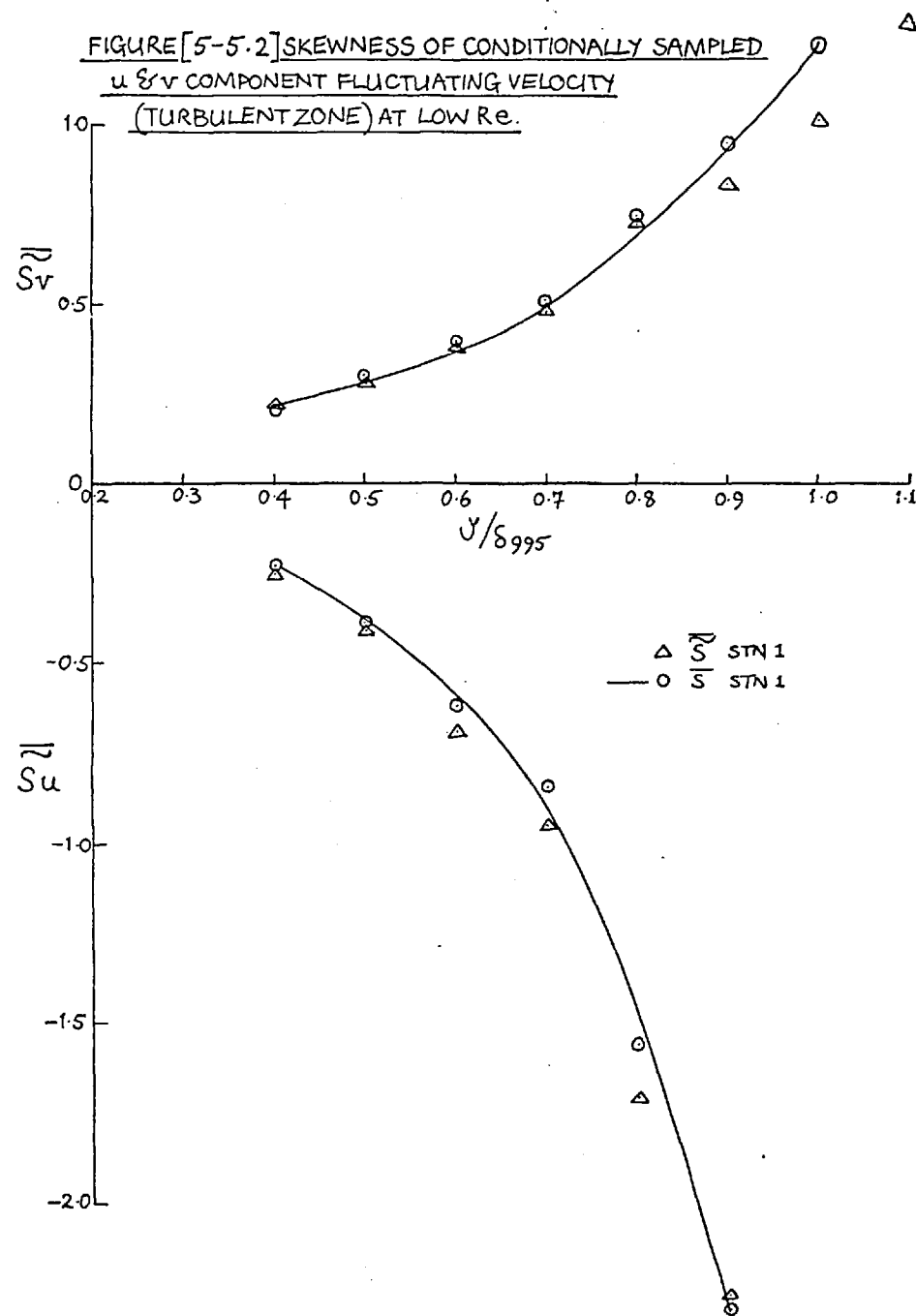


FIGURE [5-5.3] SKEWNESS OF CONDITIONALLY SAMPLED
u & v COMPONENT FLUCTUATING VELOCITY

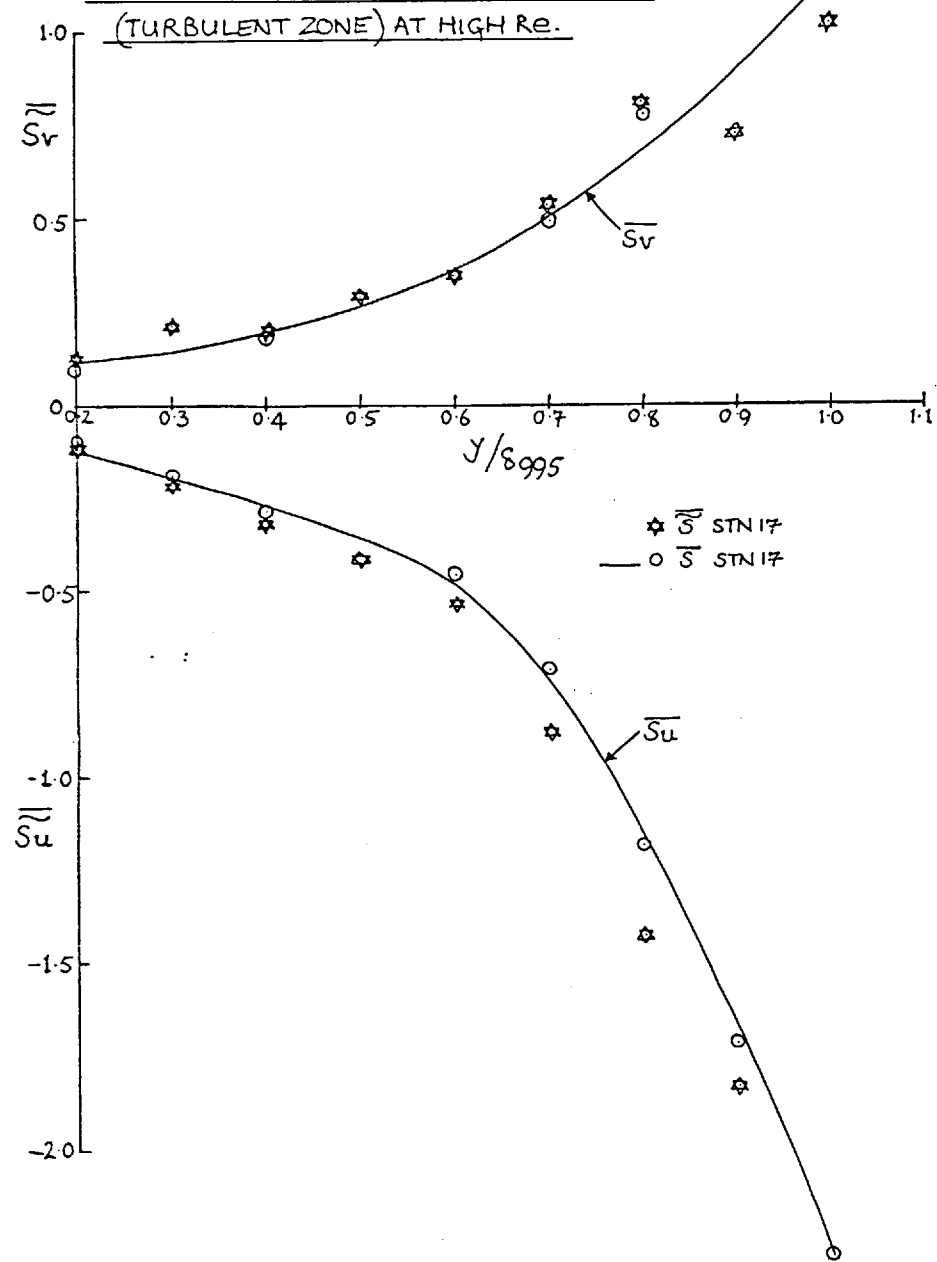


FIGURE [5-5.4] TURBULENT ZONE "SKEWNESS"

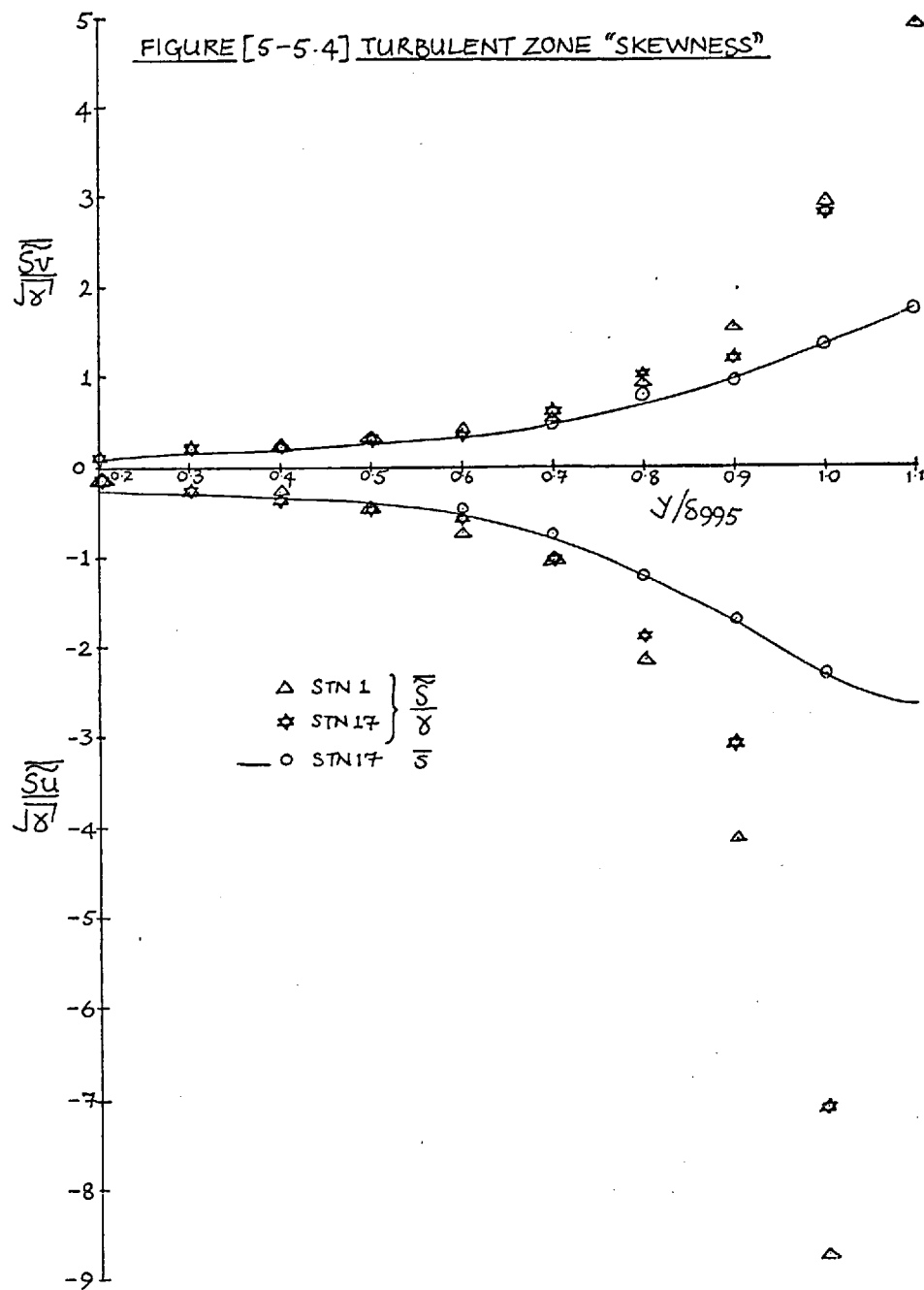


FIGURE [5-5.5] STREAMWISE DEVELOPMENT OF U COMPONENT

SKEWNESS

$\Delta y/\delta = 0.9$
 $\boxtimes y/\delta = 0.7$
 $\circ y/\delta = 0.4$

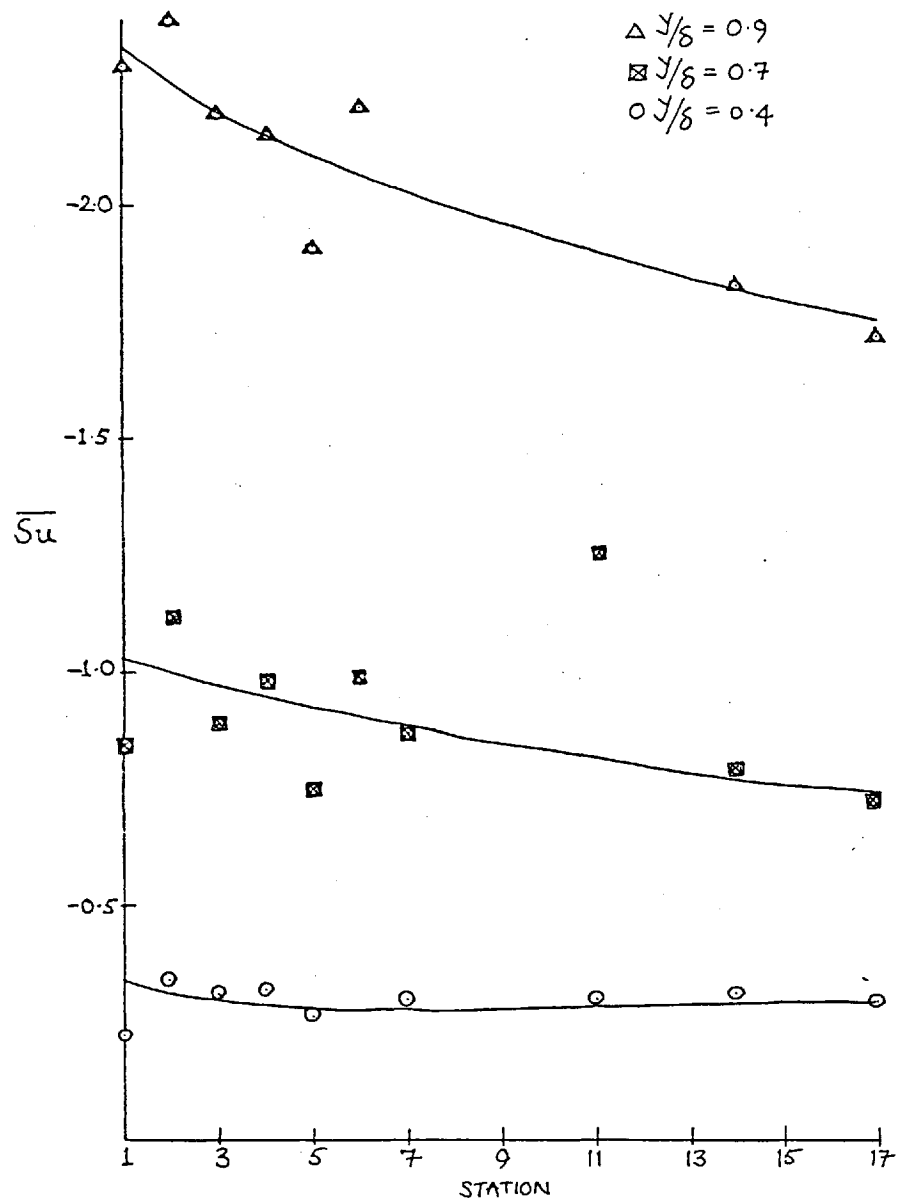


FIGURE [5-5.6] STREAMWISE DEVELOPMENT OF V COMPONENT

SKEWNESS

$\Delta y/\delta = 0.9$
 $\boxtimes y/\delta = 0.7$
 $\circ y/\delta = 0.4$

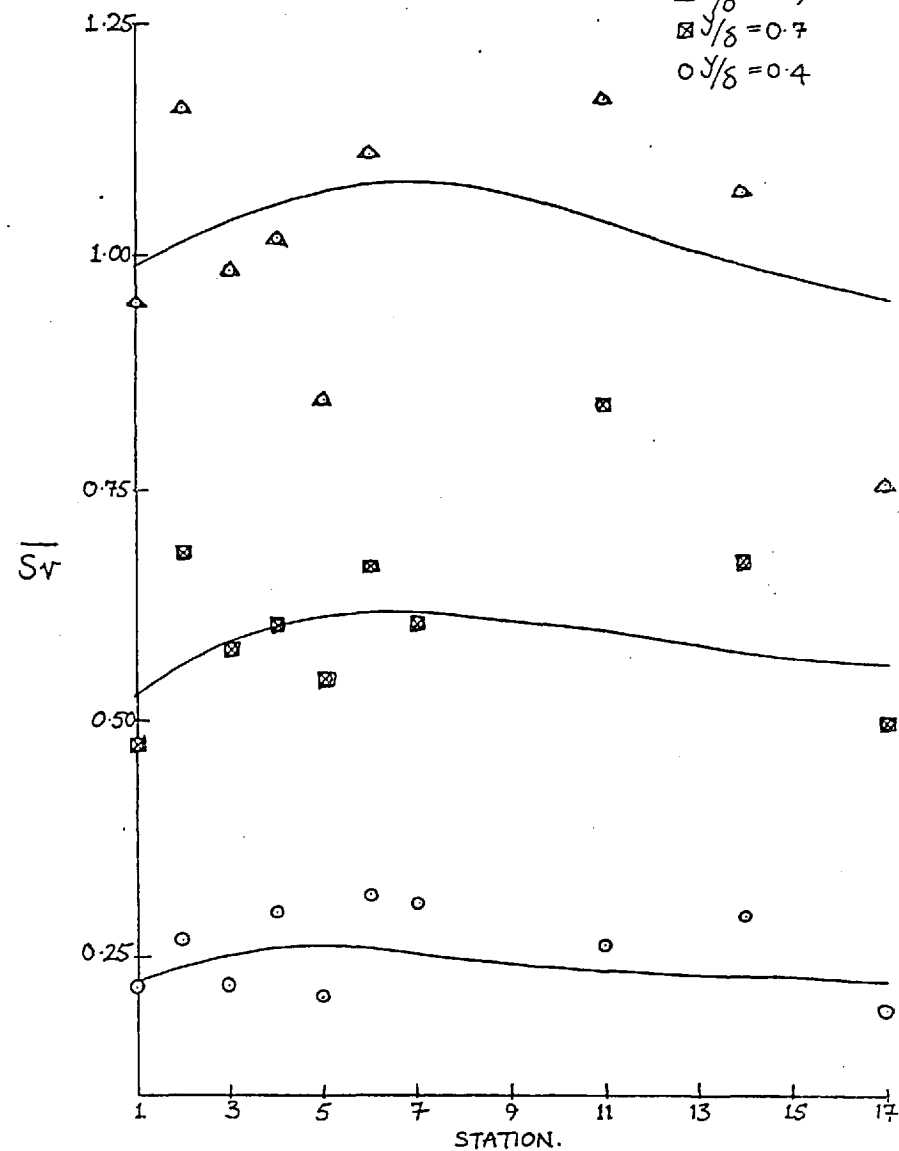


FIGURE [5-5.7] STREAMWISE DEVELOPMENT OF TURBULENT

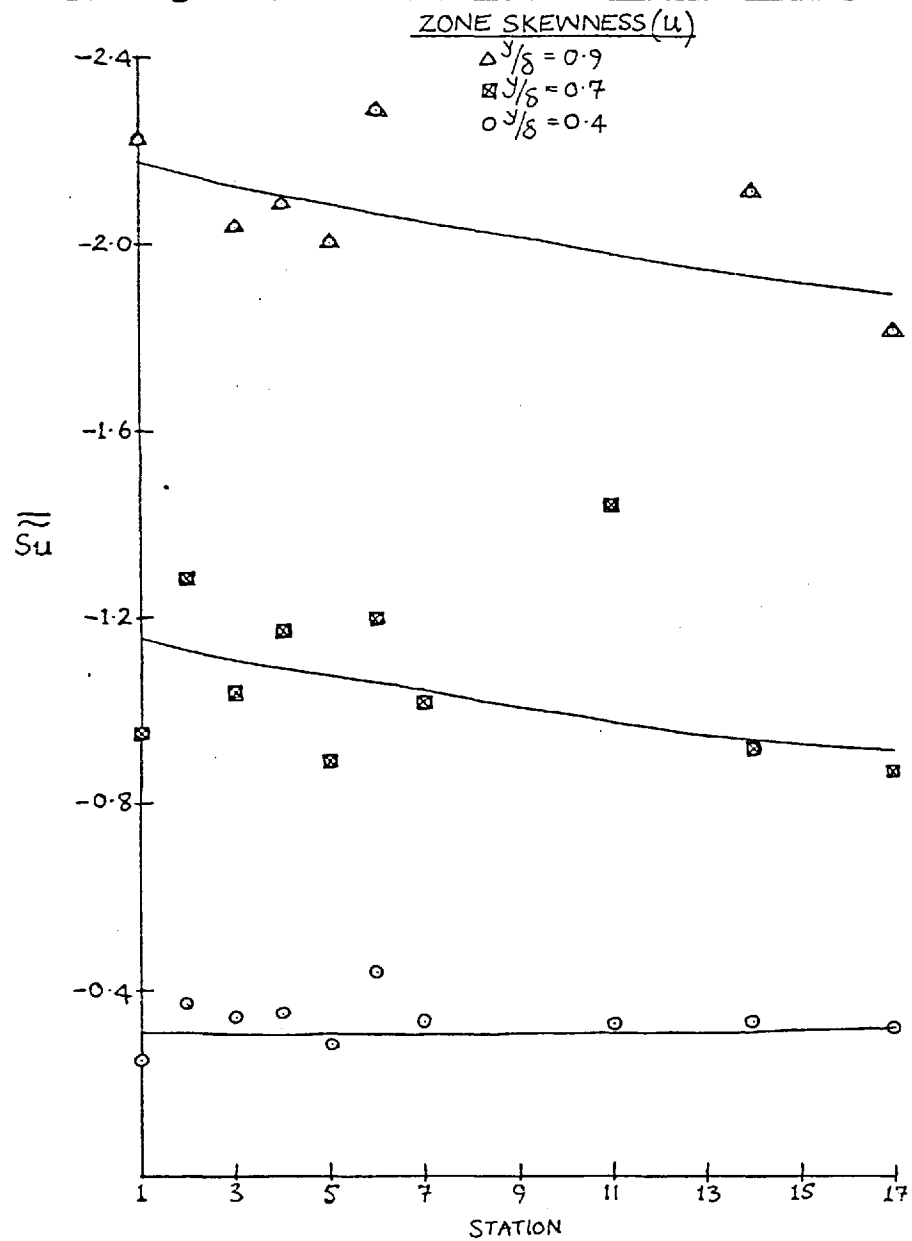


FIGURE [5-5.8] STREAMWISE DEVELOPMENT OF TURBULENT

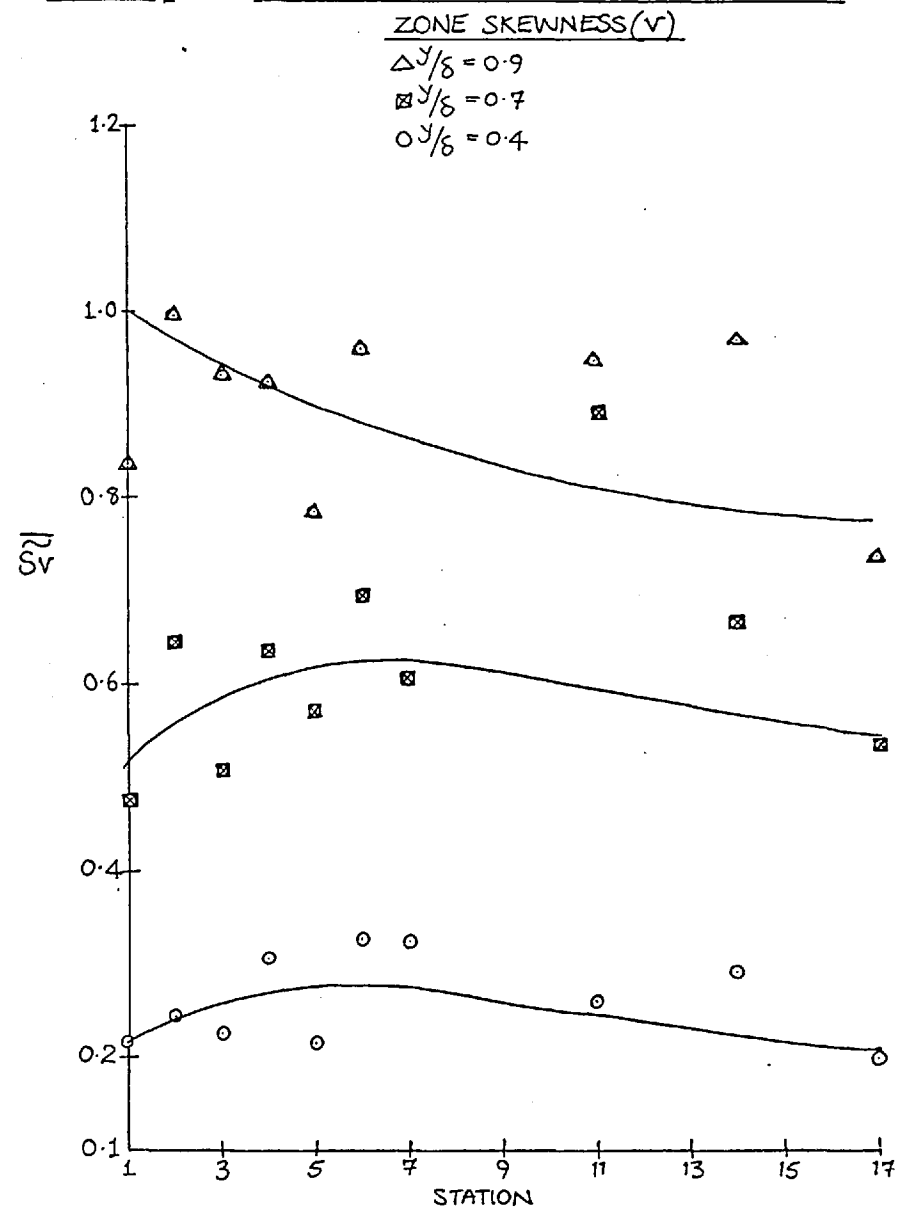


FIGURE [5-5.9] STREAMWISE DEVELOPMENT OF TURBULENT ZONE
CONTRIBUTION TO SKEWNESS (U)

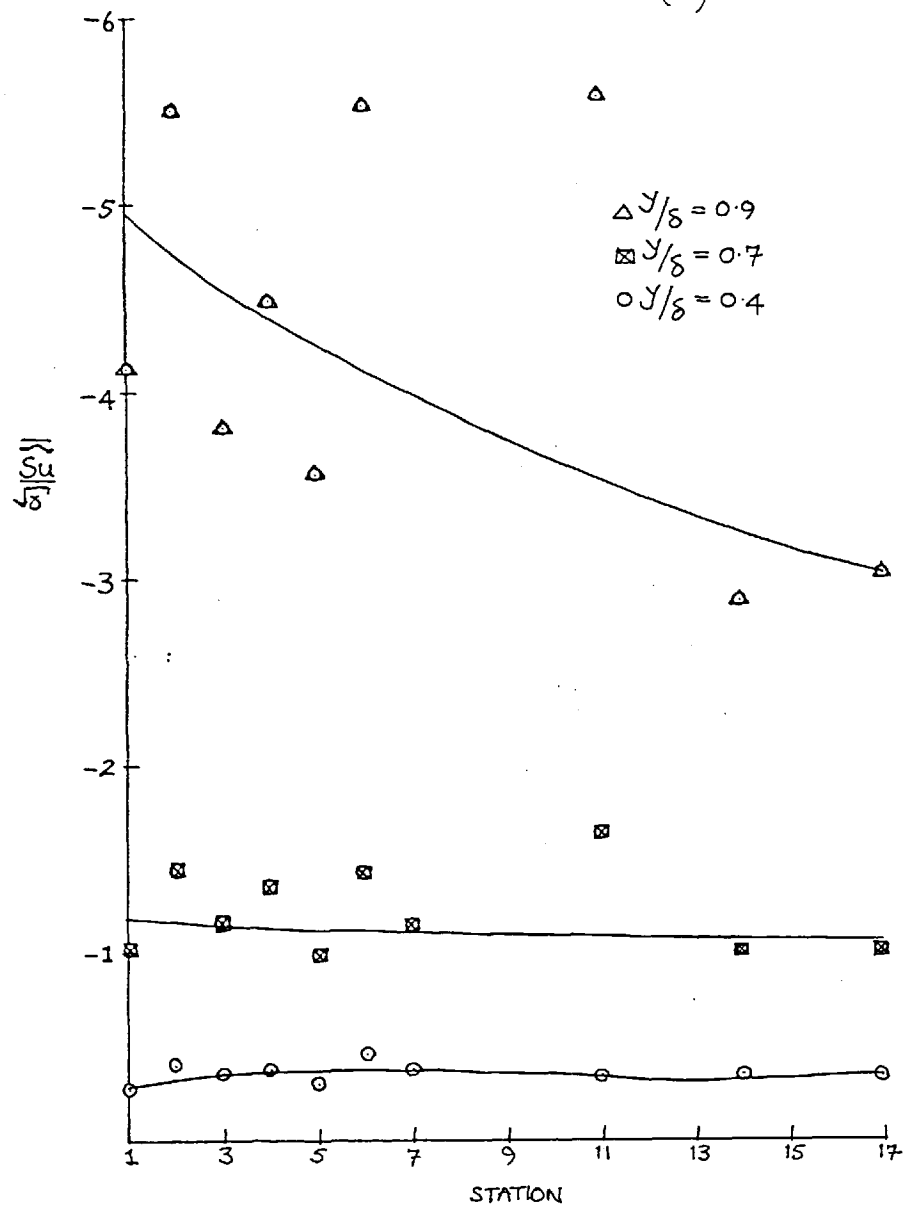


FIGURE [5-5.10] STREAMWISE DEVELOPMENT OF TURBULENT ZONE
CONTRIBUTION TO SKEWNESS (V)

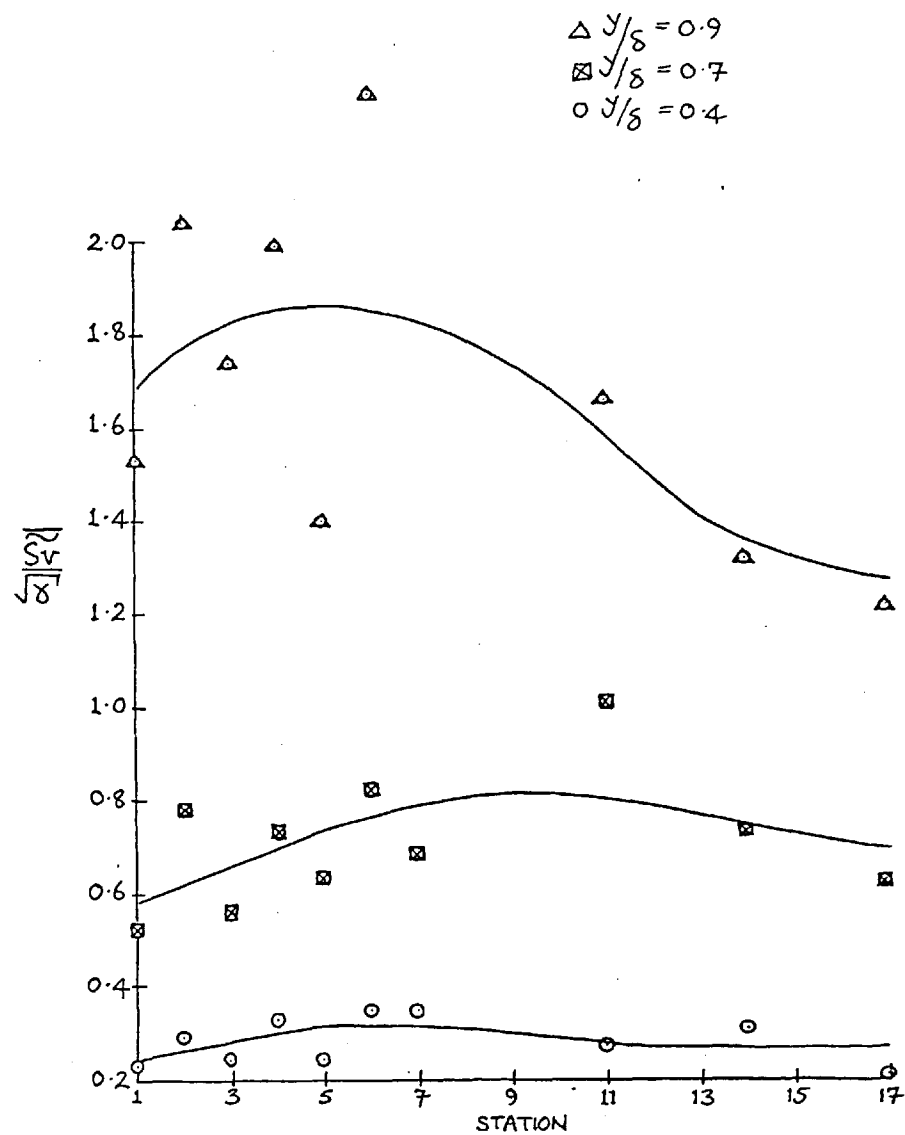


FIGURE [5-5.11] FLATNESS FACTOR OF u COMPONENT FLUCTUATING VELOCITY

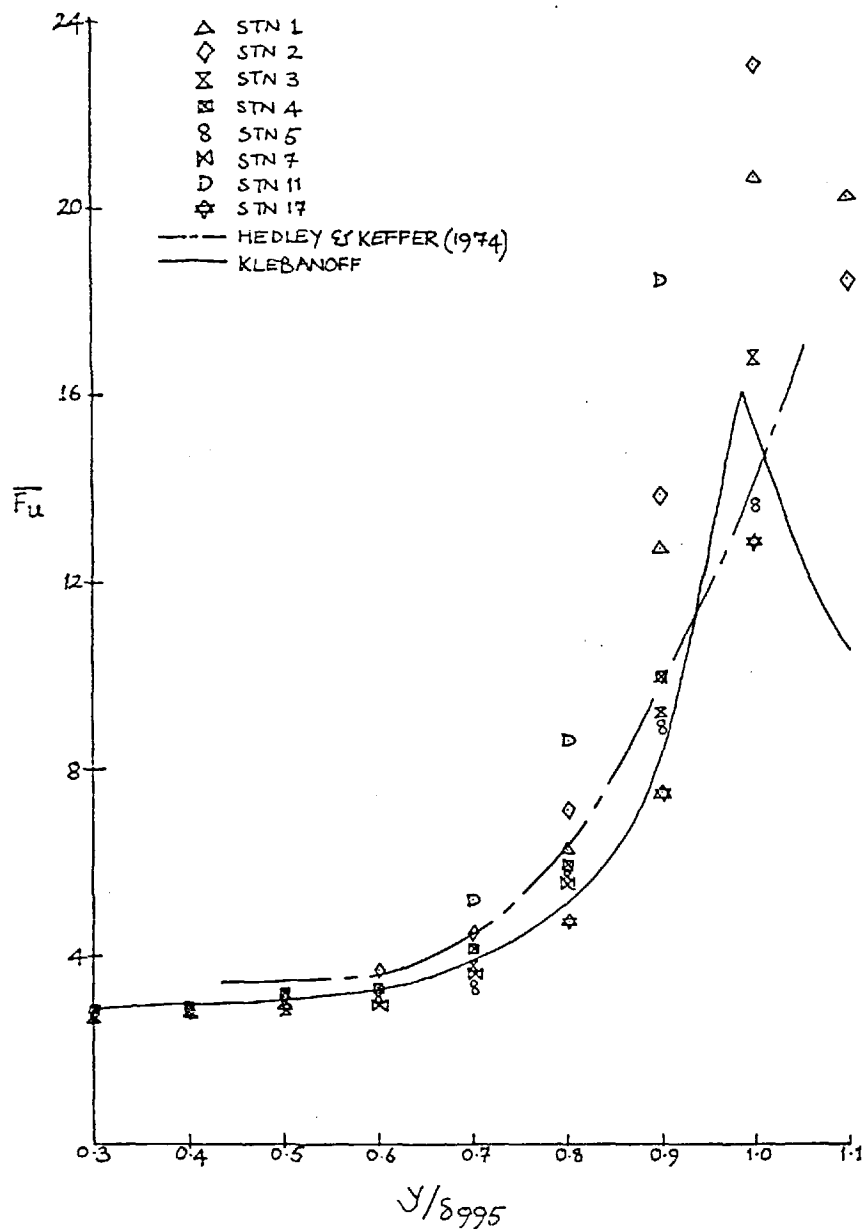


FIGURE [5-5.11b] KLEBANOFF'S FLATNESS FACTOR COMPARED WITH THE RECIPROCAL OF INTERMITTENCY FACTOR

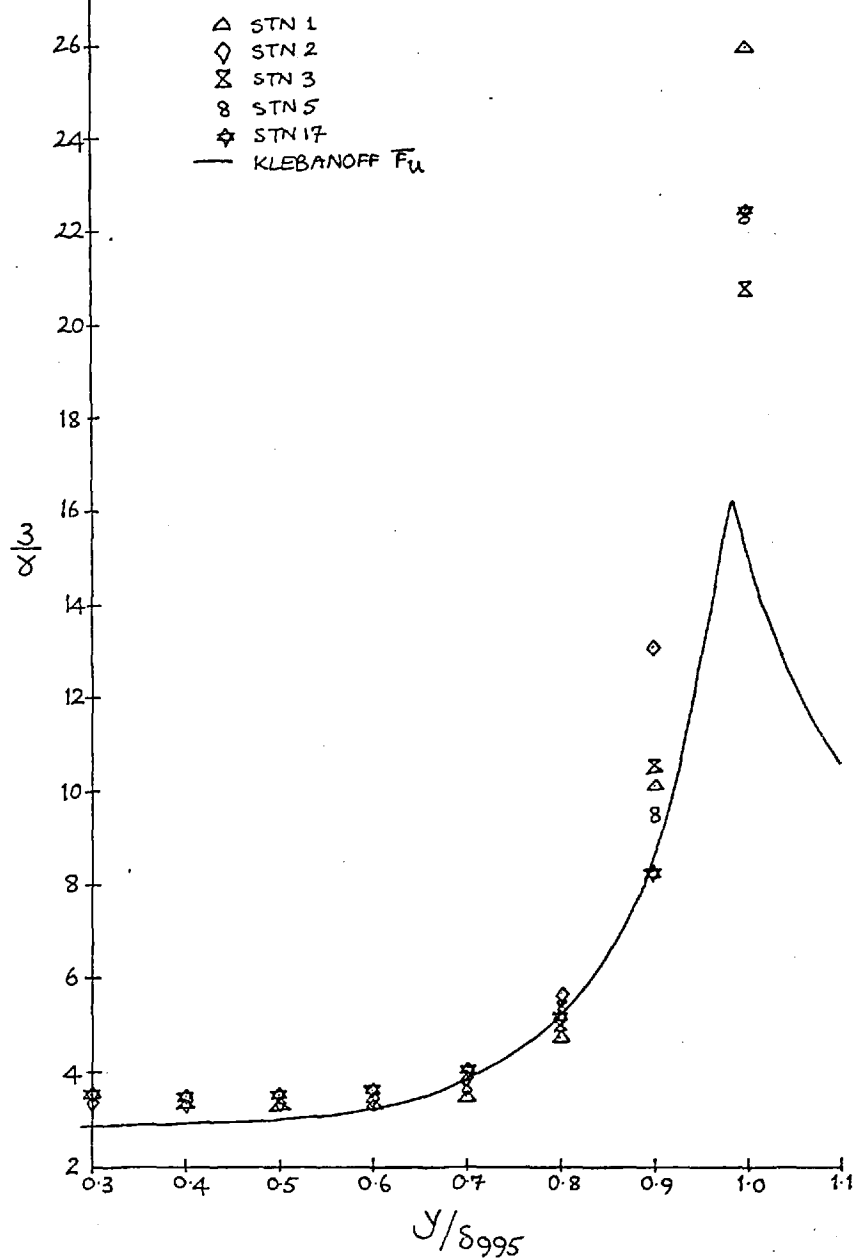


FIGURE [5-5.12] FLATNESS FACTOR - v COMPONENT

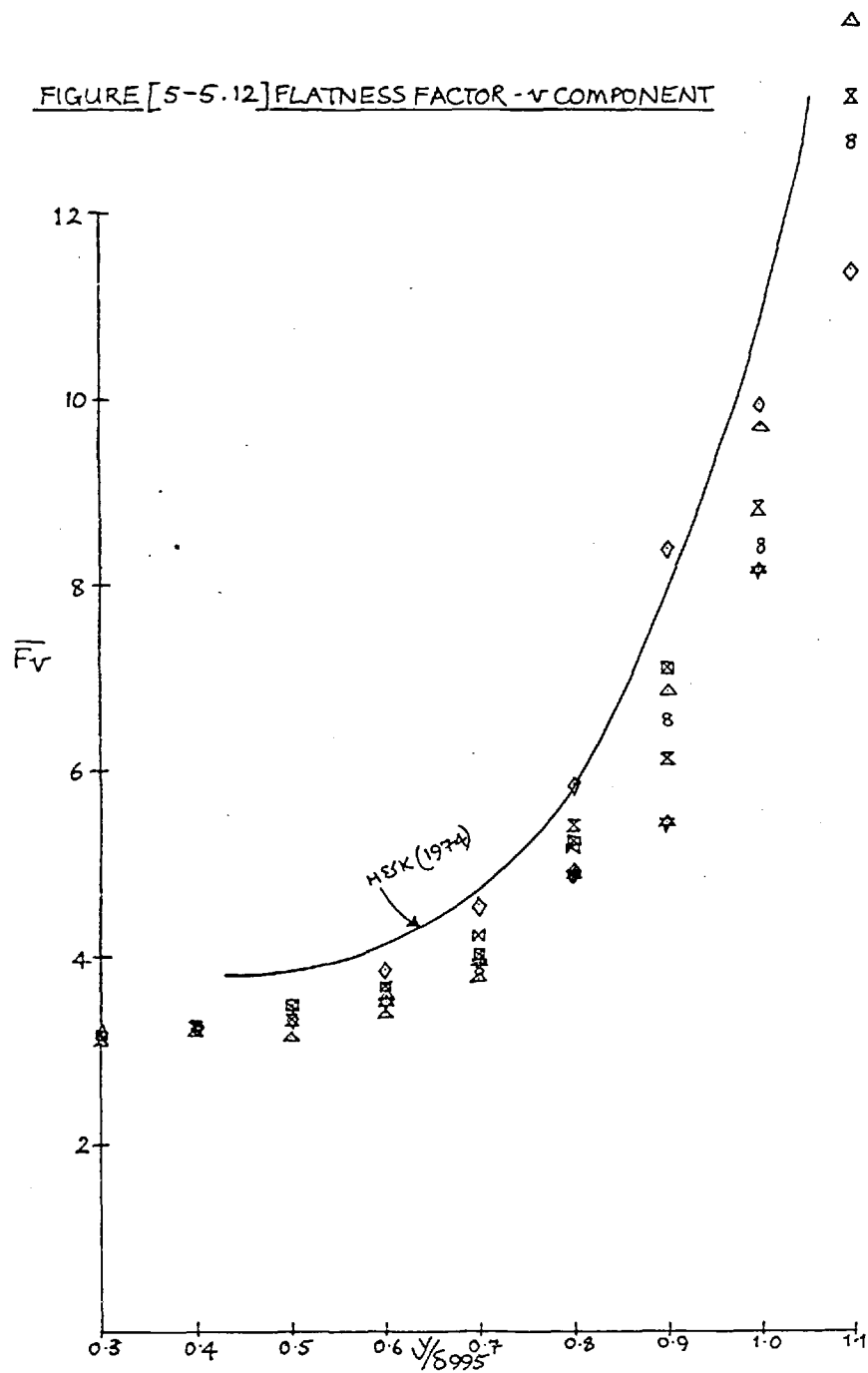
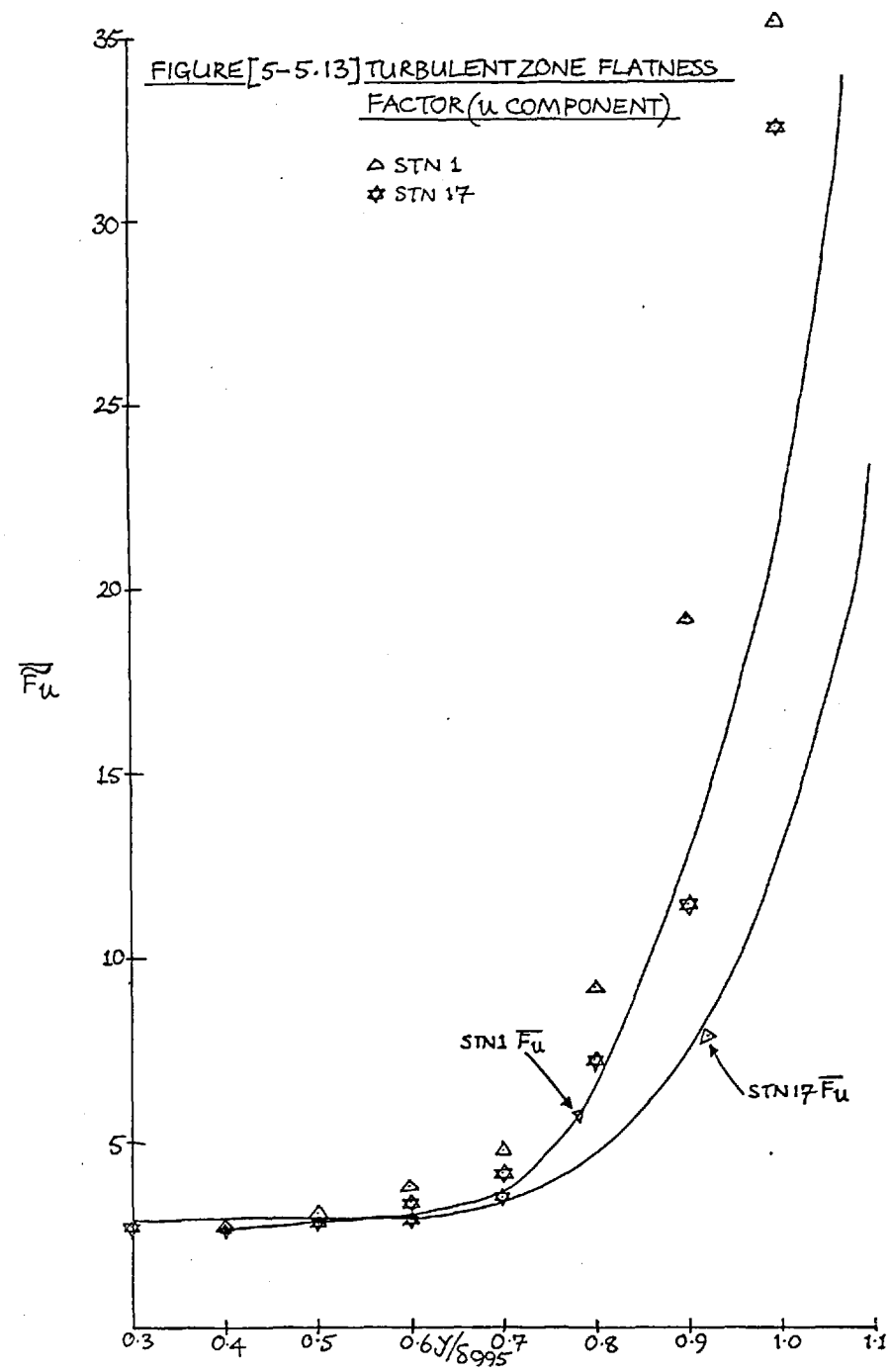
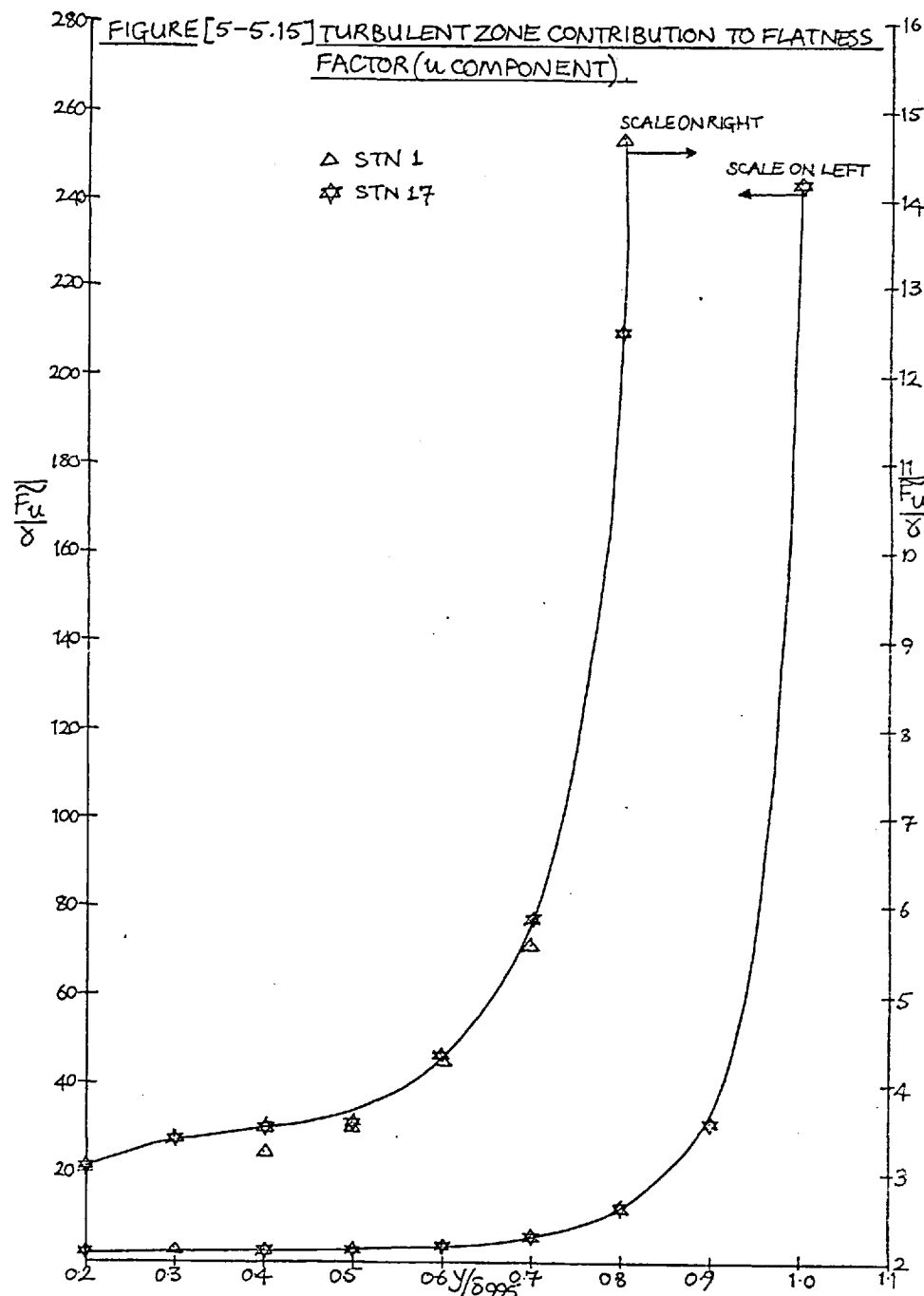
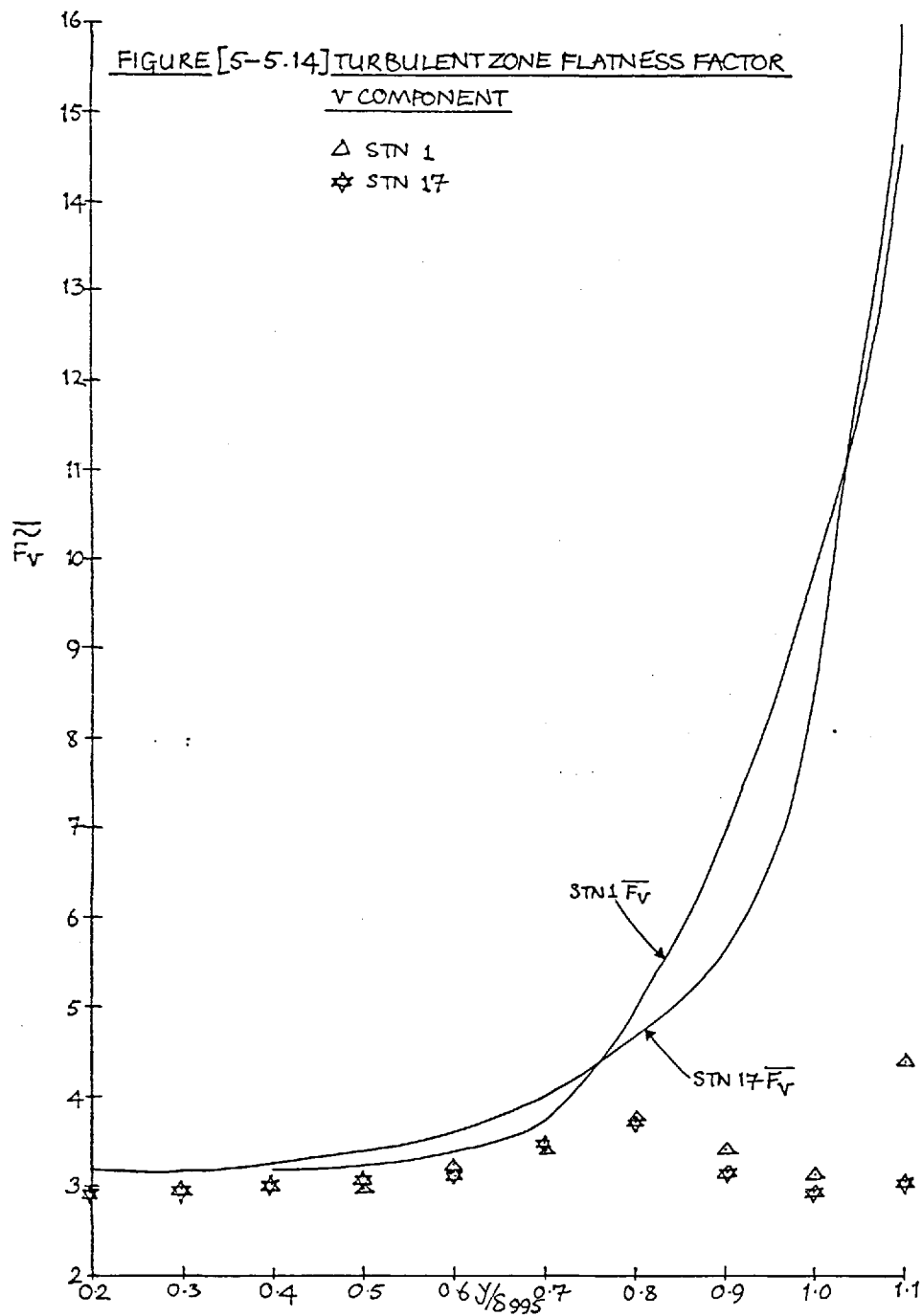


FIGURE [5-5.13] TURBULENT ZONE FLATNESS FACTOR (u COMPONENT)





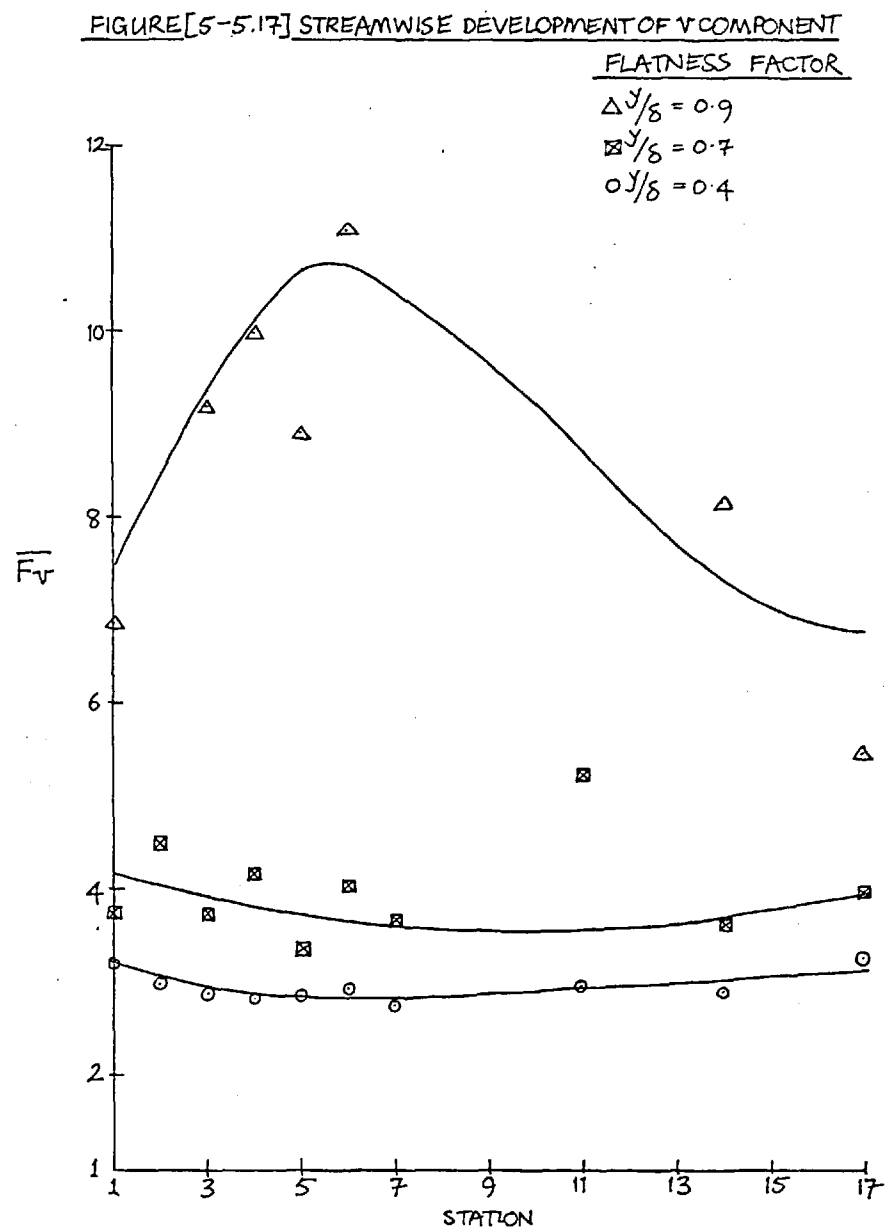
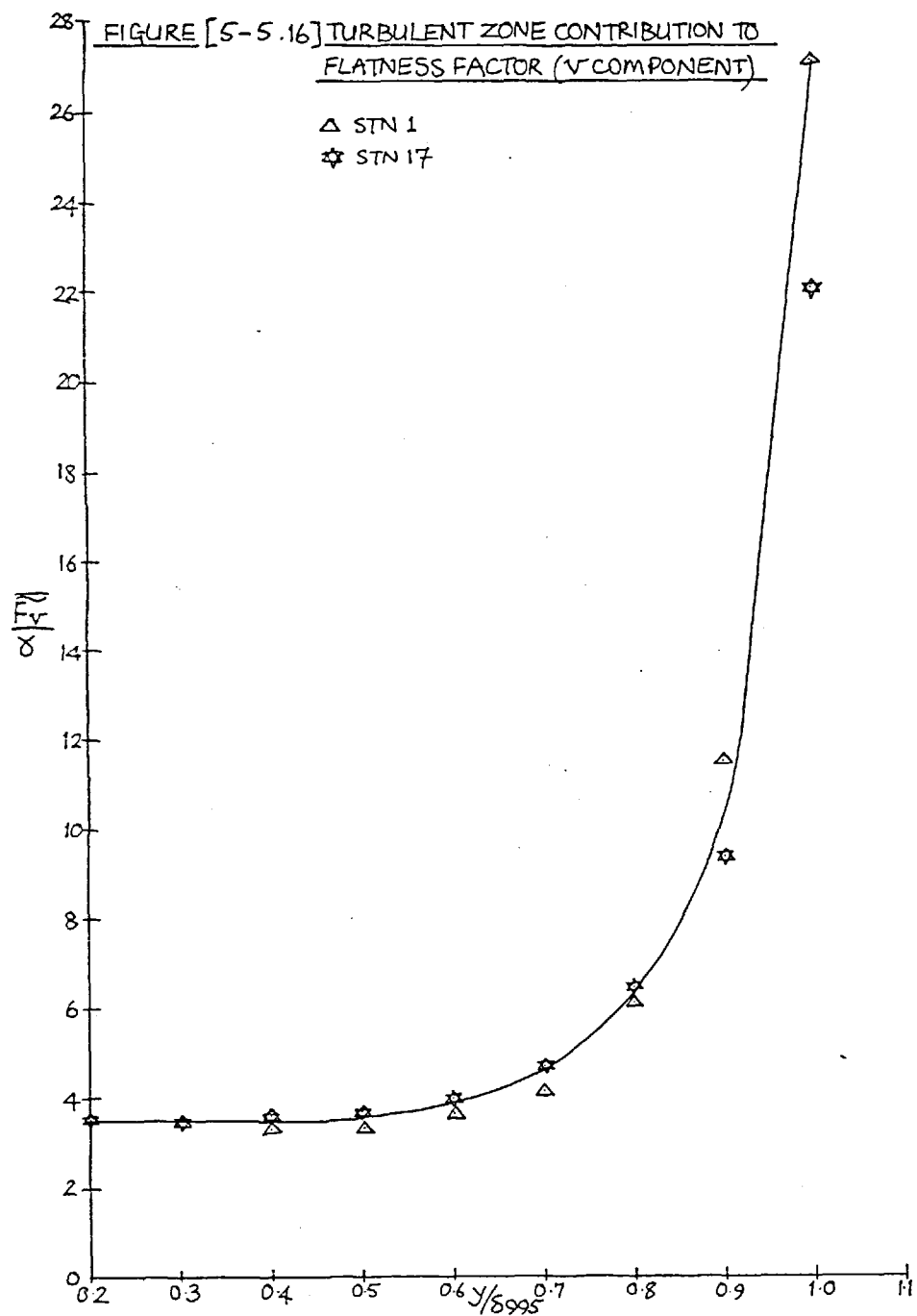


FIGURE [5-5.18a] STREAMWISE DEVELOPMENT OF U COMPONENT

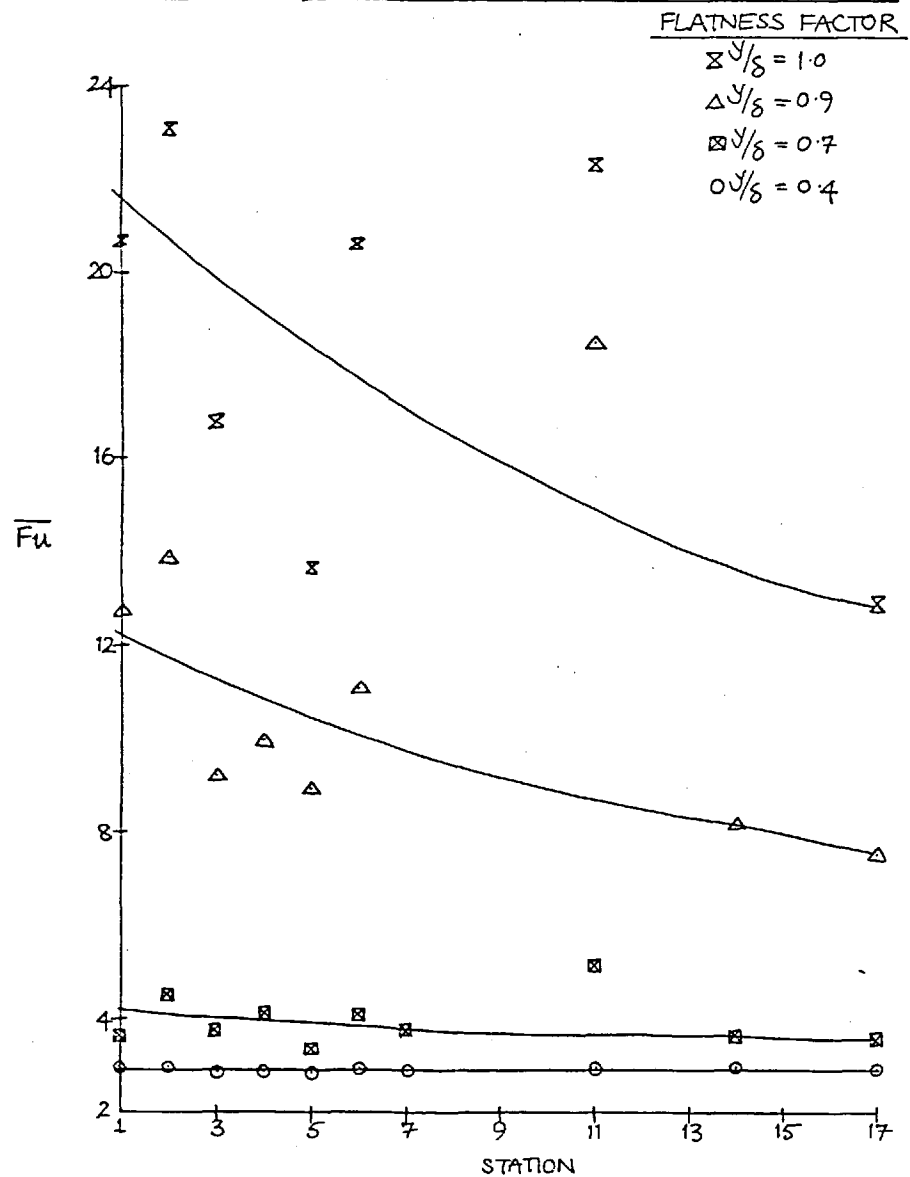


FIGURE [5-5.18b] STREAMWISE DEVELOPMENT OF INTERMITTENCY

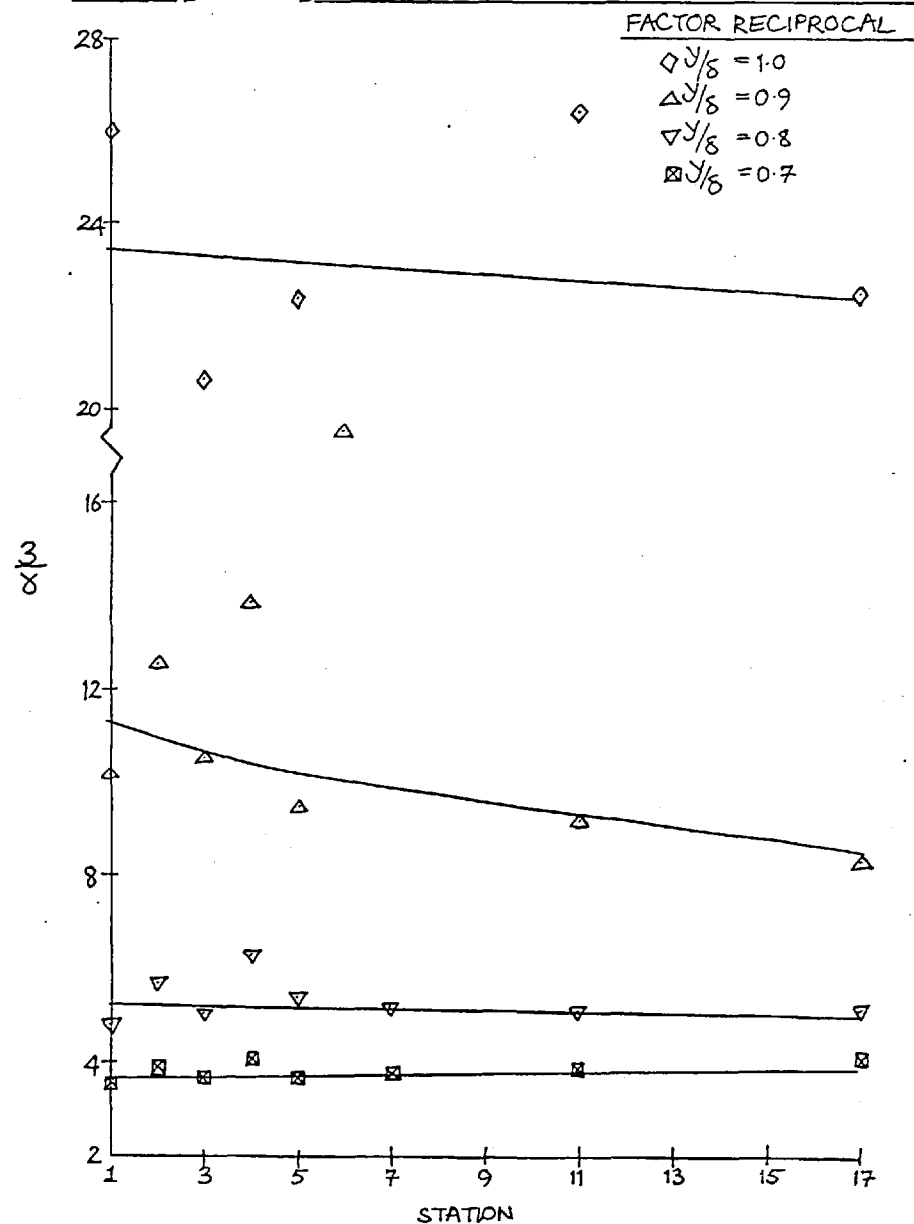


FIGURE [5-5.19] STREAMWISE DEVELOPMENT OF TURBULENT ZONE u -COMPONENT FLATNESS

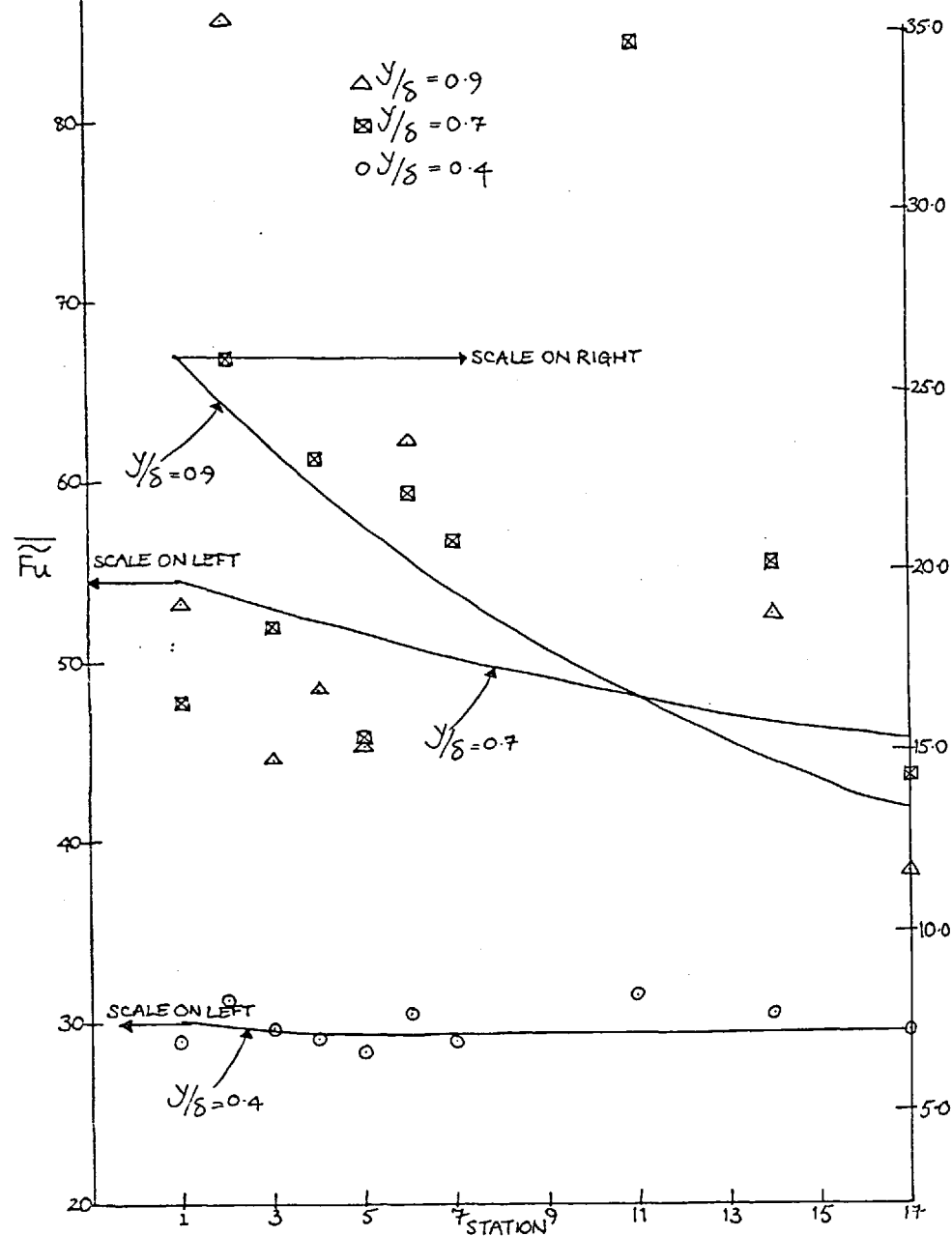


FIGURE [5-5.20] STREAMWISE DEVELOPMENT OF TURBULENT ZONE FLATNESS FACTOR (v -COMPONENT)

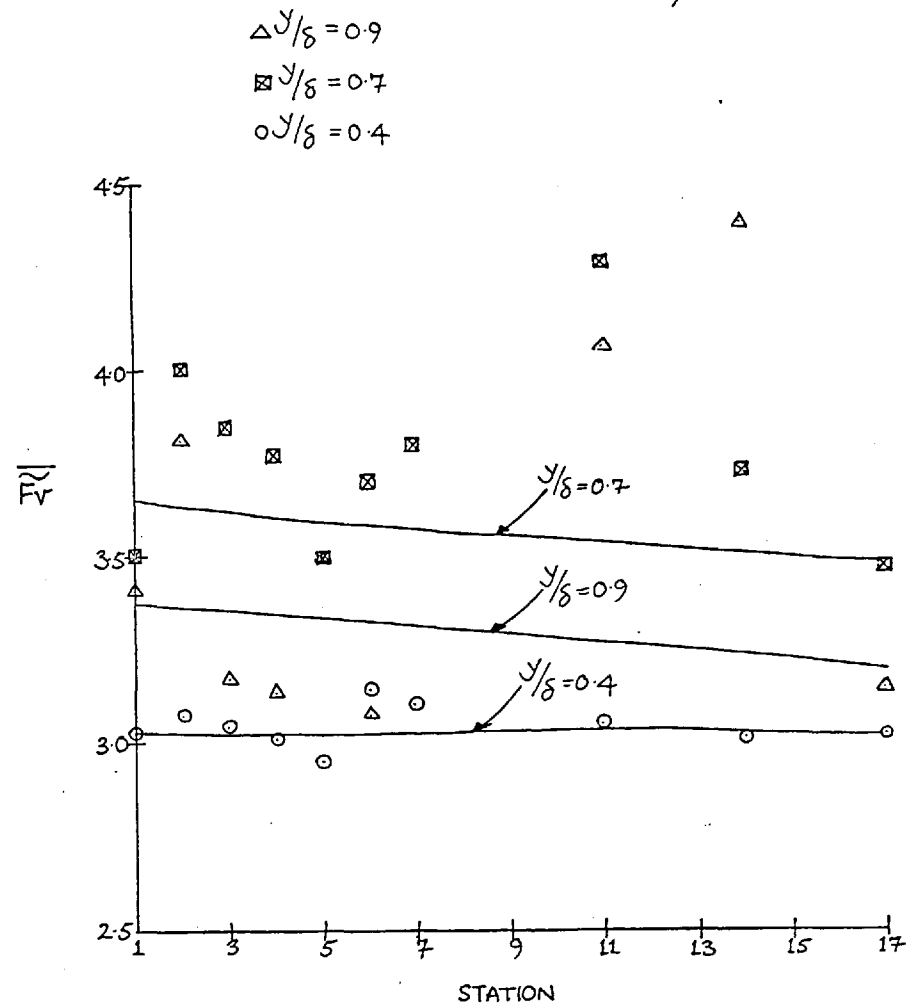


FIGURE [5-5-21] STREAMWISE DEVELOPMENT OF TURBULENT-ZONE CONTRIBUTION TO u -COMPONENT FLATNESS

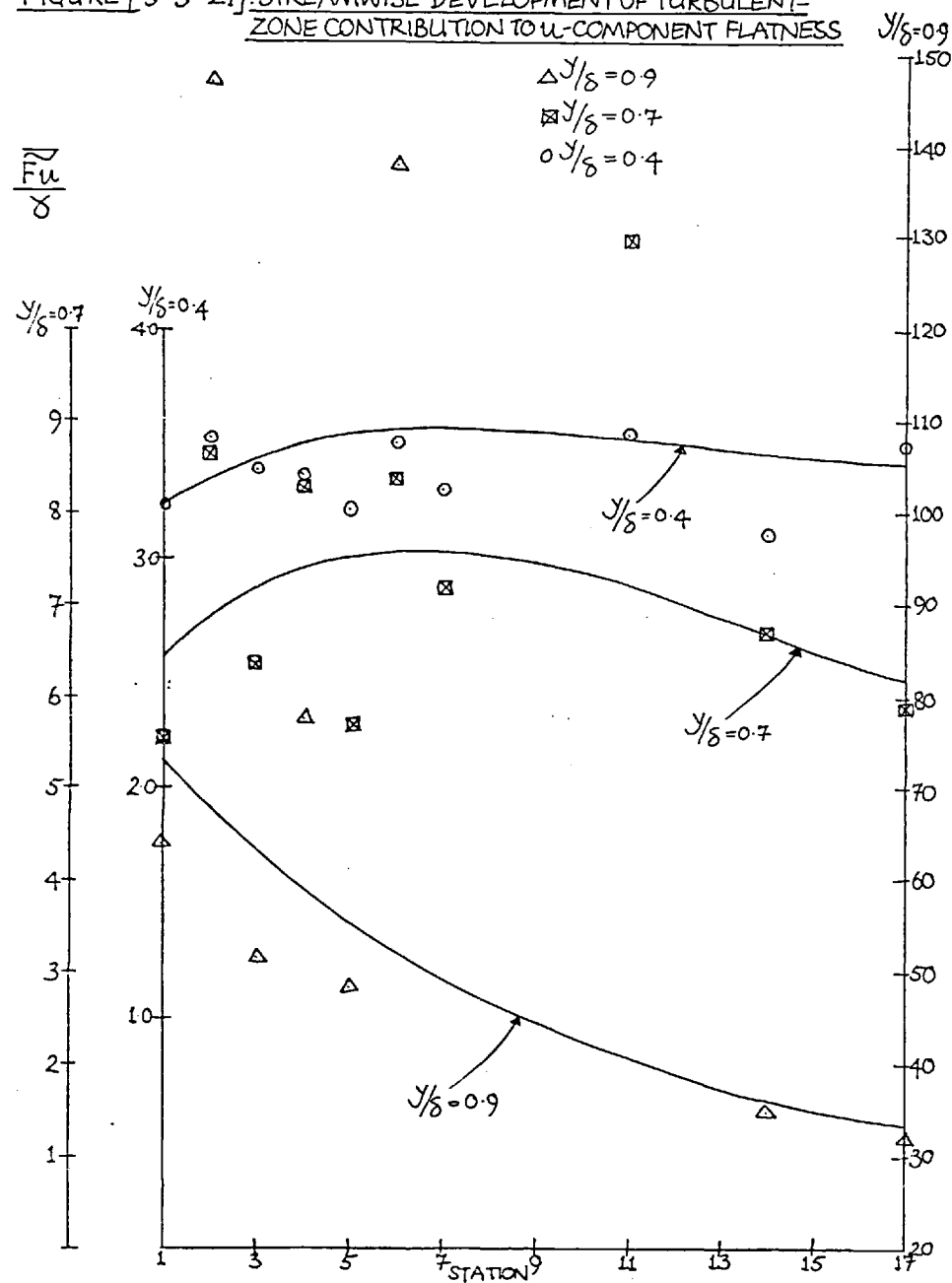


FIGURE [5-5-22] STREAMWISE DEVELOPMENT OF TURBULENT-ZONE CONTRIBUTION TO FLATNESS FACTOR (V COMP.)

