

K^+_n CHARGE EXCHANGE SCATTERING

by

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London S.W. 7,

October 1972.

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Who knows for certain? Who shall here declare it?
Whence was it born, whence came creation? The gods are later
than this world's formation; who then can know the origins of
the world?

None knows whence creation arose; And whether he
has or has not made it; He who surveys it from the lofty skies,
Only he knows - or perhaps he knows not.

The Rig Veda, X. 29.

ABSTRACT

An experiment on K^+d reactions at 2.18 GeV/c, 2.43 GeV/c, and 2.7 GeV/c is described. The film analysis system, and modifications made to it, are described.

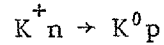
New data on the angular distribution for $K^+n \rightarrow K^0p$, at the 3 momenta above, is presented and commented upon. The data presented, together with other data on $0^{-\frac{1}{2}+} \rightarrow 0^{-\frac{1}{2}+}$ reactions, is fitted to 3 representative Regge models.

CHAPTER 1

1.1

INTRODUCTION

This thesis describes the author's work in connection with the Imperial College-Westfield College intermediate energy K^+N experiment. Chapter 2 describes the event processing system used and modifications made to it by the author. In Chapter 3 the data obtained on K^+n elastic charge exchange is described and its quality is evaluated. The final ~~three~~ chapters are devoted to an examination of the reaction:



as treated by available theoretical models.

The experiment was proposed⁽¹⁾ as a large scale survey of the K^+N system between 2.0 and 3.0 GeV/c incident beam momentum. This was motivated by the observation by Cool et al.⁽²⁾ and Bugg et al.⁽³⁾ of structure in the isospin 1, and 0, K^+N total cross sections, which was suggestive of resonance formation. If all the bumps observed in the total cross sections are interpreted as resonances, their parameters would be as tabulated below⁽⁴⁾.

Table 1:

Mass (GeV/c)	P_{LAB} (GeV/c)	I	Γ GeV
1.863	1.15	0	0.150
1.910	1.25	1	0.180
2.190	1.89	1	0.120
2.505	2.70	1	0.120

The presence of such resonances, known as Z^* 's, has also been experimentally suggested by a photoproduction experiment⁽⁵⁾

$$\gamma + p \rightarrow K^- + \left(\begin{array}{c} \text{MISSING} \\ \text{MASS} \end{array} \right)^{++}$$

which gave missing mass structure corresponding to, on a resonance interpretation:

Table 2:

Mass (GeV)	Γ (GeV)
$1.860 \pm .060$ $- .02$	0.150
$2.125 \pm .025$	0.75
$2.280 \pm .020$	0.80

and by the missing mass spectrum of

$$\pi^- p \rightarrow K^- + \left(\begin{array}{c} \text{MISSING} \\ \text{MASS} \end{array} \right)^+$$

corresponding to resonance parameters:

Table 3:

Mass (GeV)	Γ (GeV)
1.590 ± 0.030	~ 0.150
1.930 ± 0.20	~ 0.150

The energy range of the present experiment was chosen such that, if any of these structures corresponded to resonances, these resonances could be observed in the "production" mode.

So far, there has been no definite indication of Z^* resonance production in the present experiment. However, structure

has been observed⁽⁷⁾ in the 4th coefficient of a Legendre Polynomial expansion of the $K^+ \pi^+$ (892) production angular distribution in the reaction

$$K^+ p \rightarrow p K^0 \pi^+ .$$

This structure takes the form of a severe dip at $P_{LAB} \approx 2.25$ GeV/c and could be interpreted as being caused by the decay of a Z_1^* of mass corresponding to the total cross section structure at 2.19 GeV/c.

The isospin 1 and 0 data has also been examined by several groups by means of phase shift analysis⁽⁸⁾. The present situation is that $K^+ p$ elastic data can be interpreted⁽⁹⁾ on the basis of the P_{13} partial wave being resonant. The resonance parameters found:

Table 4:

Mass GeV	Γ GeV
1.890	.280
2.040	.260

corresponding to the 1st and 2nd "Cool bumps". The solutions are such that these resonances would be rather inelastic, so one would expect to see them in an analysis of the $K^+ p$ inelastic channels. In fact, the inelastic data does not seem to confirm a resonance interpretation. The rapid rise of $\sigma_{tot}(K^+ p)$ before the bump at 1.20 GeV/c is well explained by the rapid onset of single pion production from the process

$$K^+ p \rightarrow K^0 \Delta^{++}$$

The bump itself can be explained by the fact that the cross sections of the inelastic processes

$$K^+ p \rightarrow K^0 \Delta^{++}$$

$$K^+ p \rightarrow K^{*+} p$$

rise very rapidly to constant values, while the elastic cross section falls smoothly with increasing energy. Despite this non-resonant interpretation the question is still open, and at least one paper⁽¹⁰⁾ has pointed out that the rapid onset of K^* production is just the mechanism one would expect to "drive" a resonance. A final decision will require much more detailed experimental data, such as measurements of the R and A parameters.

In the isospin 0 reactions, there are prominent peaks in both the elastic and elastic charge exchange cross sections in the region of 1 GeV/c. The simplest interpretation of these is a P_{01} resonance⁽¹¹⁾ corresponding to the lower bump and a D_{01} resonance corresponding to the second. However, "speed" plots of the rate of change of phase shift through the "resonance" region do not show resonant-like behaviour.

There is thus some evidence of resonant activity in the K^+N reactions. The reason that such evidence is treated with such circumspection is that K^+N resonances would constitute a hitherto unobserved class of objects. They would be the first baryon resonances of positive strangeness to be observed. As such they would have to be classified in higher SU_3 multiplets than the well established octets and decuplets. In fact, the Z_1^* would belong to a $\{27\}$ and the Z_0^* a $\{\overline{10}\}$. Observation of either of these states would then imply, on the basis of SU_3 , the existence of many other hitherto unobserved states. The existence of these "exotic"⁽¹²⁾ baryon resonances would also have serious implications for the simple quark model. In this model, the SU_3 properties of all well established

baryon resonances can be explained on the assumption that baryons consist of a quark-antiquark triplet ($qq\bar{q}$). Z^* 's, on the other hand, would require a structure of the form ($qqq\bar{q}\bar{q}$). One could argue that if three quarks bind, why shouldn't five, but then this would imply that seven, nine, etc. could also bind; giving rise to a multitude of states for which there is no experimental evidence. The point is that the ($qq\bar{q}$) structure is well established and before any higher structure is admitted there must be very definite evidence for it.

1.2 THE EXPERIMENT

The experiment was performed using the British National Hydrogen Bubble Chamber, exposed to a separated K^+ beam from the K9 beam line at the Rutherford High Energy Laboratory 7 GeV/c proton synchrotron.

Initially, in late 1967, ~ 80,000 frames were taken, spread over the nominal beam momenta

$$2.1; 2.3; 2.5; 2.7 \text{ GeV/c}$$

using a liquid hydrogen chamber filling. This gave access to the isospin 1 reactions and was intended as a preliminary to a much larger exposure, using liquid deuterium in the chamber. The K^+p system (accessible in a hydrogen-filled chamber) is a pure isospin 1 system, whereas the K^+n system (accessible in deuterium) is a mixture of isospin 1 and 0.

Data on both systems, in principle, allows a determination of the characteristics of both isospin 1 and 0 reactions. In addition to this consideration, there is the fact that K^+n reactions in deuterium do not involve a free neutron, but one bound inside the deuteron. This leads to many problems involved with "unfolding" the free neutron parameters⁽¹³⁾.

The validity of the unfolding procedure adopted can be tested by comparing K^+p data obtained from deuterium with that obtained from hydrogen.

When the author joined the Imperial College HENP group in October 1968, the analysis of the K^+p data was at an advanced stage. It was decided that his main concern should be with the setting up of the K^+d experiment, and the analysis of the preliminary results.

The main part of the experiment commenced in November 1969 with an exposure of 390,000 frames at a nominal beam momentum of 2.2 GeV/c. A further exposure of 318,000 frames at 2.7 GeV/c and 192,000 frames at 2.45 GeV/c was carried out, starting in March 1970. These beam momenta were chosen⁽¹⁴⁾ so that the centre-of-mass energy (E^*) range covered was the same as in the K^+p experiment, taking account of the E^* spread due to the internal motion of the deuteron. The author took part in the beam tuning and data taking on both these occasions. In the following two years, the author was jointly responsible for the day-to-day running of the experiment (scanning, measuring, etc.) and the setting up and maintaining of the analysis programs.

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CHAPTER 2

THE DATA PROCESSING CHAIN

By the "Data Processing Chain" is meant the system, consisting of film measuring machines, human beings, and computers, which has the bubble chamber film as input and a magnetic tape containing the physically significant parameters of each event as output.

During the period in which the K^+ d data was processed the Imperial College system underwent severe modification. The author was closely involved in this project and was responsible, at that time, for the day-to-day running of the K^+ d part of the system. The system is described in this chapter not only because of the author's involvement, but also because certain facets of the system had a profound effect on the form of the data finally obtained.

2.1 General Outline

The first stage in the extraction of the information contained by the film was SCANNING, in which the film was visually searched, frame by frame, for events of interest. The film was passed through two independent scans. It was then subjected to a third CHECK SCAN, in which discrepancies between the first two scans were noted. This procedure not only yielded a high scanning efficiency (99% overall), but also allowed the actual efficiency to be estimated.

The main measuring device at Imperial College is an HPD which operates under a "road guidance" system⁽¹⁾. So, after scanning an experiment, the necessary guidance information is generated by a ROUGH DIGITISATION process. The rough digitisation machines operate under

computer control, the digitisations and event label information (Roll no., experiment no., frame no., etc.) being written out to a magnetic tape on an incremental tape transport.

The program controlling this process is known as ONLINE and the magnetic tape generated is termed the ONLINE TAPE; ONLINE also performs checks on the quality of the data from the machines. Several measuring machines are multiplexed onto one computer (a small HONEYWELL DDP-516) and the information from each of these machines is written out to the same magnetic tape. At any given time not all of these machines would be measuring film from the same experiment. However, the ONLINE program writes the data from all machines onto the same tape, and it services any machine which has data to transfer. This results in the data from any one machine being interspersed with data from other machines. For input to the pre-HPD program, MIST, it is necessary to separate data from different experiments, it is also necessary to order the events by roll and frame number within each experiment. This is achieved by passing the raw ONLINE tapes through a chain of editing and sorting programs. The result is a set of magnetic tapes for each experiment which contain the events from that experiment in an ordered form; these are the ONEDIT tapes.

At this point, in the K^+ d experiment, a MASTER LIST was generated from the ONEDIT tapes. This was a magnetic tape which was eventually to contain a complete record of the passage through the system of each scanned event. The ultimate fate of each event was thus recorded and losses, in what is a rather complex system, were insured against. The master list was edited and updated in an automatic fashion by a package of programs written for the purpose.

The control, or SCAN, tape for the HPD is generated using the program MIST. MIST reads the ONEDIT tape and outputs the information required by the HPD onto the SCAN tape. The HPD measures each view of each roll in a separate pass, the measurements for each view being written out to a separate magnetic tape. Tracks which are diagnosed as suspect by the HPD control program, HAZE, are later refiltered manually, using a light-pen controlled system known as RESCUE; this light-pen system is implemented on a PDP-15 computer connected to the group's main PDP-10. Thus, the HPD-RESCUE system produces three magnetic tapes for each roll of film, these are merged onto one tape by the program SMOG.

The geometrical parameters of each event are reconstructed by the program MDT⁽²⁾. This geometry information is used by GRIND, the kinematics program. GRIND, in general, finds several kinematically acceptable hypotheses for each event. In the limit of infinitesimal measurement errors, GRIND would be able to select the correct hypothesis on kinematic grounds; the presence of finite errors leads to ambiguities. In this experiment a pass through the program AUTO⁽³⁾ resolved 60% of these ambiguities by using HPD measured track ionisation and missing mass criteria. An attempt to resolve the remaining ambiguities was made by comparing the events on film with the information on each hypothesis as output by GRIND. Certain classes of ambiguities remained after this stage and, if the event was satisfactorily measured, a maximum of 3 ambiguous hypotheses were selected. When the correct hypothesis has been selected, a SLICE card is punched for the event. The SLICE card contains the Roll-Frame-Event-Measurement number and also a numerical code for the hypothesis chosen as being correct. If the event is to be remeasured, a remeasurement code is punched in place of the hypothesis code.

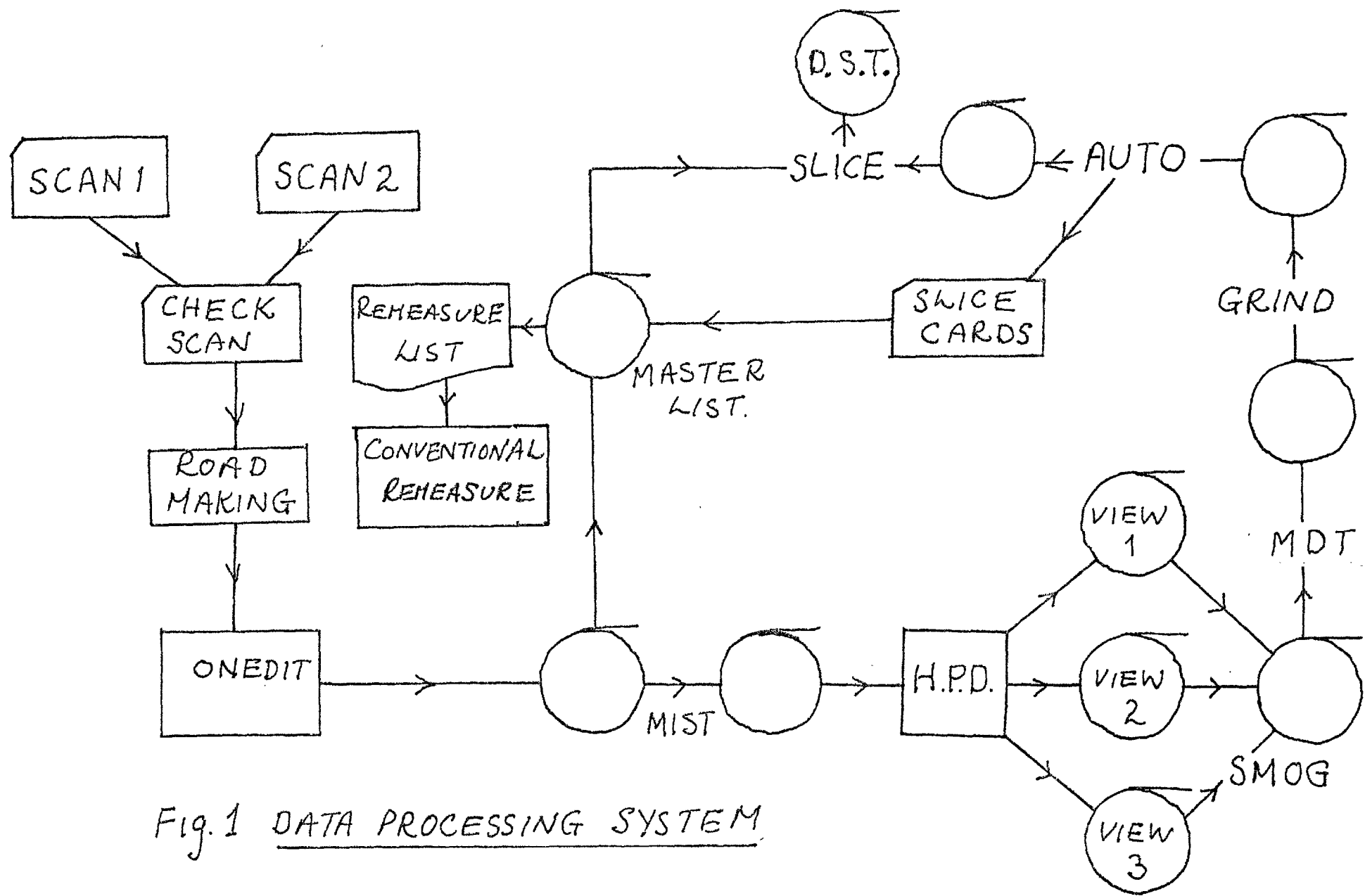


Fig.1 DATA PROCESSING SYSTEM

In the present experiment decisions made by AUTO caused a card to be punched online. The master list was updated after each measurement pass by means of the slice cards. Finally, the DATA SUMMARY TAPE was prepared, using the program SLICE. The SLICE used was a version which read the master list tape, to determine which hypotheses to transfer from the GRIND tape to the DST.

Fig. 1 is a flow chart of this process from scanning to DST production.

In the remainder of this chapter, some parts of the system are described in more detail, insofar as this detail is relevant to the present experiment.

2.2 Scanning

Due to the quantity of film involved, it was felt that an effort should be made to avoid extracting information which would not be utilised. To achieve this, it was decided to scan only for those topologies which corresponded to channels in which the most interesting, or easily accessible, reactions occurred.

A deuterium experiment is complicated by the fact that the target is a nucleus consisting of two particles. This leads to the occurrence of three general types of interactions:

- a) Coherent interactions.
- b) Interactions with the proton.
- c) Interactions with the neutron.

Coherent interactions are those in which the deuteron remains intact in the final state:

$$K^+ d \rightarrow d + \left\{ \begin{array}{c} \text{other} \\ \text{particles} \end{array} \right\}.$$

The other two classes assume the impulse approximation, in which the interaction is visualised as occurring between the incident particle and one of the nucleons. The remaining nucleon is termed a "spectator" and is assumed to have not taken part in the interaction. Then interactions with the proton have the neutron as spectator:

$$K^+ p (n) \rightarrow \{ \text{final state} \} + (n)$$

whereas interactions with the neutron have the proton as spectator

$$K^+ n (p) \rightarrow \{ \text{final state} \} + (p)$$

Most interest in this experiment was focused upon $K^+ n$ interactions.

In the light of this, it was decided not to scan for 1-prongs, 2-prongs, and ≥ 6 -prongs. 1 and 2-prongs with a seen neutral decay were scanned for, as were all other topologies.

Neglecting 1-prongs meant that no analysis was done on:

$$K^+ n (p) \rightarrow K^+ n (p)$$

or

$$K^+ n (p) \rightarrow K^+ n (p) + \{ \pi^0 \text{'s} \}$$

Neglecting 2-prongs rendered

$$K^+ p (n) \rightarrow K^+ p (n) + \{ \pi^0 \text{'s} \}$$

inaccessible. The former would have added greatly to the scanning and measuring load, and the latter was better covered in the $K^+ p$ experiment.

Greater than 6-prongs were not scanned for as the known cross sections indicated that they would be too rare for their study to be worth while.

In addition, it was decided that only 4-prongs with at least one stopping track (corresponding to a proton) would be scanned for.

The scanning instructions actually asked only for 4-prongs with a stopping track of less than 20 cm in projected length. The stop was not checked for at the scanning stage and "pseudo stops" were detected in MIST.

This criterion was set to include all proton spectator events. Not all stopping tracks of up to 20 cm were spectators, but essentially all spectators were included. Again this course of action excluded channels which were better covered in the K^+p experiment.

These decisions were estimated to have cut the scanning and measuring load by ~30%.

The relevant information on each scanned event was recorded on a computer card (SCAN CARD).

2.2.1 Scanning Efficiency

Assume p_1 and p_2 are the probabilities of finding a particular event in two independent scans. Then if n_1 and n_2 are the numbers found in these two scans, and if N is the actual number of events present

$$p_1 N = n_1 \quad (2.1)$$

$$p_2 N = n_2 \quad (2.2)$$

Some events will be seen on both scans, so:

$$p_1 p_2 = n_{12} \quad (2.3)$$

n_{12} being the number seen on both scans.

It follows that:

$$N = \frac{n_1 n_2}{n_{12}} \quad (2.4)$$

The total number of events seen is

$$(n_1 + n_2 - n_{12}) \quad (2.5)$$

and from (2.4) the number of potential events is

$$\frac{n_1 n_2}{n_{12}}$$

The overall scanning efficiency is then

$$E = \frac{(n_1 + n_2 - n_{12}) \cdot n_{12}}{n_1 n_2} \quad (2.6)$$

Events were only scanned for within a definite fiducial region.

The limits of this region were set at the scanning stage to be such that all tracks in an event at the extreme limits of the region would still be of sufficient length to be well measured. The limits were set at $x = -43.0$ cm to $x = +40.0$ cm; where x is the axis of the chamber along which the beam enters. If a neutral decay outside these limits was apparently associated with a primary vertex within the region, the event was accepted.

Table 5 gives the scanning efficiency for various topologies as calculated according to (2.6).

Table 5.

Topology	Scan Eff. %
101	98.7
201	98.9
301	99.2
300	99.0
400	98.8
401	98.6
98.9 % overall	

2.3 Rough Digitisation - MIST - HPD

When the HPD concept was first proposed, it was hoped that the device would eventually operate in a "Zero Guidance" mode. That is, it was hoped that the HPD would be able to accept raw film and filter interesting events from the background of beam tracks, uninteresting events, film scratches, and other noise. The HPD hardware is, in principle, capable of extracting all this information from the film, thus zero guidance is a problem in computer pattern recognition. In fact, this problem of pattern recognition has proved difficult to solve, and most HPD's operate in the "Road Guidance" mode. Under this system the pattern recognition is actually done by human operators and corresponds to the process of scanning. The HPD control program, HAZE, is then given the rough positions of the events to be measured. The HAZE program still has to do some pattern recognition in separating wanted tracks from noise in the form of crossing tracks, scratches, etc. The guidance information supplied to HAZE consists of the position of the vertex (or vertices) and two points on each track. These "Road Points" enable HAZE to set up a circular "road" about each track, and only to process digitisations within these roads.

In addition to the road points, the HPD also requires the positions of 2 fiducials on the frame, this fixes the coordinate system for the measurements.

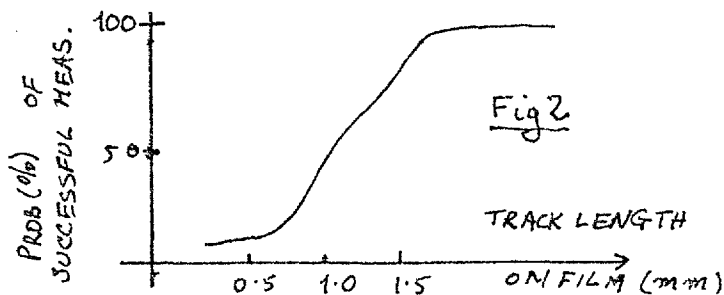
The HPD also accurately measures as many (~20 in this case) fiducials as possible, thus accurately defining the coordinate system in which the tracks are measured. As is well known the HPD digitises the information on the film by scanning a spot of light across the film to build up a raster pattern. From the point of view of this experiment,

it is important to note that this scanning can be done in two orthogonal directions. The scan perpendicular to the major axis of the chamber is known as the NORMAL scan, while the orthogonal scan is the ABNORMAL.

The normal scan is satisfactory for most tracks in a conventional bubble chamber, as they tend to make a small angle with the major axis of the chamber (corresponding to the beam direction). For tracks which make an angle $> 45^\circ$ with the major axis, the normal scan becomes very inaccurate, and the abnormal is used. The presence of spectator protons in this experiment lead to many such tracks.

The normal and abnormal scans are done sequentially, thus it is necessary to be able to transform measurements made on an abnormal scan into the same coordinate system as those made on a normal scan. The necessary transformation parameters are obtained by measuring a grid of points on both normal and abnormal scans; this process is known as calibrating the HPD. Because of its importance this calibration procedure was carried out every 24 hours during the measurement of this film.

The large number of short spectator protons present in this experiment presented a major problem for HPD measurement. The performance of the I.C. HPD is summarised in Fig. 2.



0.5 mm — 0% PROB. SUCCESS
 1.0 mm — 50% " "
 1.5 mm $\approx$ 100% " "

Due to the momentum distribution of spectators, at least 30% of events would have tracks whose length resulted in a $\leq 50\%$ probability of

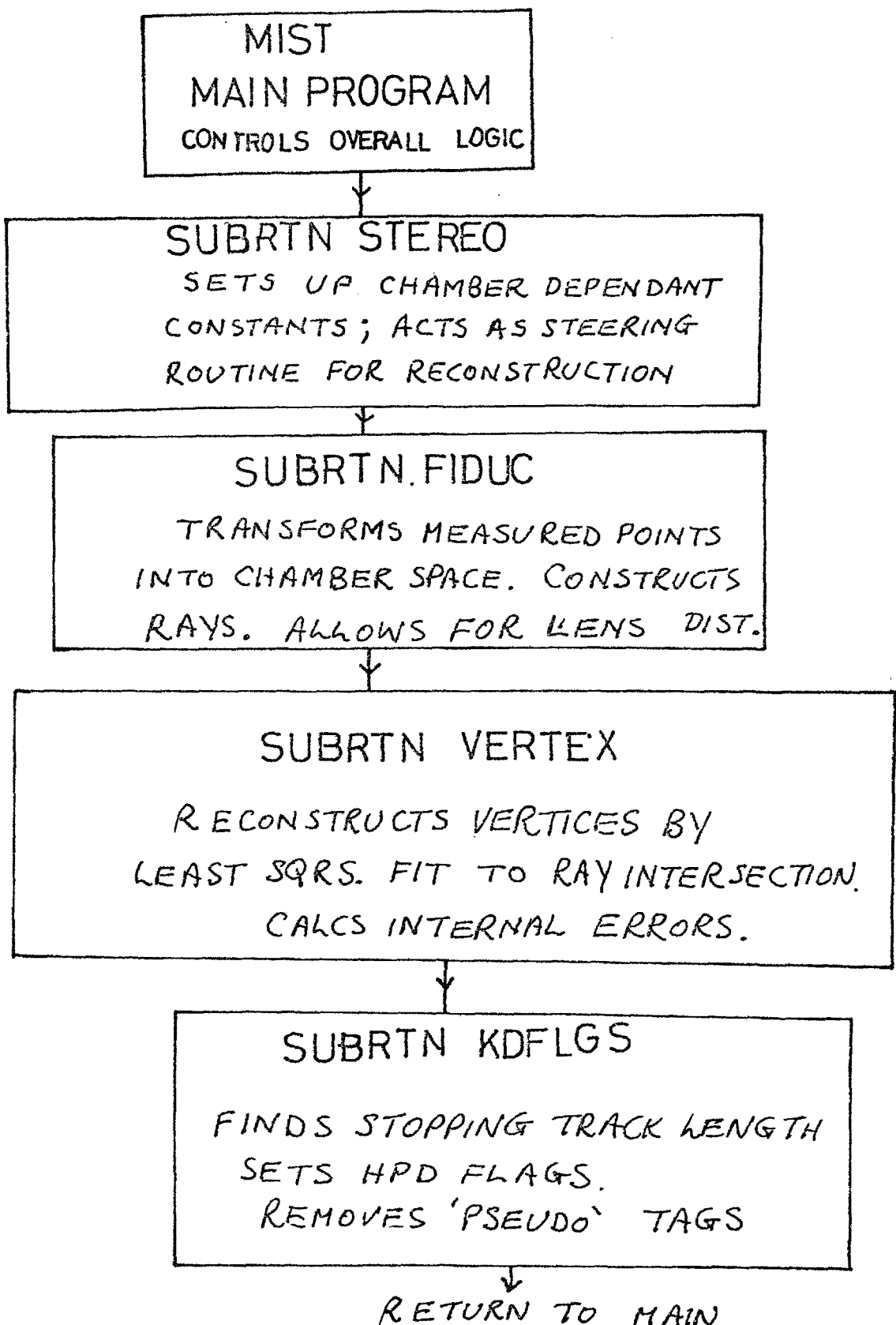
successful measurement. It thus seemed useful to attempt to improve the performance of the HPD in this respect.

The most desirable state of affairs would be that in which the HPD at least measured the end point of the shortest tracks; this would enable MDT to reconstruct the tracks as straight. Now the ultimate limit on the accuracy of the HPD is determined by the size of the scan spot, which is 20 μm . In normal operation the scan line spacing is 60 μm . So, to increase the accuracy of measurement, and so make the measurement of short track end points more likely, it is useful to decrease the scan line spacing. Decreasing the scan line spacing leads to an increase in the time required to scan a frame. If the scan spacing were reduced to its lowest ($\sim 8\mu$), the overall time taken to measure a frame would increase by a factor of 7. Hence, it was decided to implement a slow scan only for short tracks, and, in addition, to scan at low speed only over the region of the short track.

The information necessary to schedule a slow scan must be generated in MIST. The necessary modifications were carried out by the author and are described below.

2.3.1 Modifications to MIST

In order to decide whether to schedule a slow scan, it is necessary to know the length of each track in space. The simplest approach would have been to test against the projected length of the track on each individual view, this information being inherent in the road points. This would have had the disadvantage that a track with very different projected lengths on each view would perhaps be slowly



FLOW CHART OF MIST MODIFICATIONS

Fig: 3

scanned on one view but not the others. To overcome this, it was decided to reconstruct the 3-dimensional length of each track, and if this were below a certain limit to schedule a slow scan on all three views.

MIST, as modified, reconstructs the length of only those tracks which have been judged as stopping at the scanning stage. These tracks are distinguished from non-stopping tracks by the fact that their label contains a stopping flag, or "tag". On tagged tracks, at I.C., the rough digitiser operator measures three points in the usual fashion, except that the last point is placed on the end of the track. In cases where the track is too short to have three points measured, the end point is measured twice. The modifications inserted in MIST are derived from the Rutherford Laboratory geometry program RGEOM⁽⁴⁾. The flow chart in Fig. 3 indicates the general method.

MIST is supplied with the known positions, in bubble chamber space, of the rough digitised fiducials, and also with a set of coefficients to allow for camera lens distortions. This information allows it to transform rough measurements into bubble chamber space. The reconstruction proceeds as follows.

All tracks of an event are tested to ascertain if any stop. If a stopping track is present, the last digitisation on this track is inserted at the end of the vertex bank in MIST. This vertex bank is an array containing the coordinates of all vertices in the event. Next, all these coordinates are transformed into bubble chamber space, this being done by finding the transformation which takes the coordinates of the fiducials, as rough digitised, into the known coordinates. This transformation is of the form:

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix} \begin{pmatrix} f'x - X_0 \\ f'y - Y_0 \end{pmatrix}$$

where

θ - angle of rotation between the two coordinate systems.

f' - scale factor (magnification).

(X_0, Y_0) - translation relating the origins of the two frames.

(x, y) - rough digitisations.

The transformed coordinates are corrected for camera lens distortions by

$$\begin{pmatrix} x^{cor} \\ y^{cor} \end{pmatrix} = \begin{pmatrix} x^{dist} \\ y^{dist} \end{pmatrix} \begin{pmatrix} 1 + \beta_1 x + \beta_2 y + \beta_3 xy + \beta_4 x^2 + \beta_5 y^2 \\ + \beta_6 (x^2 + y^2) \end{pmatrix}$$

Next the Rays (Reconstruction Lines) are found for each vertex on each view (Fig. 4). The equations of these Rays are of the form:

$$x = X + U.z$$

$$y = Y + V.z$$

where $(X, Y, 0)$ are the coordinates of the point at which the ray passes through the liquid-glass interface ($Z = 0$), (U, V) are the direction cosines.

As shown in ref. (4) the ray parameters (X, Y, U, V) are of the form

$$X = \frac{X'}{R} \sum_{i=1}^{n-1} \delta_i \left(\frac{\tan \alpha_i}{\tan \alpha_1} \right)$$

$$U = -\frac{X'}{R} \left(\frac{\tan \alpha_n}{\tan \alpha_1} \right)$$

δ_i , and $\tan \alpha_i$ can be expressed in terms of the known quantities μ_n , R , f , e.g.

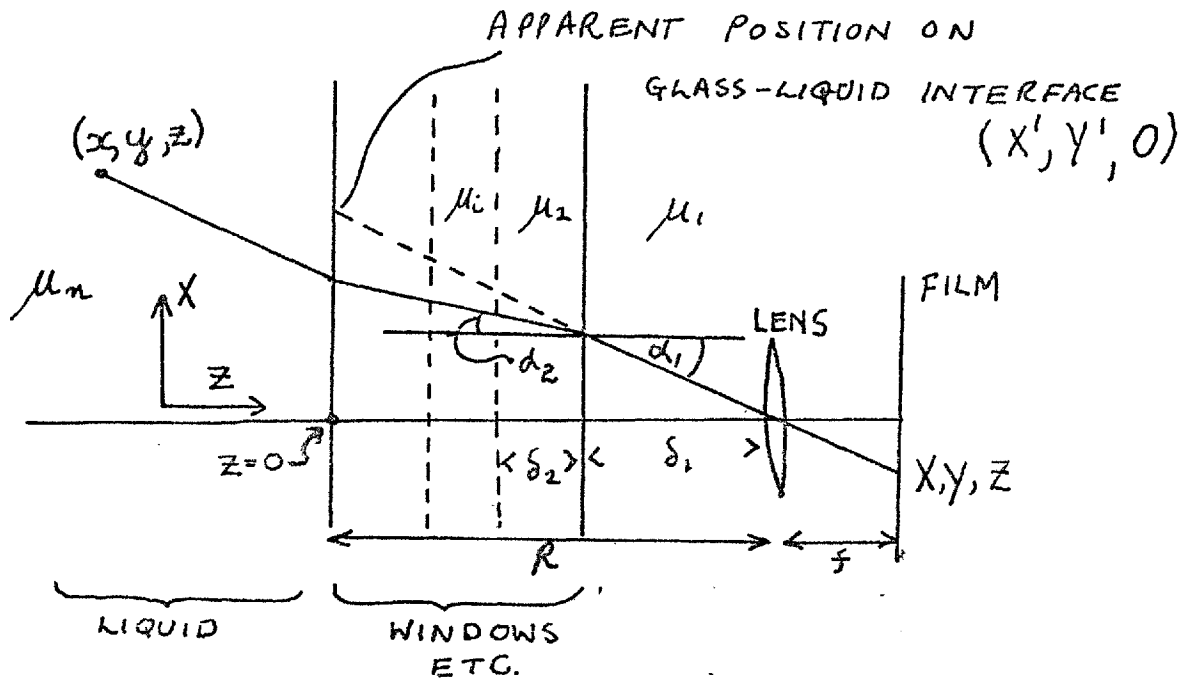


Fig: 5

SECTION THROUGH SPACE POINT
AND OPTIC AXIS

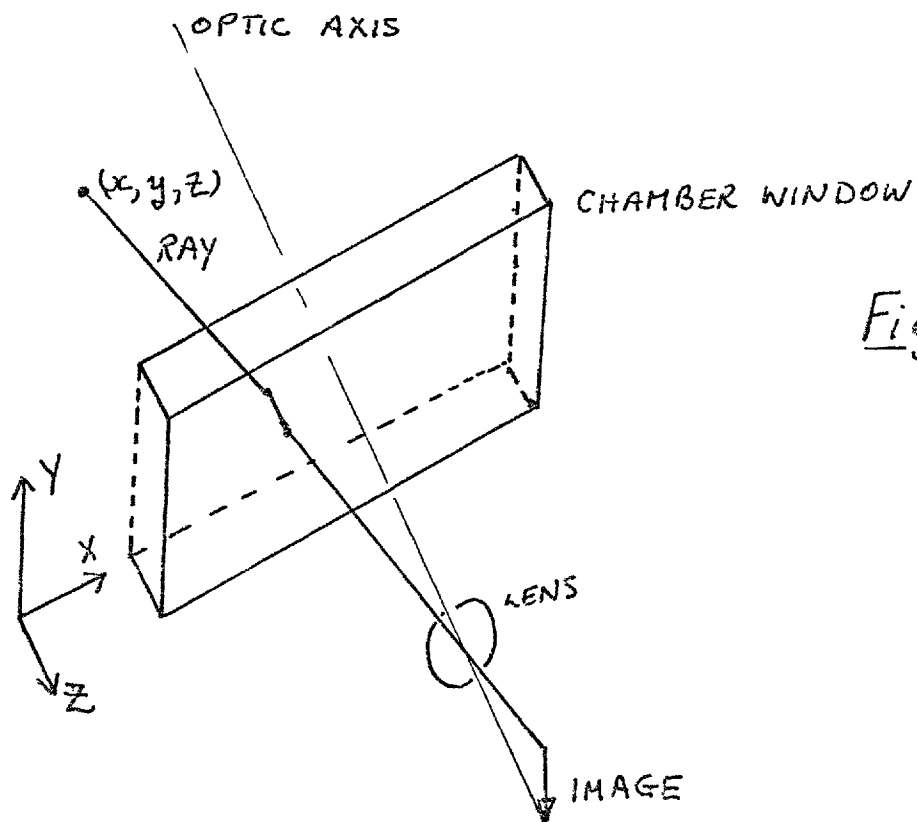


Fig: 4

$$U = \frac{-X'}{R\mu_n} \left(1 + \left[\frac{\mu_n^2 - 1}{\mu_n^2} \right] \frac{R^2}{f^2} \right)^{-\frac{1}{2}}$$

where the symbols are defined in Fig. 5.

In the ideal situation the intersection of the rays corresponding to the three views would define the space point (x, y, z) . Actually the three rays do not intersect at a point; the space point is then taken as that point closest to the three rays. The space point is projected on the film and the distance from this to the point where the ray cuts the film is calculated as

$$\Delta^2 = \omega^2 \left[(x^{\text{proj}} - X - U z)^2 + (y^{\text{proj}} - Y - V z)^2 \right]$$

where ω = demagnification space-film.

The sum of Δ^2 on all three views is then minimised in a least squares fit.

The reconstructed vertices are used to calculate the length of the stopping track, which is assumed straight. This is invalid for long tracks, but all that is needed is an accurate length for very short tracks.

The intention was to have a slow scan for tracks shorter than 1.0 mm. on film. On the film measured to date, this facility has not been used because of modifications necessary to the HPD.

The reconstruction of points, however, was used to detect and remove spurious tags, i.e. tracks tagged as stopping which actually leave the chamber. Short tracks are now specially treated in MDT in an attempt to utilise the road points as well as possible in the event of HPD measurement failure.

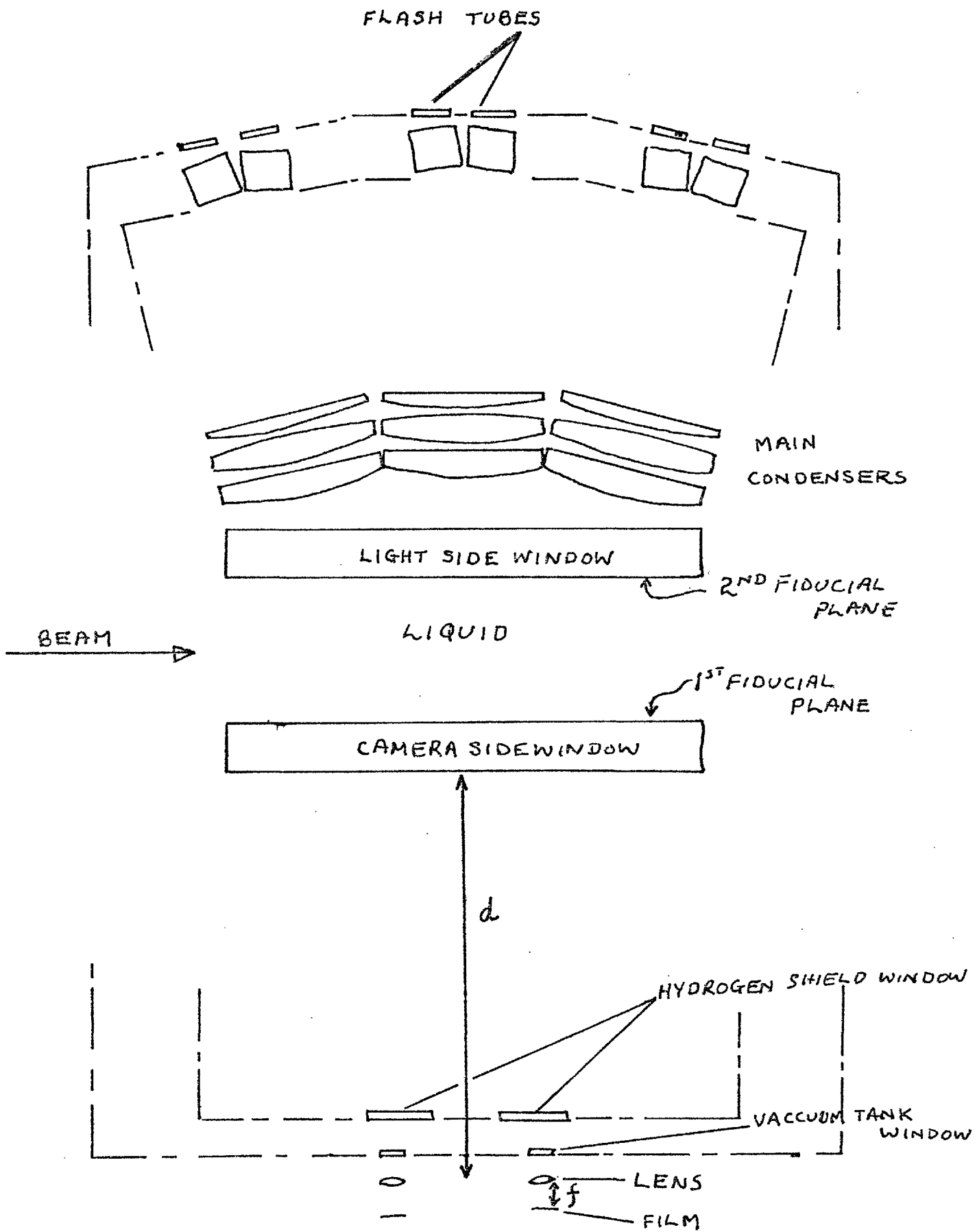


Fig 6 PLAN OF CHAMBER OPTICS

2.4 SMOG - MDT

SMOG, at I.C., is standard for all experiments and requires no further comment. Mass Dependent Thresh is a standard CERN program, it is a three view geometrical reconstruction program which includes a correction to the reconstructed track parameters to allow for energy loss by ionisation along the track. This correction depends on the mass of the particle involved, hence the program name. The quality of the reconstructed track parameters depends strongly on the degree of accuracy with which the optical configuration of the bubble chamber is known. This optical information is transmitted to MDT as a deck of cards and these constitute the MDT TITLE. Since the chamber had been warmed up and cooled down several times since the K^+p experiment, it was necessary to rederive the TITLE for this experiment. This was done by the author.

