

**Finite Element Solutions for Transient
Fluid-Structure Interaction**

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by

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To
my Mother and Father,
for their
care and support

Errata

<i>Page No.</i>	<i>Line (From Top)</i>	<i>Reads</i>	<i>Replace with</i>
1-2, 2-8, 2-14, 3-28, R-11	7, 17, 33, 14, 20	Bettes	Betess
2-1	17	DuBatt	Dubuat
2-1	28	Dubuatt	Dubuat
2-3	12	resolved	solved
2-5	6	later	latter
2-9	6	a solution of equations	an inversion of the global matrix
2-17	25	poisson	Poisson
2-24	15, 20	Bleigh	Bleich
2-26	7	approach	approaches
2-28	19	Recently	recently
3-3	10	referential system	Referential System (R.S.)
3-3	26	$\Omega_x \subset \mathfrak{R}^{\mathbb{H}}$	$\Omega_x \subset \mathfrak{R}^{n_{sd}}$
3-11	5	the continuity Equation	Equation 3.34
3-14	6	R	
3-17	7	elements	elements'
3-17	10	configuration	configurations
3-18	19	is a	may be expressed as a
3-24	9	in literature	in the literature
3-28	4	time	times
3-51	4	which	these
3-51	14, 15	Eucledian	Euclidean
3-65	1	analysis	analyses
4-1	4	numerical	numerically
4-10	10	heavy side	Heaviside
4-24	15	quarter	half
4-25	1	$d_{st} = \frac{r_0^2 P_0}{t E_s} \frac{6 - v_s}{2}$	$d_{st} = \frac{r_0^2 P_0}{t E_s} \frac{1 - v_s}{2}$
4-25	27, 28	quarter	half
6-6	16	included	include
R-1	3	Wood W.L.	Wood W.L. (1983)
R-7	13	Gvilds	Gvildys
R-8	11	Geers T.L.	Geers T.L. (1987)

ABSTRACT

The objective of this thesis is to develop numerical tools capable of modelling the nonlinear behaviour of bounded and unbounded transient fluid-structure interaction systems. Towards this end, a numerical approach based on the finite element method was developed and implemented into a general purpose computer program ASAS-NL.

Four main developments are reported :

- (i) The Mixed-Eulerian-Lagrangian description of the continuum has been developed to account for the nonlinear effects of fluid-structure interaction systems and a mesh rezoning scheme derived to be used with it. In addition a predictor-multi-corrector time marching scheme has been used for nonlinear dynamic analysis and implicit temporal integration schemes based on Newmark and α -Bossak methods have been reviewed and implemented. Nonlinear iterative schemes based on the Modified Newton Raphson and full Newton Raphson methods have also been included.
- (ii) Two-dimensional and axi-symmetric fluid finite elements were developed. The elements are compatible with the Mixed-Eulerian-Lagrangian description of the continuum. In addition the free surface gravity wave (sloshing effect) has been addressed. The purpose of these elements is to model the bounded fluid medium in fluid-structure interaction problems.
- (iii) Two-dimensional and axi-symmetric Mixed-Eulerian-Lagrangian four-noded sliding interface elements have been developed. The purpose of these elements is to prevent artificial penetration of the fluid into the structure during analysis of fluid-structure interaction problems.
- (iv) A non-radiating boundary has been developed. This is to be used for modelling the unbounded fluid medium in fluid-structure interaction problems. Furthermore, numerical techniques for modelling shock waves were reviewed and included in the analysis.

This new analytical formulation has been applied to several problems for which solutions are available to prove its versatility, accuracy and efficiency and has been shown to give satisfactory results for the cases examined.

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NOTATION

General Rules

- i- All symbols and operators are at least defined where they first appear. Usually only one meaning is assigned to each symbol; however, in cases where more than one meaning is possible the correct one will be evident from the context in which it is used.
- ii- Generic symbols of matrices and vectors are represented with a tilde below the variable name (e.g. $\underline{M}_s$, $\underline{C}_s$, $\underline{K}_s$).
- iii- Left hand superscripts denote time while right hand superscripts usually denote iteration number (e.g. ${}^t + \Delta t \delta \underline{y}^j$).
- iv- Unknown values in the final set of equations are represented by **bold font-type** characters with left and right side superscripts denoting time and iteration number respectively (e.g. ${}^t + \Delta t \delta \underline{a}^j$).
- v- Equations are identified by numbers located on the right-most margin and composed of up to three entries (e.g. 3.17a) with the first entry indicating the chapter in which the equations appear.

Operators

Δ	: Total incremental operator for variables, vectors and matrices.
$\underline{\nabla}$	: Gradient of a vector.
$\tilde{\partial}$	: Partial differentiation.
δ	: Iterative incremental operator for variables, vectors and matrices.
δ_{ij}	: Kronecker delta.
$\times$	: Vector cross product.

$[\]$	: Encloses terms of a matrix.
$\int_a^b$	: Integration operator with limits varying from a to b.
$\subset$	: Subset symbol.
$\sum_{i=1}^n$	: Summation over variable (i) ranging from (1) to (n).
$\ \underline{a} \ $	: Norm of vector $\underline{a}$.
$\ \delta \underline{a}_{EUC} \ $	: Euclidean norm of vector $\underline{a}$
$\ \delta \underline{a}_{MAX} \ $	: Maximum norm of vector $\underline{a}$.
$\cdot$	: Partial material time derivative of a function.
$\cdot_t$	: Partial spatial time derivative of a function.
$\cdot'$	: Partial referential time derivative .
$\cdot_i$	: Partial material spatial derivative in the i^{th} direction.
$\cdot_i$	: Partial mesh spatial derivative in the i^{th} direction.
$\cdot_i'$	: Partial referential spatial derivative in the i^{th} direction.
H	: Heavyside operator.
T	: Right side superscript, transpose sign.
-1	: Right side superscript, inverse sign.

Symbols

α	: Lagrange-Euler parameter.
$\alpha_B, \beta_B, \gamma_B$	: Time integration parameters for α -Bossak integration scheme.
β	: Bulk modulus of fluid.
β_N, γ_N	: Time integration parameters for Newmark's integration scheme.
$\epsilon_{x1}, \epsilon_{x2}, \epsilon_{x3}$	: Strain in the three global directions.
ϕ	: Function describing material motion.
$\hat{\phi}$	: Function describing mesh motion.
Γ_{ST}	: Fluid-structure interface boundary.
Γ_{FS}	: Free surface fluid boundary.
$\Gamma_{\bar{d}}$	: Boundary with prescribed displacements.
$\Gamma_{\bar{t}}$	: Boundary with prescribed traction.
Γ_{RA}	: Non-radiating boundary.
τ_{ij}	: Components of Cauchy stress tensor.

Φ	: Displacement potential.
Φ_{inc}	: Displacement potential of the incident step wave.
κ	: Parameter defining penalty on rotation.
κ_c	: Average curvature.
κ_x, κ_y	: Curvatures in the two global directions.
λ, G	: Lamé constants.
μ	: Kinematic Viscosity.
ρ	: Density.
$\sigma_{\text{inc}}, \varepsilon_{\text{inc}}$	: Stress and strain due to incident step wave.
τ_{nd}	: Non-dimensional time.
ϖ	: Rotation spin tensor.
$\Omega_x, \Omega_y, \Omega_z$	: Material, spatial and referential domains respectively.
ξ, η	: Local coordinates of two-dimensional and axi-symmetric finite elements.
ψ	: Function describing mapping between the material and the referential coordinates.
ψ_a	: Weighting functions at node a.
ζ	: Local coordinates of one-dimensional elements.
A	: Matrix of Lagrange-Euler parameters.
$\dot{a}$	: Referential time derivative of the material velocity vector.
B_a	: Strain displacement matrix at node a.
B_v	: Vorticity transformation matrix.
b	: Prescribed body force.
C, C_{x_1}, C_{x_2}	: Independent coefficients matrix used in the derivation of the shape functions.
c	: Convective velocity.
c_i	: Speed of sound in the i^{th} direction.
c_o	: Speed of sound.
D_μ	: Material Damping matrix.
$\underline{d}, \underline{v}, \underline{\dot{v}}$	: Material displacement, velocity and acceleration respectively.
$\hat{\underline{d}}, \hat{\underline{v}}, \hat{\underline{a}}$	: Mesh displacement, velocity and acceleration respectively.
$d_{\text{FN}}, d_{\text{SN}}$	: Normal fluid material and structural velocity respectively.
e	: Volume expansion.
F_b, F_t, F_{σ_0}	: Forces due to body, traction and initial stress respectively.
F_{ext}	: Sum of forces due to applied body, traction and initial stresses.

$E_{\text{int.str}}$	: Internal residual force vector due to structural nonlinearity.
E_{int}	: Internally balanced load vector.
E_{un}	: Out of balance load vector.
$f_{\text{inf}}, p_{\text{inf}}, v_{\text{inf}}$	: Total fluid force, pressure and velocity vectors acting on the wet surface of the submerged structure.
$f_{\text{sc}}, p_{\text{sc}}, v_{\text{sc}}$	: Scattered force, pressure and velocity vectors respectively.
g	: Gravitational acceleration.
∇v	: Velocity gradient matrix.
h_a	: Matrix of shape functions at node a for Plane Wave Approximation elements.
I	: Identity array.
$I_{\xi\eta}, I_{\xi}$	: Integral expression in local coordinates.
$I_{x_1x_2}$	: Integral expression in global coordinates.
J, J^{-1}	: Jacobian matrix and its inverse.
j_a^y	: The Jacobian of the ϕ mapping evaluated at node a.
K_c	: Fluid compressibility (stiffness) matrix.
K_{local}	: Local sliding element matrix.
K_{sl}	: Parameter defining penalty on normal movement between fluid and structure at fluid structure interface.
K_{sl}^G	: Global sliding element matrix.
K_{slf}	: Sum of fluid and sliding element stiffness matrices.
K_T'	: Sum of fluid, structural and sliding element stiffness matrices.
K_V^K	: Rotational stiffness matrix.
K_μ	: Viscosity matrix.
$L, \mathcal{L}$	: Matrices used in the rezoning scheme between the material and mesh velocities.
$l_{ab}, \mathcal{L}_a$	: Nodal matrices
$l_e, \mathcal{L}_e$	: Element matrices
$M_{\text{eff}}, F_{\text{eff}}$	: Effective mass and force matrices respectively.
M_f, C_f, K_f	: Fluid mass, damping and stiffness matrices, respectively.
M_s, C_s, K_s	: Structural mass, damping and stiffness matrices, respectively.
M_T, C_T, K_T	: Sum of fluid and structure mass, damping and stiffness matrices, respectively.
n	: Unit normal (positive outward).
NELEM	: Total number of elements in a finite element mesh.
NEN	: Number of element nodes.
NNP	: Total number of nodal points in a finite element mesh.
N_a	: Matrix of shape functions at node a.

N_a, N_a^g	: Shape functions, at node a , used in the interpolation of the element kinematic and geometric properties respectively.
$\underline{N}_v$	: Convective matrix.
n_{sd}	: Number of spatial dimensions.
O	: Origin.
PF_a	: Position functions at node a .
$\underline{p}$	: Hydrodynamic pressure in fluid.
$\underline{p}_{ff}, \underline{v}_{ff}$	: Free field pressure and velocity vectors respectively.
$\underline{p}_{inc}, \underline{v}_{inc}, \underline{d}_{inc}$	: Incident pressure, velocity and displacement vectors respectively.
p_o	: Pressure jump of incident pressure wave.
$\underline{p}_q, \underline{p}_0$	: Polynomial matrices used in the derivation of the shape functions.
R	: Error or residual function of the governing differential equation.
R_1, R_2, R_3, R_4	: Required normal reactions between fluid and structure.
R_c, d_{2c}	: Local normal reaction and displacement at point c of sliding element.
S	: Wet surface of structure.
T	: Absolute temperature.
$\underline{T}_t$	: Transformation matrix between local and global coordinates.
t	: Time variable.
$\underline{f}$	: Prescribed traction force.
$\underline{l}_n$	: Matrix of normal cosine directions.
V_o	: Magnitude of velocity jump of incident velocity wave.
$\underline{w}, \dot{\underline{w}}$	: Referential displacement and velocity respectively.
w_i	: Weighting functions used in the numerical integration of integrands.
x_1, x_2, x_3	: Material coordinates in the three global normal directions.
x_{ij}	: x_i coordinate of the j^{th} node.
x_{nd}	: Non-dimensional coordinate in direction of wave propagation.
y_1, y_2, y_3	: Spatial coordinates in the three global normal directions.
z_1, z_2, z_3	: Referential coordinates in the three global normal directions.

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CHAPTER ONE

INTRODUCTION

The past few decades have seen the emergence of ever more complex structures built to satisfy mankind's expanding needs and desires. Advanced structures not only on land and in the air, but floating on the oceans and submerged beneath them, are being conceived and built in increasing quantities.

These developments have increased the demand for more versatile and efficient tools for structural analysis and design. Design, in turn, needs to be both economical and safe. In many cases, economy is achieved by the utilisation of the reserve strength well beyond the elastic limit. Safety, on the other hand, requires the accurate estimation of the plastic capacity and large displacement response. New analytical tools, therefore, should be capable of dealing with these aspects of behaviour.

Increasingly, these tools are also required to deal with the various kinds of transient loading (seismic, impact or shock forces) acting on the different kinds of structures (e.g. nuclear power plants, dam-reservoir systems, pipe-fluid systems, pressure vessels, aerospace structures, submerged and floating structures) to which they are being applied. Many of these problems may be classified as transient fluid-structure interaction problems.

The satisfactory solution of such dynamic fluid-structure interaction problems requires significant developments in structural and fluid modelling as well as in nonlinear solution strategies and techniques. Such is the general background to the nature of the complex problems treated within this thesis.

In this first chapter, the applications of fluid-structure analysis are briefly discussed. The general ingredients for the numerical modelling of such systems are outlined and outstanding problems are highlighted. Finally, the objective and lay-out of this thesis is presented.

1.1 DEFINITION AND CLASSIFICATIONS OF TRANSIENT FLUID-STRUCTURE INTERACTION PROBLEMS

Fluid-structure interaction occurs whenever relative motion of the two phases occurs. Clearly the treatment of such problems in the broadest sense involves all aspects of solid and fluid mechanics knowledge. In practice, however, it is possible to categorise the behaviour into groups in which various simplifications can be made. Following Zienkiewicz and Bettles (1978), it is possible to classify fluid-structure interaction problems into three main categories:

- (i)- Problems of long duration with limited fluid motion such as the response of structures to waves or earthquakes. In such cases fluid wave effects are important.
- (ii)- Problems of short duration with limited fluid motion such as those involving an explosion or impact. Usually in such cases fluid inertia is much greater than that due to the structure. An example is an explosion in a reactor vessel or an incident shock wave impinging on a submerged structure.
- (iii)- Problems of large relative motion, usually governed by flow characteristics. A typical example is the interaction occurring in flutter of aircraft wings .

In this investigation, only problems lying within the first two categories will be considered.

1.2 EXAMPLES OF TRANSIENT FLUID-STRUCTURE INTERACTION

Recently, the field of fluid-structure interaction has been attracting wide attention and the range of examples of practical interest in the civil engineering field is expanding rapidly. Some examples are given below.

1.2.1 Liquid Storage Tanks

Analysis of liquid storage tanks under earthquake loading includes the estimation of the pressure acting on the walls of the tank due to the fluid sloshing and embraces the actual nonlinear response of the container. Fluid-structure interaction analysis has proved to be necessary for determining an accurate transient response of such systems. Liu (1981) and Liu and Ma (1982) developed and used fluid-structure

interaction programs to study such problems, while Maheri and Severn (1986) carried out several tests to investigate the hydrodynamic effects in steel, liquid storage tanks.

1.2.2 Dam Reservoir Systems

Analysis of dams under the effect of shock and earthquake loading is a particularly important problem in view of the catastrophic consequences of dam failure. The state of knowledge of the earthquake behaviour of dams is not as advanced as our knowledge of earthquake effect on buildings. Furthermore, very few major dams, in comparison to buildings, have been subjected to earthquakes, so there is little experience to serve as a guide.

Fluid-structure interaction analysis has been widely used to determine the seismic response of dam reservoir systems. Recently El-Aidi and Hall (1989) studied the nonlinear response of concrete gravity dams, whereas Sarma (1990) studied the two-dimensional response of an earth dam subjected to horizontal ground motion. More recently, Dominguez and Maeso (1992) studied the seismic response of arch dams.

1.2.3 Nuclear Reactors and Pressure Vessels

Tanks in nuclear power plants may be analysed under different loading scenarios (e.g. explosion, impact, or earthquake). In this field, due to the disastrous consequences of failure as well as the cost of such projects, it is very important to obtain an accurate prediction of the behaviour of the structure.

In many such problems, there is a possibility that the spectral content of the fluid and the structure overlap. In such a case, resonance may occur. Belytschko and Kennedy (1978) studied the response of the subassembly of a nuclear reactor, whereas Shantaram et al (1976) and Donea et al (1982) studied the elasto-plastic response of a fluid filled pressure vessel.

1.2.4 Floating and Submerged Structures

The response calculation of submarines and other submerged and/or floating structures subjected to underwater explosive shocks is amongst the most important applications of fluid-structure interaction analysis. In this case, it is essential to have an accurate and efficient tool for modelling the far-field domain since only then can the designer arrive at reliable values of the pressure acting on the structure.

Ranlet et al (1977) studied the elastic response of submerged shells with internally attached structures to shock loading, while Rehak et al (1985) developed interactive approximations for a cavitating fluid around a floating structure.

More recently, Huang (1986) developed numerical tools to model the interaction of pressure pulses with submerged structures. Finally Dow (1992) presented experimental and theoretical response predictions of a steel stiffened plastic ship deckhouse subjected to blast loading.

In this investigation, problems lying in the first, third and fourth application categories described above are analysed.

1.3 ASPECTS OF FLUID-STRUCTURE ANALYSIS

The analysis of the general transient response of structures containing fluids as well as floating and submerged structures subjected to shock waves may be divided into the following four components:

- i- Modelling of the structure including large deformation and plasticity effects.
- ii- Modelling of the fluid medium contained within the structure or surrounding it.
- iii- Modelling of the interface between the fluid and the structure.
- iv- Modelling of the far-field domain.

Over the last two decades, significant attention has been devoted to the development of computational methods for interactive fields such as bounded and unbounded fluid-structure interaction problems. Both finite difference and finite element methods have been used to predict coupled fluid-structure interaction response. Notwithstanding recent advances in this field, various aspects of the problem are not adequately modelled and therefore require further research.

1.3.1 Outstanding Problems

- i- Development of fluid elements based on an accurate and efficient kinematic description of motion, capable of taking into account:
 - a- Sloshing effects as in the case of water tanks subjected to earthquake loading, or any fluid free surface.
 - b- Large mesh distortions due to the fluid motion.

- c- Cavitation in the fluid medium.
- ii- Development of efficient and accurate sliding elements to model the interface between the fluid and the structure.
- iii- Development of an efficient and accurate non-radiating boundary capable of modelling the far-field fluid medium, taking into account:
 - a- Incident waves impinging on the exterior of the boundary due to underwater explosions (Figure 1.1).
 - b- Waves radiating away from the structure and impinging on the inside of the boundary due to the vibration of the structure in the infinite medium (Figure 1.1).
- iv- Development of efficient and accurate solution procedures in transient fluid-structure interaction problems.
- v- Development of efficient and accurate formulations to model thin and thick shells and other structural elements.

In this study, contributions are made to the first, second, third and fourth of the above categories of problems and implemented within a computer code.

1.4 OBJECTIVE AND LAYOUT OF THESIS

1.4.1 Objectives of Thesis

The general objective of this thesis is to contribute to the development of knowledge and understanding of dynamic fluid-structure interaction problems pertaining to the field of civil engineering applications. In particular to develop numerically based techniques which can be incorporated within the nonlinear ASAS code developed jointly by WS Atkins and the Civil Engineering Department of Imperial College.

Information about the various numerical techniques used for solving fluid-structure interaction problems is relatively plentiful if somewhat scattered throughout the literature. Furthermore, several fluid-structure interaction codes exist and the need for a new one is not immediately obvious. Thus the first particular objective of this thesis is to review the various approaches currently used to solve fluid-structure interaction problems and identify any areas of weakness.

Most fluid-structure interaction programs employ either a Lagrangian or a Eulerian kinematic description of motion for the fluid. For many problems, neither approach is accurate enough. Hence, the second particular objective is to develop an efficient and

accurate kinematic description of motion for the fluid medium and to implement it into a computer code.

For modelling submerged structures in an infinite fluid medium, most codes extend the finite element mesh to a large distance away from the structure to avoid artificial reflections. This leads to excessive computational time for the transient nonlinear analysis of fluid-structure systems. Therefore, the final particular objective is to develop an accurate and efficient non-radiating boundary to be used in the analysis of fluid-structure interaction systems.

These formulations will be applied to several fluid-structure interaction problems in order to demonstrate their efficiency, versatility and applicability.

1.4.2 Layout of Thesis

Chapter Two contains an introduction to the analysis of fluid-structure problems with particular reference to the parameters affecting the numerical treatment of such systems. This is followed by an extensive literature review on near-field and far-field fluid-structure interaction formulations. In the near-field the various formulations are classified according to kinematic description of motion, i.e. Lagrangian, Eulerian or Mixed-Eulerian-Lagrangian. Furthermore, a summary of existing software is presented and areas where research should be directed are outlined.

Chapter Three presents the formulation of the Mixed-Eulerian-Lagrangian kinematic descriptions of motion for the fluid medium. Two-dimensional and axi-symmetric fluid elements are developed. To avoid spurious oscillations of the fluid element, a penalty on the rotation term is introduced. In addition two-dimensional and axi-symmetric sliding elements are developed to model the fluid-structure interface. In order to model sloshing, free surface gravity waves are implemented. The developed formulation is validated by testing several wave propagation and bounded fluid-structure interaction problems.

Chapter Four presents the formulation of the Plane Wave Approximation to model the infinite fluid medium. Again, two-dimensional and axi-symmetric elements are developed. Numerical techniques for modelling shock waves are developed and implemented into the finite element code. The proposed formulation is validated by testing several unbounded fluid-structure interaction problems.

Chapter Five contains several examples of practical fluid-structure interaction problems to demonstrate the applicability of the program.

Finally, in Chapter Six some general conclusions are drawn from the work . Areas where future research would be beneficial are also identified.

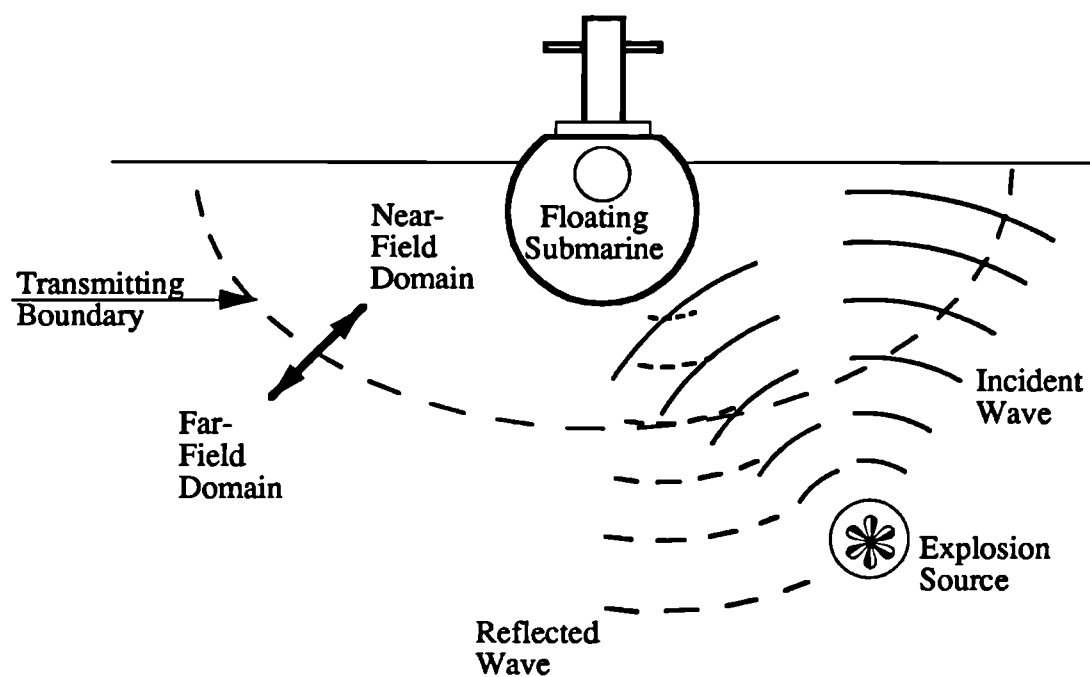


Figure 1.1 Ingredients of a General Fluid-Structure Interaction Model

CHAPTER TWO

PROBLEM DEFINITION AND LITERATURE REVIEW

In this chapter, the basic options encountered in the numerical treatment of fluid-structure systems are reviewed. Furthermore, the various numerical formulations are reviewed and their advantages and disadvantages are highlighted. Special emphasis is placed on the kinematic description of motion for the near-field fluid continuum and on modelling the infinite fluid medium. In attempting to highlight where current research should be directed, only the most widely used computer codes are reviewed.

2.1 HISTORICAL BACKGROUND

2.1.1 Early Research Methods

Over the last two decades, significant attention has been devoted to the development of computational methods for interactive fields such as bounded and unbounded fluid-structure interaction problems. In addition, the problem of calculating fluid pressures acting on solid bodies has been attracting wide attention. Although the methods used today are relatively recent, the problem itself is old.

One of the earliest studies of fluid-structure interaction dates back to 1779 when Pierre Louis Gabriel Dubatt, (Stelson and Mavis, 1957), published the results of his observations on submerged spherical pendulum bobs of lead, glass and wood vibrating in water. He noticed that the fluid effect on the solid was to increase its mass by approximately one-half the mass of the fluid that has been displaced. Since that time the effect of the surrounding fluid medium on the response of a submerged body has been the subject of many analytical and experimental investigations. In 1843, Stokes (Lamb, 1932) arrived at a qualitatively similar conclusion while studying the uniform acceleration of an infinitely long cylinder in an infinite fluid medium. He observed that the effect of the fluid on the response of the structure was identical to increasing its mass by an amount equal to the mass of the fluid displaced. Both authors derived formulas to calculate the added mass of the fluid. Despite the simplicity and efficiency of DuBuatts' and Stokes' results their equations of virtual

mass were limited to the geometry, loading and boundary conditions they were considering.

Based on the concept of added mass, Westergaard (1933), developed a procedure to determine the response of a rectangular dam subjected to earthquake excitation. The reservoir was assumed to extend to infinity and the flexibility of the dam was not taken into account. The shape of the pressure distribution in the water was found to be parabolic. Furthermore, the inertia forces exerted by an unknown added mass of water were considered to be identical to the water pressure acting on the structure. The results obtained indicated that the dynamic action of the water is neither excessively large nor negligible.

In a discussion that followed the publication of the above mentioned study, Brahtz and Heilborn, Jr. (Westergaard, 1933) examined, amongst other factors, the effects of the length of the reservoir and the flexibility of the dam on the response. By assuming the reservoir to have a finite length with a rigid wall at its end, the effect of the length of the reservoir on the response was assessed. Results showed that when a reservoir of finite length is assumed to be infinite, water pressures acting at the face of the dam are over-estimated. However as the length of the reservoir approaches twice its depth this effect becomes negligible. In order to study the effect of the flexibility of the dam on the response, only the base of the dam was assumed to move with the prescribed motion of the earth's crust. Results indicated that the flexibility of the dam does not play a significant role in the response calculation. Although the results obtained by the authors were incorrect, this was the first attempt to study the response of a dam-reservoir system taking into account the flexibility of the former.

Based on Brahtz and Heilborn Jr. method, Hoskins and Jacobsen (1934) published experimental and numerical results for water pressures in a rectangular tank subjected to earthquake loading. Jacobsen (1949) extended the above work to cylindrical geometries. He presented results for a cylindrical tank containing fluid and a cylindrical pier surrounded by fluid.

Housner (1957) developed an approximate method for calculating the pressures and free surface wave heights of liquids contained in rigid rectangular and cylindrical tanks excited by horizontal ground motion. In this approach, the total hydrodynamic pressure is assumed to be composed of two parts: impulsive pressure caused by a given portion of the fluid moving together with the rigid tank, and convective pressure caused by the sloshing motion of a given portion of the fluid. Since the tank is treated as rigid, the impulsive pressure was assumed to be in direct response to the

base acceleration. This rigid wall assumption enabled Housner to provide a simple and fast procedure for determining the seismically induced hydrodynamic forces exerted in the tank. He arrived at simplified formulas for containers having two fold symmetry, dams with sloping faces and flexible retaining walls.

Most of the methods cited above can be summarised as follows:

- i- The fluid pressures are calculated from hydrodynamic considerations, assuming the structure to be rigid.
- ii- The pressures calculated in the previous step are applied as forces on the structure in order to calculate its response.

2.1.2 Some Historical Events

At this point, the problem of near-field fluid-structure interaction appeared to be resolved and the overall validity of this simplified approach was accepted in design practice. However, as will be discussed in this section, several seismic structural failures forced designers and researchers to review their assumptions.

The scope of the seismic damage problem is broadly illustrated from various published reports relating to earthquake engineering (for example, Housner and Jennings, 1972; Steinbrugge, 1970 and Hanson, 1973).

In the Alaska earthquake of 1964 several petrochemical tank failures resulted in severe damage. At Anchorage, sixteen tanks were damaged of which eleven collapsed releasing combustible and aviation fuels. In the community of Whitter, the failure of several oil storage tanks resulted in the release of combustible liquids which caused large uncontrolled fires that burned for three days.

More significantly, in 1971, a destructive earthquake measuring 6.6 on the Richter scale struck San Fernando, California resulting in severe damage to several storage water tanks. Buckling of tank walls and lift-off of the anchored tanks from foundations were observed. Also extensive damage occurred to two major earth fill storage reservoirs which resulted in a partial loss of water supply.

2.1.3 Observations on Historical Events

Most of the above mentioned failures were suspected to be the consequence of dynamic buckling caused by excessive overturning moments of seismically induced

liquid inertia and surface sloshing. It was clear that the design of tanks based on Housner's and other similar methods, which uncouples the fluid and structure motion, may result in an unsafe structure especially if resonance between the fluid and the structure occurs. As a consequence of this, several coupled fluid-structure analysis techniques which use the finite element method emerged.

Before reviewing the various formulations employed in modelling the near-field and far-field fluid mediums, it is essential to discuss the various parameters to be considered when developing a computer code for the transient analysis of fluid-structure systems.

2.2 FLUID-STRUCTURE INTERACTION: AN INTRODUCTION TO NUMERICAL TREATMENT

In determining a method for the numerical solution of transient fluid-structure interaction problems, several fundamentally important decisions should be made. Amongst the major factors to be considered by the analyst are:

- i- Modelling of the near-field fluid medium.
- ii- Choice of the kinematic description of motion for the near-field fluid.
- iii- Modelling of the compressibility of the near-field fluid medium.
- iv- Choice of the independent nodal variables for the fluid mesh.
- v- Selection of appropriate time integration schemes.
- vi- Choice of an appropriate method to couple the solution of the fluid and the structure governing equations of motion.
- vii- Modelling of the infinite fluid medium.
- viii- Selection of type of program (general purpose versus special purpose computer codes) for the implementation of the proposed developments.

Other parameters such as the modelling of the structure and the choice of structural material and geometric nonlinearities are not mentioned above since they are not treated in detail in this study. In what follows a brief discussion on each of the above mentioned factors is presented. Although the effect of each of the parameters is discussed separately, their interdependence will become obvious from the discussion.

2.2.1 Modelling the Near-Field Fluid Medium

Semidiscretisation methods

Semidiscretisation techniques include both finite element and finite difference methods. In this approach the governing partial differential equations are discretised in space via an approximation to obtain a set of ordinary differential equations. The unknown variables are then determined by integrating the later equations in time.

Finite Difference Method

The major disadvantage of this approach is its requirement for uniform and regular meshes for an efficient modelling of a domain. Therefore, problems such as curved boundaries, gaps and slippage remain difficult to treat.

Finite Element Method

This method provides a greater versatility in modelling complex boundaries and geometries. In addition it is more easily coupled to a finite element structural code. However, it tends to be computationally more expensive (in terms of computer time and storage requirements) than finite difference methods.

Boundary integral techniques

In boundary integral methods, either Green's theorem or weighted residual techniques are used to reduce the dimension of the problem by one thereby reducing the computer processing time and storage requirements. Among the disadvantages of this approach are the following:

- i- Although the dimension of the problem is reduced the resulting equations are full, so the advantages of bandedness are lost.
- ii- In the case of geometric and material nonlinearities it becomes necessary to discretise the domain as well as the boundary. Therefore, the advantages in the reduction of the dimensionality of the problem are partially lost.

2.2.2 Kinematic Description of Motion for the Fluid Continuum

Either the Lagrangian or the Eulerian kinematic descriptions of motion may be used to model the fluid continuum. However, both possess some undesirable features. Recently, several finite element fluid formulations have been proposed based on a Mixed-Eulerian-Lagrangian description of motion. This method offers more

versatility but, unfortunately, it is more complex to develop and implement in a computer code.

Eulerian description of motion

In the Eulerian description of motion, the mesh is fixed in space and different sets of material particles pass through it. This method seems to be preferred for any flow in which the mesh would be highly distorted if required to follow the motion of the material. The major advantages of this approach are:

- i- Because of the fixed mesh, this approach can simulate very large fluid motion with accuracy and efficiency.
- ii- Most Eulerian fluid codes use scalar nodal variables which result in one unknown per node. This leads to a reduction in processing and storage requirements.

and the disadvantages are:

- i- Due to the spatial nature of the mesh, modelling of material boundary conditions such as slippage and contact surfaces is very difficult.
- ii- Convective terms, due to the relative motion between the fixed nodes and the moving material, result in unsymmetric matrices and introduce numerical difficulties.
- iii- Coupling with standard Lagrangian structural finite element programs is difficult.

Lagrangian description of motion

In this method, the mesh and the fluid material coincide; therefore, the nodal points refer to a unique set of material points. This method seems to be preferred for fluids contained within structures in which there is only small motion and when interfaces and free boundaries are important. The main advantages of this approach are:

- i- Lagrangian fluid elements are readily incorporated in structural computer codes.
- ii- The resulting global coefficient matrix is symmetric and positive definite.
- iii- For most Lagrangian codes where both the fluid and the structural variables are displacements, the interaction is solved through normal compatibility of the fundamental unknowns in both fields.

- iv- Modelling of arbitrary material boundary conditions is relatively easy.
- v- Stress and strain states can be accurately defined since the material points coincide with the finite element mesh and Gauss quadrature points throughout the deformation.

and the disadvantages are:

- i- Large displacements result in severe mesh distortions and reduce considerably the accuracy of the solution.
- ii- A complete technique of rezoning or remeshing may be needed at every time step to maintain a realistic shape for a highly distorted grid.
- iii- The use of vector nodal quantities can render the method prohibitively expensive for three-dimensional analysis.
- iv- Special numerical techniques have to be used to enforce the irrotationality of the fluid motion.

Mixed-Eulerian-Lagrangian description of motion

In this method the analyst can choose which mesh nodes should follow the motion of the material (i.e. Lagrangian), and which should be fixed (i.e. Eulerian). Alternatively, the nodes can be moved in any arbitrary manner or in such a way as to minimise distortion in the elements. Although this approach offers a more versatile description of the fluid continuum, it has several disadvantages:

- i- Convective terms are difficult to treat.
- ii- For each particular problem, suitable nodal grid descriptions should be specified.
- iii- The introduction of both material and mesh quantities as variables in the governing equations, results in more storage and processing requirements.

2.2.3 Compressibility of the Fluid

The appropriateness of the fluid-incompressibility assumption in fluid-structure interaction problems is mainly based on the time scale of the loading. If most of the pressure is generated within an interval significantly smaller than the acoustic wave transversal time over the fluid domain, then wave propagation effects may arise and a compressible fluid assumption is desirable. Whereas, if the time scale over which the load varies is greater than the acoustic transversal time, most of the movement of the fluid is inertial and an incompressible solution is sufficient.

It is essential to adopt a formulation capable of modelling, to a sufficient degree of accuracy, both types of phenomena.

2.2.4 Nodal Variables of Fluid Mesh

Among the first choices that an analyst faces is whether to adopt a vector formulation with the fluid-particle displacement as nodal unknown, or a scalar formulation based on the pressure, velocity potential or displacement potential as variables. The first option has the dual advantage of ease of integration into a structural finite element package and a symmetric form of equations of motion. However, it has several drawbacks: it does not enforce motion irrotationality and it requires three degrees of freedom per node (for three-dimensional structures).

On the other hand, the second option has the advantages of enforcing motion irrotationality and reducing the number of degrees of freedom per node. However, it results in a wide front of interaction which occurs without the characteristic banding of the algebraic equations for all nodal points on the fluid-structure interface. In addition, it yields unsymmetric equations of motion.

Regarding the choice between pressure, velocity potential or displacement potential no substantial differences exist for linear problems. Zienkiewicz and Bettles (1978) recommended the use of the pressure formulation due to its physical transparency. However, subsequent studies by Newton (1978) and (1980) showed that the displacement potential formulation, as well as the displacement method, excelled in the treatment of transient response problems in which cavitation occurs.

More recently, mixed pressure-displacement (Fenves and Vargas-Loli, 1988), pressure-velocity potential (Olson and Bathe, 1985a) and density-velocity potential (Nitikitpaiboon and Bathe, 1993) formulations have been developed. These have the advantages of resulting in symmetric matrices; however, the principal disadvantage of this method is that the number of fluid unknowns is increased.

2.2.5 Time Integration Schemes

Time integration schemes for the solution of the coupled equations of motion may be divided into two main categories: explicit and implicit methods. In both cases, a difference expression is used to relate the variables at discrete time steps to their time derivatives.

Explicit methods

If the finite difference expression for the unknown variables at a time step does not contain the time derivative of the variable at that time step, the equation is referred to as explicit. Amongst the frequently used explicit methods is the central difference method. An important advantage of this approach, when used in conjunction with lumped mass matrices, is that it does not involve a solution of equations, which results in a substantial reduction in storage and computation requirements. However, if the time step is very large, the computed response may grow in an unbounded manner.

Implicit methods

Stability difficulties can be alleviated by the use of implicit methods where the finite difference expression for a variable at a time step does contain its rate of change at that time step. However, this method requires the solution of the equations at every time step which results in large storage and computation requirements. Amongst the most popular and earliest implicit integration techniques is the trapezoidal method.

Explicit-explicit and implicit-implicit methods

In many cases it is desirable to use different time steps for the fluid medium and the structural mesh. The fluid mesh would have to be divided into two partitions to be treated separately in an implicit staggered manner. This procedure is often referred to as implicit-implicit partitioning. Another alternative is to integrate both partitions explicitly (explicit-explicit partitioning) with two different time steps.

A significant feature of implicit-implicit and explicit-explicit partitions is that the difference equations do not suffice to establish all the required variables in the time integration procedure. In an implicit-implicit technique, an extrapolation is required on the nodes of the interface between partitions, while in an explicit-explicit approach interpolation is required along the interface. These extrapolations and interpolations affect the stability of the time integration by imposing critical time step criteria.

Implicit-explicit methods

Although any fluid-structure mesh can in principle be integrated entirely using one method with a single time step, such computations are often computationally uneconomical. Mixed implicit-explicit methods can reduce the cost of the analysis by

treating the fluid mesh explicitly. The principal advantage of such time integration of the dynamic response is that no stiffness matrix need be calculated for the fluid and a relatively large time step can be employed in the incremental analysis. Another approach is to treat particular terms in the fluid governing equation, usually the free surface and convective matrices, as explicit and all the remaining terms as implicit. This results in a symmetric set of equations which leads to substantial savings in terms of computational and storage requirements.

2.2.6 Coupling Approaches in Fluid-Structure Interaction

Introduction

Until the early seventies, the state of the art approach for the analysis of most fluid-structure systems was to use an uncoupled two step procedure, as outlined in Section 2.1.1. The rationale behind this two step approach was its conservatism : it was widely believed that the pressures predicted on the rigid walls are greater than those acting on a flexible one. Therefore, if the structure can sustain the pressure calculated assuming rigid walls, it should be safe. However, this method suffers from serious drawbacks:

- In many cases it leads to overdesign due to the following reasons: (1) it does not take into account the reduction of pressure due to the flexibility of structure and (2) it does not account for the enhanced effective mass of the coupled fluid-structure system.
- On the other hand, if resonance between the fluid and the structure occurs, it can lead to unsound designs, (e.g. when this method was applied to the seismic analysis of liquid storage tanks, low loads on the fluid-tank system were predicted).

In response to these needs, several coupling procedures have emerged over the past decades. These are determined by the time integration scheme, nodal variables and the mesh description method used for the fluid mesh.

Complete coupled solution procedures

This method is mainly used in conjunction with an implicit time integration treatment of the governing fluid-structure equations. It originated by recognising that the essence of a stable numerical tool for fluid-structure interaction analysis lies in simultaneously solving the coupled equations with an implicit time integration

scheme. However, when analysing a typical three-dimensional fluid-structure system, the computer memory required by this approach may become prohibitively large. Several staggered solution procedures have been introduced to achieve an integrated treatment for the coupled fields by two single-field computer programs.

Weakly coupled staggered solution procedures

Weak couplings are those which are usually implemented in two completely separate programs, interconnected only through normal input output routines. This method was developed primarily for submerged structures in infinite acoustic mediums when the structural response is the major concern. However, for most of the combinations of time integration, it is not possible to maintain the numerical stability of the individual algorithms with weak couplings.

Strongly coupled staggered solution procedures

Strong coupling procedures are those in which there is sufficient interaction between the programs to maintain the same stability as that of the least stable part. In most cases this procedure cannot be implemented in completely separate programs.

The choice between the above three coupling approaches depends mainly on the programmer/client preference of stability versus minimal storage and computational processing time requirements as well as the characteristics (in terms of speed and available storage) of the available computational tools.

2.2.7 Modelling of the Infinite Fluid Medium

If an infinite fluid medium is modelled by a finite mesh, the presence of artificial boundaries may produce spurious reflections which contaminate the solution. It is desirable to be able to eliminate these reflections and thus simulate the effect of an infinite medium. The problem may be overcome by extending the mesh to a sufficient distance such that the response is obtained before the reflections arrive. However, this is not always possible, since the model size tends to render the analysis prohibitively expensive. Hence, the analyst must work with a smaller model and ensure that the results are not contaminated by artificial wave reflections.

Most of the methods used in simulating the effect of an infinite medium domain are based on one of the following approaches :

- i- Infinite element method.
- ii- Boundary element method.
- iii- Other methods based on acoustic approximations.

In the following, a brief discussion of each of the above mentioned methods will be presented with special emphasis given to techniques based on acoustic approximations.

Infinite element method

This method was originally based on using shape functions analogous to Lagrange polynomials but including an exponential decay term for elements which extend to infinity. Another approach is the mapping of the exterior domain onto the interior one by local mapping functions defined for each element. In this approach the only change required to a finite element routine is the derivation of the appropriate shape functions and corresponding Jacobian matrix.

Amongst the advantages of this technique is the ease of inclusion into a finite element program whilst still retaining a narrow bandwidth of the set of equations. However, its use in general transient problems is still limited.

Boundary element method

The boundary element method as a technique for modelling the near-field fluid has been discussed in Section 2.2.1. The advantages and disadvantages discussed in that section still apply when using the method for modelling an infinite fluid medium. Here it will merely be stated that this method is frequently used for modelling infinite mediums in frequency domain analysis, while its use in the transient analysis of fluid-structure systems is still limited.

Acoustic approximations

Acoustic approximations are differential equations based on the Retarded Kirchhoff Potential formulation. Three such approximations are in common use.

Early time approximations

The response predicted by this method approaches the exact one for early times in transient interactions. It has been established that this response also characterises the behaviour of high frequency surface motions; that is, motions whose period are small

compared to the acoustic wave lengths. Due to the local nature of this formulation, its implementation as boundary elements into a finite element code does not affect the bandedness of the resulting set of equations.

Late time approximations

The response predicted by this method approaches the exact one for late times in the transient response. In this approach, an added mass matrix of the infinite fluid medium is constructed using boundary elements. The major drawback of this approach is that the resulting added mass matrix is unsymmetric and fully populated.

Doubly asymptotic approximations

The response calculated from this method approaches the exact one in the limit of high and low frequency. Two one sided asymptotic expressions, one early time approximation and one late time approximation, are added together to form a corresponding Doubly Asymptotic Approximation. This method can be refined by using a wet surface fluid frequency matrix to enable it to model the response at a particular intermediate frequency or time. The major disadvantage of this method, as with the late time approximations, is that the resulting matrices are unsymmetric and fully populated.

2.2.8 General Purpose versus Special Purpose Computer Codes

The practical value and success of modern numerical techniques; such as finite elements, finite differences and boundary elements, are associated with the availability and use of high-speed electronic digital computers. The nonuniformity of the characteristics (in terms of speed, incore memory and disk space) of the available computational tools caused the development of several computer programs with various capabilities. These programs may be divided into two main categories:

- i- Large, general purpose computer codes capable of treating a wide variety of problems.
- ii- Comparatively small specialised computer codes which are able to analyse a particular class of problems.

As computer systems and numerical techniques develop, the criteria governing the advantages and disadvantages of the above methods may change and thus the need to review them becomes necessary. Comparing the two approaches the following comments may be made :

- i- General purpose programs are user oriented. An engineer does not have to be familiar with the finite element method, for example, to use such a code. Whereas special purpose programs, in most cases, do not have such a facility and require more knowledge about the finite element method from their potential user.
- ii- General purpose programs are able to solve several types of problems, whereas special purpose programs cannot.
- iii- General purpose programs are modular in nature which facilitates the addition of new analysis capabilities to the main programs. This is a desirable quality, since it permits the continuous improvement and extension of the program. On the other hand it is more difficult to add new analysis facilities to special purpose programs.
- iv- The size of general purpose programs decreases computer efficiency, since many unnecessary logical steps might be performed due to the architecture of the program. This does not happen with special purpose programs.

2.3 NEAR-FIELD FLUID-STRUCTURE INTERACTION

In this section, a literature review on near-field fluid-structure interaction will be presented. The various studies are classified according to the fluid formulation used. Special emphasis will be placed on the Mixed-Eulerian-Lagrangian Formulation.

2.3.1 Formulations Based on the Eulerian Description of Motion

The use of the finite element method for modelling fluids originated from a study in 1965 when Zienkiewicz and Cheung (Zienkiewicz and Taylor, 1989) presented a paper on the use of the finite element method in the solution of field problems which paved the way for its application in fluid problems. Selby and Severn (1972) developed a finite element formulation to calculate the hydrodynamic pressures induced in a reservoir. By ignoring the compressibility of the water, the hydrodynamic pressure in the reservoir was calculated from two- and three-dimensional solutions of the Laplace equation. These, in turn were used to determine the natural frequencies and mode shapes of the wall-reservoir system. The best agreement between experimental and theoretical results was obtained when the hydrodynamic pressures were determined from the three-dimensional solution of Laplace equation.

Zienkiewicz and Bettles (1978) derived the equations which govern the fluid

behaviour in the Eulerian formulation with the pressure used as the independent nodal variable. The fluid formulation was based on the Navier-Stokes equation and the continuity relation, while ignoring convective and viscous terms. Appropriate boundary conditions for modelling the fluid-structure interface and the free surface were derived. Several approaches for dealing with the non-symmetric form of the system of equations were suggested. Furthermore, possible simplifications due to the incompressibility of the fluid were discussed. Finally, the discretised coupled forms of the governing equations for the time domain, frequency domain and free vibration analysis were obtained. However, the proposed formulations were not validated.

Newton (1978) compared different finite element formulations using the displacement, pressure, displacement potential or velocity potential respectively as the main independent nodal variable. A one-dimensional example was used to assess the behaviour of each method. Numerical results showed that the accuracy of the finite element method in modelling cavitation was strongly dependent on the independent nodal variable used. Both the displacement potential and the displacement formulations were found to be most effective in treating problems with fluid cavitation.

Zienkiewicz et al (1983) developed and used the displacement potential formulation in studying the dynamic response of fluid-structure systems. Both the fluid and the structure were integrated in time using an implicit scheme. The proposed formulation was validated by studying the transient response of a two-dimensional dam-reservoir system subjected to earthquake excitation and blast loading. Numerical results showed that ignoring cavitation would not seriously alter safety considerations for most dams. However, it was concluded that the importance of cavitation effects increases with the height of the dam.

Felippa and DeRuntz (1984) developed a displacement potential finite element formulation to model explosion effects in a cavitating fluid continuum. The code developed was referred to as Cavitating Fluid Analysis (CFA). This fluid code was coupled to a structural code (STAGS-C1) and to the Underwater Shock Analysis code (USA). The discrete models for the three subsystems were prepared independently and coupled by a staggered solution procedure. An explicit time integration procedure was used for the fluid, and an implicit scheme was adopted for the structure. The proposed formulation was validated by testing several two- and three-dimensional fluid-structure systems. Numerical results showed that if the time step in the fluid medium is below a critical time step, adequate results can be obtained.

Olson and Bathe (1985a) proposed an alternative Eulerian fluid formulation which no longer produces unsymmetric systems. The velocity potential was used as the main independent nodal variable along with a hydrostatic pressure variable modelled at only one node per fluid region. Two-dimensional, axi-symmetric and three-dimensional fluid elements were developed. The proposed formulation was validated against a variety of examples. Numerical tests indicated that this method is accurate in predicting the vibratory response of structures submerged in fluids.

In a series of studies, Singh et al (1989), (1990), (1991) and (1992) developed two special purpose fluid-structure interaction programs, FLUSOL and FLUSHEL, for two- and three-dimensional geometries respectively. In both codes, the fluid algorithm was based on the Eulerian pressure formulation. Both implicit-implicit and explicit-explicit time integration schemes were developed. A wide variety of fluid-shell interaction problems were used to validate the codes. Numerical tests showed that the optimum choice between implicit-explicit and implicit-implicit partitioning is problem dependent.

Felippa and DeRuntz (1991) developed and applied the displacement potential formulation to study the three-dimensional response of fluid-structure systems under shock loading. A novel, three-dimensional, wedge-shaped fluid element was developed and implemented in the coupled code USA-STAGS-CFA referred to previously. The new element was validated by testing the response of a half plate initially floating on a half-space fluid and subjected to a shock wave. Their results were in good agreement with those reported in other analytical studies.

More recently, Weber (1992) developed a mixed velocity-pressure formulation which was implemented in a computer code, called DANAID. The proposed formulation was applied to the study of the seismic response of two- and three-dimensional dam-reservoir systems. Numerical results were in good agreement with those reported in other analytical studies.

Galindo et al (1992) used the pressure Eulerian formulation to study the response of a two-dimensional dam reservoir systems in the time domain. The fluid and the structure were modelled using two separate programs. Furthermore the codes were coupled and solved by a staggered solution procedure. Numerical tests showed that the size of the time step in staggered analyses is limited by accuracy considerations, especially if nonlinear behaviour is taken into account.

2.3.2 Formulations Based on the Lagrangian Description of Motion

Eulerian fluid descriptions reviewed in Section 2.3.1, are well suited to the treatment of large flows and flows around corners. Since the mesh is fixed in space, there is no possibility for mesh distortion. However, if the structure undergoes large deflection, modelling the interface between the structural and the Eulerian fluid mesh becomes quite difficult : the structural nodes lying at the interface may pass through the middle of the Eulerian mesh or cut across a corner, or a corner of the structure mesh may fall within the fluid mesh, etc.. The treatment of all these possibilities requires considerable programming, and it is difficult to make the resulting program applicable to completely arbitrary structures. This leads to the next approach which will be reviewed: the Lagrangian fluid finite element formulation.

One of the first studies on fluid-structure interaction employing the direct Lagrangian formulation to model the fluid continuum was by Chopra et al (1969). A two-dimensional, displacement based, fluid finite element was developed as a special case of an isotropic solid. The constitutive properties were expressed in terms of the fluid bulk and shear moduli, with the latter taken as zero. Furthermore, an implicit time integration procedure was used and the coupled equations of motion were solved simultaneously. One-dimensional spring elements were used at the fluid-structure interface to prevent artificial penetration of the fluid into the structure. The proposed formulation was tested by studying the transient seismic response of a two-dimensional dam reservoir system. Comparison between numerical results obtained from the proposed formulation and analytical methods showed that significant errors may be involved in the finite element solution as time progresses.

Shantram et al (1976) used an isotropic solid element with a zero shear modulus and a poisson ratio equal to 0.5 to model the fluid medium. In order to model fluid damping and to stabilise the element behaviour fluid viscosity effects were taken into account. Two-dimensional and axi-symmetric fluid finite elements were developed. The formulation was assessed by studying the response of a pipe, closed at one end and subjected to a step pressure loading at the other end. Numerical results indicated that the finite element solution correctly reproduces the pressure wave propagation along the pipe and is in reasonable agreement with available analytical solutions. By introducing separate nodal points for corresponding positions on the fluid/solid interface and by using a horizontal coupling stiffness between such adjacent nodal pairs, an axi-symmetric "sliding" element which simulates a frictionless boundary condition at the interface was developed. The developed sliding and fluid elements were tested against a variety of problems, including an elasto-plastic transient analysis

of a pressure vessel subjected to a short pressure pulse. In order to quantify the inertia effect of the fluid two cases were studied, with the vessel empty and full. It was observed that the horizontal displacement at the bottom of the vessel was significantly reduced when the fluid was present.

Wilson (1977) extended the two-dimensional formulation used by Chopra et al to axis-symmetric and three-dimensional geometries. Reduced integration was used in calculating the element stiffness matrices. Based on the principle of zero relative normal displacements, two- and three-dimensional sliding elements were developed to model the fluid-structure interface. The formulation was tested by studying the seismic response of a two-dimensional rectangular reservoir with both rigid and flexible walls. It was observed that the interaction between the liquid and the solid significantly alters the response. Reasonable agreement with analytical results was obtained, although discrepancies were observed, especially after the first two seconds of the earthquake. This was attributed to the size of the time step used in the analysis.

Kiefling and Feng (1976) studied the vibration characteristics of several fluid-structure systems. Displacement based fluid elements were developed and used. Numerical tests showed that spurious modes with quasi zero frequencies occur. It was observed that the number of these 'circulation modes' increases rapidly as the mesh becomes finer. Hamdi et al (1978) employed a penalty function on the displacement spin tensor to satisfy the irrotationality condition and to eliminate the circulation modes. A two-dimensional fluid finite element with a penalty parameter was developed. The element stiffness matrix was nonsingular and the resulting eigenvalue matrix was positive definite, symmetric and banded. In order to test the proposed enhancements, several linear frequency analyses were carried out. Numerical results showed that by choosing an appropriate value of the penalty parameter it is possible to eliminate the circulation modes. Furthermore, it was concluded that the circulation modes were very sensitive to the choice of the penalty parameter, while the real modes were almost insensitive to it.

Bathe and Hahn (1979) used an isotropic solid element with zero shear modulus to model the fluid medium. In order to eliminate spurious oscillations, the stiffness matrix of the fluid element was integrated using full integration formulae. Based on the updated Lagrangian formulation, two-dimensional, axis-symmetric and three-dimensional nonlinear fluid elements were developed. The method was validated against a variety of two-dimensional and axis-symmetric problems. Numerical results demonstrated that it is possible to use the updated Lagrangian formulation to model the nonlinear, transient response of fluid-structure systems. Wilson and Khalvati

(1983) studied the effects of irrotational constraints, free surface potential and reduced integration parameters on the eigen frequencies of two-dimensional fluid-structure systems. Observations based on numerical tests showed that the effect of the above parameters is to eliminate zero energy modes.

Sundqvist (1983) developed displacement based fluid finite elements and implemented them in the code ADINA. The proposed formulation was tested by studying the response of several fluid-structure systems in the frequency domain. Numerical results showed good agreement with experimental data. Based on the updated Lagrangian formulation, Chelghoum (1986) developed two-dimensional, axis-symmetric and three-dimensional fluid elements. A penalty formulation was used to impose the irrotationality condition. The developed formulation was integrated into a general purpose finite element code, FINAS, developed at the Department of Civil Engineering at Imperial College. An implicit time integration scheme for the fluid and the structure was adopted. The proposed developments were validated against a multitude of transient problems. The obtained results showed good agreement with those reported in other studies.

Fenves and Vargas-Loli (1988) employed a Lagrangian finite element formulation for the fluid, where the fluid pressure and displacement are approximated independently. This approach, while retaining the ease of coupling fluid elements with structural elements, avoids the need for reduced integration. Two- and four-noded elements with constant pressure and a linear variation of displacements were developed for one- and two-dimensional analyses respectively. The symmetric coupled equations of motion were expressed in terms of displacements and solved by an implicit algorithm. The proposed formulation was applied to the transient analysis of a dam-reservoir system subjected to ground motion. No comparison with other studies was reported.

2.3.3 Formulations Based on the Mixed-Eulerian-Lagrangian Description of Motion

Historical background and finite difference techniques

Since the early sixties research has been oriented towards the development of a mixed Lagrangian / Eulerian description in which the mesh may be moved with the fluid in a normal Lagrangian fashion, may be held fixed in a normal Eulerian manner or may be moved in some arbitrarily specified way. Until the mid seventies these theoretical hypotheses were implemented into finite difference codes.

Harlow (1964), at Los Alamos Laboratory, developed 'The Particle-In-Cell Method', PIC, code for the solution of problems involving large distortions of the fluid, large slips or colliding interfaces, problems for which existing techniques were not adequate. In this method the fluid domain is divided into Eulerian cells with the Lagrangian mesh of fluid particles moving through the cells. At each time step the calculations necessary to advance the solution one time increment are divided into two parts:

- i- The Eulerian part in which the pressure and the tentative velocities are calculated.
- ii- The Lagrangian part in which the particle motion is accomplished by using the values obtained in the Eulerian part.

The PIC method was applied to analyse the supersonic flow of gases past cylinders, both transversely and axially; as well as the passage of a shock wave through a discontinued enlarged channel. No comparison with other studies was reported. The most serious of the difficulties and limitations in the applicability of the PIC method are the following:

- i- No part of the flow should be subsonic.
- ii- Within a large region of flow there should be no small region where detailed resolution is required.
- iii- It must not be necessary to know in detail the pointwise results at any given instant in time.

Also, Noh (1964) developed a 'Coupled Eulerian-Lagrangian' computer code, CEL, where the fluid domain is divided into separate Eulerian and Lagrangian sub-regions. This method starts with the given initial state of the fluids and the initial position of the mesh points for the different Lagrangian grids and for the Eulerian grid. At each time step the calculations that are performed may be divided into three main parts:

- i- Lagrangian calculation which uses the known state of the fluids and the pressures acting on the boundaries to solve the difference equations for each of the grids. The solution of the difference equations gives the current state of the Lagrangian fluids and the corresponding grid positions.
- ii- By using the new state of the Lagrangian subregions, determined in the above step, the corresponding Eulerian grid positions are determined. The solution of the Eulerian difference equations gives the current state of the Eulerian fluids.
- iii- Having determined the current state of the Eulerian region the pressures acting

on the current Lagrangian grid are determined.

This method was applied to study the flow of a perfect gas past a dense metal sphere (rigid or flexible). Another application was to study the behaviour and interaction of three different fluids. However the CEL code suffered from one serious drawback which is that a subregion must be modelled as either Lagrangian or Eulerian but not a combination of both.

Hirt et al (1974) developed a method in which the finite difference mesh have vertices that may move with the fluid, be held fixed, or be moved in any other prescribed way. Because of this flexibility the method was referred to as an Arbitrary Lagrangian-Eulerian , ALE, technique. In this method the calculations that are necessary to advance a solution one time step are separated into three phases:

- i- Lagrangian calculation where the material mesh vertices do not move.
- ii- Movement of material mesh vertices to their new Lagrangian position.
- iii- Movement of the spatial mesh to its new configuration. This is accomplished by the calculation of the convective fluxes to account for the movement of fluid between cells as the mesh moves.

The proposed formulation was applied to analyse a variety of two-dimensional and axi-symmetric problems including an explosion in the atmosphere.

Pracht (1975) extended Hirt's work and developed a code for calculating the fully three-dimensional dynamics of a flow contained within arbitrarily shaped boundaries. The calculation sequence necessary to advance the solution one time step is as follows:

- i- New velocities and energies are calculated from the pressure and densities given by the previous increment. This stage of the calculation is performed on the basis of a pure Lagrangian approach.
- ii- Iteration involving velocities, pressures and densities is performed to obtain advanced time-pressure forces.
- iii- Spatial cell vertices are moved and the appropriate convective fluxes calculated using either of the following three options:
 - a- With the fluid velocity (Lagrangian).
 - b- Back to the beginning of the cycle (Eulerian).
 - c- In a prescribed manner which would satisfy certain criteria.

The method was successfully applied to study the interaction of two large heated spheres rising side by side in an ambient atmosphere and the flow of an incompressible fluid through a curved tube.

Developments in the Mixed-Eulerian-Lagrangian method using finite elements

Among the earliest researchers to have developed an Arbitrary Lagrangian-Eulerian formulation to be implemented into a finite element code were Donea et al (1977). The proposed two-dimensional finite element method in which the element nodes can be moved in a Lagrangian way, be held fixed in a Eulerian way or be moved in some arbitrary way is similar to the Arbitrary Lagrangian-Eulerian finite difference technique proposed by Hirt et al (1974). The calculations required to advance the solution one time step are divided into two parts:

- i- A first quasi-Lagrangian step yields intermediate values of the velocities and specific energies.
- ii- The velocities and specific energies calculated in the first step are used to evaluate the transport effects and to update the moving nodes.

An alternative to the above scheme was also implemented, where the first step is avoided by extrapolating the velocities of the previous time step. The method was used to test a shock tube problem and an elasto-plastic shock cylinder. In both cases numerical results showed good agreement with those based on analytical solutions.

Belytschko and Kennedy (1978) developed and used a two-dimensional Arbitrary Lagrangian-Eulerian program to test wave problems and various sub-assembly simulation problems. The method was based on the same solution scheme as that proposed by Donea et al (1977). The results were compared to other finite element studies based on the Lagrangian and Eulerian formulations. Numerical tests showed that when the maximum velocity is relatively low the momentum transport term plays a small role and all three types of mesh descriptions gave almost identical results. However, for higher velocities only the mixed description attained the desired accuracy.

Hughes et al (1981) were amongst the first to develop an Arbitrary Lagrangian-Eulerian finite element formulation without decoupling the Lagrangian and Eulerian calculations. They developed the necessary formulation for a four-noded two-dimensional, four-noded axi-symmetric and eight-noded three-dimensional elements. The proposed method was tested by studying a two-dimensional free surface wave

propagation problem. Numerical results showed good agreement with experimental data. Liu and Ma (1982) employed the above formulation in developing a computer code, FLUSTRII, for the transient analysis of liquid filled storage tanks. The proposed formulation was applied to study the seismic response of two- and three-dimensional tank-fluid systems.

The freedom in moving the fluid mesh offered by the Mixed-Eulerian-Lagrangian kinematic description proved to be very attractive. However, it can be overshadowed by the large time required for specifying grid velocities suited to a particular problem. Giuliani (1982) was among the first to develop a computer code, EURO-DYN-1M, with a facility for automatic and continuous rezoning. The main criterion of this scheme was minimising the squeeze and distortion of the finite elements. The versatility of this scheme was demonstrated by applying it to determine the response of several two-dimensional problems involving significant distortions of the fluid mesh. Observations from numerical tests showed that while such a scheme significantly reduces the mesh distortion and the amount of time spent on preparing grid velocities, it has a modest impact on the global computing time required to reach a solution. The extension of the rezoning procedure to three-dimensional meshes was also outlined. Donea et al (1982) improved the above work by introducing a corrective term which controls the grid velocity when the distance between adjacent nodes tends to become too short.

Liu and Chang (1984) and (1985) employed an Arbitrary Lagrangian-Eulerian finite element formulation for the fluid, where the pressure and displacements are approximated independently. The coupled equations of motion were solved by a staggered solution procedure. To avoid stringent time step limitations, the fluid and structure displacements were integrated implicitly. Furthermore, two different classes of transient algorithms, implicit-implicit and implicit-explicit were developed to integrate the fluid pressures. The accuracy and stability of these methods were tested by studying several one- and two-dimensional examples. Good agreement with other numerical and analytical methods was observed.

Kulak (1985) developed a computer program, NEPTUNE, for three-dimensional impact fluid-structure problems. Fluid and structural domains were solved independently and coupled by a staggered solution procedure. In addition, interface conditions were non-iteratively enforced to complete each time step. A penalty type gap-impact element was developed to model the impact region between the fluid and the structure. The formulation was validated by testing a three-dimensional shock loading problem. Numerical results showed good agreement with experimental and

analytical work.

The versatility allowed by the Mixed-Eulerian-Lagrangian approach in fluid-structure interaction problems prompted several researchers to adopt it in structural and solid mechanics problems. Haber (1984) derived such a formulation for path dependent solid materials with both geometric and material nonlinearities. The formulation was implemented into a computer code to study an elastic plastic wave problem. Several other researchers have continued to work along these lines (e.g. Liu et al 1986; Benson, 1989; Huetink et. al., 1990; Liu et al 1991 and Hu and Liu 1992).

2.4 FAR-FIELD FLUID-STRUCTURE INTERACTION

The various acoustic approximation methods described previously for modelling the infinite fluid medium are reviewed in this section. The formulations are classified under three main categories: early time approximations, late time approximations and doubly asymptotic approximations.

2.4.1 Early Time Approximations

Mindlin and Bleigh (1952) were amongst the first researchers to develop an early time approximation, the Plane Wave Approximation, which simulates the effect of the infinite fluid medium. The proposed analytical method was applied to the study of the elastic response of a submerged cylindrical shell. Bedrosian and DiMaggio (1972a) and (1972b) introduced an approximation analogous to that of Mindlin and Bleigh for spherical and spheroidal shells. The proposed formulation was implemented in a finite difference code. Numerical results showed that the proposed acoustic approximation can give good results for spherical and spheroidal shells made of steel and surrounded by an infinite body of water. Based on Kirchhoff's retarded-potential method, Felippa (1980) derived a hierarchical family of early time surface interaction approximations for a submerged structure in an infinite acoustic fluid.

Wright et al (1981) implemented the Plane Wave Approximation in a finite difference code and applied it to the calculation of the pressures in a submerged fluid-filled cylindrical shell subjected to a step pressure wave. They studied the effect of shell parameters (elastic modulus, thickness and radius) on the shape of the pressure pulse transmitted inside the shell. Numerical results showed that as the thickness increases, strong shell vibrations become important and distort the pulse. DiMaggio et al (1981) implemented and applied the Plane Wave Approximation to determine the response

characteristics of submerged structures surrounded by a cavitating fluid. The structure and the cavitating fluid were modelled by finite elements. The formulation was validated by testing several linear and nonlinear one-dimensional examples. In all cases good agreement with analytical results was reported.

Rehak et al (1985) employed the Plane Wave Approximation to study the response of a floating structure subjected to an exponentially decaying plane wave. Numerical tests showed that this formulation, applied directly to the wet surface of the structure, is sufficiently accurate to determine the structural response.

More recently, O'Reagan and DiMaggio (1990) used the Plane Wave Approximation to study the dynamic response of submerged cylindrical shells with appendages. The finite difference method was used to discretise the structure and the near-field fluid. The non-radiating boundary was first applied directly on the structural wet surface and then on a convex fluid surface. Numerical results showed that for structures with external appendages, when the Plane Wave Approximation is applied on the wet surface of the structure, reflections of radiated waves would be lost. To overcome this problem, the non-radiating boundary should be applied on a convex cylindrical fluid surface circumscribing the combined structure.

2.4.2 Late Time Approximations

Geers (1969) developed an analytical method based on a Virtual Mass Approximation of the infinite acoustic medium. The method was validated by studying the elastic response of a cylindrical shell excited by a transient acoustic wave. The proposed method was compared with the Plane Wave Approximation described previously. Numerical results demonstrated the superior performance of this formulation for late time behaviour and low frequencies. However, substantial discrepancies were observed at the early stages of the analysis. Geers (1970) employed the same analytical method to study the response of an elastic cylindrical shell to a transverse acoustic shock wave submerged in a light fluid medium. Numerical results showed that this formulation can accurately predict the post-envelopment response. Chertock (1970) applied this method to determine the transient flexural vibrations of ship-like structures exposed to underwater explosions. Only low frequency motions of the structure were considered. Comparisons of the analytical results with experimental data verified the theory as applied to low frequency modes; however, discrepancies were observed in the higher modes.

Felippa (1977) presented a top-down derivation of late time approximations based on

the exact Retarded Potential Formulation. Deruntz and Geers (1978) presented a method to calculate the virtual mass by the boundary integral method. The resulting mass matrix was unsymmetric and fully banded.

2.4.3 Doubly Asymptotic Approximations

In this approach, the formulations of two one sided asymptotic expressions, one early time approximation and one late time approximation, are superimposed. Hence, an approximate solution that approach exactness in the limit of high and low frequency motions is reached.

Everstine (1976) developed a Doubly Asymptotic Approximation formulation and implemented it into the general purpose finite element code, NASTRAN. By treating the non-symmetric fluid-structure coupling terms in an explicit manner, the resulting system of equations was symmetric and banded. However, the accuracy implications of treating the low frequency matrix explicitly could not be assessed since no numerical example was presented.

By superimposing the Plane Wave Approximation and the Virtual Mass Approximation discussed previously, Ranlet et al (1977) used the Doubly Asymptotic Approximation to model the infinite fluid medium, while modal analysis was employed for the structure. The method was validated by studying the elastic response of submerged shells with internally attached structures subjected to shock loading. Good agreement with analytical results was obtained.

Based on the superposition of one late time asymptote and one early time asymptote, the Doubly Asymptotic Approximation proved to be accurate for both early and late time behaviour. However, discrepancies with analytical solutions were often observed at intermediate times. Based on fundamental fluid boundary modes, Geers (1978) introduced fitting matrices to enhance the performance of the Doubly Asymptotic Approximation performance in the intermediate transient range. The proposed method was validated by studying the transient response of a submerged spherical shell excited by a step plane wave. Numerical results showed that this formulation is more accurate than other numerical methods for intermediate times. However, it requires more computational effort.

Zilliaccus et al (1979) developed a Doubly Asymptotic Approximation formulation and implemented it in the general purpose finite element code ADINA. The added mass matrix was calculated by a boundary integral approach. Furthermore, axi-

symmetric finite elements were used to model the structure. Numerical results showed that the formulation can accurately predict the response of submerged spherical shells subjected to plane waves. The implementation details of the method into the general purpose finite element code ADINA were presented. The method was applied to study the response of a submerged spherical shell subjected to an incident plane step wave. Results showed good agreement with the theory. Zilliacus (1983) employed the above approach to study the response of a submerged fluid filled cylinder subjected to a step incident plane wave. Results showed good agreement with other numerical results based on a finite element / Doubly Asymptotic Approximation approach.

Atkatsh et al (1983) developed a nonlinear, large deflection, elasto-plastic code (EPSA) for the analysis of shells in an acoustic medium subjected to dynamic loading. The nonlinear equations of the shell were discretised using the finite difference method, while the infinite fluid medium was modelled by the Doubly Asymptotic Approximation expressed in terms of orthogonal fluid expansion functions. The coupled equations of motion were integrated in time explicitly. The proposed formulation was verified by studying the transient nonlinear response of stiffened cylindrical panels. Numerical results showed good agreement with experimental data. However, the effect of the material nonlinearity on the solution was not discussed.

Based on an infinite element approach to calculate the added mass matrix, Olson and Bathe (1985b) developed an alternative Doubly Asymptotic Approximation formulation. Furthermore, the structure and the near-fluid were modelled by finite elements, whereas the high frequency matrix was modelled by a consistent damping term. This method offered several advantages over the boundary integral formulation; the most important of which were outlined in Section 2.2.8. Several two-dimensional and axi-symmetric problems were studied to validate the correctness of the formulation. Numerical results demonstrated the efficiency of this method as compared to the Doubly Asymptotic Approximation based on the boundary integral approach. However, it was not clear whether it is possible to develop it further to account for intermediate frequencies and times. Furthermore, the behaviour of the elements was not tested under shock loading conditions.

Felippa and DeRuntz (1984) developed a Doubly Asymptotic Approximation formulation based on a boundary integral approach and implemented it in an Underwater Shock Analysis, USA, code. This was in turn coupled to a structural code (STAGS-C1) and a fluid code (CFA). The discrete models for the three

subsystems were prepared independently and coupled using a staggered solution procedure. The proposed formulation was tested by studying the response of one- and two-dimensional submerged structures excited by step-exponential plane waves. Numerical results showed good agreement with analytical solutions. Rehak et al (1985) studied the response of floating structures subjected to shock waves. The Plane Wave Approximation and the Doubly Asymptotic Approximation were applied both on the wet surface of the structure and on a fluid boundary surrounding the structure. Numerical results showed that the former method, applied directly on the wet surface of the structure is sufficient to determine the response characteristics for this class of problems. Furthermore, it was found that the latter method offered no additional accuracy in this case.

Huang (1986) applied the Doubly Asymptotic Approximation to study the response of a submerged spherical shell excited by plane step waves. Numerical results showed good agreement with analytical methods, however the comparison revealed that this method overestimates the fluid damping force and the oscillations in the response are damped out prematurely. The author stated that by augmenting this method with an intermediate frequency matrix, it becomes possible to alleviate this deficiency, however examples were not provided to justify this statement.

More Recently, O'Reagan and DiMaggio (1990) compared the performance of the Doubly Asymptotic Approximation and the Plane Wave Approximation when applied to the study of the dynamic response of an infinite circular cylindrical shell with plate appendages and excited by plane waves. Numerical results showed that both methods give good results only if applied on a convex surface surrounding the structure. This was attributed to the occurrence of multiple reflections between the structure and the external appendages, which cannot be modelled if the non-radiating boundary is placed directly on the surface of the submerged structure.