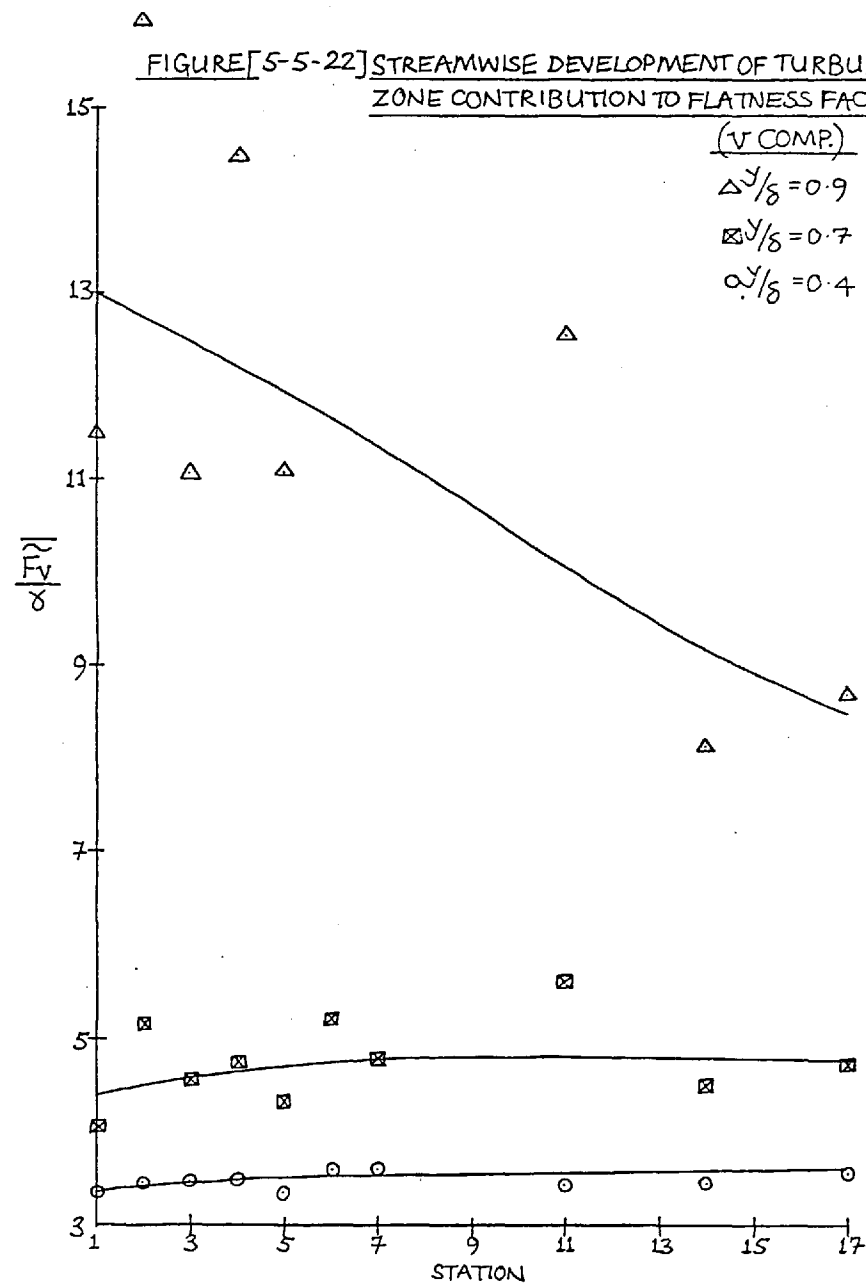
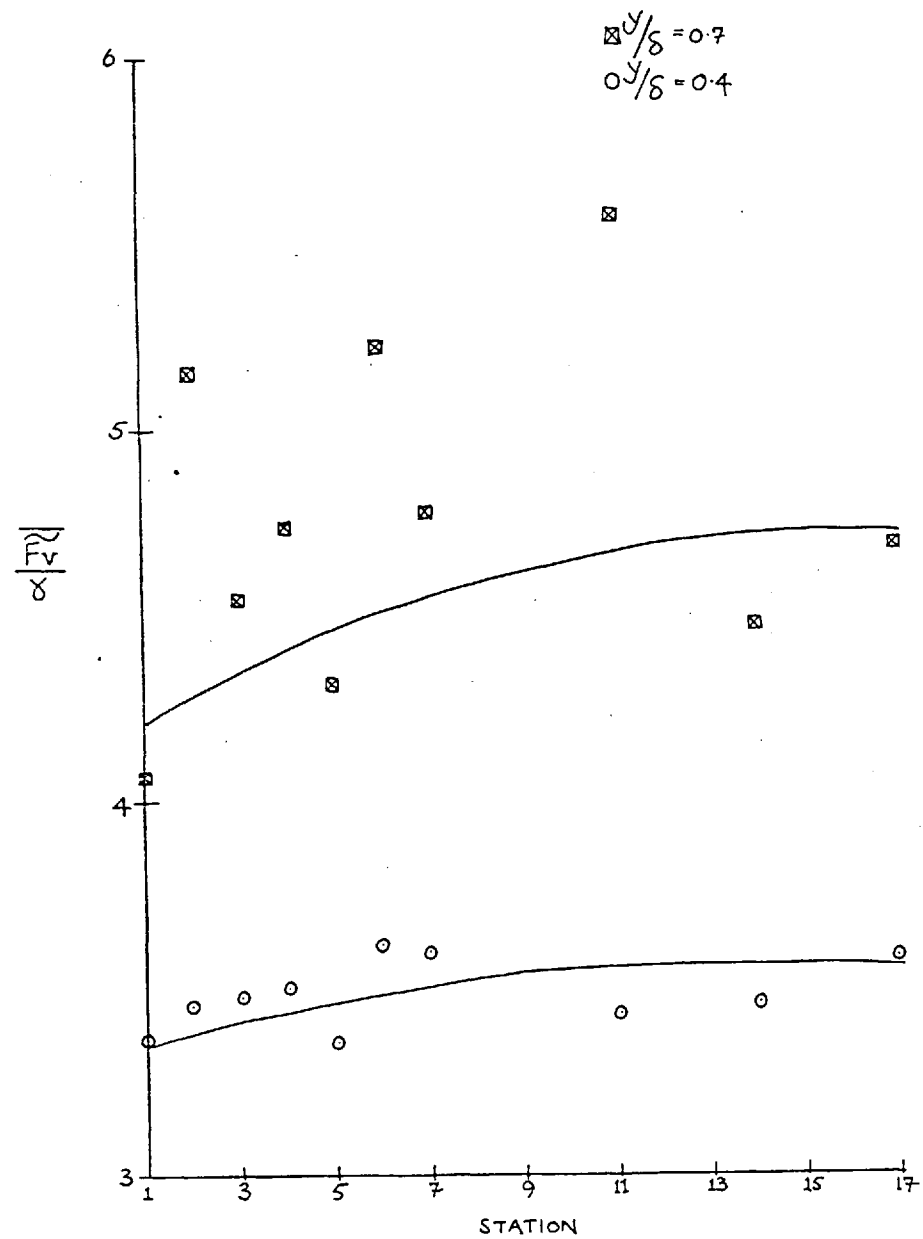
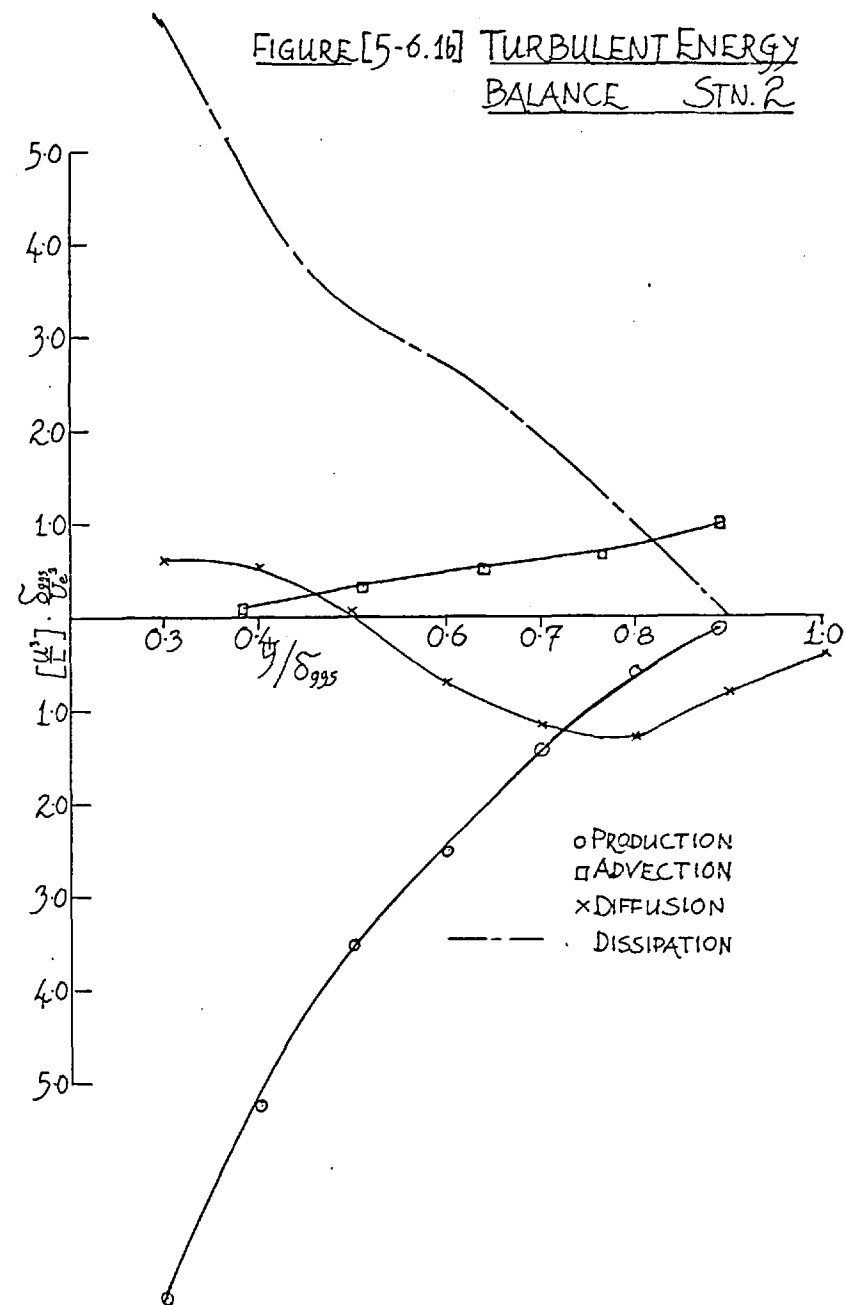
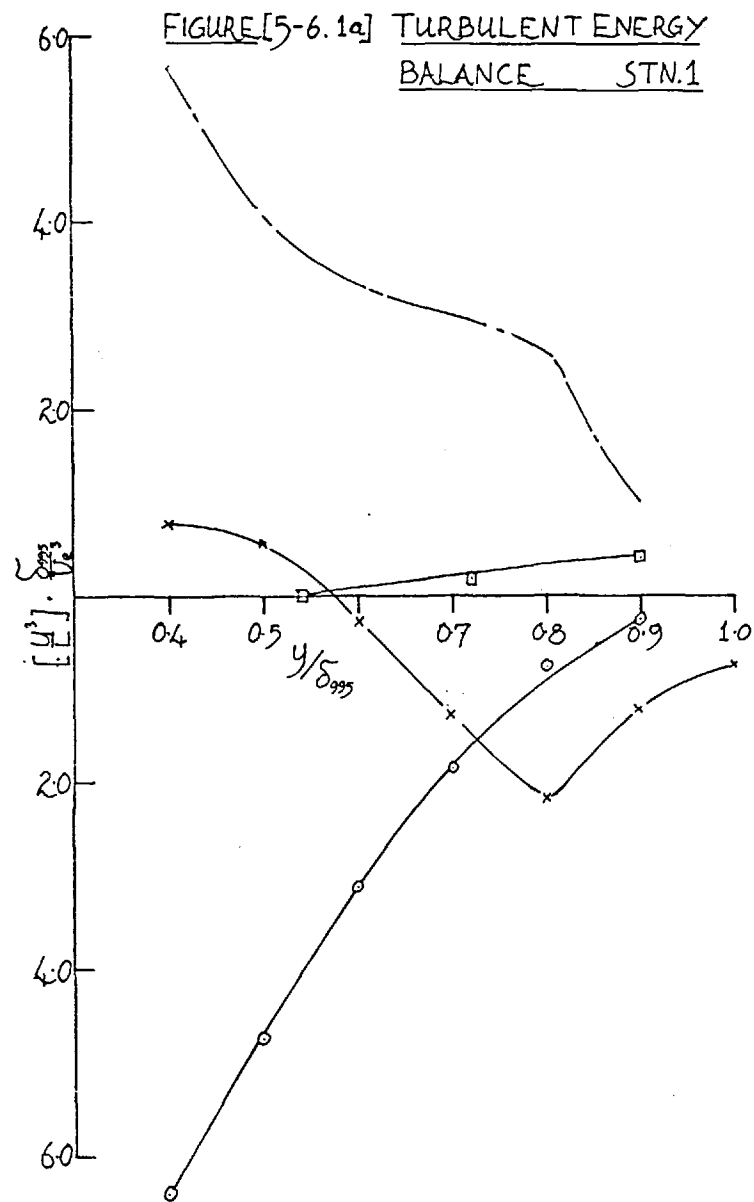
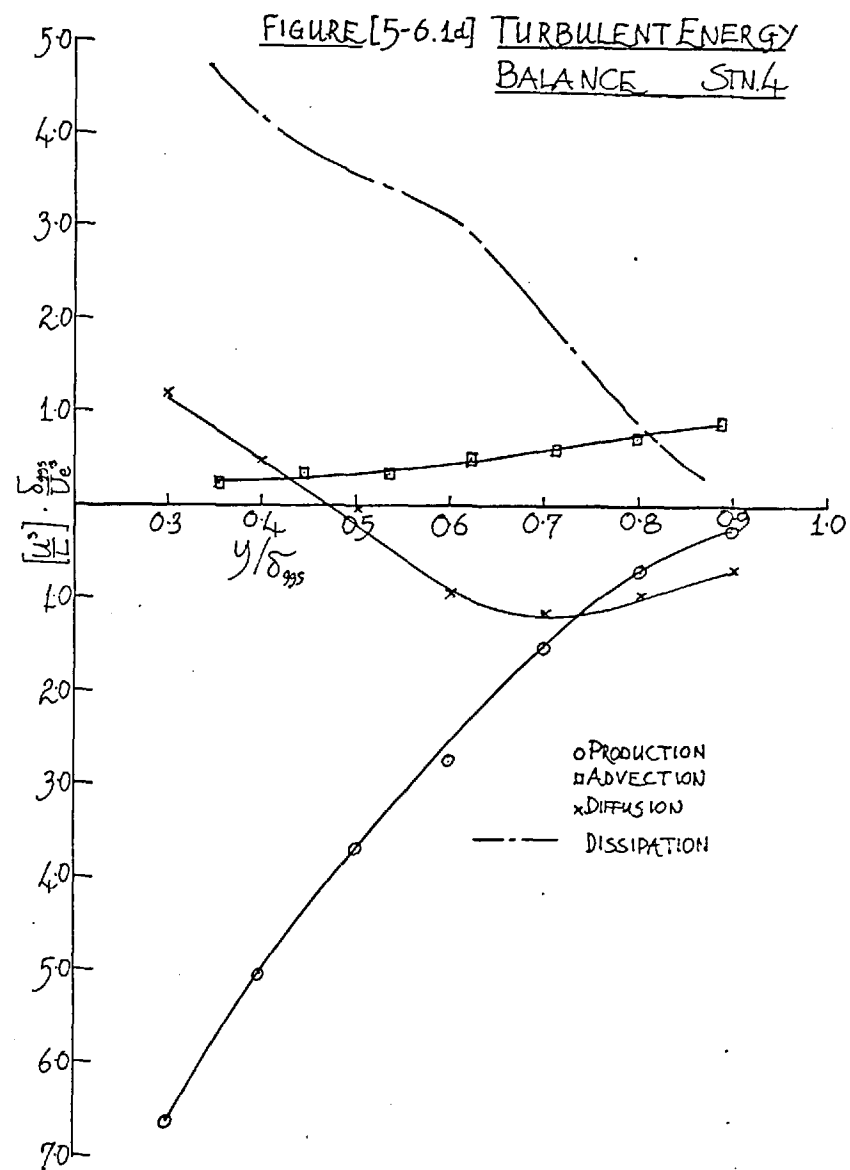
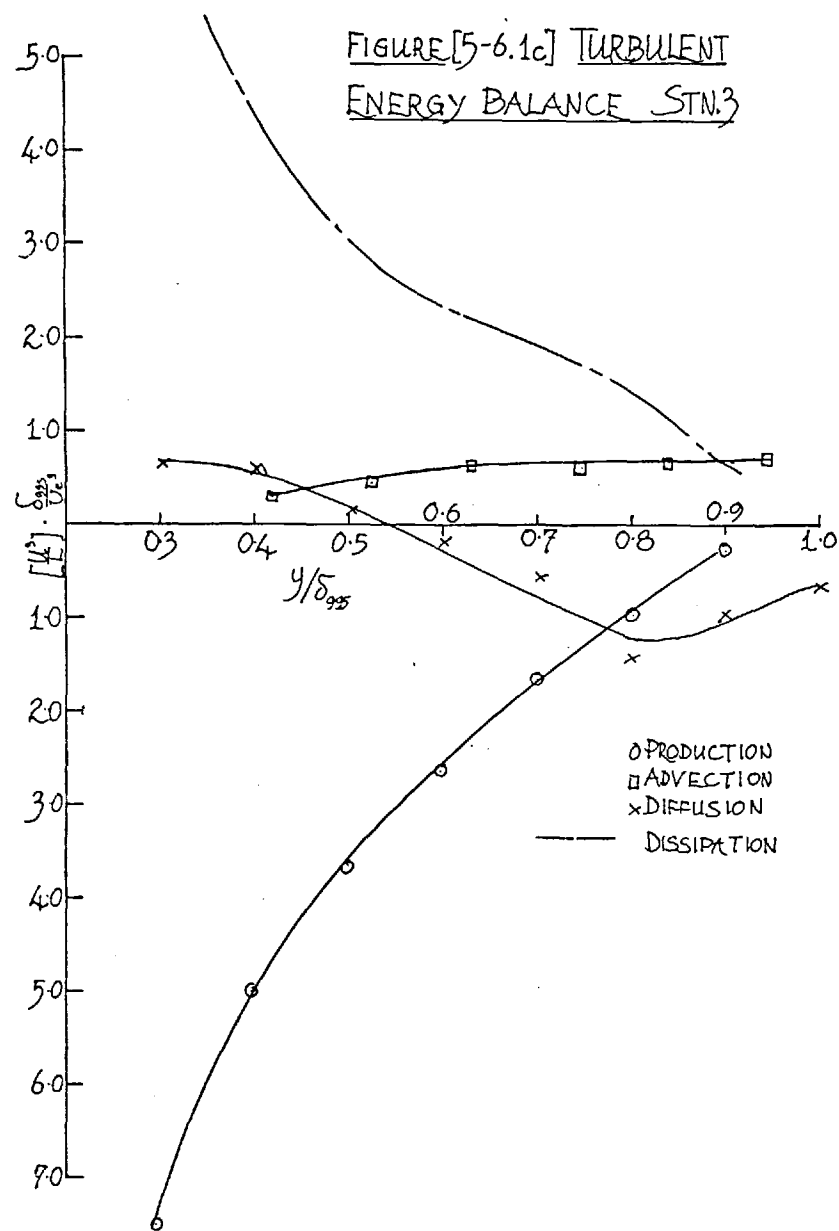
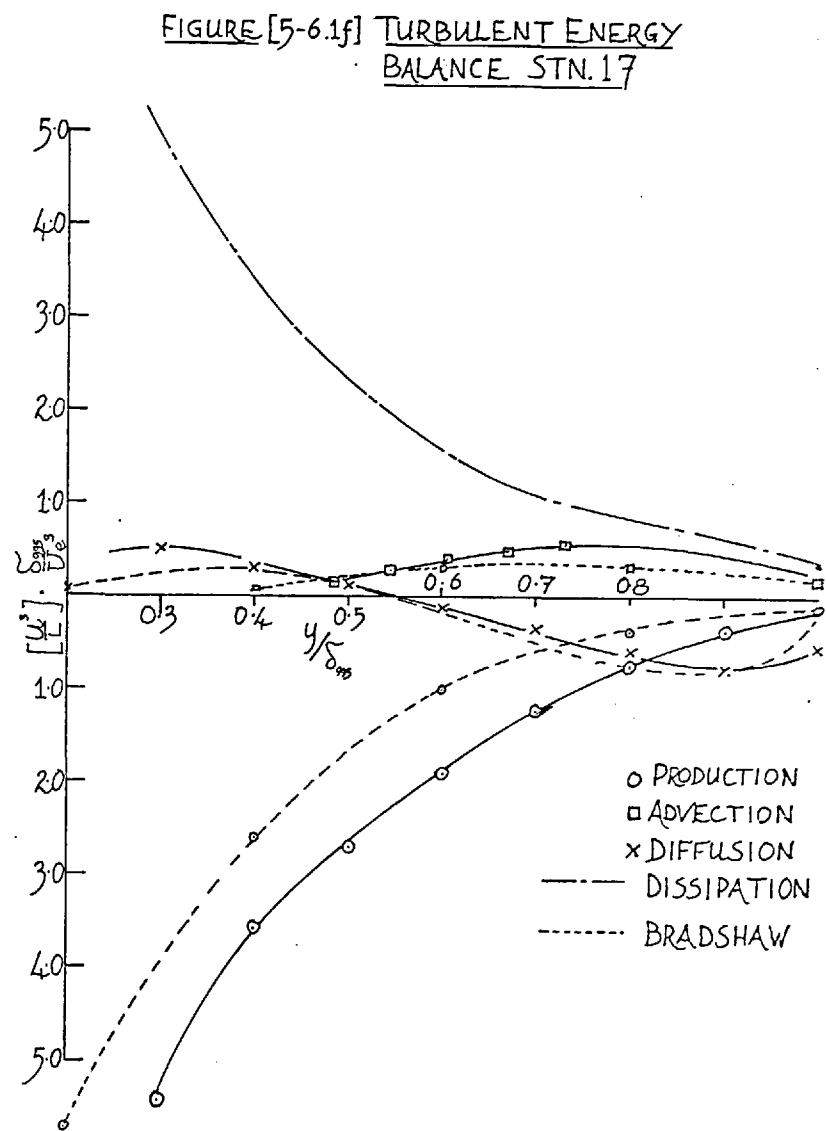
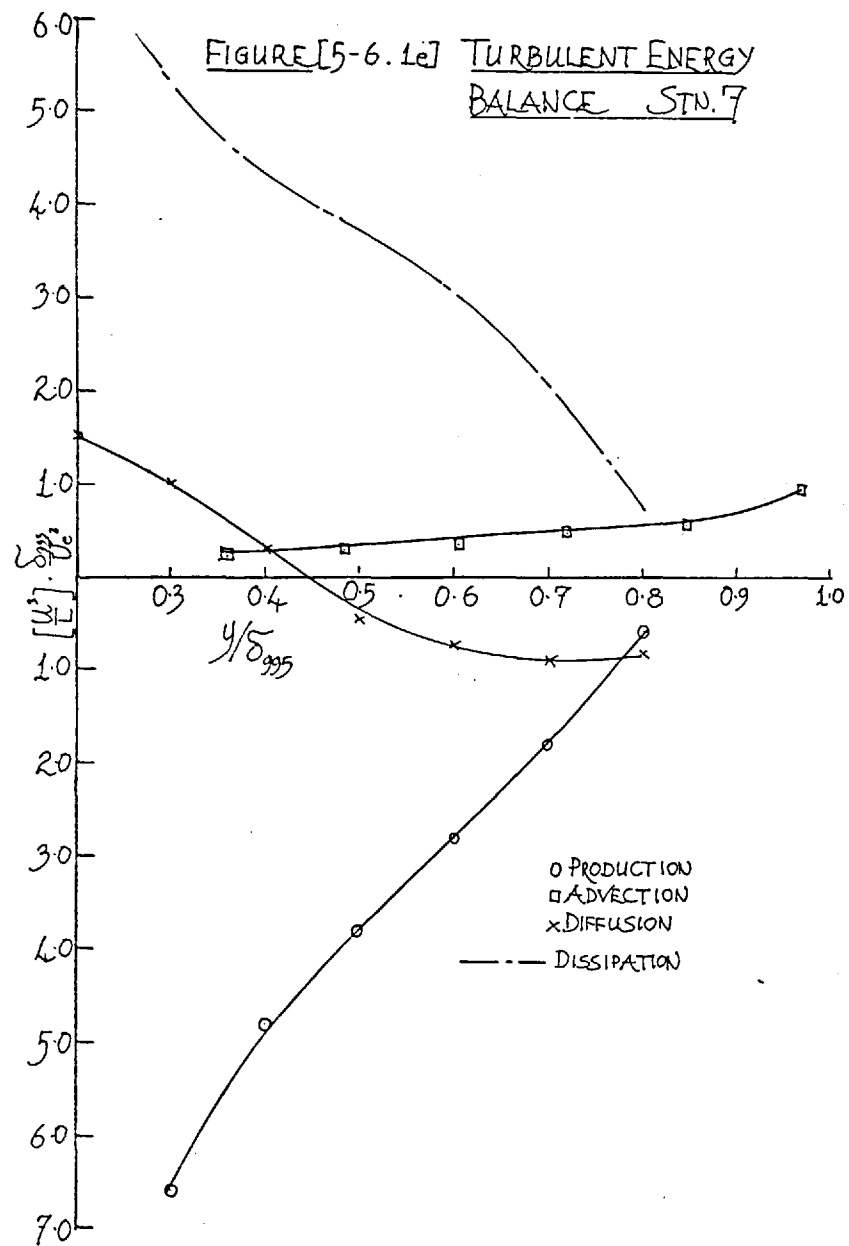


FIGURE [5-5-22a] MAGNIFIED SCALE









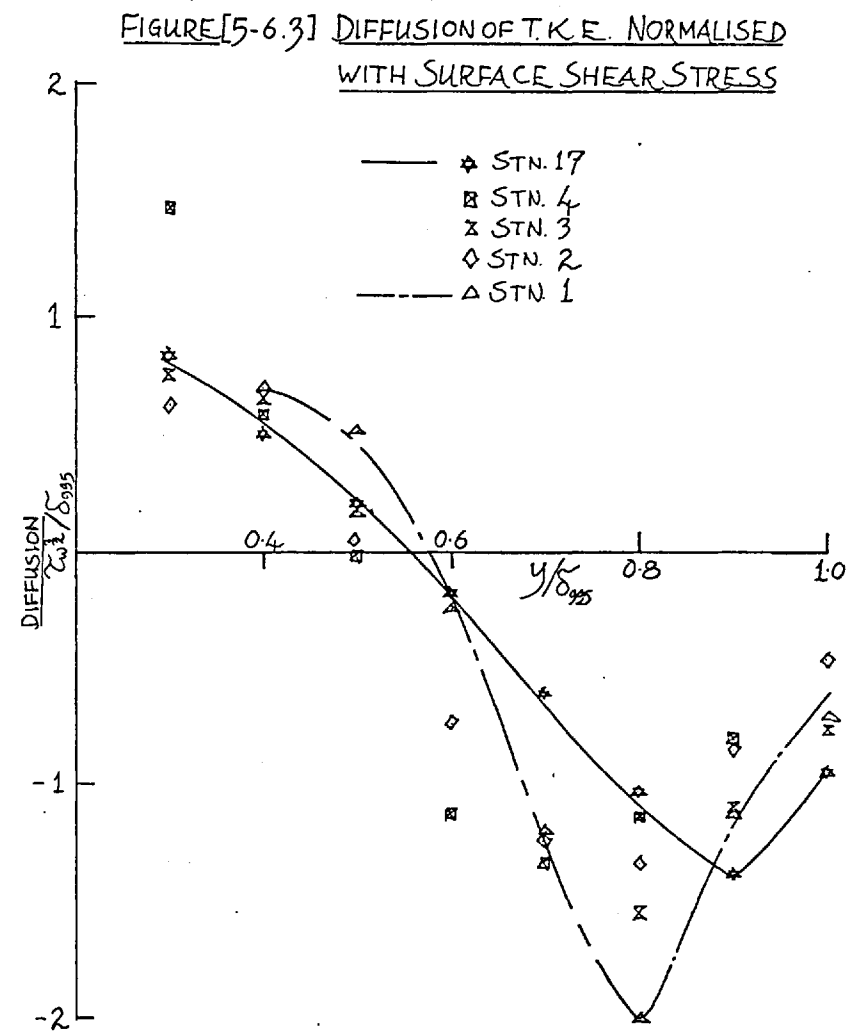
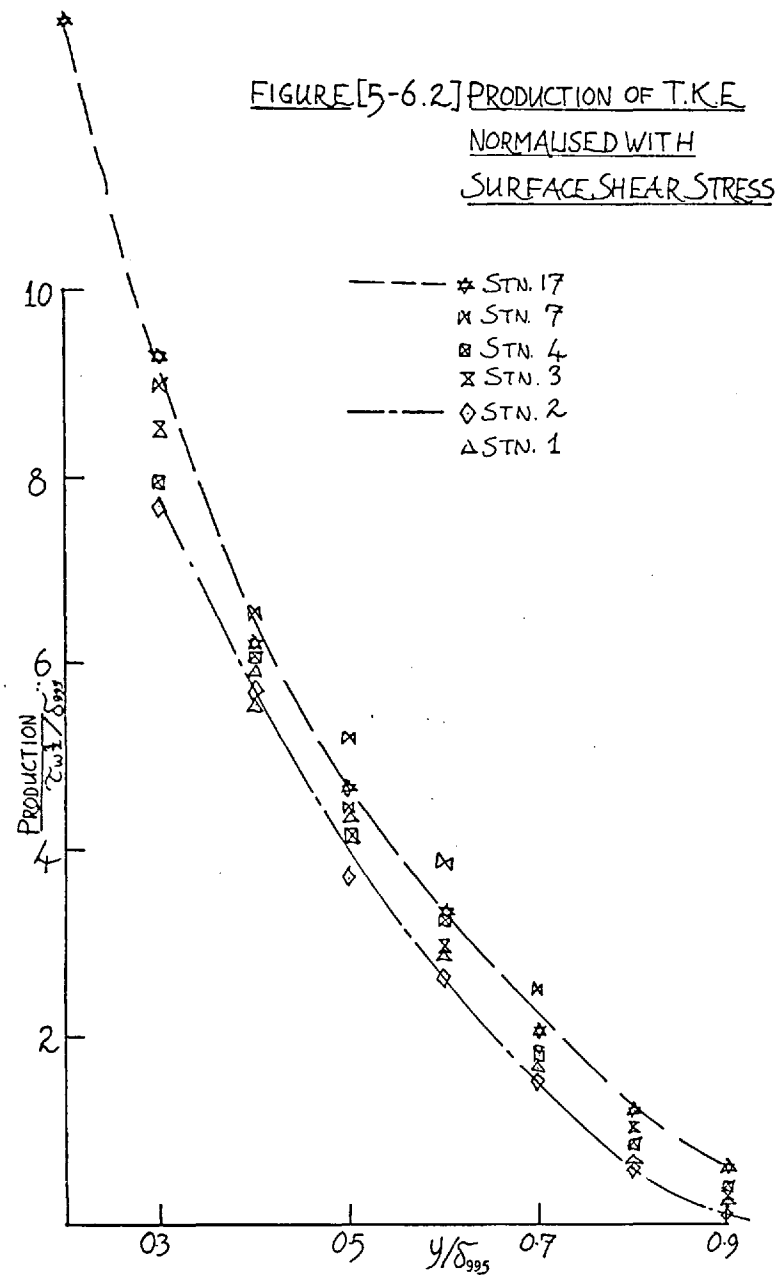


FIGURE [5-6.4] ADVECTION OF T.K.E NORMALISED
WITH SURFACE SHEAR STRESS

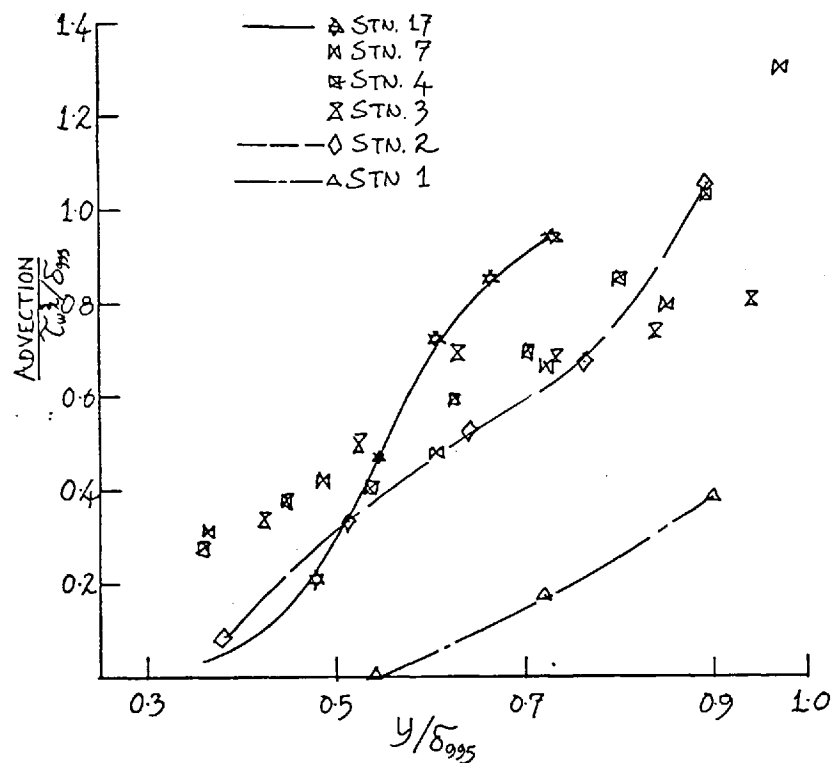


FIGURE [5-6.5] STREAMWISE DEVELOPMENT OF TURBULENT
ENERGY PRODUCTION

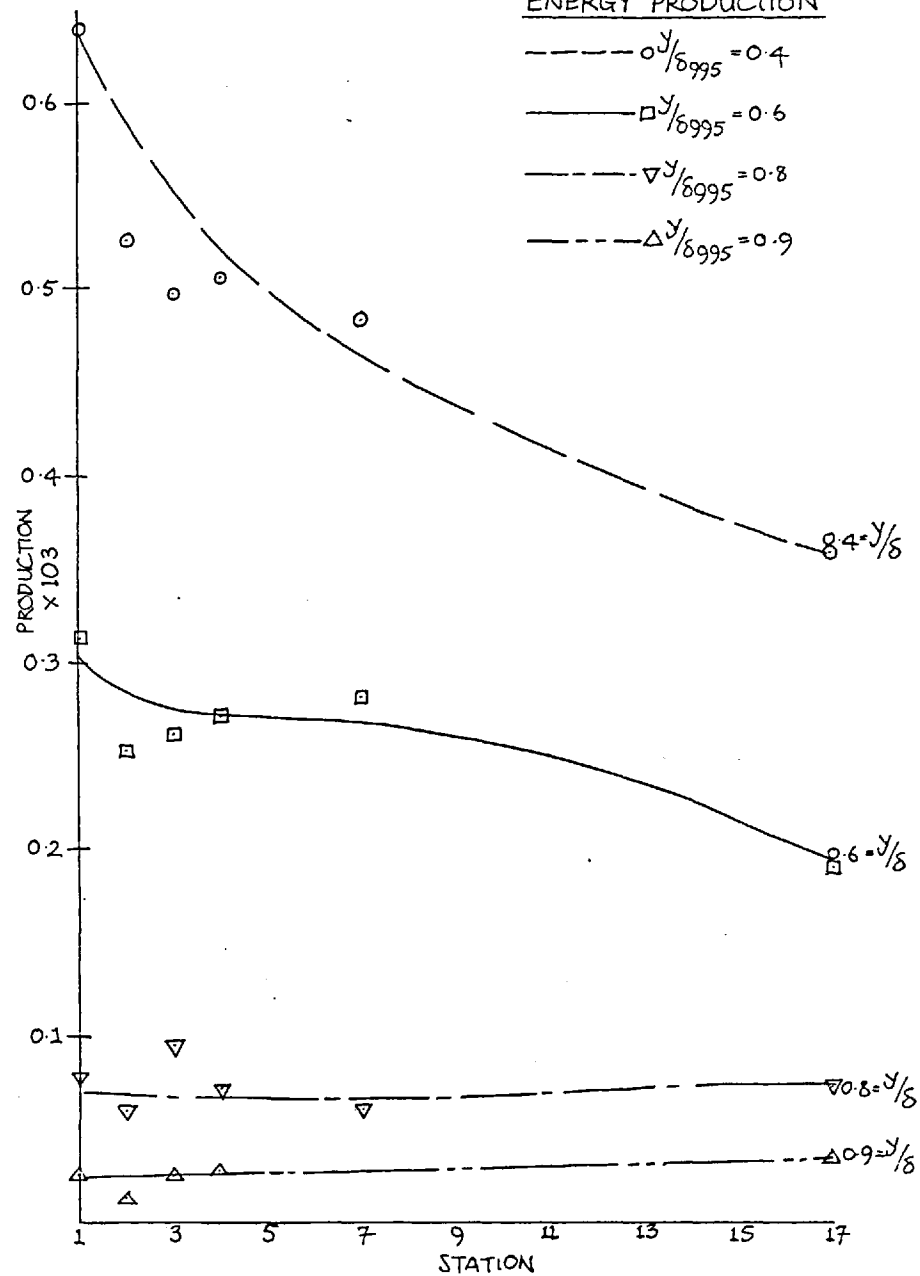


FIGURE [5-6.6] STREAMWISE DEVELOPMENT OF NORMALISED
PRODUCTION

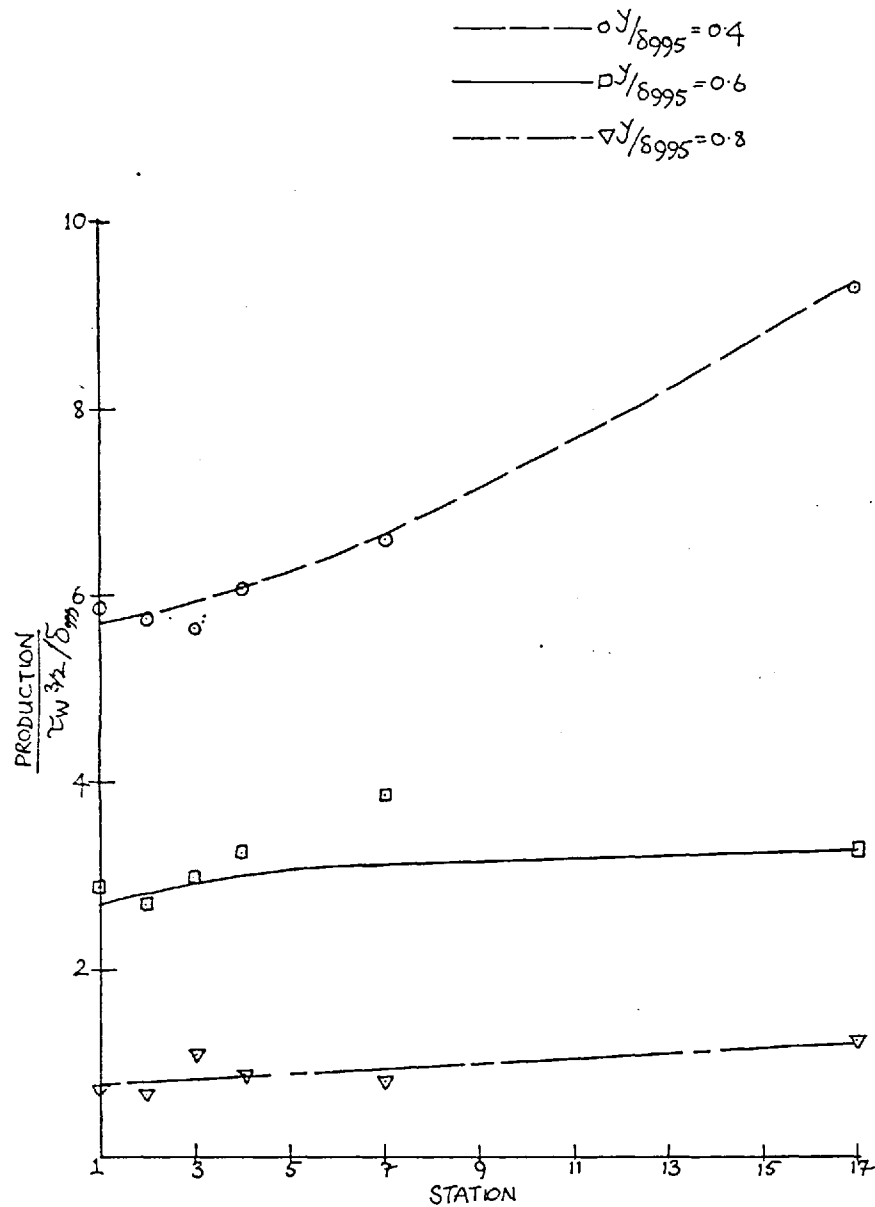


FIGURE [5-6.7] STREAMWISE DEVELOPMENT
OF NORMALISED TURBULENT ENERGY PRODUCTION

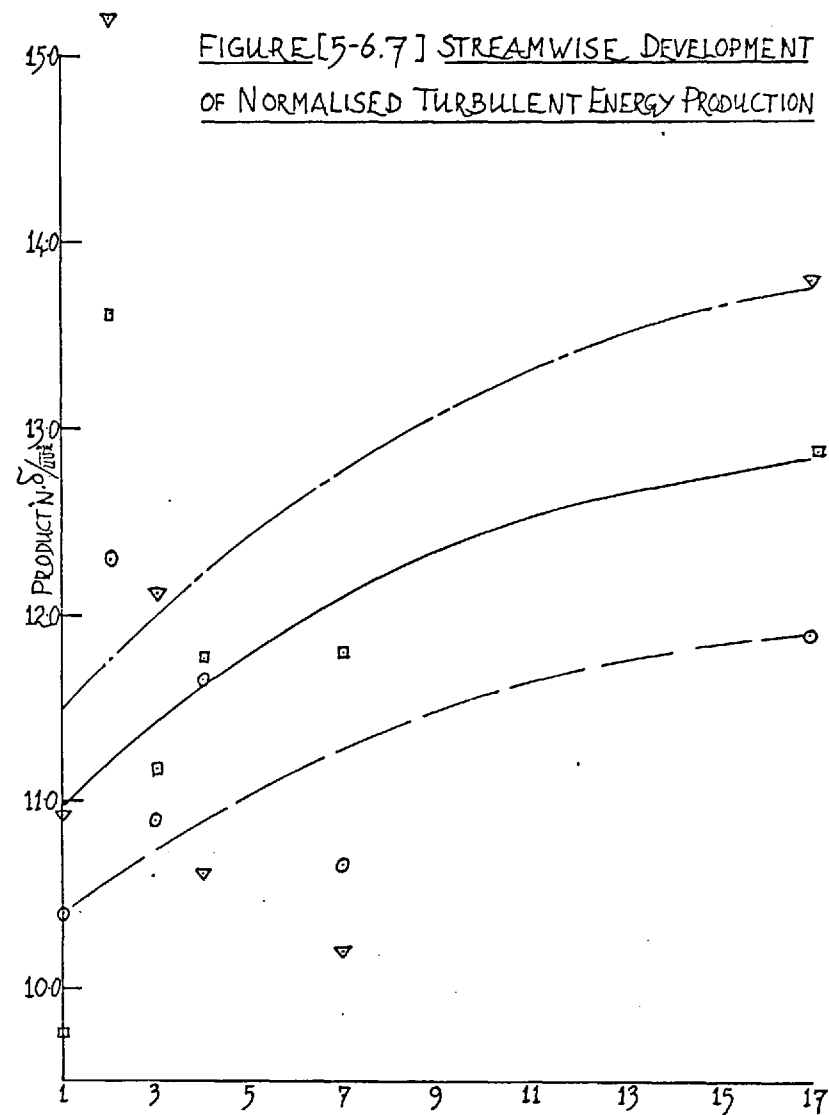


FIGURE [5-6.8] STREAMWISE DEVELOPMENT OF TURBULENT ENERGY
DIFFUSION

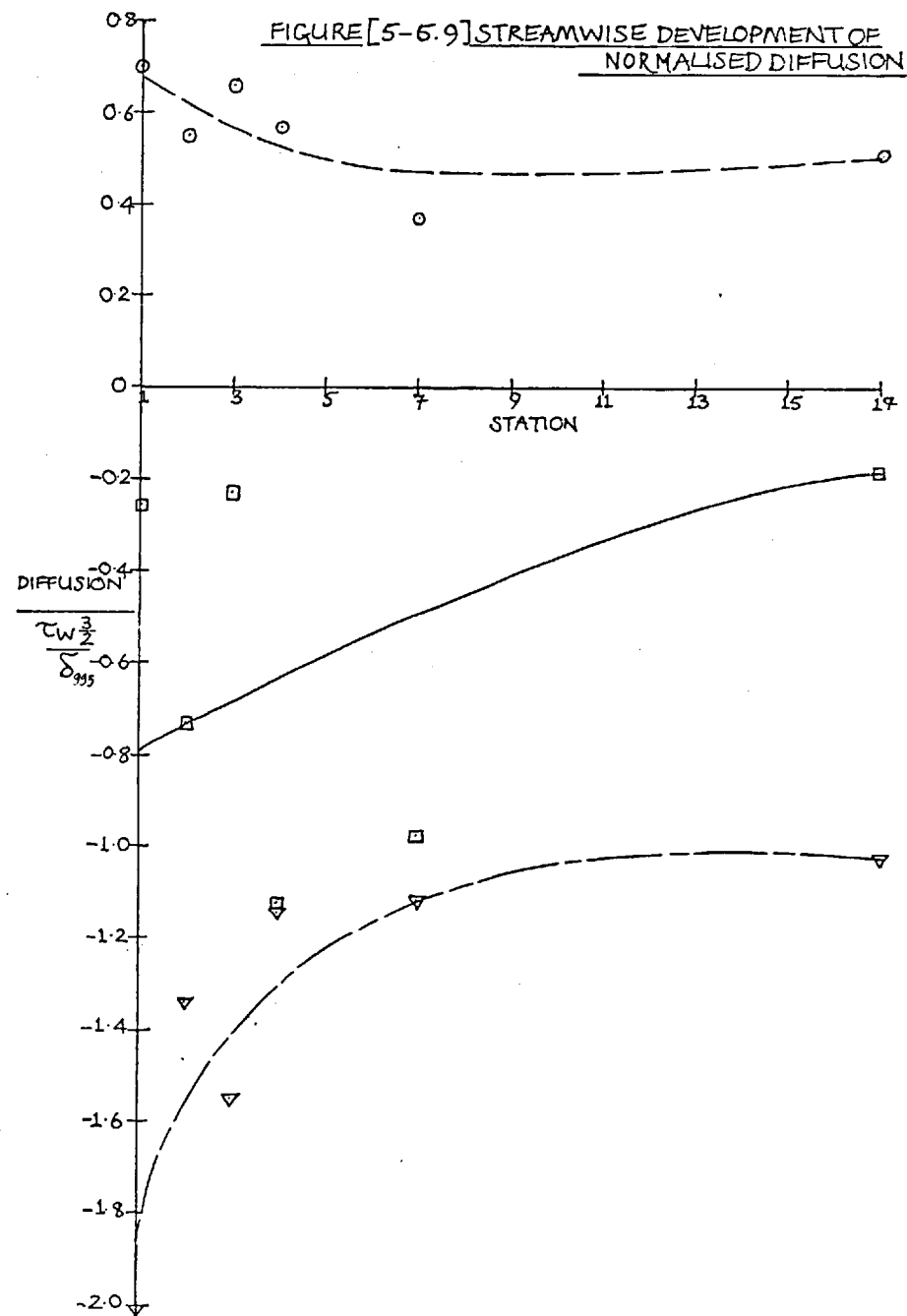
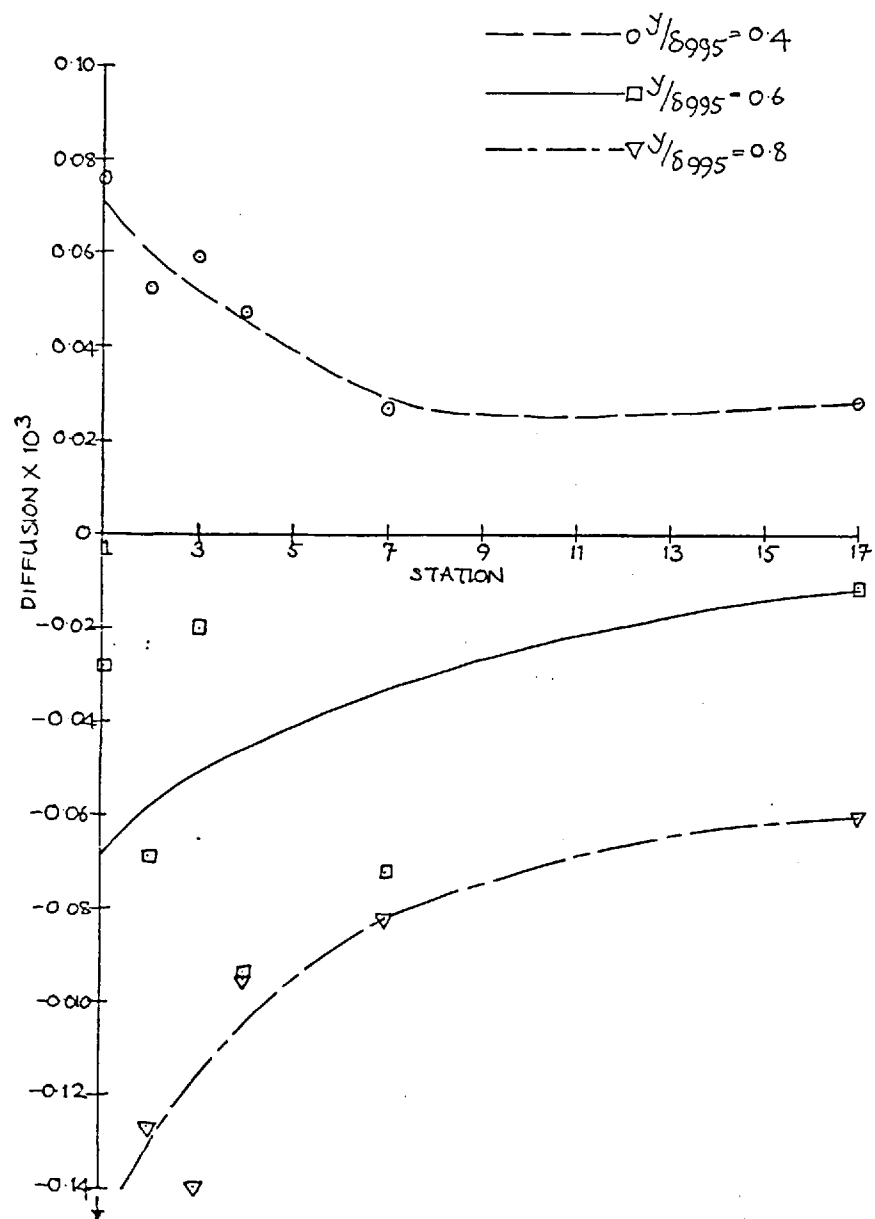


FIGURE [5-6.10] STREAMWISE DEVELOPMENT OF
NORMALISED TURBULENT ENERGY DIFFUSION

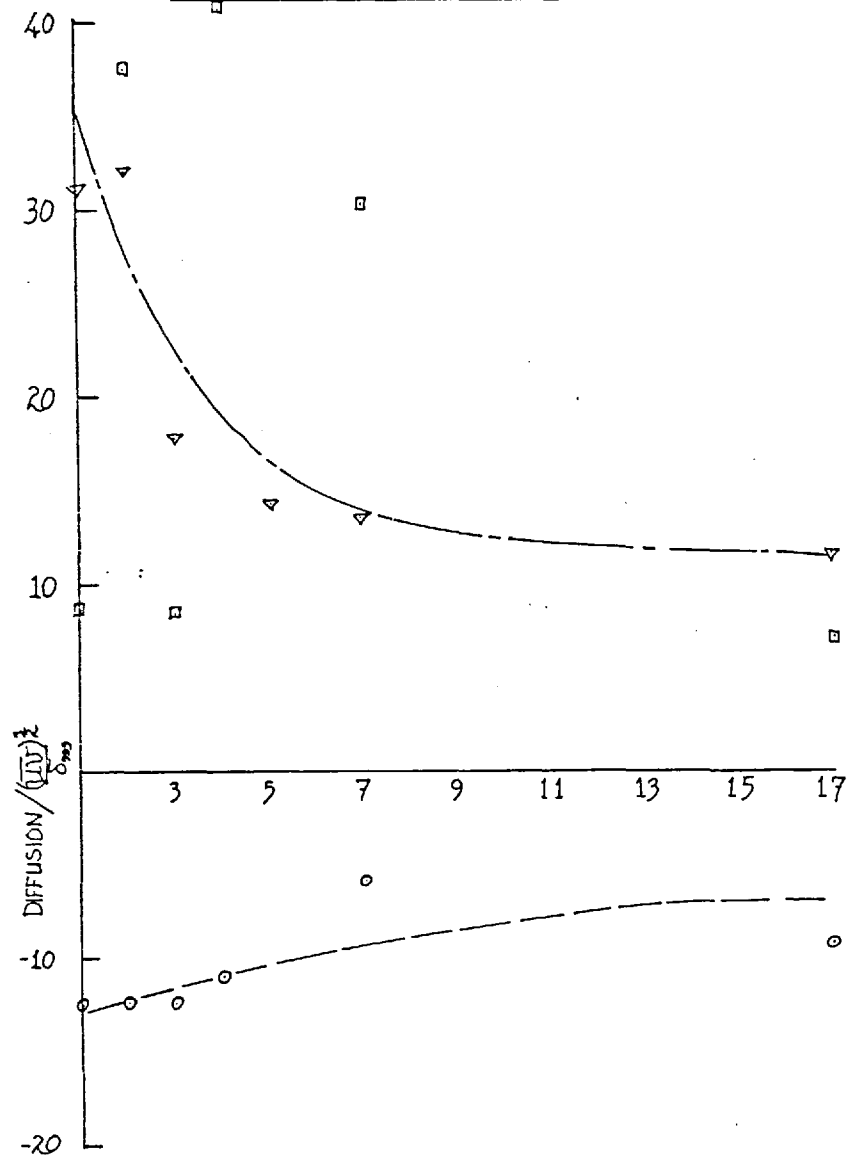
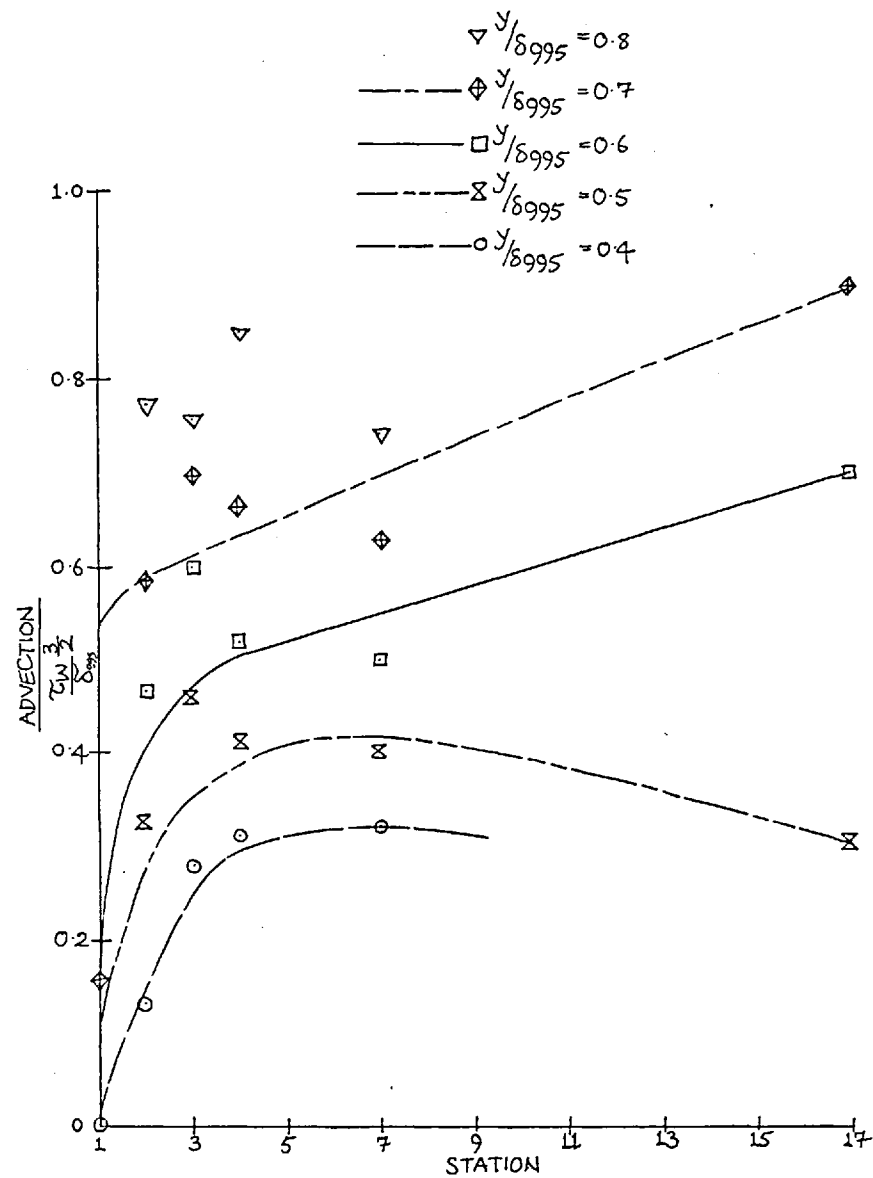
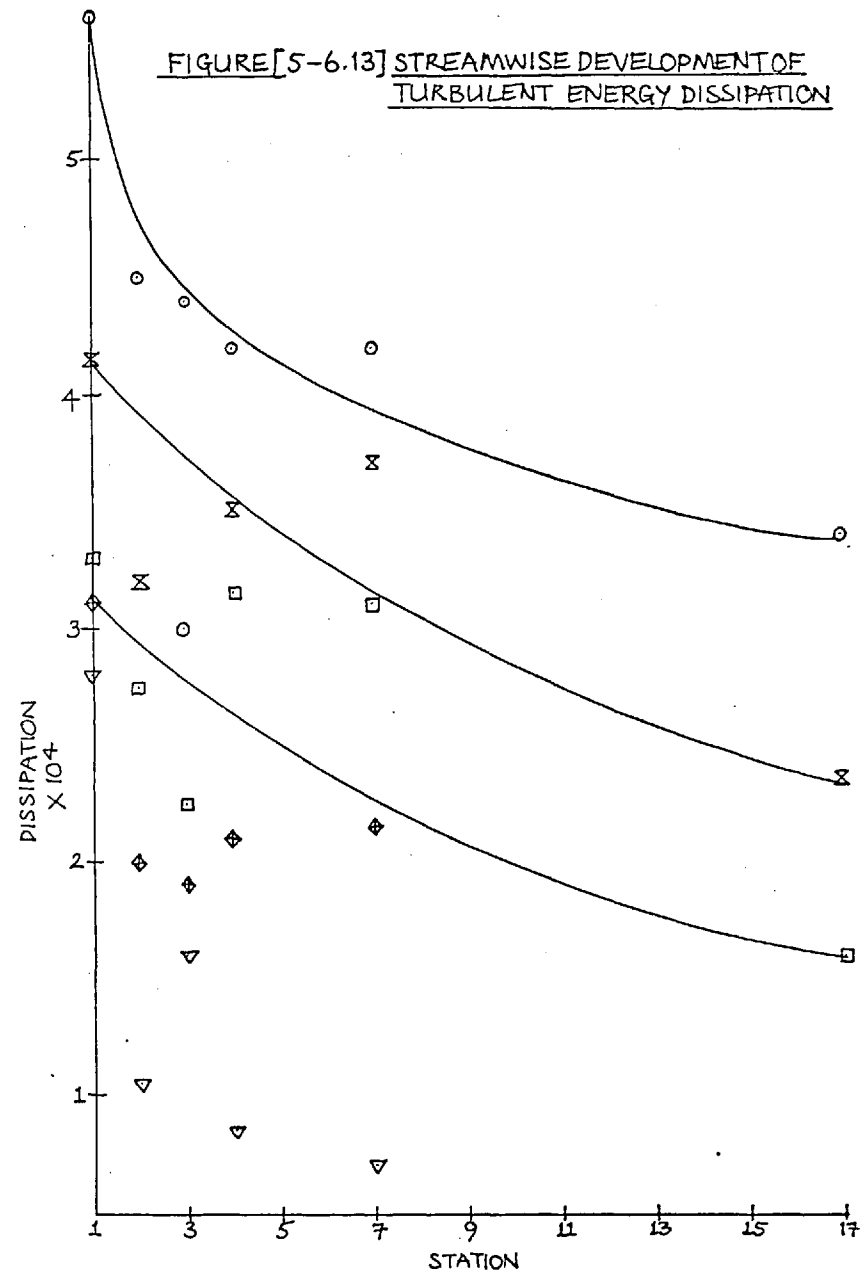
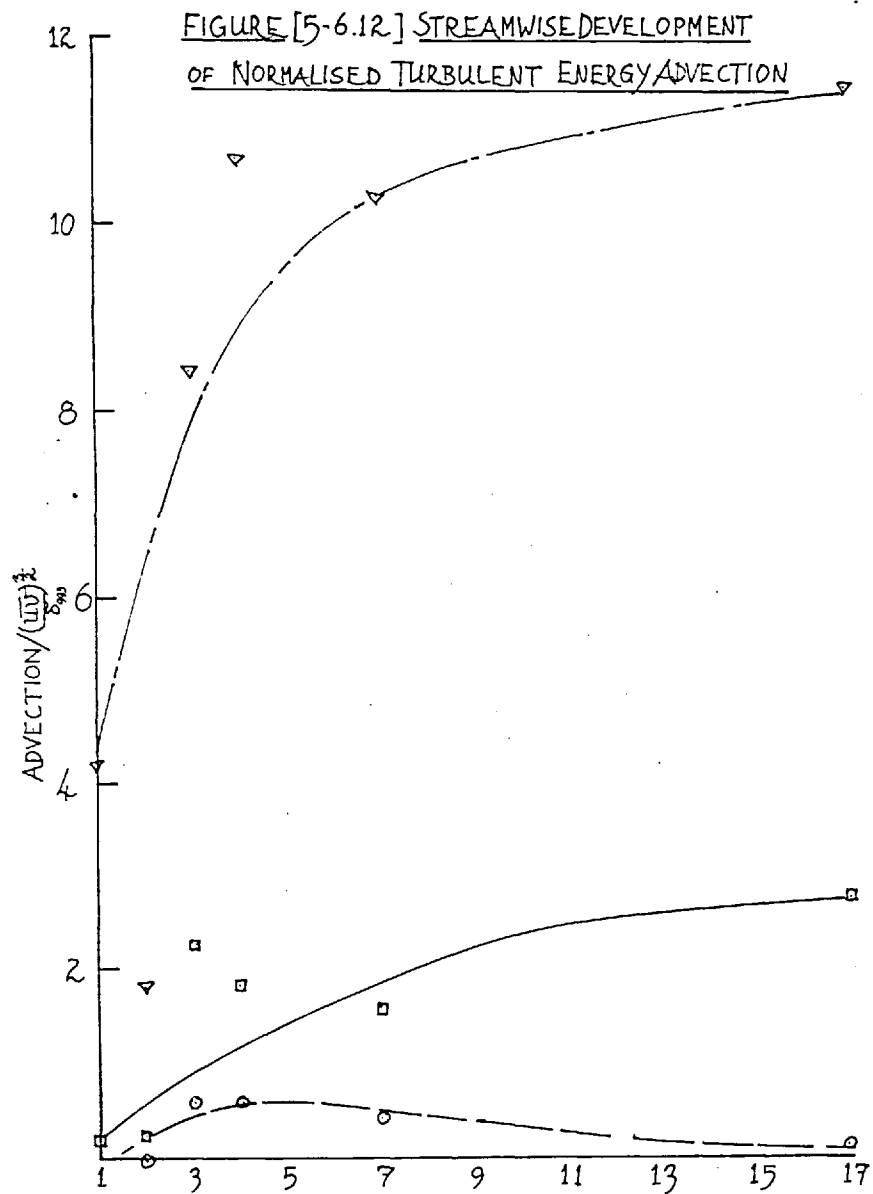
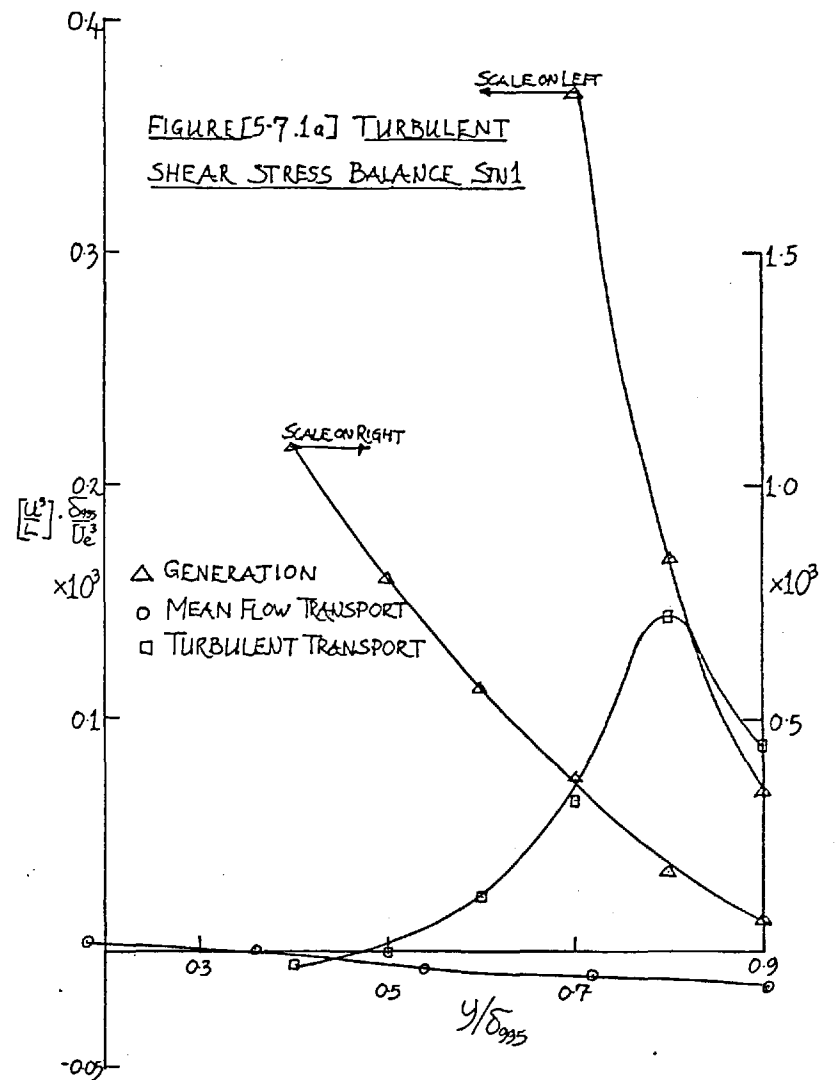
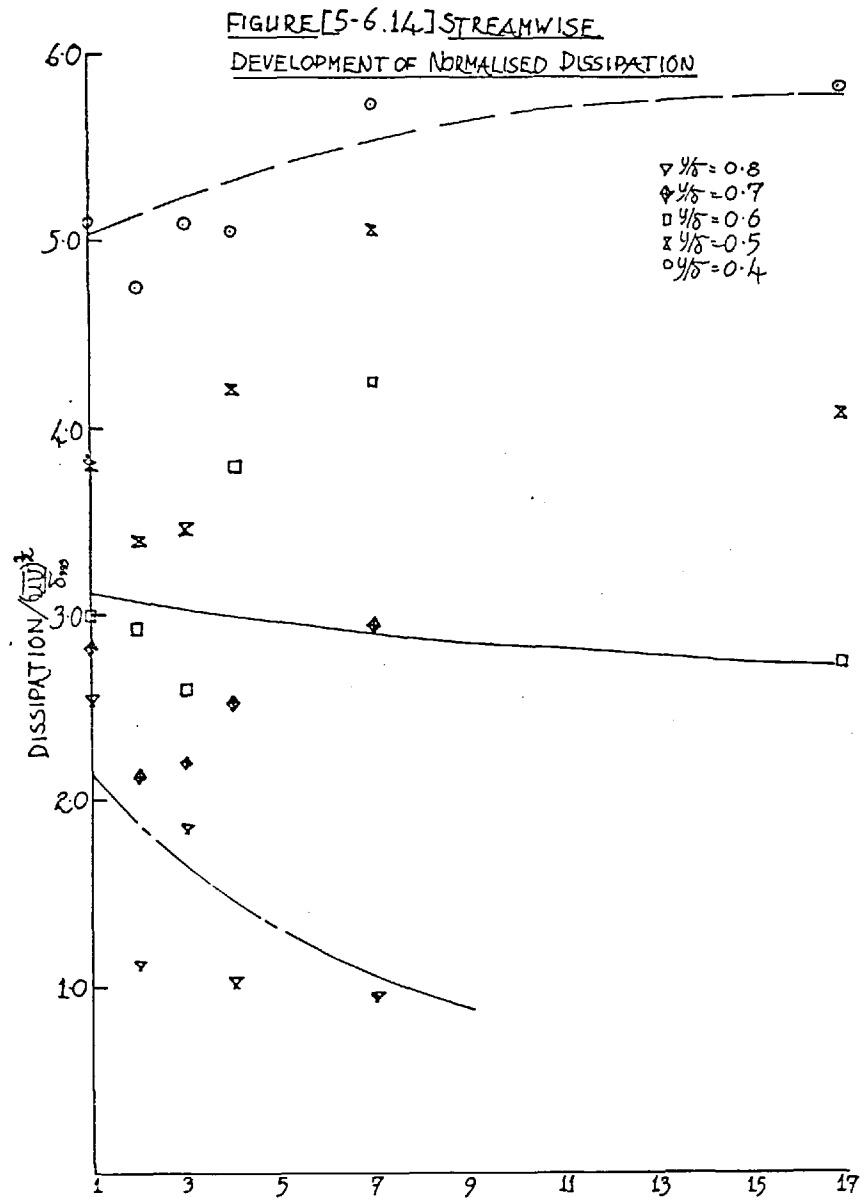


