Performance Analysis of Simultaneous Location and Power Estimation Using WLS Method for Cognitive Radio

M. Kazemi, B. Mahboobi, M. Ardebilipour

***Abstract*— Precise estimation of spatial spectrum holes is of great importance in future cognitive radio (CR) networks. This information can help the secondary users to employ the spatial resources more efficiently and limit the interference to the primary users to the admissible level. In order to measure the performance of a location and power estimation algorithm, obtaining the statistical characteristics of estimation error is of great importance. In this paper, the performance of RSS-WLS approach which is an important linear position and power estimation algorithm in cognitive radio systems has been analyzed by obtaining the closed form expression for mean and covariance of its estimation error.**

***Index Terms*— cognitive radio; simultaneous location and power estimation; received signal strength (RSS); weighted least square (WLS)**

I. INTRODUCTION

In recent years, low efficient use of licensed bands plus an increasing demand of radio bandwidth has encouraged the researchers to devise a mechanism to allocate the radio bandwidth in a more efficient way. This effort has led to creation of the idea of CR networks. A CR secondary network tries to coexist with a primary network. The primary network is a network that owns a licensed radio band and the secondary network is a network which wants to use the same band without causing any harmful interference to the primary network.

Position and power information of primary network users can help secondary users to reduce interference to the primary network by efficient utilization of this information in beamforming and power control algorithms. To choose the best estimator when implementing the cognitive system, or to utilize the hybrid estimators, the error of each position and power estimation as a performance measure, should be analyzed to define the confidence intervals and decide the best estimator.

In this paper it is assumed that there is no cooperation or known signaling between secondary and primary users. As a result, algorithms which need cooperation between primary and secondary networks cannot be used. For example, because of the need of synchronization between primary and secondary networks, time-based algorithms like Time Of Arrival (TOA) algorithm cannot be used in the CR case [3]. Hence among all of the common power and position estimation algorithms only a few algorithms, like Received Signal Strength (RSS) and Angle Of Arrival (AOA), can be used practically in the CR networks because of the lack of primary users' information in the secondary network. To estimate the position of the primary user, AOA algorithm uses the angle of arrival of the primary user's signal at each of the secondary users' receivers while RSS algorithm measures the received power of the primary user's signal at the secondary users' receivers.

There have not been a lot of works in the field of simultaneous estimation of position and power especially in CR networks. In [1], a linear AOA algorithm based on Least Square (LS) method is introduced and its statistical characteristics are analyzed. In [2], a linear RSS algorithm based on Weighted Least Square (WLS) method is introduced and analyzed which is called RSS-WLS. In WLS as a generalization of LS method, the matrix of coefficients is multiplied by a weighting matrix to let the system of linear equations be solved using LS method. In [3, 6], a spatial holes estimation algorithm is proposed based on RSS-WLS algorithm which can estimate primary users' powers and positions. This algorithm rearranges RSS-WLS algorithm to add the capability of estimating the power of the primary user in a CR network setup. Although [3,6] propose an RSS-WLS based simultaneous location and power estimation algorithm for cognitive radio setup, but it does not obtain any exact or approximate performance analysis for the estimation error.

In this paper, the closed form mean and covariance of estimation error of RSS-WLS positioning algorithm in the CR network setup is obtained which can help a radio network designer to use more statistical information in his plan.

II. OUR WORK

At each secondary user, the received signal strength is formulated by the following propagation model

$$RSS = K\frac{p_T}{d^{\alpha}} + n \tag{1}$$

Where d is the distance between the primary user and the secondary user and p_T is the transmission power of primary

user which due to lack of cooperation between primary and secondary networks, is assumed unknown. in [1], α is the path loss exponent, n is the RSS measurement error which can be generated by channel noise or any other random perturbations to the measurement system and is assumed sufficiently small compared to the received signal power and K is all other factors that affect the received signal power including the antennas gain and height and the signal carrier frequency.