2.5 REVIEW OF EXISTING SOFTWARE

A wide variety of computer codes is available for treating fluid-structure interaction systems. In this section, the codes to which reference is most commonly made (particularly in journal papers) will be reviewed with emphasis on highlighting their main features and limitations. Tables 2.1 and 2.2 show the main features of the reviewed codes. In addition, Rao (1989) refers to a computer code, PICES, for the analysis of ship structures subjected to shock wave loading. However, Rao (1989) did not present any detailed description of the general capabilities of the program.

Most of the codes, reviewed in this section, offer a solution to a particular type of fluid-structure system. Furthermore, many of them are limited to certain geometry types.

2.6 AREAS OF WEAKNESS IN CURRENT RESEARCH

The review presented in this chapter has highlighted some aspects of the problem which need to be developed further. Towards the development of an efficient and accurate fluid-structure interaction program, the following main features should be considered.

- i- Developments of fluid finite elements should not be based on the Eulerian or Lagrangian kinematic descriptions of motion but on the Mixed-Eulerian-Lagrangian description which offers more versatility in treating the various fluid-structure interaction problems.
- ii- In order to enforce the irrotationality condition and to avoid spurious oscillations a penalty on the rotational stiffness of the fluid element should be introduced.
- iii- Modelling of the infinite fluid medium should not be done with a large finite element mesh, but with a radiating boundary based on acoustic approximations.
- iv- In order to provide a versatile fluid-structure interaction program with the ease of incorporating future developments, all the above developments should be implemented in a general purpose structural finite element code.

The reason for the development of other features, which necessarily follow from the recommendations made in this section, will be presented and discussed in detail in relevant chapters. Furthermore, other minor decisions, regarding the choice between different computational techniques, will be explained and defended as they are presented.

2.7 CONCLUDING REMARKS

This chapter has defined and discussed the basic difficulties encountered in the

numerical treatment of fluid-structure interaction systems. A review of the literature on near-field and far-field fluid-structure interaction was also presented. Greater emphasis was placed on the Mixed-Eulerian-Lagrangian kinematic description of motion for the near-field fluid and on the acoustic approximations for the far-field fluid.

The available kinematic descriptions of motion for the near-field fluid were presented. The advantages and disadvantages of each method were discussed in detail, taking into account accuracy, efficiency and versatility requirements. In addition, large deformation fluid response and efficient fluid-structure interface representation have been discussed.

Next, the various numerical techniques employed to model the infinite fluid medium were presented and the limitations of each of the methods were highlighted.

Finally, a review of the most commonly used fluid-structure interaction codes have been presented. The main weaknesses which remain to be solved in near-field and far-field fluid-structure interaction problems have been outlined.

The derivation, implementation, validation and application of the proposed developments will be discussed in Chapters Three, Four and Five.

Program	Problem Geometry	Structure			Kinematic Description	Discretisation	Time Integration Structure/Fluid
		Geometric Nonlinearity	Material Nonlinearity				
USA-STAGS-CFA	Three-Dimensional	Yes	Yes		Eulerian	Finite Elements	Implicit/Explicit
NASTRAN	General	Yes	Yes		-	Finite Elements	-
FLUSHELL	Three-Dimensional	No	No		Eulerian	Finite Elements	Implicit/Explicit and Implicit/Implicit
ADINA	General	Yes	Yes		Updated Lagrangian	Finite Elements	Implicit
EURODYN-1M	Two-Dimensional and Axi-Symmetric	No	Yes		Mixed-Eulerian-Lagrangian	Finite Elements	Explicit
FLUSOL	Two-Dimensional	No	No		Eulerian	Finite Elements	Implicit/Explicit and Implicit/Implicit

Table 2.1 Fluid-Structure Interaction Computer Codes

Program	Problem Geometry	Structure			Kinematic Description	Discretisation	Time Integration Structure/Fluid
		Geometric Nonlinearity	Material Nonlinearity				
EPSA	Shells	Yes	Yes		No Near-Field Fluid.	Finite Elements and Finite Differences	Explicit/Explicit
DANAID	Two and Three-Dimensional	Yes	Yes		Eulerian	Finite Elements	Implicit-Implicit
FLUSTRII	Two and Three-Dimensional	Yes	Yes		Mixed-Eulerian-Lagrangian	Finite Elements	Implicit-Explicit
NEPTUNE	Two and Three-Dimensional	Yes	Yes		Quasi-Eulerian	Finite Elements	Explicit

Table 2.1 (Cont'd) Fluid-Structure Interaction Computer Codes

Program	Fluid Compressibility	Coupling Procedure	Nodal Fluid Variable	Non-Radiating Boundary	Reported Use
USA-STAGS-CFA	Compressible	Staggered	Displacement Potential	DAA	Hull Cavitation in Underwater Shock Problems.
NASTRAN	Slightly Compressible	Complete	Displacement; Pressure	DAA	Transient analysis of submerged cylindrical shells
FLUSHELL	Compressible	Staggered	Pressure	NO	Transient analysis of submerged spherical shells
ADINA	Slightly Compressible	Complete	Displacement	DAA	Transient analysis of submerged structures
EURODYN-1M	Compressible	-	Velocity	NO	Transient analysis of fluid filled pressure vessels and nuclear reactors
FLUSOL	Compressible	Staggered	Pressure	NO	Variety of FSI problems
EPSA	-	Uncoupled	-	DAA	Transient Analysis of cylindrical shells
DANAID	Compressible	Complete	Mixed Pressure-Velocity	Frequency dependant	Transient dam-reservoir analysis

Table 2.2 Fluid-Structure Interaction Computer Codes

Program	Fluid Compressibility	Coupling Procedure	Nodal Fluid Variable	Non-Radiating Boundary	Reported Use
FLUSTRII	Slightly Compressible	Complete	Mixed Pressure-Velocity	NO	Seismic analysis of liquid filled containers.
NEPTUNE	-	Staggered Weak Couplings	Velocity	NO	Transient analysis of fluid filled nuclear reactors.

Table 2.2 (Cont'd) Fluid-Structure Interaction Computer Codes

CHAPTER THREE

NEAR-FIELD FLUID-STRUCTURE INTERACTION

This chapter presents both the theory and the formulation for the fluid elements based on the Mixed-Eulerian-Lagrangian kinematic description of motion.

This is used to develop two-dimensional and axi-symmetric finite elements capable of undergoing large displacements and deformation. These elements are four- and eight-noded, with two translation degrees of freedom per node. Isoparametric mapping shape functions are used for the modelling of the geometry and the displacement field of the fluid elements. To enforce the irrotationality condition, a penalty on the rotation terms in the fluid stiffness matrix is introduced.

One of the main advantages of the proposed formulation remains its capability to prescribe suitable nodal mesh velocities at any stage of the solution process. In the context of this study, an efficient mesh velocity prescription scheme is formulated and implemented in the developed computer code.

At the fluid-structure interface, adequate normal stiffnesses should be employed to prevent artificial penetration of the fluid into the structure and to model the sliding boundary condition. Towards this end, two-dimensional and axi-symmetric sliding elements are developed.

Transient analysis techniques for the numerical integration of the equations of motion are then presented. Iterative methods such as the Newton Raphson approach, combined with a series of implicit time integration operators, are formulated to tackle nonlinear dynamic fluid-structure systems.

Finally, the proposed formulation is validated by testing it against several wave propagation and fluid-structure interaction problems for which there are existing solutions.

3.1 INTRODUCTION

Problems in continuum mechanics are usually formulated in terms of three spatial field variables namely: displacements, strains and stresses.

Furthermore, the equations describing the behaviour of a continuum can generally be divided into four major categories :

- i- Kinematics, otherwise referred to as strain-displacement relations.
- ii- Kinetic, also known as the balance laws of mass and momentum.
- iii- Constitutive (material) laws which describe the stress-strain relationships.
- iv- Thermodynamic, specifically the first law of thermodynamics which results in the energy balance equation.

In this chapter, emphasis is placed on developing an accurate and efficient kinematic description of motion for the fluid continuum. The three possible approaches are: the Lagrangian, Eulerian and Mixed-Eulerian-Lagrangian.

In the Lagrangian method, the equations of motion along with the kinematic and constitutive relationships are expressed with respect to the system defined by the fluid body. This system is known as the material system and will be denoted in the context of this study as (M.S.). Let Ω_x denote the corresponding domain and x_i ($i=1,2,3$) the cartesian coordinates of a vector position x as illustrated in Figure 3.1. In this configuration, the motion of the material and the mesh generic points are identical, so that at every state, the material particles coincide with the nodal and quadrature points. The coincidence of the mesh and material points in the Lagrangian approach facilitates the modelling of complex material boundary conditions.

In contrast with the previous approach, the Eulerian formulation describes implicitly the current position of a material particle with respect to the current time t of the motion, so that the state, not the history, of the material is described. In this case the referential system, which is stationary in space, is also known as the spatial frame and will be denoted in the present study by the initials (S.S.) . Let Ω_y denote the corresponding domain and y_i ($i=1,2,3$) the cartesian coordinates of a position vector y displayed in Figure 3.2. Because of the fixed spatial mesh, the Eulerian approach is very effective for modelling hydrodynamic flow problems with large fluid motions.

In view of the above description, the advantages and disadvantages of these two formulations, reviewed in the last chapter, become clear when applied to the

simulation of both bounded and unbounded fluid-structure interaction problems. The Lagrangian formulation seems to be the natural approach for predicting the transient response of fluid-structure interactive systems with no gross fluid motion. On the other hand, the Eulerian description appears appropriate for modelling fluid-structure systems involving large fluid motion. Due to the limitations related to each computational method, a more versatile method combining the merits of each configuration would be required to handle more challenging fluid-structure dynamic problems. Towards this end, the concept of Mixed-Eulerian-Lagrangian formulation will be adopted and developed in this study.

In this approach, the equations of motion are stated with respect to a referential system which is neither a priori attached to the fluid body nor fixed in space. The mesh points can follow the material motion, in a Lagrangian fashion, remain fixed in a Eulerian manner, or move in any arbitrary way defined by the user. Efficient treatment of arbitrary boundary conditions can be achieved by choosing the (R.S.) and (M.S.) generic points on the boundary to be coincident. Furthermore, at interior points where the fluid is expected to undergo large motion, the (R.S.) and (S.S.) generic points are coincident. Let Ω_z represent the fixed referential domain, shown along with the kinematic mapping concepts of the Mixed-Eulerian-Lagrangian description in Figure 3.3.

3.2 THE MIXED-EULERIAN-LAGRANGIAN KINEMATIC DESCRIPTION OF MOTION

3.2.1 Basic Definitions

This section describes all possible mappings between the previously defined (M.S.), (S.S.) and (R.S.) systems. A derivation of the basic relations of the mesh motion is presented along with the basic definitions and notations.

Consider a fluid body occupying domain $\Omega_x \subset \mathcal{R}^{n_{sd}}$ where n_{sd} is the number of spatial dimensions. The motion of the fluid is described by a mapping ϕ . The image of Ω_x at time t is denoted by Ω_y and the image of x is denoted by y (Figure 3.3) :

$$y = \phi(x, t) \quad (3.1)$$

Consider Ω_y to be the spatial domain upon which the fluid mechanics problem takes place. Ω_x is the domain occupied at $t = 0$ by the material particles which occupy Ω_y

at time t . Thus ϕ may be thought of as the material motion.

Introduce a third domain $\Omega_z \subset \mathcal{R}^{nd}$. Call Ω_z the reference domain which is fixed throughout. Assume that Ω_y is the image of Ω_z at time t under mapping $\hat{\phi}$ and that y is the image of z . This implies

$$y = \hat{\phi}(z, t) \quad (3.2)$$

By composition it is possible to arrive at a third mapping between z and x :

$$z = \hat{\phi}^{-1}(y, t) = \hat{\phi}^{-1}(\phi(x, t), t) \quad (3.3)$$

Alternatively, the above equation can be written as:

$$z = \psi(x, t) \quad (3.4)$$

where, at any time t

$${}^t\psi = {}^t\hat{\phi}^{-1} {}^t\phi$$

The general mapping relationships described above are illustrated in Figure 3.3.

Since Ω_z is always fixed generally both Ω_y and Ω_x vary with time. By specialising the definitions of Ω_y and Ω_z , the classical Eulerian and Lagrangian descriptions are obtained.

If $\phi = \underline{I}$ and $\Omega_x = \Omega_z$ where $\underline{I}$ is the identity mapping array, we have the Lagrangian description of motion (Figure 3.1).

If $\hat{\phi} = \underline{I}$ and $\Omega_y = \Omega_z$, we have the Eulerian description of motion (Figure 3.2).

More general kinematic descriptions of motion, namely the Mixed-Eulerian-Lagrangian, may be obtained by appropriate selection of Ω_z and $\hat{\phi}$.

In order to study the relationship between the kinematic descriptions discussed above, it is convenient to introduce some definitions given in the following section.

3.2.2 Partial Differential Operators and Kinematic Quantities

In the Lagrangian configuration, the material time derivative of a function h is given by:

$$\dot{h} = \frac{\partial h}{\partial t} \text{ (} x \text{ fixed)} \quad (3.5)$$

The spatial time derivative of a function g defined in the spatial domain is defined as:

$$g_{,t} = \frac{\partial g}{\partial t} \text{ (} y \text{ fixed)} \quad (3.6)$$

The time derivative of a function f with the referential coordinates held fixed is given as:

$$f' = \frac{\partial f}{\partial t} \text{ (} z \text{ fixed)} \quad (3.7)$$

The partial derivatives of the above defined functions with the time variable held fixed are :

$$h_{,i} = \frac{\partial h}{\partial x_i} \quad (3.8)$$

$$g_{,i} = \frac{\partial g}{\partial y_i} \quad (3.9)$$

$$f_i = \frac{\partial f}{\partial z_i} \quad (3.10)$$

where the subscript i refers to the spatial dimension (i.e. x_1 , x_2 or x_3).

In the Lagrangian system the material motion, displacement, velocity and acceleration are defined, in that order, as:

$$y_i = \phi_i(x,t) \quad (3.11)$$

$$d_i = y_i - x_i \quad (3.12)$$

$$v_i = \dot{y}_i = \frac{\partial \phi_i}{\partial t} \quad (3.13)$$

$$\dot{v}_i = \ddot{y}_i = \frac{\partial^2 \phi_i}{\partial t^2} \quad (3.14)$$

In the spatial description, the kinematic quantities are defined as:

$$y_i = \hat{\phi}_i(z, t) \quad (3.15)$$

$$\hat{d}_i = y_i - z_i \quad (3.16)$$

$$\hat{v}_i = y'_i = \frac{\partial \hat{\phi}_i}{\partial t} \quad (3.17)$$

$$\hat{a}_i = y''_i = \frac{\partial^2 \hat{\phi}_i}{\partial t^2} \quad (3.18)$$

Finally, in the referential system the kinematic quantities relevant to this study are defined as:

$$z_i = \psi_i(x, t) \quad (3.19)$$

$$w_i = x_i - z_i \quad (3.20)$$

$$\dot{w}_i = \dot{z}_i = \frac{\partial \psi_i}{\partial t} \quad (3.21)$$

3.2.3 Time Derivative in Referential Coordinates

The material time derivative which is usually used in the derivation of the classical (for example mass and momentum) continuum balance laws is the time derivative with the material coordinate held fixed as in Equations 3.11 to 3.14. However, the

Mixed-Eulerian-Lagrangian description of motion focus neither on material points nor on a fixed region in space, but on "reference" points which may be moving with the continuum, but independently from the motion of its material points. In order to derive the laws of motion in the Mixed-Eulerian-Lagrangian formulation, it is essential to establish a relationship between the material time derivative and the referential time derivative.

If f is a scalar function defined on any one of the regions Ω_x , Ω_y or Ω_z , then by decomposition, f can be defined in the other regions by expressing it in terms of two other functions using Equations 3.1 to 3.4 :

$$g(y,t) = g(\hat{\phi}(z,t),t) = f(z,t) \quad (3.22)$$

$$h(x,t) = h(\psi^{-1}(z,t),t) = f(z,t) \quad (3.23)$$

Differentiating Equations (3.22) and (3.23) with respect to t , holding the material coordinate x fixed, yields:

$$\frac{\partial g}{\partial t}_{y \text{ fixed}} + \frac{\partial g}{\partial y} \frac{\partial y}{\partial t}_{x \text{ fixed}} = \frac{\partial f}{\partial t}_{z \text{ fixed}} + \frac{\partial f}{\partial z} \frac{\partial z}{\partial t}_{x \text{ fixed}} \quad (3.24)$$

$$\frac{\partial h}{\partial t}_{x \text{ fixed}} = \frac{\partial f}{\partial t}_{z \text{ fixed}} + \frac{\partial f}{\partial z} \frac{\partial z}{\partial t}_{x \text{ fixed}} \quad (3.25)$$

Using the definitions introduced earlier, and equating Equations 3.24 and 3.25 gives:

$$\dot{h} = f' + f_{,i} \dot{w}_i = g_{,t} + g_{,i} v_i \quad (3.26)$$

Combining Equations 3.11 and 3.15, and differentiating with respect to time holding the material coordinate fixed gives after rearranging:

$$\frac{\partial z_{,i}}{\partial t}_{x \text{ fixed}} = \frac{\partial z_{,i}}{\partial y_{,j}} \left(\frac{\partial y_{,j}}{\partial t}_{x \text{ fixed}} - \frac{\partial y_{,j}}{\partial t}_{z \text{ fixed}} \right) \quad (3.27a)$$

Differentiating the function f with respect to y holding t fixed gives:

$$\frac{\partial f}{\partial y} = \frac{\partial f}{\partial z} \frac{\partial z}{\partial y} \quad (3.27b)$$

Substituting Equations 3.27 into 3.26, an important relationship relating the material and referential time derivatives is obtained:

$$\dot{h} = f' + f_{,i} c_i \quad (3.28)$$

where c_i is the convective velocity, defined as:

$$c_i = v_i - \hat{v}_i \quad (3.29)$$

Using the above equation the particle acceleration, which appears in the balance of momentum equation, may be expressed as follows:

$$\dot{v}_i = v'_i + (v_{,i})_j c_j \quad (3.30)$$

3.3 GOVERNING EQUATIONS IN THE REFERENTIAL SYSTEM

In preceding sections, the Mixed-Eulerian-Lagrangian kinematic description of motion has been discussed. In this section, the above mentioned information will be used in deriving differential equations which describe the motion of the fluid continuum in the Mixed-Eulerian-Lagrangian system under specific boundary conditions.

3.3.1 Balance Laws

The general governing equations for a continuum in the Lagrangian (M.S.) coordinate system have been derived and presented by several authors including Malvern (1969), Benson (1989) and Fung (1977). These are rewritten as follows:

Conservation of mass in the Lagrangian system:

$$\frac{\partial \rho}{\partial t} + \rho \frac{\partial v_i}{\partial y_i} = 0 \quad (3.31)$$

where ρ is the density of the material

The above equation is alternatively referred to as the continuity equation. If the material is incompressible so that the density of each material particle remains constant as it moves, the continuity equation takes the simple form :

$$\frac{\partial v_i}{\partial y_i} = 0 \quad (3.32)$$

The above equation is usually referred to as the incompressibility constraint which has importance in the classical hydrodynamic theory.

Conservation of momentum in the Lagrangian system:

$$\tau_{ij,j} (y,t) + b_i (y,t) + t_i - \rho(y,t) \dot{v}_i (y,t) = 0 \quad (3.33)$$

where

t_i is the prescribed traction force acting on the boundary Γ_y ,

τ_{ij} is the cauchy stress, and

$b_i (y,t)$ is the prescribed body force which is a function of position y and time t .

The governing equations for a continuum in the Mixed-Eulerian-Lagrangian system are derived by substituting the relationship between the material time derivative and the reference configuration time derivative of the velocity, i.e. Equation 3.28, into the governing equations in the Lagrangian system, i.e. Equations 3.31 and 3.33:

Conservation of mass in the Mixed-Eulerian-Lagrangian system:

$$\rho' + \rho_{,i} c_i + \rho v_{k,k} = 0 \quad (3.34)$$

Conservation of momentum in the Mixed-Eulerian-Lagrangian system:

$$\rho v_i' + \rho v_{i,j} c_j = \tau_{ij,j} + b_i + t_i \quad (3.35)$$

3.3.2 Constitutive Equation and Equation of State for the Fluid

Constitutive equations for the fluid, may be divided into two broad classes according to the type of flow :

- Viscous fluid flow.
- Inviscid fluid flow.

Inviscid flows are frictionless flows which are characterised by zero viscosity.

Although no real flow is inviscid, numerous situations arise in which viscous effects can be neglected. The formulation developed and implemented in this work can model viscous fluids (by inputting an appropriate value for the viscosity) and inviscid fluids (by setting the value of the viscosity to zero). The general constitutive equation for a Newtonian fluid, where it is assumed that the shear stress is linearly proportional to the rate of deformation, may be expressed as:

$$\tau_{ij} = -p \delta_{ij} + \mu (v_{i,j} + v_{j,i}) \quad (3.36)$$

where:

p is the hydrodynamic pressure in the fluid,

μ is the kinematic viscosity, and

δ_{ij} is the Kronecker delta defined as

$$\delta_{ij} = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{if } i \neq j \end{cases} \quad (3.37)$$

In order to formulate the governing equation in terms of accelerations, or any other kinematic variable, one more equation relating the hydrodynamic pressure to the displacements or velocities is required. Such a relationship is described by the equation of state, where, in its most general form, the pressure is assumed to be dependent on the density and temperature of the fluid according to the expression:

$$p = f(T, \rho) \quad (3.38)$$

where T is absolute temperature in the fluid.

The equation of state employed in this study does not account for cavitation and therefore allows the development of artificial negative pressure in the fluid. The full implications of this assumption will be discussed in Chapter Four.

For problems considered in this study, the fluid medium is assumed to be barotropic and isentropic. Under these conditions, and using techniques described by Malvern (1969) and Fung (1969), the corresponding equation of state expresses the hydrodynamic pressure as a unique function of the change in the density:

$$p = f(\rho) \quad (3.39)$$

or

$$p' = \frac{\partial p}{\partial \rho} \rho' \quad (3.40)$$

For barotropic flows, $\frac{\partial p}{\partial \rho}$ is further assumed constant and equal to c_0^2

where c_0 is the speed of sound in the fluid medium.

Alternatively, this can be expressed as:

$$\frac{\partial p}{\partial \rho} = \text{constant} = \frac{\beta}{\rho} \quad (3.41)$$

Also, for the purpose this work is devoted, the term $\rho_{,i} c_i$ in the continuity equation is relatively small compared to ρ' ; hence, Equation 3.34 becomes:

$$\rho' + \rho v_{k,k} = 0 \quad (3.42)$$

Substituting Equations 3.41 and 3.42 in Equation 3.40 and integrating over the time domain yields:

$$p = p_0 - \int_0^t \beta v_{k,k} dt \quad (3.43a)$$

where p_0 is the pressure at time $t = 0$.

Alternatively, analysts often express the fluid pressure in terms of the volumetric strain tensor, for example Bathe and Hahn (1979), where the fluid pressure may be expressed as:

$$p = p_0 - \beta d_{k,k} \quad (3.43b)$$

Using Equation 3.43, the constitutive equation (Equation 3.36) becomes:

$$\tau_{ij} = - \left[p_0 - \int_0^t \beta v_{k,k} dt \right] \delta_{ij} + \mu [v_{i,j} + v_{j,i}] \quad (3.44a)$$

or in terms of displacements:

$$\tau_{ij} = -[p_0 - \beta d_{k,k}] \delta_{ij} + \mu [v_{i,j} + v_{j,i}] \quad (3.44b)$$

3.3.3 Governing Equations

Substituting Equation 3.44 into 3.35, the governing equation in the Mixed-Eulerian-Lagrangian system is obtained:

$$\rho v_i' + \rho v_{i,j} c_j = -\delta_{ij} \left[p_0 - \int_0^t \beta v_{k,k} dt \right]_j + \mu [v_{i,j} + v_{j,i}]_j + b_i + t_i \quad (3.45)$$

Boundary conditions

The boundary conditions in the Mixed-Eulerian-Lagrangian case are identical to those imposed by the Lagrangian or Eulerian calculations. Therefore, the boundary can be split into five separate parts:

$$\Gamma = \Gamma_{FS} + \Gamma_{ST} + \Gamma_{RA} + \Gamma_{\bar{d}} + \Gamma_{\bar{t}} \quad (3.46)$$

where

$\Gamma_{\bar{d}}$ is the boundary where the prescribed displacements are acting.

$\Gamma_{\bar{t}}$ is the boundary where the prescribed tractions are acting.

Γ_{FS} is the free surface boundary when the free surface gravity wave is included.

Γ_{ST} is the fluid structure interface boundary.

Γ_{RA} is the radiating boundary used to model the far-field fluid domain.

The free surface conditions and the fluid-structure boundary will be discussed in Sections 3.7 and 3.10 respectively, whereas the formulation for the non-radiating boundary is the subject of Chapter Four.

3.4 GALERKIN / FINITE ELEMENT FORMULATION

The characteristic matrices and vectors of finite elements are usually arrived at by using one of the following three approaches:

- **Direct Approach**

In this method, physical reasoning is employed to establish element properties in terms of pertinent variables. Although this approach enhances the physical understanding of the finite element method, it is limited to simple types of elements and problems, mainly where the Total Lagrangian formulation is applicable.

- **Variational Approach**

In this method, the finite element analysis is interpreted as an approximate means for solving variational problems. Several finite element formulations found in the literature are based on this approach. (Hughes, 1987; and Rao 1989). Most Total and Updated Lagrangian formulations in structural mechanics are based on this method.

- **Galerkin (Weighted Residual) Approach**

In this method, the elements matrices and vectors are derived directly from the governing differential equation. This technique offers the most versatility for deriving finite element equations and can be applied to almost all practical problems of science and engineering. In most Eulerian and Mixed-Eulerian-Lagrangian formulations, this method is employed to derive the finite element equations.

3.4.1 Galerkin Weighted Residual Method

In the context of this study, finite element equations will be presented based on the Galerkin approach. The matrix form of the Galerkin finite element formulation of the governing equation, Equation 3.45, is arrived at by the following procedure:

- i- The region over which the problem takes place, such as Ω_y , is discretised into non-overlapping subregions called elements. Each element has NEN nodal points so that the discretisation of Ω_y also consist of NNP nodal points.
- ii- An approximate form of variation for the kinematic quantities (d, v and v') is assumed in each element and expressed as:

$$w_i = \sum_{b=1}^{NEN} N_b w_{ib}^e \quad (3.47)$$

w_i is any kinematic quantity such as acceleration, velocity or displacement,
 N_b are functions describing the variation of the corresponding kinematic

variable throughout the element domain, and

w_{ib}^e is the kinematic quantity in the i^{th} direction associated with the b^{th} node of the e^{th} element.

- iii- The approximation for v , d and v' , Equation 3.47, is introduced into the governing differential equations, thereby producing an error or residual function, in the domain R , defined as:

$$R = \sum_{e=1}^{NELEM} \left[\begin{array}{c} \rho N_b v_{ib}^e + \delta_{ij} \{ p_0 - \beta N_{b,k} d_{kb}^e \}_j + \\ \rho v_{ij} N_b c_{jb}^e - \\ \mu \{ N_{b,j} v_{ib}^e + N_{b,i} v_{jb}^e \}_j - b_i - t_i \end{array} \right] \neq 0 \quad (3.48)$$

where NELEM is the total number of elements in the domain and bold quantities refer to the unknown approximate variables which should be determined.

In domain methods such as finite element and finite difference methods it is assumed that the approximate solution satisfies all boundary conditions, therefore:

$$\begin{aligned} R_1 &= d - \bar{d} = 0 \text{ on } \Gamma_d \\ R_2 &= t - \bar{t} = 0 \text{ on } \Gamma_t \end{aligned} \quad (3.49)$$

- iv- Weighted Residual Methods are employed to distribute the error throughout the domain. This is done by forcing the integral of the residuals, weighted by a certain function to be zero at NNP points throughout the domain.

$$\int_{\Omega} R \psi_a = 0 \text{ in } \Omega \quad a = 1, 2, \dots, NNP \quad (3.50a)$$

Alternatively, the above equation may be expressed as:

$$\sum_{e=1}^{NELEM} \sum_{a=1}^{NNP} \int_{\Omega_e} R \tilde{\psi}_a = 0 \quad (3.50b)$$

where a tilde denotes a vector quantity and $\tilde{\psi}_a = \begin{bmatrix} \psi_a & 0 \\ 0 & \psi_a \end{bmatrix}$ for both two-

dimensional and axi-symmetric geometries.

It should be noted that different types of weighting functions, ψ_a , will define different approximation methods.

- v- In order to reduce the required order of continuity of the pressure across element boundaries, the pressure term in Equation 3.50 is integrated by parts resulting in the following governing equation:

$$\begin{aligned} \sum_{e=1}^{NELEM} \sum_{a=1}^{NEN} \left[\int_{\Omega_y^e} \rho \psi_a N_b v_{ib}^e d\Omega_y^e + \int_{\Omega_y^e} \rho \psi_a v_{ij} N_b c_{jb}^e d\Omega_y^e - \right. \\ \left. \int_{\Omega_y^e} \psi_{aj} \{ p_0 - \beta N_{b,k} d_{kb}^e \} \delta_{ij} d\Omega_y^e + \right. \\ \left. \int_{\Omega_y^e} \psi_{aj} \mu \{ N_{b,j} v_{ib}^e + N_{b,i} v_{jb}^e \} d\Omega_y^e \right. \\ \left. = \int_{\Omega_y^e} \psi_a b_i d\Omega_y^e + \int_{\Gamma_y^e} \psi_a t_i d\Gamma_y^e \right] \end{aligned} \quad (3.51a)$$

- vi- Using techniques described in Rao(1989), Hughes(1987) and Zienkiewicz and Taylor (1989), the Galerkin formulation is arrived at by equating the weighting functions to the interpolation functions:

$$\begin{aligned} \sum_{a=1}^{NNP} \sum_{e=1}^{NELEM} \int_{\Omega_y^e} \rho N_a^T N_b d\Omega_y^e a_b^e + \int_{\Omega_y^e} N_a^T \rho QV N_b d\Omega_y^e x_b^e \\ + \int_{\Omega_y^e} (B_a^e)^T \beta B_b^e d\Omega_y^e d_b^e + \int_{\Omega_y^e} (G_a^e)^T D_\mu G_b^e d\Omega_y^e x_b^e \\ = \int_{\Omega_y^e} N_a b d\Omega_y^e + \int_{\Gamma_y^e} N_a t d\Gamma_y^e + \int_{\Omega_y^e} (B_a^e)^T p_0 d\Omega_y^e \end{aligned} \quad (3.51b)$$

where for the two-dimensional case:

$$G_a^e = \begin{bmatrix} N_{a,1}^e & 0 \\ 0 & N_{a,2}^e \\ N_{a,2}^e & N_{a,1}^e \end{bmatrix}, B_a^e = \begin{bmatrix} N_{a,1}^e & 0 \\ 0 & N_{a,2}^e \end{bmatrix}, N_a = \begin{bmatrix} N_a & 0 \\ 0 & N_a \end{bmatrix},$$

$$\underline{G}\underline{V} = \begin{bmatrix} v_{1,1} + v_{2,1} & 0 \\ 0 & v_{1,2} + v_{2,2} \end{bmatrix} \text{ and } \underline{D}_\mu = \mu \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

and for the axi-symmetric case:

$$\underline{B}_a^e = \begin{bmatrix} N_{a,1} & 0 \\ 0 & N_{a,2} \\ \frac{N_a}{R} & 0 \end{bmatrix}, \underline{G}_a^e = \begin{bmatrix} N_{a,1} & 0 \\ 0 & N_{a,2} \\ \frac{N_a}{R} & 0 \\ N_{a,2} & N_{a,1} \end{bmatrix} \text{ and } \underline{D}_\mu = \mu \begin{bmatrix} 2 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

while all other matrices are identical to those used in the two-dimensional case.

Alternatively, the matrix form of Equation 3.51b can be expressed as follows:

$$\underline{M}_F \underline{a} + (\underline{N}_v + \underline{K}_\mu) \underline{v} + \underline{K}_c \underline{d} = \underline{F}_b + \underline{F}_t + \underline{F}_{\sigma_0} \quad (3.51c)$$

or

$$\underline{M}_F \underline{a} + \underline{K}_\mu \underline{v} + \underline{K}_c \underline{d} = \underline{F}_b + \underline{F}_t + \underline{F}_{\sigma_0} - \underline{N}(v) \quad (3.51d)$$

where the above global matrices are assembled from the element matrices defined as follows:

$$\underline{M}_F^e \text{ is the fluid mass matrix} = \int_{\Omega_y^e} \rho \underline{N}_a^T \underline{N}_b d\Omega_y^e,$$

$$\underline{N}_v^e \text{ is the convective term due to the transport of fluid through the grid} = \int_{\Omega_y^e} \underline{N}_a^T \rho \underline{G}\underline{V} \underline{N}_b d\Omega_y^e,$$

$$\underline{K}_\mu^e \text{ is the viscosity matrix} = \int_{\Omega_y^e} (\underline{G}_a^e)^T \underline{D}_\mu \underline{G}_b^e d\Omega_y^e,$$

$$\underline{K}_c^e \text{ is the compressibility or fluid stiffness matrix} = \int_{\Omega_y^e} (\underline{B}_a^e)^T \beta \underline{B}_b^e d\Omega_y^e,$$

$\mathbf{F}_b^e$ is the applied external body force $= \int_{\Omega_y^e} \mathbf{N}_a \mathbf{b} d\Omega_y^e$,

$\mathbf{F}_t^e$ is the applied external traction force $= \int_{\Gamma_y^e} \mathbf{N}_a \mathbf{t} d\Gamma_y^e$,

$\mathbf{F}_{s0}^e$ is the applied initial pressure force $= \int_{\Gamma_y^e} (\mathbf{B}_a^e)^T p_0 d\Omega_y^e$, and

$\mathbf{a}$ is the referential time derivative of the material velocity.

3.5 ISOPARAMETRIC FLUID ELEMENTS

A family of two-dimensional and axi-symmetric fluid finite elements is developed in this section. The elements formulation as well as their computer implementation are derived and discussed. Both linear and quadratic elements are developed with four and eight nodes respectively. The node numbering convention adopted in this study is shown in Figure 3.4. Figures 3.4 and 3.5 show the various geometric configuration of the two-dimensional and axi-symmetric elements respectively.

3.5.1 Isoparametric Concept

Basic definitions

Using Equation 3.47, the mesh and material kinematic quantities, including displacement, velocity and acceleration, at a generic point within an element are interpolated using the following expressions:

$$\mathbf{d}_i = \sum_{j=1}^{NEN} \mathbf{N}_j \mathbf{d}_{ij}^e \quad ; \quad \hat{\mathbf{d}}_i = \sum_{j=1}^{NEN} \mathbf{N}_j \hat{\mathbf{d}}_{ij}^e \quad (3.52a)$$

$$\mathbf{v}_i = \sum_{j=1}^{NEN} \mathbf{N}_j \mathbf{v}_{ij}^e \quad ; \quad \hat{\mathbf{v}}_i = \sum_{j=1}^{NEN} \mathbf{N}_j \hat{\mathbf{v}}_{ij}^e \quad (3.52b)$$

$$\mathbf{a}_i = \sum_{j=1}^{NEN} \mathbf{N}_j \mathbf{a}_{ij}^e \quad ; \quad \hat{\mathbf{a}}_i = \sum_{j=1}^{NEN} \mathbf{N}_j \hat{\mathbf{a}}_{ij}^e \quad (3.52c)$$

In addition, the geometry of the element may be expressed in a similar fashion as:

$$\mathbf{x}_i = \sum_{j=1}^{NEN} \mathbf{N}_j^g \mathbf{x}_{ij}^e \quad (3.53a)$$

$$y_i = \sum_{j=1}^{NEN} N_j^g y_{ij}^e \quad (3.53b)$$

$$z_i = \sum_{j=1}^{NEN} N_j^g z_{ij}^e \quad (3.53c)$$

where N_j^g and N_j are the shape functions used to interpolate the element geometry and kinematic quantities respectively.

If the geometric interpolation functions are of a lower order than the kinematic interpolation functions, the element is considered subparametric; whereas if the reverse is true, the element is referred to as superparametric. A finite element is considered isoparametric if the same shape functions are used to interpolate both the geometric and kinematic quantities, that is:

$$N_j = N_j^g \quad \text{for } j = 1, 2, \dots, NEN \quad (3.54)$$

Because isoparametric elements are usually curved, they tend to be more suitable than subparametric and superparametric elements for modelling complicated geometries and boundary conditions. Furthermore, they are computationally less expensive than subparametric elements since the same shape functions are used in the interpolation of the geometric and kinematic quantities of the element. In the present study, Equation 3.54 is used to develop isoparametric four- and eight-noded quadrilateral elements.

Shape functions

The selection of appropriate shape functions is fundamental to the finite element method, since it is incorporated in the computation of all finite element matrices, such as load, mass and stiffness). Therefore, it is important to consider the explicit representation of such approximating polynomials.

The shape function for a particular node is a polynomial which has the value of one at that node and zero at all other nodes. In deriving appropriate shape functions, two major factors should be considered.

Completeness Requirement:

The degree of continuity of the derivatives of the element kinematic quantities depends mainly on the polynomial degree of the approximating shape functions.

Therefore to ensure the same degree of continuity in all directions it is essential to have the same order of variation of the kinematic quantities in all directions. For example, to have second order continuity of the element kinematic quantities in all directions, it is necessary and sufficient to have a complete quadratic polynomial expression for the shape functions.

A quadratic polynomial expression $x_i = x_{ia} N_a$ can be written in the expanded form:

$$x_i = N_1 x_{i1} + N_2 x_{i2} + N_3 x_{i3} + N_4 x_{i4} + N_5 x_{i5} + N_6 x_{i6} \quad (3.55)$$

where x_{ia} is the x_i coordinate of the a^{th} node.

Alternatively, the above equation may be expressed as:

$$x_i = C_1 + C_2 x_1 + C_3 x_2 + C_4 x_1^2 + C_5 x_1 x_2 + C_6 x_2^2 \quad (3.56)$$

Since there are six independent coefficients C_i , the $x = x_i N_i$ form will require six independent nodal values x_i , where the shape functions N_i are quadratic polynomials.

Continuity Condition

In many situations, the number of nodes required to satisfy the completeness requirement (and therefore ensure a predetermined order of continuity of the kinematic variables within an element) is not sufficient to provide continuity of the independent variable across element boundaries. For example, two nodes on a boundary section are insufficient to ensure that two connected elements have the same quadratic variation along either side of the common boundary.

To ensure continuity along all points on a boundary, when the variable varies quadratically within each element, requires continuity of value at three points in the segment. This implies that a quadratic element should have three nodes on each boundary segment. This condition is met by a quadrilateral element with nodes at the vertices and at the midpoints of the edges, which in turn would result in an incomplete cubic polynomial with eight constants. In this study, an eight-noded quadrilateral element with eight constants is employed. It is convenient to define the shape functions in term of local non-dimensional coordinates ξ, η which vary from -1 to 1, as shown in the Figure 3.4.

$$x_1 = C_1 + C_2 \xi + C_3 \eta + C_4 \xi^2 + C_5 \xi \eta + C_6 \eta^2 + C_7 \xi^2 \eta + C_8 \xi \eta^2 \quad (3.57a)$$

A quadratic polynomial expansion of the element geometry can be expressed as:

$$x_2 = C_9 + C_{10} \xi + C_{11} \eta + C_{12} \xi^2 + C_{13} \xi \eta + C_{14} \eta^2 + C_{15} \xi^2 \eta + C_{16} \xi \eta^2 \quad (3.57b)$$

By defining the two matrices:

$$\mathbf{C} = [\mathbf{C}_{x_1} \quad \mathbf{C}_{x_2}] \quad (3.58)$$

and

$$\mathbf{p}_q = [1 \quad \xi \quad \eta \quad \xi^2 \quad \xi \eta \quad \eta^2 \quad \xi^2 \eta \quad \xi \eta^2] \quad (3.59)$$

where

$$\mathbf{C}_{x_1} = \begin{bmatrix} C_1 \\ C_2 \\ C_3 \\ C_4 \\ C_5 \\ C_6 \\ C_7 \\ C_8 \end{bmatrix} \quad \text{and} \quad \mathbf{C}_{x_2} = \begin{bmatrix} C_9 \\ C_{10} \\ C_{11} \\ C_{12} \\ C_{13} \\ C_{14} \\ C_{15} \\ C_{16} \end{bmatrix}$$

Equation 3.57 may be expressed in matrix form as:

$$x_1 = \mathbf{p}_q \mathbf{C}_{x_1} \quad (3.60a)$$

$$x_2 = \mathbf{p}_q \mathbf{C}_{x_2} \quad (3.60b)$$

or:

$$\begin{Bmatrix} x_1 \\ x_2 \end{Bmatrix} = \begin{bmatrix} \mathbf{p}_q & \mathbf{0} \\ \mathbf{0} & \mathbf{p}_q \end{bmatrix} \mathbf{C} \quad (3.61)$$

The sixteen constants can be evaluated easily by solving the two sets of eight simultaneous equations which will arise if the nodal coordinates and the corresponding values of the local coordinates are substituted in the above equation.

The resulting system of equations can be written as:

$$\underline{x}_{1a} = \underline{p}_0 \underline{C}_{x_1} \quad (3.62a)$$

$$\underline{x}_{2a} = \underline{p}_0 \underline{C}_{x_2} \quad (3.62b)$$

where

$$\underline{x}_{1a} = \begin{bmatrix} x_{11} \\ x_{12} \\ x_{13} \\ x_{14} \\ x_{15} \\ x_{16} \\ x_{17} \\ x_{18} \end{bmatrix}, \quad \underline{x}_{2a} = \begin{bmatrix} x_{21} \\ x_{22} \\ x_{23} \\ x_{24} \\ x_{25} \\ x_{26} \\ x_{27} \\ x_{28} \end{bmatrix} \quad \text{and} \quad \underline{p}_0 = \begin{bmatrix} 1 & -1 & -1 & 1 & 1 & 1 & -1 & -1 \\ 1 & 1 & -1 & 1 & -1 & 1 & 1 & -1 \\ 1 & 1 & 1 & 1 & 1 & 1 & 1 & 1 \\ 1 & -1 & 1 & 1 & -1 & 1 & -1 & 1 \\ 1 & 0 & -1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & -1 & 0 & 1 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Using Equation 3.62, the C constants appearing in Equation 3.56 may be expressed as:

$$\underline{C}_{x_1} = \underline{p}_0^{-1} \underline{x}_{1a} \quad (3.63a)$$

$$\underline{C}_{x_2} = \underline{p}_0^{-1} \underline{x}_{2a} \quad (3.63b)$$

Substituting Equations 3.63 into Equations 3.60, results in the following expression for the coordinates:

$$x_1 = \underline{p}_q \underline{p}_0^{-1} \underline{x}_{1a} \quad (3.64a)$$

$$x_2 = \underline{p}_q \underline{p}_0^{-1} \underline{x}_{2a} \quad (3.64b)$$

Alternatively, using Equation 3.53a and 3.54, the material coordinate in an element may be expressed as:

$$x_1 = \underline{N}^T \underline{x}_{1a} \quad (3.65a)$$

$$x_2 = \underline{N}^T \underline{x}_{2a} \quad (3.65b)$$

where

$$\underline{N}^T = [N_1 \quad N_2 \quad N_3 \quad N_4 \quad N_5 \quad N_6 \quad N_7 \quad N_8]$$

By Equating Equations 3.65 and 3.64, the shape functions are determined :

$$\underline{N}^T = \underline{p}_q \underline{p}_0^{-1} \quad (3.66)$$

The above equation defines the expressions of the shape functions employed often in finite element analysis. The shape functions for the four-noded quadrilateral element may be derived in a similar fashion. For the four-noded and eight-noded two-dimensional and axi-symmetric isoparametric fluid element shown in Figures 3.4 and 3.5 respectively, the shape function and its derivatives are presented in Tables 3.1 and 3.2.

i	Shape Function N_i	$\partial N_i / \partial \xi$	$\partial N_i / \partial \eta$
1	$(1-\xi)(1-\eta)/4$	$-(1-\eta)/4$	$-(1-\xi)/4$
2	$(1+\xi)(1-\eta)/4$	$(1-\eta)/4$	$-(1+\xi)/4$
3	$(1+\xi)(1+\eta)/4$	$(1+\eta)/4$	$(1+\xi)/4$
4	$(1-\xi)(1+\eta)/4$	$-(1+\eta)/4$	$(1-\xi)/4$

Table 3.1 Shape Functions and Derivatives for a Four-Noded Isoparametric Quadrilateral Element

i	Shape Function N_i	$\partial N_i / \partial \xi$	$\partial N_i / \partial \eta$
1	$-(1+\xi+\eta)(1-\xi)(1-\eta)/4$	$(2\xi+\eta)(1-\eta)/4$	$(1-\xi)(\xi+2\eta)/4$
2	$-(1-\xi+\eta)(1+\xi)(1-\eta)/4$	$(2\xi-\eta)(1-\eta)/4$	$(1+\xi)(2\eta-\xi)/4$
3	$-(1-\xi-\eta)(1+\xi)(1+\eta)/4$	$(2\xi+\eta)(1+\eta)/4$	$(1+\xi)(\xi+2\eta)/4$
4	$-(1+\xi-\eta)(1-\xi)(1+\eta)/4$	$(2\xi-\eta)(1+\eta)/4$	$(1-\xi)(2\eta-\xi)/4$
5	$(1-\xi^2)(1-\eta)/2$	$(1-\eta^2)/2$	$-\eta(1+\xi)$
6	$(1+\xi)(1-\eta^2)/2$	$-\xi(1+\eta)$	$(1-\xi^2)/2$
7	$(1-\xi^2)(1+\eta)/2$	$-(1-\eta^2)/2$	$-\eta(1-\xi)$
8	$(1-\xi)(1-\eta^2)/2$	$-\xi(1-\eta)$	$-(1-\xi^2)/2$

Table 3.2 Shape Functions and Derivatives for an Eight-Noded Isoparametric Quadrilateral Element

3.5.2 Mapping between Local and Global Domains

In the isoparametric formulation discussed above, the shape functions are used as mapping functions to produce elements of a more general shape. In particular, the

quadratic functions can be used to develop elements with curved boundaries.

In order to map an element with straight boundaries to one with curved boundaries, a nonlinear mapping function is necessary. The most important requirement of that mapping is that it transforms the positions of the nodes correctly. This is satisfied by

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \sum_{k=1}^{NEN} N_k(\xi, \eta) \begin{bmatrix} x_{1k} \\ x_{2k} \end{bmatrix} \quad (3.67)$$

N_k are the shape functions defined previously.

The use of shape functions with local coordinates will lead to element matrices defined over local domains. In order to transfer the element matrices from local to global coordinates, the Jacobian matrix of the mapping must be used. This is defined as:

$$J = \begin{bmatrix} \frac{\partial x_1}{\partial \xi} & \frac{\partial x_2}{\partial \xi} \\ \frac{\partial x_1}{\partial \eta} & \frac{\partial x_2}{\partial \eta} \end{bmatrix} = \sum_{k=1}^{NEN} \begin{bmatrix} \frac{\partial N_k}{\partial \xi} \\ \frac{\partial N_k}{\partial \eta} \end{bmatrix} \begin{bmatrix} x_{1k} & x_{2k} \end{bmatrix} \quad (3.68)$$

Having expressed x and y in terms of the coordinates ξ and η , the operator ∇ , which occurs frequently in the governing equation, should be expressed in a similar fashion:

$$\nabla = \begin{bmatrix} \frac{\partial}{\partial x_1} \\ \frac{\partial}{\partial x_2} \end{bmatrix} = \begin{bmatrix} \frac{\partial \xi}{\partial x_1} \frac{\partial}{\partial \xi} & \frac{\partial \eta}{\partial x_1} \frac{\partial}{\partial \eta} \\ \frac{\partial \xi}{\partial x_2} \frac{\partial}{\partial \xi} & \frac{\partial \eta}{\partial x_2} \frac{\partial}{\partial \eta} \end{bmatrix} = \begin{bmatrix} \frac{\partial \xi}{\partial x_1} & \frac{\partial \eta}{\partial x_1} \\ \frac{\partial \xi}{\partial x_2} & \frac{\partial \eta}{\partial x_2} \end{bmatrix} \begin{bmatrix} \frac{\partial}{\partial \xi} \\ \frac{\partial}{\partial \eta} \end{bmatrix} \quad (3.69)$$

The inverse of the Jacobian matrix, J^{-1} , is calculated from Equation 3.68 as follows:

$$J^{-1} = \frac{1}{|J|} \begin{bmatrix} J_{22} & -J_{12} \\ -J_{21} & J_{11} \end{bmatrix} \quad (3.70)$$

where

$|J|$ is the determinant of the Jacobian defined as $(J_{11} J_{22} - J_{12} J_{21})$, the first subscript for the matrix entry denotes the row, and

the second subscript denotes the column in which the given entry stands.

By substituting Equation 3.70 into 3.69, the ∇ operator is transferred from local to global coordinates as follows:

$$\nabla = J^{-1} \begin{bmatrix} \frac{\partial}{\partial \xi} \\ \frac{\partial}{\partial \eta} \end{bmatrix} \quad (3.71)$$

3.5.3 Numerical Integration

With most types of finite elements, analytical integration of the stiffness, mass, damping and equivalent nodal load matrices is impossible. This is true for most formulations based on the concept of isoparametric mapping. In such cases, it becomes necessary to use numerical integration techniques. There is a large number of numerical integration schemes in literature, many of them designed for special problems. Gaussian integration formulae, discussed thoroughly by Atkinson (1989), are general, simple and accurate. Furthermore, they are extensively used in the evaluation of finite element matrices in both structural and fluid mechanics. In the following, Gaussian integration formulae for non-singular functions are briefly discussed.

For a quadrilateral element in local coordinates, the integrals can be expressed as:

$$I_{\xi\eta} = \int_{-1}^1 \int_{-1}^1 f(\xi, \eta) d\xi d\eta \approx \sum_{j=1}^n \sum_{i=1}^n f(\xi_i, \eta_j) w_i w_j \quad (3.72)$$

where

n is the number of the local ξ or η coordinates at which the integral is evaluated numerically,

ξ_i is the coordinate of the i^{th} integration point along the ξ axes,

η_i is the coordinate of the i^{th} integration point along the η axes, and

w_i, w_j are the weighting functions corresponding to the integration points.

In order to transfer the integrals from local to global coordinates, the determinant of

the Jacobian of the mapping, $|J|$, is employed. Therefore, the integral evaluated over the actual area of the element takes the following form:

$$I_y = \int_{\Omega_{y_e}} f(\underline{y}) d\Omega_{y_e} \approx \sum_{j=1}^n \sum_{i=1}^n f(\xi_i, \eta_j) w_i w_j |J(\xi_i, \eta_j)| \quad (3.73)$$

Using Equation 3.73, the integrals occurring in the finite element governing equation, Equation 3.51, can be determined numerically to a sufficient degree of accuracy. However, the nonlinear nature of the mapping of curvilinear isoparametric elements leads to integrands, which are no longer simple polynomials. Thus the Gaussian integration formulae do not result in exact answers, regardless of the number of integration points used. Furthermore, according to several researchers (for example Livesley, 1983) even when exact integration is possible and is employed, elements with a continuous displacement field, as the ones discussed previously, will always result in an overstiff solution. Therefore, it is often advantageous to use an integration formula with a lower order of integration points than required by the form of variation of the kinematic variables, thereby reducing the excessive stiffness of the element.

Table 3.3 shows the exact and reduced number of integration points required for the linear and quadratic quadrilateral elements, while the values of the corresponding local coordinate and weighting functions are listed in Table 3.4.

Element Type	No. of Integration points (Exact)	No. of Integration points (Reduced)
Linear (4 Nodes)	4	1
Quadratic (8 nodes)	9	4

**Table 3.3 Gaussian Integration Points for
Two-Dimensional and Axi-Symmetric Quadrilaterals**

Note that in Table 3.4 the values of ξ_i and η_i are symmetric with respect to zero, with the corresponding weighting functions being identical for two symmetric values.

No. of Integration points (Exact)	n	$\pm\xi_i$	w_i	$\pm\eta_i$	w_j
1	1	0.0	2.0	0.0	2.0
4	2	$\sqrt{3}/3$	1.0	$\sqrt{3}/3$	1.0
9	3	0.0	8/9	0.0	8/9
		$\sqrt{3}/5$	5/9	$\sqrt{3}/5$	5/9

Table 3.4 Local Coordinates and Weighting Functions for Gaussian Quadrature Schemes

3.6 IRROTATIONALITY CONDITION

From an implementation point of view, Mixed-Eulerian-Lagrangian formulations may be considered as either Lagrangian or Eulerian fluid formulations, augmented with rezoning schemes to account for the duality of the mesh and material quantities. Usually those formulations based on a Eulerian approach have the pressure, displacement potential or velocity potential as the main independent variable, while Mixed-Eulerian-Lagrangian formulations based on a Lagrangian approach have the displacement as the main independent variable.

A problem arises when modelling inviscid fluids using a displacement based Mixed-Eulerian-Lagrangian formulation. Due to the absence of shear stresses spurious zero energy modes may contaminate the numerical results. Such problems have been reported in the past by various researchers using a displacement based Lagrangian formulation for the modelling of the fluid continuum in fluid-structure interaction problems. Solutions to this problem were suggested by Kiefling and Feng (1976) and Hamdi et al (1978) amongst others, where the spurious zero energy modes were eliminated by introducing a penalty on the rotation spin tensor of the displacement field.

However, the problem of spurious rotation modes arising in displacement based Mixed-Eulerian-Lagrangian formulations has not been addressed yet. In this study, the proposed displacement based Mixed-Eulerian-Lagrangian formulation will be augmented with an irrotationality constraint to eliminate spurious rotational modes and enforce irrotational motion.

The rotation spin vector, ω , resulting from the fluid displacements field, $\underline{d}$, can be

expressed as:

$$\omega = \frac{1}{2} \text{curl}(\underline{d}) = \frac{1}{2} \nabla \times \underline{d} \quad (3.74)$$

where the ∇ operator has been defined by Equation 3.69 and a $\times$ denotes vector cross product.

Alternatively, the above equation can be expressed in component form as:

$$\omega_{ij} = \frac{1}{2} (d_{ij} - d_{ji}) \quad i, j = 1, 2 \quad (3.75)$$

Rotational flows are those where the spin tensor generated by the displacement (or velocity) field remains existent at all stages of the solution process. In some physical problems, the use of irrotational fluid element types becomes necessary to reach a satisfactory stable solution. While the irrotationality of viscous fluid motion is accounted for by the presence of the shear stresses, the use of a zero shear modulus and zero viscosity to model inviscid fluids necessitates an explicit introduction of a penalty on the rotation terms of the fluid displacement field. Towards this end, the isoparametric description, Equations 3.54 and 3.55, is substituted into Equation 3.75 which results in the following expression for the spin tensor:

$$\omega = \underline{B}_v \underline{d} \quad (3.76)$$

where $\underline{B}_v$ is the vorticity transformation matrix which can be expressed in condensed form as:

$$\underline{B}_v = \begin{pmatrix} N_{1,2} & -N_{1,1} & N_{2,2} & -N_{2,1} & \dots & N_{NEN,2} - N_{NEN,1} \end{pmatrix}$$

The condition of irrotational motion can be achieved numerically by augmenting the compressibility matrix with a penalty constraint expressed as:

$$\underline{K}_v = \int_{\Omega_y} \kappa \underline{B}_v^T \underline{B}_v d\Omega_y^e \quad (3.77a)$$

where κ is the penalty (on rotation) parameter.

Thus the total fluid stiffness matrix becomes the sum of the compressibility and the

vorticity matrices:

$$\underline{K}_f = \underline{K}_c + \underline{K}_v^* \quad (3.77b)$$

The appropriate choice of the penalty parameter depends mainly on the rotational modes of vibration generated by the spin tensor and related to the very high eigen-frequencies. In general, values of a over hundred time the fluid bulk modulus give good separation of compressional and rotational modes, so that no effect on the dynamic solution will occur.

3.7 FREE SURFACE GRAVITY WAVE CONDITION

At the liquid free surface, two boundary conditions must be satisfied; the kinematic condition which states that a fluid particle on the free surface at some time will always remain on the free surface, and the dynamic one specifying that the pressure on the free surface is derived from gravity field effects. Following Currie (1974), these may be expressed in terms of the velocity potential ϕ as:

$$\frac{\partial \phi}{\partial x_1} \frac{\partial d_2}{\partial x_1} - \frac{\partial \phi}{\partial x_2} + \frac{\partial d_2}{\partial t} = 0 \quad (3.78a)$$

$$g d_2 + \frac{\partial \phi}{\partial t} + \frac{1}{2} \left[\left(\frac{\partial \phi}{\partial x_1} \right)^2 + \left(\frac{\partial \phi}{\partial x_2} \right)^2 \right] = 0 \quad (3.78b)$$

where ϕ is defined as

$$\underline{v} = \underline{\nabla} \phi$$

For a Lagrangian formulation, following techniques described by Zienkiewicz and Bettles (1978), these can be combined into one condition as:

$$p = \rho g d(x_2) \quad (3.79)$$

In the Mixed-Eulerian-Lagrangian formulation developed in this study, where the nodes lying on the free surface are considered to be Lagrangian, the above equation becomes:

$$p = \rho g \hat{d}(y_2) \quad (3.80)$$

where

g is the gravitational acceleration, and $\hat{d}(y_2)$ is the vertical elevation of Γ_{FS} measured from an arbitrary datum represented by the fluid at rest, and where positive y_2 is in the direction opposite to gravity (Figure 3.6).

The above equation provides a relationship between the change in the hydrostatic fluid pressure and the displacement, $\hat{d}(y_2)$, of the fluid particles on the free surface.

For implementation purposes, it is convenient to express the above equation in vector notation as:

$$\underline{p} = \rho \underline{g} \hat{d}(y_2) \quad (3.81)$$

where $\underline{g} = - \underline{n}^T g$ and $\underline{n}$ is the unit outward normal.

Using Equation 3.28, the mesh velocity of the free surface may be expressed as:

$$\hat{v}_2 = \hat{d}'_2 = v_2 - c_{2i} \hat{d}_{2,i} \quad (3.82)$$

Differentiating Equation 3.81 with respect to time, holding the referential coordinate fixed and substituting the expression for the mesh velocity in the resulting equation, the free surface boundary condition in the Mixed-Eulerian-Lagrangian formulation becomes:

$$p' = - \rho (\underline{n}^T g) (v_2 - c_{2i} \hat{d}_{2,i}) \quad (3.83)$$

If in the Mixed-Eulerian-Lagrangian configuration, the nodes lying on the free surface are chosen to move with the corresponding fluid material, the above equation reduces to:

$$p' = - \rho (\underline{n}^T g) v_2 \quad (3.84)$$

Alternatively, the sloshing effects may be modelled in the code through the segment of applied time dependent load-function routines in the form of edge or nodal loads.

3.8 MAPPING BETWEEN FLUID MATERIAL AND FLUID MESH

One of the main advantages of the Mixed-Eulerian-Lagrangian fluid formulation is its capability to prescribe suitable nodal mesh velocities at any stage of the solution process. This facility, commonly known as rezoning, may be overshadowed by the amount of time spent in specifying mesh velocities suitable to a particular problem. Towards the development of a versatile rezoning method, special schemes which would allow the user to prescribe the mesh velocity in a simple manner, should be developed and implemented in the computer code.

Another consideration is the cost of such rezoning schemes. Since the mapping of the mesh between the material and spatial domains must be carried out frequently, at every time step in most cases, efficient schemes in terms of both computer time and storage should be developed and implemented into the program.

In order to avoid codes which require large amounts of computer processing time as well as central memory and disk space, an efficient and simple mesh rezoning scheme is desirable. Such a method may be developed and formulated in a variety of ways.

3.8.1 Background

The concepts of minimising mesh distortion and prescribing mesh displacements are not unique to the Mixed-Eulerian-Lagrangian technique. Lagrangian formulations for the fluid, when employed for fluid-structure interaction problems where the fluid is expected to undergo large deformations, are often augmented with rezoning schemes. These procedures, which must be carried out several times during the computation process, add considerable expense to the calculation. Furthermore, Lagrangian rezones introduce errors into momentum and energy balance. According to Belytschko and Kennedy (1978), even with rezoning, Lagrangian formulations fail in several cases.

Mixed-Eulerian-Lagrangian formulations were developed to overcome the deficiencies of the above method. Guiliani (1982) proposed an empirical rezoning scheme with the objective of preventing excessive squeeze in the elements. Based on geometric considerations, Donea et al (1982) refined the above method rendering it less empirical.

Hughes et al (1981) employed a split description to relate uniquely and explicitly the mesh nodal velocity to the mesh displacement and the material velocity. A weakened formulation was adopted to ensure continuity across element interfaces.

3.8.2 Mixed-Eulerian-Lagrangian Mesh Rezoning Scheme

In the context of this study, an automatic and continuous mesh rezoning scheme, proposed earlier by Hughes et al (1981) for the four-noded two-dimensional fluid element, is reformulated and implemented for both four- and eight-noded two-dimensional and axi-symmetric fluid elements.

Using Equation 3.82, $\hat{v}$ is determined from v and $\hat{d}$ by:

$$\hat{v}_i = v_j - c_j \frac{\partial \hat{d}_i}{\partial y_j} \quad (3.85)$$

Introduce a "Lagrange-Euler parameter Array" $\mathbb{A}$ defined by

$$\mathbb{A} = [\alpha_{ij}] = \begin{bmatrix} \alpha_{ij} & \delta_{ij} \end{bmatrix} \quad (3.86)$$

The size of the square diagonal array is equal to the total number of the fluid mesh degrees of freedom.

The purpose of the Lagrange-Euler parameter, is to offer the user control over each particular degree of freedom of every node in the fluid mesh. Towards this end, the following values are defined:

- i- If $\alpha_{ii} = 1$, the degree of freedom is Lagrangian.
- ii- If $\alpha_{ii} = 0$, the degree of freedom is Eulerian.

Using the above definitions, the convective velocity defined by Equation 3.29 becomes:

$$c_k = v_k - \hat{v}_k = (\delta_{kj} - \alpha_{kj}) v_k \quad (3.87)$$

Using Equation 3.87 and pre-multiplying the left hand side of Equation 3.85 by Equation 3.86, the mesh velocity is expressed as:

$$\hat{v}_1 = \alpha_{11} v_1 - \alpha_{11} c_2 \frac{\partial(\hat{d}_1 + \hat{y}_1)}{\partial \hat{y}_2} \quad (3.88)$$

$$\hat{v}_2 = \alpha_{22} v_2 - \alpha_{22} c_1 \frac{\partial(\hat{d}_2 + \hat{y}_2)}{\partial \hat{y}_1} \quad (3.89)$$

In the case of the eight-noded quadrilateral element, Equations 3.88 and 3.89 can be successfully applied to relate the mesh and material velocities. However, when used in conjunction with the four-noded quadrilateral element with C_0 continuity, the terms on the left hand side are continuous, but because of the derivatives those on the right hand side are not. In order to satisfy the equation in an average sense for both linear and quadratic elements, an alternative form of Equations 3.88 and 3.89 is desirable. A weakened form of the above equation is obtained by a two step procedure:

- i- The integral counterpart of Equation 3.88 and 3.89 is multiplied by weighting functions :

$$\int_{\Omega} \hat{w} \hat{v}_i = \int_{\Omega} \hat{w} \sum_{j=1}^2 \sum_{\substack{k=1 \\ k \neq j}}^2 \left[\alpha_{ij} v_j - \alpha_{ij} c_k \frac{\partial(\hat{d}_j + \hat{y}_j)}{\partial \hat{y}_k} \right] \quad (3.90)$$

where $\hat{w}$ is the weighting function

- ii- By choosing the weighting functions to be the shape functions employed in the interpolation of the element kinematic quantities, and performing integration by parts, the following weak form is obtained:

$$\mathbf{L} \hat{\mathbf{v}} = \mathbf{A} (\mathbf{L} \mathbf{v} - \mathbf{S}) \quad (3.91)$$

where

$$\mathbf{L} = \sum_{e=1}^{NELEM} \int_{\Omega_y^e} \mathbf{N}^T \mathbf{N} d\Omega_y^e \quad (3.92)$$

$$\mathbf{S} = \sum_{e=1}^{NELEM} \int_{\Omega_y^e} \sum_{i=1}^{NEN} N_i c_{ki} \left[\sum_{j=1}^{NEN} N_{i,k} (\hat{d}_j + \hat{y}_j) d\Omega_y^e \right] \quad (3.93)$$

By inverting the $\mathbf{L}$ matrix, the mesh velocities are obtained:

$$\hat{\underline{\gamma}} = \underline{\Lambda} \left(\underline{\gamma} - \underline{\mathbb{L}}^{-1} \underline{\mathbb{S}} \right) \quad (3.94)$$

The implementation details of Equations 3.92 to 3.94 are presented in Appendix A.

3.9 SLIDING ELEMENT FORMULATION

3.9.1 Introduction

Recently, the growing application of nonlinear transient fluid-structure interaction analysis has created a new sub-discipline of research. Efficient and accurate modelling of the fluid-structure interface is becoming an increasingly important consideration. From a mathematical point of view, two bodies can be assumed in contact or interacting if the basic principles of continuum mechanics are respected on the common contact surface. These are:

- Impenetrability Condition: The corresponding bodies at any state of the deformation history must not penetrate each other.
- Contact condition: The two interacting bodies must be in contact at least through a pair of nodal points.

In Section 3.4, the discretisation of the equation of motion, Equation 3.45, with respect to the space variables led to the matrix equation of motion for the liquid, Equation 3.51d. This may be reformulated as:

$$\underline{\mathbb{M}}_f \underline{\ddot{\mathbf{a}}} + \underline{\mathbb{K}}_\mu \underline{\dot{\mathbf{v}}} + \underline{\mathbb{K}}_f \underline{\mathbf{d}} = \underline{\mathbb{F}}_{\text{ext}} - \underline{\mathbb{N}}(\mathbf{v}) \quad (3.95)$$

where $\underline{\mathbb{F}}_{\text{ext}}$ is the sum of the forces due to the traction, body and initial stresses acting on the fluid medium.

The discretisation of the structures equation of motion using finite elements have been described by many authors (for example Zienkiewicz and Taylor, 1989 and Rao, 1989). The final form of the discretised equation of motion may be expressed as

$$\underline{\mathbb{M}}_s \underline{\ddot{\mathbf{a}}} + \underline{\mathbb{C}}_s \underline{\dot{\mathbf{v}}} + \underline{\mathbb{K}}_s \underline{\mathbf{d}} = \underline{\mathbb{F}}_{\text{ext}} \quad (3.96)$$

where

$\underline{\mathbb{M}}_s$ is the structure mass matrix.

$\mathbf{C}_s$ is the structure damping matrix.

$\mathbf{K}_s$ is the structure stiffness matrix including geometric and material nonlinearities.

Equation 3.95 supplemented with the discretised form of the structure equation of motion, form a single matrix for the fluid-structure system:

$$\mathbf{M}_T \ddot{\mathbf{u}} + \mathbf{C}_T \dot{\mathbf{u}} + \mathbf{K}_T \mathbf{u} = \mathbf{F}_{\text{ext}} - \mathbf{N}(\mathbf{v}) \quad (3.97)$$

where

$$\begin{aligned} \mathbf{M}_T &: \mathbf{M}_f + \mathbf{M}_s \\ \mathbf{C}_T &: \mathbf{C}_\mu + \mathbf{C}_s \\ \mathbf{K}_T &: \mathbf{K}_f + \mathbf{K}_s \end{aligned}$$

An inherent assumption is that the velocity vector varies continuously in the region occupied by the fluid and the structure. Due to the interpolation shape functions used in the interpolation of the kinematic quantities, this assumption is well justified for the structure and within the fluid volume. Also, at the liquid structure interface the velocity vector varies continuously in the case of viscous liquids.

However for liquids with low or zero viscosity, the velocity gradient may be very large on the interface while the associated shear stress is very low. Bathe and Hahn (1979) showed that a fine mesh is required to solve for the velocity vector in the vicinity of the interface. Furthermore, when due to low viscosity a large velocity gradient is associated with very low shear stresses in a thin layer of fluid on the interface, the liquid can be thought to slide on the interface so that only the normal component of velocity is continuous across the interface.

In order to avoid fine element meshes in the liquid layers adjacent to the interface and the associated burden in computations, Liu (1981) and Sundqvist (1983) developed a sliding boundary condition which can model reliably both tangential discontinuity and normal compatibility of stiffness. This condition may be imposed by using a sliding interface element to connect the fluid and structure nodes on the boundary.

3.9.2 Background

In the field of finite element analysis, the modelling of the interaction at the fluid-structure interface has been investigated by several researchers. A common feature to most of the proposed coupling methods is the use of separate nodal points for

corresponding fluid and structure positions on the interface. Shantaram et al (1976) introduced a normal (horizontal) coupling stiffness between coincident fluid-structure nodal pairs. Using this method, each fluid node on the interface was coupled to a corresponding structural node. Akkas et al (1979) used short and axially rigid truss elements in the normal direction of the interface. Bathe and Hahn (1979) employed a thin layer of fluid elements at the interface, in order to release the axial displacement of the fluid and therefore allowing it to slip. El Aidi and Hall (1989) developed and used stiff spring elements normal to the interface to analyse a dam reservoir system. The above proposed formulations were developed and implemented in Lagrangian fluid-structure computer codes.

Liu (1981) proposed a novel contact element defined by three nodes and a spring constant to model the sliding condition at the interface. The element consisted of two structural nodes which determine the sliding direction and a fluid node. The developed formulation was implemented in a Mixed-Eulerian-Lagrangian fluid-structure computer code. Liu and Gvildys (1986) used the sliding element mentioned above in the analysis of water tanks subjected to earthquake excitation. Chelghoum (1986) extended Liu's work to a four-noded sliding element and implemented it in a Lagrangian fluid-structure computer code.

3.9.3 Four-Noded Sliding Element

In the context of this study efficient two-dimensional and axi-symmetric Mixed-Eulerian-Lagrangian sliding elements will be developed. In this model, the liquid is allowed to slide without friction on the fluid-structure interface. This is achieved by a four-noded sliding interface element which provides for tangential discontinuity at the interface along with coupling of the normal stiffnesses of both mediums at the contact surface.

The geometry of the sliding element is shown in Figure 3.7, where nodes 1, 2 and 3 are the structure nodes and define the sliding direction, whereas node 4 is the fluid node in contact with the structure. For the sake of clarity, node 4 on the liquid and point c on the structure are shown apart from each other; however, they coincide in the model (i.e. node 4 lies on the curve defined by points 1, 2 and 3).

To facilitate the use of the proposed sliding elements in the modelling of complex fluid-structure systems, two important features are introduced in the formulation. First, node 2 of the structure need not coincide with the midpoint of the line passing through nodes 1 and 3. Second, the fluid node, node 4, need not coincide with node 2

of the structure. This facilitates the construction of complex finite element meshes with a varying size of fluid and structural elements.

The following conditions are prescribed at each point of the interface:

- The grid velocity of the fluid coincides with the material velocity of the solid.
- The normal material displacement of the fluid coincides with the normal displacement of the solid.
- The tangential material displacement of the fluid and the solid are unconstrained.

In order to express the above conditions in mathematical form, it is convenient to introduce a local coordinate system (t,n) at each node of the shell interface; where t lie in a plane tangent to the sliding interface and n is normal to it positive in the outward direction. At any point on the interface, the first condition may be written as:

$$\hat{v}_f = v_s \quad (3.98)$$

where $\hat{v}_f$ and v_s are the fluid mesh velocity and the structural velocity respectively.

Whereas the second condition takes the following form:

$$d_{fN} = d_{sN} \quad (3.99)$$

where d_{fN} and d_{sN} are the normal fluid material and structure velocities respectively.

The versatility of the Mixed-Eulerian-Lagrangian sliding element can be seen through the following discussion: if node 4 is a free surface Lagrangian node, it can slide on curve 1-2-3, if it is a fixed Eulerian node, then it is fixed on curve 1-2-3. However, regardless of node 4 being fixed or sliding on curve 1-2-3, the liquid material can slide without friction in a direction perpendicular to the normal vector $\underline{n}$.

3.9.4 Local Stiffness Matrix

Regardless of node 4 being sliding or fixed, the main requirement is that the local normal relative displacement between node 4 and point c must be zero. In order to fulfil this condition, the following self-equilibrating local equations should be satisfied on the contact surface:

$$\begin{bmatrix} R_c \\ R_4 \end{bmatrix} = K_{sl} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} d_{2c} \\ d_4 \end{bmatrix} \quad (3.100)$$

where

R_c and R_4 are the required local normal actions at point c and node 4 respectively,
 K_{sl} is the penalty parameter, and
 d_{2c} and d_4 are the normal local displacements at point c and node 4 respectively.

By introducing position functions at each node, PF_1 , PF_2 and PF_3 , which depend on the position of point c as defined in Figure 3.7, and the position of the intermediate structure node 2 (λ_2), it is possible to express the normal displacements, defined above, as follows:

$$d_{2c} = PF_1 d_{21} + PF_2 d_{22} + PF_3 d_{23} \quad (3.101)$$

where

$$PF_1 = \frac{(\lambda - 1)(\lambda - \lambda_2)}{\lambda_2} \quad (3.102)$$

$$PF_2 = \frac{\lambda(\lambda - 1)}{\lambda_2(\lambda_2 - 1)} \quad (3.103)$$

$$PF_3 = \frac{\lambda(\lambda - \lambda_2)}{1 - \lambda_2} \quad (3.104)$$

where λ varies from 0 to 1 as shown in Figure 3.7.

