

Detector Limited Resolution in Ultrasound
Visualization

by

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ABSTRACT

A linear systems analysis approach to ultrasonic holography has brought out the importance of a detector coherent transfer function defined as the product of a directivity function and a modulation transfer function and its role in determining the resolution of the visualization system. By further application of the theory, an impulse response technique and a sinusoidal standing wave technique were developed which enable the coherent transfer function to be obtained quantitatively. These were used successfully on a range of polarised lead zirconate titanate ceramic discs. It was considered that these materials could form the basis of a practical ultrasound hologram detector. The nature of the results is such that both ceramic material and temporal frequency can be selected to give optimum performance of a visualization system.

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List of Symbols

a	(i)	An arbitrary constant.
	(ii)	Radius of a sample ceramic disc.
	(iii)	x position of sender in sinusoidal standing wave technique.
	(iv)	Half width of rectangular detector aperture in x-direction.
$A_o(x,y)$		Pupil function of a detector or imaging system.
$A_i(x,y)$		Pupil function of a visualized hologram.
b	(i)	An arbitrary constant.
	(ii)	Thickness of a sample ceramic disc.
	(iii)	Half width of rectangular detector aperture in y-direction.
B		Spatial frequency bandwidth of an object spectrum.
c		Phase velocity of a longitudinal ultrasound wave in water.
c_d		Phase velocity of a dilatational wave propagating along the Z axis of a piezoelectric ceramic material.
c_{ijkl}^D		Elastic stiffness tensor for D constant.
$C(p, \xi, \xi')$		Cross correlation function of two spread functions which originate from line sources at ξ and ξ' .
D_n		Electric displacement vector.
$D_p(f_x, f_y)$		Directivity function for a piezoelectric ceramic detector.
E_m		Electric field strength vector.

f_x, f_y	Spatial frequency variables referred to x or y rectangular co-ordinates respectively.
$f_1(x_1, y_1)$	An arbitrary time independent scalar potential function of x_1 and y_1 .
$f_2(x_2, y_2)$	An arbitrary time independent scalar potential function of x_2 and y_2 .
f_{θ_i}	Spatial frequency of a plane reconstructing wavefront incident at θ_i to a visualized hologram normal.
f_{θ_r}	Spatial frequency of a plane reference wavefront incident at θ_r to a detector normal.
f_o	The cut-off or maximum spatial frequency passed by a detector.
$F(f_x, f_y)$	Spatial frequency spectrum which is the Fourier transform of $f(x, y)$.
$\mathcal{F}$	Symbolic representation of the Fourier transform operation.
$\mathcal{F}^{-1}$	Symbolic representation of the inverse Fourier transform operation.
$g(x, y)$	A Green's function, the response of a linear system to a delta function.
$g_s(x_2, y_2)$	Green's function for the free space propagation problem.
$g_{os}(x_2, y_2)$	Green's function for object space propagation.
$g_{is}(x_6, y_6)$	Green's function for image space propagation.
$G(f_x, f_y)$	A coherent spatial frequency transfer function or coherent spatial frequency response function, the Fourier transform of $g(x, y)$.
$G_m(f_x, f_y)$	Real part of $G(f_x, f_y)$.
$G_p(f_x, f_y)$	A coherent transfer function characteristic of a piezoelectric detector.

$h_v(f_x, f_y)$ Optical transfer function of a visualized hologram.

h_{kln} The tensor giving the piezoelectric constants of a material which relate open circuit voltage to a specified strain.

$H(f_x, f_y)$ Optical transfer function.

J_0 Bessel function of the first kind, order zero.

J_1 Bessel function of the first kind, order one.

k Wavenumber $\frac{2\pi}{\lambda}$ of longitudinal ultrasound in water.

k_d Wavenumber $\frac{2\pi}{\lambda_d}$ of dilatational wave propagating along

the Z axis of a piezoelectric ceramic material.

k' Mean wavenumber for quasi-monochromatic radiation.

k_o Wavenumber $\frac{2\pi}{\lambda_o}$ of longitudinal ultrasound wave in

a fluid.

k_i Wavenumber $\frac{2\pi}{\lambda_i}$ of coherent monochromatic light used

to reconstruct a hologram.

L A linear differential operator.

$\bar{L}_i$ An arbitrary differential operator.

m Scale change factor accounting for a change in size of a visualized hologram prior to reconstruction.

$M_p(f_x, f_y)$ Modulation transfer function of a piezoelectric ceramic detector.

$\underline{n}$ Unit normal to a wavefront.

p	Time independent sound pressure.
P	Time dependent sound pressure.
q_x	The input spatial frequency in the sinusoidal standing wave technique.
r_1	Distance between an arbitrary source point and a point in the diffraction field of the object.
R	Radial cylindrical co-ordinate.
s_{ijkl}^D	Elastic compliance tensor with D constant.
s_{ij}	Strain tensor.
$S_i(\rho)$	Spatial frequency spectrum of an input strain function (Impulse response technique).
T_{ij}	Stress tensor.
u_n	Particle displacement vector.
$\underline{u}$	Time independent particle velocity.
$\underline{U}$	Time dependent particle velocity.
v_r	Complex amplitude of electronic reference wave.
v_{ro}	Real amplitude of electronic reference wave.
v_H	Complex amplitude of virtual hologram.
V	Electrical potential difference.
x, y, z	Rectangular cartesian co-ordinates.
R, θ, Z	Cylindrical co-ordinates in spatial domain.
ρ, β	Plane polar co-ordinates in spatial frequency domain.

β_{mn}^s	Dielectric impermeability tensor for constant strain.
β	Slope of transmission - Exposure (t - E) curve at the optimum light intensity level for recording a hologram on photographic film.
$\Gamma_{12}(\tau)$	Mutual coherence function for light.
$\delta(x - \xi, y - \eta)$	Delta function of position.
η	A variable of integration
θ_i	Angle of incidence.
θ_r	Angle of refraction.
K	Wave number for the general propagation problem.
λ_d	Dilatational wavelength of ultrasound propagating along the Z axis of a piezoelectric ceramic material.
λ_i	Free field wavelength of coherent light used to reconstruct a hologram.
λ_o	Free field wavelength of a longitudinal sound wave in a fluid.
λ, μ	Lamé elastic stiffness constants.
ν'	Mean temporal frequency for quasi-monochromatic radiation.
ξ	A variable of integration.
ρ_o	Density of a fluid.
σ	Poisson's Ratio.
$\sigma(\xi, \xi')$	The degree of space variance.
$d\sigma$	An area element of a wavefront.
τ	A variable time delay.
$\phi(f_x, f_y)$	The phase of $G(f_x, f_y)$.
$\phi(x, y)$	A time independent velocity potential (scalar point function).
$\bar{\phi}(x, y)$	A time dependent velocity potential function (scalar point function).
ψ	Time independent scalar wave function for light.

ψ_p	Complex amplitude of an optical holographic reference wave.
ψ_i	Complex amplitude of the Fresnel diffraction pattern obtained on reconstructing a hologram.
$\mathcal{I}_i$	Spatial frequency spectrum of a reconstructed hologram.
$\mathcal{I}_T$	Spatial frequency spectrum of a reconstructed True Image.
ω	Angular frequency of a sound wave.
$\otimes$	Symbolic representation of the convolution operation.

Glossary of Terms

1. Space-invariance

In the context of an image forming system, space invariance refers to the property that the conjugate image of a point source is independent of the position of the source in the object plane.

2. Optical transfer function

The transfer function of an optical system which relates an intensity spatial frequency spectrum in the object plane to an intensity spatial frequency spectrum in the image plane.

3. Spatial frequency

A concept used in the description of spatial information (ie shape and position) which is analogous to temporal frequency and has dimensions of reciprocal length.

4. Pupil function

The function which describes the amplitude transmission property of the entrance pupil of an imaging system.

5. Coherent transfer function

The transfer function of an optical system which relates a complex amplitude spatial frequency spectrum in the object plane to a complex amplitude spatial frequency spectrum in the image plane.

6. Modulation transfer function

The function which describes how contrast is transmitted through a system and is given by the modulus of the appropriate coherent transfer function.

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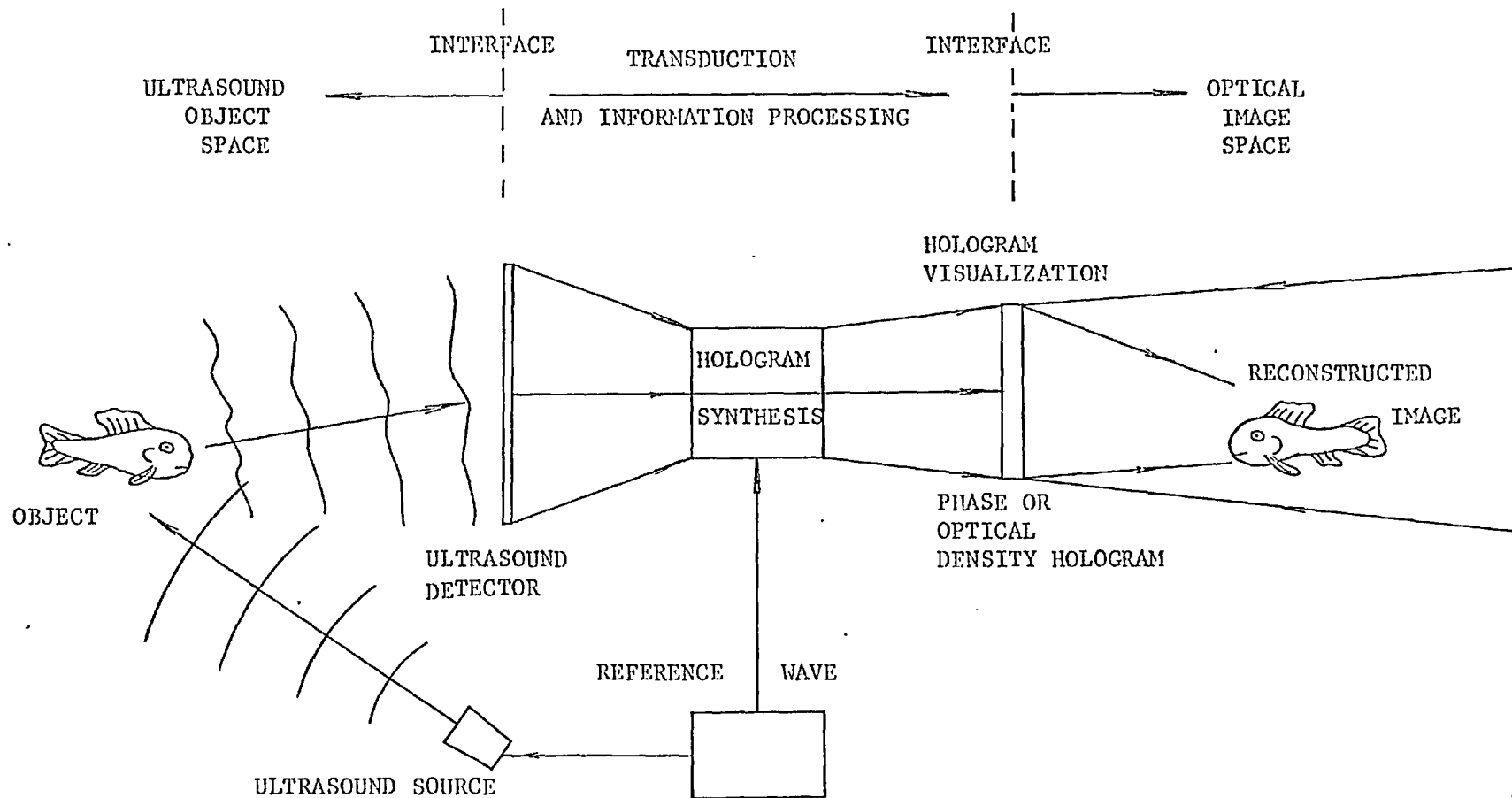
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I N T R O D U C T I O N

This thesis is concerned with the limitations on object resolution which are imposed by the characteristics of an ultrasound detector used in a particular type of ultrasound holographic visualization system. Ultrasound holography is a branch of the field of ultrasound visualization which has received a great deal of attention during recent years. Much of the work has, however, been of a qualitative nature. Although this investigation is orientated towards one branch of the subject, the results which are obtained can equally well be applied to the whole field and in particular to the more conventional system employing discrete imaging elements followed by visualization of the resulting ultrasound image.

Ultrasound holography in its present form is simply a technique by which an ultrasound diffraction pattern can be recorded so that when the record is visualized and illuminated with coherent light an image of the original ultrasonically irradiated object is obtained. The technique in its most refined form can simultaneously visualize and image. Hence moving objects can be observed. Such a system is indicated schematically in the figure.

The general behaviour of holographic systems employing one wavelength to record the hologram and another to reconstruct, of which ultrasound holography is an example, has been fully dealt with in the scientific literature and several excellent text books are now available. Consequently no attempt has been made to discuss ultrasound holography other than from the aspect of resolution.



Schematic diagram of a real time ultrasound holographic underwater visualization system.

Initially the motivation for this research was the desire to produce a high resolution underwater ultrasound imaging system to fill in the 'range gap' from zero to perhaps 100 m which at the time was not adequately covered by scanning sonar systems. Absorption of ultrasound in water restricts the range at high frequencies so it is necessary to compromise between high resolution at short wavelengths and short range, and poorer resolution at longer range with longer wavelengths. Frequencies around 1 MHz would seem to offer the best compromise between range and resolution. Range is obviously limited by the ultrasound power which can be beamed in the direction of the target and the sensitivity of the ultrasound detector. There is an upper limit to the power which can be radiated continuously by an underwater transducer, imposed by excessive heating from dielectric losses. It is, therefore, necessary to use a detector with as high a sensitivity as possible to obtain the maximum range. Of all the types of detector which have been studied, only those based on the piezoelectric effect have sufficient sensitivity for use in the underwater viewing application and also sufficiently rapid response time to enable moving objects to be observed. Work with the ultrasound camera has indicated that in an ultrasound holographic system using a detector consisting of a plate of piezoelectric material or an array of such plates, the resolution would be limited by the piezoelectric material itself. A program of research was, therefore, initiated to investigate any limitations on the resolution so imposed. Although claims have been made for the ultrasound camera of a maximum resolution in the order of the detector thickness, these are not substantiated by the visual quality of the images produced.

The linear systems approach, that has been applied so fruitfully to the elucidation of many problems in optics and optical holography,

was applied to this problem as described fully in Chapter 2. As a result, two experimental techniques were devised to measure the coherent transfer function for the piezoelectric ceramic detector. This transfer function, for a detector which can be considered as a linear system, describes how spatial (ie shape and structure) information is passed, and hence determines the resolution capabilities in much the same way that the frequency response of an audio amplifier determines the quality of sound reproduction.

The justification for applying linear systems theory to the problem is that the behaviour of a piezoelectric ultrasound detector can in principle be described by a linear wave equation, for small amplitude waves. In this respect the theory has greater validity than when applied to a photographic film or plate where non-linear chemical effects are present. The coherent transfer function, or spatial frequency response function as it will sometimes be called, is defined as the Fourier transform of the point spread function for the system. It is much simpler to compare frequency response functions than impulse response functions because these are 'smoothed out' in the Fourier transforming process. Hence theoretical resolution of a detector at one temporal frequency can be compared much more easily with that at a second temporal frequency, using the spatial frequency response function rather than using the point spread function. It is, therefore, a relatively easy matter to determine the temporal frequency at which an ultrasound detector should be used in a holographic system to produce the best object resolution.

In order to make this investigation easier to follow, as it embraces several branches of physics, the development of the research project is outlined in four stages:-

- 1) Rationalization of the detector limited resolution problem, leading to its description in terms of linear systems theory and the specification of a coherent transfer function for the detector itself.
- 2) Development and use of experimental techniques for measuring the coherent transfer function of a piezoelectric plate ultrasound detector.
- 3) Investigation of the change in coherent transfer function with temporal frequency in order to determine the frequency at which optimum performance of the detector is obtained.
- 4) Elucidation of the factors which determine the shape of the coherent transfer function so that ultimately a detector having a predictable spatial frequency bandwidth at a selected temporal frequency can be designed.

CHAPTER I

A Historical Survey of Ultrasound Visualization

1.1 Ultrasound Image Conversion

It would not be appropriate to recount in detail the numerous ultrasound image conversion systems which have been developed. Instead it is proposed to discuss improvements which have taken place since 1967 (see paper in Appendix). This paper contains a list of detectors for ultrasound imaging systems based on an earlier review (1).

Much of the research and development work has concentrated on electrical methods of ultrasound image conversion for these still possess the highest sensitivity and great potential in specific areas of non-destructive testing and underwater viewing. In parallel with this, work has been carried out to assess the usefulness of the ultrasound camera for routine inspection of flat plates and other similar objects of a simple geometry. Berger (2) has published a detailed review of 'Ultrasonic Imaging Systems for Non-destructive Testing' which includes results obtained with his ultrasound camera system when used in routine inspection of flat superheater fuel plates from a nuclear reactor. The Lamb-wave mode of inspection was employed to improve energy transmission through the plates. Typical diameters of the piezoelectric detectors employed were 5 cm at 2 MHz, 10 cm at 1 MHz and 2.5 cm at 5 MHz.

Jacobs (3) has taken the interesting step of adapting a commercially available colour television system to display coloured

images where the colours are coded to indicate the acoustical impedance over the image area. The technique is essentially one of phase sensitive detection. As each 'resolution element' on the quartz detector is scanned the phase of the signal is compared with that of the ultrasound source and the magnitude of the phase difference converted into a colour signal, in accordance with the convention in the N.T.S.C. system which is used. The colour signals are fed to a conventional colour television display tube and so the resulting display exhibits colours corresponding to the acoustic impedance of the object along a path between the image point and the ultrasound source. Fig. 1.1 shows a block diagram of the system. Impedance changes as small as 1 part in 10^8 can be detected and displayed by this technique and it is thought that there should be many applications in the field of biology.

In an effort to overcome the limitations on performance of the ultrasound camera resulting from its small aperture, several solid state switching systems have been developed to carry out scanning of arrays of small discrete piezoelectric transducing elements. This development has been stimulated in particular by underwater viewing applications where a large aperture is necessary for both high resolution and large field of view. The ultrasound camera is not suitable because the detector element closes one end of a vacuum tube. To exclude the possibility of implosion the diameter is restricted to a few centimetres with only 1 atmosphere pressure differential across it. Use on the sea bed where this pressure differential could be hundreds of atmospheres is, therefore, out of the question. Some very encouraging work has been reported by Green et al (4). Their detector takes the form of a 20 cm long strip of 2.5 MHz thickness resonant piezoelectric ceramic, the unexposed side of which is segmented with parallel cuts, into the

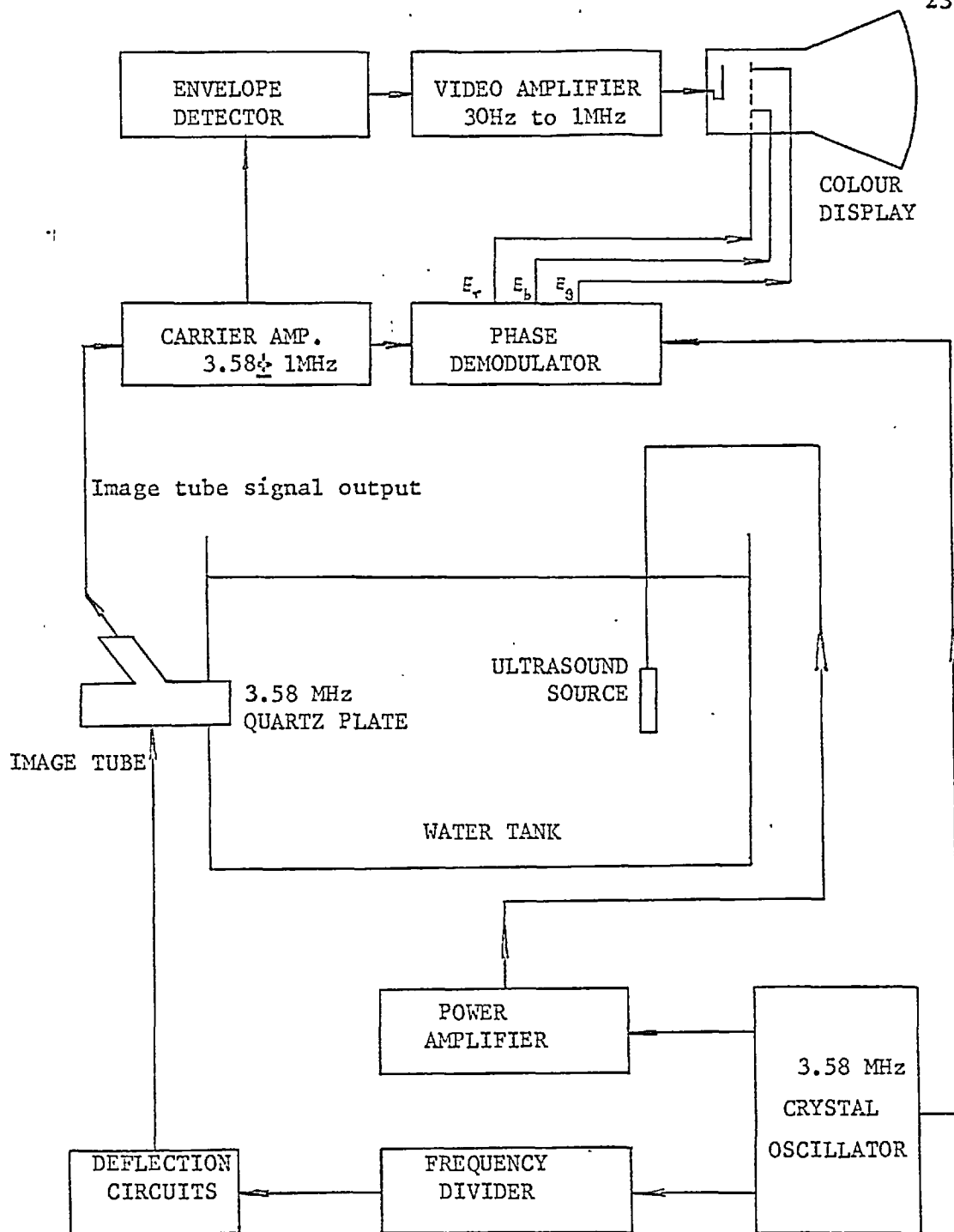


Figure 1.1 An ultrasound camera system using phase sensitive detection and a colour television display (after Jacobs)

material but not through it, to make 100 collinear detector elements. The electrical output from each of these elements can be successively switched using either diode or transistor circuitry. A neat way of separating one element signal from the leakage signals of all the other elements is employed. A non-linear switching characteristic is exploited which, together with a local oscillator, applied to each element sequentially, results in sum and difference frequency terms for that element output. These are then filtered out of the leakage signals from the remaining 99 elements. The device has a measured sensitivity of $7 \times 10^{-11} \text{ W cm}^{-2}$ for a 12 kHz bandwidth. Sensitivity is defined as the minimum detectable ultrasound intensity. This limit is imposed by the amplifier noise and so can be increased by improvements in the electronics to the limit set by the radiation resistance of each element. The maximum image resolution is set at 1.5 mm by the detector elements which are 1.5 mm square. Images are formed by means of an ultrasound lens and visualized by scanning the collinear array mechanically across the image plane. Recognisable images of simple objects imaged by back reflection have been obtained at ranges up to 4 m in turbid water containing as many as 10^3 ppm of 1μ mean particle diameter diatomaceous earth. Linear systems theory has also been used to distinguish between the pressure amplitude distribution in acoustic images of diffusely reflecting and specularly reflecting objects.

Harrold (5) has also investigated the possibilities of detector arrays. He has built an image conversion system using a 1" square array of $\frac{1}{10}$ " x $\frac{1}{10}$ " contact points on the back of a three inch diameter piezoelectric ceramic disc. Signals are taken out from each of the 100 elements by coaxial cables via spring loaded contacts. The array is scanned electronically and the output displayed on an oscilloscope screen as an intensity picture. No attempt has been made to

mechanically isolate one element from its neighbours, consequently image quality with this technique leaves much to be desired.

A technique which is in a state of development about half-way between that of Harrold and that of Green et al is described by Suckling and Hendrickson (6). They have used a narrow strip of piezoelectric material on which are mounted a line of 64 reversed biased diodes. An ultrasonic image is swept across the strip using a rotating plane mirror and at the same time the 64 diodes are sequentially biased on with an appropriate pulse at a rate of 20,000 pulses per second. Hence with an appropriately triggered oscilloscope an array of dots can be built up which are intensity modulated with the diode output as the image is swept over the detector. A picture rate of 5 frames per second is considered, but as yet only single line scans with a stationary image have been recorded. One of the main difficulties is distinguishing between the signal from the conducting diode and the leakage signal from the rest. There does not appear to be any reason why the technique employed so neatly by Green et al cannot be used here also.

Prokhorov (7) has put into practice the proposal of Turner (8) (see also Ref. (1)) for an ionic displacement potential transducer. Electrokinetic voltage is developed across the ends of capillaries in a glass filter plate as a result of the passage of an ultrasound wave. These are sensed with an array of conductors, each insulated from its neighbours, which lead through the wall of a camera tube where the voltage distribution is scanned with an electron beam in the conventional manner. This transducer is still in its early development stages but it has a sensitivity of approximately $10^{-5} \text{ W cm}^{-2}$ and can be constructed to have a large aperture which justifies continued interest in the technique.

The surface levitation technique has been subjected to some careful development work, particularly for its use in real-time ultrasonic holography, but Gericke and Grubinskas (9) have used it directly for the visualization of ultrasound diffraction patterns. This investigation is intended to assess the potentiality of the technique in non-destructive testing. Results obtained with CW, FM and pulsed ultrasound are compared and discussed. The general conclusions are that CW diffraction patterns are capable of indicating objects of the order of the ultrasonic wavelength but interpretation is difficult because of the presence of the near field transducer pattern. FM and pulsed ultrasound give cleaner looking 'shadows' but do not show such high resolution. No mention is made of difficulties due to external vibrations as experienced by other workers in this field so it must be concluded that these were of a sufficiently low level compared with the required signal so as not to be a problem.

Two new optical techniques for directly measuring the amplitude distribution across a surface forced by an ultrasonic wave have recently been described. Massey (10) has demonstrated an optical heterodyne system and Korpel (11) has successfully used the optical lever principle, to measure the extremely small surface displacements involved. Fig. 1.2 is a schematic diagram of Massey's arrangement. It is a high frequency adaption of an earlier system (12) used to measure vibration amplitudes at audio frequencies. The system shown is a true optical heterodyne as the 'local oscillator' is derived from the optical carrier wave coherent light source and so has the same frequency. With this system it is apparently possible to measure surface displacements as small as 10^{-3} Å which corresponds to a sound intensity of less than 10^{-11} W cm⁻² at 2.6 MHz in glass, provided all sources of electrical noise can be reduced to their respective statistical minima. The resolution of the optical

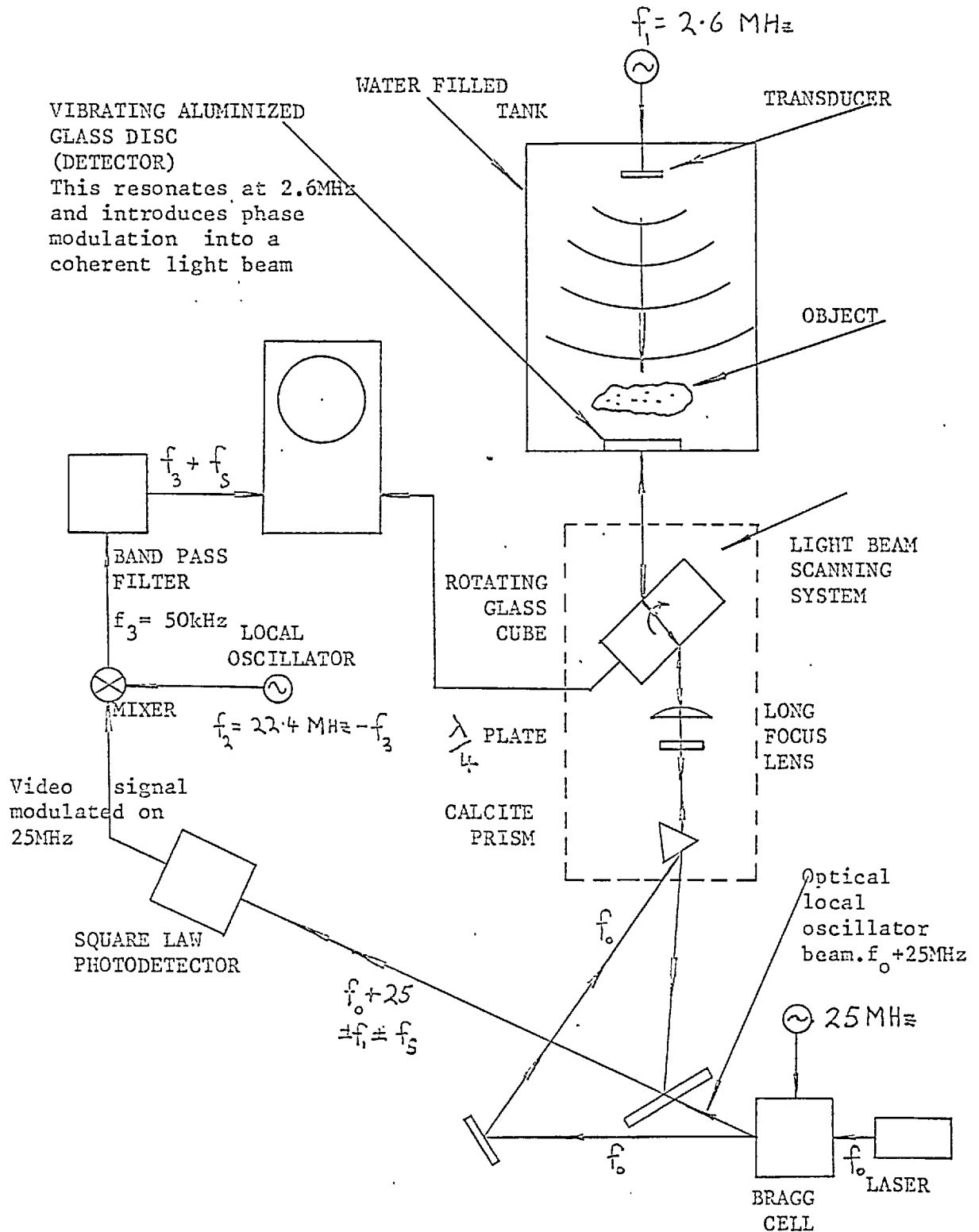


Figure 1.2 An ultrasonic visualization system using optical heterodyning. (After Massey)

heterodyne system is determined by the size of the scanning spot, which can be made very much smaller than the ultrasound wavelength at all frequencies of interest in non-destructive testing. Consequently the resolution of the ultrasound image converter is determined by the mechanical properties of the glass disc on which the ultrasound image is projected for visualization.

Korpel's technique in contrast uses spatial filtering and a photo-detector to produce an electrical signal with frequency terms at the ultrasound frequency plus the doppler frequency shift from the scanning speed of the light spot. Fig. 1.3 shows the experimental set-up. The laser scanning system is based on that developed earlier by Korpel (13) for a laser television display. Light is reflected from the highly polished surface of the converter and imaged on to a knife edge which converts the periodic deflections resulting from the surface displacement into intensity modulations which in turn produce an r.f signal from the photo-detector. The technique in fact responds to surface amplitude gradients and not directly to normal displacement components in direct contrast with Massey's technique. At a frequency of 2 MHz and with a perspex converter the theoretical sensitivity of Korpel's system is $3.7 \cdot 10^{-8} \text{ W cm}^{-2}$. In practice this was found to be $10^{-7} \text{ W cm}^{-2}$ which is still comparable with the sensitivity of the ultrasound camera. The technique does not have the same limitations on aperture as the camera and so has great potential application in ultrasound holography, Korpel (14).

A recent extension of this technique has been described by Korpel (15) which enables the amplitude and phase of a sound field to be sampled point-by-point within the bulk of an optically transparent material. This is achieved by focussing a laser beam down to a spot very much smaller than the ultrasound wavelength

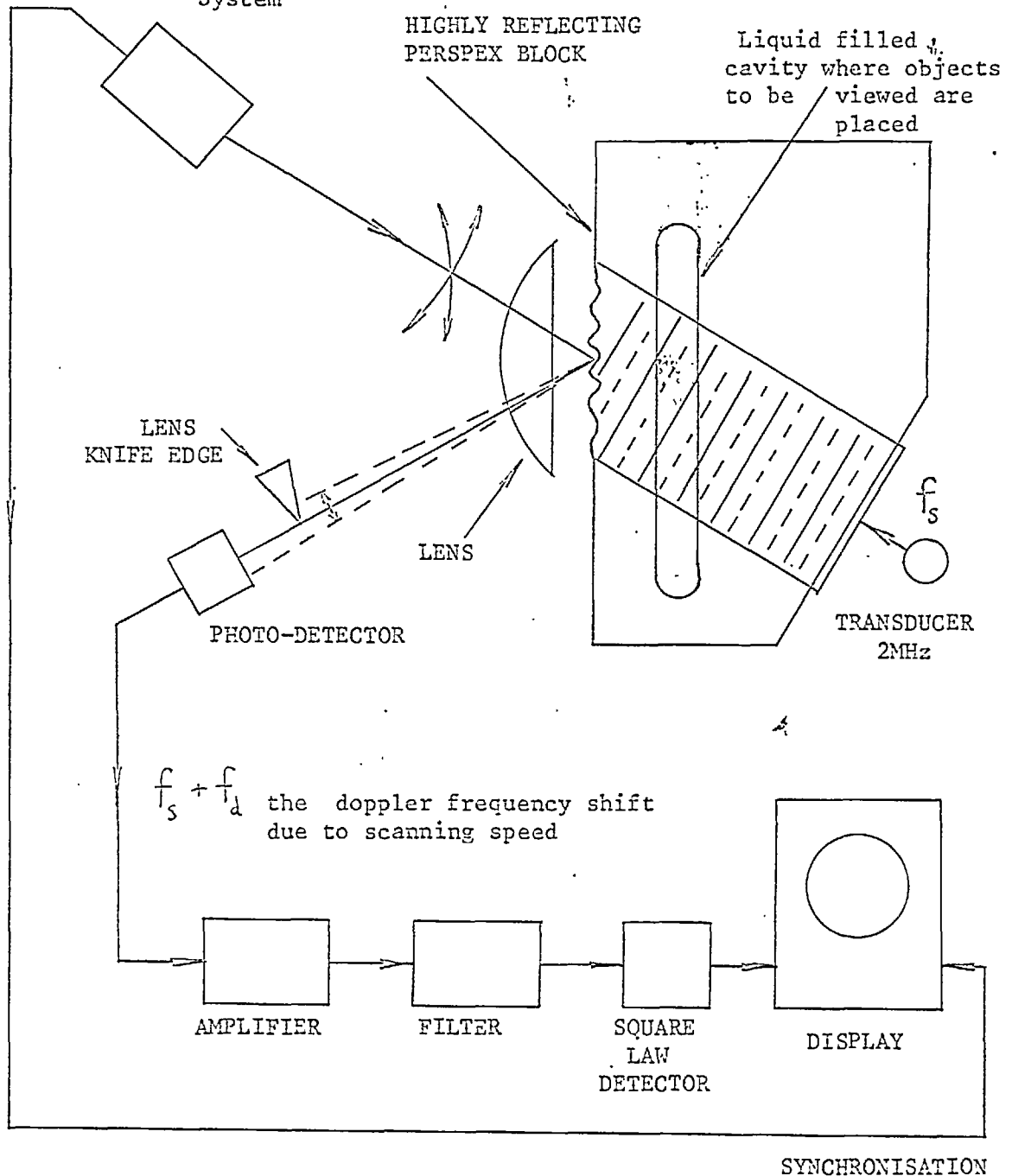


Figure 1.3 An ultrasonic surface deformation visualization system using the optical lever principle. (After Adler and Korpel).

within the field and scanning in the same manner as for the surface gradient technique described above. The emerging light beam is refracted at the ultrasonic frequency and an electrical signal whose amplitude and phase are related to that of the sound field at the focal point can be obtained by spatial filtering and detection. The technique will have far reaching consequences in ultrasound holography particularly where maximum resolution is desired. The sound field may be sampled in a fluid where the spatial frequency transfer function can be expected to have a wide bandwidth, owing to the absence of any other modes of propagation except a single longitudinal.

Amongst one of the more novel techniques which has been investigated recently is the use of liquid crystals to visualize ultrasound diffraction patterns. These, Ferguson claims (16), can be expected to show measurable optical changes with energy inputs as small as 10^{-10} W sec cm⁻² but sensitivities as high as this have not yet been achieved in practice. Once the experimental difficulties associated with the use of these materials has been overcome, they should find immediate application in ultrasonic holography. They combine high sensitivity and resolution capability with the possibility of real-time operation.

1.2 Ultrasonic Holography

There has been substantial progress in ultrasonic holography since my 1967 review (17). It is difficult to separate those steps which are a result of progress in image converter technology as described in 1.1 and those which represent steps forward in the technique of holography. There is also, of course, constant feed-back from developments in optical holography. The importance

of this field of study and its wide range of applications has only recently been accepted by the scientific community outside a small number of pioneer workers. To illustrate this, part of the last paragraph of Korpel's excellent review article is reproduced (18). "Acoustic holography seems more relevant and necessary because, in the final analysis, it may not be just another useful method, but the only method that will provide a solution to certain well-defined problems".

With considerable foresight Metherell organised the 'First International Symposium on Acoustical Holography' at Douglas Advanced Research Laboratories, California, on 14th and 15th December 1967. The proceedings have been published in book form (19) along with an extensive bibliography compiled by Metherell. Volume 2 of this work is to be published in 1970 and will contain the proceedings of the 'Second International Symposium on Acoustical Holography' held on 6th and 7th March 1969. Together these two volumes comprise the most comprehensive source of information on acoustical holography which is available. No further useful contribution can be made by reproducing here all the available information so only the most significant developments in ultrasonic holography will be discussed.

The credit for many of the innovations in ultrasound holography lies with the group at Battelle Memorial Institute, Pacific Northwest Laboratory. It is here that the techniques of liquid surface deformation real-time ultrasound holography and mechanically scanned ultrasound holography have found their greatest refinement. The results obtained with these two methods and their possible applications are compared by Smith and Brenden (20). Fig. 1.4 shows the basic arrangement used by Brenden (21), (22) for liquid surface

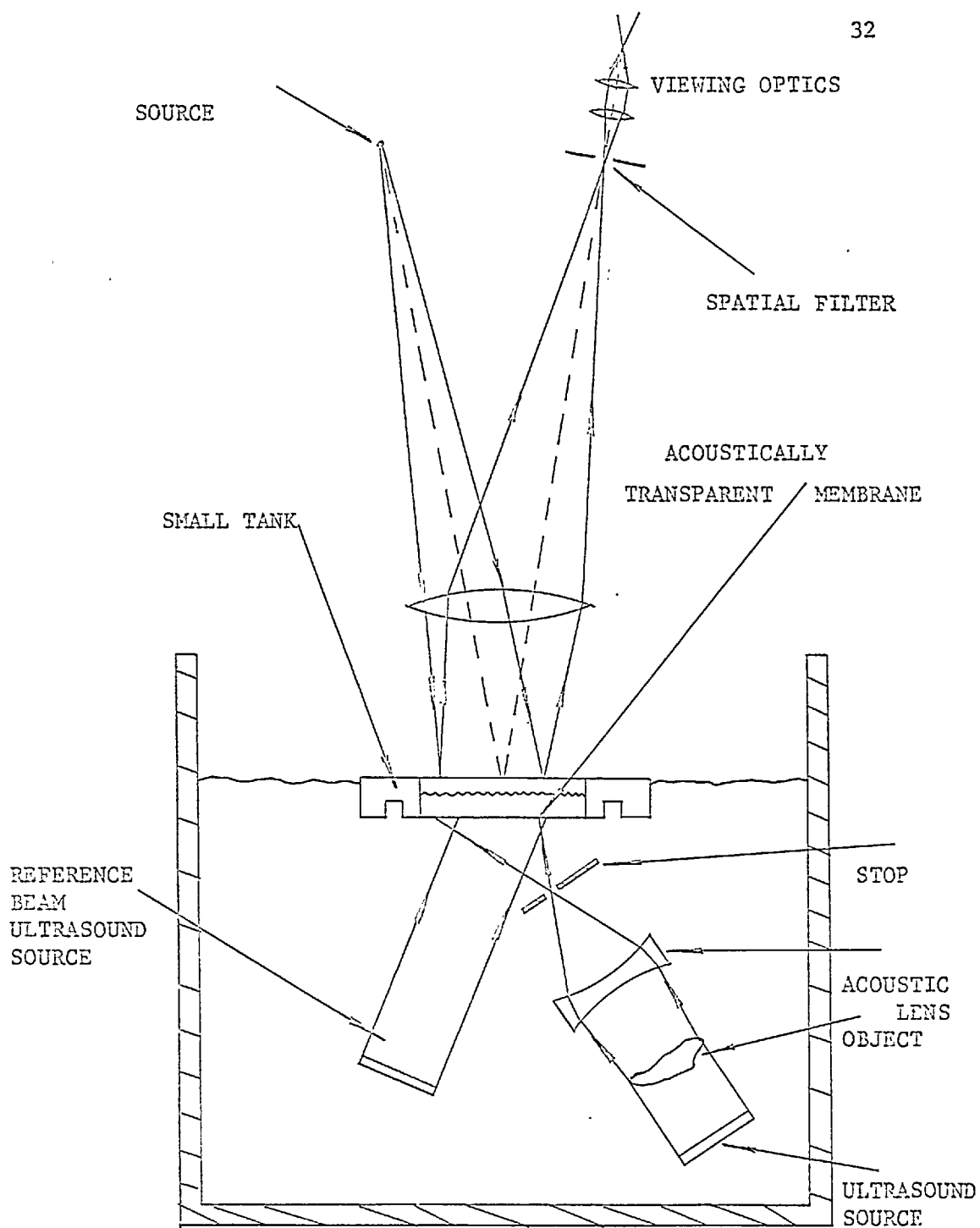


Figure 1.4 An experimental arrangement for liquid surface deformation ultrasound holography. (After Brenden).

deformation holography. In order to get good real-time reconstructions with this technique the system must be carefully designed. The conversion cell takes the form of a small tank situated inside a larger water tank and separated from it by an acoustically transparent membrane. This arrangement minimises transient disturbances of the hologram from movement of the main body of water in the large tank. Of particular importance is the liquid within the smaller tank; it is not water but Brenden declines to state specifically what it is. He does, however, indicate in general terms its optimum physical properties as follows. In order to maximise the sensitivity of the system the height of the information carrying surface deformations in the hologram must be as large as possible. Over the range of frequencies where this technique is used, 3 MHz to 50 MHz, the deformation amplitude is inversely proportional to the liquid surface tension. The average sound intensity over the hologram tends to push the surface outwards in a uniform manner; this is probably a major source of imperfection and can only be reduced by increasing the density of the liquid. To minimise streaming and resulting transient disturbances, pulsed ultrasound is used. Pulse lengths of 80 microseconds with a pulse repetition time of 3 milliseconds is near the optimum in the frequency range from 3 to 10 MHz. These periods are related to the time necessary to form the hologram, the time the hologram is stable before transients begin to disrupt it, and the time necessary to restore the undisturbed surface. Viscosity plays an important role in the surface damping effect, especially if the liquid is only several ultrasound wavelengths deep. It has been found that liquids with viscosities in the range from 2 to 10 cp give good results. In view of the conflicting requirements of low surface tension and high viscosity, Brenden implies that he chooses a liquid having the best viscosity and modifies the surface tension with additives.

It is not generally necessary to focus an ultrasound image on to the liquid surface to form a hologram as shown in Fig. 1.4 but this arrangement gives the best reconstructions. Also, with this technique, which is analogous to focused image holography in optics, the coherence requirements for the reconstructing source are much less stringent. Coherence has only to be maintained over the image 'depth' of a few millimetres. Brenden has found that an unfiltered, 'super-pressure', mercury vapour source $\frac{1}{2}$ mm in diameter gives good reconstructions. High pass spatial filtering of the reconstruction is also employed to remove any transient disturbances which remain after all the above precautions have been taken as these are usually of a lower spatial frequency than the object information.

One disadvantage of the technique is its relatively low sensitivity (approximately 10^{-3} W cm⁻²) which limits its use to transinsonified objects. This has not, however, prevented its possible application in the field of medical diagnosis as an instrument to detect breast tumors in women. Owing to the real-time nature of the technique and the rapid rate at which holograms are formed with a pulsed system, it is possible to alternately form holograms at high and low ultrasonic frequencies or 'colours' (23). These may be harmonics generated from a single pair of transducers operating at the same frequency. The reconstructions are colour coded by inserting appropriate colour filters in the focal plane of the high pressure light source. Hence coloured images are obtained where the colours give an indication of differences in acoustical properties of the object at different frequencies.

Refinements in mechanically scanned ultrasound holography have been somewhat less dramatic but none the less of considerable importance. In most reports the systems described have been

analogues of the configurations used in optical holography in which the results are entirely predictable from sampling theory and conventional holographic image equations. However, some scanning systems have closer ties with coherent radar than optical holography. The general problem of simultaneous source and receiver scanning has been analysed theoretically by Hildebrand and Haines (24). They have shown that it is possible to obtain an increase in resolution by simultaneously scanning source and receiver. In the limit when the source and receiver occupy the same position, the resolution is twice that which would be obtained if either were fixed whilst the other was scanned. The reason for this is that when both source and receiver are moving with respect to the object, the total path length to any particular object point is changing at twice the rate which would occur for fixed illumination. Consequently the rate of change of phase with position will be twice as great and so the spatial frequencies recorded in the hologram will be doubled. Another practical advantage of the technique is that the object is illuminated more uniformly when the source position is not fixed. This minimises the effect of 'highlighting' which results from specular reflection of sound by the object. By utilizing differential rates of scanning for the source and receiver, it is possible to obtain undistorted magnification in the reconstructed image. These theoretical predictions have been verified by Smith and Brenden (20).

A technique for obtaining coloured reconstructions of acoustic holograms recorded by the mechanical scanning system has been described by Pisa, Spinak and Metherell (25). Three separate holograms of the letter R were recorded at 15, 18 and 21 kHz. The three resulting images were superimposed to give a multi-colour reconstruction.

Although the mechanical scanning technique has interesting properties, it does not have a sufficiently rapid frame time to immediately guarantee its use commercially. As a result the ultrasound camera has been used to visualize ultrasound holograms (26), (27), (28). The quality of the reconstructions obtained from photographs of the ultrasound camera television display is much worse than that obtained by the mechanical scanning technique. This is largely due to the resolution limitation imposed by the converter plate of the ultrasound camera, which is the subject of the investigations described in the main part of this thesis.

Attempts are being made to couple the ultrasound camera output to a display system which will enable reconstructions to be obtained in the camera frame time. One possibility using an electro-optic projection system first designed for television has been put forward by Boutin and Mueller (29). By means of the electro-optic effect the electrical signal derived from the ultrasound camera is converted into an optical phase hologram. This is then reconstructed by illumination with coherent light as in the surface deformation technique. A feasibility study of this system has been undertaken by Goetz (30). He concludes that it is possible, provided polishing techniques are developed which yield flat surfaces on deuterated potassium dihydrogen phosphate (KD^*P) crystals sufficiently thin to produce the required resolution.

Computer reconstruction of acoustic holograms has recently been described by Powers, Wade and Landry (31) and others (19). Although this technique is capable of removing unwanted noise and correcting for image distortion, it seems to be a step away from the elegant simplicity of parallel optical processing of which optical hologram reconstruction is an example. There are already

in existence, particularly in the field of underwater acoustics, successful techniques of signal processing which enable images of objects to be produced. (See for example Tucker's review (32)).

1.3 Investigations of Detector Limited Resolution

To an acoustician familiar with normal modes of rooms and vibrating panels, it seems surprising that a continuous plate of piezoelectric material like that of the ultrasound camera detector can respond point-by-point over its area to a spatially complicated sound pressure field. A finite plate can vibrate in many different modes so he would suggest that the standing stress distribution across the plate thickness results from the linear superposition of contributions from every mode which may be driven at that frequency. He might go further and point out that the resulting stress pattern could not be expected to bear any spatial relationship to the incident pressure field. The ultrasound camera results show unequivocally that this latter statement is not true, provided that the ultrasound frequency is carefully selected to be close to the thickness resonant frequency of the detector plate. Although the visualized images under these conditions are similar in general to the pressure image on the detector face, it is still clear that the detector does limit the resolution.

The most detailed investigation into the response of quartz and barium titanate discs for use in ultrasound image converters, to be found in the literature, has been conducted by Hartwig (33). He has measured the transverse extent of the voltage distribution across a sequence of piezoelectric discs, as a function of time frequency, resulting from a 2 mm diameter sound beam incident normally

along the Z axis of the barium titanate and the X axis of the quartz. The frequencies were in the range extending from 3.9 MHz to 9.5 MHz, using discs of diameters from 12 to 26 mm. By measuring the spread of signal outside the ideal 2 mm diameter region over this frequency range, he obtained a qualitative idea of the detector resolution and its frequency dependence. The main conclusion was that the spreading is a minimum for a frequency just below thickness resonance. In the majority of cases the point spread functions obtained by Hartwig were asymmetrical, even with the rotationally symmetric barium titanate and so it was difficult for him to compare spread functions at one frequency with those at another. The method he chose was to compare amplitudes at 2.5 mm either side of the centre of the incident sound beam. Such a method would only be accurate if the point spread functions were smooth monotonically decreasing functions of radius but as they were generally of oscillatory form the comparison is of necessity extremely qualitative. Unfortunately Hartwig places a severe restriction on his experiment by fixing the direction of propagation of the incident sound field along the plate normal and at the same time measuring the voltage across the disc thickness. This automatically removes the angle of incidence as a parameter in the investigation which is not zero in any imaging or holographic application.

Several designers of ultrasound cameras have also conducted experiments to determine the resolution limits of the piezoelectric detector. Smyth, Poynton and Sayers (34), using an aperture mask with small holes in it, concluded that at normal incidence, when the mask was against the detector, the resolution limit was approximately equal to the detector thickness.

Jacobs (35) also experimentally investigated the resolution of his ultrasound camera and concluded that the point spread function under normal incidence was a minimum just below thickness resonance.

A theoretical analysis of the spread of vibration from a forced to an unforced region of a free, infinite, isotropic plate has been carried out by Lyon (36). The motivation for this was a desire to predict the resolution limit of the ultrasound camera detector and to choose a frequency of operation where the resolution would be a maximum. In an elegant manner the characteristic frequency equation for symmetric and anti-symmetric Lamb waves is calculated. The mode spectrum is then evaluated numerically for an isotropic material with a Poisson's ratio of 0.31, the same as that for barium titanate. Both propagating and evanescent modes are included. Using as a boundary condition a periodic normal stress applied to a strip across the infinite plate, contour integration is used to determine the resulting particle displacement amplitude normal to the surface. This is seen to be the linear superposition of contributions from all the modes which are driven at the chosen frequency of the applied stress. As an example Lyon calculates the normal particle displacement along a plate of barium titanate. The plate is chosen to have a thickness resonance of 1 MHz and it is driven over a 1 cm wide strip at 2.5 MHz. This frequency is chosen by reference to the mode spectrum, selecting a point where there are no strong cross-resonances (ie a point where the modes are widely spaced in frequency). Near the fundamental thickness mode (1 MHz) there is a strong cross-resonance and hence the vibration spreading should be large. At 2.5 MHz Lyon predicts that the particle displacement will fall to $1/e$ of its initial value at a point 0.5 mm outside the strip boundary, indicating a spread of 10%. Experimental confirmation of this result does not seem to have been carried out, possibly because

of the loss in sensitivity of an ultrasound detector away from thickness resonance.

CHAPTER 2

Theory

2.1 Fundamental Linear Systems Theory

The linear systems approach to image formation has been developed over the last few decades from the marriage of classical physical optics and electrical communications theory. There are now several excellent books on this subject, in particular those of Goodman (37) and O'Neil (38), so it is only necessary to outline the fundamental ideas to provide a basis for the later sections of this chapter. The historical background is given in Section 9.5 of the treatise by Born and Wolf (39).

Any physically realizable system may be represented by a differential operator $\mathcal{L}_1$. If the system is now subjected to a deterministic forcing function or input, represented in two dimensions by $f_1(x_1, y_1)$ although the ideas are applicable in any number of dimensions, then the system response or output $f_2(x_2, y_2)$ is given by:

$$\mathcal{L}_1 \left[f_2(x_2, y_2) \right] = f_1(x_1, y_1) \quad (2.1)$$

Equation (2.1) is a general differential equation; it is often more convenient in linear systems theory to work with integral equations. By transposing the operator, (2.1) can be written as an integral equation. For example see Sneddon (40).

$$f_2(x_2, y_2) = \mathcal{L}_1^{-1} \left[f_1(x_1, y_1) \right] \quad (2.2)$$

Little further can be said unless more is known about the system, ie, some of the properties of L are specified.

When the system under consideration is linear, particularly simple relationships between $f_2(x_2, y_2)$ and $f_1(x_1, y_1)$ can be derived. For a system to be linear the sum of its responses to individual stimuli must be the same as the total response from the same stimuli acting simultaneously. If $p(x_1, y_1)$ and $q(x_1, y_1)$ are two input functions and a and b are two complex constants, then a system will be linear if:

$$L^{-1} \left[ap(x_1, y_1) + bq(x_1, y_1) \right] = aL^{-1} \left[p(x_1, y_1) \right] + bL^{-1} \left[q(x_1, y_1) \right] \quad (2.3)$$

L is a linear differential operator.

An example of a linear differential operator which is fundamental to the study of scalar diffraction is $\left[\nabla^2 - \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \right]$. This describes the propagation of a scalar disturbance through a region of space which does not contain sources or sinks. As a consequence of the linearity property, if an arbitrary input function can be decomposed into a series of unit or elemental functions, these can then be passed through the system (ie operated on by L^{-1}) and the response synthesised as the sum of the elemental responses. This decomposition can be accomplished by using the so-called sifting property of the delta function; it enables a function $f_1(x_1, y_1)$ to be represented as a series of weighted and displaced delta functions.

$$f_1(x_1, y_1) = \iint_{-\infty}^{\infty} f_1(\xi, \eta) \delta(x_1 - \xi, y_1 - \eta) d\xi d\eta \quad (2.4)$$

Where ξ, η are variables of integration in the (x_1, y_1) plane or in the (x_2, y_2) plane when appropriate.

The response of a linear system to a delta function input is called the impulse response in communications theory, the spread function in optics and the Green's function in mathematical physics. Here it will be represented by the function $g(x_2, y_2; \xi, \eta)$ so that

$$L \left[g(x_2, y_2; \xi, \eta) \right] = \delta(x_1 - \xi, y_1 - \eta) \quad (2.5)$$

Using (2.1), (2.4) and (2.5) in turn the response of a linear system in terms of the input and the system Green's function is obtained as a superposition integral.

$$f_2(x_2, y_2) = \iint_{-\infty}^{\infty} f_1(\xi, \eta) g(x_2, y_2; \xi, \eta) d\xi d\eta \quad (2.6)$$

If the linear system under consideration is an imaging system whose point spread function depends only on the distances $(x_2 - \xi)$ and $(y_2 - \eta)$ then it is called space-invariant. Usually a real imaging system will be space-invariant only for small regions of the object field, for example, the region close to the principal axis. It is however possible to divide the object field into regions such that the system is space-invariant in each region individually, with a corresponding point spread function for each.

For a space-invariant system the superposition integral then takes the form of a two dimensional convolution of the object function

with the point spread function of the system.

$$f_2(x_2, y_2) = \iint_{-\infty}^{\infty} f_1(\xi, \eta) g(x_2 - \xi, y_2 - \eta) d\xi d\eta \quad (2.7)$$

Equation (2.7) will be used frequently and so it is convenient to write it in a shorthand notation:

$$f_2 = f_1 \otimes g \quad (2.7a)$$

Here the symbol $\otimes$ indicates that the functions on either side are to be convolved.

Equation (2.7) takes on a particularly simple form after Fourier transformation of both sides. This operation reveals the great power of the linear systems approach to optical image formation. The convolution operation in the spatial domain is replaced by a simple multiplication in spatial frequency space.

Let $F_2(f_x, f_y)$ and $F_1(f_x, f_y)$ be the output and input spectra of the linear system. $F_1(f_x, f_y)$ is given by the Fourier transform of $f_1(x_1, y_1)$ and conversely $f_1(x_1, y_1)$ is given by the inverse Fourier transform of $F_1(f_x, f_y)$. Mathematically the Fourier transform of $f_1(x_1, y_1)$ is written as

$$\mathcal{F}[f_1(x_1, y_1)] = F_1(f_x, f_y) = \iint_{-\infty}^{\infty} f_1(x_1, y_1) \exp[-j2\pi(f_x x_1 + f_y y_1)] dx_1 dy_1 \quad (2.8)$$

The inverse Fourier transform of $F_1(f_x, f_y)$ is written

$$\mathcal{F}^{-1} \left[F_1(f_x, f_y) \right] = f_1(x_1, y_1) = \iint_{-\infty}^{\infty} F_1(f_x, f_y) \exp \left[j2\pi(f_x x_1 + f_y y_1) \right] df_x df_y \quad (2.9)$$

Similarly the spectrum of F_2 is given by its Fourier transform.

(Functions of frequency are represented by capital letters whereas small letters are used to indicate functions of time or position.)

Mathematically f_x and f_y are defined as spatial frequency variables in a wave number space corresponding to the cartesian co-ordinates x and y . Spatial frequency, therefore, has dimensions of reciprocal length. A more physical interpretation of spatial frequency will be given later when diffraction is discussed. (Section 2.4).

On taking the Fourier transform of each side of (2.7) it can be shown that:

$$F_2(f_x, f_y) = G(f_x, f_y) \cdot F_1(f_x, f_y) \quad (2.10)$$

where $G(f_x, f_y) = \mathcal{F} \left[g(x_2, y_2) \right]$.

The function $G(f_x, f_y)$ is called the transfer function of the system.

Consequently when the spectrum of an input signal to a space invariant linear system is known, the spectrum of the system response is given simply by the multiplication of the input spectrum and the system transfer function. An inverse Fourier transform of the product then gives the system response:-

$$f_2(x_2, y_2) = \mathcal{F}^{-1} \left[G(f_x, f_y) \cdot F_1(f_x, f_y) \right] \quad (2.11)$$

This integral equation is to be compared with equation (2.7) which gives the same result. Consequently $G(f_x, f_y)$ must contain as much information characteristic of the system as $g(x_2 - \xi, y_2 - \eta)$.

The property of linearity allows the response of a series of linear systems cascaded together to be calculated from successive multiplications of the individual transfer functions of the systems. Thus if $G_1(f_x, f_y)$, $G_2(f_x, f_y)$, $G_3(f_x, f_y)$... are the transfer functions of a series of linear systems (1), (2), (3)

$$f_{n+1}(x_{n+1}, y_{n+1}) = \mathcal{F}^{-1} \left[\prod_n G_n(f_x, f_y) \cdot F_1(f_x, f_y) \right] \quad (2.12)$$

The mathematical processes are shown schematically in Fig. 2.1.

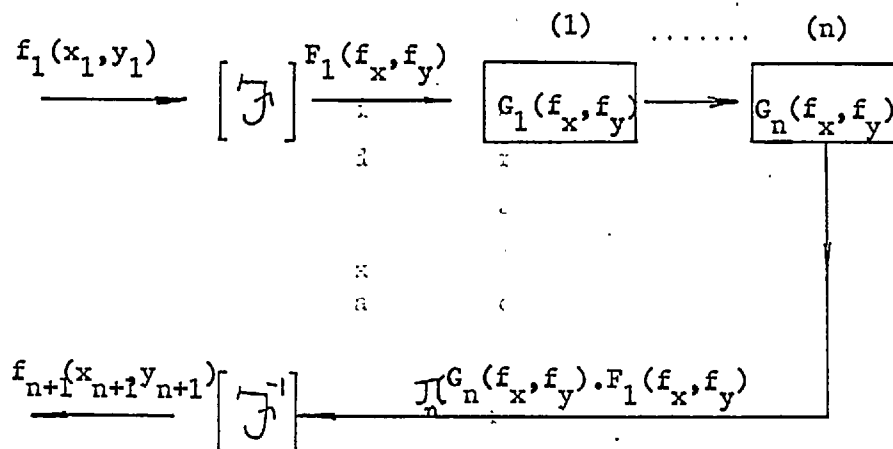


Figure 2.1

2.2 Transfer Functions for Linear Systems

The transfer function is determined for a specified parameter in which the system is linear. For example an incoherently illuminated optical imaging system is linear for light intensity mapping from object point to image point and hence will have an intensity transfer function. This is usually known as the optical transfer function (O.T.F.). On the other hand, the same system coherently illuminated is linear for complex amplitude mapping and will have a complex amplitude transfer function which is usually called a coherent transfer function. Consequently an ultrasonic imaging system which is linear in velocity potential will have a complex velocity potential transfer function (V.T.F.).

A passive electronic circuit such as a linear filter will possess a transfer function which relates voltage output, in amplitude and phase to the voltage input, in amplitude and phase. It will, therefore, have a so called filter function which is analogous to the coherent transfer function but the independent variable will be temporal frequency and not spatial frequency. On many occasions it will be useful to draw analogies from electronics to assist in the description of coherent optical and ultrasonic systems.

If $g(x_2, y_2)$ represents the space invariant impulse response of an optical system, equation (2.7) gives the response of the system to a complex amplitude $f_1(x_1, y_1)$ and hence $G(f_x, f_y)$ is the coherent transfer function of the system.

With incoherent illumination the system is linear in intensity, so equation (2.7) may be written

$$f_2(x_2, y_2) f_2^*(x_2, y_2) = \iint_{-\infty}^{\infty} f_1(\xi, \eta) f_1^*(\xi, \eta) |g(x_2 - \xi, y_2 - \eta)|^2 d\xi d\eta$$

$$= I_2(x_2, y_2) \quad (2.13)$$

where $I_2(x_2, y_2)$ is the intensity distribution in the response and * indicates a complex conjugate.

$$|g(x_2 - \xi, y_2 - \eta)|^2 = g(x_2 - \xi, y_2 - \eta) g^*(x_2 - \xi, y_2 - \eta) \quad (2.13a)$$

Intensity is always non-negative so it is usual to normalize an incoherent spectrum with respect to the zero spatial frequencies. The zero spatial frequency term in the spectrum represents the "d.c." or background intensity level on which information carrying intensity fluctuations are superimposed. The intensity of these variations relative to that of the constant background determines the contrast in the input or output distributions. The transfer function derived from $|g(x_2 - \xi, y_2 - \eta)|^2$ normalized at $f_x = 0$ and $f_y = 0$ is the optical transfer function $\mathcal{H}(f_x, f_y)$. Hence

$$\mathcal{H}(f_x, f_y) = \frac{\iint_{-\infty}^{\infty} |g(x_2, y_2)|^2 \exp[-j2\pi(f_x x_2 + f_y y_2)] dx_2 dy_2}{\iint_{-\infty}^{\infty} |g(x_2, y_2)|^2 dx_2 dy_2} \quad (2.14)$$

The modulus of $\mathcal{H}(f_x, f_y)$ is called the modulation transfer function (M.T.F.) and is therefore the function which describes how contrast is transmitted through an incoherently illuminated system.

It is similarly possible to define a modulation transfer function which describes how contrast is transmitted through a coherent system. This is of particular importance in holography where it is usual for the input functions to take the form of a spatial carrier, amplitude modulated with the object information. The amplitude of the modulation cannot be greater than the carrier amplitude otherwise the depth of modulation exceeds unity. Hence for a modulated coherent signal we have the same sort of situation as for an incoherent signal, ie the information is carried by a non-negative signal. There is, however, no need to use an impulse response function squared, it is necessary to simply define the modulation transfer function for a coherent system as the modulus of the coherent transfer function. Therefore the coherent modulation transfer function is given by $|G(f_x, f_y)|$.

$G(f_x, f_y)$ is normally a complex quantity with amplitude $G_m(f_x, f_y)$ and phase $\phi(f_x, f_y)$ written as

$$G(f_x, f_y) = G_m(f_x, f_y) \exp j\phi(f_x, f_y)$$

Therefore

$$|G(f_x, f_y)| = G_m(f_x, f_y) \quad (2.15)$$

$G_m(f_x, f_y)$ is exactly analogous to the gain function of a linear electrical circuit.

$G_m(f_x, f_y)$ is always positive but if the phase ϕ changes by π then $G(f_x, f_y)$ will suffer a change of sign.

An illustrative example of the signs of $G(f_x)$ and $G_m(f_x)$ is shown for an arbitrary function of $G(f_x)$ in Fig. 2.2.

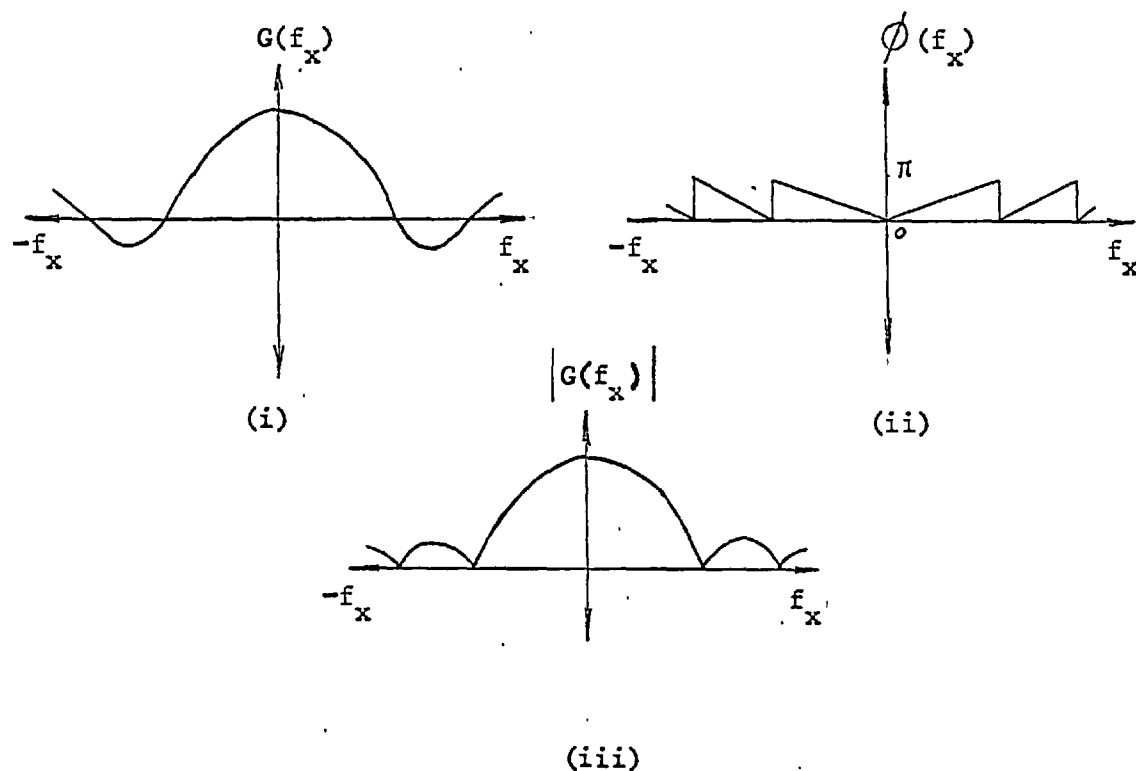


Figure 2.2

A figure of the form $|G(f_x)|$ would be recorded if $G(f_x)$ were converted into an electrical analogue signal which was subsequently passed to a detector giving the R.M.S. value of the analogue signal, ie a detector which performed the operation of converting $G(f_x)$ into $[G(f_x)G^*(f_x)]^{1/2}$. In general $G_m(f_x, f_y)$ changes the contrast in a response function whereas $\phi(f_x, f_y)$ introduces a lateral shift in its position relative to the input function.

2.3 Space Variant Linear Systems

The ability to replace the superposition integral (2.6) by the convolution integral (2.7) is a special case of the general theory of space variant linear systems. As the name implies, the impulse response of a space variant linear system is a function of the position in the system of the point source. Therefore if $\delta(x_1 - \xi, y_1 - \eta)$ represents a point source situated at (ξ, η) then the point spread function is a function of both $(x_2 - \xi)$ and ξ and $(y_2 - \eta)$ and η .

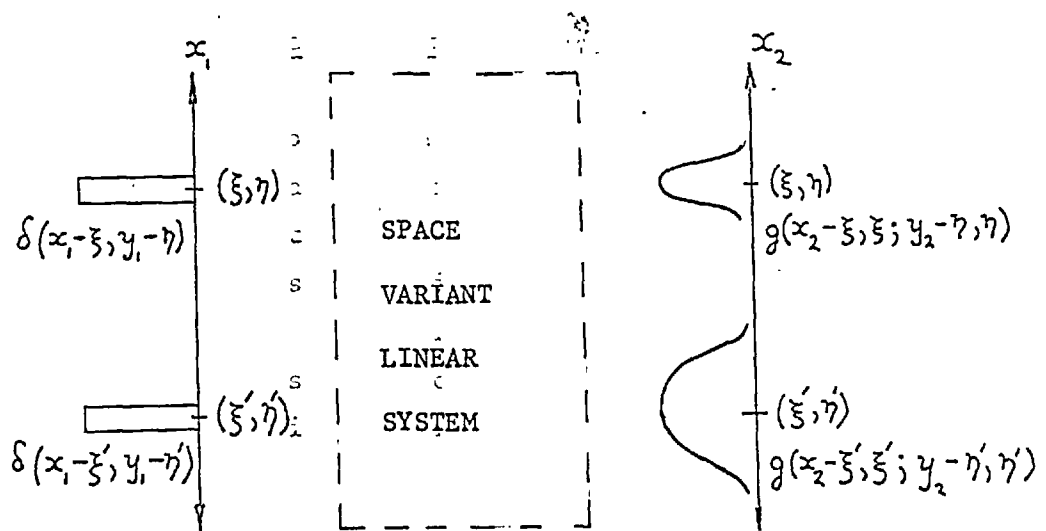


Figure 2.3

(x_1, y_1) defines the spatial plane of the input signal and (x_2, y_2) defines the spatial plane of the system response. As the point source explores the input plane, the corresponding spread function changes shape, see Fig. 2.3. Therefore if the space variant linear system was an element of an image forming system, such as the detector, then the shape of the image would depend on the position in the object plane, (x_1, y_1) of the object. This behaviour contrasts sharply with that of a linear space invariant imaging system where the image does not change shape as the lateral position of the object is changed. At first sight space variance does not seem to be a useful property of imaging systems, but as Lohmann and Paris (41) have pointed out, for certain classes of permissible object a space variant image forming system can have resolution greater than the classical Rayleigh limit. One class of permissible objects consists of all objects that do not extend outside a given region of the object plane. Provided that an error free determination of the spatial distribution of image intensity can be carried out, then with a system whose space variance is known the object distribution can be determined uniquely. This is of particular interest in ultrasonic imaging systems where resolution is usually at a premium and so the ability to obtain resolution greater than the classical limit is very desirable. As the detector used in the ultrasound camera is a space variant linear system (see Chapter 5), the possibility of utilizing this to advantage offers an additional degree of freedom which has not yet been realised.

