

AN INVESTIGATION OF THE SEPARATED
FLOW INDUCED BY A SHARP BEND IN A
PIPE.

by

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SUMMARY

An experimental investigation into the flow field associated with a fully developed turbulent flow passing through a 90° mitred pipe bend is described. The results of flow visualisation in both air and water and other techniques are presented, demonstrating that the downstream secondary flow does not conform to the anticipated twin-circulatory type, but consists of a single unsteady circulation. The circulation can assume either a clockwise or an anticlockwise sense and switches between these at a low, apparently random frequency. The resultant asymmetry and unsteadiness affects the majority of the flow field and severely complicates it. A suggested reason for this behaviour is given, based on the structure of turbulent pipe flow.

Details of the strongly peaked spectra and high intensities of the fluctuating wall pressure field generated by the separation are presented.

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1. INTRODUCTION

1.1. Secondary Flows in Pipe Bends

Bends are essential features of piping and ducting installations, and a knowledge of their flow characteristics is therefore necessary to ensure their efficient use. Nineteenth century hydraulicians observed that the friction factor for a curved pipe exceeded that for a straight pipe, even for a small curvature. No satisfactory explanation of this phenomenon was put forward until 1876 when Thompson (reference 1) gave a theoretical exposition of the meandering of rivers and related his result to curved pipe flows. He argued that a pressure gradient must be set up across the bend to balance the centrifugal force exerted on the fluid travelling in a curved path in order to establish radial equilibrium, the higher pressure being at the outside wall of the bend. Considering the plane of the bend to be horizontal, the fluid near the top and bottom walls of the pipe is moving more slowly than that near the central plane and therefore requires a smaller pressure gradient to balance its smaller centrifugal force. Thus, under the influence of the excess centripetal pressure gradient imposed upon it by the remainder of the flow, this slower fluid near the top and bottom moves inwards towards the centre of curvature, and continuity is satisfied by an outward crossflow in the central portion. Thus two longitudinal vortices, symmetrically disposed about the plane of the bend, as shown in figure 1.1, are set up and are termed 'the secondary flow'; it will be noticed that the secondary flow has four stagnation points. Thomson revealed the presence of the secondary flow in a channel model by means of threads. This explanation was based on a laminar flow model, but clearly the argument applies to the mean motion in a turbulent flow, the turbulence modifying the velocity profiles.

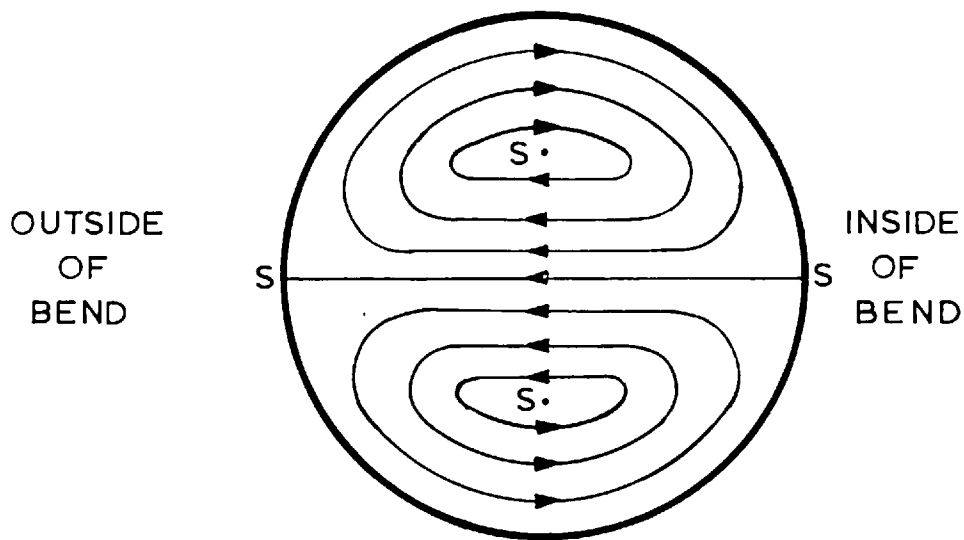


FIGURE 1.1. LAMINAR, SECONDARY FLOW STREAMLINES IN THE CROSS SECTION OF A PIPE BEND.

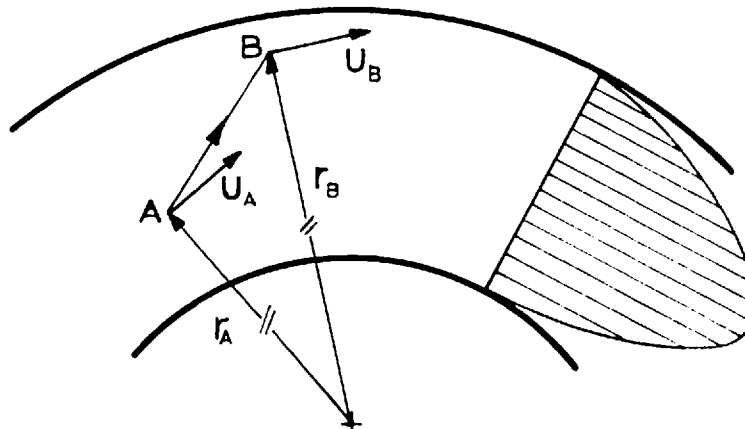


FIGURE 1.2. RADIAL PERTURBATION OF A FLUID ELEMENT IN A TWO-DIMENSIONAL, CURVED, LAMINAR CHANNEL FLOW.

Although Thomson is generally credited with the first explanation of the production of this secondary flow, in fact Boussinesq (reference 2) proved its existence theoretically in 1868, but his work appears to have been almost completely overlooked in the subsequent study of this topic. In a paper devoted to approximate solutions of the Navier-Stokes equations for various boundary conditions, he considered fully developed ($\partial/\partial s = 0$, where s is the axial co-ordinate) laminar flow through a curved rectangular section closed channel and solved these equations for the velocity distributions to the first power of the curvature of the channel axis; his solution is limited to the region away from the vertical walls. On considering the motion predicted by these results, he demonstrated that it consisted of a pair of symmetrical vortices.

In a two-dimensional, curved channel flow, such a secondary flow is not set up since there is no boundary layer at the top and bottom of the curve. The curvature produces two main effects. The first of these is best considered by referring to a potential flow entering a curved channel. The resultant velocity distribution will be governed by the equation for the angular momentum: $U r = \text{constant}$. Thus the maximum velocity will occur at the inner wall and the minimum velocity at the outer wall. Returning now to a real fluid, the laminar fully developed profile in a straight two-dimensional channel is parabolic and hence bears no resemblance to the potential profile. On entering a curved channel the laminar profile is affected very little unless the curvature is high, in which case the maximum velocity point is disturbed towards the inner wall. The effect of increasing curvature on a laminar profile is shown in Goldstein (reference 3).

However, the corresponding turbulent profile in a straight duct is a much better approximation to that of potential flow, and the effect of curvature is to produce a large

portion in the centre of the profile which corresponds to $U_r = \text{constant}$ (reference 4), the turbulence assisting in the uniform distribution of angular momentum.

At this point it is worth noting that an alternative and, for some flows, a more illuminating argument for the production of secondary flows is to consider the motion of the vortex lines. For the three-dimensional pipe bend, the vortex rings approaching the bend are deformed by the axial velocity profile $U_r = \text{constant}$ which occurs in the initial stages of the curved flow producing axial vorticity components. If the bend continues, however, the secondary flow modifies this axial profile and the maximum velocity moves to the outer wall.

The second effect of two-dimensional curvature is to produce Rayleigh or centrifugal instability (reference 5). Consider the flow through such a channel shown in figure 1.2. An element of fluid at A, radius r_A , on being displaced by a radial perturbation will carry with it its angular momentum $U_A r_A$ (neglecting viscous loss). The centrifugal force acting on it at its new position B, radius r_B , will be:

$$f_c = -U^2/r_B = \rho/r_B \left(\frac{U_A r_A}{r_B} \right)^2 = \rho U_A^2 r_A^2 / r_B^3.$$

The centripetal pressure gradient acting on the element at B, however, will be determined by the basic flow, that is:

$$\frac{dp}{dr} = -\rho \frac{U_B^2}{r_B}$$

$$\text{Thus } f_c / -dp/dr = (U_A r_A / U_B r_B)^2$$

In a potential flow therefore, $f_c = -dp/dr$ and neutral radial equilibrium persists throughout the flow. If, however, $U_B r_B < U_A r_A$, which is clearly the case for a boundary layer or a fully developed flow near the outer channel wall, either positive or negative radial perturbations are unstable. However, at the inner wall, $U_B r_B > U_A r_A$ and the flow is radially stable. Thus, if in figure 1.2 the inner boundary were now made into a rotating circular cylinder, the resultant flow would be radially unstable everywhere. Bakinazi and Kehn (reference 4)

verified that stability and instability existed at curved duct walls in turbulent flow by examining the radial turbulence components: these were found to be amplified at the outer and suppressed at the inner wall.

The non-existence of a secondary flow in the two-dimensional flow suggests a means for suppressing its undesired generation in the three-dimensional case. Thus, by splitting the bend into a series of narrow passages by means of curved vanes, two-dimensional flow is approached in each passage and although a secondary circulation is not eliminated, it is confined to the upper and lower passage walls and reduced in magnitude. This well known technique is employed in wind tunnel design (reference 6). For a sufficiently high curvature, however, the high pressure region produced at the outer wall can cause the separation of the outer boundary layer, whereas the presence of the secondary flow in the three-dimensional case can alleviate the separation by convecting faster moving fluid towards the wall.

It was not until 1927 - 1928 that what is generally considered the first analysis of secondary flow in a curved pipe was published by Dean (references 7 and 8) with no reference to the work of Boussinesq. Dean considered the steady, incompressible, laminar flow through a pipe of circular cross-section whose axis was bent on a circular arc; the secondary flow was assumed to achieve a fully developed state. He deduced that the similarity parameter for the flow is:

$$k = (2a/R)^{1/2} R_e$$

Where $2a$ is the diameter of the pipe whose axis is bent on an arc of radius R , and R_e is the Reynolds number based on $2a$ and the mean axial velocity; k is commonly called the Dean number. His solution for the velocity distributions takes the form of a series expansion of the curvature up to the fourth power. The measurements of White (reference 9) and Adler (reference 10) confirm that the friction factor of a curved pipe depends on k while the

motion is laminar but that this is no longer true at the onset of turbulence. The second restriction on Dean's solution is that k must be small (≤ 14) and thus for flows where Re is $O[10^2]$, a/R must be $O[10^{-2}]$, in comparison with the average, domestic water pipe bend which has an a/R of $O[1]$. Dean's work has been extended to elliptical and rectangular cross-section pipes by Cuming (reference 11).

The problem of laminar flow for high values of the Dean number has been studied by Barua (reference 12). His solution is based on the assumption that for high values of k , the secondary circulation is confined to a boundary layer, outside which the flow is parallel to the plane of the bend. This assumption is substantially supported by the experimental results of Squire, (reference 13). It is obvious, however, that such an assumption becomes very approximate near the edge of this boundary layer where curvature of the central cross flow is inevitable. The solution is obtained using Pohlhausen's method and leads to values of the friction factor in good qualitative agreement with the results of White and Adler.

As k increases, the strength of the secondary flow increases and separation of the secondary flow which is to be distinguished from separation of the primary flow, becomes increasingly likely. Barua's results do exhibit the presence of separation, but are very approximate in this respect since he evaluates an unknown on the assumption that the rate of change of the boundary layer thickness over the cross section is not rapid; this is very marginal as the inner wall is approached and is not true near separation.

Hawthorne (reference 14) has shown that in the rotational flow of a perfect fluid, secondary circulation is generated (in the sense that, $|\Gamma_0|$, the total original circulation is directed by the curvature into a preferred direction, i.e. along a streamline). His

analysis, while limited to inviscid flow is very concise and the part relevant to the generation of the secondary circulation is abbreviated and reproduced here.

Consider the steady flow of an inviscid, incompressible fluid: the vorticity vector, $\underline{\Omega}$, is defined by the curl of the velocity vector, $\underline{v}$.

$$\text{That is, } \underline{\Omega} = \nabla \times \underline{v} \quad 1.11$$

Resolving $\underline{\Omega}$ along a streamline yields:

$$(\underline{\Omega} \cdot \underline{v}/q) \underline{v}/q = \mathfrak{F}/q \underline{v} \quad 1.12$$

where q is the scalar of $\underline{v}$ and $\mathfrak{F}$ is the scalar of the component of $\underline{\Omega}$ along a streamline.

Resolving $\underline{\Omega}$ normal to a streamline yields:

$$-(\underline{\Omega} \times \underline{v}/q) \times \underline{v}/q = - \frac{(\underline{v} \times \underline{\Omega}) \times \underline{v}}{\underline{v} \cdot \underline{v}} \quad 1.13$$

Now $\nabla \cdot \underline{\Omega} = 0$, hence expressing $\underline{\Omega}$ in terms of its components, expanding and employing the continuity equation $\nabla \cdot \underline{v} = 0$, gives:

$$0 = \underline{v} \cdot \nabla (\mathfrak{F}/q) - \underline{v} (\nabla \times \underline{\Omega}) \cdot \nabla \left(\frac{1}{\underline{v} \cdot \underline{v}} \right) - \frac{1}{q^2} \nabla \cdot [\underline{v} \times (\underline{v} \times \underline{\Omega})] \quad 1.14$$

Expanding again by the standard identity for the divergence of a vector product and employing the relationship:

$$\underline{v} \times \underline{\Omega} = \text{grad } P/\rho \quad 1.15$$

derived by Lamb, equation 1.14 becomes

$$\underline{v} \cdot \nabla (\mathfrak{F}/q) = - \frac{\underline{v} \times (\underline{v} \times \underline{\Omega}) \cdot \nabla (\underline{v} \cdot \underline{v})}{q^2} \quad 1.16$$

Expanding $\nabla (\underline{v} \cdot \underline{v})$ leads to:

$$\underline{v} \cdot \nabla (\mathfrak{F}/q) = - \frac{1}{q^2} [\underline{v} \times (\underline{v} \times \underline{\Omega}) \cdot 2(\underline{v} \cdot \nabla) \underline{v}] \quad 1.17$$

The vector $(\underline{v} \cdot \nabla) \underline{v}$ represents the acceleration and can be resolved tangential to the streamline and along the principal normal. The component along the

principal normal is v^2/R where $1/R$ is the curvature of the streamline. As shown by equation 1.15, $\nabla \times \Omega$ represents the gradient of the total head: if the angle between this gradient vector and the principal normal is ϕ , then the only component entering into the scalar triple product of equation 1.17 is $v^2/R \sin \phi$, since the other components have the direction of ∇ and $\nabla \times \Omega$. On integration along the streamline, equation 1.17 becomes:

$$\left(\frac{3}{2}h\right)_2 - \left(\frac{3}{2}h\right)_1 = - \frac{2}{\rho} \int_1^2 \frac{v^2}{R} \sin \phi \, ds. \quad 1.18$$

Applying Hawthorne's result to a circular pipe bend (figure 1.3), it can be seen that there is no secondary circulation generated at the inside and outside of the bend where $\phi = 0$ and π respectively, and these are stagnation points of the secondary flow. The sign of the circulation can be determined from equation 1.18 by applying the right hand screw rule.

Hawthorne goes on to show that equation 1.18 has an oscillatory solution, the secondary circulation being a sinusoidal function of the length of the bend. Applying this result to a pipe and a duct bend with a linear inlet velocity profile, experimentally produced by a grid, he demonstrates a satisfactory agreement with experiment. Oscillations of the secondary flow are apparent in Squire's experimental results (reference 13), having a fully developed turbulent profile at entry, but are damped out after about 180° of bend deflection by viscosity and a fully developed secondary flow is established. The oscillation of a swirling flow of water through a 90° bend is also demonstrated by Binnie, (reference 15).

The problem of fully developed turbulent flow

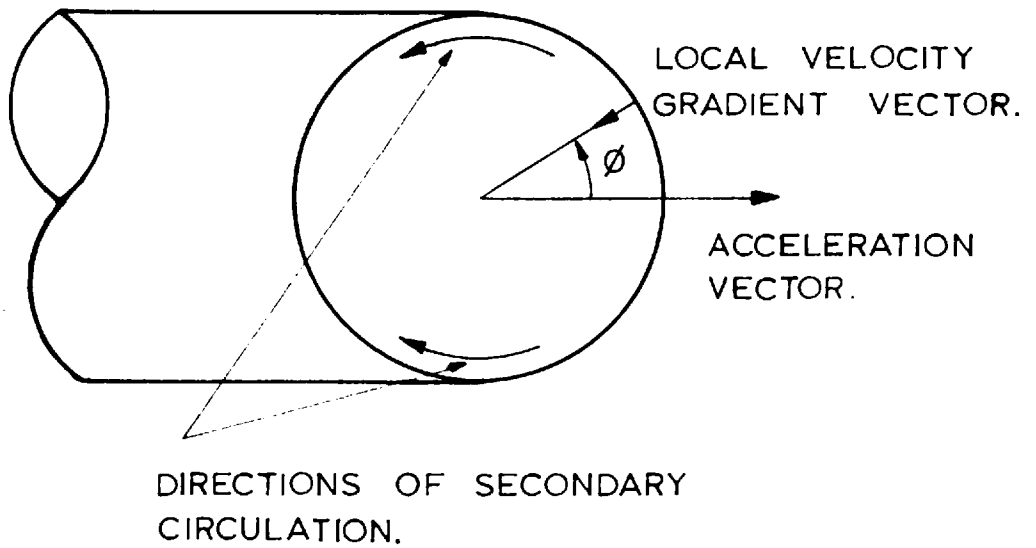


FIGURE 1.3.

through a curved pipe has been considered in detail by Ito (reference 16). The flow model which he employs is similar to Barua's, that is, all the secondary flow is assumed to take place within a boundary layer, termed the shedding layer, whose edge is defined by the point where the peripheral velocity component changes its sense. This model has been verified experimentally and the concept of a layer differs from that of the ordinary boundary layer inasmuch as its edge is not asymptotic but can be precisely defined. He derives two semi-empirical resistance formulae, the first based on a one-seventh power law velocity distribution and the second on a logarithmic distribution in the shedding layer. Both these formulae provide a sufficient degree of accuracy for engineering calculation and prediction of the shedding layer thicknesses are good for large values of $(a/R)^2$. Re, implying a large secondary circulation. Separation either of the primary or secondary flow is not discussed.

By far the greatest practical application involving curved pipe flows is the small radius ratio, 90° bend. As Hawthorne points out in a later paper (reference 17), in these bends the secondary flow does not complete a single oscillation, and although a large proportion of the circulation is developed, the cross flow is not set up within the bend. Thus on leaving the bend, the secondary flow can have achieved a variety of possible states, depending on the radius ratio, Reynolds number and inlet profile, and this accounts for the oscillatory variation of the friction factors of pipe bends with radius ratio. However, a satisfactory correlation is yet to be obtained.

Weske (reference 18) carried out a detailed study of the flow in a variety of small radius ratio bends, as opposed to the many measurements of steady-state head

losses that have been made (for example references 19 and 20). Weske measured the mean profiles of axial and peripheral velocity at outlet for a fully developed turbulent flow entering bends of a variety of cross sectional geometries, radius ratios and angles of turn, many of which produced separation of the primary flow. In some cases where the angle of turn exceeds 90° , the oscillation predicted by Hawthorne is in evidence from the measured values of the peripheral velocity component. Up to the exit of these bends the major effect governing the axial profile was the two-dimensional, constant angular momentum distribution, the maximum velocity occurring near the inner wall. From the outlet to some four diameters downstream, the three dimensional cross flow redistributed the profile such that the maximum velocity moved to the outer wall. The flow then reverted to a straight pipe flow. Many of Weske's bends produced flow reversal (primary flow separation) at the inner wall. All flows had an eddying region near the inner wall which presumably corresponds to a separation of the secondary flow near the inner stagnation point. All bends produced symmetrical, twin secondary circulations except for those which had an upstream, strongly asymmetric velocity profile artificially produced. Weske's results are referred to later in this thesis.

Curtiss, Fiel & Liquormik (reference 21) have examined turbulent flows through small radius ratio (including a mitre), square and rectangular, 90° duct bends which caused primary flow separation. Their interest in this flow was in the ability to modulate the dynamic head of the primary flow by secondary injection into the separation, and hence use the system for fluid amplification. Their measurements of mean

resultant velocity profiles at outlet indicate the presence of a symmetrical, twin circulation, secondary flow, confirmed by oil film, flow visualization of the separation size and position. The profiles have the constant angular momentum form but with a peak near the outer wall which increases with decreasing radius ratio and hence corresponds to the peripheral secondary flow component. Only mean components were measured.

The preceeding survey may be summarised as follows:

1. The existence of secondary flows within primary flows having three-dimensional curved boundaries is well known, and there is a sound understanding of its mode of production in curved pipe flow.
2. In a fully developed laminar, curved pipe flow, for low values of the Dean number, the secondary circulation occupies the whole cross-section. For high values of the Dean number and the corresponding turbulent flow, the secondary flow is restricted to the shedding layer and the flow in the central portion is approximately parallel to the plane of the bend.
3. For fully developed secondary flow, both laminar and turbulent, theoretical or semi-empirical solutions exist which provide a good description of the flow and a sufficient degree of accuracy for working estimates of friction coefficients to be made. Such results, however, are not applicable to short, small radius ratio bends. No useful predictions of separation of either the primary or secondary flow are given.
4. During development, the secondary circulation is an oscillatory function of the length of the bend. This oscillation is damped out by viscosity.
5. In small radius ratio, 90° bends, the axial velocity

distribution away from the wall is governed by the two-dimensional consideration of constant angular momentum. Secondary circulation is generated but does not complete a single oscillation in the bend. The cross-flow is set up in the downstream flow and this redistributes the profile towards the outer wall. There is no published work on the nature of the fluctuating components associated with the flow separation in such bends.

1.2 SEPARATION OF FLOW.

Of all the unsolved problems of fluid mechanics, separation of flow, or stall, is perhaps the most important, since it has such a direct application to the design of all forms of fluid devices. A survey conducted by the Fluid Mechanics Sub-committee of the Hydraulics Division of A.S.M.E. (reference 22) in order to determine on which topic general attention should be focussed, revealed that over fifty per cent of the various research establishments that were questioned considered separation to be of the most importance. It is surprising, therefore, that it is only of recent years in which the field of fluid mechanics has expanded so widely that a thorough knowledge of this phenomenon has been considered necessary. Prior to this century, fluid mechanics was virtually synonymous with hydraulics, and hydraulicians concerned themselves with mean flows, largely neglecting turbulent motion and separation. Thus there are many common fluid devices and hydraulic structures such as plumbing fixtures, bridge abutments and channel sections whose geometries facilitate separation rather than attempt to eliminate it and hence suffer the penalties of vibration, eddy loss and erosion that are characteristic of separated flows.

With the advent and rapid growth of both experimental

and theoretical aerodynamics, many investigators turned their attention to the problem of formulating a criterion for the separation of flow over an aerofoil surface. What is now termed the "classical separation model" was first put forward by Prandtl and serves as a definition of the phenomenon; it is shown in figure 1.4, taken from the transcription of Prandtl's lectures by Tietjens (reference 23).

Viscous losses at the aerofoil surface modify the boundary layer profile until momentum of the wall layer is unable to overcome the adverse pressure gradient that exists from B to E and comes to rest at D. Downstream of D, the flow is accelerated down the pressure gradient, that is, upstream, producing the separated profile at E. Eddies are formed within this region separated from the main flow by a thick shear layer with its attendant instabilities. At the onset of separation, a small zone of reverse flow occurs and grows in size, deflecting the main stream and moving upstream to its final relatively stable position. Such a model leads to the formulation of the analytical criterion $(\partial u / \partial y)_{y=0} = 0$ which exists at D, and which necessarily implies a wall shear stress of zero, since using Newton's equation:

$$\tau_w = \mu (\partial u / \partial y)_{y=0}$$

The assumptions inherent in this model are that the flow is steady and two-dimensional.

Using this criterion, many attempts have been made to predict the occurrence of separation using Prandtl's boundary layer equations, and a summary of these appears in a paper by Chang (reference 24). The results indicate that for the laminar boundary layer, good

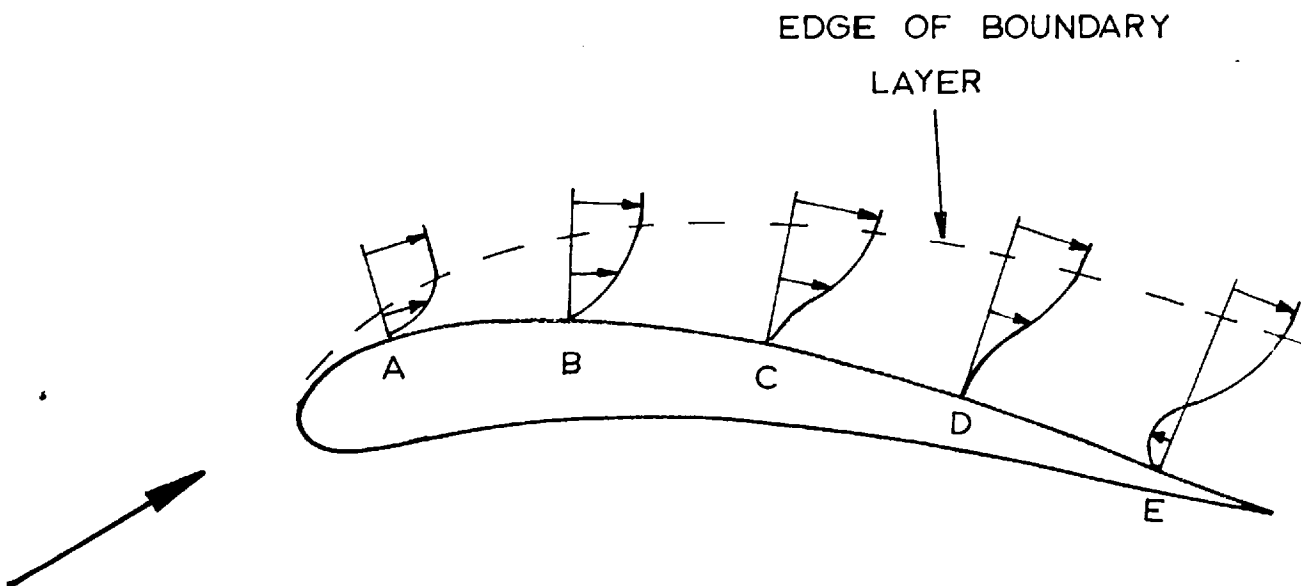


FIGURE 1.4. CLASSICAL SEPARATION MODEL.

predictions of the separation point as a function of free stream Reynolds number and boundary geometry can be obtained provided that when the separation occurs, it does not greatly disturb the external potential flow or does so in a manner capable of analysis by free streamline methods. The results are improved if the pressure distribution employed in the computation is that measured in the flow under consideration rather than determined by a potential flow analysis. Apart from the assumption of steady flow which is always marginal where separation is present, there is a further possible source of discrepancy introduced by the usual boundary layer assumption that $\partial p / \partial y$ is negligible. Any recirculatory flow in the separated region must be accompanied by a significant $\partial p / \partial y$ term and the resulting curvature of the separating streamline will also incur such a term. This consideration enforces the stipulation that the separated region must be small.

The application of boundary layer theory to turbulent separation has also been attempted by many investigators. The essential problem in turbulent boundary layer theory is that at present there is no known universal relationship between the Reynolds stresses and the mean velocity distribution, and hence either semi-empirical or approximate relationships must be used. Most methods of prediction of separation have employed the turbulent momentum integral equation:

$$\frac{d\theta}{dx} + (H+2)\frac{\theta}{U}\frac{dU}{dx} = \frac{\tau_w}{\rho U^2} - \frac{1}{U^2} \frac{d}{dx} \int_0^\infty (\bar{u}^2 - \bar{v}^2) dy$$

using measurements of pressure gradient and momentum thickness to determine the wall shear stress, neglecting contributions from the normal turbulent stresses. While this equation may be balanced for zero, negative and slightly positive pressure gradients, it becomes difficult for strongly positive gradients, in some cases the wall stress apparently increases towards separation. Such results are attributable to three main causes:

- (a) as τ_w decreases the normal Reynolds stress term is no longer negligible.
- (b) neglect of the $\partial^2 v / \partial y^2$ term.
- (c) three dimensionality in the boundary layer under investigation.

Allowance for the $\partial^2 v / \partial y^2$ term was made by Hewson (reference 25) who found quite large values in and just upstream of a separation, but some 30% error between the integral equation and experiment remained. A review by Thompson (reference 26) of turbulent boundary layer theories concluded that three-dimensional effects were the major source of error and that measurements in an accurately two-dimensional flow were badly needed.

Despite these objections, semi-empirical methods such as Maskell's (reference 27) and Stratford's approach (reference 28) using mixing length theory to relate shear stress to mean velocity profile are capable of predicting two-dimensional turbulent separation to within ten per cent. A more recent work by Bradshaw (reference 29) has used empirical relationships between shear stress intensity and turbulence structure, and this appears to give quite good results for separation, though once again normal

stresses and $\partial^2/\partial y^2$ terms have been neglected.

The approach to the problem of separation via the boundary layer equations can be considered legitimate if a criterion for the existence of separation is the only result demanded of them, and from the standpoint of designing an efficient lifting surface this is, perhaps, sufficient. If, however, a detailed knowledge of the flow within the separation, the shape of the separation and its downstream effects, including periodicities, are important, it is clear that such an approach is of little value since the boundary layer equations are no longer valid. The theoretical approach must therefore revert to the full unsteady, viscous Navier-Stokes equations. Since these equations remain unsolved, careful observation and measurement must be employed to gain an understanding of the basic mechanisms of separation, so that approximations to these equations based on physical fact may be made. Empirical correlations, for example, velocity profile similarities, are of great importance since they necessarily imply the existence of a common physical process: thus Coles (reference 30) has achieved a correlation for a few cases of a separating and re-attaching turbulent boundary layer using a wake function, lending credence to the concept that outside the sub-layer, the flow is dominated by large scale, turbulent mixing, governed primarily by inertia.

The foregoing considerations have led Kline, (reference 31), Frey and Vasuki (reference 32) et al., particularly workers in the field of fluid amplifiers and other jet control devices, to undertake very careful studies of simple geometries which induce separation,

placing particular emphasis on the use of flow visualisation techniques in order that those measurements which were taken might be, as far as possible, directly related to observation. Mean flow measurements alone are inadequate, since most experimenters have found that transient phenomena play a very important role in deciding the flow configuration, for example, the variation of the oscillatory lift coefficient of a circular cylinder by an order of magnitude produced by free stream turbulence or tunnel vibration (reference 33). Stability observation, argument and theory are, therefore, essential parts of the study of separation.

Broadly speaking, separations can be classified in three main groups:

(a) A transient separation, existing for short periods of time which is convected away by the main stream and its eddy structure broken down and dissipated.

(b) A steady separation corresponding to the classical model, such as exists behind a circular cylinder in the approximate Reynolds number range 5 to 40.

(c) A shedding wake type separation, portions of which periodically break off, are convected away and dissipated, characteristic of the wake of a circular cylinder at higher Reynolds number.

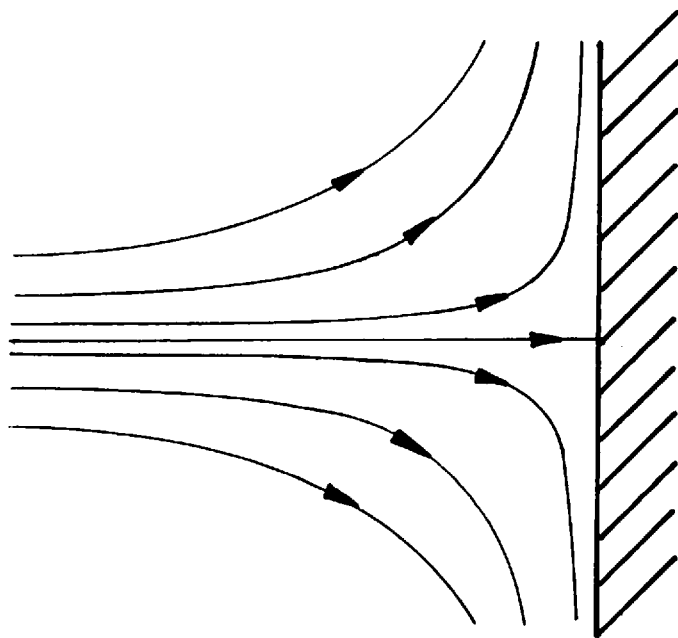
It is significant of the present knowledge of separated flows that, while it is known that the flow over, for example, a downstream facing step will separate and trapped eddies will be formed, it cannot be predicted with certainty whether a separation

corresponding to groups (b) or (c) will occur. Knowledge is limited to particular geometries and correlating parameters are rare. There are apparent divergencies of opinion as to the basic nature of separation, that is, whether it is essentially viscous or an inertial effect. Thus Chang (reference 3) refers to a 'viscous fluid phenomenon, while Housé (reference 34) observes

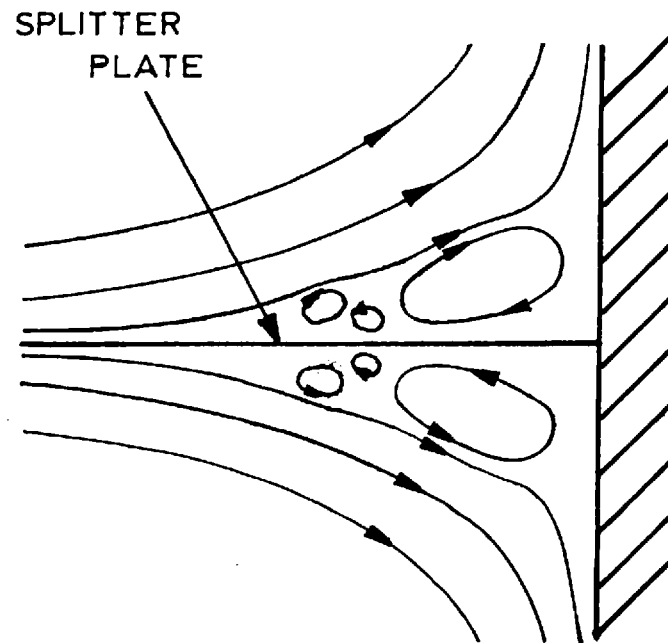
'separation that is started by the inertial characteristics of the mean flow ----- without the necessity of viscous action'. The sketches of free and decelerated stagnation flows shown in figures 1.5, taken from photographs by Foettinger and published in Schlichting (reference 35), appear to confirm the viscous argument and the classical model of separation.

Figure 1.5a is approached very closely indeed by potential flow theory because viscosity plays virtually no part in the flow approaching the wall, until it can form the boundary layers on the wall either side of the line of symmetry; these do not separate because the pressure is decreasing in both directions. There is, therefore, a very accurate balance between momentum loss and pressure recovery along the separation streamline.

The insertion of the splitter plate in figure 1.5b would not alter this flow if it merely replaced the stagnation streamline of the potential flow picture. However, the no-slip condition that applies to a real fluid calls viscous effects into play in the stream approaching the plane wall. The fluid in the boundary layer which grows on the splitter plate thus loses momentum through friction and is unable to overcome the adverse pressure gradient created by the normal wall, resulting in a flow reversal and the growth of the final separation.



(a)



(b)

FIGURE 1.5. STAGNATION FLOW AGAINST A PLANE WALL: (a) WITHOUT AND (b) WITH A SPLITTER PLATE.

It has been shown, however, by Fraenkel (Reference 36) that the solution of the general, inviscid, rotational equation: $\nabla^2 \psi = f(\psi)$ for boundaries with sharp corners includes stationary eddies, and its solution for a channel flow terminating in a sink (figure 1.6) bears at least qualitative similarity to figure 1.5b, in that separation and reverse flow are evident.

Thus although the $f(\psi)$ term represents a vorticity, which in any real fluid must be generated by viscosity, it is not necessary to consider continuous viscous loss at the wall for separation to occur. Hence figure 1.5b may be considered an inertial separation. Clearly, in a rotational, inviscid flow with zero velocity at the wall, reverse flow and hence separation must occur immediately at a pressure minimum since the fluid infinitely close to the wall is at the mercy of the applied pressure gradient. Viscosity provides a means of supplying momentum to the wall layer and hence, in this sense, acts to prevent separation. Thus a laminar layer can penetrate a small distance beyond a pressure minimum, whilst, because of the more efficient transport of momentum by eddies, a turbulent layer penetrates still further.

Considering an element of fluid, close to a curved wall, which has just been brought to rest by the adverse pressure gradient, it can be seen that it will immediately tend to move upstream thus presenting a locally more adverse gradient to the downstream flow. It therefore proceeds upstream until the downstream flow has sufficient momentum to produce a stagnation

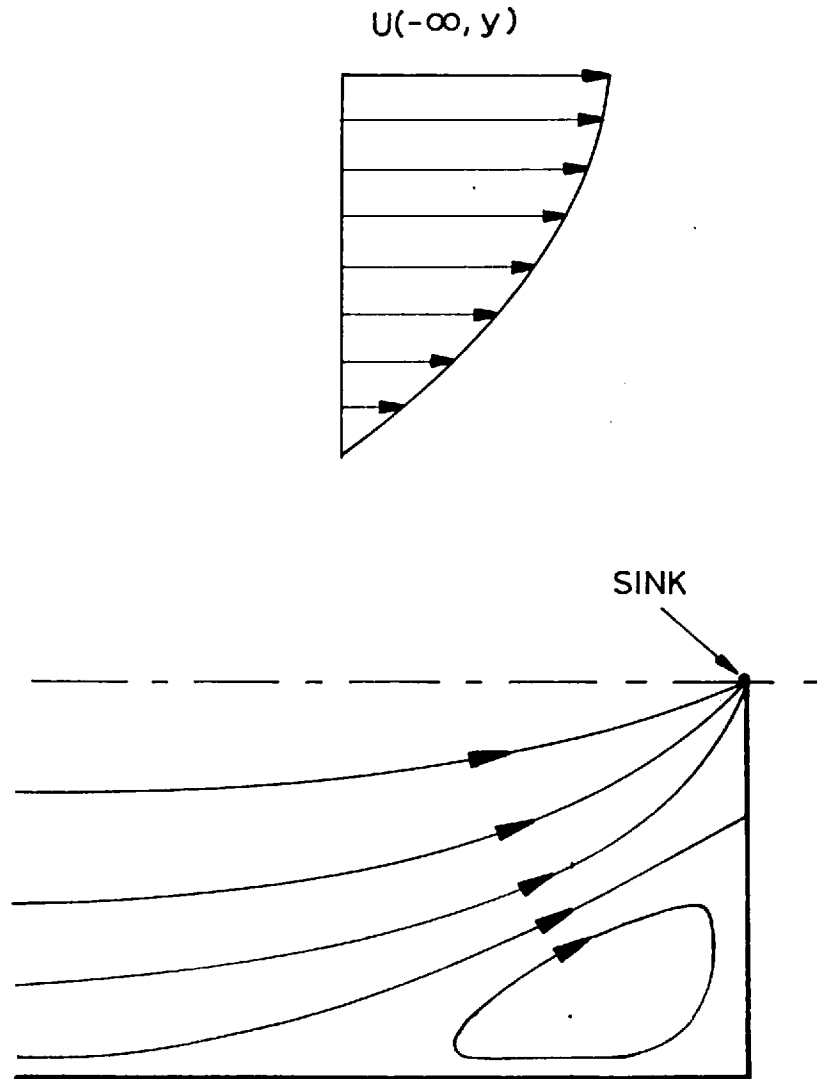


FIGURE 1.6. ROTATIONAL,
INVISCID CHANNEL FLOW.

point. The resulting separation point is obviously readily affected by upstream or downstream fluctuations and can therefore oscillate. If the surface has a discontinuity, then the wall pressure gradient is necessarily infinite at the discontinuity. This does not imply that separation must occur since the gradient effectively acts for zero time and very close to the wall the gradient is finite. Thus a thin boundary layer over a shallow discontinuity will not separate. If the layer has reached incipient stall ahead of the discontinuity then the separation can once again oscillate. However, a normal boundary layer will separate at a sufficiently large discontinuity and the separation point will be fixed there. Growth of the reverse flow upstream is not possible, since the boundary layer can overcome it. Such discontinuities are termed "salient edges" and are employed where separation exists but oscillation of its position or intermittent separation cannot be tolerated.

The preceding considerations of the nature of separation has been based on a laminar flow concept. The effect of turbulence on the separation point is well known: turbulent transfer of momentum to the fluid near the wall enables it to penetrate further downstream against the adverse pressure gradient before separating. A turbulent flow will, however, still separate at a salient edge. The work of Kline and Rundstadler, (reference 37) on separation in plane wall diffusers has revealed something of the nature of incipient, turbulent separation. Injecting dye into the sub-layer with a syringe revealed that filaments moved upstream as small "streaks of stall" and were then

caught by the main stream and convected downstream; the streaks formed a transient, three-dimensional pattern over the entire wall. This phenomenon does not occur in the sub-layer of a laminar flow which thus remains two-dimensional. Since the pressure gradient was constant over any cross-section of the duct, the streaks correspond to a three-dimensional separation of momentum of the sub-layer produced by instabilities about which there is little information. As the adverse pressure gradient was increased, the streaks increased in width and 'seeded' the transitory form of separation. Further increase produced steady separation on one wall, followed by steady separation on both walls. The production of a final, steady separation in an internal flow may be argued to be more likely than a shedding wake, because any growth of the separation produces an effective contraction in duct cross-sectional area and hence a drop in pressure. The prevention of the separation by the addition of short vanes is quite consistent with this view. Thus the presence of another surface may be considered to alter the transient response of the original flow, suggesting that in some important stability aspects, internal flow separation differs from external flow separation and, certainly, predictions based on external flow correlations have failed when applied to internal flows.

The general study of the instabilities of separated flows is still in its youth. The inviscid solutions of the Orr-Sommerfeld equations that apply to free shear layers provides a useful means of visualizing local instabilities and their rapid amplification out of the linear range. However, the prediction of frequencies,

the development of three-dimensionality and the subsequent randomization and breakdown to full turbulence remains something of a mystery. Measurement and analysis of data of separated flows is difficult. The hot-wire anemometer, which has proved such a useful tool for the study of turbulent boundary layers, becomes non-linear when the turbulence level exceeds approximately ten per cent of the mean velocity and loses its directional sensitivity when large amplitude, three-dimensional

fluctuations are present. Its application to separated regions is, therefore, limited. The use of an electromagnetic technique in an ionized fluid, such as acidified water, has been considered (reference 48). The main difficulty that must be surmounted is the production of a high field strength over a small region of the fluid, but it has the great advantages of linearity and directional sensitivity; it is a possible direction of useful development. The measurement of wall pressure fluctuations has been made possible over the last ten years by the development of fast response, sensitive transducers. The main application so far has been to the study of turbulent boundary layers, but their application to separation studies is now being realised. The measurement of fluctuating total pressure over a significant frequency range is, however, impossible at present, since no available transducer is small enough to place in the flow and the use of a pitot tube extension introduces a prohibitively long, time constant into the response. However, the use of the various flow visualization techniques is an essential part of any separation study, despite advancements in measuring

instruments and, to some extent, because of such advancements, in order that measured data may be correlated with the physical flow.

The preceding pages may be summarised as follows:

- (1) The ability of boundary layer theory to predict the separation point for both laminar and turbulent, two-dimensional flow, is generally adequate for practical applications. The neglect of $\partial p / \partial y$ and normal Reynolds stresses and three-dimensional effects in the measured data are considered to cause the discrepancies.
- (2) Detailed description of separated regions cannot be obtained from the boundary layer equations. Since the relevant Navier-Stokes equations remain insoluble, theoretical models must be formulated based on careful observation and measurement, placing particular emphasis on stability considerations.
- (3) Separation of a sheared layer is essential although the initiation of the shear is necessarily viscous. An insufficient body of knowledge is available on all forms of separation in order to formulate models or extract general correlating principles.
- (4) Studies indicate that incipient turbulent separation differs radically from the laminar case, and internal flow separation differs from that of an external flow.
- (5) Measurements within separated flows are difficult to make and interpret. Where possible, flow visualisation techniques should always be employed to assist interpretations.

1.3 THE SCOPE OF THIS THESIS.

The vast expansion of the engineering sciences that has taken place in the last fifty years, the extension of old and the development of new techniques, has brought with it, in its wake as one might say, an host of problems for the fluid dynamicist. Unfortunately, the greater part of the studies of fluid flow to date have been directed towards aeronautics which, in general, is concerned with flow around slender bodies and has led to the wide use of small perturbation methods of analysis. The problems of industrial fluid dynamics, however, are usually associated with geometries which cannot be made slender, either through difficulties in construction, variable mean flow direction, as in the case of the natural wind, or because their basic purpose does not permit it. The fluid dynamicist is allowed, therefore, to adjust only minor features of the geometry, but must have a sufficient knowledge of the flow in order to ensure that the fluid dynamics of the situation will allow the object to fulfil its major design purpose. Separation is thus the rule rather than the exception.

While the estimation of steady or time average loads from model tests has usually been sufficient, with noticeable exceptions, it has become increasingly obvious that for many structures a knowledge of the fluctuating loads that can arise and their relationship to the dynamic response of the structure is essential. The mode of production of such fluctuating loads can, in general, be categorised into the following three groups:

(a) large scale fluid dynamic instability, such as

the vibration of an isolated cylinder by vortex shedding, or the oscillation of a row of cylinders by jet switching.