This leads to the following matrix representation between the normal forces and displacements:

$$\begin{bmatrix} R_c \\ R_4 \end{bmatrix} = K_{sl} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} PF_1 & PF_2 & PF_3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} d_{21} \\ d_{22} \\ d_{23} \\ d_{24} \end{bmatrix} \quad (3.105)$$

The normal force R_c is distributed to the structure nodes 1, 2 and 3 using the principal of equivalent nodal loads which is based on the virtual load method:

$$\begin{bmatrix} R_1 \\ R_2 \\ R_3 \\ R_4 \end{bmatrix} = \begin{bmatrix} PF_1 & 0 \\ PF_2 & 0 \\ PF_3 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} R_c \\ R_4 \end{bmatrix} \quad (3.106)$$

Therefore, the normal forces required to act at the structure nodes to enforce the sliding condition may be expressed as:

$$\begin{bmatrix} R_1 \\ R_2 \\ R_3 \\ R_4 \end{bmatrix} = K_{sl} \begin{bmatrix} PF_1 & 0 \\ PF_2 & 0 \\ PF_3 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix} \begin{bmatrix} PF_1 & PF_2 & PF_3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} d_{21} \\ d_{22} \\ d_{23} \\ d_{24} \end{bmatrix} \quad (3.107)$$

The local stiffness matrix of the sliding element is defined as:

$$\begin{bmatrix} R_1 \\ R_2 \\ R_3 \\ R_4 \end{bmatrix} = K_{local} \begin{bmatrix} d_{21} \\ d_{22} \\ d_{23} \\ d_{24} \end{bmatrix} \quad (3.108)$$

By Equating Equations 3.107 and 3.108 and performing the relevant matrix multiplication, K_{local} is determined as:

$$K_{local} = K_{sl} \begin{bmatrix} PF_1^2 & PF_1 PF_2 & PF_1 PF_3 & -PF_1 \\ PF_1 PF_2 & PF_2^2 & PF_2 PF_3 & -PF_2 \\ PF_1 PF_3 & PF_2 PF_3 & PF_3^2 & -PF_3 \\ -PF_1 & -PF_2 & -PF_3 & 1 \end{bmatrix} \quad (3.109)$$

3.9.5 Congruence Transformation

The local stiffness matrix in Equation 3.109 should be rotated to the global axes prior to the element stiffness assembly process. The transformation is applied as follows:

$$K_{sl}^G = T_1^T K_{local} T_1 \quad (3.110)$$

In the present study, only normal stiffnesses must be coupled on the interface region, the other tangential direction must remain uncoupled. Therefore, the transformation matrix T_1 is written as follows:

$$\underline{T}_t = \begin{bmatrix} \underline{t}_n^T & \underline{Q} & \underline{Q} & \underline{Q} \\ \underline{Q} & \underline{t}_n^T & \underline{Q} & \underline{Q} \\ \underline{Q} & \underline{Q} & \underline{t}_n^T & \underline{Q} \\ \underline{Q} & \underline{Q} & \underline{Q} & \underline{t}_n^T \end{bmatrix} \quad (3.111)$$

where

$$\underline{t}_n^T = [n_1 \ n_2] \quad (3.112)$$

$$\underline{Q} = [0 \ 0] \quad (3.113)$$

where n_1 and n_2 are the normal cosine directions.

3.10 TRANSIENT ANALYSIS

In Section 3.8, the matrix form of the governing equation of motion for the fluid-structure system was expressed as:

$$\underline{M}_T \underline{\ddot{a}} + \underline{C}_T \underline{\dot{v}} + \underline{K}_T \underline{d} = \underline{F}_{ext} - \underline{N}(v) \quad (3.114)$$

Equation 3.114 supplemented with the matrices of the sliding interface element, becomes:

$$\underline{M}_T \underline{\ddot{a}} + \underline{C}_T \underline{\dot{v}} + \underline{K}_T' \underline{d} = \underline{F}_{ext} - \underline{N}(v) \quad (3.115)$$

where

$$\underline{K}_T' = \underline{K}_T + \sum_{i=1}^{NELEM} (\underline{K}_{sl}^G)_i$$

The above equation supplemented with the mesh material velocity equation

$$\hat{\underline{v}} = \underline{A}(\underline{v} - \underline{L}^{-1} \underline{S})$$

defines the fluid-structure system of matrix equations discretised with respect to the space variables.

The transient solution of the governing equations normally involves the use of time

integration methods. These can be classified into Mode Superposition and Direct Integration techniques. The former is usually restricted to linear dynamic analysis, while the latter is considered best suited for nonlinear transient problems. In the context of the present study, Equation 3.115 is nonlinear due to the presence of the nonlinear convective term, free surface sloshing, and material and geometric nonlinearities in the structure. Therefore, direct transient integration methods should be developed and used to solve for the stresses, displacements, velocities and accelerations appearing in the governing equation. Also, an iterative solution scheme is required to enforce equilibrium at each time step. In general, this is accomplished by the use of a predictor-multi-corrector direct time integration scheme.

3.10.1 Background

As discussed in Chapter Two, there are two general classes of direct time integration schemes: implicit and explicit methods. Implicit algorithms, permit larger time steps, but the cost per step is high and storage requirements tend to increase dramatically with the size of the finite element mesh. Such implicit integration methods have been used by Bathe and Sonnad (1980) to study the transient response of several fluid-structure systems.

On the other hand, explicit algorithms require smaller time steps to achieve numerical stability; however, they tend to be less expensive per time step and use less storage than implicit algorithms. Explicit schemes of the type mentioned above have been extensively used by Belytschko and Hsieh (1973) as well as Key and Kreig (1973).

Belytschko and Mullen (1977) and (1978) proposed implicit-explicit schemes, where the fluid is treated explicitly. They showed that the stability limit on the time step is determined strictly by the highest frequency in the explicit partition. Hughes and Liu (1978a) and (1978b), developed implicit-explicit mesh partition schemes which treat the fluid explicitly, whereas the structure is integrated in an implicit manner. Felippa and Park (1980) developed staggered analysis procedures for coupled systems.

Among the various applications of these schemes may be cited the work of Shantaram et al (1976) and Belytschko and Kennedy (1978), where the equations of motion for the fluid were integrated via an explicit method. More recently, Singh et al (1990) developed both implicit-implicit and implicit-explicit mesh partitioning schemes and applied them to a variety of fluid-structure interaction problems.

Liu (1981), Liu and Ma (1982) and Ma et al (1982) extensively employed the

implicit-explicit integration method developed by Hughes and Liu (1978a) to study the transient behaviour of a variety of fluid-structure systems.

An implicit time integration treatment of the free surface pressure and the convective force would result in their presence on the left hand side of the equation of motion for the fluid-structure system. Due to the unsymmetric nature of the two terms mentioned above, their presence on the left hand side of the equation would lead to an unsymmetric system of equations which would result in high storage and CPU requirements. On the other hand, an explicit time integration treatment of the above two terms which would result in their presence on the right hand side of the equations with substantial savings in both storage and computational requirements, has been adopted by several researchers. For example, Liu and Gvildys (1986) developed a predictor-multi-corrector time marching algorithm for analysing nonlinear fluid-structure systems. In this method, the convective and sloshing terms were treated explicitly. It was demonstrated that, for most free surface flows, the algorithm is practically an unconditionally stable method.

In the context of this study, both the convective and the free surface sloshing terms are treated explicitly.

3.10.2 Direct Time Integration

The essence of direct numerical integration is based on two ideas:

- The governing equations are satisfied only at discrete time intervals Δt apart, and not at any time t .
- A variation of displacements, velocities and accelerations within each time interval Δt is assumed.

The form of the assumption regarding the variation of the kinetic quantities within each time interval, determines the accuracy, stability and cost of the solution procedure. In general, using the variation relationships, the displacements, velocities and accelerations at time $t + \Delta t$ are computed given the previous deformation history at time t .

Newmark (1959) was amongst the first to develop a family of step by step time integration methods for the solution of transient problems. The Newmark family, which will be discussed in this section, incorporates a wide variety of schemes each being associated with a particular pair of time integration parameters β and γ . For

example, the trapezoidal rule corresponds to the choice of $\beta = 0.25$ and $\gamma = 0.5$. According to Bathe and Wilson (1973), this family of methods results in stable and dissipative schemes for parameter values of $\gamma \geq 0.5$.

Wilson et al (1973) modified the linear acceleration method (member of the Newmark family with $\beta = 1/6$ and $\gamma = 1/2$) in an attempt to compensate for the spurious oscillations inherent in discrete approximations to continuum problems. The basic objective was that the acceleration field ought to be most accurate at the midpoint of the time interval. However, it was shown by Hilber (1976) that the dissipative properties of the Newmark family possess undesirable features, since the low frequency modes are affected too strongly when only dissipation in the high frequency modes is required.

Hilber et al (1977) showed that the Wilson method possess excessive dissipation, which in many practical situations requires a smaller step size to be used than would be required for stability. In addition, it was observed that when the step size is large in comparison with the shortest period of the system, the discrete solution generated by the Wilson method can significantly overshoot the exact solution in the first couple of time steps, thus causing large peak amplitude errors.

Towards this end, Hilber et al (1977) and Wood et al (1980) developed step by step direct time integration schemes which avoid the undesirable features of the methods discussed above. The main feature of both methods was the introduction of a third time integration parameter, α , in the governing equation of motion.

Adams and Wood (1983) carried out an appraisal of the Hilber-Hughes-Taylor scheme suggested by Hilber et al (1977) and the α -Bossak method proposed by Wood et al (1980), where it was shown that the methods are exactly the same and identical to Newmark's method when α is set to zero. Furthermore, it was concluded that for small values of α , where both methods possessed satisfactory dissipative properties, the results using the two methods are similar. On the other hand, for large values of α the difference between the results obtained from the two methods becomes more distinguishable.

In the context of this study, the Newmark and the α -Bossak methods, both used in ASAS-NL for the time integration of the structural equations of motion, are employed for the implicit time integration of the coupled fluid-structure equations of motion.

Newmark β - γ method

In this method, the following assumptions, regarding the variation of the material displacement, velocity and acceleration, in each time increment are made:

$${}^{t+\Delta t}\underline{d} = {}^t\underline{d} + {}^t\underline{v} \Delta t + \left[\left(\frac{1}{2} - \beta_N \right) {}^t\underline{a} + \beta_N {}^{t+\Delta t}\underline{a} \right] \Delta t^2 \quad (3.116)$$

$${}^{t+\Delta t}\underline{v} = {}^t\underline{v} + \left[(1 - \gamma_N) {}^t\underline{a} + \gamma_N {}^{t+\Delta t}\underline{a} \right] \Delta t \quad (3.117)$$

where the superscript at the top left hand side denotes the time.

Newmark (1959) originally proposed the constant average acceleration method in which $\gamma = 0.5$ and $\beta = 0.25$ (Figure 3.8). For nonlinear transient fluid-structure problems, it is convenient to express the governing equation of motion in terms of the incremental acceleration vector. Towards this end, using Equations 3.116 and 3.117 the incremental velocity and displacement vectors can be related to the incremental acceleration vector as follows:

$${}^{t+\Delta t}\Delta\underline{d} = \Delta t {}^t\underline{v} + \frac{\Delta t^2}{2} {}^t\underline{a} + \beta_N \Delta t^2 {}^{t+\Delta t}\Delta\underline{a} \quad (3.118a)$$

$${}^{t+\Delta t}\Delta\underline{v} = \Delta t {}^t\underline{a} + \gamma_N \Delta t {}^{t+\Delta t}\Delta\underline{a} \quad (3.118b)$$

where

$${}^{t+\Delta t}\Delta\underline{d} = {}^{t+\Delta t}\underline{d} - {}^t\underline{d}$$

$${}^{t+\Delta t}\Delta\underline{v} = {}^{t+\Delta t}\underline{v} - {}^t\underline{v}$$

$${}^{t+\Delta t}\Delta\underline{a} = {}^{t+\Delta t}\underline{a} - {}^t\underline{a}$$

Similarly, the incremental mesh displacement and velocity vectors are related to the incremental mesh acceleration vector as:

$${}^{t+\Delta t}\Delta\hat{\underline{d}} = \Delta t {}^t\hat{\underline{v}} + \frac{\Delta t^2}{2} {}^t\hat{\underline{a}} + \beta_N \Delta t^2 {}^{t+\Delta t}\Delta\hat{\underline{a}} \quad (3.119a)$$

$${}^{t+\Delta t}\Delta\hat{\underline{v}} = \Delta t {}^t\hat{\underline{a}} + \gamma_N \Delta t {}^{t+\Delta t}\Delta\hat{\underline{a}} \quad (3.119b)$$

where

$${}^{t+\Delta t}\Delta \hat{\underline{d}} = {}^{t+\Delta t}\hat{\underline{d}} - {}^t\hat{\underline{d}}$$

$${}^{t+\Delta t}\Delta \hat{\underline{y}} = {}^{t+\Delta t}\hat{\underline{y}} - {}^t\hat{\underline{y}}$$

$${}^{t+\Delta t}\Delta \hat{\underline{a}} = {}^{t+\Delta t}\hat{\underline{a}} - {}^t\hat{\underline{a}}$$

The equation of motion of the fluid-structure system may be written as:

$$\begin{aligned} & {}^{t+\Delta t}\underline{M}_T \left({}^t\hat{\underline{a}} + {}^{t+\Delta t}\Delta \hat{\underline{a}} \right) + {}^{t+\Delta t}\underline{C}_T \left({}^t\hat{\underline{y}} + {}^{t+\Delta t}\Delta \hat{\underline{y}} \right) + \\ & {}^{t+\Delta t}\underline{K}'_T \left({}^t\hat{\underline{d}} + {}^{t+\Delta t}\Delta \hat{\underline{d}} \right) = {}^{t+\Delta t}\underline{F}_{\text{ext}} - \underline{N}({}^t\hat{\underline{v}}) \end{aligned} \quad (3.120a)$$

However, since the current geometry of the fluid-structure system is not known, the coordinates of the previous time step are used in the evaluation of the element and global matrices.

$$\begin{aligned} & {}^t\underline{M}_T \left({}^t\hat{\underline{a}} + {}^{t+\Delta t}\Delta \hat{\underline{a}} \right) + {}^t\underline{C}_T \left({}^t\hat{\underline{y}} + {}^{t+\Delta t}\Delta \hat{\underline{y}} \right) + \\ & {}^t\underline{K}'_T \left({}^t\hat{\underline{d}} + {}^{t+\Delta t}\Delta \hat{\underline{d}} \right) = {}^{t+\Delta t}\underline{F}_{\text{ext}} - \underline{N}({}^t\hat{\underline{v}}) \end{aligned} \quad (3.120b)$$

By substituting Equation 3.118 into 3.120, the resulting equation of motion expressed in terms of the acceleration increment is obtained as:

$$\begin{aligned} & \left({}^t\underline{M}_T + \gamma_N \Delta t {}^t\underline{C}_T + \beta_N \Delta t^2 {}^t\underline{K}'_T \right) {}^{t+\Delta t}\Delta \hat{\underline{a}} = {}^{t+\Delta t}\underline{F}_{\text{ext}} - {}^t\underline{N}({}^t\hat{\underline{v}}) - \\ & {}^t\underline{M} {}^t\hat{\underline{a}} - {}^t\underline{C}_T \left({}^t\hat{\underline{y}} + \Delta t {}^t\hat{\underline{a}} \right) - {}^t\underline{K}'_T \left({}^t\hat{\underline{d}} + \Delta t {}^t\hat{\underline{v}} + \frac{\Delta t^2}{2} {}^t\hat{\underline{a}} \right) \end{aligned} \quad (3.121)$$

Alternatively, the above equation may be expressed as:

$${}^{t+\Delta t}\underline{M}_{\text{eff}} {}^{t+\Delta t}\Delta \hat{\underline{a}} = {}^{t+\Delta t}\underline{F}_{\text{eff}} \quad (3.122)$$

where

$${}^{t+\Delta t}\underline{M}_{\text{eff}} = \left({}^t\underline{M}_T + \gamma_N \Delta t {}^t\underline{C}_T + \beta_N \Delta t^2 {}^t\underline{K}'_T \right)$$

$${}^{t+\Delta t}\mathbf{F}_{\text{eff}} = {}^{t+\Delta t}\mathbf{F}_{\text{ext}} - {}^t\mathbf{N}(\mathbf{v}) - {}^t\mathbf{M}_T \dot{\mathbf{a}} - {}^t\mathbf{C}_T \left(\dot{\mathbf{v}} + \Delta t \ddot{\mathbf{a}} \right) - {}^t\mathbf{K}_T \left(\mathbf{d} + \Delta t \dot{\mathbf{v}} + \frac{\Delta t^2}{2} \ddot{\mathbf{a}} \right)$$

α -Bossak method

Wood et al (1980) introduced an additional parameter in the governing differential equation for controlling the damping properties of Newmark's time integration algorithm. This method is defined by the following equations:

$${}^{t+\Delta t}\mathbf{d} = \mathbf{d} + \dot{\mathbf{v}} \Delta t + \left[\left(\frac{1}{2} - \beta_\beta \right) \ddot{\mathbf{a}} + \beta_\beta {}^{t+\Delta t}\ddot{\mathbf{a}} \right] \Delta t^2 \quad (3.123a)$$

$${}^{t+\Delta t}\dot{\mathbf{v}} = \dot{\mathbf{v}} + \left[(1 - \gamma_\beta) \ddot{\mathbf{a}} + \gamma_\beta {}^{t+\Delta t}\ddot{\mathbf{a}} \right] \Delta t \quad (3.123b)$$

and

$$\begin{aligned} (1 - \alpha_\beta) {}^t\mathbf{M}_T {}^{t+\Delta t}\ddot{\mathbf{a}} + \alpha_\beta {}^t\mathbf{M}_T \ddot{\mathbf{a}} + {}^t\mathbf{C}_T {}^{t+\Delta t}\dot{\mathbf{v}} + {}^t\mathbf{K}_T {}^{t+\Delta t}\mathbf{d} \\ = {}^{t+\Delta t}\mathbf{F} - {}^t\mathbf{N}(\mathbf{v}) \end{aligned} \quad (3.124)$$

It should be noted that if α_β is set to zero, $\gamma_\beta = \gamma_N$ and $\beta_\beta = \beta_N$, the above method reduces to the Newmark time integration scheme.

The incremental material displacement and velocity vectors may be expressed in terms of the incremental acceleration vector as:

$${}^{t+\Delta t}\Delta \mathbf{d} = \Delta t \dot{\mathbf{v}} + \frac{\Delta t^2}{2} \ddot{\mathbf{a}} + \beta_\beta \Delta t^2 {}^{t+\Delta t}\ddot{\mathbf{a}} \quad (3.125a)$$

$${}^{t+\Delta t}\Delta \dot{\mathbf{v}} = \Delta t \ddot{\mathbf{a}} + \gamma_\beta \Delta t {}^{t+\Delta t}\ddot{\mathbf{a}} \quad (3.125b)$$

Similarly, the incremental mesh displacement and velocity vectors are expressed in terms of the incremental acceleration vector as:

$${}^{t+\Delta t}\Delta \hat{\mathbf{d}} = \Delta t \hat{\dot{\mathbf{v}}} + \frac{\Delta t^2}{2} \hat{\ddot{\mathbf{a}}} + \beta_\beta \Delta t^2 {}^{t+\Delta t}\hat{\ddot{\mathbf{a}}} \quad (3.126a)$$

$${}^{t+\Delta t}\Delta \hat{\underline{y}} = \Delta t \hat{\underline{v}}_a + \gamma_\beta \Delta t {}^{t+\Delta t}\Delta \hat{\underline{a}} \quad (3.126b)$$

By substituting the expressions for the material incremental displacement and acceleration vectors (Equation 3.125) into the governing equation of motion (Equation 3.124), the following form is obtained in terms of the incremental acceleration vector:

$$\begin{aligned} \left((1 - \alpha_\beta) {}^t\mathbf{M}_T + \gamma_\beta \Delta t {}^t\mathbf{C}_T + \beta_\beta \Delta t^2 {}^t\mathbf{K}_T' \right) {}^{t+\Delta t}\Delta \underline{a} = & {}^{t+\Delta t}\underline{F}_{\text{ext}} - \\ & {}^t\mathbf{N}(\underline{v}) - {}^t\mathbf{M}_T \underline{v}_a - {}^t\mathbf{C}_T \left(\underline{v} + \Delta t \underline{v}_a \right) - {}^t\mathbf{K}_T' \left(\underline{d} + \Delta t \underline{v} + \frac{\Delta t^2}{2} \underline{v}_a \right) \end{aligned} \quad (3.127)$$

Alternatively, as for the Newmark case, the above equation can be expressed in the form of Equation 3.122:

$${}^{t+\Delta t}\underline{\mathbf{M}}_{\text{eff}} {}^{t+\Delta t}\Delta \underline{a} = {}^{t+\Delta t}\underline{F}_{\text{eff}}$$

where for this method:

$$\begin{aligned} {}^{t+\Delta t}\underline{\mathbf{M}}_{\text{eff}} &= \left((1 - \alpha_\beta) {}^t\mathbf{M}_T + \gamma_\beta \Delta t {}^t\mathbf{C}_T + \beta_\beta \Delta t^2 {}^t\mathbf{K}_T' \right) \\ {}^{t+\Delta t}\underline{F}_{\text{eff}} &= {}^{t+\Delta t}\underline{F}_{\text{ext}} - {}^t\mathbf{N}(\underline{v}) - {}^t\mathbf{M}_T \underline{v}_a - {}^t\mathbf{C}_T \left(\underline{v} + \Delta t \underline{v}_a \right) - \\ & \quad {}^t\mathbf{K}_T' \left(\underline{d} + \Delta t \underline{v} + \frac{\Delta t^2}{2} \underline{v}_a \right) \end{aligned}$$

3.10.3 Nonlinear iterative schemes

In a finite element formulation, the following equations for a dynamic problem are solved:

$${}^{t+\Delta t}\underline{\mathbf{M}}_{\text{eff}} {}^{t+\Delta t}\Delta \underline{a} = {}^{t+\Delta t}\underline{F}_{\text{eff}}$$

where $\underline{\mathbf{M}}_{\text{eff}}$ and $\underline{F}_{\text{eff}}$ are determined by the choice of the time integration scheme used.

For linear problems the solution of the above equation results in a unique set of

incremental accelerations which are used to advance the solution one time step. However, this is no longer the case in many nonlinear situations. Physical insight into the problem combined with small-step iterative incremental approaches are usually essential to obtain physically significant answers.

The development of an efficient and elegant incremental iterative solution algorithm has drawn much research attention. An extensive summary of the most popular techniques used for the numerical solution of nonlinear dynamic equations can be found in the work by Rao (1989) and Zienkiewicz and Taylor (1991).

In the context of this study, both Newton Raphson (N-R) and Modified Newton Raphson (MN-R) methods, available in the general purpose finite element package ASAS-NL, have been used to solve the nonlinear set of equations. The proposed mesh and material time integration schemes, as well as the pressure time integration method, have been implemented within a N-R or (MN-R) iterative loop. These will be briefly discussed in this section.

Newton Raphson algorithm

In the case of a nonlinear fluid-structure system, Equations 3.121 and 3.128 give a first approximation to the acceleration increment Δa . It is convenient to rewrite the equation of motion, Equation 3.122, at iteration j and time step n corresponding to time $t + \Delta t$, in terms of an incremental iterative acceleration vector defined as:

$${}^{t+\Delta t}\delta \ddot{a}^j = {}^{t+\Delta t}\ddot{a}^j - {}^{t+\Delta t}\ddot{a}^{j-1} \quad (3.128)$$

with a superscript on the right hand side denoting the value of the iteration counter.

Using Equations 3.116 (or 3.123a for α -Bossak), the linearised form of the displacement vector can be related to the linearised acceleration vector as follows:

$${}^{t+\Delta t}\ddot{d}^{j-1} = \ddot{d} + \ddot{v} \Delta t + \left[\left(\frac{1}{2} - \beta_N \right) \ddot{a} + \beta_N {}^{t+\Delta t}\ddot{a}^{j-1} \right] \Delta t^2 \quad (3.129a)$$

$${}^{t+\Delta t}\ddot{d}^j = \ddot{d} + \ddot{v} \Delta t + \left[\left(\frac{1}{2} - \beta \right) \ddot{a} + \beta_N {}^{t+\Delta t}\ddot{a}^j \right] \Delta t^2 \quad (3.129b)$$

By subtracting Equation 3.129a from 3.129b, the increment in displacement can be written as:

$${}^{t+\Delta t}\delta_{\underline{d}}^j = \beta_N \Delta t^2 {}^{t+\Delta t}\delta_{\underline{a}}^j \quad (3.130)$$

Similarly, the material velocity vector, as well as the pressure, mesh velocity, and mesh displacement vectors may be expressed as:

$${}^{t+\Delta t}\delta_{\underline{v}}^j = \gamma_N \Delta t {}^{t+\Delta t}\delta_{\underline{a}}^j \quad (3.131a)$$

$${}^{t+\Delta t}\widehat{\delta_{\underline{v}}^j} = \gamma_N \Delta t {}^{t+\Delta t}\widehat{\delta_{\underline{a}}^j} \quad (3.131b)$$

$${}^{t+\Delta t}\widehat{\delta_{\underline{d}}^j} = \beta_N \Delta t^2 {}^{t+\Delta t}\widehat{\delta_{\underline{a}}^j} \quad (3.131c)$$

In order to express the governing equation in terms of the linearised acceleration vector, Equation 3.120a is rewritten as:

$$\begin{aligned} {}^{t+\Delta t}\underline{M}_T^j \left({}^{t+\Delta t}\underline{d}^{j-1} + {}^{t+\Delta t}\delta_{\underline{a}}^j \right) + {}^{t+\Delta t}\underline{C}_T^j \left({}^{t+\Delta t}\underline{v}^{j-1} + {}^{t+\Delta t}\delta_{\underline{v}}^j \right) + \\ {}^{t+\Delta t}\underline{K}_T'^j \left({}^{t+\Delta t}\underline{d}^{j-1} + {}^{t+\Delta t}\delta_{\underline{d}}^j \right) = {}^{t+\Delta t}\underline{F}_{ext} - \underline{N}({}^{t+\Delta t}\underline{v}^{j-1}) \end{aligned} \quad (3.132)$$

However, since the geometry at iteration j is not known before hand, the element and global matrices are evaluated using the geometry of iteration $j-1$. Thus the above equation becomes:

$$\begin{aligned} {}^{t+\Delta t}\underline{M}_T^{j-1} \left({}^{t+\Delta t}\underline{d}^{j-1} + {}^{t+\Delta t}\delta_{\underline{a}}^j \right) + {}^{t+\Delta t}\underline{C}_T^{j-1} \left({}^{t+\Delta t}\underline{v}^{j-1} + {}^{t+\Delta t}\delta_{\underline{v}}^j \right) + \\ {}^{t+\Delta t}\underline{K}_T'^{j-1} \left({}^{t+\Delta t}\underline{d}^{j-1} + {}^{t+\Delta t}\delta_{\underline{d}}^j \right) = {}^{t+\Delta t}\underline{F}_{ext} - \underline{N}({}^{t+\Delta t}\underline{v}^{j-1}) \end{aligned} \quad (3.133)$$

By substituting Equations 3.133 and 3.134a into Equation 3.135b, the equation of motion corresponding to the Newmark method is obtained:

$$\begin{aligned} \left({}^{t+\Delta t}\underline{M}_T^{j-1} + \gamma_N \Delta t {}^{t+\Delta t}\underline{C}_T^{j-1} + \beta_N \Delta t^2 {}^{t+\Delta t}\underline{K}_T'^{j-1} \right) {}^{t+\Delta t}\delta_{\underline{a}}^j = \\ {}^{t+\Delta t}\underline{F}_{ext} - {}^{t+\Delta t}\underline{N}(\underline{v})^{j-1} - {}^{t+\Delta t}\underline{M}_T^{j-1} {}^{t+\Delta t}\underline{d}^{j-1} - \\ {}^{t+\Delta t}\underline{C}_T^{j-1} \left({}^{t+\Delta t}\underline{v}^{j-1} + \Delta t {}^{t+\Delta t}\underline{d}^{j-1} \right) - \\ {}^{t+\Delta t}\underline{K}_{slf}^{G,j-1} \left({}^{t+\Delta t}\underline{d}^{j-1} + \Delta t {}^{t+\Delta t}\underline{v}^{j-1} + \frac{\Delta t^2}{2} {}^{t+\Delta t}\underline{a}^{j-1} \right) - {}^{t+\Delta t}\underline{F}_{int.str}^{j-1} \end{aligned} \quad (3.134)$$

while those corresponding to the α -Bossak method can be expressed as:

$$\begin{aligned} & \left((1 - \alpha_\beta) {}^{t+\Delta t}\underline{M}_T^{j-1} + \gamma_\beta \Delta t {}^{t+\Delta t}\underline{C}_T^{j-1} + \beta_\beta \Delta t^2 {}^{t+\Delta t}\underline{K}_T^{j-1} \right) {}^{t+\Delta t}\delta \underline{a}^j = \\ & {}^{t+\Delta t}\underline{F}_{ext} - {}^{t+\Delta t}\underline{N}(v)^{j-1} - {}^{t+\Delta t}\underline{M}_T^{j-1} \left((1 - \alpha_\beta) {}^{t+\Delta t}\underline{a}^{j-1} + \alpha_\beta {}^t\underline{a} \right) - \\ & {}^{t+\Delta t}\underline{C}_T^{j-1} \left({}^{t+\Delta t}\underline{v}^{j-1} + \Delta t {}^{t+\Delta t}\underline{a}^{j-1} \right) - {}^{t+\Delta t}\underline{p}^{j-1}(v) - {}^{t+\Delta t}\underline{F}_{int.str}^{j-1} \\ & {}^{t+\Delta t}\underline{K}_{slf}^{j-1} \left({}^{t+\Delta t}\underline{a}^{j-1} + \Delta t {}^{t+\Delta t}\underline{v}^{j-1} + \frac{\Delta t^2}{2} {}^t\underline{a}^{j-1} \right) \end{aligned} \quad (3.135)$$

where

$\underline{K}_{slf}$ is the total of the sliding and fluid global stiffness matrices

$$\underline{K}_{slf} = \underline{K}_f + \sum_{i=1}^{NELEM} (\underline{K}_{sl}^G)_i, \text{ and}$$

${}^{t+\Delta t}\underline{F}_{int.str}^{j-1}$ is the internal residual force for the structure which depends on the kind of structural nonlinearity under consideration

A detailed discussion on the various types of structural nonlinearity may be found by Zienkiewicz and Taylor (1991).

Alternatively, Equations 3.134 and 3.135 may be expressed as:

$${}^{t+\Delta t}\underline{M}_{eff}^{j-1} {}^{t+\Delta t}\delta \underline{a}^j = {}^{t+\Delta t}\underline{F}_{un}^j \quad (3.136)$$

This new approximate acceleration increment gives rise to new displacement and velocity fields, which in turn result in new incremental strains and stresses. Using the above quantities, computed at iteration j , a new set of effective applied loads and internal equivalent loads are recomputed.

However, due to the presence of nonlinearities in the discretised equation, the internal equivalent loads will not, in general, balance the effective applied loads. In other words, if:

${}^{t+\Delta t}\underline{F}_{eff}^j$ is the effective applied load vector in current configuration at iteration j , and
 ${}^{t+\Delta t}\underline{F}_{int}^j$ is the internally balanced load vector in the current configuration at iteration j ,



then the effective out of balance load vector at iteration j is given by :

$${}^{t+\Delta t}\underline{F}_{un}^j = {}^{t+\Delta t}\underline{F}_{eff}^j - {}^{t+\Delta t}\underline{F}_{int}^j \quad (3.137)$$

where for the Newmark method:

$$\begin{aligned} {}^{t+\Delta t}\underline{F}_{eff}^j = & {}^{t+\Delta t}\underline{F}_{ext} - \underline{M} {}^{t+\Delta t}\underline{\ddot{a}}_{j-1} - {}^{t+\Delta t}\underline{C} {}^{t+\Delta t}\underline{\dot{y}}_{j-1} - \\ & {}^{t+\Delta t}\underline{N} ({}^{t+\Delta t}\underline{y}_{j-1}) - {}^{t+\Delta t}\underline{K}_{slf} {}^{t+\Delta t}\underline{d}_{j-1} \end{aligned}$$

In addition to the last term appearing on the right hand side of Equation 3.134, which accounts for the material and geometric structural nonlinearities, it should be noted that the explicit time integration treatment of the convective and free surface fluid terms as well as the nature of the mesh rezoning equation also requires the use of an iterative scheme.

The new increment of acceleration can be computed as follows:

$$\delta \underline{a}^j = {}^{t+\Delta t}\underline{M}_{eff}^{j-1-1} {}^{t+\Delta t}\underline{F}_{un}^j \quad (3.138)$$

The new approximation for the acceleration field will be:

$${}^{t+\Delta t}\underline{a}^{j+1} = {}^{t+\Delta t}\underline{a}^j + {}^{t+\Delta t}\delta \underline{a}^{j+1} \quad (3.139)$$

After a series of successive iterations, the acceleration increments are accumulated until convergence occurs, as follows:

$${}^{t+\Delta t}\underline{a}^j = {}^{t+\Delta t}\underline{a}^{(0)} + \sum_{k=1}^j {}^{t+\Delta t}\delta \underline{a}^k \quad (3.140)$$

Modified Newton Raphson

This method differs from the N-R method only in that the effective mass matrix either is not updated or is updated infrequently, for example at the beginning of every time increment. Thus, the expensive repetitions of forming and reducing the effective tangent stiffness matrix is avoided; however, more iterative cycles are needed in order to reach a prescribed degree of accuracy. Figures 3.9 and 3.10 illustrate the full Newton-Raphson and modified Newton-Raphson nonlinear iterative schemes.

Convergence criteria

When iterating and trying to reach a solution which satisfies the equilibrium conditions at time step $t + \Delta t$ and iteration j , different convergence criteria may be used as guidelines to satisfy the equilibrium conditions. The most important of which are mentioned below:

Displacement

$${}^{t+\Delta t} \|\delta \underline{d}\|^j \leq \epsilon \quad (3.141)$$

where

${}^{t+\Delta t} \|\delta \underline{d}\|^j$: is the norm of the incremental displacement vector.

ϵ : is the tolerance

Residual Force

$${}^{t+\Delta t} \|\delta \underline{F}_{un}\|^j \leq \epsilon \quad (3.142)$$

${}^{t+\Delta t} \|\delta \underline{F}_{un}\|^j$: is the norm of the incremental residual force vector.

Usually, either the maximum or the Euclidean norm are used in conjunction with the above convergence criteria. These are defined as:

Euclidian Norm

The Euclidian norm of a vector $\delta \underline{d}$ is defined as:

$${}^{t+\Delta t} \|\delta \underline{d}\|_{\text{Euc}}^j = \sqrt{\sum_{i=1}^{\text{ndof}} ({}^{t+\Delta t} \delta d_i^j)^2} \quad (3.143)$$

where ndof is the total number of degrees of freedom of the fluid-structure mesh.

Maximum Norm

The maximum norm of a vector $\delta \underline{d}$ is defined as the element in the vector with the largest absolute value:

$$\|\delta \underline{d}\|_{\text{Max}} = \max |\delta d_i| \text{ for } i = 1, 2, \dots, \text{ndof} \quad (3.144)$$

The choice of the convergence criteria is problem dependent. The one used most often is that relating to residual forces with the Euclidean norm. However, for special classes of problems experiencing large displacements, a displacement convergence criterion is necessary.

3.10.4 Predictor-Multi-Corrector Time Marching Algorithm

A combined time integration, incremental iterative scheme for the solution of nonlinear fluid-structure systems has been fully developed and used in this study. The predictor corrector algorithm used in this work is based on the one available in ASAS-NL, however it has been modified to accommodate the fluid mesh and material quantities as well as the sliding element matrices.

i- Set initial conditions :

Specify $\underline{x}_0, \underline{d}_0, \underline{a}_0, \hat{\underline{x}}_0, \hat{\underline{d}}_0, \hat{\underline{a}}_0$ and p_0 .

Specify time integration parameters α, β and γ .

Specify time increment(s).

ii- Increment time by the specified time increment.

iii- Initialise iteration counter:

$i=0$, (iteration counter)

iv- Set predictor for material motion:

$$\underline{t} + \Delta t \underline{d}^0 = \underline{t}_d + \Delta t \underline{v} + \frac{1}{2} \Delta t^2 (1-2\beta) \underline{a} \quad (3.145)$$

$$\underline{t} + \Delta t \underline{v}^0 = \underline{t}_v + \Delta t (1-\gamma) \underline{a} \quad (3.146)$$

v- Set predictor for mesh quantities:

$${}^{t+\Delta t}\hat{\underline{q}}^0 = {}^t\hat{\underline{q}} + \Delta t {}^t\hat{\underline{v}} + \frac{1}{2}\Delta t^2 (1-2\beta) {}^t\hat{\underline{a}} \quad (3.147)$$

$${}^{t+\Delta t}\hat{\underline{v}}^0 = {}^t\hat{\underline{v}} + \Delta t (1-\gamma) {}^t\hat{\underline{a}} \quad (3.148)$$

vi- Check which matrices are required to be updated, then assemble and solve the governing equations:

$${}^{t+\Delta t}\underline{M}_{\text{eff}}^{i-1} {}^{t+\Delta t}\delta \underline{a}^i = {}^{t+\Delta t}\underline{F}_{\text{un}}^i \quad (3.149)$$

vii- Set corrector for material variables

$${}^{t+\Delta t}\underline{q}^{i+1} = {}^{t+\Delta t}\underline{q}^0 + \Delta t^2 \beta {}^{t+\Delta t}\underline{a}^{i+1} \quad (3.150)$$

$${}^{t+\Delta t}\underline{v}^{i+1} = {}^{t+\Delta t}\underline{v}^0 + \Delta t \gamma {}^{t+\Delta t}\underline{a}^{i+1} \quad (3.151)$$

$${}^{t+\Delta t}\underline{a}^{i+1} = {}^{t+\Delta t}\underline{a}^0 + {}^{t+\Delta t}\underline{\Delta a}^{i+1} \quad (3.152)$$

viii- Solve for mesh velocity from current material velocity and previous mesh displacement:

$${}^{t+\Delta t}\hat{\underline{v}}^{i+1} = \underline{A} {}^{t+\Delta t}\underline{v}^{i+1} - \underline{L}^{-1} {}^{t+\Delta t}\underline{g}^i \quad (3.153)$$

ix- Set corrector for mesh variables:

$${}^{t+\Delta t}\hat{\underline{a}}^{i+1} = \frac{({}^{t+\Delta t}\hat{\underline{v}}^{i+1} - {}^{t+\Delta t}\hat{\underline{v}}^0)}{\Delta t \gamma} \quad (3.154)$$

$${}^{t+\Delta t}\hat{\underline{q}}^{i+1} = {}^{t+\Delta t}\hat{\underline{q}}^0 + \Delta t^2 \beta {}^{t+\Delta t}\hat{\underline{a}}^{i+1} \quad (3.155)$$

x- Check if convergence has been achieved.

If $i \leq I$ where I is the maximum number of iterations, and convergence has not been achieved, increment i by 1 and go to (vi).

If $i > I$ and convergence has not been achieved, abort the analysis.

If convergence has been achieved, then go to (xi).

- xi- Check for termination of analysis
 If $t + \Delta t < t_{\text{final}}$, where t_{final} is the time specified by the user for the termination of the analysis, go to (ii).
 If t_{final} has been reached then terminate analysis.

Stability considerations

Stability criteria have not yet been established for the algorithm defined by Equations 3.145 to 3.155. This task is complicated by the presence of the equation for the moving mesh as well as the explicit treatment of the convective and free surface terms.

Accuracy as well as computational efficiency represent the fundamental rule for the selection of an appropriate time step. In order to avoid unnecessary computation, the time step size Δt must not be too large or too small. Hence, the selection of time step will depend on the user's experience in structural dynamics, in general, and the finite element technique, in particular.

3.11 PARTICULARS OF A GENERAL PURPOSE ANALYSIS PACKAGE

In order to achieve the objectives set out in Chapter One, it was decided to use and develop a general purpose finite element package for modelling both the structure and fluid domains. The chosen package should meet the following requirements:

- The developed computer program should be capable of modelling a variety of two-dimensional, axi-symmetric and three-dimensional fluid-structure systems, including geometric and material nonlinearities.
- The proposed code should include static, modal and dynamic analysis facilities.
- The developed code should have a facility for solving large problems.
- The code should be modular in nature, to facilitate proposed and future developments.
- The code should be user friendly with adequate pre- and post-processing facilities.

By implementing the fluid-structure interaction analysis facility into an already existing general purpose structural finite element code, significant advantages are gained, namely:

- Various structural elements would already have been formulated, implemented and validated.
- Solution algorithms would have been developed and validated.
- Data structure and program architecture would have been developed.
- Format and use of the package would be known to others.

The general purpose structural finite element code ASAS-NL was chosen since it met the requirements outlined above. Furthermore, Lagrangian fluid and sliding elements had already been implemented into it by a team of Imperial College researchers (Chelghoum et al, 1988). The developed analysis package has been subsequently designated ASAS-NL-FSI.

3.11.1 General ASAS-NL Analysis Facilities

ASAS-NL was developed by W.S. Atkins, as a general linear / nonlinear, static / dynamic response analysis tool. The program is widely used by both the academic and the industrial communities. It is written in a selected subset of FORTRAN IV and FORTRAN 77, intended to make it easily transportable to different computer types. The present version is available on VAX, CYBER, IBM, DEC, CRAY and several other main frame machines and workstations.

The current element library of ASAS-NL contains truss, plane stress, plane strain, axi-symmetric solid, three-dimensional solid, gap, plate, shell and beam elements amongst others.

Several constitutive laws are available for material modelling. They are divided into three groups; elasticity, plasticity and creep. Elastic models cover both linear and nonlinear, isotropic and anisotropic materials. A number of plastic models exist with a wide choice of yield criteria and hardening material rules.

A variety of loading types are available for both static and dynamic behaviour. Furthermore, by using load functions, a number of load cases may be combined together and their proportion may vary over the load history. Amongst others, may be cited nodal loads, prescribed displacements (velocities or acceleration), pressure and body forces.

The analysis capabilities include static, modal and dynamic analysis.

Clearly, the above mentioned features, provide the framework for satisfying the

specification requirements detailed in Section 3.11 above. A more detailed description of ASAS-NL capabilities may be found in the User's manual (ASAS-NL, 1988).

3.11.2 Structure of ASAS-NL

The data structure and organisation of ASAS-NL is well documented in the programmers manual (ASAS-NL, 1986); therefore, only a brief description will be given here.

The program is divided into a series of computation phases, concerned with data input, bandwidth optimisation, matrix formulation, response analysis, presentation of results and post-processing.

In order to be able to solve large problems, the only limitation imposed on the size of the problem is the amount of disk space available on the computer used. Furthermore, for large problems, not all the data need to be stored within the in-core memory of the computer, some being temporarily stored on disk. The temporary files used are presented and discussed in the programmers manual. The development of the fluid-structure interaction analysis facility must be compatible with this structure to allow the analysis of large coupled problems.

3.11.3 Proposed Modifications to ASAS-NL Structure

The procedures employed for the data input and solution of fluid-structure systems are identical to those used for structural problems. However, there exist important differences between the structure and the fluid and interface domains, which in some instances require modifications to the input and solution phases of the program. The main modifications can be summarised as follows:

- i- In addition to the basic input data (nodal coordinates, element topology and material data), the Lagrange-Euler parameter at each fluid node and the free surface element flag for each fluid element is required.
- ii- Although the fluid and sliding elements have the same nodal variables as those of the structure, (displacements, velocities and accelerations), there exist an important difference between them. Whereas the structure elements have only material nodal variables, the fluid and sliding elements have both mesh and material nodal variables. This requires the use of additional arrays and files to

store the various mesh kinematic quantities.

- iii- Although the solution process is similar in fluid-structure and structural problems, one additional equation is required to describe the complete problem in the former case. The mesh and material quantities should be related by Equation 3.94, which requires the introduction of a new routine at the control level.
- iv- The time integration scheme, applicable to the structure and material fluid kinematic quantities, should be extended to time integrate the fluid mesh kinematic quantities.

In addition to the above mentioned modifications, sliding and fluid elements must be added to the element library. This involves implementing the formulation of all the relevant element matrices mentioned in previous sections.

The implementation of all of the above mentioned modifications is explained in detail in Appendix B.

3.12 VALIDATION

In order to validate the accuracy of the proposed formulations, a variety of tests have been selected in such a way as to assess the following factors:

- Accuracy of fluid elements including the penalty on rotation term.
- Veracity of the free surface sloshing formulation.
- Correctness of the sliding element formulation.
- Versatility of the developed mesh rezoning scheme.

Towards this end, three categories of free surface wave propagation and fluid-structure interaction problems are illustrated:

- Hydrodynamic pressure and displacement history of a variety of fluid domains with rigid boundary conditions such as reservoir, water column and pipe systems. This serves as a first check on the validity of the fluid element, as well as the developed transient integration scheme.
- Transient fluid-structure interaction problems, such as fluid-plate and fluid-

piston systems. This gives an indication of the accuracy and versatility of the proposed sliding element.

- Free surface wave propagation problems, such as the propagation of solitary waves in a fluid medium.

Both two-dimensional and axi-symmetric examples are presented and discussed hereafter.

3.12.1 Fluid-Filled Pipe Closed at One End and Subjected to a Step Pressure

The purpose of this example is to test the accuracy of the two-dimensional fluid finite element formulation. The finite element model, geometric properties loading and boundary conditions are shown in Figure 3.11, where a pipe which is closed at one end is subjected to a step pressure loading at the other. Fluid-structure interaction effects are ignored; hence, the walls of the pipe are assumed rigid and it is not necessary to use either sliding or solid elements in the modelling of the problem. Furthermore, plane strain conditions are assumed and the fluid is modelled using five four-noded Mixed-Eulerian-Lagrangian fluid finite elements. The degrees of freedom corresponding to the loading and boundary conditions were treated as Lagrangian while all other degrees of freedom were considered to be Eulerian.

The same problem was studied by Akkas et al (1979) who monitored the element pressure variation with time. The fluid was modelled by finite elements with a mesh consisting of twenty four-noded Lagrangian fluid elements. One point Gaussian integration was used in the evaluation of the element matrices, while the Wilson- θ implicit method was employed for the time integration of the equation of motion.

Shantaram et al (1976) analysed the same problem by using a finite element model consisting of five eight-noded Lagrangian plane strain fluid elements with a zero shear modulus. Furthermore a four-point integration rule was employed in the evaluation of the element matrices. The fluid was assumed viscid and incompressible with a Poisson ratio of 0.5, while an explicit central difference scheme with diagonal damping and mass matrices was employed for the time integration of the equations of motion. In all analyses the size of the time step used was 0.011 msec.

The numerical results obtained, shown in Figures 3.12 and 3.13, are in reasonable agreement with the analytical solution except for some spurious pressure oscillations. Similar oscillations marked the response obtained in the above mentioned studies.

The discrepancy between the numerical results shown in Figures 3.12 and 3.13 may be attributed to the different kinematic descriptions of motion used in the formulations.

3.12.2 Reservoir System Subjected to Ground Acceleration

In this example, the response of a plane strain rigid wall reservoir system which is subjected to a base motion in form of a step function is studied. The purpose of this example is to provide another test for the accuracy of the proposed fluid element formulation. The geometry, material properties and finite element model are shown in Figure 3.14. Plane strain fluid elements were used in the analysis, along with the Newmark time integration scheme. The fluid finite element mesh consisted of sixty four, four-noded plane strain elements. Along the edges of the mesh, the degrees of freedom corresponding to the boundary conditions and to the free surface were treated as Lagrangian, while all other degrees of freedom were treated as Eulerian. For nodes lying inside the fluid mesh, all degrees of freedom were considered to be Eulerian.

The same problem has been solved by Akkas et al (1979) and Chelghoum (1986) who used a finite element model consisting of twenty five four-noded plane strain fluid elements. In the latter study, the wilson- θ time integration scheme was used for the integration of the equations of motion, while in the former study the implicit Newmark scheme was employed. In both cases the size of time step used in the analysis was 2.5 msec and the pressure variation with time was monitored.

In Figure 3.15, the pressure variation with time is shown, along with the results from the above mentioned studies. It can be seen that the predicted dynamic response agrees well with other numerical studies. The small discrepancy between the results may be attributed to the different kinematic descriptions of motion used for the fluid domain and the different time integration schemes used.

3.12.3 Tall Water Column Under Step Pressure

In this example an axi-symmetric rigidly contained water column, fixed at one end and subjected to a step pressure applied at its free end is considered. The water column is idealised using five four-noded fluid elements, shown in Figure 3.16 along with the geometric and material properties as well as the loading and boundary conditions. The degrees of freedom corresponding to the boundary conditions were treated as Lagrangian, while all others were treated as Eulerian. The objective of this example is to assess the accuracy of the proposed axi-symmetric fluid element in

determining the response of the column. Since the water column is assumed to be rigidly contained, the degrees of freedom in the radial direction are all suppressed.

This problem has been previously analysed by Bathe and Hahn (1979) and Chelghoum (1986) who used ten four-noded finite elements to model the fluid column. In both studies an Updated Lagrangian formulation with a large displacement option was used along with the implicit Newmark time integration scheme. In the former study full numerical integration was used, while in the latter study reduced integration with a penalty on rotation was employed in the calculation of the element stiffness matrix. In both studies, the size of the time step used in the analysis was .001 msec.

In Figure 3.17 the longitudinal displacement at the free end of the column is plotted against time, along with the results reported in the above mentioned studies. Very good agreement between all sets of results is demonstrated, although only half the number of elements was used in this study. The small discrepancy between the results may be attributed to the different kinematic descriptions of motion used in the modelling of the fluid medium.

3.12.4 Reservoir-Plate System Subjected to Step Pressure

In this example, the wave propagation in a reservoir, closed at one end and with a steel plate at the other is studied. The purpose of this problem is to provide a check on the proper use of the proposed sliding elements. Furthermore, by considering two cases for the plate, rigid and flexible, the effect of an uncoupled solution of the fluid-structure equations on the peak fluid pressures is assessed. First, for the rigid case the fluid pressure is determined from hydrodynamic considerations only. Next, a more accurate analysis is undertaken with the fluid-structure equations being solved simultaneously thus accounting for fluid-structure interaction effects. Along the edges of the mesh, the degrees of freedom corresponding to the free surface and the boundary conditions are treated as Lagrangian, while all others are considered to be Eulerian. In addition, all degrees of freedom corresponding to nodes lying inside the fluid mesh are treated as Eulerian.

The finite element model, geometry and material properties of this problem are depicted in Figure 3.18. The fluid is modelled using sixty four four-noded, two-dimensional plane strain elements. In addition for the flexible case, the wall is modelled using eight eight-noded two-dimensional plane strain solid elements, whereas seventeen, four-noded, two-dimensional sliding elements are used to impose

the sliding condition on the fluid-structure interface.

This problem has been studied by Akkas et al (1979) who modelled the fluid using one hundred four-noded plane strain fluid elements. The fluid-structure interface was modelled using two-noded horizontal truss elements; however, it was not mentioned how the wall at the end of the reservoir was modelled. The pressure variation with time was monitored.

Figures 3.19 and 3.20 show the variation of the pressure at element X with time, for the rigid and flexible cases respectively. A fair correlation can be seen between all results. The small deviation between the peak values may be explained by the different kinematic approaches adopted in the finite element formulation as well as the different schemes employed to model the incompressibility constraint. It can be seen that by ignoring fluid-structure interaction, the maximum fluid pressure acting on the walls of the reservoir is underestimated by 20%.

3.12.5 Fluid-Piston-Spring System Subjected to Step Pressure

This example is concerned with the interaction of an elastic piston with a slightly compressible fluid. At the end opposite to the elastic piston, the fluid is subjected to a step pressure P_0 . The purpose of this example is to test the accuracy of the sliding element and to try to demonstrate the importance of fluid-structure interaction effects.

The fluid-piston system along with the finite element model and boundary conditions are shown in Figure 3.21, where the fluid is modelled using five four-noded plane strain elements, whereas the mass is modelled by an eight-noded plane strain element with a large bulk modulus. A two-noded straight axial element with constant strain was used to model the spring. Since this problem is a classical one-dimensional fluid-structure system, all the degrees of freedom in the y direction were suppressed. At the fluid-structure interface two four-noded sliding elements were used to impose the sliding condition. The size of the time step used in the analysis was chosen as 0.02 msec with the analysis being terminated after 500 time steps. The fluid degrees of freedom corresponding to the boundary conditions as well as those corresponding to the fluid-structure interface are treated as Lagrangian, while all others are considered to be Eulerian.

To further the behavioural understanding of fluid-structure interaction effects, three cases are studied. Case 1 simulates the interaction between an incompressible fluid (modelled by setting the fluid bulk modulus to a very large value, such as 1.0×10^{12})

and a piston. In Cases 2 and 3 the mass ratio of the fluid to that of the piston is set to 1.25 and 10.0 respectively. This ratio represents various degrees of interaction between the fluid and the piston. Table 3.5 shows the various material and geometric properties used in all three cases.

A similar study was carried out by Liu and Chang (1985), who modelled the fluid by forty two-noded constant pressure fluid elements. In addition, the elastic piston was modelled by one spring element, while the mass was modelled by a lumped mass. The structural displacement, fluid velocity and fluid pressure were used as main independent variables, while the Mixed-Eulerian-Lagrangian kinematic description of motion was used for the fluid medium. Both implicit-implicit and implicit-explicit schemes were used to time-integrate the coupled fluid-structure equation of motion.

It should be stressed that the unavailability of a lumped mass option in ASAS-NL at the time of testing meant that a two-dimensional solid element had to be used. Therefore, the required input for the mass changed from an explicit value of total mass, which is the case for the lumped mass as reported by Liu and Chang (1985), to an explicit value of the density of the solid element. Due to the different methods employed in the modelling of the mass it was not possible to get exactly identical fluid to mass ratios which made any quantitative comparison meaningless. However, it should be mentioned that the results reported in the above reference exhibited similar trends to those obtained from this study.

Figure 3.22 shows the displacement of the piston for all three cases. The displacement of the piston is normalised with respect to the static piston displacement defined by:

$$P_0/K_s$$

where

K_s is the spring constant, and

P_0 is the magnitude of the applied pressure.

The time is normalised with respect to:

$$L_{FE}/C_F$$

where

L_{FE} is the length of the fluid element and, and
 C_F is the speed of sound propagation in the medium.

Material Property and Parameters	Case 1	Case 2	Case 3
ρ_f (density of fluid) (lb-sec ² /in ⁴)	1.0	1.0	8.0
β (bulk modulus of fluid) (lb/in ²)	$1.0 \cdot 10^{12}$	$2.5 \cdot 10^7$	$2.0 \cdot 10^8$
l (length of fluid) (in)	5.0	5.0	5.0
ρ_s (density of piston) (lb)	4.0	4.0	4.0
k (spring constant) (lb/in ²)	$1.0 \cdot 10^7$	$1.0 \cdot 10^7$	$1.0 \cdot 10^7$
P_0 (applied of pressure) (lb/in ²)	$1.0 \cdot 10^7$	$1.0 \cdot 10^7$	$1.0 \cdot 10^7$
Area of Piston (in ²)	1	1	1
Fluid-Piston mass ratio	1.25	1.25	10.0

**Table 3.5 Material Properties and Computation Parameters
for Fluid-Piston System**

The importance of fluid-structure interaction effects can be seen from the following observation: as the mass ratio increases, the amplitude of the normalised displacement decreases and the period of vibration elongates.

3.12.6 Solitary Pressure Wave Propagation in a Fluid Medium

The accurate prediction of the propagation of the solitary wave is fundamental to the design of marine structures in general, such as offshore platforms, breakwaters and ships. Therefore, the robustness and applicability of the proposed computational fluid dynamics formulation depends on its adaptability to solve flow problems involving wave propagation effects. Towards this end, this example will test the capability of the proposed formulation in modelling free surface wave propagation phenomena. In addition, it will provide a check on the free surface wave gravity formulation.

The medium under consideration is idealised as a two-dimensional plane strain problem in which the wave propagates in the x direction. The finite element model, geometric and material properties as well as the boundary conditions are shown in Figure 3.23. The finite element model consisted of eighty eight-noded plane strain elements. The solitary wave is generated by prescribing the displacement-time history of the fluid along the left hand side of the domain. According to Hughes et al (1981), this input produces a solitary wave in which the nonlinear and dispersive effects are balanced so that the wave propagates without distortion. Along the edges

of the fluid mesh, degrees of freedom corresponding to boundary and loading conditions as well as those corresponding to nodes lying on the free surface are treated as Lagrangian, while all others are considered to be Eulerian. In addition, the degrees of freedom corresponding to the nodes lying inside the fluid mesh are treated as Eulerian.

Hughes et al (1981) studied a similar problem where the fluid medium was modelled using one hundred and sixty four-noded Mixed-Eulerian Lagrangian fluid elements. The Newmark time integration scheme was used in the direct time integration of the equations of motion.

Figure 3.24 shows the variation of the free surface displacement along the finite element mesh. It can be seen from the results that the proposed formulation can model, to a sufficient degree of accuracy, solitary wave propagation phenomena. Furthermore, good agreement between both sets of results is obtained. The small discrepancy between both sets of results may be attributed to the different ways the incompressibility constraint are treated as well as the "upwinding procedure" used by Hughes et al (1981) in the calculation of the element compressibility matrix.

3.13 CONCLUSION

The work presented in this chapter has been concerned with the development of a general fluid-structure analysis package. Towards this end, the Mixed-Eulerian-Lagrangian fluid formulation has been derived and discussed.

Before using the Mixed-Eulerian-Lagrangian fluid and sliding finite elements in the general analysis of bounded and unbounded fluid-structure systems, it is required to demonstrate the elements' capabilities and identify their limitations. In this chapter, the Mixed-Eulerian-Lagrangian sliding and fluid elements have been validated for a variety of linear and nonlinear problems. To investigate the effects of the interaction at the fluid-structure boundary, both rigid and flexible fluid-structure systems have been studied. The effects of the free surface gravity wave were examined by analysing a solitary wave propagation problem. In addition, the importance of fluid-structure interaction effects have been demonstrated by analysing a one-dimensional fluid-piston-spring system.

Finite element models of rigid and flexible fluid-structure systems under a variety of dynamic loading have been compared with analytical and numerical solutions from

previous studies. In addition to the problems mentioned above, dynamic analysis of rigid and flexible reservoir systems have been carried out using the developed finite element formulation and have compared favourably with other methods.

By treating the fluid degrees of freedom corresponding to the boundary conditions as Lagrangian, while considering all those corresponding to the nodes lying inside the fluid mesh as Eulerian, the versatility of the proposed mesh rezoning scheme was demonstrated.

All the results of rigid and flexible fluid-structure systems for various loading conditions have shown good agreement with reference solutions. It is worth noting that the response of fluid-structure systems is particularly sensitive to certain modelling assumptions, such as, edge boundary conditions, time step size and choice of time integration scheme and associated parameters. This has made comparisons with published work somewhat difficult.

In general the Mixed-Eulerian-Lagrangian fluid and sliding elements, developed in the present work, can be used with confidence for linear and nonlinear analysis of rigid and flexible fluid-structure systems. Based on this validation of the fluid and sliding elements, it is now possible to apply them in the analysis of practical bounded fluid-structure problems such as fluid filled tanks and pressure vessels. This is the objective of the work presented in Chapter 5.

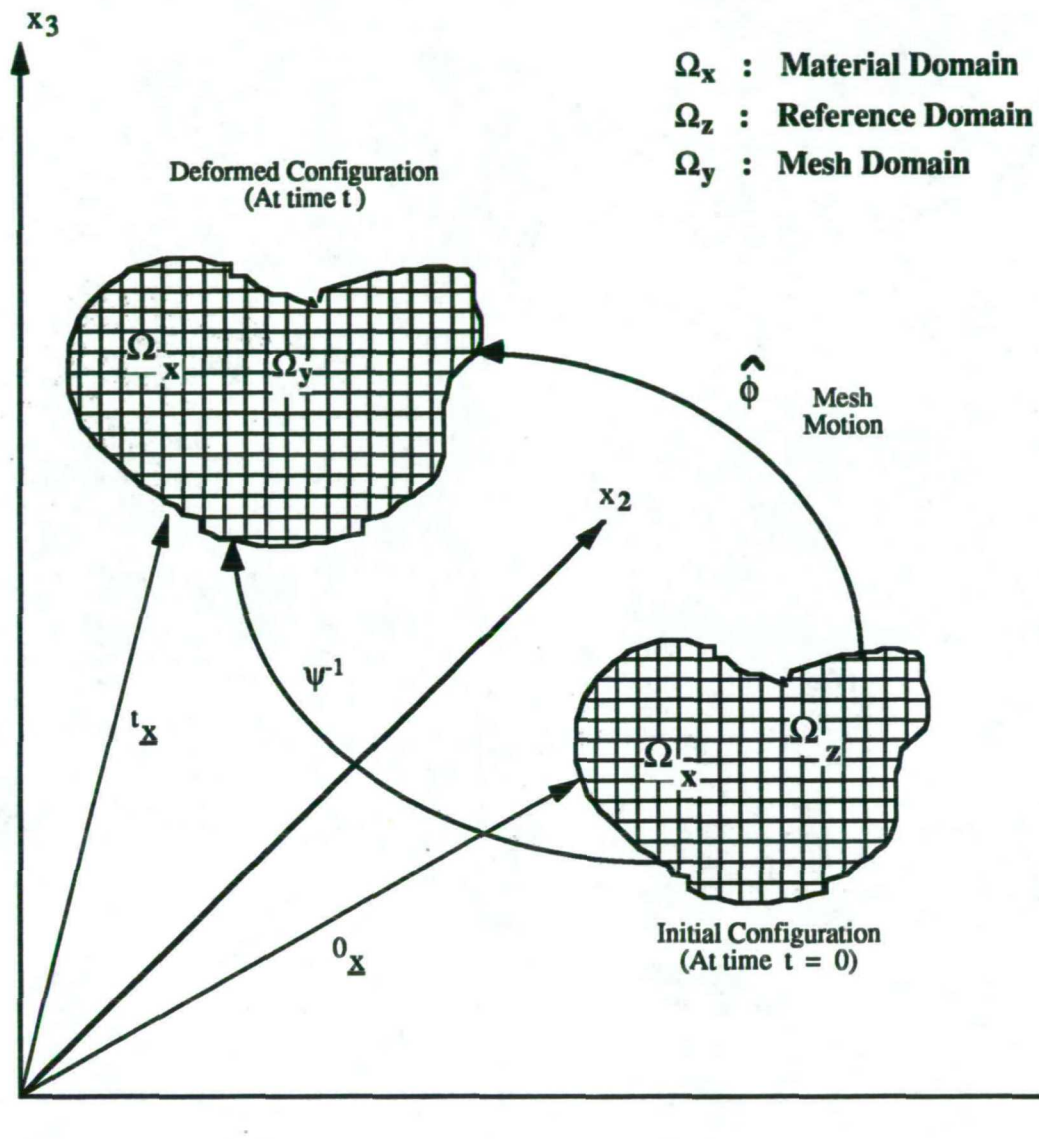


Figure 3.1 : Lagrangian Representation of Motion

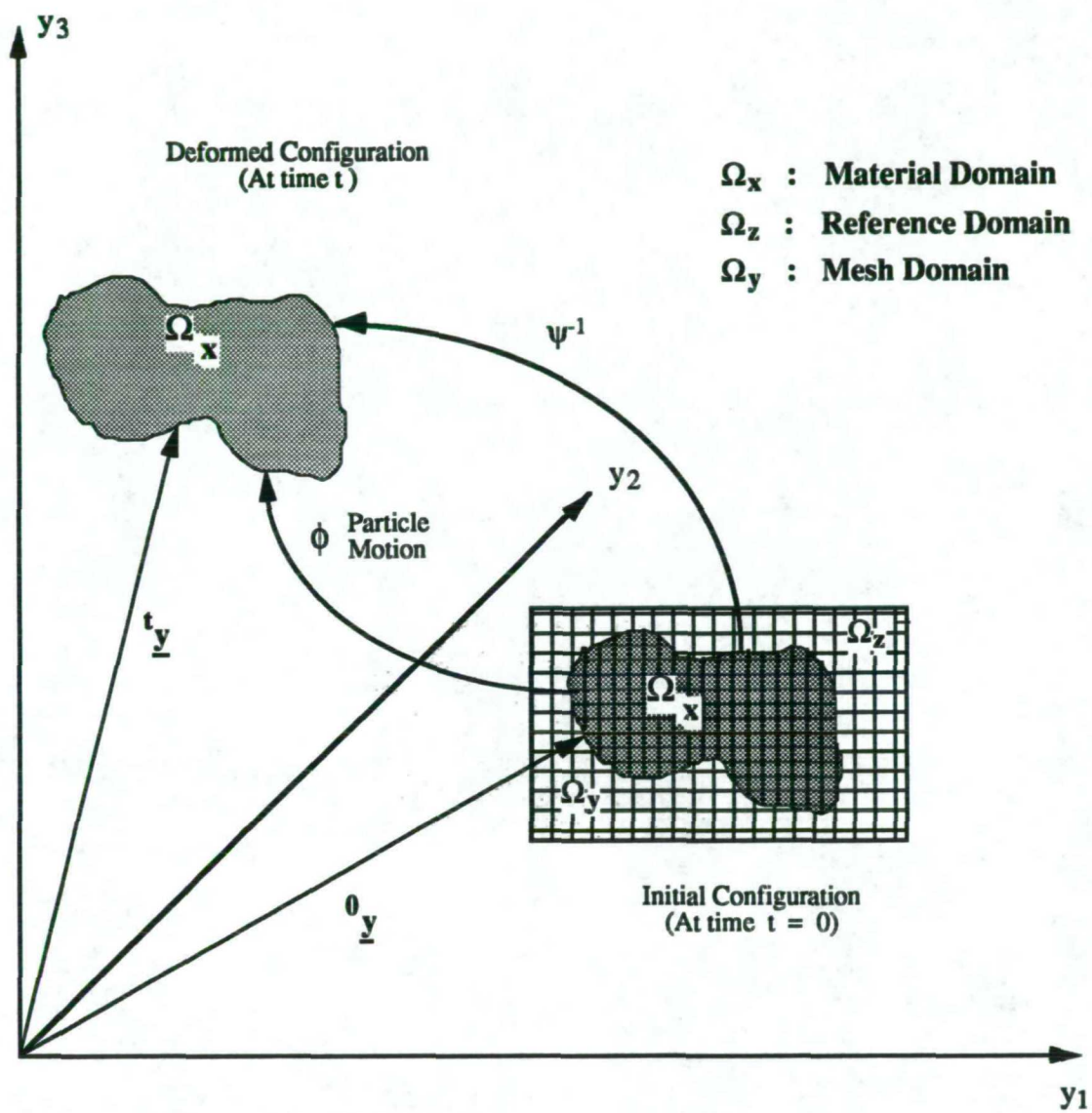


Figure 3.2 : Eulerian Representation of Motion

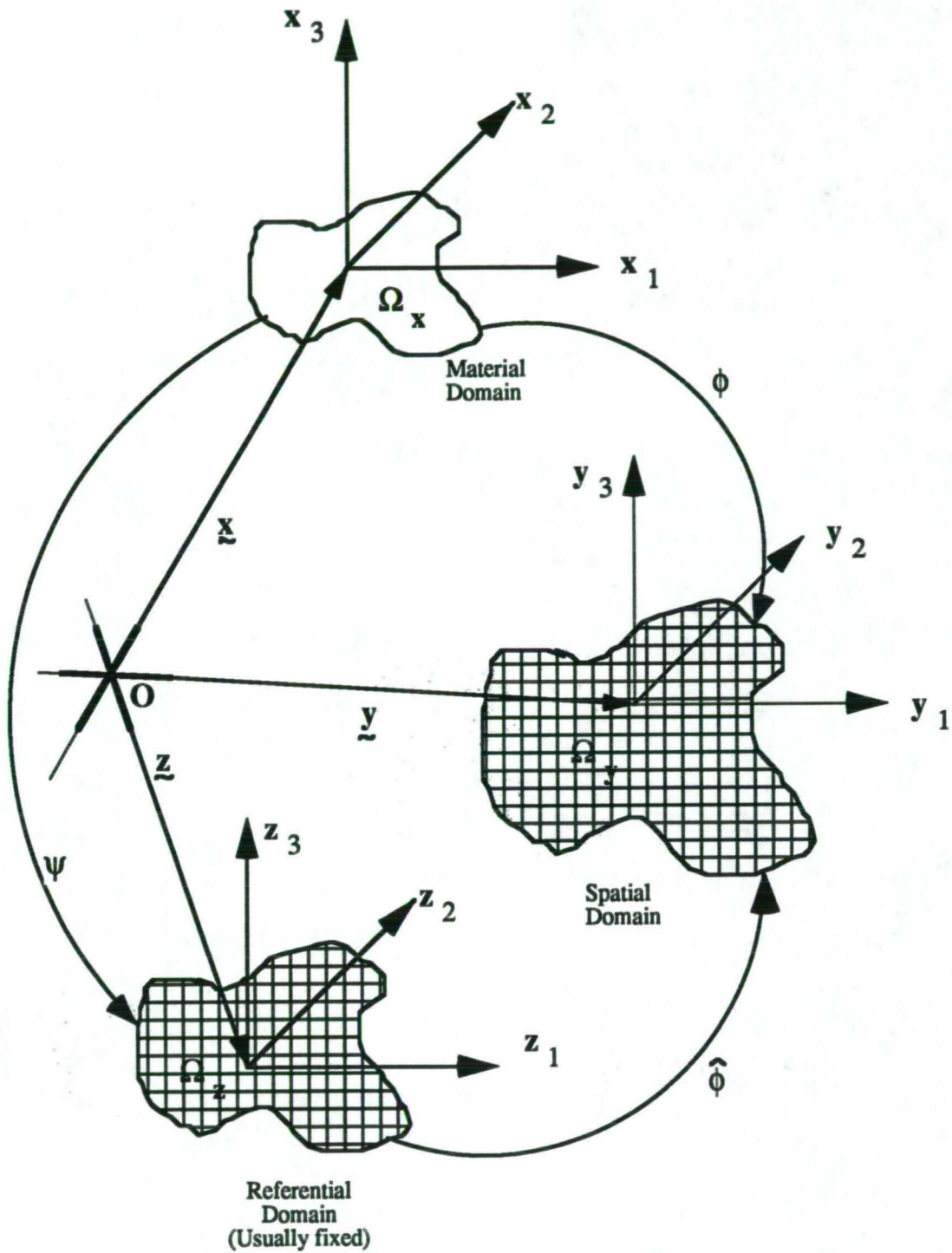


Figure 3.3 : The MEL Mapping Concept of a Fluid Body

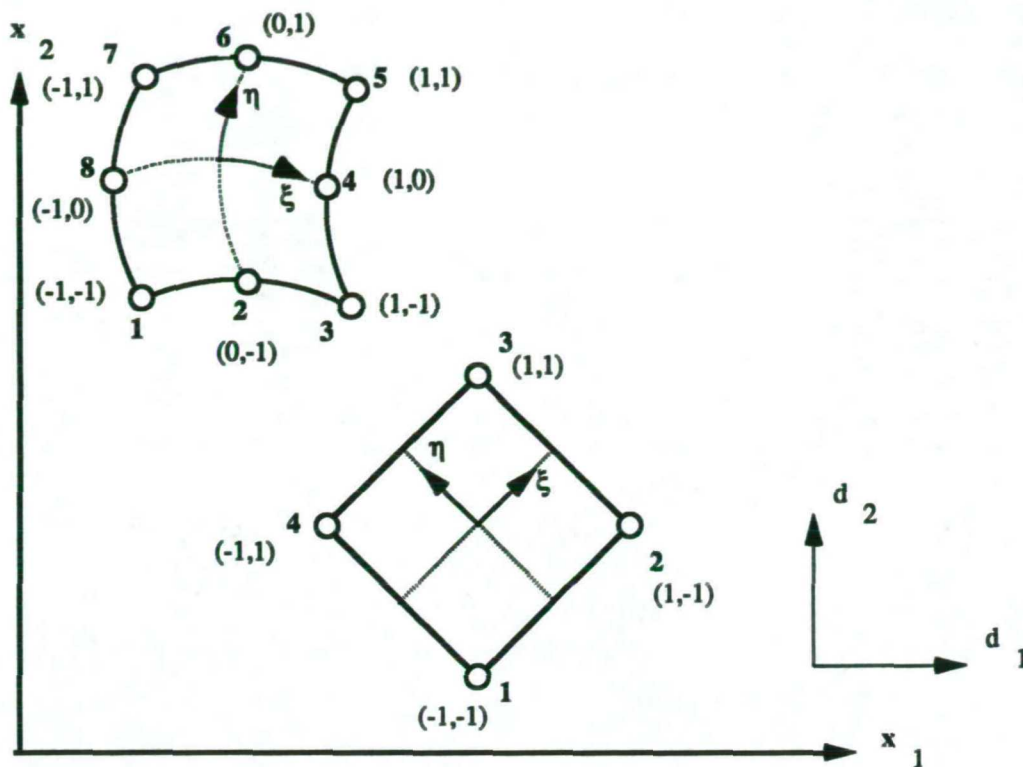


Figure 3.4 Numbering Convention and Geometric Configuration for Two-Dimensional Fluid Finite Elements