FIGURE [5-6.11] STREAMWISE DEVELOPMENT OF NORMALISED ADVECTION







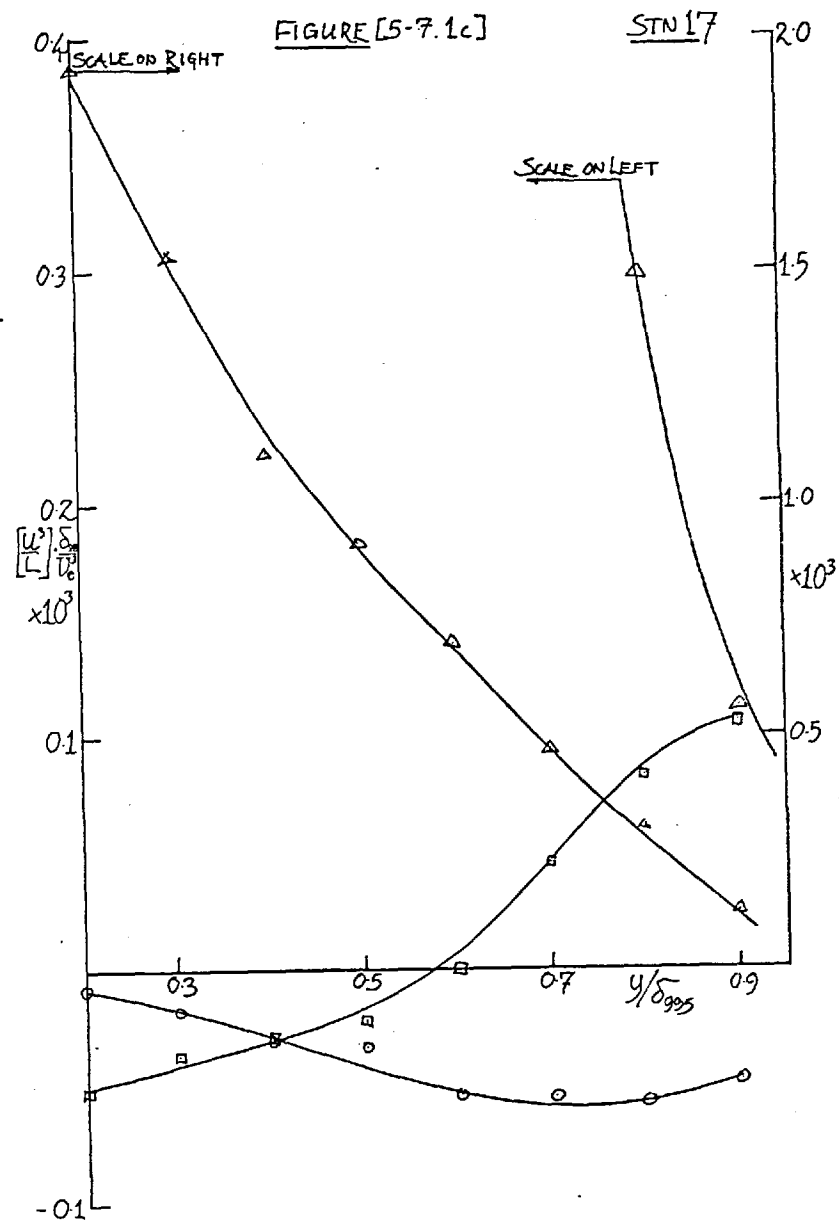
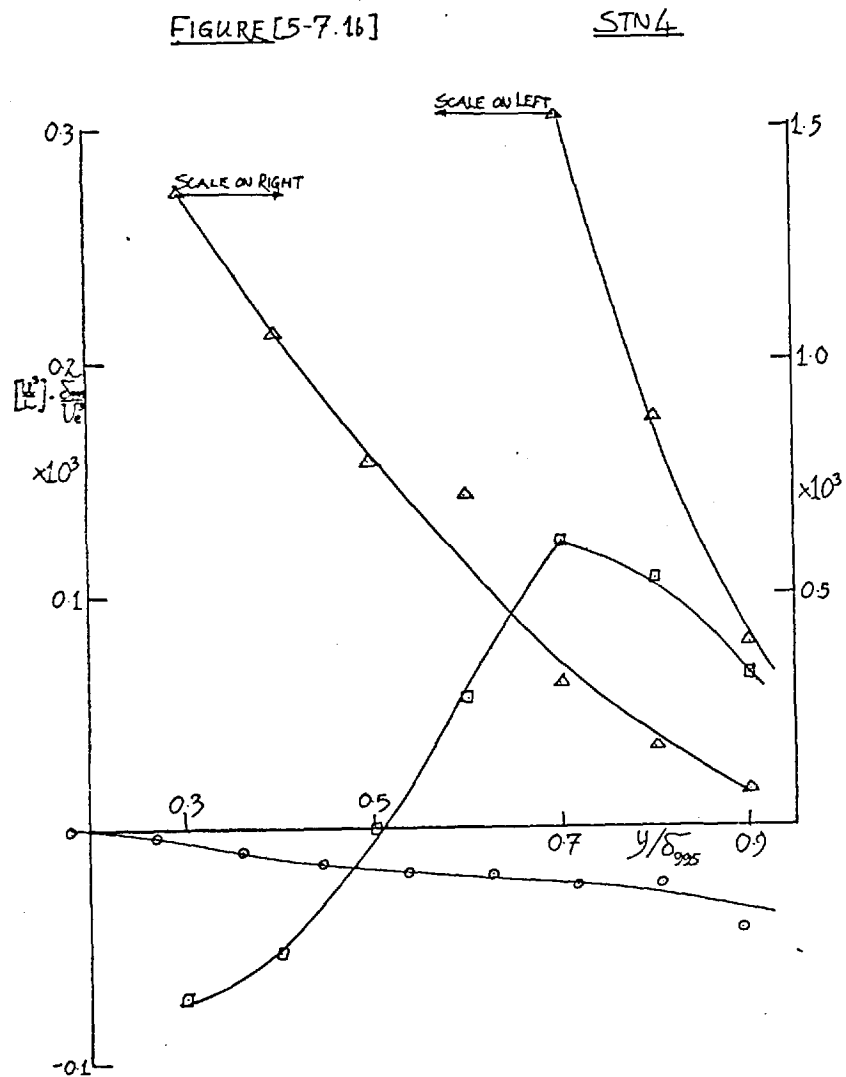


FIGURE [5-7.2] NORMALISED GENERATION OF
TURBULENT SHEAR STRESS

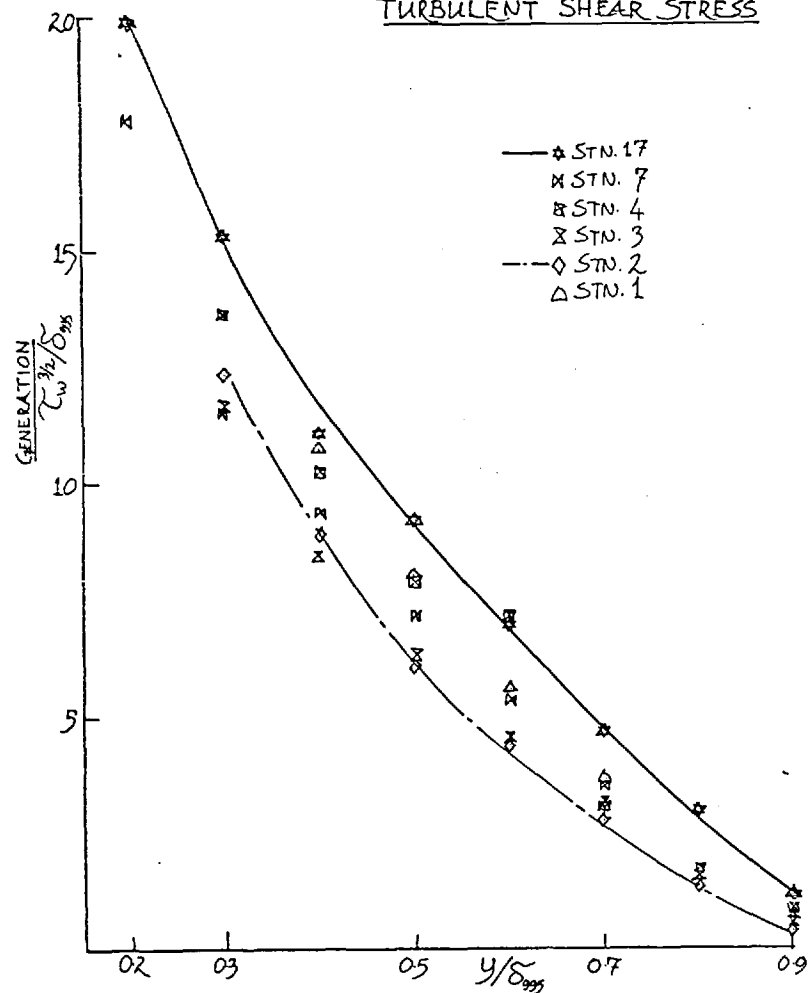


FIGURE [5-7.3] NORMALISED MEAN FLOW TRANSPORT
OF SHEAR STRESS

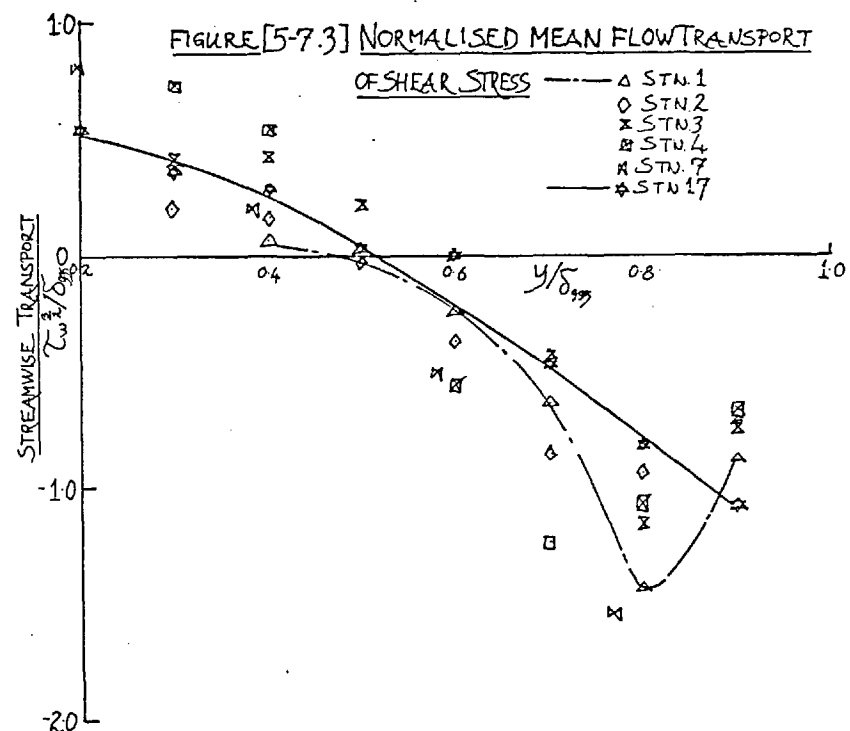


FIGURE [5-7.4] NORMALISED TURBULENT TRANSPORT OF
TURBULENT SHEAR STRESS

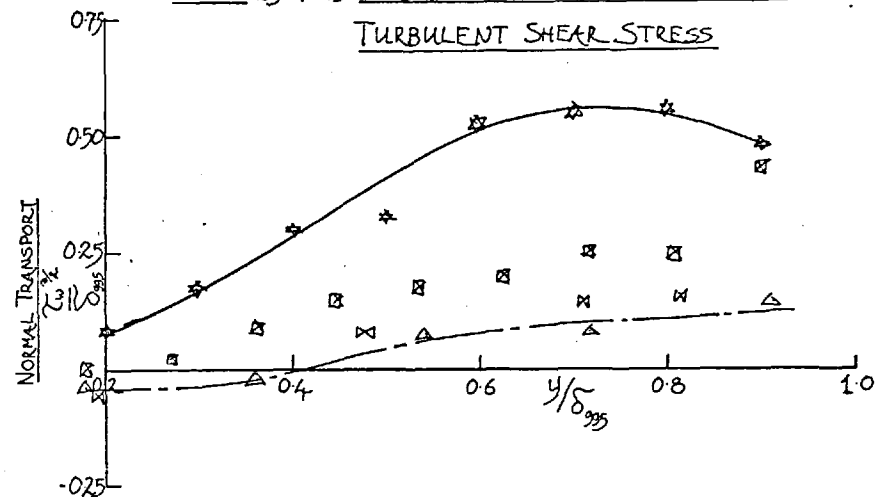


FIGURE [5-7.5] STREAMWISE DEVELOPMENT OF NORMALISED SHEAR STRESS GENERATION

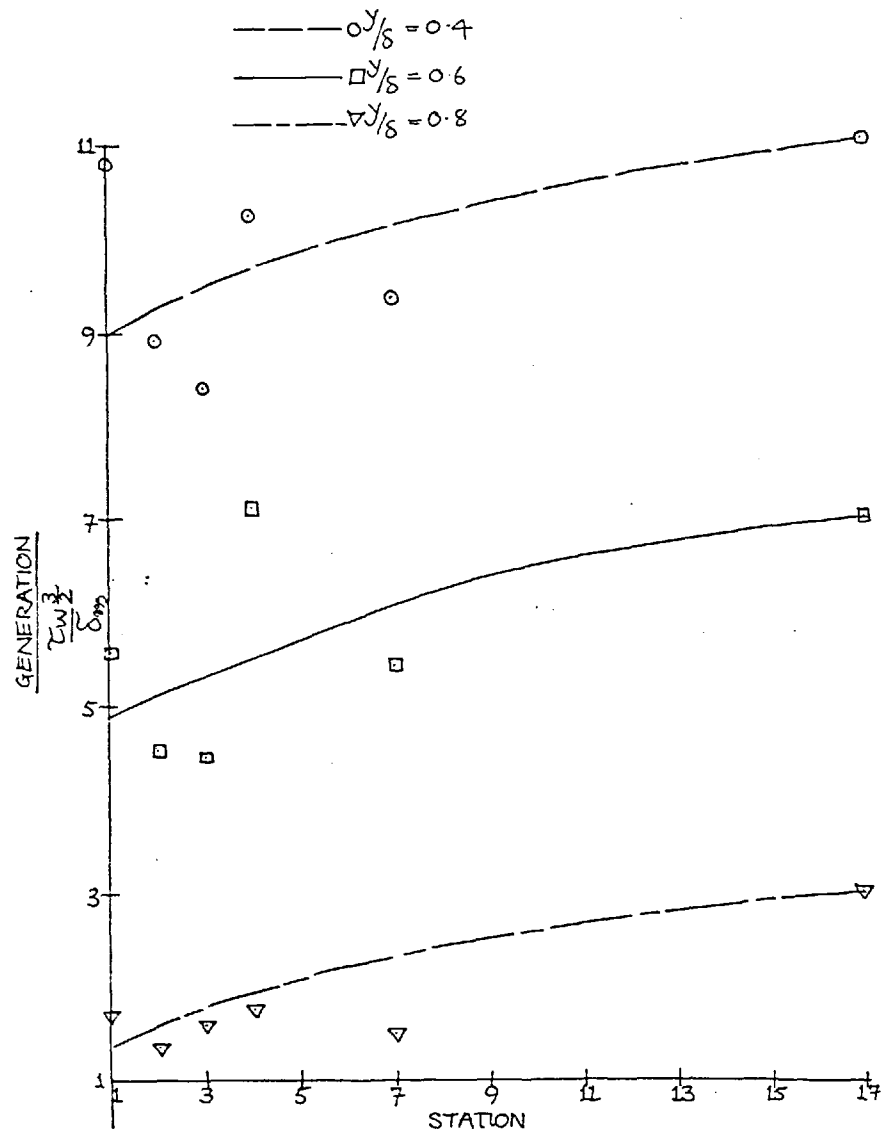


FIGURE [5-7.6] STREAMWISE DEVELOPMENT OF NORMALISED SHEAR STRESS GENERATION

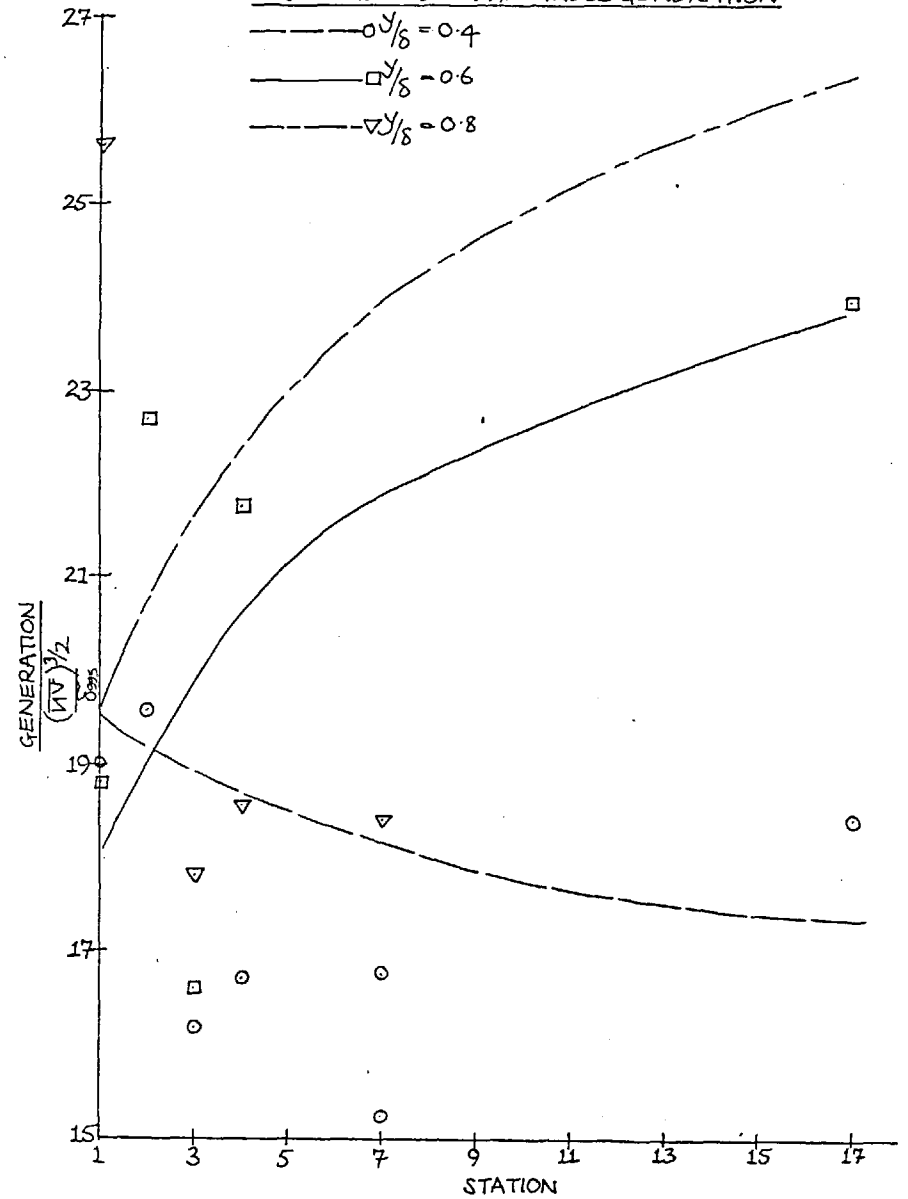


FIGURE [5-7.7] STREAMWISE DEVELOPMENT OF NORMALISED SHEAR STRESS TURBULENT TRANSPORT

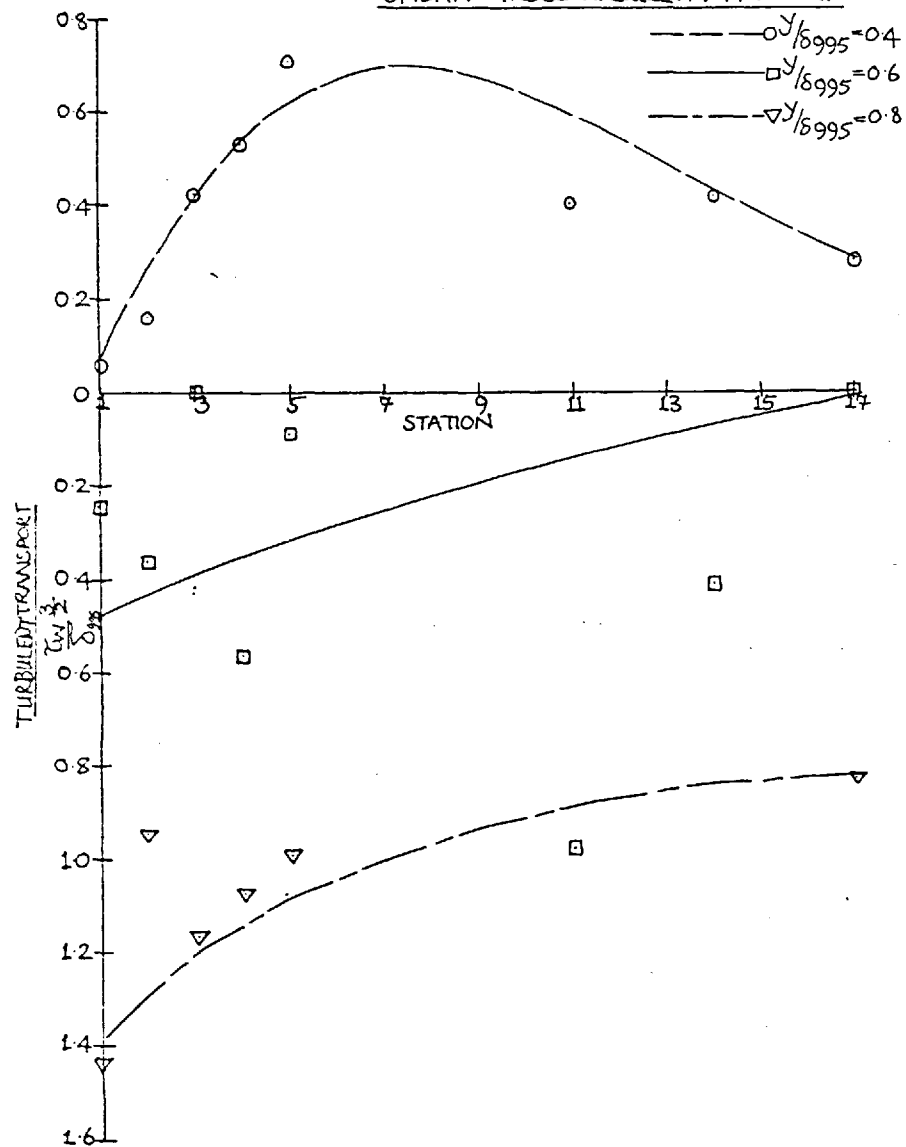


FIGURE [5-7.8] STREAMWISE DEVELOPMENT OF NORMALISED SHEAR STRESS TURBULENT TRANSPORT

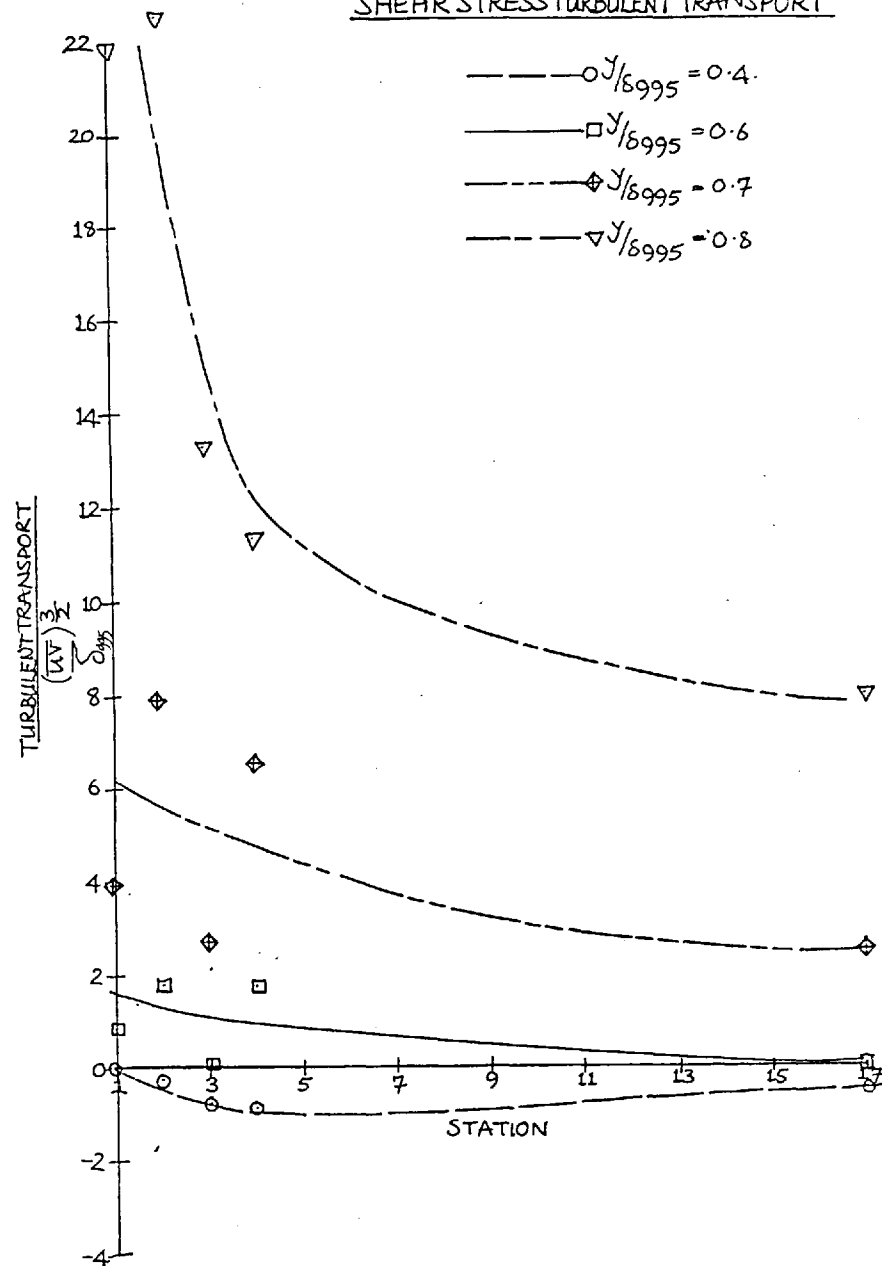


FIGURE [5-7.9] STREAMWISE DEVELOPMENT OF NORMALISED
SHEAR STRESS MEAN FLOW TRANSPORT

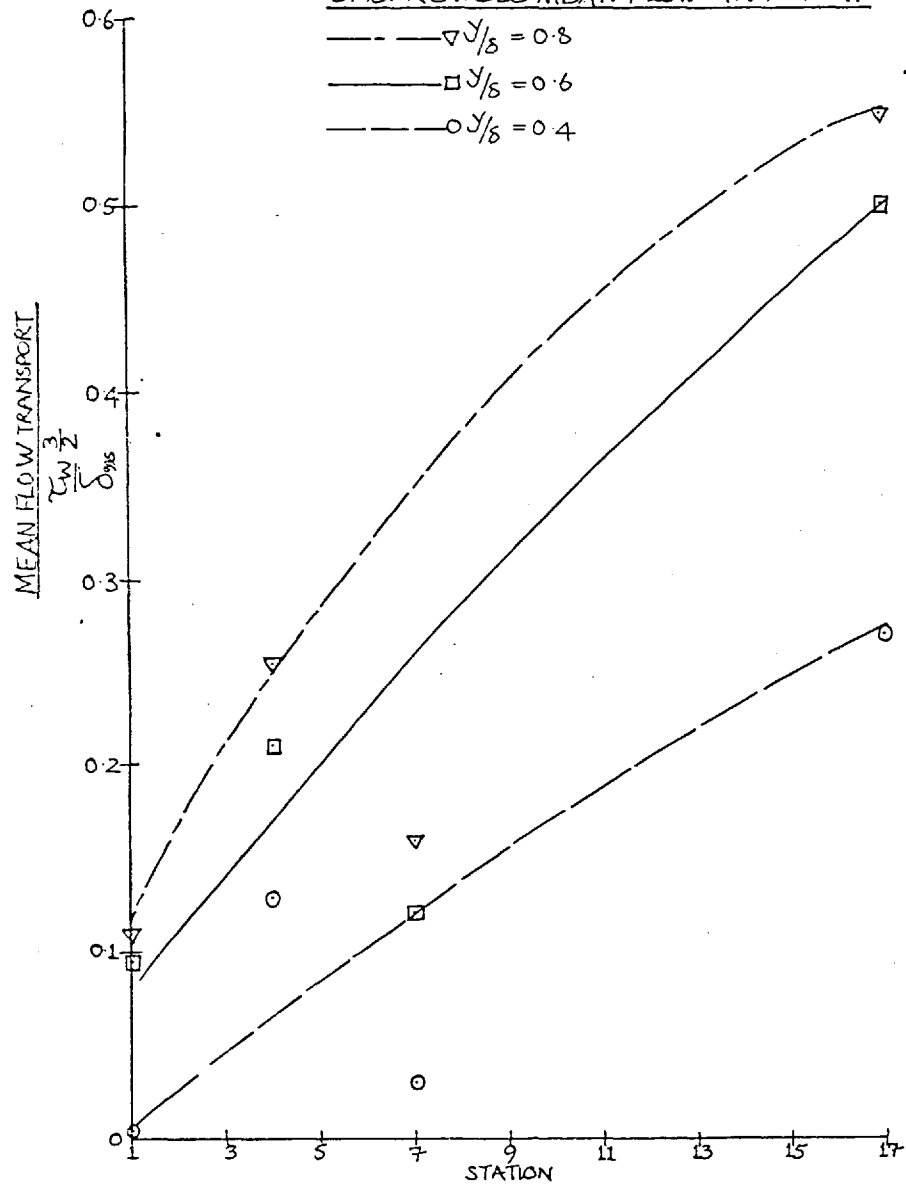


FIGURE [5-7.10] STREAMWISE DEVELOPMENT OF NORMALISED
SHEAR STRESS MEAN FLOW TRANSPORT

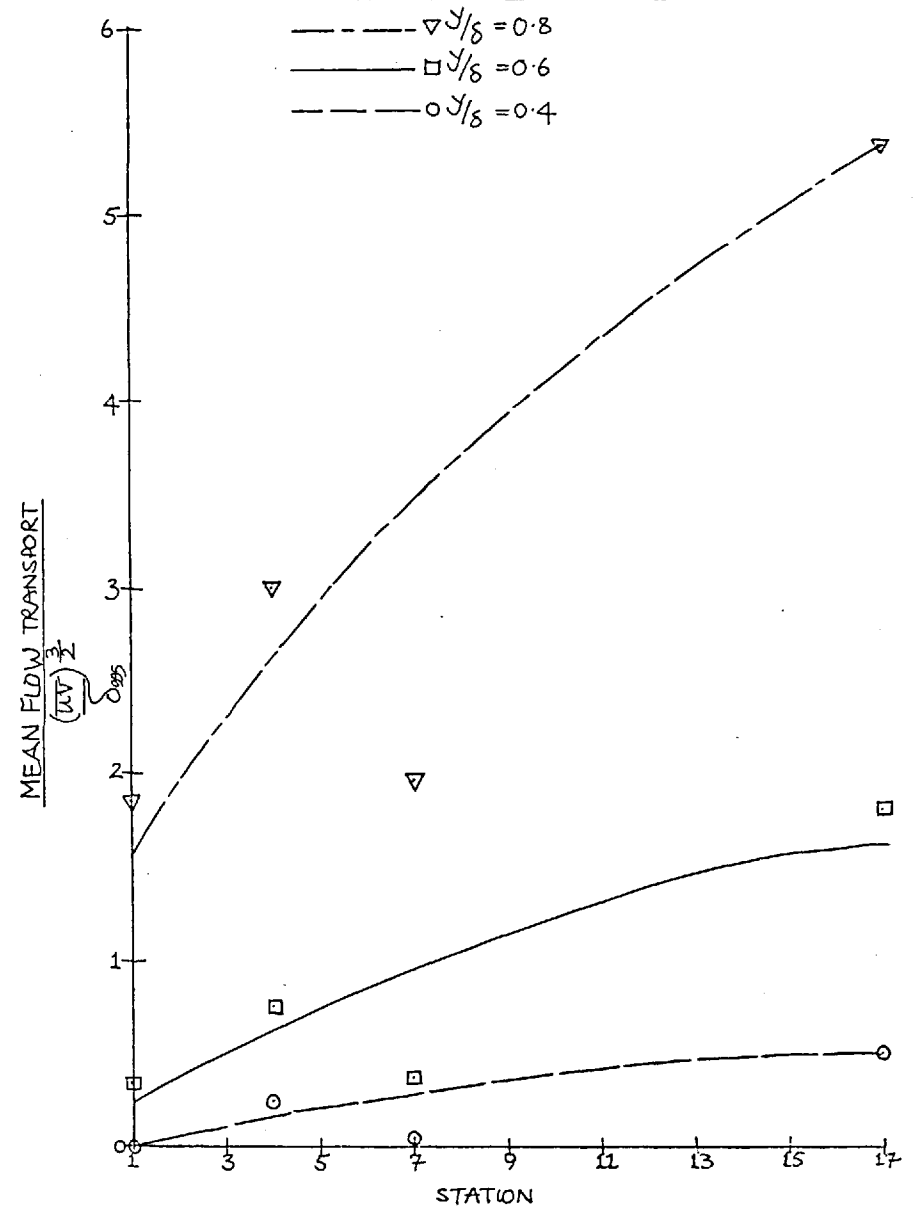


FIGURE [5-7.11] GENERATION/PRODUCTION RATIO

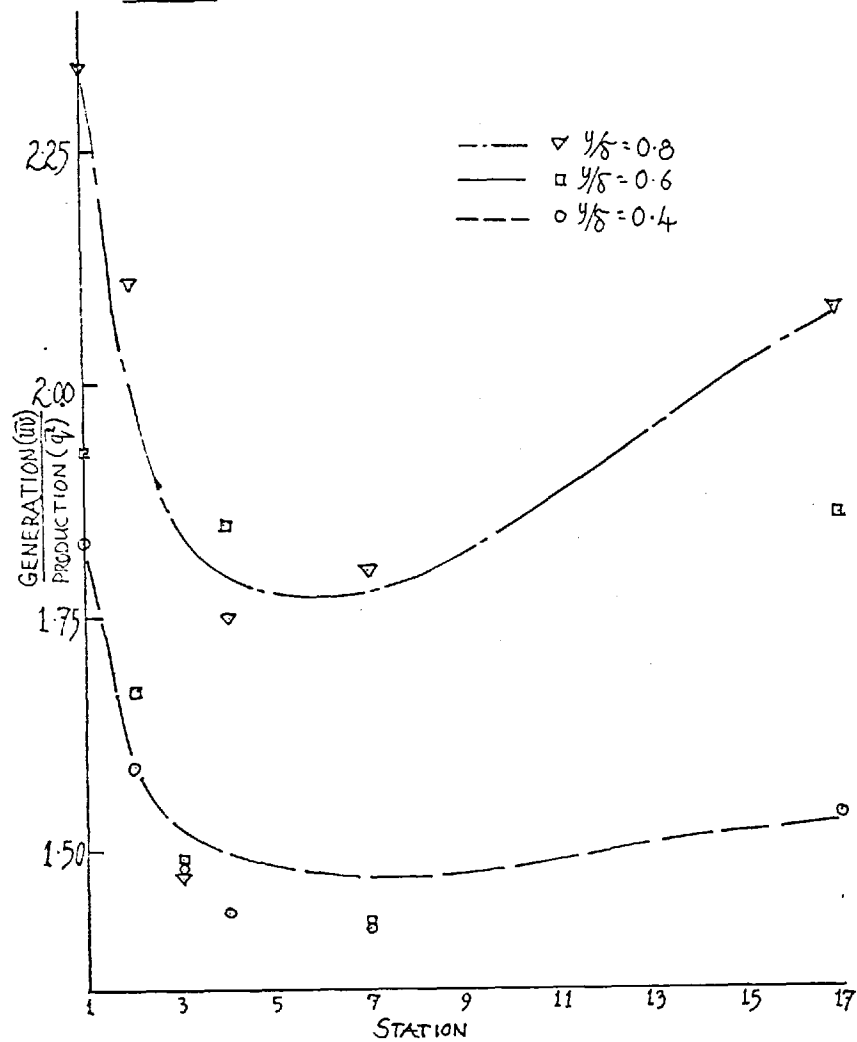
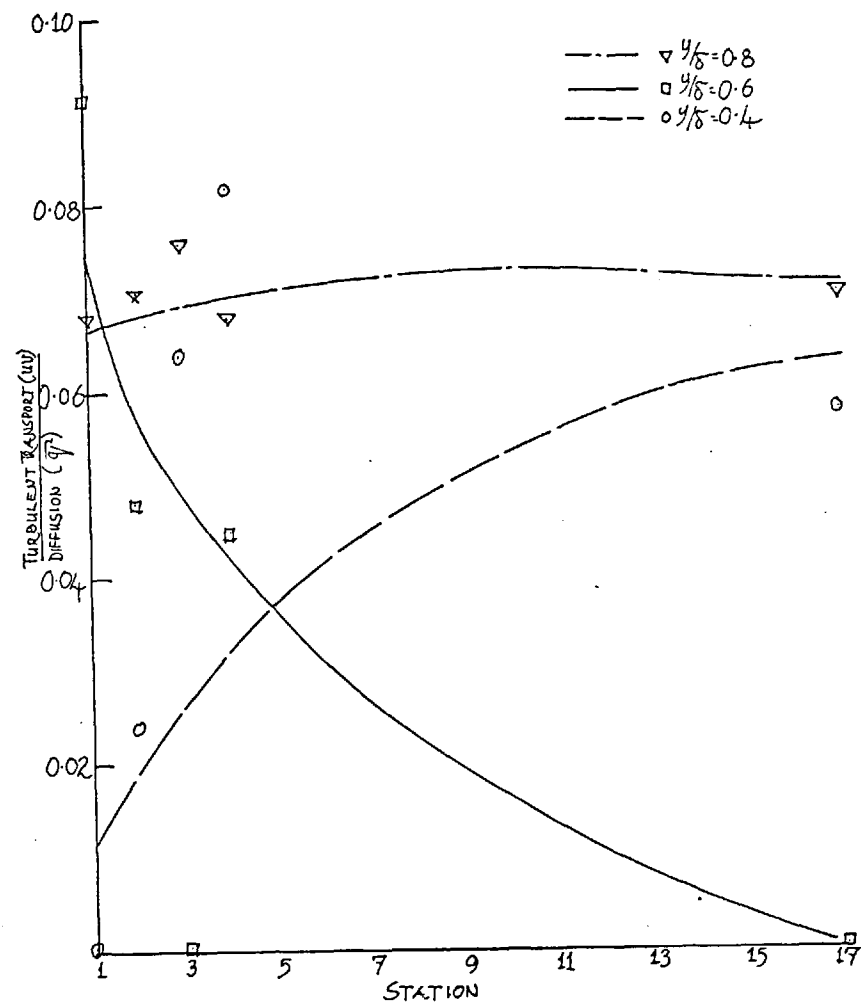


FIGURE [5-7.12] TURBULENT (NORMAL) TRANSPORT RATIO



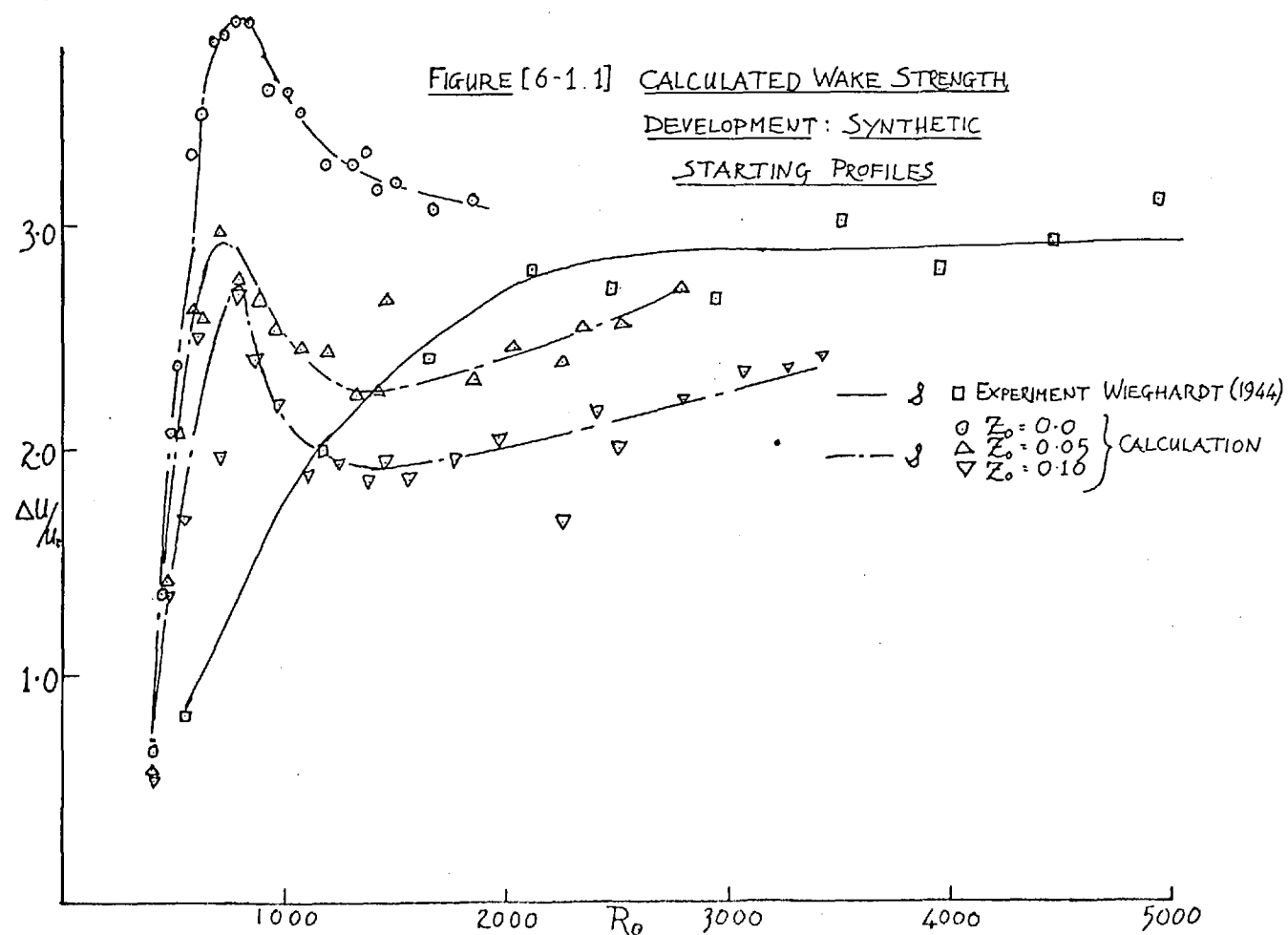
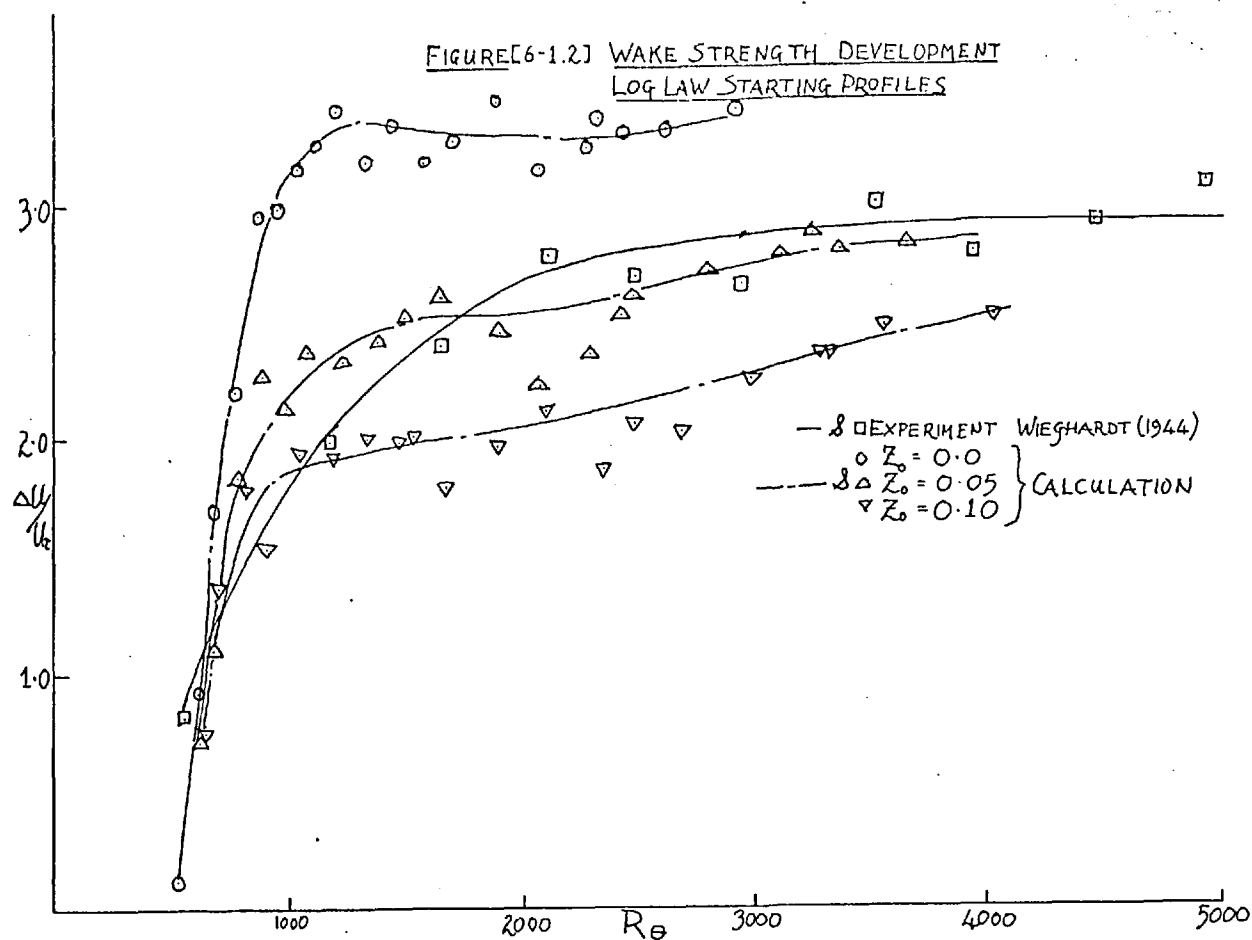
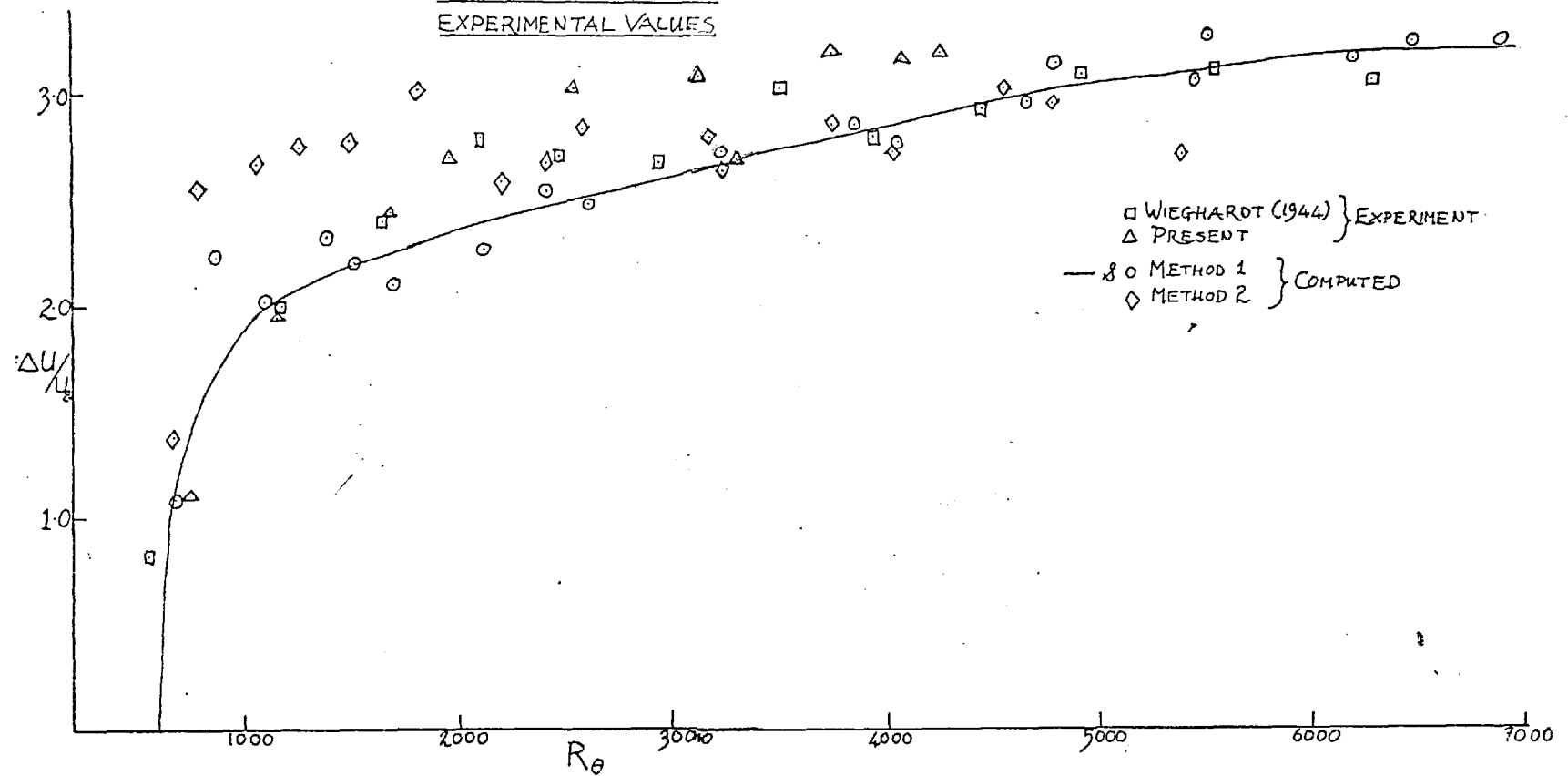


FIGURE [6-1.3] COMPARISON OF COMPUTED
WAKE STRENGTH WITH
EXPERIMENTAL VALUES



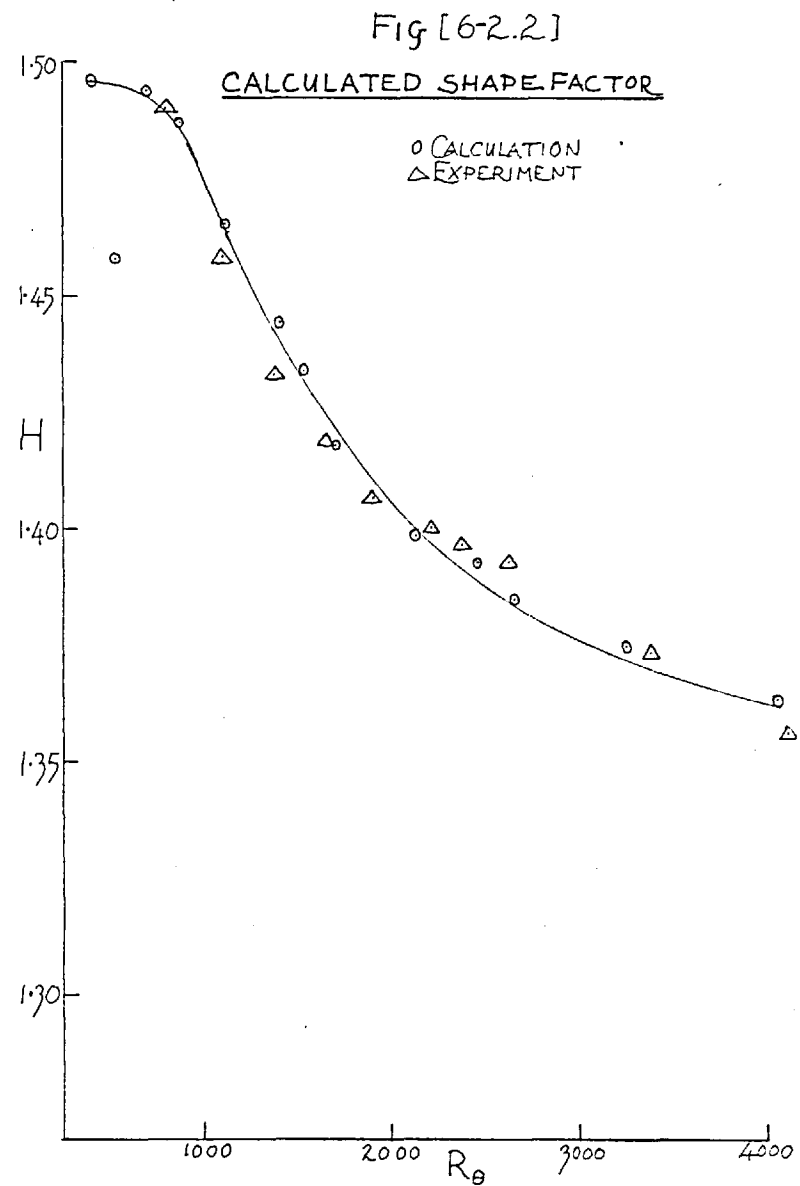
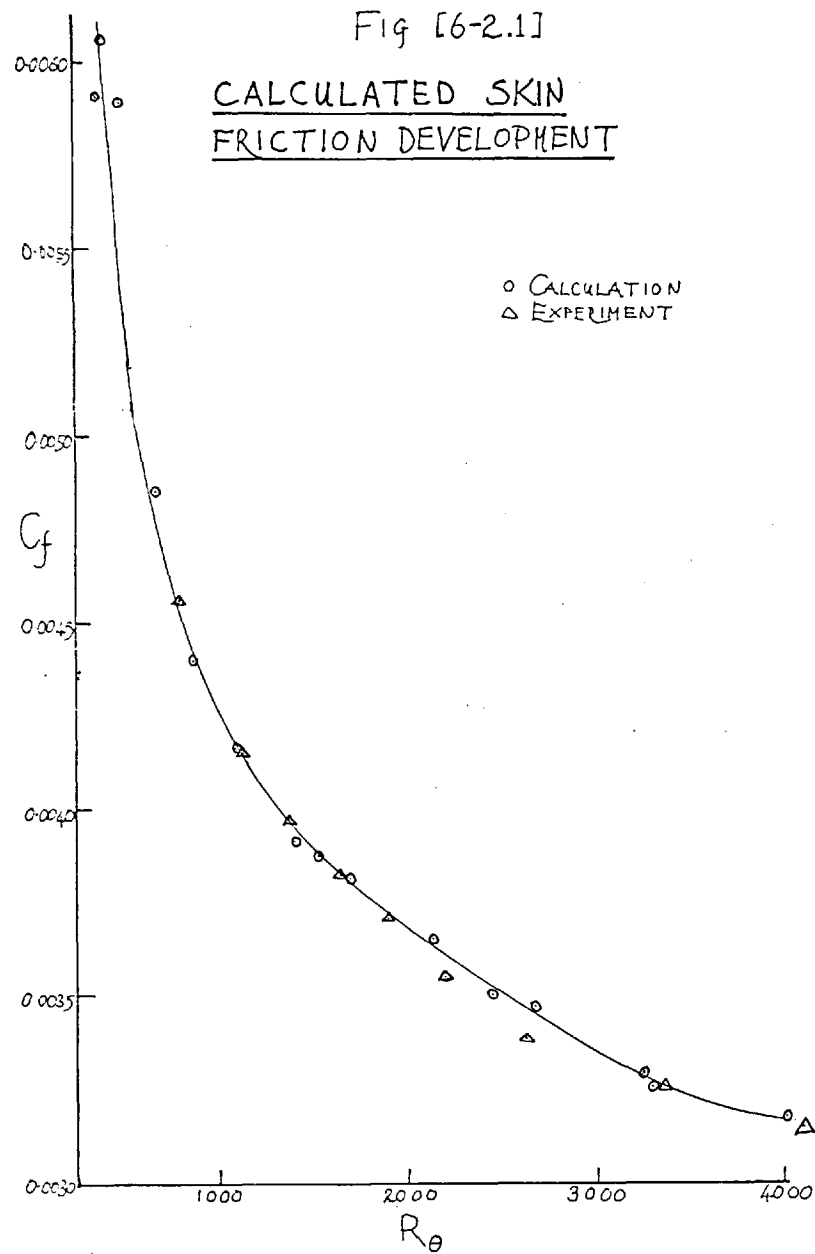
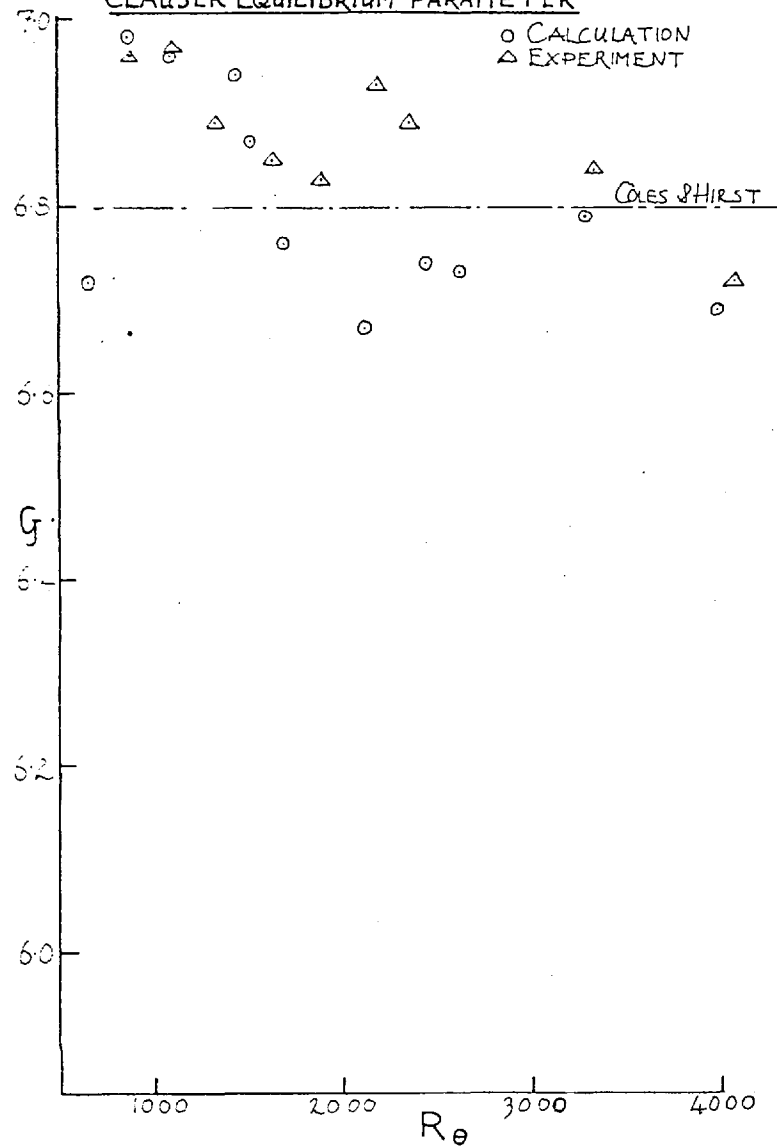


FIG [6-2.3]

CLAUSER EQUILIBRIUM PARAMETER



MEAN VELOCITY FIG [6-3.1]

PROFILE: CALCULATION

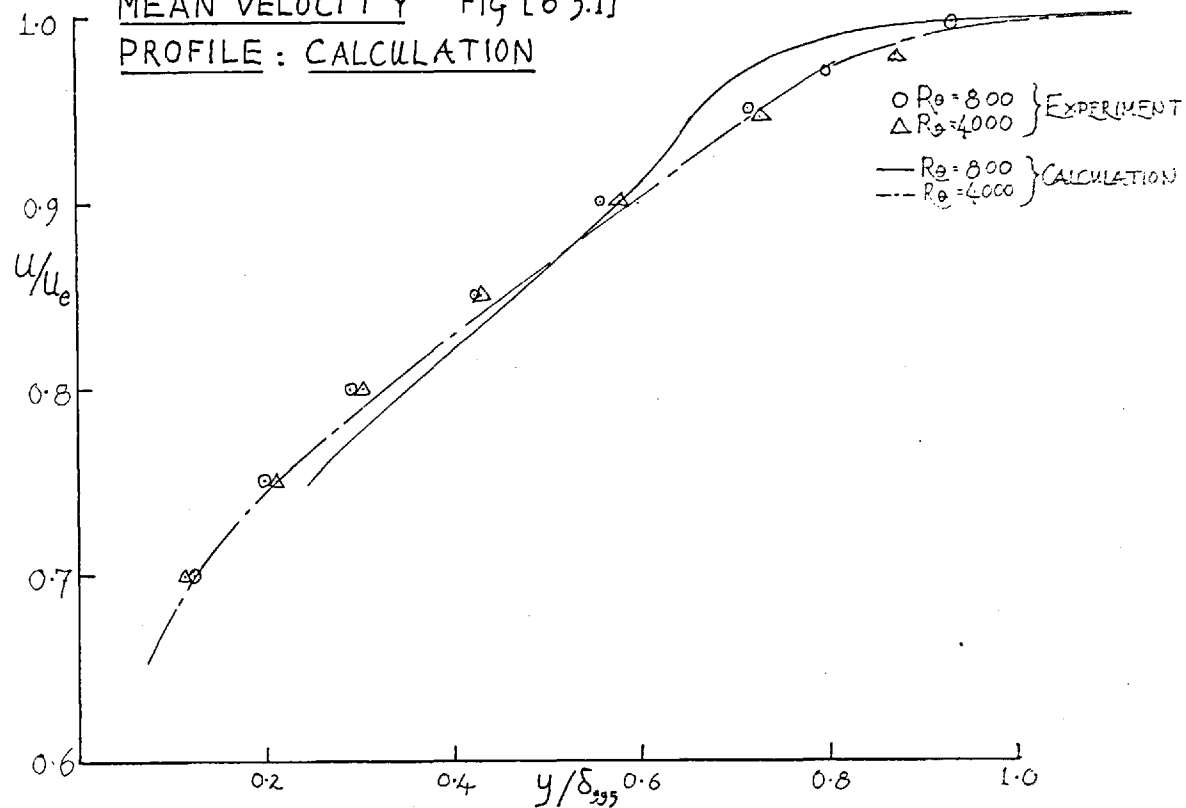


FIGURE [6-3.2] COMPUTED SHEAR
STRESS PROFILES

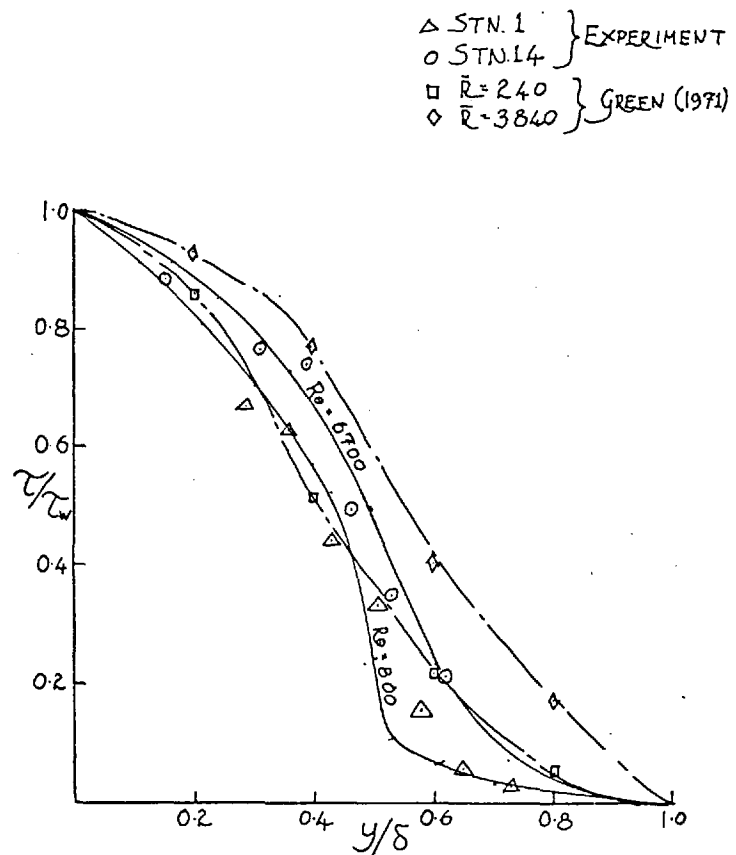


FIGURE [6-3.3] MIXING LENGTH
CALCULATION COMPARED WITH EXPERIMENT

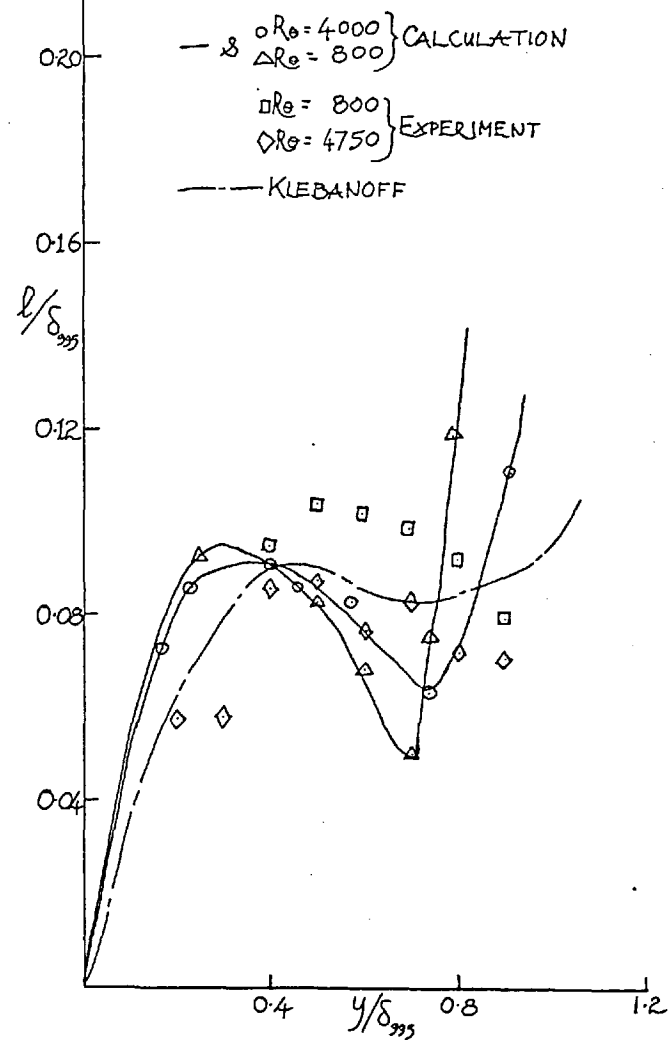


FIGURE [6-3.4] DISSIPATION LENGTH PARAMETER
AND MIXING LENGTH AT HIGH & LOW Re

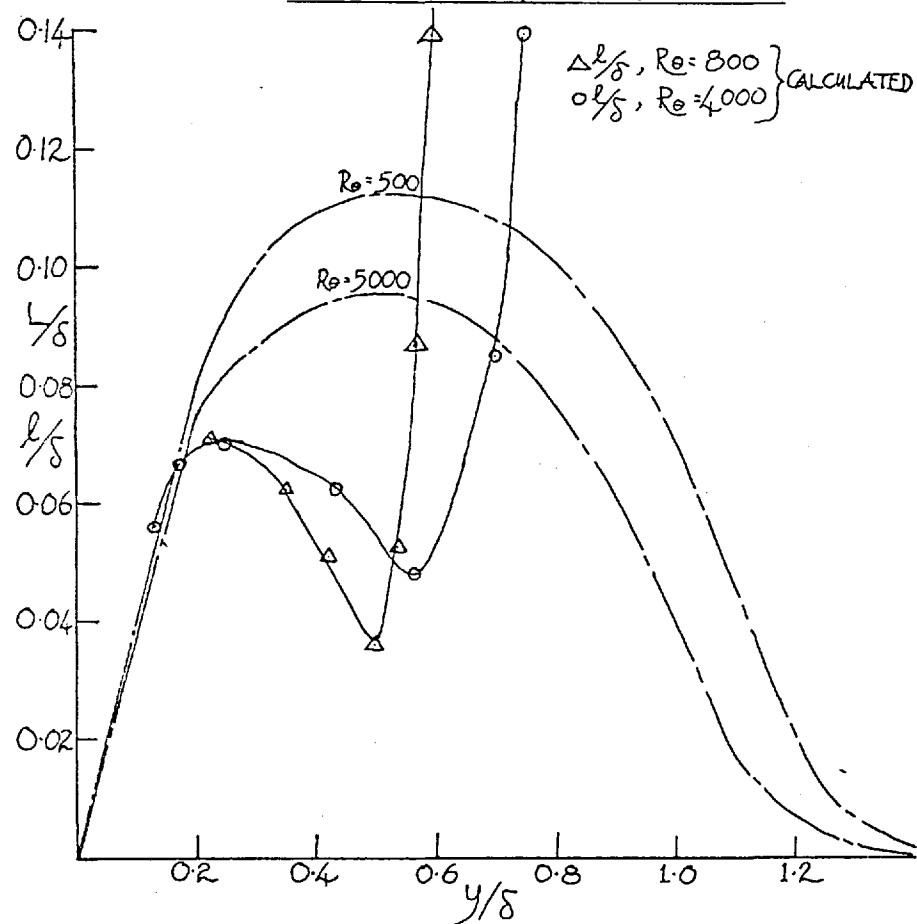


FIGURE [6-3.5] MIXING LENGTH: CALCULATIONS
COMPARED WITH GREEN'S ANALYSIS

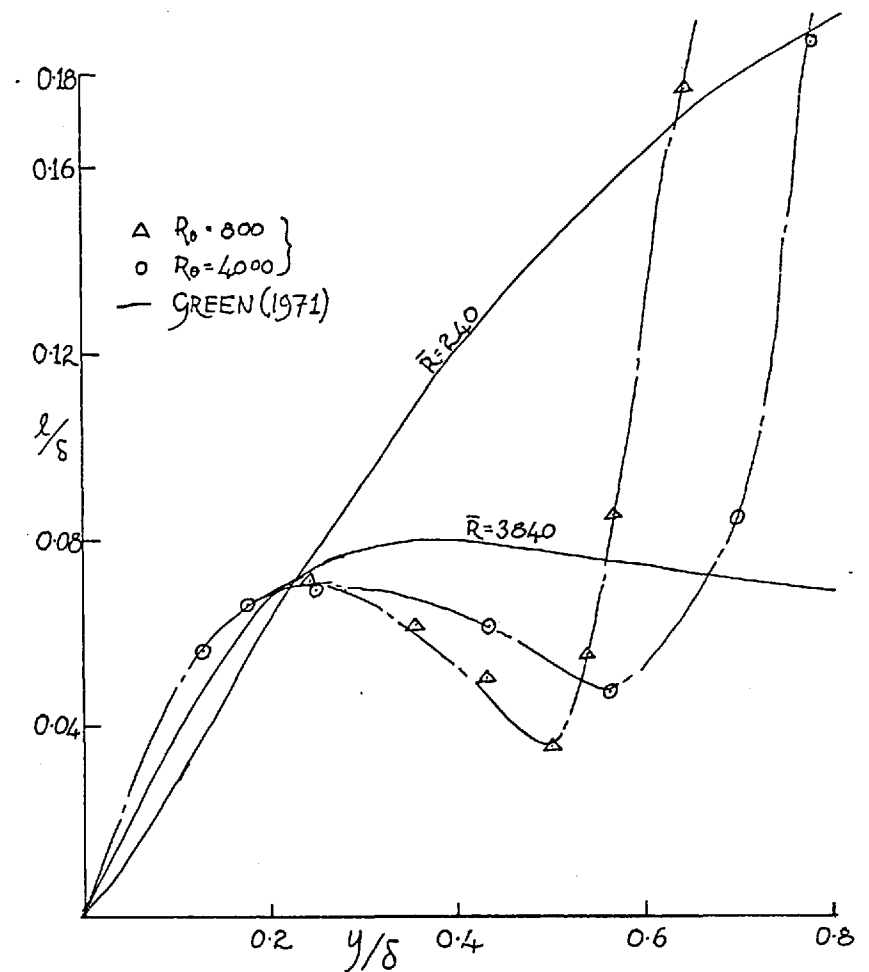


FIGURE [6-3.6] MIXING LENGTH: CALCULATION
COMPARED WITH McDONALD'S ANALYSIS

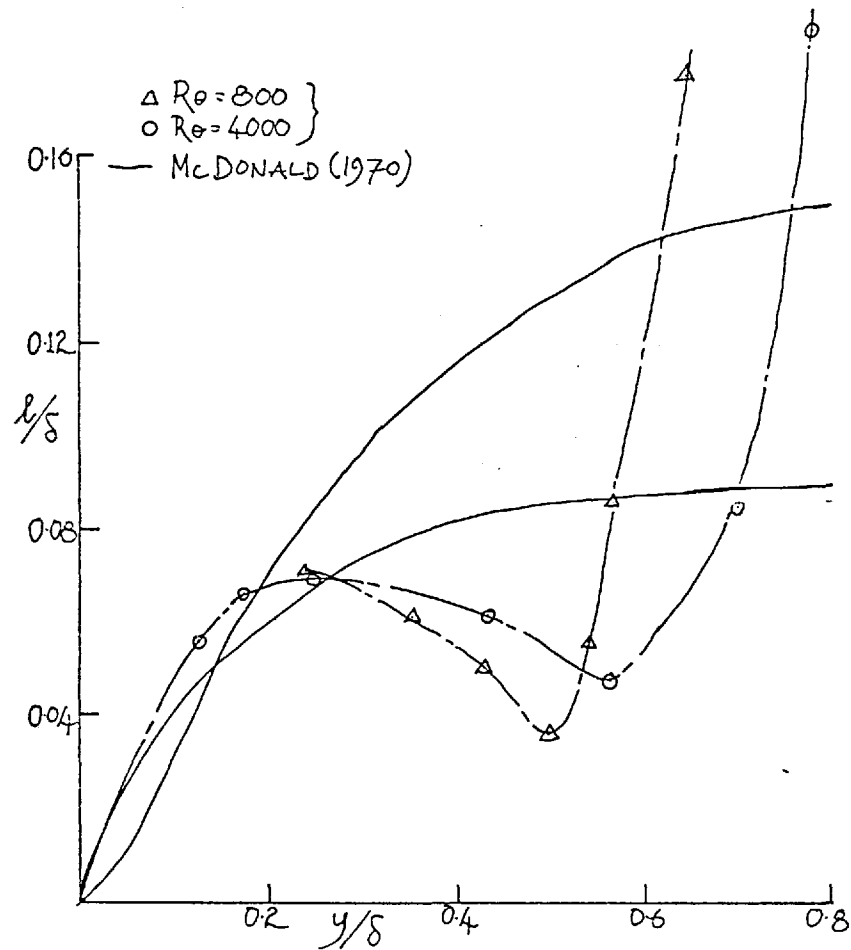


FIGURE [6-3.8] EDDY VISCOSITY: CALCULATION
COMPARED WITH EXPERIMENT

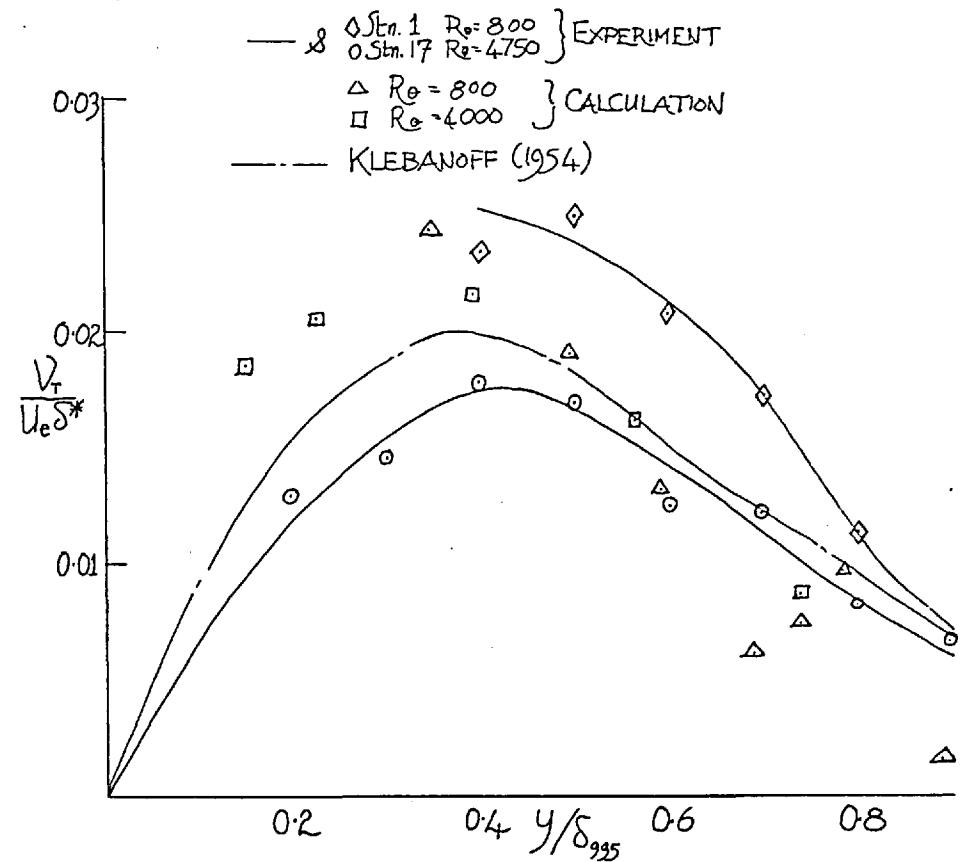
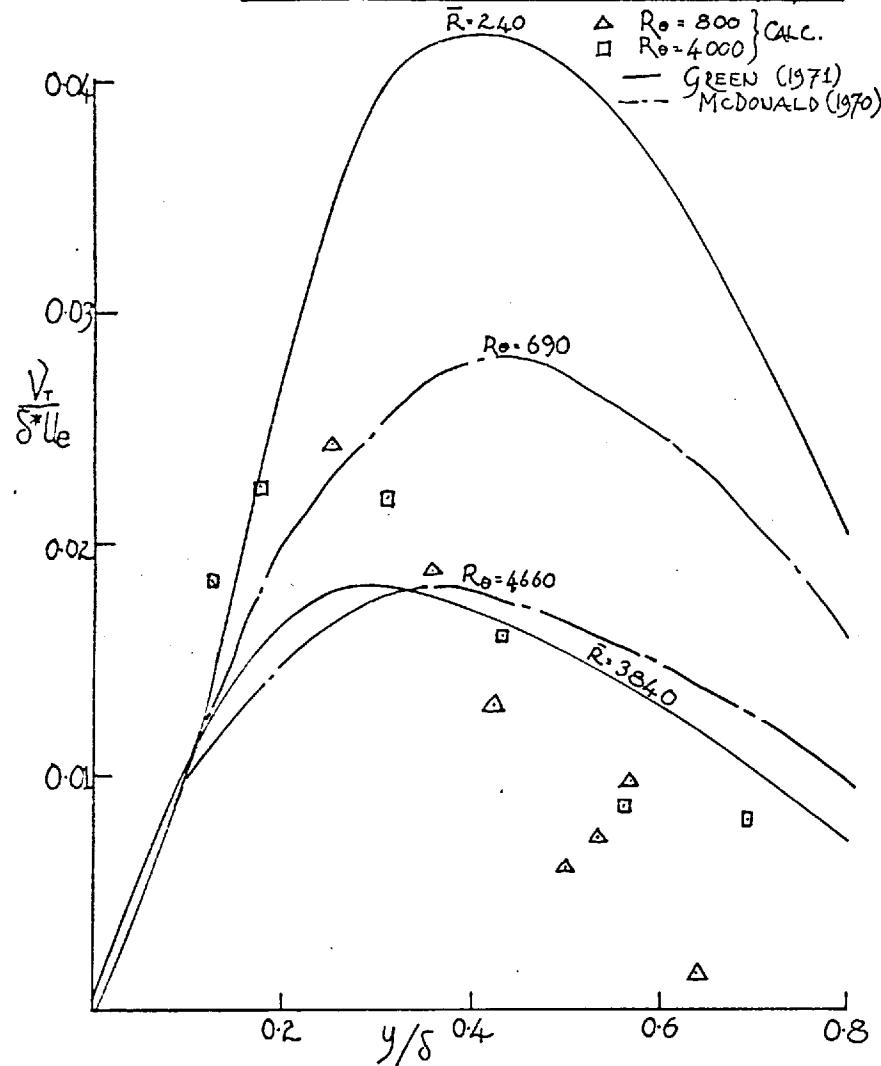