(b) an aeroelastic type instability where a fluid dynamic characteristic becomes coupled to one or more mechanical degrees of freedom.

(c) the rapidly changing properties of the flow associated with its eddy structure generating pressure fluctuations at the wall or radiating sound.

The present work is concerned with the last of these three and, in particular, with the wall pressure fluctuations generated by a sub-sonic separation. It was hoped that such an investigation could be made to yield useful information on both the general nature of the fluctuating wall pressure field associated with sub-sonic separation, and on the particular nature of a flow having a direct, practical application. In order to do this, it was necessary to obtain a suitable pressure transducer with a fast response and a high sensitivity. Since no commercial device was readily available it was decided to construct one.

The evident lack of information concerning flow through a sharp pipe bend ($R/a < 2$, say), despite its common use, and the knowledge that the fluctuating wall pressure field associated with the primary flow separation occurring in such bends can cause the structural vibration of a pipe line, promoted its choice. Because of the inevitably highly complex nature of such a flow and the conclusion of section 1.2 that detailed study of separation is essential, it was decided to examine one particular geometry thoroughly and to ensure, as far as possible, that any measurements taken could be

interpreted in the light of the actual flow, rather than to take mean measurements related to a nebulous mean flow. The particular geometry chosen is that of a mitre bend. While such a bend is not the most commonly used in practice, it is closely approached by many standard hydraulic fittings, and is now to be found in much larger installations, particularly nuclear reactor coolant circuits, where its ease of construction is a major consideration, and long reliable service essential. The reason for its choice for this study was the property of a salient edge to stabilise the separation point, thus removing one unsteady mode. This geometry also avoids double curvature, thus facilitating its construction and instrumentation. It was decided to fabricate the test section from acrylic tube so that flow visualisation techniques might be employed. Turbulent flow is more realistic of practical pipe flows and a fully developed flow to some extent fixes the upstream boundary conditions.

It was assumed in the light of the literature survey carried out on pipe bend flows that the present mean flow would be steady, with a steady symmetrical secondary flow downstream of the bend and some fluctuating field generated by the separation superimposed upon it. This, however, was not found to be the case, the secondary flow is inherently unsteady and of a form which has not been recorded elsewhere to the author's knowledge. Thus a major part of the work was involved with determining the nature of the basic flow. The extreme complexity and unsteadiness of the flow ruled out many intended measurements and, to some extent, frustrated a general interpretation of those that were made.

2. EXPERIMENTAL MODELS AND INSTRUMENTATION.

2.1 MODEL FOR TESTS IN AIR.

The layout of the model is shown diagrammatically in figure 2.1. The upstream and downstream arms were fabricated from flanged lengths of $3\frac{1}{2}$ inch nominal internal diameter, smooth bore, drawn copper tube.

The upstream arm was 13 feet long (45 diameters) and had a bell-mouth intake of moulded fibre glass; an instrumentation port was provided at 16 diameters upstream of the test section. The first four diameters of the pipe wall were roughened with coarse carborundum paper to encourage boundary layer transition.

The downstream arm was $9\frac{1}{2}$ feet long (33 diameters) and was made up of five sections, the shortest of which had an instrumentation port. The lengths were chosen in order that surveys might be made at two diameter intervals downstream if required.

The test section bend was fabricated from $\frac{1}{4}$ inch nominal wall thickness, cast perspex tube; this was machined and repolished inside and out to ensure circularity and a sharp regular edge at the join. It was found impossible to obtain a copper tube and a perspex tube from stockists such that the bores matched after machining, the perspex always having the larger bore; the final mismatch on assembly was 0.050 inch on the diameter. One arm of the test section contained a 19 inch long, indexed, ^{rotatable} ~~rotatable~~ section. A corresponding vernier scale on the fixed portion provided a resolution of 1° . The rotatable section was indexed at both ends so that it could be reversed, thus reducing to a minimum the length of the instrumentation ports required. The initial flow visualization tests were conducted

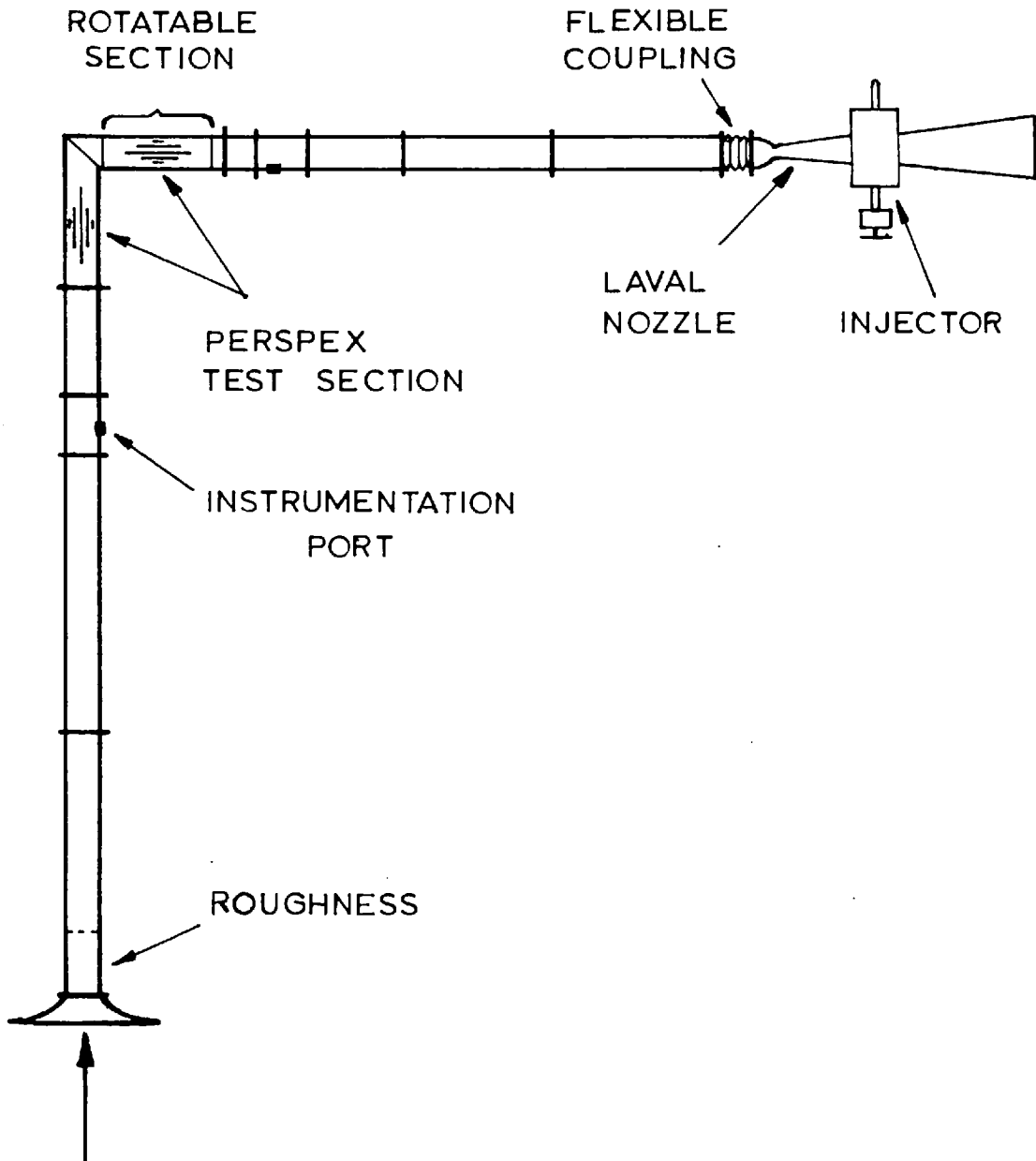


FIGURE 2.1. LAYOUT OF
AIR RIG.

without the ports having been cut; subsequently three ports were cut to provide access for the pressure transducer and hot-wire probes. The gaps either side of the transducer and probe heads, together with the ports that were not in use, were filled with an interchangeable set of radiused inserts of various lengths. The complete bend was symmetrical, thus allowing the rotatable section to be installed either upstream or downstream of the bend. The section was both retained at the desired orientation and sealed at its ends with "sellotape".

The flow was induced through the model by a 2 inch internal diameter, two slot supersonic injector. Upstream of the injector was a fibre glass Laval nozzle having a throat diameter of 1.32 inches to produce a design mean velocity of 100 feet/sec through the model. The nozzle was run in the choked condition, thus preventing pressure fluctuations, apart from those transmitted through the boundary layer, propagating upstream and affecting pressure measurements made in the test section. While such a system restricted the measurements to one particular Reynolds number, it had the great advantage over a fan system in that no swirl was generated; such swirl would be difficult to remove completely and might have critically affected the secondary circulation generated by the bend. Since the purpose of the investigation was to conduct a thorough study of one particular flow, rather than to vary many parameters, fixed Reynolds number was not considered a serious limitation and, in fact, only applied to measurements and not to flow visualization tests since these did not demand a choked nozzle.

The efflux from the injector passed through a

short diffuser and was then ducted out of the laboratory. Since supersonic injectors are essentially noisy devices, the inside of the exhaust duct and the outsides of the nozzle, injector and diffuser, were lined with expanded polystyrene to reduce the room noise to a level acceptable to both the experiment and the experimenter. The induction system was coupled to the model via a flexible P.V.C. "corrugaiter" (Plysu Industrial Limited) which effectively isolated the model from the vibrations of the injector.

2.2. MODEL FOR TESTS IN WATER.

The model was a single 90° mitre bend with two 12 inch arms fabricated from $1\frac{1}{4}$ inch nominal internal diameter, cast perspex tube and dye injection points provided at various points along them. The upstream limb was connected to a 36 inch length (29 diameters) of pyrex tube of similar diameter, which was fitted with two 2 inch lengths of aluminium honeycomb at its upstream end in order to reduce the swirl content of the flow from the reservoir. The model was submerged on a water table and illuminated from beneath to improve visability.

2.3. INSTRUMENTATION.

All mean velocity and turbulence measurements were made with a D.I.S.A. Constant Temperature Anemometer. The probes themselves, however, were not D.I.S.A. equipment, but were constructed from $1/16$ inch outside diameter stainless steel tube. The wire supports were formed by an 0.017 inch diameter needle (Sharps No. 10), insulated and mounted down the centre of the tube, and the wall of the tube filed and bent to a suitable shape; the stem of the probe could then be bent to any desired geometry.

The hot-wire was made from Wollaston wire with a platinum core diameter of 0.0001 inch and an etched length of approximately 0.020 inch.

All measurements of pressure fluctuations were made with the transducer described in Section 3.

Two spectral analysers were used:

- (a) A Brüel and Kjaer audio frequency spectrometer type 2112. This enabled r.m.s. measurements to be made covering the range 2c/s - 40 kc/s in one band or 25 c/s - 40 kc/s in 1/3 - octave bands. Automatic read out was not available and all readings were taken from the meter. Two meter dampings were available and the fast response was always used.
- (b) A set of variable analogue band pass filters covering the range 2c/s - 20c/s. These were designed by Mr. B.J. Belcher as a prototype for a more comprehensive spectral analyser. They were calibrated with a low frequency oscillator, the r.m.s. output being measured on the Brüel and Kjaer analyzer using the 2c/s - 40 kc/s band. Their calibration curves are shown in figure 2.2, and seen to be of the simple L-C filter type. Insufficient time was available to adjust their characteristics to the desired "flat topped" form of the Brüel and Kjaer filters, an example of which is shown for comparison in figure 2.2. Variable damping of the characteristics was provided, however, and this was adjusted such that their response was 3dB's down at the 1/3 octave points to match the Brüel and Kjaer filters.

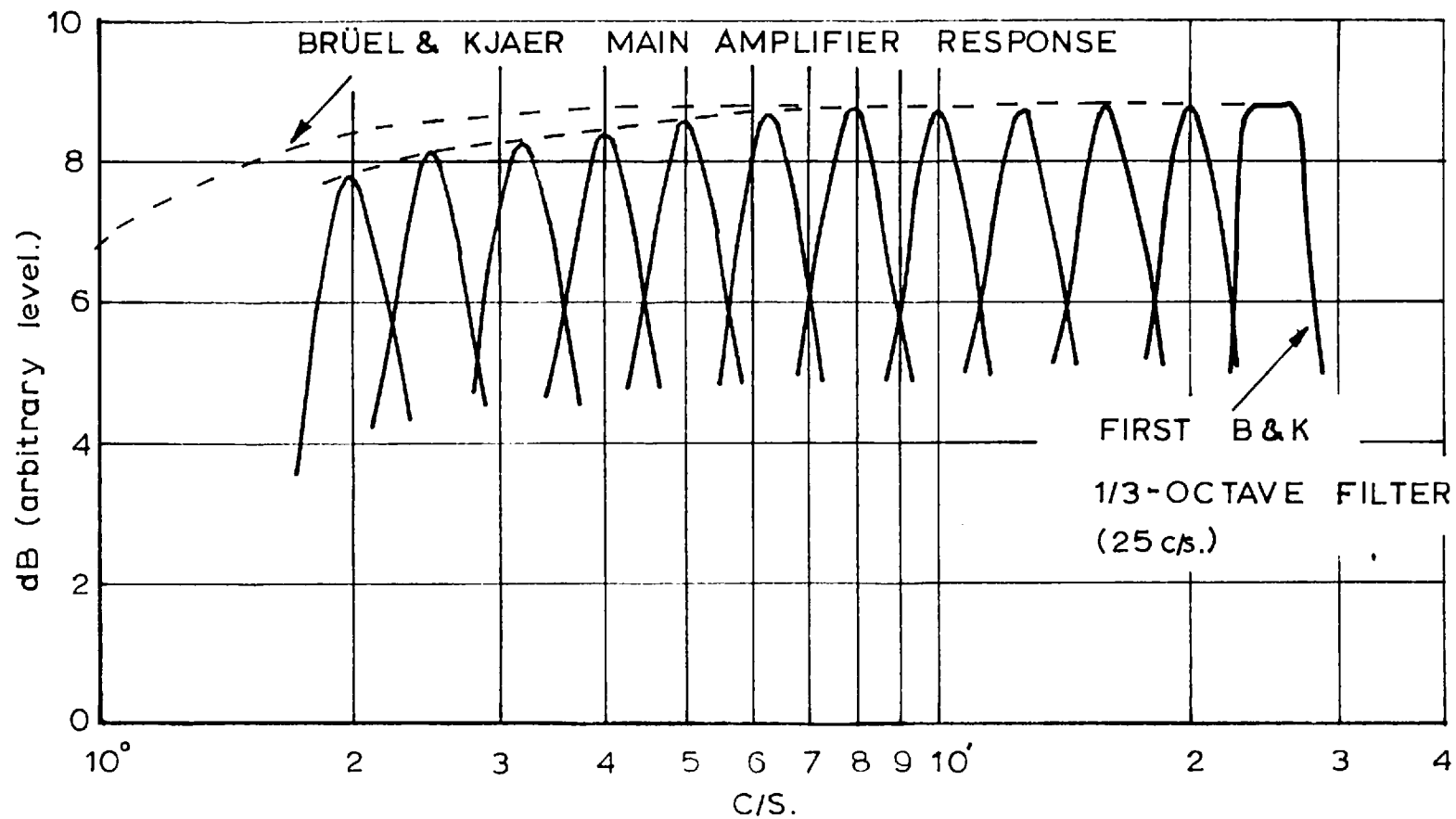


FIGURE 2.2. CALIBRATION CURVES OF L/F FILTERS.

3. THE PRESSURE TRANSDUCER.

3.1. TRANSDUCER CONSTRUCTION.

The two basic requirements for a transducer suitable for measuring the pressure fluctuations associated with a subsonic separation are that:-

- (a) it should be capable of resolving pressures of the order of 10^{-2} p.s.i., and
- (b) its frequency response should be approximately linear from approximately 0c/S to approximately 10 kc/S.

A review of the currently available commercial transducers showed that no suitable device was available.

Piezo-electric transducers of the types reported in references 38 and 39 satisfy the above requirements, but need careful mounting to overcome their high vibration sensitivity and such mountings can lead to low frequency resonances (see reference 38). When undergoing an acceleration of one "g", a pressure transducer records a force equal to the weight of its pressure sensitive element. Thus a device relying on the deflection of a crystal of significant mass will be much more sensitive to vibration than a device relying on the displacement of a very thin diaphragm. Such a device is a capacitance microphone.

The mode of operation of the capacitance microphone is as follows: a pressure difference across the diaphragm causes it to deflect; the diaphragm is made one plate of a capacitor and the resulting change of capacitance with respect to a fixed back plate is detected. The general theory of the microphone is published in Morse (reference 40). For a simple microphone consisting of a diaphragm stretched across a closed volume, the frequency response is only

approximately linear, well below the natural frequency of the diaphragm. However, various methods of adjusting the response have been devised, e.g. introducing viscous damping in the rear cavity, making use of the stiffness of the air within the cavity, and a linear response beyond the first diaphragm resonance is possible. A review of various microphone designs showed that the one published in a paper by Schultz (reference 41) had a suitable frequency and dynamic response for aerodynamic application.

The typical diameter of the active portion of a small capacitance microphone is, however, too large for aerodynamic measurements in general, and it is therefore necessary to improve the obtainable spatial resolution by mounting the device in a cavity behind a static pressure hole. This cavity then acts as an Helmholtz resonator which usually limits the high frequency end of the linear response of the coupled system. The natural frequency of such a resonator (figure 3.1) is given by:-

$$f \approx \frac{rc}{2} \sqrt{\frac{3}{V(3\pi L + 16r)}}$$

where c = velocity of sound in air
 f = natural frequency
 r = radius of static pressure hole
 V = volume of cavity
 L = entry length

For the cylindrical cavity shown in figure 3.1, this equation becomes:-

$$f \approx \frac{c}{2\pi} \cdot \frac{r}{R} \cdot \sqrt{\frac{1}{L(h + 1.7r)}}$$

' r ' should be at least an order of magnitude less than the smallest wavelength to be measured in order to avoid the averaging of these fluctuations. Assuming a minimum speed of convection of these fluctuations of

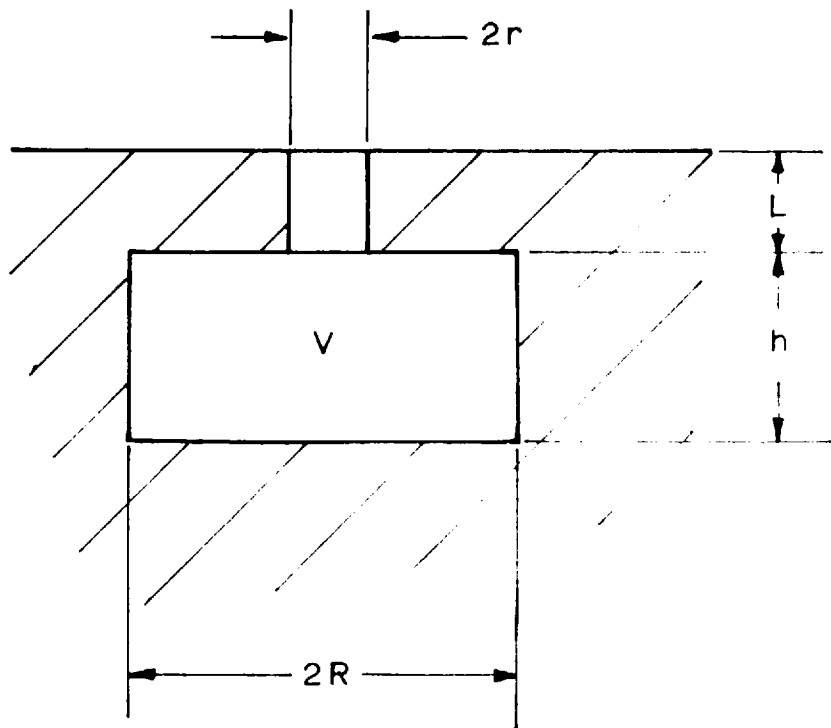


FIGURE 3.1.

0 (10) feet/sec leads to an 'r' of 0 (10^{-2}) inch. Thus for a cavity of given radius, R, L and h must be chosen to provide a suitable natural frequency.

The acoustical design of the microphone used in this work is exactly that of Schultz; however, it was found desirable to alter the mechanical design, in particular to suit the type of mounting dictated by the present application. The final design and construction is shown in figure 3.2. The centre supported diaphragm has an active diameter of 3/16 inch and is made from 0.00025 inch "Meculon" sheet. This is a polyethylene terephthalate film with an aluminium sputtering which provides a lighter and more uniform conducting surface than that achieved by the method of painting with colloidal graphite adopted in reference 4. The method of sticking the diaphragm to the body of the element reported in reference 4 was found unsatisfactory, because the diaphragm tension changed as the adhesive aged. It was abandoned in favour of a clamping ring and adhesive. The diaphragm was pretensioned before clamping and brought up to its final tension by screwing up the back plate. No measurements of the tension were made, but by trial and error it was found possible to adjust the back plate to the desired position and achieve a tension a little below the yield point of the film, thus achieving the highest natural frequency. The fillet of silver conducting paint shown in figure 3.2 serves the dual purpose of reducing the volume of the cavity and improving the conductivity of the sputtering where it has become strained at the edge of the back plate; poor conductivity appears effectively as a "loss" in the capacitor representing the microphone and, as is

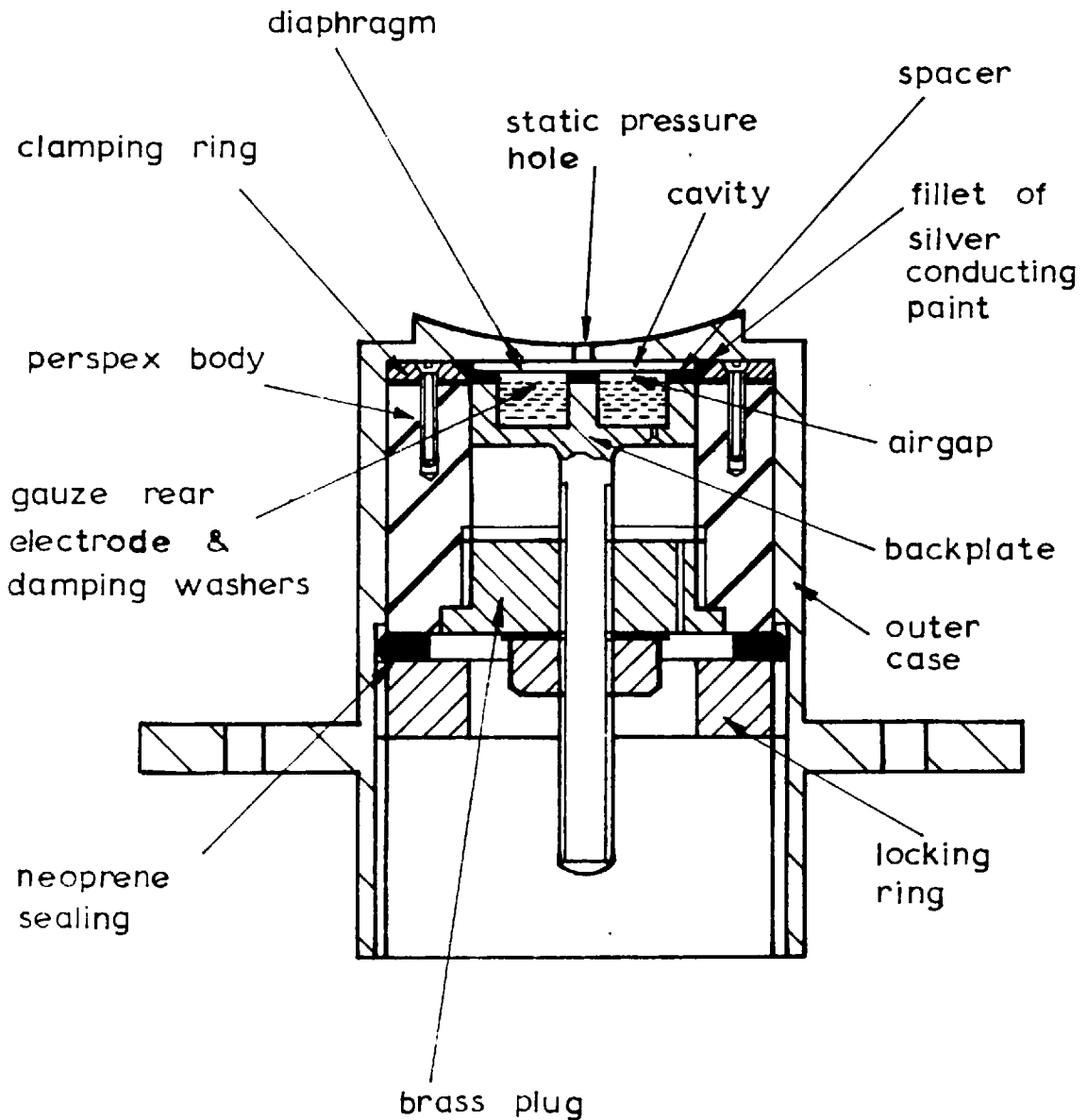


FIGURE 3.2. CROSS-SECTION
OF TRANSDUCER.

(NOT TO SCALE)

mentioned later, is undesirable.

The radiused head of the outer case of the microphone, demanded by its application to a pipe flow, is a particularly fortuitous shape since it allows the entry length to be minimised without reducing the mechanical strength of the case. An entry length of 0.015 inch, together with the small cavity in front of the diaphragm and an 0.030 inch diameter static pressure hole, produced a cavity resonance of 12.4 kc/S.

It is necessary to leak the back of the diaphragm to the front such that the static pressure difference which exists across the diaphragm under model working conditions is removed. Apart from avoiding possible damage to the diaphragm, this ensures that too large a deflection is not incurred and thus the dynamic response remains linear. The resulting drop in pressure of the air inside the rear cavity of the microphone is, however, too small to affect the density and hence the stiffness of the air, otherwise this would necessitate calibration under working conditions. The back of the microphone case is connected to the rear cavity of the microphone element by an 0.005 inch hole. The case is then connected via a tap to a static pressure tapping in the pipe model wall. The tap is opened while stopping and starting the flow and closed while taking readings of pressure fluctuations so that spurious signals do not reach the rear of the diaphragm.

3.2. TRANSDUCER BRIDGE CIRCUIT.

The normal mode of operation of a capacitance microphone is shown in figure 3.3.

Variations in the capacitance, C , of the microphone produced by pressure fluctuations acting on the diaphragm

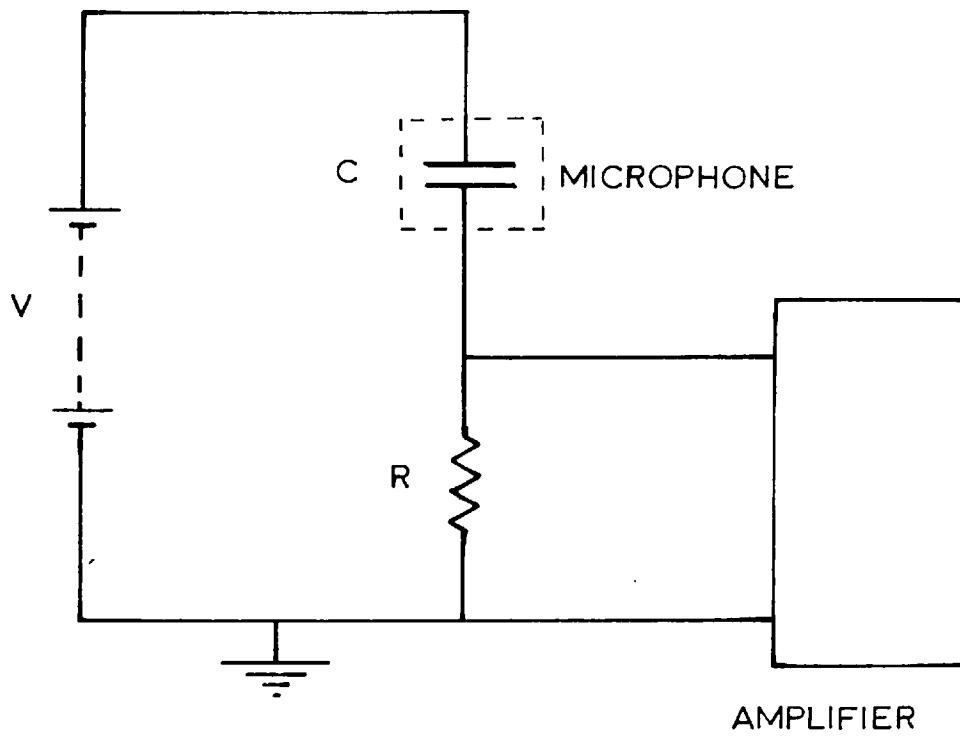


FIGURE 3.3 .

induce a fluctuating voltage across the resistance, R , provided R is large enough to maintain a constance charge. However, at very low frequencies the signal voltage will have time to leak across the resistance, and the low frequency response will be attenuated. This attenuation is governed by the time constant of the circuit, $2\pi RC$; thus at a frequency equal to $1/2\pi RC$, the response will be 3dB's down. In order to extend the response to, for example, 1 c/S with a typical microphone capacitance of 35 pfs, a resistance of $0(10^9)$ ohms is required. This imposes a severe strain on the insulation of the microphone element itself, introducing a possible sensitivity to atmospheric humidity.

These considerations led to the decision to build a capacitance bridge and thus extend the frequency response to d.c. A survey of the relevant literature revealed only one such system in a paper by J.J. Zaalberg van Zelst, (reference 42). This circuit makes use of valves and it was felt that a transistorized bridge would have advantages in size, noise level and power supply requirements. The design of an r.f. bridge was therefore undertaken. However, while this design was quite far advanced but not completed, P.J. Baxandall published his excellent paper in reference 43. This design fitted the requirements exactly and was therefore adopted. Minor modifications were necessary and the final circuit is shown in figure 3.4. The circuit is constructed on a perspex plate and is mounted inside a 6 inch by 1 inch outside diameter brass tube. The application of the transducer did not impose a restriction on its length and therefore no great pains were taken to minimise this dimension. In order to balance the circuit, the output from the filters are applied to an oscilloscope difference amplifier or centre zero millivolt meter, and

the trimmer capacitor adjusted for zero output. The outputs are then fed into a low gain difference amplifier whose output is applied to the recording device. Both the transducer and difference amplifier are battery driven.

The overall noise level of the microphone and bridge system is inferior to that of reference 6 because the microphone element has a small loss at the 1 mc/s bridge frequency. This appears as an out-of-balance resistive component in the active arm of the bridge which has to be balanced out by altering the capacitance trim of the bridge. This would restrict the usefulness of the microphone for the measurement of low level acoustic pressures, but does not impair its ability to fulfil its design purpose of measuring aerodynamic fluctuating pressures.

3.3. TRANSDUCER CALIBRATION.

An acoustical method of calibration over the entire frequency range of the transducer (0 c/s-10 kc/S) was not possible since the available calibrated standard microphones all utilise a system based on the circuit of figure 3.3 and hence the response is attenuated at low frequencies. However, the capacitance bridge circuit described have enabled one to employ an electrical method which does cover the full range. A d.c. polarizing voltage is applied to the terminals of the bridge, producing an attractive force between the microphone diaphragm and the rear electrode, representing an applied steady pressure difference. This pressure difference is then made to oscillate at any desired frequency by superimposing a signal from an oscillator on the applied voltage. The peak oscillating voltage must not exceed the steady voltage, otherwise the microphone will $\frac{1}{2}$ - wave rectify the applied signal. The output from the difference

amplifier is then measured on an r.m.s. voltmeter, at constant input voltage but varying frequency. This, however, does not provide an absolute calibration in terms of volts per p.s.i.; to obtain this an acoustical method must be employed but it is no longer necessary for it to cover the entire frequency range. The circuit used for the electrical calibration is taken from reference 6 and is shown in figure 3.5.

To perform the acoustical calibration, a plane wave tube was built. Two sound sources were used:

- (a) A Goodmans 25 watt high frequency pressure unit, covering the range 2.5 kc/S - 10 kc/S.
- (b) A Goodmans vibration unit driving an aluminium disc, covering the range 20 c/s - 2.5 kc/S.

The transducer was mounted opposite a Brüel and Kjaer Type 4132 microphone calibrated in the range 20 c/S-18 kc/S; the calibration curve provided was adjusted for grazing incidence waves.

In using the plane wave tube, it is assumed that the waves are axially symmetric. Reflections from the end, in the case of a closed tube, or from external objects, in the case of an open tube, could cause asymmetry. An absorbent cone of perforated zinc filled with glass fibre was fitted in the end of the tube to avoid such reflections.

The calibration curve obtained by the above methods is shown in figure 3.6.

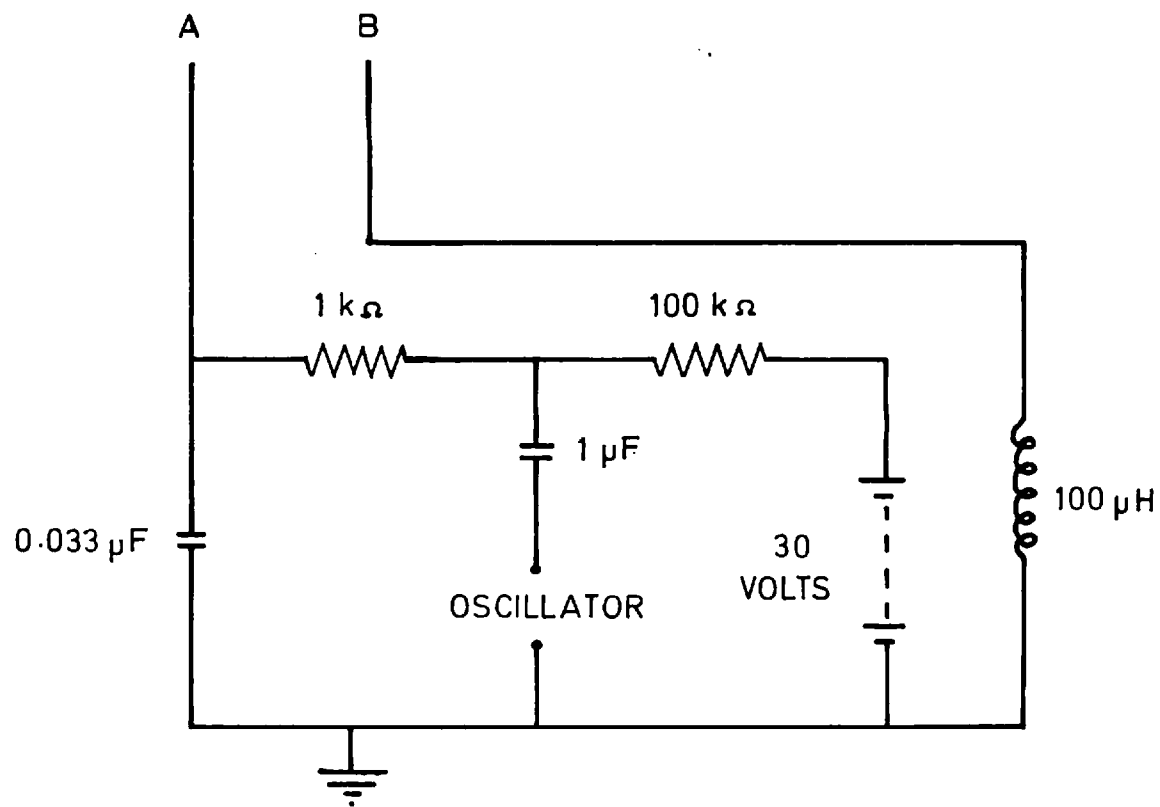


FIGURE 3.5. CALIBRATION CIRCUIT.

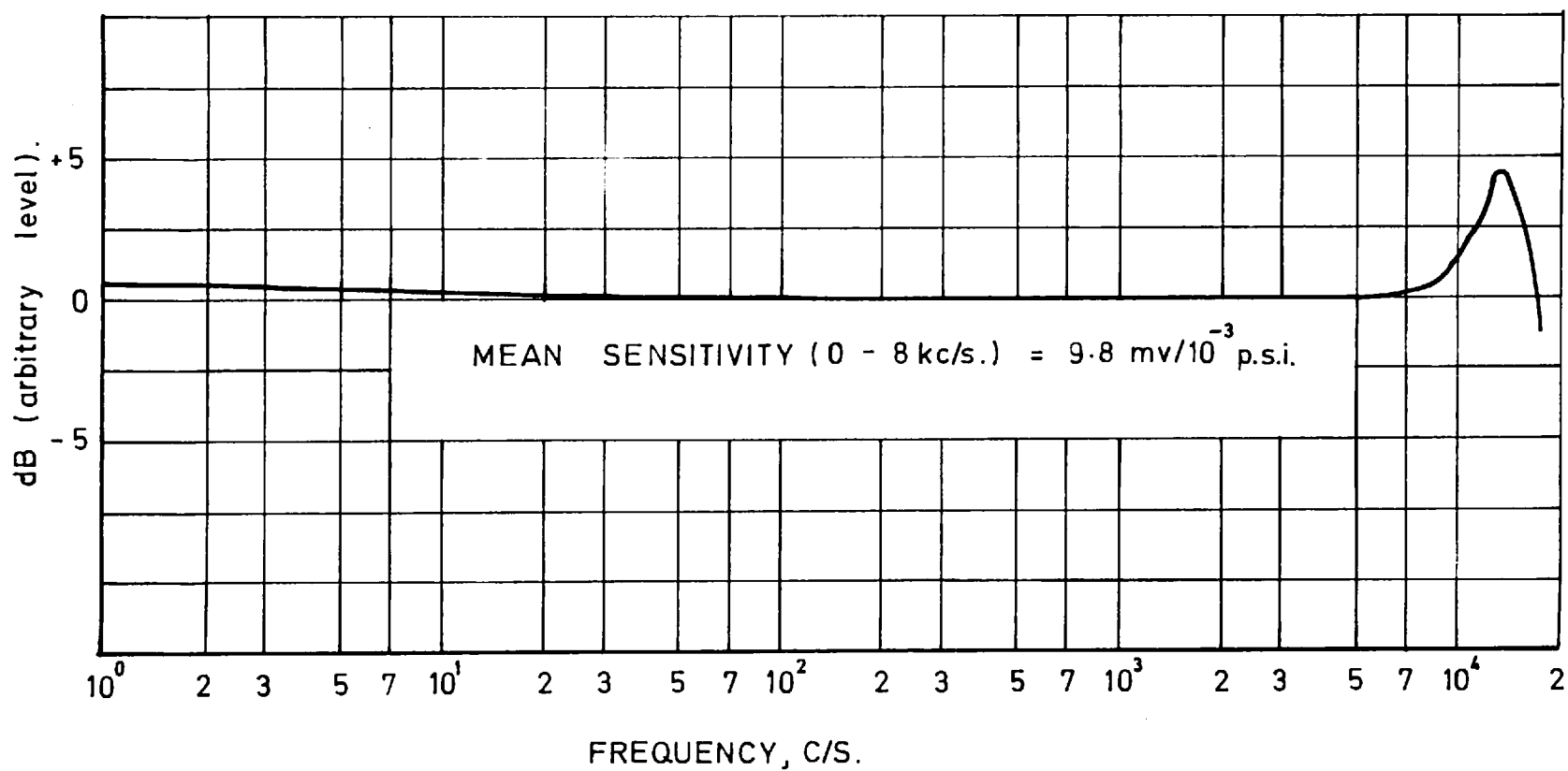


FIGURE 3.6. PRESSURE TRANSDUCER
CALIBRATION CURVE.

4. INITIAL TESTS.

Tests were conducted on the apparatus to determine whether:

- (a) the nozzle would choke
- (b) fully developed, turbulent flow was entering the bend test section.

4.1. CHECK ON THE EFFECTIVENESS OF THE NOZZLE.

The test section was removed and the bell-mouth intake attached to the downstream arm of the rig, as shown in figure 4.1. The instrumented section, in which the pressure transducer had been mounted, was fitted approximately six diameters upstream of the nozzle throat. On increasing the injector, plenum chamber pressure, the total r.m.s. pressure fluctuation was seen to drop suddenly at 80 p.s.i. and was virtually unaffected by further increase. A $1/3$ -octave, spectral analysis was then performed at an unchoked pressure (40 p.s.i.) and at a choked pressure (110 p.s.i.) and these are plotted in figure 4.2. At the date when these spectra were taken, the low frequency filters were not available. However, it is clear from figure 4.2 that the nozzle does choke and that it does adequately isolate the nozzle and injector acoustically from the remainder of the rig. The total r.m.s. level for the choked flow was at least 30 times smaller than the smallest total r.m.s. level subsequently encountered with the bend in position.

All subsequent tests were conducted at an injector pressure of 110 p.s.i., thus allowing a safety margin of 30 p.s.i. before the test section static pressure is affected by a drop in injector supply pressure which, if undetected, might damage the transducer diaphragm.

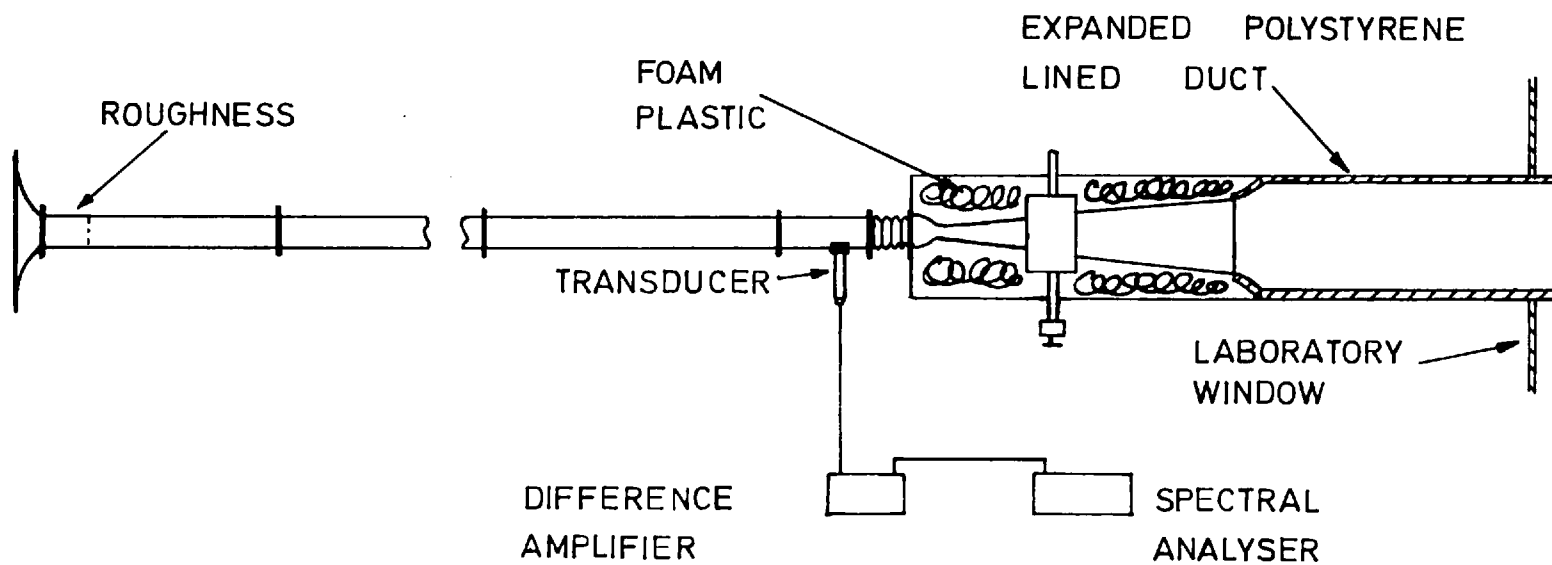


FIGURE 4.1. NOZZLE TEST
CONFIGURATION.