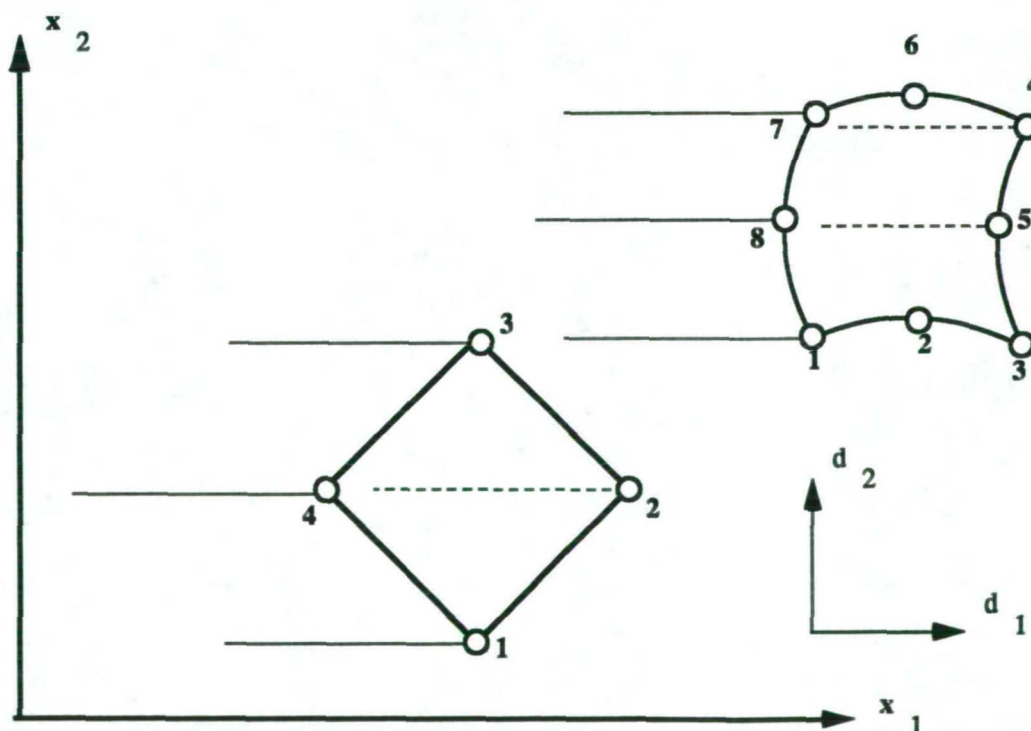


Figure 3.5 Numbering Convention and Geometric Configuration for Axi-Symmetric Fluid Finite Elements

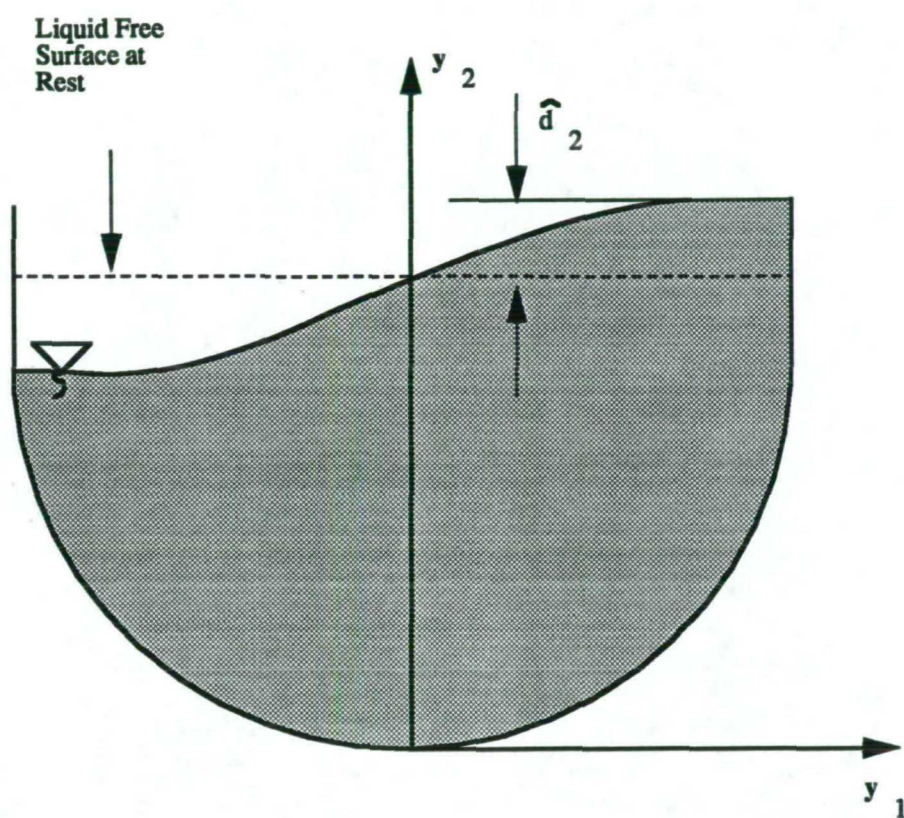
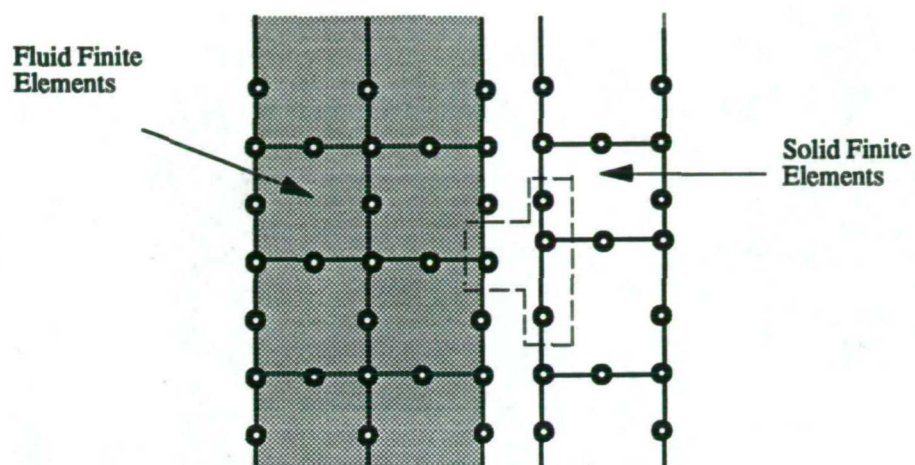
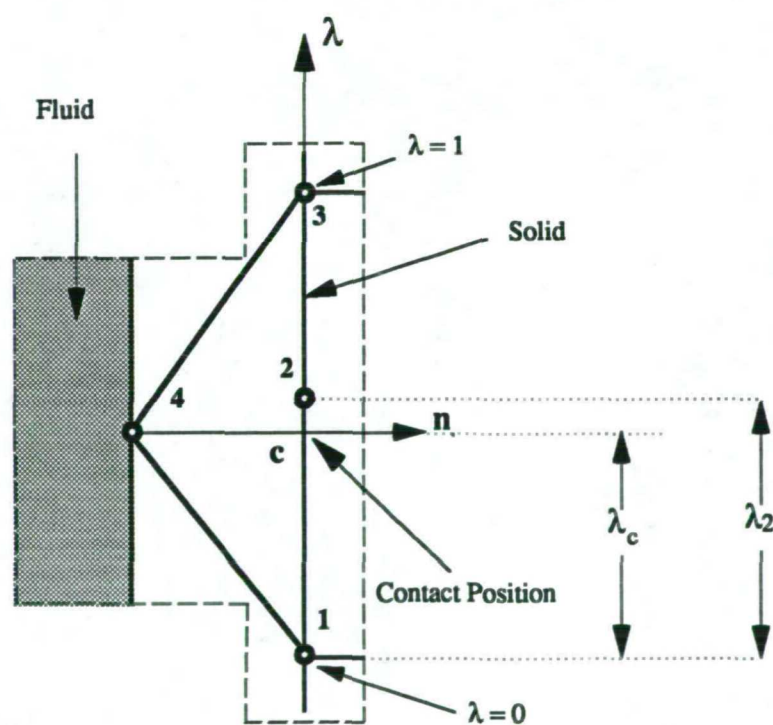


Figure 3.6 Free Surface Gravity Wave



(a) Solid-Fluid Boundary



(b) Sliding Element Geometry

Figure 3.7 (a) Solid-Fluid Boundary and
(b) Sliding Element Geometry

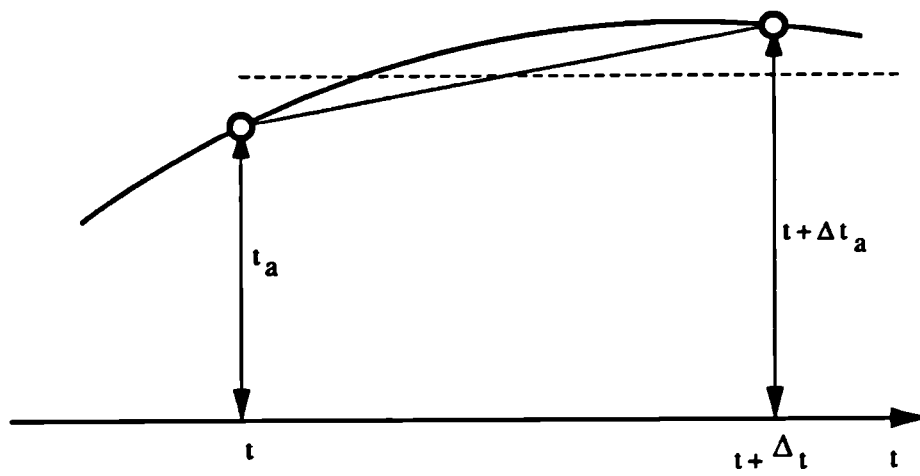


Figure 3.8 Newmark's Constant Average-Acceleration Time Integration Scheme

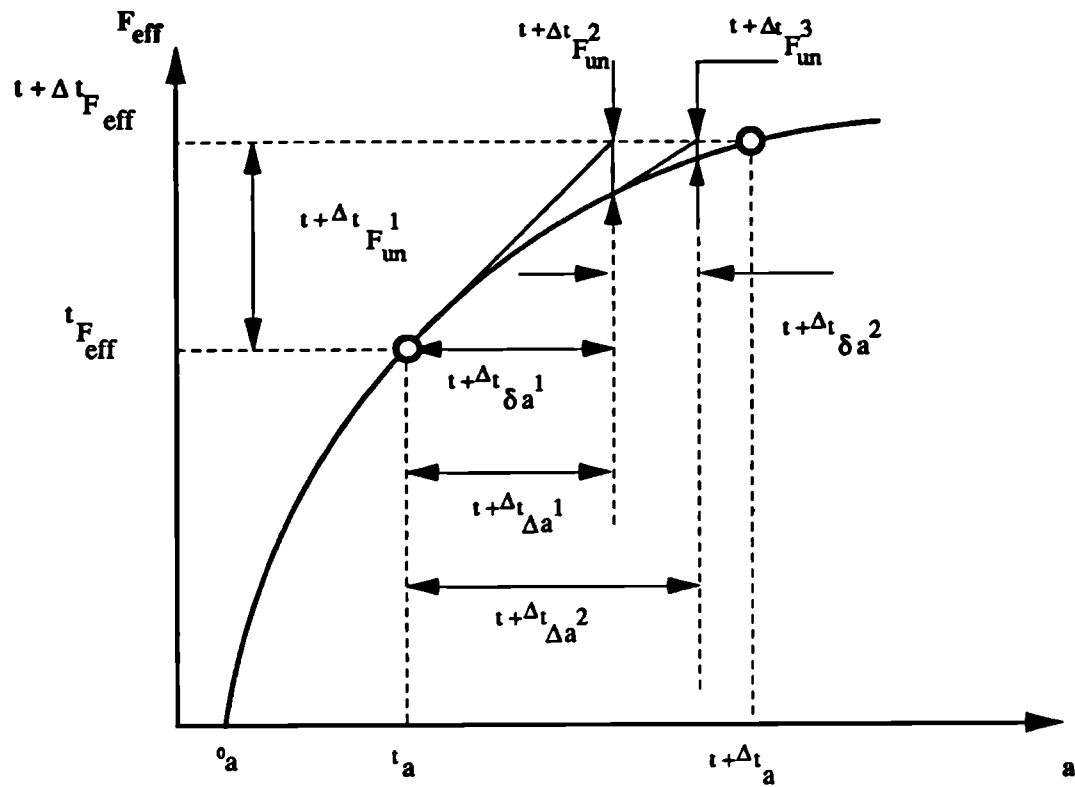


Figure 3.9 : Full Newton Raphson Iteration Scheme

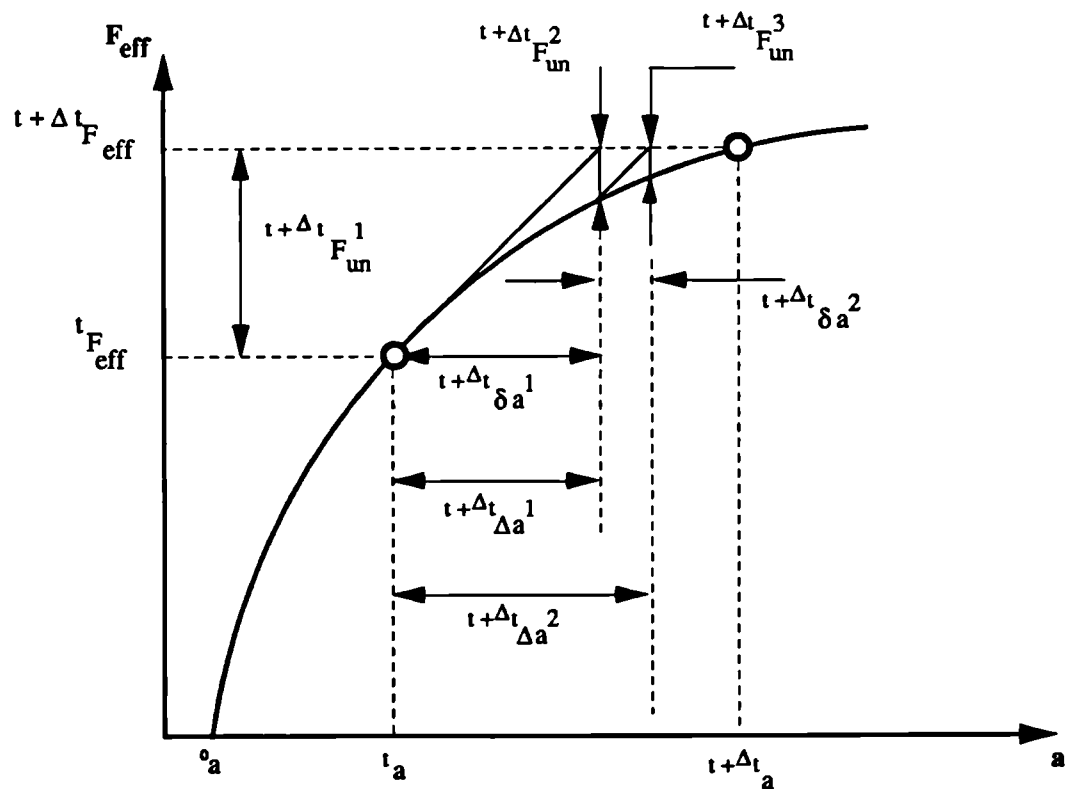
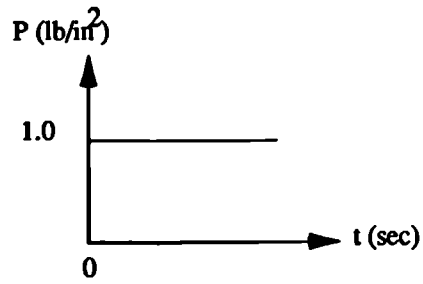
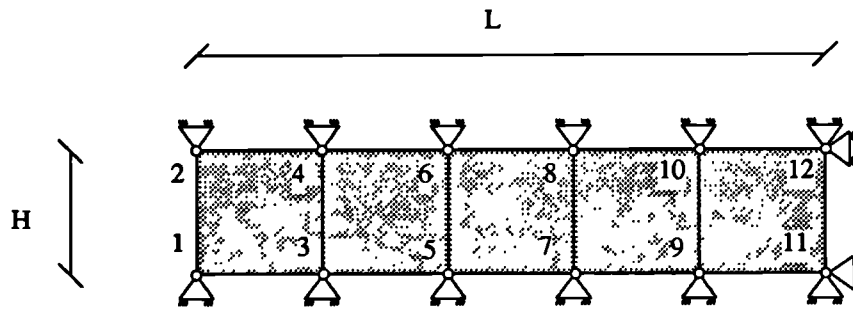


Figure 3.10 : Modified Newton Raphson Iteration Scheme



Uniform Horizontal Applied Pressure At Nodes 1 And 2



Five Two-Dimensional Four-Noded Fluid Finite Elements.

LEGEND

$$L = 20 \text{ in } H = 4 \text{ in}$$

$$\beta = 1.2 \text{ E}+05 \text{ lb/in}^2$$

$$\rho = 9.35 \text{ E-05 lb-sec}^2/\text{in}^4$$

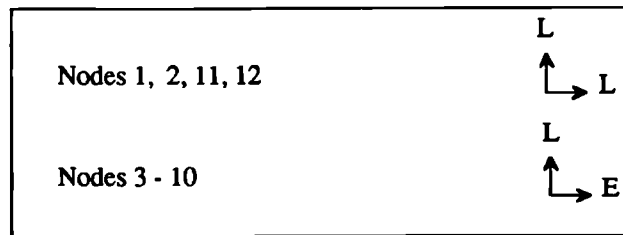


Figure 3.11 Fluid Filled Pipe, Closed at One End and Subjected to a Step Pressure

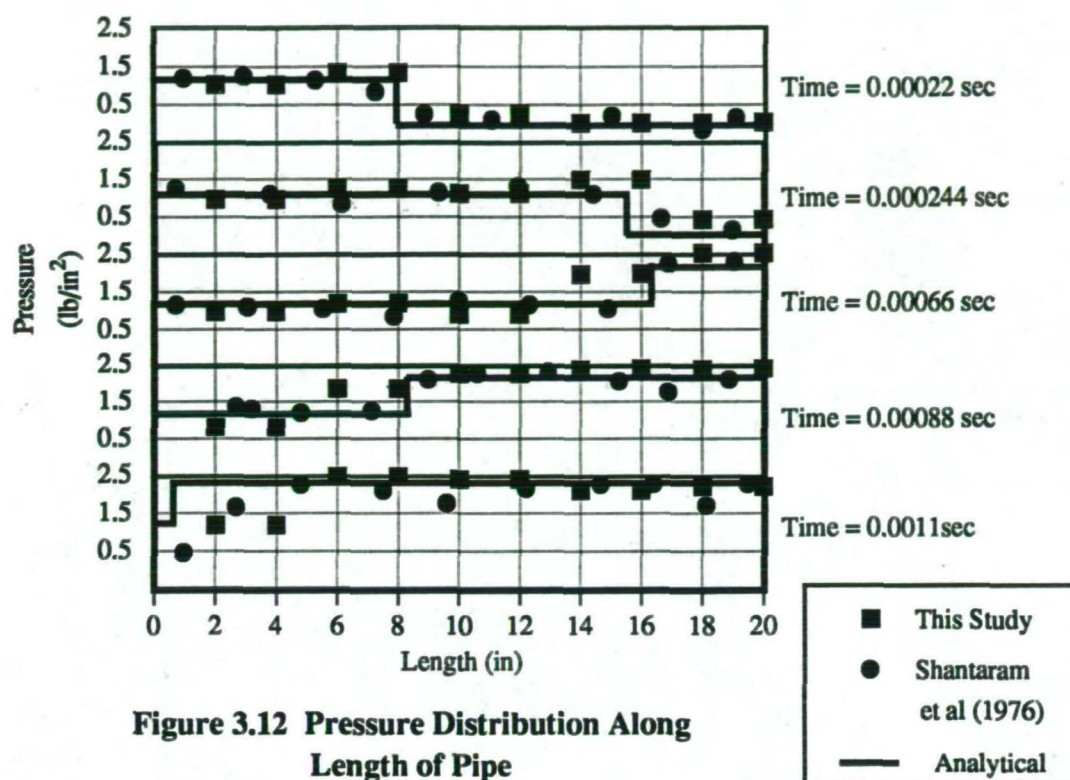


Figure 3.12 Pressure Distribution Along Length of Pipe

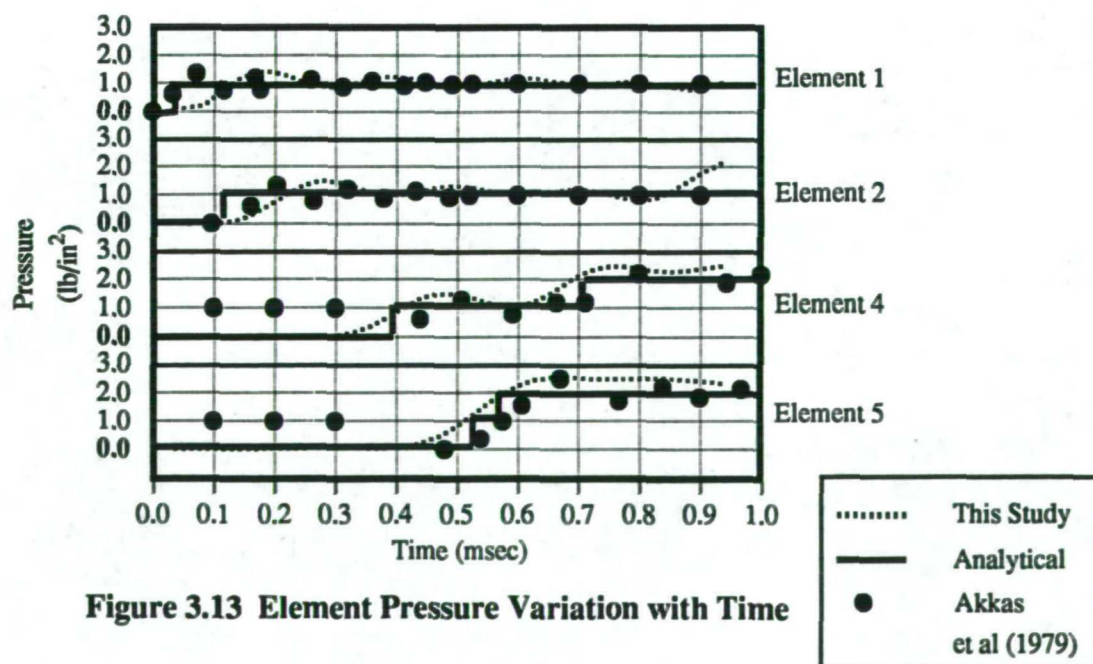


Figure 3.13 Element Pressure Variation with Time

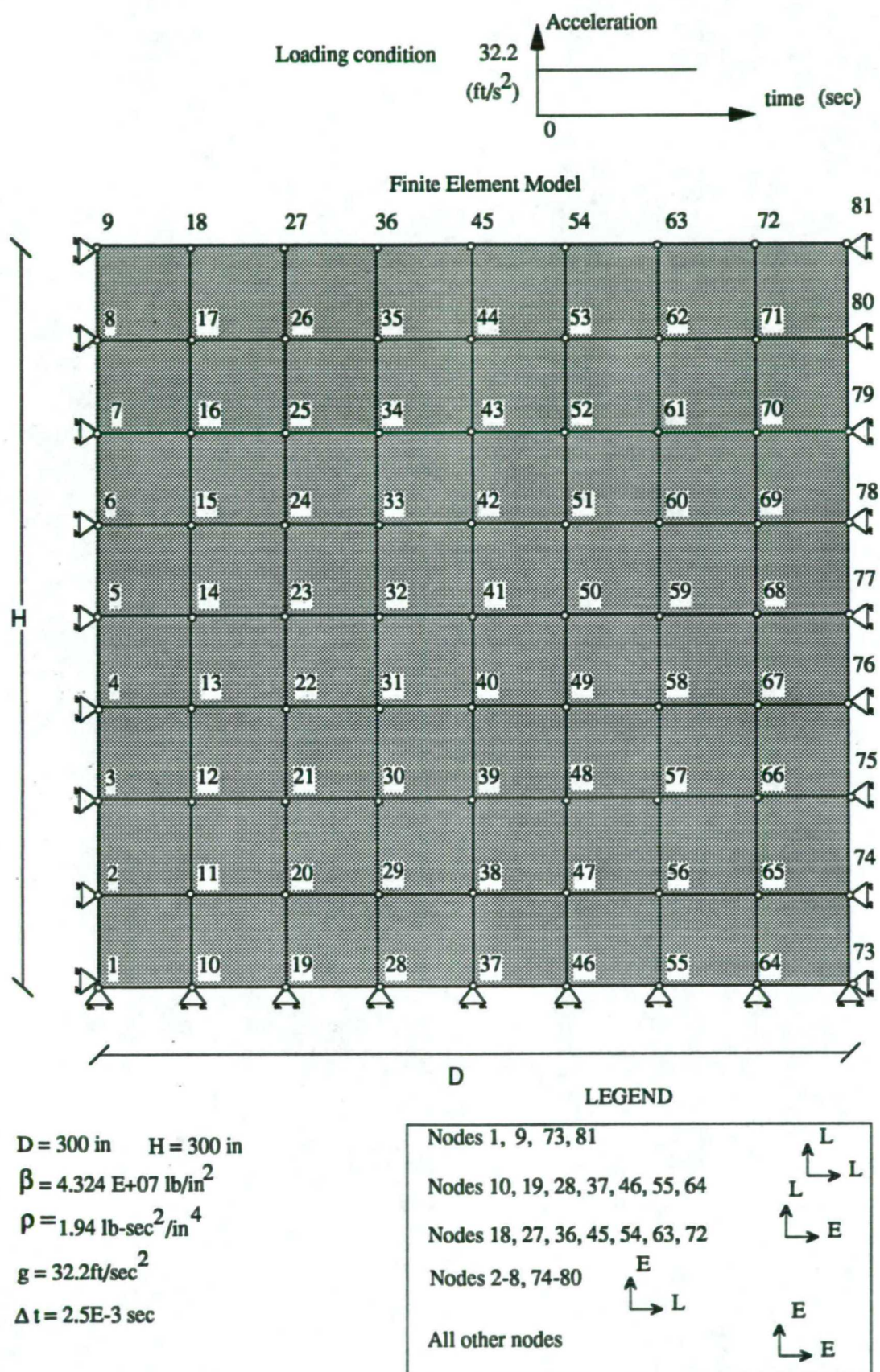


Figure 3.14 Reservoir Subjected to Ground Acceleration

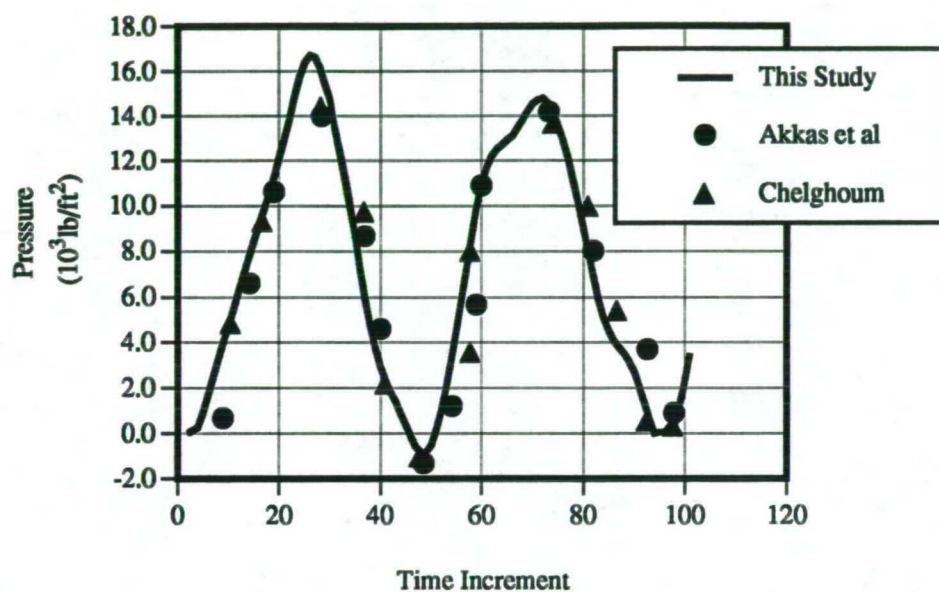
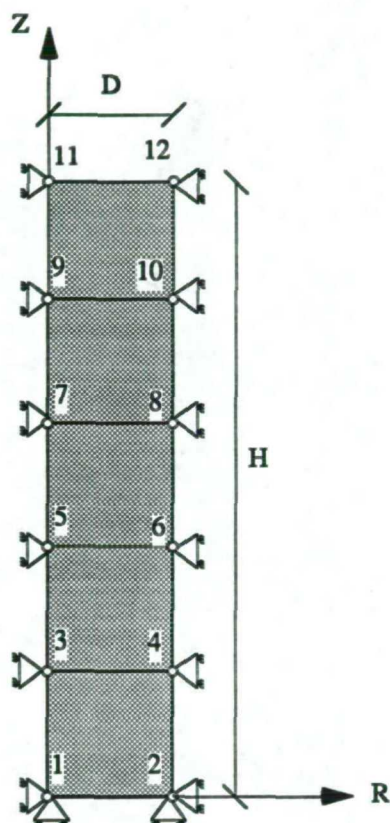


Figure 3.15 Pressure Variation in a Reservoir System Subjected to Ground Acceleration



Uniform Vertical Applied Pressure At Edge 11 - 12



Five Four-Noded Axi-symmetric Fluid Finite Elements

$D = 2 \text{ in}$ $H = 20 \text{ in}$
 $\beta = 3.0 \text{ E}+05 \text{ lb/in}^2$
 $\rho = 9.35 \text{ E}-05 \text{ lb-sec}^2/\text{in}^4$
 $\Delta t = .02 \text{ sec}$

LEGEND

Nodes 1, 2, 11, 12	$\begin{array}{c} \uparrow L \\ \rightarrow L \end{array}$
Nodes 3 - 10	$\begin{array}{c} \uparrow E \\ \rightarrow L \end{array}$

Figure 3.16 Water Column Under Step Pressure

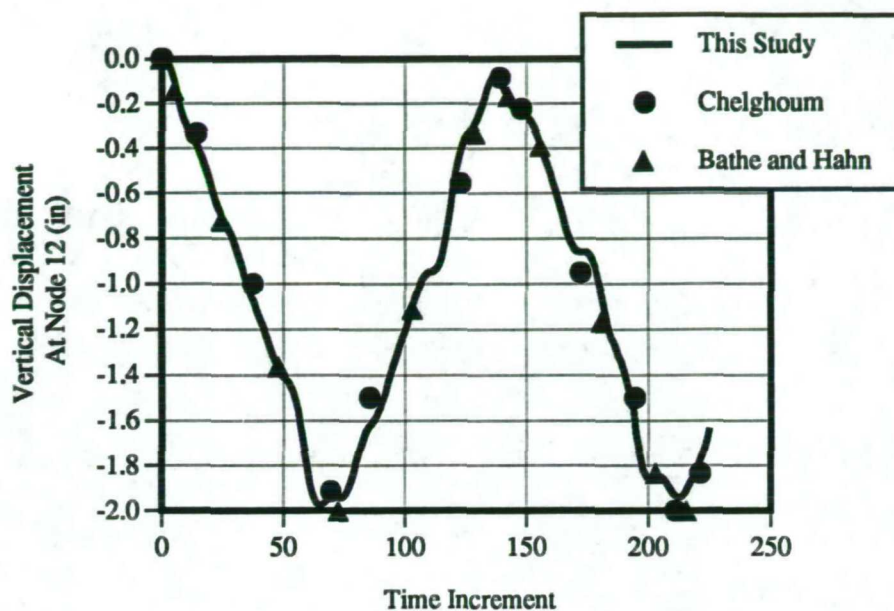


Figure 3.17 Vertical Displacement at the Top of a Water Column Subjected to a Step Pressure

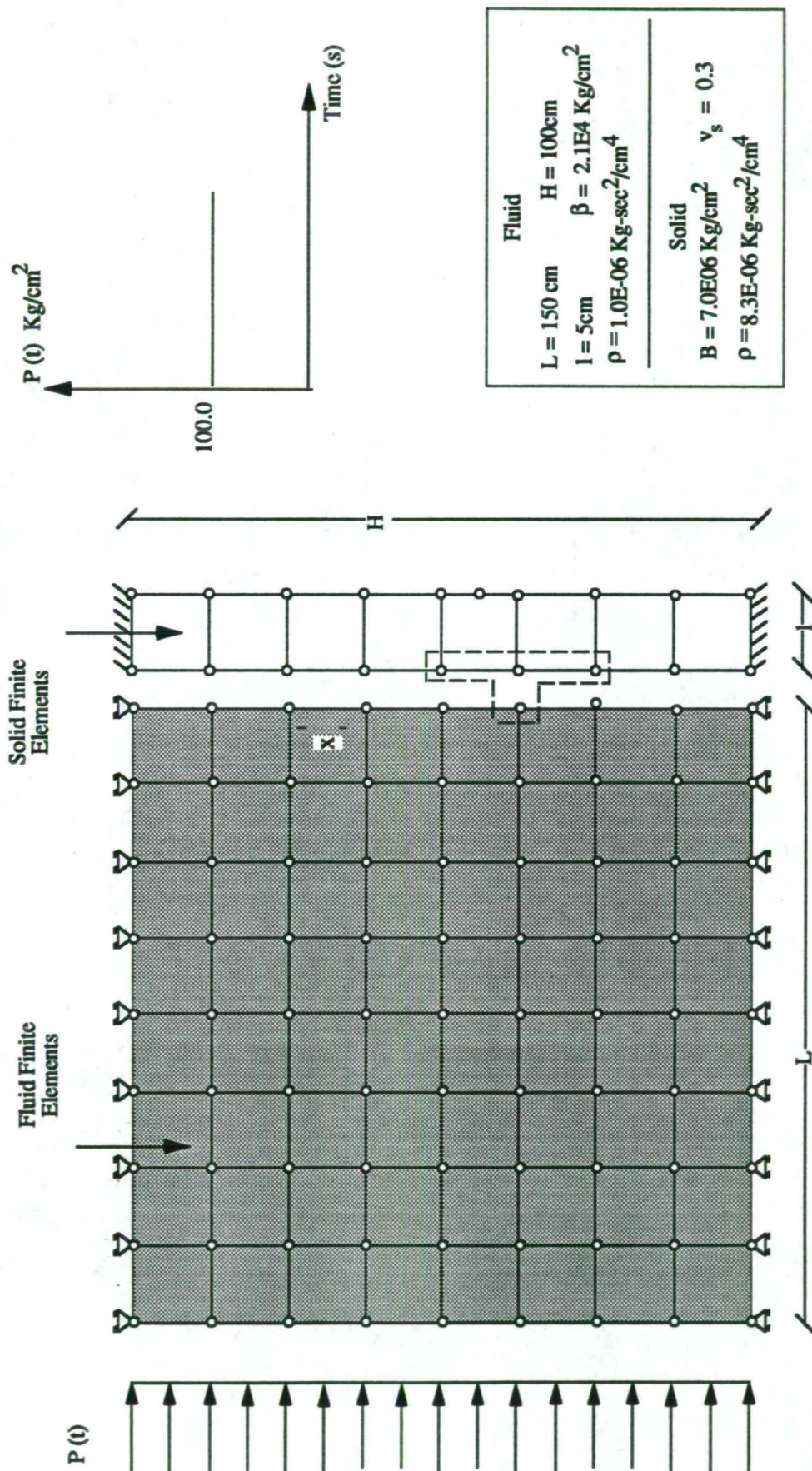


Figure 3.18 Fluid Plate System Subjected to Step Pressure

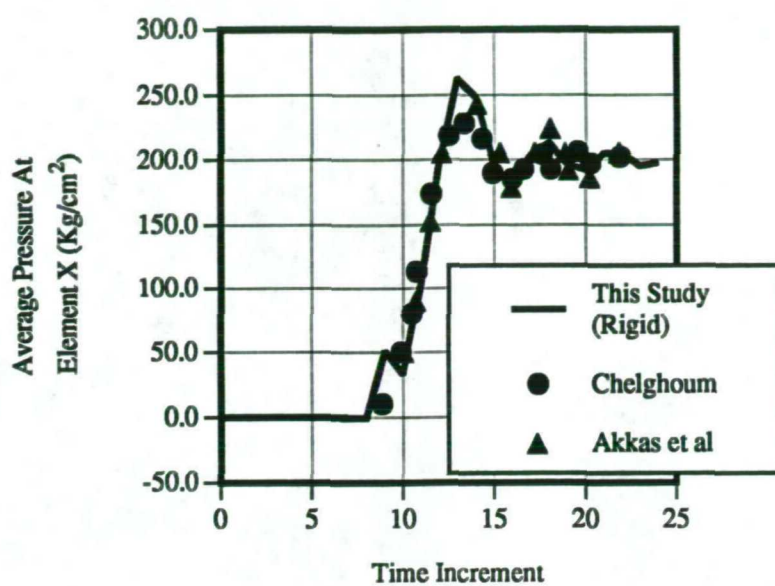


Figure 3.19 Element Pressure Variation for a Rigid Fluid-Plate System

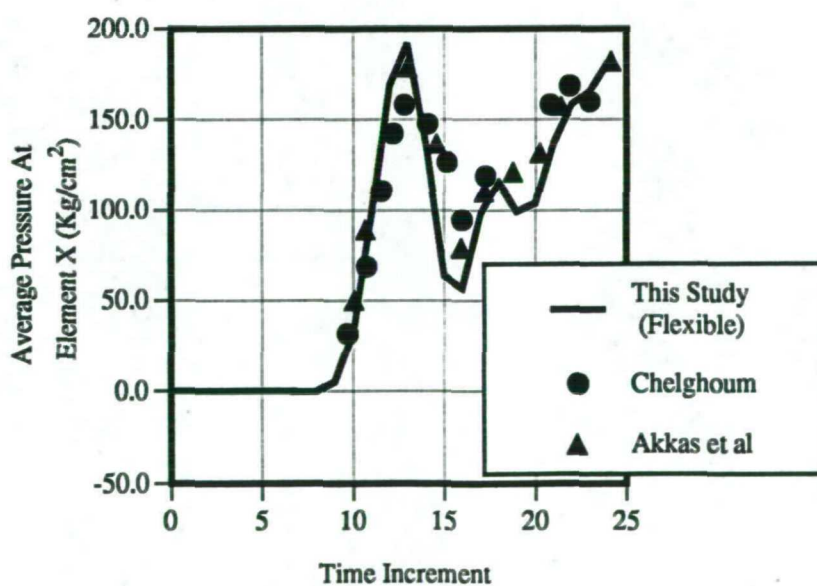
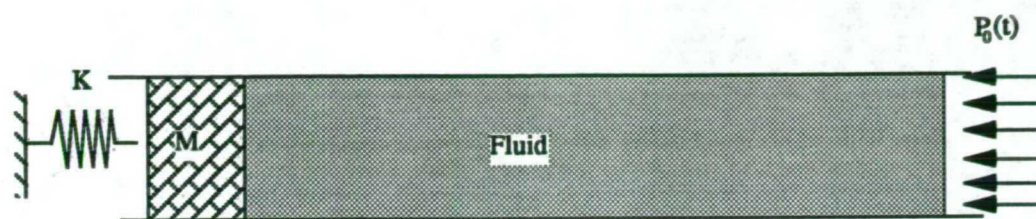
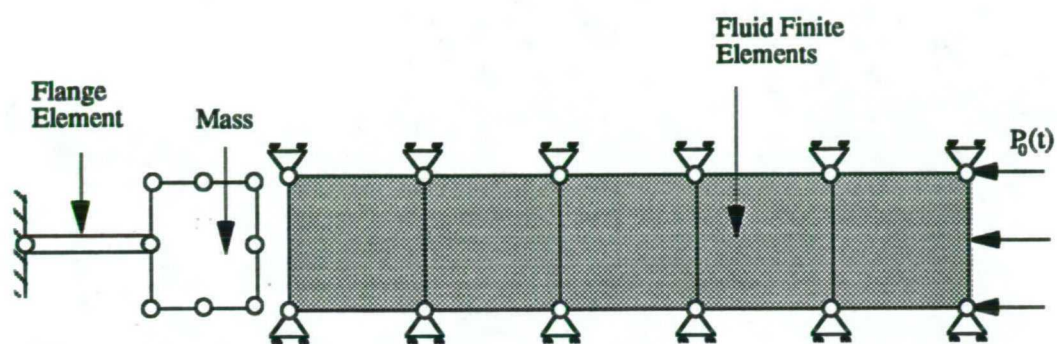


Figure 3.20 Element Pressure Variation for a Flexible Fluid-Plate System

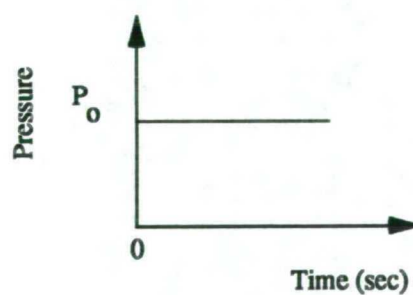
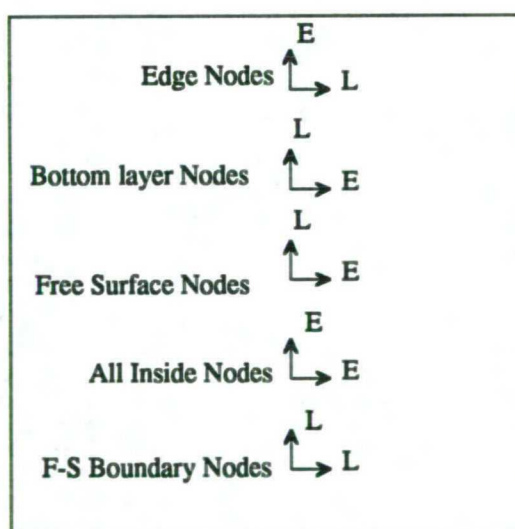


Fluid-Piston System



Finite Element Model

LEGEND



Loading Condition

Figure 3.21 Fluid Piston System, Finite Element Idealisation, Material and Geometric Properties

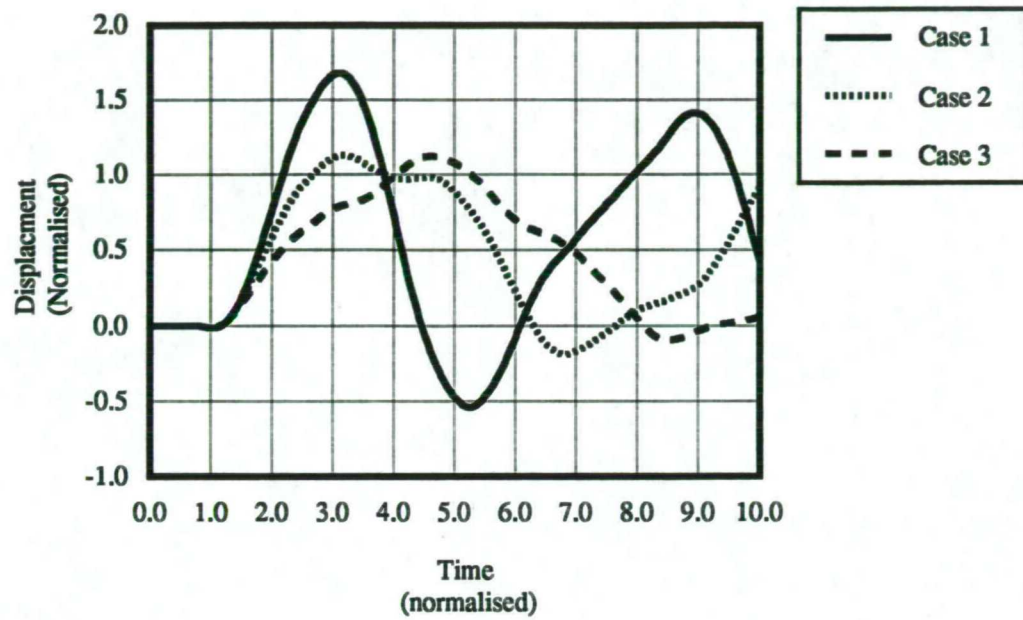
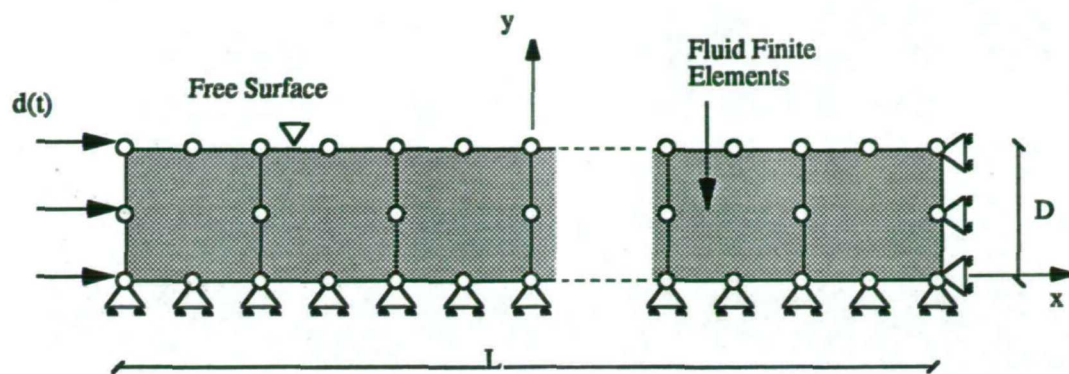


Figure 3.22 Effect of Compressibility on Displacement of Piston (See Table 3.5)



Finite Element Model

LEGEND

Geometric and
Material Properties

$D = 10$	$L = 949.095$
$\beta = 2.6E07$	$g = 1$
$\rho = 1$	

Edge Nodes



Bottom layer Nodes



Free Surface Nodes



All Inside Nodes

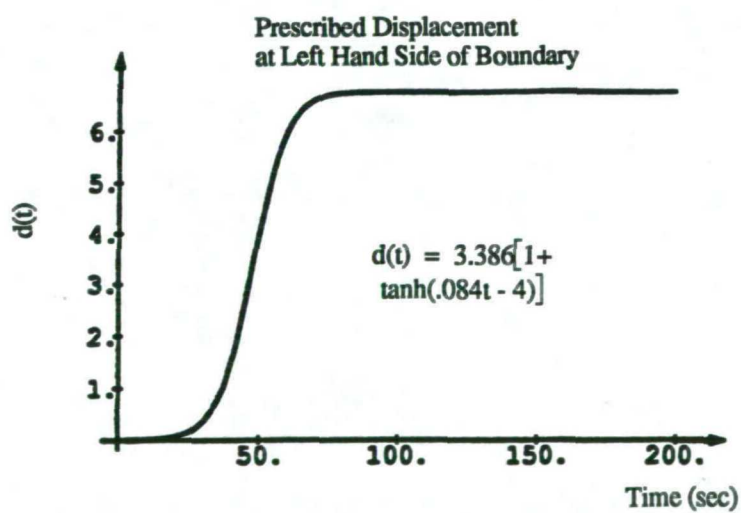


Figure 3.23 Propagation of a Solitary Wave in a Fluid Medium

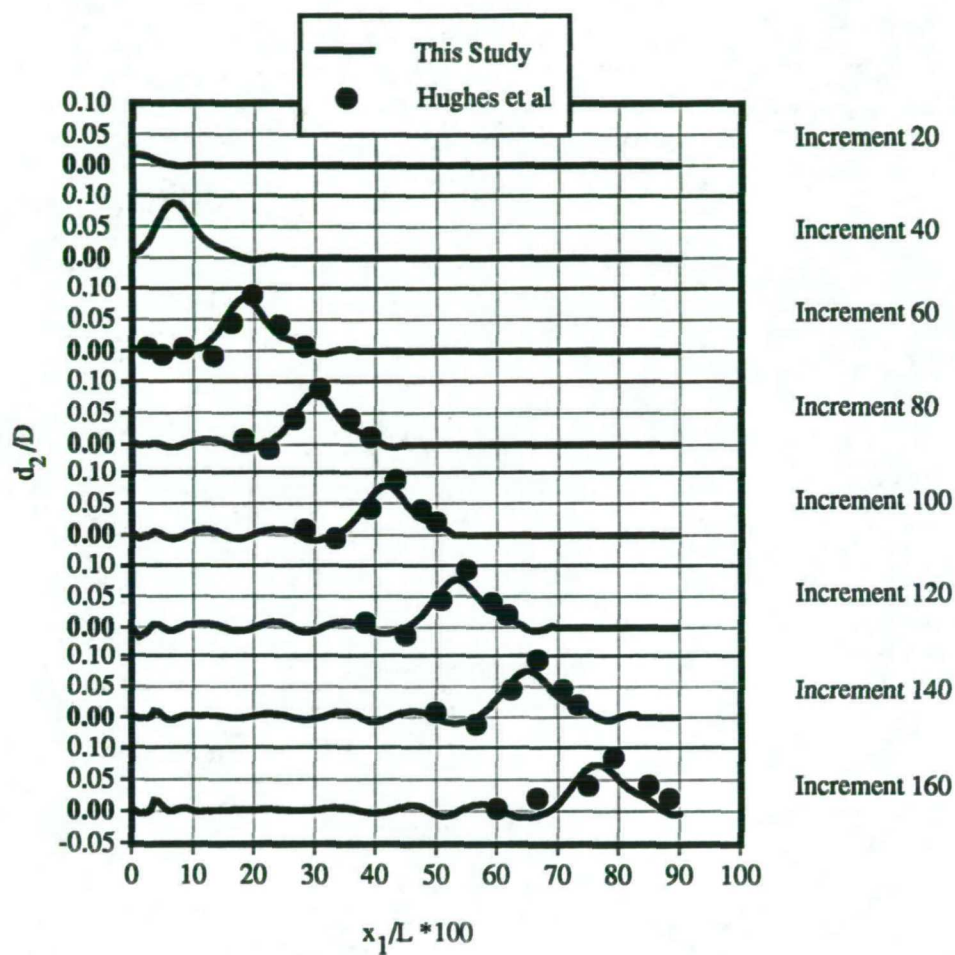


Figure 3.24 Solitary Wave Propagation in a Fluid Medium

CHAPTER FOUR

FAR-FIELD FLUID-STRUCTURE INTERACTION

This chapter is concerned with the development of numerical-based solutions for the problem of dynamic fluid-structure interaction of submerged structures subjected to underwater explosions.

Both the theory and the formulation for the non-radiating boundary based on acoustic approximations will be presented. This is used to develop elements capable of modelling the infinite fluid medium. As in Chapter Three, displacements are chosen as the main independent nodal variable, and an isoparametric formulation is employed. Two-dimensional and axi-symmetric elements are developed.

The effect of explosions on the structure may be simulated using wave equations. Both exponential and step plane waves are employed to model explosions occurring at a distance far away from the structure.

The developed formulation is coupled to the near-field fluid-structure governing equation derived in the previous chapter. Furthermore, direct time integration techniques combined with nonlinear iterative methods are employed in the transient analysis of the governing equation of motion.

Finally, the proposed formulation is validated by testing several two-dimensional and axi-symmetric submerged fluid-structure interaction problems for which there are existing numerical and analytical solutions.

4.1 INTRODUCTION

As mentioned in Chapter One, the transient analysis of floating and submerged structures subjected to shock wave loading may be divided into the following four major components:

- i- Modelling the structure, while catering for large deformation and plasticity effects.
- ii- Modelling the fluid medium contained within the structure, taking into account large mesh distortions.
- iii- Modelling the surrounding fluid medium taking into account the development of cavitation in the fluid.
- iv- Simulating the effect of shock waves impinging on the structure.

The solution to the first problem is readily available using well-established general purpose finite element programs such as ASAS-NL, while the solution to the second problem may be achieved by using the fluid finite elements developed in Chapter Three. The solution to the third problem is usually achieved by using finite elements or finite differences to model the fluid in the vicinity of the structure. These formulations are based on a bi-linear constitutive relationship, as in Figure 4.1, which takes into account the change in the equation of state due to the occurrence of cavitation. However, economic considerations dictate that the fluid mesh must be terminated at a "reasonable" distance away from the structure, thereby creating an artificial boundary which may trap the energy and produce spurious reflections which contaminate the numerical solution.

By ignoring the development of cavitation in the fluid medium, a non-radiating boundary which would allow incident shock waves to pass through, while being "transparent" to waves radiating away from the structure, may be developed. In the context of this study, a non-radiating boundary based on acoustic approximations is used in this manner to model the infinite fluid medium.

Finally, underwater explosions may be modelled by shock waves propagating through the fluid medium. Such shock waves are usually classified under two categories, plane and spherical, according to the proximity of the explosion source to the structure. In this study, it is assumed that the structure is at a great distance from the centre of disturbance and the waves radiating from the disturbance are considered as plane waves.

4.2 DECOMPOSITION OF PRESSURE AND VELOCITY FIELDS

In this section, the relationship between the various waves (and the corresponding velocities and pressures) acting on the wet surface of a submerged structure will be discussed.

When a submerged structure, subjected to incident waves, vibrates in an infinite fluid medium, the pressure acting on the structure surface in contact with the fluid, commonly known as the wet surface, must be determined. In general, the total fluid pressure acting on the wet surface may be decomposed into a free-field component and a scattered component:

$$p_{inf} = p_{ff} + p_{sc} \quad (4.1)$$

The velocity vector at the fluid-structure boundary may be decomposed in a similar manner:

$$v_{inf} = v_{ff} + v_{sc} \quad (4.2)$$

where

- p_{inf} and v_{inf} are the total fluid pressure and velocity vectors acting on the wet surface,
- p_{ff} and v_{ff} are the fluid pressure and velocity vectors caused by the incident wave in the absence of the structure (usually referred to as the free-field response), and
- p_{sc} and v_{sc} are the fluid pressure and velocity vectors corresponding to the difference between the total and free-field solutions.

For problems where the exterior fluid medium has a bi-linear constitutive relationship, as in Figure 4.1, cavitation occurs when the value of the total pressure, p_{inf} , first reaches a critical value p_{cr} . In such situations, the free-field solution may be obtained in the manner described by Bleich and Sandler (1970), DiMaggio et al (1981) and Rehak et al (1985) amongst others.

For the purpose of this study, the exterior fluid medium is assumed to have a linear constitutive relationship which does not cater for cavitation effects. In this case, the free-field solution reduces to that caused by the incident shock wave:

$$p_{ff} = p_{inc} \quad (4.3a)$$

$$v_{ff} = v_{inc} \quad (4.3b)$$

where p_{inc} and v_{inc} are the fluid pressure and velocity vectors caused by the incident wave.

The calculation of the incident pressure and velocity vectors caused by step and exponential shock waves will be discussed in Section 4.4.

The relationship between the free-field and total pressure may be expressed as a function of the difference between the corresponding velocities:

$$p_{inf} - p_{inc} = f(v_{inf} - v_{inc}) \quad (4.4a)$$

where f is a functional which is determined according to the type of formulation used. Alternatively, by substituting Equations 4.1, 4.2 and 4.3 into 4.4a, the above relationship may be expressed as:

$$p_{sc} = f(v_{sc}) \quad (4.4b)$$

The effects of the scattered waves radiating from the wet surface of a submerged structure are usually taken into account using one of the following three approaches (Figure 4.2):

- i- The infinite fluid medium is modelled by a finite element mesh of sufficient length such that the analysis is terminated before the scattered wave arrives at the exterior boundary of the fluid mesh; hence the effects of the scattered waves are implicitly taken into account and no further complications arise. However, for most practical cases, the size of such a fluid finite element mesh is prohibitively expensive from a computational point of view.
- ii- The infinite fluid medium is modelled by a finite element mesh terminated at a "reasonable" distance away from the structure; hence in most cases the scattered waves will reach the exterior boundary of the fluid mesh before the termination of the analysis. This results in artificial waves reflecting back into the fluid mesh and contaminating the response. This problem is overcome by using an explicit expression for the scattered wave at the exterior boundary of the fluid mesh. In this manner, a non-radiating boundary which would be "transparent" to scattered waves, while allowing incident waves to pass through, is formulated.
- iii- Cavitation is ignored in the fluid medium and the non-radiating boundary is placed on the wet surface of the submerged structure. In this case, since fluid finite elements are not used in the modelling of the surrounding fluid medium, substantial savings in computer processing time and storage are obtained.

In the context of this study, the third approach will be adopted and implemented in the computer code.

It can be seen from the above discussion that the main requirement for the development of a non-radiating boundary is an explicit expression for the scattered pressure and velocity vectors.

4.3 SCATTERED PRESSURE-VELOCITY FORMULATION

The exact functional f , appearing in Equation 4.4, may be determined analytically using various methods, including:

- i- Integral (time) Transform-Modal (Spatial) Decomposition: This method is usually used in cases where the structure wet surface can be described by a separable coordinate system. Huang (1969), used this method to determine the response of a submerged spherical shell subjected to a step wave.
- ii- Kirchoff Retarded Potential Formulation: Based on this method Geers (1969) determined an expression for the scattered pressure and applied it to study the response of a submerged, infinitely long cylinder subjected to an incident step pressure wave. This formulation is not only useful in its own right, but also provides insights into the differences between the various methods used in approximating the functional f . Therefore, a brief summary will be presented in what follows, while a more detailed discussion is given in Appendix C.

This method is based on the consideration of an infinite acoustic fluid governed by the isotropic wave equation:

$$\frac{\partial^2 \phi}{\partial t^2} = c^2 \frac{\partial^2 \phi}{\partial x^2} \quad (4.5a)$$

where c is the acoustic wave speed and ϕ is the velocity potential defined as :

$$\underline{v} = \underline{\nabla} \phi$$

Assuming the potential ϕ has no singularity outside a closed interaction surface S (wet surface of the structure), then according to Sobolev (1964) and Felippa (1980) the velocity potential may be expressed as:

$$2 \pi \phi_p = - \int_S \left\{ R^{-1} v_n - \cos \beta R^{-2} \left[\phi_p + \frac{R}{c} \dot{\phi}_p \right] \right\} ds \quad (4.5b)$$

where:

ϕ_p is the velocity potential at point p, defined by position vector $\underline{x}$,

$R = \underline{x} - \underline{x}_s$, is the distance from the field point p to a typical point s defined by position vector $\underline{x}_s$ on the surface S,

$\underline{n}$ is the unit outward normal (positive going into the fluid) at point s,

v_n is the normal fluid particle velocity at point s,

β is the angle formed by vectors $\underline{l}$ and $\underline{n}$, and

$\underline{l}$ is the vector defined by $\underline{x} - \underline{x}_s$.

All the above parameters are shown in Figure 4.3.

The application of the above methods to complex fluid-structure interaction problems requires extensive computations. According to Felippa (1980), one of the main difficulties is the need for preserving large amount of 'historical' data as the governing equations are numerically integrated. Furthermore, the amount of data required to be stored increases linearly with the total number of time steps of the analysis. Towards overcoming this difficulty, a 'memory-less' interaction approximation is required. Based on Kirchhoff Retarded Potential formulation, Geers (1970) and Felippa (1980) derived a family of such interaction approximations of differential form. These may be classified into three main categories:

- i- Early time approximations which predict an accurate response of the structure during the early stages of the analysis. These methods are derived from Equation 4.5b by limiting the contributing surface for each field point p to a region s_p defined by the condition $R < c t$, where t is the time variable. The most commonly used early time approximation is the Plane Wave Approximation where the counterpart of Equation 4.4 becomes:

$$p_{inf}^l - p_{ff}^l = \rho c (v_{inf}^l - v_{ff}^l) \quad (4.5c)$$

where:

ρ is the density of the fluid medium, and

the superscript l denotes variables expressed in the local coordinates.

- ii- Late time approximations which predict an accurate response of the structure during late stages of the analysis. These are derived from Equation 4.5b by extending the surface integral over the complete surface S . The most commonly used late time approximation is the Virtual Mass Approximation, for which, Equation 4.4 becomes:

$$\underline{A} (\underline{p}_{inf}^1 - \underline{p}_{ff}^1) = \underline{M} (\underline{a}_{inf}^1 - \underline{a}_{ff}^1) \quad (4.5d)$$

where:

$\underline{A}$ is the diagonal matrix which converts diagonal pressures to forces,
 $\underline{M}$ is the fluid added mass matrix, which couples the normal motion of all discrete structure surface elements at the fluid-structure interface, and
 $\underline{a}_{inf}^1, \underline{a}_{ff}^1$ are the fluid total and free-field acceleration vectors acting on the structure wet surface.

The added mass matrix obtained from Equation 4.5d is often used in conjunction with Equation 4.5c to produce an approximate relationship between the scattered pressure and velocity vectors which is accurate both at late and early times:

- iii- Doubly Asymptotic Approximations which predict an accurate response during both early and late times. In this approach, the scattered pressure and velocity vectors are related as:

$$\frac{1}{\rho c} (\dot{\underline{p}}_{inf}^1 - \dot{\underline{p}}_{ff}^1) - \underline{M}^{-1} \underline{A} (\underline{p}_{inf}^1 - \underline{p}_{ff}^1) = \underline{a}_{inf}^1 - \underline{a}_{ff}^1 \quad (4.5e)$$

where:

a dot denotes differentiation with respect to time, and
 $\underline{M}^{-1}$ is the inverse of the fluid added mass matrix.

It should be mentioned that several authors, e.g. Geers(1978), augmented Equation 4.5e by fitting matrices which result in a more accurate response at one particular intermediate frequency or time interval.

An important and common assumption in all these approximations is that the outgoing waves propagate to infinity without any additional reflections. The

implications of this assumption will be discussed in Chapter Five.

In the context of this study, the Plane Wave Approximation will be reformulated and implemented in the computer code. The full derivation of the Plane Wave Approximation is presented in Appendix C. This method is chosen for the following reasons:

- i- In many instances the response of submerged structures excited by shock waves peak at an early time in the transient analysis. Therefore, it is essential to have an accurate prediction of the early time behaviour of the structure.
- ii- The added mass matrix appearing in Equations 4.5d and 4.5e is fully populated and unsymmetric which leads to difficulties when integrating it in a structural finite element code. An implicit time integration treatment requires several modifications to the storage and solution schemes for the global set of equations, while a symmetrisation process of the added mass matrix is reported to produce inferior numerical results (Mathews and Geers, 1987). Furthermore, an explicit time integration treatment would require considerable computation and storage time as well as leading to stringent time step restrictions.

However, it should be mentioned that the type of the loading and the geometry of the structure, amongst other factors, influence the peak position, magnitude and general shape of the response. This leads to some cases where the peak response occurs at an intermediate or late time. In such cases early time approximations cannot predict an accurate peak response of the structure. Therefore, it becomes essential to augment them with matrices which would predict the behaviour of the structure at late and intermediate times.

4.4 INCIDENT SHOCK WAVE FORMULATION

An underwater explosion is a chemical reaction which releases very high temperature and pressure, producing disturbances and net flow change in the state surrounding the fluid medium. The detonation of such high explosive charges usually results in the transmission, through the medium, of shock waves consisting of very rapid increase of instantaneous pressure followed by a much slower rate of decay. These shock waves can be classified under two categories: explosions which are at a sufficiently far distance away from the structure or those which occur in its vicinity, as shown in Figure 4.4. In the first case, the motion is governed by the plane wave equation,

while in the second by its spherical counterpart. An accurate simulation of these waves caused by explosions is a fundamental pre-requisite to an accurate fluid-structure interaction analysis tool. In the context of this study only explosions occurring at a far distance away from the structure will be considered.

4.4.1 Introduction

When an explosion is produced at a point in an elastic medium, waves radiate from the source of disturbance in all directions. The general equations governing the motion of waves in an elastic medium, which have been derived by many researchers, such as Timoshenko and Goodier (1970) and Achenbach (1973), may be expressed as:

$$(\lambda + 2 G) \nabla^2 d - \rho \frac{\partial^2 d}{\partial t^2} = 0 \quad (4.6)$$

where λ and G are Lamé constants, d is the displacement of the fluid particles and t is the time.

At a sufficiently large distance from the source of the explosion, the radiating waves may be treated as plane waves, where it is assumed that the fluid particles are parallel to the direction of wave propagation, say x_1 , and the above equation reduces to:

$$\frac{\partial^2 d}{\partial t^2} = c_1^2 \frac{\partial^2 d}{\partial x_1^2} \quad (4.7a)$$

where c_1 is the horizontal speed of propagation of sound in the fluid medium. The above equation is valid for inviscid flows and where the pressure is determined from Equation 3.43b. Alternatively, Equation 4.7a may be derived from the Lagrangian equation of conservation of momentum, Equation 3.33.

A more common form of the above equation is in terms of the displacement potential ϕ which is defined by:

$$\tilde{\nabla} \phi = \underline{d}$$

By using the definition of the displacement potential and substituting Equation 3.43b into Equation 3.33, the governing equation of motion for the incident wave becomes:

$$\frac{\partial^2 \phi}{\partial t^2} = c_1^2 \frac{\partial^2 \phi}{\partial x_1^2} \quad (4.7b)$$

Wave equations satisfying Equation 4.7 are usually referred to as plane dilational waves.

Following techniques described by Achenbach (1973), an arbitrary incident plane wave may be expressed in terms of the potential, ϕ_I , as:

$$\phi_I = -\frac{1}{2} P_0 (\tau_{nd} - x_{nd} - 1)^2 H(\tau_{nd} - x_{nd} - 1) \quad (4.8)$$

where

P_0 is the initial pressure jump, normalised with respect to ρc^2 ,
 τ_{nd} is the non-dimensional time normalised with respect to a/c ,
 x_{nd} is the non-dimensional x_1 position, normalised with respect to a ,
 a is the position of the wave-front node where the wave first hits the structure, and
 H is the heavy side operator which is defined as :

$$H(\tau_{nd} - x_{nd} - 1) = \begin{cases} 0 & \text{if } (\tau_{nd} - x_{nd} - 1) < 0 \\ 1 & \text{if } (\tau_{nd} - x_{nd} - 1) > 0 \end{cases}$$

Clearly, Equation 4.8 satisfies the one-dimensional wave equation, Equation 4.7b.

The above equation applies for plane dilational step waves characterised by a compressive pressure P_0 and moving in the positive x_1 direction. If the incident wave is characterised by a velocity jump rather than a pressure jump, the equation for the incident potential becomes:

$$\phi_I = -\frac{1}{2} v_0 (\tau_{nd} - x_{nd} - 1)^2 H(\tau_{nd} - x_{nd} - 1) \quad (4.9)$$

where v_0 is the velocity jump.

4.4.2 Exponential and Step Plane Waves

The incident stress, displacement and velocity may be computed from the incident displacement potential function, Equation 4.8, by the application of classical

continuum mechanics theory:

$$\sigma_{inc\ i}^1 = -(\lambda + 2G\delta_{1i}) P_o H(\tau_{nd} - x_{nd} - 1) \quad (4.10)$$

By using the definition of the displacement potential, the incident displacement is determined from Equation 4.8:

$$d_{inc\ i} = \frac{\partial \phi}{\partial x_1} = \delta_{1i} P_o (\tau_{nd} - x_{nd} - 1) H(\tau_{nd} - x_{nd} - 1) \quad (4.11)$$

Therefore, the incident velocity is expressed as:

$$v_{inc\ i} = \delta_{1i} P_o H(\tau_{nd} - x_{nd} - 1) \quad (4.12)$$

Equations 4.8 to 4.12 apply to step plane waves, in which the initial pressure or (velocity jump) remain constant throughout the duration of the analysis. Another form of loading scenario occurs when the pressure jump P_o (or the velocity jump) decays with time. These latter kinds of plane waves are often referred to as exponential plane waves. In such a situation the value of the pressure jump, occurring in Equations 4.8 to 4.12, is changed to take into account the exponential nature of the wave:

$$\phi_I = -\frac{1}{2} P_o e^{-t/t_d} (\tau_{nd} - x_{nd} - 1)^2 H(\tau_{nd} - x_{nd} - 1) \quad (4.13)$$

$$\sigma_{inc\ i}^1 = -(\lambda + 2G\delta_{1i}) P_o e^{-t/t_d} H(\tau_{nd} - x_{nd} - 1) \quad (4.14)$$

$$d_{inc\ i} = \frac{\partial \phi}{\partial x_1} = \delta_{1i} P_o e^{-t/t_d} (\tau_{nd} - x_{nd} - 1) H(\tau_{nd} - x_{nd} - 1) \quad (4.15)$$

$$v_{inc\ i} = \delta_{1i} e^{-t/t_d} P_o H(\tau_{nd} - x_{nd} - 1) \quad (4.16)$$

where t is the time variable and t_d is the decay time of the exponential wave.

Alternatively the above equations may be expressed in terms of the wave decay constant $c_d = 1/t_d$.

In Section 4.6, the isoparametric concept is employed to evaluate Equations 4.8 to 4.12 (or 4.13 to 4.16) over each element of the fluid-structure boundary. The full derivation of the incident wave equations may be found in Appendix D.

4.4.3 Modelling of the Wave-Front

In the field of numerical analysis, the modelling of the propagation of shock waves in fluid-structure systems has been investigated by many researchers. A feature common to most of the proposed methods is the use of the Heavyside operator as the only factor in determining the position of the wave-front. Wright et al (1981) introduced a shock wave modelling facility based on the Heavyside operator into a finite difference fluid-structure interaction program. The program was used to study the plane strain response of a submerged cylindrical shell. Various researchers introduced similar shock wave modelling facilities into finite element and finite difference fluid-structure interaction programs (DiMaggio et al, 1981 and O'Reagan and DiMaggio, 1990). However, the modelling of the propagation of the wave-front in the manner described above results in a discontinuous wave-front which in turn may cause numerical instabilities.

Zilliagus (1983) proposed to distribute the incident wave pressure equitably to nodes ahead of and behind the wave-front for the wet surface nodes being traversed by the wave at any particular time increment. This wave loading facility was introduced into a finite element fluid-structure interaction code, ADINA, and was used to study the response of three-dimensional submerged fluid-structure systems. Felippa and Deruntz (1984) smoothened the wave-front so that its value at the front is one half of the jump. The proposed wave loading facility was introduced into the finite element code USA-STAGS-CFA and was used to study the transient response of several one- and three-dimensional fluid-structure systems.

In the context of this study, the pressure corresponding to the incident wave is distributed, using the theorem of virtual work, to nodes ahead of and behind of the wave-front.

4.5 PLANE WAVE APPROXIMATION FORMULATION

The equation of motion of the structure, discretised with respect to the spatial variables, may be written as:

$$\underline{M}_s \underline{\ddot{a}}_s + \underline{C}_s \underline{\dot{v}}_s + \underline{K}_s \underline{d}_s = \underline{F}_{ext} + \underline{F}_{inf} \quad (4.17)$$

where

$\mathbf{F}_{ext}$ is the sum of the body and traction forces

$\mathbf{F}_{inf}$ is the forcing term due to the infinite fluid medium,

$\mathbf{M}_s$, $\mathbf{C}_s$, and $\mathbf{K}_s$ are the structures' mass, damping and stiffness matrices respectively, and

$\mathbf{a}_s$, $\mathbf{v}_s$, and $\mathbf{d}_s$ are the structure's acceleration, velocity and displacement vectors respectively.

4.5.1 Introduction

From an implementation point of view, the Plane Wave Approximation formulation may be introduced into a finite element fluid-structure interaction code either as a boundary condition imposed on the nodes lying on the wet surface of the structure or as a new element type to be added to the element library of the code.

Most researchers adopt the first approach when implementing a non-radiating boundary formulation into a finite element fluid-structure interaction code (Felippa and DeRuntz, 1991). However, the independence offered by the latter approach between the Plane Wave Approximation elements and the structural elements lying on the wet surface of the submerged structure has several attractive features:

- i Different time integration schemes may be easily used for the treatment of the structure and infinite fluid terms. This is particularly useful when attempting to assess and further develop the current predictor-multi-corrector time marching scheme.
- ii- Further development of the fluid, structural and non-radiating boundary elements is easier. For example the non-radiating boundary may be directly used in conjunction of the fluid finite elements proposed in the previous chapter without any modification to either formulation.

In this study Plane Wave Approximation elements will be developed and implemented in the proposed code.

4.5.2 Compatibility Conditions

At the surface of the non-radiating boundary, two conditions should be enforced. Firstly, kinematic continuity across the wet surface requires the normal velocity of the structure to be identical to that of the fluid:

$$\underline{v}_{inf} = \underline{v}_s \quad (4.18)$$

Secondly, the corresponding dynamic compatibility requires the compressive force on the structural component to be equal to the fluid pressure:

$$\underline{F}_{inf} = - (\underline{f}_{inc} + \underline{f}_s) \quad (4.19)$$

where

$\underline{f}_{inc}$ is the force due to the incident wave, and
 $\underline{f}_{sc}$ is the force due to the scattered wave.

Substituting Equations 4.18 and 4.3 into 4.5a, the Plane Wave Approximation may be expressed as:

$$\underline{p}_{inf}^l - \underline{p}_{ff}^l = \rho c \underline{v}_{sc}^l = \rho c (\underline{v}_s^l - \underline{v}_{inc}^l) \quad (4.20)$$

4.5.3 Congruence Transformation

The velocity and pressure vectors appearing in Equation 4.20 are expressed in local coordinates. They are transformed to global coordinates in a manner similar to that described in Section 3.9.5:

$$\underline{p}_{inf}^l - \underline{p}_{ff}^l = \underline{T}_t (\underline{p}_{inf} - \underline{p}_{ff}) \quad (4.21a)$$

$$\underline{v}_{sc}^l = \underline{T}_t \underline{v}_{sc} \quad (4.21b)$$

where the transformation matrix $\underline{T}_t$ is defined for the three-noded element by:

$$\underline{T}_t = \begin{bmatrix} \underline{t}_n^T & 0 & 0 \\ 0 & \underline{t}_n^T & 0 \\ 0 & 0 & \underline{t}_n^T \end{bmatrix} \quad (4.22a)$$

and for the two-noded element by:

$$\underline{T}_t = \begin{bmatrix} \underline{t}_n^T & 0 \\ 0 & \underline{t}_n^T \end{bmatrix} \quad (4.22b)$$

with $\underline{t}_n^T$ and Q defined by Equations 3.112 and 3.113 as: $\underline{t}_n^T = [n_1 \ n_2]$ and $Q = [0 \ 0]$, where n_1 and n_2 are the normal direction cosines.