Let the real position of the primary user be $[x, y]$ and the coordinate of the ith secondary user be $[x_i , y_i]$, $i=1,2,..,M$, where M is the total number of the secondary receiver users. The distance between the primary user and the ith secondary user, d_i, can be calculated by the following equation:

$$d_i = \sqrt{(x_i - x)^2 + (y_i - y)^2}, \quad i = 1,2,...,M . \tag{2}$$

By Combining equations (1) and (2), and neglecting the received noise power, the following equation is obtained:

$$2xx_i + 2yy_i + \left(\frac{K_i}{RSS_i}\right)^{\frac{2}{\alpha}} \text{P-R}_2 = x_i^2 + y_i^2, \quad i = 1,2,..,M . \tag{3}$$

where $R_2 = x^2 + y^2$ and $P = (P_T)^{2/\alpha}$ are auxiliary variables to linearize our model. The matrix form of (3) can be written as:

$$A\theta = b \tag{4}$$

where,

$$\mathbf{A} = \begin{bmatrix} 2x_1 & 2y_1 & (K_1/RSS_1)^{\frac{2}{\alpha}} & -1 \\ 2x_2 & 2y_2 & (K_2/RSS_2)^{\frac{2}{\alpha}} & -1 \\ \vdots & \vdots & \vdots & \vdots \\ 2x_M & 2y_M & (K_M/RSS_M)^{\frac{2}{\alpha}} & -1 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} x_1^2 + y_1^2 \\ x_2^2 + y_2^2 \\ \vdots \\ x_M^2 + y_M^2 \end{bmatrix}$$

and $\boldsymbol{\theta} = [x, y, P, R_2]^T$.

Equation (4) is a system of linear equations in matrix form where A is the coefficients matrix, θ is the column vector of variables, and b is the column vector of solutions. Although path loss exponent is naturally an unknown variable, depending on the chosen system model, it is considered to be known based on a predefined table. For example, in log-distance system model for urban area cellular radio, its value is set to 3. As a result, path loss exponent is not taken as an unknown variable in this paper. In presence of the RSS measurement error, variable vector, θ, in (4) can be obtained using WLS method, which the details are discussed in [3,6]. In brief, in WLS method the weighted squared error due to inequality of both sides of (4) is minimized.

Since variable R_2 is dependent to $[x,y]$, the following auxiliary matrices are defined to separate the main variables from the dependent variable R_2 and also reformulate (4) to the following vector form.

$$\mathbf{S} = \mathbf{S^T} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \Rightarrow \text{R}_2 = \text{x}^2 + \text{y}^2 = \mathbf{x^T S x},$$

$$\mathbf{x} = [\text{x} \quad \text{y} \quad \text{P}]^{\text{T}}, \quad \mathbf{1_M} \triangleq [1 \quad \cdots \quad 1]^T_{1\times M},$$

$$\mathbf{X_B} = \begin{bmatrix} 2\text{x}_1 & 2y_1 & \left(\frac{K_1}{RSS_1}\right)^{\frac{2}{\alpha}} \\ \vdots & \vdots & \vdots \\ 2\text{x}_\text{M} & 2y_M & \left(\frac{K_M}{RSS_M}\right)^{\frac{2}{\alpha}} \end{bmatrix}_{M\times 3} . \tag{5}$$

Applying the above auxiliary variables definition to (4), the error of equations, $\boldsymbol{\delta}$ (i.e. difference of the sides of equations in the noisy channel), can be obtained versus auxiliary variables as

$$\boldsymbol{\delta} = \begin{bmatrix} \delta_1 \\ \vdots \\ \delta_M \end{bmatrix} \triangleq \mathbf{A\theta - b} = 2\text{x}\text{x}_\text{i} + 2\text{y}\text{y}_\text{i} + \left(\frac{\text{K}_\text{i}}{\text{RSS}_\text{i}}\right)^{\frac{2}{\alpha}} \text{P-R}_2 - \text{x}_\text{i}^2 - \text{y}_\text{i}^2 \tag{6}$$

$$= \mathbf{X_B x - 1_M x^T S x - b}$$

The square error function in the WLS method is as follows [4],

$$\text{J}(\boldsymbol{\theta}) \triangleq \mathbf{(A\theta - b)^T W (A\theta - b)} \tag{7}$$

where $\mathbf{W}$ is the weighting matrix. Denoting the covariance matrix of $\boldsymbol{\delta}$, by $\boldsymbol{\Psi}$, it is shown in [4] that the optimal weighting matrix which minimize $\text{J}(\boldsymbol{\theta})$ is

$$\mathbf{W = \Psi^{-1} = C_X^{-1} = } E\left[\boldsymbol{\delta}\boldsymbol{\delta}^{\mathbf{T}}\right]^{-1}$$

where covariance of equations errors, Ψ, is affected by the factors like error of path loss exponent estimation, shadowing factor and the input noise. By substituting the above formula for $\mathbf{W}$ and (6) into (7), the square error function is obtained as:

$$\text{J}(\mathbf{x}) = \mathbf{(X_B x - 1_M x^T S x - b)^T \Psi^{-1} (X_B x - 1_M x^T S x - b)} \tag{8}$$

