

Structural steel design using second-order inelastic analysis with strain limits

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Abstract

Steel framed structures are affected, to greater or lesser extent, by (i) geometrical nonlinearity associated with the change in geometry of the structure under load and (ii) material nonlinearity related to the onset and spread of plasticity. In traditional design approaches, the design forces and moments within structural members are usually determined from simplified structural analyses (e.g. first or second-order elastic analysis), after which member design checks are performed to assess the strength and stability of the individual members. The extent to which the strength and deformation capacity of cross-sections is affected by local buckling is typically assessed through the concept of cross-section classification. For example, only compact (Class 1) cross-sections are considered to possess sufficient rotation capacity for plastic hinges to develop and for inelastic analysis methods to be used. This approach results in step-wise capacity predictions and is considered to be overly simplistic. Since the structural analysis of steel framed structures is typically performed using beam finite elements, which are unable to explicitly capture local buckling, a more sophisticated treatment of the available deformation capacity is required if inelastic analysis methods are to be used for all cross-section classes. A novel method of design by advanced inelastic analysis has recently been developed [1–4], in which strain limits are employed to represent the effects of local buckling in beam finite element models and thereby control the spread of plasticity and level of force/moment redistribution within a structure. It is thus possible to use a consistent advanced analysis framework to design structures composed of cross-sections of any class. In the present paper, application of the proposed design method to continuous beams and planar frames is illustrated and assessed. Ultimate load capacity predictions are compared against results obtained from benchmark shell finite element models that explicitly capture local buckling and to conventional steel design. It is found that the proposed method predicts safe-sided ultimate capacities that are consistently more accurate

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than current design methods, particularly for structures benefiting from strain hardening and for structures composed of non-compact cross-sections where inelastic redistribution is typically ignored.

1 Introduction

In the context of steel structures, the term 'advanced analysis' has typically been used to describe an analysis that includes the influence of geometric nonlinearity, initial imperfections and, in most instances, material nonlinearity [5, 6]. A designation system that abbreviates the features included in an analysis has been developed [7, 8], in which GMNIA signifies an advanced geometrically and materially nonlinear analysis with imperfections. The benefits of advanced analysis have been widely recognised [9–15] – for example, member stability is captured directly in the analysis, thus making subsequent member design checks and the determination of the associated effective lengths unnecessary. Furthermore, if plasticity is included, moment redistribution is allowed for in indeterminate structures and failure mechanisms are represented more accurately. However, modern steel design specification such as EN 1993-1-1 [16], AISC 360 [17] and AS 4100 [18], rely on the concept of cross-section classification to assess the extent to which local buckling affects the resistance and deformation capacity of cross-sections. As a result, the use of inelastic analysis methods is limited to structures composed of compact cross-sections that possess sufficient deformation capacity for plastic hinges to develop. Furthermore, when inelastic analysis is permitted, a simplified elastic-perfectly plastic material model is usually employed, thereby neglecting the beneficial effects of strain hardening, while the rotation capacity of plastic hinges is typically assumed to be unrestricted.

A novel method of structural steel design based on advanced (second-order) inelastic (plastic-zone) analysis (GMNIA) with strain limits has recently been developed to address some of the aforementioned shortcomings [1–4]. The method utilises a standardised quad-linear material model to represent the full range stress-strain behaviour of hot-rolled steel [19], with strain limits applied to simulate the effects of local buckling in beam finite elements. The strain limits are obtained from the continuous strength method (CSM) [20].

The accuracy of this design approach has been successfully demonstrated on isolated I-section and square/rectangular hollow section members in [2]. In the present paper, application of the proposed method of design is extended to continuous beams and planar frames. Comparisons are made between the capacity predictions obtained from the proposed method and those obtained from more sophisticated benchmark shell finite element models, which are able to explicitly capture local buckling; comparisons are also made against the conventional EN 1993-1-1 [16] design methods.

2 Traditional steel design

The design of steel frames has traditionally been a two-step process. First, a structural analysis is performed to determine the internal forces and moments within each member of the structure. Then, design checks are performed to verify the local capacity (cross-section checks) and stability (member buckling checks) of the individual members. At the frame level, imperfections are either directly modelled in the analysis as an initial out-of-plumbness or included through a system of equivalent horizontal forces. Global sway second-order effects are either ignored if they are small or accounted for through a second-order or amplified first-order analysis. The sensitivity of a structure to second-order effects is assessed using the critical load factor α_{cr} , which is defined as the ratio of the elastic critical buckling load F_{cr} leading to global instability of the whole frame divided by the applied design load F_{Ed} on the structure, as given by Eq. (1). According to EN 1993-1-1, a frame designed using elastic analysis methods is considered to be non-sway (i.e. a first-order analysis may be used) if α_{cr} is greater than 10.

$$\alpha_{cr} = \frac{F_{cr}}{F_{Ed}} \quad (1)$$

The effects of imperfections (i.e. initial out-of-straightness and residual stresses) and second-order effects at the member level are usually accounted for through member buckling checks. While this simplifies the analysis, the subsequent member checks require the determination of effective lengths. Though design specifications have relied on the effective length method for many years, the limitations of this approach have been highlighted in several studies [6, 10, 21, 22].

The material stress-strain properties of hot-rolled carbon steel lend themselves to plastic design [23]. The long yield plateau, high ultimate strains ϵ_u and moderate levels of strain hardening allow internal force and moment redistribution to take place, particularly in structures composed of compact cross-sections, and be captured through plastic design. Plastic design offers the possibility for improved economy, particularly in statically indeterminate, bending dominated structures such as continuous beams and portal frames. However, to ensure that plastic hinges are able to develop and possess sufficient rotation capacity, Eurocode 3 [16] restricts the use of plastic design to structures composed of compact Class 1 cross-sections.

The traditional approach to structural steel design has evolved over many decades and, as described above, now relies on a mixture of simplified structural analysis results and empirical member design checks to verify the resistance of a structure. In the remainder of this paper, an integrated system-level design approach that embraces modern computational capabilities is described to bring about safe, more efficient and more rapid structural design.

3 Design by advanced inelastic analysis using beam FE with strain limits

The proposed method of structural design [1,2] is based upon a beam element geometrically and materially nonlinear (plastic-zone) analysis including imperfections (GMNIA) with the stress-strain response of hot-rolled carbon steel represented by a quad-linear strain hardening material model (see Section 3.1). Member stability and the spread of plasticity are directly captured in the analysis. However, conventional beam finite elements cannot explicitly capture local buckling. Hence, in the proposed approach, strain limits, obtained from the continuous strength method (CSM) [20], are used to define the point of cross-section failure due to local buckling [2]. It is therefore possible to use a consistent advanced analysis framework for the design of steel frames composed of cross-sections of any class. Note that the method is currently limited to in-plane design; separate design checks are required in instances where out-of-plane instability can arise. In the following subsections, a summary of the proposed method of design by advanced inelastic analysis with strain limits is provided. Further details and application of the proposed method to individual structural members may be found in [1, 2].

3.1 The continuous strength method (CSM): strain limits, cross-section slenderness and material model

The continuous strength method (CSM) is a deformation based design approach that relates the cross-section deformation capacity to the local cross-section slenderness [20]. The CSM consists of two fundamental components: (1) an experimentally derived base curve that predicts the normalised peak compressive strain $\epsilon_{\text{CSM}}/\epsilon_y$ that a cross-section of given local slenderness $\bar{\lambda}_p$ can resist prior to failure and (2) a material stress-strain model that can incorporate strain hardening. Currently, the continuous strength method features in the AISC Design Guide 27 for Structural Stainless Steel [24] and has been accepted for inclusion in the next revision of EN1993-1-4 [25].

The CSM base curve is shown in Fig. 1(a) and is defined by Eqs. (2) and (3) for non-slender and slender cross-sections respectively, where $\epsilon_y = f_y/E$ is the yield strain. A maximum local slenderness of $\bar{\lambda}_p$ of 1.0 is currently recommended for use of the CSM strain limits in an advanced analysis framework. Two upper limits for the cross-section deformation capacity $\epsilon_{\text{CSM}}/\epsilon_y$ are defined in Eq. (2). The first limit on Ω defines the permissible level of plastic deformation and may be defined on a project-by-project basis. For example, if a suitably ductile steel is used and extensive plasticity is tolerable at the considered limit state, a high value of say $\Omega = 30$ may be specified. Conversely, a value of $\Omega = 1$ can be used to prevent material yielding, as is typically desirable for serviceability limit checks. The second strain limit of $C_1\epsilon_u/\epsilon_y$ confines the cross-section response to the first three segments of the quad-linear material

model given by Eq. (5) [26, 27]. This limit exists primarily to simplify the CSM resistance functions for hand calculations and may be omitted if the full stress-strain curve (i.e. all 4 stages of the quad-linear model) is used in the advanced inelastic analysis, as is the case in the present study.

$$\frac{\epsilon_{\text{csm}}}{\epsilon_y} = \frac{0.25}{\bar{\lambda}_p^{3.6}} \quad \text{but} \leq \left(\Omega, \frac{C_1 \epsilon_u}{\epsilon_y} \right) \quad \text{for} \quad \bar{\lambda}_p \leq 0.68 \quad (2)$$

$$\frac{\epsilon_{\text{csm}}}{\epsilon_y} = \left(1 - \frac{0.222}{\bar{\lambda}_p^{1.05}} \right) \frac{1}{\bar{\lambda}_p^{1.05}} \quad \text{for} \quad \bar{\lambda}_p > 0.68 \quad (3)$$

In the CSM design framework, the process of cross-section classification becomes unnecessary. Instead, the cross-section slenderness $\bar{\lambda}_p$ is used to determine the deformation capacity of the cross-section. The cross-section slenderness is a dimensionless parameter that quantifies susceptibility to local buckling and is defined by Eq. (4), where f_y is the yield stress and $\sigma_{\text{cr,cs}}$ is the elastic local buckling stress of the full cross-section including the beneficial effects of element interaction. As a convenient alternative to the use of numerical tools such as CUFSM [28], closed-form expressions have recently been developed to predict the full cross-section elastic local buckling stress $\sigma_{\text{cr,cs}}$ of standard steel profiles subjected to compression, bending and combined loading [29]. In the present study, these expressions are used to predict $\sigma_{\text{cr,cs}}$, which is used, in return, to determine the local cross-section slenderness $\bar{\lambda}_p$ from Eq. (4) and ultimately the CSM strain limit ϵ_{csm} from Eq. (2) or Eq. (3).

$$\bar{\lambda}_p = \sqrt{\frac{f_y}{\sigma_{\text{cr,cs}}}} \quad (4)$$

Yun and Gardner [19] collated coupon test data from around the world to develop a standardised material model for hot-rolled carbon steels. The quad-linear material model is shown in Fig. 1(b) and is defined over the full strain range by Eq. (5). Only three commonly available material parameters, namely the modulus of elasticity E , the yield stress f_y and the ultimate stress f_u , are required to define the full stress-strain curve including the length of yield plateau and the strain hardening response for a given grade of steel.

$$\epsilon = \begin{cases} E\epsilon & \text{for } \epsilon \leq \epsilon_y \\ f_y & \text{for } \epsilon_y < \epsilon \leq \epsilon_{\text{sh}} \\ f_y + E_{\text{sh}}(\epsilon - \epsilon_{\text{sh}}) & \text{for } \epsilon_{\text{sh}} < \epsilon \leq C_1 \epsilon_u \\ f_{C_1 \epsilon_u} + \frac{f_u - f_{C_1 \epsilon_u}}{\epsilon_u - C_1 \epsilon_u}(\epsilon - C_1 \epsilon_u) & \text{for } C_1 \epsilon_u < \epsilon \leq \epsilon_u \end{cases} \quad (5)$$

The strain at which strain hardening commences ϵ_{sh} , the ultimate strain ϵ_u , the strain hardening modulus E_{sh} and two experimentally derived constants C_1 and C_2 , can be predicted from the three input parameters (i.e. E , f_y and f_u) using Eqs. (6) to (10) respectively. Note that $f_{C_1 \epsilon_u}$ corresponds to the

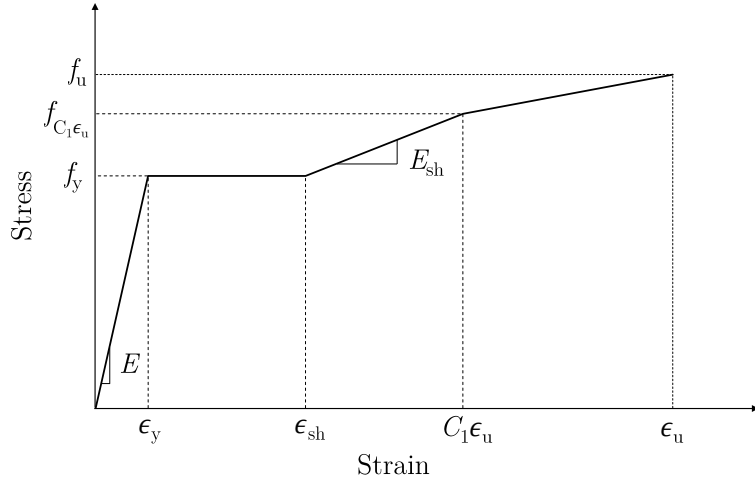
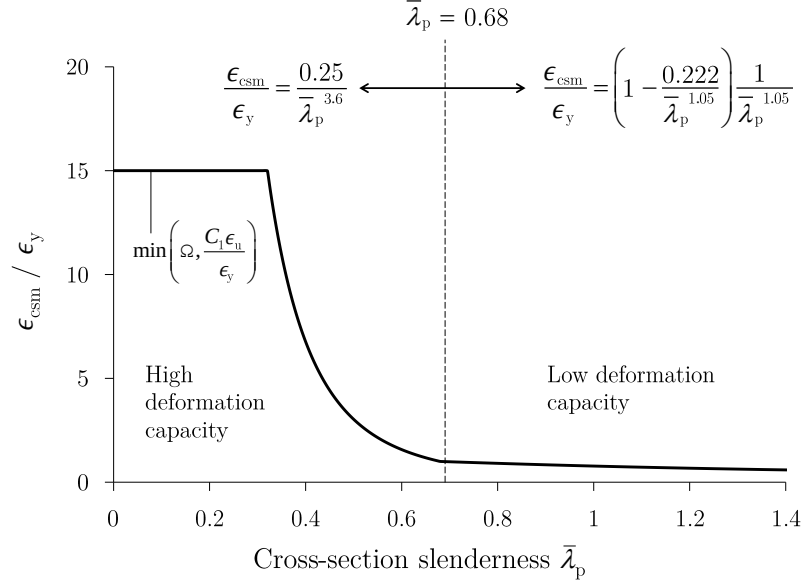


Figure 1: Two key components of the continuous strength method (CSM): (a) the base curve relating the normalised cross-section deformation capacity $\epsilon_{\text{csm}}/\epsilon_y$ to the local slenderness $\bar{\lambda}_p$ and (b) a material model. For hot-rolled steels, the quad-linear model developed by Yun and Gardner [19] is used.

stress at which the third and fourth segments of the material model intersect (i.e. when the strain equals $C_1\epsilon_u$), as shown in Fig. 1(b).

$$\epsilon_{sh} = 0.1 \frac{f_y}{f_u} - 0.055 \quad \text{but} \quad 0.015 \leq \epsilon_{sh} \leq 0.03 \quad (6)$$

$$\epsilon_u = 0.6 \left(1 - \frac{f_y}{f_u} \right) \quad \text{but} \quad \epsilon_u \geq 0.06 \quad (7)$$

$$E_{sh} = \frac{f_u - f_y}{C_2\epsilon_u - \epsilon_{sh}} \quad (8)$$

$$C_1 = \frac{\epsilon_{sh} + 0.25(\epsilon_u - \epsilon_{sh})}{\epsilon_u} \quad (9)$$

$$C_2 = \frac{\epsilon_{sh} + 0.4(\epsilon_u - \epsilon_{sh})}{\epsilon_u} \quad (10)$$

3.2 Cross-section resistance and application of strain limits

The deformation based design philosophy of the continuous strength method (CSM) lends itself to an advanced inelastic analysis framework, since numerical integration is inherently performed at each increment of the analysis. In the CSM framework, the cross-section capacity is defined by the peak compressive strain ϵ_{csm} that a cross-section can resist. Assuming a linear strain distribution through the depth of the cross-section, the corresponding stress distribution is directly obtained from the material model. The compression and bending resistances of a cross-section can then be determined by integrating the stress distribution over the area of the cross-section. Consequently, in an advanced inelastic analysis performed using beam finite elements, a cross-section can withstand the required loads if the design strain ϵ_{Ed} is less than the CSM strain limit ϵ_{csm} , as given by Eq. (11). It should be emphasised that, as explained in [2], the strain limit may be applied either at the extreme fibre or at the centreline of the wall thickness, provided the compatible compressive design strain is used, with negligible influence on the overall capacity prediction.

$$\frac{\epsilon_{Ed}}{\epsilon_{csm}} \leq 1.0 \quad (11)$$

In practice, local buckling and plastic hinges require a finite length over which to develop. If there exists a moment gradient over this finite length, which is typically the case in structural members, the less heavily stressed material provides beneficial restraint to the critical cross-section. To account for this beneficial influence of local moment gradients on the strength and stability of cross-sections both in the

elastic and inelastic range, a strain averaging approach has been proposed [2]. Instead of applying the CSM strain limit ϵ_{csm} to the peak compressive strain along the length of a member, as given by Eq. (11), the maximum compressive strain contributions ϵ_i of all n elements wholly located within the critical local buckling half-wavelength $L_{\text{b,cs}}$ are considered. The local buckling half-wavelength of the full cross-section $L_{\text{b,cs}}$ can be determined from explicit expressions presented in [30] or from numerical methods (e.g. the finite strip method).

$$\frac{\epsilon_{\text{Ed,Lb}}}{\epsilon_{\text{csm}}} \leq 1.0$$

$$\text{where } \epsilon_{\text{Ed,Lb}} = \frac{1}{n} \sum_{i=1}^n \epsilon_i \quad \text{and} \quad n \geq 1 \quad (12)$$

For the case of equal length beam elements, the strain averaging approach is expressed by Eq. (12), where $\epsilon_{\text{Ed,Lb}}$ is the averaged design strain. The location of the critical local buckling half-wavelength $L_{\text{b,cs}}$ over which to perform the strain averaging typically corresponds to the location of the maximum moment within a member. The effect of vertical web stiffeners that prevent local cross-section deformations at their location can be accounted for by averaging the strains to one side of the stiffener or the other, typically the side with the shallower moment gradient [2].