By substituting Equations 4.21 into 4.20 and pre-multiplying Equation 4.21a by T^T , the Plane Wave Approximation in global coordinates is obtained:

$$\underline{p}_{inf} - \underline{p}_{ff} = \underline{T}_t^T \rho c \underline{T}_t \underline{v}_{sc} \quad (4.23a)$$

Similarly, the incident force is transferred from local to global coordinates using the direction cosines:

$$\underline{Q}_{inc}^l = \underline{T}_t \underline{Q}_{inc}^g \quad (4.23b)$$

4.5.4 Element Matrices

By using the principle of virtual work, the nodal forces are determined from the element stresses. Therefore, Equations 4.23a and 4.23b become:

$$\underline{f}_{sc} = \int_{\Gamma_s^e} \underline{h}^T \underline{T}_t^T \rho c \underline{T}_t \underline{v}_{sc} \quad (4.24a)$$

$$\underline{f}_{inc} = \int_{\Gamma_s^e} \underline{h}^T \underline{Q}_{inc}^l \underline{t}_n \quad (4.24b)$$

with $\underline{h}$ defined as $\begin{bmatrix} h_1 & 0 \\ 0 & h_2 \end{bmatrix}$ for the two-noded element and as $\begin{bmatrix} h_1 & 0 & 0 \\ 0 & h_2 & 0 \\ 0 & 0 & h_3 \end{bmatrix}$ for the three-noded element,

where h_a is the shape function associated with the a^{th} node, defined in the next section for the two- and three-noded Plane Wave Approximation elements.

Alternatively, the above equation may be expressed in matrix form as:

$$\underline{f}_{sc} = \underline{H}_f \underline{v}_{sc} ; \underline{H}_f = \int_{\Gamma_s^e} \underline{h}^T \underline{T}_t^T \rho c \underline{T}_t \quad (4.24c)$$

Substituting Equation 4.19 into 4.17, the governing equation of motion for the structure-infinite fluid system is obtained:

$$\underline{M}_s \underline{\ddot{d}}_s + \underline{C}_s \underline{\dot{d}}_s + \underline{K}_s \underline{d}_s = \underline{F}_{ext} - \underline{f}_{inc} - \underline{f}_{sc} \quad (4.25)$$

where the scattered and incident force vectors are defined by Equations 4.24.

4.6 ISOPARAMETRIC PLANE WAVE APPROXIMATION ELEMENTS

A family of two-dimensional and axi-symmetric Plane Wave Approximation elements is developed in this section. Both linear and quadratic elements are derived with two and three nodes respectively. The node numbering convention adopted in this study is shown in Figure 4.5. Figures 4.5 and 4.6 show the various geometric configurations for the two-dimensional and axi-symmetric elements respectively.

4.6.1 Isoparametric Formulation and Shape Functions

The kinematic quantities at a generic point on an element lying at the fluid-structure interface are interpolated using the following expression:

$$d_i = \sum_{j=1}^{NEN} h_j d_{ij}^e \quad (4.26a)$$

$$v_i = \sum_{j=1}^{NEN} h_j v_{ij}^e \quad (4.26b)$$

$$a_i = \sum_{j=1}^{NEN} h_j a_{ij}^e \quad (4.26c)$$

Unlike their two-dimensional counterparts in Chapter Three, the above shape functions are one-dimensional. Towards the use of an isoparametric formulation, the geometry of the element is interpolated using the same shape functions:

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \sum_{k=1}^{NEN} h_k(\zeta) \begin{bmatrix} x_{1k} \\ x_{2k} \end{bmatrix} \quad (4.27)$$

where ζ is the non-dimensional coordinate varying from -1 to 1 as shown in Figure 4.5.

It is possible to derive shape functions which satisfy both the continuity and

completeness requirements discussed in Section 3.5.1. The derivation of the linear and quadratic shape functions follows the same steps outlined in that section.

The shape functions for the quadratic and linear elements are given in Tables 4.1 and 4.2 respectively:

i	Shape Function h_i	$\partial h_i / \partial \zeta$
1	$-\zeta (1-\zeta) / 2$	$(2 \zeta - 1) / 2$
2	$(1+\zeta) (1-\zeta)/2$	$-\zeta$
3	$\zeta (1+\zeta) / 2$	$(2 \zeta + 1) / 2$

Table 4.1 Shape Functions and Derivatives for a Three-Noded Isoparametric One-Dimensional Element

i	Shape Function h_i	$\partial h_i / \partial \zeta$
1	$(1 - \zeta) / 2$	$-1 / 2$
2	$(1 + \zeta) / 2$	$1 / 2$

Table 4.2 Shape Functions and Derivatives for a Two-Noded Isoparametric One-Dimensional Element

4.6.2 Jacobian Matrix

In a manner analogous to that described in Chapter Three, the quadratic shape functions are used as mapping functions to develop elements with curved boundaries. The determinant of the Jacobian matrix of the mapping is defined by:

$$|J| = \sqrt{\left(\frac{\partial x_1}{\partial \zeta}\right)^2 + \left(\frac{\partial x_2}{\partial \zeta}\right)^2} \quad (4.28)$$

4.6.3 Numerical Integration

The integrals appearing in Equations 4.24, are evaluated using linear and quadratic one-dimensional Gaussian integration rules. For a one-dimensional element in local coordinates, the integrals can be expressed as:

$$I_{\zeta} = \int_{-1}^1 f(\zeta) d\zeta \approx \sum_{i=1}^n f(\zeta_i) w_i \quad (4.29a)$$

where n is the number of points used in the numerical integration of the integral.

The determinant of the Jacobian of the mapping $|J|$ is employed in order to transfer the integrals from local to global coordinates. Therefore, the integral evaluated over the actual length of the element becomes:

$$I_{x_1 x_2} = \int_{\Gamma_{se}} f(x_1, x_2) ds \approx \sum_{i=1}^n f(\zeta_i) w_i |J(\zeta_i)| \quad (4.29b)$$

where $ds = \sqrt{\left(\frac{\partial x_1}{\partial \zeta}\right)^2 + \left(\frac{\partial x_2}{\partial \zeta}\right)^2}$, with all other terms defined in the previous chapter.

The values of the local coordinates and the corresponding weighting functions appearing in Equation 4.29b are identical to those listed in Table 3.4

In evaluating the integrals appearing in Equations 4.24, exact one-dimensional numerical integration formulae are used. Table 4.3 show the exact and reduced number of integration points required for the linear and quadratic one-dimensional elements.

Element Type	No. of Integration points (Exact)	No. of Integration points (Reduced)
Linear (2 Nodes)	2	1
Quadratic (3 nodes)	3	2

Table 4.3 Gaussian Integration Points for Linear and Quadratic One-Dimensional Elements.

4.7 DIRECT TIME INTEGRATION

The finite difference time integration schemes, discussed in detail in Section 3.10,

will be used to augment the predictor-multi-corrector scheme, developed in the previous chapter, with the Plane Wave Approximation formulation. The general equation of motion, Equation 4.16, may be expressed as:

$$\underline{M}_s \underline{a}_s + \underline{C}_s \underline{v}_s + \underline{K}_s \underline{d}_s = \underline{F}_{\text{ext}} - \underline{f}_{\text{inc}} + \underline{H}_f (\underline{v}_{\text{inc}} - \underline{v}_s) \quad (4.29)$$

In order to avoid complex subscript notation, the subscript pertaining to the kinematic quantities appearing in the above equation will be dropped. Furthermore, it is implied that the displacement, velocity and acceleration vectors refer to the unknown nodal quantities regardless of whether it is associated with the fluid or the structural nodes. That is $\underline{a}_s = \underline{a}$, $\underline{v}_s = \underline{v}$ and $\underline{d}_s = \underline{d}$

Equation 4.29 is discretised in the time domain using Newmark's time integration scheme:

$${}^{t+\Delta t}\underline{d} = {}^t\underline{d} + {}^t\underline{v} \Delta t + \left[\left(\frac{1}{2} - \beta_N \right) {}^t\underline{a} + \beta_N {}^{t+\Delta t}\underline{a} \right] \Delta t^2 \quad (4.30)$$

$${}^{t+\Delta t}\underline{v} = {}^t\underline{v} + \left[(1 - \gamma_N) {}^t\underline{a} + \gamma_N {}^{t+\Delta t}\underline{a} \right] \Delta t \quad (4.31)$$

Substituting Equations 4.30 and 4.31 into 4.29, yields the following equation of motion at time $t + \Delta t$:

$$\begin{aligned} & \left({}^{t+\Delta t}\underline{M}_s + \gamma_N \Delta t {}^{t+\Delta t}\underline{C}_s + \beta_N \Delta t^2 {}^{t+\Delta t}\underline{K}_s \right) {}^{t+\Delta t}\underline{a} = {}^{t+\Delta t}\underline{F}_{\text{ext}} - \\ & {}^{t+\Delta t}\underline{f}_{\text{inc}} + {}^{t+\Delta t}\underline{H}_f \left({}^{t+\Delta t}\underline{v}_{\text{inc}} - {}^t\underline{v} \right) - \\ & {}^{t+\Delta t}\underline{C}_s \left({}^t\underline{v} + (1 - \gamma_N) \Delta t {}^t\underline{a} \right) \\ & - {}^{t+\Delta t}\underline{K}_s \left({}^t\underline{d} + \Delta t {}^t\underline{v} + \left(\frac{1}{2} - \beta_N \right) \Delta t^2 {}^t\underline{a} \right) \end{aligned} \quad (4.32a)$$

However, since the current geometry of the fluid-structure system is not known, the coordinates of the previous time step are used in the evaluation of the element and global matrices.

$$\begin{aligned} & \left({}^t\underline{M}_s + \gamma_N \Delta t {}^t\underline{C}_s + \beta_N \Delta t^2 {}^t\underline{K}_s \right) {}^{t+\Delta t}\underline{a} = {}^{t+\Delta t}\underline{F}_{\text{ext}} - {}^{t+\Delta t}\underline{f}_{\text{inc}} + \\ & {}^t\underline{H}_f \left({}^{t+\Delta t}\underline{v}_{\text{inc}} - {}^t\underline{v} \right) - {}^t\underline{C}_s \left({}^t\underline{v} + (1 - \gamma_N) \Delta t {}^t\underline{a} \right) \end{aligned} \quad (4.32b)$$

$$- {}^t\mathbf{K}_s \left({}^t\mathbf{d} + \Delta t {}^t\mathbf{v} + \left(\frac{1}{2} - \beta_N \right) \Delta t^2 {}^t\mathbf{a} \right)$$

Alternatively, using the incremental form of Newmark's integration scheme, Equations 3.118, Equation 4.32 becomes:

$$\begin{aligned} \left({}^t\mathbf{M}_s + \gamma_N \Delta t {}^t\mathbf{C}_s + \beta_N \Delta t^2 {}^t\mathbf{K}_s \right) {}^{t+\Delta t}\Delta \mathbf{a} = & {}^{t+\Delta t}\mathbf{F}_{\text{ext}} - {}^{t+\Delta t}\mathbf{f}_{\text{inc}} + \\ & {}^t\mathbf{H}_f \left({}^{t+\Delta t}\mathbf{v}_{\text{inc}} - {}^t\mathbf{v} \right) - {}^t\mathbf{M}_s {}^t\mathbf{a} - {}^t\mathbf{C}_s \left({}^t\mathbf{v} + \Delta t {}^t\mathbf{a} \right) \quad (4.33) \\ & - {}^t\mathbf{K}_s \left({}^t\mathbf{d} + \Delta t {}^t\mathbf{v} + \frac{\Delta t^2}{2} {}^t\mathbf{a} \right) \end{aligned}$$

In Equations 4.32 and 4.33, the acoustic approximation terms are treated in a linear manner, since the velocity of the previous time step, ${}^t\mathbf{v}$, is used for the Plane Wave Approximation term, ${}^{t+\Delta t}\mathbf{v}_{\text{inc}}$. In some cases, it may be desirable to include these terms into a global Newton-Raphson iteration scheme. Towards this end, the equation of motion becomes:

$$\begin{aligned} \left({}^{t+\Delta t}\mathbf{M}_s^{j-1} + \gamma_N \Delta t {}^{t+\Delta t}\mathbf{C}_s^{j-1} + \beta_N \Delta t^2 {}^{t+\Delta t}\mathbf{K}_s^{j-1} \right) {}^{t+\Delta t}\delta \mathbf{a}^j = & {}^{t+\Delta t}\mathbf{F}_{\text{ext}} \\ - {}^{t+\Delta t}\mathbf{f}_{\text{inc}} + {}^{t+\Delta t}\mathbf{H}_f^{j-1} \left({}^{t+\Delta t}\mathbf{v}_{\text{inc}} - {}^{t+\Delta t}\mathbf{v}^{j-1} \right) - & {}^{t+\Delta t}\mathbf{M}_s^{j-1} {}^{t+\Delta t}\mathbf{a}^{j-1} \quad (4.34) \\ - {}^{t+\Delta t}\mathbf{C}_s^{j-1} \left({}^t\mathbf{v} + \Delta t (1 - \gamma_N) {}^t\mathbf{a} + \Delta t \gamma_N & {}^{t+\Delta t}\mathbf{a}^{j-1} \right) - {}^{t+\Delta t}\mathbf{F}_{\text{int.str}}^{j-1} \end{aligned}$$

where

$\mathbf{F}_{\text{int.str}}$ is the internal residual force for the structure which depends on the kind of structural nonlinearity, and

$\mathbf{K}_{\text{slf}}$ is the sum of the sliding and fluid global stiffness matrices.

It can be noticed that the global matrices are evaluated at the previous iteration $j - 1$ since the current geometry at iteration j is not known.

The explicit treatment of the Plane Wave Approximation terms results in a substantial saving in computer processing time and disk storage; however, care should be taken when choosing the size of the time step. On the other hand, an implicit integration of the Plane Wave Approximation terms would lead to the following equation of motion in terms of the total acceleration vector:

$$\begin{aligned}
& \left({}^t\mathbf{M}_s + \gamma_N \Delta t \left({}^t\mathbf{C}_s + {}^t\mathbf{H}_f \right) + \beta_N \Delta t^2 {}^t\mathbf{K}_s \right) {}^{t+\Delta t}\mathbf{a} = {}^{t+\Delta t}\mathbf{F}_{\text{ext}} - \\
& {}^{t+\Delta t}\mathbf{f}_{\text{inc}} + {}^t\mathbf{H}_f {}^{t+\Delta t}\mathbf{v}_{\text{inc}} - \left({}^t\mathbf{C}_s + {}^t\mathbf{H}_f \right) \left({}^t\mathbf{v} + (1 - \gamma_N) \Delta t {}^t\mathbf{a} \right) - \quad (4.35) \\
& {}^t\mathbf{K}_s \left({}^t\mathbf{d} + \Delta t {}^t\mathbf{v} + \left(\frac{1}{2} - \beta_N \right) \Delta t^2 {}^t\mathbf{a} \right)
\end{aligned}$$

The above equation may be expressed in terms of the incremental acceleration as :

$$\begin{aligned}
& \left({}^t\mathbf{M}_s + \gamma_N \Delta t \left({}^t\mathbf{C}_s + {}^t\mathbf{H}_f \right) + \beta_N \Delta t^2 {}^t\mathbf{K}_s \right) {}^{t+\Delta t}\Delta\mathbf{a} = {}^{t+\Delta t}\mathbf{F}_{\text{ext}} - {}^{t+\Delta t}\mathbf{f}_{\text{inc}} + \\
& {}^t\mathbf{H}_f {}^{t+\Delta t}\mathbf{v}_{\text{inc}} - {}^t\mathbf{M}_s {}^t\mathbf{a} - \left({}^t\mathbf{C}_s + {}^t\mathbf{H}_f \right) \left({}^t\mathbf{v} + \Delta t {}^t\mathbf{a} \right) - \quad (4.36) \\
& {}^t\mathbf{K}_s \left({}^t\mathbf{d} + \Delta t {}^t\mathbf{v} + \frac{\Delta t^2}{2} {}^t\mathbf{a} \right)
\end{aligned}$$

Equations 4.31 to 4.36 describe the behaviour of structures surrounded by an infinite fluid medium under the assumptions stated previously. For structures containing fluids and submerged in an infinite fluid medium the governing equation becomes:

$$\begin{aligned}
& \left({}^t\mathbf{M}_T + \gamma_N \Delta t \left({}^t\mathbf{C}_T + {}^t\mathbf{H}_f \right) + \beta_N \Delta t^2 {}^t\mathbf{K}_T \right) {}^{t+\Delta t}\Delta\mathbf{a} = {}^{t+\Delta t}\mathbf{F}_{\text{ext}} - {}^{t+\Delta t}\mathbf{f}_{\text{inc}} + \\
& {}^t\mathbf{H}_f {}^{t+\Delta t}\mathbf{v}_{\text{inc}} - {}^t\mathbf{M}_T {}^t\mathbf{a} - \left({}^t\mathbf{C}_T + {}^t\mathbf{H}_f \right) \left({}^t\mathbf{v} + \Delta t {}^t\mathbf{a} \right) - \quad (4.37) \\
& {}^t\mathbf{K}_T \left({}^t\mathbf{d} + \Delta t {}^t\mathbf{v} + \frac{\Delta t^2}{2} {}^t\mathbf{a} \right) - N({}^t\mathbf{v})
\end{aligned}$$

Finally, the nonlinear equation for a structure-interior fluid-structure-exterior fluid may be expressed as:

$$\begin{aligned}
& \left({}^{t+\Delta t}\mathbf{M}_T^j + \gamma_N \Delta t \left({}^{t+\Delta t}\mathbf{C}_T^j + {}^{t+\Delta t}\mathbf{H}_f^j \right) + \beta_N \Delta t^2 {}^{t+\Delta t}\mathbf{K}_T^j \right) {}^{t+\Delta t}\delta\mathbf{a}^{j+1} = \\
& {}^{t+\Delta t}\mathbf{F}_{\text{ext}}^j - {}^{t+\Delta t}\mathbf{f}_{\text{inc}}^j + {}^{t+\Delta t}\mathbf{H}_f^j {}^{t+\Delta t}\mathbf{v}_{\text{inc}}^j - {}^{t+\Delta t}\mathbf{M}_T^j {}^{t+\Delta t}\mathbf{a}^j - N({}^{t+\Delta t}\mathbf{v}^j) \\
& - \left(\mathbf{C}_T + \mathbf{H}_f \right) \left({}^t\mathbf{v} + \Delta t(1 - \gamma) {}^t\mathbf{a} + \Delta t \gamma {}^{t+\Delta t}\mathbf{a}^{j-1} \right) - {}^{t+\Delta t}\mathbf{F}_{\text{int.str}}^{j-1} \quad (4.38) \\
& - {}^{t+\Delta t}\mathbf{K}_{\text{slf}}^j \left({}^t\mathbf{d} + \Delta t {}^t\mathbf{v} + \Delta t^2 \left(\frac{1}{2} - \beta_N \right) {}^t\mathbf{a} + \Delta t^2 \beta_N {}^{t+\Delta t}\mathbf{a}^{j-1} \right)
\end{aligned}$$

By following the same steps outlined in Chapter Three, the governing equation of motion, Equation 4.29, may also be integrated in time using the α -Bossak method. The incremental form of the governing equation, using explicit time integration for the Plane Wave Approximation terms and α -Bossak for the structural terms, is:

$$\begin{aligned} \left((1 - \alpha_\beta) {}^t\mathbf{M}_s + \gamma_\beta \Delta t {}^t\mathbf{C}_s + \beta_\beta \Delta t^2 {}^t\mathbf{K}_s \right) {}^{t+\Delta t}\Delta \mathbf{a} = {}^{t+\Delta t}\mathbf{F}_{\text{ext}} - \\ {}^{t+\Delta t}\mathbf{f}_{\text{inc}} + {}^t\mathbf{H}_f \left({}^{t+\Delta t}\mathbf{v}_{\text{inc}} - {}^t\mathbf{v} \right) - {}^t\mathbf{M}_s {}^t\mathbf{a} - {}^t\mathbf{C}_s \left({}^t\mathbf{v} + \Delta t {}^t\mathbf{a} \right) \\ - {}^t\mathbf{K}_s \left({}^t\mathbf{d} + \Delta t {}^t\mathbf{v} + \frac{\Delta t^2}{2} {}^t\mathbf{a} \right) \end{aligned} \quad (4.39)$$

Using implicit time integration and α -Bossak method for both structural and Plane Wave Approximation terms, the counterpart of Equation 4.36, is given by:

$$\begin{aligned} \left((1 - \alpha_\beta) {}^t\mathbf{M}_s + \gamma_\beta \Delta t ({}^t\mathbf{C}_s + {}^t\mathbf{H}_f) + \beta_\beta \Delta t^2 {}^t\mathbf{K}_s \right) {}^{t+\Delta t}\Delta \mathbf{a} = {}^{t+\Delta t}\mathbf{F}_{\text{ext}} - \\ {}^t\mathbf{H}_f {}^{t+\Delta t}\mathbf{v}_{\text{inc}} - {}^t\mathbf{M}_s {}^t\mathbf{a} - ({}^t\mathbf{C}_s + {}^t\mathbf{H}_f) ({}^t\mathbf{v} + \Delta t {}^t\mathbf{a}) - \\ {}^{t+\Delta t}\mathbf{f}_{\text{inc}} + {}^t\mathbf{K}_s \left({}^t\mathbf{d} + \Delta t {}^t\mathbf{v} + \frac{\Delta t^2}{2} {}^t\mathbf{a} \right) \end{aligned} \quad (4.40)$$

Finally, the governing equation of motion using implicit nonlinear time integration, based on the α -Bossak method, for the structural, near-field fluid and Plane Wave Approximation terms, the counterpart of Equation 4.38, is given by:

$$\begin{aligned} \left((1 - \alpha_\beta) {}^{t+\Delta t}\mathbf{M}_T^{j-1} + \gamma_\beta \Delta t ({}^{t+\Delta t}\mathbf{C}_T^{j-1} + {}^{t+\Delta t}\mathbf{H}_f^{j-1}) + \beta_\beta \Delta t^2 {}^{t+\Delta t}\mathbf{K}_T \right) \\ {}^{t+\Delta t}\delta \mathbf{a}^{j+1} = {}^{t+\Delta t}\mathbf{F}_{\text{ext}} - {}^{t+\Delta t}\mathbf{f}_{\text{inc}} - {}^{t+\Delta t}\mathbf{F}_{\text{int.str}}^{j-1} + {}^{t+\Delta t}\mathbf{H}_f^{j-1} {}^{t+\Delta t}\mathbf{v}_{\text{inc}} \\ - {}^{t+\Delta t}\mathbf{M}_T^{j-1} \left((1 - \alpha_\beta) {}^{t+\Delta t}\mathbf{a}^{j-1} + \alpha_\beta {}^t\mathbf{a} \right) - N({}^{t+\Delta t}\mathbf{v}^{j-1}) \\ - \left({}^{t+\Delta t}\mathbf{C}_T^{j-1} + {}^{t+\Delta t}\mathbf{H}_f^{j-1} \right) \left({}^t\mathbf{v} + \Delta t (1 - \gamma_\beta) {}^t\mathbf{a} + \Delta t \gamma_\beta {}^{t+\Delta t}\mathbf{a}^{j-1} \right) \\ - {}^{t+\Delta t}\mathbf{K}_{\text{slf}}^{j-1} \left({}^t\mathbf{d} + \Delta t {}^t\mathbf{v} + \Delta t^2 \left(\frac{1}{2} - \beta_\beta \right) {}^t\mathbf{a} + \Delta t^2 \beta_\beta {}^{t+\Delta t}\mathbf{a}^{j-1} \right) \end{aligned} \quad (4.41)$$

The predictor-multi-corrector time marching algorithm, presented in detail in Section 3.10.4, is applicable to the proposed developments in this chapter. The only change required being the governing equation, Equation 3.149, which must be replaced either by Equation 4.38 or 4.41, depending on the time integration scheme employed.

4.8 IMPLEMENTATION ASPECTS

The advantages of implementing the proposed fluid finite elements in the general purpose finite element program ASAS-NL were outlined in Chapter Three. These arguments remain valid when considering the proposed non-radiating boundary formulation developed in this chapter. Therefore, the Plane Wave Approximation formulation will be implemented in the above mentioned code. The main modifications to ASAS-NL may be classified into three main categories:

- In addition to the basic input data required for the newly developed elements, the user must input the shock wave data (magnitude of pressure or velocity jump, exponential decay time, position of shock wave at the beginning of the analysis and speed of sound in the infinite fluid medium).
- Addition of new load types to model the exponential and step plane waves. This must be carried out for both two-dimensional and axi-symmetric geometries.
- Addition of new two-dimensional and axi-symmetric elements to model the infinite fluid medium.

The implementation of all of the above mentioned modifications is explained in full in Appendix E.

4.9 VALIDATION

A variety of tests are considered to examine the accuracy of the developed formulation. The sample problems are selected in a manner to assess the following factors:

- Accuracy of the Plane Wave Approximation elements.
- Accuracy of the incident exponential and step shock wave formulation.

Towards this end, two categories of far-field fluid-structure interaction problems are illustrated:

- Response of structures submerged in infinite fluids and subjected to internal pressure. This should give an indication of the accuracy of the scattered pressure-velocity formulation.
- Response of structures submerged in infinite fluid mediums and subjected to incident waves. This should give an indication of the accuracy of the incident wave formulation as well as the overall usability and stability of the method.

4.9.1 Steel Sphere Submerged in Water and Subjected to an Internal Pressure

The purpose of this example is to test the accuracy of the axi-symmetric Plane Wave Approximation formulation. The problem under consideration is shown in Figure 4.7, where a submerged steel sphere is subjected to an internal pressure, assuming plane strain conditions. Since the loading conditions and the geometry of the sphere are point symmetric, the response of the sphere is point symmetric. Therefore, it is sufficient to model one quarter of the sphere. The finite element model consists of fifteen eight-noded axi-symmetric elements for the quarter sphere, while the non-radiating boundary is modelled using fifteen three-noded axi-symmetric Plane Wave Approximation elements. The size of the time step used in the analysis is 0.01627 msec.

The same problem was studied by Soliman and DiMaggio (1983) where both the fluid and the shell were discretised by finite differences. The grid for the shell equations consisted of fifteen equal subdivisions in the circumferential direction, while that of the fluid was subdivided into fifteen equal subdivisions in the circumferential direction and twenty five unequal subdivisions in the radial direction. The finite difference grid in the fluid extended to a radius of forty five inches, where a non-radiating boundary based on the Doubly Asymptotic Approximation was used. Another analytical solution to this problem was presented by Bedrosian and DiMaggio (1972b). The numerical results obtained from this study along with those reported in the above mentioned papers are shown in Figure 4.8, where the radial displacement is considered positive in the outward direction. In addition, the displacement is normalised with respect to the static radial displacement under uniform radial pressure given by Bedrosian and DiMaggio (1972b) as:

$$d_{st} = \frac{r_o^2 P_o}{t E_s} \frac{6 - \nu_s}{2}$$

where r_o is the radius of the sphere middle surface and all other quantities have been defined in Figure 4.7.

It can be seen that the results obtained from using the Plane Wave Approximation on the shell surface are in reasonable agreement with those obtained from the use of the Doubly Asymptotic Approximation developed by Soliman and DiMaggio (1983). Figure 4.8 also shows the effect of the presence of the infinite fluid medium on the response of the shell. The infinite fluid medium has the effect of damping the response of the shell.

The response obtained by using the Plane Wave Approximation, developed in this study, on the surface of a finite element mesh extending into the fluid is not considered since a major reason for employing the procedure presented herein is to avoid discretising the infinite fluid medium using finite elements. Finally, it should be stressed that by placing the non-radiating boundary on the wet surface of the submerged structure, substantial savings in computer processing time and storage are obtained.

4.9.2 Steel Sphere Submerged in Water and Subjected to a Step Internal Pressure

This example also deals with the transient response of a submerged sphere subjected to an internal pressure. However, it differs in three ways from the one presented in Section 4.9.1. Firstly, the internal pressure has a step profile (instead of step profile with an initial rise) in the time domain. Secondly the radius to thickness ratio of the sphere is different (75 instead of 52.5). Finally, the result obtained is compared to other numerical studies which use finite elements (instead of finite differences) to model the infinite fluid medium.

The finite element model is shown in Figure 4.9, along with the geometric and material properties, boundary conditions and applied load. For the same reasons outlined in the previous section, it is sufficient to model one quarter of the sphere. The quarter sphere is modelled using six eight-noded axi-symmetric solid elements, while the non-radiating boundary is modelled using six three-noded Plane Wave Approximation elements. The time step employed in the transient analysis is 0.025 msec. The magnitude of the internal, uniformly distributed, step pressure is taken as

1.0186 psi.

The same problem was studied by Singh et al (1990), where both the fluid and the shell were discretised using finite elements. The grid for the sphere was divided into six equal subdivisions in the circumferential direction, while that for the fluid was subdivided into six equal subdivisions in the circumferential direction and 15 unequal subdivisions in the radial direction. The finite element mesh in the fluid extended to a length of twenty nine and a half inches from the outer sphere radius, where a non-radiating boundary was located.

Figure 4.10, shows the numerical results obtained from this study along with the analytical and numerical results reported in the paper mentioned above. It can be seen that all the results are in reasonable agreement. Since the amount of CPU required in the analysis described by Singh et al (1990) was not given, it is not possible to comment on the savings in computer time due to the placement of the non-radiating boundary on the sphere outer surface. However, it should be stressed that in this study, the non-radiating boundary is placed directly on the surface of the sphere, thus avoiding the need to discretise the fluid using a finite element mesh.

4.9.3 Steel Sphere Submerged in Water and Subjected to Incident Exponential and Step Waves

In this example, the response of a steel sphere submerged in water and subjected to an incident step wave is studied. The purpose of this example is to provide a first check on the accuracy of the proposed shock wave formulation. The problem is shown in Figure 4.11, where the finite element model, geometry, material properties and loading conditions are presented. Since the loading conditions and the geometry of the sphere are symmetric about the z axis, it is sufficient to model one half of the sphere. The sphere is modelled by thirty six eight-noded axi-symmetric solid elements, while the non-radiating boundary is modelled by thirty six three-noded Plane Wave Approximation elements. The time step used in the analysis is $4.345\text{E-}3$ msec. The magnitude of the incident step pressure wave was taken as ρc^2 , where ρ is the density of the fluid medium and c is the speed of propagation of sound in the fluid medium.

This problem was studied by Vasudevan and DiMaggio (1981) where an analytical solution was reported. The variation of the radial velocity of the cylinder with time was monitored at $\theta = 0^\circ$ and $\theta = 180^\circ$. The analytical solution reported in the above paper is shown in Figure 4.12, along with the numerical one obtained from this study.

The velocity was non-dimensionalised with respect to the speed of sound in the fluid medium, c , while the time was normalised with respect to r/c where r is the radius of the cylinder. The early time performance of the Plane Wave Approximation is seen to be good, but its tendency to overestimate the fluid radiation damping causes an undesirable attenuation of the late time response. This may be attributed to the lack of a late time approximation in the present study, which decreases the accuracy of the response after early times. Also shown in Figure 4.12 is the non-dimensional radial velocity for two exponential waves with decay constants of 900 and 2000, where the decay of the response with time can be observed. It should be mentioned that in the above mentioned paper the actual geometry of the sphere was not presented; however, a thickness to radius ratio was given which is identical to the one used in this study.

4.9.4 Cylinder Submerged in Water and Subjected to an Incident Step Wave

This example is concerned with the interaction of an elastic infinite cylinder with a plane acoustic wave. The purpose of this problem is to check the accuracy of the two-dimensional shock wave formulation and the two-dimensional non-radiating boundary. The problem under consideration is shown in Figure 4.13, where a submerged cylinder is subjected to a step incident wave. Also shown in Figure 4.13, are the geometric and material properties of the cylinder, where the loading conditions and the geometry of the cylinder are symmetric about the x axis. Therefore, it is sufficient to model half the cylinder. The cylinder is modelled using twelve eight-noded two-dimensional solid elements, while the non-radiating boundary is modelled using twelve three-noded two-dimensional Plane Wave Approximation elements. The time step chosen for the analysis is 0.0135266 msec. The magnitude of the incident pressure wave impinging on the structure is taken to be equal to pc^2 .

The analytical solution to the same problem was determined by Huang in 1975 and reported by Zilliacus (1983), who plotted the variation of the non-dimensional velocity of the cylinder with time.

Figure 4.14 shows the non-dimensional radial velocity of the cylinder, obtained from this study, along with the analytical solution reported by Zilliacus (1983). The velocity was non-dimensionalised with respect to the speed of sound in the fluid medium, c , while the time was normalised with respect to r/c where r is the radius of the cylinder. Good agreement between both sets of results is observed. This may be explained by the fact that the proposed formulation is based on early time approximations, while the response of the cylinder peaks at an early stage of the analysis. Also shown in the figure, is the response of the cylinder to a plane wave of

the same magnitude as the step wave, but having an exponential decay constant of magnitude equal to 750. The decay of the response with time can be observed.

4.9.5 Elastic Response of a Submerged Cylinder with Internally Attached Structures to Shock Loading

In this example, the proposed formulation is applied to determine the elastic response of a cylinder with internal structures submerged in an infinite acoustic fluid and subjected to a plane step pressure wave. The purpose of this example is to study the effect of the internal structure on the response of the cylinder and to further validate the proposed formulation. Due to the lack of published data concerning such a two-dimensional problem, the response will be validated by carrying out several analyses (while varying some geometric and material properties) and assessing the results obtained. The problem is shown in Figure 4.15, along with the geometric and material properties as well as the boundary and loading conditions, where a submerged infinite cylinder with internal structures is subjected to a step incident wave. It should be noted that the geometric and material properties of the cylinder are identical to those used in the previous problem. Since this is a symmetric problem about the x axis, it is sufficient to model one half of the cylinder. The cylinder is modelled using twelve eight-noded two-dimensional plane strain elements, while the non-radiating boundary is modelled using twelve three-noded Plane Wave Approximation elements. A single mass is elastically attached to the cylinder at $\theta = 90^\circ$ by means of four eight-noded two-dimensional plane strain solid elements which behave as a two-dimensional spring. The time step chosen for the transient analysis and the magnitude of the step incident wave are identical to those used in the previous example.

To study the effect of the spring stiffness and the weight of the mass three test cases are considered, as shown in Table 4.4.

Figure 4.16 shows the spring displacement and velocity for all the cases. To study the effect of the mass on the structure, the radial displacement and radial velocity of the cylinder at $\theta = 90^\circ$ are plotted in the absence and presence of the mass, where the kinematic quantities are considered positive in the outward normal direction. The velocity is normalised with respect to speed of sound in the fluid, c , while time is normalised with respect to r/c where r is the radius of the cylinder. The effect of the density of the mass on the response of the cylinder is investigated by comparing Case a (stiff spring with light mass) with Case b (stiff spring with heavy mass). As expected, it can be seen that for a stiff spring as the density of the attached mass

increases the velocity of the cylinder decreases.

Material Property and Parameters	Case a	Case b	Case c
ρ (density of fluid) (lb-sec ² /in ⁴)	9.3×10^{-4}	9.3×10^{-4}	9.3×10^{-4}
β (bulk modulus of fluid) (lb/in ²)	3.0×10^5	3.0×10^5	3.0×10^5
ρ_{mass} (density of mass) (lb-sec ² /in ⁴)	7.28×10^{-4}	1.0	1.0
k (spring constant) (lb/in ²)	3.0×10^5	3.0×10^5	3.0×10^2
ρ_{cyl} (density of cylinder) (lb-sec ² /in ⁴)	7.28×10^{-4}	7.28×10^{-4}	7.28×10^{-4}
β (bulk modulus of cylinder) (lb/in ²)	3.0×10^7	3.0×10^7	3.0×10^7
ν_s (Poisson ratio of cylinder)	0.3	0.3	0.3

**Table 4.4 Material Properties and Computation Parameters
for Fluid-Piston System**

The effect of the stiffness of the spring on the response of the cylinder is investigated by comparing Case a or Case b (stiff spring) with Case c (flexible spring). As expected it can be seen that the force acting on the cylinder is much lower for a flexible spring than that for a stiff spring.

4.10 CONCLUSION

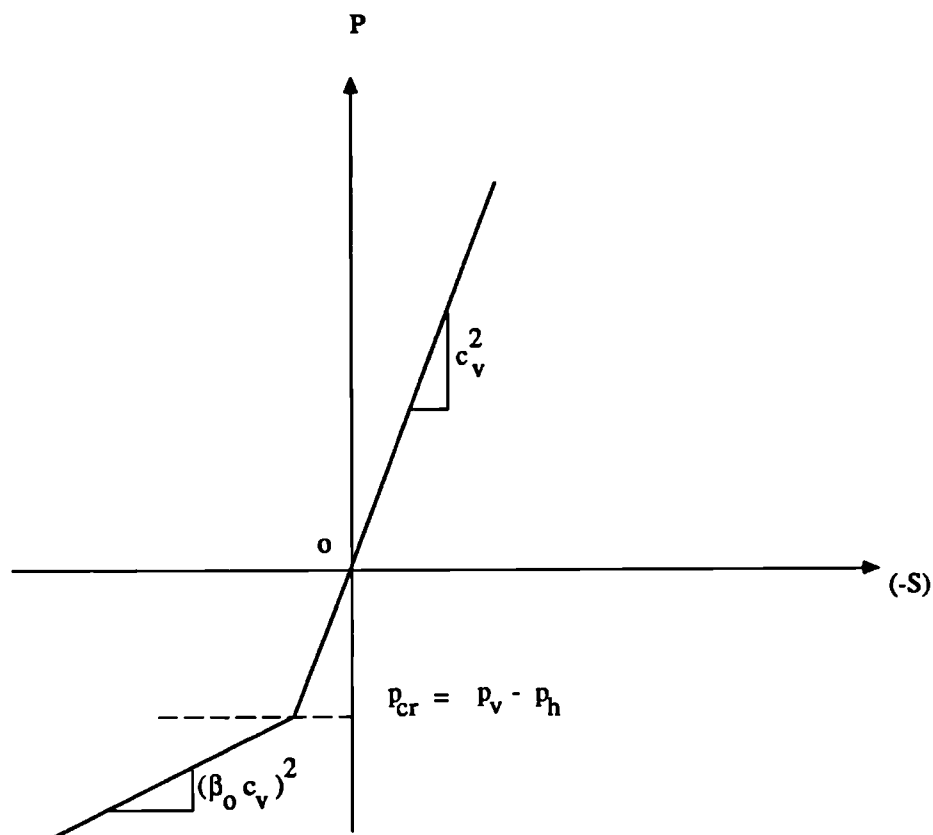
The work presented in this chapter has been concerned with the development of a non-radiating boundary based on acoustic approximations. Towards this end, the Plane Wave Approximation formulation was presented and discussed. The proposed method was implemented into the general purpose finite element package ASAS-NL.

Before using the developed formulation in the general analysis of unbounded fluid-structure interaction problems, it is required to demonstrate the elements' capabilities and identify their limitations. In this chapter, the Plane Wave Approximation elements and plane shock wave loading were assessed for a variety of problems. The accuracy of the radiated wave was examined by studying the response of submerged

spheres subjected to an internal uniformly distributed pressure. The results obtained were compared with other published numerical solutions, based on finite differences and finite elements, as well as analytical solutions. It was seen that the proposed formulation produces good results without any need for the discretisation of the infinite fluid medium.

The response of two-dimensional and axi-symmetric fluid-structure systems, comprising submerged cylinders and spheres subjected to incident shock waves, were further studied and compared to other numerical studies based on the Plane Wave Approximation and Doubly Asymptotic Approximation. The early time performance of the proposed formulation was seen to be good. However, for the axi-symmetric case, its tendency to overestimate the fluid radiation damping causes an undesirable attenuation of the response at late times. This inaccuracy did not appear in the two-dimensional plane strain response calculations for a wave-excited, infinite, circular cylindrical shell, because the exact response histories for this problem were much less oscillatory. To investigate the effect of the shock wave profile, both step and exponential wave loading scenarios were considered.

The above examples served as a qualitative appraisal of the suitability of the Plane Wave Approximation as a non-radiating boundary for underwater fluid-structure interaction analysis. It may be concluded that by placing the non-radiating boundary on the surface of the submerged structure, the proposed formulation can be used in confidence for the determination of the early time response of unbounded fluid-structure systems subjected to a variety of loading scenarios. The use of the Plane Wave Approximation, instead of the Doubly Asymptotic Approximation, results in substantial savings in computer storage and processing time due to the absence of the fully populated and unsymmetric added mass matrix. Furthermore, as long as the peak response of the structure occurs at an early stage of the analysis, the Plane Wave Approximation will also result in an accurate response. Based on this validation of the proposed developments, it is now possible to apply them to the general analysis of more practical unbounded fluid-structure systems such as cylinders with external appendages. This is the objective of the work presented in Chapter Five.



- S : Densified relative density
 c_v : Velocity of the pressure wave in the fluid
 p_v : Absolute or total pressure
 p_h : Hydrostatic pressure
 p_{cr} : Nominal cavitation pressure p_h : Hydrostatic pressure
 β_o : Resistance to expansion of the cavitated region

Figure 4.1 Equation of State for Cavitated Fluid

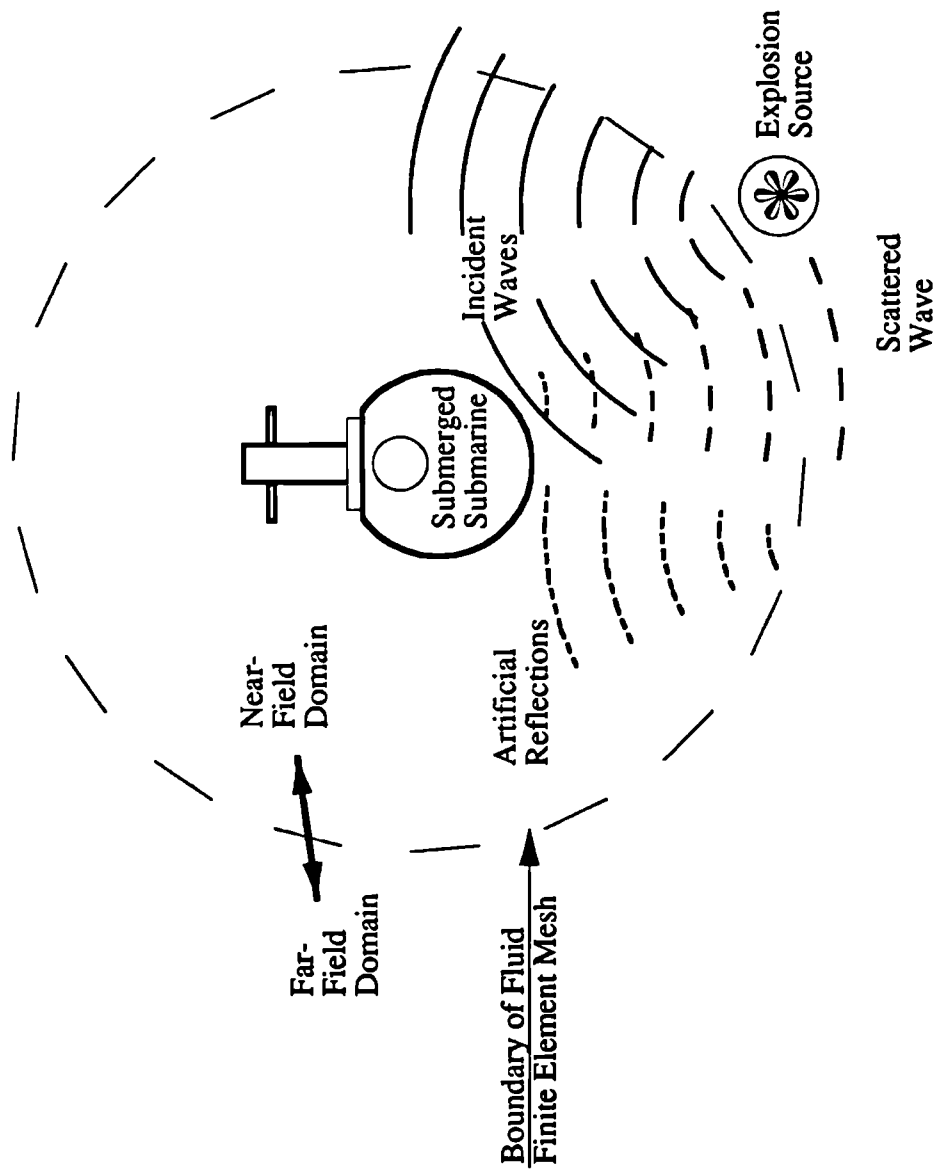


Figure 4.2 Wave Propagation in the Vicinity of a Submerged Structure

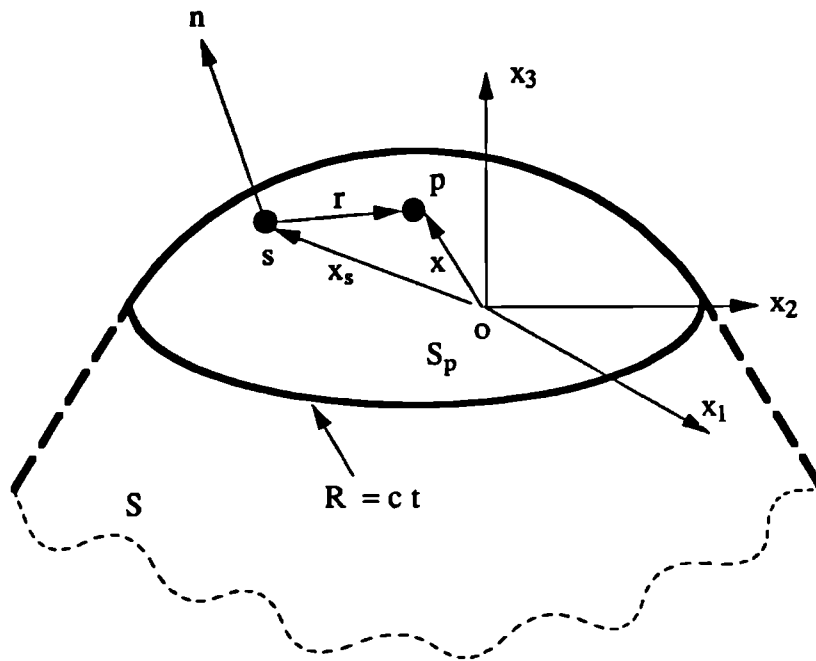


Figure 4.3 Local Coordinate System for the Derivation of the Plane Wave Approximation Formulation

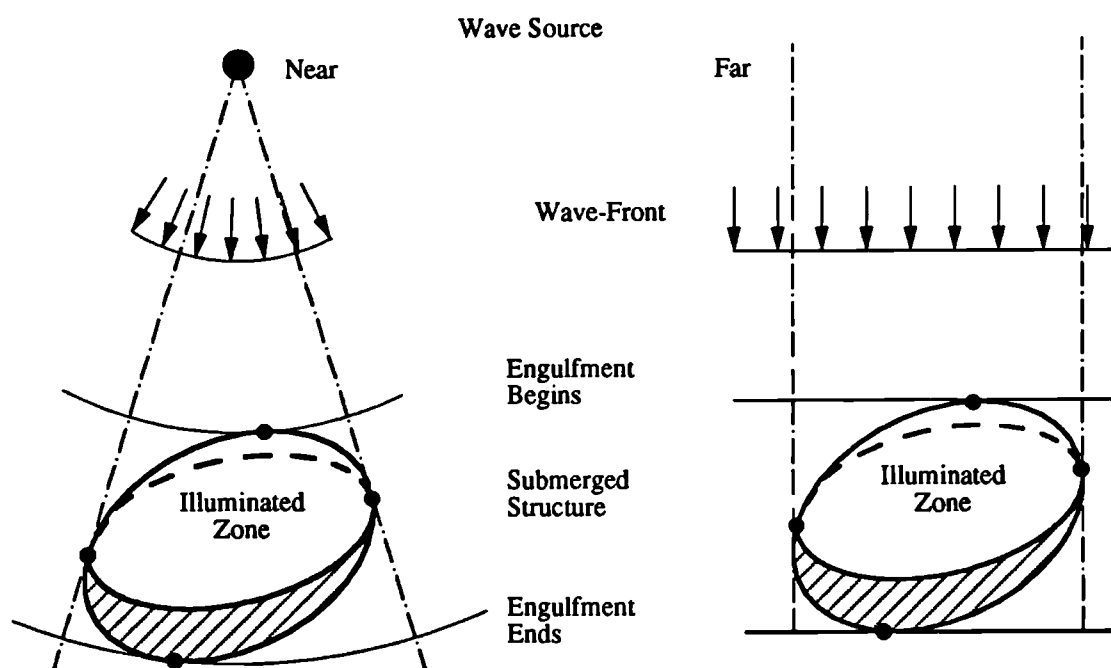


Figure 4.4 Incident (Free Field) Wave Structure Engulfment Geometry

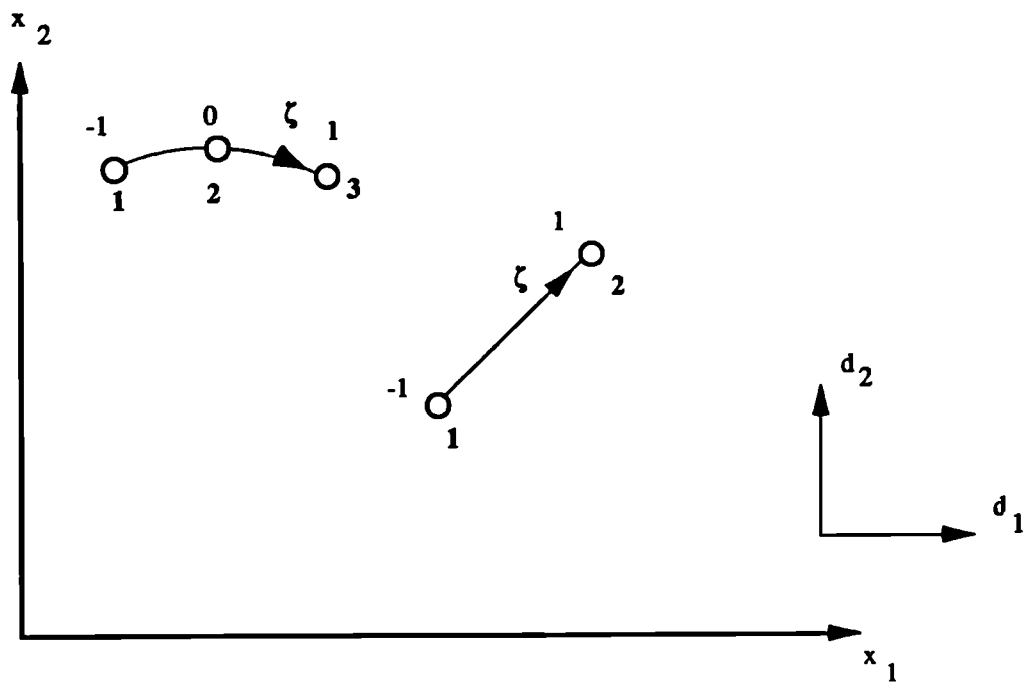


Figure 4.5 Numbering Convention and Geometric Configuration for Two-Dimensional Fluid Finite Elements

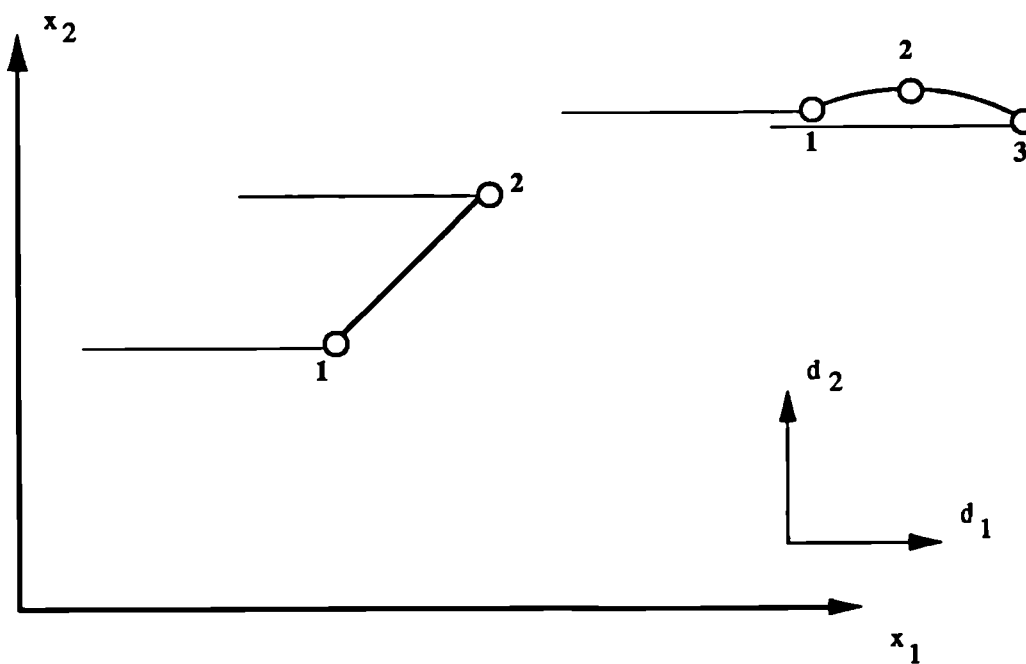
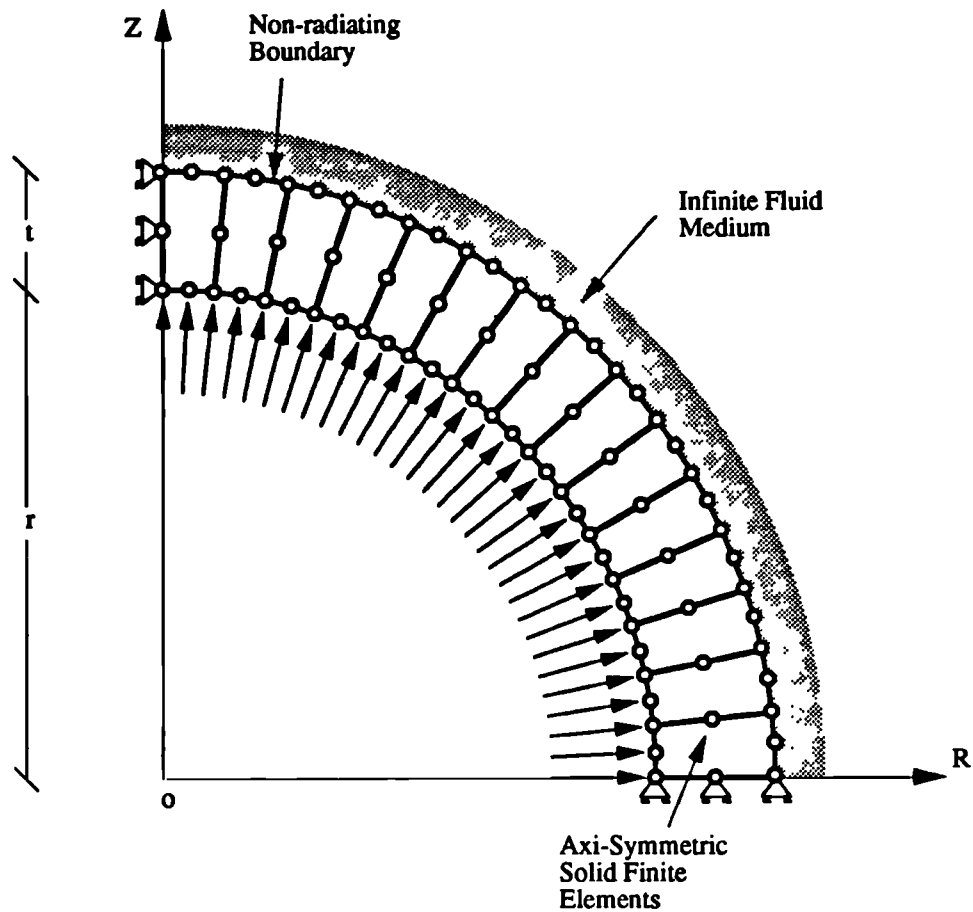


Figure 4.6 Numbering Convention and Geometric Configuration for Axi-Symmetric Fluid Finite Elements



MATERIAL AND GEOMETRIC PROPERTIES

$\nu_s = 0.3$
 $\rho_s = .735 \text{ E-3 lb sec}^2/\text{in}^4$
 $\rho = 0.958 \text{ E-4 lb sec}^2/\text{in}^4$
 $c = 0.59 \text{ E+5 in/sec}$
 $E_s = 2.900 \text{ E+7 lb/in}^2$
 $r = 21 \text{ in}$
 $t = 0.4 \text{ in}$
 $P_0 = 1.0$

LOADING CONDITIONS

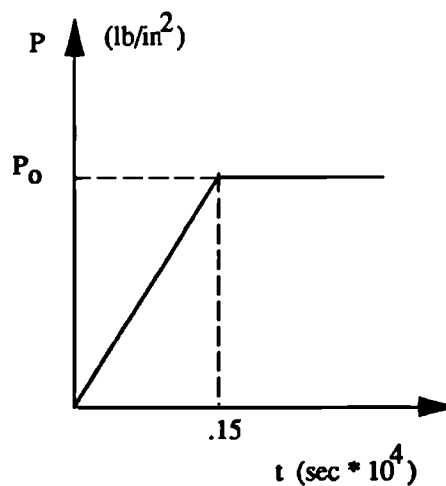


Figure 4.7 Sphere Submerged in an Infinite Fluid Medium and Subjected to internal Pressure: Finite Element Model, Loading Conditions and Geometric and Material Properties

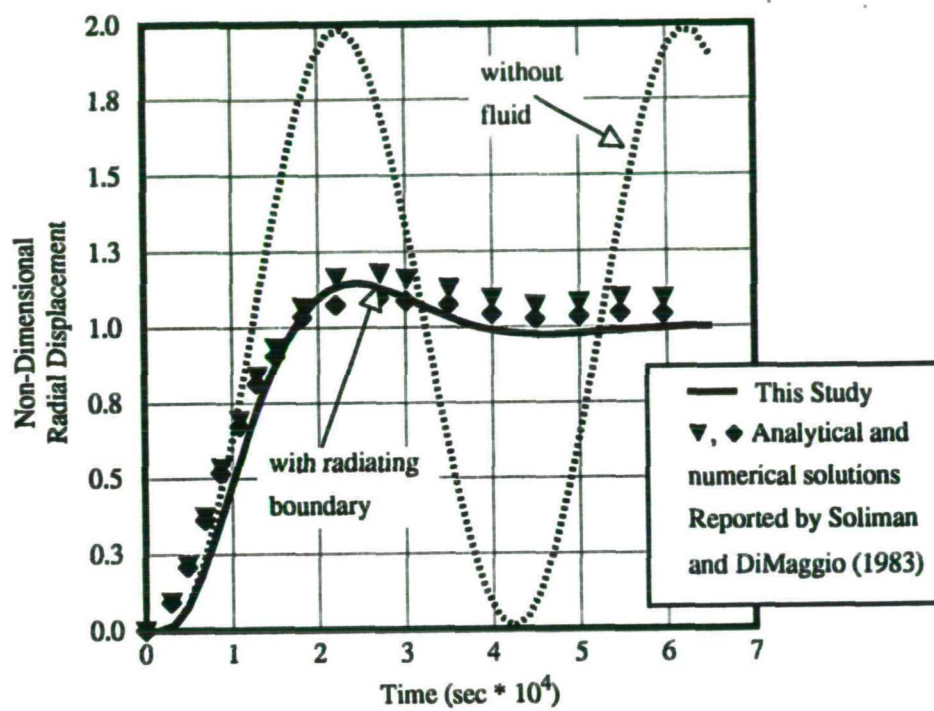
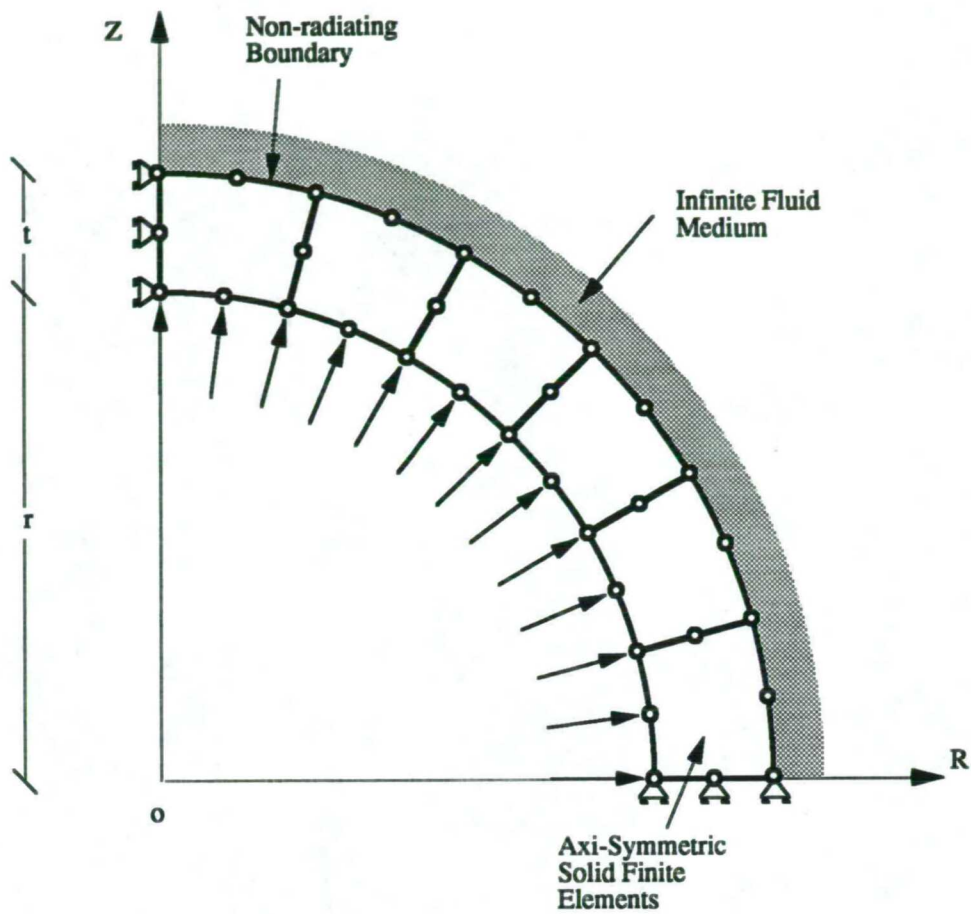


Figure 4.8 Sphere Displacement Variation with Time - Comparison with Analytical and Numerical Results Reported by Soliman and DiMaggio (1983)



MATERIAL AND GEOMETRIC PROPERTIES

$\nu = 0.3$
 $\rho_s = .7345 \text{ E-}3 \text{ lb sec}^2/\text{in}^4$
 $E = 29\text{E}+6 \text{ lb/in}^2$
 $\rho = 0.9582 \text{ E-}4 \text{ lb sec}^2/\text{in}^4$
 $c = 0.58590\text{E}+5 \text{ in/sec}$
 $r = 75 \text{ in}$
 $t = 1 \text{ in}$

LOADING CONDITIONS

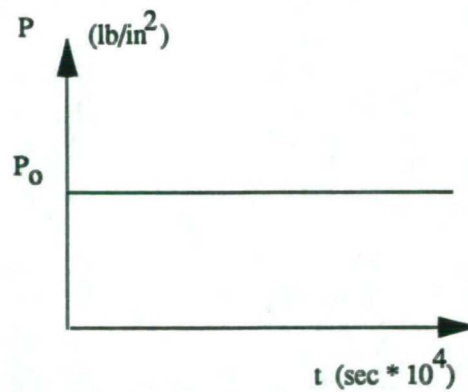
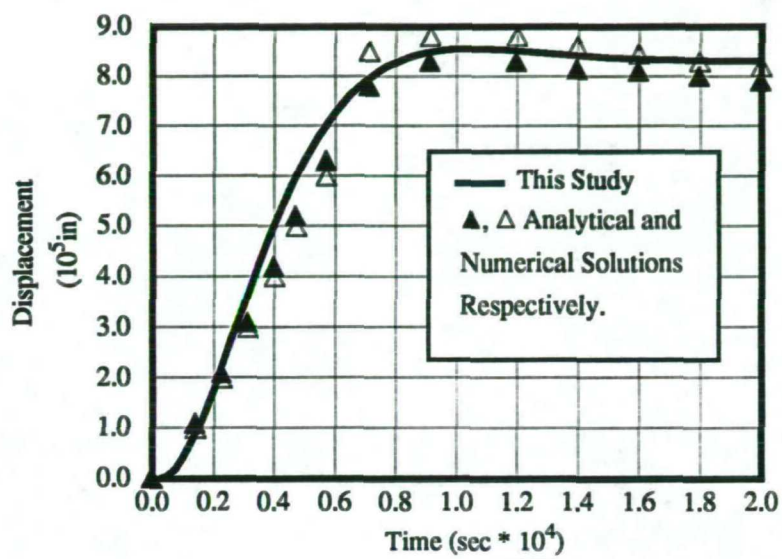
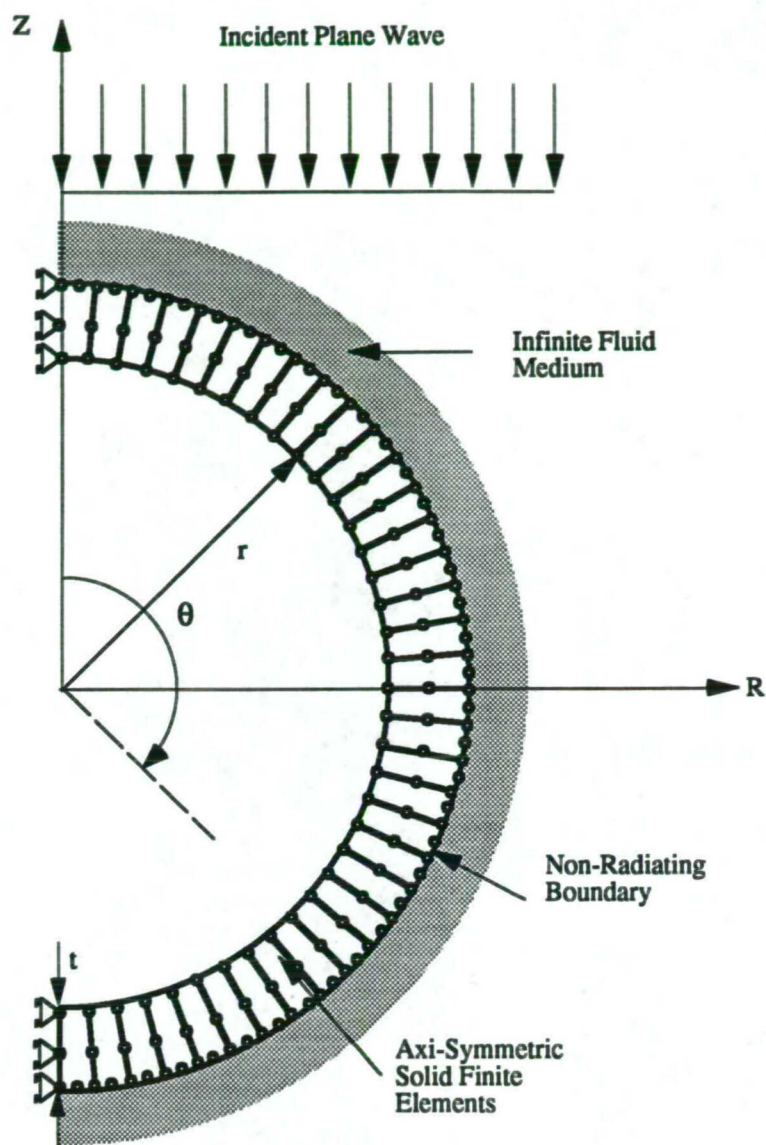


Figure 4.9 Sphere Submerged in an Infinite Fluid Medium and Subjected to internal Pressure: Finite Element Model, Loading Conditions and Geometric and Material Properties



? 0.1 msec

Figure 4.10 Sphere Displacement Variation with Time - Comparison with Analytical and Numerical Results Reported by Singh et al (1990)



MATERIAL AND GEOMETRIC PROPERTIES

$\nu = 0.3$
 $\rho_s = .728 \text{ E-}3 \text{ lb sec}^2/\text{in}^4$
 $\rho = 0.93 \text{ 10E-}4 \text{ lb sec}^2/\text{in}^4$
 $c = 0.57528\text{E+}5 \text{ in/sec}$
 $r = 10 \text{ in}$
 $t = 0.2 \text{ in}$

LOADING CONDITIONS

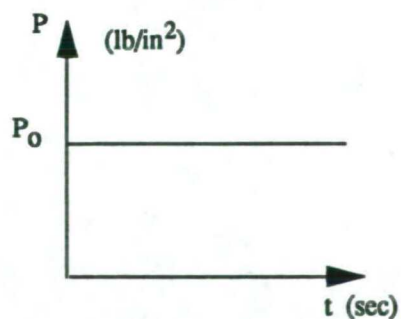


Figure 4.11 Sphere Submerged in an Infinite Fluid Medium and Subjected to Exponential and Step Pressure Waves: Finite Element Model, Loading Conditions and Geometric and Material Properties

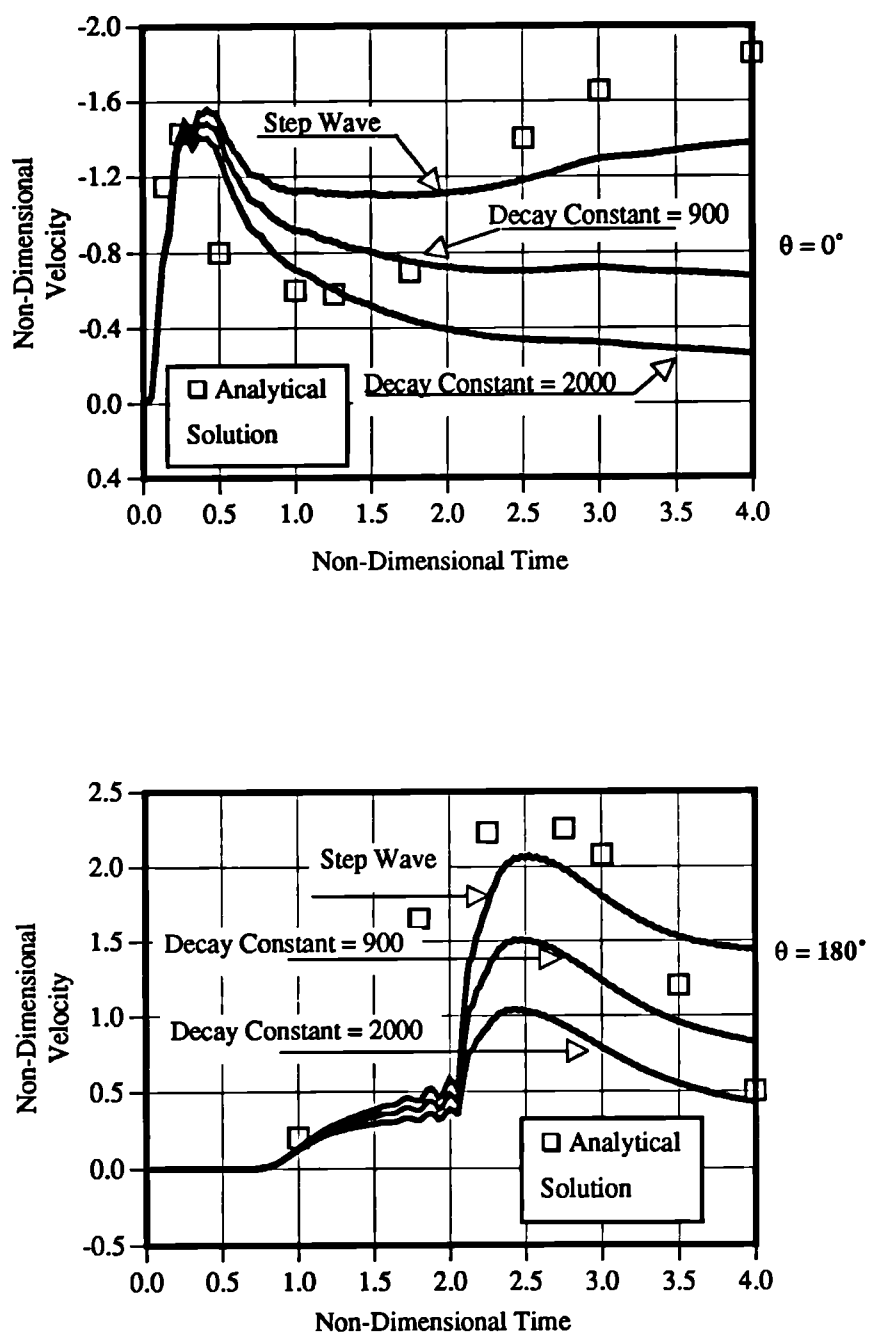


Figure 4.12 Non-Dimensional Velocity of a Submerged Sphere Subjected to Step and Exponential Waves - Comparison with Analytical Solution by Vasudevan and DiMaggio (1981) for Step Incident Wave