The degree of space variance of a linear system can be defined with reference to the cross-correlation of two line spread functions. A one dimensional system will be considered for simplicity. The cross correlation function $C(p, \xi, \xi')$ of two spread functions which originate from line sources at ξ and ξ' is

defined by the usual integral

$$C(p, \xi, \xi') = \int_{x_2} g(x_2 + \frac{p}{2} - \xi; \xi) g^*(x_2 - \frac{p}{2} - \xi'; \xi') dx_2 \quad (2.16)$$

The correlation function C is the integral of the product of the two line spread functions after one of them has been displaced by a distance p along the x_2 axis. The degree of space variance $\sigma(\xi, \xi')$ is defined as

$$\sigma(\xi, \xi') = \frac{C(\xi - \xi', \xi, \xi')}{[C(0, \xi', \xi') C(0, \xi, \xi)]^{\frac{1}{2}}} \quad (2.17)$$

2.4 Scalar Diffraction Theory in Optics and Acoustics

The propagation of acoustic waves in fluids is usually described by means of a scalar point function known as the velocity potential Φ , provided that the displacement from equilibrium position and the velocity of the medium particles are small. In regions distant from sources Φ is a solution of the time dependent homogeneous wave equation

$$\nabla^2 \Phi - \frac{1}{c^2} \frac{\partial^2 \Phi}{\partial t^2} = 0 \quad (2.18)$$

where c is the phase velocity of the acoustic radiation.

The sound pressure P is then given by:-

$$P = \rho_0 \frac{\partial \Phi}{\partial t} \quad (2.19)$$

where ρ_0 is the density of the medium in equilibrium.

The particle velocity $\underline{U}$ is given by :-

$$\underline{U} = \text{grad } \phi \quad (2.20)$$

If it is assumed that ϕ is a harmonic function of angular frequency ω , then ϕ , P and $\underline{U}$ become:-

$$\left. \begin{aligned} \phi &= \phi e^{-j\omega t} \\ P &= p e^{-j\omega t} \\ \underline{U} &= \underline{u} e^{-j\omega t} \end{aligned} \right\} \quad (2.21)$$

ϕ , p and $\underline{u}$ are thus independent of time and so are functions only of position.

ϕ satisfies the time independent acoustical wave equation, Helmholtz's equation.

$$\left[\nabla^2 + k^2 \right] \phi = 0 \quad (2.22)$$

where k is the wave number related to the free-field wavelength λ_0 by

$$k = \frac{\omega}{c} = \frac{2\pi}{\lambda_0} \quad (2.23)$$

p and $\underline{u}$ are related to ϕ by the equations:-

$$p = \rho_0 c j k \phi \quad (2.24)$$

$$\underline{u} = \text{grad } \phi \quad (2.25)$$

ϕ is a scalar potential function because the velocity field is considered to be irrotational and hence $\text{curl } \underline{u} = 0$. The Helmholtz equation is a linear differential equation and hence it is expected that linear systems theory can be applied to the scalar wave diffraction problem. The linear 'system' in this case will be the propagating medium.

The formulation of the scalar wave diffraction problem has been the subject of discussion for over one hundred and fifty years. Under carefully prescribed conditions an exact integral solution of (2.22) can be obtained subject to a given set of boundary conditions. See for example Baker and Copson (42). The number of diffraction problems which can be solved exactly is very small.

Of particular interest in both the general theory of an ultrasound holographic system and in the investigation of the resolution of an ultrasound detector is the relation between an arbitrary source distribution and the resulting radiation field. This is important both in the calculation of the holographically reconstructed image intensity and in the experimental technique devised to measure the modulus of the coherent transfer function of the detector. In the case of acoustical diffraction the velocity potential has already been defined as a solution of the Helmholtz equation (2.22).

In optical diffraction the situation is more complicated in the general case because of the physical nature of light which results in intensity being the observable quantity. Intensity does not satisfy a wave equation. A measurable quantity which satisfies a pair of coupled wave equations and hence can be used

to describe the propagation of light is called the mutual coherence function $\Gamma_{12}(\tau)$. This is defined as the complex cross-correlation of the optical disturbance at two typical field points (x_1, y_1) and (x_2, y_2) taken over a long time average

$$\Gamma_{12}(\tau) = \langle f_1(x_1, y_1, t + \tau) f_2^*(x_2, y_2, t) \rangle \quad (2.26)$$

f_1 and f_2 describe the light disturbances at (x_1, y_1) and (x_2, y_2) respectively.

$\tau = (t_2 - t_1)$ = optical path difference between the two field points divided by the velocity of light and $\langle \rangle$ denotes a time average.

It can be shown (43) that in the case of quasi-monochromatic, coherent light

$$\Gamma_{12}(\tau) = \psi(x_1, y_1) \psi^*(x_2, y_2) \exp j2\pi v' \tau \quad (2.27)$$

where $\psi(x, y)$ satisfies the Helmholtz equation.

$$[\nabla^2 + k'^2] \psi = 0$$

v' and k' are the mean frequency and wave number respectively for the quasi-monochromatic radiation. Consequently the propagation problem for quasi-monochromatic coherent light is solved by a solution of Helmholtz's equation in a fashion similar to acoustical propagation problems.

It is, therefore, possible to discuss the scalar wave diffraction problem for both the acoustical and optical cases in

terms of a general potential function f and then substituting for ϕ or ψ to fit the appropriate acoustical or optical problem. f must satisfy Helmholtz's equation:

$$\left[\nabla^2 + \kappa^2 \right] f = 0. \quad (2.28)$$

κ is a wave number for the general propagation problem.

It is next necessary to define a source of potential f situated at an arbitrary point in space and represented by a delta function $\delta(\underline{r} - \underline{r}')$, which is a three dimensional function

$$\delta(\underline{r} - \underline{r}') = \delta(x - x')\delta(y - y')\delta(z - z') \quad (2.29)$$

An appropriate Green's function $g(x, y, z,)$ is defined as the solution of (2.28) for a delta function source (2.29). Hence

$$\left[\nabla^2 + \kappa^2 \right] g = -\delta(\underline{r} - \underline{r}') \quad (2.30)$$

Now if a point Q is defined such that it is completely surrounded by a closed surface S on which f is known, by applying Green's theorem it can be shown that the potential at Q , $F(Q)$ is given by:

$$F(Q) = \frac{1}{4\pi} \int_S (g \text{ grad } f - f \text{ grad } g) \cdot \underline{n} \, d\sigma \quad (2.31)$$

where $\underline{n}$ is the unit normal to the surface element $d\sigma$.

Fig. 2.4 shows a situation commonly encountered in diffraction problems. A source of f is situated in the left hand half space and the value of f is required at a point in the right hand half space

when a diffracting obstacle, in this case a screen with an aperture, is placed between the two half spaces.

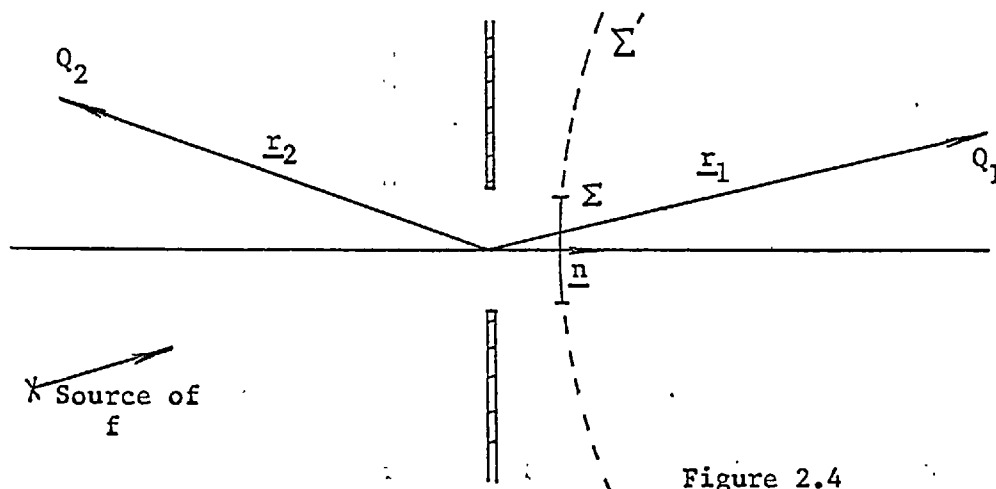


Figure 2.4

It cannot be assumed that both f and $\frac{\partial f}{\partial n}$ are zero only over the region of the screen Σ' facing the right hand half space because this implies that f is zero over the entire surface $\Sigma + \Sigma'$. It is necessary to restrict either f or $\frac{\partial f}{\partial n}$ and in addition choose g so that either g or $\frac{\partial g}{\partial n}$ is zero over the entire surface. Using (2.31)

$$F(Q_1) = \frac{1}{4\pi} \int_{\Sigma} \left(g \frac{\partial f}{\partial n} - f \frac{\partial g}{\partial n} \right) d\sigma \quad (2.32)$$

Let f be zero over the surface Σ' outside the region of the aperture.

Following Sommerfeld's image source method (44), Q_1 is the 'image' of Q_2 in the left hand half space. A Green's function is then constructed so that $g = 0$ (2.32) which conveniently removes $\frac{\partial f}{\partial n}$ from the equation as this is not specified. A suitable

Green's function has the form:-

$$g = \frac{e^{jkr_1}}{r_1} - \frac{e^{jkr_2}}{r_2} \quad (2.33)$$

which is zero when $r_1 = r_2$ (no restriction is placed on r_2 so this condition will be generally true). Substituting (2.33) in (2.32) in the case of small angle diffraction where $\frac{\partial r}{\partial n} = \cos(\underline{r}_1, \underline{r}) \approx 1$ and therefore $\frac{\partial g}{\partial n} = \frac{\partial g}{\partial r} \frac{\partial r}{\partial n} \approx \frac{\partial g}{\partial r}$, it can be shown that:-

$$F(Q_1) = \frac{jK}{2\pi} \int_{\Sigma} f \frac{e^{jkr_1}}{r_1} d\sigma \quad (2.34)$$

This form of the diffraction integral is used when f can be specified in the region of interest and not $\frac{\partial f}{\partial n}$. If g is chosen so that

$$g' = \frac{e^{jkr_1}}{r_1} + \frac{e^{jkr_2}}{r_2} \quad (2.35)$$

then $\frac{\partial g'}{\partial n} = \text{zero}$ over all Σ and Σ' when $r_1 = r_2$, but g' is not zero. Hence it can be shown on substituting (2.35) in (2.32) that:-

$$F'(Q_1) = \frac{1}{2\pi} \int_{\Sigma} \frac{\partial f}{\partial n} \frac{e^{jkr_1}}{r_1} d\sigma \quad (2.36)$$

This form of the diffraction integral is used when $\frac{\partial f}{\partial n}$ can be specified in the region of interest but not f .

Equation (2.34) fits the conditions of light diffraction by an aperture containing an arbitrary intensity distribution. This occurs for example during the reconstruction stage of a holographic imaging process.

Equation (2.36) is used for example to derive the velocity potential field produced by a plate vibrating in an infinite rigid baffle.

Fig. 2.5 illustrates the typical situation found in diffraction problems. Both rectangular and cylindrical co-ordinates are illustrated.

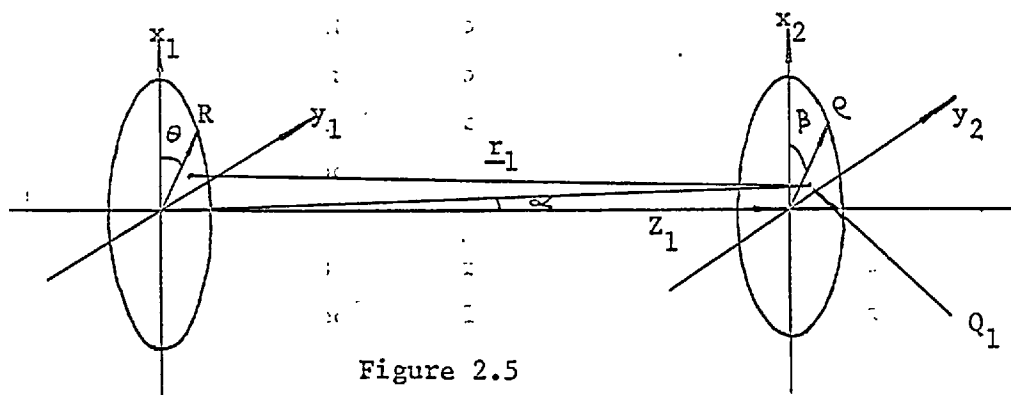


Figure 2.5

Let f or $\frac{\partial f}{\partial n}$ be specified over a region Σ of the (x_1, y_1) plane in accordance with the boundary conditions used in the derivation of (2.34) and (2.36). The value of f at Q_1 in the (x_2, y_2) plane is then calculated by applying the appropriate integral.

Co-ordinates in the (x_1, y_1) plane are related to those in the (x_2, y_2) plane via r_1

$$r_1^2 = (x_2 - x_1)^2 + (y_2 - y_1)^2 + z_1^2 \quad (2.37)$$

Setting $\cos(\underline{r}, \underline{n}) = 1$ in the derivation of (2.34) and (2.36) introduces a slight error which amounts to only 5% when the angle between $\underline{r}$ and $\underline{n}$ is 8 degrees and hence is justifiable for small angle or Fraunhofer diffraction. Under the same criterion little extra error is introduced by approximating (2.37) to

$$r_1 = z_1 - \frac{x_1 x_2}{z_1} - \frac{y_1 y_2}{z_1} \quad (2.38)$$

Because $\frac{1}{r_1}$ is slowly varying compared with $e^{jk r_1}$ it is also

possible to set $\frac{1}{r_1}$ equal to $\frac{1}{z_1}$ under the small angle approximation.

Then

$$\frac{e^{jk r_1}}{r_1} = \frac{e^{jk z_1}}{z_1} e^{-jk \left[\frac{x_1 x_2}{z_1} + \frac{y_1 y_2}{z_1} \right]}$$

On making the substitutions

$$f_x = \frac{x_2}{\lambda_0 z_1} \text{ and } f_y = \frac{y_2}{\lambda_0 z_1}$$

the equations (2.34) and (2.36) take the forms

$$F(Q_1) = \frac{jk}{2\pi} \frac{e^{jk z_1}}{z_1} \int_{\Sigma} f e^{-j2\pi(f_x x_1 + f_y y_1)} dx_1 dy_1$$

(2.39)

and

$$F'(Q_1) = \frac{1}{2\pi} \frac{e}{Z_1} \int_{\Sigma} \frac{\partial f}{\partial n} e^{-j2\pi(f_x x_1 + f_y y_1)} dx_1 dy_1 \quad (2.40)$$

On comparison with (2.8) it is clear that under the conditions of Fraunhofer diffraction, equations (2.34) and (2.36) become Fourier transform relations, multiplied by a constant amplitude and phase factor. Hence

$$F(f_x, f_y) = \frac{j\kappa}{2\pi} \frac{e}{Z_1} \mathcal{F} [f(x_1, y_1)] \quad (2.39a)$$

and

$$F'(f_x, f_y) = \frac{1}{2\pi} \frac{e}{Z_1} \mathcal{F} \left[\frac{\partial f}{\partial n} (x_1, y_1) \right] \quad (2.40a)$$

f_x and f_y are the spatial frequency variables defined in (2.8). Their physical significance in diffraction is seen by their relation to the diffraction angle. Taking f_x as an example

$$f_x = \frac{x_2}{\lambda_0 Z_1} \approx \frac{\sin \alpha}{\lambda_0}$$

α can be considered as the angle at which a plane wave is diffracted on passing through a cosinusoidal diffraction grating of period $\frac{1}{f_x}$. Using this model the diffraction process can be considered to produce an angular spectrum of plane waves each associated with a (hypothetical) diffraction grating in the object. It is easy to see

that waves diffracted through large angles are associated with small periodicity (ie fine detail) in the object and could be easily lost by a 'low pass spatial filter' in the form of a detector having a finite aperture.

For diffraction problems having circular symmetry it is more convenient to work in cylindrical co-ordinates. These are defined in the (x_1, y_1) plane so that

$$\left. \begin{aligned} x_1 &= R \cos \theta \\ y_1 &= R \sin \theta \end{aligned} \right\}$$

and in the (f_x, f_y) plane so that

$$\left. \begin{aligned} f_x &= \rho \cos \beta \\ f_y &= \rho \sin \beta \end{aligned} \right\}$$

and

$$\rho^2 = f_x^2 + f_y^2$$

$f(x_1, y_1)$ becomes $f(R)$ and $F(f_x, f_y)$ becomes $F(\rho)$.

For a circularly symmetrical source distribution the Fourier transform is replaced by a one-dimensional Hankel transformation
(40) Page 62 . Equations (2.39) and (2.40) then become

$$F(\rho) = j\kappa \frac{e}{Z_1} \int_R^{\infty} f(R) J_0(2\pi R\rho) R dR \quad (2.39b)$$

and

$$F'(\rho) = \frac{e}{Z_1} e^{jkZ_1} \int_R \frac{\partial f}{\partial n}(R) J_0(2\pi R \rho) R dR \quad (2.40b)$$

$J_0(2\pi R \rho)$ is a Bessel Function of the first kind of zero order. Note that $J_0(2\pi R \rho)$ is real and even so that if $f(R)$ and $\frac{\partial f}{\partial n}(R)$ are real and even, then their Hankel transforms are real and even.

Equation (2.38) is a first order approximation to equation (2.37). The second order approximation retains quadratic terms in the binomial expansion of r_1 and is used to define the conditions under which so called Fresnel diffraction is applicable. r_1 is then written from (2.37):

$$r_1 = Z_1 + \frac{(x_2 - x_1)^2}{2Z_1} + \frac{(y_2 - y_1)^2}{2Z_1} \quad (2.41)$$

Also $\frac{1}{r_1} = \frac{1}{Z_1}$ as before.

Hence equations (2.34) and (2.36) become

$$F(x_2, y_2) = \frac{jk}{2\pi} \frac{e}{Z_1} e^{jkZ_1} \int_{\Sigma} f(x_1, y_1) e^{jk \left[\frac{(x_2 - x_1)^2}{2Z_1} + \frac{(y_2 - y_1)^2}{2Z_1} \right]} dx_1 dy_1 \quad (2.42)$$

$$F'(x_2, y_2) = \frac{e^{jkZ_1}}{2\pi Z_1} \int_{\Sigma} \frac{\partial f}{\partial n}(x_1, y_1) e^{jk \left[\frac{(x_2 - x_1)^2}{2Z_1} + \frac{(y_2 - y_1)^2}{2Z_1} \right]} dx_1 dy_1 \quad (2.43)$$

Comparing (2.42) and (2.43) with (2.7)

$$\text{if } g_s(x_2, y_2) = e^{j\frac{\kappa}{2Z_1}(x_2^2 + y_2^2)}$$

$$\text{then } F(x_2, y_2) = \frac{j\kappa}{2\pi} \frac{e^{jkZ_1}}{Z_1} (f \otimes g_s) \quad (2.42a)$$

$$\text{and } F'(x_2, y_2) = \frac{e^{jkZ_1}}{2\pi Z_1} \left(\frac{\partial f}{\partial n} \otimes g_s \right) \quad (2.43a)$$

$g_s(x_2, y_2)$ is defined, therefore, as the point spread function of the space between the (x_1, y_1) plane and the (x_2, y_2) plane.

Equations (2.42a) and (2.42b) are extremely useful because they enable linear systems theory to be applied directly to diffraction problems. This leads to considerable simplification in successive diffraction problems which are encountered, for example, in ultrasonic holography.

2.5 An Analysis of Side-Band Fresnel Ultrasound Holography

In principle all the techniques of optical holography can be applied to ultrasonic holography. It would, however, be inappropriate to transcribe into ultrasonic holography terms the multitude of techniques which have been employed in optical holography without first of all justifying the transcription. Three important points require particular attention.

In most conceivable applications the wavelength of ultrasound is very much longer than that of visible light. All practicable ultrasound receivers, therefore, have apertures which are only a relatively small number of wavelengths across compared with their optical equivalent. This has important implications for the light diffraction efficiency and signal to noise ratio of visualized ultrasonic holograms which tend to make ultrasound holography appear unattractive.

The second point is that, because ultrasound holography inherently involves at least two stages, reception and visualization, it is possible to manipulate the hologram before it is visualized. This introduces an extra degree of freedom into ultrasonic holography compared with optical holography.

The third point is of a more fundamental nature. In comparison with visible light the frequency of ultrasound typically used in holography is very low, and the coherence time is extremely long. Consequently it is possible, at the reception stage, to use a transduction process where the amplitude and phase of the ultrasound are preserved. This process could utilize conversion from mechanical radiation to an electrical analogue signal by the direct

piezoelectric effect, although other techniques are possible (1). Immediately one is presented with the question: "If it is so easy to obtain an electrical analogue of an ultrasonic wave, what is the justification for carrying out the process of holography rather than some other type of completely electronic signal processing?" The answer to this question lies in the relative simplicity of the information processor used in holography. This is the parallel processing obtained by diffraction. The alternative involves series processing by electronic means which is both complicated and expensive. The speed and simplicity of optical information processing is well known, but it has been extensively developed only recently in certain areas of pattern recognition and information processing. Other reasons of a practical nature justifying the use of ultrasonic holography have been discussed in an earlier work (17).

The side-band Fresnel technique has been applied most frequently to ultrasound holography. With this technique no particular restrictions are placed on the type of object to be imaged and the source of ultrasound can be positioned at some experimentally convenient spot remote from the object (particularly important in non-destructive testing and medical diagnosis). Also when using an electronically synthesised hologram technique a plane reference wave inclined to the receiver normal can easily be synthesised. Finally the two images obtained on reconstruction are angularly separated in space, provided certain limitations are imposed on the reference wave angle.

Fig. 2.6 illustrates the typical arrangement for an object viewed in reflection and also serves to define the various co-ordinate systems used in the analysis.

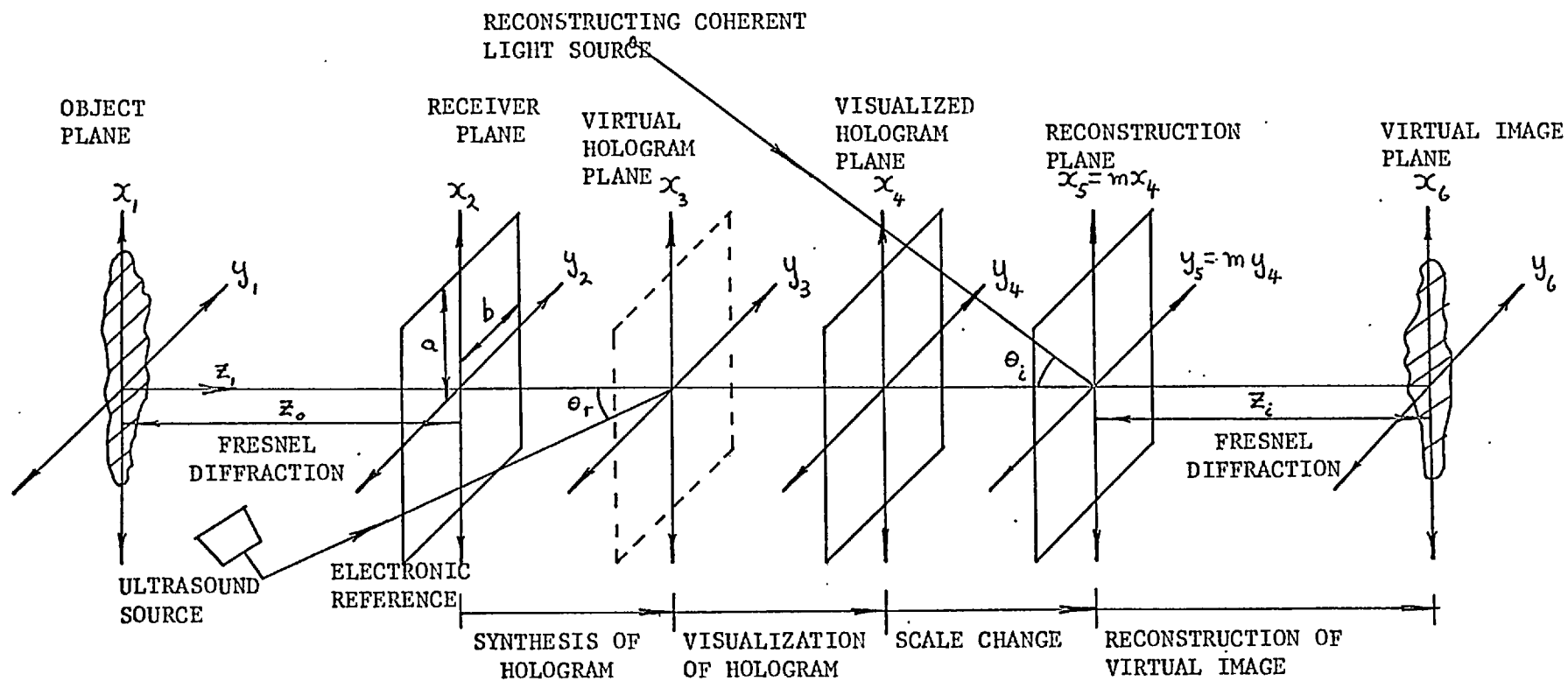


Figure 2.6 Schematic diagram of hologram recording and visualization

The ultrasound source is situated off axis in such a position that any specularly reflected ultrasound does not enter the receiver aperture. The ultrasound that enters the receiver aperture will consist of scattered and diffracted contributions. Consider the diffracted ultrasound only as being characteristic of the object. Common targets are rarely opaque to ultrasound so the particle velocity distribution over the surface will consist of terms from the incident ultrasound which forces the surface into motion and also terms from ultrasound which has penetrated the body and been reflected from internal impedance discontinuities. The nett effect will produce a particle velocity distribution whose amplitude and phase is a function of position over the surface. In order to simplify the mathematics a two dimensional region of the body in the (x_1, y_1) plane will be considered.

Let $f_o(x_1, y_1)$ be the complex velocity distribution over the prescribed region of the object in the (x_1, y_1) plane. From equation (2.25) the normal component of complex velocity is equal to the gradient of the complex velocity potential.

$$f_o(x_1, y_1) = \frac{\partial \phi}{\partial n} \quad (2.44)$$

The velocity potential in the receiver plane under the conditions of Fresnel diffraction is obtained by applying equation (2.43a).

$$\phi(x_2, y_2) = \frac{e^{jk_o Z_o}}{2\pi Z_o} \left\{ f_o(x_1, y_1) \otimes g_{os}(x_2, y_2) \right\} \quad (2.45)$$

$$\text{where } k_o = \frac{2\pi}{\lambda_o}.$$

Let the receiver be a linear piezoelectric detector having a coherent transfer function $G_1(f_x, f_y)$. This will consist of a coherent transfer function $G_p(f_x, f_y)$ characteristic of the piezoelectric detector multiplied by a pupil function for the aperture of the receiver.

If $A_o(x_2, y_2)$ is the pupil function, then

$A_o(x_2, y_2) = 1$ inside the aperture, ie $a > |x_2|; b > |y_2|$ and

$A_o(x_2, y_2) = 0$ outside the aperture, ie $a < |x_2|; b < |y_2|$.

As $f_x = \frac{x_2}{\lambda_o Z_o}$ and $f_y = \frac{y_2}{\lambda_o Z_o}$ the coherent transfer function is

$$G_1(f_x, f_y) = A_o(\lambda_o Z_o f_x, \lambda_o Z_o f_y) G_p(f_x, f_y) \quad (2.46)$$

From (2.45) and (2.44) the ultrasound pressure at the receiver is:

$$p(x_2, y_2) = j\rho_o c k_o \frac{e^{jk_o Z_o}}{2\pi Z_o} \left\{ f_o(x_1, y_1) \otimes g_{os}(x_2, y_2) \right\} \quad (2.47)$$

The piezoelectric detector is assumed to be space invariant so taking the Fourier transform of each side of (2.47) and using equation (2.10), the object spectrum is

$$P(f_x, f_y) = j\rho_o c k_o \frac{e^{jk_o Z_o}}{2\pi Z_o} F_o(f_x, f_y) G_{os}(f_x, f_y) \quad (2.48)$$

The spectrum passed by the detector is obtained by multiplying (2.48) and (2.46). Contained within $G_p(f_x, f_y)$ will be the appropriate constant relating the spatial distribution of pressure over the piezoelectric detector to the corresponding spatial distribution of voltage, consequently the voltage spatial spectrum will be

$$V(f_x, f_y) = j\rho_o c k_o \frac{e}{2\pi Z_o} F_o(f_x, f_y) G_{os}(f_x, f_y) A_o(\lambda_o Z_o f_x, \lambda_o Z_o f_y) G_p(f_x, f_y) \quad (2.49)$$

$V(f_x, f_y)$ is now modulated onto a spatially varying 'carrier' wave which is provided by the electronic reference signal. This is phased to simulate a plane ultrasonic wave incident at θ_r to the receiver plane normal. The simulated direction of propagation of the reference wave is completely arbitrary and no generalisation will be lost if it is chosen as propagating with the wavefront normal in the (x_3, Z_1) plane. The complex amplitude of the reference wave is then given by

$$v_r(x_3) = v_{ro} e^{-j2\pi f_{\theta_r} x_3} \quad (2.50)$$

$f_{\theta_r} = \frac{\sin \theta_r}{\lambda_o}$ and is therefore the spatial frequency of the

'carrier' wave. The amplitude distribution of voltage over the virtual hologram plane is given by

$$v_H(x_3, y_3) = \left\{ \left(v(x_3, y_3) + v_r(x_3) \right) \left(v^*(x_3, y_3) + v_r^*(x_3) \right) \right\}^{\frac{1}{2}} \quad (2.51)$$

$$\text{where } v(x_3, y_3) = \mathcal{F}_1^{-1} \left[v(f_x, f_y) \right] .$$

Therefore

$$v_H(x_3, y_3) = \left\{ vv^* + v_{ro}^2 + vv_{ro} e^{j2\pi f_{\theta} x_3} + v^* v_{ro} e^{-j2\pi f_{\theta} x_3} \right\}^{\frac{1}{2}} \quad (2.52)$$

The visualization processes outlined in Chapter 1, which have been used in ultrasound holography, produce light intensity displays derived from either the squares of the particle velocity or the pressure. The reason for this is that these displays are viewed visually and the eye is, of course, an energy detector. Consequently the distribution of complex voltage amplitude in the virtual hologram is converted into an intensity distribution $e_H(x_3, y_3)$ which is given by:

$$|v_H|^2 = e_H(x_3, y_3) = v_H v_H^* = \left\{ vv^* + v_{ro}^2 + vv_{ro} e^{j2\pi f_{\theta} x_3} + v^* v_{ro} e^{-j2\pi f_{\theta} x_3} \right\} \quad (2.53)$$

This is achieved electrically by passing the signal corresponding to (2.52) through a square law detector. In the synthetic hologram technique the voltage distribution derived from the ultrasound field is scanned in a T.V. like raster to produce a time varying signal, amplitude modulated with a 'video signal' which has a bandwidth determined by (2.49) and the scanning rate. The addition of (2.50) to this signal before passing through the

square law detector is achieved simply by a resistive summing network as in a conventional analogue computer.

Many techniques have been used to visualize the intensity distribution e_H of the synthetic hologram. They range from photographic recording of the television picture produced by a conventional ultrasound camera system (26) to a quasi-real time technique using a specially designed electro-optic projection television system (29). Each of these techniques merits special attention as each has its own individual transfer function.

However, it is sufficient to realise the existence of limited frequency response in all visualization systems and simply specify a general transfer function for the visualization process $g_v(f_x, f_y)$. This is analogous to the optical transfer function defined in (2.14) because intensity mapping is being considered. When a hologram is recorded with radiation having a wavelength λ_0 and reconstructed with radiation having another wavelength λ_1 , there will be considerable gains in image quality if the hologram size can be linearly scaled by an appropriate factor before reconstruction (17). Consequently the scale change m defined by (2.54) is introduced.

$$\left. \begin{aligned} x_5 &= mx_4 \\ y_5 &= my_4 \end{aligned} \right\} \quad (2.54)$$

In terms of the scaled co-ordinate system $g_v(f_x, f_y)$ becomes $g_v(mf_x, mf_y)$.

At this point a fundamental assumption concerning the nature of the visualization system is made. It is assumed that there is a linear conversion of virtual hologram intensity into coherent light complex amplitude transmittance in the visualized hologram. In the case of photographic recording of the hologram Kozma (45), (46) has shown that such a linear transference can be achieved by choosing the reference beam intensity so that the v_{ro}^2 term in (2.53) fixes the operating point for the recording on the linear portion of the transmittance-exposure characteristic of the photographic plate.

If β is the slope of the $(t - E)$ curve at the operating point, the visualized hologram transmittance is, from equations (2.53) and incorporating the scale change (2.54)

$$t(x_5, y_5) = \beta e_H(x_3, y_3) \mathcal{F}^{-1} \left[g_v(mf_x, mf_y) \right] \quad (2.55)$$

In the optical reconstruction process the amplitude transmittance described by (2.55) becomes a boundary value for the optical scalar diffraction problem. Let the transparency be illuminated by a plane coherent light wave inclined at an angle θ_i to the Z axis in the (x_5, Z) plane.

The complex amplitude of this plane wave is represented by the scalar wave function defined in (2.27) and given by

$$\psi_p(x_5) = \psi_{po} e^{-j2\pi f_{\theta_i} x_5} \quad (2.56)$$

$f_{\theta_i} = \frac{\sin \theta_i}{\lambda_i}$; λ_i is the free space wavelength of the reconstructing light.

In order to allow for the effects of the finite size of the visualized hologram transparency on the resolution of the reconstructed images, a second pupil function $A_i(\lambda_i Z_i m f_x, \lambda_i Z_i m f_y)$ is defined in the same manner as that for the aperture of the receiver.

The complex amplitude of the light transmitted by the hologram is

$$\psi_t(x_5, y_5) = A_i(mx_4, my_4) \psi_p(x_5) t(x_5, y_5) \quad (2.57)$$

Utilizing (2.42a) the Fresnel diffraction pattern derived from the visualized hologram is given by

$$\psi_i(x_6, y_6) = jk_i \frac{e^{jk_i Z_i}}{2\pi Z_i} \left\{ A_i(mx_4, my_4) \psi_p(x_5) t(x_5, y_5) \otimes g_{is}(x_6, y_6) \right\} \quad (2.58)$$

where $g_{is}(x_6, y_6)$ is the Green's function for the image space.

The spatial frequency spectrum of the reconstructed hologram is $\Psi_i(f_x, f_y)$ given by the Fourier transform of $\psi_i(x_6, y_6)$

$$\Psi_i = jk_i \frac{e^{jk_i Z_i}}{2\pi Z_i} \mathcal{F} \left[A_i \psi_p t \otimes g_{is} \right] \quad (2.59)$$

Using (2.55) and (2.56)

$$\begin{aligned} \psi_i(f_x, f_y) = & jk_i \frac{e^{jk_i Z_i}}{2\pi Z_i} A_i(\lambda_i Z_i m f_x, \lambda_i Z_i m f_y) \beta_{0i}(m f_x, m f_y) \psi_{po} \int \left[g_{is}(mx_4, my_4) \right. \\ & \cdot \int \left[v v^* e^{-j2\pi m f_{\theta_i} x_4} + v_{ro}^2 e^{-j2\pi f_{\theta_i} mx_4} + v v_{ro} e^{j2\pi m(f_{\theta_i} x_3 - m f_{\theta_i} x_4)} \right. \\ & \left. \left. + v^* v_{ro} e^{-j2\pi m(f_{\theta_i} x_3 + f_{\theta_i} x_4)} \right] \right] \end{aligned} \quad (2.60)$$

2.6 Object Resolution in Side-Band Fresnel Ultrasound Holography

Assuming that there is no scale change between the (x_2, y_2) plane, the (x_3, y_3) plane and the (x_4, y_4) plane, the frequency spectrum $\psi_i(f_x, f_y)$ can be evaluated term by term without distinguishing between the frequency variables at each successive transformation.

With appropriate changes of variables (2.49) can be substituted into each of the frequency spectra S_1, S_2, S_3, S_4 and so the complete spectrum of the reconstructed hologram obtained in terms of the object spectrum and the system transfer functions. Equations (2.61) to (2.64) effectively give the complete frequency spectrum of the signal in the visualized hologram before the effects of the various system transfer functions have been introduced, ie for a perfect system. Fig. 2.7 shows the real parts of these spectra on the spatial frequency plane.

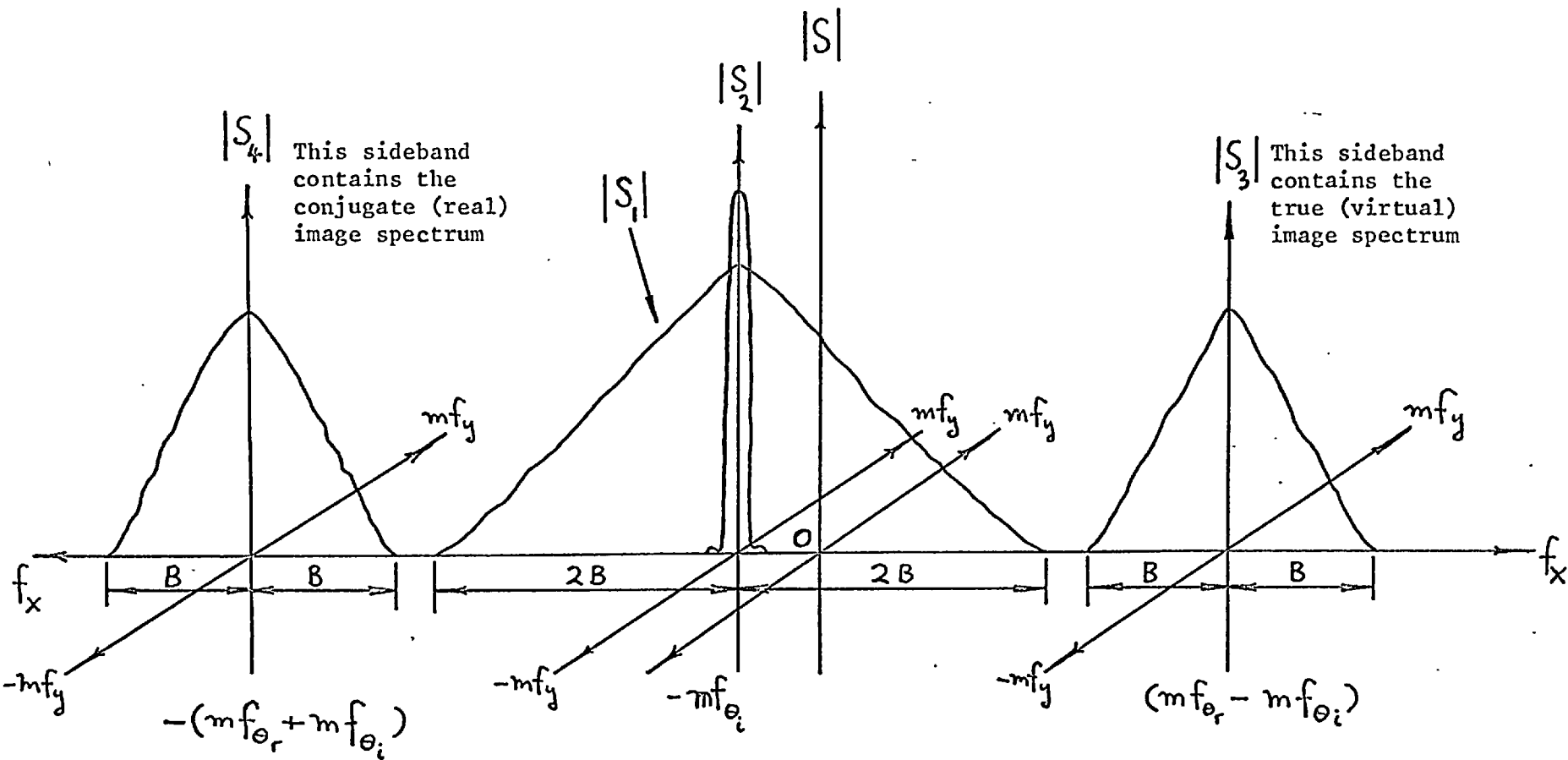


Figure 2.7 The spectrum of a visualized hologram assuming that the recording system does not modify the object bandwidth B .

$$\begin{aligned} \mathcal{F} \left[v v^* e^{-j2\pi f_{\theta_i} x_4} \right] &= V(mf_x + mf_{\theta_i}, mf_y) V^*(-mf_{\theta_i} - mf_x, mf_y) \\ &= S_1 \end{aligned} \quad (2.61)$$

$$\begin{aligned} \mathcal{F} \left[v_{ro}^2 e^{-j2\pi f_{\theta_i} x_4} \right] &= v_{ro}^2 \delta(mf_x + mf_{\theta_i}, mf_y) \\ &= S_2 \end{aligned} \quad (2.62)$$

$$\begin{aligned} \mathcal{F} \left[v v_{ro} e^{j2\pi m(f_{\theta_r} x_3 - f_{\theta_i} x_4)} \right] &= v_{ro} V(mf_x - mf_{\theta_r} + mf_{\theta_i}, mf_y) \\ &= S_3 \end{aligned} \quad (2.63)$$

$$\begin{aligned} \mathcal{F} \left[v_{ro} v^* e^{-j2\pi m(f_{\theta_r} x_3 + f_{\theta_i} x_4)} \right] &= v_{ro} V^*(-mf_x - mf_{\theta_r} - mf_{\theta_i}, mf_y) \\ &= S_4 \end{aligned} \quad (2.64)$$

The 'shift theorem' of Fourier transforms has been used in the above derivations.

Let the bandwidth of the object spectrum be B in the f_x frequencies. In order that the two images are completely angularly separated on reconstruction it can be seen that f_{θ_r} must be greater than $3B$.

The effects of the systems transfer functions will now be discussed in detail. Provided that B is less than $\frac{a}{\lambda_o Z_o}$ the object spectrum will be received without modification. This implies that

a perfect image of the object would be obtained. In practice the object spectrum will be wider than $\frac{a}{\lambda_o Z_o}$ and so the pupil function

will truncate the spectrum at $f_{\max} = \frac{a}{\lambda_o Z_o}$. On detection the

object spectrum will be converted into a voltage distribution having a bandwidth $f_{\max}$ provided that the bandwidth of $G_p(f_x)$ is greater than $f_{\max}$. A similar argument applies to f_y of course. The bandwidth of $G_p(f_x, f_y)$ is however observed to be severely limited as described in Chapter 5. Consequently the recorded bandwidth of the object spectrum is usually determined by the bandwidth of G_p . The synthetic hologram technique with simulated reference beam has important advantages over the same technique using an ultrasonic reference beam for the case where the detector transfer function has smaller bandwidth than the pupil function. Figs. 2.8 and 2.9 are used to illustrate this point. In Fig. 2.8 (i) an object spectrum having a bandwidth B is received by an aperture having a rectangular pupil function which truncates the spectrum at $f_{\max}$ as in Fig. 2.8 (ii). The detector having a bandwidth f_p transmits a spectrum shown in Fig. 2.8 (iv). This is modulated on to an electronically simulated reference wave prior to visualization, giving a synthetic hologram with a total bandwidth of at least $4f_p$. The reference wave spatial frequency must be at least $3f_p$ in order to achieve separation of the twin images on reconstruction. In Fig. 2.9 the aperture again truncates the object spectrum at $f_{\max}$ but in this case the truncated spectrum is modulated on to the ultrasonic carrier before being passed through the detector. Consequently the modulated carrier spectrum has a bandwidth of $4f_{\max}$, see Fig. 2.9 (iii). The detector has a bandwidth of f_p which is less than $f_{\max}$ so the sidebands of the modulated spectrum will not be passed and no information about the object is

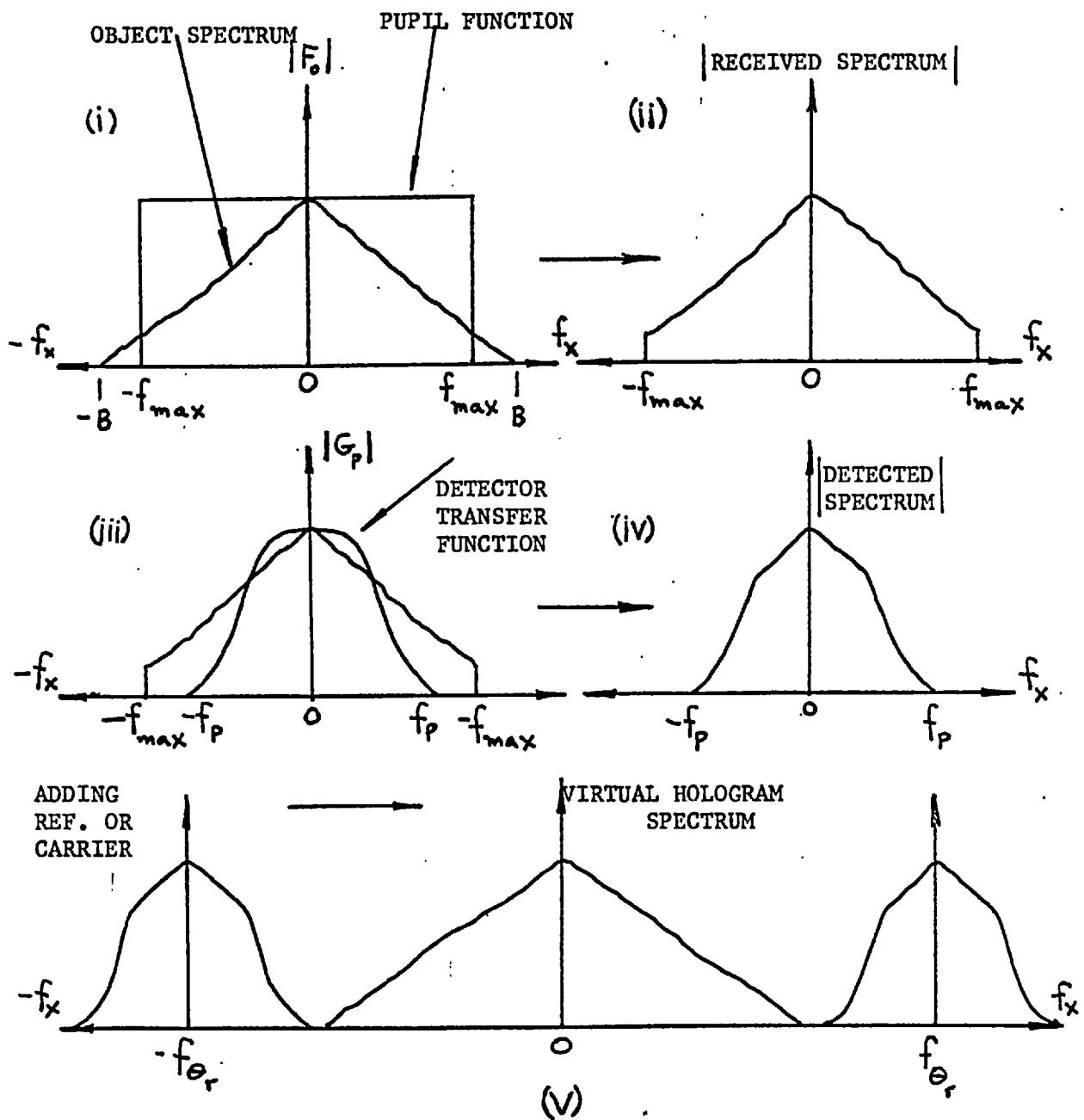


Figure 2.8 Stages in recording the object spectrum in the synthetic hologram technique using a simulated reference beam.

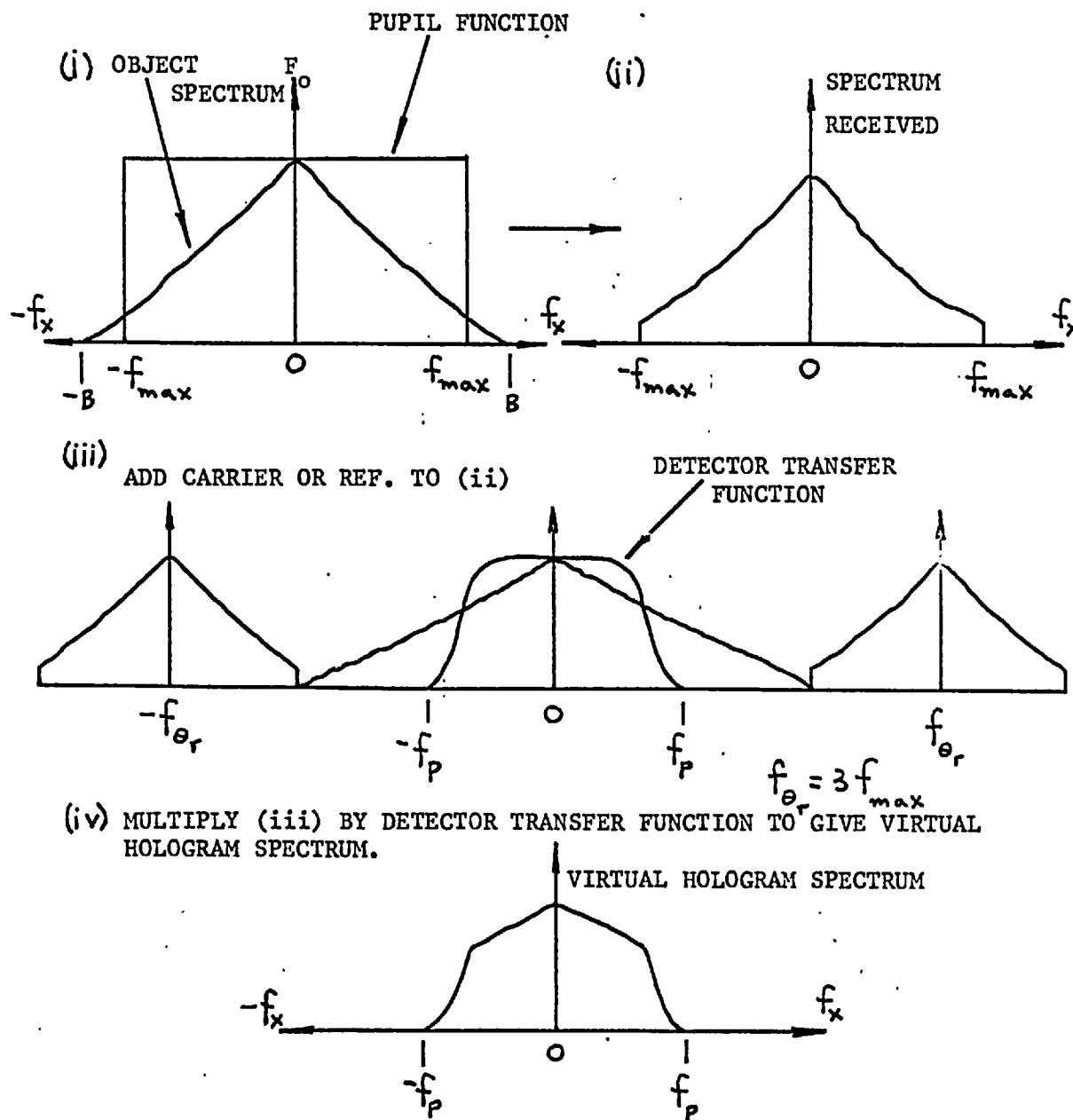


Figure 2.9 Stages in recording the object spectrum in the synthetic hologram technique using a real ultrasonic reference beam

recorded in a recoverable manner, see Fig. 2.9 (iv). If f_p is greater than $4f_{\max}$ then both types of recording technique will produce a virtual hologram having the same spectrum which contains both sidebands.

During visualization the virtual hologram spectrum is multiplied by the modulus of $g_v(f_x, f_y)$ which will usually have a bandwidth very much greater than that of the virtual hologram, particularly for photographic recording of the visualized hologram. If the scale of the visualized hologram is reduced by the ratio of the reconstructing to recording wavelengths, then the spatial frequencies are scaled accordingly. Unless the film on which the scaled hologram is recorded has a very large bandwidth, the bandwidth of the scaled hologram will be limited with a consequent loss of resolution in the final image. In the majority of the practical ultrasound holographic systems described in Chapter 1, the hologram size is reduced by less than a factor of 1/10 so the bandwidth of the usual hologram recording medium will not seriously limit the hologram spectrum. This also applies to the pupil function for the reconstruction stage.

Re-introducing equations (2.61) to (2.64) into equation (2.60) produces a linear combination of four equations for the frequency spectrum of the Fresnel diffraction pattern obtained on reconstructing the visualized hologram.

$$\begin{aligned} \psi_i(f_x, f_y) = & jk_i \frac{e^{jk_i Z_i}}{2\pi Z_i} A_i(\lambda_i Z_i m f_x, \lambda_i Z_i m f_y) \beta g_v(m f_x, m f_y) \\ & \cdot \psi_{po} G_{is}(m f_x, m f_y) (S_1 + S_2 + S_3 + S_4) \end{aligned} \quad (2.65)$$

If the cut-off frequency of $\mathcal{H}_v(mf_x, mf_y)$ is very much larger than mf_p the constructed hologram spectrum will be the same as that of Fig. 2.7 apart from a constant amplitude factor.

The terms in S_3 and S_4 give the True and Conjugate visualized images respectively of the ultrasonically illuminated object ((17) Page 81). The terms in S_1 and S_2 are coherent noise terms and do not contribute to the images. If f_θ is not large enough, S_1 and S_2 are not separated from S_3 and S_4 on the spatial frequency plane, then the noise terms appear as background to the image terms and result in decreased image contrast.

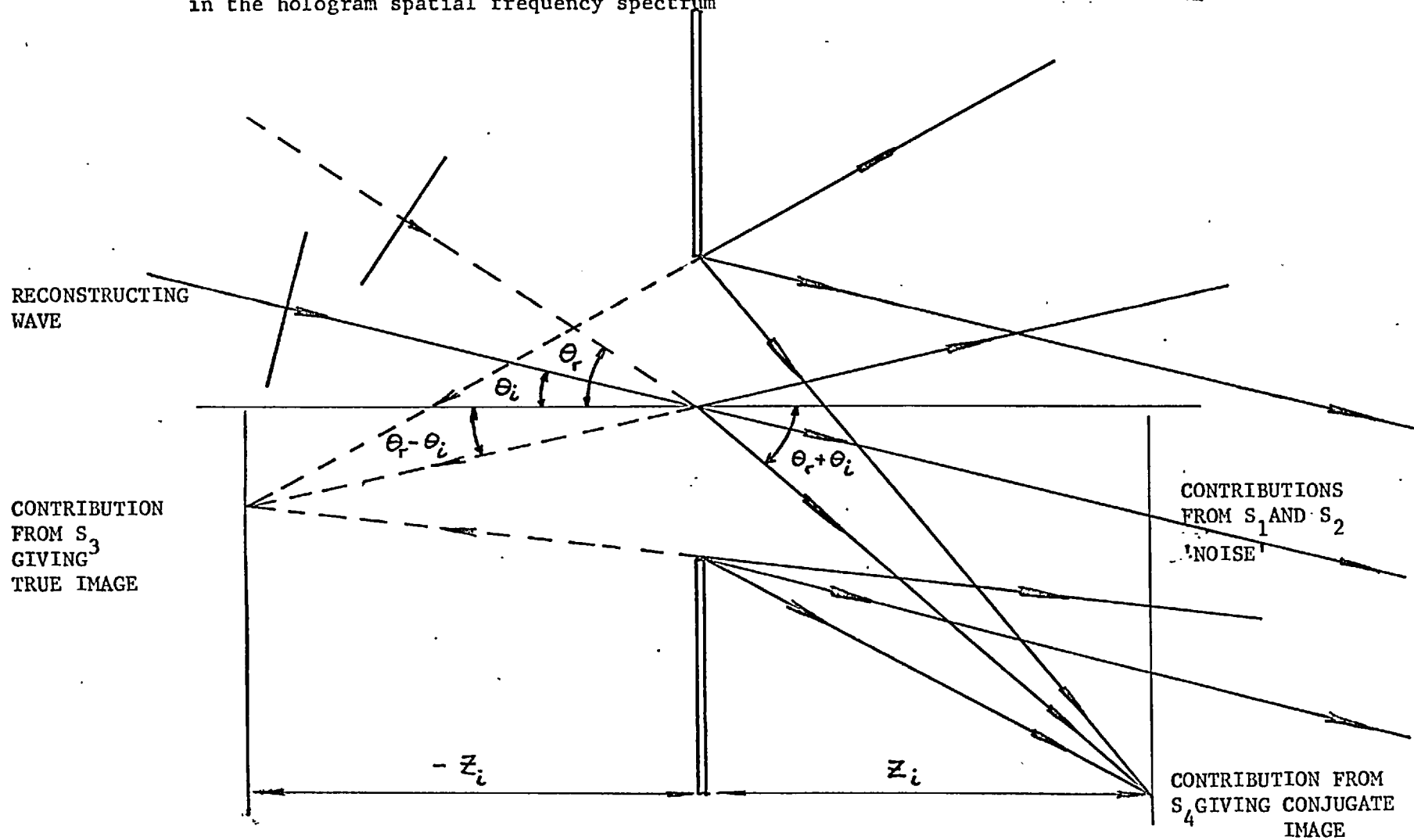
The term from equation (2.65) in S_3 is selected for particular attention, although similar reasoning to that following could be applied equally well to the term in S_4 . The image corresponding to the spectrum S_3 is that which is obtained on looking through the hologram towards the reconstructing source for the recording and reconstructing geometry described in section 2.5 (Fig. 2.10).

Let $\Psi_T(f_x, f_y)$ represent this true image spectrum.

Provided that the coherent transfer function of the medium on which the virtual hologram is recorded has a bandwidth large in comparison with $G_p(mf_x - mf_{\theta_r} + mf_{\theta_i}, mf_y)$ then the image spectrum

will be determined by the three terms $F_o \cdot A_o \cdot G_p$, where F_o is the object spectrum, A_o the pupil function for the receiver and G_p the transfer function for the material of the detector.

Figure 2.10 Reconstruction of a hologram with a plane wave showing the contributions from the four terms in the hologram spatial frequency spectrum



$$\psi_T(f_x, f_y) = - \frac{k_i}{2\pi} \frac{e}{Z_i} v_{ro} \rho_o c \frac{k_o}{2\pi} \frac{e}{Z_o} \psi_{po}^{\beta A_i}(\lambda_i Z_i m f_x, \lambda_i Z_i m f_y)$$

$$\cdot A_o(\lambda_o Z_o(m f_x - m f_{\theta_r} + m f_{\theta_i}), \lambda_o Z_o m f_y) \psi_v(m f_x, m f_y)$$

$$\cdot G_{is}(m f_x, m f_y) G_{os}(m f_x - m f_{\theta_r} + m f_{\theta_i}, m f_y)$$

$$\cdot G_p(m f_x - m f_{\theta_r} + m f_{\theta_i}, m f_y) \cdot F_o(m f_x - m f_{\theta_r} + m f_{\theta_i}, m f_y)$$

(2.66)

The resolution of the visualized reconstructed ultrasound image will therefore be determined by the product $A_o \cdot G_p$. Consequently it is essential to know the form of G_p in order to calculate the performance of an ultrasound holographic system using a piezoelectric detector (or any other detector where an electrical output analogous to the ultrasound signal is produced). In order to maximise the resolution of the object, the product $A_o \cdot G_p$ must be maximised.

2.7 The Coherent Transfer Function for a Piezoelectric Ceramic Ultrasound Detector

In section 2.6 it was shown that for the case of Fresnel side-band ultrasound holography the object resolution is determined by the product of the receiver pupil function and the coherent transfer function for the detector. The pupil function is immediately determined by the receiver aperture, the problem of the determination of the coherent transfer function for the detector remains.

For reasons outlined in the introduction a lead zirconate titanate ferroelectric ceramic plate is a potentially useful ultrasound detector. This material is polarized during manufacture and exhibits electro-mechanical behaviour similar to a hexagonal crystalline material of point group 6 m m. In the case of the ceramic, however, the polarization axis, which is conventionally called the Z axis, is an axis of infinite rotational symmetry and not six-fold symmetry. Consequently the ceramic has circular symmetry about the Z axis and therefore behaves as an isotropic material in the Z plane. These properties imply that the description of the mechanical behaviour of the material should be considerably simplified if the shape chosen for investigation is that of a disc. The Z axis must be normal to the plane of the disc because it is only then that a longitudinal piezoelectric effect is obtained.

Conversion of a spatially varying pressure distribution over the area of the disc into a correspondingly varying charge distribution is only obtained through a longitudinal piezoelectric effect.

In the detection mode charge read-out is, of course, obtained from an unelectroded face of the disc.

Having defined the detector geometry it is necessary to discuss its response to an arbitrary ultrasound pressure distribution incident on one face of the disc. This face is therefore loaded by the medium which couples the object acoustically to the detector.

The factors which can be expected to influence the detector transfer function are just those which determine the solutions of the homogeneous wave equation for the ceramic material. These are the boundary conditions at the edges and faces of the disc, the geometry of the disc, the physical properties of the material and the temporal frequency of oscillation. Naturally the detector response will also be determined by the ultrasound forcing function but the coherent transfer function will be independent of this provided the material is driven in a linear region of the stress-strain characteristic.

The system in which a detector is to be used imposes practical constraints on the arbitrary choice of boundary conditions. Consequently the coherent transfer function will most usefully be determined for commonly imposed constraints on the detector boundary conditions.

A mounting arrangement commonly found in underwater viewing, medical diagnosis and non-destructive testing has been used in the present investigation; the edge of the disc is clamped, the electroded face is water loaded and the opposite unelectroded face vibrates in air.

To relate the physical properties of the ceramic material to the detector coherent transfer function is a problem which has analogies in optics. The modulation transfer function of the photographic recording process can be related to characteristics of the photographic material and the physics of the development process. Kelly (47) has applied systems analysis to the photographic process and shown that it can be represented by a three stage model, optical diffusion, followed by conventional sensitometry and finally chemical diffusion. Vanligten (48) successfully used Kelly's model to show how the photographic recording process can limit object resolution in optical holography. There are two fundamental differences between the ceramic ultrasound detector and the photographic plate. The ultrasound detector behaves as a continuum whereas the photographic recording medium behaves as an array of light energy sensitive points dispersed throughout a matrix of different material. Secondly the ultrasound detector is usually less than one ultrasound wavelength thick and so the boundary conditions at the faces are extremely important in determining the coherent transfer function as discussed later. The photographic emulsion, however, is many light wavelengths thick and also provided with an anti-halation backing so the boundaries can usually be ignored.

Although the coherent transfer function completely determines how the detector responds to spatial information, it is easier to interpret when considered as the product of two functions. These are a modulation transfer function which describes how contrast is transmitted through the detector and a directivity function which describes the angular dependence of the detector gain. The modulation transfer function will exhibit a maximum or cut-off frequency f_0 above which spatial information is not passed. From

the work described in 1.3, f_0 is determined by the detector thickness. The directivity function will be determined by the geometry, ultrasound frequency and mounting conditions of the detector. It is to be expected that this latter function will be of great importance in determining the useful bandwidth of the coherent transfer function.

With a ferroelectric ceramic detector in the form of a disc the coherent transfer function can be expected to show rotational symmetry about the Z axis, consequently only one spatial frequency is required to describe it. Without loss of generality f_x can be chosen.

Hence
$$|G_p(f_x)| = |D_p(f_x)| \cdot |M_p(f_x)| \quad (2.67)$$

where
$$D_p(f_x) = D_p\left(\frac{\sin \theta}{\lambda}\right) \quad \text{the directivity function}$$

and
$$|M_p(f_x)| = \text{the modulation transfer function.}$$

Fig. 2.11 illustrates how $|D_p(f_x)|$ and $|M_p(f_x)|$ might be expected to determine $|G_p(f_x)|$.

Using these concepts the following idea of a continuous detector is put forward: Let the detector be considered as an array of hypothetical resolution elements of side length equal to $\frac{1}{f_0}$ but mechanically and electrically connected as in the continuous detector. Each element is then considered to have a directivity function $D_p(f_x, x - \xi)$ because of its position in the 'continuous array'. This formulation allows the directivity to change if the

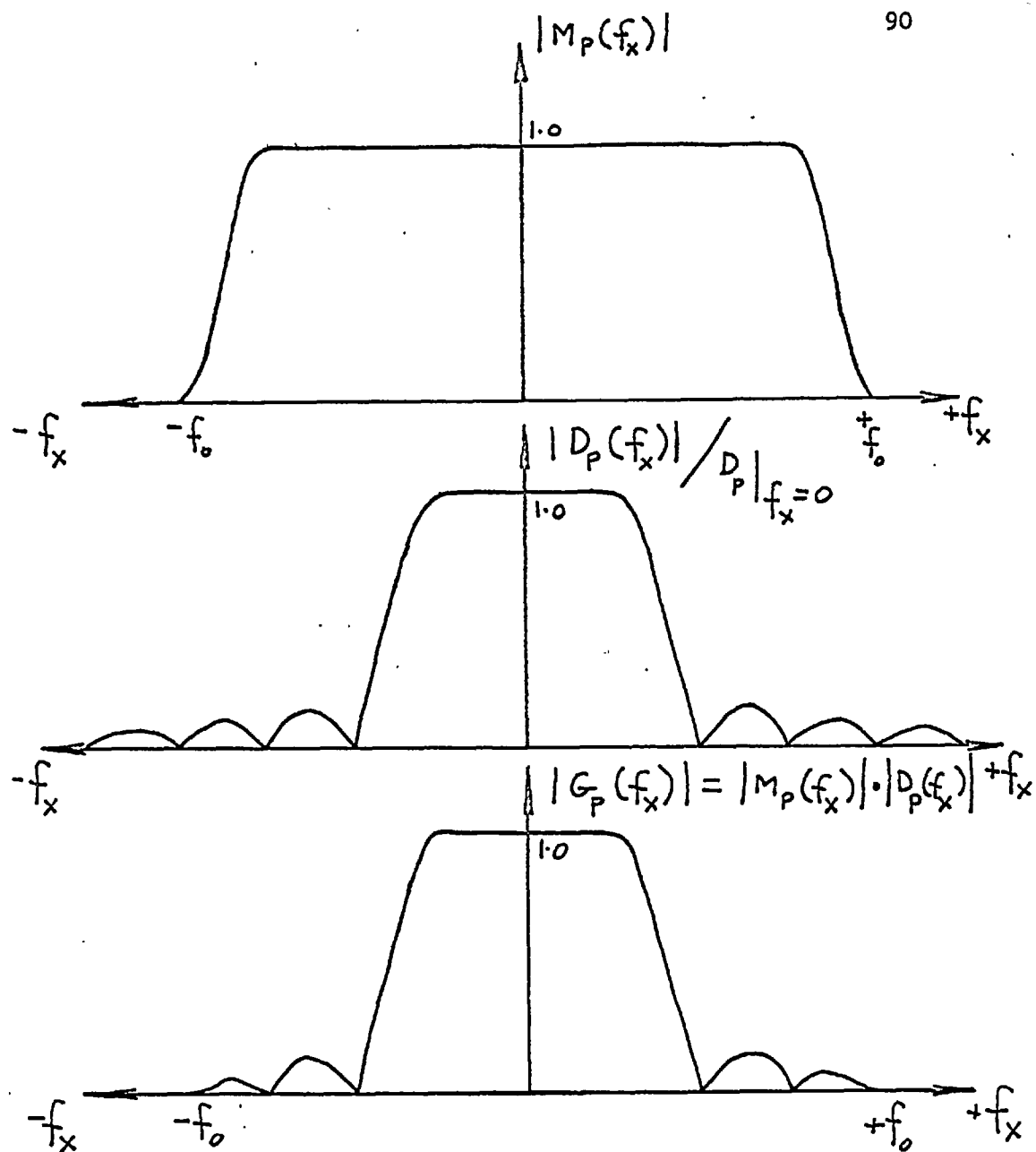
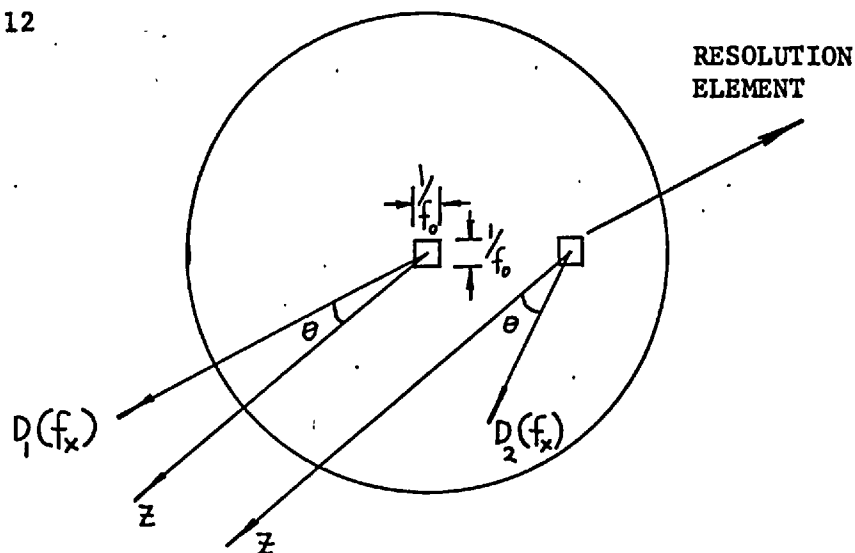


Figure 2.11 An illustration of how $|D_p(f_x)|$ and $|M_p(f_x)|$ determine $|G_p(f_x)|$

properties of the detector change as a result of the proximity of boundaries or because the properties of the material are a function of position. In other words $D_p(f_x)$ is space variant. Fig. 2.12 illustrates this concept.

