

CHAPTER 9

BIAXIAL COMPRESSION - TENSION TEST SERIES

9.1. Introduction

The desirability of being able to extend the uniaxial compression and tension, and biaxial compression work into states of biaxial compression-tension was referred to in section 1.7. Clearly a more complete picture of the behaviour of mortar and concrete under biaxial stress states would result together with a better idea of the failure criterion for concrete materials.

9.1.1. State of Stress

It was anticipated that, despite the successful development of the combined stress testing machine, a true biaxial compression-tension state of stress would be difficult to achieve. The results of the proving trials obtained in section 4.5. confirm this. Nevertheless, the proving trials established a certain degree of confidence in the technique and it was considered that some test results, although perhaps less reliable than those for biaxial compression, would provide additional information on the biaxial stress behaviour of concrete.

9.1.2. Strain and Cracking Measurements

The test series was mainly concerned with the stress behaviour although strain measurements were taken on some specimens as specified in section 9.3. using 30 mm. electrical resistance strain gauges.

Ultrasonic pulse velocity measurements were made on all specimens to determine the discontinuity stress as shown in fig.9.3.1.

9.1.3. Concrete Mixes

Two mortars, three normal aggregate concretes, and one lightweight concrete mixes were selected and tested as being representative of aggregate volume concentrations, type, and size, taking into account the limited testing time available.

The mixes used in this test series were listed in section 6.4. but are quoted again below,

MBC	MA
61B	61C
6AB	6LB

Eight tension specimens and three prisms were cast from each mix.

9.1.4. Cleavage Strains at Discontinuity

These strains were measured by the electrical resistance strain gauges referred to above. However a much longer gauge length would have been preferred for these measurements since cracking might easily occur outside the 30 mm. gauge length yet within the uniformly stressed central zone of the specimen.

9.1.5. Rates of Loading

Sequence loading was used in this test series for the reasons explained in section 9.4. Hence the rates of loading specified in section 7.1.5. were used, i.e. 700 p.s.i./min. for compression and 110 p.s.i./min. for tension loads.

9.1.6. Age and Conditions of Specimens at Test

All biaxial compression-tension specimens were tested at 28 days after casting. They were tested in a saturated condition which was maintained by the use of a curing compound during the preparation and testing as described in section 6.9.

9.2. Machines and Equipment Used

The testing machine used was the confined stress testing machine with tension jacks mounted on the horizontal sub-frame as shown in fig.2.7.6. The tension grips from the Ward uniaxial tension machine were used to grip the specimen and were supported on a specially made set of rollers and tracks.

Much of the arrangement set up in the machine can be seen in fig.9.2.1. with a tension jack, a set of grips, part of a proving trial specimen, and the support frame clearly visible.

The test specimen used was the modified tension specimen described in section 7.2. and arranged in the manner shown in fig.9.9.2. The grips held the specimen in the usual way as explained in Chapter 7 for uniaxial tension tests but were now pulled by a jack at each end. Hence they had to be supported and the frame supplying the support is shown in figs.9.2.3. and 9.2.4. A ball race roller was fixed to the ends of the central spindle of the grips and ran on a Dexion angle track.

The track and support frame were all made from Dexion angle which proved satisfactory for the small number of tests performed; it was not considered worth making a more rigid or sophisticated frame until the testing technique could be proved to work. The track was levelled using the nut and bolt arrangement shown in fig.9.2.5. and the columns of the horizontal sub-frame were used as the datum level. Finer adjustments were made using a proving trial specimen in direct uniaxial tension until the strain pattern was uniform. The grips were then considered to be level and ready for testing; this levelling procedure was carried out each time the tension facility was put into the testing machine.

4 in. concrete cubes were used to apply the compression stress from the vertical sub-frame across the 3 in. thickness of the specimen as shown in fig.9.2.2. A pad, comprising two sheets of acetate with a thin layer of a 2:1 mixture of tallow and black

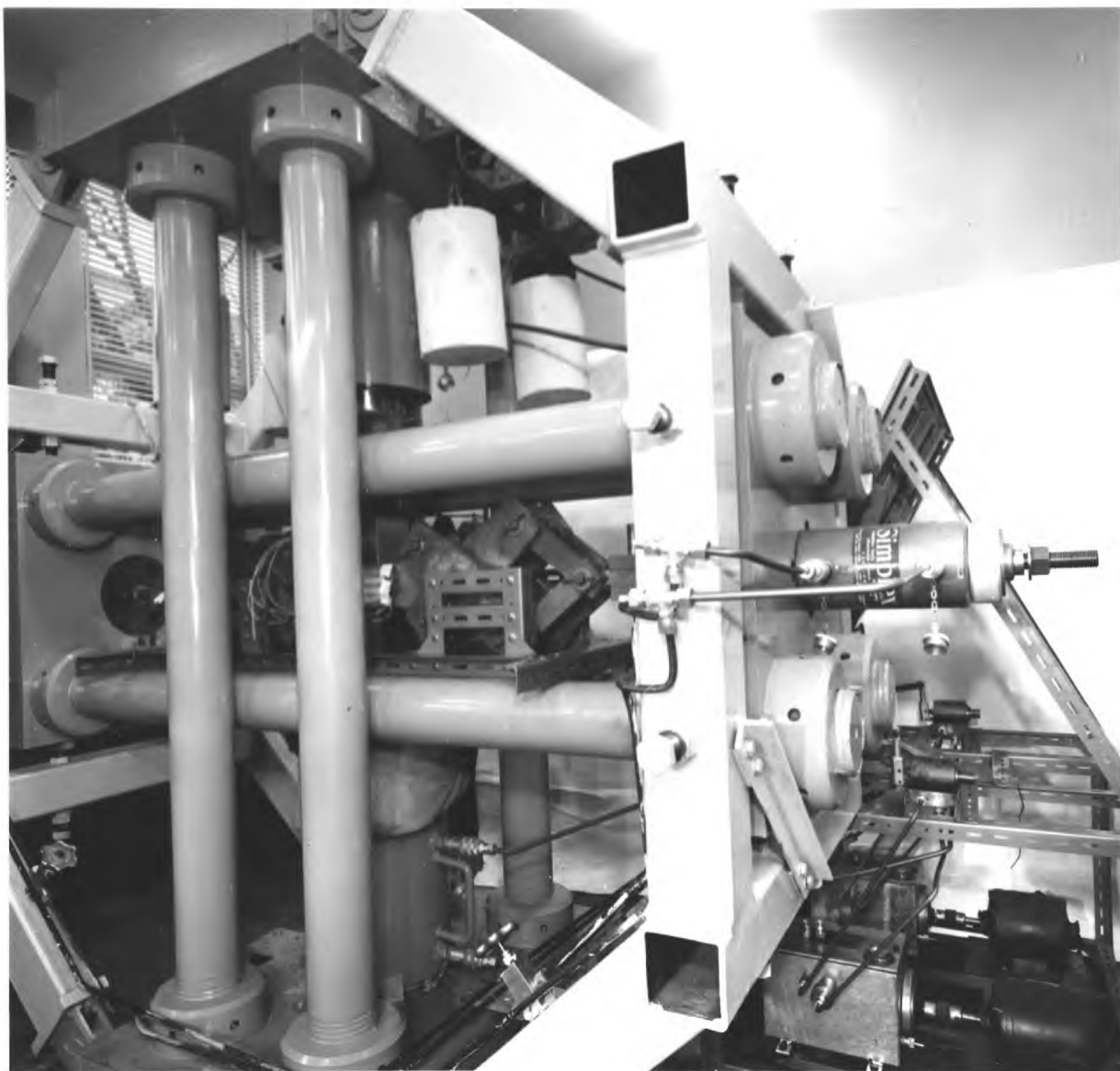


Fig. 9.2.1. Machine set-up for biaxial compression-tension tests showing tension calipers and jack.

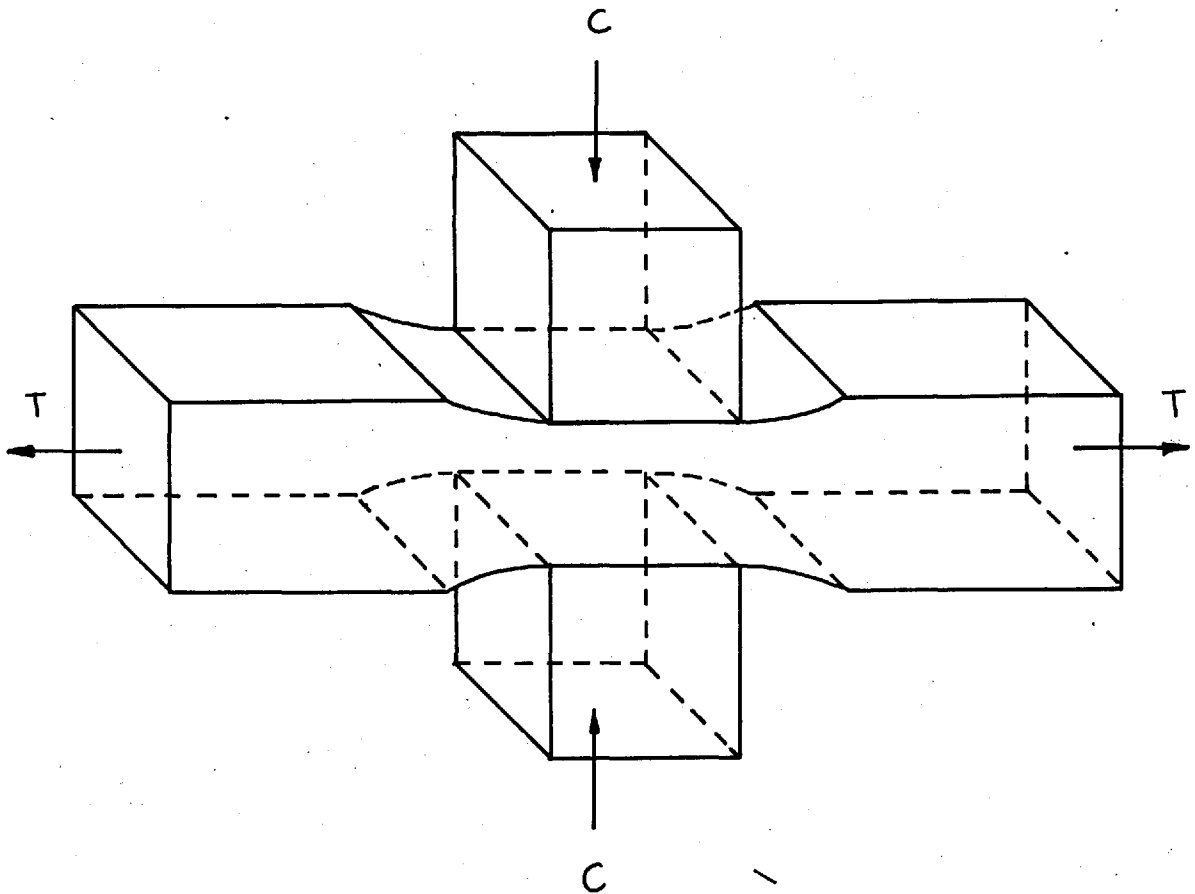


FIG. 9.2.2. USE OF CONCRETE CUBES TO APPLY COMPRESSIVE STRESSES TO TENSION SPECIMEN.



Fig. 9.2.3. General arrangement of Tension Calipers and support frame for horizontal operation.

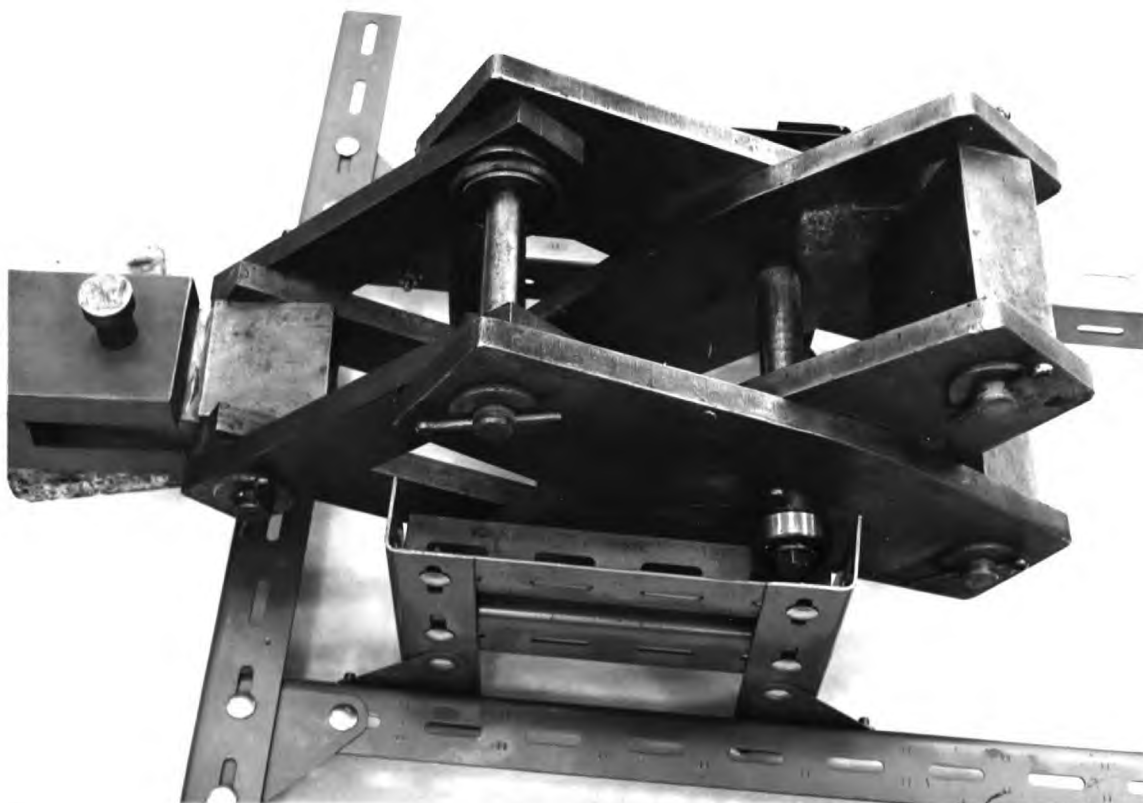


Fig. 9.2.4. Roller Support on Track for Tension Calipers.



Fig. 9.2.5. Levelling Screw Arrangement for Roller Support Track.

lead between them, was placed between each cube and the test specimen to reduce the frictional restraint effects of the cube on the test specimen. These frictional restraint effects were reduced somewhat by the lateral expansion of the cube but the magnitude of this expansion was always less than the direct tensile strain of the specimen and therefore the restraint effects could never be entirely eliminated. A concentration of tensile strain would tend to build up on the test specimen at the edges of the cube and cause a premature failure at that section. However, the friction reducing pads described above reduced this strain concentration and many failures were reported under the cubes as noted in section 9.5.

Ancillary equipment consisted of the Solartron automatic data logger for recording the surface strains and the ultrasonic pulse velocity equipment (see Chapter 5).

9.3. Testing Arrangement

The arrangement of the test specimen in the testing machine has been shown in figs. 9.2.1. and 9.2.2.

However, a variation in the loading arrangement in the vertical sub-frame was the replacement of the bottom loading platen/solid seating unit with a platen/spherical seating unit (described in section 3.4.7.). This change was made to accommodate any misalignment between the 4 in. concrete loading cube and the underside of the test specimen whose inclination, if any, to the horizontal would be dictated by the tension grips in the horizontal sub-frame and not by the bottom loading platen as in the case of biaxial compression tests.

Figure 9.3.1. shows the location of the electrical resistance strain gauges within the 4" x 3" central zone and the location pulse velocity transducers which is similar to the arrangement adopted in the uniaxial tension tests.

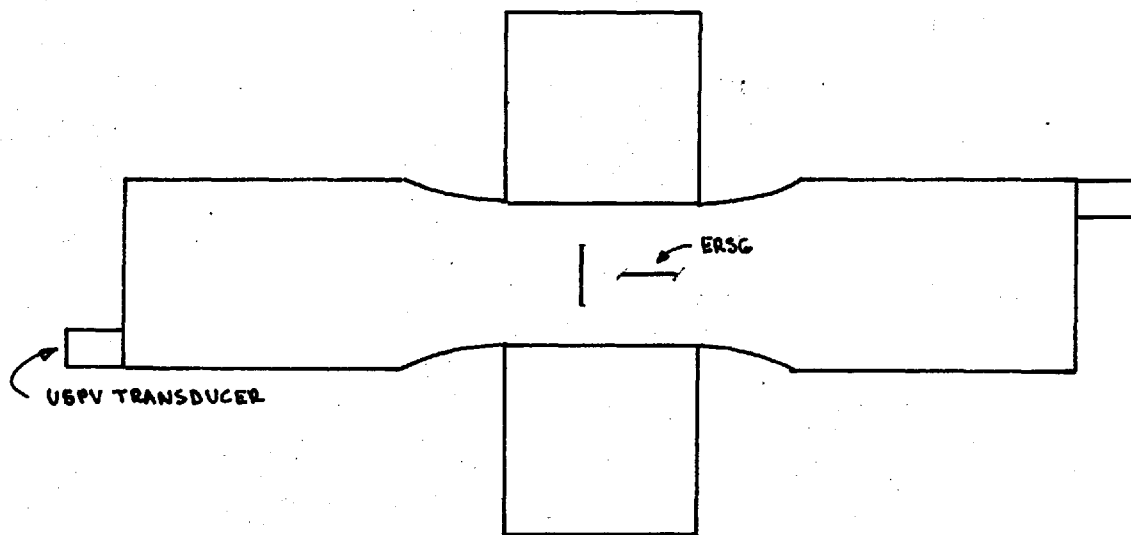


FIG. 9.3.1. STRAIN GAUGE ARRANGEMENT AS TESTED.

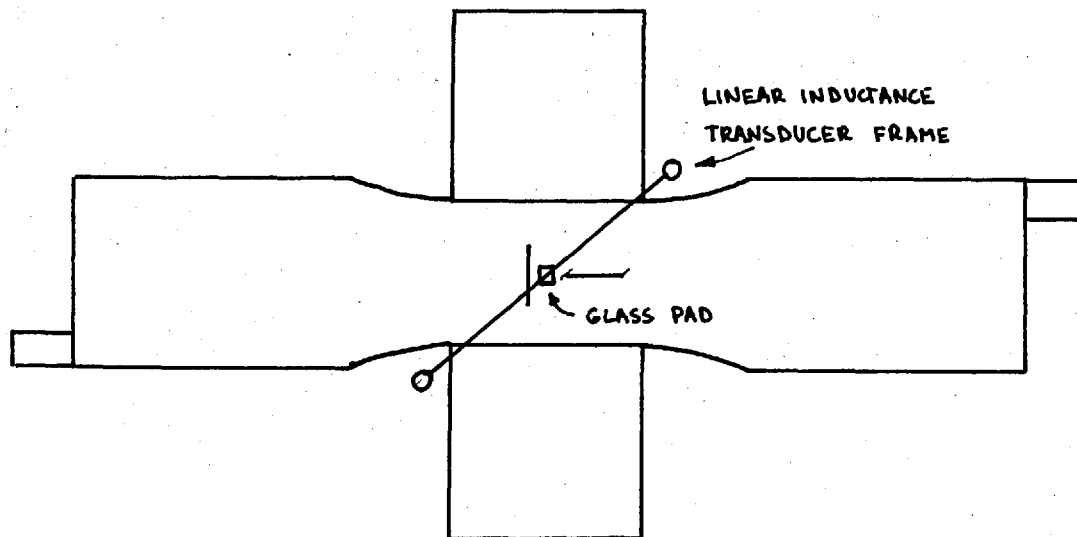


FIG. 9.3.2. 1) ADDITION OF LINEAR INDUCTANCE TRANSDUCER FRAME FOR COMPLETE THREE DIMENSIONAL STRAIN MEASUREMENTS.

9.4. Test Procedure

The setting up of the test specimen in the machine and its alignment with the axes of loading involved a number of new operations and these are described here in detail.

The test specimen was firstly offered up to the tension grips and a process of clamping and reclamping took place until the specimen was correctly positioned within the grips. The positioning was checked on the three points laid down by Ward⁽⁷⁾ which are that the specimen touches the central spindle; that is is square to the central spindle; and that it is in between the two gripping platens an equal amount. Understandably, this proved to be quite a difficult task as the access to the specimen and grips was limited by the machine columns. However, a system of adjustment developes in which each end was unclamped, adjusted, and re-clamped until both ends of the test specimen were satisfactory judged by the checks above.

A nominal tensile stress was then put on the test specimen of about 30 p.s.i. and the bottom platen with its 4 in. concrete cube and tallow/black lead lubricant pad was jacked up to the underside of the specimen. Care was taken to ensure that the cube just stopped short of the underside of the specimen by observing visually the light through the gap between the two surfaces. The danger was that if the cube touched the specimen it might fail or crack it in flexure whilst if it stopped too far away, the other top loading cube would cause failure in the final adjustment described below.

With the bottom cube in this position and the bottom jack locked using the stop valve, the top platen was jacked down and the top 4 in. concrete cube and lubricant pad aligned on the specimen. As with the bottom cube, the top platen was lowered until it stopped just short of the cube as explained above.

At this stage an additional check was made on the position of the centre of the test specimen relative to the axis of the compression platens. Any misalignment was eliminated by moving the specimen along its horizontal axis by turning the nuts on the tension rods on the faces of the jacks. Both nuts were tightened finger tight after satisfactory alignment had been achieved.

The alignment of the concrete cubes with the centre of the test specimen was again checked. Then the two vertical jacks were released and jacked onto the specimen from both sides simultaneously and very slowly. As soon as the hydraulic pressure gauges indicated that the compression load had come onto the specimen, the pressure was very gradually increased until it equalled about 150 p.s.i. The specimen now being under the influence of nominal compression and tension stresses was under the complete control of the testing machine.

Clearly, with this arrangement, an unequal effort by one jack would have a disastrous effect on the specimen loading and state of stress. The double counterweighting of the top loading platen reduced the difference in effort of the two vertical jacks to a minimum whilst the effects of any unequal effort by the two horizontal tension jacks would be reduced by the clamping action of the compression cubes and their hydraulic link. No effects such as flexural failure mechanisms or the pulling of the specimen out of central alignment were noticed in any of the tests.

At this stage the gauge leads were soldered on, if the specimen had electrical resistance strain gauges, and the ultrasonic pulse velocity transducers were fixed to the ends of the specimen. Three loading and unloading cycles at the rates of loading specified below in both tension and compression to a stress of 20% below the estimated ultimate load were then carried out to check that all the ancillary equipment was functioning properly. The specimen was thus loaded to failure.

As noted in section 4.5. more consistend results were obtained if sequence loading was used in which the compressive stress was applied first at 700 p.s.i./min. followed by the tension stress at 110 p.s.i./min. as specified in section 8.1.5. In section 4.4. it was recalled that both McHenry and Karni⁽¹²⁴⁾ and Bresler and Pister⁽⁴⁴⁾ reported that no significant difference between the results of sequential and proportional loading could be found on the ultimate strengths obtained under biaxial compression-tension states of stress. The possibility of significant variations in the discontinuity stress are thrrefore even more impeobable and so the sequence of loading adopted should yield meaningful results. Sequence loading also made it much easier to obtain different points on the compression-tension envelope.

The testing arrangement for the eight compression-tension specimens and three prisms was outlined in section 9.3. in which one compression-tension specimen was tested in uniaxial compression as a link with the results of the three uniaxial prism test results. With most mixes, one of the T5-T8 specimens was also tested under uniaxial tension, to provide a check on the uhiaxial tension results obtained from one of the T1-T4 specimens. This left a total of five specimens to define the slope of the discontinuity and ultimate stress envelopes between the uniaxial axes.

9.5. Results

9.5.1. Tabulated Results

A tabulated summary of the experimental results is given in table 9.5.1.

9.5.2. Stress-Strain Characteristics

As sequence loading was used in preference to proportional loading because of testing technique requirements, the stress-strain

MIX	PRINCIPAL STRESS	ULTIMATE STRESS psi								DISCONTINUITY STRESS psi							
		T ₁	T ₂	T ₃	T ₄	T ₅	T ₆	T ₇	T ₈	T ₁	T ₂	T ₃	T ₄	T ₅	T ₆	T ₇	T ₈
MA	σ_3	0	0	175	450	1270	2280	3150	5100	0	0	175	450	770	1450	3150	2750
	σ_1	508	419	254	200	238	90	108	0	254	200	200	35	144	0	0	0
MBC	σ_3	0	0	350	667	1667	2500	3333	5000	0	0	333	667	1667	1667	2000	2333
	σ_1	310	310	375	120	125	100	40	0	225	250	250	100	75	0	0	0
GIB	σ_3	0	0	667	1667	2000	2667		4333	0		667	850	1000			1500
	σ_1	360	360	180	235	175	115		0	250		100	125	100			0
GIC	σ_3	0		667	1667	2333	3000	3850	4000	0		667	850	1667		2000	2000
	σ_1	290		280	195	180	145	0	0	175		150	125	100		0	0
GAB	σ_3	0	0	333	667	1667	2500	3333	4550	0	0	333	667	1667	1667	2000	2000
	σ_1	400	445	285	250	215	140	110	0	300	300	150	150	50	0	0	0
CLB	σ_3	0	1667	333	667	667	1000	1333	2000	0	1000	333	667	667	1000	1000	1000
	σ_1	275	20	225	205	75	100	40	0	175	0	150	125	50	50	0	0

SIGN CONVENTION SUSPENDED FOR CLARITY — $\begin{cases} \sigma_3 \text{ COMPRESSION} \\ \sigma_1 \text{ TENSION} \end{cases}$

TABLE 9.5.1. BIAXIAL COMPRESSION - TENSION TEST RESULTS.

ULTIMATE STRESS.

- INDICATES TENSION SPECIMEN
- X INDICATES UNIAXIAL COMPRESSION PRISM

DISCONTINUITY STRESS.

- INDICATES TENSION SPECIMEN
- ⊙ INDICATES UNIAXIAL COMPRESSION PRISM

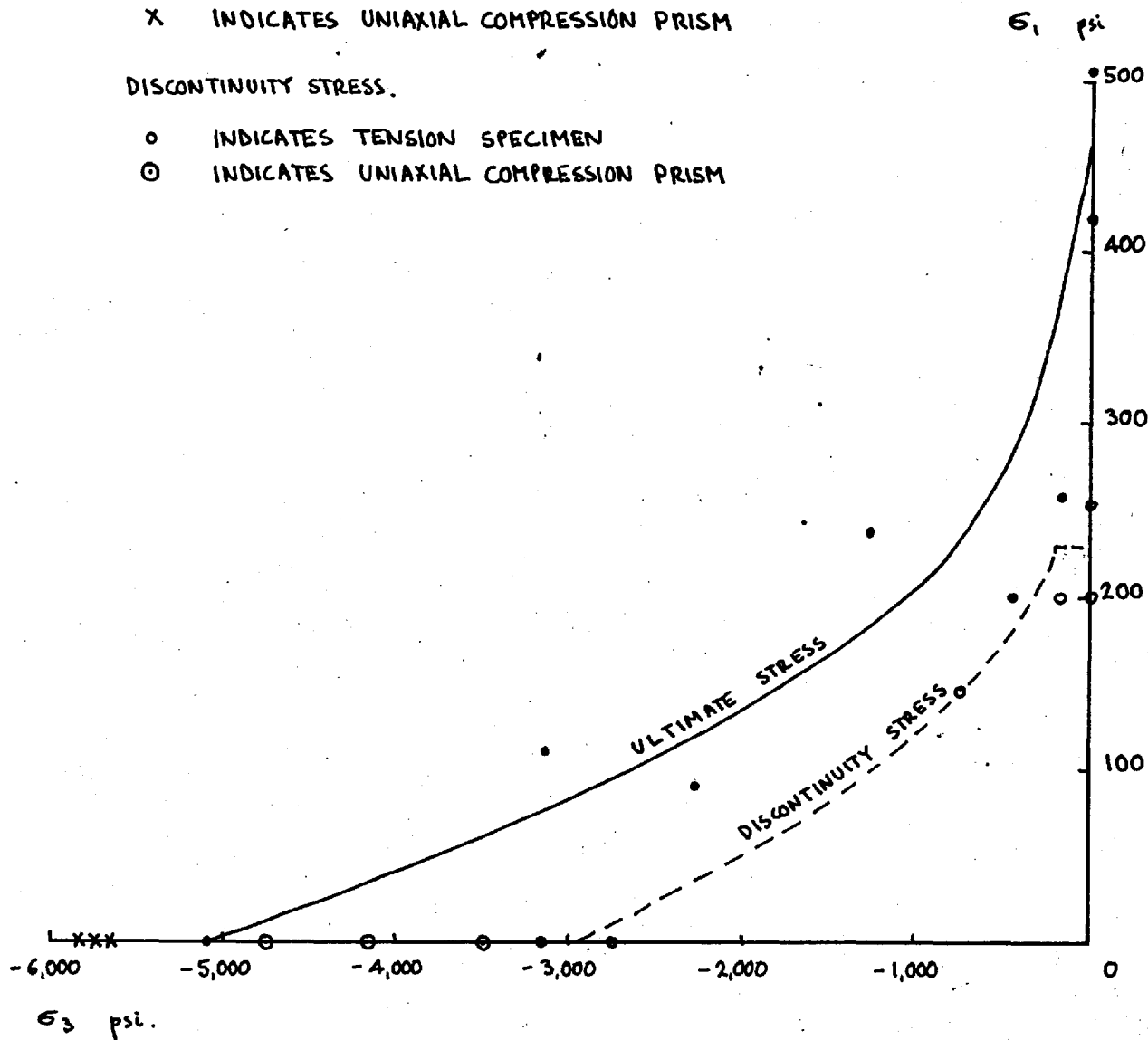


FIG. 9.5.1. BIAxIAL COMPRESSION-TENSION STRESS ENVELOPES
FOR MORTAR MIX MA.

SEE NOTES ON FIG. 9.5.1.

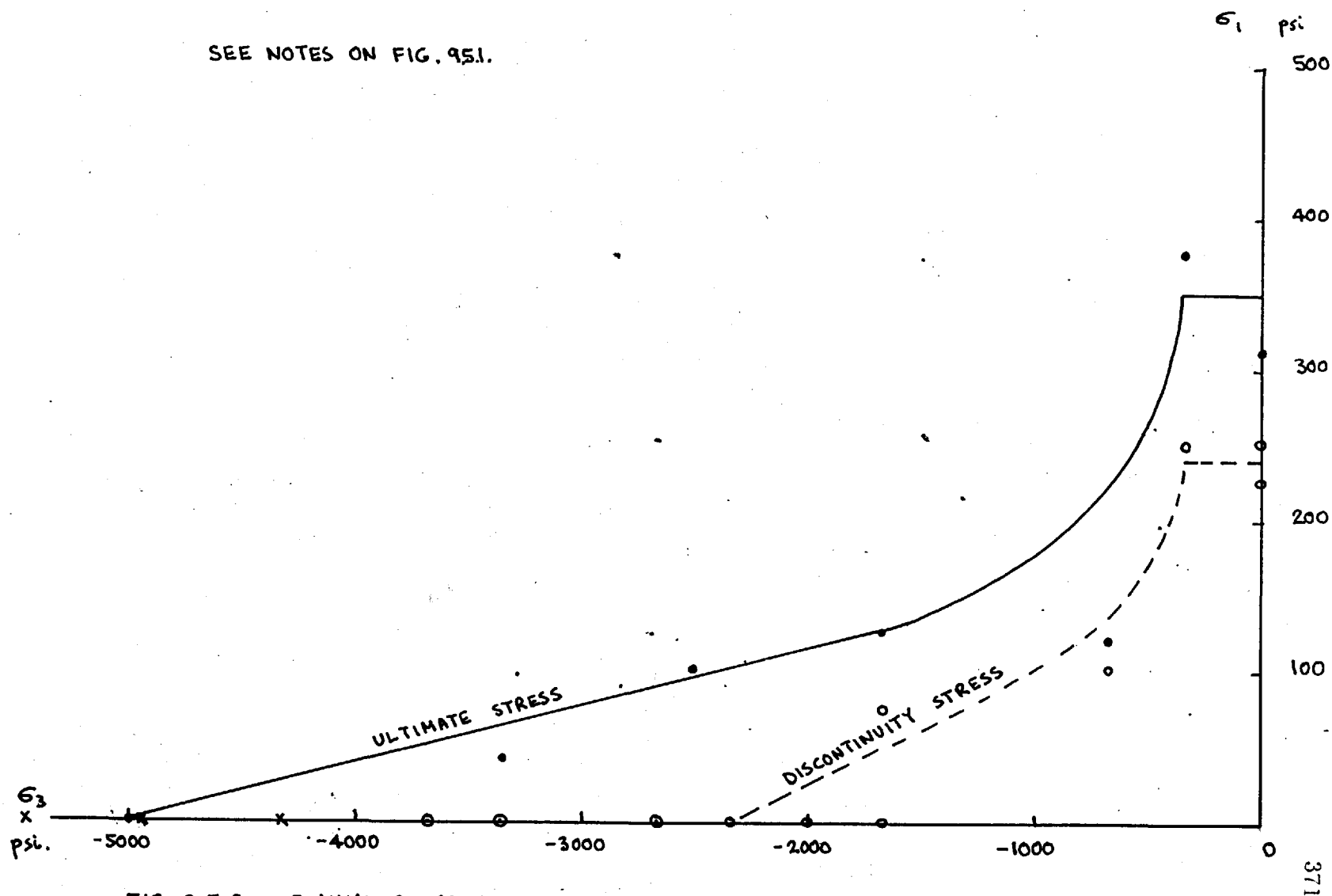


FIG. 9.5.2. BIAxIAL COMPRESSION-TENSION STRESS ENVELOPES FOR MORTAR MIX MBC

SEE NOTES ON FIG. 9.5.1.

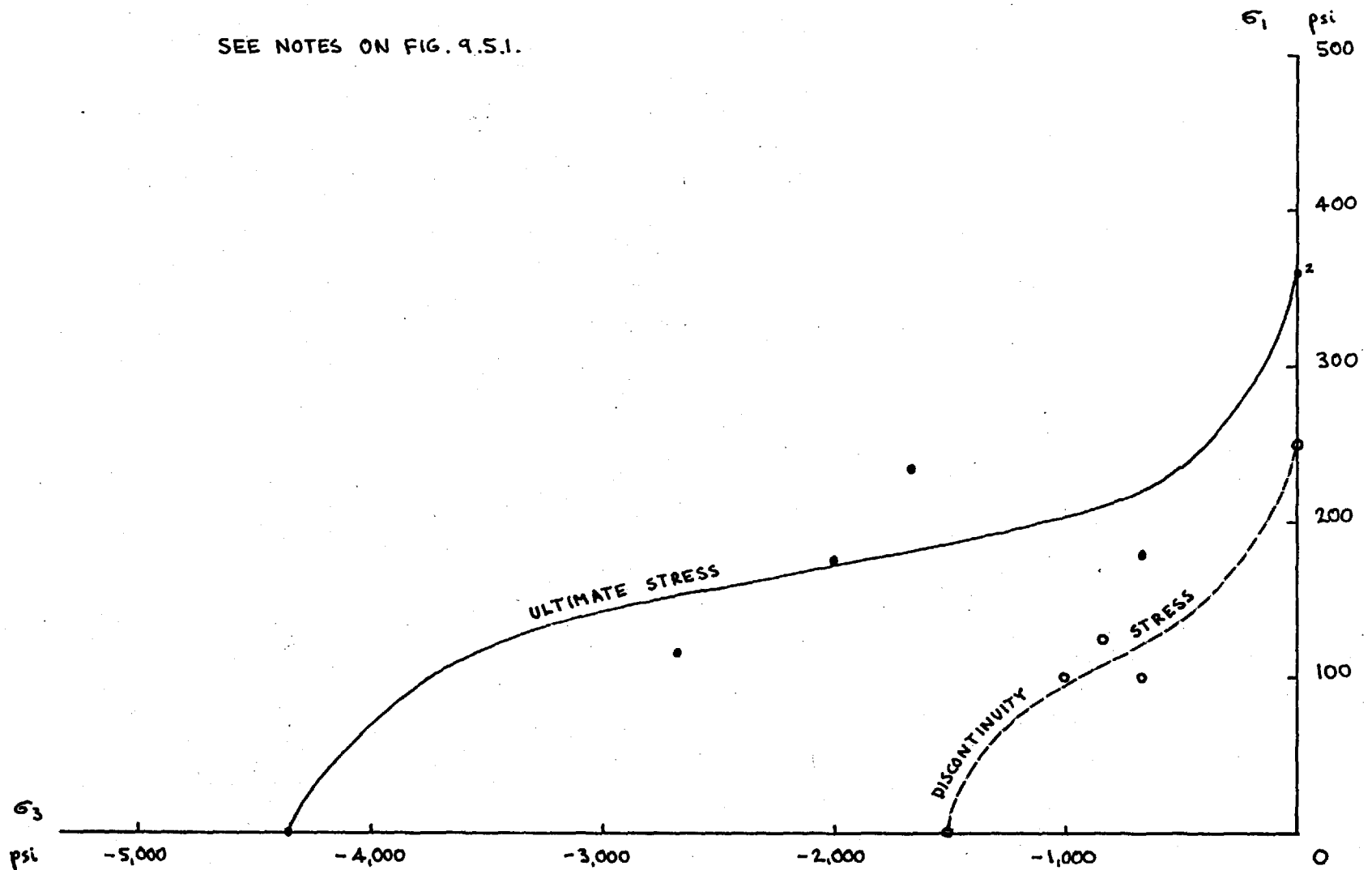


FIG. 9.5.3. BIAxIAL COMPRESSION-TENSION STRESS ENVELOPES FOR CONCRETE MIX 618.

SEE NOTES ON FIG. 9.5.1.

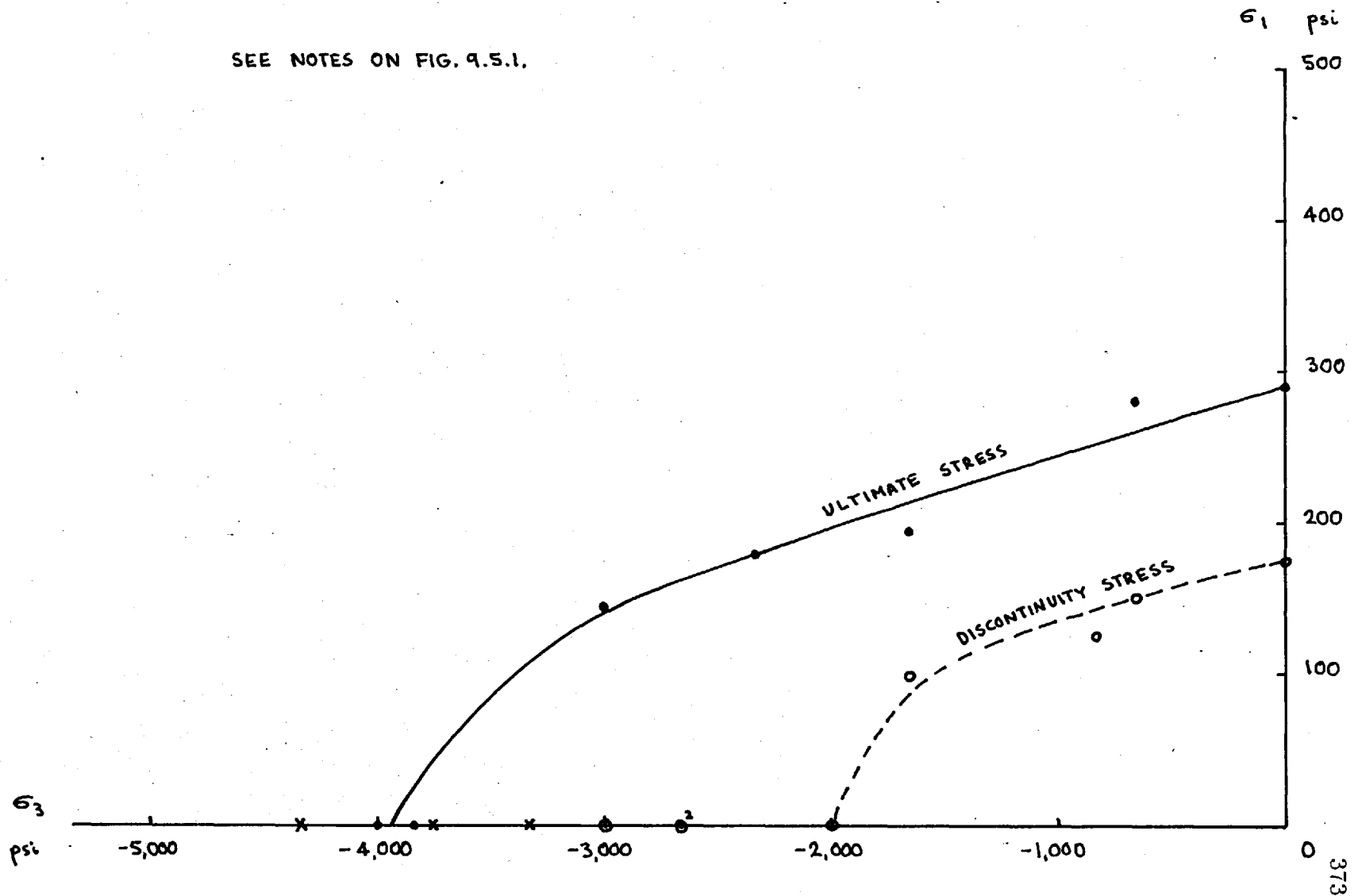


FIG. 9.5.4. BIAXIAL COMPRESSION-TENSION STRESS ENVELOPES FOR CONCRETE MIX 61C.

SEE NOTES ON FIG. 9.5.1.

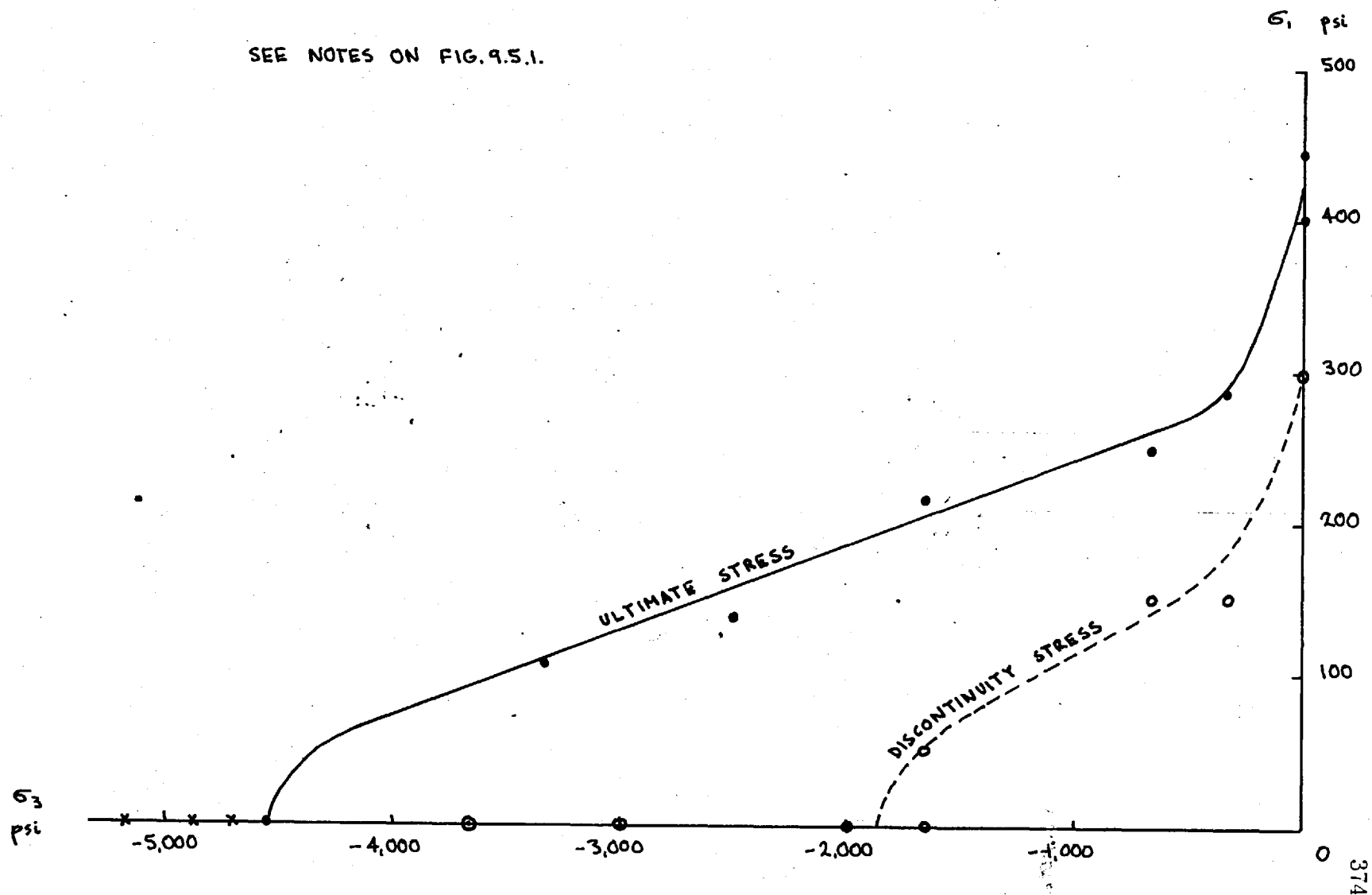


FIG. 9.5.5. BIAxIAL COMPRESSION-TENSION STRESS ENVELOPES FOR CONCRETE MIX GAB.

SEE NOTES ON FIG. 9.5.1.

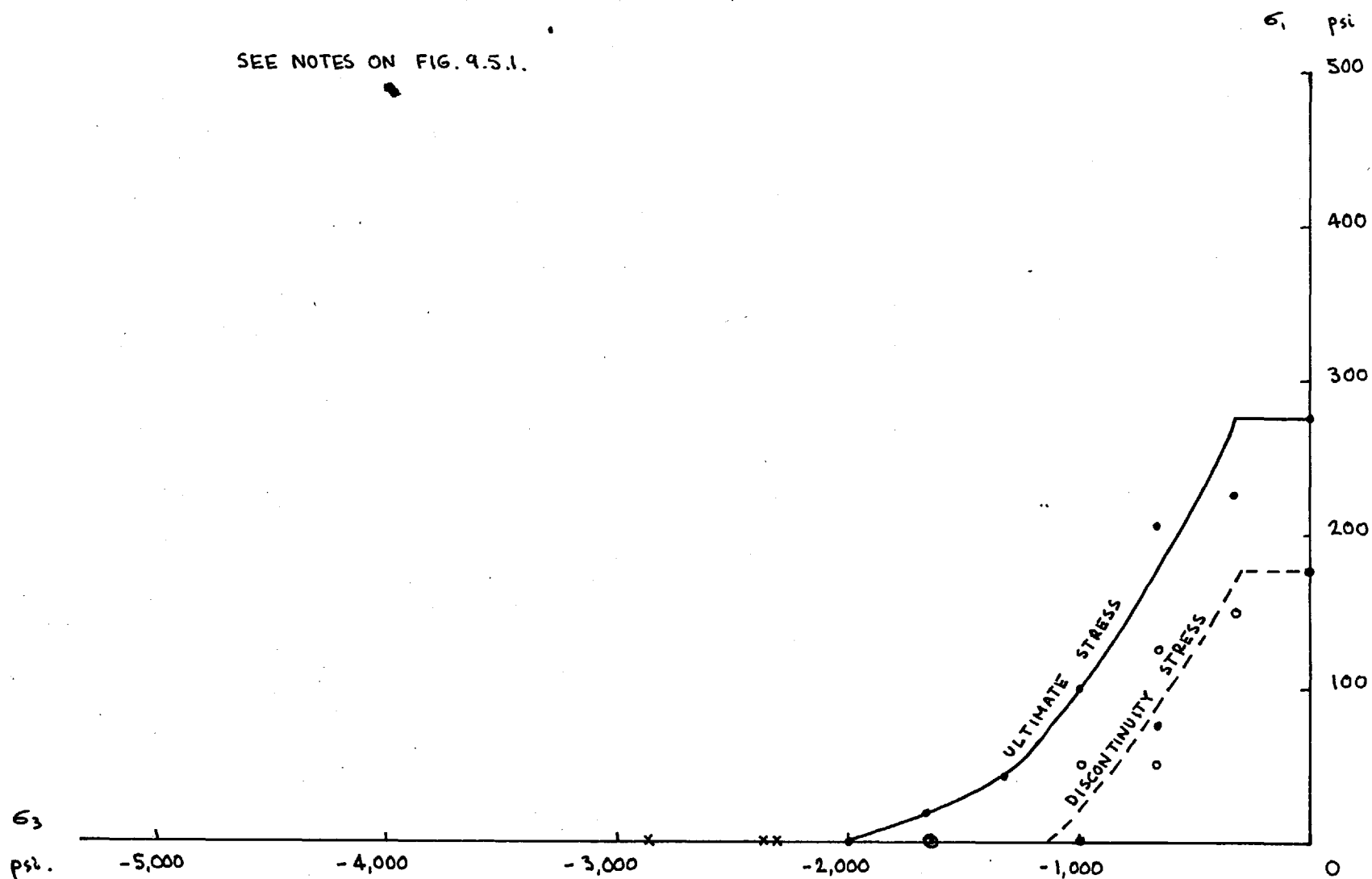


FIG. 9.5.6. BIAxIAL COMPRESSION-TENSION STRESS ENVELOPES FOR LIGHTWEIGHT CONCRETE MIX 6LB.

ULTIMATE STRESSES

NB. POSITION OF BRESLER AND PISTER ENVELOPE MAY BE TOO HIGH AS UNIAXIAL TENSION STRENGTH ESTIMATED.

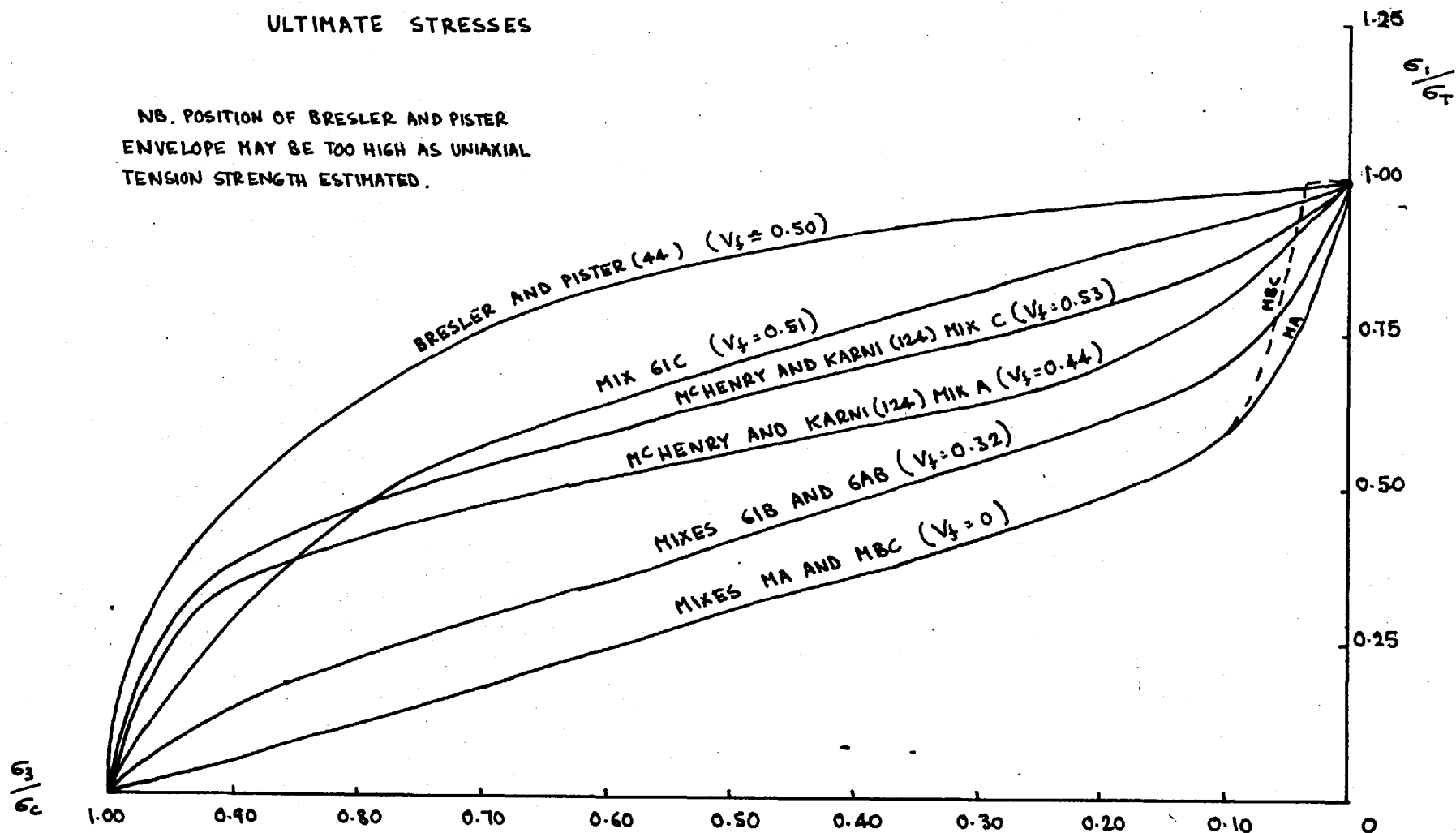


FIG. 9.5.7. UNIT DIAGRAM OF ULTIMATE STRESS

DISCONTINUITY STRESSES

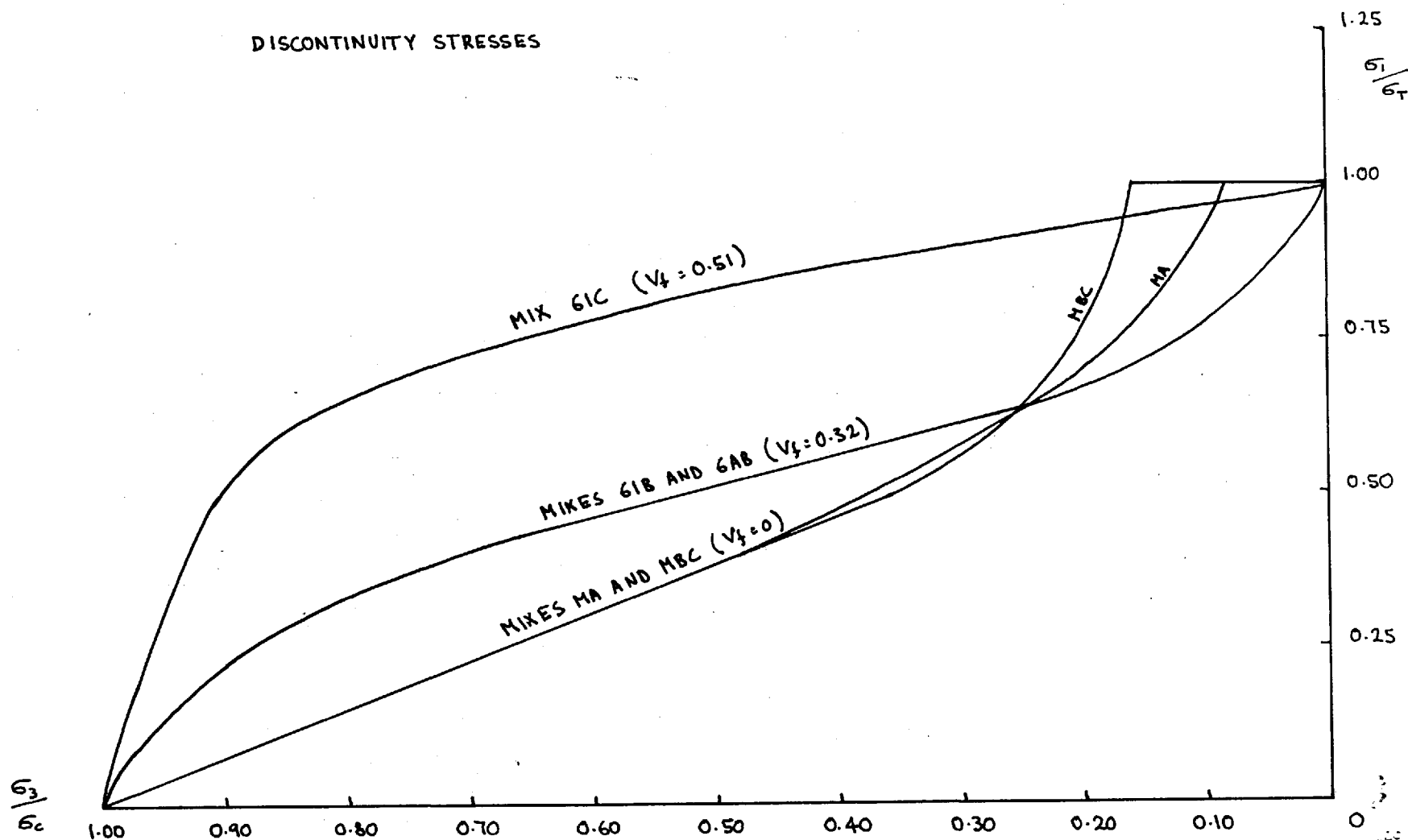


FIG. 9.5.8. UNIT DIAGRAM OF DISCONTINUITY STRESS

characteristics recorded have little significance as they are in part due to uniaxial compression and then to uniaxial tension applied on top of compression. As stated in section 9.1., the object of the tests was to obtain the ultimate and discontinuity stresses primarily with some measurements of the cleavage strain at discontinuity. However, in general, the shape of the curves was similar to those obtained in uniaxial tension and compression.

9.5.3. Discontinuity and Ultimate Stress

The envelopes of discontinuity and ultimate stress are shown separately for each mix in figs. 9.5.1. to 9.5.6. from which it can be seen that several different shapes of envelope exist. The shapes obtained are summarised in fig. 9.5.7. and 9.5.8. in which unit diagrams are shown for ultimate and discontinuity stresses for which both the uniaxial tension and compression stresses are plotted as unity. The results of Bresler and Pister⁽⁴⁴⁾ and McHenry and Karni⁽¹²⁴⁾ have been superimposed on the ultimate stress unit diagram (fig. 9.5.7.) for comparison.

The characteristic obtained from mix 61C, which had the highest volume fraction of coarse aggregate, resembled the results of Bresler and Pister whilst that from mixes 61B and 6AB with a lower volume fraction was similar to the curves of McHenry and Karni. The envelope obtained with the mortars MA and MBC were quite unusual and will be discussed in detail in section 12.2. The volume concentrations of coarse aggregate have been marked on the unit diagram and there appears to be a gradual change of behaviour dependent on this parameter from the convex envelope of the high aggregate content concretes to the concave envelope of the mortars.

The tension test specimen did not produce entirely satisfactory results when tested in uniaxial compression giving ultimate and discontinuity stress values which were consistently $15 \pm 3\%$ lower

than those obtained from the prism tests. This result may have been due to the problems of applying the compression stress in a uniform manner as highlighted in section 4.5. It may also have been due to the greased packing acting as a soft packing and causing premature failure.

9.5.4. Discontinuity Strain

The cleavage strains at discontinuity were recorded on those specimens containing strain gauges but the results as shown in table 9.5.1. were rather scattered due probably to the short strain gauge length. The results have been plotted in Chapter 11 with further discussion.

9.5.5. Failure Modes

The failure modes for both mortars and concretes were single fracture cleavages orthogonal to the direction of the principal tensile stress with a transition to multi-fracture cleavages associated with compression failures starting at about 85% of the uniaxial compression strength.

The failure modes are discussed in full in Chapter 11 together with the results from the uniaxial and biaxial compression test series.

CHAPTER 10

TORSIONAL AND TRIAXIAL TESTS

10.1 Introduction

With problems concerning the degree of reliability of the biaxial compression-tension facility still to be solved, the advisability of investigating other combined states of stress in a research project of limited duration might well be questioned. However, for a number of reasons, the opportunity to develop these other facilities was taken.

Firstly, the design and development of these two facilities of torsion and triaxial compression were a logical extension of the overall development of the combined stress testing machine. Claims were made in the design stage of the machine which need substantiation. Furthermore, as was observed in the review of previous research in section 1.3., there are several ways in which some of the stress combinations can be achieved and this was a unique opportunity to compare different testing techniques designed to obtain the same stress combinations in the same testing machine.

A number of problems are inherent in torsional and triaxial tests and the actual significance of results depends to some extent on the testing machine to be used. Hence information, valuable both in the development of the facilities and in the direction of future research into the mechanical properties of concrete under complex states of stress, could be obtained from these preliminary tests.

The torsional facility was chosen as the next priority after the development of the compression and tension facilities since its combination with compressive stresses enables an alternative biaxial compression-tension testing technique to be investigated.

Triaxial compression tests were the other type of test to be tried out, as the machine frame had been designed to allow for a subsequent extension to triaxial testing at a later date. The triaxial compression test has another attraction, however, in that beyond a certain degree of triaxial compression, the cleavage type of failure observed in biaxial states of stress almost certainly changes to some other form of failure. Preliminary tests could provide some qualitative information on this aspect of the problem.

As the purpose of these tests was mainly to develop the testing facility in the machine and investigate the problems associated with these types of test, only a few specimens were tested with each facility and large testing programmes were not planned.

10.2. Torsional Tests

10.2.1. Design of Facility

As shown in section 2.7., torsional tests can be carried out in the machine since torque can be applied to a cylindrical specimen using two angled beams and the vertical compression jacks as shown in fig.10.2.1.1. A stage of biaxial compression-torsion can be produced by applying compression to the ends of the cylinder and hence a complete biaxial compression-torsion facility was designed, the general arrangement of which is given in fig.10.2.1.2.