Use of the CSM strain limits enables accurate predictions of the compression, bending and combined compression plus bending resistances of structural cross-sections and members [2,26,31]. However, currently the CSM does not provide guidance on the shear capacity of cross-sections and hence the conventional shear check from EN 1993-1-1 [16] is still required in the proposed design method, as given by Eq. (13) where V_{Ed} is the design shear force and $V_{\text{pl,Rd}}$ is the plastic design shear resistance of the cross-section. For cross-sections that are slender in shear, shear buckling must also be checked separately.

$$\frac{V_{\text{Ed}}}{V_{\text{pl,Rd}}} \leq 1.0 \quad (13)$$

In the application of the CSM strain limits, the interaction between bending moments and shear is accounted for through a shear reduction factor ρ_{csm} [2]. The CSM shear reduction factor ρ_{csm} is given by Eq. (14), where ρ is the familiar Eurocode 3 [16] shear reduction factor given by Eq. (15).

$$\rho_{\text{csm}} = \begin{cases} 1 & \text{for } V_{\text{Ed}} \leq 0.5V_{\text{pl,Rd}} \\ \frac{0.5}{0.5 + \rho} & \text{for } V_{\text{Ed}} > 0.5V_{\text{pl,Rd}} \end{cases} \quad (14)$$

$$\rho = \left(\frac{2V_{\text{Ed}}}{V_{\text{pl,Rd}}} - 1 \right)^2 \quad (15)$$

It follows that in the case of high shear forces (i.e. $V_{\text{Ed}} > 0.5V_{\text{pl,Rd}}$), the strain averaging approach given by Eq. (12) is modified to the shear-reduced strain limit check given by Eq. (16).

$$\frac{\epsilon_{\text{Ed,Lb}}}{\rho_{\text{csm}}\epsilon_{\text{csm}}} \leq 1.0 \quad \text{for} \quad V_{\text{Ed}} > 0.5V_{\text{pl,Rd}} \quad (16)$$

3.3 Beam FE modelling considerations: mesh size and imperfections

The finite element method is inherently approximate. The results of a finite element analysis depend on the adopted mesh density and, in the case of stability governed problems, also the assumed imperfection magnitude. For consistent and accurate predictions of ultimate capacity, a standardised approach for treating these two parameters is needed.

The strain averaging approach described in Section 3.2 provides explicit guidance for the required mesh density of the beam FE model used in the proposed design method. The maximum permissible element size to be used for the mesh is directly defined by the local buckling half-wavelength $L_{\text{b,cs}}$ over which the strain averaging is performed [2]. For computational efficiency, it is recommended that the element size is set equal to the local buckling half-wavelength $L_{\text{b,cs}}$. The adoption of a finer mesh results in no significant improvements in accuracy [2], since the results of more elements are included in the strain averaging approach, but the ultimate resistance predictions remain almost constant. Conversely, a coarser mesh can result in capacity predictions on the unsafe side, particularly in bending dominated cases. In the present study, a mesh with equal length elements is used, though the strain averaging approach is equally applicable to other mesh configurations (e.g. a more refined mesh in the anticipated regions of plasticity), provided that the strain limits are applied to the weighted average design strain.

The influence of the finite element mesh size on the ultimate capacity predictions obtained from the proposed method of design by advanced inelastic analysis and from EN 1993-1-1 are investigated with reference to a single bay portal frame made from IPE 400 profiles in grade S355 steel (see Table 5 for further details, where $H/2V = 0.14$), as shown in Fig. 2. It can be seen that when the element length L_{elem} is less than the local buckling half-wavelength ($L_{\text{b,cs}} = 313$ mm for the studied case), the ultimate load factor predictions obtained using the proposed design method α_{ult} remain relatively constant and closely predict the ultimate load factor of the benchmark shell FE model α_{shell} . The most accurate capacity predictions are obtained when an integer number of elements fit into the local buckling half-wavelength. In contrast, the extent to which the Eurocode 3 ultimate capacity predictions (unintentionally) account for local moment gradients depends on the size of element used in the analysis, since the employed finite element mesh has a direct influence on the magnitude of the design forces and moments of each member. Without the strain averaging approach, the proposed method of design by advanced inelastic analysis with strain limits is subject to the same fluctuations in capacity predictions as traditional design since the design strain depends on the element length, though the benefits of the Continuous Strength Method at the cross-sectional level remain. Similar observations have been made at the member level [2].

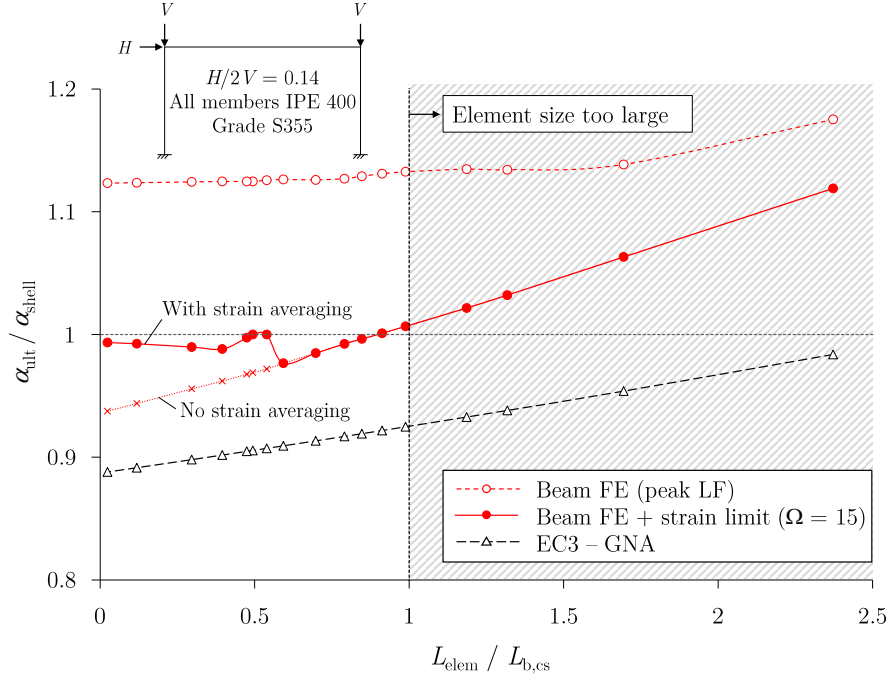


Figure 2: Effects of finite element length L_{elem} on normalised capacity predictions $\alpha_{\text{ult}}/\alpha_{\text{shell}}$ for a single bay portal frame made from IPE 400 profiles in grade S355 steel (see Table 5 for further details, where $H/2V = 0.14$).

In the proposed method of design by advanced inelastic analysis, the influence of initial member out-of-straightness and residual stresses is modelled through a single equivalent geometric member imperfection. The magnitude of the member imperfection is given by Eq. (17), where l is the member length, α is the imperfection factor for the associated Eurocode 3 column buckling curve, $\epsilon = \sqrt{235/f_y}$ accounts for the material strength and β is the elastic reference bow imperfection equal to $\beta = 1/110$ for major axis buckling and $\beta = 1/200$ for minor axis buckling [32]. As explained in [2], in GMNIA the elastic reference bow imperfection magnitudes are appropriate for cross-sections of all classes, because the spread of plasticity and associated loss in stiffness is explicitly captured during the analysis. Using the larger plastic reference bow imperfections would mean that this effect is double-counted and result in overly conservative capacity predictions.

$$\frac{e_0}{l} = \frac{\alpha}{\epsilon} \beta \quad (17)$$

It should be emphasised that the effects of residual stresses and local geometric imperfections at the cross-sectional level are accounted for by the CSM strain limits. Furthermore, for frames, the recommended out-of-plumbness imperfection specified in EN 1993-1-1 [16] is used. This imperfection may be modelled explicitly, as done in the present study, or introduced through equivalent horizontal forces (EHF).

3.4 Comparison with traditional design approaches

An overview of the treatment of the key structural phenomena that influence the ultimate capacity of steel framed structures in the proposed method of design by advanced inelastic analysis with strain limits and in EN 1993-1-1 [16] is provided in Table 1. It may be seen that the proposed method of design by advanced inelastic analysis offers a more consistent and comprehensive approach to structural steel design compared to EN 1993-1-1, thereby making the design process simpler, more efficient and less prone to errors.

Table 1: Overview of treatment of key structural phenomena in EN 1993-1-1 [16] design approaches, as well as in the proposed method of design by advanced analysis using beam finite elements (FE) with strain limits.

	Design method						
	EN 1993-1-1 [16]						Advanced analysis using beam FE with strain limits
	Elastic design			Plastic design			
Global analysis*	LA	GNA	GNIA	MNA	GMNA	GMNIA	GMNIA
Frame imperfections	Imperfect geometry or EHF			Imperfect geometry or EHF			Imp. geometry or EHF
Member imperfections	Member buckling checks		Equivalent imperfections	Member buckling checks		Equivalent imperfections	Equivalent imperfections
Local imperfections and local buckling	Cross-section classification; effective width method for Class 4			Cross-section classification; plastic analysis generally limited to Class 1			Strain limits
$P - \Delta$ effects (frame buckling)	Ignored if $\alpha_{cr} \geq 10$; Otherwise approx. methods	Global analysis		Ignored if $\alpha_{cr} \geq 15$; Otherwise approx. methods	Global analysis		Global analysis
$P - \delta$ effects (member buckling)	Member buckling checks		Global analysis + cross-section checks	Member buckling checks		Global analysis	Global analysis
Moment gradient effect on member stability	Member buckling checks with C_m factors		Global analysis	Member buckling checks with C_m factors		Global analysis	Global analysis
Moment gradient effect on local stability	Ignored			Ignored			Strain averaging approach
Force/moment redistribution due to member buckling	Ignored	Captured in global analysis**		Ignored	Captured in global analysis**		Captured in global analysis
Force/moment redistribution due to material yielding	Ignored			Captured in global analysis			Captured in global analysis and controlled by strain limits
Spread of plasticity	Ignored in analysis, approximated in design checks for Class 1 and 2 cross-sections			Captured in global analysis using elastic, perfectly plastic material model			Captured in global analysis and controlled by strain limits
Strain hardening	Ignored			Ignored			Quad-linear material model

* Different analysis types: (i) linear elastic analysis (LA), (ii) geometrically nonlinear analysis (GNA), (iii) geometrically nonlinear analysis with imperfections (GNIA), (iv) first-order materially nonlinear analysis (MNA), (v) geometrically and materially nonlinear analysis (GMNA), (vi) geometrically and materially nonlinear analysis with imperfections (GMNIA)

** Except for perfect pin-ended members subjected to pure axial compression (e.g. leaning columns).

4 Finite element modelling

The ultimate load predictions obtained from the proposed method of design by advanced inelastic analysis using beam finite elements and strain limits are compared against the results from calibrated benchmark shell finite element models, which are assumed to represent the true behaviour of the structure, as well as current EN 1993-1-1 design predictions. The adopted finite element (FE) modelling approaches used to assess the proposed design method are presented in this section. First, the benchmark shell FE models are described. The shell FE models presented herein are an extension to the shell FE models developed for individual members in [2]; the same modelling approach has also been implemented successfully in previous studies [26, 33, 34]. In the present paper, the shell FE models are further validated against experiments on continuous beams [35] and full-scale frames [36–39] from the literature. Afterwards, the beam FE modelling approach used in the proposed method of design by advanced inelastic analysis is outlined. Simplified versions of the beam FE models are used to determine the ultimate capacity through conventional steel design methods according to EN 1993-1-1 [16].

4.1 Description of benchmark shell FE modelling

Benchmark shell finite element (FE) models of continuous beams made from I-sections and square/rectangular hollow sections (SHS/RHS) and frames made from I-sections were developed using the finite element package Abaqus [40]. The steel cross-sections were modelled using four-noded shell elements with reduced integration, denoted as S4R in Abaqus [40]. A mesh with elements characterised by an aspect ratio close to unity was adopted, where each steel plate forming the member cross-section was discretised by 12 elements along its width. The full load-deformation responses of the structural systems were captured through geometrically and materially nonlinear analyses with imperfections (GMNIA), which were solved using the modified Riks arc length algorithm [41].

The quad-linear material model developed by Yun and Gardner [19] was used to represent the full range stress-strain behaviour of hot-rolled steel. Unless stated otherwise, grade S355 steel is used, with a Young’s modulus of $E = 210000$ MPa, a yield stress of $f_y = 355$ MPa and an ultimate stress of $f_u = 510$ MPa taken as input parameters for the quad-linear material model described in Section 3.1. The engineering material stress-strain response was converted into true stresses and strains to account for the change in cross-sectional area captured by the shell FE models during the analysis. A Poisson’s ratio of $\nu = 0.3$ was assumed in the elastic range, while the Abaqus default value of $\nu = 0.5$ was used in the plastic range. For the I- and H-shaped cross-sections, residual stresses were modelled explicitly as an initial condition prior to loading, assuming the ECCS [42] distribution for hot-rolled sections as shown in Fig. 3. Following the experimental observations from Wang et al. [43], residual stresses were not included in the hot-rolled SHS/RHS models.

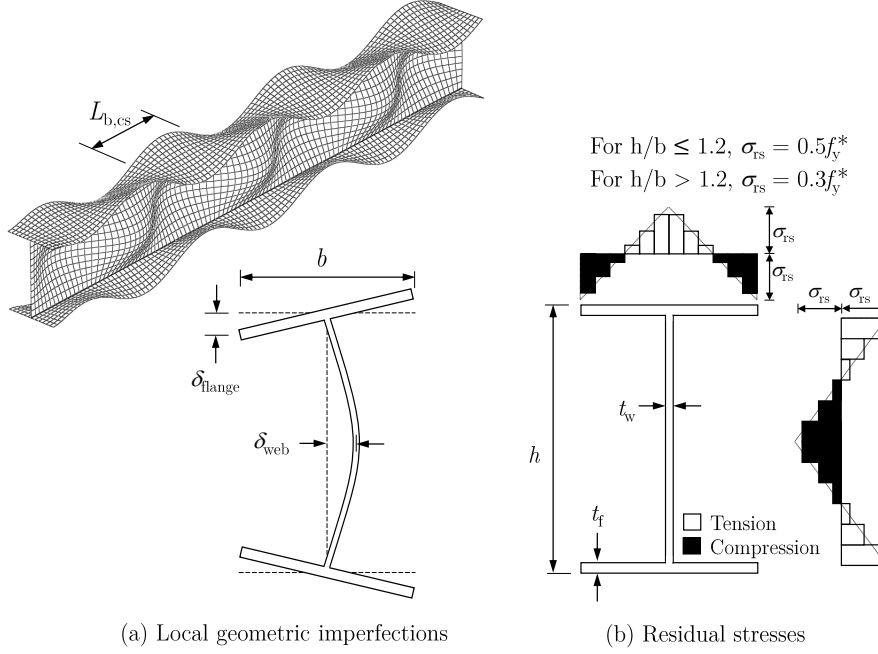


Figure 3: Geometry, local geometric imperfections and residual stress pattern adopted in shell FE models of I-sections. Note that $f_y^* = 235 \text{ N/mm}^2$, regardless of the strength of the modelled material, in line with [42].

Pin and roller support conditions were achieved by coupling all nodes of the member end cross-section to a master node. In the continuous beam models, the master node was specified at the centre of the bottom flange, while for the frames, the master node was located at the centroid of the cross-section. Fixed support conditions were achieved by restraining all degrees of freedom of the nodes at the relevant end cross-section. Concentrated loads were applied uniformly over the depth of the web for the I-sections and over the depth of both webs for the SHS/RHS. To prevent web crushing, web stiffeners were included in the numerical models at the locations of point loads. In the case of the continuous beams, web stiffeners were also modelled at the supports.

Relevant local, member and frame imperfections were included in the benchmark shell FE models to trigger the corresponding buckling phenomena. Sinusoidal local cross-section imperfections δ_{flange} and δ_{web} were included, as shown in Figs. 3 and 4 for I-sections and SHS/RHS respectively. The recommended imperfection magnitude of $c/200$, where c is the respective height or width of the cross-section (centreline dimensions), from Annex C of EN 1993-1-5 [44] was assumed. Note that the imperfection magnitude was defined such that the 90° angle at the web-flange junctions was retained (i.e. the plate imperfection magnitude of either the web or the flanges was scaled such that both δ_{flange} and δ_{web} were less than or equal to $c/200$). The local buckling half-wavelength $L_{b,cs}$ for the local imperfections was calculated using the expressions provided in [30], with the nearest integer number of half-wavelengths fitted into the individual member lengths. The sinusoidal member imperfections had a magnitude of $l/1000$, where l is the nominal member length, and the frame out-of-plumbness imperfection magnitude ψ_0 was set equal

to the recommended value from EN 1993-1-1 [16].