Figure 2.12



Now $D_p(f_x)$ can be determined theoretically for an infinite homogeneous plate under certain approximations, but $M_p(f_x)$ has defied all attempts so far at theoretical analysis. It can, however, be measured experimentally, as can $D_p(f_x)$.

The directivity function will be obtained as the far field diffraction pattern when the detector is driven over an area the size of a resolution element, as described in 2.9.

The modulation transfer function can be determined by a discrete sinusoidal standing wave method as described in 2.8. This

method in fact determines the coherent transfer function directly as both the modulation depth and gain, as a function of angle, can be measured.

Fig. 2.13 shows schematically the disc mounting arrangement.

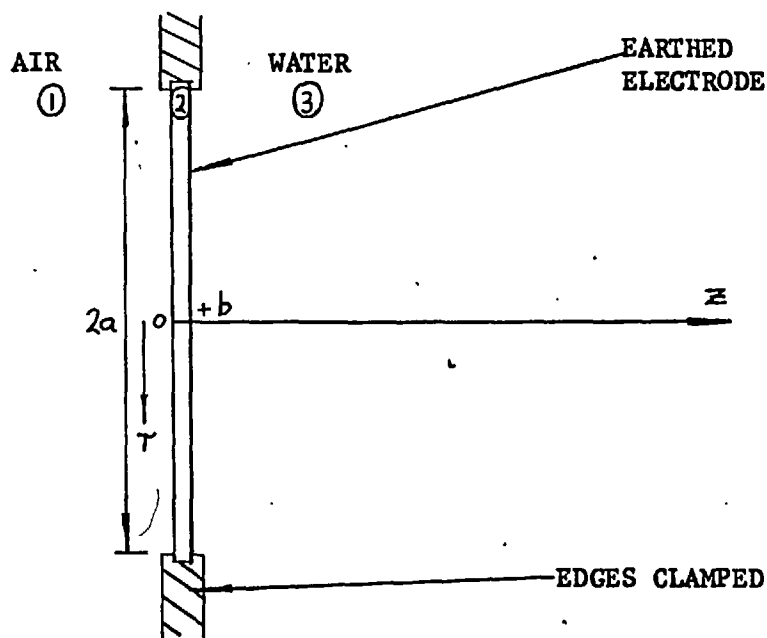


Figure 2.13

The mechanical boundary conditions are listed as follows:

- (i) The (1)/(2) face is free from tractions, therefore normal and shear stresses are zero.

$$\text{At } z = 0, T_{ij} = T_{zz} = 0 \quad (2.68)$$

(ii) The ②/③ face vibrates in water which, it is assumed, can only sustain longitudinal waves.

$$\text{At } Z = b, T_{ZZ} = p$$

$$T_{ij} = 0 \quad i \neq j \quad (2.69)$$

where p is the excess pressure in the ultrasonic wave.

(iii) The disc edges are clamped, therefore radial and shear strains are zero at the edges.

$$\text{At } r = a, S_{ij} = S_{rr} = 0 \quad (2.70)$$

(iv) At $Z = b$, the normal component of particle displacement is continuous across the boundary.

$$(u_Z)_{\text{fluid}} = (u_Z)_{\text{solid}} \quad (2.71)$$

The electrical boundary conditions are:

(i) The ceramic transducer is electroded only on one face and is therefore considered to be open circuit. It is assumed that the ceramic is a perfect insulator so there is no free charge, hence

$$D_n = 0 \quad (2.72)$$

Consideration of these boundary conditions leads to the choice of D_n and S_{ij} as independent variables for the formulation of the characteristic piezoelectric equations. Following the notation standardized by the I.R.E. (49) the piezoelectric equations are:

$$T_{kl} = c_{ijkl}^D S_{ij} - h_{kln} D_n \quad (2.73)$$

$$E_m = - h_{mij} S_{ij} + \beta_{mn}^S D_n \quad (2.74)$$

i, j, k, l, n, m can have values 1, 2, 3

A standard sign convention has also been adopted by the I.R.E. A normal strain is positive when extensional and negative when compressional. A shearing stress and also a shearing strain is positive when a rectangle becomes deformed so that an acute angle lies in the quadrant between the positive directions of the axes that form the sides of the rectangle. It is important to realize that the 'constants' in the piezoelectric equations are specified for the material in bulk and so values measured on a finite body are dependent on the shape of the specimen.

The standard measurement procedures and appropriate specimen shapes are described in an I.R.E. Report (50). The significance of this is that one must be careful when inserting values into theoretical equations that the piezoelectric constants used are measured under conditions which are nearest to those applicable to the derivation.

In order to derive the directivity function for the detector subject to the above constraints, it is necessary to relate the resulting voltage distribution over the ①/② face to the pressure distribution over the ②/③ face produced by a plane wave incident at an arbitrary angle to the disc normal. Any arbitrary pressure

field can be Fourier analysed into an equivalent spectrum of plane waves. If the response of the detector to one arbitrary plane wave is known, the total response can be calculated using the linear systems theory of section 2.1.

The normal procedure in mathematical physics would be to solve the acoustical wave equation for the disc subject to the boundary conditions. For the problem as specified this is such a formidable task that a general solution has yet to be obtained. The major difficulty is to obtain a solution which satisfies the boundary conditions at both faces of the disc and at the edges simultaneously. It is, therefore, necessary to make several simplifying assumptions in order to obtain a general idea of the disc response.

The diameter to thickness ratio of the disc is assumed to be very large. Consequently the disc can be considered as an infinite plate. The relative contributions of the transverse strains and longitudinal strain to the piezoelectric voltage generated across the disc by a normal stress will now be calculated. This result will indicate the degree of approximation necessary in order to neglect transverse strains in any further analysis.

Using boundary conditions (2.72) equations (2.73) and (2.74) become:

$$T_{kl} = c_{ijkl}^D S_{ij} \quad (2.75)$$

$$E_m = -h_{mij} S_{ij} \quad (2.76)$$

Adopting the standard reduced suffix notation (49) the six independent stresses indicated by appropriate values of k_l are specified by a single integer having values 1 to 6. The double suffix and single suffix notation as applied to the stress tensor are displayed side by side for comparison.

<u>Tensor Form</u>	<u>Abbreviated Form</u>	
T_{11}	T_1	} Normal stresses
T_{22}	T_2	
T_{33}	T_3	
T_{23}	T_4	} Shearing stresses
T_{31}	T_5	
T_{12}	T_6	

T_3 represents the normal stress in the Z direction acting over the Z face and will be equal to the pressure in a normally incident ultrasonic wave by (2.69).

The symmetry of the elastic stiffness tensor for a material of point group 6 m m limits the number of strains produced by a normal stress T_3 to three (51)

$$T_3 = c_{13}^D S_1 + c_{13}^D S_2 + c_{33}^D S_3 \quad (2.77)$$

For an isotropic body the analogous equation would be:

$$T_3 = c_{12} S_1 + c_{12} S_2 + c_{11} S_3 \quad (2.78)$$

In terms of the Lamé constants (2.78) is written:

$$T_3 = \lambda(S_1 + S_2 + S_3) + 2\mu S_3 \quad (2.78a)$$

Comparing (2.78) and (2.78a) the Lamé constants are related to the elastic stiffnesses as follows:

$$\lambda = c_{12} , \mu = \frac{c_{11} - c_{12}}{2} \quad (2.79)$$

Poisson's ratio σ is then related to the compliance constants s_{ij} by:

$$\sigma = - \frac{s_{12}}{s_{11}} \quad (2.79a)$$

Using equation (2.76) the electric field across the plate, E_3 , and hence the potential difference can be calculated when the plate is subjected to a stress T_3 .

$$E_3 = - h_{31} S_1 - h_{31} S_2 - h_{33} S_3 \quad (2.80)$$

The number of components of strain contributing to E_3 is reduced to 3 because of the symmetry of the piezoelectric constant tensor for the ceramic material (51).

Using equation (2.75) the components of strain equal to stress T_3 , T_2 and T_1 are obtained. Then on applying the boundary condition (2.77) and assuming that stresses are constant across the thickness of the disc, ie assuming that the wavelength of the ultrasonic wave is long in comparison with b , it is easy to show that:

$$S_1 = S_2 = - \frac{S_3 (c_{13}^D c_{11}^D - c_{13}^D c_{12}^D)}{c_{11}^{D^2} - c_{12}^{D^2}} \quad (2.81)$$

Inserting (2.81) into (2.80)

$$E_3 = 2h_{31} \left(\frac{c_{13}^D c_{11}^D - c_{13}^D c_{12}^D}{c_{11}^{D^2} - c_{12}^{D^2}} \right) S_3 - h_{33} S_3 \quad (2.82)$$

Mason (52) quotes values of the elastic and piezoelectric constants for a lead zirconate titanate ceramic manufactured by the Brush-Clevite Company (P.Z.T.4.). The values of these constants which appear in (2.82) are listed below.

$$c_{13}^D = 6.0 \cdot 10^{10} \text{ N m}^{-2}$$

$$h_{33} = 26.8 \cdot 10^8 \text{ V m}^{-1}$$

$$c_{12}^D = 8.4 \cdot 10^{10} \text{ N m}^{-2}$$

$$h_{31} = -9.2 \cdot 10^8 \text{ V m}^{-1}$$

$$c_{11}^D = 14.5 \cdot 10^{10} \text{ N m}^{-2}$$

$$c_{33}^D = 15.9 \cdot 10^{10} \text{ N m}^{-2}$$

Using these values it is found from (2.82) that for a material like P.Z.T.4. the transverse strains contribute only 15% of the total field strength in the Z direction.

Now in a finite disc S_3 will be the thickness strain and S_1 and S_2 will become S_r , the radial strain, giving rise to radial extensional modes of vibration. Consequently unless a strong radial resonance exists at the frequency of operation, S_r may also be neglected to a first approximation. This approximation will, therefore, be particularly well justified for the case of an infinite plate at frequencies close to thickness resonance.

Consider a plane longitudinal wave having a particle displacement amplitude A to be incident on the ceramic transducer so that the wavefront is parallel to the ceramic/water interface, (Fig. 2.14).

The particle displacement of the incident wave is represented by

$$u_3 = A e^{-j(kZ + \omega t)}$$

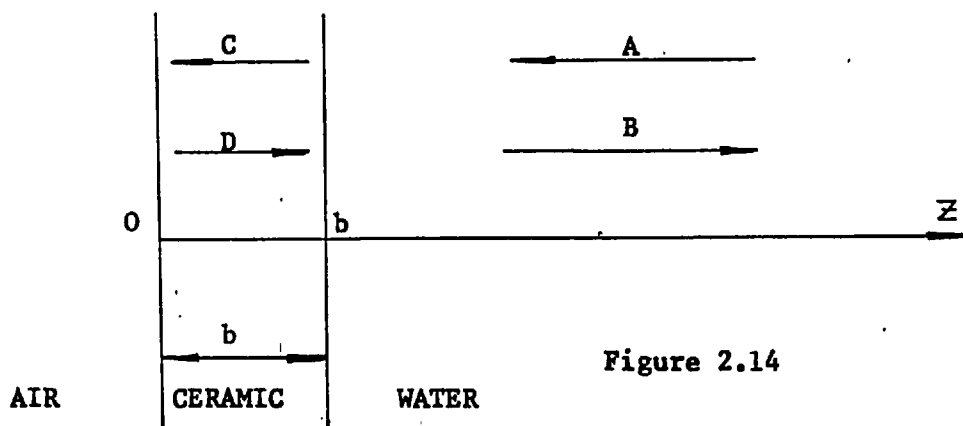


Figure 2.14

The one dimensional acoustic wave equation allows solutions corresponding to waves travelling in the direction of + Z increasing and + Z decreasing in both ceramic and water.

The particle displacement in the reflected wave is given by

$$u_3 = B e^{j(kZ - \omega t)}$$

Within the ceramic the total particle displacement from both waves is

$$u_3 = (C e^{-jk_d Z} + D e^{jk_d Z}) e^{-j\omega t}$$

where k_d is the wave number for a dilatational wave propagating along the Z axis.

Using (2.71) at $Z = b$

$$A e^{-jkb} + B e^{jkb} = C e^{-jk_d b} + D e^{jk_d b} \quad (2.83)$$

Now using the assumption that S_3 only is significant in determining the transducer output

$$T_3 = c_{33}^D S_3 = c_{33}^D \frac{\partial u_3}{\partial Z}$$

Hence equating normal stresses at $Z = b$

$$jk_d c_{33}^D \left(D e^{jk_d b} - C e^{-jk_d b} \right) = j \rho_o c^2 k \left(B e^{jkb} - A e^{-jkb} \right) \quad (2.84)$$

Applying (2.68) at $Z = 0$ it is found that $C = D$. On applying this result to (2.83) and (2.84) and subtracting to eliminate $B e^{jkb}$ it can be shown that

$$C = \frac{A e^{-jkb}}{\cos k_d b - jR \sin k_d b}$$

where

$$\frac{1}{R} = \frac{\rho_o c^2 k}{c_{33}^D k_d}$$

The particle displacement in the transducer can thus be written in terms of A the displacement amplitude of the incident wave.

$$u_3 = \frac{2A e^{-jkb} \cos k_d Z}{\cos k_d b - jR \sin k_d b} \quad (2.85)$$

The electric field across the transducer is calculated from (2.80) with $S_1 = S_2 = 0$ in accordance with the earlier approximation

$$E_3 = -h_{33} S_3 = -h_{33} \frac{\partial u_3}{\partial Z} \quad (2.86)$$

Hence the output potential difference V measured across the transducer thickness is given by the integral of E_3 .

$$V = - \int_0^b E_3 \, dz$$

Thus substituting (2.85) into equation (2.86), integrating and taking the real part:

$$|V| = \frac{h_{33} \, 2A \, |\cos k_d b - 1|}{(\cos^2 k_d b + R^2 \sin^2 k_d b)^{\frac{1}{2}}} \quad (2.87)$$

Consider the the case where the plane wave is incident at an angle θ_i to the plate normal, as shown in Fig. 2.15.

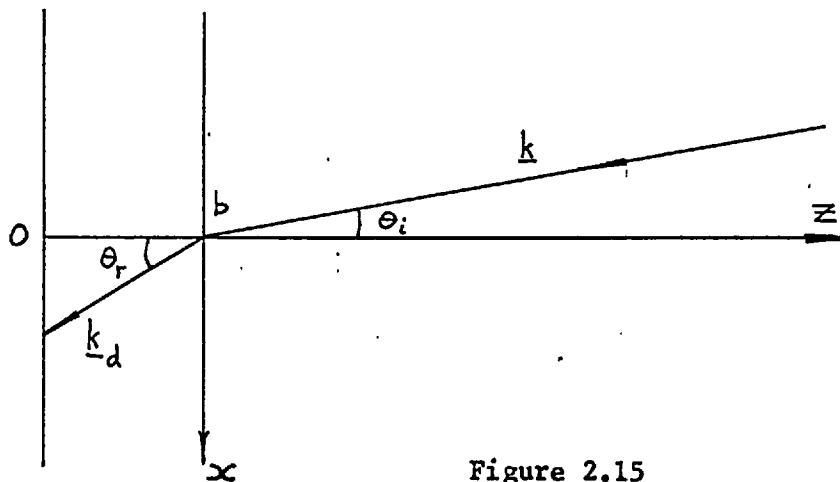


Figure 2.15

The incident wave will be refracted on passing through the water/ceramic boundary and will therefore propagate at a new angle θ_r to the normal. The earlier decision to ignore S_1 and S_2 is equivalent to considering the material of the plate as an ideal fluid. Mode conversion at the boundaries must therefore also be ignored. This approximation becomes increasingly less valid as θ_i increases because S_1 and S_2 will increase until they cannot be neglected in comparison with S_3 .

The longitudinal component of particle displacement is obtained by substituting $k_d \cos \theta_r$ for k_d in equation (2.85). Consequently the potential difference across the transducer is given by

$$|V| = \frac{h_{33} 2A |\cos k'_d b - 1|}{(\cos^2 k'_d b + R^2 \sin^2 k'_d b)^{\frac{1}{2}}} \quad (2.88)$$

$$\text{where } k'_d = k_d \cos \theta_r = k_d \left(\frac{c_d}{c} \right) \left[\left(\frac{c}{c_d} - \sin \theta_i \right) \left(\frac{c}{c_d} + \sin \theta_i \right) \right]^{\frac{1}{2}}$$

Snell's law has been assumed for the longitudinal wave

$$\frac{\sin \theta_i}{c} = \frac{\sin \theta_r}{c_d}$$

Now for a very thick plate $|V|$ can be expected to tend to zero as $\sin \theta_i \rightarrow \frac{c}{c_d}$, that is $\theta_i \rightarrow \theta_c$ the critical angle of incidence for longitudinal waves. However, for a plate whose thickness may be less

than half a wavelength, the situation is much more complicated and is discussed in detail by Brekhovskikh (53).

At angles of incidence greater than θ_c the angle of refraction is complex and so called inhomogeneous waves are propagated in the plate. These are waves which propagate in the x direction and have an amplitude which falls off exponentially in the Z direction. In a thin piezoelectric plate they will contribute to E_3 and hence to the output voltage.

The acoustic input impedance of the plate is a minimum at normal incidence when the plate is an integral number of half wavelengths thick.

$$\text{i.e. } k_d b = m\pi, m = 1, 2, 3, \dots$$

However, the piezoelectrically generated output voltage $|V|$ is a maximum only for b equal to an odd integral number of half wavelengths, ie when $m = 1, 3, 5, \dots$. Hence the output voltage will be a maximum for an obliquely incident plane longitudinal wave when

$$b k_d \cos \theta_r = m\pi, m = 1, 3, 5 \quad (2.88a)$$

An object diffraction pattern can be replaced by an equivalent plane wave spectrum from the idea of Fourier decomposition. Hence (2.88) gives the amplitude of the plate response for a single plane wave in the spectrum, incident at an angle θ_i . The phase velocity of the wave along the ceramic/water interface is $\frac{c}{\sin \theta_i}$ and therefore, from (2.39), represents a spatial frequency $f_x = \frac{\sin \theta_i}{\lambda}$

in the object spectrum. As long as θ_r is sufficiently small so that the main contribution to E_3 is directly through $h_{33} S_3$ then the information contained in f_x will be registered by the detector as a voltage distribution with spatial frequency f_x . There will, however, be a condition when the contributions to E_3 by $h_{33} S_3$ and $h_{31}(S_1 + S_2)$ are equal and it will not be possible to distinguish the frequency f_x from the spatial frequency in $h_{31}(S_1 + S_2)$ which will be different from f_x . In other words the required signal will be immersed in a 'noise' function represented by $h_{31}(S_1 + S_2)$ and similar terms. This phenomenon will therefore set a maximum limit on the object spatial frequency which can be detected.

In terms of spatial frequency, k'_d becomes, after slight re-arrangement:

$$k'_d = \left[(k_d - 2\pi f_x)(k_d + 2\pi f_x) \right]^{\frac{1}{2}} \quad (2.89)$$

The ratio of detector output voltage to particle displacement amplitude input for a plane wave spectrum $A(f_x)$ is obtained by substituting (2.89) into (2.88)

$$\frac{|V(f_x)|}{|A(f_x)|} = 2h_{33} \frac{|\cos\{(k_d - 2\pi f_x)(k_d + 2\pi f_x)\}^{\frac{1}{2}} b - 1|}{\left[\cos^2\{(k_d - 2\pi f_x)(k_d + 2\pi f_x)\}^{\frac{1}{2}} + R^2 \sin^2\{(k_d - 2\pi f_x)(k_d + 2\pi f_x)\}^{\frac{1}{2}} b \right]^{\frac{1}{2}}} \quad (2.90)$$

(2.90) has the form of a system transfer function as defined by (2.10).

The coherent transfer function for the piezoelectric detector defined in (2.46) relates an input pressure distribution to an output voltage distribution. In terms of pressure $p(f_x)$, equation (2.90) becomes:

$$\frac{|V(f_x)|}{|p(f_x)|} = \rho_o c^2 \frac{|V(f_x)|}{|A(f_x)|} = |G_p(f_x)| \quad (2.90a)$$

In order to determine $|G_p(f_x)|$ a suitable input function $p(f_x)$ must be found. The fringe system obtained with a two beam interferometer has a constant spatial period over the region of interference and is stationary in time, enabling the spatial period to be measured over a time scale much longer than the period of oscillation of the ultrasound wave.

2.8 Theory of a Sinusoidal Standing Wave Technique for Determining the Coherent Transfer Function of a Piezoelectric Ultrasonic Detector

The sinusoidal standing wave technique was devised directly from (2.10) thinking of the analogy between $G_m(f_x, f_y)$ equation (2.15) and the gain function of an electronic circuit. The simplest technique for obtaining the gain function is to compare the amplitude of the output with that of a sinusoidal input and plot this ratio against the input frequency. In terms of time varying signals and temporal frequencies this is easy to imagine; it is a little more difficult when spatially varying signals and spatial frequencies are considered. The input signal must have a fixed constant period which is, ideally, variable from infinity to zero. These requirements are fulfilled by cosinusoidal two beam interference fringes. Two discrete ultrasound

sources may be used provided they are driven from the same oscillator. Fig. 2.16 illustrates a convenient system. The two sources are positioned symmetrically on either side of the Z axis and orientated to produce interference fringes, over the plane of the detector face, which are parallel to the y axis. The period of these fringes then determines a spatial frequency q_x . Consequently the input spatial frequency is a one dimensional function only. After passing through the detector, the spatial frequency function which has been converted into a voltage of spatial frequency q_x is recorded electrically.

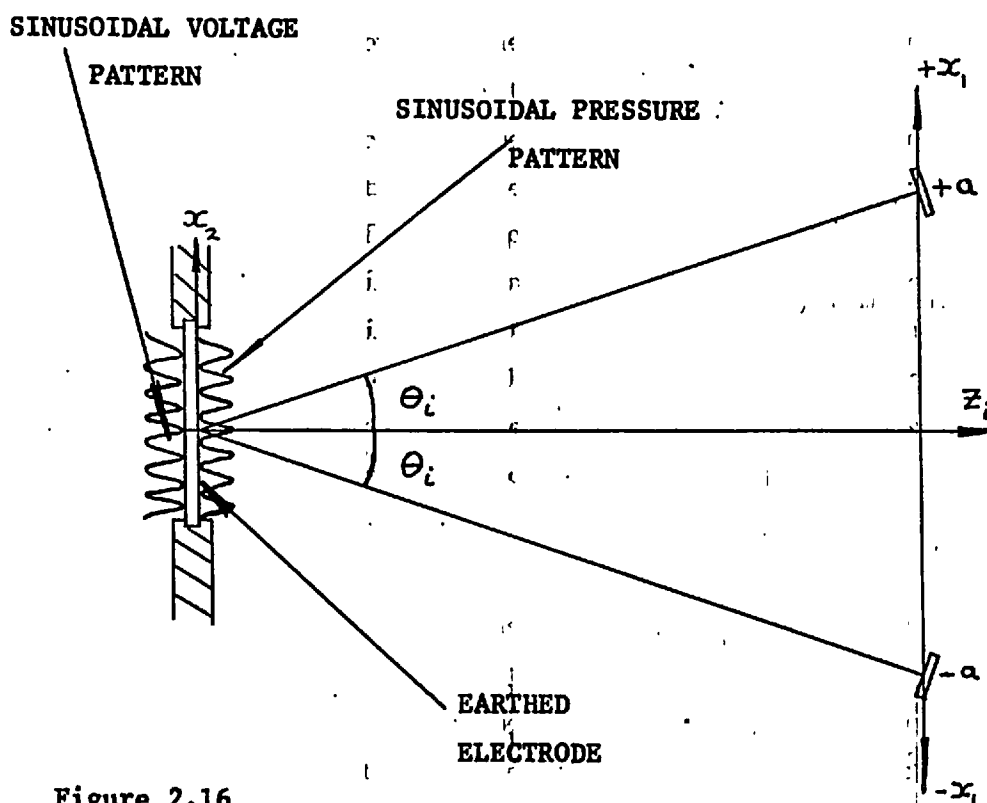


Figure 2.16

Consider a single source located at $x_1 = +a$ in the (x_1, y_1) plane. This source will have a characteristic source velocity distribution $\frac{\partial \phi}{\partial n}(x_1 - a, y_1)$ using the notation of section 2.4. The velocity potential in the far field diffraction

pattern is then given by equation (2.40a) with $f_x = \frac{x_2}{\lambda Z_1}$ and

$$f_y = \frac{y_2}{\lambda Z_1}$$

$$\phi_1 \left(\frac{x_2}{\lambda Z_1}, \frac{y_2}{\lambda Z_1} \right) = \frac{e^{jkZ_1}}{2\pi Z_1} \mathcal{H} \left[\frac{\partial \phi}{\partial n}(x_1 - a, y_1) \right]$$

Now if
$$\mathcal{H} \left[\frac{\partial \phi}{\partial n}(x_1, y_1) \right] = \phi_0 \left(\frac{x_2}{\lambda Z_1}, \frac{y_2}{\lambda Z_1} \right)$$

then
$$\phi_1 \left(\frac{x_2}{\lambda Z_1}, \frac{y_2}{\lambda Z_1} \right) = K \phi_0 \left(\frac{x_2}{\lambda Z_1}, \frac{y_2}{\lambda Z_1} \right) \exp \left(j2\pi a \frac{x_2}{\lambda Z_1} \right)$$

where $K = \frac{1}{2\pi} \frac{e^{jkZ_1}}{Z_1}$ and is a constant.

Similarly the far field diffraction pattern produced by a second identical source located at $x_1 = -a$ is:

$$\phi_2 \left(\frac{x_2}{\lambda Z_1}, \frac{y_2}{\lambda Z_1} \right) = K \phi_0 \left(\frac{x_2}{\lambda Z_1}, \frac{y_2}{\lambda Z_1} \right) \exp - \left(j2\pi a \frac{x_2}{\lambda Z_1} \right)$$

The interference pattern produced by the two sources radiating together in phase is given by:

$$\phi_2 \left(\frac{x_2}{\lambda Z_1}, \frac{y_2}{\lambda Z_1} \right) = \phi_1 + \phi_2 = K\phi_0 \left(\frac{x_2}{\lambda Z_1}, \frac{y_2}{\lambda Z_1} \right) \cos \left(2\pi a \frac{x_2}{\lambda Z_1} \right)$$

Hence the pressure amplitude distribution in front of the detector is given by:

$$|p(x_2, y_2)| = \rho_0 c k \left| \phi \left(\frac{x_2}{\lambda Z_1}, \frac{y_2}{\lambda Z_1} \right) \right| = \frac{\rho_0 c}{Z_1 \lambda} \left\{ \frac{1}{2} \left| \phi_0 \left(\frac{x_2}{\lambda Z_1}, \frac{y_2}{\lambda Z_1} \right) \right|^2 \left(1 + \cos 4\pi a \frac{x_2}{\lambda Z_1} \right) \right\}^{\frac{1}{2}} \quad (2.91)$$

where (2.24) has been used to convert from velocity potential to pressure.

As expected the pressure amplitude distribution takes the form of a cosinusoidal fringe system parallel to the y_2 axis and amplitude modulated with the diffraction pattern of a single source. The period of the fringe pattern is $\frac{\lambda Z_1}{2a}$. This fringe system is defined

as the input function to the detector having a spatial frequency of q_x where

$$q_x = \frac{a}{\lambda Z_1} \approx \frac{\sin \theta_i}{\lambda} \quad (2.92)$$

Note that q_x is half the reciprocal of the measured fringe period. This brings it in line with the standard definition of spatial frequency as used in diffraction theory.

If a is varied, a whole range of input spatial frequencies may be obtained. The input pressure function is assumed to be a cosinusoidal function of q_x only and therefore results in an output voltage distribution which is also a cosinusoidal function of q_x . Only the real parts of $V(q_x)$ and $p(q_x)$ are recorded with the present technique, therefore the modulus of the coherent transfer function will be determined. Using (2.10):

$$V(q_x) = G_p(q_x) p(q_x)$$

$$\text{Therefore } |G_p(q_x)| = \frac{|V(q_x)|}{|p(q_x)|} \quad (2.93)$$

where $|G_p(q_x)|$ is the modulus of the piezoelectric coherent transfer function defined in (2.46).

It is only necessary to determine G_p as a function of one spatial frequency variable because the ceramic material possesses rotational symmetry around the Z axis.

Because the two sources are arranged symmetrically about the Z axis in the (Z, x) plane, the transfer function measured by this technique will only be applicable to points on an object which lie on or close to the Z axis.

Now recalling (2.67)

$$|G_p(q_x)| = |D_p(q_x)| \cdot |M_p(q_x)|$$

Consequently a graph of the amplitude of $V(q_x)$ divided by the amplitude of $p(q_x)$ plotted against q_x will describe $|D_p(q_x)|$.

By fixing the depth of modulation of $p(q_x)$ and noting that of $|V(q_x)|$, the modulation transfer function $|M_p(q_x)|$ can be obtained directly.

2.9 Theory of an Impulse Response Technique for Determining the Resolution Element Directivity Function of a Piezoelectric Ultrasound Detector

The derivation of equation (2.10) implies that the coherent transfer function of the detector can be determined from its impulse response by Fourier transformation. The input function to the detector in this case would ideally be a delta function of position, in other words an ideal point source. A true point source is difficult to realize in practice so the response of the detector to a small but finite stimulation point equal to the size of a resolution element will be considered. If the detector is space invariant, then the measured directivity function will be independent of the position of the source point. However, this cannot be assumed in general so the special case of a stimulation point symmetrically placed at the origin 0 will be considered. This is indicated in Fig. 2.17.

The detector properties are circularly symmetrical about 0. Consequently the two spatial frequency variables f_x and f_y can be replaced by a single radial spatial frequency ρ and the analysis carried out in cylindrical co-ordinates. It is therefore a great simplification to place the source point at 0.

CIRCULAR ELECTRODE

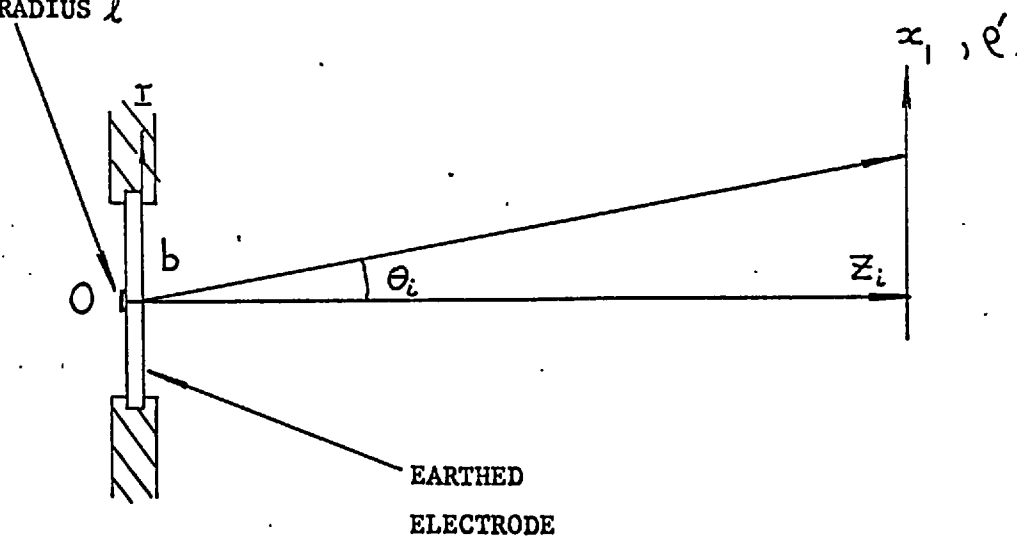
OF RADIUS ℓ 

Figure 2.17

Using the assumption that a field in the Z direction gives rise to an appreciable strain only in the same direction, the input function to the system can be considered as a constant strain function localized over a small area.

$$S_3(r) = S_3 \text{ for } r \leq \ell$$

$$S_3(r) = 0 \text{ for } r > \ell$$

The spatial frequency spectrum $S_1(\rho)$ of the symmetrical input function is given by a zero order Hankel transform which must be real and even, as the input is real and even.

$$S_i(\rho) = 2\pi \int_0^l s(r) J_0(2\pi r \rho) r dr$$

$$= 2\pi S_3 \int_0^l J_0(2\pi r \rho) r dr$$

Now $\int_0^x r J(r) dr = x J_1(x)$

Hence
$$S_i(\rho) = 2\pi S_3 l^2 \left[\frac{J_1(2\pi l \rho)}{2\pi l \rho} \right] \quad (2.94)$$

Using (2.10) the detector response spectrum $S_o(\rho)$ is given by

$$S_o(\rho) = G(\rho) 2\pi S_3 l^2 \left[\frac{J_1(2\pi l \rho)}{2\pi l \rho} \right] \quad (2.95)$$

Now $|G_p(\rho)| = |D_p(\rho)| \cdot |M_p(\rho)|$

where $|M_p(\rho)| = 1$ for an impulse response technique.

The detector response $s_o(r)$ is given by the inverse Hankel transform of (2.94)

$$s_o(r) = 2\pi \int_0^\infty D_p(\rho) 2\pi S_3 l^2 \left[\frac{J_1(2\pi l \rho)}{2\pi l \rho} \right] J_0(2\pi r \rho) \rho d\rho \quad (2.96)$$

This strain pattern produces a normal stress pattern which over the face at $Z = b$ gives rise to a pressure distribution $p(r)$ from (2.69)

$$p(r) = c_{33}^E s_o(r)$$

Constant E boundary conditions have been taken because the point source is driven from a low impedance source and so the electrodes are almost short circuited.

Using (2.25) the velocity potential over the detector face at $Z = b$ is given by:

$$\phi = \frac{-jp}{c\rho_o k}$$

Then (2.39b) with $f = \phi$ is used to take the Hankel transform of $s_o(r)$ and so obtain $S_o(\rho')$ as the far field diffraction pattern of the detector driven at the origin and allowed to radiate. ρ' is the spatial frequency variable for the far field.

$$\phi(\rho') = -\frac{c_{33}^E}{c\rho_o} \frac{e}{Z_1} \int_r s_o(r) J_o(2\pi r \rho') r dr \quad (2.97)$$

Hence if ρ' is put equal to ρ in (2.96) and using the theorem that a Hankel transform followed by an inverse Hankel transform of the same function is equal to the function alone, (2.97) reduces to

$$\phi(\rho') = -\frac{c_{33}^E}{c\rho_o} \frac{e}{Z_1} 2\pi S_3 \ell^2 \left[\frac{J_1(2\pi \ell \rho')}{2\pi \ell \rho'} \right] D_p(\rho') \quad (2.98)$$

The pressure amplitude distribution in the far field is given by $\rho_0 c k |\phi(\rho')|$.

Therefore

$$|p(\rho')| = \frac{c_{33}^E}{Z_1} 2\pi S_3 \ell^2 \left| \frac{J_1(2\pi \ell \rho')}{2\pi \ell \rho'} \right| \cdot |D_p(\rho')| \quad (2.98)$$

Now ρ' is defined by the expression $\rho'^2 = f_x^2 + f_y^2$ but if the pressure distribution in the far field diffraction pattern is recorded only along the x_1 axis

$$\rho' = f_x = \frac{x_1}{\lambda Z_1} \approx \frac{\sin \theta_i}{\lambda} \quad (2.100)$$

Therefore equating (2.100) and (2.92)

$$|p(q_x)| = K \left| \frac{J_1(2\pi \ell q_x)}{2\pi \ell q_x} \right| \cdot |D_p(q_x)| \quad (2.101)$$

where K is a real constant for a given ceramic material, point source radius and far field distance Z_1 .

Apart from an amplitude factor $|D_p(q_x)|$ will be converted into a time varying signal by driving a point hydrophone at a constant speed along the x_1 axis. This signal may then be recorded on an X - Y plotter or a chart recorder.

The amplitude factor $K \left| \frac{J_1(2\pi \ell q_x)}{2\pi \ell q_x} \right|$ could result in an erroneous

determination of $|D_p(q_x)|$ unless it is constant over the range of q_x where $|D_p(q_x)|$ is non-zero. The requirement that the amplitude factor is to be constant is the same as limiting the driving spot to the size of a 'resolution element'. The size of the resolution element is not known 'a priori' and because $|p(q_x)|$ is measured and

not $K \left| \frac{J_1(2\pi\ell q_x)}{2\pi\ell q_x} \right|$ it cannot be determined from (2.101). The first

zero of this amplitude factor is given by the first zero of J_1 which occurs when

$$\frac{2\pi\ell x_1}{\lambda Z_1} = 3.832$$

If ℓ is chosen to be 0.6λ or less the first zero of $J_1(2\pi\ell q_x)$ will occur for values of θ_i greater than $\pi/2$ and as a result the amplitude

factor $\left| \frac{J_1(2\pi\ell q_x)}{2\pi\ell q_x} \right|$ cannot significantly influence the shape of

$p(q_x)$ at values of θ_i below $\pi/10$. A graph of $2 \left| \frac{J_1(2\pi\ell q_x)}{2\pi\ell q_x} \right|$

with $\ell = \lambda/2$ for values of θ_i up to 26° is shown in Fig. 2.18.

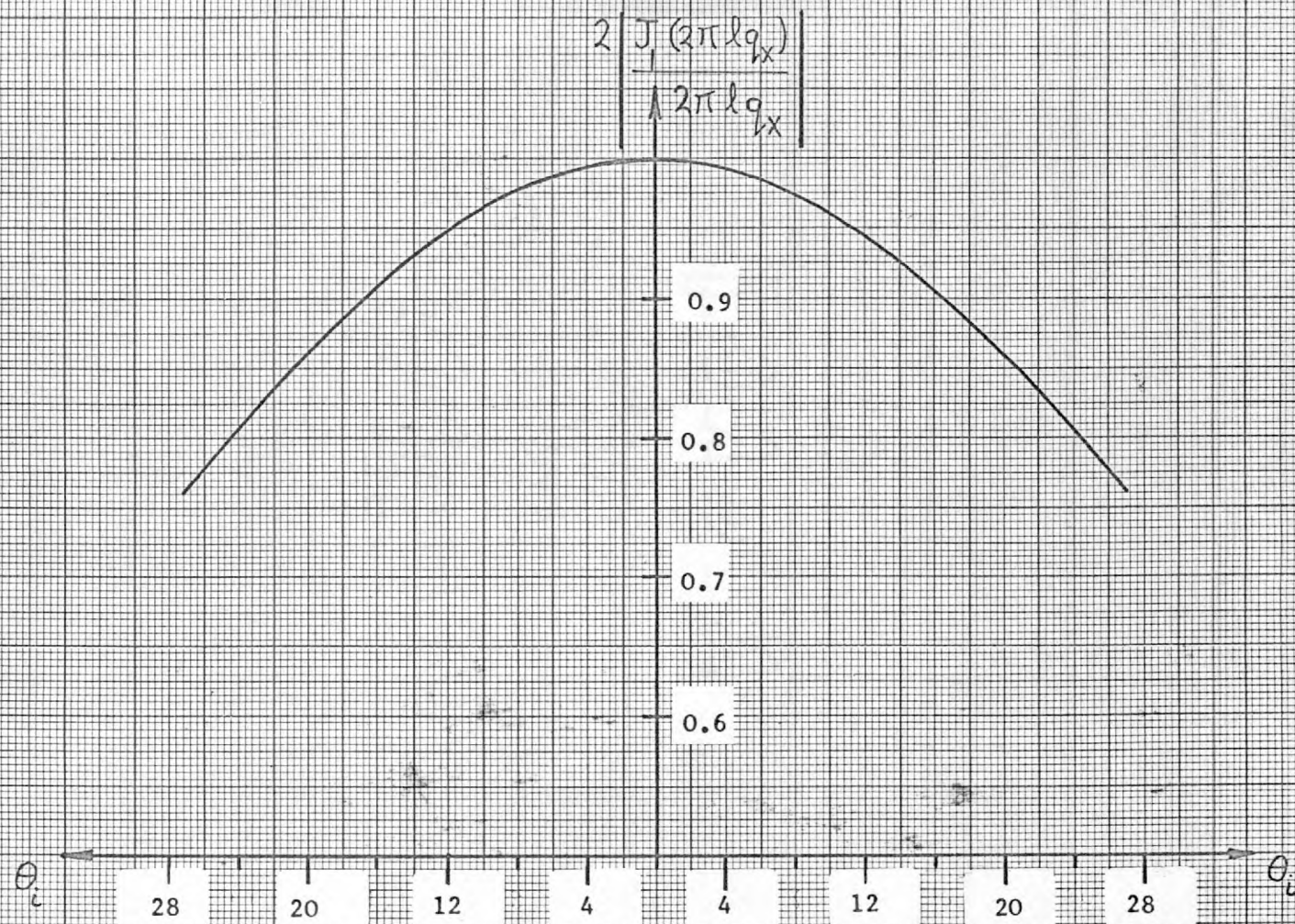


Figure 2.18 Graph of the amplitude factor $2 \left| \frac{J_1(2\pi l q_x)}{2\pi l q_x} \right|$ against the polar angle θ_i

CHAPTER 3

Description of Apparatus

Suitable equipment was not available commercially when the research project was started so the majority of the mechanical and electrical systems were specially designed and constructed.

3.1 Anechoic Tank Design

Since the entire experiment was conducted with continuous wave ultrasound, free field conditions had to be simulated within a finite working volume in order to prevent standing waves. This was achieved by using an anechoic tank. The experiments were conducted with ultrasound propagating predominantly along the longitudinal axis of the tank, so it was only necessary to fix a good ultrasound absorber to the end walls. The material used was an aluminium loaded butyl rubber produced by the Admiralty Materials Laboratory, Holton Heath, Poole, Dorset, described as AML type B1 anechoic coating. The material configuration is shown in Fig. 3.1. Its ultrasound absorption coefficient was measured at normal incidence using a modified acoustic interferometer technique as described in Appendix 1. The side walls and bottom of the tank were lined with rubber car-mats stretched over perspex and stainless steel frames. These had a sufficiently high absorption to reduce the side lobe reflections to a level below that obtained from the zero order lobe incident on the end of the tank. No attempt was made to minimise reflections from the water surface. When using the impulse response technique at frequencies of 2 MHz and above, it was found necessary

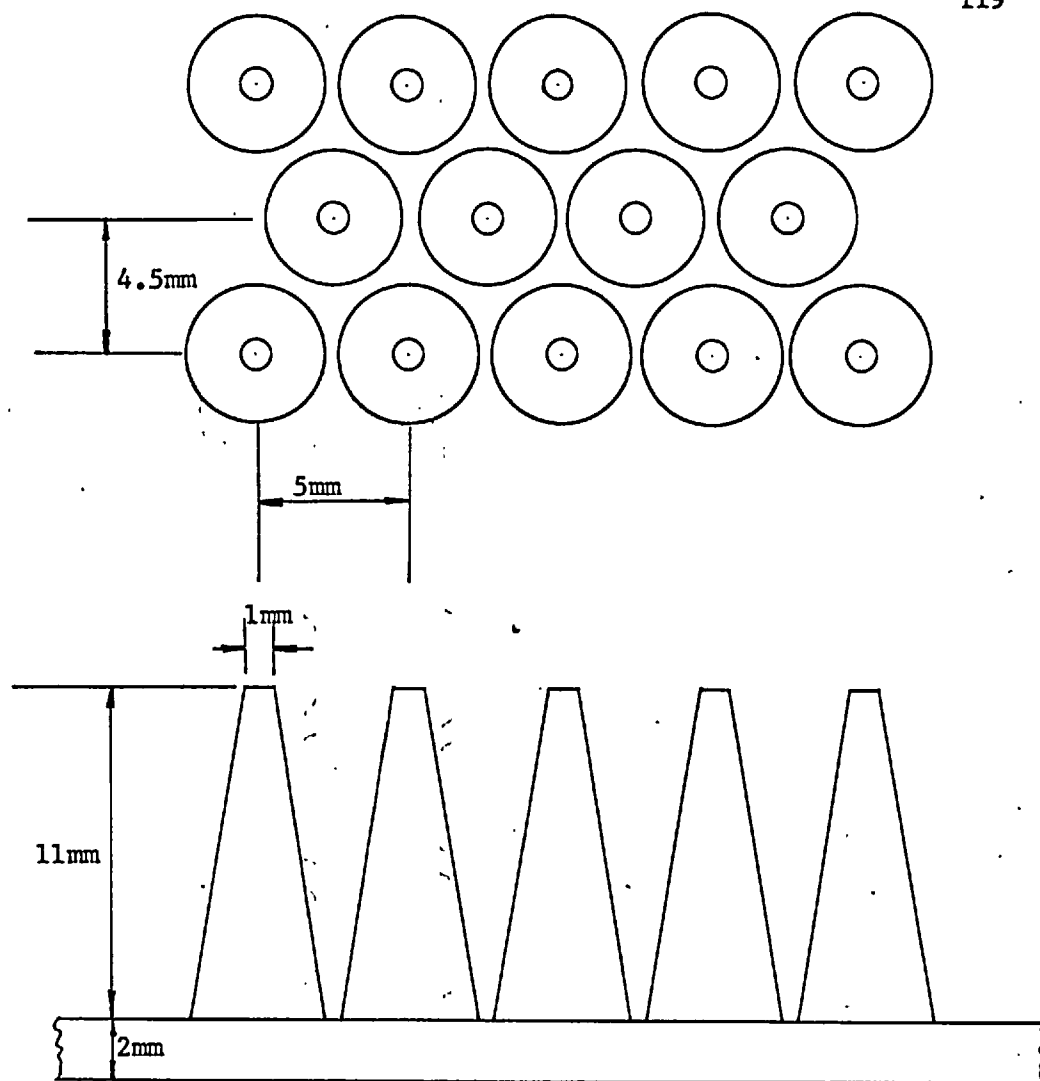


Figure 3.1 Geometric profile of A.M.L. type B1 anechoic coating.
Scale 4 times full size

to suspend obliquely at the end of the tank rectangular mats of AML type B1. This procedure reduced to an acceptable level the specularly reflected ultrasound component.

The tank itself was of a steel framed construction with plate glass sides and brass plate ends and bottom. Its overall dimensions were 20" x 20" x 36". Four adjustable feet were provided at the corners of the tank so that it could be levelled. Along the top 46" x $\frac{1}{2}$ " steel rods formed a track along which the source and hydrophone carriages could be slid manually.

3.2 Mechanical Systems

3.2.1 Hydrophone Scanning Mechanism

The hydrophone carriage consists of a brass block which is driven backwards and forwards across the tank on machined brass tubes by a rotating screw. The block is coupled to the screw thread by a free nut which rides between two skirts fixed to the block. The nut is prevented from rotating by an attached Y yoke running along a guide rail parallel to the load bearing tubes. This arrangement does not transmit any torque from the rotating screw thread to the brass block, only a longitudinal force. It is necessary to use this system to prevent the hydrophone from oscillating as the screw rotates. A key-way is also incorporated in one of the bearing tubes to help locate the centre of the block and hence keep the hydrophone moving along a straight line during a scan. The maximum length of scan is 430 mm. Motive power is provided by a 24 v d.c. motor which drives the screw thread through a worm and wheel gear having a reduction ratio of 100 to 1. The

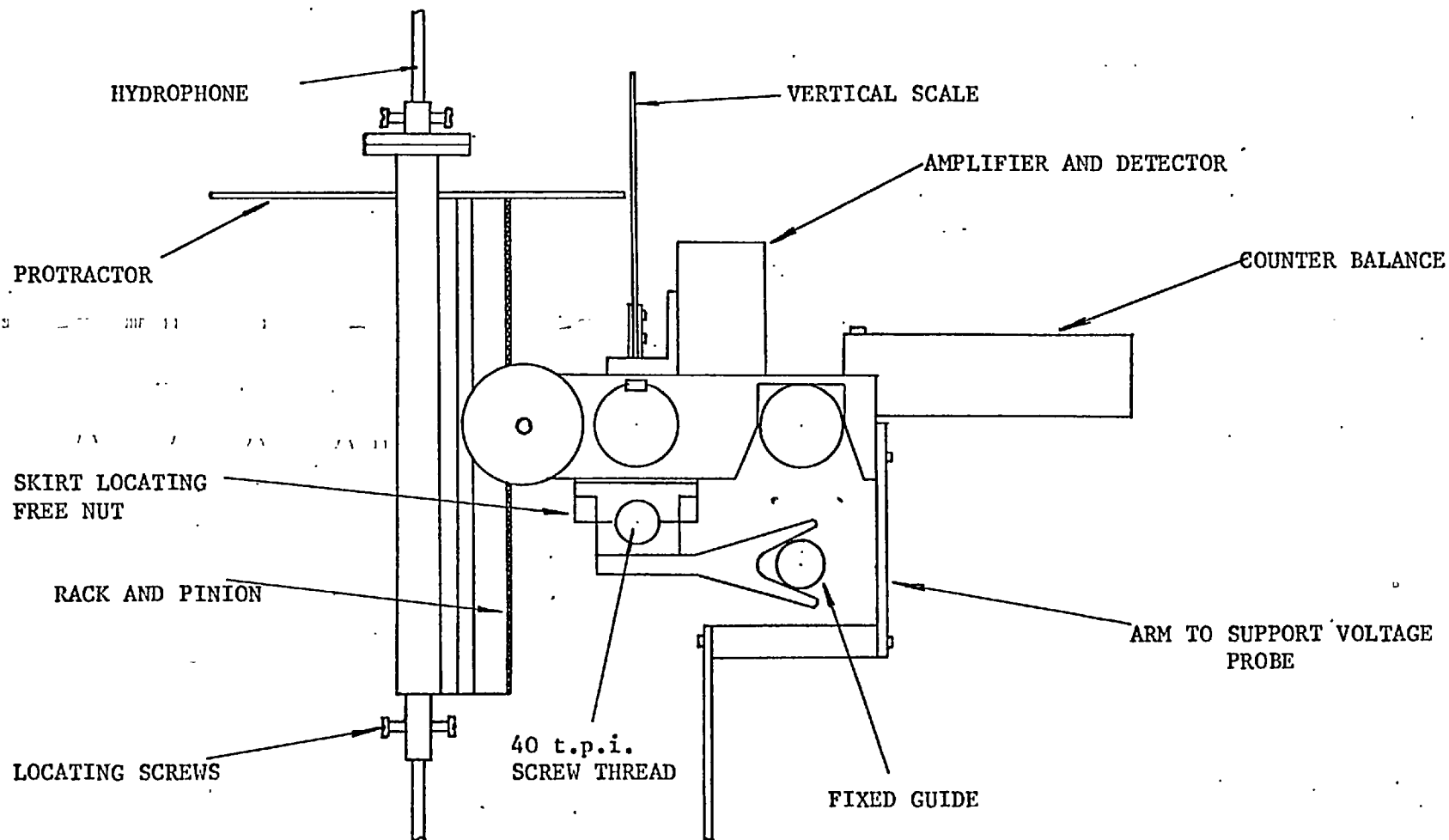


Figure 3.2 Diagram of hydrophone probe holder and scanning mechanism. Scale $\frac{1}{2}$ full size

motor speed, and hence the scanning speed, is controlled by the voltage applied to the motor. A revolution counter is attached to the screw thread to enable the length of scan undertaken in any experiment to be found from the number of revolutions. The thread is accurately machined with 40 turns per inch so that the error in recording the length of a scan is approximately ± 1 mm.

The hydrophone itself is clamped by two sets of three centering screws in a mounting which can be rotated through 360° in the horizontal plane. This in turn is fastened to a rack and pinion, on the block, for raising and lowering the hydrophone. A vertical scale is attached to the block graduated in $\frac{1}{2}$ mm divisions and a 360° protractor is fixed to the rotating part of the hydrophone mount. The brass block is provided with a counter-balance to balance the weight of the hydrophone, rack and pinion. It also carries the hydrophone signal amplifier and envelope detector circuit. Power is fed to the amplifier via a four-way screened cable which also returns the slowly varying output to the display system.

The mechanism is compact and provides the hydrophone with one angular and three translational degrees of freedom. Fig. 3.2 shows diagrammatically a vertical section through the block.

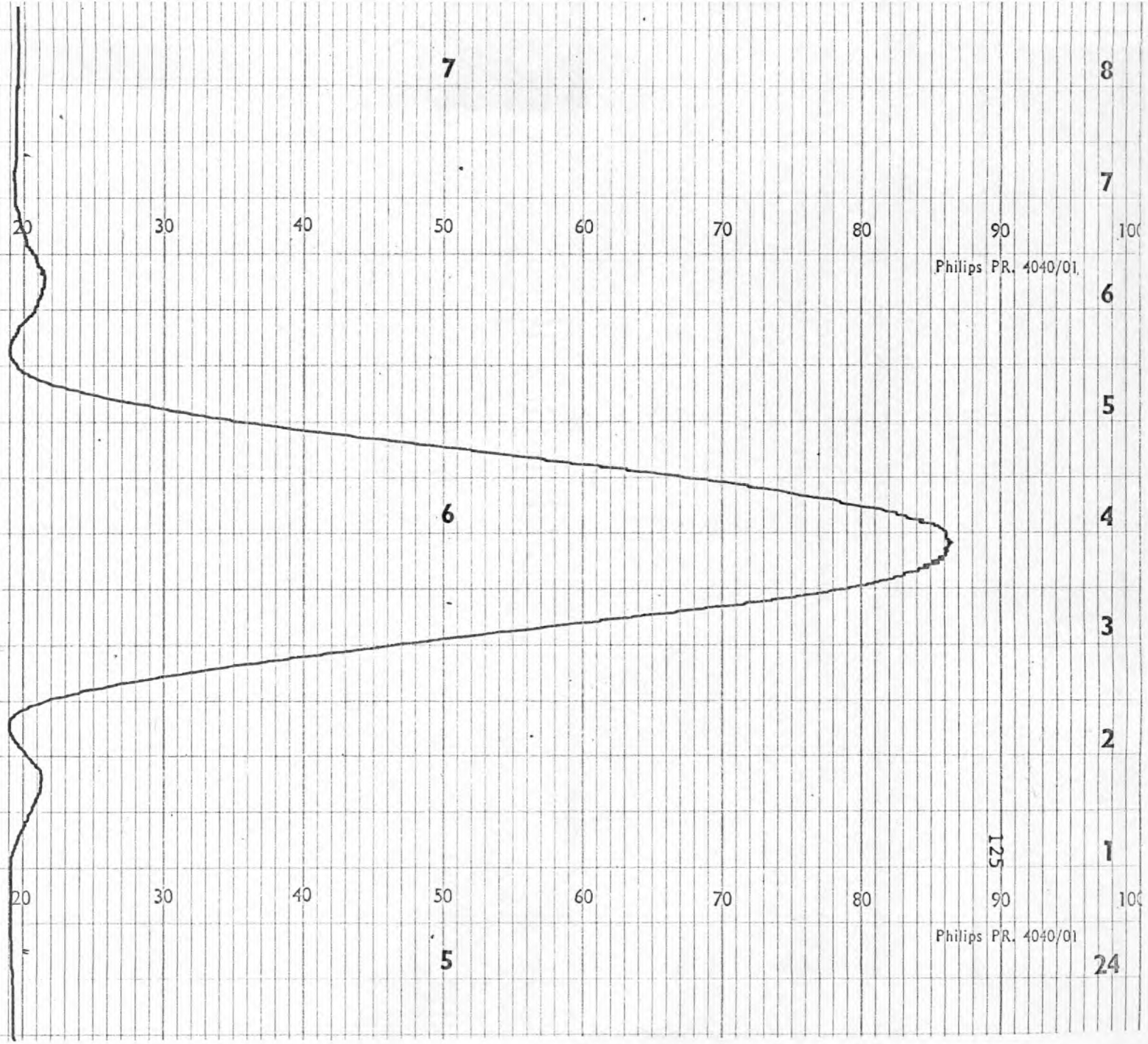
3.2.2 Ultrasound Source Holder and Carriage

The sinusoidal standing wave technique requires two spatially independent ultrasound sources. Two ceramic transducers are used, mounted in identical holders. Fig. 3.3 shows the details of these holders which are demountable so that the transducers can be easily

replaced. Space is provided in the body of the holder for a ferrite pot core transformer so that the radiation resistance of the transducer can be matched to the co-axial transmission line impedance. In practice, however, this is found to be unnecessary. "O" ring seals are used to make the holders water-tight. Electrical connection is made from the holder to the front silvered face of the transducer by means of a single turn copper spring. The driving signal is brought in by Amphenol miniature co-axial cable No. 421-106. Evo-Stick is used to make a water-tight, flexible seal between the cable and the back plate. Electrical contact with the rear face of the transducer is made via a brass ring. This is raised above the level of the surrounding insulating material to produce an air backing for the transducer. The technique of mounting the transducer is particularly interesting. One inch diameter ceramic discs are used but these are loosely 'clamped' at approximately two thirds their radius between the "O" ring and the brass contact ring. With this arrangement the central portion of the disc is very close to a piston source, unlike an edge clamped transducer which in its fundamental mode has a displacement amplitude distribution following $J_0(r)$ with the first zero at the edge. The radiating area is stopped down by the brass body of the mount to a circle of 1 cm diameter so that the beam produced by the source has sufficiently large angular width under all experimental conditions. Fig. 3.4 shows a typical source far field pattern. The source holder can be rotated about a horizontal axis and clamped at any desired azimuthal angle. This is a necessary facility for lining up.

Each source holder is clamped in a carriage which runs across the top of the tank on two accurately machined brass tubes. Only the perpendicular distance from the longitudinal axis of the tank (the x co-ordinate) is a parameter in the sinusoidal standing wave

Figure 3.4 A typical source far field amplitude pattern
recorded at 2.075 MHz



ACOUSTIC TANK — TRANSVERSE SLIDE.

SCALE : FULL SIZE. MATERIALS BRASS PLATE AND
COPPER OR BRASS TUBE.

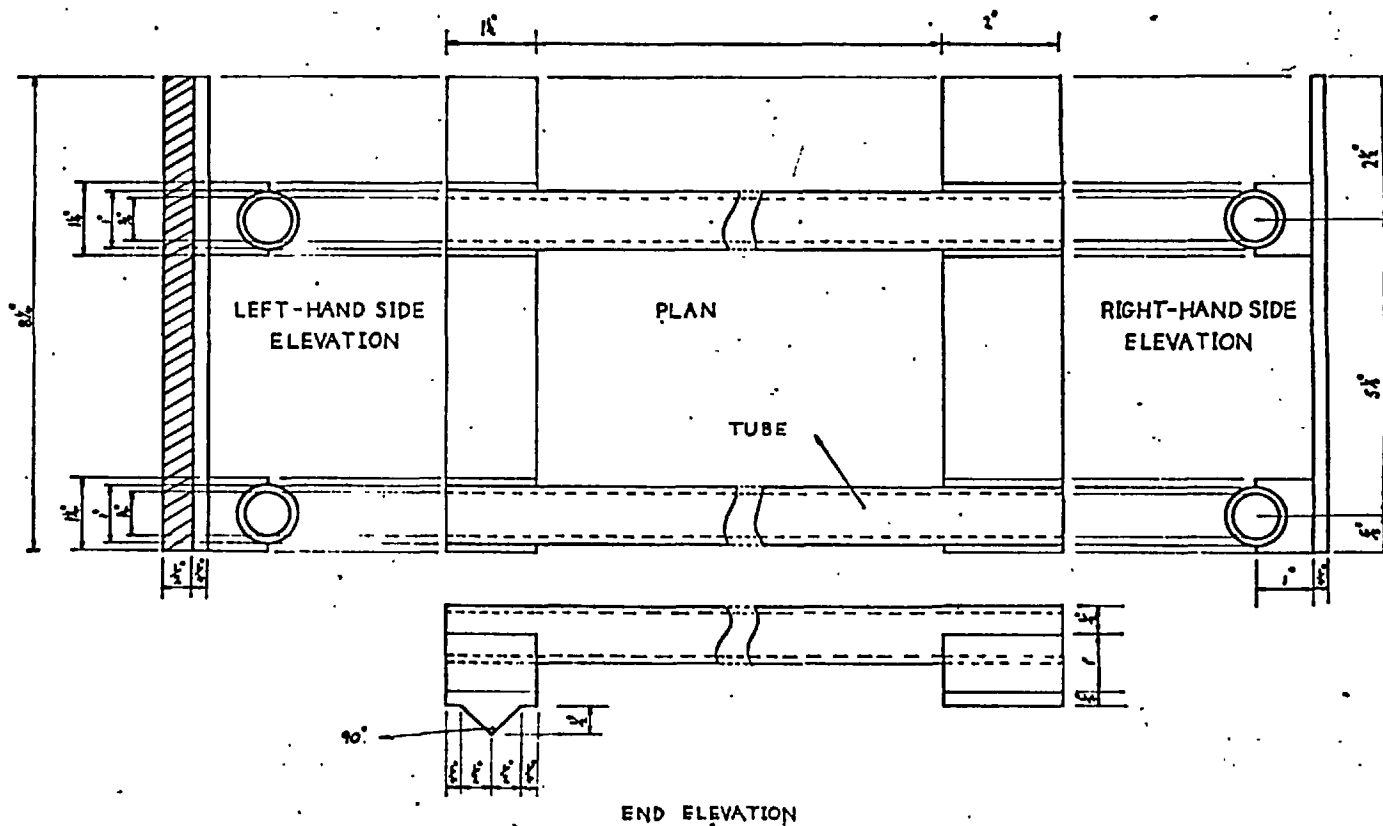
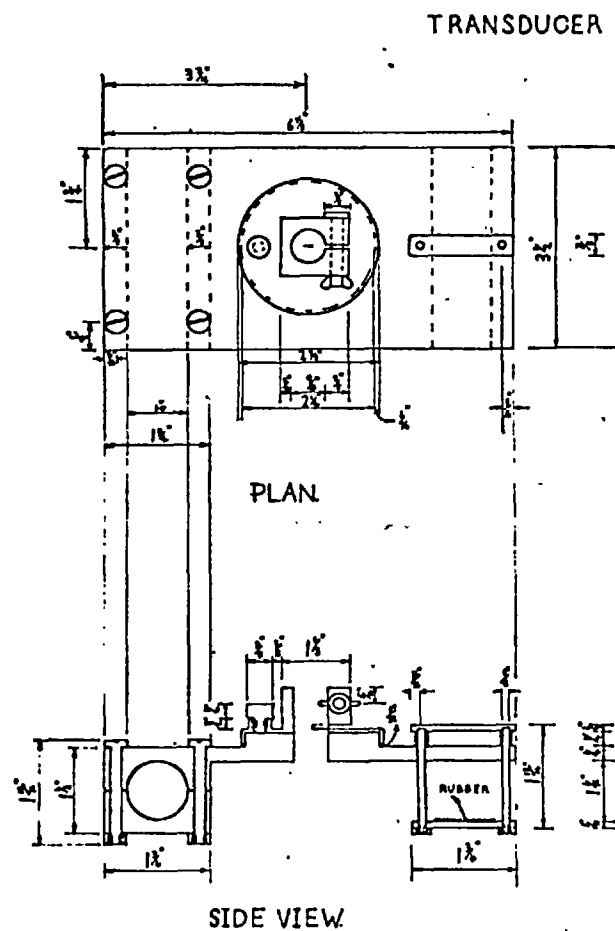
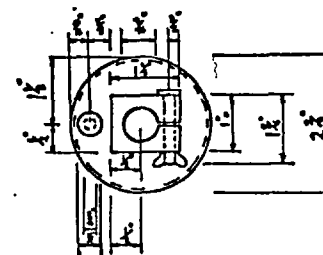


Figure 3.5 Scale drawing of the ultrasound source holder transverse slide



SCALE : FULL SIZE



DETAILS OF ROTARY MOVEMENT.

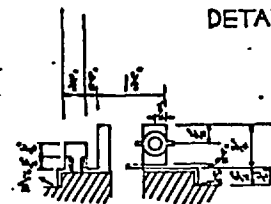


Figure 3.6 Scale drawing of the ultrasound source carriage

technique so both source carriages are fixed on to the one set of parallel tubes. Figs. 3.5 and 3.6 show the working drawings from which the carriages were made. The height of each source may be varied independently by hand and then fixed with a screw clamp. Mechanical pressure clamping is used to maintain the position of each carriage. Each source can be rotated in the horizontal plane on a two inch diameter metal-metal bearing surface. A protractor is fitted to the upper, moving part of the bearing surface so that the angle of rotation can be measured with respect to a reference mark on the carriage. A stainless steel scale graduated in millimetres is fixed parallel to the bearing tubes so that the x co-ordinate of each source may be read off easily. The whole source carriage assembly is located on the tank runners and may be positioned along the tank length manually.

3.2.3 The Voltage Probe and Scanning Mechanism

The voltage probe circuit is contained within a half inch internal diameter copper tube as shown in Fig. 3.7 which provides electrical shielding completely up to the tip of the input lead. The first field effect transistor gate lead, diameter 0.44 mm, is used to sense the voltage distribution across a sample detector. It is recessed by approximately one tenth of a millimetre from the front surface of a P.T.F.E. brush, and so is capacitatively coupled to the piezoelectric voltage when the probe slides across the detector surface.

The hydrophone scanning mechanism is used to scan the voltage probe across the detector. An aluminium strip with the probe tube holder attached is bolted to the screw thread driven brass block.

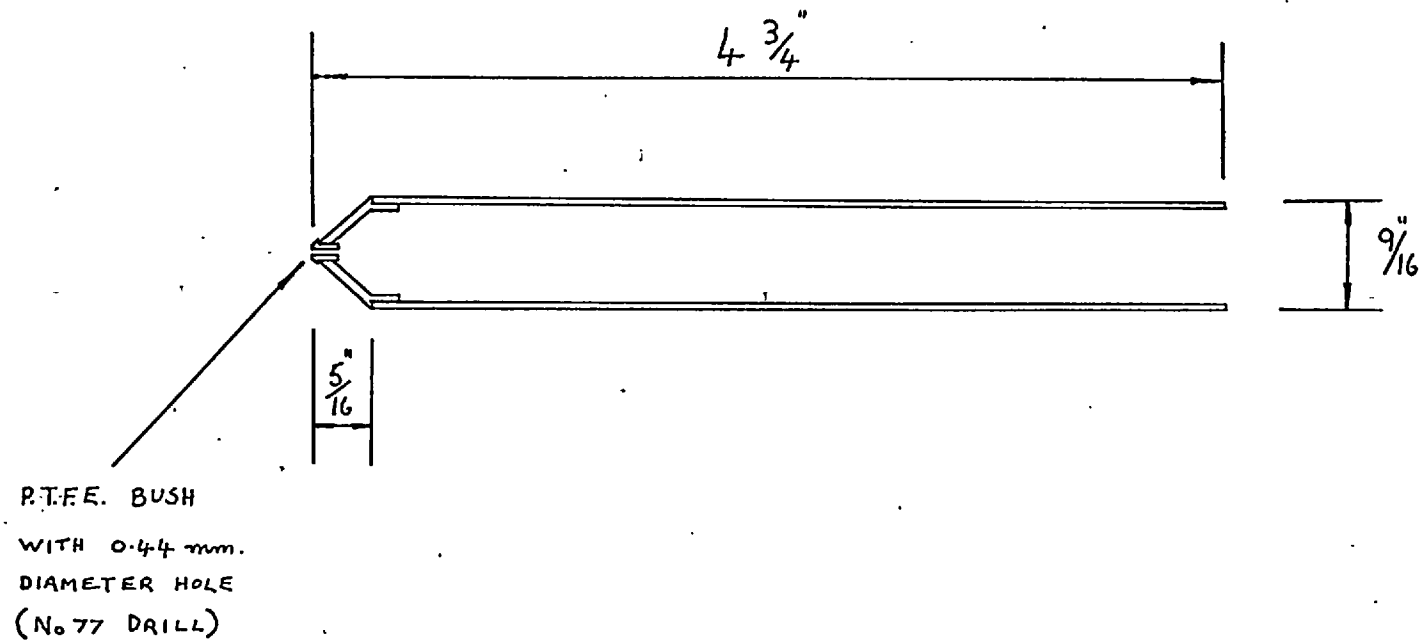


Figure 3.7 A longitudinal section of the F.E.T. voltage probe case. Scale full size

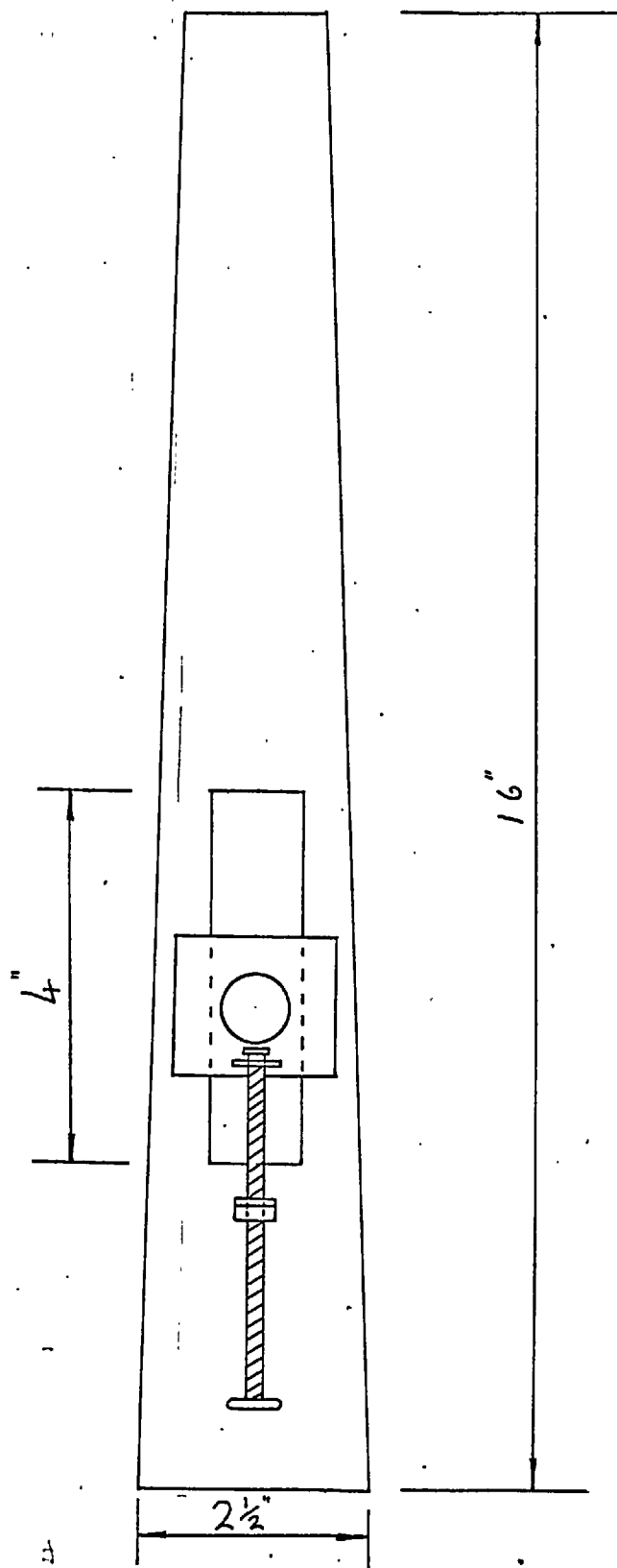


Figure 3.8 Plan view of the voltage probe scanning mechanism
Scale half full size

The other end moves along a guide rail below the detector on the outside of the tank. A milled slot in the aluminium strip contains a small brass carriage through the centre of which the copper voltage probe passes. The probe is held in light contact with the detector sample by means of an elastic band. Location in the vertical plane is achieved by a threaded rod which pushes or pulls the carriage along the milled slot. Fig. 3.8 shows a plan view of the voltage probe scanning mechanism.

