

THE TURBULENT WALL JET, ITS DEVELOPMENT  
AND FILM COOLING EFFECTIVENESS

by

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### SUMMARY

Three distinct, but related, theories for the prediction of wall jet hydrodynamic development and adiabatic film-cooling effectiveness are presented and predictions obtained with their use compared with the experimental data obtained as part of the present work and with data of other authors.

The three theories can be respectively described as a parametric entrainment theory, an explicit shear-work-integral theory, and a parametric shear-work-integral theory. The advantages and limitations of each theory are discussed.

The comparisons of prediction with experiment show generally satisfactory agreement, both with regard to hydrodynamic development and drag, and with regard to film cooling effectiveness.

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#### 1.4 Previous Work and its Relation to the Present Work

The turbulent wall jet has long found practical application in the control of surface temperatures, but it was not until fairly recently that performance requirements became stringent enough to stimulate detailed and systematic investigations of wall jet behaviour. To the author's knowledge, the earliest reported investigation of the effectiveness of a turbulent wall jet for controlling a surface temperature was that of Wieghardt (1946); the particular application of interest was aircraft de-icing. The introduction of higher operating temperatures in aircraft gas turbine engines in the 1950's (see, for example Halls, (1963) ) further stimulated interest in the effectiveness of the turbulent wall jet as a means of surface temperature control and many experimental investigations were undertaken; a review of the data available in 1963 was given by Spalding et al. (1963).

We may observe here that the methods of correlation used by the various authors up to this time were based on simple and unsophisticated jet or boundary-layer models, or some combination thereof, with substantial empirical inputs; as a consequence they neither correlated the available data well, nor were they suitable for application to flow situations which differed from the constant property, zero pressure gradient, two-dimensional configuration in which most measurements were nominally obtained. Further, two different correlations were normally required to account for flows

where the injection to freestream velocity is greater than unity and for flows where this ratio is less than unity; both cases are of interest since values of this ratio ranging from 0.2 to 1.5 are found in practice. The correlation equation suggested by Spalding, Jain and Nicoll (1963) overcame this particular deficiency, but still did not correlate the data adequately.

Another source of interest in the behaviour of turbulent wall jets were the observations of Coanda (1932, 1934, 1936) that the tangential injection of high velocity fluid into the boundary layer could be utilized to delay or prevent separation in flow over curved surfaces. A summary of the then existing state of knowledge concerning this effect, and a useful bibliography, may be found in the paper by Wille and Fernholtz (1965).

The first attempt to provide a theoretical explanation of turbulent wall jet behaviour was presented by Glauert (1956). In his theory the flow was divided into two regions; a region near the wall which was assumed to behave like a turbulent boundary layer, and an outer region which behaved like a shear-layer. In both regions the velocity profiles were assumed to conform to particular similarity profile expressions. This division of the flow into two distinct regions has been retained by most other authors; in this connection we may mention the work of Myers, Eustis, and Schauer (1961), Kruka and Eskinazi (1963), Patel (1961), Patel and Newman (1961), and Eriam and Eskinazi (1964).

In 1964 Spalding (1964a) recognized that the "wall-plus-wake" velocity profile expressions, of the sort popularized by Coles (1956) for boundary layer flows could also represent wall jet flows for suitable values of the parameter describing the wake contribution; as a consequence he was able to treat both conventional wall-jet flows (i.e. flows in which the velocity profile displays a maximum) and conventional boundary layer flows within a single theoretical hydrodynamic structure. This contribution was of major significance theoretically, and also practically, since the decay and disappearance of the velocity maximum can be anticipated in many practical applications. In his theory, Spalding also proposed a conserved property (for example, enthalpy) profile of the wall-plus wake form with the aid of which the thermal development of the flow could be predicted.

It was at this stage that the present author became associated with this work, and indeed the initial portions of the present thesis are concerned with the verification of the assumptions made by Spalding and the comparison of predictions made with his theory with experiment. Initially it appeared convenient for the present writer to confine his attention to flows with a velocity maximum, and for Dr. Escudier to examine the implications for conventional boundary layer flows. As the work progressed it became more and more apparent that this division was largely artificial and sometimes inappropriate. As a consequence

there arose the close co-operation between the present writer and Dr. Escudier that will be evident in the body of this thesis.

The examination of the implications of Spalding's original entrainment-based theory indicated certain deficiencies; specifically: the theory cannot be applied to confined flows in which there is no inviscid region, the boundary-layer separation predictions were poor, and finally the thermal development of the flow was poorly predicted. Attention was therefor diverted to methods based on the integral kinetic-energy-deficit equation; such methods constitute the bulk of the present work.

#### 1.5 Outline of the Present Thesis

The mathematical framework of the present work is examined in Chapter 2, and experimental velocity, conserved property, and shear stress profiles in Chapter 3. It is the latter examination which provides the justification for assuming, as will be done throughout the present work, that the velocity profiles may be represented by a two-parameter expression rather than the three-parameter expressions usually assumed in the multi-layer theories, that with certain exceptions the conserved property profile for adiabatic or impervious walls may be regarded as a function of a single additional parameter, and finally that the Prandtl mixing length distribution can be used with the velocity profile expression to obtain the shear stress distribution with reasonable accuracy.

In Chapter 4 the Spalding entrainment-based theory is recapitulated and the entrainment function is determined from experimental data. Initial condition correlations are presented which allow initial values of the dependent variables in the hydrodynamic equations to be determined from quantities over which the designer has control, namely, the injection to free-stream velocity ratio and the injection slot Reynolds number. Comparisons of predicted with experimental hydrodynamic development are presented, and it is shown that generally satisfactory, though by no means perfect, predictions are obtained. Finally, a critical discussion of the limitations of this approach is presented.

An explicit theory based on the integral momentum and kinetic-energy-deficit equations is put forward in Chapter 5; both hydrodynamic and  $\delta$  development are included. The generally useful contributions include a shape factor relationship for wall-jet flows, a drag law valid for flows ranging from a wall jet in still surroundings at one extreme to a separating boundary layer at the other, a shear-work-integral relation valid for wall-jet flows and initial condition correlations for the hydrodynamic variables and the film-cooling effectiveness. The comparisons of experimental and predicted hydrodynamic development are satisfactory for a wide range of flows, and the decay and disappearance of the velocity maximum is adequately determined.

Further, it is shown that, when the boundary layer upstream of the injection slot is "thin", the film-cooling effectiveness is also satisfactorily predicted.

A parametric shear-work-integral theory is presented in Chapter 6, and preliminary comparisons of predictions with the author's experimental data for impervious wall effectiveness for argon injected into air indicate that the effect of density non-uniformities may be accounted for using this approach.

A summary of the present work and suggestions for future work are presented in Chapter 7.

Finally, experimental wall-jet drag data from various sources are tabulated in Appendix A; tabulations of the present data for wall-jet velocity profile development and for impervious-wall effectiveness development are presented in Appendices B and C respectively, as are descriptions of the present experimental apparatus and procedures. It is hoped that these tabulations may prove of value to other workers.

- Coanda, H., Procédé de propulsion dan une fluide, Brevet d'Invention Gr.C1.2, No.762688, Republic Francaise (1932).
- Coanda, H., Procédé et dispositif pour faire derier une veine de fluid penetrant dans une autre fluide, Brevet d'Invention Gr.5, C1.2, No.792754, Republic Francaise (1934).
- Coanda, H., Device for deflecting a stream of elastic fluid projected into an elastic fluid, U.S. Patent Office 2,052,869 Sept (1936).
- Halls, G. H., "Some problems associated with cooling gas turbine combustion chambers, Rolls Royce Ltd., Aero. Engine Division, Reference TB/GAH/ 1/MB (1963).
- Spalding, D. B., Jain, V. K., Nicoll, W. B., "Film cooling in incompressible turbulent flow: examination of experimental data for the adiabatic wall temperature", Aero. Res. Council Rep. no.25,311 (1963).
- Spalding, D. B., Stollery, J. L., Cole, E. H., Jain, V.K., and Peerless, S. J., "Film cooling in incompressible turbulent flow: examination of existing experimental data", Mech. Eng. Dept., Report No. IC/HRJ/10, Imperial College, January (1963).
- Wille and Fernholtz, "Report on the first European Mechanics Colloquium on the Coanda effect", J. Fluid Mech. Vol. 23, pt. 4, Dec. 1965.

## CHAPTER 1

## INTRODUCTION

1.1 The Problem Considered

The physical problem to be considered is shown schematically in Fig. 1.1-1. A secondary fluid, not necessarily of the same enthalpy or composition as the mainstream fluid, is injected tangentially into a turbulent boundary layer. How can the subsequent development of the velocity, enthalpy, and composition profiles be predicted? How will this development be affected by pressure variation? How can the drag exerted on the solid boundary be computed? These questions, and methods for obtaining their answers, will be the main concern of the present work.

The practical relevance of the above questions need not be emphasized; it is sufficient to mention that such injection of fluid into a boundary layer has practical applications of considerable economic importance in combustion chamber and nozzle design, where it is used to protect a surface from the consequences of exposure to high temperature combustion products; and in boundary layer control, where it is used to energize the boundary layer fluid which might otherwise separate from the wall.

When the velocity of the injected fluid is greater than that of the mainstream fluid, the velocity profiles for some distance downstream of the injection slot will display a velocity maximum, the exact distance depending mainly on the injection to mainstream velocity

and density ratios, being greater for larger values of either, and the mainstream velocity variation. When this maximum is present, the flow is conventionally termed a "wall-jet" flow. The present work will be primarily concerned with such flows. However, since in many circumstances the velocity maximum decays and finally disappears within downstream distances of practical importance (practical injection to mainstream velocity ratios being in the range of, say, 0.2 to 2) it will also be necessary to consider also the problem of the decay of this maximum.

## 1.2 The Viewpoint Adopted

Because of the underlying complexity of turbulent phenomena in general, and turbulent shear flows in particular, and because the primary objective of the present work is the development of practical prediction methods, the viewpoint adopted will be phenomenological. That is, the methods evolved will be based on regularities observed in the time mean properties of the flow, composition, enthalpy, and stress fields. No consideration will be given explicitly to the structural properties of the turbulence per se which give rise to these regularities. It will be assumed that the usual Prandtl boundary layer equations adequately describe the actual phenomena if the turbulent contributions to the shear stress, and Prandtl/Schmidt numbers are taken into account. Since even with these approximations the resulting equations are non-linear partial differential equations involving terms which can only be approximately related to the other variables (specifically, those

terms accounting for diffusion of momentum, etc.), the solution methods adopted are approximate ones which attempt to satisfy the governing equations "on the average", rather than at every point. Such methods are known mathematically as methods of weighted residuals\*, and, in the field of fluid mechanics, usually as "profile" methods, and are ideally suited to the solution of those problems where empirical knowledge about certain aspects of the solution are known, such as regularities in the velocity profiles.

### 1.3 Restrictions

The restrictions to which the flows considered in the present work are:

- i) steady, two dimensional, turbulent flow
- ii) hydrodynamically smooth walls
- iii) impermeable, adiabatic walls
- iv) no appreciable body forces, such as bouyancy, gravitational, or centrifugal forces.

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\* For a discussion of the characteristics of such solution methods in general see Hildebrand (1952). For a discussion of their applications to fluid flow problems see Spalding (1966).

## CHAPTER 2

### THE MATHEMATICAL FRAMEWORK

In the present chapter the governing equations and integral equations derived from them are presented. Explicit and parametric methods of solution are discussed. The notation and choice of variables are largely those of Patankar and Spalding (1965). The latter are chosen so as to reduce to a minimum the numerical computation required for non-uniform density flows; specifically, they are chosen so as to reduce to a minimum the number of integral properties of the velocity and enthalpy profiles which have to be determined by numerical integration. For uniform density flows they reduce to those of Spalding (1964a).

#### 2.1 The Governing Equations

With the usual Prandtl boundary-layer approximations the equations expressing the conservation of mass and momentum are:

$$\frac{\partial}{\partial x} (\rho u) + \frac{\partial}{\partial y} (\rho v) = 0, \quad 2.1-1$$

and

$$\rho u \frac{\partial u}{\partial x} + \rho v \frac{\partial u}{\partial y} = \rho_G u_G \frac{du_G}{dx} + \frac{\partial \tau}{\partial y} \quad 2.1-2$$

where  $x$  and  $y$  are the space coordinates measured tangentially to, and normal to, the bounding surface, respectively;  $u$  and  $v$  are the

velocity components in the  $x$  and  $y$  directions respectively;  $\rho$  is the fluid density;  $\tau$  is the shear stress in the  $x$ -direction; and where the subscript  $G$  indicates the fluid state at the outer edge of the boundary layer. The pressure gradient term,  $dP/dx$ , has been eliminated from the momentum equation with the aid of Bernoulli's equation, which is assumed to apply in the invicid region just outside the boundary layer; i.e.

$$\frac{dP}{dx} = - \rho_G u_G \frac{du_G}{dx} \quad 2.1-3$$

When the density of the fluid varies due to enthalpy or composition gradients, the local value can be found from the fluid equation of state and local values of enthalpy and composition. For low speed flows the equation expressing the conservation of enthalpy and composition (or more precisely, mass fraction) have the same form. Letting  $\varphi$  stand for either we have:

$$\frac{\partial}{\partial x} (\rho u \varphi) + \frac{\partial}{\partial y} (\rho v \varphi) = - \frac{\partial J_\varphi}{\partial y}, \quad 2.1-4$$

where  $J_\varphi$  is the total (molecular plus eddy) diffusional  $\varphi$  flux in the  $y$ -direction. It has been assumed that the diffusion in the  $x$ -direction can be neglected.

If we now introduce the dimensionless variables:

$$R_x \equiv \int_0^x \frac{\rho_G u_G dx}{\mu}, \quad 2.1-5$$

$$\eta \equiv \frac{\int_0^y \rho dy}{\int_0^{y_G} \rho dy} \quad 2.1-6$$

$$z \equiv u/u_G \quad 2.1-7$$

$$\theta \equiv \frac{\varphi - \varphi_G}{\varphi_S - \varphi_G} \quad 2.1-8$$

$$\chi \equiv \rho/\rho_G \quad 2.1-9$$

$$R_G \equiv \frac{\rho_G u_G y_G}{\mu} \int_0^1 \chi^{-1} d\eta \quad 2.1-10$$

$$s \equiv \tau/\rho_G u_G^2 \quad 2.1-11$$

and

$$S \equiv J_\varphi'' / \rho_G u_G (\varphi_S - \varphi_G) \quad 2.1-12$$

[The subscript  $_S$  refers to the state of the fluid at the wall.]

the momentum and  $\varphi$  conservation equations can be written:

$$R_G z \frac{\partial z}{\partial R_x} \left\{ R_G \int_0^\eta z d\eta \right\} \frac{\partial z}{\partial \eta} +$$

$$R_G (z^2 - \chi^{-1}) \frac{d \ln u_G}{d R_x} = \frac{\partial s}{\partial \eta} \quad 2.1-13$$

and

$$\frac{R_G z}{\varphi_S - \varphi_G} \frac{\partial}{\partial R_x} \left\{ \theta (\varphi_S - \varphi_G) \right\} -$$

$$\frac{\partial \theta}{\partial \eta} \frac{\partial}{\partial R_x} \left\{ R_G \int_0^\eta z d\eta \right\} = - \frac{\partial S}{\partial \eta} \quad 2.1-14$$

In the derivation of equation (2.1-14) it has been assumed that  $\varphi_G$  does not vary with  $x$ . For the problem under consideration, and indeed for most practical situations, this assumption is valid. The algebra leading to equations (2.1-13) and (2.1-14) is straightforward if use is made of the easily verified formulae:

$$\int_0^{y_G} \rho dy = \frac{\rho_G u_G}{\int_0^1 x^{-1} d\eta} \quad 2.1-15$$

and

$$\rho dy = \frac{\rho_G u_G d\eta}{\int_0^1 x^{-1} d\eta} \quad 2.1-16$$

## 2.2 The Integral Equations

The method of solution adopted is one which has almost become standard for turbulent boundary-layer flows, namely, to reduce the governing partial differential equations to a set of ordinary differential equations by integration across the boundary layer. The dependent variables in these equations are then integral properties of the velocity, density and  $\varphi$  profiles.

When equation (2.1-13) is integrated with respect to  $\eta$  from  $\eta = 0$  to  $\eta = 1$ , with boundary conditions:  $z = 0$  at  $\eta = 0$  and  $z = 1$ ,  $s = 0$  at  $\eta = 1$ , the result, the integral momentum equation, can be written:

$$\frac{d}{dR_x} \{R_G(I_1 - I_2)\} + R_G(I_{\chi,0} - I_2) \frac{d \ln u_G}{dR_x} = s_s \quad 2.2-1$$

Similarly, when equation (2.1-13) is multiplied by  $z^*$  and then integrated the resulting equation, the kinetic-energy-deficit equation, is:

$$\frac{d}{dR_x} \{R_G(I_1 - I_3)\} + 2R_G(I_{\chi,1} - I_3) \frac{d \ln u_G}{dR_x} = 2\bar{s}, \quad 2.2-2$$

where the I's are integral properties of the velocity and density profiles defined by:

$$I_1 \equiv \int_0^1 z d\eta \quad 2.2-3$$

$$I_2 \equiv \int_0^1 z^2 d\eta \quad 2.2-4$$

$$I_3 \equiv \int_0^1 z^3 d\eta \quad 2.2-5$$

$$I_{\chi,0} \equiv \int_0^1 \chi^{-1} d\eta \quad 2.2-6$$

$$I_{\chi,1} \equiv \int_0^1 z\chi^{-1} d\eta \quad 2.2-7$$

and where  $\bar{s}$ , the shear work integral, is defined by

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\* The choice of  $z$  as the weighting function is arbitrary, although frequently used. Equally common usage is given to  $\xi$ . The advantages and disadvantages of various choices are discussed by Spalding (1960).

$$\bar{s} \equiv \int_0^1 s \frac{\partial z}{\partial \eta} d\eta \quad 2.2-8$$

When equation (2.1-14) is integrated with respect to  $\eta$  between the limits  $\eta = 0$  to  $\eta = 1$ , with the additional boundary conditions  $\theta = 0$  at  $\eta = 1$  and  $\theta = 1$ ,  $S = 0$  at  $\eta = 0$ , the result is the integral  $\varphi$ -conservation equation:

$$\frac{d}{dR_x} \{ (\varphi_S - \varphi_G) R_G I_{\theta,1} \} = 0 \quad 2.2-9$$

where

$$I_{\theta,1} \equiv \int_0^1 \theta z d\eta \quad 2.2-10$$

Equations (2.2-1), (2.2-2) and (2.2-9) can be written in more compact form as:

$$\frac{dR_2}{dR_x} + (1+H_{12})R_2 \frac{d \ln u_G}{dR_x} = s_S \quad 2.2-11$$

$$\frac{dR_3}{dR_x} + 2(H_{32}+H_X)R_2 \frac{d \ln u_G}{dR_x} = 2\bar{s} \quad 2.2-12$$

$$\frac{d}{dR_x} \{ R_{\theta 1} \epsilon \} = 0 \quad 2.2-13$$

where  $R_2$ ,  $R_3$  and  $R_{\theta 1}$  are Reynolds numbers based on the momentum, kinetic-energy, and  $\varphi$ -deficit thicknesses, viz:

$$R_2 = \frac{\rho_G u_G}{\mu} \int_0^{y_G} \frac{\rho}{\rho_G} \frac{u}{u_G} \left(1 - \frac{u}{u_G}\right) dy = R_G (I_1 - I_2) \quad 2.2-14$$

$$R_3 = \frac{\rho_G u_G}{\mu} \int_0^{y_G} \frac{\rho u}{\rho_G u_G} \left(1 - \frac{u^2}{u_G^2}\right) dy = R_G (I_1 - I_3) \quad 2.2-15$$

$$R_{\theta 1} = \frac{\rho_G u_G}{\mu} \int_0^{y_G} \frac{\rho u}{\rho_G u_G} \theta dy = R_G I_{\theta 1} \quad 2.2-16$$

and where the film cooling effectiveness  $\epsilon$  is defined by

$$\epsilon = \frac{\theta_s - \theta_G}{\theta_c - \theta_G} \quad 2.2-17$$

$H_{12}$ ,  $H_{32}$ , and  $H_X$  are shape factors defined by

$$H_{12} = R_1/R_2 \quad 2.2-18$$

$$H_{32} = R_3/R_2 \quad 2.2-19$$

$$H_X = \frac{\rho_G u_G}{\mu} \int_0^{y_G} \frac{u}{u_G} \left(1 - \frac{\rho}{\rho_G}\right) dy / R_2 \quad 2.2-20$$

where

$$R_1 = \frac{\rho_G u_G}{\mu} \int_0^{y_G} \left(1 - \frac{\rho u}{\rho_G u_G}\right) dy = R_G (I - I_1) \quad 2.2-21$$

For uniform density flows  $H_X$  is zero.

One further integral equation will be required; the integral mass conservation equation. Integration of equation (2.1-1) with respect to  $y$  between the limits  $y = 0$  to  $y = y_G$ , and substitution of the boundary condition  $v = 0$  at  $y = 0$  yields:

$$\frac{d}{dx} \int_0^{y_G} \rho u dy = -\rho_G v_G + \rho_G u_G \frac{dy_G}{dx} \quad 2.2-22$$

The right-hand side of this equation represents the total rate of entrainment of mass from the mainstream into the boundary layer. Equation 2.2-22 may be written:

$$\frac{d}{dx} \int_0^{y_G} \rho u dy = -\dot{m}_G'' \quad 2.2-23$$

where the entrainment rate,  $-\dot{m}_G''$  is defined by:

$$-\dot{m}_G'' \equiv -\rho_G v_G + \rho_G u_G \frac{dy_G}{dx} \quad 2.2-24$$

If we introduce the non-dimensional variables:

$$R_m \equiv \int_0^{y_G} \frac{\rho u dy}{\mu} = R_G I_1 \quad 2.2-25$$

$$m_G \equiv \frac{\dot{m}_G''}{\rho_G u_G} \quad 2.2-26$$

equation (2.2-22) becomes:

$$\frac{dR_m}{dR_x} = -m_G \quad 2.2-27$$

### 2.3 Some Considerations

At this point it is appropriate to ask whether or not the integral equations so far derived can serve as an adequate foundation for prediction methods. The answer depends upon the complexity of the velocity and  $\varphi$  fields to be considered. It was the contention of Spalding (1964a) that the velocity profiles for both wall jets (sufficiently far removed from the injection slot) and boundary layers can be adequately represented by a two-parameter family of curves; and can therefore be characterized by two independent quantities. Thus, for uniform density flows, equation (2.2-11) and either equation (2.2-13) or equation (2.2-27) should suffice for the prediction of hydrodynamic development. An insight into the correctness of this contention can be obtained from an examination of the experimental profiles presented in the next section. Ultimately, of course, its correctness must be judged by the extent to which predictions agree with experiment.

With regard to the prediction of the  $\varphi$  field development, equation (2.2-13) will be adequate in those circumstances in which the  $\varphi$  profile can be adequately characterized by a single quantity in addition to the hydrodynamic quantities.

Examination of the data presented in the next section indicates that, when heat transfer is absent and when the velocity and boundary layers can be considered co-extensive, only a single additional parameter is required, namely, the value of  $\varphi$  at the surface.

We shall hereafter refer to methods in which the hydrodynamic

development prediction is based on the integral momentum and mass conservation equations as "entrainment" methods; those in which the prediction is based on the integral momentum and kinetic energy equations will be termed "shear-work-integral" methods.

In order to solve the relevant differential equations we need, in addition to the initial conditions and the variation of  $u_G$  with position, auxiliary formulae relating  $s_S$ ,  $H_{12}$ ,  $\bar{s}$ , and  $R_{\theta 1}$ , to the dependent variable of the differential equations. Knowledge of these relations can be obtained from the examination of experimental velocity, shear stress, and  $\varphi$  profiles; we therefore turn our attention to these profiles in the next section.

Before doing so, however, it is useful to distinguish between what will be called "explicit" and "parametric" methods of solution. The former appellation indicates the use of explicit auxiliary formulae in terms of  $R_2$ ,  $R_3$  and  $R_{\theta 1}$ , the dependent variables of the differential equations. The latter appellation refers to the fact that, in these methods, both the auxiliary quantities, such as  $s_S$ , and the dependent variables  $R_1$ ,  $R_2$  and  $R_{\theta 1}$  are expressed not in terms of each other, but rather in terms of certain "parameters" which occur in the formulae which describe the various profiles.

Explicit methods have the advantage of computational simplicity and, because the required empirical inputs are expressed directly in the auxiliary formulae, are easy to modify to fit experimental fact. Conversely, parametric methods have the twin advantages of greater flexibility and reduced reliance on empirical inputs. Parametric methods,

however, frequently result in sets of integral equations which possess singularities that have no physical counterpart. This difficulty has been discussed by Spalding (1966); we shall encounter examples below.

We shall use both explicit and parametric methods.

### CHAPTER 3

#### THE VELOCITY, SHEAR-STRESS, AND $\phi$ PROFILES

##### 3.1 The Velocity Profile

Typical wall-jet velocity profiles are shown in Figure 3.1-1. The object here is to obtain a mathematical representation of such profiles.

In the near vicinity of the wall the velocity profile may be expected, from the usual Couette analysis, to obey the formula:

$$u^+ = \frac{1}{\kappa} \ln (E y^+) \quad 3.1-1$$

where

$$u^+ \equiv u / \bar{\tau}_S / \rho_S \quad 3.1-2$$

$$y^+ \equiv y \cdot \bar{\tau}_S \rho_S / \mu \quad 3.1-3$$

and where, for smooth impermeable walls,  $\kappa$  and  $E$  are constants.\*

Various authors Cole (1965, 1962), Chi (1962), Spalding (1964a, 1965) etc., have recommended values for these constants; we shall generally follow Spalding (1964a) and use:

$$\kappa = 0.4 \quad 3.1-4$$

$$E = 6.542 \quad 3.1-5$$

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\* [Of course, in the immediate vicinity of the wall, say  $\eta < 0.01$ , the velocity profile does not obey equation (3.1-1). In this region - the so-called "laminar sublayer" - a better approximation is  $u^+ = y^+$ . Since we shall be primarily concerned with integral properties of the profile, however, and since negligible error is introduced into these integrals by the neglect of the laminar sublayer, equation (3.1-1) will be assumed to obtain even in the immediate vicinity of the wall.]

The validity of equation (3.1-1) for boundary-layer velocity profiles is well established; its validity for wall-jet velocity profiles is less certain. The experimental measurements of several authors are shown in Figure 3.1-2; the heavy line corresponds to equation (3.1-1) with  $\kappa = 0.4$  and  $E = 6.542$ . There are considerable differences between the data of the various authors, but no discernible trend is apparent; presumably the disagreement is due to the difficulty of accurate measurement rather than any real physical effect. The measurements of Myers, Eustis, and Schauer (1961) seem to have been made with exceptional care, two separate methods of wall-shear-stress measurements having been used. The agreement between these data and equation (3.1-1) is better. All the measurements shown seem to indicate a value of  $\kappa$  somewhat greater than 0.4; in view of the uncertainties in the data, however, and because we desire the profile equation to represent both wall jets and boundary layers, it was not felt appropriate to modify the assumed value of  $\kappa$ .

In terms of the present notation equation (3.1-1) can be written:

$$z = \frac{s_S^{\frac{1}{2}}}{\kappa \chi_S^{\frac{1}{2}}} \ln \left\{ \frac{E s_S^{\frac{1}{2}} R_G \eta}{\chi_S^{\frac{1}{2}}} \right\} \quad 3.1-6$$

Now, this equation, the mathematical expression of the "law of the wall", cannot be expected to describe those regions of the velocity profile far removed from the wall since the assumptions from which it is derived are not valid in this region. Coles (1956) proposed an expression, supposedly valid for the whole of the boundary layer, which consisted of the linear sum of two components: the law of the

wall and another universal function,  $\omega(\xi)$ , called the "law of the wake".

Thus,

$$z = \frac{s^{\frac{1}{2}}}{\kappa \chi_S^{\frac{1}{2}}} \ln \left\{ \frac{E s^{\frac{1}{2}} R_G \eta}{\chi_S^{\frac{1}{2}}} \right\} + (1 - z_E) \omega(\xi) \quad 3.1-7$$

where  $z_E$  is a parameter whose meaning will be made clear later, and

where  $\xi$  is the uniform density value of  $\eta$ , viz:

$$\xi \equiv \frac{y}{y_G} \quad 3.1-8$$

Coles' attention was confined to essentially uniform density data, and hence there is no contradiction with his results in the assumption, which simplifies subsequent mathematics, that the argument of  $\omega$  is  $\eta$  rather than  $\xi$ ; in the cases Coles examined  $\eta$  and  $\xi$  are equivalent.

$$z = \frac{s^{\frac{1}{2}}}{\kappa \chi_S^{\frac{1}{2}}} \ln \left\{ \frac{E s^{\frac{1}{2}} R_G}{\chi_S^{\frac{1}{2}}} \right\} + (1 - z_E) \omega(\eta) \quad 3.1-9$$

Coles' recommended wake function,  $\omega$ , was presented in tabular form and, as was noted by Hinze (1959), differs but little from:

$$\omega(\eta) = \frac{1}{2} (1 - \cos \pi \eta) \quad 3.1-10$$

This cosine wake function is compared with Coles' in Figure 3.1-3.

Equation (3.1-9) can be simplified by the elimination of the drag coefficient,  $s_S$ . Substitution of the boundary condition  $z = 1$  at  $\eta = 1$  and the normalizing condition  $\omega(1) = 1$  yields the drag law.

\* We note that while this assumption is not contrary to available experimental evidence, neither is it supported by extensive experimental data.

$$s_S = \chi_S \left( \frac{\kappa z_E}{\ell'} \right)^2 \quad 3.1-11$$

where  $\ell'$  is a Reynolds number parameter defined by:

$$\ell' = \ln \left\{ \frac{ER_G s_S^{\frac{1}{2}}}{\chi_S} \right\} = \ln \{ \kappa ER_G z_E / \ell' \} \quad 3.1-12$$

Substitution of these equations into equation (3.1-9) yields:

$$z = z_E \left( 1 + \frac{\ln \eta}{\ell'} \right) + (1 - z_E) \omega(\eta) \quad 3.1-13$$

The meaning of the parameter  $z_E$  can now be clearly seen; its value is a measure of the relative contribution of the two components to the total velocity profile.

Spalding (1964a) proposed that equation (3.1-13) could be used to represent wall-jet velocity profiles as well as boundary-layer profiles; the difference between the two being the value of  $z_E$ . For  $z_E > 1$  the velocity profile exhibits a velocity maximum characteristic of wall-jet flows; for  $z_E < 1$  it does not. Typical profiles for  $z_E < 1$  and  $z_E > 1$  based on the cosine wake function, are shown in Figure 3.1-4.

Now, Coles' wake function (and its approximation, the cosine wake function), was obtained from the examination of experimental boundary-layer profiles (that is, from profiles without a velocity maximum) and therefore its presumed universality may be a consequence of the limited range of data examined; certainly its validity for wall-jet velocity profiles must be verified by comparison with experimental profiles. Comparisons of the uniform-density measurements of the present

author with the profiles given by equations (3.1-10) and (3.1-13) are shown in Figure 3.1-5. The values of  $z_E$  and  $\ell'$  were determined by the method of least squares fitting. This procedure gives the values of  $z_E$  and  $\ell'$  which minimize the square deviation  $R$ , defined by

$$R \equiv \sum_i (z(y_i) - z_i)^2 \quad 3.1-14$$

$z_i$  being the experimental value of  $z$  at  $y_i$ , and  $z(y_i)$  the value determined from equation (3.1-13). It may be observed that the assumed profile is indeed a fair representation of actual profiles, at least for moderate values of  $z_E$ . The deviations which do exist are consistent; the assumed profile giving values which are too low near the wall and near the outer edge, and too high in the intermediate region. Also, the position of the velocity maximum, although not its value, is in error, the predicted maximum occurring too far from the wall. These discrepancies are generally small, and appear to be proportional to  $z_E$ ; that is, proportional to the relative contribution of the wake component. This latter observation suggests that it is the wake function that is in error. In the limiting case of wall jets in still surroundings, i.e.  $z_E = \infty$ , the experimental profiles of Sigalla (1958), Bradshaw and Gee (1962) and Myers, Eustis and Schauer (1961) all are better represented by the appropriate limiting form of equation (3.1-13) with a linear wake function, viz:

$$\omega(\eta) = \eta \quad 3.1-15$$

It may be that  $\omega$  is not a universal function of  $\eta$ , but rather

a weak function of  $z_E$  as well as other variables.\* For our present purposes we shall neglect this possibility.

Escudier (see Escudier and Nicoll 1966a)) has shown that equations (3.1-10) and (3.1-13) can also represent experimental boundary-layer velocity profiles with satisfactory accuracy. Figure 3.1-16 is a reproduction of one of his comparisons.

For future reference we here present the limiting form of equation (3.1-13) for infinite Reynolds numbers. For this case  $l' = \infty$  (from equation (3.1-12)) and hence:

$$z = z_E + (1 - z_E) \omega(\eta) \quad 3.1-16$$

Values of drag computed from equation 3.1-11 are compared with uniform-density experimental values in Figure 3.1-7. In all cases  $z_E$  and  $l'$  were determined as explained previously; the experimental drag data of Kruka and Eskinazi (1964) was measured with a Preston tube while that of the present author was determined from a Clauser plot (Clauser (1954)), a typical example being shown in Figure 3.1-8. The agreement is satisfactory.

Escudier (see Escudier and Nicoll (1966a)) made the corresponding comparison for uniform density boundary-layer data ( $z_E < 1$ ) using a somewhat different procedure to obtain  $z_E$  and  $l'$ ; namely, determining their values from the experimental values of  $R_2$  and  $H_{12}$ . The result is shown in Figure 3.1-9 and again we note that except for the measurements

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\* Jayatillaka's (Jayatillaka (1966)) measurements indicate a possible dependence on surface roughness, a factor beyond the scope of the present work.

of Fage, and Falkner and Fage, the agreement between experiment and prediction is generally satisfactory.

### 3.2 The Shear Stress Profile.

In shear-work integral methods of prediction, a relation between the average (with respect to velocity) shear stress and the other profile parameters is required. Following the suggestion of Spalding (1965a), we shall examine the implications of the velocity profile formulae and Prandtl's mixing-length hypothesis. The latter can be written:

$$\tau = \rho \ell^2 \left| \frac{du}{dy} \right| \frac{du}{dy} \quad 3.2-1$$

where  $\ell$  is the mixing length. The physical arguments which underlie equation (3.2-1) do not need repeating here; we shall instead view this equation as the definition of  $\ell$ . In dimensionless form equation 3.2-1 may be written:

$$\begin{aligned} s &= \chi \lambda^2 \left| \frac{dz}{d\xi} \right| \frac{dz}{d\xi} \\ &= \chi^3 \lambda^2 I_{\chi,0}^2 \left| \frac{dz}{d\eta} \right| \frac{dz}{d\eta} \end{aligned} \quad 3.2-2$$

where the dimensionless mixing length  $\lambda$ , is defined by:

$$\lambda = \frac{\ell}{y_G} \quad 3.2-3$$

Escudier (1965), after an extensive review of uniform-density boundary-layer and wall-jet data, recommended the following  $\lambda$   $\xi$  relation:

$$\begin{aligned} \lambda &= \kappa \xi, & \xi < \lambda_1 / \kappa \\ \lambda &= \lambda_1 & 1 \geq \xi \geq \lambda_1 / \kappa \end{aligned} \quad 3.2-4$$

where:

$$\lambda_1 = 0.075 \quad 3.2-5$$

Shear stress profiles generated from the mixing-length hypothesis and the "logarithmic wall plus cosine wake" velocity profile are compared with the measured profiles of Erian and Eskinazi (1964) in Figure 3.2-1. These data are for a uniform-density wall jet with a finite mainstream velocity; the required values of  $z_E$  and  $\ell'$  were chosen to give agreement with the measured shear stress at the wall and the measured velocity maximum. Measured shear-stress profiles for wall jets in still surroundings ( $z_E \rightarrow \infty$ ) are compared with that implied by the present equations in Figure 3.2-2. The value of  $\ell'$  was taken as 10, a typical value. Although the quantitative agreement between the theoretical and experimental curves is by no means perfect, the agreement is sufficiently good to establish some confidence in the present equations as a guide to the behaviour of real systems. In particular, the shear work integral,  $\bar{s}$ , which is the area enclosed by the  $s(z)$  curve, is fairly accurately predicted. We may note here that Escudier's work shows that essentially the same comments can be made about the shear stress distribution in boundary layers. [See Escudier (1965)].

### 3.3 The $\varphi$ Profile.

As shown by Spalding (1964a), the  $\varphi$  profile may be represented as the sum of two components: one due to the  $\varphi$  transfer at the wall and one independent of this transfer; in analogy with the terminology

used for the velocity profile these components may be termed the "wall" and "wake" components. Since in the present study we are concerned only with adiabatic/impervious walls, only the latter component is of interest.

Patankar (1965) has reviewed the experimental data for wake component of the  $\varphi$  profiles in free turbulent flows, boundary layers, and wall-jets; Figures 3.3-1 are reproduced from his note. There is appreciable scatter in the data, and we may conclude that the  $\theta$  profile may either be approximated by:

$$\theta = 1 - \eta^b \quad 3.3-1$$

where the exponent  $b$  lies in the range:

$$1 \leq b \leq 1.5 \quad 3.3-2$$

or by:

$$\theta = 1 - \frac{\eta}{\Delta} \quad 3.3-3$$

where the constant  $\Delta$  lies in the range:

$$1 \leq \Delta \leq 1.25 \quad 3.3-4$$

## CHAPTER 4.

### A PARAMETRIC ENTRAINMENT THEORY FOR UNIFORM-DENSITY FLOWS.

In his Unified Theory paper, Spalding (1964a) developed a general theoretical framework for the prediction of friction, heat transfer, and mass transfer in turbulent wall jets and boundary layers. In terms of the present nomenclature his theory may be described as a parametric entrainment theory, involving the velocity-profile formula examined in the previous section (equation 3.1-13) and an empirical entrainment function. In the present chapter we examine data for uniform-density entrainment; present a recommendation for the entrainment function for wall-jet flows; and compare predicted and experimental uniform-density wall-jet hydrodynamic development. The merits and the drawbacks of the method are discussed.

#### 4.1 The governing equations.

We assume, following Spalding, that the hydrodynamic development is governed by the integral momentum and mass conservation equations, viz:

$$\frac{d}{dR_x} \{R_G(I_1 - I_2)\} + (1 - I_2) R_G \frac{d \ln u_G}{dR_x} = s_S \quad 4.1-1$$

$$\frac{dR_m}{dR_x} = -m_G \quad 4.1-2$$

In equation (4.1-1) we have set  $I_{x,0} = 1$  since we are

considering here only uniform-density flows. This equation may be rewritten as:

$$\frac{d}{dR_x} \left\{ (I_1 - I_2) \frac{R_m}{I_1} \right\} + \left( \frac{1 - I_2}{I_1} \right) R_m \frac{d \ln u_G}{dR_x} = s_S \quad 4.1-3$$

Performance of the indicated differentiation and substitution of equation (4.1-2) yields:

$$R_m \frac{d}{dR_x} \left( \frac{I_2}{I_1} \right) = \left( \frac{1 - I_2}{I_1} \right) R_m \frac{d \ln u_G}{dR_x} - s_S - m_G \left( \frac{I_1 - I_2}{I_1} \right) \quad 4.1-4$$

It follows from the velocity profile formula that

$I_1$  and  $I_2$  are functions of  $z_E$  and  $\ell'$ ; we can therefore expand

$d(I_1/I_2)/dR_x$  as:

$$\begin{aligned} \frac{d}{dR_x} \left( \frac{I_2}{I_1} \right) &= \frac{1}{I_1} \left( \frac{\partial I_2}{\partial z_E} \frac{dz_E}{dR_x} + \frac{\partial I_2}{\partial \ell'} \frac{d\ell'}{dR_x} \right) \\ &- \frac{I_2}{I_1^2} \left( \frac{\partial I_1}{\partial z_E} \frac{dz_E}{dR_x} + \frac{\partial I_1}{\partial \ell'} \frac{d\ell'}{dR_x} \right) \end{aligned} \quad 4.1-5$$

From the definition of  $\ell'$  we might anticipate that  $\ell'$  should vary slowly with  $R_x$  (since  $R_G$  increases as  $s_S$  decreases and since, under usual circumstances,  $R_G s_S^{\frac{1}{2}}$  is a large number); this is indeed found to be so and therefore the terms involving  $d\ell'/dR_x$  may be neglected.

Thus equation (4.1-4) may be written

$$R_m \frac{dz_E}{dR_x} = \frac{(1 - I_2) R_m \frac{d \ln u_G}{dR_x} - I_1 s_S - m_G (I_1 - I_2)}{\frac{\partial I_2}{\partial z_E} - \frac{I_2}{I_1} \frac{\partial I_1}{\partial z_E}}$$

4.1-6

With the exception of  $m_G$ , all the quantities appearing in equations (4.1-2) and (4.1-6) may be related to the dependent variables  $R_m$  and  $z_E$  with the aid of the velocity-profile formulae:

$$z = z_E \left(1 + \frac{\ln \eta}{\ell'}\right) + (1 - z_E) \left(\frac{1}{2} - \frac{1}{2} \cos \pi \eta\right) \quad 4.1-7$$

the drag law, equations (3.1-11), and the expression for  $\ell'$  in terms of  $R_m$  and  $z_E$ ,

$$\ell' = \ln \{ \kappa E R_m z_E / I_1 \} \quad 4.1-8$$

We obtain:

$$I_1 = \frac{1}{2}(1 + z_E) - z_E / \ell' \quad 4.1-9$$

$$I_2 = 0.375 + z_E(0.25 - 0.411/\ell') +$$

$$z_E^2 \{ 0.375 - 1.589/\ell' + 2/(\ell')^2 \} \quad 4.1-10$$

$$\frac{\partial I_1}{\partial z_E} = 0.5 - 1/\ell' \quad 4.1-11$$

and

$$\frac{\partial I_2}{\partial z_E} = 0.25 - 0.411/\ell' + z_E \{ 0.75 - 3.178/\ell' + 4/(\ell')^2 \} \quad 4.1-12$$

Thus, once the entrainment function,  $m_G$ , has been determined, the hydrodynamic development may be predicted; we therefore turn our attention to entrainment.

#### 4.2 The entrainment function.

The only previous recommendations for an entrainment function applicable to wall jets were those of Spalding (1964a; 1964b). His first proposal (1964a) was:

$$- m_G = 0.09 (1 - z_E) (1 + \frac{z_E}{3}) / (1 + z_E) ; z_E > 1 \quad 4.2-1$$

and was arrived at by considering information about free mixing layers and about the rate of spread and velocity-decay of wall jets in still surroundings. In the annotated version of this reference (1964b) his recommendation was simplified to:

$$- m_G = 0.03 (z_E - 1) ; z_E > 1 \quad 4.2-2$$

In neither case was the recommendation based on any extensive examination of the experimental data.

The basic information required for the determination of the entrainment function is the variation of  $R_m$  with  $R_x$ ; the function then being determined by differentiation of  $R_m$  with respect to  $R_x$ .

To determine  $R_m$  it is first necessary to specify the boundary layer thickness  $y_G$ . Since this quantity is to a large extent arbitrary, a consistent procedure for its determination is required if the effects

of its uncertainty, combined with the uncertainty introduced by differentiation, are not to obviate any confidence in the result. One such process has already been described: the least-square fitting process. From the values of  $z_E$  and  $\ell'$  so determined,  $R_m$  can be obtained. Where detailed velocity profiles were available, namely the present measurements and those of Kruka and Eskinazi (1964), this method was used.

There exist in the literature, however, other experimental studies of the development of wall-jet-flows in which detailed velocity profiles either are not given or are given in insufficient detail. For this data, the maximum velocity,  $u_{\max}$ , and the "half-width",  $y_{\frac{1}{2}}$ , defined as the larger value of  $y$  for which  $u = \frac{u_{\max} + u_G}{2}$ , i.e.

$$y_{\frac{1}{2}} = y \left( u = \frac{u_{\max} + u_G}{2} \right) \quad 4.2-3$$

together with the assumed velocity profile (equation 4.1-7), were used to determine  $R_m$ . Examination of the velocity profiles shown in the previous section indicates that this method and the least-squares fitting procedure should yield comparable results, since the least-squares fitted profiles agree closely with the experimental profiles with respect to maximum velocity and half-width.

The sources of data used are tabulated in Table 1. Values of  $R_m$  so obtained were plotted versus  $R_x$  and the resultant curve graphically differentiated. The resulting values of entrainment plotted versus  $z_E$ , are shown in Figure 4.2-1. There is considerable

scatter, as might have been expected in view of the method used to determine the entrainment, precluding any attempt to determine dependence on any quantities other than  $z_E$ . The data are adequately represented by the equation:

$$m_G = -0.03z_E + 0.02 ; \quad z_E > 1 \quad 4.2-4$$

We note that this formula has the same asymptote for large  $z_E$  as Spalding's proposals, but yields somewhat larger values for entrainment at low values of  $z_E$ .

In view of the uncertainty in the entrainment determined as outlined above, we might well ask if there are other methods for its determination; specifically, is there a method which eliminates the necessity of specifying  $y_G$ ? Spalding (1965)<sup>b</sup> has shown that the entrainment rate is related to the shear stress and velocity profiles by the formula:

$$-m_G = \lim_{z \rightarrow 1} \left( \frac{\partial s}{\partial z} \right)_x \quad 4.2-5$$

and hence  $y_G$  is not required if velocity and shear-stress profiles are available. Unfortunately, this gain would appear to be mainly of theoretical interest; the problems of accurate measurement of  $s$  and  $z$  at the outer edge of the flow introduce considerable uncertainty and scatter. Entrainment determined from the shear-stress and velocity profiles of Erian and Eskinazi (1964) is shown in Figure 4.2-2. These data are not inconsistent with equation (4.2-4), which will be retained at the present recommendation.

### 4.3 Initial Conditions

In the comparisons of prediction and experiment presented in Section 4.4, experimental values of  $R_m$  and  $z_E$  were used for initial conditions for the integration. This is appropriate if the object of the exercise is to test the capacity of the theory to predict subsequent development. In a practical design situation, however, such experimental initial conditions are not available. We therefore require a correlation between the initial conditions and the parameters over which the designer has control; namely, the mean injection to free-stream velocity ratio,  $\bar{u}_C/u_C$ , and the slot Reynolds number,  $R_C$ , defined by:

$$R_C \equiv \frac{\rho_C \bar{u}_C y_C}{\mu} \quad 4.3-1$$

The velocity profiles in the immediate vicinity of the slot cannot, of course, be represented by equation (4.1-7), nor can it be expected that the recommended entrainment function is valid. We are thus led to seek a correlation for the values of  $R_m$  and  $z_E$  at some downstream position. Examination of the velocity profiles indicates that  $x/y_C = 10$  is an acceptable position since at this position the velocity profile is fully developed. Correlations for  $R_m$  and  $z_E$  at this position are shown in Figures 4.3-1 and 4.3-2. The bulk of the data lie within  $\pm 2.5\%$  of the correlation formulae:

$$R_{m,10} = \left\{ 1.46 - 0.003 \frac{\bar{u}_C}{u_C} \right\} R_C \quad 4.3-2$$

and

$$z_{E,10} = 1.47 \frac{\bar{u}_G}{u_G} - 0.5 \quad \checkmark \quad 4.3-3$$

It is of interest to observe that Cole (1965) devised a calculation procedure for initial values of  $R_m$  based on a model of a free-mixing layer commencing at the upper lip of the injection slot and a boundary layer commencing at the lower surface of the injection slot. While his results cannot be compared directly with the above formulae, since they are not given for a fixed value of  $x/y_G$ , they are consistent with the above formula.

It should be noted that the data from which equations (4.3-2) and (4.3-3) were obtained covers a limited range of  $z_E$  ( $1.55 \leq z_E < 13$ ); and that these data were obtained in a wind tunnel in which the boundary layer upstream of the injection slot was relatively thin, and in which the velocity profile at the injection slot very nearly uniform. Caution should be observed in extrapolation to situations where these conditions do not obtain.

#### 4.4. Comparison of predicted and experimental hydrodynamic development.

Comparisons of predicted and observed hydrodynamic development are shown in Figures 4.4-1 to 4.4-12.

The predicted development was obtained by integration of equations 4.1-2 and 4.1-6 using a standard Runge-Kutta forward integration technique; the entrainment was determined from equation (4.2-4); and  $d \ln u_G / dR_x$  by linear interpolation between the values at the measure-

ment stations. The latter were obtained from parabolas fitted to the experimental values of  $u_G$  at the point in question and its contiguous neighbours.

The predicted quantities shown include the momentum thickness Reynolds number,  $R_2$ ; the shape factor,  $H_{12}$ ; and the dimensionless half width and velocity maximum:  $y_{1/2}/y_C$  and  $u_{\max}/u_C$ . The former pair were chosen because they are the usual variables by which boundary-layer flows are characterized; and the latter pair because they are frequently used for wall-jets. The experimental variation of  $u_G$  with  $x$  is also displayed.

The experimental data shown are those reported by Kruka and Eskinazi (1964) and those obtained by the present author; they were chosen because detailed velocity profiles were available, allowing the direct evaluation of  $R_2$  and  $H_{12}$ . The following ranges of slot Reynolds number,  $R_C$ , and mean injection to free-stream velocity ratio,  $\bar{u}_C/u_G$ , are encompassed:

$$3000 \leq R_C \leq 24,000 \quad 4.4-1$$

and

$$1.55 \leq \frac{\bar{u}_C}{u_G} \leq 18 \quad 4.4-2$$

In general the predictions are fairly satisfactory; the agreement between prediction and experiment usually being within the experimental uncertainty of the latter. Three of the comparisons, it is worth noting, are for flows with appreciable pressure gradients.

These are shown in Figures 4.4-3, 4.4-6, and 4.4-12. Although the agreement in these cases is by no means perfect, it is reasonably good and lends support to the notion that entrainment is only weakly, if at all, affected by pressure gradients.

The extent to which predicted drag coefficients agree with experimental drag coefficients can be seen in Figure 4.4-13; the ordinate is the predicted drag coefficient and the abscissa is the experimental drag coefficient. Most of the predicted values are within  $\pm 20\%$  of the experimental values; the root-mean-square deviation is about 15%, an amount not too much larger than the uncertainty in the experimental values themselves. We may therefore conclude that the drag coefficient is also fairly satisfactorily predicted.

#### 4.5 Comparison of predicted and experimental film cooling effectiveness.

The method for uniform density film cooling effectiveness prediction in which the hydrodynamic development is determined from equations (4.4-1) and (4.4-2), and the thermal development from equation (2.2-9), was examined by Cole (1965). The velocity profile used was equation (4.1-7), and the enthalpy profile used was that proposed by Spalding (1964a), viz:

$$\theta = 1 - \frac{n}{2} (1 - \cos \pi \xi) \quad \begin{array}{l} \xi \leq 1 \\ \xi > 1 \end{array} \quad \begin{array}{l} \\ 4.5-1 \end{array}$$

$$= 0$$

The value of  $n$  was taken as 0.78, initial conditions were computed from

a theoretical model for the flow development in the immediate vicinity of the slot and, as noted previously were in good agreement with those obtained from the initial condition correlations given in section 4.3.

In order to obtain good agreement between predicted and experimental film cooling effectiveness, Cole found it necessary to severely modify the entrainment law initially proposed by Spalding; equation (4.2-1); specifically, Cole assumed that a "transition" region existed just downstream of the potential core region in which the entrainment was constant and had a value lower than that given by equation (4.2-1) for the corresponding value of  $z_E$ . After  $z_E$  decreased to a value for which equation gave an entrainment rate equal to the assumed initial constant value, equation (4.2-1) was assumed to obtain.

Cole's entrainment assumption is shown graphically in Fig. 4.5-1. Also shown is the present recommendation for the entrainment function, equation (4.2-4). For large initial values of  $z_E$ , the average of the entrainment function assumed by Cole is near that of the present recommendation. Hence we can conclude that effectiveness predictions obtained using the present entrainment recommendation would be reasonable for flows in which  $z_E$  is everywhere fairly large. At low values of  $z_E$  however (that is, for low values of the injection to mainstream velocity ratio) we can anticipate serious discrepancies between the experimental values of film cooling effectiveness and those predicted with the aid of the present entrainment recommendation, since for low values of  $z_E$  the present recommendation yields much larger entrainment rates than those assumed by Cole.

#### 4.6 Discussion

Apart from the generally satisfactory accuracy of the hydrodynamic predictions, the major merit of the present method is the ease with which data for entrainment can be obtained, only mean-velocity profiles being required. The apparent drawback of the difficulty of obtaining precise entrainment data is somewhat mitigated by the relative insensitivity of the predictions to the exact entrainment function used; a given change in the entrainment function results in a much diminished change in the resulting predictions. [See Nicoll and Whitelaw (1965)].

There remain, however, several major drawbacks. First, for the practically important class of confined flows where there is no inviscid region, the entrainment is identically zero and the pressure gradient term can no longer be assumed known. In this case the equations as given are insufficient to determine a solution, since now the pressure gradient must be included in the list of unknowns.

Secondly, in the application of the present method to boundary-layer flows, difficulty can be anticipated in large adverse pressure gradients, which lead to separation, because the denominator of equation (4.1-6) possesses a zero in the vicinity of  $z_E \approx 0.1^*$ . Now, in terms of the present model, as separation is approached  $z_E$

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\* This zero is not a consequence of the neglect, in the derivation of equation (4.1-6), of the derivatives of  $l'$ ; if these derivatives are included the only effect is to slightly alter the location of the zero.

approaches zero; hence the prediction of boundary layer separation necessarily involves integration through the singular point. This observation may explain why the boundary-layer separation predictions of Escudier [see Escudier and Nicoll (1966)] obtained from the simultaneous solution of equations (4.1-2) and (4.1-6) are poor, while those obtained from the solution of the mass conservation equation alone using experimental values of  $R_2$  were satisfactory. Although the present work is concerned primarily with wall jet flows, it is a part of a larger investigation into wall-shear flows in general, the object of which is to obtain methods of prediction of wide applicability. Hence the failure to predict separation satisfactorily must be considered relevant in any assessment of the method.

A further drawback is the difficulty of extending the present approach to non-uniform density flows. Theoretical guidance concerning the effect of density non-uniformities on entrainment does not exist, and experimental data, from which the entrainment can be determined, are almost non-existent for this case.

For these reasons, the present author abandoned his investigations of entrainment methods at this stage of development and, following Spalding's suggestion, concentrated his attention on the shear-work-integral methods discussed in the following chapters.

## CHAPTER 5

### AN EXPLICIT SHEAR-WORK-INTEGRAL THEORY FOR

#### UNIFORM-DENSITY FLOWS

In the present section we present an explicit theory based on the integral momentum, kinetic energy, and  $\emptyset$  conservation equations. Explicit auxiliary relations for uniform-density flows over adiabatic/impermeable surfaces are determined and compared with available experimental data. Initial condition correlations, valid for a restricted but practical class of flow configurations are given. Finally, predictions of hydrodynamic development, including the transition of a wall-jet flow into a boundary-layer flow, and of film-cooling effectiveness are shown to be satisfactory.

Much of the material in the present section is reported in more detail in Nicoll and Escudier (1966); Escudier, Nicoll and Spalding (1966); Escudier, Nicoll, Spalding and Whitelaw (1966); Nicoll and Whitelaw (1966); and Escudier and Nicoll (1966b).

### 5.1 The Governing Equations

We assume, as before, that the velocity profiles can be characterized by two parameters; following Rotta (1953), Truckenbrodt (1955), Walz (1960), and Spalding\* (1965a), we choose the Reynolds numbers based on the momentum and kinetic energy thicknesses,  $R_2$  and

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\*

Only the last-named author proposed the use of  $R_2$  and  $R_3$  as characteristic parameters for wall-jet flows.

$R_3$ . Further, we assume that the  $\varphi$  profile can be completely characterised by the film cooling effectiveness  $\epsilon$ ,  $R_2$ , and  $R_3$ . The equations governing the development of these quantities, derived in Chapter 2, are:

$$\frac{dR_2}{dR_x} + (1 + H_{12}) R_2 \frac{d \ln u_G}{dR_x} = s_s \quad 5.1-1$$

$$\frac{dR_3}{dR_z} + 2H_{32} R_2 \frac{d \ln u_G}{dR_x} = 2\bar{s} \quad 5.1-2$$

$$\frac{d}{dR_x} (R_{\theta 1} \epsilon) = 0 \quad 5.1-3$$

[Since in the present section we are concerned with uniform density flows only,  $H_x$  has to be set to zero in equation (5.1-2)].

## 5.2 The Auxiliary Relations

In order to solve equations (5.1-1), (5.1-2), and (5.1-3), we require, in addition to the variations of  $u_G$  with  $R_x$  and appropriate initial conditions, auxiliary relations between the four quantities  $H_{12}$ ,  $s_s$ ,  $\bar{s}$ , and  $R_{\theta 1}$  and the dependent variables of the differential equations:  $R_2$ ,  $R_3$  and  $\epsilon$ . Guidance as to the form of these relations can and will be obtained from an examination of the implications of velocity-profile,  $\varphi$ -profile, and mixing-length-distribution formulae; the end result we seek, however, is not parametric formulae in terms of the profile parameters  $z_E$  and  $\ell'$ , but rather explicit formulae in terms of  $R_2$ ,  $R_3$ , and  $\epsilon$ .

### 5.2.1 The $H_{12}$ Relation.

If values of  $H_{12}$  and  $H_{32}$  are generated from the velocity profile formula:

$$z = z_E \left( 1 + \frac{\ell n \eta}{\ell'} \right) + (1 - z_E) \omega(\eta) \quad 5.2.1-1$$

for various values of  $\ell'$  and  $z_E$  and for both the "linear" or "cosine" wake function,  $\omega$ , and if the values so obtained are cross-plotted, the result is essentially a single line. This suggests that, for practical purposes, a unique  $H_{12} \sim H_{32}$  relation may be assumed to exist. Indeed the available wall-jet velocity-profile data support this contention, as may be seen in Figure 5.2.1-1. In this figure experimental values of  $H_{32} - 1^*$ , for uniform-density wall jets, are plotted versus  $H_{12}$ . The data is that of the present author, of Whitelaw (1965) and of Kruka and Eskinazi (1964).

In each case the values of  $\delta_1$ ,  $\delta_2$  and  $\delta_3$  were computed from the measured velocity profiles using the same integration procedure: Simpson's rule generalized for integration with variable-spaced pivotal points. For the range:

$$0.85 \geq H_{12} \geq 0.06 \quad 5.2.1-2$$

$$-4 \times 10^3 \geq R_2 \geq -6 \times 10^5 \quad 5.2.1-3$$

the data are well represented by the equation:

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\* The "-1" results in a greater spread of the boundary-layer data when both wall-jet and boundary-layer data are shown on the same logarithmic graph; it is of no significance in the present context.

$$H_{12} = \frac{1.1}{H_{32} - 0.82} ; \quad 0.85 \geq H_{12} \geq 0.06 \quad 5.2.1-4$$

A similar exercise was performed by Escudier [see Nicoll and Escudier (1966)] for boundary-layer data. For future reference we present here his result and, in addition, a linear relation connecting his result and the present result in the region where no data exists:

$$H_{32} = 1.431 - \frac{0.0971}{H_{12}} + \frac{0.775}{H_{12}^2} \quad 2.8 \geq H_{12} \geq 1.25 \quad 5.2.1-5$$

$$H_{32} = 2.674 - 0.660 H_{12} \quad 1.25 \geq H_{12} \geq 0.85 \quad 5.2.1-6$$

In the limiting case of a wall jet in still surroundings,  $H_{32} = \infty$  and equation (5.2.1-4) implies  $H_{12}H_{32} = 1.1$ . Jayatillaka (1966) reports an experimental value, for a radial wall jet, of 1.098. This remarkably satisfactory agreement with the present result suggests that equation 5.2.1-4 has a wider validity than the range stated above. Indeed, this is likely since, as noted above, the  $H_{12} \sim H_{32}$  relation derived from an assumed profile is so remarkably insensitive to the actual profile chosen.

The simplest possible profile which can represent the gross characteristics of wall jets is the infinite-Reynolds-number profile, derived in Chapter 3, with a linear wake component, viz:

$$z = z_E + (1 - z_E)\eta \quad 5.2.1-7$$

For this profile it is easy to show:

$$H_{12} = \frac{3}{1 + 2z_E} \quad 5.2.1-8$$

and

$$H_{32} = \frac{3}{2} \frac{(1 + z_E)^2}{(1 + 2z_E)} \quad 5.2.1-9$$

and hence:

$$H_{32} = 0.125 H_{12} + 0.75 + 1.125/H_{12} \quad 5.2.1-10$$

The values of  $H_{32}$  computed from this formula, for given values of  $H_{12}$ , differ from those computed from those computed from equation (5.2.1-4) by a maximum of 3% over the range  $0.85 \geq H_{12} \geq 0$ . The values for  $H_{12} > 0.85$  also agree well with those obtained from equations (5.2.1-5) and (5.2.1-6). The reason for this, perhaps surprising, agreement is that the regions where equation (5.2.1-7) is most in error, that is, the region near the wall, makes little contribution to the integrals defining  $H_{32}$  and  $H_{12}$ . This in turn suggests that the linear profile, because of its algebraic simplicity, may provide a useful guide in the determination of suitable correlation forms for other auxiliary formulae, and indeed this has been found to be so, as will be seen below.

### 5.2.2. The Drag Law.

The derivation presented in the present section is mainly due to D.B. Spalding and is presented in more detail in Escudier, Nicoll, and Spalding (1966).

### The Correlation Form.

In Chapter 2, we derived, from the velocity profile formula,

a drag law which, for uniform density flows, can be written:

$$s_s = \left( \frac{Kz_E}{\ell'} \right)^2 \quad 5.2.2-1$$

where

$$\ell' \equiv \ell \nu (ER_G s_s^{\frac{1}{2}}) \quad 5.2.2-2$$

Further, in Chapter 3, we showed that this drag law did yield reasonably accurate values of drag if  $z_E$  and  $\ell'$  were determined by a suitable velocity-profile fitting process. We therefore may adopt it as a starting point. The problem which we must solve can then be stated as: how can we express the experimentally ill-defined quantities  $z_E$  and  $R_G$  in terms of  $R_2$  and  $R_3^*$ ; and how can we modify the above equations so as to obtain an explicit formula? For an answer to the first of these questions we turn to the implications of the linear velocity profile. For this profile we have shown:

$$H_{32} = \frac{3}{2} \frac{(1 + z_E)^2}{(1 + 2z_E)} \quad 5.2.2-3$$

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\* In Chapter 3, the values of  $R_G$  and  $z_E$  required for the comparison of measured and experimental drag for boundary layers were determined, with the aid of an assumed velocity profile formula, from  $R_2$  and  $H_{12}$ . Since the resulting relations between  $R_G$  and  $z_E$ , and  $R_2$  and  $R_3$ , are not explicit, this procedure is not suited to our present purpose. In addition, this procedure suffers from another deficiency. It is not always possible to obtain unique values of  $z_E$  and  $R_G$ , and hence  $s_s$ , from given values of  $R_2$  and  $R_3$ . This difficulty manifests itself in the region of transition of wall jets into boundary layers, i.e. in the vicinity of  $z_E = 1$ .

Further, it is easy to show:

$$\frac{R_2}{R_G} = \frac{1}{6} (1 - z_E)(1 + 2z_E) \quad 5.2.2-4$$

The main step in the argument now follows. We can define a quantity  $\zeta$  such that\* :

$$H_{32} = \frac{3}{2} \frac{(1 + \zeta)^2}{(1 + 2\zeta)} \quad 5.2.2-5$$

that is:

$$\zeta = \frac{2}{3} H_{32} - 1 + \frac{2}{3} H_{32} \left( \frac{2}{3} H_{32} - 1 \right) \quad 5.2.2-6$$

(only the positive root is of physical interest).

and let the boundary-layer thickness Reynolds Number,  $R_G$ , which is otherwise somewhat arbitrary, be defined so that:

$$\frac{R_2}{R_G} = \frac{1}{6} (1 - \zeta)(1 + 2\zeta) \quad 5.2.2-7$$

Comparison of equations (5.2.2-5) and (5.2.2-7) with equations (5.2.2-3) and (5.2.2-4) shows that, if the velocity really was the linear profile,

\*An alternative, algebraically simpler, definition of  $\zeta$  can be obtained from the expression for  $H_{12}$ , as indeed was done by Escudier and Nicoll (1965). This procedure has not been followed here because, as may be seen from their definitions,  $H_{12}$  is more sensitive to density variations than is  $H_{32}$  and because  $H_{32}$  is<sup>12</sup> a natural variable in a theory with  $R_2$  and  $R_3$  are dependent<sup>32</sup> variables. In any event, the available data are not correlated as well when this procedure is adopted; the root-mean-square deviation between prediction and measurement being larger.

$\zeta$  and  $z_E$  would be identical. We would then expect, from equations (5.2.2-1) and (5.2.2-2), that the drag would be given by:

$$s_S = \left[ \frac{\kappa \zeta}{6ER_2} \frac{1}{\ln \left\{ \frac{6ER_2}{(1-\zeta)(1+2\zeta)} s_S^{\frac{1}{2}} \right\}} \right]^2 \quad 5.2.2-8$$

This, we observe, is very nearly the required form of the drag law, since  $\zeta$  is defined in terms of the ratio  $R_3/R_2$ ; it only remains to remove  $s_S$  from the logarithmic term. We cannot simply ignore its presence, for  $s_S$  attains very large values in wall jets. We can, however, remove it to a less influential place by substitution from the left hand side of the equation (5.2.2-8). There results:

$$s_S = \left[ \frac{\kappa \zeta}{6ER_2 \kappa \zeta} \frac{1}{\ln \left\{ \frac{6ER_2 s_S^{\frac{1}{2}}}{(1-\zeta)(1+2\zeta)} \right\}} \right]^2 \quad 5.2.2-9$$

Now, for all the wall jets (and boundary-layers) for which we have data, the value of the logarithm lies between 8 and 12. It therefore seems permissible to treat the second logarithm as a constant, i.e. to replace the argument of the first logarithm with the expression  $CR_2\zeta/(1-\zeta)(1+2\zeta)$ , where the constant  $C$  is to be determined by reference to experimental data.

Also, recognizing that equation 5.2.2-9 was obtained with the aid of a particularly crude velocity profile, we allow ourselves more flexibility in fitting the experimental data by replacing the numerator with a function of  $\zeta$ ,  $f(\zeta)$ , also to be determined by reference to

experimental data. We thus adopt, as our correlation form:

$$s_S = \frac{f(\zeta)}{\left[ \ln \left\{ \frac{CR_2\zeta}{(1-\zeta)(1+2\zeta)} \right\} \right]} \quad 5.2.2-10$$

It is appropriate to observe at this stage that:

- i) the form of equation 5.2.2-10 is suitable for both wall jets and boundary layers. Hence, if a drag law of wide applicability is desired, both types of data should be used in the determination of  $C$  and  $f(\zeta)$ . Although the present work is primarily concerned with wall jets, the utility of a single drag law for both wall jets and boundary-layers, combined with the necessity for auxiliary relations, valid for the region of transition from wall-jet to boundary-layer flow, dictated that this course be followed.
- ii) In the region of transition from a wall-jet to a boundary-layer,  $R_2$  passes through zero. The logarithmic term in equation (5.2.2-10) will only remain finite if both  $R_2$  and  $1-\zeta$  change sign simultaneously, and only if the ratio remains finite. From the first it follows that, when  $R_2 = 0$ ,  $\zeta = 1$ , or (from equation (5.2.2-6)  $H_{32} = 2$ , or finally,  $R_3 = 0$ . While not exact\* for any real velocity profile, the requirement

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\* This may be seen from the inequality, due to Mr. S.V. Patankar (private communication);

$$2\delta_2 - \delta_3 = \int_0^{Y_G} z(1-z)^2 dy > 0 \text{ for } z(-y) > 0.$$

is of minor importance since  $R_3$  is indeed small when  $R_2$  is zero. Of course, care will have to be exercised to ensure that the other hydrodynamic auxiliary function, that for the shear-work integral,  $\bar{s}$ , is such that the computed values of  $R_2$  and  $R_3$  satisfy these requirements.

iii) for the limiting case of wall-jets in still surroundings, equation (5.2.2-10) may be rearranged to yield:

$$\sigma_S = \frac{\frac{16}{9} \left( \frac{\eta'_3}{\eta'_2} \right) \lim_{\zeta \rightarrow \infty} \left\{ \frac{f(\zeta)}{\zeta^2} \right\}}{\left[ \ell^N \frac{3}{8} C \frac{\eta'_2}{\eta'_3} R_{\max} \right]^2} \quad 5.2.2-11$$

where the drag coefficient,  $\sigma_S$ , and the Reynolds number,  $R_{\max}$ , are defined by:

$$\sigma_S \equiv \frac{\tau_s}{\rho u_{\max}^2} \quad 5.2.2-12$$

and

$$R_{\max} \equiv \frac{\rho u_{\max} y_{\max}}{\mu} \quad 5.2.2-13$$

and where  $\eta'_2$  and  $\eta'_3$  are integral properties of the velocity profile defined by:

$$\eta'_2 \equiv \int_0^{\infty} \left( \frac{u}{u_{\max}} \right)^2 d\left( \frac{y}{y_{\max}} \right) \quad 5.2.2-14$$

and

$$\eta'_3 \equiv \int_0^{\infty} \left( \frac{u}{u_{\max}} \right)^3 d\left( \frac{y}{y_{\max}} \right) \quad 5.2.2-15$$

Hence, if the present drag law is to apply to this limiting case,  $f(\zeta)$

must be of degree two in  $\zeta$ , as indeed might have been anticipated from equation (5.2.2-9).

Examination of the published velocity profiles of Sigalla (1958), and of Myers, Schauer, and Eustis (1961), indicate that both  $\eta'_2$  and  $\eta'_3$  are approximately constant and have the values

$$\eta'_2 = 4.91 \quad 5.2.2-16$$

$$\eta'_3 = 4.02 \quad 5.2.2-17$$

We will adopt these values below.

#### The Data Examined.

A summary of the sources of experimental data used in the determination of  $C$ , and of  $f(\zeta)$ , is given in Table 2. The wall-jet data themselves are tabulated in the appendix; the boundary-layer data are tabulated in the paper of Escudier, Nicoll, and Spalding (1966).

We remark here only that:

- i) For each of the general groupings of shear flows along walls (namely, boundary layers,  $R_2 > 0$ ; wall jets with finite mainstream velocity,  $0 > R_2 > -\infty$ ; and wall jets in still surroundings,  $R_2 = -\infty$ ), there are one or more sets of drag data obtained by direct measurement.
- ii) In those cases where the drag was determined by "Clauser plots", the resultant values of drag were consistent with the values obtained, for similar flow conditions, by direct measurement.

iii) the data display considerable scatter; the measurements of individual authors often show irregular variations, indicating irreducible uncertainty in the data, of from five to ten percent.

#### Determination of the Constant C.

The logarithmic term,  $\ln\{CR_2\zeta/(1-\zeta)(1+2\zeta)\}$ , in equation (5.2.2-10) was obtained as an approximation to the term  $\ln(ER_G s_S^{\frac{1}{2}})$ , which appears in the profile-based drag law [equation (5.2.2-1)]. The constant C is to be determined so as to give the maximum accuracy to this approximation; thus C can be determined from:

$$\ln C = \ln(ER_G s_S^{\frac{1}{2}}) - \ln\{R_2\zeta/[1-\zeta)(1+2\zeta)]\} \quad 5.2.2-18$$

The quantity  $\ln(ER_G s_S^{\frac{1}{2}})$  on the right-hand side of equation 5.2.2-18) is not experimentally well-defined, since the boundary-layer thickness,  $y_G$ , is required for its evaluation. It is the value of the logarithm that is required, however, and not the value of its argument, and therefore great accuracy in the determination of  $y_G$  is not required; values obtained by inspection of the mean-velocity profiles will suffice. (For the range of data considered in the present paper, a change of 10% in  $y_G$  will result in a change in  $\ln(ER_G s_S^{\frac{1}{2}})$  of about 1%).

Data for 197 velocity profiles were examined; for  $\underline{E} = 7.7^*$ , the

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\* This value was chosen for consistency with the work of Escudier. The difference between 7.7 and 6.542 is of negligible consequence in the present context.

mean of the values obtained for  $\ell n(\underline{C})$  was 1.220. We have therefore adopted the corresponding result:

$$C = 3.389$$

5.2.2-19

The adequacy of the approximation of  $\ell n(\underline{E}y_G^+)$  by  $\ell n\{3.389 R_2 \zeta / [(1-\zeta)(1+2\zeta)]\}$  is demonstrated in figure 5.2.2-1. The agreement between the two quantities is clearly very good, although there is some indication of a tendency for  $\ell n\{3.389 R_2 \zeta / [(1-\zeta)(1+2\zeta)]\}$  to exceed  $\ell n(ER_G s_S^{\frac{1}{2}})$  by about 2% when  $\ell n(ER_G s_S^{\frac{1}{2}}) \approx 11$  and to fall short by about 2% when  $\ell n(ER_G s_S^{\frac{1}{2}}) \approx 9$ . This trend could easily be accounted for by simple adjustments. The deviations are small, however, compared with the scatter in the drag data, so the adjustment was not made.

#### Determination of $f(\zeta)$ .

Equation (5.2.2-10) may be rearranged as,

$$s_S \left[ \frac{\ell n \{ CR_2 \zeta / [(1-\zeta)(1+2\zeta)] \}}{\zeta} \right]^2 = g(\zeta^{-1}) \quad 5.2.2-20$$

where  $g\{\zeta^{-1}\}$  is a function to be determined empirically.  $\zeta^{-1}$  has been chosen as the argument of  $g$  because this choice allows all the data, including those for  $\zeta = \infty$  (wall-jets in still surroundings), to be presented compactly.

When  $\zeta$  equals  $\infty$ , the limiting form of equation (5.2.2-20) can be written, with the aid of equation (5.2.2-11), as:

$$\frac{9}{16} \sigma_s \frac{\eta_2'}{\eta_3'} \left[ \ln \left\{ \frac{3C}{8} \frac{\eta_2'^2}{\eta_3'} R_{\max} \right\} \right]^2 = g(0) \quad 5.2.2-21$$

Now that the constants  $C$ ,  $\eta_2$  and  $\eta_3$  have been determined, the left-hand sides of equations (5.2.2-20) and (5.2.2-21) can be evaluated for each experimental condition. The values are plotted against  $\zeta^{-1}$  in figure 5.2.2-2.