2.4.1 Optical Constants (MDT TITLE)

The information on the optical set-up of the chamber required by MDT consists of:

- a) The coordinates in the bubble chamber of as many fiducial marks as possible, on both windows of the chamber.
- b) The coordinates of the three cameras.
- c) The thicknesses and refractive indices of all media between the cameras and the "light side" window inner surface; see Fig. 6.
- d) The coefficients parametrising the optical distortions inherent in the camera lenses.

The method described here relates mainly to the CERN program PYTHON.

The positions of the fiducials in the chamber are accurately known from the results of an optical survey.⁽⁵⁾ The positions of these

same fiducials can be measured on the film. A fit between the survey positions and the film fiducials projected into the chamber is performed, the free parameters in this fit being:

- (a) The lens film distance (f).
- (b) "Vacuum path" (lens $\leftrightarrow$ chamber window)
- (c) Thickness of glass between cameras and chamber liquid.
- (d) Camera coordinates.
- (e) Lens distortion parameters.

The OPTIC AXIS of a camera is defined as the line passing through the front nodal point of the lens, perpendicular to the first fiducial carrying plane; Fig. 7.

From elementary optics the demagnification ω is

$$\omega = \frac{r}{\rho} = \frac{1}{f} \sum_i \frac{d_i}{(\mu_i^2 + (\mu_i^2 - 1) \tan^2 \alpha)^{\frac{1}{2}}}$$

i.e. $\omega = \frac{\text{PATHC}}{f}$ (2.7)

From this, it is apparent that ω depends on $\tan \alpha$. If ω were independent of α then first order optics would be applicable, i.e.

$$\omega^0 = \frac{1}{f} \sum_i \frac{d_i}{\mu_i} = \frac{\text{PATH}}{f}$$
 (2.8)

"Pseudo 1st order optics" is used in PYTHON to speed computation.

This technique consists of correcting the raw film measurements to what they would be in a first order optics system. This correction is

$$\rho' = \frac{\text{PATHC}}{\text{PATH}} \cdot \rho$$
 (2.9)

Utilisation of this method requires an approximate knowledge of all the free parameters listed above; successive approximations to the correction PATHC/PATH are obtained by an iterative method.

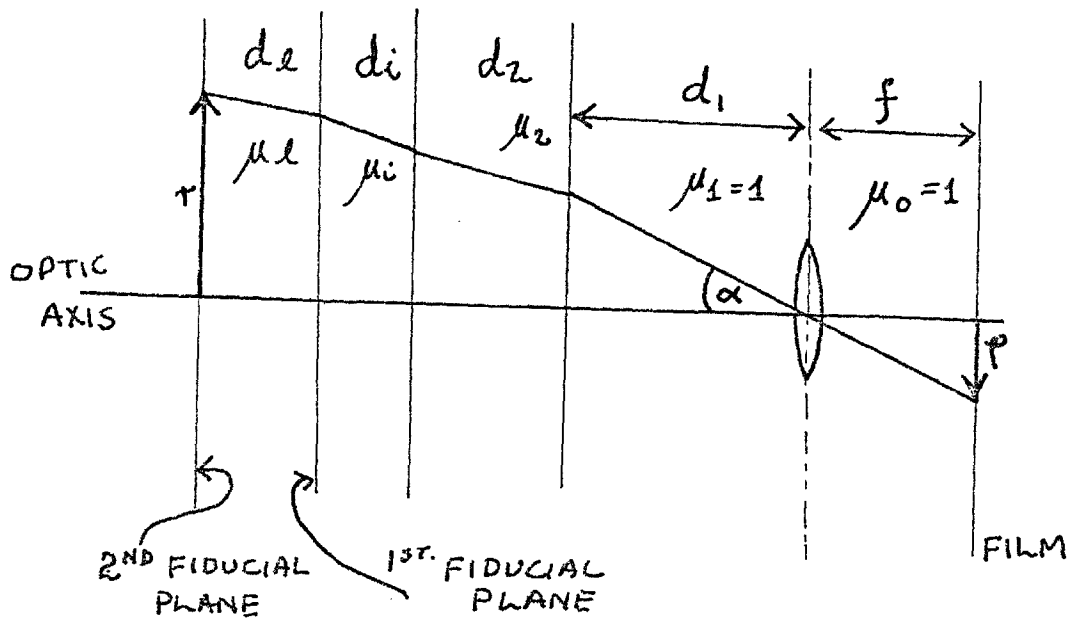


Fig: 7

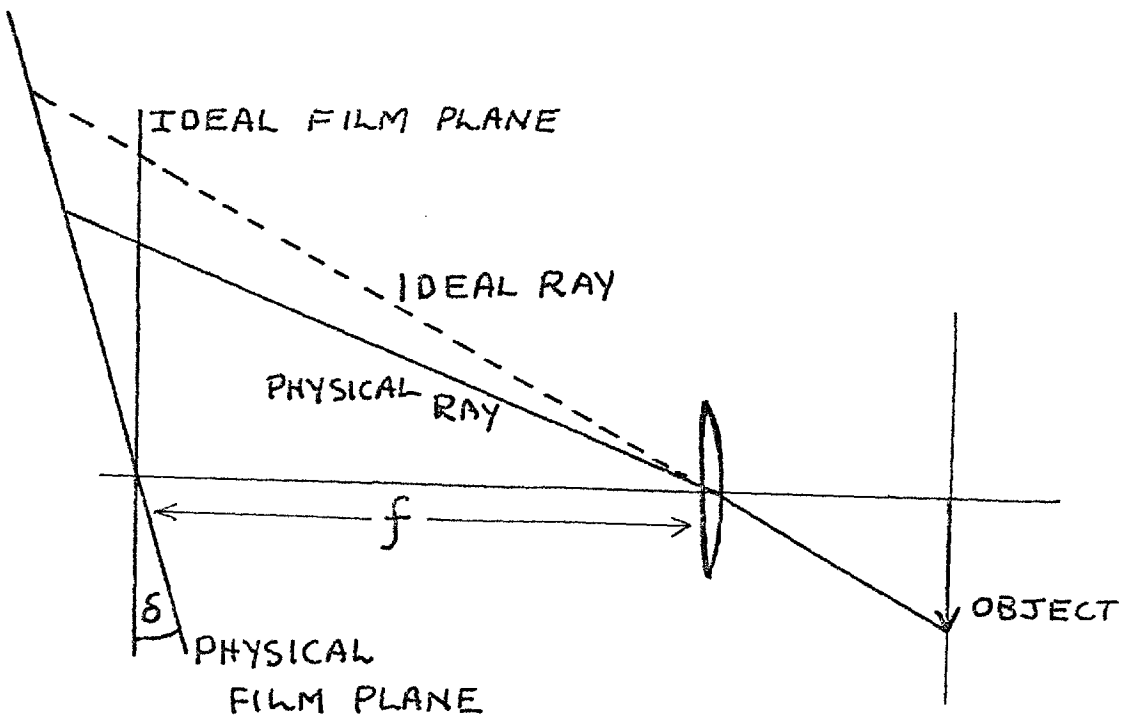


Fig: 8

The distances along the optic axis determine the sizes of the images of the 1st and 2nd fiducial planes, and the camera positions determine the relative positions on different views of the images of these two planes. Thus it is possible to work from these quantities measured on film, back to the optical axis parameters and the camera coordinates.

DEMAG	$\longleftrightarrow$	DISTANCES ALONG OPTIC AXIS.
SHIFT BETWEEN PLANES ON DIFFERENT VIEWS	$\longleftrightarrow$	CAMERA COORDINATES.

This process requires at least one distance along the optic axis to be accurately known; this was taken as the chamber depth in PYTHON. Unfortunately, this quantity is not well known for the BNHBC, but the value in ref.(5) was used.

After correction to 1st order optics, film and space are related by a transformation consisting of translation, rotation, and uniform magnification. However, there is a further consideration, that of the film not being parallel to the chamber windows and the distortions introduced by the camera lenses, see Fig. 8.

Lens distortions cause rays which would be undeviated by a perfect lens (MAJOR RAYS) to suffer a change in direction in passing through the lens system. MDT allows for such distortions, and in deriving the optical constants it is necessary to evaluate the parameters involved.

The simplest possible parametrisation of the lens distortions is

$$\begin{pmatrix} x \\ y \end{pmatrix}_{\text{undis}} = \left(1 + \alpha_1 \frac{x'}{f} + \alpha_2 \frac{y'}{f} + \alpha_3 \frac{r'^2}{f^2} \right) \begin{pmatrix} x' \\ y' \end{pmatrix}_{\text{dist.}}$$

$$r'^2 = x'^2 + y'^2$$

$(\alpha_1 \ \alpha_2 \ \alpha_3)$ then describe the deficiencies of one lens; so 9 coefficients would be needed over the three views. α_1 and α_2 are the x and y components of the film tilt angle δ

$$\delta = \sqrt{\alpha_1^2 + \alpha_2^2}$$

and α_3 parametrises the radial distortion of the lens, i.e.

$$r_{\text{undis}} = \left\{ 1 + \alpha_3 \left(\frac{r'}{f} \right)^2 \right\} r'_{\text{dist.}}$$

In most chambers more complex parametrisations have been found necessary.

The expression used in MDT is

$$\begin{pmatrix} x \\ y \end{pmatrix}_{\text{undis}} = \left\{ 1 + \alpha_1 \frac{x'}{f} + \alpha_2 \frac{y'}{f} + \alpha_3 \frac{x'y'}{f^2} + \alpha_4 \frac{x'^2}{f^2} + \alpha_5 \frac{y'^2}{f^2} + \alpha_6 \frac{r'^4}{f^4} \right\} \begin{pmatrix} x' \\ y' \end{pmatrix}_{\text{dist.}}$$

The first measurements of fiducials were carried out on a National conventional measuring machine. These were used to produce titles for a conventionally measured sub-experiment. The same titles proved adequate for use with the 2.2 GeV/c film measured on the HPD. The film at 2.45 and 2.7 GeV/c was taken much later than the 2.2 GeV/c, and so the titles were rederived. This time the fiducial measurements were carried out on the HPD using a program HAZFID⁽⁶⁾ written for the purpose by B. Penney.

On the conventional measurement, a total of 8 individual frames were used. These were distributed amongst 4 rolls and were approximately 500 frames apart.

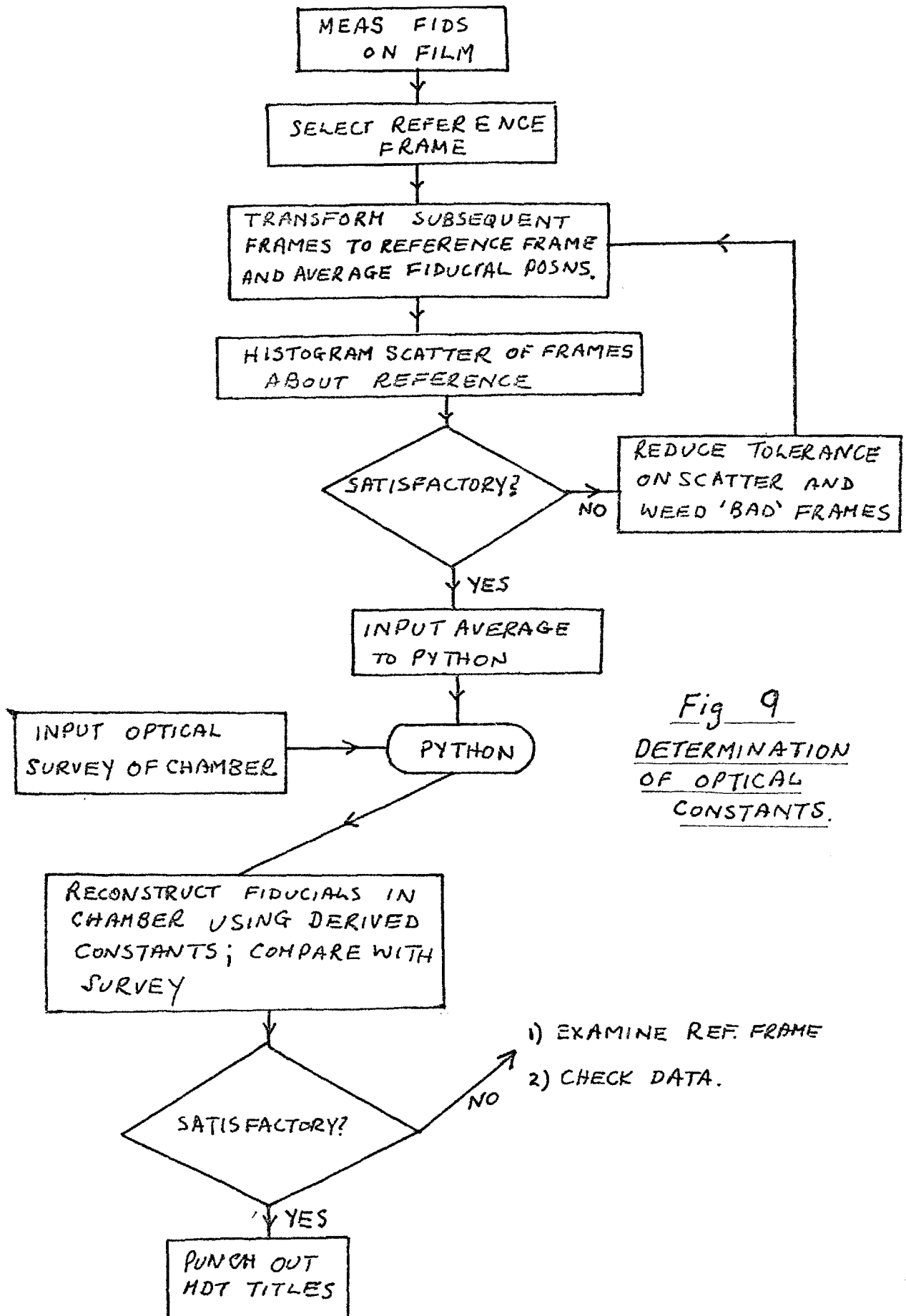


Fig 9
DETERMINATION
OF OPTICAL
CONSTANTS.

A measurement was taken on each of the 4 arms of each fiducial, and the centre of the fiducial reconstructed. This gave a useful increase in precision. Five measurements were taken of each view/frame without unclamping the film from the gate; giving 40 measurements, overall, of each fiducial. 17 fiducials were measured, an adequate number for the HPD as well as the conventional machines.

A reference frame was chosen and each subsequent frame was transformed into the reference frame coordinate system, and averaged with the reference frame, to obtain an "average" frame for input to PYTHON. As shown in the flow chart, Fig. 9, frames which would not transform into the reference frame within a certain accuracy were rejected. There is an inherent danger in this procedure in that if a "bad" frame is chosen as the reference, then subsequent "good" frames are rejected, or distorted, in the transformation. Unfortunately, there is no a priori way of distinguishing "good" from "bad".

In fact, great difficulty was experienced with the version of PYTHON used, a different version has since been released. Using the basic 3 parameter lens distortion formula, the reconstructed fiducials were within 200 μ (in space) of the survey positions on both windows; the required precision is an order of magnitude less. On using more complex lens distortion parametrisations, the χ^2 of the fit was found to behave in a very anomalous fashion, in that it increased rapidly after reaching a minimum. At the minimum, the residuals on the fiducials were $\sim 80 \mu$ on the front glass but $\sim 200 \mu$ on the back. It is known⁽⁷⁾ that there is some inconsistency in the PYTHON fitting procedure, and some trouble was experienced at I.C. with 2-metre chamber titles. Hence, a program ADDER⁽⁸⁾ was written to replace the fitting routines in PYTHON.

ADDER simultaneously constrains the three rays from a reconstructed fiducial to be close to the survey position, and constrains each ray to be close to the fiducial's image on each view. This program also allows the chamber depth to vary, and produced a completely acceptable set of titles. The reconstructed fiducials were within 10-20 μ of the survey positions.

The quality of the titles was checked in MDT by examining the residuals on the fiducials of many frames, after they had been transformed into the title coordinate system. These were found to be rarely greater than 50 μ .

The titles were further checked by examining the helix fit residuals on tracks reconstructed by MDT, these should peak at 1-2 μ (on beam tracks) for HPD measurements and 8-10 μ for conventional. The HPD helix fit residuals for a typical sample of 4-C fits are shown in Fig. 10.

A final check was made by examining the stretch functions on the fitted quantities of tracks in GRIND. These are

$1/P - P = \text{momentum.}$

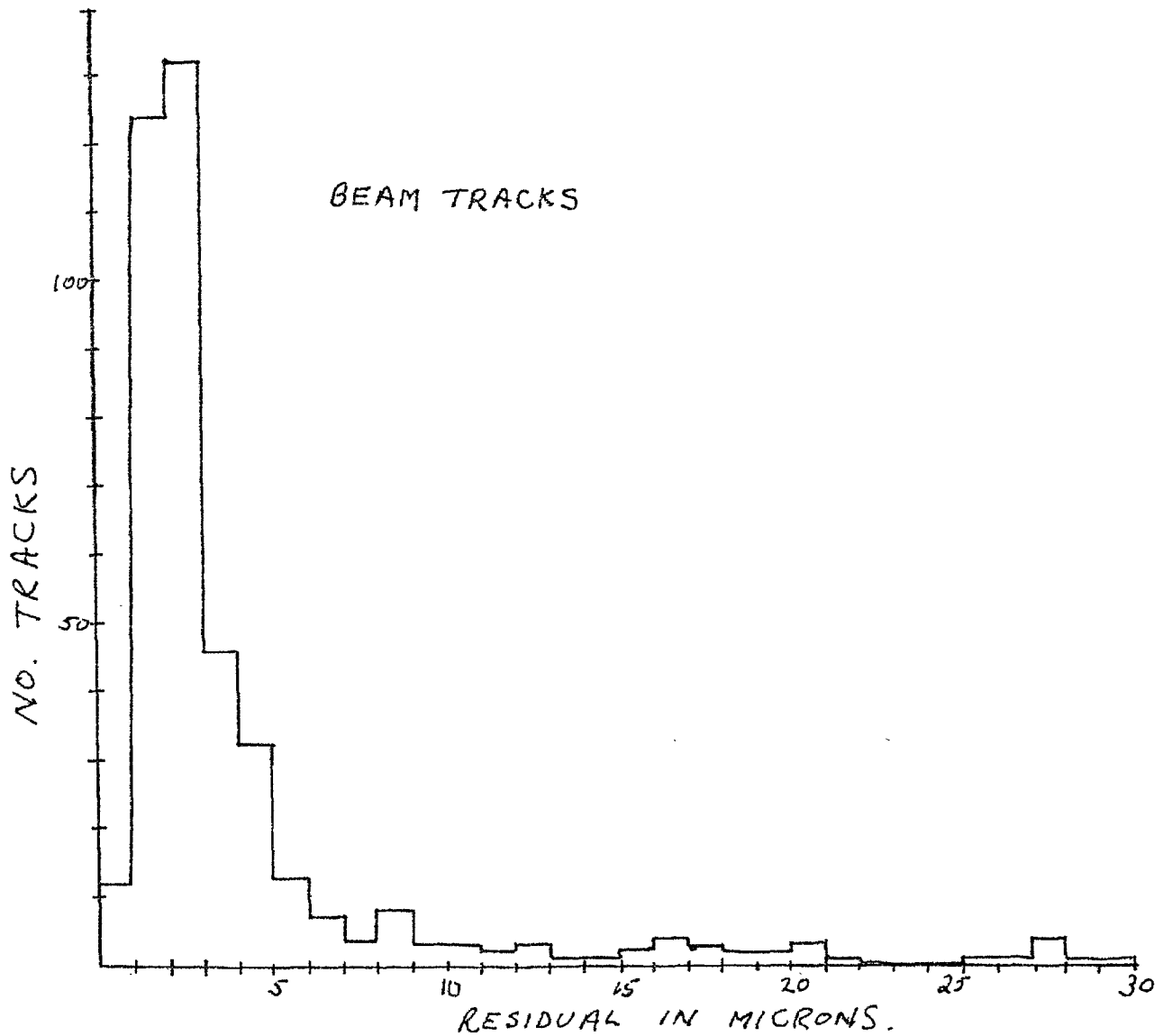
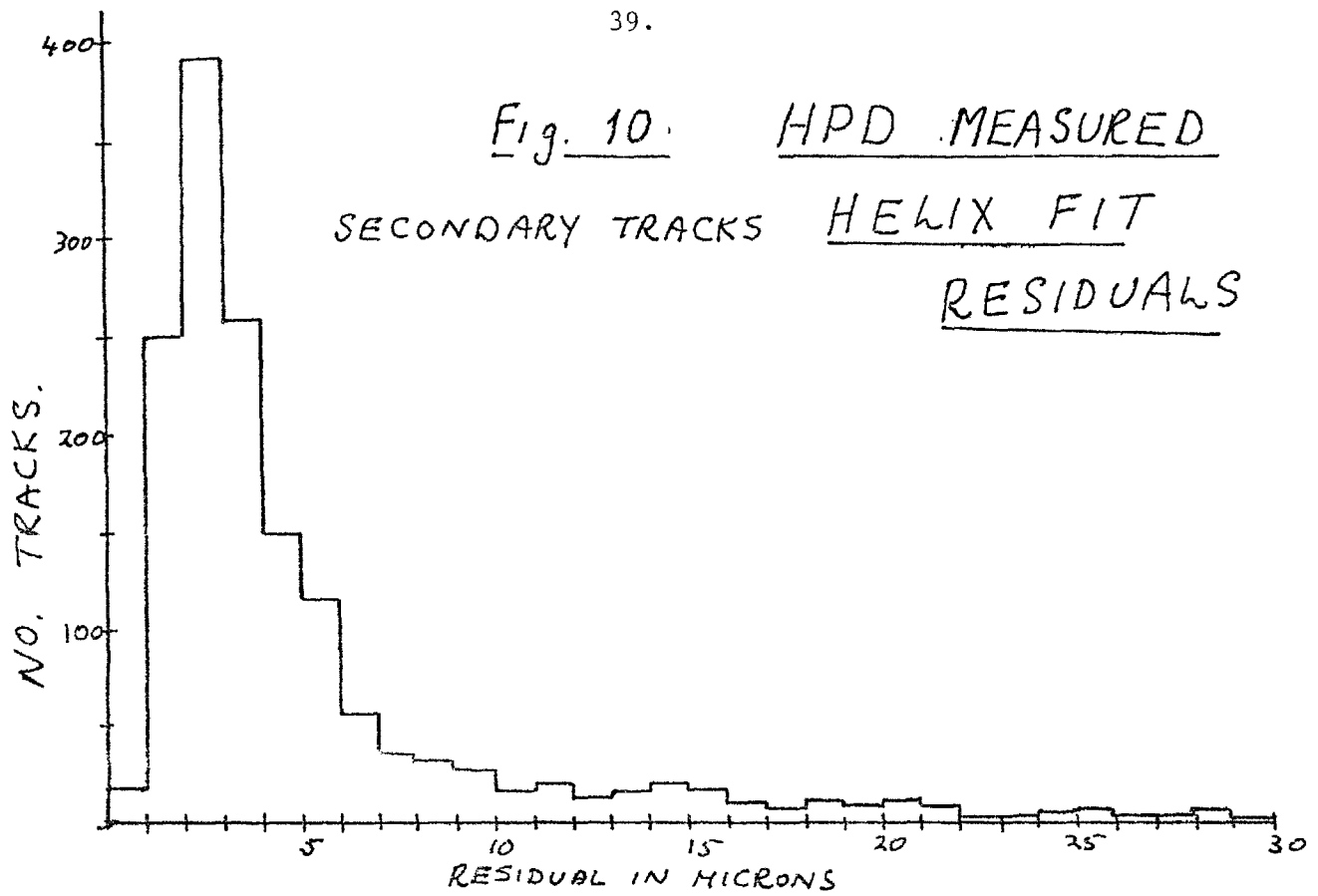
λ - dip angle from x-y plane in chamber.

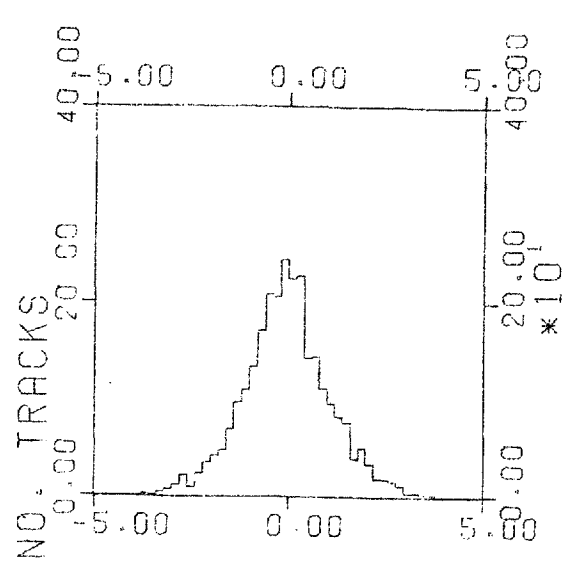
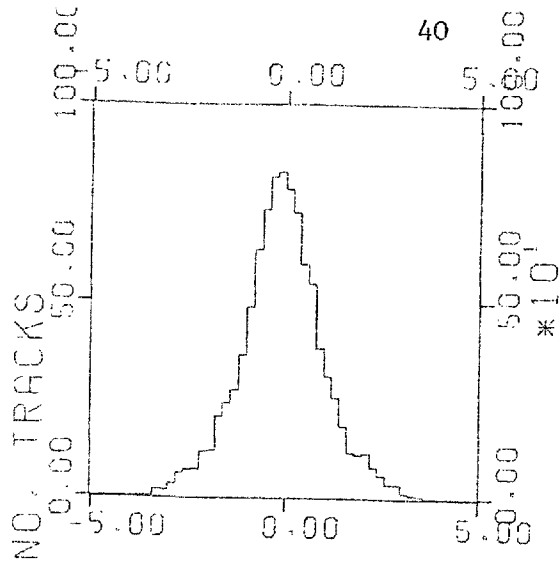
Φ - azimuth about chamber z-axis.

The back of the front chamber window is the plane (x, y, 0). These three quantities have normally distributed errors. The stretch functions are defined by

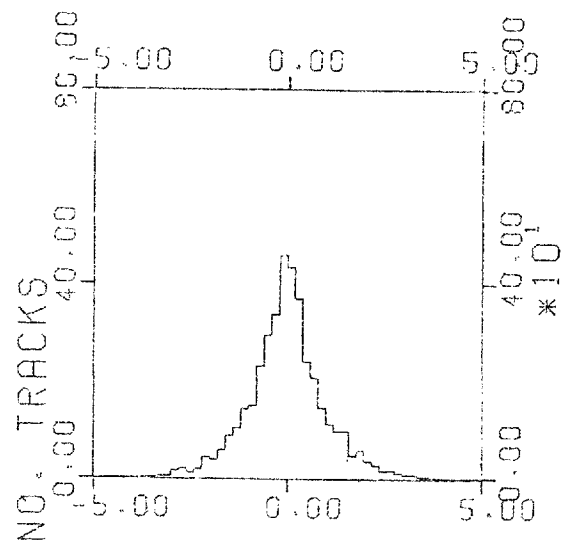
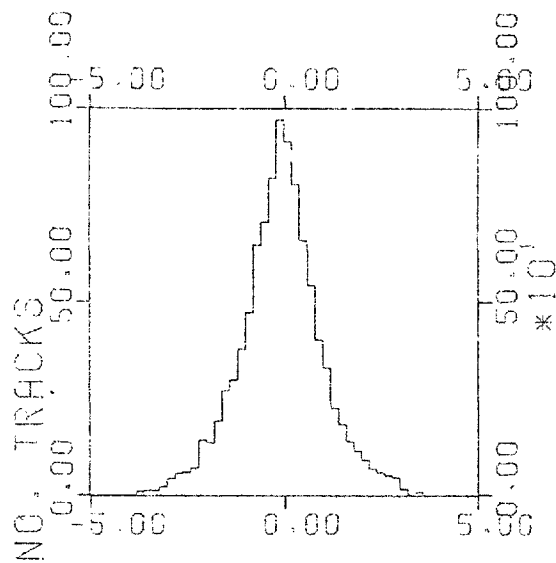
$$\frac{x_{\text{meas}} - x_{\text{fitted}}}{\sigma_{\text{meas}} - \sigma_{\text{fitted}}}$$

where x is one of the fitted quantities and σ is the error.

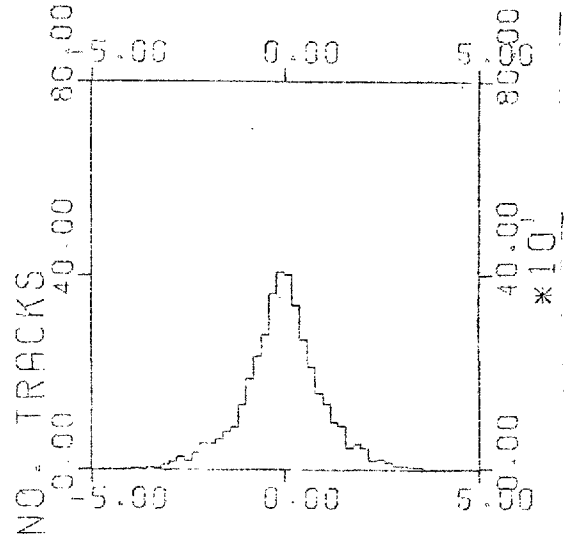
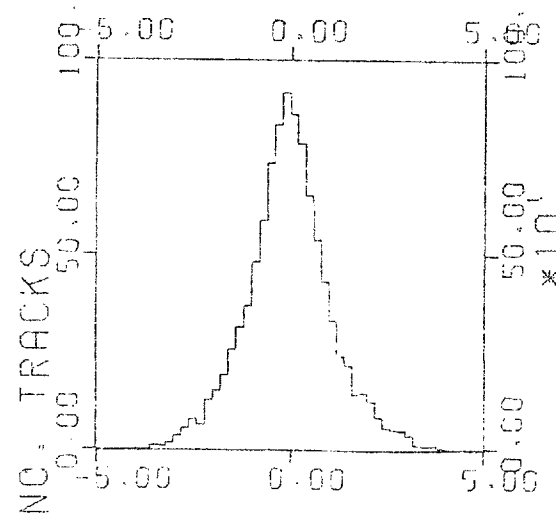




(1/P) PULLS ON ALL +VE TRACKS (1/P) PULLS ON ALL PI-



PHI PULLS ON ALL +VE TRACKS PHI PULLS ON ALL PI-



DTP PULLS ON ALL +VE TRACKS DTP PULLS ON ALL PI-

Fig 11 4-C STRETCH FUNCTIONS

These stretch functions should be normally distributed with mean zero and standard deviation unity, see fig. 11.

2.4.2 Treatment of Short Tracks in MDT

The standard CERN MDT has been modified to increase the pass rate of short stopping tracks⁽⁹⁾. The standard version handles short tracks, which have failed in HPD measurement, in a rather anomalous fashion. MDT attempts to use the road points (transmitted by SMOG). However, it reconstructs the track as a straight line between the road point at the end of the track and the vertex reconstructed from the intersection of well measured tracks. This has the drawback that, due to inaccuracies in fiducial measurements, the road points may be in a different coordinate system from the reconstructed vertex. This has been overcome by transforming the vertex as reconstructed from the road points into the MDT reconstructed vertex. The same transformation is then applied to the end point of the short track as measured by the rough digitiser.

2.5 GRIND - AUTOGRIND

Again the program used was the standard CERN version, including deuterium handling routines.

When the 4 equations of momentum-energy conservation are applied to a reaction observed in a bubble chamber, these are frequently found to be overconstrained, i.e. there are more equations than unknown variables. This enables the physical parameters to be found by fitting rather than just a simple solution of the constraint equations. The degree of constraint at a vertex is $(4-n)$, where n is the number of independent unmeasured quantities summed over all the tracks. In a channel where all

particles are charged, and thus measurable, the degree is 4, i.e. a 4-C fit results. If the channel produces a neutral particle, then the parameters $1/p$, λ , Φ are unmeasurable for that particle and the fit is reduced to a 1-C. For more than one neutral particle, no fit is possible, only a direct solution of the constraint equations.

In an experiment in a hydrogen chamber, events with, for example, a missing π^0 are 1-C and can usefully be studied. However, in a deuterium experiment one is interested in events of the type

$$K^+ n(p) \rightarrow \text{particles} + n(p) .$$

This could be handled as a conventional 1-C fit if the spectator proton were measured. In fact, the momentum distribution of nucleons inside the deuteron often leads to the proton having such a small momentum that the track is unseen. Consequently such an event would seem to be unconstrained as there are two unmeasured particles. By invoking the impulse approximation, one can assume that the spectator does not take part in the reaction. This implies that the mean values of each of the three components of the spectator momentum are zero in the deuteron centre of mass (i.e. LAB). For events in which there is the possibility of an unseen spectator, GRIND adopts the procedure of inserting a zero length proton having starting values of p_x^{LAB} , p_y^{LAB} , $p_z^{\text{LAB}} = 0$, the errors being set large enough to encompass all possible unseen spectators. These errors are

$$\begin{aligned} \Delta p_x &= \Delta p_y = \pm 30 \text{ MeV/c} \\ \Delta p_z &= \pm 40 \text{ MeV/c.} \end{aligned}$$

The larger error on p_z being due to the fact that tracks along the camera axis are intrinsically more difficult to see.

Such a procedure implicitly supplies information to GRIND about the unseen track, i.e. that it has a particular mass and a momentum in a particular range. As mentioned, the usual fitted parameters are $1/p$, λ , Φ ; since spectators are fitted with p_x, p_y, p_z , this is known as "XYZ-fitting" .

This brings one back to the situation of having 4C and 1C neutron target fits. However, it seems intuitively obvious that the amount of information contained in unseen spectator fits is less than in fits to events in hydrogen. An investigation of this problem has been performed using Monte-Carlo generated events⁽¹⁰⁾. For 4c fits both spectator momentum and angular distributions appear satisfactory, as does the centre-of-mass energy (E^*) . For 1-C fits the fitted spectator distributions deviate seriously from that expected from the Hulthén wave function. In particular the spectator momentum is strongly peaked towards zero. This in itself is no disadvantage as the spectator is, by definition, of little interest. A more serious effect shows up in the E^* distribution of 1-C fits. This appears to be distorted by the process of inserting the unseen spectator.

2.5.1 Bubble Chamber Magnetic Field

An accurate estimate of the magnetic field in the bubble chamber is necessary for momentum calculations in both MDT and GRIND. The method used in both the K^+d and K^+p experiments was that of a field survey map supplied to the program on cards, the field inside the chamber having been measured at points on a 3-dimensional cubic lattice, by the nuclear magnetic resonance method⁽⁵⁾.

The field values at these points were divided by the value at a central point, the program being supplied with this central value plus the lattice coefficients.

At the start of the K^+d experiment there was some uncertainty concerning the correct figure for the central field value.

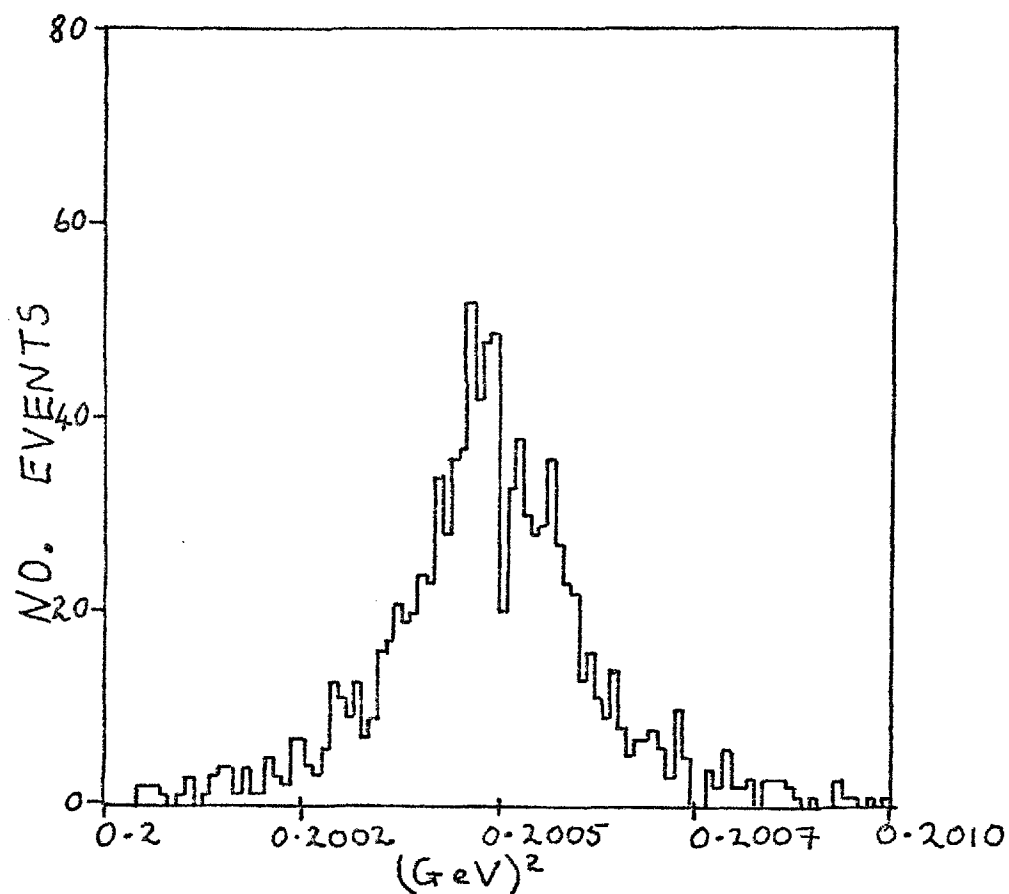
In 1967, a RHEL survey had shown the central field value to be 12.27 KG. However, in 1968, at the beginning of the K^+p experiment, using 3 C $K^0 \rightarrow \pi^+ \pi^-$ fits had shown it to be 12.67 KG. This latter was used throughout the K^+p experiment. A more recent RHEL survey in January 1970, contemporaneous with the film taking of this experiment, had produced a value of 12.311 ± 0.03 KG. It was therefore decided to redetermine the field using the K^0 mass method.

When GRIND performs a fit to a reaction in which there is a seen K^0 , the fit to the K^0 decay is 3-C since the direction, but not the momentum, of the K^0 is initially known. There is thus some "missing mass", corresponding to the mass of the K^0 . It is clear that the fitted value of this missing mass will depend on the fitted momenta of the two pions. These momenta are in turn dependent on the value of the magnetic field. So the K^0 mass obtained will depend on the field value chosen. 1222 events of the type $K^0 \rightarrow \pi^- \pi^+$ were successfully fitted in a sample of conventionally measured 2.2 GeV/c film. These fits were performed using 12.67 KG as the central value. This produced a K^0 mass of 0.5055 GeV - a serious overestimate.

The field value was then reduced to 12.30 KG, and the events refitted. This yielded a K^0 mass of

$$0.4965 \pm 0.45 \text{ MeV}$$

see Fig. 12.



$K^0 \rightarrow \pi^+ \pi^-; (\text{MISSING MASS})^2$

CENTRAL FIELD = 12.30 KG

Fig: 12

By linear interpolation it was estimated that a central field of 12.35 ± 0.05 KG would move the K^0 mass onto the accepted value. This was now completely compatible with the RHEL survey.

2.5.2 GRIND Beam Title

GRIND allows one to insert an average beam in place of the measured beam, if the latter should fail to reconstruct in MDT. This is especially useful in an HPD experiment as the beam track is most likely to fail due to noise from overlapping and crossing beam tracks. The HPD often takes segments from two intersecting beam tracks on the same view, or different tracks on different views, as being the true beam. Such errors can cause failure in reconstruction or, more subtly, to give a large helix fit residual.

All beams failing or having helix fit residuals $> 6\mu$ were replaced by title beams.

The insertion of title beams presupposes that the beam parameters are well defined in p , and that λ and ϕ vary in a simple manner in the chamber. The beam was tuned to have a momentum bite $\sim 1\%$, and to be as well focused as possible. The size of the momentum bite has the implication that the beam momentum is well defined at any point in the chamber, only varying due to ionisation loss.

The ϕ angle is a function of x and y . In the former due to curvature under the field and the latter due to the beam diverging from a focus. λ should only be a function of Z as the chamber field is assumed to be only in the Z -direction.

GRIND was supplied with p , λ , ϕ at a standard position and coefficients⁽¹¹⁾ enabling it to determine λ and ϕ at any point in the chamber. These coefficients were derived by studying successfully fitted beams. p is corrected in GRIND for ionisation loss.

The actual central beam momenta found were 2.18 GeV/c, 2.43 GeV/c, and 2.7 GeV/c.

Chapter 2 - References

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- (3) AUTOGRIND - M. Mermikides, ICHENP Physics Note.
HPD ionisation and AUTO - A.K. Islam, ICHENP Physics Note.
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- (7) Private Communication from J. Zöll.
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- (10) Kinematic fitting of neutron target reactions in a deuterium bubble chamber - Campbell, Morton, Negus; Glasgow University Preprint.
- (11) GRIND, T.C. Manual Section C 004.