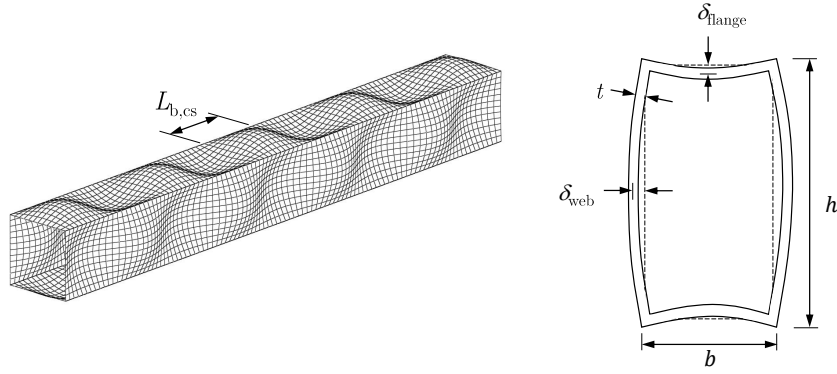


Figure 4: Geometry and local geometric imperfections adopted in shell FE models of SHS/RHS.

For the structural frames, rigid beam-to-column connections that ensure a full moment transfer were modelled by tying the nodes at the end cross-section of the beam to the nearest node on the column flange through multi-point constraints designated as *MPC, BEAM in Abaqus [40]. The mesh of the columns was adjusted in the connection regions to match that of the beam, as shown in Fig. 5. To avoid material overlap, the end of the beam was offset by half of the column flange thickness. To ensure fully rigid behaviour of the connection region and to limit failure to the structural members, horizontal plate stiffeners were utilised and arranged in the FE mesh as shown in Fig. 5. For simplicity and modelling convenience, the effects of a diagonal stiffener within the panel zone were approximated using *MPC, BEAM multi-point constraints, as also shown in Fig. 5.

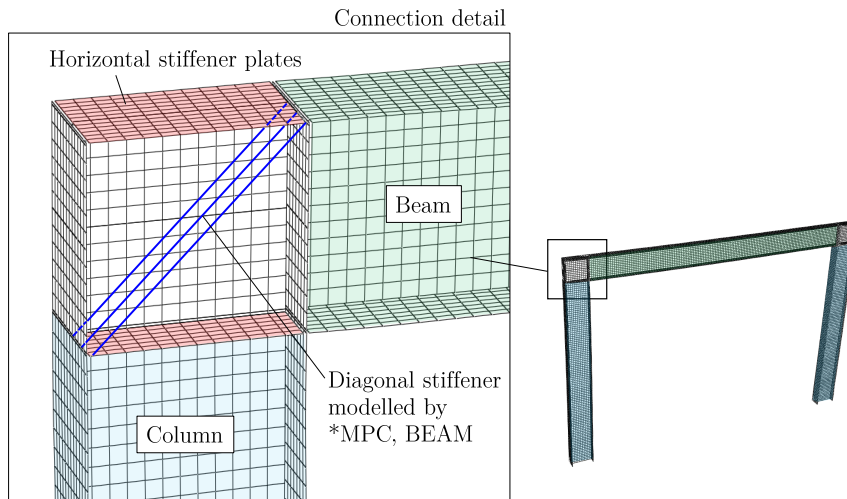


Figure 5: Modelling of fully rigid beam-to-column connections in the benchmark shell FE models.

4.2 Validation of benchmark shell FE models

The shell FE models were validated against the results of tests from the literature performed on continuous beams [35] and planar frames [36–39]. To compare the shell FE results directly with the experimental results, the measured cross-section geometry, material properties and imperfection magnitudes were used for the validation of the shell FE models. If no measured imperfections were provided, the values from Section 4.1 were assumed. Similarly, if the full stress-strain curves were not provided, the measured yield and ultimate stress values were used as input values for the quad-linear material model described in Section 3.1. In some cases, only combined material and cross-section geometry data were provided (e.g. the measured plastic moment capacity M_{pl} and elastic stiffness EI); in these cases the nominal cross-section dimensions were used to estimate the material properties. For the validation of the structural frame models, the test set-up (e.g. location of lateral bracing supports) and loading procedure (i.e. proportional loading [36] or two-step vertical and horizontal loading [37–39]) was followed closely.

A summary of the shell FE validation is given in Table 2, where FE-to-test ultimate load ratios of less than unity indicate safe-sided predictions. In the case of the continuous beams, the vertical peak loads obtained from the tests and shell FE models are shown, while the horizontal peak loads are given for the frames. Overall, the shell FE models are able to accurately predict the experimental failure loads of continuous beams and planar frames with an average FE-to-test ultimate load ratio of 1.01 and a corresponding coefficient of variation (CoV) of 0.05.

Table 2: Comparisons of experimental and shell FE ultimate capacities for hot-rolled steel continuous beams and frames.

Reference	Test label	Steel grade	Peak load		
			Test	Shell FE	FE/test
Continuous beams			<i>Vertical load</i> (kN)		
Gardner et al. [35]	RHS 60×40×4-HR1	S355	78.1	79.8	1.02
	SHS 40×40×4-HR1	S355	44.6	44.8	1.00
	SHS 40×40×3-HR1	S355	38.1	36.8	0.97
	RHS 60×40×4-HR2	S355	98.4	102.2	1.04
	SHS 40×40×4-HR2	S355	55.2	59.0	1.07
	SHS 40×40×3-HR2	S355	49.0	45.6	0.93
Frames			<i>Horizontal load</i> (kN)		
Arnold et al. [37]	-	A441 / A36	75.2	73.0	0.97
Avery & Mahendran [36]	Frame 2	S320	135.0	135.8	1.01
Yarimici [39]	Frame B	A36	7.22	7.70	1.07
	Frame C	A36	23.2	25.3	1.09
Wakabayashi et al. [38]	FM0	SS41	167.2	162.6	0.97
	FM5	SS41	83.3	83.9	1.02
	BM0	SS41	449.9	443.6	0.99
	BM5	SS41	404.0	384.7	0.95
			Mean	1.01	
			CoV	0.05	

4.2.1 Validation: continuous beam example

In Fig. 6, the normalised load-end rotation curve obtained from the shell FE model is compared against the experimental results for continuous beam RHS 60×40×3-HR1 presented in [35]. Overall, the shell FE model captures both the elastic and ultimate response of the experiment well, with a predicted peak load of $P_{\text{shell}} = 1.18P_{\text{coll}}$, where P_{coll} is the theoretical plastic collapse load (i.e. assuming zero length plastic hinges and neglecting strain hardening). The predicted peak load from the shell FE model is very close to the experimental peak load of $P_{\text{test}} = 1.15P_{\text{coll}}$. Furthermore, the collapse mechanism is accurately captured by the shell FE model, as shown in Fig. 6.

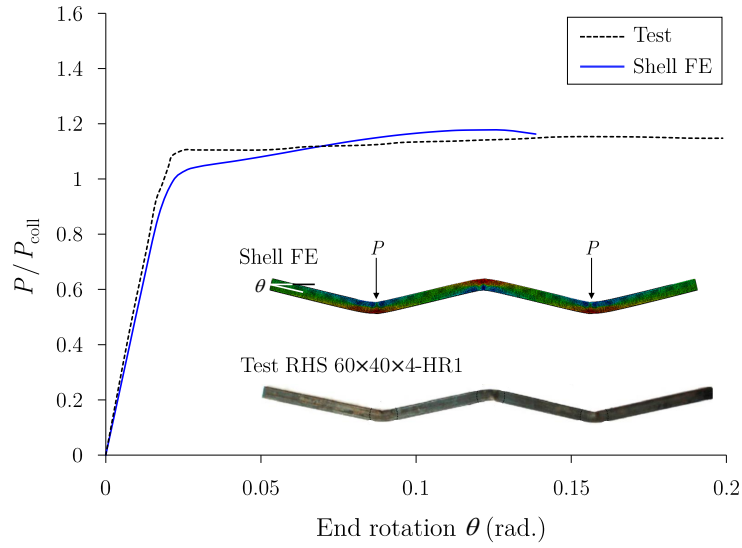


Figure 6: Shell FE model validation for a two-span continuous beam: load-deformation curves and deformed shapes from shell FE model and test on specimen RHS 60×40×4-HR1 reported by [35].

4.2.2 Validation: frame example

Arnold et al. [37] performed a full-scale test on the single span portal frame shown in Fig. 7. The columns were made from American 5WF18.5 sections in grade A441 steel, while the beam was made from an American 10I25.4 section in grade A36 steel. The beams were welded to the column flange to ensure full moment transfer at the connections. Lateral restraints were provided at the locations shown in Fig. 7 to allow a full plastic collapse mechanism to form, without premature failure due to out-of-plane deformations. During the test, the frame was first subjected to a set of vertical loads which were held constant during the test, then the horizontal load H was gradually increased until failure. The experimental load-deflection response of the single span portal frame [37] and the response of the corresponding shell FE model are shown in Fig. 7. Overall, the shell FE model accurately predicts the observed experimental behaviour, capturing both the second-order sway effects and the spread of plasticity along the members of the frame. The FE predicted horizontal failure load of $H_{\text{shell}} = 73.0$ kN is in very good agreement with the experimental peak load of $H_{\text{test}} = 75.2$ kN. The slight underprediction of the

experimental collapse load may be attributed to the conservative imperfection magnitudes assumed in the shell FE model, since no measured imperfections were provided in [37]. The progressive formation of plastic hinges in the shell FE model is shown in Fig. 7; this matched the observed experimental response, with yielding and spread of plasticity first occurring under the left internal load applied to the beam and at the top of the right column, marked with ①. A collapse mechanism eventually formed when two further hinges, marked ② and ③, developed at the base of the columns, as shown in Fig. 7.

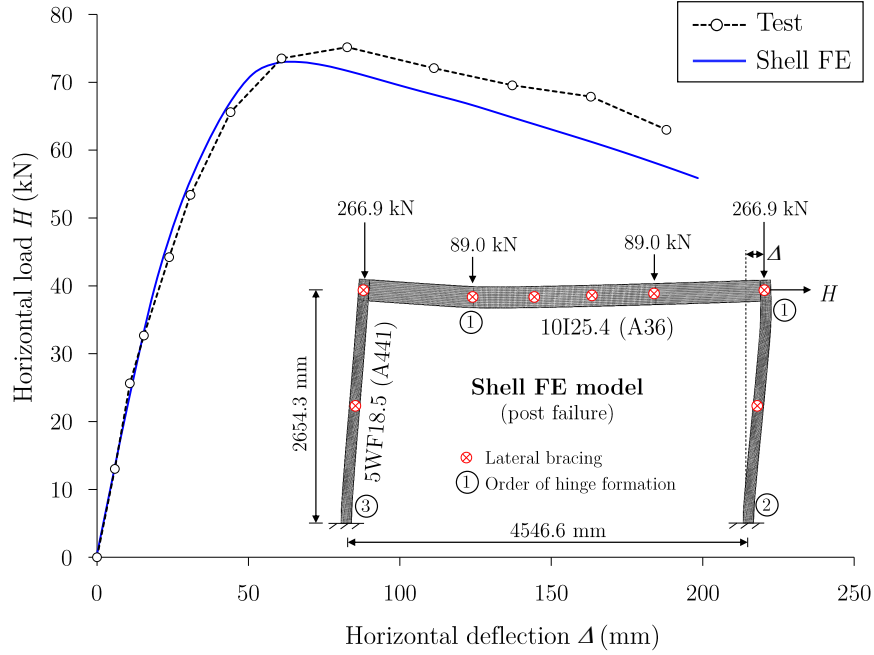


Figure 7: Shell FE model validation and geometry for single span portal frame tested by Arnold et al. [37].

4.3 Description of beam FE models

To assess the proposed method of design by advanced inelastic analysis (GMNIA) with strain limits, beam finite element models that can be compared to the benchmark shell FE models were developed. The geometrically and materially nonlinear analyses were performed in Abaqus [40] using Timoshenko (shear deformable) beam elements, denoted as B31OS and B31 in the Abaqus element library for open and closed cross-sections respectively. To capture the spread of plasticity through the depth of the cross-section accurately, 33 section points were specified along the web and flanges of each cross-section. Similar to the shell FE models, the quad-linear material model described in Section 3.1 was used to represent grade S355 steel. In the beam FE models, the material response was defined in terms of the engineering stresses and strains, since the cross-sectional area of the beam elements remained constant during the analysis. A Poisson's ratio of $\nu = 0.3$ was specified in the elastic range, while the default effective Poisson's ratio of zero was used in the plastic range. The same frame out-of-plumbness as used in the shell FE models was included in the beam FE models. Equivalent member imperfections were used to account for

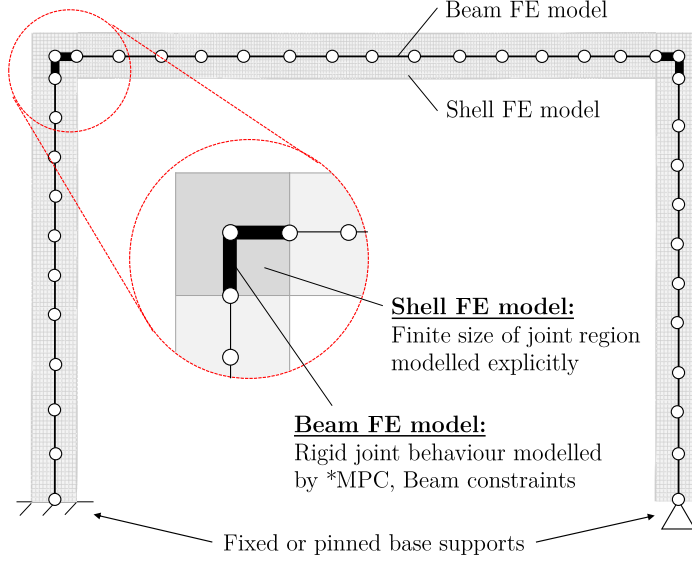
initial member out-of-straightness and residual stresses (see Section 3.3 for further details). Note that depending on the EN 1993-1-1 [16] design approach, the beam FE models were simplified accordingly (e.g. no member imperfections, no material nonlinearity or no geometrical nonlinearity).

For the frame models, it was found that the finite size of the beam-to-column connections in the benchmark shell FE simulations had a noticeable effect on the overall capacity of the structure. The beam-to-column connections in the shell FE models were close to fully rigid. In the beam FE models, this behaviour was accounted for by shortening the individual members compared to their centreline dimensions and using rigid *MPC, Beam links to represent the connection region between the beams and the columns, as shown in Fig. 8(a). Note that in both the shell and beam FE models, point loads were applied along the centreline dimensions of the frames.

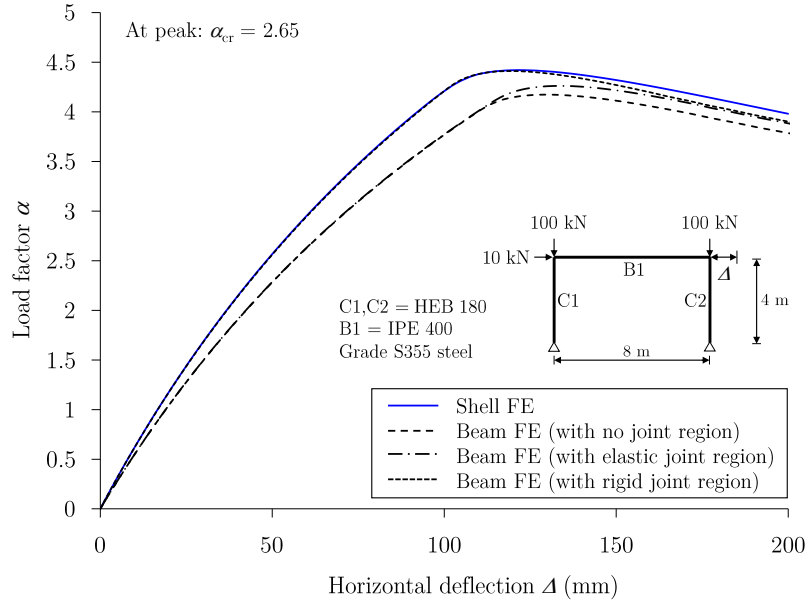
The load-deformation response of a typical stability governed portal frame composed of compact Class 1 cross-sections predicted by the shell FE and beam FE models is shown in Fig. 8(b). It can be seen that without considering the finite size of the joint region, the beam FE model underpredicts the shell FE model response by over 5%; this is primarily due to the increase in effective lengths of the columns. Likewise, in bending dominated cases, the plastic hinges in the beam FE models without modified joint regions develop earlier, since the moment at the centreline location of the joint is higher than that at the face of the joint (i.e. a distance of half the beam depth or half the column width away from the centreline node). Adjusting the beam FE models by only preventing yielding in the connection region, but still allowing for elastic deformations, approximates the shell FE failure load, but underpredicts the initial stiffness of the frame. Adjusting the beam FE models to accurately represent the rigid connection behaviour of the shell FE models results in a close prediction of the shell FE model behaviour, as shown in Fig. 8(b). A similar approach was also explored by Yarimici [39], who found that the analytical capacity prediction of a frame was significantly closer to the experimental observation when the locations of plastic hinges were offset to account for the finite size of connections.