Over most of the range of  $\zeta^{-1}$ , the data can be adequately represented by a quadratic in  $\zeta^{-1}$ . However, the ordinates for  $\zeta^{-1}$  equal to zero, that is, for wall-jets in still surroundings, have a mean which is appreciably higher than that of the data for small, but finite,  $\zeta^{-1}$ ; this requires that the  $g(\zeta^{-1})$  function must possess much greater curvature in this region than is possible with a simple quadratic. A suitable form for  $g(\zeta^{-1})$ , consistent with both these observations, is:

$$g\{\zeta^{-1}\} = C_0 + C_1\zeta^{-1} + C_2\zeta^{-2} + C_3/(1 + C_4\zeta^{-1}) \quad 5.2.2-22$$

Values for the constants  $C_0$  to  $C_4$  were obtained by minimizing the mean-square deviation,  $R$ , defined by:

$$R^* \equiv \frac{1}{N} \sum_{i=1}^N \frac{s_s(\zeta_i, R_2, i)^2}{s_i^2} - 1 \quad 5.2.2-23$$

Here  $s_s(\zeta_i, R_2, i)$  is the value of  $s_s$  computed from equation (5.2.2-20) with the use of the experimental values of  $R_2$  and  $\zeta$  for the  $i$ th profile;

$s_i$  is the corresponding measured value.

This procedure led to the values:

$$c_0 = 0.2430$$

$$c_1 = 0.03759$$

$$c_2 = 0.00106 \quad 5.2.2-24$$

$$c_3 = 0.0914$$

$$\text{and } c_4 = 65$$

$$g = 0.2430 + 0.03759\zeta^{-1} - 0.00106\zeta^{-2} + 0.0914/(1 + 65\zeta^{-1}) \quad 5.2.2-25$$

### The Result.

The recommended drag law for finite  $\zeta$  is thus:

$$s_s = \frac{0.2430\zeta^2 + 0.03759\zeta - 0.00106 + \frac{0.0914\zeta^2}{1 + 65/\zeta}}{\left[ \ln \left\{ \frac{3.389 R_2 \zeta}{(1-\zeta)(1+2\zeta)} \right\} \right]^2} \quad 5.2.2-26$$

$$\text{where } \zeta \equiv \frac{2}{3} H_{32} - 1 + \frac{2}{3} H_{32} \left( \frac{2}{3} H_{32} - 1 \right) \quad 5.2.2-27$$

For the limiting case of  $\zeta$  infinite (wall-jet in still surroundings), the recommended formula is obtained from equation (5.2.2-21). The previously given values of  $\eta'_2$  and  $\eta'_3$ , together with the other empirical constants, yield:

$$\sigma_s = \frac{0.3976}{[\ln(7.62 R_{\max})]^2} \quad 5.2.2-28$$

The implications of equations (4.2.2-26) and (5.2.2-27) for  $s_s$ , as a function of Reynolds number  $R_2$  and of the shape factors  $H_{32}$  and  $H_{12}$ , are shown graphically in Figure 5.2.2-3. The dependence on the shape factor  $H_{12}$  was determined with the aid of the  $H_{12} \sim H_{32}$

relation given earlier, i.e. equations (5.2.1-4), (5.2.1-5), and (5.2.1-6).

The parameters on the curves are  $R_2$  and  $R_2/(1-\xi)$ . The former ( $R_2$ ) is both conventional and satisfactory for flows without velocity maxima ( $R_2 > 0$ ); it is, however, a somewhat inconvenient parameter for the range of flows under consideration here since, for wall-jet flows, the values of  $R_2$  encountered in practice range from 0 to  $-\infty$ . The parameter  $R_2/(1-\xi)$  is more satisfactory; it varies relatively slowly in the streamwise direction for real flows whether they display a velocity maximum or not, and experimentally encountered values of the quantity lie in a fairly restricted range (approximately  $10^3$  to  $10^5$ ). The hatched areas show the regions covered by experimental data.

#### Comparison with Experiment.

##### $R_2$ and $H_{32}$ Dependence

The ratio of the predicted to the experimental drag, for both wall-jet and boundary-layer data, is plotted respectively versus the shape factors  $H_{32}$  and  $H_{12}$  and the Reynolds number  $R_2$  in Figures 5.2.2-5 and 4.2.2-4. The absence of detectable trends shows that the dependences of the experimental drag on  $R_2$  and on the shape factors  $H_{32}$  and  $H_{12}$  are satisfactorily represented by the present drag-law recommendation, equations (5.2.2-26) and (5.2.2-27).

The range of  $s_s$  encompassed by the above graph is masked by

the choice of the ratio of predicted to experimental drag as ordinate; the range is displayed more clearly in Figure 5.2.2-6, in which the predicted drag coefficient is plotted versus the experimental drag coefficient.

#### Wall-Jets in Still Surroundings .

The comparison of prediction with experiment for the limiting case of wall jets in still surroundings is shown in Figure 5.2.2-7; the ordinate is  $\sigma_S [\equiv \tau_S / (\rho u_{\max}^2)]$  and the abscissa is  $R_{\max}$ . The limiting form of the present recommendation (equation 5.2.2-28) is shown, along with previous recommendations; the former agrees well with the experimental data.

#### Accuracy.

One measure of the capacity of the present recommendation to correlate the experimental data is the root-mean-square (rms) deviation between the predicted and the experimental drag. The rms deviations for the present recommendation, computed from equation 5.2.2-23 were:

|                                    |                     |                  |               |
|------------------------------------|---------------------|------------------|---------------|
| Boundary layers                    | $R_2 > 0$           | 153 data points: | $R = 10.96\%$ |
| Wall-jet with<br>finite mainstream | $-\infty < R_2 < 0$ | 44 data points:  | $R = 12.91\%$ |
| Wall-jets in<br>still surroundings | $R_2 = -\infty$     | 44 data points:  | $R = 8.02\%$  |
| Overall                            |                     | 241 data points: | $R = 10.88\%$ |

Probably the irreducible experimental scatter has an rms value of not much less than 10% overall.

### Comparison with Previous Recommendations.

Comparison of the relative ability of the present drag law [equations (5.2.2-26) and (5.2.2-27), or in the limiting case, equation (5.2.2-28)] to correlate the available drag data to that of previously recommended drag laws, both explicit and implicit, was presented by Escudier, Nicoll and Spalding (1966); Table 3 is a summary of their results. We observe that the present recommendation correlates the data for all categories of flow better than any previous recommendation.

### 5.2.3 The Shear-Work-Integral Relation

#### The Correlation Form.

We have seen in the previous work that useful guidance can be obtained from the simple linear velocity profile: further, in Chapter 3, we found that the Prandtl mixing-length-distribution given by Escudier (1965) [equations (3.2-4) and (3.2-5)], when combined with a more realistic velocity profile, resulted in fairly reasonable predictions of shear-stress distribution. We therefore adopt as our starting point, the velocity profile:

$$z = z_E + (1 - z_E)\eta \quad \eta > 0^+ \quad 5.2.3-1$$

$$z = 0 \quad \eta = 0$$

and the mixing-length-distribution:

$$\begin{aligned} \lambda &= \kappa \xi & \xi < \xi_1 / \kappa \\ \lambda &= \lambda_1 & 1 < \xi < \lambda_1 / \kappa \end{aligned} \quad 5.2.3-2$$

For the present uniform-density case,  $\eta = \xi$ , and equations (5.2.3-1) and (5.2.3-2) can be combined with the definition of the shear-work-integral,  $\bar{s}$ , [equation (2.2-8)] and Prandtl's hypothesis [equation (3.2-1)] to yield:

$$\begin{aligned}\bar{s} &= \int_0^1 s \frac{\partial z}{\partial \xi} d\xi \\ &= \int_0^{z_E} s_S dz + \int_0^1 \lambda^2 \left| \frac{\partial z}{\partial \xi} \right|^3 d\xi \\ &= \int_0^{z_E} s_S dz + \int_0^{\lambda_1/\kappa} (\kappa)^2 \xi^2 \left| 1 - z_E \right|^3 dz + \int_{\lambda_1/\kappa}^1 \lambda_1^2 \left| 1 - z_E \right|^3 dz\end{aligned}$$

5.2.3-3

The first integral in the above equation represents the contribution to  $\bar{s}$  of the region between  $\xi = 0$  and  $\xi = 0^+$ . Evaluating the integrals we obtain:

$$\bar{s} = s_S \xi + \lambda_1^2 \left( 1 - \frac{2\lambda_1}{3\kappa} \right) \left| 1 - z_E \right|^3$$

5.2.3-4

Following the precedent of the drag law derivation, we can again replace  $z_E$  with  $\zeta$ , and replace the constant multiplier of the last term (i.e.  $\lambda_1^2 \left( 1 - \frac{2\lambda_1}{3\kappa} \right)$ ) with a constant,  $C'$ , to be determined empirically. Thus we have:

$$\bar{s} = s_S \xi + C' \left| \zeta - 1 \right|^3$$

5.2.3-5

Now as was noted previously (page 52) the form of the drag law imposes certain restrictions on the form of the shear-work-integral

correlation; specifically, this correlation must be such so as to ensure that the ratio  $R_2/(1 - \zeta)$  remains finite as  $R_2$  approaches zero.

This implies

$$\lim_{R_2 \rightarrow 0} \zeta = 1 \quad 5.2.3-6$$

$$R_2 \rightarrow 0$$

or, equivalently:

$$\lim_{R_2 \rightarrow 0} H_{32} = 2, \quad 5.2.3-8$$

$$R_2 \rightarrow 0$$

and

$$\lim_{R_2 \rightarrow 0} \frac{dR_2}{d\zeta} = \text{a finite quantity} \quad 5.2.3-9$$

In the region under consideration,  $R_1 \approx R_2 \approx R_3 \approx 0$ , and equations (5.1-1) and (5.1-2) reduce to:

$$\frac{dR_2}{dR_x} \approx s_s \quad 5.2.3-10$$

$$\frac{dR_3}{dR_x} \approx 2\bar{s} \quad 5.3.2-11$$

Hence equation (5.2.3-9) can be rewritten as:

$$\begin{aligned} \lim_{R_2 \rightarrow 0} \left( \frac{dR_2}{d\zeta} \right) &= \lim_{R_2 \rightarrow 0} \left( \frac{dR_2}{dH_{32}} \frac{dH_{32}}{d\zeta} \right) = \lim_{R_2 \rightarrow 0} \left( \frac{dR_2/dR_x}{dH_{32}/dR_x} \cdot \frac{dH_{32}}{d\zeta} \right) \\ &= \lim_{R_2 \rightarrow 0} \left( \frac{R_2}{\left( \frac{2\bar{s}}{s_s} - H_{32} \right)} \frac{dH_{32}}{d\zeta} \right) \end{aligned} \quad 5.2.3-12.$$

It is easily shown that the limiting value of  $\frac{dH_{32}}{d\zeta}$  is  $\frac{2}{3}$  and hence, for the above limit to remain finite as  $R_2 \rightarrow 0$  (and as  $H_{32} \rightarrow 2$ , from

above),  $\bar{s}$  must satisfy:

$$\lim_{R_2 \rightarrow 0} \left( \frac{\bar{s}}{s_s} \right) = 1$$

Further, applying l'Hopitals's rule to equation 5.2.3-12 we obtain,

$$\lim_{R_2 \rightarrow 0} \left( \frac{dR_2}{d\zeta} \right) = \frac{\lim_{R_2 \rightarrow 0} \left( \frac{dH_{32}}{d\zeta} \frac{dR_2}{d\zeta} + R_2 \frac{d^2 H_{32}}{d\zeta^2} \right)}{\lim_{R_2 \rightarrow 0} \left( \frac{d}{d\zeta} \left( \frac{2\bar{s}}{s_s} \right) - \frac{dH_{32}}{d\zeta} \right)} \quad 5.2.3-14$$

$$\lim_{R_2 \rightarrow 0} \left( \frac{d}{d\zeta} \left( \frac{2\bar{s}}{s_s} \right) - \frac{dH_{32}}{d\zeta} \right) \quad 5.2.3-13$$

Substitution of the limiting value of  $dH_{32}/d\zeta$ , and cancellation of the limiting value of  $dR_2/d\zeta$ , yields the final result:

$$\lim_{R_2 \rightarrow 0} \left( \frac{d}{d\zeta} \left( \frac{\bar{s}}{s_s} \right) \right) = \frac{2}{3} \quad 5.2.3-15$$

A suitable modification of the basic correlation form [equation (5.2.3-5)] which satisfies the restrictions imposed by equations (5.2.3-13) and (5.2.3-15) is:

$$\bar{s} = \left( \frac{1}{3} + \frac{2}{3} \zeta \right) s_s + C' |1 - \zeta|^3 \quad 5.2.3-16$$

#### The Data Examined.

The only direct (hot-wire) measurements of wall-jet shear-stress and velocity distributions, available to the present author in sufficient detail for  $\bar{s}$ ,  $s_s$ , and  $\zeta$  to be evaluated with reasonable accuracy, are those reported by Mathieu (1961), Bradshaw and Gee (1962), and Erian and Eskinazi (1964). The data of the first two references is limited to  $\zeta = \infty$  (wall-jets in still surroundings) while

the data of Erian and Eskinazi covers only the limited range  $2.5 \geq \zeta \geq 1.5$ . We note that, because little information was given by the latter authors about the velocity profile in the region between the wall and the velocity maximum, the values of  $\zeta$  for their data could not be determined with great accuracy.

To supplement this extremely meagre supply of empirical data,  $\bar{s}$  data was obtained from the present velocity profile measurements by substitution into the kinetic-energy equation, i.e. from:

$$\bar{s} = \frac{1}{2} \frac{dR_3}{dR_x} - R_3 \frac{d \ln u_G}{dR_x} \quad 5.2.3-17$$

using the experimental variation of  $R_3$ ,  $u_G$ , and  $H_{32}$  with  $R_x$ .

#### The Recommended Shear-Work-Integral.

The available  $\bar{s}$  data for finite  $\zeta$  is shown in Figure 5.2.3-1; the ordinate is  $\bar{s} - (\frac{2}{3}\zeta + \frac{1}{3})s_s$  and the abscissa is  $1 - \zeta$ . There is appreciable scatter at low values of the abscissa, as is to be expected in this region (the ordinate is computed by taking the difference of nearly equal quantities), but the data seem adequately represented equation (5.2.3-11) if  $C'$  is given the value:

$$C' = 0.01 \quad 5.2.3-18$$

Hence, our recommended expression for the shear-work-integral is:

$$\bar{s} = (\frac{1}{3} + \frac{2}{3}\zeta)s_s + 0.01(\zeta - 1)^3 \quad \zeta > 1 \quad 5.2.3-19$$

An analogous result for boundary layers,  $\zeta < 1$ , was obtained by Escudier see [Escudier and Nicoll (1966)]. His recommendation was:

$$\bar{s} = \left( \frac{1}{3} + \frac{2}{3} \zeta \right) s_s + 0.00565(1 - \zeta)^{2.715} \quad \zeta < 1 \quad 5.2.3-20$$

The limiting form of equation (5.2.3-19) for  $\zeta \rightarrow \infty$  (wall jets in still surroundings) is:

$$\bar{\sigma} = \int_0^{\infty} \frac{\tau}{\rho u_{\max}^2} \frac{d}{d\frac{y}{y_{\max}}} \left( \frac{u}{u_{\max}} \right) d \left( \frac{y}{y_{\max}} \right) = \frac{8}{9} \frac{\eta_3'}{\eta_2'} \sigma_s + 0.0237 \left( \frac{\eta_3'}{\eta_2'} \right)^2 \quad 5.2.3-21$$

Substitution of the previously determined values for  $\eta_2'$  and  $\eta_3'$  yields:

$$\bar{\sigma} = 0.73 \sigma_s + 0.013 \quad 5.2.3-22$$

If  $\sigma_s$  is evaluated from equation (5.2.2-28) using the typical value of 10 for the logarithmic term, the resultant value of the left-hand side of the above equation is 0.016. This compares satisfactorily with the value of 0.0155, obtained from the measurements of Bradshaw and Gee (1962), and the value of 0.0134, obtained from the measurements of Mathieu (1961).

#### 5.2.4 The $R_{\theta 1} \sim \zeta$ Relation

The final auxiliary relation required is that linking  $R_{\theta 1}$  to the other variables. If we again assume the linear velocity profile:

$$z = z_E + (1 - z_E) \eta \quad \eta \leq 1 \quad 5.2.4-1$$

$$z = 1 \quad \eta > 1,$$

and the linear  $\theta$  profile:

$$\begin{aligned}\theta &= 1 - \frac{\eta}{\Delta} & \eta &\leq \Delta \\ \theta &= 0 & \eta &> \Delta\end{aligned}$$

we obtain, from the definition of  $R_{\theta 1}$  (equation 2.2-16):

$$R_{\theta 1} = \frac{R_G}{6} \left[ (z_E - 1) \left( 3 - \frac{1}{\Delta} \right) + 3\Delta \right] \quad 5.2.4-3$$

Implicit in the above equation is the assumption the  $\Delta$ , which may be interpreted as the ratio of the thermal-to hydrodynamic-boundary-layer thickness, is greater than unity. As in the derivation of the drag-law correlation equation, we eliminate  $R_G$  with the aid of equation (5.2.2-4), and then replace  $z_E$  with  $\zeta$ . The result is: \*

$$R_{\theta 1} = \frac{R_2 \left[ (\zeta - 1) \left( 3 - \frac{1}{\Delta} \right) + 3\Delta \right]}{(1 - \zeta) (1 + 2\zeta)} \quad 5.2.4-4$$

A value for  $\Delta$  must be chosen before the above equation can be utilized. We have seen in Chapter 3, that the experimental  $\theta$  profiles indicate values of about 1.25. Also the agreement between predicted and experimental effectiveness, shown below is best if this value is adopted.

\* A similar, but algebraically different, equation is obtained from the assumption that the  $\theta$  profile is given by  $\theta = 1 - \eta^b$ . If we adopt the values, shown in chapter 3 to be reasonable, of 1.5 and 1.25 for  $b$  and  $\Delta$  respectively, the difference between the two  $R_{\theta 1} \sim \zeta$  relations is about 7% at  $\zeta = 0.5$ , decreases to zero at  $\zeta \approx 2.3$ , and then increases asymptotically to about 4% at  $\zeta = \infty$ .

Therefore, we have:

$$R_{\theta 1} = \frac{R_2 (2.2\zeta + 1.55)}{(1 - \zeta)(1 + 2\zeta)} \quad 5.2.4-5$$

We remark here that:

i) the assumption of a constant value of  $\Delta$  is equivalent to the assumption that the ratio of the thermal to hydrodynamic boundary-layer thickness is constant. This is certainly not always valid in film-cooling applications; most importantly it is not valid when the hydrodynamic boundary layer upstream of the coolant injection slot is thick. Thus the assumption of a constant value represents a fundamental limitation on the range of validity of equation (5.2.4-5), as will be demonstrated later.

ii) the linear  $\theta$  and  $z$  profiles, on which equation (5.2.4-5) is based, cannot be expected to be even approximately valid in the immediate vicinity of the coolant injection slot, and hence predictions obtained with its aid cannot be expected to be valid in this region. Fortunately, since in this region the film-cooling is nearly unity, this limitation is of minor consequence.

iii) were  $R_{\theta 1} \sim \zeta$  experimental data available, equation (5.2.2-5) could be tested directly and suitably modified if required. To the author's knowledge, however, such data is unavailable.

Hence the validity of equation (5.2.2-5) will be tested only indirectly by comparing predicted and experimental film-cooling effectiveness.

### 5.3 Initial Conditions

The efficacy of the present theory for prediction of hydrodynamic development and film-cooling effectiveness can be tested using experimentally determined initial conditions. As was discussed in chapter 4, however, it is desirable to have formulae relating these initial conditions to the flow at the injection slot. We will follow our previous procedure and seek appropriate formulae at the downstream position where the ratio  $x/y_C$  has the value ten.

#### 5.3.1 The Initial Value of $R_2$

In the absence of wall shear stress and pressure gradient, conservation of momentum implies:

$$R_{2,10} = R_{2,C} + R_{2,u} \quad 5.3.1-1$$

where  $R_{2,10}$  is the value of  $R_2$  at  $x/y_C = 10$ , and where  $R_{2,C}$  and  $R_{2,u}$  are Reynolds numbers based on the momentum deficit of the flow through the slot and of the boundary layer into which the slot fluid is injected, viz:

$$R_{2,C} = \frac{u_G}{\nu} \int_0^{y_C} z(1-z) dy \quad 5.3.1-2$$

$$R_{2,u} = \frac{u_G}{\nu} \int_{y_C}^{y_C} z(1-z) dy \quad 5.3.1-3$$

It is easy to show that, for a uniform velocity profile at the injection slot,  $R_{2,C}$  equals  $R_C(1 - \bar{u}_C/u_G)$  whereas, for a parabolic profile,  $R_{2,C}$

equals  $R_C(1 - 6u_C/5u_G)$ . This indicates that  $R_{2,C}$  is relatively insensitive to the exact shape of the profile and may, with only moderate error, be equated to  $R_C(1 - u_C/u_G)$ . Thus from equation (5.3.1-1):

$$\frac{R_{2,10} - R_{2,u}}{R_C(1 - \frac{u_C}{u_G})} = 1 \quad 5.3.1-4$$

Now the assumptions made in the above derivation are crude; we therefore adopt as our correlation formula:

$$\frac{R_{2,10} - R_{2,u}}{R_C(1 - \frac{u_C}{u_G})} = h(u_C/u_G, R_C) \quad 5.3.1-5$$

where the function  $h$  is to be determined empirically. In Figure 5.3.1-1 the available data are shown to be adequately represented by:

$$h = 1.1 - 0.2e^{-0.3 u_C/u_G} \quad 5.3.1-6$$

and hence the correlation formula is:

$$\frac{R_{2,10} - R_{2,u}}{R_C(1 - \frac{u_C}{u_G})} = 1.1 - 0.2e^{-0.3 u_C/u_G} \quad 5.3.1-7$$

In Figure 5.3.1-2 values of  $R_{2,10}$  computed from the above formula are compared with the experimental values. The slot velocity profile for the present measurements was nearly uniform; while for the data of Nicoll and Whitelaw (1966) it was parabolic. In both cases the agreement is good.

### 5.3.2 The Initial Value of $R_3$

In the determination of the correlation formula for  $R_{3,10}$  (the value of  $R_3$  at  $x/y_c = 10$ ) it is necessary to ensure that the ratio  $R_2/(1-\zeta)$ , which occurs both in the drag law and in the  $R_{\theta 1} \sim \zeta$  relation, remains well-behaved for all values of  $R_2$ . Consequently, the procedure by which we determined the  $R_2$  initial-condition formula would be inappropriate here; small percentage errors in  $R_{2,10}$  and  $R_{3,10}$  would result in very large errors in the ratio  $R_{2,10}/(1-\zeta_{10})$  when  $R_{2,10}$  is near zero.

From equation (5.2.2-7) we have:

$$\frac{R_{2,10}}{1-\zeta_{10}} = \frac{R_{G,10}}{6} (1 + 2\zeta_{10}) \quad 5.3.2-1$$

Thus the requirement that  $R_{2,10}/(1 - \zeta_{10})$  remains well-behaved is equivalent to the requirement that  $R_{G,10}$ , or specifically,  $y_{G,10}$ , remains well behaved.

If we neglect the presence of an appreciable upstream boundary-layer, the flow immediately downstream of the injection slot may be imagined as consisting of a boundary-layer near the wall and a free-shear layer with its origin at the upper lip of the slot. Experiment shows that the width of a free-shear layer,  $W$ , is approximately given by the formula:

$$\frac{W}{x} = 0.27 \frac{1 - \frac{u_c}{u_g}}{1 + \frac{u_c}{u_g}} \quad 5.3.2-2$$

If we neglect any inclination of this shear layer, the distance from the wall to its outer edge would be:

$$y_c = y_C + \frac{W}{2} \quad 5.3.2-3$$

or finally, from equation (5.3.2-2),

$$R_c = \frac{u_c}{\bar{u}_C} R_C \left\{ 1 + 1.35 \frac{1 - \frac{\bar{u}_C}{u_c}}{1 + \frac{\bar{u}_C}{u_c}} \right\} \quad 5.3.2-4$$

Substitution of this result into equation (5.3.2-1) yields:

$$1 + 1.35 \frac{1 - \frac{\bar{u}_C}{u_c}}{1 + \frac{\bar{u}_C}{u_c}} = 6 \frac{\bar{u}_C}{u_c} \frac{R_{2,10}}{R_c (1 - \zeta_{10}) (1 + 2\zeta_{10})} \quad 5.3.2-5$$

The constants 1 and 1.35 were obtained using crude assumptions; we therefore replace them with the constants  $a$  and  $b^*$  to be determined empirically. The available data are shown in Figure 5.3.2-1; the ordinates are experimental values of the right-hand side of the above equation and the abscissas are  $|(1 - \bar{u}_C/u_c)/(1 + \bar{u}_C/u_c)|$ . If we neglect the points at low values of the abscissa (as  $\bar{u}_C/u_c$  approaches unity, the uncertainty in the data increases) the indicated values of  $a$  and  $b$  are:

$$\begin{aligned} a &= 1.3 \\ b^* &= 1.24 \end{aligned} \quad 5.3.2-6$$

Substitution of these values into equation (5.3.2-5) and solution for  $\zeta_{10}$  yields.

$$\zeta_{10} = \frac{1}{4} + \frac{3}{4} \left[ 1 - \frac{\frac{4}{3} \frac{\bar{u}_C}{u_g} R_{2,10}}{R [1.3 + 1.24 \left| \frac{1 - \bar{u}_C/u_g}{1 + \bar{u}_C/u_g} \right|]} \right] \quad 5.3.2-7$$

The required initial value of  $R_3$  can then be obtained from

$$R_{3,10} = \frac{3}{2} R_{2,10} \frac{(1 + \zeta_{10})^2}{(1 + 2\zeta)} \quad 5.3.2-8$$

Values of  $R_{3,10}$ , computed from equations (5.3.2-8), (5.3.2-7), and (5.3.1-7), are compared with the corresponding experimental values in Figure 5.3.3-2. The agreement is satisfactory.

We should note, however, that the present  $R_{3,10}$  correlation formula is only valid when the upstream boundary is relatively thin this is, when  $R_{2,u}$  is small compared to  $R_{2,C}$ . In the present context this limitation is acceptable since the  $R_{\theta 1} \sim \zeta$  relation derived previously is also invalid when the upstream boundary layer is thick. It remains, none-the-less a fundamental limitation.

### 5.3.3 The Initial Value of Effectiveness

Equation (5.1-3) implies that  $\epsilon R_{\theta 1}$  is constant. Since this quantity is just the total excess  $\phi$  flux divided by  $\phi_C - \phi_g$ , the constant is equal to  $R_C$ . Thus the required initial value of the effectiveness,  $\epsilon_{10}$ , can be obtained from

$$\epsilon_{10} = \frac{R_C}{R_{\theta 1,10}} \quad 5.3.3-1$$

where  $R_{\theta 1,10}$  can be evaluated from equations (5.2.4-5) using the initial values of  $R_2$  and  $\zeta$  obtained from equations (5.3.1-7) and (5.3.2-7) respectively.

#### 5.4 Velocity Profile Generation

The variables normally used to describe wall-jet flows, namely  $u_{\max}$  and  $y_{1/2}$ , cannot be obtained directly from the present theory since the dependent variables are integral properties of the velocity profile. If, however, we assume that the velocity profiles obey the expression examined in Chapter 3, which in the present uniform density case can be written:

$$z = z_E \left( 1 + \frac{\ln \xi}{\ell'} \right) + (1 - z_E) \left( \frac{1}{2} - \frac{1}{2} \cos \pi \xi \right) \quad 5.4-1$$

where

$$\ell' = \ln E R_G s_S^{1/2} = \ln \kappa E R_G z_E / \ell' \quad 5.4-2$$

$$\xi = \frac{y}{y_G} \quad 5.4-3$$

$$R_G = y_G u_G / \nu, \quad 5.4-4$$

then the complete profile, including  $u_{\max}$  and  $y_{1/2}$ , can be predicted if suitable values of  $z_E$ ,  $\ell'$ , and  $y_G$  can be determined. One method for their determination, denoted Method I below, is based solely on the assumed velocity profile. When applicable, this method was used for profile generation.

It has been noted previously, however, that it is not always possible to obtain, with the aid of the assumed velocity profile alone, values of  $z_E$ ,  $\ell'$ , and  $y_G$  for given values of  $R_2$  and  $R_3$ . This difficulty manifests itself when  $|R_2| \approx 0$ , i.e.

in the vicinity of the disappearance of the maximum in the velocity profile. Since this region is of practical as well as theoretical interest an alternative procedure (denoted Method II below) will be presented for velocity profile generation when  $R_2$  is small. It is appropriate to observe that this alternative procedure is not suited for general application since it yields values for  $y_G$  which sometimes differ substantially from the observed values. Fortunately, in the region where its use is required, the resultant profiles are relatively insensitive to the value of  $y_G$ . In this region the wake component of the total velocity profile is almost absent and  $y_G$  thus affects the velocity profile only logarithmically.

#### Method I

When  $|R_2| \gg 0$  the following procedure, denoted Method I, may be used. Since  $R_1 = R_G(1 - I_2)$  and  $R_2 = R_G(I_1 - I_2)$ , the appropriate values of  $z_E$ ,  $l'$ , and  $R_G$  can be obtained, by numerical methods, from

$$0 = R_2 H_{12} - R_G(1 - I_1) \quad 5.4-5$$

$$0 = R_2 - R_G(I_1 - I_2) \quad 5.4-6$$

and equation (5.4-2).  $R_2$  and  $H_{12}$  are here understood to stand for the predicted values. The expressions for  $I_1$  and  $I_2$ , appropriate to the assumed velocity profile expression, are given by equations (4.1-9) and (4.1-10).

The value of  $y_G$  can then be obtained from equation (5.4-4).

### Method II

When  $|R_2| \approx 0$  the following alternative procedure was used for velocity profile generation.

The logarithmic term in the present drag law  $\left[ \ln 3.389 R_2 \xi / (1 - \xi) (1 + 2\xi) \right]$  was obtained as an approximation to  $\ln(E R_G s_S^{1/2})$ ; indeed the constant was chosen so as to give maximum accuracy to this approximation. Hence we shall assume

$$\ell' = \ln \left[ \frac{3.389 R_2 \xi}{(1 - \xi) (1 + 2\xi)} \right] \quad 5.4-7$$

The value of  $y_G$  can then be determined from equations (5.4-2) and (5.4-4) using this value of  $\ell'$ , the predicted value of the drag coefficient,  $s_S$ , and the known values of  $u_G$  and  $v$ . Finally, the value of  $z_E$  can be determined from the drag law, derived in Chapter 3, which is implied by the assumed velocity profile, namely,

$$z_E = \frac{s_S^{1/2} \ell'}{\kappa} \quad 5.4-8$$

### 5.5 Comparison of Prediction and Experiment

The first three subsections which follow are concerned with hydrodynamic development only; the flow situations examined are: wall jets in still surroundings; wall jets with finite mainstream velocity; and flows in which the velocity maximum decays and finally disappears. The bulk of the comparisons are made in terms of the integral profile properties for flows with finite mainstream velocities and in terms of the dimensionless maximum velocity,  $u_{\max}$ , and jet half-width,  $y_{1/2}$ , in the case of zero mainstream velocity, this procedure being more economical of space. A sufficient number of comparisons of generated with experimental velocity profiles are, however, presented to establish the adequacy of the procedures presented in the previous section. In all three subsections attention will be confined to flows in which the initial velocity profiles (at least) display a velocity maximum.

The fourth subsection contains the comparison of predicted and experimental film-cooling effectiveness. In this subsection flows both with and without initial velocity maxima, i.e. flows in which the injected to mainstream velocity ratios are both greater and less than unity, are examined.

The relevant equations were solved using a standard forward integration procedure (Runge Kutta) in all cases except wall jets in still surroundings. For these flows an approximate analytical

solution of sufficient accuracy was obtained. The required gradient of  $\ln u_G$  was obtained from the experimental variation of  $u_G$  with  $x$ , as explained in Chapter 4. For the hydrodynamic predictions experimental initial conditions were used; for the film-cooling effectiveness predictions the initial conditions were obtained from the formulae given in section 5.3. Since these formulae yield initial values which agree very closely with the experimental values, it was not thought appropriate to recalculate the hydrodynamic predictions using initial values obtained from the aforementioned formulae.

#### 5.5.1 Wall Jets in Still Surroundings

The limiting form of equations (5.1-1) and 5.1-2) for  $u_G$  approaching zero can be written

$$\frac{1}{u^{*2}} \frac{d}{dx^*} (u^{*2} y^*) = - \frac{\sigma}{\eta_2'} \quad (5.5.1-1)$$

and

$$\frac{1}{u^{*3}} \frac{d}{dx^*} (u^{*3} y^*) = - 2 \frac{\bar{\sigma}}{\eta_3'} \quad (5.5.1-2)$$

where

$$u^* = u_{\max} / u_c \quad (5.5.1-3)$$

$$y^* = y_{\max} / y_c \quad (5.5.1-4)$$

$$x^* = x/y_c \quad (5.5.1-5)$$

and where  $\eta_2'$  and  $\eta_3'$  have the definitions given previously (equations (5.2.2-14) and (5.2.2-15)). The limiting expressions

for the drag coefficient,  $\sigma_S$ , and the shear-work-integral,  $\sigma$ , (equations (5.2.2-28) and (5.2.3-22) ) can be expressed in terms of the present variables as

$$\sigma_S = \frac{0.398}{\ln^2 (7.62 R_c u^* y^*)} \quad (5.5.1-6)$$

and

$$\sigma = \left( \frac{8}{9} \frac{\eta'_3}{\eta'_2} \right) \sigma_S + 0.01 \left( \frac{4}{3} \frac{\eta'_3}{\eta'_2} \right)^2 \quad (5.5.1-7)$$

Equations (5.5.1-6) and (5.5.1-7) could be solved exactly using numerical techniques. It is possible, however, to obtain a sufficiently accurate approximate analytical solution; this is the course followed here.

#### Rate of spread

Solving the above equations for  $dy^*/dx^*$  we obtain

$$\frac{dy^*}{dx^*} = \frac{4\sigma}{\eta'_3} - \frac{3\sigma_S}{\eta'_2} \quad 5.5.1-8$$

Substitution of the expressions for  $\sigma_S$  and  $\sigma$ , together with the previously determined values of  $\eta'_2$  and  $\eta'_3$  yields:

$$\frac{dy^*}{dx^*} = 0.013 - \frac{0.045}{\ln^2 (762 R_c u^* y^*)} \quad 5.5.1-9$$

Now, as we have already noted, the value of the logarithm in the drag law, for all experiments known to the present author, lies in the range 8 to 12. The second term on the right-hand side of the above equation is thus small compared to the first. We, therefore, may adopt, with little loss of accuracy, the value of 10 for the logarithm so obtaining

$$\frac{dy^*}{dx^*} = 0.0125 \quad (5.5.1-10)$$

for which the solution is

$$y^* = 0.0125 x^* + \text{constant} \quad (5.5.1-11)$$

The constant of integration can be evaluated from data for the half width,  $y_{1/2}$ , since the ratio of  $y_{1/2}$  to  $y_{\max}$  is essentially constant (see the velocity-profile data of Sigalla (1958) and Myers, Eustis, and Schauer (1961) with the value of about 6.67. Thus, if we adopt the typical initial value of  $y_{1/2}^* = 1.2$  at  $x^* = 10$ , where

$$y_{1/2}^* = y_{1/2} / y_c$$

the resultant formulae for  $y^*$  and  $y_{1/2}^*$  are:

$$y^* = 0.0125 x^* + 0.0625$$

and

$$y_{1/2}^* = 0.0833 x^* + 0.417$$

Comparisons of the predicted rate of spread are compared with the experimental data of Sigalla (1958) and Myers, Eustis, and Schauer (1961) in Figure 5.5.1-1. Although there is a tendency for the predicted width to be somewhat larger than the experimental value in the initial regions, the general level of agreement may be regarded as satisfactory.

#### Rate of maximum-velocity decay

If the above formula for  $y^*$  is substituted into equation (5.5.1-1), and the same approximation made for the logarithmic term in the drag law, the resultant equation may be integrated to yield

$$u^* = \text{const.} (x^* + 5)^{-0.531}$$

The constant may be evaluated from the initial condition  $u^* = 1$  at  $x^* = 10$ . The resulting formula is

$$u^* = 4.22 (x^* + 5)^{-0.531}$$

Comparisons of predicted and experimental velocity maxima of Sigalla (1958) and Myers, Eustis and Schauer (1961) are shown in Figure 5.5.1-2. Again the agreement is satisfactory. We may also note here that the value of the exponent (-0.531) lies between the reported experimental values of Myers, Eustis, and Schauer (-0.49) and of Schwartz and Cosart (-0.555).

### Drag

The variation of the wall shear stress can now be computed from equation (5.5.1-6), and the above formulae for  $u^*$  and  $y^*$ . Values so obtained are compared with experimental data in Figure 5.5.1-3. There appears to be a tendency for the predicted values to be somewhat smaller than the experimental values when the drag coefficient is itself small (large values of  $x^*$ ), but in general the agreement is good.

### 5.5.2 Wall Jets with Finite Mainstream Velocity

Comparisons of predicted with experimental hydrodynamic development, for wall jets with finite mainstream velocity, are shown in Figures 5.5.2-1 and 5.5.2-2, the former displays data for flows in which the mainstream velocity was essentially constant, and the latter data for flows in which the mainstream velocity variation (and hence the pressure gradient) was significant. The quantities shown are the momentum thickness Reynolds number,  $R_2$ , and the shape factor,  $H_{12}$ . In addition, the mainstream velocity variation is shown for those flows in which it was significant, that is, for those flows displayed in Figure 5.2.2-2. The experimental data are those of Kruka and Eskinazi (1961), Erian and Eskinazi (1964) and those of the present work.

We may observe that;

i) The predictions of  $R_2$  development are generally satisfactory for flows both with and without pressure gradient; significant discrepancies being found only in Run 5 of the present work and in the measurements of Erian and Eskinazi. In the author's opinion, which is supported by an inconsistency between the measured values of  $s_g$ ,  $R_2$ ,  $H_{12}$ , and  $d \ln u_G / dR_x$ , the flow in Run 5 was significantly non-two-dimensional; also as noted previously (in section 5.2.3), the velocity profile data of Erian and Eskinazi were incomplete; thus there is some doubt as to the accuracy of the experimental data shown.

ii) The predictions of  $H_{12}$  development are quite satisfactory.

iii) The present theory yields marginally better predictions for flows without pressure gradient than did the entrainment theory, and moderately better predictions when a pressure gradient is present. (Compare Figures 4.4-3, 4.4-6, and 4.4-6 with, respectively Figures 5.5.2-2a, b, and c).

The extent to which the predicted drag coefficients agree with the experimental values is shown in Figure 5.5.2-3. As was the case in the similar comparison made in Chapter 4 for the entrainment based predictions, most of the predicted values are within  $\pm 20\%$  of the experimental values and the root-mean-square deviation is about 15%. As noted earlier, this deviation is not much larger than the uncertainty in the experimental data.

Comparisons of regenerated velocity profiles are shown in Figure 5.5.2-4. The method used is that denoted as Method I previously. The various runs were generally chosen so as to span a useful range of injection to mainstream velocity ratios, the exception being Run 5 which was chosen to satisfy the author's curiosity about the effects of poor prediction (of integral properties) on the regenerated profiles. For all data except Run 5 the general level of agreement is satisfactory; the small deviations which exist being consistent with the limitations (noted in Chapter 3) of the velocity-profile expression used. For Run 5 the agreement is less satisfactory but, in view of the apparent non-two dimensionality of the actual flow noted above, it is perhaps better than might have been expected.

### 5.5.3 The Decay of the Velocity Maximum

In the determination of the shear-work-integral correlation, it was necessary to impose certain constraints on the correlation form so as to avoid singularities in the vicinity of  $R_2 = 0$ . The efficacy of the present theory in this vicinity is tested in the present subsection.

The experimental values of  $R_1$ ,  $R_2$ , and  $s_s$  reported by Wieghardt (1946) and by Whitelaw (1966) are shown in Figure 5.5.3-1; also shown are the present predictions. Values of  $R_1$  and  $R_2$  are shown, rather than the more usual pair,  $R_2$  and  $H_{12}$  ( $R_1/R_2$ ), because the shape factor,  $H_{12}$ , loses its usual significance in the vicinity of  $R_2 = 0$ .

Comparison of the experimental and predicted values of  $R_1$  and  $R_2$  indicates that the predicted values tend to be somewhat high, comparison of the experimental values of the drag coefficient with the present predictions indicates that, despite the discrepancies in the values of  $R_1$  and  $R_2$ , the predicted shear stress is not subject to large errors. In general the predicted values of  $s_s$  tend to be higher than the experimental values, the magnitude of the disagreement being about 20% initially and decreasing to about 5% at the larger values of  $R_x$ . It should be noted here that the initial velocity profiles of Whitelaw, as was observed by Escudier, Nicoll, Whitelaw and Spalding (1966), were not of the universal form. Thus the discrepancies at low values of  $R_x$  may be in part due to errors in the experimental values obtained by Clauser's method [Clauser (1954)].

The experimental velocity profiles are compared with generated profiles, in Figure 5.5.3-2; the procedure used is that procedure previously given as Method II. As may be seen, the agreement is satisfactory, the maximum discrepancy between actual and predicted velocity being about 5%.

The generally satisfactory, if not perfect, agreement between prediction and experiment indicates that the measures adopted to avoid singularities in the vicinity of  $R_2 = 0$  are adequate.

#### 5.5.4 Film Cooling Effectiveness

Comparisons of the values of effectiveness,  $\epsilon$ , predicted by the present method with the measurements of Seban and Back (1961) and of Nicoll and Whitelaw (1966) are shown in Figure 5.5.4-1. The data of the former authors is for "adiabatic wall" effectiveness, that is, the relevant conserved property in their experiments was the fluid enthalpy; the data of Nicoll and Whitelaw is for "impervious wall" effectiveness, that is, the relevant conserved property was the mass concentration of a tracer gas. The impervious wall effectiveness and the adiabatic wall effectiveness are governed by similar equations, and may be regarded as equivalent if the total (laminar plus turbulent) Lewis number is unity. As may be seen, the comparisons of predicted and experimental effectiveness support the hypothesis of unity Lewis number.

The experimental techniques and procedure used to obtain the impervious wall effectiveness data are discussed in detail by Nicoll & Whitelaw; it suffices here to note that the use of a tracer gas (in this case Helium) allows the simultaneous attainment of an impervious wall sensitive measurement and uniform density.

The data of Seban and Back and those of Nicoll and Whitelaw were chosen for comparison because the boundary layers on the upper lip of the injection slot were, in general, relatively thin (i.e.,  $R_{2,u}$  was much less than  $R_c (1 - \frac{u_c}{u_G})$ ), the exceptions being Nicoll and Whitelaw's Runs 4 and 5. In all cases the initial conditions were obtained from the correlations described in section 5.3, using the experimental values of  $R_c$  and  $\frac{u_c}{u_G}$ . For the data of Nicoll and Whitelaw the experimental values of  $R_{2,u}$  were used in the initial condition correlations; for the data of Seban and Back,  $R_{2,u}$  was taken as zero, an assumption justified by their experimental apparatus and by the large values of the mass flow parameter,  $\frac{u_c}{u_G}$ , associated with their data.

In general the comparisons are satisfactory and require little comment. In the immediate vicinity of the slot, however, it should be noted that the predicted values are greater than unity; this is a direct consequence of the inaccuracy of the linear,  $\phi$ , profile in this region and is of little practical significance since the measured values of effectiveness in this region are close to unity.

Two prediction curves are shown for Nicoll and Whitelaw's run 4, the solid line being the prediction using the measured value of  $R_{2,u}$  in the initial condition correlation, and the dashed line being the prediction based on the assumption that  $R_{2,u}$  is zero. We observe that the latter prediction is better. While this result is undoubtedly a consequence of the inapplicability of the initial condition correlations when  $R_{2,u}$  is appreciable compared to  $R_c (1 - \frac{u_c}{u_G})$ , in Run 4 the respective values of these quantities were approximately 700 and -2,680, it does suggest that possibly reasonable effectiveness predictions may be obtained using the present method even in cases where the basic theoretical model is incorrect provided  $R_{2,u}$  is neglected. This would, of course, only be true of zero pressure gradient flows since only in this case is the predicted hydrodynamic, and hence thermal, development insensitive to the absolute value of  $R_2$ . To test the limitations of this hypothesis, the present method was applied to the data of Chin, Shervin, Hayes, and Silver. From the description of their experimental apparatus (the nominal origin of the boundary layer on the upper lip was reported to be 41.5 inches upstream of the injection slot) it may be inferred that  $R_{2,u}$  is large compared to  $R_c (1 - \frac{u_c}{u_G})$ . The resulting comparisons of prediction and measurement are shown in Figure 5.5.4-2. This figure shows that despite the inapplicability of the initial condition correlations and the resultant large error in  $R_{2,10}$  if  $R_{2,u}$  is neglected, the predictions are reasonable for values of  $\frac{u_c}{u_G}$  not in the immediate vicinity of unity.

## 5.6 Discussion

It is appropriate here to attempt to assess briefly and critically the accomplishments and limitations of the present approach, and to compare the present approach with alternative approaches. First let us consider the accomplishments.

i) We have presented an explicit, computationally simple, theoretical structure which, within the limitations of small upstream boundary layers and uniform density, appears to accurately predict the hydrodynamic development of wall-jet flows from the extreme of zero mainstream velocity to the other extreme of the disappearance of the velocity maximum; the effects of (at least moderate) pressure gradients on this development appear to be accurately predicted.

ii) In the development of the theory we have obtained certain auxiliary functions which correlate the available data better than any previous proposals, namely; the drag law; the shape factor relation; and the shear-work-integral relation.

iii) We have presented initial condition correlations which allow the designer to base his predictions directly upon the variables under his control, namely,  $R_c$  and  $u_c/u_G$ .

iv) Methods have been developed for the generation of complete velocity profiles which correspond well to the experimental velocity profiles.

v) A procedure was presented for the calculation of film cooling effectiveness, which, within the aforementioned limitations, yields accurate predictions.

It remains to examine the practical significance of the two major limitations on the present theory. The first is the limitation, in both the  $R_{3,10}$  initial condition correlation and the  $R_{0,1} \sim \xi$  relation, to "small" upstream boundary layers. In many practical applications the upstream boundary layer is not negligible. Indeed, the desire for maximum film cooling effectiveness at any given downstream station dictates a coolant injection velocity to mainstream velocity ratio of about unity. This requirement was first put forward by Spalding (1963) and is rendered apparent in Figure 5.6-1 in which the effectiveness measurements of Nicoll and Whitelaw (1966) at constant downstream distances are plotted as a function of the velocity ratio  $\frac{u_C}{u_G}$ . It thus dictates operation in circumstances in which the upstream boundary layer is likely to be significant\*.

The second limitation of practical significance is embodied in the assumption of uniform density. In many practical applications the density of the injected fluid is very much greater than the density of the mainstream fluid.

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\* An extension of the present approach, in which the thermal to hydrodynamic boundary-layer thickness parameter,  $\Delta$ , is a variable rather than a fixed quantity, shows promise of overcoming the limitation to small upstream boundary layers.

The only alternative calculation procedure known to the author, which does not suffer from the above limitations is that put forward by Patankar and Spalding (1965). A variant of their procedure will be presented in the following chapter; we shall simply observe here that this procedure is of the parametric type and shares the deficiencies of parametric theories in general, namely, computational complexity and lack of flexibility in the required auxiliary functions. Thus, within its limitations, the present theory appears to be preferable as a basis for practical prediction; when the upstream boundary layer is not negligible or when density variations are appreciable, the approach of Patankar and Spalding is preferable.

## CHAPTER 6

A Parametric Shear-Work-Integral Theory

In the present chapter we return to the parametric approach and present a preliminary examination of a theoretical structure which does not suffer from the restriction to uniform-density flows. The present theory should be regarded as a special case of the theory presented by Patankar and Spalding (1965).

The restriction to flows in which the hydrodynamic and  $\phi$  boundary layers are co-extensive will be maintained. We note, however, that this restriction is not an essential restriction nor is it a restriction imposed by Patankar and Spalding, but is rather imposed in the present circumstances because the data presented may be expected to warrant it and because it results in a substantial computational economy through a reduction in the number of independent equations required.

Comparisons of predicted and experimental impervious-wall effectiveness are presented for the case of an argon secondary stream injected into an air mainflow; the comparisons show satisfactory agreement between prediction and experiment.

### 6.1 The Governing Equations

Let us assume that the velocity profile, for non-uniform density flows as well as uniform-density flows, can be represented by the formula presented in Chapter 3, viz:

$$z = z_E + \left(1 + \frac{\ln \eta}{\ell'}\right) + (1 - z_E) \omega(\eta) \quad 6.1-1$$

where

$$\ell' = \ln \left[ \kappa E R_G z_E / \ell' \right] \quad 6.1-2$$

and where the dimensionless distance from the wall,  $\eta$ , is defined in terms of the ratio of the focal to mainstream density,  $\chi$ , and the ratio of the distance from the wall to the boundary-layer thickness,  $\xi$ , by the equation

$$\eta = \frac{\int_0^\xi \chi d\xi}{\int_0^1 \chi d\xi} \quad 6.1-3$$

We shall also assume that the wake component of the velocity profile,  $\omega(\eta)$  is adequately represented by the linear relation:

$$\omega = \eta \quad 6.1-4$$

It follows that the velocity profile is completely specified if the parameters  $z_E$  and  $R_G$  and the density distribution,  $\chi(\eta)$  are known.

Further let us assume that the  $\phi$  profiles obey the expression

$$\theta = \frac{\phi - \phi_G}{\phi_S - \phi_G} = 1 - \eta^b \quad 6.1-5$$

where  $b$  is a constant with the value  $1.5^*$ .

Then the velocity and  $\phi$  profiles are both specified when  $z_E$ ,  $R_G$ ,  $\phi_S$ , and  $X \in \eta$  are known. The last function is not, however, independent; it is specified by the assumed  $\theta$  profile,  $\phi_S$ , and the equation of state of the fluid.

For example, if the density non-uniformity arises from a temperature difference between the mainstream and injected fluids, and if both are perfect gases of the same composition, then  $X$  is given by

$$X = \frac{1}{1 + \left( \frac{\phi_S}{\phi_G} - 1 \right) \theta} \quad 6.1-6$$

where, in this case,  $\phi$  represents the enthalpy of the fluid.

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\*

Computations using values of  $b$  between 1 and 1.5 show that the exact value of  $b$  has little effect on the predictions.

It may be of interest to the reader that when Mr. S. V. Patankar and the writer examined the implications of an effectiveness prediction procedure in which the value of  $b$  was not fixed, but rather was treated as an additional variable in order to obtain prediction of flows in which the hydrodynamic and  $\phi$  boundary layers are not co-extensive, the most striking and disturbing results were obtained. The predicted value of effectiveness increased in the downstream direction! It was finally ascertained that this absurd result could be attributed to the peculiar character of the profile  $\theta = 1 - \eta^b$  when the value of  $b$  is small. While a detailed discussion would be inappropriate here, this result does serve to emphasize that apparently satisfactory gross correspondence between measured profiles and their mathematical representation is not sufficient to ensure accurate, or even reasonable, predictions.

If, on the other hand, the non-uniformity arises from a difference of composition between the injected and mainstream fluids, and if both are perfect gases of the same temperature and of respective molecular weights  $M_G$  and  $M_C$ , then the required expression is

$$\chi = \frac{1}{1 - \left( \frac{M_G}{M_C} - 1 \right) \phi_S \theta} \quad 6.1-7$$

where, in this case,  $\phi$  represents the local mass fraction of the injected fluid. We note that, by definition,  $\phi_G$  is here zero.

In the present work we shall confine our attention to flows in which the density non-uniformity arises solely from compositional variations. The extension to other cases can be accomplished with ease, requiring only the substitution of the appropriate expression.

The development of the three independent parameters,  $z_E$ ,  $R_G$ , and  $\phi_S$ , can be determined from the integral momentum, kinetic-energy-deficit, and  $\phi$  conservation equations derived in Chapter 2 equations (2.2-1), (2.2-2), and (2.2-9). For convenience of solution it is desirable to express these equations in terms of the derivatives of the independent parameters. If, as justified in Chapter 4, we neglect the derivatives of  $\phi'$ , the integral momentum and kinetic-energy-deficit equations may be written:

$$\begin{aligned}
(I_1 - I_2) \frac{dR_G}{dR_X} + R_G \left( \frac{\partial I_1}{\partial z_E} - \frac{\partial I_2}{\partial z_E} \right) \frac{dz_E}{dR_X} \\
= s_S + R_G (I_2 - I_X, 0) \frac{d \ln u_G}{dR_X}
\end{aligned} \quad 6.1-8$$

and

$$\begin{aligned}
(I_1 - I_3) \frac{dR_G}{dR_X} + R_G \left( \frac{\partial I_1}{\partial z_E} - \frac{\partial I_3}{\partial z_E} \right) \frac{dz_E}{dR_X} = \\
= 2 \bar{s} + 2R_G (I_{X,1} - I_3) \frac{d \ln u_G}{dR_X}
\end{aligned} \quad 6.1-9$$

where the partial derivative notation indicates  $\mathcal{L}'$  is held constant. The  $I$ 's have the same definitions as in Chapter 2.

The  $\emptyset$  conservation equation may be integrated directly; the result, expressed in terms of the effectiveness  $\epsilon$ , may be written:

$$\epsilon R_G I_{\theta_1} = \text{constant} \quad 6.1-10$$

## 6.2 The Auxiliary Relations

For solution of equations (6.1-8), (6.1-9) and (6.1-10), we require, in addition to the variation of  $u_G$  with  $R_X$  and initial conditions, expressions relating the integral profile properties ( the  $I$ 's ), the drag coefficient,  $s_S$ , and the shear-work integral,  $\bar{s}$ , to the independent variables,  $R_G$ ,  $z_E$ , and  $\emptyset_S$ .

The required expressions for the integral profile properties may be found from the profile expressions. The results are:

$$I_1 = \frac{1}{2} (1 + z_E) - \frac{z_E}{\ell'} \quad 6.2-1$$

$$I_2 = \frac{1}{3} + z_E \left( \frac{1}{3} - \frac{1}{2\ell'} \right) + z_E^2 \left( \frac{1}{3} - \frac{3}{2\ell'} + \frac{2}{\ell'^2} \right) \quad 6.2-2$$

$$I_3 = \frac{1}{4} + z_E \left( \frac{1}{4} - \frac{b'}{\ell'} \right) + z_E^2 \left( \frac{1}{4} - \frac{7 - 6b'}{6\ell'} + \frac{3}{4\ell'^2} \right) + z_E^3 \left( \frac{1}{4} - \frac{11}{6\ell'} + \frac{21}{4\ell'^2} - \frac{6}{\ell'^3} \right) \quad 6.2-3$$

$$I_{X,0} = 1 + \phi_S \left( \frac{M_G}{M_C} - 1 \right) \frac{b}{b+1} \quad 6.2-4$$

$$I_{X,1} = z_E \left[ 1 + \phi_S \left( \frac{M_G}{M_C} - 1 \right) \frac{b}{b+1} \right] - \frac{z_E}{\ell'} \left[ 1 + \phi_S \left( \frac{M_G}{M_C} - 1 \right) \frac{b(b+2)}{(b+1)^2} \right] + \frac{(1 - z_E)}{2} \left[ 1 + \phi_S \left( \frac{M_G}{M_C} - 1 \right) \frac{b}{b+2} \right] \quad 6.2-5$$

$$I_{\theta,1} = z_E \left[ 1 - \frac{1}{\ell'} - \frac{1}{b+1} + \frac{1}{\ell'(b+1)^2} \right] + (1 - z_E) \left( \frac{1}{2} - \frac{1}{b+2} \right) \quad 6.2-6$$

where the parameter,  $b'$ , in equation (6.2-3) is obtained from

$$\begin{aligned} b' &= 0.255 & 0 < z_E < 1 \\ b' &= -0.25 & z_E > 1 \end{aligned} \quad 6.2-7$$

It should be noted that, if the full implications of the velocity profile were utilized,  $b'$  would have the value 0.333. This value, however, leads to a singularity in the inversion matrix when  $z_E$  is near unity. The propensity of some parametric methods to suffer from such singularities has been noted earlier; in the present work we have followed the procedure proposed by Patankar and Spalding (1966) for the avoidance of this difficulty.

The required partial derivatives of  $I_1$ ,  $I_2$ , and  $I_3$  are:

$$\frac{\partial I_1}{\partial z_E} = \frac{1}{2} - \frac{1}{\ell'} \quad 6.2-8$$

$$\frac{\partial I_2}{\partial z_E} = \left( \frac{1}{3} - \frac{1}{2\ell'} \right) + z_E \left( \frac{2}{3} - \frac{3}{\ell'} + \frac{4}{\ell'^2} \right) \quad 6.2-9$$

$$\begin{aligned} \frac{\partial I_3}{\partial z_E} = & \left( \frac{1}{4} - \frac{b'}{\ell'} \right) + z_E \left( \frac{1}{2} - \frac{7 - 6b'}{3\ell'} + \frac{3}{2\ell'^2} \right) \\ & + z_E^2 \left( \frac{3}{4} - \frac{11}{2\ell'} + \frac{63}{4\ell'^2} - \frac{18}{\ell'^3} \right) \end{aligned} \quad 6.2-10$$

For the drag coefficient we shall use the expression derived in Chapter 3, viz:

$$s_S = \chi_S \left( \frac{\kappa z_E}{\ell'} \right)^2 \quad 6.2-11$$

Finally, an expression for the shear-work integral,  $\bar{s}$ , can be obtained as follows. By definition:

$$\bar{s} = \int_0^1 s \frac{\partial z}{\partial \eta} d\eta \quad 6.2-12$$

Near the wall, say  $\eta \leq 0.04$ ,  $s$  is nearly uniform and equal to  $s_S$ . Therefore we may express the above integral as:

$$\sigma = s_S z \{ \eta = 0.04 \} + \int_{0.04}^1 s \frac{\partial z}{\partial \eta} d\eta. \quad 6.2-13$$

The first term may be evaluated from the drag law and the assumed velocity profile. The second term must be evaluated numerically with the aid of the expression for  $s$  and the mixing-length distribution given in Chapter 3, i.e. from:

$$s = \lambda^2 I_{\chi,0}^2 \chi^3 \left| \frac{\partial z}{\partial \eta} \right| \frac{\partial z}{\partial n}, \quad 6.2-14$$

$$\left. \begin{aligned} \lambda &= \kappa \xi & 0 \leq \xi \leq \lambda_1/\kappa \\ \lambda &= \lambda_1 & \lambda_1/\kappa \leq \xi \leq 1 \end{aligned} \right\} 6.2-15$$

and the expression, which follows from the definitions, relating  $\eta$  and  $\xi$  :

$$\xi = \frac{\int_0^\eta \chi^{-1} d\eta}{I_{\chi 0}} \quad 6.2-16$$

### 6.3 Initial conditions

For the comparisons of predicted and experimental impervious-wall effectiveness presented in the following section, experimental velocity profiles were not available; hence it was not possible to determine by reference to experimental data, appropriate initial values of  $z_E$  and  $R_G$ . Further, for non-uniform density flows, there is not available a sufficient body of experimental data from which initial condition correlations can be determined. For these reasons the following approximate procedure was used.

The value of  $R_G$  at  $x/y_C = 10$ , was calculated from the expression derived in Chapter 5, i.e. from

$$R_{G,10} = \frac{u_G}{\bar{u}_C} R_C \quad 1.3 + 1.24 \frac{\left| 1 - \frac{u_C}{u_G} \right|}{\left| 1 + \frac{\bar{u}_C}{u_G} \right|} \quad 6.3-1$$

We note that this expression was obtained from data for uniform density flows with the aid of a linear velocity profile assumption, and thus its use in the present context is open to question. The results presented in the following section, however, suggest that whatever the inadequacies of this expression may be, its use does not lead to large errors in the predicted impervious-wall effectiveness, at least for flows in which the pressure gradient is small.

For flows in which the velocity profile displayed a maximum, that is, for  $\bar{u}_C/u_G > 1$ , an appropriate initial value of  $z_E$  was then determined by equating the maximum velocity with

the mean injection velocity. For flows in which the injection was less than the mainstream velocity this procedure of course could not be used; in this case initial values of  $z_E$  were obtained by extrapolation, as shown in Figure 6.3-1, of the  $z_E \sim \bar{u}_C/u_G$  relation obtained as described above.

Finally the value of  $\phi_{S,10}$  was chosen so as to achieve the best agreement between predicted and experimental effectiveness development. We observe here that, as in the case of the prediction procedure presented in Chapter 5, the value of  $\phi_{S,10}$  may exceed unity when the injection to mainstream velocity ratio is near unity. This is again a consequence of the fact that, for such flows, the profiles do not become fully developed until further downstream.

#### 6.4 Comparison of Predicted and Experimental Impervious-Wall Effectiveness

Comparisons of predicted and experimental impervious-wall effectiveness are shown in Figure 6.4-1. The data is that of the present author and was obtained, as explained in the appendices, for argon injected into an air mainstream. In general the comparisons are satisfactory and confirm the capacity of the present theory, given appropriate initial conditions, to predict the subsequent effectiveness development for at least moderate density non-uniformities (the ratio of the density of

the injected fluid to the mainstream density was approximately 1.40).

It is of interest to compare the present experimental results with those given by Nicoll and Whitelaw (1966) for uniform density flows. This is best done by plotting the present data versus  $\bar{u}_C/u_G$  with  $x/y_C$  as a parameter, as shown in Figure 6.4-2, and comparing this figure with Figure 5.6-1. When this is done, three features may be observed which are worthy of note. First, the optimum ratio of injected to mainstream velocity for maximum effectiveness at any particular downstream distance has about the same value, say  $\bar{u}_C/u_G \approx 1.2$ , for argon injected into air as for uniform density film cooling. Secondly, near the optimum value of  $\bar{u}_C/u_G$ , the initial decrease in effectiveness is greater for the argon injection. For example, in the range  $1 < \bar{u}_C/u_G < 2$ , the value of the uniform-density effectiveness at  $x/y_C \approx 45$  is about 0.85, whereas for the argon injection, the value is about 0.75. This is perhaps due to increased mixing, in the initial region, due to the density non-uniformity. Thirdly, near the optimum value of  $\bar{u}_C/u_G$  the rate of decrease of effectiveness for the uniform density flows is less than the rate for the argon injection, whereas for injection ratios of substantially greater values the rate of decrease for the uniform density flows appears greater. For example, over the range  $45 \leq x/y \leq 130$ , the decreases in effectiveness for argon injection and for uniform density flows are respectively 0.30 and 0.20 for  $\bar{u}_C/u_G \approx 1$ ,

while for  $u_c/u_G \approx 3$ , the respective figures, obtained by extrapolation, are 0.35 and 0.40. This observation is consistent with the hypothesis that, near the optimum injection ratio, the density non-uniformity increases the mixing between the mainstream and the injected fluid. It is also consistent with the theoretical predictions of Cole (1966), using a theory nearly identical with the present theory and the experimental data of Stollery and El-Ehwany (1966), that for  $u_c/u_G > 1$ , the effect of the greater density of the injected fluid is to decrease the rate of effectiveness decay.

## CHAPTER 7

### Summary and Suggestions for Future Work

#### 7.1 The Parametric Entrainment Theory

The general features of the parametric entrainment theory may be summarized as follows. Given the initial assumptions of a two parameter velocity profile and an entrainment function, wall-jet hydrodynamic development can be predicted from the solution of the integral momentum and mass-conservation equations. If the specific velocity profile, entrainment function, and initial condition correlations given in Chapter 4 are used in the calculations, the resultant predictions are satisfactory when there is no pressure gradient, and fair when pressure gradients are present.

The major deficiencies of this theory were its incapacity to handle internal flows when no inviscid flow region is present, and its failure (see Escudier and Nicoll (1966)) to predict boundary layer separation accurately. It is this latter deficiency which furnishes the basis for a suggestion for future work.

As was noted by Spalding (1964a), the velocity-profile expression assumed in Chapter 4 does not correspond to the experimental profiles for separating flows presented by Stratford (1959). Hence the failure to accurately predict

separation may be due to the inadequacy of the velocity-profile for such flows. In the author's opinion, an examination of the effect, on separation predictions, of a modified expression may very well lead to an improved separation prediction procedure\*. Indeed, a preliminary investigation, by one of the author's colleagues, Mr. B. R. Ramaprian, does indicate that improved prediction of shape factor and drag development may be obtained if the velocity-profile expression is modified so as to have the appropriate asymptotic form for zero wall-shear-stress flows. Only a more extensive investigation will show whether or not these preliminary indications of improvement are of general validity.

## 7.2 The Explicit Shear-Work-Integral Theory

The explicit shear-work-integral theory, presented in Chapter 5, appears to be an improvement on the parametric entrainment theory both with regard to the accuracy of prediction of wall-jet hydrodynamic development and film-cooling effectiveness, and with regard to computational simplicity.

The results presented show that accurate hydrodynamic predictions can be obtained for flows ranging from wall jets in still surroundings at the one extreme to wall-jet flows in which the velocity maximum disappears at the other. Methods

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\*

We should note here that, although the predictions of the present theory are not satisfactory, they are indeed as good as those obtained by other methods and better than most.

were presented by which the detailed velocity profiles could be accurately regenerated.

In the development of the theory, certain auxiliary relations were obtained which correlated the available data better than any previous proposals; namely, the drag law, the shape factor relation, and the shear-work-integral relation.

Further, initial condition correlations were presented which will allow the designer to base his calculations directly on the variables under his control; namely the injection velocity and mass flow rate.

The restrictions to which the theory presented was subject, i.e. the restriction to uniform density flows and to flows in which the upstream boundary layer was comparatively thin, are not inherent in the general approach but rather were imposed by the limitations of the auxiliary equations and the initial condition correlations. Thus these restrictions may be removed by suitably modifying these formulae. As was noted previously, an extension of the theory presented, in which the ratio of the  $\phi$  to hydrodynamic boundary layer thickness is treated as a variable rather than a fixed quantity, does show promise of removing the limitation of thin upstream boundary layers. This work will, of course, be continued.

A further extension to include finite heat transfer at the wall should also lead to some useful prediction procedures.

### 7.3 The Parametric Shear-Work-Integral Theory

The results presented in Chapter 6 show that a prediction procedure, based on the integral momentum, kinetic-energy-deficit, and  $\emptyset$  conservation equations, in which the required auxiliary relations are obtained from velocity-profile,  $\emptyset$ -profile, and mixing-length-distribution expressions, can be used for the prediction of film-cooling effectiveness for at least moderate density non-uniformity. These results should, however, at best be regarded as preliminary results. Future work suggestions would include:

- i) comparison of prediction and experiment for large density differences between the injected and mainstream fluids.
- ii) the development of suitable initial condition correlations relating the required initial values of the parameters to the flow issuing through the injection slot.

Nomenclature

| <u>Symbol</u>    | <u>Significance</u>                                           | <u>Equation of first mention</u> |
|------------------|---------------------------------------------------------------|----------------------------------|
| a                | constant in equation (5.3.2-6)                                | 5.3.2-6                          |
| b                | constant in the $\emptyset$ -profile expression               | 3.3-1                            |
| b'               | constant in the $I_3$ expression                              | 6.2-3                            |
| b*               | constant in equation (5.3.2-6)                                | 5.3.2-6                          |
| C                | constant in the drag law                                      | 5.2.2-10                         |
| C <sub>0</sub>   | " " " " "                                                     | 5.2.2-22                         |
| C <sub>1</sub>   | " " " " "                                                     | 5.2.2-22                         |
| C <sub>2</sub>   | " " " " "                                                     | 5.2.2-22                         |
| C <sub>3</sub>   | " " " " "                                                     | 5.2.2-22                         |
| C <sub>4</sub>   | " " " " "                                                     | 5.2.2-22                         |
| C'               | constant in the shear-work-integral expression                | 5.2.3-18                         |
| E                | constant in the velocity-profile expression                   | 3.1-1                            |
| f                | function of $\xi$                                             | 5.2.2-10                         |
| g                | " " "                                                         | 5.2.2-20                         |
| H <sub>12</sub>  | shape factor defined by eqn. (2.2-18)                         | 2.2-11                           |
| H <sub>32</sub>  | " " " " " (2.2-19)                                            | 2.2-12                           |
| H                | " " " " " (2.2-20)                                            | 2.2-12                           |
| I <sub>1</sub>   | integral property of the velocity profile                     | 2.2-1                            |
| I <sub>2</sub>   | " " " " " "                                                   | 2.2-1                            |
| I <sub>3</sub>   | " " " " " "                                                   | 2.2-2                            |
| I <sub>0</sub>   | " " " " " and<br>$\emptyset$ profiles                         | 2.2-1                            |
| I <sub>1</sub>   | integral property of the velocity and<br>$\emptyset$ profiles | 2.2-2                            |
| I <sub>0,1</sub> | " " " " " "                                                   | 2.2-9                            |

|                |                                                                                                    |          |
|----------------|----------------------------------------------------------------------------------------------------|----------|
| $J_\phi$       | total $\phi$ flux in the y direction                                                               | 2.1-4    |
| $\ell$         | Prandtl's mixing length (ft)                                                                       | 3.2-1    |
| $\ell'$        | parameter defined by eqn ( 3.1-12)                                                                 | 3.2-11   |
| $m''$          | entrainment rate into the boundary layer<br>(lbm/ft hr)                                            | 2.2-23   |
| $m_G$          | dimensionless entrainment                                                                          | 2.2-26   |
| $P$            | pressure ( $lb_f/ft^2$ )                                                                           | 2.1-3    |
| $R$            | square deviation                                                                                   | 3.1-14   |
| $R^*$          | " "                                                                                                | 5.2.2-23 |
| $R_C$          | Reynolds number of the flow through the<br>injection slot                                          | 4.3-1    |
| $R_G$          | Reynolds number based on the boundary<br>layer thickness                                           | 2.1-10   |
| $R_m$          | Reynolds number based on the mean<br>velocity within the boundary layer                            | 2.2-25   |
| $R_{max}$      | Reynolds number based on the maximum<br>velocity and its distance from the wall                    | 5.2.2-13 |
| $R_x$          | Reynolds number based on x                                                                         | 2.1-5    |
| $R_1$          | Reynolds number based on the displacement<br>thickness                                             | 2.2-18   |
| $R_2$          | Reynolds number based on the momentum<br>thickness                                                 | 2.2-11   |
| $R_3$          | Reynolds number based on the kinetic-<br>energy thickness                                          | 2.2-12   |
| $R_{2,u}$      | Reynolds number based on the momentum<br>deficit of the boundary layer above<br>the injection slot | 5.3.1-1  |
| $R_{\theta 1}$ | Reynolds number based on the $\phi$ -deficit<br>thickness                                          | 2.2-13   |
| $s$            | dimensionless shear stress                                                                         | 2.1-11   |
| $s_s$          | the drag coefficient                                                                               | 2.2-1    |
| $S$            | dimensionless $\phi$ transfer                                                                      | 2.1-12   |

|             |                                                        |          |
|-------------|--------------------------------------------------------|----------|
| $\bar{s}$   | shear-work-integral                                    | 2.2-2    |
| $u$         | x velocity component (ft/sec)                          | 2.1-1    |
| $u_{\max}$  | maximum velocity (ft/sec)                              | 4.2-3    |
| $u^*$       | dimensionless maximum velocity                         | 5.5.1-1  |
| $\bar{u}_C$ | mean injection velocity(ft/sec)                        | 4.3-1    |
| $u^+$       | dimensionless x velocity                               | 3.1-1    |
| $v$         | y velocity component (ft/sec)                          | 2.1-1    |
| $x$         | distance along the wall (ft)                           | 2.1-1    |
| $y$         | distance normal to the wall (ft)                       | 2.1-1    |
| $y_C$       | injection slot height (ft)                             | 4.3-1    |
| $y_{\max}$  | distance from the wall to the velocity<br>maximum (ft) | 5.2.2-13 |
| $y_{1/2}$   | "half-width" of the wall jet                           | 4.2-3    |
| $y^+$       | dimensionless distance from the wall                   | 3.1-1    |
| $y^*$       | dimensionless distance to velocity<br>maximum (ft)     | 5.5.1-1  |
| $z$         | ratio of local to mainstream velocity                  | 2.1-7    |
| $z_E$       | velocity profile parameter                             | 3.1-7    |

#### Greek letter symbols

|            |                                                                   |       |
|------------|-------------------------------------------------------------------|-------|
| $\delta_1$ | displacement thickness (ft)                                       | —     |
| $\delta_2$ | momentum thickness (ft)                                           | —     |
| $\delta_3$ | kinetic energy thickness (ft)                                     | —     |
| $\Delta$   | ratio of the $\delta$ to hydrodynamic<br>boundary-layer thickness | 3.3-3 |

|                |                                                                                  |          |
|----------------|----------------------------------------------------------------------------------|----------|
| $\epsilon$     | film-cooling effectiveness                                                       | 2.2-13   |
| $\zeta$        | shape factor                                                                     | 5.2.2-5  |
| $\eta$         | dimensionless distance from the surface                                          | 2.1-6    |
| $\eta'_2$      | integral property of the velocity profiles<br>of wall jets in still surroundings | 5.2.2-14 |
| $\eta'_3$      | integral property of the velocity profiles<br>of wall jets in still surroundings | 5.2.2-15 |
| $\theta$       | dimensionless $\theta$ profile                                                   | 2.1-8    |
| $\kappa$       | constant in the "law of the wall"                                                | 3.1-1    |
| $\lambda$      | dimensionless mixing length                                                      | 3.2-2    |
| $\lambda_1$    | dimensionless mixing length in the outer<br>regions of the flow                  | 3.2-4    |
| $\mu$          | dynamic viscosity ( $\text{lb}_m/\text{ft sec}$ )                                | 2.1-5    |
| $\xi$          | ratio of the distance from the surface<br>to the boundary layer thickness        | 3.1-7    |
| $\rho$         | fluid density ( $\text{lb}_m/\text{ft}^3$ )                                      | 2.1-1    |
| $\sigma$       | dimensionless shear stress                                                       | —————    |
| $\sigma_s$     | drag coefficient                                                                 | 5.2.2-11 |
| $\bar{\sigma}$ | dimensionless shear-work-integral                                                | 5.2.3-21 |
| $\tau$         | shear stress ( $\text{lb}_f/\text{ft}^2$ )                                       | 2.1-2    |
| $\emptyset$    | conserved property                                                               | 2.1-4    |
|                | ratio of local to mainstream density                                             | 2.1-9    |
| $\omega$       | wake function                                                                    | 3.1-7    |

## References

- Binckebanck, J. (1965), 'Messungen des Turbulenten Austausches in Windschatten Hinter beheizten K rpern' Int. J. Heat Mass Transfer 8, 83.
- Bradshaw, P. and Ferris, D. H. (1965) "The response of a retarded equilibrium turbulent boundary layer to the sudden removal of pressure gradient", NPL Aero Report 1145, ARC 26, 758, FM3577.
- Bradshaw, P. and Gee, M. T. (1962), "Turbulent wall jets with and without an external stream", ARC R&M 3252.
- Brunello, G. and Brun, E. A. (1957) "Contribution a l'etude la convection forcee de la chaleur sur des parois rugueuses", Publications scientifiques et techniques du Ministere de l'air, Paris No.332.
- Chi, S. W. (1962) Unpublished work at Imperial College.
- Chin, J. H., Skirvin, S. C., Hayes, L. E. and Silver, A., (1958), "Adiabatic wall temperature downstream of a single injection slot", ASME Paper No.58-A-107.
- Clauser, F. H. (1954) "Turbulent boundary layers in adverse pressure gradients", J. Aero. Sci. 21, 91.
- Cole, E. H. (1965), "A procedure for predicting the adiabatic wall temperature in the film cooling process", Imperial College, Mech. Eng. Dept. Rep. IC/HRJ/26.
- Coles, D. (1956) "The law of the wake in the turbulent boundary layer", J. Fluid Mech, 1.