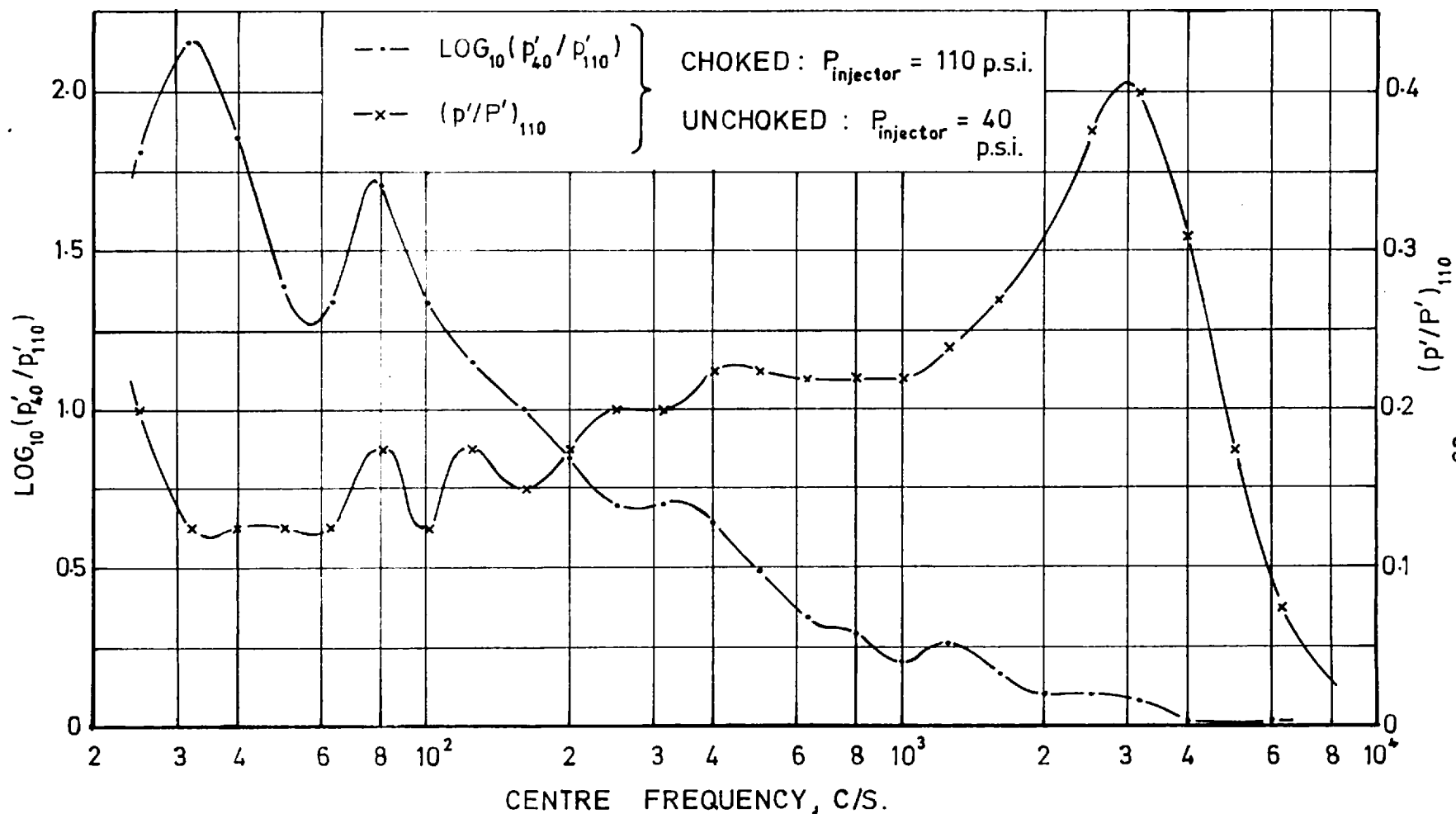


FIGURE 4.2. EFFECT OF CHOKING ON THE WALL PRESSURE FLUCTUATIONS IN A STRAIGHT PIPE.

4.2. UPSTREAM VELOCITY AND TURBULENCE DISTRIBUTIONS.

With the rig fully assembled as in figure 2.1, mean velocity and turbulence distributions were measured with the hot-wire anemometer at 33.5 and 42 diameters from the intake. It was immediately obvious that the flow was unsteady, the mean velocity appeared to have a high and a low value between which it "switched" at a low random frequency of the order of 1 c/s. The ratio of these two mean velocities was approximately 0.95. However, on restricting the readings to either state, the normal, symmetrical, fully developed turbulent profile was obtained at both stations and these are shown in figure 4.3, compared with Laufer's (reference 44) data. The maximum mean velocity was found to be 112 feet/sec, corresponding to a Reynolds number based on pipe diameter of 217,000.

The intake was checked for separation with a wool tuft, but none was found and it was concluded that the unsteadiness was probably characteristic of the flow through such a bend: this is confirmed later.

Much greater difficulty was encountered in taking consistent readings of velocity fluctuations since the r.m.s. thermocouple meter introduces a time lag. The results are tentatively presented together with Laufer's data in figure 4.4.

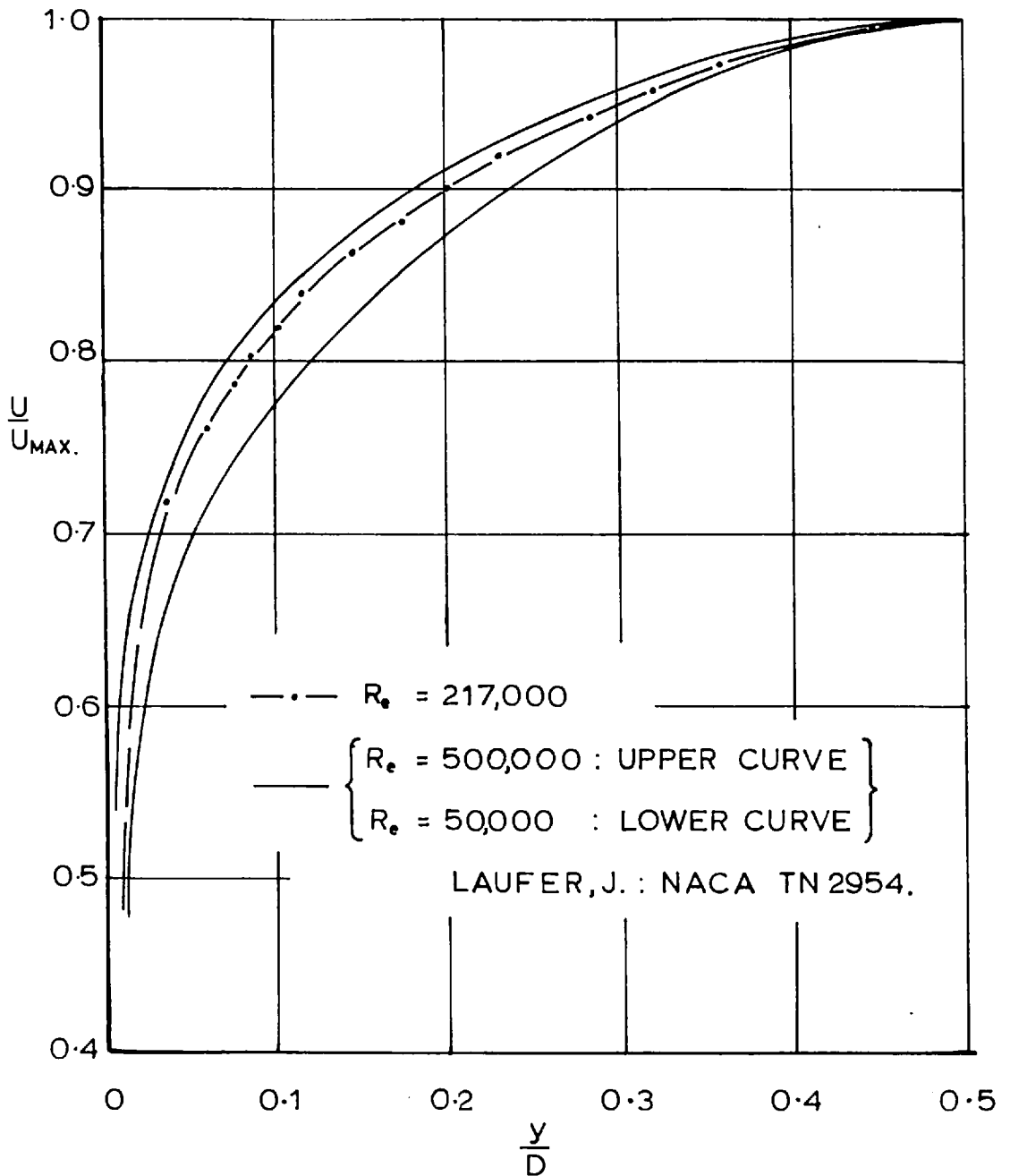


FIGURE 4.3. UPSTREAM MEAN VELOCITY DISTRIBUTION.

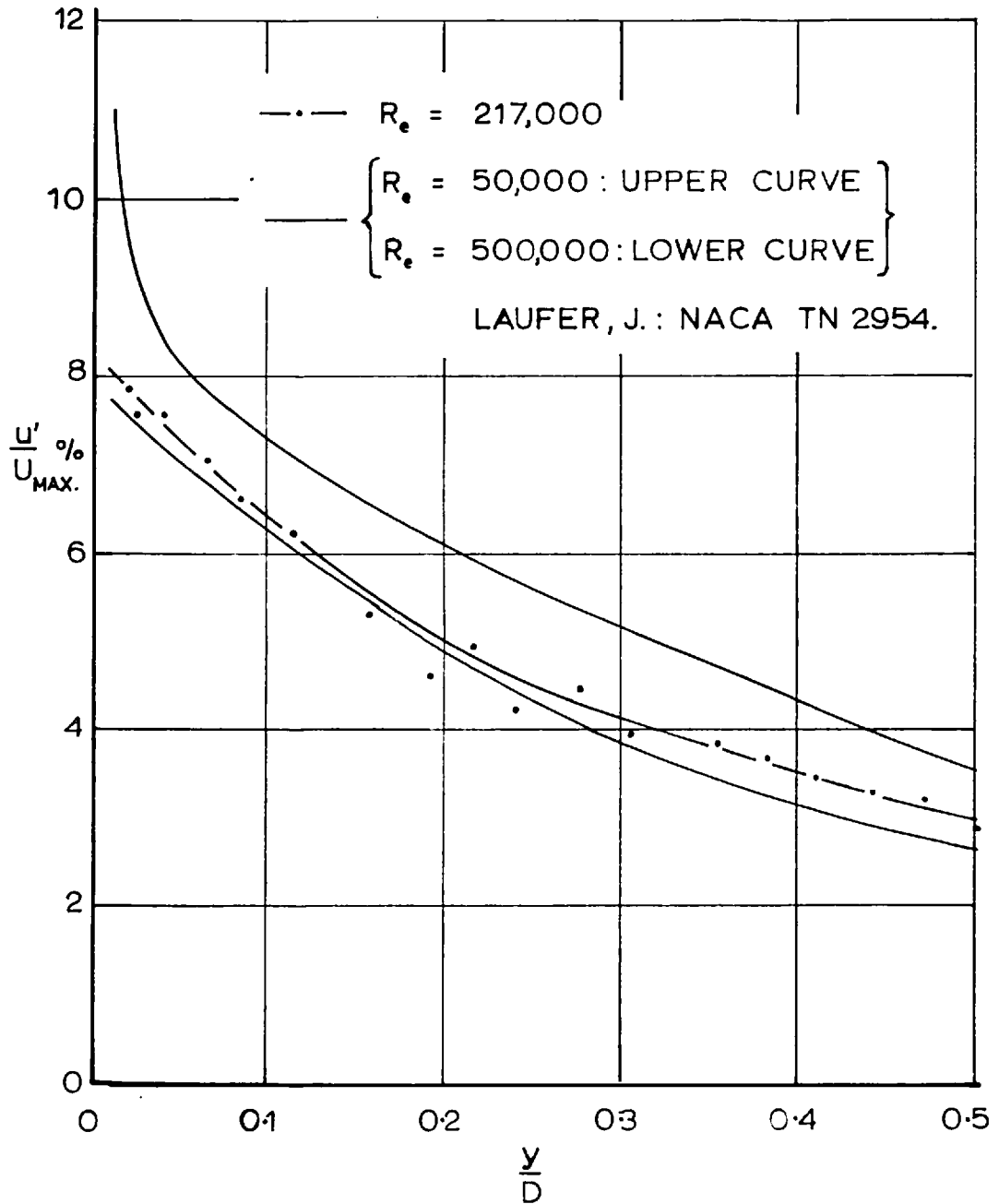


FIGURE 4.4. UPSTREAM
DISTRIBUTION OF u' .

5. FLOW VISUALIZATION.

5.1. TESTS IN AIR.

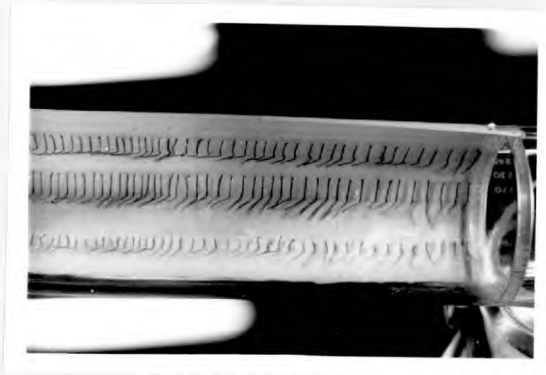
5.1.1 WOOL TUFTS: Three 12 inch rows of $\frac{3}{4}$ inch long wool tufts were sellotaped to the inside of the rotatable section 30° apart, immediately downstream of the bend. Photographs of the tufts were taken using a 1/1000 sec. exposure and are shown in figures 5.1 (a), (b) and (c). The motion of the tufts confirmed that the flow was unsteady and on rotating the section through 360° , it was apparent that, over a period of time that was large compared with the characteristic period of the unsteadiness, the flow was symmetrical about the plane of the bend. Three regions could be distinguished and are sketched in figures 5.2 (a) and (b).

(a) A region of reverse flow immediately downstream of the inside edge of the bend, approximately three-quarters of a diameter in length.

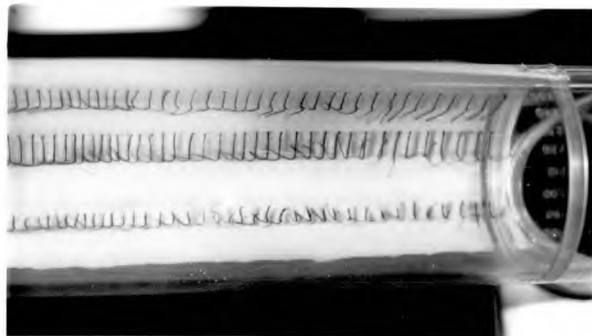
(b) A region downstream of region (a) in which the tufts "switched" from the configuration shown in figure 5.1 (a) to that in figure 5.1 (b). The switching appeared to occur at a random low frequency consistent with the fluctuations initially observed in the upstream mean velocity. The violence of the switching decreased in the direction of the bend on the outer wall until its strength was very small approximately opposite the inner edge.

(c) A region above and below region (a) extending to approximately $\pm 120^\circ$ where the tufts pointed consistently inwards towards the inner wall. Their inclination fluctuated but did not switch as did those in region (b).

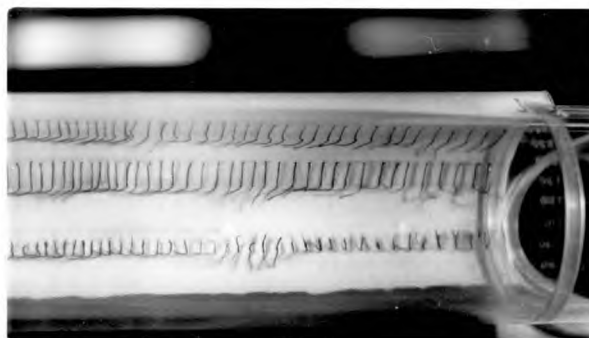
The tufts within the reverse flow region (a) were affected by the switching disturbance and assumed the same mode as those in region (b).



(a) CLOCKWISE SENSE.

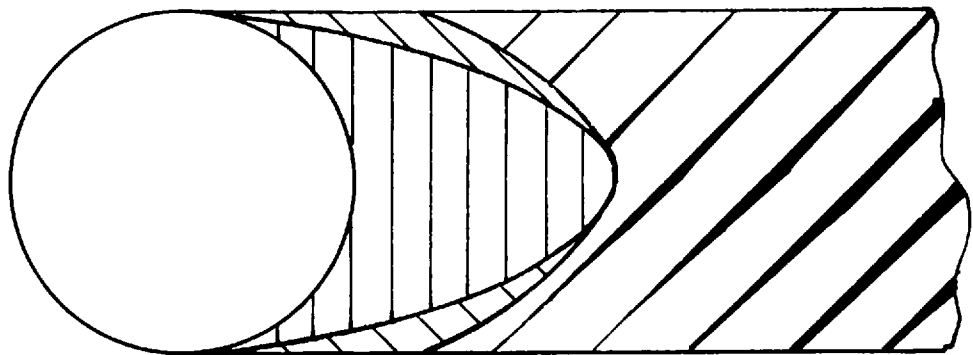
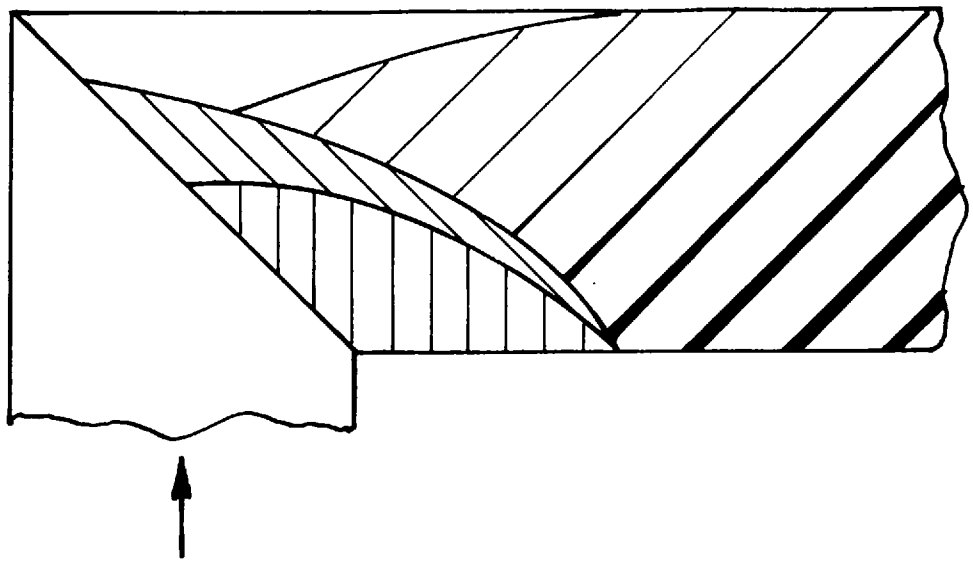


(b) ANTICLOCKWISE SENSE.



(c) UNDERGOING SWITCHING.

FIGURE 5.1. WOOL TUFTS.



REGION (a)



REGION (b)



REGION (c)

ALL REGIONS
LIMITED TO FLOW
AT THE NEAR WALL.

FIGURE 5.2.

Figure 5.1 (c) shows the tufts in the action of switching; it is clear that although the switching frequency is low (order of 1 c/S), the operation of switching is very fast (10^{-3} sec. exposure) and this must be borne in mind when interpreting the photographs.

5.12. TALC INJECTION: Two 1/16 inch diameter holes, 110° apart were drilled in the rotatable section one inch downstream of the inside edge. First attempts were made with smoke from a standard N.P.L. generator; these were a complete failure: the smoke was diffused immediately. It was clear that a very dense plume was necessary and if an acidic smoke such as titanium chloride were used, the large quantities would cause corrosion of the downstream limb and the injector. Talc injected from a polythene "squash bottle" was found successful, although it was difficult to produce a consistent stream for more than a few seconds.

The talc immediately revealed the nature of the low frequency unsteadiness when injected into regions (b) or (c) of figure 5.2 (~~(a) and (b)~~); it was entrained by a longitudinal vortex which swept the entire pipe wall downstream of the reverse flow region (region (a) of figure 5.2). Such a vortex existed for periods of approximately a second, switching abruptly to a vortex of similar strength but reverse sense; the dominant vortex fluctuated in strength. Sketches of the two vortex configurations are shown in figures 5.3 (a) and (b), both talc streams being injected into region (c) of figure 5.2. Both streams initially move inwards but the non-dominant rapidly becomes diffused and appeared to be pushed away from the wall towards the central portion of the flow. The dominant vortex appeared to be continuous through the region of impingement rather than being

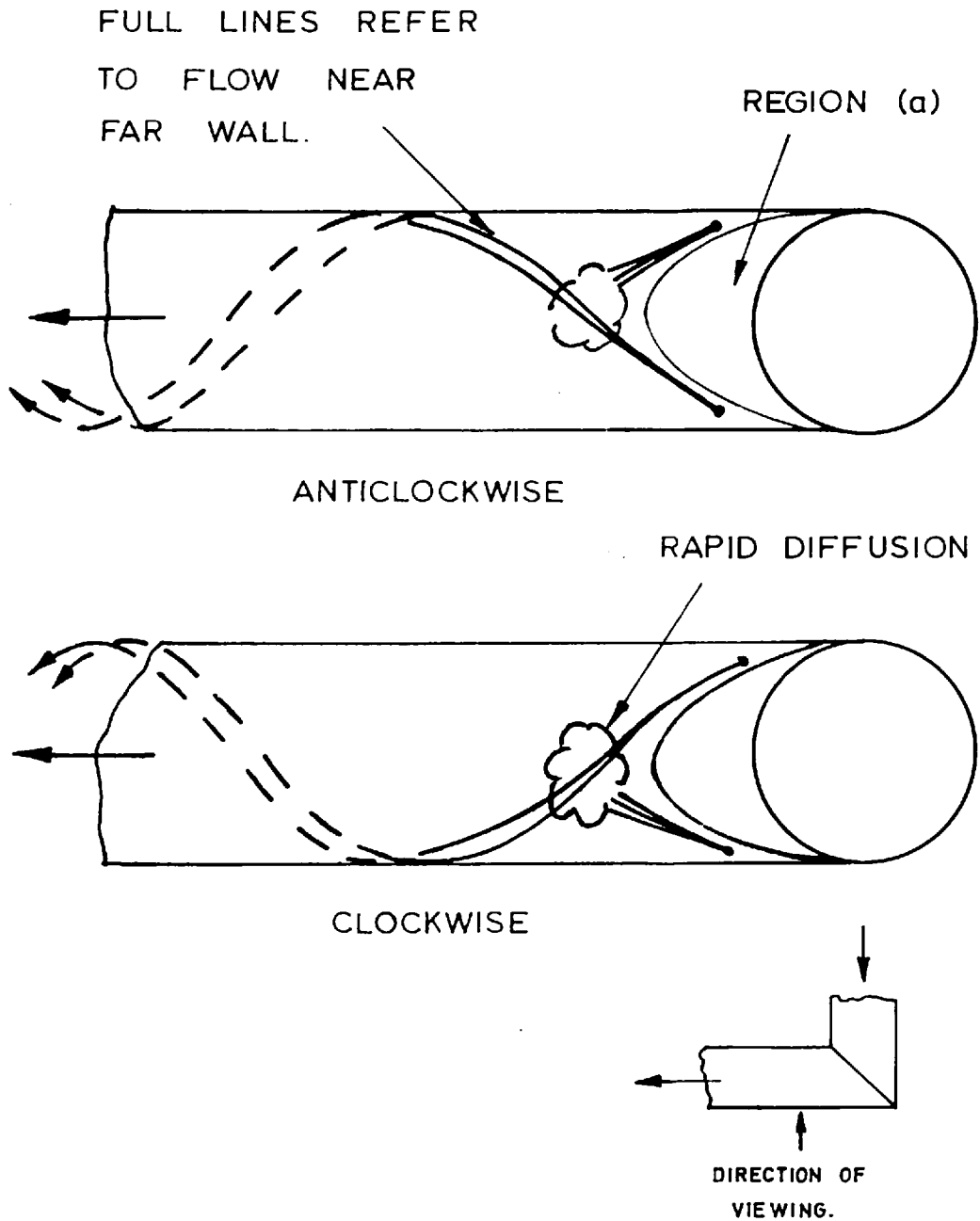


FIGURE 5.3. SKETCH OF TALC
TRACES.

partially cancelled out by the lesser vortex (see figure 5.3). Attempts at photographing the talc traces were not very rewarding: the results, however, are shown in figures 5.4 and 5.5. Figure 5.4 (c) has caught the intermediate stage during switching; both streams now diffuse rapidly. Figure 5.5 shows views looking downstream through the end wall of the bend and are particularly poor because of the long field of view required.

Talc injected into the reverse flow region revealed nothing of the nature of the flow within it and gave only a very approximate idea of its size. Rotating the section and injecting talc at the outside wall showed that the flow was approximately axial in this region and it was not until 2 - 2.5 diameters downstream that the dominant vortex swept the outer wall.

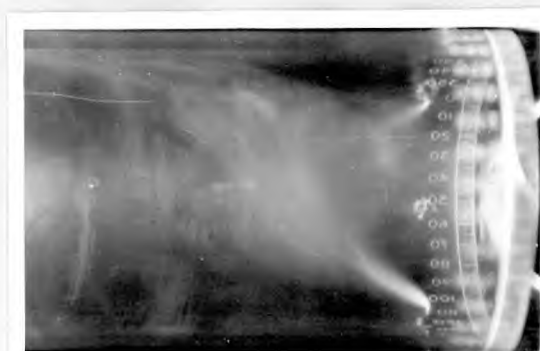
Talc was also injected upstream of the bend and the flow model based on these observations together with those of the previous photographs, is shown in figure 5.6. These sketches are drawn for the case of the dominant clockwise vortex. Only the clear details are shown: no attempt has been made to show the shear layer or the diffuse region in the centre of the flow. The effect of the outer corner appeared to "focus" the talc traces into the vortex; it was not apparent whether the flow separated in this corner.

5.2 TESTS IN WATER

Before proceeding with measurements in the air rig, it was decided to run some tests in water in a simple model of the bend in order to determine whether the phenomenon of vortex^{switching} was a function of right angle mitre bends in general, or of the particular rig that had been



(a) CLOCKWISE SENSE.

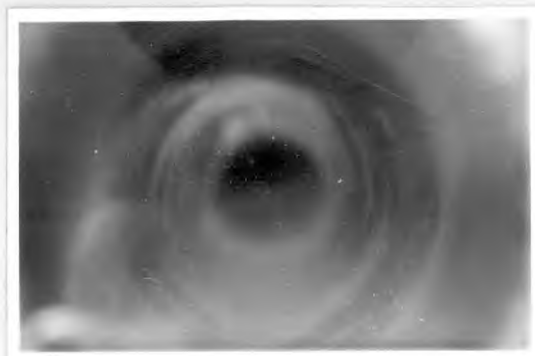


(b) ANTICLOCKWISE SENSE.

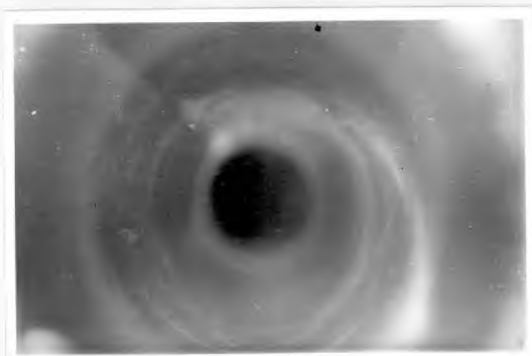


(c) UNDERGOING SWITCHING.

FIGURE 5.4 . TALC TRACES.



(a) CLOCKWISE SENSE.



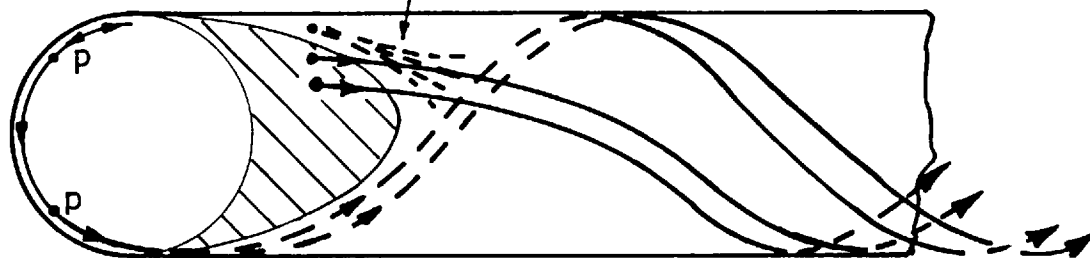
(b) ANTICLOCKWISE SENSE.

FIGURE 5.5 . TALC TRACES.

'p' indicates talc injection point.

Dotted lines refer to flow in lower half of pipe.

Talc trace moving in this sense is dispersed (omitted in other views).



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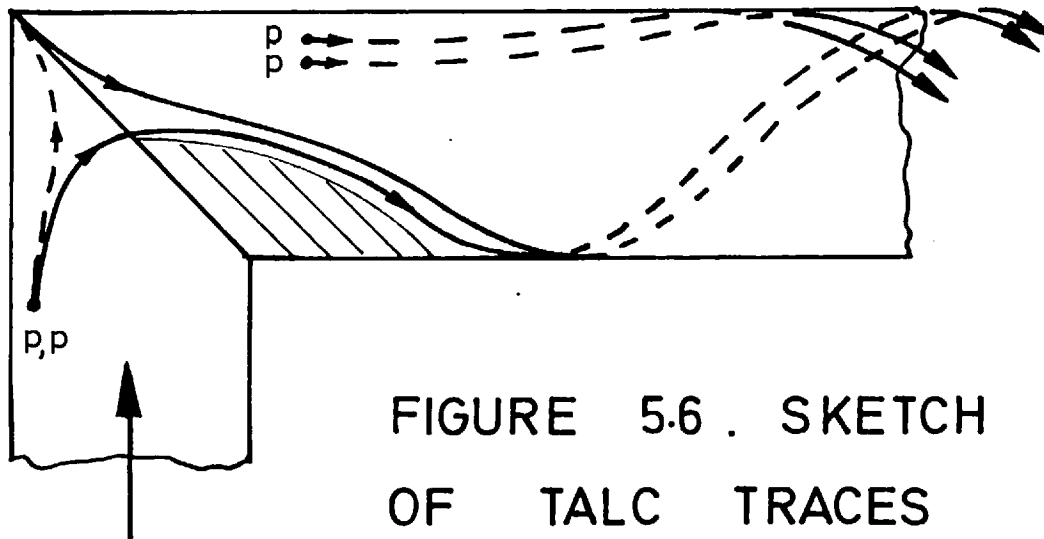
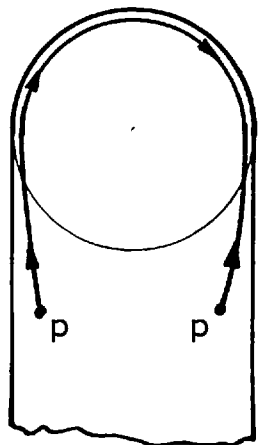


FIGURE 5.6 . SKETCH
OF TALC TRACES
(clockwise vortex).

assembled. The model is described in section 2.2; the maximum obtainable Reynolds number was 40,000. Coloured milk was injected into the flow but diffused very rapidly and defied photography. All subsequent tests were therefore performed with saturated aqueous potassium permanganate solution and commercial water soluble dyes injected by means of a peristaltic pump.

It was immediately obvious that the flow in the water model was similar to that existing in the air rig, that is, a dominant vortex and vortex switching occurred. By injecting directly into the reverse flow region the extent of the shear layer could be seen clearly over approximately the first diameter, beyond this the inwards flow either side of the region complicated the geometry beyond identification. An important observation was made: the reverse flow region and shear layer was forced to oscillate about the symmetrical position by the switching of the dominant vortex. The reverse flow region consisted essentially of a single cell horseshoe vortex, one of whose arms appeared to link up with the dominant circulation round the edge of the region. Secondary vortexes were formed in the shear layer. The frequency of switching was still low and random, but probably a little higher than in the air rig (approximately 2-3 c/S).

An additional feature of the flow observed in these tests, was the existence of a strong, single cell, horseshoe vortex in the outer corner whose arm trailed downstream. It switched about its symmetrical position in sympathy with the dominant vortex and the arm which moved the further downstream had the same sense as the dominant vortex and appeared to re-inforce it.

The effect of varying the Reynolds number was small:

this was also noticed in the air rig when running unchoked. Down to a Reynolds number of approximately 3,000, the flow remained similar although the edge of the shear layer became progressively less well defined as the size of the secondary vortices increased. Below this figure, the switching decreased in definition and frequency until the two secondary circulations merely intermingled and mixed as they proceeded downstream. The outer corner vortex oscillated between the corner and an upstream position but remained fairly symmetrical about the plane of the bend.

Injection into the flow at a point some four diameters downstream of the bend indicated a much weaker resultant vortex than the initial circulations. This contrasts markedly with the talc observations where the dominant vortex appeared of comparable strength to the initial circulation.

The flow model based on these observations is shown in figures 5.7, 5.8 and 5.9.

5.3. EFFECT OF THE OUTER SEPARATION.

In order to determine the effect of the outer corner vortex, the corner was faired out with resin on a radius approximately equal to the pipe diameter. This did not completely remove the vortex but reduced its strength and size very considerably. The resultant flow appeared almost identical to the previous one. Switching occurred at a similar frequency although the pitch of the secondary circulations had increased slightly, together with the length of the reverse flow region.

In the air rig, the outer corner was faired out with plasticene until wool tufts mounted in the plasticene ceased to indicate reverse flow. This occurred at a radius slightly greater than the pipe diameter. The

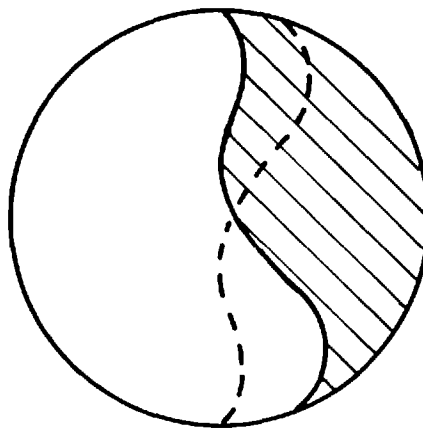
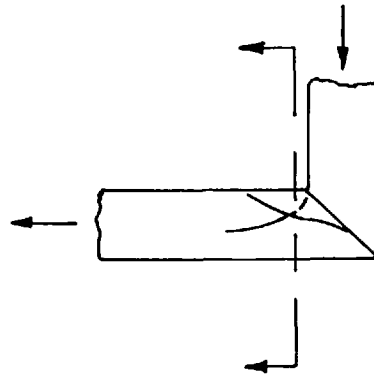


FIGURE 5.7. SIMPLIFIED
SKETCH OF OSCILLATION
OF SEPARATION.

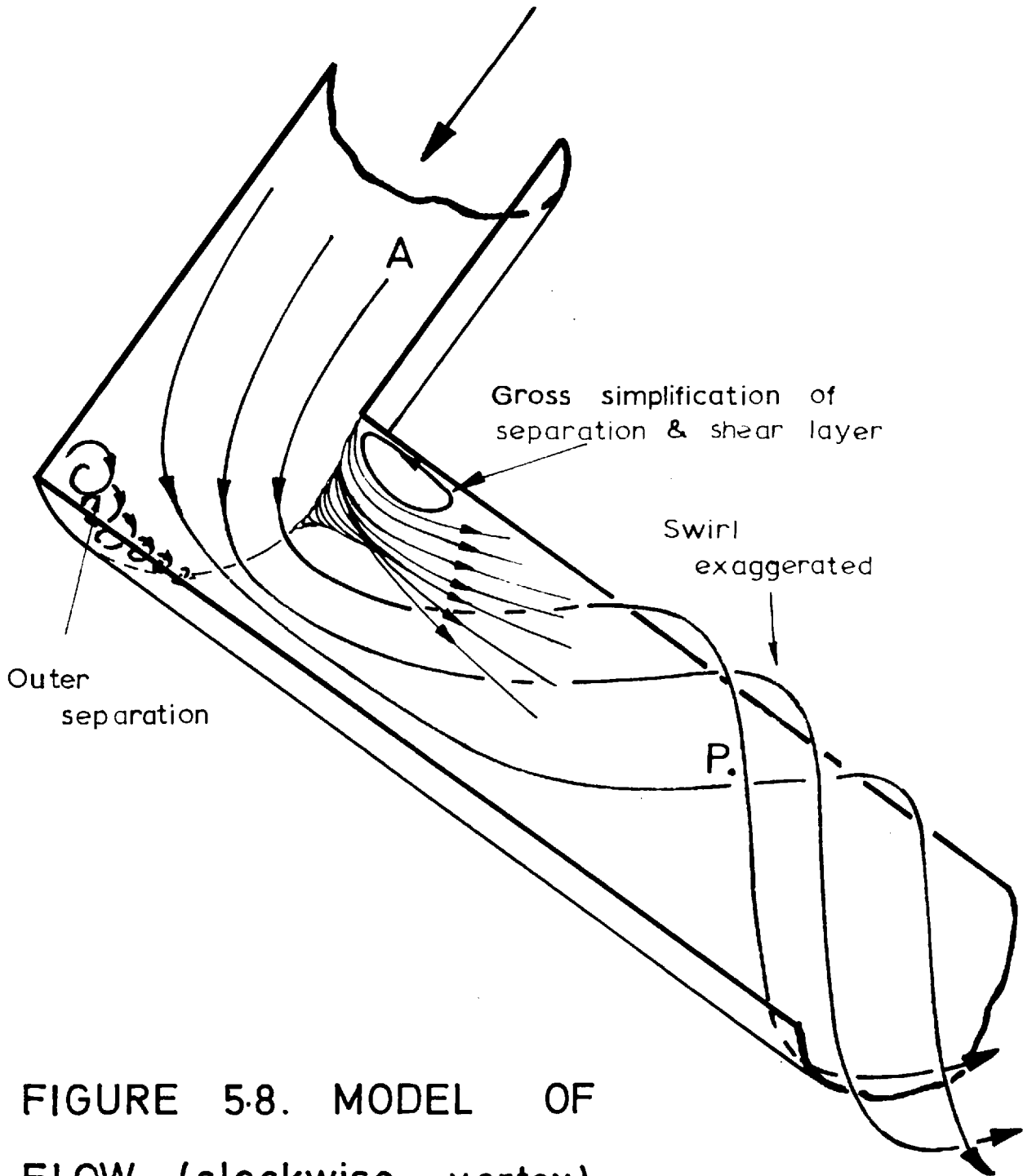


FIGURE 5.8. MODEL OF
FLOW (clockwise vortex).

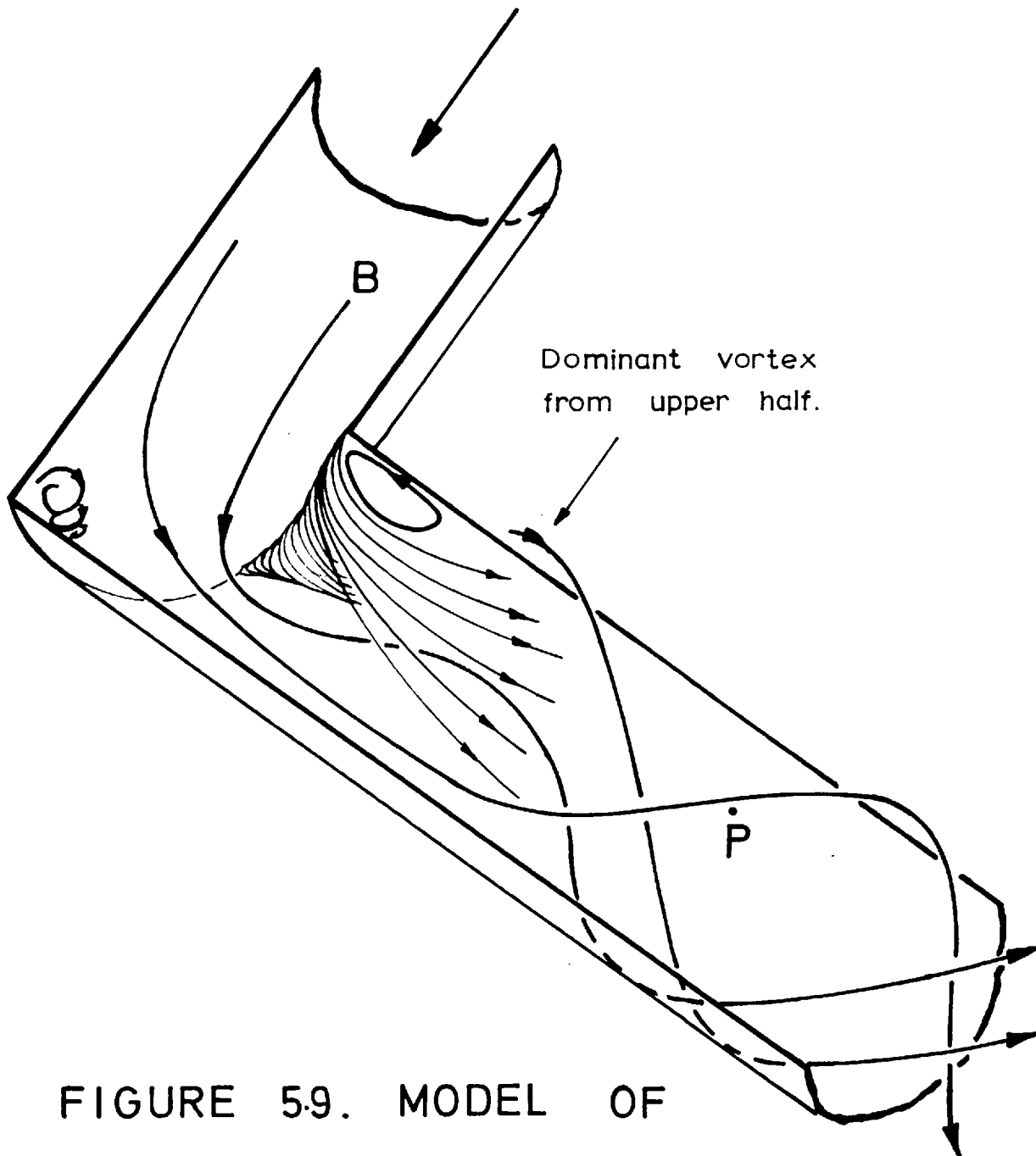


FIGURE 5.9. MODEL OF
FLOW (anticlockwise vortex).

observations of the resultant flow demonstrated trends similar to the water flow.

5.4. MODEL OF THE SECONDARY FLOW.

A simple model of the flow was set up in water. Fully developed turbulent flow through a length of $1\frac{1}{4}$ inch bore perspex tube, had two tangential jets of dyed water injected, diametrically opposed, and set at 45° to the axis. With unequal jet velocities, a resultant downstream circulation was set up which was clearly the difference between the two jet tangential velocities, but which fluctuated slightly in magnitude. A cross flow in the plane of symmetry was also produced.

6. SUMMARY AND DISCUSSION OF OBSERVATIONS.

- (1) In the region immediately downstream of the bend, fairly steady, secondary circulation was produced in both senses in the usual way. However, one vortex appeared to dominate over the other which was diffused and pushed towards the centre of the flow. Thus downstream of the impingement region a single vortex swept the wall.
- (2) The resultant vortex could be either clockwise or anticlockwise and switched between these two states.
- (3) The dominant vortex existed for periods of typically one second, but the action of switching was very rapid. The frequency was apparently random. Its strength fluctuated.
- (4) A region of reverse flow occurred immediately downstream of the edge and this region was not symmetrically disposed about the plane of symmetry but was forced a little to one side by the dominant vortex. It oscillated in sympathy with the switching.
- (5) The flow separated at the outer corner but this had little effect on the switching or flow pattern.
- (6) The flow pattern was basically independent of Reynolds number provided this exceeded approximately 3,000. Below this value the vortices merely intermingled with no domination.
- (7) In water, the resultant vortex had a much larger pitch than that indicated by the talc in air. A model of the flow downstream of the bend, using fully developed turbulent flow through a straight pipe with two tangential, impinging jets, produced a resultant vortex whose tangential velocity was clearly the difference of the two jet tangential velocities. A cross flow was set up in the plane of symmetry.

Hawthorne's analysis reproduced in section 1.2 clearly demonstrates that the boundary layer entering the bend must produce secondary circulation in both senses, and this is manifested in the inwards flow occurring around the periphery of the reverse flow region. If symmetry were preserved, these circulations would be of equal strengths, but the displacement of the inner separation destroys the symmetry. Thus an initial perturbation of the separation tends to thin it in one half (above or below the plane of symmetry) and thicken it in the other. This both allows more fluid to flow in paths of higher curvature in one half than the other, and also increases and decreases the shear respectively. Both these effects will result in the production of a greater circulation in one half which will react on the separation such as to assist the perturbation. This displacement must reach an equilibrium since the flow is still obliged to separate at the salient edge.

The observation from the talc traces, which is evident in figure 4.7(a), that the strength of the dominant vortex was unchanged after impinging with the lesser vortex, did suggest that perhaps a mechanism existed which allowed the stronger vortex to adhere to the wall in preference to the weaker. Such a result could, perhaps, be brought about by the Rayleigh instability which must exist in each vortex as it rotates around the wall of the tube. Under such conditions particles with a higher circumferential velocity tend to move towards the wall and slower particles away from the wall. Thus if, in the region of impingement of the two streams, at each point from the wall the particles of one stream are moving faster than those of the other, the faster stream would then tend to the wall.

Such conditions should exist in the simple straight pipe flow with tangential injection but the resultant downstream circulation in this case was certainly the difference of the two circumferential velocities, as was suggested by the injection downstream in the water flow through the bend. It is concluded, therefore, that the talc stream was strongly affected by the density of the talc and in this respect does not accurately represent the flow.

Weske (reference 18) demonstrated that by placing a gauze screen normal to the flow and to one side of the plane of symmetry upstream of the bend, a steady single vortex exists downstream of the bend and clearly this has similarities to the present flow. An interesting point which is suggested by his results but on which he does not comment, is that while the symmetrical profile flow through the bend separated, the screened flow at the same Reynolds number appears not to have separated. It is quite conceivable that the single vortex flow is set up within the bend and because, unlike the two vortex flow, it has no stagnation point at the inner wall, manages to overcome the separation. If this is the case, it would be worth knowing whether the resultant head loss is reduced by such a scheme, apart from the obvious advantages to be gained from suppressing the fluctuating loads produced by the separation. Alternatively, it could be used to allow the geometrical convenience of a tighter radius bend but which does not cause separation.