FIGURE [6-3.9] EDDY VISCOSITY: CALCULATION

COMPARED WITH GREEN'S & McDONALD'S ANALYSIS



Appendix I Data Analysis Programs JINF01 and JINF02

Input JINF01 Combist reading
 Position transducer reading } from data logger
 Solo

NR - number of readings

USL - slope of position transducer calibration

DELTA - position at which $U/U_e = 1$

UL - combist reading in free stream

Output JINF01

$(U/U_e)^2$
 (U/U_e)
 y
 $y/DELTA$ } punched on paper tape

Input JINF02

paper tape from JINF01

UL

CF - C_f

TEMP - Temperature °C

DS95 - δ_{995}

WPL - U/u at $y^+ = 50$

PRQ - ρ

NP - number of points

Output JINF01

U/U_e - U/u_τ

YPLUS - $u_\tau y / \nu$

LOGYPLUS - $\log u_\tau y / \nu$

DELTA* = δ^* THETA = θ

$U_e = u_\tau$ $U_e = U_e$ $U_e = U_e$ $\eta = \frac{\delta^*}{\theta}$ $\eta_{DS95} = \frac{\delta_{95}}{\theta}$ $\eta_{THETA} = \frac{\theta U_e}{\nu}$

```

C      DATA REDUCTION LOW REYNOLDS NO. F.L. CMF01
      DIMENSION IC(240),IV(240),IR(240),V(240),CODE(50),COM(50),
      IRU(50),IRAV(50),RY(50),RVF(50),RVF1(50),RYP(50),RVFM(50),
      2RVFN(50),ARY1(50),ARVPT(50),ARVF(50),ARY(50)
      ACCEPT 2,105,NCODE,Nr,YSL,USL,UIIN1,CONST,RNEU,CF
105  FORMAT (214,6F9.6)
      L=0
      N=0
      J=1
1000  J6=J+4
      ACCEPT 2,101,IC(K),IV(K),IR(K),K=J,J6
101  FORMAT (12,17,12,4(14,17,12))
      DO 1003 M=1,5
      RV=FLOAT(IV(J))
      RH=FLOAT(IR(J)-6)
      V(J)=RV*(10.0**RH)
      J=J+1
1003  CONTINUE
      CODE(L)=V(N+1)
      COM(L)=V(N+2)
      RU(L)=V(N+3)
      TRAV(L)=V(N+4)
      RY(L)=V(N+5)
      IF(CODE(L).GT.0.9) GO TO 1004
      N=N+5
      L=L+1
      IF(L.EQ.(N+1)) GO TO 1004
      GO TO 1000
1004  UONE=(SQRT(RUO*USL+UIIN1))*CONST
      UTAU=(SQRT(2.0/CF))*UONE
      TYPE 1, 0
100  FORMAT (5X,1HY,10X,4RU/01,10X,5HYPLUS,10X,6RU/UTAU)
      L=1
      DO 1005 M=1,Nr
      RVF(L)=SQRT(RU(L)/RUO)
      RY(L)=(RY(L)/TRAV(L))*YSL
      RVF1(L)=(RVF(L))*(SQRT(2.0/CF))
      RYP(L)=UTAU*RY(L)/RNEU
      TYPE 1,104,RY(L),RVF(L),RYP(L),RVF1(L)
104  FORMAT (4(5X,F9.6))
      PUNCH TAPE 1,107,RY(L),RVF(L),RYP(L),RVF1(L)
107  FORMAT (4(5X,F9.6))
1005  CONTINUE
      L=1
      IF(RYP(L).GT.50.0) GO TO 1007
      L=L+1
      GO TO 1005
1007  CONTINUE

```

```

NP=NF-L
RYP(L-1)=50.
RVPT(L-1)=RLLC
RVF(L-1)=RLLC*(SQRT(2.0/CF))/UONE
FP=NF/2.
RY(L-1)=50.*RNEU/SQRT(2./CF)
IP=NF/2.
IF((FP-IP).GT.0.0) GO TO 1008
RYP(NR+1)=RYP(NP)*1.01
RY(NR+1)=RYP(NR+1)*RNEU/SQRT(2./CF)
RVP(NR+1)=1.
NP=NP+1
1008 CONTINUE
M=L+1
L=1
DO 1009 N=M,NP
ARYP(L)=RYP(M)
ARVPT(L)=RVPT(M)
ARVP(L)=RVP(M)
RVFM(L)=(1-ARVP(L))
RVFN(L)=ARVP(L)*(1-ARVF(L))
ARY(L)=RY(M)
L=L+1
M=M+1
1009 CONTINUE
CALL SIMP(ARY,RVFM,NP,AREA1)
CALL SIMP(ARY,RVFN,NP,AREA2)
TYPE 1,102
102 FORMAT (5X,5HTHETA,10X,6HDELTA*)
DELST=AREA1+50.*(RNEU/UTAU-540.6*(RNEU/UONE))
THETA=AREA2+540.6*(RNEU/UONE)-6546.0*(UTAU*RNEU/(UONE**2.0))
TYPE 1,103,DELST,THETA
103 FORMAT (2(5X,F9.6))
CONTINUE
END
SUBROUTINE SIMP(X,Y,NV,AREA)
DIMENSION X(60),Y(60),AI(40)
J=1
N=1
AI(1)=0.0
112 N=N+1
A1=X(J+1)-X(J)
A2=X(J+2)-X(J+1)
A3=X(J+2)-X(J)
B1=Y(J+1)-Y(J)
B2=Y(J+2)-Y(J+1)
B3=Y(J+2)-Y(J)
C1=((A3**2.0)/3.0)*(((B3*A1)-(B1*A3))/((A1*A2)))
C2=((A3**2.0)/2.0)*(((B3/A3)-((B3*A1)-(B1*A3))/(A1*A2)))
C3=Y(J)*A3
AI(N)=C1+C2+C3
AI(N)=AI(N)+AI(N-1)
J=J+2
IF(J-NV)112,1006,1006
1006 AREA=AI(N)
RETURN
END

```

```

C JMF02 MARK2 COMPUTES LOG LAW PARAMETERS FROM F01 O/P
  DIMENSION RVP(100),RYP(100),RY(100),RVPT(100),
  1ARVP(100),BRVP(100)
  TYPE 1,150
150 FORMAT(5TYPE U1,CF,TEMP,D995,F9.6$)
  ACCEPT 1,151,U1,CF,TEMP,D995
151 FORMAT (4(F9.6))
  TYPE 1,152,U1,CF,TEMP,D995
152 FORMAT ($U1=$,F9.6,2X,$CF=$,F9.6,2X,$TEMP=$,F9.6
  1,2X,$D995=$,F9.66)
  TYPE 1,153
153 FORMAT (5TYPE RVP(1),RHO,NP$)
  ACCEPT 1,154,RVP(1),RH0,NP
154 FORMAT (F9.6,F9.6,I3)
  TYPE 1,155,RVP(1),RHO,NP
155 FORMAT ($RVP(1)=$,F9.6,2X,$RHO=$,F9.6,2X,$NP=$,I3)
  RNEU=.0001*(1.42+(.0096*TEMP))
  UONE=((8.2*U1)**.5)*13. 126
  UTAU=UONE*((CF/2.)**.5)
  M=0
  L=1
  RYP(1)=50.
  RY(1)=(50.*RNEU)/(UTAU/12.)
  RVP(1)=RVP(1)*(UTAU/UONE)
  RVPT(1)=RVP(1)*UONE/UTAU
  TYPE 1,110
110 FORMAT ($U/UTAU$,6X,$YPLUSE$,7X,$Y/D995$,6X,$LOG YPLUSE$)
1000 CONTINUE
  ACCEPT 2,101,RVP2,RRVP,RRY,RYD
101 FORMAT (4(2X,F7.4))
  M=M+1
  RRVPT=RRVP*UONE/UTAU
  RRYP=((RRY)*UTAU)/(RNEU*12.)
  YD=RRY/D995
  RYPL=CLOG(RRYP)
  TYPE 1,106,RRVPT,RRYP,YD,RYPL
106 FORMAT (4(1X,F11.5))
  PUNCH TAPE 2,106,RSVPT,RRYP,YD,RYPL
  IF(RRYP.GT.50.) GO TO 1001
  GO TO 1000

```

```

1001 CONTINUE
  L=L+1
  RYP(L)=RRYP
  RVPT(L)=RRVPT
  RVP(L)=RRVP
  RY(L)=RRY
  IF(M.EQ.NP) GO TO 1002
  TO TO 1000
1002 CONTINUE
  A3=0.000
  L2=L/2
  A2=L/2
  A3=A2-L2
  NP=L
  IF (A3.GT.0.0) GO TO 1003
  NP=L+1
  RVP(NP)=1
  RY(NP)=RY(L)*1.01
  RVPT(NP)=UONE/UTAU
  TYPE 1,104
1004 FORMAT (4HEVEN)
1003 CONTINUE
  L=0
  DO 1004 N=1,NP
  L=L+1
  ARVP(L)=1-RVP(L)
  BRVP(L)=RVP(L)*ARVP(L)
1004 CONTINUE
  CALL SIMP(RY,ARVP,NP,AREA1)
  CALL SIMP(RY,BRVP,NP,AREA2)
  TYPE 1,102
1002 FORMAT (5X,5HTheta,10X,6HDELTA*)
  DELST=AREA1/12.+(50.*RNEU)/UTAU-(540.6*RNEU)/UONE
  THETA=AREA2/12.+(540.6*RNEU)/UONE-(6546.*UTAU*RNEU)/(UONE**2.)
  TYPE 1,103,DELST,THETA
1003 FORMAT ($DELSTAR=$,F9.6,2X,$THETA=$,F9.6)
  TYPE 1,156,UTAU,UONE,RNEU
156 FORMAT ($UTAU=$,F9.6,2X,$UONE=$,F9.6,2X,$RNEU=$,F9.6)
  RH=DELST/THETA
  RRDE=D995*UTAU/RNEU
  RRTH=UONE*THETA/RNEU
  TYPE 1,107,RH,RRDE,RRTH
1007 FORMAT ($H=$,F6.4,2X,$RED995$,2X,F9.3,2X,$RETHET=$,F9.3)
  CONTINUE
  END

```

XXXVI

Appendix II Output from JMW02

STN 1

TYPE U1,CF,TEMP,D995,F9.6
1.934 .00464 26.5 .278
U1= 1.934000 CF= .004640 TEMP=26.500000 D995= .2780
TYPE RVP(1),RHO,NP
14.85 0.002378 64
RVP(1)=14.850000 RHO= .000000 NP= 64

U/UTAU	YPLUS	Y/D995	LOG YPLUS
13.70666	30.32399	.08705	1.48179
13.82915	33.83255	.09712	1.52933
14.03074	36.21336	.10396	1.55887
14.20078	38.59417	.11079	1.58652
14.50182	42.22204	.12122	1.62560
14.55580	43.35579	.12446	1.63705
14.63884	45.73660	.13129	1.66026
14.71358	48.11741	.13813	1.68230
14.87137	50.49822	.14496	1.70328
14.94611	52.87903	.15180	1.72328
15.05822	55.25984	.15863	1.74241
15.17449	58.26717	.16727	1.76542
15.39248	64.78307	.18597	1.81146
15.58650	69.54469	.19964	1.84226
15.69975	74.18100	.21295	1.87029
15.86169	78.94262	.22662	1.89731
16.00079	83.70424	.24029	1.92275
16.16480	88.46586	.25396	1.94678
16.34335	93.22748	.26763	1.96954
16.45754	97.98910	.28129	1.99118
16.57173	101.62296	.29173	2.00699
16.65893	105.13153	.30180	2.02173
16.77934	109.39314	.31547	2.04097
16.89145	114.65476	.32914	2.05939
17.16966	125.30575	.35971	2.09797
17.27346	129.44034	.37158	2.11207
17.39595	134.82899	.38705	2.12978
17.52883	140.71836	.40396	2.14835
17.66170	147.35956	.42302	2.16838
17.78004	153.24894	.43990	2.18540
17.92575	159.13830	.45683	2.20177
18.06447	165.15293	.47410	2.21789
18.20565	171.04235	.49101	2.23316
18.32606	177.05703	.50827	2.24811
18.44648	182.94640	.52518	2.26232
18.58558	188.83577	.54209	2.27608
18.69769	194.85044	.55935	2.28970
18.81603	200.73991	.57626	2.30263
18.92192	206.75449	.59353	2.31545
19.05064	212.64396	.61043	2.32765
19.17936	218.53323	.62734	2.33952
19.26448	224.54791	.64460	2.35131
19.37451	230.43728	.66151	2.36255
19.48870	236.45196	.67879	2.37374
19.60704	243.46908	.69892	2.38644
19.70877	250.61150	.71942	2.39900
19.81050	255.37312	.73369	2.40718
19.92261	264.89636	.76043	2.42303
20.03888	272.03878	.78094	2.43463
20.12607	279.19122	.80144	2.44589
20.20289	286.32365	.82194	2.45686
20.31293	295.84688	.84928	2.47107
20.38352	305.24421	.87626	2.48465
20.47694	313.64029	.90036	2.49643
20.54130	324.29129	.92094	2.51094
20.59528	332.56147	.94669	2.52187
20.62642	340.95695	.97078	2.53270
20.67002	350.48019	1.00612	2.54464
20.66660	362.25893	1.03993	2.55962
20.73438	375.92459	1.08777	2.57855
20.75514	387.19477	1.11151	2.58793
20.76137	402.73269	1.15612	2.60592
20.76137	416.89224	1.19676	2.62692
20.76137	446.58978	1.28291	2.64991

THETA DELTA
DELTA= .003772 THETA= .002530
UTAU= 2.517743 NONE=52.071810 RHEU= .000167
H=1.4808 PED995 4180.200 RETHET= 739.239

STN 2

TYPE U1,CF,TEMP,D995,F9.6
1.929 .00409 27. .392
U1= 1.929000 CF= .004090 TEMP=27.000000 D995= .392000
TYPE RVP(1),RHO,NP
14.85 0.002378 63
RVP(1)=14.850000 RHO= .000000 NP= 63

U/UTAU	YPLUS	Y/D995	LOG YPLUS
13.73678	28.82063	.06276	1.45970
13.78322	29.99220	.06531	1.47701
14.01383	32.80397	.07143	1.51593
14.24317	35.49858	.07730	1.55021
14.41123	37.72456	.08214	1.57662
14.55939	39.48191	.08597	1.59640
14.68986	42.17653	.09184	1.62507
14.86893	45.57408	.09923	1.65872
15.04588	48.85448	.10638	1.68890
15.16530	52.25203	.11378	1.71810
15.33115	56.58684	.12321	1.75272
15.49478	61.03881	.13291	1.78561
15.68054	66.07656	.14388	1.82065
15.88177	72.16872	.15714	1.85835
16.04762	77.79226	.16939	1.89094
16.23558	84.35305	.18367	1.92610
16.42133	91.03100	.19821	1.95919
16.61372	97.70895	.21276	1.98993
16.79505	104.97269	.22857	2.02109
16.98301	111.85064	.24311	2.04786
17.21962	120.55457	.26250	2.08118
17.40095	128.87272	.28061	2.11016
17.59555	136.60508	.29745	2.13547
17.76803	144.33745	.31429	2.15938
17.91619	151.60118	.33010	2.18070
18.06435	158.36492	.34592	2.20103
18.23904	166.01150	.36143	2.22014
18.35624	173.27523	.37730	2.23874
18.52209	180.42181	.39286	2.25629
18.66362	187.09976	.40740	2.27207
18.78524	194.34928	.42449	2.28992
18.94667	203.26743	.44260	2.30897
19.08598	210.41401	.45816	2.32307
19.23856	218.84931	.47653	2.34015
19.40884	227.63609	.49566	2.35724
19.53709	234.89982	.51143	2.37083
19.68746	243.21797	.52959	2.38600
19.84005	250.48170	.54541	2.39678
19.94840	258.21407	.56224	2.41193
20.09435	265.94643	.57908	2.42479
20.23145	274.38174	.59745	2.43836
20.36855	281.52832	.61301	2.44952
20.49239	289.37784	.63010	2.46147
20.60959	297.11020	.64694	2.47292
20.73785	306.40276	.66735	2.48641
20.87053	317.14404	.69056	2.50126
21.00542	327.68818	.71352	2.51546
21.11156	338.70094	.73750	2.52992
21.25087	349.82086	.76173	2.54386
21.36365	360.37499	.78469	2.55675
21.48749	372.67643	.81148	2.57133
21.60911	385.44659	.83929	2.58596
21.74490	400.44268	.87194	2.60254
21.88024	413.68143	.90077	2.61667
21.93418	429.26332	.93469	2.63272
21.96513	445.89461	.97092	2.64924
21.99388	459.84120	1.00128	2.66281
22.02011	471.43985	1.02653	2.67343
22.06685	489.24771	1.06531	2.68953
22.09560	511.50755	1.11378	2.70885
22.11329	533.65682	1.16199	2.72728
22.11329	547.00612	1.19107	2.73799
22.11329	567.00997	1.23469	2.75361

THETA DELTA
DELTA= .005260 THETA= .001935
UTAU= 2.560761 NONE=52.004194 RHEU= .000168
H=1.4672 PED995 5511.966 RETHET= 1114.520

Sh2

xxxvii

STN3

STN4

TYPE U1.CF.TEMP.D995.F9.6

1.940 .00386 27. .478

U1= 1.940000 CF= .003860 TEMP=27.000000 D995= .4780

TYPE RVP(1).RHO.NP

14.85 0.002378 62

RVP(1)=14.850000 RHO= .000000 NP= 62

U-UTAU YPLUS Y.D995 LOG YPLUS

13.72583	27.62170	.05063	1.44125
13.91931	29.79035	.05460	1.47408
14.08093	31.95899	.05858	1.50459
14.27669	34.12764	.06255	1.53311
14.45424	35.72559	.06548	1.55298
14.64544	38.46492	.07050	1.58506
14.70918	40.51944	.07427	1.60766
14.90453	43.82948	.08033	1.64177
15.11890	47.02538	.08619	1.67233
15.25093	51.36267	.09414	1.71065
15.47628	56.84135	.10418	1.75466
15.65610	62.20590	.11402	1.79383
15.86552	67.57045	.12385	1.82976
16.03996	73.04913	.13389	1.86362
16.16598	79.55507	.14522	1.90067
16.36174	86.06191	.15774	1.93481
16.60530	92.56694	.16967	1.96646
16.78057	98.95874	.18138	1.99545
16.92170	107.63333	.19728	2.03195
17.14022	116.30791	.21318	2.06561
17.33370	124.98250	.22908	2.09685
17.53856	133.65708	.24498	2.12599
17.71384	142.33167	.26088	2.15330
17.93008	150.99211	.27657	2.17867
18.06210	159.56665	.29247	2.20294
18.22372	168.24128	.30837	2.22593
18.38306	176.91586	.32427	2.24777
18.58564	187.75909	.34414	2.27360
18.75636	197.46093	.36192	2.29548
18.96805	208.87486	.38285	2.31999
19.12967	218.00600	.39958	2.33847
19.35681	229.87648	.42134	2.36149
19.47338	239.68246	.43933	2.37965
19.65776	250.53569	.45921	2.39887
19.82620	261.26478	.47887	2.41708
19.99920	272.10801	.49874	2.43474
20.16537	282.95124	.51862	2.45171
20.31560	293.79447	.53849	2.46804
20.49998	304.63770	.55837	2.48378
20.65476	315.36679	.57803	2.49882
20.82093	327.35141	.60000	2.51501
20.98027	339.65120	.62071	2.52975
21.22610	357.59932	.65544	2.55340
21.35130	371.63745	.68117	2.57012
21.53112	386.81797	.70900	2.58751
21.62445	392.75321	.71987	2.59412
21.73371	406.33579	.74477	2.60889
21.87939	420.37492	.77050	2.62364
22.01369	434.98474	.79728	2.63847
22.14116	449.02387	.82301	2.65227
22.27318	464.20434	.85084	2.66671
22.36195	478.81422	.87762	2.68017
22.43935	494.45130	.90628	2.69412
22.53267	511.22977	.93703	2.70862
22.59969	527.43755	.96674	2.72217
22.66242	548.09675	1.00460	2.73886
22.69884	565.33179	1.03619	2.75230
22.73981	594.55143	1.08975	2.77419
22.76258	634.04362	1.16213	2.80812
22.76940	659.13851	1.22657	2.82555
22.76940	706.96549	1.29414	2.84684
22.76258	739.48105	1.35356	2.86834

EVEN

THETA

DELTA*

DELTA*F= .006345 THETA= .004389

UTAU= 2.220952 UONE=52.05823 FNEU= .000168

H=1.4457 PED995 6547.023 PETHET= 1363.308

TYPE U1.CF.TEMP.D995.F9.6

1.95 .00373 27. .56

U1= 1.950000 CF= .003730 TEMP=27.000000 D995= .560000

TYPE RVP(1).RHO.NP

14.85 0.002378 67

RVP(1)=14.850000 RHO= .000000 NP= 67

U-UTAU YPLUS Y.D995 LOG YPLUS

13.59943	27.22248	.04321	1.43493
13.63879	29.35978	.04661	1.46775
13.88888	31.49708	.05000	1.49827
14.09496	33.63438	.05339	1.52678
14.26863	34.64679	.05500	1.53966
14.40062	36.78409	.05839	1.56566
14.50482	38.92139	.06179	1.59019
14.67386	42.07110	.06679	1.62398
14.91468	48.34571	.07357	1.66601
15.17866	50.62031	.08036	1.70432
15.28749	54.89491	.08714	1.73953
15.46579	59.16952	.09393	1.77210
15.63251	63.44412	.10071	1.80239
15.77376	67.71873	.10750	1.83071
15.91964	71.99333	.11429	1.85729
16.06553	78.40524	.12446	1.89435
16.28551	86.95444	.13804	1.93929
16.46844	95.39116	.15143	1.97951
16.69305	102.92796	.16339	2.01253
16.91303	111.92713	.17768	2.04894
17.04965	119.46393	.18964	2.07724
17.24184	129.58799	.20571	2.11256
17.45488	140.27450	.22268	2.14696
17.67254	150.39857	.23875	2.17724
17.86705	159.39773	.25304	2.20248
18.04072	169.07184	.26839	2.22807
18.20049	176.49615	.28018	2.24674
18.36490	187.18266	.29714	2.27227
18.53625	197.86917	.31411	2.29638
18.70529	208.44319	.33089	2.31899
18.85812	219.12970	.34786	2.34070
19.09273	231.39106	.36732	2.36435
19.24019	242.64002	.38518	2.38496
19.39534	253.21404	.40196	2.40349
19.55048	262.88814	.41732	2.41977
19.69636	273.57465	.43429	2.43708
19.85614	285.27357	.45286	2.45526
20.01360	295.96008	.46982	2.47128
20.16411	306.53410	.48661	2.48648
20.31231	317.32061	.50357	2.50136
20.46745	327.90712	.52054	2.51575
20.59786	338.59363	.53750	2.52968
20.70827	349.28014	.55446	2.54317
20.87268	359.85416	.57125	2.55613
21.00235	370.54067	.58821	2.56884
21.14592	382.35207	.60596	2.58246
21.28717	393.48854	.62464	2.59493
21.41684	405.74990	.64411	2.60826
21.55115	417.44883	.66268	2.62060
21.67156	429.26023	.68143	2.63272
21.78965	441.52159	.70039	2.64495
21.88453	452.65806	.71857	2.65577
22.00963	466.04432	.73982	2.66843
22.13236	480.44299	.76268	2.68164
22.27592	495.85406	.78714	2.69535
22.40228	512.95247	.81429	2.71008
22.54453	530.05089	.84143	2.72432
22.64179	548.61167	.87089	2.73926
22.76220	567.84739	.90143	2.75423
22.88724	587.53307	.93268	2.76903
22.90113	606.76878	.96321	2.78302
23.00765	621.16744	.98607	2.79321
23.07248	640.85312	1.01732	2.80676
23.10259	666.50075	1.05894	2.82380
23.13732	702.72229	1.11554	2.84679
23.14896	749.63055	1.13000	2.87425
23.15525	796.09875	1.26275	2.90096

THETA

DELTA*

DELTA*F= .007496 THETA= .005244

UTAU= 2.266710 UONE=52.48756 FNEU= .000168

H=1.4281 PED995 7559.300 PETHET= 1646.640

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STN 5

STN 6

TYPE U1,CF,TEMP,D995,F9.6

U1= 1.942 .00358 26. .648

U1= 1.942000 CF= .003580 TEMP=26.000000 D995= .6480

TYPE PVP(1),FHD,NP

14.85 0.002378 67

PVP(1)=14.850000 FHD= .000000 NP= 67

U/UTAU YPLUS Y/D995 LOG YFLUS

13.64978	26.76776	.03735	1.42761
13.67105	28.31631	.03951	1.45204
13.79632	28.86936	.04028	1.46044
13.97123	30.97096	.04321	1.49095
14.12486	33.07256	.04614	1.51947
14.30686	35.17417	.04907	1.54622
14.48413	37.27577	.05201	1.57143
14.64249	39.26676	.05478	1.59403
14.81739	41.26836	.05772	1.61667
14.89539	43.46996	.06065	1.63819
14.97103	45.57156	.06358	1.65869
15.06793	47.67316	.06651	1.67827
15.13412	49.77476	.06944	1.69701
15.23811	51.87636	.07238	1.71497
15.35866	56.07857	.07824	1.74880
15.51702	59.28727	.08272	1.77296
15.69192	64.48597	.08997	1.80947
15.87392	70.23772	.09799	1.84657
16.05355	77.09558	.10756	1.88703
16.27337	83.40038	.11636	1.92117
16.40573	89.70518	.12515	1.95282
16.58773	98.00098	.13673	1.99123
16.78627	106.40738	.14846	2.02697
16.98954	116.91539	.16312	2.06787
17.19990	127.46339	.17778	2.10525
17.39844	137.93140	.19244	2.13966
17.61826	148.32879	.20694	2.17123
17.82153	158.86860	.22160	2.20095
18.02007	169.34480	.23627	2.22877
18.20206	179.85281	.25093	2.25492
18.39824	191.35631	.26698	2.28134
18.60151	202.85981	.28302	2.30720
18.78351	215.46942	.30062	2.33339
19.01278	228.07903	.31821	2.35809
19.18532	238.58703	.33287	2.37765
19.34841	250.09053	.34892	2.39810
19.50204	261.15159	.36435	2.41639
19.67222	273.20814	.38117	2.43649
19.85895	289.35728	.40370	2.46143
20.06694	304.06849	.42423	2.49297
20.25367	318.22665	.44398	2.50274
20.42857	331.83175	.46296	2.52092
20.60348	345.54746	.48210	2.53851
20.72875	358.04646	.49954	2.55394
20.90129	372.20461	.51629	2.57078
21.08565	386.36277	.53904	2.58700
21.22038	401.07297	.55957	2.60322
21.41656	416.33824	.58086	2.61945
21.58201	433.04044	.60417	2.63653
21.73328	448.74714	.62608	2.65260
21.91055	466.11300	.65031	2.66849
22.08073	483.36826	.67439	2.68428
22.24382	500.18106	.69784	2.69913
22.38327	518.54241	.72346	2.71479
22.55108	537.34621	.74969	2.73025
22.70945	555.70757	.77531	2.74485
22.83472	573.62648	.80031	2.75863
22.96471	591.43479	.82515	2.77191
23.05926	612.34019	.85432	2.78699
23.20816	639.10795	.89167	2.80557
23.33590	665.32265	.92824	2.82303
23.45398	696.29362	.97145	2.84279
23.59199	725.80542	1.01225	2.86070
23.56270	759.67249	1.05988	2.88063
23.62416	803.25407	1.10668	2.90485
23.63548	847.83012	1.13237	2.92031
23.63598	920.81190	1.26441	2.96408

THETA DELTA*

DELTHAP= .002577 THETA= .006044

UTAU= 2.216105 UDNE=52.379903 PNEU= .000167

H=1.4190 PED995 .8601679 PETHET= 1896.317

TYPE U1,CF,TEMP,D995,F9.6

U1= 1.942 .00347 27. .77

U1= 1.942000 CF= .003470 TEMP=27.000000 D995= .770000

TYPE PVP(1),FHD,NP

14.85 0.002378 71

PVP(1)=14.850000 FHD= .000000 NP= 71

U/UTAU YPLUS Y/D995 LOG YFLUS

13.44190	26.63576	.03195	1.42546
13.55234	28.63576	.03195	1.42546
13.66037	28.25989	.03390	1.45117
13.87404	29.77574	.03571	1.47386
14.03489	31.83298	.03818	1.50288
14.22215	34.86469	.04182	1.54239
14.42862	38.92192	.04429	1.56728
14.51985	38.97915	.04675	1.59083
14.66149	41.03639	.04922	1.61317
14.79354	43.09362	.05169	1.63441
14.89197	45.15085	.05416	1.65467
14.98080	47.20809	.05662	1.67402
15.06002	49.26532	.05909	1.69254
15.12004	51.32255	.06156	1.71031
15.19687	53.37979	.06403	1.72738
15.38173	57.49425	.06896	1.75966
15.59059	62.58320	.07506	1.79646
15.73224	67.78042	.08130	1.83110
15.91950	73.41074	.08805	1.86576
16.12356	80.01554	.09597	1.90317
16.30602	86.18724	.10338	1.92544
16.47648	94.41617	.11325	1.97505
16.70935	105.67682	.12675	2.02398
16.94463	115.96298	.13909	2.06432
17.14389	126.24915	.15143	2.10123
17.33355	136.53531	.16377	2.13524
17.55442	147.25459	.17662	2.16807
17.72963	157.54074	.18896	2.19739
17.91214	167.28553	.20065	2.22346
18.06098	178.65445	.21429	2.25201
18.22424	189.37372	.22714	2.27732
18.39509	200.20127	.24013	2.30147
18.60116	211.46191	.25364	2.32523
18.79082	222.72255	.26714	2.34776
18.94637	233.55010	.28013	2.36938
19.12692	246.32660	.29545	2.39151
19.31178	259.64447	.31143	2.41438
19.51585	274.64510	.32870	2.43782
19.68150	288.44574	.34597	2.46006
19.83275	301.22223	.36130	2.47889
20.04642	315.62287	.37957	2.49917
20.26009	331.53936	.39766	2.52054
20.42094	345.93999	.41494	2.53900
20.57939	360.23235	.43208	2.55658
20.72103	374.63298	.44935	2.57361
20.91310	390.54946	.46844	2.59168
21.05474	407.98181	.48835	2.61064
21.24440	426.47691	.51156	2.62992
21.46527	444.47063	.53312	2.64784
21.62853	460.39712	.55221	2.66312
21.79898	478.73671	.57182	2.67828
21.96223	494.27732	.59286	2.69397
22.10366	510.62692	.61247	2.70810
22.26953	528.60063	.63403	2.72313
22.44239	547.65710	.65688	2.73851
22.60564	567.14668	.68026	2.75370
22.76169	588.15211	.70545	2.76949
22.88413	607.10032	.72518	2.78326
22.97536	625.07404	.74974	2.79593
23.12901	645.10500	.77377	2.80963
23.27545	665.13594	.79779	2.82291
23.42190	684.08414	.82052	2.83511
23.52270	704.11510	.84455	2.84764
23.57795	723.60468	.86792	2.85950
23.65477	744.17701	.89260	2.87168
23.75521	780.66592	.93636	2.89247
23.87084	824.19255	.98857	2.91603
23.94047	973.02477	1.04714	2.94103
23.97169	935.66593	1.12156	2.97084
24.00048	1004.06286	1.20468	3.00189
24.00769	1076.25774	1.29091	3.03192

THETA DELTA*

DELTHAP= .009893 THETA= .006225

UTAU= 2.181793 UDNE=52.379903 PNEU= .000168

H=1.4084 PED995 10004.649 PETHET= 2191.223

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STN 7

TYPE U1,CF,TEMP,D995,F9.6
 1.94 .00342 27. .837
 U1= 1.940000 CF= .003420 TEMP=27.000000 D995= .8370
 TYPE RVP(1),RHO,NP
 14.85 0.002378 65
 RVP(1)=14.850000 RHO= .000000 NP= 65
 U-UTRU YPLUS Y/D995 LOG YPLUS

13.45244	26.03777	.02991	1.41560
13.50798	28.08206	.03118	1.44843
13.74945	30.12635	.03345	1.47895
13.95471	32.17064	.03572	1.50746
14.14783	34.21493	.03799	1.53482
14.24206	36.25921	.04026	1.55942
14.46180	38.19591	.04241	1.58202
14.67671	40.24020	.04468	1.60466
14.79936	43.36043	.04815	1.63709
14.90123	46.37306	.05149	1.66627
15.03167	50.46164	.05603	1.70296
15.27798	56.59450	.06284	1.75277
15.56050	62.72737	.06965	1.79746
15.76092	68.86023	.07646	1.83797
15.91305	74.99309	.08327	1.87502
16.14486	81.12596	.09008	1.90916
16.27526	87.25882	.09689	1.94081
16.43946	95.32838	.10585	1.97922
16.63938	103.50554	.11493	2.01496
16.82823	113.72698	.12628	2.05586
17.03107	124.91676	.13871	2.09662
17.28461	138.25843	.15352	2.14069
17.55989	152.99882	.16989	2.18469
17.80378	168.81516	.18746	2.22741
18.02352	183.12518	.20335	2.26275
18.23602	197.32760	.21912	2.29519
18.50647	213.68190	.23728	2.32977
18.70689	230.03621	.25544	2.36180
18.94595	245.31457	.27240	2.39972
19.14395	260.59293	.28937	2.41596
19.35403	275.87130	.30633	2.44071
19.56170	291.15726	.32342	2.46428
19.78869	307.50397	.34146	2.48785
19.98428	322.78233	.35842	2.50891
20.11950	338.16829	.37551	2.52913
20.33441	356.45989	.39582	2.55201
20.51793	372.81359	.41398	2.57149
20.67006	389.16790	.43214	2.59014
20.85116	407.45890	.45245	2.61008
21.03710	425.85749	.47288	2.62926
21.22786	444.25608	.49331	2.64763
21.40414	462.54703	.51362	2.66516
21.57075	479.86974	.53286	2.68112
21.71081	497.19238	.55209	2.69652
21.86776	515.59097	.57252	2.71231
22.03197	533.98957	.59295	2.72753
22.20824	552.28056	.61326	2.74216
22.38210	572.18547	.63536	2.75754
22.51733	590.58406	.65579	2.77128
22.66221	608.44469	.67563	2.78422
22.81434	627.81163	.69713	2.79733
22.93266	648.25452	.71933	2.81175
23.06788	668.58900	.74241	2.82516
23.19828	690.53900	.76679	2.83919
23.32143	708.93759	.78722	2.85061
23.46873	730.77920	.81147	2.86379
23.60154	755.84862	.83931	2.87843
23.70779	784.36106	.87097	2.89458
23.82128	810.92681	.90048	2.90899
23.89855	841.49354	.93441	2.92565
23.97824	880.22742	.97742	2.94459
24.06034	921.00558	1.02270	2.96426
24.11105	980.18236	1.08241	2.99131
24.14727	1028.70729	1.14229	3.01229
24.14727	1079.16892	1.19333	3.03309

EVEN

THETA

DELTA*

DELTA* = .010725 THETA = .007661
 UTRU = 2.168064 UONE = 52.252028 FNEU = .000168
 H=1.4000 RED995 10806.752 FEHET = 2388.385

STN 11

TYPE U1,CF,TEMP,D995,F9.6
 1.936 .00320 27. 1.216
 U1= 1.936000 CF= .003200 TEMP=27.000000 D995= 1.21600
 TYPE RVP(1),RHO,NP
 14.85 0.002378 67
 RVP(1)=14.850000 RHO= .000000 NP= 67
 U-UTRU YPLUS Y/D995 LOG YPLUS

13.38000	25.53897	.02023	1.40720
13.42750	27.51149	.02179	1.43951
13.65500	29.48401	.02336	1.46959
13.86500	31.45653	.02492	1.49771
14.06000	33.42905	.02648	1.52412
14.23250	35.40158	.02804	1.54962
14.41250	37.37410	.02961	1.57257
14.56750	41.31914	.03273	1.61615
14.78750	45.26418	.03586	1.65575
15.03500	49.20923	.03898	1.69205
15.22750	55.12679	.04367	1.74136
15.44750	61.04436	.04836	1.78565
15.67500	66.96192	.05304	1.82583
15.87750	72.87949	.05773	1.86261
16.06000	79.73141	.06316	1.90163
16.25000	85.64897	.06785	1.93272
16.42750	94.47341	.07484	1.97531
16.60250	104.33602	.08265	2.01843
16.84250	115.65207	.09161	2.06315
17.05250	125.92994	.09975	2.10013
17.25000	135.27347	.10715	2.13121
17.40750	147.62768	.11694	2.16917
17.64500	160.91625	.12747	2.20660
17.84500	175.13917	.13873	2.24338
18.07000	190.40026	.15082	2.27967
18.30000	207.63387	.16447	2.31730
18.53500	225.90565	.17895	2.35392
18.73500	243.55453	.19293	2.38660
18.95750	261.30723	.20699	2.41715
19.15000	279.05993	.22105	2.44570
19.35250	296.18972	.23462	2.47157
19.51750	314.46150	.24910	2.49757
19.76250	333.66763	.26431	2.52331
19.93250	353.50812	.28026	2.54877
20.10250	371.56022	.29433	2.57003
20.29500	391.18222	.30987	2.59238
20.47500	410.50744	.32549	2.61374
20.64750	428.66013	.33956	2.63211
20.81000	448.28152	.35510	2.65155
20.95750	468.00675	.37072	2.67025
21.10250	488.14724	.38668	2.68855
21.26250	508.30680	.40304	2.70655
21.43500	531.95798	.42138	2.72588
21.64500	557.08167	.44128	2.74582
21.79250	581.16721	.46036	2.76430
22.00000	607.22527	.48100	2.78335
22.16750	629.33826	.49852	2.79888
22.29750	654.46197	.51842	2.81588
22.49250	680.52002	.53906	2.83284
22.66250	703.15211	.55699	2.84705
22.82500	727.34146	.57615	2.86174
22.95250	752.36124	.59597	2.87643
23.11250	778.00412	.61628	2.89098
23.25250	802.60874	.63577	2.90450
23.40250	830.12023	.65757	2.91914
23.59250	856.28210	.67829	2.93262
23.73250	884.31267	.70049	2.94661
23.86750	913.72668	.72385	2.96095
23.99500	942.86542	.74687	2.97445
24.14250	974.32195	.77179	2.98870
24.26750	1003.90978	.79523	3.00169
24.38000	1034.32814	.81933	3.01466
24.48500	1064.85032	.84350	3.02729
24.62250	1098.38220	.87007	3.04075
24.66250	1129.32065	.89457	3.05292
24.99250	1451.67223	1.14992	3.16137
24.99250	1486.13944	1.17722	3.17266

EVEN

THETA

DELTA*

DELTA* = .014923 THETA = .010379
 UTRU = 2.091953 UONE = 52.252028 FNEU = .000168
 H=1.3731 RED995 15146.267 FEHET = 2388.221

XL

STN 14

STN 17

TYPE U1-CF-TEMP-D995-F9.6
 1.919 0.00315 25. 1.4111
 U1= 1.918000 CF= .003150 TEMP=25.000000 D995= 1.4111
 TYPE RVP(1)-FHD-NP
 14.35 0.002373 67
 RVP(1)=14.350000 FHD= .000000 NP= 67
 U-UTAU YPLUS Y-D995 LOG YPLUS

13.27970	26.62223	.01750	1.42524
13.41662	27.80793	.01823	1.44417
13.60027	29.74791	.01956	1.47346
13.86443	32.82691	.02119	1.50822
14.03047	34.16699	.02246	1.53261
14.18141	35.64598	.02409	1.56403
14.34242	37.39337	.02495	1.57909
14.45563	39.37945	.02622	1.60075
14.58142	42.35844	.02735	1.62694
14.72262	46.77752	.03076	1.67004
14.93401	50.65758	.03231	1.70465
15.24307	55.26626	.03689	1.75022
15.47955	63.26821	.04160	1.80119
15.63805	70.81297	.04656	1.85011
15.90975	79.75990	.05244	1.90173
16.12359	89.39044	.05910	1.95371
16.34749	99.31419	.06569	1.99963
16.61663	112.63250	.07406	2.05166
16.86323	126.53642	.08320	2.10222
17.12487	142.33041	.09351	2.15345
17.39154	158.11643	.10396	2.19393
17.61796	173.85063	.11433	2.24045
17.83935	190.45132	.12522	2.27978
18.04061	206.83423	.13539	2.31562
18.26955	226.45061	.14899	2.35497
18.41294	248.00707	.16306	2.39446
18.63716	268.27014	.17639	2.42357
18.92365	292.35223	.19262	2.46680
19.15761	316.34105	.20799	2.50016
19.36394	341.26399	.22455	2.53361
19.58026	366.39873	.24130	2.56466
19.83184	394.80656	.25939	2.59633
20.04568	424.55447	.27914	2.62793
20.25952	453.80990	.29795	2.65611
20.46078	482.75691	.31741	2.68373
20.67714	512.59492	.33697	2.70970
20.87589	547.21851	.35926	2.73624
21.10231	590.83680	.38190	2.76406
21.31866	614.35909	.40294	2.78942
21.51233	647.87939	.42598	2.81149
21.70106	680.10620	.44717	2.83258
21.98220	712.97990	.46878	2.85309
22.04069	745.96129	.49047	2.87272
22.23440	779.43158	.51251	2.89181
22.41806	813.64857	.53497	2.91044
22.62933	848.46225	.55785	2.92863
22.81906	881.88254	.57990	2.94546
22.99320	916.14954	.60237	2.96197
23.14763	953.44221	.62639	2.97929
23.29606	996.51521	.64880	2.99402
23.46462	1024.30187	.67387	3.01063
23.62563	1064.13463	.69967	3.02700
23.79167	1102.72068	.72504	3.04247
23.95016	1145.33361	.75228	3.05912
24.11620	1190.67104	.77926	3.07579
24.28476	1234.96356	.81199	3.09166
24.40303	1277.33579	.83991	3.10634
24.56149	1319.50026	.86691	3.12003
24.66213	1359.78700	.89256	3.13322
24.77534	1408.39123	.92602	3.14972
24.87597	1446.97795	.95139	3.16046
24.97597	1493.10817	.98172	3.17489
25.00679	1562.73552	1.00750	3.19389
25.00679	1626.50376	1.00910	3.21112
25.07220	1701.38248	1.11299	3.23092
25.13761	1809.44422	1.13971	3.25755
25.13761	1922.71744	1.23121	3.27247

EVEN

THETA DELTA*
 DELTAP= .012556 THETA= .013871
 UTAU= 2.147023 UONE=54.014241 RHEU= .000166

EC3TYPE U1-CF-TEMP-D995-F9.6
 1.917 0.00301 27. 1.634
 U1= 1.917000 CF= .003010 TEMP=27.000000 D995= 1.63400
 TYPE RVP(1)-FHD-NP
 14.85 0.002373 61
 RVP(1)=14.850000 FHD= .000000 NP= 61
 U-UTAU YPLUS Y-D995 LOG YPLUS

13.14625	24.64734	.01506	1.39177
13.21327	26.55099	.01622	1.42402
13.38082	28.45465	.01738	1.45415
13.62313	30.35830	.01854	1.48228
13.96080	32.76292	.02001	1.51538
14.09742	35.16754	.02148	1.54614
14.24177	37.97293	.02319	1.57947
14.50728	41.79024	.02552	1.62097
14.78824	46.58948	.02846	1.66829
14.96353	51.29852	.03133	1.71010
15.25996	58.91314	.03599	1.77021
15.57444	66.52777	.04064	1.82300
15.86830	75.94586	.04639	1.88050
16.14927	86.46606	.05232	1.93685
16.41477	98.78973	.06034	1.99471
16.66481	112.11532	.06848	2.04966
16.92258	123.43706	.07540	2.09145
17.18550	136.76266	.08354	2.13597
17.41492	147.68363	.09021	2.16933
17.60051	161.51019	.09865	2.20320
17.75759	174.32481	.10649	2.24138
17.98974	190.46520	.11634	2.27982
18.17018	208.50043	.12736	2.31911
18.42537	230.34238	.14070	2.36237
18.65737	253.09606	.15459	2.40327
18.94091	276.83166	.16903	2.44232
19.18579	302.48092	.18476	2.48070
19.38170	329.13211	.20104	2.51737
19.66525	357.58675	.21842	2.55333
19.87404	380.43063	.23237	2.58028
20.06479	406.48066	.24829	2.60904
20.26069	432.63088	.26426	2.63612
20.46433	460.68476	.28140	2.66340
20.71695	494.85037	.30226	2.69447
20.94894	530.91965	.32430	2.72503
21.14484	564.18352	.34461	2.75142
21.38715	605.06203	.36953	2.78180
21.64492	652.55325	.39859	2.81462
21.92073	704.25253	.43017	2.84773
22.18108	751.74374	.45918	2.87607
22.43627	801.13861	.48935	2.90371
22.73271	854.34078	.52185	2.93163
22.97501	901.83199	.55036	2.95513
23.19927	949.82417	.58017	2.97764
23.43642	997.31537	.60918	2.99883
23.69161	1046.71024	.63935	3.01983
23.93133	1097.00682	.67007	3.04021
24.14528	1149.30727	.70202	3.06044
24.35923	1196.79848	.73103	3.07802
24.57060	1244.28969	.76004	3.09492
24.70980	1291.78090	.78905	3.11119
24.91859	1343.98116	.82003	3.12839
25.05779	1386.76331	.84706	3.14200
25.19441	1434.25452	.87607	3.15663
25.33102	1486.45478	.90796	3.17215
25.46764	1533.94599	.93696	3.18581
25.53466	1581.43718	.96597	3.19905
25.66870	1647.96495	1.00661	3.21295
25.73572	1720.31031	1.05309	3.23651
25.80274	1818.89324	1.11102	3.25991
25.80274	1913.87566	1.16903	3.28191

THETA DELTA*
 DELTAP= .020777 THETA= .015310
 UTAU= 2.010317 UONE=52.041563 RHEU= .000168
 H=1.3571 FELD95= 19645.730 PETHET= 4744.796

FIGURE [4-8.3] EFFECTS OF THRESHOLD FACTOR ON INTERMITTENCY AT STN. 17

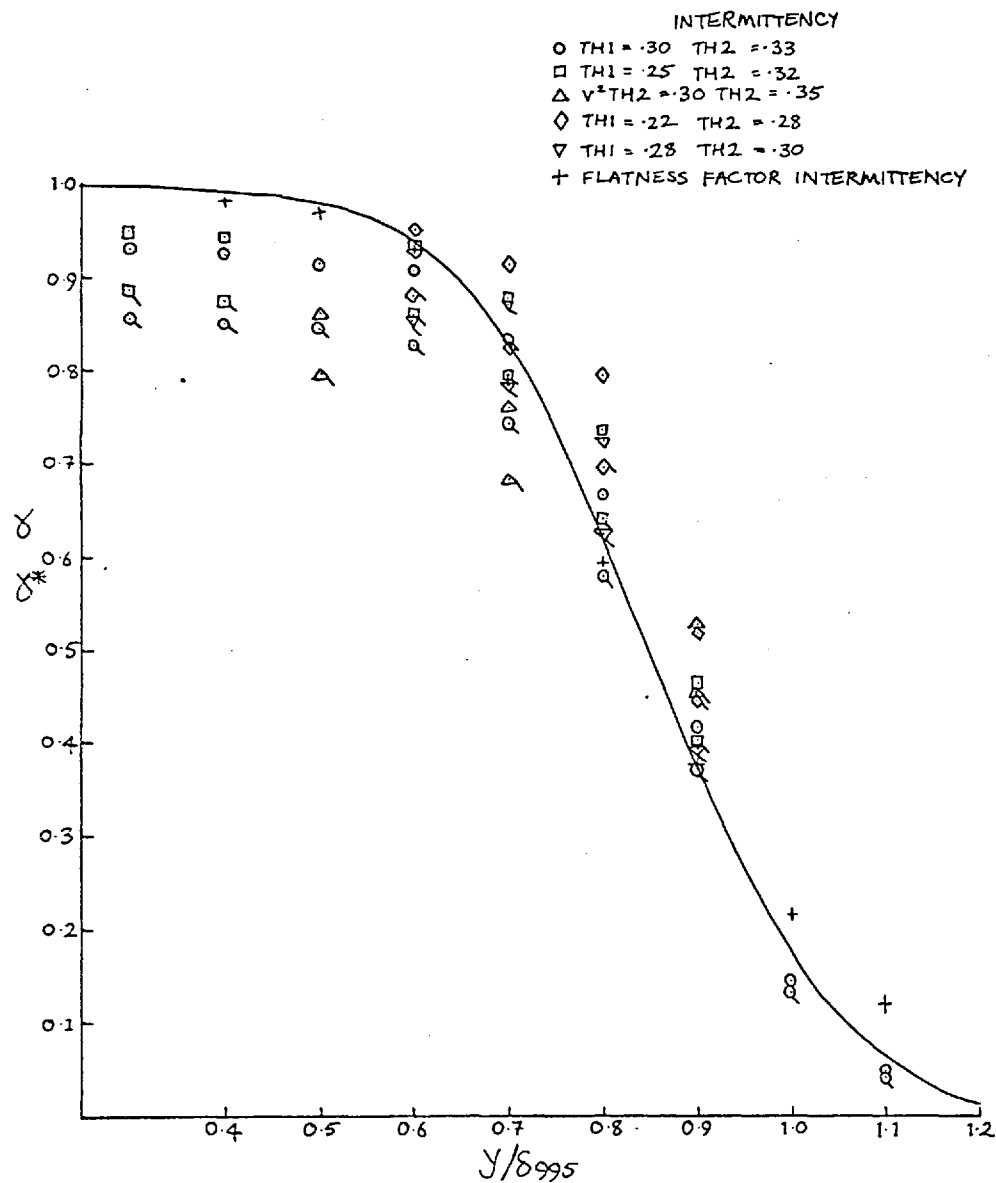


FIGURE [4-8.4] FLATNESS FACTOR INTERMITTENCIES BASED ON WALL VALUE 3

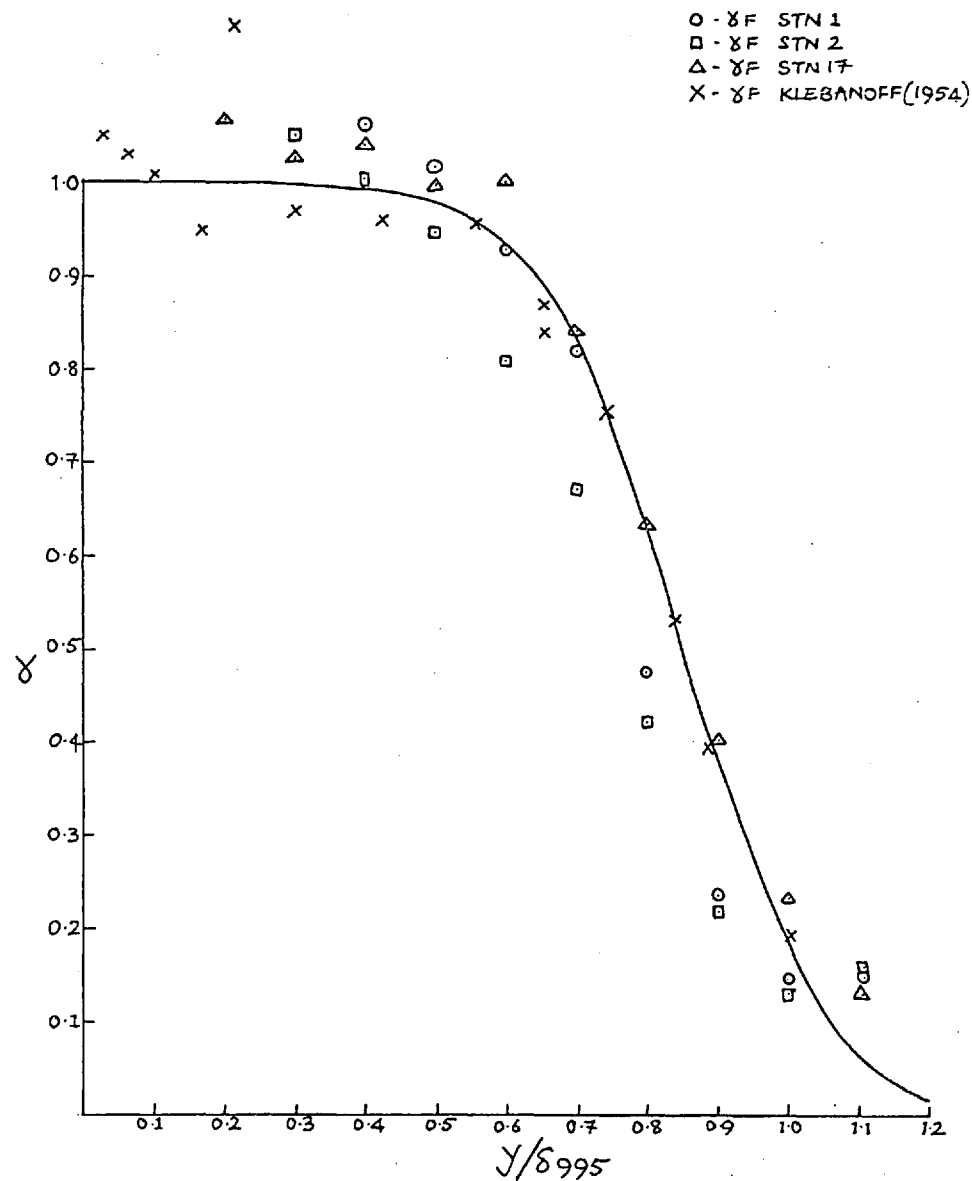


FIGURE [4-8.5] FLATNESS FACTOR INTERMITTENCY BASED ON MEASURED F_u AT WALL

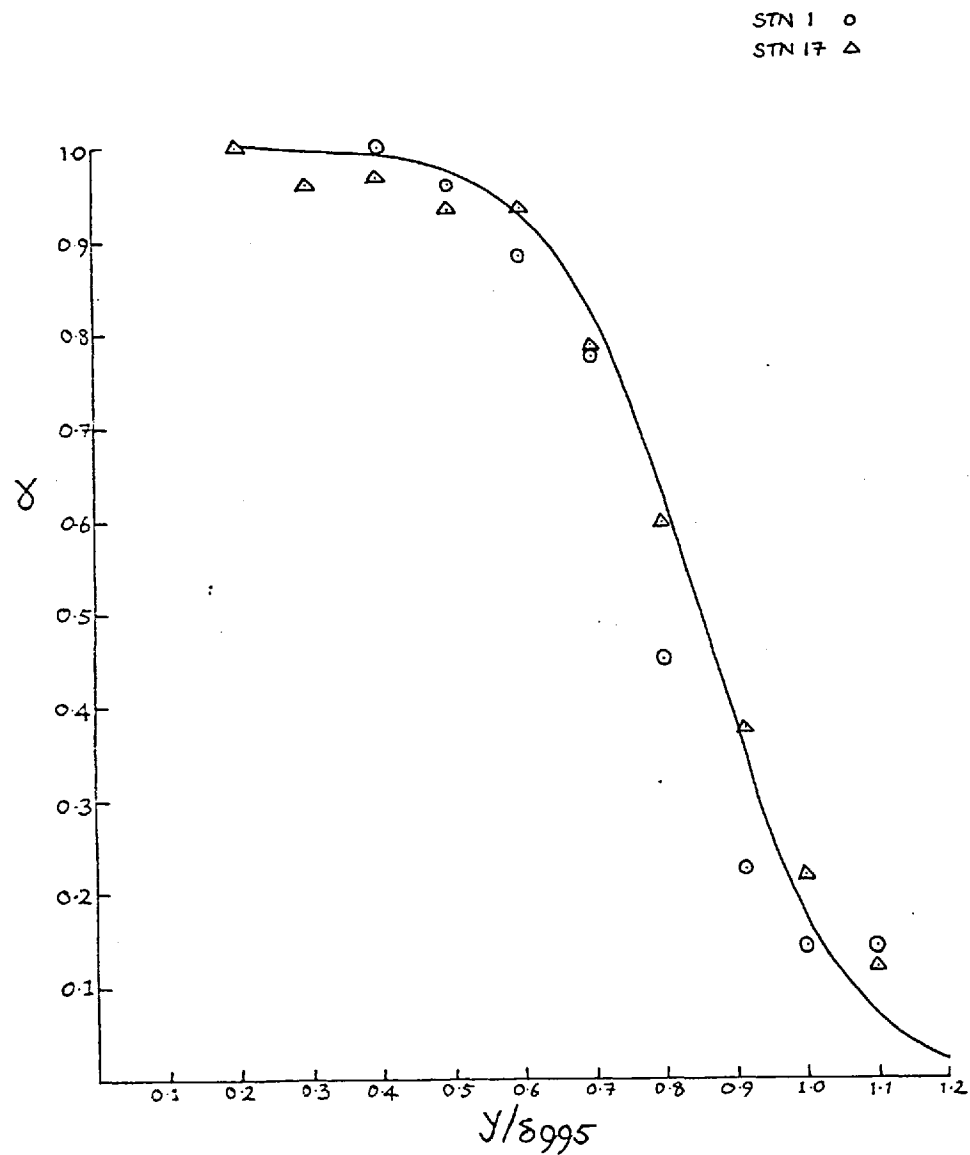


FIGURE [4-8.6] COMPUTED INTERMITTENCIES COMPARED WITH FLATNESS FACTOR INTERMITTENCY

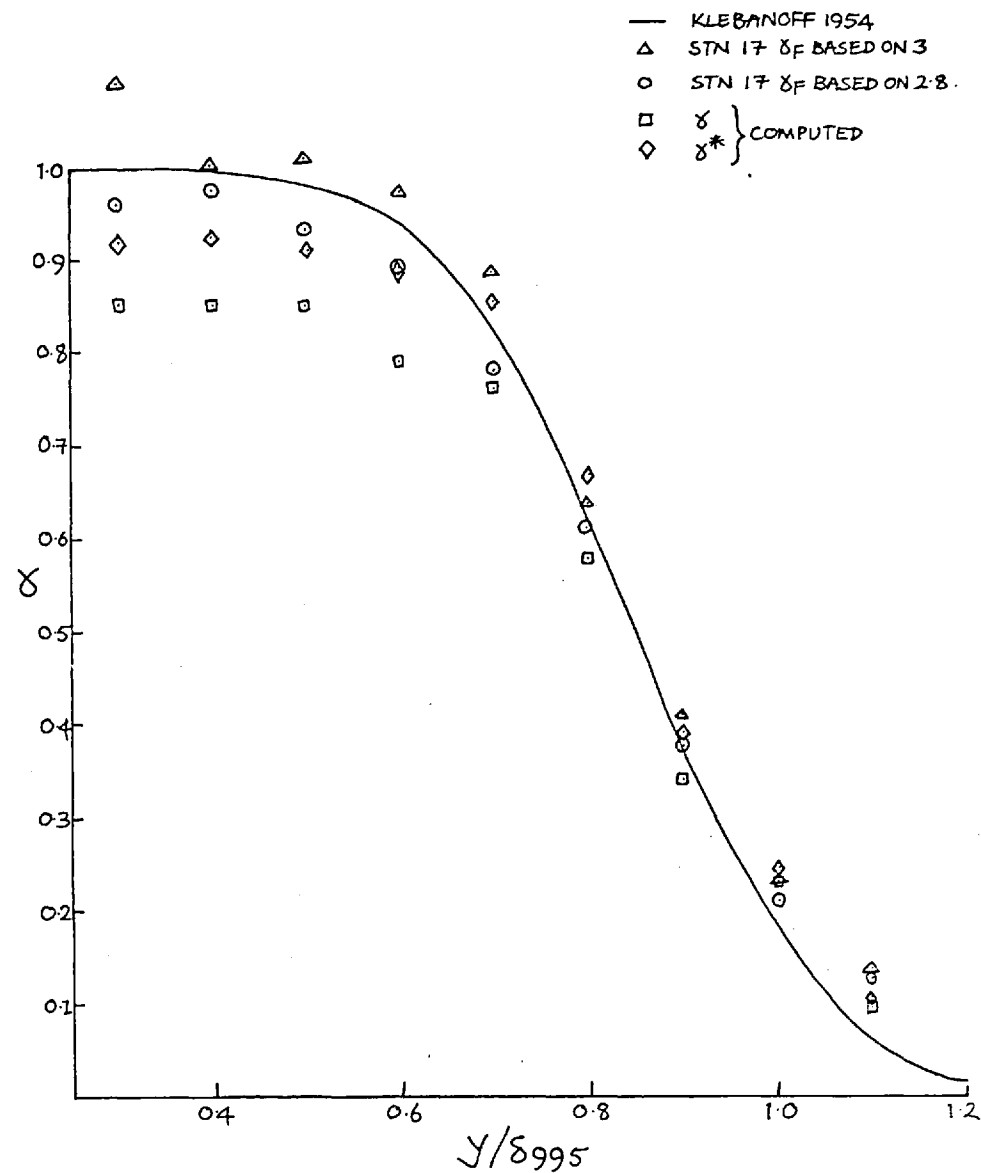


FIGURE [4-8.7] EFFECT OF δ, δ^* ON $\frac{\bar{u}^2}{Ue^2}$

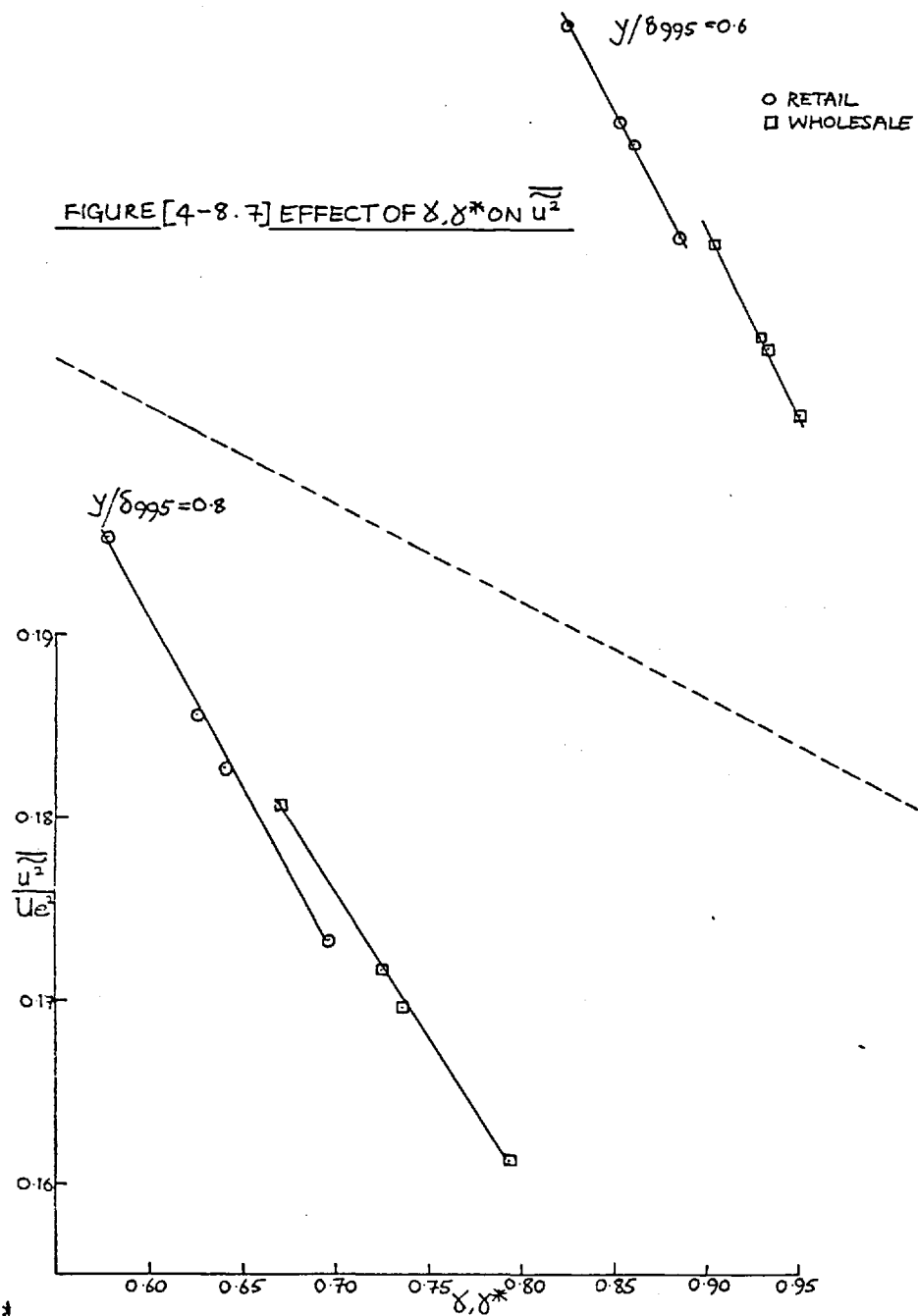


FIGURE [4-8.8] EFFECT OF δ, δ^* ON $\frac{\bar{\delta u^2}}{Ue^2}, \frac{\bar{\delta^* u^2}}{Ue^2}$