10.2.1.1. Torsion Loading

A clamp (see fig.10.2.1.3.) was provided at each end of the test specimen through which the torque was applied by the loads from the angled beams. The inside faces of the clamps were serrated to grip the outside surface of the cylinder and a stiff rubberoid material was used between the two halves of the clamp to assist the tightening of the clamp bolts. The positions of the $\frac{3}{4}$ " ball bearings, through

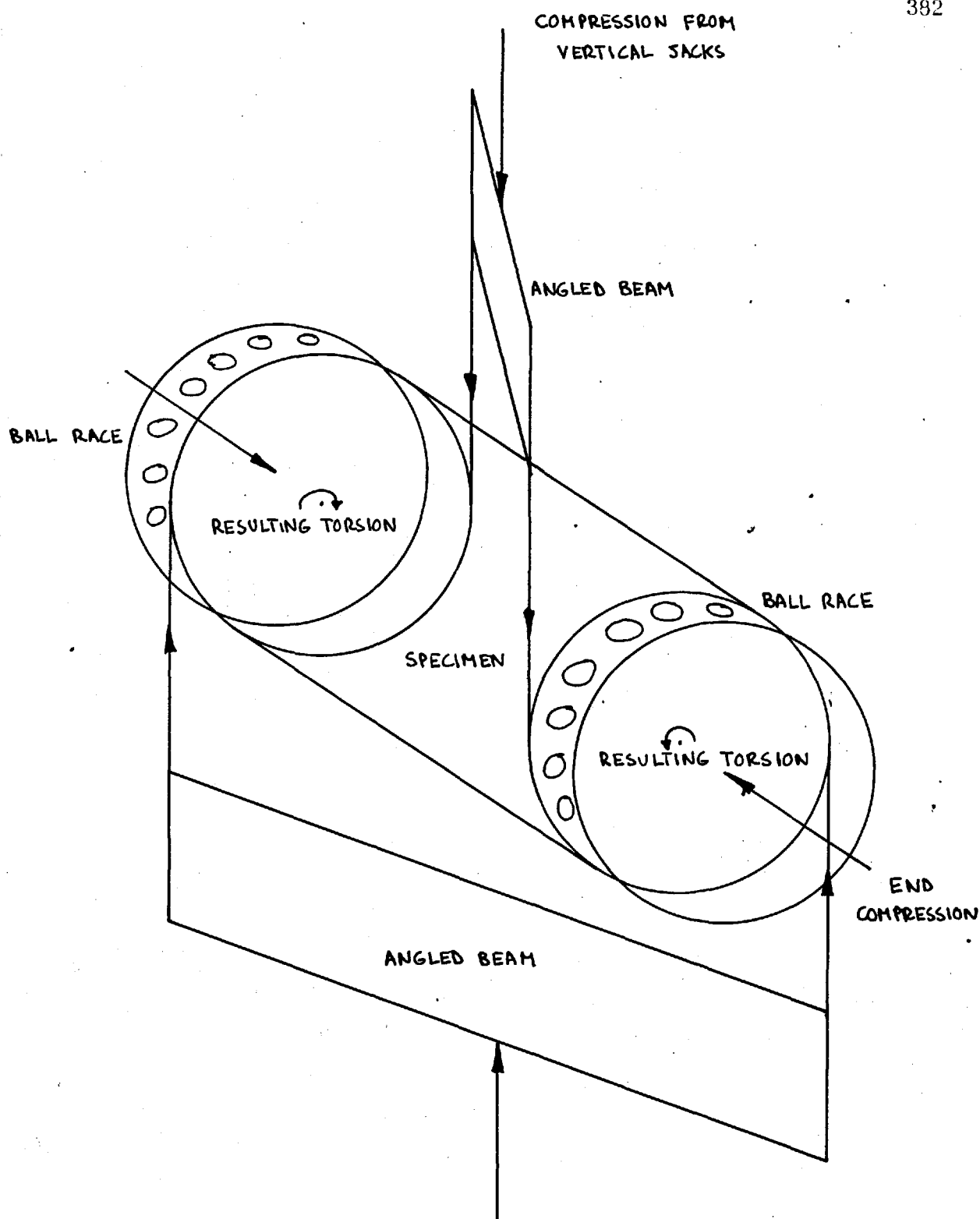


FIG. 10.2.1.1. APPLICATION OF TORSION USING ANGLED BEAMS AND END COMPRESSION THROUGH BALL RACES TO SPECIMEN

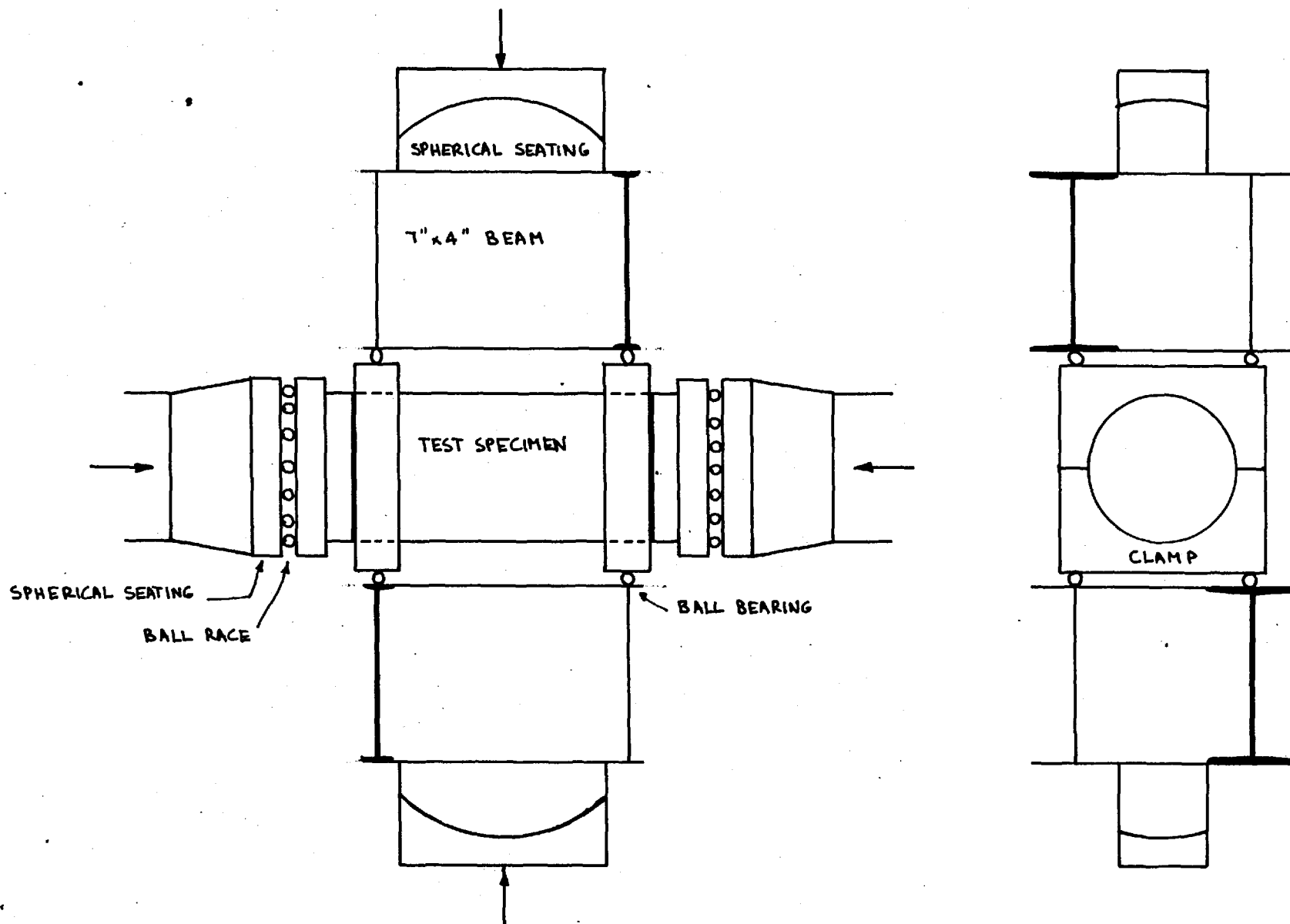


FIG. 10.2.1.2. GENERAL ARRANGEMENT OF BIAxIAL COMPRESSION-TORSION FACILITY



(19)

Fig. 10.2.1.3. End Clamp for Torsion Test Cylinder and Loading Beam showing ball bearings at loading points.

which the beam loads were transferred to the clamps, were arranged such that the beam reactions were tangential to the outside surface of the cylinder. The clamps were positioned about $\frac{1}{8}$ " or slightly less in from the ends of the cylindrical specimens so that the compression platen, described below, could bear on the cylinder ends without fouling the torsion clamps.

The angled beams were clamped to the standard 8" x 4" x 4" spherical seatings on the vertical rams of the machine to give flexibility in alignment and were made up from available 7" x 4" B.S.J. At one end of each beam was a fixed socket for the $\frac{3}{4}$ " ball bearings through which the beam load was transferred to the test specimen through the torsion clamps and at the other end was a moveable socket of similar design as shown in fig.10.2.1.3. The spherical seatings and the moveable ball socket between them enable any required adjustment to be made when aligning the angled beams on the test specimen. As with the other biaxial tests, the top spherical seating and beam unit were doubly counterweighted to ensure equal effort from the two opposed vertical jacks.

10.2.1.1. Compression Loading

End compression has to be applied through platens in such a way that it does not hinder the rotation of the test cylinder under the applied torsional stresses. Hence ball races were incorporated in the compression facility and the complete unit is illustrated in figs.10.2.1.4. and 10.2.1.5. A spherical seating was also included at each end to allow ease of alignment during setting up and was arranged so that its centre of curvature coincided with the face of the loading platen. Note that this seating will also lock under pressure as it has a small contact area and was not greased for the tests. The tapered block under the concave half of the spherical seating was to give a smooth transition of load from the ram face of the jack to the larger spherical seating of the ball race.

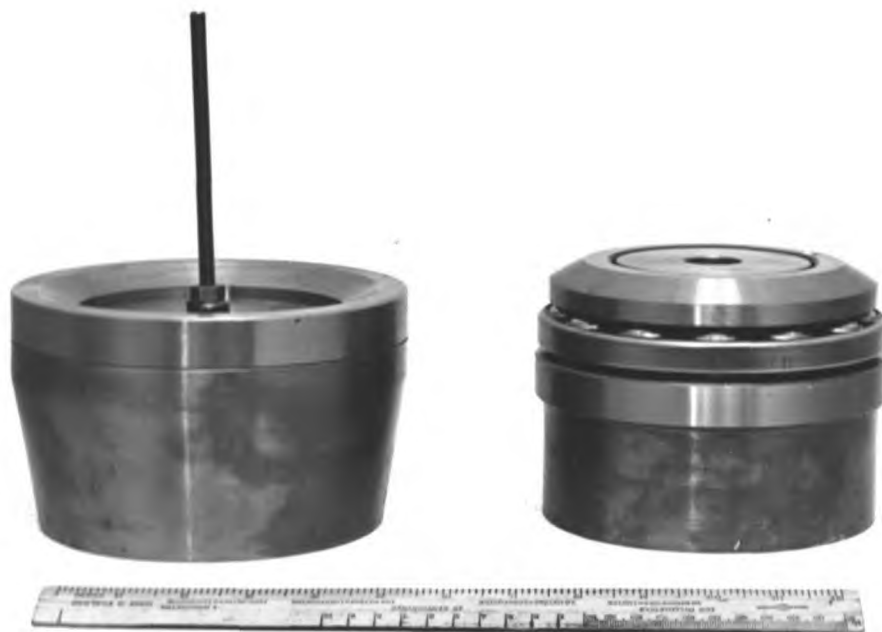


Fig. 10.2.1.4, Ball Race and Loading Platen showing Spherical Seating and Tie Rod.

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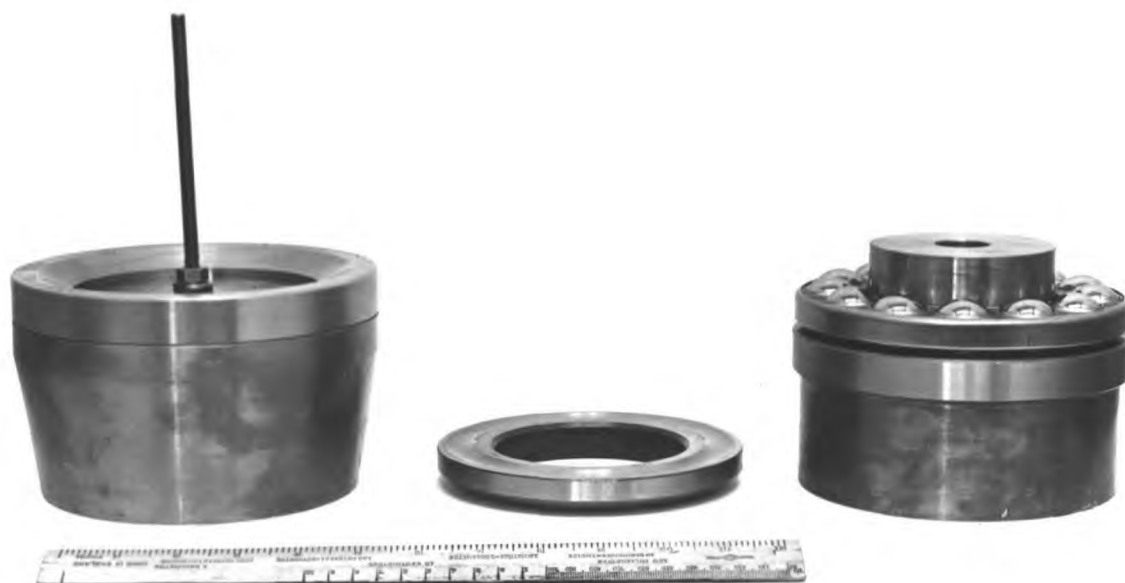


Fig. 10.2.1.5. Ball Race and Spherical Seating.

It should be noted also that the ring of balls has been positioned so that a good transfer of stress to the hollow cylinder is achieved through the very stiff loading platen. The whole assembly shown in figs.10.2.1.4. and 10.2.1.5. was linked together with a long central bolt and a tensioned spring.

A photograph of the complete machine arrangement is shown in fig.10.2.1.6. with a close-up view of the torsional specimen in fig.10.2.1.7.

10.2.2. Specimen Shape and Manufacture

The dimensions of the testing machine and necessary attachments limited the outside dimensions of the cylindrical specimen to 6 in. diameter and 12 in. long and a hollow form of cylinder was selected to investigate the problems involved in casting this type of specimen.

Given the limiting overall dimensions of the cylinder above, a wall thickness of 1 in. was selected so that aggregate sizes up to $\frac{3}{8}$ could be used in the test mixes. However, the end clamps will induce local secondary stresses and therefore the specimen required reinforcing at the ends so that the critical section of fracture should be beyond the influence of the clamps. Hence the specimen dimensions given in fig.10.2.2.2. were specified so that the central third of the specimen is of uniform cross-section whilst the wall thickness increases gradually in the end thirds to reinforce it against stress concentrations caused by the clamping action.

The virtue of maintaining a uniform profile on the outside of the cylindrical specimen is that standard cylinder moulds can be used but the shape of the inside former leads to a number of problems. These were solved by casting wax internal formers using a wooden mould as shown in fig.10.2.2.1. and described in Appendix F. The formers were positioned within the cylinder mould

(21)

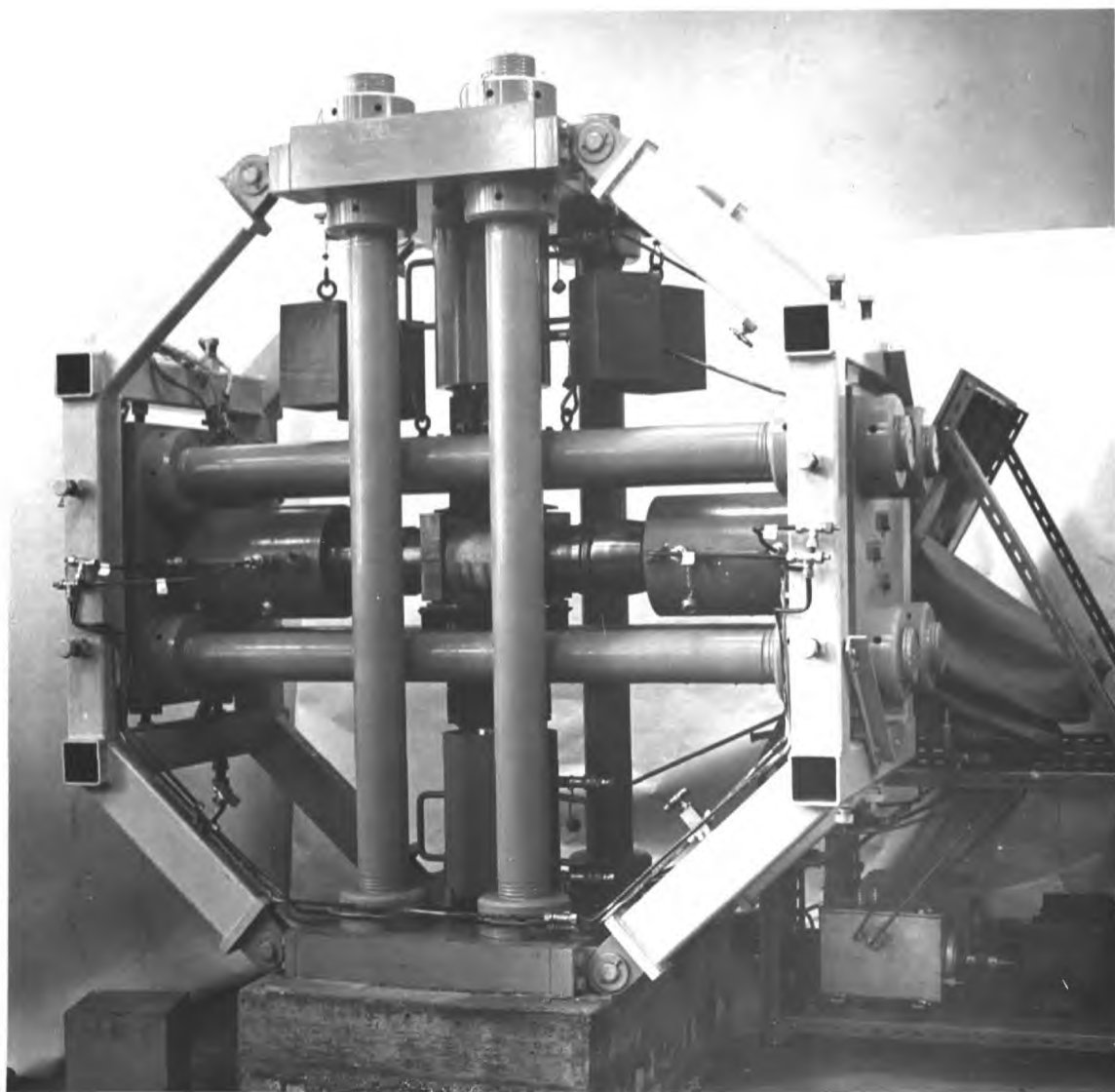
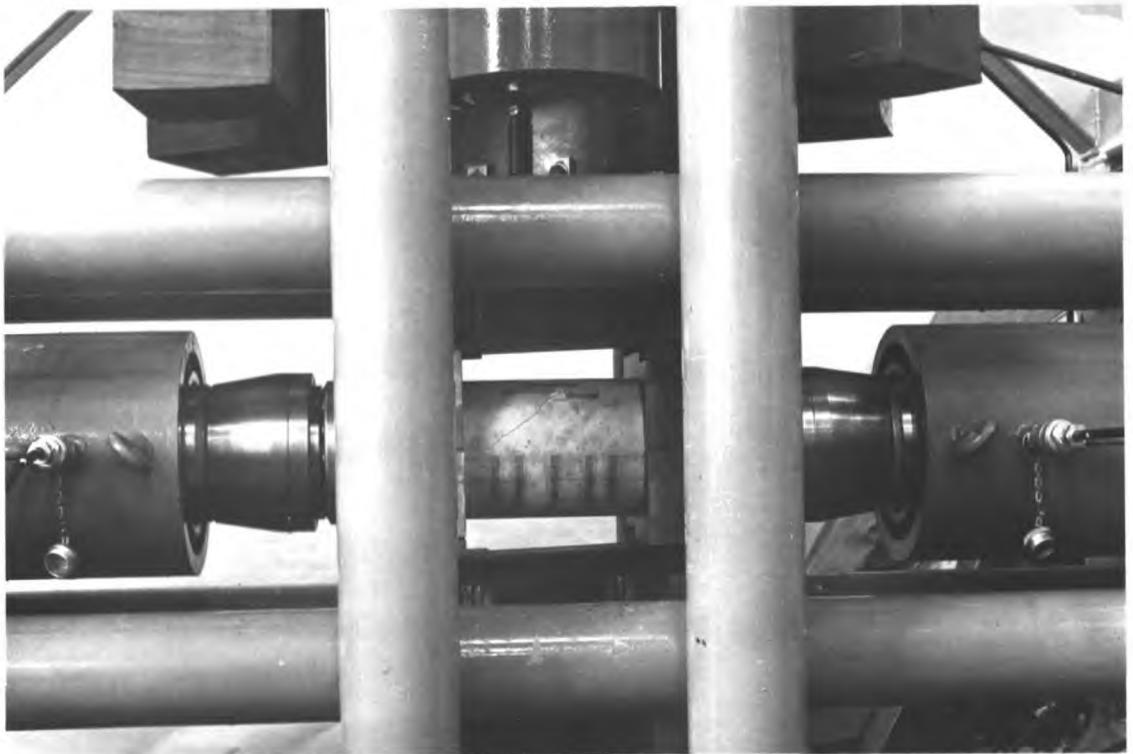


Fig. 10.2.1.6. Machine set-up for biaxial compression-torsion.



(22)

Fig. 10.2.1.7. Close-up view of Cylindral Torsion Specimen showing failure crack.

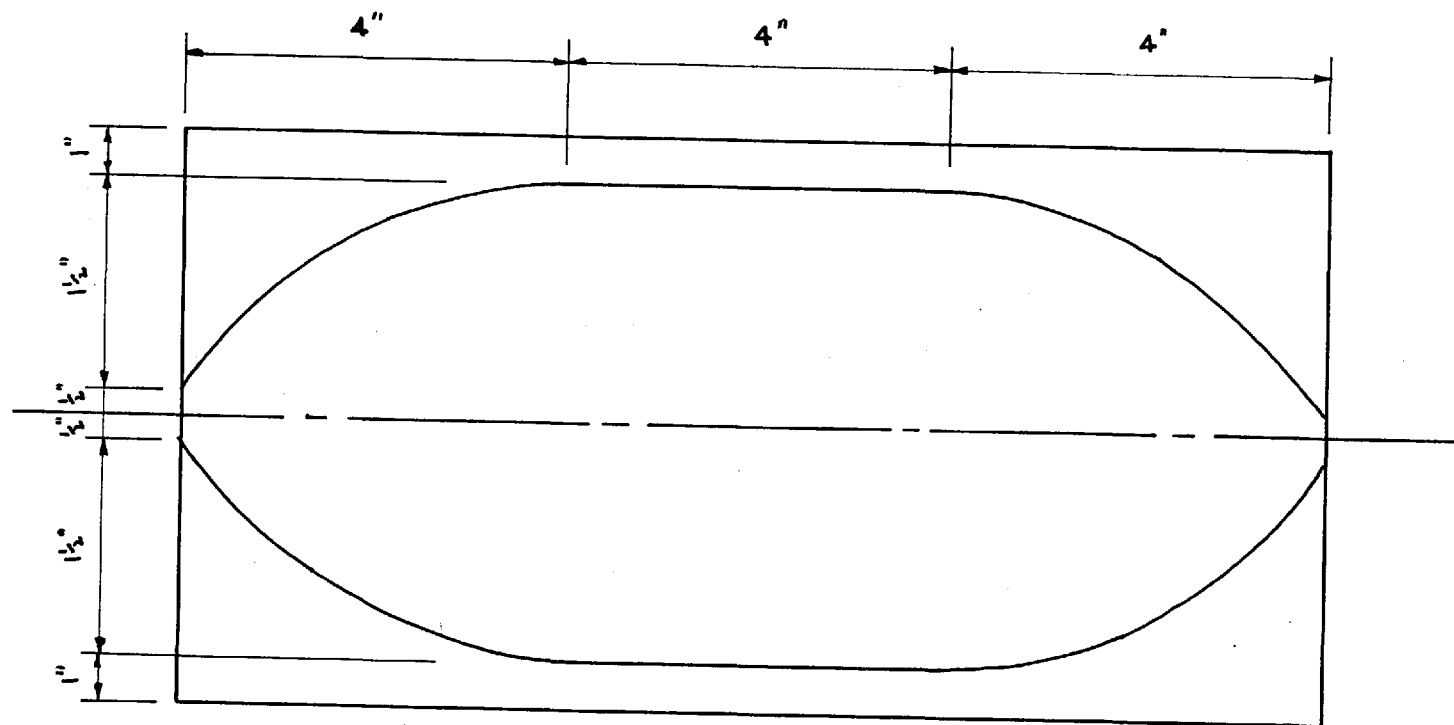


a) Wax Mould.



b) Wax Mould and Wooden Former.

Fig. 10.2.2.1. Internal Wax Mould for Hollow Torsion Cylinder.



12" x 6" ϕ CYLINDER

N.T.S.

FIG. 10.2.2.2. DIMENSIONS OF TEST SPECIMEN.

and were removed by melting the wax after the concrete had hardened. Unfortunately this process meant that the concrete was also heated up a little, thus interfering with the constant curing conditions and preventing the use of fully saturated specimens in testing.

Alternative solutions to the problem of the inside former were investigated and are described in Appendix F.

10.2.3. Testing Arrangement

The testing arrangement can be seen in figs.10.2.1.2. and 10.2.1.6. and the principles of the loading system have been described above. On only one specimen were strain gauges used in these preliminary tests and that was the proving trial specimen described in section 4.6. The other two specimens tested were to practise the setting-up procedure described below and to investigate the practical problems of this type of test.

An attempt was made to use the ultrasonic pulse velocity method of crack detection but difficulty was experienced in trying to get a good signal due probably to the problems of making a good contact on the curved surface of the specimen for the transducers.

10.2.4. Test Procedure

The test procedure for the proving trial specimen has already been described in section 4.6. The setting up procedure is described here in detail as the angled beams are incapable of holding the specimen in a state of stable equilibrium until they are both in place.

To set the test specimen up for this stress facility, the end clamps were fixed onto the specimen square to each other and the specimen was placed in the machine. The specimen was supported on a shaped timber cradle on the loading platen of the bottom jack and

raised until it was co-axial with the circular compression platens of the horizontal jacks. A nominal compression of about 100 p.s.i. (based on the smallest cross-sectional area) was applied to the specimen by the horizontal jacks and the bottom jack withdrawn. The specimen was rotated between the ball race^s until the end clamps were correctly aligned to receive the angled beams. Then the two beams were jacked onto the specimen clamps slowly following the procedure described in section 9.4. for applying compression to the tension specimen. A nominal torsion was applied and then, starting from these small nominal compression and torsion stresses, the specimen was loaded to failure. In the two specimens tested compression was applied first followed by torsion but the testing technique is such that sequence or proportional loading can be used.

Note that if torsion only is to be applied to the test specimen the above procedure is still followed but the compression stress is removed once the nominal torsion has been applied and the specimen is stable in this position.

10.2.5. Results

The results of the strain measurements made on the proving trial specimen (A) have been given in section 4.6. but the ultimate stress and crack patterns are described here. The ultimate stresses recorded for the three specimens tested are given in the table below; all three specimens were mortar mixes MBC but tested within the range 26-35 days old.

Specimen No.	Age days	Mix	ultimate compressive stress p.s.i.	ultimate tensile stress p.s.i.
A	28	MBC	725	210
B	35	MBC	1,820	170
C	26	MBC	4,350	40

The failure crack pattern for specimens A and B, comprised two cleavage cracks on a spiral around the cylindrical test specimen and visible only on the middle third of the cylinder dying out into the thickened ends. The cracks observed with specimen C were more in the nature of a multi-cleavage compression failure with the major cracks about parallel to the axis of the specimen. Again these cracks occurred mainly in the middle third of the specimen.

As only three specimens were tested no conclusions can be drawn regarding a comparison of these test results and those obtained under direct biaxial compression-tension.

10.2.6. Conclusion

The biaxial compression-torsion testing technique as developed here should produce satisfactory results. The strain platens obtained in section 4.6. and the crack patterns described above indicate that the central uniform section of the test specimen has a uniform state of stress on the outer surface. However, the problems with any testing technique containing a strain gradient through the loading of the specimen, such as the shear stress applied to this hollow cylinder, remain.

The method of manufacturing the hollow cylindrical specimen was satisfactory as far as obtaining the desired dimensions were concerned but it was slow and tedious. A better technique would be required for a large test series.

The machine behaviour was satisfactory in that no machine effects were observed in the results and it may be concluded that the testing machine is capable of conducting this type of test.

10.3. Triaxial Compression Tests

In order to check the feasibility of such a testing technique, prior to permanent modifications to the machine, the triaxial compression facility was achieved by the introduction of a third independent frame as described below.

10.3.1. Design of Facility

The test specimen chosen was the 4 in. cube so that reasonable pressure could be exerted in the third direction by the independent frame without exceeding the limit of the 200 ton capacity sub-frames of the testing machine. Though intrinsically the best specimen shape for general triaxial testing, the cube suffers the disadvantage of being particularly susceptible to the effects of platen restraint as discussed in section 1.3.18. Attempts and suggestions to overcome this problem have already been made in this thesis with greased packings in section 2.5. and hydraulic platens in section 2.4. However, this test series was purely a preliminary trial to see if the technique was possible without too much attention being paid to the state of stress on the test specimen at this stage.

The cubical specimen was loaded by the two sub-frames of the combined stress testing machine through 4" machined mild steel cubes as shown in fig.10.3.2.1. The steel cubes had a $\frac{1}{8}$ fillet all round on the loading face reducing the actual loading area to $3\frac{3}{4}$ " to avoid contact between platens with the large strains expected.

The independent frame shown in fig.10.3.1.1, was designed to accommodate one of the 30 ton push-pull hollow ram Simplex jacks used in the tension facility of the testing machine. It loaded the test specimen through loading plates of identical shape and size to the chamfered loading cubes used in the testing machine. The frame was suspended between the columns of the testing machine and its weight supported on the horizontal sub-frame.

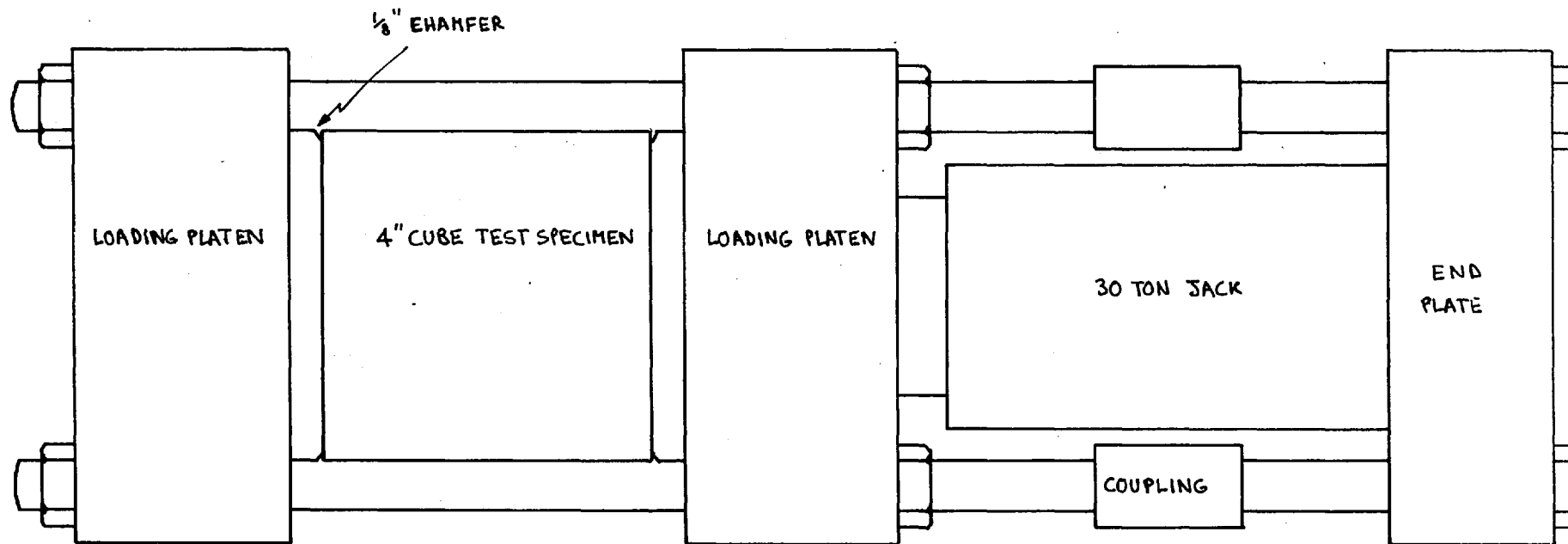


FIG. 10.3.1.1. GENERAL ARRANGEMENT OF 30 TON. INDEPENDANT COMPRESSION FRAME
USED IN TRIAXIAL COMPRESSION TESTS.

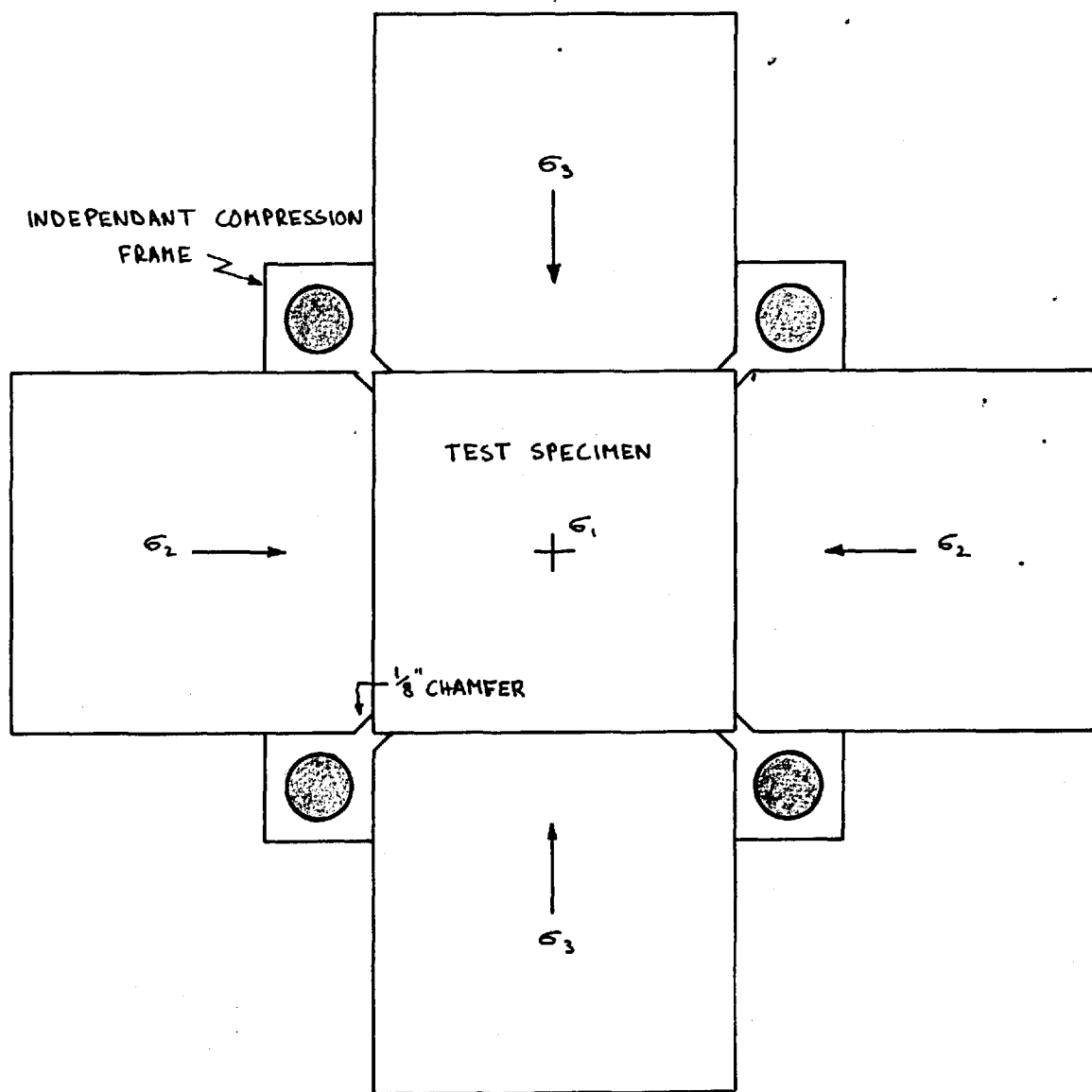


FIG. 10.3.2.1. ARRANGEMENT OF LOADING CUBES AROUND TEST SPECIMEN.

The effect of using a freely suspended frame, as opposed to the other two sub-frames which are fixed in position, is to give the test specimen an additional degree of freedom for its failure mechanism compared with the rigid permanent extension proposed for the machine.

10.3.2. Testing Arrangement

As the 4" cube test specimen is loaded on all six faces, the cast face of the cube had to be ground down to give a smooth bearing surface for the loading platen on a lapping plate.

The test specimen, once aligned in the testing machine and with the independent frame clamped on, was completely surrounded by steel loading platens. As a result, no surface strain measurements were possible and the use of linear inductance transducers would have required the development of a special support frame. An attempt was made to use the ultrasonic pulse velocity equipment in the manner shown in fig. 10.3.2.2. but a clear cut signal could not be obtained. This interference with the pulse signal may have been due to the four surfaces of contact involved in the direct path between the transducers slowing the pulse to a lower total velocity than the longer route along the columns of the independent frame.

10.3.3. Test Procedure

To ensure that the test specimen was centrally positioned in the combined stress testing machine, the cube was first aligned between the four steel cubes following a similar procedure to that used in the biaxial compression tests. The test specimen was supported on the base cube and loading platen and jacked up to the concrete level. The top cube was then placed on top and the top platen jacked down to apply a nominal compression to the specimen to hold it in place. The side cubes and platens were then jacked into position and the bottom jack released. Care had to be taken with aligning the steel cubes on the test specimen because the chamfer on the loading face of the cubes made alignment difficult.

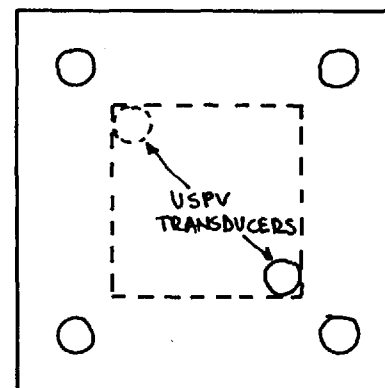
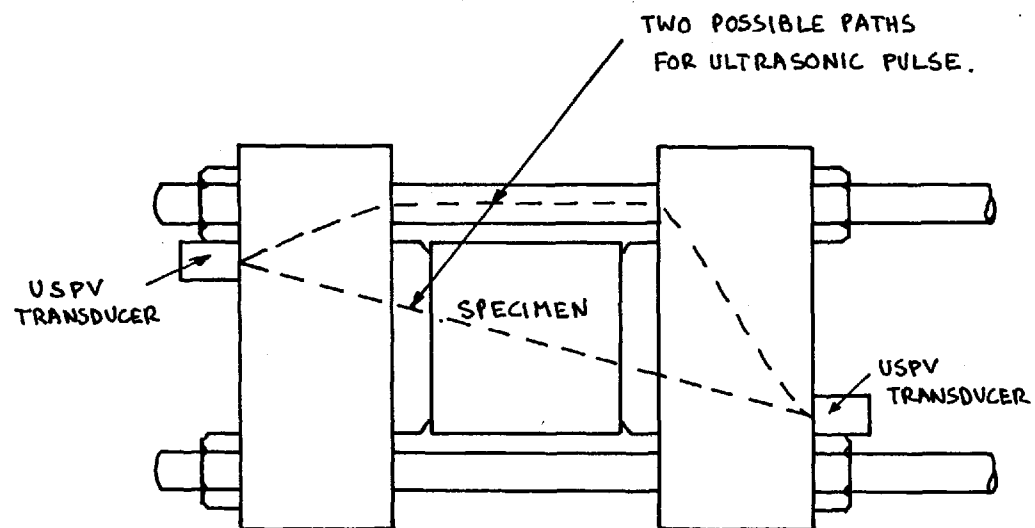


FIG. 10.3.2.2. USE OF ULTRASONIC PULSE VELOCITY IN TRIAXIAL COMPRESSION TESTS.

Once this alignment was satisfactorily completed, the independent frame was threaded between the columns of the machine and aligned on the test specimen. A pre-extension of the jack in the independent frame of at least 1" was required to offset the expansion of the test specimen in the third direction on failing.

A predetermined load was firstly applied by the independent frame in the σ_1 direction; the other two stresses (σ_2 and σ_3) having been set to about 200 p.s.i. to clamp the test specimen and loading cubes in place. The two principal stresses σ_2 and σ_3 were then increased uniformly at a loading rate of 350 p.s.i./min. each until the specimen failed. Failure in these tests was indicated by a very gradual reduction in the load supported by the specimen accompanied by excessive distortion of the specimen.

10.3.4. Results

Only a few specimens were tested as the object of the tests was to discover the problems associated with this type of triaxial compression testing. In all six 4" mortar (MBC) cubes and one 4" concrete (61B) cube were tested as follows:-

Uniaxial compression	1 mortar cube
Biaxial compression	1 mortar cube
Triaxial compression	{ 4 mortar cubes 1 concrete cube

The ultimate strength results of the mortar cubes, all of which were tested at 28-36 days in a saturated condition, are plotted in fig.10.3.4.1. together with the strains measured on the specimens after testing to failure. Two of the mortar cubes tested under triaxial compression gave unsatisfactory results because of errors in the testing technique used; the main mistake was failure to provide enough pre-extension of the jack of the independent frame to ensure sufficient room for expansion of the test specimen in the

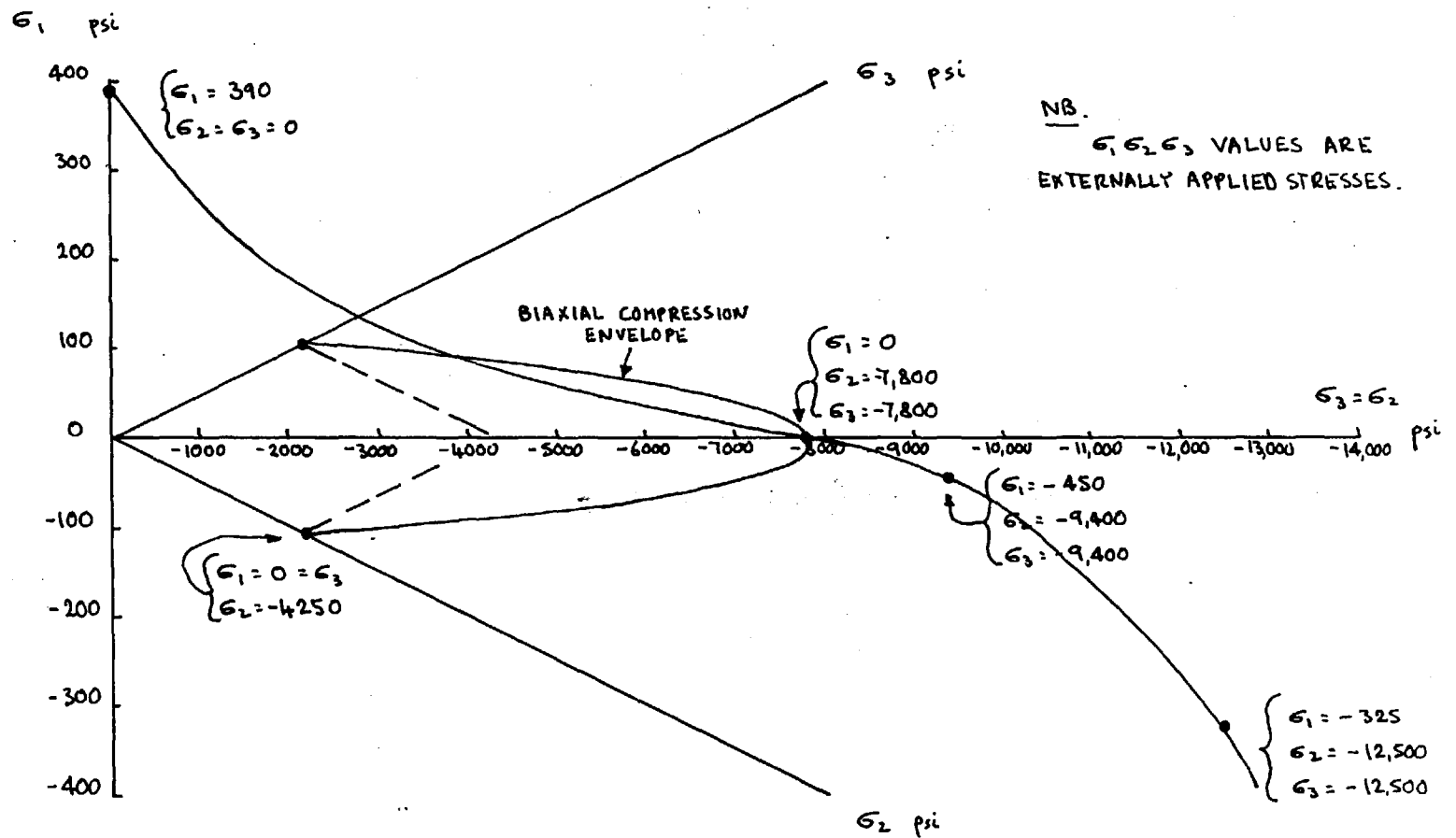


FIG. 10.3.4.1. ULTIMATE STRENGTH RESULTS OF TRIAXIAL COMPRESSION TEST SERIES.

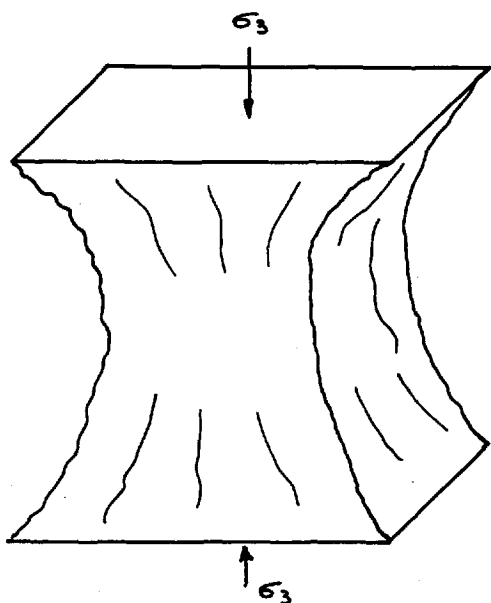
third direction (σ_1 direction). A measure of the actual state of stress induced in a 4" cube under biaxial compression is indicated by the increase of strength over the uniaxial strength of 86% whereas the increase for true biaxial compression is only of the order of 15% (see section 8.5.). This increase was on a cube coated in curing compound and compares with 220% for dry cubes reported by Sundara Raja Iyengar et al⁽⁵⁵⁾ described in section 1.3.20.

The failure mode for the uniaxial and biaxial compression specimens was the normal cleavage type as reported in this thesis. Under triaxial compression the failure was almost plastic in nature with no significant major cracking although a fine mesh pattern of shear cracking was visible on the surface; the material maintaining a high degree of continuity with very large strains up to 0.13 being reported. This plastic-shear mode of failure was very similar to that obtained by both Boker and von Karman in triaxial tests on mortar (reported in section 1.3.).

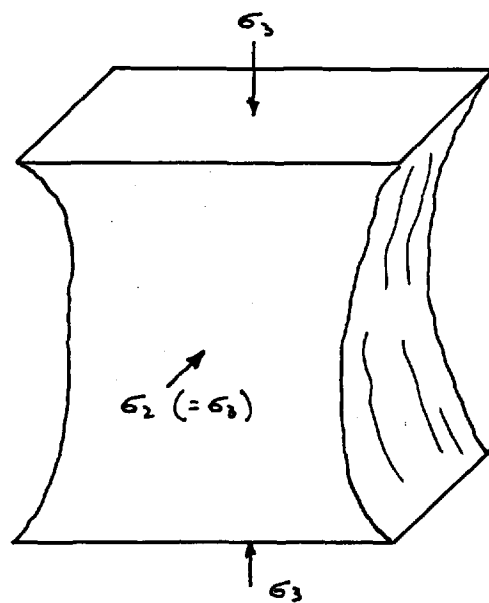
In the case of the concrete specimen tested the failure was still basically plastic but the pressure of aggregate particles in the mortar matrix were probably responsible for the lower strain (0.05), more rapid fall in the load supported after ultimate load, and the tendency for the mortar to crack and lose its continuity. Sketches of the failed specimens in fig.10.3.4.2. also show the thin plate at each end of the specimen which was separated from the main body of the cube due to the method of loading through chamfered platens.

10.3.5. Conclusions

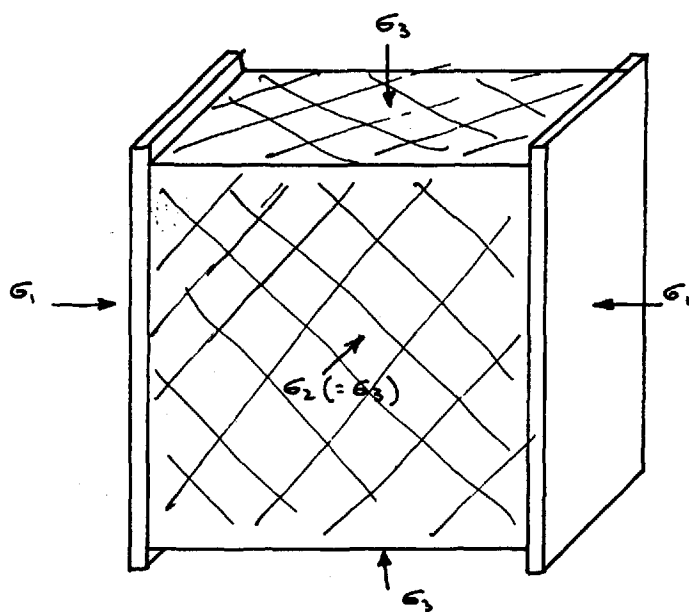
The problems of triaxial compression testing of this type were very apparent from these tests and the main difficulties are summarised below:-



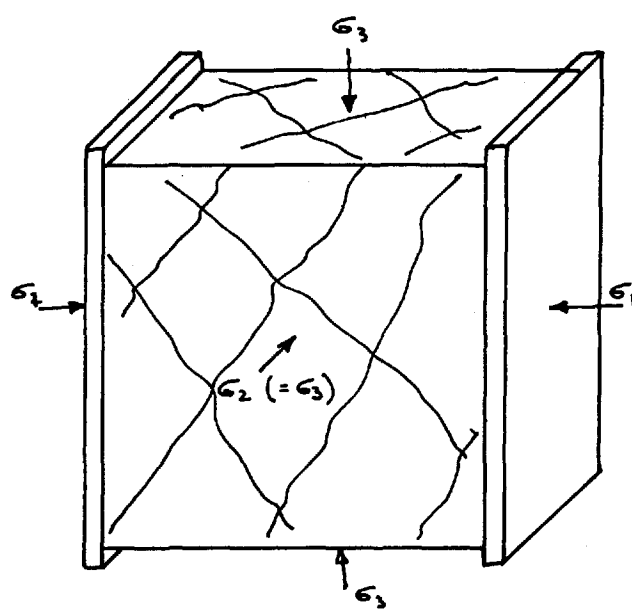
① UNIAXIAL COMPRESSION (MORTAR)



② BIAxIAL COMPRESSION (MORTAR)



③ TRIAXIAL COMPRESSION (MORTAR)



④ TRIAXIAL COMPRESSION (CONCRETE)

FIG. 10.3.4.2. SKETCHES OF FAILURE MODES OF 4" CUBE TEST SPECIMENS UNDER UNI-, BI-, AND TRIAXIAL COMPRESSION.

1) The alignment of the loading cubes and platens on the test specimen was difficult because of the chamfer on the loading surfaces. This problem would certainly be easier with a permanent triaxial facility for the testing machine as much of the misalignment would be avoided by the initial alignment of the permanent sub-frames.

2) Platen restraint showed itself in the biaxial compression strength obtained. Moreover the effect of the chamfers on the platens which forced off a thin slice of the specimen on the two faces of minor compression stress is most undesirable. Even with the best friction relieving materials, the chamfer required to avoid platen clashing will still distort the stress distribution in the test specimen.

3) Difficult access to the test specimen rules out the use of surface strain gauges and probably the only practical method of measuring the three principal strains would be by measuring the relative movements of the steel loading platens using the linear inductance transducers.

4) The problem of the conflicting requirements of a uniform pressure distribution over the whole surface of the cube and the need to avoid clashing of the loading platens (achieved in this case by chamfered edges as explained above) will be a very difficult problem to solve. This difficulty may in itself rule out the solid loading platen method of loading in favour of some form of hydraulic loading.

5) The definitions of ultimate and discontinuity stress which were satisfactory under biaxial states of stress can no longer be employed under triaxial states of stress. Whilst a suitable definition of ultimate stress would be the maximum stress sustained by the specimen, the identification of the discontinuity stress poses a number of difficulties. The selection of a method of defining the discontinuity point under triaxial states of stress

depends on the philosophy of this parameter. It has in general terms been regarded in this thesis as the stress limit of elastic or quasi-elastic behaviour and under biaxial states of stress has been indicated by the onset of rapid crack propagation. It is suggested that the 'elastic limit' philosophy should be the criterion for the determination of the discontinuity point in which case some experimental means of defining this point must be found. In view of the plastic behaviour observed in the test specimens the measurement of ultrasonic pulse velocity may be unsuitable for detecting this point and either strain readings or some new technique will have to be used. One possible, though tedious, technique would be to examine the test specimen at various load stages to see whether there was any significant change in its uniaxial elastic parameters especially Poisson's ratio as a measure of internal structural deterioration.

It is obvious that this problem needs much further research.

C H A P T E R 1 1
PRINCIPAL RESULTS OF RESEARCH

11.1 Introduction

Results from the various test series have been discussed separately in the relevant chapters. The intention in this chapter is to discuss the behaviour of mortars and concretes under all the states of stress for which experiments have been conducted in this research project in order to try to establish coherent explanations of the overall results. Such grouping of results is advantageous in the search for a reliable failure criterion and it is proposed to deal with the results under the following section headings:-

11.2 Stress-Strain Relationships

11.3 Stress Characteristics

11.4 Strain Characteristics

11.5 Failure Modes

11.6 Long-term Strength.

11.2 Stress-Strain Relationships

In Chapter 8, the stress-strain curves obtained for a mortar, a normal aggregate concrete, and a lightweight concrete under uniaxial and equal biaxial compression were compared. It was found that the elastic theory equations applied to the uniaxial results could accurately predict the biaxial behaviour of the mortars and lightweight concrete but were in error in predicting the biaxial behaviour of normal aggregate concretes. It was then shown that, whilst the value of Poisson's ratio was effectively constant below the discontinuity point for all three mixes, the elastic modulus remained constant for only the mortar and lightweight concrete; the elastic modulus for normal aggregate concrete showing a substantial increase with increasing degree of biaxial compression. The error in applying the elastic theory equations to normal aggregate concrete and the variation of its elastic modulus were shown to be directly related and the phenomenon was ascribed to particle interaction of the coarse aggregate.

Previously in Chapter 7 it had been shown that the elastic modulus of all mortars and concretes was effectively equal under uniaxial compression and tension states of stress. Therefore it would appear that the elastic theory equations are valid within practical limits under uniaxial and biaxial states of stress except under biaxial compression where a variation of elastic modulus must be included for normal aggregate concretes.

The stress-strain curves obtained under uniaxial tension have been added to the curves used in Chapter 8 of uniaxial and equal biaxial compression and are shown in figs.11.2.1.-11.2.3. In addition the variation of Poisson's ratio with tensile stress for the same three representative mixes is shown in fig.11.2.4.

As the elastic moduli under uniaxial tension and compression are equal the uniaxial tension and compression curves follow the same line. It was also noted in Chapter 7 that Poisson's ratio is

approximately equal under these two states of stress but considerable scatter was recorded in the results due probably to the small lateral strains recorded under uniaxial tension. However, the similar slope of the stress-lateral strain curves shown in figs. 11.2.1. - 11.2.3. must be due to very similar values of Poisson's ratio.

In Chapter 8 the effect of the mean normal stress on the bulk modulus K and shear rigidity modulus G was examined. On the assumption that the elastic theory applies, these elastic parameters are given by:-

$$K = \frac{E}{3(1 - 2\nu)} \quad G = \frac{E}{2(1 + \nu)}$$

It can be seen that the value of the bulk modulus K will increase in the same manner as the elastic modulus if Poisson's ratio remains reasonably constant. Moreover, if the small increase of Poisson's ratio with stress noted with normal aggregate concretes is taken into account, the increase of the bulk modulus with degree of biaxial compression becomes even more marked.

The value of the rigidity modulus G will also increase in the same manner as the elastic modulus for constant Poisson's ratio. However, in the case of the rigidity modulus as opposed to that of the bulk modulus, a small increase in Poisson's ratio with increasing stress would reduce the increase in the rigidity modulus due to the increase in elastic modulus with mean normal stress. The variation of the rigidity modulus with mean normal stress has been plotted in fig. 11.2.5. for all three representative mixes. As the elastic modulus and Poisson's ratio are effectively constant for the mortar and light-weight concrete, the value of the rigidity modulus is also constant. However, the effect of particle interaction in normal aggregate concretes still manifest itself by a variation in the rigidity modulus though to a slightly less pronounced degree than in the elastic and bulk modulus values.

FIG. II.2.1. STRESS-STRAIN CHARACTERISTICS UNDER UNIAXIAL COMPRESSION AND TENSION AND BIAXIAL COMPRESSION.

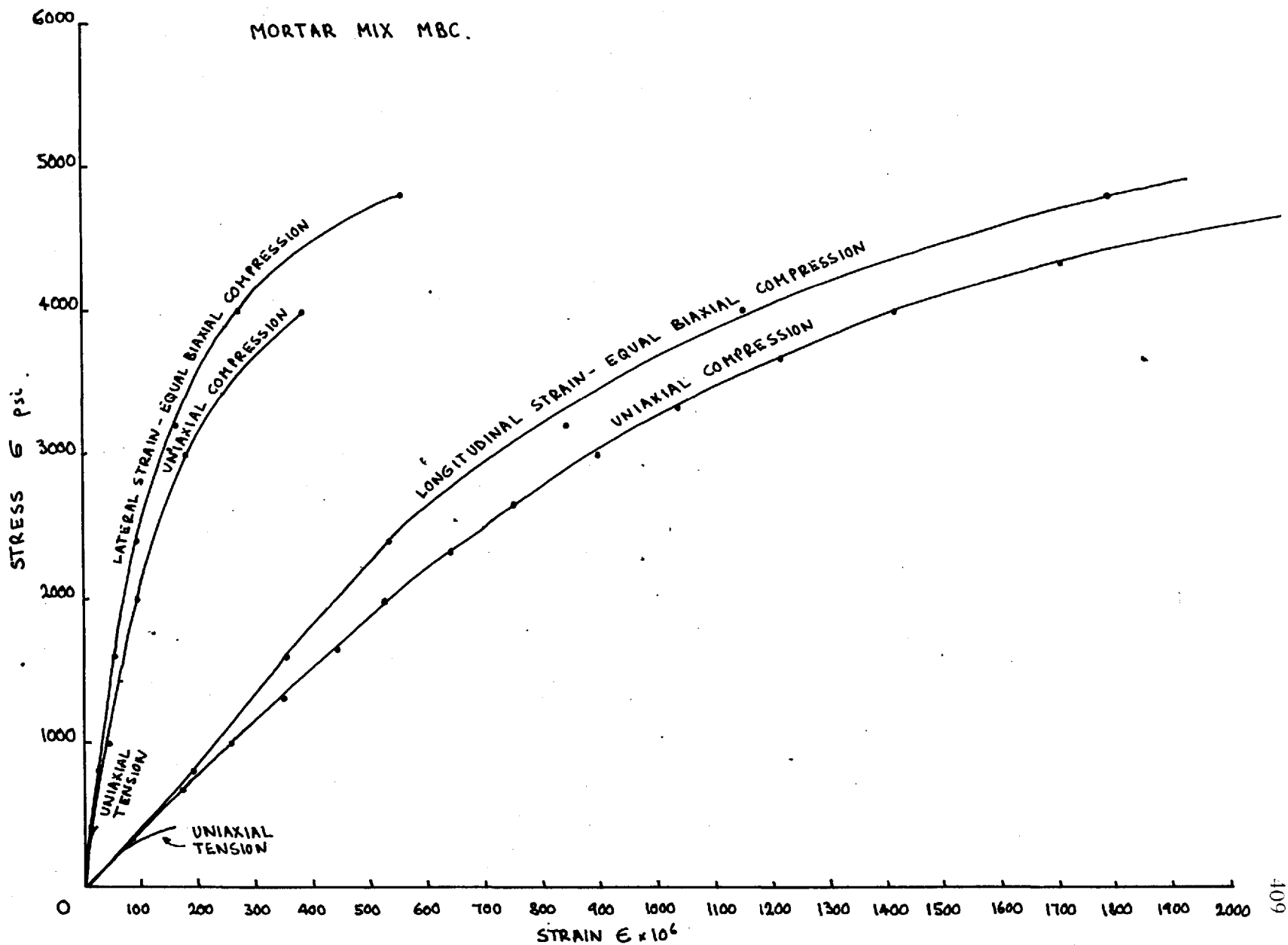


FIG. 11.2.2. STRESS-STRAIN CHARACTERISTICS UNDER UNIAXIAL COMPRESSION AND TENSION AND BIAxIAL COMPRESSION.