3.3 Hydrophone Design and Construction

Ideally an ultrasound field plotting hydrophone should be small enough to be regarded as a point, otherwise its presence perturbs the field and true measurements are not obtained. In practice this requirement would limit the hydrophone diameter to less than one tenth of the ultrasound wavelength in the field under investigation. At 1 MHz, where the wavelength in water is 1.5 mm, this limit is beginning to impose constructional difficulties owing to the small size of the hydrophone. If a point hydrophone cannot be realized in a practical example, then obviously it must be made as small as possible to minimise the measurement error. Romanenko (54) has described a technique for making hydrophones with diameters down to one fifth of a millimetre. Recent experience in this laboratory with Romanenko type hydrophones (55) has found them difficult to make and also extremely fragile. These hydrophones consist essentially of very small spherical barium titanate transducers which are effectively omnidirectional. This property is not desirable when they are to be used in a small water tank where there are always unwanted reflections from the surface and walls. Assuming a single line profile in the horizontal plane is required,

directional hydrophones should be used. The directivity function in the horizontal plane must, however, be a slowly varying function compared with the field under investigation, otherwise a weighting factor will be required to allow for the hydrophone directivity. Directionality in azimuth is desirable to cut out surface reflections.

One other point which has to be considered when undertaking the design of an ultrasound field plotting system is electromagnetic pick-up. When continuous wave ultrasound is used, there will always be a certain amount of pick-up, especially when the hydrophone is a few centimetres from an ultrasound source. The conventional technique is to use a single conductor for the piezoelectrically generated signal surrounded by an earth shield which also provides the return conduction path to the signal source. Often the earth shield has a slight resistance so cannot maintain a uniform potential over its entire length. In the presence of rapidly varying r.f. fields, this results in an unwanted electrical signal appearing across the output of the hydrophone. If an output is taken from each side of the hydrophone transducer, then the transducer may be considered as a current source. The signal from one side will therefore be 180 degrees out of phase with that from the other side, but relative to a third conductor which forms an earthed shield, any r.f. pick-up will have the same phase in each of the signal outputs. Consequently if the output from the hydrophone is fed to a differential amplifier, consisting essentially of a Darlington pair of transistors, the out-of-phase ultrasound signals will be added but the in-phase pick-up will be subtracted. The effectiveness of this system will be determined by the common mode rejection ratio of the differential amplifier. (See section 3.4).

a	b	p	q
c	d	r	s
e	f	t	u
v	w	x	y
z	aa	ab	ac
ad	ae	af	ag
ah	ai	aj	ak
al	am	an	ao
ap	aq	ar	as
at	au	av	aw
ax	ay	az	ba
bb	bc	bd	be
bf	bg	bh	bi
bj	bk	bl	bm
bn	bo	bp	bq
br	bs	bt	bu
bv	bw	bx	by
bz	ca	cb	cc
cd	ce	cf	cg
ch	ci	cj	ck
cl	cm	cn	co
cp	cq	cr	cs
ct	cu	cv	cw
cx	cy	cz	da
db	dc	dd	de
df	dg	dh	di
dj	dk	dl	dm
dn	do	dp	dq
dr	ds	dt	du
dv	dw	dx	dy
dz	ea	eb	ec
ed	ee	ef	eg
eh	ei	ej	ek
el	em	en	eo
ep	eq	er	es
et	eu	ev	ew
ex	ey	ez	fa
fb	fc	fd	fe
ff	fg	fh	fi
fj	fk	fl	fm
fn	fo	fp	fq
fr	fs	ft	fu
fv	fw	fx	fy
fz	ga	gb	gc
gd	ge	gf	gg
gh	gi	gj	gk
gl	gm	gn	go
gp	gq	gr	gs
gt	gu	gv	gw
gx	gy	gz	ha
hb	hc	hd	he
hf	hg	hh	hi
hj	hk	hl	hm
hn	ho	hp	hq
hr	hs	ht	hu
hv	hw	hx	hy
hz	ia	ib	ic
id	ie	if	ig
ih	ii	ij	ik
il	im	in	io
ip	iq	ir	is
it	iu	iv	iw
ix	iy	iz	ja
jb	jc	jd	je
jf	jg	jh	ji
jj	jk	jl	jm
jn	jo	jp	jq
jr	js	jt	ju
kv	kw	kx	ky
kz	la	lb	lc
ld	le	lf	lg
lh	li	lj	lk
ll	lm	ln	lo
lp	lq	lr	ls
lt	lu	lv	lw
lx	ly	lz	ma
mb	mc	md	me
mf	mg	mh	mi
mj	mk	ml	mm
mn	mo	mp	mq
mr	ms	mt	mu
mv	mw	mx	my
mz	na	nb	nc
nd	ne	nf	ng
nh	ni	nj	nk
nl	nm	nn	no
np	nq	nr	ns
nt	nu	nv	nw
nx	ny	nz	oa
ob	oc	od	oe
of	og	oh	oi
oj	ok	ol	om
on	oo	op	oq
or	os	ot	ou
ov	ow	ox	oy
oz	pa	pb	pc
pd	pe	pf	pg
ph	pi	pj	pk
pl	pm	pn	po
pp	pq	pr	ps
pt	pu	pv	pw
px	py	pz	qa
qb	qc	qd	qe
qf	qg	qh	qi
qj	qk	ql	qm
qn	qo	qp	qq
qr	qs	qt	qu
qv	qw	qx	qy
qz	ra	rb	rc
rd	re	rf	rg
rh	ri	rj	rk
rl	rm	rn	ro
rp	rq	rr	rs
rt	ru	rv	rw
rx	ry	rz	sa
sb	sc	sd	se
sf	sg	sh	si
sj	sk	sl	sm
sn	so	sp	sq
sr	ss	st	su
sv	sw	sx	sy
sz	ta	tb	tc
td	te	tf	tg
th	ti	tj	tk
tl	tm	tn	to
tp	tr	ts	tt
tu	tv	tw	tx
ty	tz	ua	ub
uc	ud	ue	uf
ug	uh	ui	uj
uk	ul	um	un
uo	up	uq	ur
us	ut	uu	uv
uw	ux	uy	uz
va	vb	vc	vd
ve	vf	vg	vh
vi	vj	vk	vl
vm	vn	vo	vp
vq	vr	vs	vt
vu	vv	vw	vx
vy	vz	wa	wb
wc	wd	we	wf
wg	wh	wi	wj
wk	wl	wm	wn
wo	wp	wq	wr
ws	wt	wu	wv
wx	wy	wz	xa
xb	xc	xd	xe
xf	yg	yh	yi
yj	yk	yl	ym
yn	yo	yp	yq
yr	ys	yt	yu
yv	yw	yx	yy
yz	za	zb	zc
zd	ze	zf	zg
zh	zi	zj	zk
zl	zm	zn	zo
zp	zq	zr	zs
zt	zu	zv	zw
zx	zy	zz	

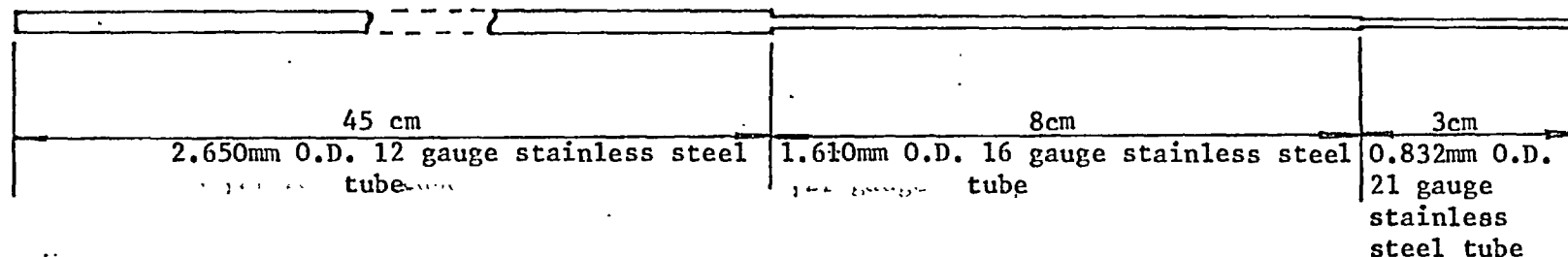


Figure 3.9b

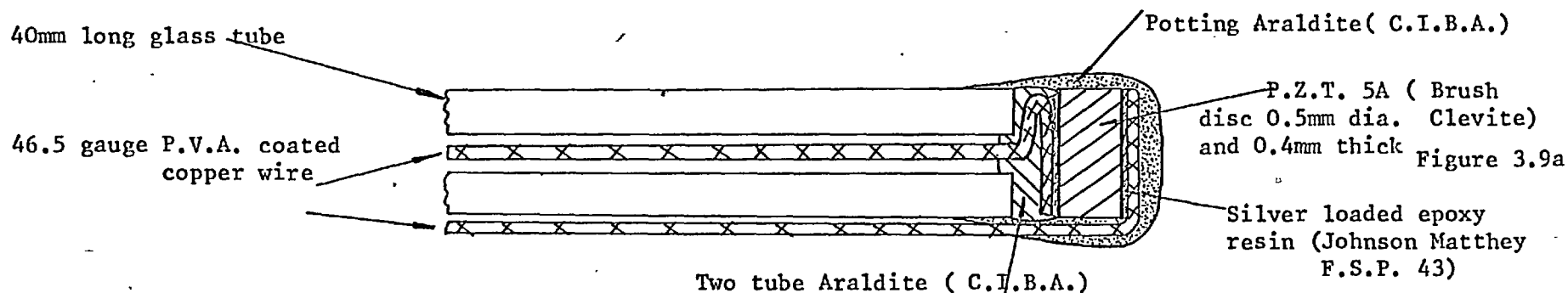


Figure 3.9a

Figure 3.9b Diagram of hydrophone probe. Scale full size.

Figure 3.9a Diagram of hydrophone sensitive element. Scale approximately 40 times full size

As a result of both ultrasonic and electrical considerations a series of hydrophone elements of the general form shown in Fig. 3.9a were constructed.

Very careful work is required to make a satisfactory hydrophone element of the design shown in Fig. 3.9. At most stages of construction the element is extremely fragile and a 50% breakage rate was experienced during manufacture, which was carried out entirely by hand. However, after the hydrophone element is sealed into a stainless steel hypodermic tubing, it is sufficiently robust to withstand light knocks.

The manufacturing procedure is outlined below:-

1. A strip of P.Z.T. 5A ceramic approximately 1 mm wide was cut from a larger ceramic transducer plate of fundamental thickness resonance 5 MHz silvered on both faces. An adjustable width drawing pen was used as a cutting tool with a carborundum and water slurry as abrasive.
2. The slice was then sectioned into one millimetre square plates.
3. The insulation was carefully removed from a short region of a length of 46.5 gauge P.V.A. coated wire which was then attached to one face of a ceramic plate using Johnson Matthey conducting epoxy adhesive F.S.P. 49. A special jig was used to hold the fine wire and the ceramic was placed in a position so that the stripped length of wire was just on the silver electrode. A needle pointed tool was used under a times ten microscope eye-piece to apply the adhesive.
4. One half of the copper wire was cut off after curing the

adhesive and the remaining half was carefully bent round in a tight 'U' until it was flat against the ceramic. It was then bent perpendicular to the transducer and inserted into a fine glass tube about 40 mm long and 0.30 mm in diameter.

5. A small spot of araldite (two tube) was placed on top of the fine tube and the transducer carefully drawn down on to it with the fine wire. The small ceramic plate was manipulated until it was centred on the cylindrical axis of the tube and lying at right angles to it.

6. After curing, the excess araldite was carefully filed away using a fine emery board and then by the same technique the ceramic was fashioned into a disc approximately half a millimetre in diameter. At this stage the transducer element has a diameter only slightly larger than the supporting glass tube.

7. The glass tube was then inserted into a jig consisting of a block of perspex having a fine hole drilled through it just large enough to allow the glass tube to pass but too small for the transducer. By this means the transducer is supported in a convenient position so that the second fine wire can be attached to the upper electrode. The same technique as described in 3. was used.

8. Excess conducting araldite was filed away from the top of the transducer and round the sides. This stage requires the most care as it is easy to break off the fine wire to the top electrode. A final check was made to ensure that there was not a d.c. path between the upper and lower electrodes of the transducer.

9. The fine wire to the upper electrode was then bent round to

lie alongside the glass tube and the complete element inserted into a gauge 21, 0.832 mm O.D. hypodermic needle tube. A check was made to ensure that the two wires were electrically isolated from the stainless steel shield.

10. The glass tube was drawn up inside the stainless steel shield until only the transducer element remained outside. It was then carefully sealed into the steel tube and waterproofed with a thin layer of potting araldite.

The hydrophone element was mounted at the end of a gauge 12, 2.650 mm O.D. stainless steel tube using an intermediary section to match external and internal diameters as shown in Fig. 3.9b. The individual sections were bonded together using F.S.P. 49 conducting epoxy adhesive.

In operation, hydrophones constructed by this technique had typical sensitivities of 20 mV/atmosphere. At frequencies in the order of 1 MHz they showed dipole like directivity patterns in the horizontal plane but at 3 MHz this had degenerated into a four lobed pattern similar to a quadrupole directivity. This behaviour is discussed more fully in Appendix 2. The azimuthal directivity pattern was not measured. From the change in output of the hydrophone in a uniform ultrasound field, when the probe was displaced slightly from the vertical it was deduced that the main lobe of the pattern was approximately 10 degrees wide. Consequently the hydrophone scanning system had to be designed so that the hydrophone did not oscillate from side to side during scanning. Using the system described in section 3.2.1 sideways oscillation was eliminated and the hydrophone output accurately followed the ultrasound field profile.

3.4 Design and Construction of Electronics

The circuit diagrams shown in this section are largely self explanatory. Any special features of the circuits are, however, discussed more fully. Calibration and frequency response curves are shown for the appropriate circuits in Appendix 3.

3.4.1 Hydrophone Signal Amplifier and Envelope Detector

Fig. 3.10 shows the circuit diagram. The integrated circuit used is intended as a video amplifier and has a gain of almost 40 dB up to 40 MHz. Unfortunately the internal biasing conditions require a d.c. current path with a resistance of approximately 50 ohms between each input lead (1 and 3) and the circuit earth. The output impedance of the hydrophone is in the order of 1000 ohms so 50 ohms across the input of the amplifier would divide the signal by 20. R.f. chokes are therefore used to provide both correct biasing and high input impedance over the band of operating frequencies (1 to 4 MHz). The calculated output impedance of the transducer element at 1 MHz is approximately 17 k ohms (element capacitance 9.0 pF). Therefore the value of 1000 ohms obtained by measurement suggests stray capacitance in the order of 150 pF in the probe tube and associated cable. As previously indicated, the differential amplifier facility of the integrated circuit is used to suppress unwanted electrical pick-up. The common mode rejection ratio is 85 dB. To ensure reliability and also to minimise stray capacitance and wiring inductance a printed layout was designed for the circuit. This was etched on to copper faced printed circuit board (p.c.b.) with dilute nitric acid. The entire circuit is mounted in a copper box on the hydrophone carriage.

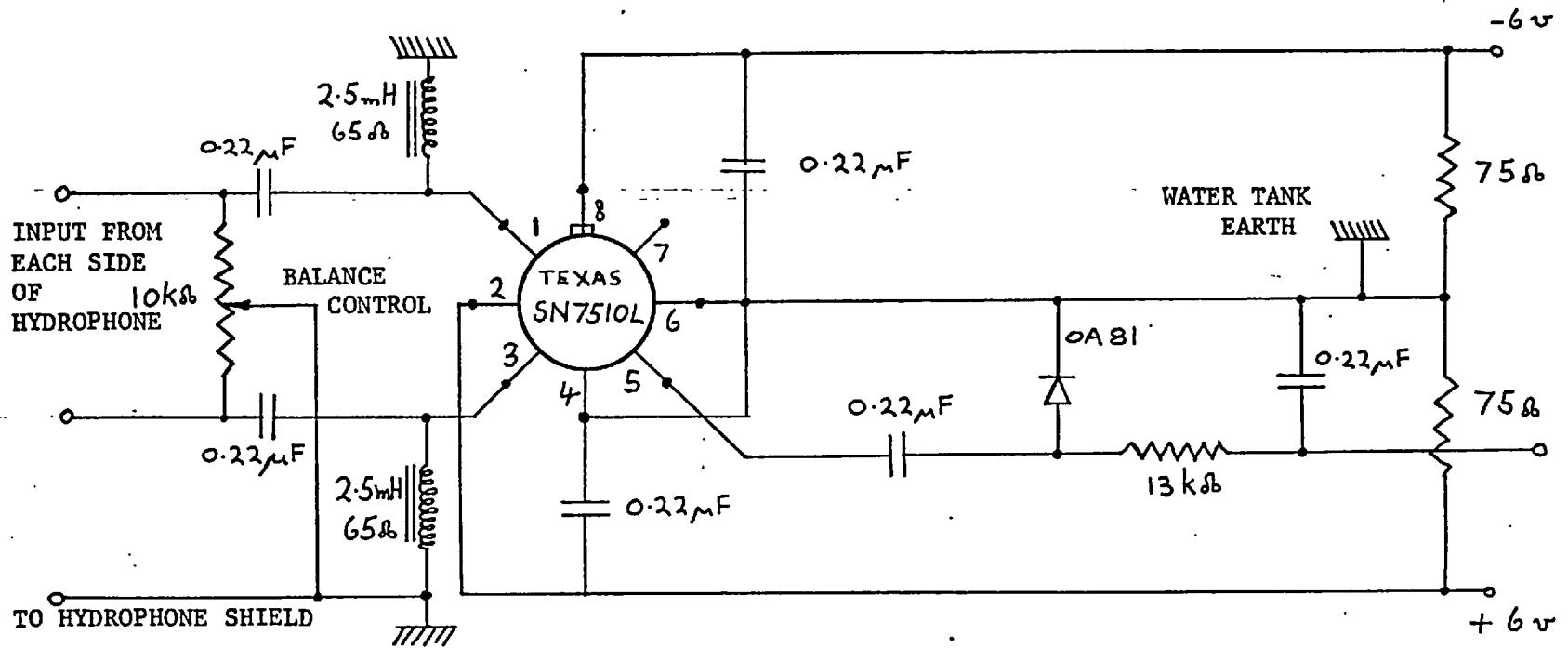


Figure 3.10 Circuit diagram of hydrophone signal amplifier and detector circuit.

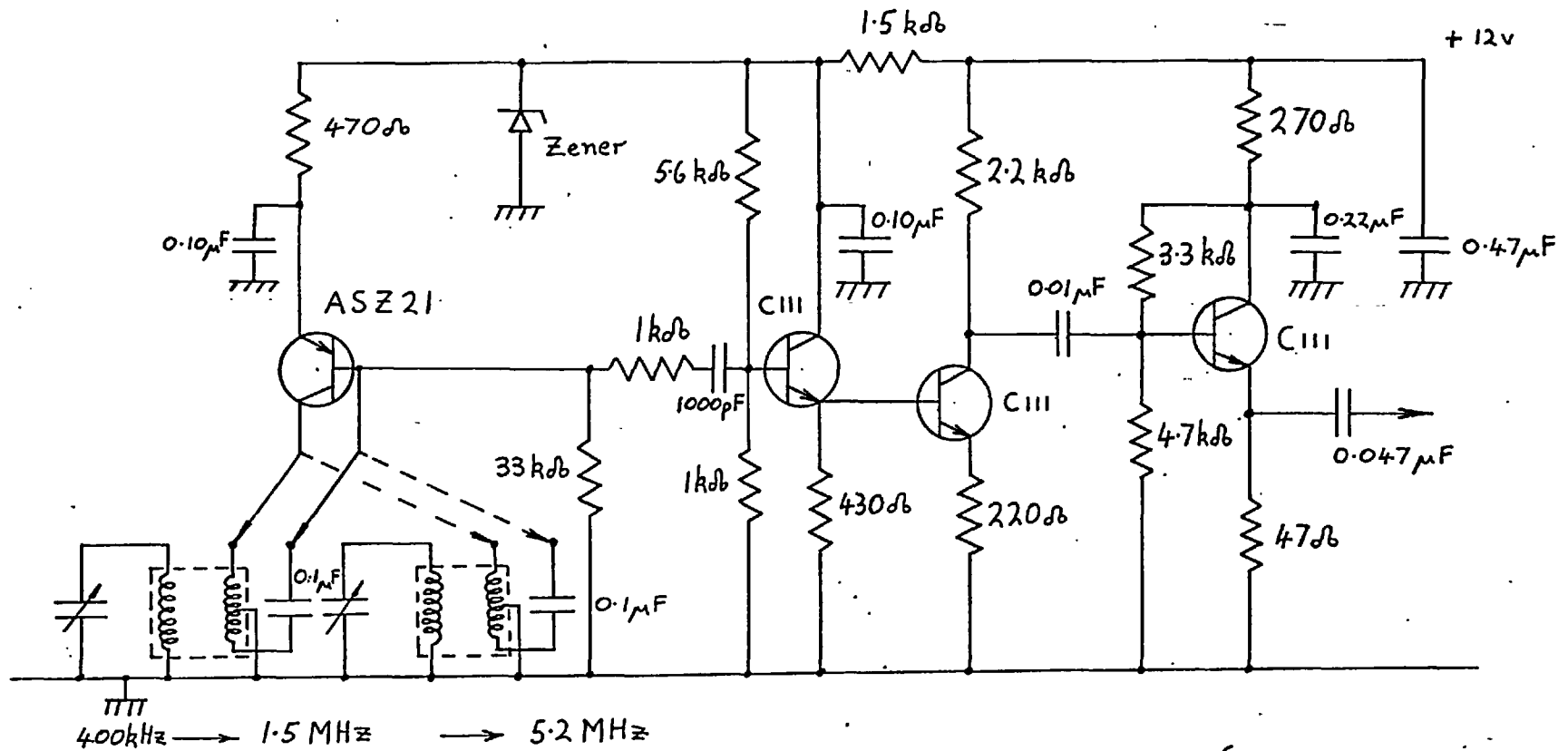


Figure 3.11 Circuit diagram of radio frequency oscillator



3.4.2 Radio Frequency Oscillator with Frequency Sweep

Fig. 3.11 shows the circuit diagram. The oscillator was designed to cover the frequency band 500 kHz to 5 MHz in two ranges. A forty-eight to one reduction gear was used on the variable capacitor to allow precise frequency selection. Provision was made for a small synchronous motor to be directly coupled to the reduction gear so that the frequency could be swept at a steady rate when required. A $2\frac{1}{2}$ watt Chamberlain and Hookham type B synchronous motor giving one revolution in 90 seconds was used. The long term stability of the oscillator frequency is 1 part in 10^4 .

3.4.3 Wide Band Power Amplifier

The circuit diagram is shown in Fig. 3.12. This amplifier was originally designed to provide an r.f. power output of up to 5 watts over a frequency band extending from a few hundred kilohertz to twenty megahertz. With the circuit shown the power output begins to fall off above 2 MHz because the transistor driving the output stage cannot provide sufficient power. If it is replaced by an XB401 then the initial specification can be met. The parallel arrangement of XB401's in the output stage is used for reasons of economy. To ensure a broad bandwidth it is necessary to use transistors with very high frequency characteristics, the power transistors themselves have an f_T of 400 MHz. However, with such a high frequency capability the layout must be carefully designed if oscillation is to be prevented. The entire amplifier was built round a copper box having dimensions 185 mm x 50 mm x 30 mm. A 5 mm thick strip of brass plate was used as a combined base and

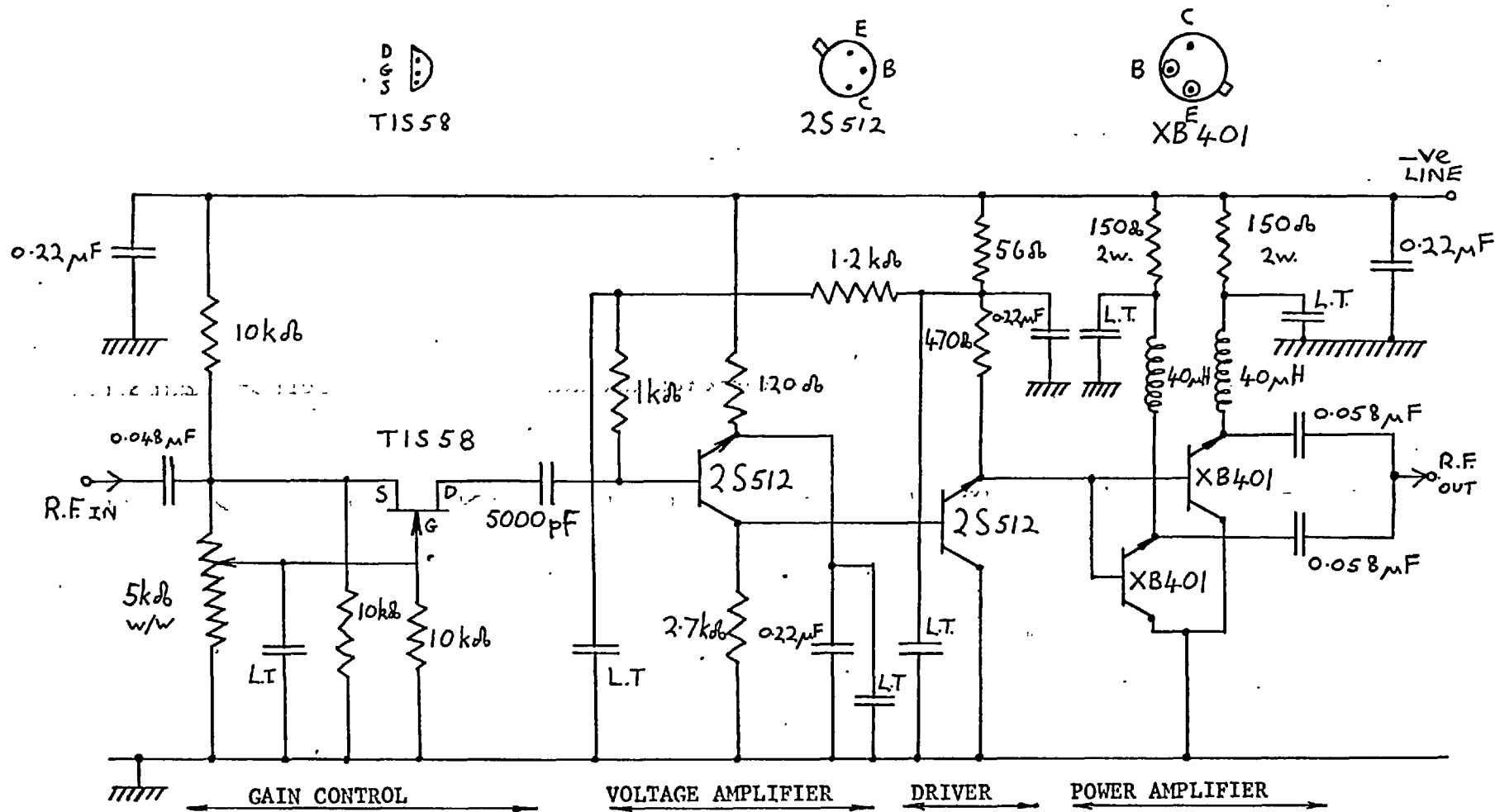


Figure 3.12 Circuit diagram of wide band power amplifier with voltage gain control

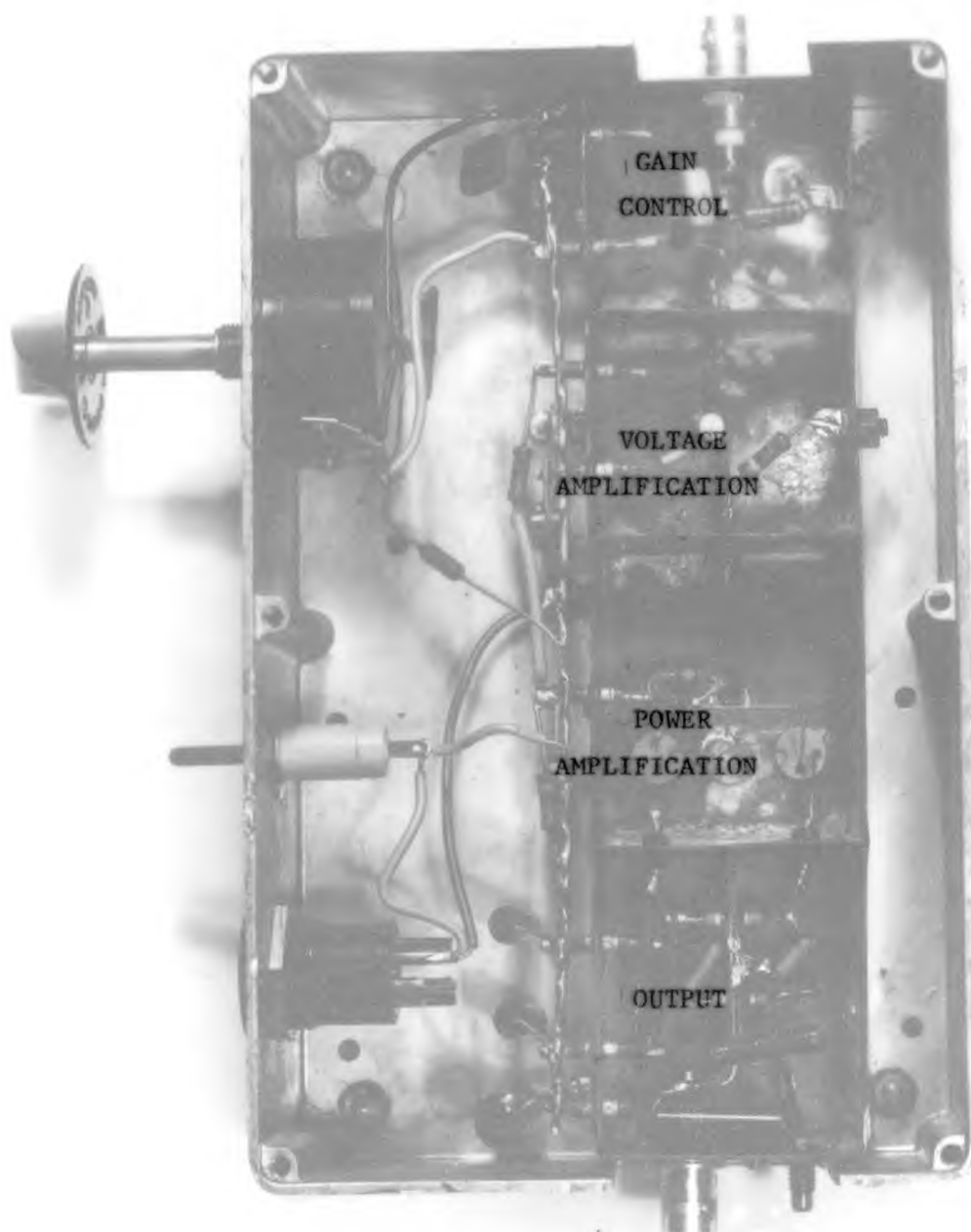
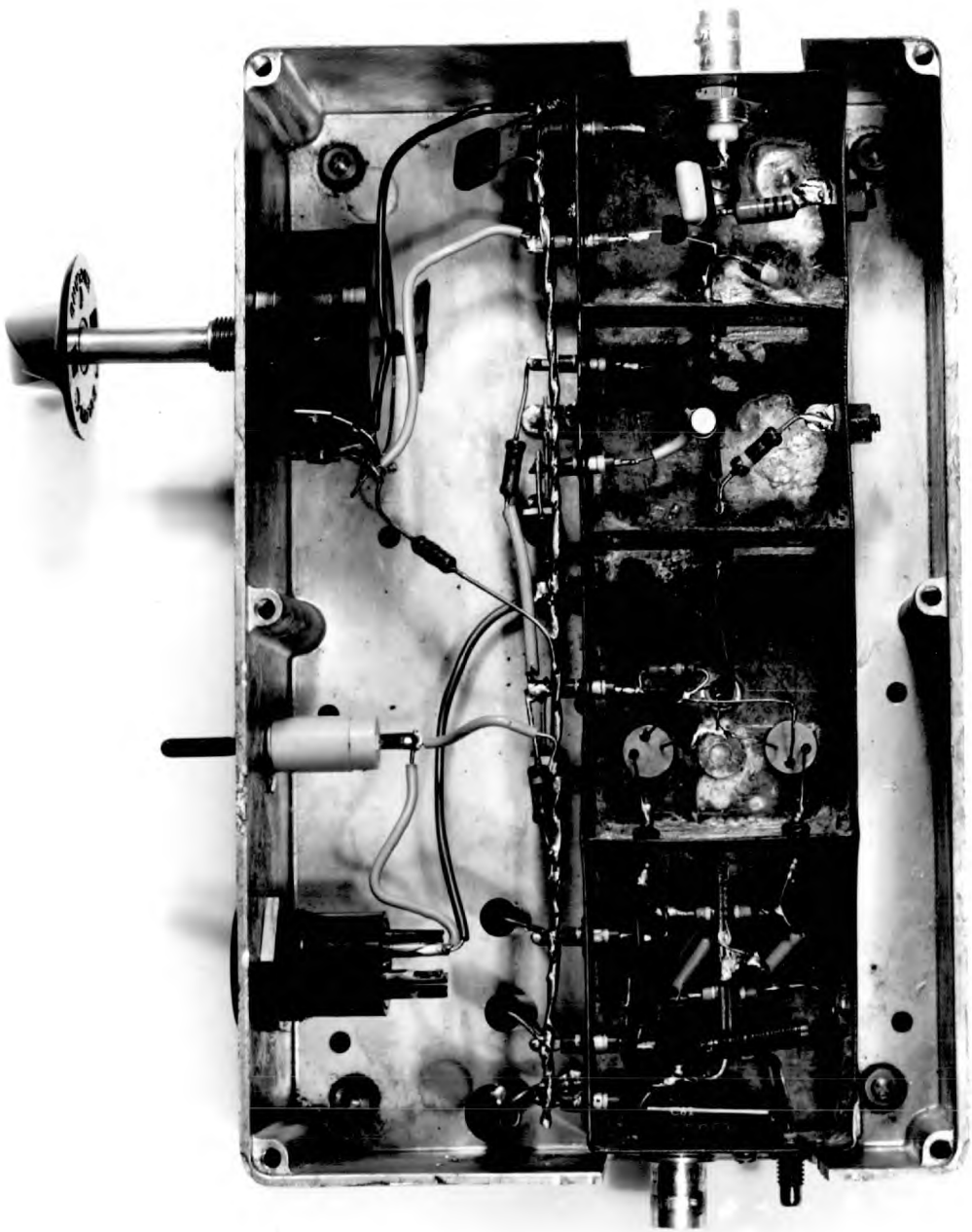


Figure 3.13 Component layout and construction of the wide band power amplifier



heat sink. Inside four compartments were sectioned off. Each contains the r.f. carrying components of one stage of the amplifier. Those components which determine the d.c. levels are mounted on the outside of the box and are connected to the inside by 'lead-through' decoupling capacitors. This technique eliminates r.f. feedback through the d.c. circuitry. Fig. 3.13 shows the amplifier construction.

3.4.4 A Stabilized Power Supply with Preset Current Trip

A stabilized power supply was designed for use with the r.f. power amplifier described in 3.4.3. Fig. 3.14 shows the circuit diagram. Under certain circumstances the output transistors in the r.f. power amplifier are driven close to their maximum dissipation. Thermal runaway then becomes a serious problem. If this should occur, it is necessary to switch off the amplifier in a few microseconds to prevent the transistors from being destroyed. A silicon controlled rectifier is used to switch off the power supply to the amplifier, should the current rise above a preset value. The output voltage is stabilized from 15 v to 30 v and a maximum current of 500 ma can be drawn. The second stabilized power supply shown having 100 ma current capability is used to supply power to the oscillator circuit of 3.4.2. At full load the voltage ripple on each of the circuits is less than 0.05%.

3.4.5 F.E.T. Voltage Probe and Signal Amplifier

The F.E.T. voltage probe circuit was adapted from that described by Macario (56). Fig. 3.15 shows the complete circuit

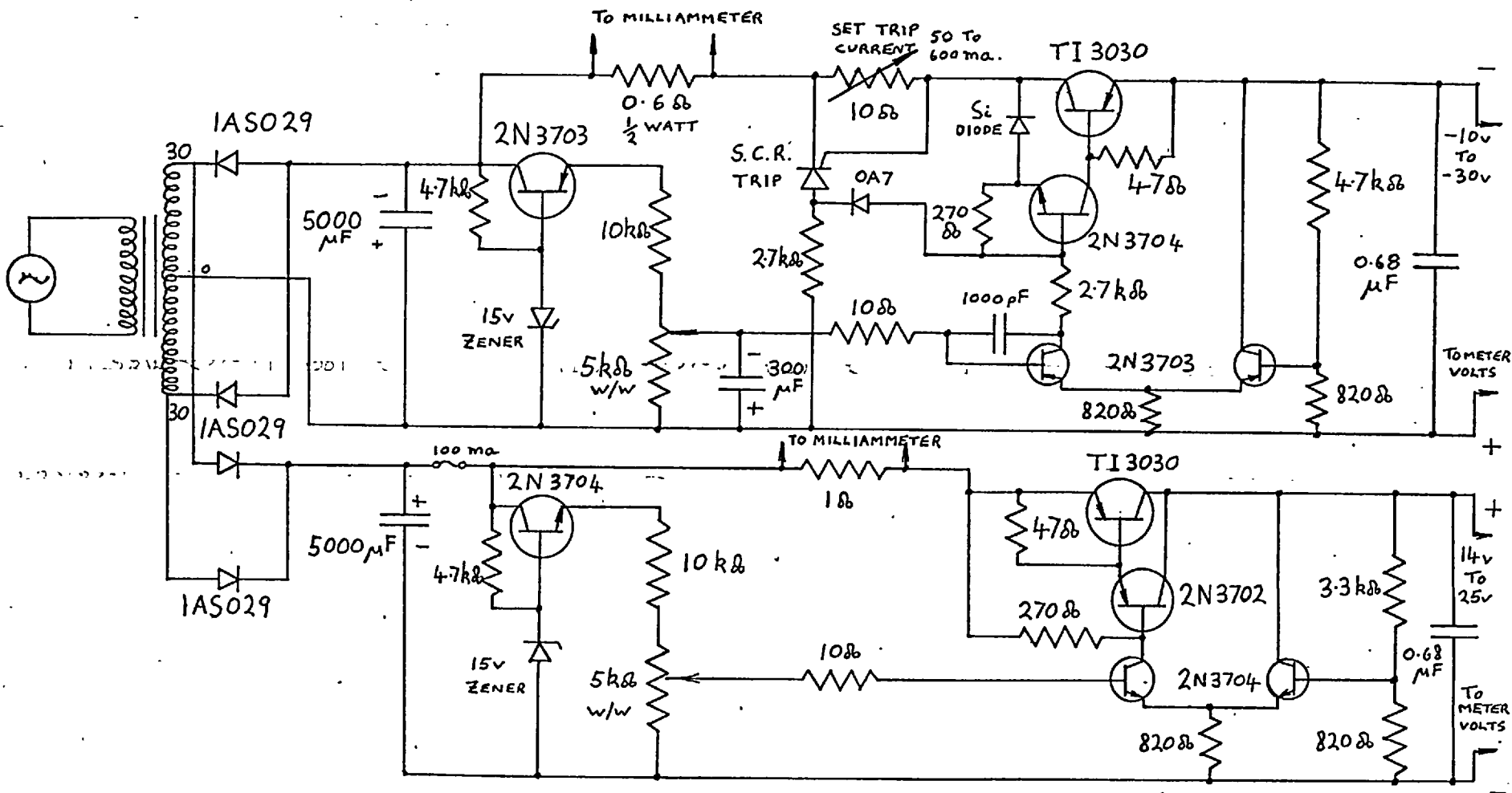
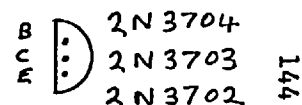
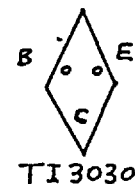


Figure 3.14 Circuit diagram of stabilized power supply for oscillator and power amplifiers



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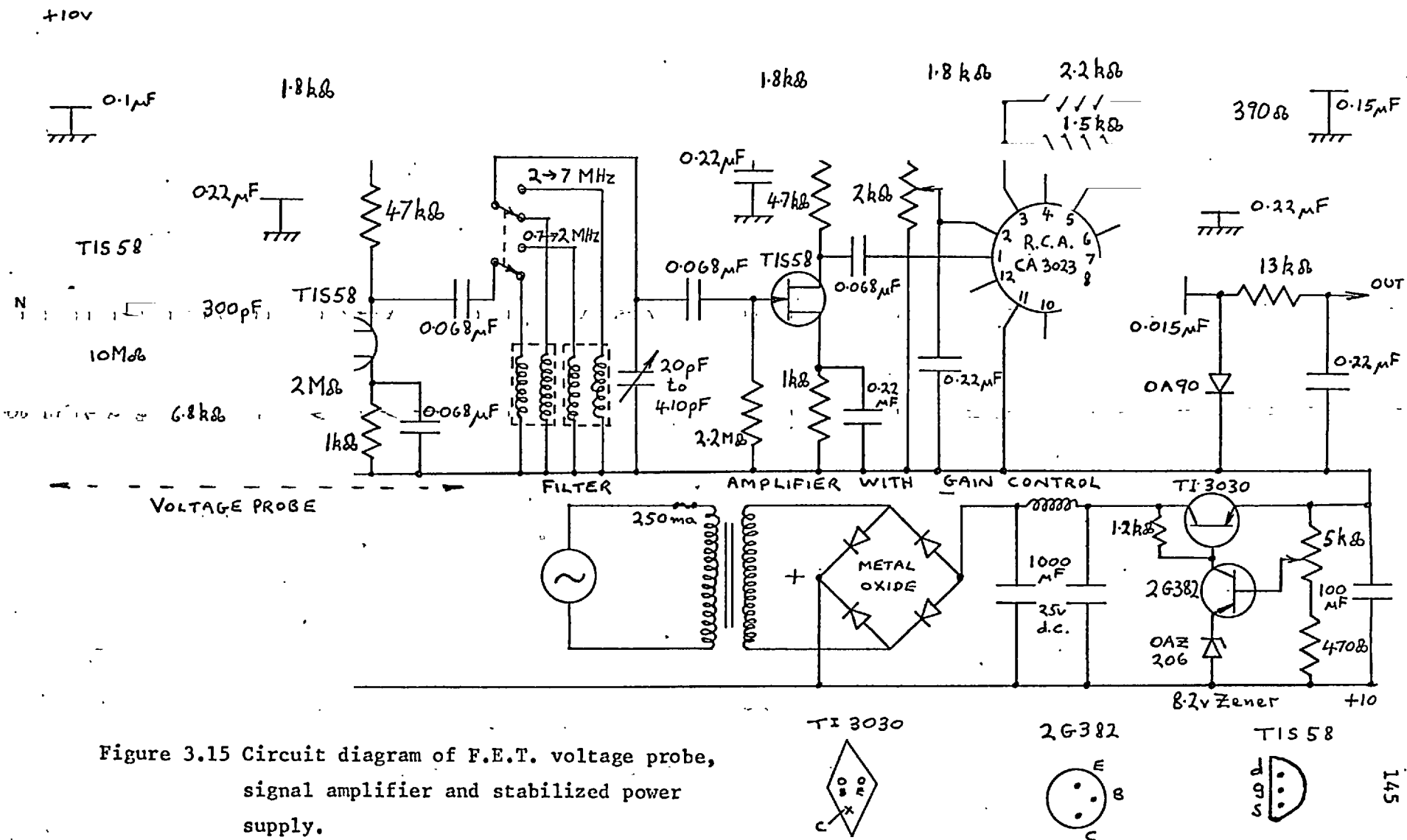


Figure 3.15 Circuit diagram of F.E.T. voltage probe, signal amplifier and stabilized power supply.

Frequency range 0.7 to 7.0 MHz in two ranges. $Z_{in} = 10\text{Megohm}$ in parallel with 4pF. Max gain 53 dB

diagram for the voltage probe, signal amplifier and stabilized power supply. Because of the high input impedance of the probe circuit, it must be carefully shielded to avoid picking up medium waveband radio broadcasts. This was achieved using a copper tube as described in 3.2.3. A narrow band tuned filter circuit was used half-way down the amplifier chain to increase the signal to noise ratio. The R.C.A. integrated circuit has a variable gain determined by the voltage applied to lead 2. Here it is derived from the supply voltage and varied using a logarithmic potentiometer. It is necessary to decouple lead 2 effectively, otherwise the amplifier oscillates. There is a point on the gain control corresponding to the change from a net attenuation to a net amplification for the I.C. at which a large noise signal appears at the output. This is a characteristic of the I.C. so, in practice this region cannot be used. The third F.E.T., integrated circuit and detector, are mounted on a printed circuit board. The high input impedance of the third F.E.T. permits its direct connection across the tuned circuit without lowering the Q value.

3.4.6 Motor Drive Unit

The speed of the d.c. motor used to drive the hydrophone carriage is controlled by the applied voltage. Using the circuit shown in Fig. 3.16 an unsmoothed direct voltage, continuously variable between 0 and 30 volts, is obtained. Coupled to a mechanical load, the d.c. motor reaches its steady state speed more rapidly when driven with an unsmoothed direct voltage than with a smoothed.

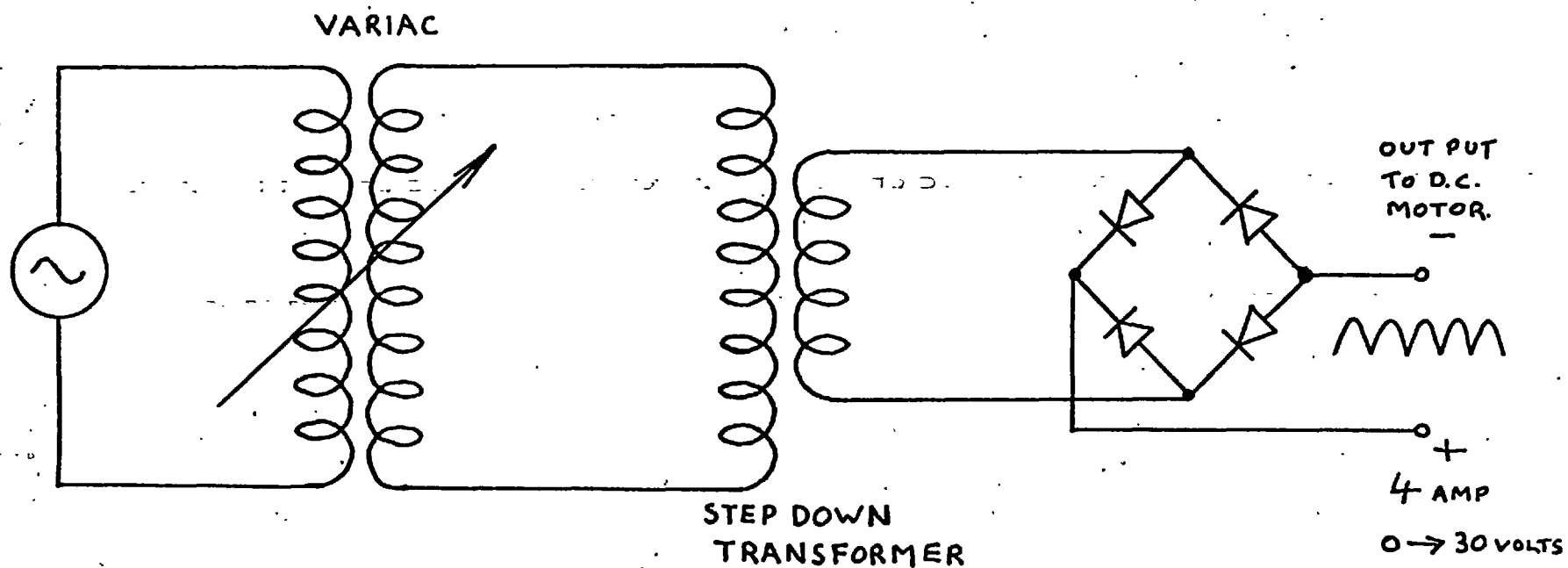


Figure 3.16 Motor drive unit

3.5 Mounting of Piezoelectric Ceramic Detector Samples

The detector samples used were 50 mm diameter discs of lead zirconate titanate piezoelectric ceramic. One face of each disc is completely coated with a silver electrode and maintained at earth potential throughout the experiments. The opposite face is not electroded. It is by scanning this latter face that the spatial voltage distribution is obtained in the sinusoidal standing wave technique 2.8. For the impulse response technique 2.9 nine spots of conducting paint are placed on the uncoated face as shown in Fig. 3.17.

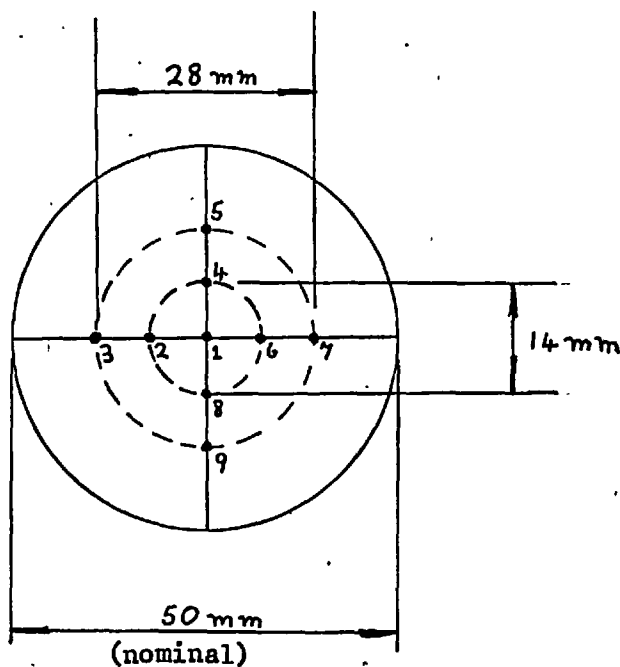


Figure 3.17

These point electrodes must have diameters equal to the ultrasound wavelength in water at the driving frequency in order to approximate to delta functions of position, as described in 2.9.

Two types of conducting paint were tried, a two part epoxy paint, Johnson Matthey F.S.P. 43, and a quick drying petroleum based paint, De Gussa conductive silver 200. The De Gussa paint is to be preferred because it can be easily removed with acetone, unlike the epoxy paint which, once cured, is extremely tough and difficult to remove.

The investigations were carried out using a series of discs having fundamental thickness resonances at approximately 1 MHz, 2 MHz and 3 MHz. Consequently the point diameters are 1.5 mm on the 1 MHz samples, 0.75 mm on the 2 MHz samples and 0.5 mm on the 3 MHz samples. Stainless steel hypodermic needle tubes of appropriate diameter were used to apply the spots of paint to the discs. This technique is quick and simple yet produces acceptable reproducibility of spot size and shape. In practice it was found that departure from a circular shape produced asymmetrical directivity functions.

The discs were edge-clamped in a brass and perspex holder as shown in Fig. 3.18 and mounted on one end wall of the anechoic tank. To ensure symmetrical boundary conditions the disc was carefully centred in the mount before tightening down the perspex clamping ring. The centring could be carried out visually by observing the relative positions of the disc and the clamping ring through the transparent perspex.

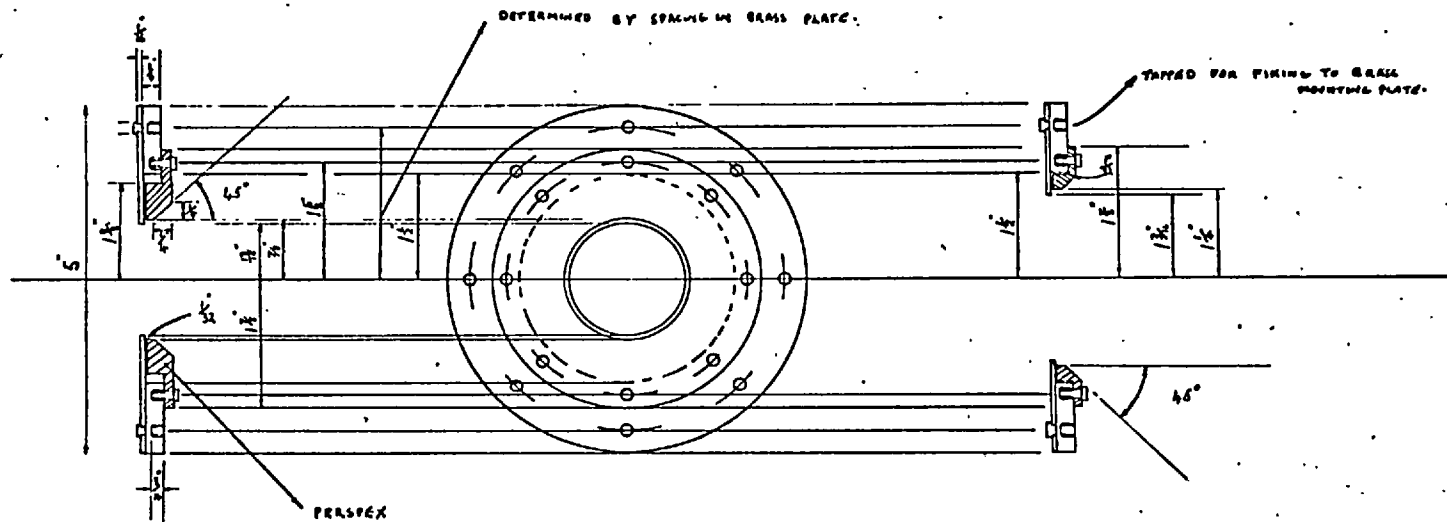
A thin layer of grease between the disc and the front brass plate was used to ensure a waterproof seal.

Several layers of De Gussa conductive silver 200 paint were applied over the join between the disc and the brass front plate to provide good electrical connection between the tank wall and the front electrode of the disc. The paint also helped to waterproof the join.

TRANSDUCER PLATE HOLDER

MATERIALS :- BRASS AND PERSPEX

SCALE : FULL SIZE.



CHAPTER 4

Experimental Procedure

4.1 Preliminary Investigations

Two preliminary experiments were carried out on each mounted detector sample before proceeding with the coherent transfer function measurements. The electrical input impedance and the ultrasound pressure amplitude at a point on the Z axis in the far field were recorded as a function of temporal frequency. The object of these experiments was to select temporal frequencies at which the coherent transfer function should be measured. For example, the frequency of thickness resonance.

4.1.1 Electrical Input Impedance of Piezoelectric Detector as a Function of Temporal Frequency

To obtain the detector impedance the voltage across it, at constant current, was recorded on a Philips PR 2210 A chart recorder as the frequency was slowly swept over the desired range. The electrical driving signal could be applied to any of the 9 contact points on the detector, (see Fig. 3.17). The static capacitance of the transducer was tuned out with a parallel inductance set to maximise the acoustic output on the Z axis at thickness resonance. A block diagram of the experimental set-up is shown in Fig. 4.1. These investigations indicate the presence of all types of mechanical resonance, not just the ones producing appreciable displacement in the thickness direction.

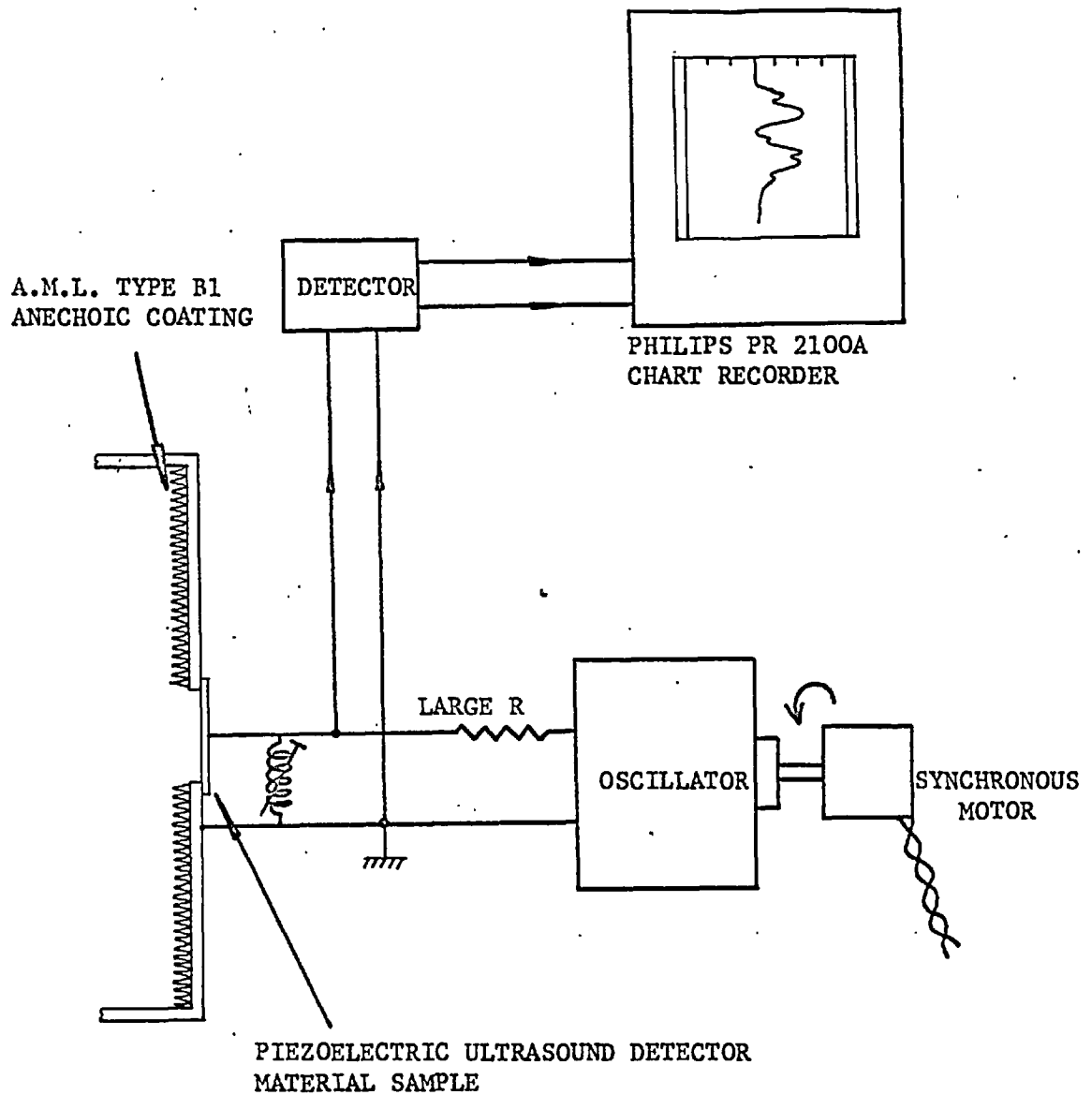


Figure 4.1 Experimental system for recording the detector input impedance as a function of temporal frequency

4.1.2 Piezoelectric Detector Far Field Ultrasound Pressure Amplitude as a Function of Temporal Frequency

A constant voltage driving signal was applied to the detector at one of the contact points. The output of the hydrophone probe was then recorded at a point on the Z axis in the far field of the detector as the driving temporal frequency was slowly swept over the desired range. Fig. 4.2 shows a block diagram of the experimental set-up. The results indicate frequencies where there is appreciable piezoelectric coupling to a thickness mode of vibration. Assuming the principle of reciprocity, the ultrasonic output distribution as a function of temporal frequency, with a constant driving voltage, will have the same shape as the voltage output distribution of the detector, as a function of frequency, with a constant driving pressure. These results, therefore, were the main guide in selecting temporal frequencies at which to record the detector coherent transfer function.

4.2 The Impulse Response Method for Measuring the Directivity Function $|D_p(f_x)|$ of Piezoelectric Detectors

Any one of the 9 contact points can be selected as the delta function of space from which the $|D_p(f_x)|$ can be determined. The procedure described below can be applied to each one equally well.

After selecting the contact point the probe hydrophone was positioned in the far field of the disc on the point's Z axis. From the preliminary experiments described in 4.1 a series of temporal frequencies at which to record $|D_p(f_x)|$ were noted. At each of these frequencies in turn the hydrophone was rotated in the horizontal plane until its directivity pattern was symmetrical with respect to the

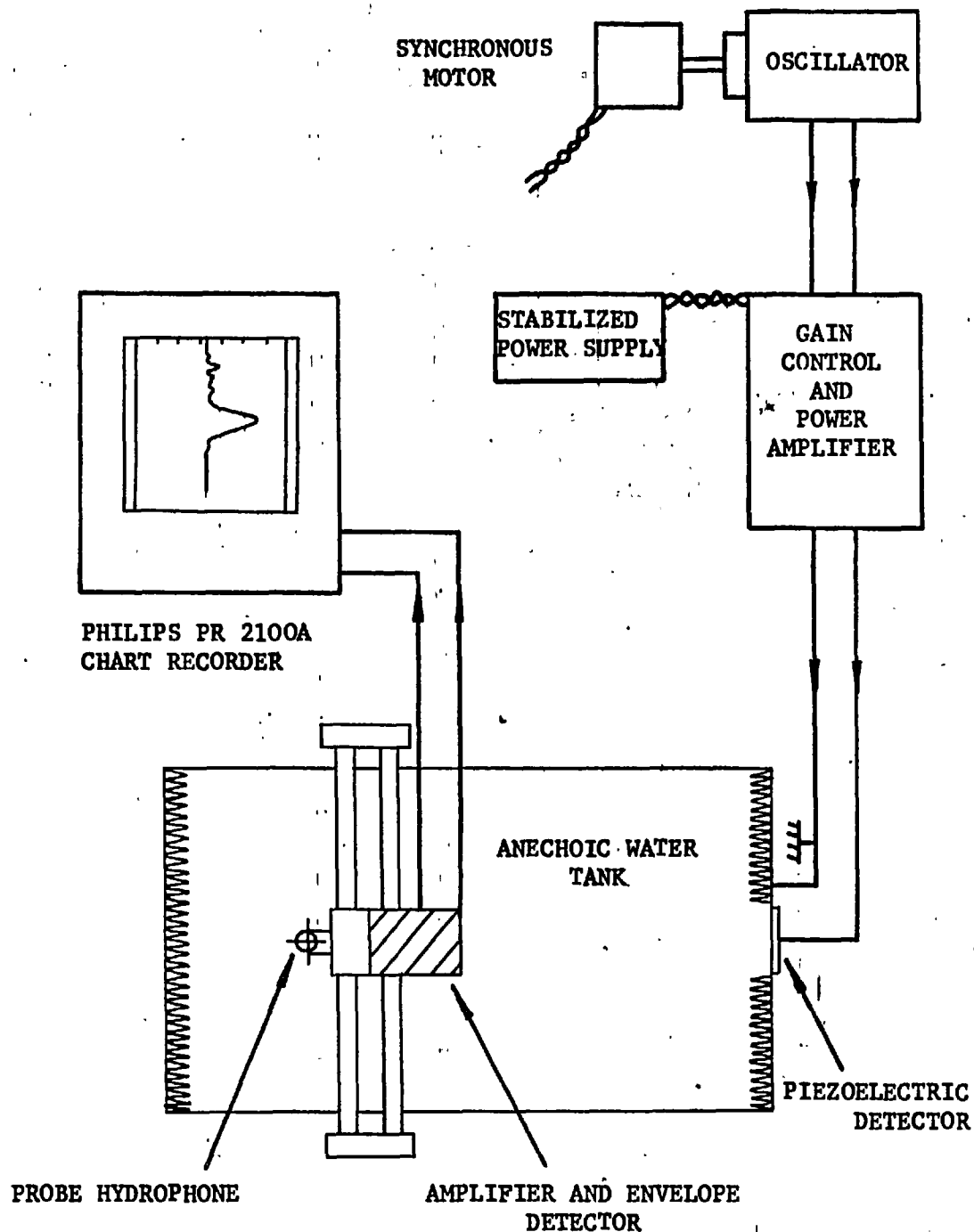


Figure 4.2 Experimental system for recording the far field ultrasound pressure from the piezoelectric detector driven at a point

contact point Z axis. The angular position in the horizontal plane of the directivity pattern relative to a set of fixed hydrophone co-ordinates changes with frequency as indicated in Appendix 2, so it is important to re-align the pattern at each frequency to avoid introducing spurious asymmetry into $|D_p(f_x)|$. The detector directivity function was then recorded at each of the selected time frequencies. In accordance with equation (2.101), $|D_p(f_x)|$ was converted into a time varying signal as the hydrophone was driven at a constant speed along the x_1 axis and recorded on the Philips chart recorder. The chart speed and hydrophone scanning speed were selected so that the record of $|D_p(f_x)|$ was of a convenient length. Fig. 4.3 shows in schematic form the entire experimental system. During investigations of the space variance of $|D_p(f_x)|$ results were obtained for all 9 contact points.

4.3 The Sinusoidal Standing Wave Method for Measuring the Coherent Transfer Function of Piezoelectric Detectors

Very careful alignment of the two ultrasound sources, probe hydrophone and detector sample was necessary to achieve satisfactory results.

The hydrophone was first positioned on the Z axis of the detector as required for the impulse response technique using contact 1. Each ultrasound source was then carefully aligned so that its axis of symmetry was the detector Z axis. This was achieved by keeping the hydrophone fixed and adjusting the height and azimuthal angle of the source until the hydrophone output was a maximum.

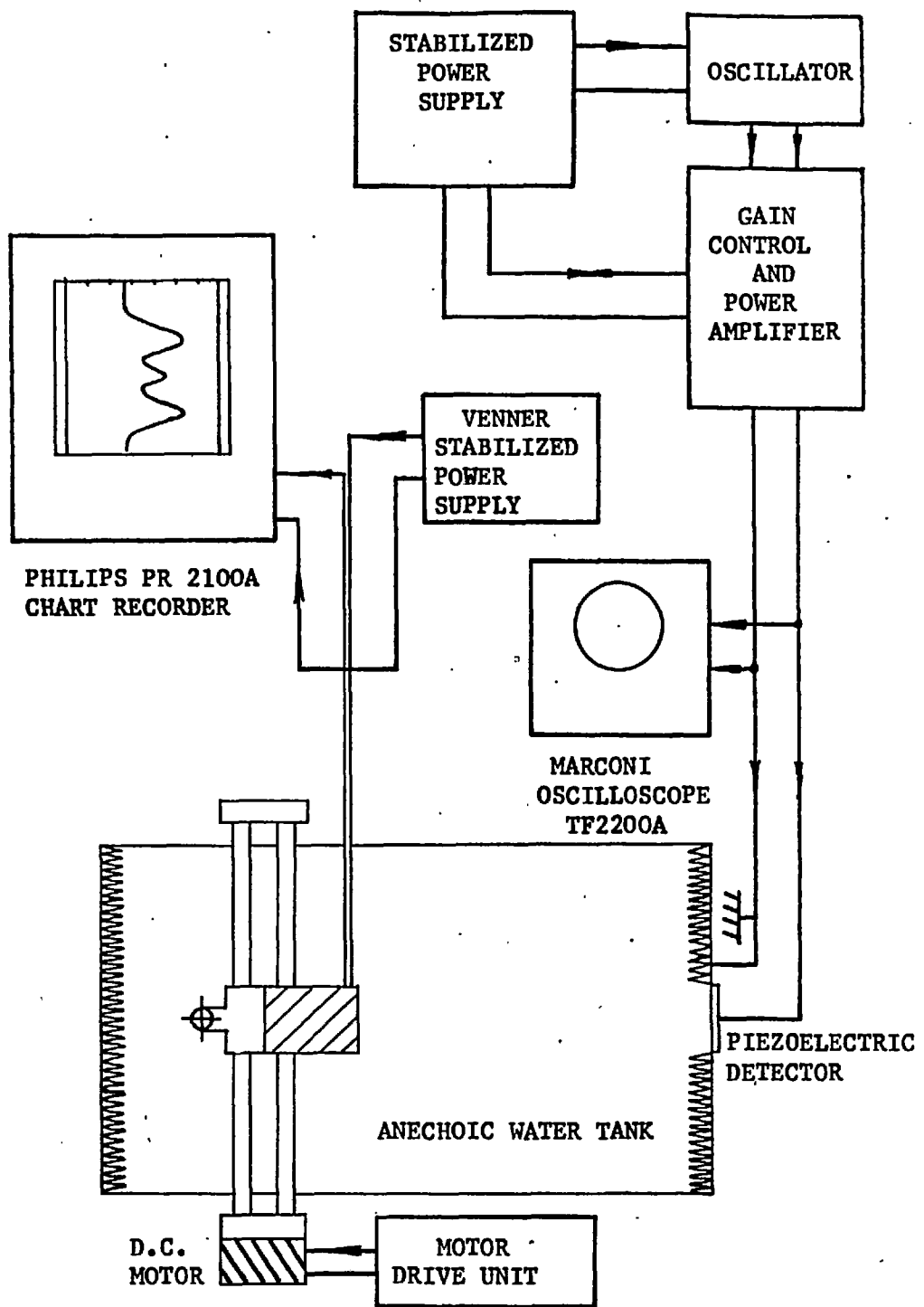


Figure 4.3 Experimental system for measuring $|D_p(f_x)|$ by the impulse response method.

The hydrophone was then placed approximately 40 mm in front of the detector on the Z axis and the two sources positioned at $x_1 = +a$ and $x_1 = -a$ on their carriage near the far end of the tank as shown in Fig. 4.4.