5 Application of proposed method to continuous beams

In this section, the accuracy of the proposed method of design by advanced inelastic analysis is assessed against benchmark shell FE models on a total of 784 continuous beams made from I-sections and square/rectangular hollow sections (SHS/RHS) of varying local slenderness. The capacity predictions from conventional design according to EN 1993-1-1 [16] are also compared, with all partial safety factors set to unity. An overview of the considered loading configurations and varied parameters is given in Table 3, where the parameter β is used to define the relative lengths of the individual beam spans and the position of the applied point loads. For the continuous beams, loading was applied proportionally, with α being the applied load factor. The loading configurations were chosen to cover a range of different



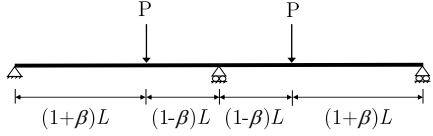
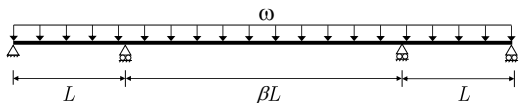
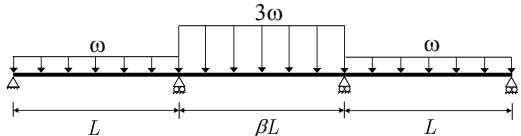
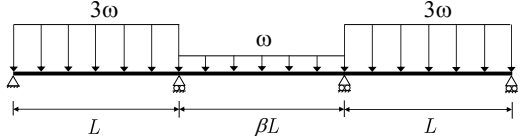
(a) Joint modelling



(b) Load-deformation response

Figure 8: Modelling of beam-to-column connections in beam FE models: (a) joint modelling and (b) load-deformation response of beam FE and shell FE models for a simple portal frame. Note that to compare the load-deformation responses, a member imperfection of $l/1000$ and no residual stresses were included in both the beam FE and shell FE models.

Table 3: Overview of continuous beam configurations considered during assessment of the proposed method of design by advanced inelastic analysis.

Load case	No. of numerical simulations	Varied parameters
LC1 	I-sections 182 SHS/RHS 156	Cross-section I-sections: $0.24 \leq \bar{\lambda}_p \leq 0.77$ SHS/RHS: $0.20 \leq \bar{\lambda}_p \leq 0.75$ Member $2L = 15L_{b,cs}$ $\beta = -0.25, 0, 0.1, 0.25$
LC2 	I-sections 260	Cross-section I-sections: $0.14 \leq \bar{\lambda}_p \leq 1.10$ Member $\beta = 1.0$: $L = 15L_{b,cs}, 20L_{b,cs}$ $\beta = 0.5$: $L = 20L_{b,cs}$ $\beta = 2.5$: $L = 10L_{b,cs}$
LC3 	I-sections 57	Cross-section I-sections: $0.14 \leq \bar{\lambda}_p \leq 1.10$ Member $\beta = 1.0$: $L = 20L_{b,cs}$
LC4 	I-sections 129	Cross-section I-sections: $0.14 \leq \bar{\lambda}_p \leq 1.10$ Member $\beta = 1.0$: $L = 15L_{b,cs}, 20L_{b,cs}$

levels of moment redistribution; this was achieved by varying the ratio of the plastic collapse load α_{coll} to the load at which the first hinge formed α_{pl} . The Eurocode 3 capacity predictions are based on plastic design (i.e. full moment redistribution) for Class 1 cross-sections and on elastic design for Class 2 to 4 cross-sections.

A summary of the normalised ultimate capacities $\alpha_{ult}/\alpha_{shell}$, where α_{shell} is the peak load factor from the shell FE model and α_{ult} is the predicted capacity according to the design method considered, is presented in Table 4 for the proposed method of design by advanced inelastic analysis and EN 1993-1-1 [16]; the results are sub-divided according to the four EN 1993-1-1 cross-section classes. The same data is presented in Fig. 9, which shows the mean normalised capacity predictions $\alpha_{ult}/\alpha_{shell}$ plotted against the respective $\alpha_{coll}/\alpha_{pl}$ ratios for each of the considered continuous beam configurations from Table 3. Note that each data point in Fig. 9 represents the mean normalised capacity prediction of analysed continuous beams with constant $\alpha_{coll}/\alpha_{pl}$ ratio and composed of cross-sections from the same class. The ratio $\alpha_{coll}/\alpha_{pl}$ depends on the specific load distribution applied to the continuous beam and

Table 4: Capacity predictions of continuous beams from the proposed method of design by advanced inelastic analysis with strain limits (AA + CSM) for $\Omega = 15$ and EN 1993-1-1 [16] versus the results from benchmark shell FE models.

Profile	Load case*	No. of cases	Design method	$\alpha_{ult}/\alpha_{shell}$			
				Class 1	Class 2	Class 3	Class 4
I	LC1	182	AA + CSM	0.912 (0.923**)	0.973	0.931	0.881
			EN 1993-1-1	0.846	0.830	0.766	0.780
I	LC2	260	AA + CSM	0.911 (0.932**)	0.980	0.945	0.866
			EN 1993-1-1	0.861	0.859	0.799	0.788
I	LC3	57	AA + CSM	0.873 (0.901**)	0.982	0.966	0.893
			EN 1993-1-1	0.819	0.894	0.835	0.817
I	LC4	129	AA + CSM	0.930 (0.962**)	0.969	0.901	0.932
			EN 1993-1-1	0.903	0.921	0.863	0.881
SHS/RHS	LC1	156	AA + CSM	0.911 (0.956**)	0.953	0.906	0.852
			EN 1993-1-1	0.851	0.908	0.793	0.785
	Total	784	AA + CSM	0.912 (0.940**)	0.972	0.942	0.880
			EN 1993-1-1	0.856	0.875	0.803	0.807

*Refer to Table 3 for details

** Results achieved with $\Omega = 30$

higher values of $\alpha_{coll}/\alpha_{pl}$ indicate grater potential for moment redistribution. In both Table 4 and Fig. 9, $\alpha_{ult}/\alpha_{shell}$ values less than unity indicate safe-sided capacity predictions compared to the shell FE model results.

For all cross-section classes and $\alpha_{coll}/\alpha_{pl}$ ratios, the developed method of design by advanced inelastic analysis with strain limits consistently provides more accurate capacity predictions than the EN 1993-1-1 methods relative to the benchmark shell FE data. The improvement in capacity predictions compared to EN 1993-1-1 is primarily due to the fact that the proposed design method allows for cross-section dependent levels of moment redistribution. Additionally, the developed method of design by advanced inelastic analysis accounts for the effects of local moment gradients and strain hardening, which results in a capacity increase of around 6% in Class 1 cross-sections compared to Eurocode 3. In the case of compact cross-sections with a local slenderness of $\bar{\lambda}_p < 0.32$, an increase in the maximum allowable deformation capacity from $\Omega = 15$ to $\Omega = 30$ results in further improvements, as shown in Fig. 9(a) and Fig. 10(a). For Class 2, 3 and 4 cross-sections, the EN 1993-1-1 capacity predictions become increasingly less accurate as the ratio $\alpha_{coll}/\alpha_{pl}$ increases, since no moment redistribution is allowed for in the stipulated elastic analysis. In contrast, the capacity predictions from the proposed method of design by advanced inelastic analysis with strain limits are noticeably less influenced by the $\alpha_{coll}/\alpha_{pl}$ ratio, particularly for Class 2 and 3 cross-sections, as shown by the more consistent $\alpha_{ult}/\alpha_{shell}$ results that are closer to unity in Fig. 9(b) and (c). A mean increase in capacity of 10% for Class 2 cross-sections and 15% for Class 3 cross-sections is achieved for the studied cases. For slender Class 4 cross-sections, the strain limits

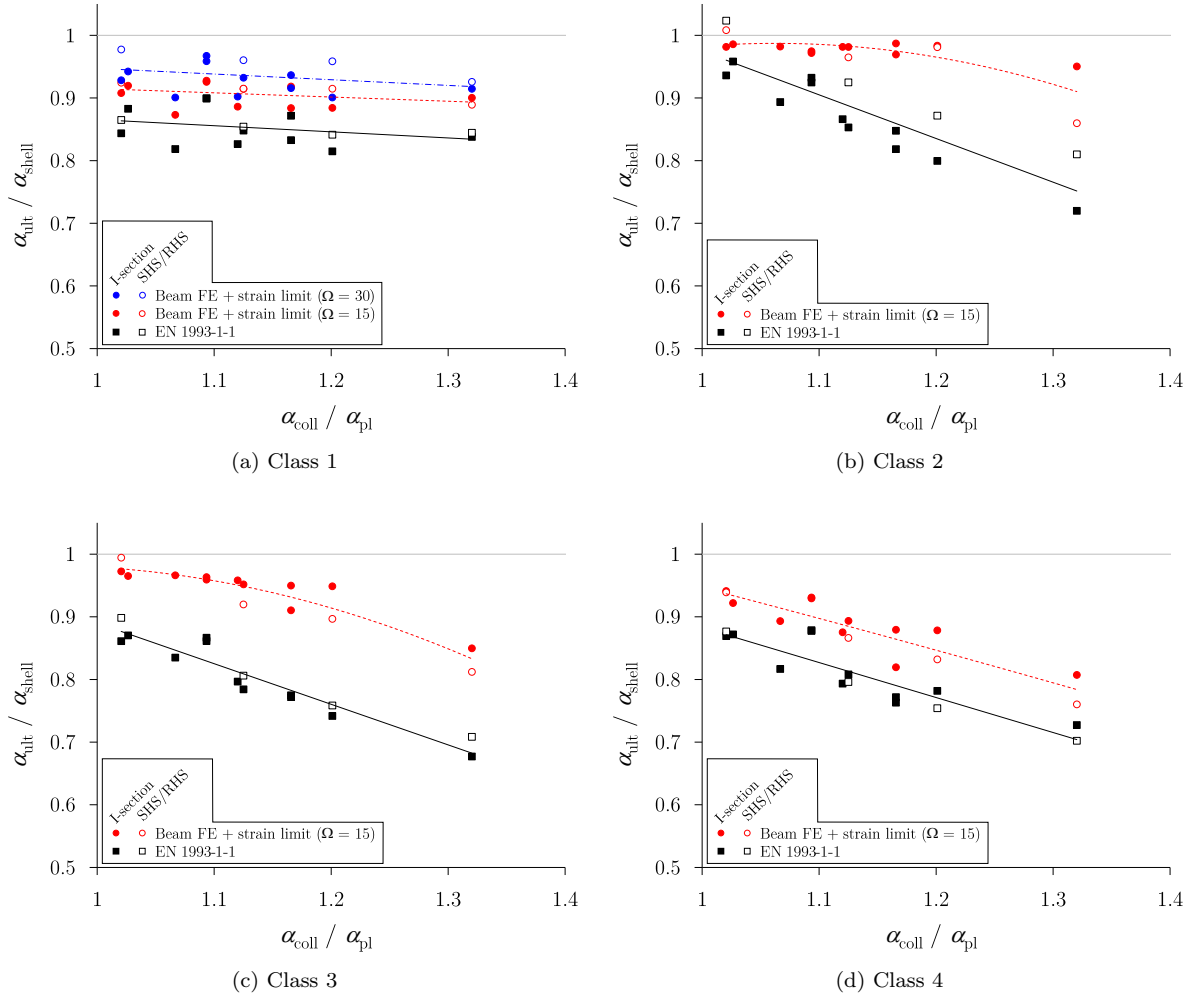


Figure 9: Capacity predictions of advanced analysis with strain limits and EN 1993-1-1 relative to benchmark shell FE results for continuous beams with different levels of plastic collapse (α_{coll}) to formation of first hinge (α_{pl}) ratios for (a) Class 1, (b) Class 2, (c) Class 3 and (d) Class 4 cross-sections, according to the EN 1993-1-1 classification system.

(see Eq. (3)) predict that failure of the system occurs in the elastic range (i.e. $\epsilon_{\text{csm}}/\epsilon_y < 1$), and hence there is no moment redistribution. In reality, however, some moment redistribution remains possible in Class 4 systems [45, 46], and the capacity predictions from the proposed method become slightly more conservative, particularly as the $\alpha_{\text{coll}}/\alpha_{\text{shell}}$ ratio increases. Nonetheless, consideration of the effects of local moment gradients through the strain averaging approach still yields a capacity increase of around 7% compared to EN 1993-1-1 design.

An overview of the normalised capacity predictions of all 784 analysed continuous beams predicted by the proposed method of design by advanced analysis and Eurocode 3 is provided in Fig. 10(a) and (b) respectively. It is observed that the scatter in the capacity predictions from the proposed method of design by advanced analysis is significantly reduced across the considered range of local cross-section slenderness compared to traditional design. The scatter in the Eurocode 3 predictions observed in Fig. 10(b), particularly in the Class 3 range, is primarily due to the different $\alpha_{\text{coll}}/\alpha_{\text{pl}}$ ratios considered herein. Note

that the Eurocode 3 results shown in Fig. 10(b) are plotted against the full cross-section slenderness $\bar{\lambda}_p$, which explains the small overlap of the cross-section classes; this is partly due to the different slenderness limits for I-sections and SHS/RHS and partly due to the element-by-element approach (i.e. the effects of element interaction are ignored) adopted in the cross-section classification process outlined in EN 1993-1-1.

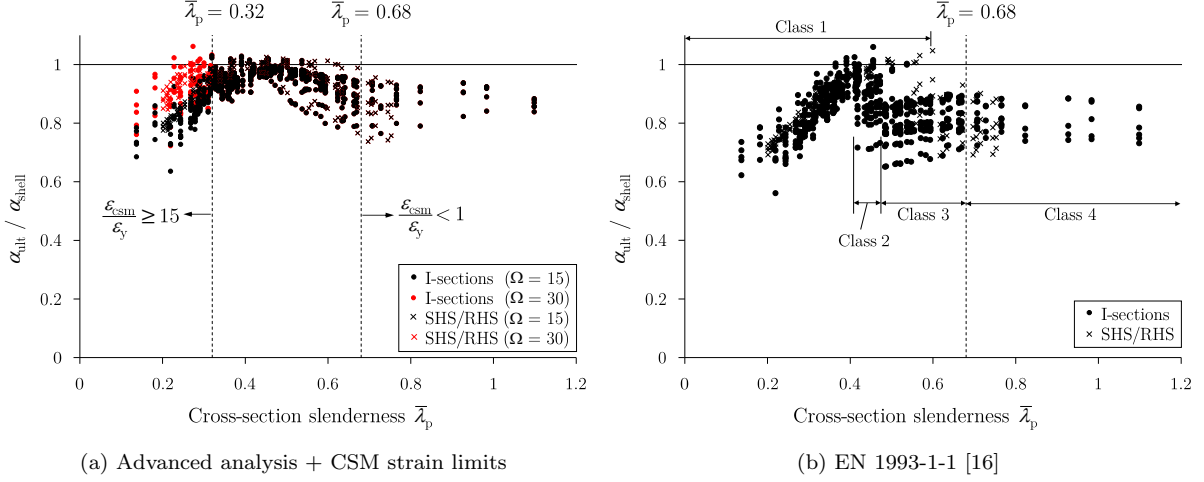


Figure 10: Predicted ultimate load factor α_{ult} versus ultimate load factor from shell FE models α_{shell} of considered continuous beam configurations for (a) design by advanced inelastic analysis with CSM strain limits and (b) EN 1993-1-1.

The results of specific continuous beam cases from Table 3 are discussed in more detail in the following three subsections. Furthermore, the ability of the proposed design approach to capture the actual collapse mechanism as predicted by the shell FE models is confirmed in Section 5.3.

5.1 Example application to 3-span continuous I-section beams under distributed loading

The normalised capacity predictions of a series of three-span continuous I-section beams of varying local slenderness and subjected to constant and non-constant uniformly distributed loads (UDL) are shown in Fig. 11, where α_{ult} is the ultimate load factor (from either the proposed design approach or the shell FE models) and α_{el} is the load factor at which first yield occurs (i.e. the first cross-section reaches its elastic moment capacity M_{el}). The results shown in Fig. 11(a) correspond to load case LC2 from Table 3 with $\beta = 1.0$, where β is a parameter used to define the length of the middle span relative to the two side spans, and $L = 20L_{b,cs}$. Fig. 11(b) corresponds to load case LC4 with $\beta = 1.0$ and $L = 20L_{b,cs}$. For the continuous beams shown in Fig. 11(a), there is considerable potential for moment redistribution with $\alpha_{coll}/\alpha_{pl} = 1.16$. For the continuous beams subjected to a non-constant UDL in Fig. 11(b), the potential for moment redistribution is lower with a ratio of $\alpha_{coll}/\alpha_{pl} = 1.09$. In Fig. 11, the potential for moment redistribution is visualised by the step in the Eurocode 3 capacity predictions between Class 1

and 2 cross-sections.

Overall, the results shown in Fig. 11(a) and (b) represent typical results and the general conclusions drawn in Section 5 apply i.e. EN 1993-1-1 tends to under-predict the system capacities of the shell FE models for the considered cases. An exception to this are cross-sections that lie just within the Class 1 range (i.e. $\bar{\lambda}_p \approx 0.4$), in which, similar to the underlying EN 1993-1-1 design assumptions, the influence of strain hardening is negligible, yet full plastic redistribution is achieved. Note that the EN 1993-1-1 capacity predictions shown in Fig. 11 are mean values; the actual capacity may vary slightly depending on the shape factor of the individual cross-sections. The CSM strain limits on the other hand accurately allow for cross-section slenderness dependent levels of redistribution, resulting in accurate capacity predictions of the shell FE models.