CHAPTER 3THE DATA

The channel examined in this thesis is

$$K^+ d \rightarrow K^0 p (p)$$

with a view to unfolding the elastic charge exchange reaction

$$K^+ n \rightarrow K^0 p$$

In this chapter, the data obtained at the time of writing is presented. The quality of the data is discussed, and a description is given of the various corrections made to allow for experimental biases.

Events of this type can be selected in an almost unambiguous fashion. This is due partly to the fact that the fits are highly constrained, and partly to the presence of a seen decay (K^0).

In fits to this reaction GRIND initially fits to the K^0 decay using information on the K^0 direction; as pointed out in Chapt. 2 this is a 3-C fit. A fit is then performed at the primary vertex, this being a 4-C fit, since the K^0 momentum is now known from the decay fit. Finally, the K^0 parameters are optimised in a 7-C multi-vertex fit.

These 7-C fits comprise the majority of the events. In some of the events the number of constraints drops to 6-C or 4-C, due to either one primary track being straight within errors, or the K^0 failing to fit in the decay fit. The latter events have been used, as the information on the K^0 is not lost. These two classes together contain 20 events out of 201 at 2.8 GeV/c.

With such a large volume of film on hand, it was decided to initially measure samples at each of the 3 momenta. The motivation

being that this would allow decisions to be made as to what momenta and/or channels would be given priority in processing the complete exposure. The results presented here are derived from a subset of the data comprising 36.4 K frames at 2.18 GeV/c, 28.6 K frames at 2.43 GeV/c and 15.6 K at 2.7 GeV/c.

The data consists of a mixture of HPD and conventionally measured events. When the film arrived at I.C. the HPD was not immediately available and a restricted topology conventional measurement was undertaken. The topology classes included 201 and 101 events and hence the charge exchange. This subset was separately processed up to, and including, DST production. The events in this subset were eventually used as a conventional "remeasurement" of the HPD pass, since the same film was involved. On merging the two data sets the HPD measurements were considered preferable. If an event passed successfully through both chains, the HPD event was utilised. If an event failed on the HPD pass, but was successful in the conventional pass, the conventional measurement was used. Events failing both measurements were remeasured on a conventional machine.

At the time of writing, no HPD measured data was available for analysis at 2.7 GeV/c, and the results at this momentum are based on a single conventional measurement.

It is worth emphasising the preliminary nature of the data presented here, both in numbers of events and quality. The number of events at each momentum are listed in Table 6.

Table 6.

Momentum	No. of Events
2.18 GeV/c	201
2.43 "	153
2.7 "	64

3.1 Ambiguities

As already pointed out, the events presented are essentially unambiguous.

The problem of π^+ contamination in the beam does not arise due to the fact that a seen K^0 decay always occurs.

The possibility of confusion with Λ^0 production as in

$$K^+ n (p) \rightarrow K^0 \Lambda^0 K^+ (p)$$

is unlikely due to kinematics.

Another source of contamination would be from

$$K^+ n (p) \rightarrow K^0 \pi^0 p (p)$$

fitting to the charge exchange hypothesis. This would only be possible if the π^0 were very slow and the event badly measured. Of the 201 events at 2.2 GeV/c only one was chosen as ambiguous with the π^0 production hypothesis, and this in a fit where one pion track in the decay failed to reconstruct. Even this ambiguity only arose from a somewhat arbitrary feature of the GRIND choicing instructions. In GRIND choicing a 4C (7C) fit was always preferred to a 1C (3C) fit except when the confidence level of the latter was >10 times that of the former; assuming both consistent with ionisation. The probability

distribution of fits is shown in Fig. 16. This would be flat if the measurement errors were correctly apportioned. The high probability peak is not, however, due to serious overestimate of the measurement errors. It is mainly due to the fact that K^0 decays usually fit with a high probability, and this is multiplied into the overall fit probability. The lack of any serious low probability peak indicates an absence of misfitted events.

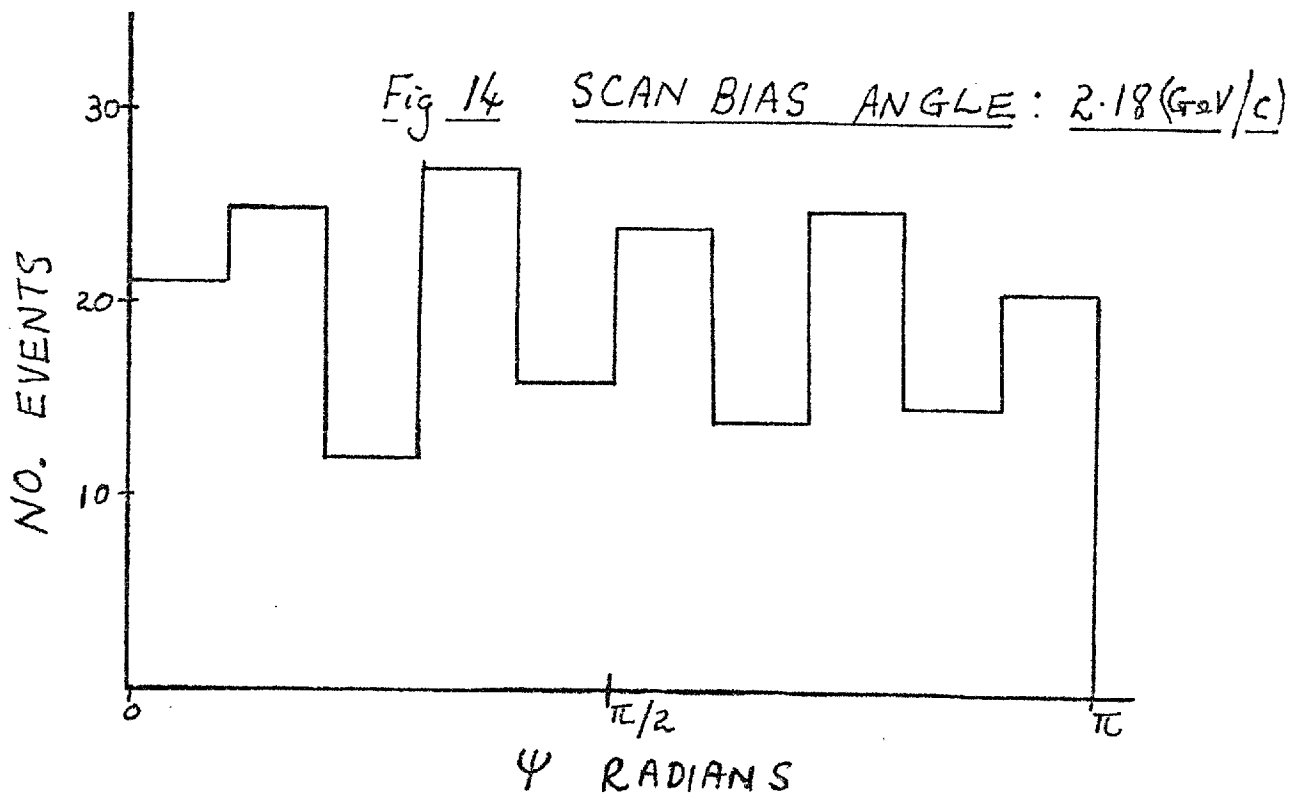
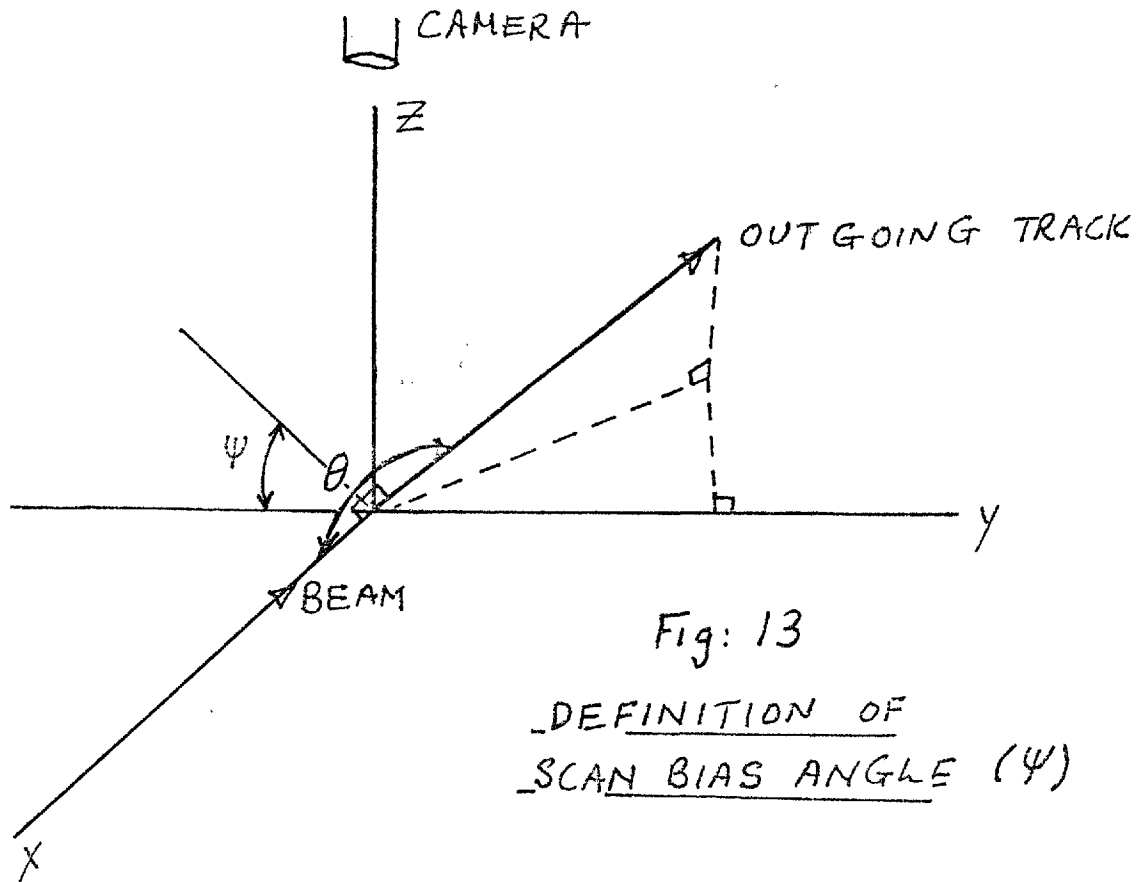
3.2 Scanning Biases and Measuring Efficiency

An overall loss of events due to scanning inefficiencies would effect the cross section obtained for the channel. The signature of this channel plus the high calculated scanning efficiency indicate that this is not an important effect.

What is more important is that certain classes of events within a topology are intrinsically more difficult to scan. In particular, events with a short proton travelling in the direction of the camera axis are difficult to scan. Now, although charge exchange events can contain such tracks, the V^0 signature makes any bias unlikely. Nonetheless, if such events were missed it would result in the depopulation of particular regions of the angular distribution, and so such an effect has been checked for.

For an unpolarised beam and target, the plane of the two outgoing particles should be isotropically distributed in the azimuthal angle (ψ) about the beam direction at the point of interaction, see Fig. 13.

The angle ψ is defined by



$$\cos \psi = \frac{\vec{K}_{in}^+ \times \vec{\hat{O}}_z}{|\vec{K}_{in}^+ \times \vec{\hat{O}}_z|} \cdot \frac{\vec{K}_{in}^+ \times \vec{K}_{out}^0}{|\vec{K}_{in}^+ \times \vec{K}_{out}^0|}$$

$\vec{\hat{O}}_z$ is a unit vector perpendicular to the front window of the bubble chamber, and $\vec{K}_{in}^+$ and $\vec{K}_{out}^0$ are the incoming and outgoing K^+ , K^0 directions.

Any scanning loss should occur for events in which the production plane contains the line of sight. This corresponds to $\Psi \rightarrow 0$ or $\cos \psi \rightarrow 1$. Within the statistics of this data set, the distribution of ψ is flat [Fig. 14], and no correction has been introduced.

The measuring efficiency is defined as the ratio of the number of successful measurements of a particular topology to the number of that topology scanned, excluding unmeasurable events. At 2.18 GeV/c, this was 90% and at 2.43 GeV/c 88%, the difference being due to the fact that the only remeasurement at 2.43 GeV/c was from the preliminary conventional pass.

At 2.18 GeV/c there was a further complication. It was found that there had been a fault in the camera for view 1. This fault had the effect of causing the view to periodically go out of focus.

Now, events often reconstruct successfully with information from only two views, but those that do not reconstruct will be of a definite geometrical configuration. To utilise "2-view" events without first showing them to be unbiased would be to run the risk of biasing the final results obtained. The approach adopted has been to calculate the cross section (and measuring efficiency) on the basis of events from rolls on which no defocusing occurred.

Fig: 15 MOMENTUM TRANSFER DIST $2.18 \text{ GeV}/c$

□ ALL ROLLS (201 EVENTS)

▨ WELL-FOCUSED ROLLS (125 EVENTS)

$0.063 \text{ (GeV}/c)^2$ BIN WIDTH

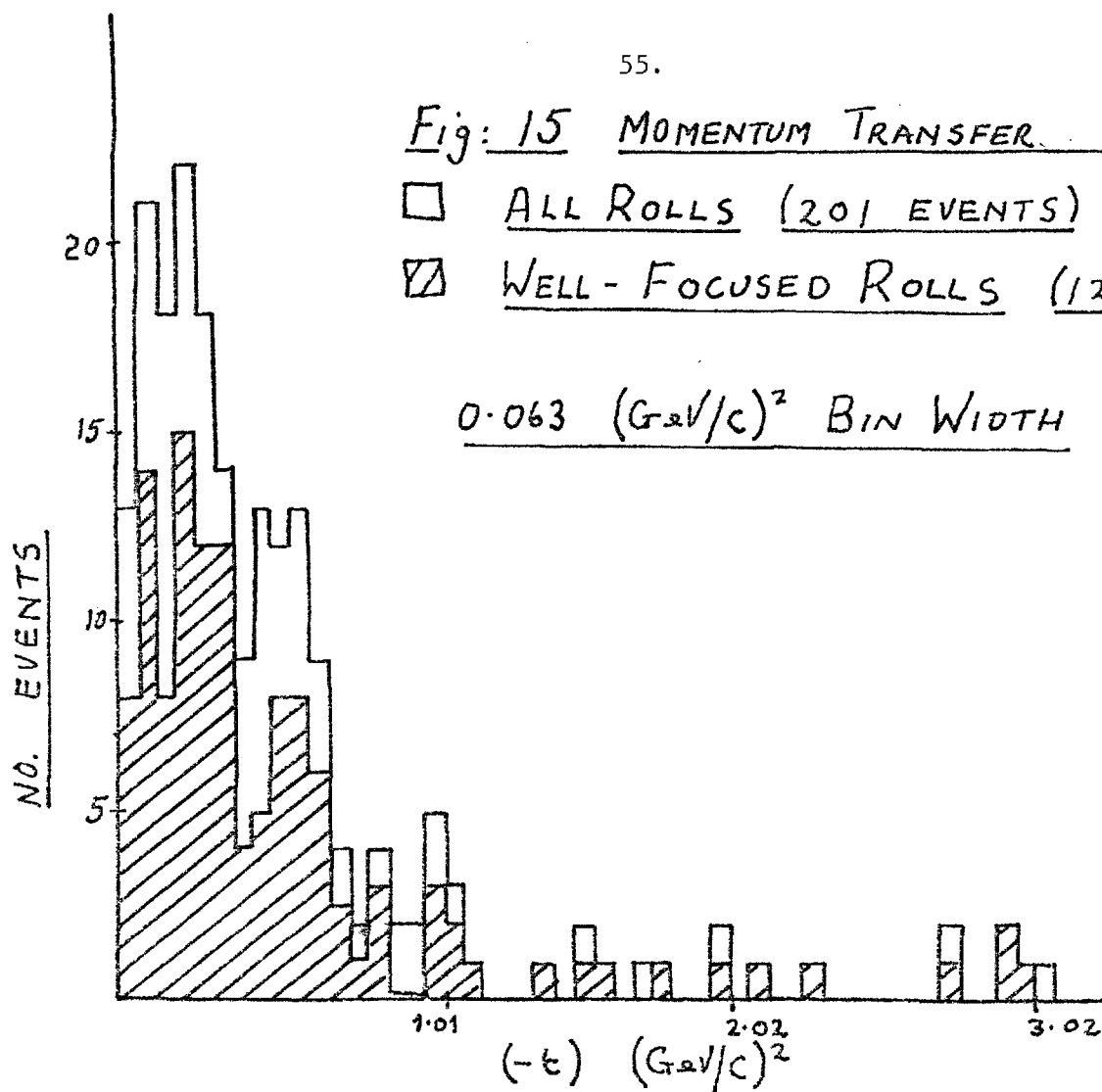
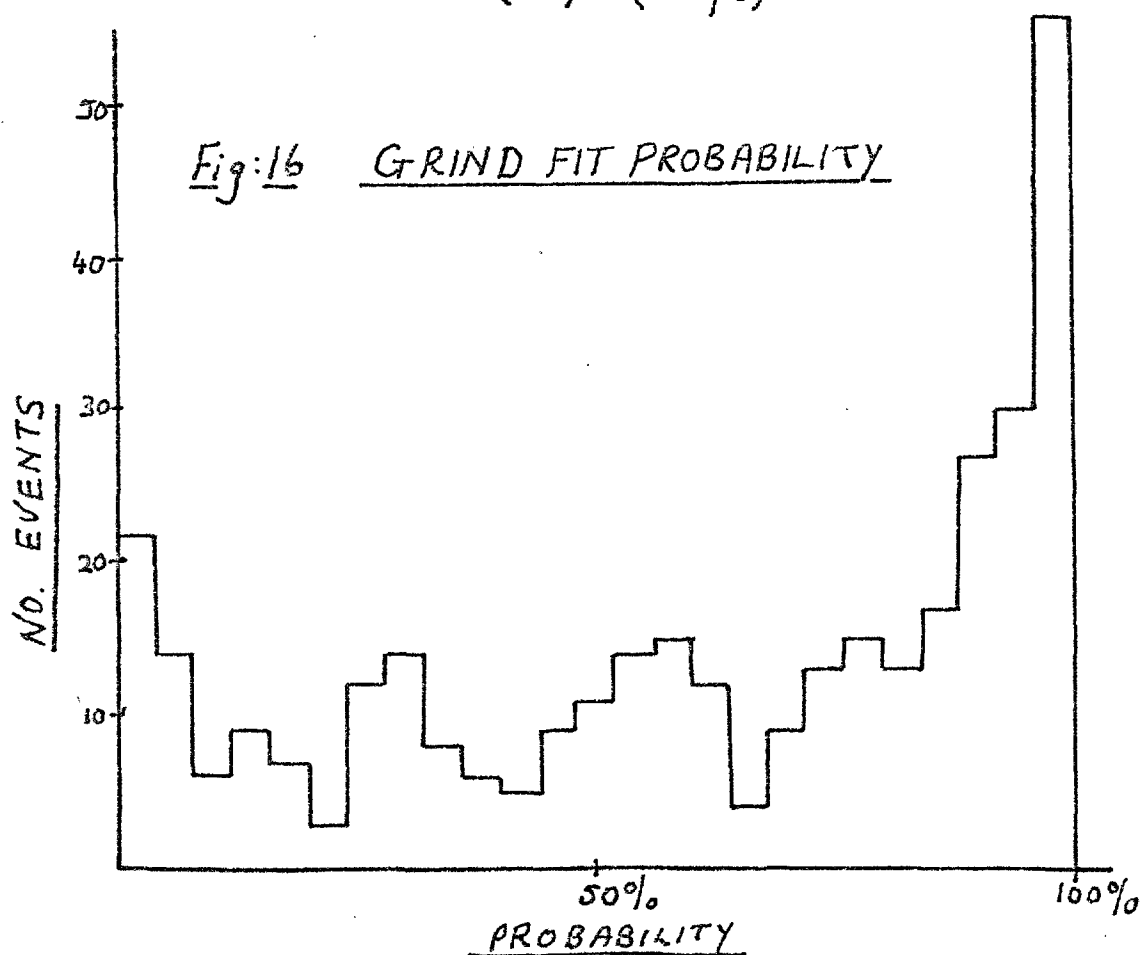


Fig: 16 GRIND FIT PROBABILITY



The events (excluding 2-view events) from the defocused rolls have been included in the angular distribution, as the defocusing was a sporadic effect and is not present on all frames. Examination of the momentum transfer distribution with and without these events showed that they had no significant effect on the shape, within the present statistics, Fig. 15.

3.3 Correction for Unseen K^0 's

By including only events in which a K^0 is seen to decay in the chamber, the data set only includes a definite fraction of all the events producing a K^0 . Events are lost in two ways. Firstly, some K^0 's decay so close to the production vertex that the event is classed in the topology 400, and so is lost from this channel. In addition, some K^0 's decay outside the chamber and the event is classed as 200, and again lost.

The form of the corrections applied to allow for these losses is governed by the behaviour of the K^0 decay. The K^0 decays via the weak, parity (P) violating, force. The idea of CPT (Charge conjugation, Parity, Time reversal) invariance is deeply imbedded in the theory of elementary particles and since K^0 decays do not conserve P they are assumed to conserve CP. The main decay modes of the K^0 are into $\pi^+ \pi^-$ and $\pi^0 \pi^0$. These final states are eigenstates of CP. However, the K^0 and its antiparticle, $\overline{K}^0$, are not eigenstates of CP, since

$$CP |K^0\rangle = |\overline{K}^0\rangle$$

$$\text{and } CP |\overline{K}^0\rangle = |K^0\rangle.$$

So, to describe the decays, it is necessary to construct eigenstates of CP, i.e.

$$\text{CP} = +1 \quad |K_1\rangle = \frac{1}{\sqrt{2}} \{ |K^0\rangle + |\overline{K}^0\rangle \} \quad (3.1)$$

$$\text{CP} = -1 \quad |K_2\rangle = \frac{1}{\sqrt{2}} \{ |K^0\rangle - |\overline{K}^0\rangle \} \quad (3.2)$$

Then the states $|K_1\rangle$ and $|K_2\rangle$ are the correct states for describing the decay.

Clearly

$$|K^0\rangle = \frac{1}{\sqrt{2}} \{ |K_1\rangle + |K_2\rangle \} \quad (3.3)$$

So, a K^0 must be considered as a quantum mechanical mixture of K_1 and K_2 . The 2-pion decay mode of the K^0 is, from CP conservation, allowed for $|K_1\rangle$, but is forbidden for $|K_2\rangle$. (3.3) implies that a K^0 has a 50% probability of decaying via the modes open to the $|K_1\rangle$, and 50% via $|K_2\rangle$ decay modes, which are

$ K_2\rangle \rightarrow \pi^0 \pi^0 \pi^0$	21.4 %
$\rightarrow \pi^+ \pi^- \pi^0$	12.6 %
$\rightarrow \pi \mu \nu$	26.8 %
$\rightarrow \pi e \nu$	39.0 %
+ rare.	

Whereas the 2-pion decay modes have a mean lifetime of 0.862×10^{-10} sec, the 3-pion modes have a mean lifetime of 5.172×10^{-8} sec. So, only $|K_1\rangle$ decays will be seen in a bubble chamber.

Thus, the first correction is to allow for the 50% of reactions which produce K^0 which decay as K_2 . Additionally, only the $\pi^+ \pi^- K_1$ decays can be seen, and these only constitute 68.85% of the K_1 decays; so again a correction must be made for this branching ratio.

In fact⁽¹⁾ the long and short-lived K^0 components are not $|K_2\rangle$ and $|K_1\rangle$ but

$$|K_L\rangle = |K_2\rangle + \varepsilon |K_1\rangle$$

and $|K_S\rangle$. Where $(\varepsilon) \sim 2 \times 10^{-3}$ and is due to CP violation. If all charged K_S^0 decays were seen, the total number of events would be 2.91 times the number of seen decays. Not all such K_S^0 's are, however, seen. Some will decay outside the illuminated volume of the chamber [INLITE volume]. The INLITE volume was found from a 2-dimensional plot of the decay vertices of all seen K^0 producing events.

The probability that a given K_S^0 will not decay before reaching the INLITE boundary is:

$$e^{-P/L}$$

L being the theoretical mean decay length, in the lab, of the K_S^0 .

P is the POTENTIAL LENGTH, defined as the distance from the primary vertex to the INLITE boundary.

To allow for the loss due to K_S^0 's decaying at an unobservably small distance from the production vertex a cut, C, was chosen as being the minimum observable projected decay length. For a particular K_S^0 this C corresponds to a length c' along the line of flight. Then

$$e^{-c'/L}$$

is the probability of that K_S^0 decaying at an observable distance from the production vertex.

So the probability of a given K_S^0 decaying in an observable fashion is

$$e^{-c'/L} - e^{-P/L}$$

To allow for unseen decays, each seen event was given a weight.

$$\frac{1}{e^{-c'/L} - e^{-P/L}}$$

Any events with K_S^{04} observed to decay closer than C to the vertex were ignored, to avoid double counting.

The value of C was found empirically, by assuming several values of the cut. The total weighted number of events was examined for cuts ranging from 1 mm - 10 mm. The value of C was chosen at the point at which the weighted number of events levelled off. This was taken as 0.3 cm, giving a mean K^0 weight of 3.1.

3.4 Deuterium Effects

In order to describe the implications of employing a compound nucleus as target, and so describe the necessary corrections, it is necessary to outline the approach of the impulse approximation.

The impulse approximation was originally discussed by G.F. Chew⁽²⁾. The method consists of superimposing contributions from single and multiple scatters of the beam off the components of the nucleus. In physical terms the method visualises the dominant term of an Eikonal series

$$\text{Total Amp.} = \underbrace{\text{I}}_{\text{single scatter}} + \underbrace{\text{II} + \text{III} + \dots}_{\text{multiple scatters}}$$

to be the leading term corresponding to the incident particle scattering off one of the nucleons. The remaining nucleon is considered to take no part in the interaction (to 1st order) and is left in the lab frame with the momentum it had inside the deuteron before the collision.

For this approximation to be valid the following conditions must be satisfied.

(a) The incident particle-nucleon force must have a range small compared with the inter-nucleon distance. This just implies that the beam can interact with one nucleon without effecting the other. As the beam energy increases, causing its "wavelength" to decrease, this requirement will be increasingly well satisfied.

(b) The nucleus as a whole must not effect the amplitude of the incoming particle to any great extent. This is in order that the amplitude falling on the nuclear constituents is similar to that which would exist if the particles were free. In effect, this requires that the deuteron be "transparent" to the incoming wave packet.

(c) The time scale of the beam-target interaction must be small compared with the time scale of the nucleon-nucleon interaction. This ensures that the spectator nucleon experiences nothing of the beam-target interaction via nucleon-nucleon coupling.

(d) Multiple scattering processes must be much less important than single scatters; i.e.

$$\begin{aligned}
 \text{Amplitude} &\approx \begin{array}{c} \text{p} \text{ --- } \text{p} \\ \text{n} \text{ --- } \text{n} \\ | \\ \text{K} \text{ --- } \text{K} \end{array} + \begin{array}{c} \text{---} \\ | \\ \text{---} \end{array} \\
 &+ \begin{array}{c} \text{---} \\ | \\ \text{---} \end{array} + \begin{array}{c} \text{---} \\ | \\ \text{---} \end{array} + \dots \\
 &\approx \begin{array}{c} \text{p} \text{ --- } \text{p} \\ \text{m} \text{ --- } \text{m} \\ | \\ \text{K} \text{ --- } \text{K} \end{array} + \begin{array}{c} \text{p} \text{ --- } \text{p} \\ \text{m} \text{ --- } \text{m} \\ | \\ \text{K} \text{ --- } \text{K} \end{array}
 \end{aligned}$$

The diagrams are Feynman diagrams representing particle interactions. The first set of diagrams shows a deuteron (p and n) interacting with a kaon (K) via a vertical line, with a plus sign and a diagram showing a single interaction. The second set shows a deuteron (p and m) interacting with a kaon (K) via a vertical line, with a plus sign and a diagram showing a single interaction.

Point (c) above implies that the inter-nucleon binding force is much weaker than the beam-target force. Since the deuteron has a binding energy⁽⁴⁾ of 2.225 MeV, this condition is well satisfied with a beam momentum in the region of this experiment. The nucleon-nucleon binding force only serves to give the spectator a small momentum distribution in the lab system. In practical terms the foregoing implies that of the two nucleons in the final state the faster is considered to be the one struck by the incoming meson and the slower is considered to be the spectator.

By examining the momentum and angular distributions of the spectator in the laboratory frame, it is possible to evaluate how well the impulse approximation works in the case in hand. Assuming that these distributions are satisfactory, it is then possible to unfold the free target data by allowing for the following effects.

(a) Shielding of one nucleon by the other (GLAUBER SHIELDING), this leads, particularly, to the cross section for:

$$K^+ d \rightarrow K^0 p (p)$$

being smaller than that for

$$K^+ n \rightarrow K^0 p .$$

(b) The centre-of-mass energy (E^*) smearing due to the motion of the target inside the deuteron. This being contrasted with the uniquely defined E^* in a hydrogen experiment.

(c) The effects of the Pauli exclusion principle acting between the two protons in the final state.

The treatment of these effects is now considered in detail.

3.4.1 Spectator Distributions

In order to check the validity of the impulse approximation the spectator momentum and angular distributions in the laboratory frame have been examined. This carries the implication that there is some theoretical prediction with which to compare these distributions. The theory employed here was, in common with most deuterium experiments, a description of the deuteron in terms of the Hulthén wave function⁽⁵⁾.

As already pointed out the deuterium nucleus is a weakly bound object. The ground state is predominantly an angular momentum S-state; there is some D-wave, but this is usually ignored. The isotopic spin is 0.

The Hulthén wave function is

$$\Psi_d(r) = \left(\frac{N}{r} \right) \left\{ e^{-\alpha r} - e^{-\beta r} \right\}$$

r being the internucleon distance (~ 2 fermi). The values of α, β have been chosen as⁽⁶⁾

$$\alpha = 0.0457 \text{ GeV}/c$$

$$\beta = 7 \alpha \text{ GeV}/c$$

although other values of β have been suggested (e.g. $\beta = 5.18 \alpha$ ⁽⁷⁾).

The normalisation constant, N , is

$$N = \sqrt{\frac{\alpha\beta(\alpha+\beta)}{2\pi(\beta-\alpha)^2}}$$

Performing a Fourier transform on the configuration space wave function, $\Psi_d(r)$, gives the wave function in momentum space.

$$\Phi_d(p) = N \sqrt{\frac{2}{\pi}} \left(\frac{1}{\alpha^2 + p^2} - \frac{1}{\beta^2 + p^2} \right)$$

This directly gives the momentum distribution

$$4 \pi p^2 \phi_d^2(p) dp .$$

Which is the probability of a spectator having momentum in the range $p \rightarrow p + dp$, irrespective of direction.

The spectator momentum distributions at 2.18 GeV/c and 2.43 GeV/c are presented in Fig. 17, as are the Hulthén distributions. As explained in Chapt. 2, fits in which an unseen spectator is inserted by GRIND, in an event already having a missing neutral, are known to have distorted spectator momentum distributions. In the highly constrained fits considered here, one expects the momentum distribution to be satisfactorily described by the Hulthén wave function predictions. As can be seen this is so.

The predicted distributions in Fig. 17 are normalised to the number of spectators in the experimental distribution having momenta < 300 MeV/c. This is because of the well-known problem of an excess of high momentum spectators. The Hulthén wave function predicts only 1 % of spectators with momenta > 300 MeV/c. Experimentally this proportion is found to be as high as 20% and usually of the order 10%-20%⁽³⁾. In the present case the proportions are 20% at 2.18 GeV/c and 12% at 2.43 GeV/c.

The deviation from the Hulthén prediction has been the subject of various proposed explanations. The most basic notion is that the Hulthén wave function is inadequate. However, the wave function follows from solving the Schroedinger equation, and since agreement is good below ~ 300 MeV/c it would appear that some other factor is producing the high momentum tail. The most likely explanation is scattering in the final state,

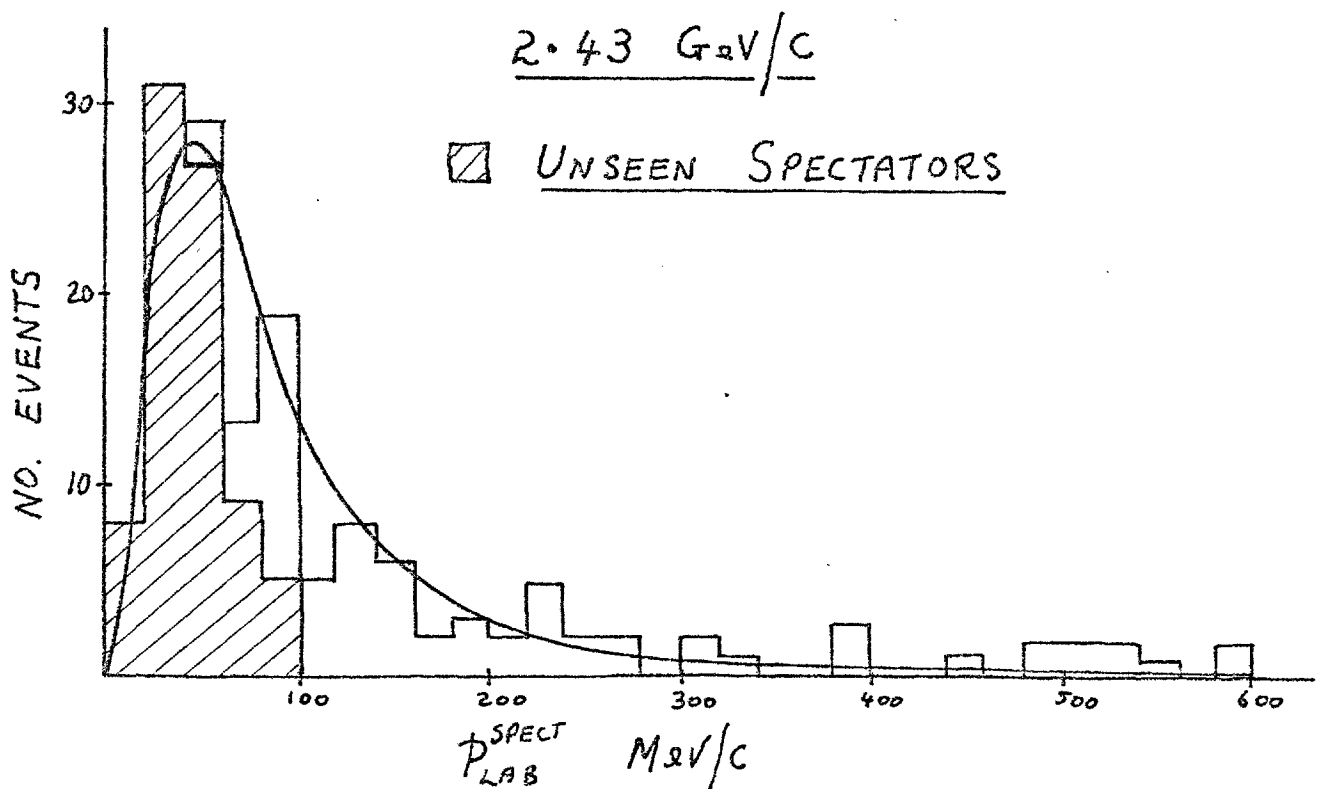
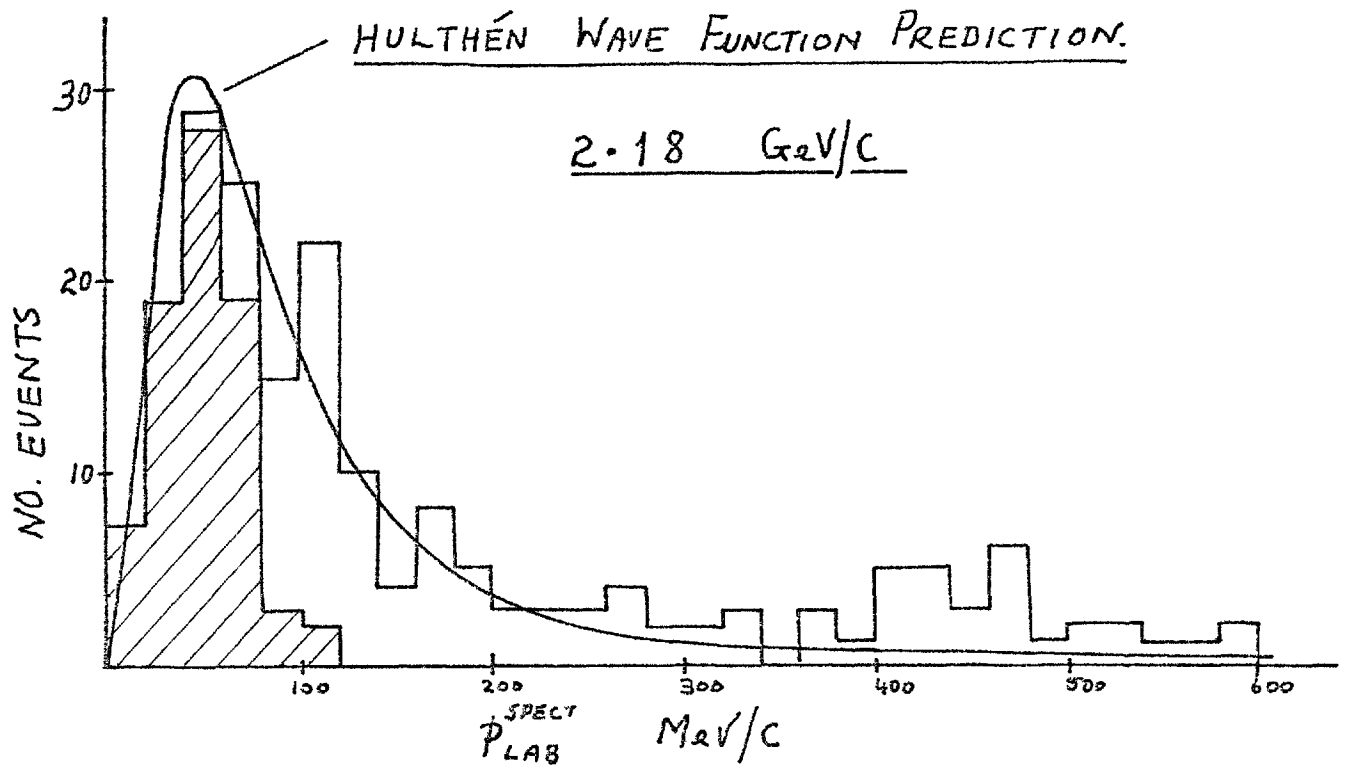


Fig:17 SPECTATOR p_{LAB} DISTRIBUTIONS.

either elastic or inelastic. It seems intuitively obvious that such a process must occur, and it was felt that this cast some doubt on the events with high momentum spectators.

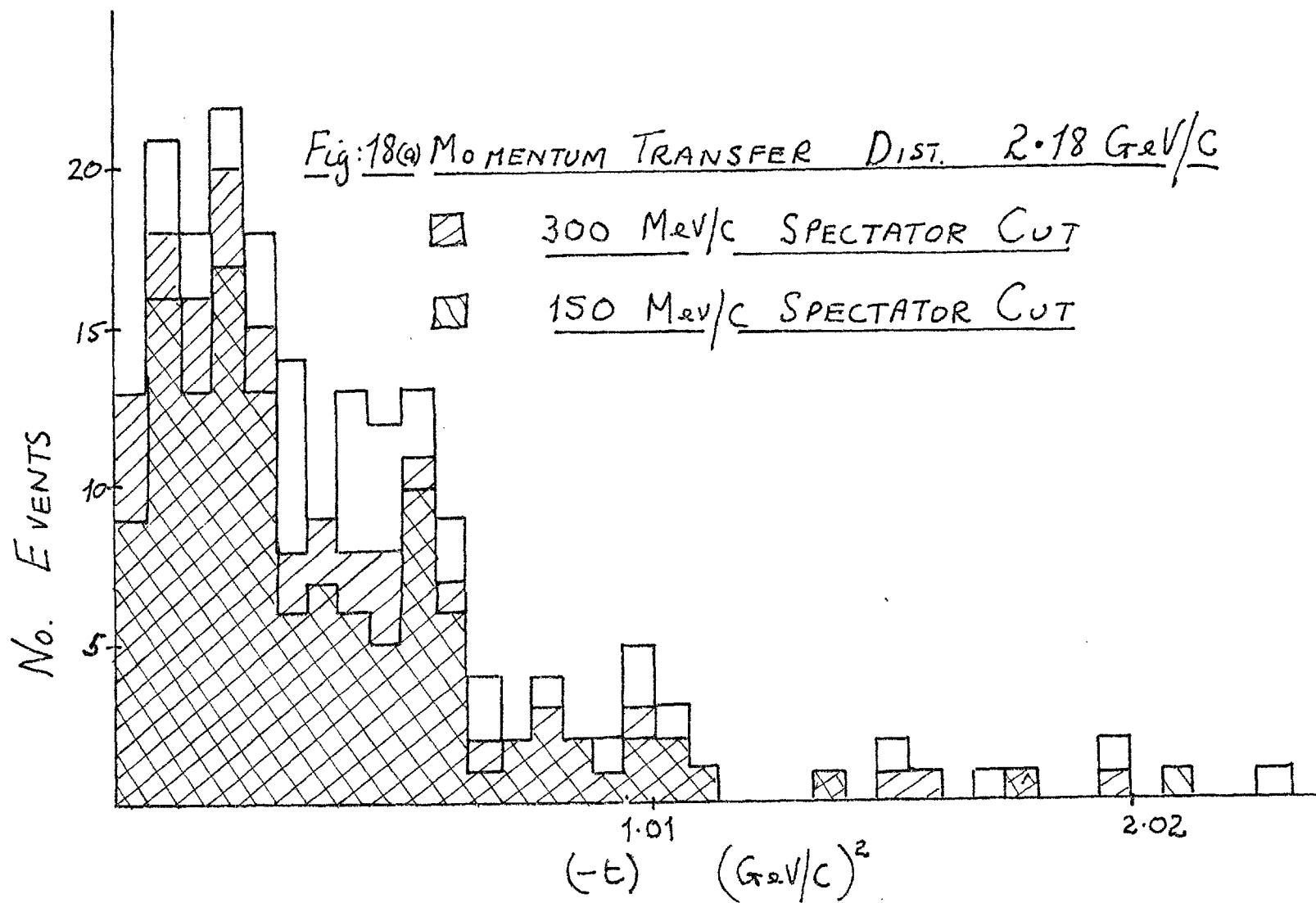
In fact, since the impulse approximation is, by its very nature, approximate, it is to be expected that its use will lead to second order disagreements.