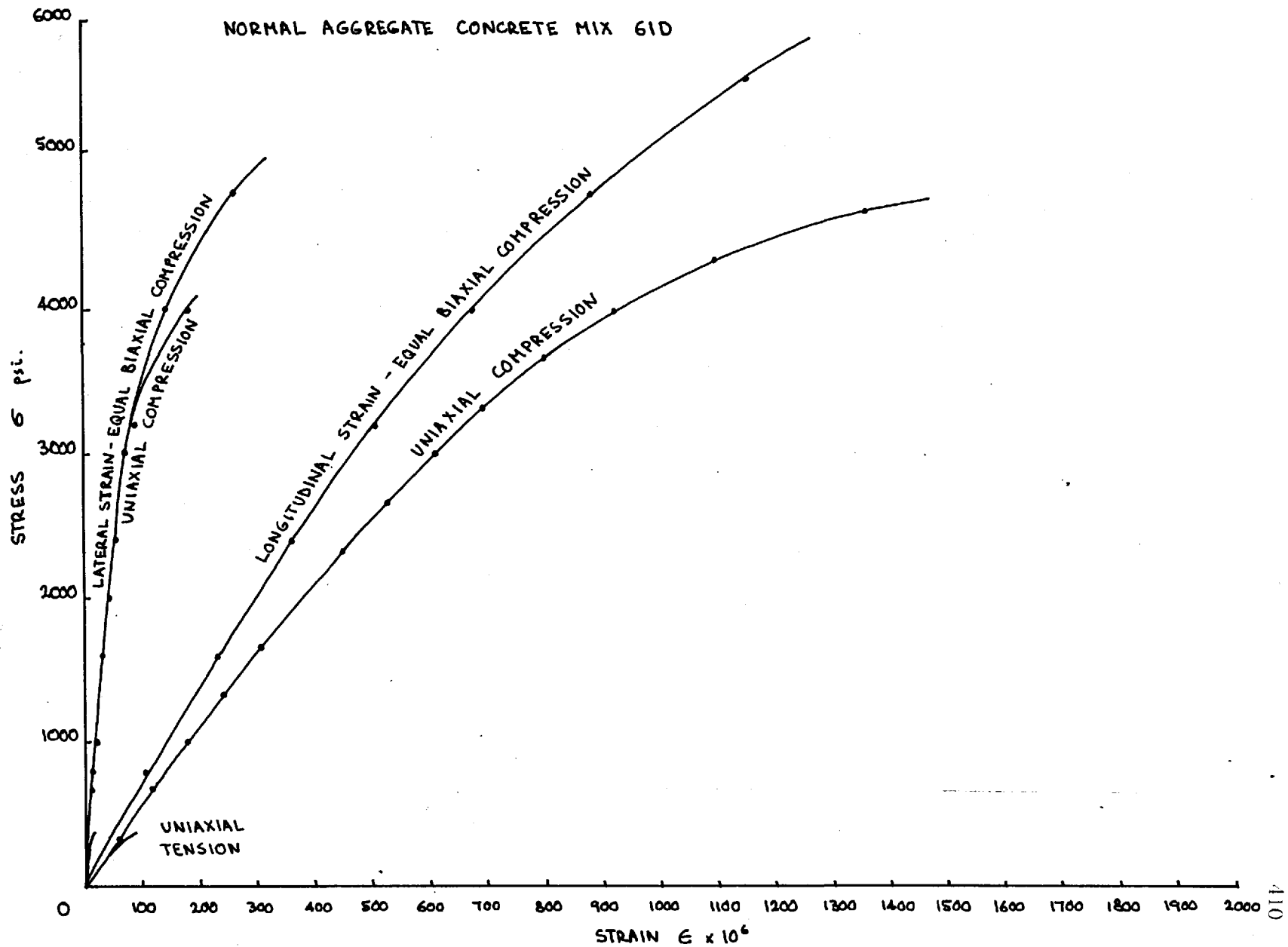
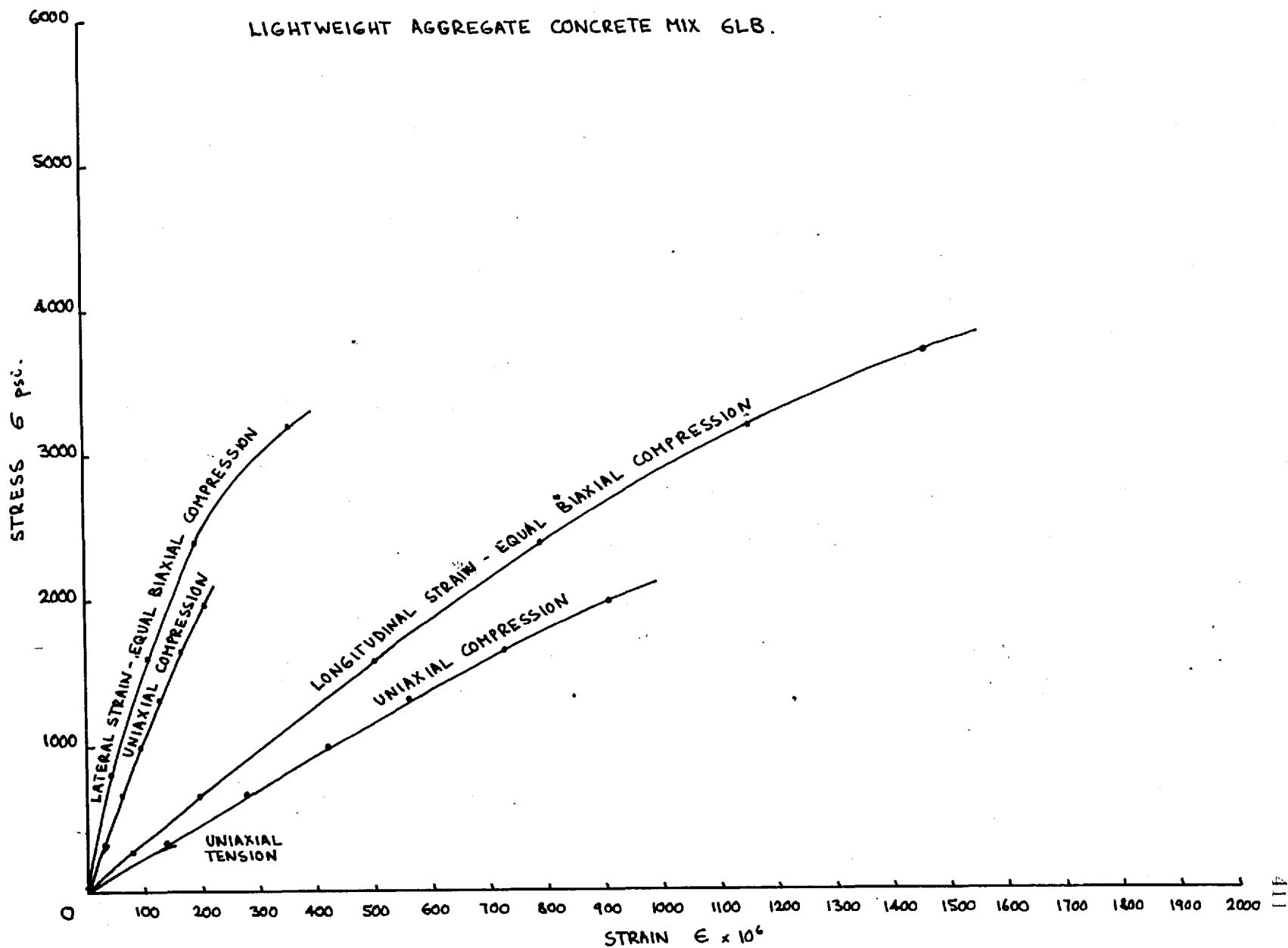


FIG. II.2.3. STRESS-STRAIN CHARACTERISTICS UNDER UNIAXIAL COMPRESSION AND TENSION AND BIAxIAL COMPRESSION.



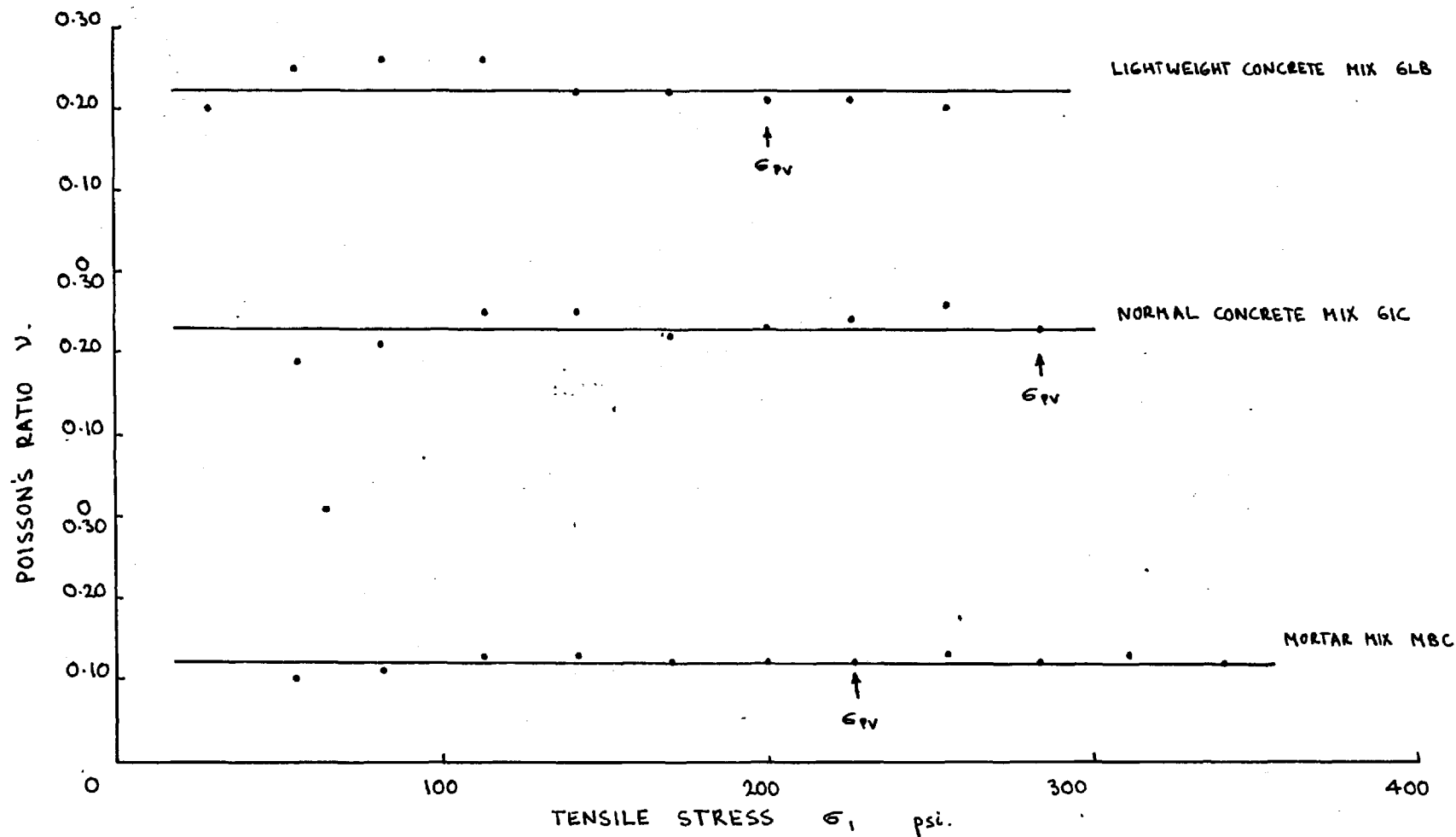
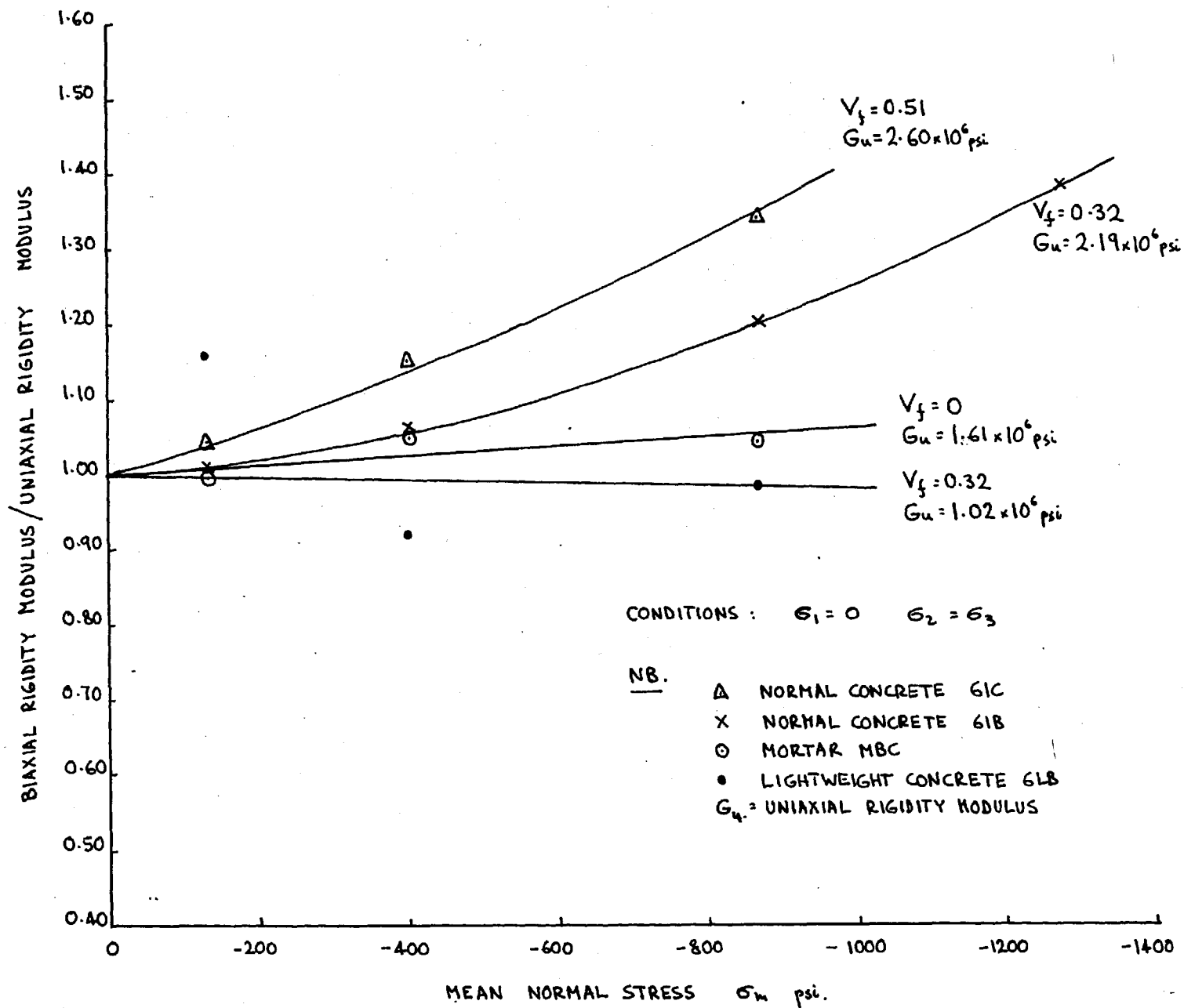


FIG. 11.2.4. VARIATION OF POISSON'S RATIO WITH UNIAXIAL TENSILE STRESS

FIG. 11.2.5. VARIATION OF RIGIDITY MODULUS WITH BIAxIAL STRESS.



It would seem, therefore, that the phenomenon of variable elastic, rigidity and bulk moduli in normal aggregate concretes must be taken into consideration as a complementary function if the elasticity theory quotations are to be used.

11.3. Stress Characteristics

The stress envelopes, under biaxial states of stress, of the discontinuity and ultimate stresses have so far been considered in one section of the stress field at a time: the biaxial compression envelope was considered in Chapter 8 and biaxial compression-tension in Chapter 9. In this section the six complete discontinuity and ultimate stress envelopes obtained are considered and have been plotted in figs. 11.3.1. - 11.3.6.

Despite the several shapes of envelope already pointed out in Chapters 8 and 9, there is a remarkable degree of continuity of curve between the two sections of biaxial compression and compression-tension. This is all the more noteworthy since these two halves of the envelope were obtained from very different testing techniques. The difference noted in Chapter 9 between the uniaxial compression strength of the biaxial compression-tension specimen and the prism strength has been allowed for by a pro-rata adjustment of the compression component of the biaxial compression-tension strength results. This adjustment has no significant effect on the shape of the envelopes and in any case never exceeded 15%. No adjustment of the biaxial compression results was necessary as the uniaxial strength of the biaxial compression specimen was always within 5% of the prism strength.

By plotting the complete envelopes on equal compression and tension scales, the significance of the biaxial compression-tension envelope diminished considerably and the variations experienced in that zone are reduced to their true proportions. In addition, the biaxial tension zone is shown to be relatively unimportant except to complete the picture of biaxial stress behaviour.

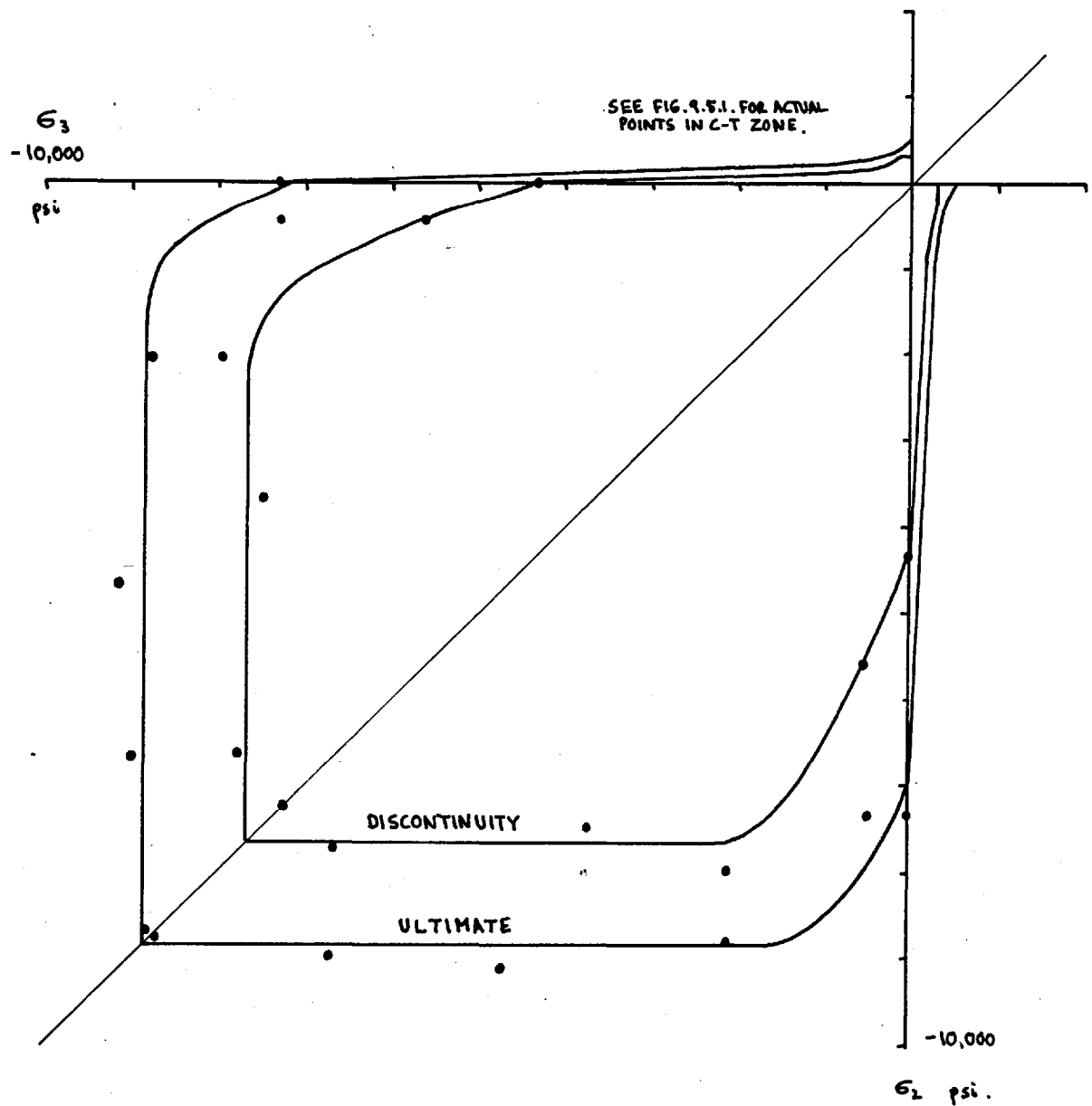


FIG. 11.3.1. BIAxIAL STRESS ENVELOPES FOR MIX MA.

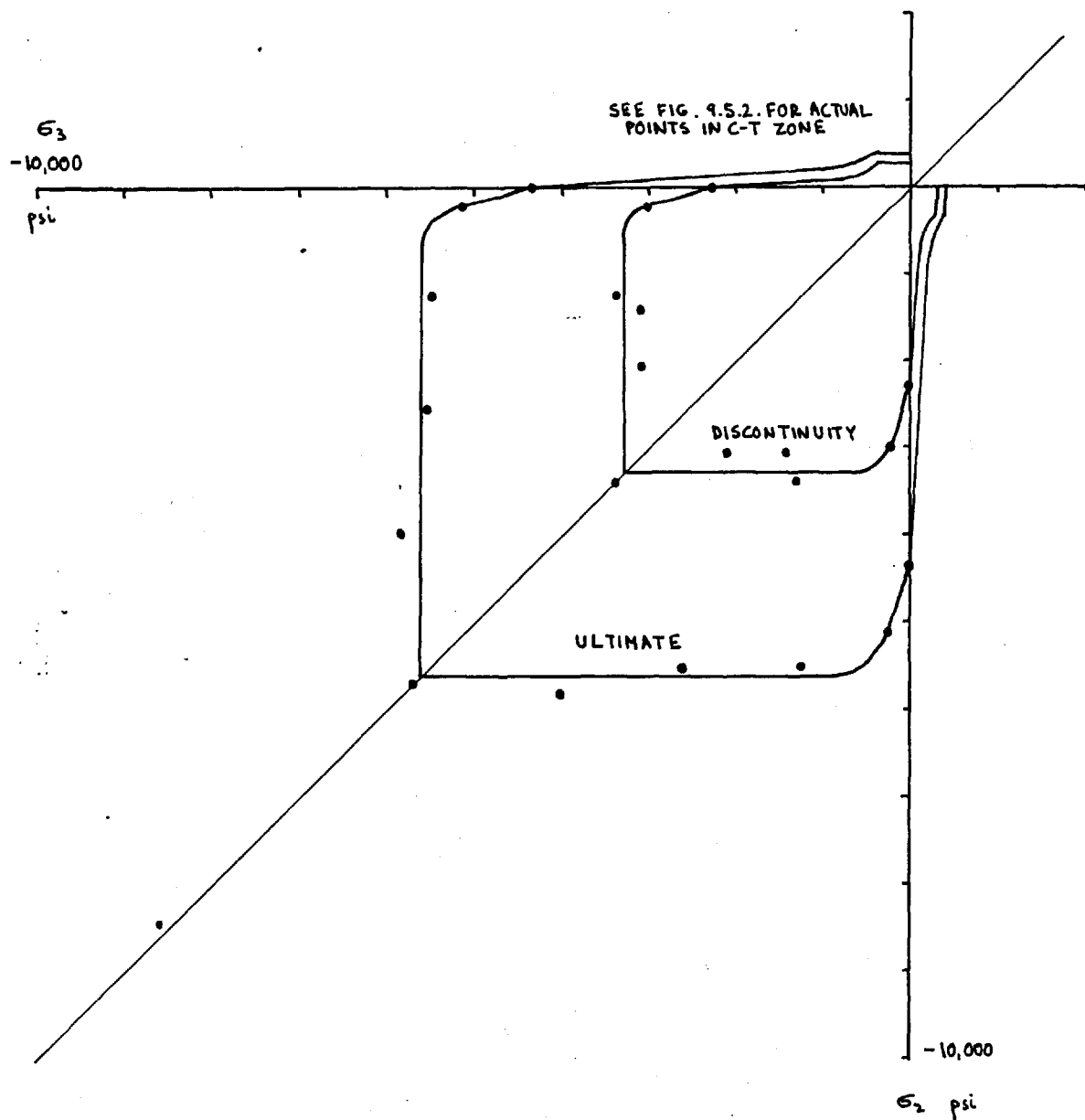


FIG. 11.3.2. BIAxIAL STRESS ENVELOPES FOR MIX MBC.

MIX 61B $\frac{1}{2}$ " 4.5 $\frac{1}{2}$ " 2.4 $\frac{1}{2}$ " 0.55 $\frac{3}{4}$ " IRREGULAR COARSE AGGREGATE

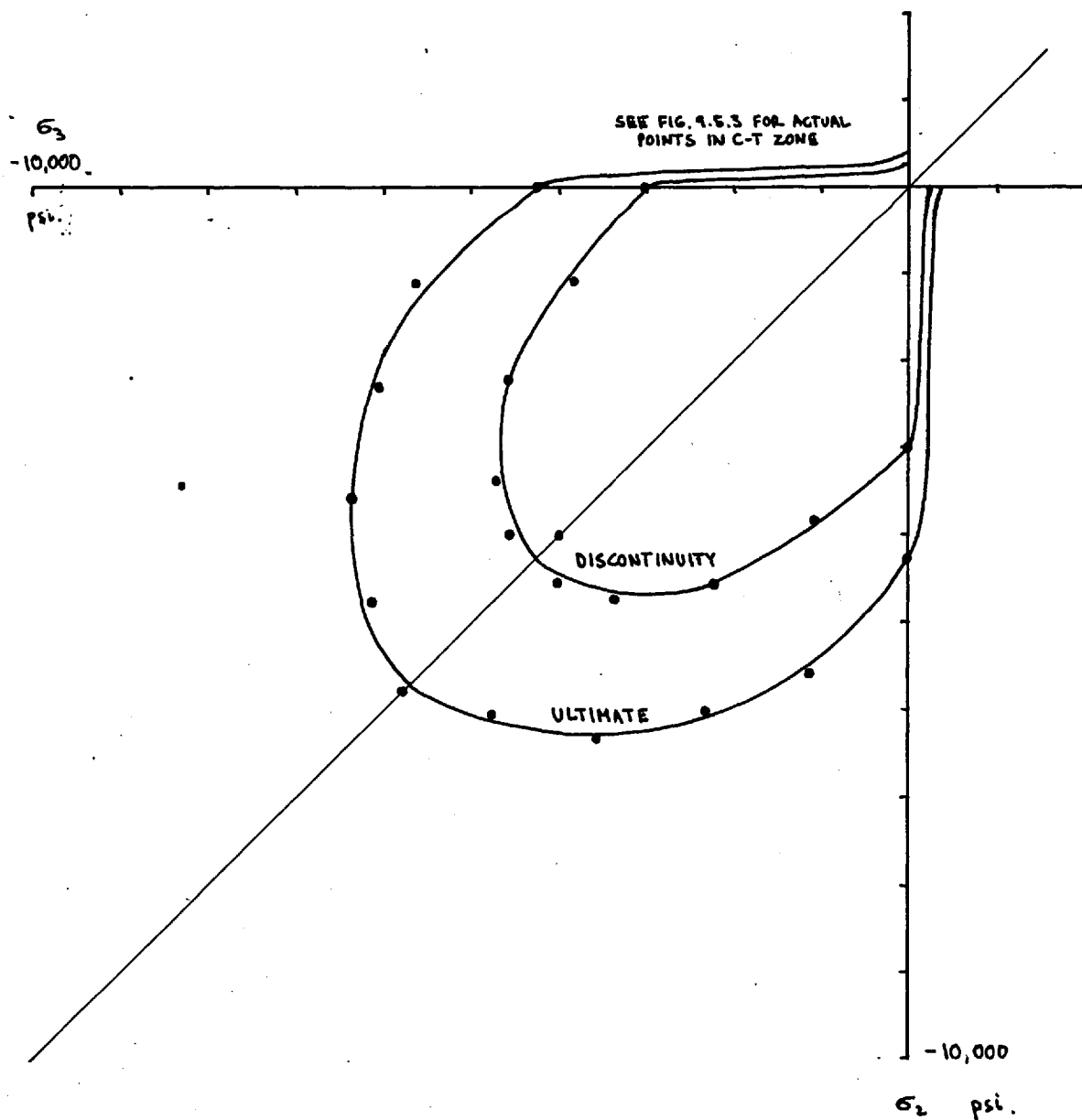


FIG. 11.3.3. BIAxIAL STRESS ENVELOPES FOR MIX 61B.

MIX 61C $\frac{1}{2}$ " 7.5 $\frac{3}{4}$ " 2.4 $\frac{1}{2}$ " 0.55 $\frac{3}{4}$ " IRREGULAR COARSE AGGREGATE

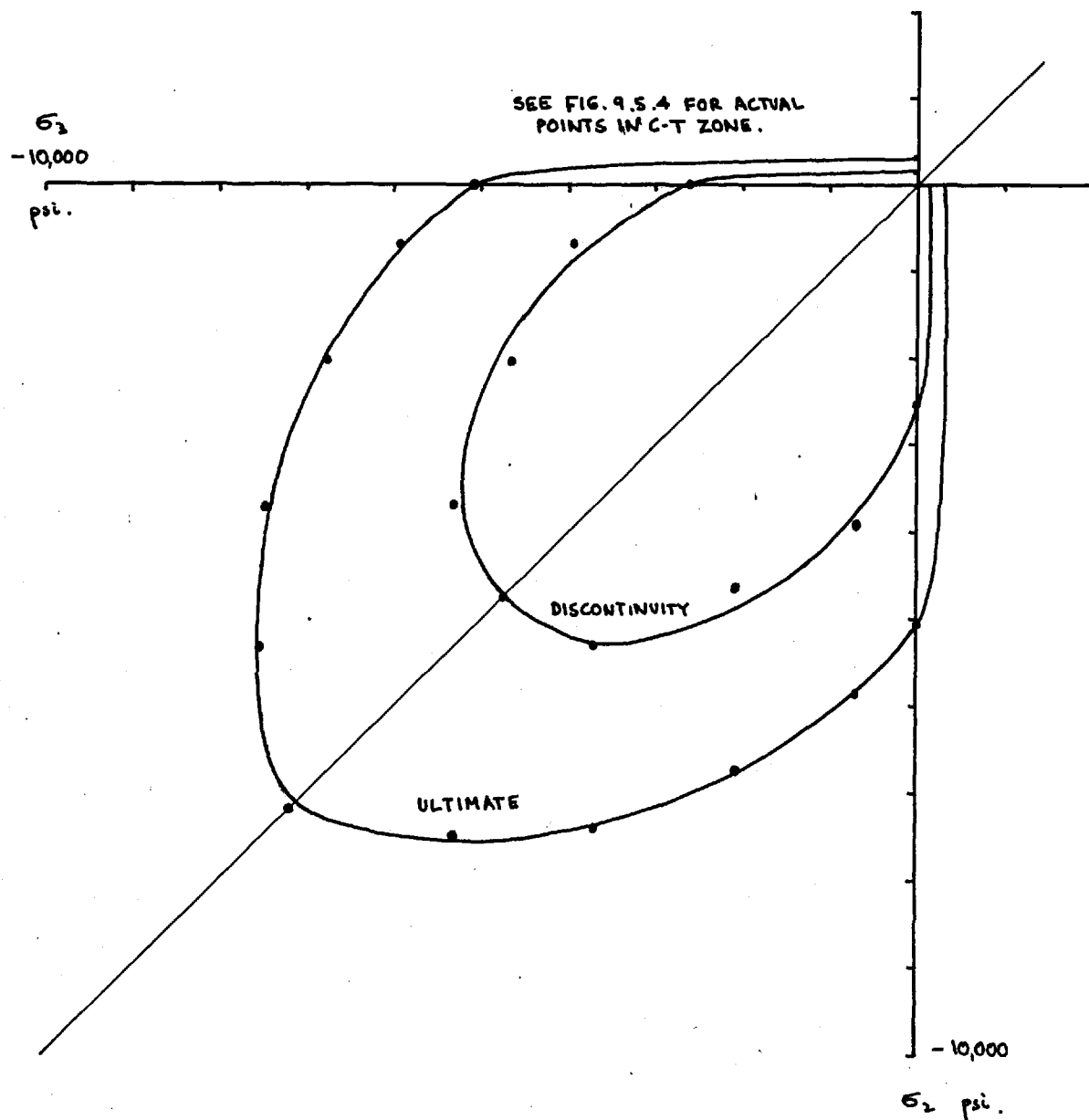


FIG. 11.3.4. BIAxIAL STRESS ENVELOPES FOR MIX 61C.

MIX GAB $\frac{A}{C}$ 4.5 $\frac{S}{C}$ 2.4 $\frac{W}{C}$ 0.55 $\frac{3}{4}$ " ANGULAR COARSE AGGREGATE

119

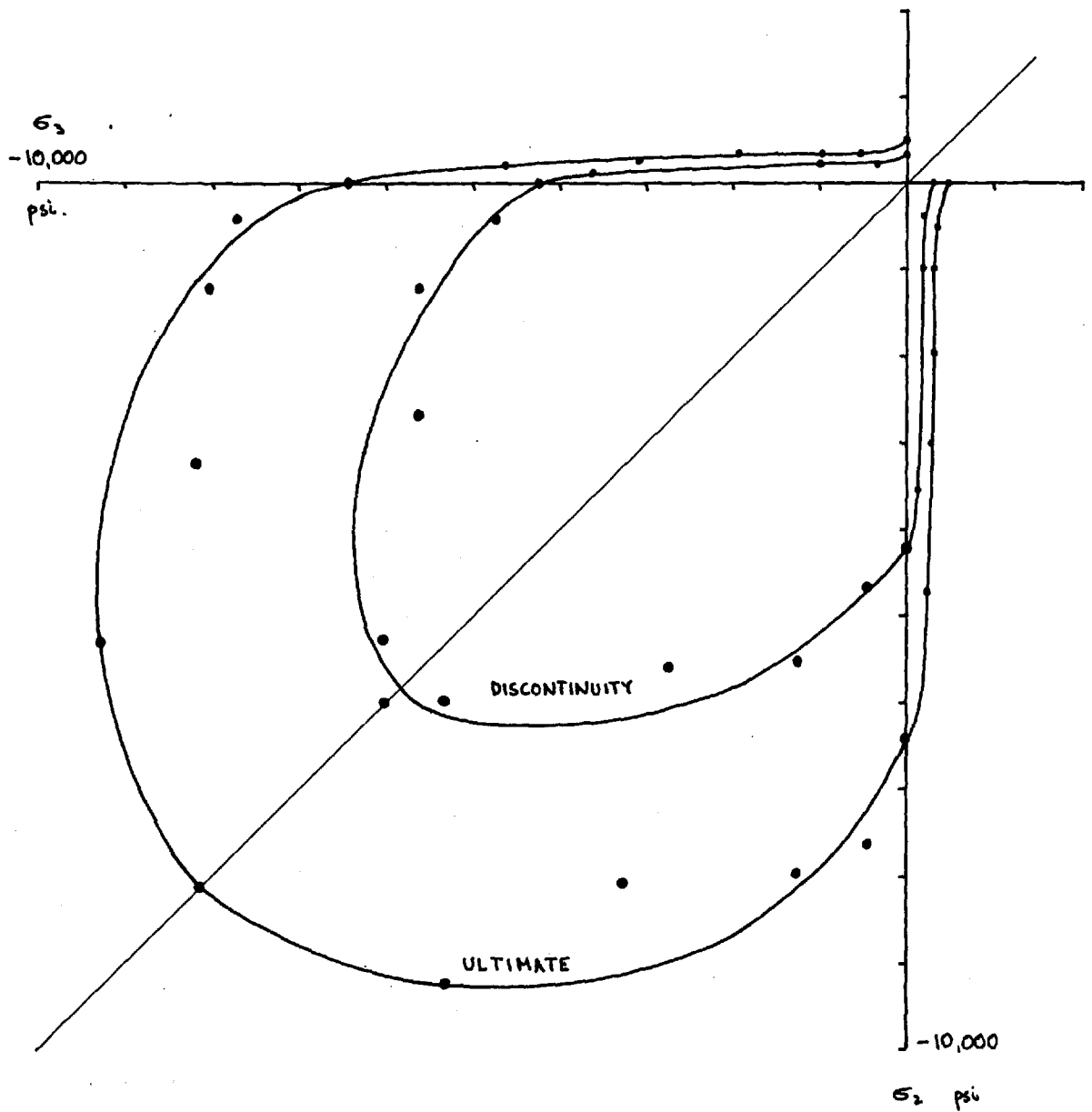


FIG. 11.3.5. BIAxIAL STRESS ENVELOPES FOR GAB.

MIX 6LB $\frac{N}{C}$ 4.5 $\frac{S}{C}$ 2.4 $\frac{W}{C}$ 0.55 $\frac{3}{4}$ " LIGHTWEIGHT COARSE AGGREGATE

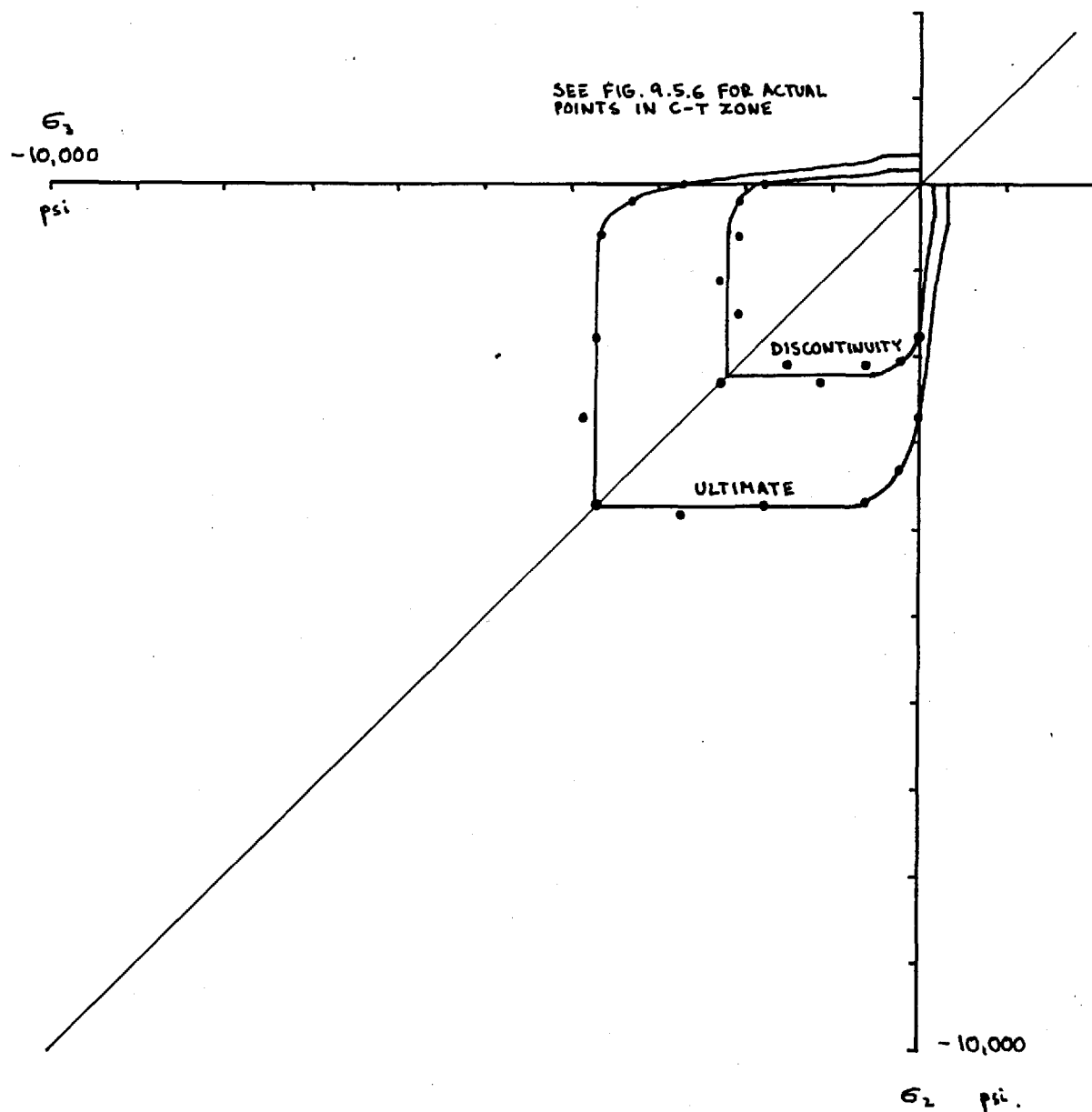


FIG. 11.3.6. BIAxIAL STRESS ENVELOPES FOR MIX 6LB.

Some considerable thought has been put into these results and their validity in view of the platen restraint problems especially in sections 4.4.6. and 8.6.1. on the biaxial compression envelopes. It is not proposed to repeat these arguments here but simply refer to these two sections and the fact that the mortar results are very similar to those obtained for cast iron which is a material with a very similar structure.

As has been pointed out in Chapter 8, many of the factors which affect the uniaxial strength properties and elastic parameters, such as aggregate stiffness, texture, shape, and water-cement ratio, appear to have no significant effect on the shape of the stress envelopes obtained. An examination of the envelopes shown in figs. 11.3.1. - 11.3.6. shows that three basic shapes of envelope exist and that these are determined by the volume fraction of normal aggregate (including and above $\frac{3}{8}$ " maximum size) in the mix. Whilst the mortars (MA and MBC) exhibit a square shaped envelope in the biaxial compression zone and a concave envelope in the biaxial compression-tension zone, a normal aggregate concrete with a nominal volume fraction V_f of 0.50 (61C) exhibits a continuously convex envelope in both sections. The intermediate mixes (61B and 6AB) have a slightly less pronounced convex envelope under biaxial compression and show an intermediate behaviour under biaxial compression-tension with a doubly curved envelope. The remaining mix which was a lightweight concrete (6LB) behaved exactly the same as the mortars except that under biaxial compression-tension there was a slight tendency to copy the intermediate mix (61B and 6AB) behaviour. These three basic envelopes are summarised in the unit diagrams in fig. 11.3.7.

These stress characteristics can be examined from the alternative viewpoint of a two-phase model of concrete in which the coarse aggregate particles are regarded as the included phase in the mortar which is the matrix plane. With this model in view, the stress envelopes of a parent mortar (MBC) and two concretes from

M = MORTAR + LIGHTWEIGHT CONCRETE
 C32 = NORMAL CONCRETE ($V_f = 0.32$)
 C51 = NORMAL CONCRETE ($V_f = 0.51$)

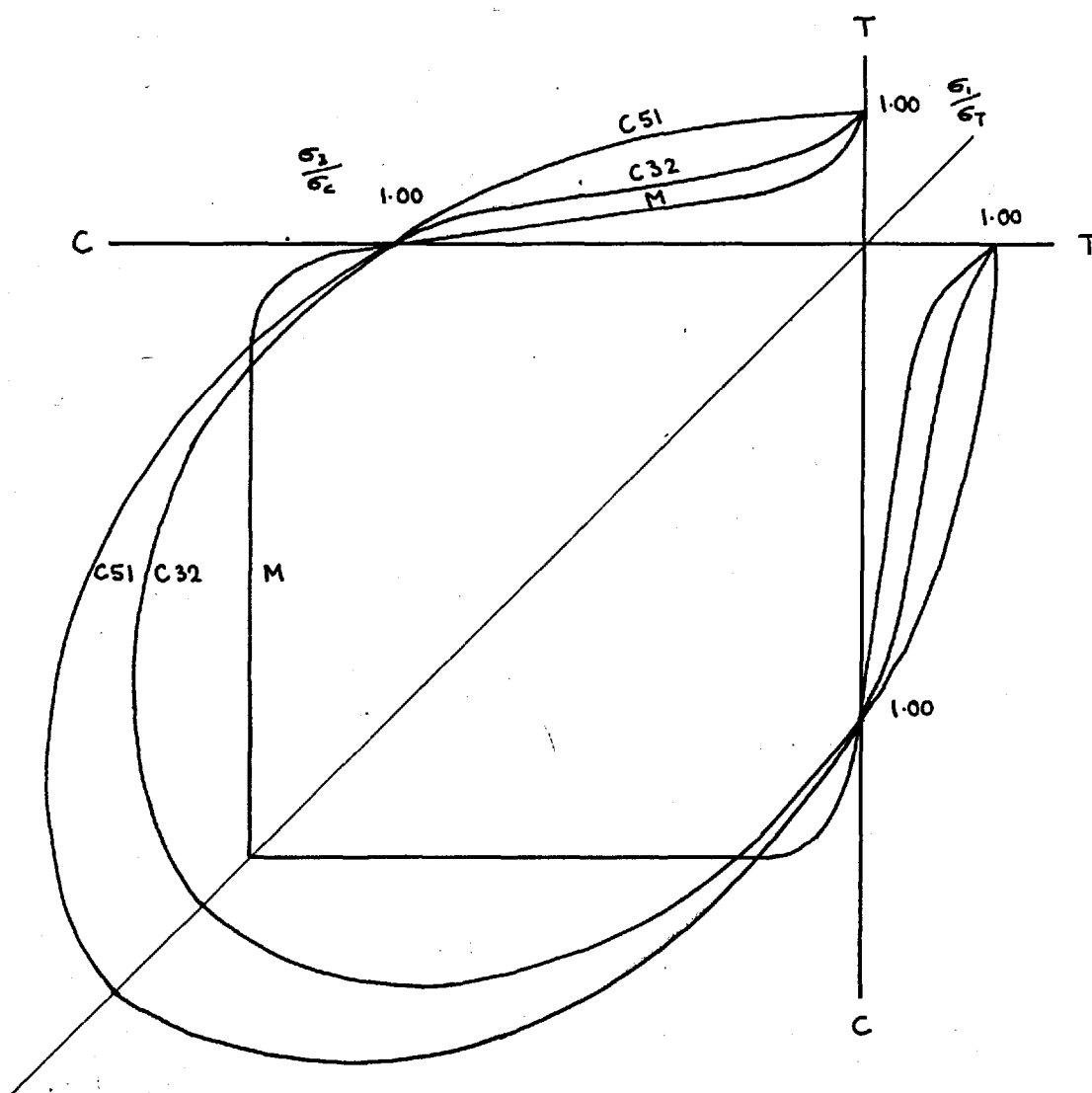


FIG. II.3.7. DIAGRAMMATIC SUMMARY OF THREE BASIC SHAPES OF STRESS ENVELOPES.

the same mortar (6AB and 6LB) have been plotted in fig.11.3.8. As the discontinuity and ultimate stress envelopes are of the same shape, the ultimate stress envelopes have been plotted only, for clarity. The concrete mix 6AB contains $\frac{3}{4}$ " crushed granite aggregate and is therefore representative of a model of hard particles ($V_f \approx 0.30$) in a soft matrix whilst the lightweight aggregate concrete 6LB contains $\frac{3}{4}$ " Leca aggregate and is therefore representative of soft particles ($V_f \approx 0.30$) in a hard matrix. The effect of inserting stiff aggregate particles into mortar can be seen to change the whole stress envelope and probably the failure mechanism as well.

Mohr's circle construction has been used as an alternative method of presenting the stress results and these are shown in figs. 8.5.2, 16-19 and are referred to in Chapter 12 when discussing a failure criterion for mortar.

11.4. Strain Characteristics

The only major strain characteristic not so far discussed in this chapter is the cleavage strain at discontinuity. It was recognised in Chapters 7, 8 and 9 that the failures under all states of biaxial stress were cleavage in nature with the fracture plane being at right angles to the principal tensile strain. From this it was concluded that the cleavage strain at discontinuity might be a possible criterion of failure.

The cleavage strains at the discontinuity point, as defined by the ultrasonic pulse velocity are plotted in fig. 11.4.1. against mean normal stress for all tests ranging from uniaxial tension to equal biaxial compression for a representative sample of mortars and concretes. In all cases it was noted that the cleavage strain increased with mean normal stress with the greatest increases being observed in the mortars. It might be suggested that this difference between the mortar and concrete cleavage strains was due to an error

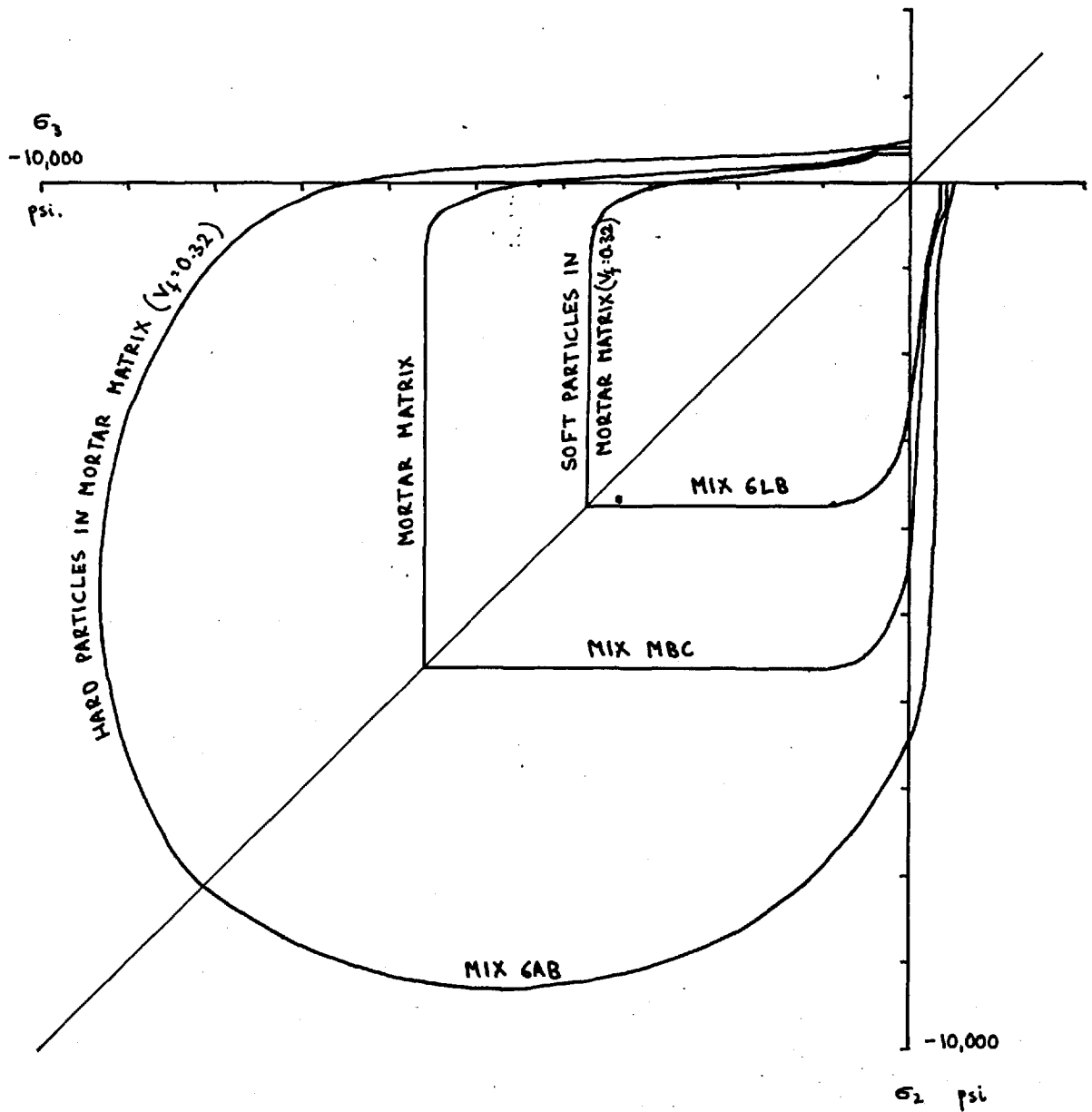
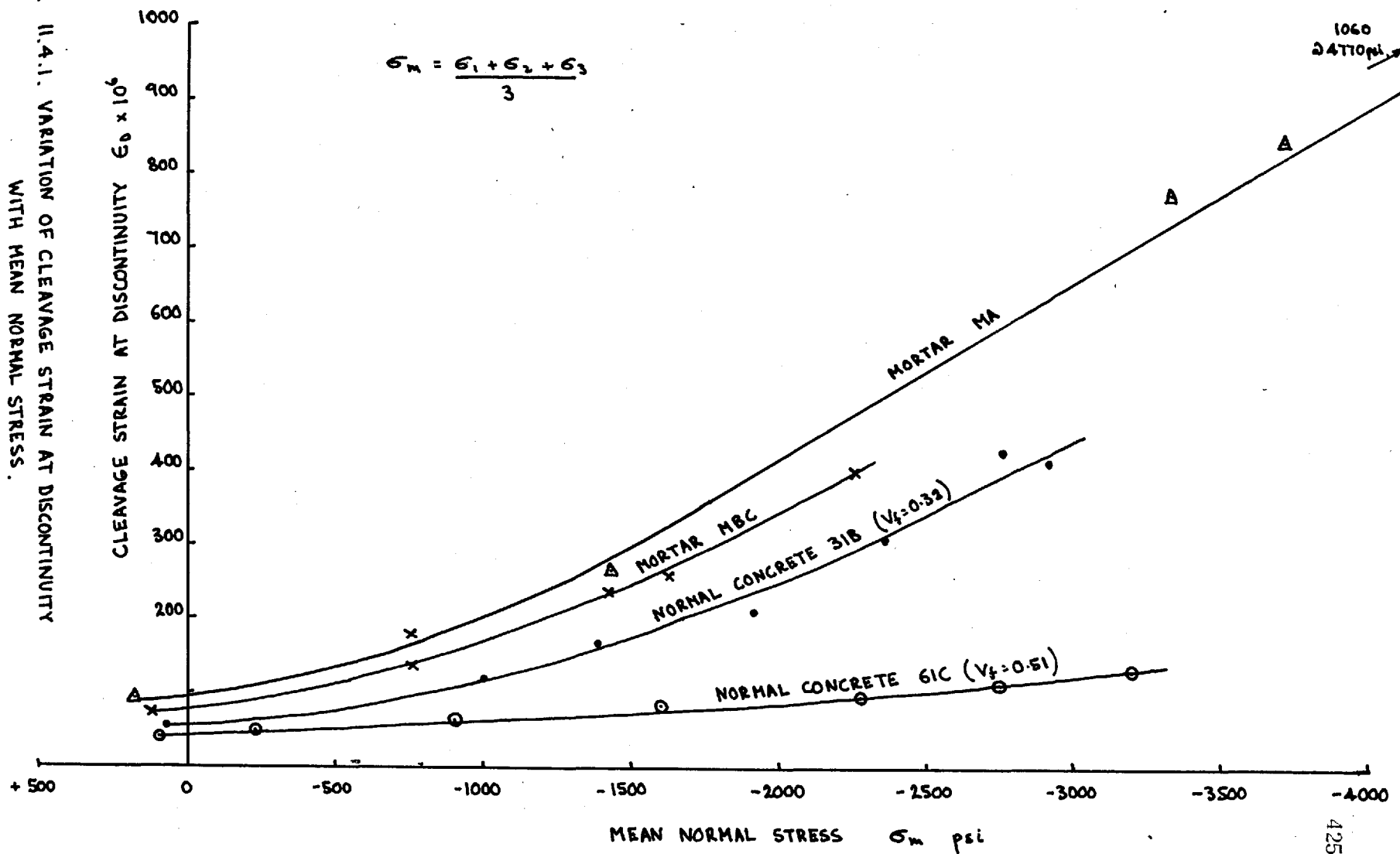


FIG. 11.3.8. ILLUSTRATION OF COARSE AGGREGATE PARTICLE EFFECTS ON STRESS ENVELOPES.

FIG. 11.4.1. VARIATION OF CLEAVAGE STRAIN AT DISCONTINUITY WITH MEAN NORMAL STRESS.



in determining the discontinuity point for one of them. However, it can be seen both in fig. 8.5.1.6. and fig. 11.2.4. that the discontinuity point for both mortars and concretes as defined by the ultrasonic pulse velocity coincides with a marked increase in the Poisson's ratio which can only be due to the onset of major cracking. Hence the increase in cleavage strains at discontinuity must be fundamental because the mortars have lower elastic moduli and higher Poisson's ratio than the concretes. The increased distortional capacity of concrete under increasing mean normal stress is very significant when related to the plastic flow experienced under triaxial compression with very large strains reported in Chapter 10. The results appear to be rather scattered but this is due to the fact that the cleavage strain is very sensitive to the actual selection of the discontinuity stress. This point is not as easily defined as the ultimate stress as it depends upon the interpretation of the ultrasonic pulse velocity measurements and the interpolation required between load stages.

Evans and Kong⁽¹⁸⁴⁾ in tests on composite prestressed concrete beams have observed strains at discontinuity (their term is microcracking), as defined by strain measurement, was within the range $70-107 \times 10^{-6}$ for concretes ranging from 1:1½:3:0.55 to 1:2:4:0.65. This discontinuity point was observed to occur at between 48% and 90% of the ultimate load. These results compare closely with the results of this research project.

11.5 Failure Modes

The crack patterns observed in all the tests have been collected together in this section so that they may be seen as a complete pattern of behaviour and not as fragmentary evidence. Indeed the results observed do form a definite picture and an idealised summary of the crack patterns is given in fig. 11.5.1. for the states of uniaxial and biaxial stress covered by this research project.

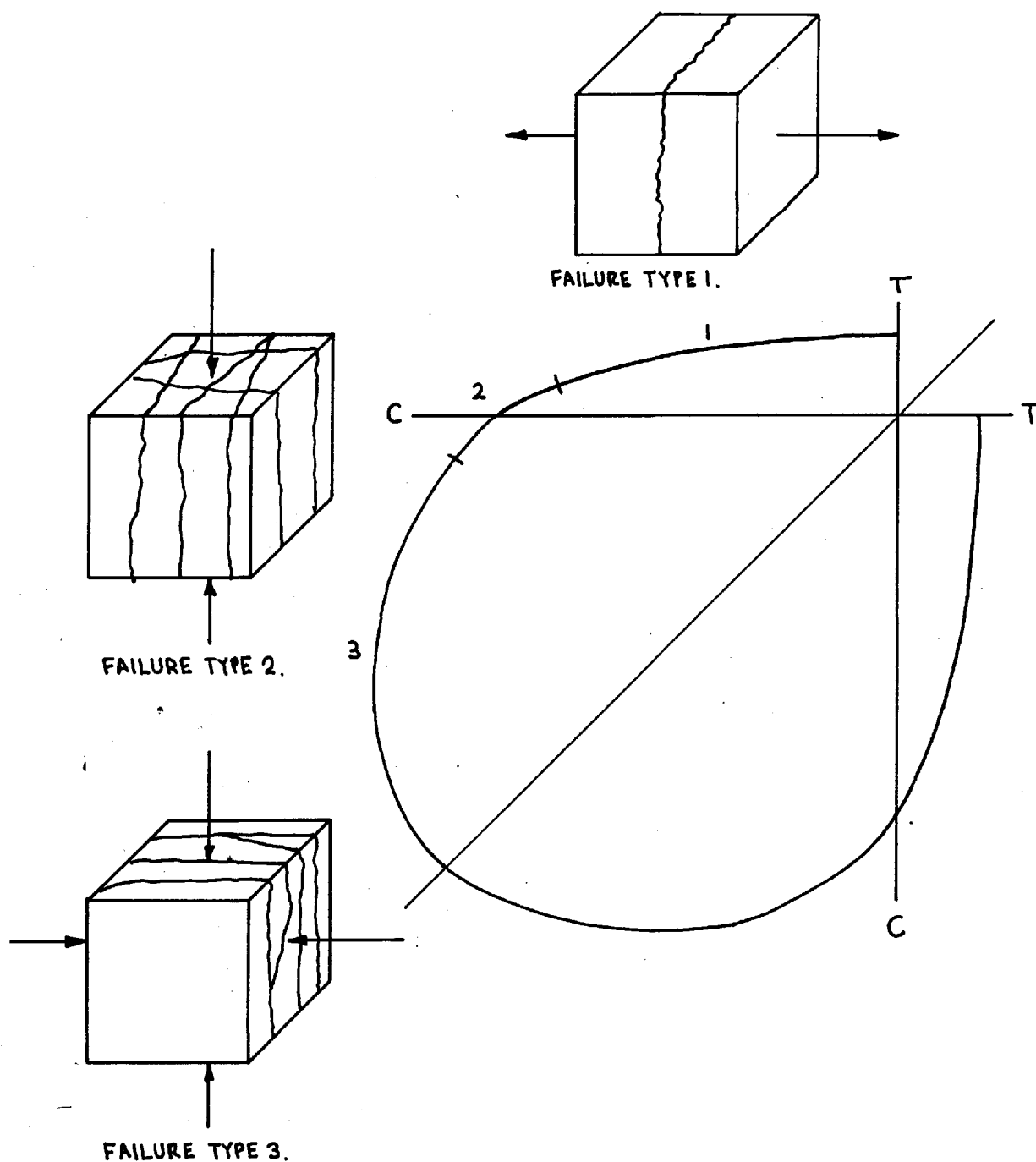


FIG. II.5.1. IDEALISATION OF THREE MAIN CATEGORIES OF BIAxIAL STRESS FAILURE CRACK PATTERNS.

There is a constant danger of over-simplification in describing the complex behaviour of concrete in order to make the topic more manageable which could lead to erroneous conclusions because significant characteristics were ignored for the sake of simplicity. However, with this qualification in mind, it is suggested that the crack behaviour observed may be divided into three main categories.

Under uniaxial tension, and biaxial compression-tension states of stress until the compression component of stress exceeded approximately 85% of the uniaxial compression strength, failure took the form of a single cleavage failure at right angles to the axis of the principal tensile stress. This behaviour was common to both mortars and concretes and in both cases the fracture was very clean with very little loose material being expelled from the fracture surface. In states of direct uniaxial tension there was often no such debris as tiny pieces of aggregate or mortar to be seen at all and only a very small quantity of fine dust.