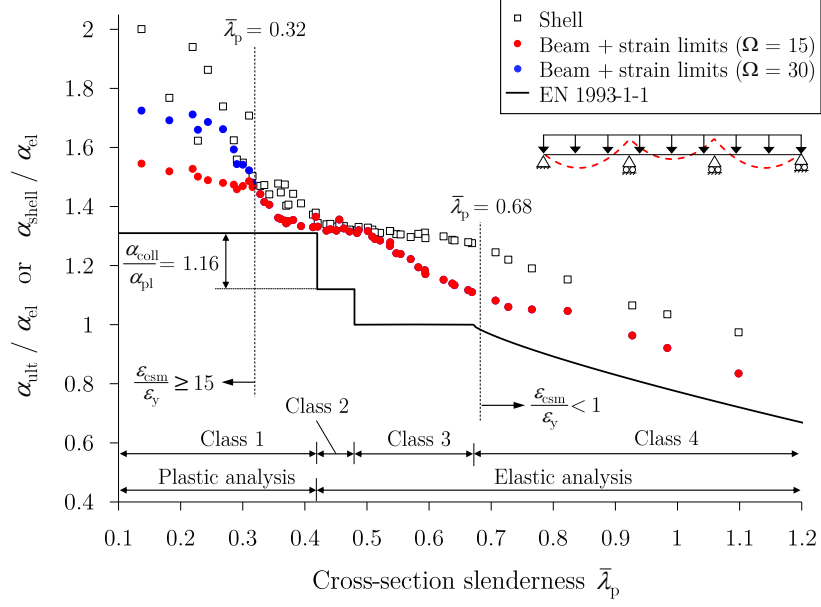
In Fig. 11(a), the maximum moment occurs above the internal supports, in a region of relatively high local moment gradient. In contrast, for the continuous beams shown in Fig. 11(b), the maximum moment occurs within the side spans. The resulting local moment gradient across the critical cross-section is considerably lower and as a result, the capacity predictions for continuous beams with cross-sections that are not benefiting from strain hardening are considerably closer together for all three design and analysis approaches, as shown in Fig. 11(b). The benefit of increasing the maximum strain limit from $\Omega = 15$ to $\Omega = 30$ is clearly demonstrated in Fig. 11(a) and (b) for cross-sections with a local slenderness $\bar{\lambda}_p < 0.32$.

5.2 Example application to 2-span continuous SHS/RHS beams with point loads

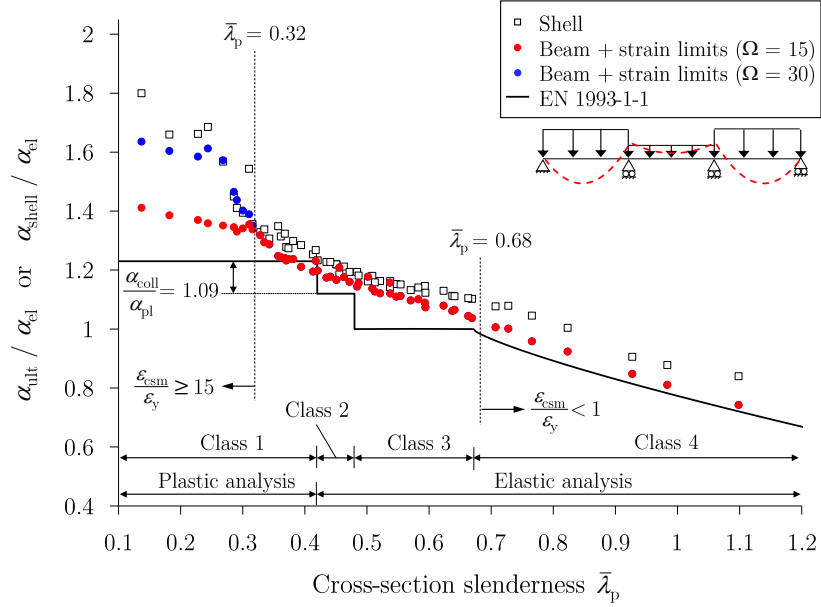
Similar observations to those made in Section 5.1 for the continuous I-section beams also apply to continuous SHS/RHS beams, as shown in Fig. 12 with reference to two-span continuous beams with point loads. The ratio of load levels between the formation of the plastic collapse mechanism and the formation of the first hinge $\alpha_{\text{coll}}/\alpha_{\text{pl}}$ depends on the parameter β , as given in Table 3. For $\beta = 0.1$, the ratio $\alpha_{\text{coll}}/\alpha_{\text{pl}} = 1.21$ and significant redistribution is observed in the shell FE models. For $\beta = -0.25$, the plastic collapse mechanism forms shortly after the first hinge and $\alpha_{\text{coll}}/\alpha_{\text{pl}} = 1.02$. Similar to the results shown in Fig. 11(a), the strain limits predict slightly conservative capacities for slender cross-sections when $\alpha_{\text{coll}}/\alpha_{\text{pl}} = 1.21$, since the redistribution in Class 4 systems is not captured by the strain limits. This effect is noticeably smaller in Fig. 12(b), where the level of moment redistribution is smaller and the resulting capacity predictions are in closer agreement with the shell FE models.

5.3 Prediction of failure mechanisms

The ability of advanced inelastic analysis with strain limits to accurately capture the collapse mechanism observed in the shell FE models is demonstrated in this section through two examples. The strain

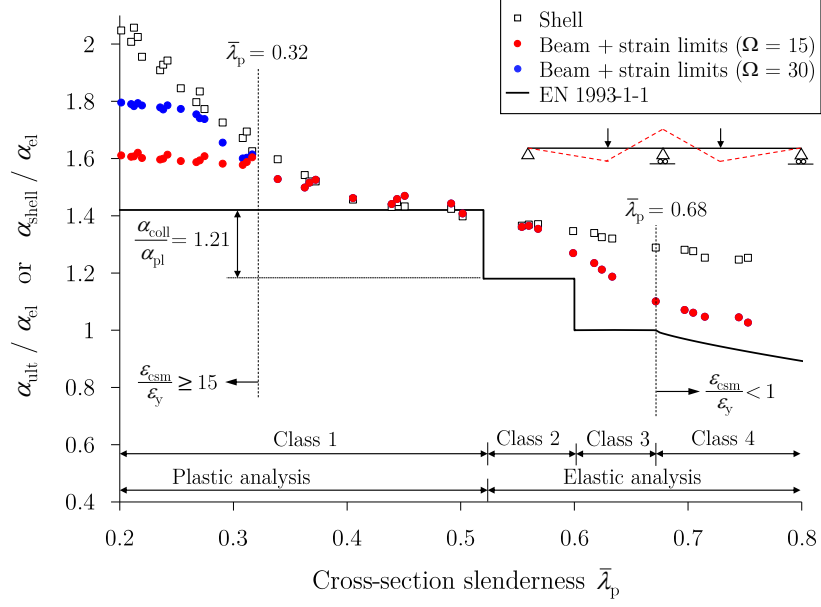


(a) LC2: $\beta = 1.0$, $L = 20L_{b,cs}$

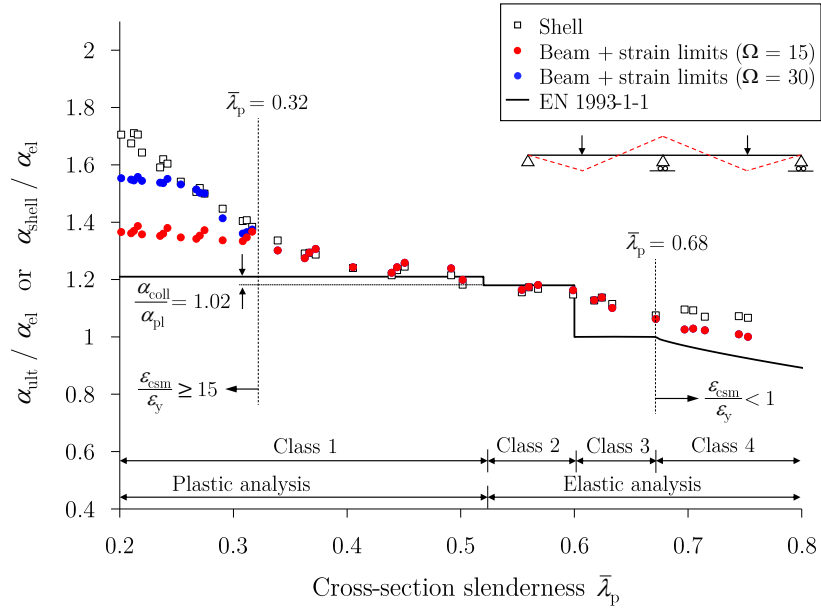


(b) LC4: $\beta = 1.0$, $L = 20L_{b,cs}$

Figure 11: Normalised capacity predictions for three-span continuous I-section beams of varying local slenderness for (a) LC2 with $\beta = 1.0$, $L = 20L_{b,cs}$ and (b) LC4 with $\beta = 1.0$, $L = 20L_{b,cs}$.



(a) $\beta = 0.1$



(b) $\beta = -0.25$

Figure 12: Normalised capacity predictions for two-span continuous SHS/RHS beams of varying local slenderness with (a) $\beta = 0.1$ and (b) $\beta = -0.25$.

distributions at failure of two three-span continuous beams with different cross-sections, namely an IPE 80 and a W8×31, are shown in Fig. 13(a) and (b) respectively. The span lengths of each of the continuous beams are defined as multiples of the local buckling half-wavelength of each cross-section, but the ratios of the spans are kept constant. Consequently, the elastic bending moment diagram takes the same shape for both cross-sections, with the peak moment occurring over the internal supports.

The distributions of maximum compressive strain along the length of the two continuous beams, corresponding to the load level at which the proposed method of design by advanced inelastic analysis with strain limits predicts failure, are shown in Fig. 13. Note that compression is taken as positive. According to EN 1993-1-1, an S355 IPE 80 section subjected to pure major axis bending is Class 1 and hence plastic design is permitted. Nevertheless, the ultimate collapse load of the shell FE model is still underpredicted by around 30%, primarily because strain hardening is not accounted for. According to the CSM, the local cross-section slenderness of an IPE 80 subjected to pure major axis bending is $\bar{\lambda}_p = 0.24$. In this case, the normalised CSM strain limit $\epsilon_{\text{csm}}/\epsilon_y = 42.6$, but is limited to $\Omega = 15$ in this example. Three potential failure regions $L_{b,1}$, $L_{b,2}$ and $L_{b,3}$ are identified from the normalised strain distributions shown in Fig. 13, corresponding to the local buckling half-wavelengths at the locations of maximum hogging moment (i.e. $L_{b,1}$ and $L_{b,2}$ to the left and right of the internal supports) and maximum sagging moment at midspan (i.e. $L_{b,3}$). Note that due to the symmetry of the continuous beam, $L_{b,1}$ and $L_{b,2}$ are mirrored at each internal support. The averaged strain at failure across the three local buckling half-wavelengths $L_{b,1}$, $L_{b,2}$ and $L_{b,3}$ is shown in red in Fig. 13. For the IPE 80 continuous beam considered in Fig. 13(a), the strain averaging approach identifies $L_{b,1}$ as critical. This corresponds to the location where local buckling is observed in the shell FE model, as shown in Fig. 13(a). The resulting ultimate capacity prediction from the proposed method of design by advanced inelastic analysis with strain limits is 12% above the EN 1993-1-1 prediction, yet still safe-sided relative to the shell FE model.

An S355 W8×31 cross-section subjected to pure major axis bending has a cross-section slenderness $\bar{\lambda}_p = 0.42$ and the associated normalised CSM strain limit is $\epsilon_{\text{csm}}/\epsilon_y = 5.8$. According to Eurocode 3, this cross-section is Class 2. Despite being subjected to the same loading configuration as the IPE 80 continuous beam Fig. 13(a), the capacity of the W8×31 beam is governed by the averaged strain at $L_{b,3}$, as shown in Fig. 13(b). This is because the local moment gradient across $L_{b,3}$ is shallower compared to that across $L_{b,1}$ or $L_{b,2}$ and the fact that the deformation capacity of a S355 W8×31 is significantly less than that of a S355 IPE80. Hence, the resulting averaged strain across the critical local buckling half-wavelength $L_{b,3}$ is higher than that across $L_{b,1}$ or $L_{b,2}$ for the W8×31 continuous beam. This accurately reflects the failure mechanism observed in the shell FE model, as shown in Fig. 13(b). Eurocode 3, on the other hand, conservatively predicts failure when the first cross-section reaches the plastic moment M_{p1} , without allowing for any moment redistribution. This results in a capacity prediction that is 15% below the shell FE model ultimate capacity. Moreover, failure is erroneously predicted to occur at the locations

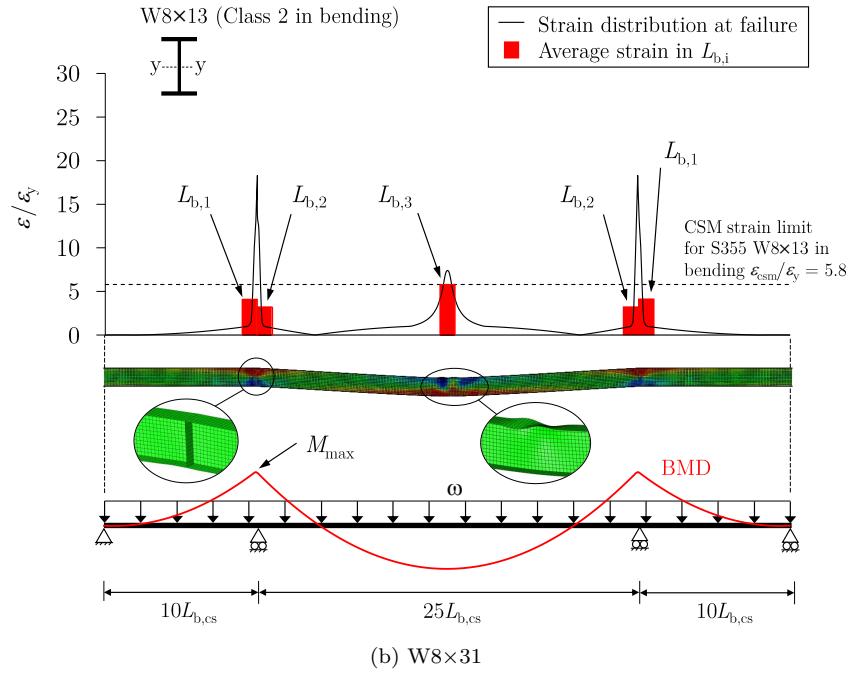
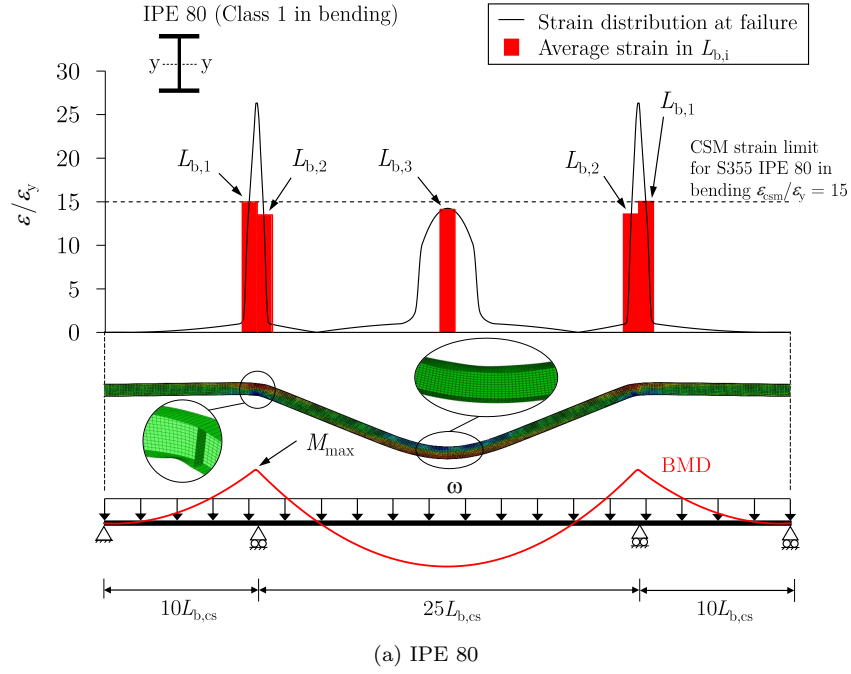


Figure 13: Strain distributions and critical failure regions in beam FE advanced inelastic analysis and shell FE models for three-span continuous beams with (a) IPE 80 and (b) W8×31 profiles under uniformly distributed loading.

of the maximum moment (i.e. over the internal supports), since the effects of local moment gradients are ignored, as is customary in traditional design.