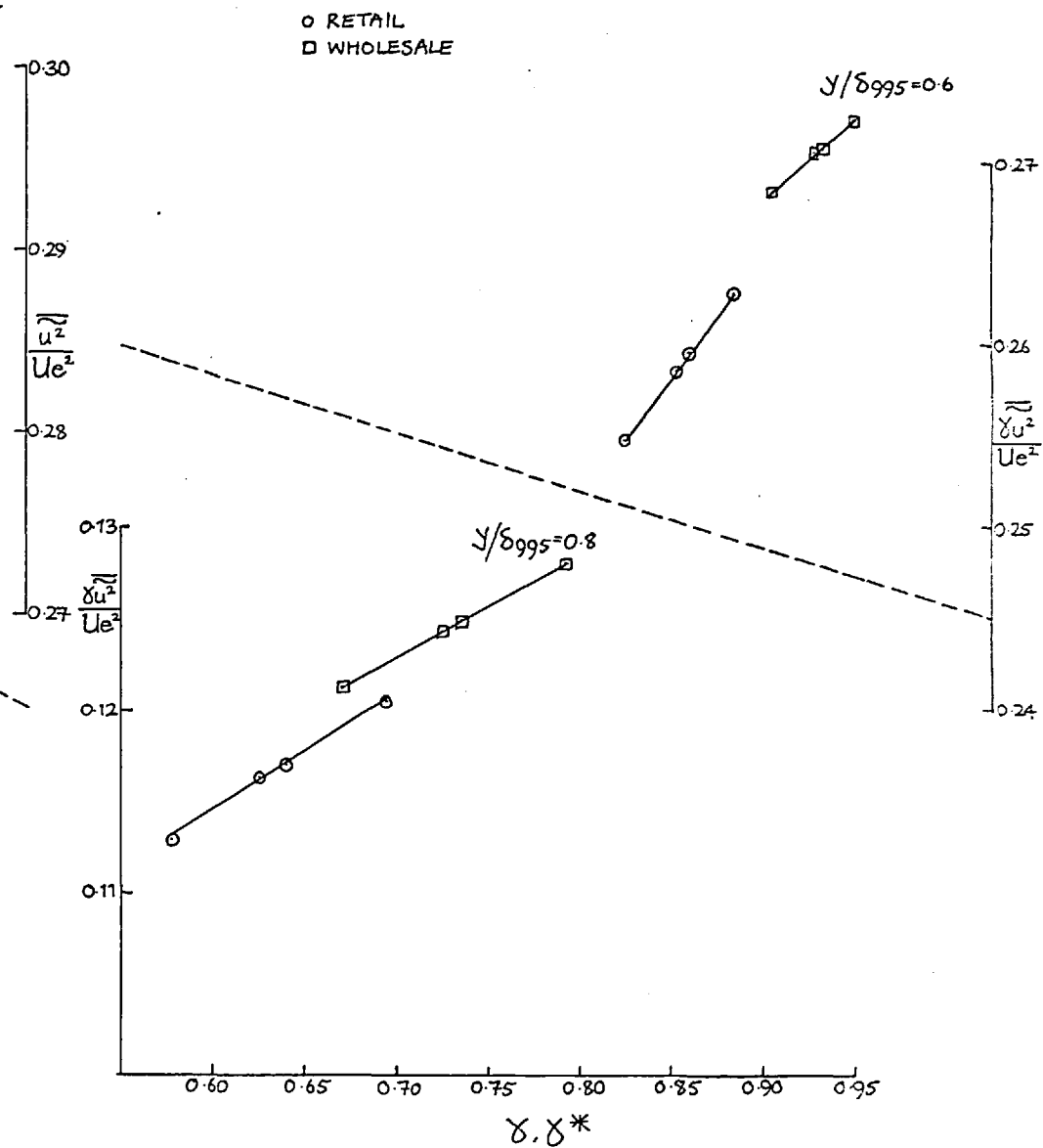


FIGURE [4-8.9] EFFECT OF δ, δ^* ON $\frac{\bar{u}^2}{Ue^2}$

○ RETAIL
□ WHOLESALE

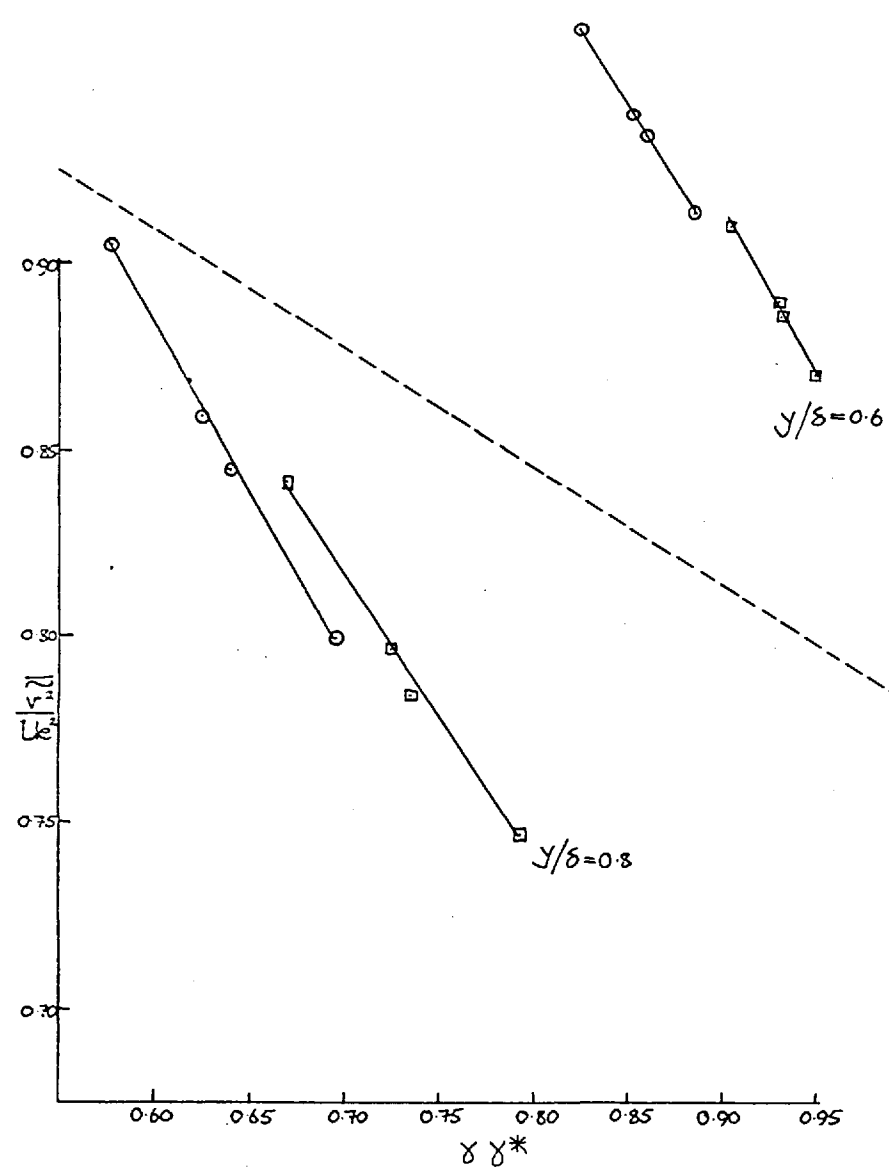
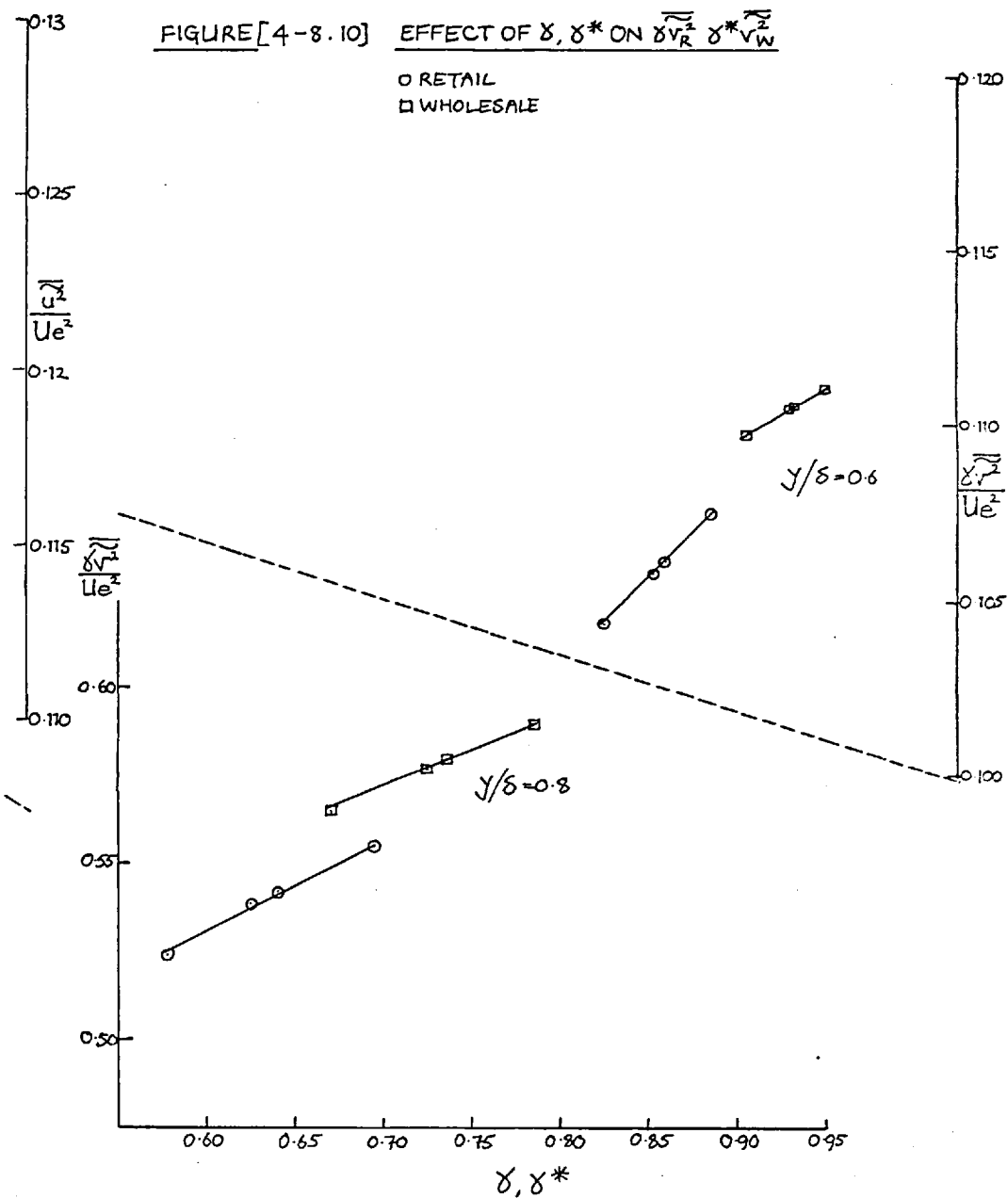


FIGURE [4-8.10] EFFECT OF δ, δ^* ON $\frac{\bar{u}^2}{Ue^2} \delta^* \sqrt{V_W}$

○ RETAIL
□ WHOLESALE



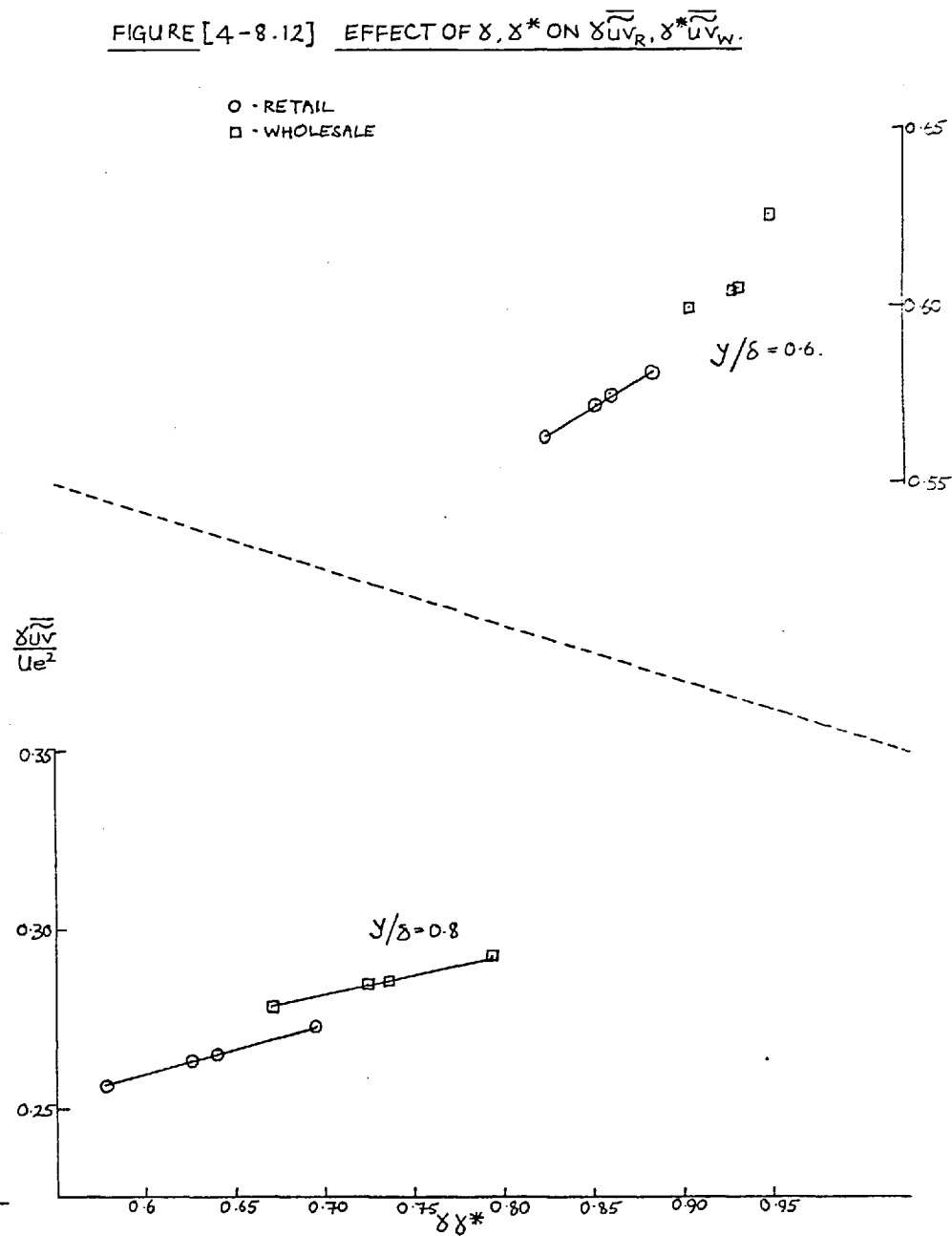
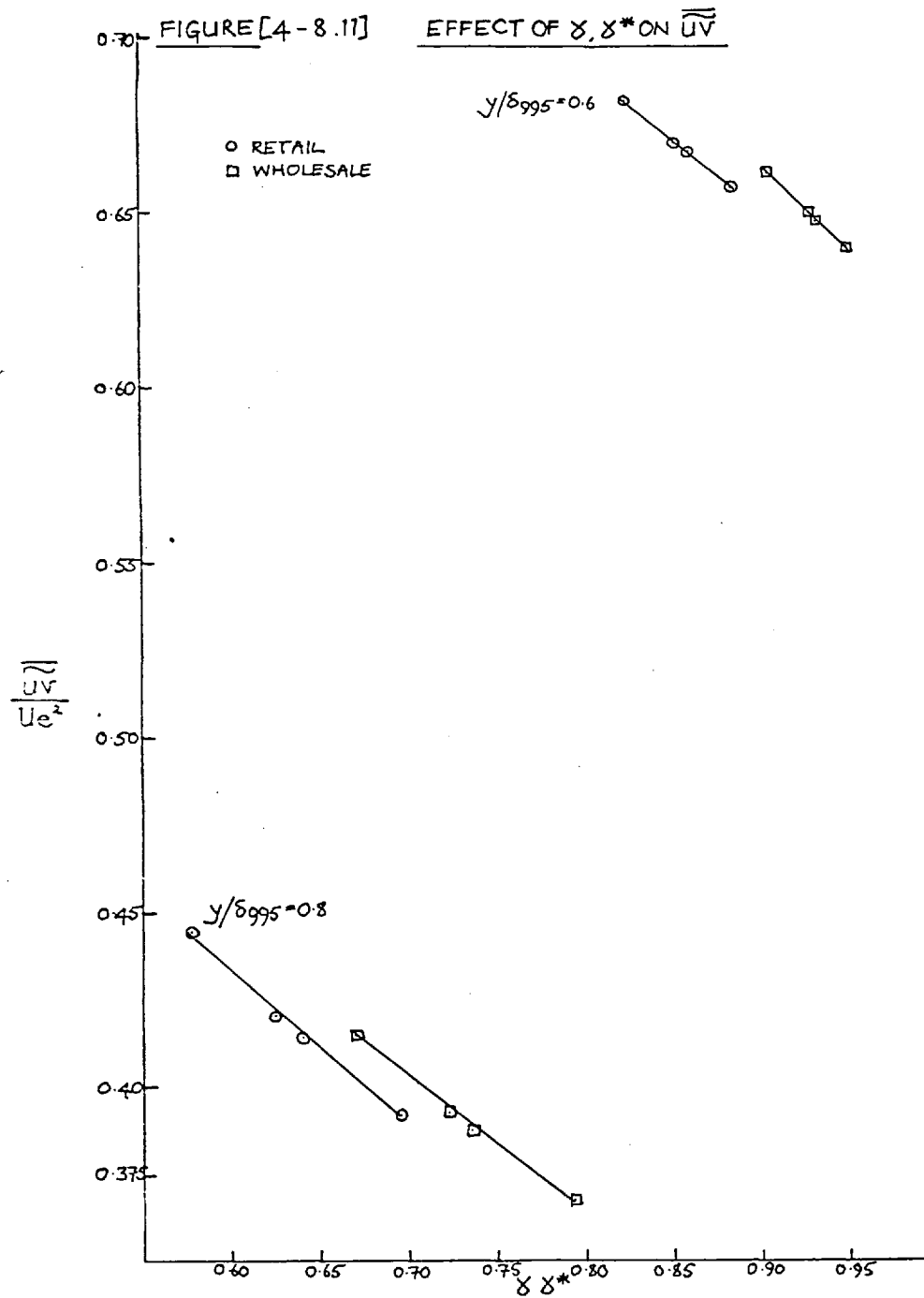


FIGURE [4-8.13] EFFECT OF γ, γ^* ON $\overline{uv^2}$

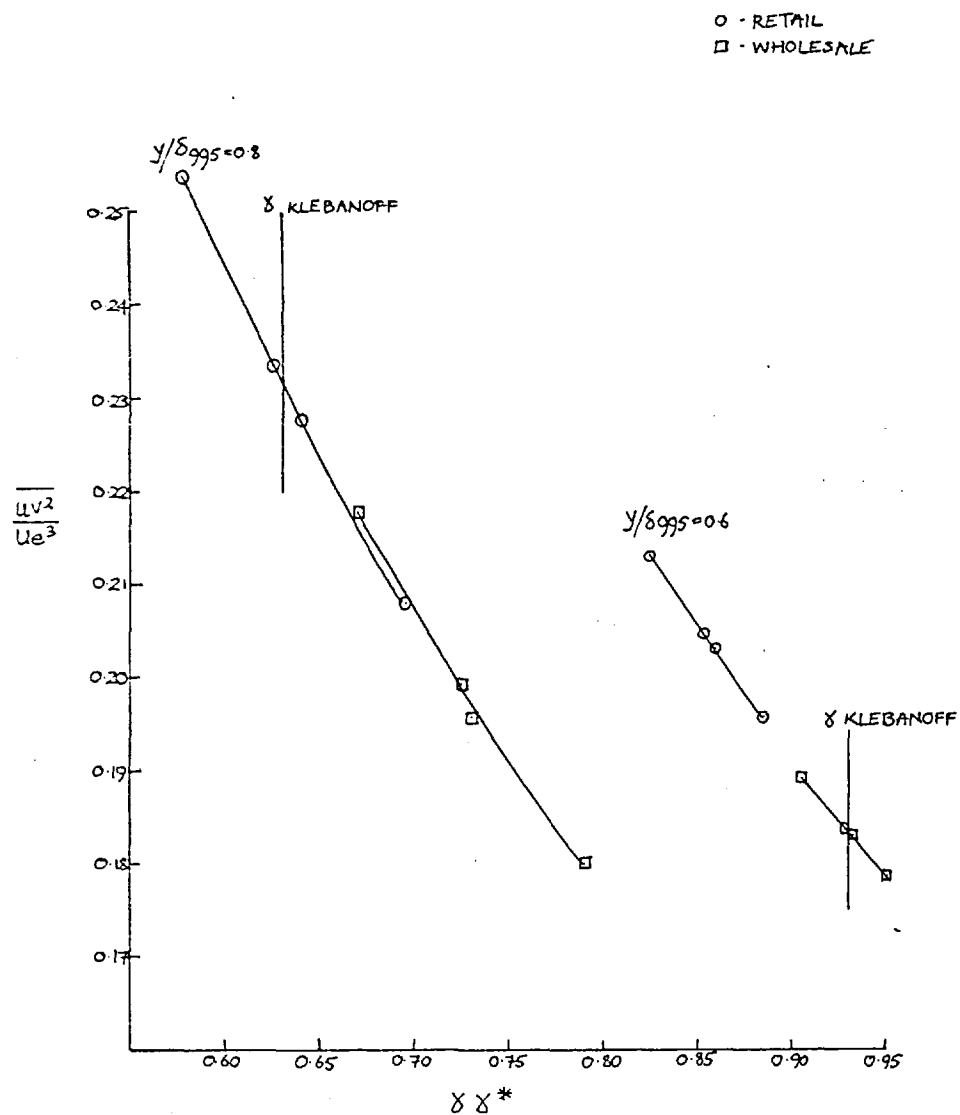
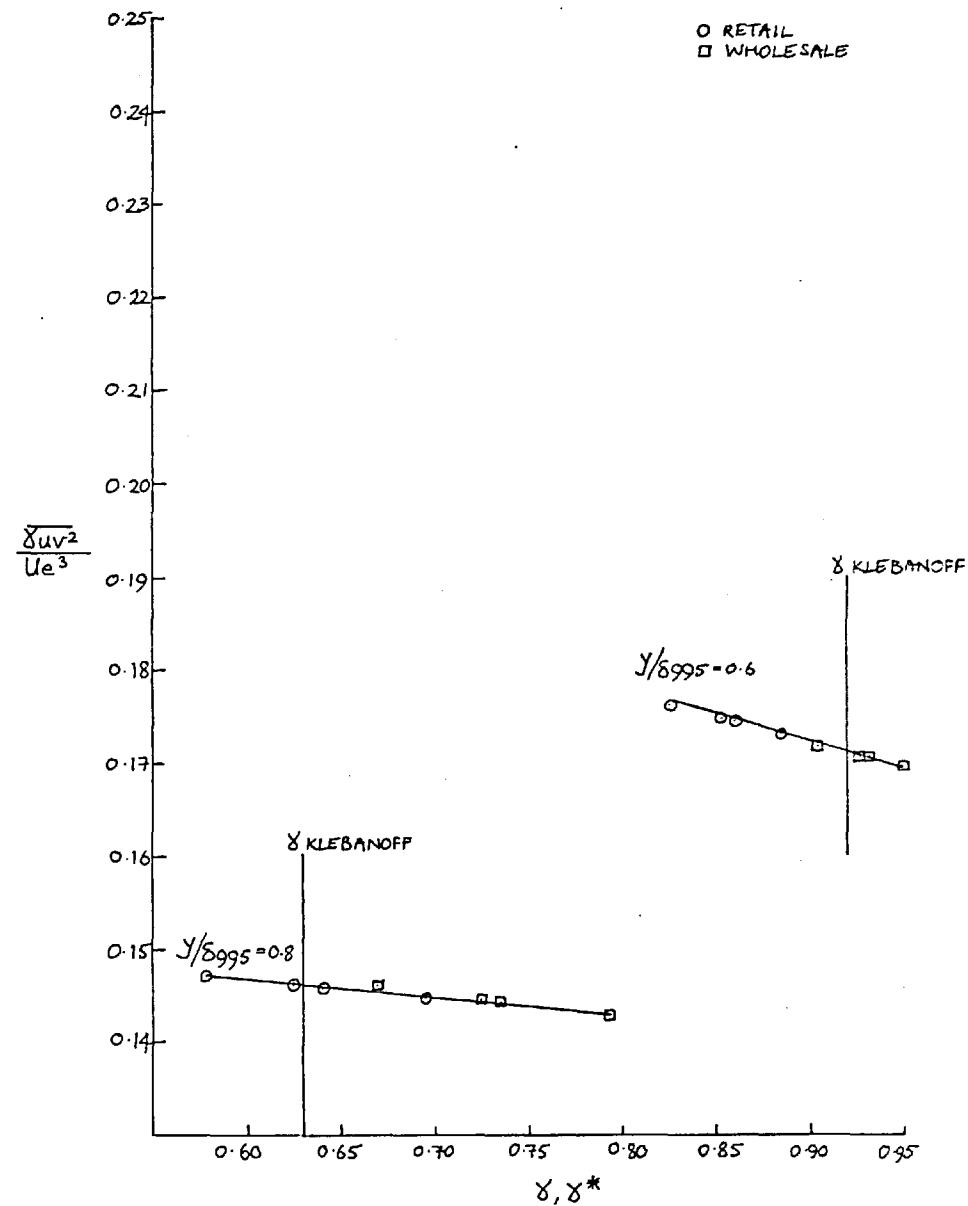


FIGURE [4-8.14] EFFECT OF γ, γ^* ON $\overline{\delta v^2 u_R}$ & $\overline{\delta v^2 u_W}$



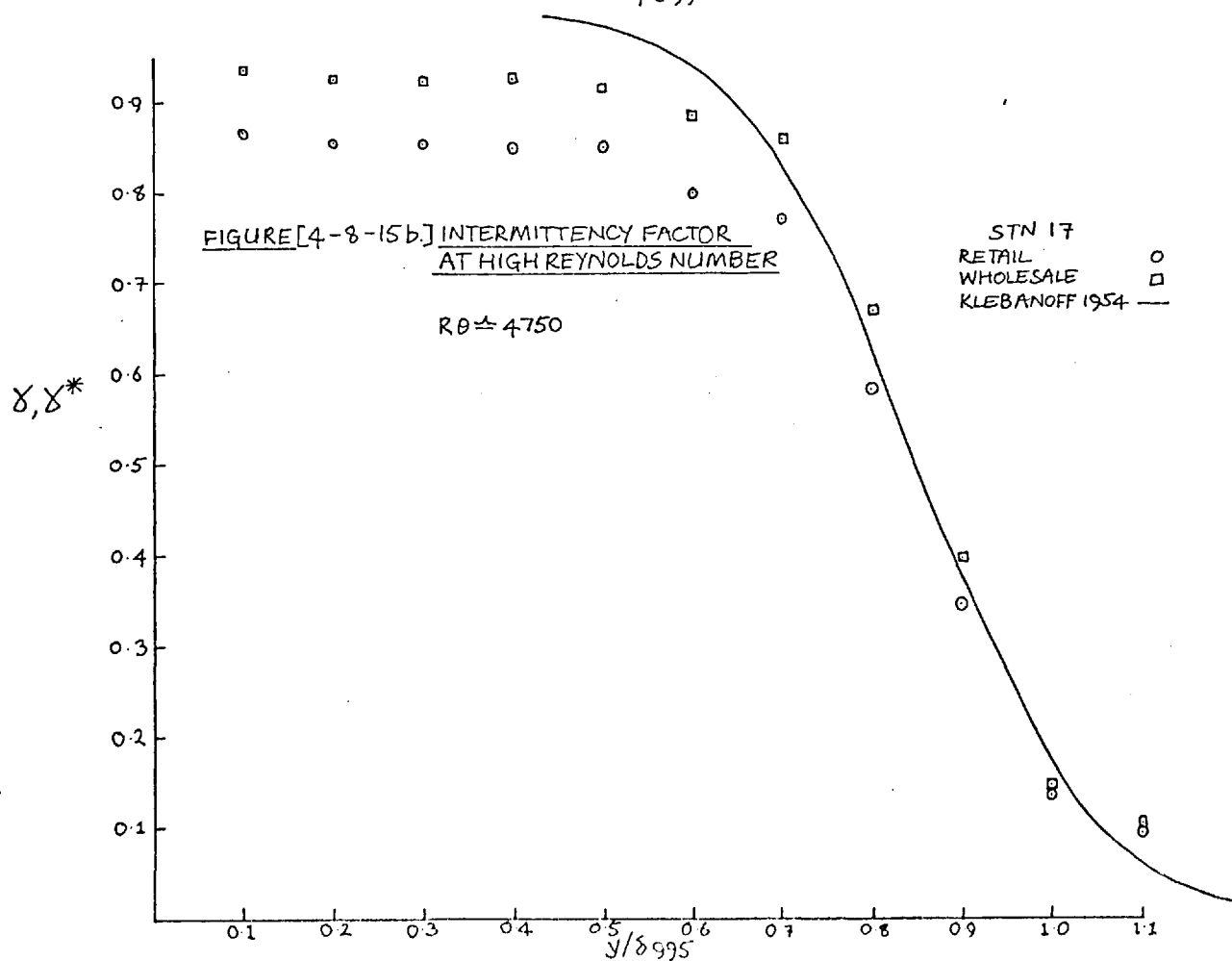
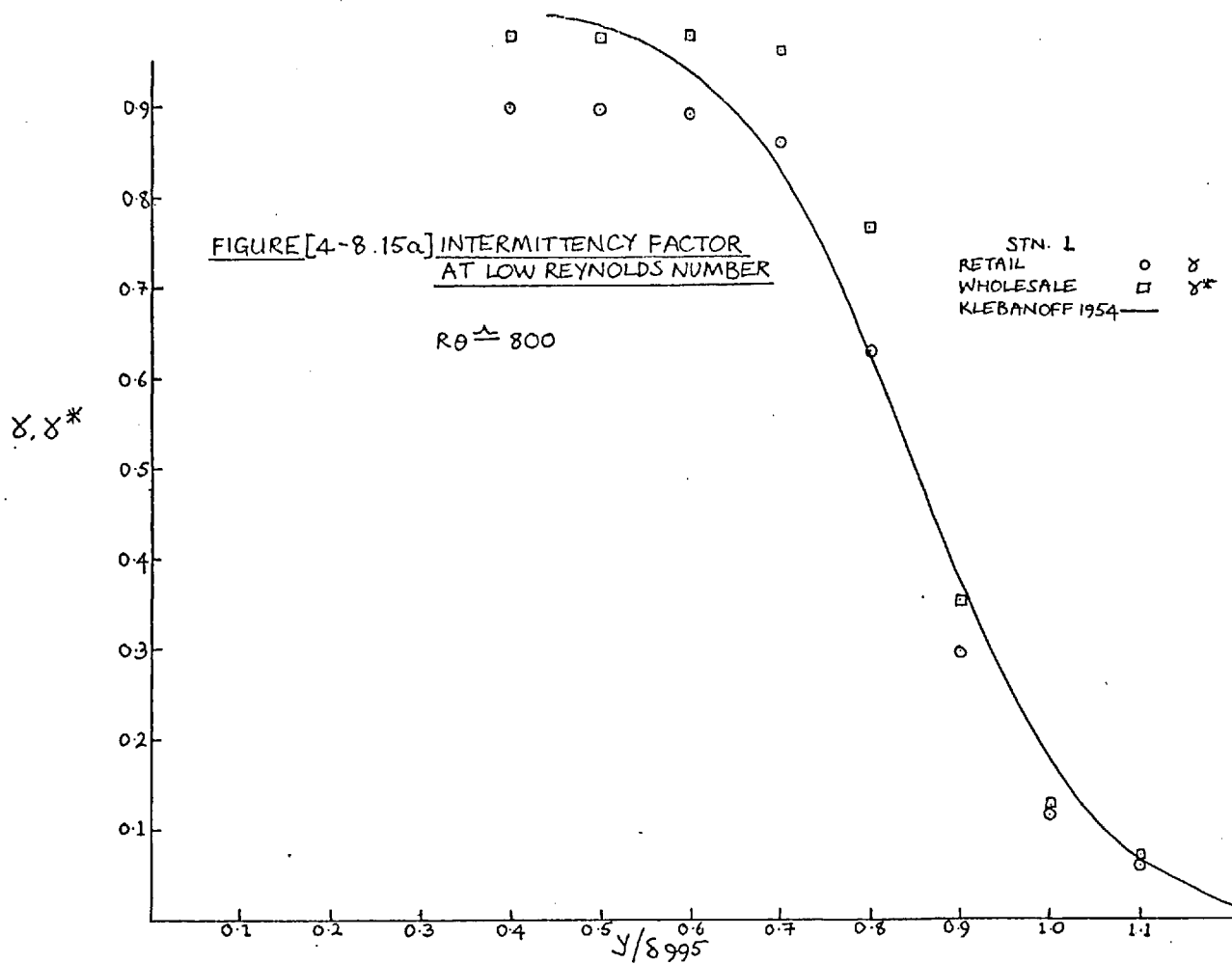


FIGURE [4-8.16b] WHOLESALE INTERMITTENCY

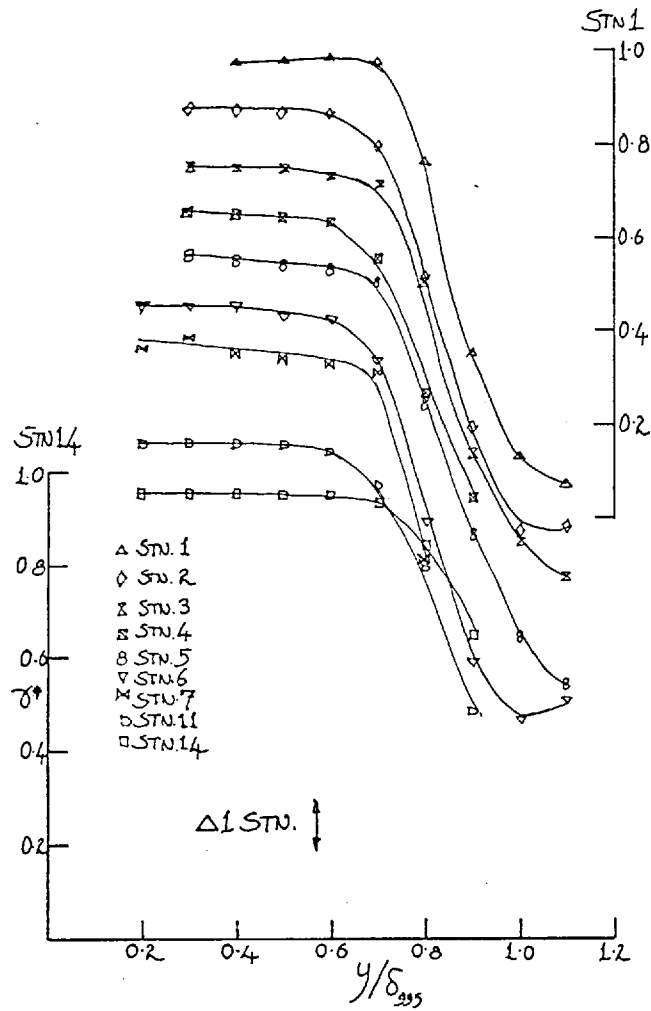


FIGURE [4-8.16a] RETAIL INTERMITTENCY

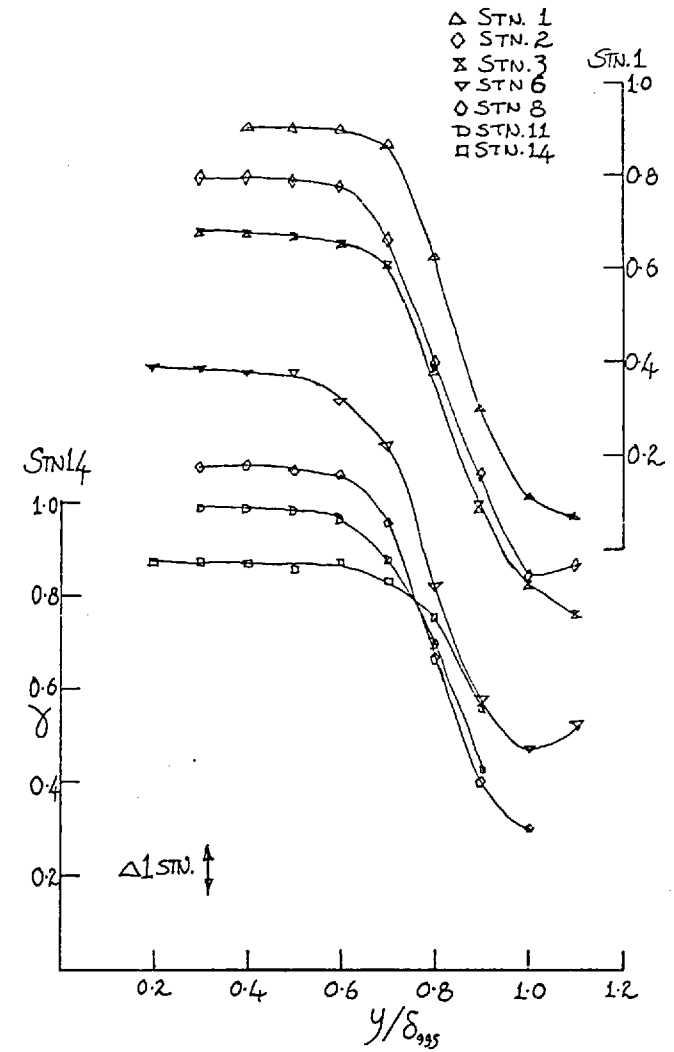


FIGURE [4-11.1] BURST LENGTH PROBABILITY
DISTRIBUTION AT LOW Re

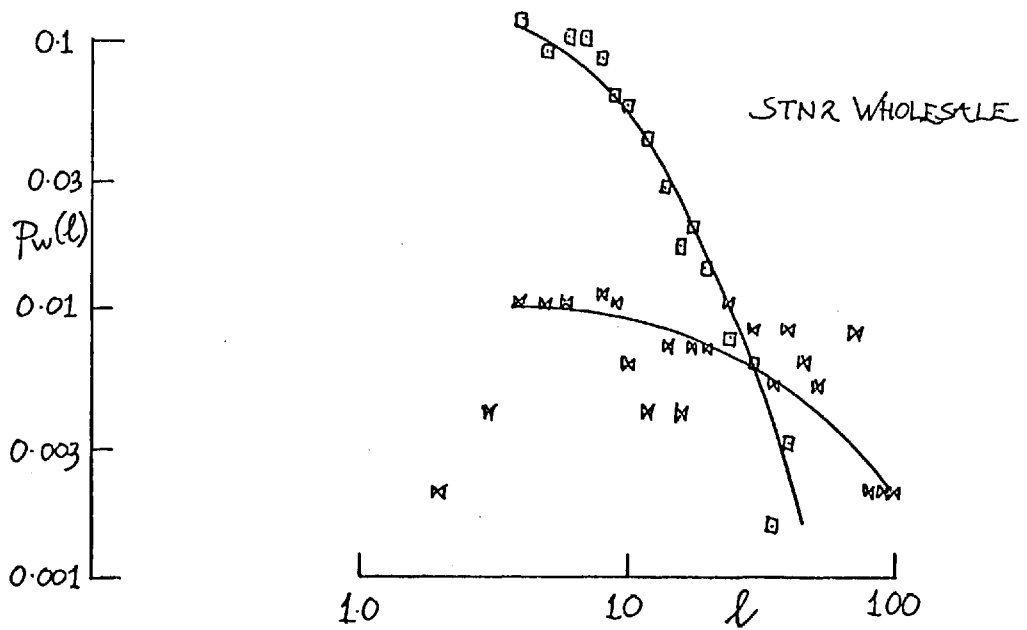
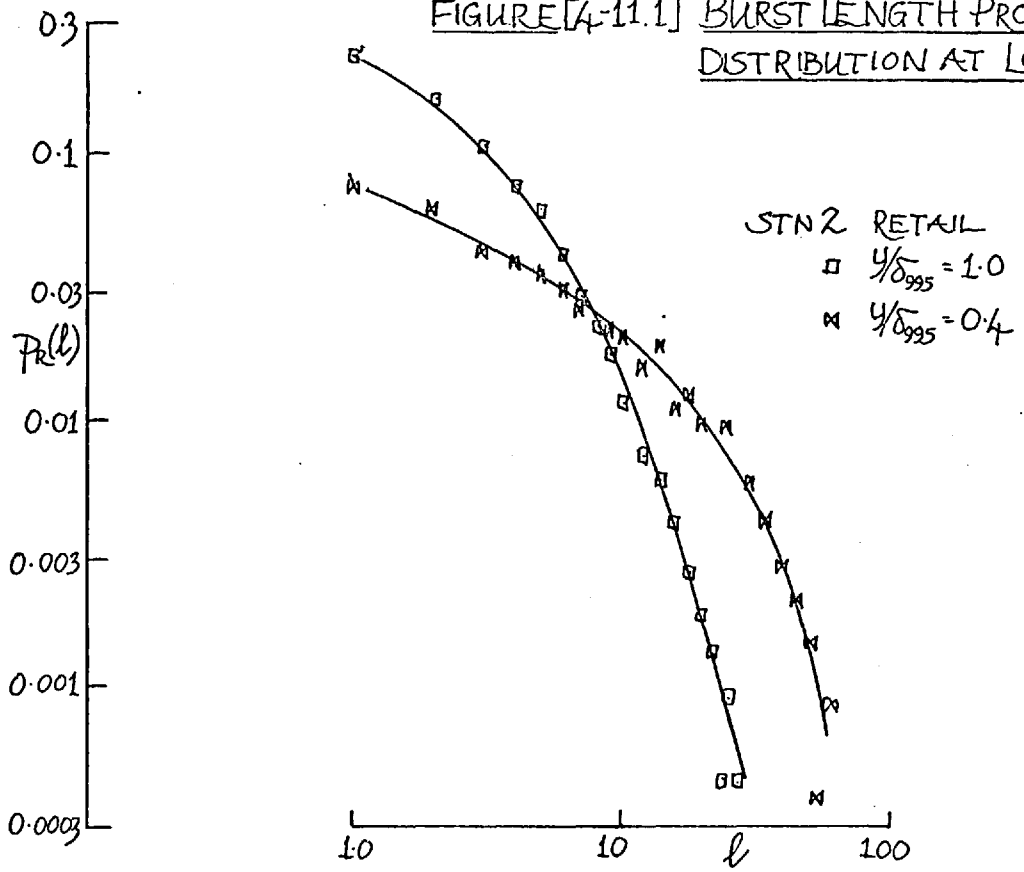


FIGURE [4-11.2] BURST LENGTH PROBABILITY DISTRIBUTION AT MODERATE R_e

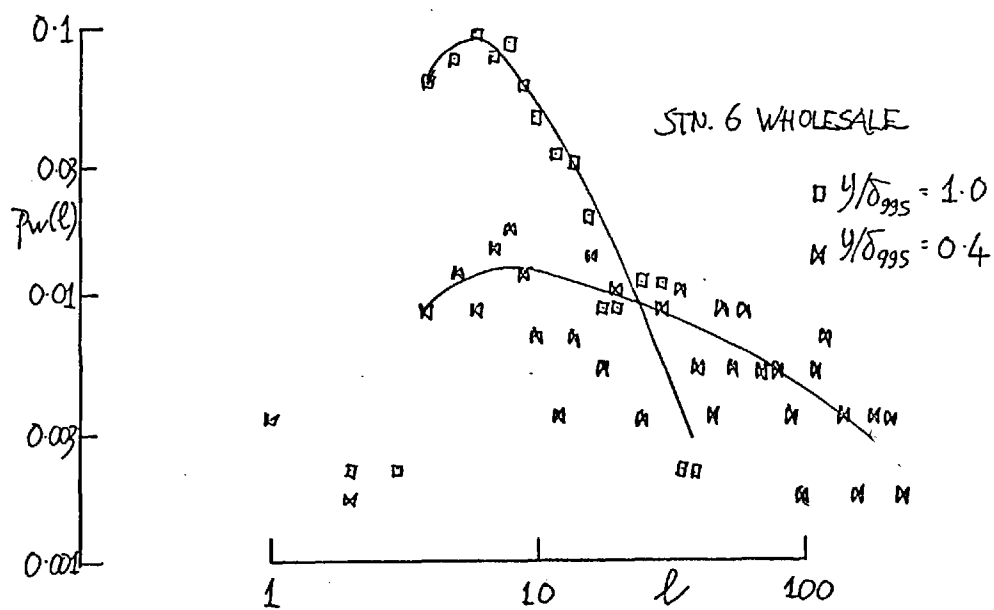
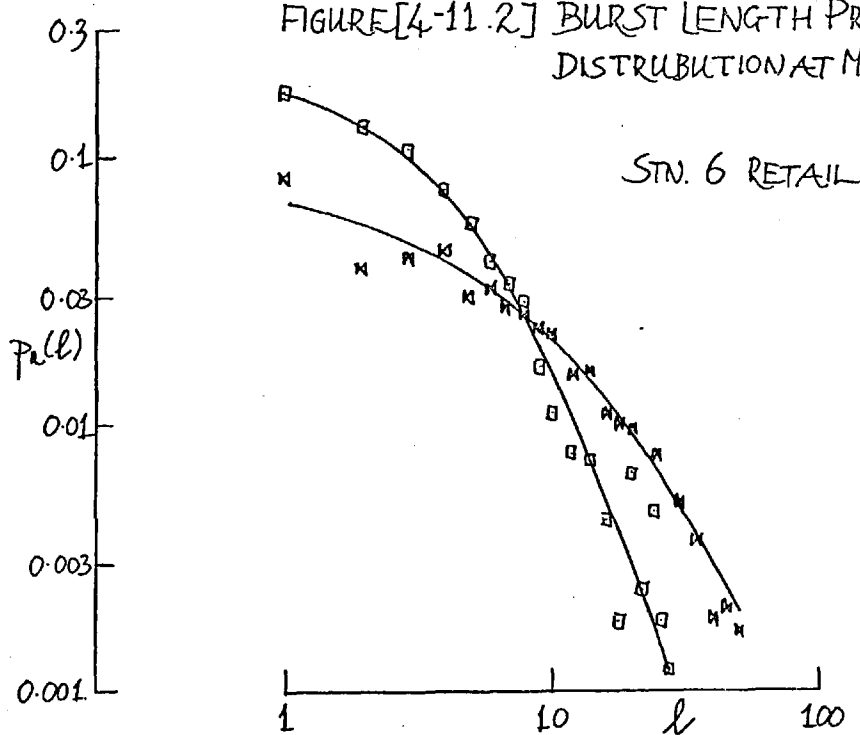
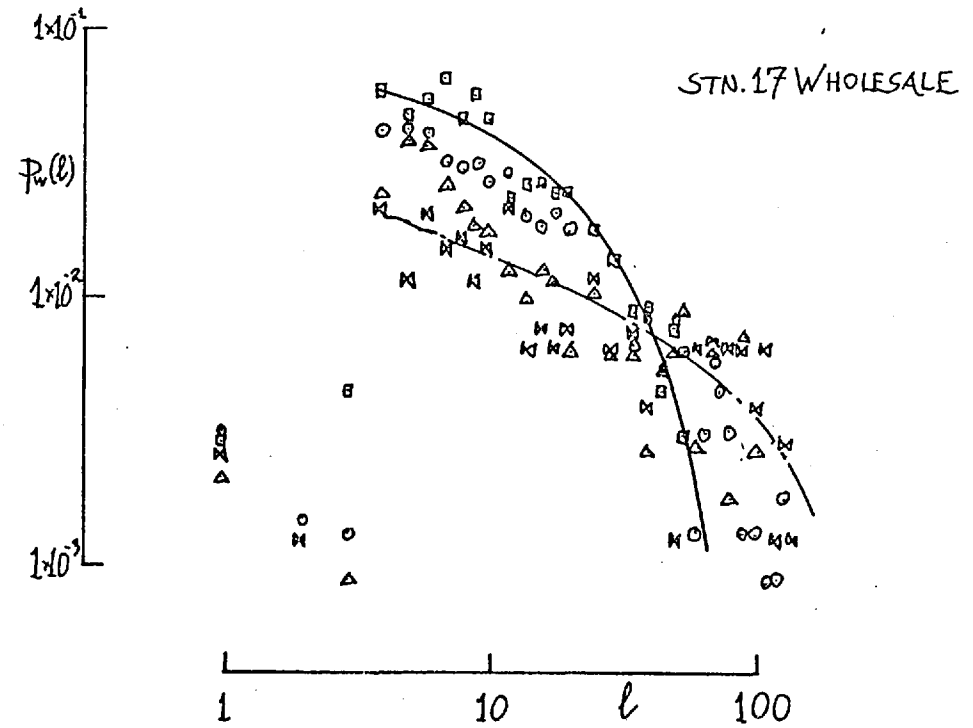
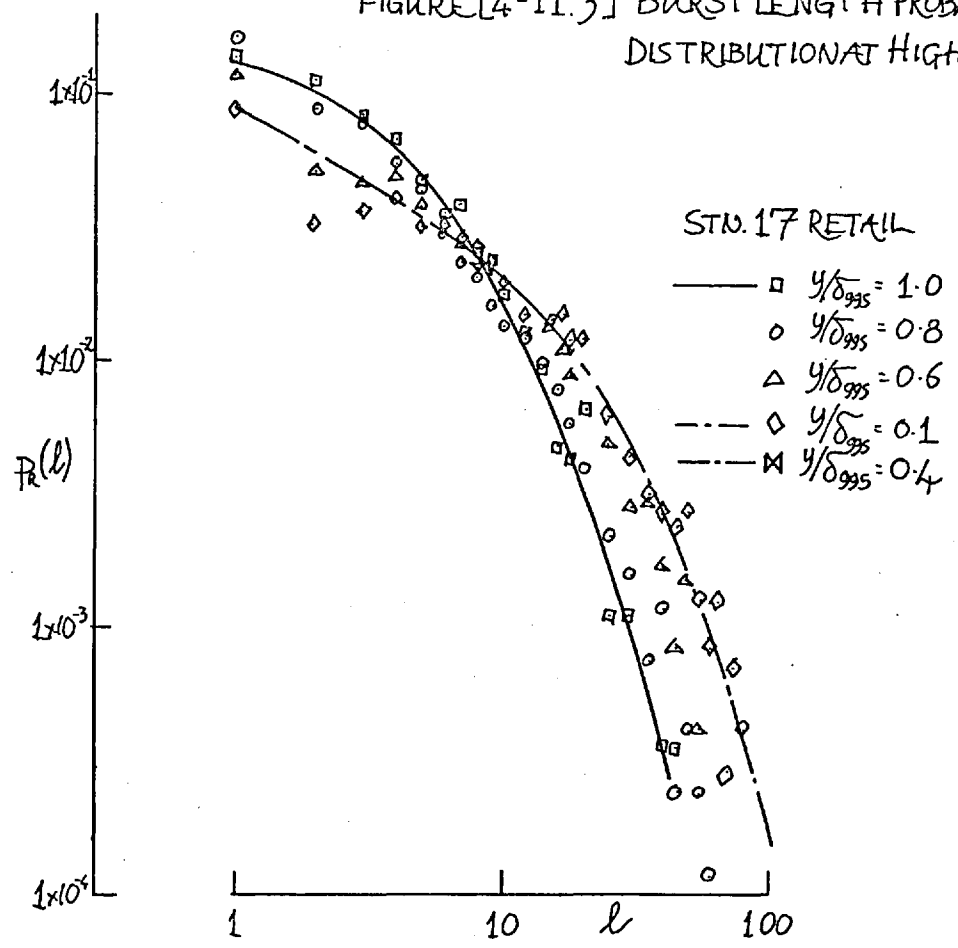


FIGURE [4-11.3] BURST LENGTH PROBABILITY DISTRIBUTION AT HIGH Re



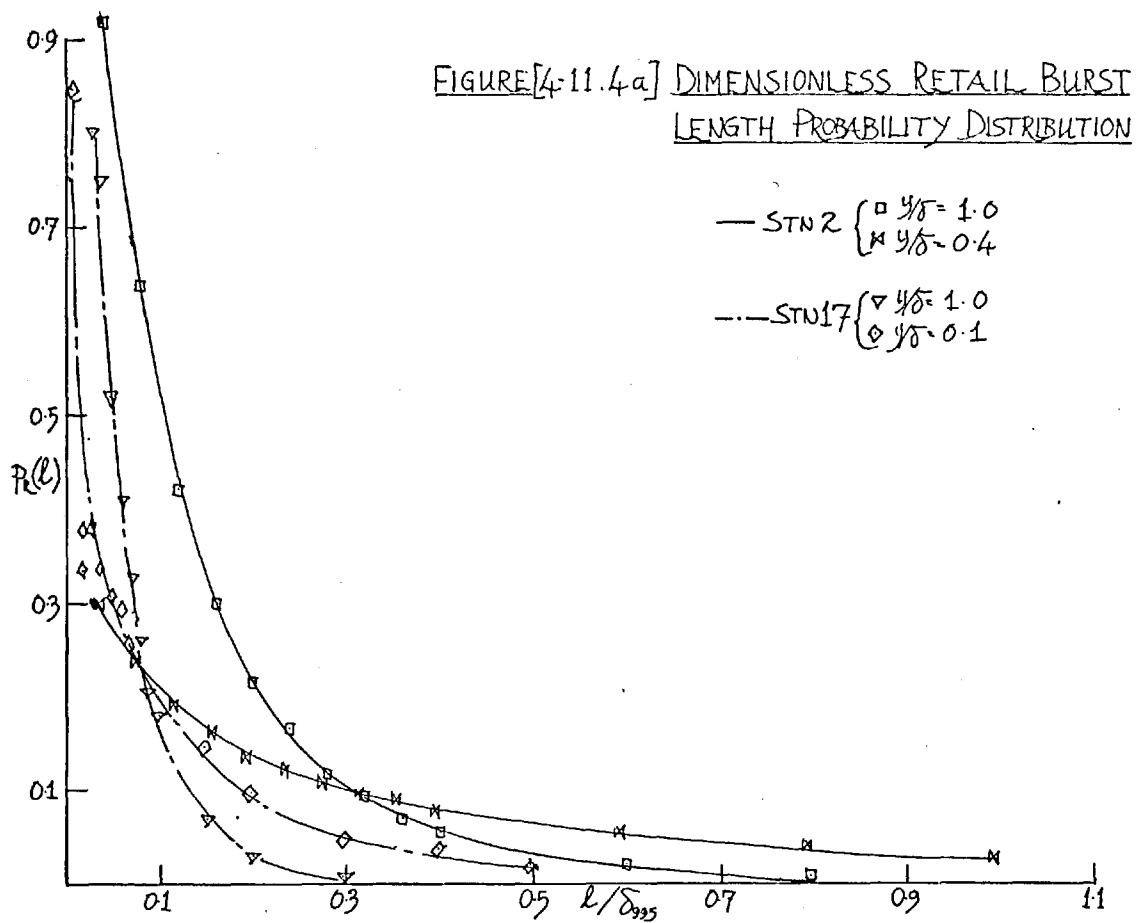
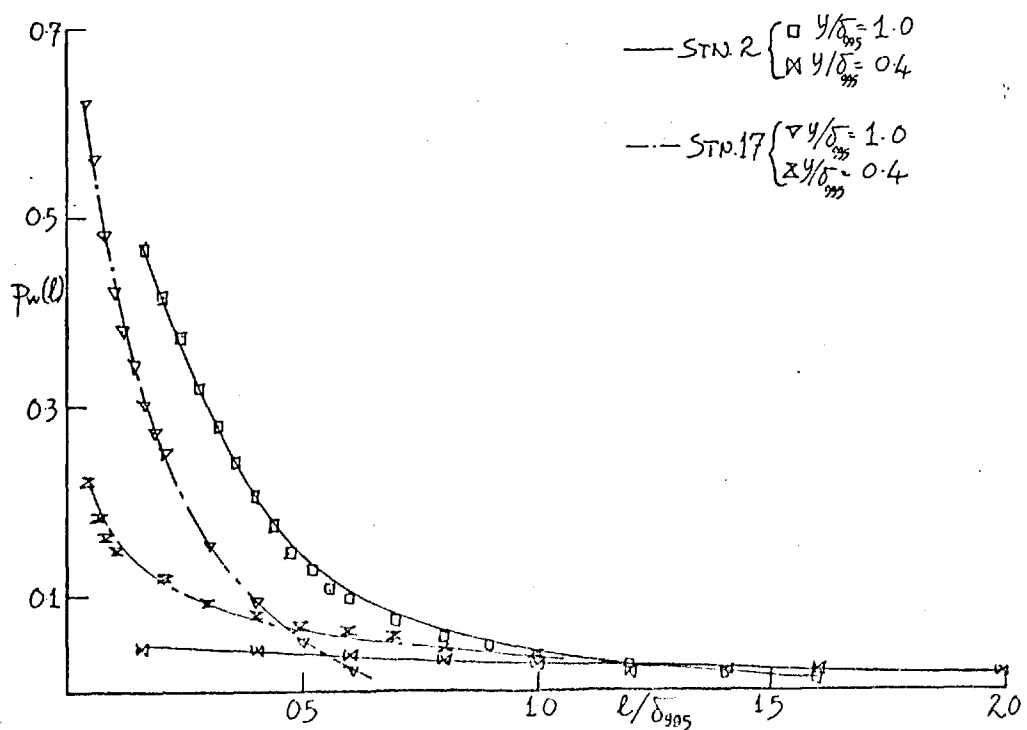


FIGURE [4-11.4b] WHOLESALE DIMENSIONLESS BURST LENGTH PROBABILITY DISTRIBUTION



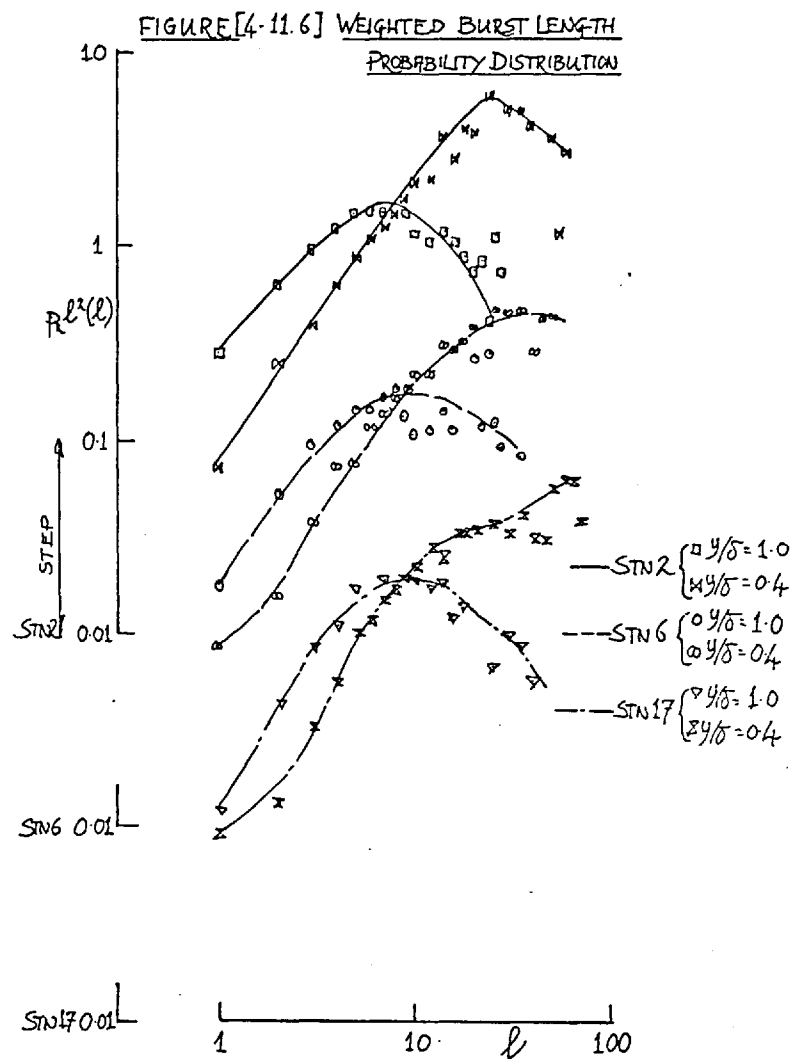
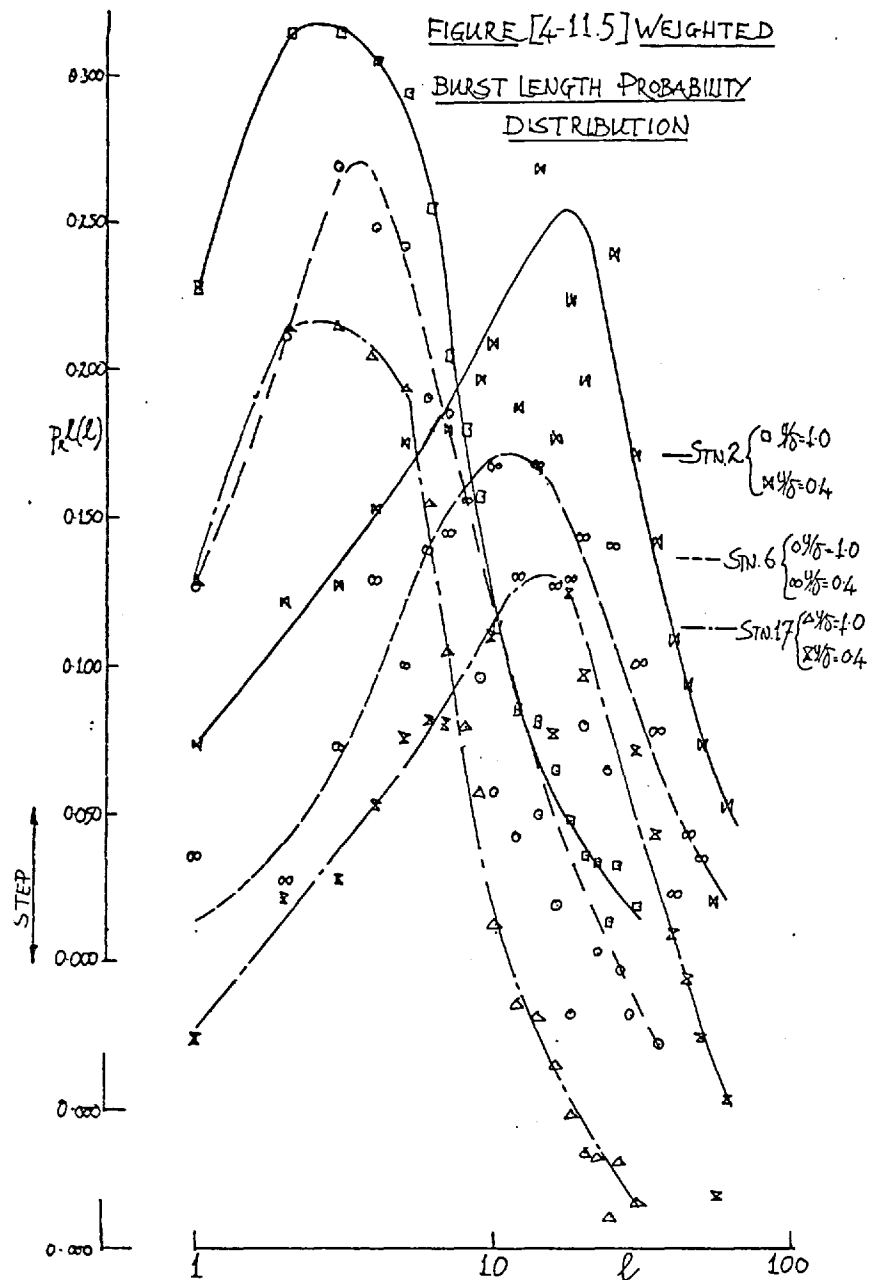


FIGURE [4-11.7] MOMENTS OF BURST LENGTH PROBABILITY
DISTRIBUTION (RETAIL)