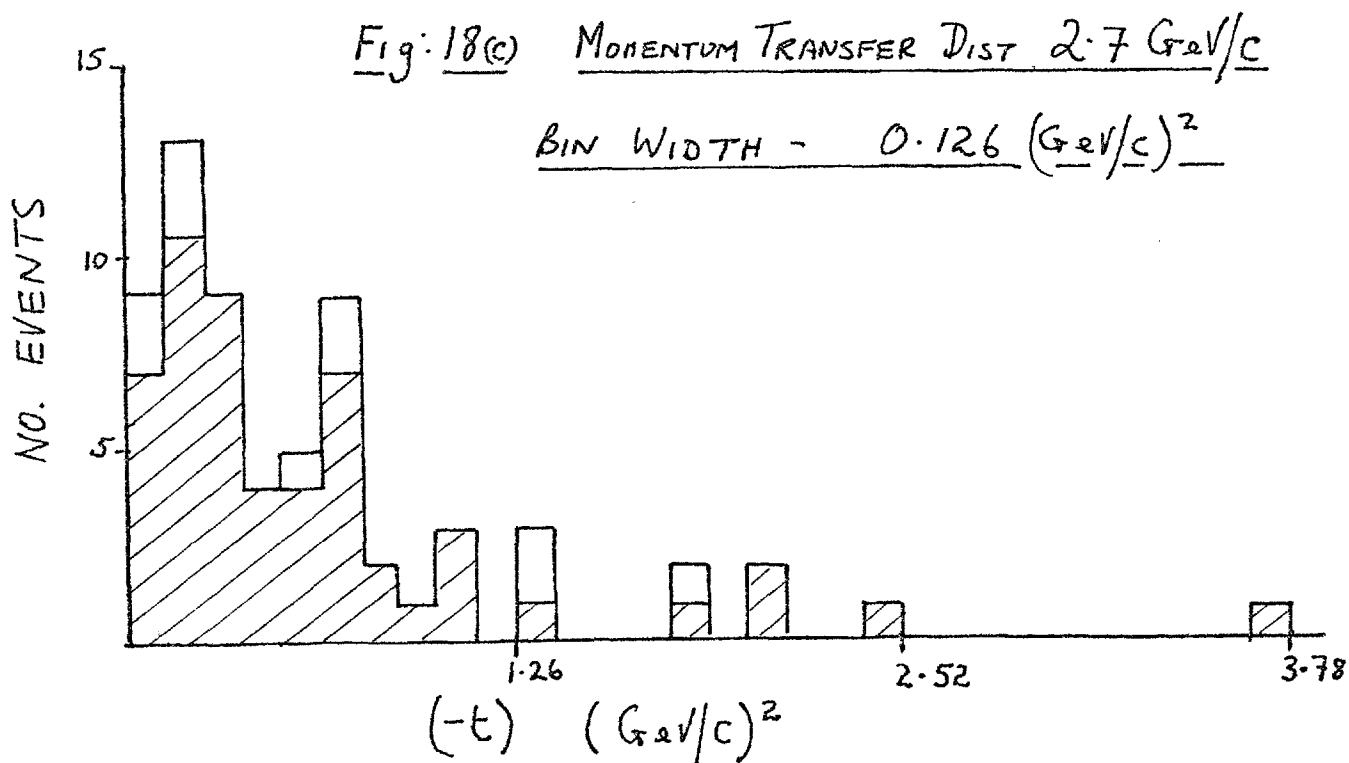
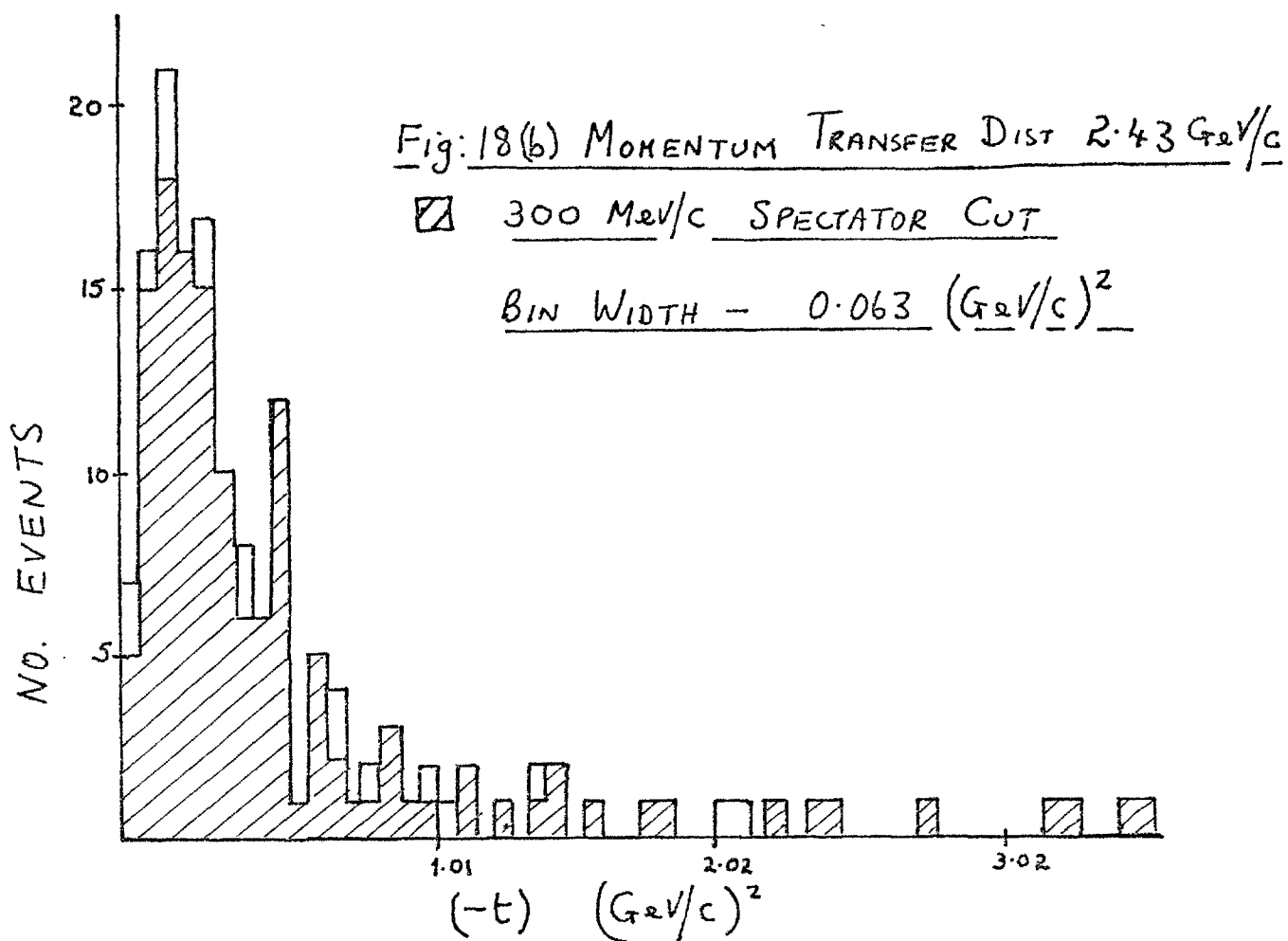
Having recognised this deficiency of the method, the problem remains of how to treat the high momentum spectator events. Some authors⁽³⁾ have included such events in both the cross section and angular distribution calculations. In the present work, the view has been taken that, since the high momentum spectators are probably due to double scattering, they should be excluded from the angular distribution. Nonetheless such events are included in the cross section calculation, as seems reasonable, the angular distribution being normalised to the uncut cross section.

Fig. 18 shows the effect of the spectator cut on the momentum transfer distribution at 2.18 GeV/c, 2.43 GeV/c and 2.7 GeV/c. Fig. 19 is a scatter plot of the laboratory momentum of the fast and slow protons in events in the elastic charge exchange channel. This serves to illustrate the extent to which true spectator events exist.

The angular distribution of spectators in the laboratory frame is shown in Fig. 20.

To a first approximation, this distribution should be flat, due to the random motion of nucleons inside the deuteron. There are, however, two factors which complicate the situation.





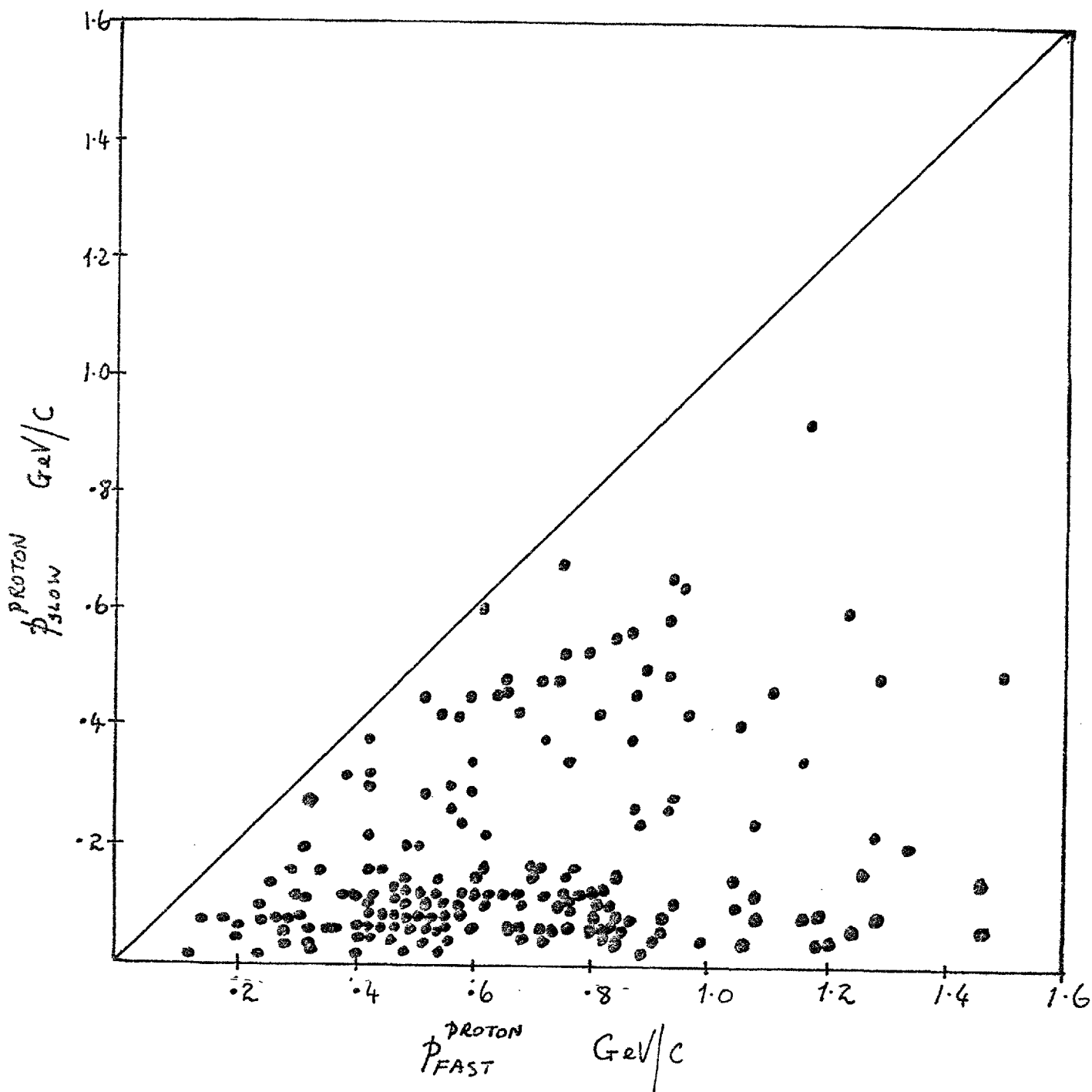


Fig 19: FAST - SLOW PROTON MOMENTUM

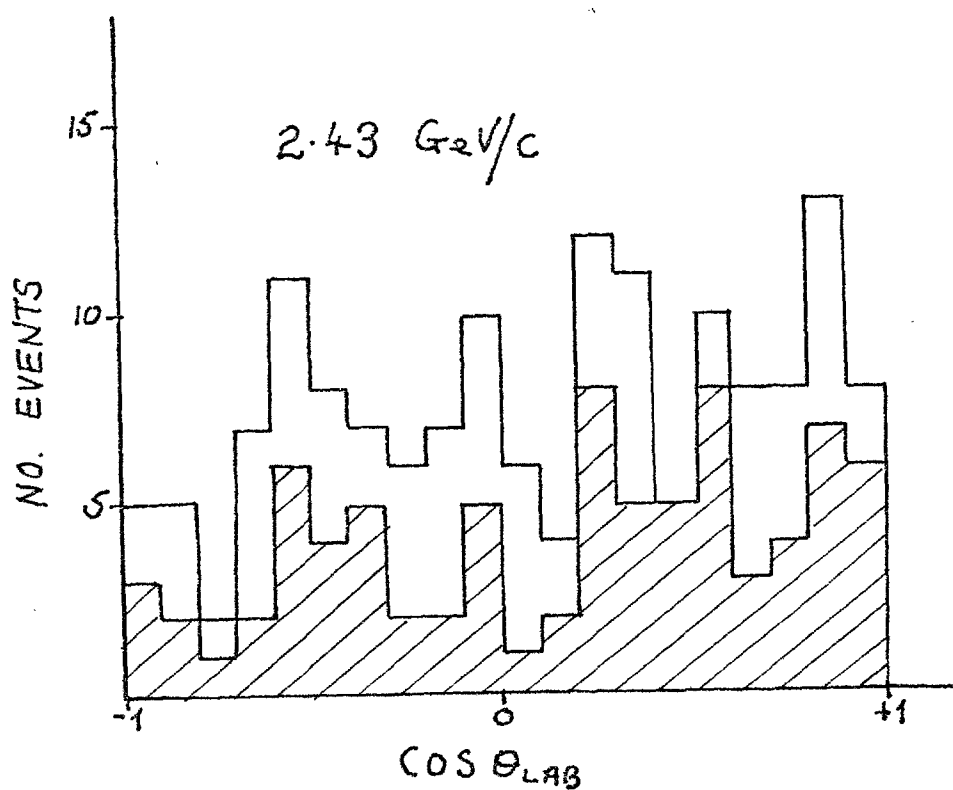
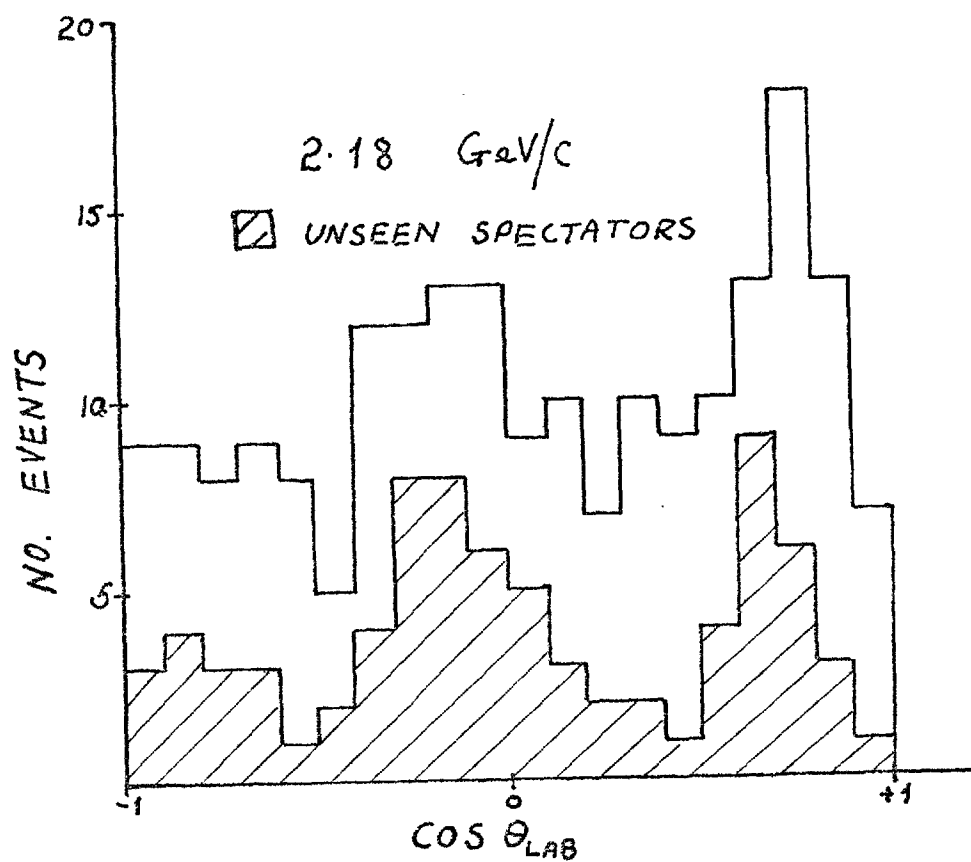


Fig 20: SPECTATOR ANGULAR DISTRIBUTIONS

The first is the fact that when the target nucleon is moving towards the beam, there is a higher particle flux than when it is moving away from the beam. This on its own would lead to an excess of events with spectators moving in the forward direction, as the spectator moves anti-parallel to the target before the collision. The full expression for this effect is⁽³⁾

$$W(\cos \theta_{sp}) \approx 1 + \left(\frac{\langle p \rangle}{\left(\frac{m_{nuc}}{m_K} \right) p_K} + 2 \left\langle \frac{p + B \frac{m_{nuc}}{m_K}}{p p_T} \right\rangle \right) \cos \theta_{sp} \quad (3.4)$$

where

$\langle p \rangle$ - Mean spectator momentum

p_K - Beam momentum

p_T - Target nucleon momentum

B - Deuteron binding energy.

The behaviour in (3.4) will be modified if the channel cross section is changing in the E^* region covered, since:

$$\text{number events} \approx \text{flux} \times \sigma_{\text{channel}}.$$

The charge exchange cross section is still falling in the region of this experiment, and this tends to give an excess of events with backward moving spectators. That is, a backward moving spectator implies a forward moving target and hence a lower E^* , with its attendant higher cross section.

The form of the angular distributions found [Fig. 20] indicate that the flux factor effect is dominant, but not significantly so.

3.4.2 Centre-of-Mass Energy Spread

The Fermi motion of the nucleons inside the deuteron has the effect of smearing out the E^* . For a hydrogen target a given beam momentum corresponds to a unique E^* . A deuterium target leads to a range of E^* for a single beam momentum. This makes it possible to examine the physical parameters of a process as a function of E^* . In fact the beam momenta for this experiment were chosen with this factor in mind. When the full data set, an order of magnitude greater than the present set, becomes available it will be useful to examine the charge exchange reaction in this way. However, the low statistics available here render any attempt at such analysis futile. Each beam momentum has been treated as a separate experiment, it being felt that this is a satisfactory procedure for extracting the broad features of the process. The E^* distributions at the two lower momenta are shown in Fig. 21, the smaller spread with the spectator cut is indicated.

3.4.3 Glauber Screening

Glauber⁽⁸⁾ has shown, by using the Eikonal approach, that the deuterium cross section is given, in terms of the free neutron and proton cross sections, by:

$$\sigma_{K^+d} = \sigma_{K^+p} + \sigma_{K^+n} - \Delta\sigma$$

where $\Delta\sigma$ is a cross section defect. In his treatment Glauber derived

$$\Delta\sigma = \frac{1}{4\pi} \left[\sigma_{K^+p} \cdot \sigma_{K^+n} \right] \langle r^{-2} \rangle \quad (3.5)$$

$\langle r^{+2} \rangle$ being the mean squared neutron-proton separation.

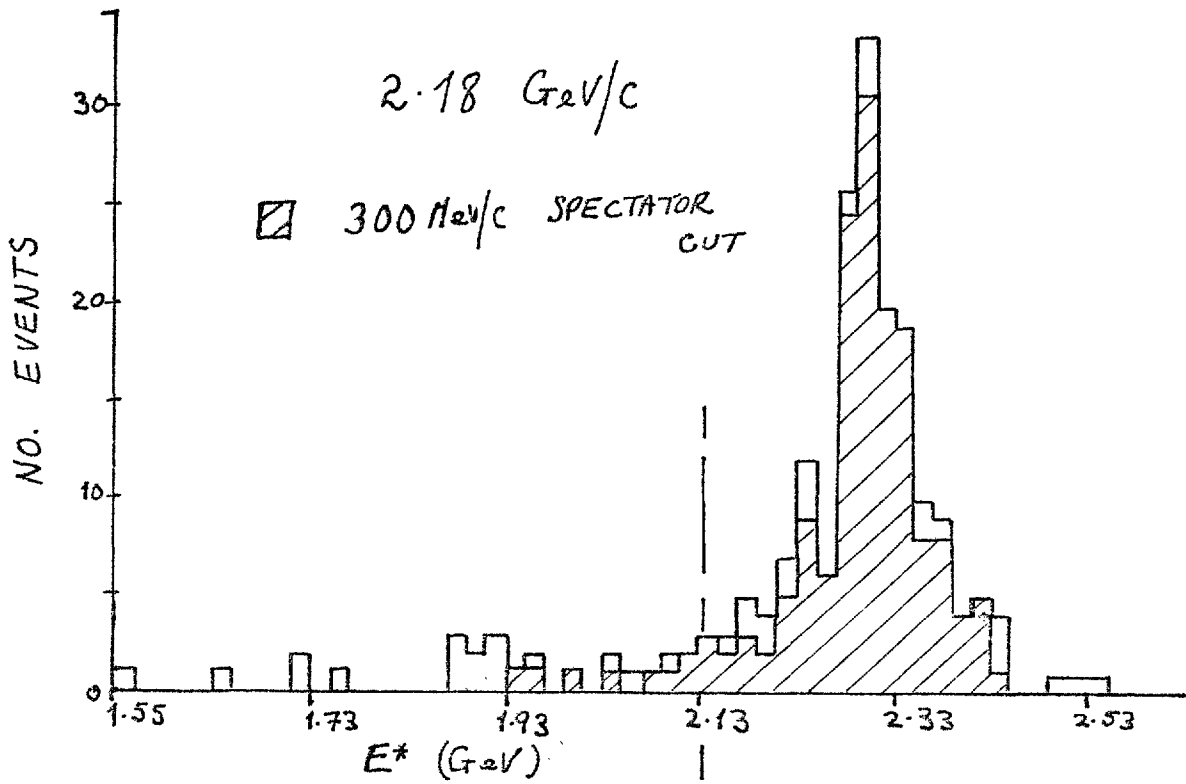
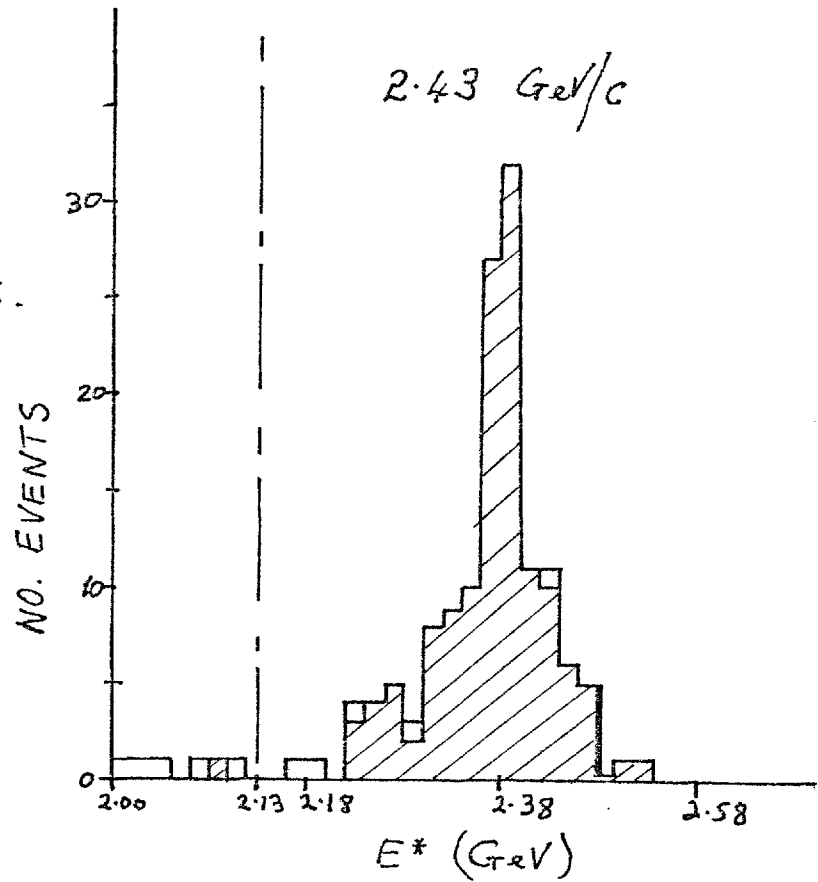


Fig. 21

CENTRE OF MASS
ENERGY DISTRIBUTIONS.



(3.5) was later shown⁽⁹⁾ to violate charge independence, a more correct expression being

$$\Delta \sigma = \frac{1}{4\pi} \left\{ (1 - \gamma_n \gamma_p) \sigma_{K^+n} \sigma_{K^+p} - \frac{1}{2} \left[(\sigma_{K^+n} - \sigma_{K^+p})^2 - (\gamma_n \sigma_{K^+n} - \gamma_p \sigma_{K^+p})^2 \right] \right\} \times \langle r^{-2} \rangle \quad (3.6)$$

where $\gamma_{\frac{n}{p}}$ is the ratio of the Real and Imaginary parts of the forward scattering amplitude. The defect, $\Delta \sigma$, is for the total cross section, and it is not clear how it should be apportioned amongst the different channels. The shielding effect basically involves multiple scattering effects, and such effects will be different for different channels. It is usual^(3,10) to assume that the defect is equally apportioned amongst all channels, and this assumption has been made in extracting the free neutron cross section presented here. Since this assumption results in some uncertainty in the unfolded cross section it has been considered sufficient to utilise the simpler expression (3.5) with $\langle r^{-2} \rangle$ taken as $0.03 \pm 0.01 \text{ mb}^{-1}$. With the full data set, it would be useful to consider final state scattering effects in greater detail, by looking for correlations between the two final state protons.

3.4.4 Pauli Exclusion Effects

A consideration of the charge exchange process in the limit of zero momentum transfer illustrates the implications of the Pauli exclusion Principle in this process.

If the interaction occurred with no flip of the target nucleon spin, this would result in two protons being close together in configuration space and both in a ^1S spin state. However, such a state (symmetric to particle exchange) is forbidden for two protons. Thus forward scattering must necessarily occur with nuclear spin flip. But angular momentum conservation requires that the spin flip amplitude vanish in the forward direction, so one expects the $K^+d \rightarrow K^0 pp$ differential cross section to vanish in the forward direction, irrespective of the behaviour of

$$K^+n \rightarrow K^0p \quad .$$

Using the Impulse and Closure approximation⁽¹¹⁾ it has been shown that:

$$\left(\frac{d\sigma}{dt}\right)_d = \left(\frac{d\sigma}{dt}\right)_n \frac{[1 - H + R(1-H/3)]}{[1 + R]} \quad (3.7)$$

where:

$$R(t) = \frac{\text{spin-flip} \left(\frac{d\sigma}{dt}\right)_n}{\left(\frac{d\sigma}{dt}\right)_n^{\text{non-flip}}} \quad (3.8)$$

and $H(t)$ is the deuteron form factor:

$$H(-t) = \int d^3 \vec{r} \quad e^{-i \vec{t} \cdot \vec{r}} \left/ \Psi_d^H(\vec{r}) \right/ ^2 .$$

Carrying out this integration, using the Hulthén wave function, gives⁽¹²⁾

$$H(-t) = \frac{\alpha\beta(\alpha+\beta)}{(\beta-\alpha)^2} \frac{2}{(-t)} \left[\tan^{-1}\left(\frac{-t}{2\alpha}\right) + \tan^{-1}\left(\frac{-t}{2\beta}\right) - 2 \tan^{-1}\left(\frac{-t}{\alpha+\beta}\right) \right] \quad (3.9)$$

The ratio $R(t)$ is unknown a priori and can only be got by the use of models fitted to the data. In correcting the unfolded cross section

for the Pauli effect $R(t)$ has been used as predicted by the Rarita-Schwartzchild Regge Pole model⁽¹³⁾. This model has deficiencies which are described in Chapter 5, but it does give reasonable fits to the differential cross section of this process. One can expect the Pauli correction derived from it to be a reasonable approximation.

The foregoing serves to illustrate the complexity resulting from the use of a deuterium target. Although the treatment here is approximate it is felt that this is justified by the small statistics available, the degree of uncertainty in how to treat some of the above effects (e.g. double scattering), and the fact that the main interest lies in the form of the differential cross section, rather than in a highly accurate cross section determination.

3.5 Cross Sections

In this section the cross sections for

$$K^+ d \rightarrow K^0 pp$$

and

$$K^+ n \rightarrow K^0 p$$

are derived at 2.18 GeV/c and 2.43 GeV/c. No cross section has been calculated at 2.7 GeV/c due to the lack of τ -decays in the conventional measurement from which the 2.7 GeV/c data is obtained.

The cross sections are displayed in Table 7. They are compatible with data from other experiments at nearby beam momenta^(14,15,16,17,18,19,20). The available data on the cross section of the reaction (up to $P_{\text{LAB}} = 5.5$ GeV/c) is shown in Fig. 22. This figure illustrates the rapid rise from threshold to a peak at ~ 800 MeV/c and subsequent fall off, which has been suggestive of resonance activity.

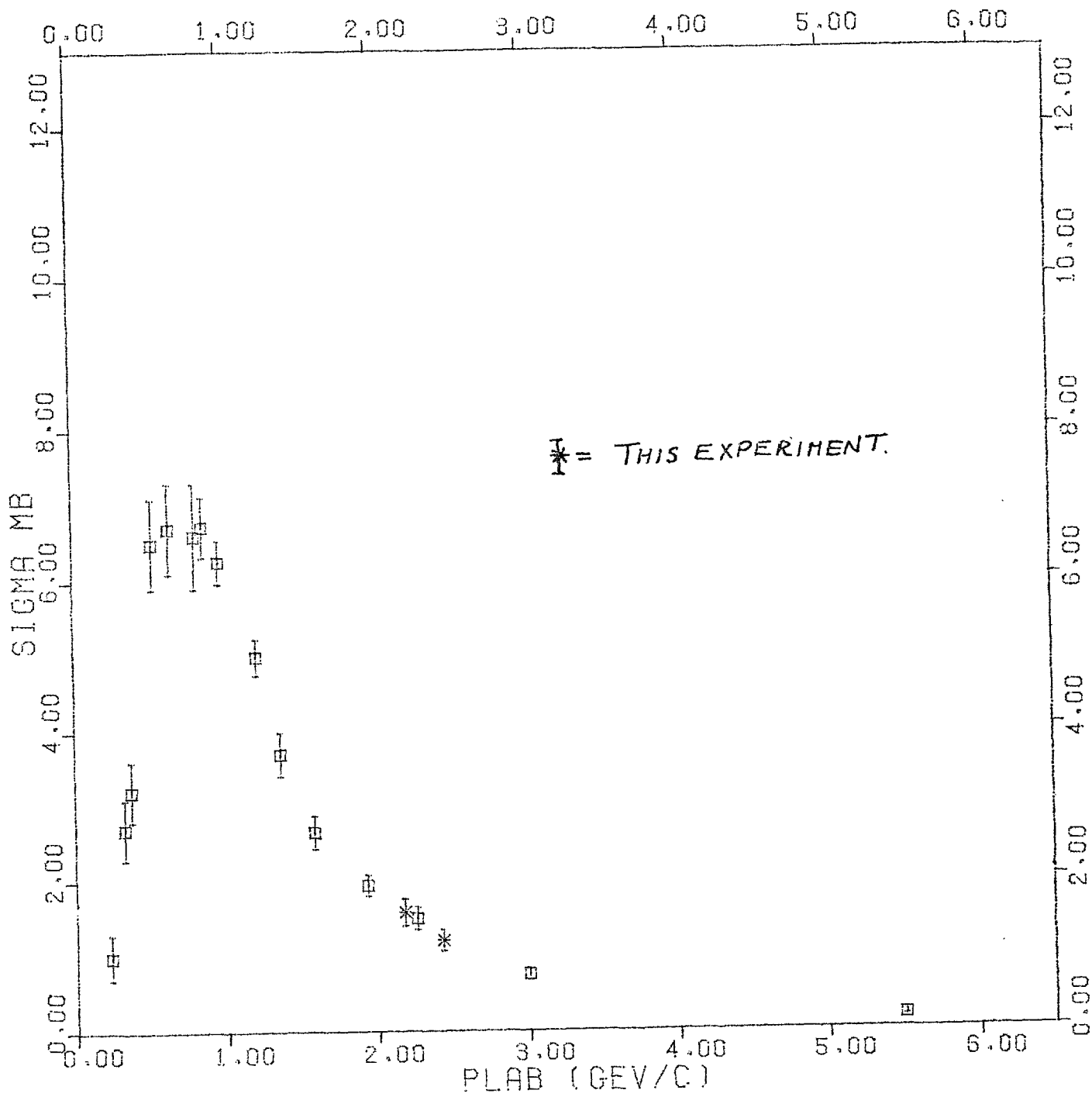


Fig: 22 $K^+d \rightarrow K^0 p p$ CROSS SECTION

The number of events observed to occur in a particular channel can be converted to a cross section in several ways.

For instance, if the total cross section has been accurately measured in a counter experiment, it is possible for a bubble chamber experiment (at the same beam momentum) to normalise the total cross section to the counter value. In the present case not all topologies were scanned for and this method was not available. Alternatively, it is possible to count the number of beam tracks and hence get the total K^+ path length; this procedure must be corrected for μ^+ beam contamination, and also π^+ contamination.

In this experiment the cross sections were normalised to the number of

$$K^+ \rightarrow \pi^+ \pi^+ \pi^- \quad \text{decays}$$

observed (τ -decays). The mean lifetime and branching ratio for this decay are well known, and these quantities together with the number of τ -decays enable the total K^+ path length to be calculated.

The mean distance travelled by a K^+ before decaying is:

$$c \beta \tau$$

reckoned in the K^+ rest frame where:

$$\tau = \text{mean lifetime}$$

$$\beta = \text{velocity in units } c = 1.$$

In the lab. the mean distance to decay is then

$$\gamma c \beta \tau = \lambda_{\tau} \quad (3.10)$$

γ is the γ in the Lorentz transformation.

λ_{τ} is essentially the "mean free path" against τ -decay.

So if N_{τ} τ -decays are observed then their mean separation will be λ_{τ} .

The total K^+ path length is then

$$N_{\tau} \lambda_{\tau} ,$$

if all K^+ decay as τ .

Correcting for the τ branching ratio (f) gives:

$$\text{Total } K^+ \text{ path length} = \frac{N_{\tau} \lambda_{\tau}}{f} = L . \quad (3.11)$$

From relativistic kinematics

$$\gamma\beta = \frac{p}{m_K}$$

p being the beam momentum, so

$$\lambda_{\tau} = \frac{p}{m_K} c \tau$$

$$\therefore L = N_{\tau} \frac{p}{m_K} \frac{c\tau}{f} = N_{\tau} \alpha . \quad (3.12)$$

If N_X interactions of type 'X' are observed, one can analogously define a mean free path for this reaction

$$\lambda_X = \frac{L}{N_X}$$

i.e.

$$\lambda_X = \frac{N_{\tau}}{N_X} \alpha .$$

But the cross section is defined by:

$$\sigma_X = \frac{1}{n \lambda_X}$$

where n is the number of scattering centres per unit volume.

$$n = \frac{A}{M} \cdot \rho$$

where:

M - Atomic weight of target

A - Avogadroes' number

ρ - Density of target

so

$$\sigma_X = \frac{1}{n} \cdot \frac{1}{\alpha} \frac{N_X}{N_T} \quad (3.13)$$

In calculating $\sigma_{K^+d \rightarrow K^0pp}$ (3.13) has been used, N_X , and N_T being corrected for estimated measuring efficiencies. The density of deuterium has been taken as 0.134 gm/cc at a chamber temperature of 32.4°K.

Fig. 23 shows all available data (references 3, and 14-20) compared with the dependence⁽¹⁰⁾

$$\sigma(p_{\text{beam}}) = A p_{\text{beam}}^{-n}$$

with

$$A = 7.3 \text{ mb}$$

$$n = 2.1.$$

3.6 Differential Cross Section

The main interest in examining K^+n elastic charge exchange lies in the differential cross section, as this can be compared with various phenomenological models. A full discussion of the interpretation of the differential cross section is given in the latter part of this thesis. In the present section the data obtained is examined.

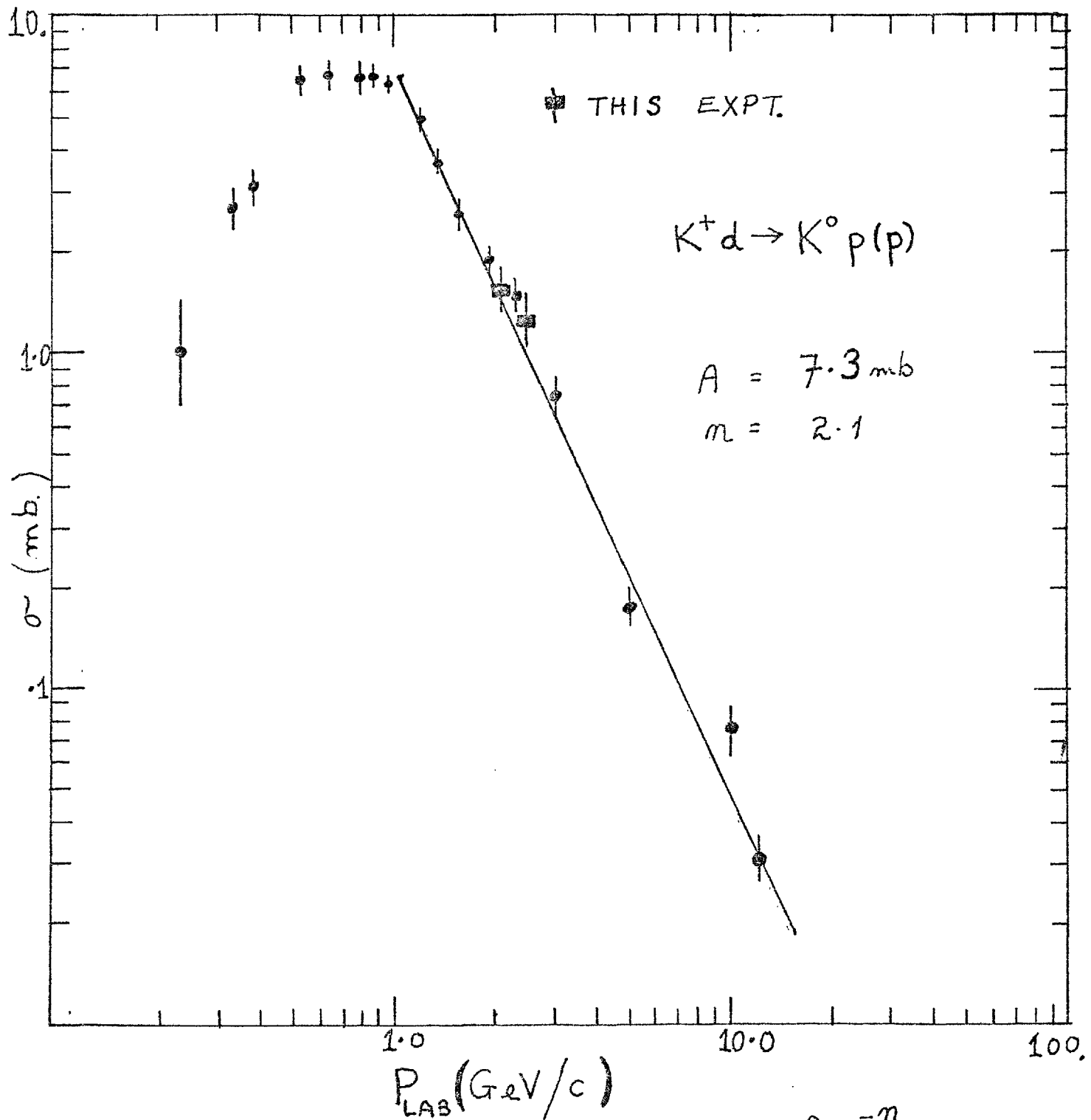


Fig 23: COMPARISON WITH $\sigma = A P_{\text{LAB}}^{-m}$

Fig. 24 shows $(d\sigma/dt)_{K^+_d \rightarrow K^0 pp}$ as determined at 2.18 GeV/c and 2.43 GeV/c. $(d\sigma/dt)$ was obtained from $(d\sigma/dN)$ using

$$\frac{d\sigma}{dt} = \frac{\sigma}{N} \frac{dN}{dt}$$

N being the number of events in the distribution.