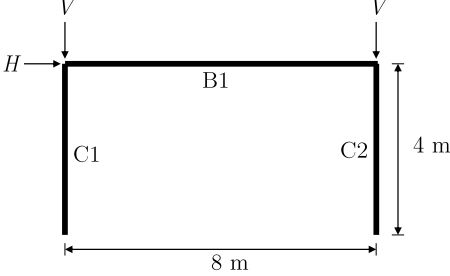
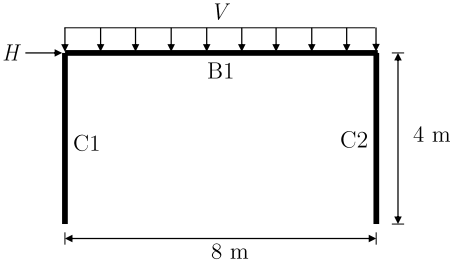
6 Application of proposed method to structural frames

In this section, the accuracy of the proposed method of design by advanced analysis is assessed on planar frames. A key parameter used to distinguish between different types of frame behaviour is α_{cr} , which was defined in Eq. (1) as the ratio of the load at which elastic buckling of the frame occurs in a global sway mode to the load applied to the frame. The frames have been chosen to cover both stability and bending dominated failure mechanisms, corresponding to low and high α_{cr} values respectively. In the following examples, the reported α_{cr} values are calculated based on the applied loads corresponding to the peak load factor reached in the analysis of the shell FE models. All frames are composed of European hot-rolled grade S355 steel HEB, HEA or IPE cross-sections. The beam-to-column connections are assumed to be fully rigid, out-of-plane behaviour is prevented and loading is applied proportionally. The ultimate capacity of each frame has been predicted using the proposed method of design by advanced analysis with strain limits, the benchmark shell FE models and different design approaches from EN 1993-1-1 [16], namely: (1) a linear elastic analysis (LEA) with member checks using the sway effective lengths (with the sway effective length factor denoted as $k_{cr,sway}$), referred to herein as the $k_{cr,sway}$ method, (2) a linear elastic analysis (LEA) with amplified sway moments using k_{amp} followed by member checks utilising non-sway effective lengths (with the non-sway effective length factor denoted as k_{cr}), referred to herein as the k_{amp} method, (3) a second-order elastic analysis (GNA) with member and cross-section checks performed using non-sway effective lengths, (4) a second-order elastic analysis with equivalent imperfections (GNIA) followed by cross-section checks, and (5) a second-order inelastic analysis (GMNA) followed by member checks performed using non-sway effective lengths. The effective lengths used in the member design checks were obtained using the limited frame method outlined in Appendix E of BS 5950 [47] and account for the relative rotational stiffness provided by the neighbouring members. Recall that, according to EN 1993-1-1, not all of these design approaches are permitted for every frame (see Section 2).

6.1 Parametric study on 8 m $\times$ 4 m sway frames

A parametric study on a series of 8 m $\times$ 4 m single bay sway frames was conducted to assess the accuracy of the proposed method of design by advanced inelastic analysis for a variety of α_{cr} values, ranging from 1.28 to 258. This was achieved by considering pinned (P) and fixed (F) support conditions at the base of the columns and by changing the ratio of horizontal load H to vertical loads V (point loads) or ω (UDL). For the frames subjected to point loads, the ratio $H/2V$ varied from 0 (i.e. gravity loads only) to 2.86 (i.e. primarily horizontal loading), where $2V$ is the total vertical load applied to the frame. For

Table 5: Overview of 8 m × 4 m portal frame configurations considered during assessment of the proposed method of design by advanced inelastic analysis. All members are grade S355 steel and N is the number of numerical simulations.

Load case	N	Varied parameters
Point loads (PL) 	HEB180 – P+F 30 HEA 180 – P+F 30 IPE 400 – P+F 30	Cross-section C1,C2 = HEB 180, HEA 180, IPE 400 B1 = IPE 400 in all cases Boundary conditions Pinned (P) or fixed (F) bases Loading range $0 \leq \frac{H}{2V} \leq 2.86$
UDL 	HEB180 – P+F 30 HEA 180 – P+F 30 IPE 400 – P+F 30	Cross-section C1,C2 = HEB 180, HEA 180, IPE 400 B1 = IPE 400 in all cases Boundary conditions Pinned (P) or fixed (F) bases Loading range $0 \leq \frac{H}{\omega L} \leq 7.16$

the frames subjected to uniformly distributed loading along beam B1, the ratio $H/\omega L$ varied from 0 to 7.16, where ωL is the total vertical load applied to the frame (i.e. the product of the UDL magnitude ω in kN/m and the frame width of $L = 8$ m). A summary of the varied parameters and frame geometries is given in Table 5.

Three cross-sections – HEB 180, HEA 180 and IPE 400, of different local slenderness, were considered for the column members, C1 and C2, while an IPE 400 was used for beam B1 in all cases. According to EN 1993-1-1, the HEB 180 cross-section in grade S355 steel is Class 1 under all loading conditions between pure compression and pure major axis bending, while the HEA 180 cross-section in grade S355 steel is Class 3 in all cases. The IPE 400 cross-section in grade S355 steel is Class 1 when subjected to pure major axis bending and Class 4 under pure compression. The normalised CSM strain limits $\epsilon_{\text{CSM}}/\epsilon_y$ for the HEB 180, HEA 180 and IPE 400 sections in grade S355 steel under different loading conditions are shown in Fig. 14, where ψ_w is the stress distribution along the centreline of the web. A value of $\psi_w = 1$ corresponds to pure compression, while $\psi_w = -1$ corresponds to pure major axis bending. It can be firstly observed from Fig. 14 that the predictive functions used to calculate the critical buckling stress $\sigma_{\text{cr,CS}}$ [29], and thus the local slenderness $\bar{\lambda}_p$ and ultimately the CSM strain limit ϵ_{CSM} , are in good agreement with the numerical predictions obtained from CUFSM [28] across the full range of applied loading from pure compression to pure major axis bending. Secondly, it can be observed that, compared

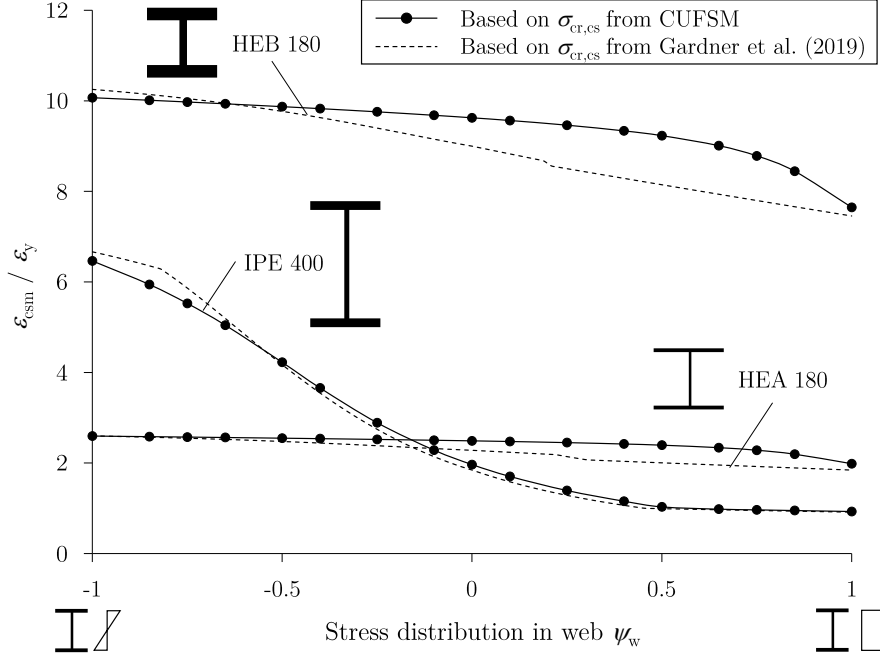


Figure 14: Normalised CSM strain limit $\epsilon_{\text{csm}}/\epsilon_y$ for HEB 180, HEA 180 and IPE 400 sections made from grade S355 steel subjected to combined compression and major axis bending, where ψ_w is the stress distribution along the centreline of the web.

to the HEB and HEA sections, there is a significant change in the CSM strain limit with ψ_w for the IPE section, primarily due to the changing slenderness of the web. Consequently, the deformation capacity of IPE cross-sections is significantly more sensitive to changes in the applied loading.

A summary of the normalised capacity predictions $\alpha_{\text{ult}}/\alpha_{\text{shell}}$ for the proposed method of design by advanced inelastic analysis with strain limits and the different EN 1993-1-1 design approaches is given in Table 6, where α_{shell} is the peak load factor reached by the shell FE model. Note that for the results of the Eurocode 3 k_{amp} method, only the 162 out of 180 frames with $\alpha_{\text{cr}} \geq 3$ are considered in Table 6. Likewise, only the results of the 104 out of 180 frames with Class 1 cross-sections are considered for the Eurocode 3 GMNA design approach. The scatter of the Eurocode 3 results is largely due to the step-wise capacity predictions resulting from the classification of cross-sections. The Eurocode 3 GNIA design approach is overly conservative for bending dominated cases because the interaction between compression and bending is not accurately captured by the assumed linear interaction cross-section check. Similar observations were made at the isolated member level [29]. The application of the CSM strain limits to the beam FE advanced analysis on the other hand allows for cross-section dependent levels of deformation and force/moment redistribution, resulting in more accurate and consistent capacity predictions. Furthermore, the same design approach can be used for all 180 analysed frames of varying α_{cr} values and differing cross-section classes.

Table 6: Summary of the normalised ultimate capacity predictions $\alpha_{\text{ult}}/\alpha_{\text{shell}}$ of the 180 analysed $8 \text{ m} \times 4 \text{ m}$ portal frames using the proposed method of design by advanced inelastic analysis with strain limits (AA+CSM) and the different Eurocode 3 (EC3) approaches, where α_{shell} is the peak load factor reached by the benchmark shell FE models.

Analysis method	$\alpha_{\text{ult}}/\alpha_{\text{shell}}$			
	Mean	CoV	Min	Max
AA + CSM	0.97	0.05	0.73	1.06
EC3 - LEA + $k_{\text{cr,sway}}$	0.83	0.10	0.57	0.98
EC3 - LEA + k_{amp}^*	0.84	0.11	0.56	1.02
EC3 - GNA	0.86	0.11	0.56	1.06
EC3 - GNIA	0.83	0.11	0.57	0.99
EC3 - GMNA**	0.94	0.06	0.78	1.06

* Results for the 162/180 frames with $\alpha_{\text{cr}} \geq 3$.

** Results for the 104/180 frames with Class 1 cross-sections.

In the following three subsections, the results from some of the frames listed in Table 5 are considered in more detail. The examples show how the proposed method of design by advanced inelastic analysis with strain limits is able to accurately predict the ultimate capacity of the shell FE models for both stability and bending dominated frames. Recall that in the proposed design method, failure is defined as the lower of (1) the peak load factor reached during the analysis, which is typically critical in stability governed scenarios, or (2) the load at which the CSM strain limit is first reached, which is typically critical in bending dominated scenarios.

6.1.1 Frame example 1: frame 8×4 -HEB180-P-PL with $H/2V = 0.042$

In the proposed method of design by advanced inelastic analysis, the ultimate load-carrying capacity of stability governed frames is directly captured in the analysis. This type of behaviour is shown through a typical example in Fig. 15 with reference to the load-deformation response of frame 8×4 -HEB180-P-PL (i.e. a frame with an HEB 180 cross-section used for column members C1 and C2, pinned supports and subjected to point loads) with $H/2V = 0.042$.

Most of the Eurocode 3 design approaches accurately predict the load-carrying capacity of the shell FE model, the exception being the $k_{\text{cr,sway}}$ method, which underpredicts the ultimate capacity, as shown in Fig. 15. At an applied load level of $\alpha/\alpha_{\text{shell}} = 0.86$, the frame reaches $\alpha_{\text{cr}} = 3$ and hence EN 1993-1-1 does not permit the use of the k_{amp} method beyond this point. Nevertheless, the capacity prediction using the k_{amp} method is found to be similar to the GNA results for this case.

The peak load factor reached by the beam FE advanced analysis accurately predicts failure at $\alpha_{\text{ult}}/\alpha_{\text{shell}} = 0.98$. The CSM strain limit is first reached after the peak load, and thus does not govern the capacity of the frame. The slightly different nonlinear responses of the various analysis methods shown in Fig. 15 are due to the different imperfection magnitudes incorporated in each analysis method. Recall that the equivalent imperfections described in Section 3.3 are used to account for the member out-

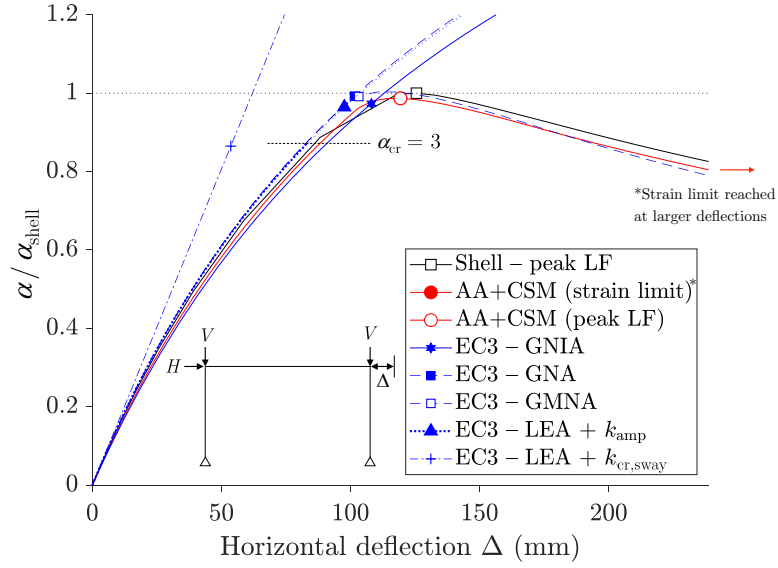


Figure 15: Load-deformation responses of frame $8 \times 4\text{-HEB180-P-PL}$ with $H/2V = 0.042$ predicted by benchmark shell FE model, advanced inelastic analysis using beam FE and various EN 1993-1-1 analysis and design methods.

of-straightness and residual stresses in the beam FE models of the proposed design by advanced inelastic analysis method, while a geometric member out-of-straightness of magnitude $l/1000$ plus residual stresses are explicitly modelled in the shell FE simulations. The same frame out-of-plumbness imperfection was included in both models though.

6.1.2 Frame example 2: frame $8 \times 4\text{-HEB180-F-UDL}$ with $H/\omega L = 0.217$

In this section, the proposed method of design by advanced inelastic analysis with strain limits is assessed on a bending dominated Class 1 cross-section frame. The normalised load-deformation response of frame $8 \times 4\text{-HEB180-F-UDL}$ with $H/\omega L = 0.217$ is shown in Fig. 16(a). The frame is not sensitive to second-order effects, with $\alpha_{cr} = 17.9$ at the peak load factor of the shell FE model. Substantial inelastic redistribution is observed in the shell FE model prior to failure. As a result, the elastic Eurocode 3 design approaches predict an overly conservative ultimate capacity of around $\alpha_{ult}/\alpha_{shell} = 0.65$ (i.e. when the first cross-section reaches M_{pl}). Inelastic analysis and design enables more accurate predictions of the ultimate capacity, though, following the member buckling checks, the predicted capacity still remains 15% below the peak shell FE model result. This is partly because strain hardening is ignored and partly because the influence of local moment gradients is not considered in the formation of the plastic hinges.

In the proposed method of design by advanced inelastic analysis, the capacity prediction of the considered frame is governed by the CSM strain limit of $\epsilon_{csm}/\epsilon_y = 10.2$ at the top of column C2. The normalised load-maximum compressive strain responses at the top of column C2 obtained from the beam FE and shell FE advanced analyses are shown in Fig. 16(b). It can be seen that there is a good agreement

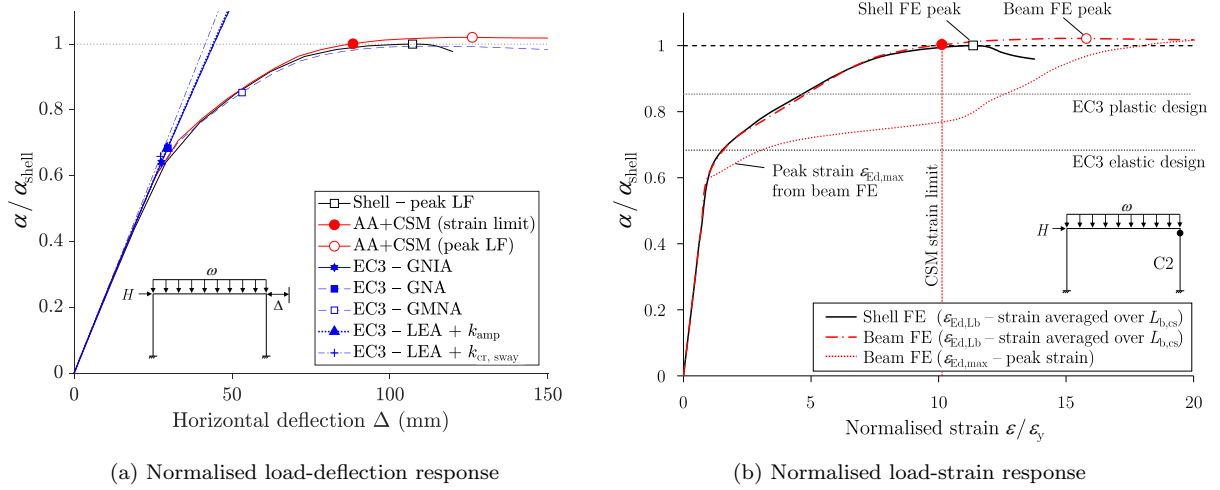


Figure 16: Load-deformation responses of frame $8 \times 4\text{-HEB180-F-UDL}$ with $H/\omega L = 0.217$ predicted by benchmark shell FE model, advanced analysis using beam FE modelling with strain limits and various EN 1993-1-1 analysis methods, showing (a) normalised load versus horizontal deflections and (b) normalised load versus normalised maximum compressive strain at the top of column C2.

between the design strains $\epsilon_{\text{Ed,Lb}}$ (i.e. the maximum compressive strains averaged over the local buckling half-wavelength $L_{\text{b,cs}} = 300$ mm at the top of column C2) predicted by the beam FE and shell FE models for most of the load-deformation path. Close to the ultimate load factor of the shell FE model, the paths slightly diverge as the shell model experiences the development of local buckling and cross-section failure. It can be seen that the application of the CSM strain limit ($\epsilon_{\text{CSM}}/\epsilon_y = 10.2$ in this case) to the beam FE model also accurately predicts cross-section failure and prevents overprediction of the shell FE model failure load.