The second division of behaviour may be defined as that under uniaxial compression in which failure occurred by cleavage splitting parallel to the direction of the principal compression stress causing a bursting action in the lateral direction. Considerable loose material was found on the fracture surfaces and the specimen tended to split into columns.

Lastly, under biaxial compression, the mode of failure observed for all degrees of biaxial compression was a cleavage splitting parallel to the plane of the principal compression stresses causing a bursting action in the third unstressed direction. This type of failure also contained a lot of loose material on the fracture planes. In this division of behaviour there did appear to be a distinct difference between mortars and concretes in that whilst the concretes contained many planes of cleavage strictly parallel to the plane of the principal compressive

stresses, the mortars only contained two or three such fracture planes and these were often at an angle of up to 40° to the plane of cleavage over sections of their surface (fig. 11.5.2.). These mortar fractures also contained a lot of fine dust in contrast to their very clean fractures under tensile states of stress.

Figure 11.5.3. shows diagrammatically the location of the photographs of crack patterns shown in figs. 11.5.4. - 11.5.9. around the biaxial stress envelope. This photographic evidence is a graphical confirmation of the generalisations made above regarding failure modes. Only under biaxial compression states of stress was any distinct difference observed between mortars and concretes; the behaviour of lightweight concrete being very similar to that of mortar under all states of stress.

The mechanism of crack formation and propagation could be observed by the ultrasonic pulse velocity measurements and often cracking sounds could also be heard some time before ultimate failure occurred. In the case of uniaxial and biaxial compression states of stress cracks could also be seen on the surfaces of the test specimens prior to failure. However, the final fracture of the specimen took place very rapidly under all states of stress due probably to the sudden releast of energy stored in the hydraulic system as discussed in section 3.3.

From loose material collected from uniaxial and biaxial compression fracture surfaces of normal aggregate concretes, it was noticed that many of the aggregate particles had small volumes of mortar stuck onto some parts of their surface whilst the remainder of the surface was completely clean of any mortar so indicating a clean break with the mortar matrix. These particles could be picked off the heavily fissured fracture surfaces and the orientation of the aggregate particles to the direction of the principal stresses was noted. A large number of these particles (about one hundred)

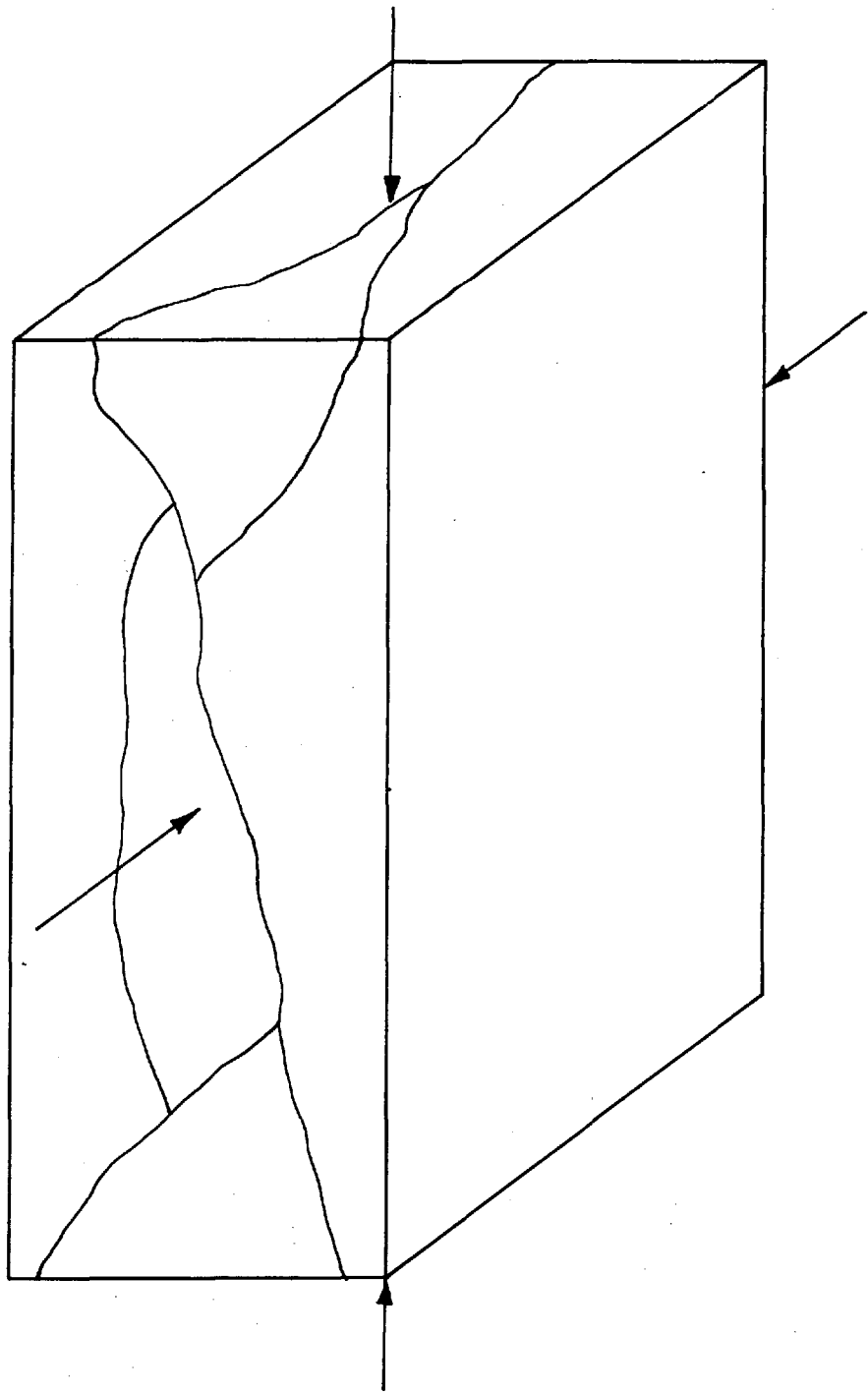


FIG. 11.5.2. SKETCH OF CRACK PATTERNS IN MORTAR BIAXIAL COMPRESSION SPECIMEN.

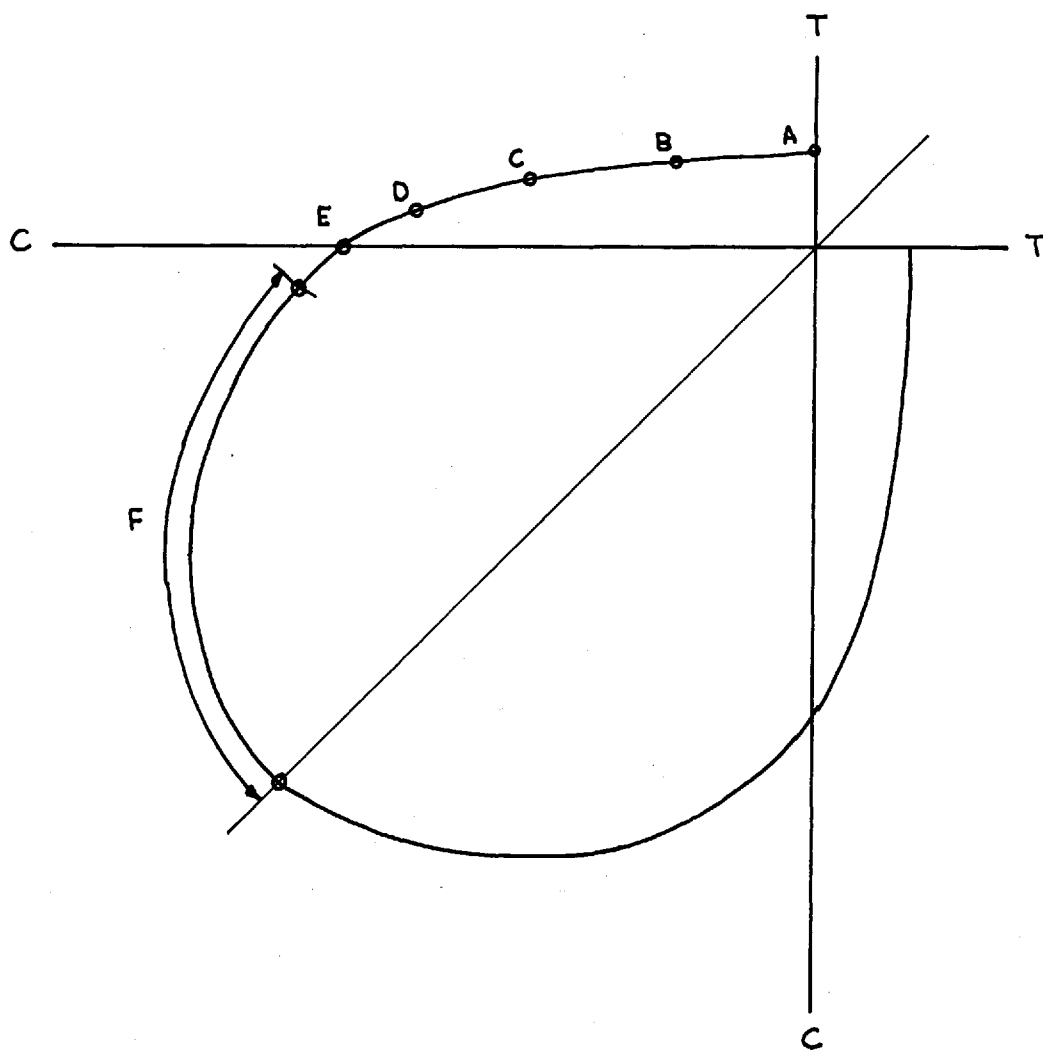


FIG. II.5.3. LOCATION OF PHOTOGRAPHS OF CRACK PATTERNS ON ULTIMATE STRESS ENVELOPE.



Fig. 11.5.4. Uniaxial Tension Fracture A.

(24)



Fig. 11.5.5. Biaxial Compression-Tension Fracture B.

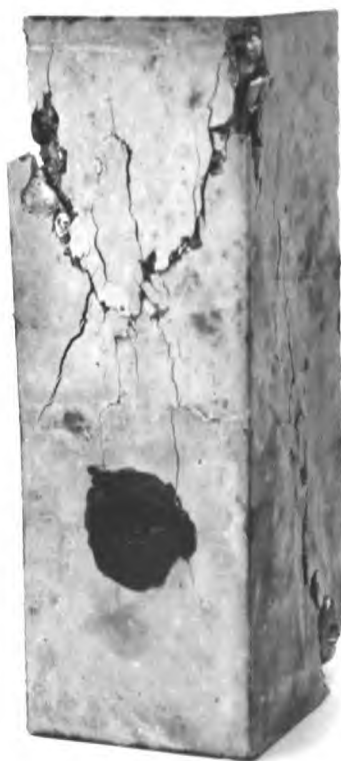


Fig. 11.5.6. Biaxial Compression-Tension Fracture C.



Fig. 11.5.7. Biaxial Compression-Tension Fracture D.

(26)



61C

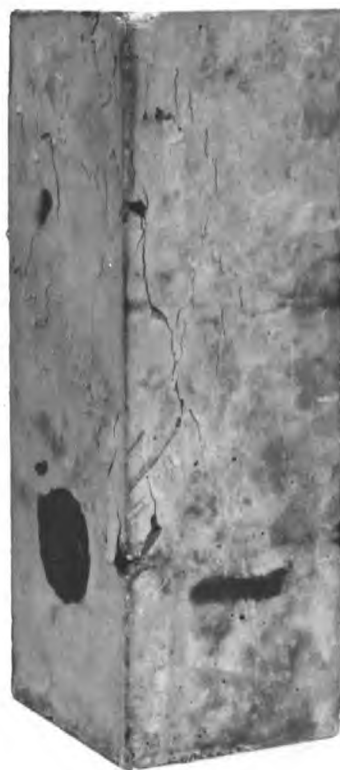


Fig. 11.5.8. Uniaxial Compression Fracture E.

(27)



Fig. 11.5.9. Biaxial Compression Fracture F.

were collected from the various mixes tested under biaxial compression and some significant trends were observed.

The particles collected from uniaxial compression tests had small cone-like pieces of mortar sticking to them. These cones of mortar were often to be found at opposite ends of an aggregate particle with the common axis of the cones parallel to the axis of the applied compression (a similar observation has been reported by Sigvaldason⁽⁹⁾). The remainder of the stone was very clean, often polished, with no particles of mortar clinging to it and only traces of cement paste visible inside cavities in the surface of the stone.

Under biaxial compression these same pieces of mortar were seen to stick to the aggregate particles but they now tended to form complete rings or haloes around the stone with the plane of the halo lying in the plane of the principal compressive stresses. For states of biaxial compression in which the intermediate stress was very low, these haloes were less continuous being more in the shape of a short crescent.

An idealised picture of these mortar shapes sticking to aggregate particles is given in fig.11.5.10. and some photographs of actual aggregate particles collected are shown in figs. 11.5.11 and 11.5.12.

Another general trend with these mortar shapes was the different dimensions of the mortar cones occurring with different aggregate surface textures. The coarse aggregate textures gave a large cone whilst the smooth rounded aggregate exhibited very small cones or none at all. The contrast between the cones found on rounded, irregular, and angular aggregate is illustrated in fig. 11.5.13.

It was observed that very few aggregate particles were split in the fracture process in uniaxial and biaxial compression tests. They amounted to 10% of the aggregate particles on the fracture surface at most.

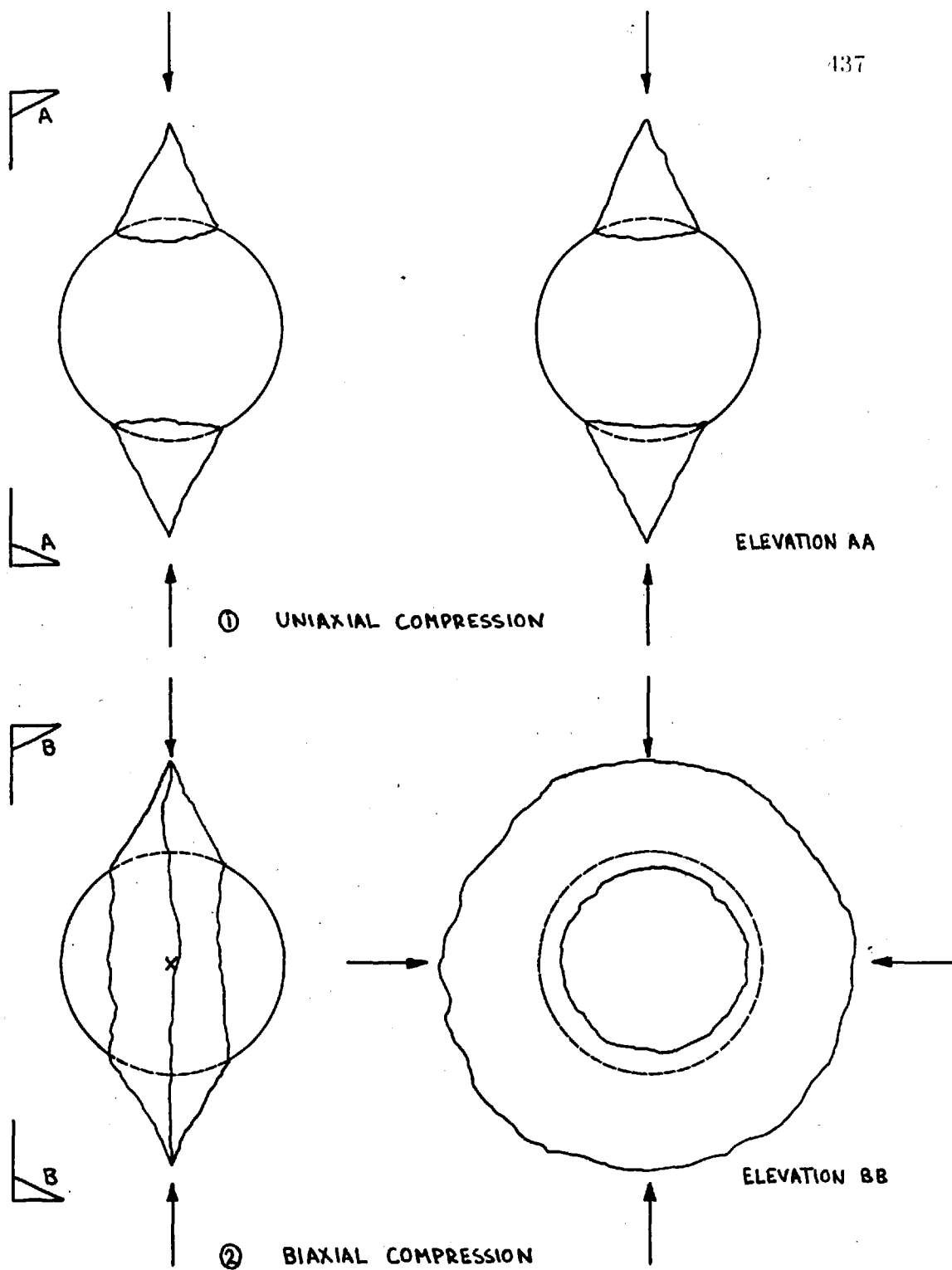


FIG. 11.5.10. IDEALISED PICTURE OF MORTAR SHAPES FOUND ON AGGREGATE PARTICLES.

(28)

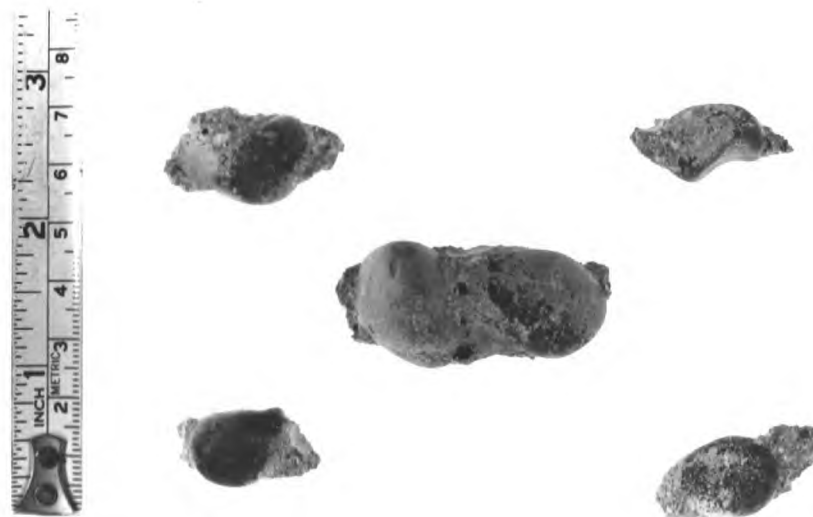
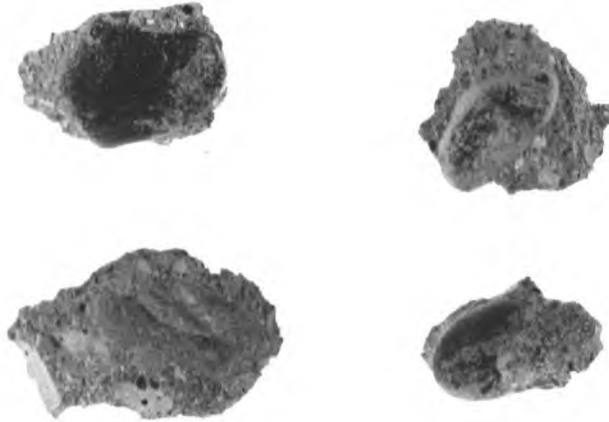


Fig. 11.5.11. Mortar Cones on Aggregate Particles
from Uniaxial Compression Tests.



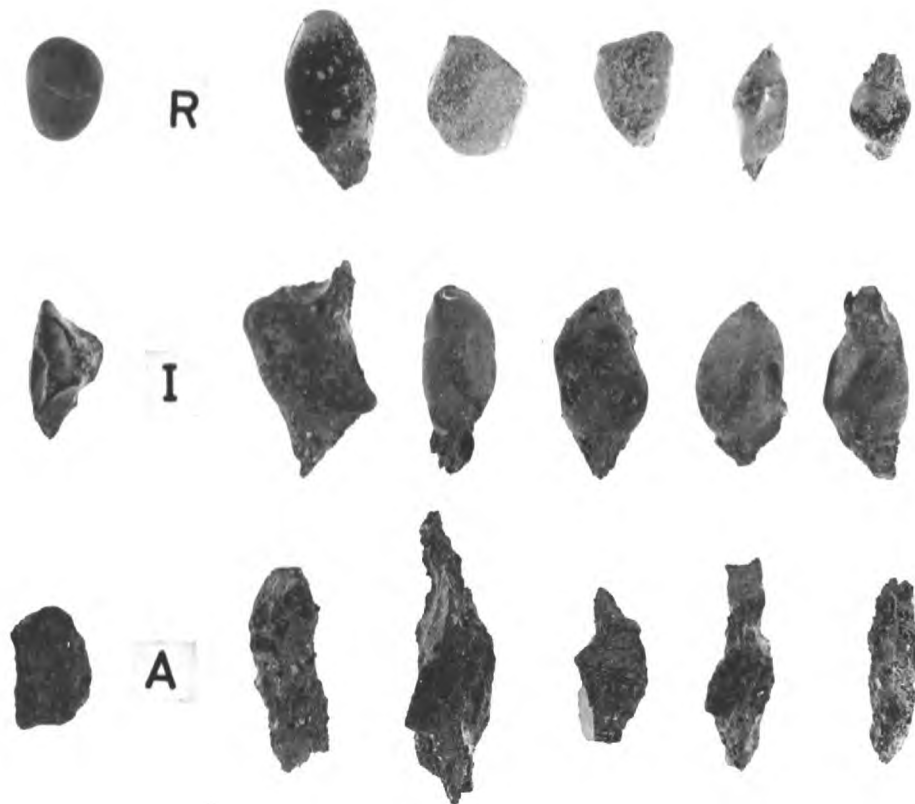
a) Front elevation.

(29)



b) Side elevation

Fig. 11.5,12. Mortar Haloes on Aggregate
Particles from Biaxial Compression Tests.



R = Rounded. I = Irregular. A = Angular.

Fig. 11.5.13. Effect of Aggregate Texture on sizes
of Mortar Cones.

The failure modes observed in the biaxial compression-torsion tests were both of the single cleavage and the multi-cleavage types similar to those observed under biaxial compression-tension. However, under triaxial compression, the material maintained a high degree of continuity unlike the uniaxial and biaxial failures and the failure was non-explosive. Tensile strains in the direction of the minor compressive stress of 0.13 were recorded with the mortar MBC and the material showed no outward signs of cracking. Even with the concrete 61B a strain of 0.05 was recorded but some cracks were evident, probably caused by the mortar in its plastic state having to 'flow' round the stiff aggregate particle.

11.6. Long-Term Strength

The principal results of this test series on uniaxial compression prisms were that failures were obtained on long-term tests at stresses below the short-term test ultimate strength and that the rate of longitudinal strain increase was dependent upon the discontinuity stress. The rate of longitudinal strain increase was found to be of a very small magnitude (seldom greater than 3×10^{-6} per minute) until the discontinuity stress was exceeded whereupon a very rapid increase in this strain rate under constant stress resulted. This behaviour once again emphasises the fundamental significance of the point of discontinuity as defined by the ultrasonic pulse velocity measurements and suggests a positive link between the discontinuity stress and long-term strength of concrete. However, a very considerable test series with good load maintaining equipment would be necessary to establish this link.

The failure modes observed in the long-term tests were the same as those obtained in short-term tests with the same debris resulting. However, the failure was less explosive and the crack formation was less noisy.

11.7. Conclusion

From the many experimental observations reported and discussed in this chapter the following more general conclusions may be drawn:-

1) The stress-strain characteristics of mortars and lightweight concrete under uniaxial tension and compression and biaxial compression obey the elastic theory equations within practical limits. The stress-strain characteristics of normal aggregate concretes under uniaxial tension and compression obey the elastic theory equations within practical limits but the behaviour under biaxial compression does not obey the elastic theory.

2) Below the discontinuity stress mortars and lightweight concrete maintained equal and constant values of elastic modulus, Poisson's ratio and hence bulk modulus and rigidity modulus under uniaxial tension, uniaxial compression, and biaxial compression states of stress. Normal aggregate concretes gave equal values of elastic modulus and Poisson's ratio under uniaxial tension and compression states of stress but under biaxial compression considerable increases in elastic modulus with degree of biaxial compression together with small increases in Poisson's ratio were noted. The values of bulk modulus and rigidity modulus also increased with the degree of biaxial compression. The effect of the small increase in Poisson's ratio was to increase further the value of the bulk modulus but to reduce the increase in rigidity modulus.

3) The biaxial stress envelopes which covered the whole biaxial stress characteristic with the exception of biaxial tension combinations showed similar shapes of discontinuity and ultimate stress envelopes for a given mix. Three basic shapes of envelope emerged in which mortars and lightweight concretes gave a square shaped envelope under biaxial compression and a

concave envelope under biaxial compression-tension. Normal aggregate concretes with a volume fraction of about 0.50 gave a rounded convex envelope in both segments whilst normal aggregate concretes with volume fractions of about 0.30 gave a convex envelope under biaxial compression and an intermediate doubly curved envelope under biaxial compression-tension. The shapes of the envelopes obtained appeared to be influenced only by the limiting aggregate size of $\frac{3}{16}$ " volume fraction, and normal or lightweight aggregate. The shapes were uninfluenced by aggregate size above $\frac{3}{8}$ ", aggregate/cement or water/cement ratios, aggregate stiffness, or aggregate surface texture.

4) The cleavage strain at the discontinuity point as defined by the ultrasonic pulse velocity measurements was observed to increase with the mean normal stress. The cleavage strain was also seen to decrease with increasing volume fraction of coarse aggregate.

5) Under triaxial compression both mortars and concretes were observed to undergo very large strains of up to 0.13, whilst maintaining a high degree of overall continuity, amounting to plastic distortion. The influence of the coarse aggregate was to obstruct this plastic deformation and cause the mortar matrix to crack.

6) Cleavage type failures were observed under all states of biaxial stress investigated, with single cleavage fractures under uniaxial tension and most of the biaxial compression-tension zone and multi-cleavage fractures under uniaxial and biaxial compression and the remainder of the biaxial compression-tension zone. All the cleavage planes were parallel to the principal compressive stress (or at right-angles to the principal tensile stress) and small yet significant differences were observed between the compressive failures of mortars and concretes. Studies were also made of the mortar remaining on aggregate particles loosened by the fracture process. The failures observed under biaxial compression-tension tests on solid specimens and biaxial compression-tension tests on hollow cylinders were similar.

C H A P T E R 12

PROPOSED THEORIES FOR THE DEFORMATIONAL BEHAVIOUR AND FAILURE OF MORTARS AND CONCRETES.

12.1. Introduction

Any theory for the deformation and failure of mortars and concrete must satisfy the following observed phenomena if it is to be acceptable as a possible criterion of mechanical behaviour:-

1. Stress-strain relationships.
2. Stress characteristics.
3. Strain characteristics
4. Fracture modes.

These four groups of observed phenomena have been described in Chapter 11 from the results of experimental work in this research project.

Considering the stress characteristics alone, it can be seen that two distinct classes of behaviour exist under biaxial compression as noted in section 11.7. The mortars and lightweight aggregate concrete yield a square shaped stress characteristic whilst the normal aggregate concretes yield a more rounded envelope of strength (see figs. 8.5.1.1 to 8.5.1.13). However, to reduce this discussion of deformational behaviour and failure criteria to a convenient fundamental basis, lightweight concrete will be considered as a separate material from mortar. Therefore three distinct "materials" will be recognised in the discussion and treated separately as follows:-

- 12.2 Deformation and failure of mortar.
- 12.3 Deformation and failure of normal concrete.
- 12.4 Deformation and failure of lightweight concrete.

It is convenient to consider the deformational behaviour and failure criteria for mortar first as it constitutes one phase of concrete when it is considered as a two-phase material. Of the two classes of concretes, the lightweight concrete is dealt with lastly

as it exhibits qualities found in both mortars and concretes. Hence a logical sequence of discussion and hypothesis should result.

The terms of reference for this project (section 1.1) defined its aims as ultimately to establish the failure criteria for concrete and to define the permissible working stresses that concrete can withstand under all states of biaxial stress in the form of an envelope. It was recognised that an exact mathematical expression for the failure criteria proposed might not be possible in which case the results could be expressed graphically or in the form of theoretically based empirical equations.

Typical mortar and lightweight concrete, and normal concrete stress envelopes are shown in fig. 12.1.1. together with some of the more simple failure criteria for comparison. From this comparison it would appear that the experimental results do not lend themselves to definition by any simple criterion of stress or strain. However, in the ensuing sections of this chapter, attempts will be made to establish at least the mechanisms present in the deformation and failure of the three classes of "material" listed above and hypothetical failure criteria will be proposed.

12.2. Deformation and Failure of Mortar

12.2.1. Elastic Behaviour.

The elastic range is limited by the discontinuity stress as determined by ultra-sonic pulse velocity measurements previously described (section 5.4).

Within this elastic range, mortar behaves remarkably like an ideal homogeneous elastic material under short-term loading. Both the results of this investigation (see figs. 7.5.2.1-3) and the findings of Ward (7) show some degree of non-linear elasticity below the discontinuity stress which is distinct from any creep effects. However the tangent elastic modulus and the average elastic modulus and Poissons ratio are equal under uniaxial tension and compression. Furthermore, the elastic modulus is effectively constant under varying degrees of biaxial compression.

- MAXIMUM STRAIN THEORY
- MAXIMUM STRAIN ENERGY THEORY
- · - · - INTERNAL FRICTION THEORY
- · — · — DISTORTIONAL ENERGY THEORY

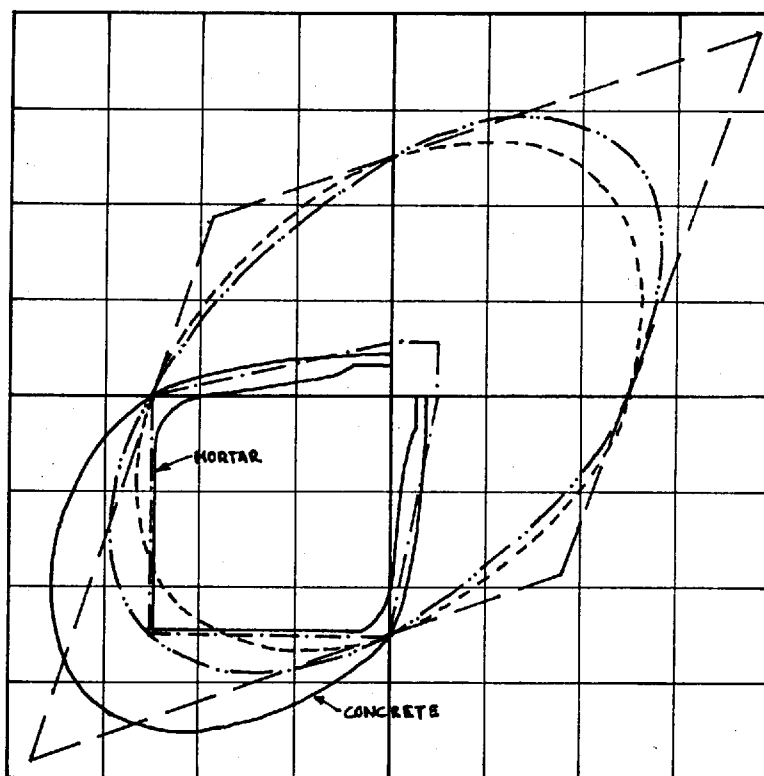


FIG. 12.1.1. COMPARISON OF SOME OF THE MAJOR CLASSICAL FAILURE THEORIES AND EXPERIMENTAL RESULTS.

(CLASSICAL FAILURE THEORIES FROM MARIN (60))

Though drying shrinkage together with entrapped air and water produce flaws in mortar, they are very small and evenly distributed throughout the mass.

Hence, for most practical purposes, mortar may be regarded as an effectively homogeneous and elastic material below the discontinuity stress for short-term static loading.

12.2.2. Criterion of Failure

It can be seen from fig. 12.1.1. that the stress envelopes for mortar, both for ultimate and discontinuity stress, do not appear to fit any of the major classical failure theories, nor do they fit any of the other theories proposed for concrete (see Chapter 1). However, the shape of the curves does have a certain likeness to that predicted by the Coulomb-Mohr internal friction theory and the results of Mohr circle plots given in figs. 5.2.16-17 are quite encouraging.

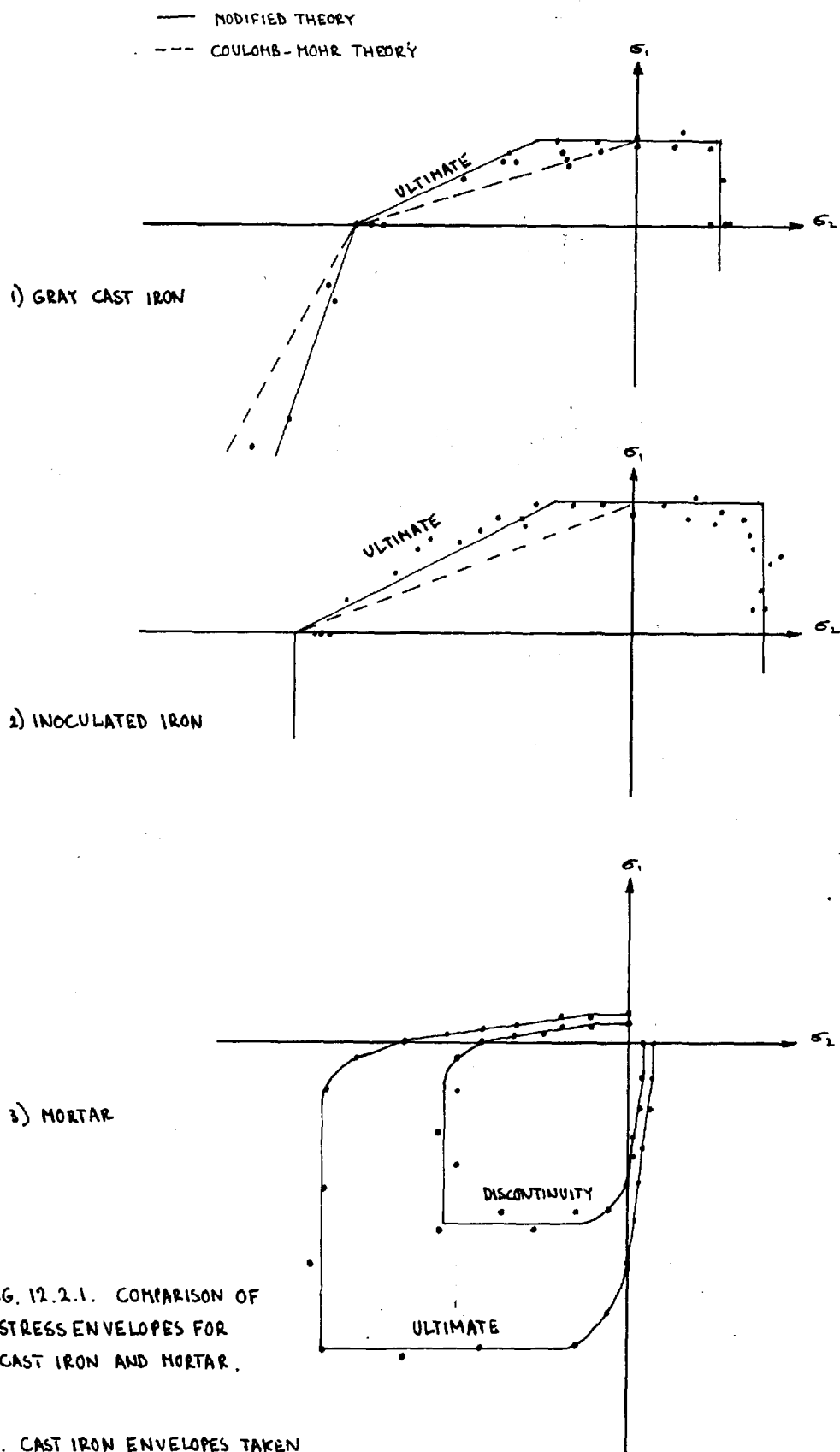
a) Application of Coulomb Mohr Theory

Therefore, it is worth comparing these results with those obtained from cast iron which like mortar is a cemented granular material containing small flaws and air pockets.

The Coulomb-Mohr internal friction theory has been suggested as the failure criterion for cast iron and therefore the comparison is particularly apt. Experimental results obtained by Coffin (185) on gray cast iron and Cornet and Grani (186) on inoculated iron are shown in fig. 12.2.1. together with those for a typical mortar. It can be seen that there are several similarities in the biaxial stress envelopes.

Burton Paul (72), in examining the application of the Coulomb-Mohr theory to intrinsically brittle materials (defined by the author as those materials which fail by cleavage in both tension and torsion tests with very little plastic flow prior to failure), suggested a modification in the form of a three-parameter theory.

The shortcomings of the Coulomb-Mohr theory are that it predicts that failure in tension or torsion occurs on a slip plane



inclined to the axis of the principal tensile stress and maintains that the ratio of tensile to compressive strength is a fixed quantity for all strength grades of the material. For both mortar and cast iron these two claims are incorrect. Simple cleavage fractures are observed under tension or torsion stress with the fracture plane orthogonal to the axis of the principal tensile stress for both cast iron and mortar. Furthermore the ratio of tensile to compressive strength is not a constant for either material (see fig. 12.2.2.).

b) Modifications to Coulomb-Mohr Theory by Paul

Defining the Coulomb-Mohr theory as a two-parameter theory because its two experimentally determined constants are the cohesion strength and the angle of friction, Burton Paul introduced a third parameter of limiting tensile stress. Thus his modified theory states that a brittle material will fracture on either that plane where the virtual shear stress reaches a critical value S_s , or on that plane where the maximum tensile stress reaches a critical value S_T , whichever occurs first.

The introduction of a limiting tensile stress into the Coulomb-Mohr theory is not new. Roš (32) and Cowan (69) have both proposed similar modifications to the basic theory but the tension limit or cut-offs suggested by both workers were restricted to the triaxial tension zone in such a manner that they had no effect on the biaxial stress characteristic and only very limited effect on the triaxial characteristic (see fig. 12.2.3.). On the other hand, the tension cut-off proposed by Burton Paul affects all biaxial and many triaxial stress envelopes. A pictorial comparison of the limiting tensile stress criteria or tension cut-offs proposed by Cowan and Burton Paul is given in fig. 12.2.3. with the biaxial stress envelopes marked.

In applying this modified three-parameter version of the Coulomb-Mohr theory to the cast-iron experimental results quoted above, Burton Paul was able to report a better correlation than that obtained with the unmodified theory plus the fact that there was a better explanation of the failure modes (fig. 12.2.4.).

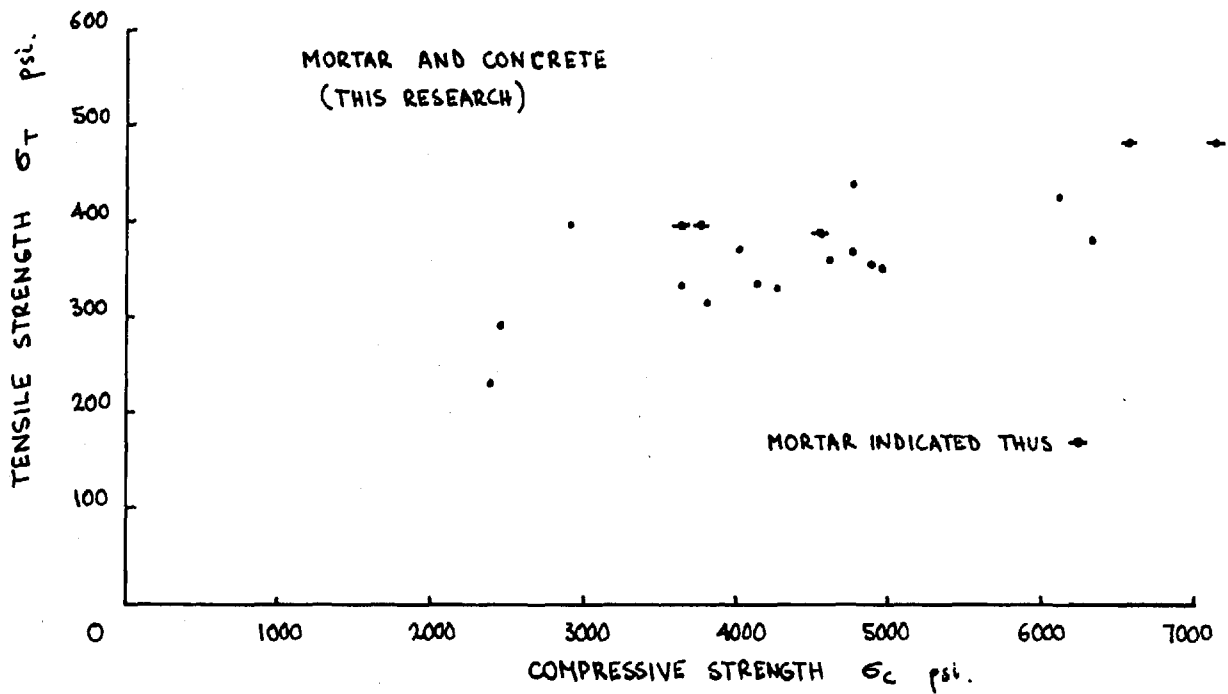
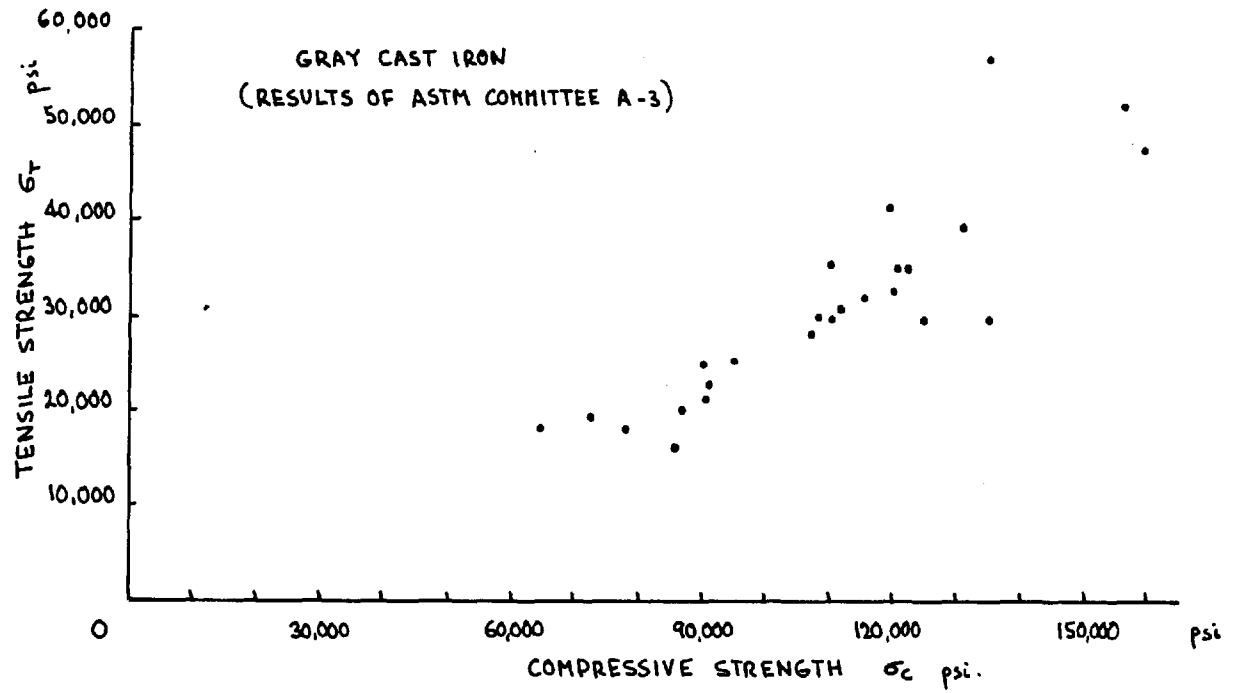


FIG. 12.2.2. COMPARISON OF TENSILE AND COMPRESSIVE STRENGTHS FOR GRAY CAST IRON, MORTAR, AND CONCRETE.

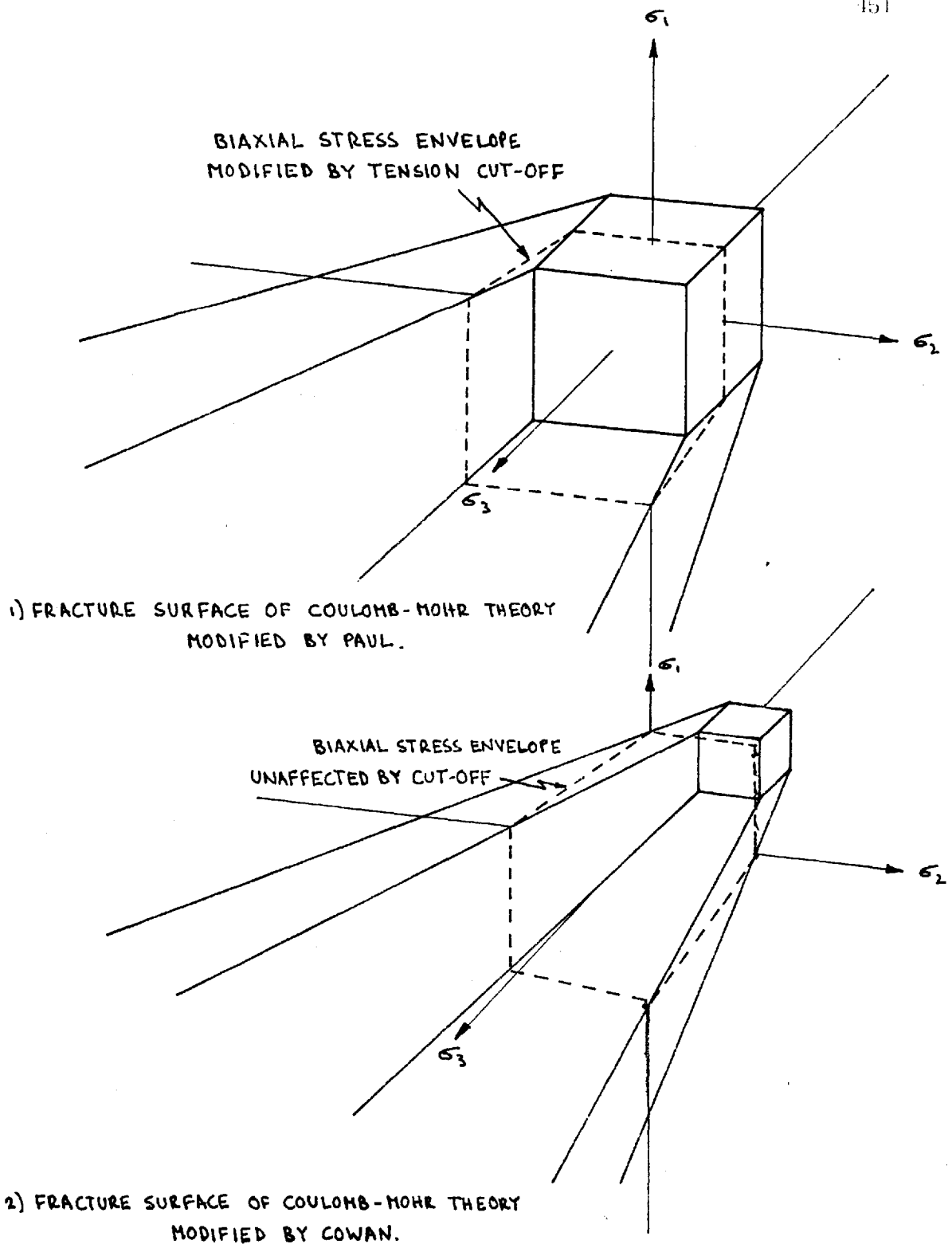


FIG. 12.2.3. COMPARISON OF MODIFICATIONS TO COULOMB-MOHR THEORY
BY PAUL AND COWAN AND THEIR EFFECT
ON THE BIAxIAL STRESS ENVELOPE.

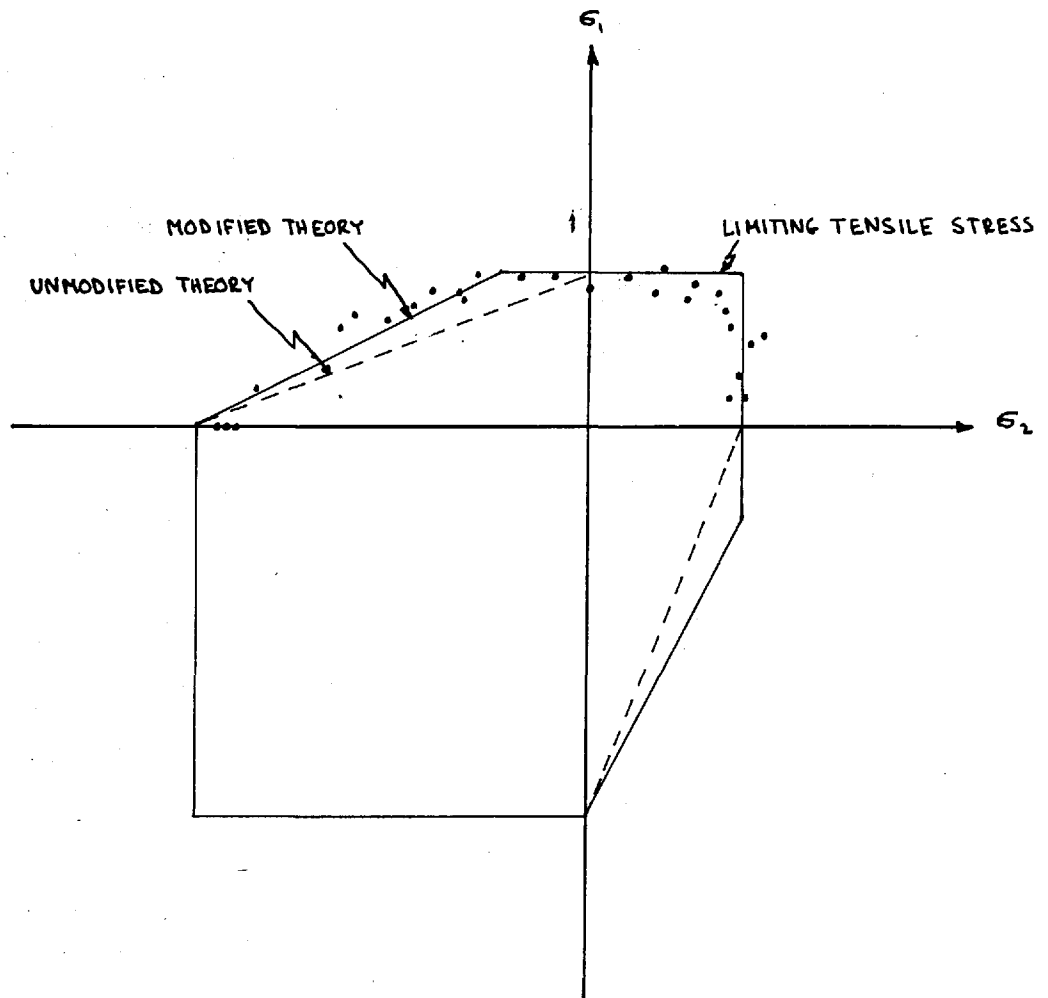


FIG. 12.2.4 MODIFICATION TO COULOMB-MOHR THEORY BY PAUL (72)
BY ADDITION OF LIMITING TENSILE STRESS CRITERIA
COMPARED WITH RESULTS FROM INOCULATED IRON.

He then proceeded to advance a hypothesis which would explain the appearance of concavities on the biaxial compression-tension envelope given certain conditions. The hypothesis was based on the argument that whilst it is easy to see that compression stress tends to strengthen the material because of the friction coefficient μ , it is difficult to see how a frictional mechanism can weaken a material when the normal stress on the plane of fracture is tensile. The hypothesis was therefore developed on the modified Coulomb-Mohr theory described above but with the new condition that the friction coefficient μ was zero under normal tensile stress.

The resulting stress envelope obtained from this hypothesis is shown in fig. 12.2.5. Although experimental scatter makes it difficult to decide which of the two modified theories (i.e. three parameter or no friction under tension plus three parameter) proposed by Burton Paul is more suited to cast iron, the resemblance between the shape of this second stress envelope from the "no friction under tension" hypothesis and the experimental results obtained from mortars is worthy of note.

Both modifications of the Coulomb-Mohr theory by Burton Paul require only the results of uniaxial compression and tension tests to establish the three parameters required (cohesion strength, friction coefficient, and limiting tensile stress) to define the biaxial and triaxial stress envelopes. However, it should be noted that the concavities predicted in the second modification are contrary to the convex envelopes required by plastic theory for ductile materials (187).

As the second modification appears to give a much better correlation with the stress envelopes obtained from mortars and also allows for a cleavage mechanism of failure over the biaxial compression-tension sector of the stress envelope, it is suggested here that this modification of the Coulomb-Mohr theory be adopted for mortar. It is worth noting that Sigvaldason (9) has suggested the use of the unmodified Coulomb-Mohr theory for mortars tested

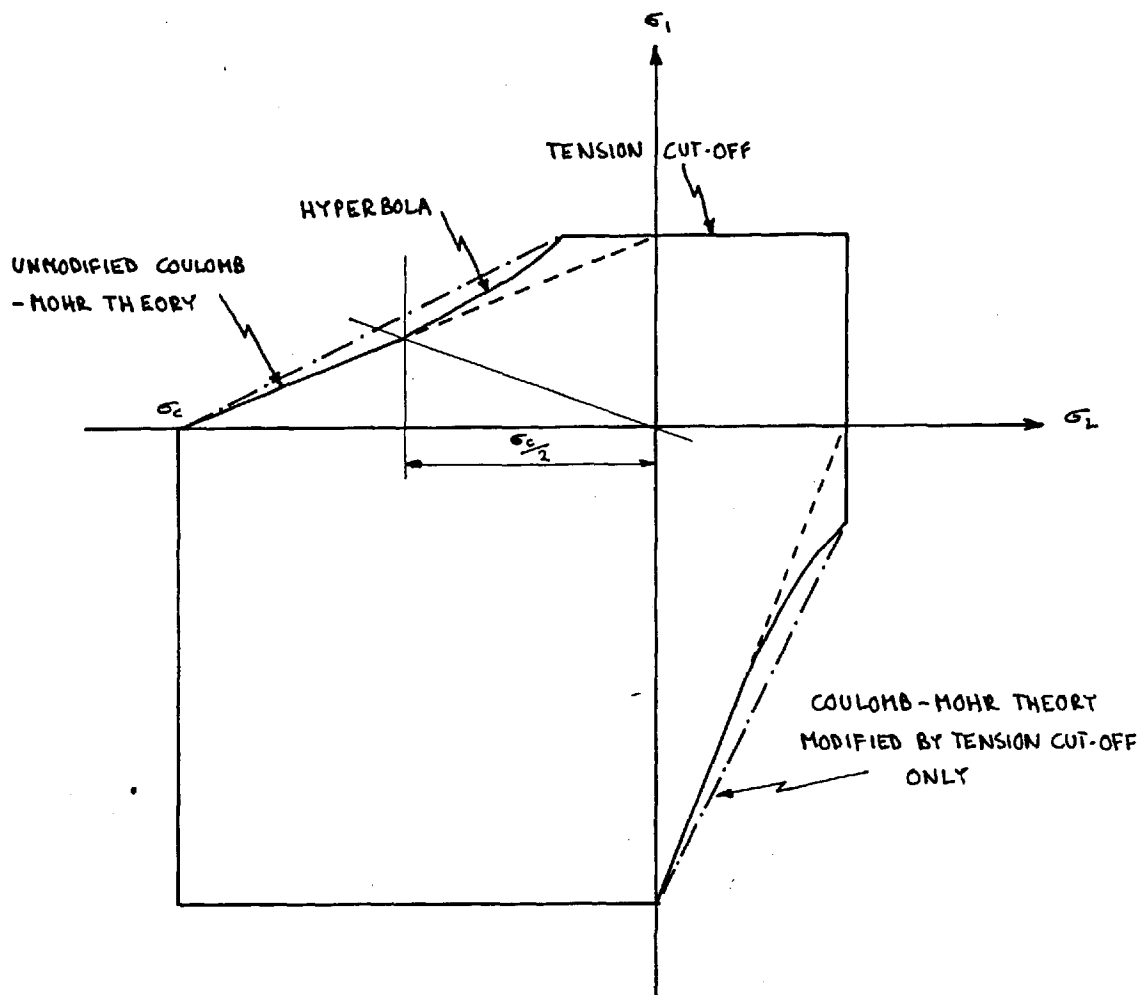


FIG. 12.2.5. COULOMB-MOHR THEORY MODIFIED BY LIMITING TENSILE STRESS AND "NO FRICTION UNDER TENSION" HYPOTHESES BY PAUL (72)

under biaxial compression-tension flexural states of stress but he had insufficient experimental data to observe the concavities or the limiting tensile stress phenomenon described above.

c) Suggested Further Modification to Coulomb-Mohr Theory for Mortars

A phenomenon common to both cast iron and mortar is a reduction in strength under both uniaxial compression and equal biaxial tension compared with the strength predicted by the modified theory. It is suggested that this can be explained by considering the alignment of the critical flaws in the material and a further refinement to the theory would be to include a modification to account for this.

It has been shown that the mechanism of failure of mortars under biaxial stresses as applied by the testing techniques employed in this project is generally one of cleavage orthogonal to the direction of the principal tensile stress. This being so, it is clear that whilst only one such principal or preferred direction of crack propagation (one degree of freedom) exists under all states of biaxial stress, two exceptions are uniaxial compression and equal biaxial tension in which two principal directions of crack propagation (two degrees of freedom) exist. In these two latter states of stress there are an infinite number of possible directions of cleavage at right angles to the axis of the principal compression stress. Hence there is a greater chance of a suitably aligned critical flaw occurring in these states of stress having two degrees of freedom and therefore failure can be expected at a lower stress than those with only one degree of freedom.

The Coulomb-Mohr theory as interpreted by Paul is basically for failure under only one degree of freedom in crack propagation and therefore the failure stresses obtained in uniaxial compression and equal biaxial tension are regarded as a reduction on the stresses predicted by the theory.

Both the results of Coffin on gray cast iron and Comet and Grassie on inoculated iron show this effect as the results of this research project (fig. 12.2.1). Furthermore work by Smith and Brown (188) on the shear strength of mortar has also revealed a significant reduction in the strength of mortar under uniaxial compression compared with that predicted by the Coulomb-Mohr theory (fig. 12.2.7).

Exactly how this new concept of strength reduction due to degrees of crack propagation can be incorporated into the mathematics of the modified Coulomb-Mohr theory now proposed will depend on the relative significance of the phenomenon for the various brittle materials. The effect seems to be more pronounced in mortar than in cast iron, since the relative reductions in strength are less for the latter material. This may be due to the relative flaw to particle size ratio or it may be related to the difference in cohesion strength. However, the effect of flaw alignment seems to be established but the manner of its incorporation into the failure criterion may be best achieved by coefficients based on experimental results.

A comparison of the actual results obtained for mortar in this research contract and the biaxial stress envelope predicted by the modified Coulomb-Mohr theory is given in fig. 12.2.6. The strength reduction coefficients referred to above for dealing with the statistical effects of the critical flaw alignment are indicated on this figure.

d) Use of Proposed Theory to predict Strains at Discontinuity

A major result of the strain measurements made on mortar specimens below the discontinuity point was the realisation that the elastic modulus and Poisson's ratio were effectively constant under all the biaxial states of stress investigated. One consequence of this result is that the cleavage strain at discontinuity can be calculated from the biaxial discontinuity stress

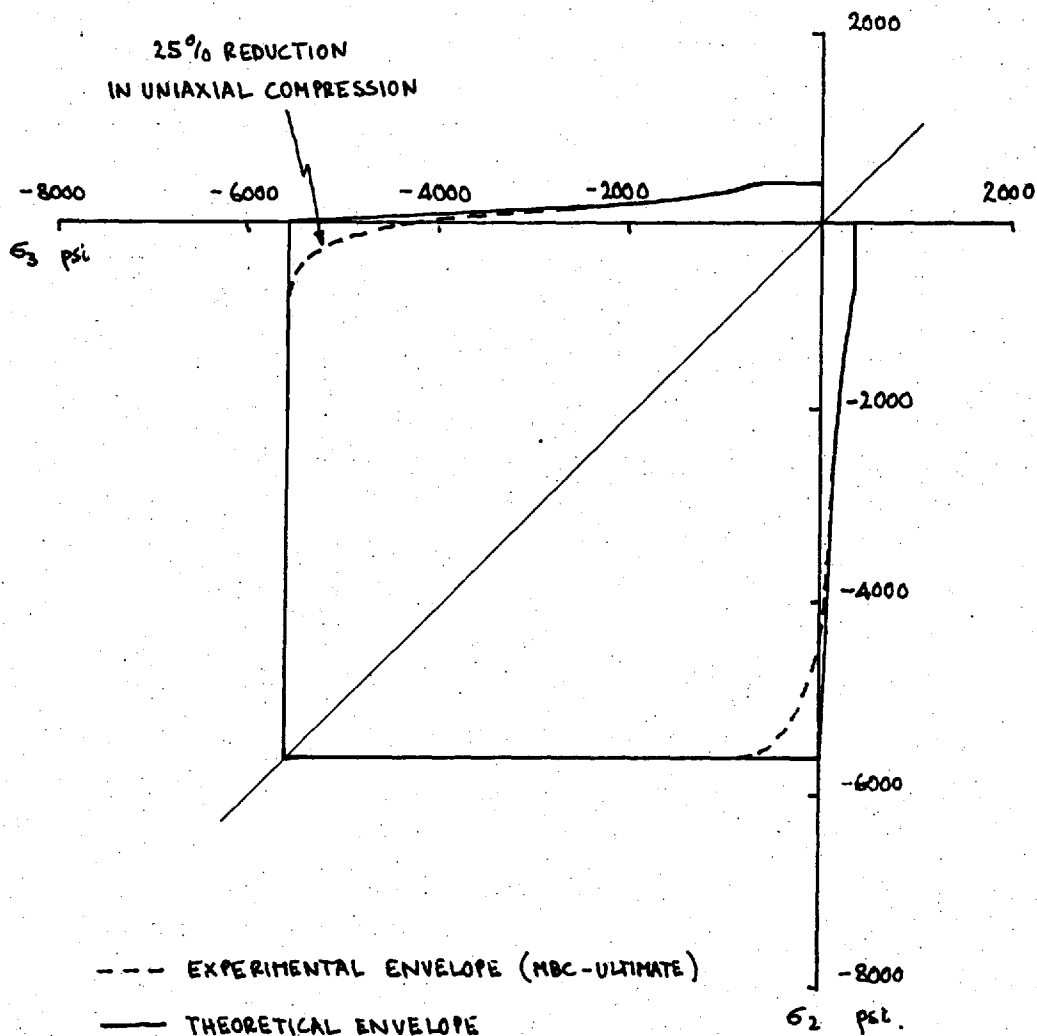


FIG. 12.2.6. COMPARISON ON EXPERIMENTAL STRESS ENVELOPE
AND MODIFIED COULOMB-MOHR THEORY.