A most prominent feature of both these momentum transfer distributions is the excess of events around $|t| \approx 0.6 \text{ (GeV/c)}^2$, the effect is more pronounced at 2.18 GeV/c than 2.43 GeV/c. This is to be compared with the smooth behaviour in, for example, Fig. 33, of a typical theoretical model. This structure does not exist in nearby data at 1.97 GeV/c⁽¹⁹⁾, or 3.0 GeV/c⁽¹⁷⁾. It does, however, appear to be present at 2.3 GeV/c⁽¹⁶⁾. Also the distributions of reference 3 at 0.87 GeV/c and 0.97 GeV/c are not as smooth as t-channel exchange models predict. There thus appears to be some suggestion of structure in the differential cross section. Since the very high (2000 events) experiment at 1.97 GeV/c shows no structure, it will be of extreme interest to see if the full statistics of the present experiment show any structure. Comparing dN/dt for all events with dN/dt excluding events with spectators $>300 \text{ MeV/c}$ (Fig. 18) indicates that the spectator cut reduces the significance of the structure. This may just be a result of diminishing statistics as a further cut at 150 MeV/c does not further reduce the structure. Which would appear to exclude the possibility of the structure being due to double scattering.

To further investigate this effect the events in the region $t = -0.5$ to -0.7 GeV/c and -0.4 to -0.6 GeV/c were examined and compared with the remainder. The following quantities were examined:

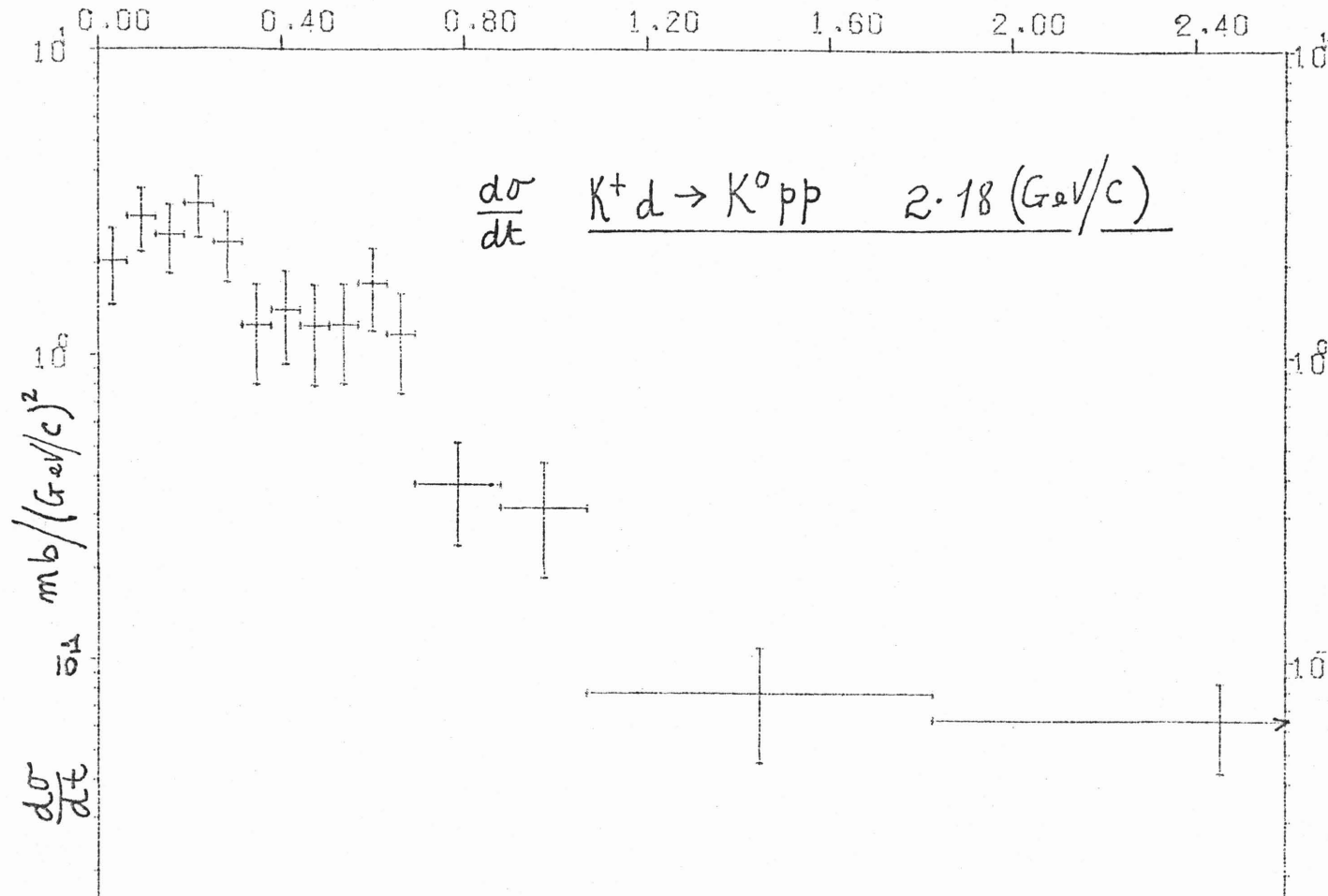


Fig: 24 (a)

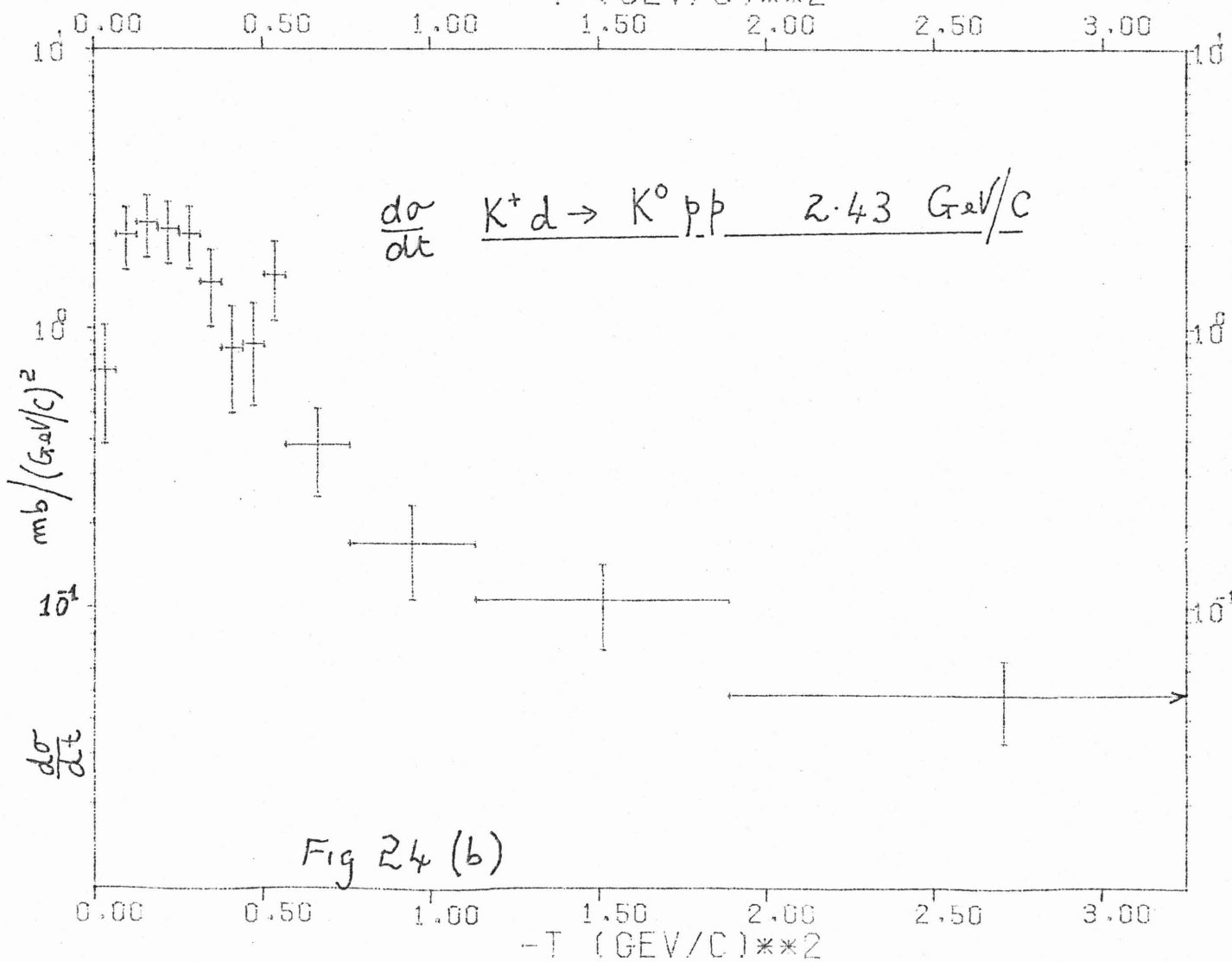


Fig 24 (b)

$\cos \theta_{\text{spect.}}^{\text{LAB}}$

E^*

Scan Bias Angle

p, λ, ϕ for all tracks

production vertex position

$p_{\text{spect.}}^{\text{LAB}}$

Degree of fit constraint

$(\text{Missing Mass})^2$

Confidence Level of fit.

None of these distributions showed any difference between the selected events and the whole sample. The level of statistics does not allow any firm conclusion to be drawn from this survey. It does indicate that the events in the selected t regions do not differ from those in the rest of the sample in any obvious way.

3.7 Legendre Polynomial Fits

A model independent investigation of the angular distribution has been carried out by expanding it as a sum of Legendre Polynomials;

$$\frac{dN}{d(\cos \theta^*)} = \sum_{\ell=0}^n A_{\ell} P_{\ell}(\cos \theta^*) \quad (3.14)$$

and performing a fit with the A_{ℓ} as free parameters.

The "Method of Moments" was used⁽²¹⁾. By invoking the orthonormality of the $P_{\ell}(\cos \theta^*)$;

$$\int_{-1}^{+1} P_{\ell} P_k d(\cos \theta^*) = \frac{2}{2\ell+1} \delta_{\ell k}$$

it is simple to show that:

$$A_{\ell} = \frac{2\ell+1}{2} \sum_{i=1}^N P_{\ell}(\cos \theta_i^*) \quad (3.15)$$

the index i running over the number of events in the distribution.

In particular

$$A_0 = \frac{1}{2} \sum_{i=1}^N P_0(\cos \theta_i^*) = \frac{N}{2}$$

The orthogonality of the $P_{\ell}(\cos \theta^*)$ is useful in that it implies that the n^{th} term of the series (3.14) will give a fit to the accuracy $(\cos \theta^*)^n$; increasing the number of terms in the series will increase the accuracy, but will not change the values of the individual A_{ℓ} .

The physical motivation for parametrising the angular distribution in this fashion lies in the fact that the coefficients A_{ℓ} are related to the partial waves contributing to the amplitude for the process. Thus structure in the s -dependence of the A_{ℓ} is often a reflection of structure in the s -channel partial waves.

A simple classical impact parameter argument⁽²¹⁾ indicates that coefficients up to A_7 would be expected to contribute at the beam momenta involved here.

In the present case, interest lies in the K^+n amplitude, rather than in the K^+d . The latter of necessity vanishes in the forward direction. Such behaviour requires high order coefficients which would have no significance for the K^+n amplitude. It has already been pointed out that to unfold the K^+n from K^+d angular distribution

requires a knowledge of the ratio of the spin flip to non-flip differential cross section. It is common to perform the fit making, in turn, the two extreme assumptions of all flip and all non-flip and then to state that the truth lies somewhere between⁽²²⁾. In the fits presented here the experimental distribution has been weighted according to the Regge Pole model of reference 13. Each event has been individually weighted, the weights being obtained by linear interpolation from a table derived from the Regge Pole model. The correction together with the deuteron form factor and the ratio $R(t)$ are shown in Fig. 25. The values of the coefficients obtained are shown in Table 8. Fig. 26 shows the s -dependence and includes nearby data⁽²³⁾. As can be seen the energy dependence is smooth. From these fits it is possible to deduce the forward $d\sigma/d\Omega$ at $\Theta=0$.

The optical theorem plus isospin relations enable one to show that

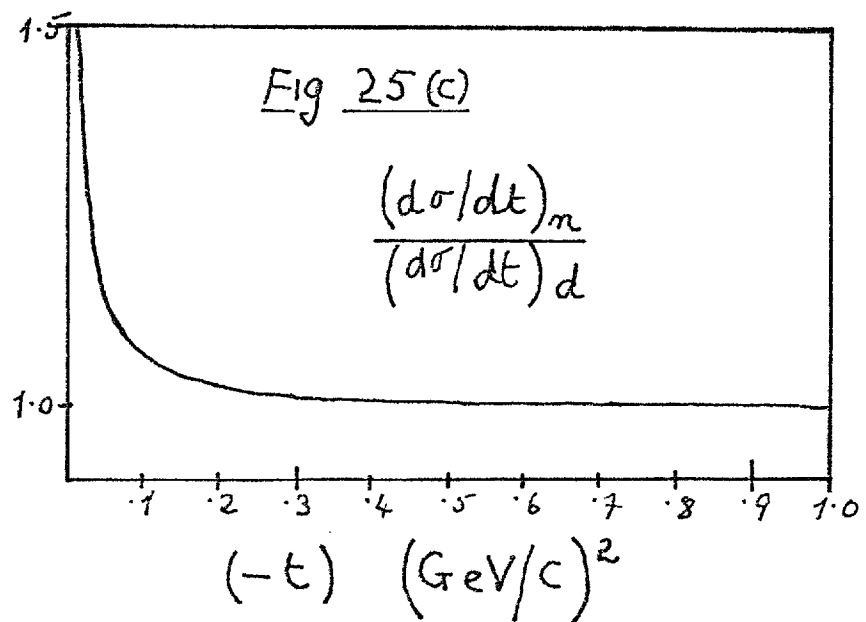
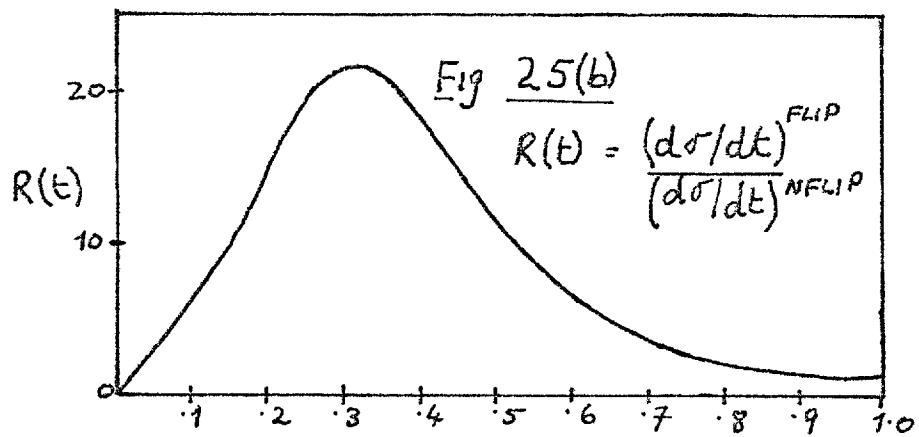
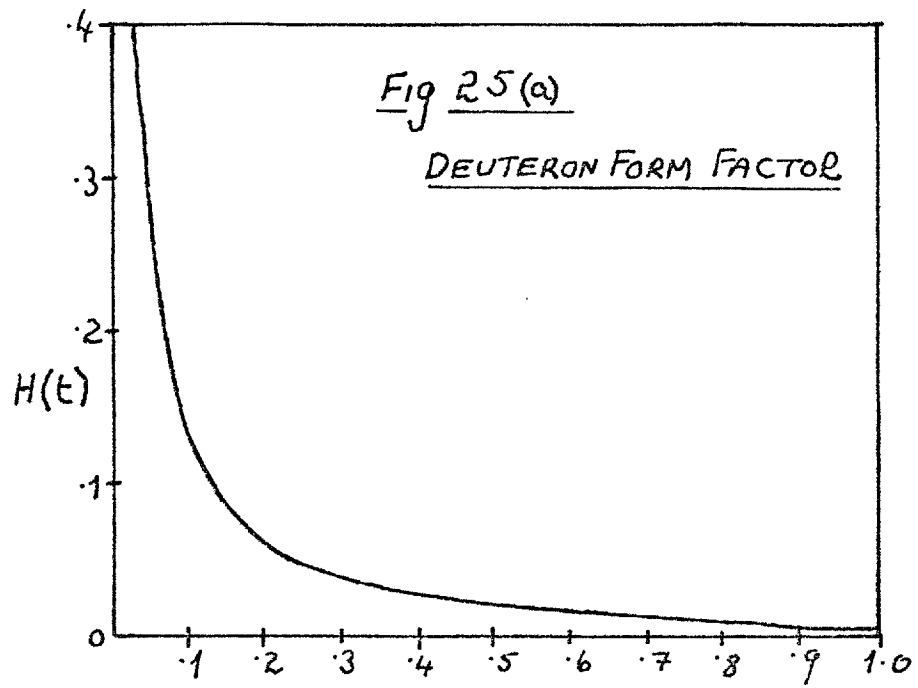
$$\left(\text{Im } A_{\text{CEX}} \right)_{\Theta=0} = \frac{q_{\text{CN}}}{4\pi} \left[\sigma_{\text{tot}}^{K^+p} - \sigma_{\text{tot}}^{K^+n} \right]$$

where A_{CEX} is the non-flip amplitude for $K^+n \rightarrow K^0p$.

This yields the inequality

$$\left(\frac{d\sigma}{d\Omega} \right)_{\Theta=0} \geq \frac{q_{\text{CN}}}{4\pi} \left[\sigma_{\text{tot}}^{K^+p} - \sigma_{\text{tot}}^{K^+n} \right] \quad (3.16)$$

The equality would hold if the amplitude for $K^+n \rightarrow K^0p$ were purely imaginary. Table 9 indicates that the amplitude is, in fact predominantly real.



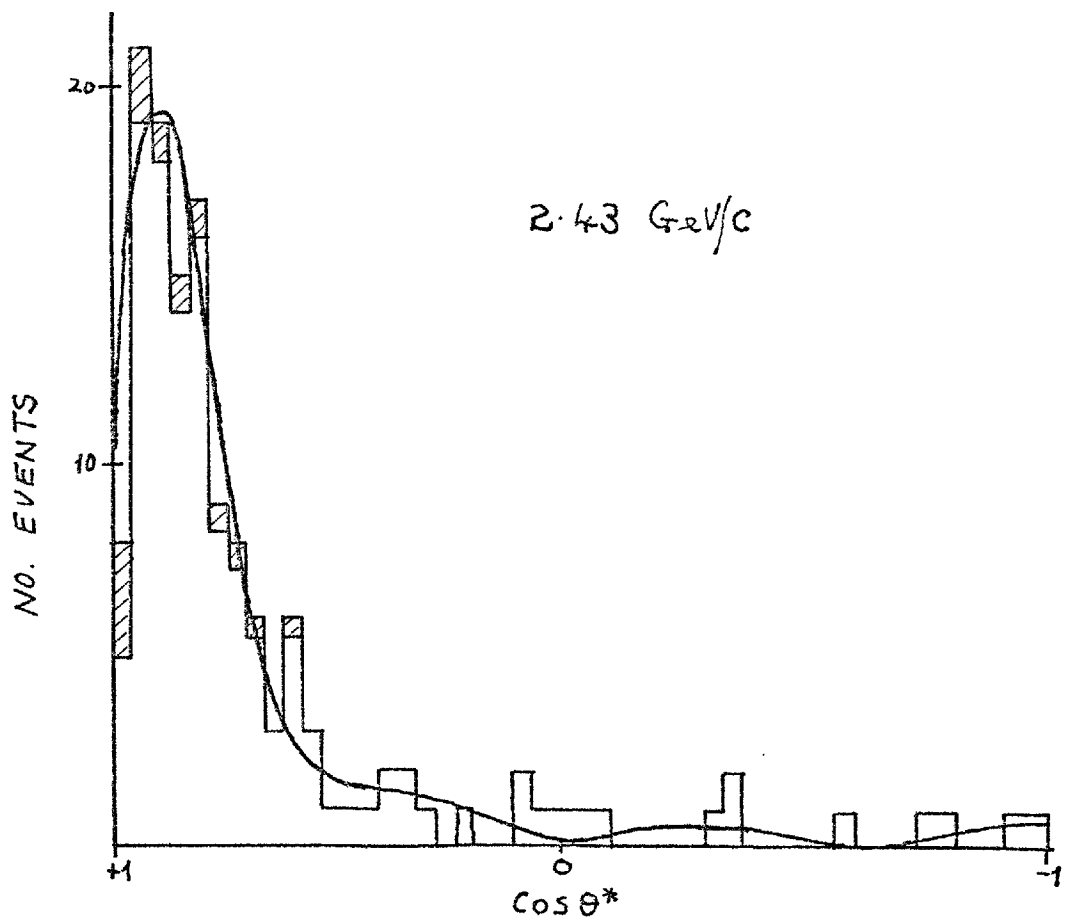
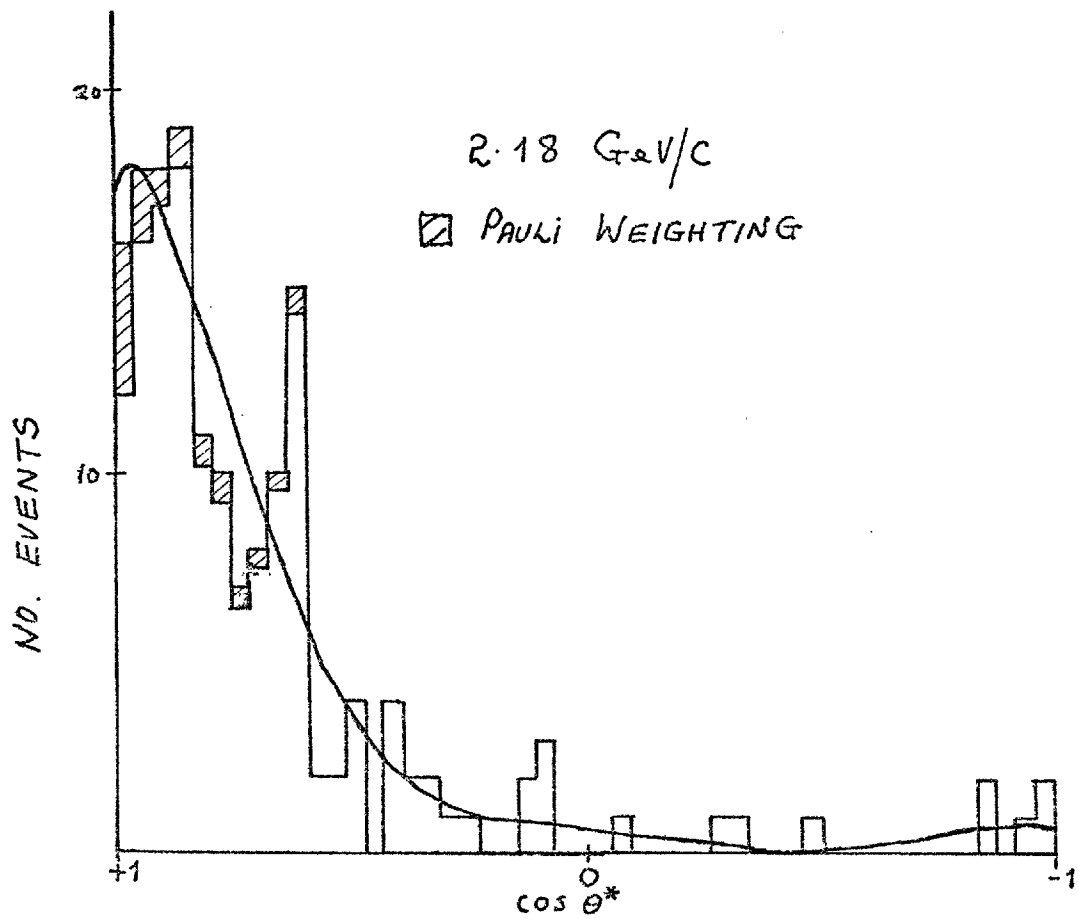


Fig 26(a) LEGENDRE POLYNOMIAL FITS

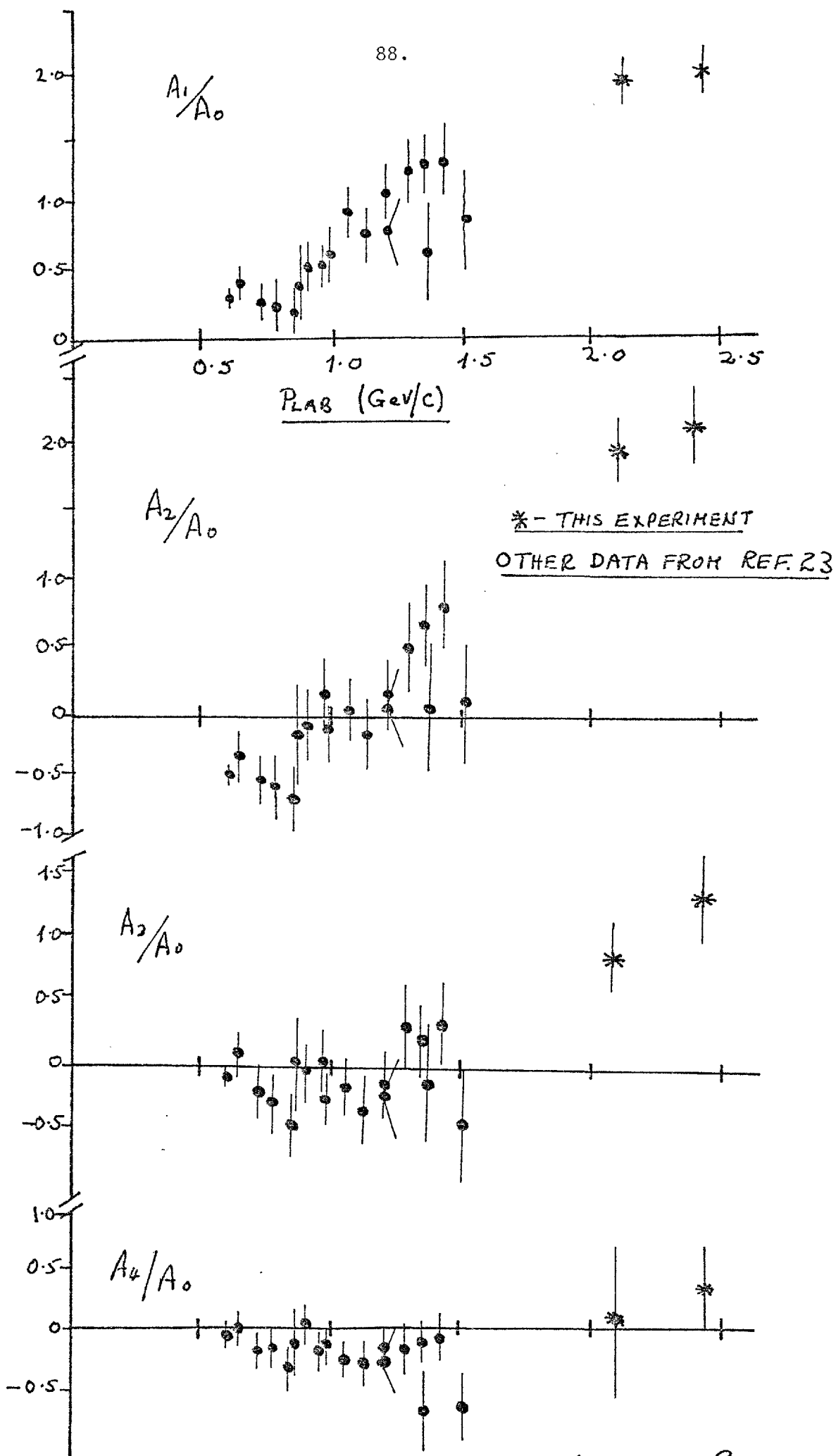


Fig: 26(b)

 P_{LAB} DEPENDANCE OF LEGENDRE COEFFICIENTS

Table 7.

Deuterium Cross Sections:

P_{beam} (GeV/c)	σ_{mb}
2.18	1.58 ± 0.18
2.43	1.22 ± 0.13

Unfolded Neutron Cross Sections:

P_{beam} (GeV/c)	σ_{mb}
2.18	1.68 ± 0.25
2.43	1.28 ± 0.19

Deuterium Corrections:

P_{beam} (GeV/c)	Glauber	Pauli
2.18	2.2% ($\sigma \times 1.02 \pm 0.09$)	4% ($\sigma \times 1.042 \pm .11$)
2.43	2.3% ($\sigma \times 1.02 \pm 0.08$)	4% ($\sigma \times 1.035 \pm .10$)

Table 8

Legendre Coefficients

$\begin{matrix} P_{\text{beam}} \\ A_n/A_0 \end{matrix}$	2.18	2.43
A_1/A_0	2.02 ± 0.09	2.08 ± 0.11
A_2/A_0	1.99 ± 0.16	2.21 ± 0.18
A_3/A_0	0.88 ± 0.24	1.39 ± 0.25
A_4/A_0	0.058 ± 0.27	0.34 ± 0.29
A_5/A_0	-0.55 ± 0.29	-0.71 ± 0.31
A_6/A_0	-0.68 ± 0.31	-1.37 ± 0.30
A_7/A_0	-0.37 ± 0.35	-1.46 ± 0.36

$\begin{matrix} P_{\text{BEAM}} \\ (\text{GeV}/c) \end{matrix}$	2.18	2.43
CONFIDENCE LEVEL	30%	10%

Table 9.

P_{beam}	Forward ($d\sigma/d\Omega$).	
	Predicted from σ_{tot}	From $K^+n \rightarrow K^0p$
2.18	0.004 ± 0.001	0.65 ± 0.02
2.43	0.003 ± 0.001	0.20 ± 0.03

Table 10 (a) $d\sigma/dt$ 2.18 GeV/c

$t \text{ (GeV/c)}^2$	$d\sigma/dt \text{ mb/GeV/c}^2$
0.032 ± 0.032	2.02 ± 0.57
0.095 ± 0.032	2.83 ± 0.67
0.158 ± 0.032	2.47 ± 0.63
0.221 ± 0.032	3.13 ± 0.71
0.284 ± 0.032	2.33 ± 0.61
0.347 ± 0.032	1.24 ± 0.45
0.410 ± 0.032	1.39 ± 0.47
0.473 ± 0.032	1.24 ± 0.45
0.536 ± 0.032	1.25 ± 0.45
0.599 ± 0.032	1.72 ± 0.52
0.662 ± 0.032	1.17 ± 0.42
0.788 ± 0.095	0.377 ± 0.140
0.977 ± 0.095	0.316 ± 0.130
1.45 ± 0.378	0.078 ± 0.032
2.46 ± 0.63	0.064 ± 0.021

Table 10 (b) $d\sigma/dt$ 2.43 GeV/c

$t \text{ (GeV/c)}^2$	$d\sigma/dt \text{ mb/GeV/c)}^2$
0.032 ± 0.032	0.71 ± 0.32
0.095 ± 0.032	2.17 ± 0.55
0.158 ± 0.032	2.39 ± 0.60
0.221 ± 0.032	2.27 ± 0.57
0.284 ± 0.032	2.18 ± 0.55
0.347 ± 0.032	1.46 ± 0.45
0.410 ± 0.032	0.85 ± 0.35
0.473 ± 0.032	0.88 ± 0.35
0.536 ± 0.032	1.56 ± 0.49
0.662 ± 0.095	0.384 ± 0.134
0.945 ± 0.189	0.170 ± 0.063
1.51 ± 0.378	0.107 ± 0.036
2.71 ± 0.82	0.049 ± 0.016

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CHAPTER 4

THEORETICAL CONSIDERATIONS

In the remainder of this thesis 3 theoretical models based on the exchange of t-channel Regge trajectories are examined, and fitted to the data on the $0^{-\frac{1}{2}+} \rightarrow 0^{-\frac{1}{2}+} \quad (J^P)$ two-body reactions

$$\begin{aligned} K^+ n &\rightarrow K^0 p \\ \bar{K}^- p &\rightarrow \bar{K}^0 n \\ \pi^- p &\rightarrow \pi^0 n \\ \pi^- p &\rightarrow \eta^0 n . \end{aligned}$$

Which, in terms of Regge theory, are dominated by ρ and A_2 exchange. Regge theory has a high degree of internal freedom in that many quantities involved in it are not a priori calculable. This feature has lead to many different models, embodying various assumptions, being fitted to the above two-body reactions. The 3 models examined here are representative of 3 broad classes, i.e. Pure Regge models, Regge models incorporating features of the Veneziano model, and Regge Pole + Absorptive cut models. In order to define the terminology used in later chapters and to give some necessary theoretical background a brief survey⁽¹⁾ of Regge Theory is given.

4.1 Background to Regge Theory

A basic observation of high energy particle interactions is that the forward or backward peaking, or non-peaking, of the differential cross section is related to the availability of particles with the quantum nos. appropriate for exchange in the crossed channel. This is

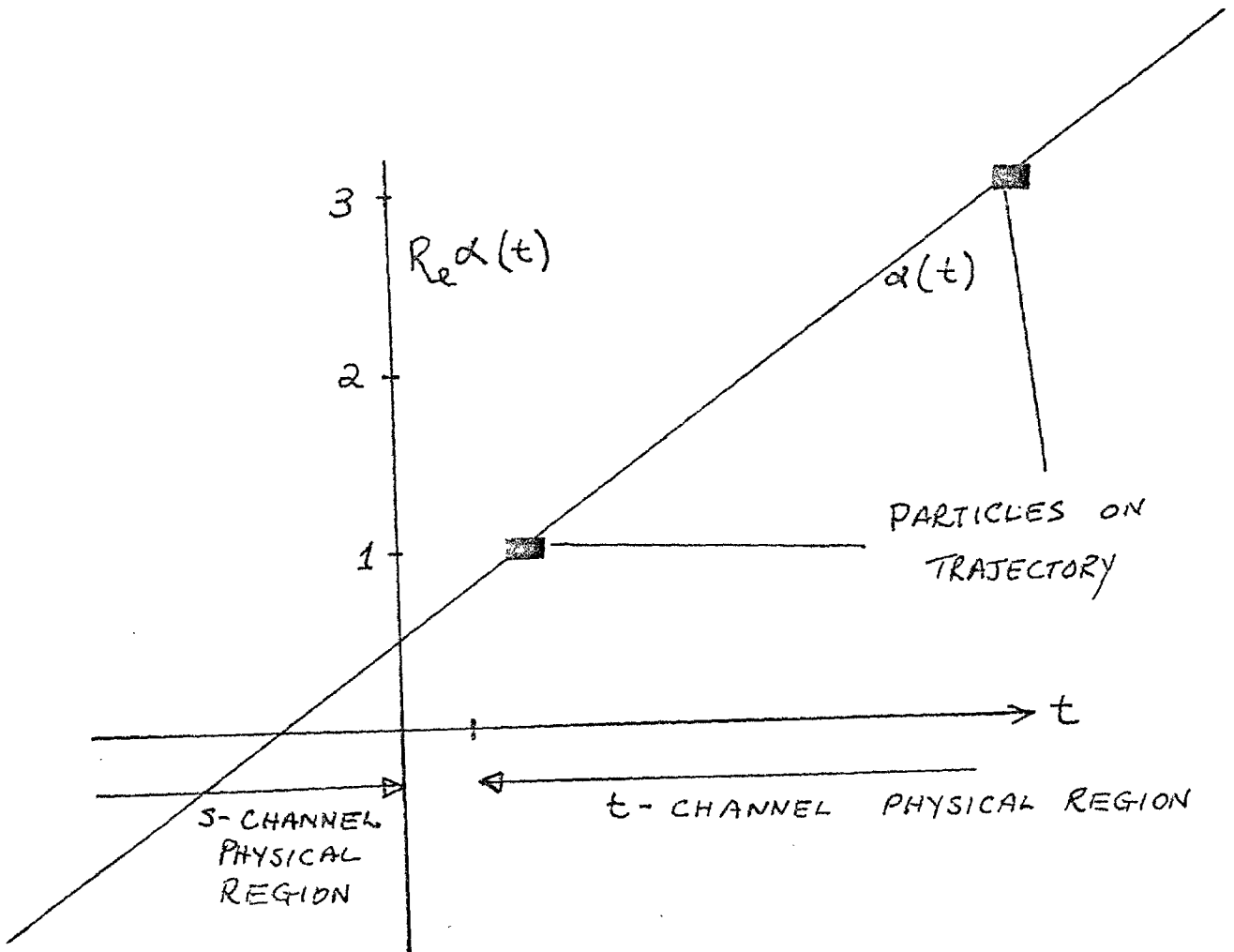
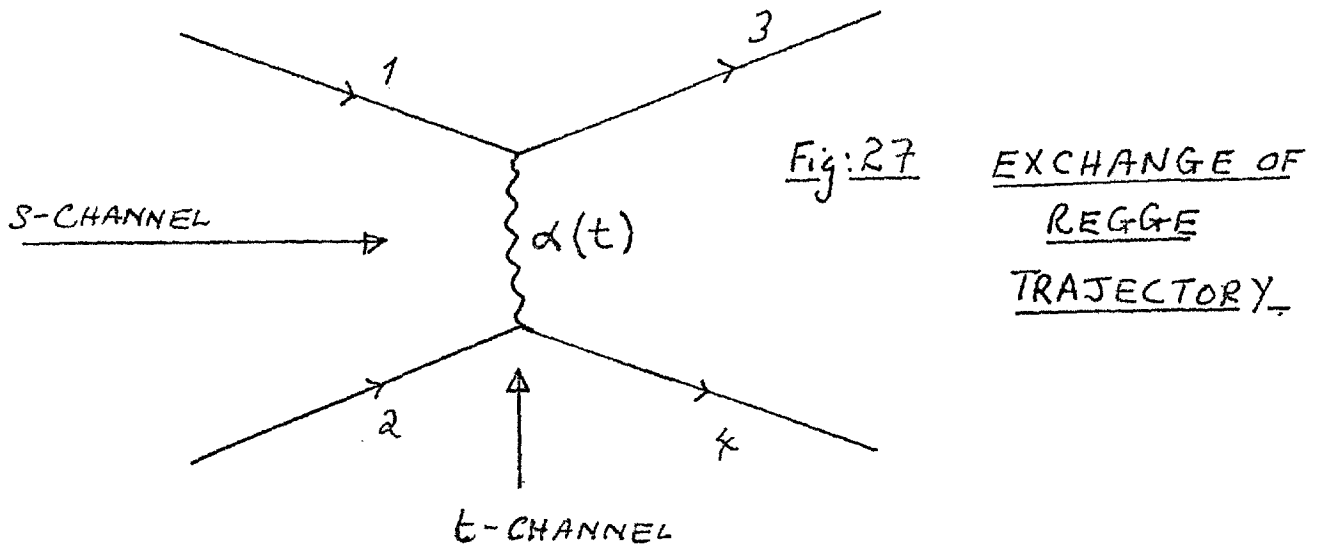


Fig: 28 CHEW-FRAUTSCHI DIAGRAM

immediately suggestive of an explanation of particle interactions at high energy in terms of particle exchange.

An early theory incorporating the concept of reactions being mediated by particle exchange was the One Particle Exchange model. This model has the deficiency⁽²⁾ that the differential cross section has the energy dependence

$$\frac{d\sigma}{dt} \propto s^{2j-2}.$$

j is spin of exchange particle. For $j > 2$ this diverges with increasing s . The Regge Pole Model overcomes this difficulty by postulating that particles of different spin, but the same internal quantum nos., can be grouped together. These groups of particles are postulated to be associated with a pole in the complex angular momentum plane. Such Regge Poles are then assumed to trace out trajectories in the complex angular momentum plane as a function of t , this trajectory being defined by a function $\alpha(t)$. Physical particles are then associated with +ve integral values of $\alpha(t)$; $\alpha(t)$ at such points being the "spin" of the particle. Particle reactions are assumed to be mediated by the exchange of such trajectories; however, in the scattering domain $\alpha(t)$ takes values such that the $d\sigma/dt$ does not $\rightarrow \infty$ with increasing s

$$\frac{d\sigma}{dt} \propto s^{2\alpha(t)-2}.$$

This can be seen from diagram Fig. 28 which assumes a linear rising trajectory. A diagram such as Fig. 28 is known as a Chew-Frautschi Plot.

4.2 Origin of Regge Pole Amplitude

In this section a brief sketch of the mathematical origin of the Regge Pole amplitude is given. The treatment of potential scattering is indicated and it is later shown how this is extended to the relativistic case.

4.2.1 Regge Poles in Potential Theory

A Regge Pole is formally defined as a pole in the analytic continuation of the partial wave scattering amplitude at unphysical (i.e. Complex) values of the angular momentum. In the non-relativistic potential case the trajectory is a function of the energy E ; $\alpha(E)$.