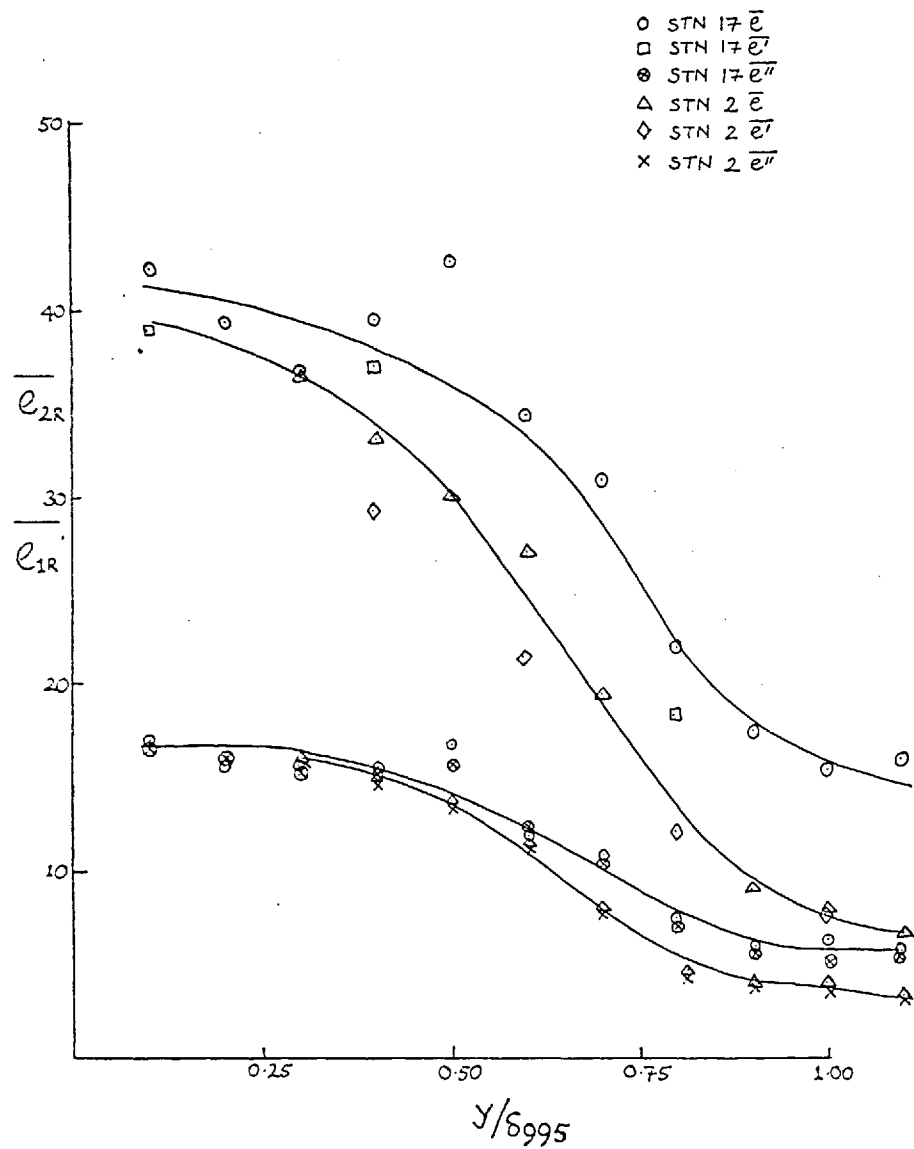


FIGURE [4-11.8] RETAIL AVERAGE BURST LENGTHS

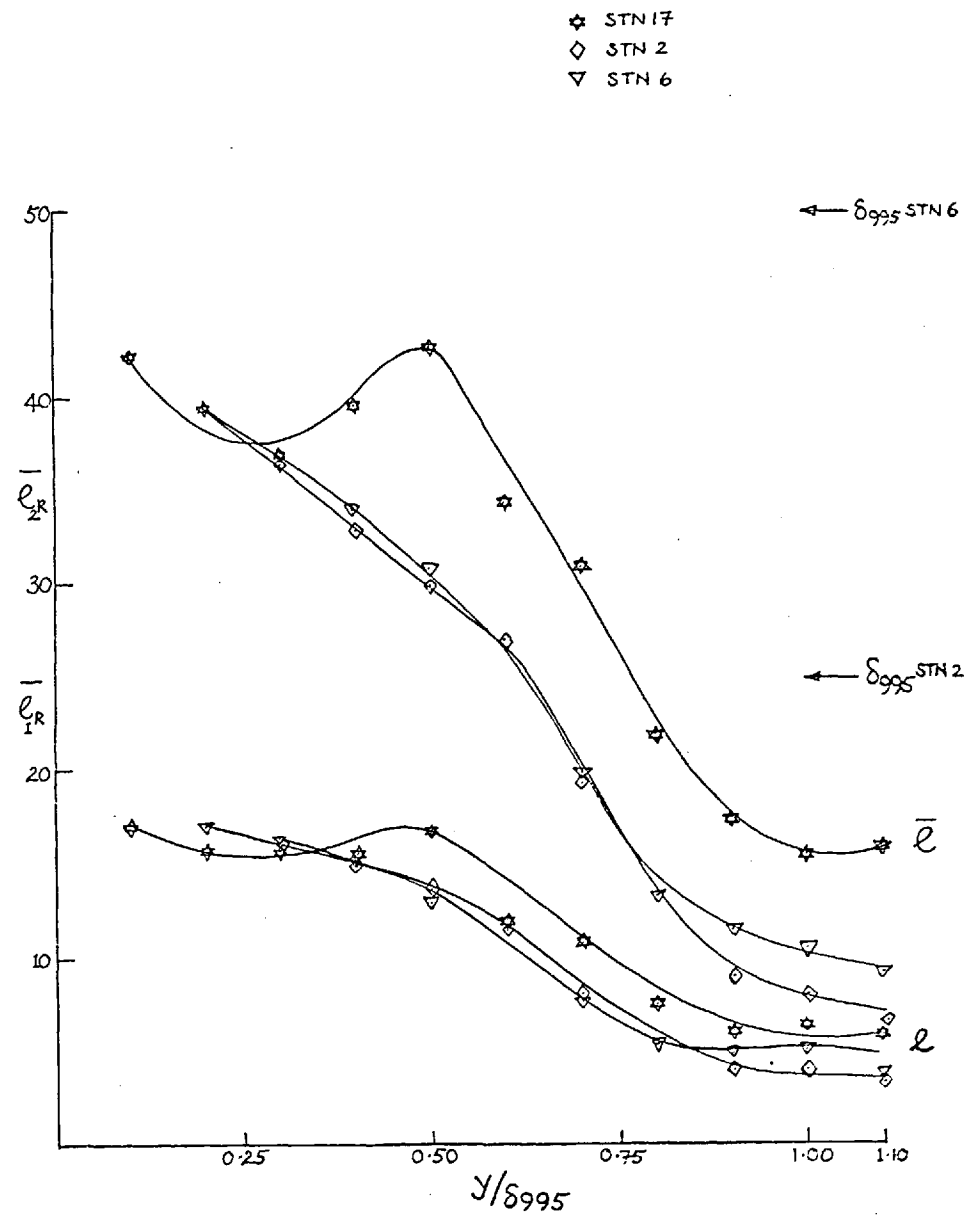


FIGURE [4-11.9a] WHOLESALE AVERAGE BURST LENGTH

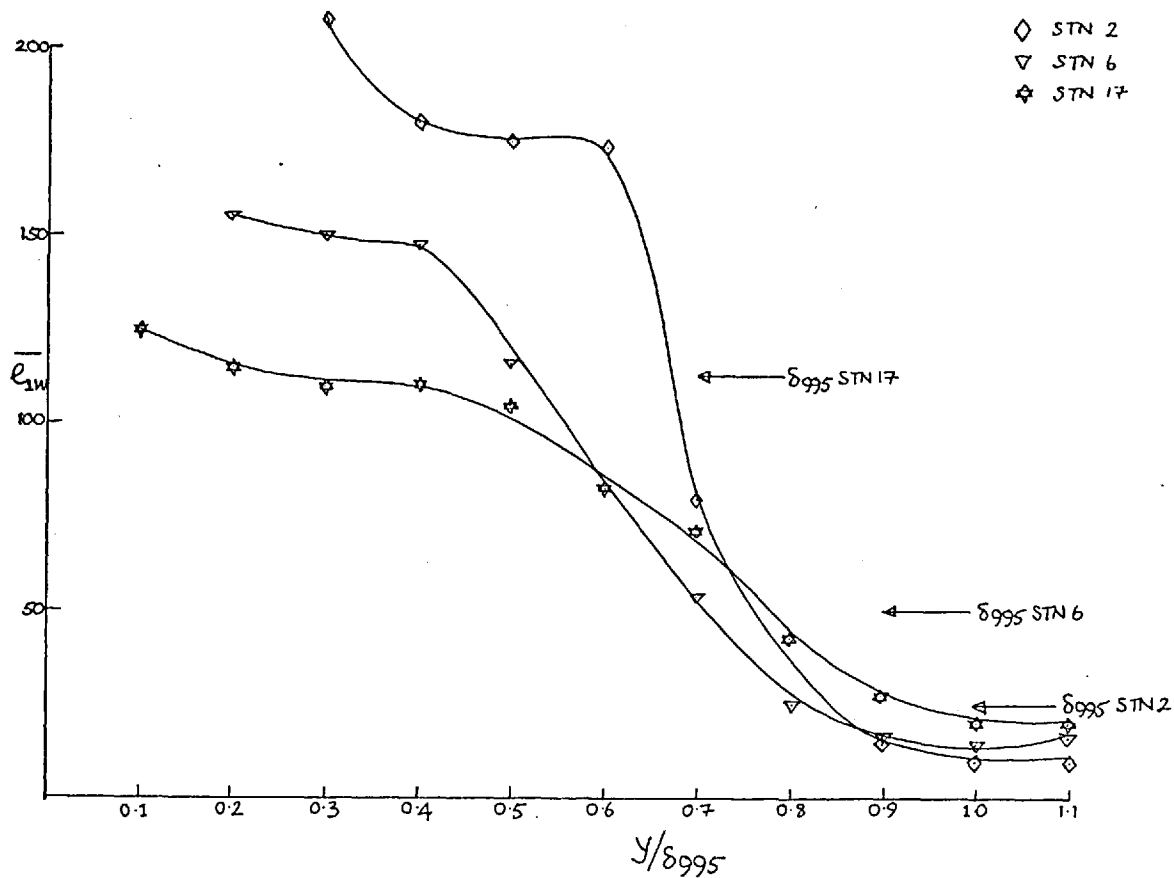


FIGURE [4-11.9b] WHOLESALE AVERAGE BURST LENGTH

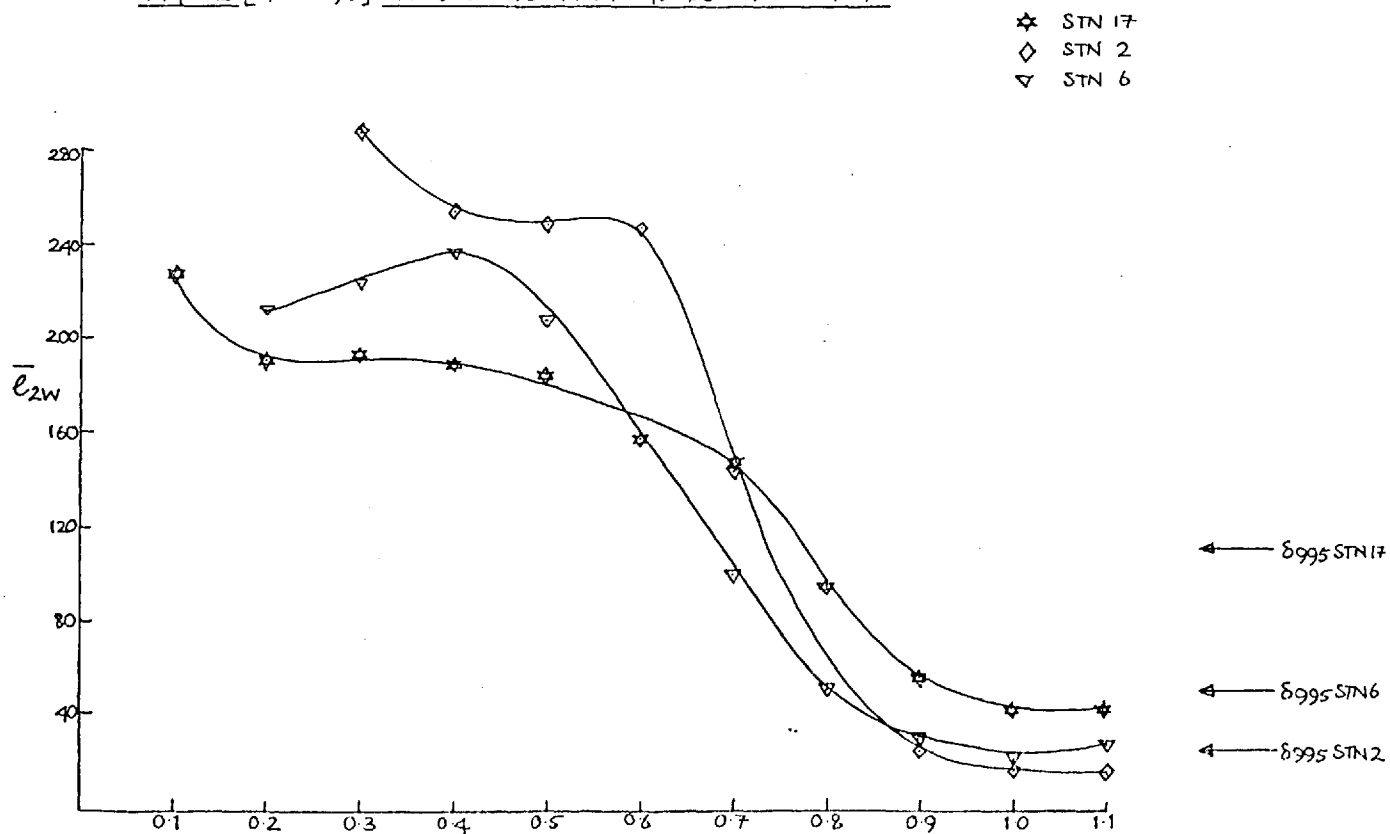


FIGURE [4-11-10] WHOLESALE AVERAGE BURST LENGTH COMPARED WITH INTERMITTENCY

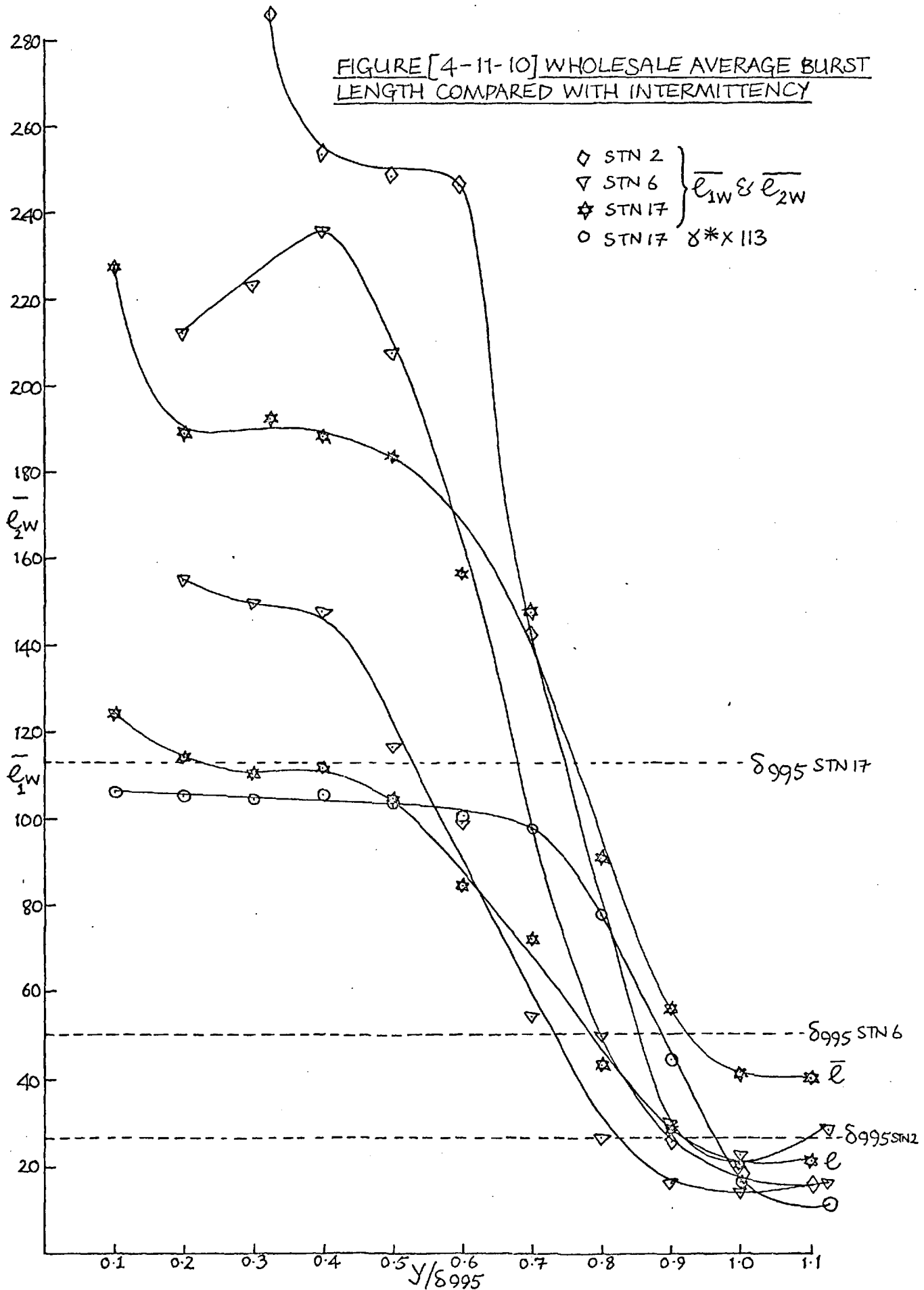


FIGURE [4-11.11b] STREAMWISE CHANGE IN WHOLESALE AVERAGE BURST LENGTH

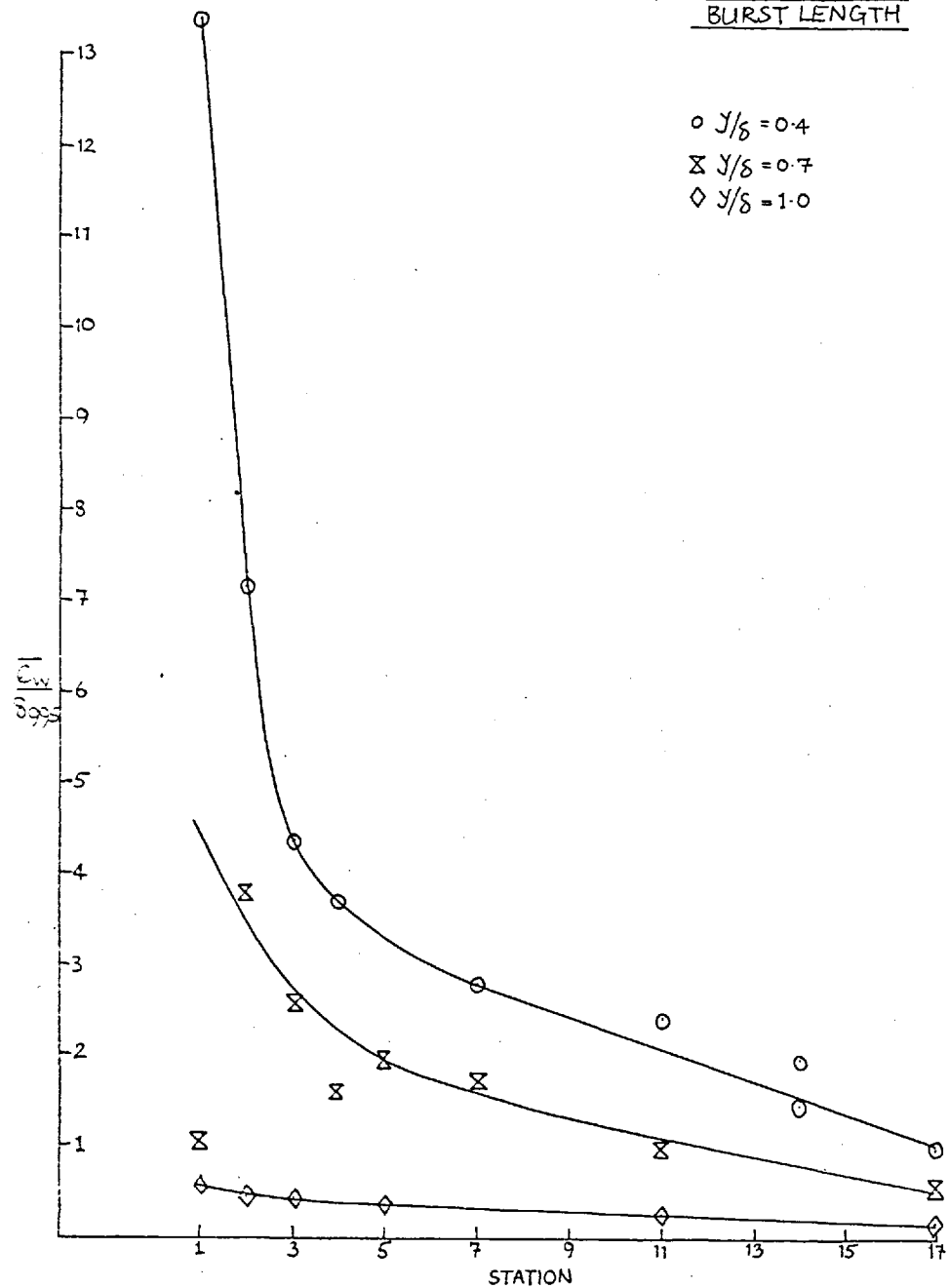
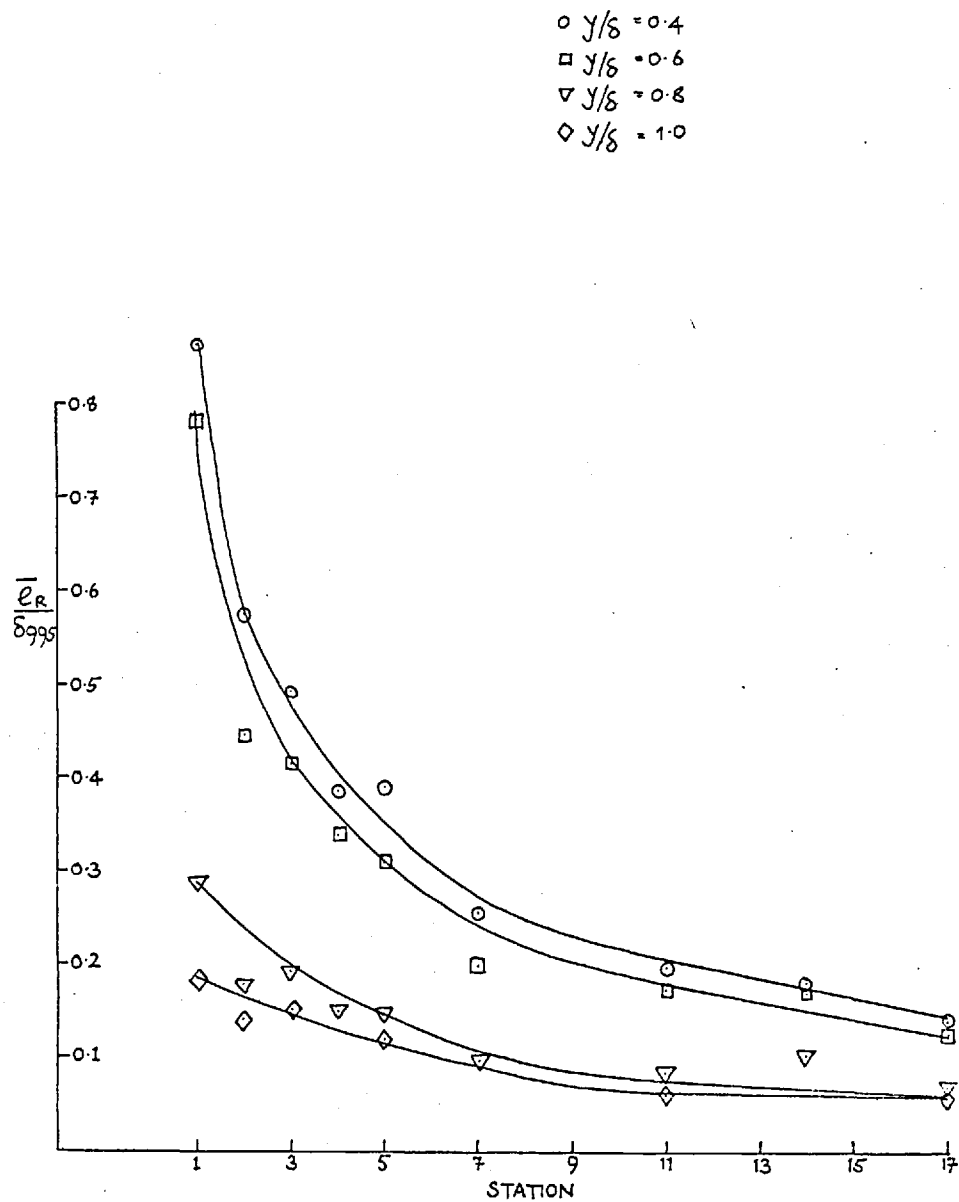


FIGURE [4-11.11a.] STREAMWISE CHANGE IN RETAIL AVERAGE BURST LENGTH



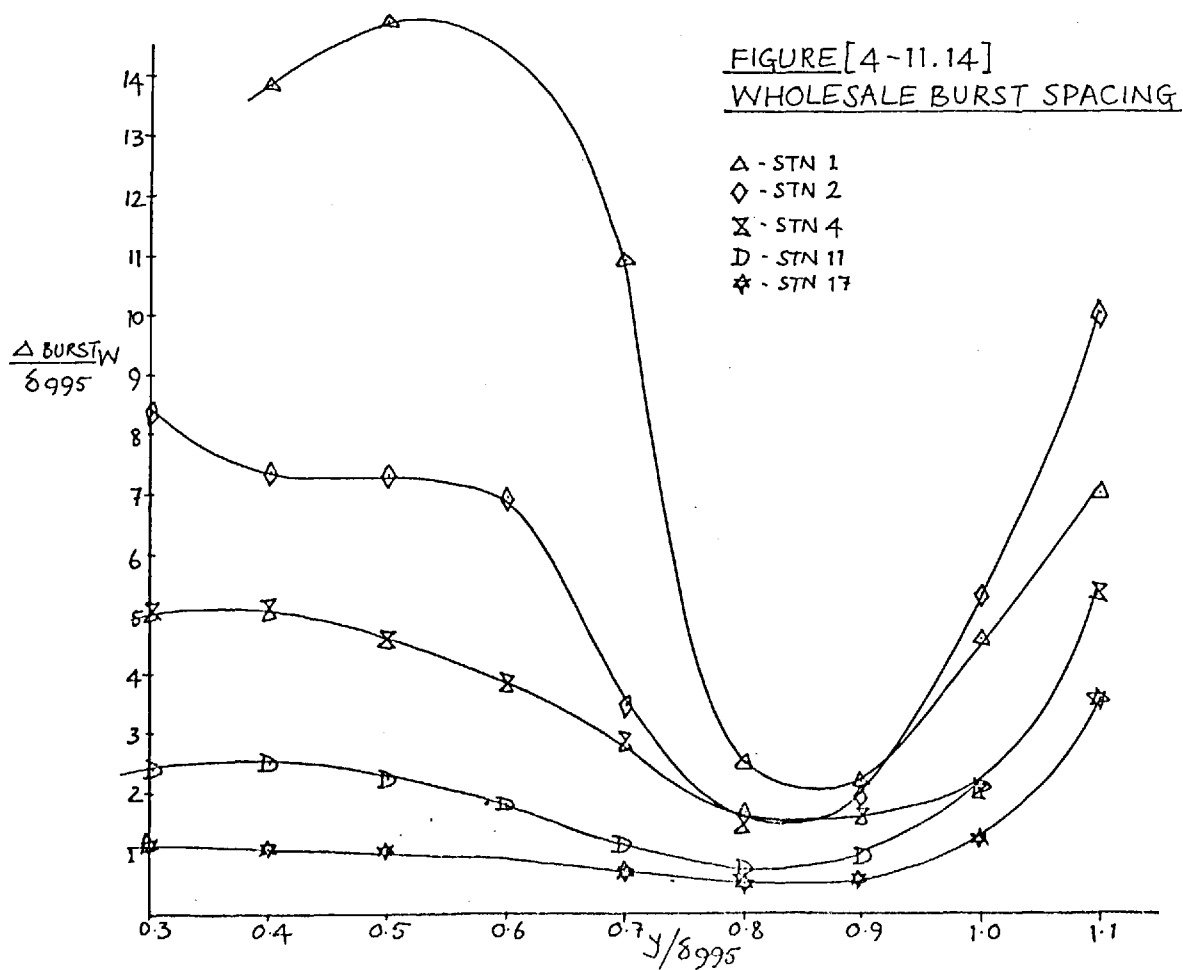
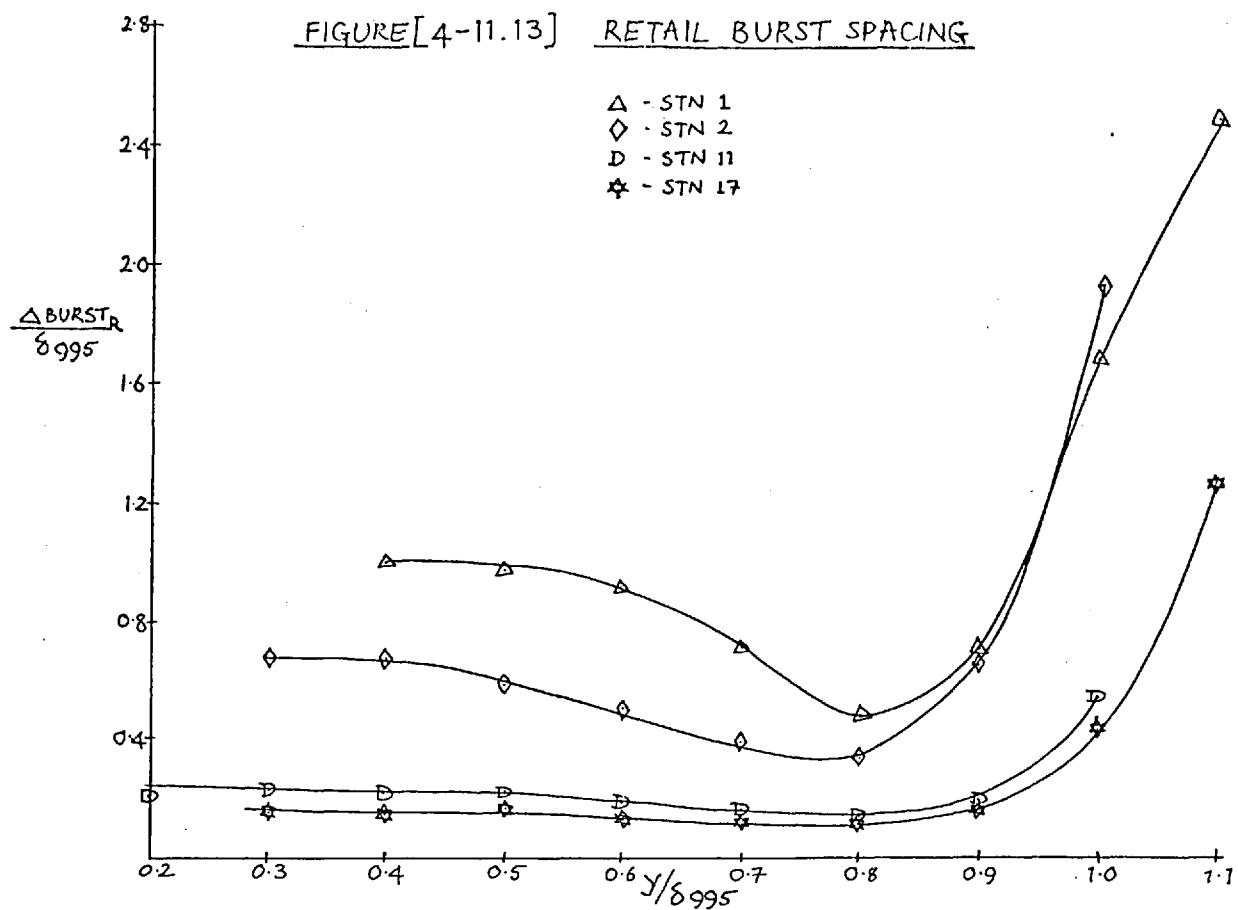


FIGURE [4-11.12] AVERAGE
BURST LENGTHS OF VARIOUS
EXPERIMENTERS COMPARED

- KOVASZNAV $Re = 3300$
- CORRSIN & KISTLER $Re = 8000$
- △ JMW STN 2 $Re = 1000$
- ▽ JMW STN 17 $Re = 4750$
- ◇ JMW STN 11 $Re = 3360$

- KOVASZNAV ET AL. (1971) $Re = 3300$
 - CORRSIN & KISTLER (1955) $Re = 8000$
 - △ $Re = 1000$
 - ▽ $Re = 4750$
 - ◇ $Re = 3360$
- } PRESENT EXPERIMENTS

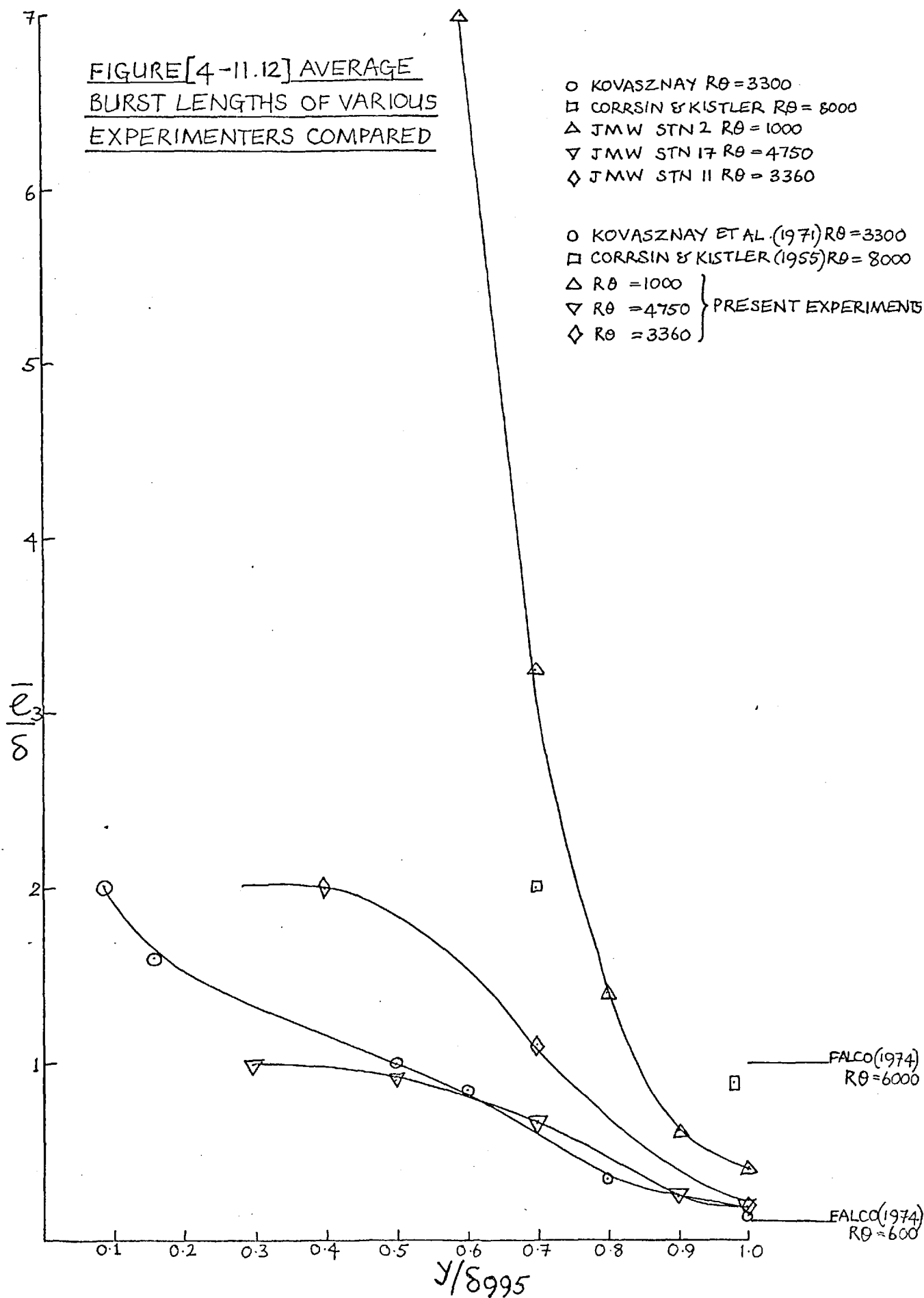
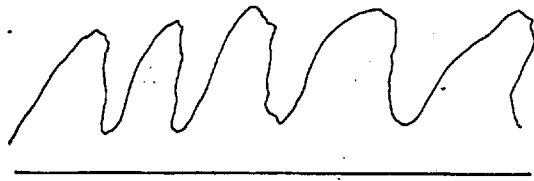


FIGURE [4-11-15] BURST SHAPE

① SOLITARY

HIGH Re

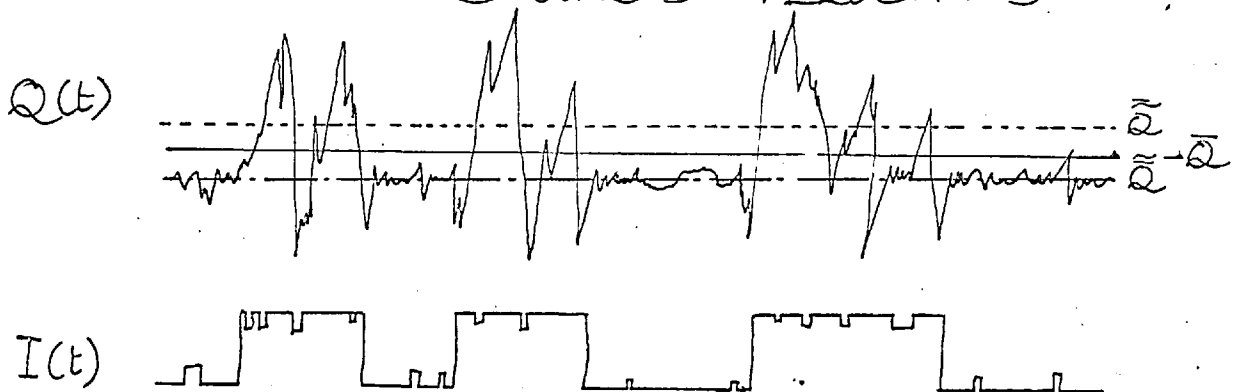


② SPLIT

LOW Re



FIGURE [5-1] BASELINES FOR CONDITIONALLY SAMPLED VELOCITIES



BASELINE

$$\bar{Q} = \sigma \tilde{Q} + (1-\sigma) \hat{Q}$$

"TURBULENT" ----- $\tilde{Q} \equiv$ MEAN TURBULENT ZONE VELOCITY

"IRROTATIONAL" --- $\hat{Q} \equiv$ MEAN IRROTATIONAL ZONE VELOCITY

"TOTAL" ——— $\bar{Q} \equiv$ OVERALL MEAN VELOCITY

$$q = Q - \bar{Q} \equiv \text{VELOCITY FLUCTUATION W.R.T. "TOTAL" BASELINE}$$

$$\tilde{q} = \tilde{Q} - \bar{Q} \equiv \text{TURBULENT ZONE CONDITIONAL MEAN VELOCITY WRT. TOTAL BASELINE}$$

$$\hat{q} = \hat{Q} - \bar{Q} \equiv \text{IRROTATIONAL ZONE CONDITIONAL MEAN VELOCITY WRT. TOTAL BASELINE}$$

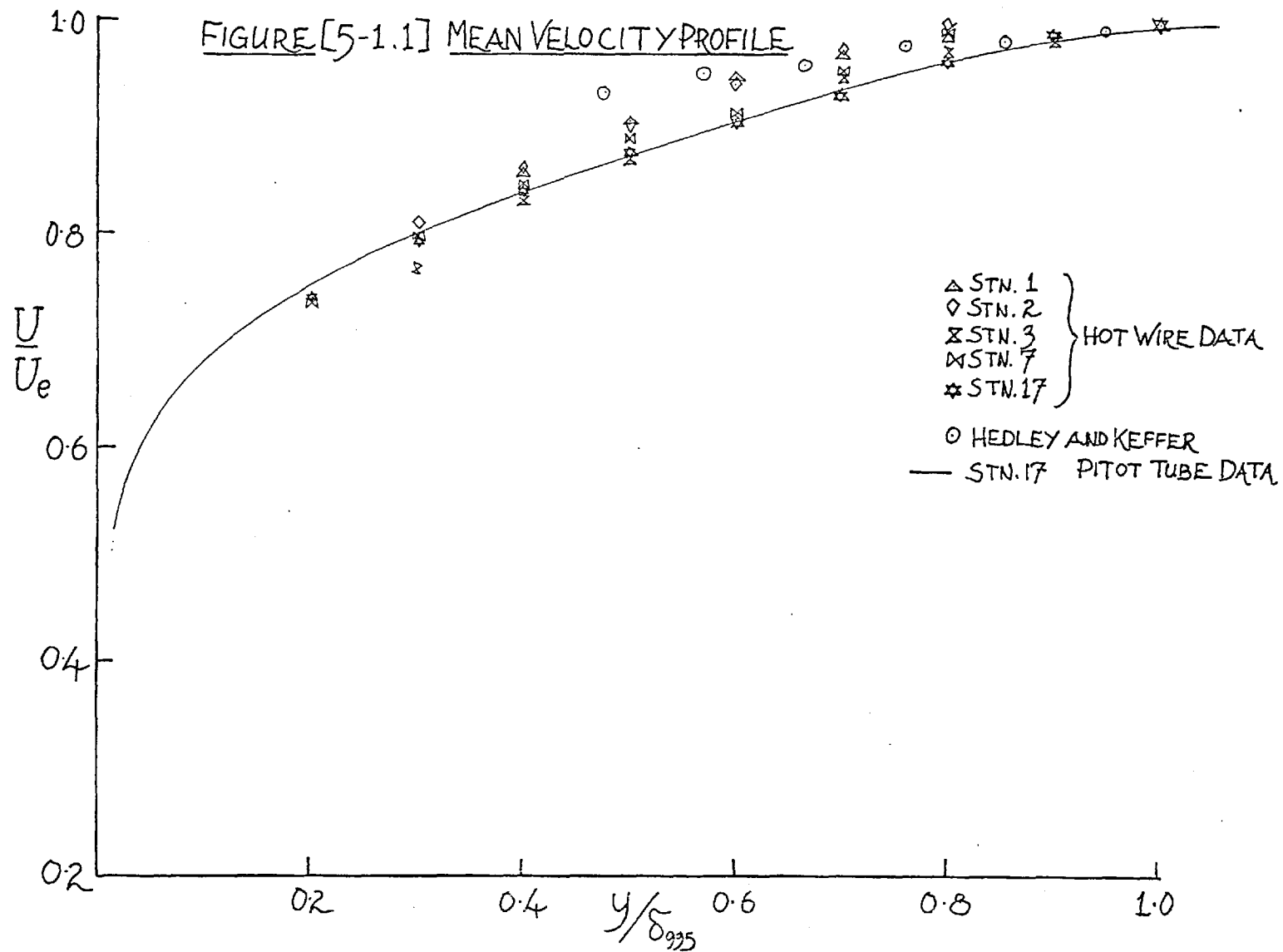


FIGURE [5-1.2] ZONE AVERAGED
STREAMWISE MEAN VELOCITY

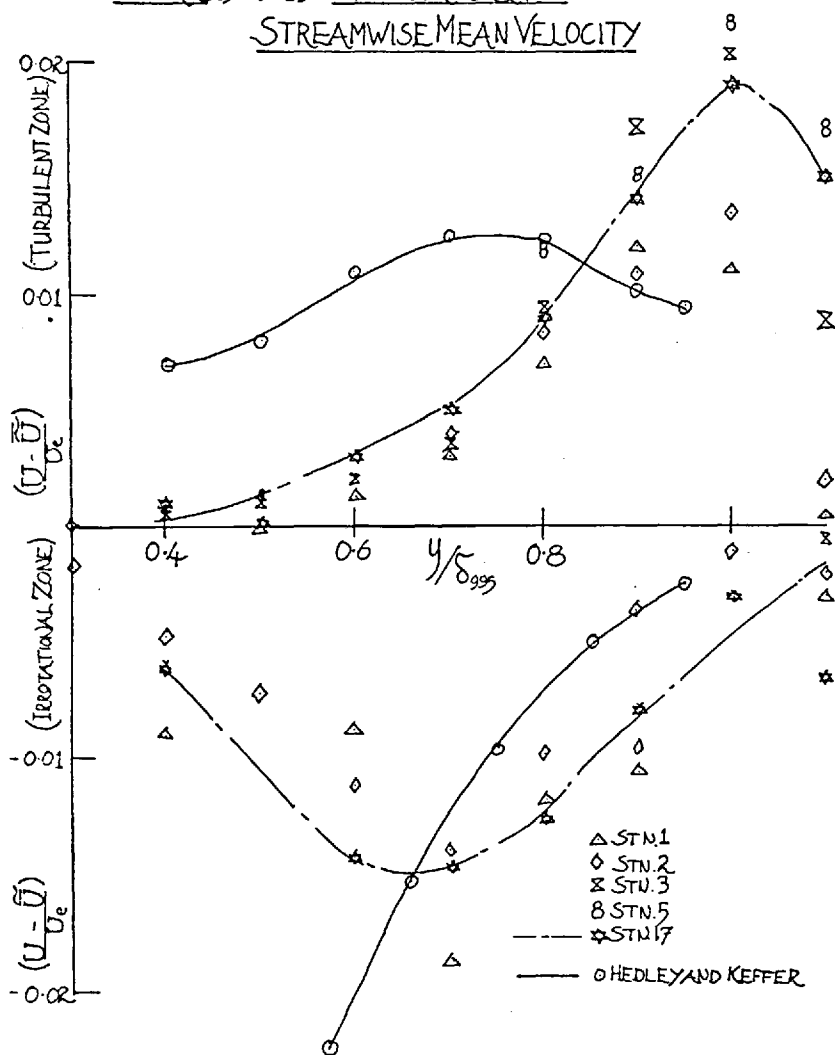
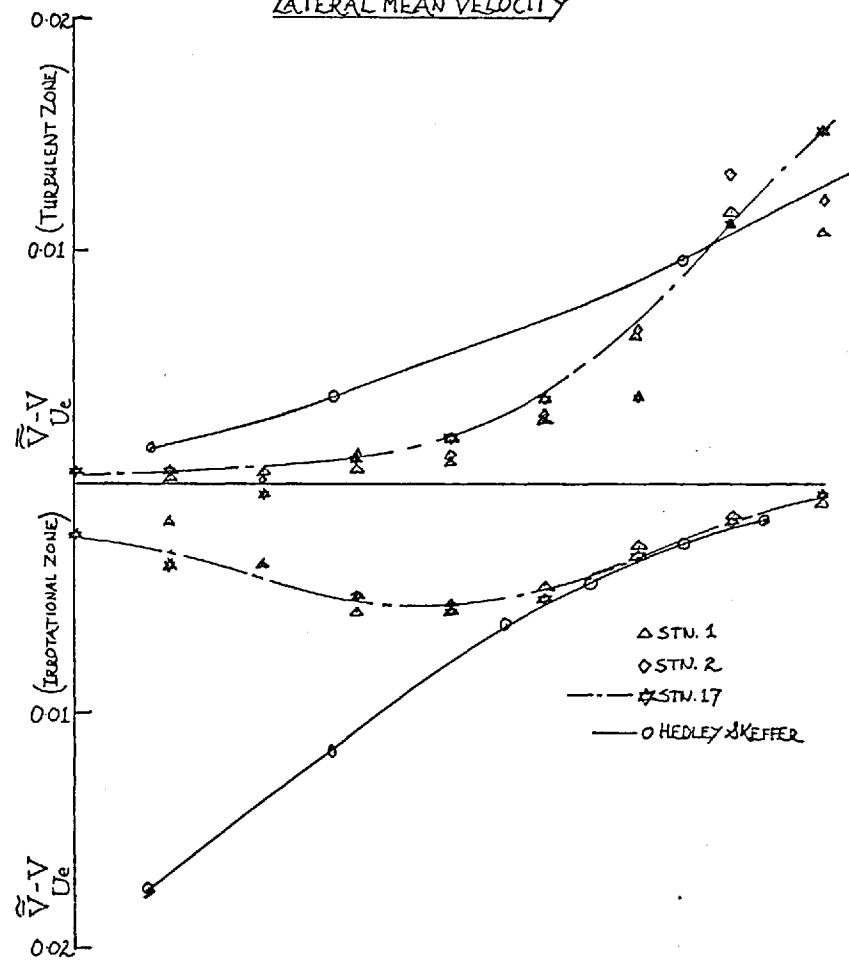
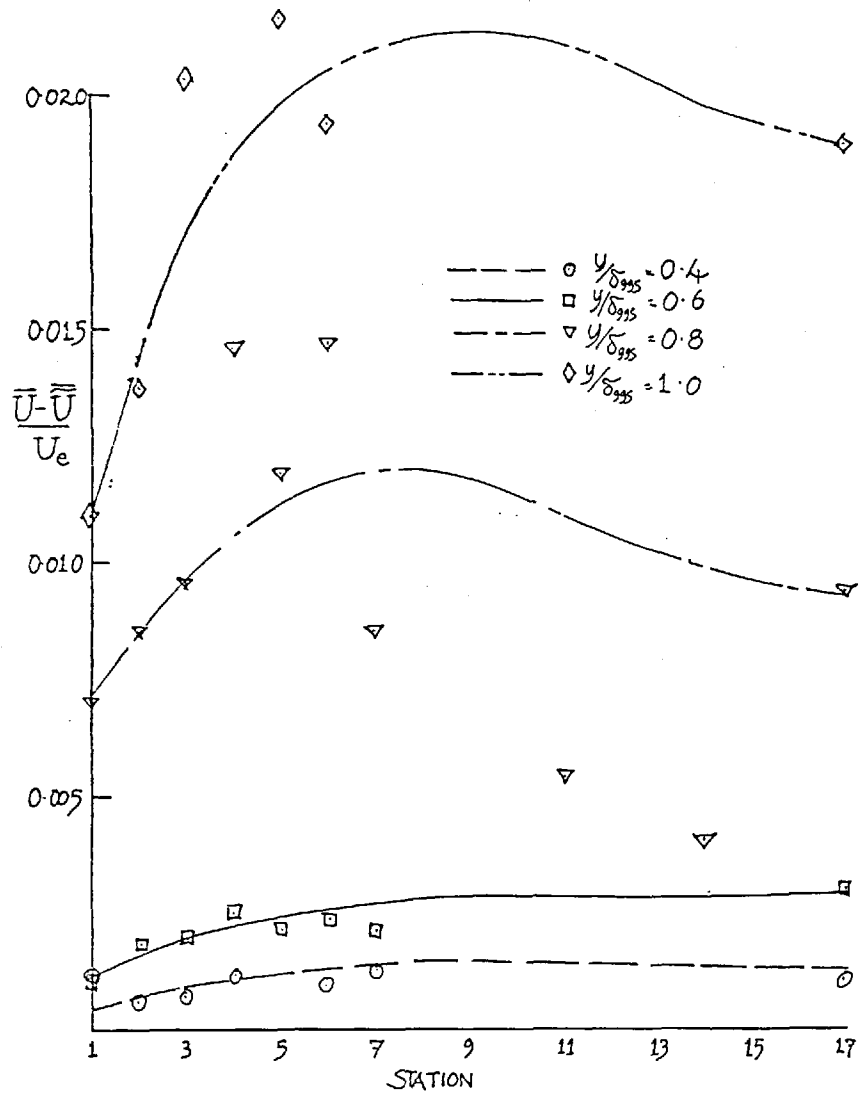


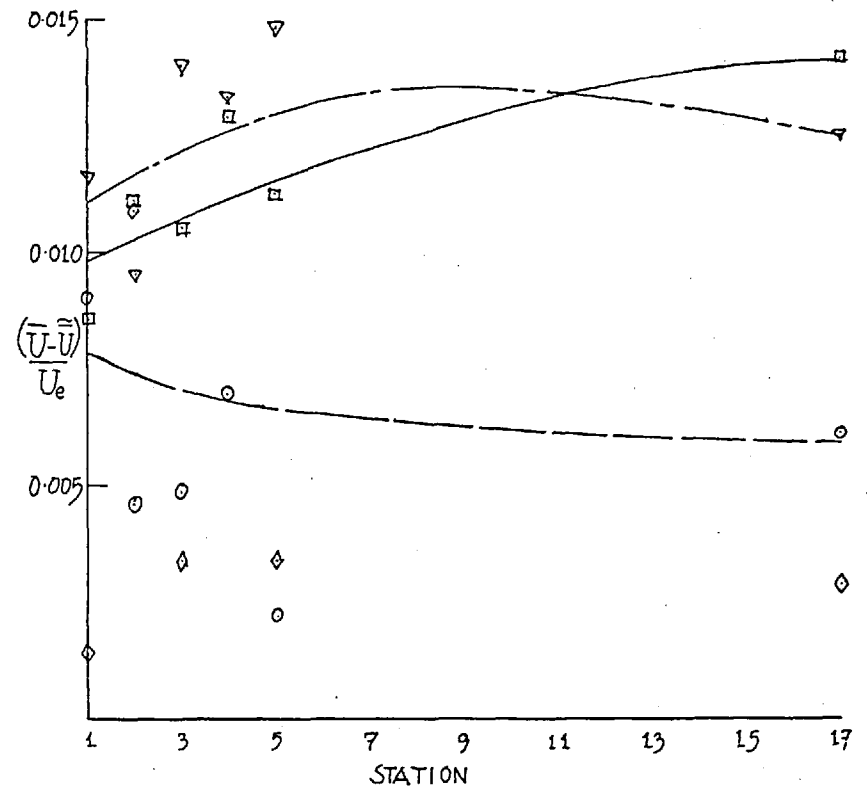
FIGURE [5-1.3] ZONE AVERAGED
LATERAL MEAN VELOCITY

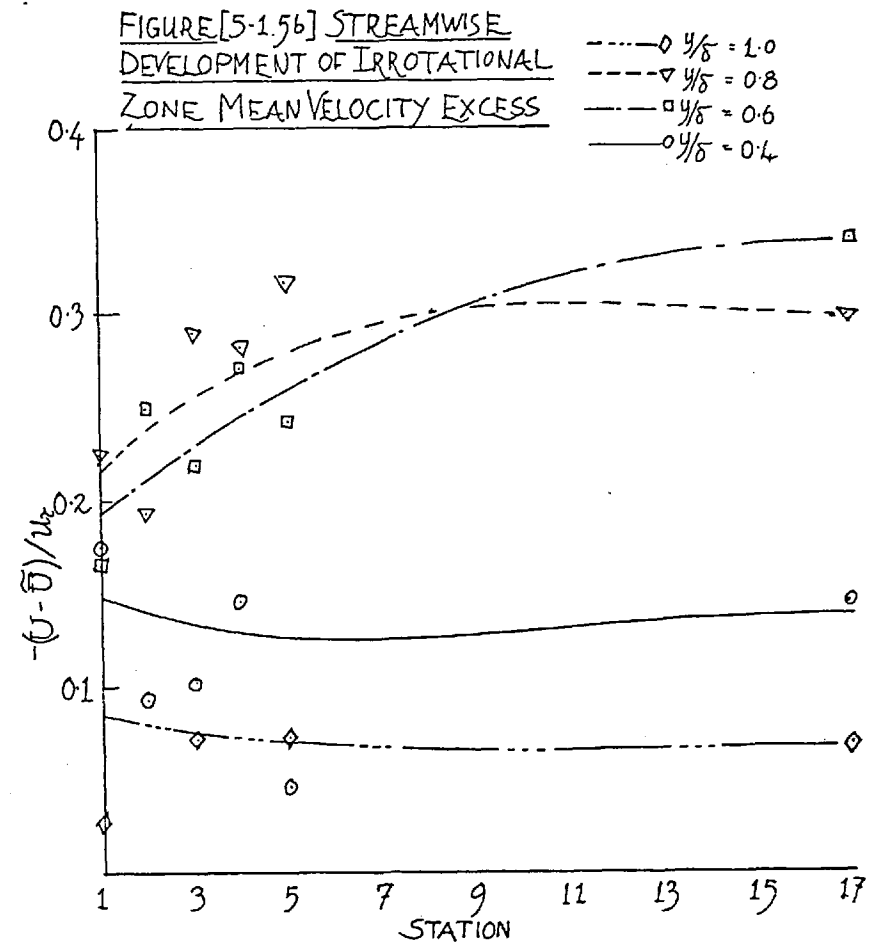
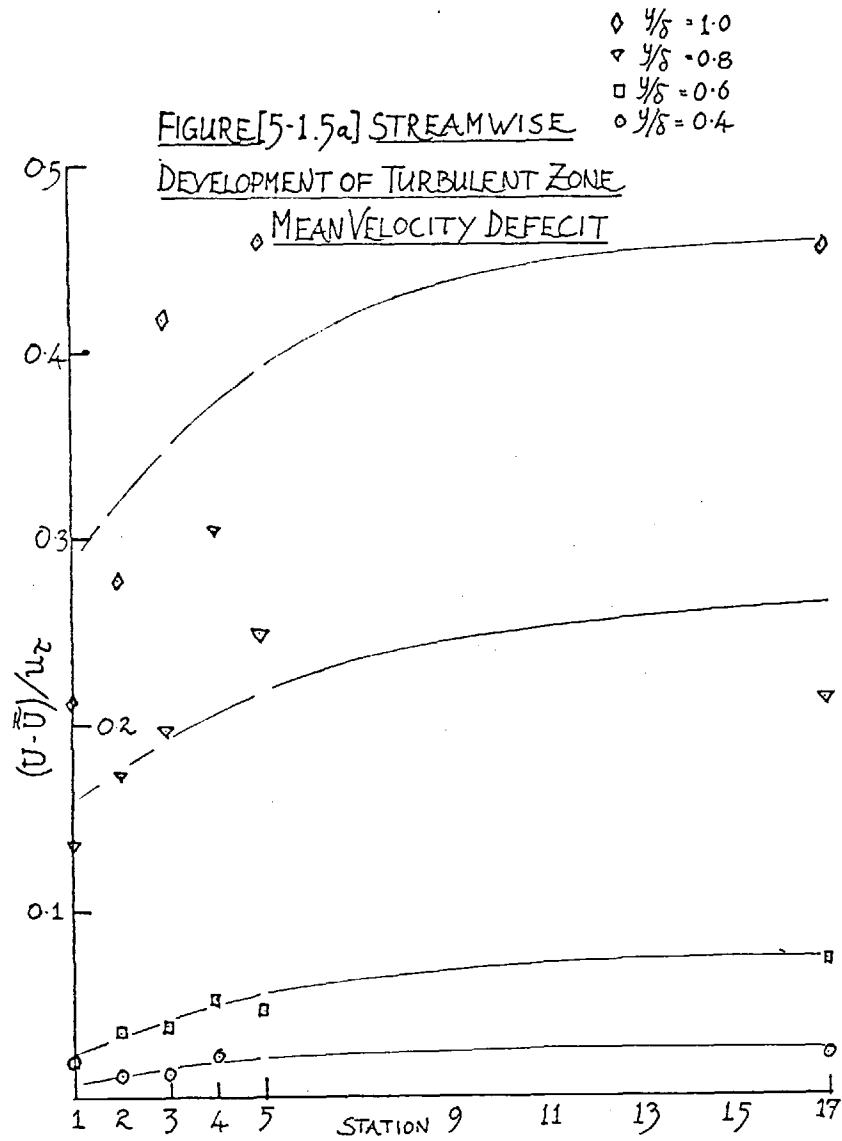


FIGURE[5-1.4a] STREAMWISE VARIATION
OF TURBULENT ZONE MEAN VELOCITY



FIGURE[5-1.4b] STREAMWISE VARIATION
OF IRROTATIONAL ZONE MEAN VELOCITY





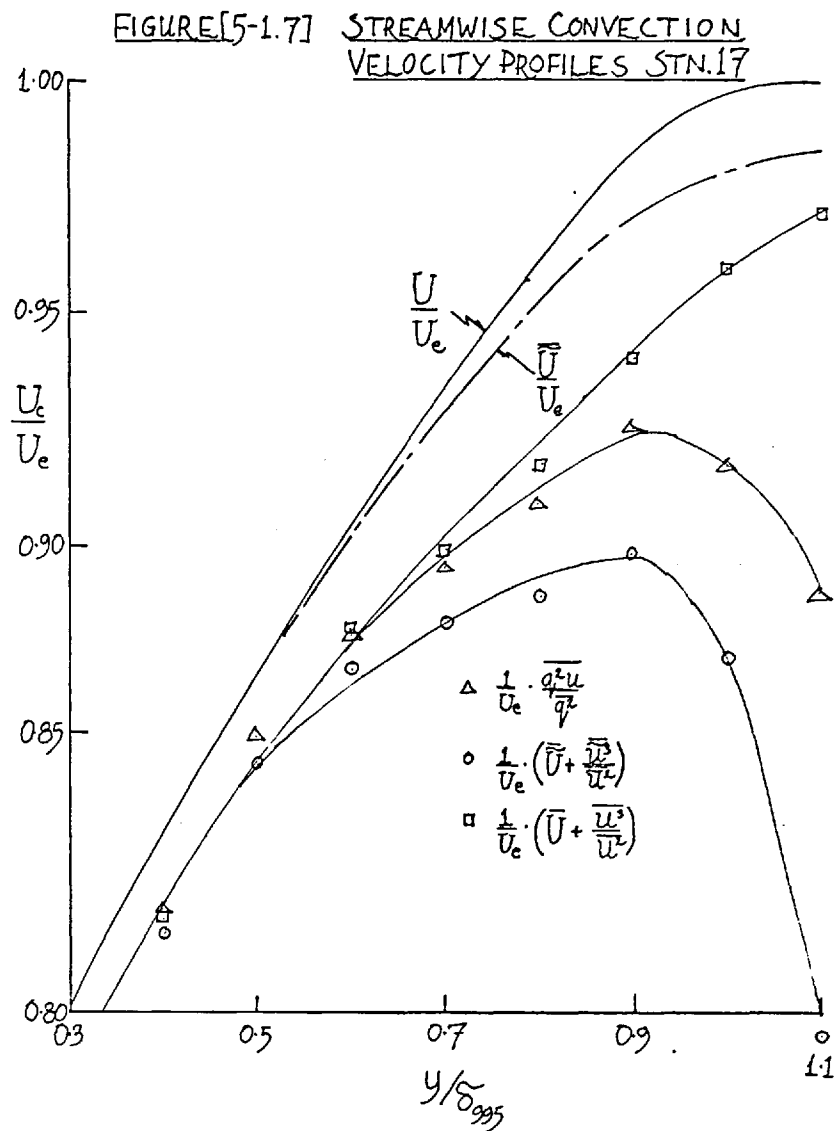
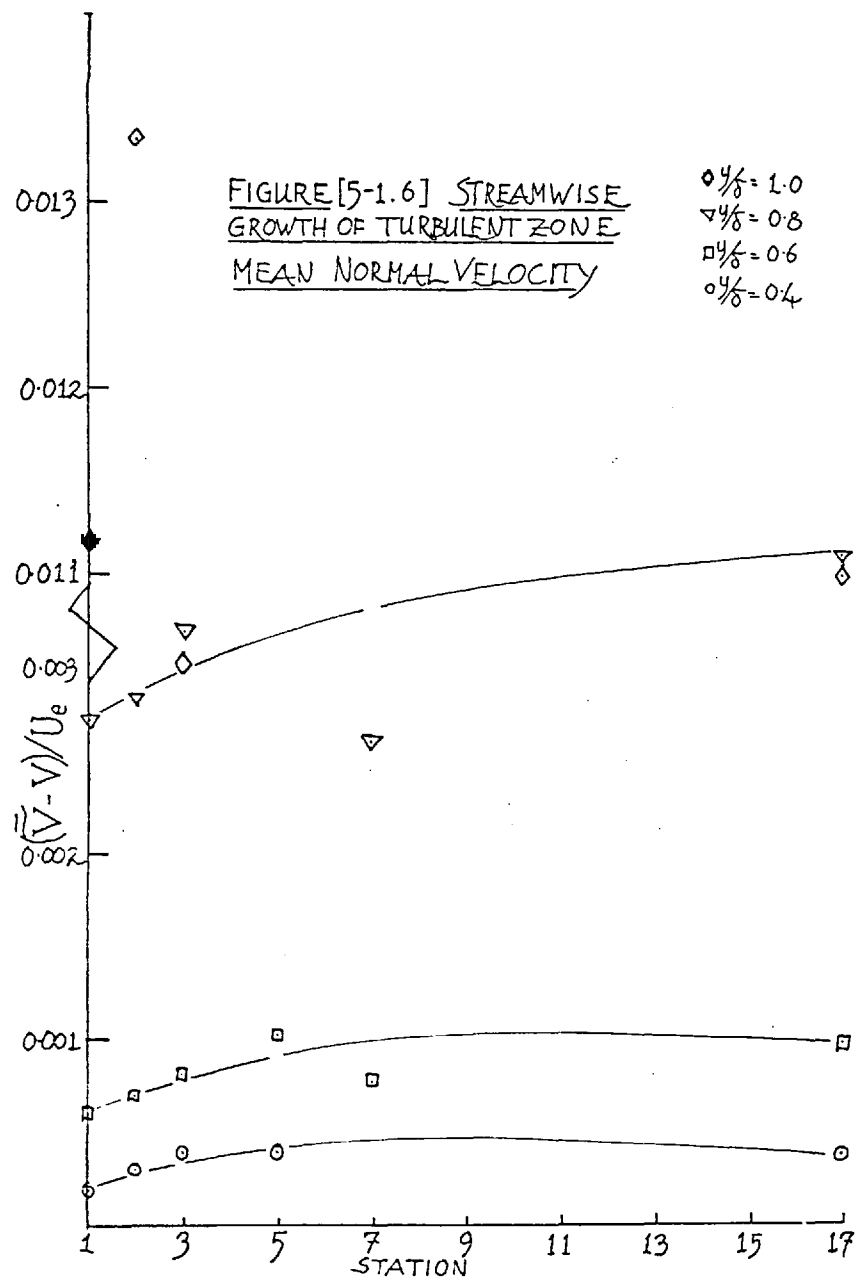


FIGURE [5-1.8] STREAMWISE CONVECTION VELOCITY
PROFILES AT VARIOUS STREAMWISE POSITIONS

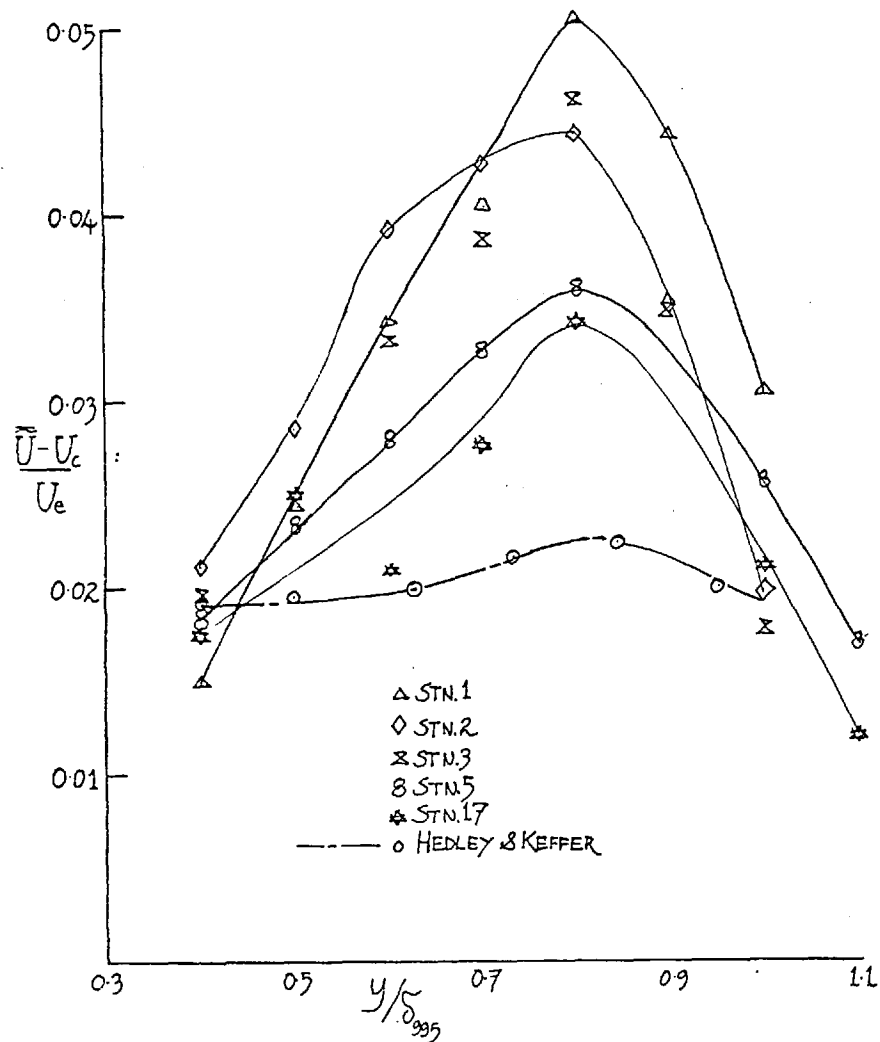
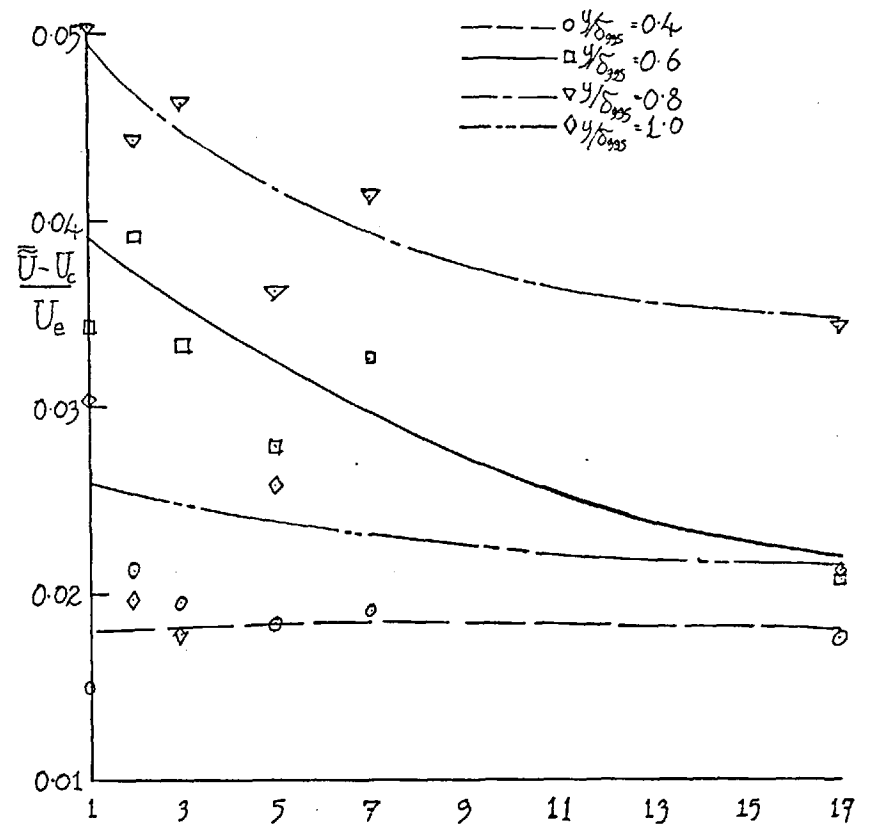


FIGURE [5-1.9a] STREAMWISE DEVELOPMENT OF
CONVECTION VELOCITY



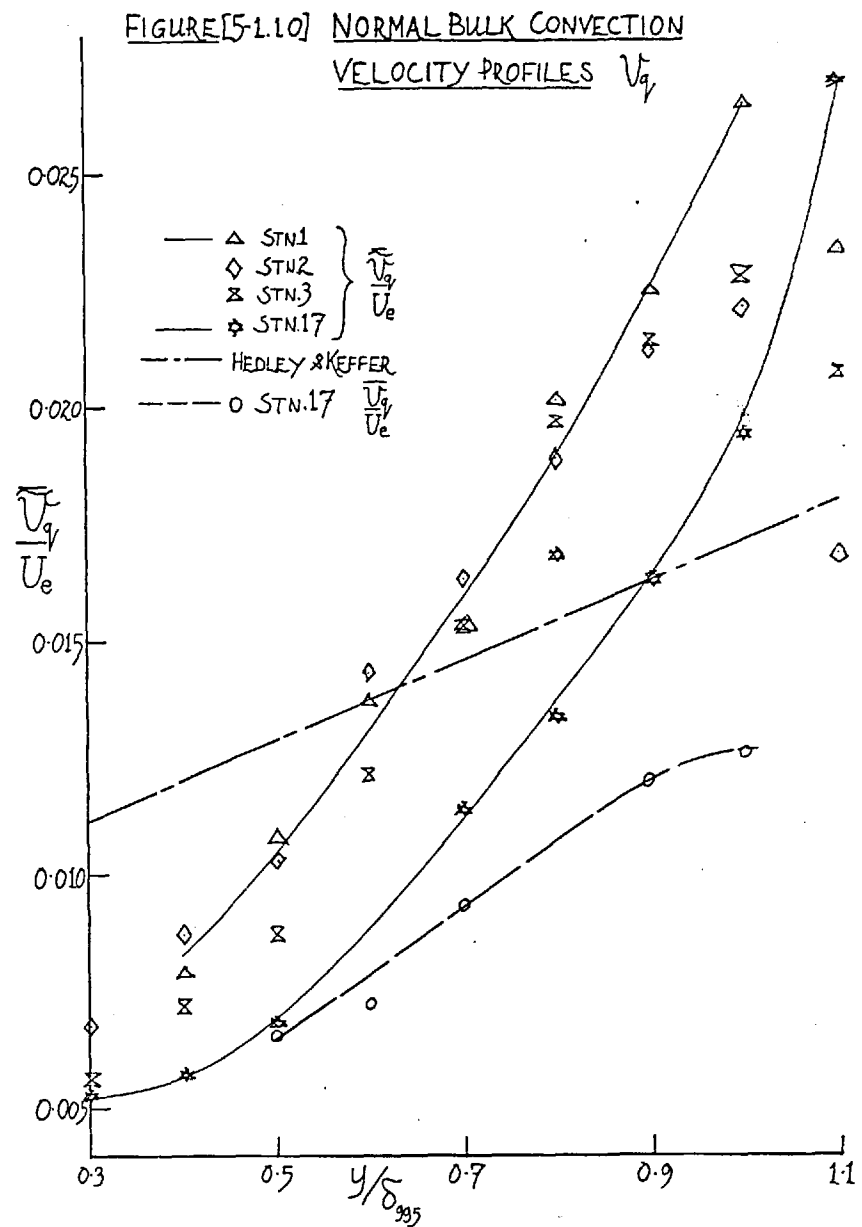
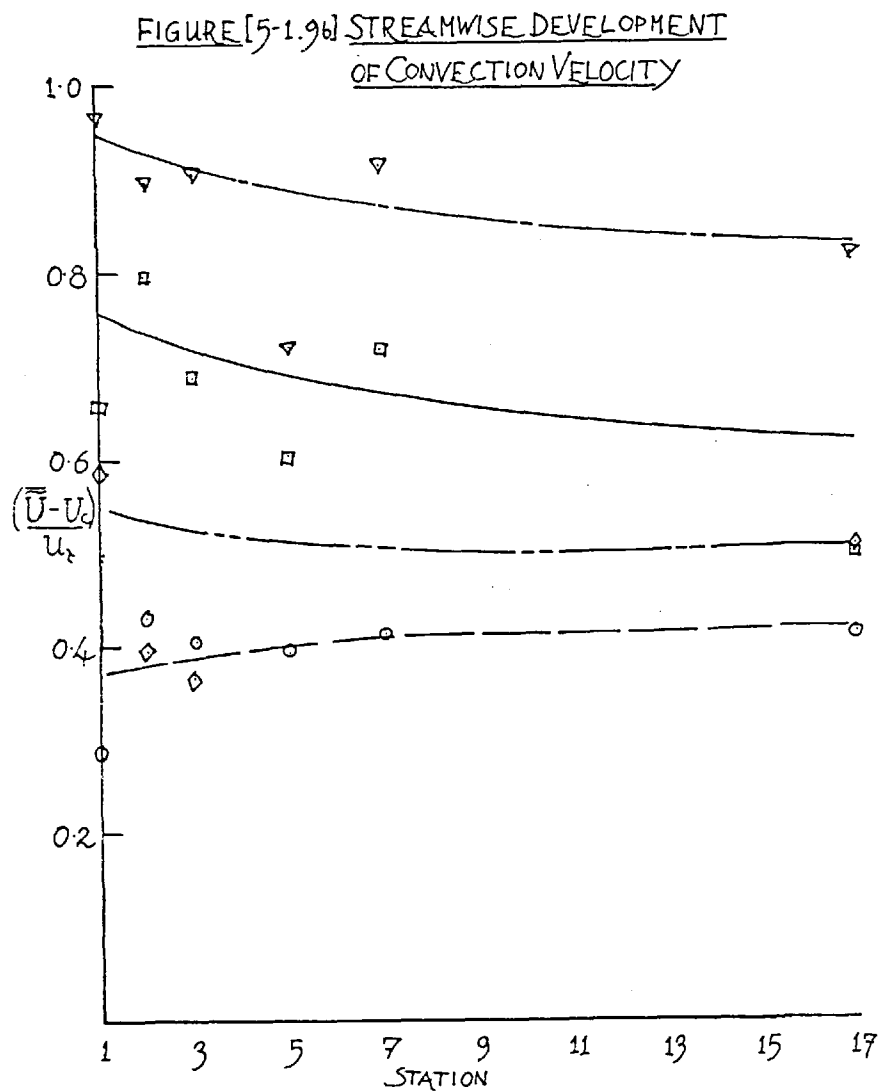


FIGURE [5-1.11] NORMAL BULK CONVECTION

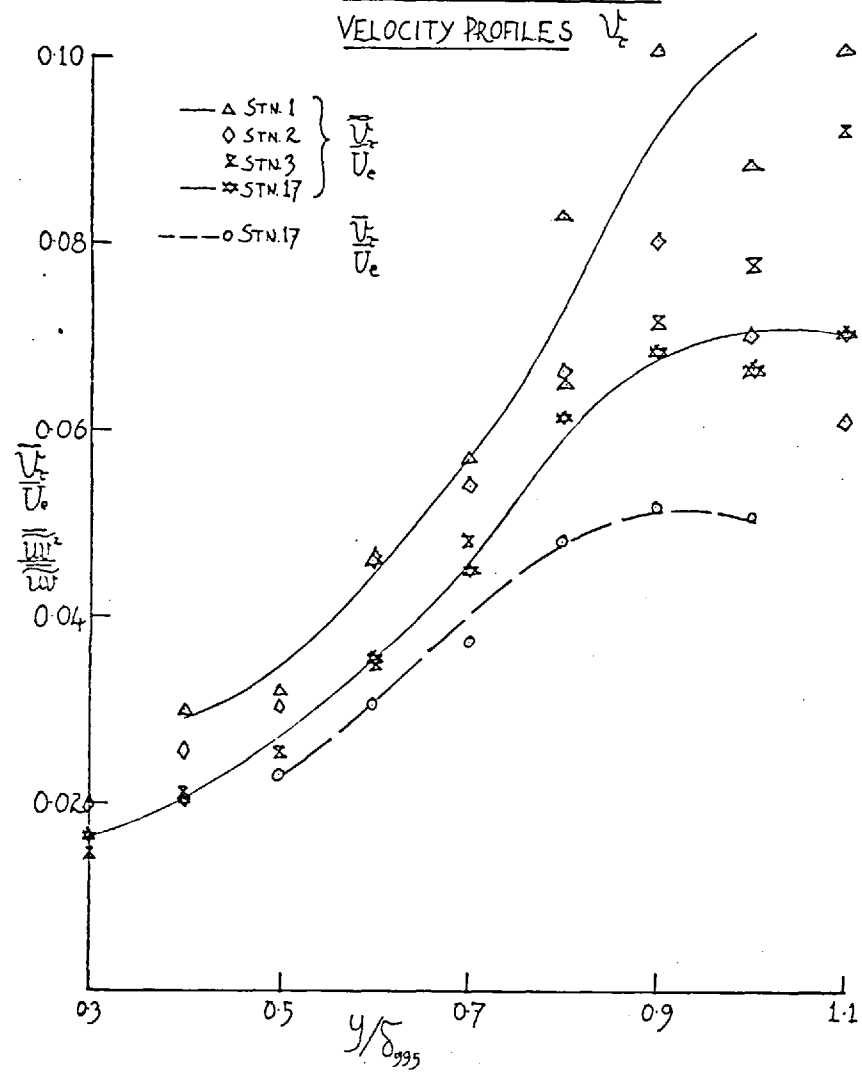
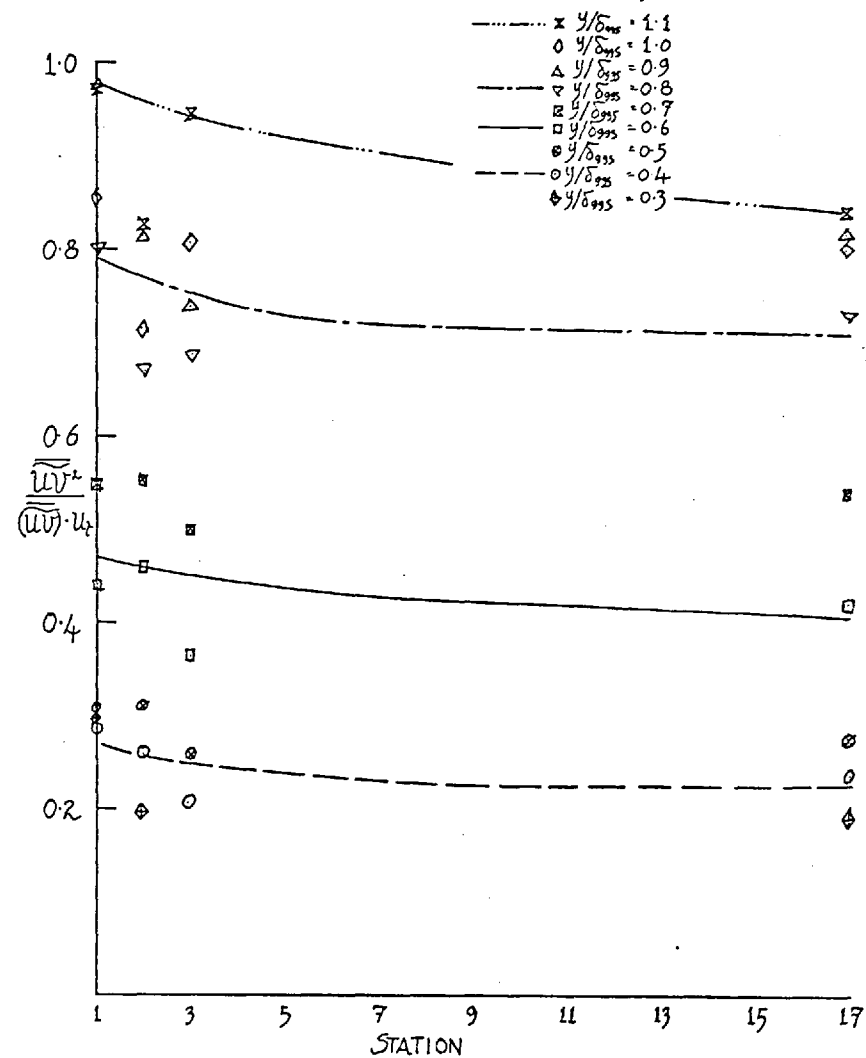