Returning to the present flow, it remains to decide the nature of the switching disturbance. The outer separation can be ruled out as a source since a similar flow was in evidence when this separation was removed. Considering the dominant vortex, it is known that flows

with combined axial and swirl components can exhibit "vortex breakdown" or "bursting"; this has been studied experimentally by Harvey (reference 45) and theoretically by Brooke Benjamin (reference 46).

It is possible to consider, therefore, that initially, by some means unspecified, one vortex dominates over the other, displaces the separation such that the vortex increases in strength, subsequently breaking down and allowing the other vortex to dominate. The characteristic feature of breakdown is an abrupt change of structure of the vortex at a point on its axis which has been shown to be analagous to the hydraulic jump in an open channel. It appears to occur under conditions of increasing swirl and a swirl velocity which increases radially outwards. Comparing these observations with those made in the present work suggests that vortex breakdown does not occur since:

- (a) The growth of the dominant vortex is necessarily associated with the time taken to "switch" which is very short; thus the resultant switching frequency would be high.
- (b) No abrupt change in the structure of the dominant vortex was observed.
- (c) The measurements of Ito et al. in a symmetrical, twin vortex, pipe bend flow show that the swirl velocity in the shedding layer increases slowly outwards then decreases rapidly as the wall is approached.

If the disturbance producing switching were caused by an instability of the shear layer associated with the separation, such as vortex shedding, this might be expected to have a time scale associated with the diameter of the pipe and the mean velocity and hence be of order 10^{-2} seconds in air, that is, much shorter than the switching

scale. Such a scale would be directly affected by Reynolds number which was not apparent. The fact that below a Reynolds number of approximately 3,000 the switching ceased necessarily suggests that it is associated with turbulent flow: either the mean velocity profile or the turbulence structure and the latter, being a source of random unsteadiness, is the more likely.

In order to effect a switch, the disturbance would need to have a circulation of the magnitude of the resultant downstream vortex. It is, therefore, necessary to determine whether the upstream turbulence could generate such a circulation and how often it would be expected to occur. The nature of the flow made the measurement of the downstream, circumferential velocity extremely difficult and this was not done. Weske's results show the maximum circumferential velocity in the shedding layer for a fully developed turbulent flow entering a 90° bend of radius ratio, $a/R = 0.67$, and at a comparable Reynolds number in air to be 0.18 of the upstream mean velocity. Observations in water of the pitch of the two initial circulations would suggest a ratio of roughly two for their circumferential velocities, hence, using Weske's figure, an estimate of the difference between these two velocities and the circumferential velocity of the resultant vortex, is 0.1 of the upstream mean velocity, that is: $w/U_m = 0.1$.

A relationship for the mean square circulation around a circular path in isotropic turbulence (reference 47) is:-

$$\overline{\kappa^2} = 4\pi r L \overline{u_T^2} \quad \text{provided } r \gg L$$

where r is the radius of the path, L is the lateral integral scale and $\overline{u_T^2}$ is the mean square tangential

velocity component of the turbulence. This result represents the production of large eddies from a chance configuration of the turbulence and while it is an isotropic result, it is not critically affected by this.

Thus the r.m.s. tangential velocity around the circuit is given by:

$$\frac{(\overline{K^2})^{1/2}}{2\pi R} = W_T' = \left(\frac{L}{\pi R}\right)^{1/2} u_T'$$

Taking a circuit of radius approximately the radius of the pipe, that is, $r \simeq a$, then from Laufer's results:

$$u_T'/u_m \simeq 6\%$$

The structure of turbulent pipe flow discussed in Hinze (reference 48) sets the value of L at roughly $0.25a$. The condition $r \gg L$ is more strictly the condition that the correlation between the tangential velocity components at two points on the circuit shall fall to zero for a separation that is small in comparison with r . Such a correlation distance will be larger than L by a factor depending on the shape of the correlation function $R_{22}(y; 0, \frac{r}{2}, 0)$, being equal to L only for a rectangular function. While measurements of this function have not been made, the general shapes of correlation functions would suggest a ratio of two to three. Thus the required condition is not strictly observed; however, such functions fall off rapidly with separation and the error is therefore small and certainly acceptable for the present argument.

Substitution of these estimated values leads to:

$$\frac{W_T'}{u_m} \approx 1.7 \times 10^{-2}$$

The turbulent circulation around this circuit of approximately the pipe radius will re-inforce one

secondary circulation and oppose the other. Thus the required tangential velocity around the circuit need be of only half the value of that of the resultant vortex or $W/U_m \approx 5 \times 10^{-2}$. Hence the required value of W_T is roughly three times its r.m.s. value, W_T^0 . Assuming a Gaussian distribution for the velocity fluctuation, u_T , the probability of a deviate lying between three times the standard deviation and infinity can be determined. The probability distribution curve, however, is very sensitive and to apply the result of the preceding estimates to it is, perhaps, facetious. However, if it is assumed that the required deviate lies between roughly 2.5 and 3.5 times the standard deviation and infinity respectively, then the corresponding probability lies between 6.2×10^{-3} and 2.3×10^{-4} .

Approaching now the problem from the opposite direction: if the number of switches per second is n , then, since switching will occur only for consecutive disturbances of opposite sense and the probability of this occurring is a half, assuming the upstream flow has no bias, the number of disturbances per second of a magnitude capable of producing switching is $2n$. Thus during one second, the length of time for which a disturbance of such a magnitude exists at a point is $\frac{2nl}{U}$, where l is the characteristic axial length and U is the convection velocity of the disturbance. This is also equal to the probability, P .

For a fully developed, turbulent pipe flow l is $\approx 0.8A$ and assuming that U is roughly U_m , then:

$$P \approx 2.3 \times 10^{-3} n.$$

Equating the probabilities gives the value of n ranging from 2.7 to 0.1 c/S, to be compared with the

characteristic frequencies of 1 c/s in air and 2-3 c/s in water. Clearly one would also expect the turbulent circulation to produce continuous fluctuation in the strength of the resultant vortex, as was observed.

Nothing more can be claimed for the above argument than that it does suggest that the turbulence can produce disturbances of the required magnitude with a comparable time scale to the switching.

The latter part of the argument expresses the probability as $2 \frac{n_l}{U}$ which is clearly a Strouhal number type term and hence might be expected to remain fairly constant over some range of Reynolds number. Present observations would suggest that it does not since there was no clear variation of n with U . This point can only really be resolved by a spectral analysis of n at widely different values of U , demanding a technique for handling low frequencies. It must be emphasized that n appeared random and that the characteristic values ascribed to it are merely estimates.

Considering the other quantities, the effect of Reynolds number on U_r/U_m is small, in Laufer's data, its value increased by less than 10% for a decrease by an order of magnitude in U_m and thus assuming the turbulence structure remains similar, w_T/U_m might be expected to remain fairly constant. The downstream secondary circulations are proportional to the upstream shear, which $7/4$ for a turbulent pipe flow will vary approximately as U_m , assuming the Blasius result. This would suggest that w/U_m will vary as $U_m^{3/4}$, but since w is the difference of the two circulations and is governed by the relative dimensions and shears of the separation, this is something of an inponderable. Clearly measurements of w are needed but this presents a very substantial experimental problem in itself.

The production of such a turbulent circulation obviously applies to other pipe flows and thus any pipe bend producing separation might be expected to also produce switching under such conditions. For a square section bend, the top and bottom walls will constrain the separation, preventing it from oscillating. The turbulent circulation will therefore appear as fluctuations in the strength of the twin vortices. The measurements of Curtis, Feil and Lignormik in such a flow confirm that switching does not occur but fluctuations in the secondary flow are not mentioned. Their flow does not strictly conform, however, since it was not fully developed.

Weske's work includes flow through circular section, 90° bends producing separation with an upstream fully developed turbulent flow and hence switching might be anticipated, but none was recorded. No flow visualization techniques were employed, but the small yaw meter used for measuring the tangential velocity components in the shedding layer would presumably have indicated such a fluctuation unless the time constant of the meter was very large ($\sim .030$ inch bore but of unknown length). The text implies that the tangential flow at the outer wall was quite steady and only on approaching the inner wall was unsteadiness, produced by the separation and "eddy region", recorded. These results seem incompatible with the present work, but future experiments which are envisaged on bends with small radii at the inner walls should provide decisive information.

The discussion is summarised in the following conclusions:

- (1) A lateral perturbation of the separation from the symmetrical position is unstable since it allows one secondary circulation to grow at the expense of the other and hence

re-inforce the perturbation. This process is limited by the salient edge which forces the flow to separate.

(2) The difference between the two circulations produces a downstream single vortex.

(3) In order to effect a switch, a disturbance is required to perturb the separation to the other side of the plane of symmetry.

(4) The frequency of switching is much lower than that related to the characteristic velocity and length scale of the flow.

(5) Turbulent flow is essential to the switching phenomenon.

(6) Estimation of the turbulent circulations in the fully developed pipe flow upstream of the bend suggests that the magnitude required to effect switching would occur at a time scale comparable to the observed magnitude.

(7) The effect of Reynolds number on this phenomenon requires quantitative measurements of the switching frequency and downstream tangential velocity.

(8) Such results should apply to other separated, turbulent pipe bend flows, but Weske's results appear contradictory at present. Future work should resolve this.

(9) It may be possible to suppress separation in a given bend geometry by forcing a single vortex, secondary flow to be established.

7. EXPERIMENTAL MEASUREMENTS.

As may be anticipated from the preceding discussion of the nature of the flow, it was exceedingly difficult to make sensible measurements of it. Any transducers placed within the flow were, in general, operating under conditions of widely variable yaw and high turbulence levels. While time means of any measurements could always have been taken, they would not have been meaningful nor would they have reflected the real flow. Because the switching caused the separation and shear layer to oscillate, such considerations applied to virtually the entire flow field for several diameters downstream of the bend, thus the initial intention of a careful examination of the separated flow field was frustrated.

7.1. HOT-WIRE MEASUREMENTS.

Although the instantaneous flow pattern was not symmetrical with respect to the plane of the bend, the two flow configurations were mirror images about this plane. Hence the velocity components to which a simple u-wire is sensitivity were less affected in this plane than elsewhere in the flow. Four mean velocity and u - component turbulence profiles in this plane are shown in figures A1 - A7.

Off the plane of symmetry, the turbulence level indicated by a u-wire was in general, except within the reverse flow region, of a distinctly two level nature. A typical output trace is shown in figure 7.1. At the left and right of this trace are comparatively low turbulence levels, while the centre of the trace reveals large departures from the mean. From the knowledge of the flow gained from the previous experiments, it can be reasoned that, off the plane of symmetry, the fluid

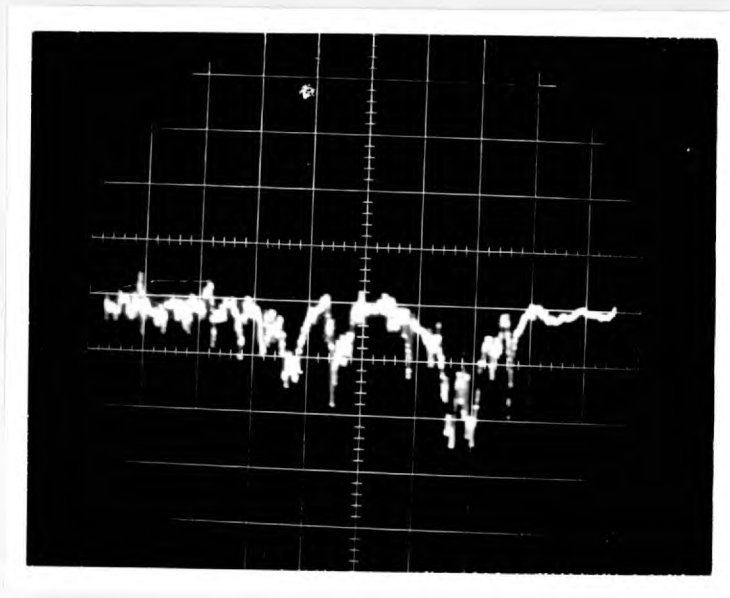


FIGURE 7.1. TYPICAL HOT-WIRE
OUTPUT TRACE.

reaching a point when the clockwise vortex exists will have had a very different history from that when the anti-clockwise vortex dominates. In particular, referring to the sketches in figures 5.8 and 5.9 when the clockwise vortex exists, fluid convected to the point 'P' near the upper wall will have come from near the separation and hence might be expected to be very turbulent, while the anti-clockwise vortex will bring fluid which has not been contaminated by the separation.

In order to test this argument and, hence if it is found valid, confirm the nature of the flow field, it was decided to attempt to detect the instantaneous flow state by some means and thus to examine the turbulence in the two states separately.

First attempts were made with a u-wire set such that it was approximately parallel to the estimated mean path of one of the vortices, as shown in figure 7.2. While a change in mean bridge voltage was detectable, the level was too small and fluctuations superimposed on the mean voltages were frequently in excess of this difference.

A device which was used very successfully is the small flag shown in figure 7.3. The flag was made of 0.002 inch gold shim and the contacts of 0.008 inch gold-plated wire. It was considered advisable to use gold in order to ensure a good, low resistance contact and to avoid arcing which, even with a low voltage (6 volts), would rapidly destroy the contact points. The flag was connected up and mounted as shown in figure 7.3. Examination of the flag output revealed that a clean "switch" was not being achieved, but that as the vortex drove the flag against a contact, the flag bounced on the contact to produce "grass" made up of discreet pulses characteristically 0.5 ms in length and at 4 ms intervals. It was also clear that while predominantly in one state,

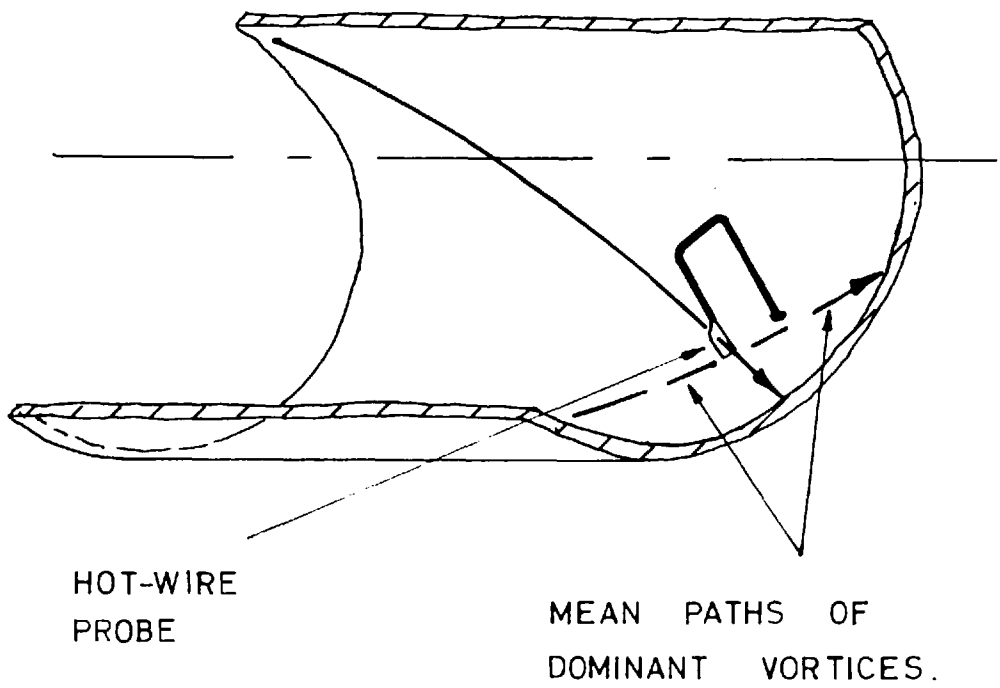


FIGURE 7.2 .

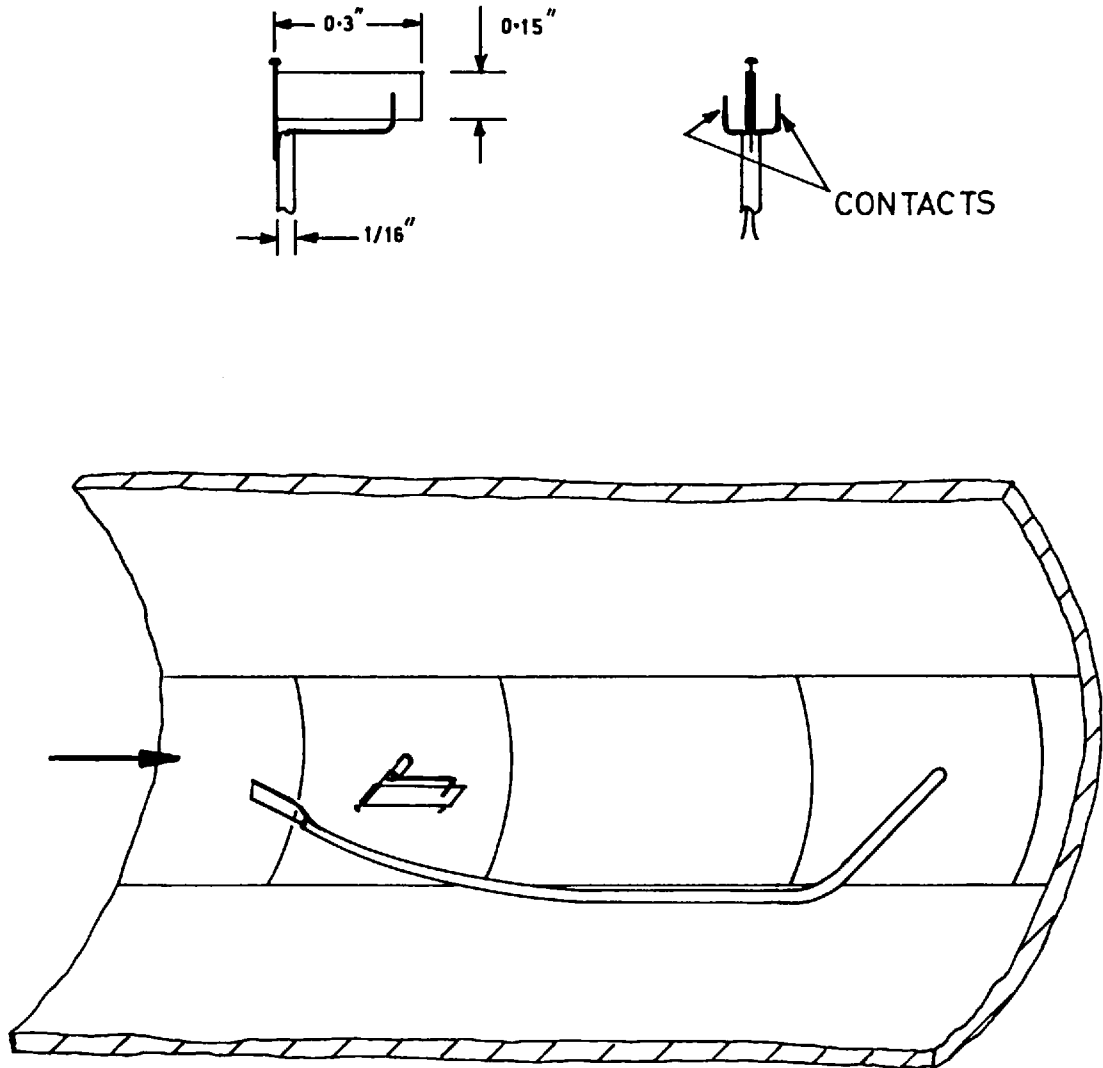


FIGURE 7.3. FLAG and MOUNTING DETAILS.

switches to the other state indicated by a single or double pulse were occurring; an example of the flag output is shown in figure 7.4 where three major vortex switches occur together with several single pulse switches. Such switches are of a much shorter time scale than that associated with the vortex switching and may, perhaps, have been produced by the turbulence content of the flow. Moreover, since a mechanical device was being used, it was likely that there would be a significant phase difference between such short period disturbances and the corresponding motion of the flag. Hence, using the flag output to switch an analogue signal derived from the flow, large errors could be introduced. It was, therefore, decided to ignore these short period switches.

In order to derive from the flag a signal which could be used to switch an analogue signal, obtained from either a hot-wire or pressure transducer, and which would also ignore short switching intervals, the logic circuit of figure 7.5 was devised and built up using "Farnel Logic Instruction" equipment which, being a "building brick" system, is very suitable for such applications. Since the pulses produced by the flag were very regular, it was decided that any switching interval less than some small arbitrary number of pulses would be neglected. Examination of the flag output indicated that although there were many switches of one or two pulses, there were few in the range three to ten pulses. The number four was chosen as the limit. Thus the logic would inevitably lag behind the flag by approximately 15 ms, but in view of the average switching interval, this was acceptable.

The action of the logic circuit was, therefore,

CLOCKWISE



ANTI -
CLOCKWISE

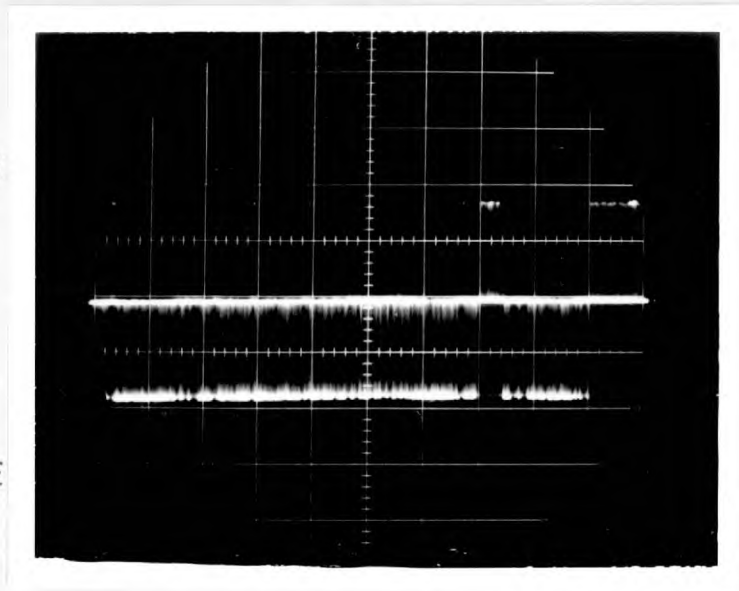


FIGURE 7.4. FLAG OUTPUT :
HORIZONTAL SCALE = 0.5 cm/sec..

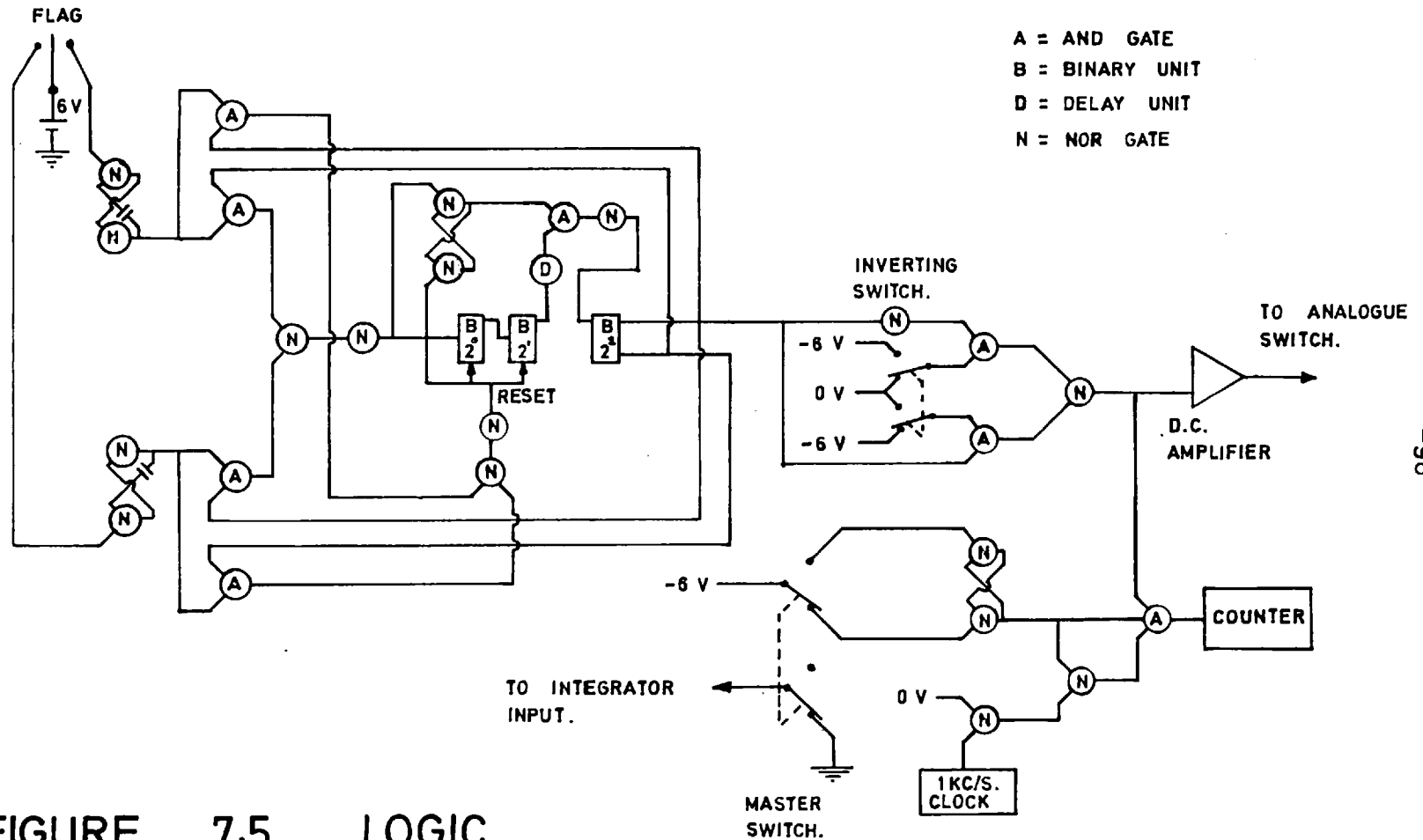


FIGURE 7.5. LOGIC
CIRCUIT TO PROVIDE SWITCHING FUNCTION.

as follows: with the logic output in the state corresponding to a clockwise vortex (say), pulses from the flag in the anti-clockwise sense were counted and those in the clockwise sense neglected. If the number of consecutive anti-clockwise pulses reached four, the logic output switched over to indicate the anti-clockwise state and the procedures were reversed. If, however, the number of consecutive anti-clockwise pulses did not reach four, the next clockwise pulse reset the count to zero. An example of the logic output, that is, the switching function, following the flag output is shown in figure 7.6 and the switching function compared with the hot-wire bridge output trace in figure 7.7. The switching function was used to drive an analogue switch operating on the transducer output and also to gate a clock, thus allowing the interval for which a particular state existed to be timed. The gated transducer output was then taken to the spectral analyser, the mean square of whose output was amplified and integrated by an operational amplifier. A master switch enabled the integrating and timing operations to be started and stopped coincidentally and an inverting switch incorporated in the logic allowed the switching function to be inverted and thus either state could be selected for measurement.

The whole chain from the analogue switch to the integration was calibrated by applying a sine wave of known amplitude at the input and measuring the integrated voltage on a digital voltmeter over a period measured on the counter. In order to determine the mean d.c. hot-wire bridge voltages relevant to the two states, the bridge output was integrated directly after being divided down by a potentiometer to enable a suitable integration period to be attained; this was

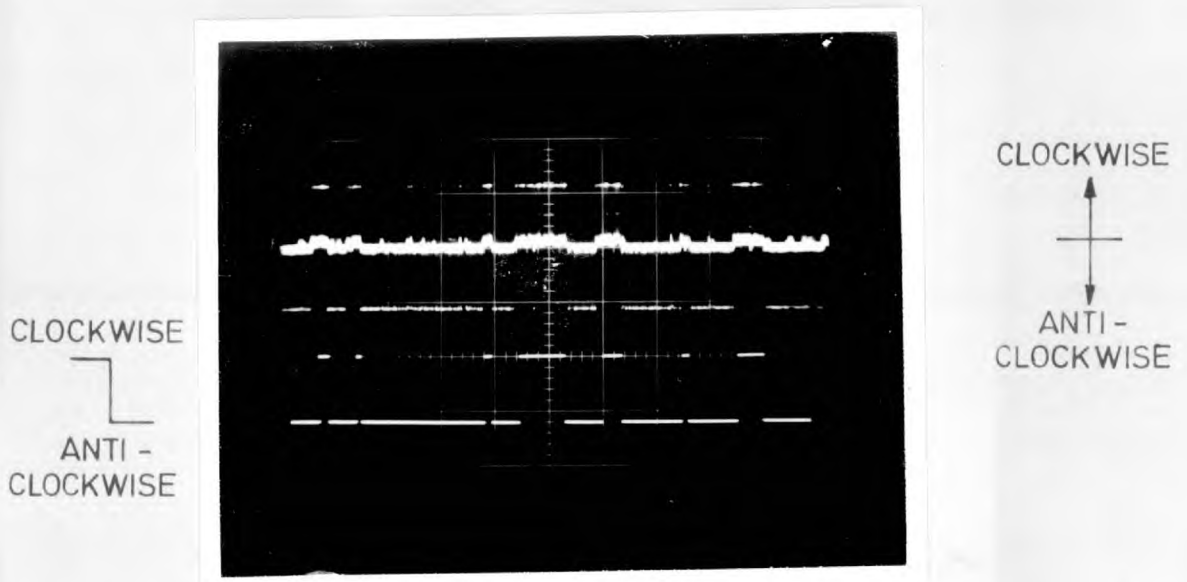


FIGURE 7.6. COMPARISON OF SWITCHING FUNCTION (LOWER TRACE) WITH FLAG OUTPUT.

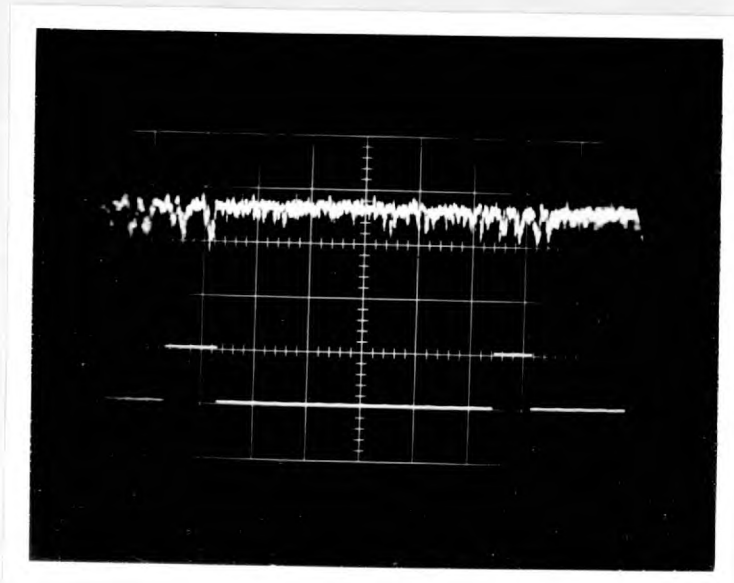


FIGURE 7.7. COMPARISON OF
SWITCHING FUNCTION WITH
TURBULENCE TRACE.

governed by the time constant of the integration (5.5 secs.) and the linearity of its integrated voltage against time curve (linear up to an integrated voltage of 20 volts). The block diagrams are shown in figure 7.8.

The hot-wire probe was mounted in conjunction with the flag as shown in figure 7.3. Four spectral analyses of the two flow states were performed, all at a fixed distance downstream from the edge and at a fixed distance from the pipe wall. Two were symmetrically disposed about the axis at $\pm 120^\circ$ and the other two were situated in the plane of symmetry at the inner and outer wall; the distributions are shown in figures A8 - A11. All integrations were performed over a period of at least a minute, that is, an integrated period of a minute.

Whilst taking these measurements, it became apparent that over long periods of perhaps half an hour, a preferred direction would exist such that the flag would, on average, remain up to four times longer in one state than the other. It was thought that any residual swirl in the laboratory might be generating a "bath-tub" type of vortex in the bell-mouth intake, thus biasing the flow. However, smoke filaments gave no indication of its existence and a screen stretched across the intake did not affect the ratio. If, indeed, the flow is very sensitive to residual swirl, then a redesigned intake with a large provision for flow straighteners would be necessary to remove its effect. However, the measurements made in the present work would not be affected by this since the instantaneous flow patterns would remain the same. Any future work aimed at analysing the switching frequency spectrum and the factors affecting would, however, necessitate a close examination of such swirl.

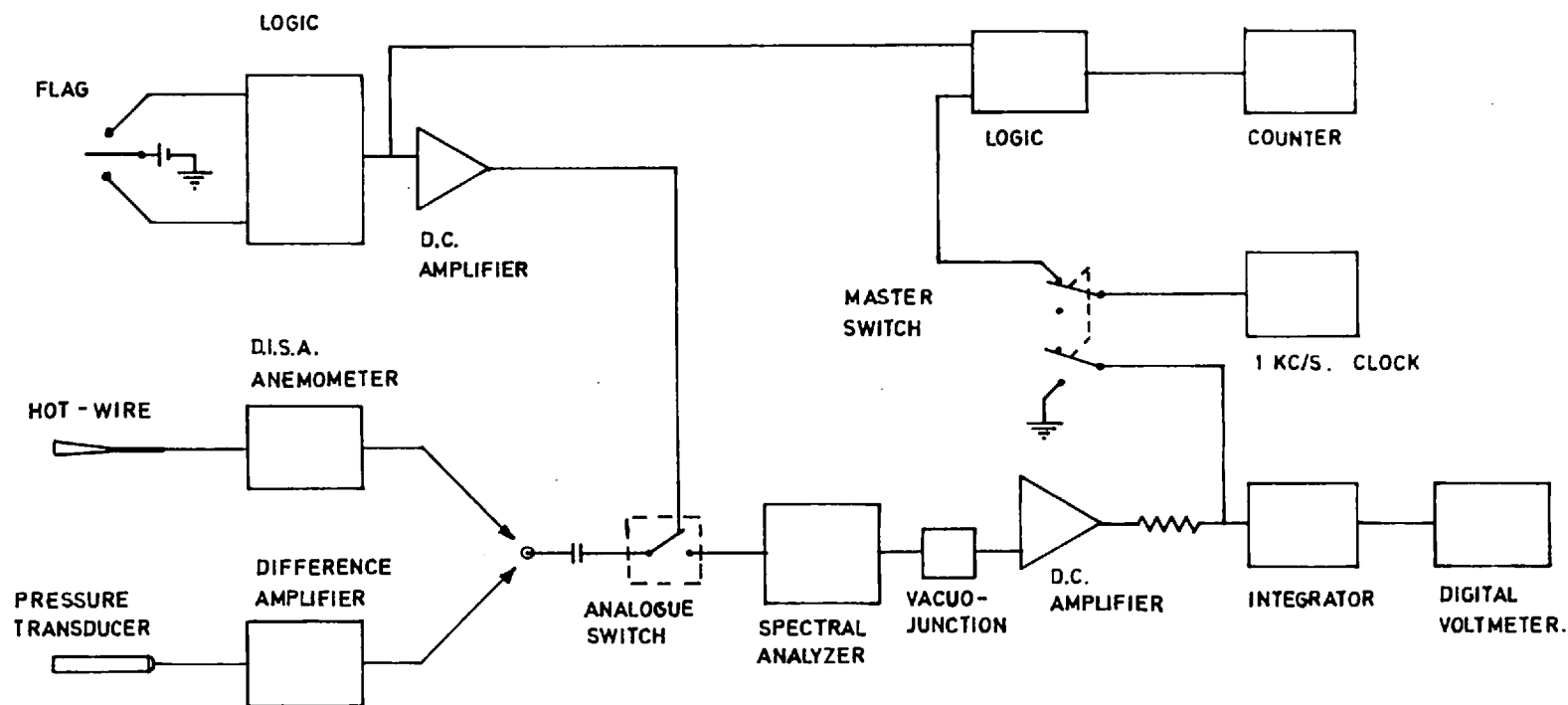


FIGURE 7.8. BLOCK DIAGRAM FOR MEASUREMENTS WITH FLAG.

7.2 FURTHER TESTS IN WATER.

Tests of an exploratory nature were made to see whether the techniques of injecting saline solution and detecting its presence downstream with conductance probes would be of value. Details of the probes and their a.c. bridge circuit are shown in figure 7.9. No attempts were made to calibrate the probes, other than to ensure that their sensitivities to saline solution concentration were similar to within five percent.

In general, the rapid diffusion of the solution prevented any useful results from being obtained in the centre and reverse flow regions. However, by positioning the probes close to the wall in the plane of symmetry, horizontally opposed, the vortex switching could be detected very clearly. A typical trace with the corresponding sketch of probe positions is shown in figure 7.10. The trace shows a marked tendency for the saline solution to be convected to the inner probe (clockwise vortex). This is because the injection point was near the region (c) of figure 5.2, where the flow in both halves of the pipe is fairly steady inwards. Strong anti-clockwise vortices, however, were capable of convecting the solution across to the outer probe.

7.3. FLUCTUATING WALL PRESSURE MEASUREMENTS.

The transducer's r.m.s. output voltage recorded by the Brüel and Kjaer analyser fluctuated wildly but seemed to consist of a high level with rapid "dips" to a much lower level. Again, the time scale was comparable with vortex switching. Typical examples of the analyser output are shown in the traces of figure 7.11. Unfortunately, the long time scale put these fluctuations beneath the range of the available analysers, although an occasional unexpectedly large reading was recorded in the

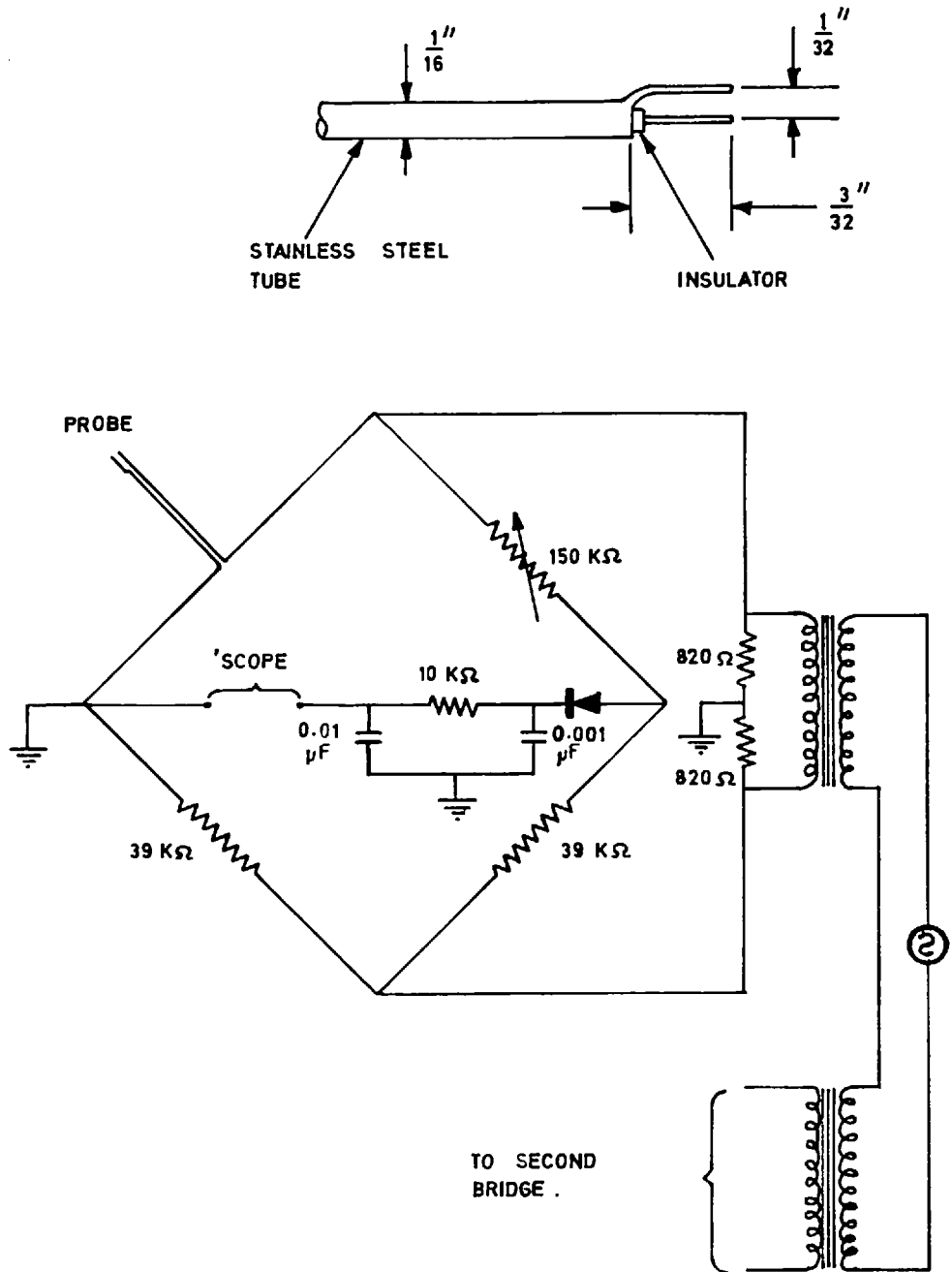
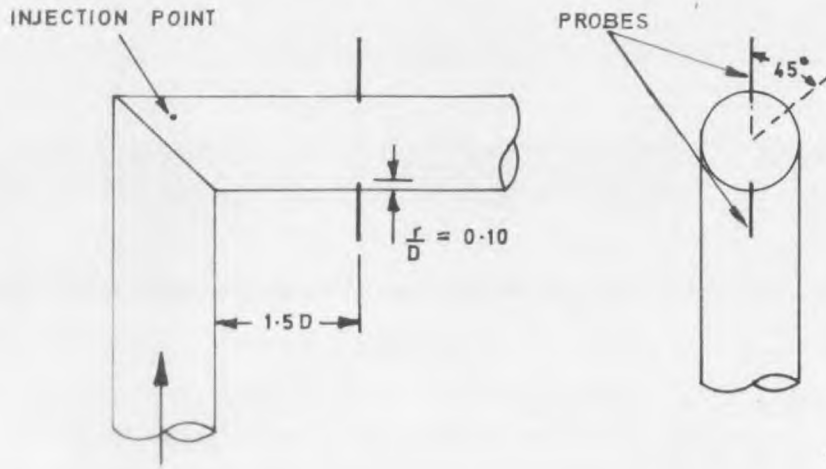


FIGURE 7.9 . DETAILS OF
CONDUCTANCE PROBE & BRIDGE.



OUTER
PROBE

INNER
PROBE

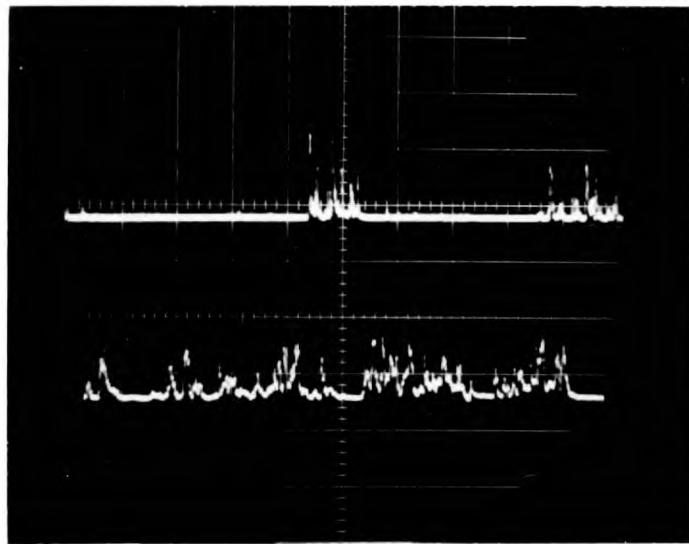


FIGURE 7.10. OUTPUT TRACE
FROM CONDUCTANCE PROBE
BRIDGES.

2.5 c/S 1/3-octave frequency band. The two level nature of these readings was not of the same character as those of the turbulence, however, since the latter readings spent, on average, equal periods in each state. It was decided to record a maximum, minimum and a mean reading to define each measurement in order that the spectra might be compared.

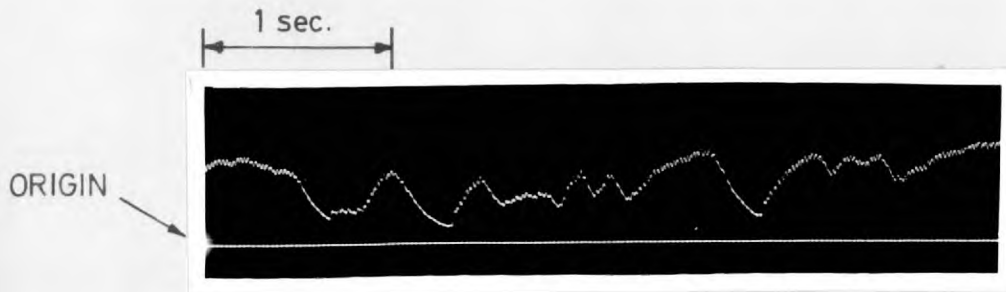
A survey around the circumference of the tube confirmed that the flow was symmetrical over a sufficient period of time and that the three readings elected to be taken were repeatable at the symmetrical positions: further surveys were restricted to the upper half of the model.

Four axial surveys of the r.m.s. pressure fluctuations in the band 2 c/S to 40 kc/S, which will be termed the total r.m.s. fluctuation, p' , were conducted and are presented in figures A16, 17 and 18. Circumferential spectral analyses were then carried out at six stations downstream, together with the spectra upstream and far downstream (figures A19 - A41).