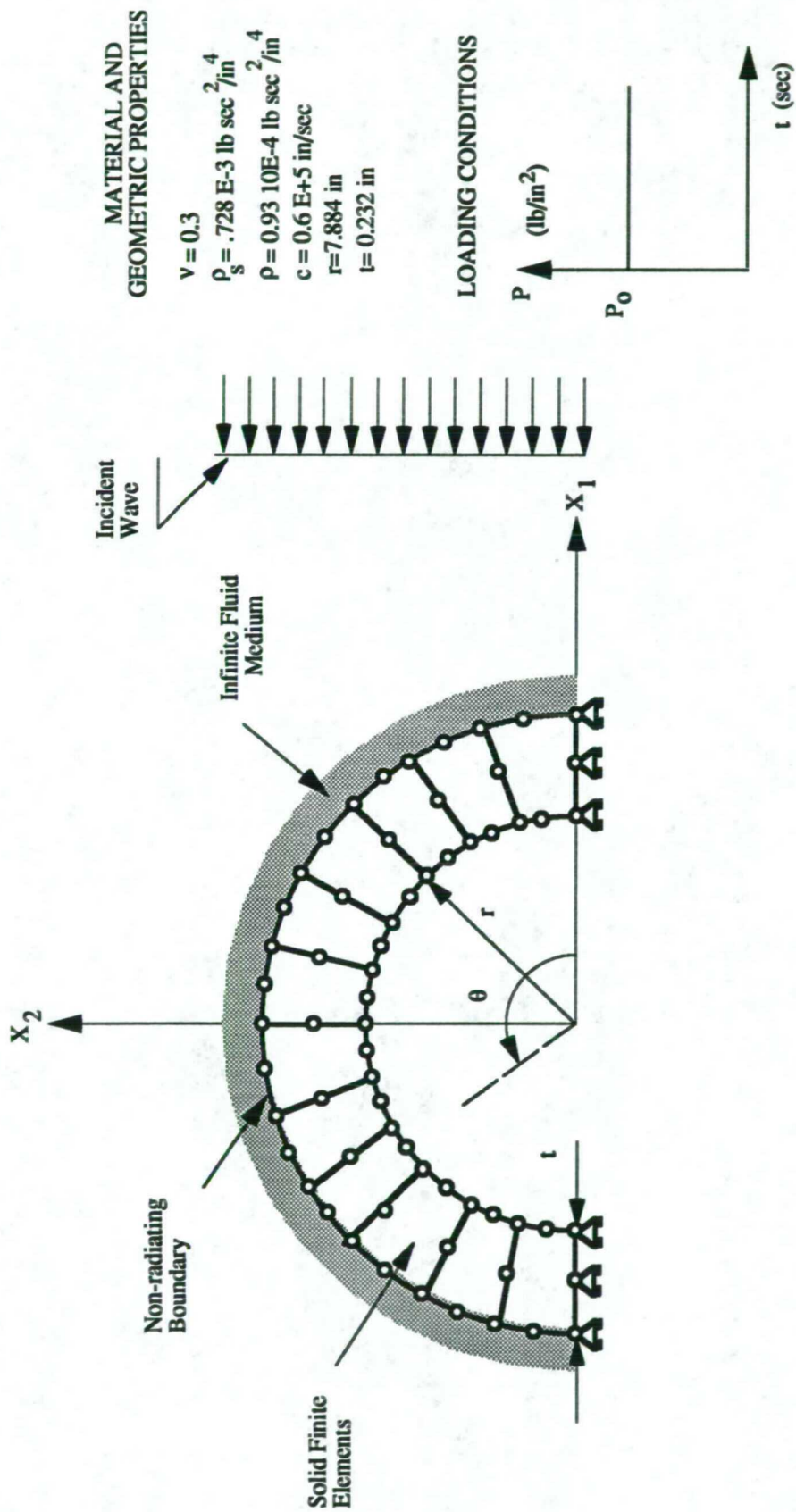


Figure 4.13 Submerged Cylinder Subjected to a Shock Wave

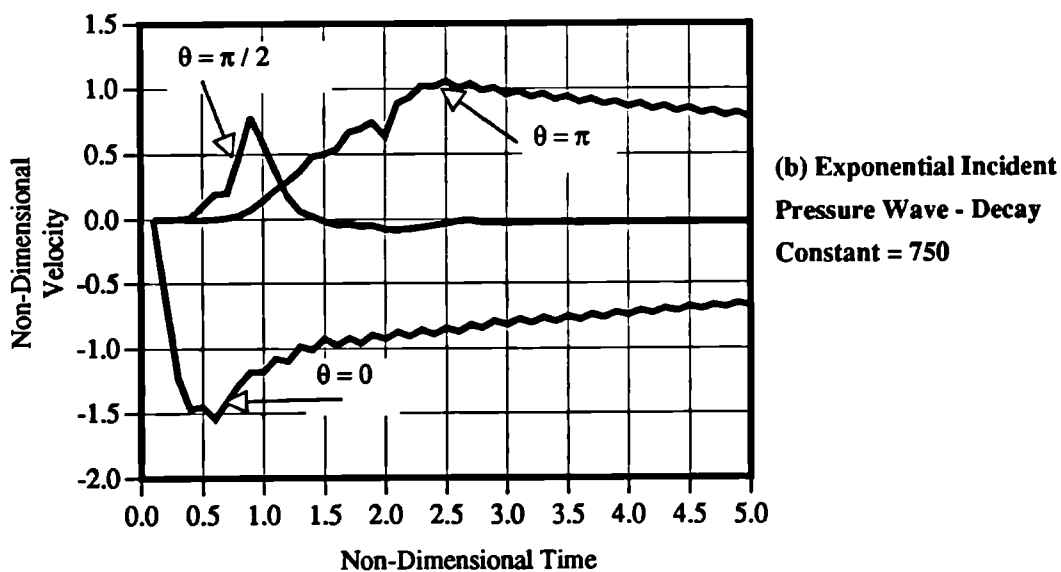
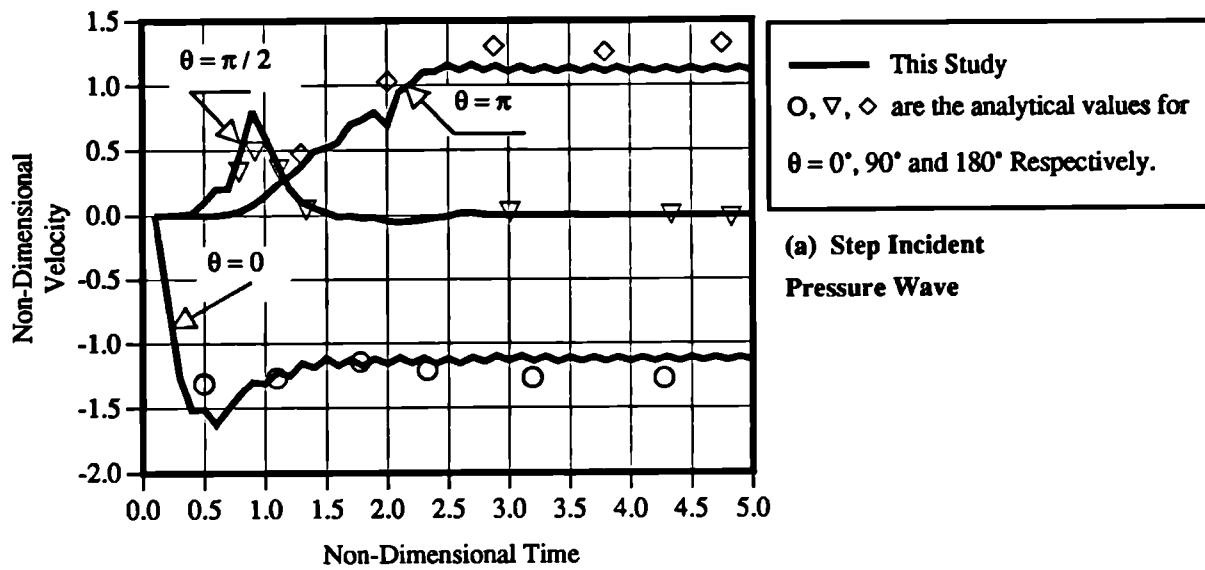
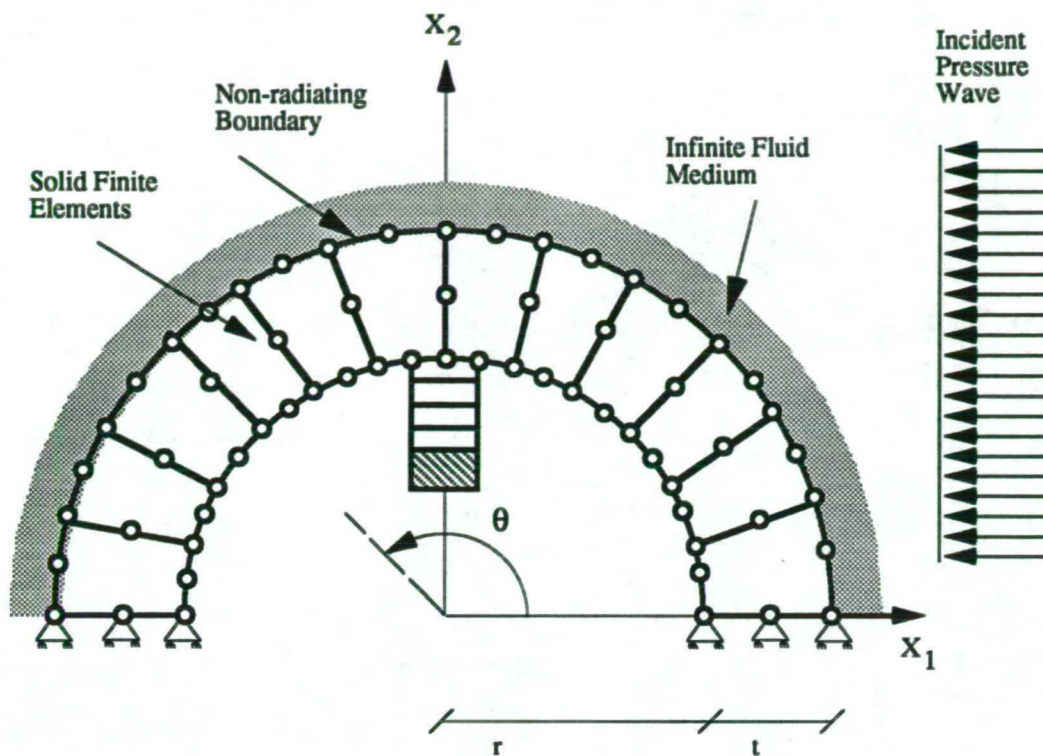


Figure 4.14 Non-Dimensional Radial Velocity of a Submerged Cylinder in an Infinite Fluid and Subjected to Step and Exponential Waves - Comparison with Analytical Results Reported by Zilliacus (1983), for the Step Incident Wave



MATERIAL AND GEOMETRIC PROPERTIES

$$\begin{aligned} \nu &= 0.3 \\ \rho_s &= .728 \text{ E-3 lb sec}^2/\text{in}^4 \\ \rho &= 0.93 \text{ 10E-4 lb sec}^2/\text{in}^4 \\ c &= 0.6 \text{ E+5 in/sec} \\ r &= 7.884 \text{ in} \\ t &= 0.232 \text{ in} \\ P_I &= \rho c^2 \end{aligned}$$

LOADING CONDITIONS

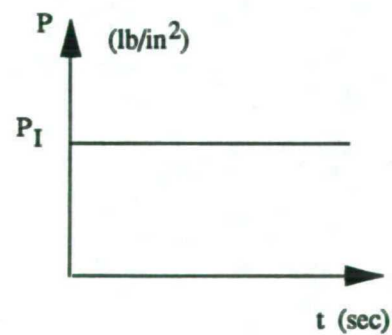


Figure 4.15 Submerged Cylinder in an Infinite Fluid Medium with Internally Attached Structures: Finite Element Model and Geometric and Material Properties

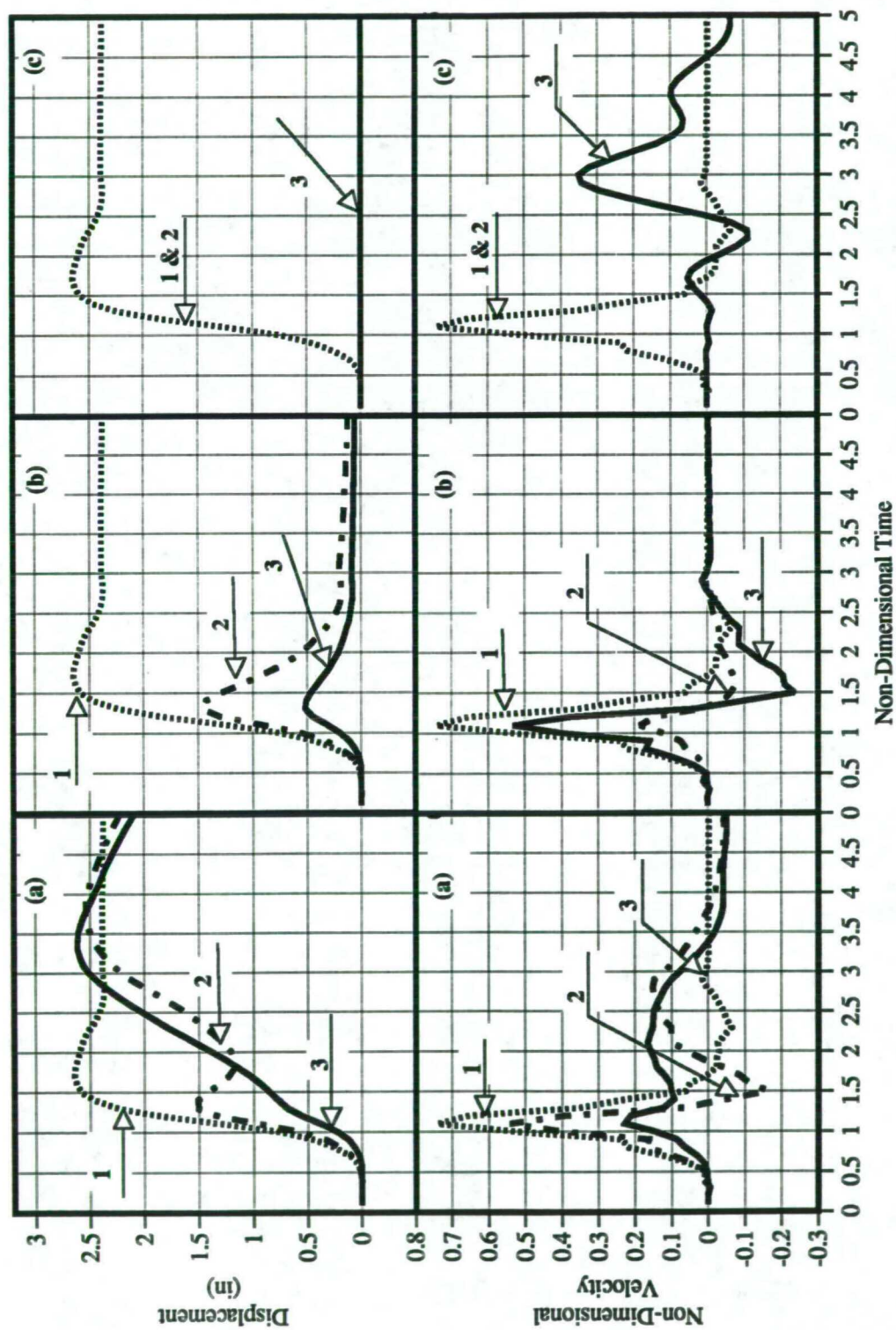


Figure 4.16 Submerged Cylinder with Internally Attached Structures- 1: Displacement of Cylinder w/o Internal Structures, 2: Displacement of Cylinder w/ Attached Structures, 3: Spring Displacement

CHAPTER FIVE

PRACTICAL APPLICATIONS

In Chapter One, practical applications of fluid-structure interaction analysis were classified under four main categories: liquid storage tanks, dam reservoir systems, nuclear reactors and pressure vessels, and floating and submerged structures.

In Chapter Three, a general two-dimensional and axi-symmetric finite element formulation was developed for transient near-field fluid-structure analysis. This was integrated into the general purpose finite element package ASAS-NL. Numerical examples demonstrated the accuracy of the proposed fluid elements where, as part of the validation process, the response of a rigid reservoir system was analysed.

In Chapter Four, a non-radiating boundary based on the Plane Wave Approximation formulation was developed and implemented into ASAS-NL. Step and exponential plane, shock wave formulations were implemented into the package to model the effect of explosions. Finally, the responses of several two-dimensional and axi-symmetric submerged structures were studied.

The purpose of this chapter is to demonstrate the applicability of the above mentioned formulations. For this purpose, the transient analysis of fluid-structure systems, including both near-field and far-field problems such as water tanks, pressure vessels and double-hulled cylindrical shells, will be considered. Where possible, the results obtained will be compared to those reported in other numerical and analytical studies.

Since this thesis is concerned with the development of numerically based tools for the analysis of general fluid-structure systems, it does not concentrate on a particular application but tries to highlight the various fields where the proposed formulation may be applied. Therefore parametric studies of particular fluid-structure systems will not be carried out. However, several test cases of each problem are considered to provide results which would assist in creating a database to be used in the assessment and validation of general numerical methods for the analysis of fluid-structure interaction systems.

5.1 ELASTO-PLASTIC RESPONSE OF A FLUID FILLED PRESSURE VESSEL

In this section, the practical problem of an axi-symmetric pressure vessel, surrounded by a fluid medium and contained within an external wall will be studied. The finite element model, geometry and boundary conditions are shown in Figure 5.1, while the three-dimensional pressure vessel geometry, material properties and loading conditions are given in Figure 5.2. The structure is discretised using eight-noded axi-symmetric solid elements while the fluid is modelled using eight-noded axi-symmetric Mixed-Eulerian-Lagrangian fluid elements. Fluid nodes lying on the free surface and on the fluid-structure interface are treated as Lagrangian, while those lying inside the fluid mesh are considered Eulerian. At the fluid-structure interface boundary, four-noded axi-symmetric sliding elements are used.

Shantaram et al (1976) and Chelghoum (1986) studied a similar problem where the fluid extended to a height of 14 inches, the external wall was assumed rigid and the magnitude of the internal applied pressure corresponds to load Case A (Figure 5.2) of this study. They used Lagrangian elements to model the fluid. In both publications, the vertical displacement at node 17 and the horizontal displacement at node 1 were monitored. Figure 5.3 shows the results obtained from this study, using the above mentioned assumptions, along with those reported in the above references. Reasonable agreement between both sets of results can be observed. The deviation between the peak values may be attributed to the different approaches adopted in the temporal discretisation, fluid element formulation and sliding interface condition. In the present study, an implicit time integration scheme with Mixed-Eulerian-Lagrangian fluid and sliding elements was used, while Shantaram et al (1976) adopted an explicit time integration scheme with Lagrangian fluid elements and horizontal coupling stiffnesses at the interface. Furthermore, Chelghoum used an implicit time integration scheme with Lagrangian fluid and sliding elements.

To study the effect of the fluid inertia on the structure, three analyses were carried out. In the first, the pressure vessel was considered to be empty while in the other two the fluid height was taken to be fourteen and eighteen inches respectively. In all cases, the external wall was assumed rigid and the horizontal degrees of freedom of all the fluid nodes lying on the vertical line defined by $r = 34$ inches were suppressed. Figure 5.4 shows the displacements obtained from the three sets of results at nodes 1 and 17. The effect of the fluid inertia is apparent from the increase of the vertical displacement of node 17 and the decrease in the horizontal displacement of node 1. Figure 5.5 shows the fluid pressure at elements 9 and 10 for the two cases where fluid is present, where it can be seen that the fluid undergoes negative pressure. In the

absence of published analytical and numerical solutions for the fluid pressure it is not possible to assess the effect of ignoring cavitation on the response. Figures 5.6 and 5.7 show the free surface profiles of the fluid.

In an attempt to study the effect of the pressure intensity on the response, two impulsive loading conditions, shown in Figure 5.2, are applied to the internal wall of the pressure vessel. For each loading case, the pulse intensity and duration are chosen so that the total impulse remains constant. Figure 5.8 shows the horizontal and vertical deflections at nodes 1 and 17 respectively, while Figure 5.9 shows the fluid pressures at elements 9 and 10. As expected, it is observed that for a constant impulse the fluid pressures are not affected by the variation in the load magnitude and duration, while the displacement response is slightly different. This may be attributed to the displacement based convergence criteria employed in this example.

In all sets of runs, the analyses were carried out for approximately 2.11 msec. Table 5.1 shows the different time steps used in all the analyses.

Time Interval (sec)	0.0 to 2.0E-5	2.0 E-5 to 6.0E-5	6.0 E-5 to 1.4 E-4	1.4 E-4 to 2.11 E-3
Type of Analysis				
Empty Pressure Vessel	1.0 E-6	2.0 E-6	2.0 E-6	2.0E-6
Fluid Height = 14 in	1.0 E-6	2.0 E-6	4.0 E-6	4.0E-6
Fluid Height = 18 in	1.0 E-6	1.0 E-6	2.0 E-6	4.0E-6
Load Case B	1.0 E-6	1.0 E-6	2.0 E-6	4.0E-6

Table 5.1 Time Steps for Different Intervals in the Transient Analysis of a Pressure Vessel

5.2 DYNAMIC RESPONSE OF A WATER TANK SUBJECTED TO EARTHQUAKE LOADING

In this section, the seismic response of a two-dimensional water tank will be studied. The finite element model, geometry and boundary conditions are shown in Figure 5.10. The tank is discretised using nineteen eight-noded plane strain two-dimensional solid elements, while the fluid is modelled using sixty four eight-noded Mixed-Eulerian-Lagrangian elements. Fluid nodes lying on the free surface and on the fluid-

structure interface are treated as Lagrangian, while those lying inside the fluid mesh are considered Eulerian. At the fluid-structure interface, seventeen four-noded sliding elements are used to enforce the sliding condition. Throughout the analysis, the tank base is assumed to be rigid. The earthquake excitation input, a five second duration, modified N-S component of the 1940 El Centro acceleration time history is applied at the tank base. The time history of the loading is shown in Figure 5.11. A maximum acceleration of 0.5 g is used throughout this study.

Ma et al (1982) studied the same problem, where the tank was modelled by sixteen four-noded shell elements, while the fluid mesh consisted of forty eight four-noded Mixed-Eulerian-Lagrangian elements. Eighteen contact elements were used to model the sliding boundary condition at the fluid-structure interface. Chelghoum (1986) also studied the seismic response of a fluid filled tank. The fluid was modelled using forty eight-noded, five-noded and four-noded Lagrangian elements, while the tank was discretised using sixteen five-noded and four-noded solid elements. Twenty contact elements were used to model the sliding boundary condition at the fluid-structure interface. In both references, a modified N-S component of the 1940 El Centro acceleration time history, with a maximum acceleration of 0.5g, was applied at the tank base which was assumed rigid. Plots showing the free surface displacement response were presented in both studies. Figure 5.12 shows the fluid displacement at the free surface, assuming the tank to be rigid, along with the results reported in the above references. Reasonable agreement between all sets of results may be seen. The deviation between the peak values produced in this study and those reported by Chelghoum (1986) may be attributed to the different time step size and fluid element formulation. In the present study an implicit Newmark integration scheme is used with a time step of magnitude equal to 0.02 seconds, while Chelghoum (1986) used a time step of 0.01 seconds. In addition, Chelghoum (1986) used Lagrangian sliding and fluid elements while the Mixed-Eulerian-Lagrangian formulation is used in this study.

To study the effects of the flexibility of the vertical tank wall on the response, Young's modulus of the wall was varied, and a total of three cases were considered. Table 5.2 shows the different material constants used in all the analyses.

Figure 5.13 shows the tank displacement at node 19 for Cases B and C, while Figure 5.14 shows the fluid displacements at nodes 55, 81, 107 and 133 on the free surface. It can be seen that, as the flexibility of the tank wall increases, both the fluid displacement at the free surface and the structural displacement increase. Table 5.3 shows the position and magnitude of the maximum wave height for all three cases.

Material Property	ρ_f lb - sec ² /in ⁴	β_f lb/in ²	ρ_s lb/in ²	β_s lb - sec ² /in ⁴
Flexibility				
Rigid Tank (Case A)	9.34E-5	1.45E05	-	-
Case B	9.34E-5	1.45E05	7.324E-4	3.0E08
Case C	9.34E-5	1.45E05	7.324E-4	1.0E07

Table 5.2 Material Properties of a Tank Fluid system

Flexibility	t max sec	d max in	t min sec	d min in
Rigid Tank (Case A)	1.82	11.821	3.76	-35.513
Case B	2.4	16.93	3.76	-47.366
Case C	2.58	21.098	4.36	-58.909

Table 5.3 Maximum / Minimum Fluid Displacement

Figure 5.15 shows the free surface sloshing response at ten points in time, where it can be seen that the free surface motion is anti-symmetric with respect to the centre line.

Plots of the horizontal stress developed in the tank walls at elements 1, 10 and 18 are shown in Figure 5.16, while plots of the fluid pressure variation at elements 4, 5, 6 and 7 are shown in Figure 5.17. The relationship between the base acceleration input and the total pressure may be examined by comparing Figure 5.17 with Figure 5.14. At the bottom of the tank (element 4 in the fluid mesh), the total pressure is dominated by the acceleration input, while at the top of the tank (element 7 in the fluid mesh), the pressure is dominated by sloshing behaviour. The maximum fluid pressure for each case is shown in Table 5.4, where it can be seen that the fluid pressure across the flexible tank is much higher than that of the rigid tank case. It should be noted that the fluid pressures do not always increase as the flexibility of the tank increases; in fact the highest fluid pressures occurs for the intermediate Case B. This may be due to the fact that the excitation frequency of the earthquake loading is closer to the natural frequency of the fluid-structure system of Case B rather than to that corresponding to Case C.

Element Number	4	5	6	7
Flexibility				
Rigid Tank (Case A)	-5.364	-4.855	6.189	2.729
Case B	-8.96	-14.12	11.31	19.19
Case C	-9.867	-5.976	9.836	-7.072

Table 5.4 Maximum / Minimum Fluid Pressure (lb / in²)

It can be seen that negative fluid pressure is occurring in the fluid medium due to the linear constitutive equation of the fluid employed in this study. Both Chelghoum (1986) and Ma et al (1982) used a similar constitutive relationship. However, neither presented fluid pressure plots for the problem under consideration. Towards an understanding of the effect of cavitation on the response of water tanks subjected to earthquake loading, it is envisaged that the development, implementation and use of a bi-linear constitutive equation for the fluid is necessary.

5.3 DYNAMIC RESPONSE OF SUBMERGED CYLINDERS WITH EXTERNAL APPENDAGES

In this example, the Plane Wave Approximation is used as a non-radiating boundary on the surface of submerged structures subjected to external loading. As mentioned in Chapter Four, this approximation assumes that the outgoing waves propagate to infinity with no additional reflections. However, if the Plane Wave Approximation is applied directly on the wet surface of submerged structures with appendages, multiple reflections may occur. The purpose of this section is to test the accuracy and the applicability of the Plane Wave Approximation in such situations, where due to the limitations of the proposed formulation, multiple reflections occurring between the submerged structure and the external appendages are ignored. A practical application of this category is the transient analysis of the hull of a submarine with externally attached appendages, subjected to external shock loading.

The problem statement, boundary conditions as well as the material and geometric properties are shown in Figures 5.18, where a submerged cylinder with an externally attached plate-shaped structure is subjected to an incident step pressure wave of magnitude equal to p_c^2 . The material properties are chosen to approximate those of a steel cylinder in water. In order to test the accuracy of the Plane Wave Approximation, a total of four runs are carried out for various appendage lengths and

thicknesses. The model configuration corresponding to the various test cases mentioned above are given in Table 5.5.

Idealisation Flexibility	Shell	plate	Infinite Fluid Medium
Case A	Twelve eight-noded solid elements	-	Twelve three-noded infinite fluid elements
Case B	Twelve eight-noded solid elements	Three four-noded plane strain elements	Eleven three-noded and eight two-noded infinite fluid elements
Case C	Twelve eight-noded solid elements	Seven four-noded plane strain elements	Eleven three-noded and sixteen two-noded infinite fluid elements
Case D	Twelve eight-noded solid elements	Seven eight-noded plane strain elements	Twenty five three-noded and two two-noded infinite fluid elements

Table 5.5 Model Configuration

Case A corresponds to a cylinder without external appendages, whilst Cases B, C and D consider appendages of various lengths and thicknesses. The finite element model for Cases B and C is shown in Figure 5.18a, while that corresponding to Case D is given in Figure 5.18b. The finite element model for Case A is identical to the one used for the above cases in the absence of the external appendage.

The geometric properties corresponding to the various test cases are given in Table 5.6, where l and t_p (the length and thickness of the appendage) are defined in Figures 5.18.

Figures 5.19 to 5.21 show the displacement and velocity response of the cylinder and the plate for the three Cases B, C and D. These are compared with those reported in Section 4.9.4 for a submerged cylinder, with no appendage (Case A), subjected to a step pressure wave. In all the plots, the velocity is non-dimensionalised with respect to the speed of sound in the fluid medium. In Figure 5.19, the horizontal

displacement and non-dimensional velocity of the cylinder at $\theta = 0^\circ$, 97.5° and 180° are shown for all cases. Comparing the results obtained for Case A with those for Cases B, C and D it can be seen that both the displacement and velocity of the cylinder are not drastically affected by the presence of the external appendage. This may be explained by the fact that the force transmitted by the plate to the shell does not cause significant horizontal response of the cylinder and because waves reflected off the appendage are not considered to reload the cylinder. Table 5.7 shows the maximum displacement and non-dimensional velocity for all the cases considered.

Geometric Property Case	Length of Plate (in) l	Thickness of Plate (in) t_p	x position of right edge of Plate (in)
Case A	-	-	-
Case B	1/2	2.1006	0
Case C	1	2.1006	0
Case D	1	1.0503	1.0503

Table 5.6 Geometric Properties

Kinematic Property	Horizontal Displacement (in)			Horizontal Non-Dimensional Velocity		
θ	0°	97.5°	180°	0°	97.5°	180°
Case A	-46.122	-37.403	-29.19	-1.748	-1.21	-1.102
Case B	-42.568	-33.211	-25.770	-1.747	-0.976	-0.975
Case C	-41.676	-30.842	-25.033	-1.747	-0.974	-1.019
Case D	-39.398	-28.086	-23.495	-1.742	-0.919	-0.98

Table 5.7 Maximum / Minimum Displacement and Velocity of Cylinder

Figure 5.20 shows the displacement and non-dimensional velocity of the plate at its tip, right edge and left edge, while Figure 5.21 shows the vertical velocity and displacement of the plate tip and centre right edge for all runs. Tables 5.8 and 5.9 show the maximum displacement and velocity for all the cases considered.

A qualitatively similar study was carried out by O'Reagan and DiMaggio (1990),

where the structure was discretised by finite differences and the Plane Wave Approximation was used to model the non-radiating boundary. Similar conclusions regarding the shell displacement at $\theta = 0^\circ$ and the tip displacement of the plate were drawn, however it is not possible to compare both sets of results quantitatively since the geometric properties of the cylinder and external appendage were not given.

Kinematic Property	Horizontal Displacement (in)			Horizontal Non-Dimensional Velocity		
	Centre Left	Centre Right	Tip	Centre Left	Centre Right	Tip
Case A	-	-	-	-	-	-
Case B	-9.574	-8.258	-	-0.522	-0.496	-
Case C	-12.285	-11.276	-	-0.430	-0.399	-
Case D	-15.219	-15.207	-2.415	-0.522	-0.522	0.276

Table 5.8 Maximum / Minimum Horizontal Displacement & Velocity of Plate

Kinematic Property	Vertical Displacement (in)		Vertical Non-Dimensional Velocity	
	Centre Right	Tip	Centre Right	Tip
Case A	-	-	-	-
Case B	-3.518	-	-0.142	-
Case C	-3.008	-	-0.119	-
Case D	-3.128	1.304	-0.108	0.108

Table 5.9 Maximum / Minimum Vertical Displacement & Velocity of Plate

In this study all analyses were carried out with the non-radiating boundary placed directly on the surface of the submerged structure. However, O'Reagan and DiMaggio showed that by discretising the fluid medium in the vicinity of the cylinder-plate system by finite differences and applying the non-radiating boundary at a convex surface surrounding the structure, a qualitatively different set of results is obtained, especially when the length of the plate approaches the characteristic length of the cylinder. The discrepancy between these results is attributed to the multiple reflections occurring between the plate and the cylinder, which cannot be accounted

for when the Plane Wave Approximation is applied at the surface of the plate-cylinder system.

In view of the results obtained from this study and those reported in the above mentioned reference, it is possible to make the following observations:

- When the Plane Wave Approximation is applied on the structure surface, the results obtained from this study are qualitatively similar to those obtained by O'Reagan and DiMaggio.
- The results mentioned above differ significantly from those reported by O'Reagan and DiMaggio, when the non-radiating boundary is applied at a convex fluid surface circumscribing the structure. This is attributed to the occurrence of multiple reflections which cannot be accounted for if the non-radiating boundary is placed directly on the surface of submerged structures.
- To draw a final conclusion regarding the comments mentioned above, further validation of the fluid finite elements to situations where the fluid is surrounding the structure is necessary.

5.4 TRANSIENT RESPONSE OF TWO FLUID-COUPLED CYLINDRICAL SHELLS SUBJECTED TO AN INCIDENT PRESSURE WAVE

To gain physical insight into the underwater explosion response of fluid-structure coupled systems, such as the double hull section of a submarine, the transient response of such systems was investigated by Zilliacus (1983). The case of two three-dimensional concentric elastic cylinders of infinite length and subjected to an incident step wave was considered. In his paper, Zilliacus presented plots of the velocity response of the fluid coupled cylindrical system. Singh et al (1992), studied the response of a three-dimensional coupled problem consisting of two adjacent cylindrical shells submerged in an infinite fluid medium and subjected to both surface and line waves of magnitude equal to 100 psi. Plots showing the velocity, pressure and moment variation with time were presented. Finally, the authors concluded by outlining the extension of the method for the safety analysis of submerged tubes.

In this section, the three-dimensional problem studied by Zilliacus (1983) will be analysed as a two-dimensional problem with the aim of demonstrating the applicability of the proposed method and testing the coupled structure / near-field

fluid / far-field fluid formulations. The problem under consideration applies for the case in which both shells are made of steel and both fluids are water. The incident wave is a step pressure wave with an initial pressure jump equivalent to $p c^2$. The finite element model, geometric and loading configurations are shown in Figure 5.22, where each cylindrical shell is modelled with twelve four-noded two-dimensional plane strain elements, while the contained fluid is modelled with twelve four-noded two-dimensional elements. The infinite fluid medium is modelled using twelve two-noded Plane Wave Approximation elements.

To study the effect of the Young's modulus of the cylinder on the general response two cases are considered. Table 5.10 shows the material properties for these cases. It should be noted that the material properties of Case A corresponds to those used by Zilliacus et al (1983).

Material Property	ρ_f lb - sec ² /in ⁴	β_f lb/in ²	ρ_s lb/in ²	β_s lb - sec ² /in ⁴
Case				
A	9.3E-5	1.45E05	7.28E-4	3.0E06
B	9.3E-5	1.45E05	7.28E-4	3.0E08

**Table 5.10 Material Properties of a
Fluid Coupled Double-hulled Cylindrical System**

Figure 5.23 shows the non-dimensional radial velocity obtained from this study along with that reported in the above mentioned reference. In this plot and all subsequent plots, the velocity is normalised with respect to the speed of sound in the fluid, while the time is normalised with respect to c/a . From Figure 5.23, good agreement can be seen between all sets of results. The difference between both sets of values may be attributed to three factors: first, four-noded linear elements are used in this study to model the structure and the contained fluid, while Zilliacus used quadratic elements in his model. Second, the infinite fluid medium is modelled in this study by linear two-noded Plane Wave Approximation elements, while Zilliacus used a quadratic Doubly Asymptotic Approximation formulation. Finally, two-dimensional analysis was used in this study while Zilliacus carried out a three-dimensional analysis.

Figures 5.24a and 5.24c show the horizontal displacement of the inner and outer cylinders for Cases A and B at $\theta = 0^\circ$ and 180° respectively, while Figure 5.24b shows the vertical displacement of the inner and outer cylinders at $\theta = 90^\circ$. It can be

seen that as the stiffnesses of both cylinders increase, the point at which the inner cylinder displaces horizontally more than the outer shifts backward in time. This may be due to a larger proportion of the fluid added mass contributing to the outer cylinder effective mass. Figures 5.25 shows the radial velocity of the fluid coupled cylinder system for both cases, while Figure 5.26 shows the horizontal pressure at the inner and outer cylinders for Cases A and B at $\theta = 0^\circ, 90^\circ$ and 180° .

The figures mentioned above are provided to be used for comparison with and validation of other numerical-based tools. However, to assess the effect of the outer cylinder flexibility, the presence of the outer cylinder and the fluid coupling medium on the general response of submerged cylinders an extensive parametric study is required.

5.5 CONCLUSION

Results have been presented from several studies of near-field and far-field fluid-structure systems. It has been shown that the developed near-field formulation can be successfully applied to the solution of dynamic transient problems, where material non-linear effects are present. In particular elasto-plastic and large sloshing response of pressure vessels and water tanks have been considered, and good agreement with solutions reported in other studies has been obtained. The extension of the method to three-dimensional fluid-structure systems remains to be done.

The far-field formulation has been applied for the linear analysis of submerged cylinders with external appendages. In addition, by considering the case of a double-hulled fluid coupled cylinder system, it has been demonstrated that the method can be applied to realistic two-dimensional problems. In general, further work is required in this area, particularly in the application of the proposed formulation to complex three-dimensional fluid-structure systems where structural, geometric and material non-linearities arise as well as fluid material non-linearity (cavitation). Furthermore, analytical and numerical work is required for the development of a database. Both of the above requirements are essential ingredients to further behavioural understanding of the response of submerged fluid-structure systems.

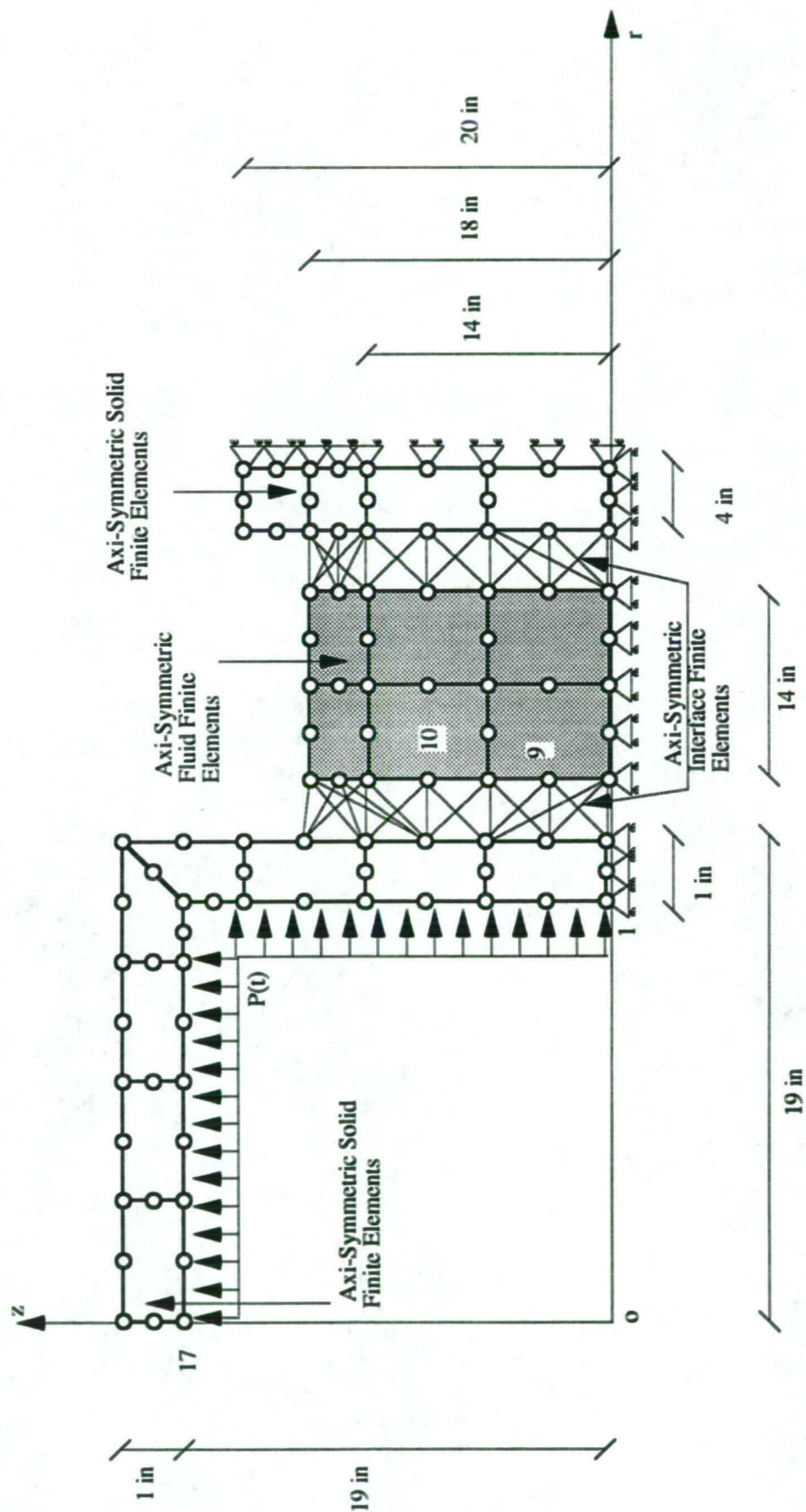
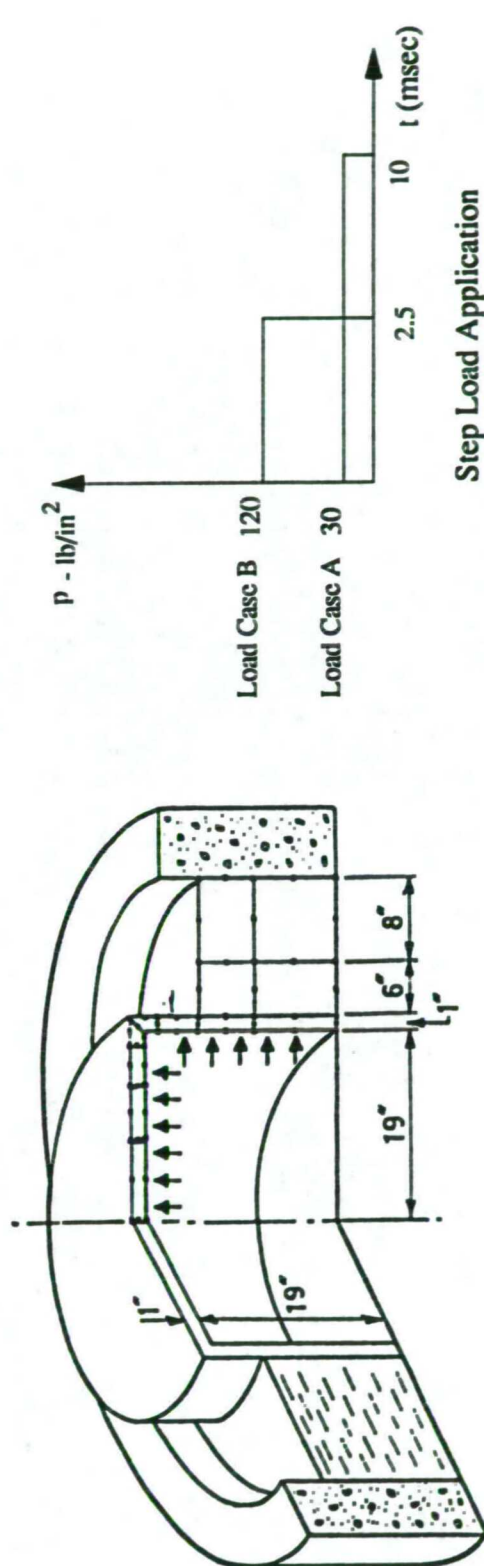


Figure 5.1 A Fluid-Filled Vessel Subjected to Step Pressure Loading - Finite Element Model and Geometric Properties



Three-Dimensional
Geometry of a Pressure Vessel

Structure

$$E = 30.0E+6 \text{ lb/in}^2$$

$$\nu = 0.3$$

$$\sigma_y = 36.0E+3 \text{ lb/in}^2$$

$$\rho = 0.259 \text{ E-3 lb-sec}^2/\text{in}^4$$

Fluid

$$\beta_f = 30.0E+6 \text{ lb/in}^2$$

$$\nu = 0.5$$

$$\rho_f = 0.33 \text{ E-3 lb-sec}^2/\text{in}^4$$

Figure 5.2 Fluid-Filled Pressure Vessel - Loading Conditions
, Material Properties and Three-Dimensional Geometry

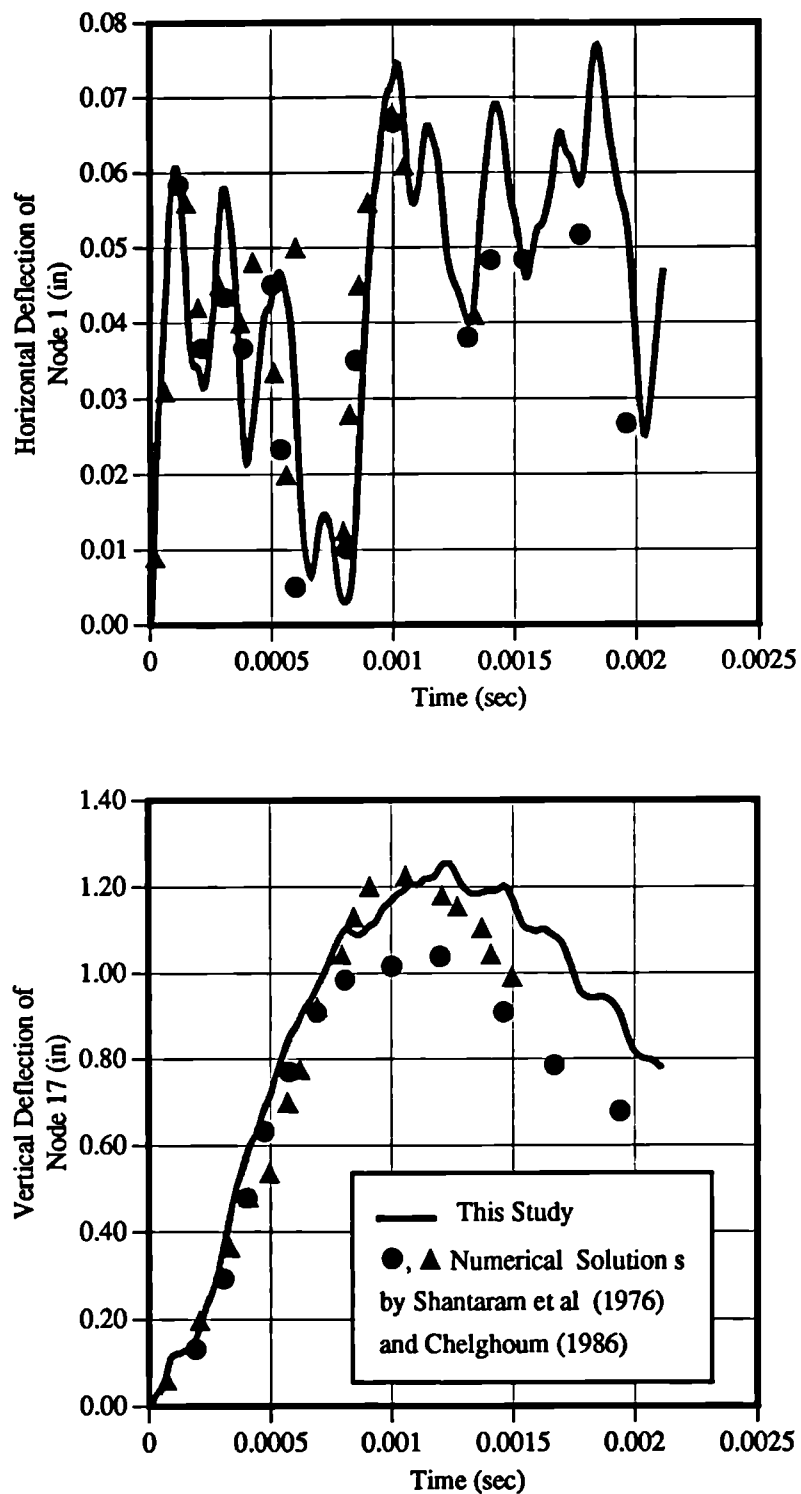


Figure 5.3 Elasto-Plastic Response of an Axi-Symmetric Pressure Vessel - Comparison with Numerical Work Reported by Shantaram et al (1976) and Chelghoum (1986)

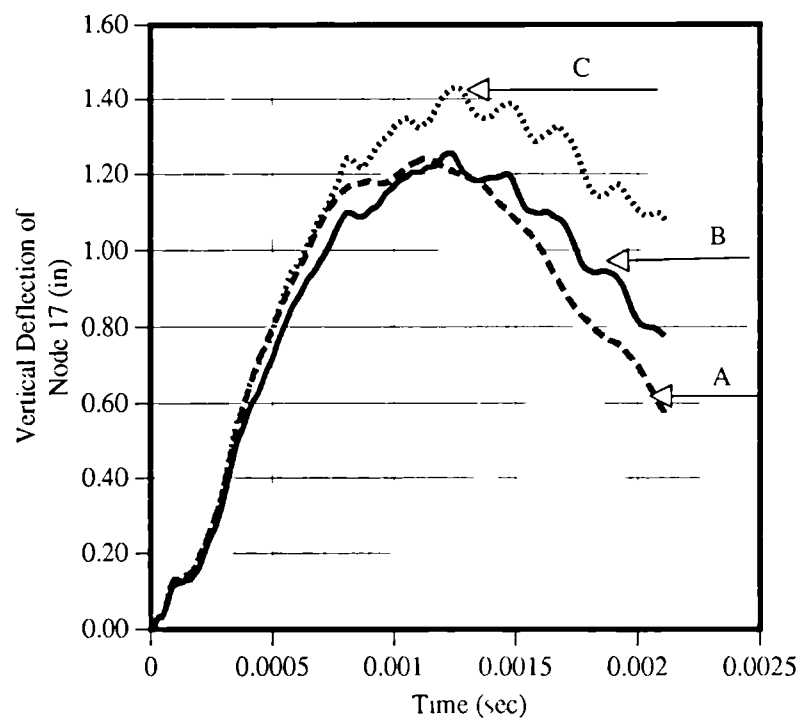
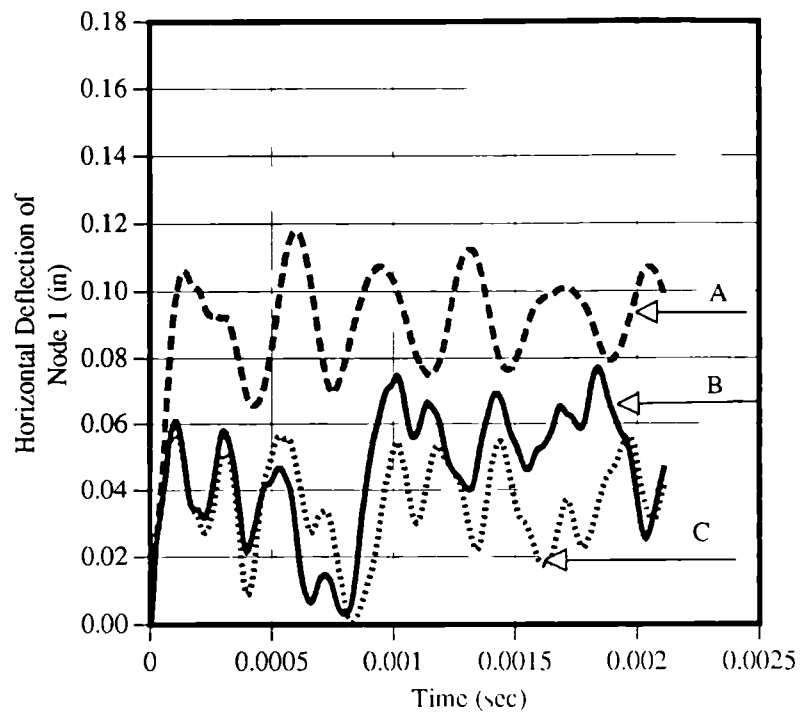


Figure 5.4 Elasto-Plastic Displacement Response of a Pressure Vessel - A: without Fluid, B: Fluid height = 14in, C: Fluid height = 18 in

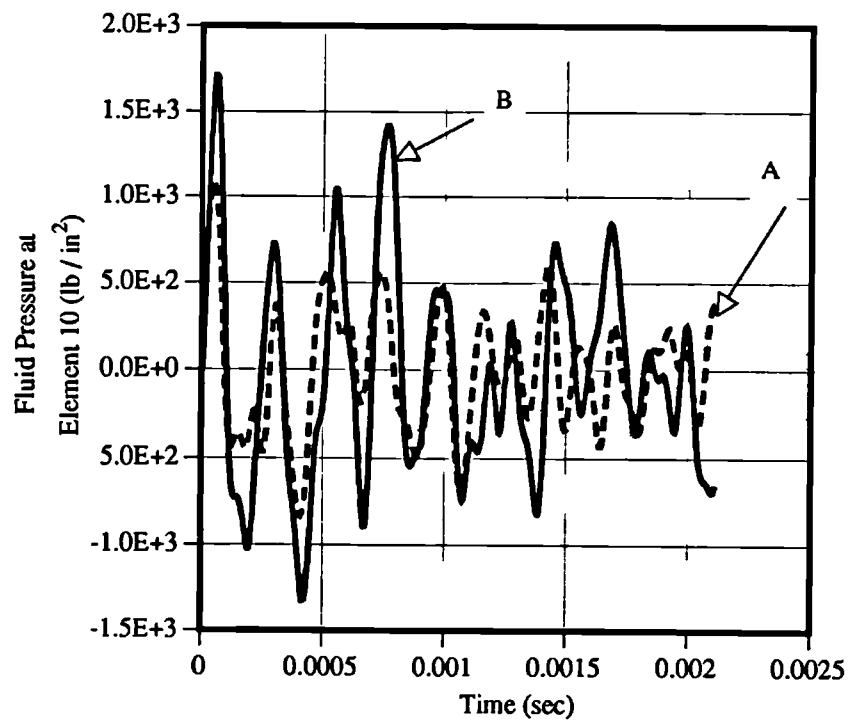
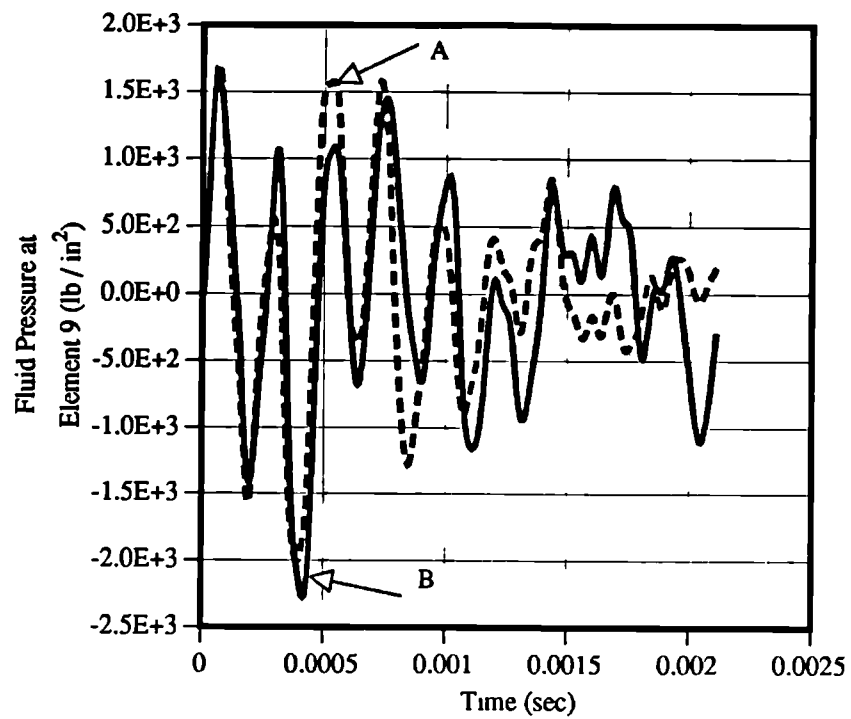


Figure 5.5 Elasto-Plastic Response of an Axi-Symmetric Pressure Vessel - Fluid Pressure in Elements 9 and 10 - A: Fluid Height = 14 in, B : Fluid Height = 18 in

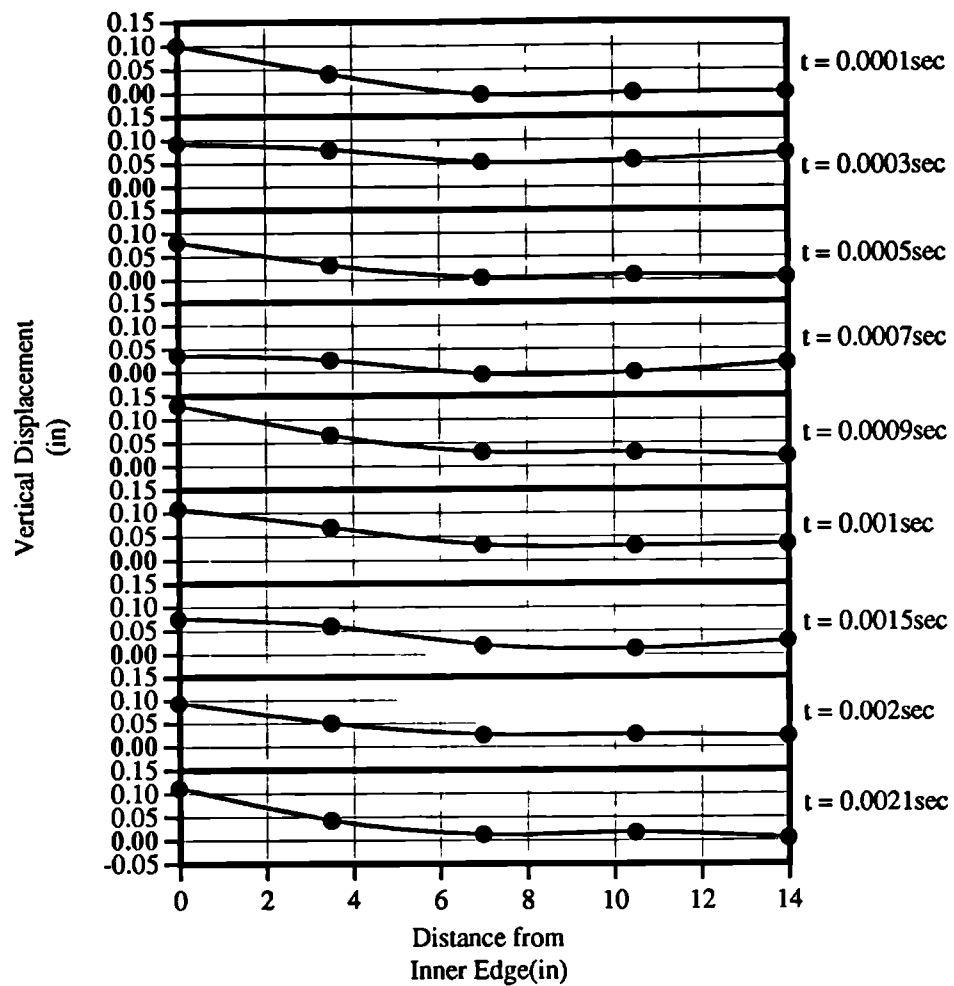


Figure 5.6 Elasto-Plastic Response of an Axi-Symmetric Pressure Vessel - Fluid Height = 14 in - Free Surface Profiles of Surrounding Fluid at Nine Discrete Times

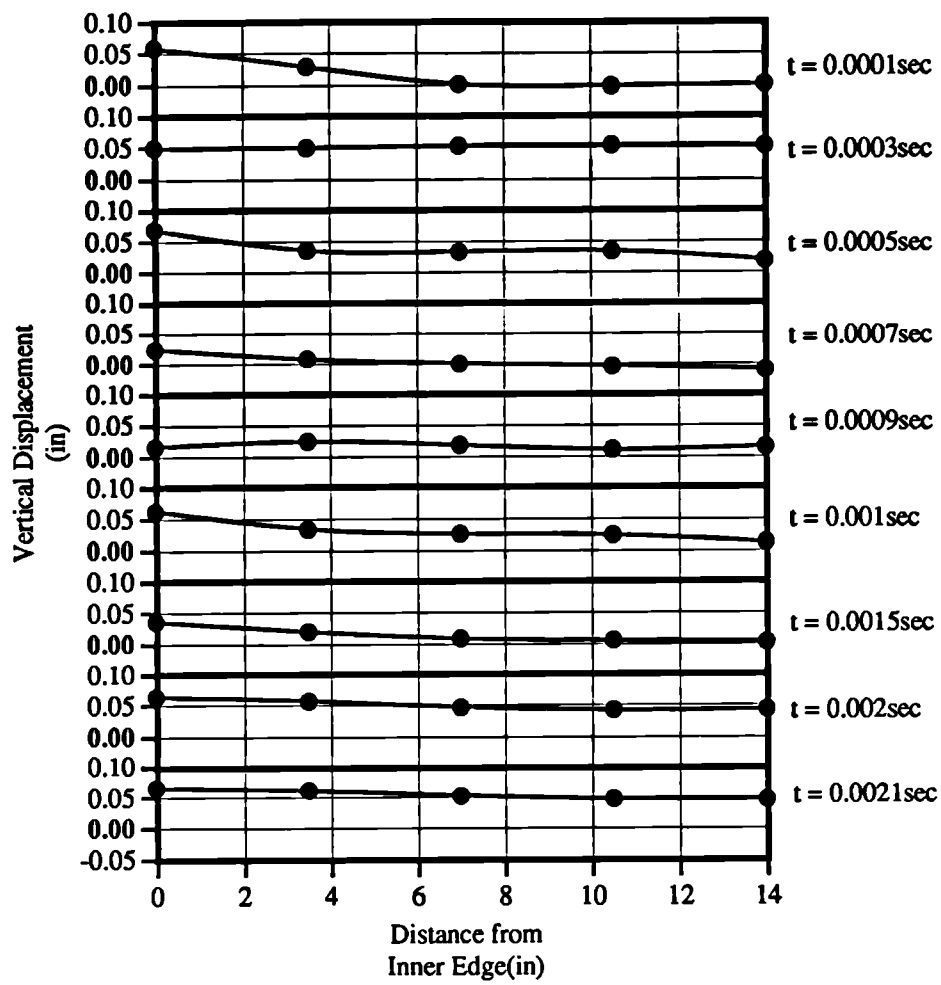


Figure 5.7 Elasto-Plastic Response of an Axi-Symmetric Pressure Vessel - Fluid Height = 18 in - Free Surface Profiles of Surrounding Fluid at Nine Discrete Times

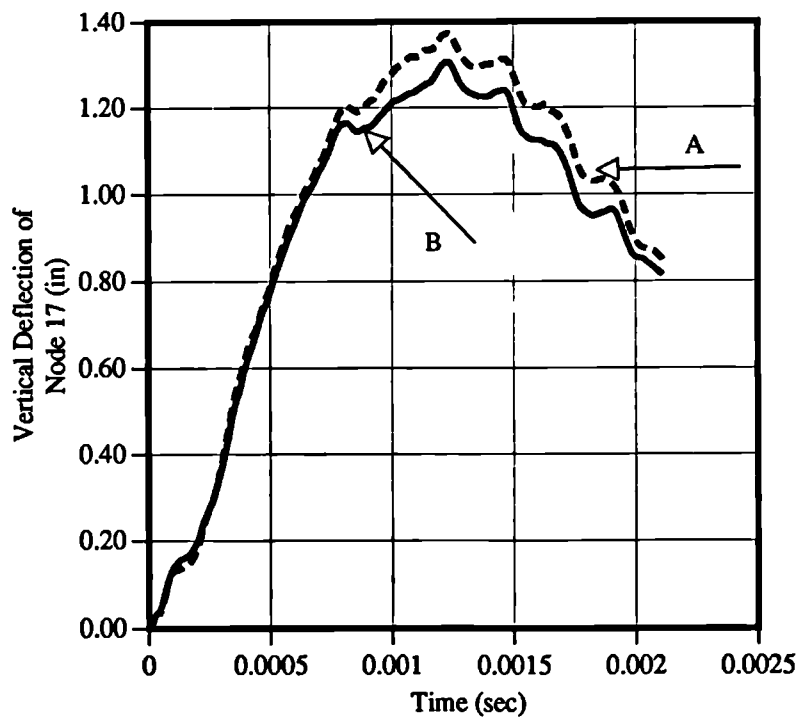
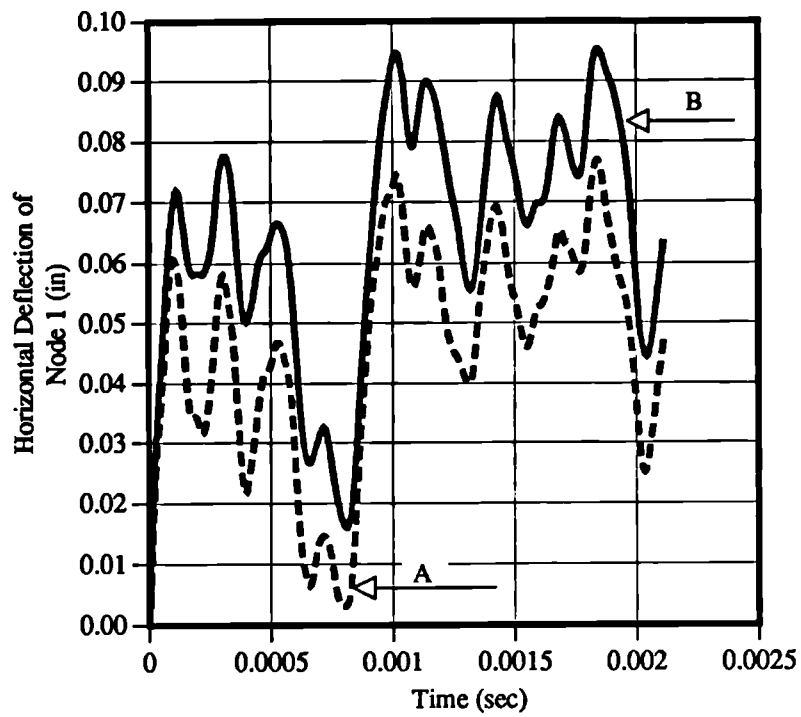


Figure 5.8 Elasto-Plastic Displacement of a Pressure Vessel - Effect of Load Pulse- A: Load Case A, B: Load Case B (See Figure 5.2)

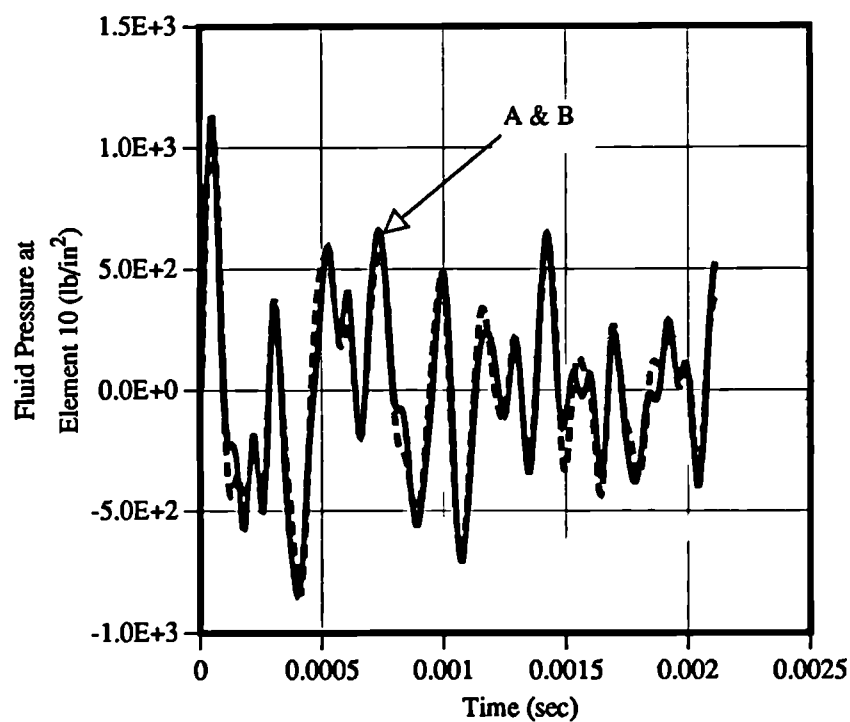
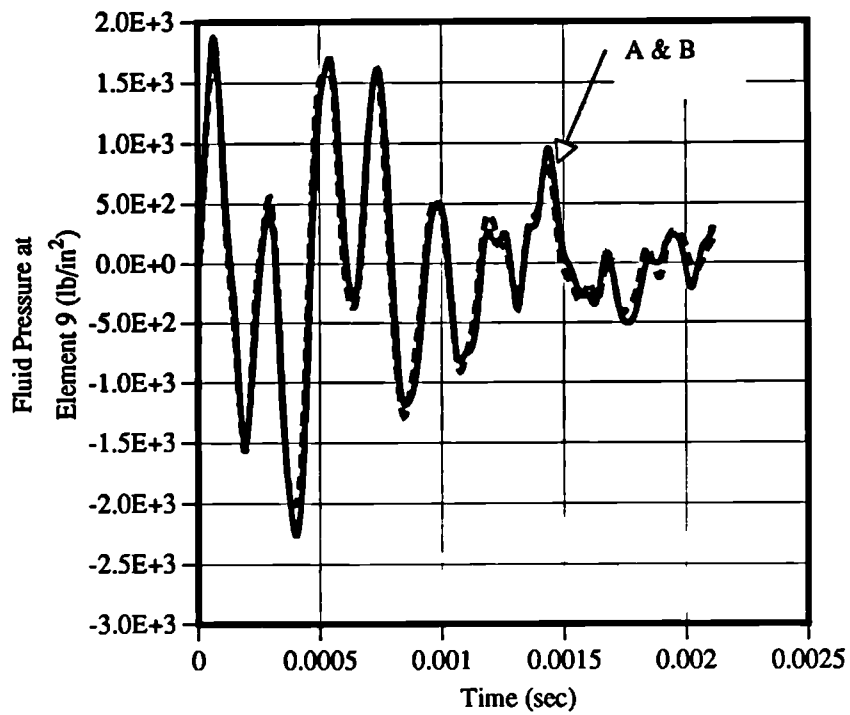


Figure 5.9 Elasto-Plastic Response of an Axi-Symmetric Pressure Vessel - Effect of Pulse on Fluid Pressure - A : Load Case A, B : Load Case B (See Figure 5.2)

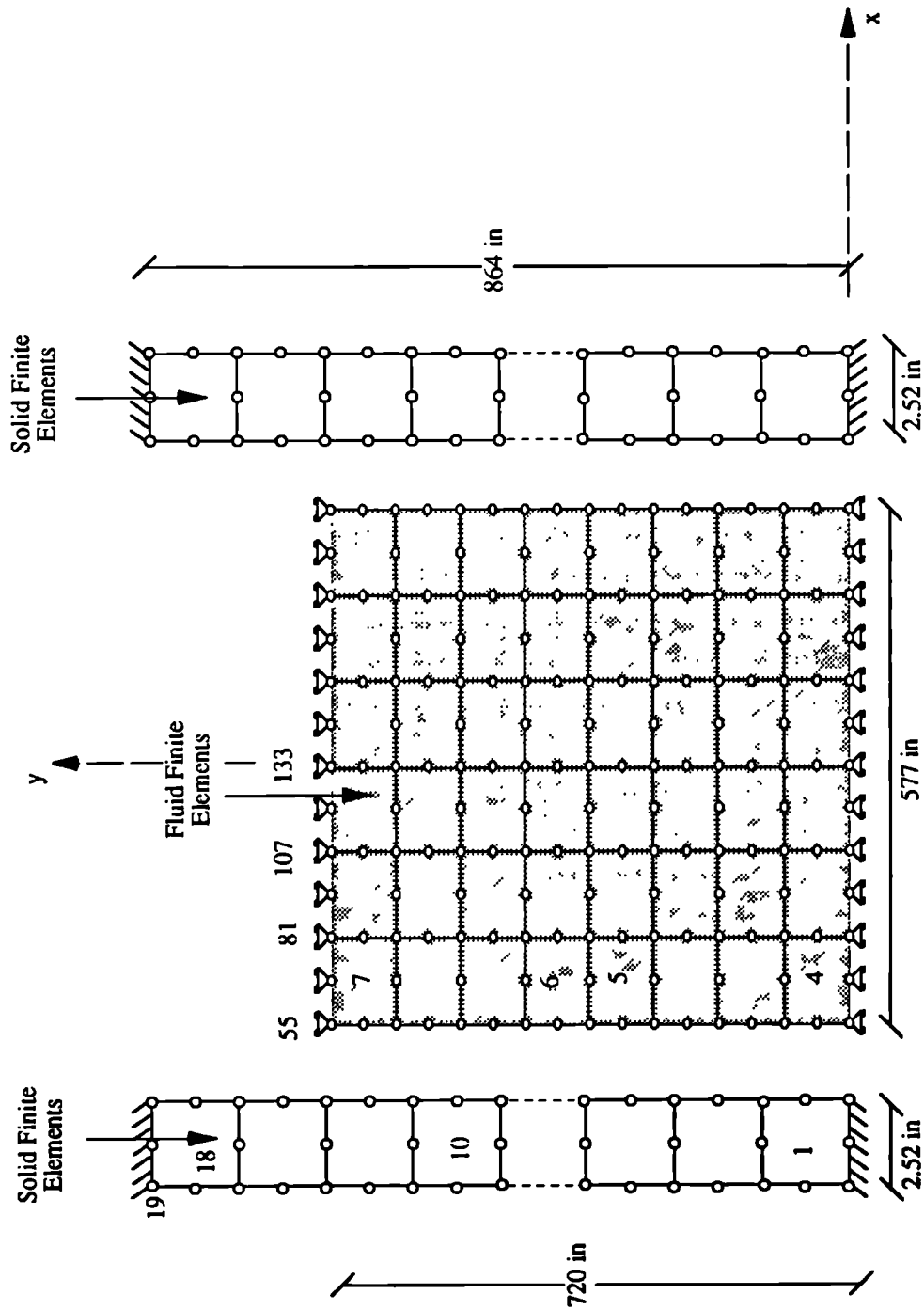


Figure 5.10 Water Tank Subjected to Earthquake Excitation - Finite Element Model and Geometric Properties