It is shown in [5] that bias and covariance of a general optimization problem

$$\hat{\mathbf{x}} = \arg\min_{\mathbf{x}} \text{J}(\mathbf{x}) \tag{9}$$

can be approximated as

$$\text{bias}(\mathbf{x}) \cong -E\left[\frac{\partial^2 \text{J}(\mathbf{x})}{\partial \mathbf{x} \partial \mathbf{x}^{\mathbf{T}}}\right]^{-1} E\left[\frac{\partial \text{J}(\mathbf{x})}{\partial \mathbf{x}}\right] \tag{10}$$

$$\mathbf{C_x} = E\left[\frac{\partial^2 \text{J}(\mathbf{x})}{\partial \mathbf{x} \partial \mathbf{x}^{\mathbf{T}}}\right]^{-1} E\left[\left(\frac{\partial \text{J}(\mathbf{x})}{\partial \mathbf{x}}\right)\left(\frac{\partial \text{J}(\mathbf{x})}{\partial \mathbf{x}}\right)^T\right] E\left[\frac{\partial^2 \text{J}(\mathbf{x})}{\partial \mathbf{x} \partial \mathbf{x}^{\mathbf{T}}}\right]^{-1} \tag{11}$$

where $\text{J}(\mathbf{x})$ is a continuous function of $\mathbf{x}$.

The approximations in (10) and (11) are for sufficiently small noise power variances, means the variance of error of RSS measurements is small enough which is considered in this work. To derive the final relation for (10) and (11) the following relation needs to be computed for each of the expected value term used in (10) and (11).

Taking the derivative of (8) with respect to $\mathbf{x}$ leads to

$$\frac{\partial \text{J}(\mathbf{x})}{\partial \mathbf{x}} = \mathbf{2\,(X_B - 1_M x^T} \underbrace{\left(\mathbf{S^T + S}\right)}_{\mathbf{2S}}\mathbf{)^T \Psi^{-1} (X_B x - 1_M x^T S x - b)}$$

$$= \mathbf{2\,(X_B - 21_M x^T S)^T \Psi^{-1} (X_B x - 1_M x^T S x - b)} \tag{12}$$

Hence the expected value of (12) is as follows.

$$\mathrm{E}\left[\frac{\partial \mathrm{J}(\mathbf{x})}{\partial \mathbf{x}}\right]=2\,(\mathbf{X_B}-\mathbf{21_M}\,\mathbf{x^T S})^{\mathbf{T}}\,\boldsymbol{\Psi}^{-1}\mathrm{E}\left[\boldsymbol{\delta}\right]=0 \tag{13}$$

Now using (12), the second expected value in (11) is

$$\mathrm{E}\left[\left(\frac{\partial \mathrm{J}(\mathbf{x})}{\partial \mathbf{x}}\right)\left(\frac{\partial \mathrm{J}(\mathbf{x})}{\partial \mathbf{x}}\right)^{T}\right]=4(\mathbf{X_B}-2\mathbf{1_M}\,\mathbf{x^T S})^{\mathbf{T}}\,\boldsymbol{\Psi}^{-1}(\mathbf{X_B}-2\mathbf{1_M}\,\mathbf{x^T S}) \tag{14}$$

To compute the first matrix term in (11), it needs to be partitioned to 3 column vectors as follows.

$$E\left[\frac{\partial^2 \mathrm{J}(\mathbf{x})}{\partial \mathbf{x}\partial \mathbf{x}^{\mathbf{T}}}\right]=\left[\mathbf{g_1}\quad \mathbf{g_2}\quad \mathbf{g_3}\right] \tag{15}$$