The normal partial wave expansion, for spinless particles, is

$$f(E, \cos \theta) = \frac{1}{q} \sum_{\ell=0}^{\infty} (2\ell + 1) a_{\ell}(E) P_{\ell}(\cos \theta) \quad (4.1.)$$

E and q are the centre-of-mass energy and momentum, θ is the centre-of-mass scattering angle. $P_{\ell}(\cos \theta)$ is a Legendre function; this is no longer a polynomial for non-integral ' ℓ ', but can be expressed in terms of a product of Γ functions and a hypergeometric series. In this form $P_{\ell}(\cos \theta)$ has a branch cut running from $-1 \rightarrow -\infty$. $a_{\ell}(E)$ is a partial wave amplitude, it can be expressed as:

$$a_{\ell}(\cos \theta) = \frac{1}{2i} \left[e^{2i\delta} - 1 \right] \quad (4.2)$$

δ_{ℓ} is the phase shift; also

$$a_{\ell}(\cos \theta) = \frac{1}{2} \int_{-1}^{+1} q f(E, \cos \theta) P_{\ell}(\cos \theta) d(\cos \theta) \quad (4.3)$$

The central point of the Regge approach is to assume that there is a function $a(\ell, E)$, where ℓ is a continuous complex variable, such that;

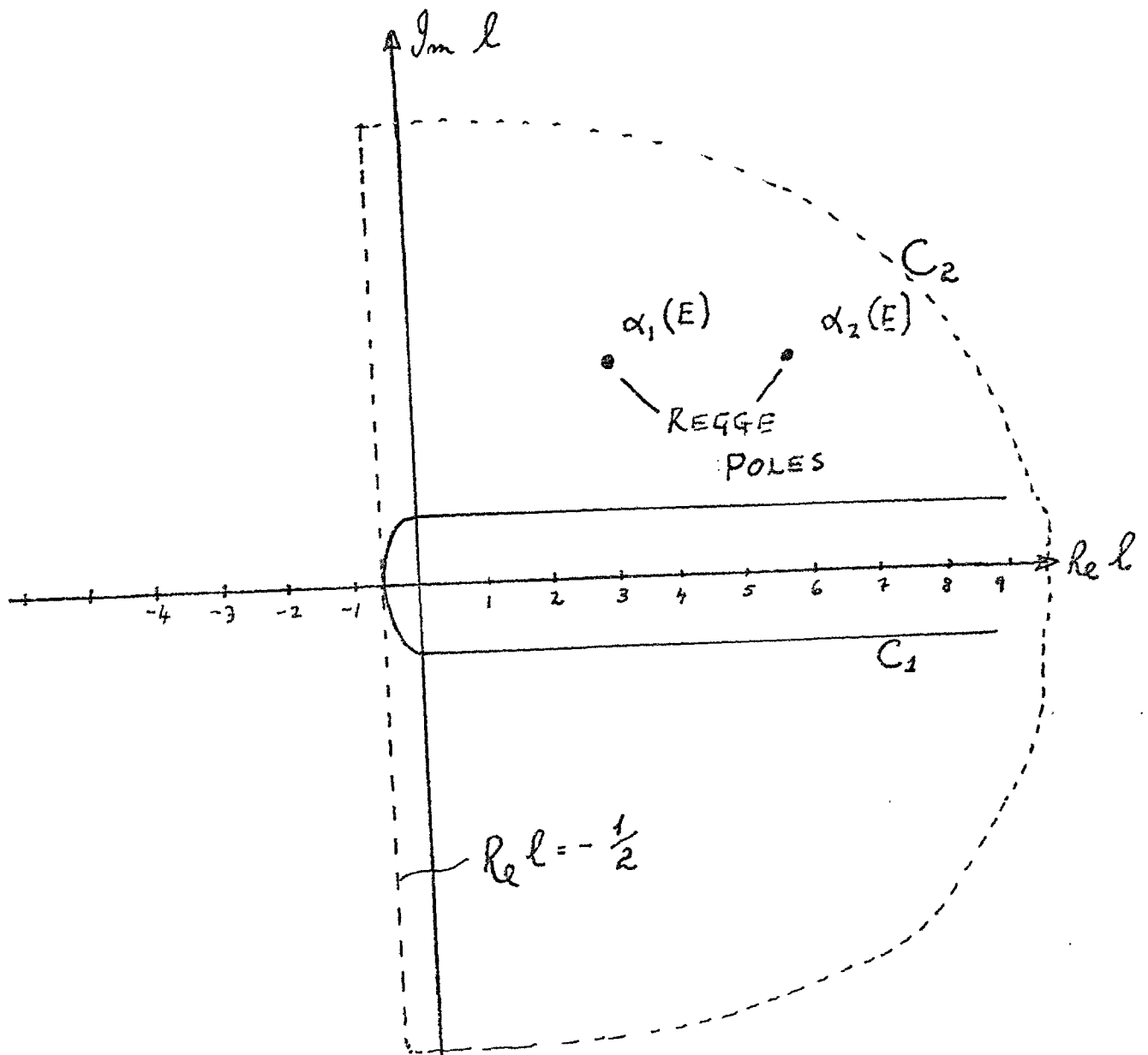


Fig 29: INTEGRATION CONTOURS IN
SOMMERFELD - WATSON TRANSFORM.

$$a(\ell, E) \equiv a_\ell(E) \quad \text{for } \ell = 0, 1, 2, \dots$$

One can then apply the Sommerfeld-Watson transform to (4.1). to give

$$f(E, \cos \Theta) = \frac{i}{2q} \int_{C_1} \frac{(2\ell+1)}{\sin \pi \ell} \cdot a(\ell, E) P_\ell(-\cos \Theta) d(\cos \Theta) \quad (4.4)$$

The integration is around the counter C_1 shown in Fig. 29.

For a potential consisting of a superposition of Yukawa terms:-

$$V(r) = \sum_n g_n \frac{e^{-\mu_n r}}{r} \quad (4.5)$$

Regge proved the following:-

i) The continued function $a(\ell, E)$ contains a finite number of singularities, these singularities being poles whose position in the complex ℓ -plane is a function of E . For $E < 0$ the trajectory of such a pole is purely real. For $E > 0$ the trajectory lies in the region $\text{Im } \alpha(E) > 0$ and ultimately curves back beyond $\text{Re } \alpha(E) < -\frac{1}{2}$ with increasing E .

ii) If the contour of integration C_2 is used in the Sommerfeld-Watson transform, the contribution from the infinite semi-circle vanishes. The vanishing of this contribution is a requirement for the uniqueness of the analytic continuation $a_\ell(E) \rightarrow a(\ell, E)$ (Carlson's Theorem).

Using this latter fact one obtains:-

$$\frac{i}{2q} \int_{C_2} \frac{(2\ell+1)}{\sin \pi \ell} a(\ell, E) P_\ell(-\cos \Theta) d\ell = f(E, \cos \Theta) + \frac{\pi}{q} \sum_i \frac{(2\alpha_i(E) + 1)}{\sin \pi \alpha_i(E)} \times \beta_i(E) P_{\alpha_i}(-\cos \Theta) \quad (4.6)$$

$\beta_i(E)$ being the Residue of the i^{th} Regge Pole. Then

$$f(E, \cos \Theta) = \frac{-\pi}{q} \sum_i \frac{(2\alpha_i(E)+1)}{\sin \pi \alpha_i(E)} \beta_i(E) P_{\alpha_i}(-\cos \Theta) \\ + \frac{i}{2q} \int_{-\frac{1}{2}-i\infty}^{-\frac{1}{2}+i\infty} \frac{(2\ell+1)}{\sin \pi \ell} a(\ell, E) P_{\ell}(-\cos \Theta) d\ell \quad (4.7)$$

(4.7) is central to the Regge model and shows that the scattering amplitude can be expressed as a sum over Regge Pole residues + a background integral. It can be shown that at a physical value of ℓ in (4.7) the contribution from a Regge Pole has a Breit-Wigner form; which suggests that physical values of ℓ can be identified with resonances.

Now suppose that $\cos \Theta$ is allowed to increase towards ∞ . Since $P_{\ell}(\cos \Theta) \sim \cos \Theta^{\ell}$ the normal partial wave expansion (4.1) diverges, and is thus meaningless. (4.7), however, defines the amplitude for just such asymptotic values of $\cos \Theta$. For $\cos \Theta \rightarrow \infty$ the Regge Pole with the largest Real part (highest on Chew-Frautschi plot) will dominate, while the background integral $\rightarrow 0$. For the relativistic case crossing implies that the unphysical limit in one channel corresponds to physical scattering in the crossed channel. So if (4.7) defines a t-channel amplitude it may be that its unphysical limit will govern scattering in the s-channel, i.e. s-channel scattering will be governed by the "exchange" of t-channel Regge Trajectories.

Some important features of the extension to Relativistic scattering are now considered.

4.2.2 Regge Poles in Relativistic Scattering

Regge Poles in potential theory do not yield any new information. Nonetheless the asymptotic behaviour suggests applications in the study of relativistic strong interactions. The vital expression (4.7) depends on the fact that there is a dynamical theory for potential scattering viz. the Schrödinger equation. No such dynamical theory exists for strong interactions and it is not at present possible to show that hadron scattering amplitudes contain Regge Poles. However the Mandelstam representation of the amplitude;

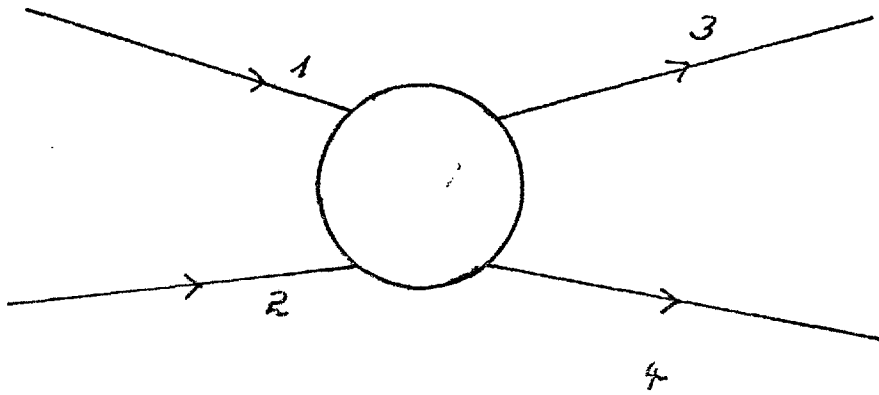
$$\begin{aligned}
 A(s, t, u) = & \frac{1}{\pi} \int \frac{ds' \rho_s(s')}{(s-s')} \\
 & + \frac{1}{\pi} \int \frac{dt' \rho_t(t')}{(t-t')} \\
 & + \frac{1}{\pi} \int \frac{du' \rho_u(u')}{(u-u')} \\
 & + \frac{1}{\pi^2} \iint ds' dt' \frac{\rho_{st}(s', t')}{(s-s')(t-t')}
 \end{aligned}$$

+ similar terms in $\rho_{su} + \rho_{tu}$, suggests that there is a unique amplitude that describes a process and its related processes in the cross channels, in the appropriate physical regions. The central hypothesis of the Regge Model of strong interactions is that such an amplitude exists and contains Regge Poles.

The Mandelstam variables for equal mass scattering are defined in Fig. 30. Then in the t-channel

$$s = -2 q_t^2 (1 + \cos \theta_t)$$

(q_t is CM. 3-momentum) and hence;



$$s = (p_1 + p_2)^2 = (p_3 + p_4)^2$$

$$t = (p_1 - p_3)^2 = (p_2 - p_4)^2$$

$$u = (p_1 - p_4)^2 = (p_2 - p_3)^2$$

$$s + t + u = \sum_i m_i^2$$

Fig 30: DEFINITION OF
MANDELSTAM VARIABLES

$$\cos \theta_t = -1 + \frac{2s}{4m^2 - t} \quad (4.8)$$

If in (4.8) $\cos \theta_t \rightarrow \infty$ with t -ve then one moves into the s -channel physical region with $s \rightarrow \infty$. Crossing symmetry implies that the t -channel amplitude in this limit will govern high energy s -channel scattering. So the Regge Model consists of writing the t -channel partial wave expansion;

$$f^t(t, \cos \theta_t) = \frac{1}{q_t} \sum_{\ell=0}^{\infty} (2\ell+1) a_{\ell}^t(t) P_{\ell}(\cos \theta_t)$$

and obtaining a Regge expansion, assuming that $a^t(\ell, t)$ contains

Regge Poles

$$f^t(t, \cos \theta_t) = \frac{-\pi}{q_t} \sum_i \frac{2\alpha_i(t)+1}{\sin \pi \alpha_i(t)} \beta_i(t) P_{\alpha_i}(-\cos \theta_t) \quad (4.9)$$

+ background integral .

Then the unphysical limit of (4.9) as $\cos \theta_t \rightarrow \infty$ enables one to describe high energy scattering in the s -channel in terms of t -channel Regge Poles.

4.2.3 Signature

The property of signature of Regge trajectories is an implication of the relativistic case and is due to the existence of the u -channel. It is clear that it is possible to reach the s -channel by taking the unphysical limit of the u -channel, instead of the t -channel. For the equal mass, spinless, case the u and t channels are related by

$$\begin{aligned} t &\longleftrightarrow u \\ \cos \theta_t &\longleftrightarrow -\cos \theta_u \end{aligned} \quad (4.10)$$

To include u-channel effects, as well as t-channel, the partial wave amplitude becomes:

$$a_{\ell}(t) = \frac{q_t}{2} \int_{-1}^{+1} [f^t(t, \cos \theta_t) + f^u(u, \cos \theta_u)] \times P_{\ell}(\cos \theta_t) d(\cos \theta_t)$$

cf. (4.3). Then using (4.10),

$$a_{\ell}(t) = \frac{q_t}{2} \int_{-1}^{+1} [f^t(t, \cos \theta_t) + f^u(t, -\cos \theta_t)] \times P_{\ell}(\cos \theta_t) d(\cos \theta_t) .$$

This can reasonably be written as:

$$a_{\ell}(t) = b_{\ell}(t) + (-1)^{\ell} c_{\ell}(t)$$

At this point ℓ is still a discrete number, no analytic continuation has been made. Now the factor $(-1)^{\ell} \equiv e^{+i\pi\ell}$ oscillates at ∞ . This violates the requirement that the infinite semi-circle in Fig. 29 vanishes at ∞ . So $a_{\ell}(t)$ cannot be usefully analytically continued. This is overcome by separating the partial wave amplitude, $a_{\ell}(t)$, into parts for even and odd ℓ , and continuing these two parts separately.

$$\begin{aligned} a_{\ell}^{+} & \quad (\ell \text{ even}) \\ a_{\ell}^{-} & \quad (\ell \text{ odd}) . \end{aligned}$$

Poles in a_{ℓ}^{+} now have no a priori connection with poles in a_{ℓ}^{-} . The form of the partial wave expansion suitable for applying the Sommerfeld-Watson transform to is now;

$$f^t(t, \cos \theta_t) \equiv f_t^{+}(t, \cos \theta_t) + f_t^{-}(t, \cos \theta_t)$$

where

$$f_t^{\pm}(t, \cos \theta_t) = \frac{1}{q_t} \sum_{\ell=0}^{\infty} (2\ell + 1) a_{\ell}^{\pm}(\ell, t) \times [P_{\ell}(\cos \theta_t) \pm P_{\ell}(-\cos \theta_t)] / 2 . \quad (4.11)$$

4.2.4 Unphysical Limit of t-Channel Amplitude

Various approximations are used for the unphysical limit of the t-channel amplitude, those used in the models examined later are detailed here.

The final step in obtaining a relativistic Regge expansion is the performing of the Sommerfeld-Watson transform on (4.11) giving

$$f^t(\cos \theta_t, t) = \frac{-\pi}{q_t} \sum_i \frac{2\alpha_i(t)+1}{2} \beta_i(t) \times \left\{ \frac{P_{\alpha_i(t)}(-\cos \theta_t) \pm P_{\alpha_i(t)}(\cos \theta_t)}{\sin \pi \alpha_i(t)} \right\} \quad (4.12)$$

+ background integral

cf. (4.7). Taking (4.12) to the limit $\cos \theta_t \rightarrow \infty$ one hopes that one pole will dominate and that the background integral will fall off.

As noted before, $P_\alpha(\cos \theta_t)$ has a branch cut extending from $\cos \theta_t = -1 \rightarrow \cos \theta_t = -\infty$, such that:

$$P_\alpha(-\cos \theta_t \pm i\epsilon) \Big|_{\cos \theta_t > +1} = e^{\pm i \pi \alpha} P_\alpha(\cos \theta_t) - \frac{2}{\pi} \sin \pi \alpha Q_\alpha(\cos \theta_t) \quad (4.13)$$

where $Q_\alpha(\cos \theta_t)$ is a Legendre function of the second kind.

Adopting the convention that the physical amplitude corresponds to approaching the Real energy axis from positive imaginary values;

$$\cos \theta_t = -1 + \frac{2(s+i\epsilon)}{4m^2 - t}.$$

Applying this to (4.13) yields; $P_\alpha(-\cos \theta_t) = e^{-i \pi \alpha} P_\alpha(\cos \theta_t).$

The Legendre function, $Q_\alpha(\cos \theta_t)$, has been dropped as it diverges as $\cos \theta \rightarrow \infty$.

In the same limit,

$$P_{\alpha}(\cos \Theta_t) \sim \frac{2^{\alpha}}{\sqrt{\pi}} \frac{\Gamma(\alpha + \frac{1}{2})}{\Gamma(\alpha + 1)} (\cos \Theta)^{\alpha}$$

Since $\cos \Theta_t = 2(s + t/2 - \frac{1}{2} \sum_i m_i^2)$;

where m_i are the external masses, one can approximate this by,

$$P_{\alpha}(\cos \Theta_t) \sim \frac{2^{\alpha}}{\sqrt{\pi}} \frac{\Gamma(\alpha + \frac{1}{2})}{\Gamma(\alpha + 1)} (s/s_0)^{\alpha}$$

where s_0 is a scaling factor. Then the Regge amplitude for a process dominated by a single pole at high s is;

$$f_t^{\pm}(\cos \Theta_t, t) = -\sqrt{\pi} \left[\frac{2\alpha(t)+1}{2} \right] \cdot \beta(t) \left(\frac{1 + e^{-i\pi\alpha}}{\sin \pi\alpha} \right) \times 2^{\alpha} \frac{\Gamma(\alpha + \frac{1}{2})}{\Gamma(\alpha + 1)} (s/s_0)^{\alpha(t)} \quad (4.14)$$

The Residue function $\beta(t)$ is completely undetermined, and various factors are often absorbed into it.

The cut in $P_{\alpha}(\cos \Theta_t)$ restricts the range of t over which the Regge expansion (4.14) is valid. This can be overcome⁽³⁾ by extending the integration contour, in Fig. 29, out past $\text{Re } \alpha < -\frac{1}{2}$ and using

$$M_{\alpha}(\cos \Theta_t) = \frac{\tan \pi \alpha}{\pi} Q_{-\alpha-1}(\cos \Theta_t) \text{ in place of } P_{\alpha}(\cos \Theta_t), \text{ for}$$

$\text{Re } \alpha(t) < 0$. At $\text{Re } \alpha(t) = 0$, $M_{\alpha}(\cos \Theta_t) \equiv P_{\alpha}(\cos \Theta_t)$ thus the

integration can be extended throughout the complex plane by using

the appropriate function in the appropriate region. Further instead of approximating

$$P_{\alpha}(\cos \Theta_t) \sim (\cos \Theta_t)^{\alpha} \sim (s/s_0)^{\alpha}$$

it is possible to explicitly calculate $P_{\alpha}(\cos \Theta_t)$ and $M_{\alpha}(\cos \Theta_t)$.

This approach is used in one of the models examined in this thesis.

4.2.5 Helicity and Invariant Amplitudes

Because of complexities introduced by spin the simple partial wave amplitudes are not normally used in Regge Models. It is more usual to employ either Helicity amplitudes, or invariant amplitudes.

Helicity Amplitudes:

The helicity of a particle, λ , is defined as the projection of its spin $\vec{S}$ on its direction of motion:

$$\lambda = \frac{\vec{p} \cdot \vec{S}}{|\vec{p}|}$$

The discussion here is restricted to a spin $\frac{1}{2}$ particle scattering from a spin 0 particle as in,

$$0^{-\frac{1}{2}+} \rightarrow 0^{-\frac{1}{2}+} .$$

In this case there are two normal "partial wave" amplitudes, f_1 and f_2 .

The differential cross section can be expressed in terms of a 2×2 scattering matrix, M ,

$$\frac{d\sigma}{d\Omega} = \sum_{\alpha, \beta} |\langle \beta | M | \alpha \rangle|^2$$

$|\beta\rangle$ - final state

$|\alpha\rangle$ - initial state.

The T-matrix is defined by

$$\langle \beta | M | \alpha \rangle = \frac{m}{4\pi\sqrt{s}} \bar{u}_\beta T_{\beta\alpha} u_\alpha \quad (4.15)$$

where the u are Dirac spinors.

Then in terms of the amplitudes f_1 and f_2 ,

$$M = f_1(s,t) + (\vec{\sigma} \cdot \hat{q}_2) (\vec{\sigma} \cdot \hat{q}_1) f_2(s,t)$$

$\vec{\sigma}$ is the Pauli matrix and $\hat{q}_{1,2}$ are unit vectors along the initial and final meson 3-momenta, in the centre of mass.

Of 4 possible helicity amplitudes only 2 are independent, corresponding to helicity non-flip and flip. These are defined by,

$$\begin{aligned} \text{Non-flip} \quad F_{++} &= F_{--} = (f_1 + f_2) \cos \theta/2 \\ \text{Flip} \quad F_{+-} &= -F_{-+} = (f_1 - f_2) \sin \theta/2 . \end{aligned}$$

In terms of the helicity amplitudes, the physically interesting quantities are:

1) Unpolarised $d\sigma/dt$.

$$\frac{d\sigma}{dt} = \frac{1}{64\pi s^2 q_{1,2}^2} \cdot \frac{1}{(2S_1+1)} \cdot \frac{1}{(2S_2+1)} \sum_{\lambda} |F_{\lambda}(s,t)|^2$$

or for the particular case

$$\begin{aligned} & 0^{-\frac{1}{2}} \rightarrow 0^{-\frac{1}{2}} \\ \frac{d\sigma}{dt} &= \frac{1}{64\pi s^2 q_{1,2}^2} \cdot \frac{1}{2} \left\{ |F_{++}|^2 + |F_{+-}|^2 \right\} \end{aligned} \quad (4.16)$$

since

$$S_1 = \frac{1}{2} - \text{nucleon}$$

$$S_2 = 0 - \text{nucleon} .$$

2) Polarisation.

$$P(t) = \frac{2 \operatorname{Im} (F_{++} F_{+-}^*)}{|F_{++}|^2 + |F_{+-}|^2} \quad (4.17)$$

The helicity amplitudes used can be either s-channel or t-channel.

The form of the t-channel helicity amplitudes is:-

$$F_{\lambda\lambda'}^t(s,t) = -16 \pi^2 [2\alpha(t) + 1] \beta(t) \left\{ \frac{1 \pm e^{-i\pi\alpha}}{\sin \pi\alpha(t)} \right\} \\ \times d_{\lambda\lambda'}^\alpha(\cos \theta_t)$$

where the rotation function $d_{\lambda\lambda'}^\alpha(\cos \theta_t)$ has replaced the Legendre function $P_\alpha(\cos \theta_t)$. The $\cos \theta_t \rightarrow \infty$ limit is taken in a similar fashion to that described above.

Invariant Amplitudes:

The scattering amplitude $T_{\alpha\beta}$ as defined in (4.15) can, for a parity conserving process, be expressed in terms of two invariant amplitudes $A'(s,t)$ and $B(s,t)$;

$$T_{\alpha\beta} = A'(s,t) - \frac{1}{2} i B(s,t) \gamma_\mu(q_1+q_2)^\mu.$$

The invariant amplitudes are useful in that they have simple analytic properties, e.g. they satisfy the Mandelstam representation. Their use also overcomes problems involved with kinematic singularities in t . The Reggeized forms are:

$$A(s,t) = (2\alpha(t) + 1) \beta(t) \left[\frac{1 \pm e^{-i\pi\alpha}}{\sin \pi\alpha} \right] (s/s_0)^\alpha$$

$$B(s,t) = (2\alpha(t) + 1) \beta(t) \left[\frac{1 \pm e^{-i\pi\alpha}}{\sin \pi\alpha} \right] (s/s_0)^{\alpha-1}$$

The physically interesting quantities are given by⁽⁴⁾

$$\frac{d\sigma}{dt}(s,t) = \frac{1}{\pi} \left(\frac{1}{4p^2} \right) \left[\left(1 - \frac{t}{4m^2} \right) |A|^2 - \frac{t}{4m^2} \left(\frac{s+p^2}{1 - \frac{t}{4m^2}} - s \right) |B|^2 \right] \quad (4.18)$$

$$\begin{aligned}
 P(t) \frac{d\sigma}{dt}(s, t) &= \frac{1}{\pi} \left(\frac{1}{8p^2} \right) \left[1 - \frac{t}{4m^2} \right]^{\frac{1}{2}} \\
 &\times \left[\left(\frac{s+p^2}{1 - \frac{t}{4m^2}} \right) \frac{(-t)}{4m^2} \right]^{\frac{1}{2}} \\
 &\times [\operatorname{Re}(A) \operatorname{Im}(B) - \operatorname{Re}(B) \operatorname{Im}(A)]
 \end{aligned} \tag{4.19}$$

p = lab momentum if incoming meson.

m = nucleon mass

$$A = A' + \frac{(E_L + \frac{t}{4m})}{(1 - \frac{t}{4m^2})} \quad B ,$$

(where E_L = energy in lab of incoming meson) has been introduced to remove terms in $A'B$.

The following expressions relate the invariant amps to the helicity amps .

$$f_1 = \frac{(E_L + m)}{8\pi w} [A + (w - m) B]$$

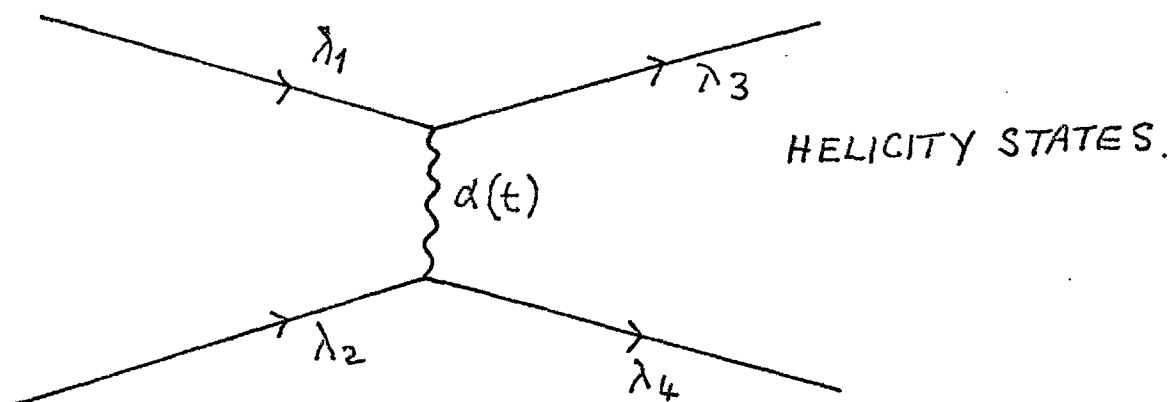
$$f_2 = \frac{(E_L - m)}{8\pi w} [-A + (w + m) B]$$

$$w = \sqrt{s} .$$

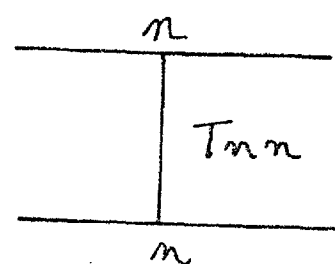
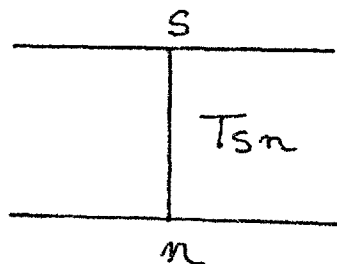
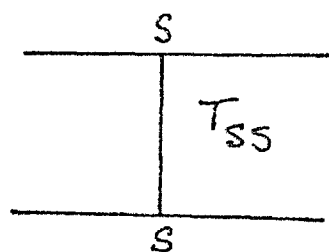
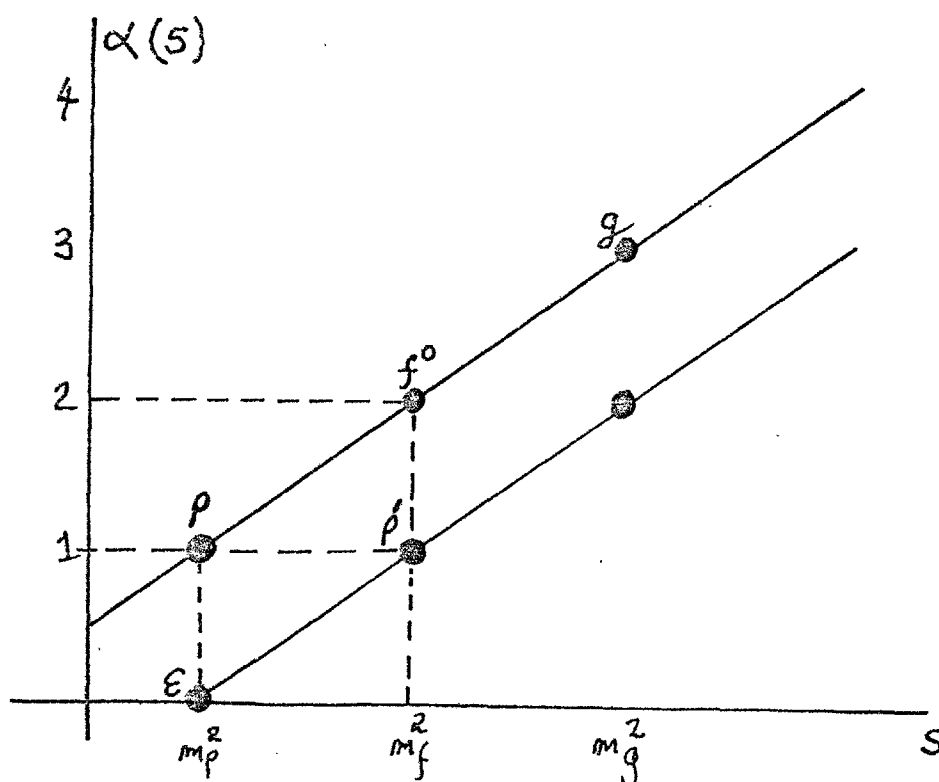
$$\text{also} \quad f = f_1 + f_2 \cos \Theta$$

$$g = f_2 \cos \Theta$$

where f and g are the spin non-flip and spin flip amplitudes.



HELICITY STATES.

Fig:31SENSE AND NONSENSE GRAPHS.Fig:32VENEZIANO DAUGHTER OF ρ

4.2.6 Sense and Nonsense Choosing

The introduction of spin has a further important result in that it has implications for the behaviour of the residue function, $\beta(t)$, at certain values of t .

Consider the process

$$1 + 2 \rightarrow 3 + 4$$

where the helicities are

$$\lambda_1, \lambda_2, \lambda_3, \lambda_4 .$$

The net helicity flip is defined by;

$$\bar{\lambda} = |(\lambda_3 - \lambda_4) - (\lambda_1 - \lambda_2)|$$

The simplest example of sense-nonsense choosing is to consider the behaviour of the Regge Pole amplitude near $\alpha(t) = 0$.

The upper vertex in Fig. 31 is called a SENSE vertex if,

$$\lambda_1 - \lambda_3 = 0 = \alpha(t)$$

and a NONSENSE vertex if,

$$|\lambda_1 - \lambda_3| > 0 = \alpha(t) .$$

By combining sense and nonsense vertices the 3 exchange diagrams in Fig. 31 can be constructed, corresponding to 3 possible amplitudes,

$$T_{ss} \propto \beta_{ss} \xi_{\alpha} s^{\alpha} \quad (4.20)$$

$$T_{sn} \propto \sqrt{\alpha} \beta_{sn} \xi_{\alpha} s^{\alpha-1} \quad (4.21)$$

$$T_{nn} \propto \alpha \beta_{nn} \xi_{\alpha} s^{\alpha-1} \quad (4.22)$$

ξ is the signature factor $\left(\frac{1 \pm e^{-i\pi\alpha}}{\sin \pi \alpha} \right)$

The $\sqrt{\alpha}$ and α terms come from the behaviour of the rotation functions $d_{\lambda\lambda}^{\alpha}(\cos \theta_t)$ near $\alpha = 0$.

Unitarity implies that the Regge residues factorise, so

$$(\beta_{sn})^2 = \beta_{ss} \cdot \beta_{nn} \quad (4.23)$$

It is further necessary to define the concepts of Right Signature Points (RSP) and Wrong Signature Points (WSP). For a trajectory of signature $+1$ even values of α are RSP and odd values of α WSP; and vice versa for a trajectory of signature -1 .

Now the signature function

$$\xi(t) = \frac{1 \pm e^{-i\pi\alpha(t)}}{\sin \pi \alpha}$$

can be expressed as:

$$\xi(t) = i - \frac{\cot \pi \alpha(t)}{2} \quad \text{+ve sig.}$$

$$\xi(t) = i + \frac{\tan \pi \alpha(t)}{2} \quad \text{-ve sig.}$$

$$\text{so} \quad \xi_{\alpha} \sim \alpha^{-1} \quad \text{at RSP.}$$

$$\xi_{\alpha} \sim \text{constant at WSP.}$$

If, for example, $\alpha = 0$ is an RSP for an exchanged trajectory, then β_{sn} must contain a factor $\alpha^{\frac{1}{2}}$ to avoid a singularity in T_{sn} ; and β_{ss} must not contain factor $\alpha^{\frac{1}{2}}$. Then the factorisation condition (4.23) can be satisfied in two possible ways.

$$\left. \begin{array}{l} \beta_{ss} \sim \text{constant} \\ \beta_{nn} \sim \alpha \end{array} \right\} \begin{array}{l} \text{SENSE} \\ \text{CHOOSING.} \end{array}$$

or

$$\left. \begin{array}{l} \beta_{ss} \sim \alpha \\ \beta_{nn} \sim \text{constant} \end{array} \right\} \begin{array}{l} \text{NONSENSE} \\ \text{CHOOSING.} \end{array}$$

If sense choosing is the physical mechanism,

$T_{ss} \sim C \xi_{\alpha} s^{\alpha}$, would have a pole at $\alpha = 0$; i.e. when t is -ve. A pole in this region would correspond to a physical state of negative (mass)² known as a GHOST state. This GHOST can be "killed" by multiplying the residues by α , causing the amplitudes to vanish at $\alpha = 0$; this is known as sense choosing with the Chew ghost killing mechanism.

The second choice (nonsense) is known as the Gell-Mann mechanism. This however has a pole in a non-leading term of T_{nn} , this is assumed to be cancelled by a compensating trajectory.

Which of these mechanisms occurs in nature has important implications for whether there is structure in $d\sigma/dt$ at certain t -values.

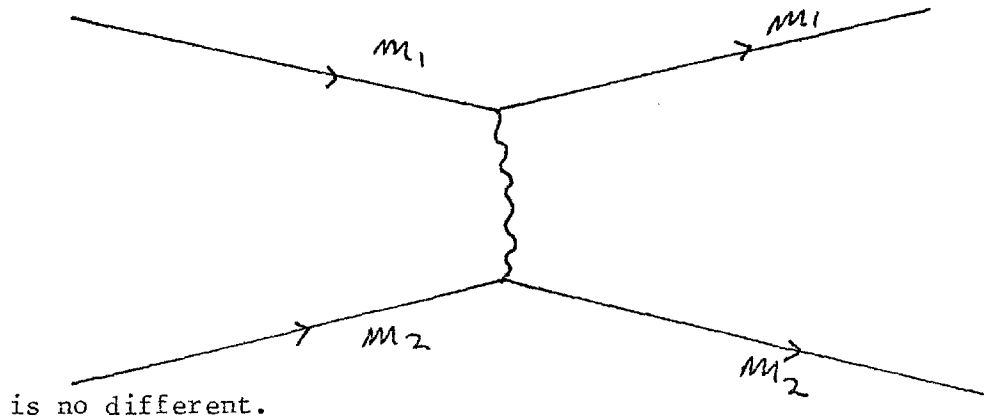
For instance, both mechanisms predict dips at WSP, so both will predict a dip in $d\sigma/dt$ for $\pi^{-}p \rightarrow \pi^0 n$ at $-t = 0.6$ (process dominated by ρ). For $\pi^{-}p \rightarrow \eta^0 n$ (dominated by A_2) sense choosing again predicts a dip at $\alpha = 0$ whereas nonsense choosing predicts no dip.

Both sense and nonsense mechanisms are used in models examined in this thesis.

4.2.7 Unequal Mass Scattering

Although the outline of Regge theory given above is for equal mass scattering, it can be shown that pairwise equal mass scattering,

i.e. $m_1 + m_2 \rightarrow m_1 + m_2$



is no different.

For the general unequal mass case certain difficulties occur. In fact normal Regge theory gives a singularity in the unequal mass scattering amplitude at $t = 0$. Since no such singularity is observed experimentally, it must be cancelled. The concept of subsidiary daughter trajectories can be introduced to cancel this singularity.

In the unequal mass case these daughters are merely a formal artefact and there is no reason why they should correspond to physical particles. However, even daughters have the same quantum nos. as the parent trajectory and thus one would expect them to contribute even to pair wise and equal mass scattering.

Although a parent trajectory and its associated daughters have been hypothesised as corresponding to a single Lorentz pole, there is little indication as to what the properties of the daughter trajectories should be. The relevance of these daughters to the present work is

that they give some justification for the use of secondary trajectories, which are required by the fits, but are not well confirmed as far as seeing particles on them goes.

4.2.8 Duality

A survey of the Regge pole model, especially within the context of K^+N reactions, would be incomplete without mentioning the concept of DUALITY.

The Regge Pole Model is essentially concerned with the high energy domain. That is, reactions at beam momenta above the region in which s -channel resonance production is the dominant feature. The concept of Duality forms a link between the low energy description of processes in terms of s -channel resonance production, and the high energy description in terms of Regge exchange in the t -channel.

By considering fixed t dispersion relations like

$$f(v, t) = \frac{1}{\pi} \int_{-\infty}^{+\infty} \text{Im} \frac{f(v', t)}{v' - v} dv'$$

where $v = \frac{s-u}{4}$

one can show that:

$$\begin{aligned} \frac{1}{N} \int_0^N \text{Im} f(v, t) dv \\ = \sum_{i=1, 2, \dots} \beta_i(t) \frac{\alpha_i(t)}{\alpha_i(t)+1} \end{aligned}$$

This is known as a Finite Energy Sum Rule. N is some upper limit to v , above which f is well described in terms of Regge theory, and below which resonance production is a good description.

The FESR essentially implies that the low energy resonance amplitude integrated over a finite range is equal to the amplitude described as a sum of Regge exchanges. The necessity of the integration is the reason for the FESR being considered an expression of "Global", or average, Duality.

This viewpoint, of the Regge and Resonance pictures being equivalent, is contrary to the former idea of interference, i.e.

$$A^{\text{TOTAL}} = A^{\text{RESONANCE}} + A^{\text{Regge}}.$$

Duality leads one to expect that the interference model contains double counting.

At present the concept of Duality is still somewhat uncertain, but the outline given contains the essential feature.

4.2.9 Exchange Degeneracy

It is an experimental fact that there seems to be no strong resonance formation in K^+N processes, while resonance formation is the dominant feature of low energy K^-N processes. This observation leads to the expectation that the K^+p (for example) amplitude is predominantly real, while the K^-p contains a large, and rapidly varying, imaginary part.

The exchanged trajectories involved are

$$K^-p \Rightarrow \mathbb{P} + f^0 + \omega + A_2 + \rho$$

$$K^+p \Rightarrow \mathbb{P} + f^0 - \omega + A_2 - \rho$$

($\mathbb{P}$ is Pomeron).

The sign of each contribution comes from the signature of the trajectory; the important point being that only trajectories of odd signature change sign between the two processes.

The requirement that the residue and trajectory functions for (ω, f_0) and (ρ, A_2) be degenerate ensures that $K^- N$ is complex and $K^+ N$ real. This requirement is known as "strong exchange degeneracy".

Experimentally these pairs of trajectories do indeed appear to have similar trajectories; if this were true, even though the residues were not equal, the condition would be weak exchange degeneracy. This latter would not exclude $K^+ N$ having a imaginary part.

The Pomeron is separate from this degeneracy. According to Harari⁽⁵⁾ the Pomeron is dual to the non-resonant background at low energy, whereas the Regge trajectories are dual to the resonances.

4.3 Veneziano Model

The Veneziano model is the best known example of an amplitude which exhibits explicit Duality and crossing symmetry. It is written as

$$F(s, t) = \frac{\Gamma(m - \alpha(s)) \Gamma(n - \alpha(t))}{\Gamma(p - \alpha(s) - \alpha(t))} + (m \leftrightarrow n)$$

where m, n, p are integers.

It exhibits Regge asymptotic behaviour.

$$F(s, t) \xrightarrow{s \rightarrow \infty} \frac{\beta}{\Gamma(\alpha)} \left\{ \frac{1 \pm e^{-i\pi\alpha}}{\sin \pi\alpha} \right\} (as)^\alpha$$

where $a = \frac{1}{\alpha_0}$.

The model implies a grid of real linearly rising trajectories, spaced by 1 unit of $\alpha(s)$, this feature being in accord with the experimentally observed trajectories.

The low lying trajectories are Veneziano daughters, and are of interest in the present case, since the 1st daughter implies the existence of a ρ' particle of the same mass as the f_0 , see Fig. 32. It is important to note that these secondary trajectories have no a priori connection with the kinematic daughters in unequal mass scattering. Also the Veneziano daughters have a definite form predicted, unlike the kinematic ones.

A further feature is that the Veneziano amplitude chooses the Gell-Mann residue mechanism, in that $\Gamma(\alpha(t))$ in the denominator causes the amplitude to vanish at W.S.P. and tend to a constant at R.S.P.

Chapter 4, REFERENCES

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CHAPTER 5

REGGE POLE MODELS5.1 Introduction

In this chapter two Regge Pole models are examined. Due to the number of free parameters involved in these Regge Models there is no point in considering the data produced by this experiment in isolation. Even to consider the available data on K^+n elastic charge exchange would be insufficient to constrain the models. Instead SU_3 symmetry of the Regge pole residues is invoked and all the $0^{-\frac{1}{2}+} \rightarrow 0^{-\frac{1}{2}+}$ charge exchange reactions are fitted to.