- Coles, D. E. (1962) "The turbulent boundary layer in a compressible fluid", Rand Corp. Report R-403-PR.
- Cornish, J. J. (1960) "A universal description of turbulent boundary layer profiles with or without transpiration", Research Report 29, Aerophysics Dept. Mississippi State University.
- Elias, F. (1931) "The transference of heat from a hot plate to an air stream" NACA techn. memo No. 614.
- Eriam, F. and Eskinazi, S (1964) "The wall jet in a longitudinal pressure gradient", Syracuse Univ. Research Inst. Mech and Aerospace Eng. Dept. Rep. No. ME 937-6410F.
- Escudier, M. P. (1965) "The distribution of the mixing length in turbulent flows near walls", Imperial College, Mech. Eng. Dept. Rep. No. TWF/TN/1.
- Escudier, M. P. and Nicoll, W. B. (1965) "A single explicit drag law for uniform-density turbulent boundary layers and wall jets on smooth impermeable wall", Imperial College, Mech. Eng. Dept. Report TWF/TN/5.
- Escudier, M. P. and Nicoll, W. B., (1966a) "The entrainment function in turbulent boundary-layer and wall-jet calculations", J. Fluid Mech. 25, 337.
- Escudier, M. P., and Nicoll, W. B. (1966b) "The shear-work-integral relation in calculations of turbulent boundary layers", Imperial College, Mech. Eng. Dept. Report No. TWF/TN/18.

Escudier, M. P., Nicoll, W. B., and Spalding, D. B. (1966), "An explicit drag law for uniform-density turbulent boundary layers on smooth impermeable walls", Imperial College, Mech. Eng. Dept. Report No. TWF/TN/12.

Escudier, M. P., Nicoll, W. B., Spalding, D. B. and Whitelaw, J. H., (1966) "Decay of a velocity maximum in a turbulent boundary layer", Imperial College Mech. Eng. Dept. Report No. TWF/TN/13. To be published in Aeronautical Quarterly, November (1967).

Fage, A. (1938) "Profile and skin-friction aerofoil drags", ARC R&M 1852.

Fage, A. and Falkner, V. M. (1930) "An experimental determination of the intensity of friction on the surface of an aerofoil" ARC R&M 1315.

Fage, A. and Falkner, V. M. (1932) 'Note on experiments on the temperature and velocity in the wake of a heated cylindrical obstacle', Proc. Roy. Soc. A135 702.

Falkner, V. M. (1943) "The resistance of a smooth flat plate with turbulent boundary layer", Aircraft Eng., 15.

Felsch, K. O (1965) "Beitrag zur Berechnung turbulenter Grenzschichten in zweidimensionaler inkompressibler Strömung" Dr. -Ing. Dissert., Technischen Hochschule, Karlsruhe.

Fernholz, H. von (1964) "Halbempirische Gesetze zur Berechnung turbulenter Grenzschichten nach der Methode der Integralbedingungen" Ing. Arch 33, pp 384-395.

Gartshore, I. S. (1964) Jets and Wall jets in uniform streaming flow, McGill University, Mech. Eng. Dept. Rep. No.64-4.

Glauert, M. B. (1956) "The wall jet", J. Fluid Mech., 1, pp. 625-643.

Hartnett, J. P., Birkeback, R. C. and Eckert, E.R. G. (1961)  
"Velocity distributions, temperature distributions and heat transfer for air injected through a tangential slot into a turbulent boundary layer", J. Heat Transfer C83, 293.

Hartnett, J. P. , Birkeback, R. C. and Eckert, E.R.G. (1961)  
"Velocity distributions, temperature distributions, effectiveness and heat transfer in cooling of a surface with a pressure gradient", Int. Developments in Heat Transfer part IV, 682.

Herring, H. J. and Norbury, J. F. (1963) "Some experiments on equilibrium turbulent boundary layers in favourable pressure gradients", Dept. of Aerospace and Mech. Sci., Princeton University, FLD No.15.

Hildebrand, F. G. (1952) "Methods of Applied Mathematics"  
Prentice Hall.

Hinze, J. O. (1959) Turbulence, New York, McGraw-Hill.

Hinze, J. O. and B. G. Van der Hegge zijnen (1948) 'Heat - and mass transfer in the turbulent mixing zone of an axially symmetrical jet', Proc. of the 7th Int. Congress for Applied Mechanics, 2 part 1, 286.

- Jayatillaka, C. L. V. (1966) The influence of Prandtl number and surface roughness on the resistance of the laminar sub-layer to momentum and heat transfer, Imperial College, Mech. Eng. Dept. Rep. TWF/R/2.
- Klebanoff, P. S. (1955) "Characteristics of turbulence in a boundary layer with zero pressure gradient", NACA Rep. 1247.
- Kruka, V. and Eskinazi, S. (1963) "The wall jet in a moving stream" Mech. Eng. Dept. Report ME 937-630F, Syracuse University Research Institute.
- Ludwig, H. and Tillmann, W (1949) "Untersuchungen über die Wandschubspannung in turbulenten Reibungsschichten" Ing.-Arch. 17 pp 288-299; translated as "Investigations of the wall shearing stress in turbulent boundary layers", NACA TM 1285 (1950).
- Mathieu, J. (1961a) "Contribution a l'Etude Aerothermique d'un Jet Plan Evoluant En Presence d'un Paroi, Publ. Sci. Techn. Ministere de l'Air, No.374.
- Mathieu, J. 'Contribution a l'etude aerothermique d'un jet plan frappant une paroi sous des incidences diverses', Institut Francais des combustibles et de l'energie, Journees de la Transmission de la chaleur 3.15 (1961b).
- Moses, H. L. (1964) "The behaviour of turbulent boundary layers in adverse pressure gradients", MIT GTL Report No.73.

- Mueller, T. J., Korst, H. H. and Chow, W. L. (1964) "On the separation, reattachment, and redevelopment of incompressible turbulent shear flow" J. Basic Eng., Trans. ASME 94D, pp 221-226.
- Myers, G. E., Schauer, J. J. and Eustis, R. H. (1961) "The plane turbulent wall jet, Part 1. Jet development and friction factor." Tech. Rep. No. 1, Dept. of Mech. Eng. Stanford Univ.
- Nash, J. F. (1965) "A note on skin-friction laws for the incompressible turbulent boundary layer", NPL Aero Report 1135, ARC 26, 632 FM 2372.
- Newman, B. G. (1951) "Some contributions to the study of the turbulent boundary layer near separation", ARCC Report ACA-53.
- Nicoll, W. B. (1965) "Wall jet velocity profiles and entrainment", Report IC/HRJ/23, Mech. Eng. Dept. Imperial College.
- Nicoll, W. B. and Escudier, M. P. (1966) "Empirical relationships between the shape factors  $H_{32}$  and  $H_{12}$  for uniform-density turbulent boundary layers and wall jets" AIAA J., 4, (5), pp 940-942.
- Nicoll, W. B. and Whitelaw, J. H. (1965) "The solution of the unified theory equations and their application to problems involving heat transfer", Imperial College, Mech. Eng. Dept. Report IC/HRJ/25.
- Nicoll, W. B., and Whitelaw, J. H. (1966) "The effectiveness of the uniform density, two-dimensional wall jet", Imperial College, Mech. Eng. Dept. Report TWF/TN/17. To be published in the Int. J. of Heat and Mass Transfer, May (1967).

- Patankar, S. V. (1965) An investigation to determine the best expression for the wake component of the  $\delta$ -profile.  
Imperial College, Mech. Eng. Dept. Rep. TWF/TN/2.
- Patankar, S. V., and Spalding, D. B. (1965) Personal communication  
See also Spalding (1966).
- Patel, P. O. (1962) Self-Preserving Two-Dimensional Turbulent Jets and Wall Jets in a Moving Stream, McGill Univ. M. Eng. Thesis.
- Patel, R. P. and Newman, B. G. (1961) Self-preserving, two-dimensional turbulent jets and wall jets in a moving stream. McGill University, Mech. Eng. Dept. Rep. Ae5.
- Poreh, M. and Cermak, J. E. (1964) "Study of diffusion from a line source in a turbulent boundary layer", Int. J. Heat Mass Transfer 7, 1083.
- Reichardt, H. (1942) 'Gesetzma Bigkeiten der freien Turbulenz'  
VDI - Forschungsheft, 414, 2 ed 1951.
- Reynolds, W. C., Kays, W. D. and Kline, S. J. (1958) 'Heat transfer in the turbulent incompressible boundary layer: I-constant wall temperature' NASA Memo 12-1-58 W.
- Ross, D. (1953) "A study of incompressible turbulent boundary layers", Ph.D. Thesis, Harvard University.
- Rotta, J. (1950) "Über die Theorie der turbulenten Grenzschiechten"  
Mitteilungen aus dem Max-Planck-Institut für Strömungsforschung (Göttingen) Nr. 1; translated as "On the theory of the turbulent boundary layer" NACA TM 1344 (1953).

- Rotta, J. (1953) "Näherungsverfahren zur Berechnung turbulenter Grenzschichten unter Benutzung des Energiesatzes", Mitteilungen aus dem Max-Planck-Institut für Strömungsforschung (Göttingen), Nr. 8.
- Sandborn, V. A. and Slogar, R. J. (1955) "Study of the momentum distribution of turbulent boundary layers in adverse pressure gradients" NACA TN 3264.
- Schubauer, G. B. and Klebanoff, P. S. (1951) "Investigation of separation of the turbulent boundary layer" NACA Rep. 1030.
- Schubauer, G. B. and Spangenberg, W. (1960) "Forced mixing in boundary layers", J. Fluid Mech. 8, (1) pp 10-32.
- Schwarz, W. H. and Cosart, W. P. (1961) "The Two-Dimensional Turbulent Wall Jet, Journal of Fluid Mech. Vol 10, part 4, p 481.
- Seban, R. A. and Back, L. H. (1961) Velocity and temperature profiles in a wall jet, Int. J. of Heat and Mass Transfer Vol 3, No.4.
- Sigalla, A. (1958) "Experimental data on turbulent wall jets", Aircraft Engineering 30 pp 13-134.
- Spalding, D. B. (1963) "Recommendations for the choice of the velocity of injection of a film coolant", Imperial College, Mech. Eng. Dept. Report IC/HRJ/21.
- Spalding, D. B. (1964a) "A unified theory of friction, heat transfer and mass transfer in the turbulent boundary layer and wall jet," Aero. Res. Council. Rep. No. 25, 925.

- Spalding, D. B. (1964b) "A unified theory of friction, heat transfer and mass transfer in the turbulent boundary layer and wall jet, Imperial College, Mech. Eng. Dept. Rep. (Annotated version of the previous reference)
- Spalding, D. B. (1965a) The kinetic-energy-deficit equation of the turbulent boundary layer AGARDograph, No. 97, part 1, 191.
- Spalding, D. B. (1965b) A relation between entrainment rate, shear-stress gradient and velocity gradient", Imperial College, Mech. Eng. Dept. Report TWF/TN/8.
- Spalding, D. B. (1966) "Progress in the development of a unified theory of friction, heat transfer, and mass transfer in boundary layers and jets", TWF/TN/10 Mech Eng. Dept., Imperial College.
- Stollery, J. L., and El-Ehwany, A. A. M., Personal communication.
- Stratford, B. S. (1959) "An experimental flow with zero skin friction throughout its regions of pressure rise", J. Fluid Mech 5, 17.
- Townsend, A. A. (1949) "The fully developed turbulent wake of a circular cylinder", Australian J. Sci. Research series A 2, 451.
- Truckenbrodt, E. (1951) "Ein Quadraturverfahren zur Berechnung der laminaren und turbulenten Reibungsschicht bei ebener und rotationssymmetrischer Strömung Ing.-Arch. 20, - 211. Translated as "A method of quadrature for calculation of the laminar and turbulent boundary layer in the case of plane and rotationally symmetrical flow" NACA TM 1379(1955).

- Uram, E. M. (1960) "Skin-friction calculation for turbulent boundary layers in adverse pressure distributions", J. Aero/Space Sci. 27, pp 75-76
- Walz, A. (1965) "Über Fortschritte in Näherungstheorie und Praxis der Berechnung kompressibler laminarer und turbulenter Grenzschichten mit Wärmeübergang", Zeitschrift für Flugwissenschaften, 13, (3) pp 89-102.
- Wieghardt, K. (1948) "Über Ausbreitungsvorgänge in Turbulenten Reibungsschichten", Z. Angew. Math. Mech. 27, 346.

TABLE 1 : A summary of the sources of velocity profile data used for the determination of entrainment

| Author(s)          | Date | Run | $u_c/u_G$ | $R_C$  | $(x/y_c)_{\max}$ |
|--------------------|------|-----|-----------|--------|------------------|
| Gartshore          | 1964 | 1   | 3.00      | 20,200 | 147              |
|                    |      | 2   | 5.07      | 20,200 | 167              |
| Seban & Back       | 1961 | 1   | 7.60      | 3,030  | 250              |
|                    |      | 2   | 9         | 6,050  | 250              |
| Kruka and Eskinazi | 1964 | 1   | 3.8       | 10,740 | 275              |
|                    |      | 2   | 10        | 13,060 | 275              |
|                    |      | 3   | 18        | 26,270 | 275              |
| Present Work       |      | 1   | 4.16      | 22,076 | 75               |
|                    |      | 2   | 2.71      | 23,660 | 100              |
|                    |      | 3   | 13.15     | 23,070 | 50               |
|                    |      | 4   | 6.22      | 22,795 | 100              |
|                    |      | 5   | 2.10      | 5,590  | 50               |
|                    |      | 7   | 2.43      | 11,373 | 100              |
|                    |      | 8   | 3.37      | 11,392 | 75               |
|                    |      | 9   | 4.81      | 11,350 | 100              |
|                    |      | 10  | 12.92     | 11,180 | 100              |
|                    |      | 11  | 1.55      | 14,750 | 50               |
| Patel and Newman   | 1961 | 1   | 4.05      |        | 50               |
|                    |      | 2   | 3.0       |        | 100              |

TABLE 2 : A summary of the sources of drag data

| Author(s)                 | Date | $s_s$ determination                           | Comments                                                                                                                                                                                          |
|---------------------------|------|-----------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Fage and Falkner          | 1930 | Stanton surface                               | Because of large shear-stress gradients near the wall, and because of experimental scatter, the $s_s$ values obtained by these authors by extrapolating shear-stress profiles to wall unreliable. |
| Fage                      | 1938 | tubes                                         |                                                                                                                                                                                                   |
| Ludwig and Tillmann       | 1949 | Heated Element                                |                                                                                                                                                                                                   |
| Newman                    | 1951 | Clauser plot                                  |                                                                                                                                                                                                   |
| Schubauer and Kebanoff    | 1951 | Clauser plot                                  |                                                                                                                                                                                                   |
| Clauser                   | 1954 | Clauser plot                                  |                                                                                                                                                                                                   |
| Klebanoff                 | 1955 | Extrapolation of shear-stress profile to wall |                                                                                                                                                                                                   |
| Sandborn and Slogar       | 1955 | Clauser plot                                  |                                                                                                                                                                                                   |
| Schubauer and Spangenberg | 1960 | Clauser plot                                  |                                                                                                                                                                                                   |
| Herring and Norbury       | 1963 | Clauser plot                                  |                                                                                                                                                                                                   |
| Moses                     | 1964 | Clauser plot                                  | Authors reported $s_s$ values not used because velocity profiles were not reported at the shear-stress-measurements stations.                                                                     |
| Bradshaw and Ferriss      | 1965 | Preston Tubes                                 |                                                                                                                                                                                                   |
| present work              |      | Clauser plot                                  | $-\infty < R_2 < 0$                                                                                                                                                                               |
| Kruka and Eskinazi        | 1963 | Preston tube                                  | $-\infty < R_2 < 0$                                                                                                                                                                               |
| Bradshaw and Gee          | 1962 | Surface tube                                  | $R_2 = -\infty$                                                                                                                                                                                   |
| Sigalla                   | 1958 | Preston Tube                                  | $R_2 = -\infty$ ; used only for graphical comparison                                                                                                                                              |
| Myers, Schauer and Eustis | 1961 | Clauser plot                                  | $R_2 = -\infty$ ; $s_s$ also measured by heated-film technique. Agreement between results obtained by both methods good.                                                                          |

TABLE 3 : Comparison of present drag law with previous recommendations

| Author(s)                   | Date | Type     | Root-mean-square deviations<br>(per cent) |              |                 |         |
|-----------------------------|------|----------|-------------------------------------------|--------------|-----------------|---------|
|                             |      |          | $R_2 > 0$                                 | $0 \geq R_2$ | $R_2 = -\infty$ | Overall |
| Ludwig and Tillmann         | 1949 | Explicit | 19.21                                     | 1            | 1               | 1       |
| Ross                        | 1953 | "        | 19.71                                     | 1            | 1               | 1       |
| Uram                        | 1960 | "        | 19.40                                     | 1            | 1               | 1       |
| Fernholz                    | 1964 | "        | 14.95                                     | 1            | 1               | 1       |
| Mueller, Korst and Chow     | 1964 | "        | 15.84                                     | 1            | 1               | 1       |
| Walz                        | 1965 | "        | 20.60                                     | 1            | 1               | 1       |
| Felsch                      | 1965 | "        | 14.45                                     | 1            | 1               | 1       |
| Glauert                     | 1956 | "        | 2                                         | 2            | 29.05           | 2       |
| Sigalla                     | 1958 | "        | 2                                         | 2            | 13.22           | 2       |
| Bradshaw and Gee            | 1962 | "        | 2                                         | 2            | 9.61            | 2       |
| Rotta (1)                   | 1950 | Implicit | 24.12                                     | 20.15        | 3               | 3       |
| Rotta (2)                   | 1953 | "        | 21.43                                     | 1            | 1               | 1       |
| Coles                       | 1956 | "        | 15.35                                     | 15.93        | 3               | 3       |
| Cornish                     | 1960 | "        | 13.74                                     | 16.37        | 3               | 3       |
| Moses                       | 1964 | "        | 12.38                                     | 15.71        | 16.80           | 13.93   |
| Spalding                    | 1964 | "        | 13.83                                     | 15.79        | 14.03           | 14.24   |
| Nash                        | 1965 | "        | 15.41                                     | 1            | 1               | 1       |
| "log-wall plus linear-wake" | -    | "        | 18.48                                     | 30.07        | 9.48            | 19.96   |
| Present                     | -    | Explicit | 10.96                                     | 12.91        | 8.02            | 10.88   |

Notes:

- 1) Not applicable to flows with negative  $R_2$
- 2) Restricted to flows with negative infinite  $R_2$
- 3) Restricted to flows with finite  $R_2$

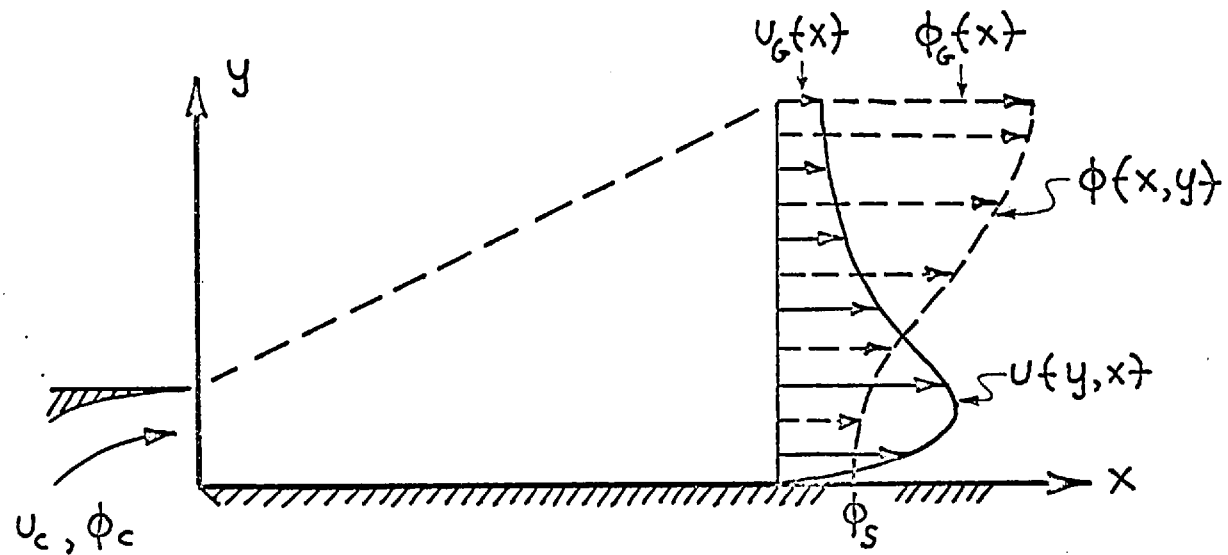


FIGURE I.1-1: A SCHEMATIC REPRESENTATION OF THE PHYSICAL PROBLEM CONSIDERED

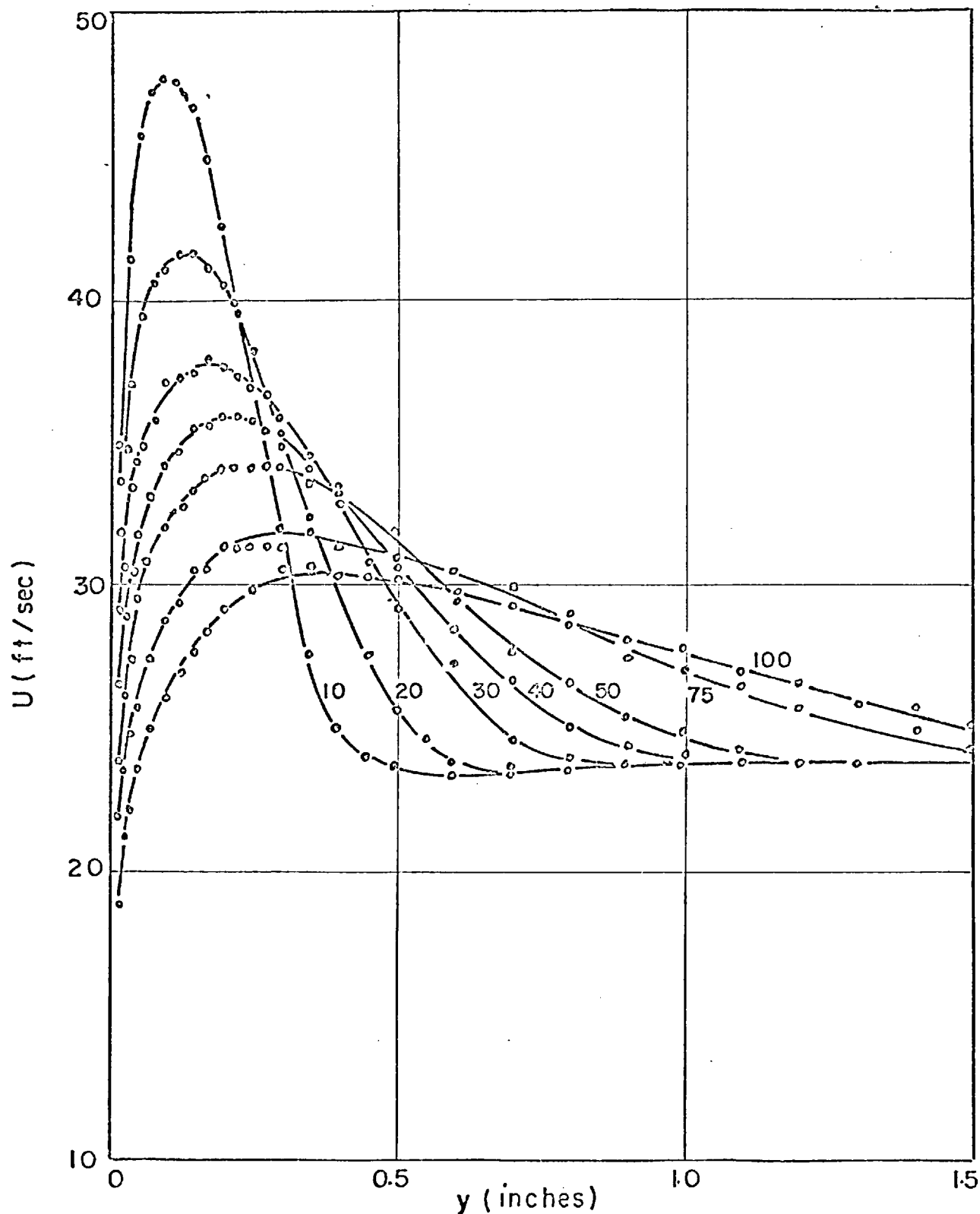
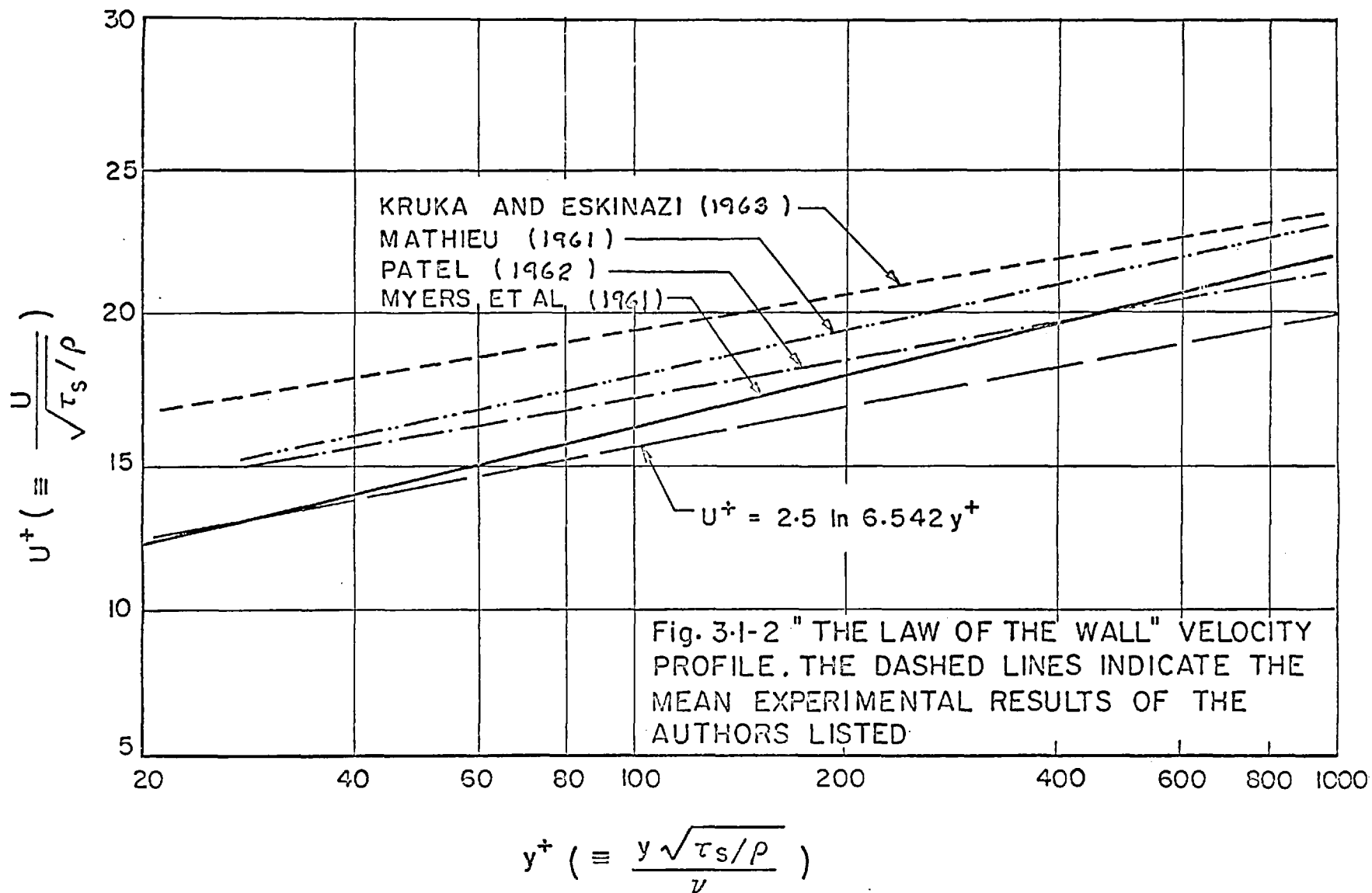


Fig. 3.1-1 TYPICAL VELOCITY PROFILE DEVELOPMENT.  
(THE PARAMETER ON THE PROFILES IS  $X/Y_c$ )



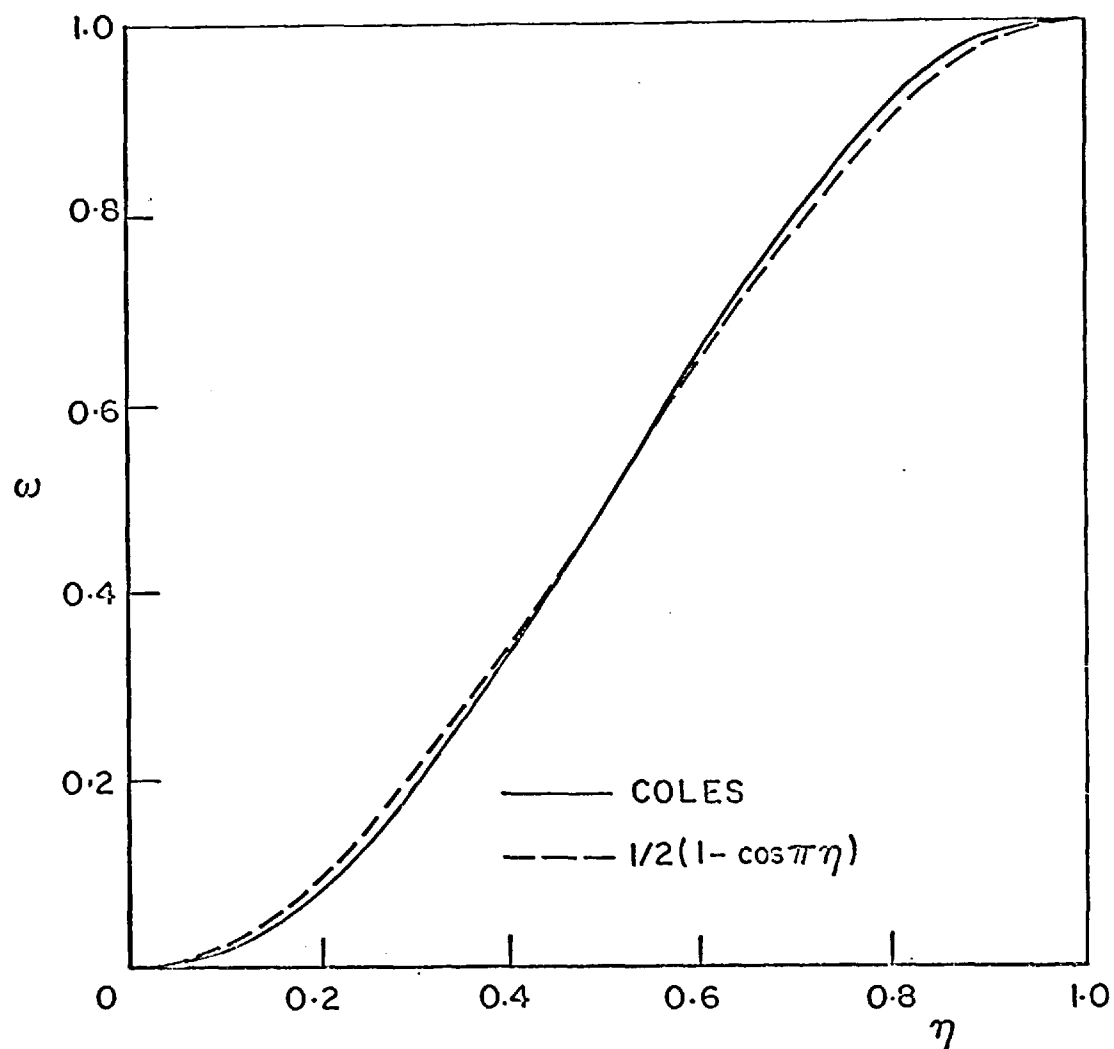
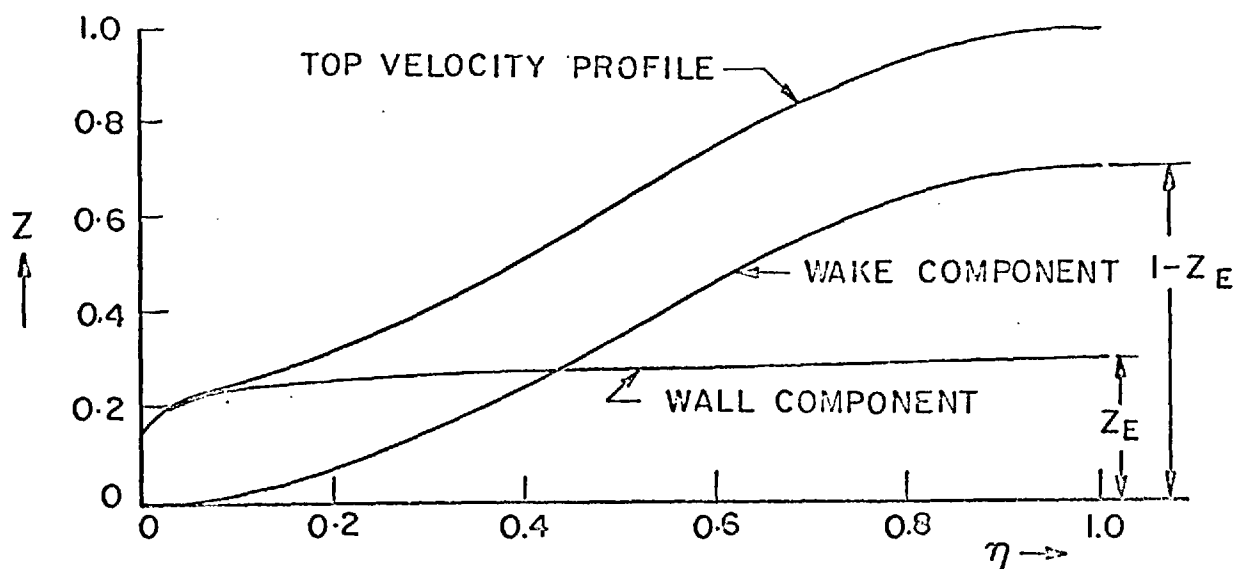
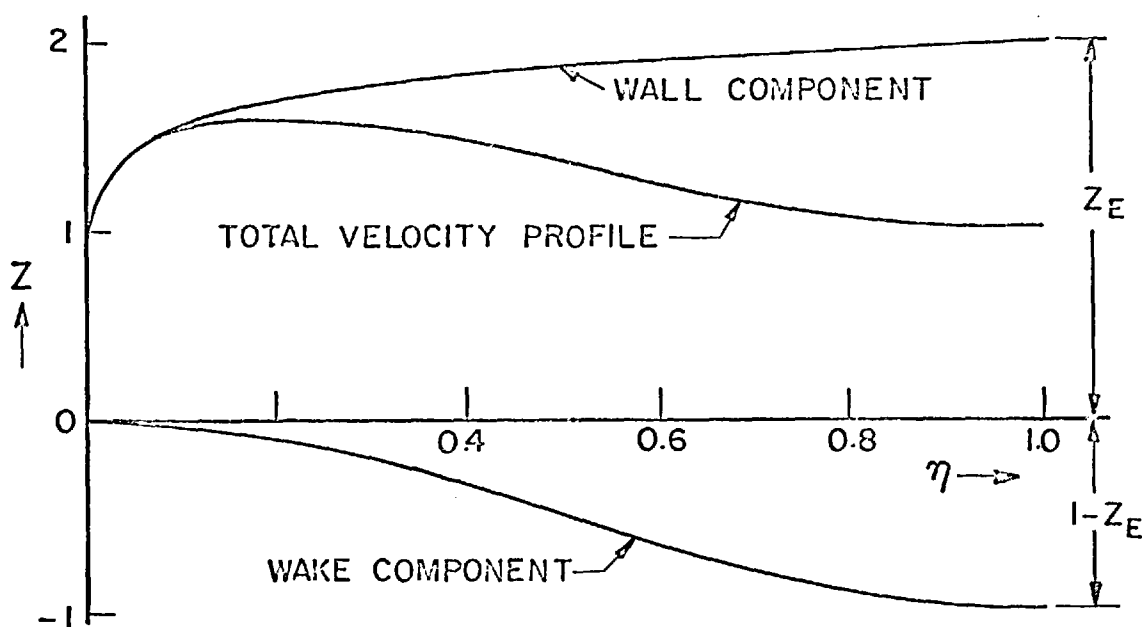


Fig. 3.1-3 COMPARISON OF  $\omega = 1/2(1 - \cos \pi \eta)$  WITH COLE'S LAW OF THE WAKE



a) BOUNDARY-LAYER VELOCITY PROFILE  $Z_E < 1$



b) WALL-JET VELOCITY PROFILE  $Z_E > 1$

Fig. 3.1-4 TYPICAL VELOCITY PROFILES AS GIVEN BY EQUATION (3.1-13)

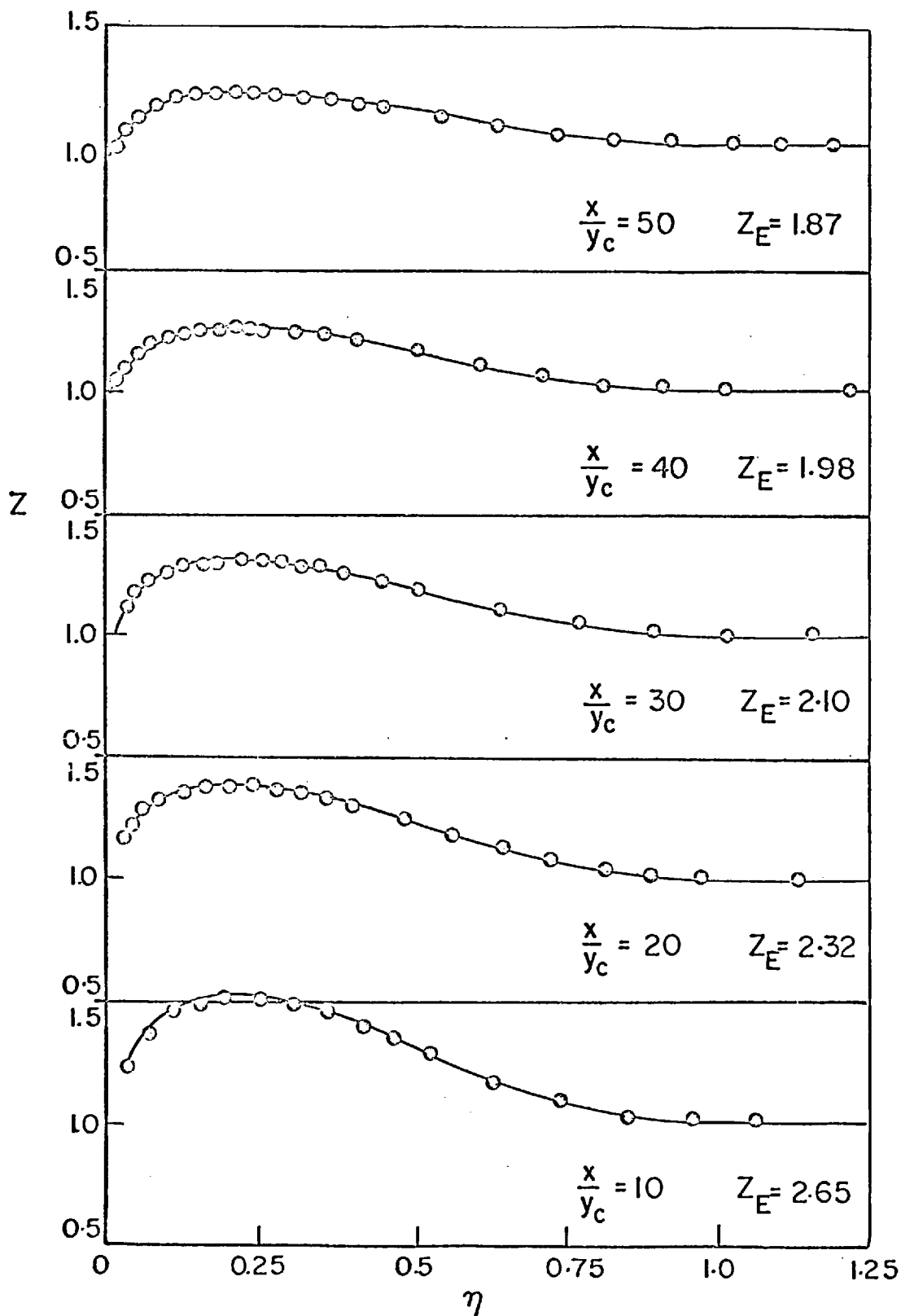


Fig. 3.1-5a COMPARISON OF PRESENT EXPERIMENTAL VELOCITY PROFILES WITH EQUATION (3.1-13)

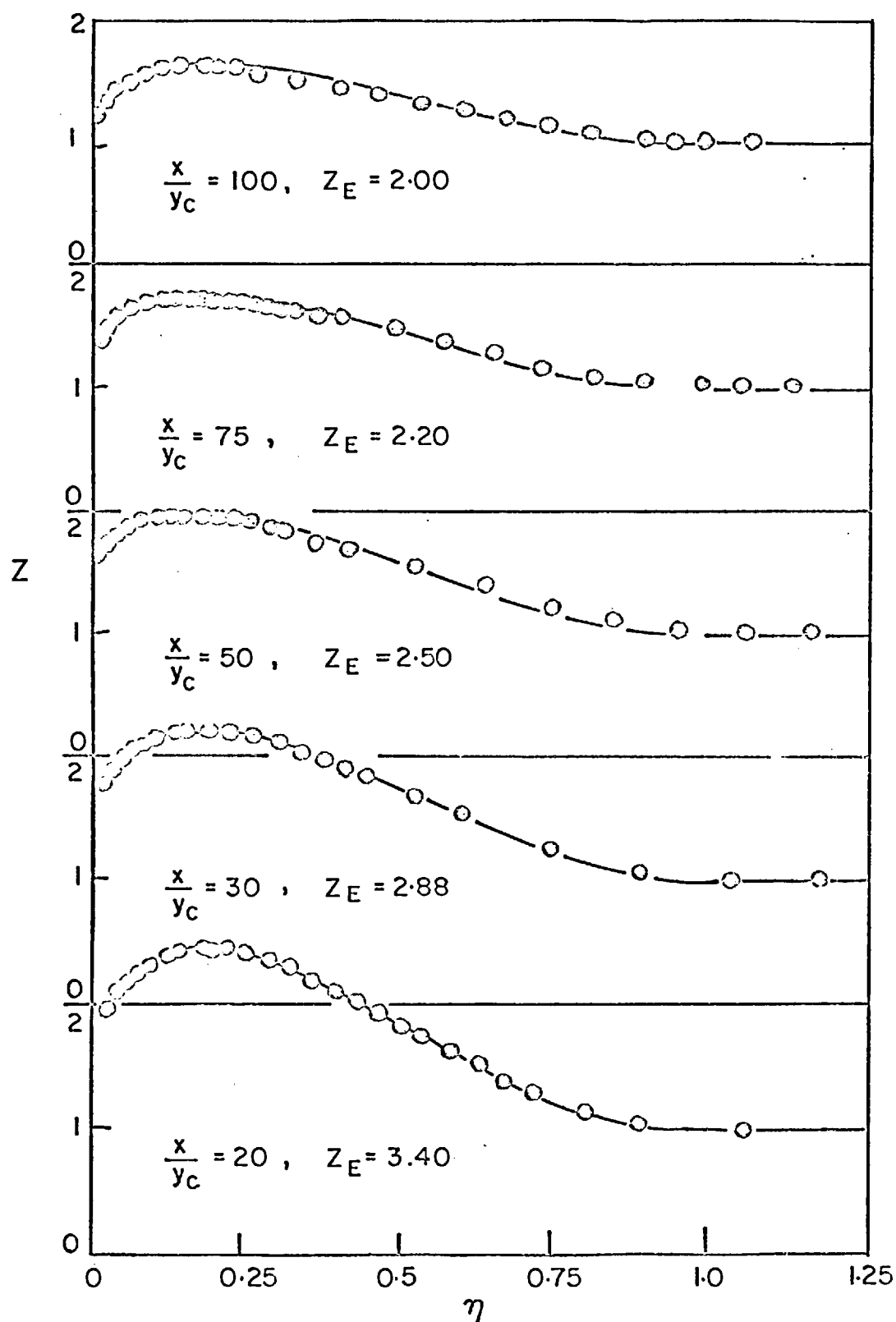


Fig. 3.1-5b COMPARISON OF PRESENT EXPERIMENTAL VELOCITY PROFILES WITH EQUATION (3.1-13)

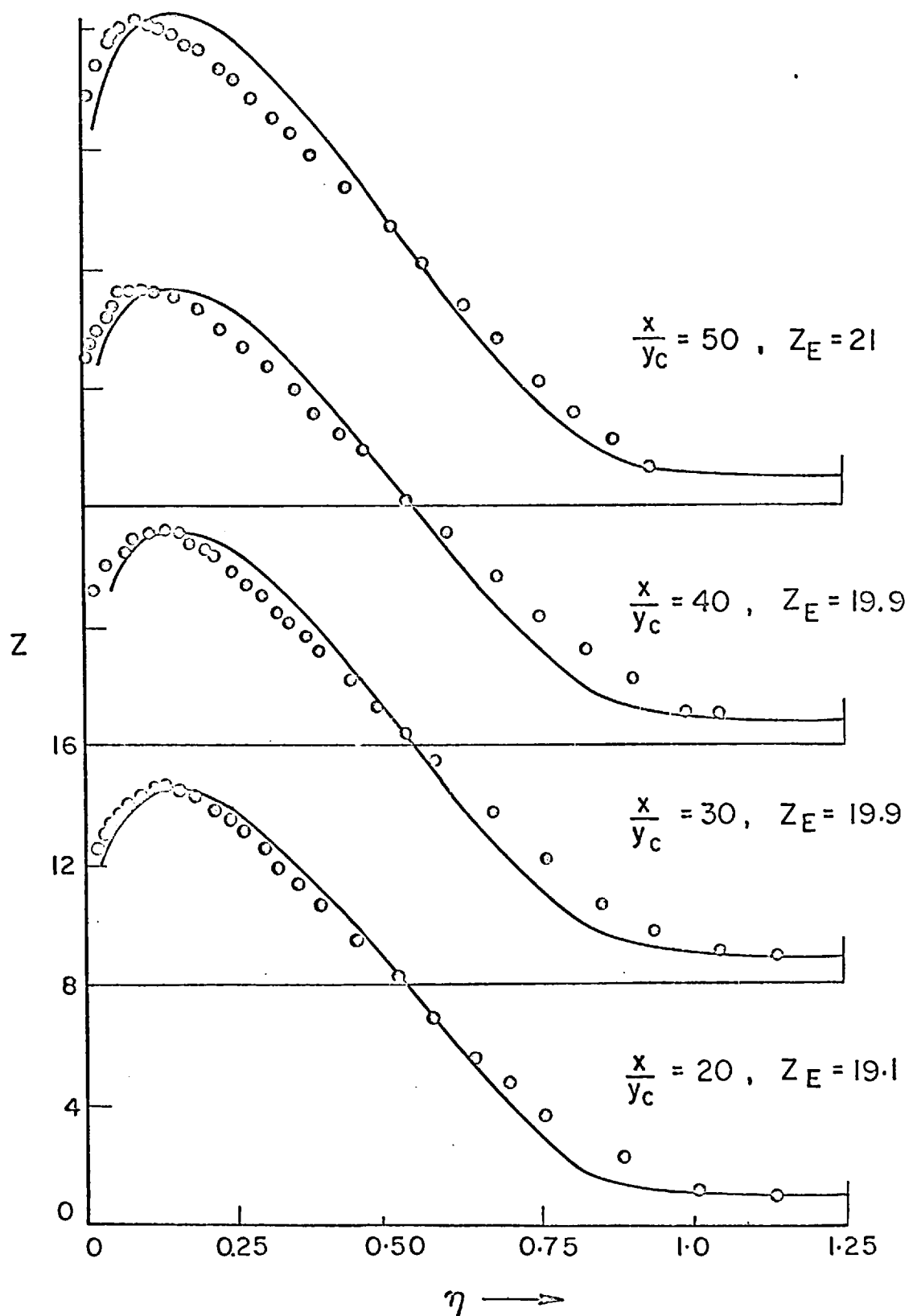


Fig. 3.1-5c COMPARISON OF PRESENT EXPERIMENTAL VELOCITY PROFILES WITH EQUATION (3.1-13)

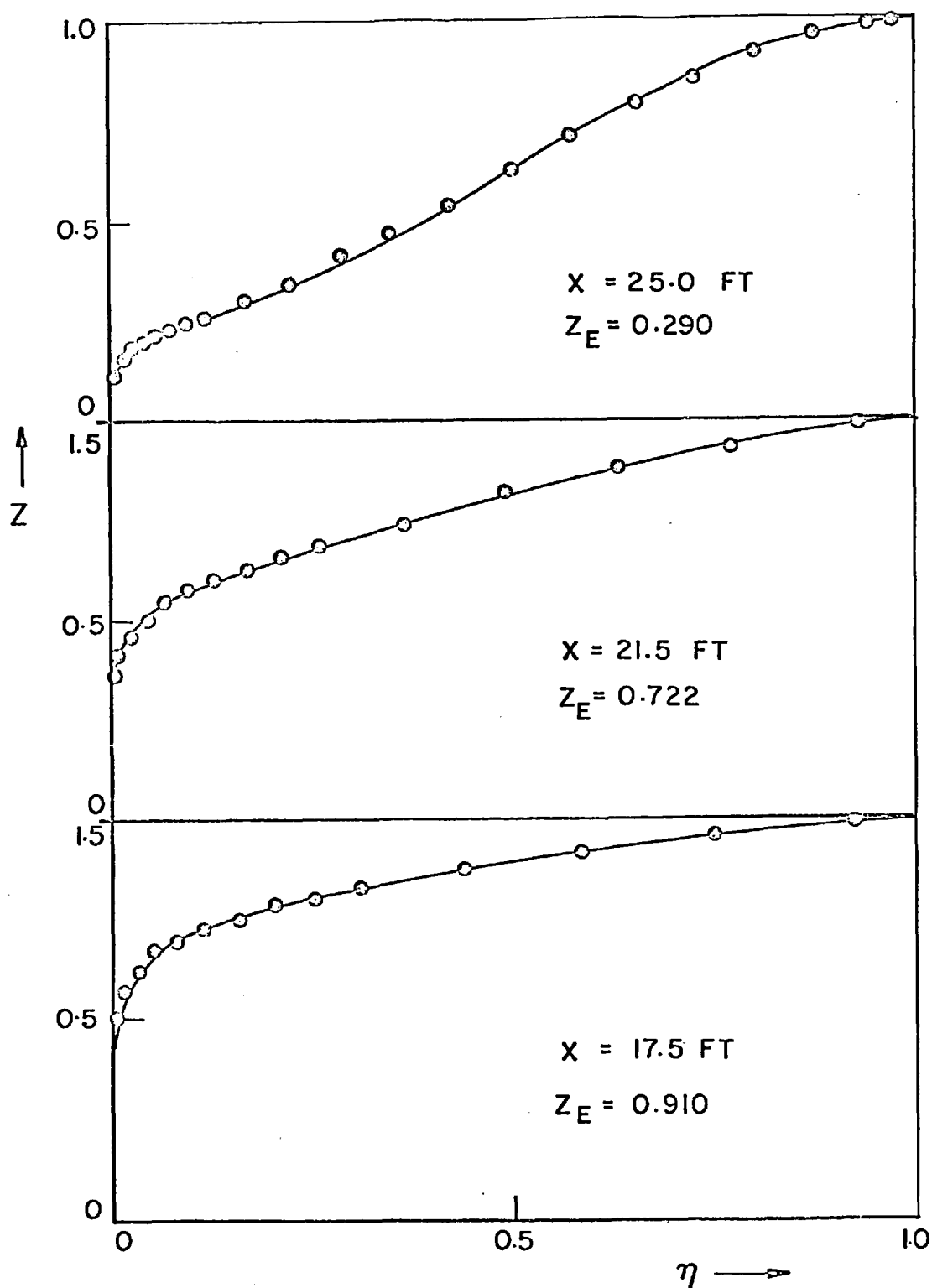


Fig. 3.1-6 COMPARISON OF EXPERIMENTAL VELOCITY PROFILES OF SCHUBAUER AND KLEBANOFF (1951) WITH EQUATION (3.1-13) [FROM ESCUDIER AND NICOLL (1966)]

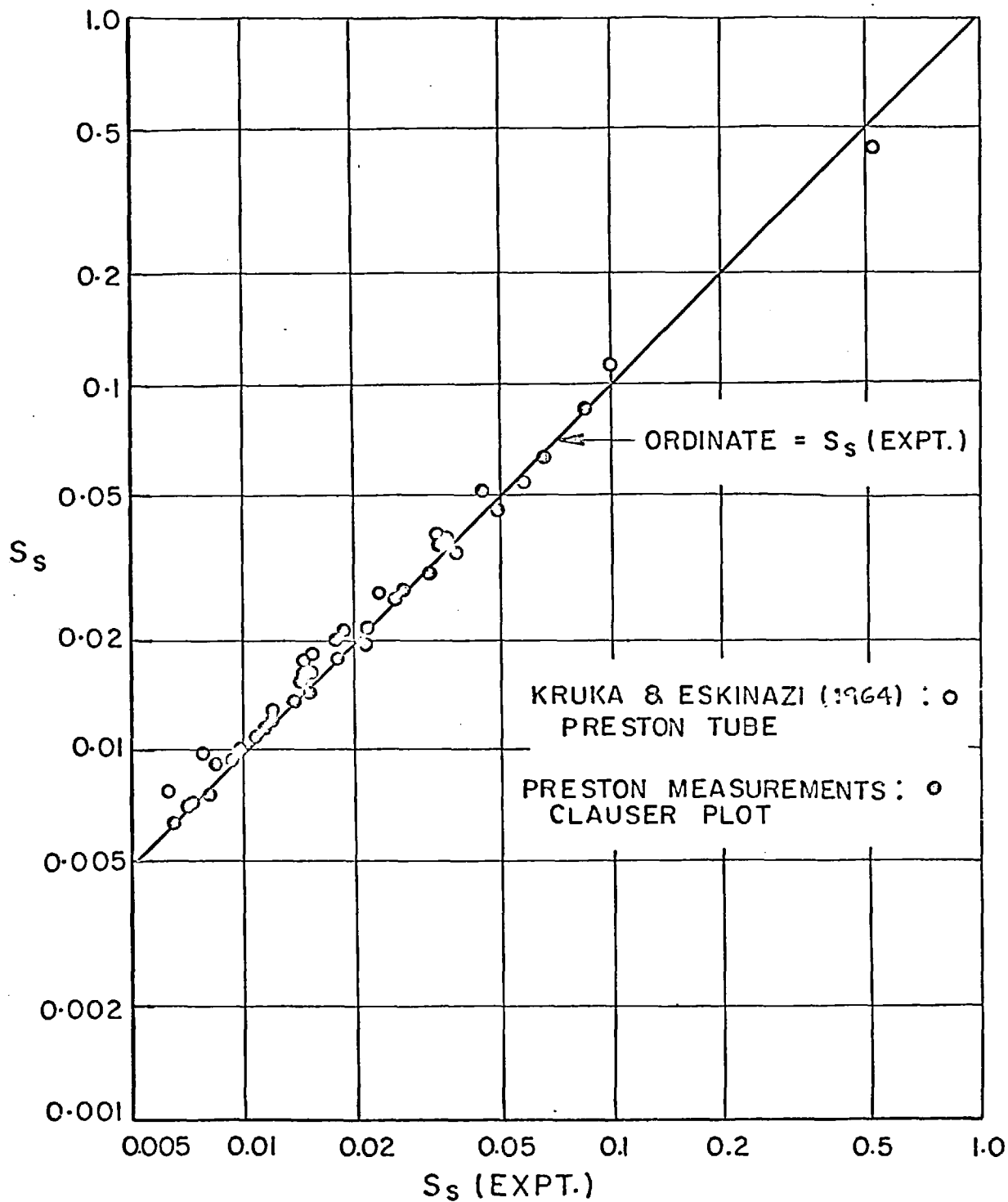


Fig. 3-1-7 COMPARISON OF EXPERIMENTAL UNIFORM-DENSITY DRAG COEFFICIENTS WITH VALUES COMPUTED FROM EQUATION (3.1-11) ; WALL-JET FLOWS

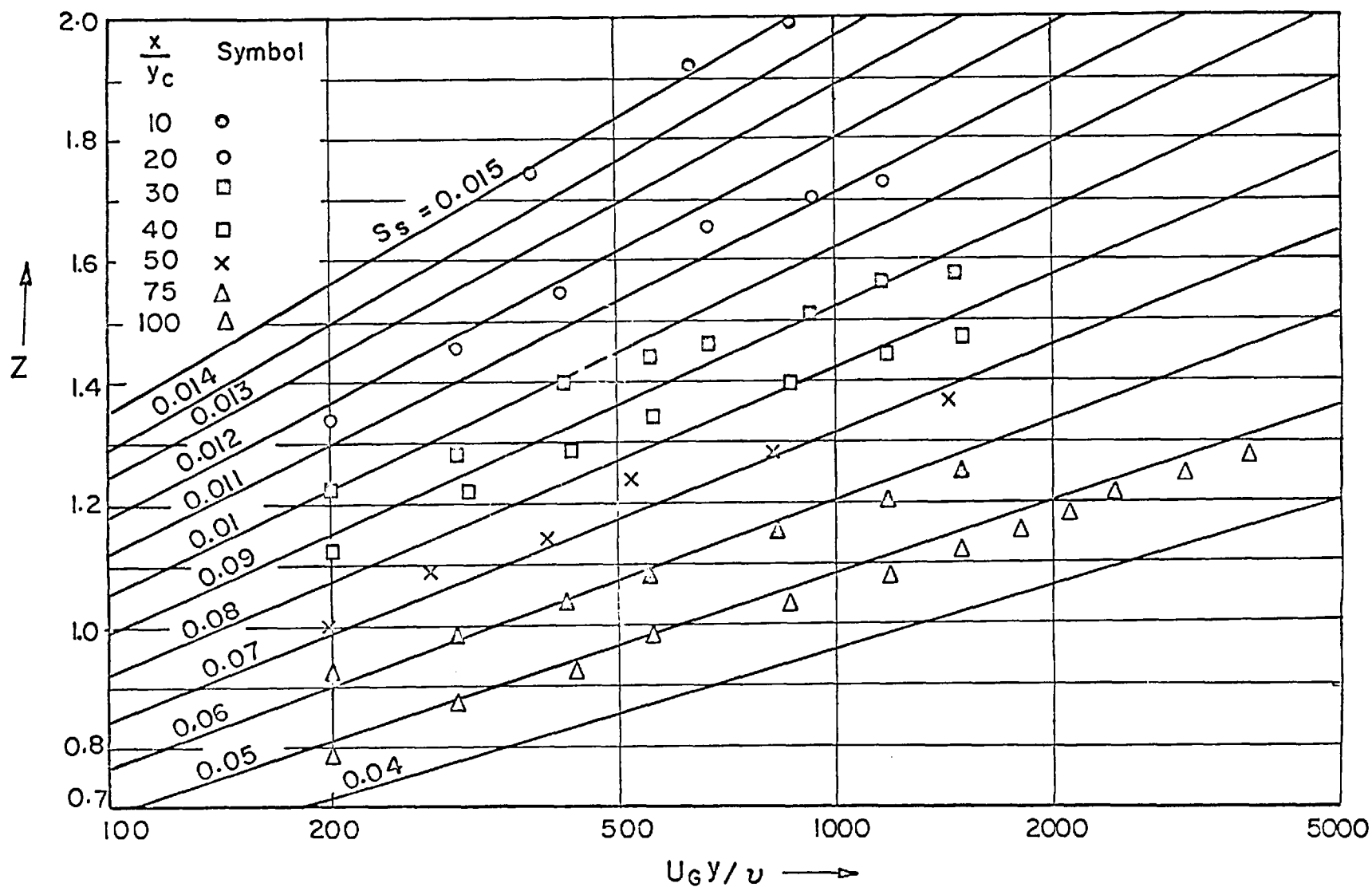


Fig. 3.1-8 A TYPICAL CLAUSER PLOT ( PRESENT MEASUREMENTS -RUN 5 )

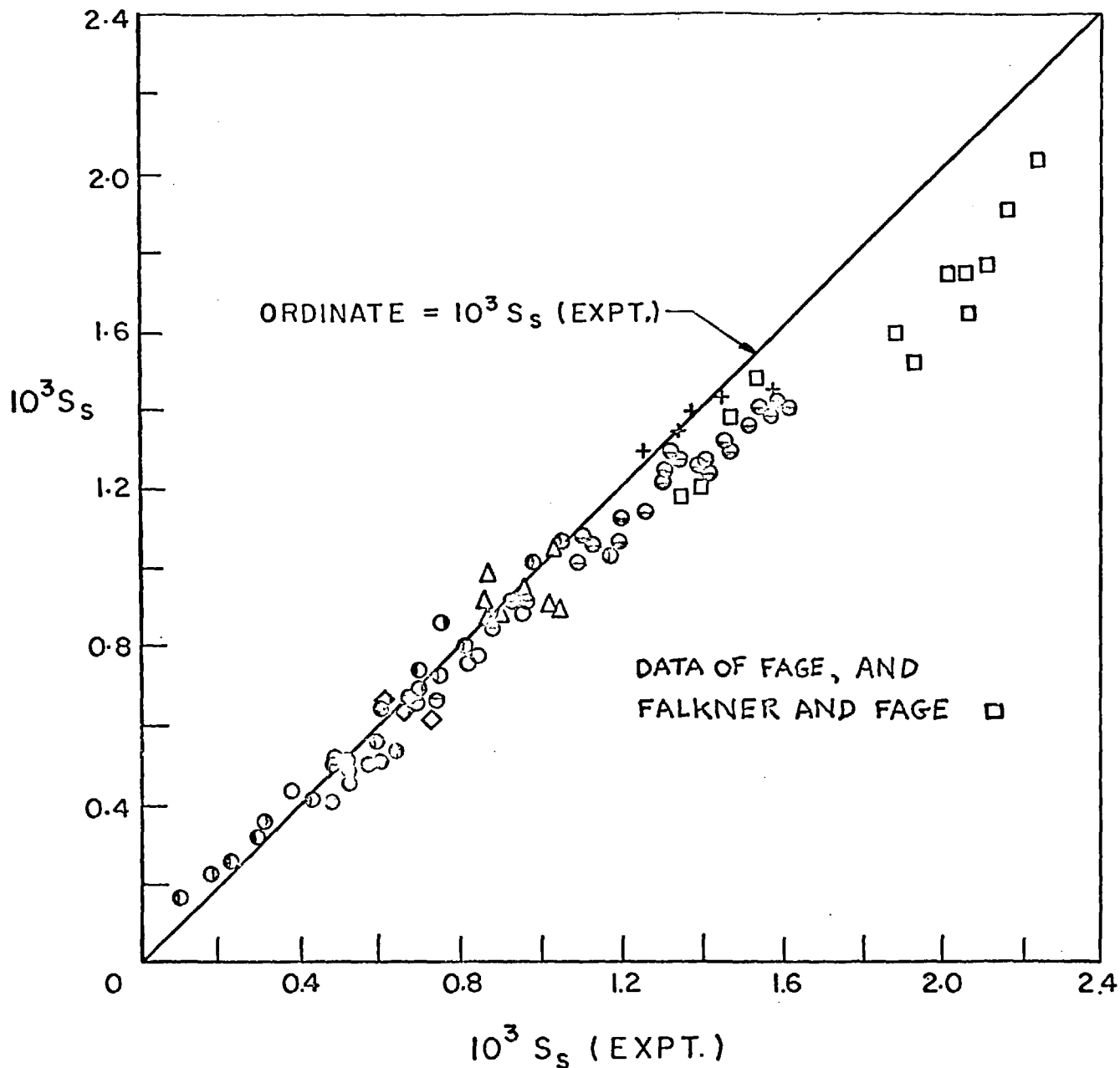


Fig. 3.1-9 COMPARISON OF EXPERIMENTAL UNIFORM-DENSITY DRAG COEFFICIENTS WITH VALUES COMPUTED FROM EQUATION (3.1-11) ; BOUNDARY-LAYER FLOWS [FROM ESCUDIER AND NICOLL (1966)]

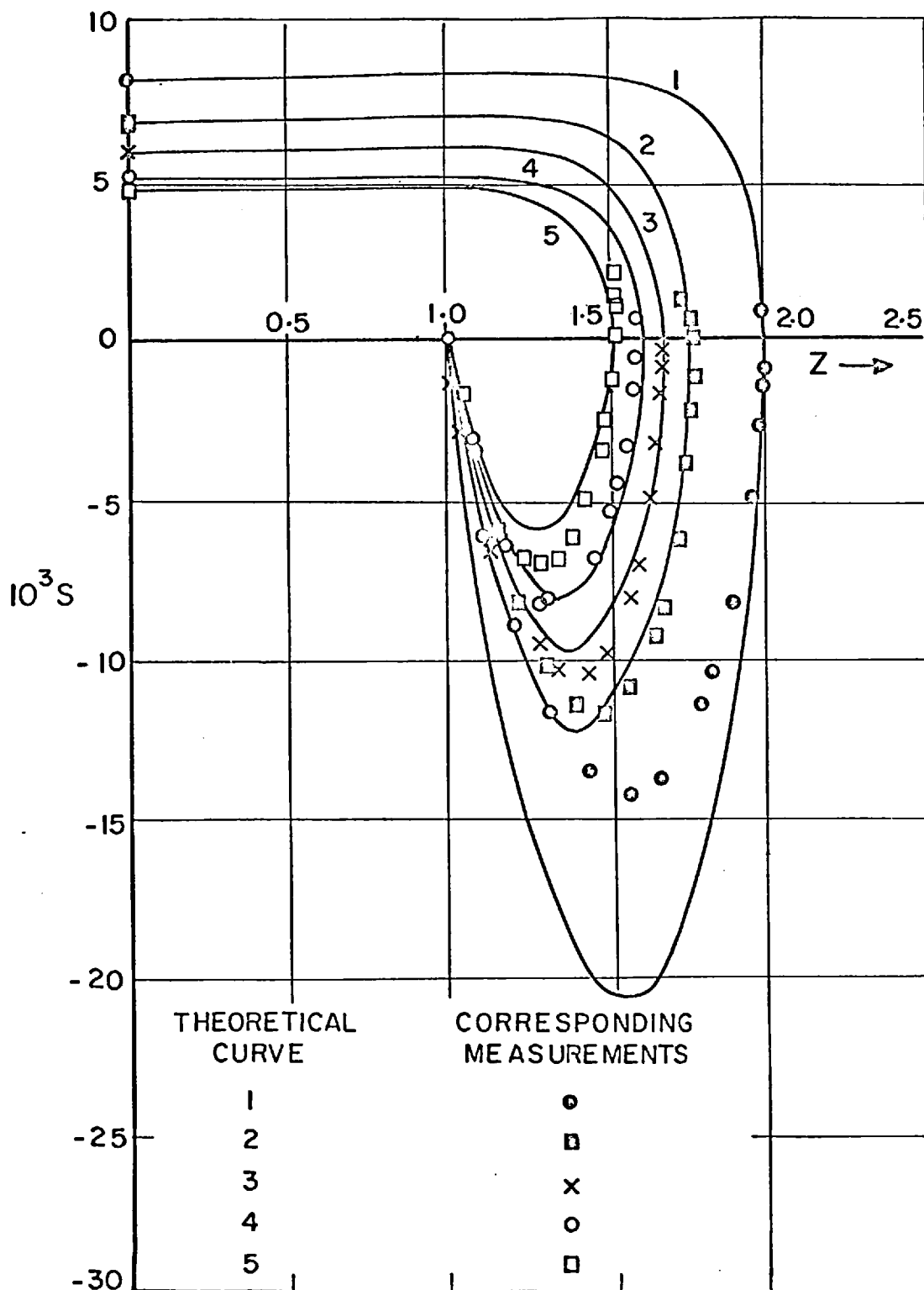


Fig. 3.2-1 COMPARISON OF EXPERIMENTAL WITH THEORETICAL SHEAR-STRESS DISTRIBUTIONS. DATA OF ERIAN & ESKINAZI (1964)

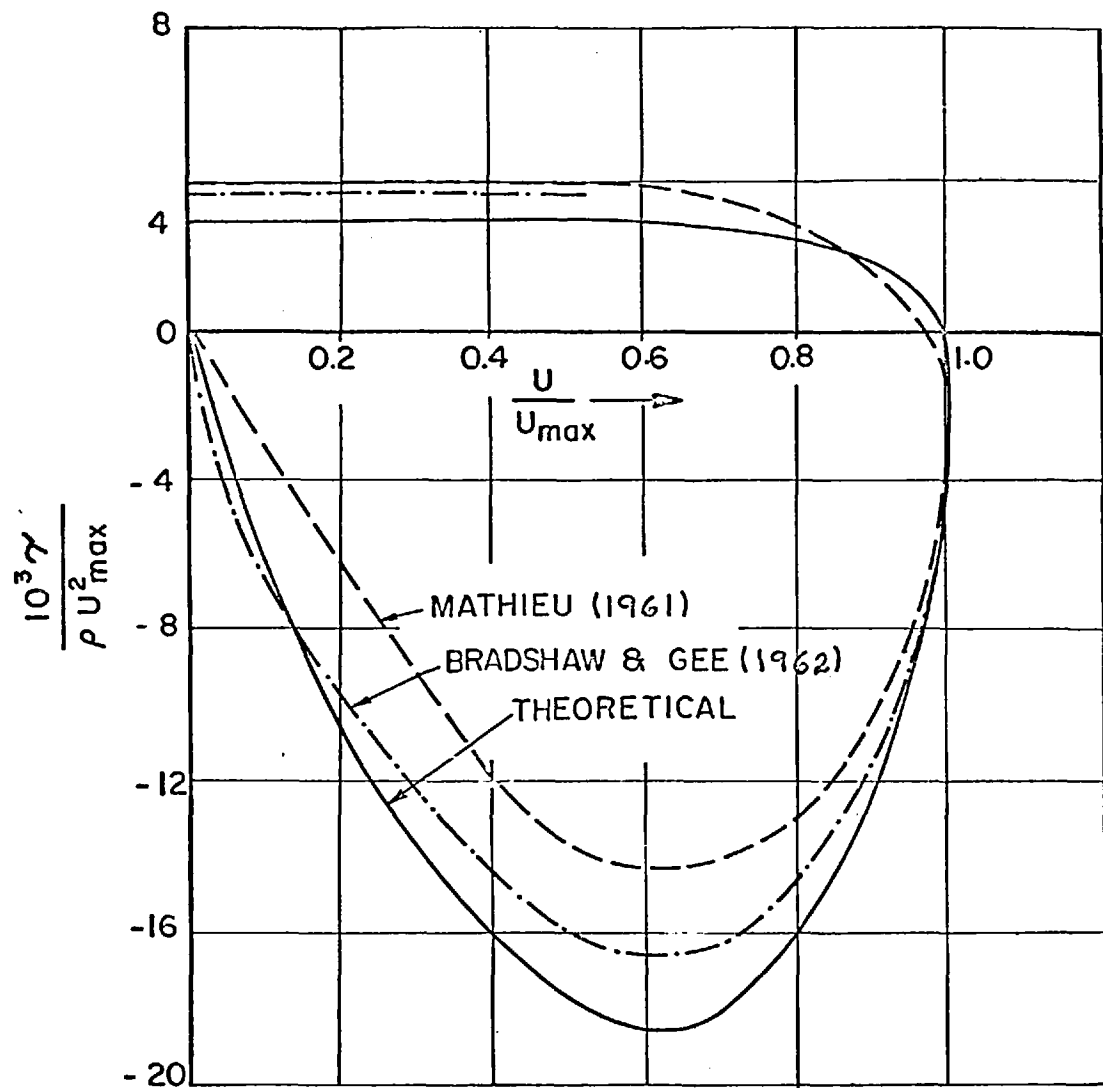


Fig. 3.2 -2 COMPARISON OF EXPERIMENTAL AND THEORETICAL SHEAR-STRESS DISTRIBUTION FOR WALL JETS IN STILL SURROUNDINGS

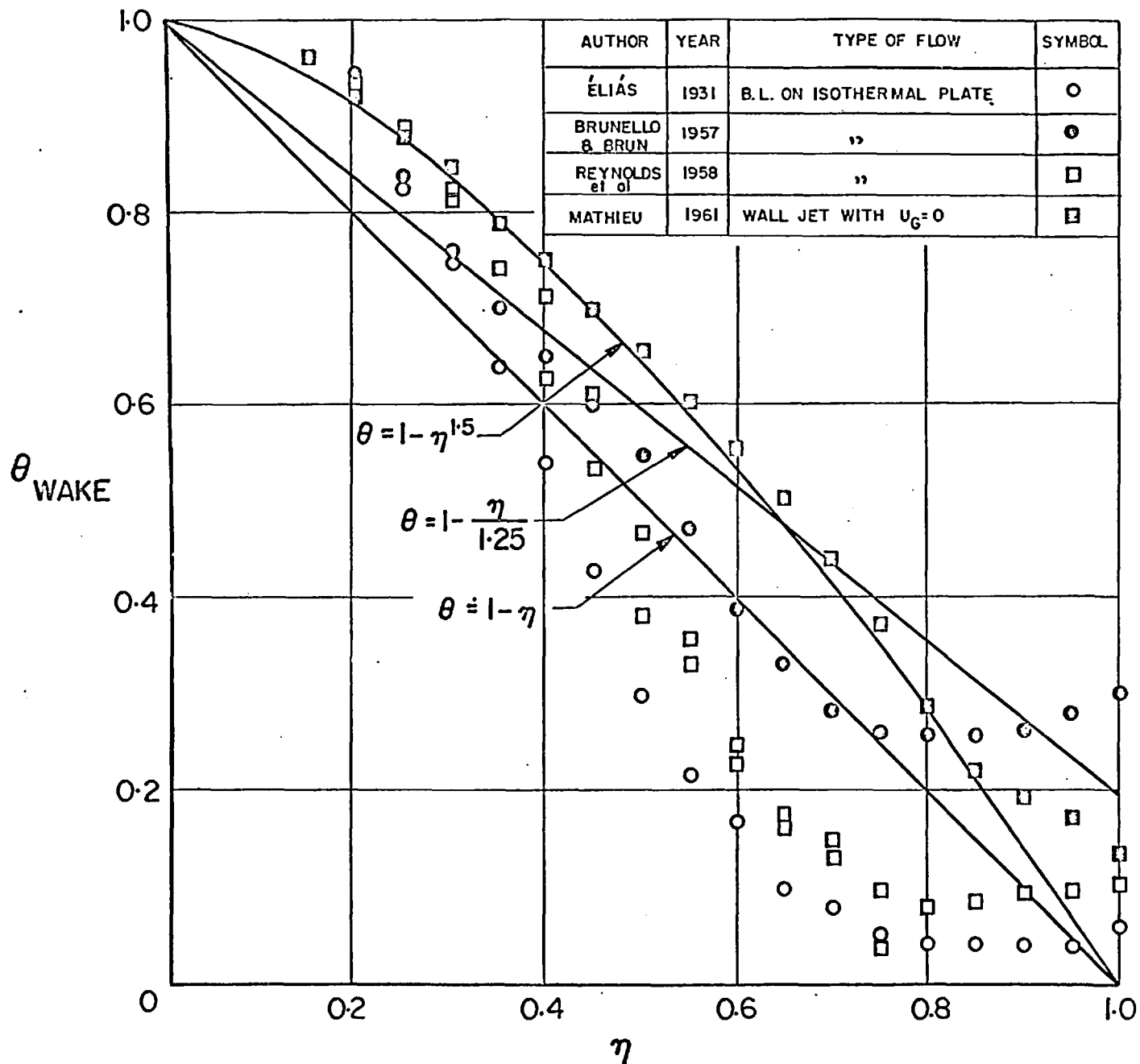


Fig. 3.3-1(a) THE  $\phi$  PROFILE WAKE COMPONENT : WALL-SHEAR FLOWS WITH  $(\partial\phi/\partial\eta) \neq 0$ , [FROM PATANKAR (1965)]