Where

$$\mathbf{g_1}\triangleq E\left[\frac{\partial}{\partial x}\left(\frac{\partial J(\mathbf{x})}{\partial \mathbf{x}}\right)\right],\mathbf{g_2}\triangleq E\left[\frac{\partial}{\partial y}\left(\frac{\partial J(\mathbf{x})}{\partial \mathbf{x}}\right)\right],\mathbf{g_3}\triangleq E\left[\frac{\partial}{\partial P}\left(\frac{\partial J(\mathbf{x})}{\partial \mathbf{x}}\right)\right]$$

Using(12), $\mathbf{g_1}$ can be decomposed into two additive terms as follows

$$\mathbf{g_1}=\boldsymbol{\Xi}_1+\boldsymbol{\Xi}_2 \tag{16}$$

where,

$$\begin{aligned}\boldsymbol{\Xi}_1 &= E\left(2\,\frac{\partial}{\partial x}\left[(\mathbf{X_B}-\mathbf{21_M}\,\mathbf{x^T S})^{\mathbf{T}}\right]\boldsymbol{\Psi}^{-1}(\mathbf{X_B x}-\mathbf{1_M}\,\mathbf{x^T S x}-\mathbf{b})\right)\\ &=2\,\frac{\partial}{\partial x}\left[(\mathbf{X_B}-\mathbf{21_M}\,\mathbf{x^T S})^{\mathbf{T}}\right]\boldsymbol{\Psi}^{-1}E\left[(\mathbf{X_B x}-\mathbf{1_M}\,\mathbf{x^T S x}-\mathbf{b})\right] \qquad (17)\\ &=2\,\frac{\partial}{\partial x}\left[(\mathbf{X_B}-\mathbf{21_M}\,\mathbf{x^T S})^{\mathbf{T}}\right]\boldsymbol{\Psi}^{-1}E\left[\boldsymbol{\delta}\right]=0,\end{aligned}$$

$$\boldsymbol{\Xi}_2=E\left(2\,(\mathbf{X_B}-\mathbf{21_M}\,\mathbf{x^T S})^{\mathbf{T}}\,\boldsymbol{\Psi}^{-1}\,\frac{\partial}{\partial x}\left[(\mathbf{X_B x}-\mathbf{1_M}\,\mathbf{x^T S x}-\mathbf{b})\right]\right)$$

$$=2\,(\mathbf{X_B}-\mathbf{21_M}\,\mathbf{x^T S})^{\mathbf{T}}\,\boldsymbol{\Psi}^{-1}E\left[\frac{\partial}{\partial x}\left[(\mathbf{X_B x}-\mathbf{1_M}\,\underbrace{\mathbf{x^T S x}}_{x^2+y^2}-\mathbf{b})\right]\right] \tag{18}$$

$$=2\,(\mathbf{X_B}-\mathbf{21_M}\,\mathbf{x^T S})^{\mathbf{T}}\,\boldsymbol{\Psi}^{-1}\left(\mathbf{X_B}\left[1\quad 0\quad 0\right]^{\mathrm{T}}-\mathbf{21_M}\,x\right)$$

By substituting (17) and (18) into (16), $\mathbf{g_1}$ is as follows

$$\mathbf{g_1}=\mathbf{2}\,(\mathbf{X_B}-\mathbf{21_M}\,\mathbf{x^T S})^{\mathbf{T}}\,\boldsymbol{\Psi}^{-1}\left(\mathbf{X_B}\left[1\quad 0\quad 0\right]^{\mathrm{T}}-\mathbf{21_M}\,x\right) \tag{19}$$

In a similar way used to obtain the above expression for $\mathbf{g_1}$, the following relations for $\mathbf{g_2}$ and $\mathbf{g_3}$ can be obtained as

$$\mathbf{g_2}=2\,(\mathbf{X_B}-\mathbf{21_M}\,\mathbf{x^T S})^{\mathbf{T}}\,\boldsymbol{\Psi}^{-1}\left(\mathbf{X_B}\left[0\quad 1\quad 0\right]^{\mathrm{T}}-2\mathbf{1_M}\,y\right) \tag{20}$$

$$\mathbf{g_3}=2\,(\mathbf{X_B}-\mathbf{21_M}\,\mathbf{x^T S})^{\mathbf{T}}\,\boldsymbol{\Psi}^{-1}\left(\mathbf{X_B}\left[0\quad 0\quad 1\right]^{\mathrm{T}}\right) \tag{21}$$