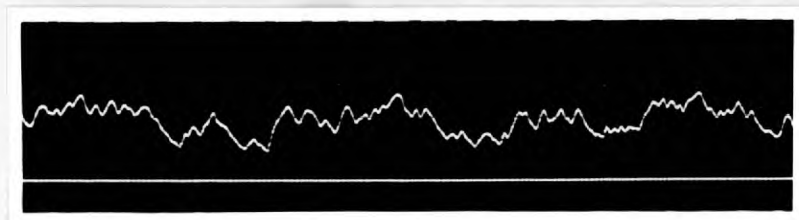
Examination of the repeatability of these measurements indicated that dimensional quantities were repeatable to within 10 per cent, but the ratios $p'/p'_{\max}$ and p'_f/p'_{mean} were repeatable to within 5 per cent. The p'_{mean} referred to here and used for the plotting of the spectra is its maximum value occurring around the periphery of the tube at that value of L/D . The rotation at which it occurred together with its value is recorded on each graph.

Measurements using the flag and analogue switch system previously described were performed at the same positions as the corresponding hot-wire measurements. The nature of the construction of the pressure transducer necessitated a slight modification to the flag.

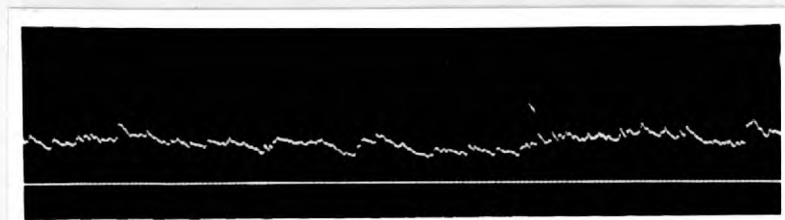
The frequencies at which measurements could be taken were restricted to the fixed 1/3-octave bands of the analyser, and thus in the regions of peaks and troughs of the spectra a variety of curves may be drawn through the experimental points. It has been attempted to provide a coherent set.



CENTRE FREQUENCY = 25 C/S.



CENTRE FREQUENCY = 100 C/S.



CENTRE FREQUENCY = 1 KC/S.

(vertical scale $\times 5$)

FIGURE 7.11. TYPICAL PRESSURE
FLUCTUATION TRACES (horiz. scale :
1 inch $\equiv$ 1 sec.).

8. DISCUSSION.

The experimental results discussed in this section are collected in the Appendix.

8.1 MEAN VELOCITY AND TURBULENCE PROFILES.

No evidence of reverse flow is to be seen in figures A.1, A2 and A3 since the hot-wire probe employed was incapable of distinguishing it. While a minimum might be expected at the point of reversal, the unsteadiness of the flow and the high turbulence level in this region prevented its identification, although it certainly should have existed in the first two profiles. Thus the portion of the profiles near the inner wall must be treated with reserve.

The profiles are qualitatively similar to Weske's, the peak is seen to move across to the outer wall as the shear layer spreads, assisted by the cross flow, reaching its closest approach at about $L/D = 1.64$ and subsequently returning as the profile reverts to that of a straight pipe flow. By estimating the curvature of the mean streamlines at $L/D = 0.0052$, a plot of mean angular momentum distribution is shown in figure A2, compared with that for two-dimensional, curved channel flow. The major portion of the two dimensional profile is virtually the potential distribution $UR = \text{constant}$, but, as in Weske's and Curtiss' results there is a portion of the three-dimensional profile uninfluenced by the separations or a cross flow which also corresponds to $UR = \text{constant}$. The velocity profiles are, again, qualitatively similar to those obtained by Graves and Ránov (reference 49) in a two-dimensional, mitred channel bend, although naturally there is no crossflow to assist the spread of the shear layer. However, the extent to which the cross-flow assists the re-attachment in the three-dimensional flow can be appreciated from the

conclusion of Graves and Ranov that re-attachment was not achieved within the two-dimensional channel, some 40 channel widths long.

The mean shear is necessarily very intense since the growth of the separation accelerates the mean flow to a much greater extent than in, for example, an external wake flow. In the first two profiles, the turbulence is seen to reach a sharp peak over the central portion of the shear layer, since any convection of fluid up and down the shear by the component in the direction of the shear contributes greatly to the u' term. The last two profiles show the rapid spread of the turbulence across the wake under the influence of the cross-flow established at the impingement of the two inward circulations. The rapid growth of the turbulence level near the outer wall in the final profile indicates that the dominant vortex has now started to sweep the outer wall, convecting with it highly turbulent fluid from near the separation.

That this is the action of the dominant vortex is shown very clearly by the switched spectra plotted in figures A8 - A11. The off-axis plane spectra at $\pm 120^\circ$ reveal the observed two level nature of the turbulence and confirm that the two levels are related to the structure of the two dominant vortices. Thus at the $+ 120^\circ$ position the highly turbulent state is associated with an anti-clockwise vortex, that is, the vortex convecting fluid from near the separation, while the clockwise vortex brings "uncontaminated" fluid to this point and hence has a much lower level. This description of the flow demands that the relationship between vortex sense and turbulence level should reverse for the symmetric position at $- 120^\circ$, but that the spectra of the respective levels should remain similar and this is

seen to be so. Again, one would expect that on the plane of symmetry the spectra would be identical in both vortex senses, and this is shown in figures A8 and A11 within the limits of the experimental accuracy. The low levels at the outer wall show that the dominant vortex does not yet sweep the outer wall and has a level similar to that recorded in the traverse at $L/D = 1.64$.

There is a striking difference in the peak frequencies of the spectra. Thus while, as might be expected, the spectra of the low level states at $\pm 120^\circ$ and the spectra at the outer wall are all fairly similar, the spectra of the high level states at $\pm 120^\circ$ and those at the inner wall are very different, even though the fluid is coming from the separated region for all the later cases. Thus the peak at $\pm 120^\circ$ is at roughly 60 c/S, while at the inner wall it is at roughly twice this frequency, 120 c/S. This point is referred to in the discussion of the switched pressure spectra.

8.2 FLUCTUATING WALL PRESSURE MEASUREMENTS.

On examination of the "switched" pressure spectra in figures A12 - A15, two features are immediately obvious: all the spectra are very similar and the spectra at $\pm 120^\circ$, while qualitatively exhibiting the same trend as the turbulence spectra, do not have two distinct intensity levels. These features reflect that the fluctuating pressure at a point in an incompressible flow is determined by the entire velocity field and may be represented by an integral over-all space, whereas the turbulent velocities are local. Thus the turbulence spectra are governed by the portions of the shear layer convected locally by the dominant vortex, while the pressure spectra are governed by the fluctuating vorticity

in the shear layer, most of which passes down the core of the flow and gives rise to the high turbulence levels in the central regions of figures A3, A4 and A5. Hence the pressure spectra at $\pm 120^\circ$ are almost independent of vortex state.

All the "switched" pressure spectra exhibit the two peak frequencies characteristic of all the pressure spectra, whereas the switched turbulent velocity spectra separated these peaks, the higher frequency being predominant at the 0° position, although the spectrum is broad enough to include a large component at the lower frequency. This suggests that the shear layer is producing essentially two instabilities associated with different regions of the layer. Thus the streamline labelled "A" in figure 5.8 will tend to convect instabilities associated with the smaller region of the shear layer to the inner wall, while the streamline labelled "B" in figure 5.9 will tend to convect, in addition, instabilities from the broader part to the wall regions off the plane of symmetry. Intuitively, these would be associated with a higher and lower frequency respectively.

Despite the similarity of the "switched" pressure spectra, the r.m.s. pressure fluctuations did indicate a two level nature, though by no means of equal period as previously mentioned. Since the switched measurements did not separate these levels, these measurements must be averages over the two levels and, in fact, the total r.m.s. fluctuation p' , was significantly smaller than the mean readings observed in the circumferential spectra at $L/D = 1.96$. The time scale associated with the two levels was again that of the vortex switching which suggests that the re-organisation of the flow during switching suppresses, or randomises, the instabilities

of the shear layer: the spectra of the minimum readings tends to be broader and far less peaked than the corresponding maximum or mean spectra. The effect is, perhaps, similar to the loss of a pronounced shedding frequency from a circular cylinder during transition, when the turbulence in the shear layer is sufficient to disorganise it, followed by the recovery of a dominant frequency at much higher Reynolds numbers as observed by Roshko.

At $L/D = 0.35$, reverse flow exists at the inner wall and the spectra in this region (figures A19 - A24) are seen to be broad. The intensity increases with angle of rotation, that is, as the shear layer begins to develop a more peaked nature. The intensity drops off as the rotation is increased, passing out of the separation, and the major peak at roughly 50 c/S is reduced to about 35 c/S, which is presumably a Doppler type effect produced by propagation across the mean flow. The two major peaks in the upstream pressure spectrum (figure A41) also show this effect to a similar extent and the frequency of both scale by the same factor.

Proceeding downstream, the spectra become more peaked as the two major instabilities predominate over the less organised components. There is evidence of a lower frequency component at roughly 10 c/S being present and is strongly in evidence at $L/D = 4.75$ (figure A39). The origin of this peak is unknown. The spectra become progressively similar with respect to circumferential position as the shear layer is distributed throughout the core of the flow by the cross-flow. Far downstream (figure A40), the spectrum retains its peaked nature and contrasts markedly with the spectrum of the minimum readings. The 50 c/S peak has again reduced to roughly

32 c/S while the higher frequency peak has not been significantly affected (the low frequency filters were not available at this point).

This modification of peak frequency is likely to be brought about by distortion of the wavelength and changes in the convection velocity of the disturbance.

The axial distributions of figures A16, A17 and A18 have a maximum at $L/D \approx 0.6$ corresponding to the region in which the predominant shear layer instability occurs. Thus, although the flow at the inner wall is still separated in this region, the spectra at $L/D = 0.62$ have developed a large peak at roughly 45 c/S compared with the broader spectra at $L/D = 0.75$ (c.f. also, the large peak in the turbulent velocity distributions at $L/D = 0.57$). The secondary peak in the two inner axial distributions is probably associated with re-attachment since it does not occur in the outer, unseparated distributions at 120° and 180° . The inner distributions decay rapidly after $L/D \approx 0.9$, while the outer distributions retain a more constant level since the shear layer is spreading towards the outer wall.

A significant feature of the pressure fluctuations in the maximum r.m.s. level of 38.7×10^{-3} p.s.i., corresponding to a pressure coefficient, based on the upstream mean dynamic head, of 0.47. The maximum spectral component, measured at $L/D = 0.62$ and at 40 c/S, was equivalent to a coefficient of 0.29. Unfortunately, no spectral analyser was available for examining the frequency range of the vortex switching (~ 1 c/S), but a significant contribution to the pressure fluctuations might be anticipated from the oscillation of the separation.

9. GENERAL SUMMARY.

9.1. CONCLUSIONS.

- (1) The flow through the mitre bend separated at the inner salient edge, re-attaching after approximately three-quarters of a diameter. The separation was not symmetrically disposed about the plane of symmetry and thus the two secondary circulations generated in the two halves of the pipe were not equal.
- (2) The two secondary circulations flowed around the edge of the reverse flow region meeting immediately after re-attachment. A resultant downstream circulation, termed the dominant vortex, equal to the difference between the two circulations was set up together with a cross-flow which assisted in spreading the shear layer across the flow.
- (3) The separation was not stable but switched about the plane of symmetry, thus causing the dominant vortex to reverse its sense. The frequency of switching appeared random but of low frequency, typically of order 1 c/S; the action of switching was very much more rapid (of order 10^{-3} seconds). The dominant vortex, while maintaining one sense fluctuated in strength.
- (4) Decreasing the Reynolds number did not significantly affect the flow pattern until a value of roughly 3,000 when switching became less well defined

(5) The dominant vortex was of insufficient strength to sweep across the plane of symmetry at the outer wall until 2.5-3 diameters downstream of the inner edge. In general, therefore, in regions off the plane of symmetry immediately downstream of the bend, but not within the reverse flow, the fluid convected to a point near the wall was either "contaminated" by the separation or not, depending on which vortex dominated. Thus the turbulence levels in these regions were of a distinctly two level nature.

(6) Switched hot-wire measurements in this region, confined to a particular vortex sense, confirmed the two level nature of the turbulence and the suggested flow model. The turbulent velocity spectra indicate that the two predominant instabilities of the shear layer are related to two separate regions of the layer: the broad central portion corresponding to the lower frequency (60 c/S) and the smaller portion at either side to the higher frequency (~ 120 c/S).

(7) Fluctuating wall pressure measurements were also of a two level nature but did not reside at each level for equal periods: the lower level was more transient but switching between these levels occurred with the time scale of the vortex switching.

(8) Switched pressure fluctuation measurements did not separate these two levels as in the case of the turbulent velocity measurements. It seems likely that the lower level is produced by the disorganisation of the shear layer during switching randomising the shear layer instabilities: the spectra of the minimum readings tended to be broader and less peaked. The pressure fluctuations were dominated by the shear layer fluctuations and not by the structure of the fluid convected

locally. Thus all the switched pressure spectra were similar.

(9). Pressure spectra under the reverse flow region of the separation tend to be broad. The predominant instabilities occur at $L/D \approx 0.6$ after which the spectra take on a strongly peaked form. On moving around the circumference of the tube near the reverse flow region, the intensity and peaked nature increases as the shear layer approaches the wall. Spectra at the outer wall in this region indicate a frequency shift probably caused by convection against the mean flow. On proceeding downstream, the spectra become progressively more peaked as the less organised fluctuations decay more rapidly than the dominant ones. The spread of the shear layer and its distribution through the core of the flow cause the spectra to become similar at all circumferential positions after approximately two diameters downstream. A lower frequency peak of unknown origin becomes evident at roughly 10 c/S. The three peak frequencies have mean Strouhal numbers, based on upstream mean velocity and pipe diameter of 0.03, 0.14 and 2.37, the most powerful corresponding to 0.14. The maximum level of the total pressure fluctuation corresponds to a pressure coefficient of 0.44.

(10) Upstream, the peaks of the pressure spectra have moved to lower frequencies, presumably through convection of the pressure disturbances against the mean flow since they scale by similar factors. Far downstream, distortion of the disturbances and changes in their convection velocities cause scaling of the peak frequencies by different factors.

(11) The time scale associated with vortex switching was much longer than that associated with the instabilities of the shear layer (of order 10^{-2} seconds). Turbulent flow appeared essential to the phenomenon. An order of magnitude argument based on the structure of fully developed turbulent pipe flow suggests that the production of a turbulent circulation of a magnitude sufficient to effect switching would occur with a comparable time scale. Thus the switching disturbance is considered to arise from a chance orientation of the turbulent eddies in the pipe flow, such that a turbulent circulation of comparable magnitude but opposite sense to the resultant downstream circulation is generated in the flow entering the bend. This is sufficient to perturb the separation to the opposite side of the plane of symmetry and hence reverse the sense of the dominant vortex. No sensible estimate of the effect of Reynolds number on switching frequency can be made using this argument, since it depends critically on the dependence of the resultant circulation on Reynolds number, which is not known.

(12) No reason can be foreseen why the above considerations should not apply to a sharp pipe bend without a salient edge but which does produce separation. However, Weske's result in such a bend appears anomalous at present.

9.2 CONSIDERATIONS OF FURTHER WORK

The nature of the flow field which has been established in the present work raises a number of points for further study.

- (1) The frequency spectrum of the vortex switching and the effect of upstream swirl and Reynolds number. This would require either a very low frequency filter analysing a transducer output or an extension of the "flag" and pulse counter technique.
- (2) The variation of downstream circulation with Reynolds number, again requiring a "flag" technique or possibly a method which fixes the dominant vortex in a particular sense, for example, by injecting upstream swirl, provided this does not significantly change the other features of the flow.
- (3) The effect on vortex switching of radiusing the inner wall. This would establish whether switching existed in Jeske's flow or whether a salient edge is essential to its existence.
- (4) The effect of Reynolds number and radius ratio on wall pressure fluctuation peak frequency and intensity. It may be anticipated that frequencies will be governed by a rough constancy of Strouhal number, and intensity by some power of the upstream mean velocity (approximately the fourth power for the surface pressure fluctuation on a circular cylinder). It is unlikely that small changes of the inner wall radius will effect these quantities, except that the narrower regions of the separation which gave rise to the highest frequency component in the mitre bend flow might not be present since there is no salient edge. Oscillations of the

separation point would also be possible, which, if they became coupled to another mode, might change the flow field.

(5) The question raised by Weske's work of whether a pronounced asymmetry, with respect to the plane of symmetry of the bend, of the upstream mean velocity profile can overcome or alleviate separation in a sharp bend.

APPENDIX.

SYMBOLS RELEVANT TO THE EXPERIMENTAL RESULTS

D	Pipe diameter (inches)
L	Distance downstream of Mitre at inner wall of bend (inches).
p	Instantaneous wall pressure fluctuation in the band 2 c/S - 40 kc/S
p_f	Spectral component of p in a 1/3-octave band.
r	Radial distance from pipe wall (inches)
R	Estimated radius of curvature of mean streamlines (inches).
u	Instantaneous fluctuating axial velocity component in the band 2 c/S - 40 kc/S.
u_f	Spectral component of u in a 1/3-octave band.
U	Local Mean Axial Velocity.
y	Normal distance from inner wall in the plane of the bend (inches).

Prime indicates r.m.s. value.

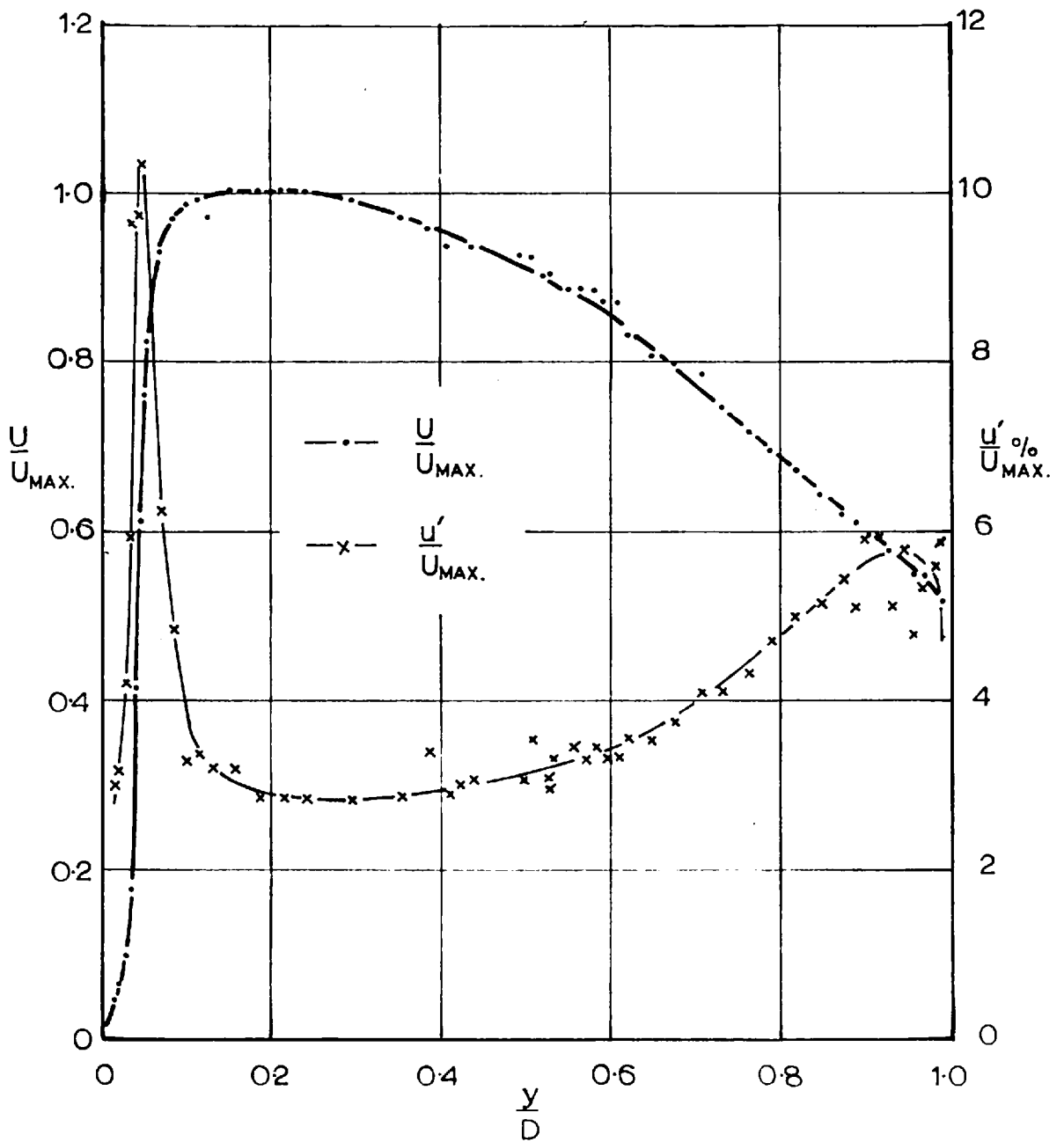


FIGURE A1. MEAN VELOCITY
and TURBULENCE DISTRIBUTIONS
at $L/D = 0.006$.

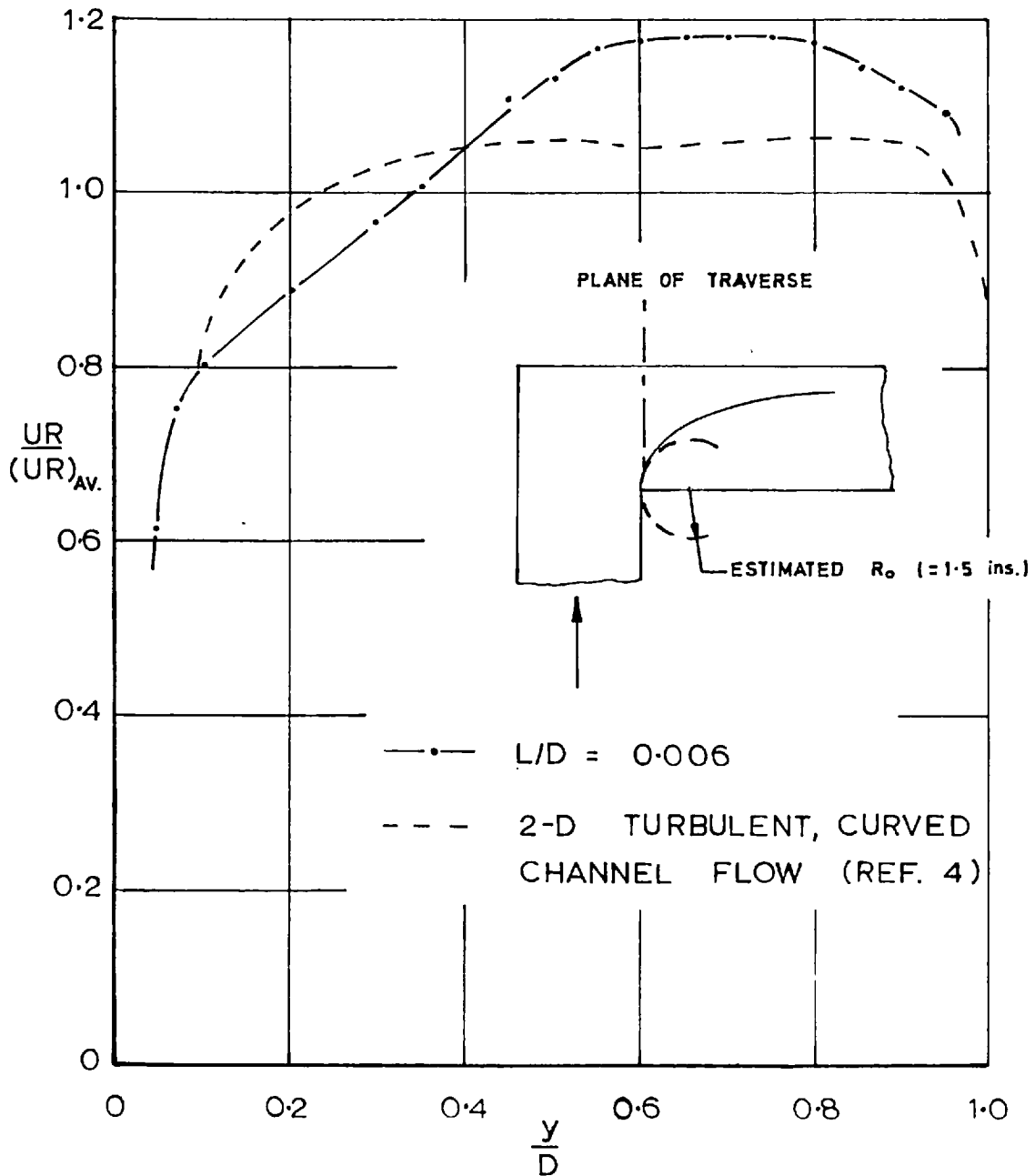


FIGURE A2. MEAN ANGULAR
MOMENTUM DISTRIBUTION at
 $L/D = 0.006$.

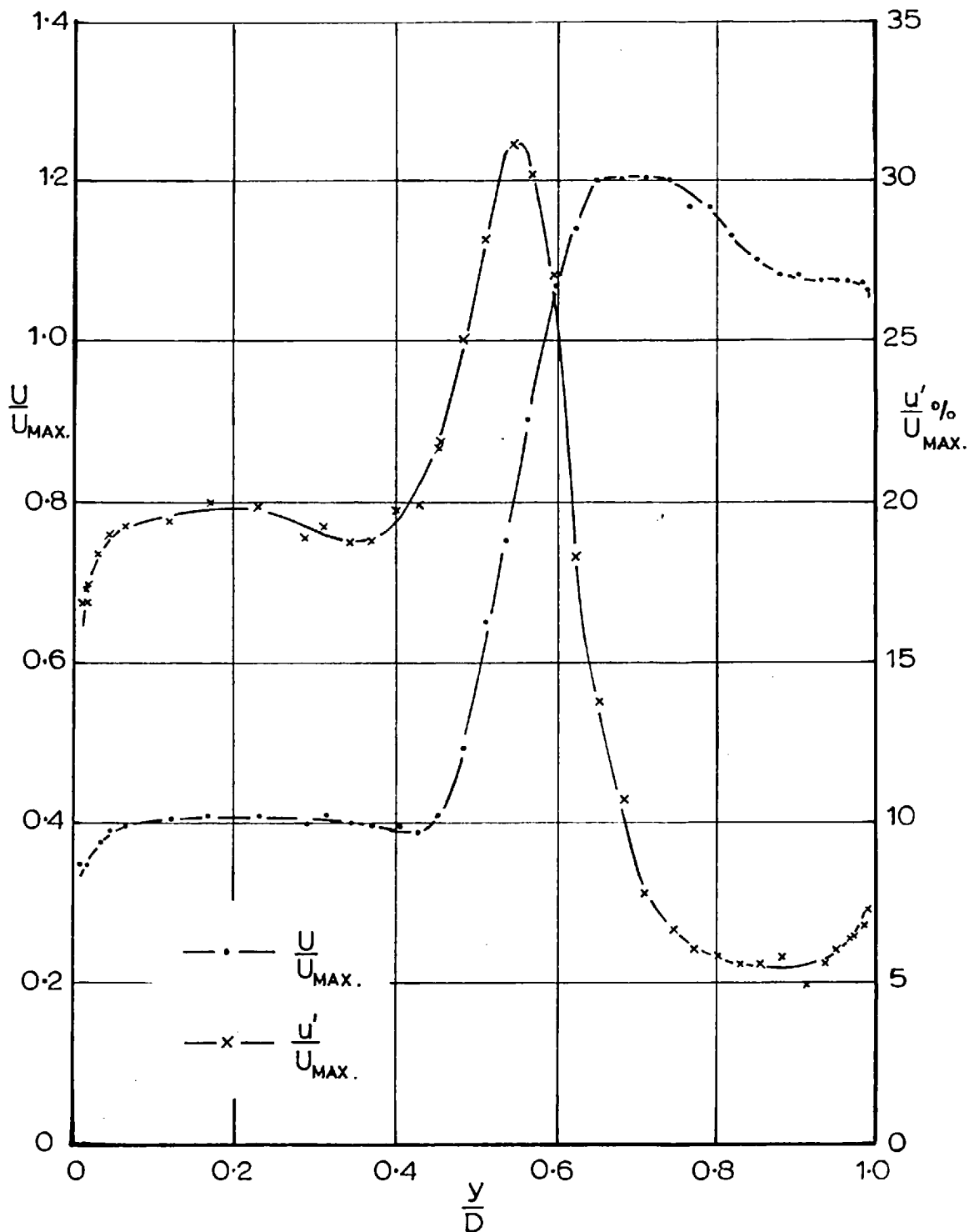


FIGURE A3. MEAN VELOCITY and
TURBULENCE DISTRIBUTIONS at
 $L/D = 0.57$.

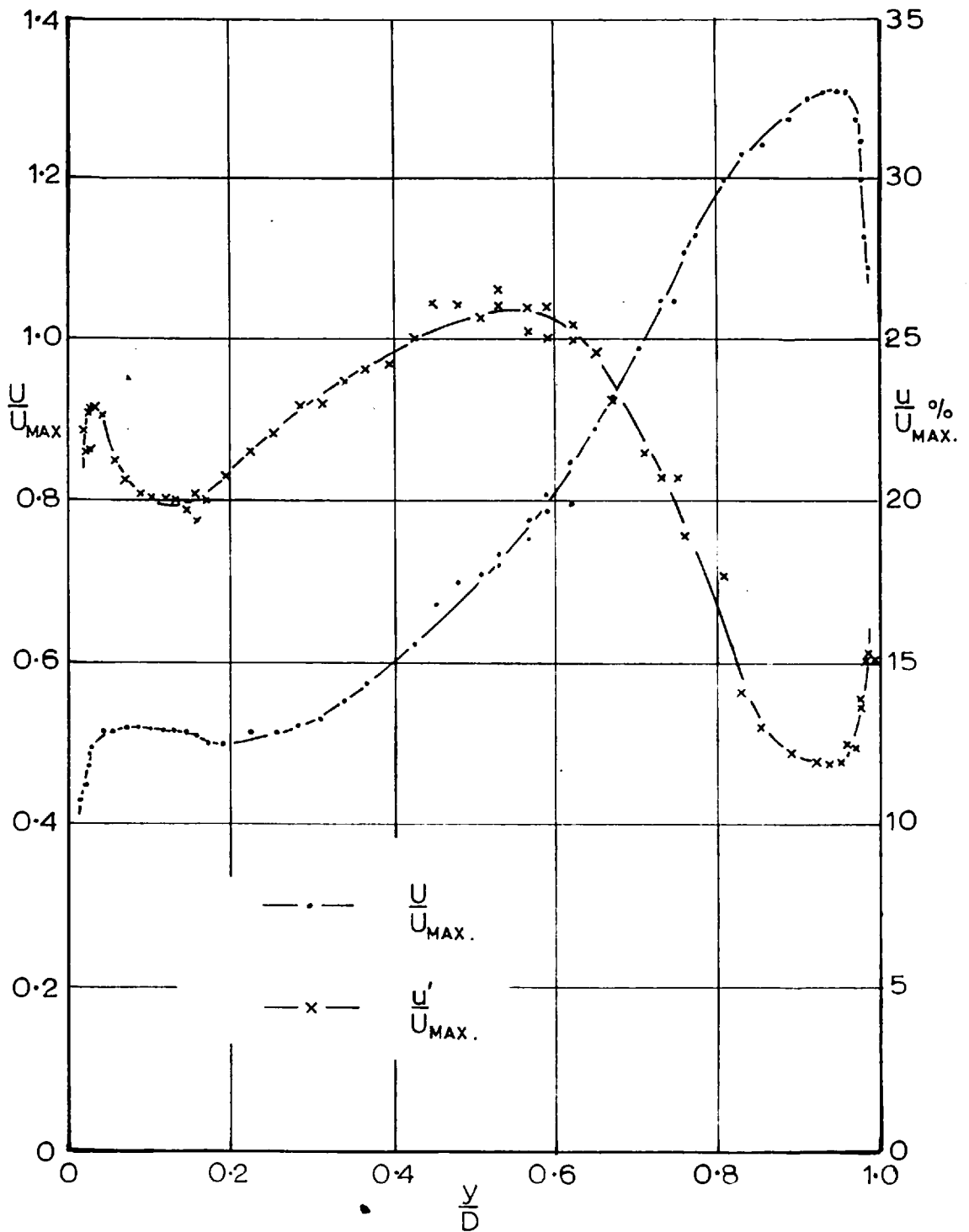


FIGURE A4. MEAN VELOCITY and
TURBULENCE DISTRIBUTIONS at
 $L/D = 1.64$.

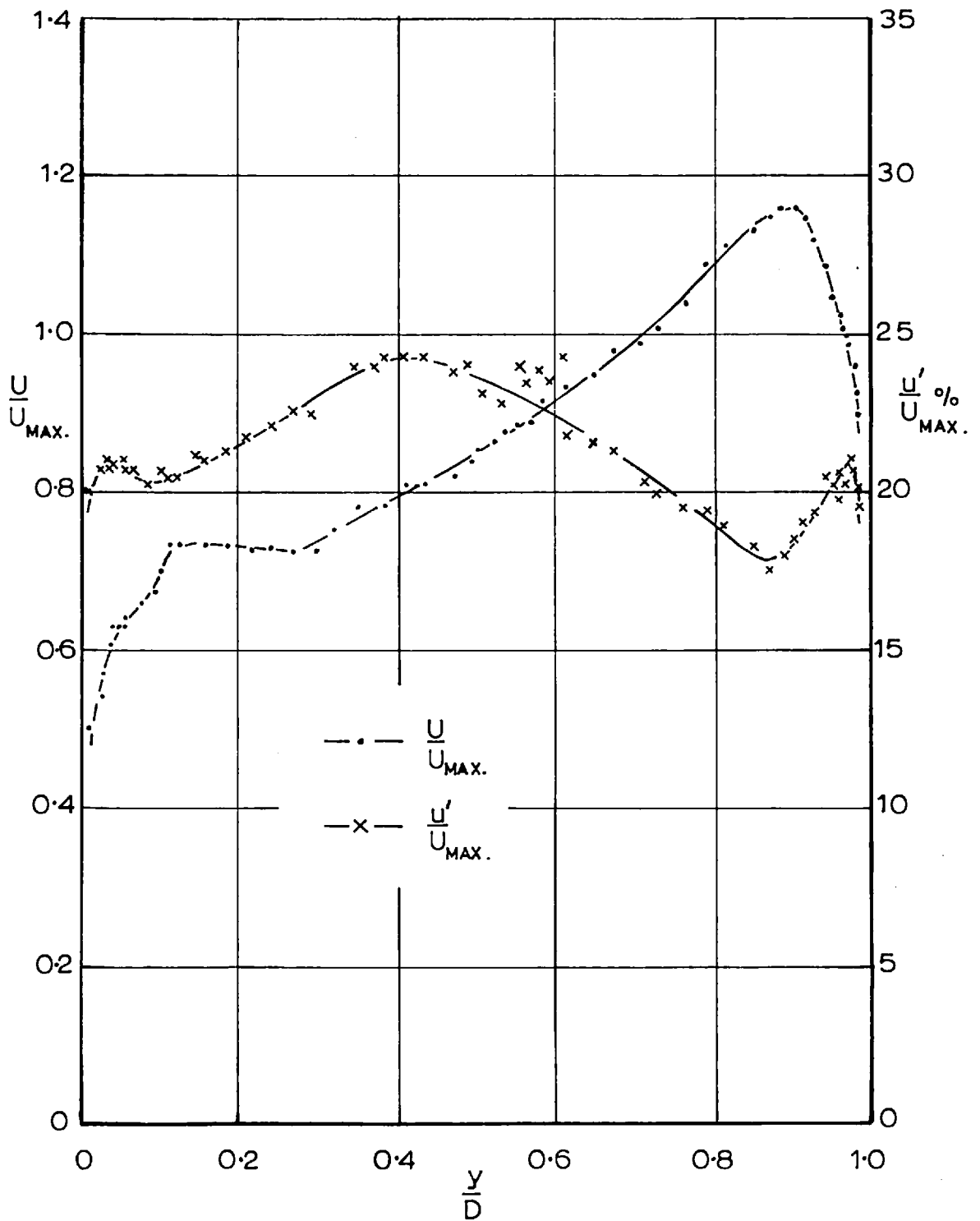


FIGURE A5. MEAN VELOCITY and
TURBULENCE DISTRIBUTIONS at
 $L/D = 2.68$.

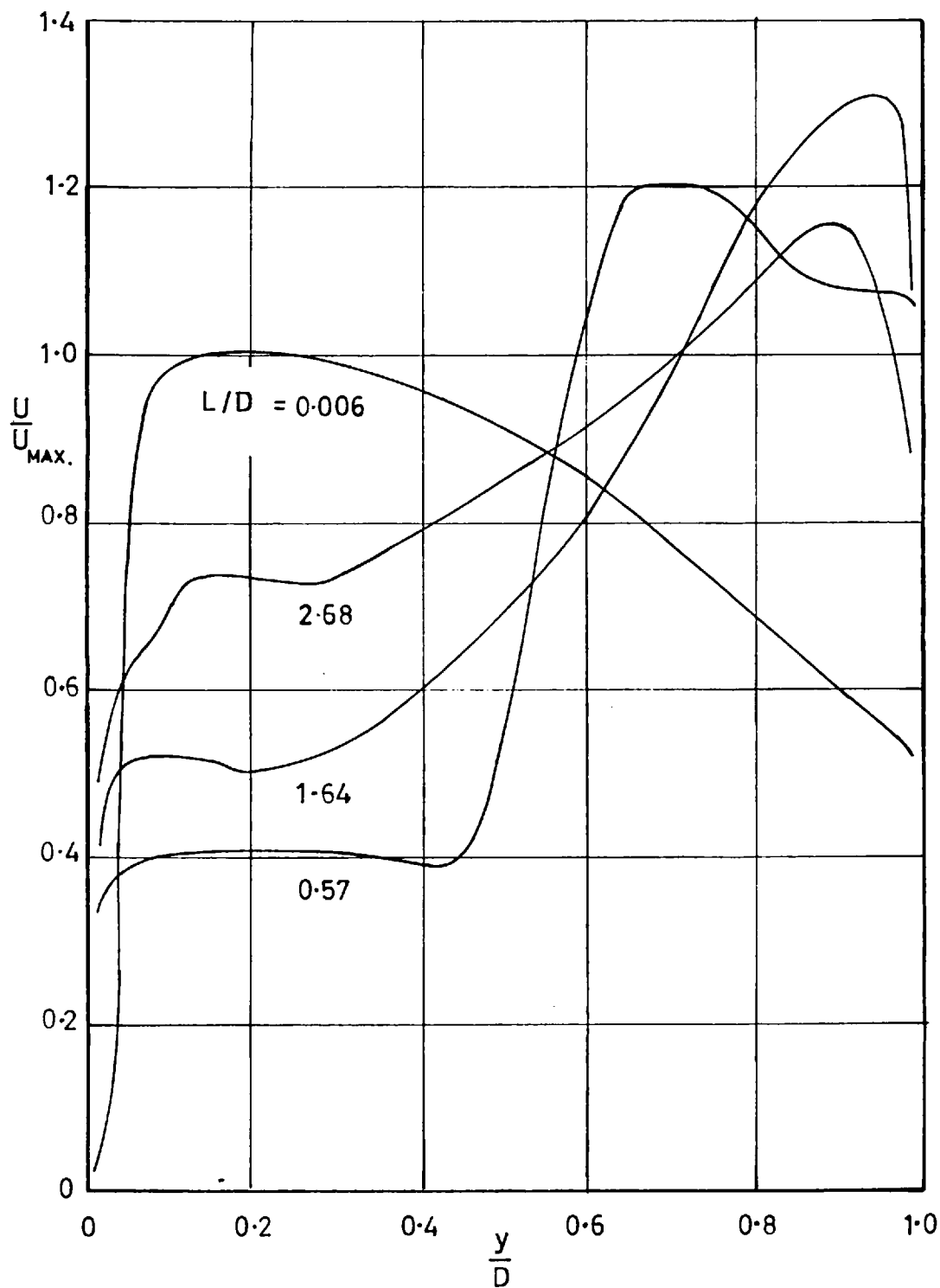


FIGURE A6. COMPARISON OF
MEAN VELOCITY PROFILES.

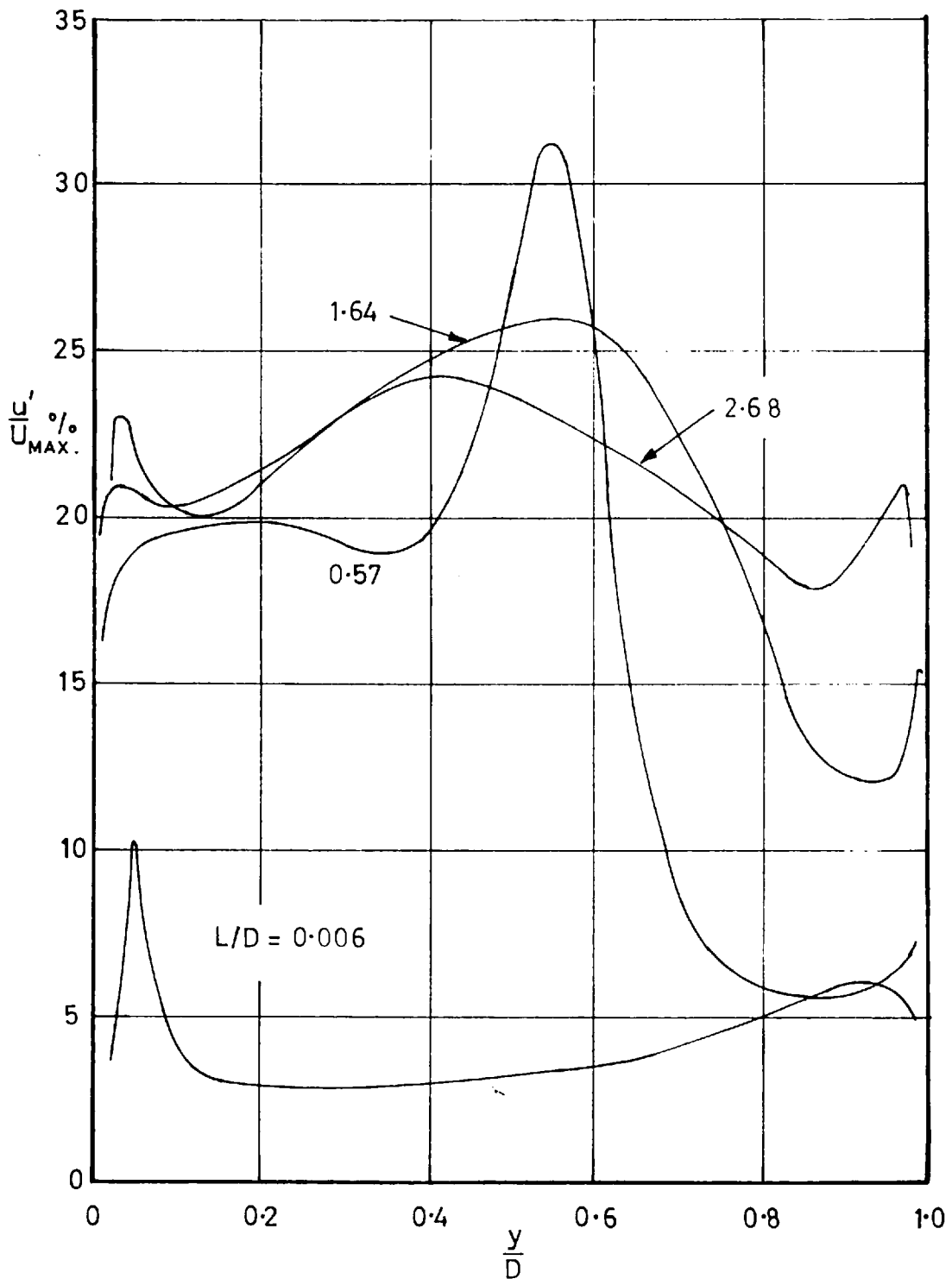


FIGURE A7. COMPARISON OF
TURBULENCE PROFILES.