The effect of accounting for local moment gradients by using the strain averaging approach described in Section 3.2 is also shown in Fig. 16(b). Application of the CSM strain limit to the peak design strain $\epsilon_{\text{Ed,max}}$ at the top of column C2, as compared to the strain $\epsilon_{\text{Ed,Lb}}$ that is averaged over the local buckling half-wavelength $L_{\text{b,cs}}$ at the top of column C2, results in a capacity prediction that is 23% lower than that of the shell FE model, as shown in Fig. 16(b). Nevertheless, limiting the peak strain to the CSM strain limit still results in better capacity predictions compared to those obtained from the elastic Eurocode 3 design approaches, since some redistribution and, in some cases, strain hardening are allowed for. Eurocode 3 plastic design clearly provides a better capacity prediction than elastic design for the Class 1 frame considered in this example, as shown in Fig. 16, since the deformation capacity of the critical hinge is unrestricted until the member buckling check reaches full utilisation. Overall though, by far the most accurate representation of the shell FE model is obtained using the proposed design method with the developed strain averaging approach, which is able to capture the beneficial effects of local moment gradients and strain hardening.

6.1.3 Frame example 3: 8×4-IPE400-F-PL frame series

The 15 frames in the 8×4-IPE400-F-PL series (i.e. fixed base supports, an IPE 400 cross-section for C1 and C2 and applied point loads) are used to show the benefits of the proposed method of design by advanced inelastic analysis over current EN 1993-1-1 across the range of all four cross-section classes defined by EN 1993-1-1 [16]. For the considered frames, as the ratio of horizontal to vertical load decreases, the value of α_{cr} reduces and the classification of the IPE 400 column cross-section varies from Class 1 (under bending dominated loading) to Class 4 (under compression dominated loading). A summary of the normalised capacity predictions $\alpha_{ult}/\alpha_{shell}$ using alternative design methods is shown in Fig. 17. Firstly, it can be seen that the peak load factor reached in the beam FE advanced analysis overpredicts the capacity obtained from the shell FE model by around 10% in most cases. For very high α_{cr} values (i.e. almost only horizontal loading), the frame behaviour is bending dominated. As a result, the load-deformation curve of the beam FE model does not reach a peak load factor and is monotonically increasing. Application of the CSM strain limits consistently predicts the failure load of the shell FE models to within 5% for most loading conditions, as shown in Fig. 17. Conversely, the accuracy of the Eurocode 3 capacity predictions varies significantly, depending on the classification of column C2. Inelastic analysis and design provides accurate predictions for bending dominated frames, as shown for the frames with high α_{cr} values in Fig. 17. For lower α_{cr} values, the benefit of using plastic design are seen to be less significant, since the member buckling checks govern the capacity of the frame. Once column C2 is classified as Class 2, inelastic analysis is not permitted, as shown in Fig. 17. The step-wise capacity predictions resulting from the cross-section classification process are also clearly visible towards the lower range of α_{cr} values in Fig. 17.

The ability of the strain limits to allow for levels of redistribution in structures composed of IPE cross-sections that are commensurate with the local slenderness is shown in Fig. 18 with reference to frame 8 × 4-IPE400-F-PL with $H/2V = 0.95$. This frame is bending dominated and the column cross-section is Class 1. The development of the maximum section moment M at the bottom (hinge 1) and top (hinge 2) of column C2 according to different analysis methods are shown in Fig. 18. After the formation of hinge 1 at the base of column C2, the internal moments are redistributed as further load is applied to the frame. This is visualised in Fig. 18 by the change in gradient of the normalised load factor-moment curve for hinge 2 obtained from inelastic analysis methods. Eventually, the shell FE model fails in side-sway. Clearly, elastic analysis cannot capture this effect and failure is conservatively predicted by the member design checks just before the first hinge has formed.

The peak load reached in the beam FE advanced analysis model exceeds the peak shell FE result by 12%. The CSM strain limit, equal to $\epsilon_{csm} = 6.32\epsilon_y$ for this studied case, is reached at hinge 1 at an applied load level of $\alpha/\alpha_{shell} = 1.00$ and accurately predicts the failure load of the shell FE model. Note that the strain limit at the non-critical hinge 2 is eventually reached at an applied load level of

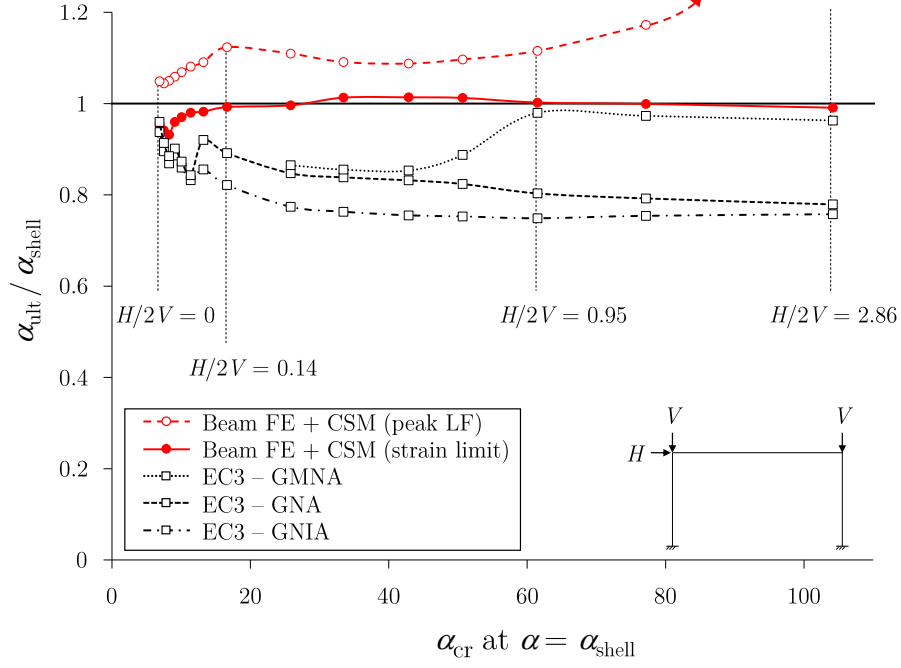


Figure 17: Normalised ultimate capacity predictions for 15 frames in 8×4 -IPE400-F-PL frame series.

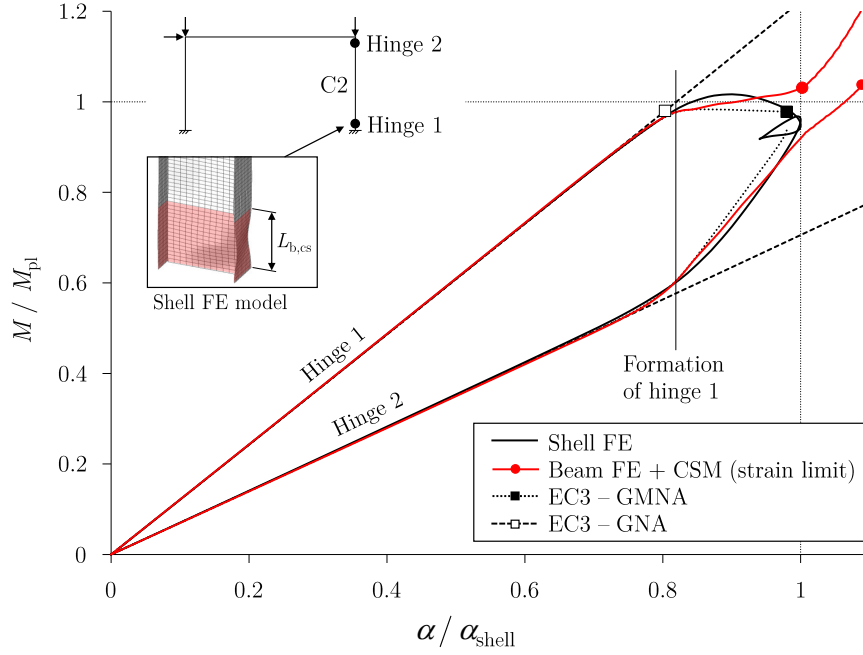


Figure 18: Development of maximum section moments at the bottom (hinge 1) and top (hinge 2) of column C2 with applied loading for frame 8×4 -IPE400-F-PL with $H/2V = 0.95$.

$\alpha/\alpha_{\text{shell}} = 1.09$, as shown in Fig. 18.

The full cross-section elastic local buckling stress $\sigma_{\text{cr,cs}}$ is typically determined with reference to the elastic stress distribution $\psi = \sigma_2/\sigma_1$, where the stresses σ_1 and σ_2 correspond to the maximum and minimum stresses across the cross-section and are determined from the applied section force and moment assuming elastic material behaviour. In steel frame structures, the forces and moments may be

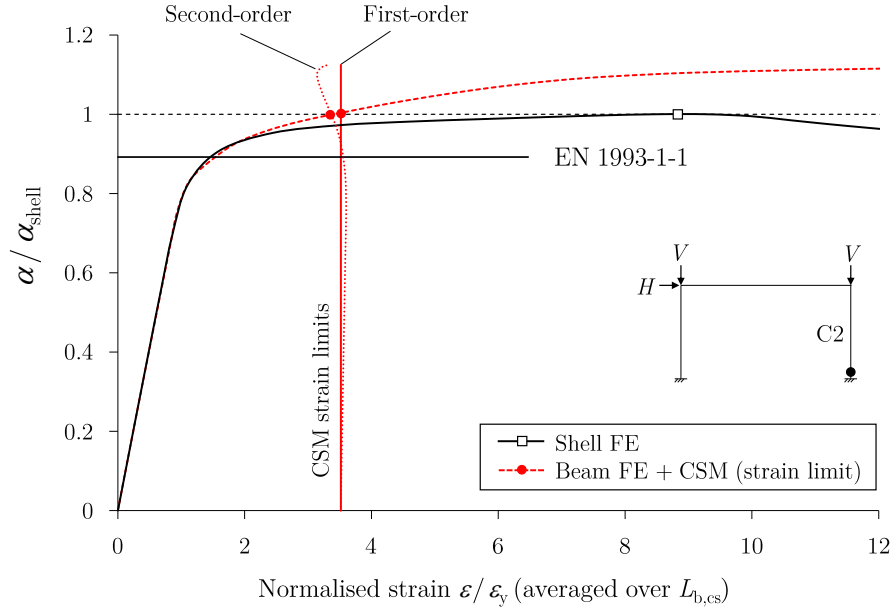


Figure 19: Normalised load-deformation response for frame $8 \times 4\text{-IPE400-F-PL}$ with $H/2V = 0.14$ from the shell FE and beam FE models, where the strains are averaged over the local buckling half-wavelength $L_{b,cs}$ at the base of column C2.

redistributed due to member buckling and/or the spread of plasticity. As a result, the stress distribution across the critical cross-section may change as the load level increases, which in turn would have a direct influence on the local slenderness and the available deformation capacity of the considered cross-section. Application of the first-order elastic CSM strain limits, though analogous to the current EN 1993-1-1 cross-section classification process, does not account for this effect. A slightly better representation of the deformation capacity of the cross-section can be achieved by considering the instantaneous stress state of the cross-section.

The effect of the changing stress distribution, due to force and moment redistribution, on the cross-section slenderness and deformation capacity is generally small for HEA and HEB profiles, which retain a relatively constant strain limit across the range of loading conditions from pure compression to pure bending, as was shown in Fig. 14. Depending on the frame configuration and loading, the change in applied stress distribution can have slightly more visible effects on the behaviour of IPE sections, as shown in Fig. 19 with respect to frame $8 \times 4\text{-IPE400-F-PL}$ with $H/2V = 0.14$. Failure of the frame is governed by local cross-section failure at the base of column C2 and eventually side-sway. According to EN 1993-1-1, the critical cross-section at the base of column C2 is Class 2. Hence, full plasticity can develop at this location, with the bending resistance limited by the plastic moment capacity, but the cross-section is considered to possess insufficient rotation capacity for plastic design to be used. This results in a capacity prediction that is 11% conservative compared to the shell FE model when using GNA with EN 1993-1-1 member buckling checks.

On the other hand, application of the CSM strain limit of $\epsilon_{\text{csm}}/\epsilon_y = 3.52$, obtained based on the first-order distribution of internal forces and moments at the base of column C2 in the beam FE model, accurately predicts the ultimate capacity of the shell FE model. Application of the CSM strain limit of $\epsilon_{\text{csm}}/\epsilon_y = 3.37$, based on the instantaneous (i.e. second-order) elastic stress distribution, accounts for the small increase in axial compression after the plastic hinge has formed, resulting in a small decrease in strain limit, as shown in Fig. 19. Note that the influence of considering second-order effects for the determination of the cross-section deformation capacity on the ultimate resistance prediction of the structure was found to be small for the analysed frames.

6.2 Multi-storey frame examples

6.2.1 Multi-storey frame example 1: Class 3 asymmetric frame

The proposed method of design by advanced inelastic analysis with strain limits is particularly useful for the design of frames composed of Class 3 cross-sections because allowance is made for partial plastic redistribution. This is illustrated with reference to the asymmetric frame shown in Fig. 20(a). The corresponding normalised load-displacement responses of the frame obtained using different analysis methods are shown in Fig. 20(b). The frame is sensitive to second-order sway effects with an elastic critical buckling load factor of $\alpha_{\text{cr}} = 3.10$ at the peak load factor of the shell FE model. Failure in the shell FE model is governed by inelastic local buckling at the top of column C4 and eventually side-sway. The deformed shape in the critical region of the shell FE model at the peak load factor is shown in Fig. 20(b).

According to EN 1993-1-1 [16], the column cross-sections are Class 3 and the beam cross-sections are Class 1, which requires the use of elastic global analysis of the frame. A similar load-carrying capacity is predicted by the different Eurocode 3 analysis and design methods. A maximum normalised load factor of $\alpha_{\text{ult}}/\alpha_{\text{shell}} = 0.84$ is predicted by GNA design, where the non-sway effective length factor of column C4 is taken as $k_{\text{cr}} = 0.745$, based on the limited frame method outlined in Appendix E of BS 5950 [47]. Using the results of a linear elastic analysis (LEA) and accounting for second-order effects through the sway effective length factor, taken as $k_{\text{cr,sway}} = 2.1$ for column C4, results in an overly conservative structural capacity prediction of $\alpha_{\text{ult}}/\alpha_{\text{shell}} = 0.68$.

In the proposed method of design by advanced inelastic analysis, the CSM strain limit is first reached at the top of column C4, the same location at which local buckling is first observed in the shell FE model. At this location, the CSM strain limit is $\epsilon_{\text{csm}}/\epsilon_y = 1.67$. Thus, in contrast to Eurocode 3, some plastification is allowed and failure is predicted at a load factor of $\alpha_{\text{ult}}/\alpha_{\text{shell}} = 0.94$, just 6% short of the peak load factor reached by the shell FE model and 12% above the best Eurocode 3 prediction. Note that if no strain limits were applied to the beam FE advanced analysis, the deformation capacity of the Class 3 cross-sections would be overpredicted and an unconservative peak load factor of $\alpha_{\text{ult}}/\alpha_{\text{shell}} = 1.05$ would be reached.