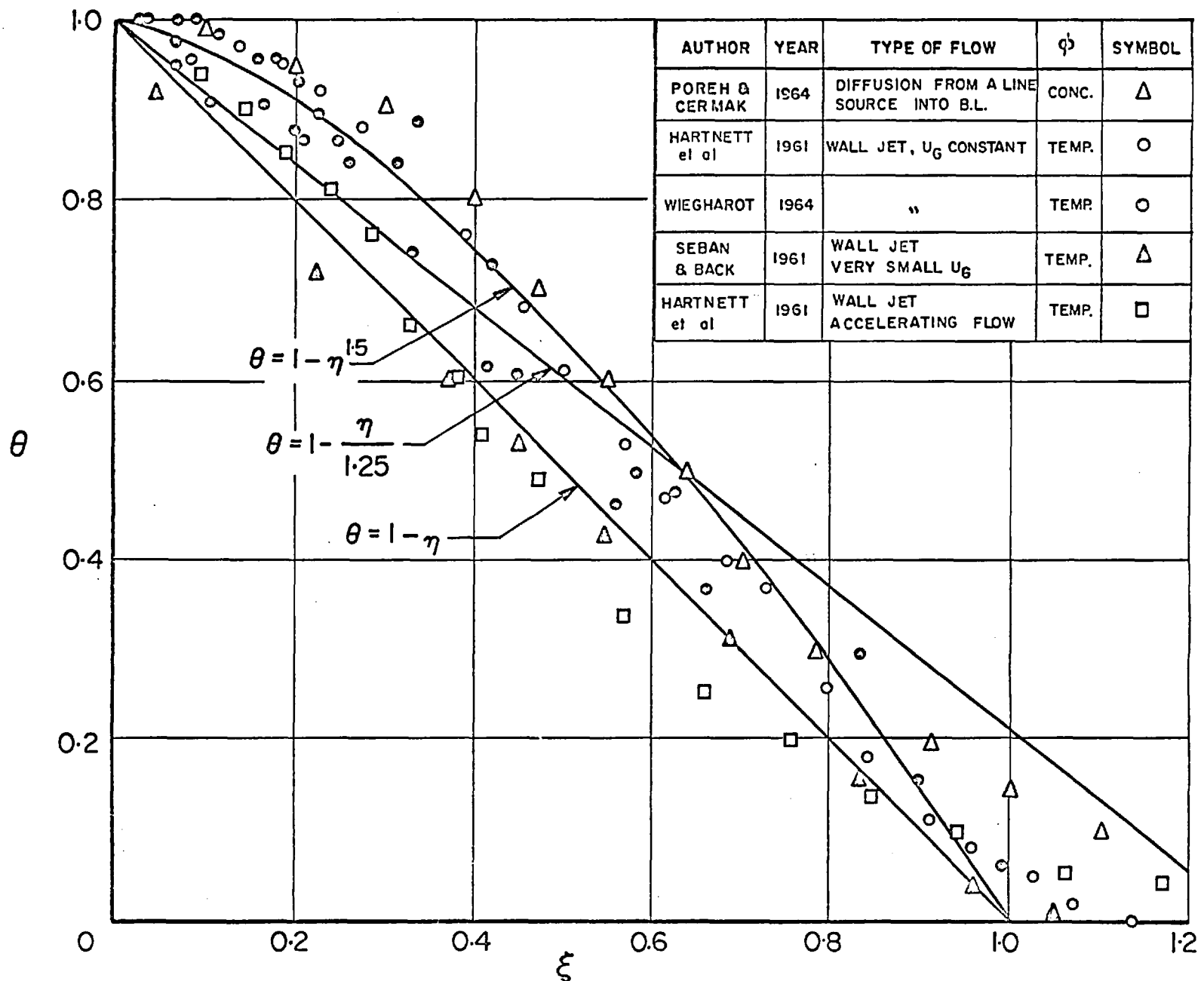


Fig. 33-1(b) THE  $\phi$  PROFILE WAKE COMPONENT: WALL-SHEAR FLOWS WITH  $(\partial \phi / \partial \eta)_0 = 0$ , FROM PATANKAR (1965)

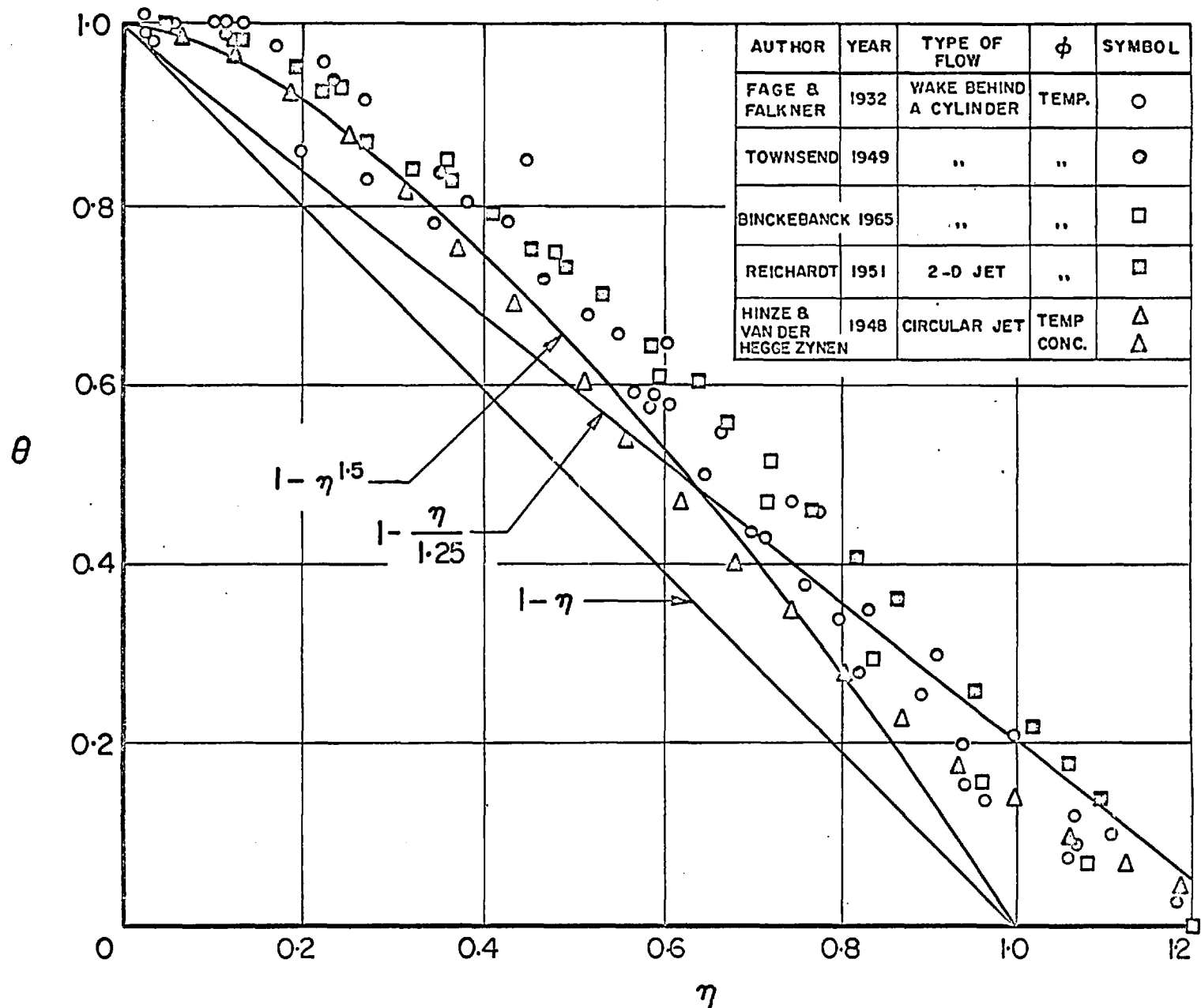


Fig. 3.3-1(c) THE  $\theta$ -PROFILE WAKE COMPONENT : FREE TURBULENT FLOWS  
[FROM PATANKAR (1965)]

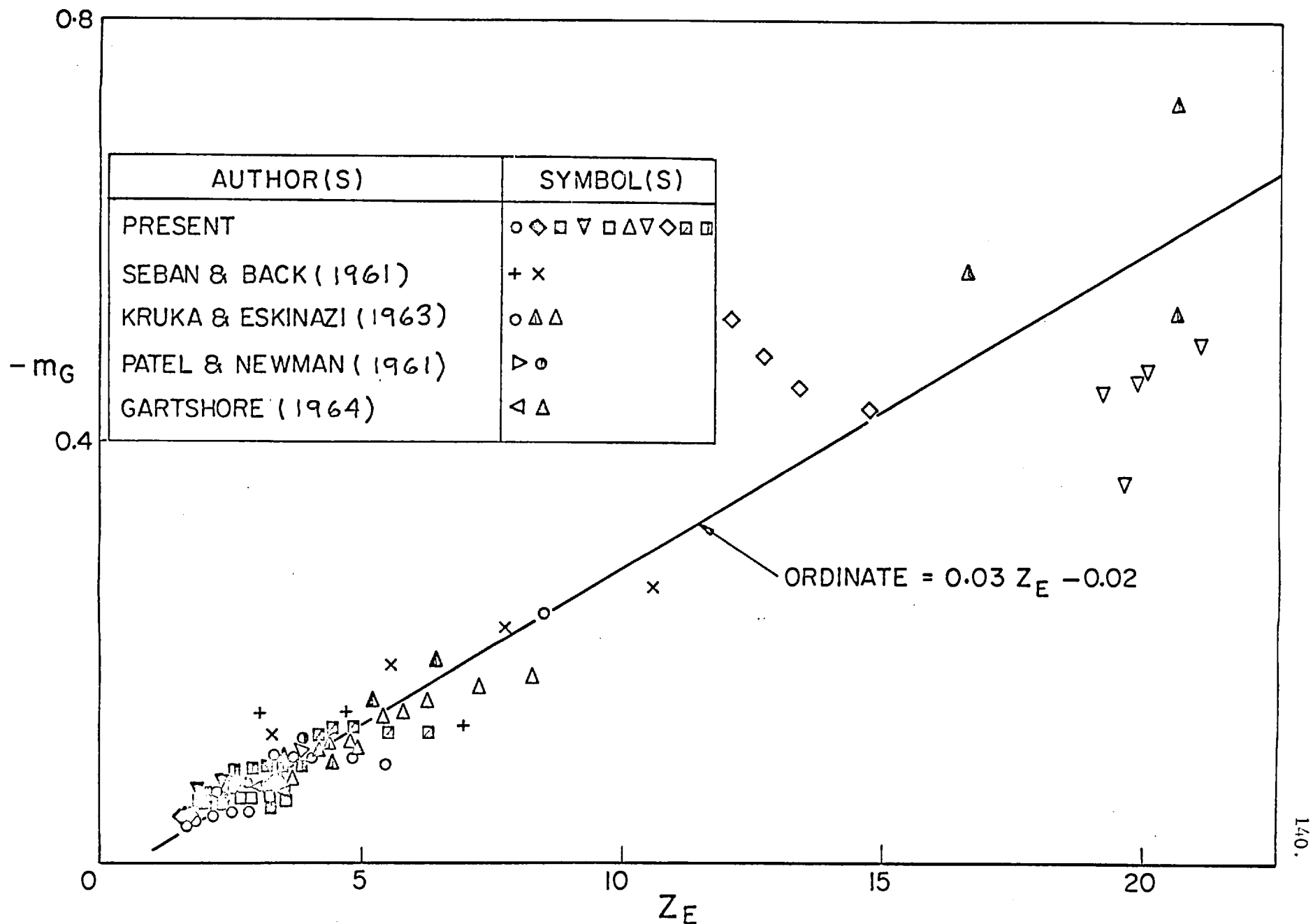


Fig. 4-2-1 ENTRAINMENT DEDUCED FROM EXPERIMENTAL VELOCITY PROFILES

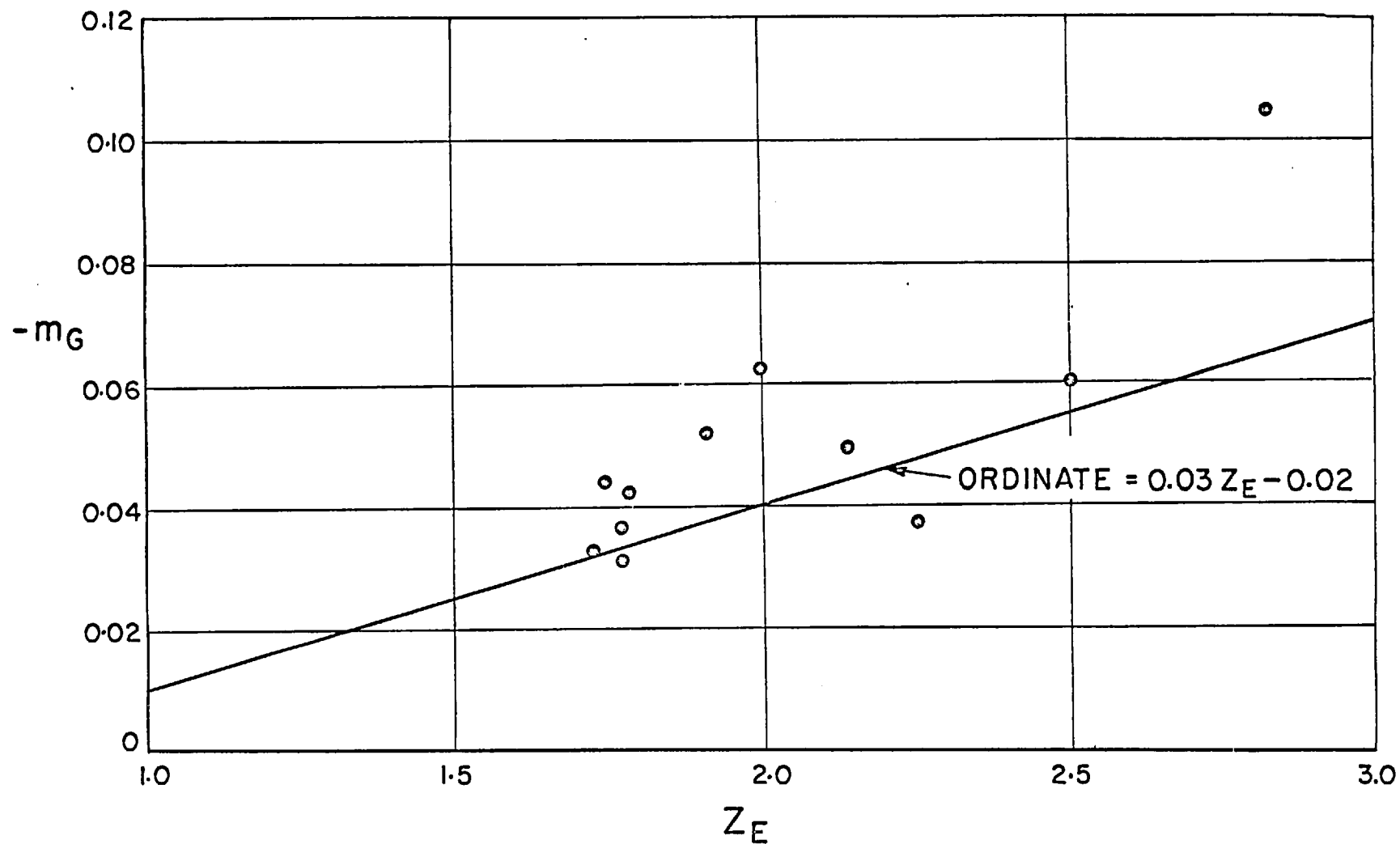


Fig. 4.2-2 ENTRAINMENT DEDUCED FROM SHEAR-STRESS DISTRIBUTION  
DATA OF ERIAN AND ESKINAZI (1964)

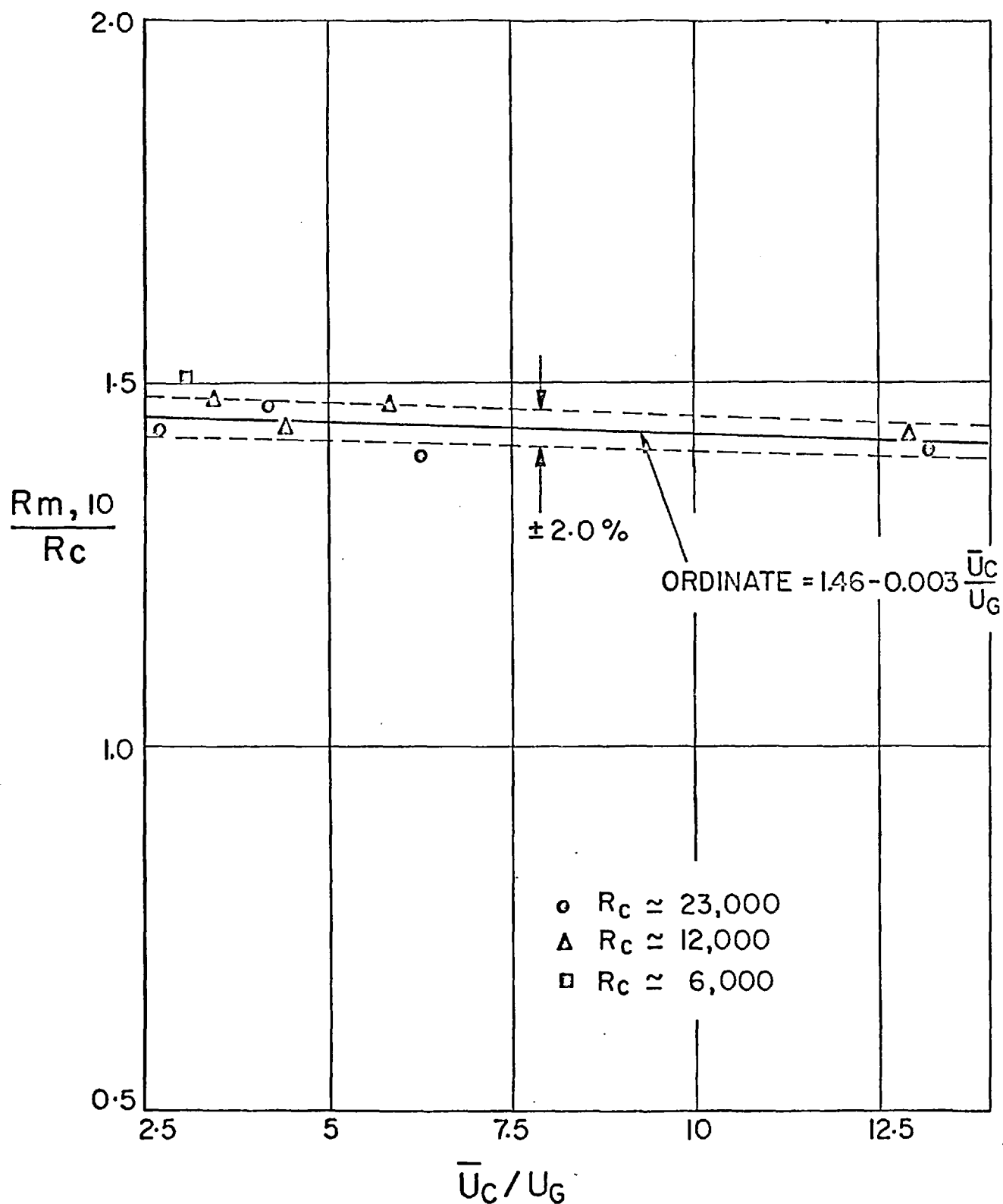


Fig. 4.3-1 THE  $R_m$  INITIAL VALUE CORRELATION ( PRESENT MEASUREMENTS )

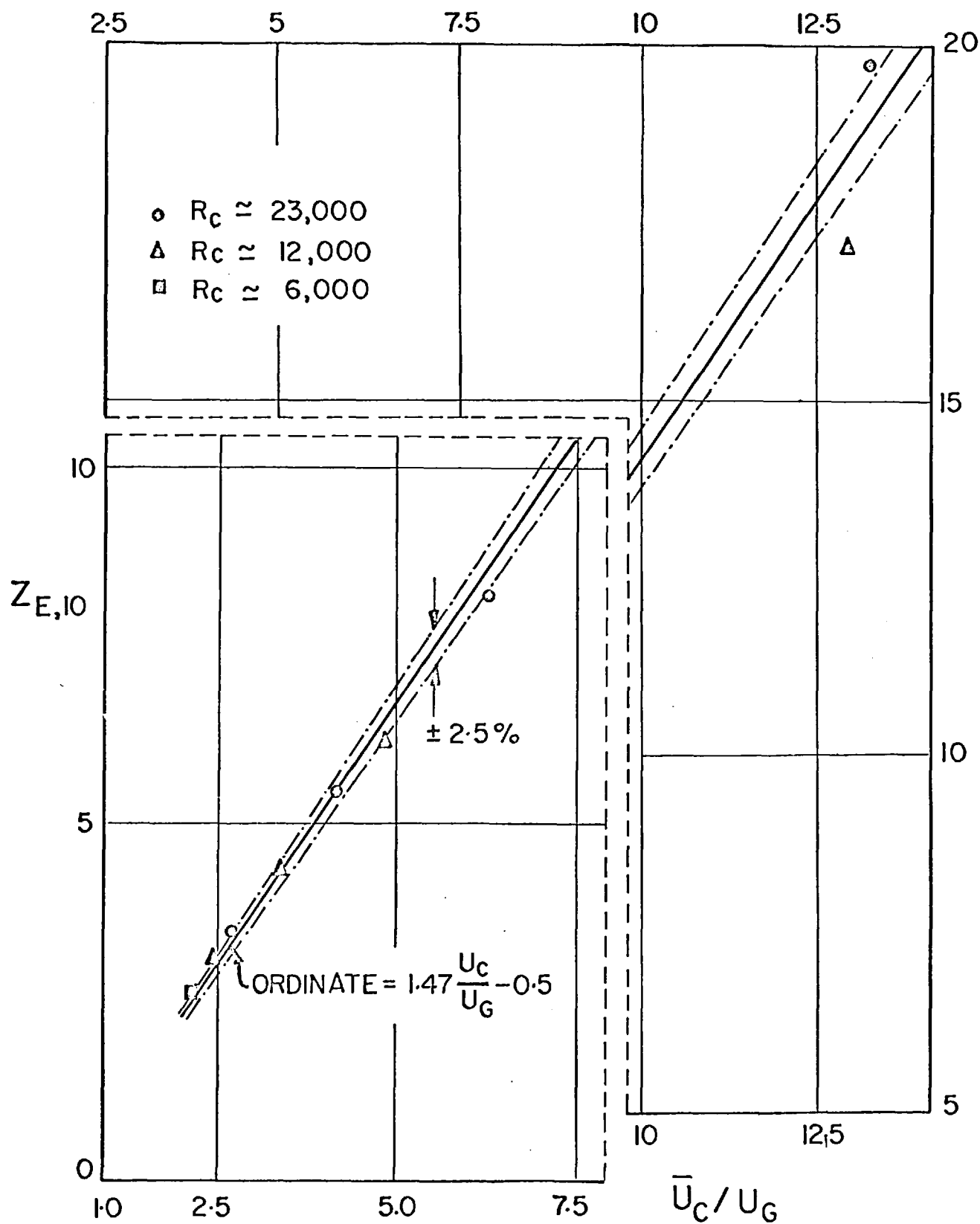


Fig. 4.3 -2 THE  $Z_E$  INITIAL VALUE CORRELATION (PRESENT MEASUREMENT)

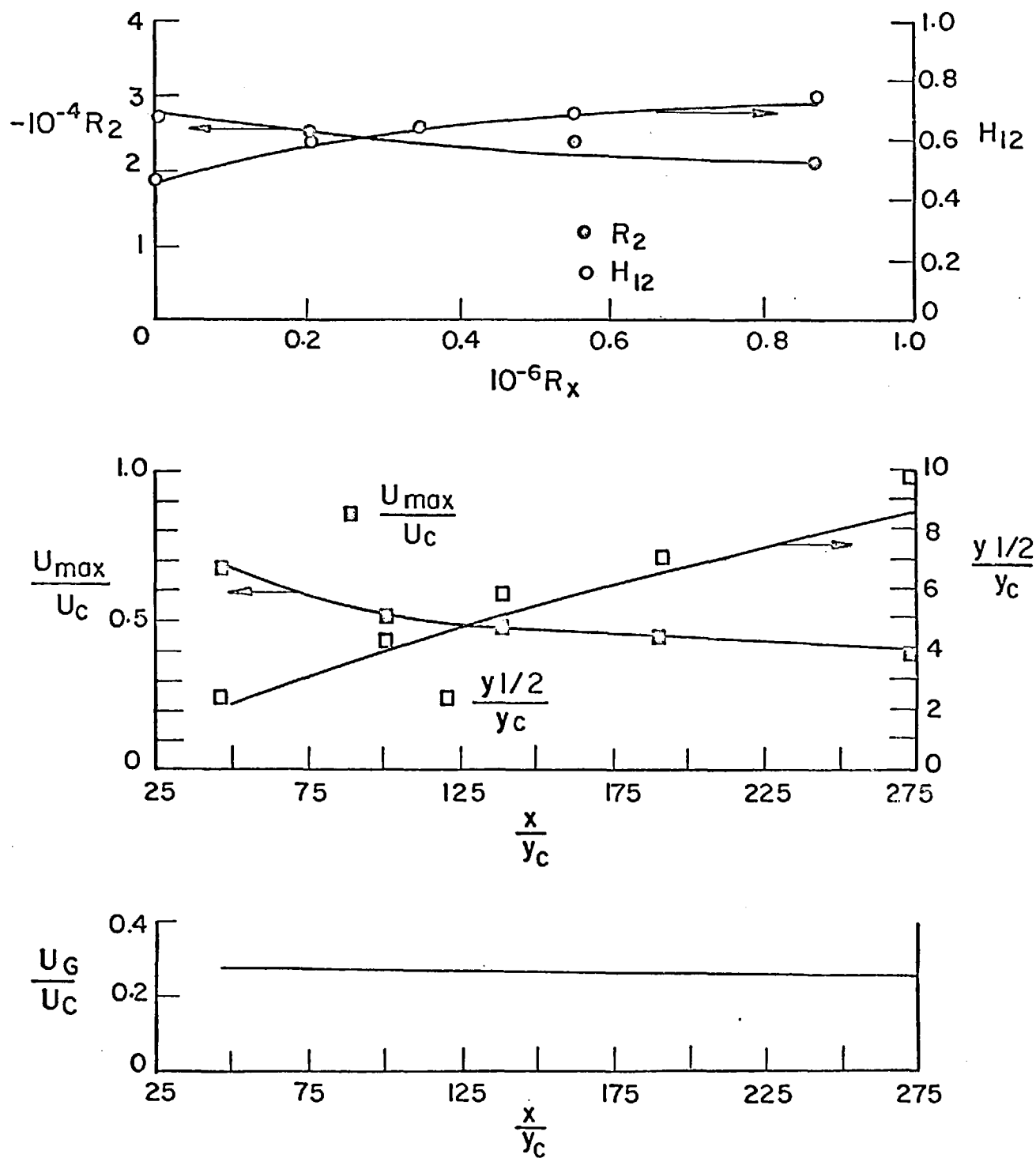


Fig. 4.4-1 COMPARISON OF PREDICTED WITH EXPERIMENTAL WALL - JET DEVELOPMENT, KRUKA & ESKINAZI RUN 1

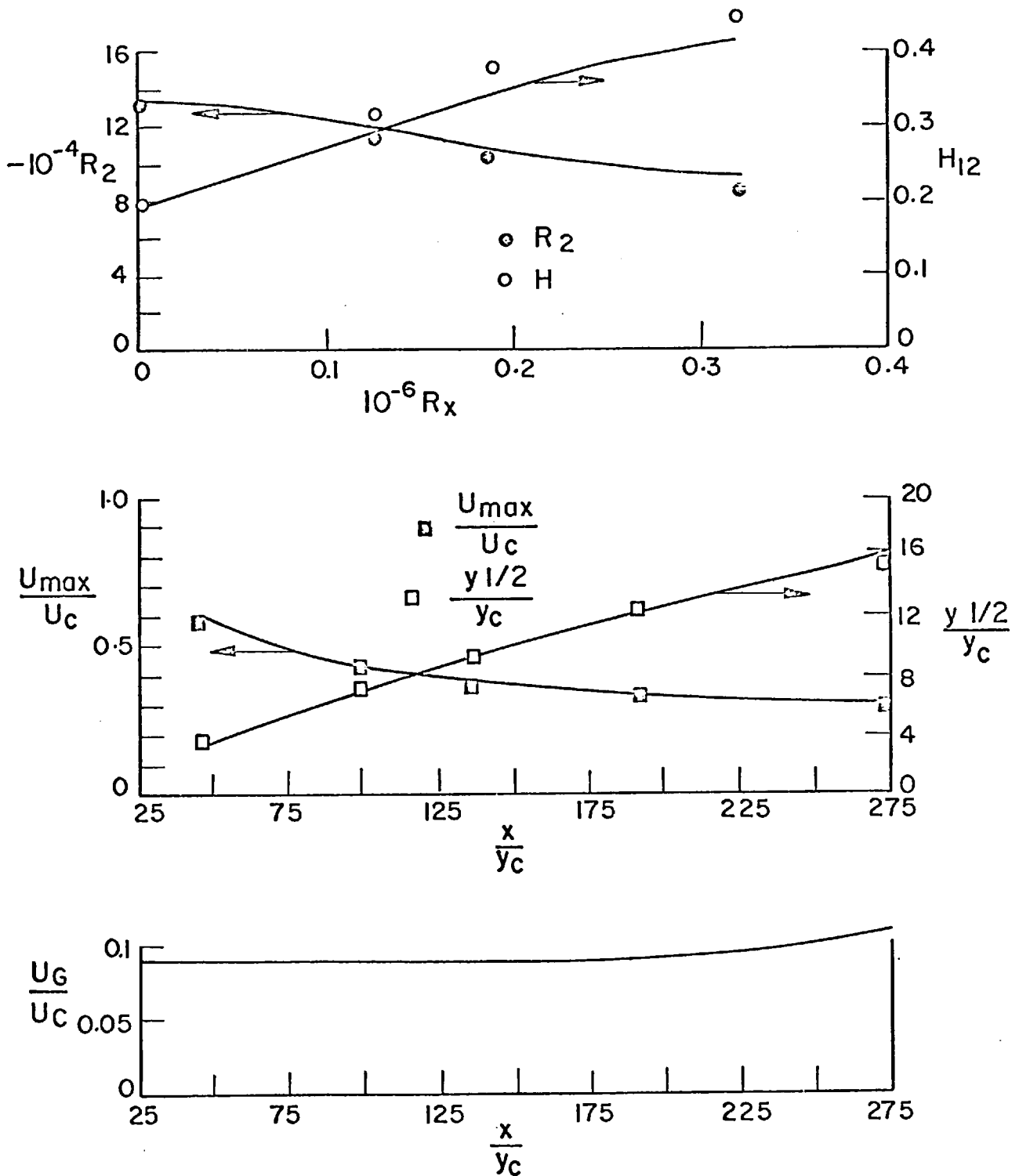


Fig. 4.4 -2 COMPARISON OF PREDICTED WITH EXPERIMENTAL  
WALL-JET DEVELOPMENT KRUKA & ESKINAZI RUN 2

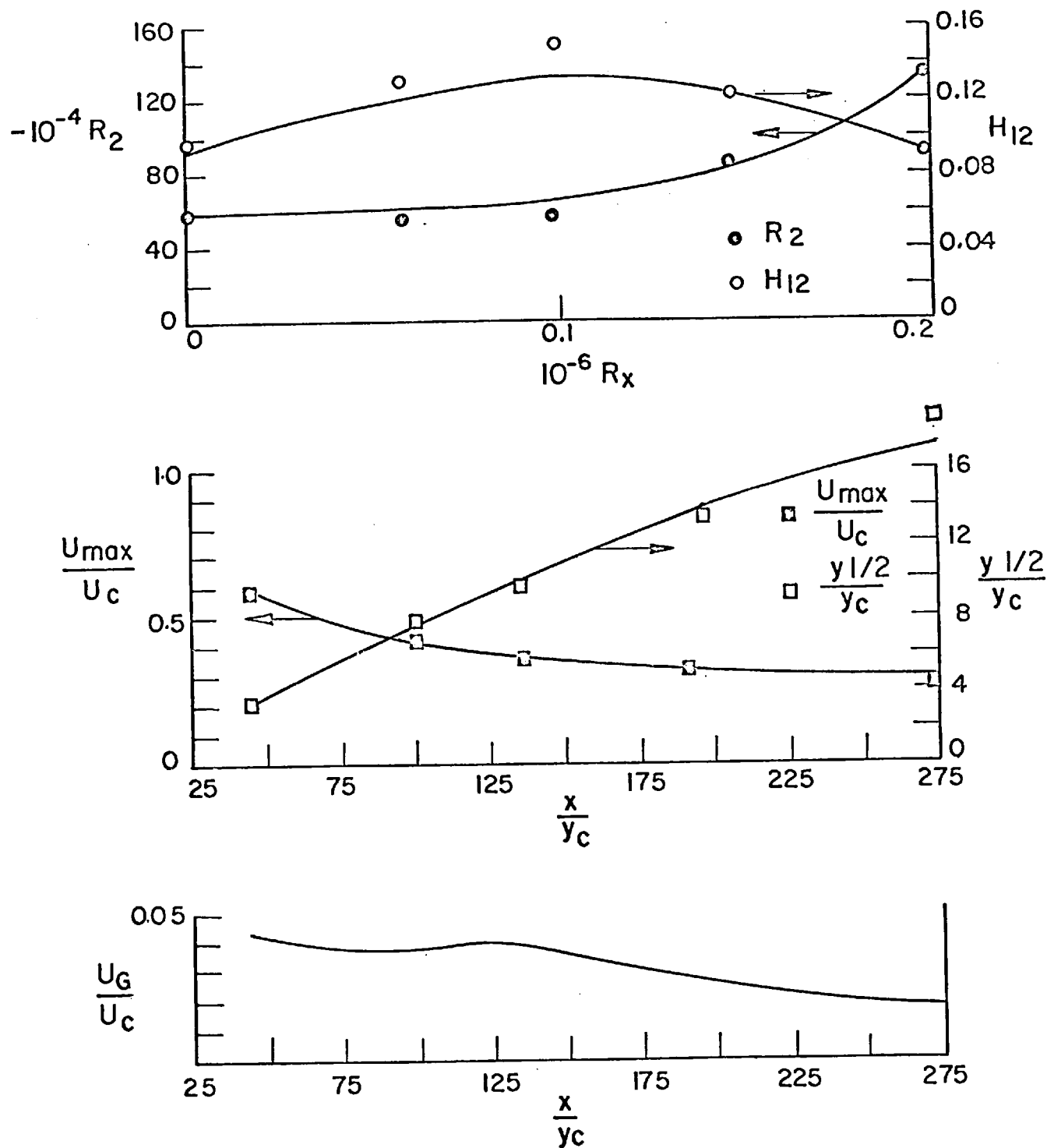


Fig. 4.4-3 COMPARISON OF PREDICTED WITH EXPERIMENTAL WALL-JET DEVELOPMENT KRUKA & ESKINAZI RUN 3

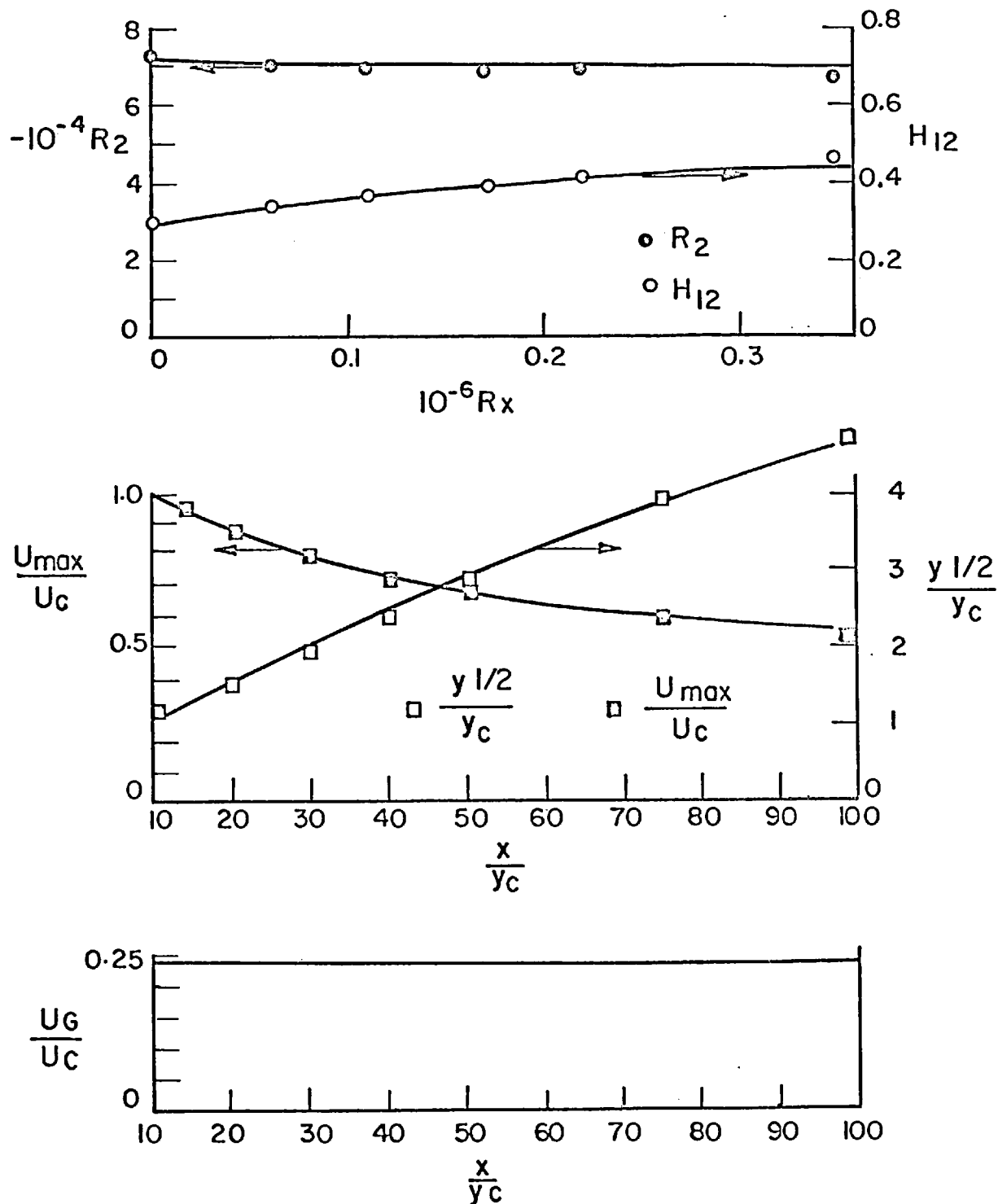


Fig. 4-4-4 COMPARISON OF PREDICTED WITH EXPERIMENTAL WALL-JET DEVELOPMENT PRESENT MEASUREMENTS RUN I

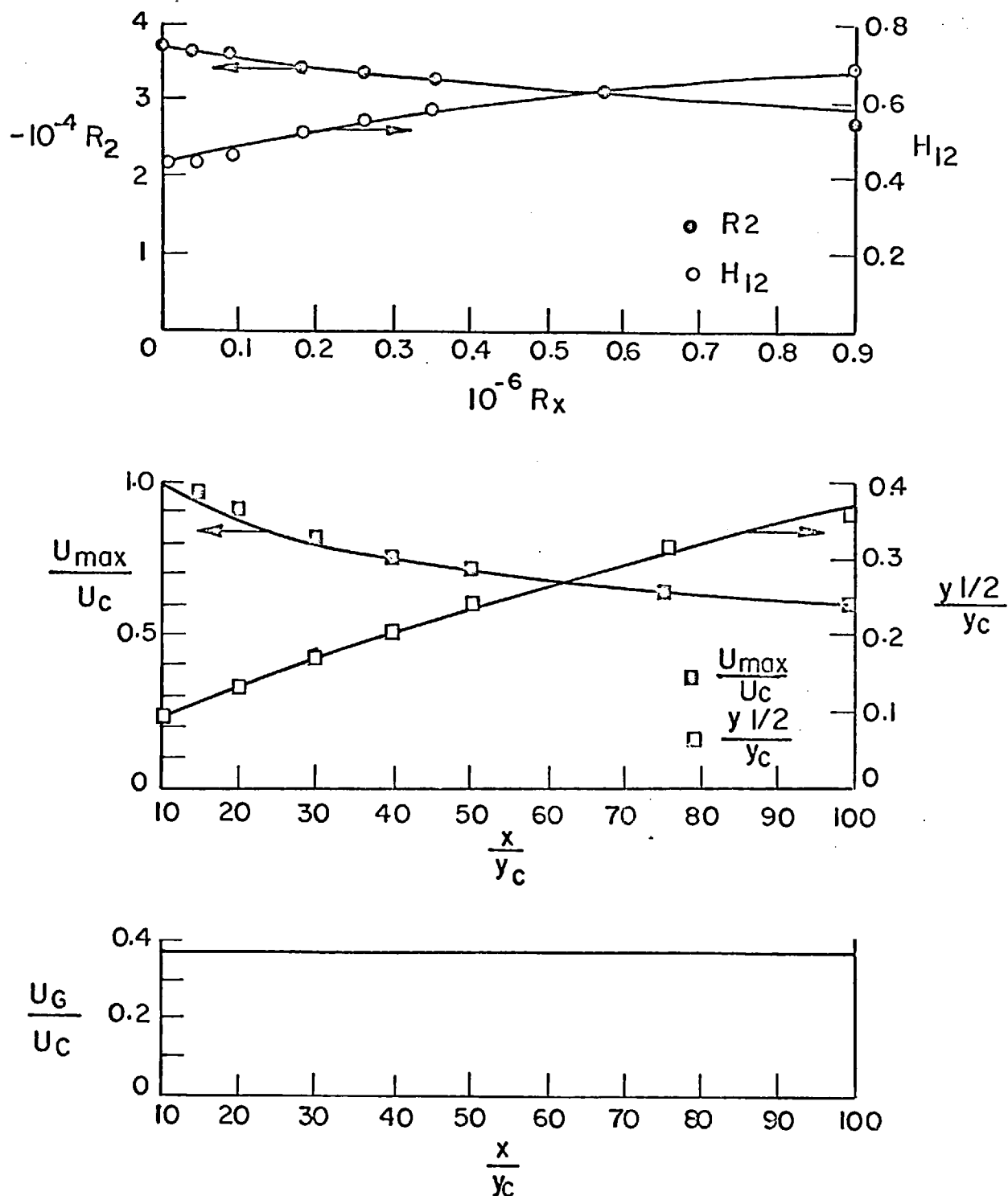


Fig. 4-4-5 COMPARISON OF PREDICTED WITH EXPERIMENTAL WALL-JET DEVELOPMENT PRESENT MEASUREMENTS RUN 2

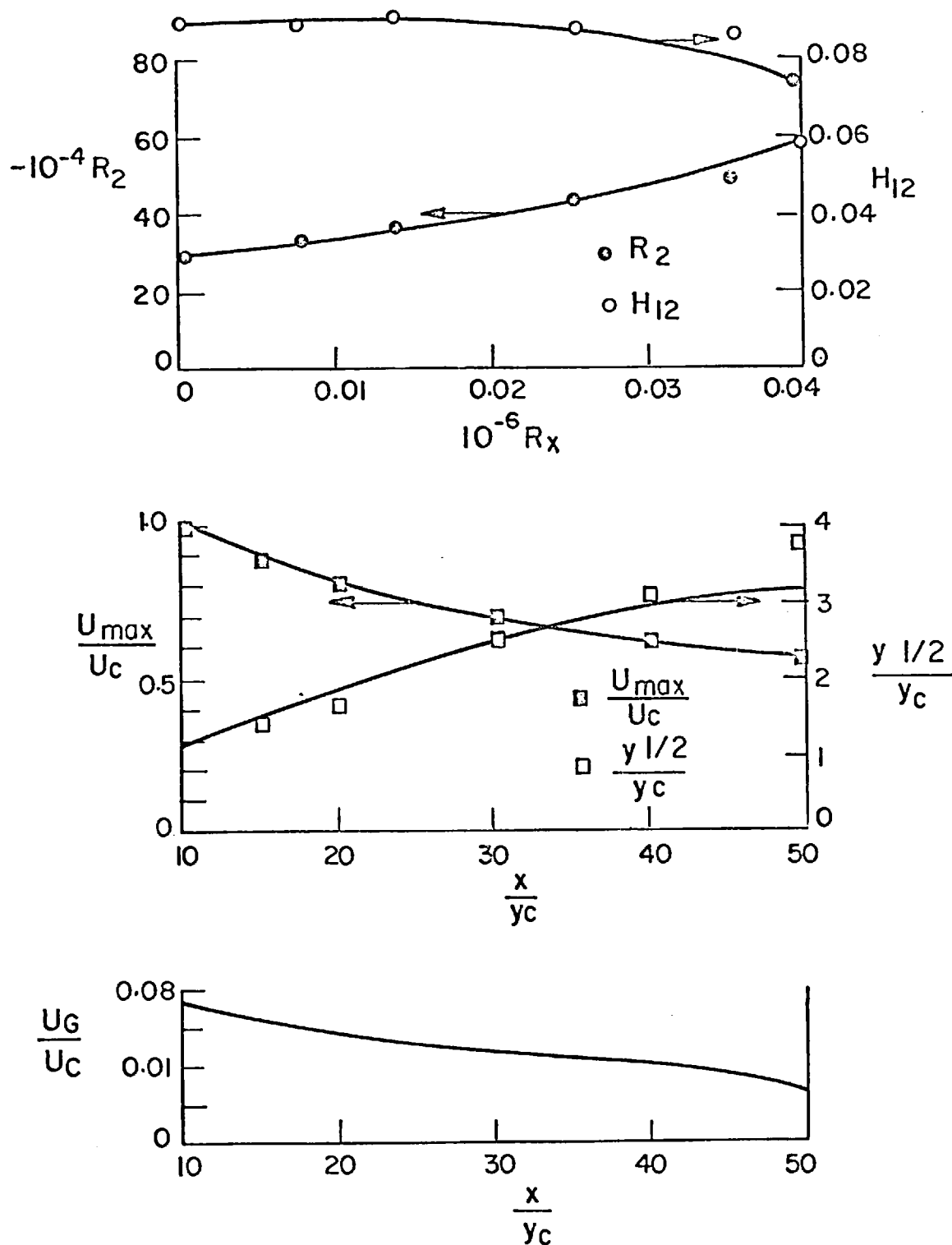


Fig. 4-4-6 COMPARISON OF PREDICTED WITH EXPERIMENTAL WALL-JET DEVELOPMENT PRESENT MEASUREMENTS RUN 3

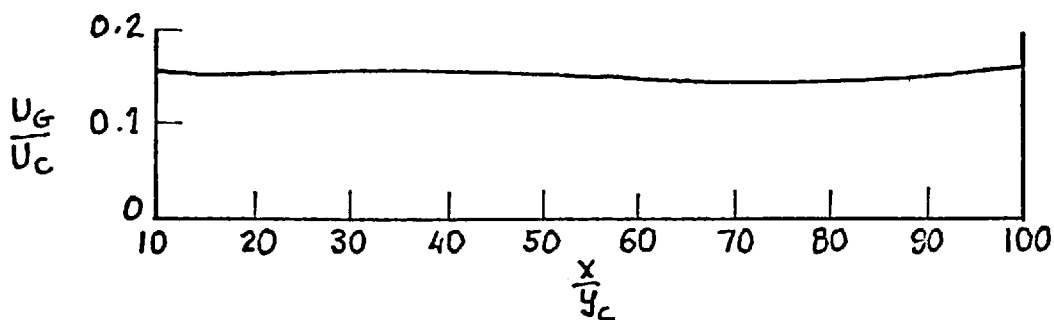
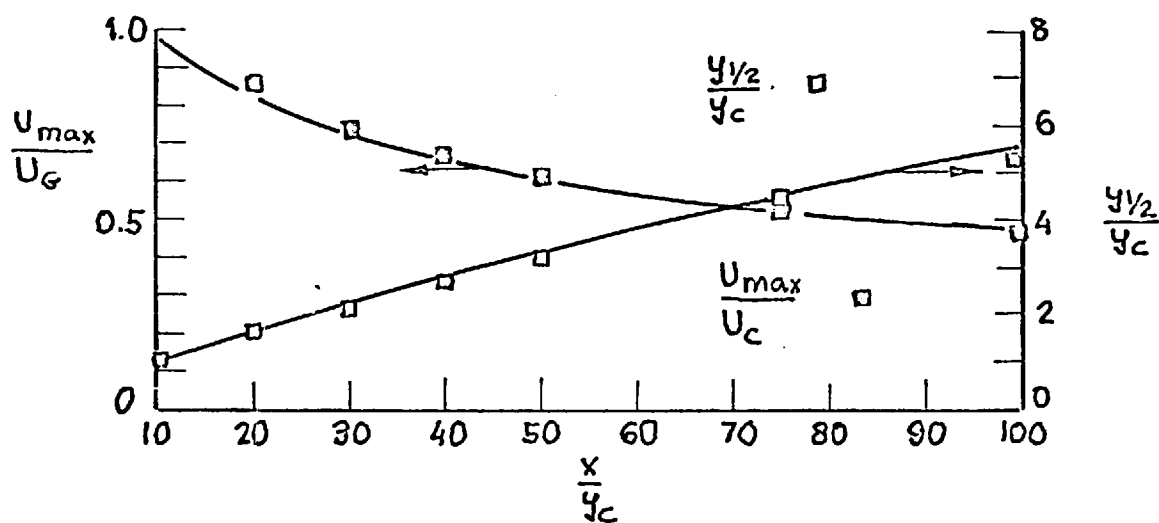
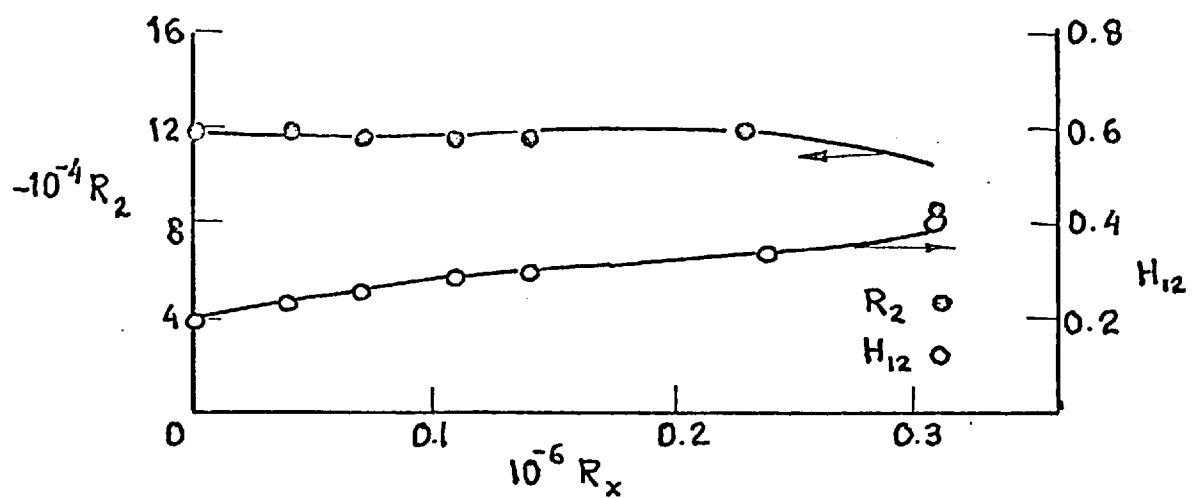


Fig. 4.4-7: COMPARISON OF PREDICTED WITH EXPERIMENTAL WALL-JET DEVELOPMENT PRESENT MEASUREMENTS RUN 4

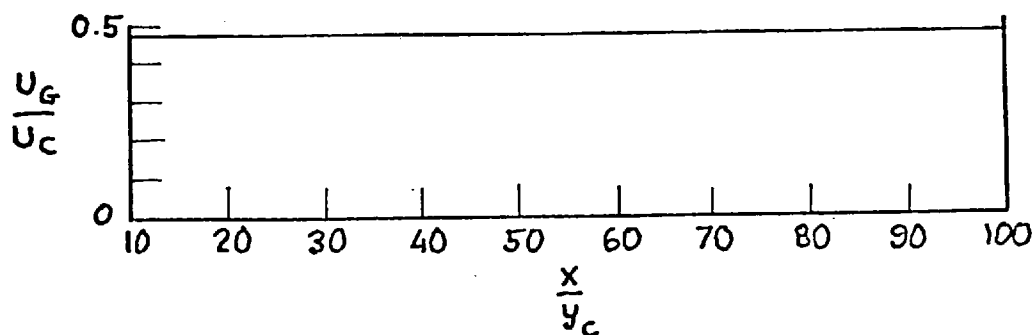
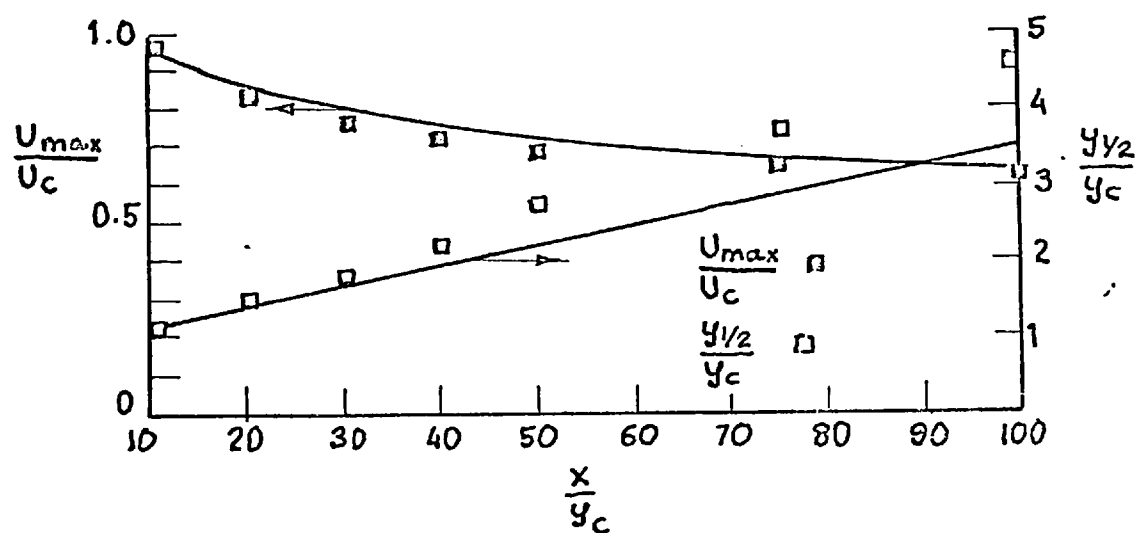
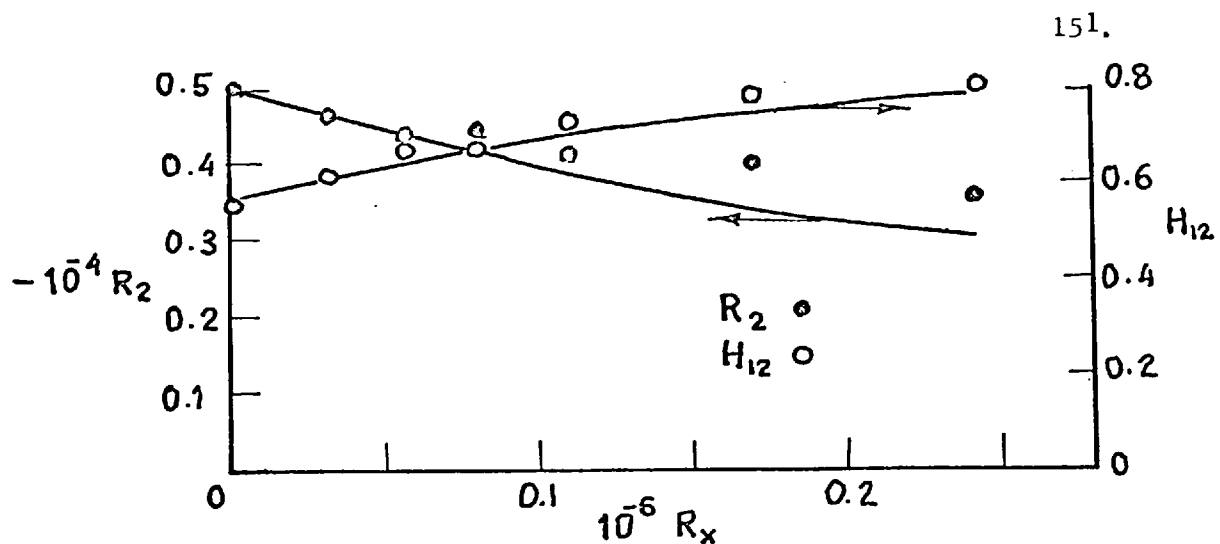


Fig. 4.4-8: COMPARISON OF PREDICTED WITH EXPERIMENTAL WALL-JET DEVELOPMENT PRESENT MEASUREMENTS RUN 5

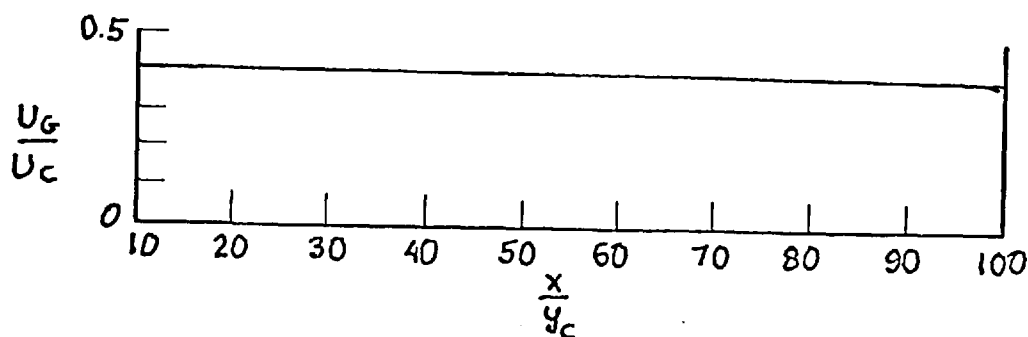
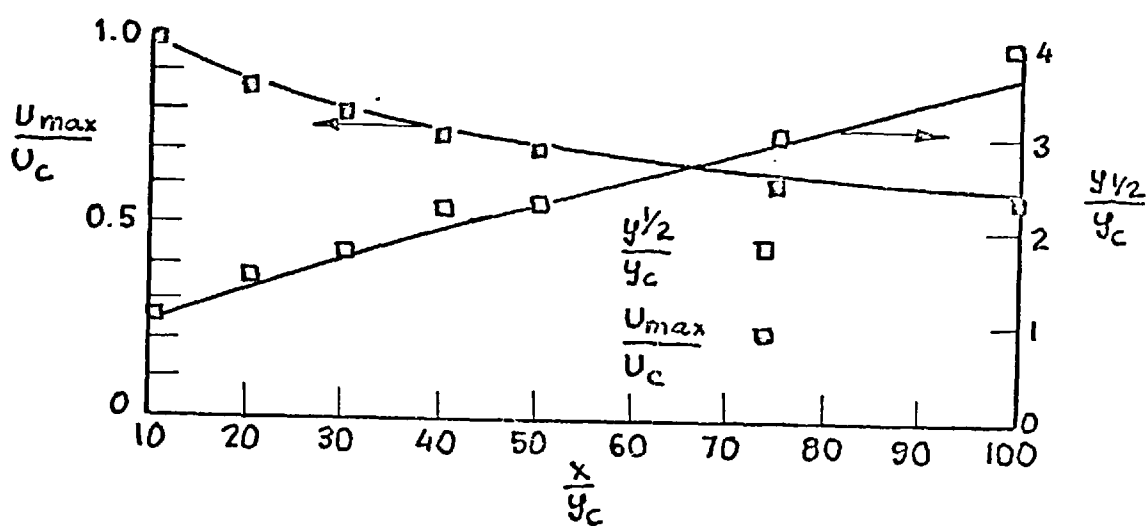
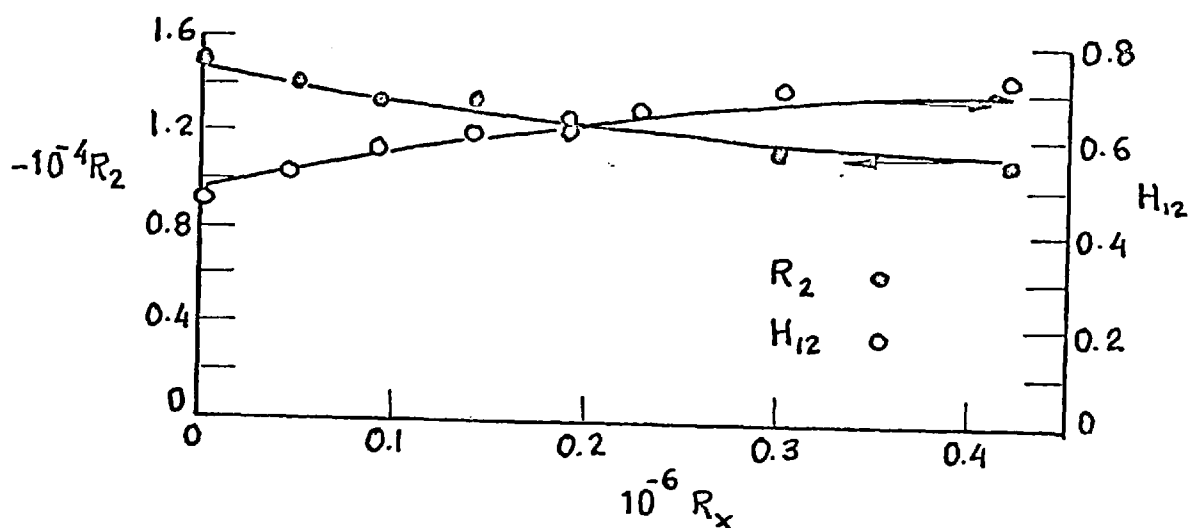


Fig. 4.4-9: COMPARISON OF PREDICTED WITH EXPERIMENTAL WALL-JET DEVELOPMENT PRESENT MEASUREMENTS RUN 7

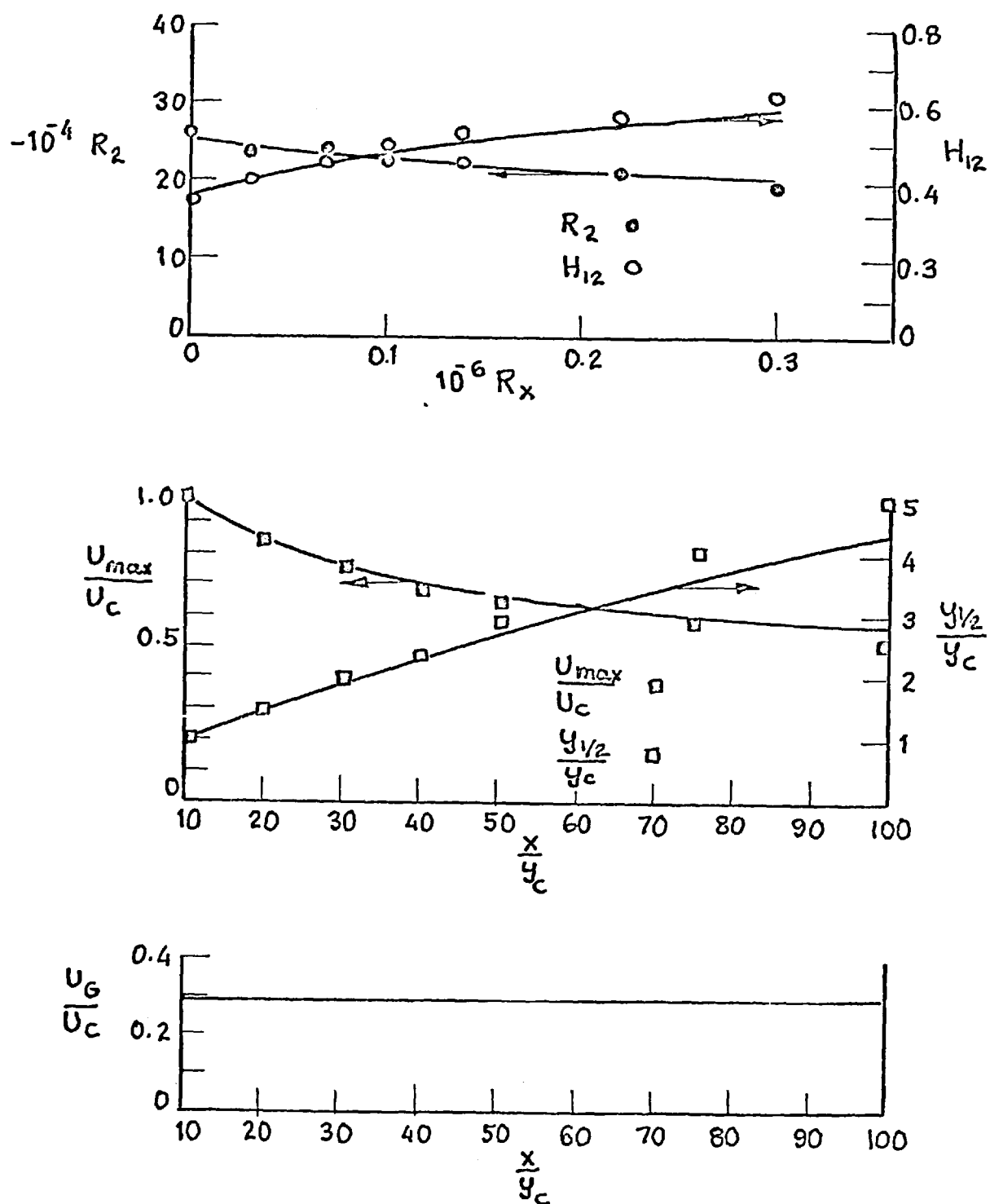


Fig. 4.4-10: COMPARISON OF PREDICTED WITH EXPERIMENTAL WALL-JET DEVELOPMENT PRESENT MEASUREMENTS RUN 8

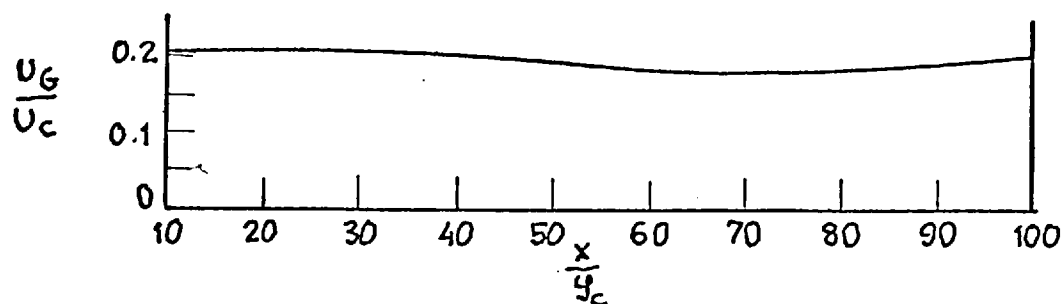
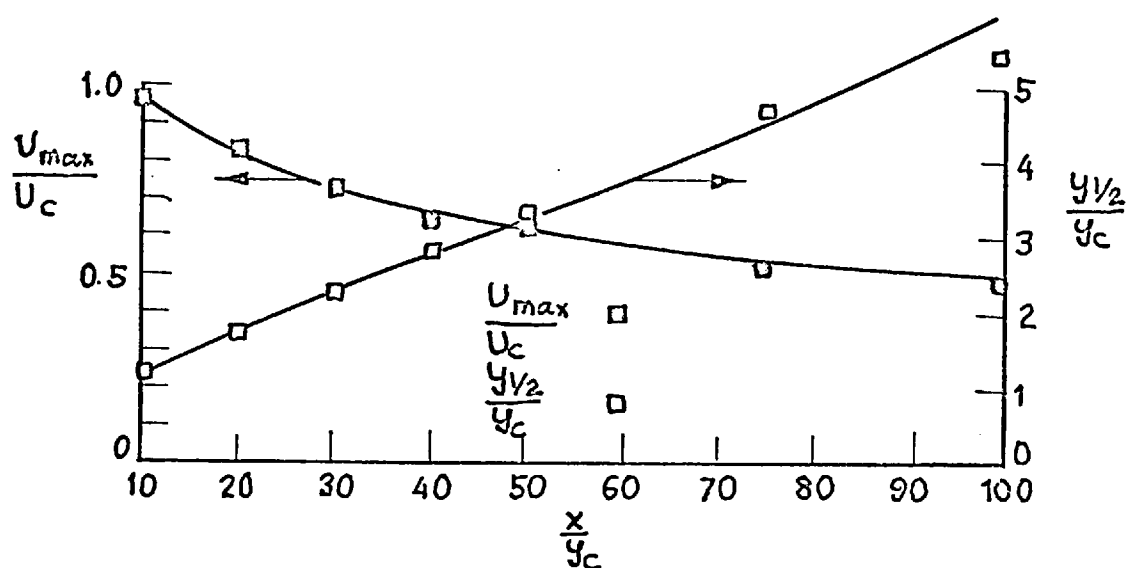
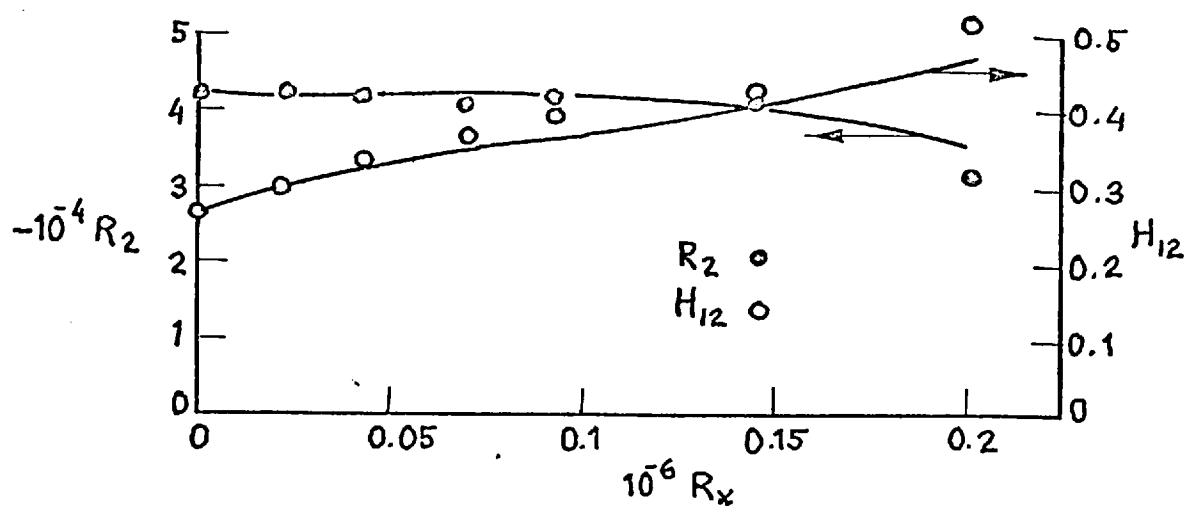