A disc of AML type B₁ anechoic coating was suspended behind the hydrophone, directly in front of the detector sample so that it was completely masked. Each source was then aligned in the direction of the hydrophone by rotating the holder on its bearing surface until the hydrophone output was a maximum. The alignment was carried^{out} for one source at a time with the other turned down to a minimum. The electrical driving signal for each source was set to give the same output from the hydrophone. When both sources were driven in phase, the depth of modulation of the interference fringes formed in the region of the hydrophone was 100%. The hydrophone was then scanned across the interference pattern, over a fixed distance and at a convenient speed, and the electrical output recorded. From the recorded trace the spatial frequency of the ultrasonic input function to the piezoelectric detector was determined. The masking disc of absorbing material was removed from in front of the piezoelectric detector and with the same source output and orientation the piezoelectric output voltage was recorded by scanning the F.E.T. voltage probe across the x axis of the disc. During the recording of the output function the hydrophone was raised well above the level of the detector to prevent scattered ultrasound from the hydrophone probe tube introducing spurious signals. A series of results at particular temporal frequencies was recorded by repeating the above procedure for a range of values of source position.

Fig. 4.5 shows a general view of the anechoic tank, source holders and hydrophone scanning system.

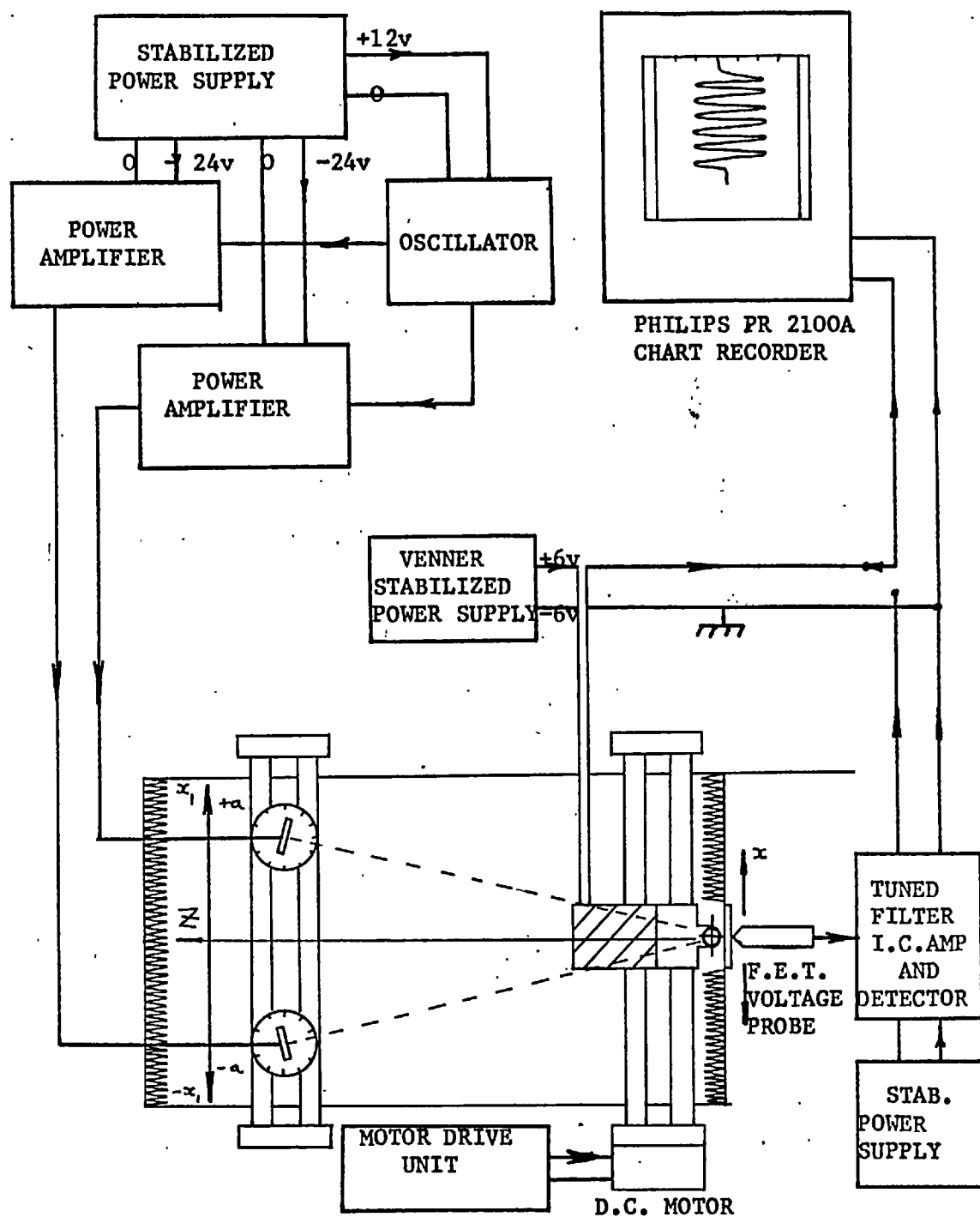


Figure 4.4 Experimental system for measuring $|G_p(f_x)|$ by the sinusoidal standing wave method

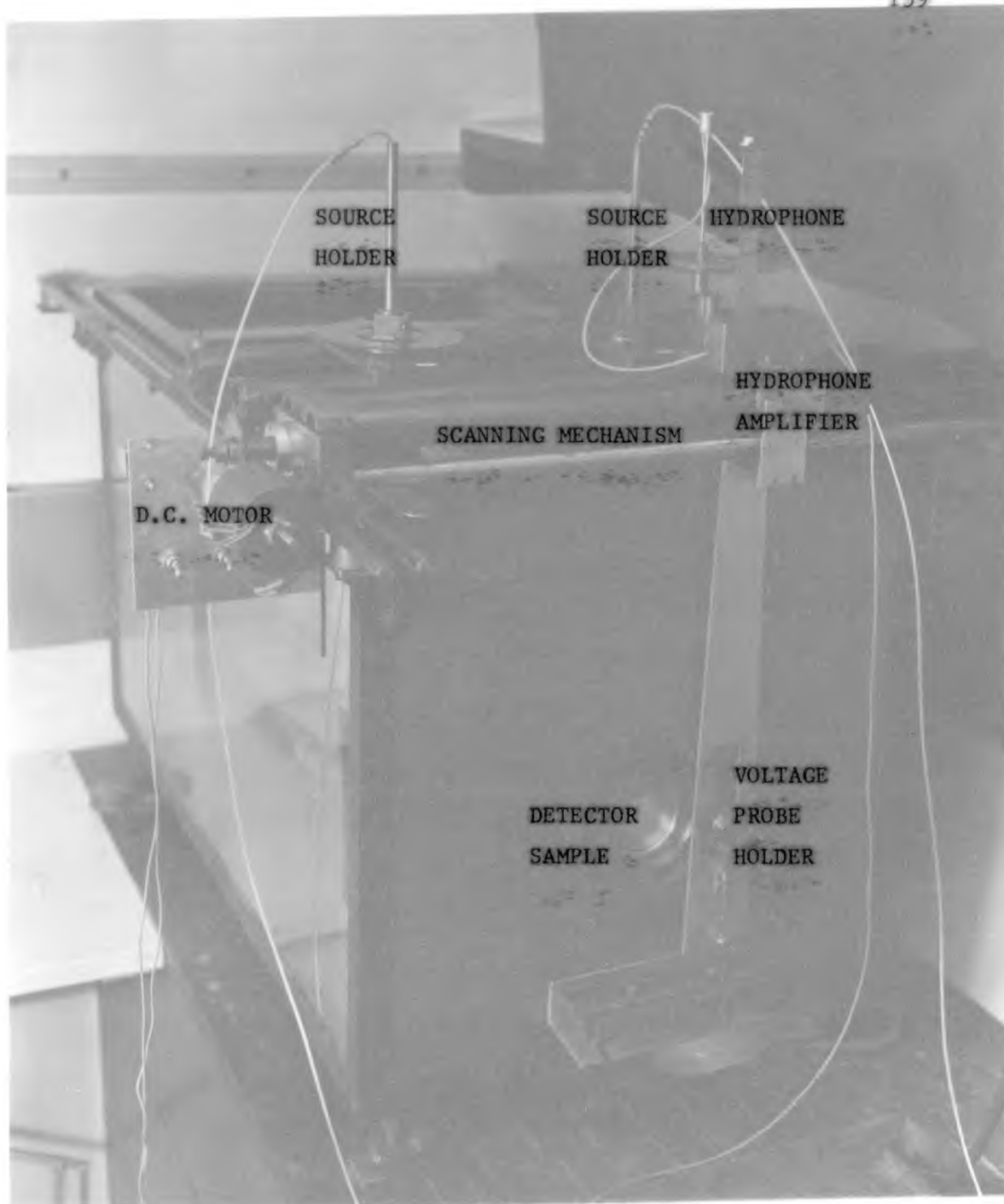
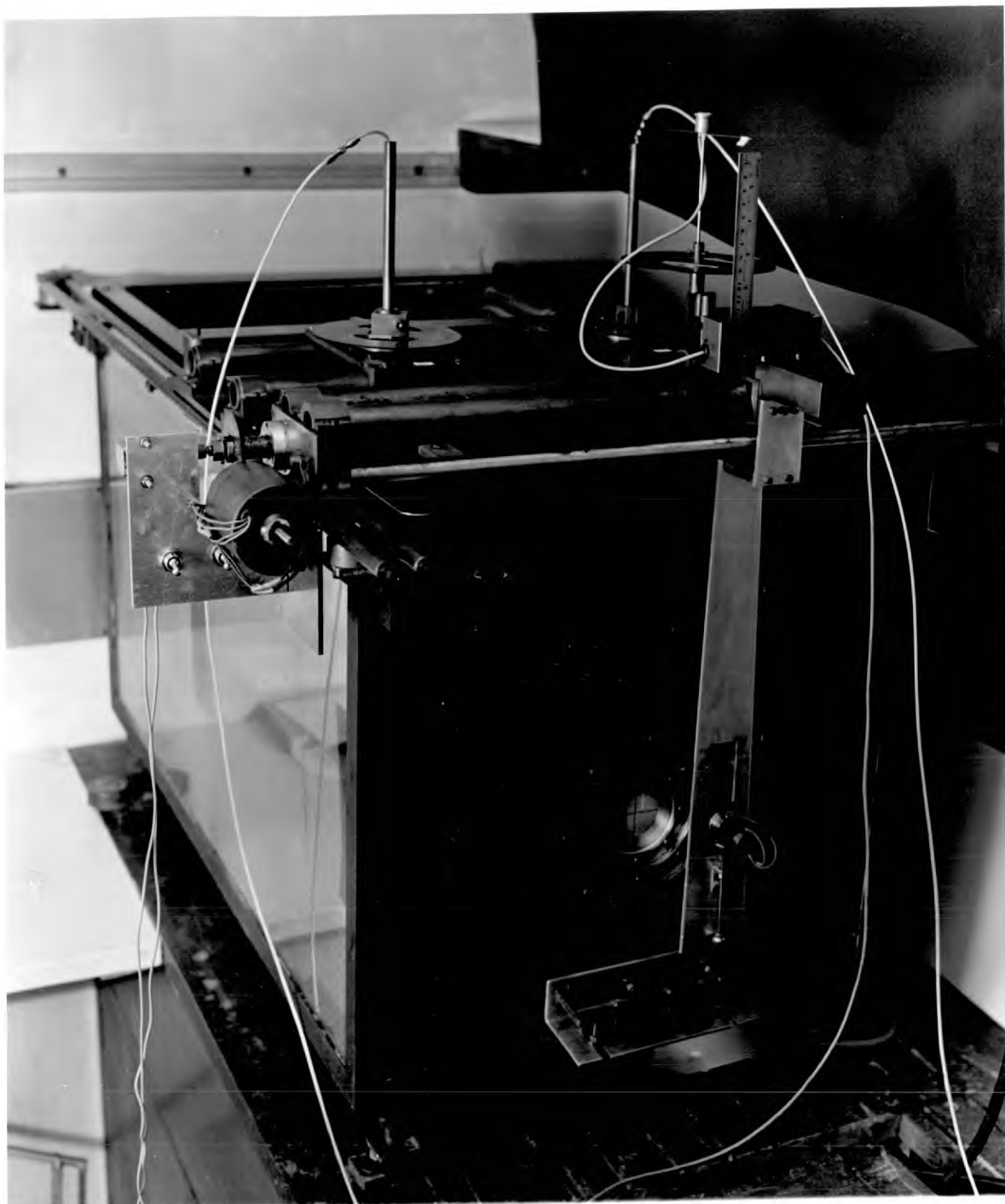


Figure 4.5 A view of the anechoic tank showing the hydrophone, source holders and probe scanning mechanism



CHAPTER 5

Discussion of Results

During the course of these investigations the procedures described in Chapter 4 produced over one thousand individual results, in the form of pen recorder traces, for 15 different lead zirconate titanate ceramic discs. Six different ceramic compositions were investigated over a range of diameter to thickness ratios. Because of limited availability of certain compositions, it was not possible to cover the same series of diameter to thickness ratios in each case. Specifically only 1 MHz fundamental thickness resonant discs of the Brush Clevite materials P.Z.T. 4 and P.Z.T. 5H were available. These had a diameter to thickness ratio of 25. Similarly measurements were made only on a single 1 MHz Admiralty Materials Laboratory (AML) ceramic AM 510/430 disc having a diameter to thickness ratio of 25. This material was specially requested towards the end of the research program because of its high degree of homogeneity.

Each disc of ceramic material was given a code number for identification purposes, which has been adhered to in the presentation of the results. The code comprises the material type followed by a disc number, for example AM 510 No. 4A describes disc Number 4A of AML ceramic material type AM 510.

Complete piezoelectric, dielectric and elastic data is available for the Brush Clevite materials so in principle it is possible to relate the measurements to these physical properties, as suggested in the introduction. Only a limited amount of

piezoelectric, dielectric and elastic data has, however, been obtained for the AML ceramics. Measurements of the coherent transfer function cannot, therefore, be directly related to the physical properties of these materials. Any correlations of properties can only be deduced from corresponding indications obtained using the Brush Clevite materials. It must, therefore, be emphasised that there is no reason why the relevant quantities for the AML materials could not be measured in the future, opening the way for direct comparison.

It is impractical to display all the information available from the large number of pen recorder traces obtained. Results obtained with two materials, P.Z.T. 5H and AM 522/558, have therefore been chosen to show the general form of these traces. A large reduction in volume has been achieved by displaying the spatial frequency bandwidth of the coherent transfer function curves as a function of $\frac{\lambda_d}{2b}$, where λ_d is the dilatational

wavelength in the ceramic material. The spatial frequency bandwidth was measured between half amplitude points on the coherent transfer function curves. From these figures one can immediately choose the optimum operating frequency for a detector made from a particular ceramic material, achieving one of the main objects of this research work as outlined in the introduction.

5.1 Results of electrical input impedance and far field ultrasound pressure measurement, as a function of temporal frequency

Results of these two preliminary investigation techniques for P.Z.T. 5H and AM 522/558 are shown in Figs. 5.1 to 5.9. They are mainly qualitative because of the non-linear nature of the frequency sweep and the measuring electronics. However, the temporal frequencies of points of interest on the curves were obtained accurately by manually re-locating each point after the first trace was recorded. The error in the temporal frequency labelling is, therefore, that resulting from inability to re-set the frequency precisely at a particular point on the curve. It is not more than ± 0.002 MHz. In the case of the far field pressure amplitude results the hydrophone output may be linearized by using the appropriate calibration curve in Appendix 3. It was not considered worthwhile to calibrate the electrical input impedance system in view of the difficulty associated with interpretation of an impedance / frequency plot. In contrast the interpretation of the more usual circle diagram obtained by plotting resistive and reactive components of the impedance is straightforward and if quantitative information is required, this method is recommended.

The Figures are grouped in pairs, for example Fig. 5.1 and Fig. 5.2, with the exception of Fig. 5.9 which is unusually complex and will be discussed separately.

The electrical input impedance results show many turning points, as expected, owing to the large number of modes of vibration which are possible for a finite disc. In each case the curves show a single strong resonance, thickness resonance, with a number of weaker subsidiary resonances superimposed. These resonances will

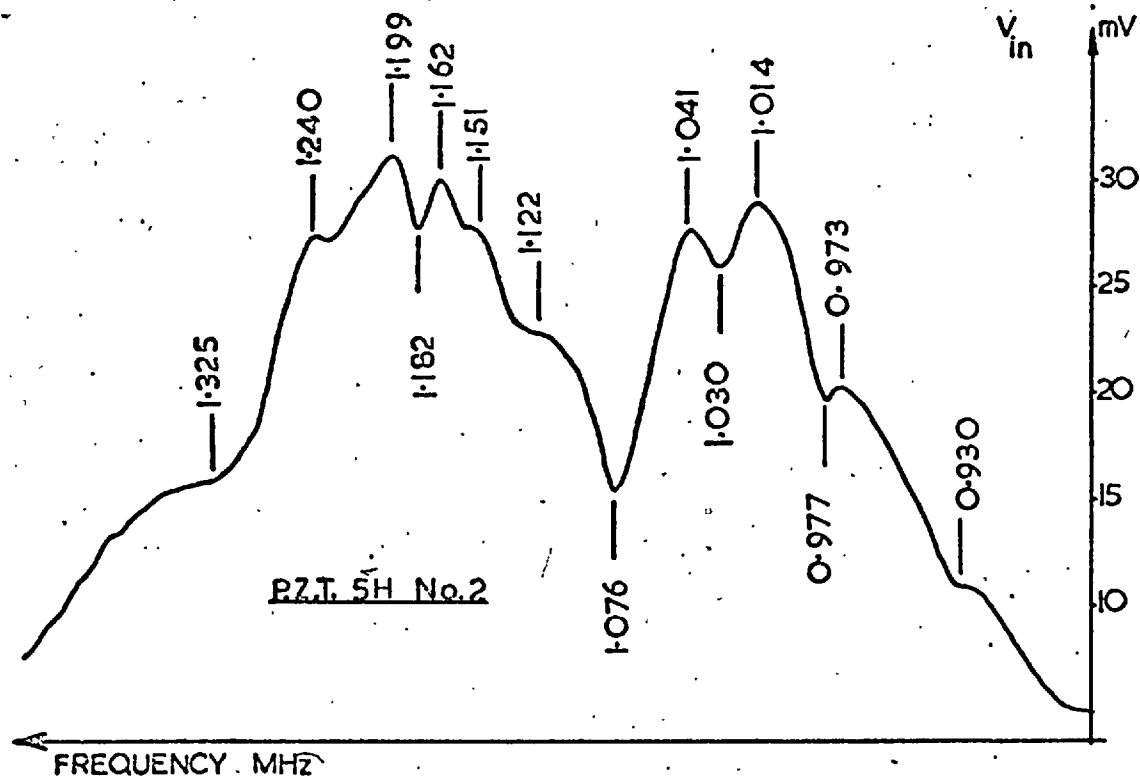


Figure 5.1 Electrical input impedance of the transducer against temporal frequency. Signal applied at contact 1.

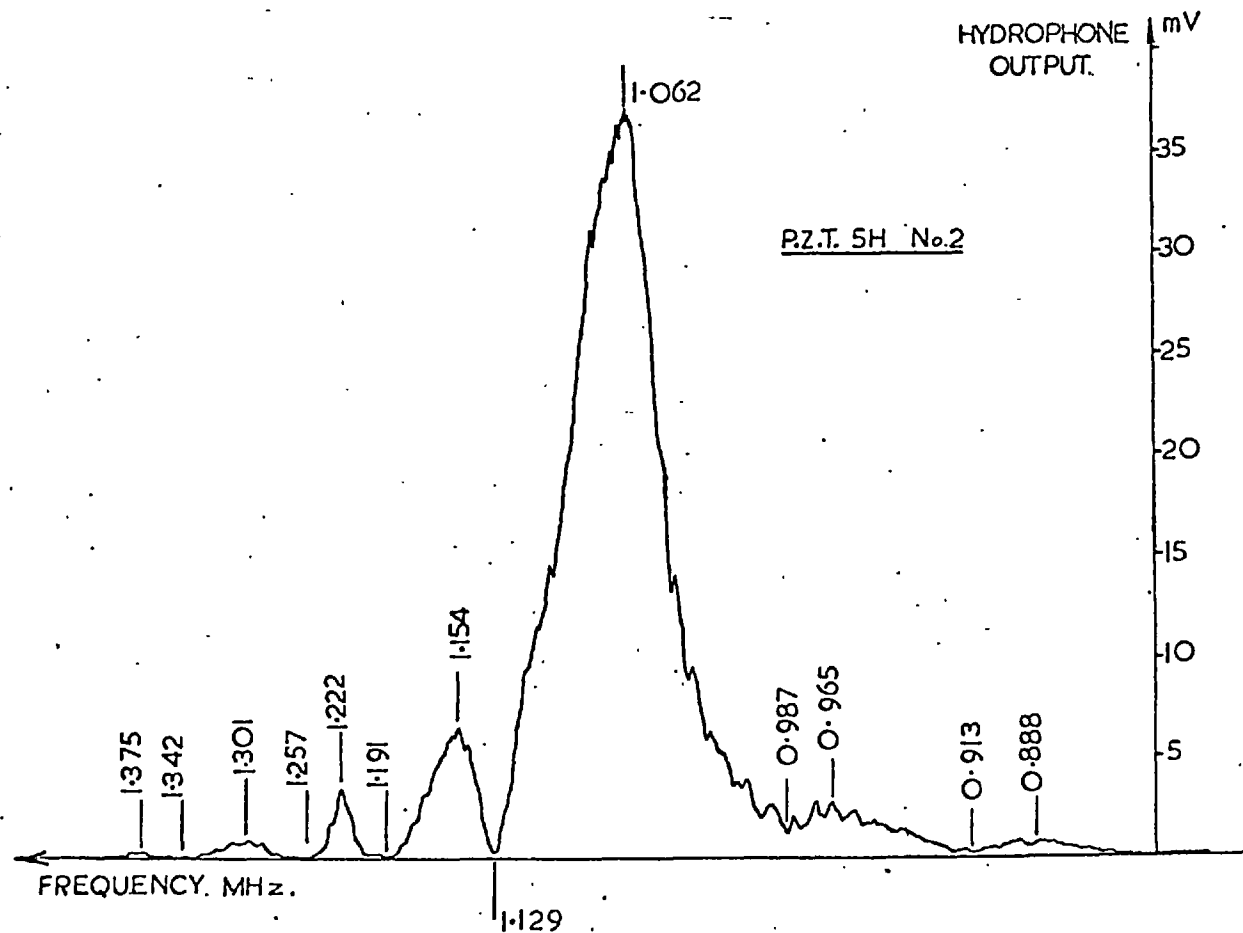


Figure 5.2 Far field pressure amplitude on the transducer Z axis against temporal frequency. Drive voltage applied at contact 1.

not necessarily have appreciable particle displacement along the thickness direction (Z axis).

For a source in water only longitudinal waves will propagate to the far field. Consequently a range of far field pressure amplitudes corresponding to the appropriate Z axis displacement amplitude over the radiating disc will be observed. The far field pressure amplitude results, therefore, indicate the frequencies and the relative strength of those modes of the disc having displacement in the Z direction. When acting as a receiver the electrical potential difference across the disc thickness must show a similar amplitude variation with frequency for a source in the far field on the Z axis, assuming the principle of reciprocity. The far field pressure amplitude results are, therefore, of much greater value in predicting the performance of a piezoelectric ultrasound hologram detector than the electrical input impedance curves.

Equations (2.75) and (2.76) indicate that for an infinite plate of piezoelectric material driven with an electric field parallel to the Z axis, only a single mode having a particle displacement along the Z axis can exist. Consequently one would expect the far field pressure amplitude to show a peak at thickness resonance and its odd harmonics only. Fig. 5.2 shows clearly the presence of subsidiary peaks at frequencies above, but close to, the fundamental thickness resonance frequency. A disc having a diameter to thickness ratio of 25 or more is, however, known to be a close approximation to an infinite plate (50). Therefore it is reasonable to suppose that any radial resonances which can be excited directly by an electric field E_3 must be weak compared with the fundamental thickness resonance. The fundamental radial resonance frequency for the Admiralty Materials Laboratory discs studied was in the range

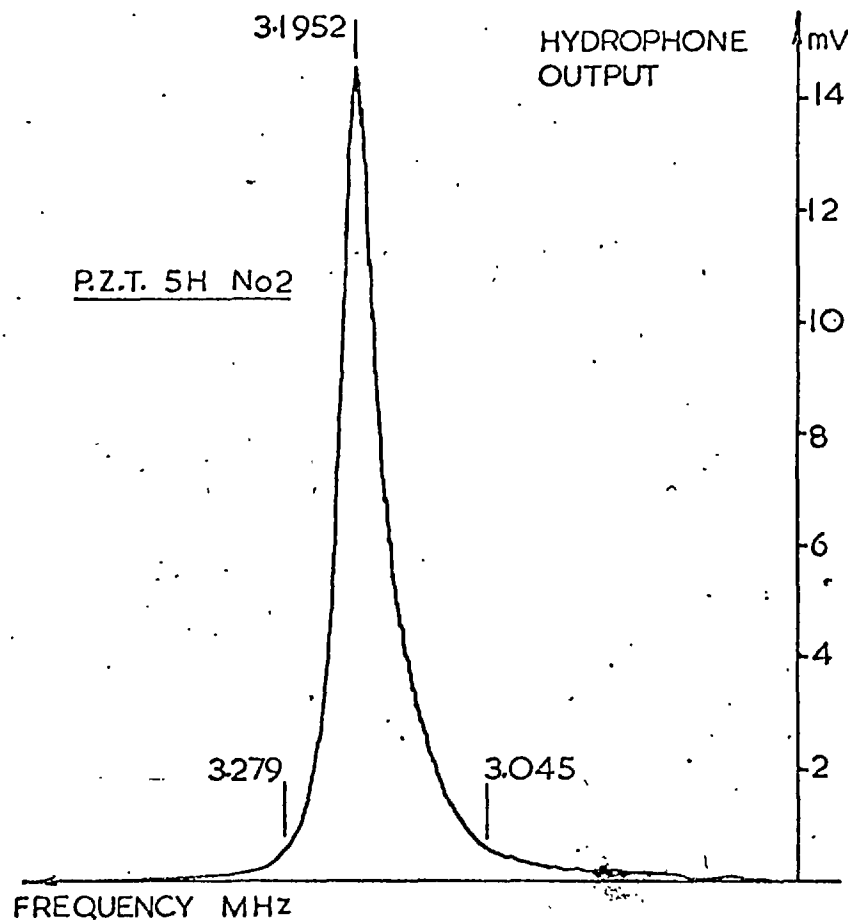


Figure 5.3 1st overtone far field pressure amplitude on the transducer Z axis. Drive voltage applied at contact 1.

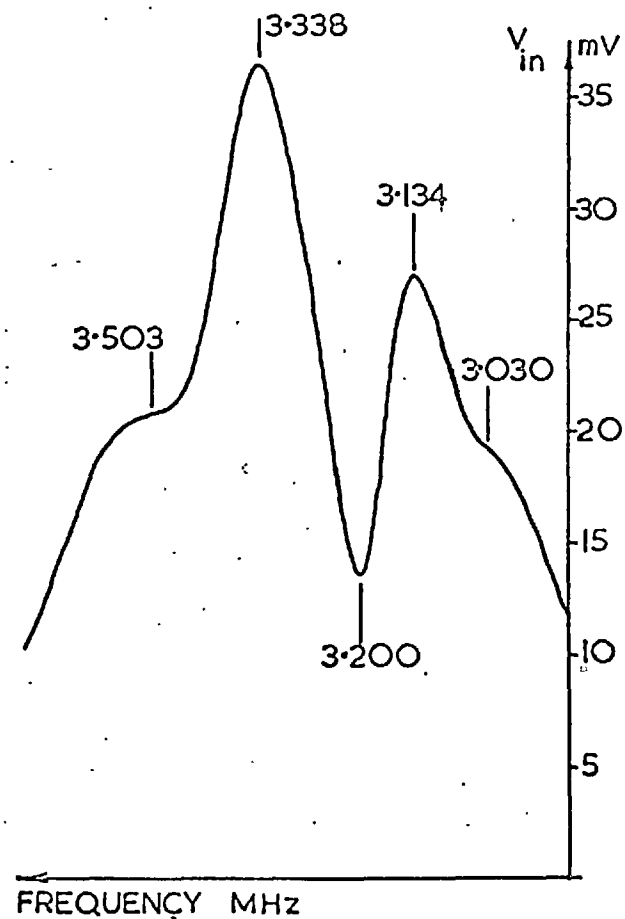


Figure 5.4 1st overtone electrical input impedance of the transducer. Signal applied at contact 1.

44 kHz to 48 kHz, the exact value being dependent on the material composition. Consequently it seems probable that other modes of vibration are excited in the disc as a result of divergence of the electric field within the material. For the point electrical contact representation of a spatial delta function electrical fringing fields will be present and $\underline{E}$ will have components other than along the Z axis. Any inhomogeneities such as grain boundaries within the material will further increase the divergence of $\underline{E}$.

For a complete set of results on one particular ceramic the general trend was for a decrease in the amplitude of detectable subsidiary peaks as the diameter to thickness ratio was increased from 25 to 50 and then 75. This undoubtedly indicates a progressive decrease in the effects of the edges of the disc on its response to a spatial delta function. The number of peaks for a given frequency interval, for example 200 kHz, remains constant at four. This is consistent with the view that they are associated with high harmonics of the radial mode of vibration. The diameter of the conducting spot representing the spatial impulse was always set equal to the thickness of the ceramic disc in accordance with the requirements set by equation (2.101). For a given material the electric field divergence could sensibly be expected to remain constant within the set of three disc diameter to thickness ratios. Hence the subsidiary peaks result from a complicated interaction between the waves set up in the disc as a result of the divergences of $\underline{E}$ and the disc boundaries.

Figs. 5.3 and 5.4 show the results obtained with a P.Z.T. 5H disc (No. 2) near its first overtone frequency. The impedance curve shows only a single thickness resonance and the far field pattern is very similar to that which would be expected from an ideal

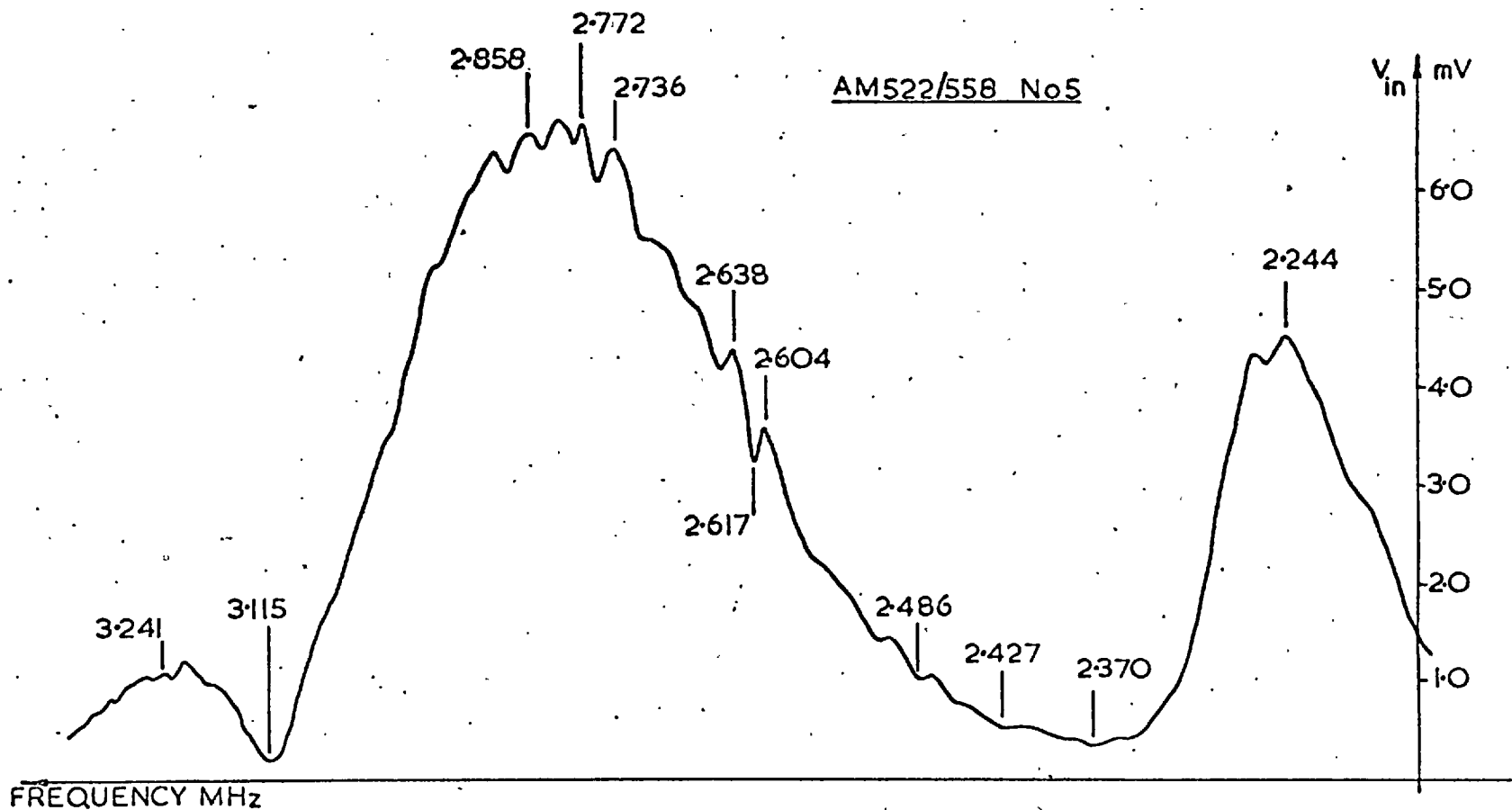


Figure 5.5 Electrical input impedance of the transducer against temporal frequency. Signal applied at contact 1.

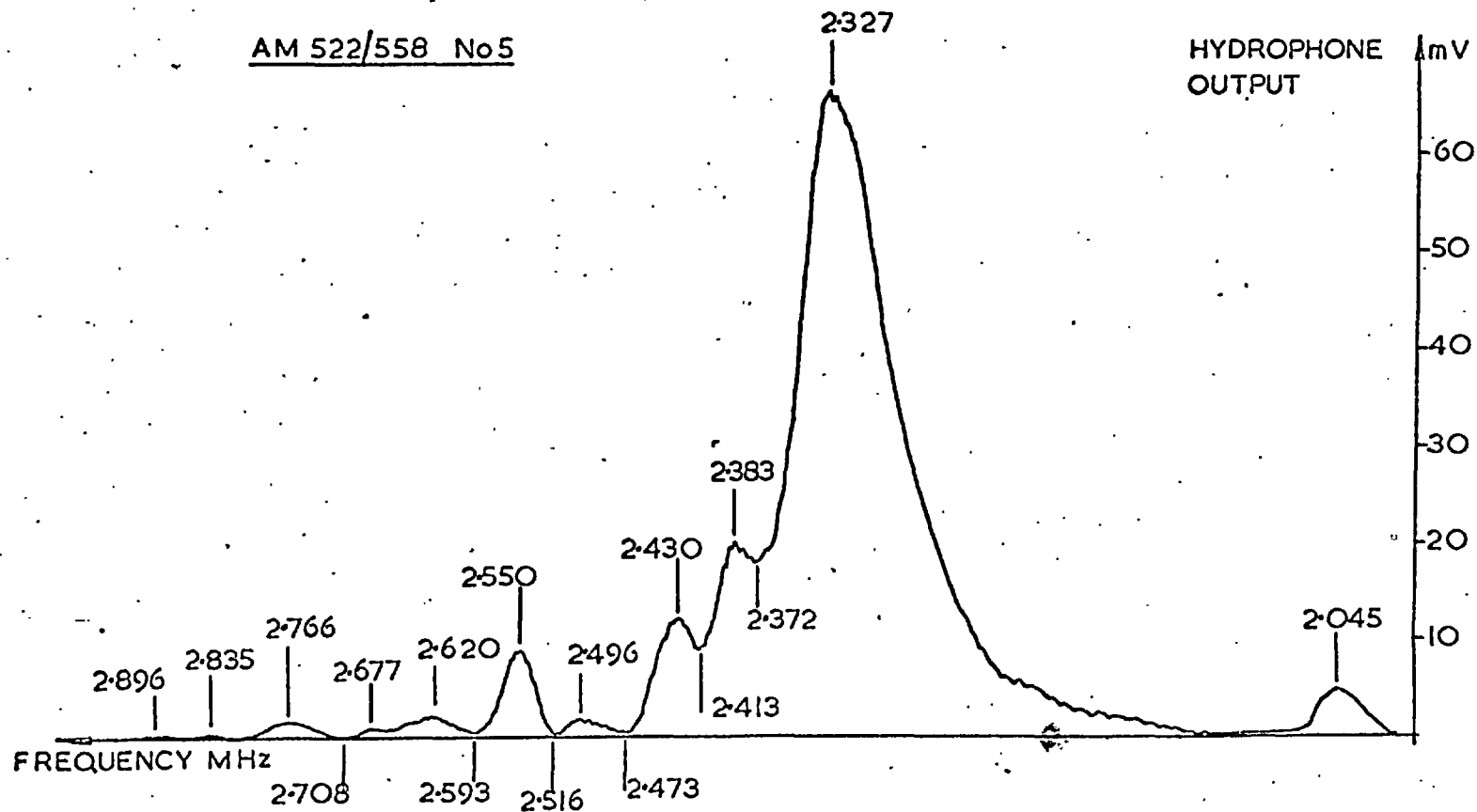


Figure 5.6 Far field pressure amplitude on the transducer Z axis against temporal frequency. Drive voltage applied at contact 1.

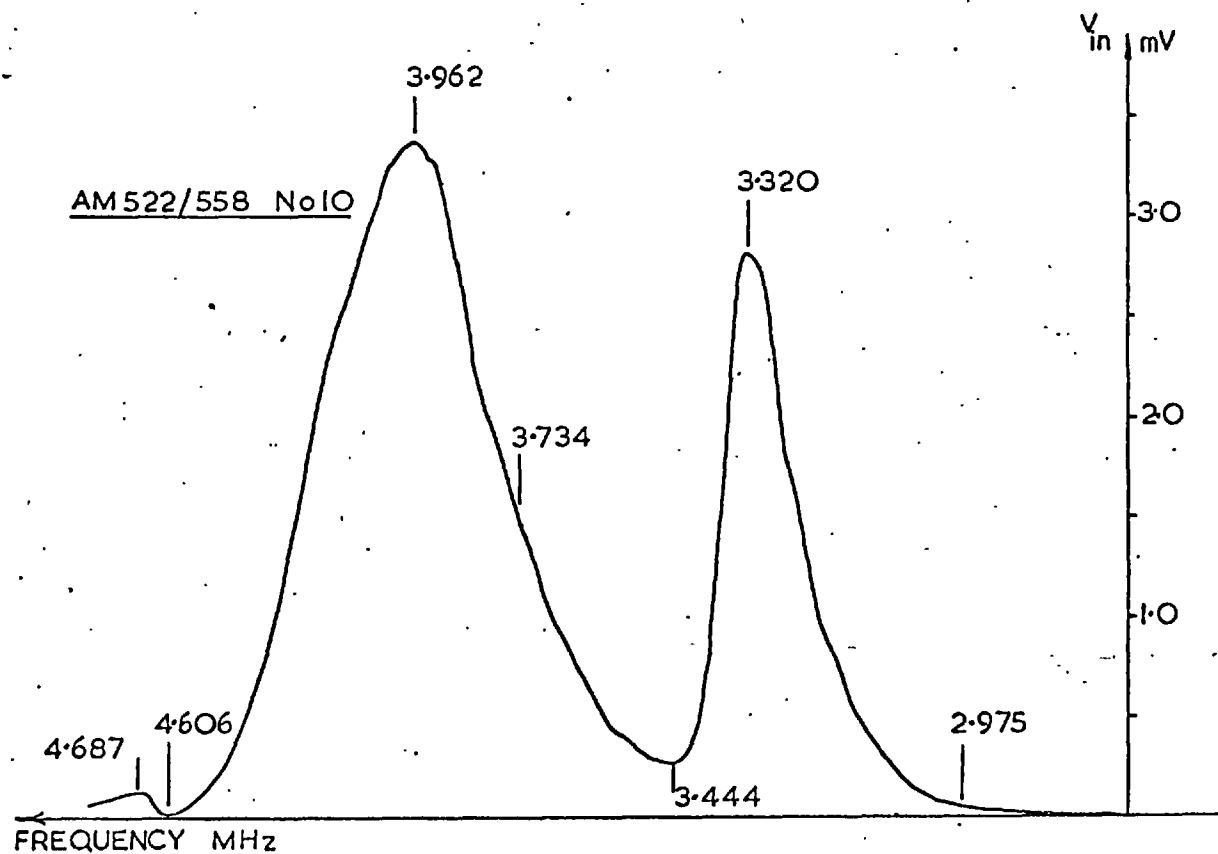


Figure 5.7 Electrical input impedance of the transducer against temporal frequency. Signal applied at contact 1.

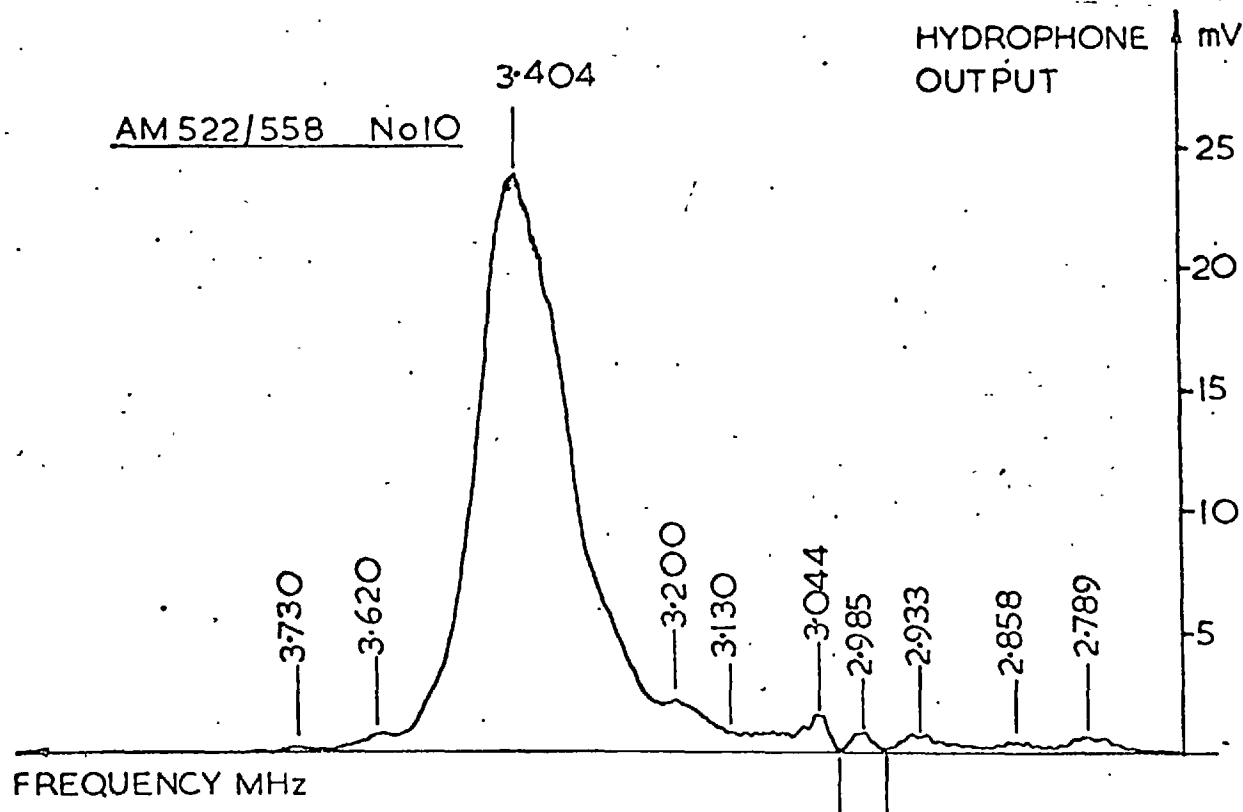


Figure 5.8 Far field pressure amplitude on the transducer Z axis against temporal frequency. Drive voltage applied at contact 1.

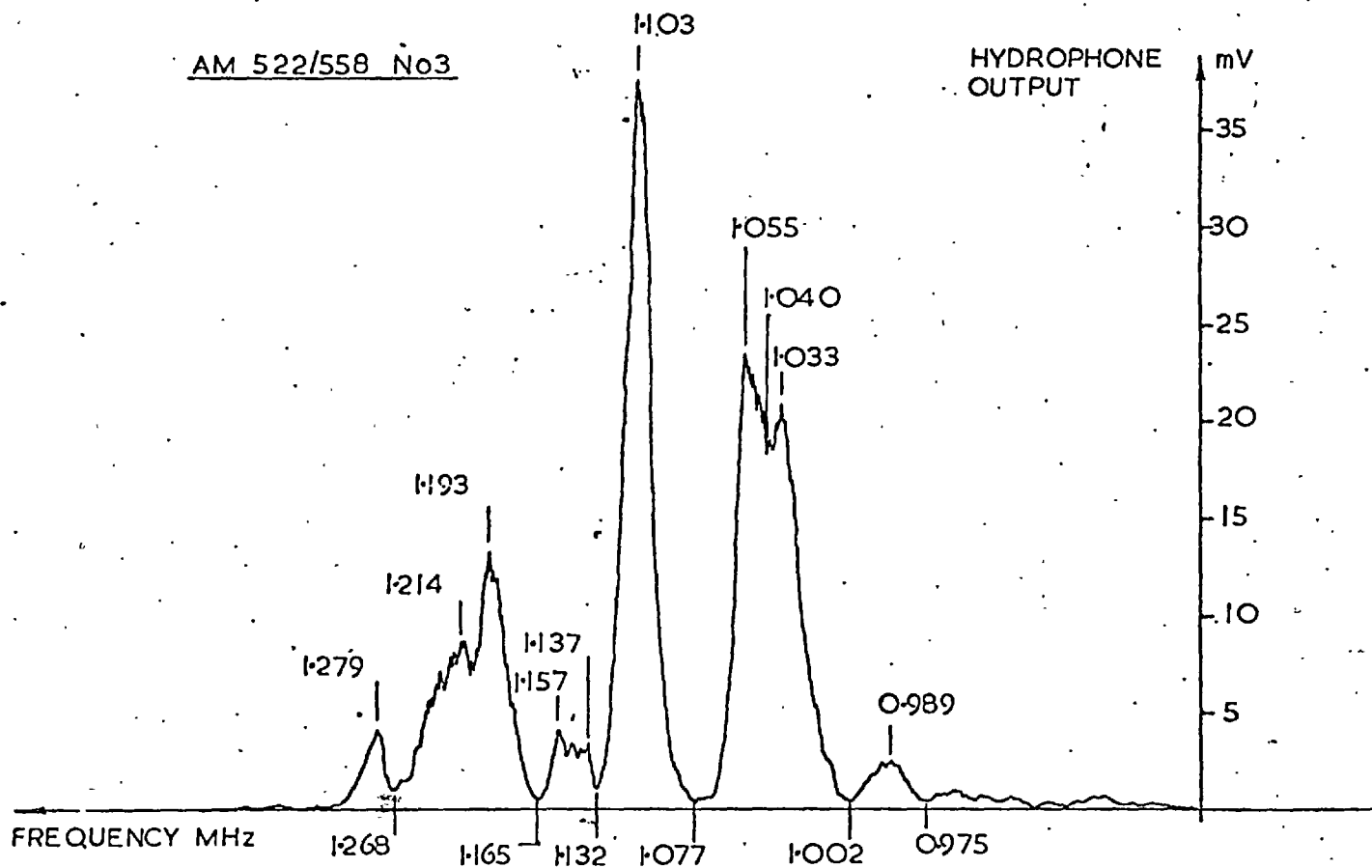


Figure 5.9 Far field pressure amplitude on the transducer Z axis against temporal frequency. Drive voltage applied at contact 1.

infinite plate. Similar behaviour was observed with all the 1 MHz discs driven at their first overtones which seems to indicate that the diameter of a disc in wavelengths, not just its diameter to thickness ratio, is the overriding factor in determining its response to point stimulation.

Fig. 5.5 and Fig. 5.6 form a pair of results for a disc of AM 522/558 having a diameter to thickness ratio of approximately 50. Figs. 5.7 and 5.8 are the corresponding results, again for AM 522/558, for a disc having a diameter to thickness ratio of approximately 75. Unfortunately the experimental system did not permit either of these two discs to be driven at its first overtone and so results similar to Fig. 5.3 could not be obtained.

Fig. 5.9 was a unique result amongst the 15 different discs investigated in that two strong thickness resonance peaks are indicated. Apparently the material in this one disc exhibits acoustic bi-refringence with two possible dilatational wave velocities along the Z axis. This may be a consequence of a polarization anomaly resulting in a polarization vector having a direction which is not perpendicular to the disc faces. The effect was observed on driving this disc at different contact points over its face and so was not a consequence of a large isolated local inhomogeneity.

5.2 Results obtained with the impulse response method for measuring the directivity function $|D_p(f_x)|$ of piezoelectric detectors

Using the results of the far field ultrasound pressure measurements as a guide, $D_p(f_x)$ was measured as described in 4.2

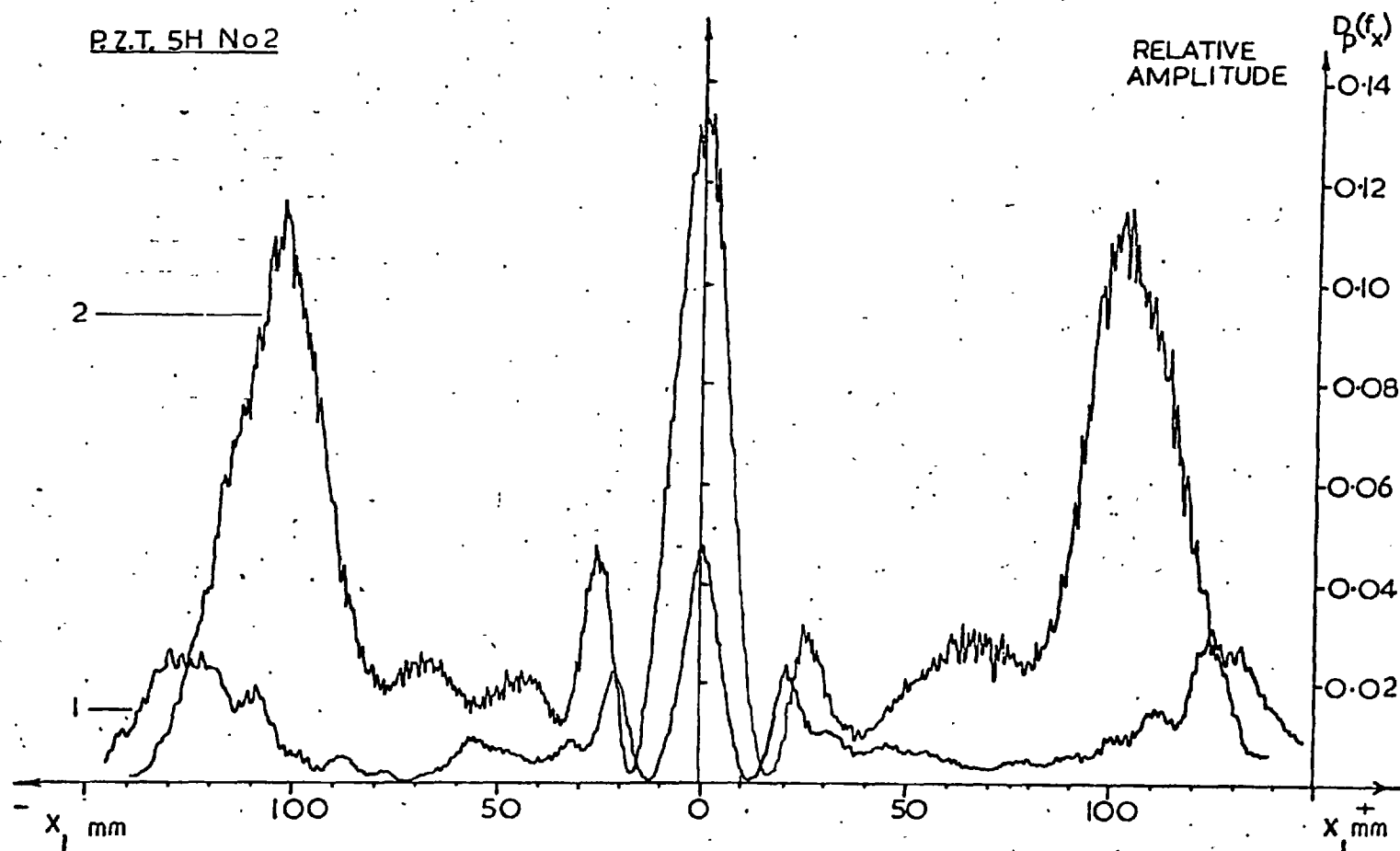


Figure 5.10 Detector directivity functions for set temporal frequencies using the impulse response technique at contact 1.

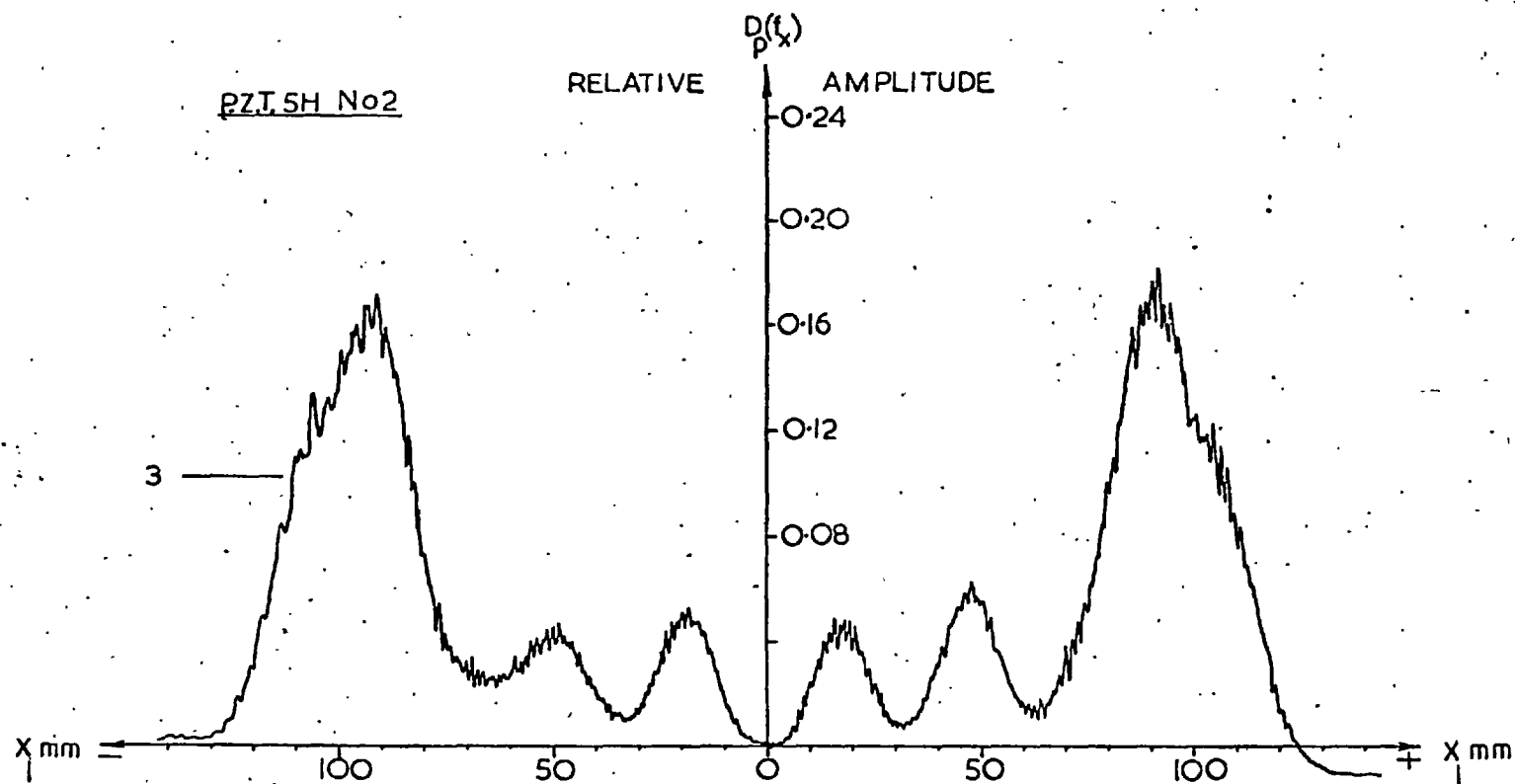


Figure 5.11 Detector directivity function for a set temporal frequency using the impulse response technique at contact 1.

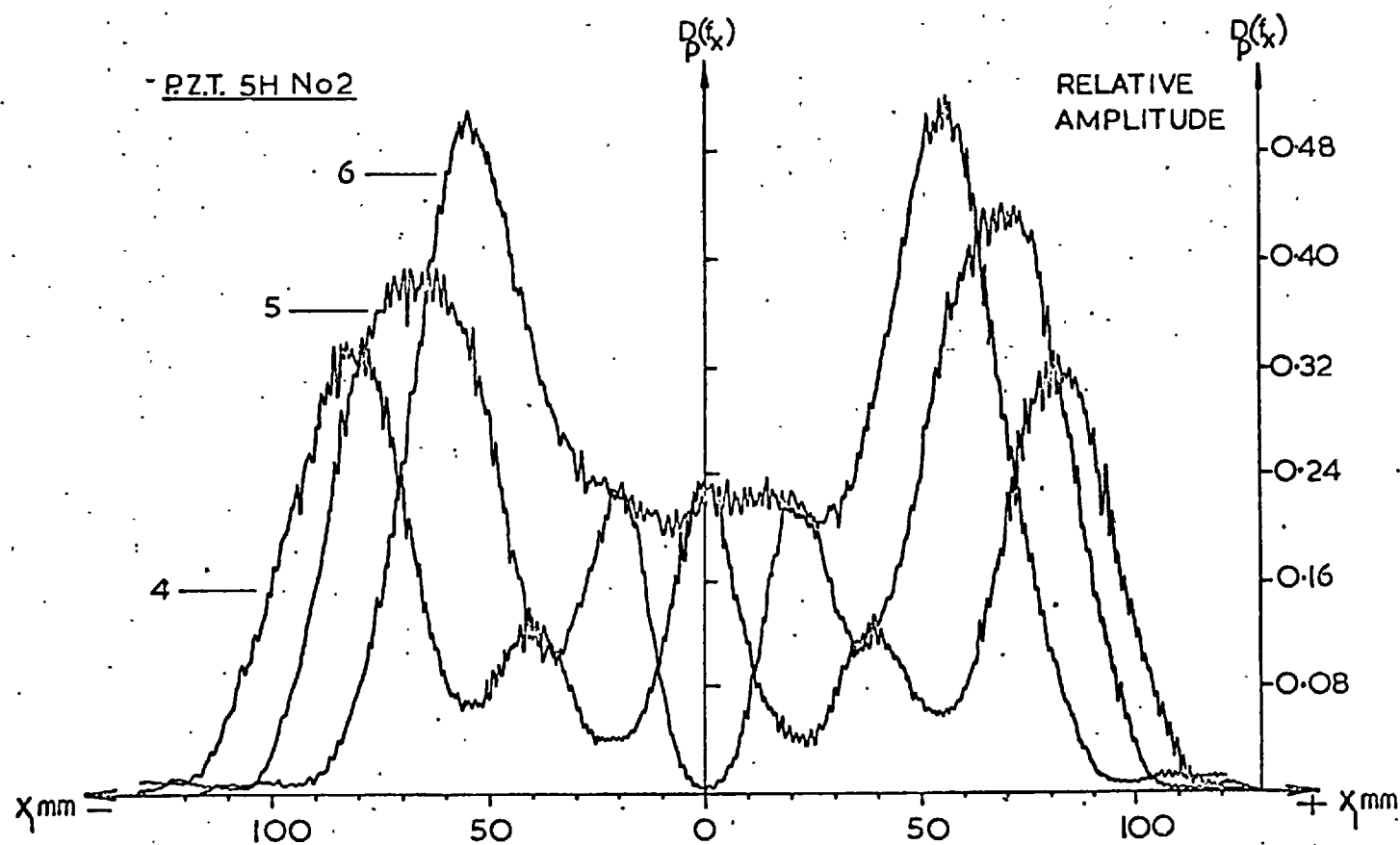


Figure 5.12 Detector directivity functions for set temporal frequencies using the impulse response technique at contact 1.

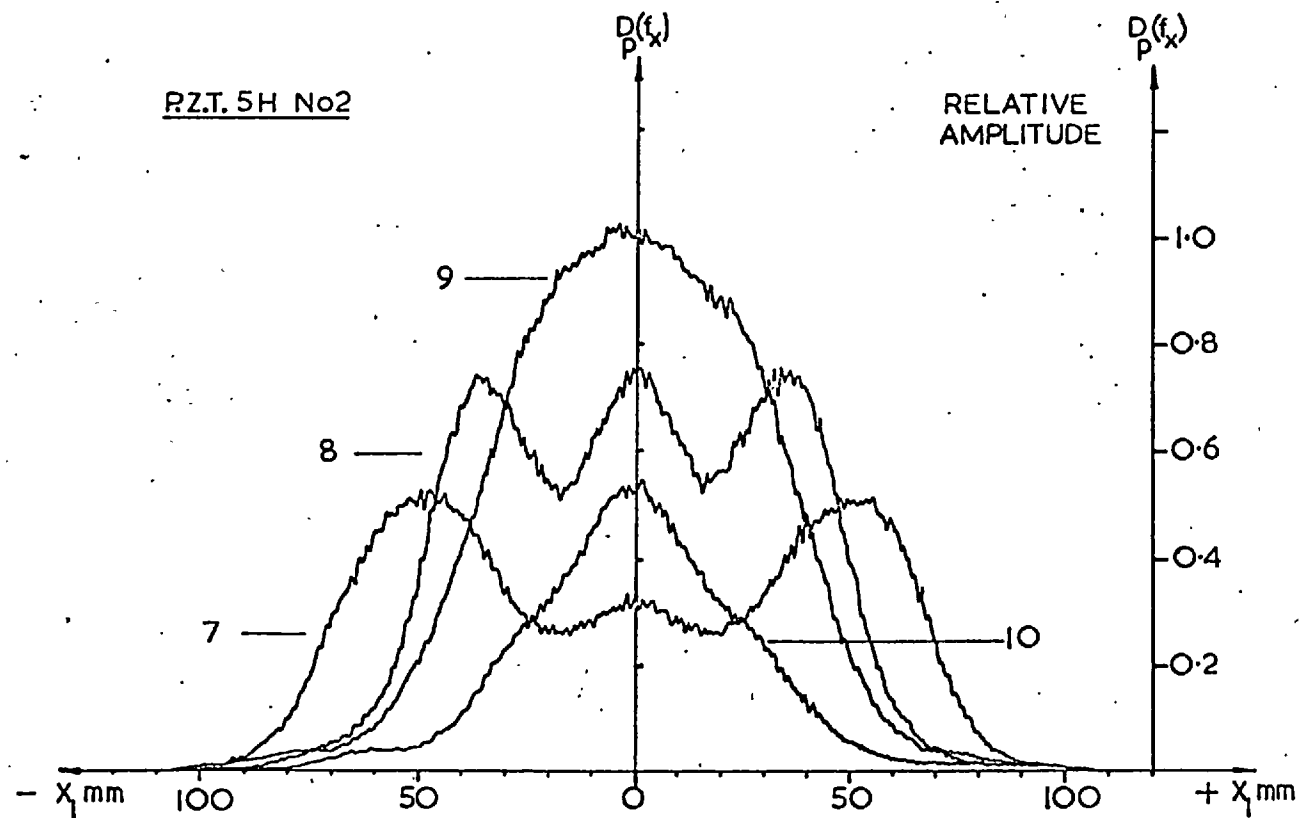


Figure 5.13 Detector directivity functions for set temporal frequencies using the impulse response technique at contact 1.

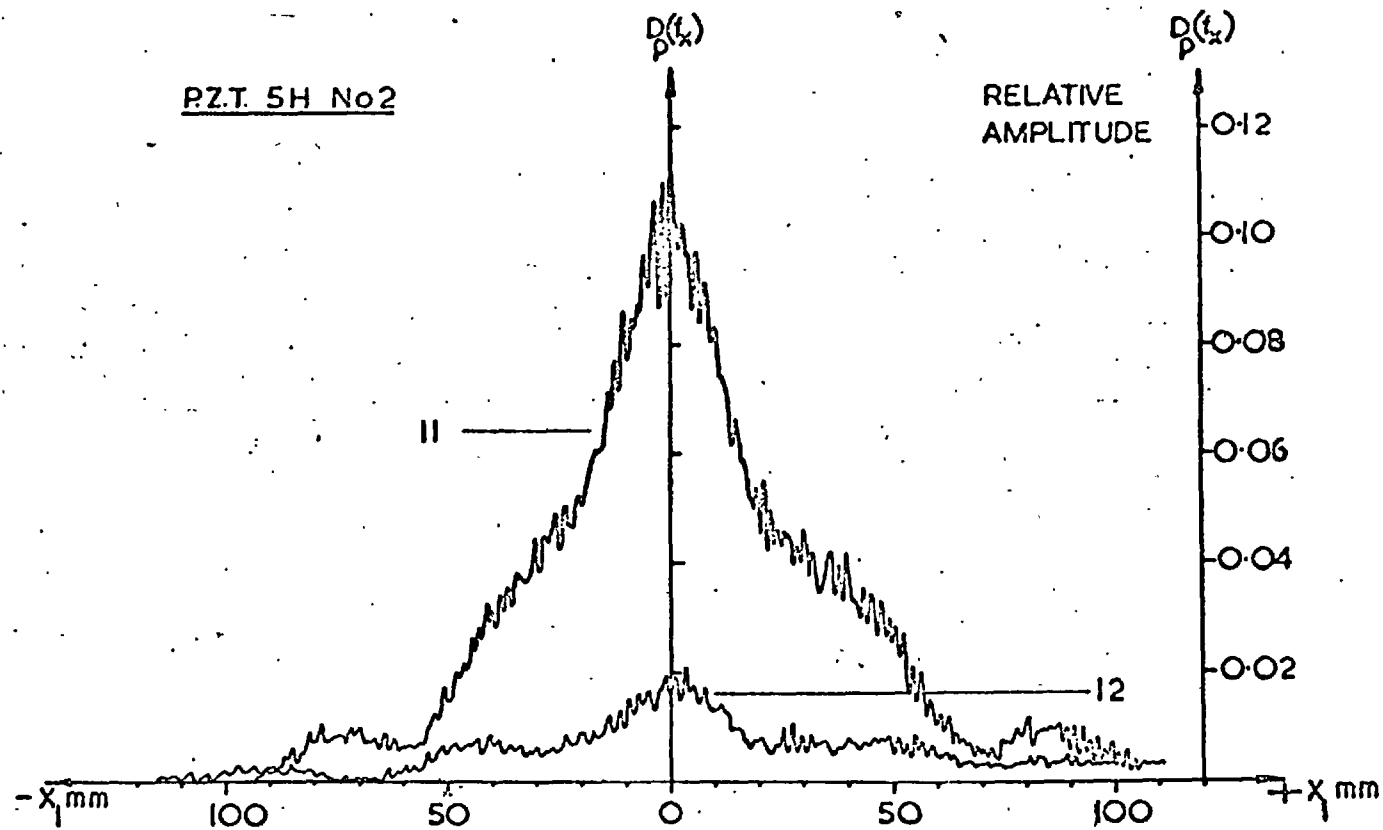


Figure 5.14 Detector directivity functions for set temporal frequencies using the impulse response technique at contact 1.

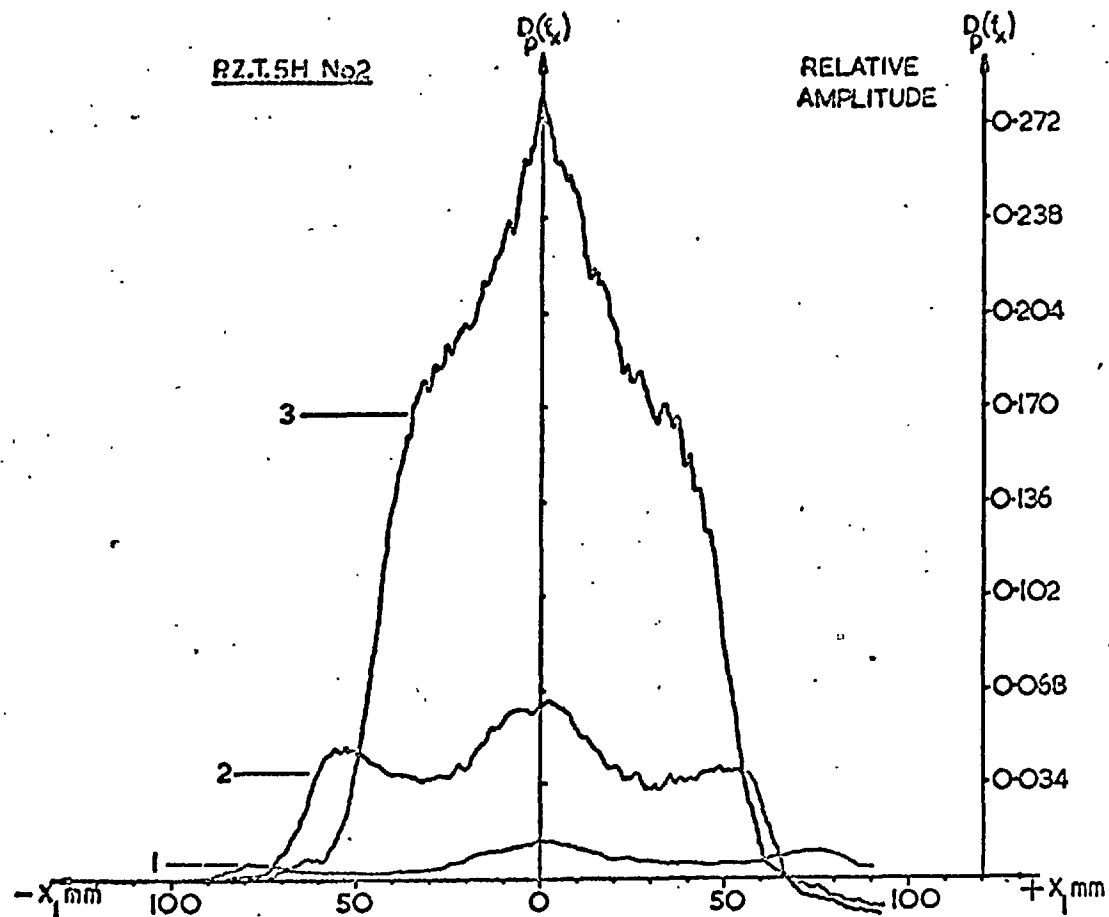


Figure 5.15 Detector directivity functions for set temporal frequencies (1st overtones) using the impulse response technique at contact 1.