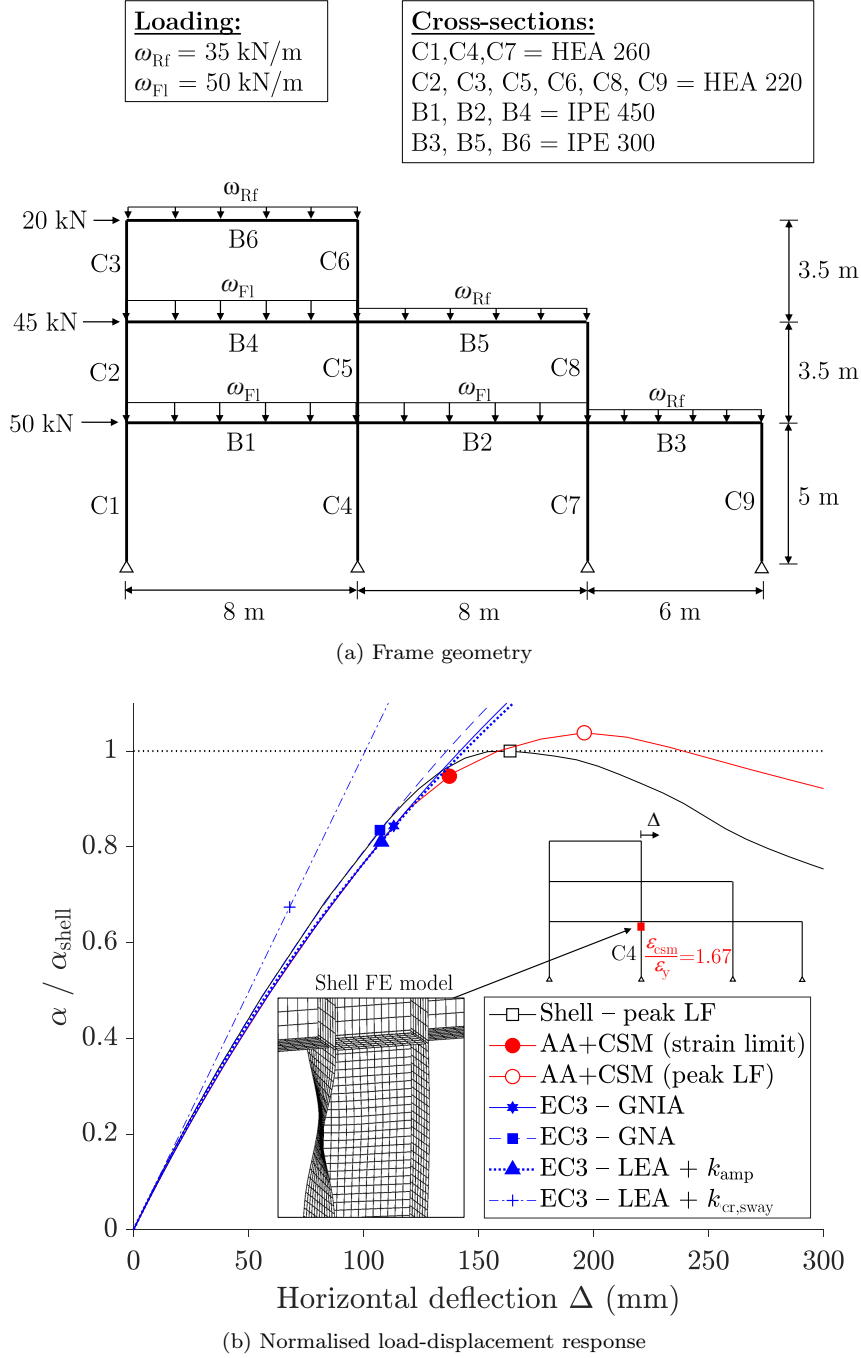


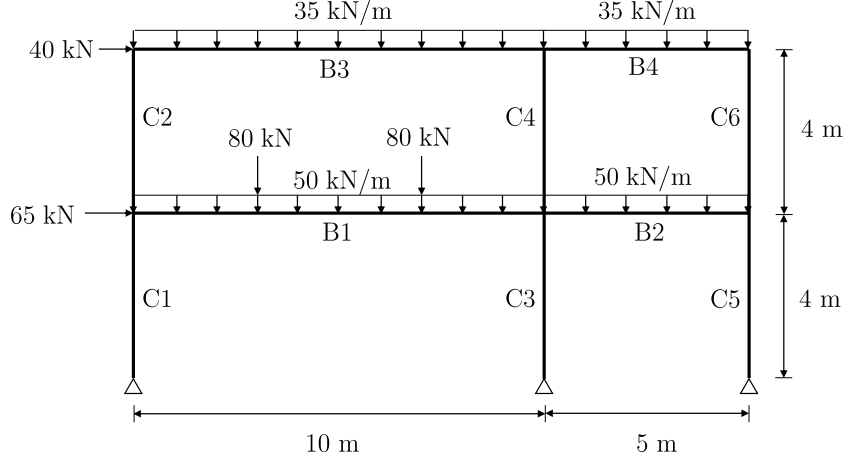
Figure 20: Application of the proposed method of design by advanced inelastic analysis with strain limits and elastic EN 1993-1-1 design approaches to an asymmetric frame with Class 3 cross-sections: (a) frame geometry and (b) normalised load-displacement response.

6.2.2 Multi-storey frame example 2: Prediction of failure mode

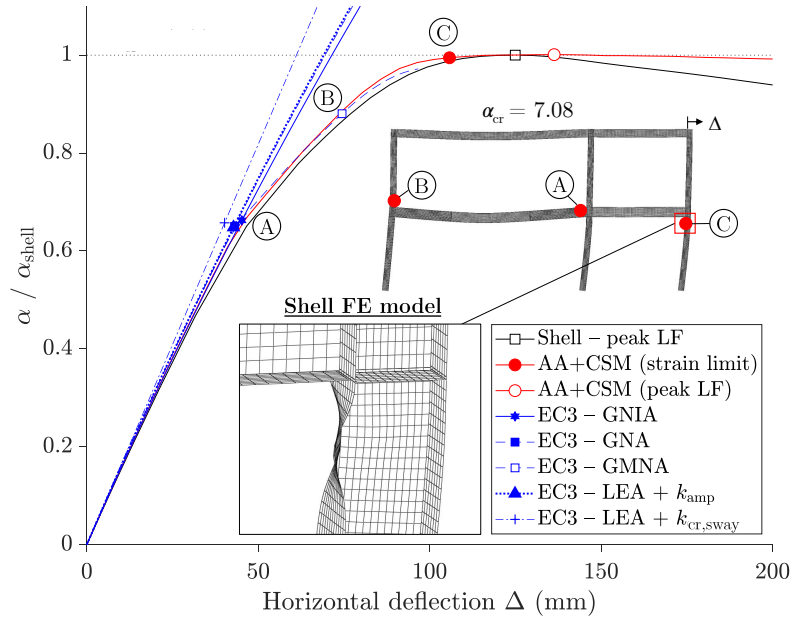
The ability of the proposed method of design by advanced inelastic analysis to capture the correct mode of failure is demonstrated with reference to the two-bay, two-storey (adapted from [48]) frame shown in Fig. 21(a). According to EN 1993-1-1, the IPE beams are Class 1, the HEB columns are Class 2 and the HEA columns are Class 3 cross-sections. Plastic design is therefore not strictly permitted for this frame

Cross-sections:

C1, C2 = HEA 320; C3, C4 = HEB 300; C5, C6 = HEA 280
 B1, B2 = IPE 500; B3, B4 = IPE 400



(a) Frame geometry



(b) Normalised load-displacement response

Figure 21: Application of the proposed method of design by advanced inelastic analysis with strain limits and EN 1993-1-1 design approaches to a two-bay, two-storey frame: (a) frame geometry and (b) normalised load-displacement response.

as not all cross-sections at locations of plastic hinges are Class 1, but since significant redistribution can take place in the frame, plastic design is still considered. The frame is influenced by both geometrical and material nonlinearity, with $\alpha_{cr} = 7.08$ at the peak load factor reached by the shell FE model. Failure in the shell FE model is through a combination of inelastic local buckling at the top of column C5 and global side-sway; the failure mechanism of the shell FE model is shown in Fig. 21(b).

According to the elastic design methods, failure is predicted to occur at the right-hand end of beam B1, designated as point ① in Fig. 21(b). However, the beam cross-section is Class 1 and has substantial deformation capacity, with a normalised CSM strain limit of $\epsilon_{\text{csm}}/\epsilon_y = 7.62$. Significant redistribution of forces/moments can therefore take place, rendering the elastic design methods overly conservative. The inelastic redistribution of forces and moments is captured using inelastic analysis and design. As a result, the formation of a plastic hinge at location ① is tolerated and instead failure of the structure is predicted when yielding first occurs at the base of column C2 at point ② (recall that the HEA section of member C2 is Class 3). This results in a significant improvement in the prediction of the ultimate capacity of the frame, as shown in Fig. 21(b). However, both the shell FE model and the proposed method of design by advanced inelastic analysis with strain limits allow for some plasticity to develop at point ②, with the normalised CSM strain limit equal to $\epsilon_{\text{csm}}/\epsilon_y = 2.24$ at this location.

The proposed method of design by advanced inelastic analysis with strain limits accurately captures all of the key phenomena and, like the shell FE model, predicts failure at the top of column C5 (i.e. point ③ in Fig. 21(b)). The deformation capacity of the cross-section at this location is relatively low with $\epsilon_{\text{csm}}/\epsilon_y = 1.76$ and local buckling is first observed at this location in the shell FE model. This example highlights the fact that the application of strain limits allows for cross-section appropriate levels of redistribution and accurately predicts the ultimate capacity and failure mode of frames with a combination of Class 1, 2 and 3 cross-sections.

6.2.3 Multi-storey frame example 3: 10 storey frame

In this section, the proposed method of design by advanced inelastic analysis with strain limits is applied to the 10-storey frame (adapted from [49]) shown in Fig. 22(a). The response of the shell FE model is characterised as follows: first, yielding initiates at the right-hand end of beam B10, closely followed by plastic hinges forming at the right-hand ends of beams B13 and B16, as well as at the base of column C32 and the top of column C31. As loading continues to increase, plasticity also spreads through column C21. Eventually, collapse occurs when the ground floor storey fails in side-sway. At the ultimate capacity of the shell FE model, $\alpha_{\text{cr}} = 4.94$.

According to EN 1993-1-1 [16], all cross-sections in the frame are Class 1 and inelastic analysis and design can be used. In this case, the member buckling check of column C32 governs the design, although yielding is first observed at the right-hand end of member B10. Inelastic analysis and design yields an ultimate capacity prediction that is almost 10% above the elastic analysis methods, but the resulting capacity prediction is still considerably below that obtained from the shell FE model and the proposed design method. The elastic analysis and design methods are not able to capture the spread of plasticity throughout the frame; instead, failure is predicted when the plastic moment is reached at the right-hand end of member B10, which occurs at a load level that is approximately 25% lower than the ultimate load

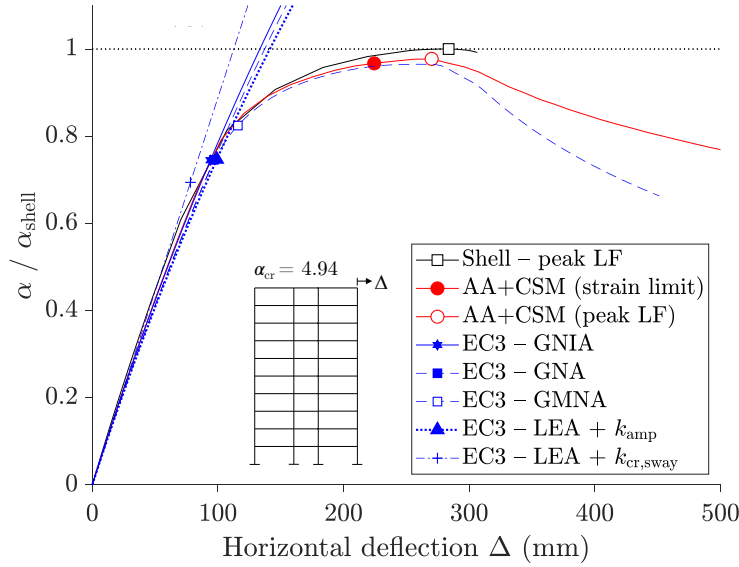
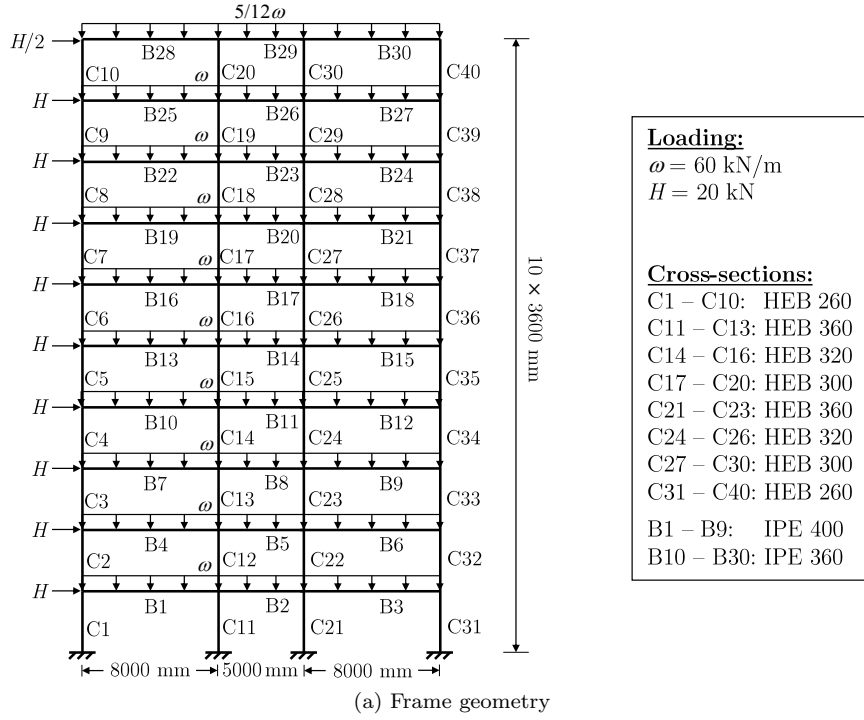


Figure 22: Application of the proposed method of design by advanced inelastic analysis with strain limits and EN 1993-1-1 design approaches to a 10-storey frame: (a) frame geometry and (b) normalised load-displacement response.

reached during the analysis of the shell FE model. In the Eurocode 3 $k_{cr,sway}$ method (i.e. performing linear elastic analysis and using increased effective lengths for the member checks), the member design check for column C32 governs the predicted frame capacity. However, when using an increased effective length factor of $k_{cr,sway} = 1.84$, in conjunction with a C_{my} factor of 0.9, as recommended in EN 1993-1-1 for members with a sway buckling mode, the predicted frame capacity is conservative by almost 30% compared to the peak load factor reached by the shell FE model, as shown in Fig. 22(b). An improved

capacity prediction is obtained when using a C_{my} factor based on the ratio of the member end moments. In this case, similar to the other elastic design approaches, column C32 is not the critical element and instead failure is predicted when the plastic moment is reached at the right-hand end of beam B10.

In the proposed method of design by advanced inelastic analysis with strain limits, failure is predicted at $\alpha_{ult}/\alpha_{shell} = 0.965$, just 3.5% below the peak shell FE result. The first cross-section in the structure to reach its strain limit of $\epsilon_{csm} = 5.14\epsilon_y$ is at the base of column C32; the strain limit of $\epsilon_{csm} = 4.87\epsilon_y$ at the top of column C31 is reached almost at the same load level, confirming the visual observations in the shell FE model, where local deformations were observed at both of these locations.

7 Summary of key features and benefits of proposed design method

A summary of the key features and benefits of the proposed method of design by advanced inelastic analysis with strain limits is given below:

- Cross-section classification is replaced by cross-section slenderness dependent strain limits obtained from the Continuous Strength Method (CSM);
- No member buckling design checks (and corresponding effective lengths) are required;
- Spread of plasticity and member buckling are captured explicitly in the advanced inelastic analysis;
- CSM strain limits are used to define the point of cross-section failure due to local buckling in beam finite elements;
- The CSM strain limits are derived from the local slenderness of the full cross-section, accounting for element interaction between the individual plates;
- No calculations of effective section properties are required for slender Class 4 cross-sections;
- The structural response is predicted more accurately compared to simplified analysis methods used in traditional design;
- Explicit modelling of residual stresses is not required in the analysis; instead the effects of residual stresses are modelled through equivalent imperfections at the member level and are accounted for through the CSM strain limits at the cross-sectional level;
- Strain hardening is exploited rationally in sufficiently stocky/compact cross-sections through a standardised quad-linear material model for hot-rolled carbon steel;
- The effects of local moment gradients on cross-section stability are accounted for through application of the CSM strain limits to a design strain that is averaged over a characteristic length of member equal to the local buckling half-wavelength;
- The interaction between bending moment and moderate shear is captured by the strain averaging approach through a shear reduction factor, while the cross-section shear check from Eurocode 3 is retained for shear governed cases;
- Plastic redistribution is allowed for in a continuous way and controlled by the CSM strain limits, increasing with reducing cross-section slenderness (as compared to the traditional approach of full redistribution for Class 1 cross-sections and none for cross-sections of other classes);

- A consistent advanced inelastic analysis design framework using computationally efficient beam finite elements can be used for structures composed of cross-sections of any class, whereas previously this was only possible using more sophisticated shell finite element models. A maximum local slenderness $\bar{\lambda}_p$ of 1.0 is currently recommended, pending further studies.

8 Conclusions

A new method of design by advanced (second-order) inelastic analysis with strain limits is presented and applied to structural steel systems. The design method is based on a geometrically and materially nonlinear analysis with imperfections and employs the Continuous Strength Method (CSM) strain limits to represent local buckling in beam finite elements. In the proposed design method, failure is defined as either the peak load reached during the advanced inelastic analysis or the load level at which the CSM strain limit is reached, whichever occurs first. The proposed method is able to capture the beneficial effects of local moment gradients and predict cross-section dependent levels of force and moment redistribution. Thus, the proposed method can be employed within a consistent advanced inelastic analysis framework for structural systems composed of any cross-section class, whereas current design specifications restrict inelastic analysis methods to compact Class 1 cross-sections. Following the successful application of the developed design method to individual structural members [2], the present paper extends this design approach to continuous beams and planar frames. It is shown that the proposed method is consistently more accurate than current Eurocode 3 design approaches in predicting the response obtained using the more computationally expensive shell FE models. Research is currently underway to extend the design method to tapered members and structures made from stainless steel members.

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