FIGURE [5-1.12] STREAMWISE DEVELOPMENT OF NORMAL BULK CONVECTION VELOCITY $\bar{u}_z$



FIGURE[5-2.1] TURBULENCE INTENSITY

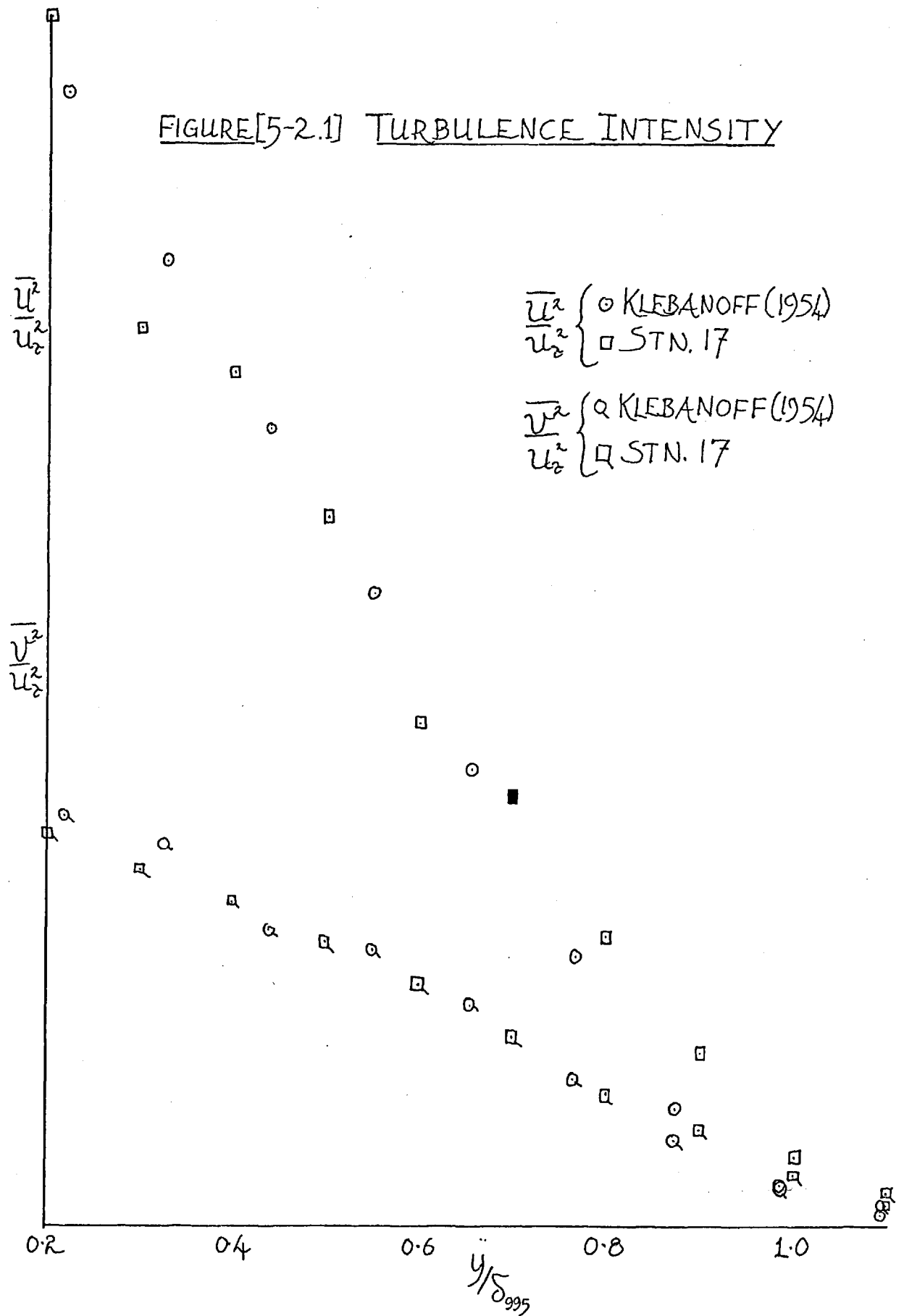


FIGURE [5-2.2b] R.M.S. TURBULENCE INTENSITY STN 17

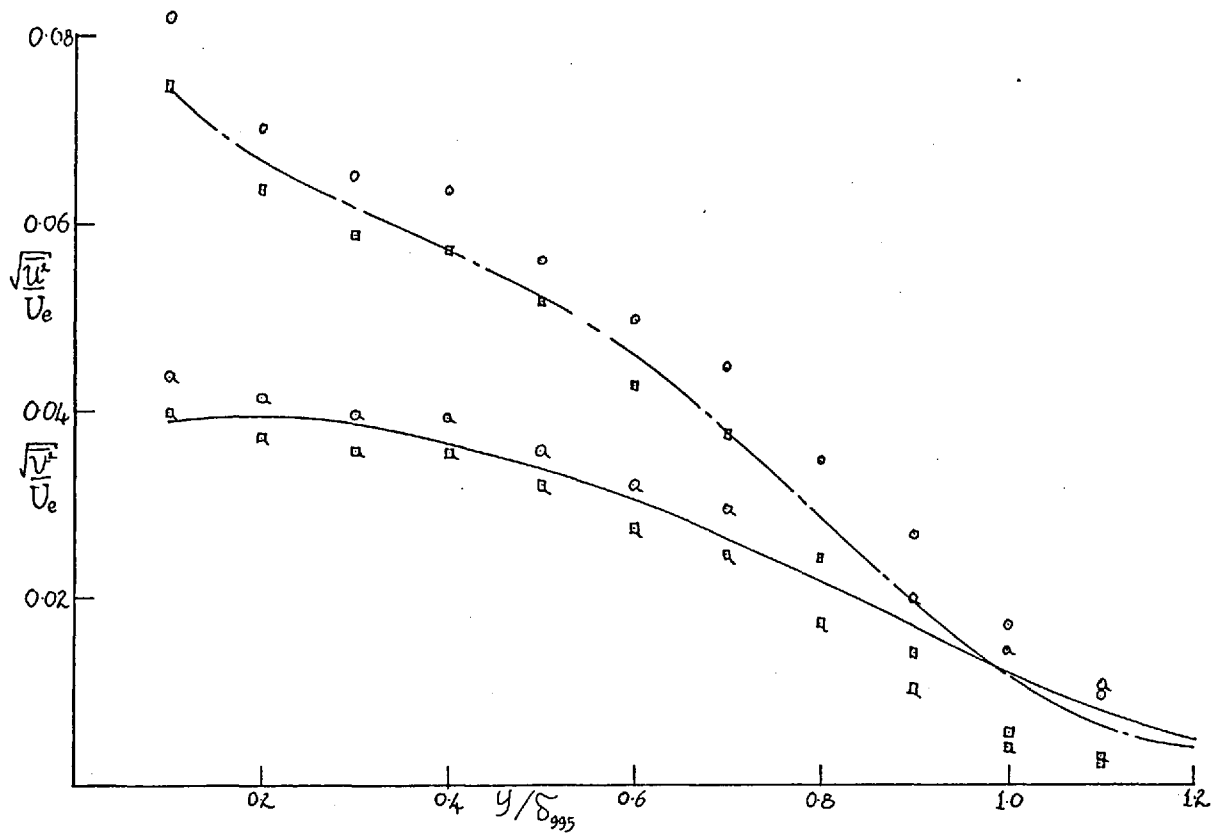


FIGURE [5-2.2a] R.M.S. TURBULENCE INTENSITY STN. 1

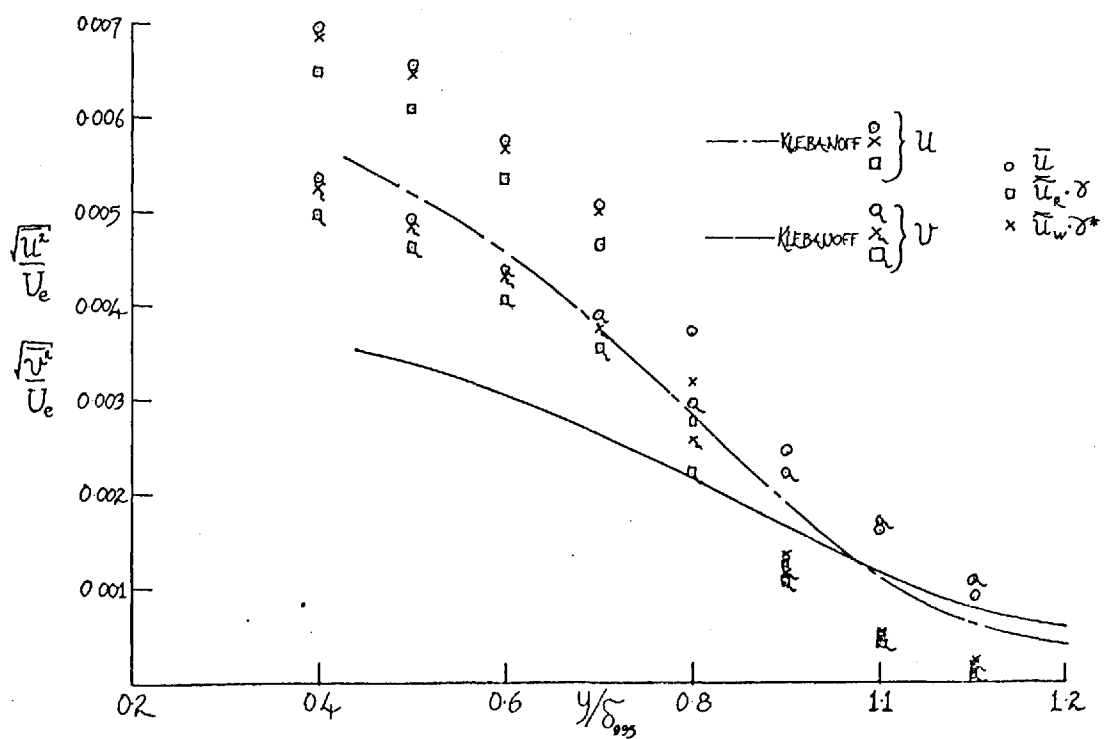


FIGURE [5-2.3a] TURBULENCE INTENSITY STN. 1

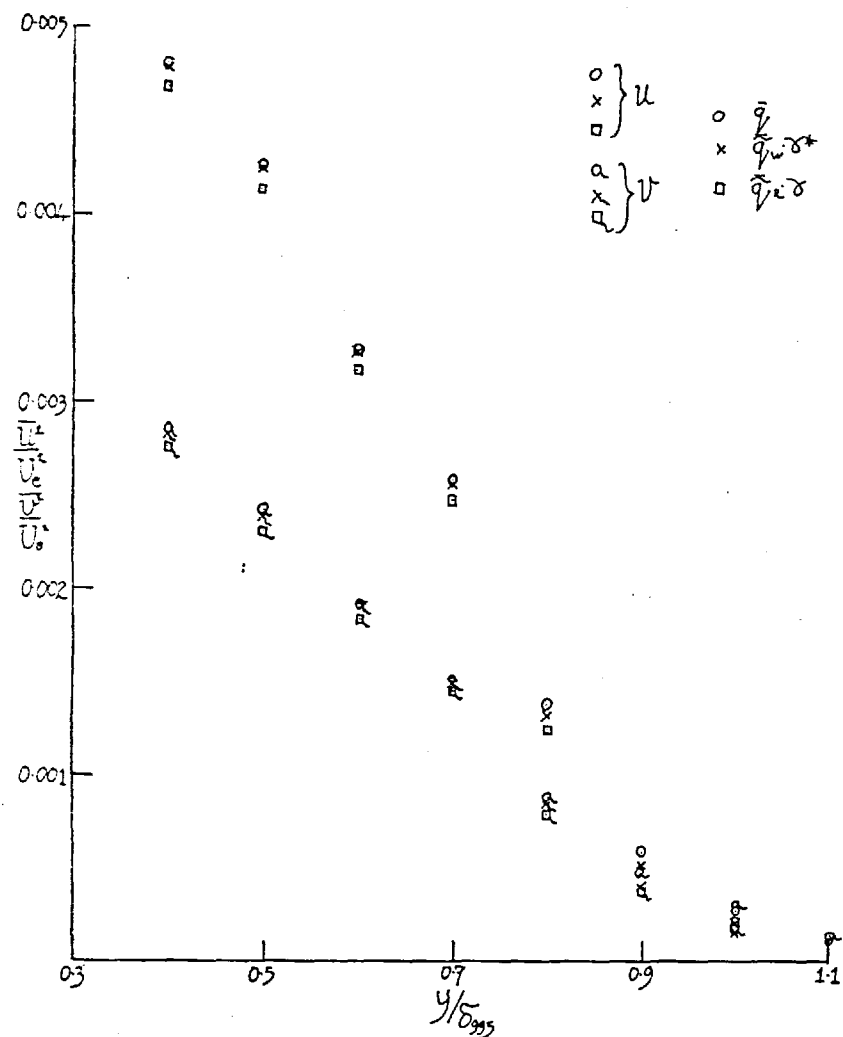
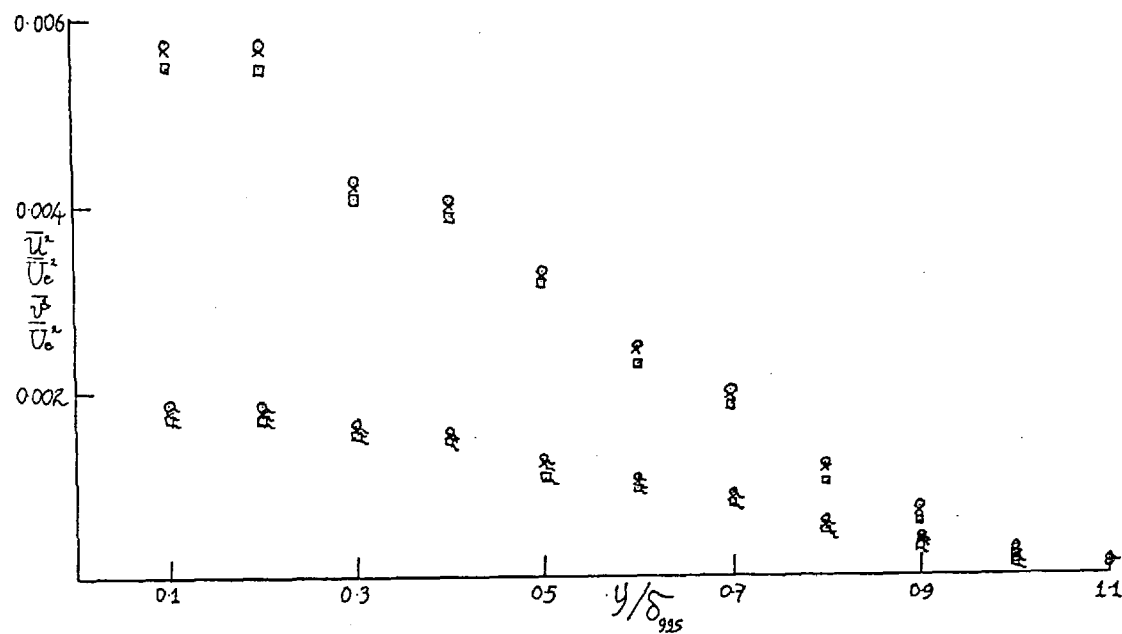


FIGURE [5-2.3b] TURBULENCE INTENSITY STN. 17



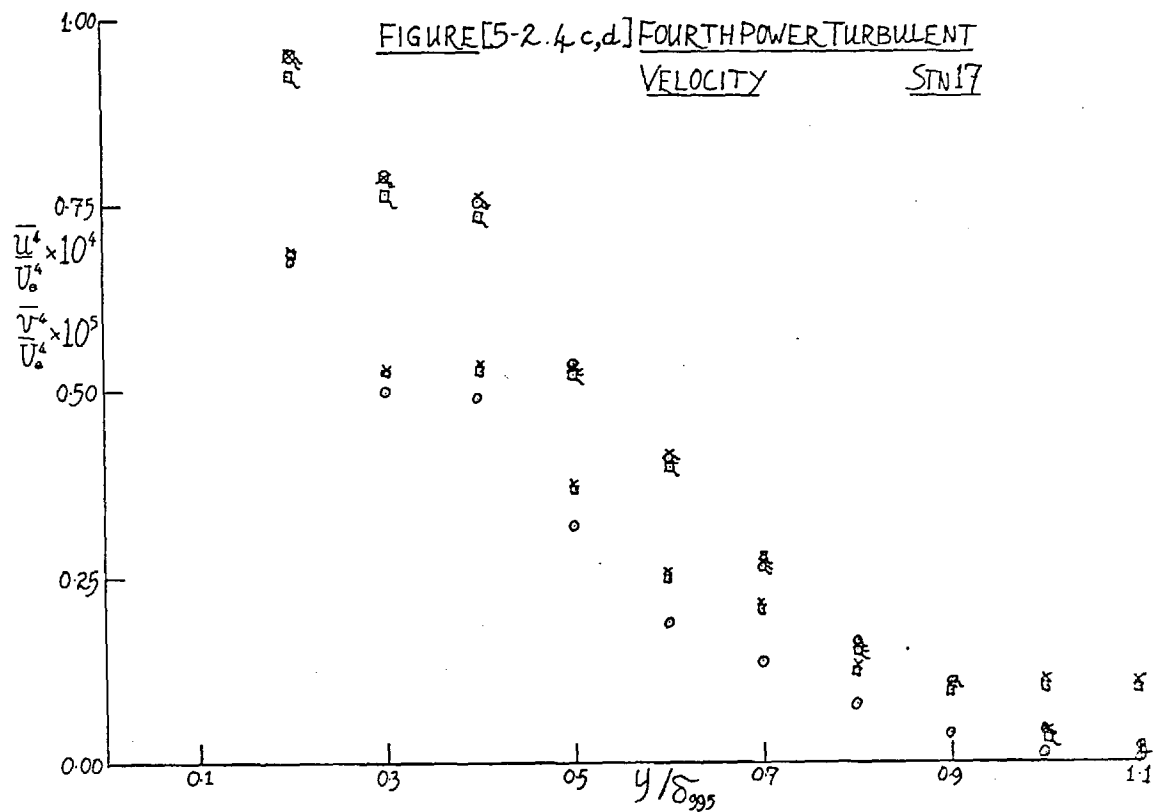
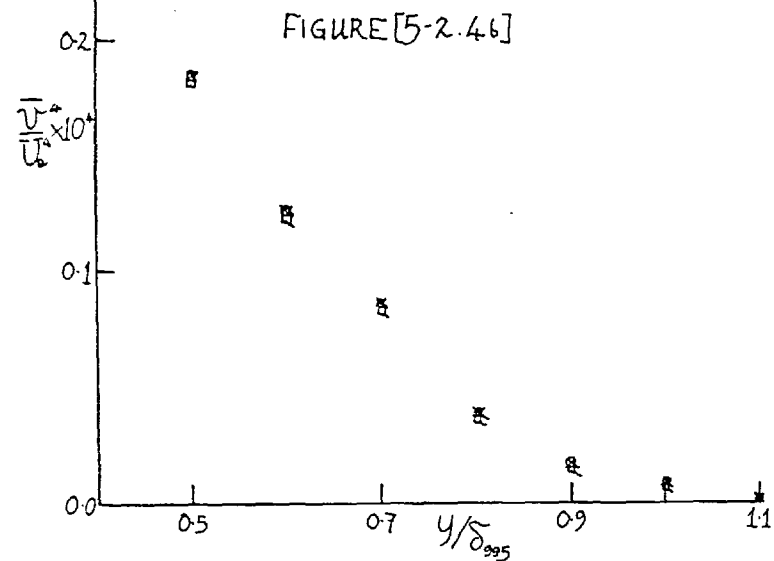
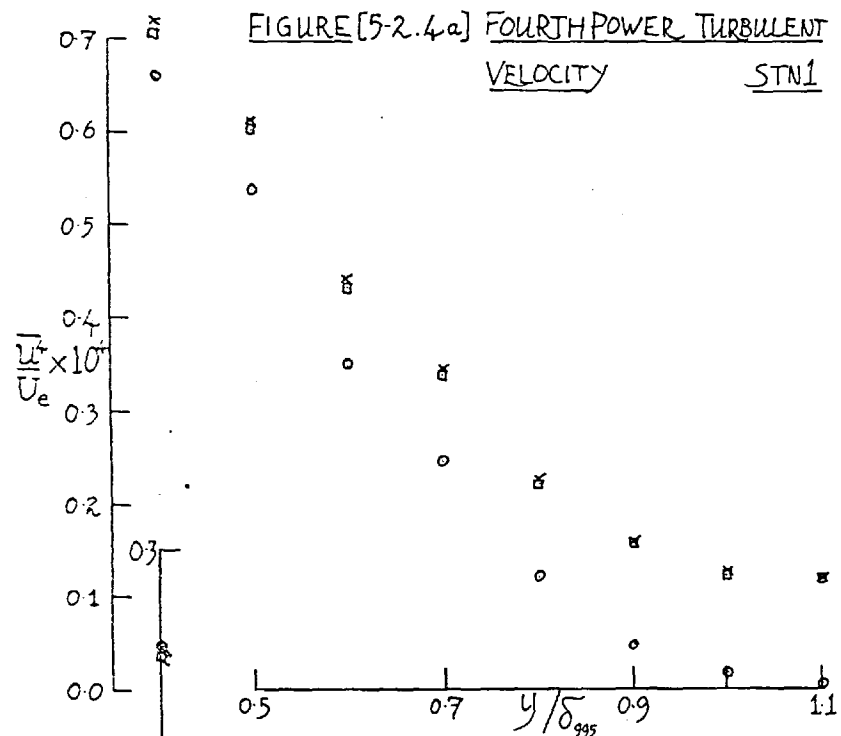


FIGURE [5-2.5a] RMS INTENSITY
LL, AT VARIOUS STNS.

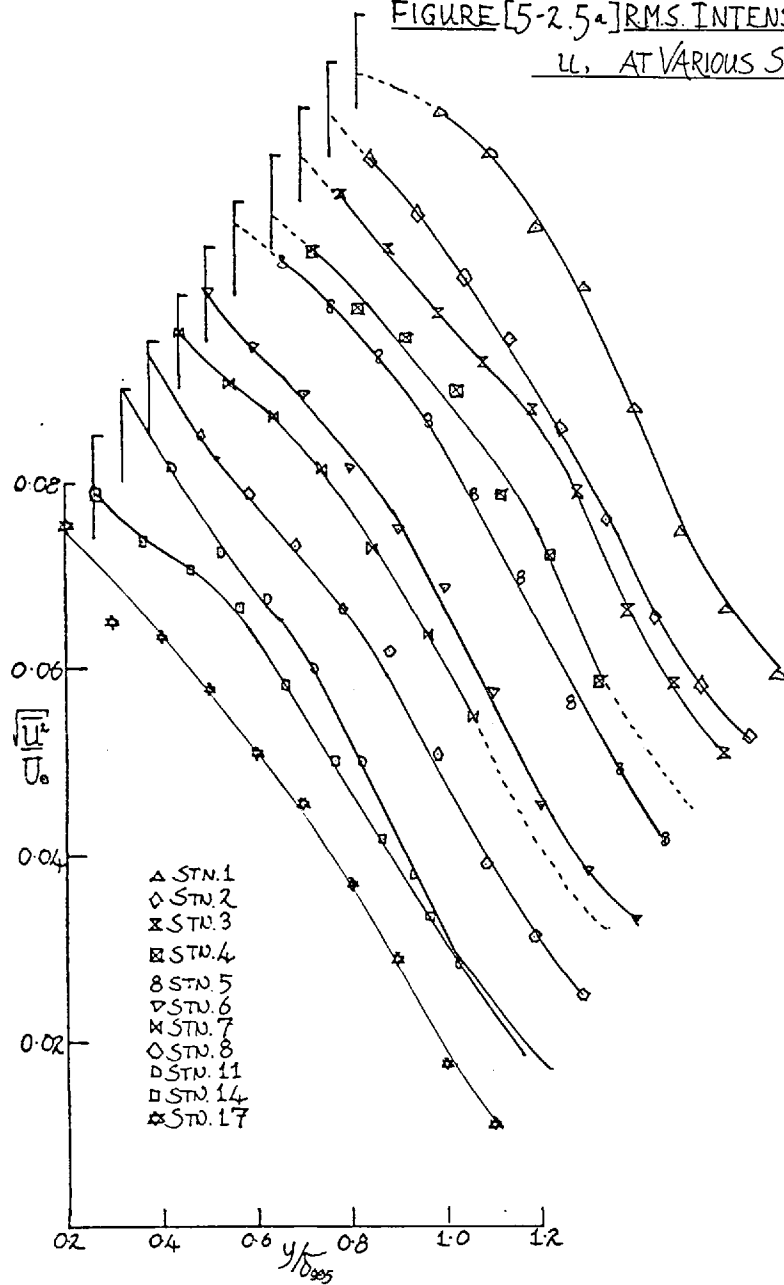
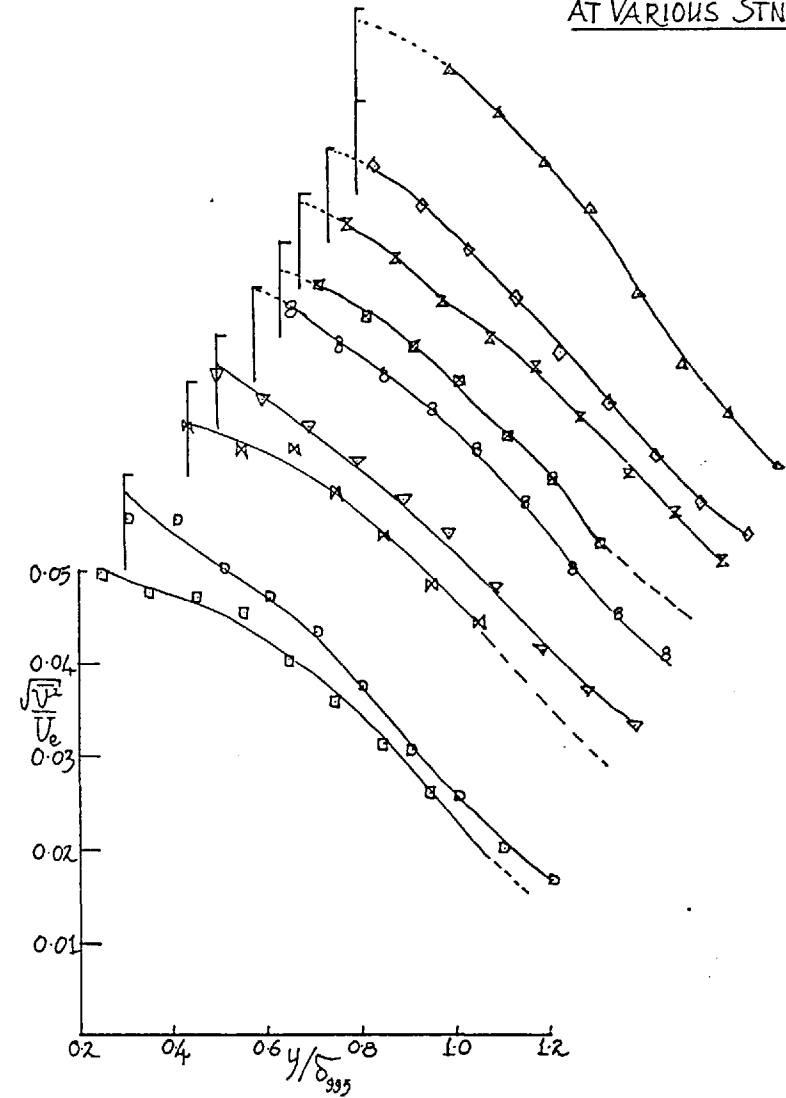


FIGURE [5-2.5b] RMS INTENSITY V
AT VARIOUS STNS.



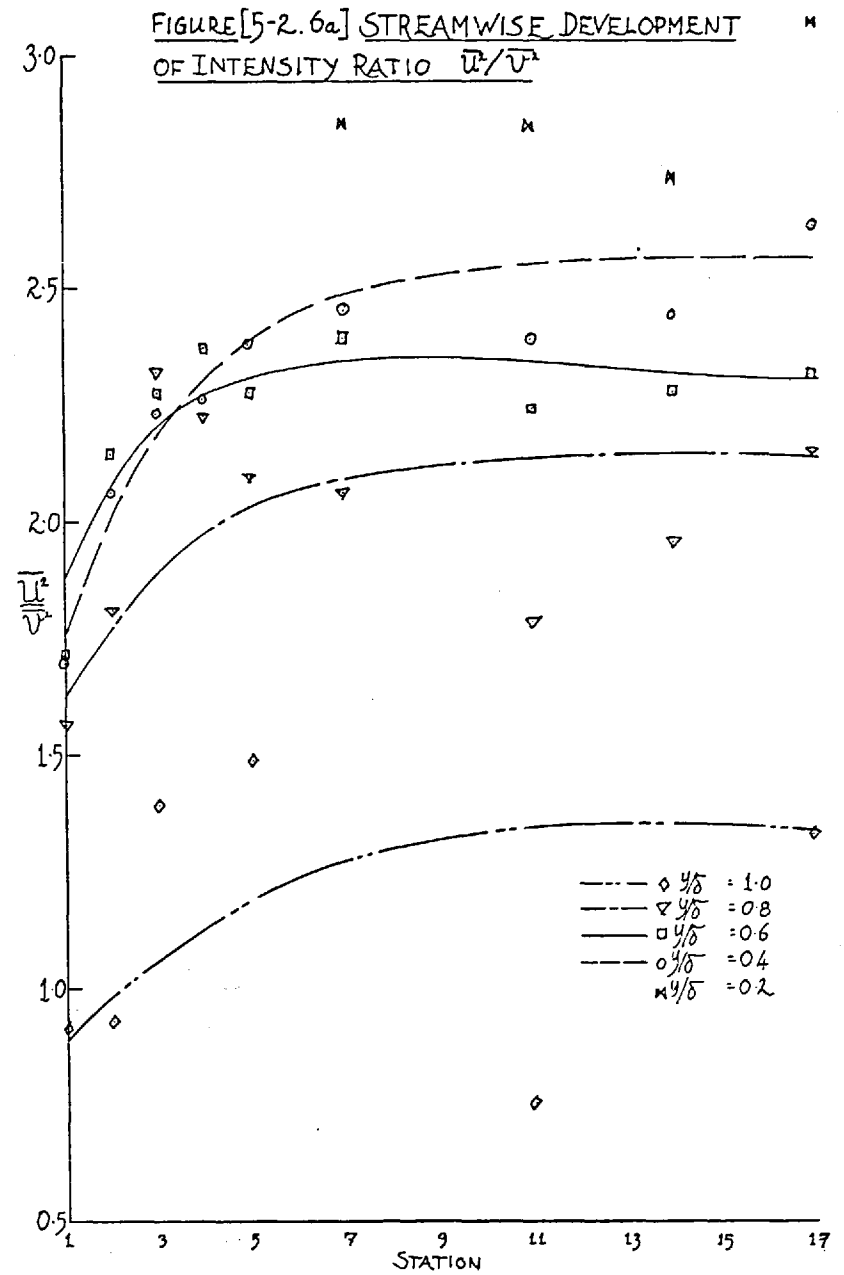
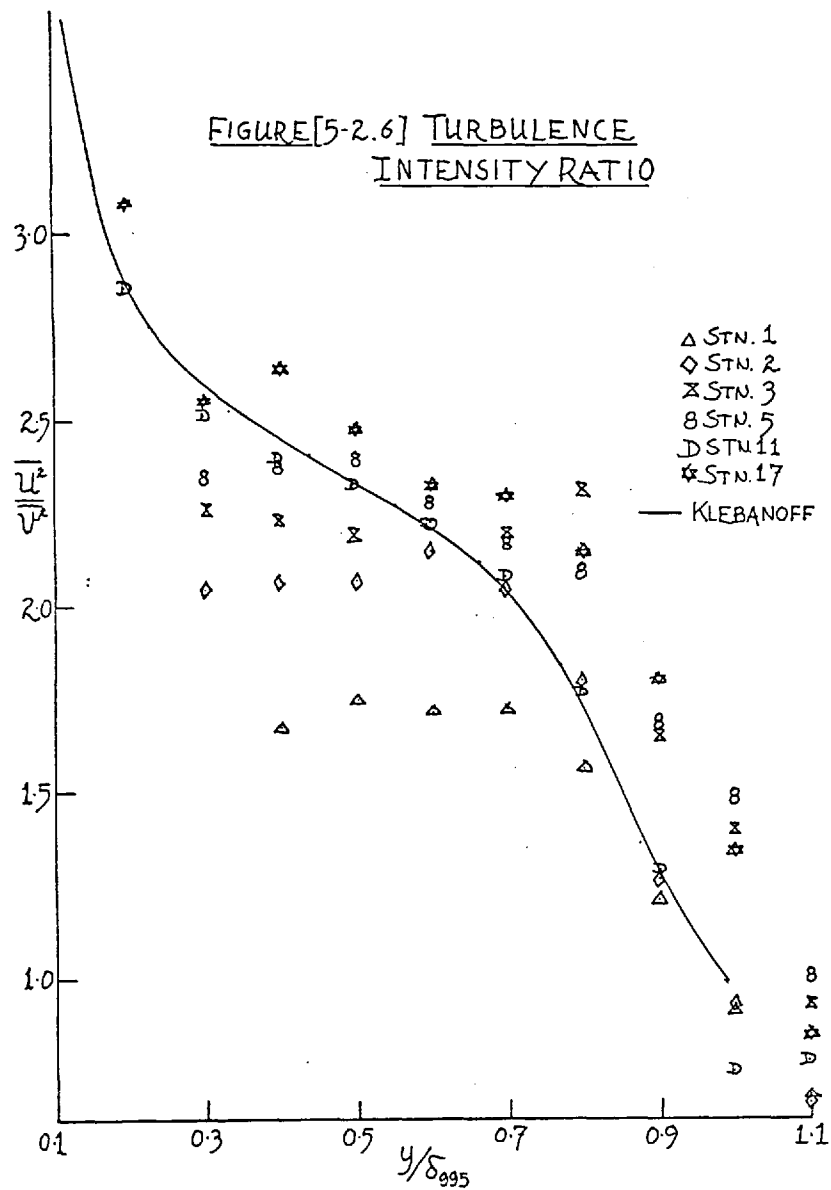


FIGURE [5-2.7] DIMENSIONLESS TURBULENCE
INTENSITY PROFILES

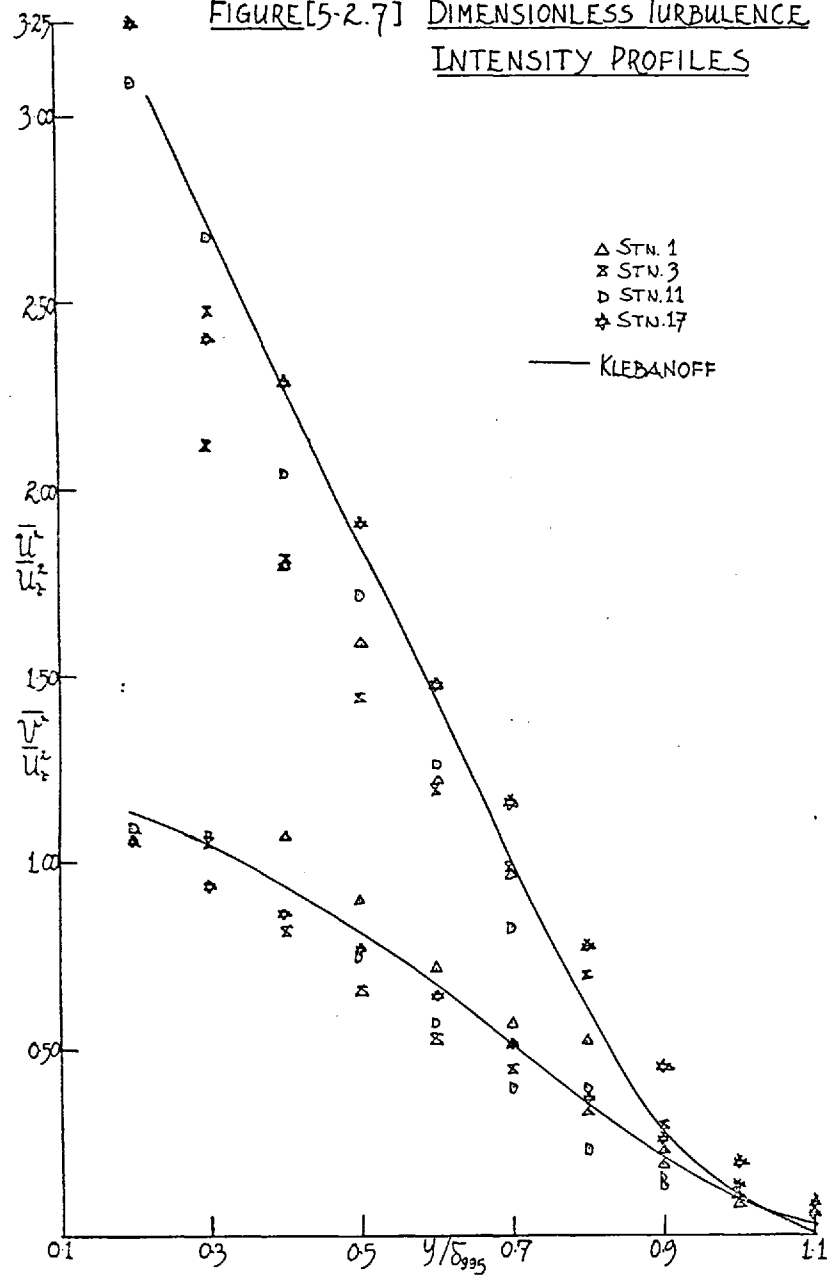


FIGURE [5-2.8] DIMENSIONLESS TURBULENCE
INTENSITY PROFILES

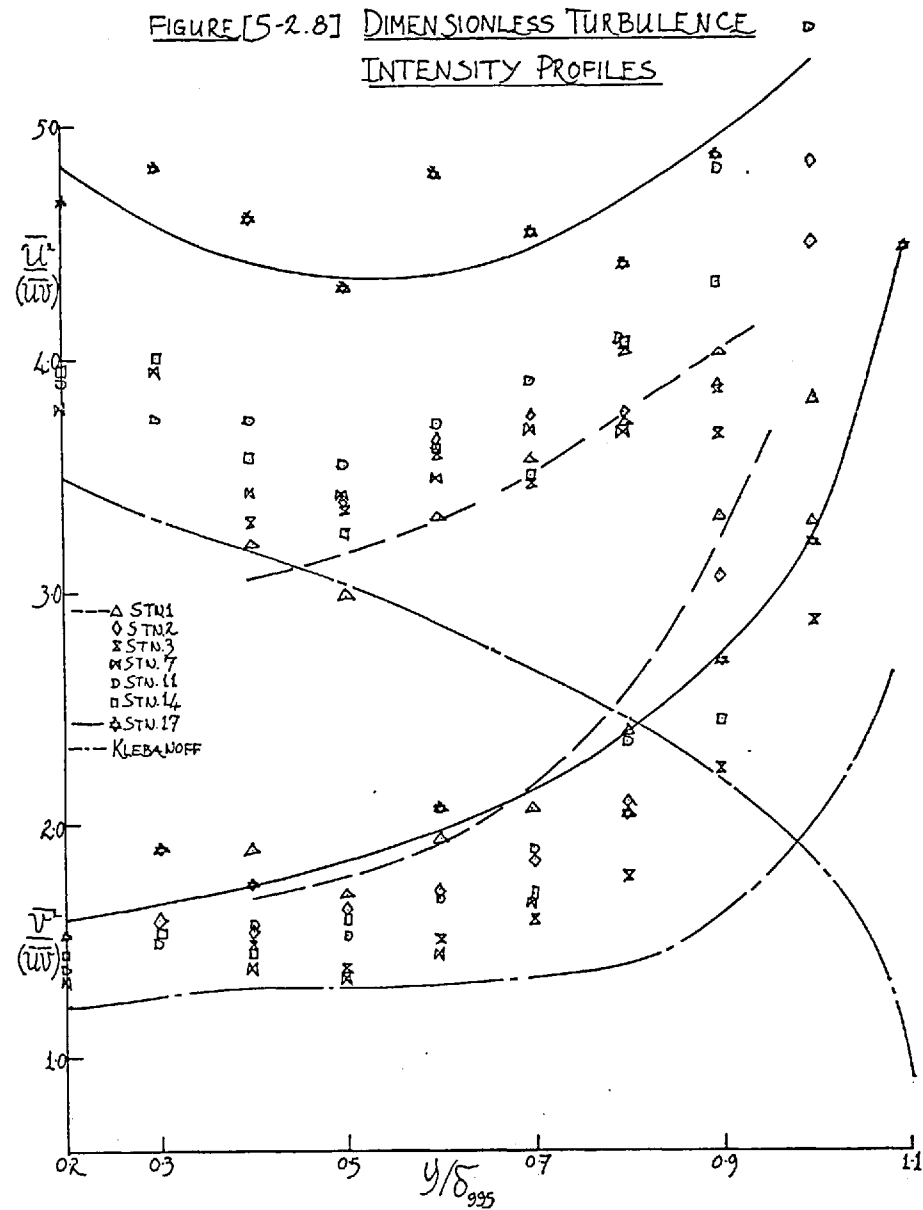


FIGURE [5-2.9] STREAMWISE
DEVELOPMENT OF u INTENSITY

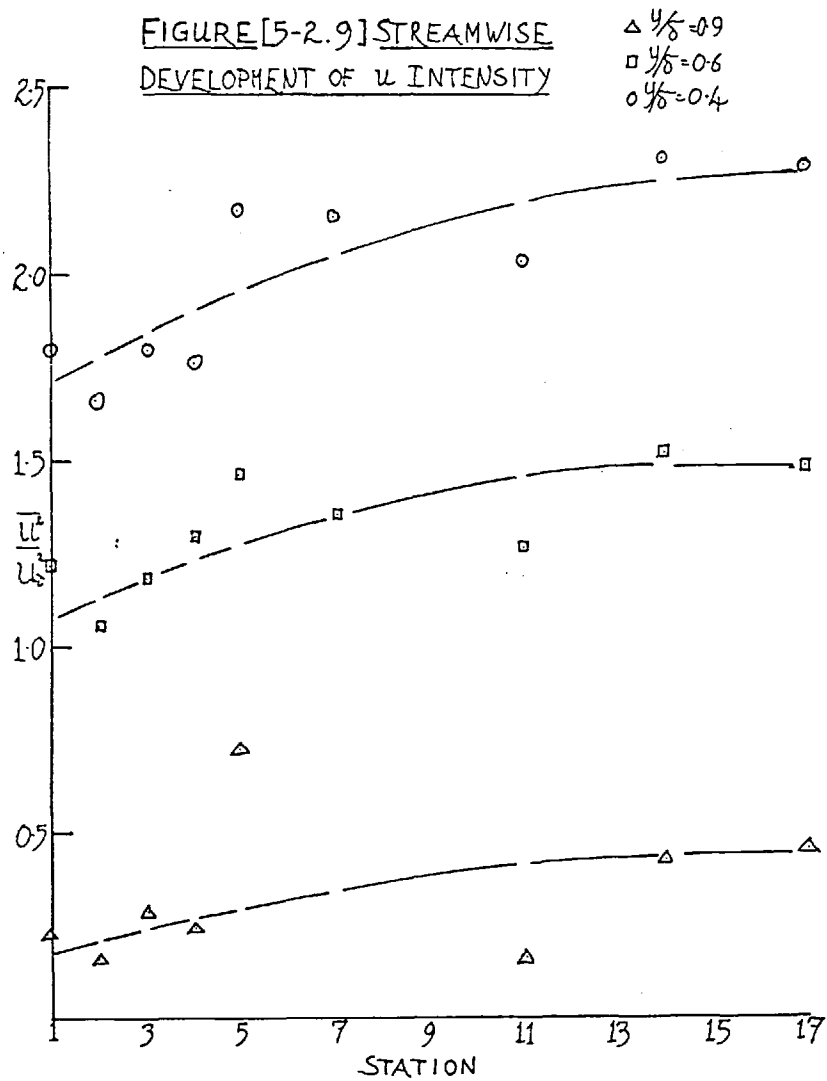
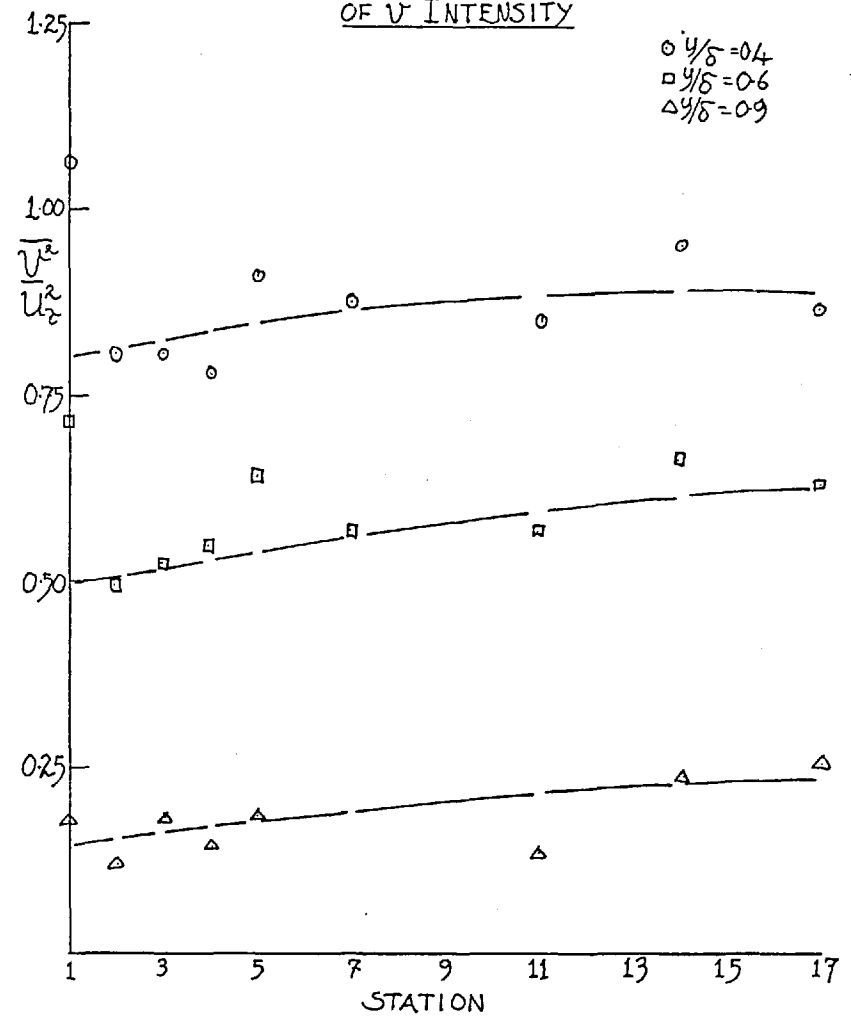


FIGURE [5-2.10] STREAMWISE DEVELOPMENT
OF v INTENSITY



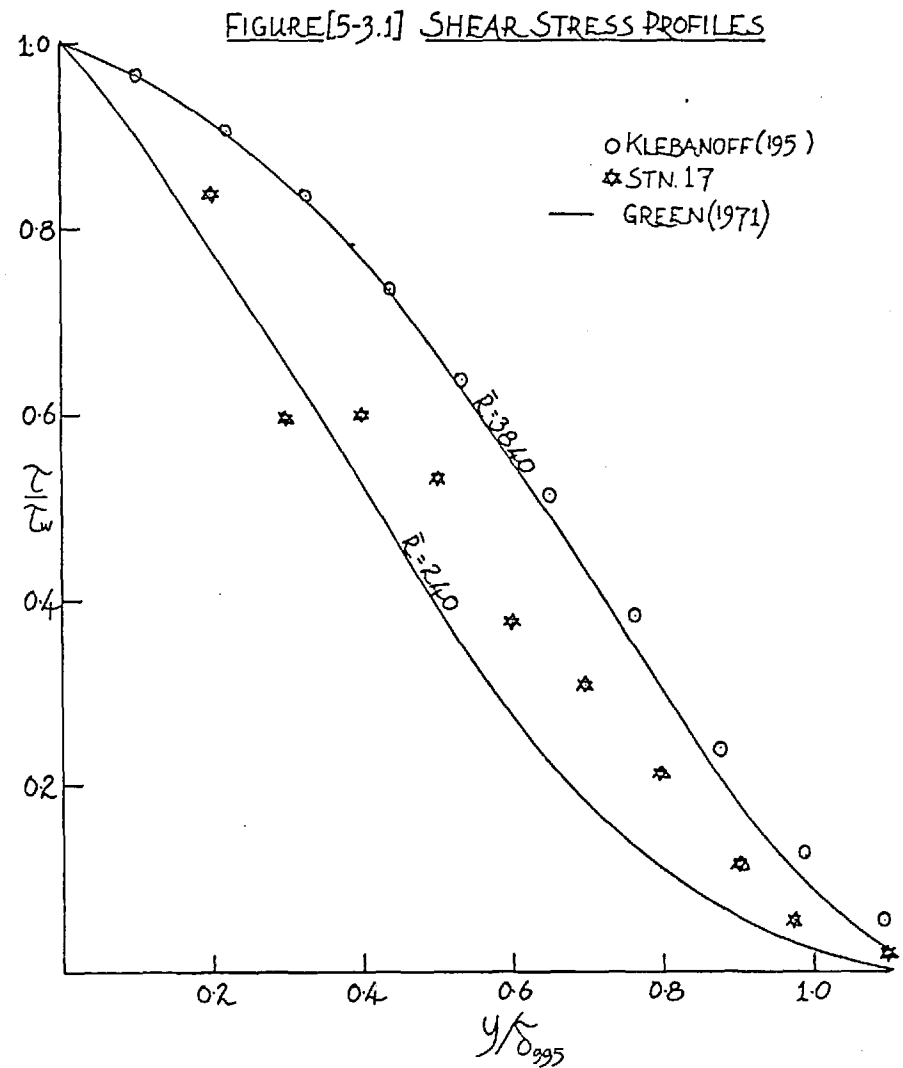
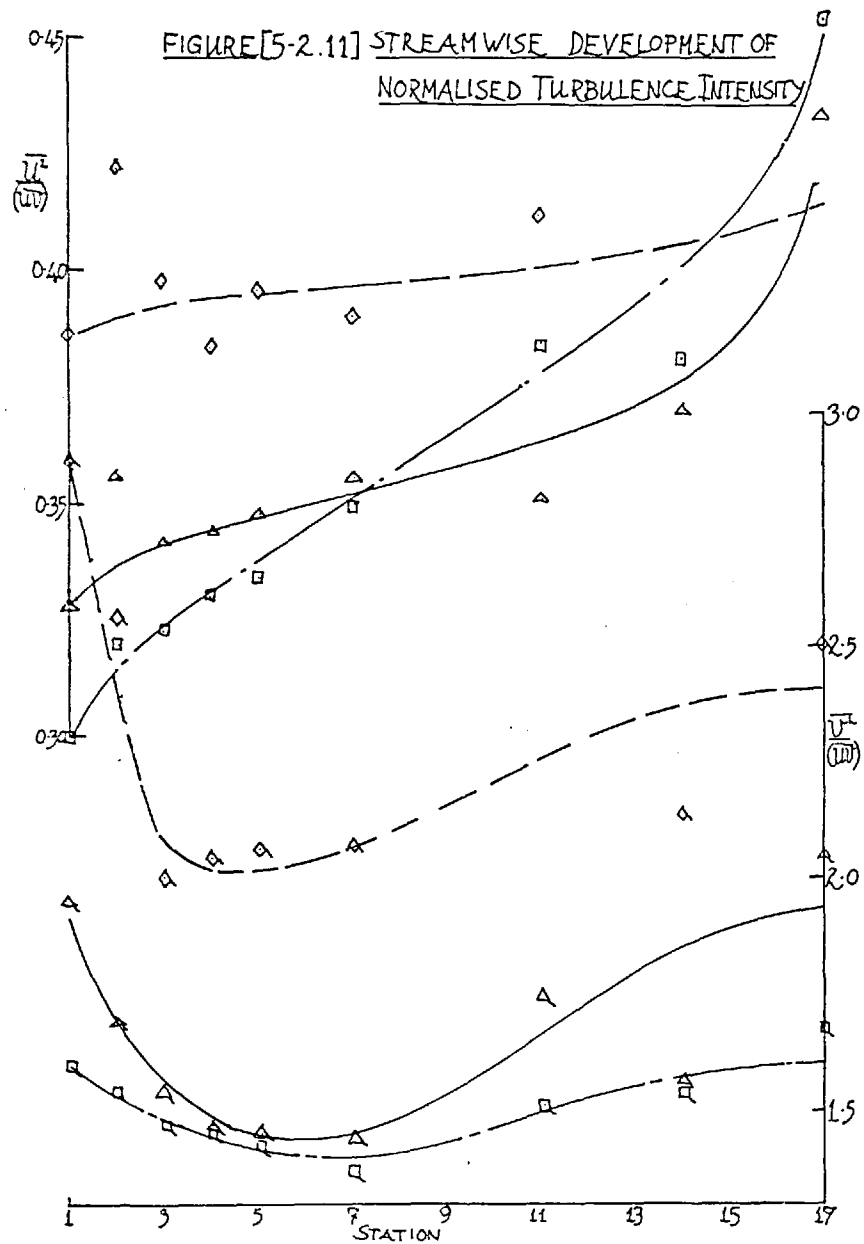


FIGURE [5-3.2a] TURBULENT SHEAR STRESS PROFILE STN 1

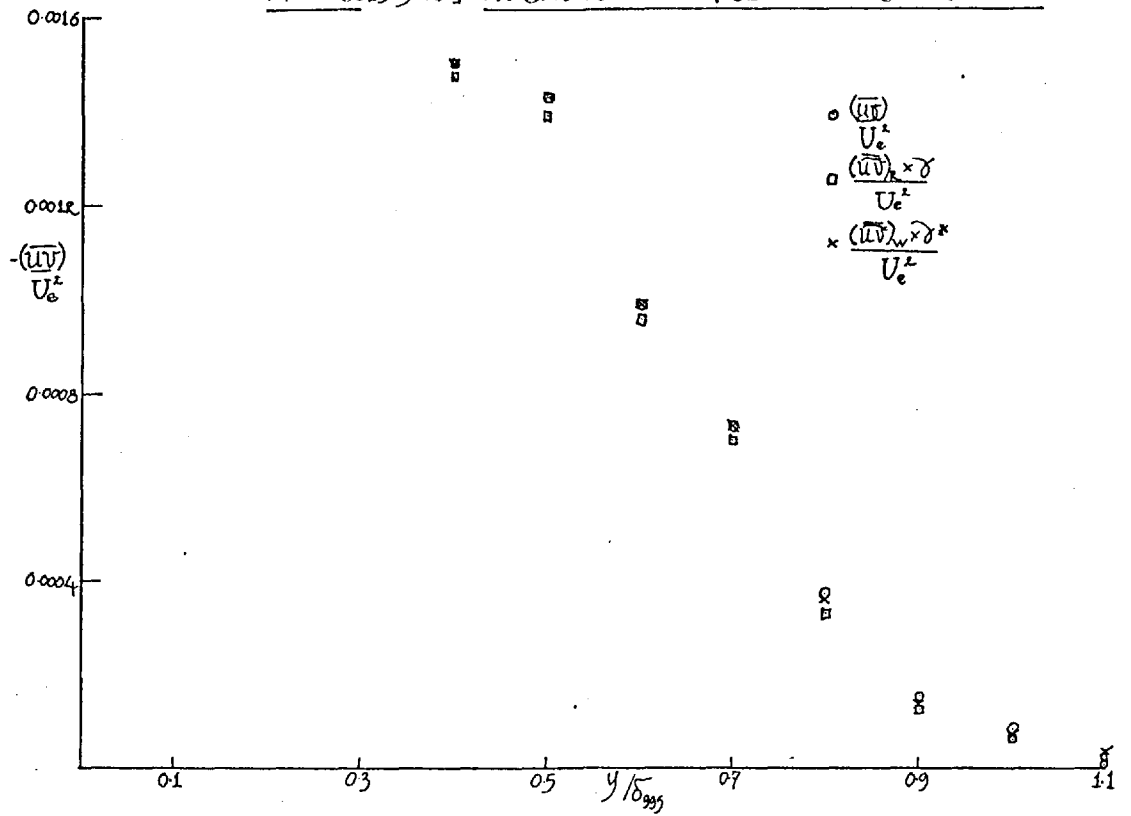


FIGURE [5-3.2b] TURBULENT SHEAR STRESS PROFILE STN 17

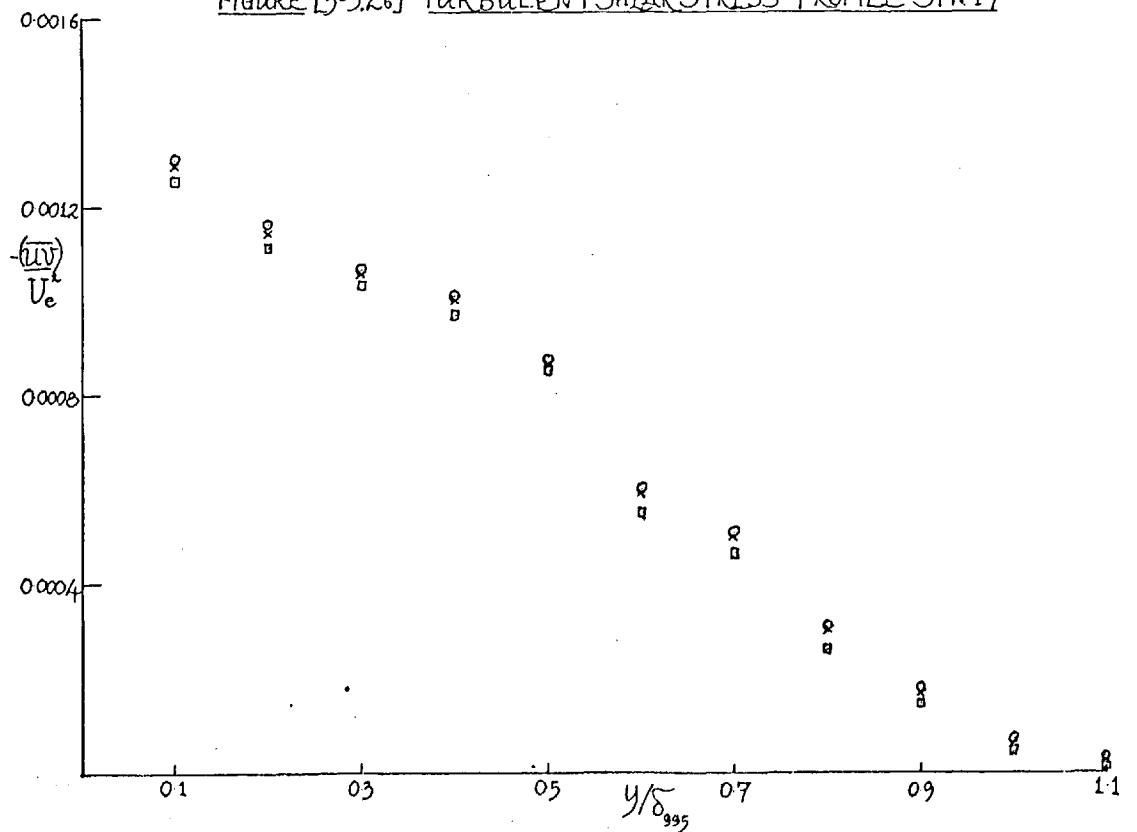


FIGURE [5-3.4] NORMALISED SHEAR STRESS PROFILES

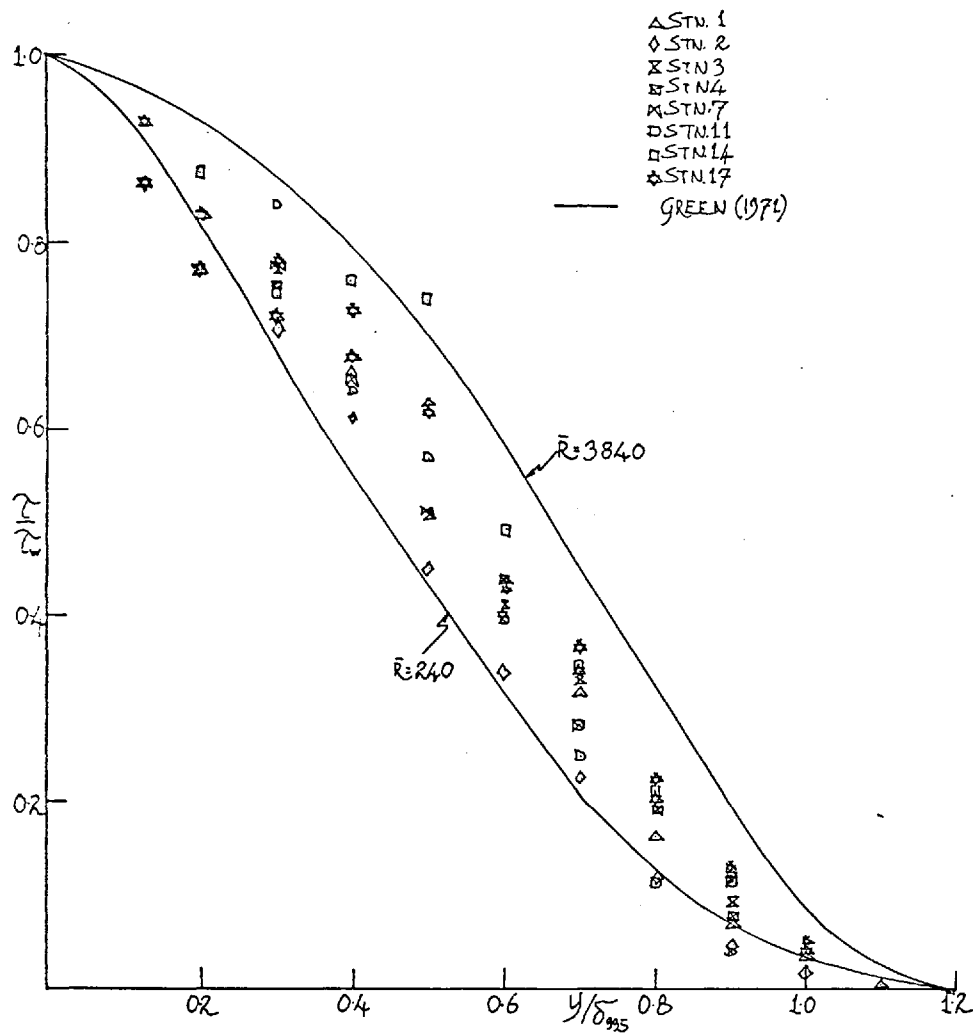


FIGURE [5-3.4a] STREAMWISE DEVELOPMENT OF NORMALISED SHEAR STRESS

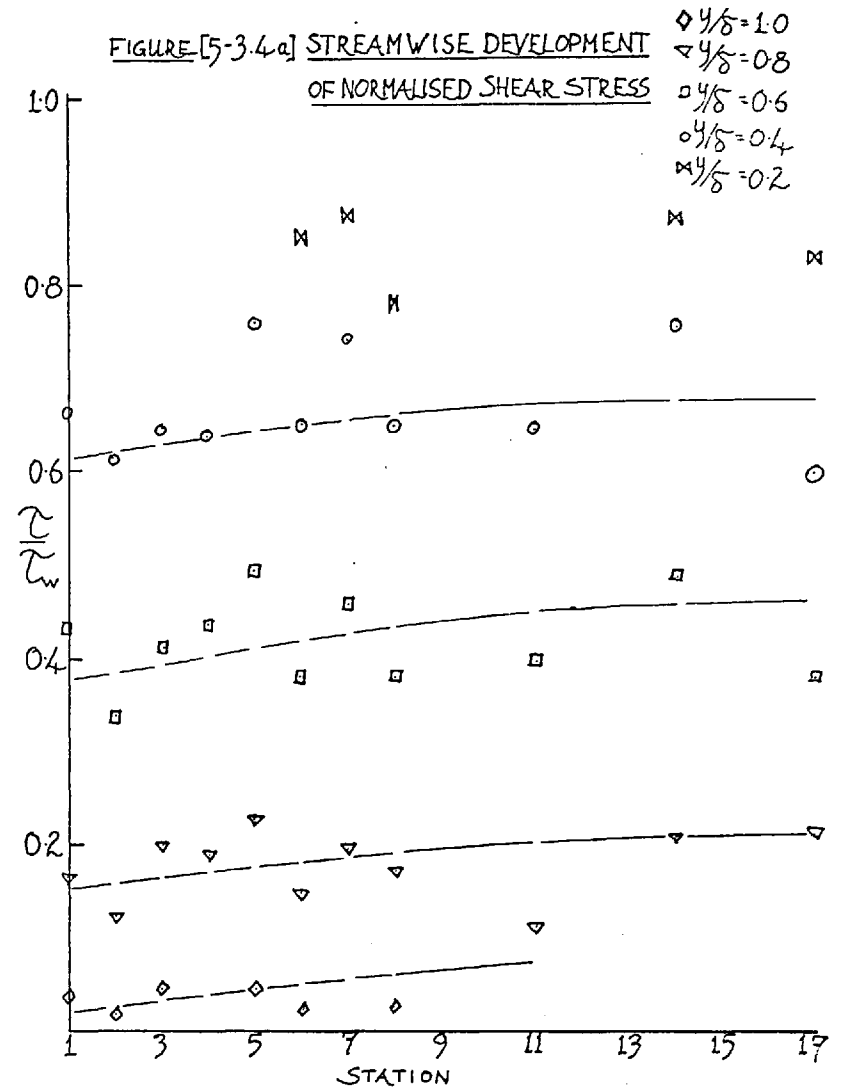


FIGURE [5-3.3] TURBULENCE SHEAR
STRESS PROFILE FOR EACH
EXPERIMENTAL STATION

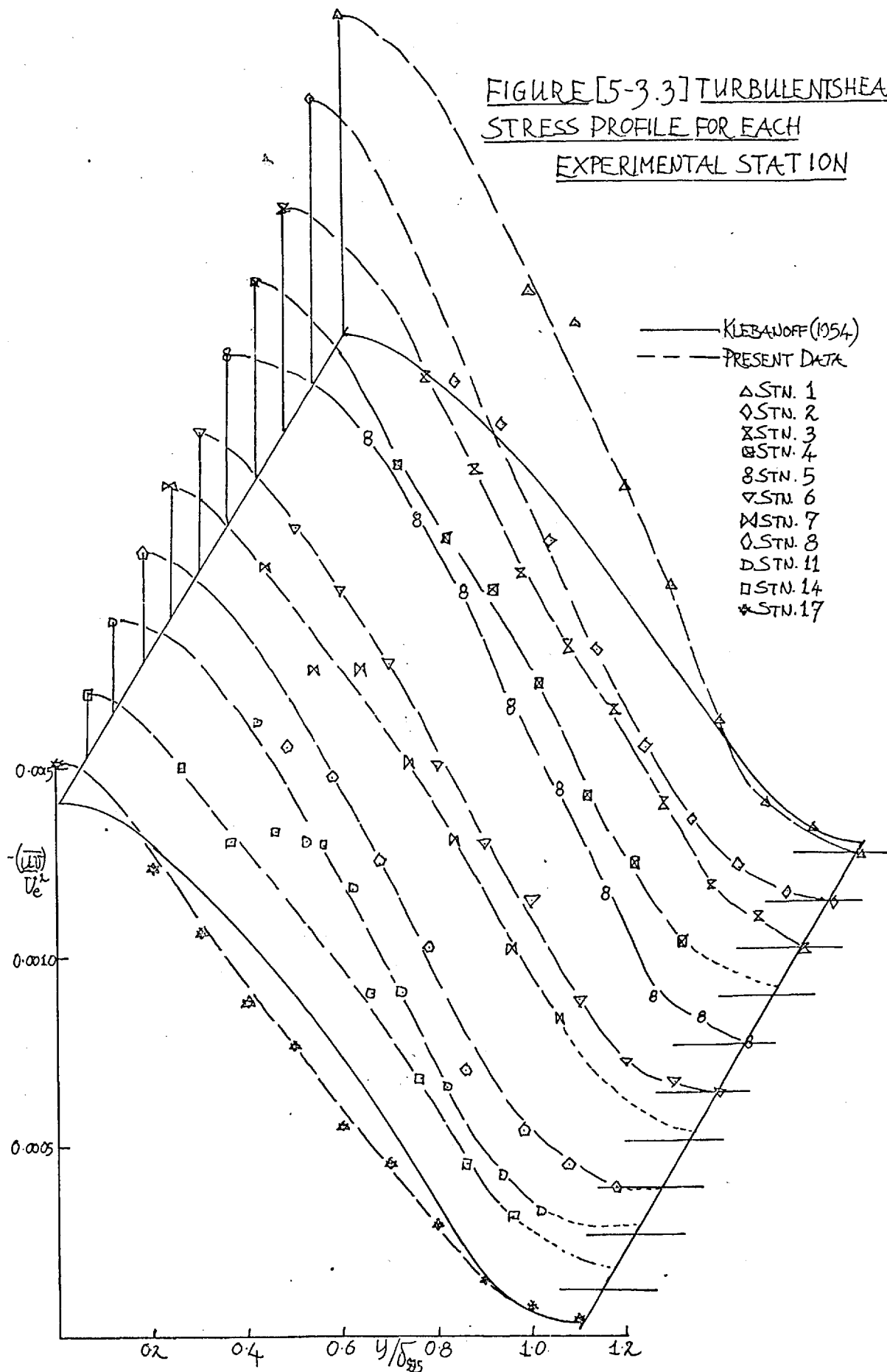


FIGURE [5-3.5] CORRELATION COEFFICIENT

$\frac{(\overline{uv})}{\sqrt{u^2} \sqrt{v^2}}$ PROFILES

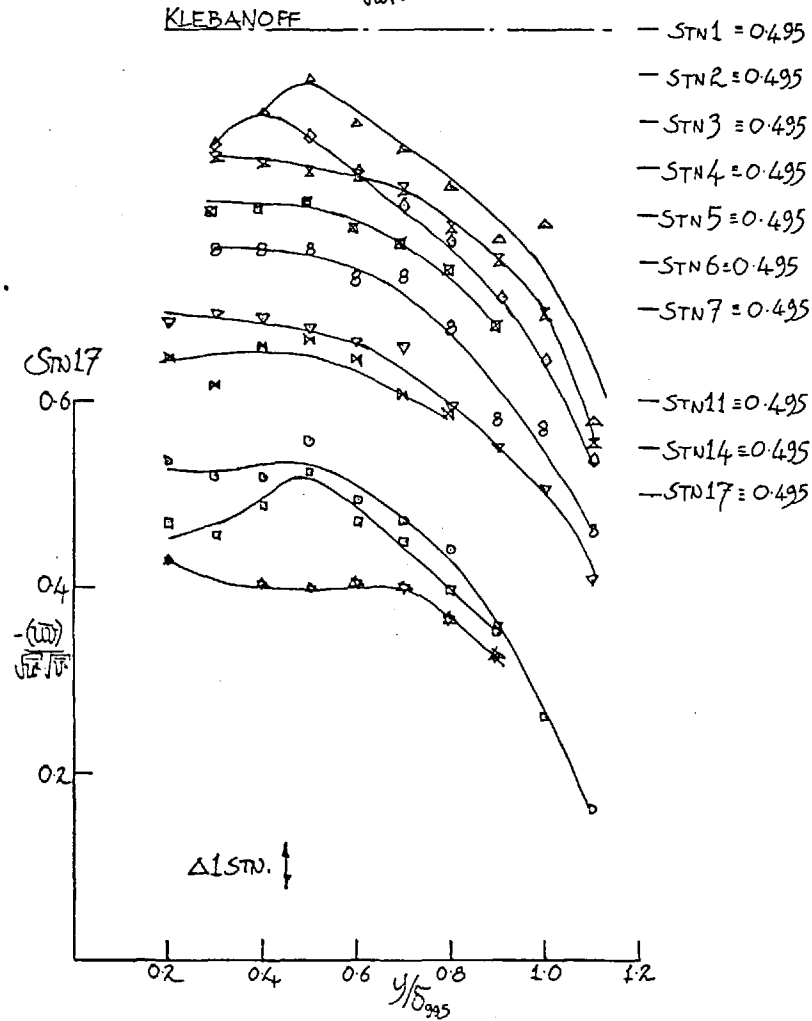
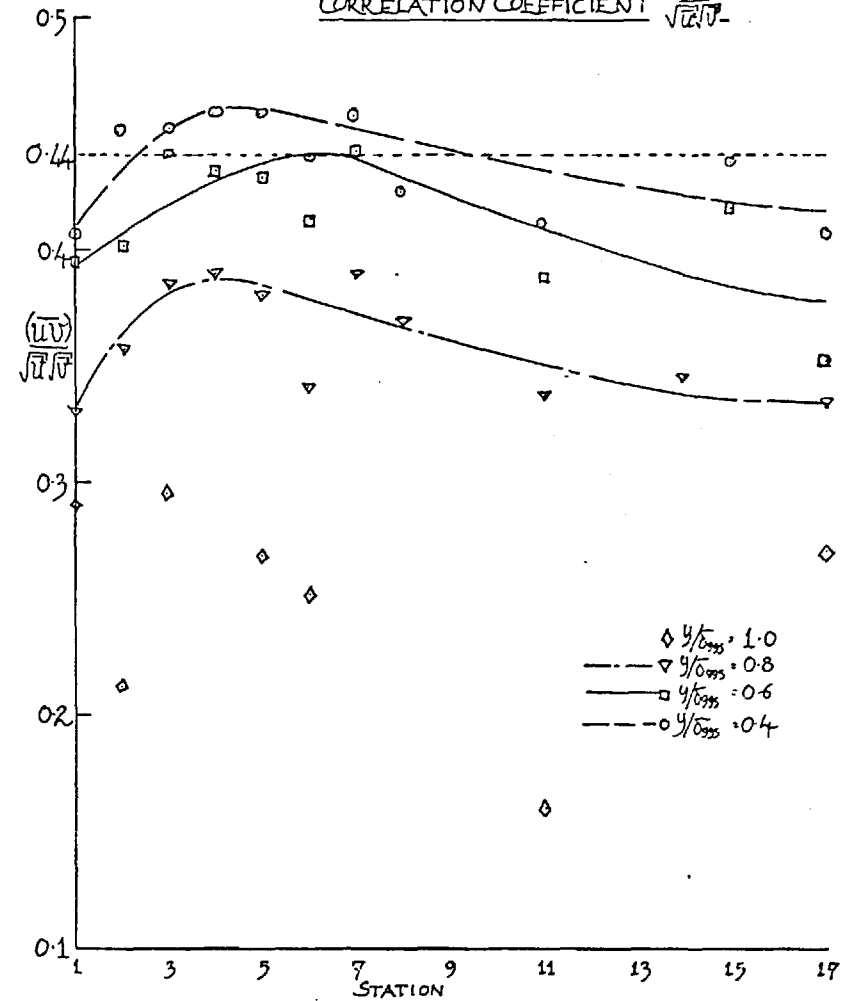