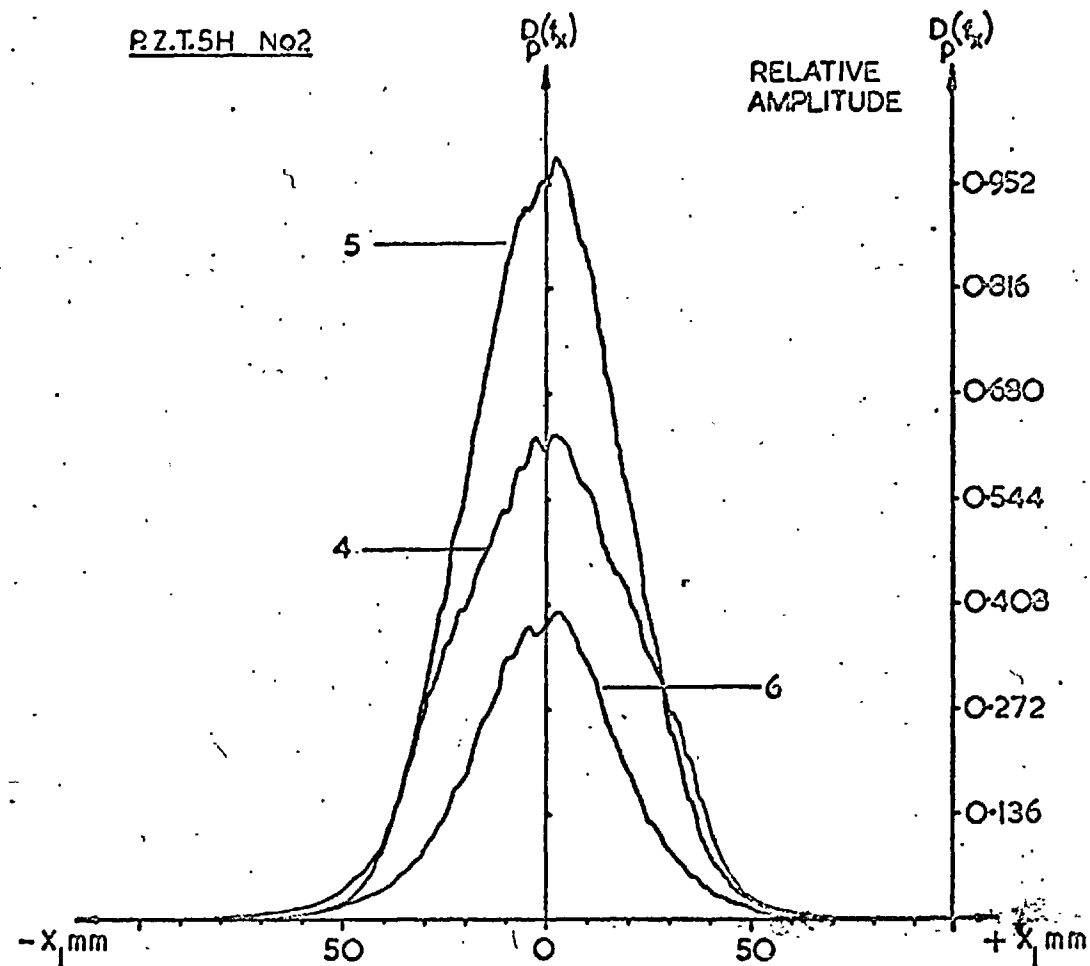


Figure 5.16 Detector directivity functions for set temporal frequencies (1st overtones) using the impulse response technique at contact 1.

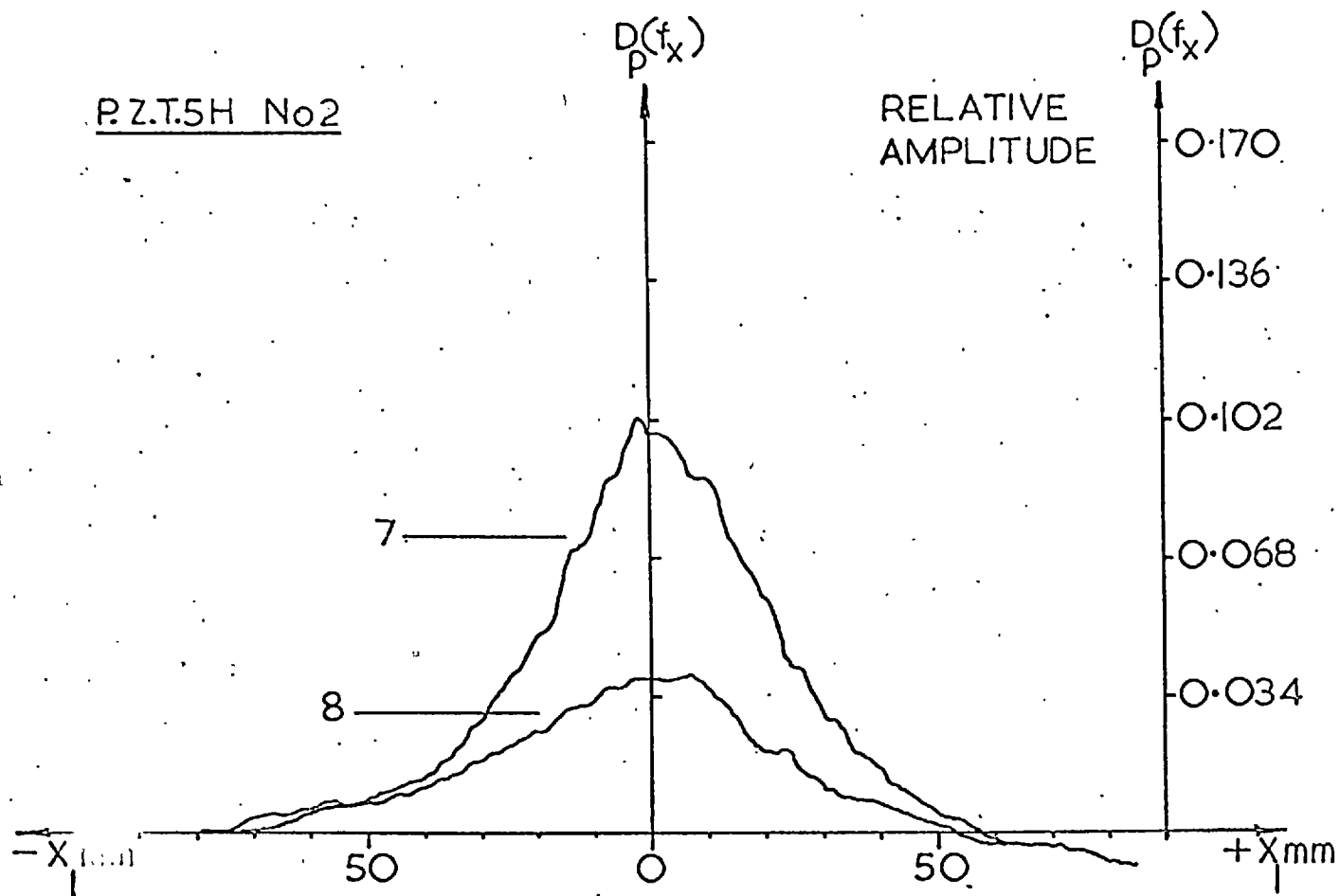


Figure 5.17 Detector directivity functions for set temporal frequencies (1st overtones) using the impulse response technique at contact 1.

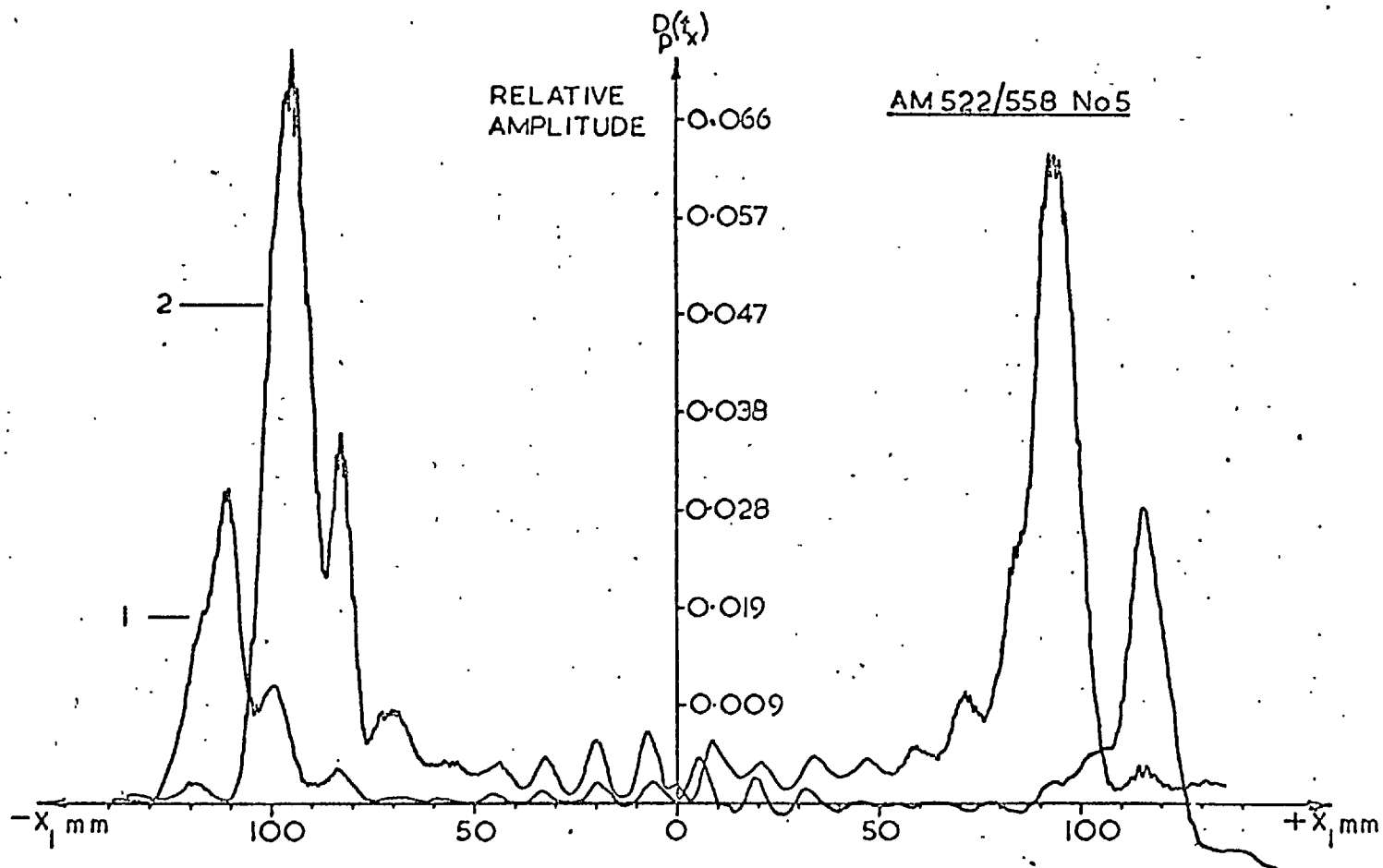


Figure 5.18 Detector directivity functions for set temporal frequencies using the impulse response technique at contact 1.

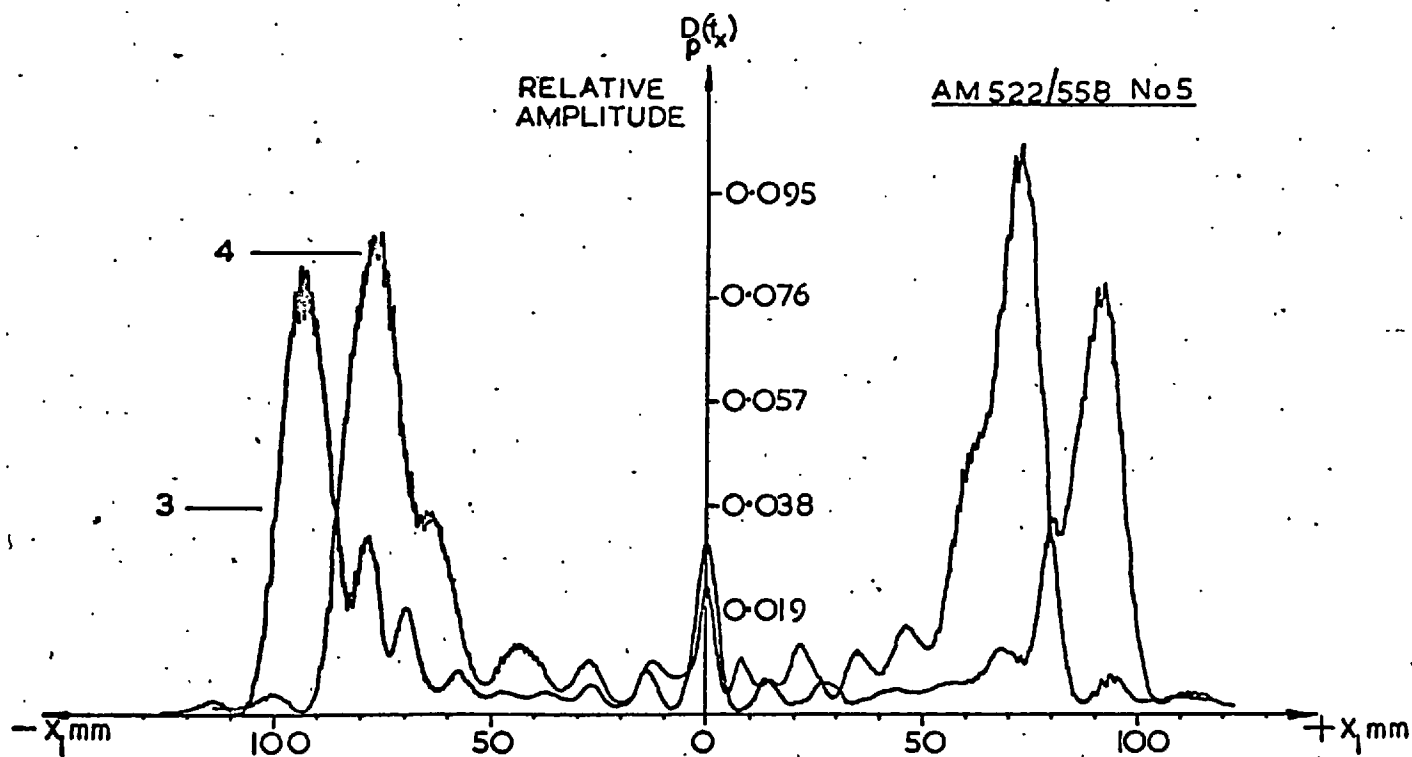


Figure 5.19 Detector directivity functions for set temporal frequencies using the impulse response technique at contact 1.

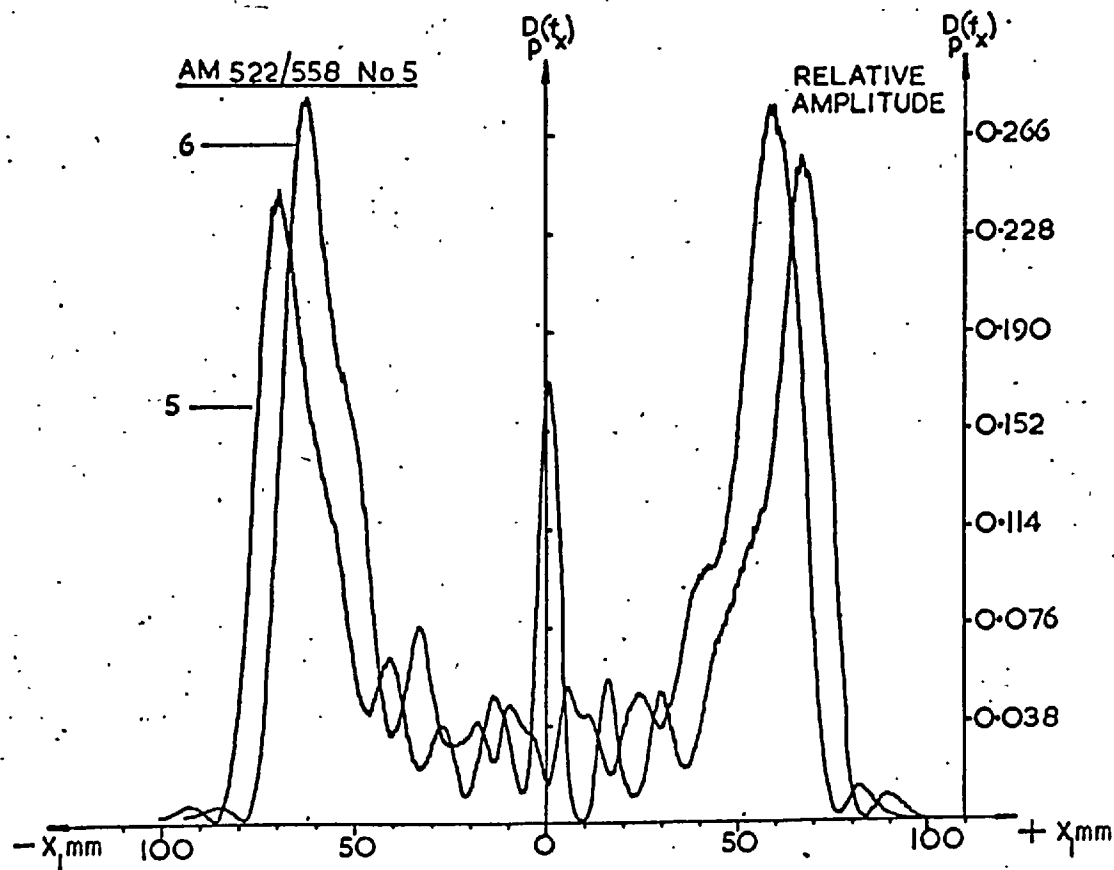


Figure 5.20 Detector directivity functions for set temporal frequencies using the impulse response technique at contact 1.

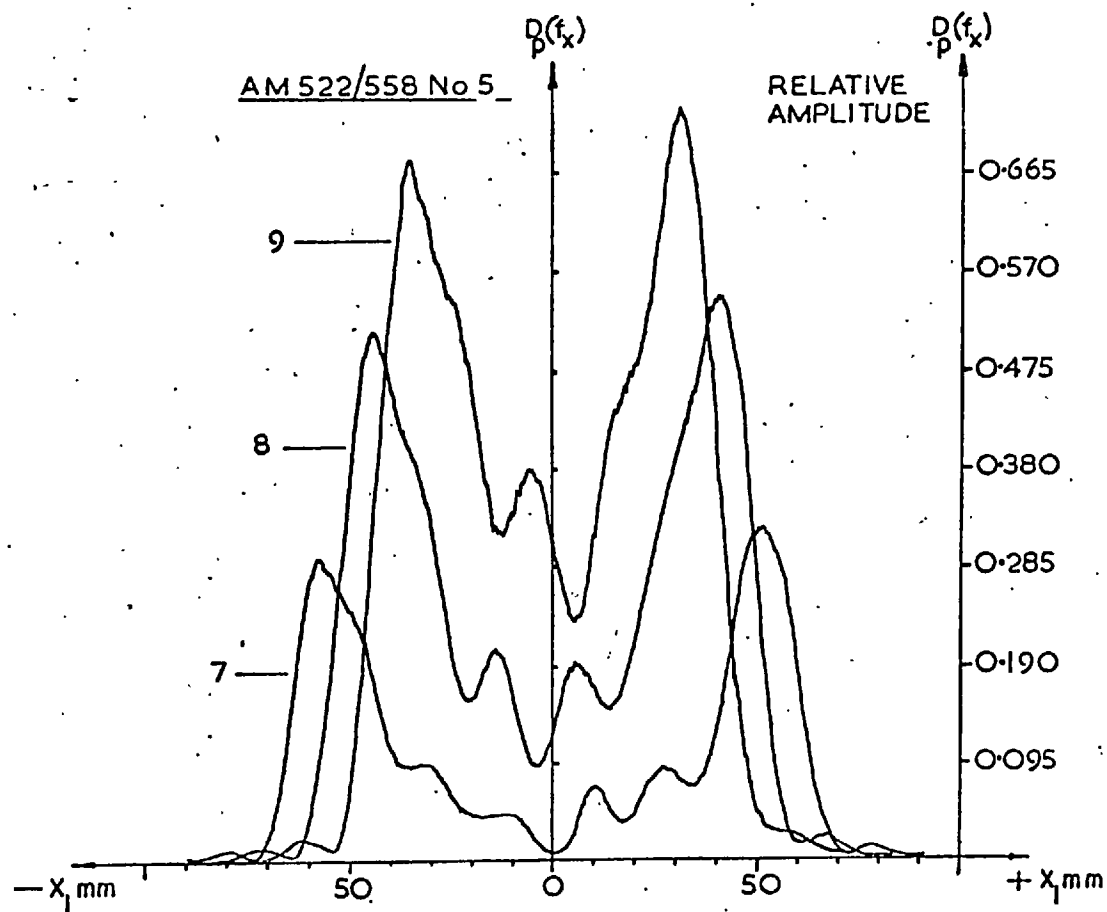


Figure 5.21 Detector directivity functions for set temporal frequencies using the impulse response technique at contact 1.

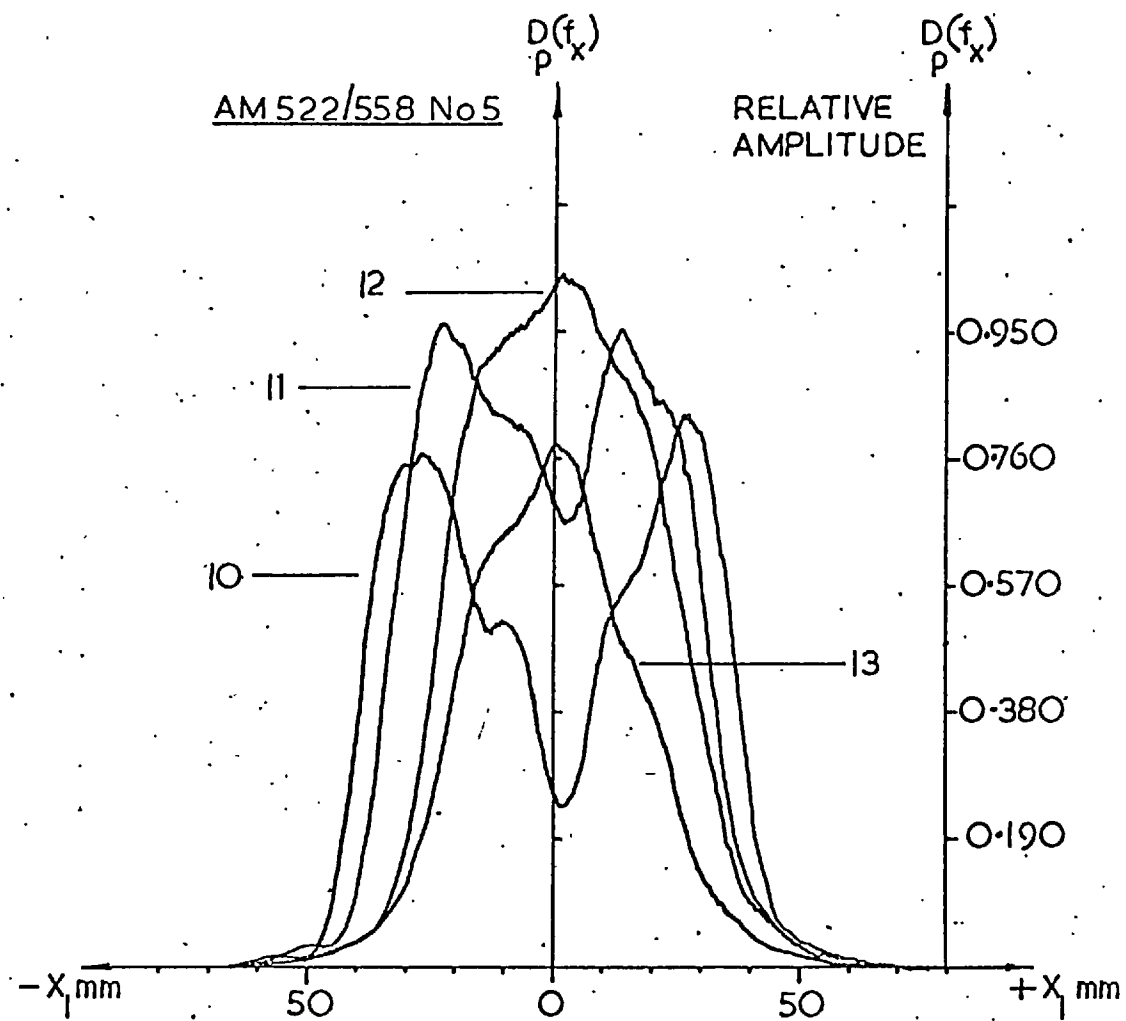


Figure 5.22 Detector directivity functions for set temporal frequencies using the impulse response technique at contact 1.

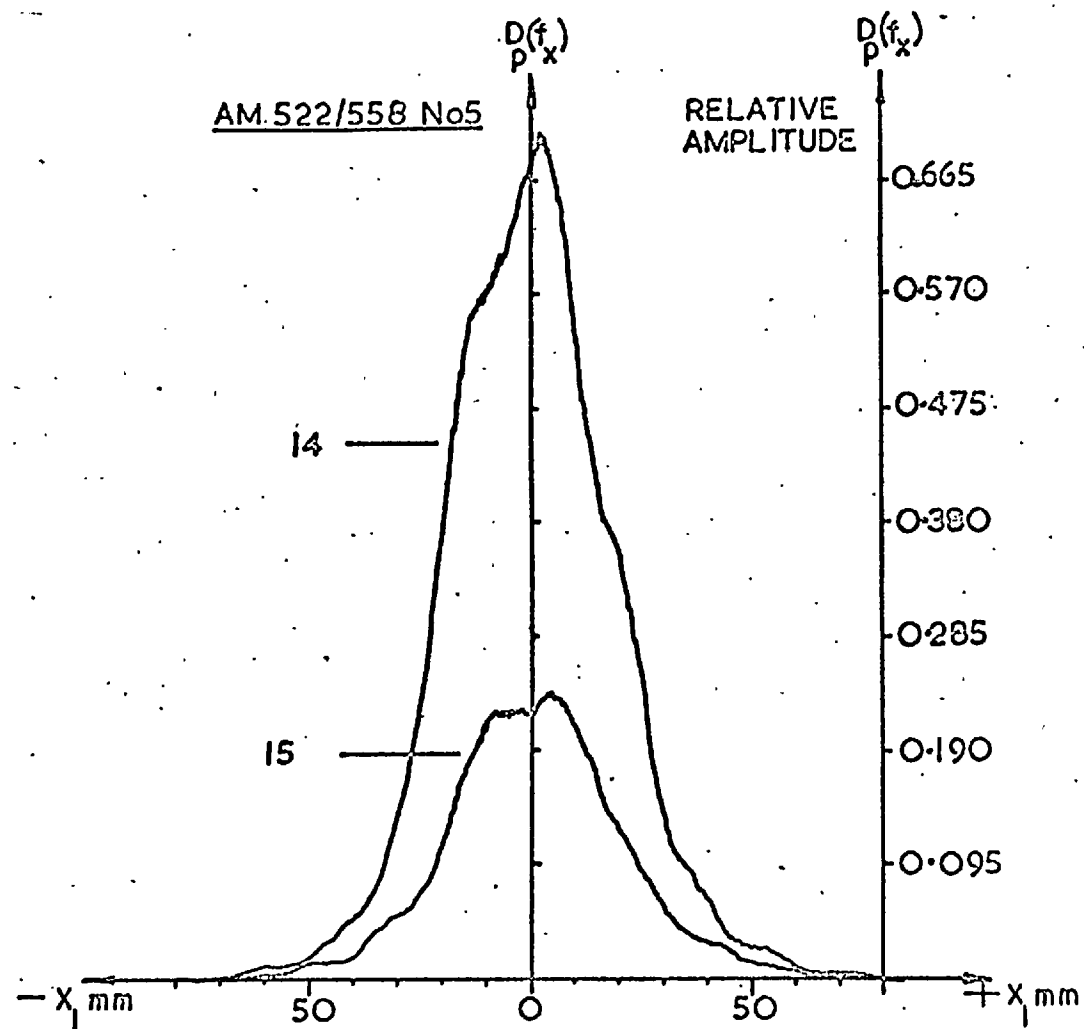


Figure 5.23 Detector directivity functions for set temporal frequencies using the impulse response technique at contact 1.

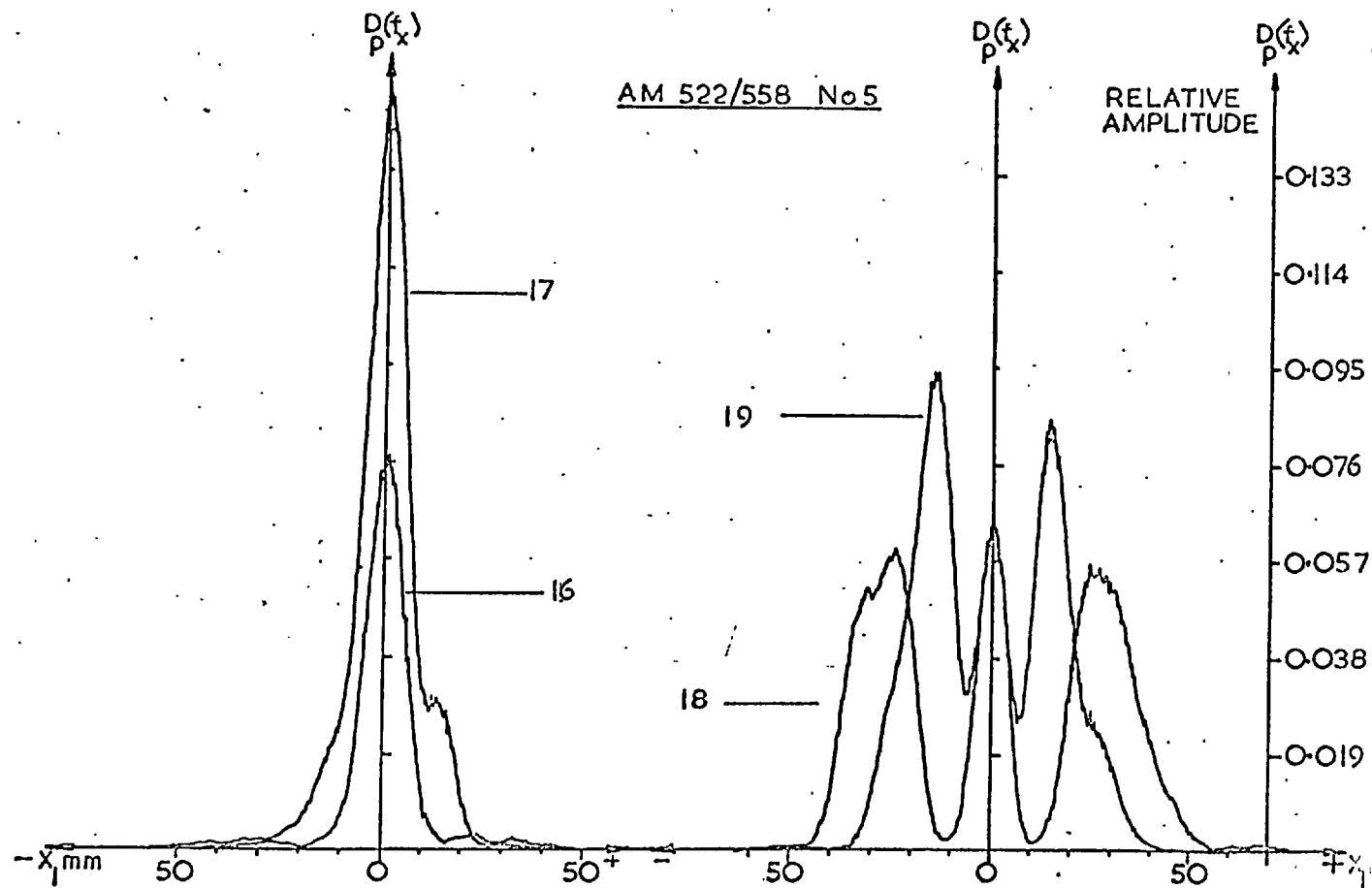


Figure 5.24 Detector directivity functions for set temporal frequencies using the impulse response technique at contact 1.

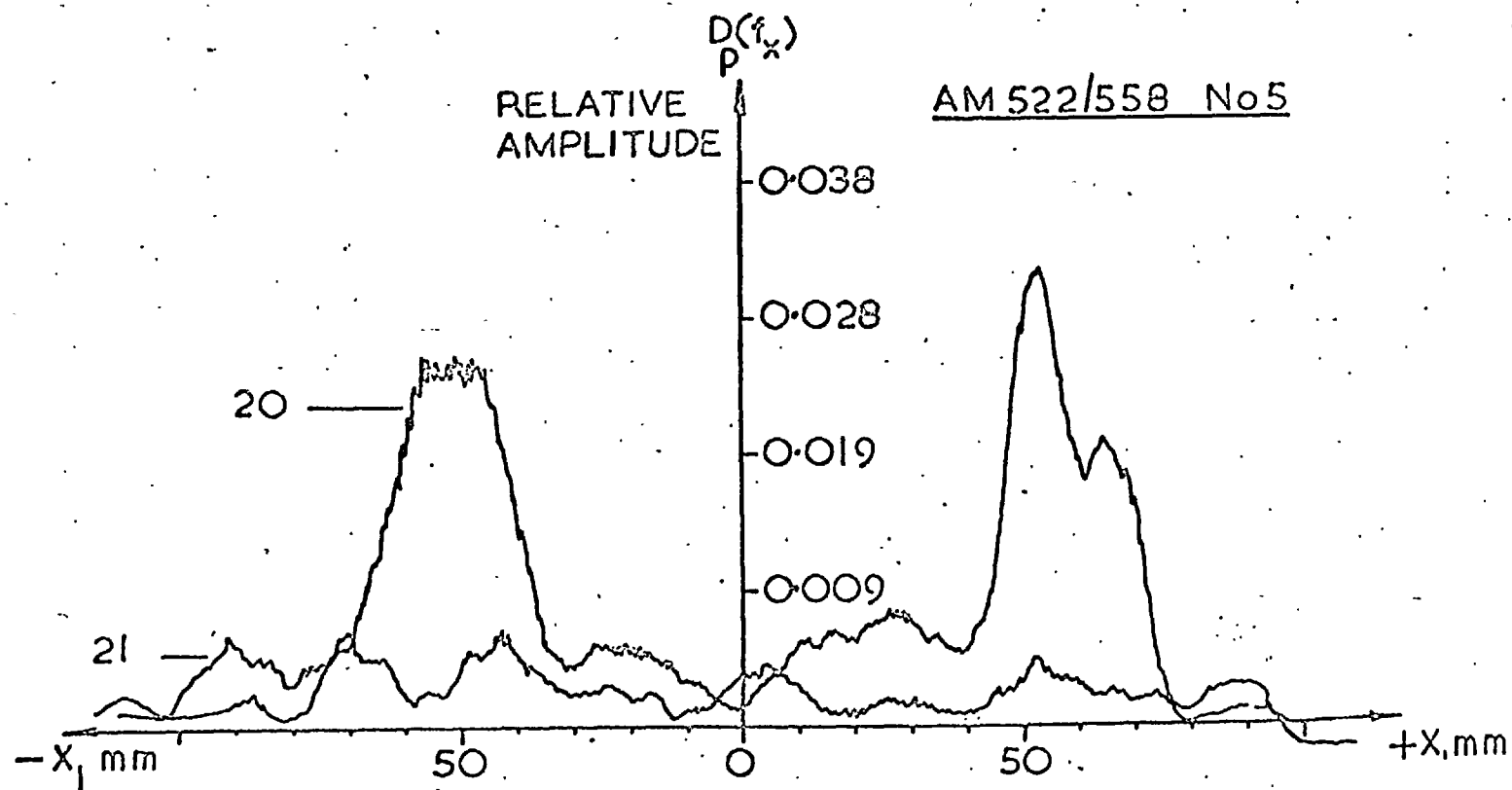


Figure 5.25 Detector directivity functions for set temporal frequencies using the impulse response technique at contact 1.

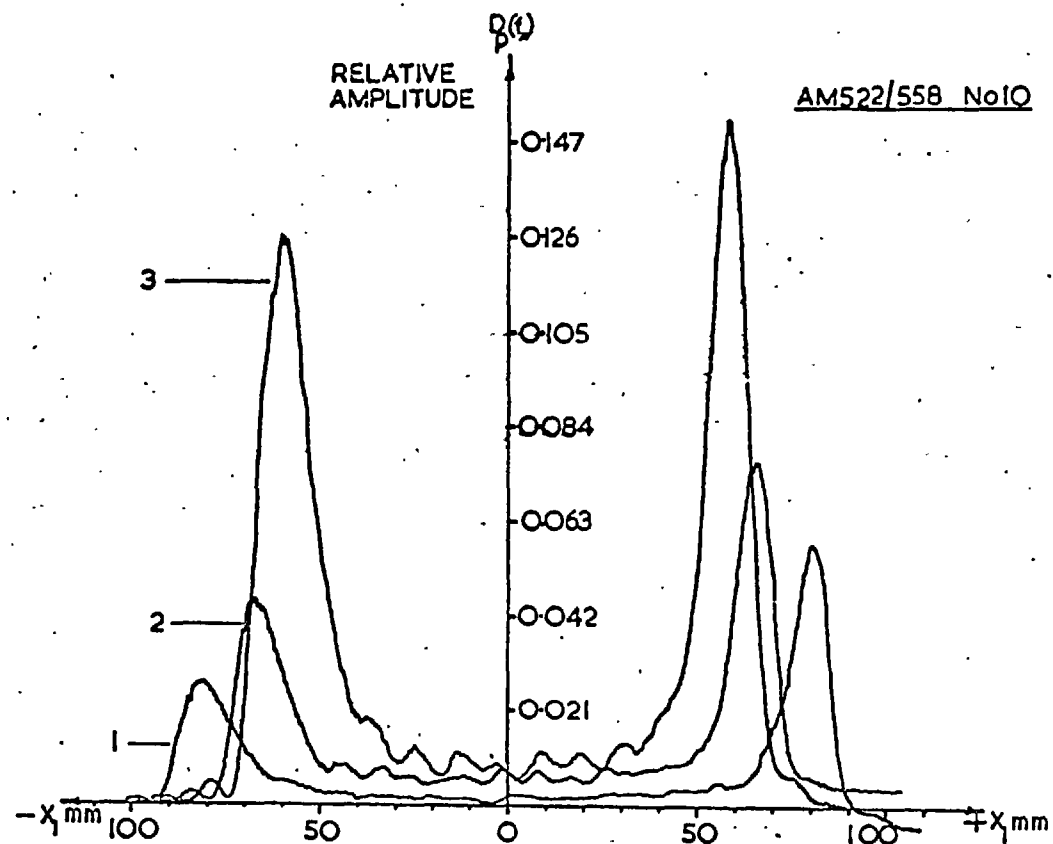


Figure 5.26 Detector directivity functions for set temporal frequencies using the impulse response technique at contact 1.

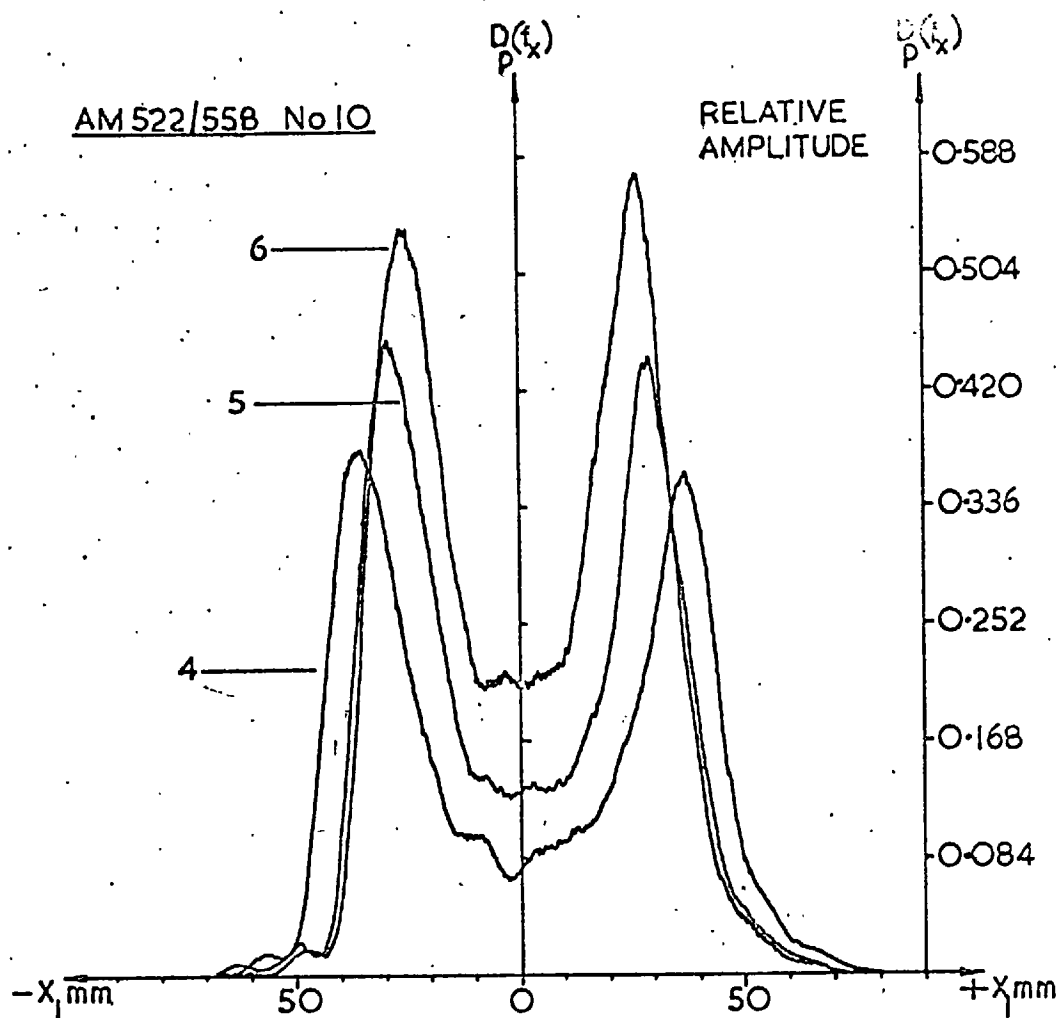


Figure 5.27 Detector directivity functions for set temporal frequencies using the impulse response technique at contact 1.

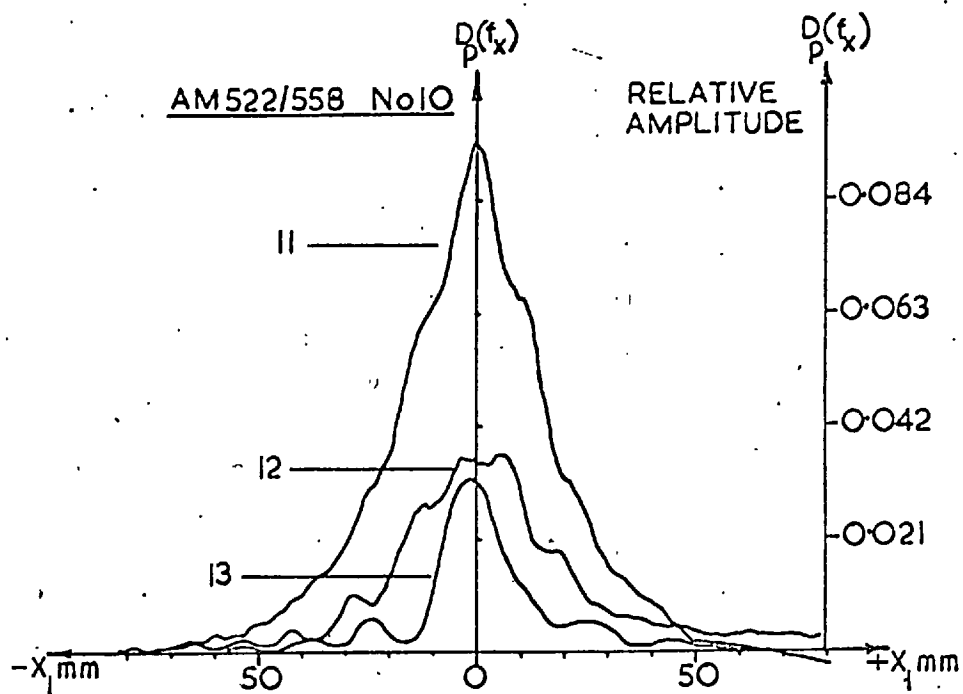
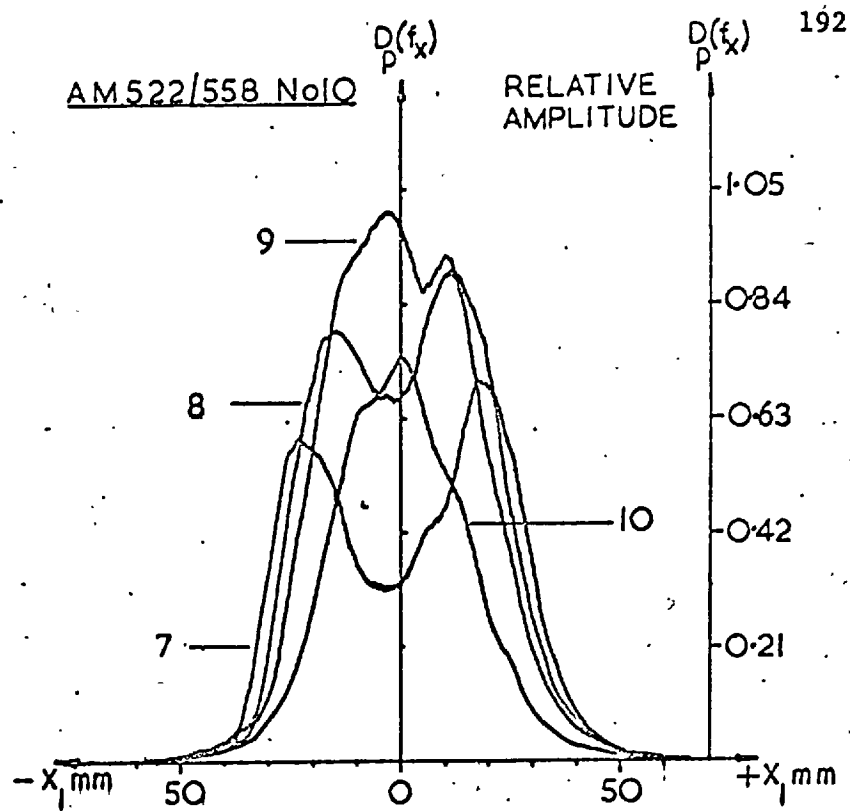


Figure 5.28 Detector directivity functions for set temporal frequencies using the impulse response technique at contact 1.

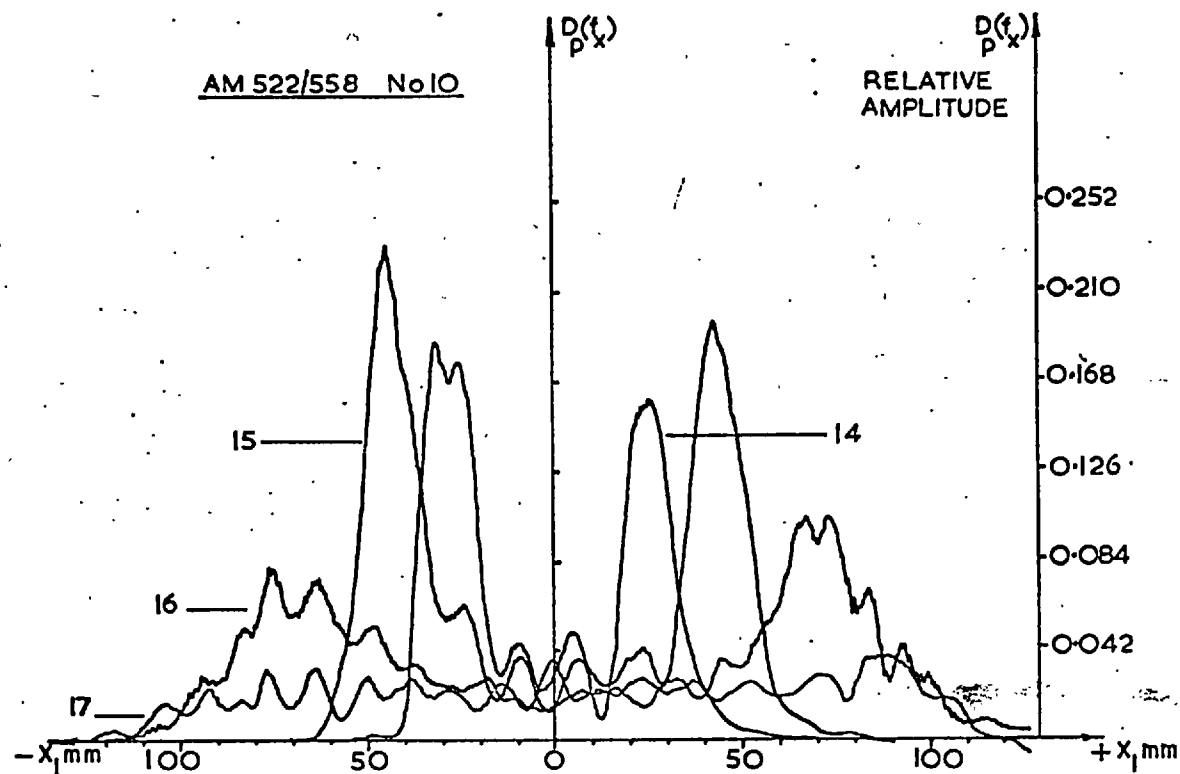


Figure 5.29 Detector directivity functions for set temporal frequencies using the impulse response technique at contact 1.

Table 5.1

Scale factors to convert from linear displacement x_1 to spatial frequency f_x for the results presented

Material	Figure No.	Curve No.	$f_x = C x_1$ $C \cdot 10^{-3}$	$\frac{\lambda_d}{2b}$	Temporal frequency MHz
P.Z.T.5HNo2	5.10	1	1.77	0.822	1.291
P.Z.T.5HNo2	5.10	2	1.66	0.868	1.222
P.Z.T.5HNo2	5.11	3	1.56	0.892	1.191
P.Z.T.5HNo2	5.12	4	1.51	0.920	1.154
P.Z.T.5HNo2	5.12	5	1.49	0.942	1.129
P.Z.T.5HNo2	5.12	6	1.55	0.958	1.110
P.Z.T.5HNo2	5.13	7	1.43	0.966	1.100
P.Z.T.5HNo2	5.13	8	1.52	0.984	1.080
P.Z.T.5HNo2	5.13	Max. 25mV	1.41	1.000	1.062res
P.Z.T.5HNo2	5.13	10	1.46	1.022	1.040
P.Z.T.5HNo2	5.14	11	1.33	1.104	0.965
P.Z.T.5HNo2	5.14	12	1.38	1.220	0.880
P.Z.T.5HNo2	5.15	1	4.52	0.318	3.350
P.Z.T.5HNo2	5.15	2	4.31	0.326	3.270
P.Z.T.5HNo2	5.15	3	4.27	0.329	3.230
P.Z.T.5HNo2	5.16	4	4.50	0.332	3.210
P.Z.T.5HNo2	5.16	Max. 14.6mV	4.20	0.333	3.193res
P.Z.T.5HNo2	5.16	6	4.15	0.338	3.155
P.Z.T.5HNo2	5.17	7	4.11	0.343	3.100
P.Z.T.5HNo2	5.17	8	4.04	0.351	3.030
AM522/558No5	5.18	1	3.66	0.802	2.900
AM522/558No5	5.18	2	3.86	0.831	2.800
AM522/558No5	5.19	3	3.80	0.841	2.766

Table 5.2

Scale factors to convert from linear displacement x_1 to spatial frequency f_x for the results presented.

Material	Figure No.	Curve No.	$f_x = \frac{C}{C \cdot 10^{-3}} x_1$	$\frac{\lambda_d}{2b}$	Temporal frequency MHz
AM522/558No5	5.19	4	3.58	0.887	2.620
AM522/558No5	5.20	5	3.30	0.912	2.550
AM522/558No5	5.20	6	3.32	0.925	2.516
AM522/558No5	5.21	7	3.19	0.942	2.473
AM522/558No5	5.21	8	3.06	0.965	2.413
AM522/558No5	5.21	9	3.16	0.975	2.383
AM522/558No5	5.22	10	3.37	0.982	2.372
AM522/558No5	5.22	11	3.07	0.991	2.350
AM522/558No5	5.22	Max. 54mV	3.24	1.000	2.327res
AM522/558No5	5.22	13	3.12	1.012	2.300
AM522/558No5	5.23	14	2.96	1.015	2.290
AM522/558No5	5.23	15	2.96	1.035	2.250
AM522/558No5	5.24	16	2.75	1.130	2.060
AM522/558No5	5.24	17	2.80	1.135	2.045
AM522/558No5	5.24	18	2.76	1.165	2.000
AM522/558No5	5.24	19	2.63	1.145	2.030
AM522/558No5	5.25	20	2.72	1.195	1.950
AM522/558No5	5.25	21	2.50	1.225	1.900

Table 5.3

Scale factors to convert from linear displacement x_1 to spatial frequency f_x for the results presented.

Material	Figure No.	Curve No.	$f_x = Cx_1$ $C \cdot 10^{-3}$	$\frac{\lambda_d}{2b}$	Temporal frequency MHz
AM522/558No10	5.26	1	4.98	0.872	3.900
AM522/558No10	5.26	2	5.10	0.895	3.800
AM522/558No10	5.26	3	4.67	0.919	3.700
AM522/558No10	5.27	4	4.38	0.962	3.530
AM522/558No10	5.27	5	4.82	0.968	3.510
AM522/558No10	5.27	6	4.58	0.973	3.490
AM522/558No10	5.28	7	4.76	0.980	3.470
AM522/558No10	5.28	8	4.76	0.988	3.440
AM522/558No10	5.28	Max. 24.5mV	4.64	1.000	3.400res
AM522/558No10	5.28	10	4.60	1.009	3.370
AM522/558No10	5.28	11	4.25	1.036	3.280
AM522/558No10	5.28	12	4.28	1.086	3.130
AM522/558No10	5.28	13	3.94	1.100	3.080
AM522/558No10	5.29	14	3.85	1.138	2.985
AM522/558No10	5.29	15	3.85	1.160	2.930
AM522/558No10	5.29	16	4.06	1.172	2.900
AM522/558No10	5.29	17	3.64	1.215	2.800

for a range of ultrasound frequencies through thickness resonance. Some of the results obtained with P.Z.T. 5H No. 2 are shown in Figs. 5.10 to 5.17 and with AM 522/558, Nos. 5 and 10, in Figs. 5.18 to 5.29. By mentally positioning the $D_p(f_x)$ results along the temporal frequency axis of the appropriate far field pressure distribution, it is easy to visualize the change in profile of $D_p(f_x)$ as the temporal frequency increases through thickness resonance.

Several points connected with the display of these results should be noted. $D_p(f_x)$ has been presented as a function of the far field spatial variable x_1 instead of f_x . If $D_p(f_x)$ were displayed as a function of f_x a separate abscissa would be necessary for each curve resulting in a three-fold increase in the number of figures. However, each set of results for a given disc was recorded in practice with the same x_1 scale so it was decided that a useful saving in space and an easy comparison would result from displaying $D_p(f_x)$ against x_1 . Tables of conversion factors are supplied so that a spatial frequency corresponding to a value of x_1 can be obtained by a simple multiplication (Table 5.1 for P.Z.T. 5H No. 2; Table 5.2 for AM 522/558 No. 5; Table 5.3 for AM 522/558 No. 10). The amplitude of $D_p(f_x)$ is normalised with respect to $D_p(f_x = 0)$ at the temporal frequency of thickness resonance. The resulting scale of relative amplitudes is shown linearly but the true output from the scanning hydrophone is a non-linear function of the input amplitude in common with that from all diode detector circuits and must be linearized using the appropriate calibration curve of Appendix 3. On certain of the results at low signal levels, for example those shown in Figs. 5.10 and 5.11, the right hand side does not apparently have the same ordinate zero as the left hand side. This is a consequence of

magnetic pick-up from the D.C. drive motor which was situated on the right hand side of the tank, in plan view. Traces obtained with the probe scanning towards the motor show a positive displacement of zero near to the motor and a negative displacement on reversal of the field coil current. The noise recorded on some of the traces, particularly Figs. 5.10 to 5.17, results from standing waves within the tank. These are mainly present because of reflections from the water surface and the tank bottom.

The symmetrical nature of the driving point and the rotational symmetry of lead zirconate titanate ceramics about the Z axis requires that $D_p(f_x)$ should be an even function of x_1 , at all temporal frequencies, having an axis of symmetry through $f_x = 0$. (See equations 2.39b and 2.97). A departure from symmetry will result if the phase of the impulse response is not an even function of x_1 . This could be caused by local inhomogeneities in the region of the driving point.

The complete series of experimental results is characterised by two peaks, symmetrically placed with respect to the origin, which are widely separated above thickness resonance. As the temporal frequency tends to thickness resonance, the peaks approach each other. At thickness resonance they merge to form a single peak with a maximum at $f_x = 0$, the amplitude of which tends to zero as the temporal frequency is further decreased. The position shape and behaviour with temporal frequency of these two peaks agrees closely with that predicted by equation (2.90) and hence must result from the thickness component of strain in the piezoelectric ceramic plate.

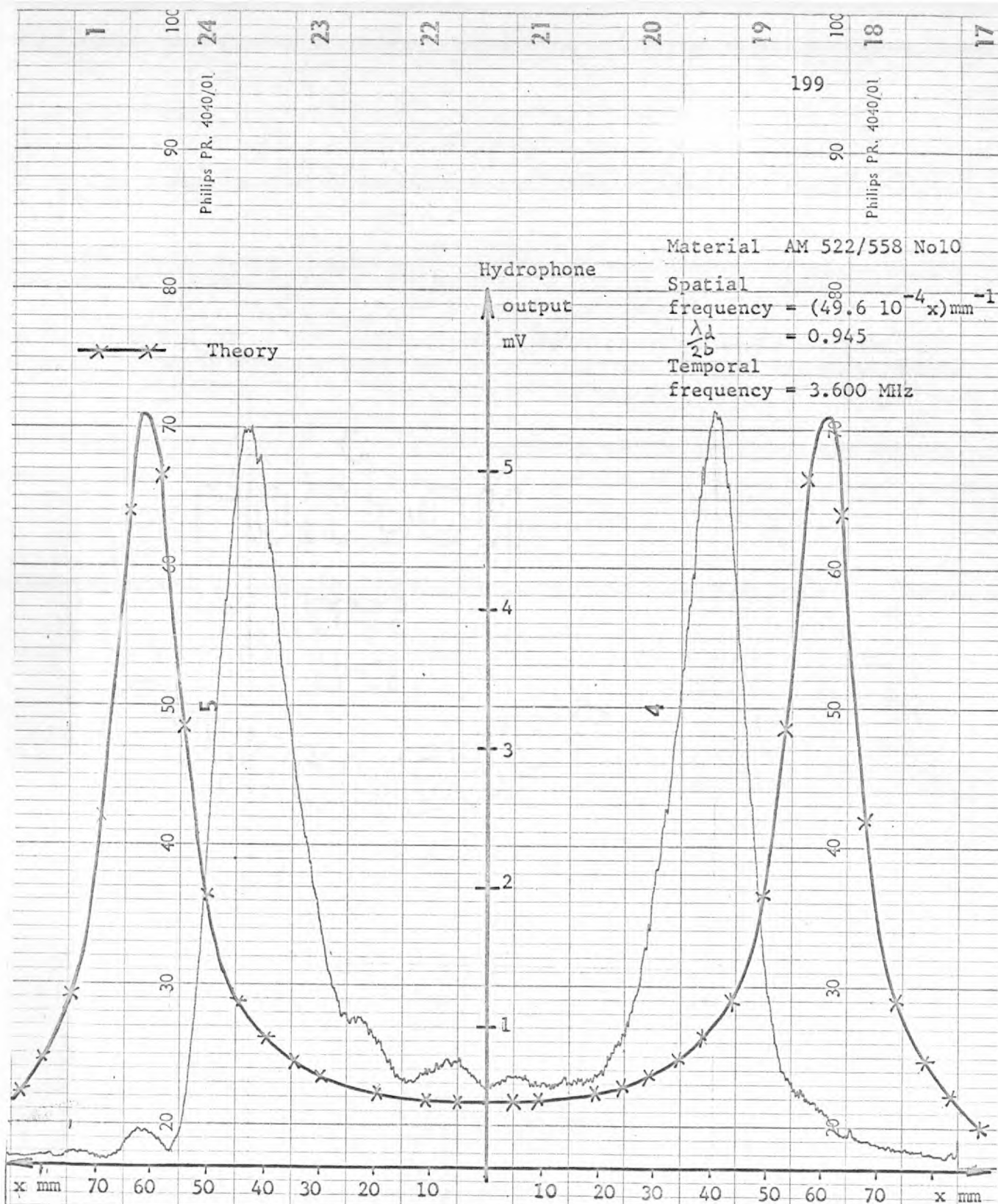


Figure 5.30 Comparison of experimental and theoretical directivity functions

On Figs. 5.31 to 5.35 the positions of these thickness component resonance peaks are indicated with a cross and the corresponding peak predicted by (2.90) with a circle for all the materials investigated.

The theoretical points lie on a curve given by $f_x^2 = \frac{f_d^2}{c_x^2} - \frac{1}{4b^2}$

for results obtained near the fundamental thickness, or dilatational wave resonance, and on a curve given by

$$f_x^2 = \frac{f_d^2}{c_d^2} - \frac{9}{4b^2}$$

near the first overtone of the dilatational wave resonance. These results are simply derived from (2.88a) where f_d is the temporal frequency of the incident ultrasound wave and c_d is the dilatational wave velocity (along the Z axis) in the piezoelectric ceramic material.

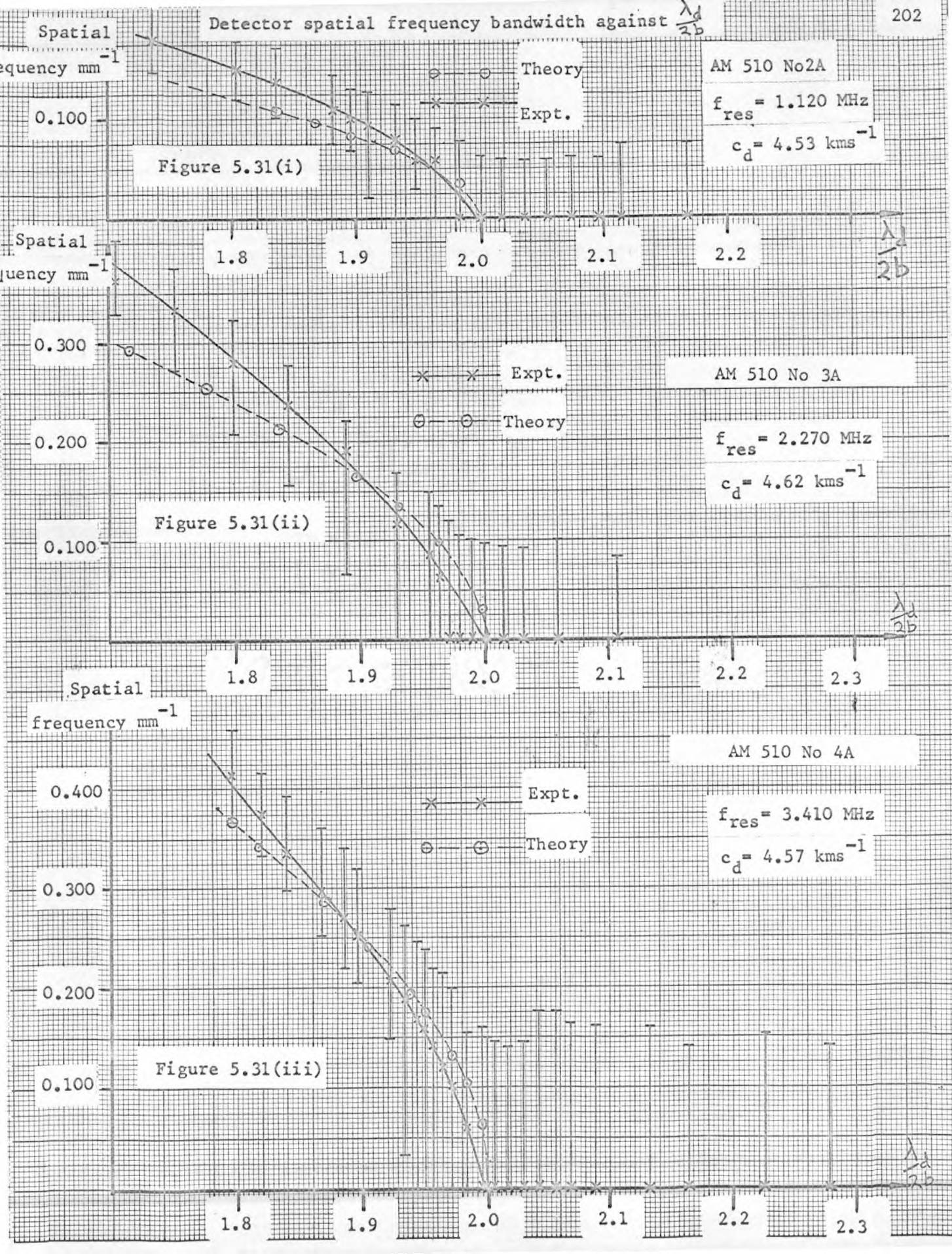
Fig. 5.30 shows the curve predicted by equation (2.90) for AM 522/558 No. 10 at 3.600 MHz compared with the experimental result. The amplitude of the theoretical curve was scaled to equal that of the experimental result at its peak value. The discrepancy between the spatial frequency of the theoretical and experimental curves results from a decrease in the thickness component of longitudinal phase velocity at the temporal frequency of observation relative to that at thickness resonance.

The theoretical curve was calculated using a value of phase velocity calculated from the thickness resonant frequency. This

velocity dispersion results from the combined effect of the finite dimensions of the disc and the anisotropic nature of lead zirconate titanate. Considering that the theory was developed for a fluid plate of infinite lateral extent, the agreement is surprisingly good.

Equation (2.90) does not indicate the presence of the peaks observed between the thickness component resonances. These other peaks are also shown as satellite peaks on the far field ultrasound pressure swept frequency responses discussed previously and can therefore be explained in the same manner. There is a progressive decrease in amplitude and number of these intermediate peaks as the diameter to thickness of the samples increases. Many peaks appear on the P.Z.T. 5H No. 2 results where the diameter to thickness ratio is 25. The intermediate peaks on the AM 522/558 No. 5 results (diameter to thickness ratio 50) are, however, noticeably smaller and they are almost non-existent on the AM 522/558 No. 10 results (diameter to thickness ratio 75).