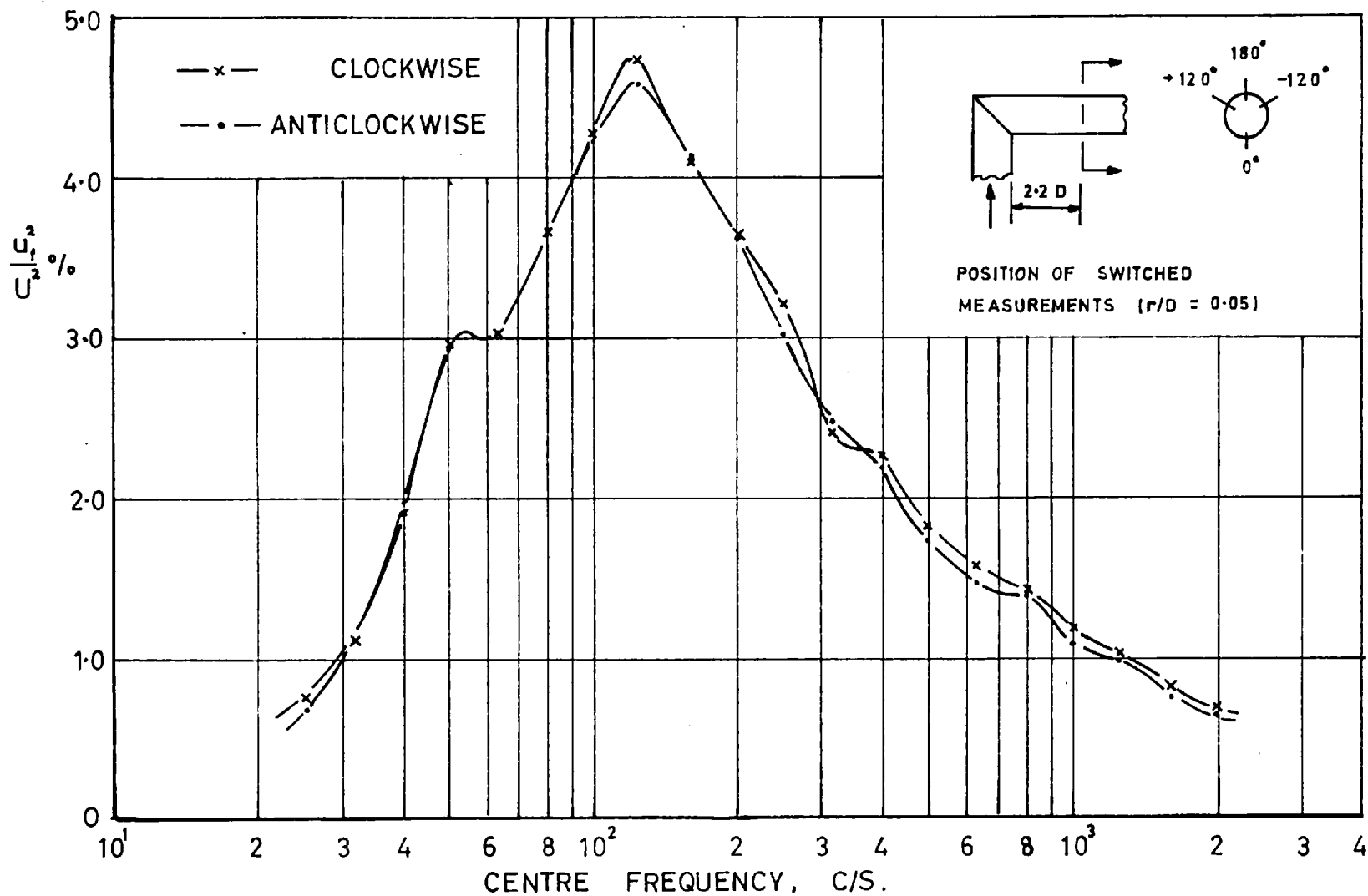


FIGURE A8. SWITCHED TURBULENCE SPECTRA,
ROTATION = 0°.

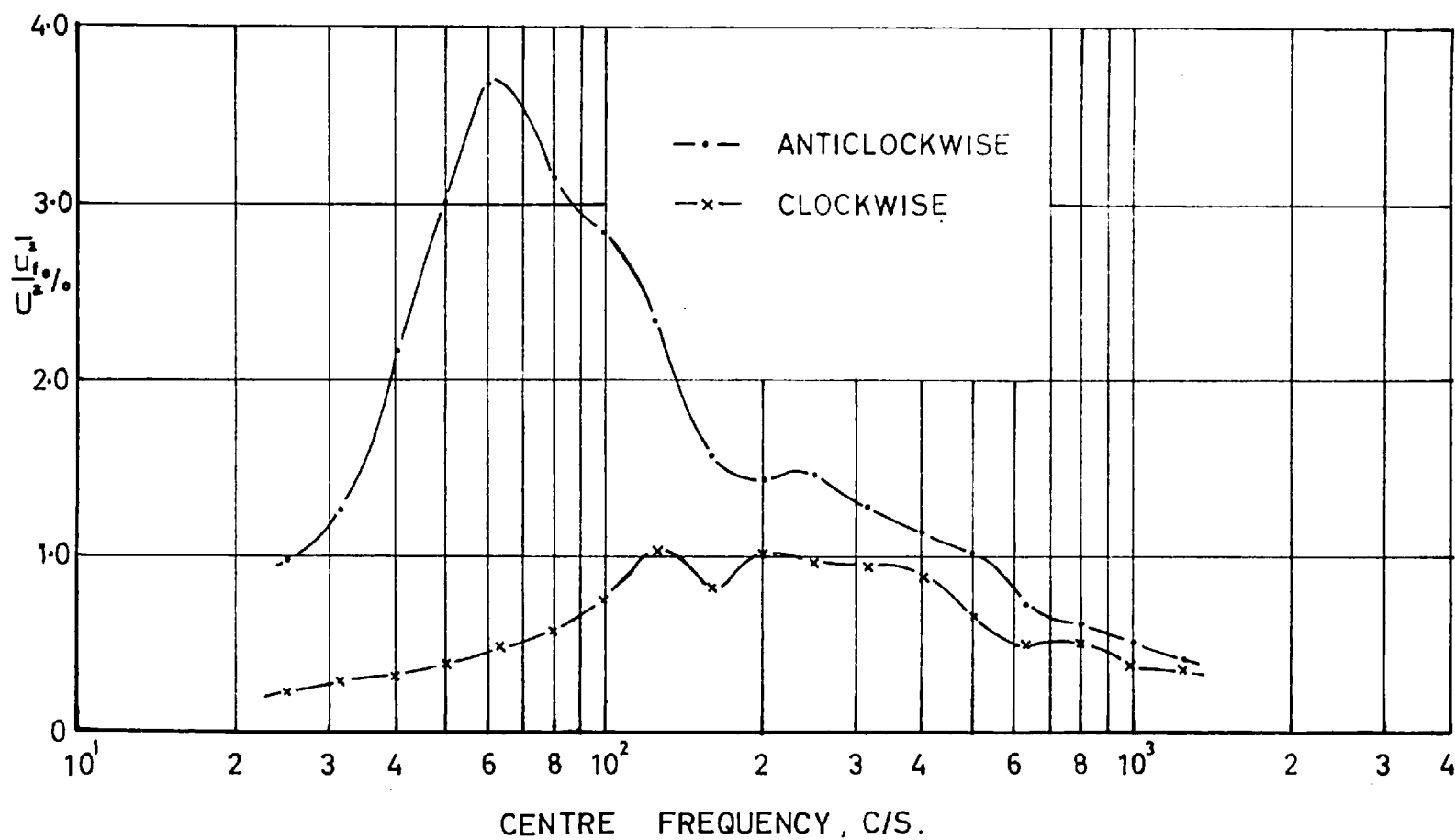


FIGURE A9. SWITCHED TURBULENCE SPECTRA,
ROTATION = +120°.

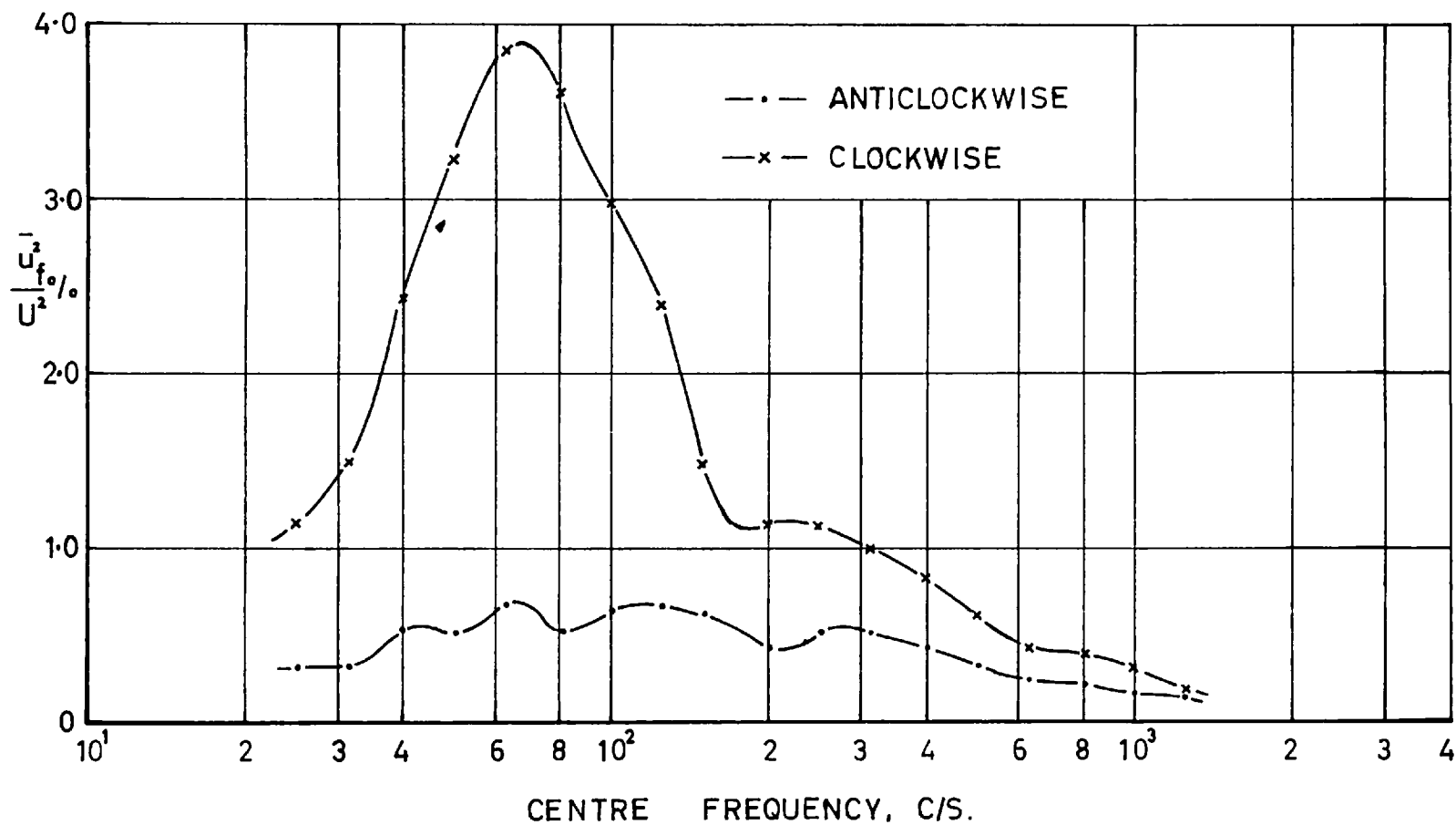


FIGURE A10. SWITCHED TURBULENCE SPECTRA,
ROTATION = -120° .

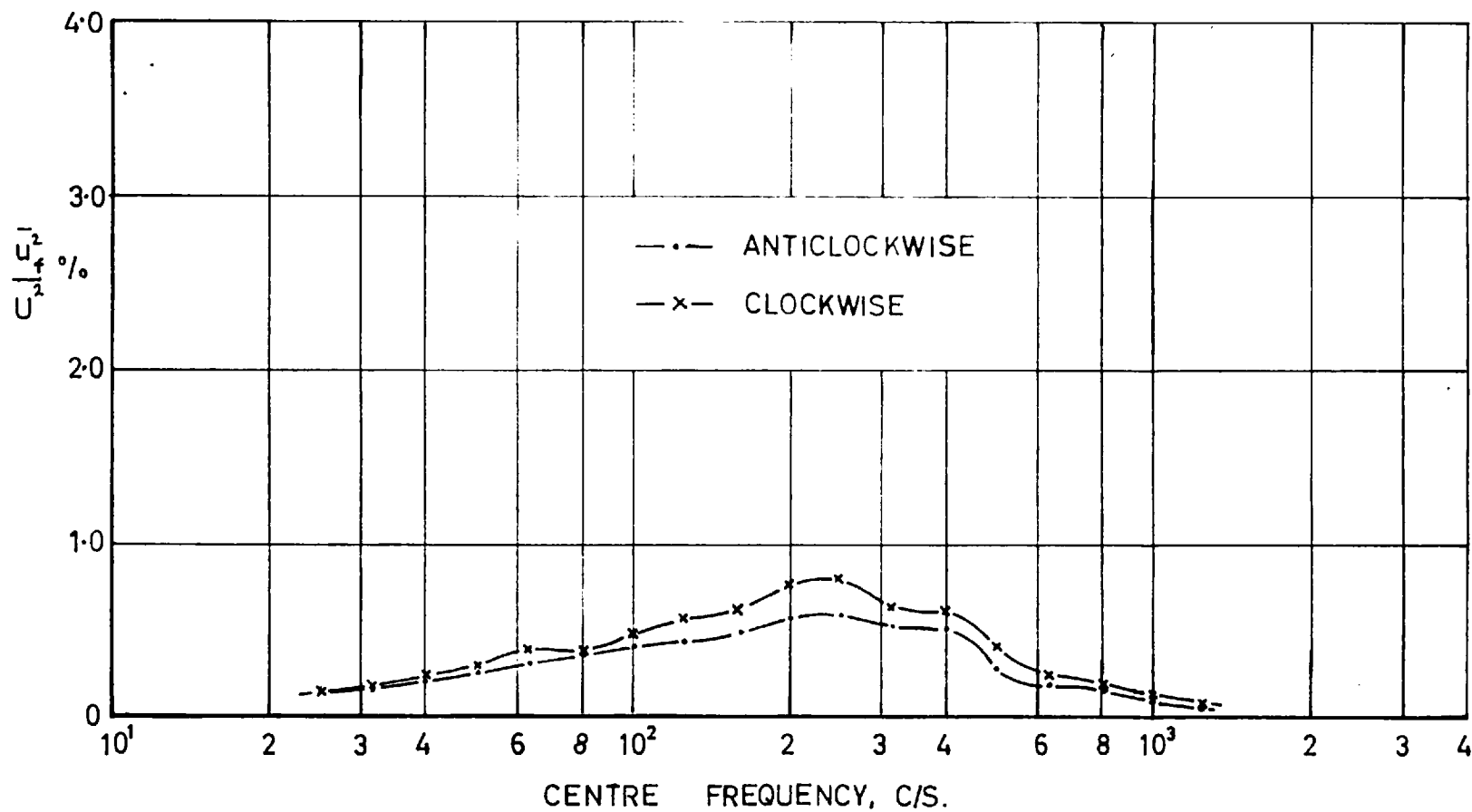


FIGURE A11. SWITCHED TURBULENCE SPECTRA,
ROTATION = 180° .

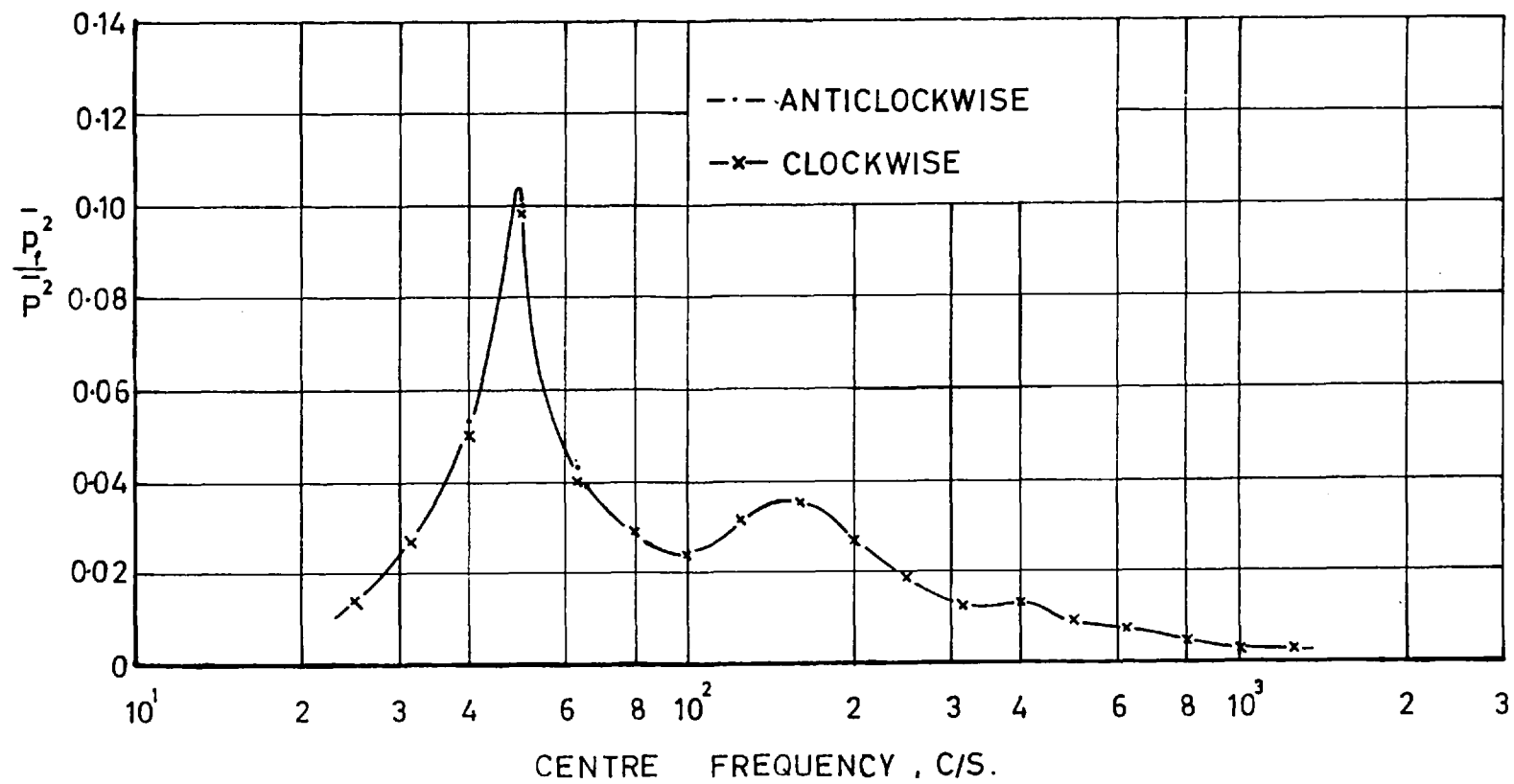


FIGURE A12. SWITCHED PRESSURE
FLUCTUATION SPECTRA, ROTATION = 0° .

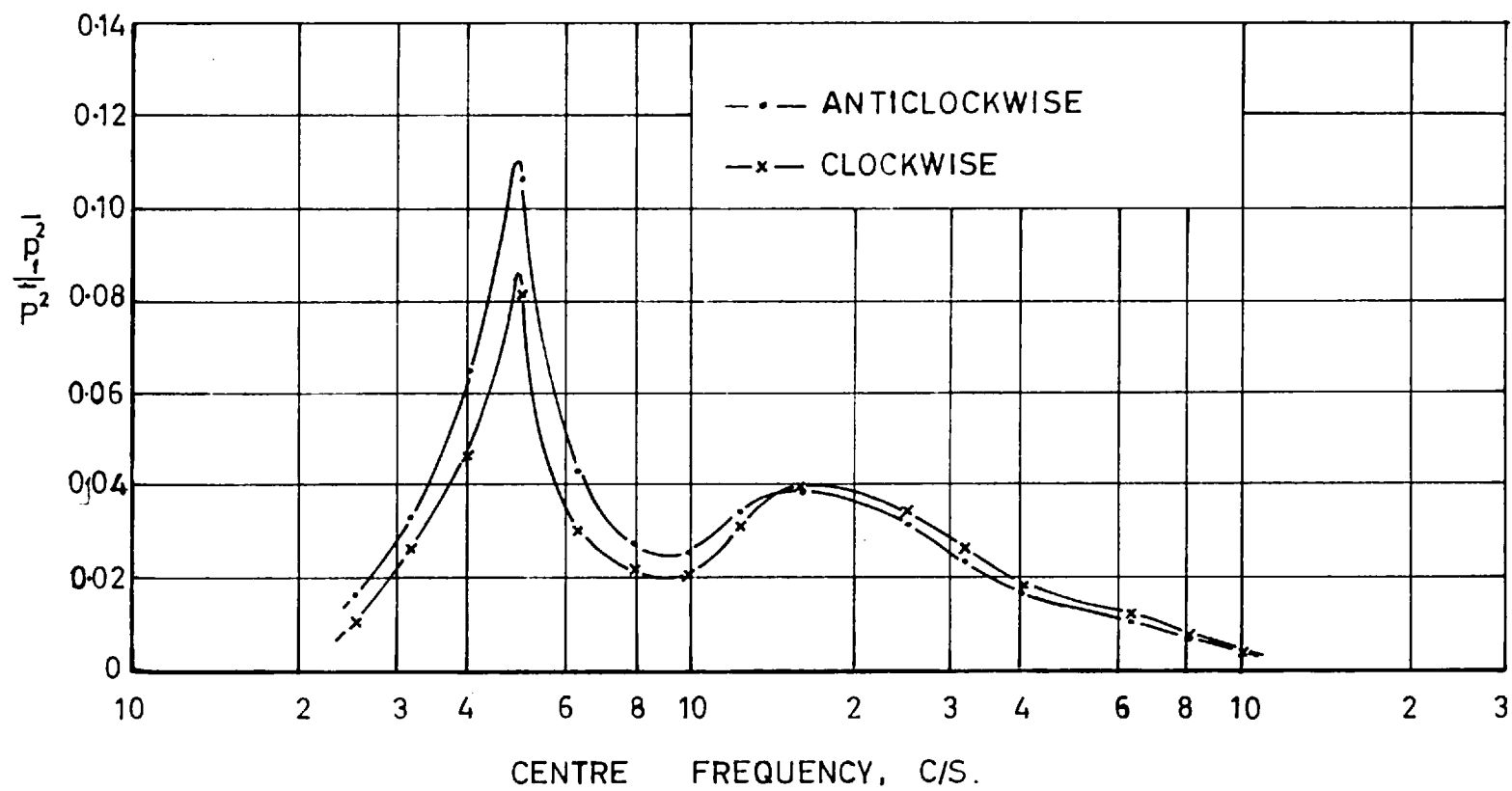


FIGURE A13. SWITCHED PRESSURE
FLUCTUATION SPECTRA, ROTATION = 120° .

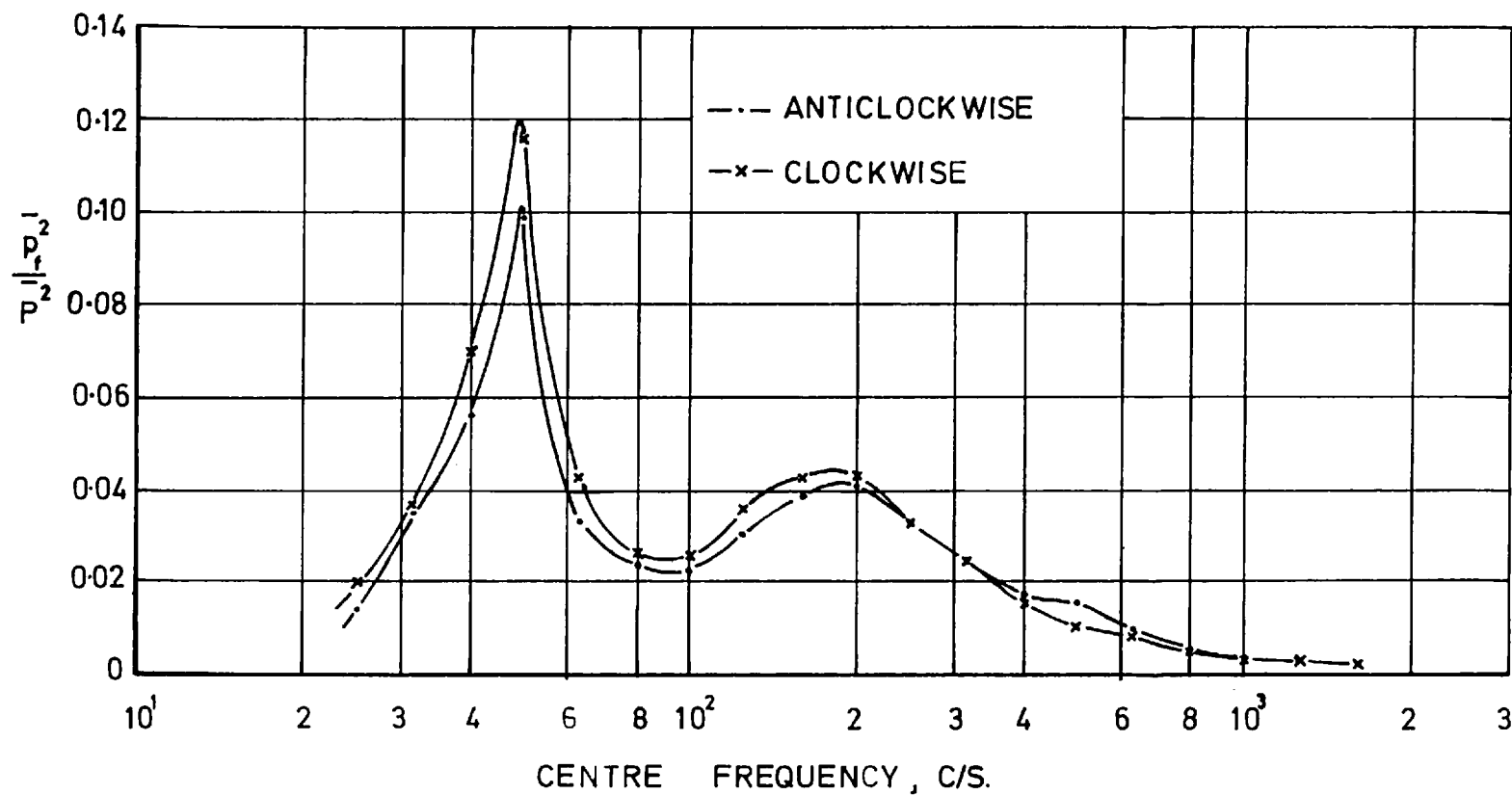


FIGURE A14. SWITCHED PRESSURE
FLUCTUATION SPECTRA, ROTATION = -120° .

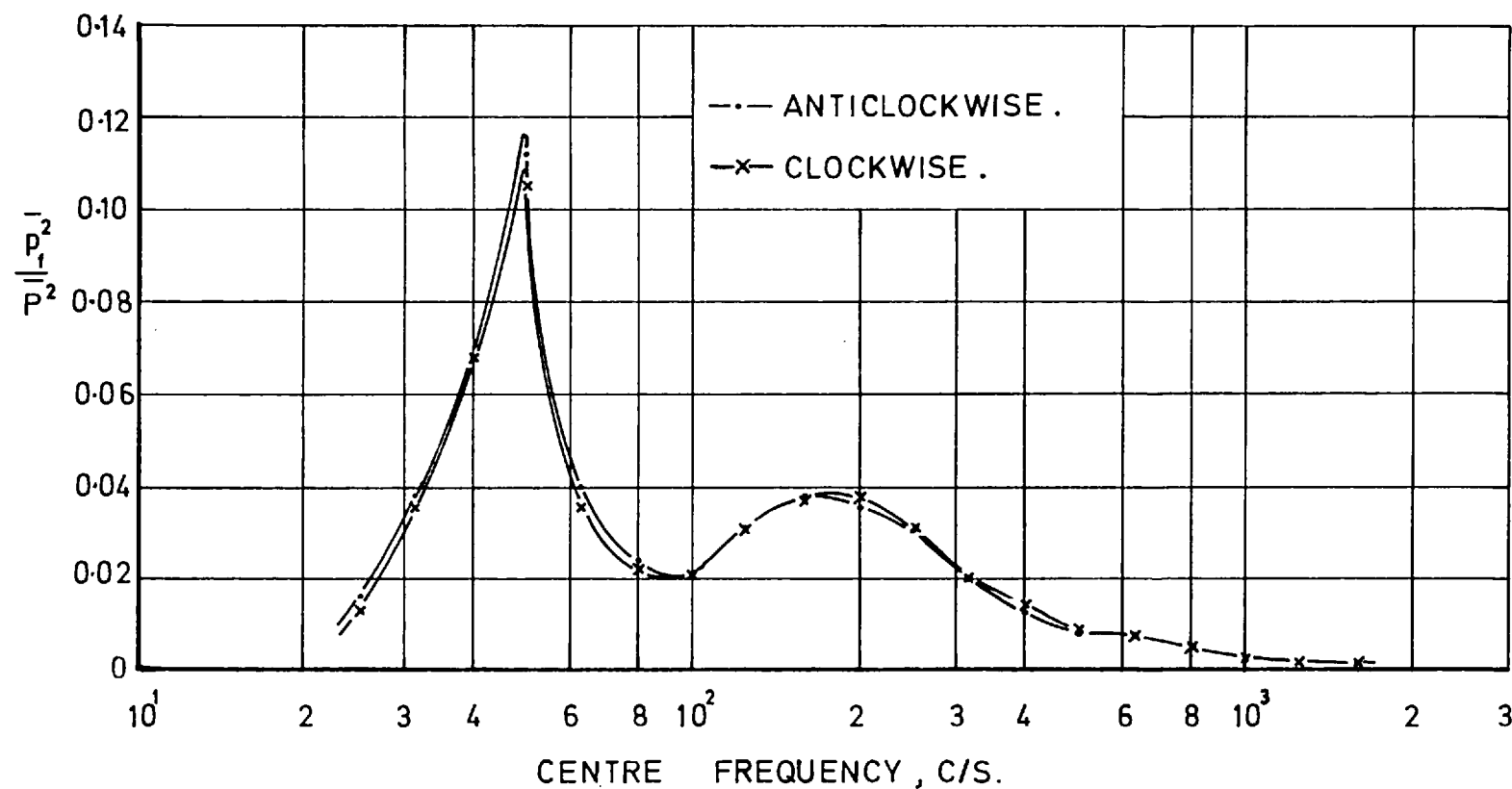


FIGURE A15. SWITCHED PRESSURE
FLUCTUATION SPECTRA, ROTATION = 180°.

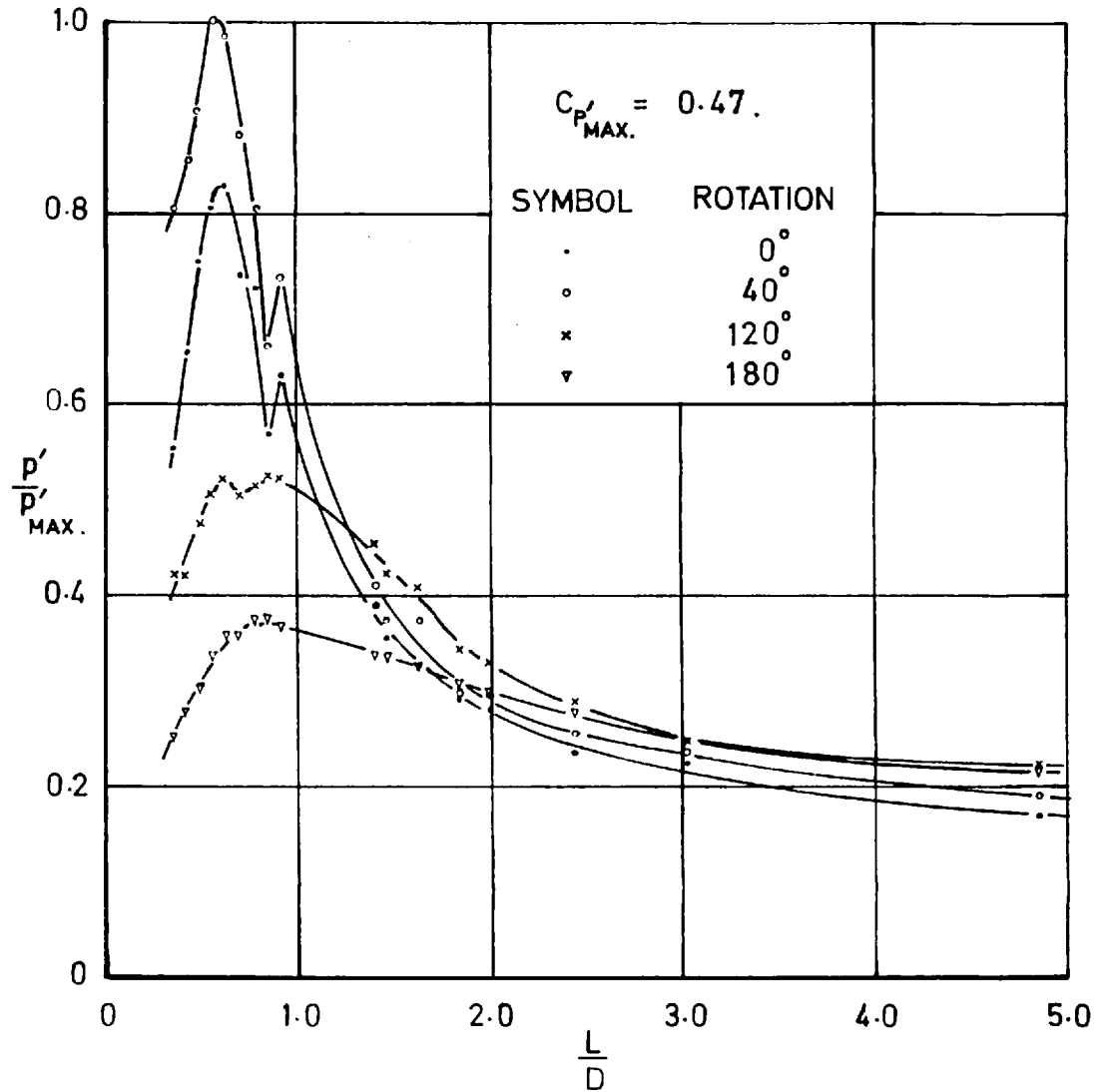


FIGURE A16. AXIAL DISTRIBUTION OF TOTAL R.M.S. WALL PRESSURE FLUCTUATION (maximum readings).

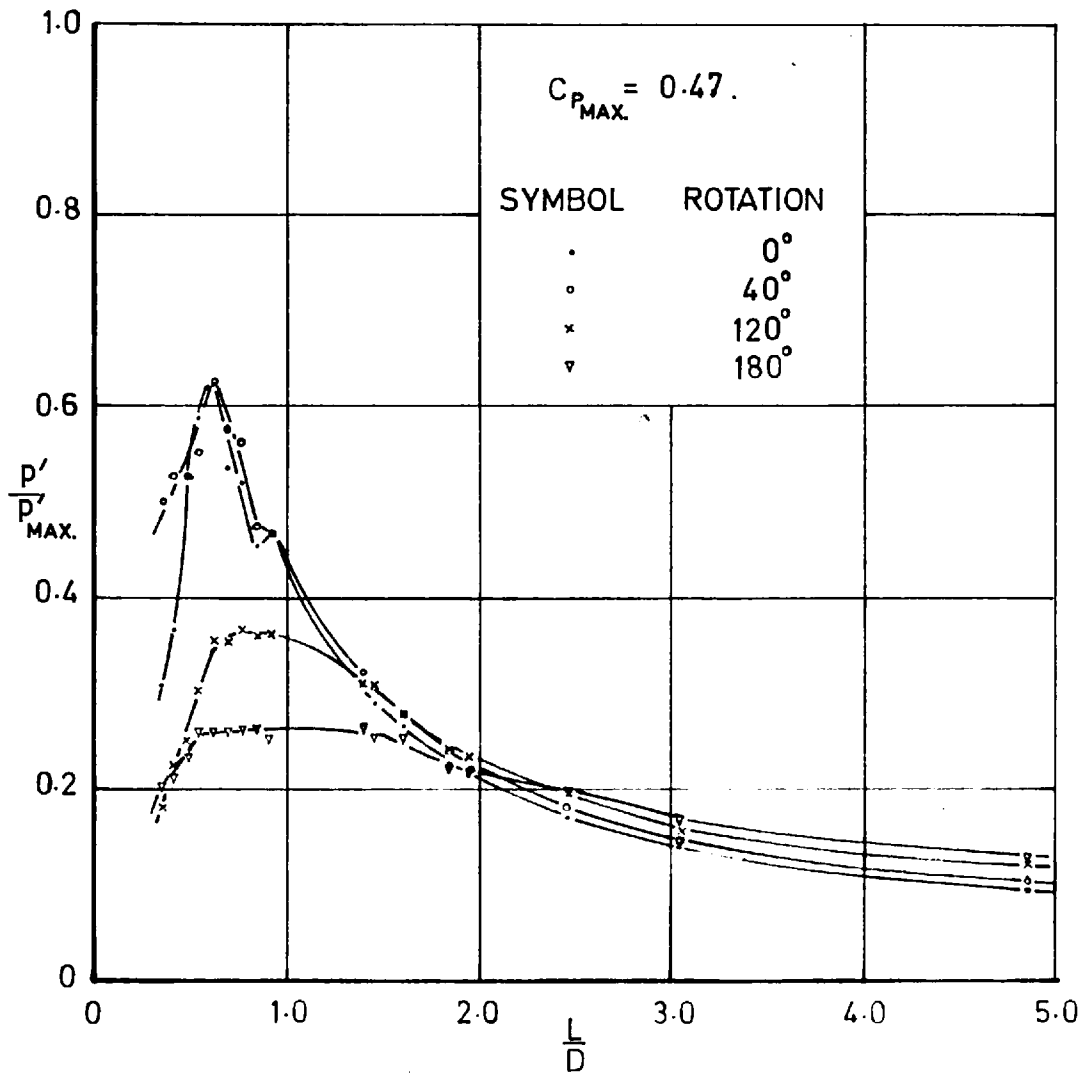


FIGURE A17. AXIAL DISTRIBUTION OF TOTAL R.M.S. WALL PRESSURE FLUCTUATION (minimum readings).

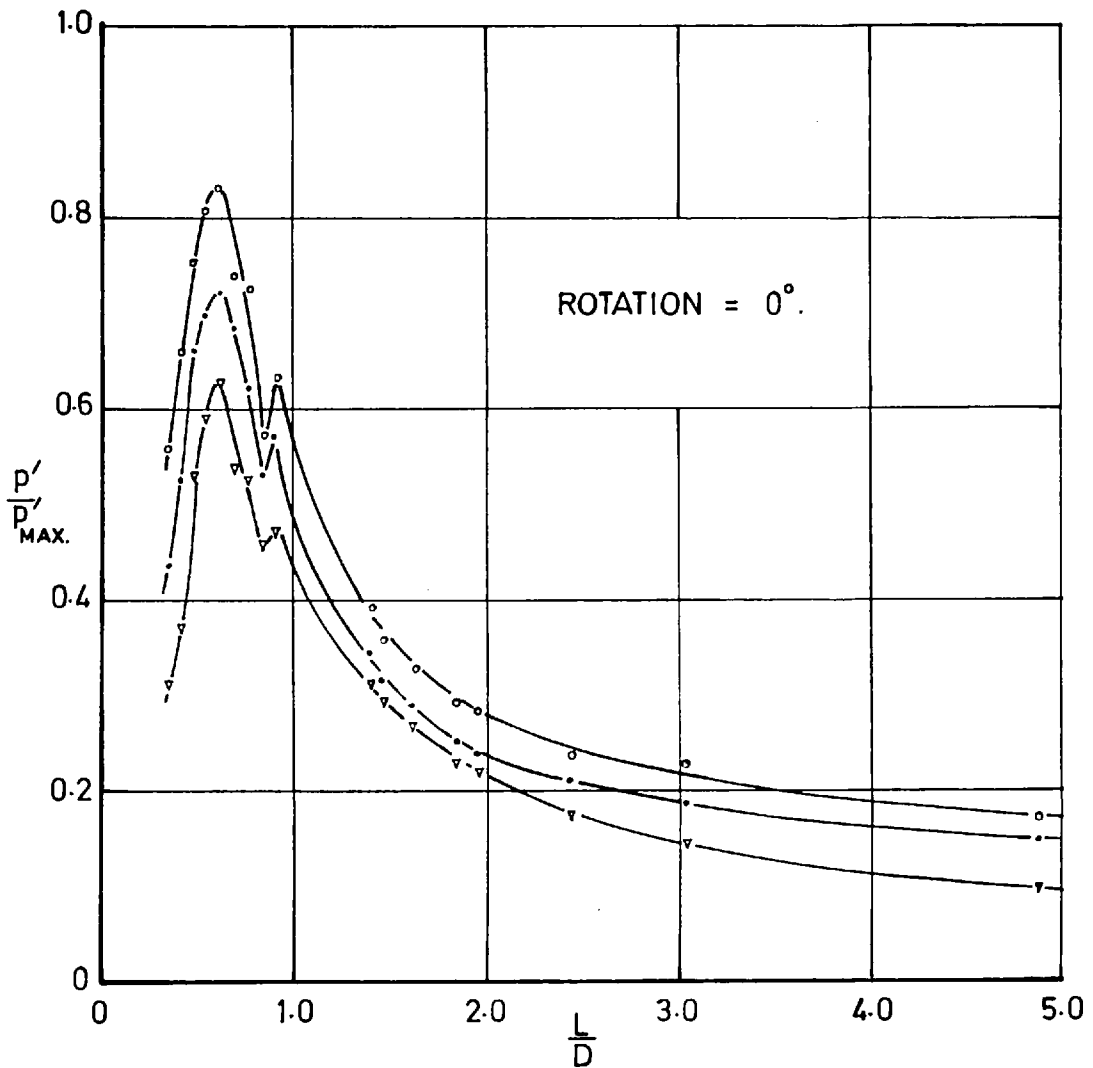


FIGURE A18. COMPARISON OF AXIAL DISTRIBUTIONS OBTAINED FROM MAX., MEAN & MIN. READINGS .