From a consideration of the quantum numbers which are exchanged it can be shown that the amplitudes governing these processes are;

$$A(K^+n \rightarrow K^0p) = A_{K A_2} - A_{K P} \quad (5.1)$$

$$A(K^-p \rightarrow \bar{K}^0n) = A_{K A_2} + A_{K P} \quad (5.2)$$

$$A(\pi^-p \rightarrow \pi^0n) = A_{\pi P} \quad (5.3)$$

$$A(\pi^-p \rightarrow \eta^0n) = A_{\pi A_2} \quad (5.4)$$

where, for example, A_{A_2} is the invariant amplitude due to A_2 exchange; similar expressions are true for the B amplitudes.

The A_K and A_π for a given trajectory are related⁽¹⁾ via SU_3

$$\frac{\pi_i^{A_i}}{K_i^{A_i}} = \frac{\pi_i^{B_i}}{K_i^{B_i}} \equiv F_i(t).$$

For exact SU_3 symmetry $F(t)$ is not a function of t and one has,

$$C = +1 ; F_{A_2} = \sqrt{2}/3 \quad (5.5)$$

$$C = +1 ; F_P = -\sqrt{2} \quad (5.6)$$

where C is the charge conjugation quantum number of the trajectory involved.

The data fitted to in each model comprises:

- (a) $K^+n \rightarrow K^0p$: $d\sigma/dt$ 800 MeV/c - 12 GeV/c
- (b) $K^-p \rightarrow \bar{K}^0n$: $d\sigma/dt$ 5 GeV/c - 9.5 GeV/c
- (c) $\pi^-p \rightarrow \pi^0n$: $d\sigma/dt$ 5.9 GeV/c - 18.2 GeV/c
- (d) $\pi^-p \rightarrow \eta^0n$: $d\sigma/dt$ 5.9 GeV/c - 18.2 GeV/c
- (e) $(K^\pm p \rightarrow K^\pm n)$ σ_{TOTAL}
- (f) $(\pi^\pm p \rightarrow \pi^\pm p)$ σ_{TOTAL}
- (g) $\pi^-p \rightarrow \pi^0n$ Polarisation 5 GeV/c - 11.2 GeV/c.

Not all the data available on all of these processes is utilised, but the $d\sigma/dt$ for $K^+n \rightarrow K^0p$ and $\pi^-p \rightarrow \pi^0n$ $|\vec{P}|$ data are considerably greater than has been used before in either of the models considered⁽²⁾.

To enable fits to be performed with various models a generalised χ^2 fitting program was written by the author, encompassing processes (a) $\rightarrow$ (g). The program takes the form of a package of subroutines called from the CERN minimising program MINUIT via the "user" subroutine FCN. The program is general insofar as only one subroutine (DCS1) needs to be changed in order to change the model being fitted.

Two useful features are, that the program can produce plots of fitted curves (on line printer or CALCOMP) at any point during the minimising process, and that fits to a particular process are only triggered by the presence of data for that process in the input string. The former is useful in monitoring the progress of fitting. The latter gives a high degree of flexibility in that any subset of the processes (a) $\rightarrow$ (g) can be fitted to just as readily as the complete set.

5.2 Rarita-Schwartzchild Model⁽¹⁾

This model has been examined in some detail, despite the fact that it is somewhat outmoded. The motivation for this was two-fold. Firstly, several authors^(3,4,5,6,7) have compared their data on $K^+n \rightarrow K^0p$ with the predictions of the model. This despite the fact that the original fits were performed including only $K^+n \rightarrow K^0p$ data at 2.3 GeV/c, and also that the fit is incompatible with recent data on $\pi^-p \rightarrow \pi^0n$ polarisation. In the light of this it was considered worthwhile to refit the model to a wider range of data on $K^+n \rightarrow K^0p$ and also to examine the possibility of a better fit to the polarisation data.

Secondly an examination of the model is useful for comparison with the behaviour of more modern models.

5.2.1 The Formalism

The model was proposed before the advent of Duality. The assumption is made that all trajectories choose SENSE, and the CHEW ghost killing mechanism is employed. In the light of recent ideas this is a deficiency of the model, in that it breaks strong exchange degeneracy. This is due to the fact that the Chew mechanism requires the A_2 residue to have an extra factor of $\alpha(t)$ compared with the ρ residue, while exchange degeneracy requires the residues be equal.

Additionally, the lack of theoretical constraints stemming from Duality leads to a large number of free parameters in the fit, since nothing is assumed concerning any of the trajectory or residue functions.

A further noteworthy feature of the model is its introduction of the ill-established ρ' trajectory. This was introduced primarily to give non-zero $\pi^- p \rightarrow \pi^0 n$ polarisation, since ρ exchange alone would predict identically zero polarisation from (4.17). The ρ' was also required by the $K^+ n \rightarrow K^0 p$ data, as $\rho + A_2$ exchange predicted a cross section too small by a factor of 2.

5.2.2 Parametrisation of Amplitude

For a Kaon process the invariant amplitudes, for each trajectory, are parametrised as

$$K A_i^{\pm} = - C_i(t) (\alpha_i + 1) \left(\frac{e^{-i \pi \alpha_i \pm 1}}{\sin \pi \alpha_i} \right) \left(\frac{E}{E_0} \right)^{\alpha_i(t)} \quad (5.7)$$

$$K B_i^{\pm} = - D_i(t) \alpha_i (\alpha_i + 1) \left\{ \frac{e^{-i \pi \alpha_i \pm 1}}{\sin \pi \alpha_i} \right\} \left(\frac{E}{E_0} \right)^{\alpha_i(t)-1} \quad (5.8)$$

$\pm$ referring to the signature of the trajectory involved.

E is the laboratory energy of the incident meson, E_0 is a scale factor (taken equal to 1), and $C_i(t)$, $D_i(t)$ are the residue functions.

The total amplitude for a particular process is

$$A = \sum_{\text{TRAJECTORIES}} A_i$$

using the sign convention defined by (5.1) $\rightarrow$ (5.4).

The ρ' is assumed to be a member of the same SU_3 multiplet as the ρ .

The residue functions are parametrised in a t -dependent fashion;

$$\begin{aligned}
C_{\rho}(t) &= C_{\rho}^0 (1 + C_{\rho}^1 t) & D_{\rho}(t) &= D_{\rho}^0 e^{D_{\rho}^1 t} \\
C_{\rho'}(t) &= C_{\rho'}^0 e^{C_{\rho'}^1 t} & D_{\rho'}(t) &= D_{\rho'}^0 e^{D_{\rho'}^1 t} \\
C_{A_2}(t) &= \alpha_{A_2} C_{A_2}^0 e^{C_{A_2}^1 t} & D_{A_2}(t) &= \alpha_{A_2} D_{A_2}^0 e^{D_{A_2}^1 t}
\end{aligned}$$

The trajectory functions are assumed to be linearly rising, as suggested by experiment:

$$\alpha_i(t) = \alpha_i^0 + \alpha_i^1 t$$

SU_3 is assumed broken, again as a function of t ;

$$F_i(t) = F_i^0 e^{F_i^1 t}.$$

This parametrisation scheme leads to 24 free parameters.

In the original model some constraint was introduced by employing a superconvergent sum rule⁽⁸⁾ to relate

$\alpha_{\rho}^0, \alpha_{\rho'}^0, C_{\rho}^0, C_{\rho'}^0, F_{\rho}^0, F_{\rho'}^0$, and also by limiting SU_3 breaking to 25%.

The amplitudes (5.7) and (5.8) are inserted in (4.18) and (4.19) to yield differential cross sections and polarisation.

The cross section differences are given by

$$\sigma(K^{\pm}p) - \sigma(K^{\pm}n) = \frac{\text{Im}(A_K^{+} + A_K^{-})_{t=0}}{P_{\text{LAB}}}$$

and

$$\sigma(\pi^{+}p) - \sigma(\pi^{-}p) = \sqrt{2} \frac{\text{Im}(\pi A^{-})_{t=0}}{P_{\text{LAB}}}. \quad (5.9)$$

The factor of $\sqrt{2}$ in (5.9) is a result of the normalisation chosen for $d\sigma/dt$ ($\pi^{-}p \rightarrow \pi^0 n$).

The amplitudes for $\pi^\pm p \rightarrow \pi^\pm p$ are ⁽⁹⁾ usually written

$$\begin{aligned} A(\pi^- p \rightarrow \pi^- p) &= A_\rho + A_{f^0} + A_\rho \\ A(\pi^+ p \rightarrow \pi^+ p) &= A_\rho + A_{f^0} - A_\rho. \end{aligned}$$

Since $\pi^+ p \rightarrow \pi^+ p$ is a pure isospin $3/2$ process, one can write

$$A(\pi^+ p \rightarrow \pi^+ p) = a_{3/2} \quad (5.10)$$

$\pi^- p \rightarrow \pi^- p$ is a mixture of isospin $1/2$ and $3/2$ so similarly, using Gellish-Gordon coefficients,

$$A(\pi^- p \rightarrow \pi^- p) = \frac{1}{3} a_{3/2} + \frac{2}{3} a_{1/2} \quad (5.11)$$

and also,
$$A(\pi^- p \rightarrow \pi^0 n) = \frac{\sqrt{2}}{3} a_{3/2} - \frac{\sqrt{2}}{3} a_{1/2}$$

and subtracting (5.11) from (5.10)

$$A(\pi^+ p) - A(\pi^- p) = \frac{2}{3} a_{3/2} - \frac{2}{3} a_{1/2} = \sqrt{2} A(\pi^- p \rightarrow \pi^0 n)$$

so

$$A(\pi^- p \rightarrow \pi^0 n) = -\frac{\sqrt{2}}{2} A_\rho$$

But, by defining $A(\pi^- p \rightarrow \pi^0 n) = A_\rho$ the $-\sqrt{2}$ factor has been pushed back into the total amplitudes, i.e.

$$A(\pi^+ p \rightarrow \pi^+ p) = -\frac{1}{\sqrt{2}} (A_\rho + A_{f^0} - A_\rho)$$

$$A(\pi^- p \rightarrow \pi^- p) = -\frac{1}{\sqrt{2}} (A_\rho + A_{f^0} + A_\rho)$$

subtracting gives

$$\sigma(\pi^+ p) - \sigma(\pi^- p) = \sqrt{2} A_\rho.$$

5.2.3 $K^+ n \rightarrow K^0 p$ below 5 GeV/c

When this model was first constructed the apparent lack of s-channel resonances in the $K^+ N$ system made it seem plausible that the Regge Pole model could be used at a much lower energy than normal. In processes rich in resonance formation ($K^- p$, $\pi^\pm p$) the Regge model is normally only used beyond the resonance region (i.e. above approximately 5 GeV/c). The concept of Duality makes this extension of the Regge model even more plausible. However, the parametrisation of the amplitudes (5.7), (5.8) assumes high s and low t [see 4.2.4]. In an effort to extend the validity of the Regge approach down to the low s region the exact Legendre form of the amplitudes, as described in the last chapter, was used.

Before the asymptotic approximation $s \rightarrow \infty$, $\cos \theta_t \rightarrow \infty$, at fixed t, the exact form of the Regge Pole amplitude is (from 4.12)

$$A_i^\pm = \frac{-\pi}{q_t} \frac{2\alpha_i + 1}{2} \beta_i(t) L_{\alpha_i}(-\cos \theta_t) \left\{ \frac{e^{-i\pi\alpha_i}}{\sin \pi\alpha_i} \right\}$$

where

$$L_\alpha(-\cos \theta_t) = P_\alpha(-\cos \theta_t) \quad \text{for } \alpha \geq 0$$

$$\text{and } L_\alpha(-\cos \theta_t) = M_\alpha(-\cos \theta) \quad \text{for } \alpha < 0.$$

In reference (1) this was written as

$$A_i^\pm = -a(t) \left(\frac{-q_t^i q_t^f}{m_n E_b} \right)^{\alpha_i} (\alpha_i + \frac{1}{2}) L_{\alpha_i}(-\cos \theta_t) \times \left\{ \frac{e^{-i\pi\alpha_i}}{\sin \pi\alpha_i} \right\} \quad (5.12)$$

where $a(t) = \frac{\pi}{q_t} \beta_i(t)$, and $q_t^{i,f}$ is t-channel centre-of-mass 3-momentum.

The factor,

$$\left(\frac{q_t^{i,f}}{m_n E_0} \right)^{\alpha_i} = \left\{ \frac{[(4 m_n^2 - t)(4 m_K^2 - t)]^{\frac{1}{2}}}{4 m_n} \right\}^{\alpha_i}$$

is factorised out of the residue in order to cancel t-channel kinematic singularities at,

$$t = 4 m_n^2$$

and

$$t = 4 m_K^2.$$

These singularities are in the t-channel physical region.

At asymptotic (i.e. non-physical) t-values these singularities are far away and can be ignored. However, at low s-values it may be that the t-channel Regge Pole amplitudes are influenced by these singularities, and the above approach will counteract this influence.

The extraction of such a factor is valid in that the residue $\beta_i(t)$ is completely undetermined.

From (4.14) the asymptotic Regge amplitude is

$$A_{s \rightarrow \infty}^{\pm} = -\pi \frac{2\alpha+1}{2} \beta(t) \left\{ \frac{1 + e^{-i\pi\alpha}}{\sin \pi\alpha} \right\} \\ \times \frac{2^\alpha}{\sqrt{\pi}} \frac{\Gamma(\alpha+\frac{1}{2})}{\Gamma(\alpha+1)} (s/s_0)^\alpha (t)$$

In ref. 1 this was written as

$$A_{s \rightarrow \infty}^t = -a(t) \left(\frac{2E}{E_0} \right)^\alpha \frac{\alpha+\frac{1}{2}}{\sqrt{\pi}} \frac{\Gamma(\alpha+\frac{1}{2})}{\Gamma(\alpha+1)} \left\{ \frac{e^{-i\pi\alpha} \pm 1}{\sin \pi\alpha} \right\} \quad (5.13)$$

comparing (5.13) with (5.7) allows one to express (5.12) in terms of the same parameters as (5.7) i.e.

$$A_i^{\pm} = -C_i(t) (\alpha_i+1) \frac{\Gamma(\alpha_i+1)}{\Gamma(\alpha_i+\frac{1}{2})} \frac{\sqrt{\pi}}{2^{\alpha_i}} \left(\frac{-q_t^i q_t^f}{m_n E_0} \right)^{\alpha_i} \\ \times L_{\alpha_i}(-\cos \theta_t) \left\{ \frac{e^{-i\pi\alpha_i} \pm 1}{\sin \pi\alpha_i} \right\} \quad (5.14)$$

The B invariant amplitude is, before the asymptotic limit is taken,

$$B_i^{\pm} = -b_i(t) \left(\frac{-q_t^i q_t^f}{m_n E_0} \right)^{\alpha_i-1} (\alpha_i+\frac{1}{2}) \frac{d}{d(-\cos \theta_t)} L_{\alpha_i}(-\cos \theta_t) \\ \times \left\{ \frac{e^{-i\pi\alpha_i} \pm 1}{\sin \pi\alpha_i} \right\} \quad (5.15)$$

and the asymptotic form is;

$$B_i^{\pm} \underset{s \rightarrow \infty}{=} -b_i(t) \left(\frac{2E}{E_0} \right)^{\alpha_i - 1} \frac{2\alpha_i (\alpha_i + \frac{1}{2})}{i\sqrt{\pi}} \frac{\Gamma(\alpha_i + \frac{1}{2})}{\Gamma(\alpha_i + 1)} \times \left\{ \frac{e^{-i\pi\alpha_{i+1}}}{\sin \pi\alpha_i} \right\} \quad (5.16)$$

As before, comparing (5.16) with (5.8) gives (5.15) in terms of the same fitted parameters as (5.8), i.e.:-

$$B_i^{\pm} = -D_i(t) (\alpha_i + 1) \frac{\Gamma(\alpha_i + 1)}{\Gamma(\alpha_i + \frac{1}{2})} \frac{\sqrt{\pi}}{2^{\alpha_i}} \left(\frac{-q_t^i q_t^f}{m_n E_0} \right)^{\alpha_i - 1} \times \frac{d}{d(-\cos \theta_t)} P_{\alpha_i}(\cos \theta_t) \left\{ \frac{e^{-i\pi\alpha_{i+1}}}{\sin \pi\alpha_i} \right\}$$

The Legendre functions in the exact amplitudes were computed using standard CERN Program Library routines⁽¹⁰⁾. Also, in order to reduce computing time, the recursion relation⁽¹¹⁾

$$P_{\alpha-1}(Z) = \frac{\tan \alpha\pi}{\pi} \left\{ Q_{\alpha-1}(Z) - Q_{-\alpha-2}(Z) \right\}$$

was employed.

In the correct expression for a Regge Pole amplitude the factor $[\Gamma(\alpha + 1)]^{-1}$ serves to remove poles, due to the signature factor, along the -ve Real axis. In the asymptotic expressions (5.7), (5.8) this Γ function has been replaced by $(\alpha + 1)$. This can be justified by writing

$$\Gamma(1 + \alpha) = \frac{1}{1 + \alpha} \prod_{m=1}^{\infty} \left[\frac{\left(1 + \frac{1}{m}\right)^{1+\alpha}}{1 + \frac{\alpha+1}{m}} \right]$$

which, for small α

$$\sim \frac{1}{1+\alpha}$$

The scattering angle cosine in the t-channel is⁽¹²⁾

$$\cos \theta_t = \frac{2(s - m_n^2 - m_K^2) + t}{(4m_n^2 - t)^{1/2}(4m_K^2 - t)^{1/2}}$$

where s, t refer to the s-channel.

In addition to using the exact Legendre formalism below 5.0 GeV/c, the K^+n data at all energies has been corrected for the Pauli effect (3.7) in the forward direction. No other deuteron effect has been allowed for, since these are small compared with the normalisation errors between different energies.

In the expression (3.8)

$$R(t) = \frac{(\frac{d\sigma}{dt})_n^{\text{spin flip}}}{(\frac{d\sigma}{dt})_n^{\text{non flip}}}$$

R is in terms of spin flip and non flip. Since this model employs invariant amplitudes, which refer to helicities, it is necessary to connect the spin and helicity representations.

Since $d\sigma/dt \propto (\text{amplitude})^2$

$$R = \frac{g^2}{f^2}$$

where $f(\theta) = f + g \underline{\sigma} \cdot \underline{\hat{n}}$.

Ref. 1 shows that R can be expressed in terms of the s-channel helicity amplitudes. G_1 and G_2 by;

$$R = \left/ \frac{G_2 \sin \theta_s}{G_1 + G_2 \cos \theta_s} \right/ ^2$$

where $f(\theta) = G_1 + G_2 \underline{\sigma} \cdot \hat{q}_f \underline{\sigma} \cdot \hat{q}_i$.

Where G_1 and G_2 are connected with the t-channel invariant amplitudes via;

$$G_2 = \frac{(q_s^2 + m_n^2)^{\frac{1}{2}} \pm m_n}{8\pi \sqrt{s}} \left\{ \pm A' + (\sqrt{s} \mp m_n) B \right\}$$

$$\text{where} \quad A = A' + \frac{E + (t/4m_n)}{1 - (t/4m_n^2)} \cdot B \quad (5.17)$$

5.2.4 Units and Conversion Factors

It is necessary to ensure that the theoretically computed quantities are in the same units as the experimental data.

Considering the optical theorem

$$\sigma_{\text{TOTAL}}(s) = \frac{1}{p_{\text{LAB}}} \text{Im } A(s, t = 0)$$

then from (5.7),

$$\begin{aligned} \text{Im } A_{t=0} &\sim C^0 \\ \therefore \sigma_{\text{TOT}} &\sim \frac{C_0}{p_{\text{LAB}}} \end{aligned}$$

so C_0 has the dimensions $\text{mb} \times \text{GeV}$.

A similar consideration of (5.17) shows that D_0 has the dimensions of mb . Considering the dimensions of (4.18) at $t = 0$ indicates that $d\sigma/dt$ then has the units $\text{mb} \cdot \text{mb}$, whereas the data has units $\text{mb}(\text{GeV}/c)^{-2}$. To get the units correct the calculated $d\sigma/dt$ was multiplied by 2.571 since

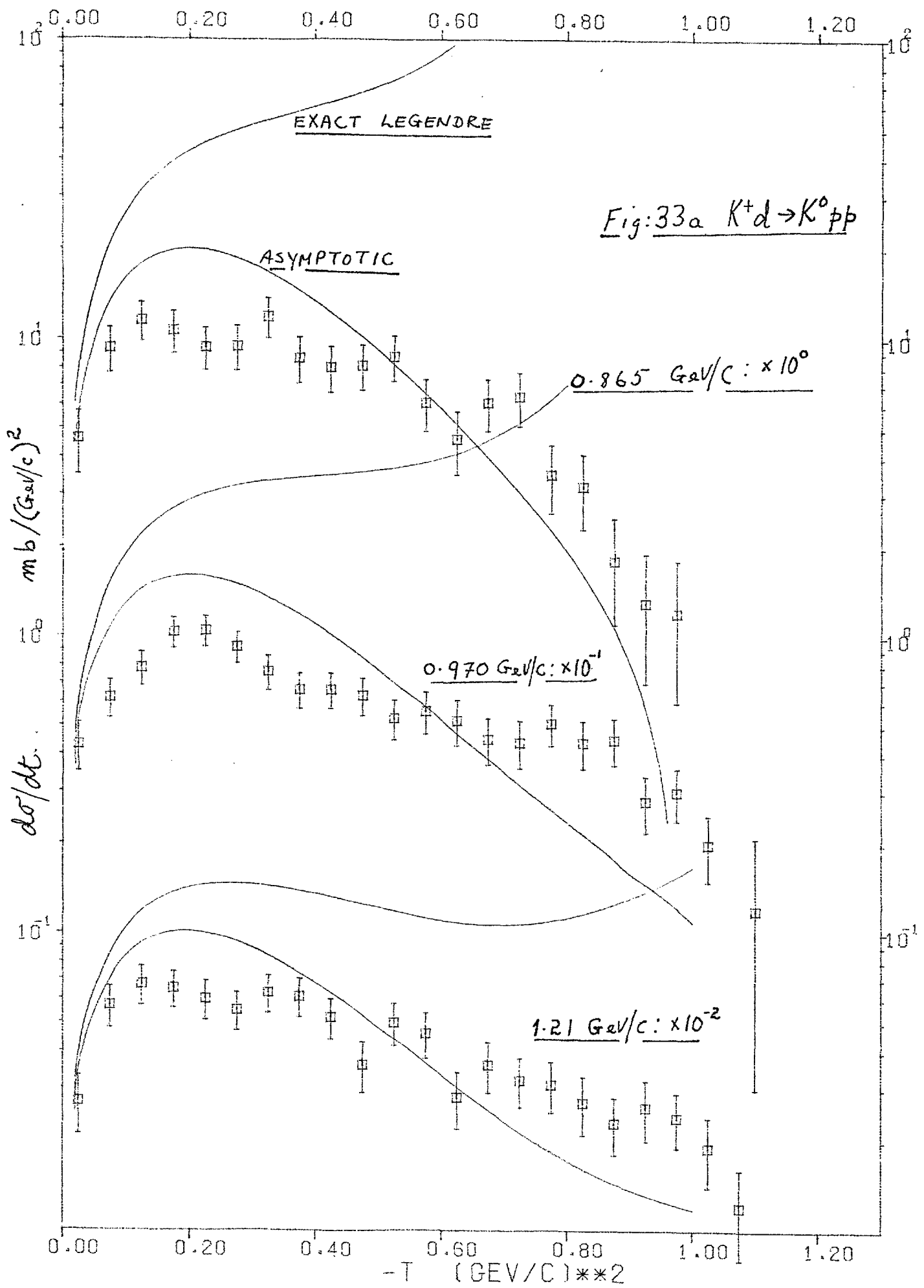
$$[\text{mb}] = 0.389 [\text{GeV}]^{-1}.$$

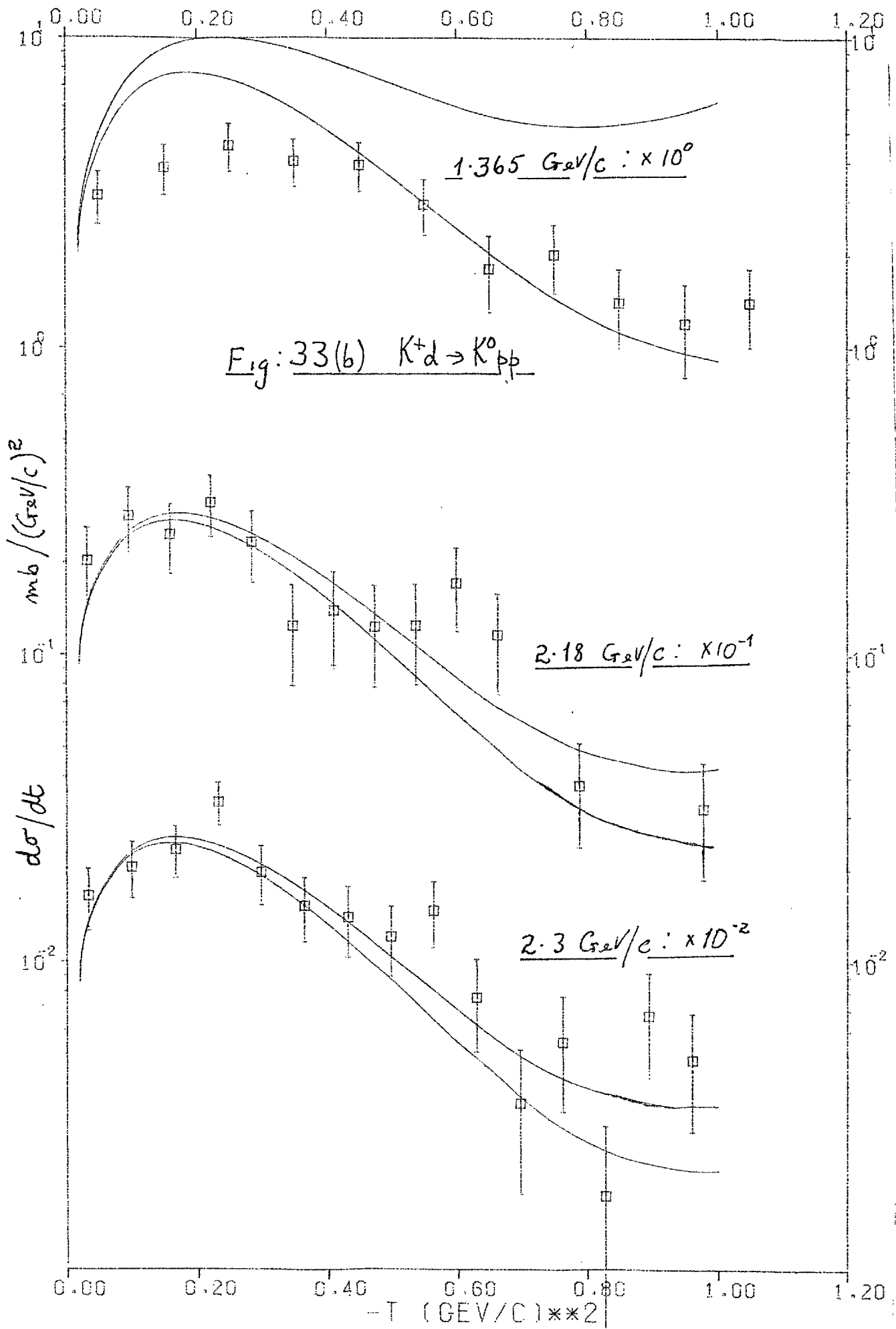
5.2.5 The Fits

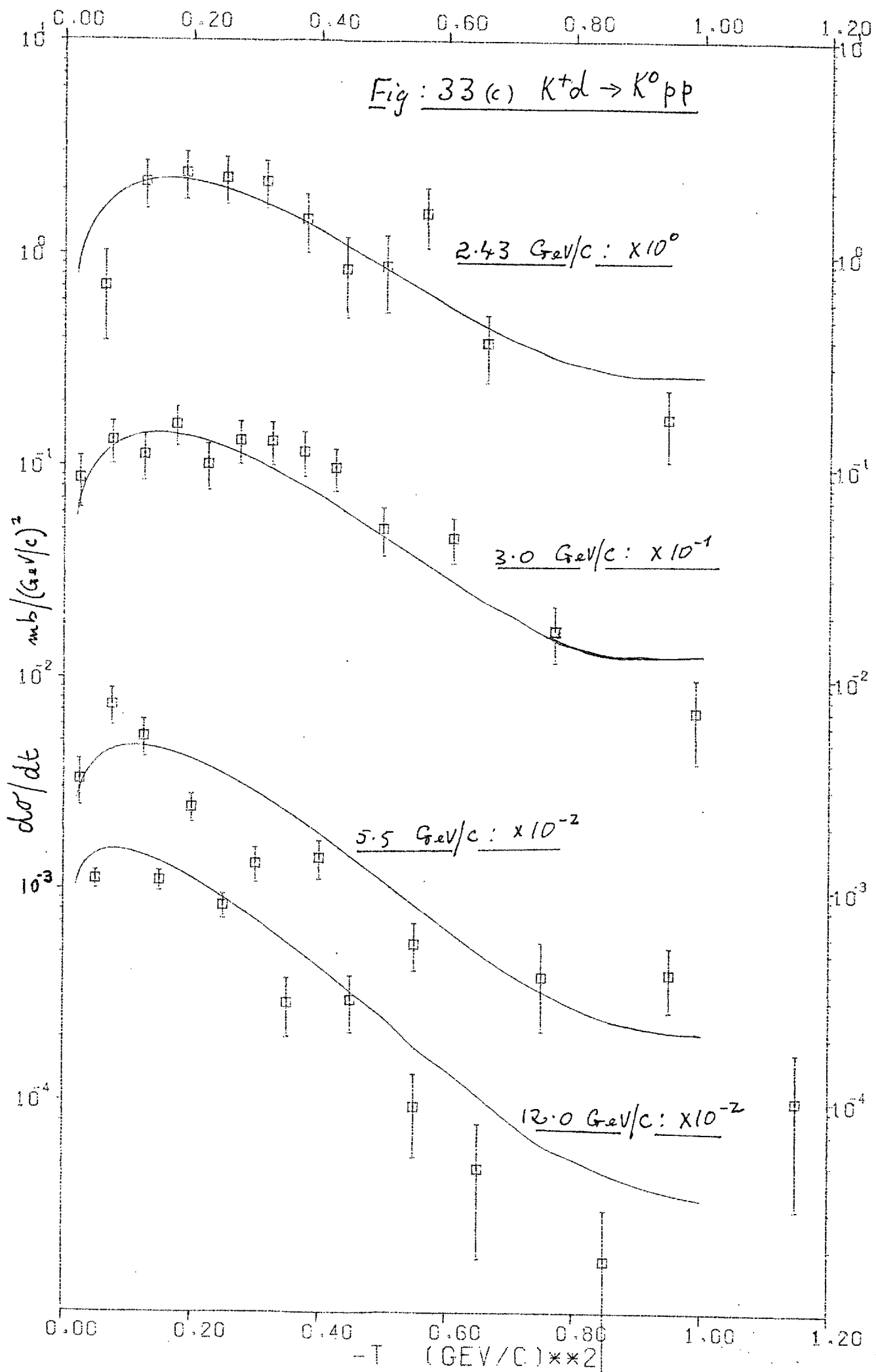
For comparison the fitted curves from ref. 1 parameters (Table 11) are shown in Fig. 33. The predictions for new $K^+n \rightarrow K^0p$ data and also $\pi^-p \rightarrow \pi^0n$ polarisation are shown. As can be seen the fit to the data, available at that time, is rather good. However, the polarisation has the wrong sign in $\pi^-p \rightarrow \pi^0n$, and the low energy exact Legendre predictions for $d\sigma/dt$ ($K^+n \rightarrow K^0p$) are unsatisfactory, as are the asymptotic curves.

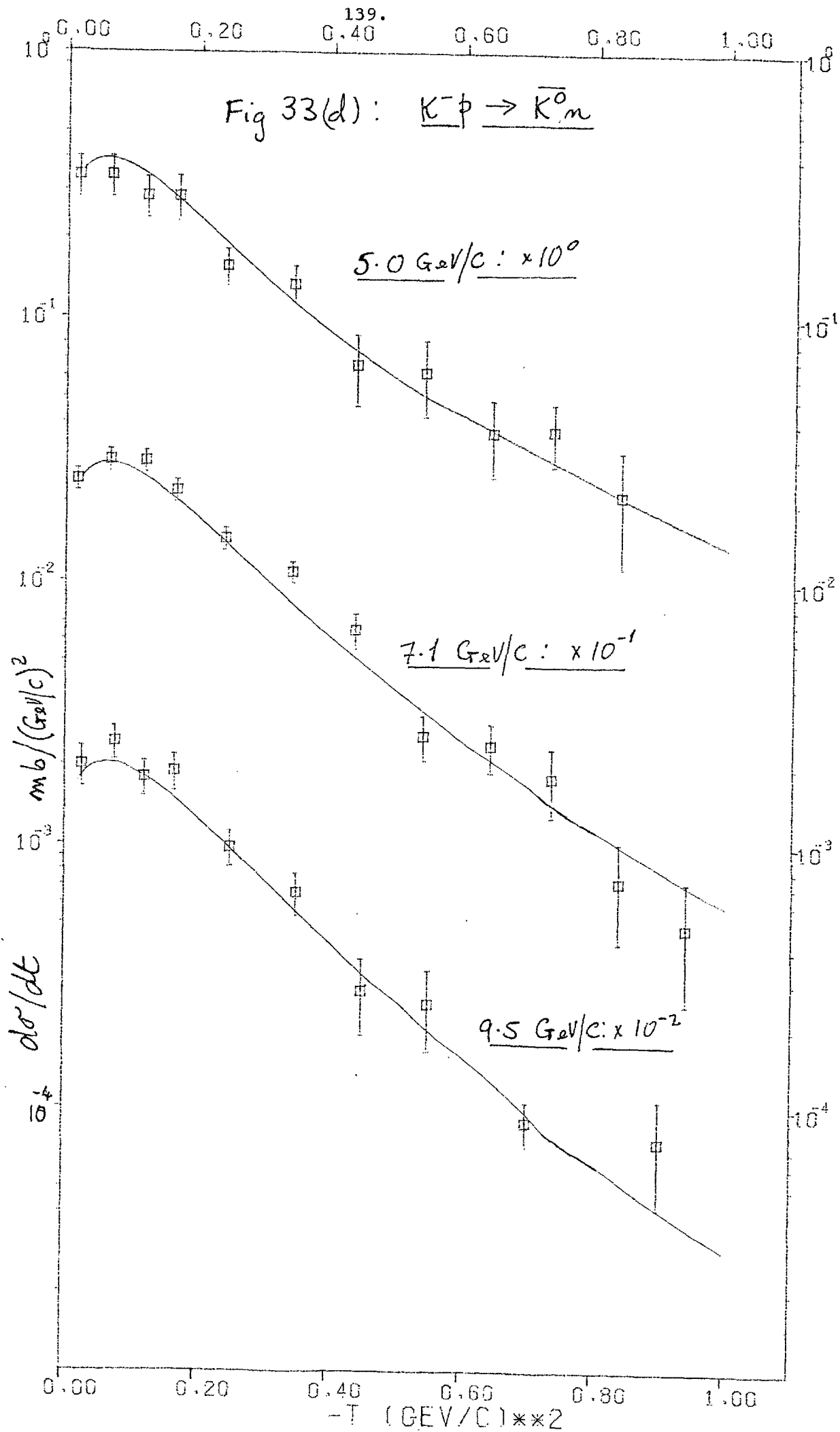
The $\pi^-p \rightarrow \pi^0n$ exhibits a WSP dip, and although a similar dip is predicted in $\pi^-p \rightarrow \eta^0n$ the fitted A_2 parameters result in this being beyond the range of the experimental data. The large helicity flip residues in this fit are responsible for the forward turnover in all the differential cross sections.

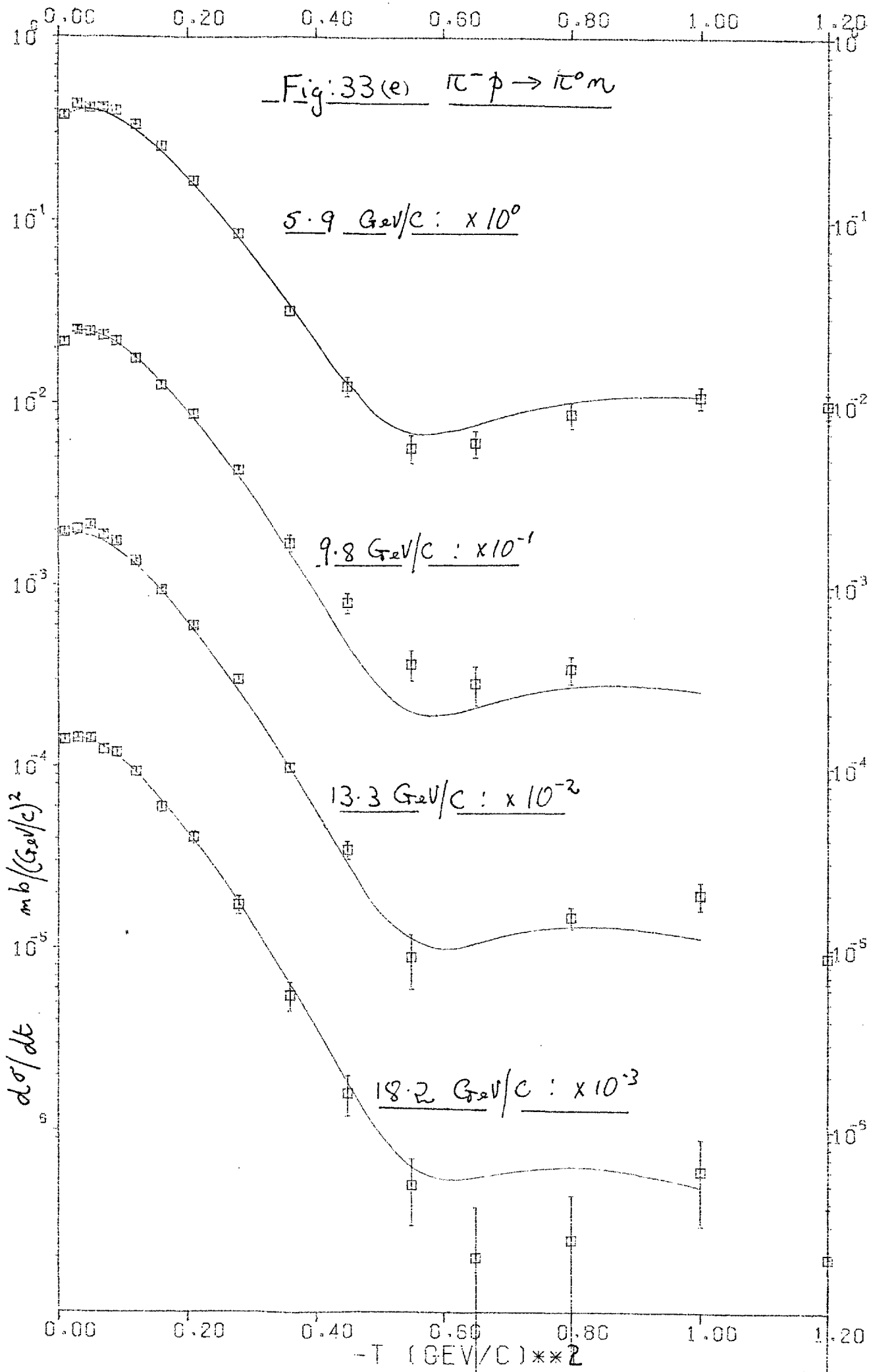
Because of the behaviour of the exact Legendre fits at beam momenta below 2.0 GeV/c a preliminary fit was performed omitting this data. The parameter values of ref. 1 were used as starting values, but exact SU_3 symmetry was assumed. This was partly to reduce the number of free parameters, but also to investigate how important the SU_3 breaking was with respect to the general features of the fit. The results obtained are shown by the full lines in Fig. 34, and by Table 12.