Certain of the directivity functions which were observed had an entirely unexpected shape. The series of Figs. 5.18 to 5.23 show the usual progression of the thickness component resonances down to curve 15 at 2.250 MHz. However, at 2.060 MHz the directivity function becomes very narrow and its amplitude increases as the temporal frequency is decreased until a sharp maximum is reached at 2.045 MHz as shown on Fig. 5.24 curve 17. On further decreasing the temporal frequency as curves 18 and 19 show, peaks appear which gradually separate and diminish until at 1.900 MHz a broad, almost flat, directivity function is observed. It seems most likely that this behaviour results from a strong thickness shear mode at 2.045 MHz which produces sufficient displacement in the Z direction to enable the whole disc to radiate in a piston mode. At a



Spatial
frequency mm^{-1}

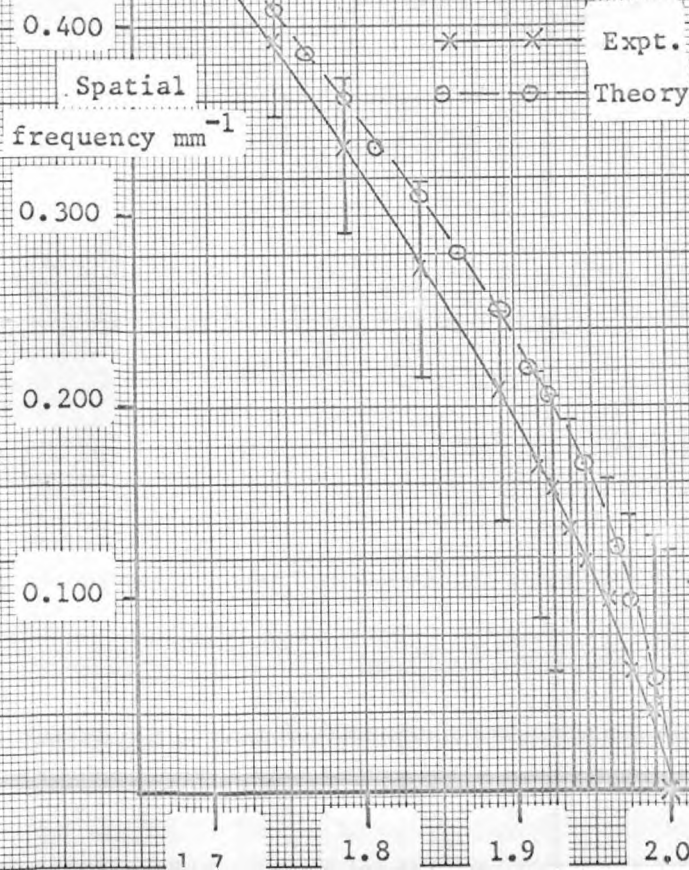


AM 522/558 No5

$f_{\text{res}} = 2.3270 \text{ MHz}$

$c_d = 4.65 \text{ kms}^{-1}$

Figure 5.32(i)

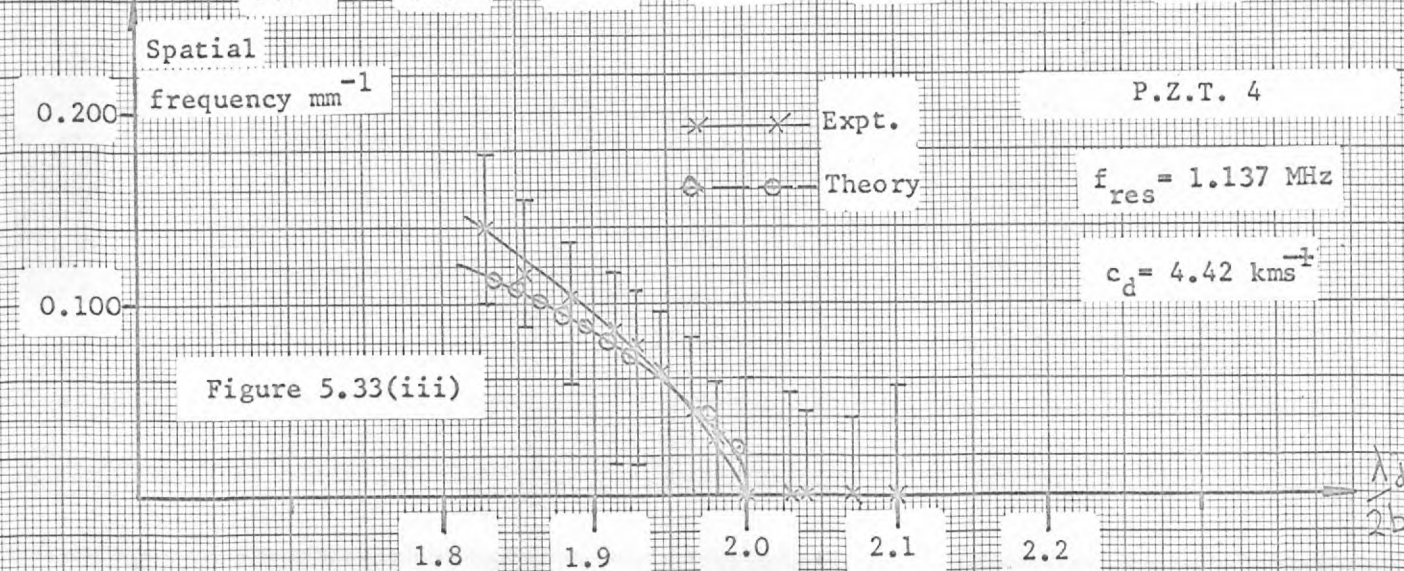
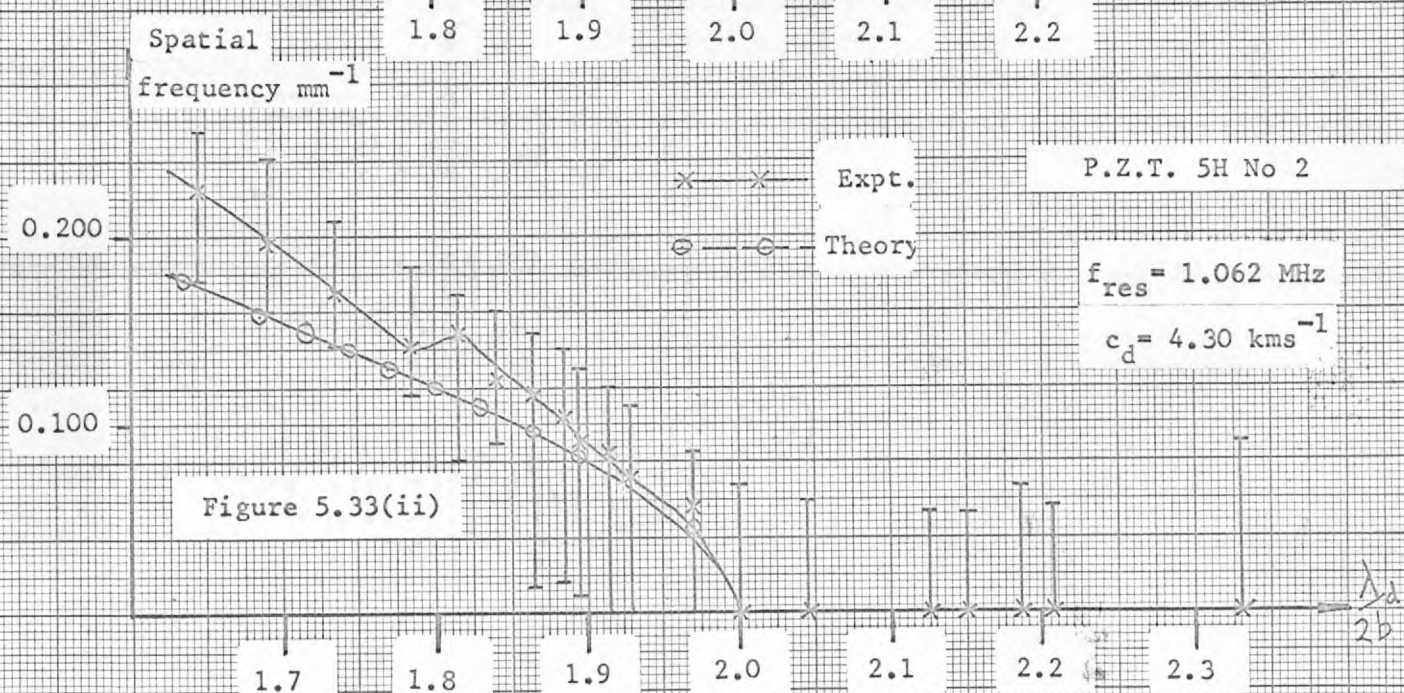
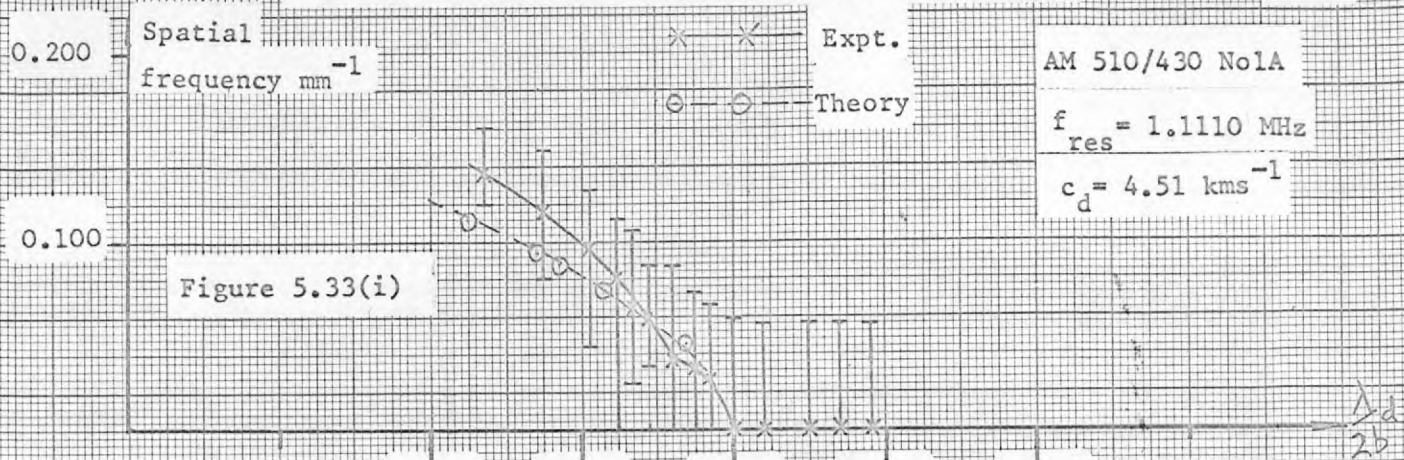


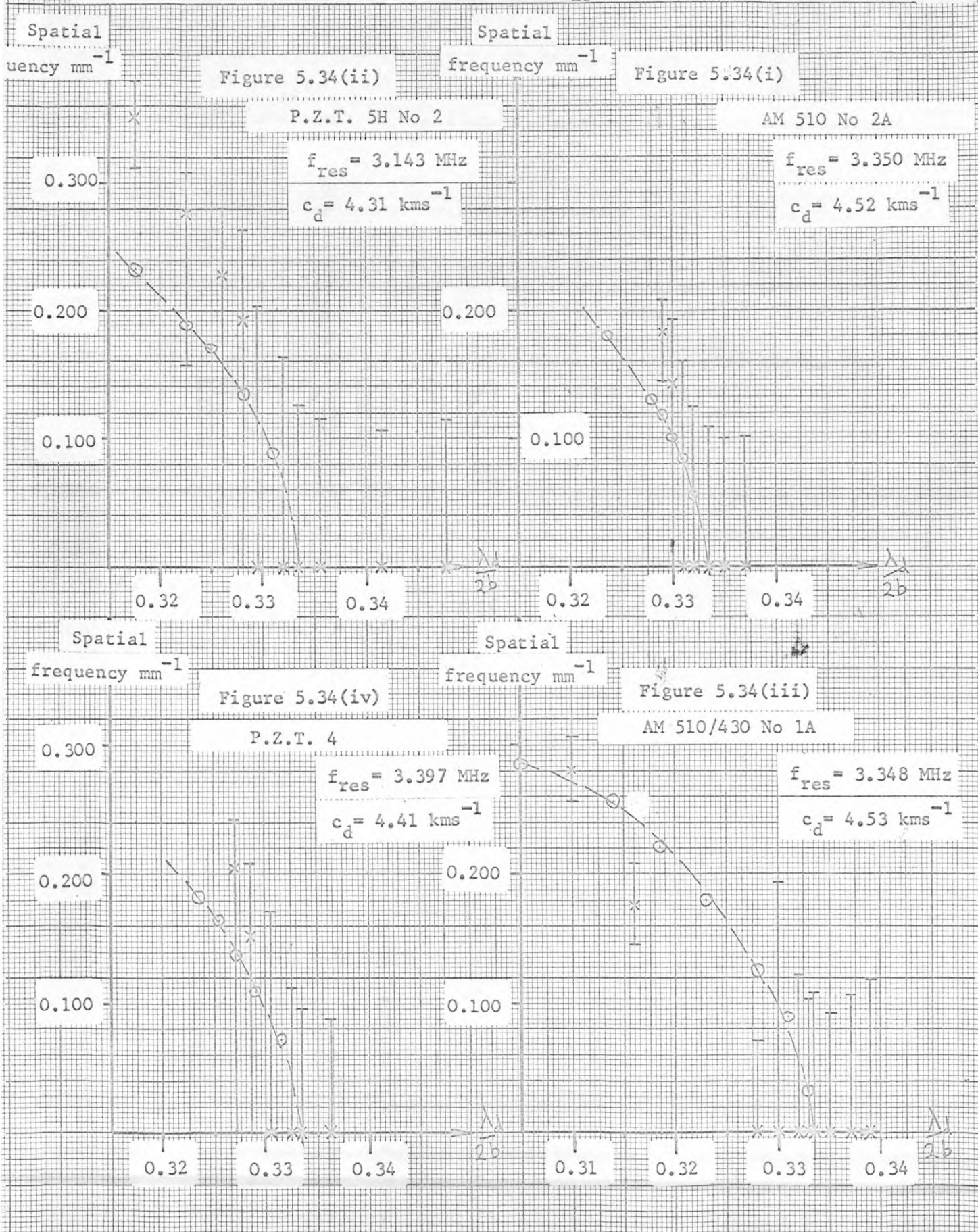
AM 522/558 No 10

$f_{\text{res}} = 3.410 \text{ MHz}$

$c_d = 4.64 \text{ kms}^{-1}$

Figure 5.32(ii)





temporal frequency below this shear resonance it seems to be possible to find a broad directivity pattern as shown by curve 21, Fig. 5.25, at which maximum possible detector resolution can be achieved.

A similar type of behaviour is shown in Fig. 5.29 for AM 522/558 No. 10, where maximum resolution would be achieved at 2.800 MHz with directivity function number 17.

As well as indicating the agreement between the fluid plate theory and the experimental observations, Figs. 5.31 to 5.34 provide the basic quantitative information from which the detector resolution can be obtained, as a function of temporal frequency, for all the materials investigated. Taking $\frac{\lambda_d}{2b}$ as the abscissa

enables results obtained from samples with a range of thickness to be displayed on the same axes and hence makes direct comparison possible. Thickness resonance occurs when $\frac{\lambda_d}{2b} = 1$. Below resonance

$\frac{\lambda_d}{2b}$ is greater than unity and above thickness resonance it is less

than unity. Consequently selection of the temporal frequency at which a chosen detector material has the greatest resolution is straightforward. Alternatively the material having the greatest resolution at a particular temporal frequency can easily be chosen. The information displayed in Figs. 5.31 to 5.34 was extracted from the directivity functions by measuring the spatial frequency bandwidth between the half amplitude points. As most of the curves were symmetrical about $f_x = 0$ only half the total bandwidth has been shown so it must be remembered that a mirror image in the $\frac{\lambda_d}{2b}$ axis also exists when assessing the detector resolution. The

length of the vertical line at a given value of $\frac{\lambda_d}{2b}$ is equal to the

spatial frequency bandwidth. When the directivity function exhibits the two peaked form above thickness resonance the pass band about the spatial frequency of the longitudinal thickness resonance peak is displayed. This results in a systematic displacement of the spatial frequency pass band up the spatial frequency axis as $\frac{\lambda_d}{2b}$

increases above unity and produces the characteristic trend towards the top left hand corner of the page. The spatial frequency values of the thickness resonance component peaks are indicated by crosses; kinks in the smooth upward trend are accompanied by strong on axis peaks in the directivity function. These are therefore probably associated with closely coupled modes of the disc which may exist at that particular temporal frequency. This is shown clearly by Fig. 5.33 (ii) for P.Z.T. 5H No. 2.

The significance of the band pass and low pass spatial frequency responses of the detectors is readily ascertained. The broadest low pass spatial frequency response will result in a final image having the closest resemblance to the original object. However, the image which is obtained from a detector having a pass band about a high spatial frequency will tend to emphasise the fine structure in the object and in particular its edges.

It is, therefore, possible, knowing the requirements of a visualization system, to select the most appropriate detector material. For example, P.Z.T. 5H has a broader pass band for $\frac{\lambda_d}{2b} = 0.932$ than any of the other materials investigated with a

diameter to thickness ratio of 25. It also has a slightly broader

bandwidth at $\frac{\lambda_d}{2b} = 1.00$. There is insufficient elastic data

available for the AML materials to draw firm conclusions about the reason for this, but complete physical data is available for P.Z.T 4 which suggests that the fine grain size of P.Z.T 5H is the important factor.

The set of results obtained using AM 510 covers the three diameter to thickness ratios investigated in the present work. Fig. 5.31 (i) shows results obtained with a disc of diameter to thickness ratio 25 having a fundamental thickness resonance at 1.120 MHz. Figs. 5.31 (ii) and 5.31 (iii) were obtained with diameter to thickness ratios of 50 and 75 having fundamental thickness resonances at 2.277 MHz and 3.410 MHz respectively. As expected from the change in temporal frequency, the spatial frequency bandwidth at $\frac{\lambda_d}{2b} = 1.00$ for AM 510 No. 4A is approximately

three times that for AM 510 No. 2A and that for AM 510 No. 3A approximately twice that for AM 510 No. 2A.

Comparing Fig. 5.31(i) with Fig. 5.34 (i), it is clear that operating a detector at its first overtone does not increase the bandwidth by a factor of three as one would expect on temporal frequency grounds. Also the first overtone bandwidth is less than that obtained from a disc with a fundamental thickness frequency close to that of the first overtone as is shown by comparing Fig. 5.31(iii) with Fig. 5.34(i). This indicates that the impulse response of the thicker disc has a greater width in wavelengths than that of a thinner disc at the same wavelength, as one would expect intuitively. The first overtone results shown in Fig. 5.34 are in poor agreement with the theoretical points. The directivity

functions themselves, shown in Figs. 5.15 to 5.17 for P.Z.T. 5H No.2 have a much smoother appearance than the corresponding curves obtained near the fundamental. The thickness component resonances are relatively inconspicuous because of the broadening of the impulse response.

Figs. 5.32 (i) and (ii) for AM 522/558 No. 5 and No. 10 show a similar trend to the results obtained with AM 510 in Figs. 5.31(ii) and (iii) but the corresponding bandwidths are slightly smaller. However, for $\frac{\lambda_d}{2b}$ approximately equal to 1.22 both AM 522/558 No. 5 and

No. 10 show a very broad low pass directivity function.

A detector using AM 522/558 working at this value of $\frac{\lambda_d}{2b}$ would give very much superior image quality than could be obtained with any of the other materials investigated.

5.3 Investigations of the space variance of the directivity function of piezoelectric detectors

The results discussed in 5.2 were all obtained using contact point 1 (see Fig. 3.17). The space variance of the directivity function can in principle be investigated by comparing the results of method 4.2 obtained separately at all nine contact points. To carry out such an investigation on every sample at every temporal frequency would obviously have multiplied the number of results by a factor of nine. To reduce all this information to a form suitable for display would have taken a prohibitively long time. Consequently a set of directivity functions were recorded at

3.510 MHz at all nine contact points using AM 522/558 No. 10. These are displayed in Fig. 5.35. On other materials a record was made of the thickness resonant frequency at all nine contact points. As the change in thickness from point to point over the disc is less than 1% a change in thickness resonance from point to point by more than this indicates the variation of c_d over the area of the disc and hence is a measure of the homogeneity of the material. The change in resonant frequency observed over a disc was seldom more than 3%, so although some inhomogeneity exists, it is not very large. The shape of the directivity function is a sensitive function of temporal frequency, but as can be shown from Fig. 5.35, the spatial frequency bandwidth remains appreciably constant over the area of the disc. The temporal frequency chosen for the results of Fig. 5.35, if anything, exaggerates the space variance of the directivity function for if a temporal frequency below the lowest resonant frequency of any of the nine contact points had been chosen, then all the directivity functions would have been of the low pass variety and hence very similar in bandwidth. (In Figs. 5.31 to 5.34 for $\frac{\lambda_d}{2b} > 1$ the spatial frequency bandwidth is almost constant

as the frequency decreases). It can be concluded from this that if inhomogeneities influence the average performance of the entire disc, then they will have the least disturbing effect on the quality of the visualized image if the detector is operated below the minimum thickness resonance frequency. This may account for the usual statement that the ultrasound camera produces the best images at temporal frequencies just below the thickness resonant frequency.

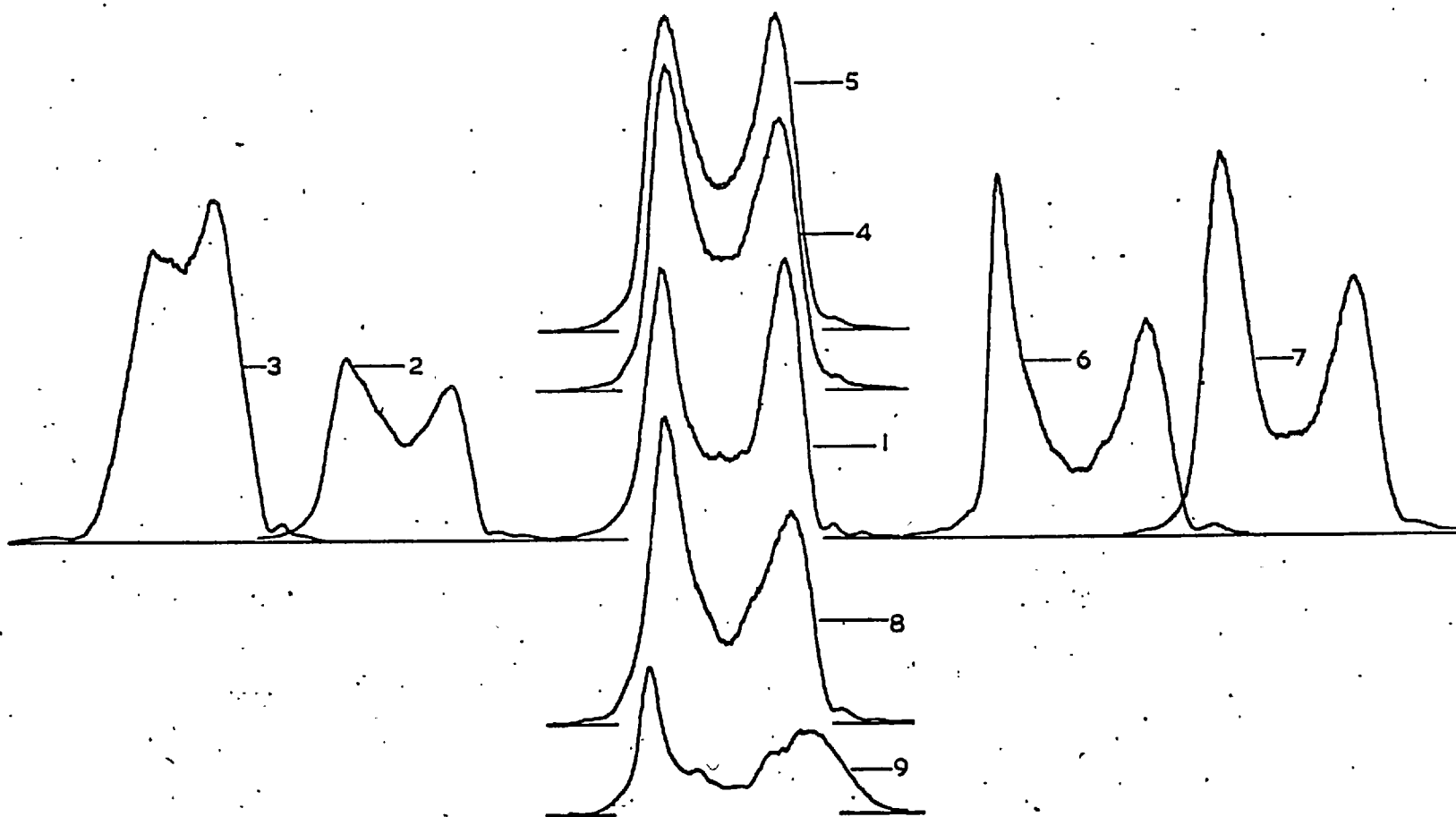


Figure 5.35 Directivity functions at all nine contacts to illustrate space variance.

Material AM 522/558 No 10. Temporal frequency 3.5100 MHz

5.4 Results obtained with the sinusoidal standing wave method for measuring the coherent transfer function of piezoelectric detectors

By comparing the amplitudes of the input and output cosinusoidal standing waves, the directivity function $D_p(f_x)$ is obtained point by point as described in section 2.8. The modulation transfer function $M_p(f_x)$ is obtained from the relative depth of modulation of the output and input standing waves. The coherent transfer function is then defined as the product of $M_p(f_x)$ and $D_p(f_x)$.

In order to obtain a coherent transfer function by this technique, pairs of input and output functions must be recorded over a range of spatial frequencies and for a known depth of modulation of the input standing wave. For convenience this was set equal to unity (100% modulation). Up to twenty pairs of results must be recorded at each temporal frequency at which a coherent transfer function is required. Consequently the technique is not suitable for the investigation of $D_p(f_x)$ as a function of temporal frequency as many hours would be required to record a set of directivity functions such as those shown in Figs. 5.18 to 5.25.

There are, however, two important points concerning the sinusoidal standing wave technique which must be considered. Firstly, it is the only method by which the modulation transfer function can be obtained. Hence it enables the 'cut-off' spatial frequency, f_0 , of the detector material to be measured. The significance of f_0 will be discussed in a later section. Secondly, it provides an alternative independent method of obtaining $D_p(f_x)$

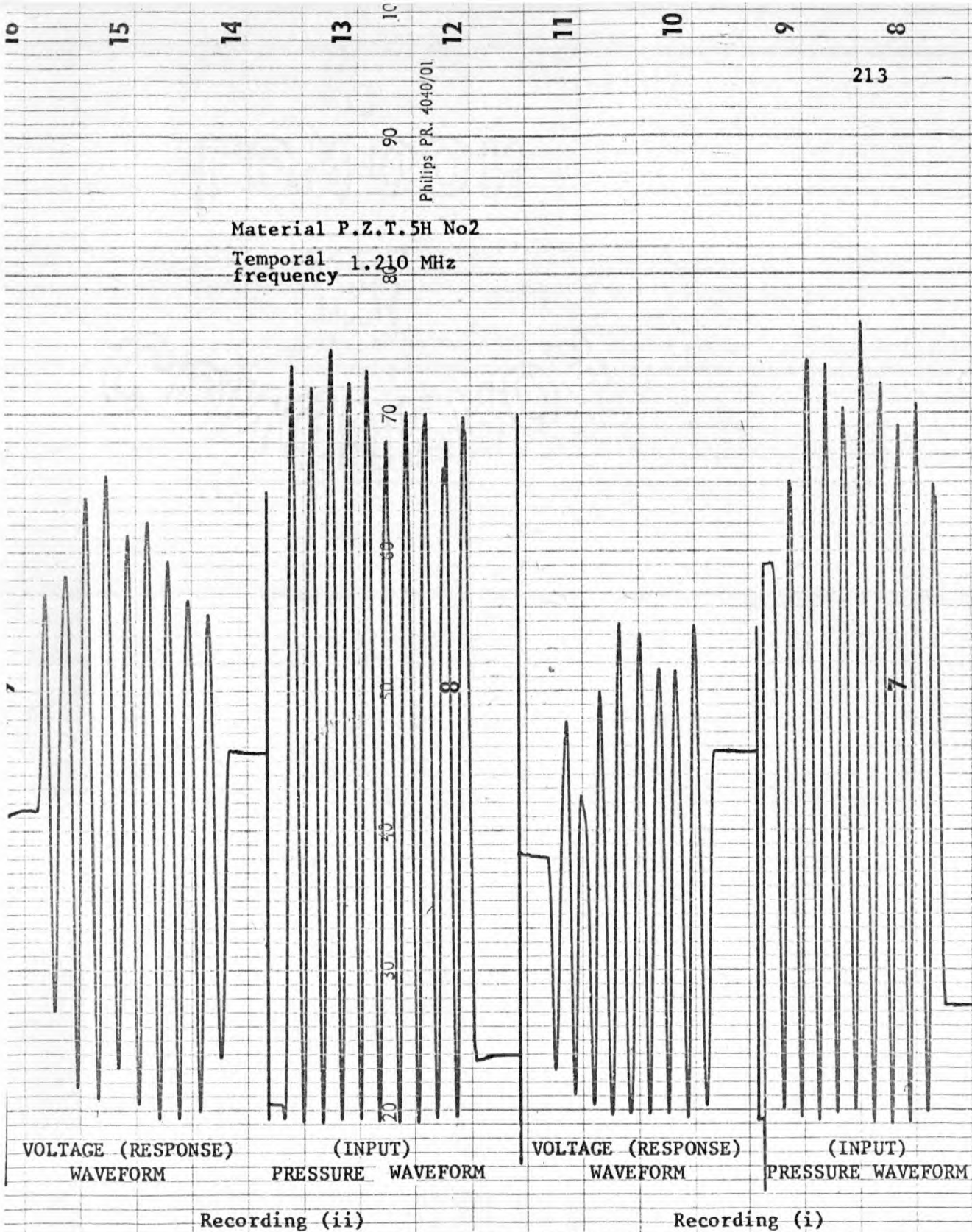


Figure 5.36 Typical examples of two pairs of traces recorded consecutively using the sinusoidal standing wave technique

which is probably easier to justify physically and hence is a useful check on the validity of results obtained by the response technique.

5.4.1 Directivity functions from the sinusoidal standing wave technique and comparison with the impulse response technique

Figs. 5.36(i) and 5.36(ii) are two typical pairs of results recorded consecutively by the sinusoidal standing wave technique. They were obtained during investigations on P.Z.T. 5H No. 2 at a temporal frequency of 1.210 MHz. The depth of modulation of both input pressure waveform and output voltage waveform is approximately one, but there is a significant increase in the amplitude of the voltage waveform in (ii) compared with that of (i) for a constant pressure waveform amplitude. The amplitude of the output voltage waveform relative to that of the input pressure waveform, when plotted against the spatial frequency of the input waveform traces out the spatial gain function or directivity function of the detector transducer. These results are in good agreement with those obtained by the impulse response technique at the same temporal frequency and for the same material as Figs. 5.37 and 5.38 show. Fig. 5.37 illustrates the agreement between the two methods using P.Z.T 5H No. 2 at 1.210 MHz and Fig. 5.38 using AM 522/558 No. 5 at 2.516 MHz.

It is very satisfying that the results obtained by two independent techniques are in such close agreement as this vindicates both the experimental techniques and their underlying physical principles.

I Sinusoidal standing wave results

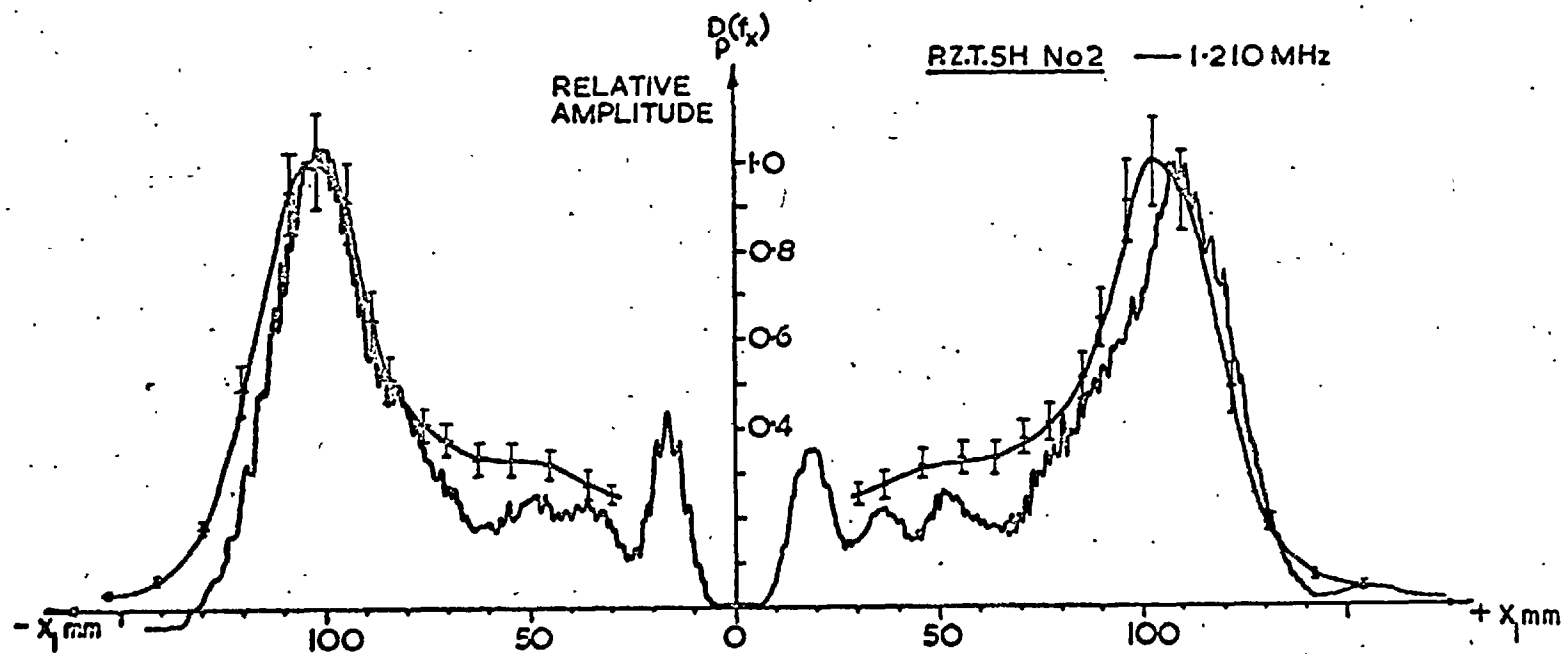


Figure 5.37 Comparison of directivity functions obtained with the impulse response Technique and the sinusoidal standing wave technique

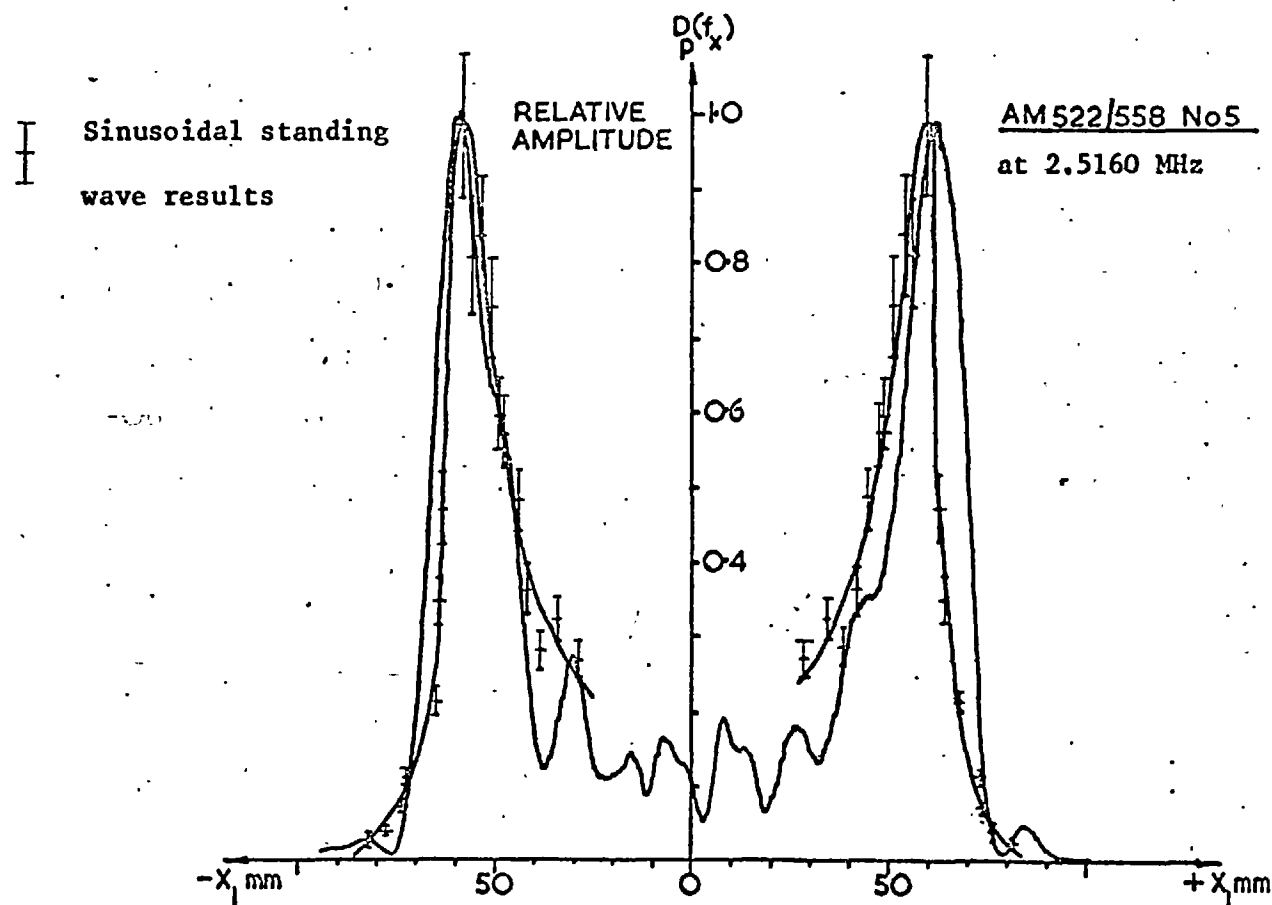


Figure 5.38 Comparison of directivity functions obtained with the impulse response technique and the sinusoidal standing wave technique

The sinusoidal standing wave technique results were scaled to fit those of the impulse response method at the maximum amplitude of $D_p(f_x)$. The difference in levels between the two sets of results shown, particularly on Fig. 5.37 near $x_1 = 50$ mm, is probably a consequence of a difference in the non-linear response of the pressure waveform recording system and the voltage waveform recording system. There is a slight displacement of the sinusoidal standing wave results in the negative x_1 direction relative to the impulse response curve. It is due to a decrease in the unwinding speed of the Phillips pen recorder on which the results were recorded as the length of record increased. This undesirable characteristic of the recorder was not noticed until a large proportion of the results had been obtained. Results could not be obtained with the standing wave technique at values of x_1 less than 20 mm because the size of the source holders prevented them from being brought sufficiently close together at the maximum value of Z_1 which was determined by the size of the tank.

A significant feature of the sinusoidal standing wave results is that they do not follow the small amplitude oscillations between the two thickness component resonance peaks shown on the impulse response results. This is probably a consequence of the fact that the relative amplitude value obtained from the sinusoidal standing wave technique is an average of the whole disc response. If the small amplitude peaks are a result of local divergence of electric field in the vicinity of the driving point, they could reasonably be expected to average out over the area of the disc. A certain amount of smoothing will also result from the finite size of the sources used to form the standing ultrasound pressure field.

5.4.2 Modulation transfer functions from the sinusoidal standing wave technique

The use of the sinusoidal standing wave technique to obtain the modulation transfer function for a ceramic detector is a most significant development. Not only does it enable the coherent transfer function to be obtained, but it also indicates the maximum spatial frequency that the detector can pass and hence the natural limit to its resolution capability.

Figs. 5.39 (i) and 5.39 (ii) are two typical pairs of results obtained by the sinusoidal standing wave technique recorded at a temporal frequency of 2.516 MHz. using AM 522/558 No. 5. On Fig. 5.39 (i) the depth of modulation of both the input pressure waveform and the output voltage waveform is unity giving a value of unity to the amplitude of the modulation transfer function. On Fig. 5.39 (ii), however, the input waveform has a depth of modulation of unity, but the response waveform has a depth of modulation of 0.55, giving a value of 0.55 to the amplitude of the modulation transfer function. The complete modulation transfer function is shown in Fig. 5.40 (i).

Close inspection of Fig. 5.39 (ii) reveals that there are two more cycles in the pressure waveform than in the corresponding voltage waveform, when in each case the length of scan was the same. Two complete cycles have, therefore, been absorbed during the transduction from pressure to voltage. In a standing wave field the phase of one loop is opposite to that of the adjacent loop. However, in the ceramic material each loop is coupled with the adjacent loops through the rigidity of the material. Consequently they cannot vibrate independently and intuitively one can say that a stage must

Figure 5.39(i) Low spatial frequency

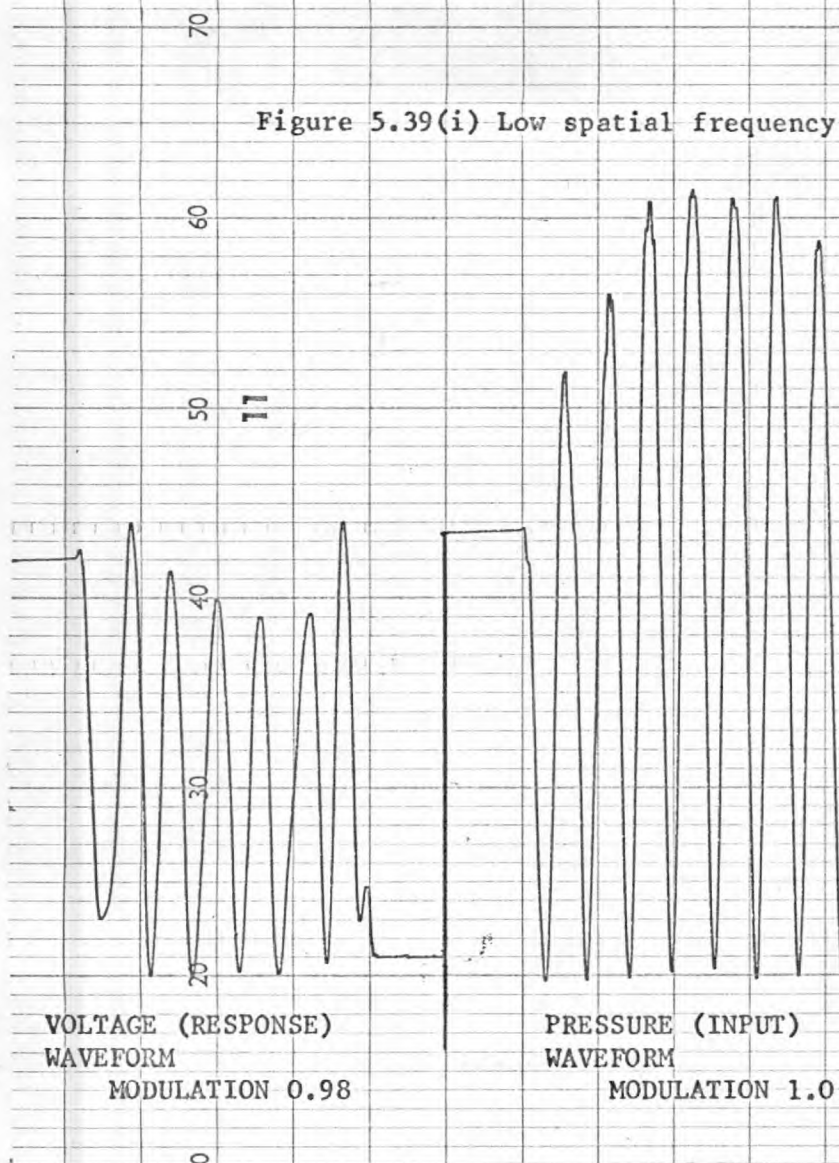


Figure 5.39(ii) High spatial frequency

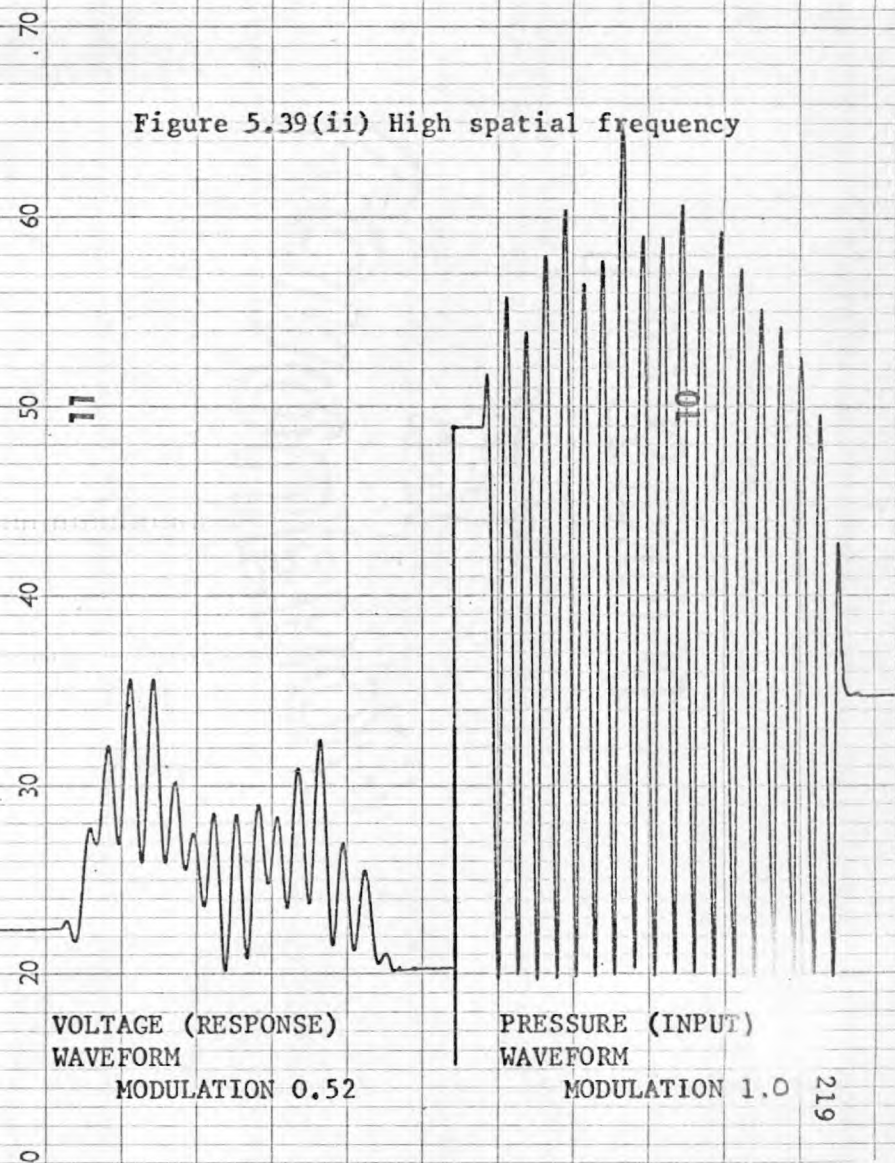


Figure 5.39 Illustrative examples of the use of the sinusoidal standing wave technique to obtain the modulation transfer function. Material AM522/558 No5 at 2.5160 MHz

be reached when the spatial period of the standing wave will be so small that adjacent loops will be forced to vibrate in phase. This will occur when the spatial period of the standing wave pattern is of the order of the disc thickness corresponding to a spatial frequency of $1/2 b$ and the resulting spatial frequency of the voltage waveform will be zero. The spatial frequency of the input waveform could then be sensibly called the cut off spatial frequency f_o .

This behaviour is not observed in practice. A clearly defined cut off frequency does not exist because of mode conversion as mentioned in 2.7, but the response spatial frequency gradually decreases relative to the input spatial frequency until there is no similarity between the two. This situation is shown in Fig. 5.42 using AM 522/558 No. 5 at 2.327 MHz, and is considered to be the practical cut off spatial frequency of the detector. A precise cut off criterion was not used to decide when the practical cut off was reached as it could be located to within 10% by visual observation of the similarity between pairs of traces! When calculating the modulation of the response waveform the missing cycles were considered to have a modulation of zero.

In order to investigate the shape of the modulation transfer function, sets of results were recorded at different temporal frequencies. AM 522/558 No. 5 and P.Z.T. 5H No. 2 were again selected for these investigations and the results are shown in Figs. 5.40 and 5.41.

The form of the modulation transfer functions are very similar to those usually obtained for photographic film. These usually show a plateau region over the low spatial frequency range where the

Modulation

Figure 5.40(i)

 $f = 2.5160 \text{ MHz}$ $\frac{\lambda_d}{2b} = 0.925$

AM522/558 No5

1.0
0.8
0.6
0.4
0.2Spatial frequency mm^{-1}

Modulation

Figure 5.40(ii)

 $f = 1.900 \text{ MHz}$ $\frac{\lambda_d}{2b} = 1.225$

AM522/558 No5

1.0
0.8
0.6
0.4
0.2Spatial frequency mm^{-1}

Modulation

Figure 5.40(iii)

 $f = 2.3270 \text{ MHz}$ $\frac{\lambda_d}{2b} = 1.000$

AM522/558 No5

1.0
0.8
0.6
0.4
0.2Spatial frequency mm^{-1}

Modulation transfer functions obtained using the sinusoidal standing
wave technique

Figure 5.41(ii)

P.Z.T.5H No2

 $f = 3.1950 \text{ MHz}$

$$\frac{\lambda_d}{2b} = 0.333$$

Modulation

1.0
0.8
0.6
0.4
0.2

Spatial frequency mm^{-1}

0.100 0.150 0.200 0.250 0.300 0.350

Modulation

1.0
0.8
0.6
0.4
0.2

Figure 5.41(i)

P.Z.T.5H No2

 $f = 1.210 \text{ MHz}$

$$\frac{\lambda_d}{2b} = 0.878$$

Spatial frequency mm^{-1}

0.50 0.100 0.150 0.200 0.250 0.300

Modulation transfer functions obtained using the sinusoidal standing
wave technique

modulation transfer function is unity and then a steady decline at higher spatial frequencies until the modulation transfer drops to zero.

In order to obtain the coherent transfer function the modulation transfer functions must be multiplied by the corresponding directivity function. The directivity function corresponding to Fig. 5.40 (i) is displayed in Fig. 5.20 curve No. 6, that corresponding to Fig. 5.40 (ii) in Fig. 5.25 curve No. 21, and that corresponding to Fig. 5.40 (iii) in Fig. 5.22 curve No. 12. For the P.Z.T. 5H No. 2 results the directivity function corresponding to Fig. 5.41 (i) is displayed in Fig. 5.16 curve No. 5 but that corresponding to Fig. 5.41 (ii) is displayed in Fig. 5.37.

Consider as an example the M.T.F. curve for AM 522/558 No. 5 shown in Fig. 5.40 (ii). The directivity function at 1.900 MHz is relatively even and quite broad. Using Fig. 5.32 (i) at $\frac{\lambda_d}{2b} = 1.225$ the spatial frequency bandwidth is 0.244 mm^{-1} centred on

zero. Hence it is clear from Fig. 5.40 (ii) that throughout this bandwidth the depth of modulation is not less than 0.9 and consequently image contrast will be preserved during visualization. This will not be the case for imaging at 2.516 MHz. The two peaked directivity function corresponding to the M.T.F. of Fig. 5.40 (i) has a pass band from 0.126 mm^{-1} to 0.240 mm^{-1} with the peak at a spatial frequency of 0.200 mm^{-1} , as is shown by Fig. 5.32 (i). Over this spatial frequency range the M.T.F. decreases from 0.9 to 0.6, hence the contrast falls off at the higher spatial frequencies which will result in a loss of visibility of the fine detail in the image. These two examples serve to illustrate the importance of both the directivity function and the modulation transfer function when

considering limitations on resolution imposed by a detector.

With the exception of Fig. 5.40 (i) the modulation transfer functions show a cut off frequency which tends to substantiate the heuristic argument that this should be $\frac{1}{2b}$. For AM 522/558 No. 5

$\frac{1}{2b}$ is equal to 0.500 mm^{-1} and for P.Z.T. 5H No. 2 it is equal to

0.250 mm^{-1} . The M.T.F. obtained at the first overtone frequency of P.Z.T. 5H No. 2, shown in Fig. 5.41 (i), shows a significantly higher cut off than expected, and that for AM 522/558 No. 5, shown in Fig. 5.40 (i) a significantly lower one. These empirical results suggested that the cut off spatial frequency was not simply determined by the disc thickness but was also a function of the temporal frequency.

5.4.3 Modulation transfer function cut off spatial frequency as a function of temporal frequency

Measurements were carried out to investigate specifically the change in f_o with temporal frequency after the modulation transfer function results indicated that it was not independent of temporal frequency. The sinusoidal standing wave was again used and f_o determined by comparing successive pairs of traces until a response such as that shown in Fig. 5.42 obtained. Two discs of different materials were used in the measurements. They were P.Z.T. 5H No. 2 and AM 522/558 No. 5. The results are shown in Figs. 5.43 and 5.44, plotted against $\frac{\lambda_d}{2b}$ for easy comparison with the directivity

function bandwidth results.

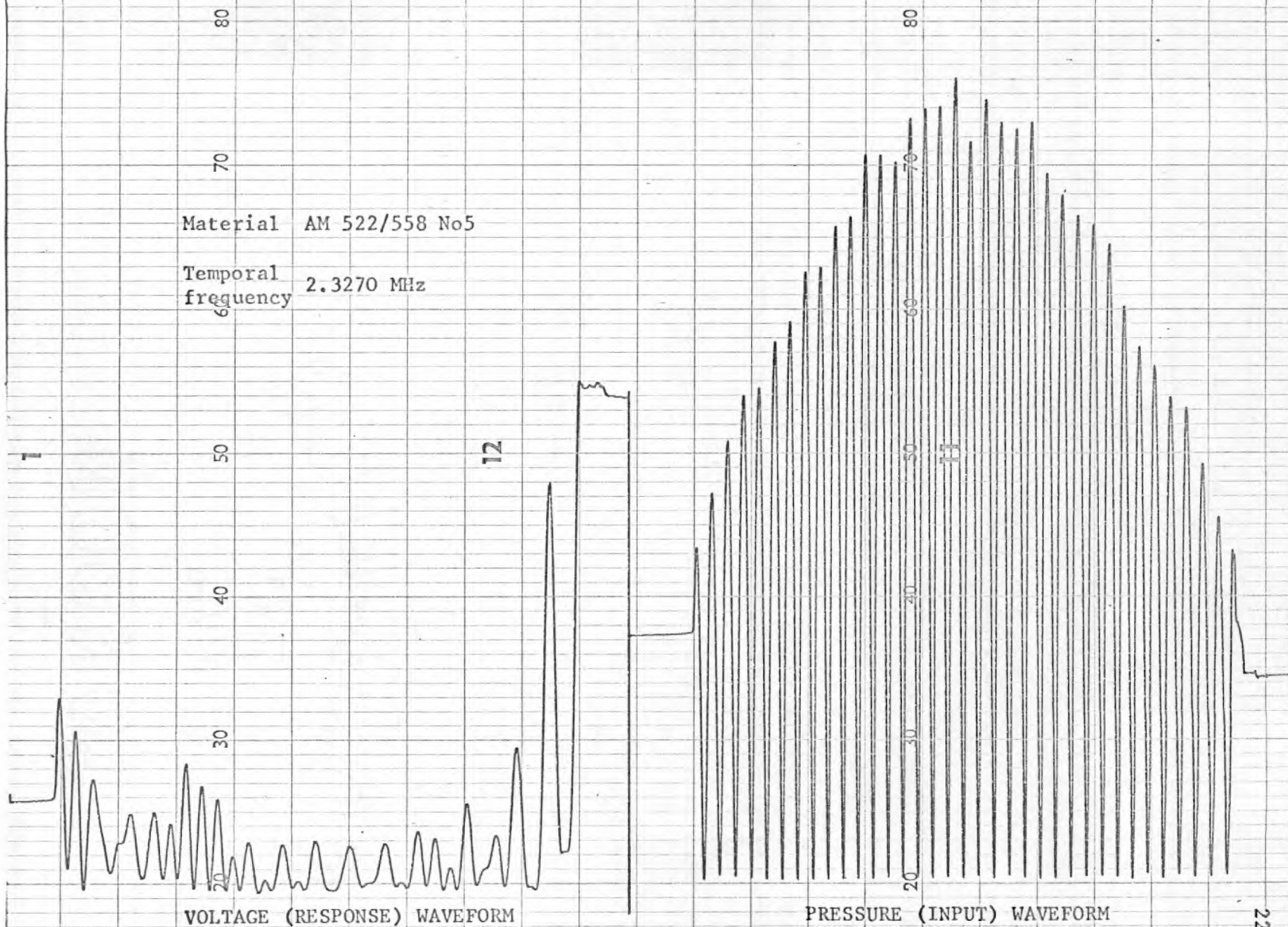


Figure 5.42 Atypical pair of results obtained using the sinusoidal standing wave technique at the modulation cut-off spatial frequency f_c

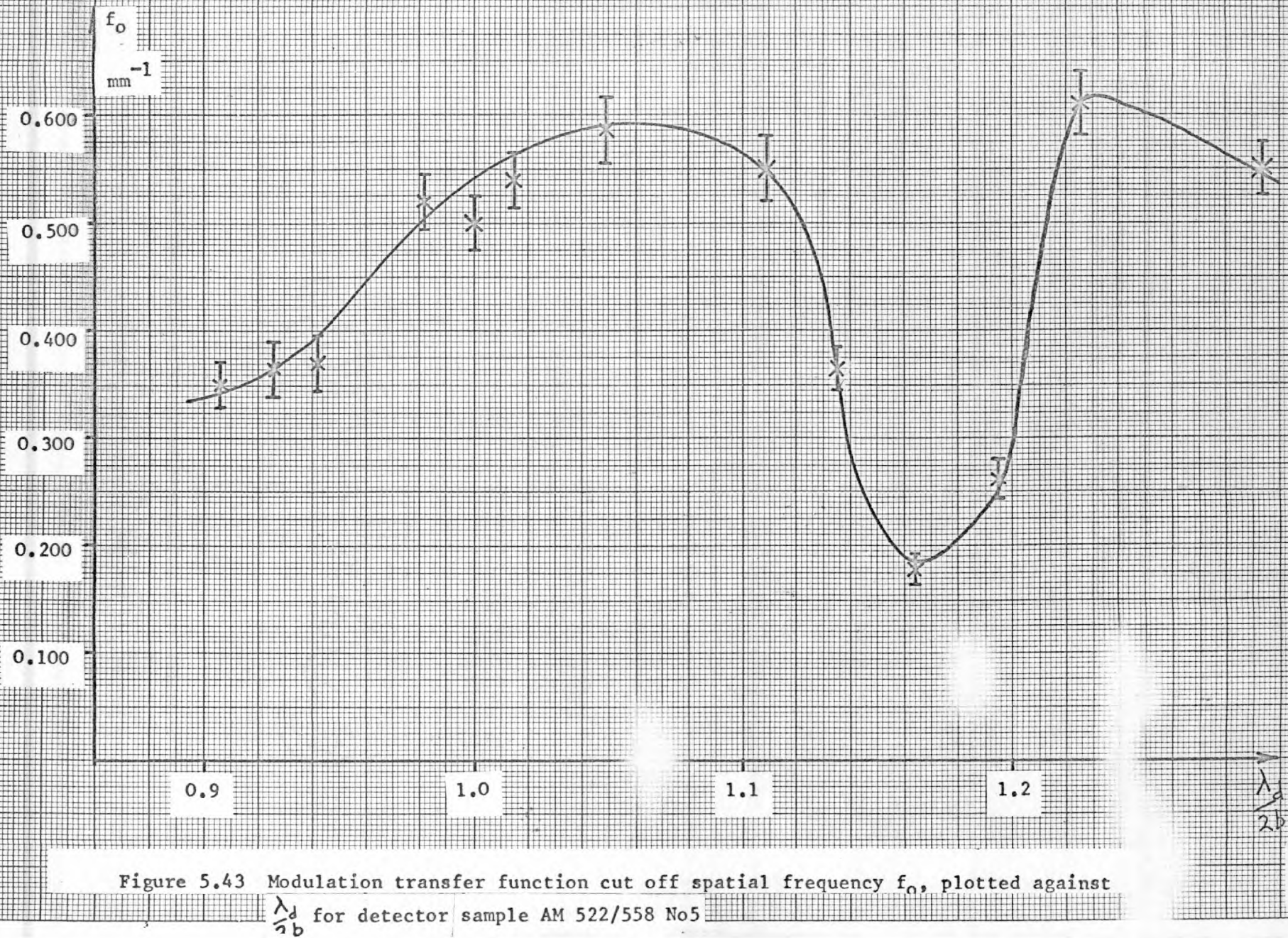


Figure 5.43 Modulation transfer function cut off spatial frequency f_0 , plotted against $\lambda_d / 2b$ for detector sample AM 522/558 No5

$f_o \text{ mm}^{-1}$

0.600

0.500

0.400

0.300

0.200

0.100

0.8

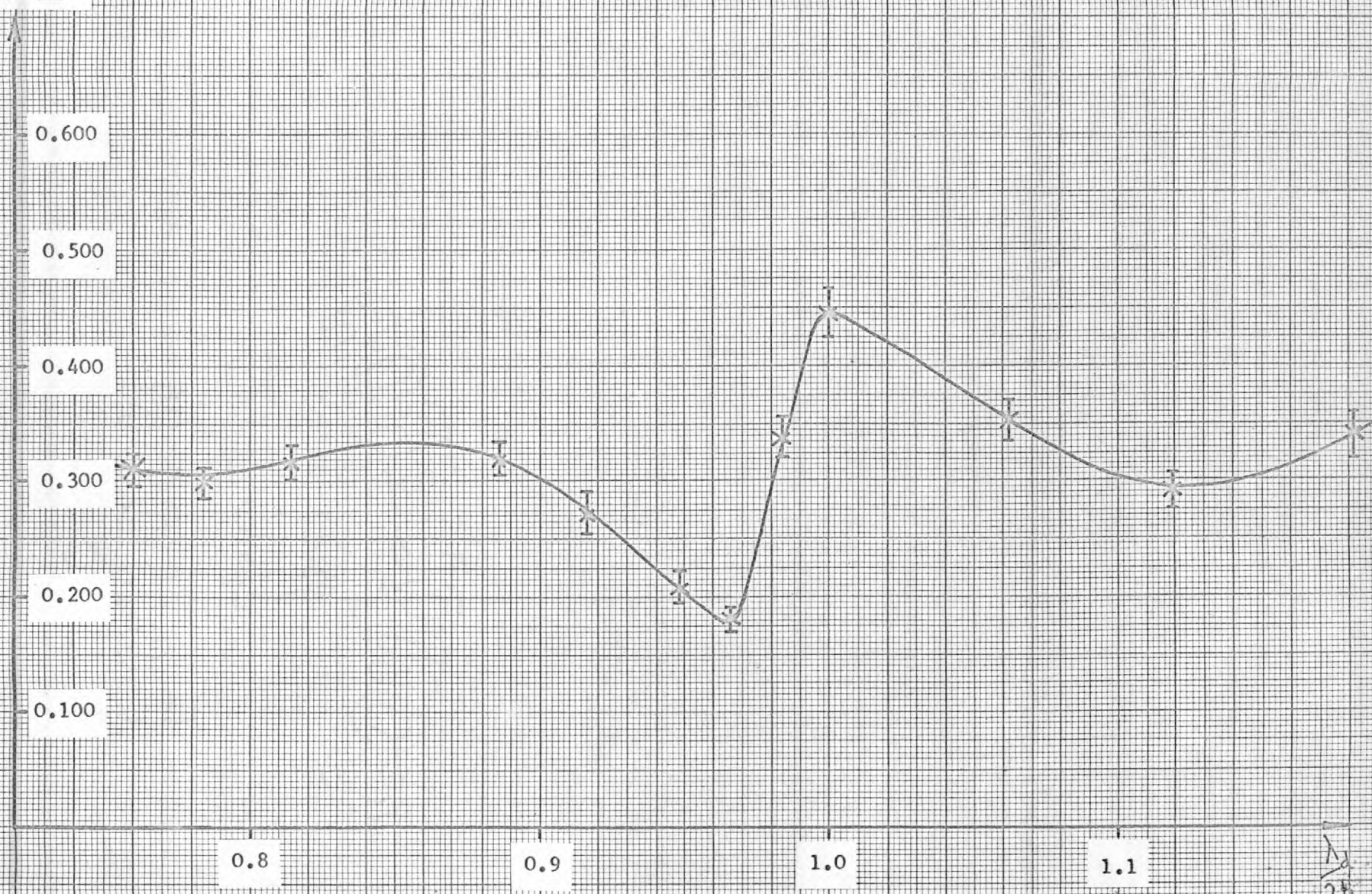
0.9

1.0

1.1

$\frac{\lambda_d}{2b}$

Figure 5.44 Modulation transfer function cut off spatial frequency f_o , plotted against $\frac{\lambda_d}{2b}$ for detector sample P.Z.T. 5H No2



Both curves show similar features. The most probable value of f_0 over the range of temporal frequencies covered is approximately 20% higher than that given by $\frac{1}{2b}$ and both curves show a definite

minimum in f_0 at a particular temporal frequency. This minimum for the AM 522/558 No. 5 results is comparable to the temporal frequency range of the directivity functions shown in Fig. 5.24 which were described as resulting from a strong thickness shear resonance at $\frac{\lambda_d}{2b} = 1.135$. On this basis one would expect a strong

lateral coupling between adjacent loops in the sinusoidal standing wave field and a corresponding decrease in f_0 . Why the minimum value of f_0 does not occur precisely at $\frac{\lambda_d}{2b} = 1.135$ is not known.

The minimum of f_0 for the P.Z.T. 5H No. 2 results occurs at $\frac{\lambda_d}{2b} = 0.965$ which corresponds to curve 7 in Fig. 5.13. A strong

shear resonance is not apparent at this frequency; only a prominent peak at $f_x = 0$ which again suggests that a high degree of lateral coupling exists.

It is quite clear from the difference in form of the two sets of results that they can only be considered as empirical values. A far more detailed investigation is required before the real significance of the shapes of these curves can be ascertained.

CHAPTER 6

Conclusions and Suggestions for Further Work

There are two main conclusions to be drawn from the discussion of the theoretical and experimental results. Firstly, that a coherent transfer function can sensibly be defined for a ceramic detector and that its form may be determined using either an impulse response technique or a sinusoidal standing wave technique. Secondly, from the results it is clear that only at certain temporal frequencies will a plate of lead zirconate titanate ceramic perform well as an ultrasound hologram detector. For example, consider a detector constructed from AM 522/558 having a fundamental thickness resonance at 2.327 MHz used to receive an ultrasound hologram at 1.900 MHz. Using the information from Fig. 5.32 (i) and Fig. 5.40 (ii), it is easy to see that the contrast will be unimpaired over the entire pass band of the detector, and the maximum spatial frequency passed will be 0.244 mm^{-1} . Consequently the maximum object resolution will be 4 mm. In an underwater viewing system using a detector with an aperture of 0.5 m the closest distance at which this object resolution will be obtained is 1.3 m. If the object were to be brought nearer to the detector, no further increase in resolution would be observed. For object distances greater than 1.3 m the pupil function would limit the resolution to more than 4 mm. Increasing the aperture of the detector further would increase the object distance at which the pupil function limit and the detector transfer function limit were again equal. A maximum resolution of 4 mm at 1.3 m should be adequate for any underwater viewing system. If this is not the

case the system resolution could be increased by using conventional imaging and projecting a magnified image on to the detector. In this case, provided an aberration free ultrasonic image element could be obtained, improved resolution would result at the expense of a reduced field of view.

In contrast to this a severe limitation is placed on the performance of a non-destructive testing system by the detector spatial frequency transfer function. A 10 cm diameter detector of AM 510 with a fundamental resonant frequency of 3.410 MHz working at 3.490 MHz will pass a maximum spatial frequency of 0.245 mm^{-1} . This corresponds to a maximum object resolution of 4 mm at a distance of 13 cms which will not be improved by increasing the detector aperture or by reducing the object distance. Typical object distances in non-destructive testing may be as small as 5 cms and the required resolution could be less than 2 mm, half that available using the AM 510 detector.

It is, therefore, clear that the detector must be re-designed for it to be of use in non-destructive testing. The cut off spatial frequency investigations clearly showed that the modulation transfer function at the optimum temporal frequency only influences the resolution at spatial frequencies greater than $\frac{1}{2b}$. Provided

that the directivity function can be broadened out considerably, a detector limited only by the modulation transfer function would be obtained. A 0.7 mm thick detector would then be capable of resolving object dimensions down to 1.4 mm. In order to flatten the directivity function, it is necessary to either increase the mechanical losses in the detector material or decrease the value of R in equation (2.90).

Decreasing R is synonymous with reducing the acoustic impedance mismatch between the detector material and the acoustic loading material. This increases the fraction of mechanical energy which is re-radiated into the loading medium and hence lowers the mechanical Q factor of the detector. If the detector is coupled directly on to a steel specimen for a non-destructive testing application, then R could easily be nearly one and the directivity function should not impair the detector resolution limit set by the M.T.F. However, there is no information available of the effects on the M.T.F. of bonding a ceramic plate directly on to a solid acoustic loading medium. It could easily reduce the bandwidth because of mechanical coupling through both the loading medium and the detector ceramic. This is a possibility which deserves further investigation.

S

In many applications the detector cannot be attached directly on to the object so a coupling medium, usually a fluid such as water, must be used. A matching layer with a suitable characteristic impedance could then be used to reduce R . Again this arrangement could influence the bandwidth of the modulation transfer function and deserves further investigation.

n

Another alternative would be to decrease the characteristic impedance of the detector material. This could be achieved by setting grains of lead zirconate titanate in a matrix of plastics material along the lines of the 'monograin layer' recently developed for solar cells in space vehicles by the Phillips Research Laboratories, Eindhoven (57). A lossy type of matrix would also have the advantage of minimising thickness shear resonance amplitude.

n

p

n

n

v

n

It has not been possible to relate the modulation transfer function results or the directivity functions, in detail, to a theory of disc vibrations and hence more generally to the physical properties of the material. This was one of the objects outlined in the introduction. There is great scope for further theoretical work and it is hoped that this investigation will provide an incentive for future researchers.

Appendix 1

Measurement of the amplitude reflection coefficient of AML type B1 anechoic coating at normal incidence

A simple technique was devised for measuring the amplitude reflection coefficient of the anechoic coating which was used on the ends of the water tank. Fig. A.1 (i) shows a schematic diagram of the arrangement. At the hydrophone, which is assumed to be small enough not to significantly affect the acoustic field, the pressure amplitude is determined by the relative phases of the incident wave and reflected wave. By slowly sweeping the frequency of the ultrasound source, the relative phase of the two waves will sweep from 0 to 180 degrees and the corresponding time averaged pressure amplitude from a maximum to a minimum. It is a simple matter to calculate the amplitude reflection coefficient of the reflecting material from the pressure maxima and minima which are recorded on the electrical output from the hydrophone.

Let the incident pressure wave be represented by

$$p_i = p_o \sin(kz + \omega t)$$

and let the reflected wave be represented by

$$p_r = a p_o \sin(kz - \omega t)$$

where a is the amplitude reflection coefficient of the absorbing material. For simplicity, it is assumed that there is no change of phase on reflection.