By substituting (19), (20) and (21) into (15), the expected value of the second derivative of (8) with respect to *X* is obtained as:

$$\begin{aligned}E\left[\frac{\partial^2 \mathrm{J}(\mathbf{x})}{\partial \mathbf{x}\partial \mathbf{x}^{\mathbf{T}}}\right] &= 2\,(\mathbf{X_B}-\mathbf{21_M}\,\mathbf{x^T S})^{\mathbf{T}}\,\boldsymbol{\Psi}^{-1}\cdot\\ &\left(\mathbf{X_B}\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}-\left(2\mathbf{1_M}\,x+2\mathbf{1_M}\,y\right)\right) \qquad (22)\\ &=2\,(\mathbf{X_B}-\mathbf{21_M}\,\mathbf{x^T S})^{\mathbf{T}}\,\boldsymbol{\Psi}^{-1}\left(\mathbf{X_B}-\mathbf{21_M}\,\mathbf{x^T S}\right)\end{aligned}$$

Finally, by substituting (14) and (22) into (11), the 3 by 3 covariance matrix of estimation error is obtained as

$$\mathbf{C_X}=\left(\left(\mathbf{X_B}-\mathbf{21_M}\,\mathbf{x^T S}\right)^{\mathbf{T}}\,\boldsymbol{\Psi}^{-1}\left(\mathbf{X_B}-\mathbf{21_M}\,\mathbf{x^T S}\right)\right)^{-1} \tag{23}$$

where the first two rows and columns of C_X consist of covariance matrix of positioning error, while the last diagonal element is the covariance of power estimation error. By substituting (13) and (22) into (10), the bias of estimation error is obtained as:

$$\mathrm{bias}(\mathbf{x})=-E\left[\frac{\partial^2 \mathrm{J}(\mathbf{x})}{\partial \mathbf{x}\partial \mathbf{x}^{\mathbf{T}}}\right]^{-1}\left(2\,(\mathbf{X_B}-\mathbf{21_M}\,\mathbf{x^T S})^{\mathbf{T}}\,\boldsymbol{\Psi}^{-1}E\left[\boldsymbol{\delta}\right]\right)=0 \tag{24}$$

By the above, it can be concluded that RSS-WLS algorithm is an unbiased location and power estimation algorithm. It should be noted that since WLS is a generalization of LS method, the error statistics of RSS-LS method for the same scenario can be obtained by substituting the weighting matrix W with the identity matrix in (23) and (24).

III. Conclusion

RSS-WLS algorithm can be used to simultaneously estimate location and power of the primary user in cognitive radio system. Through this paper, the closed form expression for the bias and covariance of estimation error of the location and power estimation of the primary user by RSS-WLS algorithm has been computed where negligible error in measuring the received signal power is assumed as a practical approximation to RSS-based measurements to let our analysis cast into a closed form formulation.

References

[1] A. Pages-Zamora, J. Vidal, and D. R. Brooks, "Closed-form solution for positioning based on angle of arrival measurements," in Proceedings of the 13th IEEE International Symposium on Personal, Indoor and Mobile Radio Communications (PIMRC '02), vol.4, pp. 1522–1526, Lisbon, Portugal, September 2002.

[2] K. W. Cheung, H. C. So, W. K. Ma, and Y. T. Chan,"Received signal strength based mobile positioning via constrained weighted least squares," in Proceedings of the IEEE International Conference on Acoustic, Speech and Signal Processing (ICASSP '03), vol.5, pp. 137–140, Hong Kong, April 2003.

[3] S. Kim, H. Jeon, and J. Ma, "Robust localization with unknown transmission power for cognitive radio," in Proc. IEEE Int. Conf. on Military Communications, October 2007.

[4] A. C. Aitken, "On least squares and linear combinations of observations," Proceedings of the Royal Society of Edinburgh, vol. 55, pp. 42-48, 1935.

[5] K. W. Cheung and H. C. So, "A multi-dimensional scaling framework for mobile location using time-of-arrival measurements," IEEE Transactions on Signal Processing, vol.53, no.2, pp. 460–470, 2005.

[6] M. Kazemi, M. Ardebilipour, and N. Nouri, "Improved weighted RSS positioning algorithm for cognitive radio," International Conference on Signal Processing (ICSP '10), 2010.