Fig. 4.4-11: COMPARISON OF PREDICTED WITH EXPERIMENTAL WALL-JET DEVELOPMENT PRESENT MEASUREMENTS RUN 9

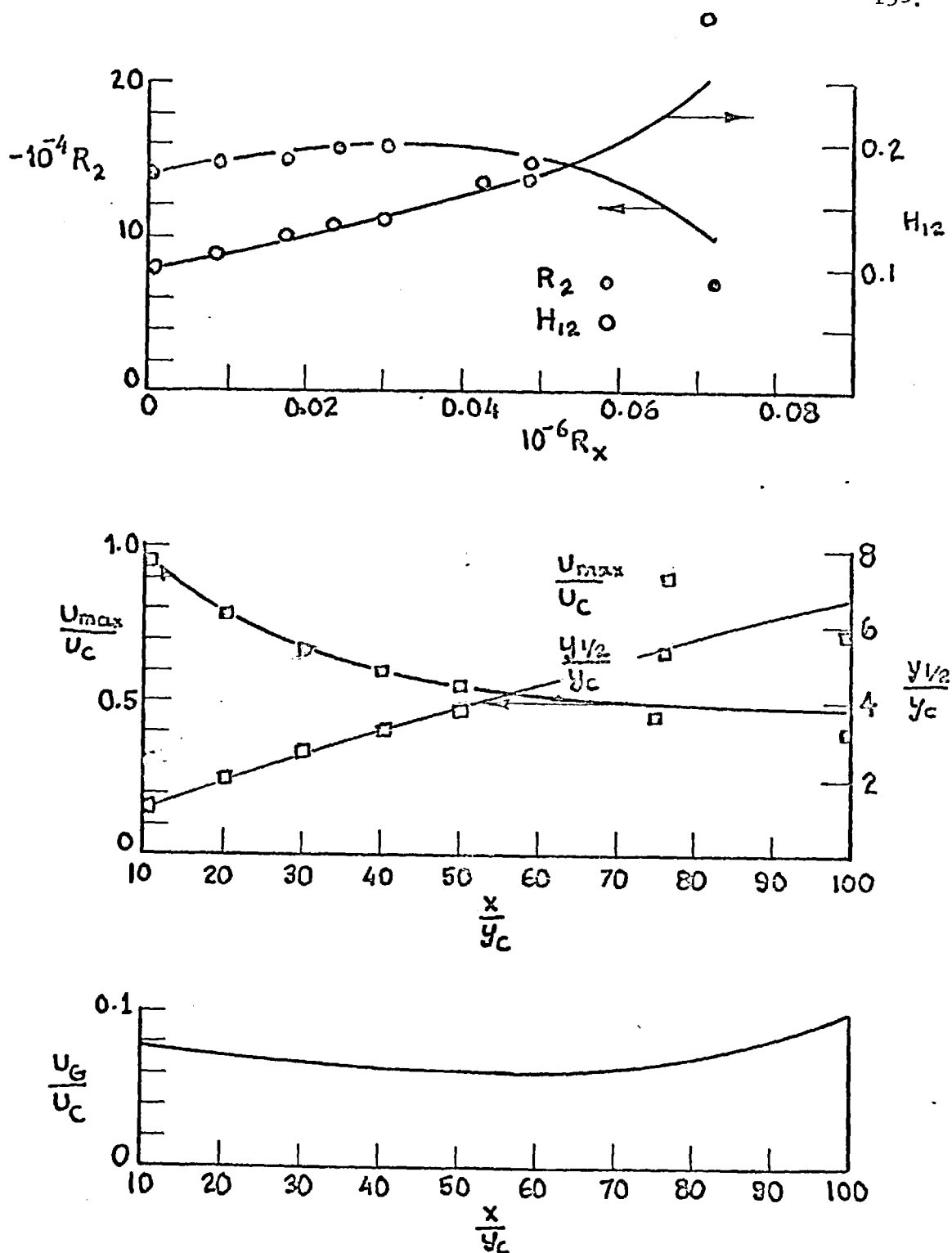


Fig. 4.4-12: COMPARISON OF PREDICTED WITH EXPERIMENTAL WALL-JET DEVELOPMENT PRESENT MEASUREMENTS RUN 10

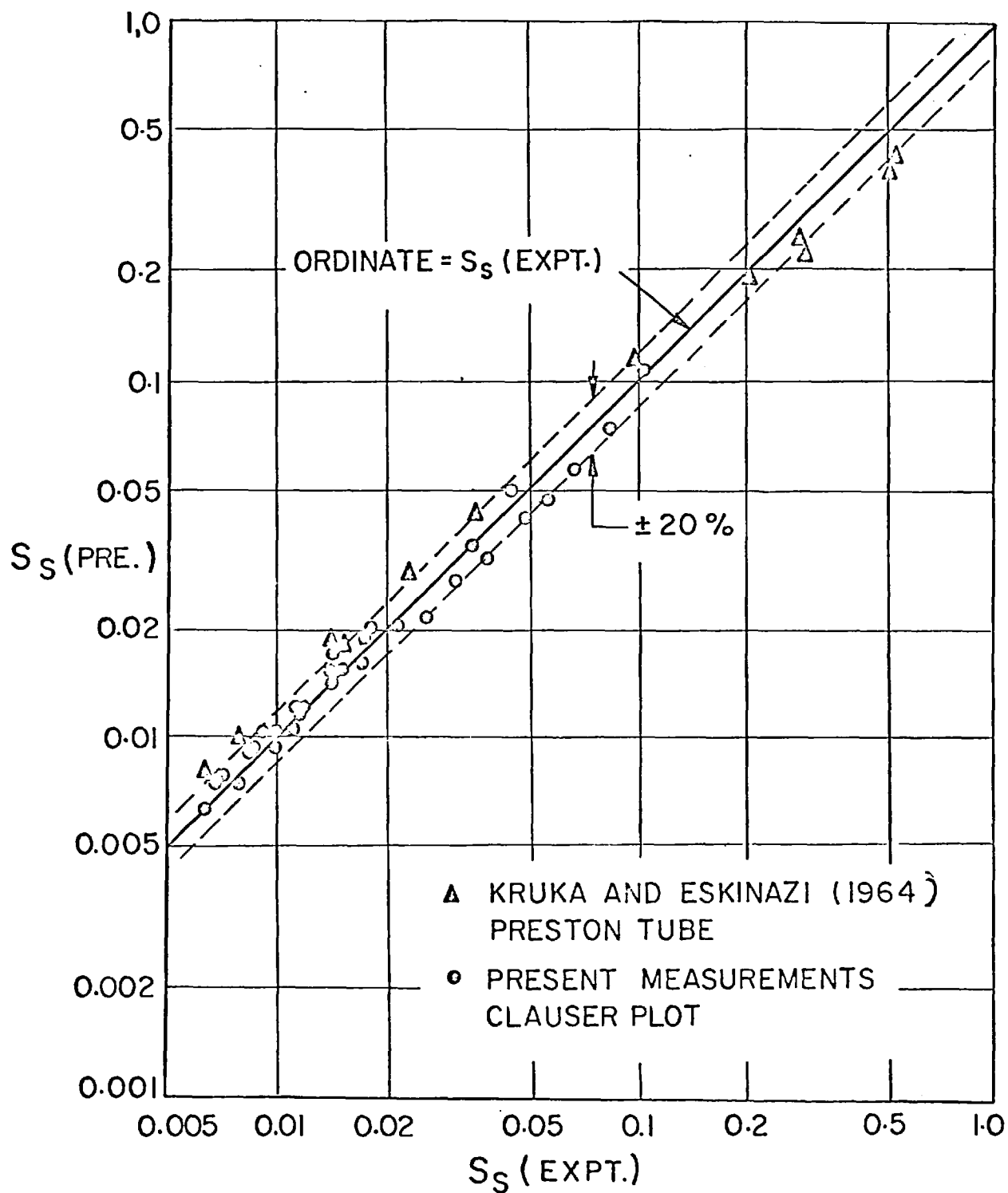


Fig. 4-4-13 COMPARISON OF PREDICTED WITH EXPERIMENTAL DRAG

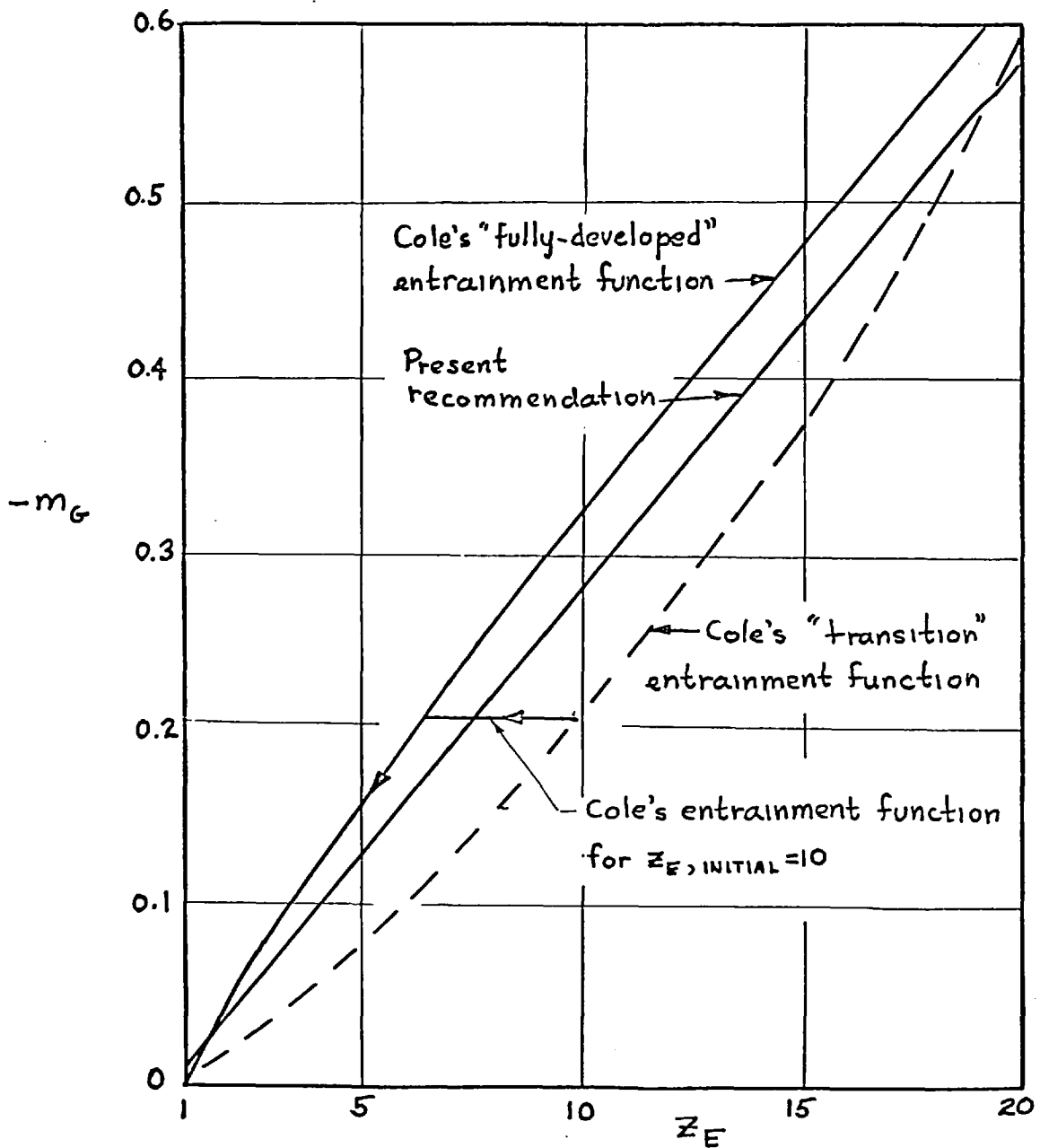


FIGURE 4.5-1: Comparison of Cole's (1965) entrainment function with the present recommendation [Eqn. (4.2-4)]

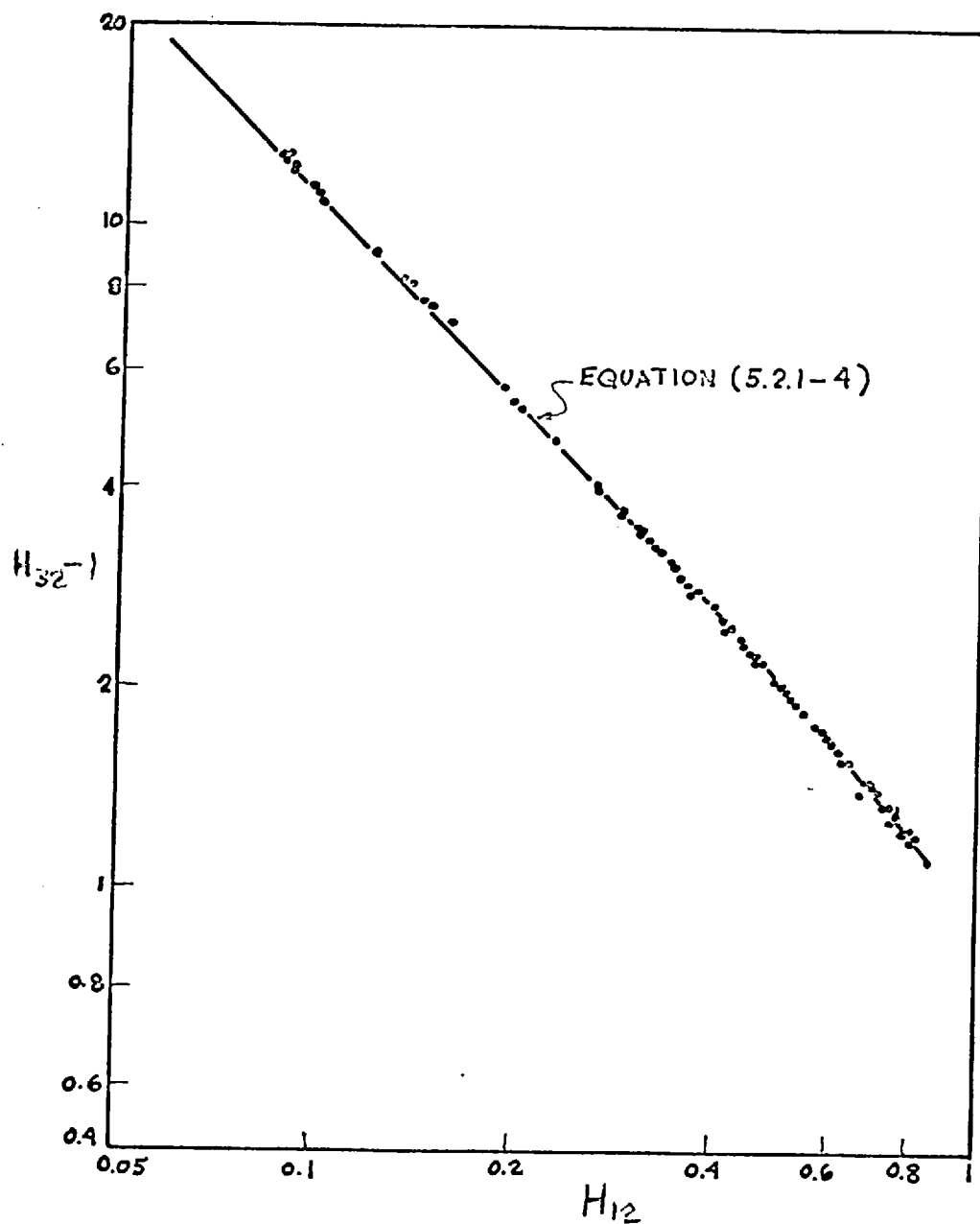


FIGURE 5.2.1-1:  $H_{32}-H_{12}$  DATA FOR WALL JETS

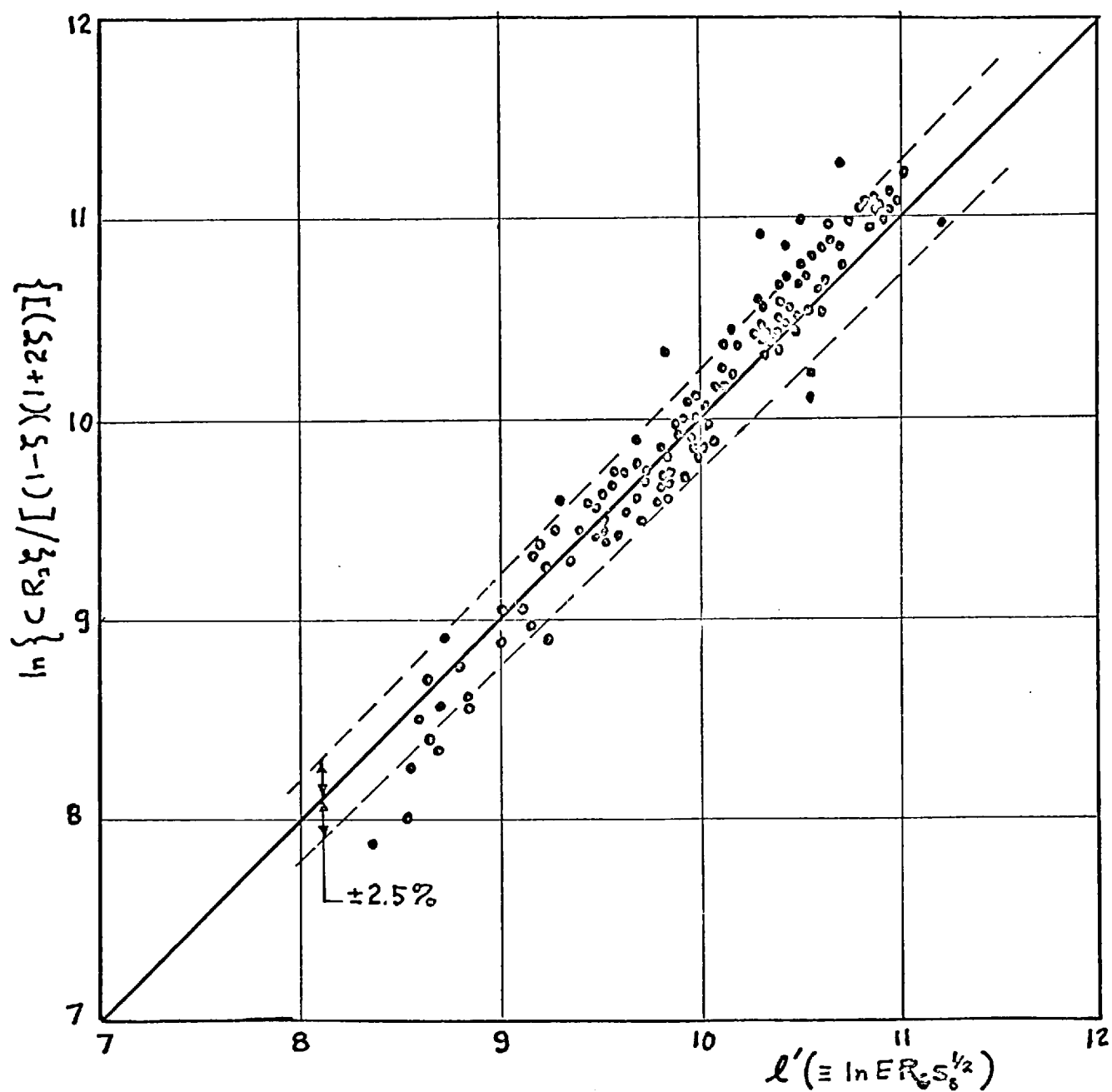


FIGURE 5.2.2-1: COMPARISON OF  $\ln \left\{ 3.389 R_2 \xi / [(1-\xi)(1+2\xi)] \right\}$   
WITH EXPERIMENTAL VALUES OF  $l'$

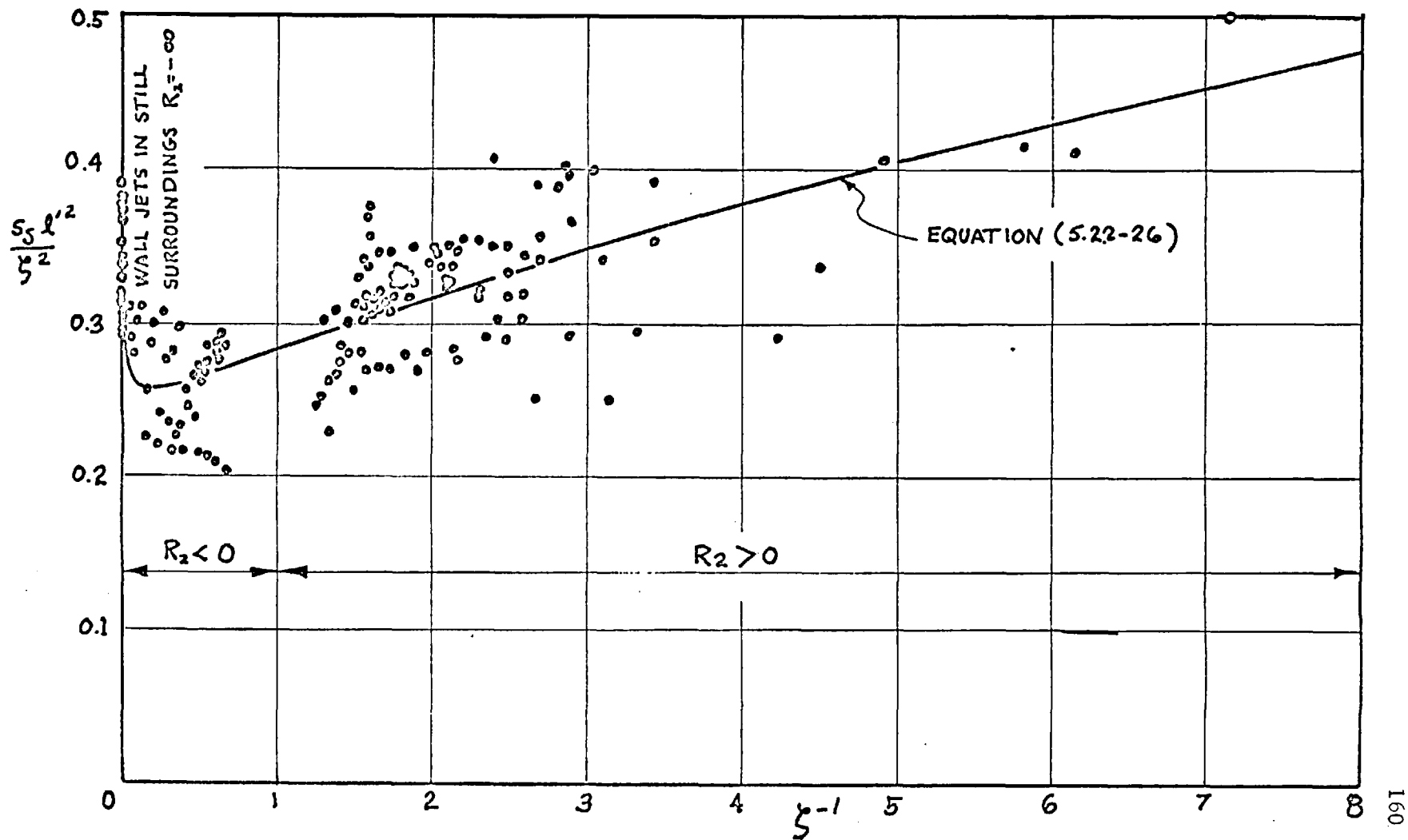


FIGURE 5.2.2-2: DATA USED IN DETERMINATION OF EMPIRICAL CONSTANTS IN EQUATION (5.2.2-22)

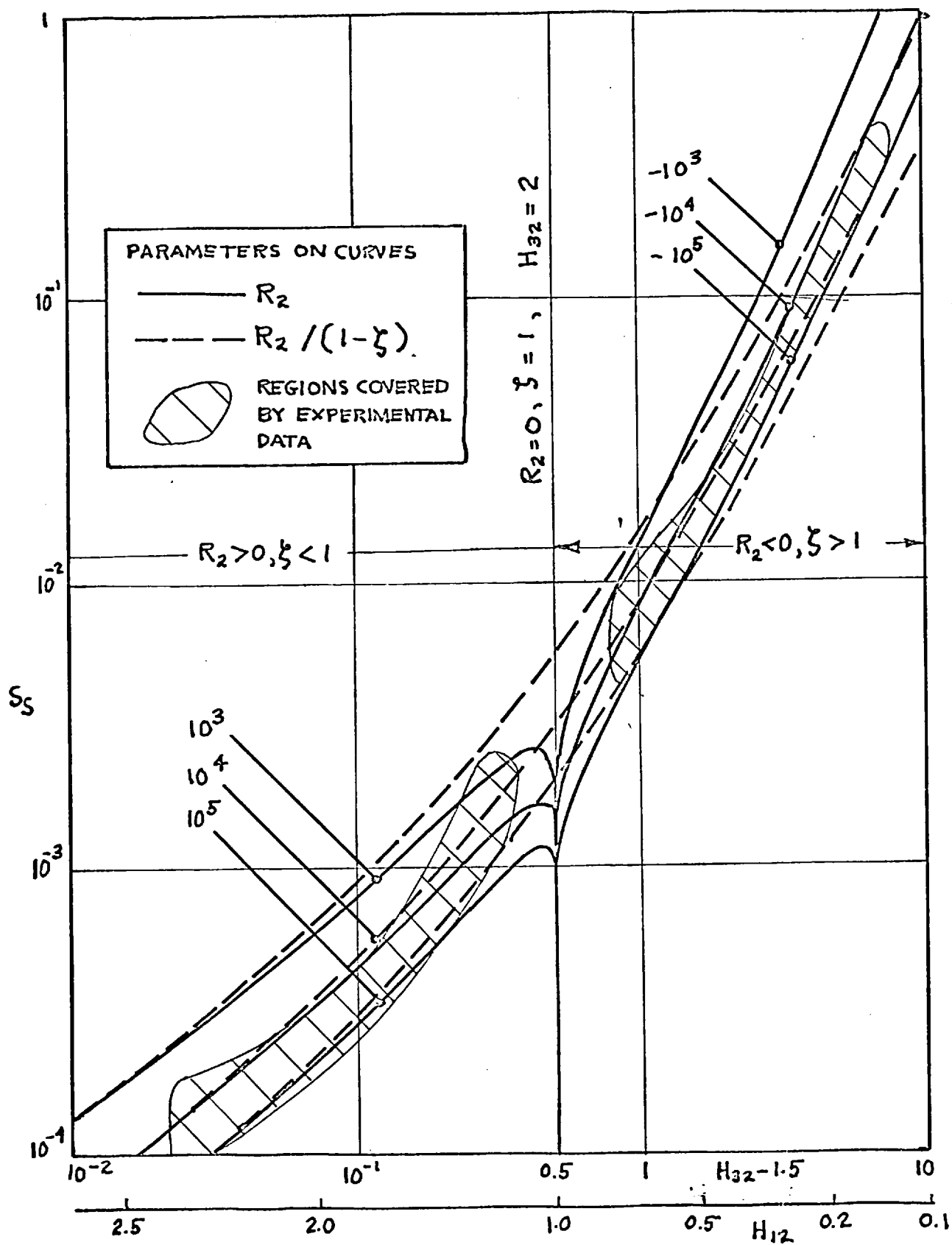


FIGURE 5.2.2-3: THE DRAG LAW

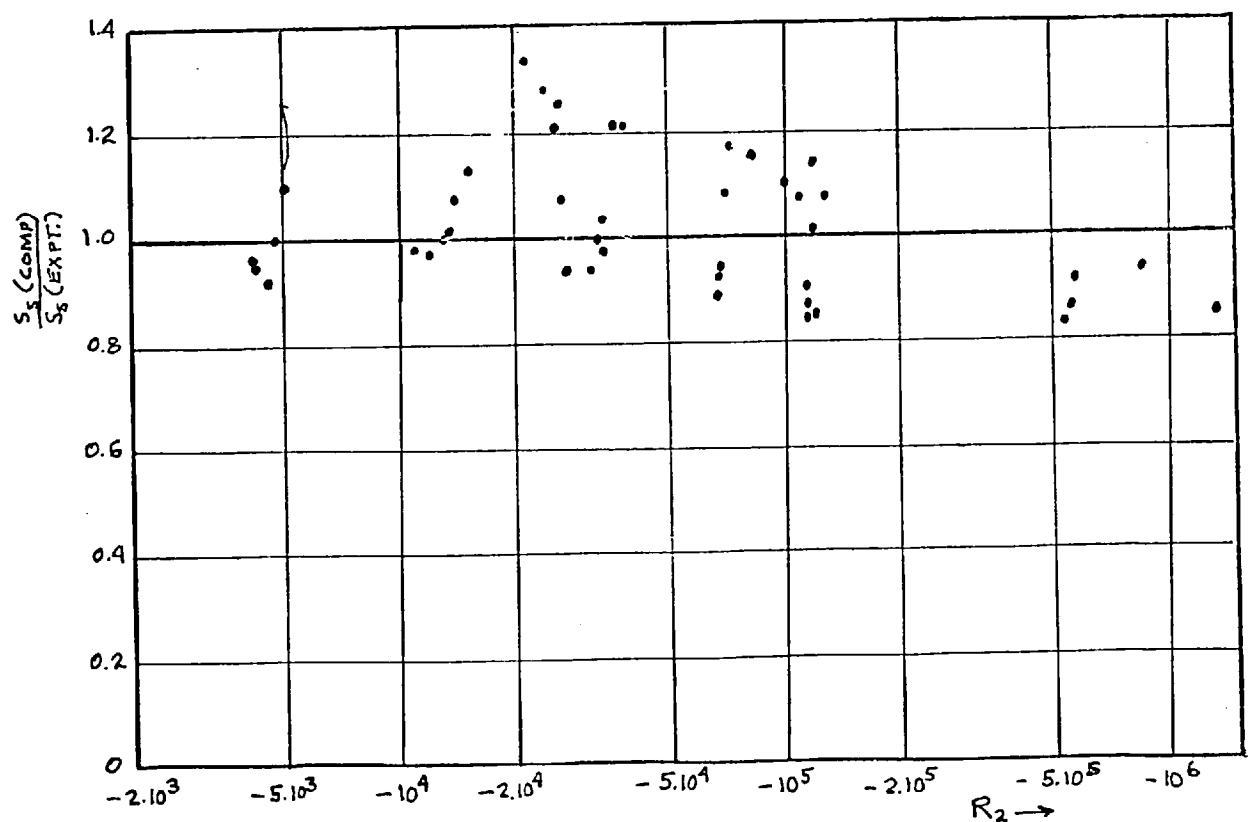
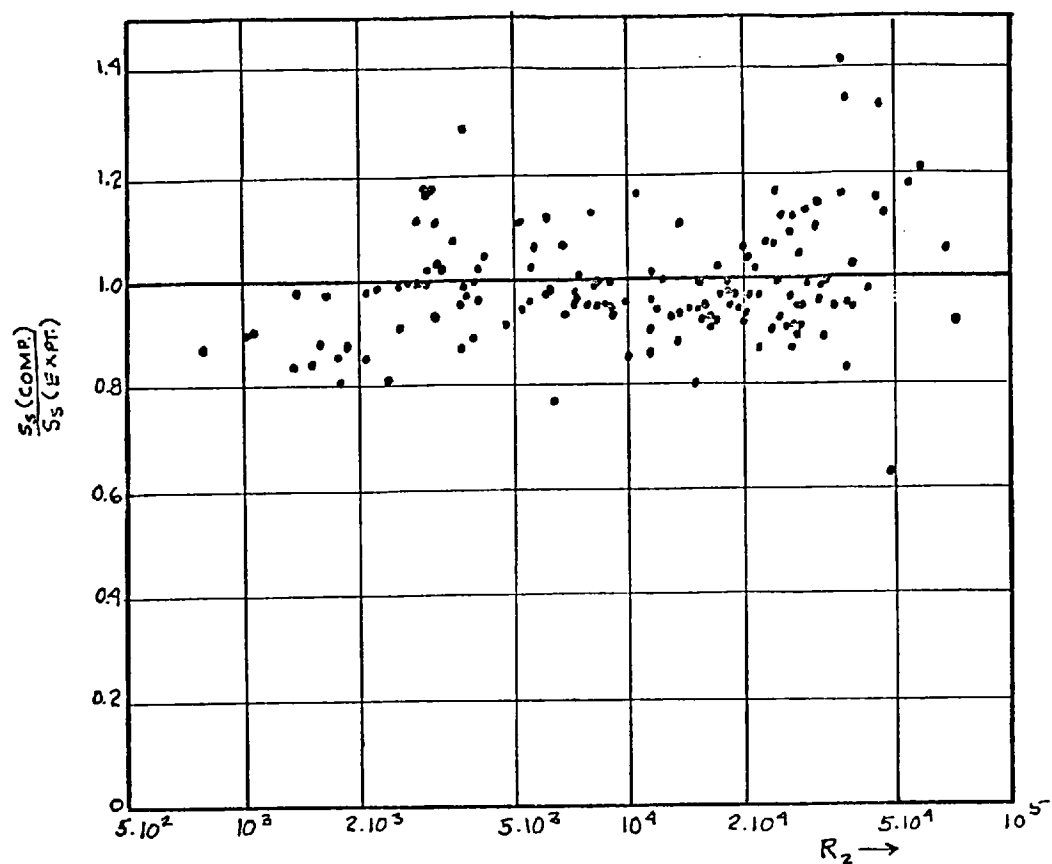


FIGURE 5.2.2-4: DRAG LAW REYNOLDS NUMBER DEPENDENCE

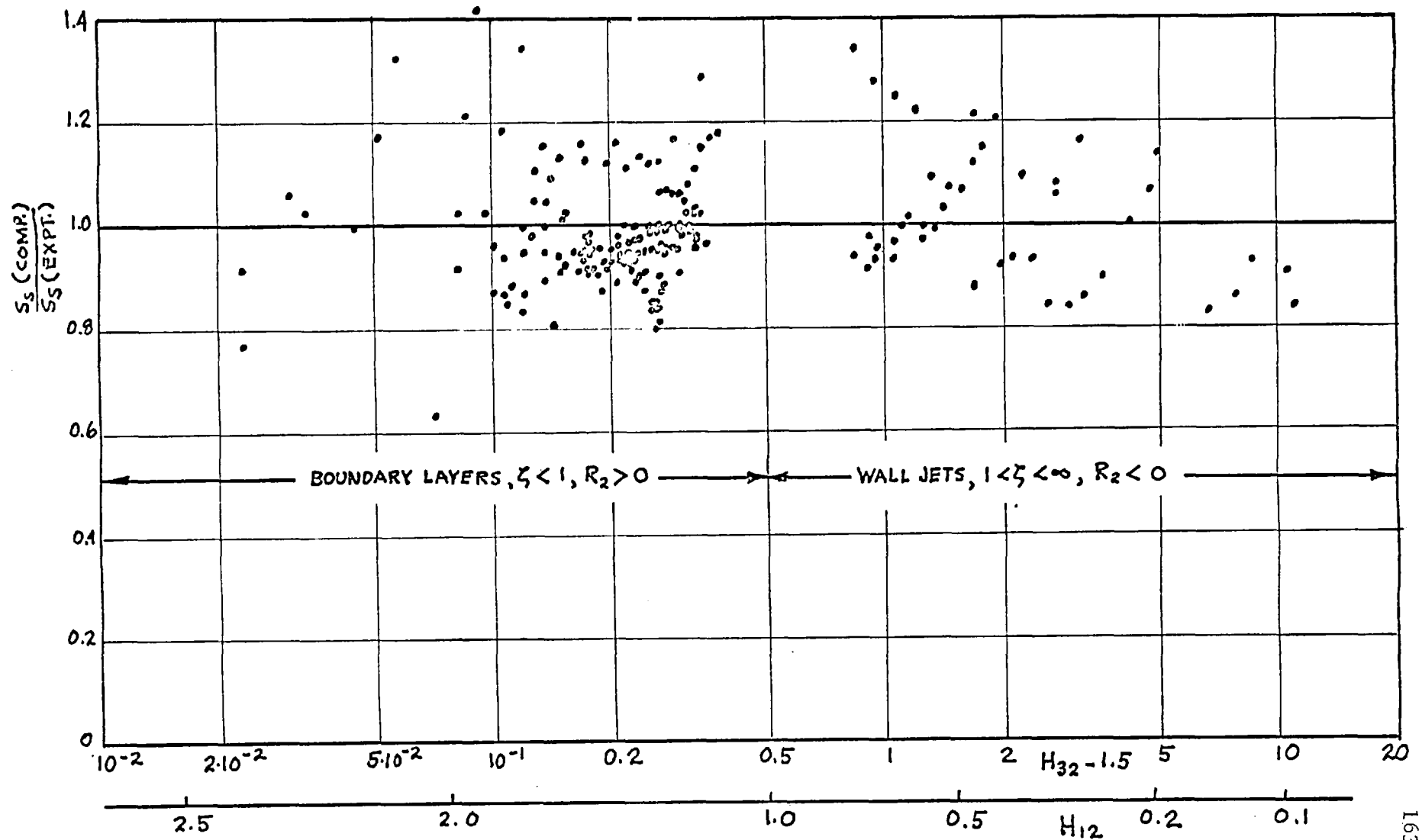


FIGURE 5.2.2-5: DRAG LAW SHAPE FACTOR DEPENDENCE

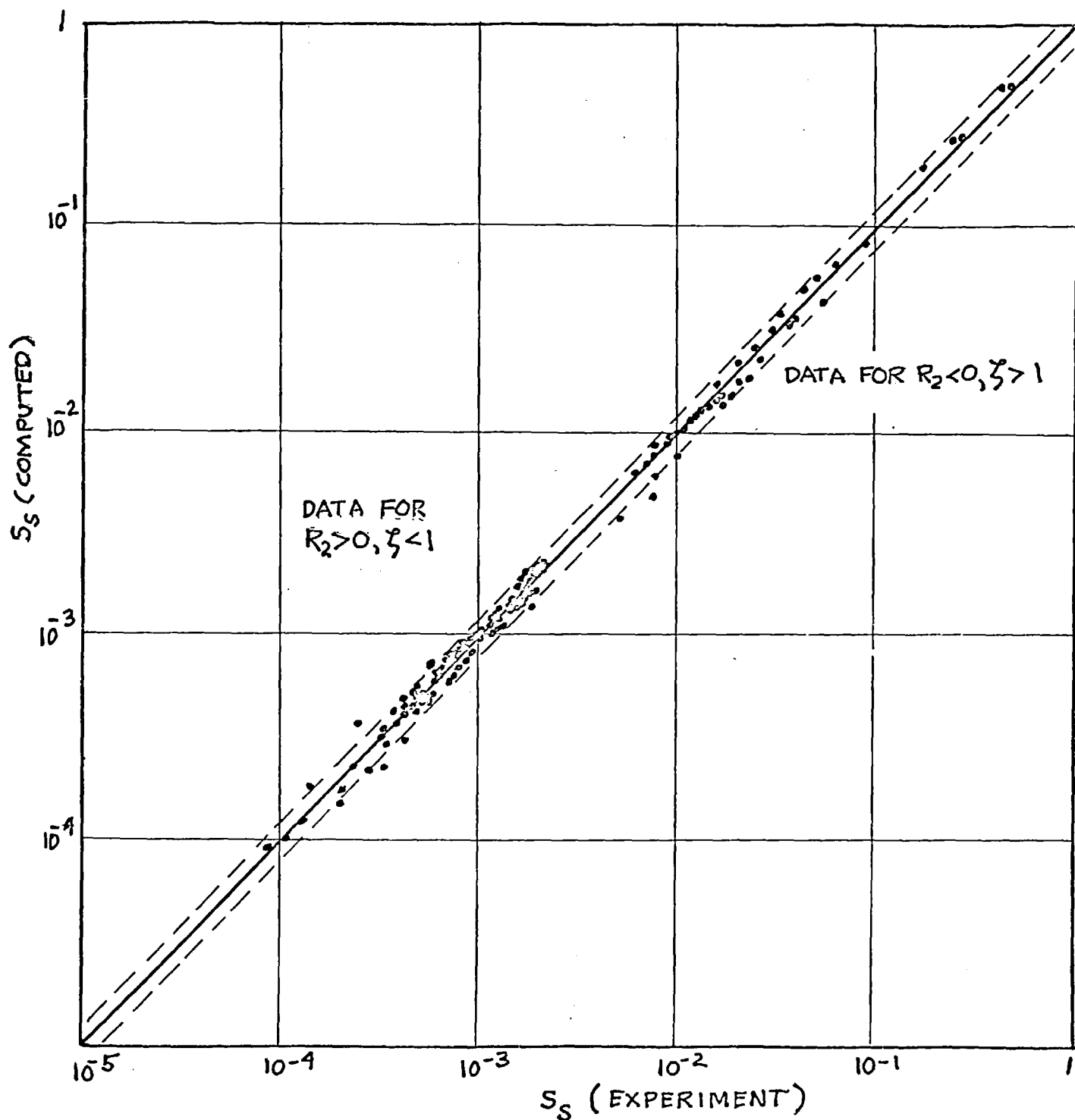


FIGURE 5.2.2-6: COMPARISON OF DRAG COEFFICIENTS  
COMPUTED FROM EQUATION (5.2.2-26) WITH  
EXPERIMENTAL DATA

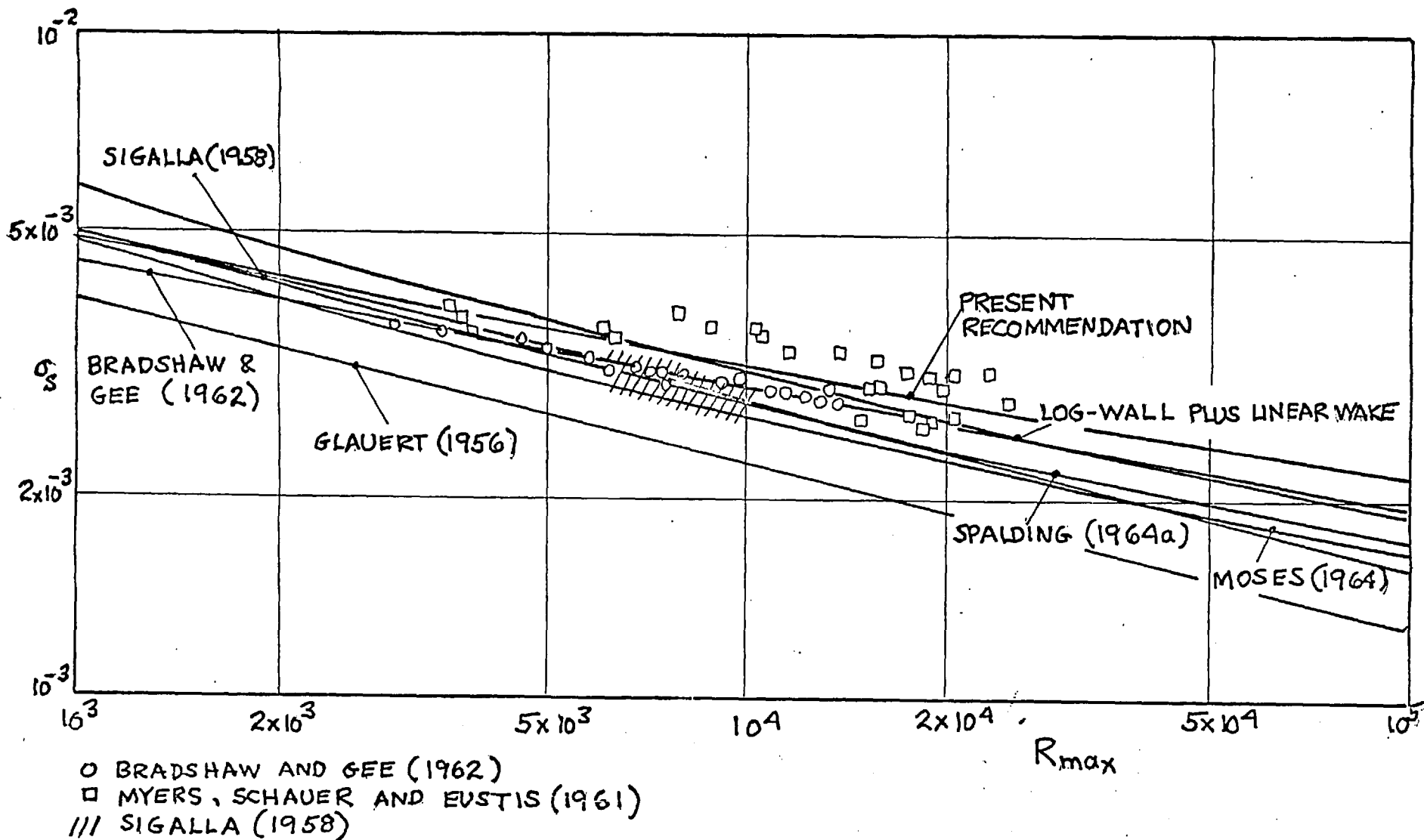


FIGURE 5.2.2-7 ; COMPARISON OF DRAG LAWS WITH EXPERIMENTAL DATA FOR WALL JETS IN STILL SURROUNDINGS. ( $R_2 = -\infty$ )

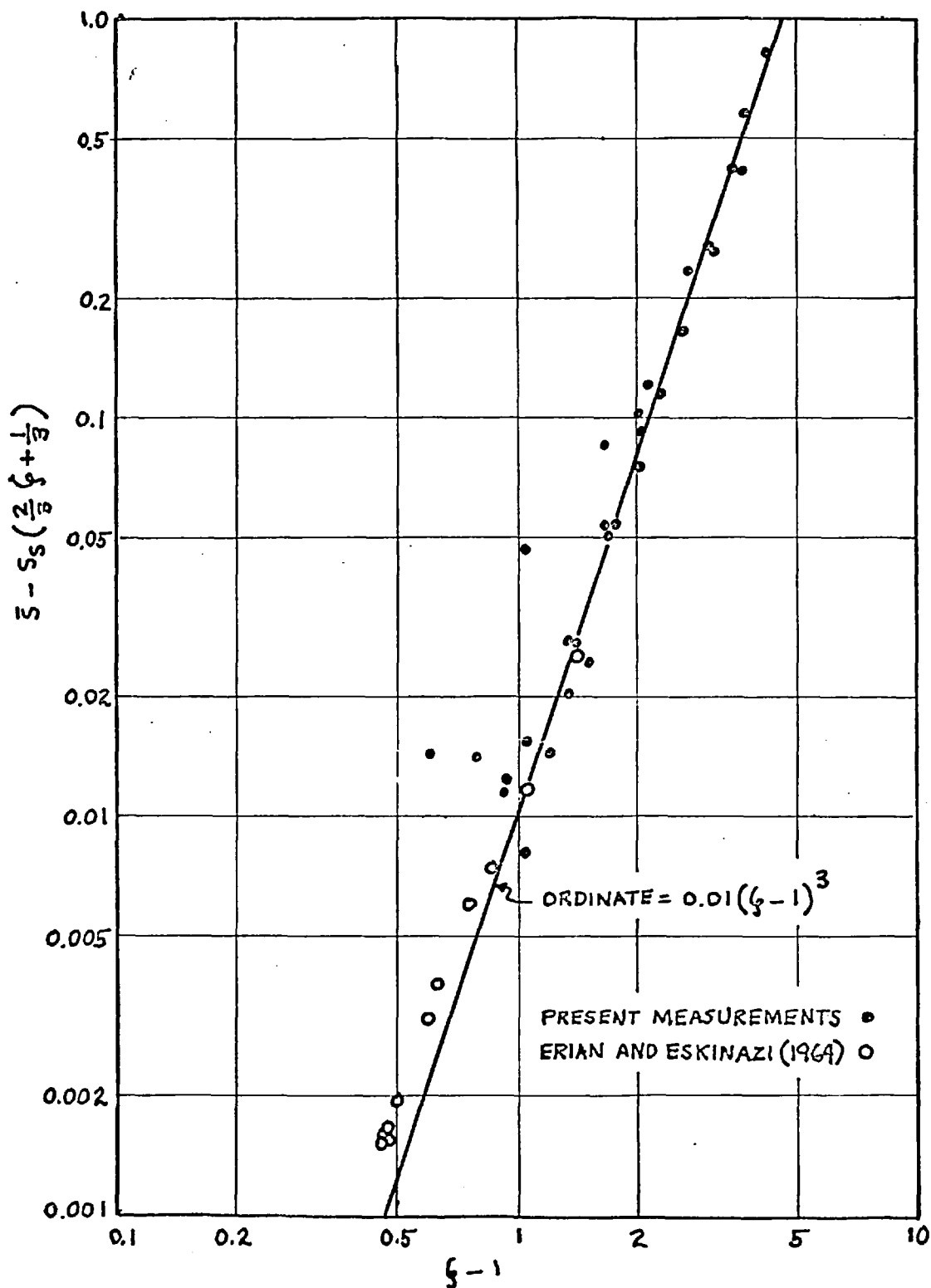


FIGURE 5.2.3-1: EXPERIMENTAL SHEAR-WORK-INTEGRAL DATA.

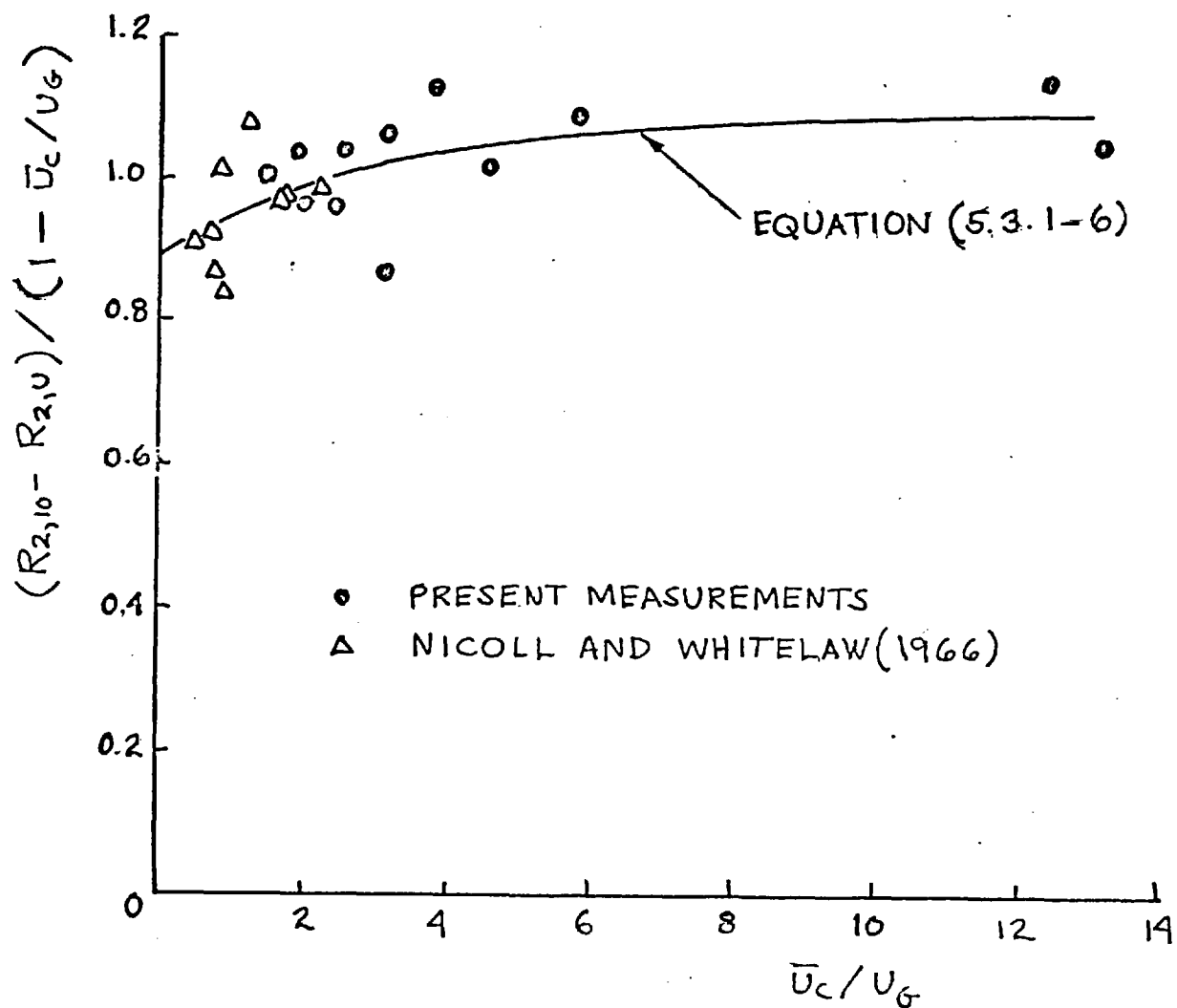
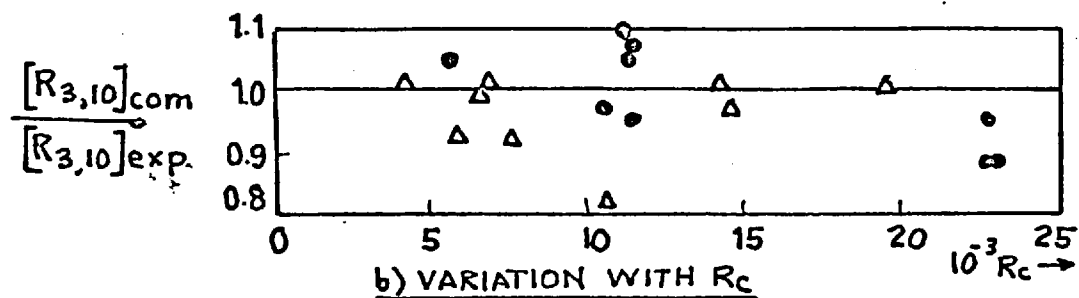
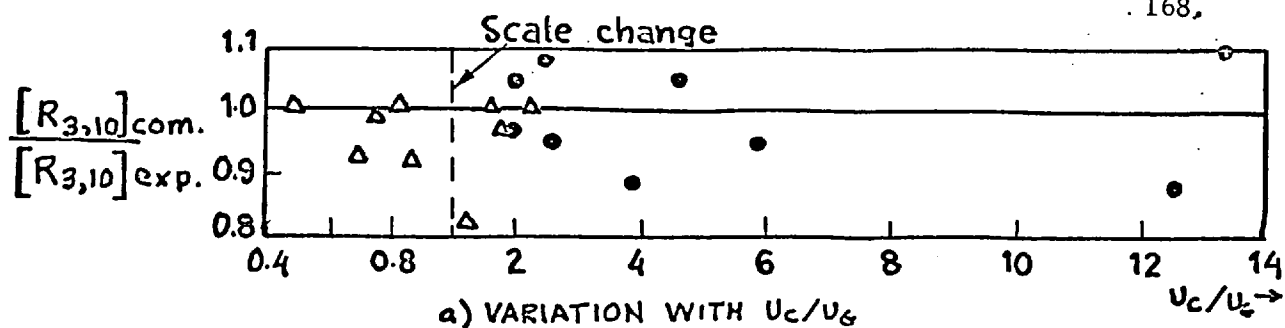


FIGURE 5.3.1-1: The CORRELATION FOR  $R_{2,10}$



PRESENT DATA: ●  
NICOLL & WHITELAW: Δ

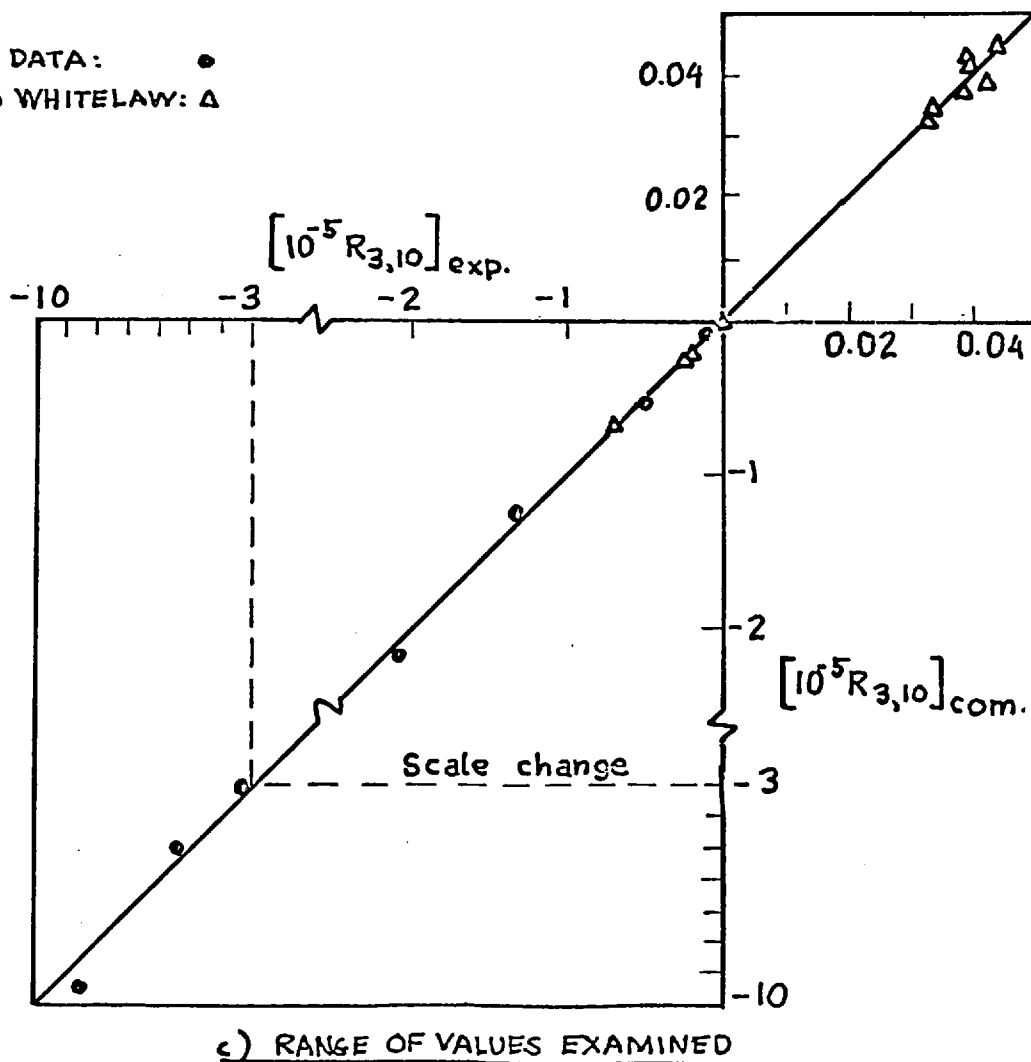


FIGURE 5.3.1-2: COMPARISON OF COMPUTED WITH EXPERIMENTAL VALUES OF  $R_{2,10}$

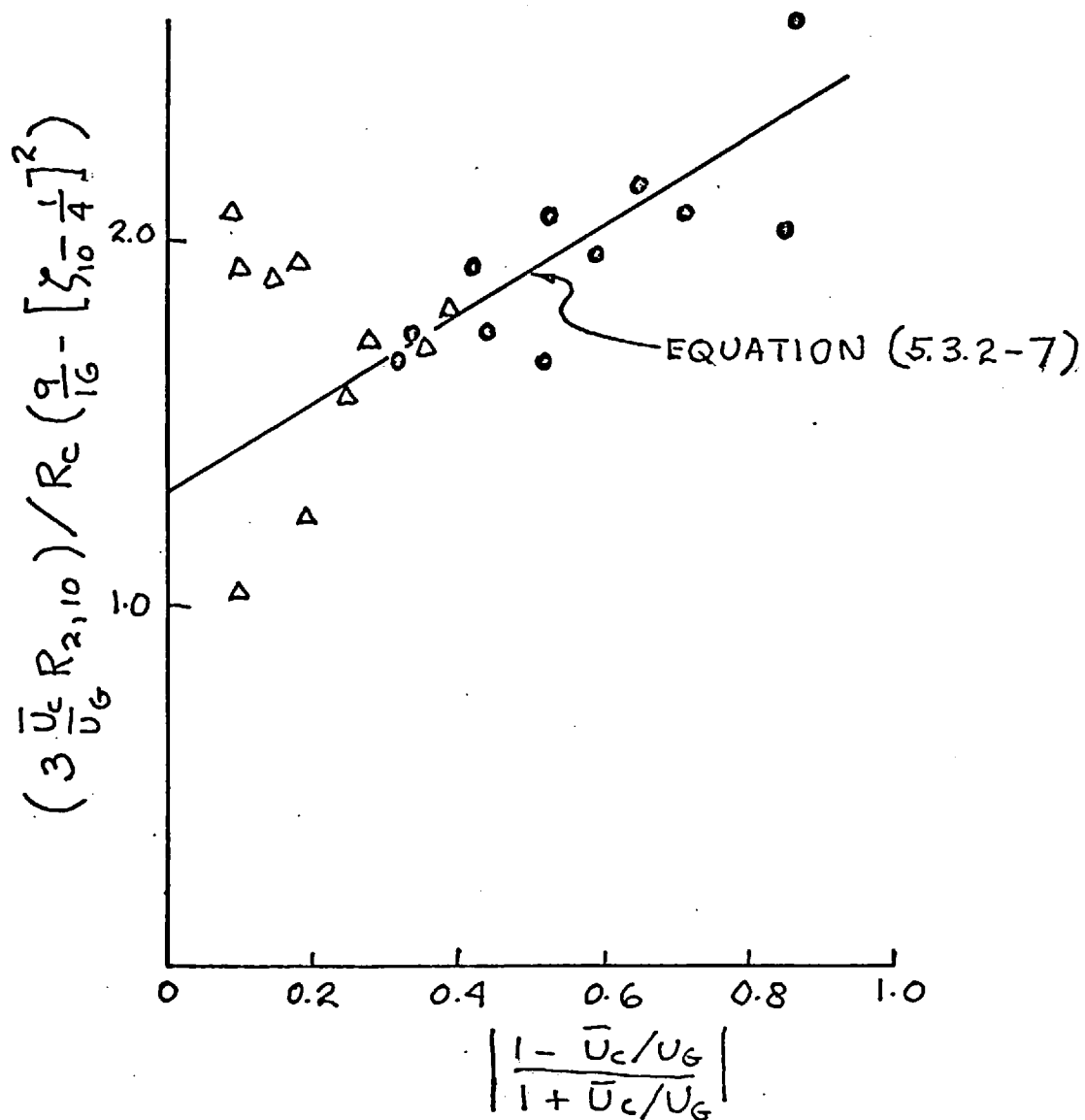
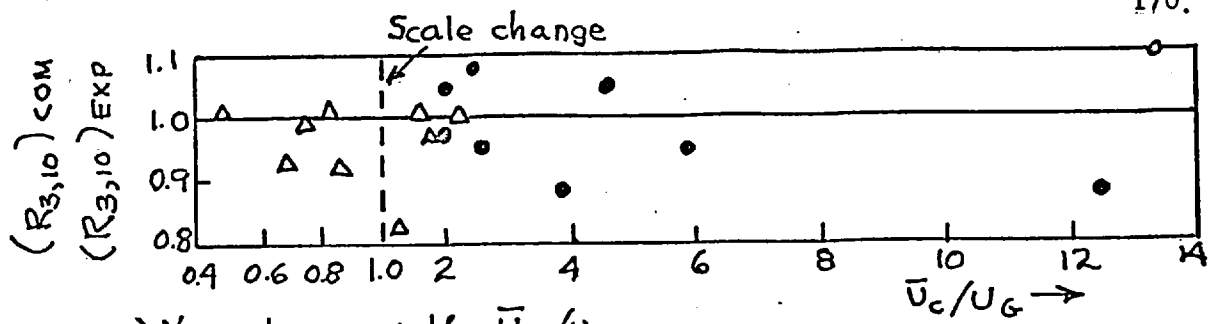
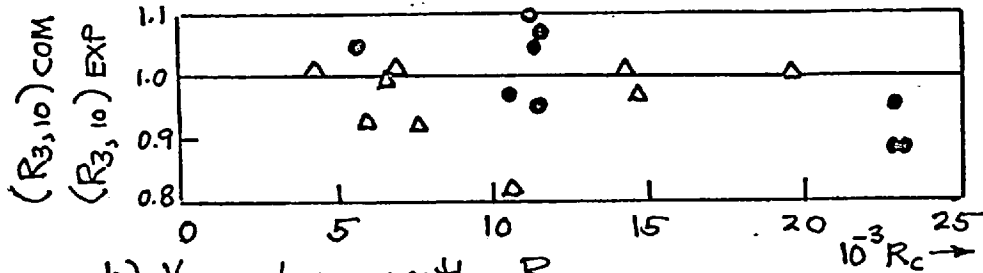


FIGURE 5.3.2-1 : THE CORRELATION  
FOR  $R_{3,10}$

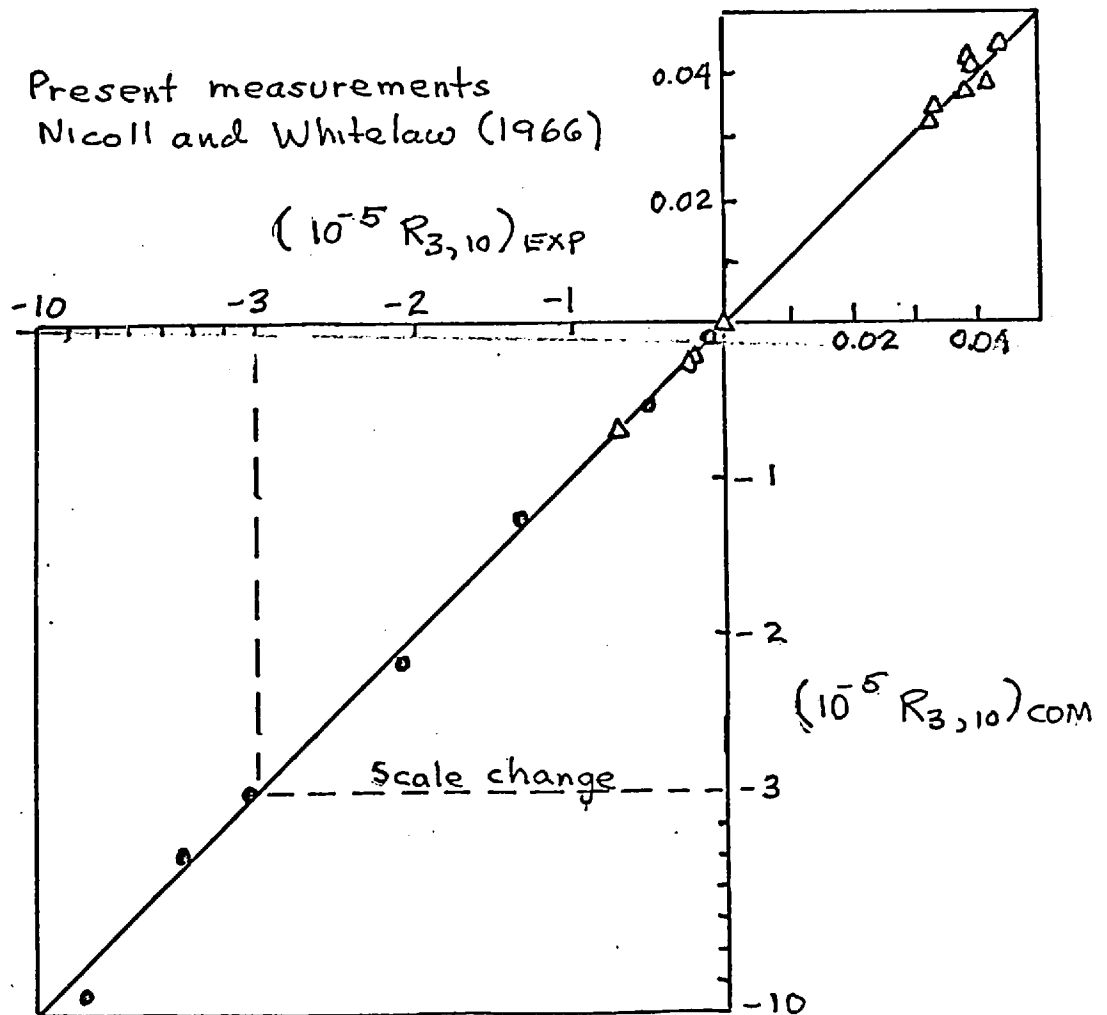


a) Variation with  $\bar{U}_c/U_G$



b) Variation with  $R_c$

• Present measurements  
 $\Delta$  Nicoll and Whitelaw (1966)



c) Range of values of  $R_{3,10}$  examined

FIGURE 5.3.2-2: COMPARISON OF COMPUTED WITH EXPERIMENTAL VALUES OF  $R_{3,10}$

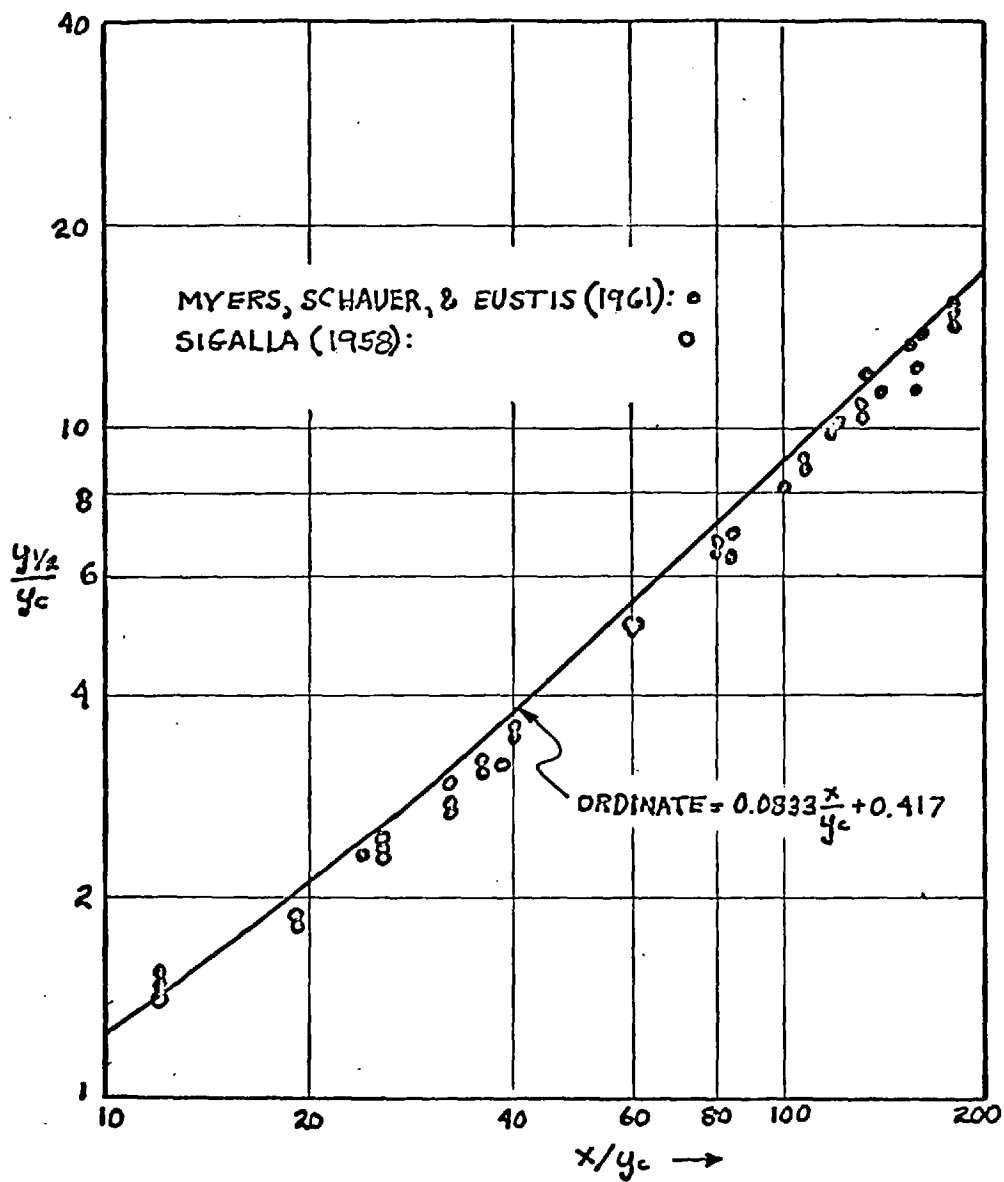


FIGURE 5.5.1-1 : COMPARISON OF PREDICTED  
WITH EXPERIMENTAL RATE OF SPREAD  
FOR WALL JETS IN STILL SURROUNDINGS

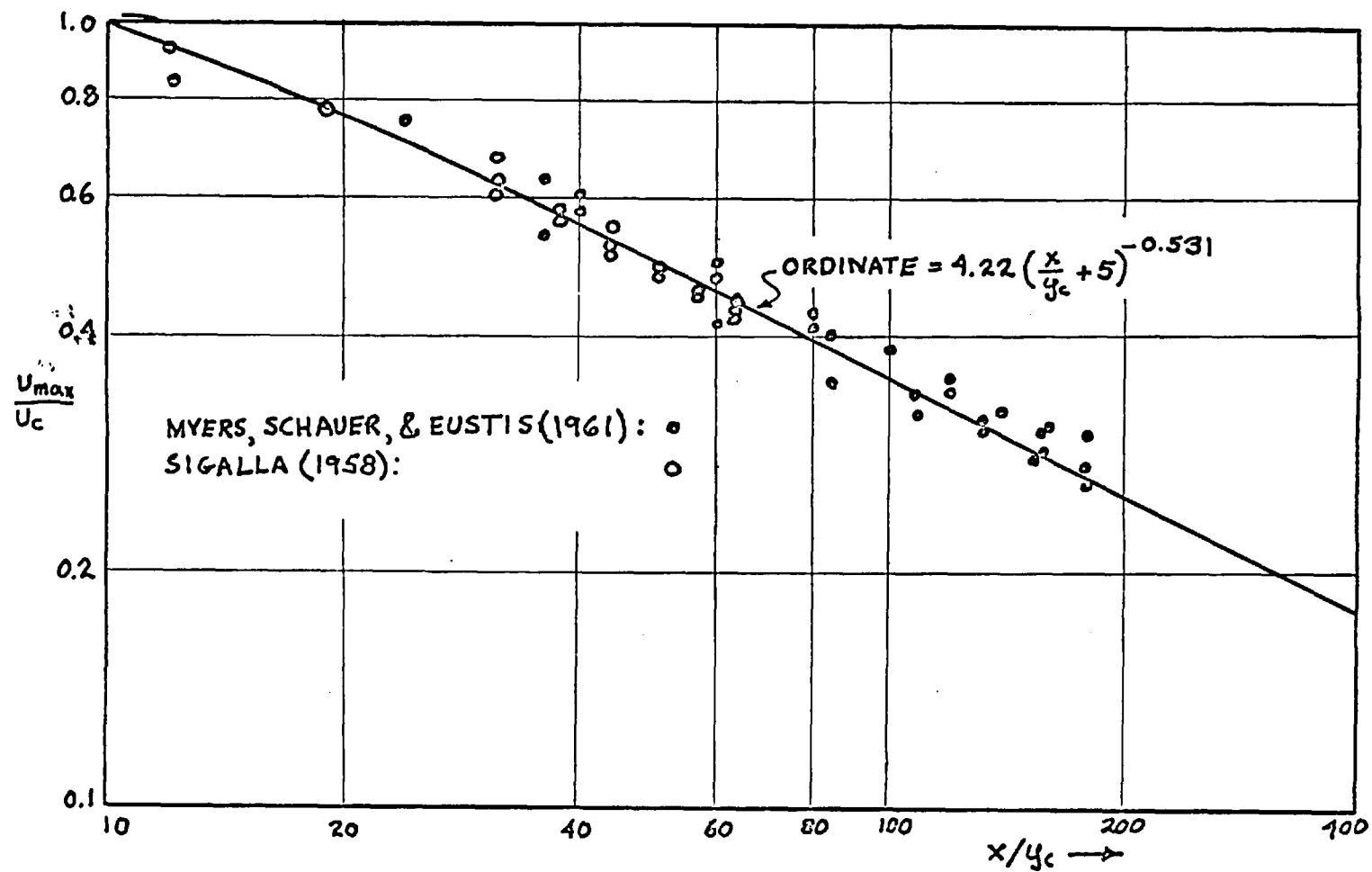


FIGURE 5.5.1-2: COMPARISON OF PREDICTED WITH  
 EXPERIMENTAL VALUES OF VELOCITY MAXIMA  
 FOR WALL JETS IN STILL SURROUNDINGS

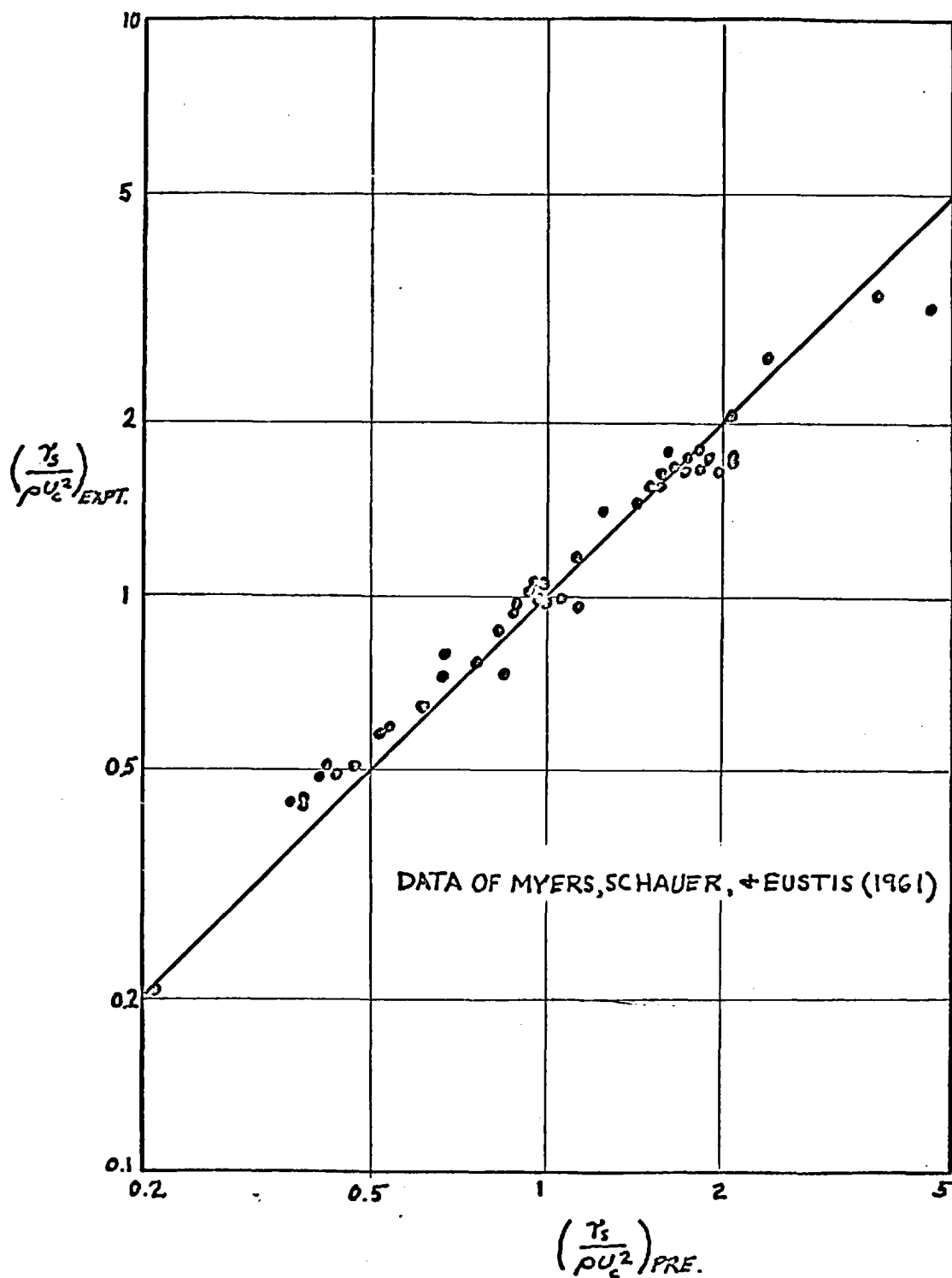


FIGURE 5.5.1- 3: COMPARISON OF PREDICTED WITH EXPERIMENTAL DRAG COEFFICIENTS FOR WALL JETS IN STILL SURROUNDINGS

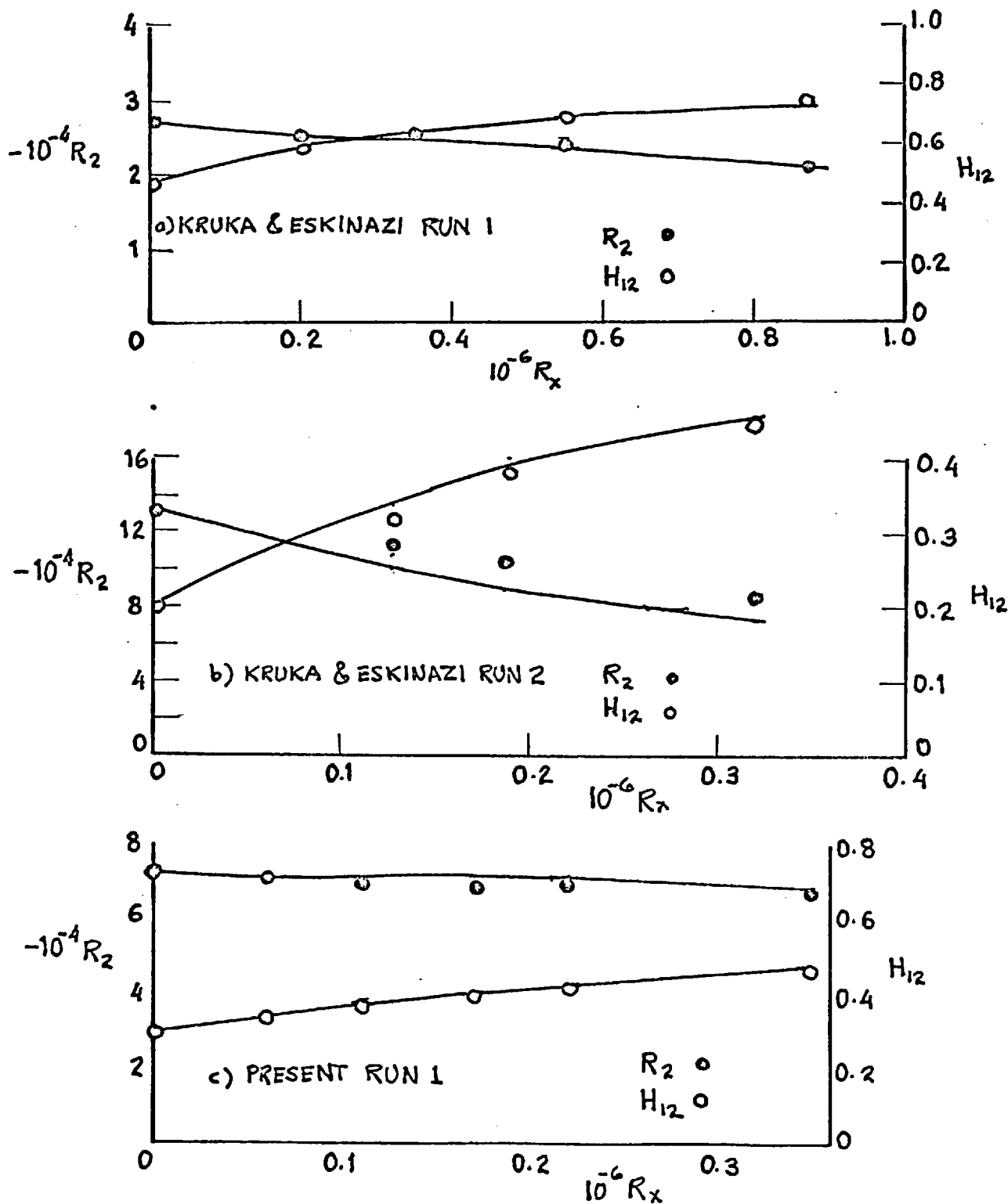


FIGURE 5.5.2-1: COMPARISON OF PREDICTED WITH EXPERIMENTAL WALL JET DEVELOPMENT FOR WALL JETS WITH FINITE CONSTANT MAINSTREAM VELOCITY.