$-20\log_{10} a$

Figure A.1(ii) Amplitude reflection coefficient against frequency for A.M.L. type B1 anechoic material

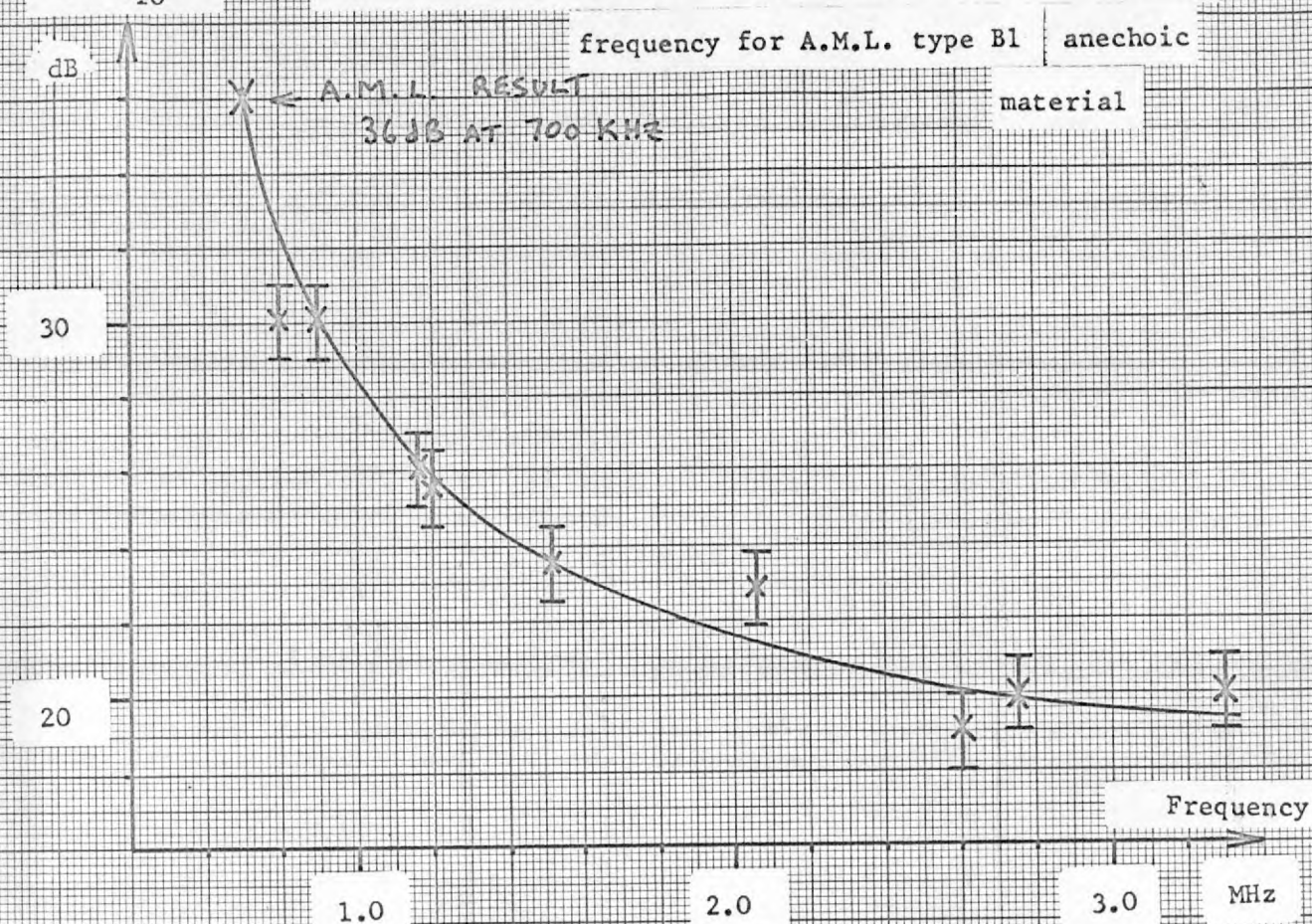
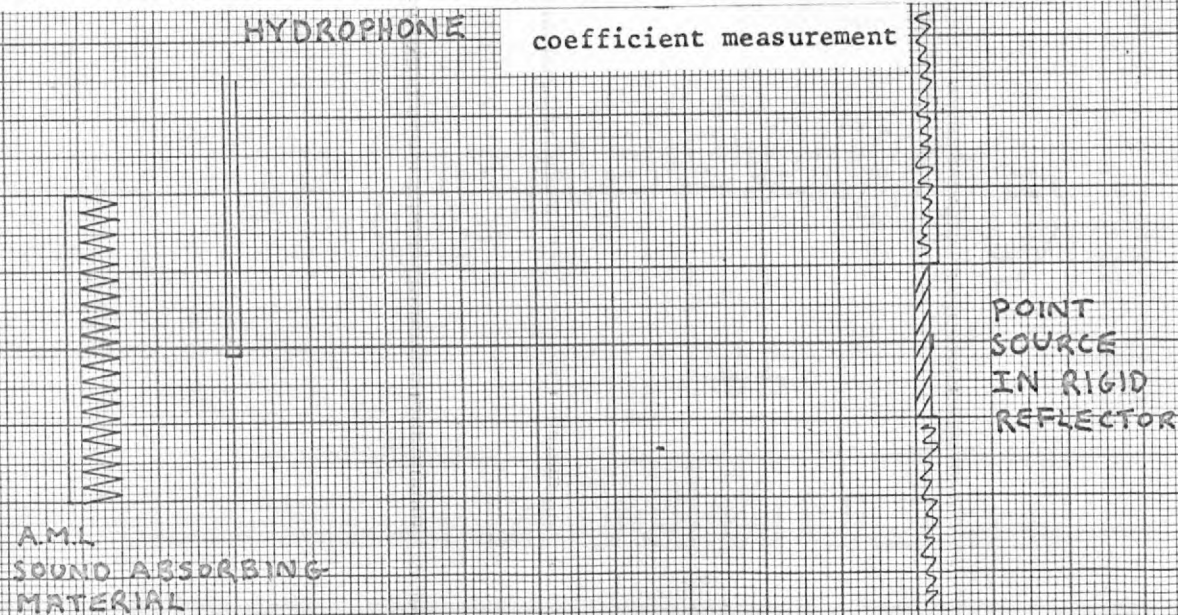


Figure A.1(i) Experimental arrangement used for amplitude reflection



At the hydrophone where $z = z_H$ the total pressure p_T is given by

$$p_T = p_o \left[\sin(kz_H + \omega t) + a \sin(kz_H - \omega t) \right]$$

Therefore

$$p_T = p_o \left[(1 + a) \sin kz_H \cos \omega t + (1 - a) \cos kz_H \sin \omega t \right] \quad (\text{A1.1})$$

z_H is fixed but k varies with the frequency.

If $\overline{p_T}$ is the time average of the pressure signal, averaged over a time which is long compared with the ultrasound period but short compared with the sweep period, then when $kz_H = \frac{\pi}{2}$

$$\left. \begin{aligned} \overline{p_T} &= p_o(1 + a) \\ \overline{p_T} &= p_o(1 - a) \end{aligned} \right\} \quad \text{and when } kz_H = 0 \quad (\text{A1.2})$$

Assuming that the electrical output from the hydrophone is proportional to $\overline{p_T}$, $V_{\max} \propto p_o(1 + a)$ and $V_{\min} \propto p_o(1 - a)$.

$$\text{Hence} \quad a = \frac{V_{\max} - V_{\min}}{V_{\max} + V_{\min}} \quad (\text{A1.3})$$

The results obtained are shown in Fig. A1 (ii). In accordance with the usual convention, the reflection coefficient is displayed on a logarithmic scale.

The reflection coefficient at 700 kHz was obtained by the Admiralty Materials Laboratory with a pulse-echo method at the upper frequency limit of their measuring technique. This work, therefore, extends the range of known data on AML type B1 anechoic coating.

The rapid increase in the reflection coefficient with increase in frequency results from a progressive decrease in the effectiveness of the spikes. These were designed to match the characteristic impedance of aluminium loaded butyl rubber to that of water at about 100 kHz. They cannot be expected to do this at wavelengths very much smaller than the spacing between the spikes. The reflection coefficient, therefore, tends to that expected for the butyl rubber alone which is - 19 dB.

Appendix 2Miniature Probe Hydrophone Directivity and
Sensitivity MeasurementsDirectivity

A series of preliminary investigations were carried out on the probe hydrophones to assess their suitability for use in ultrasonic field plotting. The design suggested that the hydrophones should show circular symmetry in the plane of the transducing element. Consequently at frequencies below the thickness resonant frequency of the element (5 MHz), it was expected that the directivity patterns of the hydrophones would be circular and centred on the transducer.

The technique used to measure the directivity pattern of a hydrophone involved rotating it in a uniform sound field with the Z axis accurately aligned normal to the direction of sound propagation.

Contrary to expectations the directivity patterns were not circularly symmetric. At frequencies below 1 MHz dipole directivity patterns were obtained. As the frequency was increased a systematic development of the directivity pattern was observed, as shown in Fig. A2.1.

The dipole form gradually changes into something similar to a lateral quadrupole pattern.

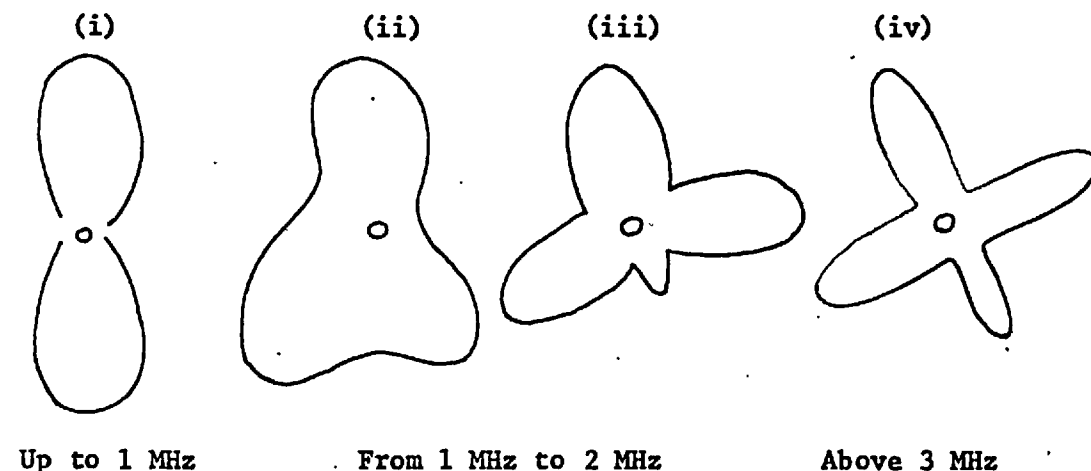


Figure A2.1

The hydrophone transducer element is approximately one third of a wavelength in diameter at 1 MHz and one wavelength at 3 MHz. Consequently the alternating stress pattern in the transducer produced by the incident ultrasonic wave will have both longitudinal and shear components. It is shown in Stephens and Bate (58) that shear and longitudinal stress systems acting on an isolated volume give rise to both lateral and longitudinal quadrupole radiation. Many radiation patterns can be synthesised using linear combinations of lateral and longitudinal quadrupole radiation patterns. The radiation pattern must, however, show the same form as the reception directivity pattern by the principle of reciprocity. Hence the observed directivity of the hydrophones can be sensibly ascribed to the stress system within the transducing element.

Sensitivity

The acoustic power radiated by a thickness resonant air backed piezoelectric transducer is easily calculated using the well known relationship:

$$\text{Acoustic power radiated} = \frac{1}{2} \frac{A e^2 V^2}{l^2 \rho c} \quad (\text{m}^{-2}) \quad (\text{A2.1})$$

A is the radiating area of the source (m^2)

V is the peak voltage driving the transducer (volts)

l is thickness of the transducer (m)

ρc is the characteristic impedance of the loading fluid
($\text{kg m}^{-2} \text{sec}^{-1}$)

e is the piezoelectric constant of the transducer material
(Cm^{-2})

In order to determine the sensitivity of the hydrophone, its response was measured when situated in the near field of a half wave thickness resonant source, radiating a known acoustic intensity. The intensity was determined from (A2.1) knowing the driving voltage.

The source used was a disc of AM 510 piezoelectric ceramic, mounted so that it was airbacked and radiating from a circular area 10 mm in diameter.

$$\begin{aligned} e &= 15 \text{ C m}^{-2} \\ l &= 2.2 \cdot 10^{-3} \text{ m} \\ \rho c &= 1.5 \cdot 10^6 \text{ kg m}^{-2} \text{ sec}^{-1} \\ A &= 25 \cdot 10^{-6} \text{ m}^2 \\ V &= 7 \text{ volts peak to peak} \end{aligned}$$

The acoustic power radiated by the source was then 80 mW cm^{-2} which gave a measured hydrophone output of 20 mV peak to peak. The sensitivity of the hydrophone was therefore 20 mV R.M.S. / atmosphere.

Appendix 3

Calibration and Frequency Response Curves

Calibration curves were obtained for the hydrophone amplifier and detector and also the F.E.T. voltage probe amplifier and detector. These curves are necessary for the extraction of linear amplitude information from the experimental results discussed in Chapter 5. They give the output signal amplitude as a function of input signal amplitude for a range of representative signal frequencies. The curves are shown in Figs. A3.1 and A3.2.

Frequency response curves were also recorded for the hydrophone amplifier circuit and the power amplifier circuit. These are shown in Figs. A3.3 (i) and (ii) where circuit gain is plotted against frequency. The hydrophone amplifier frequency response shows an interesting peak at 3.84 MHz which is caused by the self resonance of the r.f. chokes used to determine the input impedance and biasing conditions (see Fig. 3.10).

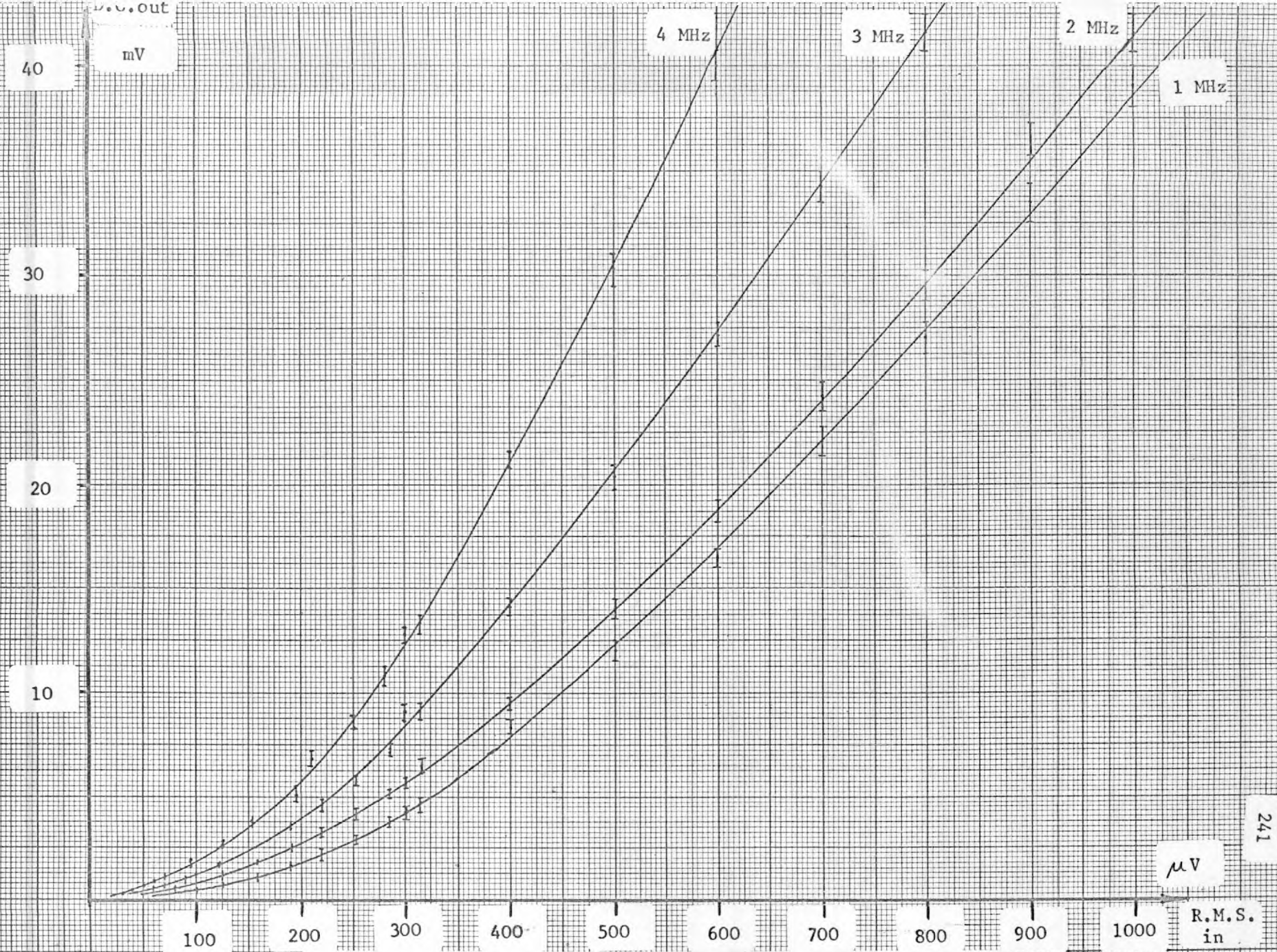


Figure A.3.1 Calibration curve for the hydrophone amplifier

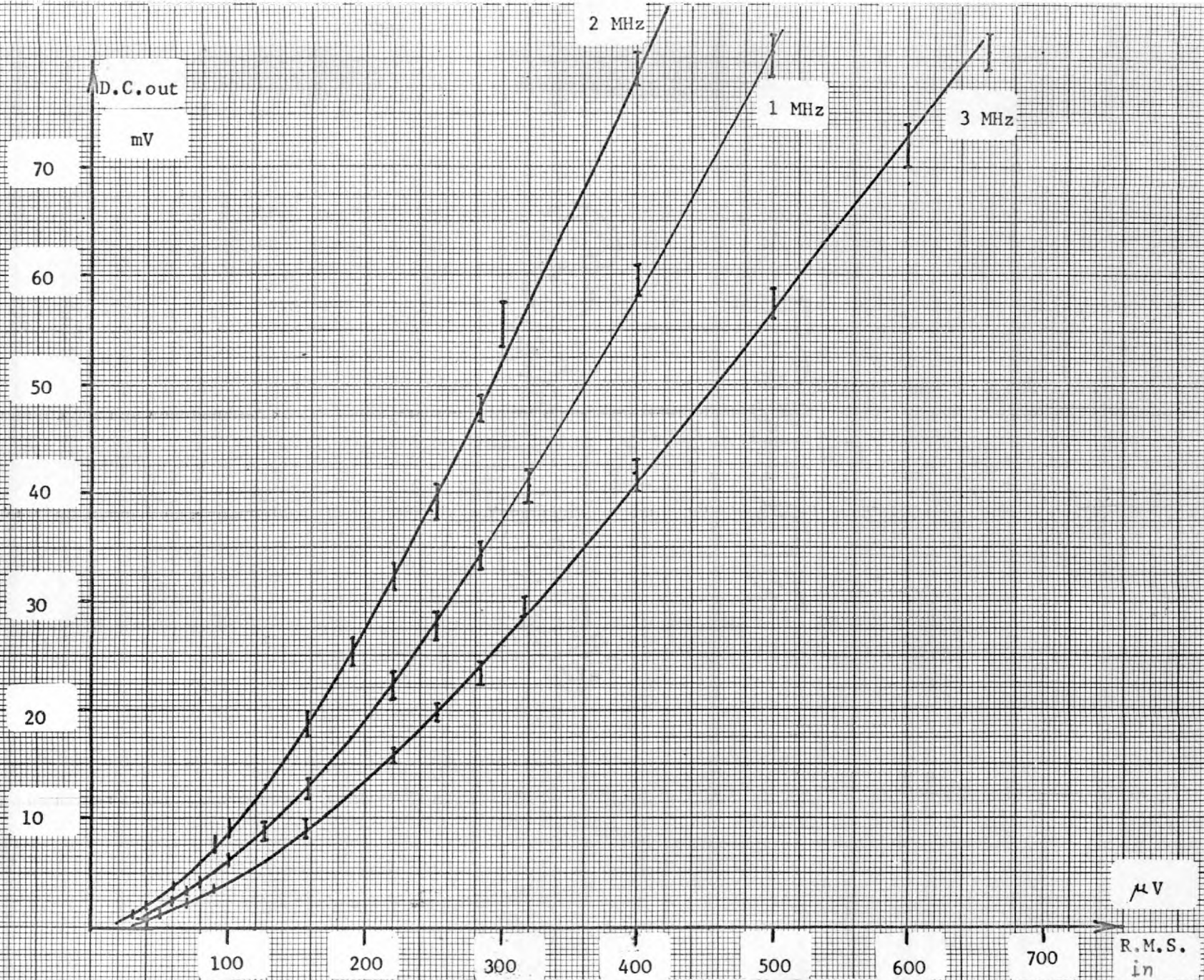


Figure A.3.2 Calibration curve for the F.E.T. voltage probe

Voltage gain

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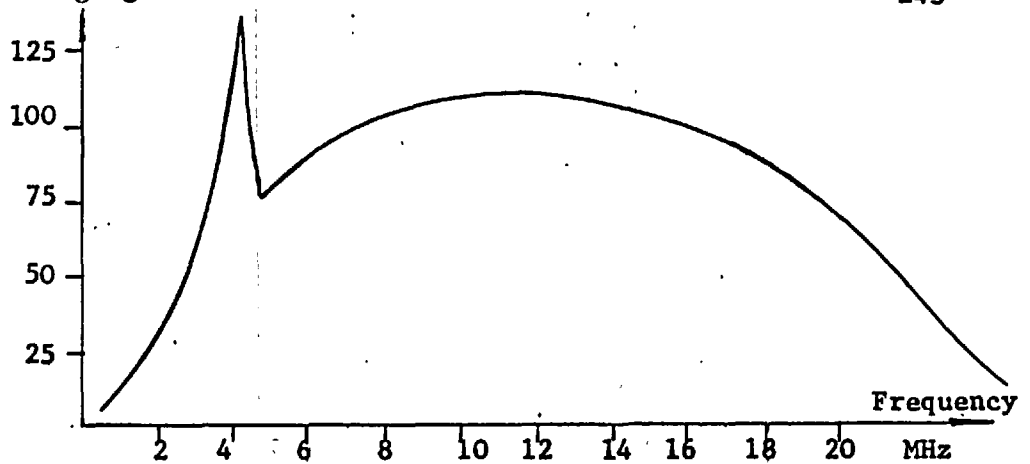


Figure A.3.3(i) Frequency response of the hydrophone amplifier and detector circuit.

Voltage gain

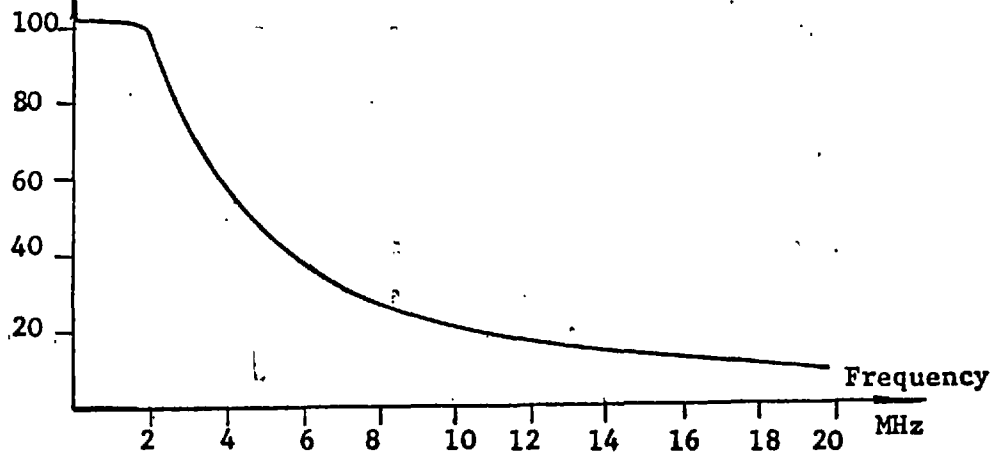


Figure A.3.3(ii) Frequency response of the power amplifier

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ULTRASOUND HOLOGRAPHY

J. Halstead*

Ultrasound to optical image converters can visualize objects by ultrasonic waves in optically opaque media. Ultrasound holography displaces the difficult production of conventional images by ultrasound lenses. Five examples of ultrasound object-image relations are derived from the principles of holography.

The absence of a suitable ultrasound image converter restricts practical applications. Possible ultrasound detectors are outlined; for example, a potentially useful ultrasound camera with a wide aperture modification. Also, current literature of ultrasound holography is reviewed.

INTRODUCTION

Many useful industrial and technological services could be performed by a device capable of producing images of objects immersed in media through which light cannot propagate. Situations which come to mind are viewing of objects immersed in dirty water, such as may be found in navigable rivers and harbours, observation of weld defects and viewing of internal organs in the living body. In principle it is possible to use mechanical radiation as the information carrier in such imaging devices. In comparison with electromagnetic radiation it is possible to obtain a given ultrasound wavelength at much lower frequencies because of the low velocity of propagation of mechanical radiation in these media. Ultrasound wavelengths in the submillimetre range are obtained with frequencies 1–10MHz in liquids. The choice of this range of frequencies is significant. The quality of an imaging system is judged on several criteria, the most important of which is resolution. If the aperture of the system cannot be considered a variable, then high resolution is synonymous with short wavelength. Unfortunately, the attenuation of mechanical radiation is a function of frequency and is of an order such that high frequency propagation is possible only over short distances. Consequently distant objects can only be imaged with low resolution. With frequencies above about 10MHz in liquids, resolution is high but propagation distance is becoming inconveniently small, and below about 1MHz resolution is becoming impracticably low. For propagation in solids, this frequency band may be multiplied by a factor of three or four.

AN ANALYSIS OF THE PROBLEMS

Until very recently ultrasonic imaging systems have used lenses or mirrors in the conventional optical manner to form real images. The success of these techniques has been limited because resulting image quality was worse than that indicated simply by resolution considerations. For most practical imaging systems the ultrasound wavelength is appreciable compared with the aperture and so diffraction effects are pronounced. Ultrasonic radiation can easily be made both spatially and temporally coherent, so any two wavefronts are capable of interfering to produce a system of fringes. This latter effect is very noticeable in continuous wave imaging systems, and is probably the major factor contributing to poor image quality and consequent interpretation difficulties. Multiple reflections inside imaging components, or between these and the image converter, an integral part of

any ultrasound imaging system, will introduce phase differences between incident and reflected waves. These phase differences produce sets of interference fringes over the image plane.

Fig 1 shows a series of circular interference fringes, ultrasonic Newton's rings, formed as a result of reflections at a lens-water interface in a simple imaging system. A plano-concave perspex lens was used in water, and the ultrasonic field in the focal plane visualized by means of a piezoelectric image converter. This instrument is similar to a television camera but the photoelectric phosphor is replaced by a piezoelectric plate which forms a part of the tube wall. The ultrasound image is projected onto the outer face of this plate and the resulting charge pattern, characteristic of the ultrasound pressure distribution, scanned off the inner face to provide the video signal. Fortunately interference difficulties may be eliminated to a large extent by using pulsed ultrasound and appropriate electronic gating of the image converter.

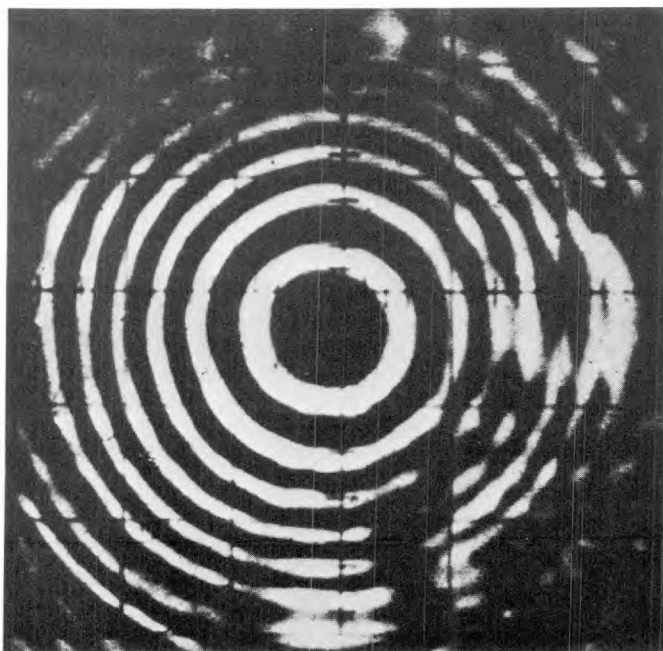


Fig 1 Ultrasonic Newton's rings. The ultrasonic wave was transmitted through a plano-concave lucite lens to produce the rings

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This is illustrated by Fig 2 which shows two images of the same object, one taken with continuous ultrasound and the other with pulsed ultrasound. Picture (a) shows the continuous wave ultrasound image of four holes in a metal plate, using a single lens in water. It is obvious that interference effects have completely masked the images of the holes. In picture (b), pulsed ultrasound and range gating have been used with the same object and lens imaging system. The holes in the plate are clearly visible.

These two illustrations are taken from work by W. E. Lawrie of the Illinois Institute of Technology Research Institute.¹

Apart from interference effects, poor image quality and loss of resolution have contributions from several other factors, which are not as easy to eliminate. The usual aberrations present in an optical imaging system are also present in the ultrasonic analogue. Spherical aberration, in particular, may be minimized by the use of elliptical refracting surfaces.

A second factor peculiar to mechanical wave systems is mode conversion at, for example, a water-perspex interface. This is a phenomenon where energy from longitudinal wave motion is wholly or partially transferred into a different mode of propagation, at the interface, and has a new propagation velocity. Consequently a phase difference is introduced, and this difference ultimately produces a reduction of resolution in the final image.

Absorption and inhomogeneities in the lens material can also be significant factors contributing to poor image quality.

These latter problems, with the exception of mode conversion, can be eliminated by using a reflecting instead of a refracting imaging system but for many purposes this is inconvenient.

ULTRASONIC HOLOGRAPHY—GENERAL PRINCIPLES

If imaging could be performed without the use of lenses or mirrors many of the above difficulties would no longer exist. In 1947, a similar state of affairs existed in electron imaging which led Gabor to invent the technique of holography²; this technique has been very widely studied and exploited optically, but has only recently been applied to ultrasonic imaging.

The technique as applied to ultrasonic imaging does not require lenses or mirrors. The ultrasound diffraction pattern of an object 'illuminated' with ultrasound is itself recorded in such a form that it can be reconstructed, with the original amplitude and phase distribution, from the recording. This object wavefront is reconstructed optically and hence is an optical copy of the ultrasonic wavefront. In fact two copies of the original wavefront are always reconstructed, one having the same form as the original and the other being its complex conjugate. These wavefronts may be considered as either diverging from virtual images or converging to real images, depending on how the recording and reconstruction are carried out. Consequently the technique simultaneously effects both ultrasound visualization and imaging.

Some insight into the process may be obtained by considering an object viewed in coherent light through an aperture in a large screen. Amplitude and phase modulations of the light waves carry all the information necessary for us to experience the three-dimensional nature of the object and the different perspectives obtainable by moving our observation position relative to the aperture. The technique of holography enables this amplitude and phase information to be recorded. If such a record or hologram, in the form of a suitable photographic transparency, is placed in the aperture and illuminated with a coherent light source the light transmitted will have the same amplitude and phase distribution as the original object wave. Consequently an observer looking through the transparency will see the original object just as if it were actually there, and by changing his position will see that the aspect of the object will change accordingly. The crucial stage in the technique is the recording of the phase information. Amplitude information can be recorded optically by a photographic plate and in the case of ultrasound by any one of several detection systems, which will be discussed later. Phase information cannot be recorded directly but must first be converted to an amplitude distribution by causing the object wave to interfere with a second wave which

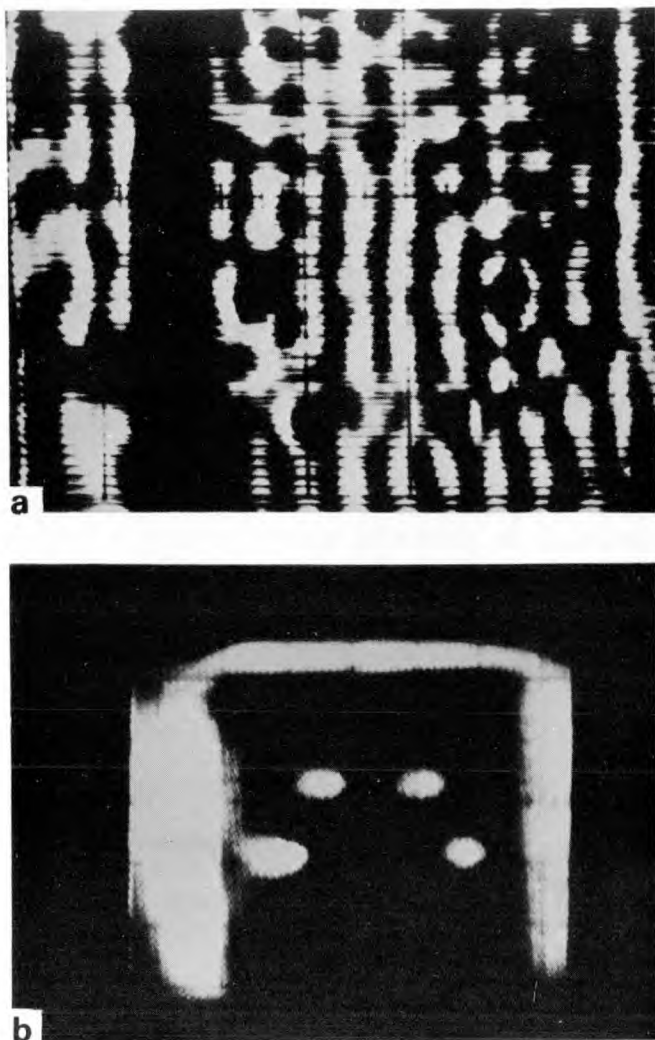


Fig 2 Comparison of images obtained using (a) continuous and (b) pulsed ultrasonic energy. The continuous wave image was electronically enlarged. The images were made by blocking the sound beam by a rectangular steel plate with a distinctive hole pattern. The holes in the continuous image are obscured by interference

has a uniform distribution of phase. This second wave is called the reference wave.

The principle of the recording of the hologram and its subsequent reconstruction may be illustrated by a simple one-dimensional theory. A more complete treatment of ultrasound holography would require Scalar Wave theory and involve integration of the Huygens-Fresnel relation over the relevant areas of the wavefronts involved.

Amplitude and phase terms in a propagating wavefront may be represented mathematically as an expression known as the complex amplitude. This is the product of the real wave amplitude and a complex phase term.

$O(x, y)$ is the complex amplitude of the object wavefront having a real amplitude $O_0(x, y)$ and a phase $\exp[-i\phi_0(x, y)]$ at time $t = 0$

$$O(x, y) = O_0(x, y) \exp[-i\phi_0(x, y)]$$

A negative sign in the exponential is used to represent a diverging wave, that is, a wavefront whose curvature is convex to the direction of propagation, propagating along the direction of $+z$ increasing. This will be called the forward direction of propagation. A positive exponential can be taken as representing a converging wave propagating in the forward direction, that is a wavefront whose curvature is concave to the direction of propagation.

For simplicity we will consider a plane reference wave, which is inclined at an angle θ_r to the recording medium, and which has a complex amplitude $R(x, y)$.

$$R(x, y) = R_0(x, y) \exp [-i\beta_r x]$$

$$\beta_r = \frac{2\pi}{\lambda_0} \sin \theta_r = k_0 \sin \theta_r$$

λ_0 is the wavelength of the radiation used in the formation of the hologram.

Inclining the reference beam to the direction of the object wave causes each object wavefront to be angularly separated from the others on reconstruction. This realization represented a major step forward in optical holography, and was first put into practice by Leith and Upatnieks³. Prior to this the 'in line' method of Gabor² utilized undiffracted light, passing through the object, as the reference beam, which consequently was not inclined to the object wave. On reconstruction, the object waves were not angularly separated and each image had an out-of-focus background provided by the other reconstructed object waves.

In the skew arrangement, the reference beam is obtained by a beam-splitting arrangement from the same laser source used to illuminate the object, to ensure spatial and temporal coherence. Ultrasonically the arrangements are more flexible. Two separate transducers may be used for the object illuminating beam and the reference beam, because, since they are electrically driven, it is easy to arrange for them to radiate in phase. Alternatively with electrical methods of detecting and recording the ultrasound hologram, it is possible to supply the reference wave by an internal electrical signal. With appropriate phasing, a plane or spherical reference wavefront may be simulated. This is a useful facility as it is difficult to produce an ultrasonic reference beam which has a uniform curvature. The uniformity is important because any differences of aberration between the reference wavefront and reconstruction wavefront are carried through the holographic transformation as aberrations of the reconstructed images.

Over the detection plane (the aperture mentioned in the previous illustration) the object wavefront and the reference wavefront interfere to produce an ultrasound amplitude distribution which is recorded as the ultrasound hologram.

The amplitude distribution is given by

$$\begin{aligned} [U_A U_A^*]^{1/2} &= [(O + R)(O^* + R^*)]^{1/2} \\ &= [OO^* + RR^* + RO^* + OR^*]^{1/2} \\ &= [O_0^2 + R_0^2 + 2O_0 R_0 \cos (\beta_r x - \phi_0)]^{1/2} \end{aligned}$$

The first two terms combine to form the background ultrasound amplitude. The third term represents a set of parallel interference fringes of period $1/\beta_r$ spatially modulated by the object wavefront phase distribution ϕ_0 . Consequently it is obvious that as a result of the interference of the two wavefronts both object amplitude and phase information are present over the hologram plane as a recordable amplitude distribution. Notice that the fringe amplitude contains the factor $O_0 R_0$. R_0 is usually greater than O_0 and is a constant. The process, therefore, results in a heterodyne amplification of the object wave.

The detection system placed in the aperture will record the ultrasound amplitude to the power γ . The value of γ will depend on the type of detector used and on the nature of the recording process. There may be several intermediary stages before the ultrasound hologram is in the form of a photographic transparency which can be used in the reconstruction process. Each stage will have its own γ but the overall value alone is significant.

The transmission function of the transparency will be given by

$$t_p = [O_0^2 + R_0^2 + 2O_0 R_0 \cos (\beta_r x - \phi_0)]^{\gamma/2}$$

Putting $A^2 = O_0^2 + R_0^2$ and expanding in a binomial series we obtain:

$$\begin{aligned} t_p &= A^\gamma \left[1 + \frac{\gamma}{2} \left(\frac{O_0 R_0}{A} \right) \left(\exp [i(\beta_r x - \phi_0)] \right. \right. \\ &\quad \left. \left. + \exp [-i(\beta_r x - \phi_0)] \right) + \frac{1}{8} \gamma(\gamma - 2) \left(\frac{O_0 R_0}{A} \right)^2 \right. \\ &\quad \left(\exp [2i(\beta_r x - \phi_0)] + \exp [-2i(\beta_r x - \phi_0)] + 2 \right) \\ &\quad \left. + \frac{1}{48} \gamma(\gamma - 2)(\gamma - 4) \left(\frac{O_0 R_0}{A} \right)^3 \left(\exp [3i(\beta_r x - \phi_0)] \right. \right. \\ &\quad \left. \left. + \exp [-3i(\beta_r x - \phi_0)] + 3 \exp [-i(\beta_r x - \phi_0)] \right. \right. \\ &\quad \left. \left. + 3 \exp [i(\beta_r x - \phi_0)] \right) \right. \\ &\quad \left. + \dots \right] \end{aligned}$$

In order to reconstruct the original object, the hologram transparency is illuminated with coherent light. For simplicity we will assume that the hologram is illuminated with a plane wave incident at some angle θ_c to the hologram. As before, the light propagates in the forward direction. The reconstruction wavefront is represented by:

$$C = C_0 \exp [-i\beta_c x]$$

where

$$\beta_c = \frac{2\pi}{\lambda_c} \sin \theta_c$$

λ_c is the wavelength of the reconstructing radiation and for most applications will be very much less than the hologram-forming radiation. For example if we are considering 1MHz ultrasound propagating in water the wavelength is approximately 1.5mm whereas the wavelength of the He-Ne laser light which may be used in reconstruction is 6328Å giving a ratio of

$$\frac{\lambda_c}{\lambda_0} = \frac{1}{2500}$$

The transmitted wavefront has a complex amplitude of the form

$$H = C[OO^* + RR^* + O^*R + OR^*]^{\gamma/2}$$

If we put $\gamma = 2$ only the first term in the binomial expansion of t_p is finite and only two images of the object are reconstructed. This may be difficult to achieve in practice but it will reduce the mathematical complexity here.

Therefore we can write

$$\begin{aligned} H &= C(OO^* + RR^*) + CO^*R + COR^* \\ &= H_D + H_C + H_T \\ H_D &= C_0(O_0^2 + R_0^2) \exp (-i\beta_c x) \\ H_C &= C_0 O_0 R_0 \exp (-i[\beta_r + \beta_c]x) \exp (i\phi_0) \\ H_T &= C_0 O_0 R_0 \exp (+i[\beta_r - \beta_c]x) \exp (-i\phi_0) \end{aligned}$$

H_D is the undiffracted portion of the reconstruction wave which is simply attenuated on passing through the transparency.

H_C is a diffracted wave containing the same amplitude and phase distribution as the original object wave, apart from multiplication by a constant factor. The sign change in the phase term means that the wavefront is converging and will produce a real image. Because this term reconstructs the complex conjugate of the object wave, the image is called the conjugate image. The wave is diffracted at an angle

$(\theta_r + \theta_c)$ if θ_r and θ_c are small, otherwise the diffraction angle is $\arcsin(\sin \theta_c + \sin \theta_r)$ ignoring wavelength differences.

H_T is a diffracted wave containing the same amplitude and phase distribution as the original object wave. The sign of the exponential $\exp(-i\phi_0)$ is negative so the wave is diverging and therefore is a complete replica of the original object wave, apart from multiplication by a constant. An observer looking along the direction of propagation of $H_T[(\theta_r - \theta_c)]$ if these are both small] through the transparency, will see a virtual image of the object. Because the complex amplitude of the object wave appears in its original form the image produced is called the true image.

In general, images produced by H_c and H_T may be real or virtual depending on the curvature of the reference wavefront and the reconstructing wavefront but the nomenclature remains the same.

Fig 3 illustrates the arrangement for hologram formation and Fig 4 shows roughly the positions of the two images relative to the hologram for the case just discussed but ignoring the effects of wavelength differences. The magnification of each image will be the same but they will be located at different distances below the (y, z) plane. Each will have the same magnitude of Z co-ordinate for True and Conjugate image co-ordinates (Table 1).

OBJECT—IMAGE RELATIONS

The more general problem of the relations between object and image distances for any positions of object point, reference point and reconstruction point sources is now considered. Any differences in wavelength between the hologram and image reconstruction radiation, as occurs in ultrasound holography, will be included and the possibility of changing the physical size of the hologram allowed for. Change of size is a very important parameter which is necessary for removal of intrinsic aberrations of the holographic process.

The appropriate co-ordinate system and object source point, reference source point, reconstruction source point and corresponding image points are introduced with the help of Fig 5. By considering the first order terms of the phases of the reference wave, object wave and reconstruction wave in the hologram plane, as they appear in H_T and H_c , it is possible, following Meier's analysis,⁴ to derive expressions for the corresponding image points. The cartesian system is specified for the reconstruction stage, to allow for a change of size of the hologram between recording and reconstruction. In this case the reconstruction co-ordinate system is related to the recording system by the following linear transformations:

$$x = mx', y = my', z = z'$$

It is assumed that the hologram stores information only in two dimensions so a change of scale of the Z co-ordinate is unnecessary. The thickness dimension of all real holograms can, however, be used to store further information as, for example, in the techniques of colour optical holography. Variations in thickness of the hologram resulting from emulsion shrinkage in preparation can produce phase variations in the reconstructed wavefront which seriously impair the image quality. In the case of an optical transparency these can be minimized by wetting the emulsion with a liquid having the same refractive index and then sandwiching the plate between optical flats.

Table 1 gives the image co-ordinates in terms of the parameters introduced in Fig 5. m is the scale change factor, and μ the ratio of the reconstruction wavelength to the formation wavelength.

By dividing numerator and denominator of each term in Table 1 with $z_c z_0 z_r$, and re-arranging, it is possible to obtain reciprocal relations for the holographic process similar to those which occur in conventional imaging. It is also easy to include the situation where the reconstructing wave and/or the reference wave is a plane wave inclined to the hologram. In such a case z_c or z_r goes to ∞ but x_c or x_r remains finite and the ratios x_r/z_r and x_c/z_c go to $\tan \theta_r$ and $\tan \theta_c$ respectively.

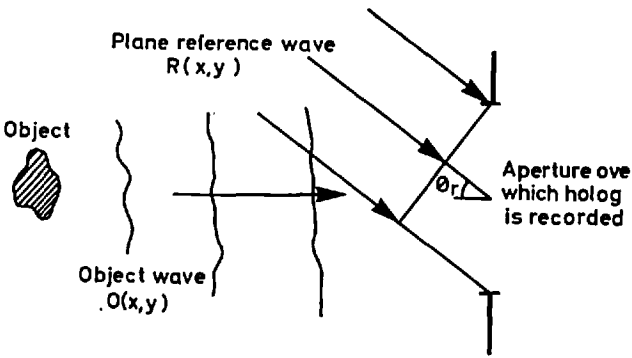


Fig 3 The skew arrangement for hologram formation in the case of a plane reference wave which is inclined at an angle θ_r to the recording medium, and which has a complex amplitude $R(x, y)$

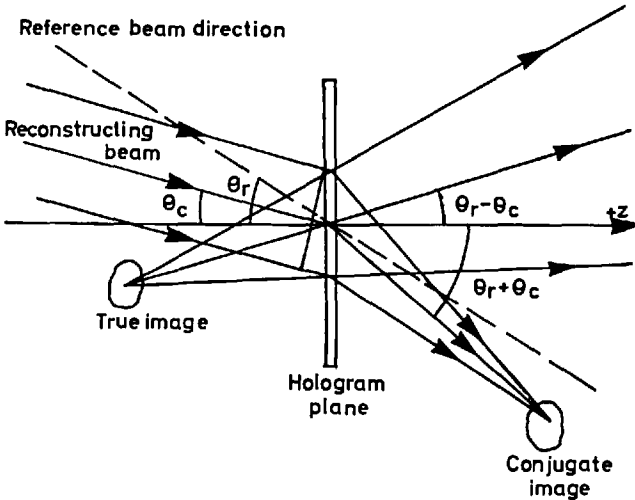


Fig 4 True and conjugate image positions relative to the hologram, for the same case as Fig 3

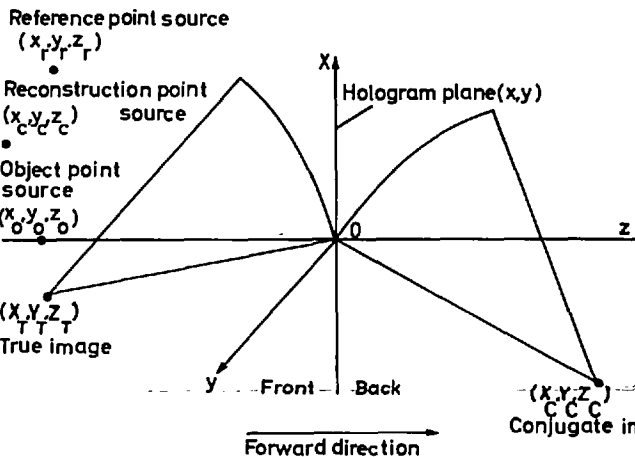


Fig 5 The approximate co-ordinate system for plotting the relations between object and image positions for any positions of object, reference and reconstruction point sources

Table 1 Object co-ordinate—image co-ordinate relations

$$\begin{aligned}
 X_T &= \frac{m^2 x_C z_O z_R + \mu m x_O z_R z_C - \mu m x_R z_C z_O}{m^2 z_O z_R + \mu z_C z_R - \mu z_C z_O} \\
 Y_T &= \frac{m^2 y_C z_O z_R + \mu m y_O z_R z_C - \mu m y_R z_C z_O}{m^2 z_O z_R + \mu z_C z_R - \mu z_C z_O} \\
 Z_T &= \frac{m^2 z_C z_O z_R}{m^2 z_O z_R + \mu z_C z_R - \mu z_C z_O} \\
 X_C &= \frac{m^2 x_C z_O z_R - \mu m x_O z_R z_C + \mu m x_R z_C z_O}{m^2 z_O z_R - \mu z_C z_R + \mu z_C z_O} \\
 Y_C &= \frac{m^2 y_C z_O z_R - \mu m y_O z_R z_C + \mu m y_R z_C z_O}{m^2 z_O z_R - \mu z_C z_R + \mu z_C z_O} \\
 Z_C &= \frac{m^2 z_C z_O z_R}{m^2 z_O z_R - \mu z_C z_R + \mu z_C z_O}
 \end{aligned}$$

where $\mu = \lambda_C / \lambda_O$

For the case of forward illumination z_O is always negative but the other co-ordinates may take on any sign. If Z is negative the image is virtual, while if it is positive the image is real, for forward illumination. A positive Z co-ordinate indicates a converging wavefront, whereas a negative Z co-ordinate indicates a diverging wavefront.

Table 2 shows the True image and Conjugate image co-ordinates respectively, for five different sets of initial conditions. Lateral and longitudinal magnifications are also shown. Y co-ordinates are not given for either of the two images but they may be obtained simply by changing x coordinates in the appropriate expressions.

Longitudinal magnification is defined by: $\frac{dZ}{dz_O}$

and

Lateral magnification is defined by: $\frac{dX}{dx_O}$

It can be shown that:

$$(M_T)_{LONG} = \frac{1}{\mu} (M_T)_{LAT}^2$$

$$(M_C)_{LONG} = \frac{-1}{\mu} (M_C)_{LAT}^2$$

Consequently if $M_{LAT} = \mu$ then $M_{LONG} = M_{LAT}$ and perfect three-dimensional imaging will result.

Example A illustrates the general case where both reference and reconstructing beams are plane waves inclined at θ_R and θ_C to the hologram respectively. If $m = 1$, that is, the size of the hologram is not changed before reconstruction, X_T will have a large positive value and X_C will have a large negative value. Z_T and Z_C will both be very small so the angular position of the True and Conjugate images as seen from the origin are almost $+\pi/2$ and $-\pi/2$, making observation difficult.

Consequently, θ_C must be set equal to zero by putting $x_C = 0$ to produce an observable image, as in example B. Alternatively the scale factor m can be equated to μ . In this latter condition all the Seidel aberrations of the hologram are zero, but the image is greatly demagnified making a further magnification stage necessary.

Example B is very similar to Example A, except that the reconstruction beam propagates along the normal to the hologram plane. With $m = 1$ the diffraction angles of the two images are small.

Example C gives the conditions for maximum magnification, that is, object point source distance equal to reference beam point source distance. The real magnification is not infinite but is limited by the resolution of the system. With this arrangement the object wavefront and the reference wavefront have almost the same curvature, and hence interfere to produce a fringe system, which has relatively large periodicity. If the recording medium has a low intrinsic resolution, as discussed in the following section, the arrangement of Example C will not encounter this resolution limit as rapidly as the arrangement of Example A, because of the larger periodicity of the fringes to be recorded.

Example D illustrates the case of reconstruction with a converging light beam. In this case both True and Conjugate images are real. If $m^2 = \mu$ then X_C and Z_C are finite but X_T and Z_T are infinite, and only the Conjugate image will be reconstructed. In practice it is not possible to set m^2 exactly equal to μ so two images will be reconstructed which are widely separated longitudinally, the True image forming an out-of-focus background to the Conjugate image.

Example E This arrangement is intermediate between Examples B and C indicating the trend of magnification from m , where $Z_T = -\infty$, to ∞ , where $Z_T = -u$.

INTRINSIC ABERRATIONS OF THE HOLOGRAM

By considering higher order terms of the relative phase distribution across the hologram, it is possible to obtain expressions for the intrinsic aberrations of the holographic process. These aberrations are introduced by the technique itself, and are independent of any aberrations of the reference or reconstruction wavefronts. Terms of the third order give the aberrations which are also common to conventional optical imaging systems—the five Seidel aberrations—spherical aberration, coma, astigmatism, field curvature and distortion. The expressions for these aberrations are somewhat complicated and it would be superfluous to reproduce them here. In any practical system, we are more interested in eliminating them in order to maximize the image quality. The conditions under which the five Seidel aberrations of the hologram may be minimized are as follows.

- 1 **Spherical aberration.** If $\mu = m$ when $z_R = z_C = \infty$ for both true and conjugate images (H_T and H_C), spherical aberration is removed. It is also removed for both images if $z_T = z_O$, regardless of other parameters.
- 2 **Coma.** If $z_C = z_R = \infty$, and the ratios x_C/z_C and x_R/z_R are finite, then coma due to skewness of the reference beam is minimized for one image at a time, if $x_C/z_C = -\mu x_R/mz_R$. Coma is removed if $\mu = m$. It is also removed for both images, if $z_T = z_O$ and $z_C = \pm mz_O$.
- 3 **Astigmatism.** If $z_C = z_R = \infty$, $\mu = m$ and $x_C/z_C = -\mu x_R/mz_R$ (which made coma disappear), astigmatism is also removed for one image at a time (H_C), and positive signs replace negative signs in minimal conditions for H_T . If $z_T = z_O$ and $x_C/z_C = -\mu x_R/mz_R$, astigmatism is removed if $z_C = -\mu z_O$ for the conjugate image; this converging wave condition is incompatible with the condition for removing coma, unless $m = \mu$.
- 4 **Field curvature.** The conditions which remove astigmatism also remove field curvature.
- 5 **Distortion.** If $x_C/z_C = -\mu x_R/mz_R$ and $m = \mu$, distortion is removed for all values of z_O , z_T and z_C .

To summarize, it can be seen that all aberrations can be made to disappear simultaneously for one image by using parallel reference and reconstructing beams, by giving the latter an angle of inclination to the Z axis equal to that of the reference beam, but in the opposite direction, and by scaling the hologram by the same factor as the wavelength ratio. The lateral magnification is then equal to μ for both images and therefore perfect three-dimensional imaging will always be achieved. If $z_O = z_T$, spherical aberration is always zero, but coma and astigmatism cannot be removed simultaneously.

Table 2 True and Conjugate image co-ordinates for initial conditions in ultrasound holography

Initial conditions	X_T	X_C	Z_T	$\frac{Z}{X_C}$	$\frac{dX_T}{dx_0}$	$\frac{dX_C}{dx_0}$	$\frac{dZ_T}{dz_0}$	$\frac{dZ_C}{dz_0}$
A $z_C = -\infty = z_T$ $z_0 = -u$ x_T, x_0, x_C are finite	mx_0 $+ \frac{m^2u}{\mu} \tan \theta_C$ $-mu \tan \theta_T$	mx_0 $\frac{-m^2u}{\mu} \tan \theta_C$ $-mu \tan \theta_T$	$\frac{-m^2u}{\mu}$	$\frac{m^2u}{\mu}$	m	m	$\frac{m^2}{\mu}$	$\frac{-m^2}{\mu}$
			(virtual)	(real)				
B $z_C = -\infty = z_T$ $z_0 = -u, x = 0$ x_T, x_0 are finite	mx_0 $-mu \tan \theta_T$	mx_0 $-mu \tan \theta_T$	$\frac{-m^2u}{\mu}$	$\frac{m^2u}{\mu}$	m	m	$\frac{m^2}{\mu}$	$\frac{-m^2}{\mu}$
			(virtual)	(real)				
C $z_T = -u = z_0$ $z_C = -\infty, x_C = 0$ x_T, x_0 are finite	∞	∞	∞	∞	∞	∞	∞	∞
			(If $ Z_0 > Z_T $, Z_T Z_C are positive. If $ Z_0 < Z_T $, Z_T , Z_C are negative.					
D $z_C = u, z_0 = -u$ $z_T = -\infty, x_C = 0$ x_T, x_0 are finite	$\frac{-m\mu(x_0 - u \tan \theta_T)}{m^2 - \mu}$	$\frac{m\mu(x_0 - u \tan \theta_T)}{m^2 - \mu}$	$\frac{m^2u}{m^2 - \mu}$	$\frac{m^2u}{m^2 + \mu}$	$\frac{-m^2\mu}{m^2 - \mu}$	$\frac{m\mu}{m^2 + \mu}$	$\frac{m^2\mu}{(m^2 - \mu)^2}$	$\frac{-m^2\mu}{(m^2 + \mu)^2}$
			(real)	(real)				
E $z_C = -\infty, z_T = -2u$ $z_0 = -u, x_C = 0$ x_T, x_0 are finite	$m(2x_0 - x_T)$	$m(2x_0 - x_T)$	$\frac{-2m^2u}{\mu}$	$\frac{2m^2u}{\mu}$	$2m$	$\frac{2m}{\mu}$	$\frac{4m^2}{\mu}$	$\frac{-4m^2}{\mu}$
			(virtual)	(real)				

LIMITS OF RESOLUTION OF HOLOGRAPHIC TECHNIQUE

The resolution of an ultrasonic imaging system is the most important factor, by reference to which its usefulness may be determined. Consequently it is important to arrange for an ultrasonic imaging system to have maximum possible resolution. There are five major factors which influence resolution of an ultrasonic holographic system:

- 1 Aperture of the ultrasound hologram, that is, the lateral dimensions of the ultrasound field over which the pressure distribution is recorded to form the hologram
- 2 Intrinsic resolution of the ultrasound recording technique
- 3 Ultrasound reference beam source size
- 4 Intrinsic aberrations of the hologram
- 5 Aberrations introduced by the reference beam and re-construction beam

Points 1 and 2 can be discussed in terms of the Abbé theory of image formation in coherent radiation.⁵ Briefly, the theory states that as far as its diffraction characteristics are concerned an object can be analysed into an equivalent series of sinusoidal diffraction gratings each with a different period. The object diffraction pattern is then thought of as the superposition of the individual diffraction patterns for the whole series of gratings. Before any object detail appears in an image at least two orders of diffraction from a component grating must be accepted by the aperture of the imaging system. The resolution will then be equal to that grating

period. For one particular wavelength of radiation λ , the angle of diffraction θ of the spectra produced by a diffractive grating of period p is given by $\lambda = p \sin \theta$. Consequently as p tends to λ the value of θ tends to $\pi/2$. Any holographic system can only form a perfect image of an object if it records all the diffraction spectra. This would require the recording medium to be infinite in extent. Consequently any real system can only accept spectra at diffraction angles up to a maximum value determined by the linear aperture of the system and its distance from the object. The grating period corresponding to this maximum angle of diffraction is then defined as the smallest object dimension resolvable by the system. It is given by:

$$p_{min} = \frac{\lambda}{\sin \theta_{max}}$$

Fig 6 illustrates how the resolution of a holographic system is limited by its aperture. There may, however, be a resolution limit lower than that defined by the aperture, because of the intrinsic resolution of the hologram recording medium. The diffraction spectra from the sinusoidal grating of period p_{min} are plane waves incident on the hologram recording medium at angles $\pm \theta$ and consequently produce groups of spatial fringes of period p_{min} on the hologram, as shown in Fig 6. If the recording medium has an intrinsic resolution such that two fringes separated by a distance p_{min} are not recorded separately then the smallest resolvable object dimension will be greater than p_{min} and equal to the intrinsic resolution of the recording medium, independent of the aperture. The minimum possible value of p is equal to λ .

Therefore, if the intrinsic resolution of the recording medium is equal to the ultrasound wavelength, or less, the recording medium will not limit the resolution. In the skew reference beam system, the angle of incidence may be adjusted so that the carrier fringe spacing is not limited by the intrinsic resolution of the hologram recording medium. The same resolution limits apply to the object in this case, as the object information is recorded as a modulation of the carrier fringe system. There are several possible ultrasound detection methods which could be used to record the ultrasound hologram, and which have an intrinsic resolution of the order of the ultrasound wavelength. The resolution of such a holographic system would then depend on factors 1, 3, 4 and 5.

3 The effect of the reference beam source size on the reconstructed image is to 'spread out' the size of an object point image, until it is the same size as the source point image. Provided that the angle subtended at the hologram by the source is smaller than that subtended by the smallest resolvable dimension of the object, the resolution of the image will not be impaired.

4 The effects of hologram aberrations will be more conspicuous for objects close to the hologram recording plane, so it is important to minimize them, as indicated in the previous section.

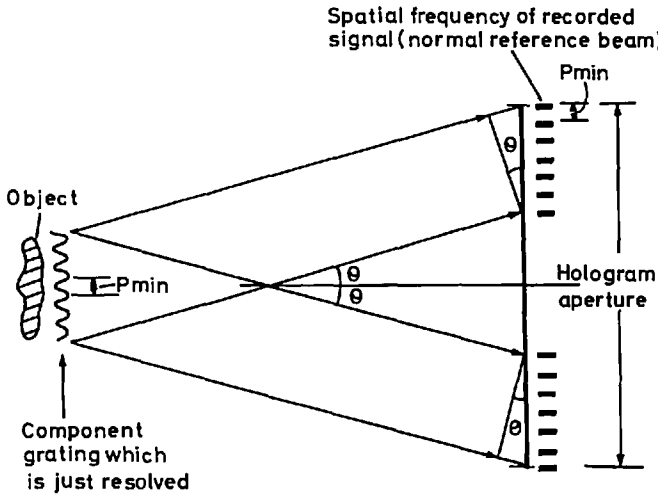


Fig 6 The limitations imposed on the resolution of a holographic system by its aperture

Table 3 List of electrical ultrasound detectors

Detector	Sensitivity [W/cm ²]	Resolution	Image formation time [secs]
Ultrasound cameras:			
1 Quartz plate converter ⁶	(2 × 10 ⁻⁸) 7 × 10 ⁻⁷	Approximately 1 to 2 times thickness of converter plate in ideal conditions. Resolution of quartz is less than BaTiO ₃ , because of greater lateral mechanical coupling of quartz.	Charge image formed in approximately 10 cycles of the ultrasonic field. The image is displayed at the frame rate of the system used.
2 BaTiO ₃ converter plate ^{7, 8}	(6 × 10 ⁻¹⁰) 3 × 10 ⁻⁹		
3 Lead zirconate titanate converter plate ⁹	(10 ⁻¹¹)		
Pulsed ultrasound camera ^{1, 10}	[Similar to continuous wave camera]	[Approaching wave theory limit]	As above
Koch electron mirror camera ¹¹	0.05	Several ultrasonic wavelengths	[Less than 0.02]
Mechanically scanned quartz plate ¹²	3 × 10 ⁻¹²	Approximately 1 to 2 times thickness of converter plate	3-120, depending on method of scanning

Table 4 List of thermal, mechanical and chemical ultrasound detectors

Detector	Sensitivity [W/cm ²]	Resolution	Image formation time [secs]
Pohlman cell ^{13, 14}	2.5 × 10 ⁻³ (2.8 × 10 ⁻⁷)	At least several ultrasonic wavelengths. Typically a few mm	Less than 1.0, but up to 1200, at maximum theoretical sensitivity
*Surface deformation ¹⁵	1.5 × 10 ⁻³ , by light reflection	Typically a few mm	[Less than 1.0]
Stimulation and exhaustion of phosphorescence	0.05	Approximately 0.2mm	Approximately 60
*Photodiffusion methods:	0.05	Limited to several mm by streaming	30-130
1 Developer solution ¹⁷ (exposed and undeveloped film)			
2 Reversal bath and developed film ¹⁸	[greater than 1.0]	As 1 above	40-50
3 Fixer bath ¹⁸	1.0	As 1, 2 above	Several 10's
Starch plate in iodine solution ¹⁹	0.5	As 1, 2, 3 above	60-600

5 Any differences in wavefront aberrations between the reference and reconstruction beams will appear as aberrations of the holographic images. It is difficult, in practice, to obtain an ultrasound wavefront of uniform curvature, and so a recording technique, where it is possible to electronically simulate the reference beam, has a distinct advantage over a technique where this is not possible. With a laser source of reconstructing light, production of a plane wave or one of uniform curvature presents no problem.

DETECTORS FOR ULTRASOUND HOLOGRAPHIC SYSTEMS

After discussing the principles and some of the limits of the holographic technique, it is now relevant to discuss particular parts of the system. Most important is the type of ultrasound detector which may be used to record and visualize the ultrasound hologram.

Tables 3 and 4 give a list of some of the techniques which have been employed in the past for visualizing ultrasound images and fields over a two dimensional area. The list is not complete but the methods excluded have low sensitivities and long image formation times.

The three most important parameters of an experimental system are, sensitivity, resolution, and image formation time. In Tables 3 and 4 experimental values are shown without any form of brackets, square brackets indicate estimated value and round brackets theoretical values. An asterisk denotes a method which has been used as part of a holographic system.

A REVIEW OF ULTRASOUND HOLOGRAPHY

Several ultrasound holographic systems have been reported in the literature. Quite a few of these are substantially the same, although they have been developed by different organizations. These techniques²⁰⁻²⁶ use a mechanically scanned transducer probe to plot out the ultrasonic field. The output from the probe is either used to modulate a small light source, scanned with the probe, whose intensity variations are then recorded photographically, or, alternatively, the probe output is used to Z modulate an oscilloscope, from which an intensity picture is recorded, line by line, by a Polaroid camera. In other reports, the techniques of surface deformation²⁷ and photodiffusion²⁸ have been used.

Fig 7 shows a block diagram of the type of arrangement for producing and visualizing ultrasound holograms using the mechanical scanning technique. This type of system has been used successfully to demonstrate ultrasound holography.²⁴ Several objects have been imaged in this manner by K. Preston and J. L. Kreuzer,²⁴ including a steel bolt, as shown in Fig 8. The bolt is 12.7mm in diameter and has one thread every 2mm at the end. The wavelength of the ultrasound used was 0.3mm and was propagated as 100μsec pulses of a 5MHz carrier. As can be seen by comparing photograph (b) with the reconstructed image (a), the resolution is better than 0.5mm. Visualization and recording of the hologram was performed photographically with a 1/10 reduction in scale. No further reduction in size of the transparency was found to be necessary before reconstructing, which was carried out with a converging laser beam. This produced two real images of the object in a manner similar to example D in Tables 2 and 3, dealing with object-image relations.

The surface deformation method has been used to visualize ultrasound holograms.²⁷ This method is interesting, because it could be used for real-time holography, by reflecting a laser beam directly from the phase hologram formed by the surface deformation and automatically reconstructing the image. It was found, however, that the quality of images produced by this real-time method was not as good as that produced by first photographing the surface deformation pattern with incoherent light, and reconstructing the images from the resulting transparency. The images suffer from aberrations introduced by the ultrasonic reference beam, and the method is susceptible to extraneous vibrations.

P. Greguss of Hungary was the first person to publish pictures of actual ultrasound holograms.²⁸ Both object wave and reference wave were produced ultrasonically and the holograms were visualized through the effect of ultrasound intensity on the rate of solution of silver halide from a uniformly

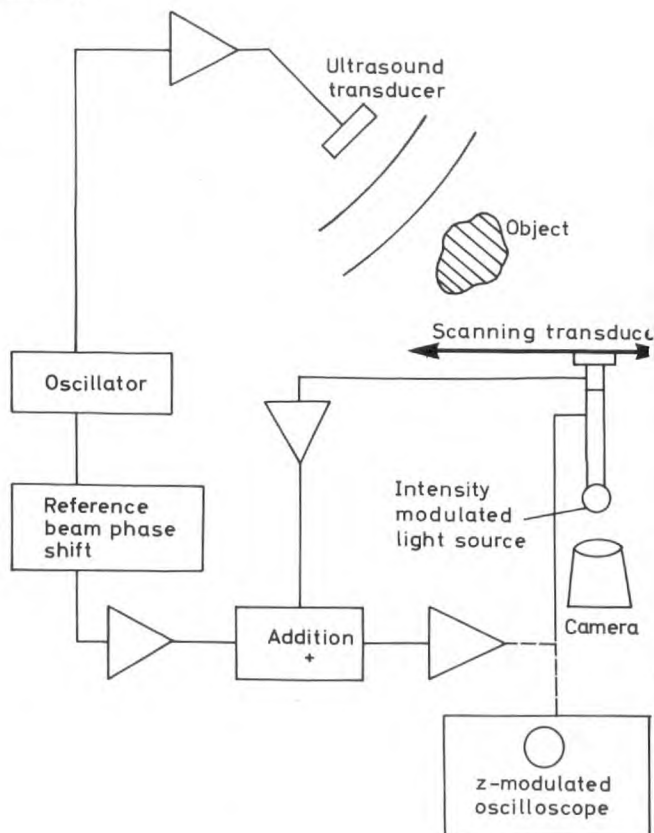


Fig 7 A block diagram of the mechanical scanning technique for producing and visualizing ultrasound holograms

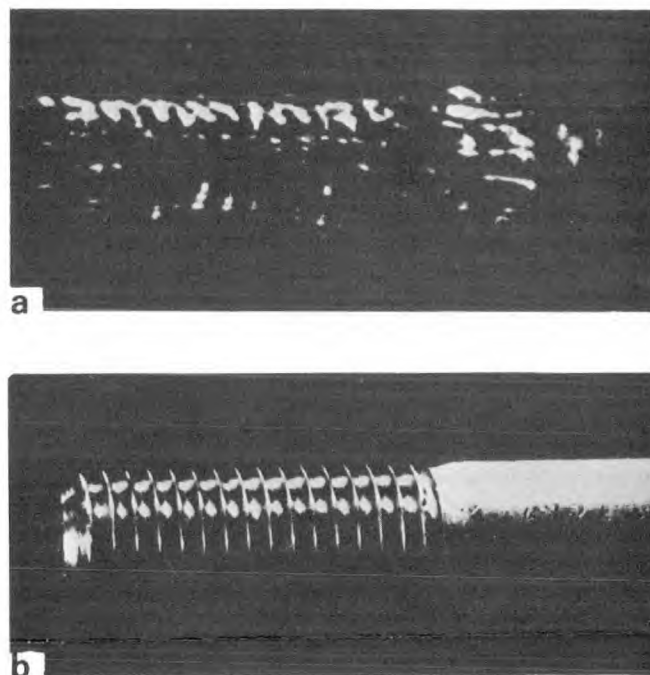


Fig 8 (a) The reconstructed image of the ultrasound hologram, obtained from a steel bolt by the mechanical scanning technique. The resolution is better than 0.5mm, as can be seen by comparing it with (b), the ordinary photograph of the steel bolt, which has one thread every 2mm

exposed film, which was insonified in a bath of fixer solution. The silver halide is dissolved away more quickly at regions of high intensity than at regions of low intensity, because of the effect of ultrasound on the rate of diffusion of fixer molecules to the film. After several minutes exposure at quite high levels, the film is removed and developed in daylight to produce a permanent record. Although Greguss has demonstrated that ultrasound holograms can be visualized by this technique, he has not published results of any attempts to reconstruct images of the original objects holographed.

CONCLUSION

It is possible to indicate future trends, although the technique of ultrasound holography is still in its infancy. Sensitivity and the real advantages of an electronic reference beam seem to demand an electronic detection system, fundamentally based on the piezoelectric or electrostrictive effects. Electronic scanning, as in the ultrasound camera, can obviously result in rapid hologram visualization and potentially in real-time reconstruction, once the problem of conversion of a TV picture into a suitable amplitude or phase hologram has been solved. For the resolution to be useful with this system the receiver aperture, however, must be increased to 20 or 30cm, that is, by a factor of three over those currently available. Recent work in Germany by E. Schätzer,²⁹ has shown that it is possible, in principle, to build wide aperture ultrasound cameras by using as the sensitive element a transducer in the form of a shell, which is very resistant to external pressures. For example, an ultrasound camera, with an aperture of 60cm and resonant at 1MHz, could be built with this new technique and still have a pressure safety factor of eight atmospheres.

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