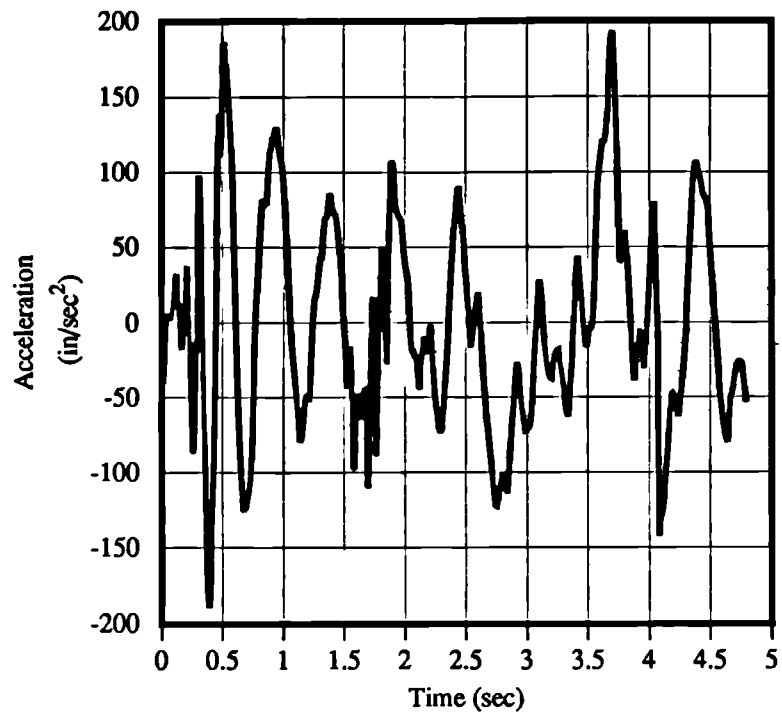


Figure 5.11 Modified N-S El-Centro Earthquake Acceleration Input

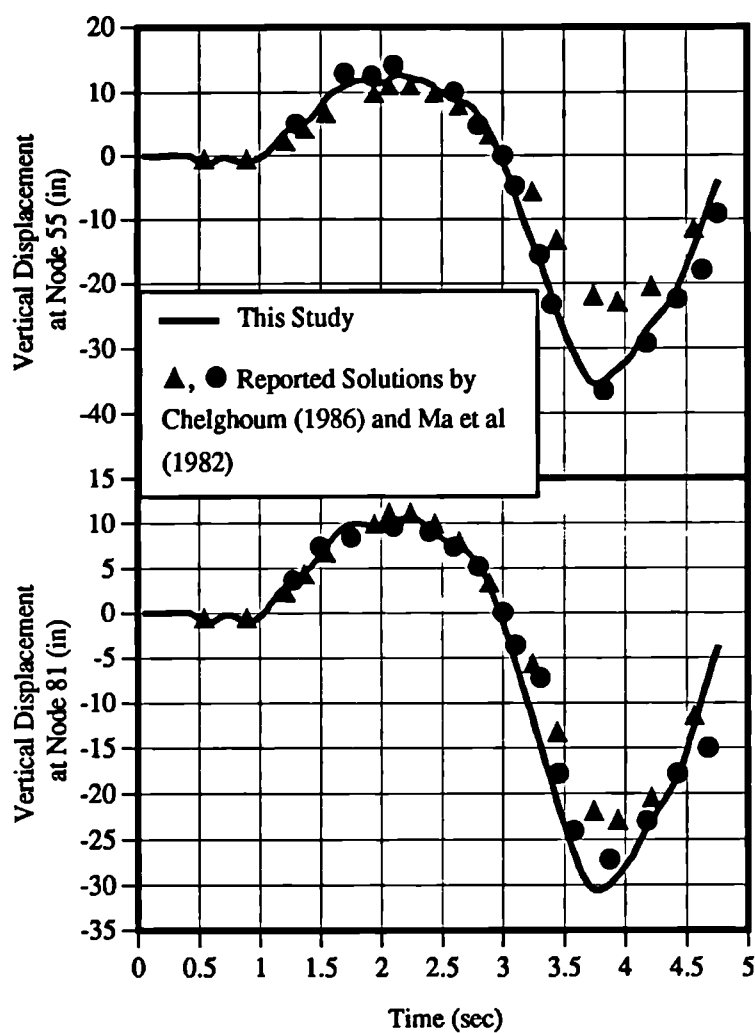


Figure 5.12 Sloshing Response of a Rigid Water Tank - Comparison of Free Surface Displacement with Other Numerical Work

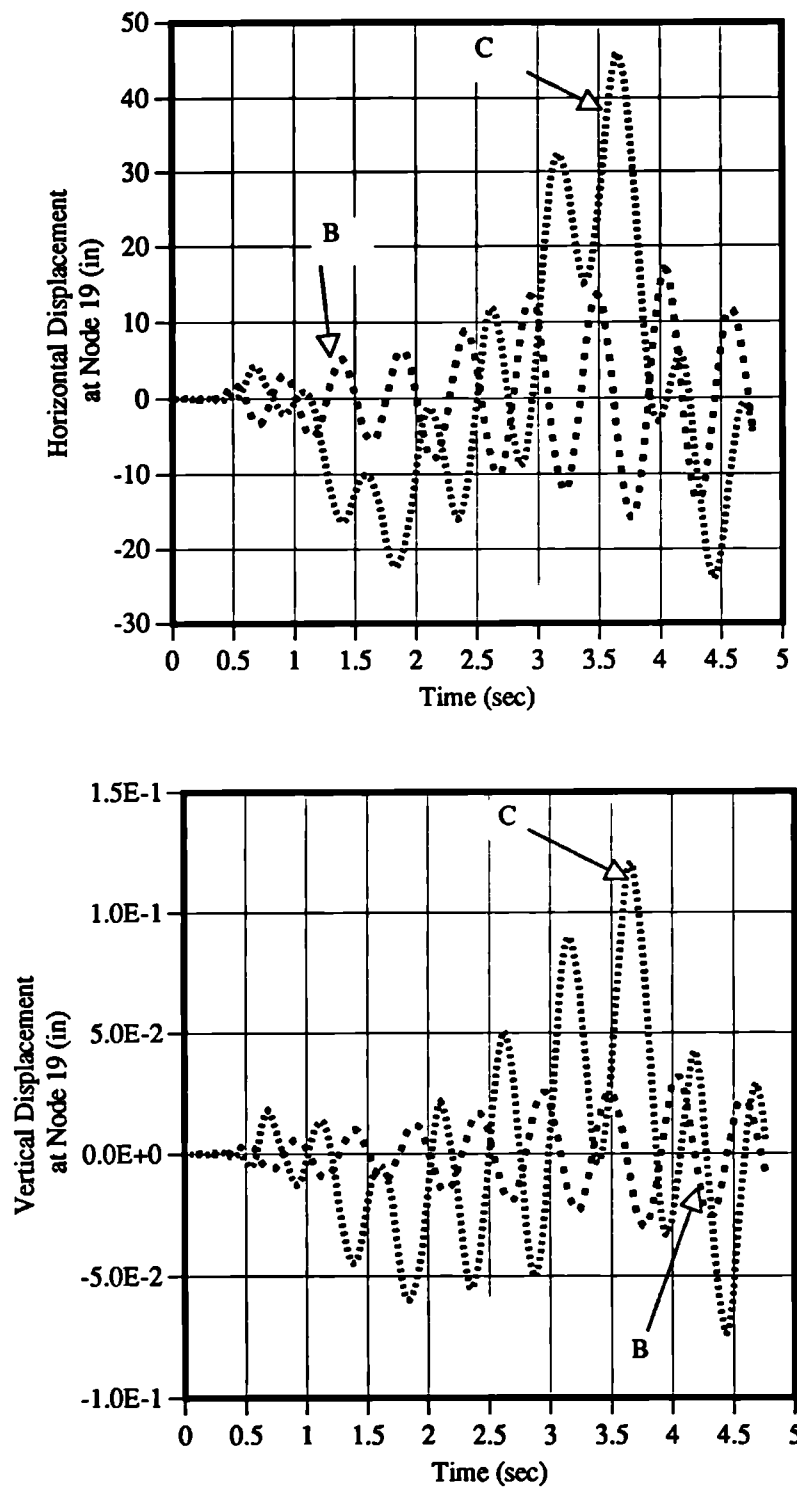


**Figure 5.13 Response of Tank Subjected to Earthquake
Excitation - Displacement of Tank at Node 19
for Cases B and C, (See Table 5.2)**

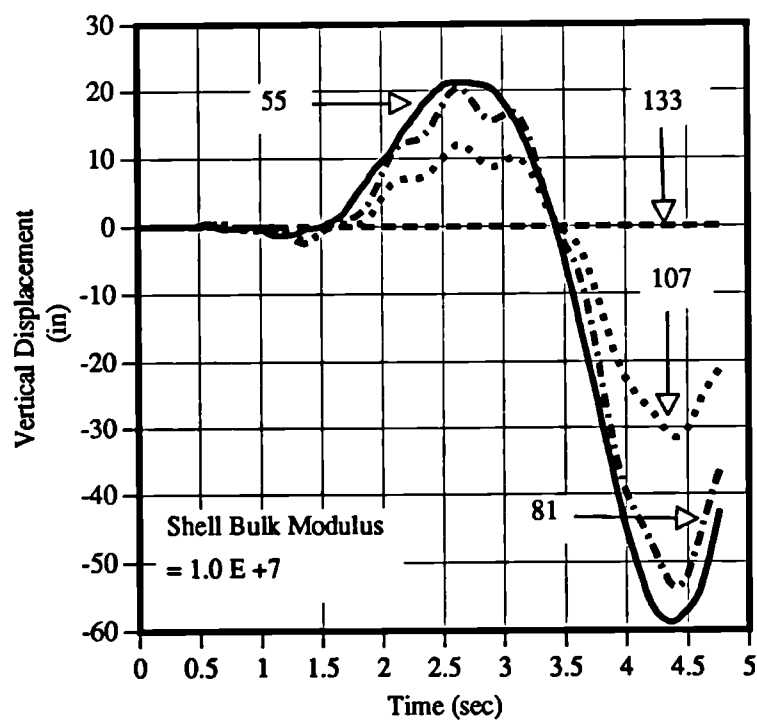
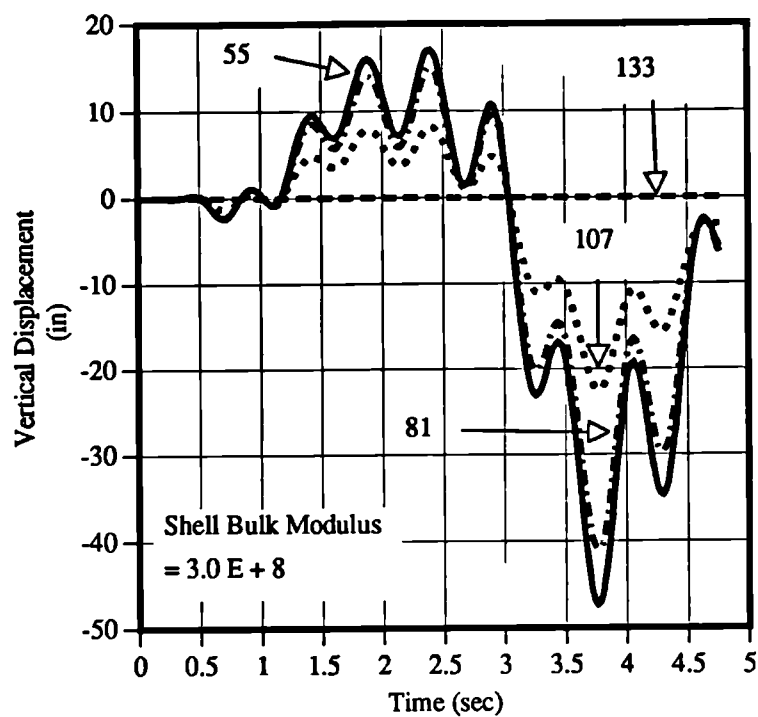


Figure 5.14 Response of a Tank Subjected to Earthquake Excitation - Free Surface Sloshing Response at Four Nodes (See Figure 5.10)

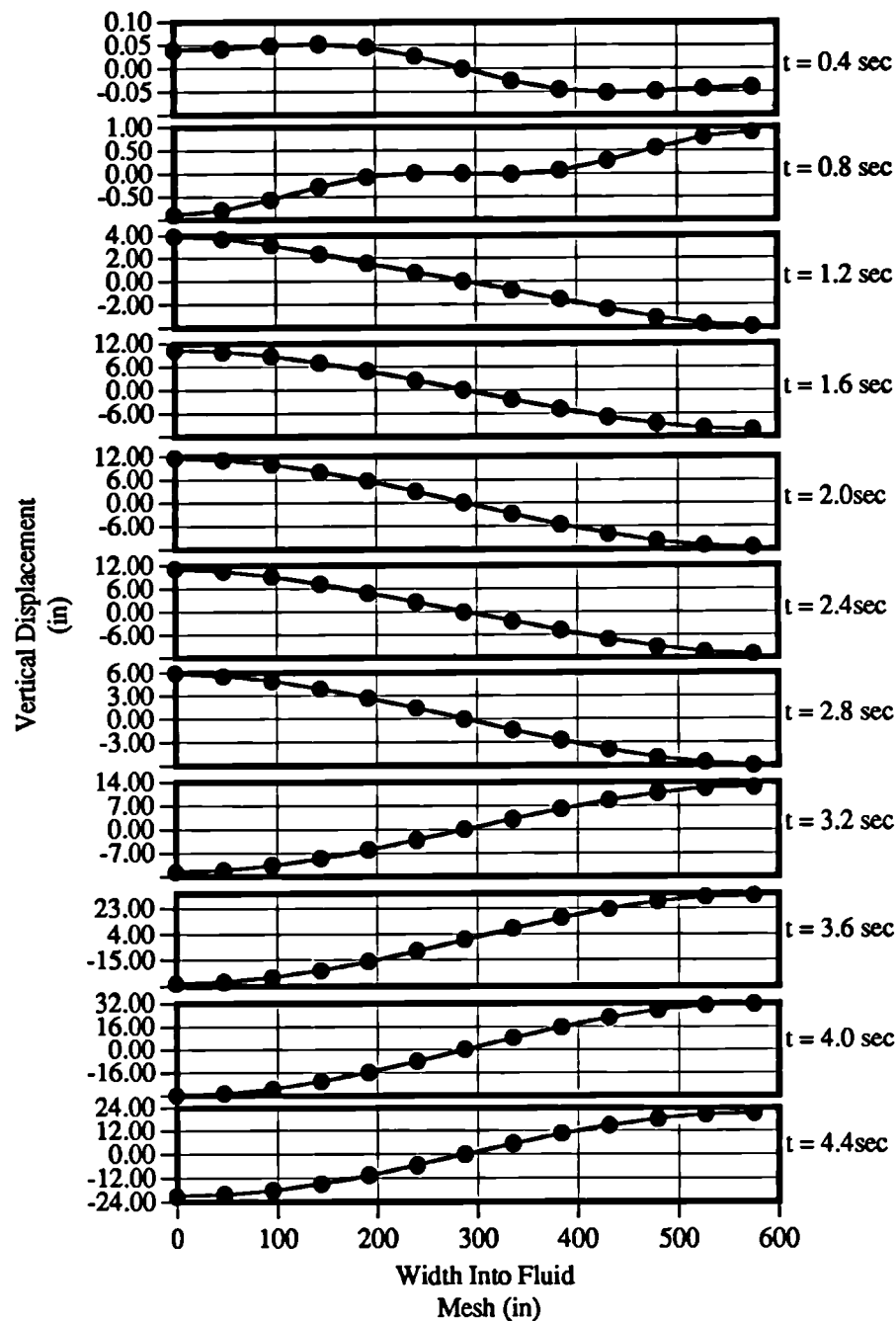


Figure 5.15a Response of a Tank Subjected to Earthquake Excitation - Free Surface Sloshing Response - Rigid Tank (Case A) (See Table 5.2)

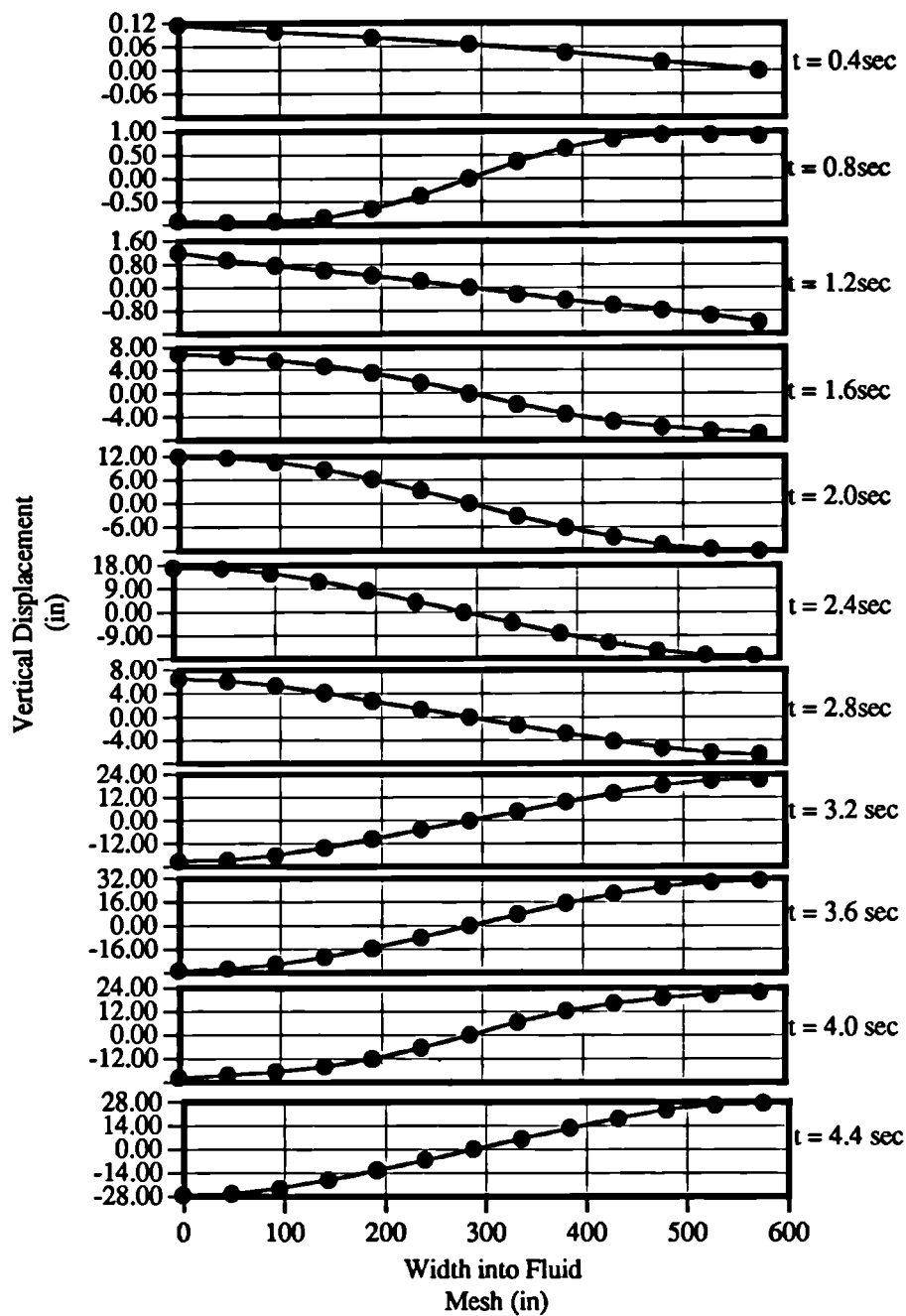


Figure 5.15b Response of a Tank Subjected to Earthquake Excitation - Free Surface Sloshing Response Case B (See Table 5.2)

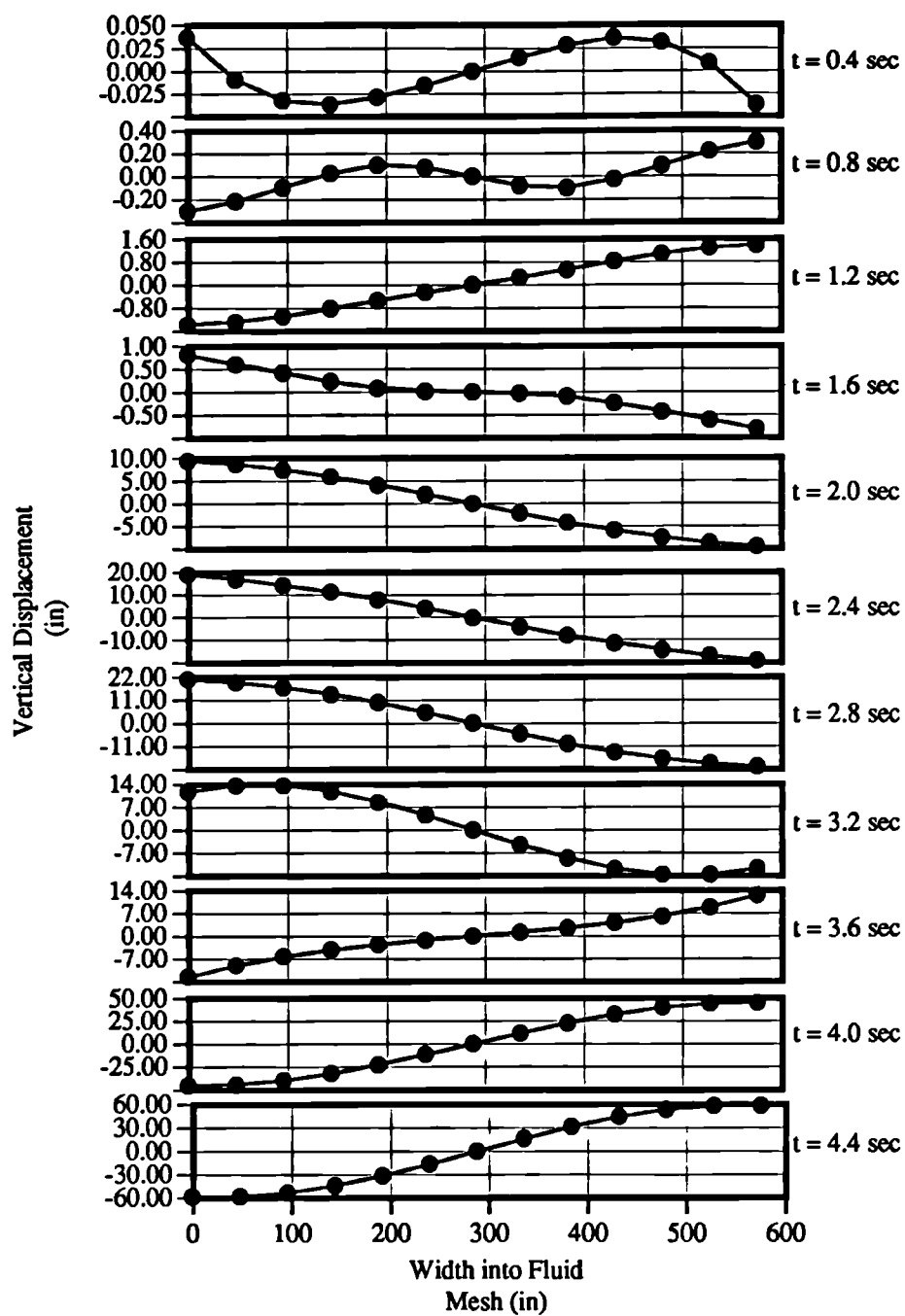


**Figure 5.15c Response of a Tank Subject to Earthquake
Excitation - Free Surface Sloshing Response -
Case C (See Table 5.2)**

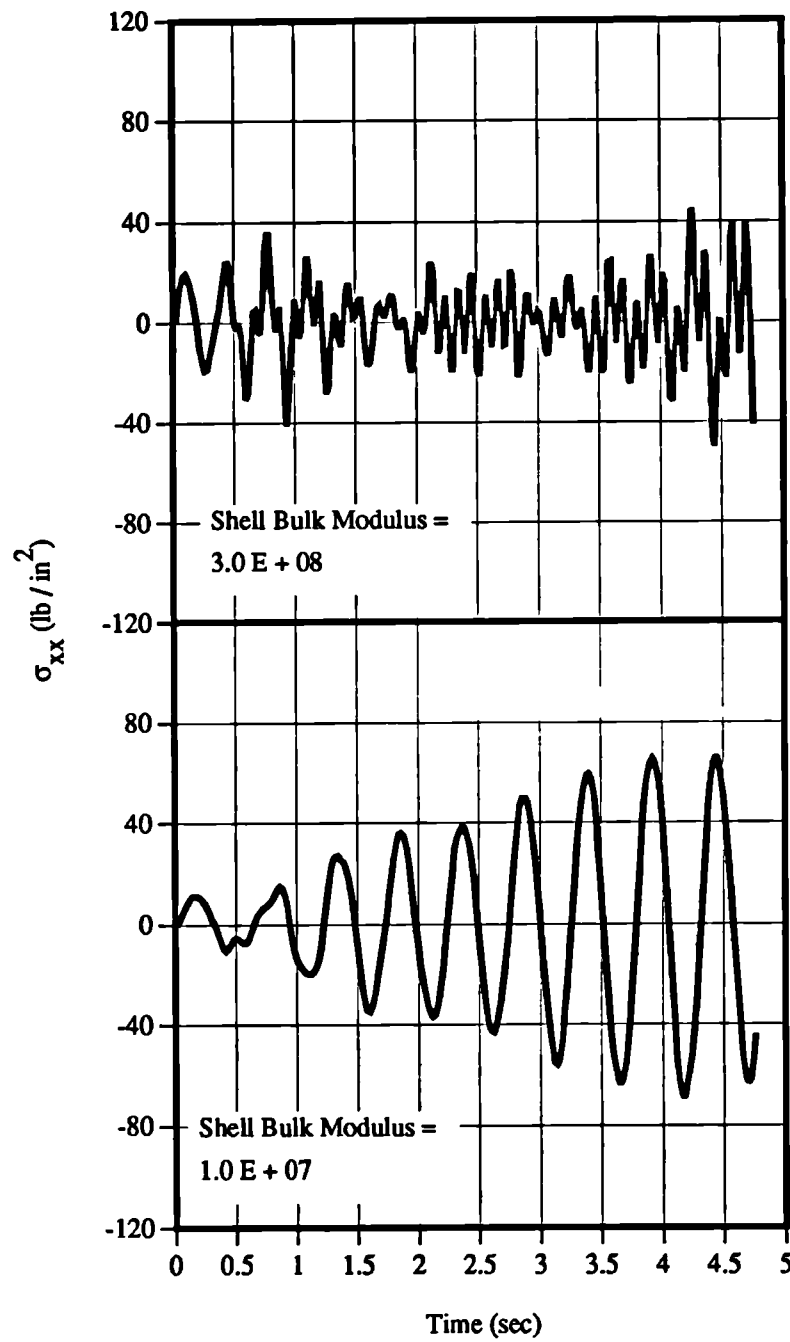


Figure 5.16a Response of a Tank Subjected to Earthquake Excitation - Total Stress at Element 1

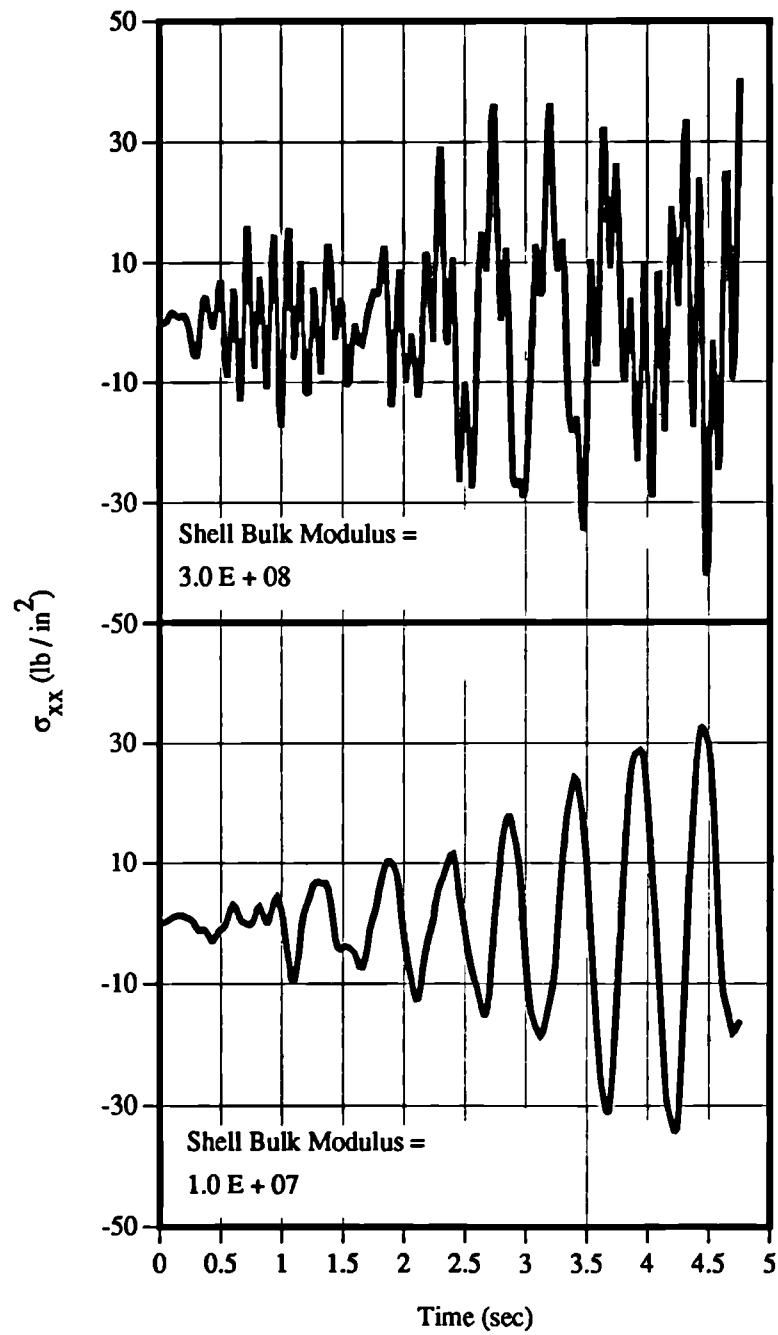


Figure 5.16b Response of a Tank Subjected to Earthquake Excitation - Total Stress at Element 10

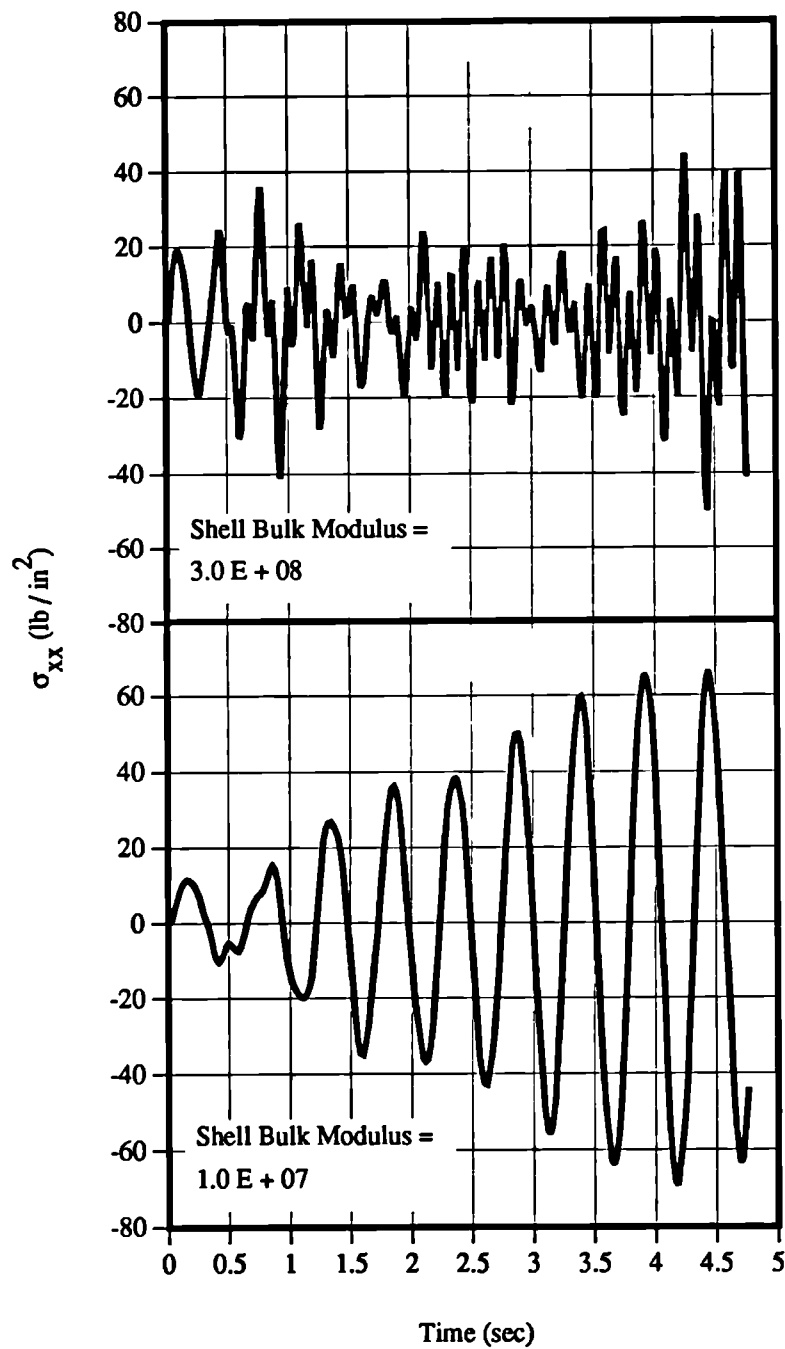


Figure 5.16c Response of a Tank Subjected to Earthquake Excitation - Total Stress at Element 18

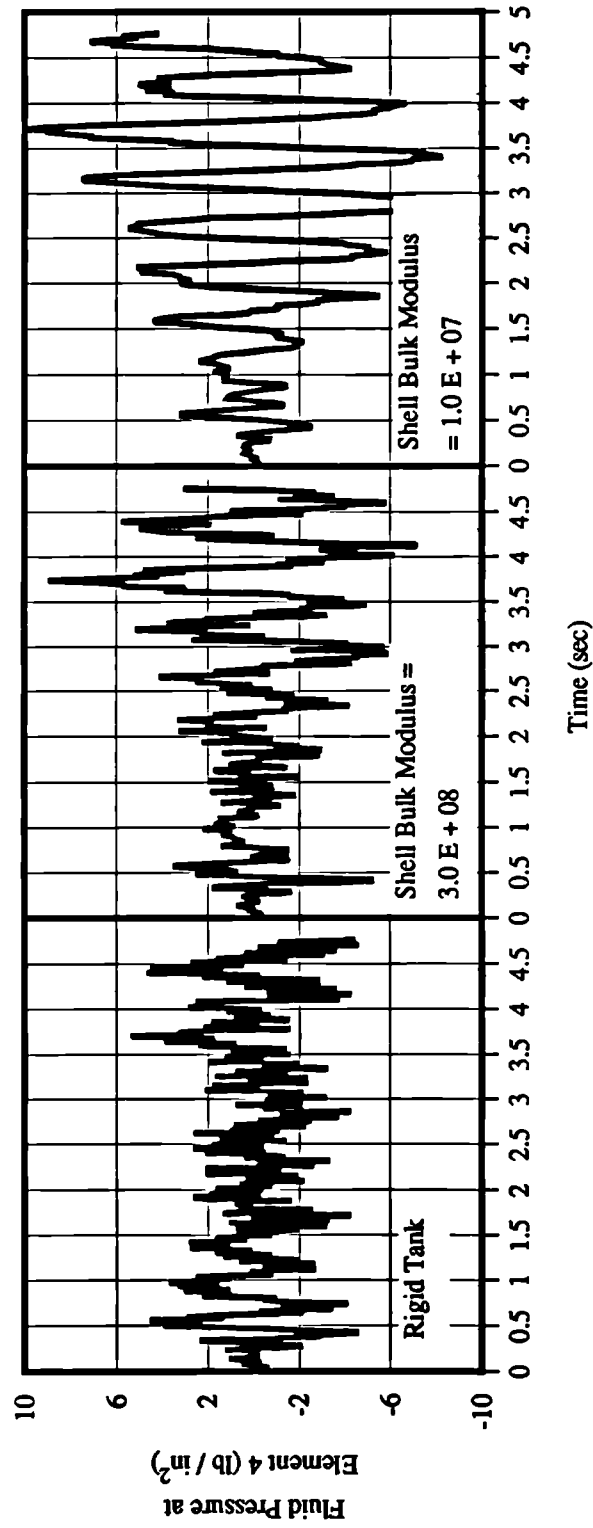


Figure 5.17a Response of a Tank Subjected to Earthquake Excitation - Fluid Pressure Variation at Element 4

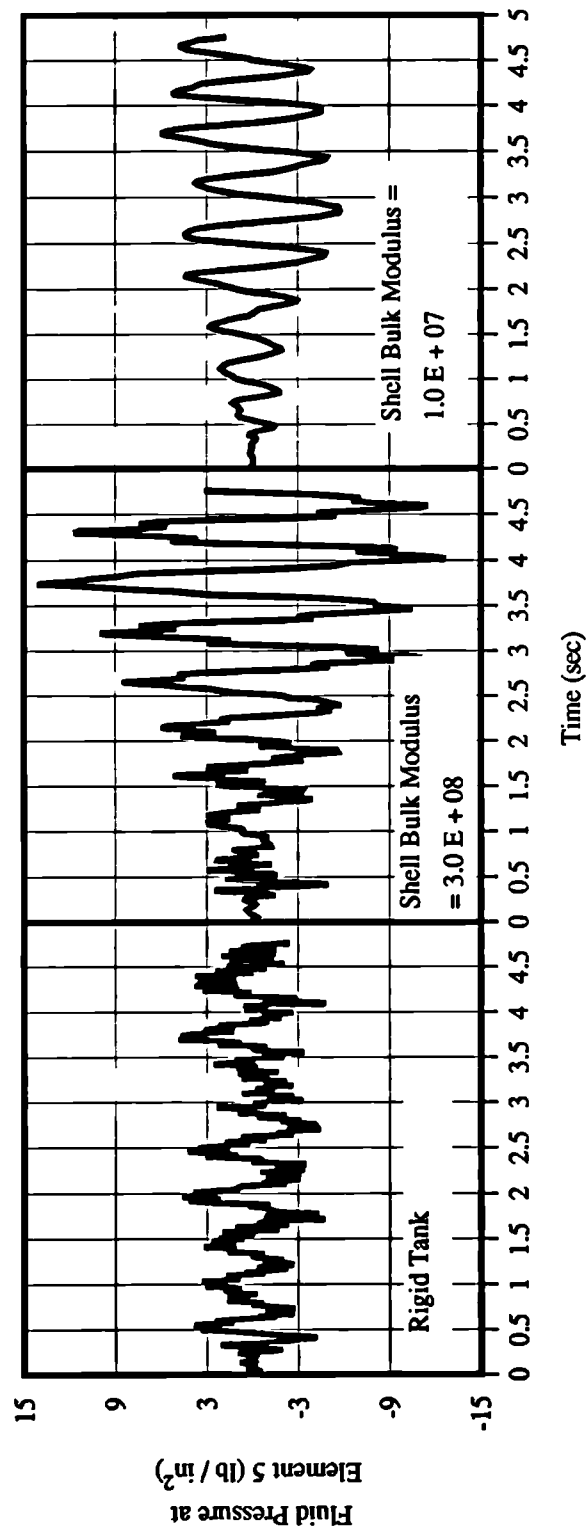


Figure 5.17b Response of a Tank Subjected to Earthquake Excitation - Fluid Pressure Variation at Element 5

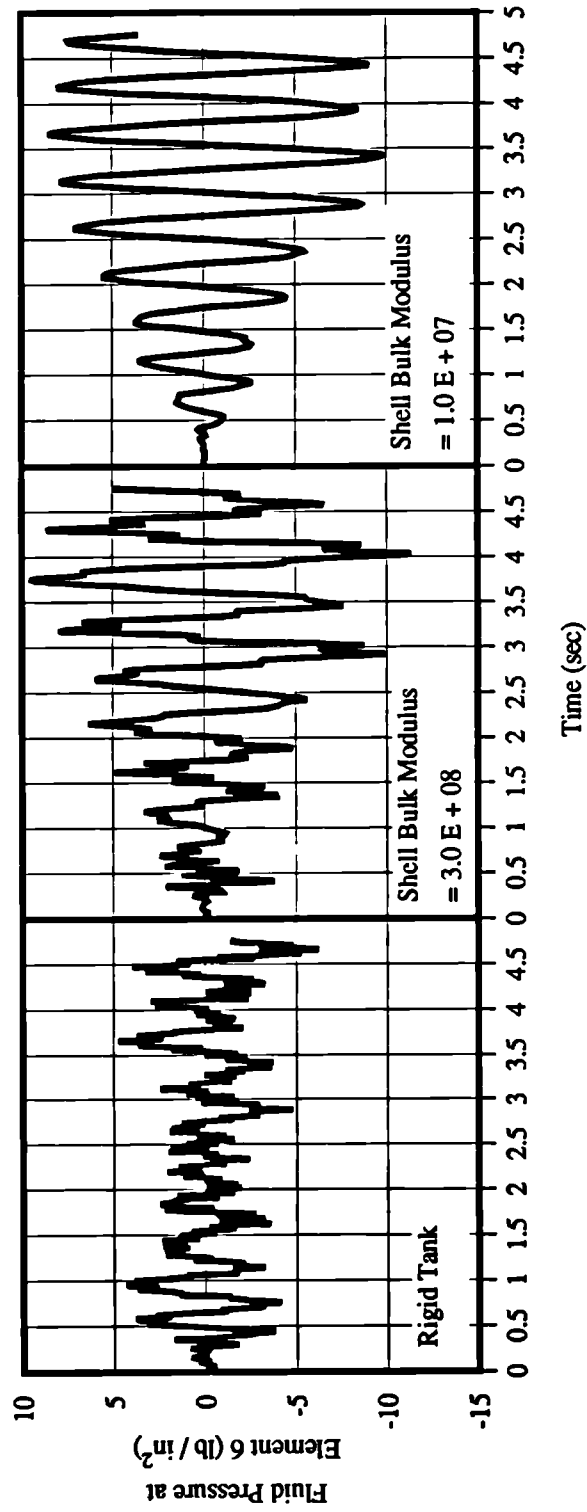


Figure 5.17c Response of a Tank Subjected to Earthquake Excitation - Fluid Pressure Variation at Element 6

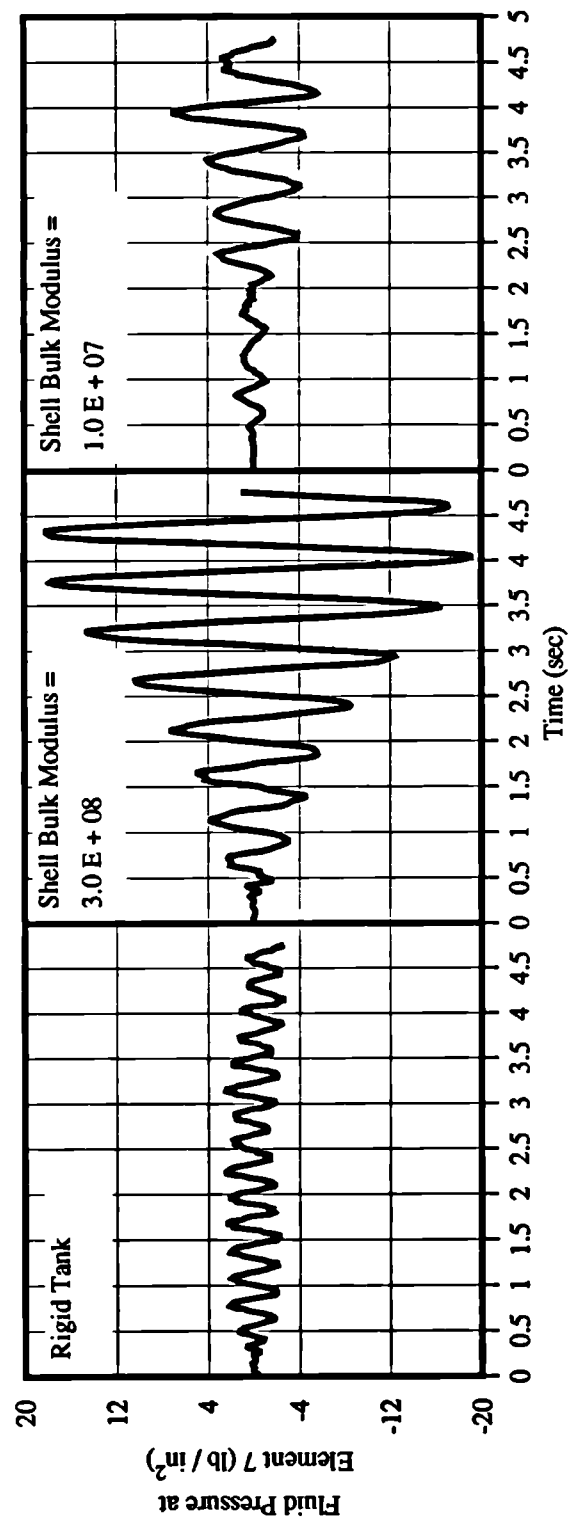
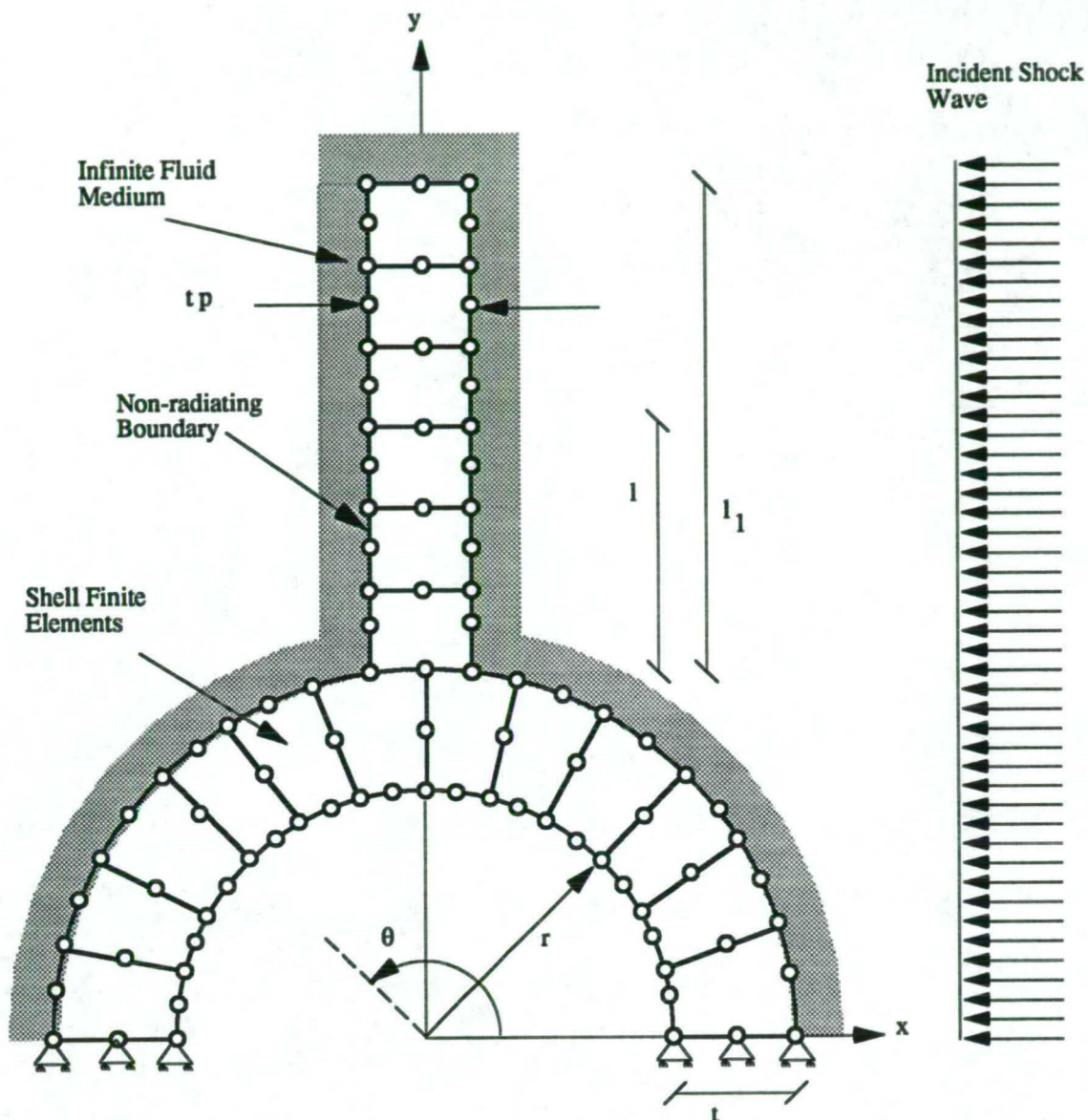


Figure 5.17d Response of a Tank Subjected to Earthquake Excitation - Fluid Pressure Variation at Element 7



MATERIAL AND GEOMETRIC PROPERTIES

$\nu = 0.3$
 $\rho_s = .728 \text{ E-3 lb-sec}^2/\text{in}^4$
 $\rho = 0.93 \text{ E-4 lb-sec}^2/\text{in}^4$
 $c = 0.6 \text{ E+5 in/sec}$ $l = r / 2$
 $r = 7.884 \text{ in}$ $l_1 = r$
 $t = 0.232 \text{ in}$ $t_p = 2.1006 \text{ in}$

LOADING CONDITIONS

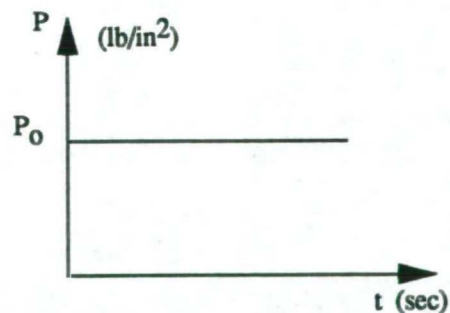
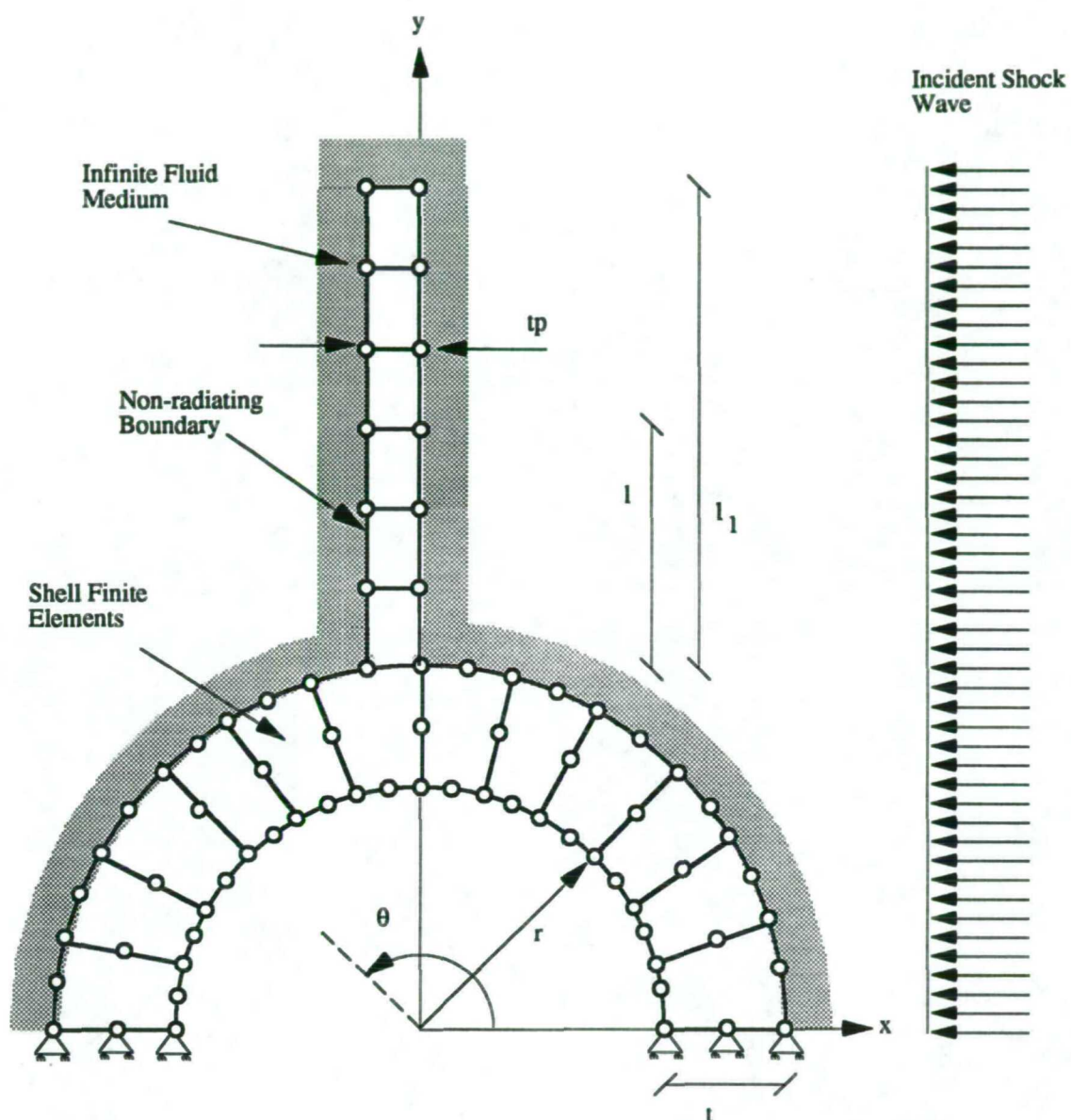


Figure 5.18a Submerged Cylinder in an Infinite Fluid Medium with External Appendages: Finite Element Model, and Geometric and Material Properties



MATERIAL AND GEOMETRIC PROPERTIES

$\nu = 0.3$
 $\rho_s = .728 \text{ E-}3 \text{ lb sec}^2/\text{in}^4$
 $\rho = 0.93 \text{ 10E-}4 \text{ lb sec}^2/\text{in}^4$
 $c = 0.6 \text{ E+}5 \text{ in/sec}$ $l = r/2$
 $r = 7.884 \text{ in}$ $l_1 = r$
 $t = 0.232 \text{ in}$ $t_p = 1.0503 \text{ in}$

LOADING CONDITIONS

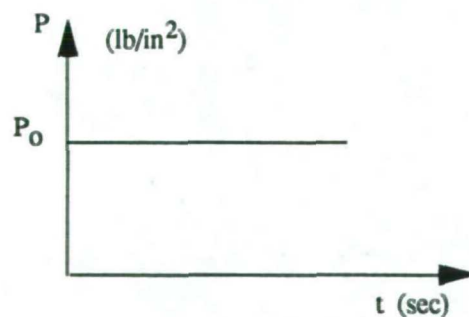


Figure 5.18b Submerged Cylinder in an Infinite Fluid Medium with External Appendages: Finite Element Model, and Geometric and Material Properties

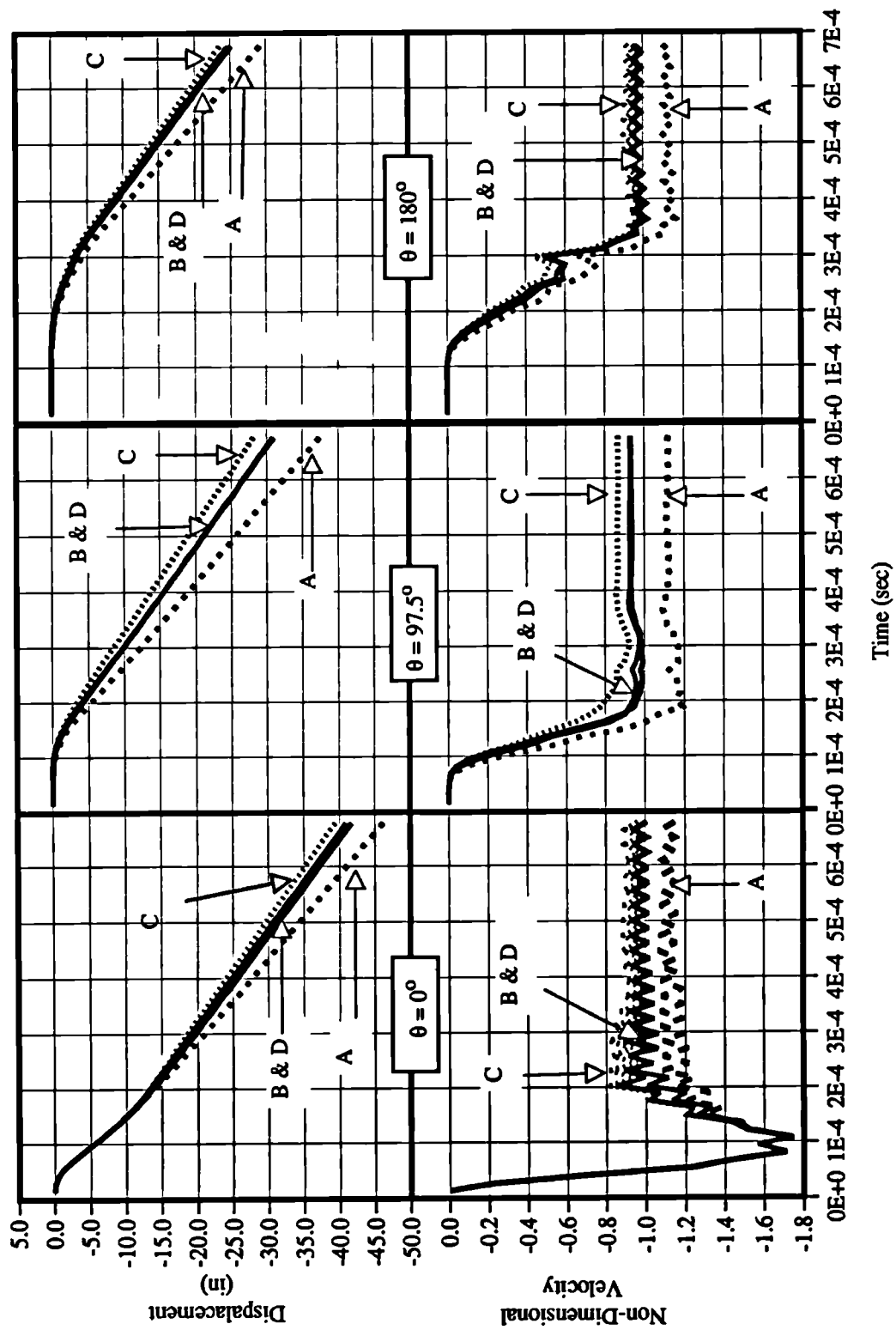


Figure 5.19 Horizontal Displacement and Velocity Response for Submerged Cylinder with External Appendages

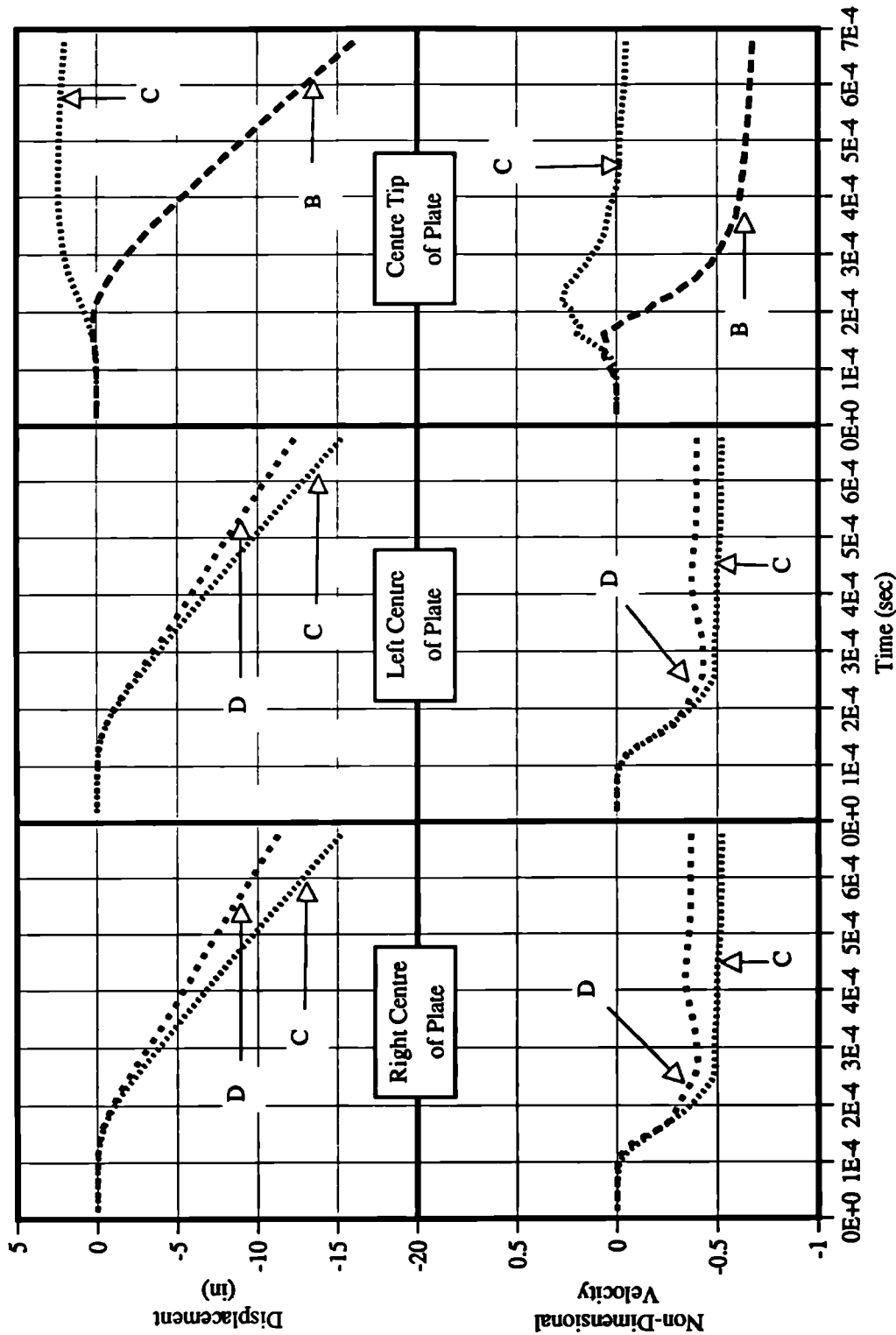


Figure 5.20 Submerged Cylinder with External Appendages - Horizontal Displacement and Velocity of Plate (See Table 5.5)

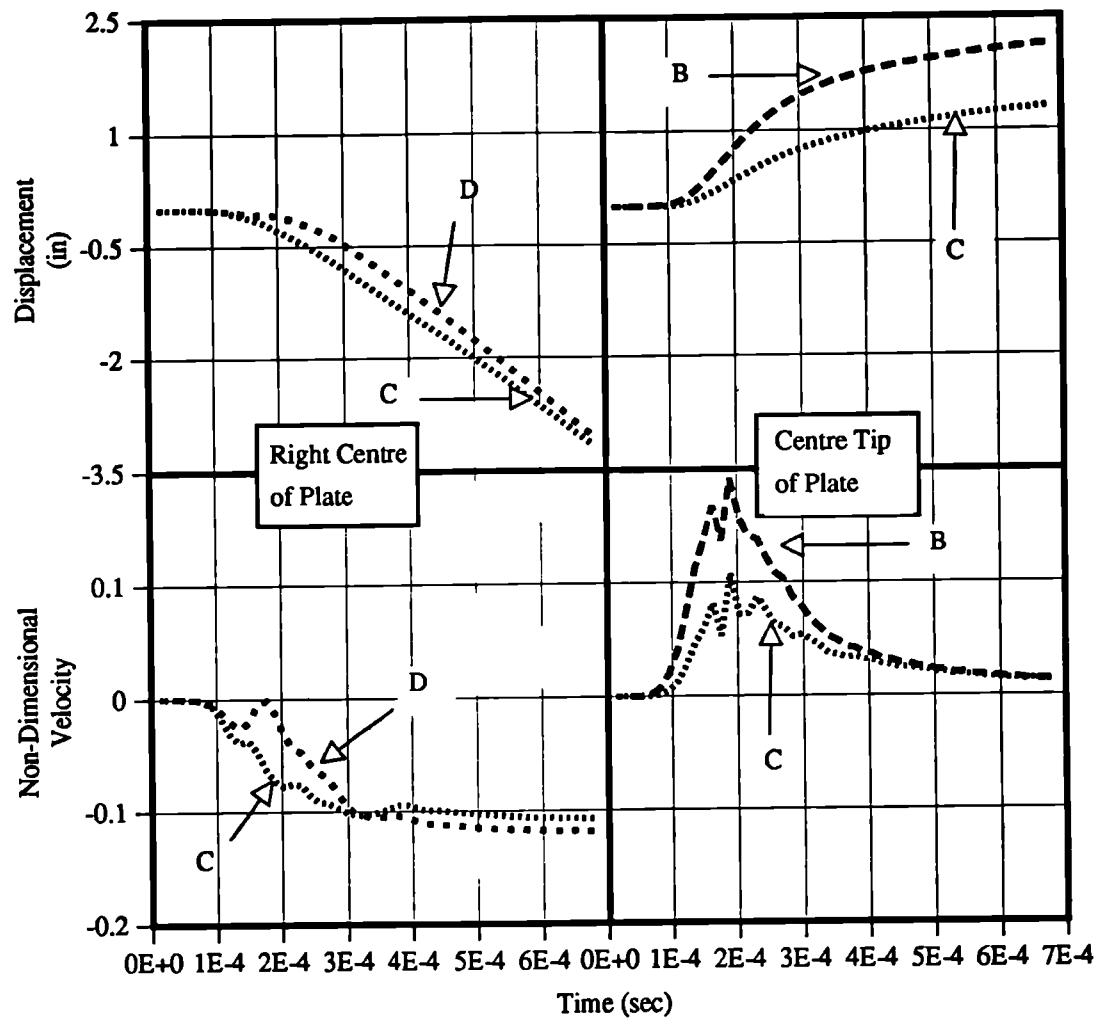


Figure 5.21 Submerged Cylinder with External Appendages - Vertical Displacement and Velocity of Plate (See Table 5.5)

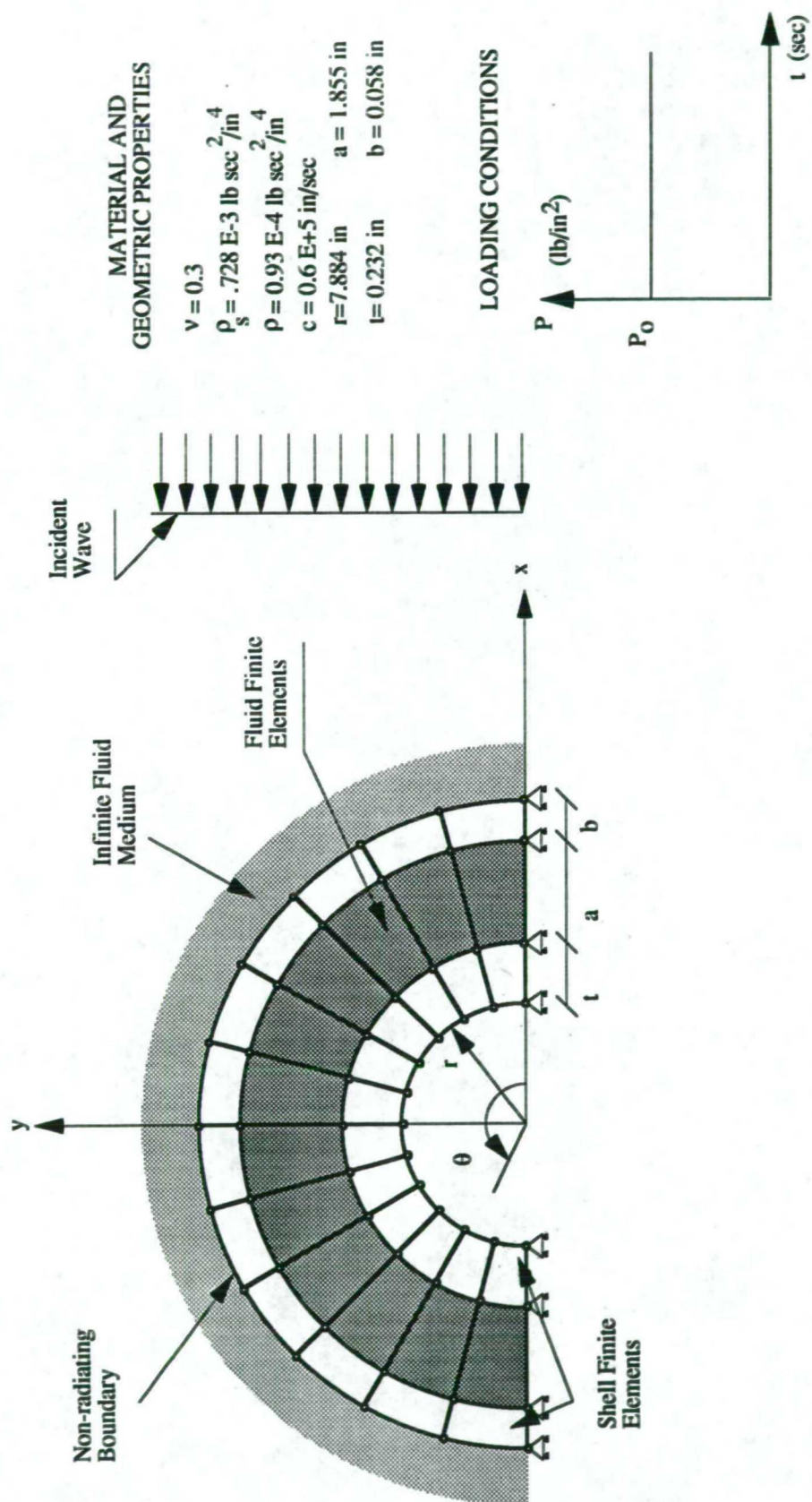


Figure 5.22 Submerged Double-Hulled Cylinder Subjected to a Shock Wave

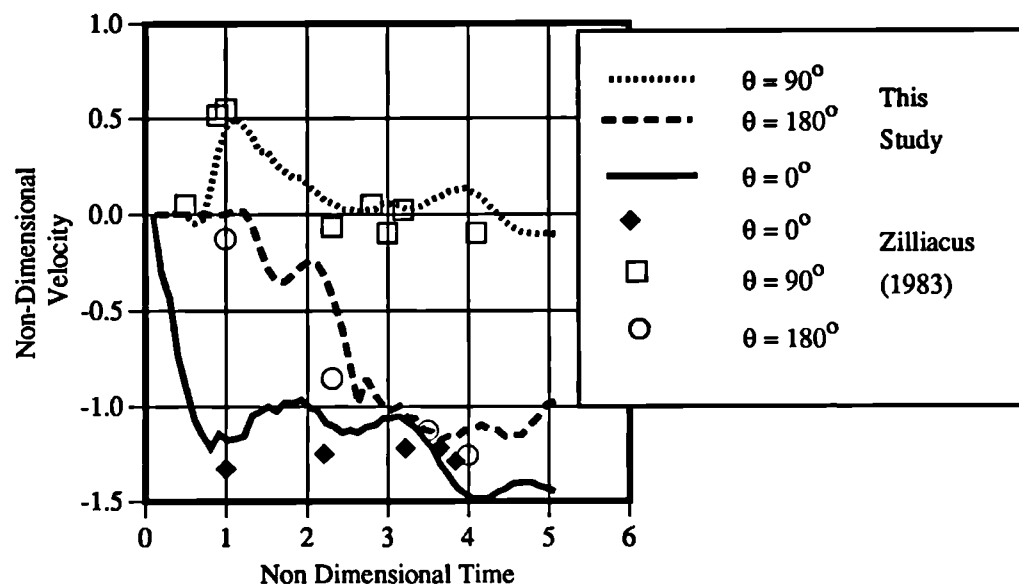


Figure 5.23 Response of a Double Hulled Cylinder Subjected to a Step Pressure Wave - Comparison with Numerical Work Reported by Zilliacus (1983)