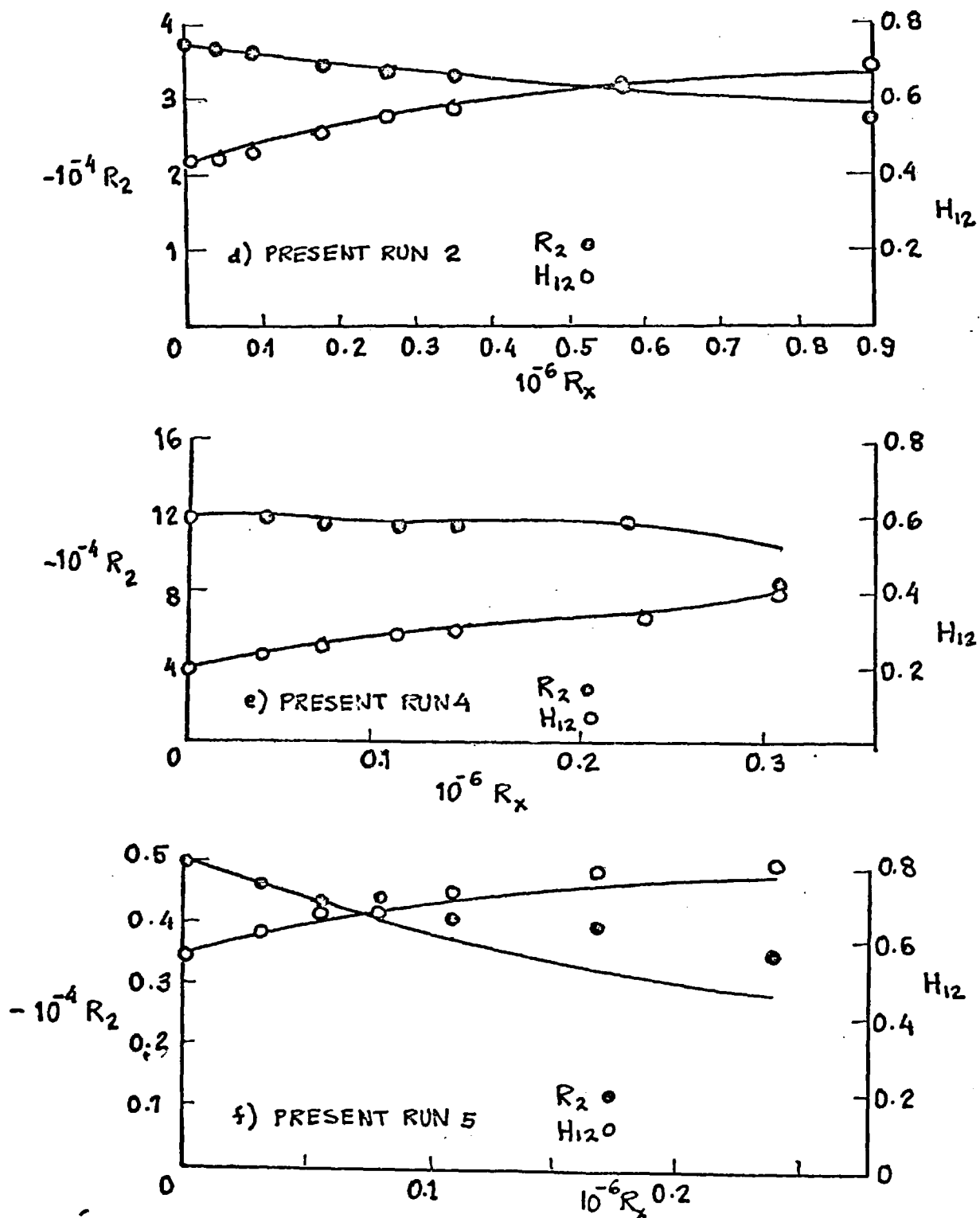


FIGURE 5.5.2-1: COMPARISON OF PREDICTED WITH EXPERIMENTAL WALL JET DEVELOPMENT FOR WALL JETS WITH FINITE CONSTANT MAINSTREAM VELOCITY

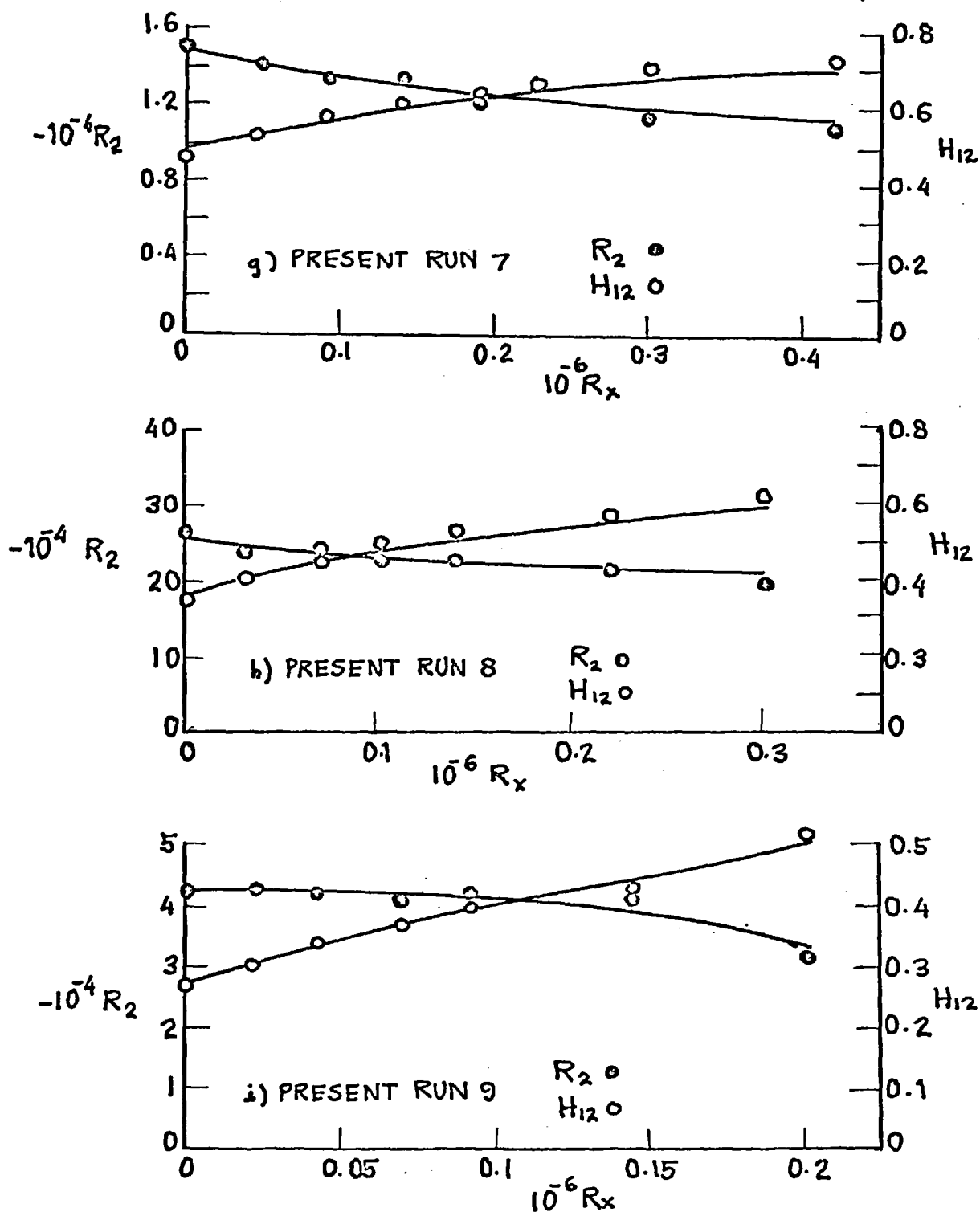


FIGURE 5.5.2-1: COMPARISON OF PREDICTED WITH EXPERIMENTAL WALL JET DEVELOPMENT FOR WALL JETS WITH FINITE CONSTANT MAINSTREAM VELOCITY

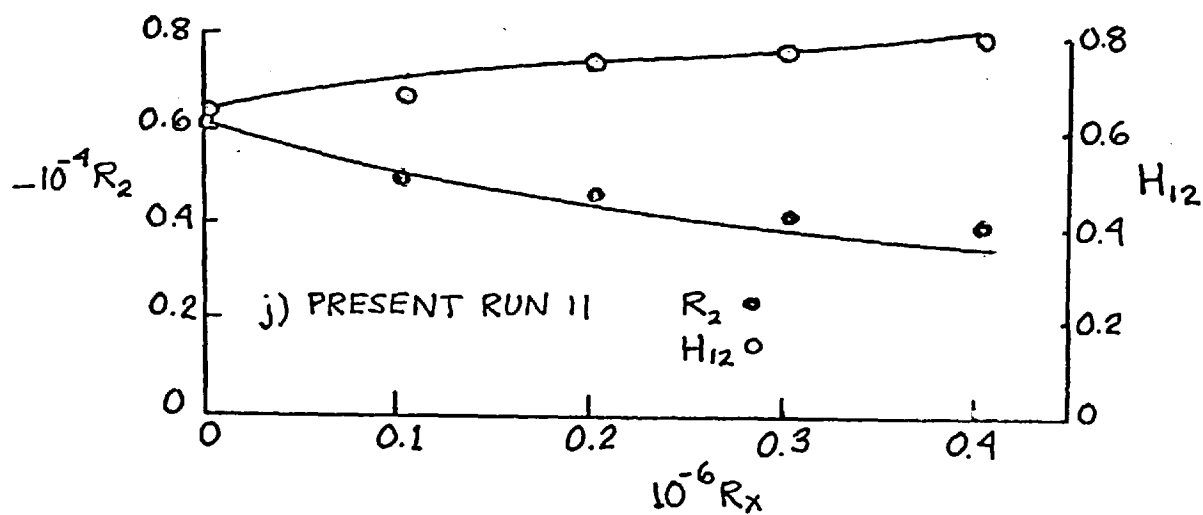


FIGURE 5.5.2-1: COMPARISON OF PREDICTED WITH EXPERIMENTAL WALL JET DEVELOPMENT FOR WALL JETS WITH FINITE CONSTANT MAINSTREAM VELOCITY

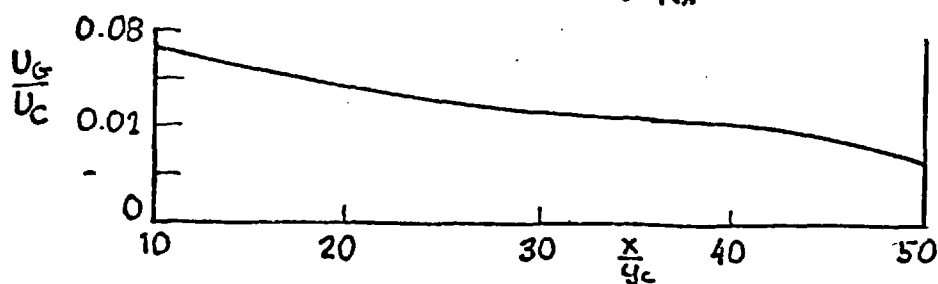
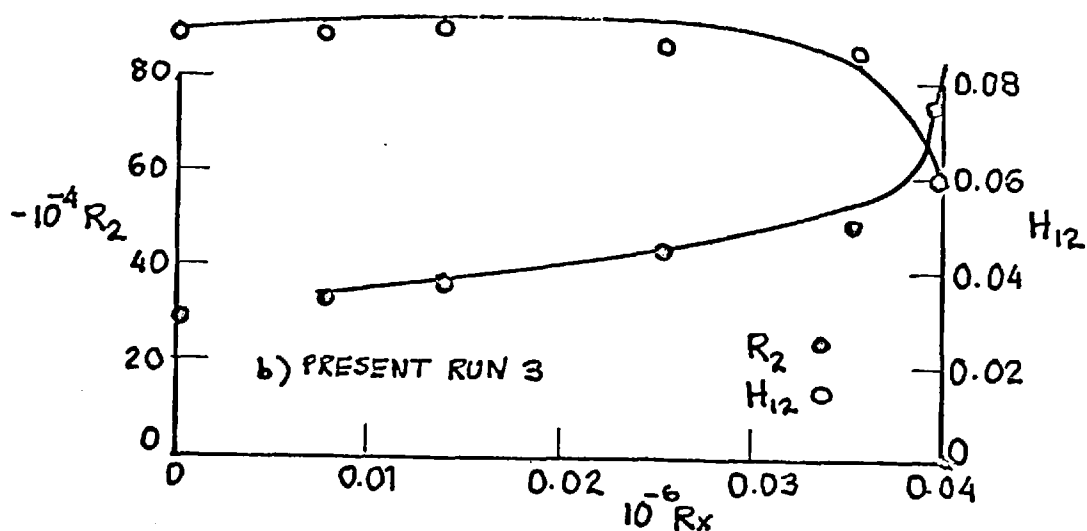
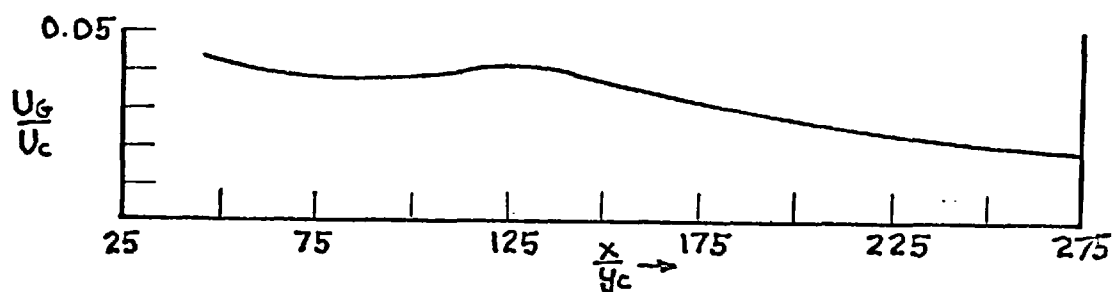
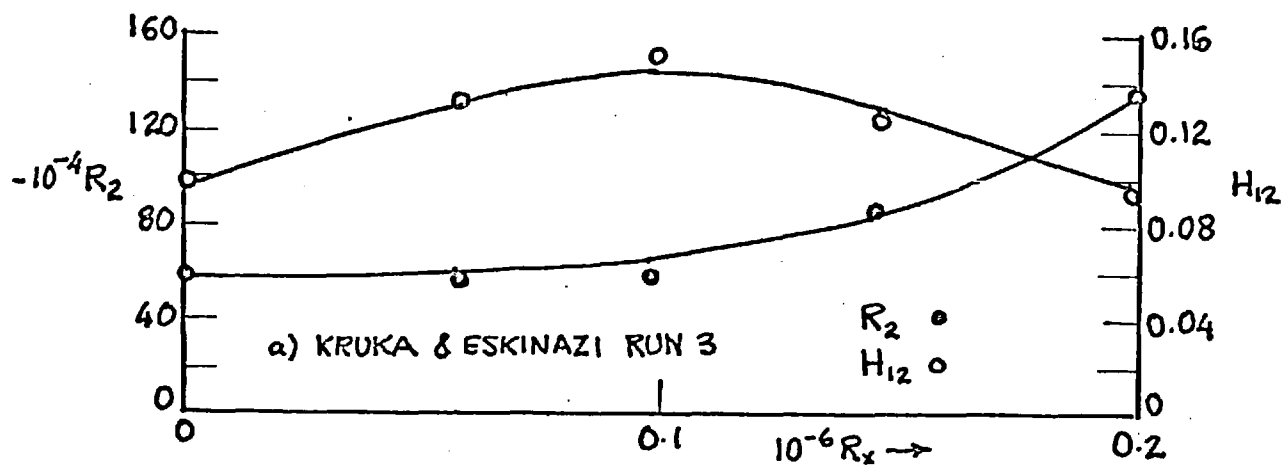


FIGURE 5.5.2- 2 : COMPARISON OF PREDICTED WITH EXPERIMENTAL WALL JET DEVELOPMENT FOR FLOWS WITH PRESSURE GRADIENTS

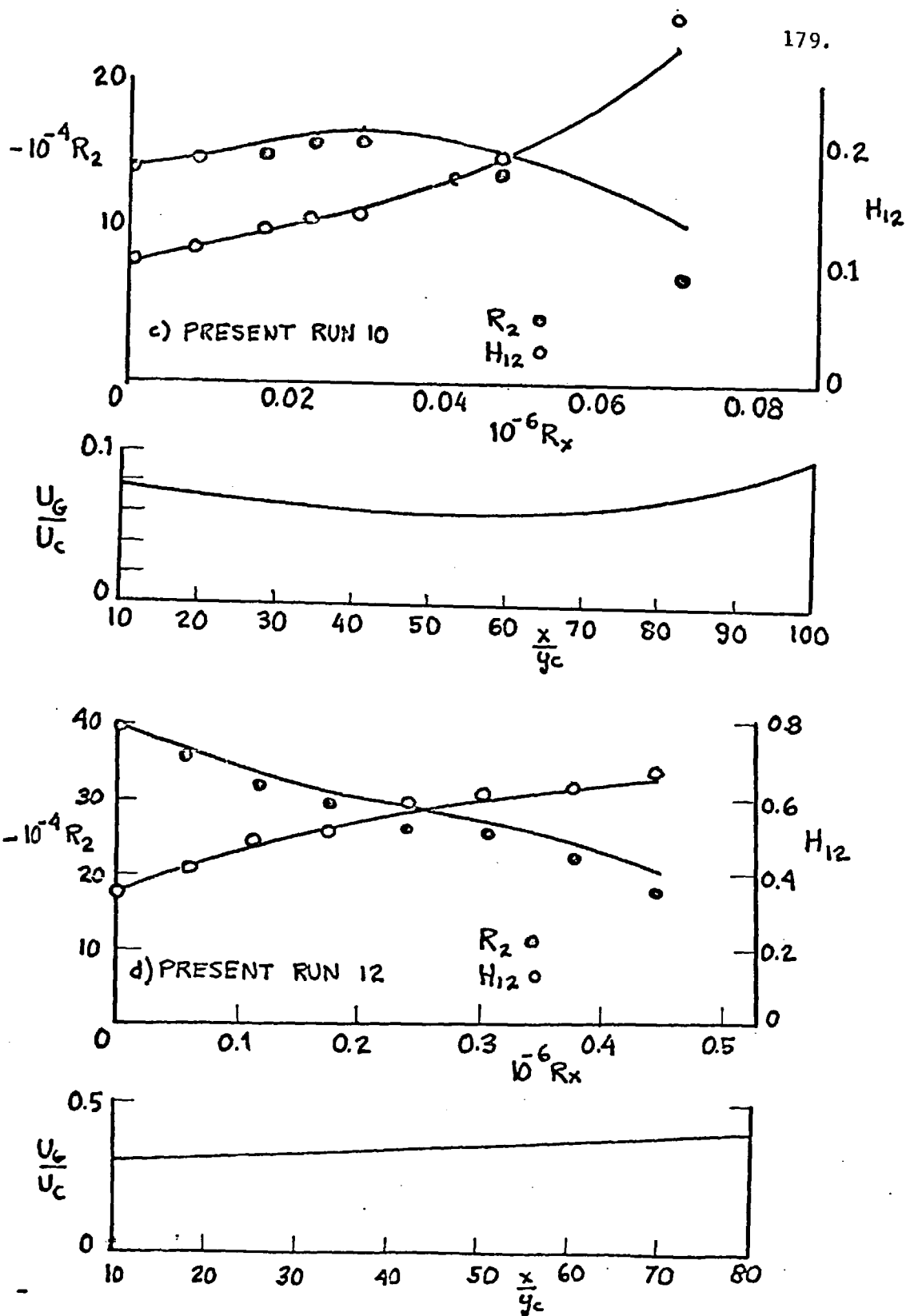


FIGURE 5.5.2-2: COMPARISON OF PREDICTED WITH EXPERIMENTAL WALL-JET DEVELOPMENT FOR FLOWS WITH PRESSURE GRADIENTS

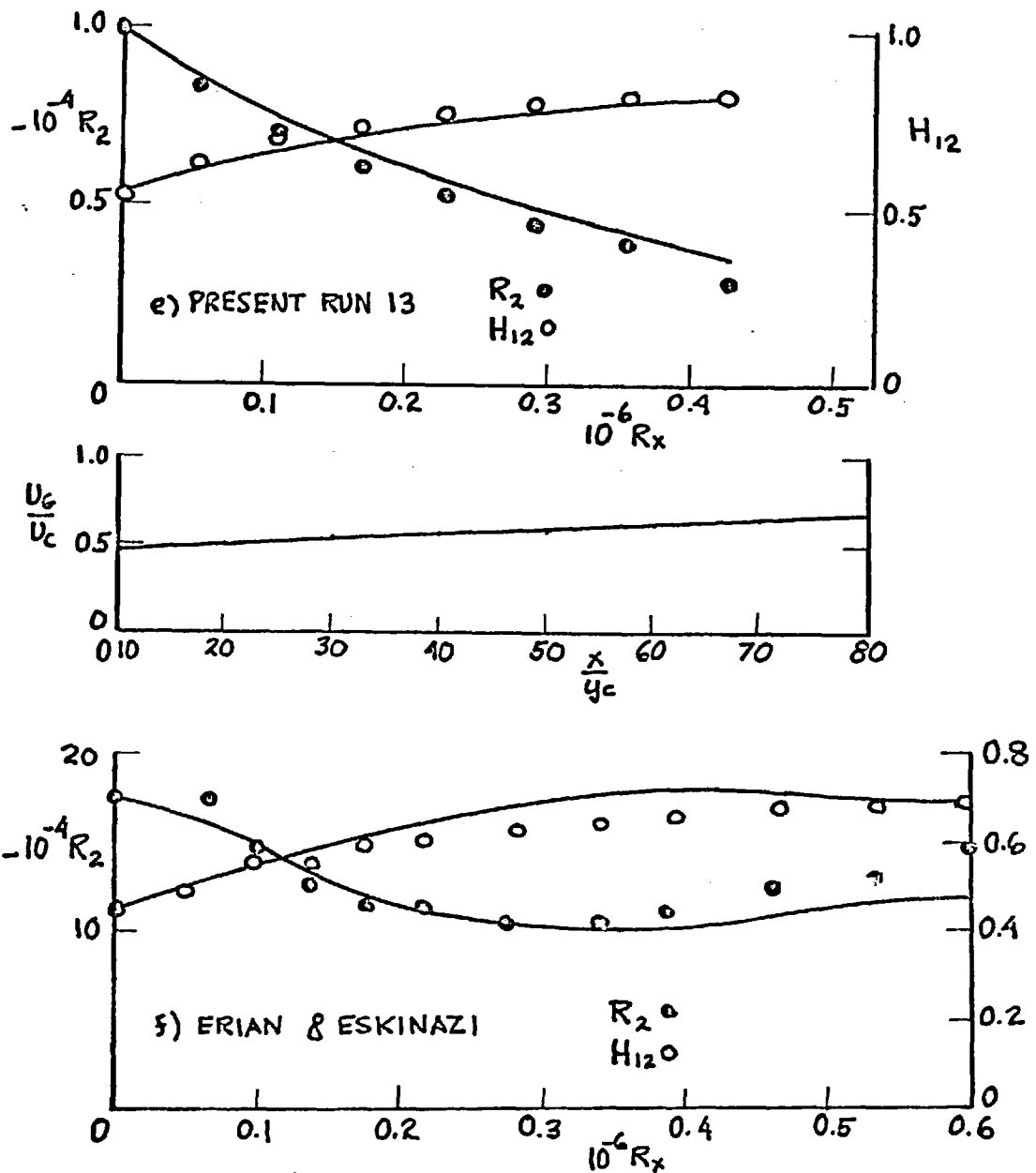


FIGURE 5.5.2-2: COMPARISON OF PREDICTED WITH EXPERIMENTAL WALL-JET DEVELOPMENT FOR FLOWS WITH PRESSURE GRADIENTS

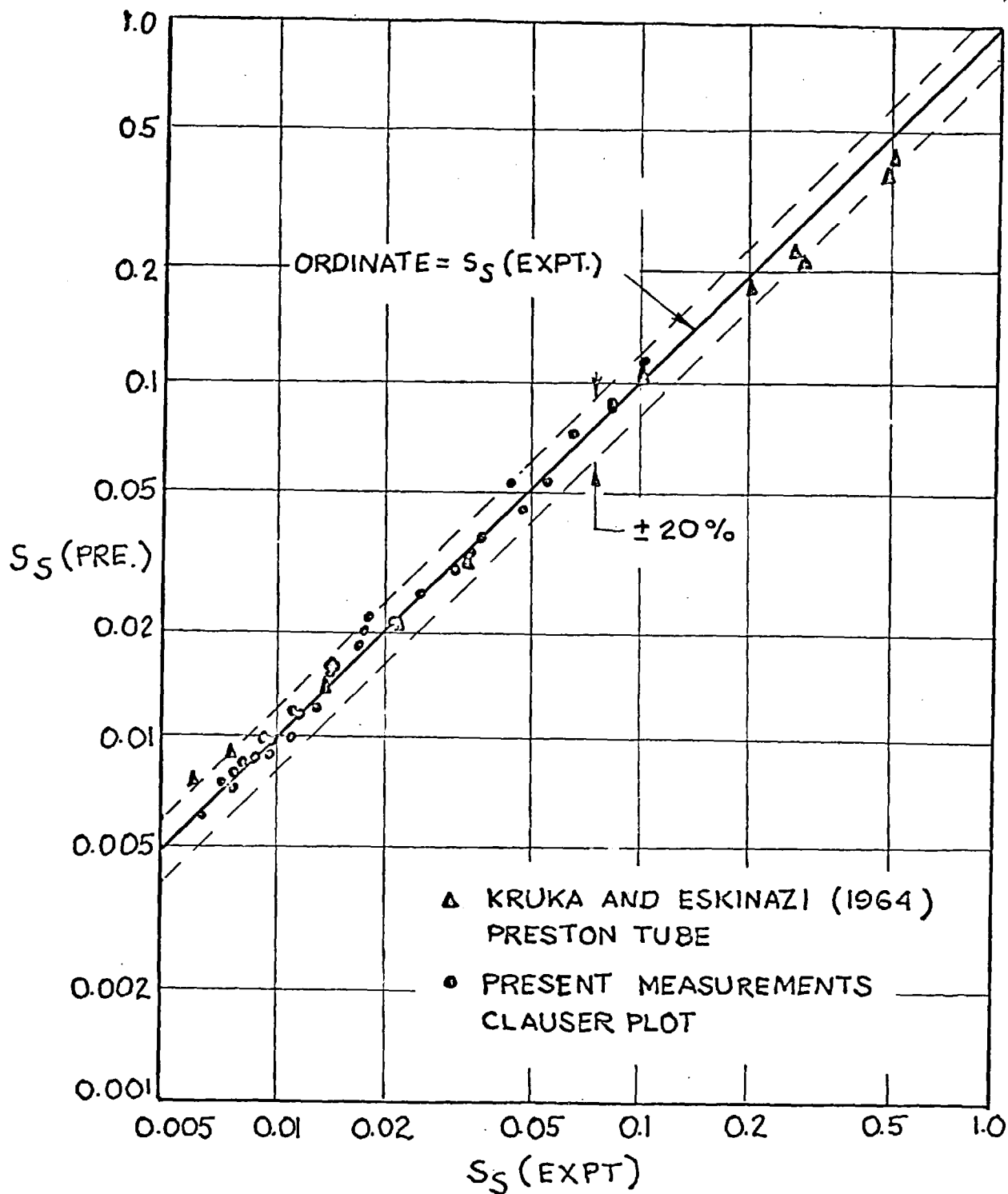


Fig. 5.5.2-3: COMPARISON OF PREDICTED WITH  
EXPERIMENTAL DRAG

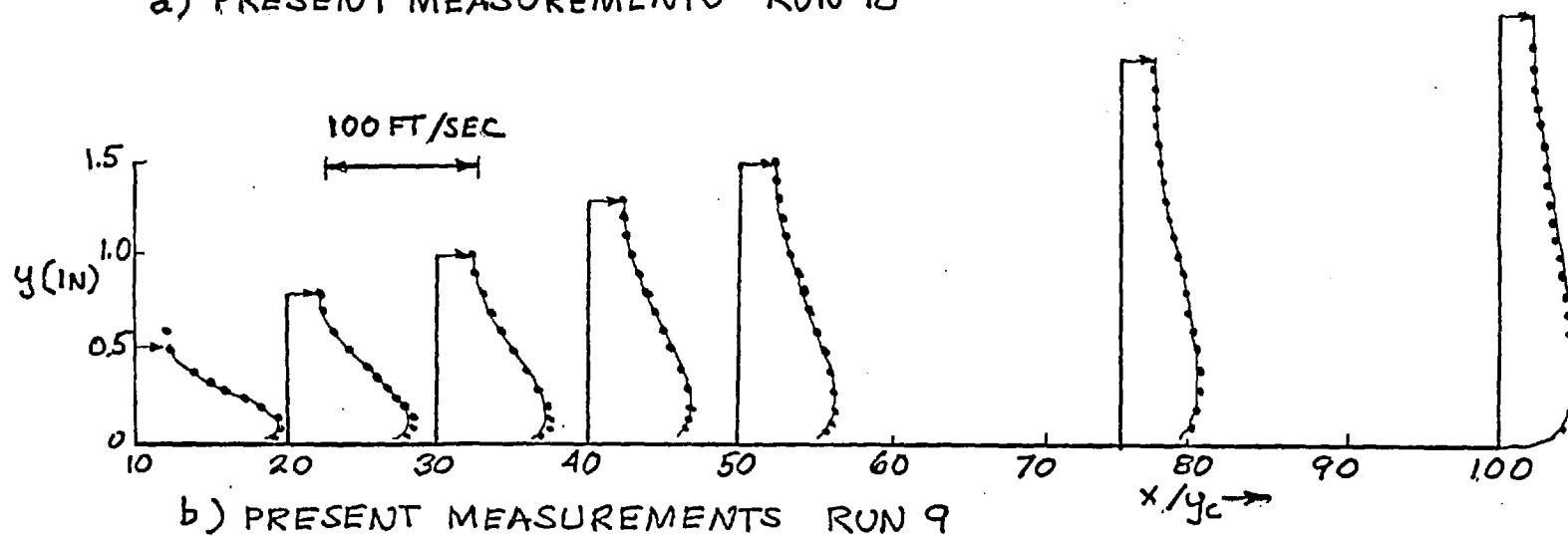
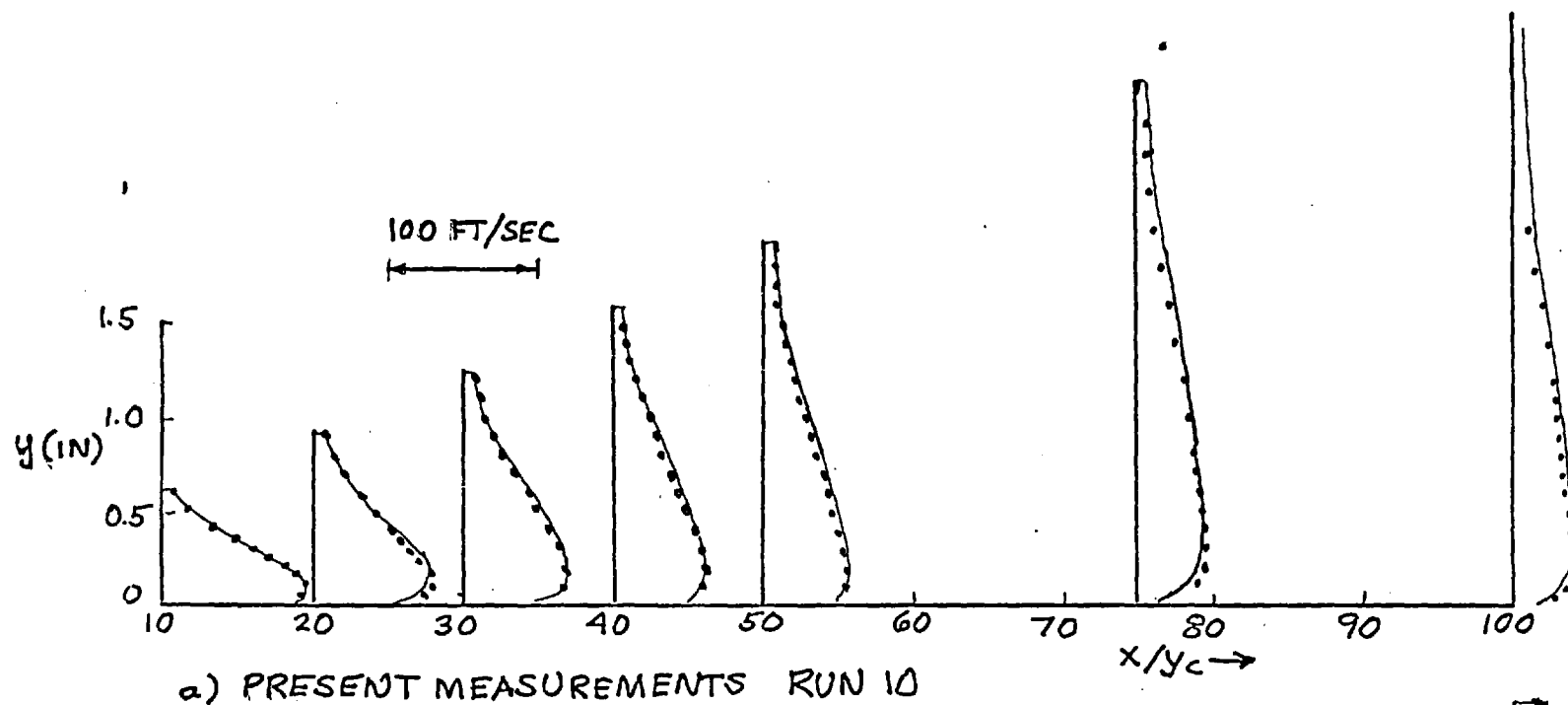


FIGURE 5.5.2-4: COMPARISON OF EXPERIMENTAL VELOCITY PROFILES WITH REGENERATED PROFILES (METHOD I)

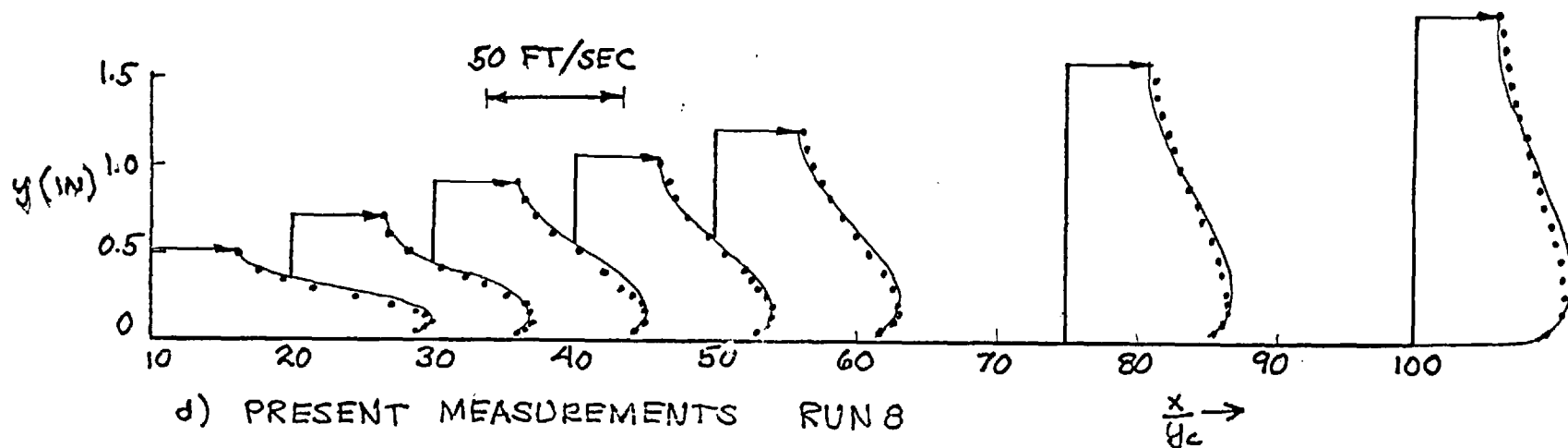
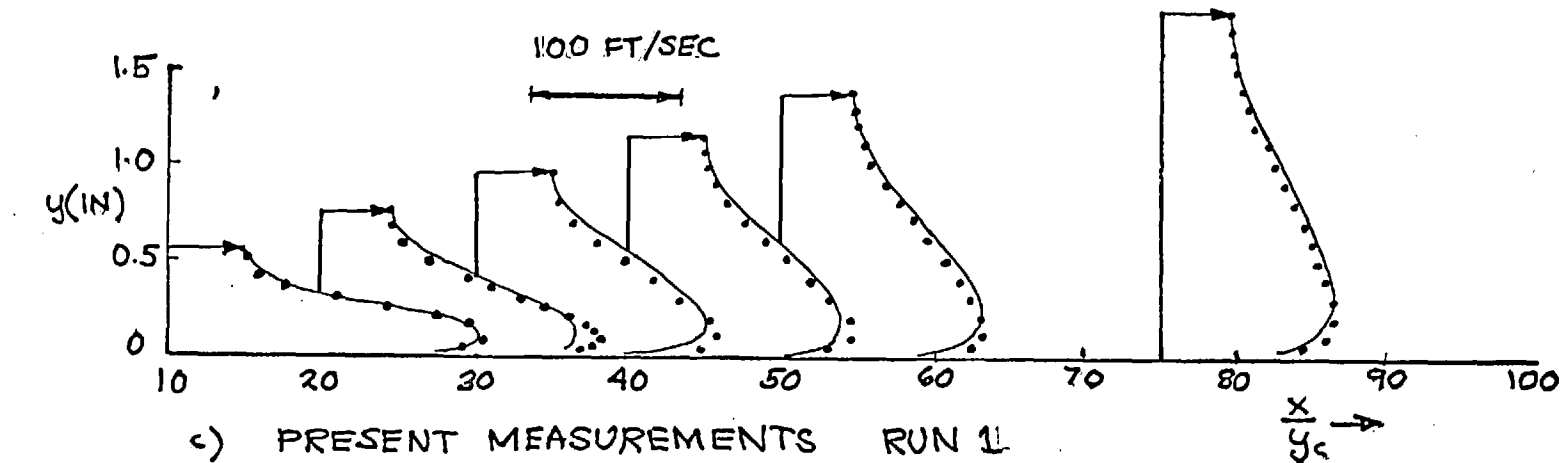


FIGURE 5.5.2-4: COMPARISON OF EXPERIMENTAL VELOCITY PROFILES WITH REGENERATED PROFILES (METHOD I)

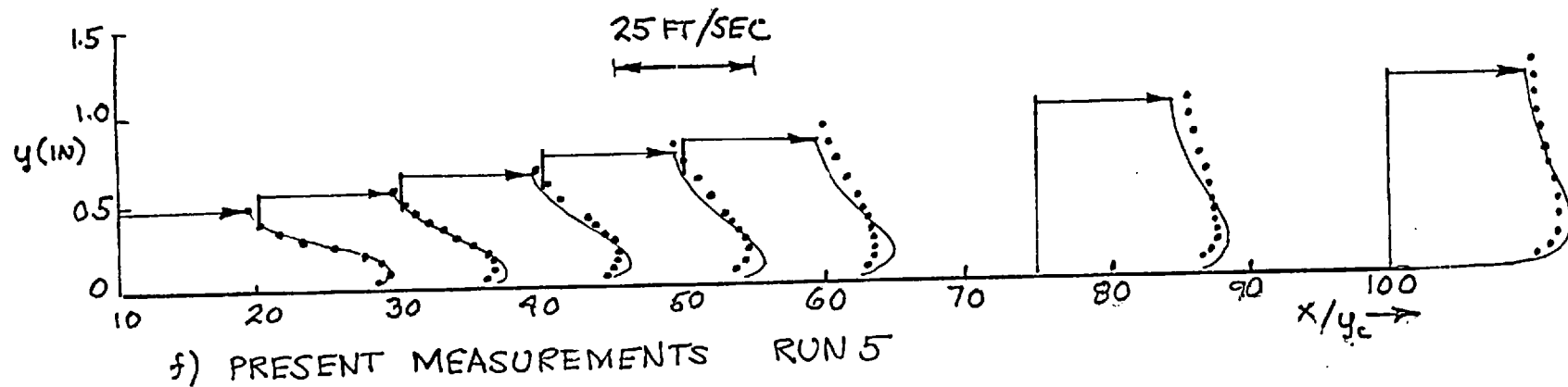
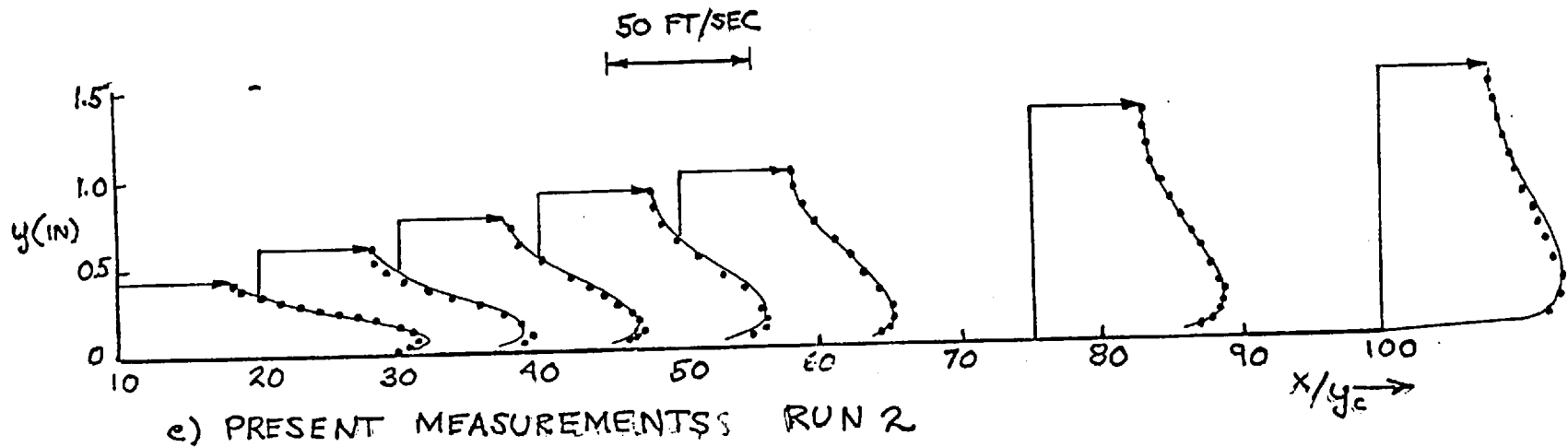
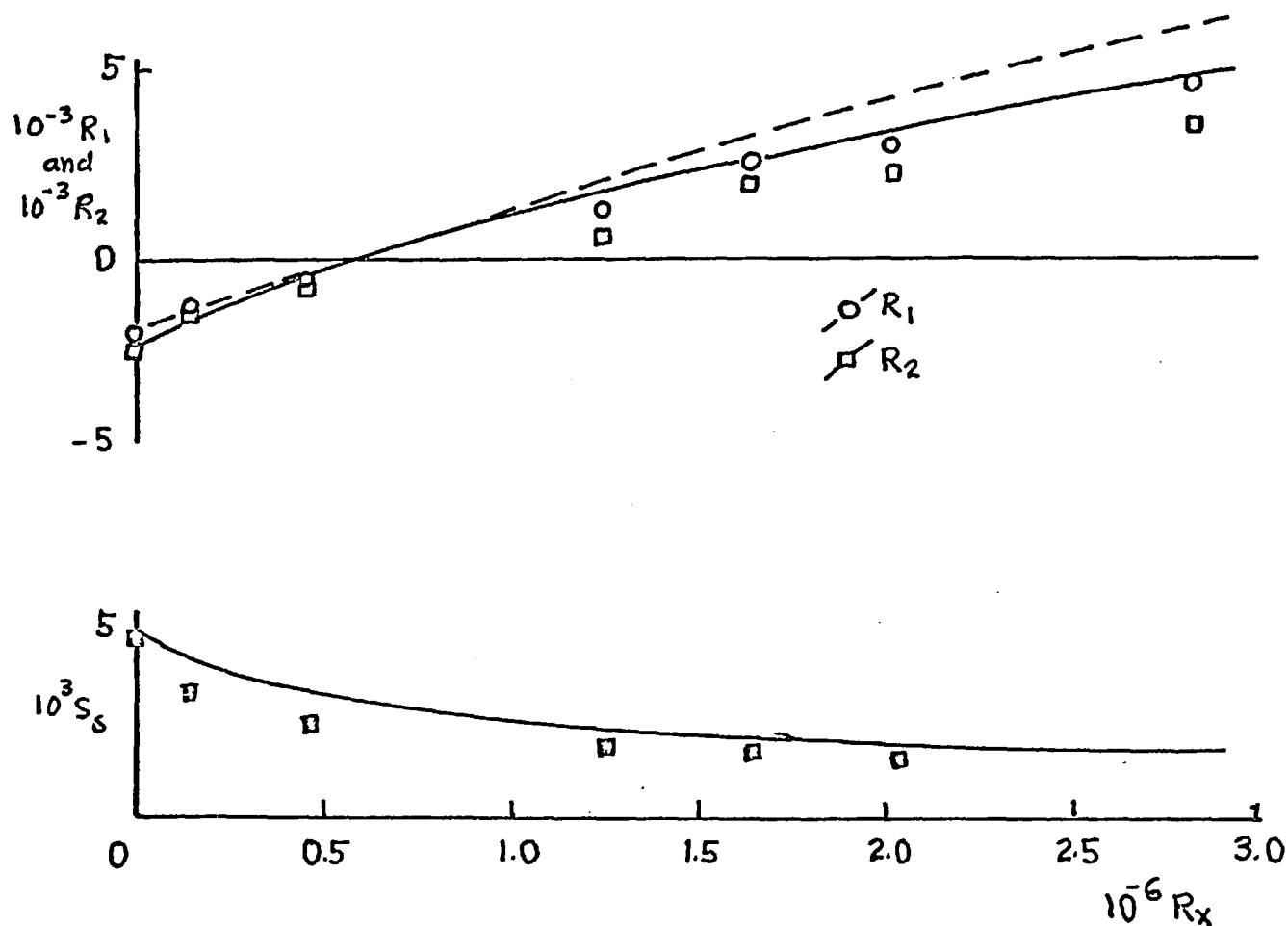
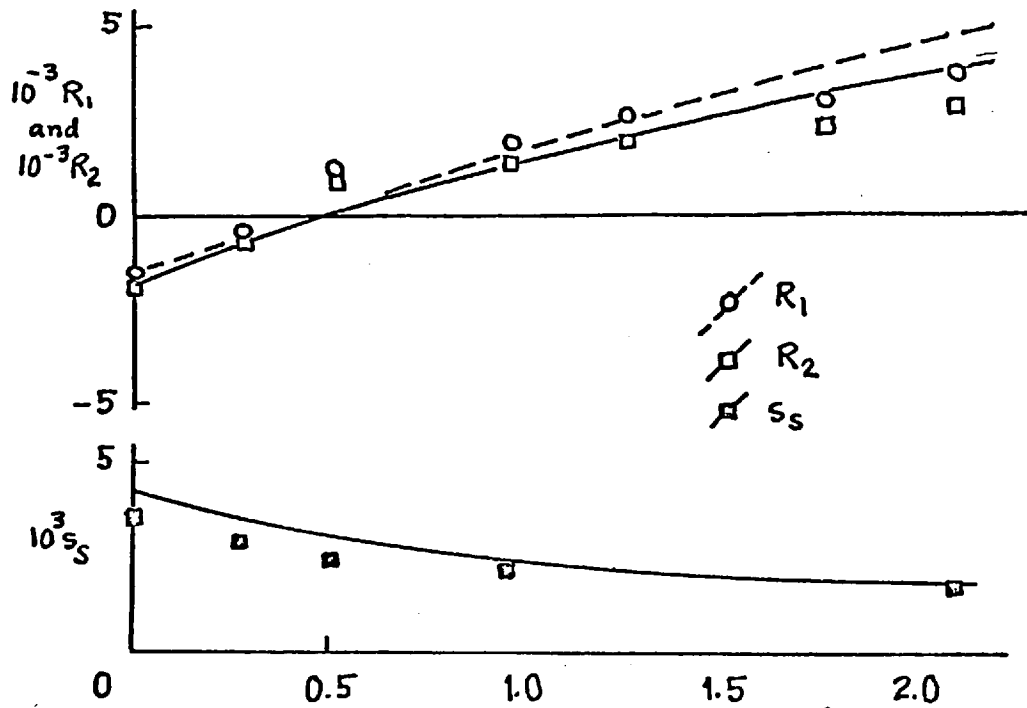


FIGURE 5.5.2-4: COMPARISON OF EXPERIMENTAL VELOCITY PROFILES WITH REGENERATED PROFILES (METHOD I)

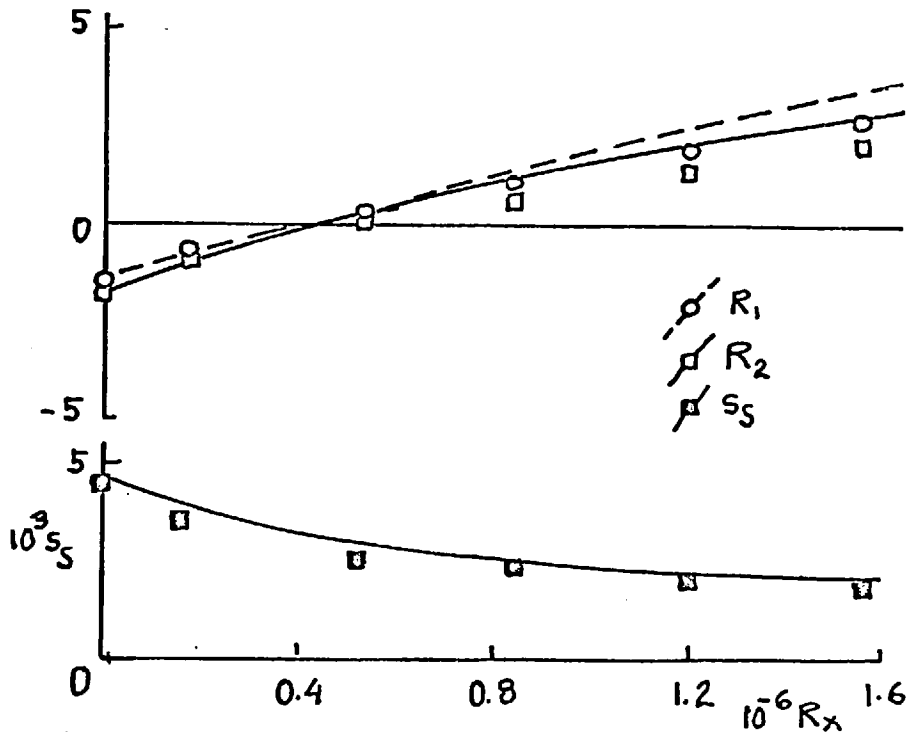


a) DATA OF WIEGHARDT

FIGURE 5.5.3-1: COMPARISON OF MEASURED AND PREDICTED WALL-JET DEVELOPMENT FOR FLOWS IN WHICH THE VELOCITY MAXIMUM DISAPPEARS



b) DATA OF ESCUDIER ETAL (1966)  $10^{-6} R_x$



c) DATA OF ESCUDIER ETAL (1966)

FIGURE 5.5.3-1: COMPARISON OF MEASURED AND PREDICTED WALL-JET DEVELOPMENT FOR FLOWS IN WHICH THE VELOCITY MAXIMUM DISAPPEARS

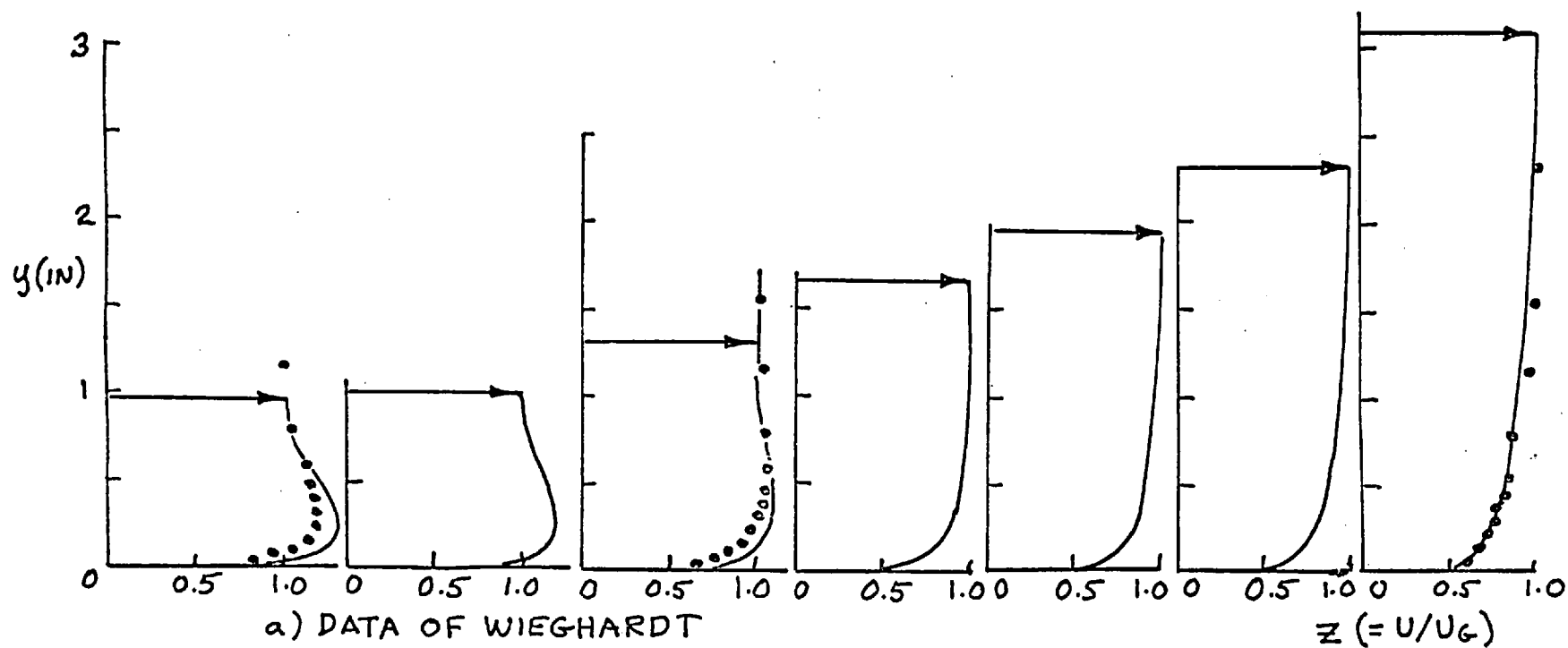


FIGURE 5.5.3-2 : COMPARISON OF EXPERIMENTAL VELOCITY PROFILES  
WITH REGENERATED PROFILES (METHOD II)

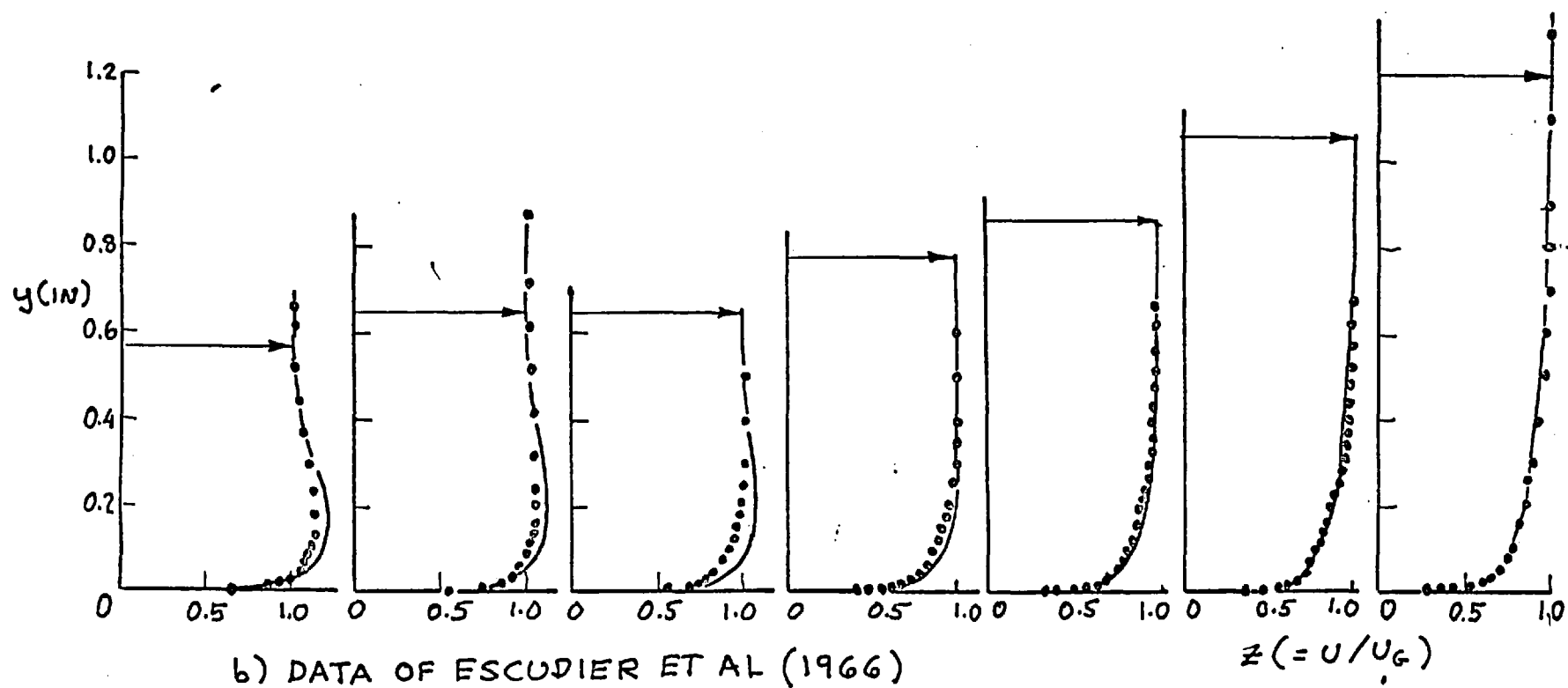


FIGURE 5.5.3-2: COMPARISON OF EXPERIMENTAL VELOCITY PROFILES WITH REGENERATED PROFILES (METHOD II)

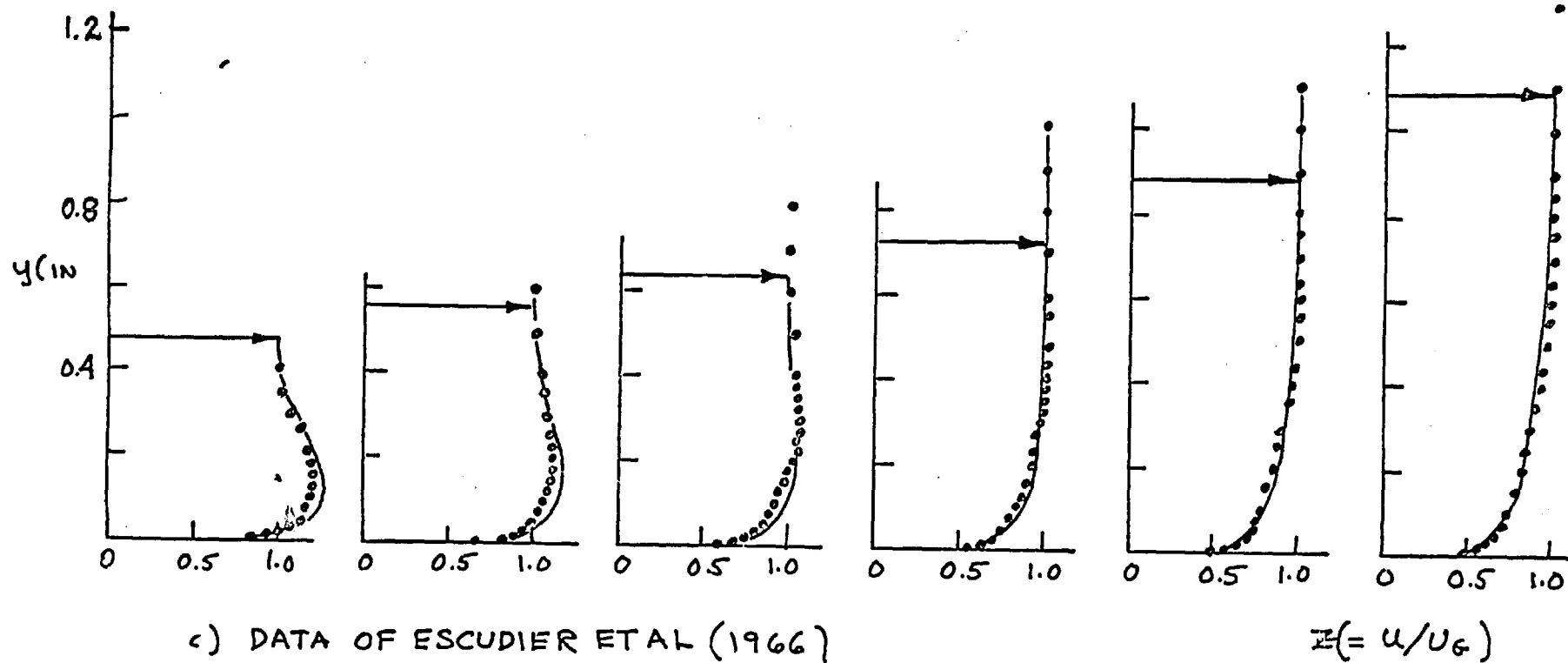


FIGURE 5.5.3-2: COMPARISON OF EXPERIMENTAL VELOCITY PROFILES WITH REGENERATED PROFILES (METHOD II)

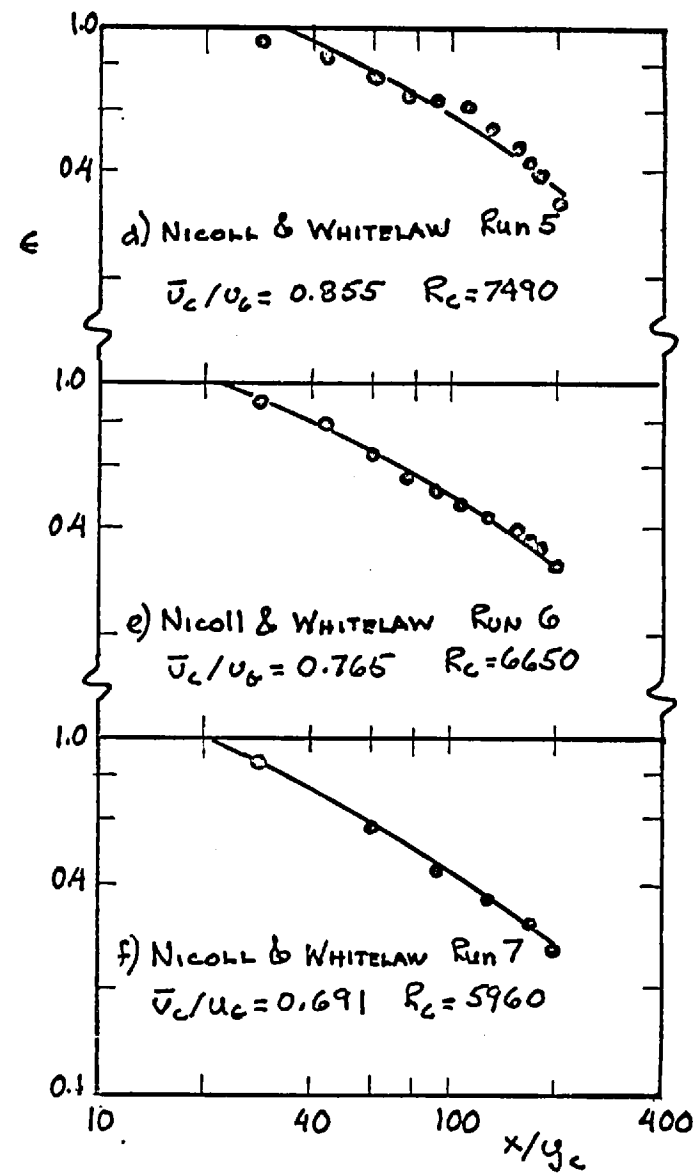
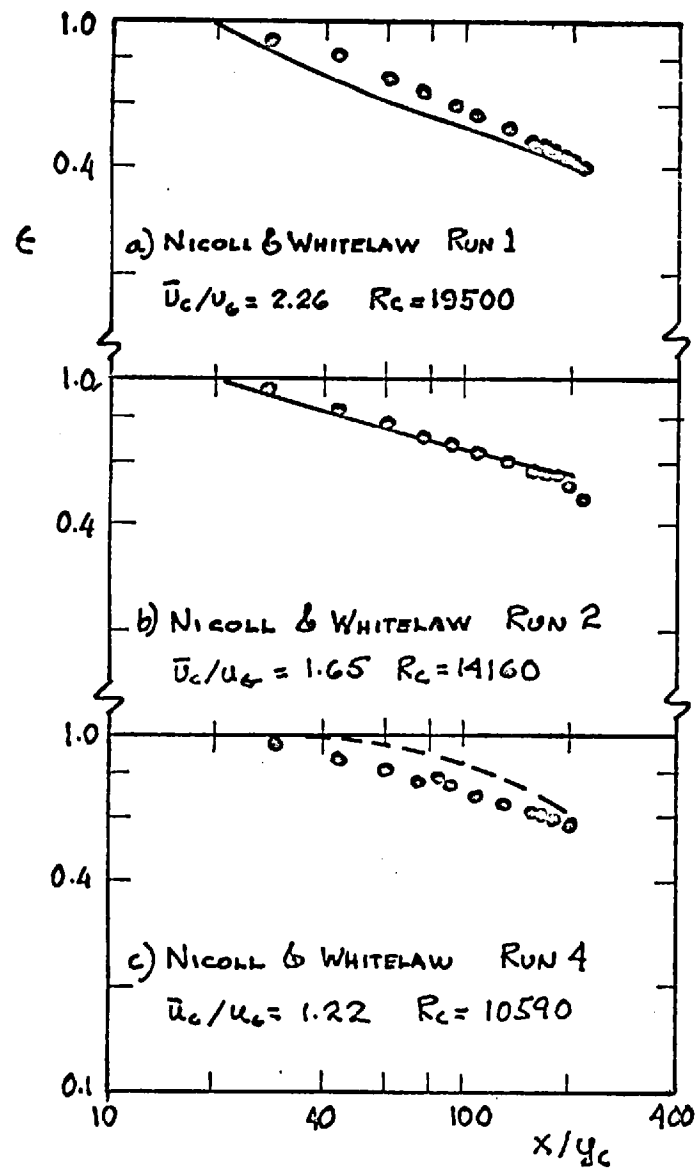