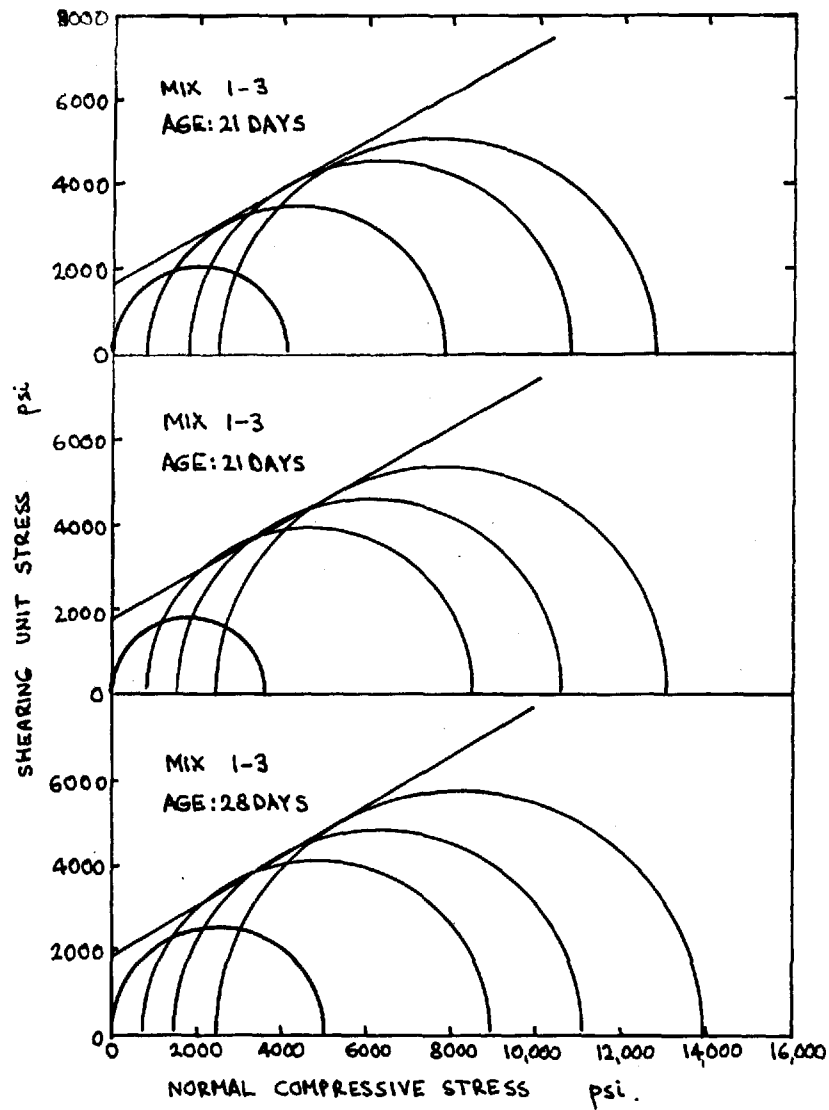


FIG. 12.2.7. TYPICAL REDUCTIONS IN UNIAXIAL STRENGTH OF MORTAR OBTAINED BY SMITH AND BROWN (188).

envelope. This exercise has been performed in fig. 12.2.8 in which the envelope selected was the one used in fig. 12.2.6. for comparison with the actual results obtained. This envelope complete with its tension cut-offs and statistical flaw strength reductions was used to calculate the cleavage strain at discontinuity. Then the resulting curve was compared with the actual results obtained from mortar mix MBC which was used in fig. 12.2.6 to establish the theoretical biaxial stress envelope. The correlation obtained in this comparison is very encouraging.

e) Mode of Failure of Mortars

Although by adopting the modifications to the Coulomb-Mohr theory recommended by Burton Paul, it has been possible to explain cleavage failures in the biaxial compression-tension zone, the theory even in its modified form still predicts a shear failure in the biaxial compression zone. Now at a phenomenological level, the biaxial compression failures observed with mortars in this research project were generally although not always of the regular cleavage type. Although the cleavage fractures appeared to be almost orthogonal to the direction of the principal tensile stress (in this case, zero stress), they were very irregular and often inclined by as much as 40° to the orthogonal plane over areas in excess of 4 sq. in.

However, because of the apparent cleavage failure of mortars under biaxial compression and the shear failure predicted by the theory, the Coulomb-Mohr theory has often been rejected as a valid failure criterion for mortar (10).

It is important, nevertheless, not to lose sight of the effects of the loading system used to obtain these test results. In chapters 2 and 3 the design of the combined stress testing machine was described and particular note was made of the fact that the loading platens locked into position once load was applied giving an effectively fixed end condition on the loaded faces of the test specimen.

VALUES USED. -

$$\nu = 0.18$$

$$E = 3.60 \times 10^6 \text{ psi} - \text{CONSTANT}$$

460

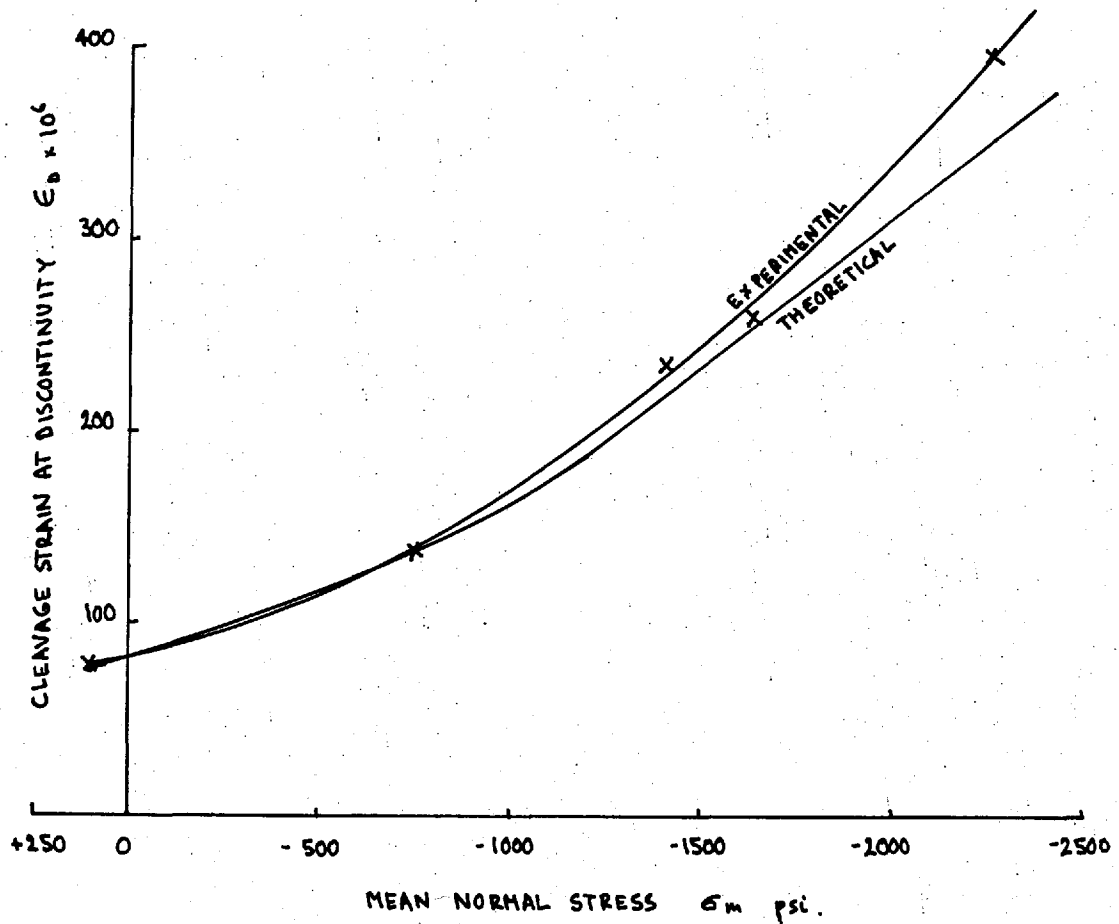


FIG. 12.2.8. COMPARISON OF EXPERIMENTAL RESULTS OBTAINED FOR MIX MBC WITH CURVE PREDICTED FROM THEORETICAL STRESS ENVELOPE FOR MIX MBC GIVEN ON FIG. 12.2.6.

This method of loading with fixed ended conditions may have an overriding effect on the direction of the crack propagation once the discontinuity point has been passed and the cracks start to propagate rapidly. Indeed it is possible that at the discontinuity point, the cracks start initially as shear cracks as predicted by the theory but, after reaching a certain critical length or velocity, are dominated by the system of loading which would encourage cleavage failures.

This explanation could account for both the remarkable agreement between the experimental and theoretical biaxial compression stress envelopes and the irregular shape of the 'cleavage' fractures observed under biaxial compression. The existence of an initial shear or sliding mechanism of failure would also account for the relatively large quantities of fine dust given off from the biaxial compression failures of mortar compared with the clean cleavage fractures observed under biaxial compression-tension.

f) Summary

In conclusion, the failure criterion proposed for mortar is the Coulomb-Mohr internal friction theory modified as suggested by Burton Paul by the addition of a maximum tensile stress criterion and the elimination of the 'friction' component of strength in the biaxial compression-tension zone. It is further modified by a consideration of the degrees of freedom of crack propagation governed by the state of stress which causes a reduction of strength under uniaxial compression and equal biaxial tension. The resulting theory predicts the variation of 'cleavage' strain at discontinuity with mean normal stress. The fracture modes predicted by the theory are illustrated in fig. 12.2.9. and are compatible with the experimental observations reported in Chapter 11.

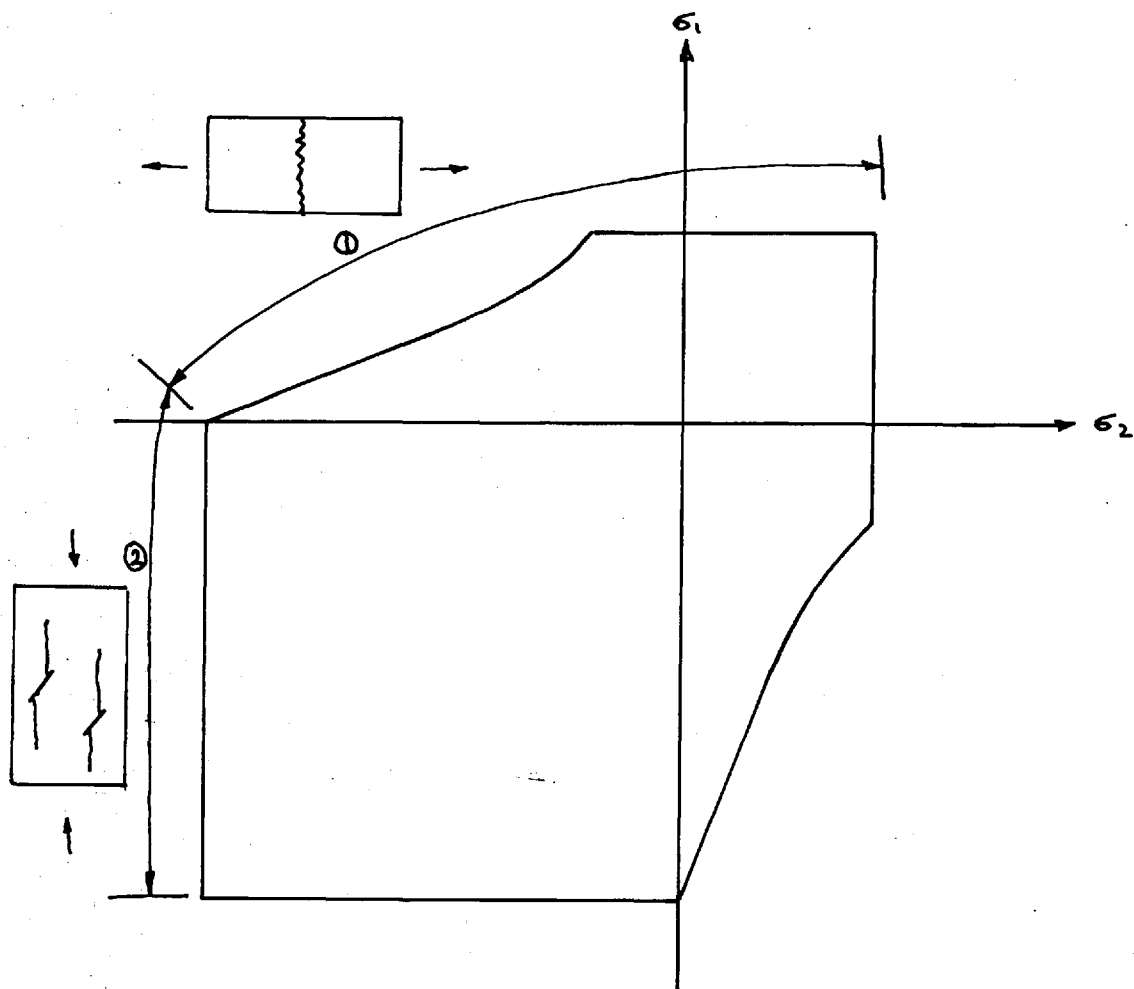


FIG. 12.2.9. FRACTURE MODES PREDICTED BY MODIFIED COULOMB-MOHR THEORY.

12.3. Deformation and Failure of Normal Concrete

12.3.1. Introduction

It is clear from the biaxial stress envelopes that normal aggregate concrete cannot be considered in the same manner as that adopted for mortar in the previous section. The influence of the particles of coarse aggregate must account for the differences in the biaxial stress envelopes and hence this two phase nature of concrete must be incorporated into the discussion from the start.

In conceiving a two phase model to simulate concrete behaviour a number of simplifying assumptions have to be made in order to produce a structure which is manageable from the mathematical aspect without losing sight of the basic mechanisms involved. It is important, however, that the basic assumptions are not forgotten when extending the analysis of the model.

An attempt is made in the following section to define the general levels of complexity present in the assumptions on which simplified models of concrete are based with a view to differentiating between the various contributions in this field and to show how this present work is an extension of this process to a new level.

12.3.2. Elastic Behaviour

The simplest and most usual two-phase model of concrete is one in which the aggregate, represented by uniform spheres is dispersed in a regular pattern through a homogeneous, isotropic cement paste or mortar matrix.

Before considering the elastic behaviour of concrete, the levels of complexity in the treatment of such two-phase models of concrete will be defined and amplified. The levels selected in this discussion are as follows:-

- STAGE 1. Plane sections remain plane - continuity at phase boundaries.
- STAGE 2. Plane sections do not remain plane - continuity at phase boundaries.

STAGE 3. Plane sections do not remain plane - total discontinuity at phase boundaries.

STAGE 4. Plane sections do not remain plane - partial discontinuity at phase boundaries.

These levels could alternatively be regarded as degrees of effect, i.e. primary effect; secondary effect, tertiary effect, etc. However, as their significance is not always in the same relative order in discussion of a given phenomenon, this classification by degree of complexity was considered more satisfactory.

To reduce the problem of the two-phase model to manageable proportions, the two phases are treated as homogeneous and isotropic. Though having different elastic moduli - normal concrete by definition consists of hard particles in a soft matrix - the two phases can be assumed to have very similar values of Poisson's ratio.

12.3.2.1. Stage 1

This is the most simple way in which the two phase material can be considered. Full continuity at the phase boundaries and plane sections remaining plane lead to simple expressions for strain and stress distributions. The integration of stress on any plane orthogonal to the direction of loading must be equal to the total applied load. This has also been interpreted as the phases being subjected to uniform strain.

For such a material Hansen (189) has shown that the overall elastic modulus of the material is given by the simple expression:-

$$E = V_1 E_1 + V_2 E_2$$

where E_1 = elastic modulus of matrix phase

E_2 = elastic modulus of included phase

V_1 = volume fraction of matrix phase

V_2 = volume fraction of included phase.

From the same assumptions regarding the two phase model of concrete, Baker (4,6) derived his Lattice Theory. By considering two adjacent planes in a stressed concrete member and the stress distribution on those planes assuming plane sections remained plane (see fig. 12.3.1.), he concluded that equilibrium could only be maintained in the material by lateral tension ties and that these must be provided by the mortar matrix. From this it followed that failure would occur in most cases by progressive failure of the local tension ties resulting in cleavage types of failure under all states of stress with the possible exception of triaxial compression.

The Lattice Theory provided an explanation for a number of observed phenomena in concrete particularly the cleavage modes of failure observed in uniaxial and biaxial states of stress (10), the effects of platen restraint on the failure mode (4), and some thermal effects in concrete (190).

The success of the Lattice Theory illustrates the progress that can be made by simply considering concrete as a two phase material even with the very restrictive assumptions of the plane sections remaining plane and full continuity at the phase boundaries. It should be noted that a form of lattice action will still take place if these two conditions are waived provided only that a state of uniform stress distribution is not achieved.

It is of interest to note that a similar diamond shaped lattice model was used by Bleich (73) to explain the disintegration of bodies of granular materials governed by the Coulomb rule under dynamic loads. However, in place of the nodal ties used by Baker, Bleich introduced friction at the nodal pin joints to maintain equilibrium. This friction was composed of a constant value to allow for the cohesion strength plus a constant coefficient multiplied by the lattice member force to include the friction component of strength. This model, though of little use under static loads, can be used to explain observed behaviour under dynamic loads.

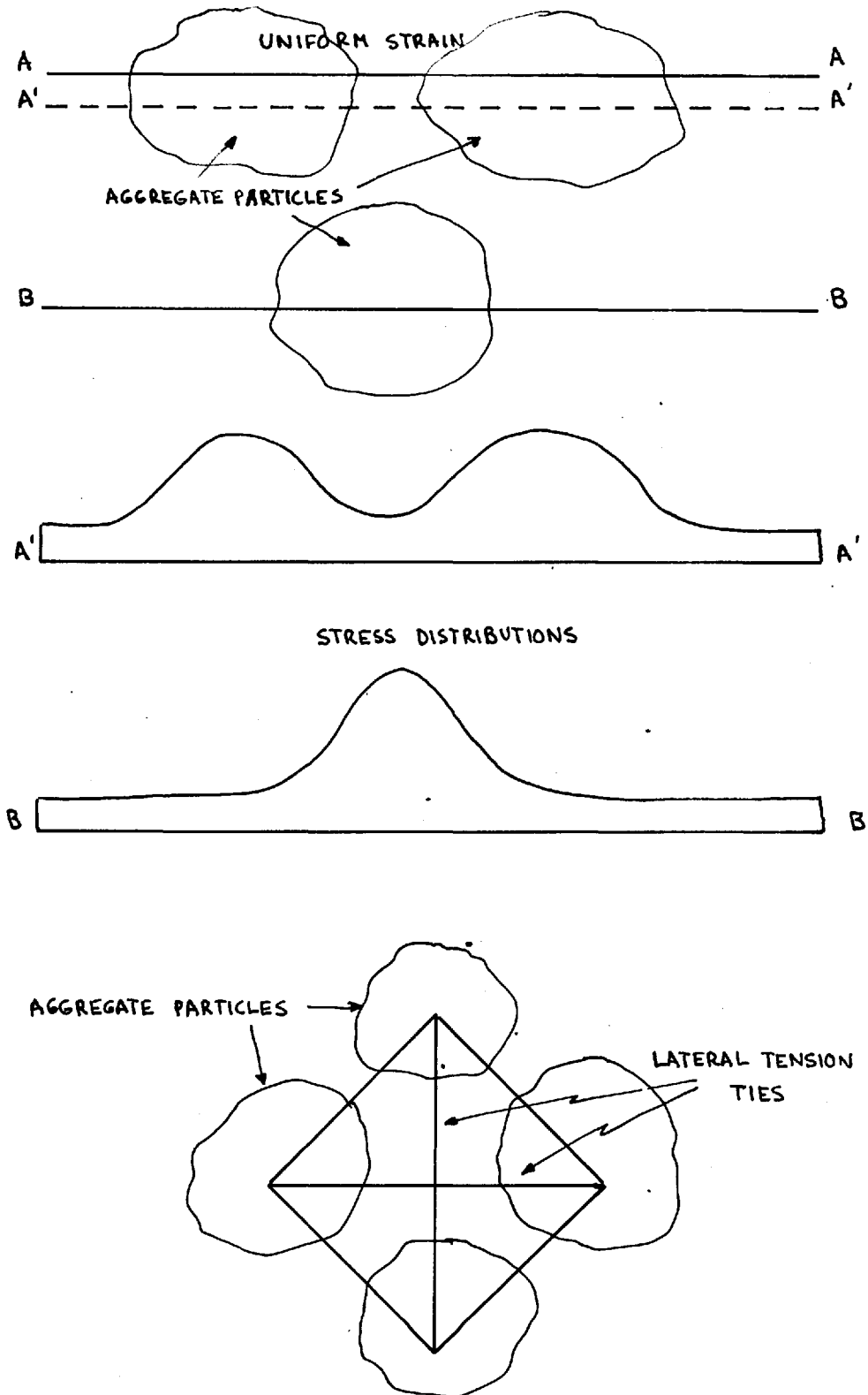


FIG. 12.3.1. STAGE.I. PLANE STRAIN . RESULTING STRESS DISTRIBUTIONS AND BAKER'S LATTICE THEORY.

Other models proposed for concrete include that of Reinius (43) described in section 1.4, and a cubic space model by Roy and Sozen (87).

12.3.2.2. Stage 2

The first major complication is now introduced by ^{different} recognising that if the two phases are of different elastic moduli then under load plane sections cannot remain plane. It is possible to make this assertion and still maintain full continuity at the phase boundaries by assuming adequate shear bond strength at those boundaries (see fig. 12.3.2.).

However, the mechanism of shear has now been introduced into the state of simple compressions and tensions of stage 1 and it is no longer possible to consider the model in terms of a pin-jointed lattice structure. To maintain compatibility of stress and strain the members of the lattice must now be subjected to total restraint at the structural joints.

This stage in the two-phase model of concrete has been analysed mathematically by Dantu (192-194) and by Hashin (195). Whilst Dantu was mainly concerned with deriving expressions for the local normal and shear stresses at the phase boundaries, Hashin obtained an expression for the overall elastic modulus of the model which, by assuming equal Poisson's ratios for the two phases and the combined material (196), reduces to:-

$$E = \left\{ \frac{(1-V_2) E_1 + (1+V_2) E_2}{(1+V_2) E_1 + (1-V_2) E_2} \right\} E_1$$

The elastic moduli predicted by this stage in the two phase model are much closer to the observed values than those predicted by stage 1 and represent a more realistic upper bound than Stage 1 on the zone within which the actual values must lie. Thus the release of the condition regarding plane sections can be seen as a progressive step.

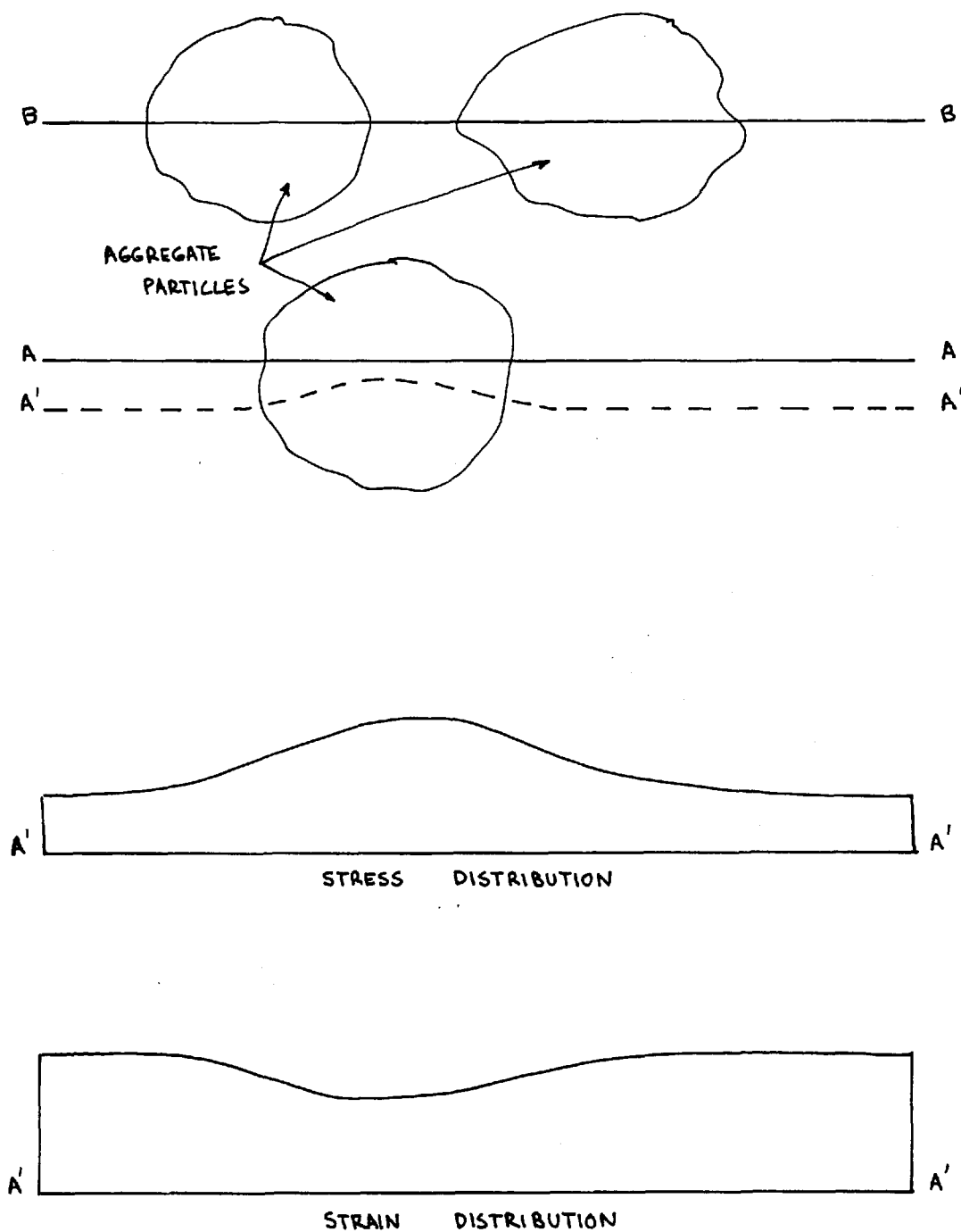


FIG. 12.3.2. STAGE 2. PLANE SECTIONS DO NOT REMAIN PLANE.
FULL CONTINUITY AT PHASE BOUNDARIES.

12.3.2.3. Stage 3

The opposite extreme to stage 2 is to assume that there is total discontinuity at the phase boundaries. This is equivalent to a state of plane stress throughout the material; at least the average stresses in the two phases would be the same (fig. 12.3.3.)

Based on this assumption, the expression for the overall elastic modulus is,

$$E = \frac{1}{\frac{V_1}{E_1} + \frac{V_2}{E_2}}$$

Whilst the assumptions of stage 2 gave an upper bound to the elastic modulus values, stage 3 provides the corresponding lower bound. Note that this is the only condition under which there can be no lattice action.

Clearly the actual behaviour of concrete must lie between these two which leads us on to stage 4 in order to build up a realistic structural model for deformation and to be able to understand the failure modes.

12.3.2.4. Stage 4

This stage is the next logical step in this process of increasing refinement of the two-phase model but to the best of the author's knowledge it has not yet been fully investigated. The recognition that plane sections do not remain plane under load is retained but now only partial discontinuity is allowed around the phase boundaries.

These two assumptions lead to a complicated situation in which both shear and tensile stresses can cause failure both in the matrix phase and at the phase boundaries. In addition the whole science of fracture mechanics can be invoked as the partial discontinuities at the phase boundaries represent critical flaws in the material. Finally particle interaction effects as represented by the Lattice Theory are superimposed upon the above stress conditions.

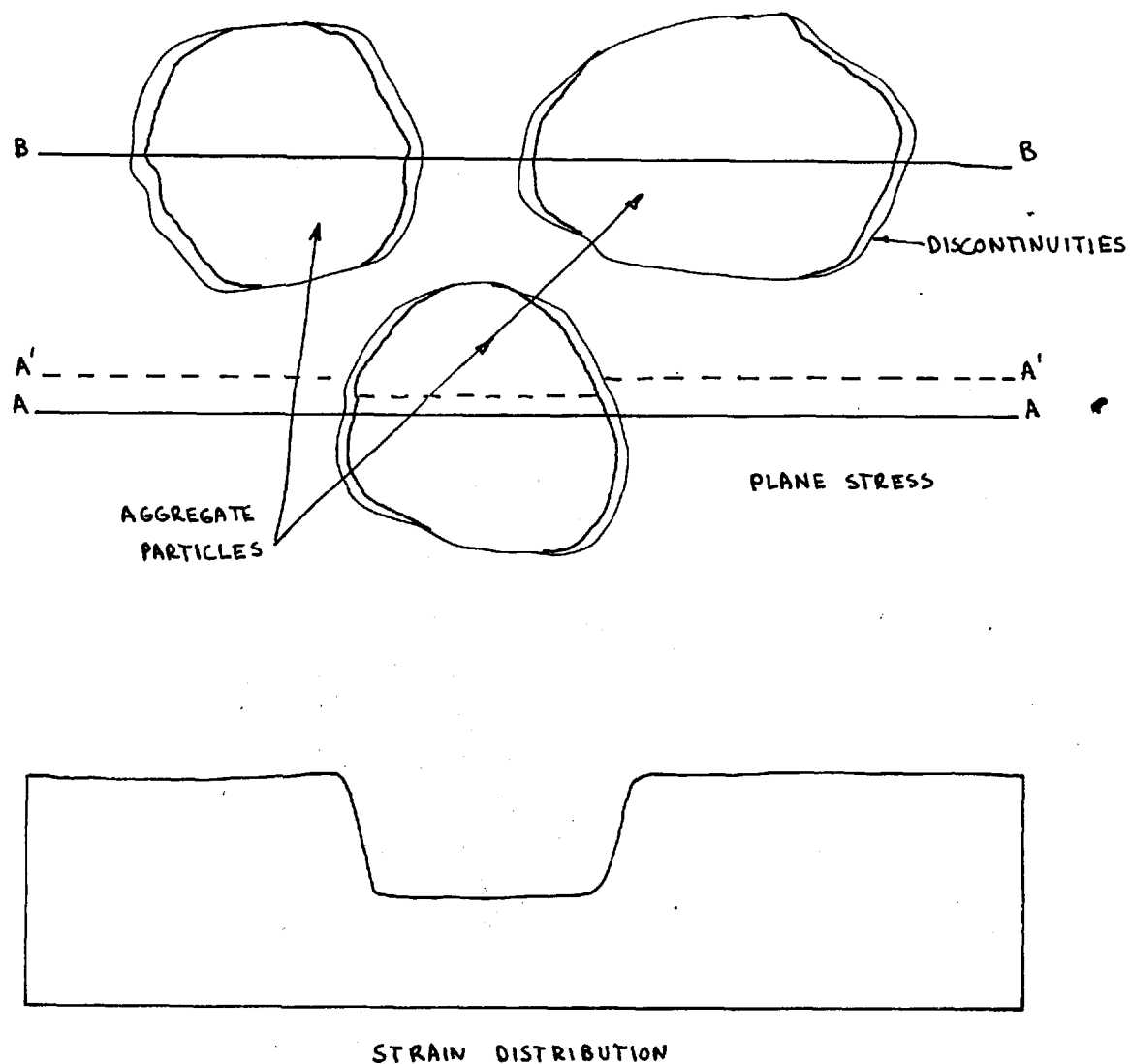


FIG. 12.3.3. STAGE 3. PLANE SECTIONS DO NOT REMAIN PLANE.
FULL DISCONTINUITY AT PHASE BOUNDARIES - PLANE STRESS.

Therefore the build up of this situation from the casting of fresh concrete to its behaviour in the hardened state below the discontinuity stress will be described in detail.

When concrete is cast small pockets of air and water collect on the aggregate-mortar phase boundary. They are either trapped there during the compaction of the concrete or some of the air voids are due to the absorption of water by dry aggregate. An examination of the aggregate-mortar phase boundaries both by slicing prior to loading or after failure shows ample evidence for the existence of these small pockets on the surface of the aggregate. These pockets form local discontinuities in isolated positions on the phase boundary (fig.12.3.4.).

The effect for example of applying a uniaxial compressive load to the hardened concrete is to deform the soft mortar matrix more than the stiff aggregate particles with the result that tensile stresses exist on the phase boundaries at right-angles to the applied load. The mathematical analysis of Biezeno and Grammel of the stress distribution around a single spherical inclusion used by both Weigler and Becker (50,51) and Robinson (10) predicts these tensile stresses concentrated on the phase boundary.

As the tensile load strength is very low, bond breakdown occurs in these directions (fig. 12.3.4). However, the extent of bond breakdown due to tensile stress is limited by the subsequent stress redistribution and a point is reached beyond which the discontinuity cannot be extended without the failure of the bond by shear.

Work on the strength of the bond between aggregate and cement by Alexander (197), Taylor and Broms (198) and others has recognised the mechanisms of both tension and shear in bringing about this partial breakdown of the bond between the two phases. As the discontinuity point has been defined in this research project as that point at which there is a significant change in the ultrasonic pulse velocity through the material, it would appear

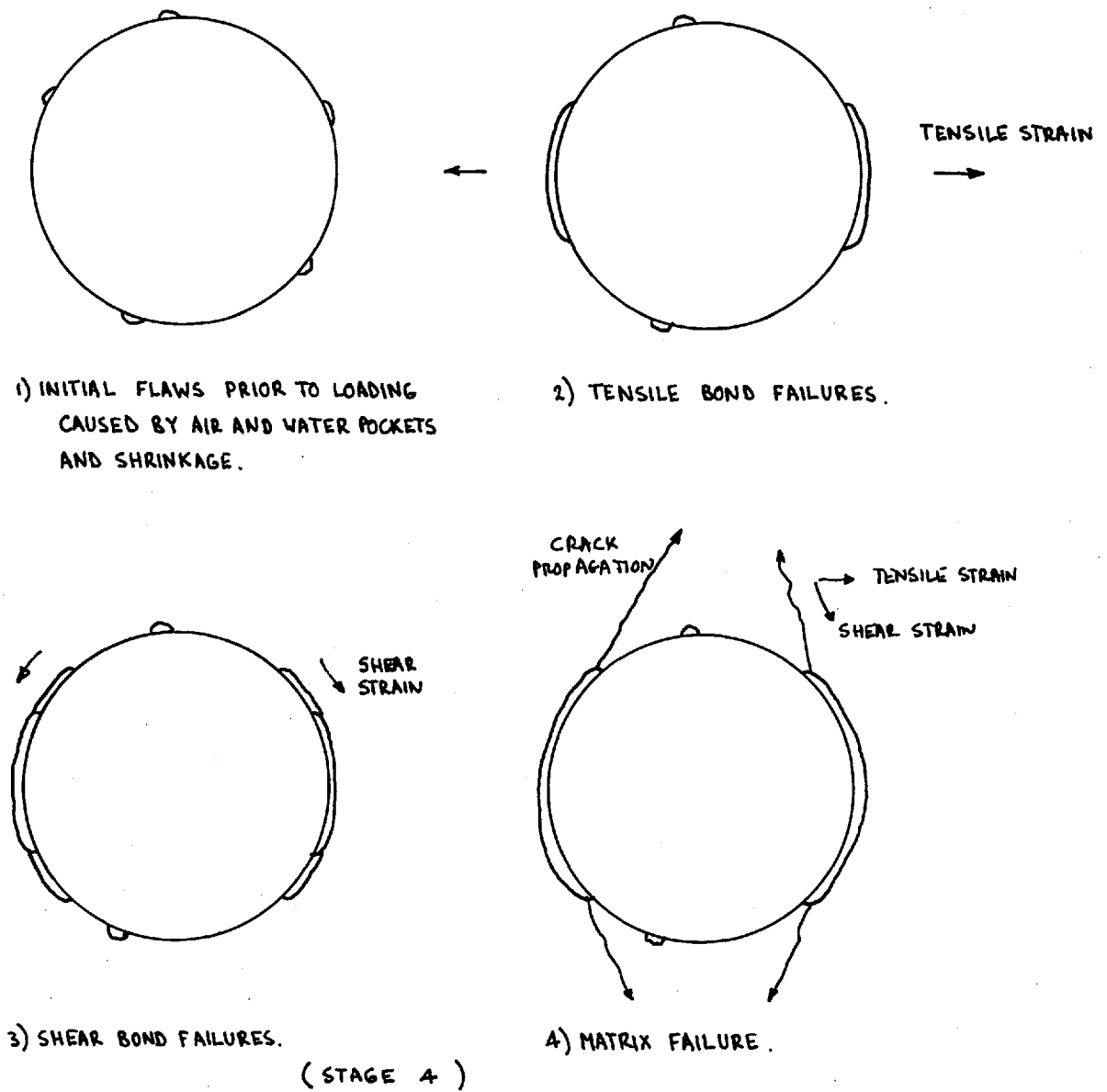


FIG. 12.3.4 IDEALISATION OF SEQUENCE OF CRACKING IN CONCRETE.

that both tensile bond and shear bond failures could occur below the discontinuity stress.

Evidence for the existence of bond breakdown below the discontinuity point can be seen in the difference between the initial stress-strain curve of a virgin specimen up to a point just below the discontinuity stress which tends to have a definite curvature and subsequent loading curves which are much more linear. (fig. 12.3.5)

Over this section of the stress-strain curve, i.e. below the discontinuity stress, the bond cracks have no significant effect on the ultrasonic pulse velocity. Therefore it is concluded that the shear bond cracks which occur after the tensile bond cracks have formed can likewise have no effect on the ultrasonic pulse velocity and therefore occur below the discontinuity level.

Furthermore it may be claimed that these cracks, although causing a reduction in the elastic modulus compared with the virgin state of the hardened concrete described above, are stable once formed provided the discontinuity stress is not exceeded. Evidence for this is provided by the results of the long-term loading tests on prisms conducted during this research (see section 7.5) in which there was no increase in the longitudinal strain rate, i.e. no crack propagation or extension, until the discontinuity stress, as defined by the ultrasonic pulse velocity, was exceeded.

Hence the material envisaged by the stage 4 assumption of partial yet stable discontinuities around the phase boundaries has been realised. Once the process of bond breakdown is complete, the behaviour of the concrete is elastic in that its deformations are repeatable (unless creep effects become significant) and crack formations are stable provided the discontinuity stress is not exceeded.

Several points are worth noting about this two phase model of concrete at this stage.

Finally, although the case of uniaxial compression was used to illustrate the formation of bond cracks due to load on

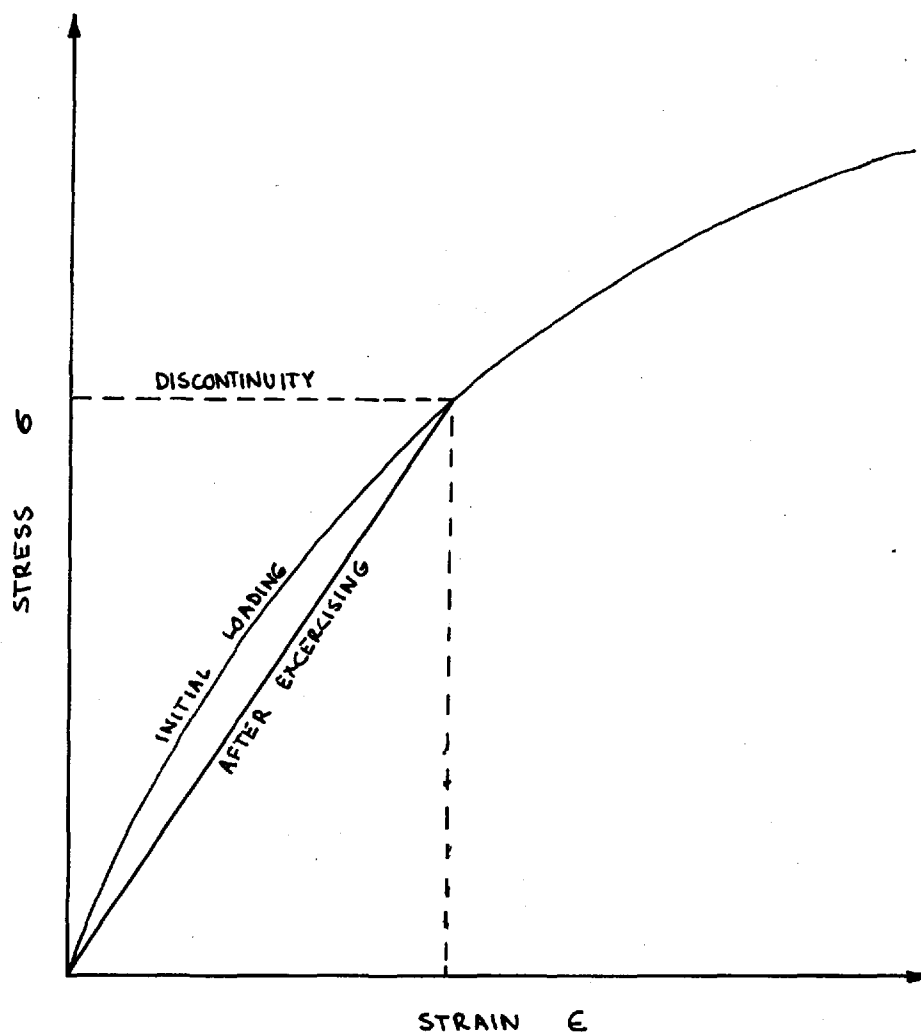


FIG. 12.3.5. EFFECT OF EXERCISING SPECIMEN ON STRESS-STRAIN CURVE.

the concrete, the same basic reasoning could be used for bond breakdown under all uniaxial and biaxial states of stress and even some triaxial states of stress given adequate overall tension strain in one direction at least. However, the extent of the bond breakdown will depend on the state of stress and will be less under uniaxial tension and biaxial compression than under uniaxial compression and biaxial tension as shown in fig. 12.3.6. This fact has already been established in section 11.5 in the examination of fracture modes and mortar deposits on aggregate particles.

Secondly the effective equality of the elastic modulus under uniaxial tension and compression can be explained. Assuming the mortar matrix and aggregate phase to be elastic and the bond crack formations to be of approximately the same extent, the strain distributions for given stresses in the two cases of uniaxial tension and compression illustrated in fig. 12.3.7 are identical and hence the measured elastic moduli will be equal. However, it should be noted that the variation in particle interaction between the particles of coarse aggregate caused by other states of stress will result in changes in the elastic modulus. Biaxial compressive stresses on this model will result in increases in the elastic modulus, due to the greater particle interaction under this state of stress, whilst biaxial tensile stresses will cause a corresponding decrease. Robinson (10) has likened this action to a shortening of the compression arms of Baker's lattice giving a stiffened thrust ring effect.

Lastly, the difference between the uniaxial compressive and tensile strengths of concrete, although strictly the subject of the next section, can be seen in the replacement of tensile bond failures by shear (and hence higher strengths) under compressive loads compared with the predominantly tensile nature of the failures under tensile loads. This aspect is discussed in more detail below.

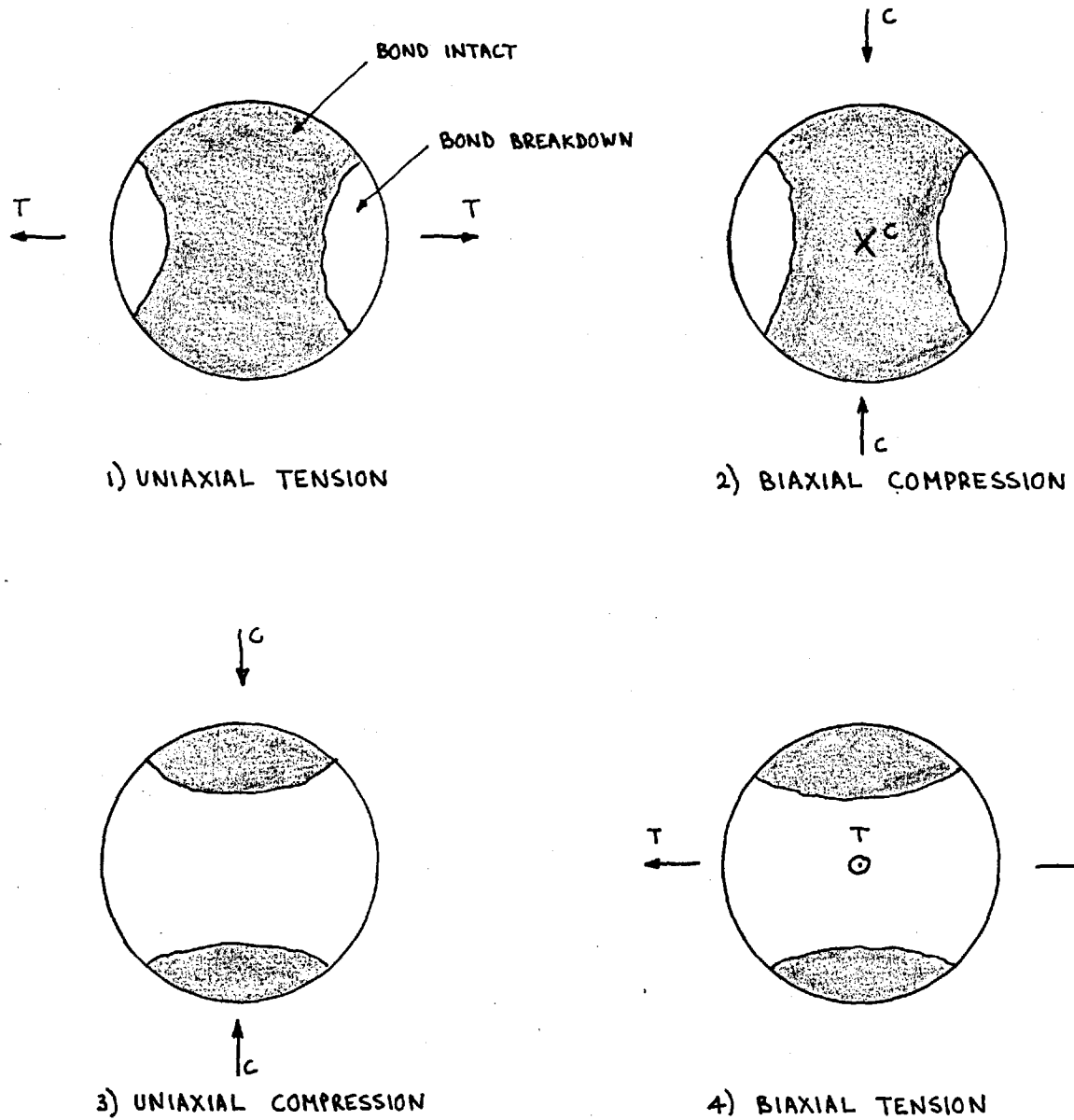


FIG. 12.3.6. IDEALISATION OF EXTENT OF BOND FAILURE PRIOR TO DISCONTINUITY UNDER UNIAXIAL AND BIAxIAL STATES OF STRESS.

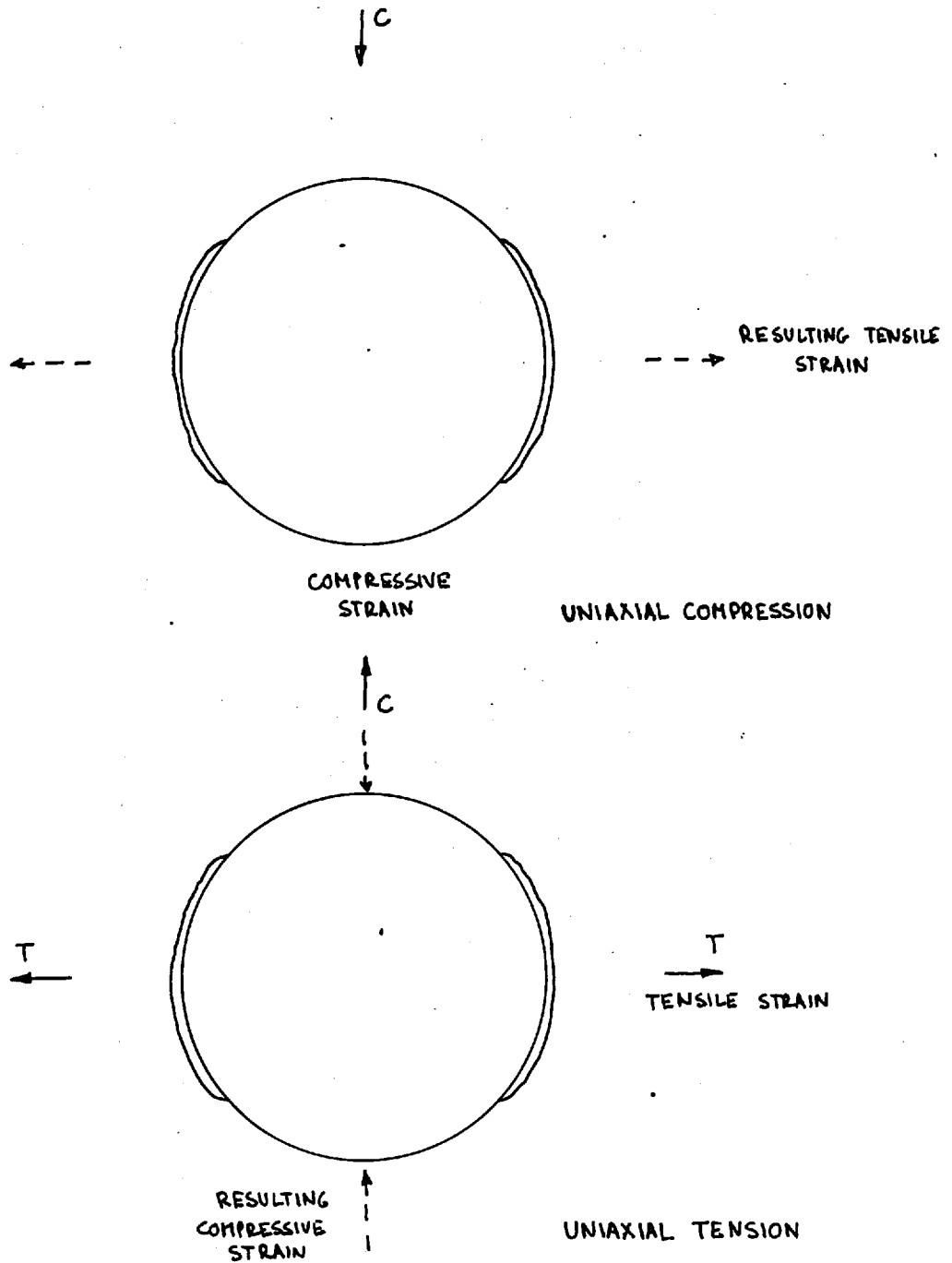


FIG. 12.3.7. COMPARISON OF STRAIN DISTRIBUTIONS UNDER UNIAXIAL COMPRESSION AND TENSION.

12.3.3. Criterion of Failure

In the model of concrete described above, the bond breakdown process was assumed to be completed prior to the discontinuity point being reached. It is therefore necessary to examine the resulting stress and strain distributions on completion of the bond breakdown in order to predict the next stage of crack propagation which commences at the discontinuity point.

As has already been noted above, there appear to be two conditions of stress to consider; the one with tensile stresses predominating and the other with shear stresses taking over from the tensile stresses (20). These two conditions are typified by the uniaxial compressive and tensile tests. These two failure mechanisms will now be considered and then their places in the general biaxial stress states will be established,

a) Modes of Failure

Under uniaxial tension stress, the bond cracks behave as flaws containing a concentration of tensile stress at the crack tip and at right angles to the axis of the crack. This is the weakest situation possible with the mortar matrix being parted as the crack spreads under ideal stress conditions (fig. 12.3.8.). The crack as it propagates may either pass through or round aggregate particles depending on the energy requirements of either route. The net result of this mechanism on a phenomenological level is a single cleavage plane orthogonal to the axis of the tensile load. A single critical plane is always observed as, once a crack starts to propagate, the reduction in area of the specimen at that section causes higher stresses to develop on that particular plane hence giving added encouragement to the failure to remain on that plane.

The situation under uniaxial compression on the other hand is very different. The state of stress around a single aggregate particle can be seen in fig. 12.3.9. in which the softer mortar phase is tending to flow past the aggregate particle.

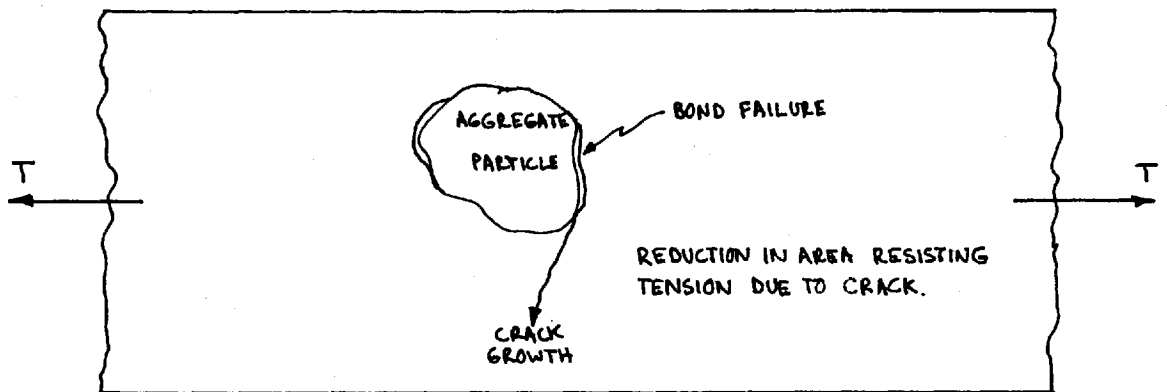


FIG. 12.3.8. REDUCTION OF AREA RESISTING TENSION
DUE TO CRACK GROWTH.

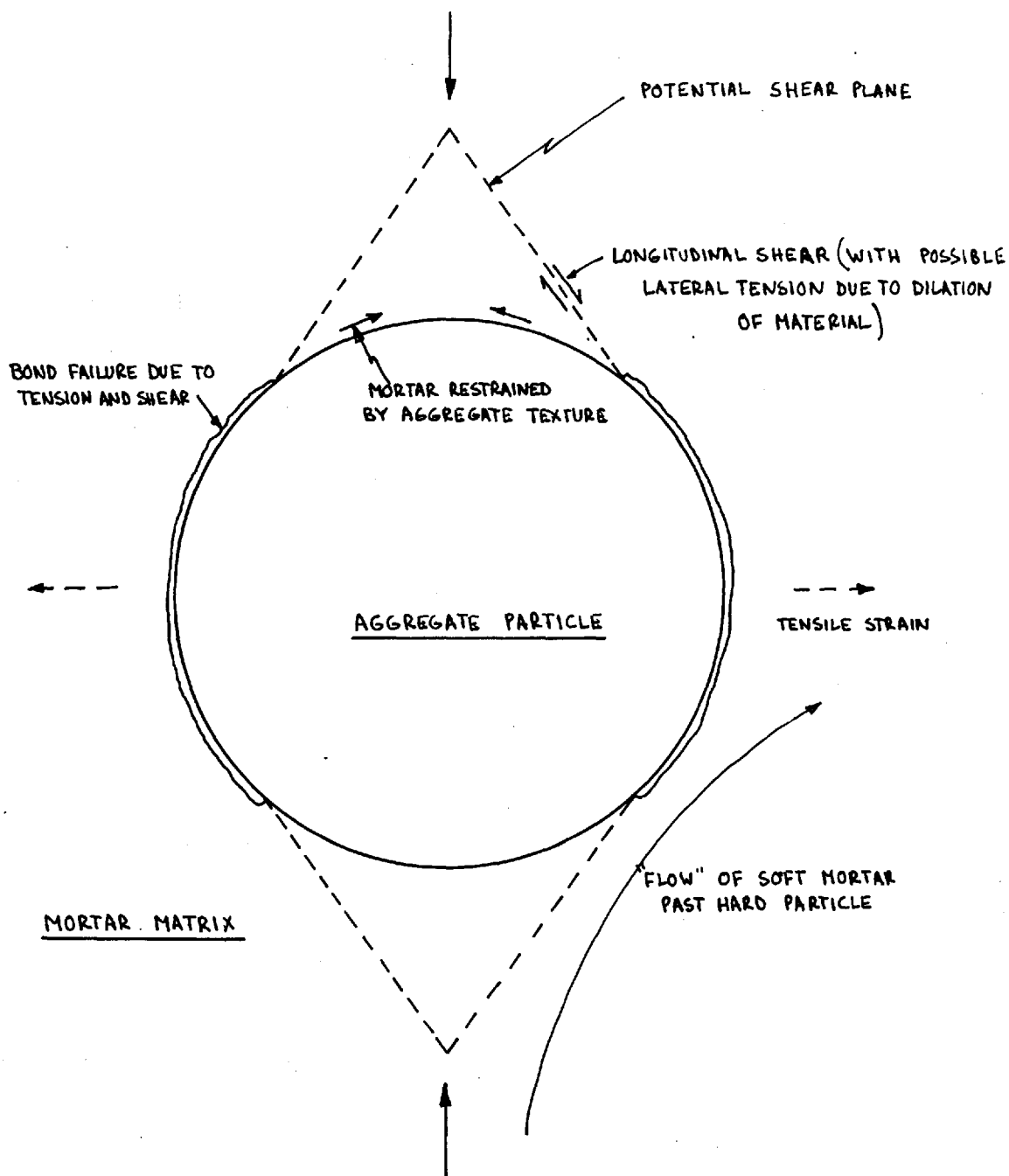


FIG. 12.3.9. IDEALISATION OF STRESSES AROUND AGGREGATE PARTICLE AT THE DISCONTINUITY POINT UNDER UNIAXIAL COMPRESSION.

The texture of the aggregate will retain a cone of the mortar phase on the axis of the applied compression and thus the bond crack will propagate along the sides of this cone under a combination of longitudinal shear and tension. The tension in this case would be due to the lattice or interparticle action of the coarse aggregate particles and would cancel the small normal compression acting on the plane of the crack.

The crack can either link up with the bond cracks on an adjacent aggregate particle (1), propagate through an adjacent aggregate particle (2) or, on reaching the peak of the cone, the natural dilation of the mortar matrix plus the lattice action of the coarse aggregate will force the crack to propagate in the direction of the axis of loading (3). The several possible paths nevertheless combine to give an overall cleavage fracture (fig. 12.3.10).

Numerous cleavage cracks propagate in this type of failure as the formation of a crack in one section of the concrete does not reduce the cross-sectional area resisting load and has no significant effect on the distribution of stress in other sections of the concrete. Hence a number of cracks can form relatively independent of each other yielding a multi-cleavage fracture.

This type of fracture which splits the concrete into a system of columns of material has a much greater reserve of strength compared with the single cleavage failure under tension stresses and use is made of this in the ultimate load analysis of reinforced concrete structures.

The dimensions of the cones of mortar observed under uniaxial compression are determined by the aggregate texture with very small cones or no cones at all with smooth rounded aggregate compared with large cones with rough textured aggregate. Newman (PC) has compared these cones which result from a state of triaxial compression with the 'shear' cones observed in the cube crushing test.