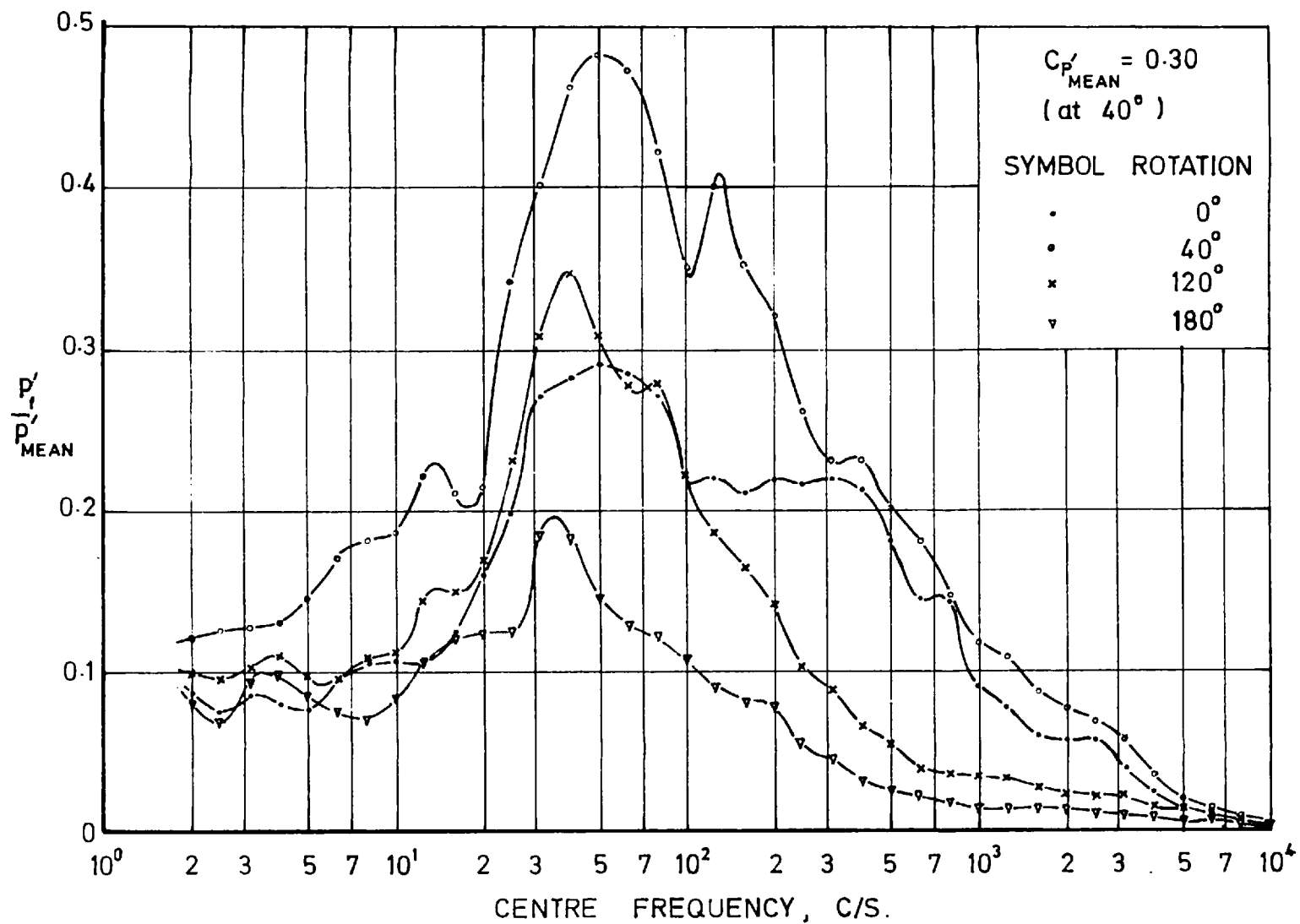


FIGURE A19. PRESSURE SPECTRA AT $L/D = 0.35$.
(max. readings)

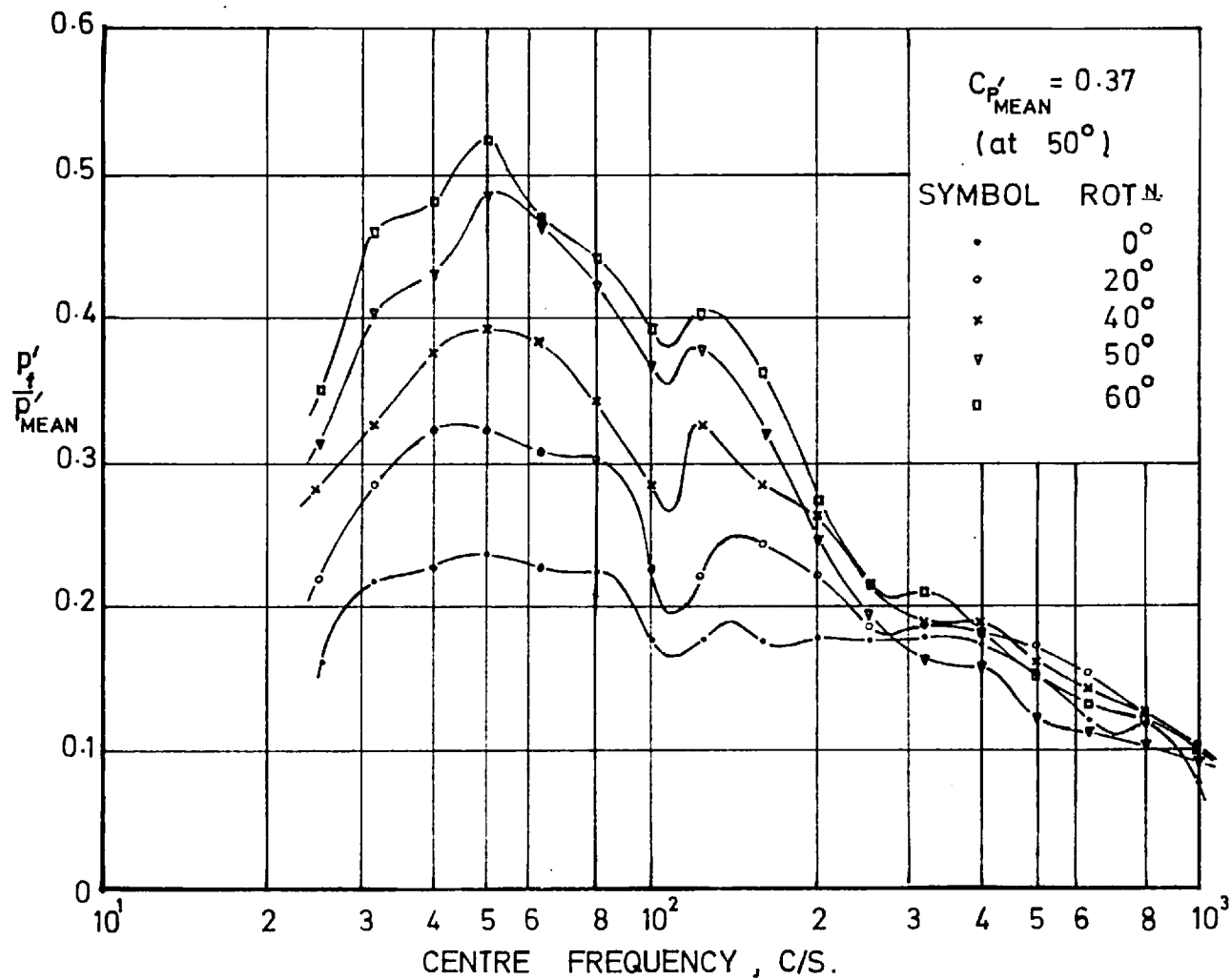


FIGURE A 20. PRESSURE SPECTRA AT $L/D = 0.35$.
(maximum readings)

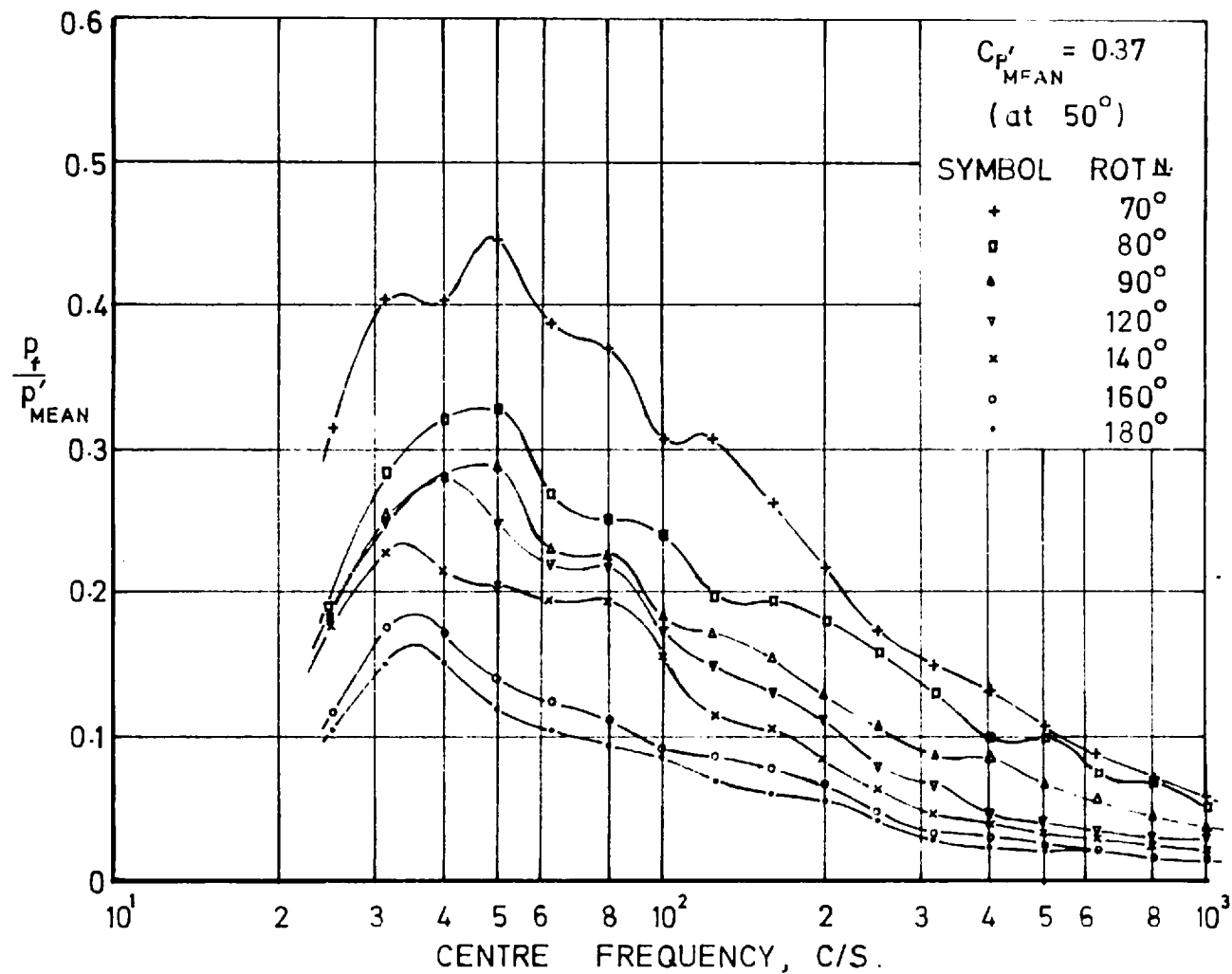


FIGURE A21. PRESSURE SPECTRA AT $L/D = 0.35$.
(maximum readings)

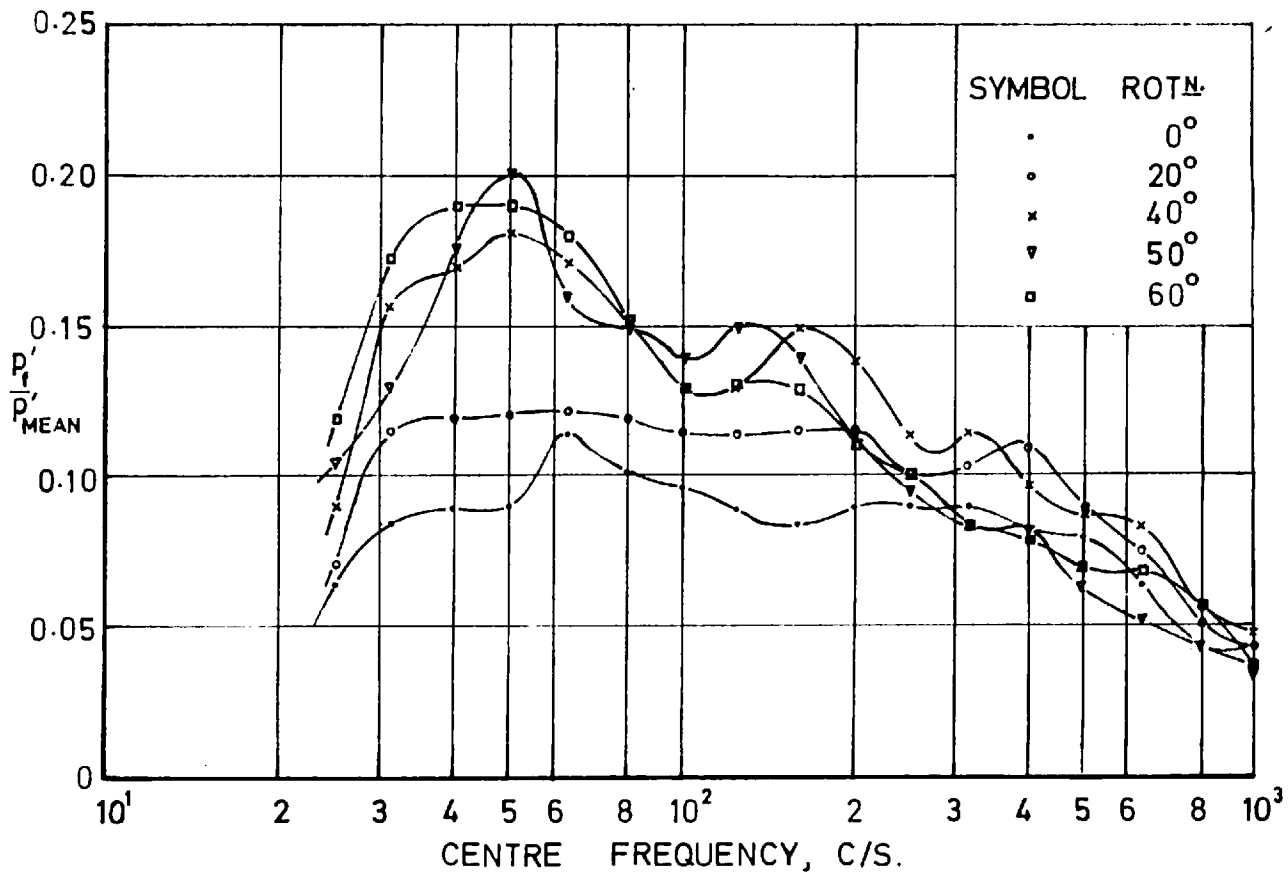


FIGURE A22. PRESSURE SPECTRA AT $L/D = 0.35$.
(minimum readings)

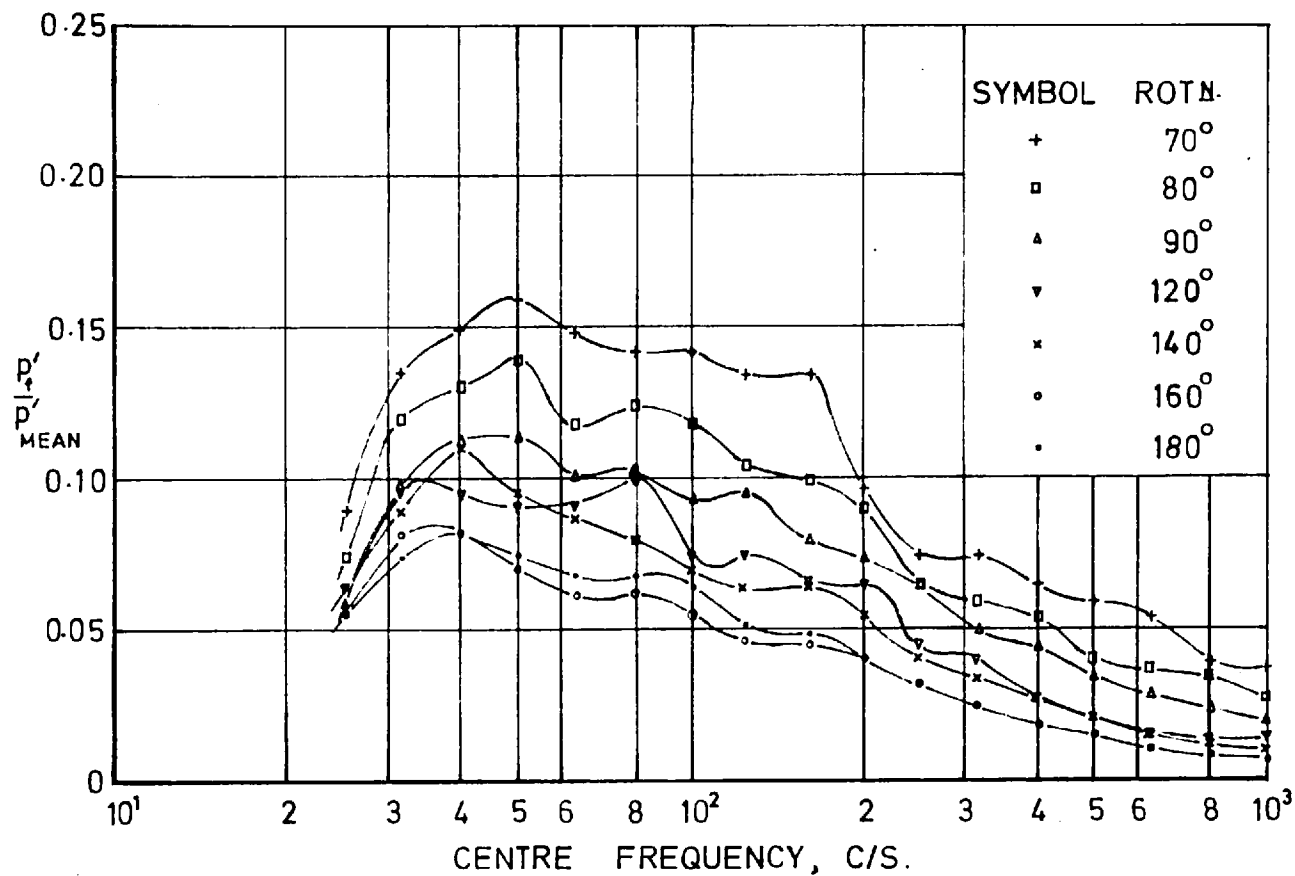


FIGURE A23. PRESSURE SPECTRA AT $L/D = 0.35$.
(minimum readings)

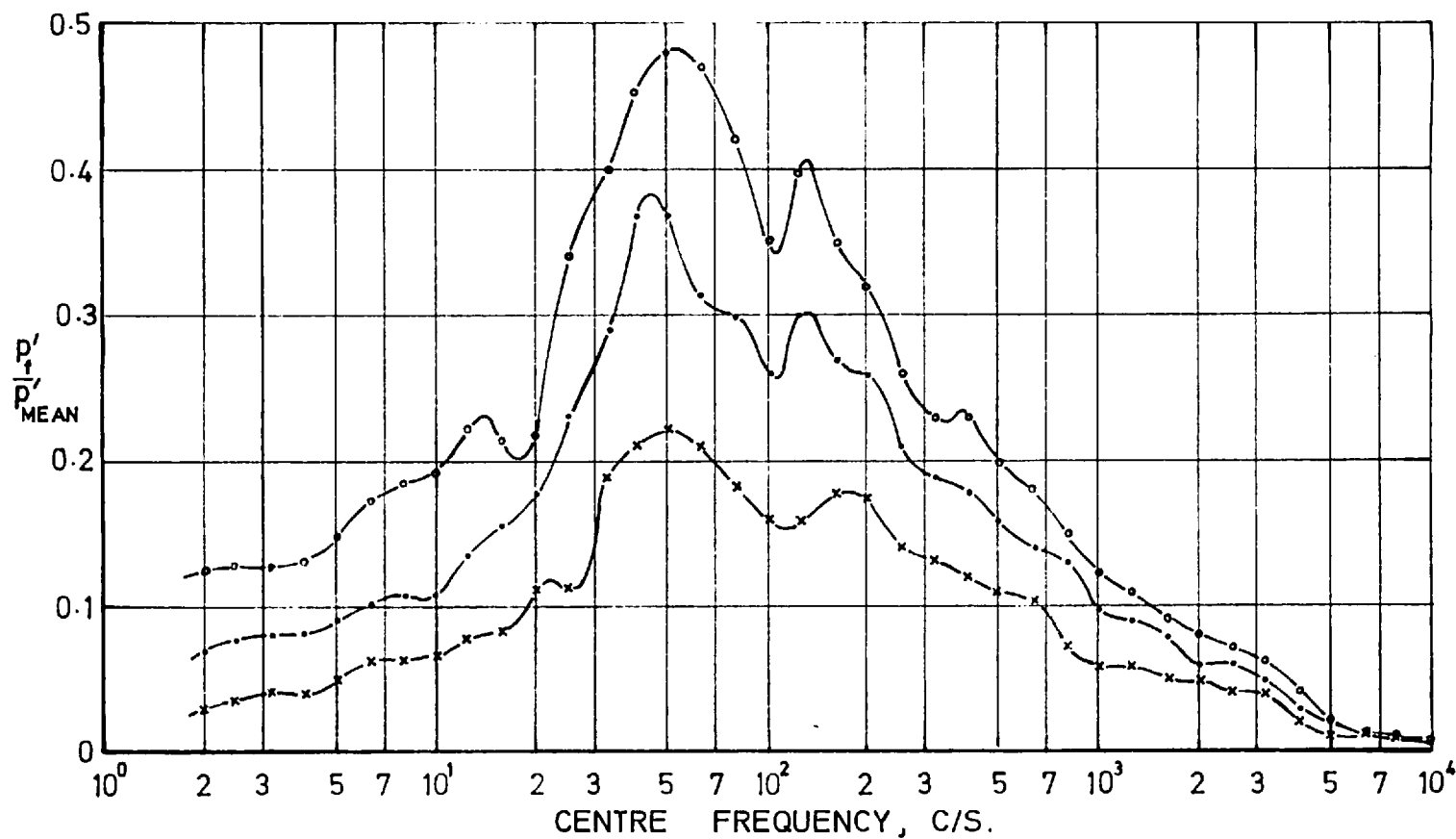


FIGURE A24. COMPARISON OF SPECTRA OBTAINED FROM MAX., MEAN & MIN. READINGS AT $L/D = 0.35$, ROTATION = 40° .

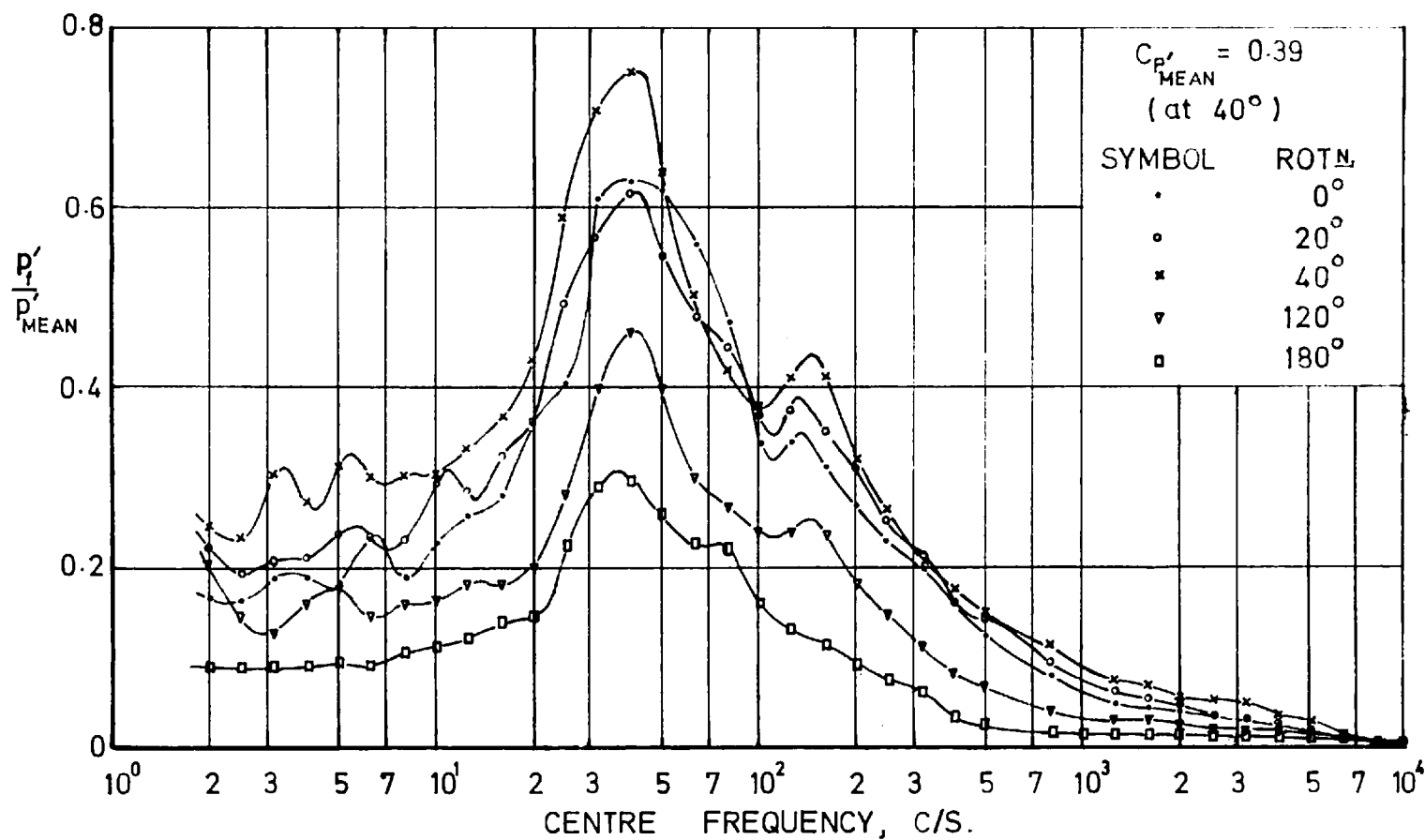


FIGURE A25. PRESSURE SPECTRA AT $L/D = 0.62$.
(maximum readings)

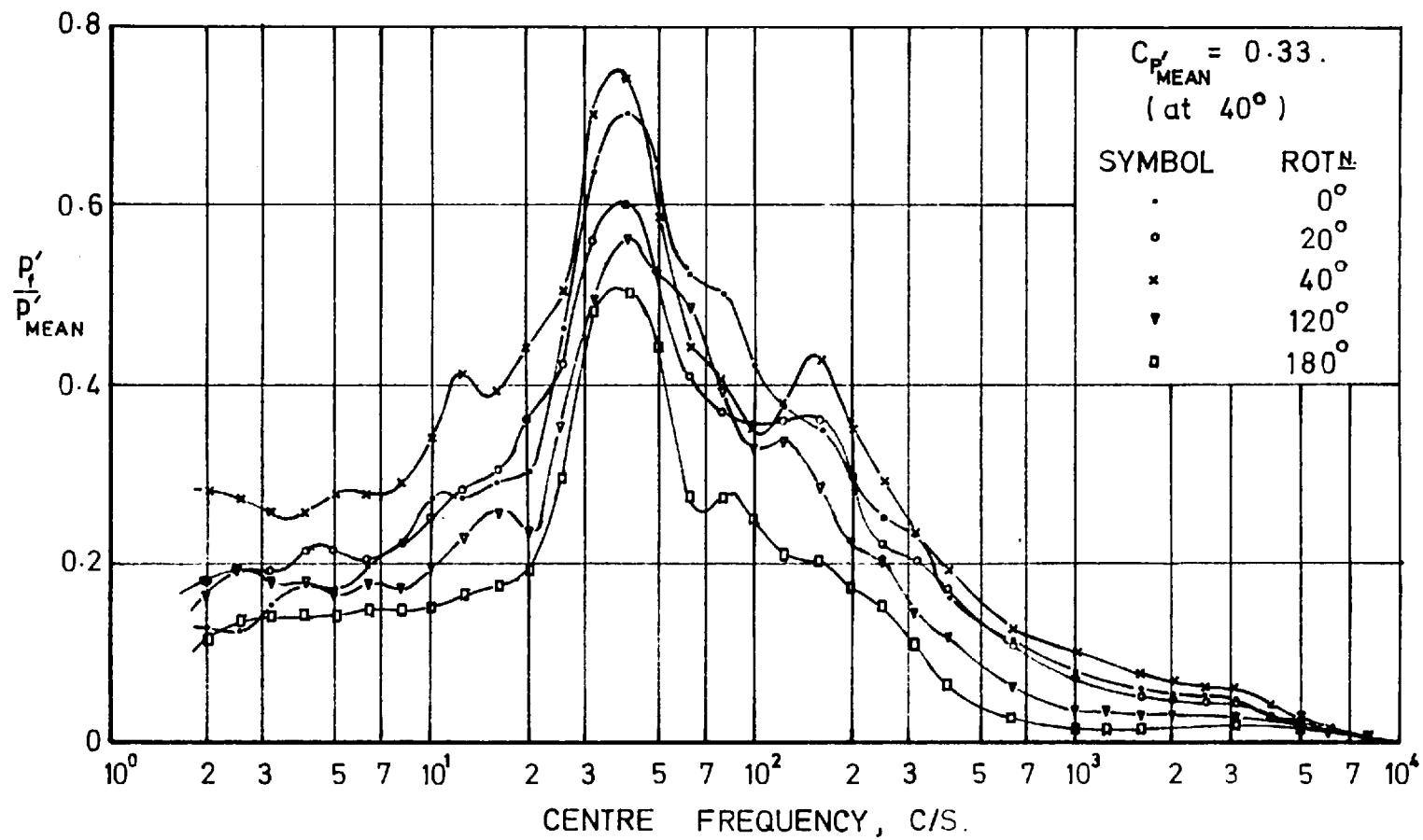


FIGURE A26. PRESSURE SPECTRA AT $L/D = 0.76$.
 (maximum readings)

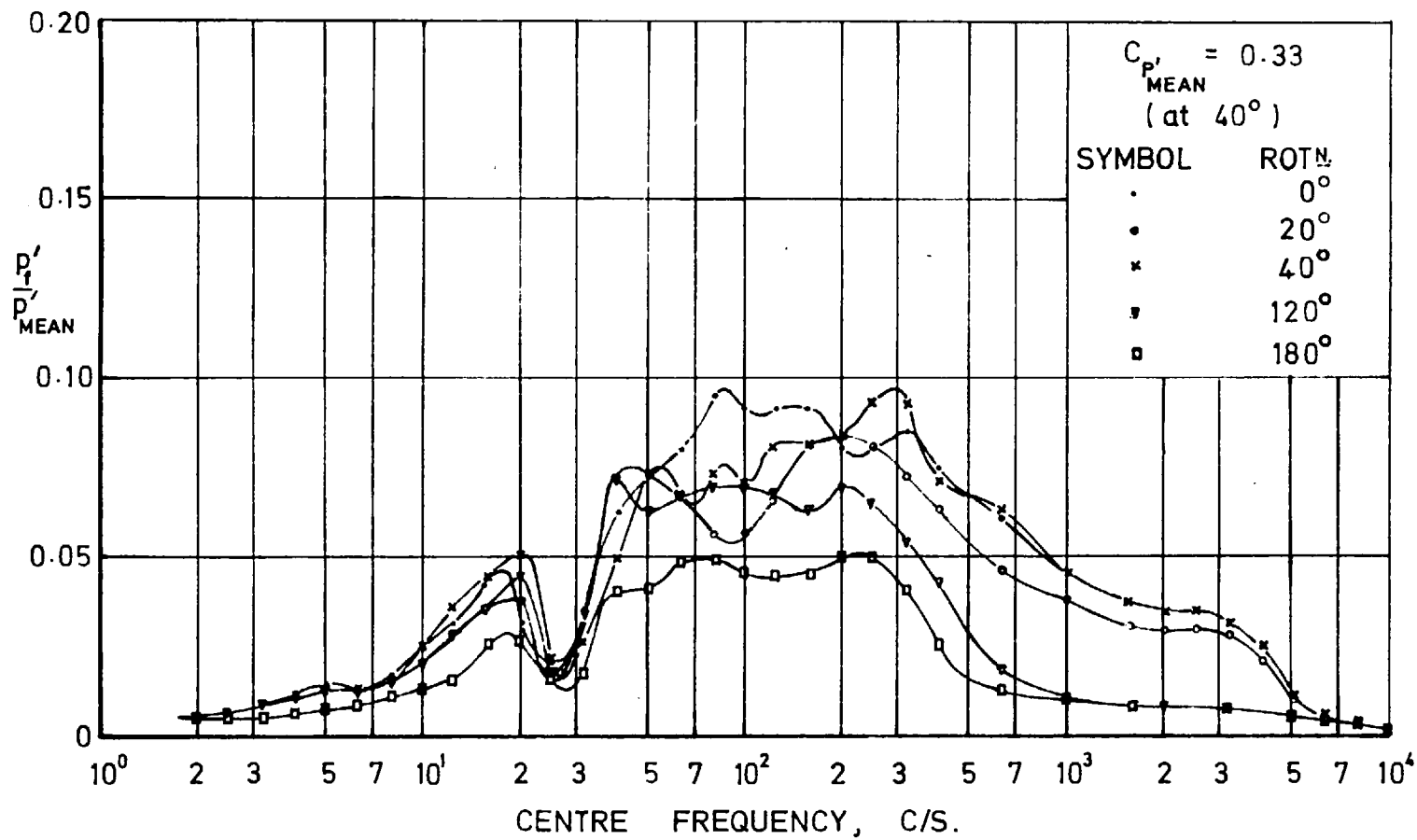


FIGURE A27. PRESSURE SPECTRA AT $L/D = 0.76$.
 (minimum readings)

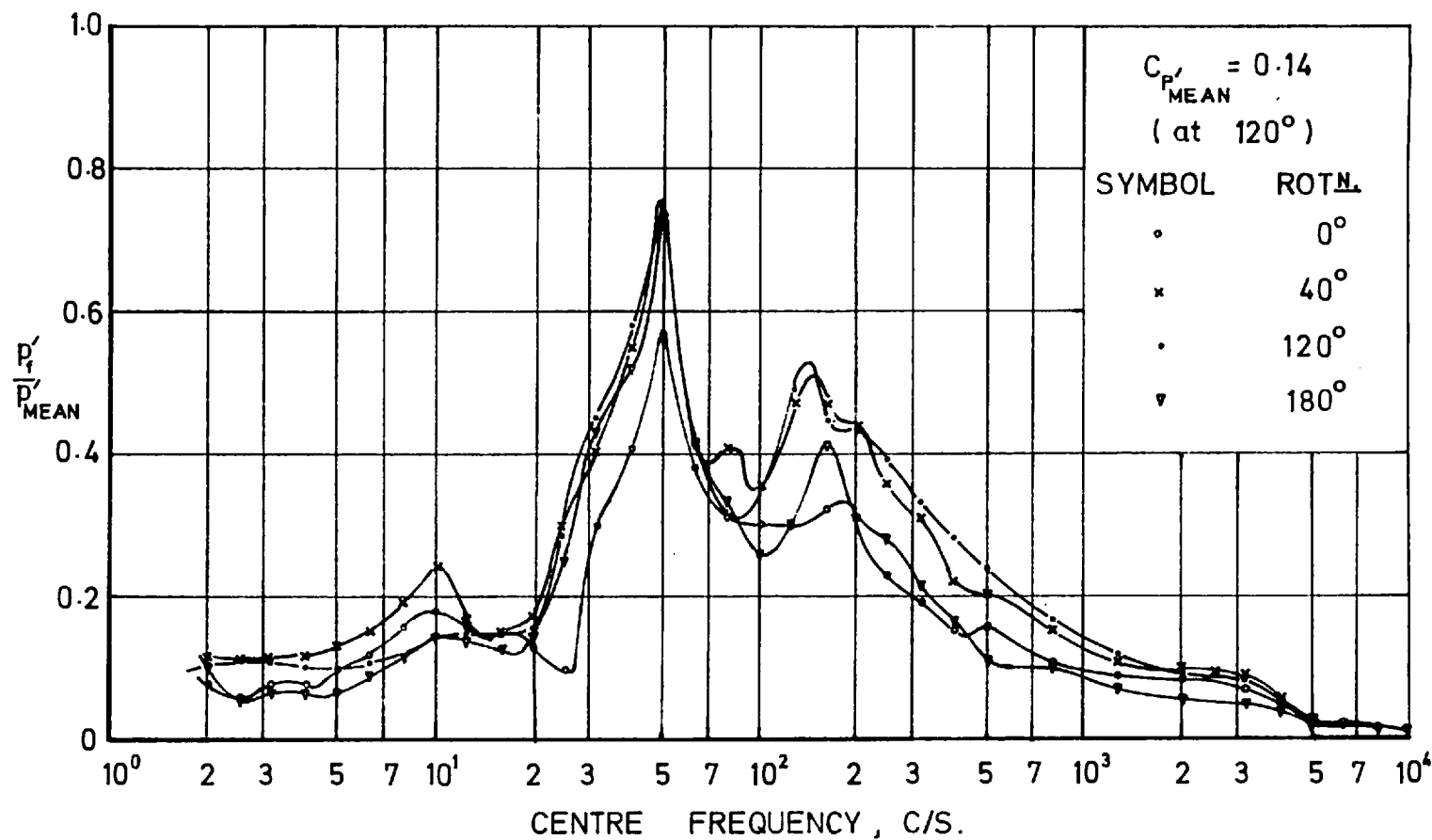


FIGURE A28. PRESSURE SPECTRA AT $L/D = 1.96$.
 (maximum reading).

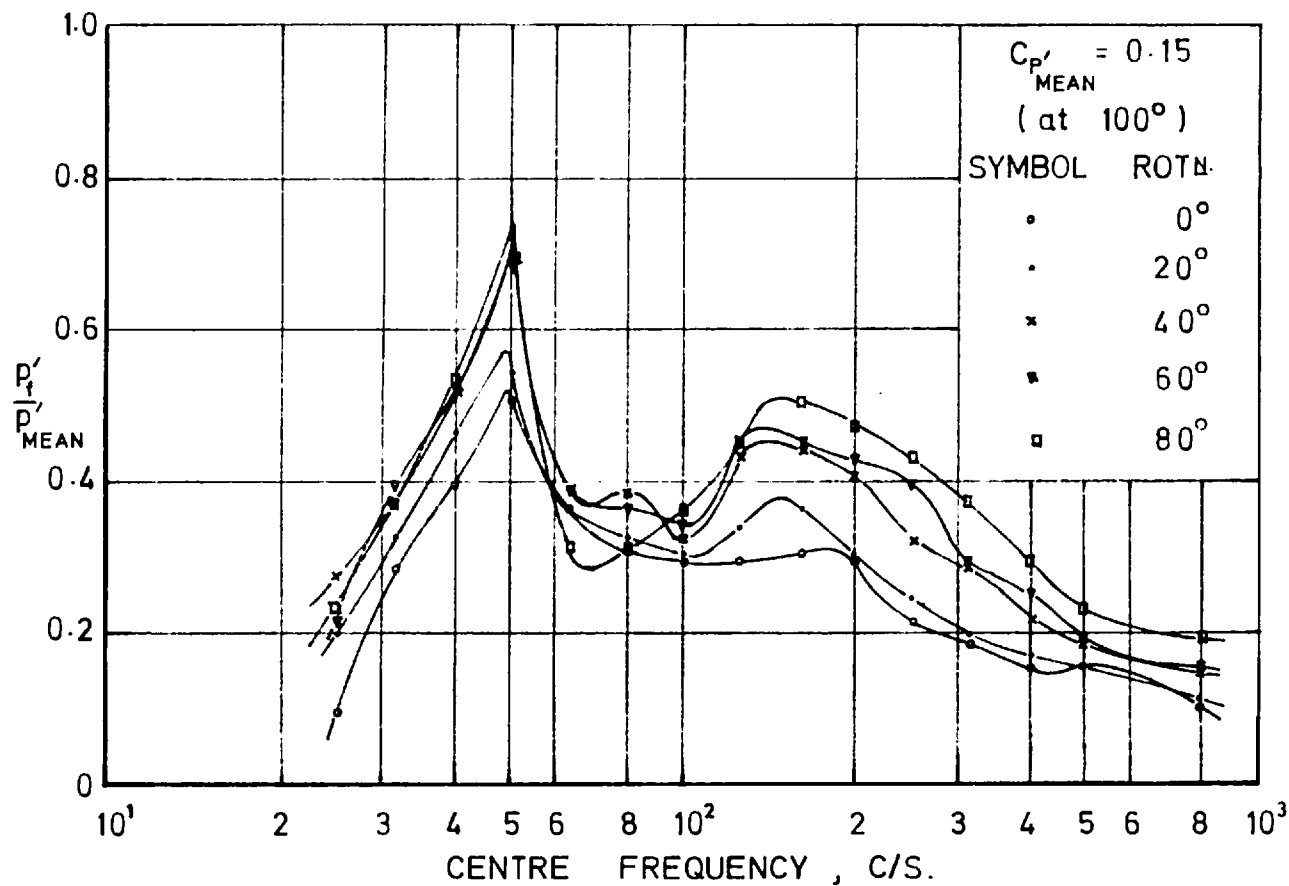


FIGURE A29. PRESSURE SPECTRA AT $L/D = 1.96$.
(maximum reading)

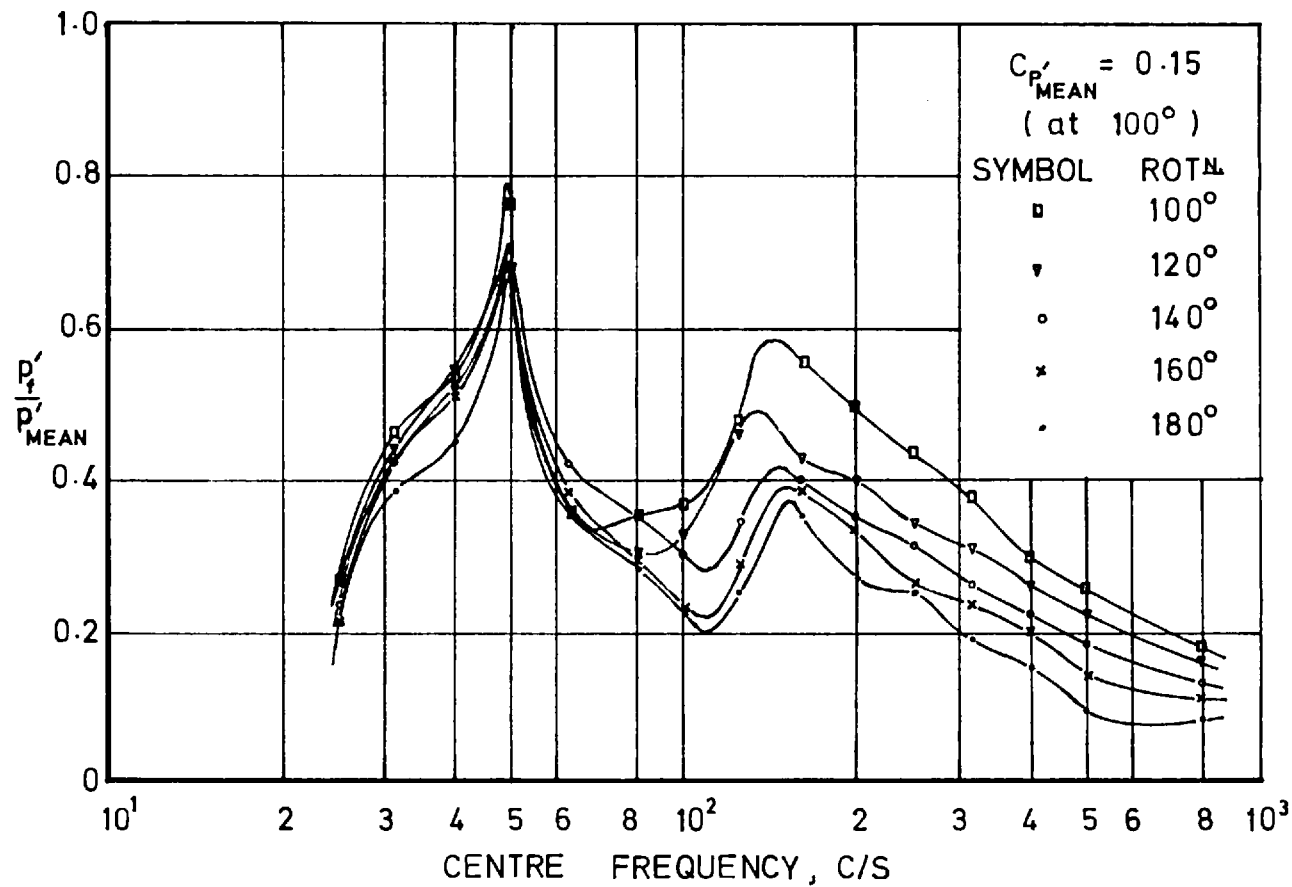


FIGURE A30. PRESSURE SPECTRA AT $L/D = 1.96$.
 (maximum reading)

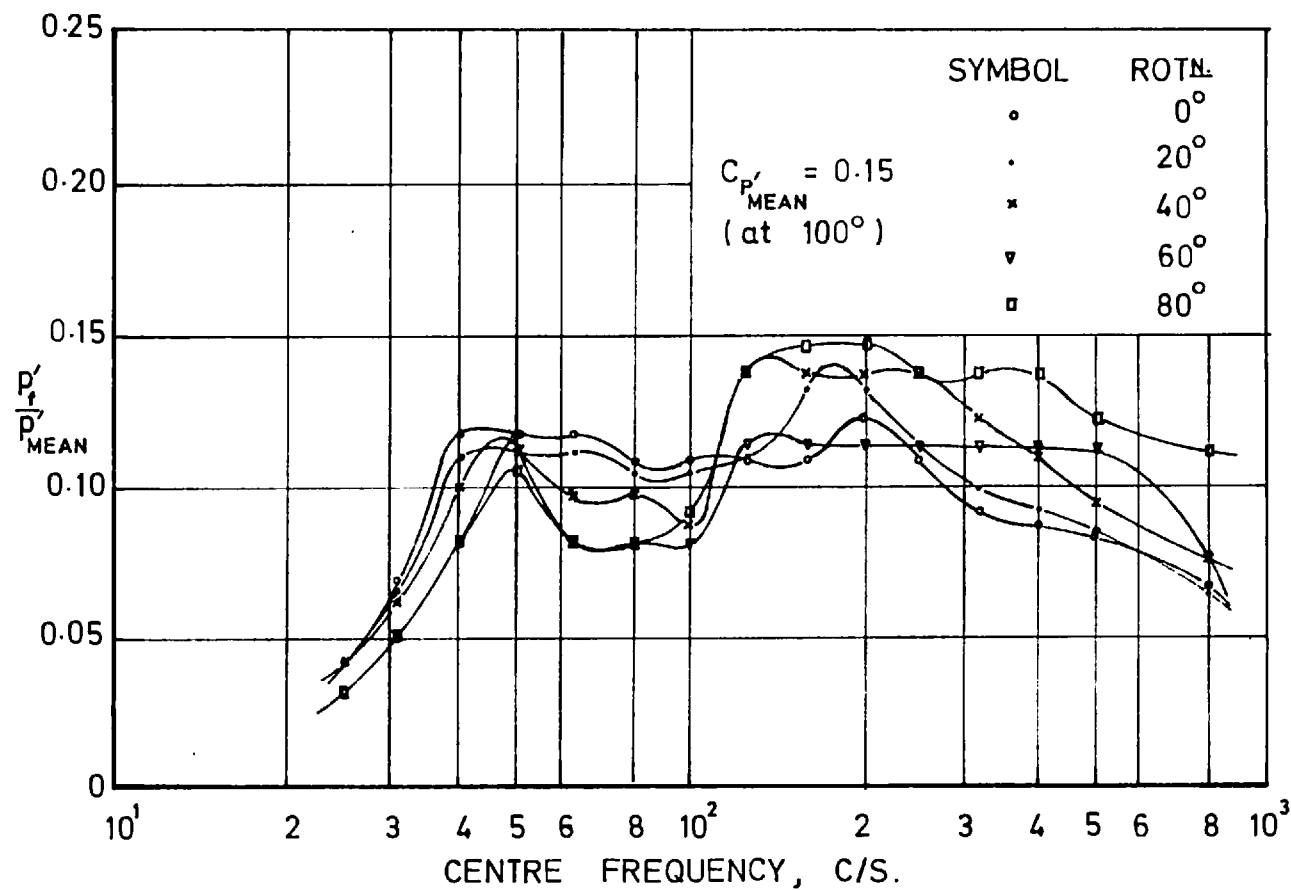


FIGURE A31. PRESSURE SPECTRA AT $L/D = 1.96$.
 (minimum readings)