FIGURE 5.5.4-1: COMPARISON OF PREDICTED WITH EXPERIMENTAL EFFECTIVENESS

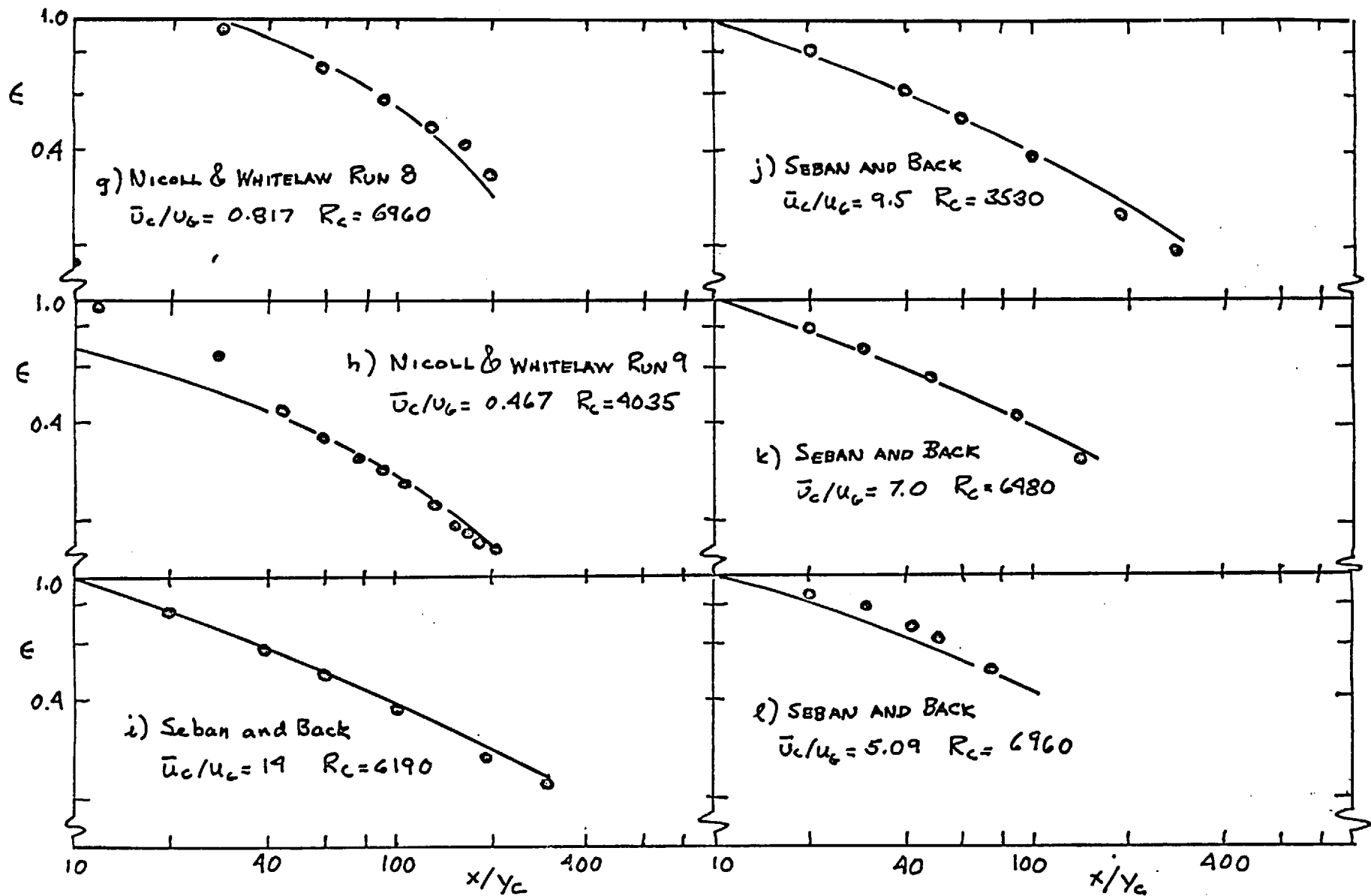


FIGURE 5.5.4-1: COMPARISON OF PREDICTED WITH EXPERIMENTAL EFFECTIVENESS

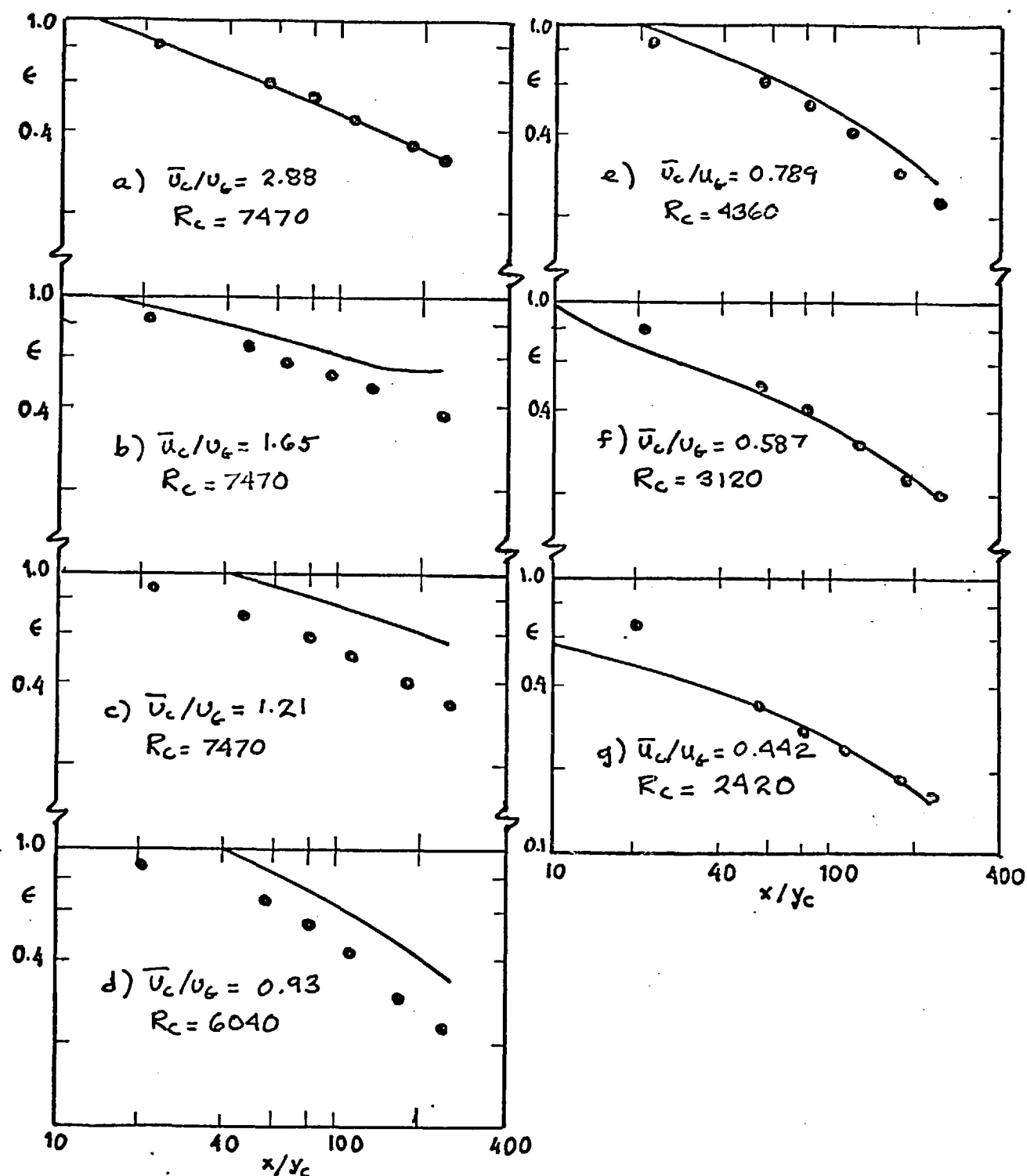


FIGURE 5.5.4-2: COMPARISON OF MEASURED AND PREDICTED VALUES OF EFFECTIVENESS. MEASUREMENTS OF CHIN ET AL (1958)

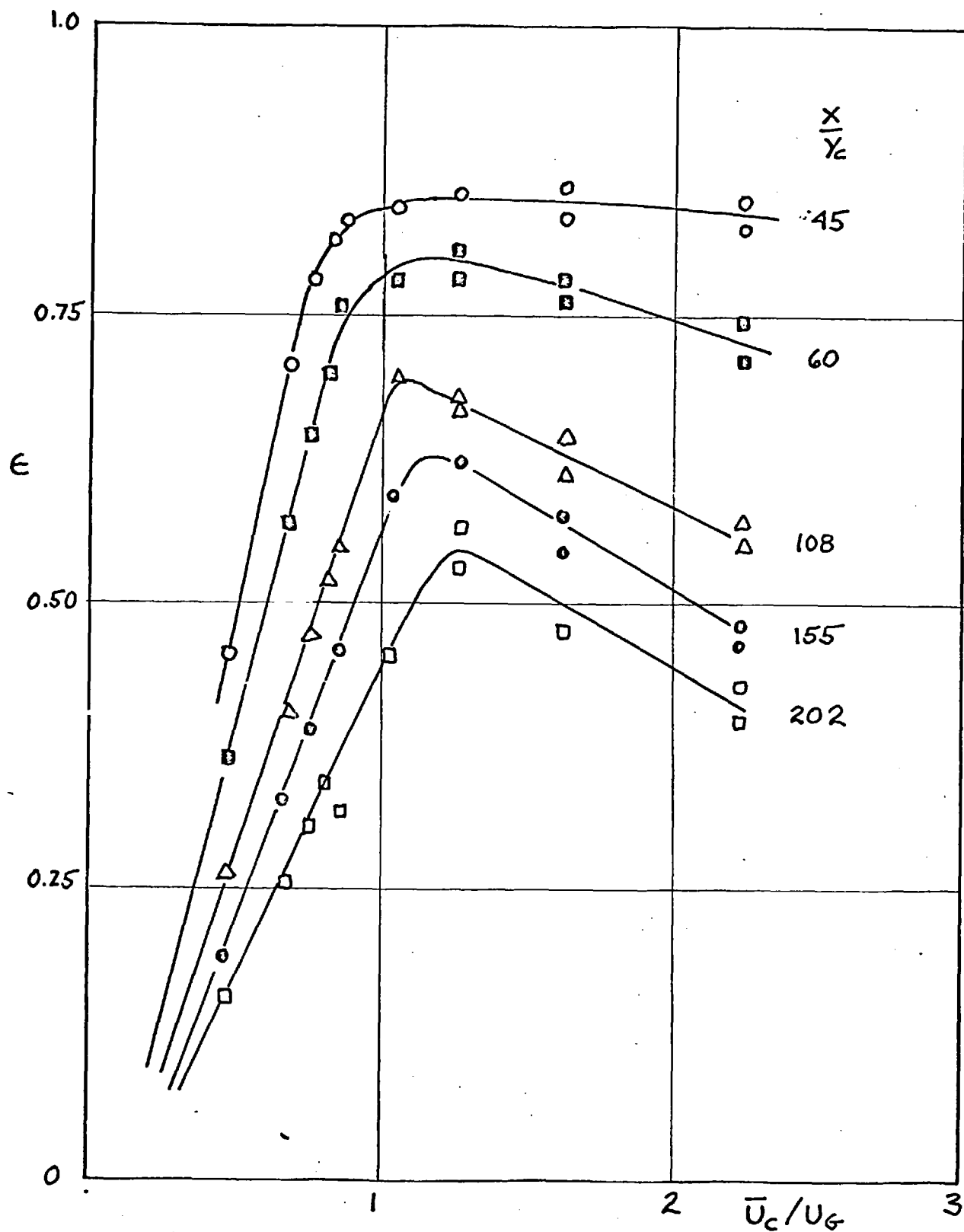
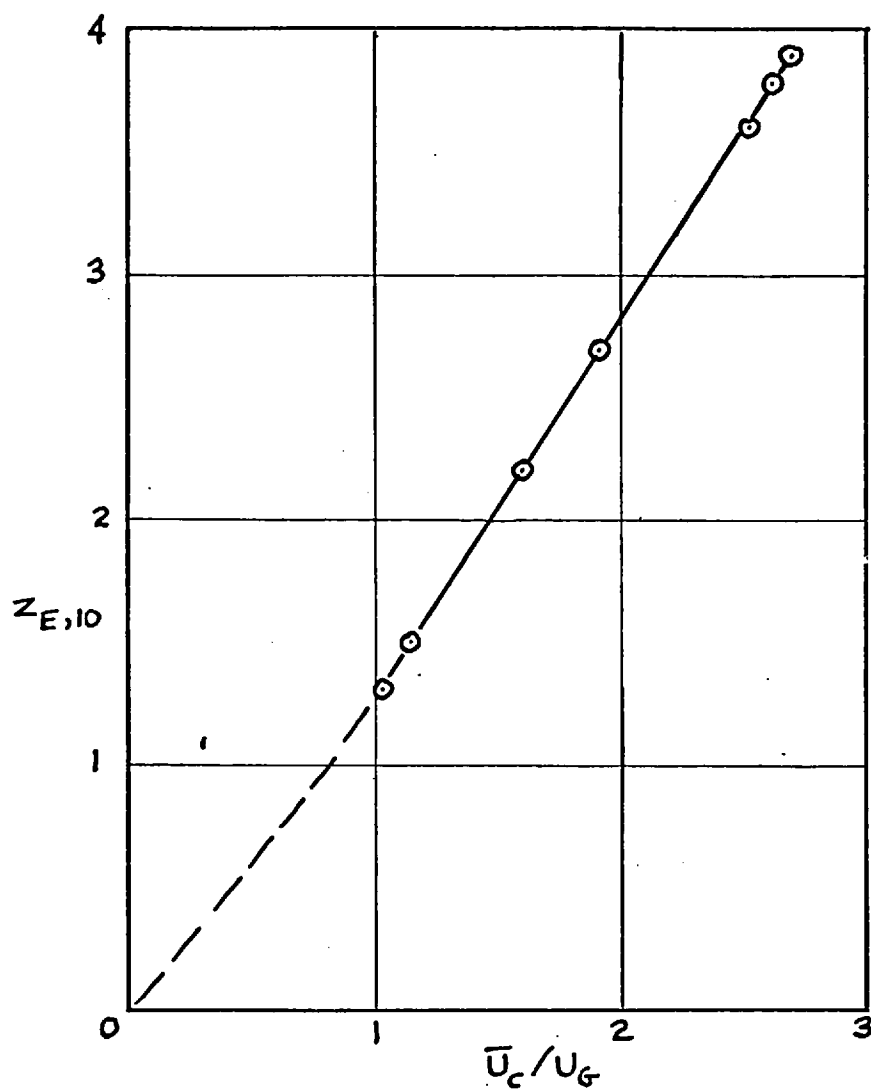


FIGURE 5.6-1: THE EFFECT OF  $\bar{U}_c/U_G$  ON THE EFFECTIVENESS AT FIXED POSITIONS

FIGURE 6.3-1: THE INITIAL VALUES OF  $Z_E$

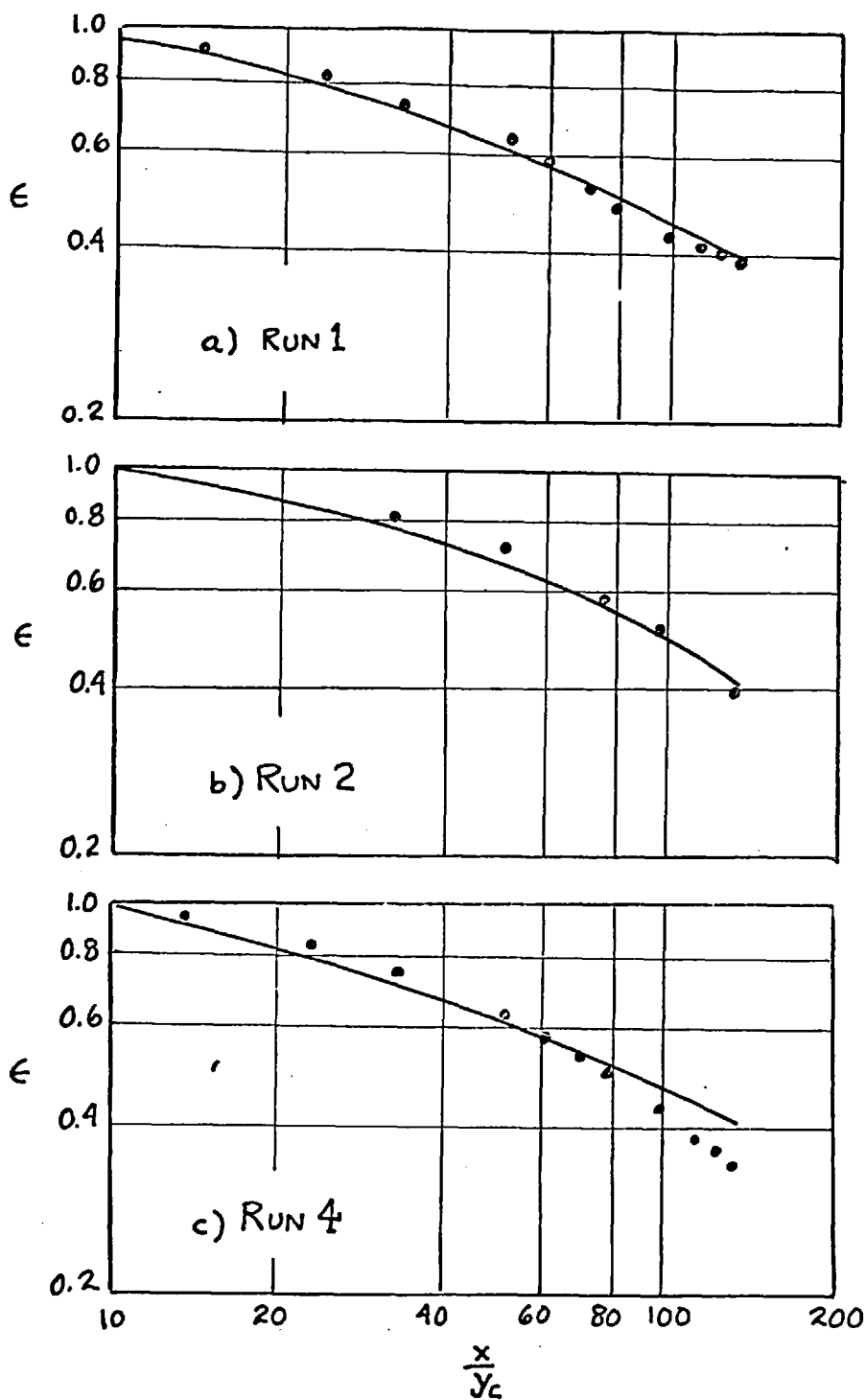


FIGURE 6.4-1: COMPARISON OF PREDICTED WITH EXPERIMENTAL EFFECTIVENESS FOR ARGON INJECTED INTO AIR. PRESENT MEASUREMENTS.

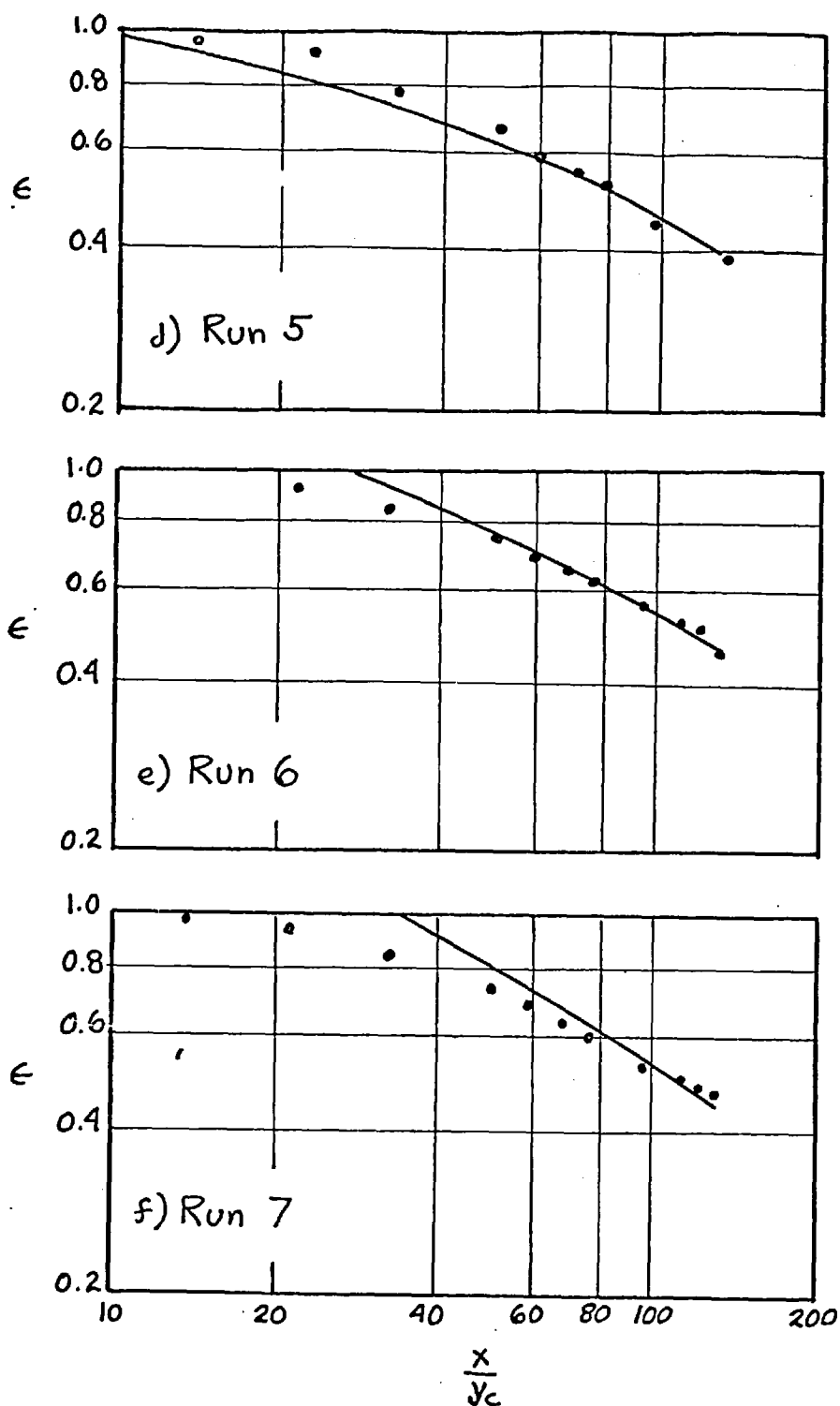


FIGURE 6.4-1: COMPARISON OF PREDICTED WITH EXPERIMENTAL EFFECTIVENESS DEVELOPMENT FOR ARGON INJECTED INTO AIR. PRESENT MEASUREMENTS

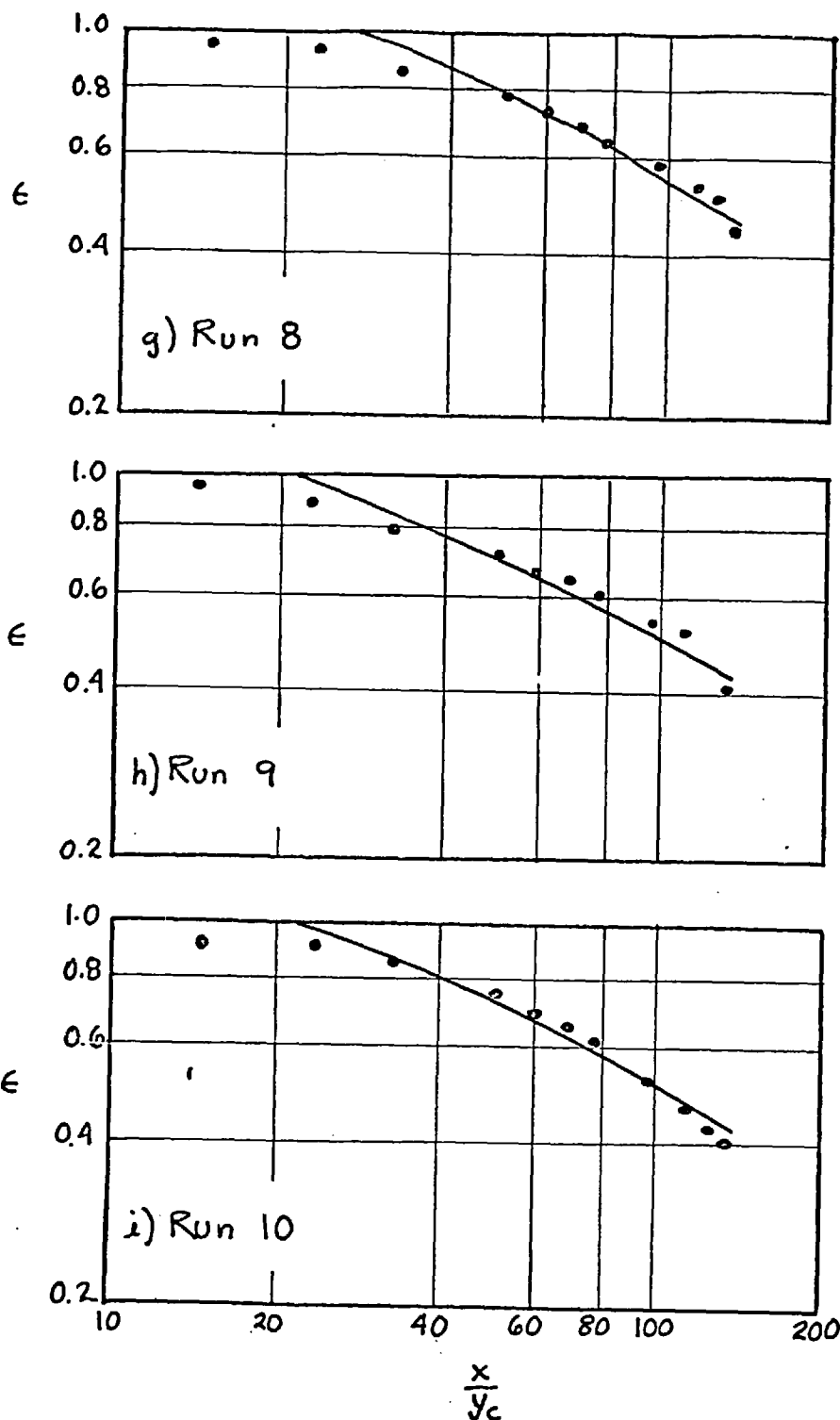


FIGURE 6.4-1: COMPARISON OF PREDICTED WITH EXPERIMENTAL EFFECTIVENESS DEVELOPMENT FOR ARGON INJECTED INTO AIR. PRESENT MEASUREMENTS

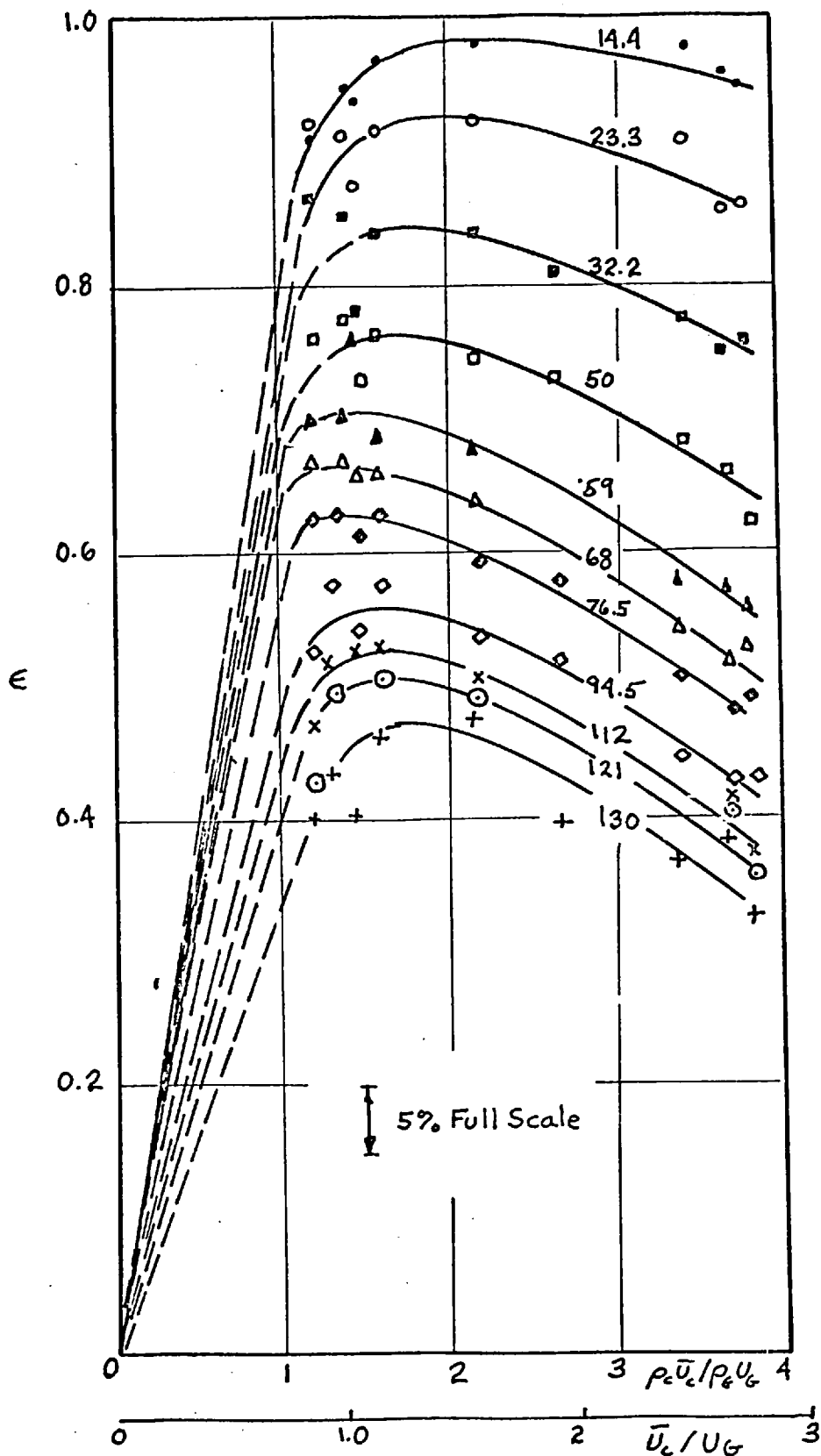


FIGURE 6.4-2: THE EFFECT OF  $\bar{U}_c/U_g$  ON THE EFFECTIVENESS AT FIXED POSITIONS FOR ARGON INJECTION INTO AIR

## APPENDIX A

A SUMMARY OF DRAG DATA FOR WALL JETS

## a) Wall jets with finite mainstream velocity

| Author(s)          | Date | Run | $-R_2$  | $H_{12}$ | $H_{32}$ | $\ln(ER_B s_s)$ | $10^3 s_s$ |
|--------------------|------|-----|---------|----------|----------|-----------------|------------|
| Kruka and Eskinazi | 1963 | 1   | 26597   | 0.492    | 3.058    |                 | 14.82      |
|                    |      |     | 25356   | 0.595    | 2.690    |                 | 7.80       |
|                    |      |     | 25474   | 0.626    | 2.556    |                 | 6.18       |
|                    |      |     | 24132   | 0.688    | 2.445    |                 | 4.85       |
|                    |      |     | 21383   | 0.741    | 2.339    |                 | 3.78       |
|                    |      | 2   | 129579  | 0.199    | 6.267    |                 | 102.0      |
|                    |      |     | 111372  | 0.322    | 4.236    |                 | 34.1       |
|                    |      |     | 103796  | 0.375    | 3.768    |                 | 23.2       |
|                    |      |     | 83722   | 0.453    | 3.265    |                 | 14.17      |
|                    |      | 3   | 573334  | 0.098    | 12.182   |                 | 506.5      |
|                    |      |     | 554735  | 0.131    | 9.312    |                 | 274.5      |
|                    |      |     | 543903  | 0.156    | 8.177    |                 | 205.0      |
|                    |      |     | 852441  | 0.121    | 10.104   |                 | 290.2      |
|                    |      |     | 1328295 | 0.096    | 12.510   |                 | 500.0      |
| Present Work       |      | 1   | 73180   | 0.283    | 4.659    | 9.82            | 45.0       |
|                    |      |     | 70056   | 0.326    | 4.205    | 10.15           | 36.0       |
|                    |      |     | 69320   | 0.362    | 3.873    | 10.29           | 32.0       |
|                    |      |     | 68030   | 0.396    | 3.633    | 10.52           | 26.0       |
|                    |      |     | 68951   | 0.422    | 3.446    | 10.58           | 22.0       |
|                    |      |     | 66860   | 0.470    | 3.182    | 10.64           | 17.5       |
|                    |      | 2   | 37938   | 0.414    | 3.429    | 10.56           | 18.5       |
|                    |      |     | 36350   | 0.459    | 3.196    | 10.41           | 14.5       |
|                    |      |     | 33630   | 0.508    | 2.991    | 10.29           | 13.5       |
|                    |      |     | 32840   | 0.550    | 2.839    | 10.31           | 11.5       |
|                    |      |     | 33680   | 0.580    | 2.736    | 10.45           | 10.0       |
|                    |      |     | 31059   | 0.639    | 2.568    | 10.58           | 8.0        |
|                    |      |     | 27018   | 0.691    | 2.444    | 10.30           | 6.5        |
|                    |      | 4   | 120632  | 0.194    | 6.442    | 10.12           | 105.0      |
|                    |      |     | 119347  | 0.225    | 5.706    | 10.41           | 85.0       |
|                    |      |     | 115209  | 0.261    | 5.055    | 10.41           | 68.0       |
|                    |      |     | 114531  | 0.285    | 4.717    | 10.52           | 58.0       |
|                    |      |     | 114429  | 0.306    | 4.454    | 10.61           | 50.0       |
|                    |      |     | 119209  | 0.340    | 4.086    | 10.78           | 38.0       |
|                    |      | 5   | 5099    | 0.546    | 2.825    | 8.64            | 15.0       |
|                    |      |     | 4766    | 0.616    | 2.603    | 8.71            | 11.7       |
|                    |      |     | 4218    | 0.677    | 2.457    | 9.01            | 9.5        |
|                    |      |     | 4581    | 0.707    | 2.396    | 9.21            | 8.5        |
|                    |      |     | 4224    | 0.736    | 2.334    | 9.18            | 7.3        |
|                    |      | 7   | 15246   | 0.460    | 3.168    | 9.71            | 18.0       |
|                    |      |     | 14045   | 0.515    | 2.943    | 9.83            | 14.5       |
|                    |      |     | 13328   | 0.571    | 2.752    | 9.57            | 12.0       |
|                    |      |     | 13691   | 0.602    | 2.652    | 9.69            | 10.0       |
|                    |      |     | 12039   | 0.644    | 2.549    | 9.89            | 9.0        |
|                    |      |     | 11200   | 0.701    | 2.419    | 9.94            | 7.0        |

## APPENDIX A

A SUMMARY OF DRAG DATA FOR WALL JETS

## b) Wall jets in still surroundings

| Author(s)                    | Date | $R_{\max}$ | $10^3 \sigma_s$ |
|------------------------------|------|------------|-----------------|
| Myers, Schauer<br>and Eustis | 1961 | 11500      | 2.91            |
|                              |      | 13300      | 2.98            |
|                              |      | 14800      | 2.62            |
|                              |      | 15100      | 2.97            |
|                              |      | 16000      | 2.97            |
|                              |      | 18600      | 2.58            |
|                              |      | 17600      | 2.70            |
|                              |      | 19000      | 2.60            |
|                              |      | 18700      | 3.08            |
|                              |      | 20600      | 2.66            |
|                              |      | 3760       | 3.70            |
|                              |      | 3600       | 3.90            |
|                              |      | 6050       | 3.61            |
|                              |      | 6420       | 3.49            |
|                              |      | 7900       | 3.82            |
|                              |      | 8900       | 3.64            |
|                              |      | 10300      | 3.66            |
|                              |      | 10700      | 3.55            |
|                              |      | 11700      | 3.36            |
|                              |      | 13900      | 3.33            |
|                              |      | 15800      | 3.24            |
|                              |      | 17500      | 3.13            |
|                              |      | 19700      | 2.95            |
|                              |      | 20400      | 3.09            |
|                              |      | 23400      | 3.10            |
|                              |      | 25000      | 2.82            |
| Bradshaw and<br>Gee          | 1962 | 2980       | 3.67            |
|                              |      | 3520       | 3.57            |
|                              |      | 4600       | 3.45            |
|                              |      | 5050       | 3.34            |
|                              |      | 5850       | 3.27            |
|                              |      | 6220       | 3.11            |
|                              |      | 6870       | 3.14            |
|                              |      | 7145       | 3.13            |
|                              |      | 7328       | 3.10            |
|                              |      | 7499       | 3.12            |
|                              |      | 8166       | 3.08            |
|                              |      | 7621       | 2.91            |
|                              |      | 9722       | 3.05            |
|                              |      | 11040      | 2.92            |
|                              |      | 12390      | 2.84            |
|                              |      | 12850      | 2.84            |
|                              |      | 13090      | 2.83            |
|                              |      | 13800      | 2.78            |

## APPENDIX B

HYDRODYNAMIC MEASUREMENTS

A schematic diagram of the wind tunnel used to obtain the present data for wall-jet hydrodynamic development is shown in Figure B-1; a more detailed drawing of the injection slot is shown in Figure B-2. The major components of the tunnel are: the test section, the transition and dilution section, a main centrifugal blower driven by a variable speed D.C. motor, the injected fluid plenum chamber, and a secondary centrifugal blower driven by a constant speed A.C. motor.

The main dimensions of the test section are:

|                           |       |
|---------------------------|-------|
| Internal height (in)      | 6     |
| Internal width (in)       | 6     |
| Total length (ft)         | 6     |
| Injection slot height(in) | 0.225 |

Length from entry fairing to injection slot (in): 6.5

The overall design represents a compromise between the conflicting requirements of a sufficiently large size to allow reasonably detailed velocity profile measurements, and a sufficiently small size to allow economic operation with injected gases other than air. Only the first 30 inches downstream from the injection slot were used for measurements; the additional length served to ensure maximum mixing between the injected gas and the mainstream (original plans included the use of town gas, as an injected fluid, which required dilution below the combustion limit before entry into the exhaust duct) and to increase the

damping between the test section and the main blower. For several of the tests an adjustable panel, as shown in Figure B-1 was fitted to the top of the test section to accelerate the mainstream flow. The mainstream flow rate was controlled by a sliding panel which, when moved across the test section exit, served to choke the flow.

The transition section, apart from its function of connecting the test section with the main blower, was designed to dilute the flow from the test section sufficiently to allow operation with town's gas and to allow sufficient flow through the main blower to ensure stable operation. The flow rate of the diluent air is controlled by a sliding panel at the entry.

The injection air passes through the secondary blower, through a diffuser and cascade into the plenum chamber, and thence through the injection slot, its flow rate was controlled by choking the inlet to the blower. The flow at the exit of the injection slot was two dimensional within the capacity of the instrumentation to detect deviations. The slot velocity profile was very nearly uniform, as shown in Figure B-3.

The static pressure measurements were obtained from variably spaced static pressure taps, of 0.020 inch diameter, along the bottom plate and one side plate of the test section which were connected to an inclined manometer. The total pressure measurements were obtained from a round impact probe of 0.032 inch outside diameter and 0.016 inch inside diameter.

This probe was supported by a traversing mechanism which allowed free movement along the axis of the test section, transverse motion within  $\pm 1.5$  inches of the test section center, and free vertical motion; the vertical position could be measured, with the aid of a vernier scale, to an accuracy of 0.001 inch.

Preliminary tests indicated that the flow over the central 3 inches was two dimensional to within 0.5%.

The test procedure was as follows. The main and secondary blowers were started and the various flows adjusted to the desired values. After a suitable warm up period the static pressures were recorded. Then, beginning at the slot exit, consecutive total pressure profiles were measured. Finally, after the final downstream total pressure profile was obtained, the total pressure profile at the slot exit and the static pressures were again recorded. These latter measurements served to check the long-term stability of the flow.

The data was then recorded on tape and the velocity profiles computed automatically, using a curve fitted to the measured static pressures. The resultant velocity profiles are tabulated in Table B-1.

TABLE B-1: Tabulated Velocity Profile Data: Run 1  
(U in feet/second; X and Y in inches)

| X = 2.25 |       | X = 3.375 |       | X = 4.50 |       | X = 6.75 |       | X = 9.0 |       | X = 11.25 |       | X = 16.875 |       | X = 22.5 |       |
|----------|-------|-----------|-------|----------|-------|----------|-------|---------|-------|-----------|-------|------------|-------|----------|-------|
| U        | Y     | U         | Y     | U        | Y     | U        | Y     | U       | Y     | U         | Y     | U          | Y     | U        | Y     |
| 160.60   | 0.016 | 154.18    | 0.016 | 145.54   | 0.016 | 137.32   | 0.016 | 123.28  | 0.016 | 113.62    | 0.016 | 93.62      | 0.016 | 81.47    | 0.018 |
| 166.81   | 0.024 | 159.34    | 0.024 | 156.86   | 0.025 | 143.28   | 0.025 | 127.52  | 0.024 | 115.11    | 0.024 | 97.59      | 0.026 | 84.49    | 0.026 |
| 179.92   | 0.034 | 170.85    | 0.034 | 163.16   | 0.035 | 148.23   | 0.035 | 133.35  | 0.034 | 119.65    | 0.034 | 102.35     | 0.036 | 89.04    | 0.036 |
| 190.14   | 0.044 | 179.32    | 0.044 | 169.67   | 0.045 | 153.15   | 0.045 | 138.11  | 0.044 | 125.39    | 0.044 | 107.25     | 0.046 | 90.63    | 0.046 |
| 197.45   | 0.054 | 184.60    | 0.054 | 174.09   | 0.055 | 156.10   | 0.055 | 142.18  | 0.054 | 128.38    | 0.054 | 107.78     | 0.056 | 95.26    | 0.066 |
| 201.45   | 0.064 | 189.83    | 0.064 | 178.83   | 0.065 | 157.92   | 0.065 | 142.58  | 0.064 | 130.59    | 0.064 | 111.25     | 0.066 | 97.49    | 0.086 |
| 206.39   | 0.074 | 196.76    | 0.084 | 181.16   | 0.075 | 160.31   | 0.075 | 144.96  | 0.074 | 131.75    | 0.074 | 110.39     | 0.076 | 100.21   | 0.106 |
| 206.62   | 0.084 | 198.54    | 0.104 | 183.45   | 0.085 | 161.73   | 0.085 | 147.56  | 0.084 | 132.89    | 0.084 | 112.10     | 0.086 | 104.07   | 0.126 |
| 207.22   | 0.094 | 197.00    | 0.124 | 184.79   | 0.095 | 162.90   | 0.095 | 148.08  | 0.094 | 134.88    | 0.094 | 113.78     | 0.096 | 101.63   | 0.146 |
| 207.04   | 0.104 | 189.93    | 0.144 | 185.00   | 0.105 | 163.48   | 0.105 | 148.21  | 0.104 | 136.15    | 0.104 | 114.61     | 0.106 | 105.61   | 0.166 |
| 206.58   | 0.114 | 183.57    | 0.164 | 185.00   | 0.115 | 163.01   | 0.115 | 148.72  | 0.114 | 135.45    | 0.114 | 114.61     | 0.116 | 105.95   | 0.186 |
| 205.28   | 0.124 | 176.33    | 0.184 | 183.66   | 0.125 | 162.90   | 0.125 | 148.08  | 0.124 | 136.56    | 0.124 | 115.44     | 0.126 | 106.79   | 0.211 |
| 202.91   | 0.134 | 166.34    | 0.204 | 180.21   | 0.145 | 161.49   | 0.145 | 148.21  | 0.144 | 137.53    | 0.134 | 115.44     | 0.136 | 105.10   | 0.236 |
| 201.17   | 0.144 | 157.54    | 0.224 | 174.75   | 0.165 | 160.31   | 0.165 | 148.33  | 0.164 | 136.56    | 0.144 | 116.75     | 0.146 | 105.10   | 0.261 |
| 195.95   | 0.154 | 147.31    | 0.244 | 168.54   | 0.185 | 156.10   | 0.185 | 146.27  | 0.184 | 136.70    | 0.154 | 116.26     | 0.156 | 105.95   | 0.286 |
| 193.36   | 0.164 | 138.53    | 0.264 | 161.40   | 0.205 | 152.16   | 0.210 | 143.91  | 0.209 | 134.88    | 0.164 | 117.88     | 0.166 | 105.10   | 0.336 |
| 188.78   | 0.174 | 129.74    | 0.284 | 155.15   | 0.225 | 147.85   | 0.235 | 141.38  | 0.234 | 134.60    | 0.174 | 117.88     | 0.176 | 103.90   | 0.386 |
| 183.47   | 0.184 | 120.47    | 0.304 | 147.88   | 0.245 | 143.41   | 0.260 | 137.15  | 0.259 | 134.88    | 0.184 | 116.26     | 0.186 | 103.03   | 0.436 |
| 176.39   | 0.194 | 112.31    | 0.324 | 140.90   | 0.265 | 136.90   | 0.285 | 132.92  | 0.284 | 133.61    | 0.209 | 116.59     | 0.211 | 98.77    | 0.486 |
| 171.59   | 0.204 | 102.39    | 0.344 | 133.13   | 0.285 | 132.38   | 0.310 | 127.08  | 0.334 | 130.59    | 0.234 | 116.26     | 0.236 | 96.20    | 0.536 |
| 166.41   | 0.214 | 93.66     | 0.364 | 124.26   | 0.310 | 126.51   | 0.335 | 120.94  | 0.384 | 129.86    | 0.259 | 115.44     | 0.261 | 96.20    | 0.586 |
| 160.48   | 0.224 | 88.01     | 0.384 | 114.88   | 0.335 | 123.00   | 0.360 | 111.11  | 0.434 | 129.12    | 0.284 | 115.11     | 0.286 | 93.36    | 0.686 |
| 153.21   | 0.234 | 78.66     | 0.404 | 109.63   | 0.360 | 115.51   | 0.385 | 105.32  | 0.484 | 122.78    | 0.334 | 113.78     | 0.336 | 88.43    | 0.786 |
| 146.35   | 0.244 | 72.36     | 0.424 | 99.83    | 0.385 | 105.71   | 0.435 | 89.93   | 0.584 | 120.75    | 0.384 | 109.53     | 0.386 | 86.38    | 0.886 |
| 140.32   | 0.254 | 67.75     | 0.444 | 83.90    | 0.435 | 94.71    | 0.485 | 76.20   | 0.684 | 113.45    | 0.434 | 105.10     | 0.486 | 84.28    | 0.986 |
| 133.51   | 0.264 | 60.96     | 0.464 | 70.88    | 0.485 | 86.09    | 0.535 | 64.59   | 0.784 | 108.31    | 0.484 | 99.90      | 0.586 | 76.48    | 1.086 |
| 126.57   | 0.274 | 56.08     | 0.484 | 60.77    | 0.535 | 75.75    | 0.585 | 59.38   | 0.884 | 102.53    | 0.534 | 94.22      | 0.686 | 72.88    | 1.186 |
| 118.82   | 0.284 | 53.66     | 0.584 | 53.44    | 0.585 | 66.10    | 0.635 | 50.74   | 0.984 | 97.21     | 0.584 | 87.31      | 0.786 | 67.78    | 1.286 |
| 111.22   | 0.294 | 51.12     | 0.524 | 50.52    | 0.635 | 60.39    | 0.635 | 49.22   | 1.084 | 87.97     | 0.684 | 81.69      | 0.886 | 63.69    | 1.386 |
| 106.06   | 0.304 | 50.37     | 0.544 | 49.37    | 0.685 | 53.73    | 0.735 | 48.44   | 1.184 | 77.63     | 0.784 | 75.15      | 0.986 | 60.81    | 1.486 |
| 102.41   | 0.314 | 49.61     | 0.564 |          |       | 50.82    | 0.785 |         |       | 69.90     | 0.884 | 70.45      | 1.086 | 54.60    | 1.586 |
| 94.79    | 0.324 | 49.61     | 0.584 |          |       | 50.06    | 0.835 |         |       | 58.02     | 0.984 | 61.20      | 1.186 | 51.21    | 1.686 |
| 89.32    | 0.334 |           |       |          |       |          |       |         |       | 53.94     | 1.084 | 57.02      | 1.286 | 49.43    | 1.786 |
| 76.23    | 0.359 |           |       |          |       |          |       |         |       | 50.29     | 1.184 | 55.33      | 1.386 | 49.07    | 1.886 |
| 64.76    | 0.384 |           |       |          |       |          |       |         |       | 48.37     | 1.284 | 49.91      | 1.486 |          |       |
| 55.43    | 0.409 |           |       |          |       |          |       |         |       | 47.97     | 1.384 | 48.37      | 1.586 |          |       |
| 49.83    | 0.434 |           |       |          |       |          |       |         |       |           |       | 46.77      | 1.686 |          |       |
| 49.83    | 0.484 |           |       |          |       |          |       |         |       |           |       | 46.77      | 1.786 |          |       |

**TABLE B-1: Tabulated Velocity Profile Data : Run 2**  
(U in feet/second: X and Y in inches)

| X = 2.25 |       | X = 3.375 |       | X = 4.50 |       | X = 6.75 |       | X = 9.00 |       | X = 11.25 |       | X = 16.875 |       | X = 22.5 |       |
|----------|-------|-----------|-------|----------|-------|----------|-------|----------|-------|-----------|-------|------------|-------|----------|-------|
| U        | Y     | U         | Y     | U        | Y     | U        | Y     | U        | Y     | U         | Y     | U          | Y     | U        | Y     |
| 165.37   | 0.016 | 156.19    | 0.016 | 151.51   | 0.016 | 144.20   | 0.016 | 133.97   | 0.016 | 123.18    | 0.016 | 107.19     | 0.016 | 95.20    | 0.016 |
| 179.23   | 0.028 | 161.28    | 0.023 | 158.46   | 0.024 | 146.79   | 0.022 | 142.03   | 0.029 | 132.18    | 0.029 | 113.85     | 0.032 | 100.17   | 0.030 |
| 189.36   | 0.038 | 173.52    | 0.033 | 167.61   | 0.034 | 154.30   | 0.032 | 147.25   | 0.039 | 136.20    | 0.039 | 120.89     | 0.052 | 107.27   | 0.050 |
| 197.81   | 0.048 | 180.02    | 0.043 | 174.02   | 0.044 | 159.91   | 0.042 | 150.51   | 0.049 | 138.42    | 0.049 | 126.70     | 0.072 | 113.61   | 0.070 |
| 203.27   | 0.058 | 187.83    | 0.053 | 179.71   | 0.054 | 164.45   | 0.052 | 153.23   | 0.059 | 143.39    | 0.059 | 129.91     | 0.092 | 116.27   | 0.090 |
| 207.47   | 0.068 | 193.02    | 0.063 | 185.03   | 0.064 | 168.01   | 0.062 | 156.13   | 0.069 | 145.38    | 0.069 | 131.56     | 0.112 | 116.73   | 0.110 |
| 209.62   | 0.078 | 199.16    | 0.073 | 188.87   | 0.074 | 170.45   | 0.072 | 157.16   | 0.079 | 147.47    | 0.079 | 134.53     | 0.132 | 120.51   | 0.130 |
| 210.31   | 0.088 | 201.75    | 0.083 | 192.26   | 0.084 | 172.96   | 0.082 | 159.54   | 0.089 | 148.56    | 0.089 | 134.53     | 0.152 | 121.26   | 0.150 |
| 210.82   | 0.098 | 204.32    | 0.093 | 193.47   | 0.094 | 173.59   | 0.092 | 160.44   | 0.099 | 149.16    | 0.099 | 133.33     | 0.177 | 123.46   | 0.175 |
| 210.39   | 0.108 | 205.02    | 0.103 | 195.14   | 0.104 | 174.10   | 0.102 | 161.11   | 0.109 | 151.31    | 0.109 | 134.67     | 0.202 | 124.90   | 0.200 |
| 210.05   | 0.118 | 204.93    | 0.113 | 194.95   | 0.114 | 174.82   | 0.112 | 162.11   | 0.119 | 151.67    | 0.119 | 134.00     | 0.227 | 124.47   | 0.225 |
| 206.69   | 0.138 | 203.44    | 0.123 | 194.49   | 0.124 | 175.03   | 0.122 | 161.67   | 0.129 | 150.72    | 0.129 | 133.33     | 0.252 | 124.18   | 0.250 |
| 200.69   | 0.158 | 201.31    | 0.133 | 190.29   | 0.144 | 175.03   | 0.132 | 161.11   | 0.139 | 151.08    | 0.139 | 133.33     | 0.277 | 123.46   | 0.275 |
| 191.35   | 0.178 | 198.61    | 0.143 | 185.22   | 0.166 | 174.31   | 0.142 | 160.33   | 0.149 | 150.96    | 0.159 | 131.97     | 0.302 | 121.26   | 0.300 |
| 180.23   | 0.198 | 195.15    | 0.153 | 178.71   | 0.184 | 173.69   | 0.152 | 160.22   | 0.159 | 151.67    | 0.179 | 127.82     | 0.327 | 120.51   | 0.350 |
| 167.09   | 0.218 | 191.05    | 0.163 | 171.94   | 0.204 | 170.24   | 0.177 | 159.77   | 0.169 | 149.28    | 0.199 | 127.82     | 0.352 | 116.73   | 0.400 |
| 155.86   | 0.238 | 182.89    | 0.173 | 164.47   | 0.224 | 164.77   | 0.202 | 158.64   | 0.179 | 149.28    | 0.224 | 127.82     | 0.377 | 115.18   | 0.500 |
| 139.57   | 0.258 | 187.74    | 0.173 | 158.35   | 0.244 | 159.57   | 0.227 | 158.07   | 0.189 | 146.24    | 0.249 | 124.98     | 0.402 | 110.40   | 0.600 |
| 127.60   | 0.278 | 184.55    | 0.183 | 150.44   | 0.264 | 155.00   | 0.252 | 156.36   | 0.199 | 143.14    | 0.274 | 123.54     | 0.452 | 108.76   | 0.700 |
| 114.07   | 0.298 | 179.42    | 0.193 | 143.35   | 0.284 | 148.01   | 0.277 | 155.21   | 0.224 | 139.97    | 0.299 | 120.59     | 0.502 | 101.94   | 0.800 |
| 102.29   | 0.318 | 174.65    | 0.203 | 135.75   | 0.304 | 143.82   | 0.302 | 151.46   | 0.249 | 134.07    | 0.349 | 115.26     | 0.602 | 96.51    | 0.900 |
| 93.48    | 0.338 | 163.94    | 0.223 | 125.01   | 0.329 | 131.29   | 0.352 | 147.49   | 0.274 | 127.90    | 0.399 | 107.19     | 0.702 | 94.64    | 1.000 |
| 85.03    | 0.358 | 155.85    | 0.243 | 117.45   | 0.354 | 119.84   | 0.402 | 142.54   | 0.299 | 119.93    | 0.499 | 100.26     | 0.802 | 88.76    | 1.100 |
| 79.80    | 0.378 | 146.34    | 0.263 | 109.05   | 0.379 | 98.09    | 0.502 | 135.83   | 0.349 | 108.94    | 0.599 | 88.87      | 0.902 | 86.71    | 1.200 |
| 77.05    | 0.398 | 136.17    | 0.283 | 101.37   | 0.404 | 83.87    | 0.602 | 127.93   | 0.399 | 94.84     | 0.699 | 83.66      | 1.002 | 81.37    | 1.300 |
| 75.64    | 0.448 | 128.01    | 0.303 | 89.31    | 0.454 | 78.56    | 0.702 | 111.26   | 0.499 | 86.94     | 0.799 |            |       | 80.26    | 1.400 |
| 76.11    | 0.498 | 116.24    | 0.328 | 81.09    | 0.504 | 78.11    | 0.802 | 96.38    | 0.599 | 81.17     | 0.899 | 84.73      | 1.102 | 78.68    | 1.500 |
| 77.05    | 0.598 | 104.87    | 0.353 | 76.53    | 0.604 | 78.11    | 0.902 | 83.61    | 0.699 | 78.47     | 0.999 | 80.38      | 1.202 | 78.22    | 1.600 |
| 77.51    | 0.698 | 94.79     | 0.378 | 77.69    | 0.804 |          |       | 80.32    | 0.799 | 78.01     | 1.099 | 79.71      | 1.302 | 77.99    | 1.700 |
| 77.98    | 0.798 | 88.12     | 0.403 | 78.16    | 0.904 |          |       | 78.06    | 0.899 |           |       | 77.88      | 1.402 |          |       |
| 78.21    | 0.898 | 79.55     | 0.453 |          |       |          |       | 78.06    | 0.999 |           |       |            |       |          |       |
|          |       | 76.79     | 0.503 |          |       |          |       |          |       |           |       |            |       |          |       |
|          |       | 76.79     | 0.603 |          |       |          |       |          |       |           |       |            |       |          |       |
|          |       | 77.72     | 0.703 |          |       |          |       |          |       |           |       |            |       |          |       |
|          |       | 77.95     | 0.803 |          |       |          |       |          |       |           |       |            |       |          |       |
|          |       | 78.18     | 0.903 |          |       |          |       |          |       |           |       |            |       |          |       |

TABLE B-1: Tabulated Velocity Profile Data : Run 3  
(U in feet/second: X and Y in inches)

| X = 2.25 |       | X = 3.375 |       | X = 4.50 |       | X = 6.75 |       | X = 9.0 |       | X = 11.25 |       |
|----------|-------|-----------|-------|----------|-------|----------|-------|---------|-------|-----------|-------|
| U        | Y     | U         | Y     | U        | Y     | U        | Y     | U       | Y     | U         | Y     |
| 154.73   | 0.016 | 148.28    | 0.016 | 140.05   | 0.016 | 123.25   | 0.016 | 106.81  | 0.016 | 95.60     | 0.016 |
| 169.03   | 0.029 | 152.89    | 0.021 | 145.26   | 0.024 | 130.28   | 0.034 | 110.79  | 0.022 | 95.98     | 0.021 |
| 177.53   | 0.039 | 159.78    | 0.031 | 151.11   | 0.034 | 134.85   | 0.044 | 113.55  | 0.032 | 103.55    | 0.041 |
| 186.25   | 0.049 | 165.64    | 0.041 | 155.63   | 0.044 | 138.92   | 0.064 | 117.73  | 0.042 | 108.11    | 0.061 |
| 190.84   | 0.059 | 171.51    | 0.051 | 159.09   | 0.054 | 140.61   | 0.084 | 121.15  | 0.062 | 110.72    | 0.081 |
| 194.38   | 0.069 | 175.51    | 0.061 | 161.02   | 0.064 | 141.13   | 0.104 | 124.67  | 0.082 | 112.27    | 0.101 |
| 196.41   | 0.079 | 178.45    | 0.071 | 163.54   | 0.074 | 141.13   | 0.124 | 125.86  | 0.102 | 112.82    | 0.121 |
| 196.72   | 0.099 | 180.24    | 0.081 | 164.90   | 0.084 | 140.43   | 0.144 | 125.47  | 0.127 | 113.26    | 0.141 |
| 193.23   | 0.119 | 181.68    | 0.091 | 165.05   | 0.094 | 138.84   | 0.164 | 124.87  | 0.152 | 112.93    | 0.161 |
| 187.90   | 0.199 | 181.47    | 0.101 | 165.20   | 0.104 | 136.04   | 0.184 | 123.98  | 0.177 | 113.04    | 0.181 |
| 179.75   | 0.159 | 179.28    | 0.121 | 164.22   | 0.114 | 134.30   | 0.204 | 122.47  | 0.202 | 112.71    | 0.201 |
| 170.56   | 0.179 | 175.44    | 0.141 | 163.09   | 0.124 | 130.94   | 0.229 | 119.40  | 0.252 | 110.04    | 0.226 |
| 161.31   | 0.199 | 170.21    | 0.161 | 162.33   | 0.144 | 127.49   | 0.254 | 114.31  | 0.302 | 109.48    | 0.251 |
| 146.84   | 0.224 | 162.16    | 0.181 | 157.45   | 0.164 | 123.55   | 0.279 | 109.11  | 0.352 | 108.91    | 0.276 |
| 131.83   | 0.249 | 153.45    | 0.201 | 153.14   | 0.184 | 119.26   | 0.304 | 102.67  | 0.402 | 106.62    | 0.301 |
| 116.89   | 0.274 | 143.62    | 0.226 | 147.79   | 0.204 | 114.82   | 0.329 | 97.73   | 0.452 | 103.07    | 0.351 |
| 103.28   | 0.299 | 135.36    | 0.251 | 141.28   | 0.229 | 110.98   | 0.354 | 91.18   | 0.502 | 99.40     | 0.401 |
| 89.26    | 0.324 | 124.90    | 0.276 | 134.64   | 0.254 | 107.01   | 0.379 | 85.58   | 0.552 | 95.08     | 0.451 |
| 75.92    | 0.349 | 115.74    | 0.301 | 128.24   | 0.279 | 103.24   | 0.404 | 80.35   | 0.602 | 90.54     | 0.501 |
| 64.46    | 0.374 | 105.67    | 0.326 | 120.98   | 0.304 | 94.08    | 0.454 | 67.82   | 0.702 | 86.77     | 0.551 |
| 52.61    | 0.399 | 94.80     | 0.351 | 105.92   | 0.354 | 85.09    | 0.504 | 58.19   | 0.802 | 83.57     | 0.601 |
| 43.32    | 0.424 | 86.90     | 0.376 | 91.63    | 0.404 | 77.48    | 0.554 | 47.40   | 0.902 | 75.31     | 0.701 |
| 32.93    | 0.449 | 78.20     | 0.401 | 78.06    | 0.454 | 69.39    | 0.604 | 35.45   | 1.002 | 65.65     | 0.801 |
| 19.13    | 0.499 | 70.54     | 0.426 | 64.33    | 0.504 | 52.76    | 0.704 | 26.21   | 1.102 | 56.53     | 0.901 |
| 12.95    | 0.599 | 62.93     | 0.451 | 53.15    | 0.554 | 39.32    | 0.804 | 17.06   | 1.202 | 46.96     | 1.001 |
| 12.95    | 0.699 | 52.88     | 0.476 | 43.98    | 0.604 | 24.59    | 0.904 | 8.25    | 1.302 | 38.89     | 1.101 |
| 12.95    | 0.799 | 46.40     | 0.501 | 25.42    | 0.704 | 16.82    | 1.004 | 8.25    | 1.402 | 30.30     | 1.201 |
| 13.88    | 0.899 | 33.36     | 0.551 | 12.29    | 0.804 | 9.21     | 1.104 |         |       | 22.84     | 1.301 |
|          |       | 23.83     | 0.601 | 10.08    | 0.904 | 9.21     | 1.204 |         |       | 14.99     | 1.401 |
|          |       | 11.06     | 0.701 | 11.24    | 1.004 |          |       |         |       | 7.16      | 1.501 |
|          |       | 12.13     | 0.801 | 11.24    | 1.104 |          |       |         |       | 5.15      | 1.601 |
|          |       | 12.13     | 0.901 | 12.29    | 1.204 |          |       |         |       |           |       |
|          |       | 13.11     | 1.001 |          |       |          |       |         |       |           |       |

TABLE B-1: Tabulated Velocity Profile Data: Run 4  
(U in feet/second; X and Y in inches)

| X = 2.25 |       | X = 4.50 |       | X = 6.75 |       | X = 9.0 |       | X = 11.25 |       | X = 16.875 |       | X = 22.5 |       |
|----------|-------|----------|-------|----------|-------|---------|-------|-----------|-------|------------|-------|----------|-------|
| U        | Y     | U        | Y     | U        | Y     | U       | Y     | U         | Y     | U          | Y     | U        | Y     |
| 157.51   | 0.016 | 143.75   | 0.016 | 129.32   | 0.016 | 113.96  | 0.016 | 104.26    | 0.016 | 85.03      | 0.016 | 73.23    | 0.016 |
| 171.68   | 0.022 | 153.97   | 0.024 | 136.20   | 0.025 | 118.22  | 0.021 | 108.55    | 0.021 | 87.96      | 0.023 | 77.27    | 0.030 |
| 181.93   | 0.032 | 159.47   | 0.034 | 141.49   | 0.035 | 124.18  | 0.031 | 111.66    | 0.031 | 92.86      | 0.033 | 80.80    | 0.040 |
| 190.04   | 0.042 | 162.46   | 0.044 | 145.20   | 0.045 | 128.59  | 0.041 | 115.45    | 0.041 | 94.22      | 0.043 | 83.57    | 0.065 |
| 195.30   | 0.052 | 171.35   | 0.069 | 149.30   | 0.070 | 133.24  | 0.066 | 120.61    | 0.066 | 101.35     | 0.068 | 87.72    | 0.090 |
| 197.24   | 0.062 | 174.93   | 0.094 | 150.94   | 0.095 | 135.12  | 0.091 | 123.72    | 0.091 | 103.20     | 0.093 | 88.58    | 0.140 |
| 199.28   | 0.072 | 174.42   | 0.119 | 151.36   | 0.120 | 136.06  | 0.116 | 124.74    | 0.116 | 105.03     | 0.118 | 90.98    | 0.190 |
| 201.05   | 0.082 | 170.23   | 0.144 | 149.93   | 0.145 | 135.59  | 0.141 | 126.25    | 0.141 | 105.63     | 0.143 | 92.36    | 0.240 |
| 201.05   | 0.042 | 164.55   | 0.169 | 146.51   | 0.170 | 135.12  | 0.166 | 124.23    | 0.166 | 107.42     | 0.168 | 93.04    | 0.265 |
| 201.05   | 0.092 | 157.31   | 0.194 | 144.33   | 0.195 | 133.24  | 0.191 | 122.69    | 0.191 | 104.43     | 0.193 | 90.28    | 0.290 |
| 199.28   | 0.117 | 152.15   | 0.219 | 140.77   | 0.220 | 131.32  | 0.216 | 121.13    | 0.241 | 105.63     | 0.243 | 91.67    | 0.315 |
| 191.37   | 0.142 | 143.40   | 0.244 | 136.38   | 0.245 | 128.39  | 0.241 | 117.41    | 0.291 | 104.43     | 0.293 | 92.36    | 0.340 |
| 183.11   | 0.167 | 136.33   | 0.269 | 128.54   | 0.295 | 124.18  | 0.291 | 113.01    | 0.341 | 100.09     | 0.343 | 92.36    | 0.365 |
| 168.32   | 0.192 | 126.20   | 0.319 | 120.39   | 0.345 | 117.57  | 0.341 | 108.44    | 0.391 | 100.09     | 0.393 | 92.36    | 0.390 |
| 157.83   | 0.217 | 110.34   | 0.344 | 109.36   | 0.395 | 112.05  | 0.391 | 103.66    | 0.441 | 94.22      | 0.493 | 88.86    | 0.440 |
| 139.23   | 0.242 | 103.83   | 0.369 | 99.15    | 0.445 | 106.25  | 0.441 | 98.00     | 0.491 | 91.49      | 0.593 | 88.15    | 0.490 |
| 123.50   | 0.267 | 95.30    | 0.394 | 89.19    | 0.495 | 99.47   | 0.491 | 94.71     | 0.541 | 81.99      | 0.693 | 84.48    | 0.540 |
| 109.93   | 0.292 | 81.39    | 0.444 | 79.90    | 0.545 | 90.81   | 0.541 | 89.91     | 0.591 | 79.64      | 0.793 | 85.97    | 0.590 |
| 96.16    | 0.317 | 68.17    | 0.494 | 71.88    | 0.595 | 85.79   | 0.591 | 79.43     | 0.691 | 77.22      | 0.893 | 80.64    | 0.690 |
| 79.57    | 0.342 | 57.03    | 0.544 | 63.05    | 0.645 | 78.06   | 0.641 | 71.01     | 0.791 | 69.44      | 0.993 | 79.05    | 0.790 |
| 66.56    | 0.367 | 43.69    | 0.594 | 55.80    | 0.695 | 69.47   | 0.691 | 60.40     | 0.891 | 63.73      | 1.093 | 68.77    | 0.890 |
| 57.36    | 0.392 | 33.14    | 0.694 | 48.25    | 0.745 | 59.23   | 0.791 | 53.02     | 0.991 | 59.62      | 1.193 | 68.77    | 0.990 |
| 38.93    | 0.442 | 32.36    | 0.794 | 42.68    | 0.795 | 50.44   | 0.891 | 44.73     | 1.091 | 51.65      | 1.293 | 65.95    | 1.090 |
| 32.54    | 0.492 | 31.57    | 0.894 | 35.55    | 0.895 | 40.71   | 0.991 | 39.30     | 1.191 | 45.10      | 1.393 | 61.99    | 1.190 |
| 32.15    | 0.592 |          |       | 31.78    | 0.995 | 35.74   | 1.091 | 35.22     | 1.291 | 43.68      | 1.493 | 60.95    | 1.290 |
| 32.15    | 0.792 |          |       |          |       | 32.00   | 1.191 | 31.41     | 1.391 | 40.67      | 1.593 | 54.36    | 1.390 |
|          |       |          |       |          |       | 31.19   | 1.291 | 30.59     | 1.491 | 35.69      | 1.693 | 51.98    | 1.490 |
|          |       |          |       |          |       |         |       |           |       | 33.12      | 1.793 | 46.85    | 1.590 |
|          |       |          |       |          |       |         |       |           |       | 30.73      | 1.893 | 45.48    | 1.690 |
|          |       |          |       |          |       |         |       |           |       | 29.90      | 1.993 | 44.06    | 1.790 |
|          |       |          |       |          |       |         |       |           |       | 29.04      | 2.093 | 35.46    | 1.990 |
|          |       |          |       |          |       |         |       |           |       |            |       | 32.47    | 2.125 |

TABLE B-1: Tabulated Velocity Profile Data: Run 5  
(U in feet/second; X and Y in inches)

| X = 2.25 |       | X = 4.50 |       | X = 6.75 |       | X = 9.00 |       | X = 11.25 |       | X = 16.875 |       | X = 22.5 |       |
|----------|-------|----------|-------|----------|-------|----------|-------|-----------|-------|------------|-------|----------|-------|
| U        | Y     | U        | Y     | U        | Y     | U        | Y     | U         | Y     | U          | Y     | U        | Y     |
| 34.91    | 0.016 | 31.85    | 0.016 | 29.08    | 0.016 | 26.54    | 0.016 | 23.91     | 0.016 | 22.06      | 0.016 | 18.89    | 0.016 |
| 41.37    | 0.031 | 34.67    | 0.024 | 30.58    | 0.024 | 28.96    | 0.025 | 26.06     | 0.022 | 23.63      | 0.024 | 21.13    | 0.024 |
| 45.78    | 0.051 | 37.05    | 0.034 | 33.39    | 0.034 | 30.47    | 0.035 | 27.40     | 0.032 | 24.92      | 0.034 | 22.16    | 0.034 |
| 47.32    | 0.071 | 39.39    | 0.054 | 34.31    | 0.044 | 31.76    | 0.045 | 29.60     | 0.042 | 25.81      | 0.044 | 23.73    | 0.044 |
| 47.79    | 0.091 | 40.51    | 0.074 | 34.83    | 0.054 | 33.14    | 0.070 | 30.79     | 0.067 | 27.49      | 0.069 | 25.01    | 0.069 |
| 47.61    | 0.116 | 41.06    | 0.094 | 35.84    | 0.074 |          |       | 32.07     | 0.092 | 28.76      | 0.094 | 26.07    | 0.094 |
| 46.66    | 0.141 | 41.60    | 0.119 | 37.07    | 0.094 | 34.21    | 0.095 | 32.76     | 0.117 | 29.38      | 0.119 | 27.08    | 0.169 |
| 44.90    | 0.166 | 41.60    | 0.144 | 37.31    | 0.119 | 34.60    | 0.120 | 33.31     | 0.142 | 30.57      | 0.144 | 27.73    | 0.144 |
| 42.43    | 0.191 | 41.06    | 0.169 | 37.55    | 0.144 | 35.37    | 0.145 | 33.84     | 0.167 | 30.57      | 0.169 | 28.37    | 0.169 |
| 39.82    | 0.216 | 40.51    | 0.194 | 38.03    | 0.169 | 35.62    | 0.170 | 34.10     | 0.192 | 31.30      | 0.194 | 29.15    | 0.194 |
| 36.90    | 0.241 | 39.62    | 0.219 | 37.67    | 0.194 | 35.87    | 0.195 | 34.10     | 0.217 | 31.30      | 0.219 | 29.91    | 0.244 |
| 31.96    | 0.291 | 38.24    | 0.244 | 37.31    | 0.219 | 35.87    | 0.220 | 34.10     | 0.242 | 31.30      | 0.244 | 30.65    | 0.294 |
| 27.60    | 0.341 | 35.44    | 0.294 | 36.95    | 0.244 | 35.74    | 0.245 | 34.23     | 0.267 | 31.30      | 0.269 | 30.65    | 0.344 |
| 25.04    | 0.391 | 32.40    | 0.344 | 36.71    | 0.269 | 35.37    | 0.270 | 34.23     | 0.292 | 31.30      | 0.294 | 30.35    | 0.394 |
| 24.13    | 0.441 | 30.41    | 0.394 | 35.84    | 0.294 | 34.86    | 0.295 | 33.57     | 0.342 | 32.00      | 0.344 | 30.35    | 0.444 |
| 23.76    | 0.491 | 27.63    | 0.444 | 34.05    | 0.344 | 34.60    | 0.345 | 32.76     | 0.392 | 31.30      | 0.394 | 30.35    | 0.494 |
| 23.38    | 0.591 | 25.79    | 0.494 | 33.25    | 0.394 | 33.55    | 0.395 | 32.35     | 0.442 | 30.86      | 0.444 | 29.45    | 0.594 |
| 23.38    | 0.691 | 24.72    | 0.544 | 29.24    | 0.494 | 31.05    | 0.495 | 32.21     | 0.492 | 30.86      | 0.494 | 29.15    | 0.694 |
| 23.76    | 0.791 | 23.80    | 0.594 | 27.34    | 0.594 | 28.33    | 0.595 | 29.75     | 0.592 | 30.57      | 0.594 | 28.84    | 0.794 |
| 23.76    | 0.891 | 23.61    | 0.694 | 24.58    | 0.694 | 26.71    | 0.695 | 27.73     | 0.692 | 30.28      | 0.694 | 28.05    | 0.894 |
|          |       | 23.61    | 0.794 | 24.02    | 0.794 | 25.15    | 0.795 | 26.57     | 0.792 | 29.07      | 0.794 | 27.89    | 0.994 |
|          |       | 23.80    | 0.894 | 23.84    | 0.894 | 24.43    | 0.895 | 24.37     | 0.892 | 27.49      | 0.894 | 27.08    | 1.094 |
|          |       | 23.80    | 0.994 | 23.65    | 0.994 | 24.06    | 0.995 | 25.01     | 0.992 | 27.16      | 0.994 | 26.58    | 1.194 |
|          |       |          |       | 23.84    | 1.094 | 23.87    | 1.095 | 24.28     | 1.092 | 26.66      | 1.094 | 25.89    | 1.294 |
|          |       |          |       | 23.84    | 1.194 | 23.87    | 1.295 | 23.91     | 1.192 | 25.81      | 1.194 | 25.89    | 1.394 |
|          |       |          |       |          |       |          |       | 23.91     | 1.292 | 25.81      | 1.294 | 25.19    | 1.494 |
|          |       |          |       |          |       |          |       |           |       | 25.10      | 1.394 | 24.47    | 1.594 |
|          |       |          |       |          |       |          |       |           |       | 24.38      | 1.494 | 24.10    | 1.694 |
|          |       |          |       |          |       |          |       |           |       | 24.01      | 1.594 | 24.10    | 1.794 |