FIGURE [5-3.6] STREAMWISE DEVELOPMENT OF

CORRELATION COEFFICIENT $\frac{(\overline{uv})}{\sqrt{u^2} \sqrt{v^2}}$



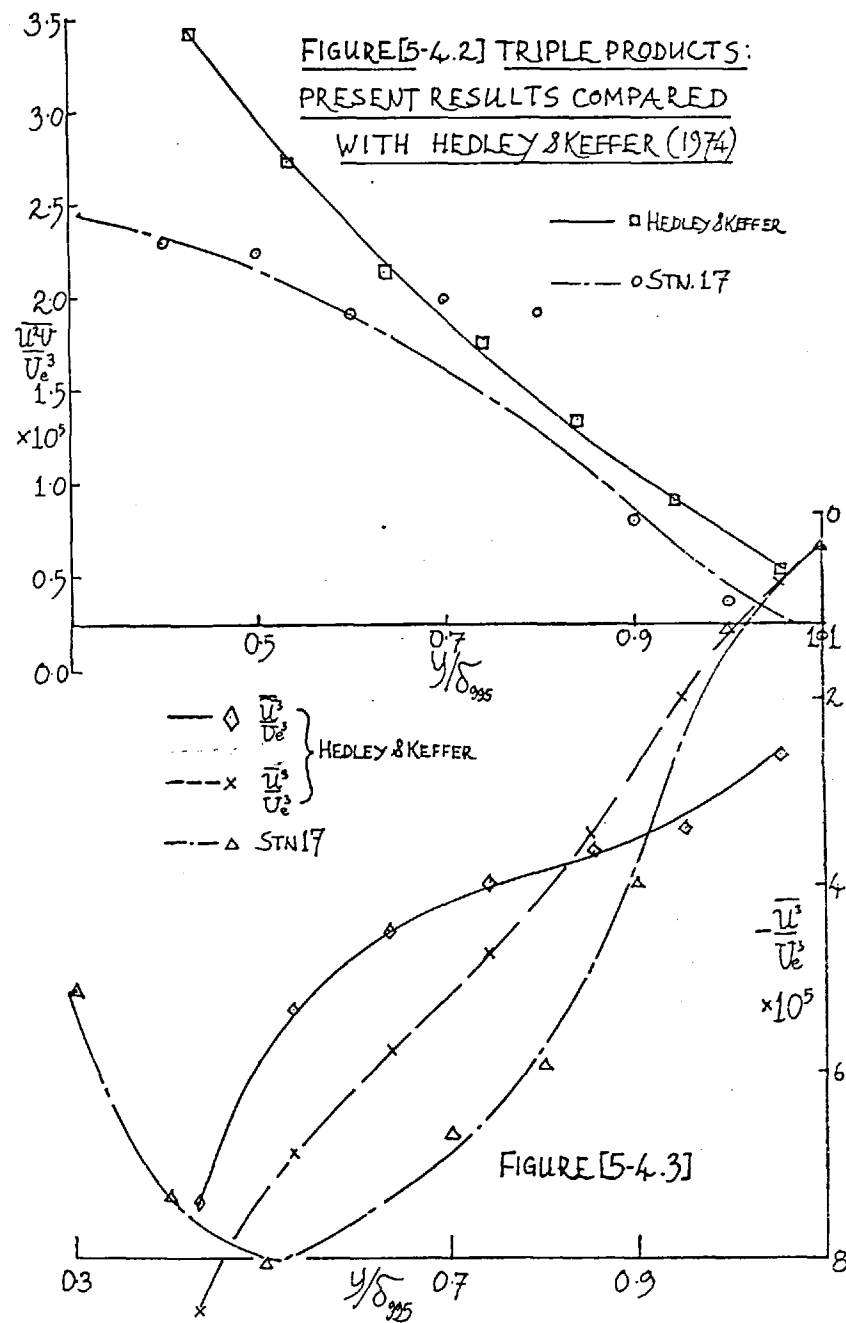
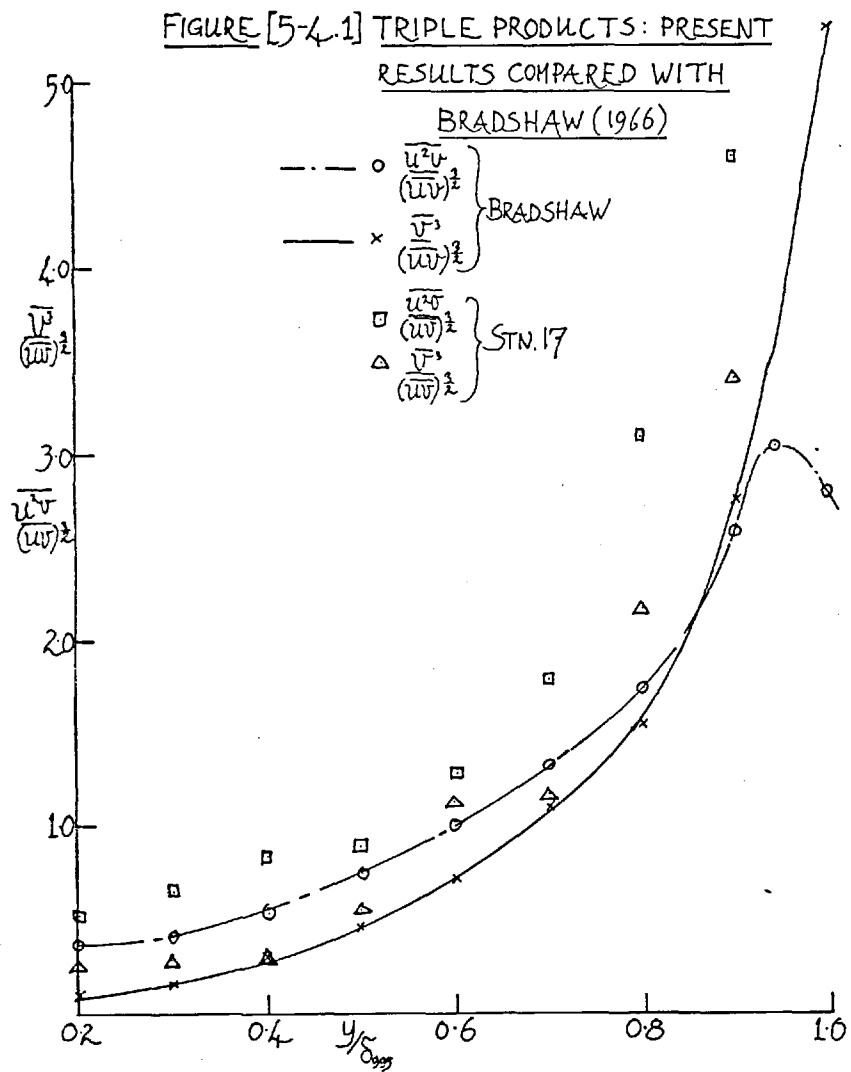


FIGURE [5-4.4] TRIPLE PRODUCTS: PRESENT
RESULTS COMPARED WITH HEDLEY
& KEFFER (1974)

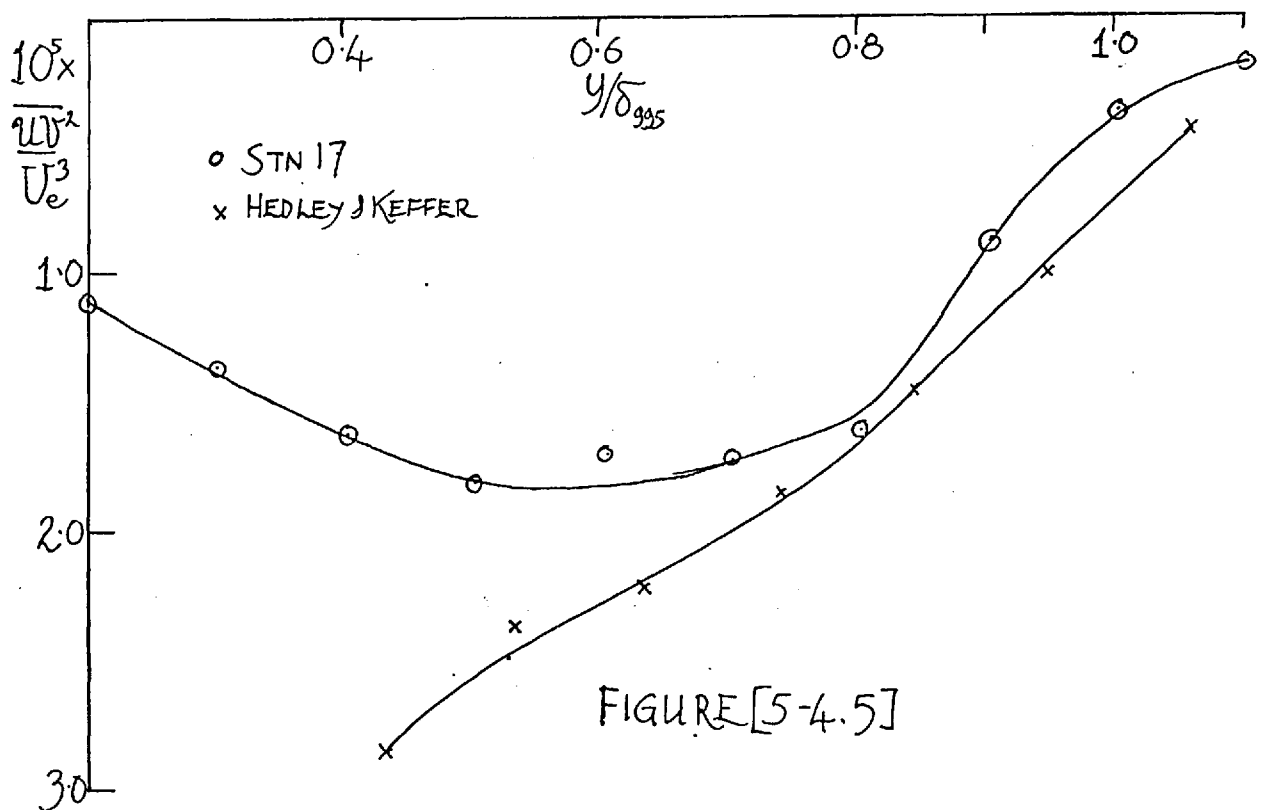
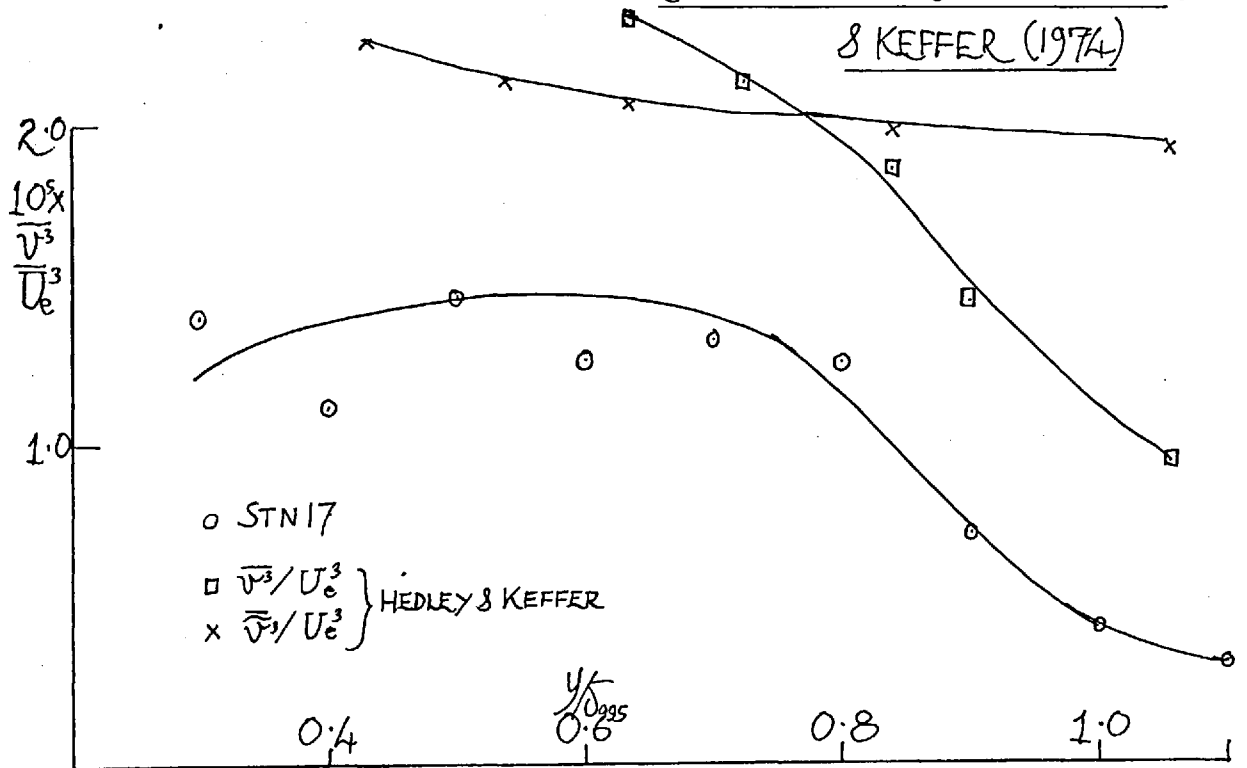
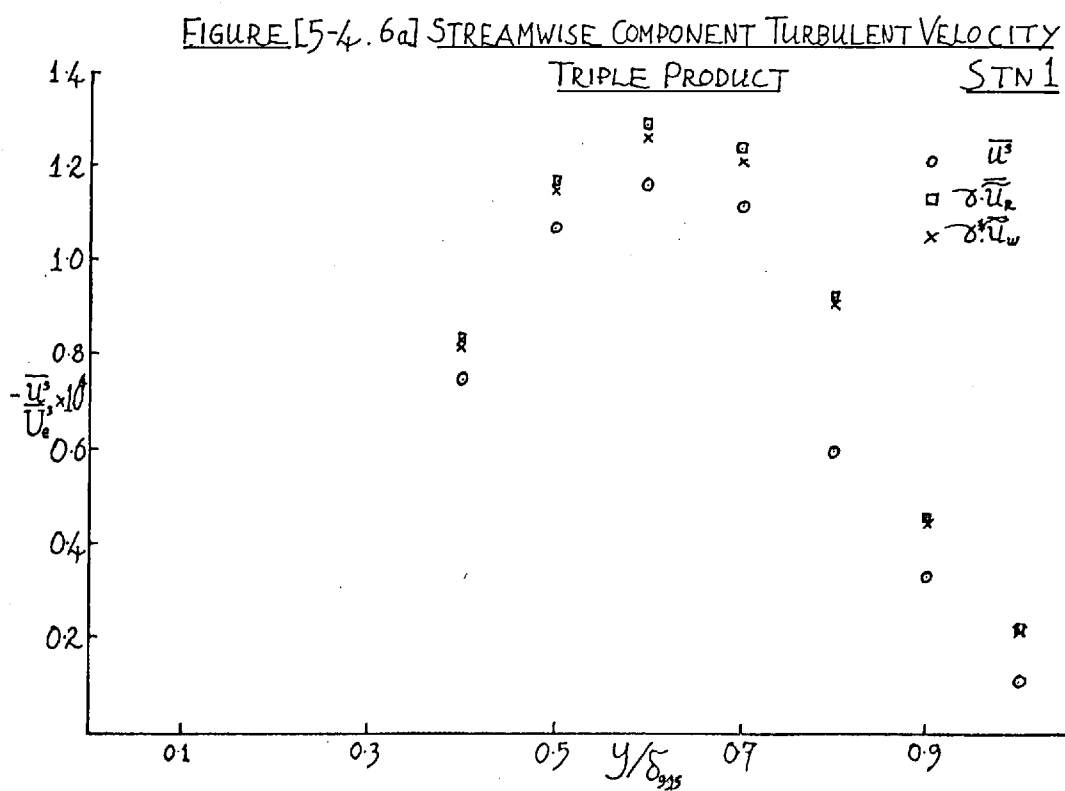
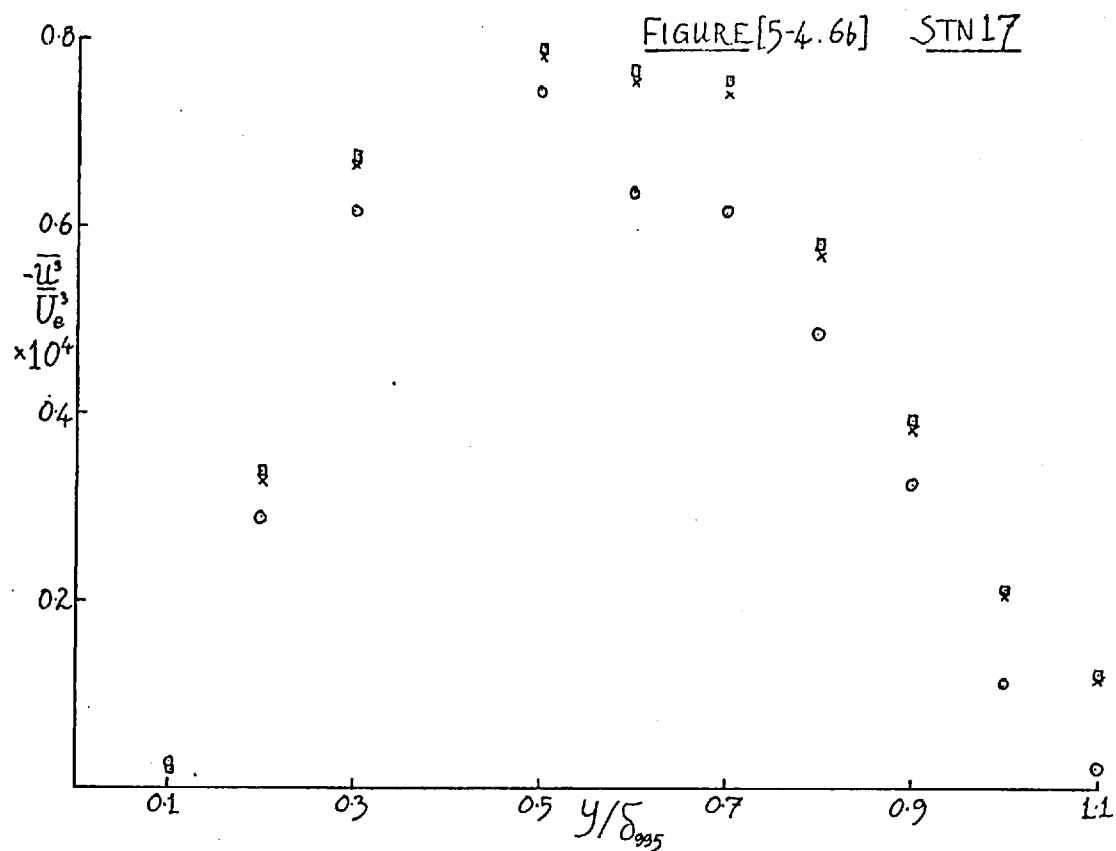


FIGURE [5-4.5]



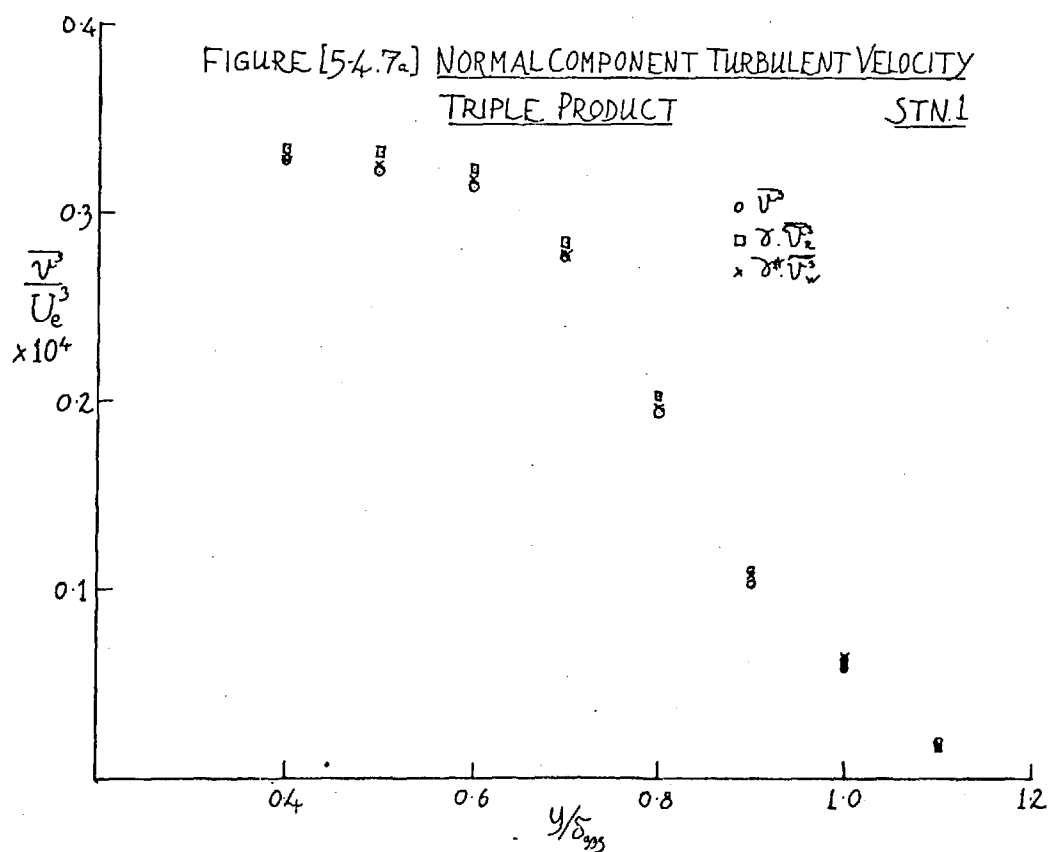
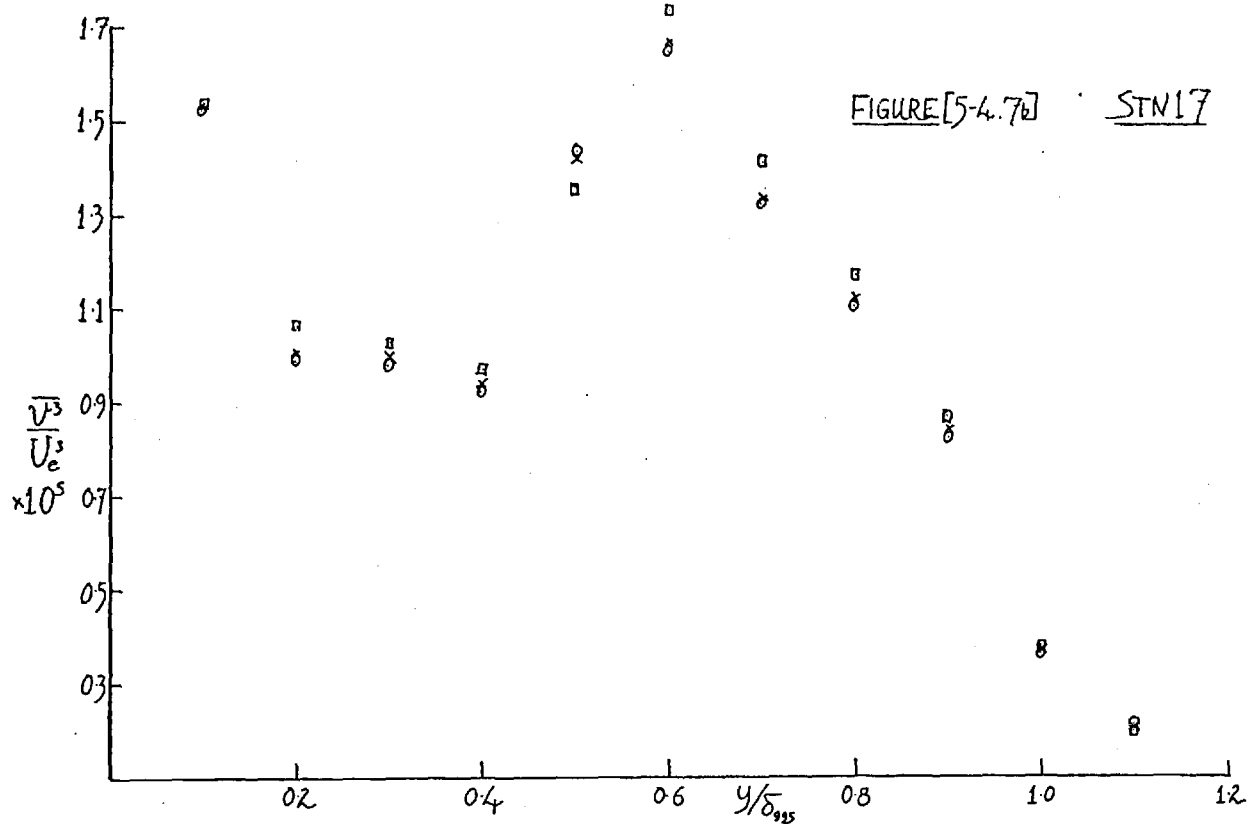


FIGURE [5-4.6c]

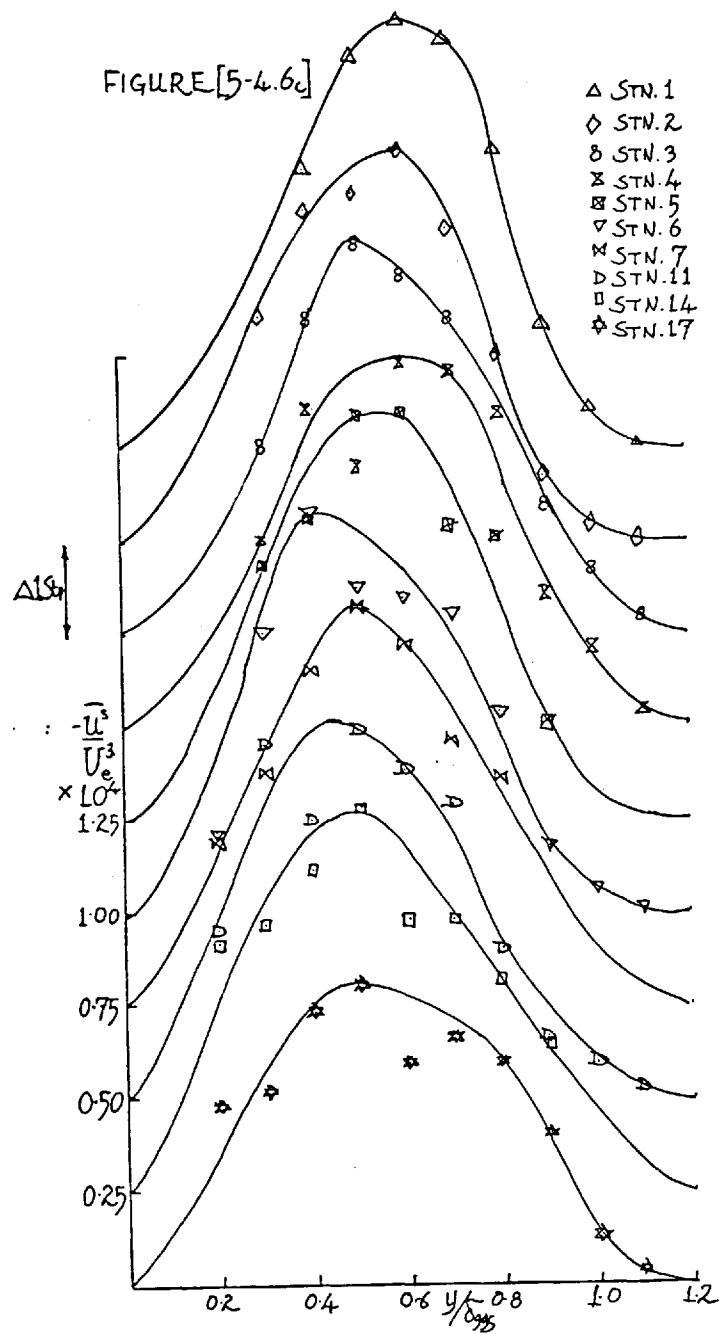
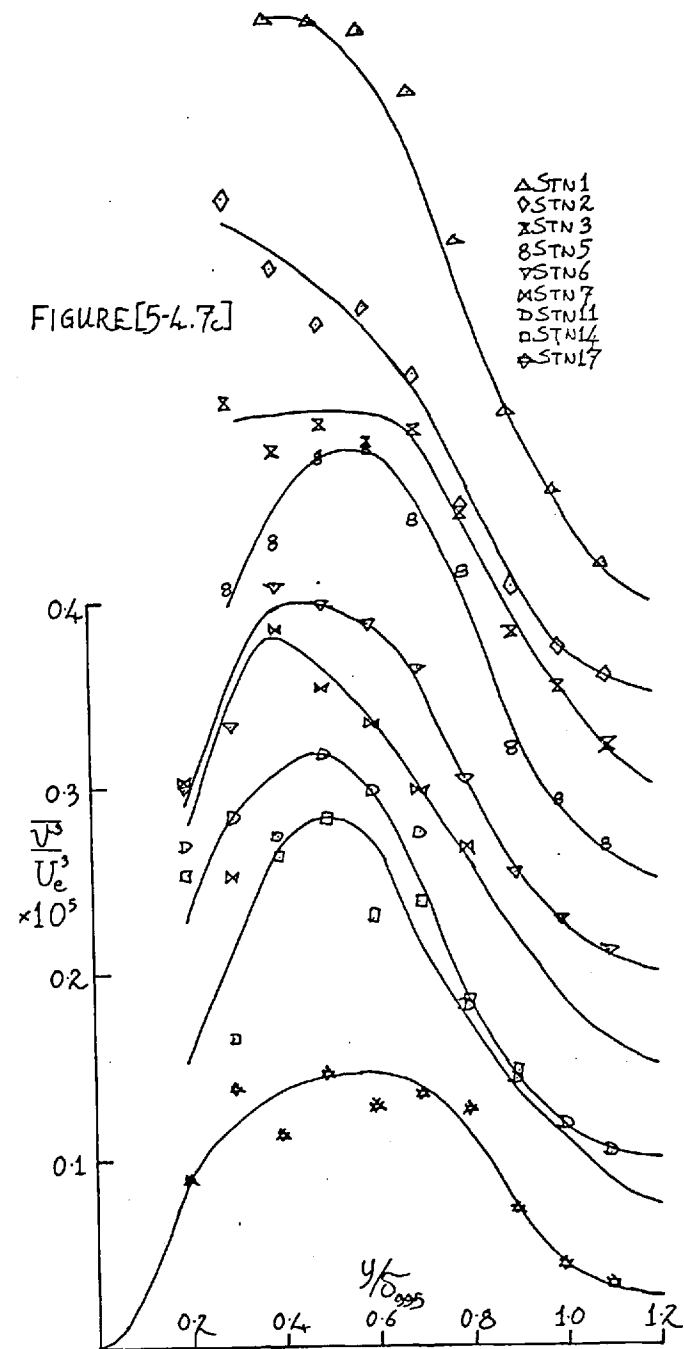


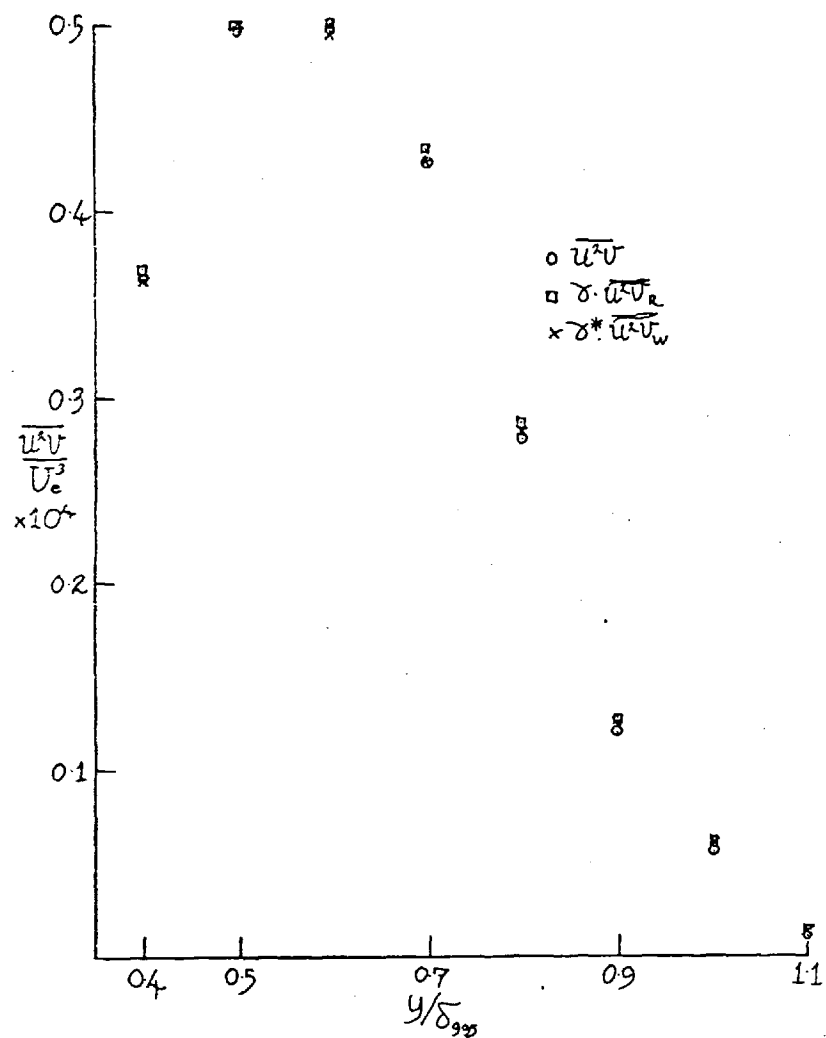
FIGURE [5-4.7c]



FIGURE[5-4.8a] MIXED TRIPLE PRODUCT

$\overline{u^2v}$

STN1



FIGURE[5-4.8b] STN17

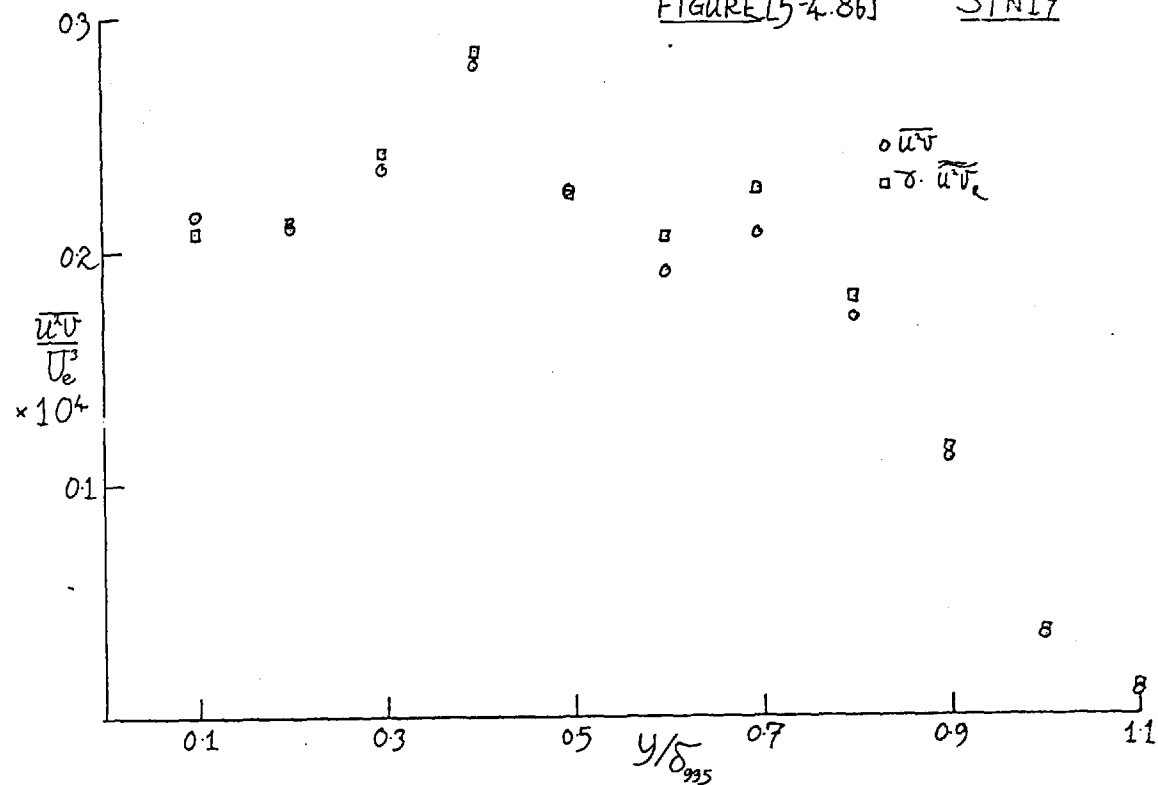


FIGURE [5-4.9a] MIXED TRIPLE PRODUCT
 $\overline{uv^2}$ STN 1

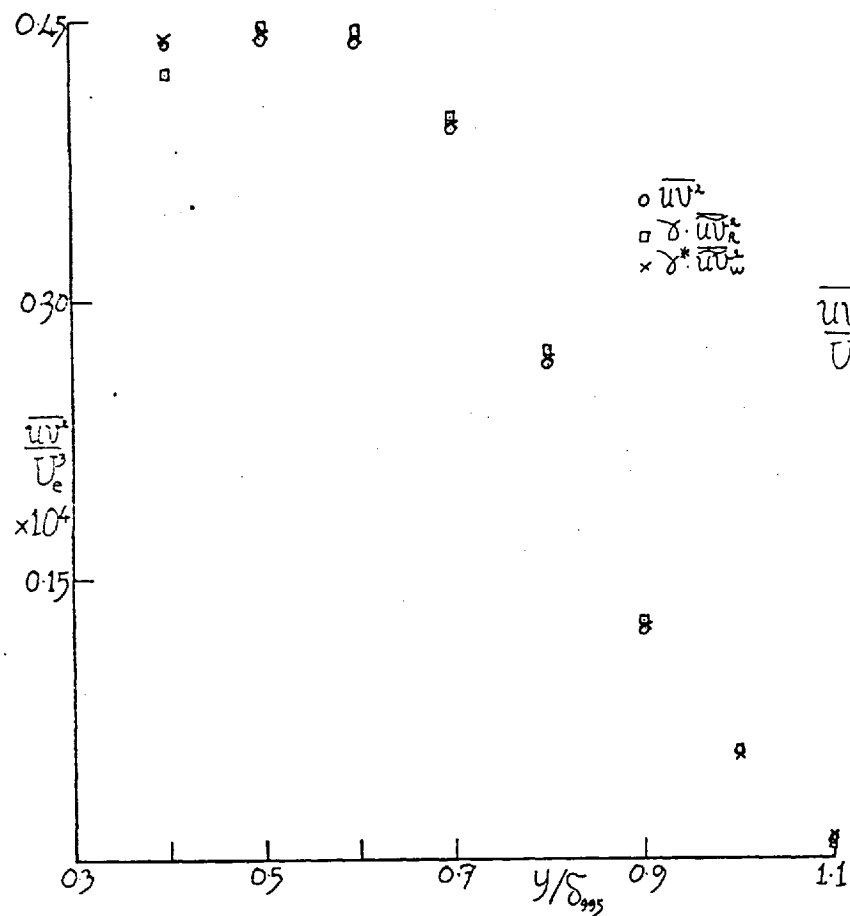
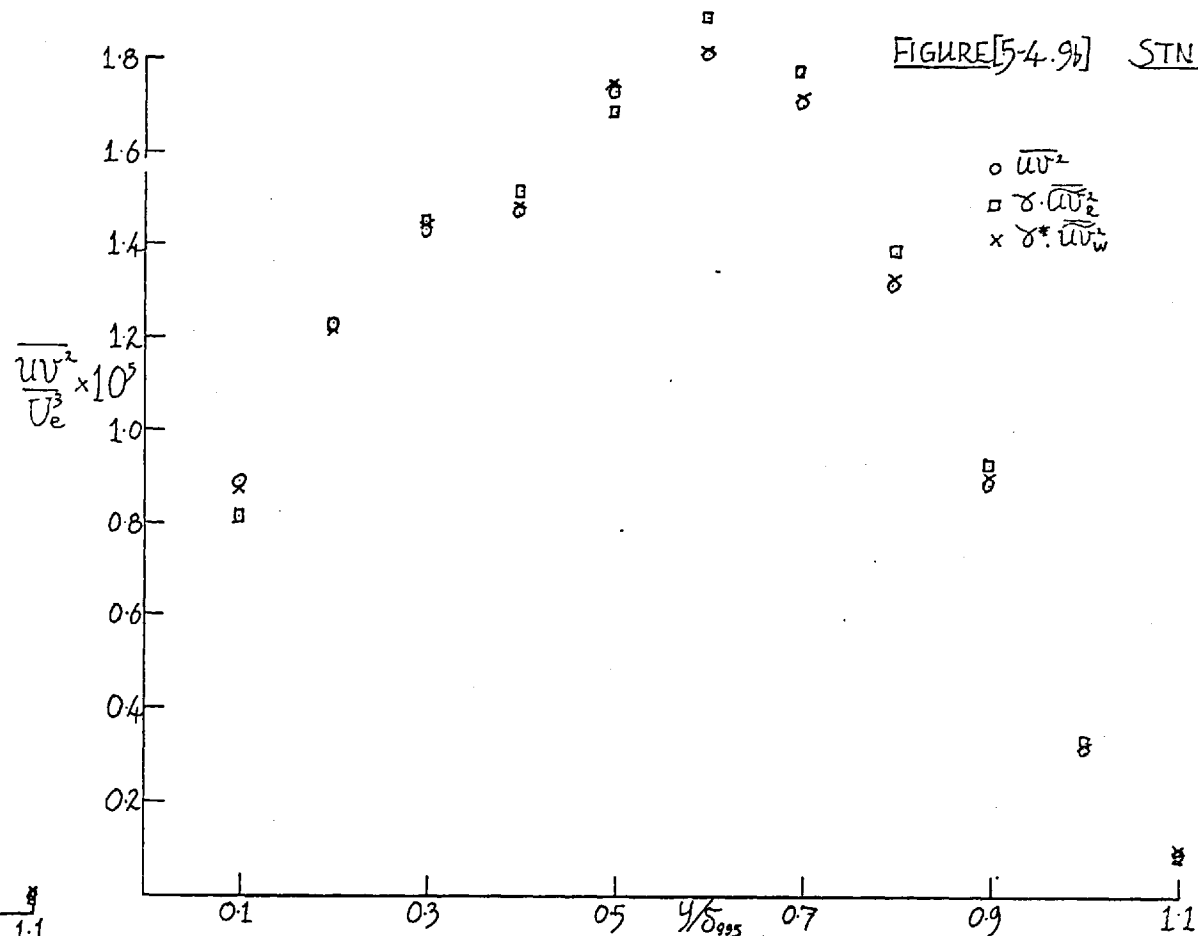


FIGURE [5-4.9b] STN 17



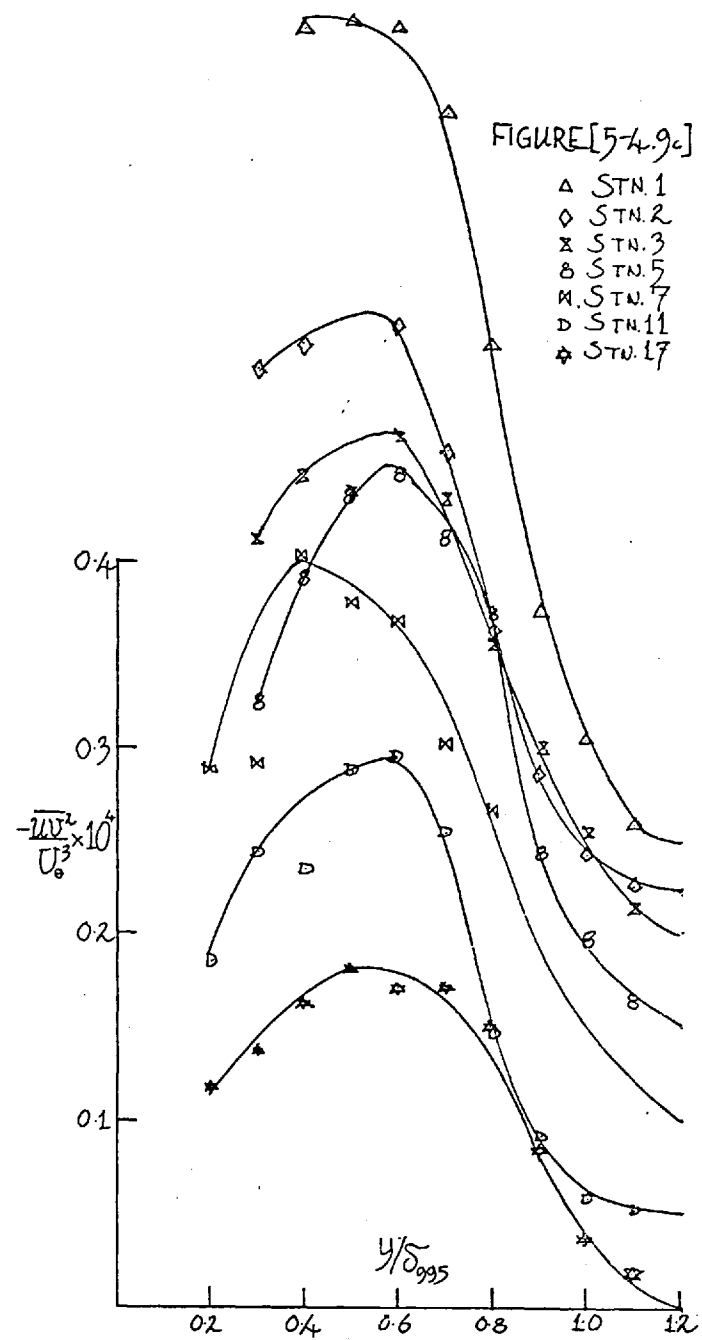
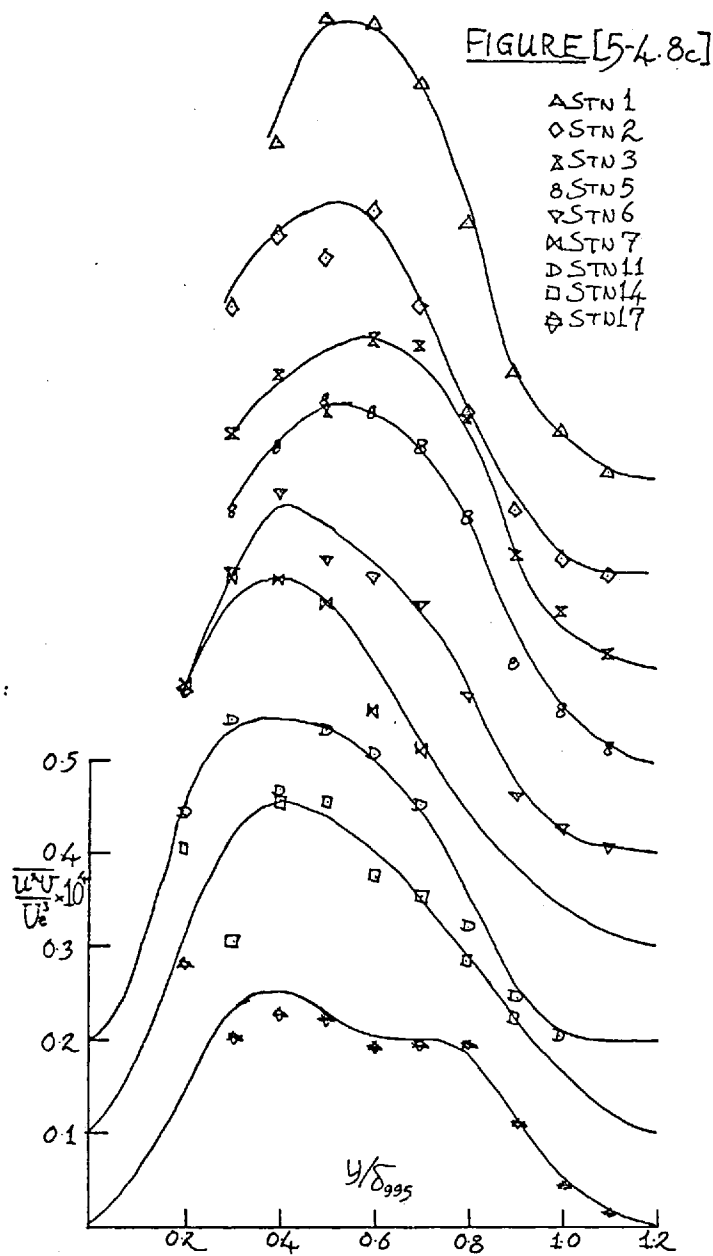


FIGURE [5-4.10a] PROFILES OF DIMENSIONLESS
TRIPLE PRODUCTS STN 1

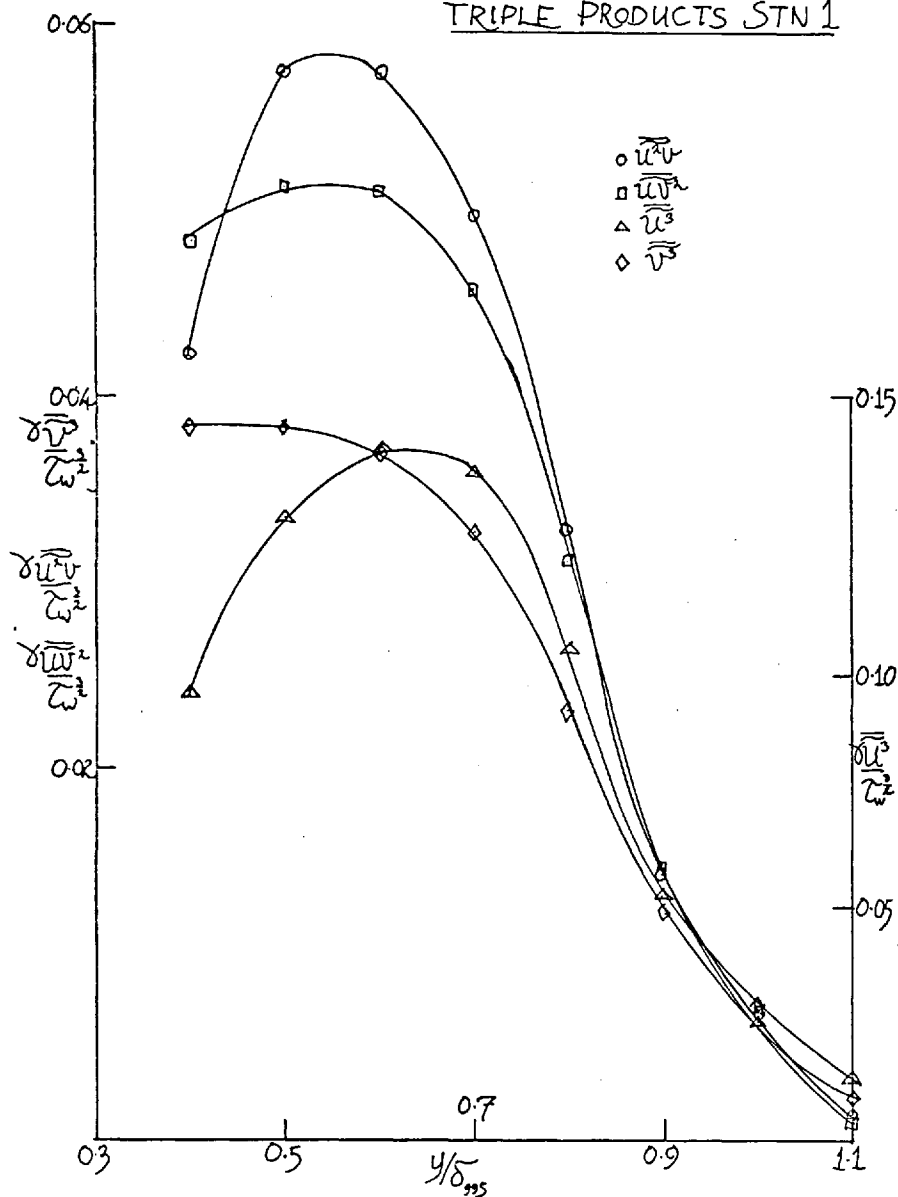
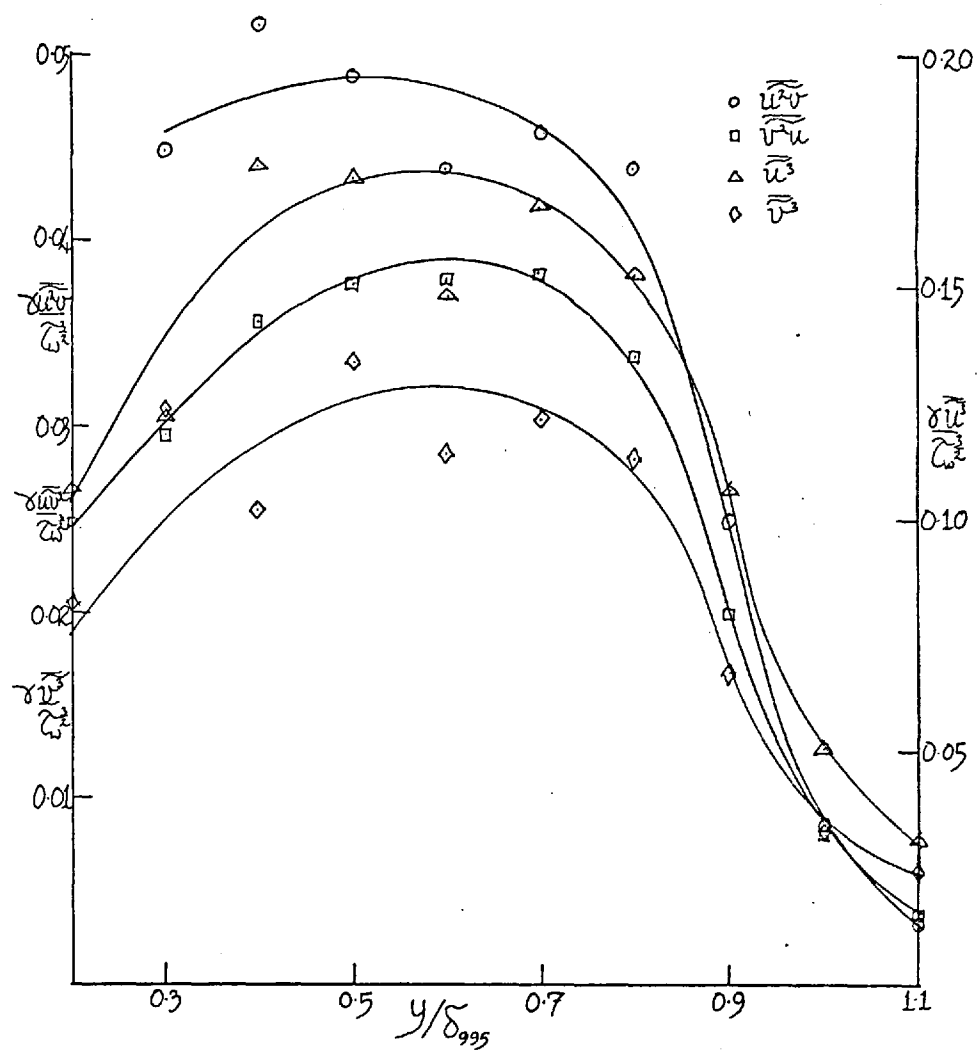
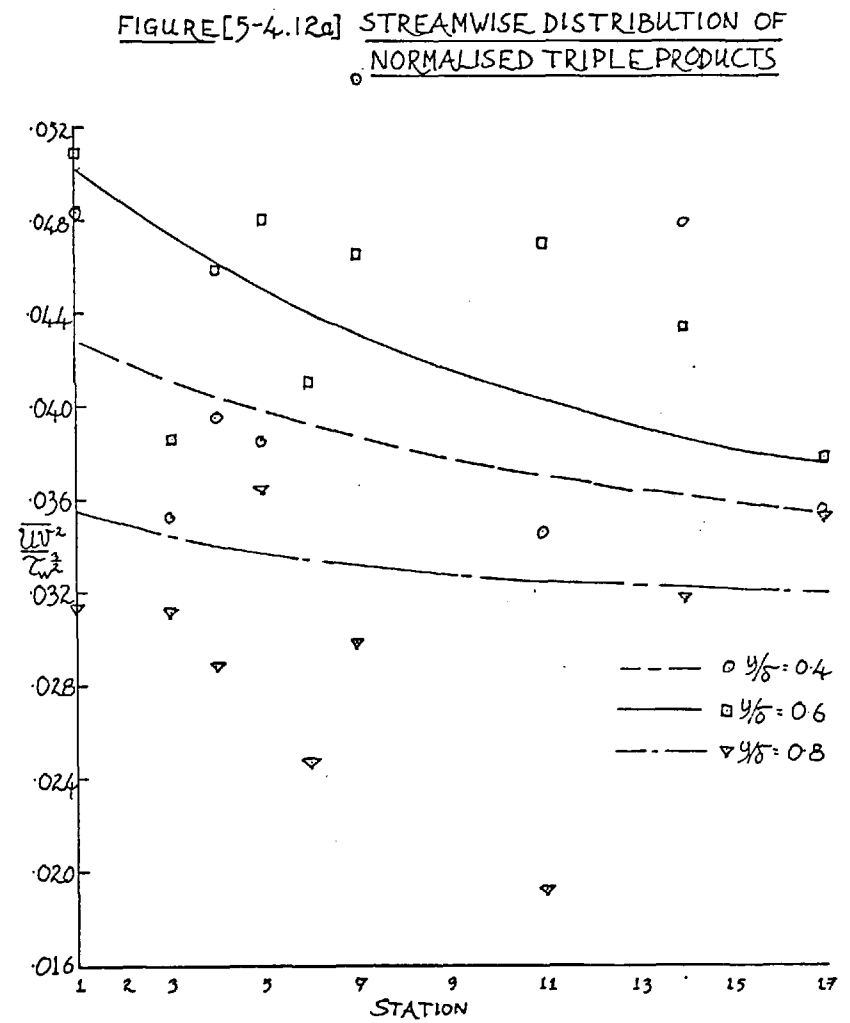
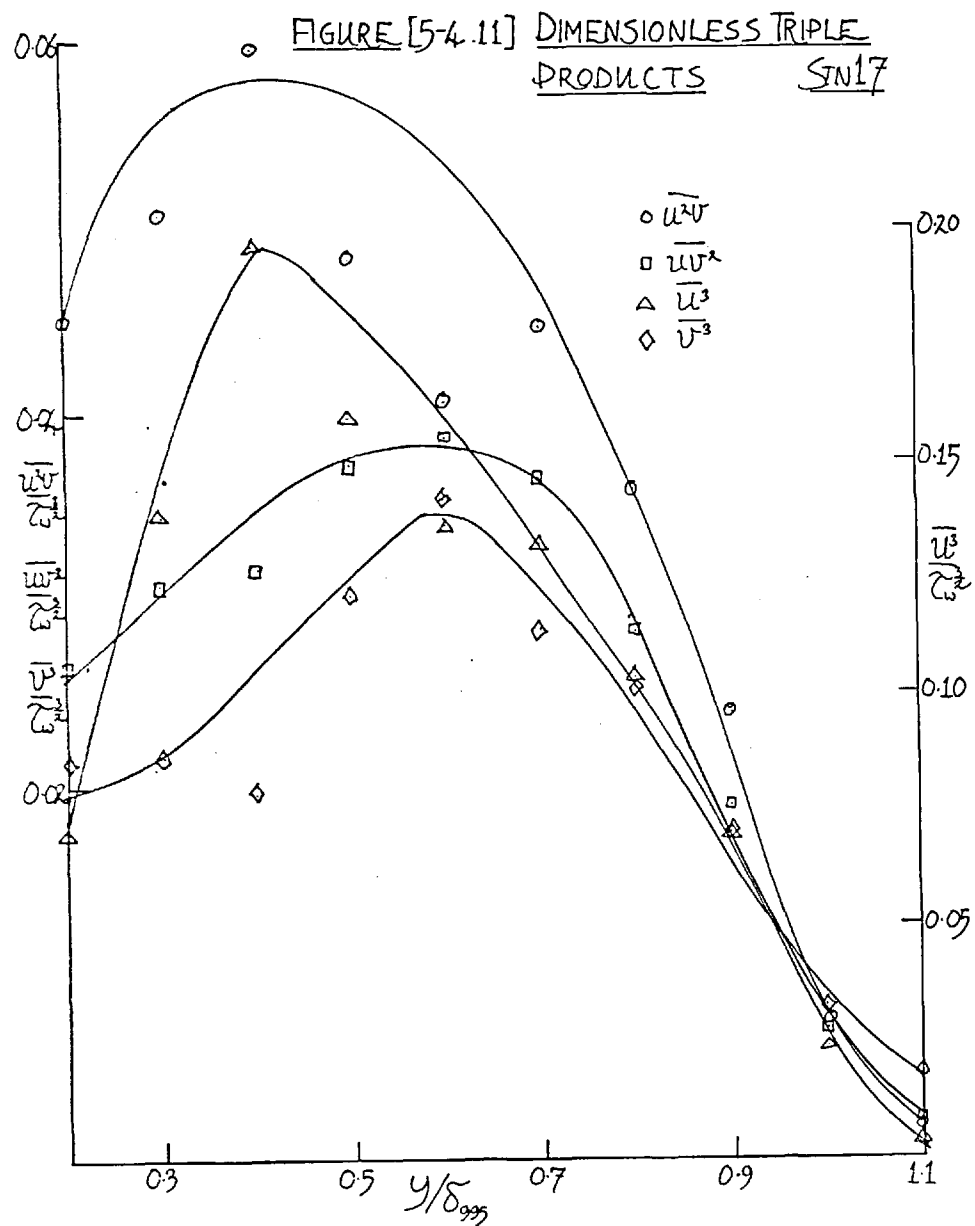
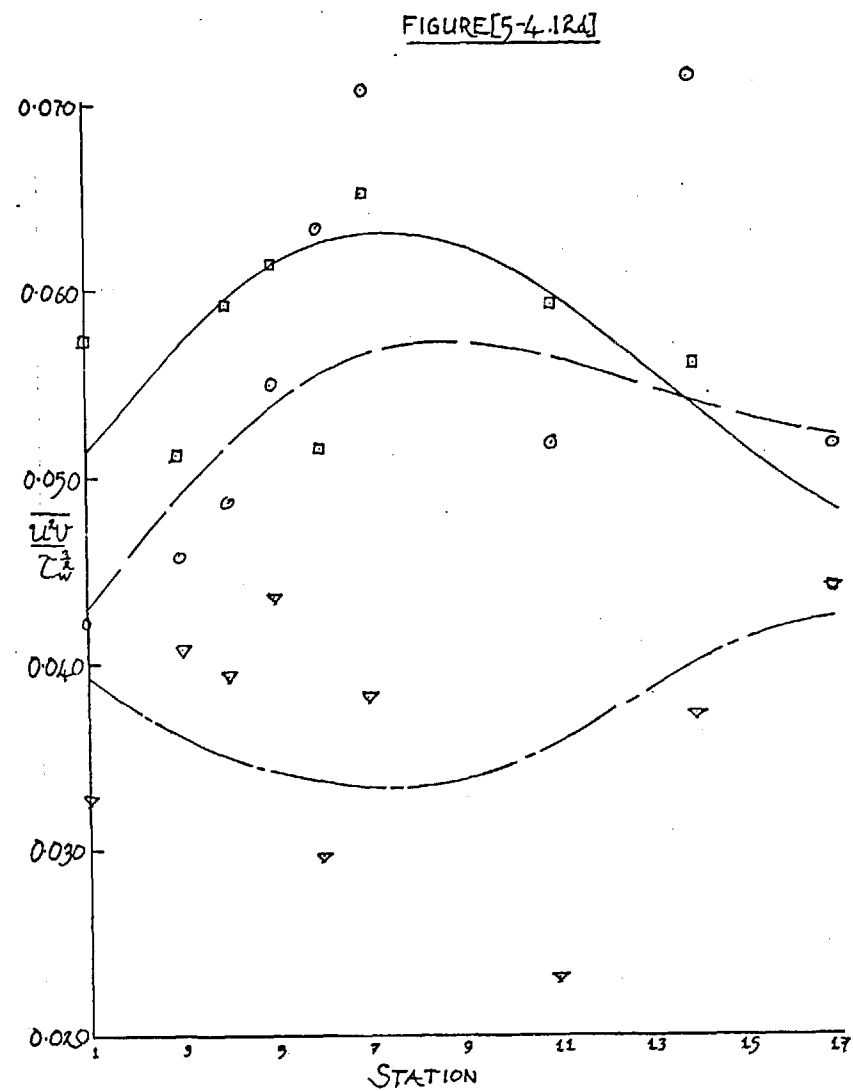
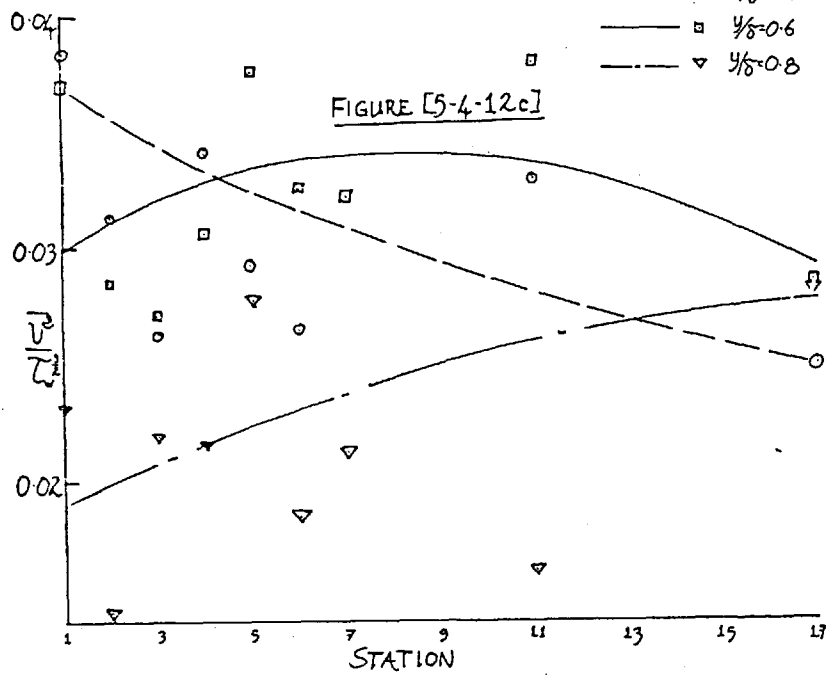
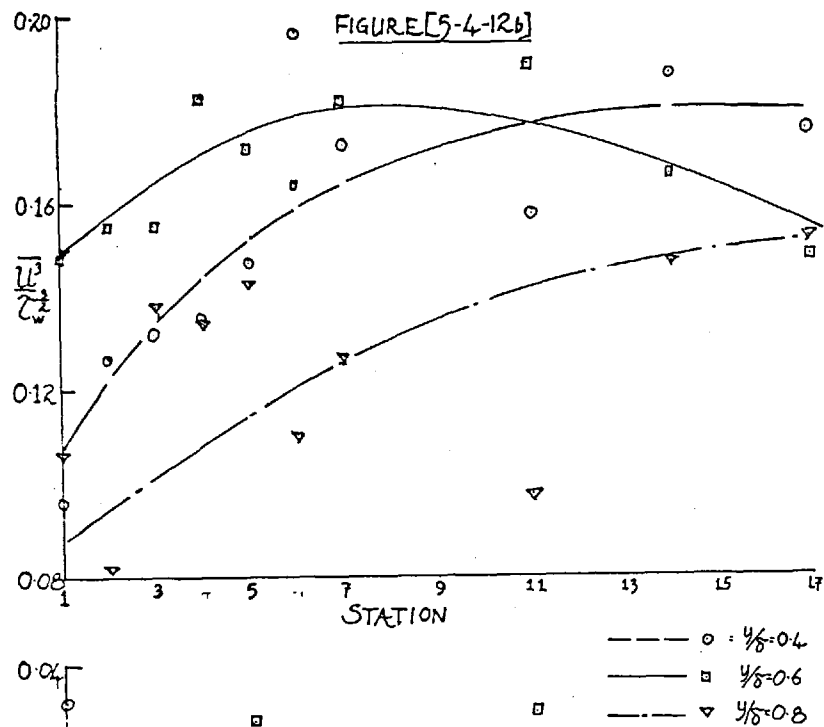


FIGURE [5-4.10b]

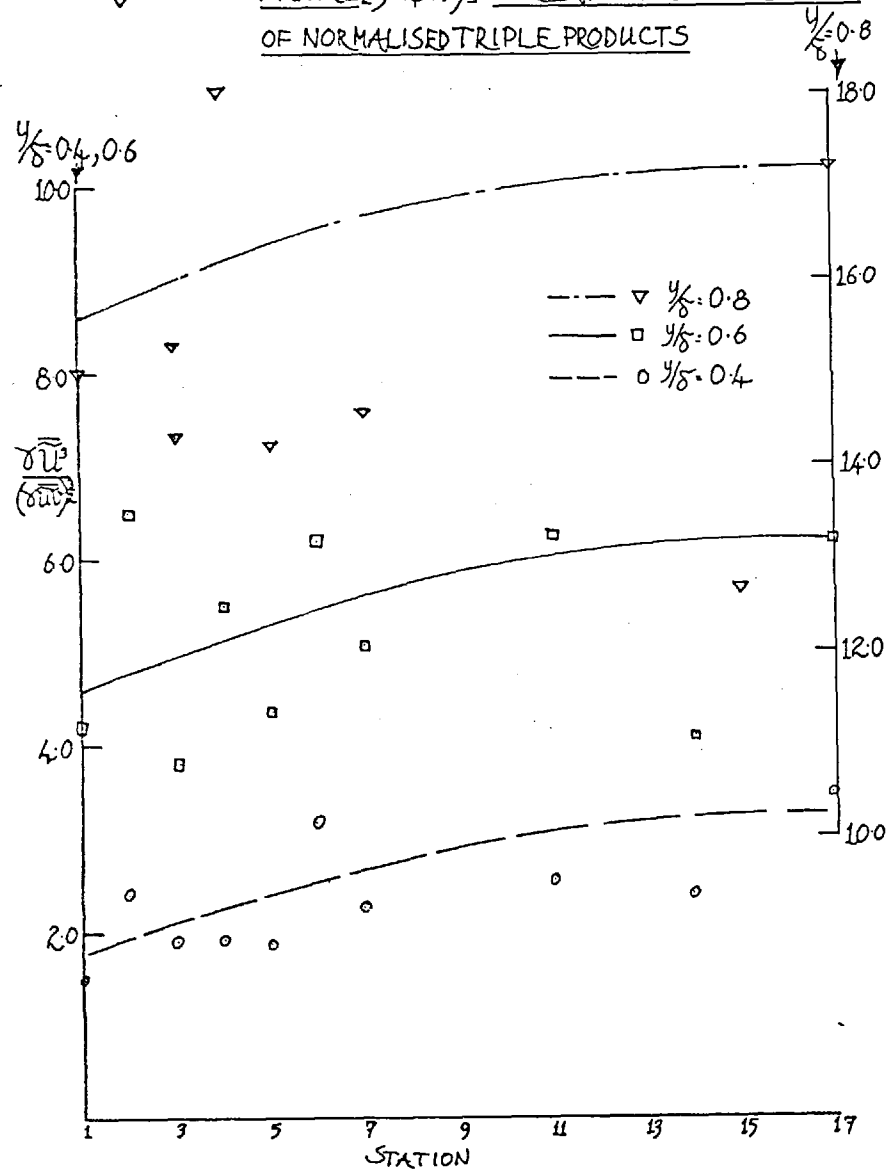
STN 17







FIGURE[5-4.13a] STREAMWISE DISTRIBUTION
OF NORMALISED TRIPLE PRODUCTS



FIGURE[5-4.13b]