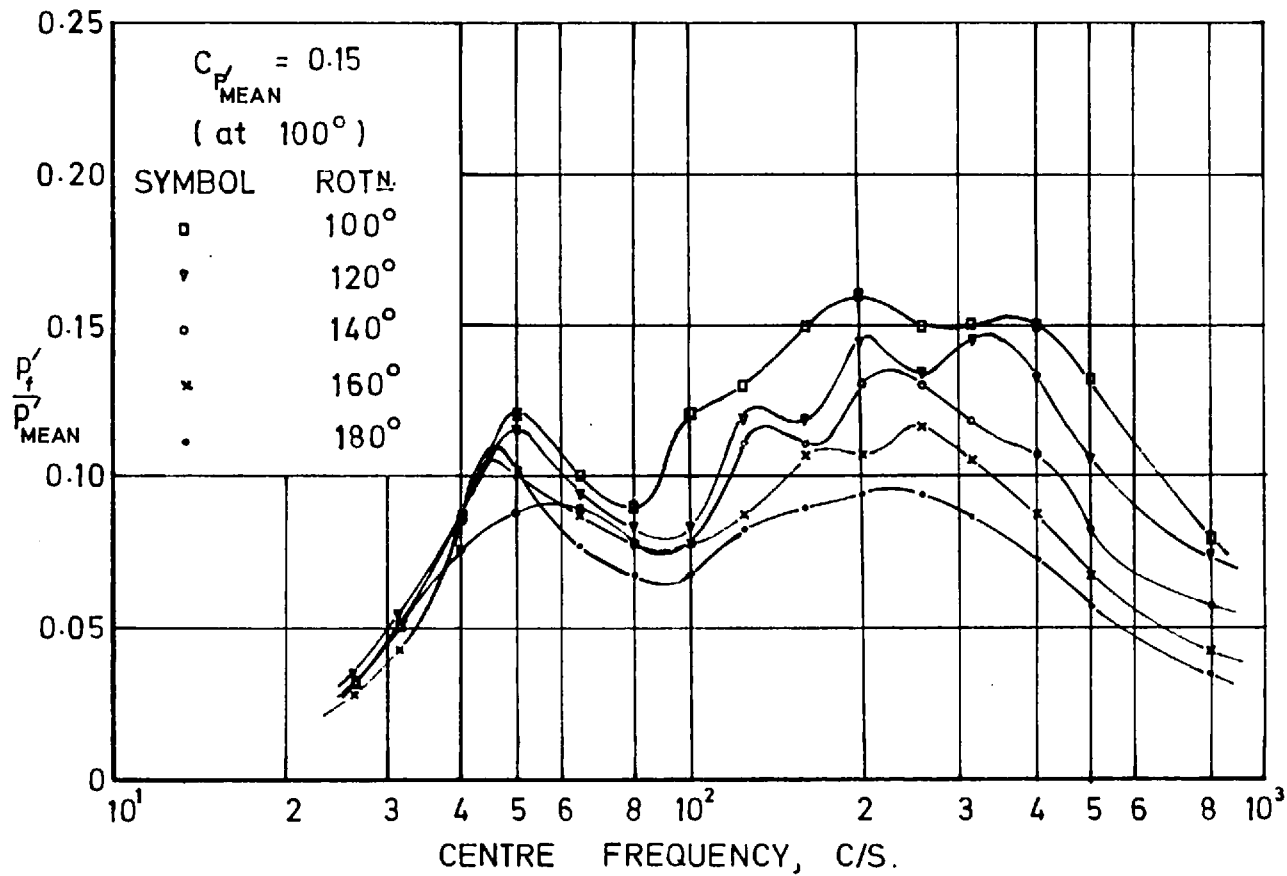


FIGURE A32. PRESSURE SPECTRA AT $L/D = 1.96$.
 (minimum readings)

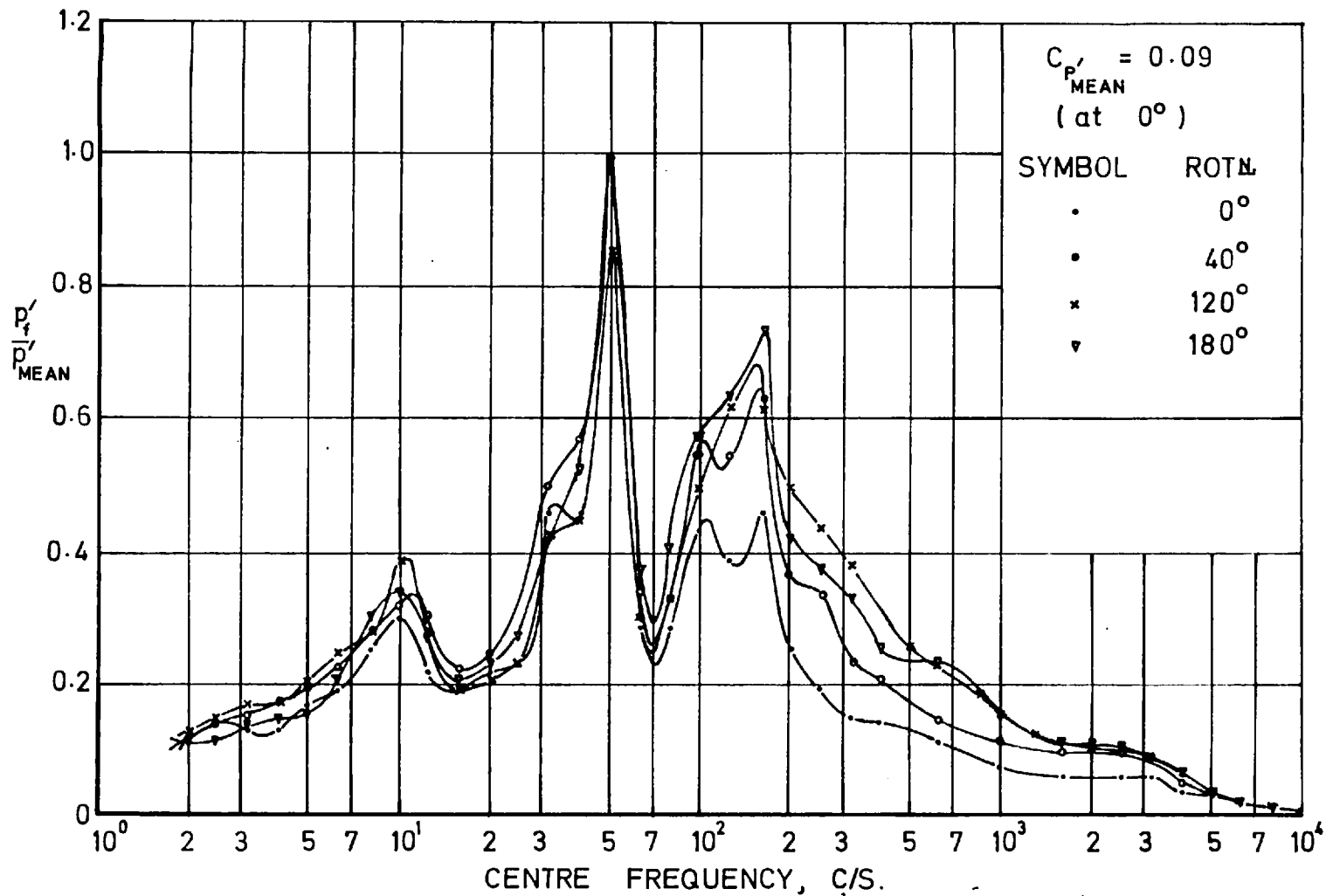


FIGURE A33. PRESSURE SPECTRA AT $L/D = 3.03$.
 (maximum readings)

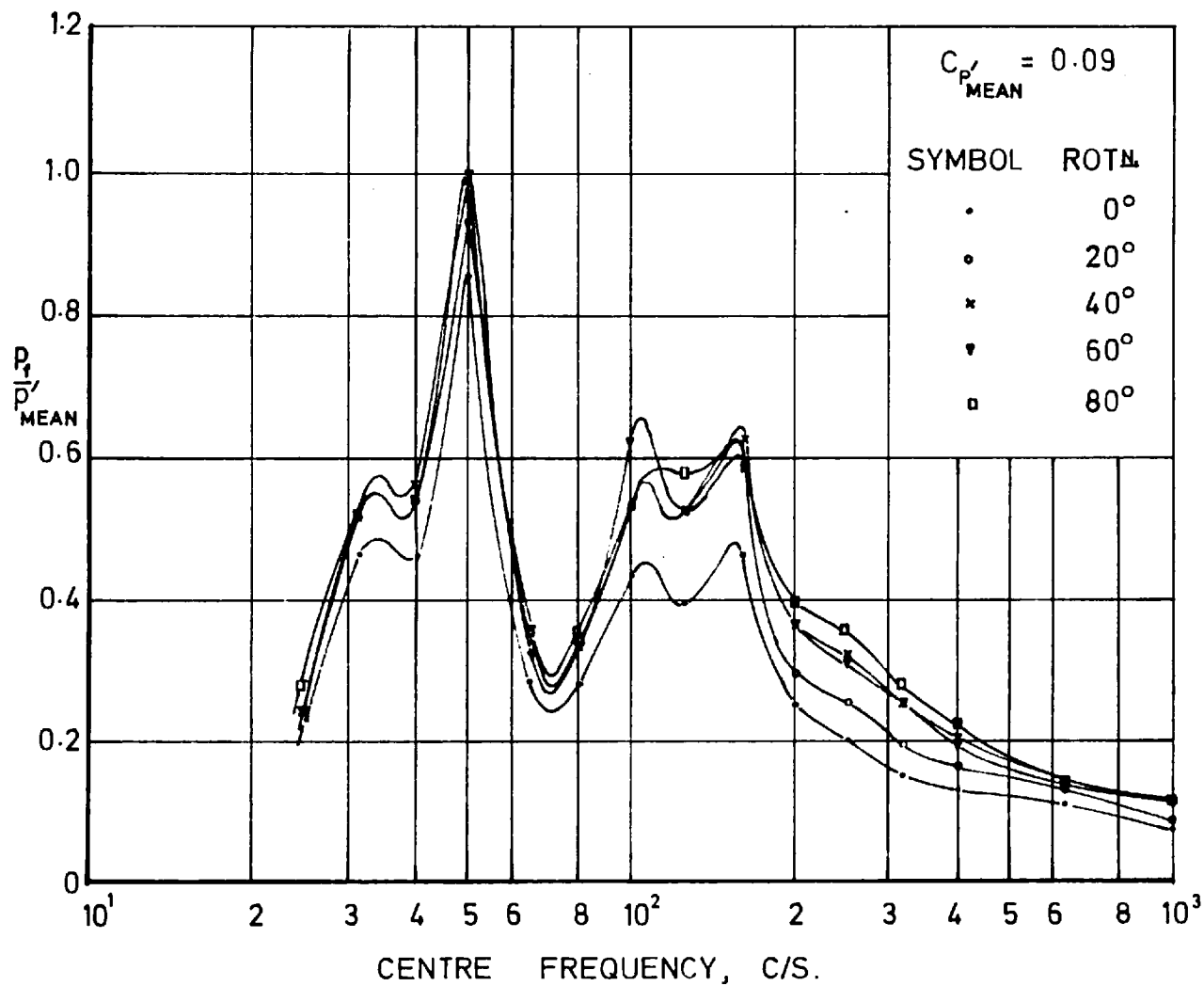


FIGURE A34. PRESSURE SPECTRA AT $L/D = 3.03$.
(maximum readings)

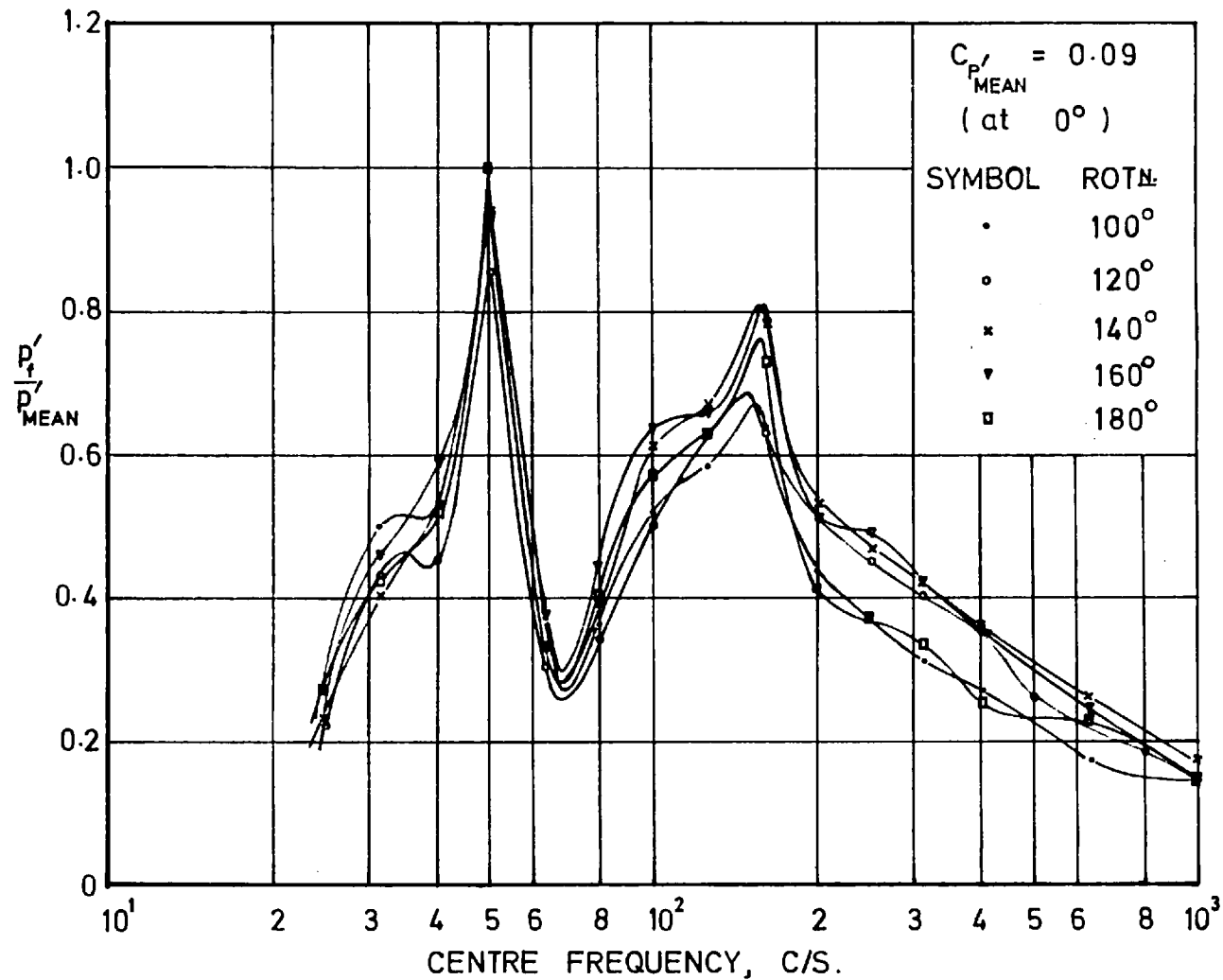


FIGURE A35. PRESSURE SPECTRA AT $L/D = 3.03$.
(maximum readings)

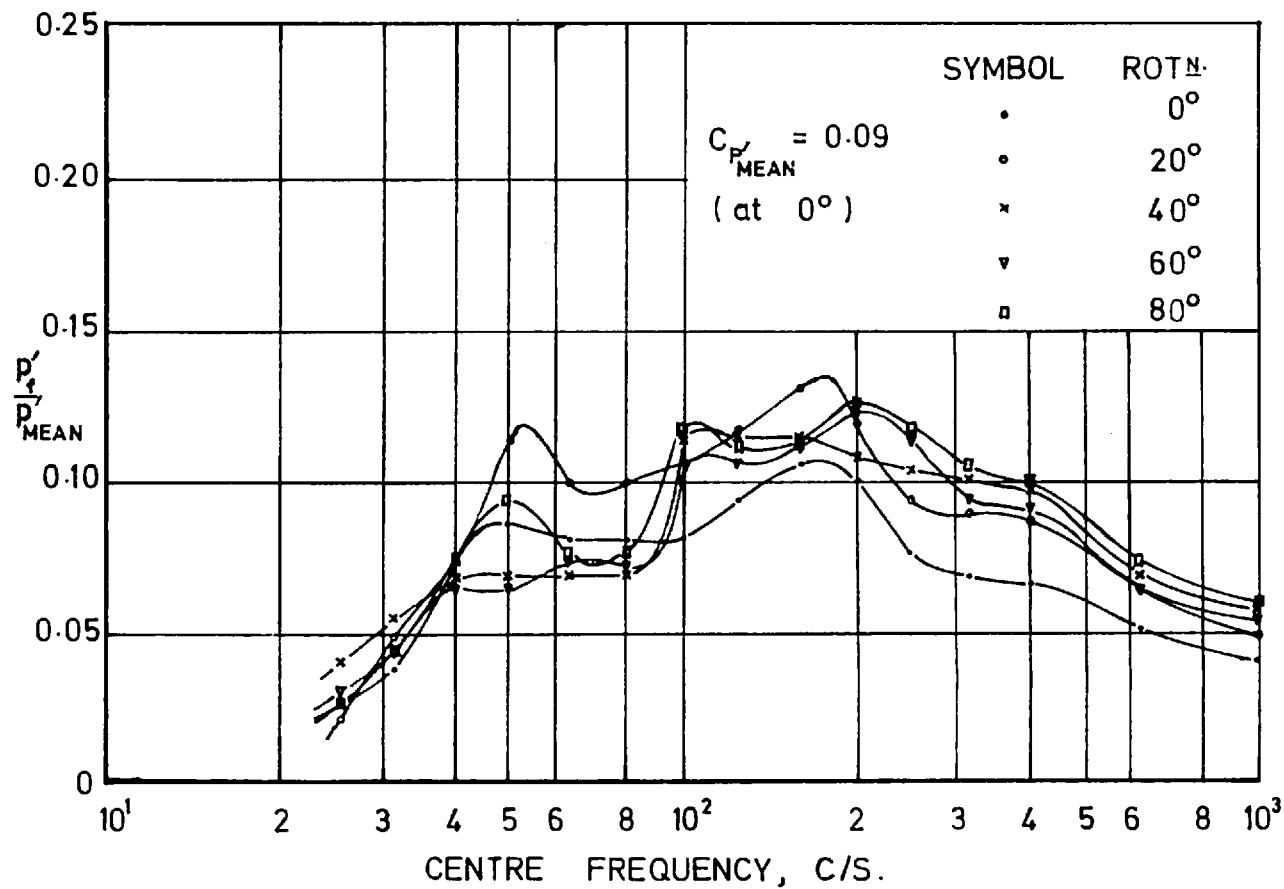


FIGURE A36. PRESSURE SPECTRA AT $L/D = 3.03$.
(minimum readings)

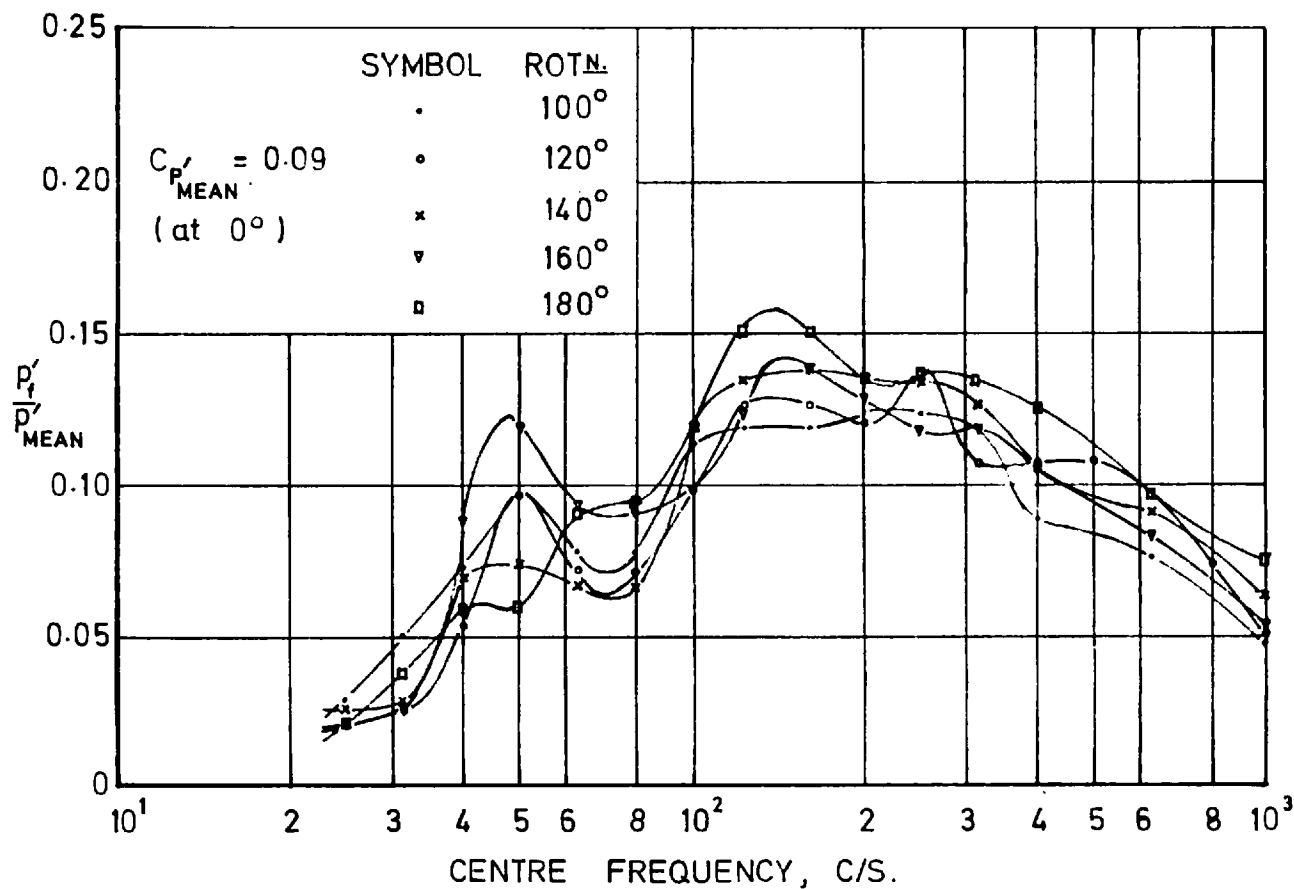


FIGURE A37. PRESSURE SPECTRA AT $L/D = 3.03$.
(minimum readings)

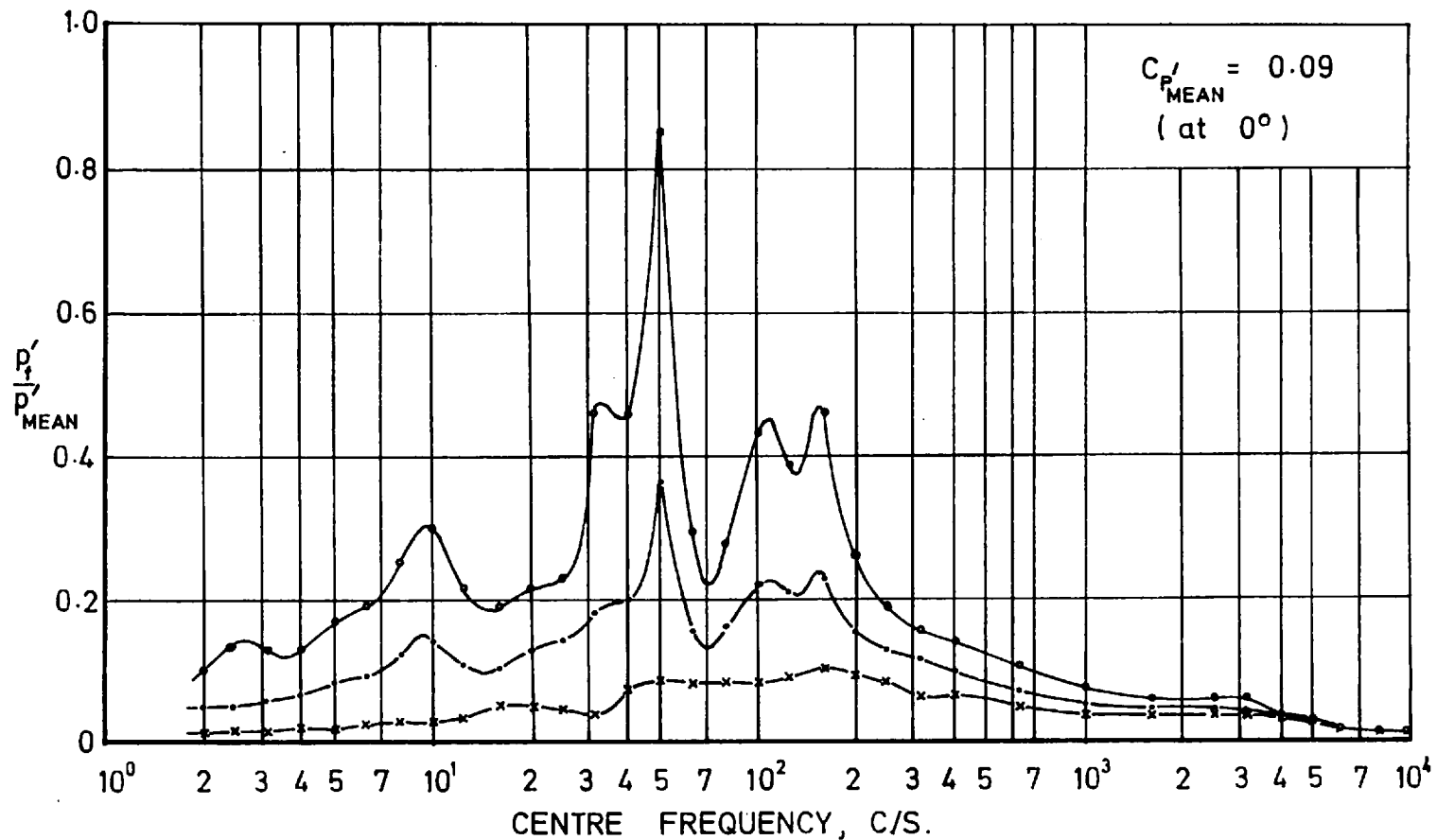


FIGURE A38. COMPARISON OF SPECTRA OBTAINED FROM MAX., MEAN & MIN. READINGS AT $L/D = 3.03$, ROTATION = 0° .

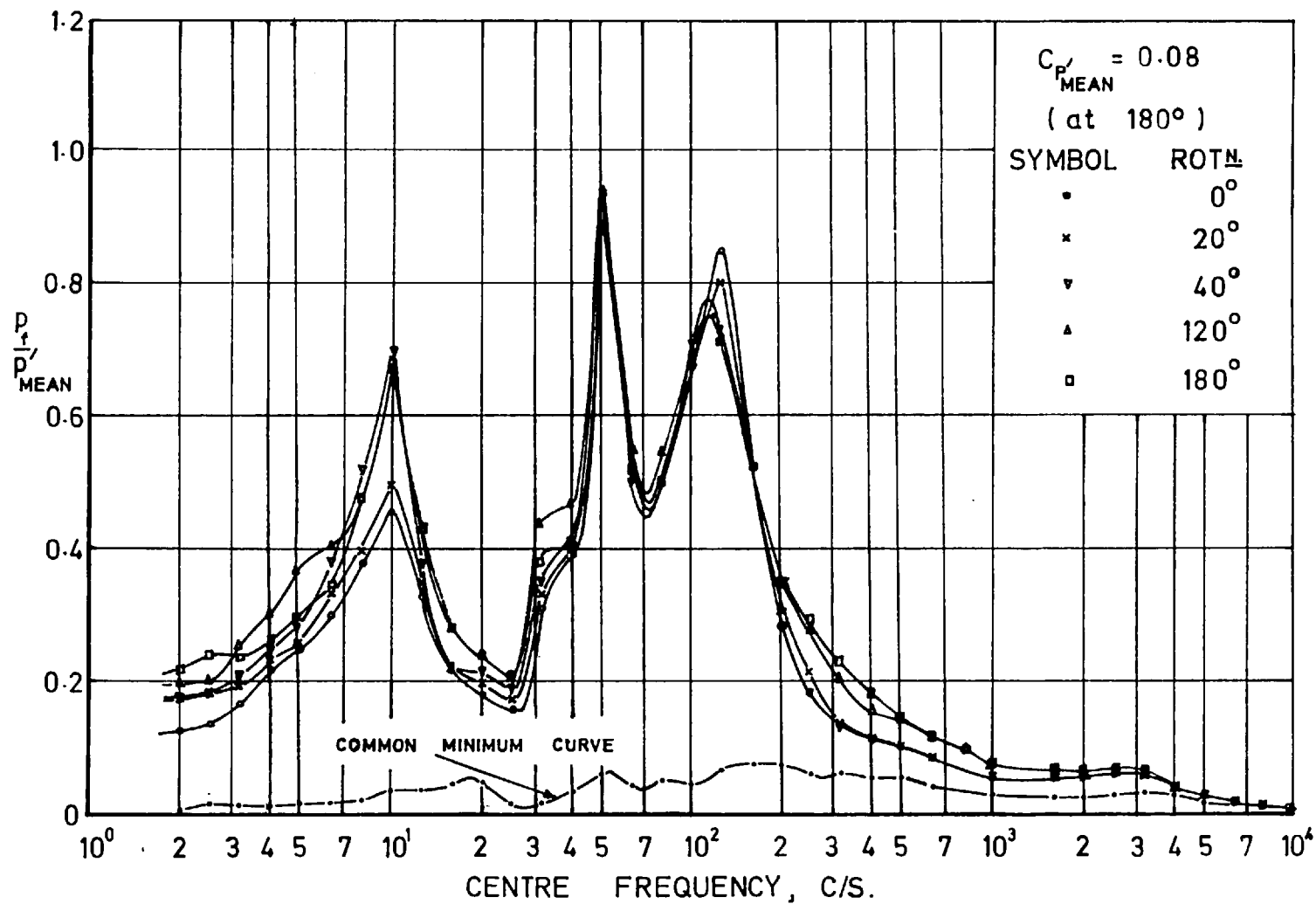


FIGURE A39. PRESSURE SPECTRA AT $L/D = 4.75$.
 (maximum readings)

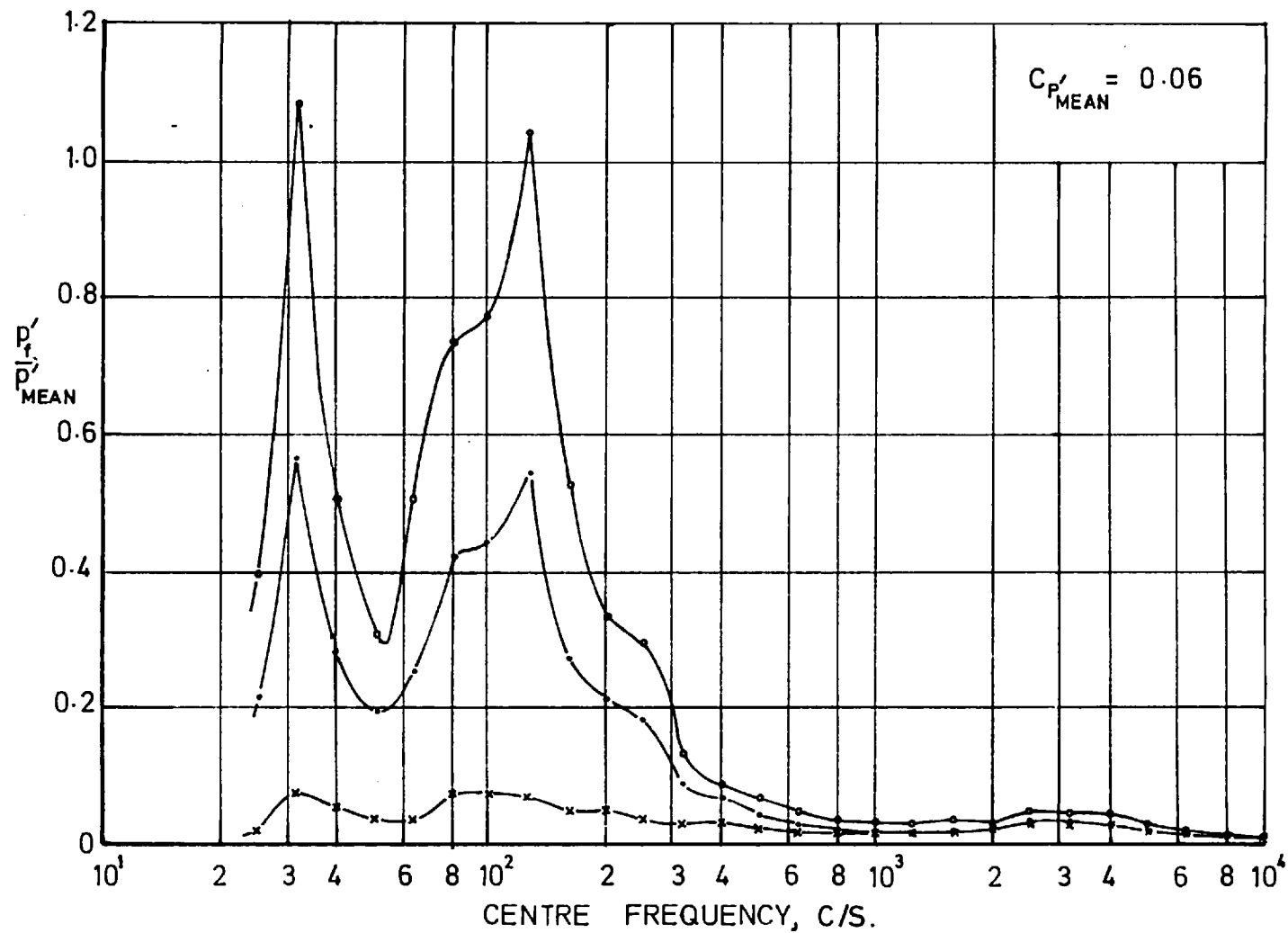


FIGURE A40. PRESSURE SPECTRA AT $L/D = 24.0$.
(max., mean & min. readings)

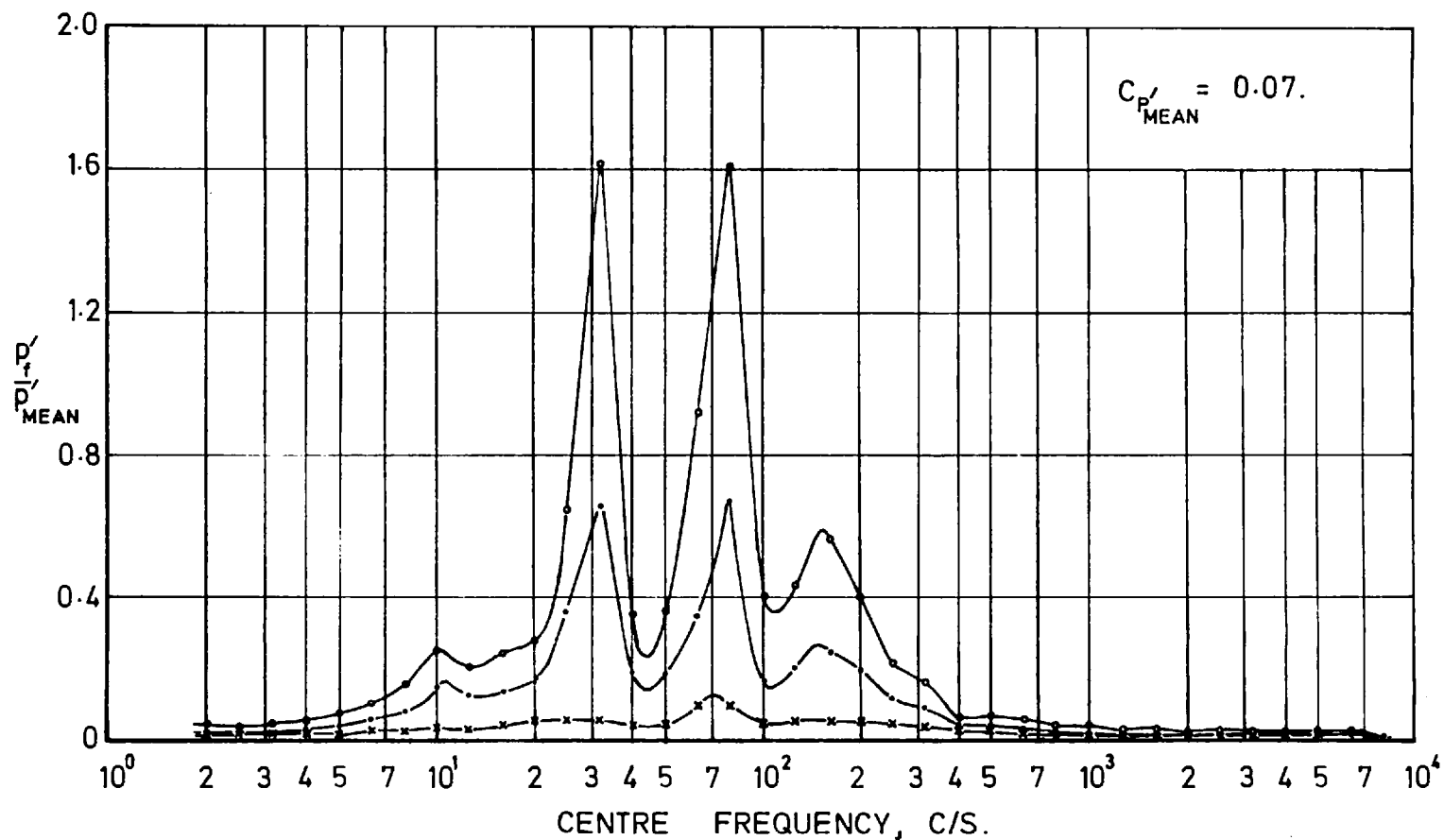


FIGURE A41. PRESSURE SPECTRA 16 DIAMETERS UPSTREAM OF BEND (max., mean & min. readings).

REFERENCES

1. THOMSON, J. Proc. Roy. Soc., Series A, 25 1879, pp. 5-8 and 356 and 357.
2. BOUSSINESQ, M.J. Memoire sur l'Influence des Frottements dans les Mouvements Reguliers des Fluides. J. Mathematique Pures et Appliquees, Series 2, 13, 1868 pp.377.
3. GOLDSTEIN, S. (Editor) Modern Developments in Fluid Dynamics, 1, pp. 316 O.U.P..
4. ESKINAZI, S. and YEH, H. An Investigation on Fully Developed Turbulent Flows in a Curved Channel. J. Ae. S., Jan. 1956, 23, pp.23.
5. RAYLEIGH, Lord. On the Dynamics of Revolving Fluids - Scientific Papers, 6, 1916.
6. SQUIRE, H.B. and WINTER, K.G. The Secondary Flow in a Cascade of Aerofoils in a Non-uniform Stream. J. Ae.S., 18, 1951, pp. 271.
7. & 8. DEAN, W.R. Note on the Motion of Fluid in a Curved Pipe; Phil. Mag. and J. of Science, 4, Series 7, 1927, pp.208 and 5, Series 7, 1928, pp. 673.
9. WHITE, C.M. Streamline Flow through Curved Pipes. Proc. Roy. Soc., Series A, 23, 1929, pp. 645.
10. ADLER, M. Stromung in Gekrümmtten Rohren, Zeitschrift J. Angewandte Math. U. Mech., 14, 1934, pp.257.
11. CUMING, H.G. The Secondary Flow in Curved Pipes. R. & M. 2880, 1955.

12. BARUA, S.N. Secondary Flows in Stationary Curved Pipes. Qu. J. Mech. Appl. Maths. 16, 1963, pp.61.
13. SQUIRE, H.B. Note on Secondary Flow in a Curved Pipe, A.R.C. 16601.
14. HAWTHORNE, W.R. Secondary Circulation in Fluid Flow, Proc. Roy. Soc., Series A, 206, 1951, pp. 374.
15. BINNIE, A.M. Experiments on Swirling Flow of Water in a Vertical Pipe and a Bend, Proc. Roy. Soc., 270, Series A, 1962, pp.452.
16. ITO, H. Friction Factors for Turbulent Flows in Curved Pipes. J. Basic Eng., 81, 1959, pp. 123.
17. HAWTHORNE, W.C. Flow in Bent Pipes. Seminar on Aeronautical Sciences, 1961.
18. WESKE, J.R. Experimental Investigation of Velocity Distributions Downstream of a Single Duct Bend. N.A.C.A. TN 1471, Jan.1948.
19. SZOMANSKI, E. Investigation of Steady-State Head Losses in a 90° Mitred Bend. G.P.D. Technical Record TD.01.15.033.079,1961.
20. PIGOTT, R.J.S. Losses in Pipes and Fittings, Trans. A.S.M.E.,1957.
21. CURTISS, H.A.
FEIL, O.G.
LIQUORMIK, D.J. Separated Flow in Curved Channel with Secondary Injection. A.F.O.S.R. Sc. Rep. Nr. 64 - 0860, 1964.
22. KLINE, S.J. and
DEAN, R.C. Unsolved Problems of Fluid Mechanics, Mech. Eng.,80, pp. 54, 1958.
23. PRANDTL, L. and
TIETJENS, O. Fundamentals of Hydro and Aero Mechanics, McGraw Hill Inc. N.Y. 1934.

24. CHANG, P.K. Separation of Flow, J. Franklin Institute, 272, pp. 433, 1961.
25. HEWSON, C.T. The Growth and Separation of a Turbulent Boundary Layer. Diss. Cambridge Univ., 1949.
26. THOMPSON, B.G.J. A Critical Review of Existing Methods of Calculating the Turbulent Boundary Layer. A.R.C. 26, 109. August 1964.
27. MASKELL, E.C. Appropriate Calculation of the Turbulent Boundary Layer in Two-Dimensional Incompressible Flow, A.R.C. 14,654, Nov. 1951.
28. STRATFORD, B.S. The Prediction of Separation of the Turbulent Boundary Layer. J.F.M., 5, 1959.
29. BRADSHAW, P.
FERRISS, D.H.
ATWELL, N.P. Calculation of Boundary Layer Development using the Turbulent Energy Equation. N.P.L. Aero Report 1182, Jan. 1966.
30. COLES, D. The Law of the Wake in the Turbulent Boundary Layer, J.F.M. 1, 1956, pp.191.
31. KLINE, S.J. On the Nature of Stall. J. Basic Eng., 81, 1959, pp.305.
32. FREY, K.P.H. and Tests on Flow Development in
VASUKI, N.C. Diffusers, A.S.M.E. Symposium on Fully Separated Flows, 1964, pp.40.
33. GERRAND, J.H. A Disturbance Sensitive Reynolds
Number Range of the Flow Past a
Circular Cylinder. J.F.M., 22, 1965, pp. 187.

34. ROUSE, H. Distribution of Energy in Regions
of Separation, *La Houille Blanche*, 1960,
pp. 235-246 and pp. 404-411.
35. SCHLICHTING, H. Boundary Layer Theory, McGraw-Hill Inc.,
N.Y. 1955.
36. FRAENKEL, L.F. On Corner Eddies in Plane Inviscid
Shear Flow. *J.F.M.* 11, 1961, pp. 400.
37. KLINE, S.J. and Some Preliminary Results of Visual
RUNDSTADLER, P.W. Studies of the Flow Model of the Wall
Layers of the Turbulent Boundary Layer.
J. Appl. Mech., 26, *Trans. A.S.M.E.*,
81, 1959, pp.166.
38. WILLMARTH, W.W. Wall Pressure Fluctuations in a
Turbulent Boundary Layer. *N.A.C.A.*
TN 4139, 1958.
39. BULL, M.K. and Some Results of Exptl. Investigations
WILLIS, J.L. of the Surface Pressure Field due to
a Turbulent Boundary Layer. *Univ. of*
Southampton, A.A.S.U. Rep. No.199, 1961.
40. MORSE, P.M. Vibration and Sound. McGraw-Hill Book
Co. Inc., 1936.
41. SCHULTZ, T.J. A Miniature Condenser Microphone
Employing a Flexible Diaphragm
Controlled by Air Stiffness, *Acoustics*
Research Lab., Harvard Univ., T.M. No.35,
1954.
42. ZAALBERG van Circuit for Condenser Microphone with
ZELST, J.J. Low Noise Level. *Philip's Technical*
Review, 2., No.12, 1947/48, pp.357-363.

- 43. BAXANDALL, P.J. New Low Noise Transistor Circuit for Electrostatic Microphones, Wireless World, Nov. 1963, pp. 538-542 and Dec. 1963.
- 44. LAUFER, J. The Structure of Turbulence in Fully Developed Pipe Flow. N.A.C.A. TN 2954, 1953.
- 45. HARVEY, J.K. Some Observations of the Vortex Breakdown Phenomenon. J.F.M. 14, 1962.
- 46. BROOKE BENJAMIN, T. Theory of the Vortex Breakdown Phenomenon. J.F.M. 14, 1962.
- 47. TOWNSEND, A.A. The Structure of Turbulent Shear Flow. Cambridge University Press, 1956.
- 48. HINZE, J.O. Turbulence. McGraw-Hill Inc., 1959.
- 49. GRAVES, K.W. and RANOV, T. Separation of Channel Flow Induced by a Right Angle Bend. A.S.M.E. Symposium on Fully Separated Flows, May, 1964, pp.10.