**TABLE B-1: Tabulated Velocity Profile Data: Run 7**  
(U in feet/second; X and Y in inches)

| X = 2.25 |       | X = 4.50 |       | X = 6.75 |       | X = .00 |       | X = 11.25 |       | X = 16.875 |       | X = 22.5 |       |
|----------|-------|----------|-------|----------|-------|---------|-------|-----------|-------|------------|-------|----------|-------|
| U        | Y     | U        | Y     | U        | Y     | U       | Y     | U         | Y     | U          | Y     | U        | Y     |
| 79.00    | 0.016 | 70.99    | 0.016 | 66.34    | 0.016 | 59.48   | 0.016 | 54.74     | 0.016 | 45.83      | 0.016 | 42.45    | 0.016 |
| 85.19    | 0.021 | 76.61    | 0.024 | 68.13    | 0.021 | 62.12   | 0.022 | 58.06     | 0.021 | 49.50      | 0.024 | 43.49    | 0.021 |
| 89.78    | 0.031 | 79.02    | 0.034 | 70.70    | 0.031 | 65.34   | 0.032 | 59.95     | 0.031 | 51.71      | 0.034 | 45.21    | 0.031 |
| 92.52    | 0.041 | 81.13    | 0.044 | 73.30    | 0.041 | 68.80   | 0.052 | 62.50     | 0.041 | 54.82      | 0.054 | 48.37    | 0.051 |
| 94.90    | 0.051 | 84.31    | 0.054 | 75.46    | 0.051 | 71.47   | 0.072 | 63.63     | 0.051 | 57.06      | 0.074 | 51.07    | 0.071 |
| 97.31    | 0.061 | 87.84    | 0.074 | 78.01    | 0.071 | 73.19   | 0.092 | 66.51     | 0.071 | 59.14      | 0.094 | 53.05    | 0.091 |
| 98.22    | 0.071 | 89.35    | 0.094 | 79.71    | 0.091 | 74.28   | 0.112 | 68.29     | 0.091 | 60.27      | 0.114 | 56.08    | 0.141 |
| 99.58    | 0.091 | 89.80    | 0.114 | 80.49    | 0.111 | 74.82   | 0.132 | 70.23     | 0.141 | 61.22      | 0.134 | 57.66    | 0.191 |
| 99.13    | 0.111 | 89.60    | 0.134 | 80.49    | 0.131 | 74.82   | 0.152 | 70.29     | 0.191 | 61.22      | 0.154 | 58.58    | 0.241 |
| 96.53    | 0.141 | 88.24    | 0.154 | 80.27    | 0.151 | 74.40   | 0.172 | 69.78     | 0.241 | 62.46      | 0.174 | 58.58    | 0.291 |
| 88.08    | 0.191 | 86.04    | 0.174 | 79.76    | 0.171 | 74.10   | 0.192 | 67.64     | 0.291 | 63.31      | 0.194 | 58.81    | 0.341 |
| 74.35    | 0.241 | 83.52    | 0.194 | 78.47    | 0.191 | 72.09   | 0.242 | 65.49     | 0.341 | 62.81      | 0.244 | 58.28    | 0.391 |
| 61.04    | 0.291 | 76.26    | 0.244 | 74.98    | 0.241 | 69.76   | 0.292 | 63.42     | 0.391 | 62.60      | 0.294 | 57.66    | 0.491 |
| 49.69    | 0.341 | 68.63    | 0.294 | 70.19    | 0.291 | 66.15   | 0.342 | 60.69     | 0.441 | 61.37      | 0.344 | 55.92    | 0.591 |
| 42.43    | 0.391 | 60.48    | 0.344 | 65.86    | 0.341 | 63.40   | 0.392 | 58.67     | 0.491 | 60.49      | 0.394 | 53.80    | 0.691 |
| 40.60    | 0.491 | 53.10    | 0.394 | 61.38    | 0.391 | 57.11   | 0.492 | 53.50     | 0.591 | 59.52      | 0.444 | 51.33    | 0.791 |
| 40.82    | 0.591 | 44.11    | 0.494 | 52.19    | 0.491 | 50.48   | 0.592 | 48.98     | 0.691 | 58.53      | 0.494 | 50.18    | 0.891 |
| 41.25    | 0.691 | 41.29    | 0.594 | 45.82    | 0.591 | 45.56   | 0.692 | 46.36     | 0.791 | 55.63      | 0.594 | 48.83    | 0.991 |
| 41.36    | 0.791 | 41.18    | 0.694 | 42.49    | 0.691 | 42.73   | 0.792 | 43.17     | 0.891 | 51.53      | 0.694 | 46.96    | 1.091 |
| 41.36    | 0.891 | 40.96    | 0.794 | 41.42    | 0.791 | 41.78   | 0.892 | 42.02     | 0.991 | 49.95      | 0.794 | 44.00    | 1.191 |
| 41.36    | 0.991 | 41.18    | 0.894 | 41.32    | 0.891 | 41.46   | 0.992 | 41.81     | 1.091 | 46.80      | 0.894 | 43.49    | 1.291 |
| 41.47    | 1.091 | 41.29    | 0.994 | 40.66    | 0.991 | 41.35   | 1.092 | 41.49     | 1.191 | 45.24      | 0.994 | 42.87    | 1.391 |
|          |       | 41.29    | 1.094 | 41.42    | 1.091 | 41.35   | 1.192 | 41.27     | 1.291 | 44.14      | 1.094 | 41.70    | 1.491 |
|          |       | 41.29    | 1.194 | 41.42    | 1.191 | 41.46   | 1.292 | 41.38     | 1.391 | 42.38      | 1.194 | 43.49    | 1.591 |
|          |       | 41.50    | 1.294 | 41.42    | 1.291 | 41.24   | 1.392 | 41.38     | 1.491 | 41.52      | 1.294 | 40.84    | 1.691 |
|          |       |          |       | 41.42    | 1.391 | 41.35   | 1.492 | 41.38     | 1.591 | 40.76      | 1.394 | 40.62    | 1.791 |
|          |       |          |       |          |       | 41.35   | 1.592 | 41.49     | 1.691 | 40.54      | 1.494 | 40.51    | 1.891 |
|          |       |          |       |          |       | 41.46   | 1.692 |           |       | 40.43      | 1.594 | 40.51    | 1.991 |
|          |       |          |       |          |       | 41.46   | 1.792 |           |       | 40.43      | 1.694 |          |       |
|          |       |          |       |          |       |         |       |           |       | 40.43      | 1.794 |          |       |

**TABLE B-1: Tabulated Velocity Profile Data; Run 8**  
(U in feet/second; X and Y in inches)

| X = 2.25 |       | X = 4.50 |       | X = 6.75 |       | X = 9.0 |       | X = 11.25 |       | X = 16.875 |       | X = 22.5 |       |
|----------|-------|----------|-------|----------|-------|---------|-------|-----------|-------|------------|-------|----------|-------|
| U        | Y     | U        | Y     | U        | Y     | U       | Y     | U         | Y     | U          | Y     | U        | Y     |
| 81.69    | 0.016 | 72.06    | 0.016 | 64.84    | 0.016 | 57.88   | 0.016 | 51.73     | 0.016 | 43.81      | 0.016 | 37.85    | 0.016 |
| 86.19    | 0.022 | 75.88    | 0.025 | 67.63    | 0.024 | 58.66   | 0.021 | 53.78     | 0.023 | 46.89      | 0.024 | 39.94    | 0.021 |
| 91.65    | 0.032 | 79.87    | 0.035 | 70.36    | 0.034 | 62.96   | 0.031 | 56.64     | 0.033 | 48.31      | 0.034 | 42.14    | 0.031 |
| 93.21    | 0.042 | 83.18    | 0.055 | 71.88    | 0.044 | 64.86   | 0.041 | 58.60     | 0.043 | 50.86      | 0.054 | 44.93    | 0.051 |
| 96.43    | 0.052 | 85.78    | 0.075 | 73.31    | 0.054 | 66.03   | 0.051 | 60.12     | 0.053 | 51.91      | 0.074 | 46.22    | 0.071 |
| 99.19    | 0.072 | 86.57    | 0.095 | 75.42    | 0.074 | 68.38   | 0.071 | 62.32     | 0.073 | 55.03      | 0.094 | 48.32    | 0.091 |
| 99.87    | 0.092 | 86.57    | 0.115 | 76.43    | 0.094 | 69.23   | 0.091 | 64.46     | 0.093 | 56.88      | 0.144 | 51.91    | 0.191 |
| 99.19    | 0.112 | 85.63    | 0.135 | 77.19    | 0.114 | 70.71   | 0.141 | 65.84     | 0.143 | 57.67      | 0.194 | 52.26    | 0.241 |
| 96.53    | 0.132 | 84.68    | 0.155 | 77.19    | 0.134 | 70.20   | 0.191 | 65.50     | 0.193 | 57.28      | 0.244 | 52.09    | 0.291 |
| 93.83    | 0.152 | 82.47    | 0.175 | 76.61    | 0.154 | 68.25   | 0.241 | 65.16     | 0.243 | 57.28      | 0.294 | 52.09    | 0.341 |
| 89.97    | 0.172 | 80.09    | 0.195 | 75.54    | 0.174 | 66.24   | 0.291 | 63.90     | 0.293 | 56.88      | 0.344 | 51.57    | 0.391 |
| 85.72    | 0.192 | 73.35    | 0.245 | 74.64    | 0.194 | 63.10   | 0.341 | 61.08     | 0.343 | 55.92      | 0.394 | 49.70    | 0.491 |
| 72.17    | 0.242 | 67.15    | 0.295 | 72.38    | 0.244 | 60.55   | 0.391 | 59.97     | 0.393 | 54.29      | 0.494 | 49.06    | 0.591 |
| 58.16    | 0.292 | 58.72    | 0.345 | 67.23    | 0.294 | 53.68   | 0.491 | 55.43     | 0.493 | 42.08      | 0.594 | 47.94    | 0.691 |
| 43.89    | 0.342 | 52.40    | 0.395 | 59.05    | 0.394 | 47.82   | 0.591 | 51.21     | 0.593 | 48.87      | 0.694 | 45.92    | 0.791 |
| 37.83    | 0.392 | 40.61    | 0.495 | 50.94    | 0.494 | 42.32   | 0.691 | 46.60     | 0.693 | 47.37      | 0.794 | 44.22    | 0.891 |
| 30.31    | 0.492 | 32.63    | 0.595 | 42.47    | 0.594 | 36.97   | 0.791 | 41.48     | 0.793 | 42.87      | 0.894 | 42.14    | 0.991 |
| 30.16    | 0.592 | 30.49    | 0.695 | 35.81    | 0.694 | 34.45   | 0.891 | 38.67     | 0.893 | 41.27      | 0.994 | 41.05    | 1.091 |
| 30.16    | 0.692 | 30.05    | 0.795 | 31.96    | 0.794 | 31.72   | 0.991 | 35.12     | 0.993 | 39.02      | 1.094 | 39.94    | 1.191 |
| 30.16    | 0.792 | 30.05    | 0.895 | 30.67    | 0.894 | 30.56   | 1.091 | 33.01     | 1.093 |            |       | 36.89    | 1.291 |
|          |       |          |       | 30.08    | 0.994 | 30.27   | 1.191 | 31.89     | 1.193 |            |       | 36.52    | 1.391 |
|          |       |          |       | 29.93    | 1.094 | 30.27   | 1.291 | 31.32     | 1.293 |            |       | 33.83    | 1.491 |
|          |       |          |       |          |       |         |       | 30.45     | 1.393 |            |       | 32.74    | 1.591 |
|          |       |          |       |          |       |         |       | 30.15     | 1.493 |            |       | 32.47    | 1.691 |
|          |       |          |       |          |       |         |       |           |       |            |       | 31.34    | 1.791 |
|          |       |          |       |          |       |         |       |           |       |            |       | 30.61    | 1.891 |
|          |       |          |       |          |       |         |       |           |       |            |       | 30.16    | 1.991 |
|          |       |          |       |          |       |         |       |           |       |            |       | 29.86    | 2.091 |
|          |       |          |       |          |       |         |       |           |       |            |       | 29.71    | 2.121 |

**TABLE B-1: Tabulated Velocity Profile Data: Run 9**  
(U in feet/second; X and Y in inches)

| X = 2.25 |       | X = 4.50 |       | X = 3.75 |       | X = 9.00 |       | X = 11.25 |       | X = 16.875 |       | X = 22.50 |       |
|----------|-------|----------|-------|----------|-------|----------|-------|-----------|-------|------------|-------|-----------|-------|
| U        | Y     | U        | Y     | U        | Y     | U        | Y     | U         | Y     | U          | Y     | U         | Y     |
| 79.96    | 0.016 | 71.35    | 0.016 | 59.77    | 0.016 | 54.12    | 0.016 | 48.56     | 0.016 | 38.72      | 0.016 | 33.44     | 0.016 |
| 84.78    | 0.021 | 75.41    | 0.024 | 64.56    | 0.023 | 56.81    | 0.021 | 51.71     | 0.021 | 42.28      | 0.024 | 35.53     | 0.012 |
| 88.84    | 0.031 | 77.77    | 0.034 | 67.23    | 0.033 | 58.91    | 0.031 | 53.60     | 0.031 | 43.85      | 0.034 | 39.16     | 0.041 |
| 94.40    | 0.051 | 80.06    | 0.044 | 68.76    | 0.043 | 61.83    | 0.051 | 55.90     | 0.051 | 45.47      | 0.054 | 43.00     | 0.091 |
| 96.58    | 0.071 | 81.62    | 0.054 | 70.38    | 0.053 | 63.70    | 0.071 | 57.73     | 0.071 | 48.73      | 0.074 | 45.55     | 0.141 |
| 97.04    | 0.091 | 82.94    | 0.074 | 71.78    | 0.073 | 65.24    | 0.091 | 58.89     | 0.091 | 49.65      | 0.094 | 46.34     | 0.191 |
| 96.25    | 0.111 | 83.69    | 0.094 | 73.09    | 0.093 | 65.38    | 0.111 | 60.55     | 0.111 | 51.17      | 0.114 | 47.49     | 0.241 |
| 94.11    | 0.131 | 83.48    | 0.114 | 72.90    | 0.113 | 65.86    | 0.131 | 60.70     | 0.131 | 51.61      | 0.134 | 47.21     | 0.291 |
| 91.44    | 0.151 | 82.55    | 0.134 | 72.78    | 0.133 | 65.79    | 0.151 | 61.15     | 0.151 | 51.78      | 0.154 | 46.73     | 0.341 |
| 87.66    | 0.171 | 81.29    | 0.154 | 72.28    | 0.153 | 65.51    | 0.171 | 61.15     | 0.151 | 51.78      | 0.174 | 46.53     | 0.391 |
| 83.55    | 0.191 | 79.77    | 0.174 | 71.65    | 0.173 | 65.38    | 0.191 | 61.15     | 0.171 | 51.61      | 0.194 | 46.24     | 0.491 |
| 71.69    | 0.241 | 76.89    | 0.194 | 71.02    | 0.193 | 63.70    | 0.241 | 61.15     | 0.191 | 51.52      | 0.244 | 43.73     | 0.591 |
| 59.89    | 0.291 | 71.86    | 0.244 | 67.90    | 0.243 | 61.39    | 0.291 | 60.10     | 0.241 | 51.26      | 0.294 | 42.48     | 0.691 |
| 49.03    | 0.341 | 65.41    | 0.294 | 64.00    | 0.293 | 56.97    | 0.391 | 59.65     | 0.291 | 50.72      | 0.344 | 41.40     | 0.791 |
| 38.98    | 0.391 | 58.57    | 0.344 | 61.19    | 0.343 | 51.30    | 0.491 | 55.90     | 0.391 | 50.55      | 0.394 | 40.96     | 0.891 |
| 24.65    | 0.491 | 52.90    | 0.394 | 56.51    | 0.393 | 46.11    | 0.591 | 52.58     | 0.491 | 49.01      | 0.494 | 37.99     | 0.991 |
| 20.89    | 0.591 | 39.41    | 0.494 | 48.61    | 0.493 | 40.59    | 0.691 | 47.62     | 0.591 | 47.03      | 0.594 | 35.79     | 1.091 |
| 20.67    | 0.691 | 30.05    | 0.594 | 40.95    | 0.593 | 34.45    | 0.791 | 44.18     | 0.691 | 44.37      | 0.694 | 34.50     | 1.191 |
| 20.89    | 0.791 | 23.48    | 0.694 | 34.10    | 0.693 | 29.67    | 0.891 | 39.54     | 0.791 | 42.60      | 0.794 | 33.71     | 1.291 |
|          |       | 21.47    | 0.794 | 27.99    | 0.793 | 26.45    | 0.991 | 35.71     | 0.891 | 39.75      | 0.894 | 30.48     | 1.391 |
|          |       | 20.61    | 0.894 | 23.43    | 0.893 | 22.79    | 1.091 | 31.55     | 0.991 | 37.53      | 0.994 | 30.03     | 1.491 |
|          |       | 20.61    | 0.994 | 22.04    | 0.993 | 21.14    | 1.191 | 28.38     | 1.091 | 34.40      | 1.094 | 28.17     | 1.591 |
|          |       |          |       | 20.99    | 1.093 | 20.71    | 1.291 | 26.58     | 1.191 | 31.95      | 1.194 | 27.19     | 1.691 |
|          |       |          |       | 20.33    | 1.193 | 20.05    | 1.391 | 22.93     | 1.291 | 30.20      | 1.294 | 25.12     | 1.791 |
|          |       |          |       |          |       | 20.05    | 1.491 | 21.51     | 1.391 | 29.29      | 1.394 | 24.39     | 1.891 |
|          |       |          |       |          |       |          |       | 20.87     | 1.491 | 25.68      | 1.494 | 23.25     | 1.991 |
|          |       |          |       |          |       |          |       | 20.43     | 1.591 | 24.79      | 1.594 | 22.06     | 2.091 |
|          |       |          |       |          |       |          |       | 19.99     | 1.691 | 22.50      | 1.694 | 21.64     | 2.121 |
|          |       |          |       |          |       |          |       | 19.76     | 1.791 | 21.26      | 1.794 |           |       |
|          |       |          |       |          |       |          |       |           |       | 20.17      | 1.894 |           |       |
|          |       |          |       |          |       |          |       |           |       | 19.72      | 1.994 |           |       |
|          |       |          |       |          |       |          |       |           |       | 19.26      | 2.094 |           |       |
|          |       |          |       |          |       |          |       |           |       | 19.02      | 2.124 |           |       |

TABLE B-1: Tabulated Velocity Profile Data: Run 10  
(U in feet/second; X and Y in inches)

| X = 2.25 |       | X = 4.50 |       | X = 6.75 |       | X = 9.00 |       | X = 11.25 |       | X = 16.875 |       | X = 22.5 |       |
|----------|-------|----------|-------|----------|-------|----------|-------|-----------|-------|------------|-------|----------|-------|
| U        | Y     | U        | Y     | U        | Y     | U        | Y     | U         | Y     | U          | Y     | U        | Y     |
| 81.06    | 0.016 | 68.06    | 0.016 | 56.43    | 0.016 | 49.45    | 0.016 | 43.01     | 0.016 | 33.78      | 0.013 | 27.85    | 0.016 |
| 83.92    | 0.021 | 71.82    | 0.023 | 57.94    | 0.022 | 51.07    | 0.021 | 44.57     | 0.021 | 35.34      | 0.023 | 29.42    | 0.021 |
| 89.21    | 0.031 | 73.82    | 0.033 | 61.22    | 0.032 | 53.16    | 0.031 | 48.75     | 0.041 | 38.03      | 0.033 | 31.34    | 0.031 |
| 91.12    | 0.041 | 76.42    | 0.053 | 63.90    | 0.052 | 56.15    | 0.051 | 53.03     | 0.091 | 40.77      | 0.053 | 33.69    | 0.051 |
| 92.90    | 0.051 | 77.53    | 0.073 | 65.86    | 0.072 | 58.13    | 0.071 | 54.04     | 0.141 | 42.07      | 0.073 | 35.26    | 0.071 |
| 94.49    | 0.061 | 78.18    | 0.093 | 66.54    | 0.092 | 59.29    | 0.091 | 54.46     | 0.191 | 42.92      | 0.093 | 35.64    | 0.091 |
| 95.35    | 0.071 | 78.00    | 0.113 | 67.22    | 0.112 | 59.67    | 0.111 | 52.94     | 0.241 | 44.66      | 0.143 | 37.48    | 0.141 |
| 95.45    | 0.081 | 77.18    | 0.133 | 66.88    | 0.132 | 60.05    | 0.131 | 52.34     | 0.291 | 45.46      | 0.193 | 38.66    | 0.191 |
| 95.45    | 0.091 | 76.12    | 0.153 | 66.41    | 0.152 | 59.67    | 0.151 | 51.02     | 0.341 | 45.16      | 0.243 | 38.66    | 0.241 |
| 94.73    | 0.111 | 74.61    | 0.173 | 65.86    | 0.172 | 59.60    | 0.171 | 49.94     | 0.391 | 44.66      | 0.293 | 39.12    | 0.291 |
| 92.16    | 0.131 | 72.45    | 0.193 | 65.16    | 0.192 | 59.29    | 0.191 | 47.33     | 0.491 | 44.36      | 0.343 | 38.66    | 0.391 |
| 89.11    | 0.151 | 67.66    | 0.243 | 63.04    | 0.242 | 57.74    | 0.241 | 43.33     | 0.591 | 43.95      | 0.393 | 37.72    | 0.491 |
| 85.68    | 0.171 | 62.65    | 0.293 | 59.72    | 0.292 | 56.23    | 0.291 | 39.15     | 0.691 | 42.29      | 0.493 | 36.26    | 0.591 |
| 80.78    | 0.191 | 57.04    | 0.343 | 56.76    | 0.342 | 54.51    | 0.341 | 35.88     | 0.791 | 40.99      | 0.593 | 35.64    | 0.691 |
| 70.07    | 0.241 | 51.17    | 0.393 | 53.72    | 0.392 | 51.95    | 0.391 | 32.57     | 0.891 | 38.03      | 0.693 | 33.69    | 0.791 |
| 57.87    | 0.291 | 45.24    | 0.443 | 46.47    | 0.492 | 47.38    | 0.491 | 28.40     | 0.991 | 36.34      | 0.793 | 31.91    | 0.891 |
| 45.60    | 0.341 | 39.91    | 0.493 | 39.74    | 0.592 | 42.65    | 0.591 | 24.64     | 1.091 | 34.05      | 0.893 | 30.47    | 0.991 |
| 35.28    | 0.391 | 29.91    | 0.593 | 32.72    | 0.692 | 38.28    | 0.691 | 21.70     | 1.191 | 31.72      | 0.993 | 29.11    | 1.091 |
| 25.05    | 0.441 | 20.34    | 0.693 | 26.61    | 0.792 | 32.65    | 0.791 | 18.30     | 1.291 | 29.06      | 1.093 | 27.85    | 1.191 |
| 16.81    | 0.491 | 12.99    | 0.793 | 20.44    | 0.892 | 28.17    | 0.891 | 15.62     | 1.391 | 27.64      | 1.193 | 24.78    | 1.291 |
| 7.47     | 0.591 | 8.83     | 0.893 | 15.37    | 0.992 | 24.74    | 0.991 | 13.10     | 1.491 | 25.26      | 1.293 | 23.48    | 1.391 |
| 6.84     | 0.691 | 7.12     | 0.993 | 11.68    | 1.092 | 19.63    | 1.091 | 10.39     | 1.591 | 22.63      | 1.393 | 22.50    | 1.491 |
| 6.84     | 0.791 | 7.12     | 1.093 | 7.40     | 1.192 | 16.35    | 1.191 | 7.92      | 1.691 | 19.89      | 1.493 | 20.84    | 1.591 |
| 7.47     | 0.891 |          |       | 6.76     | 1.292 | 12.59    | 1.291 | 6.67      | 1.791 | 18.24      | 1.593 | 18.07    | 1.691 |
|          |       |          |       | 6.76     | 1.392 | 9.27     | 1.391 | 5.95      | 1.891 | 16.15      | 1.693 | 16.24    | 1.791 |
|          |       |          |       |          |       | 8.23     | 1.491 | 5.95      | 1.991 | 14.06      | 1.793 | 14.79    | 1.891 |
|          |       |          |       |          |       | 7.04     | 1.591 |           |       | 12.36      | 1.893 | 12.84    | 1.991 |
|          |       |          |       |          |       | 6.37     | 1.691 |           |       | 9.00       | 1.993 | 10.95    | 2.091 |
|          |       |          |       |          |       | 6.37     | 1.791 |           |       | 6.71       | 2.093 | 10.10    | 2.121 |
|          |       |          |       |          |       |          |       |           |       | 6.71       | 2.123 |          |       |

**TABLE B-1: Tabulated Velocity Profile Data: Run 11**  
(U in feet second; X and Y in inches)

| X = 2.25 |       | X = 4.50 |       | X = 6.75 |       | X = 9.00 |       | X = 11.25 |       |
|----------|-------|----------|-------|----------|-------|----------|-------|-----------|-------|
| U        | Y     | U        | Y     | U        | Y     | U        | Y     | U         | Y     |
| 101.15   | 0.016 | 88.01    | 0.016 | 87.33    | 0.016 | 81.80    | 0.016 | 76.81     | 0.016 |
| 106.37   | 0.021 | 91.99    | 0.023 | 91.73    | 0.021 | 83.80    | 0.023 | 77.68     | 0.021 |
| 111.38   | 0.031 | 97.89    | 0.033 | 94.94    | 0.031 | 88.98    | 0.033 | 82.43     | 0.031 |
| 120.16   | 0.051 | 108.52   | 0.053 | 101.68   | 0.051 | 95.49    | 0.053 | 88.35     | 0.051 |
| 125.91   | 0.071 | 116.36   | 0.073 | 106.21   | 0.071 | 100.24   | 0.073 | 94.10     | 0.071 |
| 129.66   | 0.091 | 120.82   | 0.093 | 109.93   | 0.091 | 102.98   | 0.093 | 96.91     | 0.091 |
| 130.93   | 0.111 | 122.18   | 0.113 | 114.52   | 0.141 | 108.07   | 0.143 | 100.18    | 0.111 |
| 130.55   | 0.131 | 122.25   | 0.133 | 111.95   | 0.191 | 108.09   | 0.153 | 101.65    | 0.131 |
| 128.27   | 0.151 | 121.07   | 0.153 | 107.88   | 0.241 | 108.28   | 0.193 | 103.61    | 0.151 |
| 125.45   | 0.171 | 119.89   | 0.173 | 101.90   | 0.291 | 104.06   | 0.243 | 105.32    | 0.171 |
| 121.39   | 0.191 | 117.09   | 0.193 | 92.22    | 0.391 | 101.66   | 0.293 | 105.41    | 0.191 |
| 104.97   | 0.241 | 108.52   | 0.243 | 83.75    | 0.491 | 94.35    | 0.393 | 104.47    | 0.241 |
| 99.54    | 0.291 | 99.02    | 0.293 | 84.09    | 0.591 | 89.48    | 0.593 | 102.30    | 0.291 |
| 80.32    | 0.341 | 85.69    | 0.393 | 84.09    | 0.691 | 86.43    | 0.593 | 96.45     | 0.391 |
| 78.92    | 0.391 | 80.58    | 0.493 | 84.09    | 0.791 | 84.86    | 0.693 | 93.14     | 0.491 |
| 82.79    | 0.491 | 81.68    | 0.593 | 84.09    | 0.891 | 84.86    | 0.793 | 89.71     | 0.591 |
| 83.28    | 0.591 | 83.31    | 0.693 | 84.62    | 0.991 | 84.33    | 0.893 | 86.15     | 0.691 |
| 84.13    | 0.691 | 84.38    | 0.793 | 85.15    | 1.091 | 84.97    | 0.993 | 85.63     | 0.791 |
| 84.13    | 0.791 | 84.38    | 0.893 | 84.62    | 1.091 | 85.39    | 1.093 | 85.10     | 0.891 |
| 84.13    | 0.891 | 84.38    | 0.993 | 84.73    | 1.191 | 85.49    | 1.193 | 85.10     | 0.991 |
| 84.13    | 0.991 | 84.91    | 1.093 | 85.15    | 1.291 | 85.60    | 1.293 | 85.00     | 1.091 |
| 84.40    | 1.091 | 85.17    | 1.193 | 85.67    | 1.391 | 85.65    | 1.393 | 85.10     | 1.191 |
| 84.93    | 1.191 | 85.17    | 1.293 |          |       |          |       | 85.10     | 1.291 |
| 85.19    | 1.291 | 85.43    | 1.393 |          |       |          |       | 85.10     | 1.291 |
|          |       |          |       |          |       |          |       | 85.73     | 1.391 |
|          |       |          |       |          |       |          |       | 86.15     | 1.491 |

**TABLE B-1: Tabulated Velocity Profile Data: Run 12**  
 (U in feet/second; X and Y in inches)

| X = 2.25 |       | X = 4.50 |       | X = 6.75 |       | X = 9.00 |       |
|----------|-------|----------|-------|----------|-------|----------|-------|
| U        | Y     | U        | Y     | U        | Y     | U        | Y     |
| 125.2    | 0.016 | 112.7    | 0.016 | 101.2    | 0.016 | 92.7     | 0.016 |
| 125.2    | 0.021 | 118.0    | 0.021 | 103.9    | 0.021 | 94.1     | 0.021 |
| 129.2    | 0.026 | 120.6    | 0.026 | 107.3    | 0.026 | 96.8     | 0.026 |
| 131.2    | 0.031 | 122.7    | 0.031 | 109.6    | 0.031 | 99.6     | 0.031 |
| 136.4    | 0.036 | 125.2    | 0.036 | 111.3    | 0.036 | 101.2    | 0.036 |
| 139.1    | 0.041 | 127.7    | 0.046 | 112.5    | 0.041 | 103.7    | 0.041 |
| 141.8    | 0.046 | 131.2    | 0.056 | 115.2    | 0.051 | 104.5    | 0.046 |
| 143.6    | 0.051 | 133.1    | 0.066 | 117.4    | 0.061 | 105.5    | 0.051 |
| 147.1    | 0.061 | 134.5    | 0.076 | 118.7    | 0.071 | 106.7    | 0.056 |
| 150.1    | 0.071 | 135.9    | 0.096 | 119.6    | 0.081 | 109.0    | 0.066 |
| 151.7    | 0.081 | 135.0    | 0.121 | 120.2    | 0.091 | 110.8    | 0.076 |
| 153.4    | 0.101 | 132.6    | 0.146 | 120.8    | 0.101 | 110.8    | 0.086 |
| 152.0    | 0.126 | 127.7    | 0.171 | 120.6    | 0.111 | 111.3    | 0.096 |
| 148.8    | 0.151 | 125.2    | 0.196 | 120.1    | 0.131 | 112.5    | 0.106 |
| 141.8    | 0.176 | 120.1    | 0.221 | 119.0    | 0.151 | 112.5    | 0.116 |
| 135.0    | 0.201 | 114.7    | 0.246 | 118.0    | 0.171 | 112.5    | 0.126 |
| 124.4    | 0.226 | 108.5    | 0.271 | 115.8    | 0.191 | 111.3    | 0.146 |
| 114.7    | 0.251 | 102.5    | 0.296 | 113.3    | 0.216 | 110.8    | 0.156 |
| 101.8    | 0.276 | 96.8     | 0.321 | 110.8    | 0.241 | 110.2    | 0.166 |
| 92.7     | 0.301 | 91.4     | 0.346 | 107.9    | 0.266 | 109.6    | 0.176 |
| 72.9     | 0.351 | 86.4     | 0.371 | 103.7    | 0.291 | 109.0    | 0.196 |
| 57.3     | 0.401 | 81.9     | 0.396 | 100.6    | 0.316 | 107.9    | 0.221 |
| 50.3     | 0.451 | 74.6     | 0.421 | 98.0     | 0.341 | 105.5    | 0.246 |
| 47.7     | 0.501 | 70.2     | 0.446 | 92.7     | 0.366 | 104.3    | 0.271 |
| 47.7     | 0.551 | 65.6     | 0.471 | 90.7     | 0.391 | 102.5    | 0.296 |
| 47.7     | 0.601 | 63.6     | 0.496 | 86.4     | 0.416 | 98.0     | 0.346 |
|          |       | 55.1     | 0.546 | 84.9     | 0.441 | 94.8     | 0.396 |
|          |       | 50.8     | 0.596 | 83.4     | 0.466 | 89.3     | 0.446 |
|          |       | 49.3     | 0.646 | 77.1     | 0.491 | 82.6     | 0.496 |
|          |       | 48.2     | 0.696 | 73.8     | 0.516 | 77.9     | 0.546 |
|          |       | 48.2     | 0.746 | 71.1     | 0.541 | 74.6     | 0.596 |
|          |       | 48.2     | 0.796 | 68.4     | 0.566 | 69.3     | 0.646 |
|          |       |          |       | 64.6     | 0.591 | 63.6     | 0.696 |
|          |       |          |       | 62.6     | 0.616 | 59.5     | 0.746 |
|          |       |          |       | 60.6     | 0.641 | 58.4     | 0.796 |
|          |       |          |       | 58.4     | 0.666 | 55.1     | 0.846 |
|          |       |          |       | 56.7     | 0.691 | 53.9     | 0.896 |
|          |       |          |       | 55.1     | 0.716 | 52.8     | 0.946 |
|          |       |          |       | 53.5     | 0.741 | 51.5     | 0.996 |
|          |       |          |       | 52.5     | 0.766 |          |       |
|          |       |          |       | 51.8     | 0.791 |          |       |
|          |       |          |       | 51.0     | 0.841 |          |       |
|          |       |          |       | 50.0     | 0.891 |          |       |
|          |       |          |       | 50.0     | 0.991 |          |       |

TABLE B-1: Tabulated Velocity Profile Data: Run 12 cont.  
 (U in feet/second; X and Y in inches)

| X = 11.25 |       | X = 13.50 |       | X = 15.75 |       | X = 18.00 |       |
|-----------|-------|-----------|-------|-----------|-------|-----------|-------|
| U         | Y     | U         | Y     | U         | Y     | U         | Y     |
| 86.7      | 0.016 | 79.2      | 0.016 | 78.7      | 0.016 | 76.3      | 0.016 |
| 87.4      | 0.020 | 82.3      | 0.021 | 81.1      | 0.021 | 81.9      | 0.036 |
| 88.8      | 0.025 | 83.9      | 0.026 | 83.0      | 0.026 | 85.7      | 0.046 |
| 90.3      | 0.030 | 86.8      | 0.031 | 84.2      | 0.031 | 87.1      | 0.056 |
| 03.7      | 0.035 | 88.3      | 0.036 | 85.7      | 0.036 | 88.6      | 0.066 |
| 96.4      | 0.045 | 89.7      | 0.041 | 86.4      | 0.041 | 90.0      | 0.076 |
| 98.9      | 0.055 | 91.1      | 0.046 | 87.8      | 0.046 | 91.4      | 0.086 |
| 100.8     | 0.065 | 93.2      | 0.056 | 90.0      | 0.056 | 92.1      | 0.096 |
| 101.5     | 0.075 | 94.5      | 0.066 | 91.6      | 0.066 | 90.0      | 0.106 |
| 102.7     | 0.085 | 96.5      | 0.076 | 92.7      | 0.076 | 92.7      | 0.116 |
| 103.9     | 0.095 | 97.8      | 0.086 | 94.8      | 0.086 | 93.4      | 0.126 |
| 104.5     | 0.120 | 99.7      | 0.096 | 96.1      | 0.096 | 94.1      | 0.136 |
| 106.3     | 0.145 | 101.0     | 0.121 | 97.4      | 0.101 | 94.8      | 0.146 |
| 106.3     | 0.170 | 101.6     | 0.146 | 98.0      | 0.146 | 94.8      | 0.171 |
| 105.7     | 0.195 | 102.2     | 0.171 | 99.3      | 0.171 | 94.1      | 0.196 |
| 103.3     | 0.220 | 102.2     | 0.196 | 98.0      | 0.196 | 92.1      | 0.246 |
| 102.1     | 0.245 | 100.3     | 0.221 | 96.8      | 0.221 | 91.4      | 0.296 |
| 100.8     | 0.270 | 99.7      | 0.246 | 96.1      | 0.246 | 90.0      | 0.346 |
| 98.3      | 0.295 | 68.0      | 0.271 | 95.4      | 0.271 | 89.3      | 0.396 |
| 95.7      | 0.345 | 97.8      | 0.296 | 94.8      | 0.296 | 87.8      | 0.446 |
| 91.6      | 0.395 | 95.2      | 0.346 | 93.4      | 0.321 | 87.1      | 0.496 |
| 88.1      | 0.445 | 93.8      | 0.396 | 92.7      | 0.346 | 85.7      | 0.546 |
| 86.7      | 0.495 | 91.1      | 0.446 | 92.7      | 0.371 | 83.4      | 0.596 |
| 85.2      | 0.545 | 87.6      | 0.496 | 92.1      | 0.396 | 81.9      | 0.696 |
| 79.0      | 0.595 | 83.9      | 0.546 | 91.4      | 0.446 | 77.1      | 0.796 |
| 77.4      | 0.645 | 80.8      | 0.596 | 89.3      | 0.496 | 72.0      | 0.896 |
| 71.5      | 0.695 | 77.6      | 0.646 | 87.1      | 0.546 | 70.2      | 0.996 |
| 65.0      | 0.795 | 74.3      | 0.696 | 84.2      | 0.596 | 68.4      | 1.096 |
| 62.0      | 0.895 | 71.7      | 0.796 | 80.3      | 0.696 | 64.6      | 1.196 |
| 54.4      | 0.995 | 68.0      | 0.896 | 74.6      | 0.796 | 60.6      | 1.096 |
| 54.4      | 1.095 | 63.2      | 0.996 | 71.1      | 0.896 |           |       |
| 53.2      | 1.195 | 59.1      | 1.096 | 69.3      | 0.996 |           |       |
|           |       | 56.9      | 1.196 | 64.6      | 1.096 |           |       |
|           |       | 56.9      | 1.296 | 63.6      | 1.196 |           |       |
|           |       | 54.6      | 1.496 | 60.6      | 1.396 |           |       |
|           |       | 54.6      | 1.596 | 58.4      | 1.496 |           |       |

TABLE B-1: Tabulated Velocity Profile Data: Run 13  
 (U in feet/second; X and Y in inches)

| X = 2.25 |       | X = 4.50 |       | X = 6.75 |       | X = 9.00 |       |
|----------|-------|----------|-------|----------|-------|----------|-------|
| U        | Y     | U        | Y     | U        | Y     | U        | Y     |
| 72.8     | 0.016 | 65.1     | 0.016 | 61.0     | 0.016 | 56.9     | 0.016 |
| 75.2     | 0.021 | 65.7     | 0.021 | 62.4     | 0.021 | 57.8     | 0.021 |
| 78.2     | 0.026 | 67.8     | 0.026 | 64.8     | 0.031 | 59.3     | 0.026 |
| 80.1     | 0.031 | 69.6     | 0.031 | 67.7     | 0.041 | 61.9     | 0.036 |
| 82.1     | 0.036 | 72.2     | 0.041 | 69.6     | 0.051 | 63.6     | 0.046 |
| 83.2     | 0.041 | 73.9     | 0.051 | 72.6     | 0.076 | 67.7     | 0.071 |
| 83.8     | 0.046 | 77.6     | 0.076 | 74.3     | 0.101 | 69.1     | 0.096 |
| 85.6     | 0.056 | 79.3     | 0.101 | 75.1     | 0.126 | 70.9     | 0.121 |
| 87.0     | 0.066 | 79.5     | 0.126 | 75.1     | 0.151 | 71.6     | 0.146 |
| 88.0     | 0.076 | 78.8     | 0.151 | 74.9     | 0.176 | 71.6     | 0.171 |
| 88.6     | 0.086 | 77.9     | 0.176 | 73.9     | 0.201 | 70.9     | 0.196 |
| 88.9     | 0.096 | 75.8     | 0.201 | 73.0     | 0.226 | 70.5     | 0.221 |
| 88.4     | 0.121 | 74.0     | 0.226 | 71.8     | 0.251 | 70.3     | 0.246 |
| 86.6     | 0.146 | 70.9     | 0.251 | 68.2     | 0.301 | 67.7     | 0.296 |
| 84.2     | 0.171 | 68.3     | 0.276 | 64.6     | 0.351 | 65.0     | 0.346 |
| 79.9     | 0.196 | 65.7     | 0.301 | 61.7     | 0.401 | 63.6     | 0.396 |
| 74.6     | 0.221 | 61.7     | 0.326 | 58.0     | 0.451 | 61.4     | 0.446 |
| 69.6     | 0.246 | 59.6     | 0.351 | 54.8     | 0.501 | 59.2     | 0.496 |
| 63.6     | 0.271 | 56.2     | 0.376 | 53.1     | 0.551 | 56.9     | 0.546 |
| 57.6     | 0.296 | 53.6     | 0.401 | 51.4     | 0.601 | 54.4     | 0.596 |
| 52.8     | 0.321 | 50.1     | 0.451 | 49.8     | 0.651 | 53.4     | 0.646 |
| 48.7     | 0.346 | 47.4     | 0.501 | 49.2     | 0.701 | 52.4     | 0.696 |
| 45.8     | 0.371 | 46.4     | 0.551 | 48.9     | 0.751 | 51.6     | 0.746 |
| 43.8     | 0.396 | 45.5     | 0.551 | 48.7     | 0.801 | 51.0     | 0.796 |
| 43.4     | 0.421 |          |       |          |       | 50.1     | 0.846 |
| 43.6     | 0.446 |          |       |          |       | 50.1     | 0.896 |

TABLE B-1: Tabulated Velocity Profile Data: Run 13 cont.  
 (U in feet/second; X and Y in inches)

| X = 11.25 |       | X = 13.50 |       | X = 15.75 |       | X = 18.00 |       |
|-----------|-------|-----------|-------|-----------|-------|-----------|-------|
| U         | Y     | U         | Y     | U         | Y     | U         | Y     |
| 53.2      | 0.016 | 51.5      | 0.016 | 49.9      | 0.016 | 52.1      | 0.016 |
| 54.6      | 0.021 | 54.6      | 0.021 | 52.1      | 0.021 | 54.1      | 0.021 |
| 56.4      | 0.026 | 56.5      | 0.031 | 53.8      | 0.026 | 56.2      | 0.026 |
| 58.6      | 0.036 | 58.6      | 0.041 | 55.1      | 0.036 | 57.7      | 0.036 |
| 60.1      | 0.046 | 62.3      | 0.066 | 56.4      | 0.046 | 58.9      | 0.046 |
| 64.0      | 0.071 | 64.4      | 0.091 | 60.1      | 0.071 | 62.3      | 0.071 |
| 65.8      | 0.096 | 66.5      | 0.116 | 62.6      | 0.096 | 64.8      | 0.096 |
| 67.3      | 0.121 | 67.2      | 0.141 | 64.5      | 0.121 | 66.4      | 0.121 |
| 68.0      | 0.146 | 67.8      | 0.166 | 65.3      | 0.146 | 67.2      | 0.146 |
| 68.5      | 0.171 | 67.8      | 0.191 | 66.1      | 0.196 | 68.6      | 0.171 |
| 68.5      | 0.196 | 67.8      | 0.241 | 67.2      | 0.246 | 69.2      | 0.196 |
| 68.5      | 0.221 | 67.2      | 0.291 | 67.2      | 0.296 | 69.2      | 0.246 |
| 68.2      | 0.246 | 66.5      | 0.341 | 66.5      | 0.346 | 68.8      | 0.296 |
| 67.7      | 0.271 | 65.1      | 0.391 | 64.8      | 0.396 | 68.3      | 0.346 |
| 67.2      | 0.296 | 63.7      | 0.441 | 63.5      | 0.446 | 67.8      | 0.396 |
| 66.4      | 0.321 | 61.9      | 0.491 | 63.1      | 0.496 | 67.2      | 0.446 |
| 65.8      | 0.346 | 60.1      | 0.591 | 61.6      | 0.596 | 66.5      | 0.496 |
| 65.1      | 0.371 | 57.8      | 0.691 | 59.2      | 0.696 | 64.5      | 0.596 |
| 63.7      | 0.396 | 56.2      | 0.791 | 57.4      | 0.796 | 63.1      | 0.696 |
| 62.3      | 0.446 | 55.1      | 0.891 | 56.2      | 0.896 | 61.9      | 0.796 |
| 60.1      | 0.496 | 54.6      | 0.991 | 55.4      | 0.996 | 60.4      | 0.896 |
| 58.6      | 0.546 | 54.2      | 1.091 | 55.1      | 1.096 | 60.0      | 0.996 |
| 57.0      | 0.596 | 53.9      | 1.191 | 54.6      | 1.196 | 59.5      | 1.096 |
| 55.4      | 0.696 | 53.7      | 1.291 | 54.6      | 1.296 | 59.4      | 1.196 |
| 52.9      | 0.796 |           |       |           |       |           |       |
| 52.0      | 0.896 |           |       |           |       |           |       |
| 51.2      | 0.996 |           |       |           |       |           |       |
| 51.2      | 1.096 |           |       |           |       |           |       |

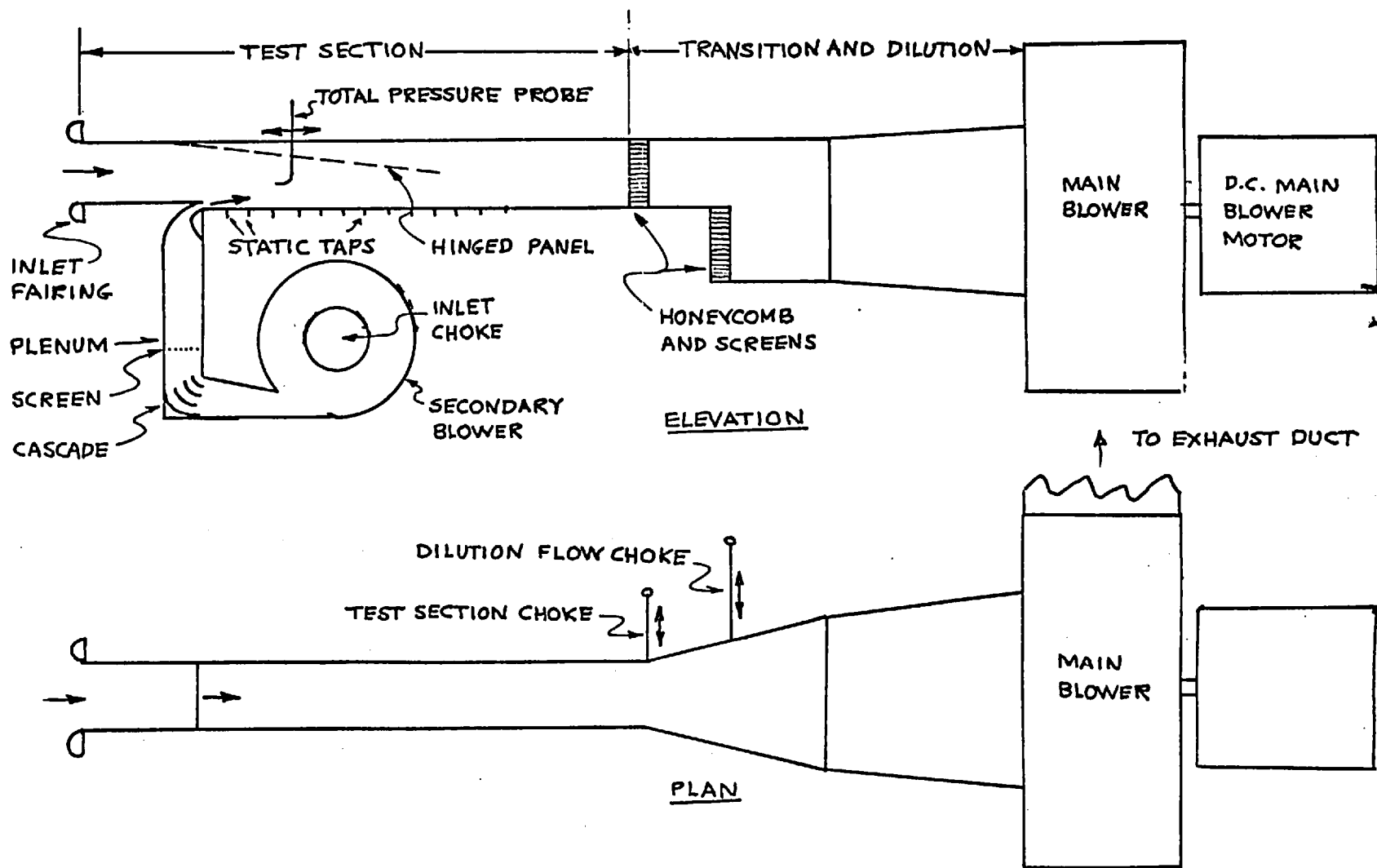


FIGURE B-1: THE WIND TUNNEL

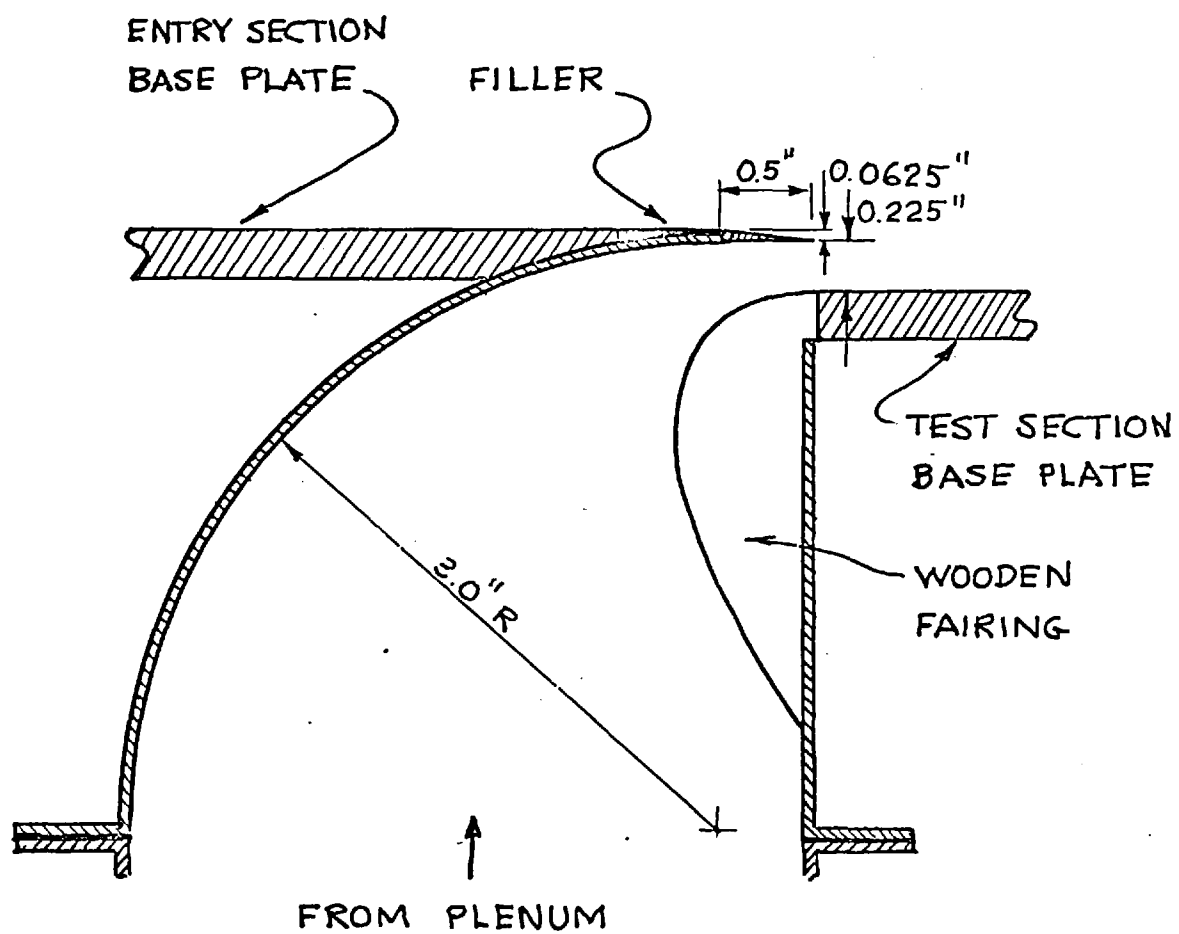


FIGURE B-2: THE INJECTION SLOT

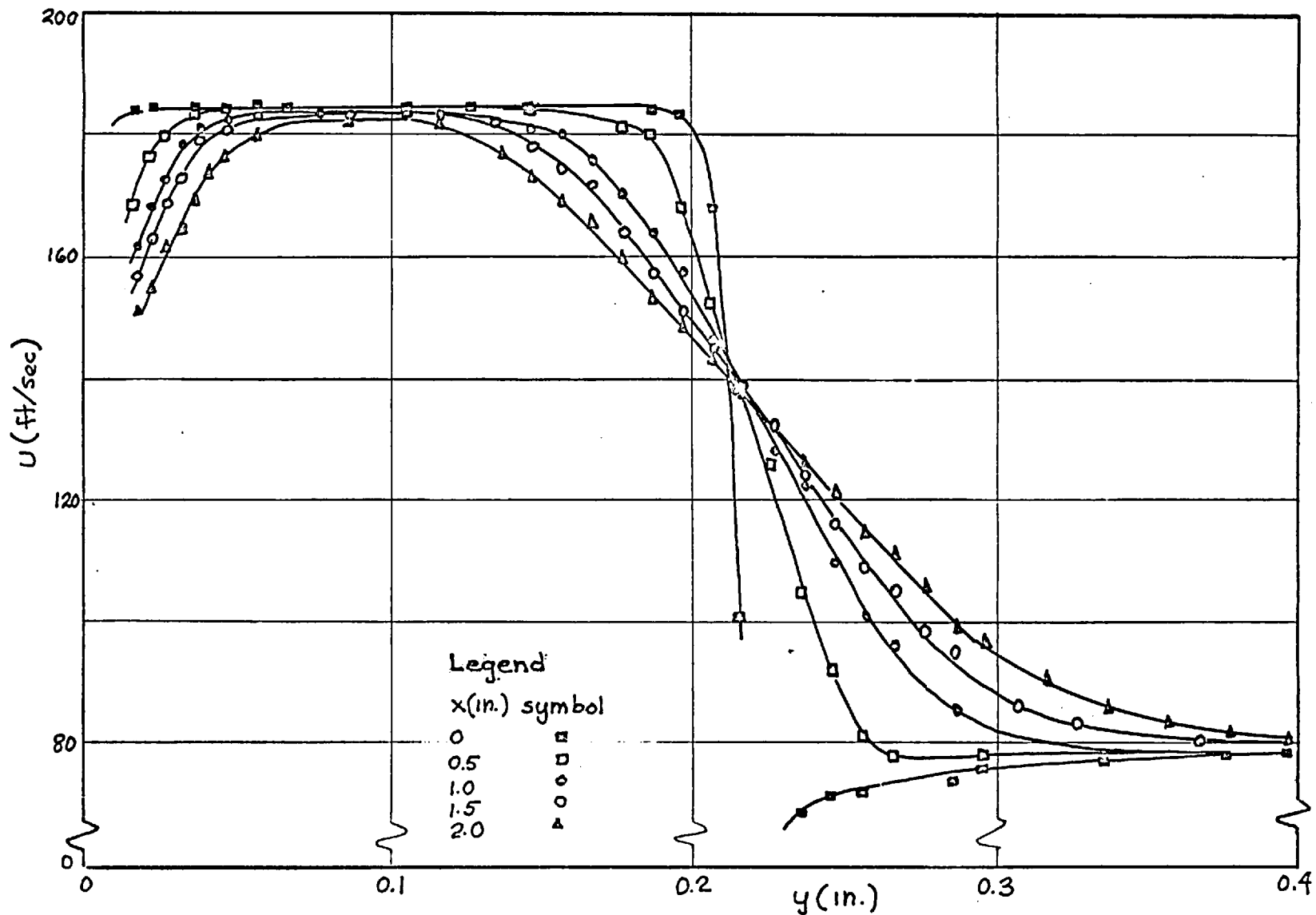


FIGURE B-3: TYPICAL VELOCITY PROFILES IN THE POTENTIAL-CORE REGION

## APPENDIX C

EFFECTIVENESS MEASUREMENTS

The impervious-wall effectiveness measurements for argon injection into an air mainflow were obtained on the same basic tunnel as were the hydrodynamic measurements with an argon supply system replacing the secondary blower, and with gas sample collection apparatus connected to the static pressure taps on the test-section base plate.

The argon supply system, shown schematically in Figure C-1, was designed by the British Oxygen Company Limited and consisted of a three-bottle manifold, a high pressure regulator, a low pressure regulator, and suitable connecting plumbing. The design specifications called for argon flow rates between 10 and 60 CFM at near ambient temperature; the actual supply temperature differed from ambient by a maximum of  $-7^{\circ}\text{C}$ , a deviation which may be regarded as negligible.

The sample collection apparatus is shown in Figure C-2. It consisted of a series of 20 ml gas sample bottles with high vacuum stopcocks at both ends. The lower stem of each bottle was sealed into a manifold and projected down into a pool of high vacuum oil. The upper stem was connected by a short length of heavy walled rubber tubing to

the appropriate static pressure top. The manifold was connected, through suitable valving, to a vacuum pump and to a bulb which could be used to pressurize the system.

The experimental procedure was as follows. With the aid of the bulb the sample bottles were filled with high vacuum oil and the stopcocks then closed. The mainstream and argon flows were then set to the desired values and the injection and mainstream total pressures, and the static pressures on the wall of the test section, recorded. The vacuum pump was then turned on and the stopcocks opened. After approximately two minutes, during which time the sample bottles and collection tubes were thoroughly flushed, the stopcocks were closed and the argon supply and the vacuum pump turned off. The total elapsed time for the complete run was approximately four minutes.

It is appropriate at this point to note that, because the composition gradient at the wall is zero for an impermeable wall, the sample composition is insensitive to the sampling rate at small sampling rates. In the case of the present measurements the sampling rate was determined by experiment to be sufficiently small.

The sample compositions were then measured with a gas chromatograph. A 1ml volume was withdrawn from a sample bottle through the thick-walled rubber tubing, after opening the lower pair of stopcocks, with a gas-tight syringe and injected into the chromatograph. The particular chromatograph used for the

present measurements employed a 2 m long column, packed with Linde type Molecular Sieve, to separate the permanent gases and a thermal conductivity cell to measure their relative concentrations. The output of the thermal conductivity cell was recorded by a strip chart recorder. Since nitrogen was used as the carrier gas, the argon provided the only significant peaks on the strip chart record; a typical example record for a complete run is shown in Figure C-3.

The argon concentration could then be determined from the peak height with the aid of the calibration chart shown in Figure C-4. The effectiveness data so obtained are tabulated in Table C-1.

TABLE C-1: Effectiveness Data for Argon Injection into Air

| Run                    | 10    | 8     | 9     | 6     | 7     | 2    | 5     | 1     | 4     |
|------------------------|-------|-------|-------|-------|-------|------|-------|-------|-------|
| $u_C/u_G$              | .86   | 1.0   | 1.06  | 1.16  | 1.61  | 1.92 | 2.52  | 2.65  | 2.73  |
| $\frac{x}{y_C} = 14.4$ | 0.91  | 0.95  | 0.94  | 0.97  | 0.98  |      | 0.98  | 0.955 | 0.95  |
| 23.2                   | 0.92  | 0.915 | 0.875 | 0.92  | 0.925 |      | 0.91  | 0.855 | 0.86  |
| 32.2                   | 0.87  | 0.85  | 0.78  | 0.84  | 0.84  | 0.81 | 0.777 | 0.75  | 0.76  |
| 50                     | 0.76  | .777  | 0.725 | 0.765 | 0.74  | 0.73 | 0.68  | 0.656 | 0.67  |
| 59                     | 0.70  | 0.705 | 0.672 | 0.69  | 0.68  |      | 0.58  | 0.576 | 0.565 |
| 68                     | 0.67  | 0.677 | 0.657 | 0.657 | 0.64  |      | 0.546 | 0.518 | 0.53  |
| 76.5                   | 0.623 | 0.63  | 0.615 | 0.634 | 0.595 | 0.58 | 0.505 | 0.478 | 0.49  |
| 94.5                   | 0.524 | 0.576 | 0.538 | 0.574 | 0.53  | 0.52 | 0.445 | 0.43  | 0.43  |
| 112                    | 0.47  | 0.523 | 0.530 | 0.535 | 0.504 |      | -     | 0.424 | 0.364 |
| 121                    | 0.424 | 0.498 | -     | 0.505 | 0.493 |      | -     | .0403 | 0.35  |
| 130                    | 0.40  | 0.432 | 0.40  | 0.463 | 0.480 | .395 | 0.372 | 0.388 | 0.325 |

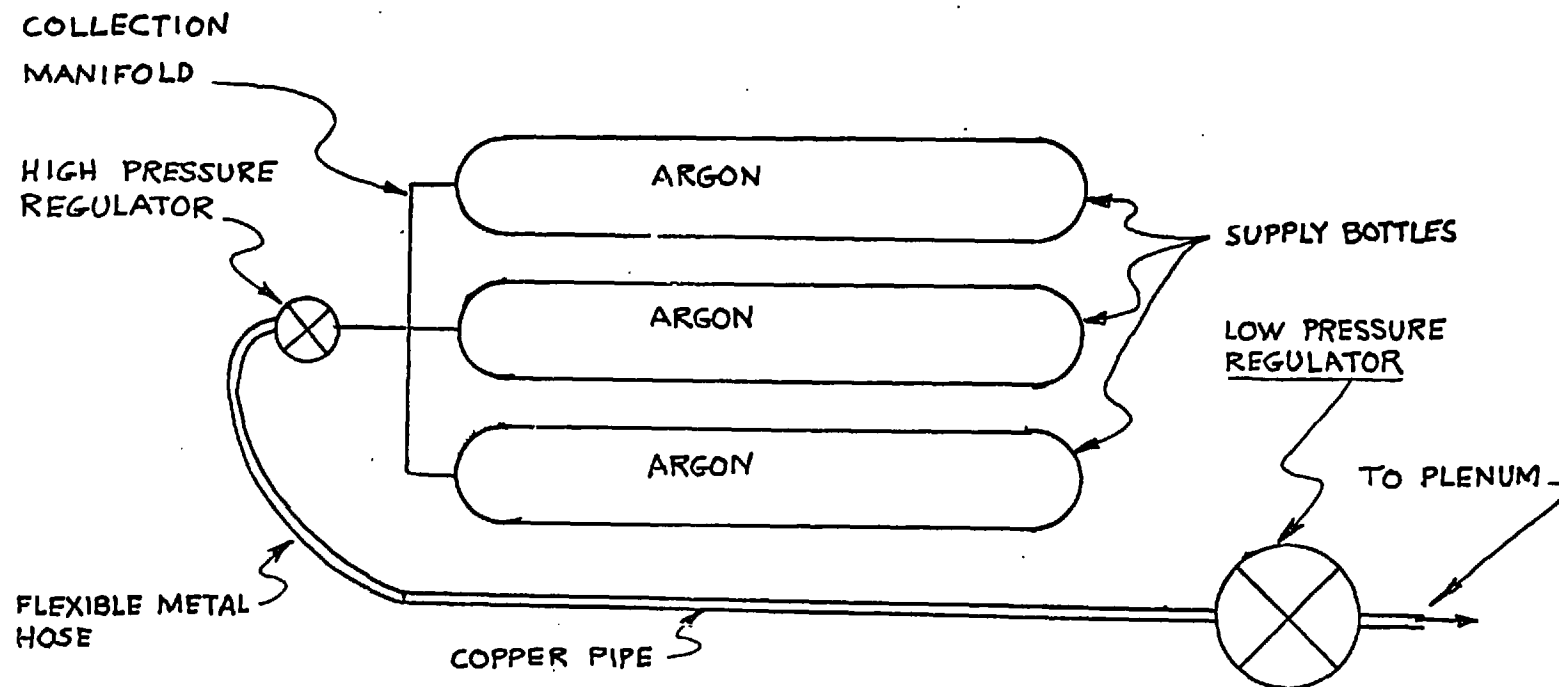


FIGURE C-1: THE ARGON SUPPLY SYSTEM

SAMPLES DRAWN THROUGH 0.020" DIA. HOLES  
IN THE BASE PLATE OF THE WIND TUNNEL

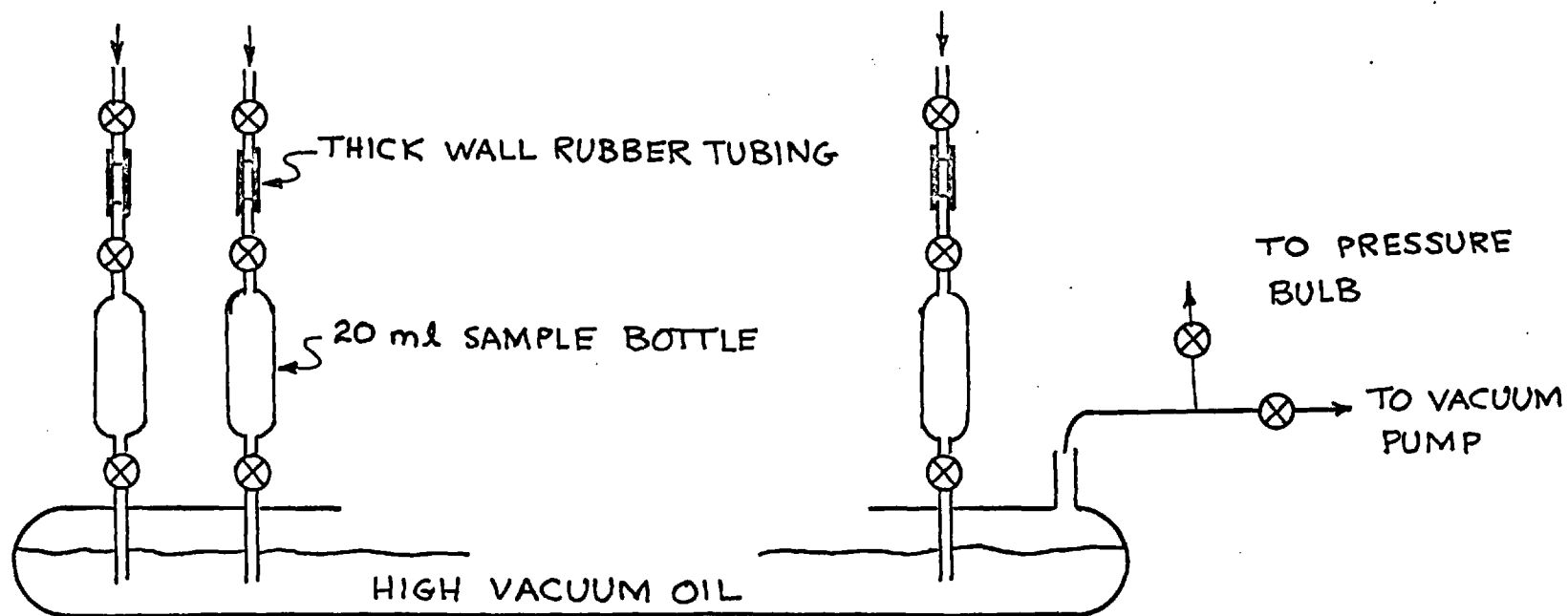


FIGURE C-2: THE SAMPLE COLLECTION APPARATUS

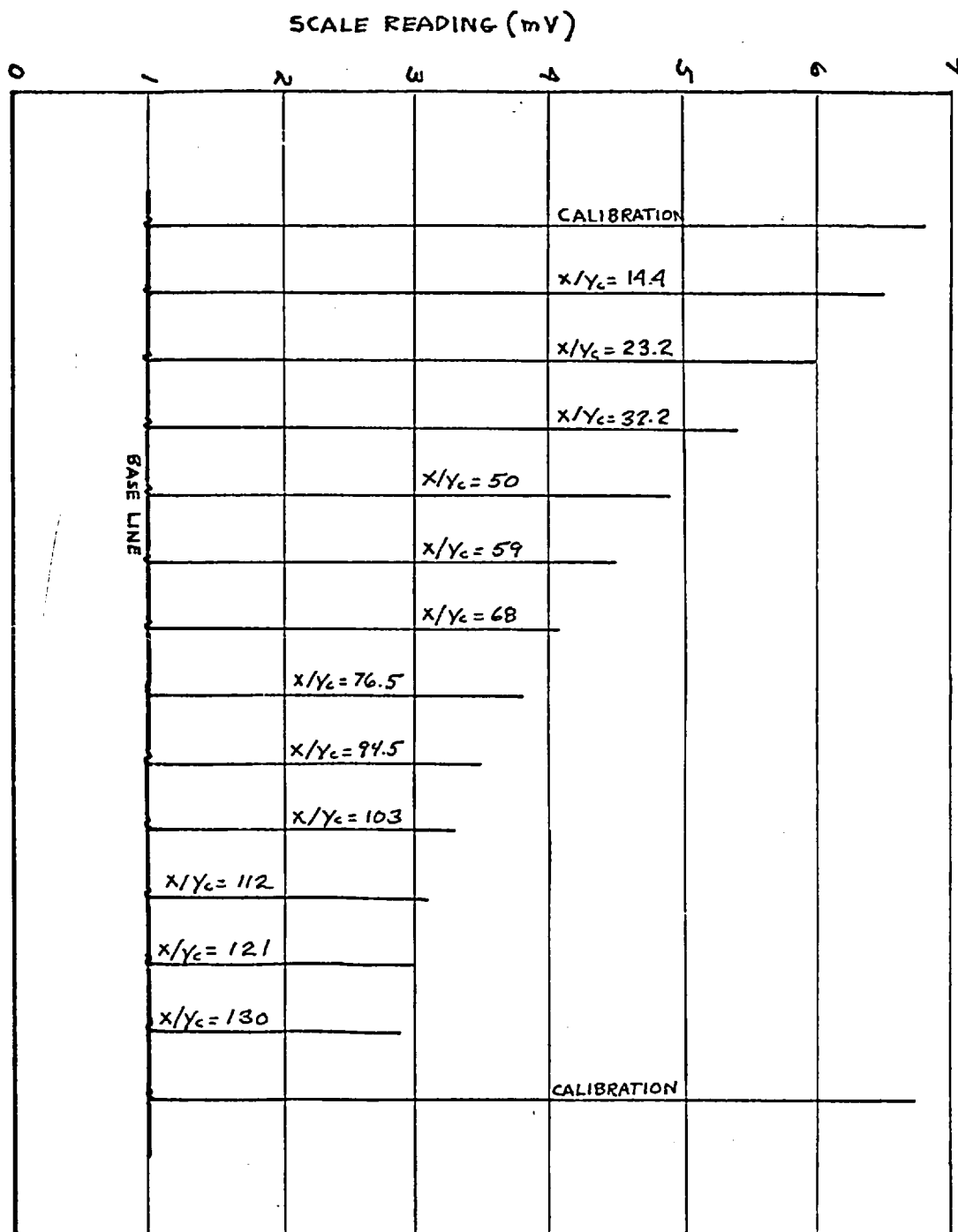


FIGURE C-3: A TYPICAL STRIP-CHART RECORD

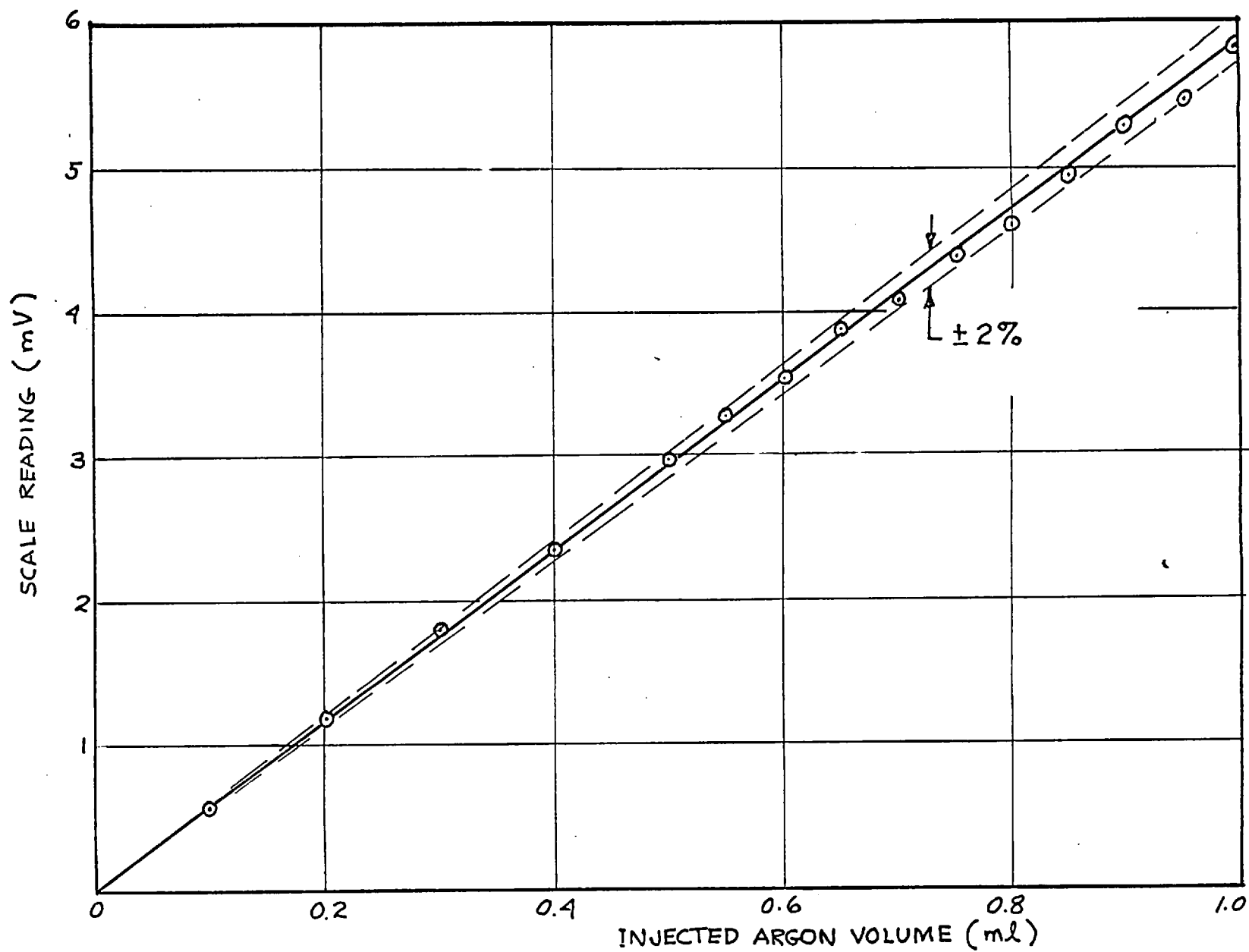


FIGURE C-4: THE CALIBRATION CHART