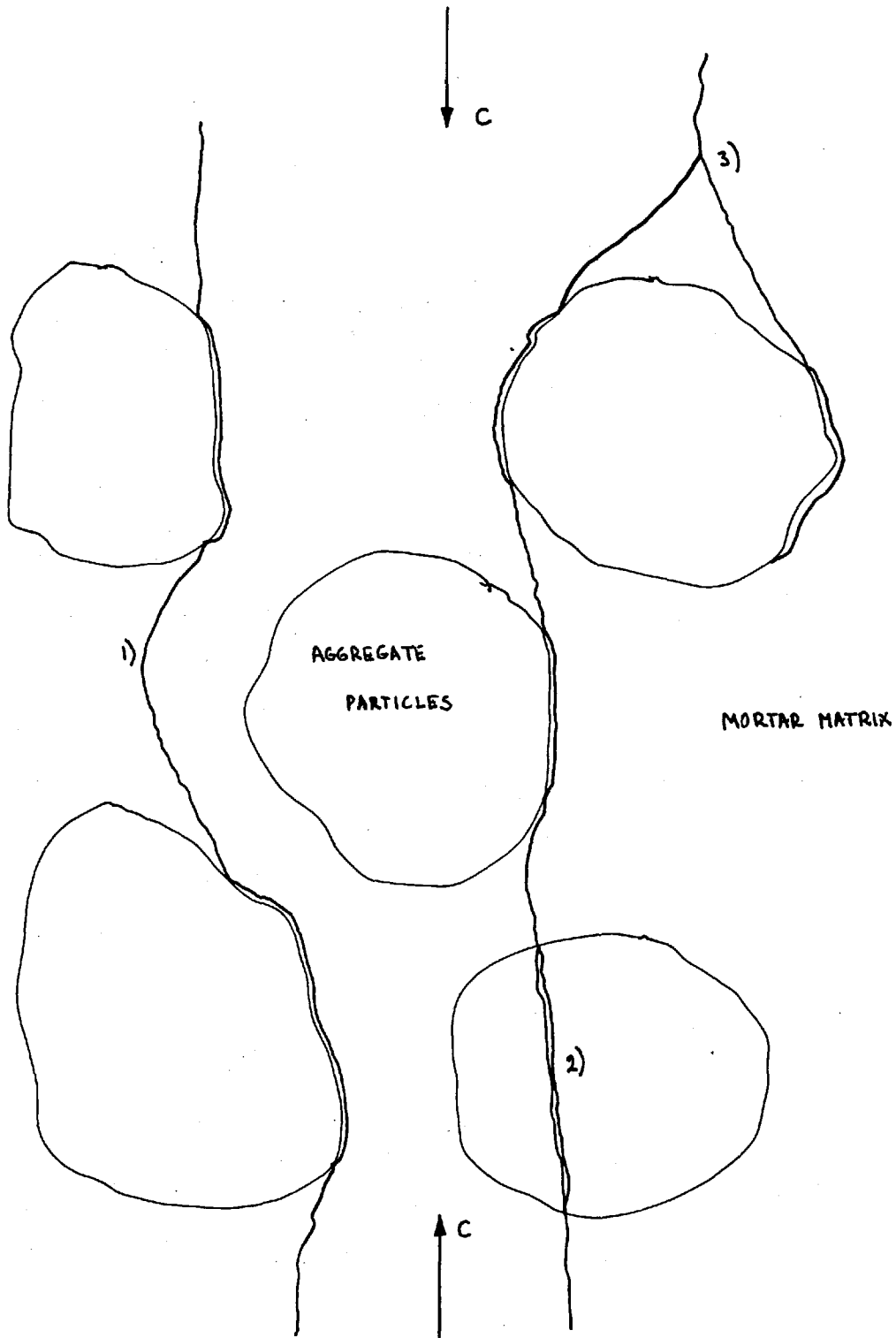


FIG. 12.3.10. THREE POSSIBLE CRACK PATHS UNDER UNIAXIAL COMPRESSION - ALL RESULTING IN A GENERAL CLEAVAGE FAILURE.

These two types of failure mechanism typified by the observed behaviour under uniaxial tensile and compressive stress explain the crack patterns described in section 11.5 and the cones of mortar observed on coarse aggregate particles.

Much of the biaxial compression-tension zone of the biaxial stress envelope exhibits the tension type of mechanism whilst the remaining section close to the uniaxial compressive strength, together with the entire biaxial compression envelope, are subject to the shear type of mechanism. The transition between the two mechanisms will occur when the tendency of the applied tensile load to open up a single major flaw is balanced by the ability of the compressive load to open up other flaws. At the actual transition the balance will be between a single flaw and two flaws.

An idealised biaxial stress envelope for concrete is sketched in fig. 12.3.11 on which the limits of the two failure types are marked. Although this is an idealised picture it is based on the results obtained from actual concretes and described in section 11.3.

Whilst the tension type of failure mechanism remains essentially a single cleavage throughout the biaxial compression-tension zone, the shear mechanism undergoes a transition through the basically compression zones. Under biaxial compression-tension stress, the shear mechanism results in cleavages orientated orthogonally to the direction of the principal tension. In the neighbourhood of the uniaxial compression state the cleavages have a random orientation and the cones of mortar on aggregate particles are observed. However, once into biaxial compression, these cones become haloes (fig.11.5.10) though the basic procedure of tensile and shear bond failures followed by crack extension under shear and tension through the mortar remains essentially the same as for uniaxial compression. In this case the failures revert to multi-cleavage fractures with the phases of cleavage orthogonal to the third unstressed principal axis (fig.12.3.12).

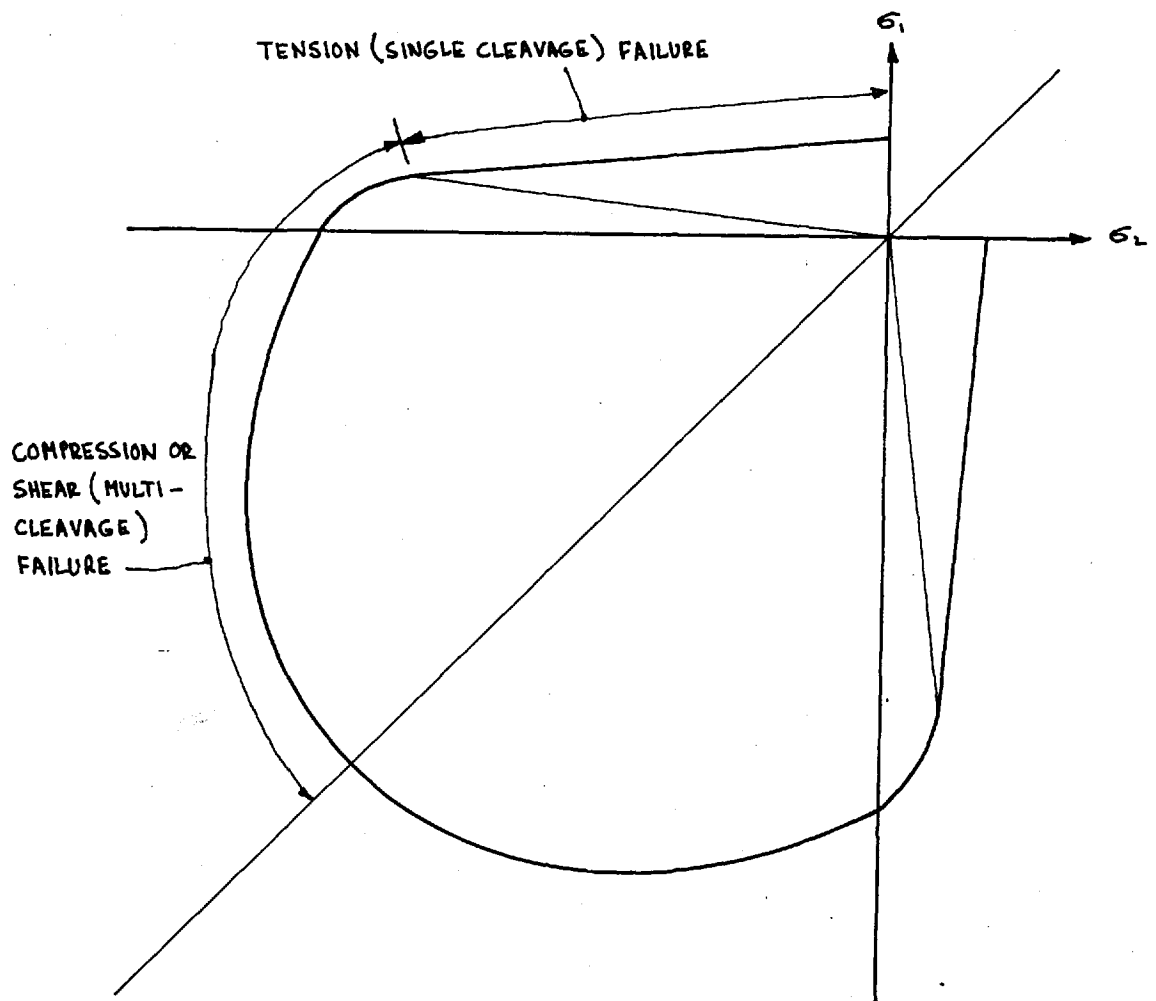


FIG. 12.3.11. FAILURE TYPES RELATED TO BIAxIAL STRESS ENVELOPE.

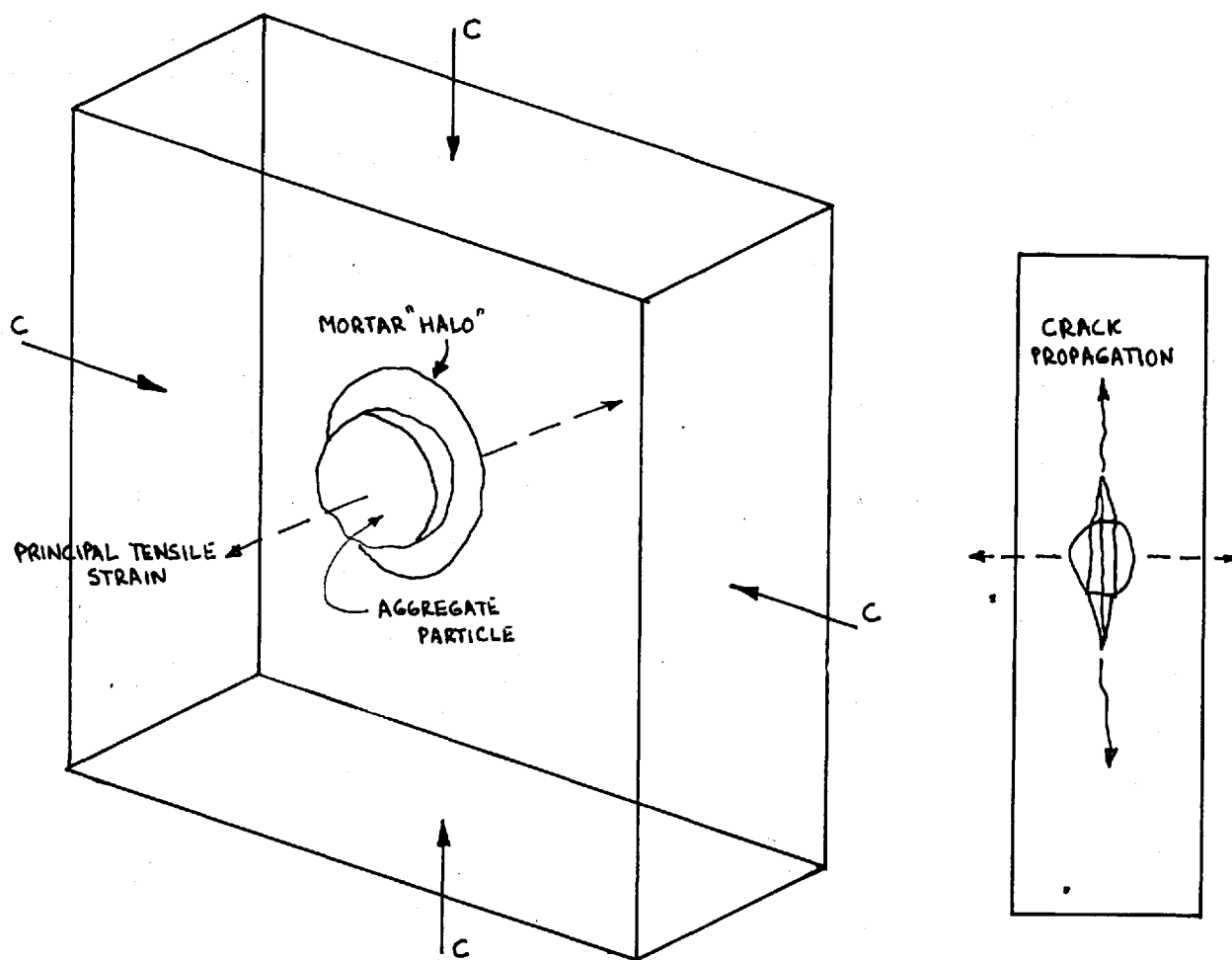


FIG. 12.3.12 PHENOMENOLOGICAL CLEAVAGE STRAIN RESULTING FROM SHEAR FAILURE OF MORTAR AROUND AGGREGATE PARTICLES UNDER BIAXIAL COMPRESSION.

Under biaxial compression-tension stress and within the zone of tension type failures, the tensile stress at the crack tip prior to discontinuity is assisted by the orthogonal compressive stress. This assistance is not in proportion to the Poisson's ratio but does have the effect of reducing the tensile strength with increasing compression stress.

The bulge in the biaxial compression strength envelopes, on the other hand, is probably due to a combination of several factors. These include the increased particle interaction with increasing biaxial compression resulting in greater stiffness and the change in the shear areas through the mortar due to the variation in the failure mechanism from that which produces cones of mortar under uniaxial compression to that producing haloes under biaxial compression. This latter factor would lead to an increase in the lateral strain capacity at discontinuity under increasing degree of biaxial compression as has in fact been observed in actual tests (fig. 11.4.1-3).

b) Proposed Failure Criteria

There remains the problem of defining these failure mechanisms in quantitative terms with the limited amount of experimental work completed in this project. Although the research described in this report is probably sufficient to justify the general features of the failure mechanisms described above, it is clear that more work will be required before a reliable failure criterion based on these mechanisms can be established. However, some possible failure criteria will now be considered with the intention of establishing an intermediate criterion of failure.

Some researchers have considered modifying the distortional energy theorem (also referred to as the octahedral shear stress theory) because of its similarity to the shape of the biaxial compression strength envelope. The envelope could be modified in the biaxial compression-tension zone by an additional criterion involving some modified form of limiting tensile stress or strain.

This modification would probably take the form of a limiting stress or strain related to the volumetric stress for mathematical convenience.

An additional modification to the biaxial envelope would be the introduction of a factor in the biaxial compression zone to allow for inter-particle behaviour. As previously noted, the interaction of the coarse aggregate particles has an overall stiffening effect on the concrete increasing the elastic modulus with increasing degree of biaxial compression. Newman (20) has suggested that the elastic modulus will be a function of the mean normal (or volumetric) stress and this would seem to be a convenient relationship to use.

Hence a criterion of failure could be constructed on the basis of the distortional energy theory but with the additional criterion of a limiting tensile strain and the effect of inter-particle behaviour under compressive states of stress; both of these latter factors being functions of the volumetric stress and determined from experiment. It is worth noting that Robinson (10) concluded that limiting tensile strain and volumetric stress were significant in the failure of concrete.

However, the significance of factors such as the volumetric stress should not be over-emphasised as, in cases such as that above for mortar in which an existing classical failure theory has been modified to fit experimental data, it is really a convenient mathematical parameter to which the empirical variables such as tensile strain can be related.

Furthermore the basic distortional energy theory is intended for homogeneous isotropic materials and predicts a phenomenological shear failure on the octahedral plane. The results of this research described in chapter 11 and the subsequent discussions above have emphasised the need to take into account the heterogeneous nature of concrete at least to the extent of recognising it as a two phase material. Although the modification to the theory suggested by incorporating a value of elastic modulus

as a function of the volumetric stress under biaxial compression is a tacit recognition of this fact, it cannot of itself account for all the aspects of the heterogeneity.

As regards the prediction of the mode of failure, the point has been made by Robinson (10) that a satisfactory failure criterion must predict the observed failure mode. Otherwise, although it may be quantitatively correct for the data considered, it may lead to dangerous applications if extended outside these narrow limits. The same standard has been adopted at the beginning of this chapter.

Now the results of this research and other work has shown that under biaxial states of stress, failure at a phenomenological level is invariably one of either single or multiple cleavage with varying orientation of the planes. Hence the shear failure predicted by the theory is a major flaw as far as its application to concrete is concerned.

Thus the use of this type of approach to concrete suffers from two of its basic assumptions, viz, homogeneity and shear failure at the phenomenological level. However, it may prove a useful quantitative approach to the problem of failure criterion prior to the establishment of a more satisfactory theorem.

c) Possible Empirical Failure Formula

An alternative approach to the problem is afforded by the results obtained in this research project. This would be based on the observed variation of the overall cleavage strain at discontinuity with the mean normal stress.

Starting with the initial assumption that the strain needed to separate two adjacent molecules of paste is directly proportioned to the total strain energy applied to the system, the relationships obtained can be extended to the phenomenological level. By equating the sum of these intermolecular strains to the overall phenomenological cleavage strain, a simple functional relationship with the applied strain energy can be established

as follows:-

$$\epsilon_d = f(u)$$

where ϵ_d = cleavage strain at discontinuity

u = total strain energy at discontinuity.

As a first approximation, this can be expressed as a simple first-order linear function of the form,

$$\epsilon_d = a + bu.$$

where a, b = arbitrary constants (to be determined from experiment).

Now these two arbitrary constants can be solved for the two unique cases of uniaxial compression and tension thus:-

$$u = \frac{1}{2E} (\sigma_1^2 - 2\nu\sigma_1\sigma_2 + \sigma_2^2) \quad \text{(for biaxial states of stress)}$$

where E = elastic modulus

ν = Poisson's ratio

$$\therefore \epsilon_d = a + \frac{b}{2E} (\sigma_1^2 - 2\nu\sigma_1\sigma_2 + \sigma_2^2) \quad \text{———— (1)}$$

For uniaxial states of stress, either $\sigma_1 = 0$ or $\sigma_2 = 0$.
Hence for a uniaxial state of stress:-

$$\epsilon_d = a + \frac{b\sigma_1^2}{2E}$$

Let ϵ_d^c = cleavage strain at discontinuity under uniaxial compression

ϵ_d^t = cleavage strain at discontinuity under uniaxial tension

c = discontinuity stress under uniaxial compression

t = discontinuity stress under uniaxial tension

$$\begin{aligned} \text{Then } \epsilon_p^c &= a + \frac{bc^2}{2E} \quad \text{and} \quad \epsilon_p^T = a + \frac{bt^2}{2E} \\ \therefore (\epsilon_p^c - \epsilon_p^T) &= \frac{b}{2E} (c^2 - t^2) \\ \text{hence } b &= \frac{2E (\epsilon_p^c - \epsilon_p^T)}{(c^2 - t^2)} \quad \text{--- (2)} \end{aligned}$$

$$\text{and } \epsilon_p^c = a + \frac{c^2}{2E} \cdot \frac{2E (\epsilon_p^c - \epsilon_p^T)}{(c^2 - t^2)}$$

$$\text{hence } a = \epsilon_p^c - \frac{c^2 (\epsilon_p^c - \epsilon_p^T)}{(c^2 - t^2)} \quad \text{--- (3)}$$

Having now established the means of determining the numerical values of the arbitrary constants a and b from the results of uniaxial tension and compression tests, the next stage is to convert the expression for cleavage strain at discontinuity into discontinuity stress.

This involves the complication of the variation of elastic modulus due to particle interaction and the following expression relating the elastic modulus to the volumetric stress is suggested:-

$$E = E_c \left\{ 1 + k \left(\frac{\epsilon_1 + \epsilon_2 + \epsilon_3}{3} \right) \right\}$$

However, as the results obtained in the experimental work were for biaxial states of stress only, it is considered more appropriate to express the variation of elastic modulus in terms of the degree of biaxial compression, i.e.:-

$$E = E_c \left\{ 1 + k \left(\frac{\epsilon_1 + \epsilon_2}{2} \right) \right\}$$

(valid under biaxial compression only)

where E = elastic modulus under biaxial stress

E_c = elastic modulus under uniaxial compression

ϵ_1, ϵ_2 = principal biaxial stresses

k = empirical function.

The empirical function k must be determined experimentally and classified according to the volume concentration of coarse aggregate and the aggregate texture. From the variation of elastic modulus with degree of biaxial compression shown in fig. 11.2.11 it would appear that k is a non-linear function.

From the elastic equations:-

$$\begin{aligned} E_0 &= \frac{\nu}{E} (\sigma_1 + \sigma_2) \\ &= \frac{2 \nu (\sigma_1 + \sigma_2)}{E_c (2 + k (\sigma_1 + \sigma_2))} \end{aligned} \quad \text{---(4)}$$

Therefore substituting for a and b from equations 2 and 3 in equation 1 and eliminating E , using equation 4,-

$$\nu (\sigma_1 + \sigma_2) = E_c \left\{ \frac{2 + k (\sigma_1 + \sigma_2)}{2} \right\} \left\{ \frac{a + b (\sigma_1^2 - 2\nu \sigma_1 \sigma_2 + \sigma_2^2)}{2E} \right\}$$

USE ONLY UNDER
BIAXIAL COMPRESSION

Which is the equation for the biaxial discontinuity stress envelope.

The equation has been plotted for mix 61C, using the results of stress and strain measurements taken in uniaxial compression and tension, in fig. 12.3.13 and the actual experimental envelope of biaxial discontinuity stress superimposed. For this comparison the function k was assumed to be of the form,-

$$k = k_1 (\sigma_1 + \sigma_2)$$

$$\text{hence } E = E_c f (\sigma_1 + \sigma_2)^2$$

The comparison from a quantitative viewpoint is quite good and it should be noted that this failure criterion predicts failure by cleavage for all states of biaxial stress. However, it must be admitted that the equation of the curve is complicated and requires the results of uniaxial and some biaxial tests.

VALUES USED:-

$$E_s^C = 63 \times 10^{-6}$$

$$c = 2643 \text{ psi}$$

$$E_s^T = 53 \times 10^{-6}$$

$$t = 237 \text{ psi}$$

$$E_c = 6.10 \times 10^6 \text{ psi}$$

$$\nu = 0.15$$

$$k = 5 \times 10^{-5}$$

for MIX 61C

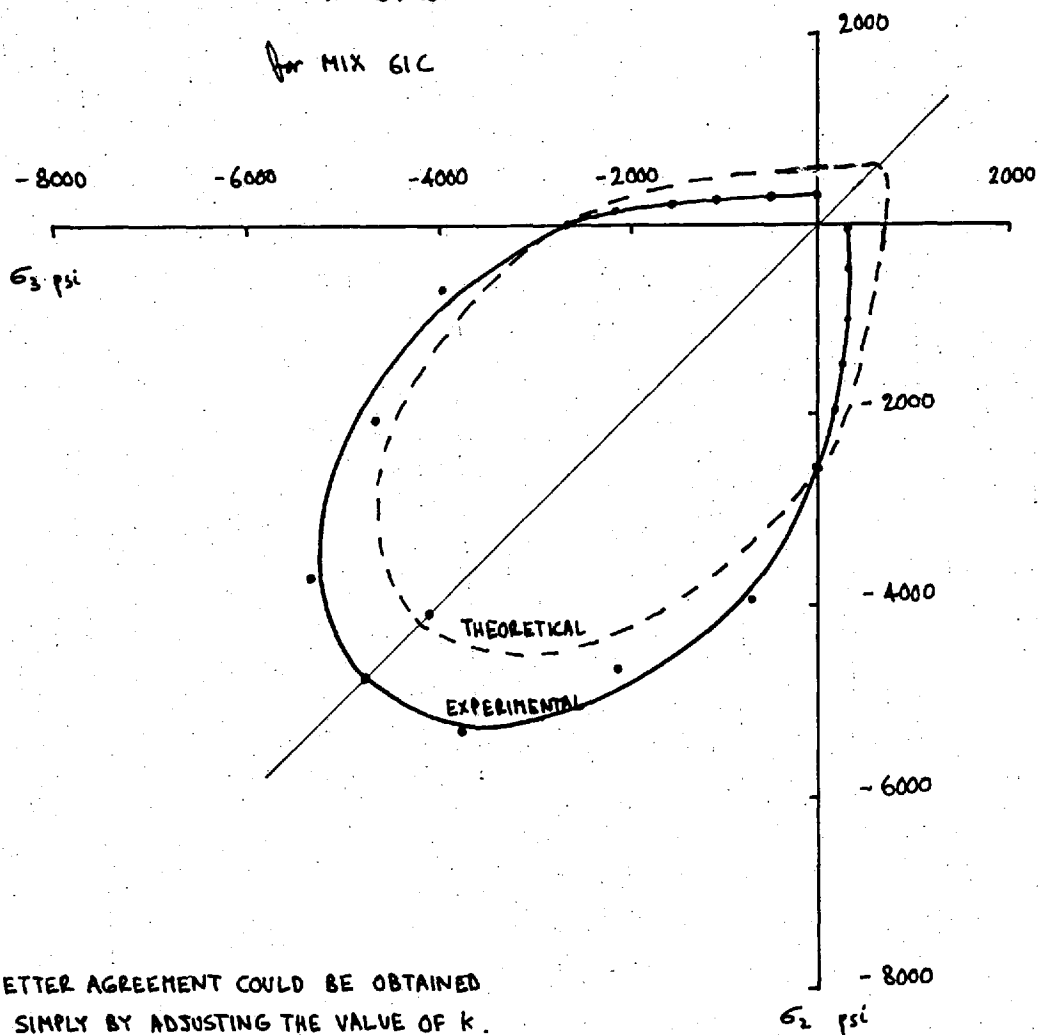


FIG. 12.3.13 COMPARISON OF DISCONTINUITY STRESS ENVELOPES
OBTAINED BY EXPERIMENT AND FROM THEORY.

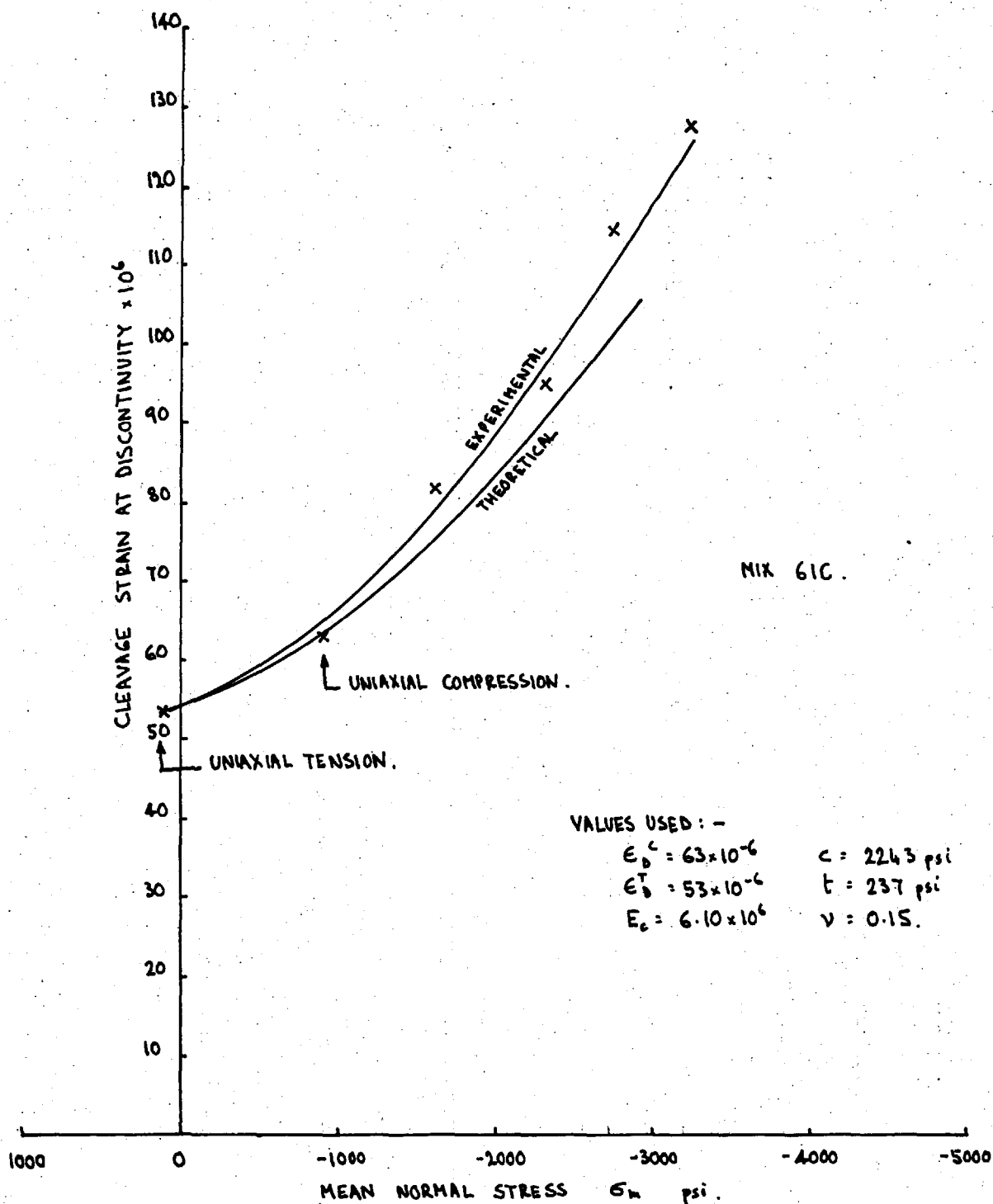


FIG. 12.3.14 COMPARISON OF EXPERIMENTAL AND THEORETICAL VARIATIONS OF CLEAVAGE STRAIN WITH MEAN NORMAL STRESS.

One important aspect of the biaxial stress envelopes which was noted in section 11.7 was the shape of the envelope for normal aggregate concretes in the biaxial compression-tension zone. The shape appeared to be dependant on the volume fraction of coarse aggregate in the mix for, whilst concrete with a volume fraction of about 0.50 yielded a distinct convex envelope, the concretes with a volume fraction of about 0.30 yielded a doubly-curved envelope which appeared to be a transition between the convex concrete curve and the concave mortar curve. From this result it must be concluded that, under tensile states of stress, concretes with unusually low volume fractions of coarse aggregate will exhibit the properties of mortar to a degree.

12.3.3. Comparison with Results

Throughout the above discussion the results of experimental work have been referred to in justification for some of the statements made. Hence this section is very much in the nature of a conclusion.

As the key results from uniaxial tension and compression tests plus the variation of elastic modulus with degree of biaxial compression have already been incorporated in the semi-empirical failure criterion proposed, only two comparisons between the criterion and test results are possible.

The first comparison with the biaxial discontinuity stress envelope has already been described above and shown in fig. 12.3.13. The comparison was satisfactory.

The second comparison possible is that of the predicted cleavage strain at discontinuity forecast by equation 1 given the uniaxial discontinuity stresses and strains and the biaxial discontinuity stress envelope. Again mix 61C has been used for the comparative exercise and the predicted experimental results are shown in fig. 12.3.14. As before, the comparison is very encouraging.

However, it is important that the basic necessary assumptions on which this criterion of failure has been based be recognised and for this purpose the main assumptions are listed below:-

- 1) Local interparticle strains at all dimensional levels are proportional to overall strain.
- 2) Cleavage strain at discontinuity is a linear function of the total applied strain energy.
- 3) Under biaxial compression the elastic modulus is a second order function of the degree of biaxial compression.
- 4) The elastic equations relating stress and strain are valid at a phenomenological level.

A consideration of the assumptions listed above plus the dependence of the equations defining the failure criterion on experimental results emphasises the limitations of the criterion proposed. However, it may provide a useful interim tool in failure criteria for plain concrete applied to the interpretation of research into structural uses of concrete.

It is to be hoped that the more fundamental work now being pursued into the shear bond strength and crack mechanics of the problem plus work suggested by this research will yield a more satisfactory theoretical basis for a failure criterion.

12.3.4. A Possible Theoretical Approach

During the course of this research project an attempt was made to analyse the model of behaviour and failure of normal aggregate concrete given in this section 12.3 in a more fundamental manner by considering the crack mechanics at each stage of crack formation and the resulting distribution of stress and strain around an aggregate particle. Unfortunately this work has not been completed as the full mathematical analyses required are not yet available in a usable form. However, it was thought worthwhile to outline the theoretical approach believed to offer the

best chances of success and list the relevant mathematical references for future use. This exercise, though incomplete as far as predicting a suitable failure criterion is concerned, does indicate how several approaches to the problem of concrete failure criteria might be interrelated.

The problem of the mechanism of crack propagation around aggregate particles has hitherto been the most difficult phenomenon to state in mathematical terms. Work by Sherman (191) in 1959 considered the problem of crack propagation at the junction of two dissimilar elastic materials, but he assumed that there were no cavities or flaws on the bond surface. Erdogan (199) investigated the effect on the stress distribution by introducing cracks in 1963 and in a very recent paper (200) has shown that it is possible to apply the Griffith-Irwin theories to the problem. In this contribution two materials with different elastic moduli and Poisson's ratio were considered and some attempt was made to include residual stresses. This latter consideration is particularly valuable if the effects of shrinkage are to be taken into account. Furthermore the crack propagation along the bond surface was investigated under several applied states of stress including tension orthogonal to the bond surface and shear along the bond surface. It would seem that a most useful link could be forged here between these mechanics of crack propagation and the work of Alexander et al (197, 198) on tensile and shear bond strengths.

On completion of the bond cracks, the cracks start to spread through the mortar matrix under the action of shear and tensile stresses though the tensile stresses may be replaced by compressive stresses in some instances. This is basically the propagation of a crack under shear forces in a homogeneous material. Whilst much work has been done on the propagation of cracks under tensile stresses and some of the results have been applied to concrete on a phenomenological level (82-84) very little work

has been published on the shear propagation of cracks. However, the problem is simpler than for cracking in normal fracture and under transverse shear and useful work has been done by McClintock and Sukhatme (201), Borenblatt and Cherepanov (202), and Field and Baker (203). The problem of friction on the crack propagation if the tensile stress is replaced by a compression stress has been investigated by McClintock (204) in work on rocks under pressure.

It would seem that on the basis of the above work on crack propagation, plus stress distribution investigations on idealised inclusion problems such as those conducted by Edwards (205), Stephen and Pirtz (206), and Willis (207), a better theoretically based failure criterion could be developed. Additional refinements could include the propagation of a crack through rather than round an aggregate particle and the effect of particle interaction in stiffening and possibly reinforcing the material against crack propagation by the closure of cracks under compressive stresses referred to above.

12.4 Deformation and Failure of Lightweight Concrete

12.4.1 Introduction

The case has been made in section 12.3 for the use of a two phase model for concrete and so the same basic model will be used in the discussions contained in this section.

The lightweight aggregates commercially available to date are all much weaker than the mortars into which they are incorporated and also have a lower elastic modulus being manufactured for the most part from foamed slag or expanded clay or shale. Therefore lightweight concretes can be said to be the complement of normal aggregate concretes being effectively soft particles in a hard matrix as opposed to the latter case of hard particles in a soft matrix. This concept of relative particle and matrix stiffness is basic to the following discussion and the two-phase model will be invoked throughout.

12.4.2. Elastic Behaviour

Analogous to the definitions of stages of complexity in the assumptions made for the two phase model of normal concrete described in section 12.3, it is possible to obtain two extreme stages with lightweight concrete between which the actual behaviour of the material will lie. The two stages may be summarised as follows:-

STAGE 1 Plane sections remain plane -
 continuity at phase boundaries

STAGE 2 Plane sections do not remain plane -
 continuity at phase boundaries,

As the lightweight aggregate is weaker than the bond strength between itself and the mortar matrix, the condition of partial or full discontinuity at the phase boundary does not arise.

12.4.2.1. Stage 1

As explained in section 12.3.2.1, this stage is equivalent to a state of uniform strain throughout the material which yields the following simple formula for the overall elastic modulus:-

$$E = V_1 E_1 + V_2 E_2$$

where E_1 = elastic modulus of matrix phase

E_2 = elastic modulus of included phase

V_1 = volume fraction of matrix phase

V_2 = volume fraction of included phase.

Clearly, as the elastic modulus of the mortar matrix is considerably greater than the elastic modulus of the included lightweight aggregate particles, the bulk of the load is carried by the mortar and the eventual failure is due to failure of the mortar matrix. Likewise, there can be no particle interaction effects.

12.4.2.2. Stage 2

This stage is the same as stage 2 in section 12.3.2.1 in that full continuity is maintained at the phase boundaries but the condition that plane sections remain plane is relaxed.

If equal Poisson's ratios are assumed for both phases and the combined material, the expression for the overall elastic modulus is as follows, -

$$E = \left\{ \frac{(1 - V_2) E_1 + (1 + V_2) E_2}{(1 + V_2) E_1 + (1 - V_2) E_2} \right\} E_1$$

This formula shows the dominating effect of the mortar elastic modulus on the overall elastic modulus as was the case in stage 1 above. As with stage 1, there can be no effective particle interaction because of the relative stiffness of the aggregate and mortar phases.

The stress-strain characteristic followed the same pattern as that of the mortars and concretes with a linear 'elastic' portion followed by an inelastic curve.

12.4.3. Criterion of Failure

Under uniaxial compression and tension, the cleavage strain at discontinuity was virtually the same as the cleavage strain of the parent mortar, irrespective of the volume fraction of coarse lightweight aggregate particles present (figs. 7.5.4.1-3).

This same adherence to the behaviour of the mortar matrix was observed under biaxial compression in which the biaxial stress envelope was identical in shape to that of the parent mortar. Indeed for the one lightweight aggregate mix tested (mix 6LB), the following strength relationship was found to apply approximately (i.e. within $\pm 10\%$) for the whole biaxial strength envelope with the exception of biaxial tension which was not investigated (fig. 12.4.1).

$$\frac{\text{Strength of Lightweight Concrete}}{\text{Strength of Parent Mortar}} = \text{Volume fraction of Mortar in Lightweight Concrete}$$

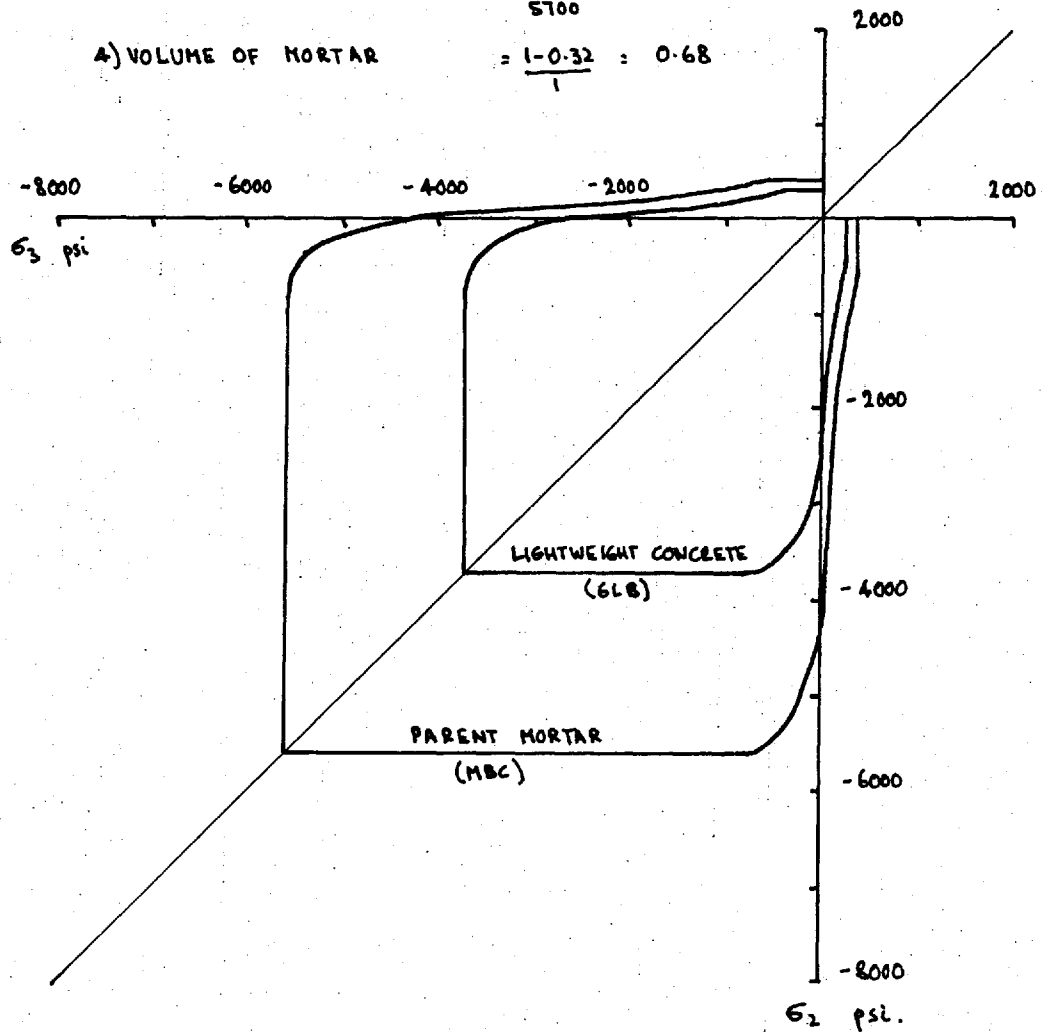
RATIOS: LIGHTWEIGHT CONCRETE / PARENT MORTAR

1) UNIAXIAL TENSION STRENGTH = $\frac{292}{390} = 0.75$

2) UNIAXIAL COMPRESSION STRENGTH = $\frac{2700}{4350} = 0.62$

3) EQUAL BIAxIAL COMP. STRENGTH = $\frac{3700}{5700} = 0.65$

4) VOLUME OF MORTAR = $\frac{1-0.32}{1} = 0.68$



ULTIMATE STRESS ENVELOPES SHOWN.
DISCONTINUITY ENVELOPES SIMILAR.

FIG. 12.4.1. COMPARISON OF STRENGTH OF LIGHTWEIGHT CONCRETE AND PARENT MORTAR.

This relationship was valid for both ultimate strength and discontinuity stresses being accurate to within $\pm 6\%$ for the latter.

Hence the modified Coulomb-Mohr theory described in section 12.2 is recommended as the failure criterion for lightweight concretes as well as mortars. Hanson's work (208) on triaxial tests on lightweight concrete gave results similar to those predicted by the Mohr theory.

CHAPTER 13

CONCLUSIONS

13.1. Introduction

The purpose of this chapter is to conclude the account of the research project given in this thesis and to examine and summarise in general terms its results and consequences. Not only are the experiment results considered but also the development and performance of the combined stress testing machine, the testing techniques developed, and the ancillary equipment used.

Mechanical behaviour and failure criteria have been the principal object of the research and these are commented upon together with the basic philosophies adopted for the project.

The conclusions are grouped under the following five general headings corresponding to a chapter or group of chapters in the thesis as noted below:-

13.2 Combined Stress Testing Machine	CHP. 2 & 3
13.3 Ancillary equipment	CHP. 5
13.4 Testing techniques	CHP. 3 & 6-10
13.5 Results of research	CHP. 7-10 & 11
13.6 Failure criteria	CHP. 12

13.2. Combined Stress Testing Machine

The development of the combined stress testing machine was considered successful in that the limited objectives of providing a variety of stress combinations in the one machine for tests under constant stress rate was achieved. The rigidity of the machine as designed was satisfactory giving remarkably low coefficients of variation in the test results obtained and the automatic machine alignment mechanism on the test specimen worked well.

The facilities of biaxial compression, biaxial compression-tension and biaxial compression-torsion were developed with varying degrees of reliability though all of them were an advance on previous work. Preliminary tests to investigate the difficulties involved in extension to triaxial compression testing were carried out.

Two major modifications to the machine were considered as promising to increase its usefulness as a research tool. These were the hydraulic compensator and the load maintaining controls.

Some comment was made on the problems of stiffness, especially longitudinal stiffness, in the machine sub-frames and how this was radically reduced by the elasticity of the hydraulic system giving undesirable explosive failures. A hydraulic compensating system, much as that designed by Hinde (138), would cure this trouble and enable the descending portion of the stress-strain curve to be investigated under combined stresses. This would represent a significant advance on existing testing facilities.

The other modification concerns the control of the rate of testing and the fact that the control is on stress rate and not on strain rate. As pointed out earlier, constant stress rate is adequate for testing up to the ultimate load but, to obtain definable results beyond the ultimate load, a constant strain rate device (such as that designed by Broch (209) at Birmingham) would be advantageous.

A primary requirement, yet one which is seldom realised in practice, for a testing machine is the elimination of operator technique or influence on the results.

Taylor (210), as an adjunct to work on prestressed anchor blocks, carried out a series of biaxial compression tests on his concrete mix. The results are shown in fig. 13.2.1 and show the consistency of the results obtainable by an operator unused to the machine using the testing machine independent of outside supervision.

Taylor was also able to use the combined stress facilities of the machine for tests on anchor blocks which were completely unforeseen at the design stage.

$\frac{3}{8}$ " A66. A/C 3.75 W/C 0.564 V_f 0.43

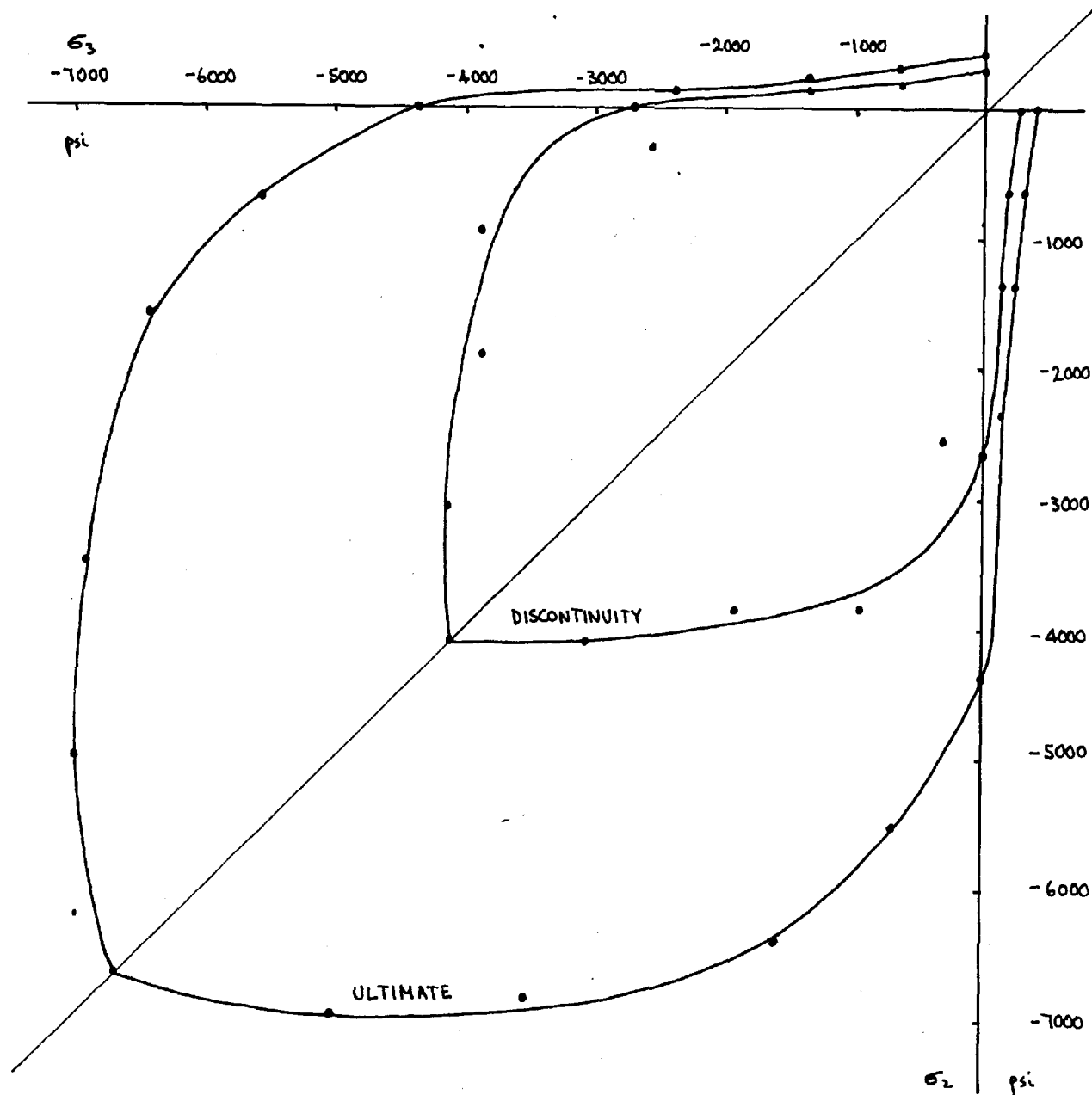


FIG. 13.2.1. BIAxIAL STRESS ENVELOPES OBTAINED BY TAYLOR (210)
USING COMBINED STRESS TESTING MACHINE.

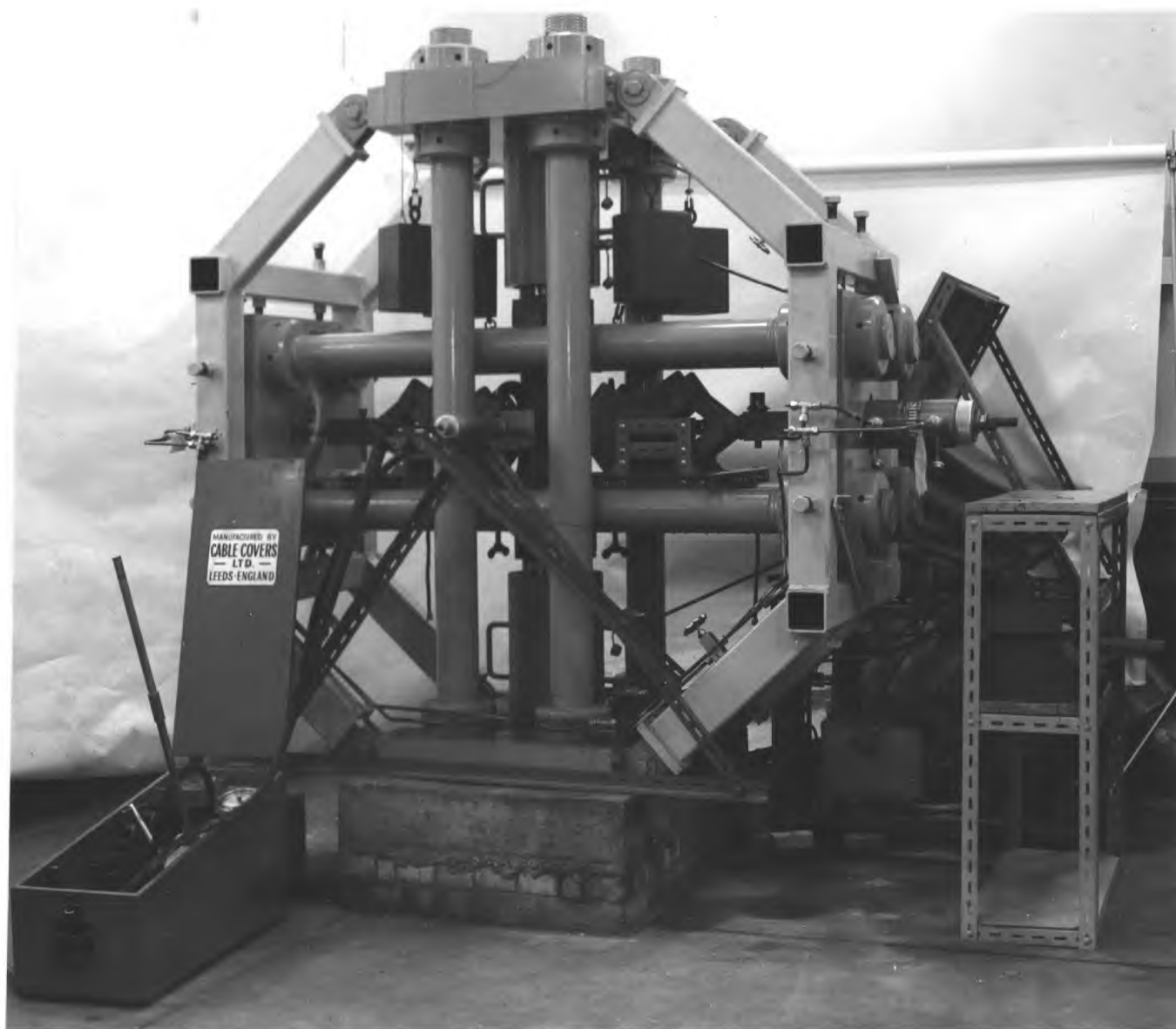


Fig.13.2.2. Investigation of Pre-stressed Concrete Anchorage Strains Orthogonal to a Biaxial Compression-Tension Stress Field (Taylor)

13.3. Ancillary Equipment

Fortunately, first-class strain measuring equipment was available for this research project.

The Peekel boxes and the Solartron data logger were reliable and time-saving tools and the performance of the Sogenque linear displacement transducers was excellent. Indeed, in most situations, the accuracy and sensitivity of the ancillary equipment were greater than the stress control and measuring gauges really warranted.

In the case of the linear displacement transducers, the problem was to find a support or datum frame of sufficient rigidity and reliability to hold such an accurate measuring device. The frame used in the biaxial compression tests gave a satisfactory calibration but the same principle could be used whilst making a more versatile frame capable of making measurements at any point on the surface of the specimen.

The ultra-sonic pulse velocity equipment worked satisfactorily and appeared to be quite capable of detecting the discontinuity point under all states of biaxial stress. It was not, however, able to give a good signal where non-plane or extra surfaces of contact were involved such as in the torsional and triaxial tests (CHP.10) nor was it entirely free from external electrical interference.

13.4. Testing Techniques

The techniques used in the uniaxial test series were only slightly modified from their earlier inception and were found to yield satisfactory results. The techniques for the biaxial compression tests were a logical development of the uniaxial compression test techniques and also gave good results.

The practice of exercising the specimen prior to elastic measurements and final load to failure was justified on the grounds that the results of the initial loading are unique whereas the

behaviour after several pre-loadings is general and continues thereafter. This was explained as being due to the formation of stable bond cracks.

Biaxial compression-tension tests yielded a number of problems associated with the state of stress obtained and the order of loading. This technique, though basically sound, needs further development and special attention should be paid to the design of a more elaborate support frame for the tension clamps and to the alignment problems with the concrete cubes used to apply compression.

The few tests conducted in torsion indicated that very little further development would be required on this facility to produce reliable results, the only problem associated with this test being in the casting of the test specimens with an inside former. This facility needs a little more development.

Triaxial compression testing was found to have many difficulties, not least of which was the problem of platen clashing. This testing facility and technique will prove very difficult to establish.

13.5 Results of Research

The results of this research have already been summarised in section 11.7 but are repeated here for completeness.

1) The stress-strain characteristics of mortars and lightweight concrete under uniaxial tension and compression and biaxial compression obey the elastic theory equations within practical limits. The stress-strain characteristics of normal aggregate concretes under uniaxial tension and compression obey the elastic theory equations within practical limits but the behaviour under biaxial compression does not obey the elastic theory.

2) Below the discontinuity stress mortars and lightweight concrete maintained equal and constant values of elastic modulus, Poisson's ratio and hence bulk modulus and rigidity

modulus under uniaxial tension, uniaxial compression, and biaxial compression states of stress. Normal aggregate concretes gave equal values of elastic modulus and Poisson's ratio under uniaxial tension and compression states of stress but under biaxial compression considerable increases in elastic modulus with degree of biaxial compression together with small increases in Poisson's ratio were noted. The values of bulk modulus and rigidity modulus also increased with the degree of biaxial compression. The effect of the small increase in Poisson's ratio was to increase further the value of the bulk modulus but to reduce the increase in rigidity modulus.

3) The biaxial stress envelopes which covered the whole biaxial stress characteristic with the exception of biaxial tension combinations showed similar shapes of discontinuity and ultimate stress envelopes for a given mix. Three basic shapes of envelope emerged in which mortars and lightweight concretes gave a square shaped envelope under biaxial compression and a concave envelope under biaxial compression-tension. Normal aggregate concretes with a volume fraction of about 0.50 gave a rounded convex envelope in both segments whilst normal aggregate concretes with volume fractions of about 0.30 gave a convex envelope under biaxial compression and an intermediate doubly curved envelope under biaxial compression-tension. The shapes of the envelopes obtained appeared to be influenced only by the limiting aggregate size of $\frac{3}{16}$ " , volume fraction, and normal or lightweight aggregate. The shapes were uninfluenced by aggregate size above $\frac{3}{8}$ " , aggregate/cement or water/cement ratios, aggregate stiffness, or aggregate surface texture.

4) The cleavage strain at the discontinuity point as defined by the ultrasonic pulse velocity measurement was observed to increase with the mean normal stress. The cleavage strain was also seen to decrease with increasing volume fraction of coarse aggregate.

5) Under triaxial compression both mortars and concretes were observed to undergo very large strains of up to 0.13, whilst maintaining a high degree of overall continuity, amounting to plastic distortion. The influence of the coarse aggregate was to obstruct this plastic deformation and cause the mortar matrix to crack.

6) Cleavage type failures were observed under all states of biaxial stress investigated, with single cleavage fracture under uniaxial tension and most of the biaxial compression-tension zone and multi-cleavage fractures under uniaxial and biaxial compression and the remainder of the biaxial compression-tension zone. All the cleavage planes were parallel to the principal compressive stress or at right-angles to the principal tensile stress and small yet significant differences were observed between the compressive failures of mortars and concretes. Studies were also made of the mortar remaining on aggregate particles loosened by the fracture process. The failures observed under biaxial compression-tension tests on solid specimens and biaxial compression-tension tests on hollow cylinders were similar.

13.6. Failure Criterion

From the results of the research reported in Chapters 7 - 11 and a consideration of concrete as approximating to a two-phase material, it was decided to examine the deformation and failure of concrete materials under the sub-divisions of mortar, normal aggregate concrete, and lightweight aggregate concrete.

It was recognised that for most engineering purposes, the short-term static load behaviour of mortar may be regarded as elastic and the material as homogeneous. A three-parameter modification of the Coulomb-Mohr internal friction-failure theory proposed by Burton Paul (72) in which the additional criterion of limiting tensile stress was introduced showed fair correlation with the experimental results obtained. A further modification

to eliminate the friction component of strength under biaxial compression-tension explained both the concave envelopes obtained and the cleavage failures observed and an additional refinement was introduced by considering the statistical alignment of critical flaws which explained the reductions in strength observed under uniaxial compression. The behaviour and failure of mortar as recorded in this research project was seen to be remarkably similar to several types of cast iron.

Normal aggregate concrete was regarded as a two phase material in which the coarse aggregate particles were the included phase and the mortar was the matrix phase. In an attempt to clarify the several approaches made to the deformation and failure of this two-phase model of concrete, the various assumptions regarding plane sections and continuity around the phase boundary were listed and their consequences discussed in detail. The stage at which plane sections did not remain plane and partial discontinuity was allowed around the phase boundary was shown as the level of complexity that should be examined if the complex behaviour of the concrete is to be explained.

Accordingly, from evidence collected both from the experimental and theoretical work of this project and from previous contributions to the subject, a general picture of behaviour was assembled showing the formation and propagation of cracks from zero load to failure. Of particular note was the mechanism of cracking above the discontinuity stress proposed in which cracks were propagated under a combination of longitudinal shear and lateral tension or even compression. A semi-empirical failure criterion was proposed based on the assumption that the cleavage strain at discontinuity was a first order function of the applied strain energy. However, this was regarded as an interim solution only and an indication was given of how recent work on crack mechanics might be applied to the failure mechanism proposed in this thesis and also provide a link

with other work on the tensile and shear bond strengths of mortar and aggregate.

Lightweight aggregate concrete followed the behaviour of mortar very closely and it was observed that the strength of the lightweight concrete under any biaxial state of stress investigated was directly proportional to the volume of mortar in the mix. Although a very slight tendency to follow the behaviour of concrete was observed in some experimental results, for all practical purposes this material can be treated in exactly the same way as a mortar both from a deformation and failure viewpoint.

CHAPTER 14

SUGGESTIONS FOR FUTURE RESEARCH

From the results obtained in this research project and their limitations it is clear that more research could be profitably conducted on some of the matters raised by the work. In particular the failure criterion proposed warrants further investigation and confirmation both at a phenomenological and at a macroscopic level.

Accordingly the following projects are suggested as subjects for future research:-

1. The main combined stress facility developed and used in this research project was that of biaxial compression. Though a wide range of mortars and concretes were tested under this state of stress, the results obtained cannot be claimed to cover biaxial compression comprehensively and much useful information could be gained by a further series of biaxial compression tests. This could include preplaced aggregates and model concretes. It could also investigate other factors such as the effect of testing with pinned-ended, i.e. lubricated, spherical seatings as recommended by Newman and Sigvaldason (15). If the recommended modifications to the machine were carried out, i.e. hydraulic compensator to increase longitudinal stiffness and constant strain rate mechanism, then tests could also be planned to investigate the post-ultimate region of the stress-strain curve.

2. As noted in the thesis, the biaxial compression-tension testing facility, although basically sound, needs further development time and effort. This development and a consequent large test series would be very useful both as part of the general development of the combined stress testing machine into

a useful research tool and as a check on the failure criterion proposed. A series of biaxial compression-torsion tests could be run parallel to this series to give a comparison between the two means of achieving states of biaxial compression-tension stress.

3. A very long-term research project, that offers most interesting results from the few obtained in this research project yet one that has more than its fair share of problems, is triaxial compression. It cannot be emphasised too much just how difficult this type of mechanical test would be and, although the combined stress testing machine has been designed for extension to triaxial testing at a later date, the idea of loading the test specimen through solid platens might eventually have to be abandoned in favour of hydraulic loading such as that designed in section 2.4. Certainly no test results should be expected from a project of this kind for at least three years as this is felt to be the minimum time required to develop this facility.

4. The long-term testing of concrete is suggested as a further research project with a view to making the result of a short term test, and especially the discontinuity point, more meaningful. Some tests were carried out in this project and a positive link was suggested by the results between the discontinuity stress and the long-term ultimate strength. In this context a load maintaining device would be very useful on the combined stress testing machine and most useful results could be obtained even if the longest test lasted only a few days to failure.

5. The significance of the degrees of freedom of movement allowed on each loaded face of a test specimen has already been touched upon both in designing the testing machine and in the suggested research under biaxial compression. This problem

might be thought sufficiently significant to warrant a special test series of its own. In particular a development of Ward's tension test could be an enlarged pin at the jacking end of the tension grips to produce an effectively locked condition under load thus making it equivalent to the present method of conducting compression tests in the combined stress testing machine. An alternative uniaxial tension specimen named the "Theta" specimen has been developed by Durelli, Morse and Parks (211) and this too is worthy of consideration in this context.

6. In the selection of a failure criterion for mortar, the view was expressed that the overall cleavage failures observed in the tests might be due to the degrees of freedom of the loading platens and not representative of the actual initial failure of the material. It was suggested that a shear crack initiated in the centre uniformly strained section would not be able to propagate very far before either the natural dilation of the material or the loading system adopted took over and dictated the extension of the crack. This of course ties in with the philosophy at stake in suggestion 5.

However a more fundamental investigation of these mechanics of crack extension would be most useful in checking the validity of the failure criterion proposed.

7. The present state of knowledge in the field of crack mechanics and stress distribution analysis was regarded as not quite sufficient to derive a theoretical failure criterion for concrete based on the model of deformation and failure proposed. However, at some time in the near future, a theoretical study of this problem by an engineer and applied mathematician together could yield some useful relationships between the several approaches to the problem of failure criterion and perhaps the final criterion for concrete failure.