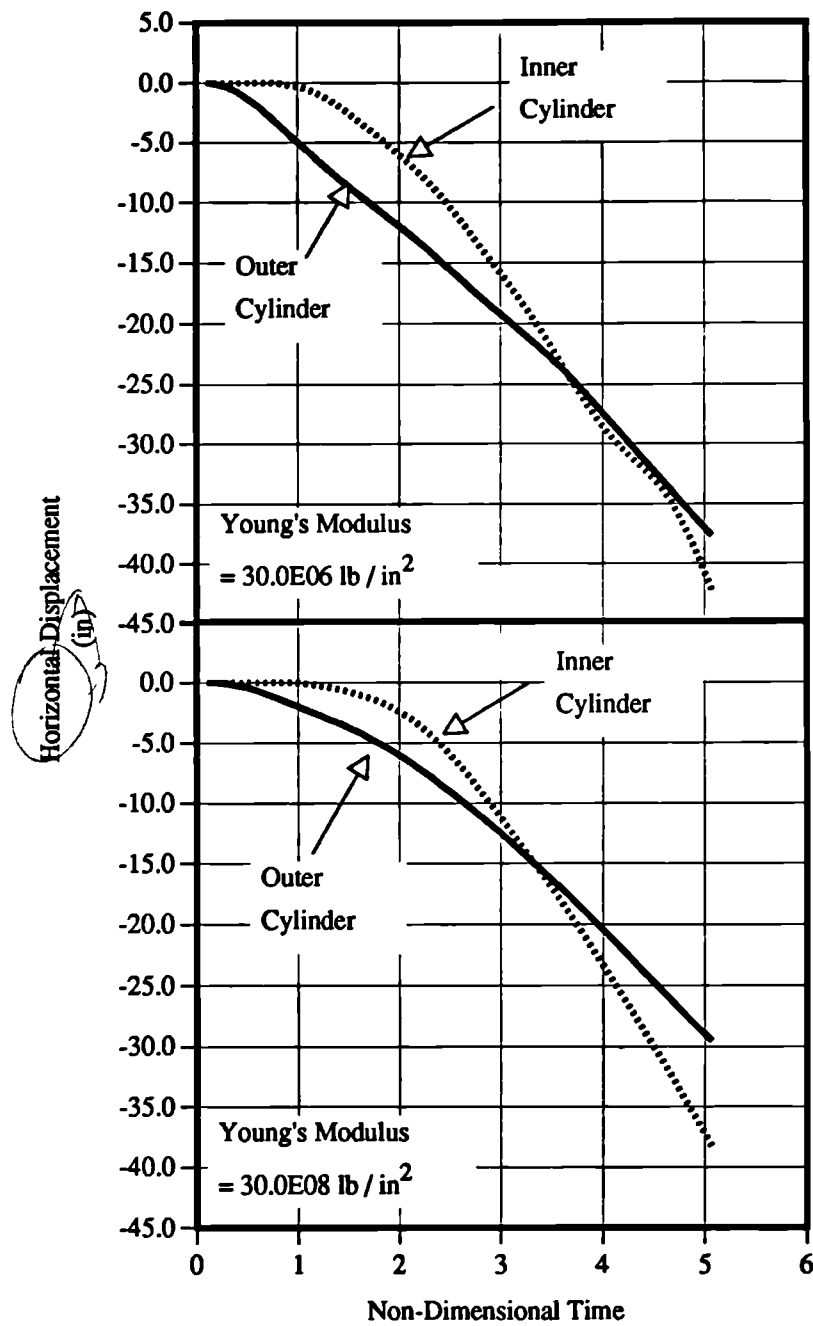


Figure 5.24a Response of a Double Hulled Cylinder Subjected to a Step Pressure - Displacement Response at $\theta = 0^\circ$

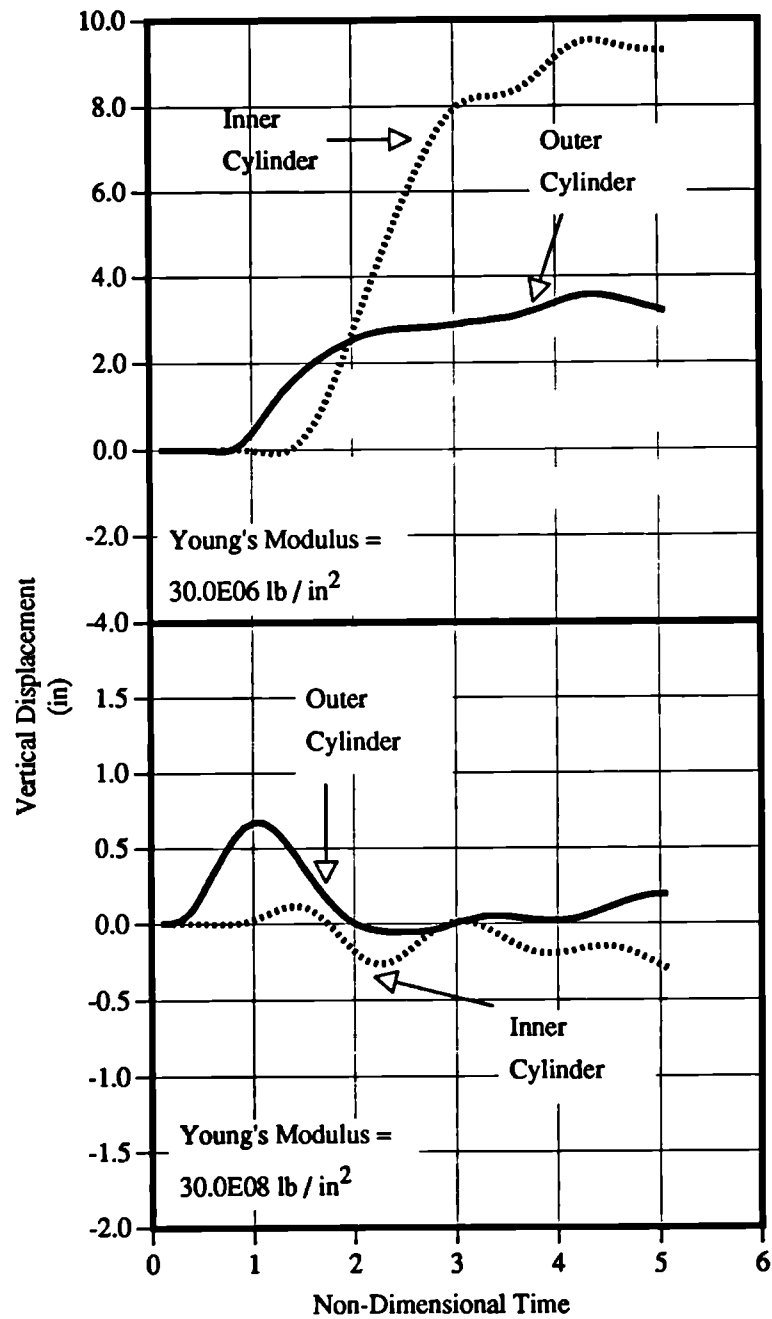


Figure 5.24b Response of a Double Hulled Cylinder Subjected to a Step Pressure - Displacement Response at $\theta = 90^\circ$

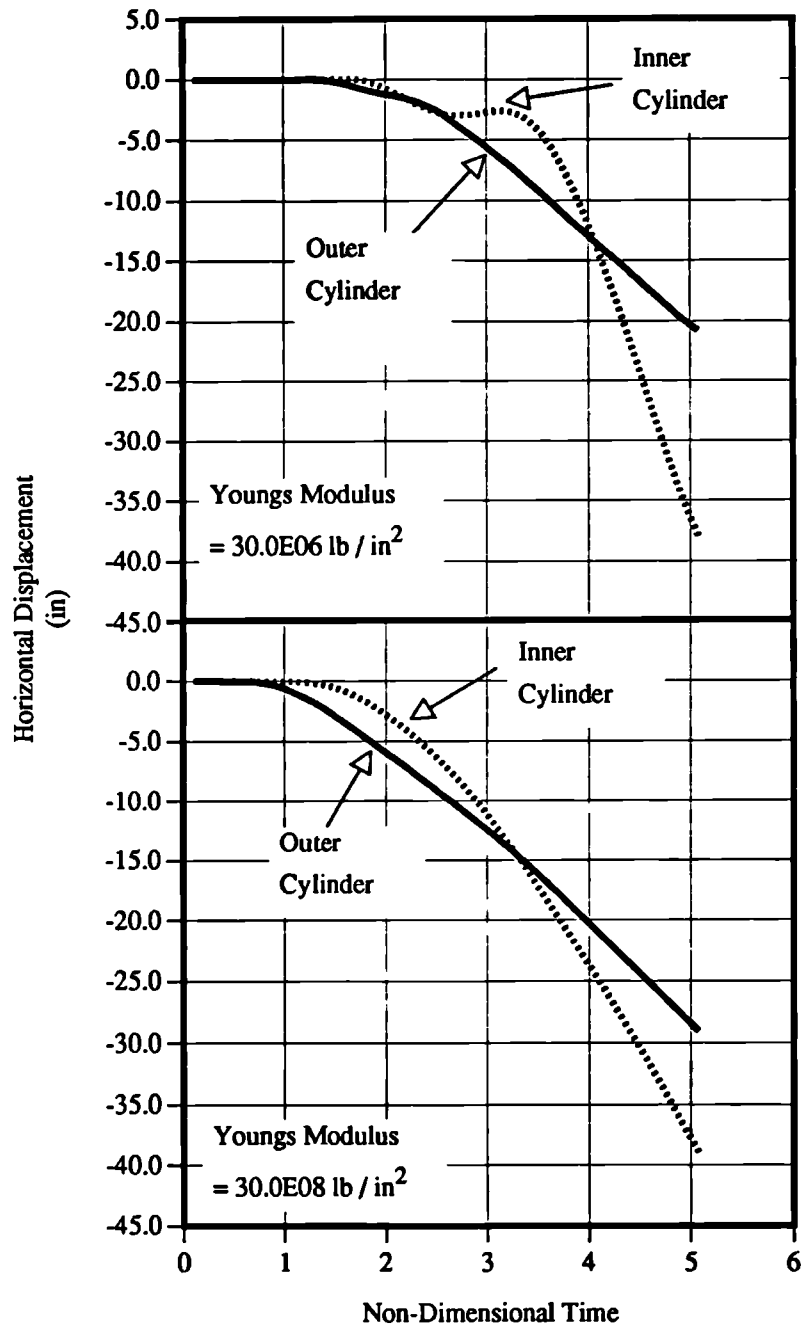


Figure 5.24c Response of a Double Hulled Cylinder Subjected to a Step Pressure - Displacement Response at $\theta = 180^\circ$

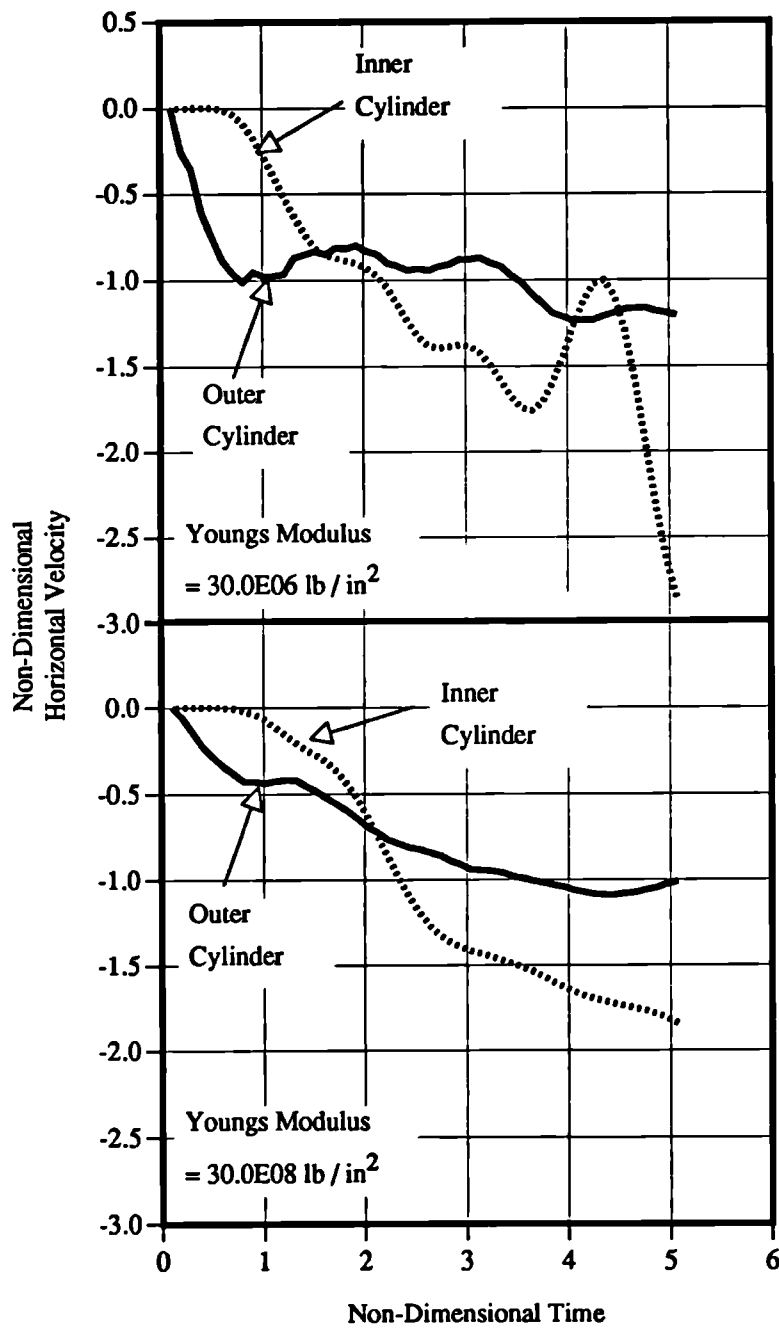


Figure 5.25a Response of a Double Hulled Cylinder Subjected to a Step Pressure - Velocity Response at $\theta = 0^\circ$

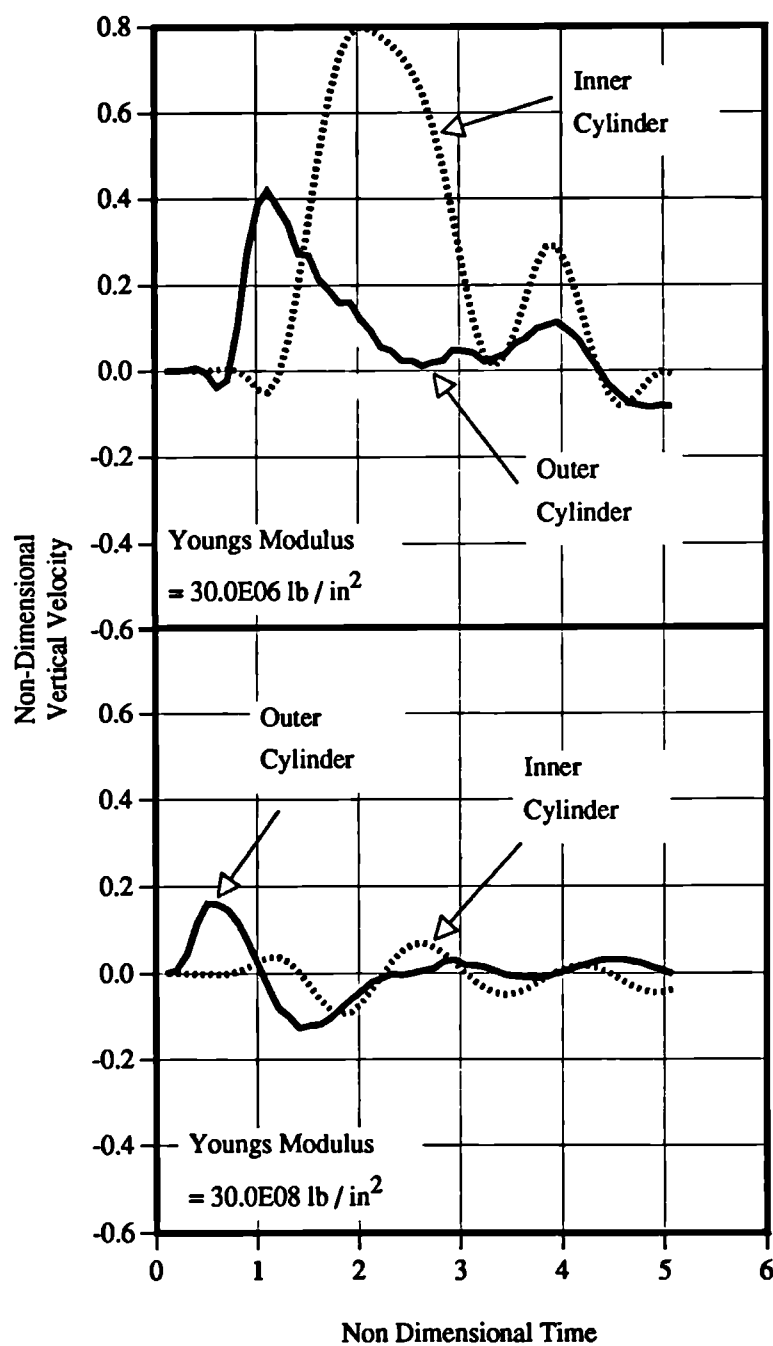


Figure 5.25b Response of a Double Hulled Cylinder Subjected to a Step Pressure - Vertical Velocity Response at $\theta = 90^\circ$

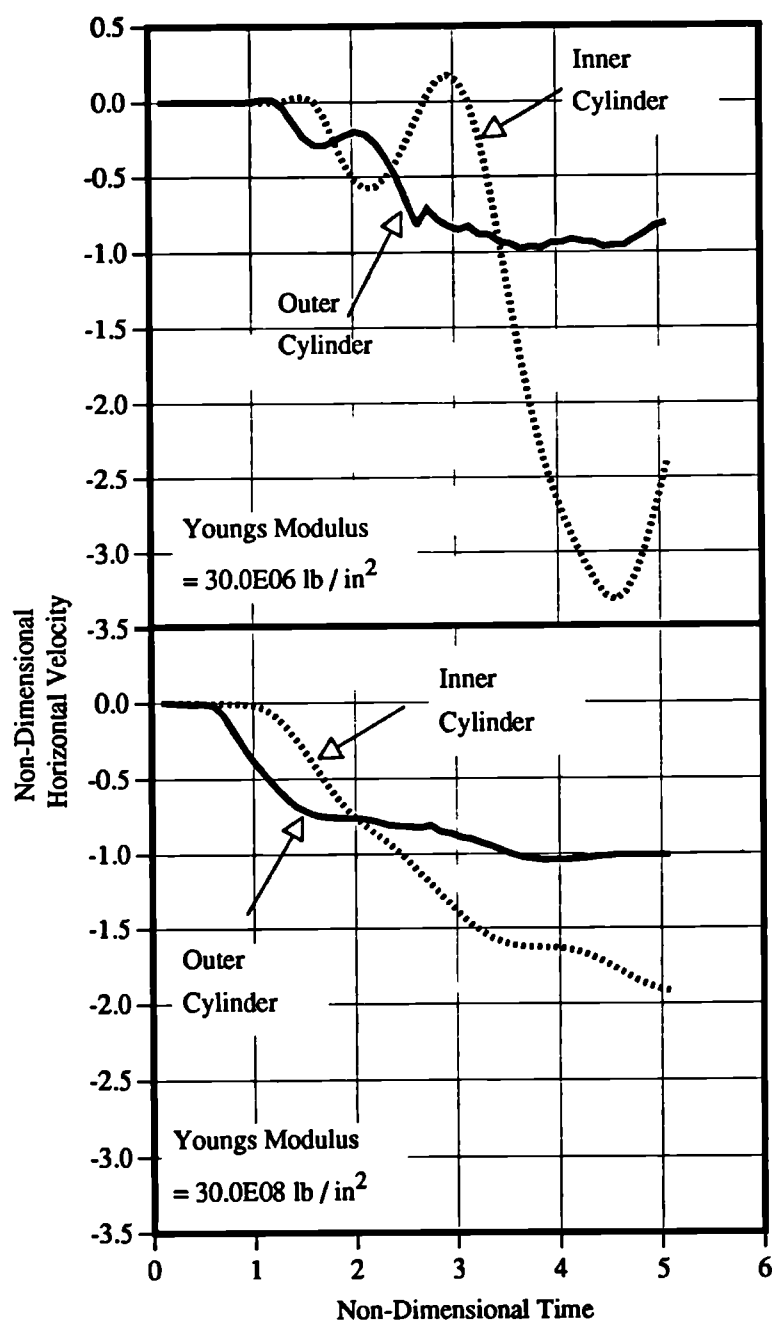


Figure 5.25c Response of a Double Hulled Cylinder Subjected to a Step Pressure - Velocity Response at $\theta = 180^\circ$

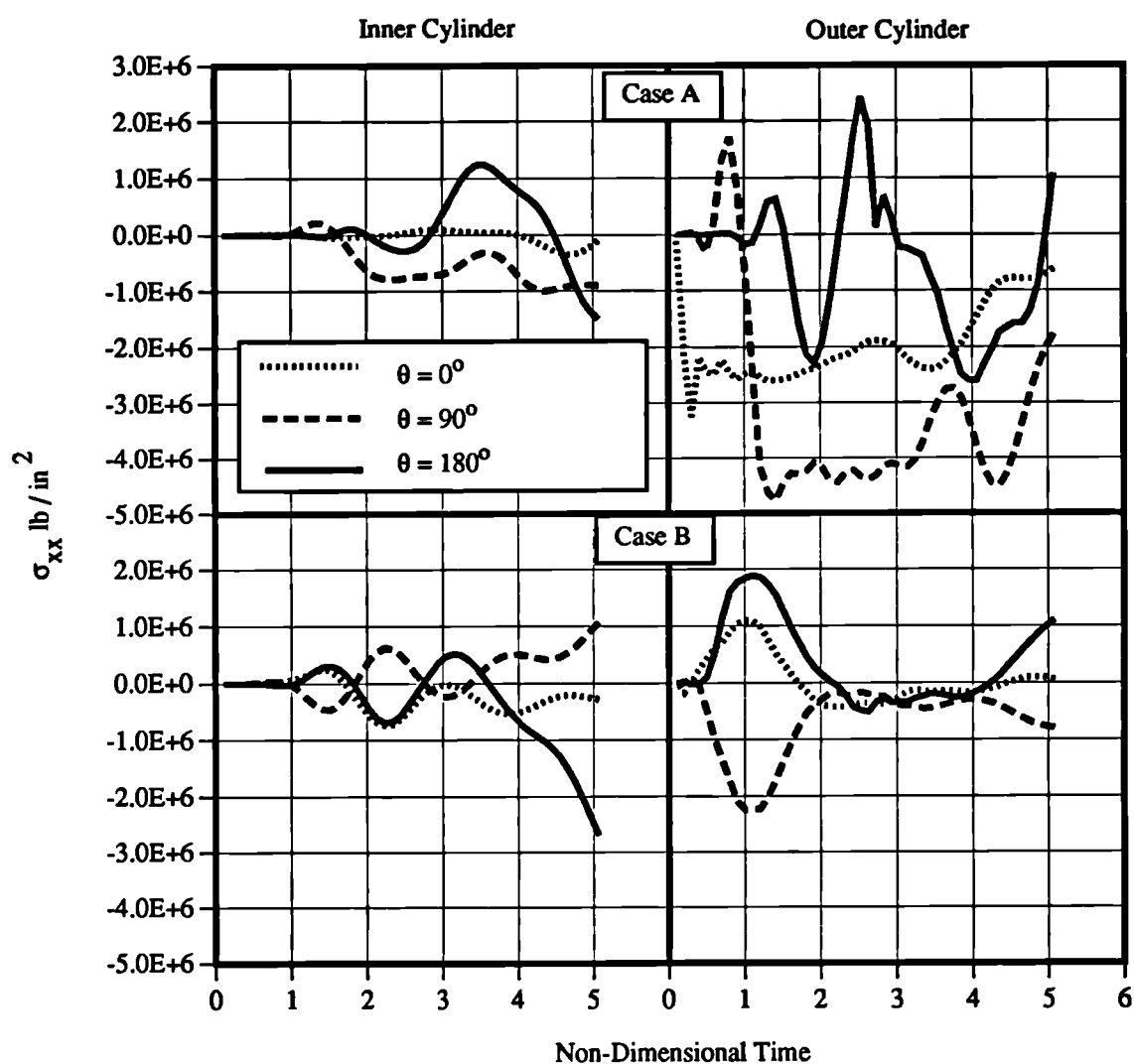


Figure 5.26 Response of a Double Hulled Cylinder Subjected to a Step Pressure Wave
- Effect of Young's Modulus on Cylinder Pressure - Pressure Plots at $\theta = 0^\circ, 90^\circ$ and 180° (See Table 5.10)

CHAPTER SIX

CONCLUSIONS AND RECOMMENDATIONS FOR FUTURE RESEARCH

6.1 CONCLUSIONS

The objective of this work was to develop accurate and versatile numerical tools for the transient analysis of practical fluid-structure interaction problems.

Towards this end, it was necessary to review existing numerical formulations. In Chapter Two a literature survey was undertaken with the aim of outlining the required ingredients of a fluid-structure interaction program and highlighting aspects of the problem which needed further development. As a result, it was concluded that:

- i- Fluid finite elements should be used in modelling the fluid medium contained within structures. Furthermore, the Mixed-Eulerian-Lagrangian formulation was considered to be the most suitable kinematic description of motion, as opposed to both the Eulerian and Lagrangian formulations. Also, a penalty on the fluid spin tensor should be introduced to be used when the fluid is assumed to be inviscid.
- ii- A non-radiating boundary capable of modelling the effect of the infinite fluid medium should be used. The Plane Wave Approximation was chosen due to its efficiency in terms of computer CPU time and storage requirements.
- iii- The proposed developments should be implemented into a general purpose, rather than specialised, structural computer code. This would result in a more user-friendly computer code capable of treating a wide variety of problems and amenable to continuous updating and development.

The necessity to test the fluid and sliding elements under a variety of loading types was also identified.

In order to fulfil the recommendations concerning bounded fluid-structure interaction problems outlined above, Chapter Three presented the theory, implementation details and the validation of the fluid and sliding elements. The following major steps were undertaken in the formulation and validation process:

- i- Both four- and eight-noded, two-dimensional and axi-symmetric, isoparametric finite elements were developed. The elements are based on the Mixed-Eulerian-Lagrangian description of the continuum, mentioned previously. In the case of the present study, the developed formulation allowed the user to input the viscosity of the fluid. Furthermore, an option to allow the user to input a penalty on fluid rotation was introduced. The gravity wave effect, particularly important in the case of water tank systems, was implemented as a pressure term.
- ii- A rezoning scheme for relating the mesh and material nodal velocities was developed and implemented in the code. Furthermore, the nonlinear transient algorithm was modified to account for the mesh kinematic quantities.
- iii- Four-noded, two-dimensional and axi-symmetric, isoparametric contact elements were developed. These were based on the provision of tangential discontinuities and the coupling of normal motion at the fluid-structure interface.
- iv- To test the accuracy of the two-dimensional and axi-symmetric formulation, the developed fluid elements were tested for a variety of problems, with different loading conditions. Results using the present formulation showed good agreement with those reported in other studies based on numerical and analytical work. In addition, the analysis of a solitary wave propagation problem showed that the developed gravity wave formulation models correctly the effect of the free surface pressure term.
- v- By combining the fluid, structural and sliding elements it was possible to study the response of fluid-structure systems such as reservoirs, fluid-plate and fluid-mass-piston systems. These problems served as a test on the accuracy of the sliding element formulation and the overall stability of the solution procedure. It was shown that the results obtained compare well with those reported in other studies. Furthermore, it was demonstrated that the developed formulation is capable of modelling accurately the transient response of fluid-structure systems.

To fulfil the recommendations concerning unbounded fluid-structure interaction problems outlined above, Chapter Four presented the theory, implementation details and the validation of the non-radiating boundary. The following major steps were undertaken in the formulation and validation process:

- i- Both two- and three-noded, two-dimensional and axi-symmetric, isoparametric elements were developed for modelling the non-radiating boundary. The elements are based on the Plane Wave Approximation. It was assumed that the waves, radiating off the wet surface of the structure, propagate to infinity without any further reflections.
- ii- Both step and exponential plane waves were developed. The shock wave formulation is based on the propagation of waves in an infinite elastic medium. It was assumed that the source of wave disturbance lies at a distance infinitely far from the structure, in comparison to the structure characteristic length.
- iii- In order to validate the non-radiating boundary formulation, several problems of submerged structures subjected to internal pressure as well as incident shock waves were studied. In most cases, the results obtained showed good agreement with those reported in other analytical and numerical-based studies which model the infinite fluid medium using finite differences and finite elements. The comparison showed that the developed formulation succeeds, to a large extent, in reproducing the same early time results as those mentioned above. Therefore, it was concluded that the present formulation results in considerable savings in computer time and storage without a corresponding reduction in accuracy. However, if the structures peak response occurs at a late stage in the analysis, the proposed formulation fails to predict an accurate peak response of the structure.

In Chapter Five, it was shown that the proposed formulation could be applied to the study of practical fluid-structure interaction problems such as pressure vessels, water tanks subjected to earthquake excitation as well as submerged double-hulled cylinders subjected to incident shock waves.

The elasto-plastic response of a pressure vessel under impulsive loading conditions was studied. The practical implication of such a problem are of significant importance, as it resembles situations which occur in nuclear and chemical industries where fail-safe conditions are essential. Such loading conditions may be caused by

impact or explosions. Time histories of displacements obtained from this study showed good agreement with those reported in other work based on numerical analysis. Hence, it was demonstrated that the developed formulation is capable of predicting, to a sufficient degree of accuracy, the response of practical fluid-structure systems while taking into account the effect of plasticity and gross deformation.

Next, the large displacement response of a water tank, subjected to the 1940 El-Centro earthquake excitation record was analysed. The free surface wave, or sloshing, induced by earthquake excitation in ground supported liquid containers was reproduced in a two-dimensional finite element tank-fluid model. Time histories of displacements obtained from the present study showed good agreement with those reported in other numerical studies.

To assess the accuracy of the Plane Wave Approximation in predicting the response of submerged structures with appendages, the transient response of a submerged cylinder with an externally attached plate-like structure was studied. Time histories of the displacements and velocities obtained from this study were examined. It was concluded that the proposed non-radiating boundary formulation, when applied on the convex wet surface of structures with appendages, fails to predict an accurate response because it ignores the occurrence of multiple reflections between the structure and the appendages.

Finally, the response of a fluid-coupled double-hulled cylinder submerged in an infinite fluid medium was studied. This problem is of significance to underwater structures such as a fluid-coupled double-hulled section of a submarine. Time histories of velocities showed good agreement with those reported in other studies. This demonstrated that the numerical techniques used in combining the sliding element, near-field fluid, far-field fluid and the structure produce a stable and accurate response.

It is therefore believed that this work has gone a considerable way in providing an analysis tool for studying the response of practical fluid-structure systems. Several aspects of the problem, where further work is needed are outlined in the next section.

6.2 SUGGESTIONS FOR FUTURE RESEARCH

In order to extend the understanding of the nonlinear response of fluid-structure interaction phenomena, further developments in the numerical field are still required. In terms of the numerical treatment, the following refinements to the software developed in this study are required:

- i- Only two-dimensional and axi-symmetric fluid-structure systems were considered in this work. To investigate the behaviour of complex three-dimensional fluid-structure systems, it is necessary to develop three-dimensional fluid, sliding and non-radiating boundary elements.
- ii- Most of the fluid terms were integrated in the time domain using an implicit scheme. However, in many fluid-structure interaction problems, it is beneficial to treat the structure with an implicit time integration operator and the fluid with an explicit one. This would result in large cuts in computational costs. Furthermore, several problems associated with the implicit treatment of the fluid would vanish. To achieve this, efficient implicit-explicit time integration schemes need to be developed.
- iii- Amongst the difficulties most commonly encountered in the transient analysis of fluid-structure systems is the selection of an appropriate time step for nonlinear problems. For example, if any plastic deflection occurs in the structure at the very beginning of the analysis then a small time step should be used. Later on, the time step size should be increased to avoid large CPU requirements, whereas at the near collapse stage the step size has to be reduced to achieve convergence. The user of a computer code needs considerable skill and experience to select a suitable time step for each stage and often only succeeds through trial and error which results in a large consumption of CPU time and the user real time. Hence there is a need for an automatic time stepping algorithm whose scaling decisions would depend on the information gained in the current time step.
- iv- At present, the developed software does not have a facility to allow for cavitation in the fluid medium. In several cases, such as for earth dams and submerged structures, cavitation does occur in the fluid medium under consideration. Towards studying such problems, it is necessary to develop a fluid material model which would be capable of modelling cavitation.

- v- The Mixed-Eulerian-Lagrangian fluid description of motion offers the user a versatile way to move the nodes in a Lagrangian fashion or to keep them fixed in a Eulerian manner. However, it suffers from one disadvantage, namely, the requirement to specify explicitly the movement of the mesh nodes. As the number of nodes increases, which often happens for three-dimensional fluid-structure systems, this may become a prohibitively time consuming task. It is suggested that this disadvantage could be eliminated by the use of automatic mesh rezoning. In such a scheme, at each time step or several time steps, the mesh nodes are moved in such a way to satisfy certain physical criteria; for example, minimising squeeze and distortion in the fluid finite elements.
- vi- The use of the Plane Wave Approximation leads to inaccurate results for the later stages of the response. For problems where the expected peak pressure may occur at late times, the drop in accuracy will have significant consequences rendering the analysis meaningless. Towards an accurate solution of such problems, it is necessary that the Plane Wave Approximation, developed in this study, be augmented to include an added mass matrix to enhance the accuracy of the non-radiating boundary formulation at late times.
- vii- The code in its present form has the ability to model the effect of explosions that have occurred at a sufficiently far distance away from the structure. In such a case, the motion of the moving fluid particles can be described by plane waves. To model the effect of explosions that occur near the structure, appropriate techniques should be used to model the spherical nature of the wave propagation into the fluid medium.

Finally, it is suggested that the numerical results obtained be validated against experimental as well as numerical and analytical solutions. To achieve this, it is hoped that experimental results held by various authorities and researchers will be made more freely available. This is especially important for the far-field formulation where in many instances numerical and analytical data are lacking.

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APPENDIX A

FLUID MESH VELOCITY REZONING SCHEME FOR TWO-DIMENSIONAL AND AXI-SYMMETRIC ISOPARAMETRIC FINITE ELEMENTS

In Chapter Three, the relationship relating the mesh and material nodal velocities was expressed as:

$$\hat{\underline{v}} = \underline{A}(\underline{v} - \underline{L}^{-1} \underline{S}) \quad (3.94)$$

where:

$$\underline{L} = \sum_{e=1}^{NELEM} \int_{\Omega_y^e} \underline{N}^T \underline{N} d\Omega_y^e \quad (3.92)$$

$$\underline{S} = \sum_{e=1}^{NELEM} \int_{\Omega_y^e} \sum_{i=1}^{NEN} N_i c_{ki} \left[\sum_{j=1}^{NEN} N_{i,k} (\hat{d}_j + \hat{y}_j) d\Omega_y^e \right] \quad (3.93)$$

The purpose of this appendix is to discuss the implementation details of the Equations 3.92 to 3.94. The global $\underline{L}$ and $\underline{S}$ matrices are assembled from the element matrices as follows:

$$\underline{L} = \sum_{e=1}^{NELEM} \underline{l}_e ; \quad \underline{S} = \sum_{e=1}^{NELEM} \underline{s}_e \quad (A.1)$$

where:

$$\underline{l}_e = [\underline{l}_{ab}] ; \quad \underline{s}_e = \{\underline{s}_a\} \quad (A.2)$$

where a and b are the nodes corresponding to the element nodal matrices appearing in the above equation. The matrices appearing in Equation A.2 are evaluated in the following manner:

$$l_{ab} = j_a^y \delta_{ij} \delta_{ab} \quad (A.3a)$$

$$\underline{\xi}_a = \begin{pmatrix} j_a^y \frac{\partial f_1}{\partial y_2} (1 - \alpha_{2a}) v_{2a} \\ j_a^y \frac{\partial f_2}{\partial y_1} (1 - \alpha_{1a}) v_{1a} \end{pmatrix} \quad (A.3b)$$

where:

$$f_j = \hat{d}_j + y_j$$

j_a^y is the jacobian determinant evaluated at node a

Note that by using Equation A.3a to evaluate the $\underline{L}$ matrix, it becomes sufficient to store only one value per node.

The derivatives of the function f with respect to the local coordinates are expressed as:

$$\begin{aligned} \frac{\partial f_j}{\partial \xi} &= \sum_{i=1}^{NEN} (\hat{d}_{ji} + y_{ji}) \frac{\partial N_i}{\partial \xi} \\ \frac{\partial f_j}{\partial \eta} &= \sum_{i=1}^{NEN} (\hat{d}_{ji} + y_{ji}) \frac{\partial N_i}{\partial \eta} \end{aligned} \quad (A.4)$$

Two special cases are in use:

i- ξ lines move with the free surface

$$\begin{aligned} \frac{\partial f_1}{\partial y_2} &= \frac{\partial f_1}{\partial \xi} \frac{\partial \xi}{\partial y_2} \\ \frac{\partial f_2}{\partial y_1} &= \frac{\partial f_2}{\partial \xi} \frac{\partial \xi}{\partial y_1} \end{aligned} \quad (A.5)$$

ii- η lines move with the free surface

$$\begin{aligned} \frac{\partial f_1}{\partial y_2} &= \frac{\partial f_1}{\partial \eta} \frac{\partial \eta}{\partial y_2} \\ \frac{\partial f_2}{\partial y_1} &= \frac{\partial f_2}{\partial \eta} \frac{\partial \eta}{\partial y_1} \end{aligned} \quad (A.6)$$

APPENDIX B**IMPLEMENTATION OF THE MIXED-EULERIAN-LAGRANGIAN
FINITE ELEMENT FLUID FORMULATION INTO THE GENERAL
PURPOSE PROGRAM ASAS-NL**

In this appendix, the main subroutines used in the implementation of the Mixed Eulerian-Lagrangian finite element technique into ASAS-NL are presented. These may be divided into two main categories: those that are either at the global level or at a local (element) level.

B.1 GLOBAL LEVEL:

In order to change the structure of ASAS-NL to account for the newly proposed developments, two main requirements were carried out:

- Modification of the global solution procedure, including the predictor-multi-corrector algorithm, to account for the duality of mesh and material properties.
- Establishment of a general relationship between material and mesh velocity fields.

B.2 LOCAL OR ELEMENT LEVEL :

- Computation of the fluid element mass and damping matrices and the corresponding residual forces.
- Computation of the fluid compressibility and vorticity matrices, and the corresponding residual forces.
- Computation of the convective matrix to account for the transportation of mass through the mesh.

Table B.1 presents a brief description of the main subroutines, modified or newly developed, in this study, while Figures B.1 through B.6 show the flow-charts of these subroutines.

Subroutine	Description
GLOCON	Controls the global assembly and solution procedures.
STRCON	Controls the assembly and formulation of the residual load vector.
STICON	Controls the assembly and formulation of the mass, stiffness and damping matrices
CORREC	Controls the correction part of the corrector-multi-predictor solution scheme.
PDTCAL	Controls the prediction part of the corrector-multi-predictor solution scheme.
PRSSUR AXPRSR	Calculates the pressure due to the compressibility of the fluid and free surface gravity wave.
RGTCNV AXRCNV	Calculates the convective term due to the transport of mass.
CVM AXCVN	Calculates the pressure due to the viscosity of the fluid.
MSHVEL	Forms the mapping relationship between the material and mesh velocities.

**Table B.1 Subroutines Used in the
Implementation Near-Field Fluid Formulation**

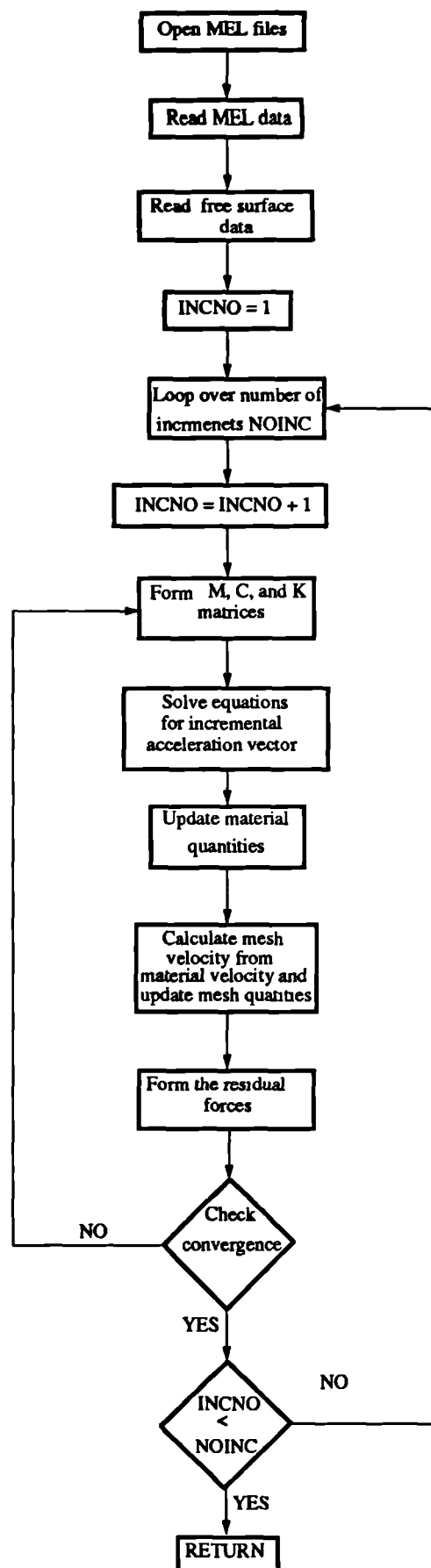


Figure B.1 Flow-Chart for Subroutine GLOCON

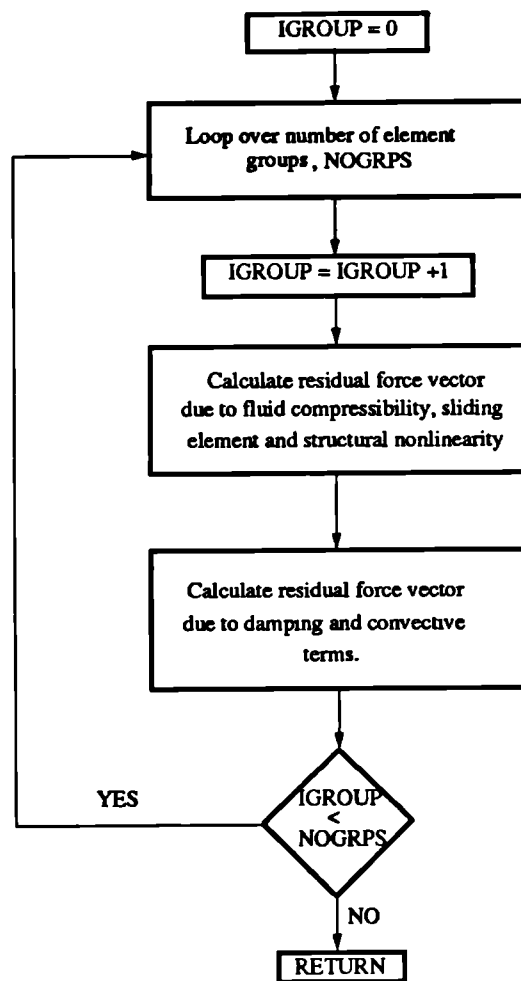


Figure B.2 Flow-Chart for Subroutine STRCON

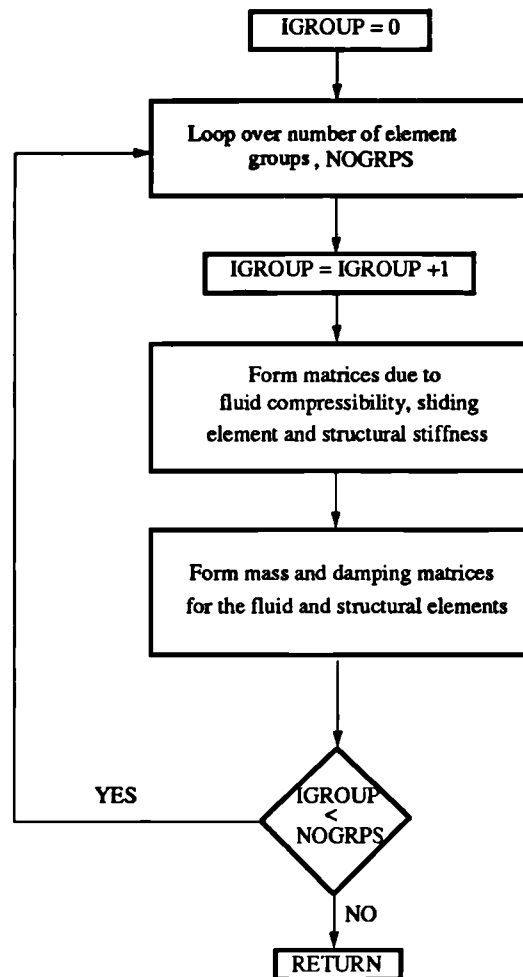


Figure B.3 Flow-Chart for Subroutine STICON

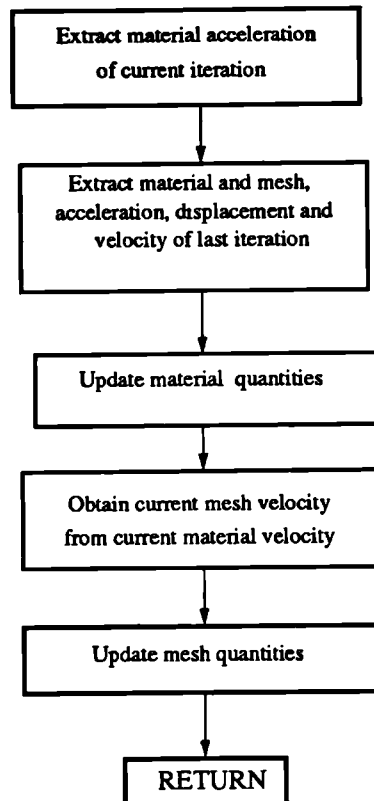


Figure B.4 Flow-Chart for Subroutine CORREC

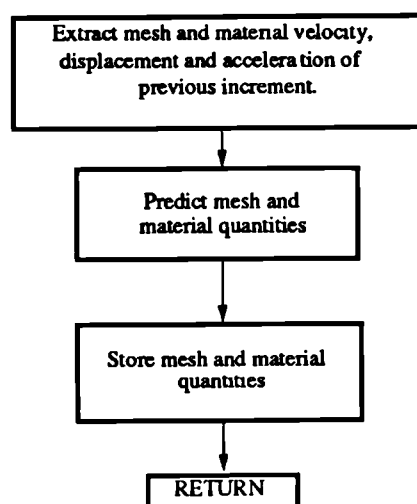


Figure B.5 Flow-Chart for Subroutine PDTCAL

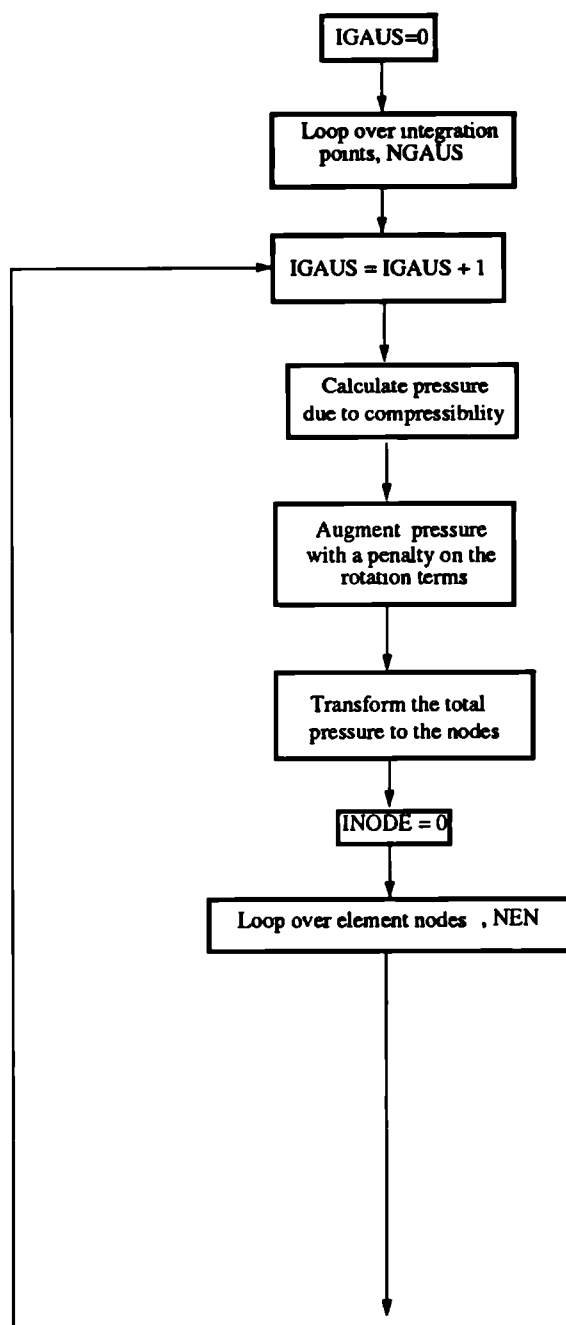


Figure B.6 Flow-Chart for Subroutines PRSSUR and AXPRSR

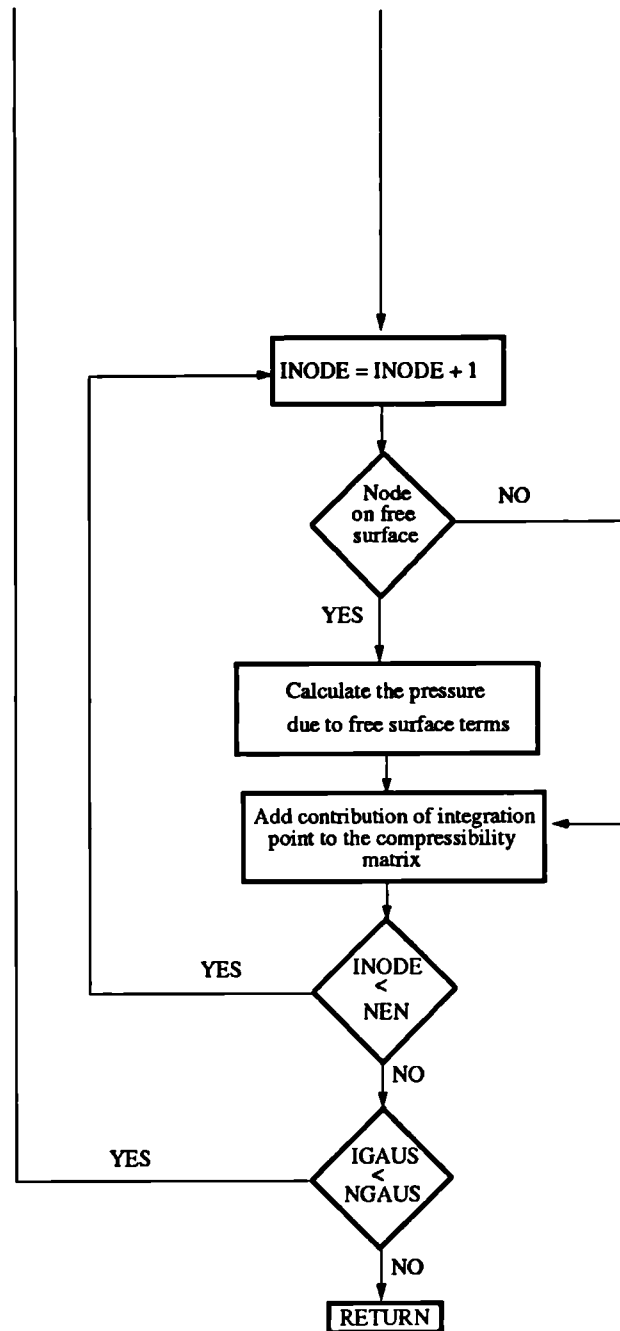


Figure B.6 (cont'd) Flow-Chart for Subroutines PRSSUR and AXPRSR

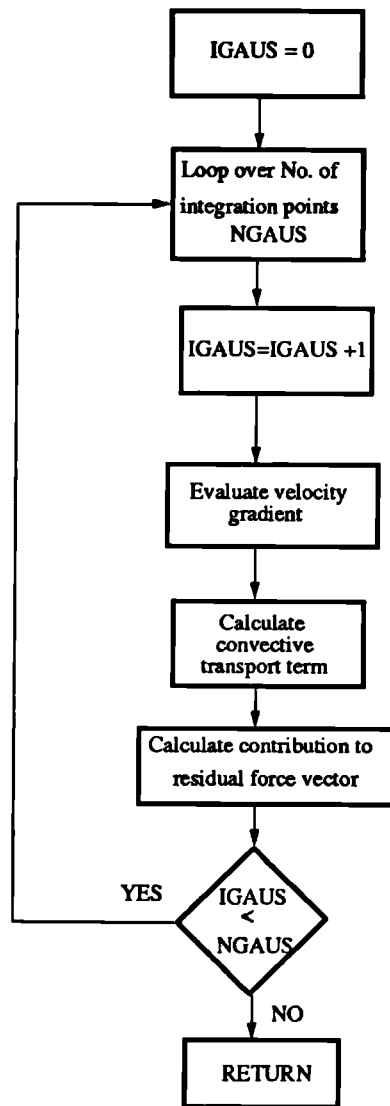


Figure B.7 Flow-Chart for Subroutines RGTCNV and AXRCNV

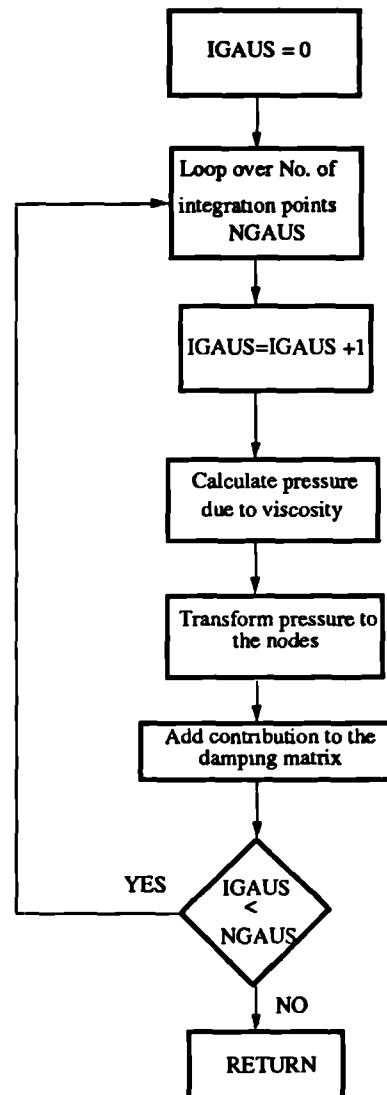


Figure B.8 Flow-Chart for Subroutines CVM and AXCVM

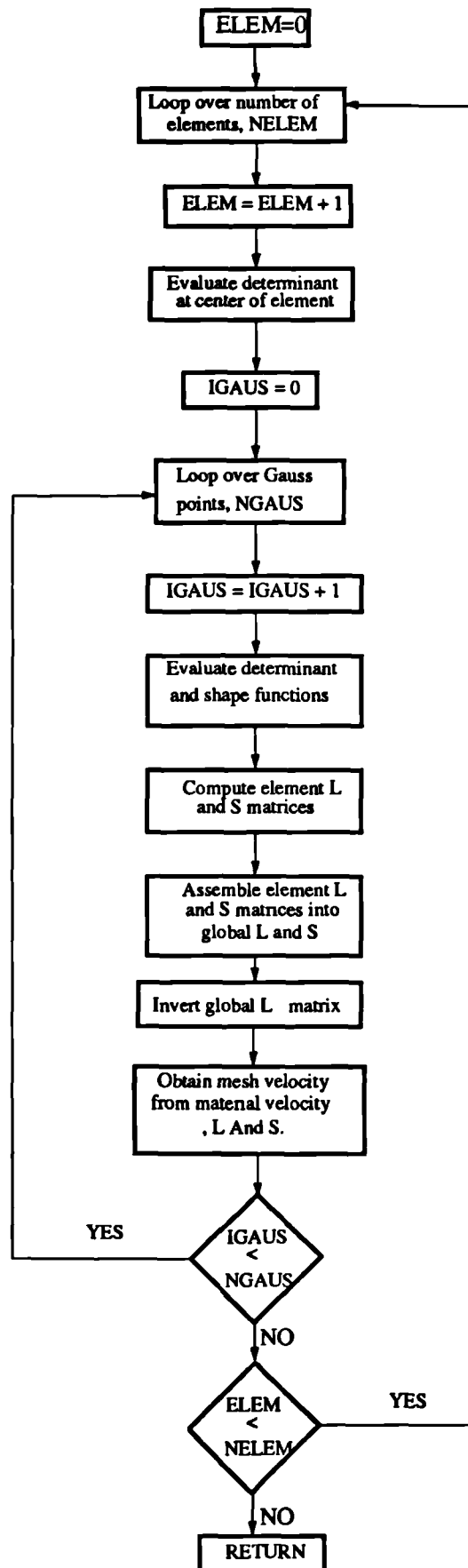


Figure B.9 Flow-Chart For Subroutine MSHVEL

APPENDIX C

DERIVATION OF THE PLANE WAVE APPROXIMATION AS A NON-RADIATING BOUNDARY FORMULATION

The motion, of an infinite elastic fluid medium, is governed by:

$$\frac{\partial^2 \phi}{\partial t^2} = c^2 \nabla^2 \phi \quad (\text{C.1})$$

where ϕ is the velocity potential defined as

$$\underline{v} = \nabla \phi$$

Assuming the potential ϕ has no singularity outside a closed interaction surface S (wet surface of the structure), then Kirchoff retarded potential formulation may be expressed as:

$$4 \pi \epsilon_c \phi_p = - \int_s \left\{ R^{-1} \frac{\partial \phi_s}{\partial n} + \beta R^{-2} \left[\phi_p + \frac{R}{c} \dot{\phi}_p \right] \right\} ds + 4 \pi \epsilon \phi_{inc} \quad (\text{C.2})$$

where

ϕ_p is the velocity potential at point p , defined by position vector $\underline{x}_p$,

$R = |\underline{x} - \underline{x}_s|$, is the distance from the field point p to a typical point s defined by position vector $\underline{x}_s$ on the surface S ,

$\underline{n}$ is the unit outward normal (positive going into the fluid) at point s ,

$\epsilon_c = 1, 1/2$, or 0 depending on whether the field point is within the fluid, on the surface S (assumed to be smooth) or inside S respectively,

β is the angle formed by vectors $\underline{l}$ and $\underline{R}$,

$\underline{l}$ is the vector defined by $\underline{x} - \underline{x}_s$, and

ϕ_{inc} is the velocity potential due to incident waves.

For deriving surface interaction approximations, the field point is considered to be on S ($\epsilon_c = 1/2$) and the incident wave potential ϕ_{inc} is dropped. Hence, only the scattered

component of the pressure field will be retained:

$$2 \pi \phi_p = - \int_s \left\{ R^{-1} v_n - \beta R^{-2} \left[\phi_p + \frac{R}{c} \dot{\phi}_p \right] \right\} ds \quad (C.3)$$

where $v_n = \frac{\partial \phi_s}{\partial n}$ is the normal velocity at point s .

Equation C.3 is a homogenous integro-differential equation in which all variables pertain to S . The initial conditions are:

$$\phi(t) = d(t) = 0 \text{ for } t \leq 0$$

$$\dot{\phi} = \frac{\partial \phi}{\partial n} = 0 \text{ for } t < 0$$

Early time approximations are differential equations whose solutions approximate that of Equation C.3 early in the transient approximation:

$$c t \ll a$$

where a is the characteristic length of the surface S , shown in Figure C.1.

The contributing surface for a field point p is defined by the surface S_p , whose boundary is defined by $r \leq c t$. A section of S_p is considered as shown in Figure C.1. The equation of S_p in the local coordinate system may be defined as:

$$z = \kappa_x x^2 + \kappa_y y^2 + O(R^3) \quad (C.4)$$

where κ_x and κ_y are principal curvatures at p . By substituting Equation C.4 into C.3, carrying out various algebraic manipulations, integration around p , integration by parts in the time domain, application of initial conditions and neglect of higher order terms as discussed by Felippa (1980), Equation C.3 becomes:

$$\underline{p} - c \kappa \bar{\underline{p}} = \rho c \underline{v}_n \quad (C.5)$$

where

a superposed dash denotes temporal integration and $\kappa = \frac{(\kappa_x + \kappa_y)}{2}$ is the mean curvature at point p .

Differentiating Equation C.5 with respect to time yields:

$$\dot{\underline{p}} - c \kappa \underline{p} = \rho c \dot{\underline{y}}_n \quad (\text{C.6})$$

The above equation is usually referred to as the Curved Wave Approximation. The Plane Wave Approximation is arrived at by setting the mean curvature in Equation C.6 to zero:

$$\dot{\underline{p}} = \rho c \dot{\underline{y}} \quad (\text{C.7})$$

A more detailed derivation may be found in the work reported by Felippa (1980) and Sobolev (1964).

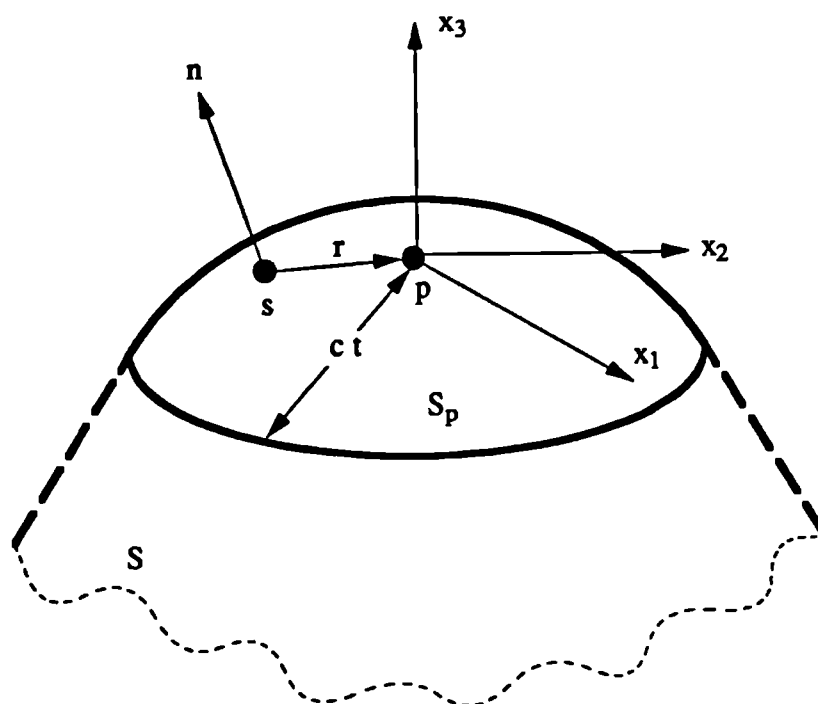


**Figure C.1 Local Coordinate System for the
Retarded Kirchhoff Potential Formulation**

APPENDIX D

GOVERNING EQUATION OF STEP AND EXPONENTIAL PLANE
SHOCK WAVES PROPAGATING IN AN ELASTIC MEDIUM

The equation of motion, for an elastic medium, in terms of displacements has been derived by many authors, e.g. Timoshenko and Goodier (1970):

$$(\lambda + G) \frac{\partial e}{\partial x_i} + G \frac{\partial^2 d_i}{\partial x_i^2} + B_i = \rho \frac{\partial^2 d_i}{\partial t^2} \quad (D.1)$$

in which $e = \epsilon_{x_1} + \epsilon_{x_2} + \epsilon_{x_3}$ is the volume expansion and ϵ_{x_i} is the strain component in the i^{th} direction.

By assuming the deformation produced by the wave is not accompanied by rotation, i.e.

$$\frac{\partial d_i}{\partial x_j} - \frac{\partial d_j}{\partial x_i} = 0 \quad (D.2)$$

the displacements are derived from a single displacement potential function φ :

$$d_i = \frac{\partial \varphi}{\partial x_i} \quad (D.3)$$

Using Equation D.2 and assuming the body forces to be zero, Equation D.1 becomes:

$$(\lambda + 2G) \nabla^2 d - \rho \frac{\partial^2 d_i}{\partial t^2} = 0 \quad (D.4)$$

Waves satisfying the above equation are called irrotational waves or waves of dilatation. At a great distance from the disturbance such waves may be treated as plane waves, where it is assumed that all particles are moving parallel to the direction of wave propagation or perpendicular to it. If the x -axis is taken as the direction of wave propagation, then

$$d_{inc2} = d_{inc3} = v_{inc2} = v_{inc3} = 0$$

while d_{inc1} and v_{inc1} are a function of x_1 only. Hence, Equation D.4 becomes:

$$\frac{\partial^2 d_1}{\partial t^2} = c^2 \frac{\partial^2 d_1}{\partial x^2} \quad (D.5a)$$

or in terms of the displacement potential:

$$\frac{\partial^2 \varphi}{\partial t^2} = c^2 \frac{\partial^2 \varphi}{\partial x^2} \quad (D.5b)$$

It can be easily seen that the wave function chosen in Chapter Four readily satisfies the above equation:

$$\varphi_I = -\frac{1}{2} P_o (\tau_{nd} - x_{nd} - 1)^2 H(\tau_{nd} - x_{nd} - 1) \quad (4.8)$$

The incident displacement and velocity vectors, corresponding to the above incident displacement potential function, are expressed as:

$$d_{inc\ i} = \frac{\partial \varphi_I}{\partial x_i} = P_o (\tau_{nd} - x_{nd} - 1) H(\tau_{nd} - x_{nd} - 1) \quad (4.11)$$

$$v_{inc\ i} = P_o H(\tau_{nd} - x_{nd} - 1) \quad (4.12)$$

The stress is obtained from the displacement potential by the following sequence:

$$\sigma_{inc}^l = (\lambda + 2 G \delta_{11}) \epsilon_{inc}^l \quad (D.6)$$

where ϵ_{inc} is the strain caused by the incident wave and the superscript l refers to the local direction.

$$\text{but } (\lambda + 2 G \delta_{11}) = \rho c^2 \text{ and } \epsilon_{inc}^l = \frac{v_{inc}^l}{c}$$

By substituting the above two expressions into Equation D.6, we get

$$\sigma_{inc}^l = \rho c v_{inc}^l \quad (D.7)$$

APPENDIX E

IMPLEMENTATION OF THE NON-RADIATING BOUNDARY FORMULATION INTO THE GENERAL PURPOSE PROGRAM ASAS-NL

In this appendix, the main subroutines used in the implementation of both the Plane Wave Approximation technique and the incident plane shock waves into ASAS-NL are presented. These may be divided into two main categories: those that are either at the global level or at a local (element) level.

E.1 GLOBAL LEVEL:

In order to change the structure of ASAS-NL to account for the newly proposed developments, two main requirements were carried out:

- Modification of the global solution procedure to read the data relating to the non-radiating boundary and the position, magnitude and profile of the shock incident wave.

E.2 LOCAL OR ELEMENT LEVEL :

- Computation of element high frequency force and the corresponding residual force.
- Computation of the element displacement and velocity vectors due to the incident plane wave.
- Computation of the element force due to the incident plane wave.

Table E.1 presents a brief description of the main subroutines, modified or newly developed, in this study; while Figures E.1 through E.4 show the flow-charts of these subroutines.

Subroutine	Description
DAAFEM	Controls the global assembly and solution procedures for the Plane Wave Approximation formulation.
HFELNC HFXLNC	Formulates the Plane Wave Approximation terms.
INCDNT	Calculates the displacement and velocity vectors due to the incident wave.
INCIDF	Calculates the force vector due to the incident wave.

**Table E.1 Subroutines Used in the
Implementation of the Far-Field Formulation**

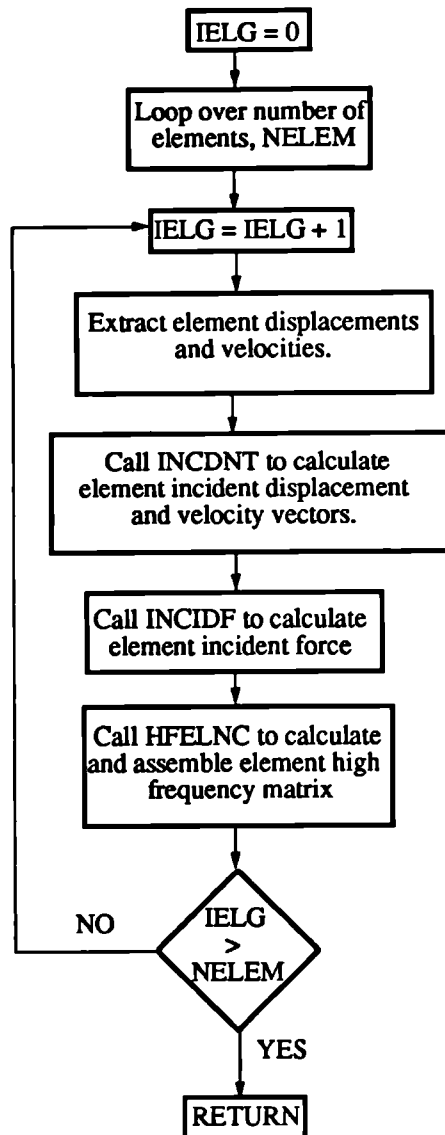


Figure E .1 Flow-Chart for Subroutine DAAFEM

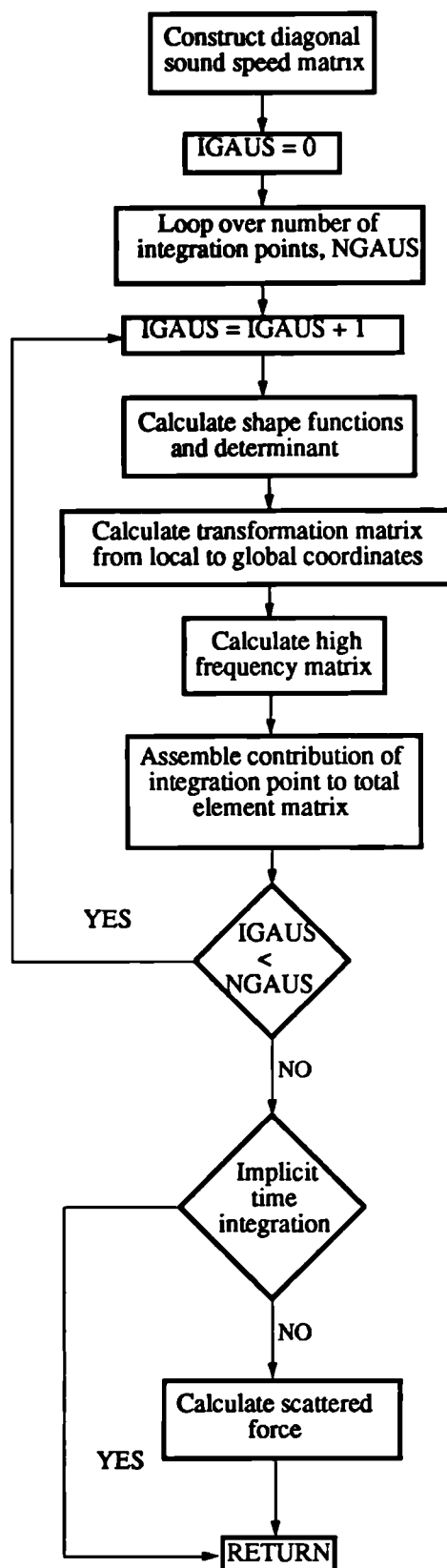


Figure E.2 Flow-Chart for Subroutine HFELNC

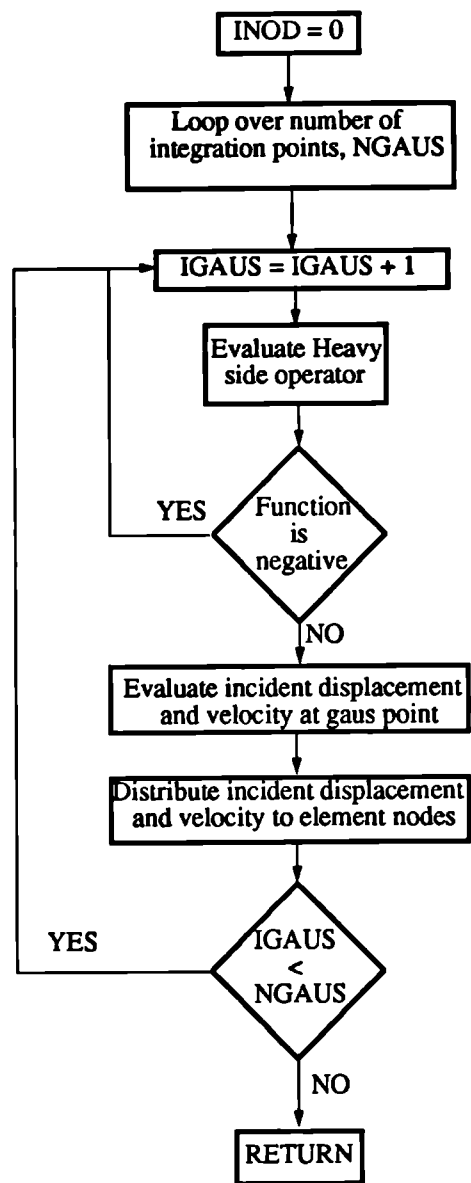


Figure E.3 Flow-Chart for Subroutine INCNT

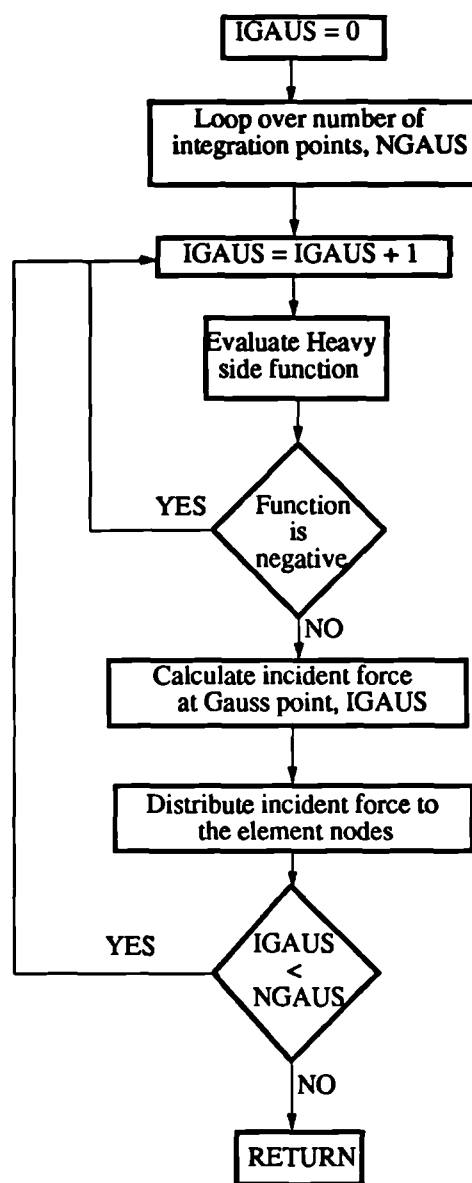


Figure E.4 Flow-Chart for Subroutine INCIDF