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PC = Personal Communication

APPENDIX A - SYMBOLS

The symbols and abbreviations used throughout the text of the thesis were as follows:-

σ	stress
σ_1	major principal stress (tension positive)
σ_2	intermediate principal stress
σ_3	minor principal stress
σ_u	ultimate axial stress
σ_c	ultimate uniaxial compressive stress
σ_T	ultimate uniaxial tensile stress
σ'	ultimate uniaxial cylinder compressive stress
σ_r	radial stress
σ_{PV}	discontinuity stress as defined by USPV measurements
c	discontinuity stress under axial compression
t	discontinuity stress under uniaxial tension
σ_o	octahedral normal stress = $\frac{\sigma_1 + \sigma_2 + \sigma_3}{3}$
τ_o	octahedral shear stress = $\frac{1}{3} \sqrt{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2}$

σ_m	mean normal stress = $\frac{\sigma_1 + \sigma_2 + \sigma_3}{3} = \sigma_o = 3I_1$
σ_B	equal biaxial compressive stress ($\sigma_2 = \sigma_3 = \sigma_B$)
I_1	first invariant of stress tensor (spherical stress tension)
ϵ	strain
ϵ_1	strain in direction of principal stress σ_1
ϵ_2	strain in direction of principal stress σ_2
ϵ_3	strain in direction of principal stress σ_3
ϵ_v	volumetric strain = $\frac{\epsilon_1 + \epsilon_2 + \epsilon_3}{3}$
ϵ_D	cleavage strain at discontinuity as defined by USPV measurements measurements
ϵ_D^C	cleavage strain at discontinuity under uniaxial compression
ϵ_D^T	cleavage strain at discontinuity under uniaxial tension
E	elastic modulus = $\frac{\sigma}{\epsilon}$
E_s	static elastic modulus
E_D	dynamic elastic modulus
E_c	static elastic modulus under uniaxial compression
E_T	static elastic modulus under uniaxial tension
E_1	elastic modulus of matrix phase
E_2	elastic modulus of included phase
G	shear modulus of rigidity = $\frac{E}{2(1 - \nu)}$
K	bulk modulus of elasticity = $\frac{E}{3(1 - 2\nu)}$

ν	Poisson's ratio
ν_s	static Poisson's ratio
ν_D	dynamic Poisson's ratio
ν_c	Poisson's ratio under uniaxial compression
ν_T	Poisson's ratio under uniaxial tension
V_f	volume fraction of coarse aggregate ($\frac{3}{16}$ ")
V_{fm}	volume fraction of mortar
V_1	volume fraction of matrix phase
V_2	volume fraction of included phase
u	strain energy
p.s.i.	pound per square inch
mm	millimetre
min.	minute
I.D.	inside diameter
O.D.	outside diameter
USPV	ultrasonic pulse velocity
ERSG	electrical resistance strain gauge
W/C	water:cement ratio (by weight)
S/C	sand:cement ratio (by weight)
A/C	aggregate:cement ratio (by weight)

APPENDIX B - TESTING MACHINE CALCULATIONS

This appendix contains an abstract from the machine design calculations, to elaborate on the criteria described in Chapter III which formed the basis of the detailed design.

The preliminary calculations on structural sizes are omitted and calculations given are simply an analysis of the final design.

1) Machine Capacity.

a) Compression. 200 TON. (U.S.) = 400,000 lb.

27,500 psi hydraulic working pressure

Overload to 10,000 psi = 533,333 lb.

on 10" x 10" x 4" slab = 10,000 psi (13,333 psi overload)

4" x 4" x 4" cube = 25,000 psi (33,333 psi overload)

b) Tension. 30 TON. (U.S.) = 60,000 lb.

29,200 psi hydraulic working pressure

Overload to 10,000 psi = 65,200 lb.

on 3" x 4" section = 5,000 psi (5,430 psi overload)

2) Longitudinal Stiffness. (Horizontal Frame)

Calculate on basis of 200 TON (U.S.) compressive effort on 10" x 10" x 4" specimen and neglect stored strain energy in hydraulic system.

a) Specimen deflection (assume $E = 6 \times 10^6$ psi)

$$= \frac{10,000 \times 10}{6 \times 10^6}$$

$$= 1.67 \times 10^{-2} \text{ in.}$$

b) Column deflection.

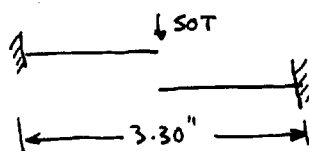
$$= \frac{400,000}{4 \times \pi r^2} \times \frac{62}{30 \times 10^6}$$

$$r = 2.25"$$

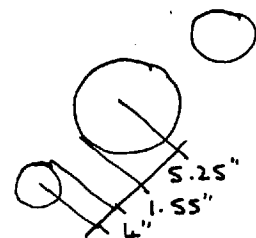
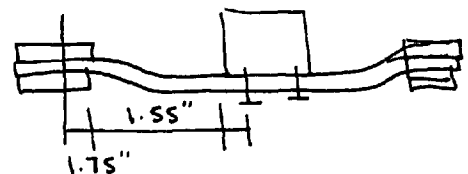
$$= 1.30 \times 10^{-2} \text{ in.}$$

c) End plate deflection.

(i) Bending.



$$\delta = \frac{Pa^3}{3EI}$$



$$\delta = \frac{Pa^3}{3EI}$$

$$= \frac{50 \times 2,000 \times 1.65^3}{3 \times 30 \times 10^6 \times 40}$$

$$= 0.115 \times 10^{-3} \text{ in.}$$

53T

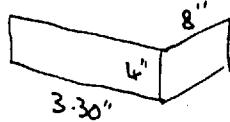
$$I = \frac{bd^3}{12} = 40 \text{ in}^4$$

but 2 cantilevers + 2 ad plates = $4 \times 0.115 \times 10^3$

$$= \underline{0.05 \times 10^{-2} \text{ in.}}$$

(2) shear.

$$(1+\nu) = 1.3$$



$$\gamma = \frac{2 \times 1.3}{30 \times 10^6} \times \frac{50 \times 2,000}{32}$$

$$= 0.27 \times 10^{-3}$$

$$\gamma = \frac{1}{G} \tau$$

$$G = \frac{E}{2(1+\nu)}$$

$$\therefore e_s = 3.30 \times 0.27 \times 10^{-3}$$

$$= 0.089 \times 10^{-2} \text{ in.}$$

total for 2 plates = $\underline{0.18 \times 10^{-2} \text{ in.}}$

Total machine deflection = $1.30 + 0.05 + 0.18 = \underline{1.53 \times 10^{-2} \text{ in.}}$
 which is less than specimen deflection of $1.67 \times 10^{-2} \text{ in.} \therefore \text{O.K.}$

Longitudinal stiffness = $\frac{4 \times 10^5}{1.53 \times 10^{-2}} = 26 \times 10^6 \text{ lb/in.}$

O.T. SIGVALDASON (12) RECOMMENDS
 $10 \times 10^6 \text{ lb/in. MINIMUM.}$

Compare with measured stiffness from chapter 4 of -

Machine $88.5 \times 10^6 \text{ lb/in.}$ (26×10^6 calculated)

Specimen $18.3 \times 10^6 \text{ lb/in.}$ (23.8×10^6 assumed on $E = 6 \times 10^6 \text{ psi}$).

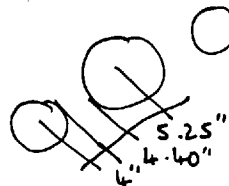
3) Longitudinal Stiffness (Vertical Frame)

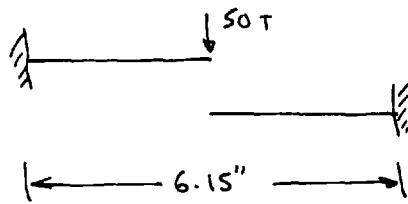
a) Specimen Deflection = 1.67×10^{-2}

b) Column Deflection = 1.30×10^{-2}

c) Top plate deflection.

(i) bending.





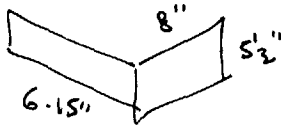
$$\delta = \frac{P a^3}{3EI}$$

$$= \frac{50 \times 2,000 \times 3.07^3}{3 \times 30 \times 10^6 \times 111}$$

$$= 0.29 \times 10^{-3}$$

for 2 cantilevers = $0.06 \times 10^{-2} \text{ in.}$

(2) shear.



$$\gamma = \frac{2 \times 1.3}{30 \times 10^6} \times \frac{50 \times 2,000}{44}$$

$$= 0.196 \times 10^{-3}$$

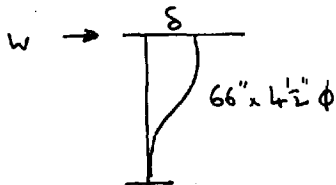
$$e_D = 6.15 \times 0.196 \times 10^{-3}$$

$$= \underline{0.12 \times 10^{-2} \text{ in.}}$$

Total machine deflection = $1.30 + 0.06 + 0.12 = \underline{1.48 \times 10^{-2} \text{ in.}}$ $\checkmark$ o.k.

Longitudinal Stiffness = $27 \times 10^6 \text{ lb/in.}$

4) Lateral Stiffness (Both Frames).



$$I = \frac{\pi R^4}{4}$$

$$\delta = \frac{2 W l^3}{3EI}$$

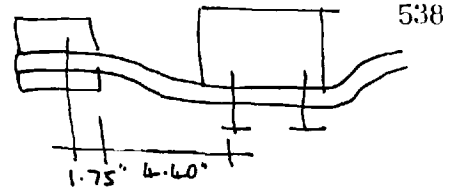
$$= \frac{2 W 33^3 \times 4}{3 \times 30 \times 10^6 \times \pi (2.25)^4}$$

$$= 0.396 W \times 10^{-4} \text{ per column.}$$

$$\therefore E_{LAT} = \frac{4}{0.396 \times 10^{-4}}$$

$$= \underline{10.1 \times 10^4 \text{ lb/in.}} \quad \checkmark \text{ o.k.}$$

as Sigurdson (12) recommends $10 \times 10^4 \text{ lb/in.}$ MINIMUM.



$$I = \frac{8 \times (5 \frac{1}{2})^3}{12}$$

$$= \underline{111 \text{ in}^4}$$

APPENDIX C - TESTING MACHINE DRAWINGS

A complete list of all the detailed machine drawings used in the design and manufacture of all the machine components and ancillary equipment is given below.

The original negatives and prints of the latest revision of each drawing has been deposited with the Concrete Materials Research Group and access to these drawings may be obtained from Mr. K. Newman.

A full-size print of the general arrangement drawing (drg.No.1.) is enclosed in the loose pocket at the back of this thesis and, together with the sketches and drawings in Chapter 3, should give a representative detail of the testing machine.

<u>Drg. No.</u>	<u>Title.</u>
1	General Arrangement
2	Details of Components
3	Details of Extra Machine Components
4	Tension Rig for Biaxial Machine
5	Tersion Rig for Biaxial Machine
6	Triaxial Rig for Biaxial Machine
7	Instrument Panel Layout

The main title of all the drawings is, "Short-Term Combined Stress Testing Machine."

APPENDIX D - ANALYSIS OF RESULTS

In this appendix the calculations and corrections involved in converting the original experimental instrument readings to absolute values of stress and strain quantities are recorded and illustrated by worked examples where necessary. The analysis is sub-divided into the various tests or groups of tests performed as follows:-

1. Non-destructive tests
2. Uniaxial prism compression tests
3. Uniaxial tension tests
4. Flexural beam tests
5. Biaxial compression tests
6. Biaxial compression-tension tests.

1. Non-destructive Tests

a) Density

The flexural beam specimen was used for this test and weighed in air and water. The weight in water was determined by weighing a tank of water with and without the specimen being suspended in it from an independent frame. The following worked example is drawn from mix 31B beam B1.

WT. IN AIR	12,062 gm.
WT. OF WATER	9,876
WT. OF WATER & CONCRETE	15,162
WT. IN WATER	= 15,162 - 9,876 = 5,286.

$$\therefore \text{density} = 62.4 \times \frac{12,062}{5,286}$$

$$= \underline{142.4 \text{ lb./cu.ft.}}$$

The average value of all three specimens is recorded.

b) Dynamic elastic modulus E_D

Again the flexural beam specimen is used and the dynamic elastic modulus is calculated for each specimen from the density of that specimen and the average of three or more measurements of its resonant frequency.

$$E_D = 5.99 n^2 l^2 \rho \times 10^{-6} \text{ psi.}$$

where n = resonant frequency (cycles/sec.)

l = length of specimen = 20"

ρ = density of concrete.

The resonant frequencies recorded in the tests are in the form of the total number of cycles measured over ten seconds. The following example is drawn from mix 31B beam B1.

resonant frequency readings, -	040284
	040279
	040282
3	120845
	040282

$$(\rho = 142.4 \text{ lb./cu.ft.})$$

$$\begin{aligned} \therefore E &= 5.99 \times (4028.2)^2 \times (20)^2 \times 142.4 \times 10^{-6} \\ &= \underline{5.55 \times 10^6 \text{ psi.}} \end{aligned}$$

c) Dynamic Poisson's ratio

This quantity is calculated from the results of both the resonant frequency and ultrasonic pulse velocity measurements on the flexural beam specimen. The equation for the dynamic Poisson's ratio is,-

$$\left(\frac{v}{2nl} \right)^2 = \frac{1 - \nu_D}{(1 + \nu_D)(1 - 2\nu_D)} = k \text{ (say).}$$

Again using the results of mix 31B and beam B1 as our example and taking the difference in the ultrasonic pulse velocity between the transducers being together and apart (i.e. at opposite ends of the beam).

$$\begin{array}{rcl} \text{APART} & 131.72 & \\ \text{TOGETHER} & \underline{14.38} & \\ & 117.34 & \times 10^{-6} \text{ secs for 20 in. path length} \\ \therefore V & = 14,200 \text{ ft./sec.} & \end{array}$$

$$\begin{aligned} \therefore k &= \left(\frac{14,200 \times 12}{2 \times 4,028.2 \times 20} \right)^2 \\ &= (1.058)^2 \\ &= \underline{1.12.} \end{aligned}$$

and from a graph of k and v_0 .

$$\underline{v_0 = 0.207}$$

2. Uniaxial Prism Compression Tests

a) Axial Stress

The pressure gauge reading was converted directly into average stress applied to the assumed 4" x 4" cross-section of the specimen from graphs prepared from the actual machine calibration charts. No correction was applied to each specimen to allow for its actual cross-section due to casting on the variation in the dimension of 4" seldom exceeded 1/16" (i.e. 1.5%).

b) Longitudinal Strain

The Peckel transistorised strain box type T200 was adjusted such that it gave readings of electrical resistance strain gauges directly in strain assuming a gauge factor of 2.00. As the gauge factor often differed from the value of 2.00 the following correction had to be applied to all the readings.

$$\text{true strain} = \frac{2.00}{\text{gauge factor}} \times \text{recorded strain.}$$

c) Lateral Strain

The frame used to measure the lateral strain used one linear inductance transducer only so that the reading on the Sogenique equipment was the total lateral expansion in micro-inches (10^{-6} inches). Hence all the increments of expansion had to be divided by 4" to give microstrain.

d) Strain from Zero Load

For practical purposes, the measurements taken on each test started not from zero load but from a small nominal load. Hence to draw stress-strain diagrams and to obtain total value of strain from zero load, the strain increment from zero load to the nominal load was estimated from the subsequent two or three increments assuming a linear relationship between stress and strain over this initial portion of the stress-strain curve.

Occasionally, e.g. for determining the total cleavage strain at discontinuity, this initial increment of strain was assessed on a pro-rata basis.

c) Worked Example

A completely analysed test on a prism is shown in fig. D1 for mix 31B and prism P3. The calculation of the elastic modulus, Poisson's ratio and longitudinal and lateral strains at discontinuity are shown as well as the discontinuity and ultimate stresses.

3. Uniaxial Tension Tests

All calculations and conversions used in the analysis of the uniaxial tension test results were identical to those used for the uniaxial prism compression test results with the obvious exception of the machine calibrations used.

	LOAD (GAUGE READING)	PEEKEL	SOGENIQUE	USPV
INTERVAL	100	14 888	78 265	
1333 psi.	500	592 ²⁹⁶	364 ⁹⁹	$\nu = \frac{97.75}{295} \times \frac{2.1}{2.0} \times \frac{1}{4} = 0.087$
	100	884 ²⁹²	265 ⁹⁹	
	500	590 ²⁹⁴	360 ⁹⁵	$E = \frac{1333}{295} \times \frac{2.1}{2.0} \times 10^6 = \frac{4.74 \times 10^6}{\text{psi}}$
	100	888 ²⁹⁸	262 ⁹⁸	
		AV. 295	AV. 97.75	
	100	888	262	1.71
	200	817	272	1.69
	300	740	300	1.67
	400	665	326	1.64
	500	588	361	1.61
	600	500	403	1.63
	700	402	456	1.62
	800	300	518	1.62
DISCONTINUITY. (CRACKING)	900	189	599	1.64
→ 3180 psi	1000	072 ⁸¹⁶	710 ⁴⁴⁸	1.65
	1100	13 940	840	1.68
	1200	753		1.75
	1300	570		1.80
ULTIMATE → 4300 psi	1360	260		

$$E_D \text{ (CLEAVAGE STRAIN AT DISCONTINUITY)} = \frac{448}{4} \times \frac{11}{10} = \underline{123 \times 10^{-6}}$$

$$E_L \text{ (LONGITUDINAL STRAIN AT DISCONTINUITY)} = \frac{816}{21} \times \frac{2.2}{2.1} = \underline{855 \times 10^{-6}}$$

$$\nu \text{ (AT DISCONTINUITY)} = \frac{448}{4 \times 816} = \underline{0.144}$$

FIG. D1. TYPICAL PRISM TEST AND ANALYSIS.

4. Flexural Beam Tests

a) Flexural Stress

Two different testing machines had to be used for these tests due to laboratory re-organization as explained in Chapter 7. Load calibration charts were available for both of these two machines and tables were drawn up in which the relevant instrument readings were converted to total load on the beam. Then, assuming plane sections to remain plane and a central neutral axis, the extreme fibre stress was calculated using the usual beam formula.

For both machines, a correction had to be applied to the stresses to account for the weight of the block and roller assembly through which the beam was loaded. This amounted to 59.5 lb. which was equivalent to an extra 15 psi. at the extreme fibre.

In addition, when the Sigvaldason flexural machine was used and the Cawkell ultrasonic pulse velocity transducers attached, the frame used to support the transducers exerted a reinforcing effect equivalent to 4 lb. of applied load which meant a reduction of 1 psi. at the extreme fibre.

No correction was made to allow for the self-weight of the beam specimen as this was negligible in terms of extreme fibre stress.

b) Flexural Strain

These results were analysed in the same manner as for the uniaxial tests above.

5. Biaxial Compression Tests

a) Applied Stress

The pressure gauge reading was converted directly into average stress applied to the annual 4" x 10" cross-section of the specimen from graphs prepared from the actual machine calibration chart. No correction was applied to each specimen

to allow for its actual cross-section due to casting as the variation in the dimension of 4" seldom exceeded $\frac{1}{16}$ " (i.e. 1.5%). The stress on the uniformly strained volume was assumed to be equal to the average applied stress as explained in Chapter 8.

b) Longitudinal strain

The Solartron automatic data logger print-out results required two corrections: one for the gauge factor and one for the output voltage. The data logger will give results directly in strain only if the gauge factor is 2.00 and the output voltage is 2.00 volts. The actual output voltage is printed out for each load stage on the tape and can be used in the following correction,-

$$\text{true strain} = \frac{2.00}{\text{gauge factor}} \times \frac{2.00}{\text{output voltage}} \times \text{recorded strain}$$

c) Lateral strain

The frame used for supporting the linear inductance transducers carried two transducers horizontally opposed over a 4 in. gauge length. The Sogenique equipment reading was the average of the two transducer readings in micro-inches and therefore the Sogenique recording had to be divided by 2" to obtain the result in microstrain. The initial zero error in the frame detected in the calibration trial was neglected in most cases being less than 1% of the total strain.

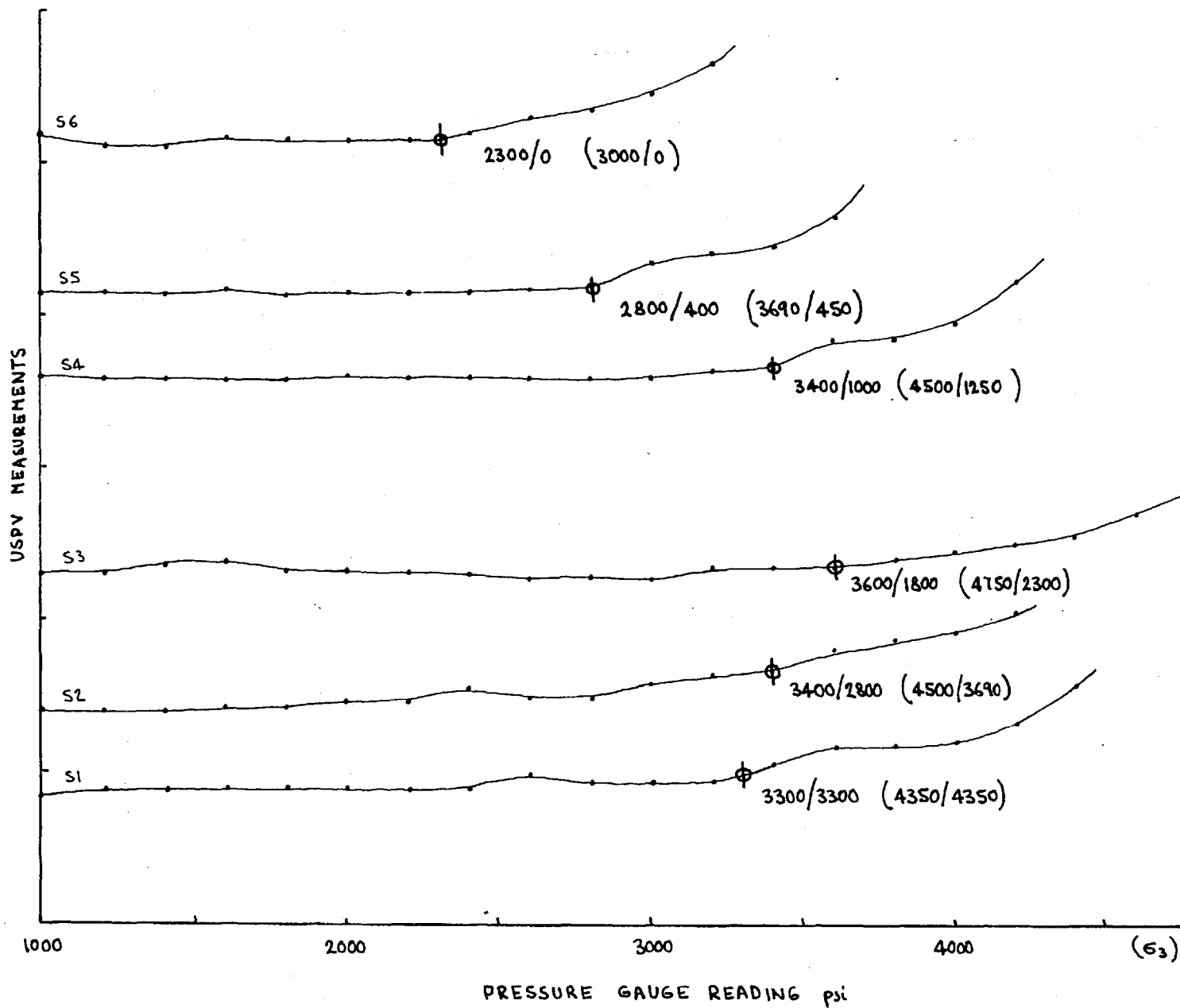
d) Strain from Zero Load

The same adjustments to the strains were made in these tests as described in section 2 d) above.

e) Determination of Discontinuity Point

As in all the other tests described, the discontinuity point was determined by a radical change in the ultrasonic pulse velocity measurements. A typical set of plots used to determine the discontinuity point are shown in fig. D2 for mix 31B.

FIG. D2. DETERMINATION OF DISCONTINUITY POINT FROM USPV READINGS.



MIX 31B

$\Sigma \sigma$	σ_m	ϵ_D
3000	1000	120

$\Sigma \sigma$	σ_m	ϵ_D
4140	1380	167

$\Sigma \sigma$	σ_m	ϵ_D
5750	1917	207

$\Sigma \sigma$	σ_m	ϵ_D
7050	2350	309

$\Sigma \sigma$	σ_m	ϵ_D
8190	2730	428

$\Sigma \sigma$	σ_m	ϵ_D
8700	2900	405

6. Biaxial Compression - Tension Tests

a) Applied Stress

The applied tensile stress was calculated by dividing the applied tensile force obtained from the calibration of the tension facility by the specimen cross-sectional area of 3" x 4".

The applied compressive stress was taken as the applied compressive load obtained from the original machine calibration divided by the area of the 4" x 4" compression loading cubes. This latter assumption is subject to error as the intensity of compressive stress will reduce as the centre of the specimen is approached but it does represent the maximum compressive stress applied to the specimen.

b) Strains

All strain measurements taken were analysed using the corrections described above.

7. Calibration and Conversion Charts

All calibration and conversion charts not contained in the text of the thesis or in this appendix have been deposited with the Concrete Materials Research Group and are in the immediate care of Mr. K. Newman.

APPENDIX E - USEFUL UNIAXIAL TEST REFERENCES

During the course of the research into the literature available on the behaviour of concrete under simple and combined states of stress a large number of references on uniaxial tests (compression, tension and flexure) were collected. Those most relevant to the work conducted in this project have already been referred to in the main text of the thesis. However, it was felt that much time and effort could be spared future research workers if a permanent record were made of these other contributions and accordingly they are listed below,-

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APPENDIX F - INSIDE FORMS FOR TORSIONAL SPECIMENS

The selection of a torsional test specimen of uniform external profile and tapering internal profile - the dimensions of which are shown in fig. F.1. - led to the problem of providing an inside form to cast this specimen shape as explained in section 10.2.2.

A number of possibilities were examined, before the selection of the wax former described in section 10.2.2. was made, and these are listed below:-

- 1) Collapsible steel mould
- 2) Air- filled rubber bag
- 3) Polystyrene former
- 4) Wax former
- 5) Sand former

Each of the above possibilities had various advantages and disadvantages and these are discussed below.

1) Collapsible Steel Mould

This mould would have taken the form of interlocking steel splines formed to the correct shape by a removable central core. However this idea was rejected on the following grounds:-

- a) Complex and precise machining would be required for the interlocking mechanism of the splines
- b) However good the interlocking system was, an impression would be left on the inside profile of the concrete giving a non-uniform cross-section to the cylinder.
- c) Cost was considered too high for even a major series of tests.

2) Air-filled Rubber Bag

The use of air-filled rubber hose to form ducts in mass concrete is widespread and the use of a rubber bag in forming the torsional cylinder seemed feasible. A specially shaped bag would have been required and the pressure needed to obtain the correct dimensions specified.

However, as only a pilot series of tests had been planned, it was uneconomic to order one purpose made mould of this type though an order for six or twelve such moulds might well have made this solution a satisfactory answer to the problem.

3) Polystyrene Former

Imperial Chemical Industries Ltd. were approached for advice on the possibility of producing inserts moulded in polystyrene to the correct shape for the torsion specimen. They advised that this was not possible unless a very large order was forthcoming and we were prepared to pay part of the cost of producing the necessary steel mould. Alternatively, they would supply expanded polystyrene 4 in. cubes from stock which could then be shaped and stuck together.

From our experience of machining polystyrene on a lathe, it was recognised that it would be very difficult to obtain a series of inserts of identical shape. An alternative method of using a hot wire bent to the required profile was also rejected for the same reason.

An inquiry to Shell Chemicals brought forth the suggestion of buying polystyrene in bulk and moulding the required shape ourselves. This idea requires a special steel mould to be made up and was therefore rejected for these pilot tests but is worthy of consideration for a major series of tests. The technique for expanding and moulding the polystyrene is described in

Shell Chemical Company Technical Bulletin "STYROCELL" No. PRP/6197/D, April 1963, obtainable from Shell Chemicals.

The question of the water absorption capacity of expanded polystyrene is pertinent to its use in the role as a former for concrete and a test was carried out on an average commercial sample (PORON 30). It was found that the expanded polystyrene absorbed its own weight of water in a week but lost it at a much slower rate. This suggests that only saturated expanded polystyrene formers should be used in casting concrete specimens.

4) Wax Former

This solution was the one eventually adopted for the pilot tests and consisted of casting the former in ships wax in a timber mould.

A timber mould, illustrated in fig. 10.2.2.1., was constructed in six sections which were dowelled and clamped together. 'Polyfilla' was used between the mating surfaces to obtain a leakproof joint and one end of the mould was blanked off so that the mould could be stood on end. The ships wax was heated and then allowed to cool until it had almost solidified again when it was poured into the open end of the mould. As the wax cooled it shrank and the mould had to be topped up several times. Care had to be taken when adding fresh wax to ensure that it mated properly with the wax already solidified in the mould. This was achieved by breaking the existing solid wax surface with a heated rod.

The other major problem associated with filling the mould with wax was that of air being trapped by formations of wax solidifying on the sides of the mould and overhanging the general wax surface. This behaviour was noted particularly during the first two trials and was prevented by the use of the heated rod referred to above.

There was also a tendency for the wax to stick to the sides of the mould when trying to strip the mould and the first trials in which Vaseline was used as a mould oil showed that this preparation was insufficient.

Accordingly a coating of banana oil supplied by Proops of London was painted onto the inside of the mould and allowed to solidify. A layer of mould grease as used in the steel moulds for concrete specimens was then smeared on top of the banana oil to act as a releasing agent. This preparation worked satisfactorily.

After the wax had hardened for at least 24 hours, the mould was stripped and the wax former placed in the 6" dia. 12" long steel cylinder mould. It was centrally located by two $\frac{1}{4}$ " steel dowels located in the baseplate of the mould and in a fitted plate across the top of the mould. This method of locating the wax former was very satisfactory and spot checks on the wall thicknesses of broken specimens showed a variation on the 1 in. specified thickness of only ± 0.015 in. Casting of the concrete specimen was carried out in the usual way.

Once the concrete mix had hardened and the steel mould been stripped the specimen was placed in the curing tank for a few days to allow the concrete to mature before removing the wax former.

The wax was removed from the inside of the specimen by gently heating it, care being taken not to overheat and crack the surrounding concrete. This made the removal of the wax a long and tedious process and had the effect of drying out sections of the concrete specimen and hence interfering with the constant curing conditions. After the wax had been removed, the specimen was placed in a bath of warm water prior to returning to the curing tank to avoid cracking of the specimen caused through rapid cooling.

If the specimen, is placed in hot water before the wax former is removed, the wax will expand and crack the specimen. This fact was discovered on one occasion when the curing tanks developed a fault and became over-heated.

5) Sand Former

Although this method has not been tried so far, a sand insert was made in the Royal School of Mines foundry shop using the moulding sand normally used for cast iron. The same mould was used as for the wax formers and the foundry technicians claimed that the filling process was quick and easy. The releasing agent used on this occasion on the inside of the mould was a graphite grease. During the filling of the mould a steel rod was inserted into one end to facilitate removal of the sand after the concrete is cast. The sand would be scraped out of the specimen after it had hardened.

This method showed considerable promise but has yet to be tried out fully.

Conclusions

From the limited experience gained so far it would appear that whilst the collapsable steel former can be rejected outright, the other possibilities offer varying chances of success.

The wax former is probably too tedious for a large series of tests whilst the inflated air-filled rubber bag would give problems with precise dimensions. The use of sand or expanded polystyrene formers look the most hopeful as these can readily be moulded in the laboratory and stripped from the specimen in a relatively short period of time.

STRENGTH OF CONCRETE UNDER SHORT-TERM STATIC
BIAXIAL STRESS

INTERNATIONAL
CONFERENCE
ON THE
STRUCTURE OF
CONCRETE

Gerald W.D. Vile

Paper F2

SUMMARY

The results of short-term static load tests on plain concrete specimens under biaxial compression and compression tension states of stress are described. Particular attention is paid to the "elastic" behaviour of the material compared with the general three dimensional elastic theory. The ultimate and discontinuity stress envelopes obtained and the crack patterns observed are shown and certain hypotheses of fracture mechanisms and failure criteria are proposed.

INTRODUCTION

The need for a criterion of deformation and failure for plain concrete to describe its behaviour under all states of stress is well known. In view of the several levels at which concrete as a material may be studied, it is not surprising that several different approaches to the problem have emerged. In this paper the results of short-term static load tests under biaxial states of stress at a phenomenological level, but with a structural bias, are described. This particular level of investigation was chosen because it complemented existing research programmes of a more fundamental nature and so could act as a moderator on the hypotheses advanced. Furthermore, the results obtained might suggest further fundamental research which could be fruitful whilst at the same time providing information of immediate use to the design engineer.

Although what is meant by deformational behaviour is generally understood, the term "strength" of concrete raises problems of definition. The dependence of the magnitude of the ultimate strength on the rate of loading has been established (1, 2); in this paper the ultimate stress is expressed in terms of specified rates of loading. However, there still remains the question of breakdown or deterioration of the structure before ultimate load.

It has been shown that deterioration in the form of cracks exists prior to load in the material (3, 4) and that these cracks grow in size as the material is stressed (5). This gradual

N.B. This paper has been condensed from the original draft, but will be published in full in the final Proceedings of the Conference.

deterioration continues until a critical point is reached when there is a marked change in the mechanical properties of the material and more severe cracking takes place which leads to ultimate failure. The existence and determination of this point has been reported by many researchers (6) and a detailed investigation into various methods which have been used is described by Robinson at this Conference (7). This critical point has been termed "onset of rapid crack propagation" by Kaplan (8) and "discontinuity" by Newman (9). The latter term is used throughout this paper.

Recognising that the discontinuity stage is a significant "strength" parameter of concrete, this point was determined in the research work reported in this paper by means of the ultrasonic pulse velocity technique developed by Jones (10).

Previous work on biaxial compression and compression-tension (11-26), with the exception of that performed by Robinson (26), ignored this distinction between ultimate and discontinuity stress and quoted ultimate strength values only. The results obtained by previous research show in general a varying bonus of strength under biaxial compression compared with uniaxial compression but conflicting and very scattered results under biaxial compression-tension. Only in Robinson's work (26) were comprehensive strain measurements taken and the scope of this work was restricted by the machine capacity to a limited portion of the biaxial compression zone. Very few attempts have been made to check the states of stress actually achieved in the test specimen and there are major objections to several of the testing techniques used (27).

Consequently a fresh approach to the problem was required and a comprehensive investigation involving the development of suitable testing techniques and machines has been carried out by the Concrete Materials Research Group at Imperial College (26, 28). As a result a new combined stress testing machine, which has been described elsewhere (29) was designed by the author for this research project. Briefly, this machine applies combinations of compressive, tensile, and torsional stress to a variety of specimen shapes through solid loading platens, grips, and clamps. At present it is used basically as a biaxial stress testing machine but provision has been made for conversion to triaxial testing at a later date.

So far, only four possible stress combinations have been investigated. Of these, the work on triaxial compression and biaxial compression-tension have been in the nature of preliminary trials. The biaxial compression facility has been fully developed and shown to give states of uniform stress and strain of known magnitude. Although experiments have been carried out using the biaxial compression-tension facility, the test results have not the same degree of reliability as those achieved under biaxial compression and more development of this technique is required. However, the results obtained from this testing technique are reported here since the trends shown are interesting and may be significant.

For biaxial compression test, the test specimen, which measures 10" x 10" x 4", is suitable for $\frac{3}{4}$ " maximum aggregate size and is loaded through solid platens as shown in Figure 1. The biaxial compression-tension specimen is a modification of the direct tension specimen developed at Imperial College (30) and the desired state of stress is applied to a 4" x 4" x 3" volume as shown in Figure 2.

The mixes selected for testing under biaxial stress were chosen to cover as wide a range of parameters as possible. Thirteen mixes were tested under biaxial compression and six under biaxial compression-tension and these included mortars, normal aggregate concretes and light-weight aggregate concretes. All specimens were tested at 27-29 days old in a fully saturated condition after curing in water at 68°F.

EXPERIMENTAL RESULTS

Stress-Strain Characteristics

The stress-strain characteristics obtained under biaxial compression in general were similar in shape to those obtained under uniaxial compression and tension, for both lateral and longitudinal strain. However, under biaxial compression, the strains in the direction of loading were smaller than those in uniaxial compression as would be expected from Poisson's ratio effect. On using the normal elastic relations to compare the strains from uniaxial and biaxial compression, the mortars and light-weight concrete showed excellent correlation (within 4% at worst) whereas those for normal aggregate concretes were in disagreement by 15% and more.

Investigations showed that Poisson's ratio remained effectively constant under uniaxial tension and compression and biaxial compression below the discontinuity stress, although, for normal aggregate concretes, there was a slight tendency for this to increase under biaxial compression (see Figure 3). However, measurements of the elastic modulus under varying degrees of biaxial compression showed that, whilst this value for mortars and light-weight concrete was constant for most practical purposes, normal aggregate concretes exhibited significant increases of elastic modulus with mean normal stress up to 40% as shown in Figure 4. The increase appears to be dependent on the volume concentration of coarse aggregate and contrasts with the equality of elastic modulus values obtained under uniaxial compressive and tensile states of stress, shown in Figure 5.

This variation of elastic modulus with mean normal stress explains the incompatibility of the stress-strain behaviour described above and confirms that a particle interaction effect occurs. From the three-dimensional equations of the elastic theory it is clear that the shear modulus and bulk modulus will show similar variations. If the small increase in Poisson's ratio observed under biaxial compression is included in the elastic theory equations, the increase in shear modulus corresponding to the increase in elastic modulus is reduced by an insignificant amount (about 5%) and likewise the bulk modulus is slightly increased. It is clear that for normal aggregate concretes the elastic constants are a function of the mean normal stress under biaxial compression.

Stress Envelopes

Both ultimate and discontinuity stress envelopes were obtained for thirteen mixes under biaxial compression and six mixes under biaxial compression-tension. An examination of these envelopes revealed two basic classes of behaviour under biaxial compression with an additional sub-division under compression tension. It is

interesting to note that the ultimate and discontinuity stress envelopes for a given mix are similar in shape (see Figures 6-8). In addition, it was found that the order of loading, either sequence or proportional, appeared to have very little effect on the shape of either envelope under biaxial compression, provided the specified rate of loading of $700 \text{ lb/in}^2/\text{min.}$ was maintained.

Mortars and light-weight concrete exhibit a square shaped stress envelope under biaxial compression reminiscent of the maximum shear stress failure theories. In the compression-tension zone, the envelope contains a marked concavity which is discussed below. A typical mortar characteristic is shown in Figure 6 and a light-weight concrete in Figure 7.

Normal aggregate concrete, by contrast, exhibits a very rounded convex envelope under both biaxial compression and compression-tension as shown in Figure 8. A sub-division of behaviour occurs under biaxial compression-tension where concretes with a low volume concentration of coarse aggregate (say, 0.30) appear as a transition between mortar and normal concretes and have both convex and concave portions of their envelope.

The main parameters which determine the different classes of behaviour are a limiting maximum aggregate size of about $\frac{1}{4}$ " for normal aggregate concretes and the volume concentration of coarse aggregate in the mix above this critical maximum size.

Strain Behaviour

An additional feature of the three-dimensional strain behaviour described above was the variation of strain at discontinuity in the direction of the principal tensile stress. Kaplan (8) has shown that this cleavage strain is proportional to the volume fraction of coarse aggregate. This general trend has been confirmed in this research but furthermore it has been found that the cleavage strain at discontinuity is dependent on the state of stress. The effect of both volume fraction of coarse aggregate and state of stress, expressed in the form of mean normal stress, is shown in Figure 9. It can be seen that the strain capacity of mortars and concretes increases with degree of biaxial compression.

Crack Patterns

The crack patterns observed at the phenomenological level were similar to those obtained by previous workers except that the patterns were more clearly defined and less distorted by testing machine effects.

For both mortars and concretes the general trend of crack patterns is shown in Figure 10. Single cleavages orthogonal to the principal tensile stress predominated in the biaxial compression-tension zone and under uniaxial tension. However, at compressive stresses close to the strength in uniaxial compression, the crack patterns in the biaxial compression-tension zone undergo a transition to multi-cleavage fractures which continue through to the uniaxial compression state of stress. In the biaxial compression zone, the failure becomes one of several parallel cleavage fractures orthogonal to the direction of the principal tensile stress.

A significant difference exists between the mortar and concrete fractures. Whilst the concrete cleavage fractures under compression stresses are extensive and regularly aligned in the directions specified above, the mortars have far fewer planes of cleavage and the angle of crack plane can vary by as much as 40° over significant proportions of the fracture surface. Photographs of concrete fractures are shown in Plates A and B (biaxial compression), Plate C (biaxial compression-tension), and Plate D (transition from single to multi-cleavage fracture).

A further observation on the fractures of normal aggregate concrete was the regular occurrence of deposits of mortar on aggregate particles located on the cleavage planes. These deposits formed cones at one or each end of the aggregate particle on the axis of the principal compression stress whilst the remainder of the aggregate particle was free of mortar deposits (Plates E and F). Under biaxial compression these cones extended to halos as shown in Plates G and H. Although these deposits were not found on all aggregate particles and some aggregate particles were split by the fracture, there were a sufficient number of them to be significant. Sigvaldason (28) and Taylor (31) have confirmed these observations, which are discussed below in connection with failure mechanisms and criteria for normal aggregate concrete.

STRENGTH CRITERIA UNDER BIAXIAL STRESS

For the convenience of this discussion, mortar, normal aggregate concrete and light-weight aggregate concrete are considered separately.

Mortar

It was observed that the stress envelopes obtained for mortar were very similar to those obtained for several types of cast iron. The behaviour of cast iron approximates to that predicted by the Coulomb-Mohr theory except for a general lack of fit and discrepancy of fracture modes in the biaxial compression-tension zone. Paul (32) has proposed the addition of a limiting tensile stress criterion to account partially for the lack of fit and also to explain the tensile cleavage failures observed. Furthermore, he has suggested that the internal friction component of strength is unrealistic under biaxial compression-tension and its omission leads to a concavity in the stress envelope and predicts tensile cleavage fractions over the whole zone. These two modifications are shown in Figure 11 where the resulting curve is compared with a set of typical mortar envelopes.

One outstanding discrepancy can be seen still to exist within the region of uniaxial compression where there is an unexplained loss of strength. A similar loss of strength is seen with cast iron under uniaxial compression and biaxial tension. Unfortunately no results of uniform biaxial tension stress tests on mortar are available due to the difficult technique involved. However, it is evident that these two particular states of stress are unique in the biaxial stress envelope in that they have two degrees of freedom for failure whilst all other biaxial stress states have only one. This being so the loss of strength observed under these two states of stress can be accepted

for any material containing flaws which ultimately cause failure. Hence a further modification to the Coulomb-Mohr theory would be to incorporate a function to predict this strength reduction.

In the biaxial compression zone the correlation of the stress envelope with the internal friction theory is good but the theory predicts a shearing mode of failure. However, it is conceivable that the initial cracking is initially of a shearing type which is then dominated by the effects of the loading system and the dilation of the material resulting in an apparent cleavage type of failure. This would explain both the irregular appearance of the mortar fracture surfaces and the shape of the stress envelopes.

It is therefore suggested that the Coulomb-Mohr theory with these three modifications is a suitable failure criterion for mortar. The comparison with cast iron is particularly apt as they are both cemented granular materials containing critical flaws.

Concrete

The behaviour of concrete both below the discontinuity point and at the ultimate load is far more complex as can be seen from the results reported. The most simple "model" of concrete that is likely to explain the main features of its behaviour is a two phase model in which the coarse aggregate particles form an included phase distributed in a mortar matrix. An additional necessary qualification is that partial discontinuity at the phase boundaries be taken into account (27).

Given this model of hard particles distributed at random in a soft matrix, a stress analysis, together with a knowledge of the aggregate-paste bond and matrix strengths, reveals the following order of structural failure:

1. Tensile bond failure
2. Shear bond failure
3. Shear and tensile matrix failure
4. Tensile (dilation) matrix (and occasional aggregate) failure

Tensile and shear bond failures are already under examination by a number of workers including Alexander, Wardlaw and Gilbert, and Shah and Slate at this Conference. The following hypothesis is proposed for the mechanism of crack propagation after completion of the bond failure process.

Mortar is retained in contact with the aggregate particle over the bond surface of the aggregate where both the aggregate texture and the shear strength of the mortar matrix are sufficient to prevent shear bond failure. This explains the existence of mortar deposits on the aggregate. Under compressive stress, the crack will propagate from the aggregate-paste interface at an angle to the principal compressive stress under the action of longitudinal shear and lateral tension as shown in Figure 12. This mechanism will form a cone shaped mortar deposit under uniaxial compression and a halo under biaxial compression.

The fracture process is completed when these cracks propagate more rapidly as the material dilation enables them to open under lateral tension alone and at this stage they will interconnect and also split some aggregate particles. The particle interaction will aid this process of cleavage splitting and accounts for the more pronounced cleavages observed in concretes compared with mortars.

Light-weight Concrete

The same two phase model described above may be used for light-weight concrete except that now soft particles are distributed throughout a hard matrix. As a result aggregate or matrix failure will precede any bond failure. Hence as long as the matrix remains intact, the material will behave like mortar and the aggregate particles have the function of forming voids in the mortar. The failure of the mortar will be unhindered by the aggregate particles and will proceed as if the material were a normal mortar. This explains why the light-weight aggregate concretes are weaker than their parent mortar yet exhibit similar stress envelopes and failure crack patterns.

POSSIBLE LINK BETWEEN CURRENT RESEARCH

Recent work in the field of fracture mechanics together with the results of this research suggest a possible link between the four of the main streams of research into concrete strength which are classified as follows:

1. Fracture mechanics
2. Shear and tensile bond strength
3. Multi-phase models
4. Phenomenological investigations

In a recent paper Erdogan (33) investigated crack propagation along the bond surface of two materials having different elastic moduli and Poisson's ratio, in which pre-existing cracks were included on the bond surface. Some attempt was made to include the effect of residual stresses and the Griffith-Irwin theories were shown to be applicable under a number of superimposed stress states including tension and shear. The link between this work and the investigation into shear and tensile bond strengths is self-evident.

The link between multi-phase models and phenomenological investigations has been attempted on a number of occasions. In this paper a simplified two phase model was used to explain the experimental results obtained and to itemise the crack mechanisms apparently causing ultimate failure. The crack mechanisms predicted by the use of models and phenomenological test results can be related to the theories of crack propagation. It would also be of interest to compare the angles of the mortar cones and halos described above with the angles predicted by the work on shear and tensile bond strength.

CONCLUSIONS

Elastic Behaviour

The expression elastic behaviour refers to that strain behaviour which is observed in the material before the discontinuity point is reached. Mortar and light-weight concrete behave effectively elastically whereas normal aggregate concrete, whilst exhibiting very small increases in Poisson's ratio with stress, does stiffen considerably with mean normal stress as shown in Figure 4.

Failure

Mortar and light-weight concrete stress envelopes and crack patterns can be predicted using a modified form of the Coulomb-Mohr internal friction theory.

Normal aggregate concretes are much more complex and only the mechanism of failure has been suggested in this paper.

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PLATE A.



PLATE B.



PLATE C.

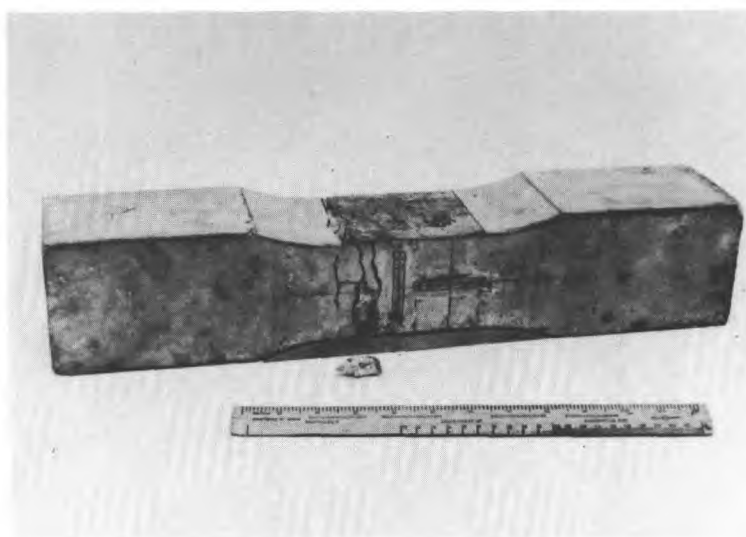


PLATE D.



PLATE E.



PLATE F.



PLATE G.



PLATE H.

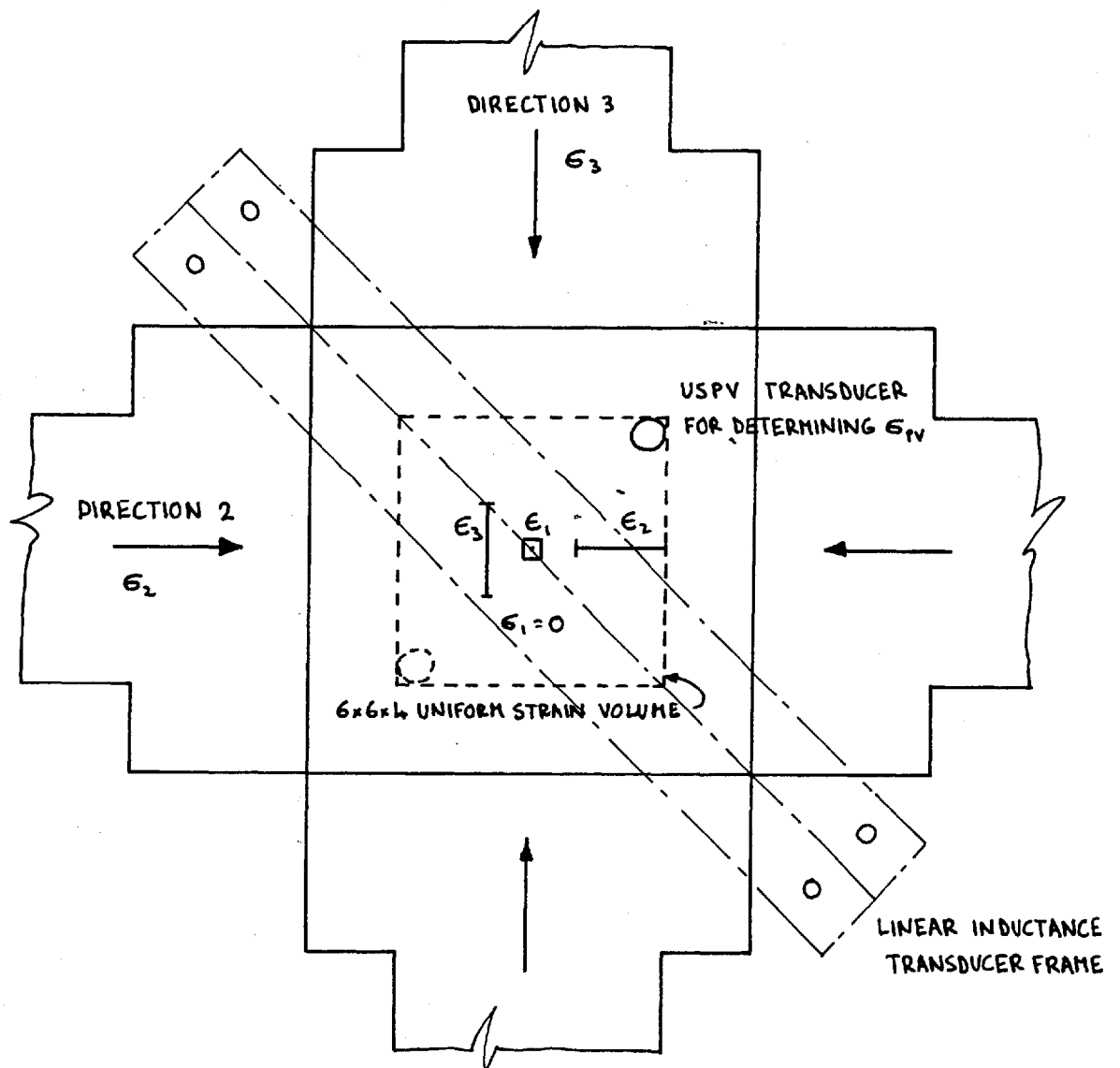


FIG. 1. GENERAL ARRANGEMENT OF MEASURING EQUIPMENT FOR BIAXIAL COMPRESSION TESTING.

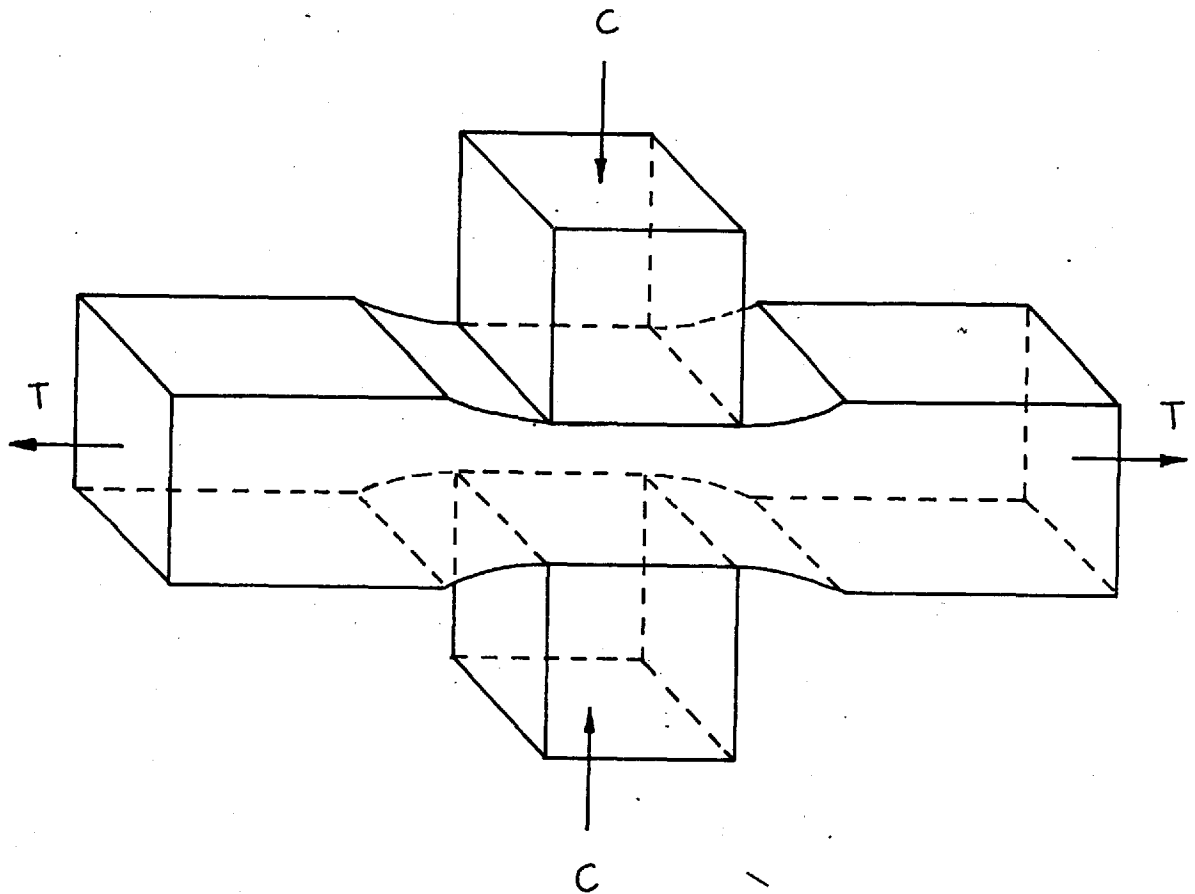


FIG. 2. USE OF CONCRETE CUBES TO APPLY COMPRESSIVE STRESSES TO TENSION SPECIMEN.

FIG. 3. VARIATION OF POISSON'S RATIO IN UNI- AND BIAXIAL COMPRESSION.

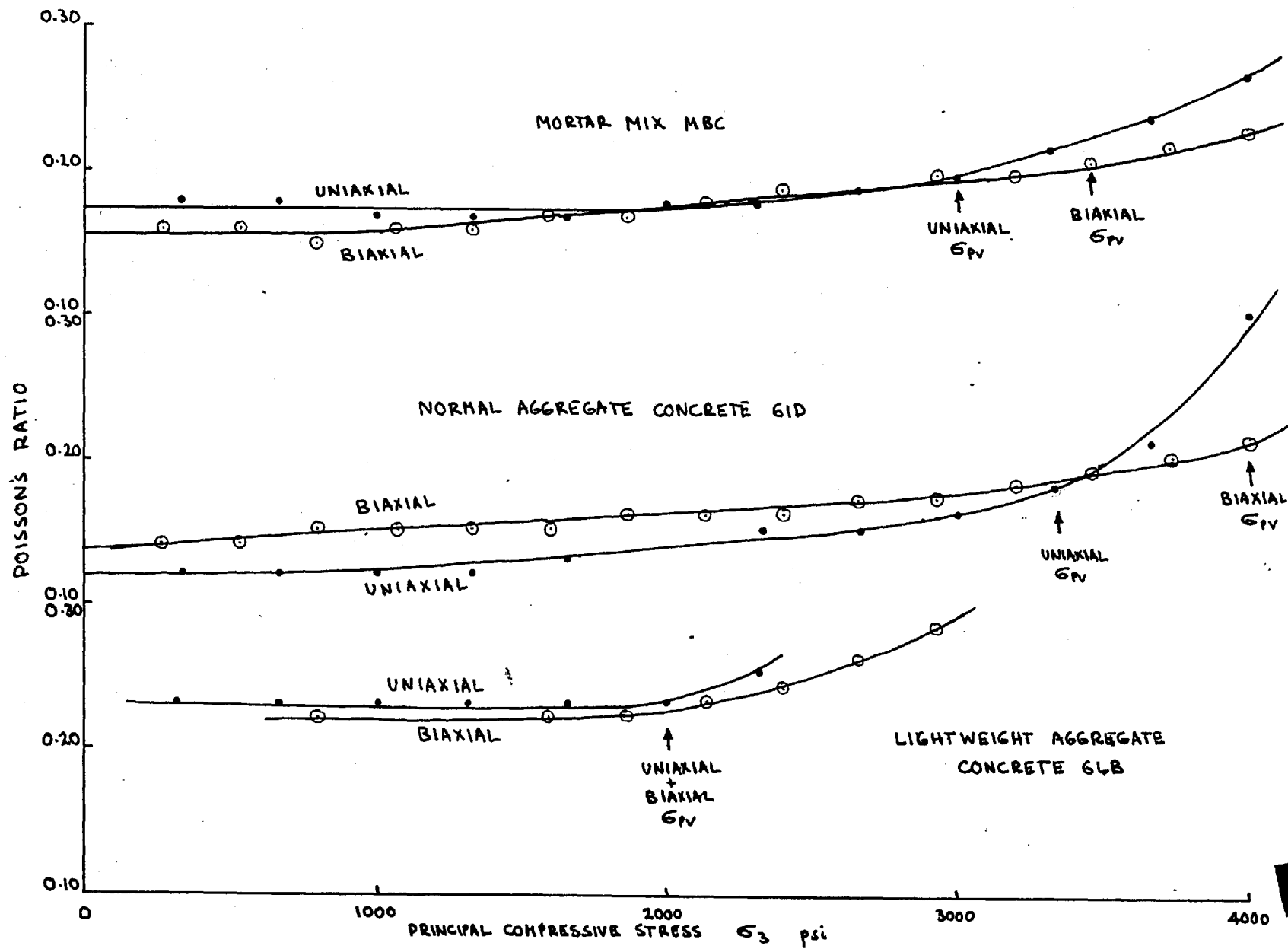
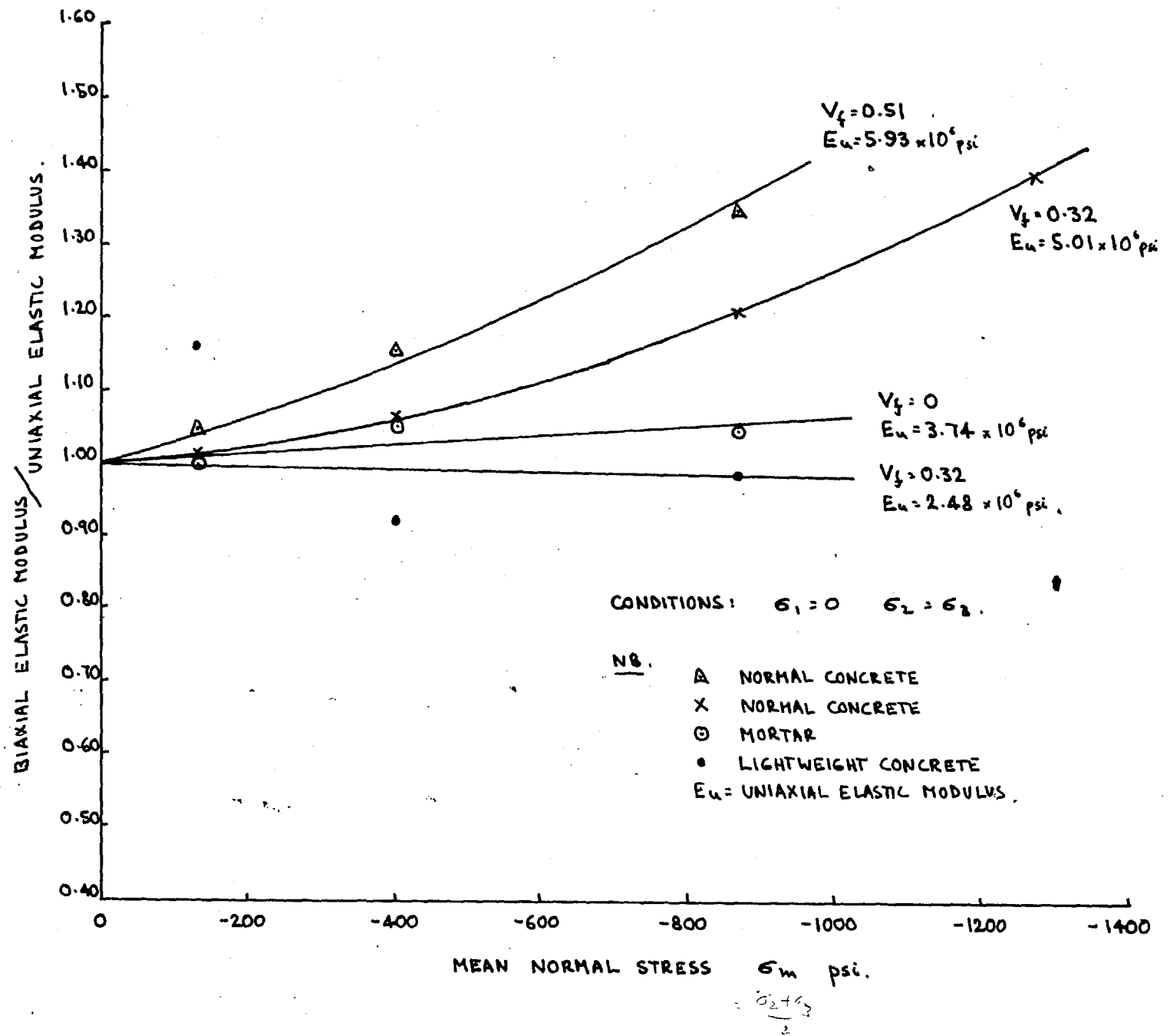


FIG. 4. VARIATION OF ELASTIC MODULUS WITH BIAxIAL STRESS.



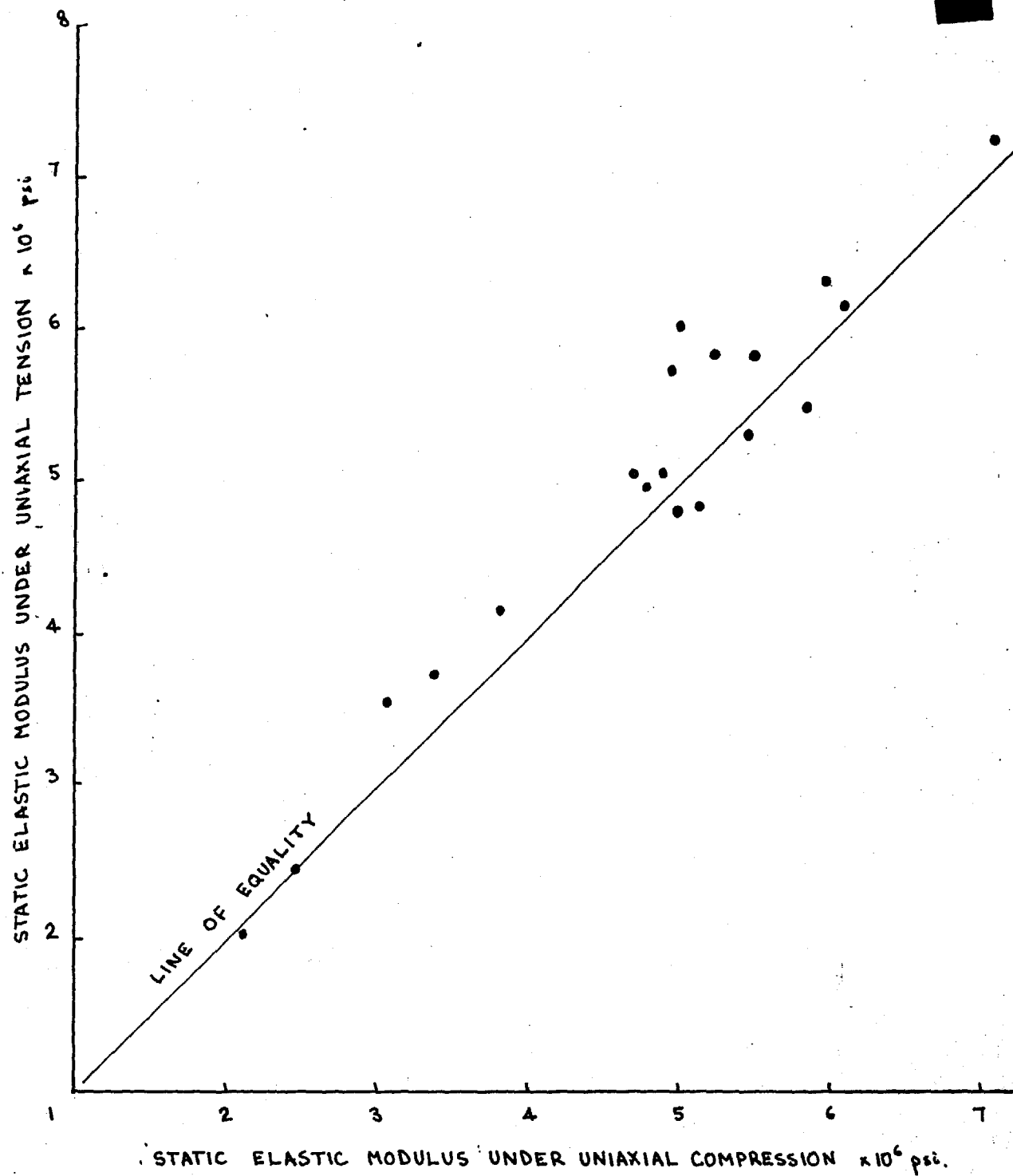


FIG. 5.12 STATIC ELASTIC MODULUS UNDER UNIAxIAL STRESS

MIX MA $\frac{s}{c}$ 2.4 $\frac{w}{c}$ 0.45

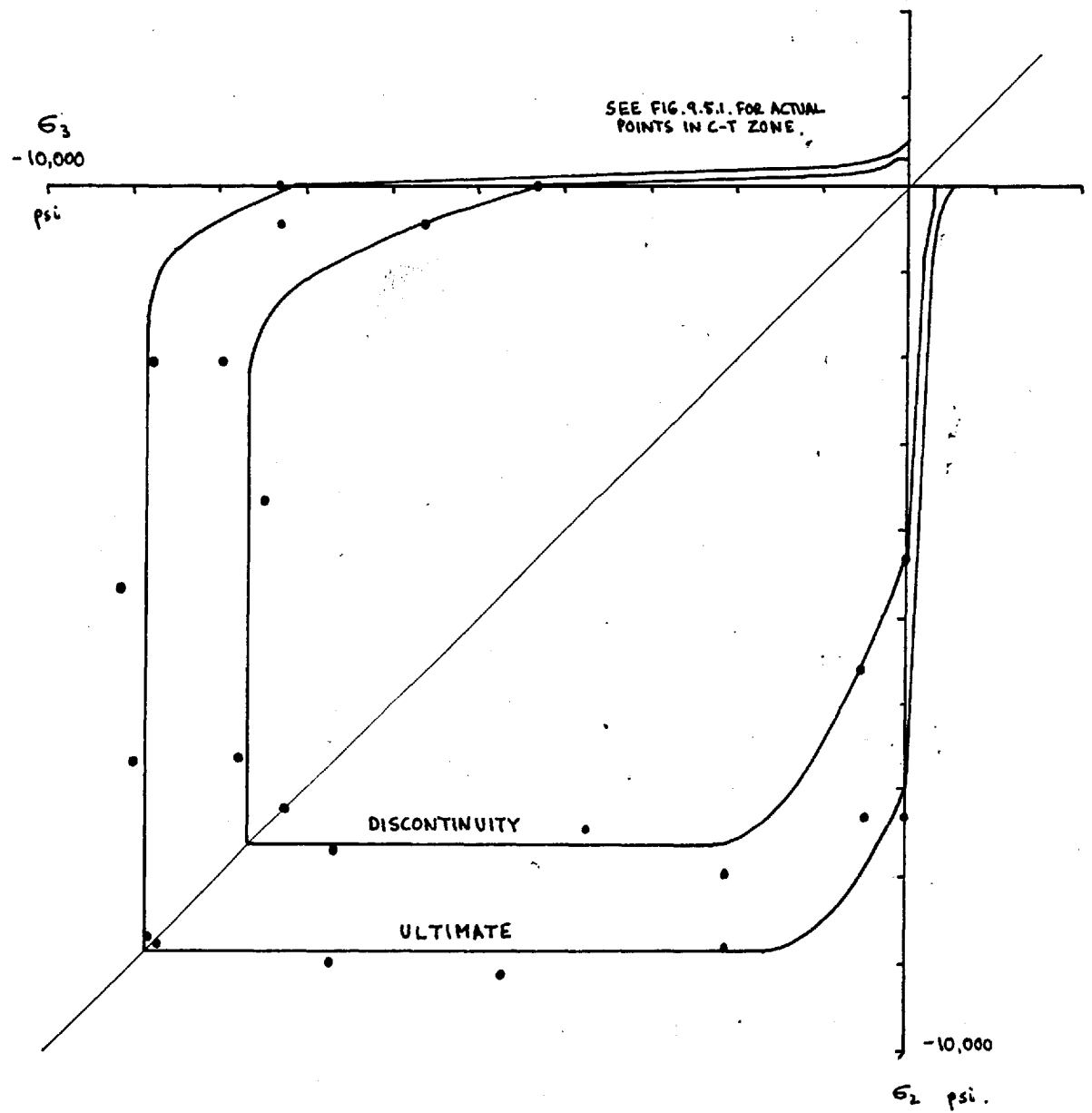


FIG. 9.5.6. BIAxIAL STRESS ENVELOPES FOR MIX MA, MORTAR.

MIX 6LB $\frac{N}{C}$ 4.5 $\frac{S}{C}$ 2.4 $\frac{W}{C}$ 0.55 $\frac{3}{4}$ " LIGHTWEIGHT COARSE AGGREGATE

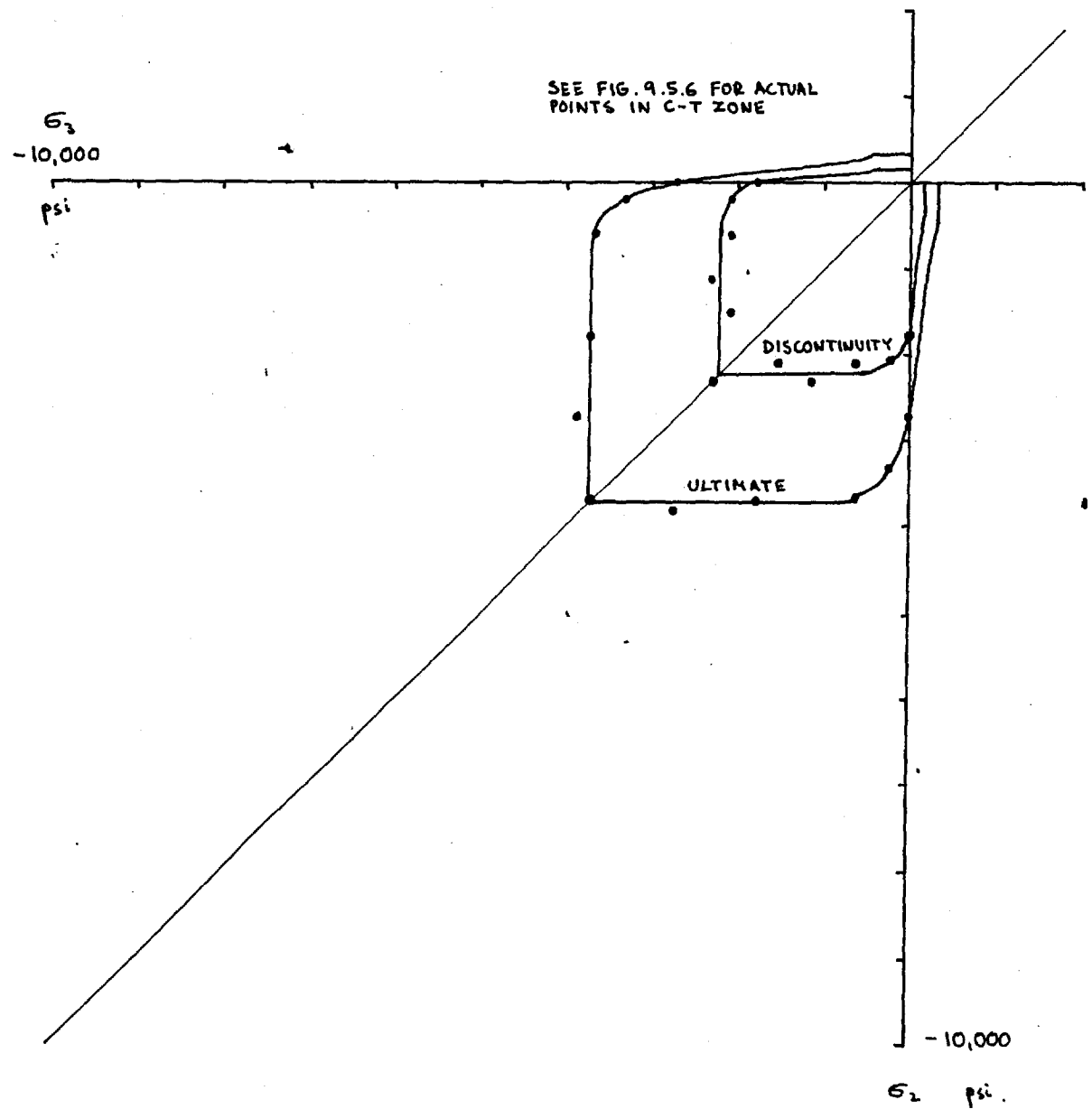


FIG. 7. BIAXIAL STRESS ENVELOPES FOR MIX 6LB.

MIX 6AB $\frac{1}{2}$ " 4.5 $\frac{3}{4}$ " 2.4 $\frac{1}{2}$ " 0.55 $\frac{3}{4}$ " ANGULAR COARSE AGGREGATE

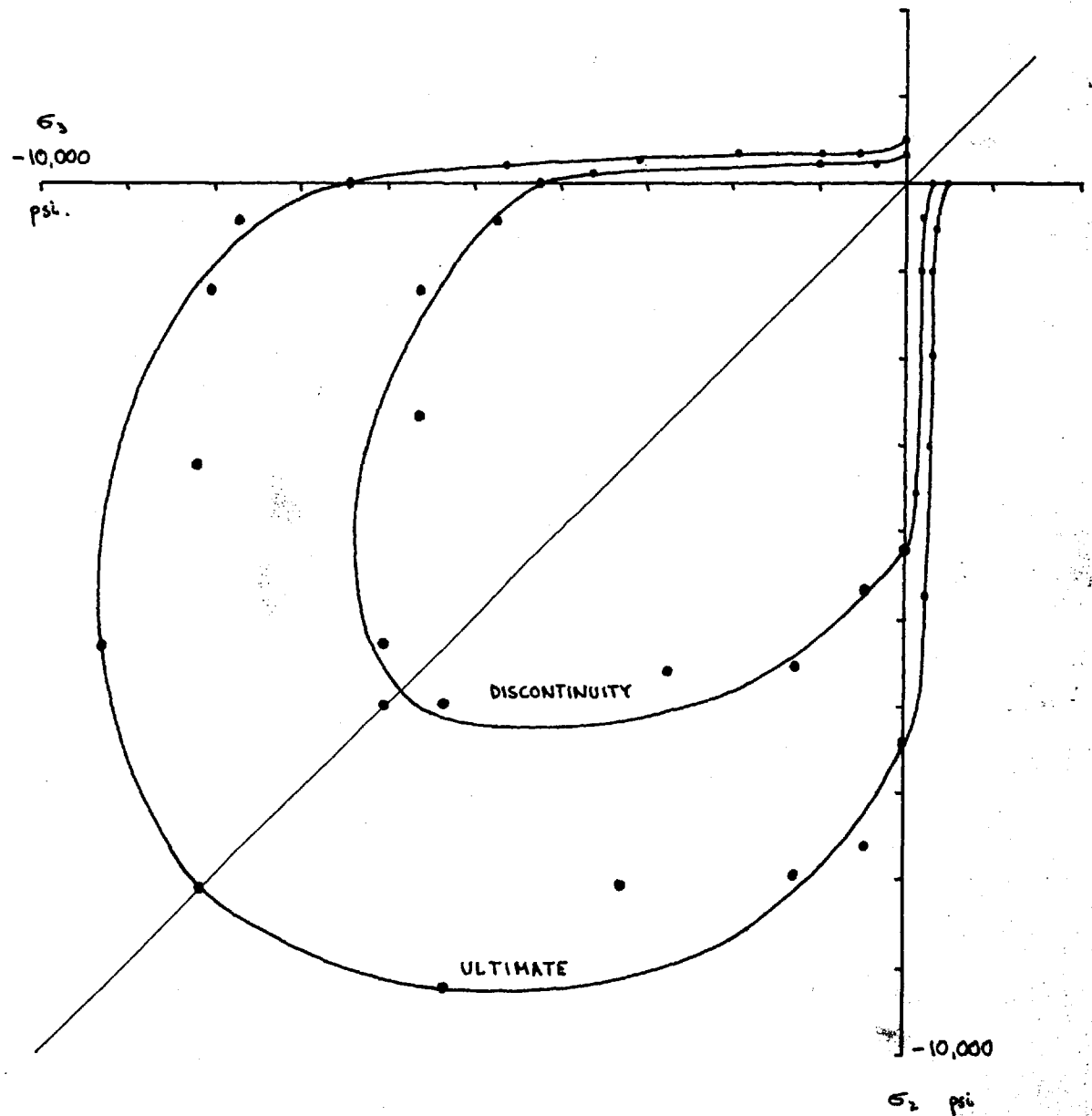
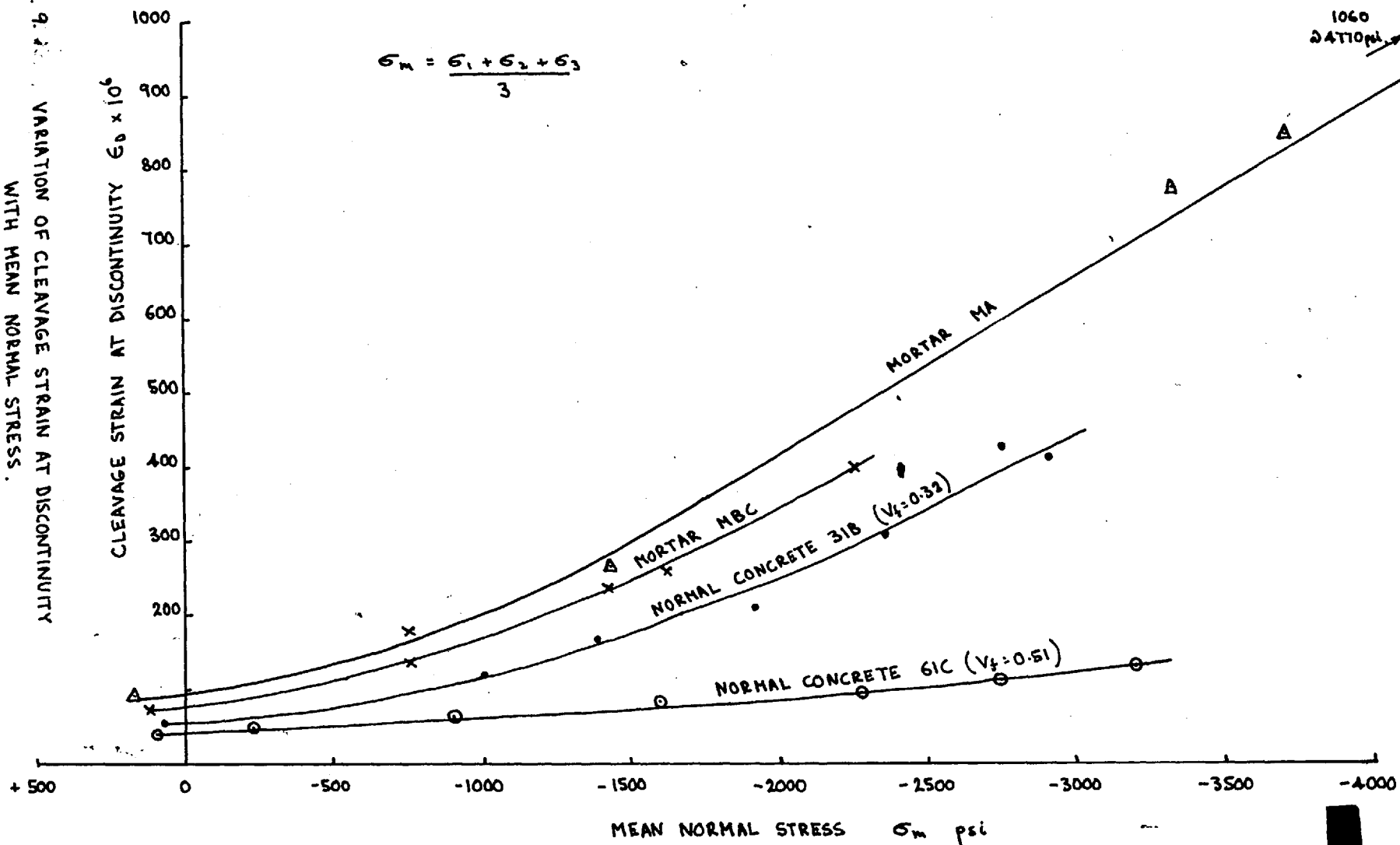


FIG. 2.20 BIAxIAL STRESS ENVELOPES FOR 6AB.

FIG. 9.23. VARIATION OF CLEAVAGE STRAIN AT DISCONTINUITY WITH MEAN NORMAL STRESS.



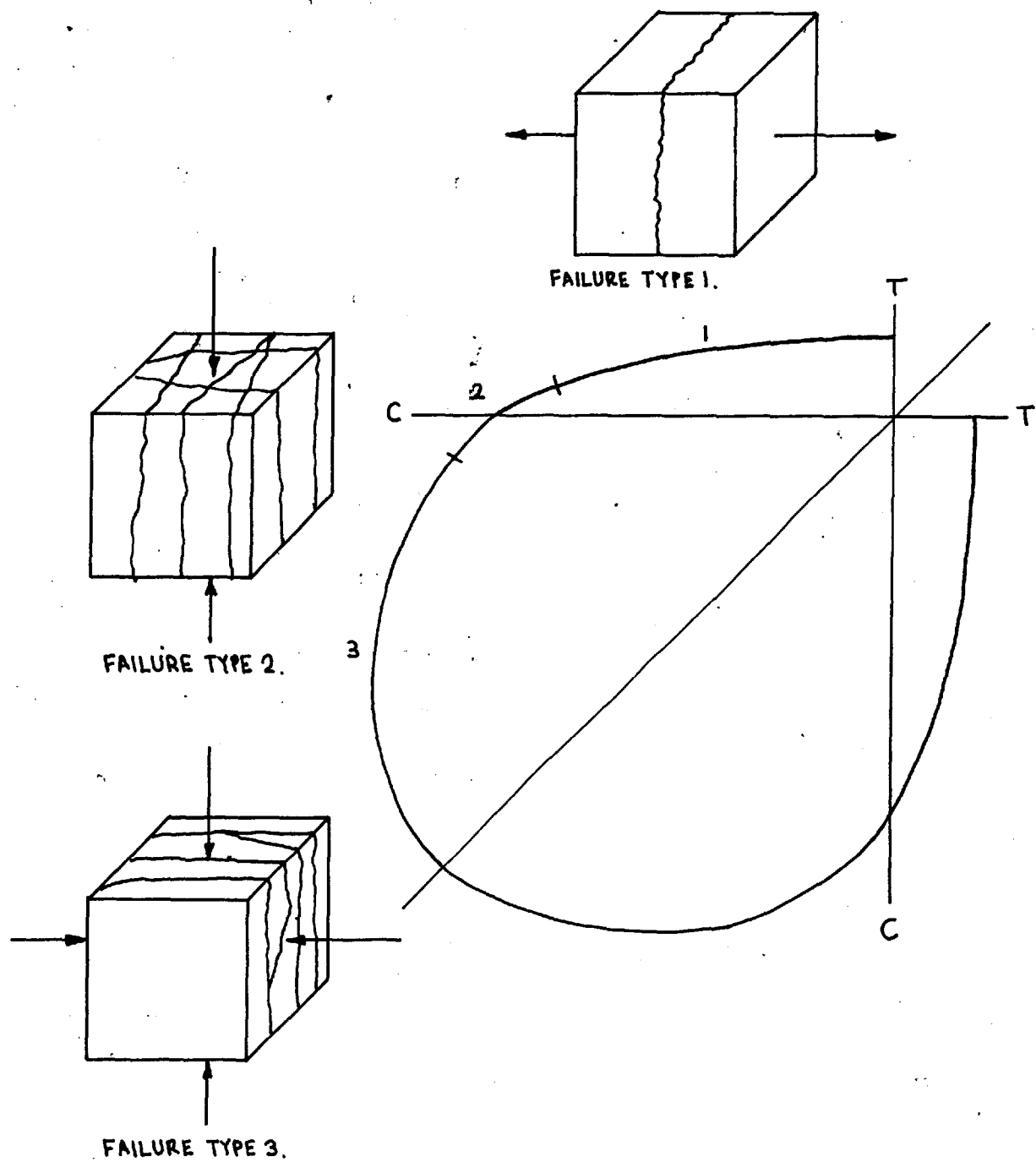


FIG. 10. IDEALISATION OF THREE MAIN CATEGORIES OF BIAXIAL STRESS FAILURE CRACK PATTERNS.

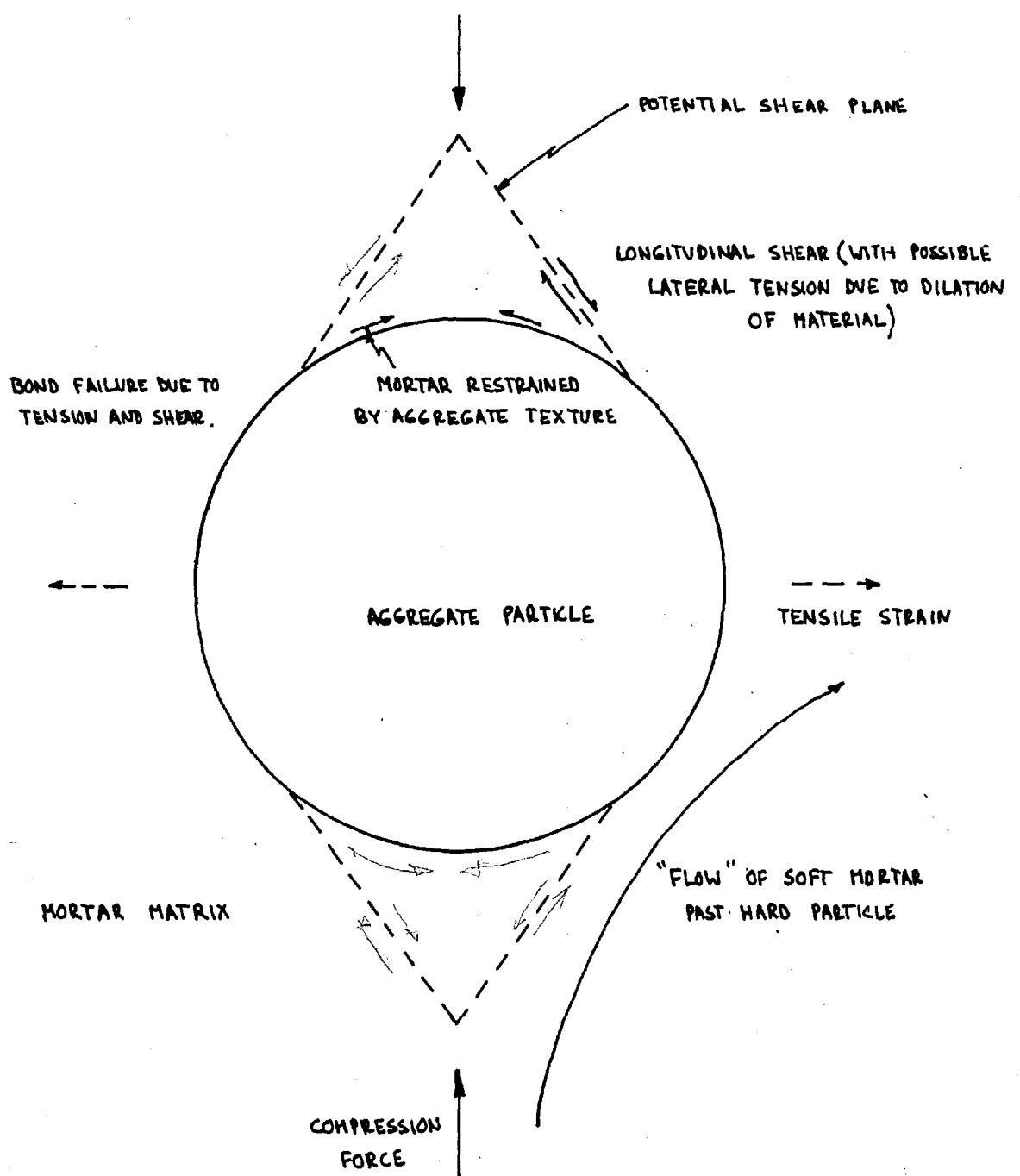


FIG. 11. IDEALISATION OF STRESSES AROUND A SINGLE AGGREGATE PARTICLE AT THE DISCONTINUITY POINT UNDER UNIAXIAL COMPRESSION.

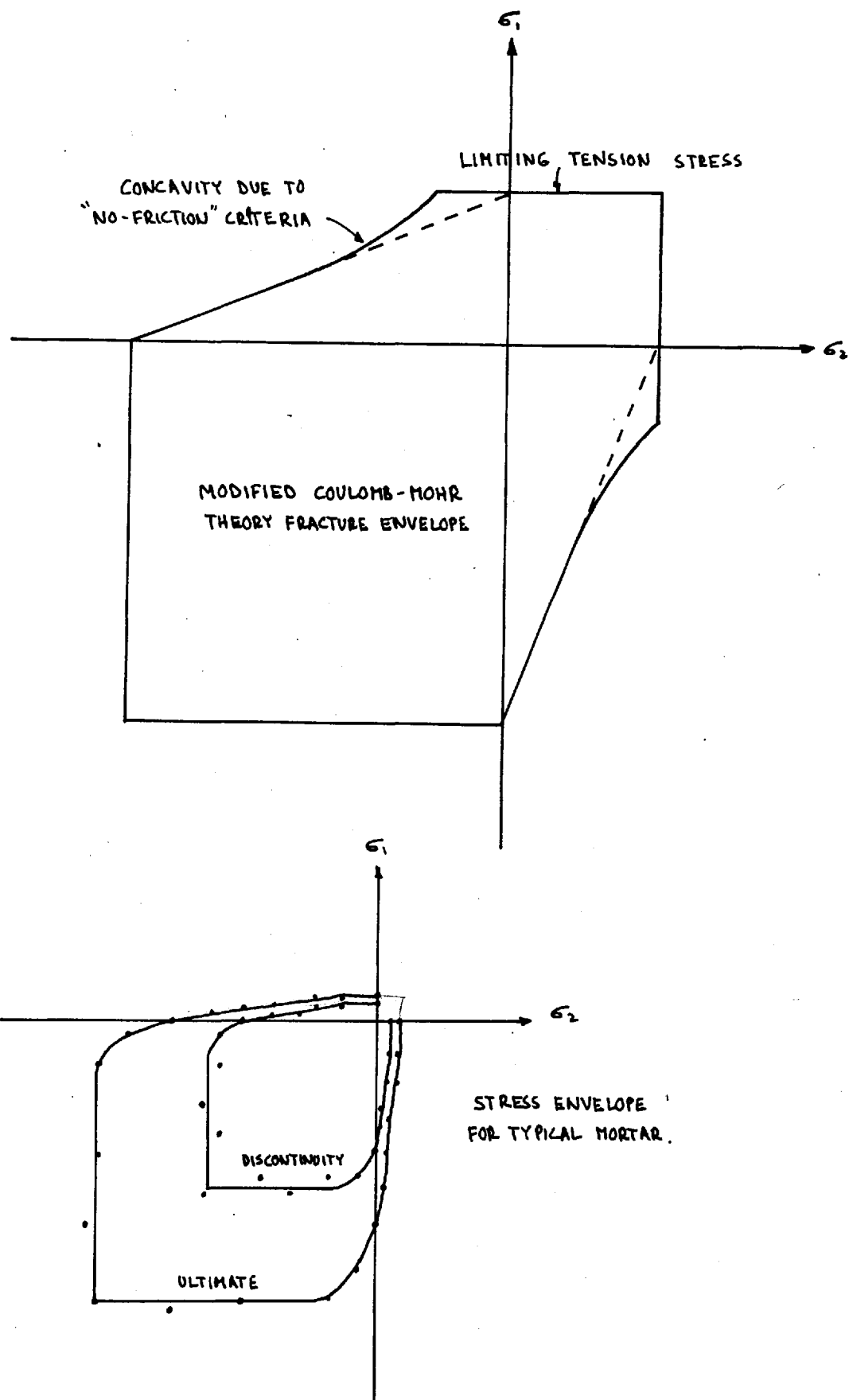


FIG. 12. COMPARISON OF STRESS ENVELOPES OF A TYPICAL MORTAR AND COULOMB-MOHR THEORY MODIFIED BY PAUL.

of being the result of specimen confinement or extension resulting from loading conditions or capping.

By G. W. D. VILE and O. T. SIGVALDASON*

The authors' work is similar to an investigation being carried out at Imperial College. There appear, however, to be substantial differences both in testing technique and in the results obtained.

The testing technique used in any experimental work is a measure of the confidence in the results obtained. Testing technique can be subdivided as follows: (1) testing machine design (2) test specimen (3) testing procedure.

Testing machine design

Testing machine characteristics have a significant influence on the strength and deformational behavior of concrete specimens.¹³ With a seating which tilts freely at only one end of the specimen, a lateral force will be induced from the bending of the specimen, thereby resulting in a large variation of stress. Hence, a machine should be such that both ends of the specimen be either effectively fixed (no tilting allowed) or effectively pinned (free to tilt). The former is recommended as the best definable method of test.

The testing machine designed and used at Imperial College is shown in Fig. D and is described in Reference 14. Essentially, it consists of two testing frames mounted at right angles with opposed pairs of 200-ton jacks and a support frame which maintains frame alignment on the center of the test specimens regardless of machine deflections.

Testing specimen selection

A suitable specimen has a representative volume of material subjected to a uniform and known state of stress.

In the uniaxial testing of cubes, the specimen is subjected to a complex stress system with a varying state of triaxial compression throughout most of its volume.¹⁵ By superposition, the degree of triaxial compression would be substantially increased under applied biaxial compression.

At Imperial College, a specimen 4 x 10 x 10 in. has been found most suitable specimen for concrete with up to ¾ in. aggregate. To satisfy requirements for packing between specimen and platen to eliminate platen effect,¹⁶ a curing compound with a thickness of about 0.002 in. was selected. It was applied and allowed to harden for a day, prior to testing.

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Proving trials on mortar specimens using surface strain, lateral strain, and photoelastic measurements showed conclusively that the central 6 x 6 x 4-in. volume was in a known and uniform state of biaxial compressive strain and stress.

Testing procedure

As ultimate strength is strongly dependent on the loading rate, it is important that the rate should be specified for any test. A more fundamental property of concrete, however, is the discontinuity level.¹⁷ For, as has been shown by Hsu, *et al.*,¹⁸ the development of continuous internal crack patterns after this load stage results in the stress-strain graphs becoming markedly discontinuous in a linear sense. This load stage can also be detected by changes in the ultrasonic pulse velocity.¹⁹ With accurate measurement of strain, it is possible not only to determine the discontinuity level, but also to obtain useful data on Poisson's ratio and modulus of elasticity under combined states of stress. Besides providing necessary data for the design engineer, the above information results in a more realistic approach to determining the correct failure criterion.

Comparison of test results

In the investigation carried out at Imperial College, 13 mixes (3 mortars and 10 concretes) were tested in biaxial compression.²⁰ The short term ultimate strengths (loaded to failure in about 10 min.) and discontinuity stresses of a concrete with $\frac{3}{4}$ in. crushed granite aggregate and a mortar with natural river sand are shown in Fig. E and F. For direct comparison, the writers have used ratios of uniaxial compres-

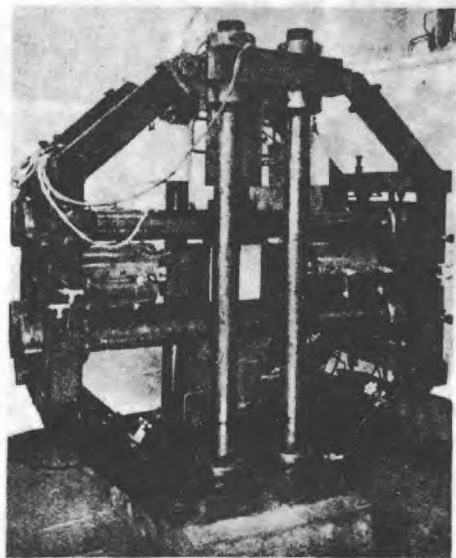


Fig. D—Biaxial compression testing machine at Imperial College

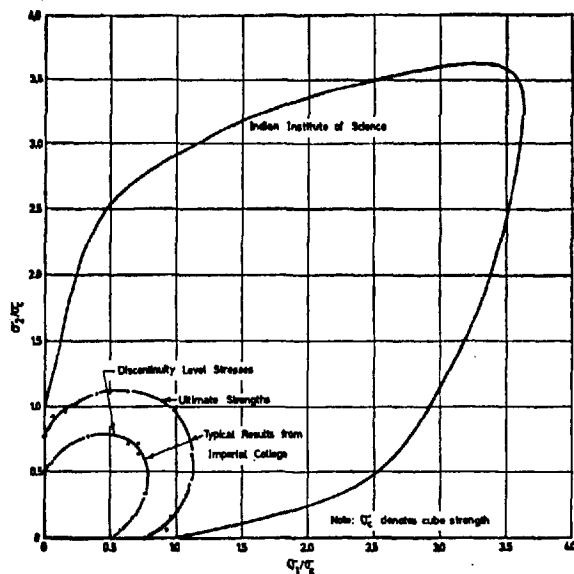


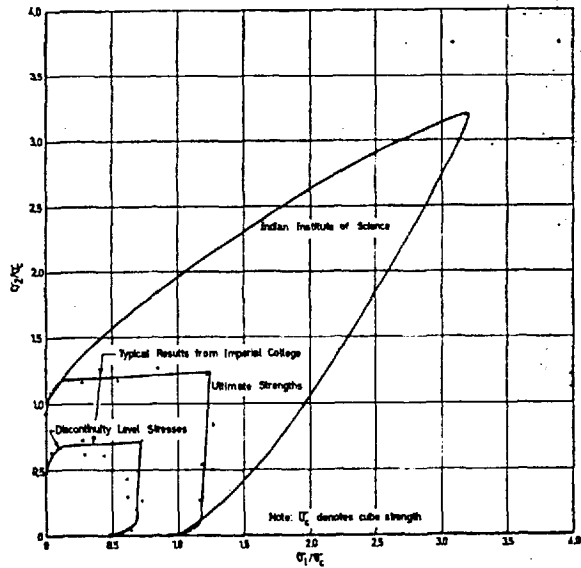
Fig. E — Comparison of biaxial compressive strengths for concrete

sive strength (4 x 4 x 12-in. prisms) to cube strength (4-in. cubes) of 0.77 and 0.93 for the concrete and mortar, respectively.

Although some differences would be expected in view of the different aggregate types used, these differences would only be minute in relation to the large differences observed between the authors' and writers' results. It is of interest to note that the results of Weigler and Becker¹² gave similar biaxial strength loops to those obtained at Imperial College. Although direct comparison has been made by the authors to the results of Bellamy's work, it should be realized that his hollow cylinders were actually in a state of triaxial compression reducing to biaxial compression at the inner surface only. In view of these comparisons, it is suggested that the high strength results obtained by the authors are due to a state of triaxial compressive stress which increases with the biaxial load.

In the tests at Imperial College, the discontinuity level was obtained with both strain gages and the measurement of ultrasonic pulse velocity in the third principal direction, i.e., perpendicular to the direction of the applied loadings. On the basis of both discontinuity level and short term ultimate strengths, it was observed that, under biaxial states of stress, greater strengths were obtained than in uniaxial states of stress. However, the influence of the lateral principal stress seldom produced a strength increase of more than 50 percent with concretes or 20 percent with mortars (see Fig. E and F).

Fig. F—Comparison of bi-axial compressive strengths for mortar



Other considerations

Although the authors report that the influence of capping and curing was investigated, no mention is made of the results obtained. Yet, they conclude that the type of curing and capping conditions have no appreciable effect on biaxial strength of concrete if the test conditions are kept on the same! Likewise, it would be of interest to know if the results of the concrete and mortar given in Fig. 2 are a representative mix or whether they are the average of all the mixes. The number of specimens tested under each combination of principal stresses should also be stated.

In Fig. 6, a smooth curve has been fitted to two different sets of data. However, over the range of 0.8 to 1.0 for the value of σ_2/σ_c , the curve does not satisfy the data. In fact, there is a lack of continuity in the principal stress plot between the biaxial compression results of the authors and the compression-tension results of McHenry and Karni.

The writers are particularly concerned about the possibility that the results presented will lead to dangerous design applications, especially where a bonus of strength is suggested.

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AUTHORS' CLOSURE

It appears that Mr. Gunasekaran has misunderstood the function of thrust bearing assembly provided in the set-up. This is provided only to control the lateral loading and not to minimize the surface frictional effects. Further, the loading arrangement used by the authors is of similar type as used by Messrs. Vile and Sigvaldason, who have used a pair of jacks instead of a single jack and thrust bearing assembly as used by the authors.

In the biaxial tests on cylinders as carried out by Professor Fumagalli, the failure of the specimen is primarily due to the extension of the specimen in the third direction since the state of stress in the cylinder is more towards plane strain condition rather than the plane stress condition.

The widely discussed point is on the testing technique adopted by the authors in their tests. It is true that the method of loading adopted in these tests develops a three-dimensional state of stress in the specimen due to surface friction. In fact it is well known that even in uniaxial testing of cubes such a state of stress prevails. It is only to study the effect of three dimensional stress condition which existed because of surface friction on the biaxial strength, that different capping materials, were used. The different types of capping materials used by the authors are such that the surface friction will either cause a confining or extensional stress in the specimen depending on the nature of the capping material used. Hence with different capping materials, the nature of the stresses in the specimen due to surface friction will be different. However, the biaxial strength obtained with different capping materials when nondimensionalized with respect to the uniaxial cube compressive strength, having the same capping material as the biaxial test specimen, was found to lie on the same curve given in Fig. 2 to 5, thus

indicating that the surface frictional effects have no effect on the biaxial strength of concrete. The experimental results for different capping materials were not given in the paper due to space restrictions.

To include the restraint or extensional effect in the expression for octahedral normal and shear stresses Professor Linger and Mr. Gillespie have introduced a constant K which in the authors' opinion is difficult to determine. In the authors' opinion the principal stress plot gives a better indication of the biaxial strength compared to an octahedral normal and shear stress plot.

The results of Messrs. Vile and Sigvaldason on the biaxial strength of concrete and mortar differ very much from those of the authors and they attribute this to the three-dimensional state of stress existing in tests conducted by the authors. The authors are not convinced by this argument in view of their results with different capping materials. They also refer to the comparison made between the results of Bellamy and authors in Fig. 2 and mention that in the hollow cylinder used by Bellamy a triaxial state of stress exists which reduces to biaxial compression at the inner surface. However it is apt to quote the following lines from Bellamy's paper: "in fact the first sign of failure in the hollow cylinder was the spalling of pieces of mortar from the inside face which indicates that failure started there" and this supports the comparison made by the authors. In the plate specimens used by Messrs. Vile and Sigvaldason the failure of the specimens may be due to buckling rather than due to combined compression. In fact the buckling effect may be more predominant especially at higher loads in which case the results cannot be directly compared with authors' results. This point in authors' opinion, needs a careful consideration before concluding that the ultimate strength does not increase much under biaxial stress condition.

Messrs. Vile and Sigvaldason refer to the lack of continuity in the principal stress plot between biaxial compression results of the authors and compression-tension results of McHenry and Karni. This is due to three dimensional nature of stress existing in the tests of McHenry and Karni. It should be noted that they have neglected the radial stress in biaxial analysis. More detailed information on the strength of concrete under combined tension-compression has been reported elsewhere.²¹

The points raised by Mr. Gunasekaran regarding the effects of sustained loading and residual stresses, etc., certainly need careful consideration. Since, even under the normal condition itself the information on biaxial strength of concrete was not fully available, the authors made detailed investigations on this aspect only.

By G. W. D. VILE*

The author, in a most informative and lucid paper, has given some useful limits to the anticipated elastic modulus of concretes made from known materials.

However, the writer would like to draw attention to the fact that the results used in the paper are from short-term, uniaxial compression tests on usually saturated specimens. It must be realized that the actual measured values of elastic modulus depend on other factors as discussed by Newman.¹⁸

Of these, the state of stress has received little attention. Although it has been shown that the elastic modulus for a given concrete is almost the same under uniaxial compressive or tensile loads,¹⁹ there is no reason to suppose that this will be so under all states of stress.

Recent work by the Concrete Materials Research Group at Imperial College on the biaxial compressive strength of concrete has shown that the elastic modulus of concretes, and to a lesser extent mortars, varies according to the degree of biaxial compression. Some representative results are shown in Fig. B from which the effect of both stress state and volume fraction of coarse aggregate can be seen. The results were

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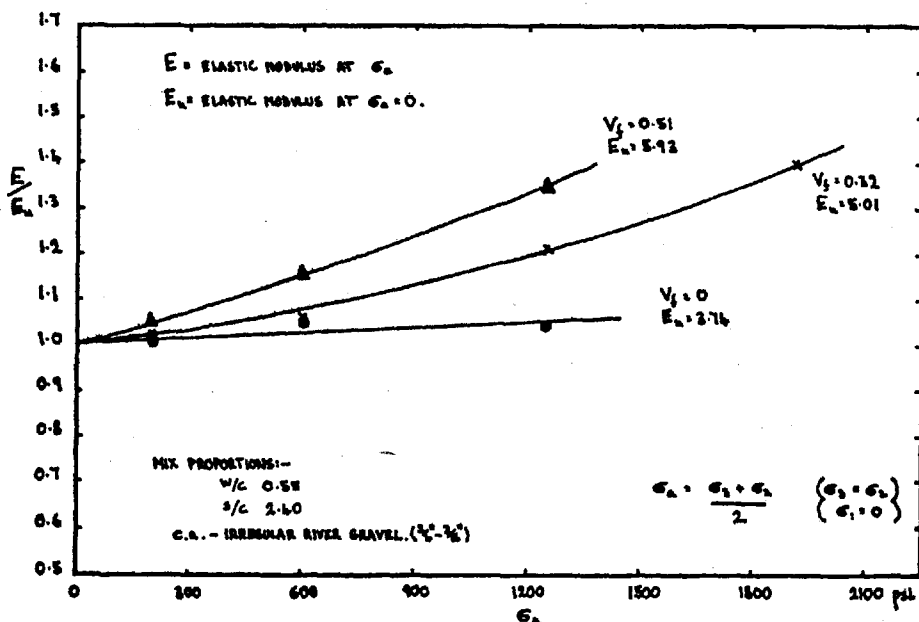


Fig. B—Variation of elastic modulus with biaxial compression

obtained from single specimens loaded within the elastic range, at load stages below which cracking would occur as defined by the ultrasonic pulse velocity measurements.

These results underline the necessity of closely defining how values of the elastic modulus are obtained.

There appear to be at least two possible mechanisms at work under the action of the lateral stress. The discontinuities along the phase boundaries are being partly closed thus increasing the continuity of the material and there seems to be an increase in particle interaction.

Though this phenomenon by no means invalidates the author's recommendations, it is a factor which should be borne in mind when selecting suitable upper and lower bounds during design studies.

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AUTHOR'S CLOSURE

The author would like to thank Messrs. Bennett and Vile for their interesting contributions to the discussion.

Mr. Bennett found that Eq. (14) is in better agreement with his experimental data than Eq. (7). This can be due to the fact that Mr. Bennett obtained his data on concretes made with crushed aggregate. There is a tendency indicated in the data presented in Fig. 9 that concretes made with crushed aggregate have somewhat higher modulus of elasticity than predicted by Eq. (7). This is probably due to the fact that the bond between crushed aggregate and cement mortar is better than the bond between gravel and cement mortar.

Mr. Bennett suggests that equal stress in particles and matrix can be obtained only when the aggregate particles creep extensively. However, the condition of equal stress may also be approached when there is no bond between cement mortar and aggregate particles or when the bond is very weak.

Mr. Vile is to be congratulated for the interesting data which he has obtained on the modulus of elasticity of concrete under biaxial compression. Perhaps it should have been emphasized in the paper that all formulas are suggested for the dynamic modulus of elasticity of concrete, mortar, and paste subjected to no external load, which is roughly the same as the static modulus of elasticity at low, uniaxial, compressive stress.

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Combined Stress Testing Machine for Concrete

by G. W. D. VILE*

The Concrete Materials Research Group at Imperial College, London, has been carrying out a programme of research work to establish the criterion of failure for concrete. In 1963 it was decided to extend this work into combined states of stress and a research contract was subsequently awarded by the Civil Engineering Research Association. This article describes the testing machine designed and built to fulfil the requirements of this research project.

THE basic requirement was to accommodate as many combinations of stress types as was practical within the one machine whilst at the same time ensuring that the desired states of stress were actually achieved. In this field of research a number of combined stress testing machines have been developed in the past but on the whole they have only been capable of applying one or two combinations of stress. Hence, whilst advantage was taken of the experience of previous researchers, it was necessary nevertheless to adopt a completely new design approach.

A picture of the final design of the machine as set up for biaxial compression testing is shown in Fig. 1. Essentially it consists of two four-column testing frames supported at right angles and containing opposed pairs of 200-ton hydraulic compression jacks. The vertical frame is fixed to the machine plinth but the horizontal frame is supported on

rollers and is free to move horizontally. The horizontal compression jacks can be replaced by 30-ton hollow ram tension jacks to provide the tensile stress facility. All the hydraulic jacks are by Simplex and fitted with special "Teflon" (p.t.f.e.) seals in place of the standard rubber seals to reduce friction problems in the horizontal operation of the jacks.

Because of the importance of the coarse aggregate size in determining the properties of concrete, one of the main research requirements was that the machine should be capable of testing concretes of up to $\frac{3}{4}$ in aggregate size. This, combined with the need to achieve a pure state of stress, resulted in a biaxial compression specimen size of 10in by 10in by 4in and fixed the capacity of the machine at a minimum of 200 tons for this facility^(1, 2, 3). As concrete is much stronger in compression than in tension, the other biaxial stress combinations did not call for machine capacities greater than this 200-ton

figure. Much preliminary work was done on the effects of loading platten restraint on the concrete specimen^(2, 3) from which it had been established that a substantial volume (6in by 6in by 4in) in the case of the biaxial compression specimen) was always in a uniform state of stress. Final trials on the test specimens to establish the zones of uniform stress state included a surface strain survey by both photo-elastic and electrical resistance strain gauge techniques and measurement of the lateral strain distribution.

Special features of the machine design include the self-alignment mechanism of the

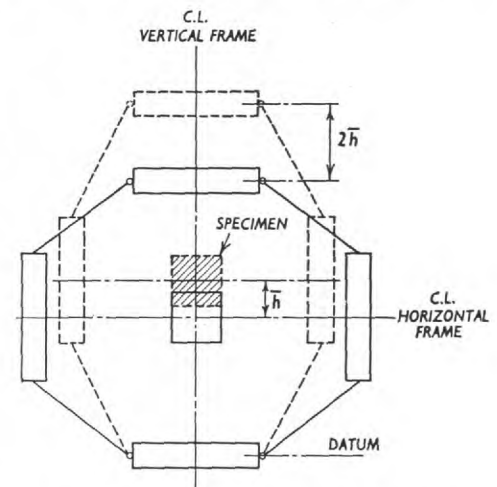


Fig. 2—Self-alignment of machine

machine illustrated in Fig. 2. If, through compressing the test specimen, the vertical frame extended a distance of $2h$ relative to the baseplate, the centreline of the test specimen would move away from the baseplate a distance h . Now the deformation of the two side portals would be $2h$ between their pinned ends hence lifting the horizontal frame a distance h relative to the baseplate and thus maintaining its alignment on the test specimen. Pairs of levelling screws were provided on all four sides of the horizontal frame end plates to facilitate final machine alignment after construction. Another feature, which is unusual in testing machines, was the manufacture of the spherical seating surfaces such that they allowed free movement during initial alignment on the specimen

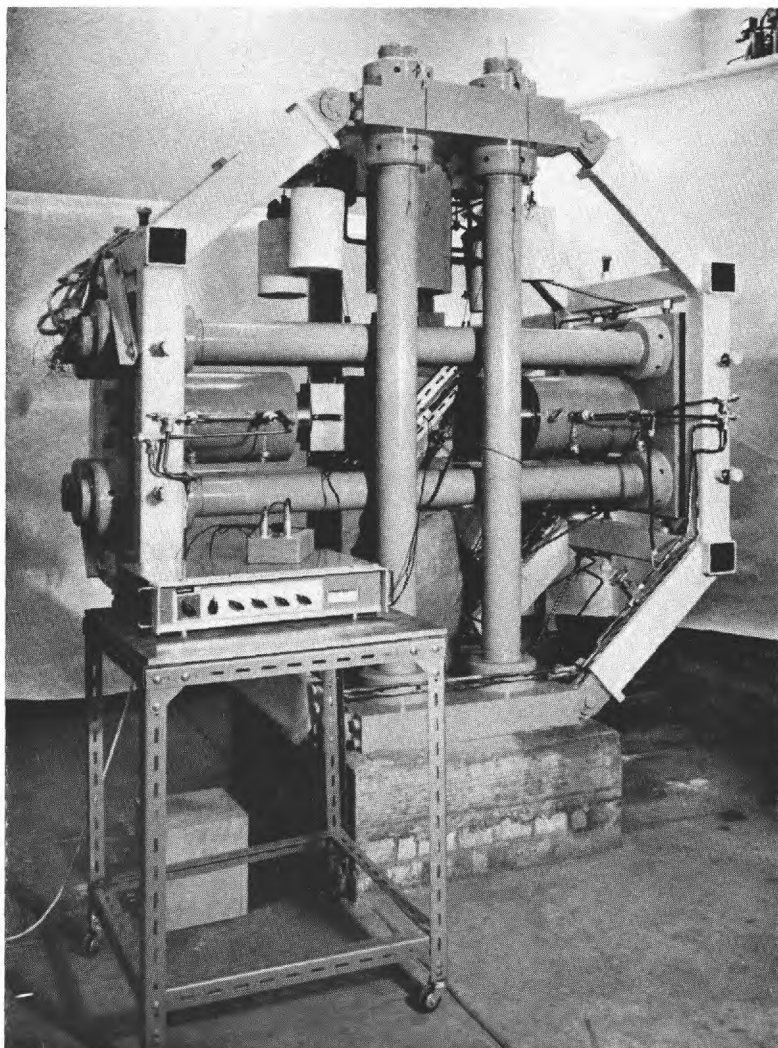
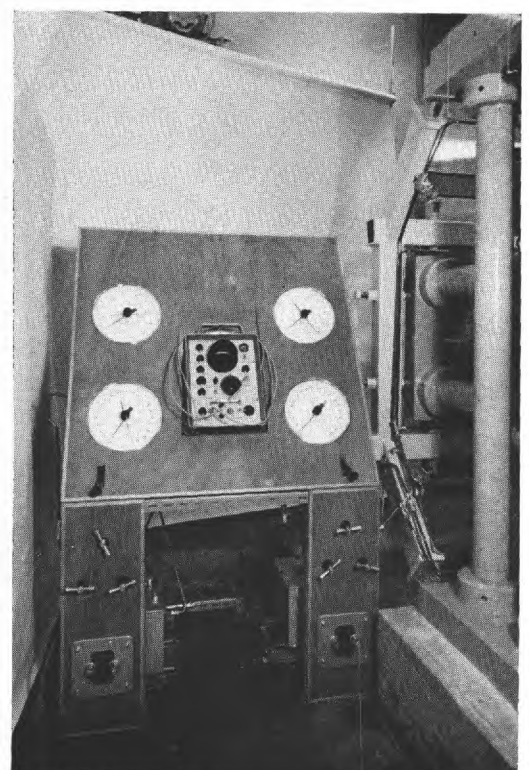


Fig. 1—(Left) Machine set up for bi-axial compression testing

Fig. 3—(Right) Control panel adjacent to the testing machine



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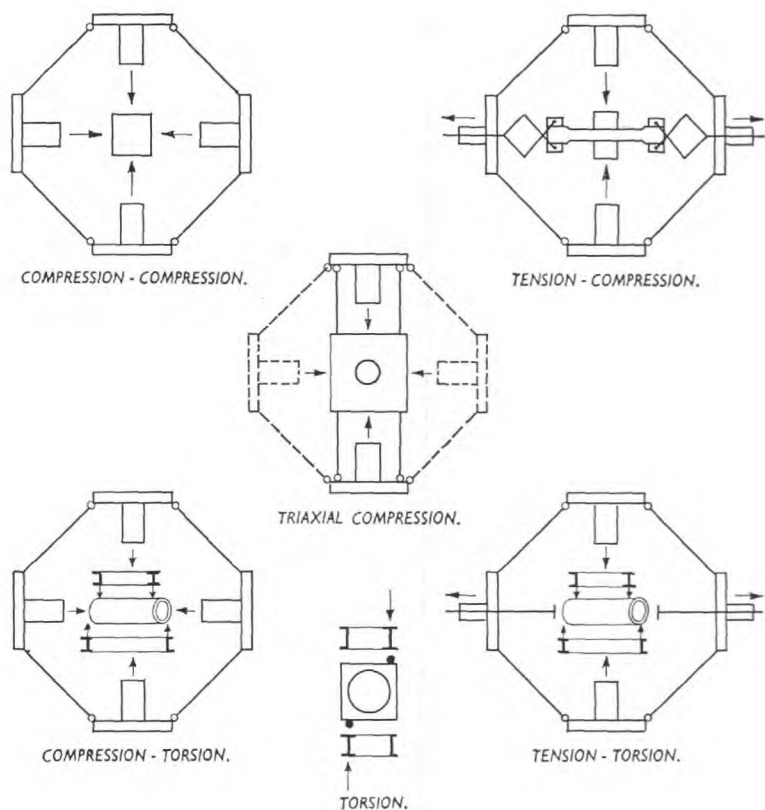


Fig. 4—Possible combinations of stress systems

but became effectively locked as soon as any load was applied. This characteristic overcame the problems associated with the testing of specimens of non-uniform stiffness such as segregated concretes.

Recommendations made by Sigvaldason⁽⁴⁾ regarding the stiffness of testing machines were adhered to in the design. The longitudinal stiffness of the two frames was designed to be greater than that of a 10in by 10in by 4in compression specimen with an elastic modulus of 8×10^6 lb/in². Lateral stiffness of the frames was increased by the use of stepped nuts in the end plates, providing a semi-rigid column to plate connection which was in any case in excess of Sigvaldason's recommendation of 10×10^4 lb/in². However, a considerable amount of energy is stored in the present hydraulic system which tends to reduce the effective longitudinal stiffness of the frames.

The hydraulic system was designed to allow completely independent operation of the vertical and horizontal pairs of jacks such that an infinite number of load com-

binations could be achieved. This was made possible by equipping each frame with its own pump and motor, control valves and pressure gauges whilst housing all this equipment in the one control panel (see Fig. 3). As the 200-ton jacks operated up to oil pressures of 7 500 lb/in² and the 30-ton jacks up to 9 200 lb/in², the whole hydraulic system of pumps, pipes and couplings, and valves were selected to withstand a safe working pressure of 10 000 lb/in².

The testing machine in its present form applies load to the test specimens at controlled rates of stress. However, it is hoped to develop a conversion to controlled rates of strain at a later date in order to obtain definable results on the post-ultimate load portion of the stress-strain curve.

Versatility in the combinations of stress systems available in the machine was one of the main considerations throughout the design. The degree of success achieved is illustrated in Fig. 4, where five distinct combinations of stress systems are shown diagrammatically and several others are possible. Already three combinations, namely biaxial compression, compression-tension, and compression-torsion have been fully developed and proved in the machine. Extension to the triaxial combinations of stress at a later date should involve only a relatively simple modification.

Typical of the results obtained from this machine are the ultimate failure and discontinuity stress envelopes shown in Fig. 5 which were obtained using the two facilities of biaxial compression and compression-tension. The final results of this research are to be published in detail elsewhere⁽⁵⁾.

ACKNOWLEDGMENTS

The author wishes to acknowledge his debt to Professor A. L. L. Baker, Mr. K. Newman, and his colleagues at Imperial College and to the Civil Engineering Research Association for supporting this project.

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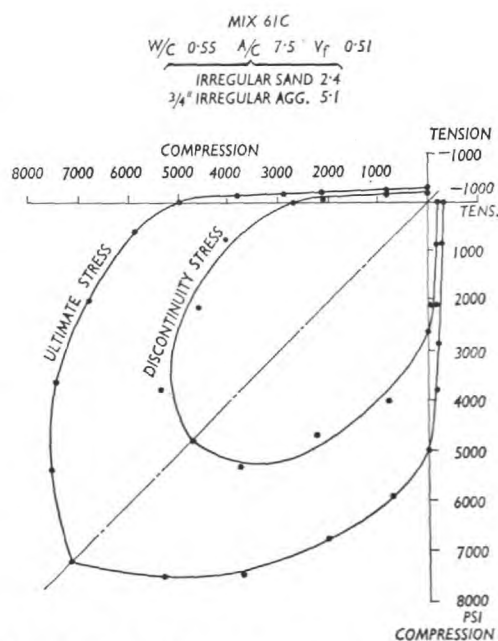


Fig. 5—Typical biaxial strength of concrete