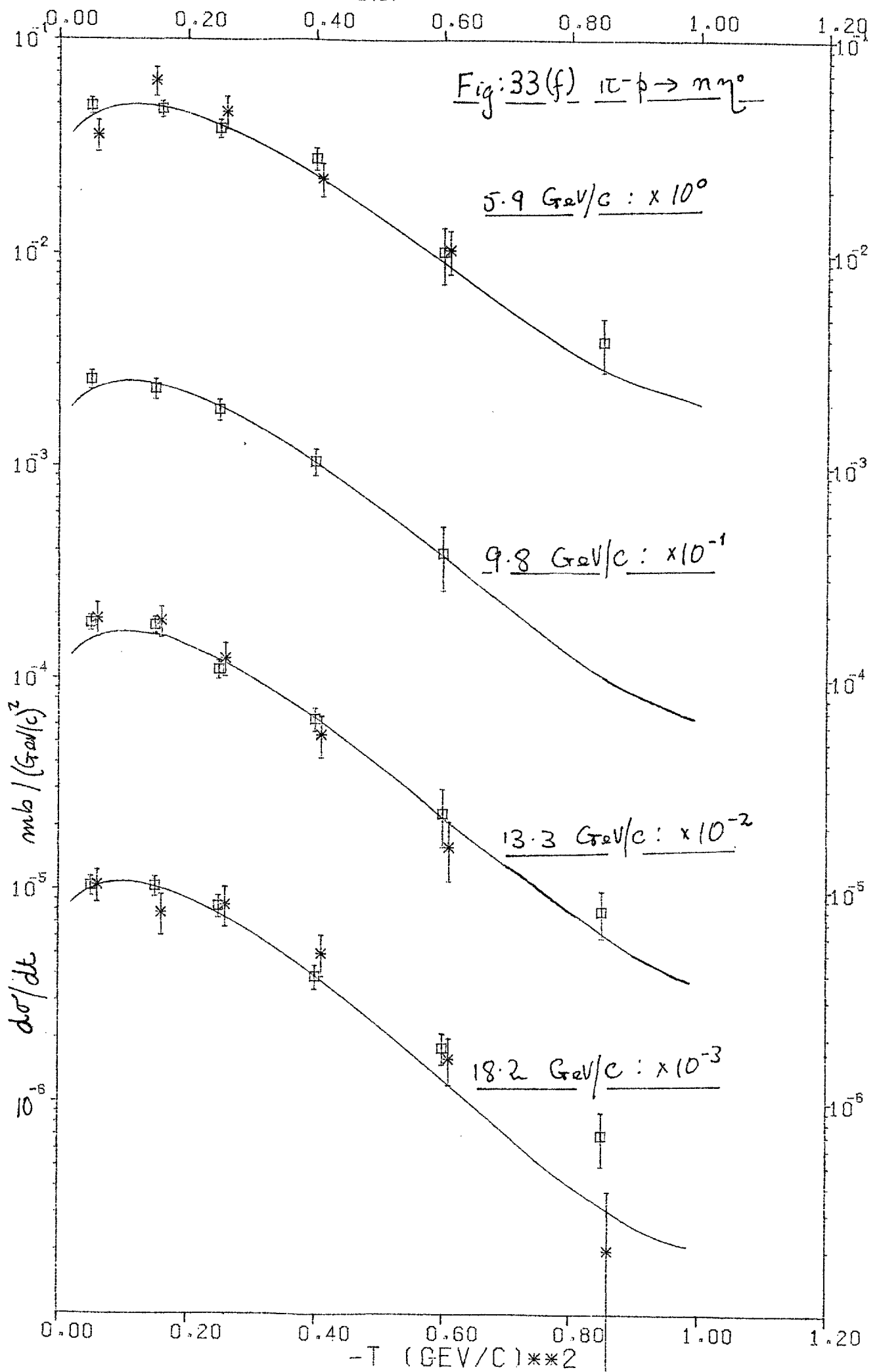


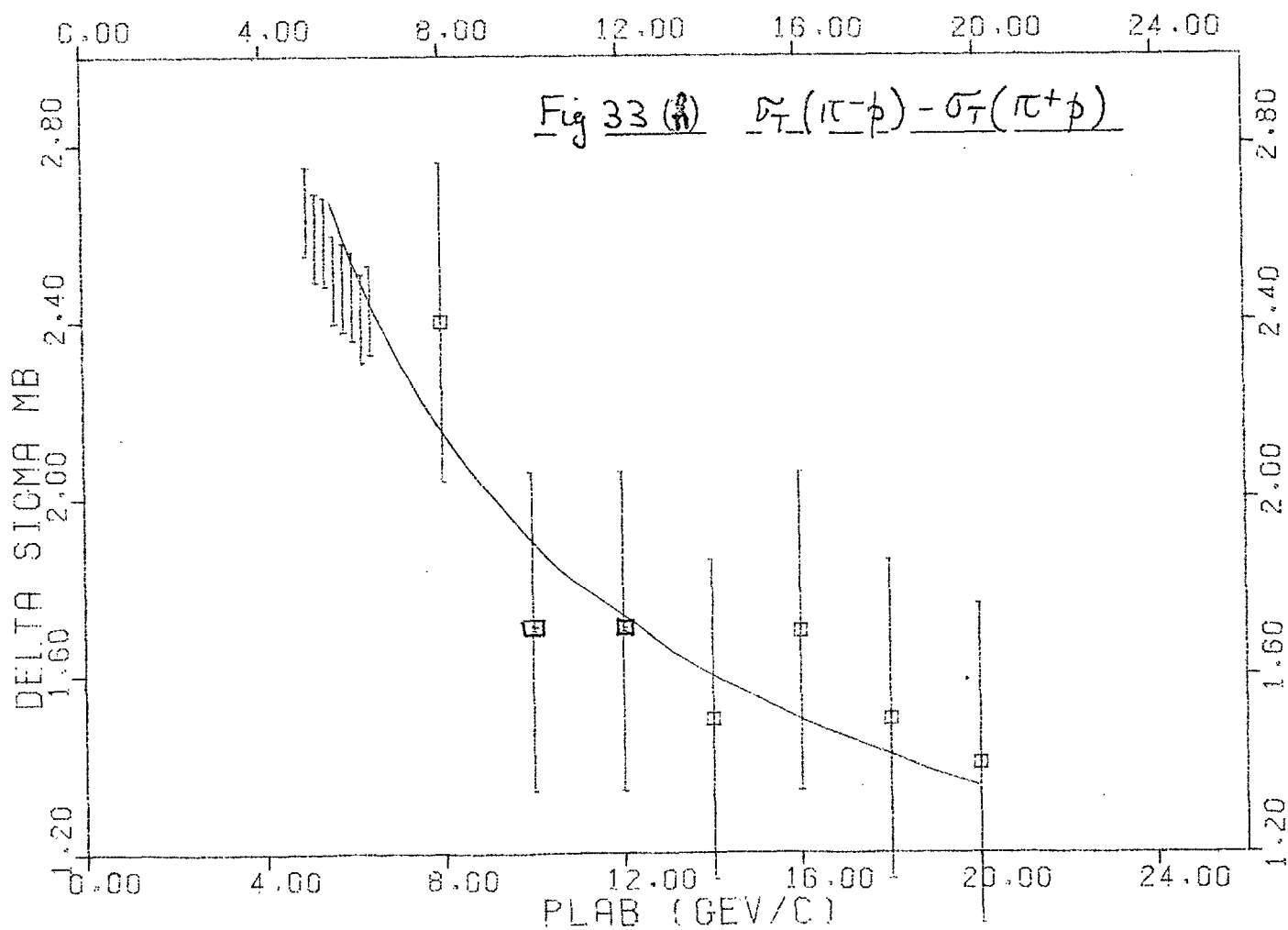
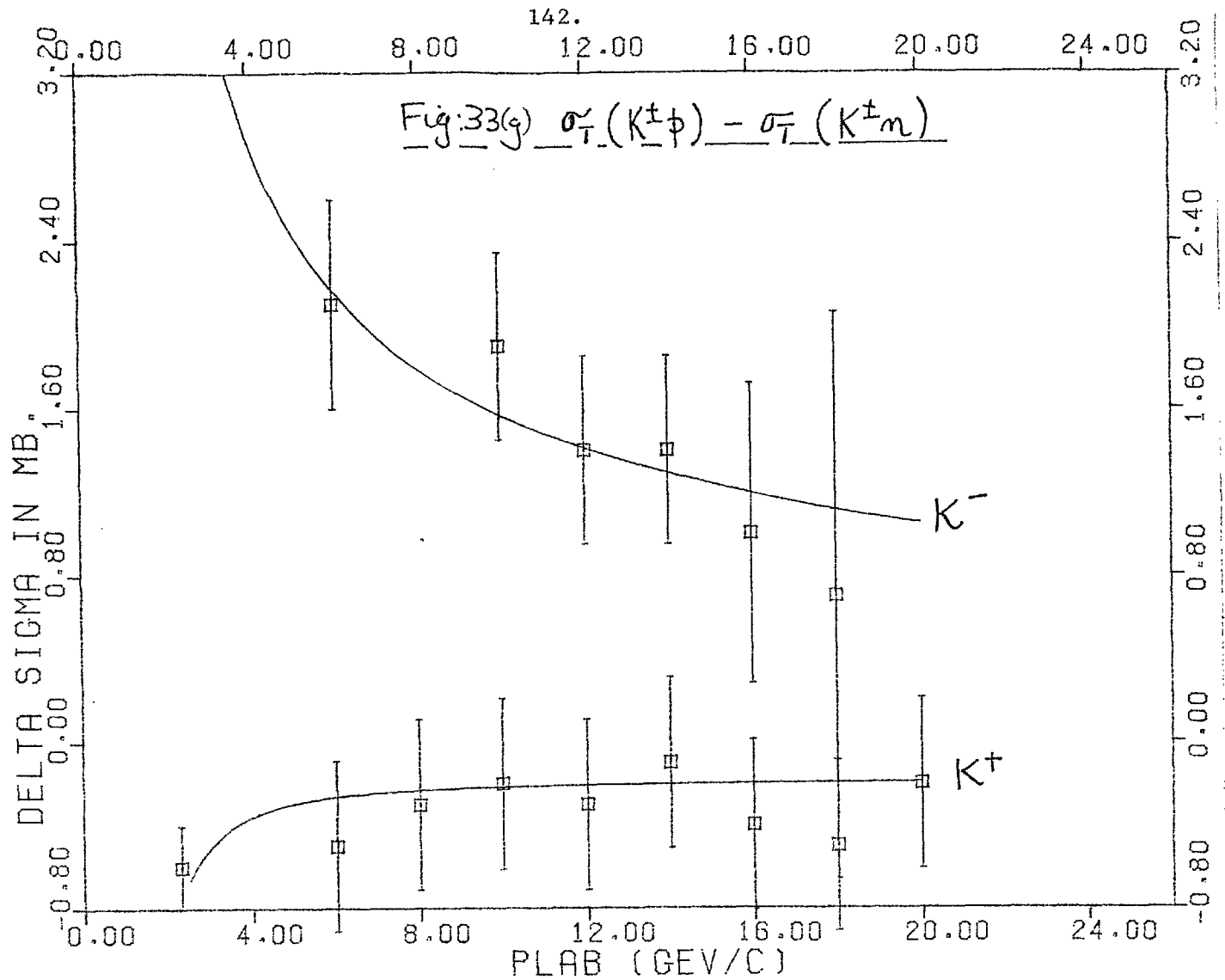












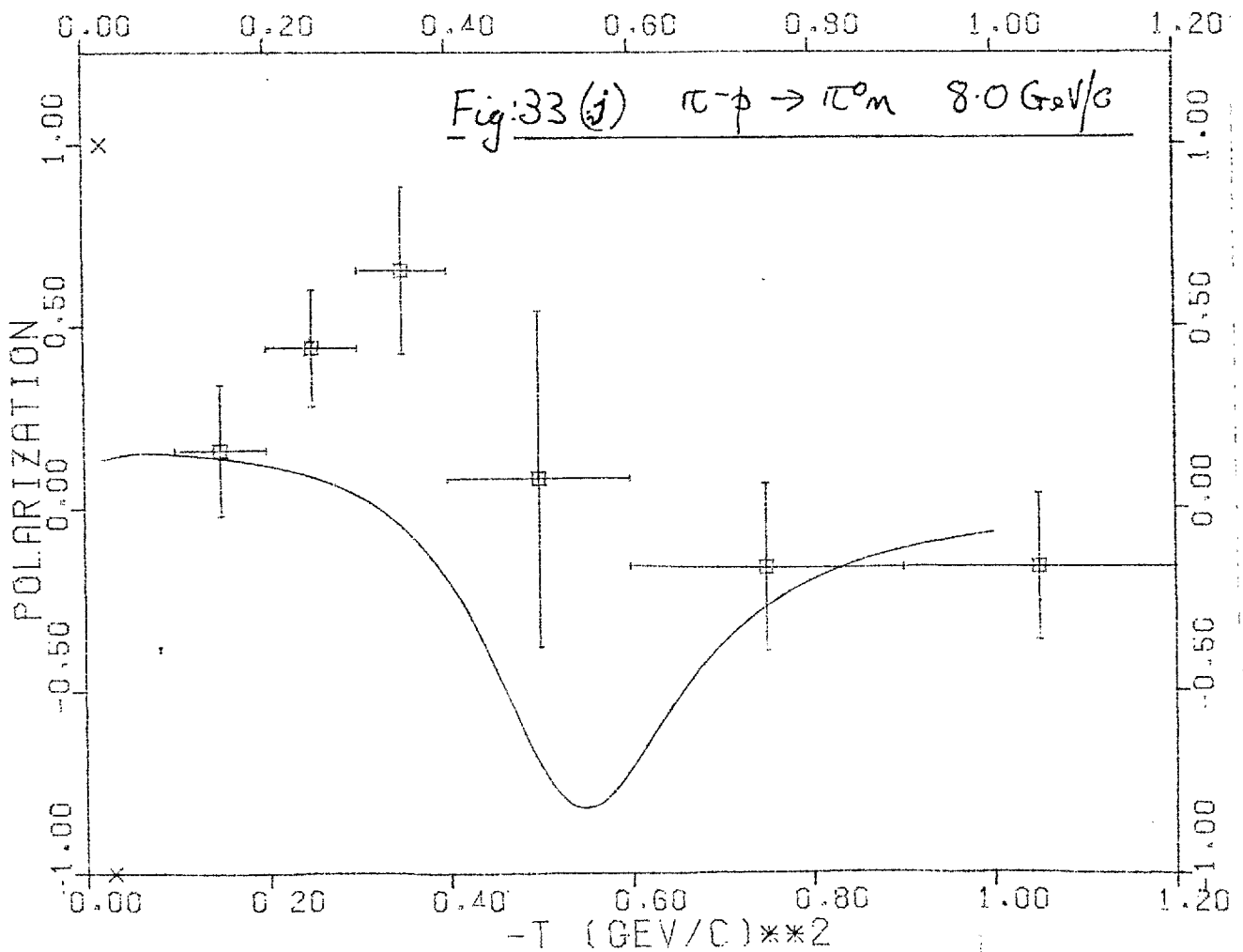
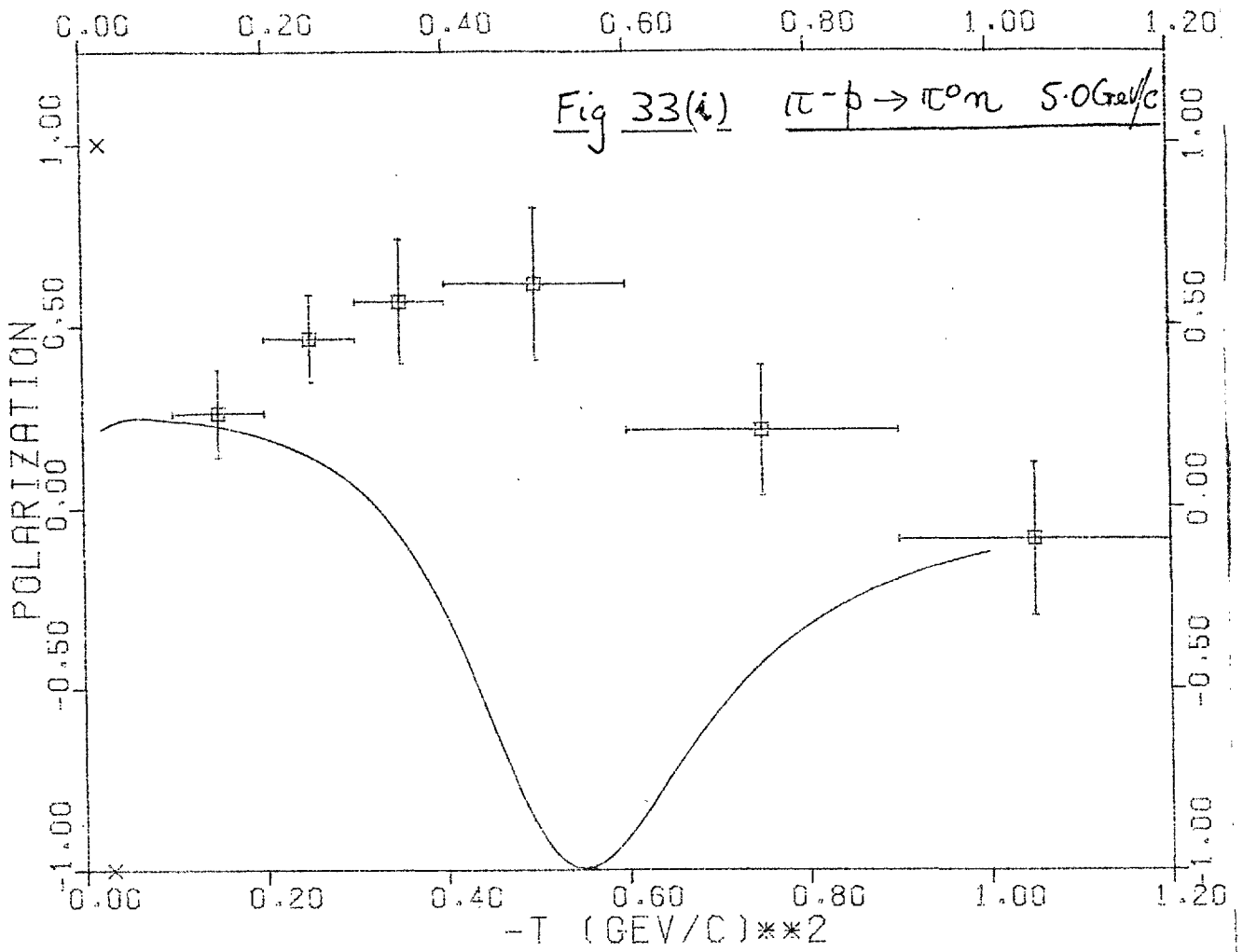


Table 11.

α_0^0	0.58	
$\alpha_0^{A_2}$	0.37	
$\alpha_0^{\rho'}$	- 0.48	
α'^ρ	0.92	
α'^{A_2}	0.41	
$\alpha'^{\rho'}$	1.44	
$C^0\rho$	1.30	mb×GeV
$C^1\rho$	2.92	(GeV) ⁻²
$D^0\rho$	22.7	mb
$D^1\rho$	0.26	(GeV) ⁻²
$C^0\rho'$	5.02	
$C^1\rho'$	4.4	
$D^0\rho'$	- 264.0	
$D^1\rho'$	2.95	
$C^0_{A_2}$	5.50	
$C^1_{A_2}$	0.42	
$D^0_{A_2}$	- 116.0	
$D^1_{A_2}$	0.66	
$F^0\rho$	1.1	
$F^1\rho$	- 0.006	(GeV) ⁻²
$F^0\rho'$	0.8	
$F^1\rho'$	0.2	
$F^0_{A_2}$	1.01	
$F^1_{A_2}$	- 0.07	

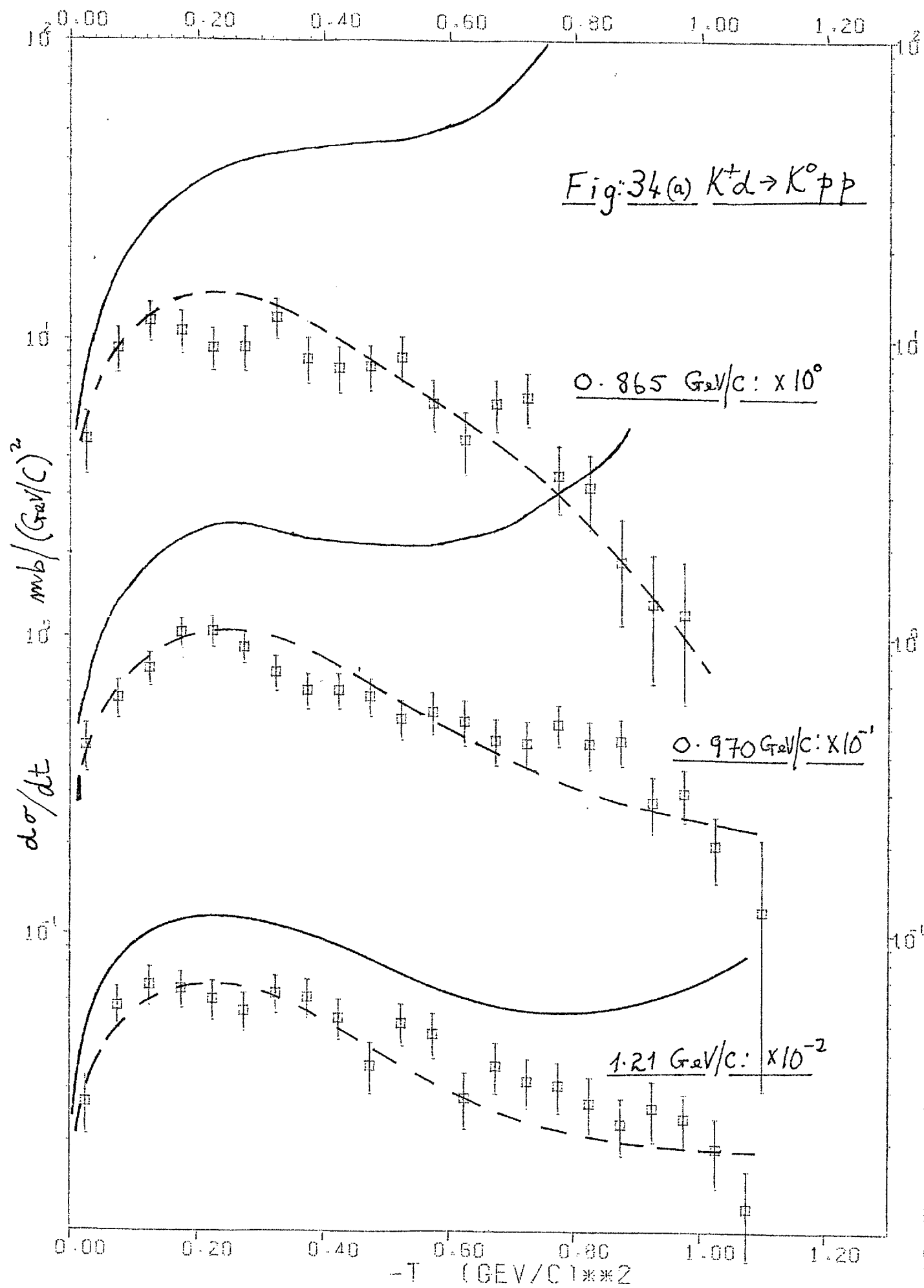
As can be seen, this fit is essentially identical to the original fit of ref. 1, the polarisation data being only a very weak constraint. It does, however, indicate that the assumption of SU_3 breaking is unnecessary.

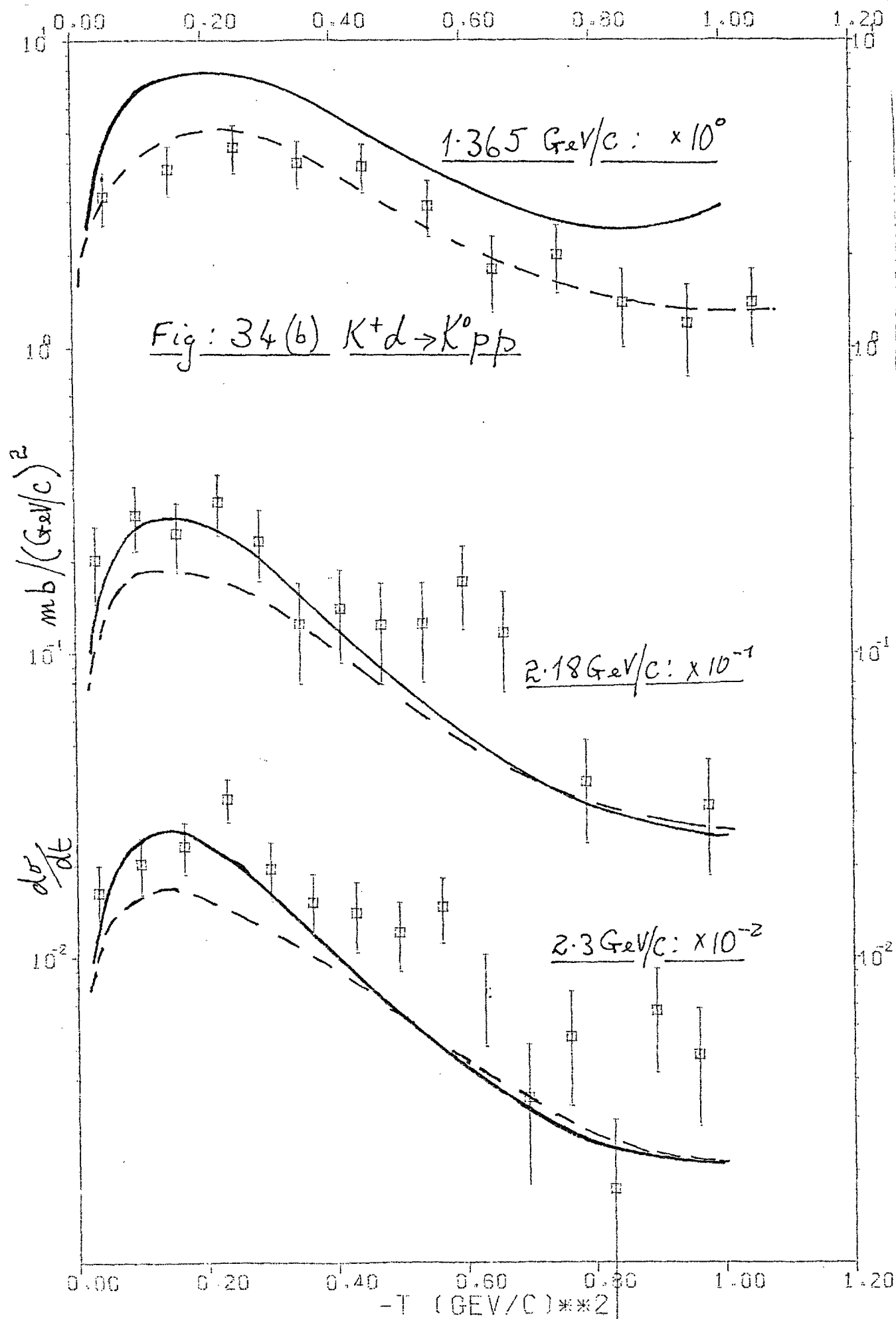
Because the use of the exact Legendre formalism plus the low energy K^+n data lead to computing times of the order of 15,000 seconds (CP time on CDC 6600) the low energy data was first used in a fit employing the asymptotic amplitudes. This again lead to very little change from the original fitted values. In fact the only result was to slightly degrade the fit to the K^+n data above 2 GeV/c. This fit is shown by the dotted lines in Fig. 34. Only the K^+n is shown as the other processes changed very little. The fitted parameters are not tabulated as they did not differ in any important respect from those of Table 12.

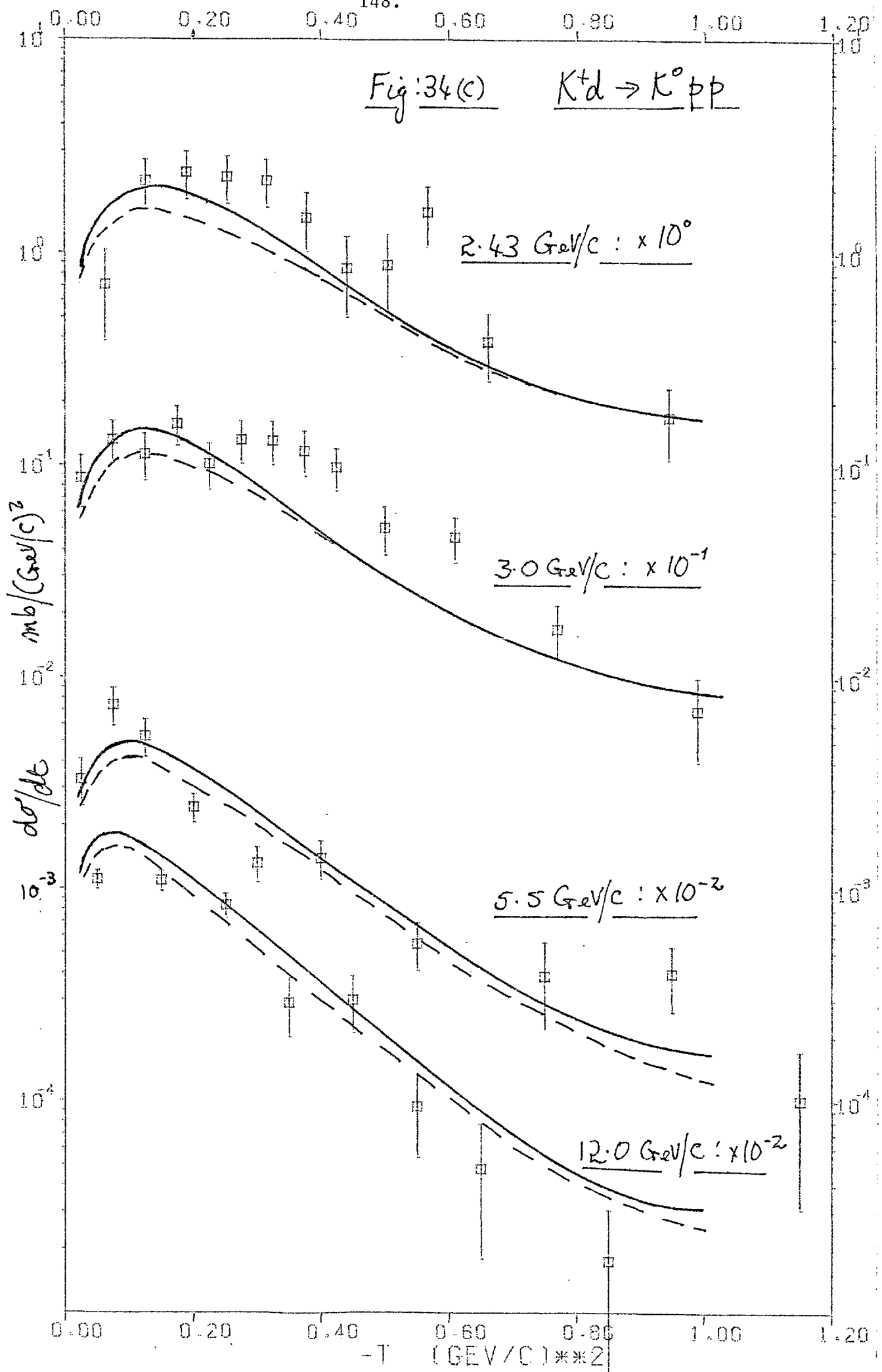
To investigate the effect of the low energy K^+n data on the full Legendre amplitudes a fit was performed to all the K^+n data alone. This is shown in Fig. 35 and Table 13.

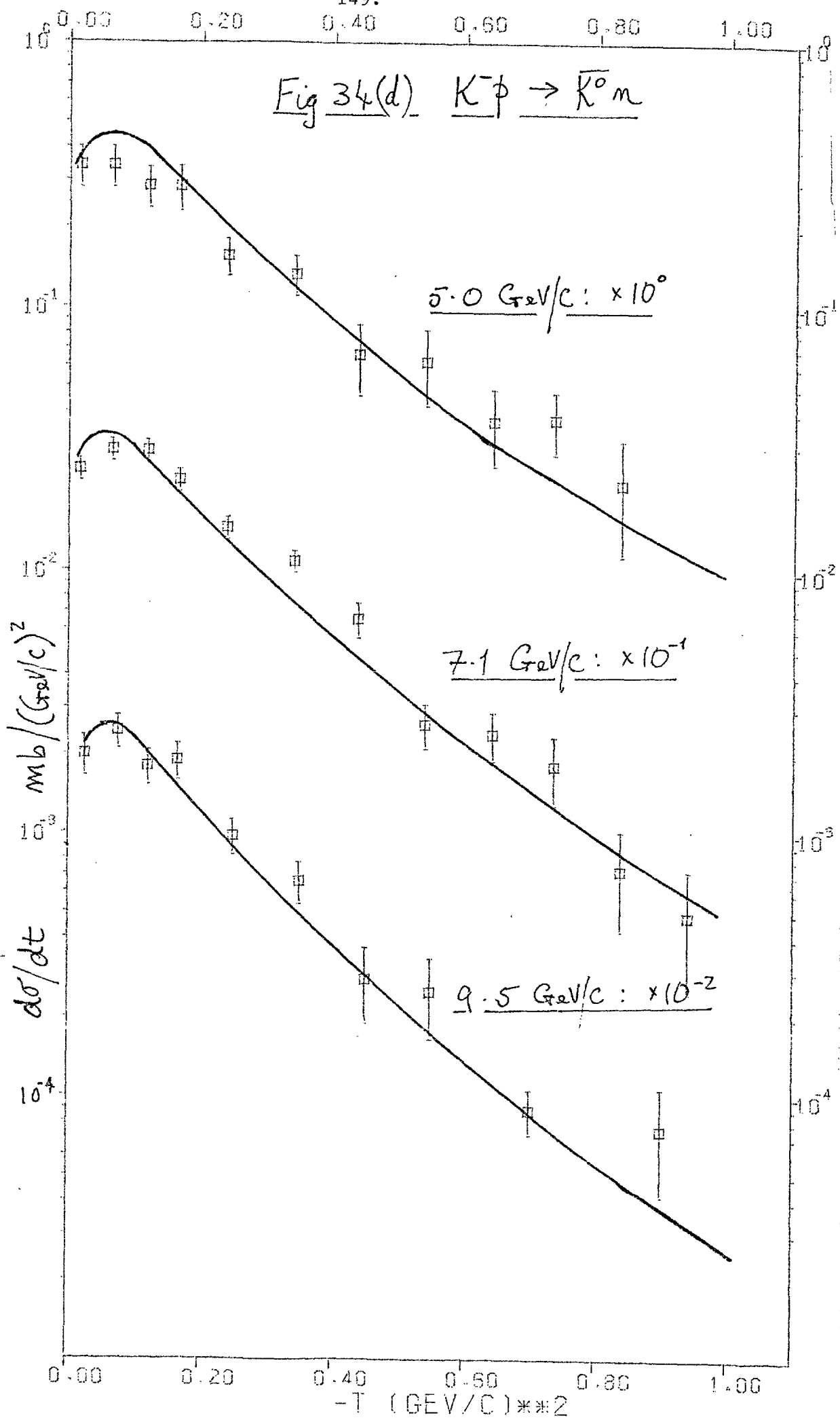
This demonstrates that the low energy data completely changes the structure of the fit.

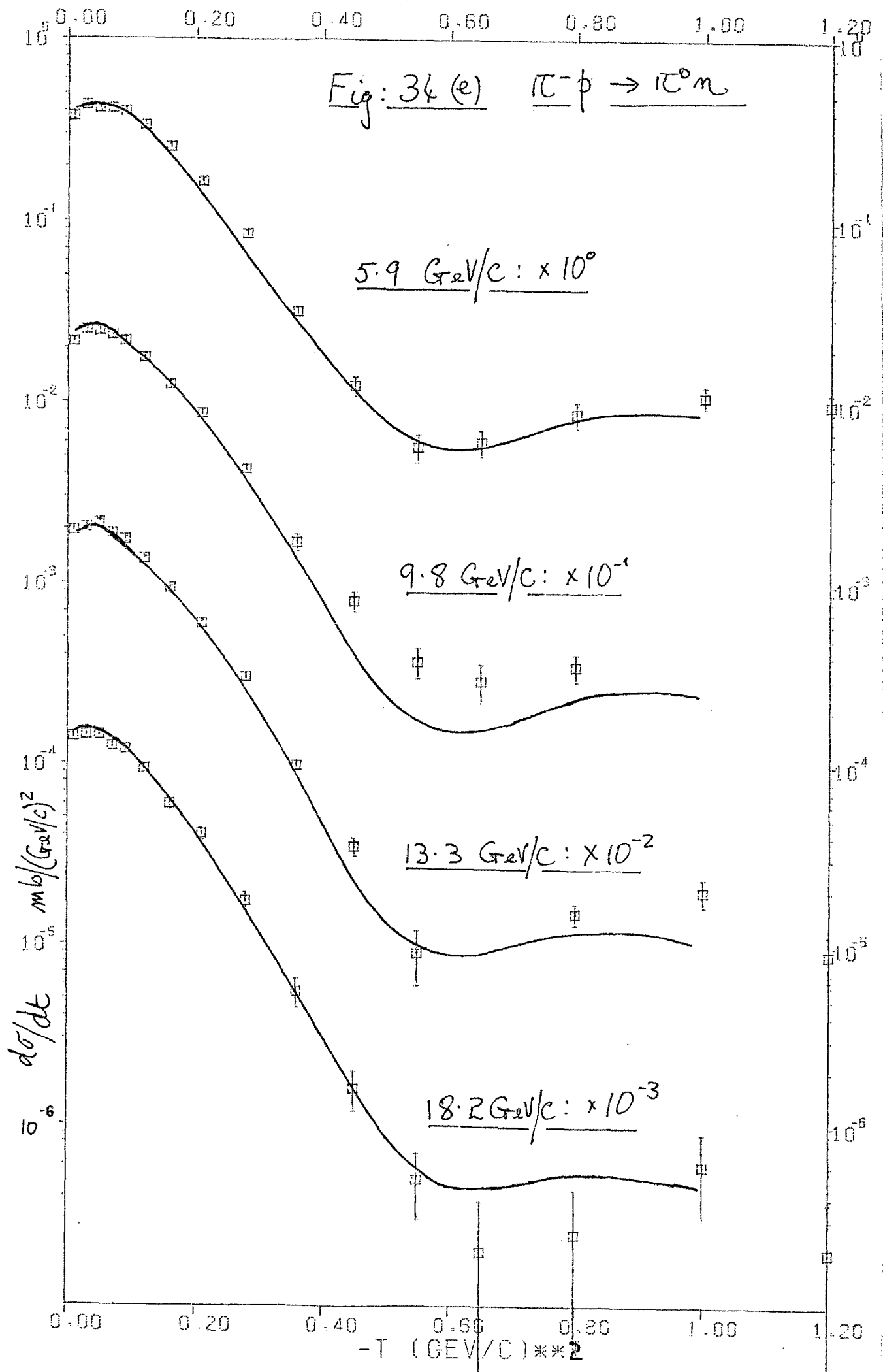
The ρ' has become much shallower and higher lying, and its non flip residue has changed sign. In addition the A_2 flip residue has become small and positive as opposed to large and negative. Of course fitting only to the K^+n data completely destroyed the fit to other processes.

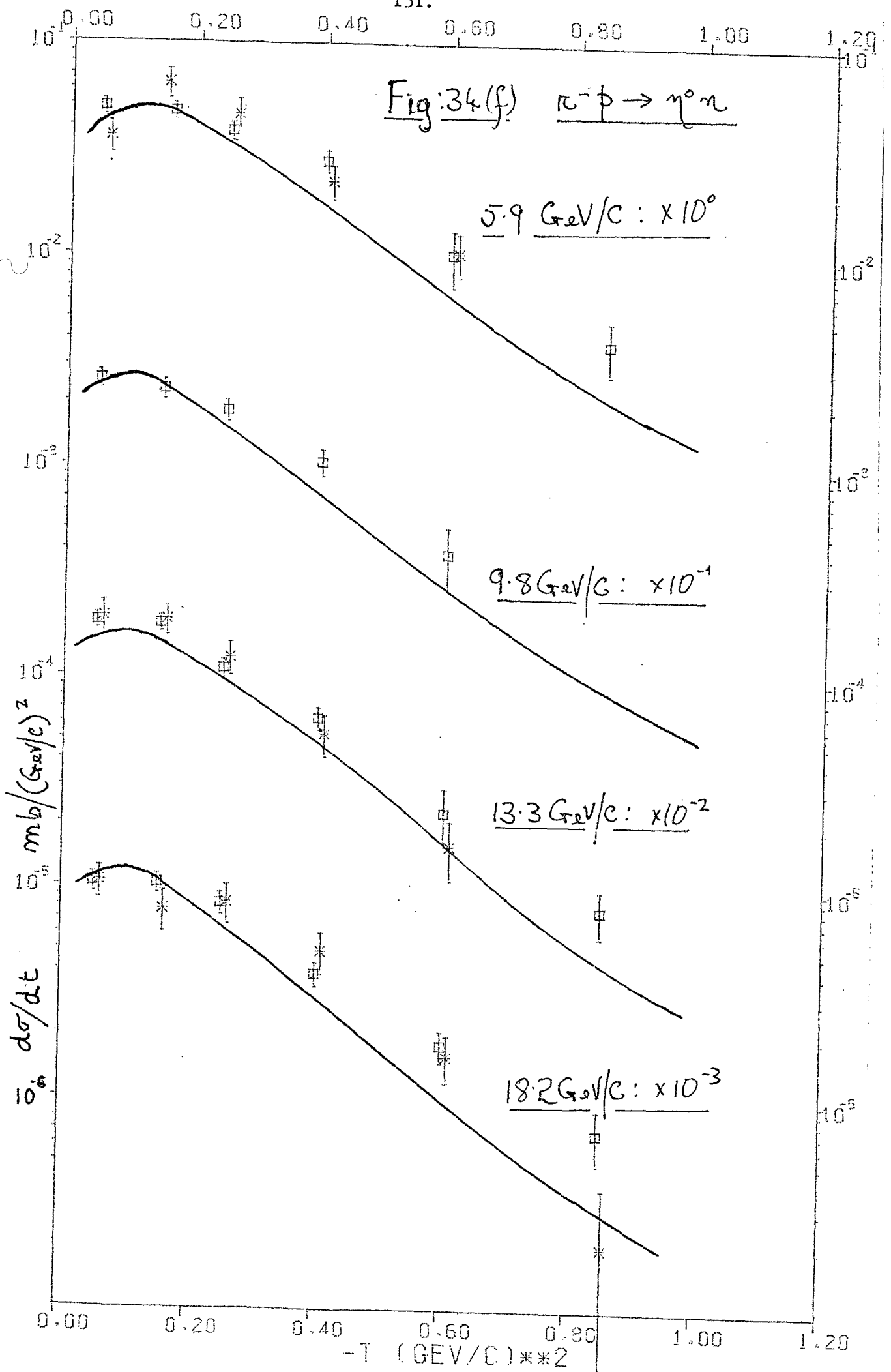


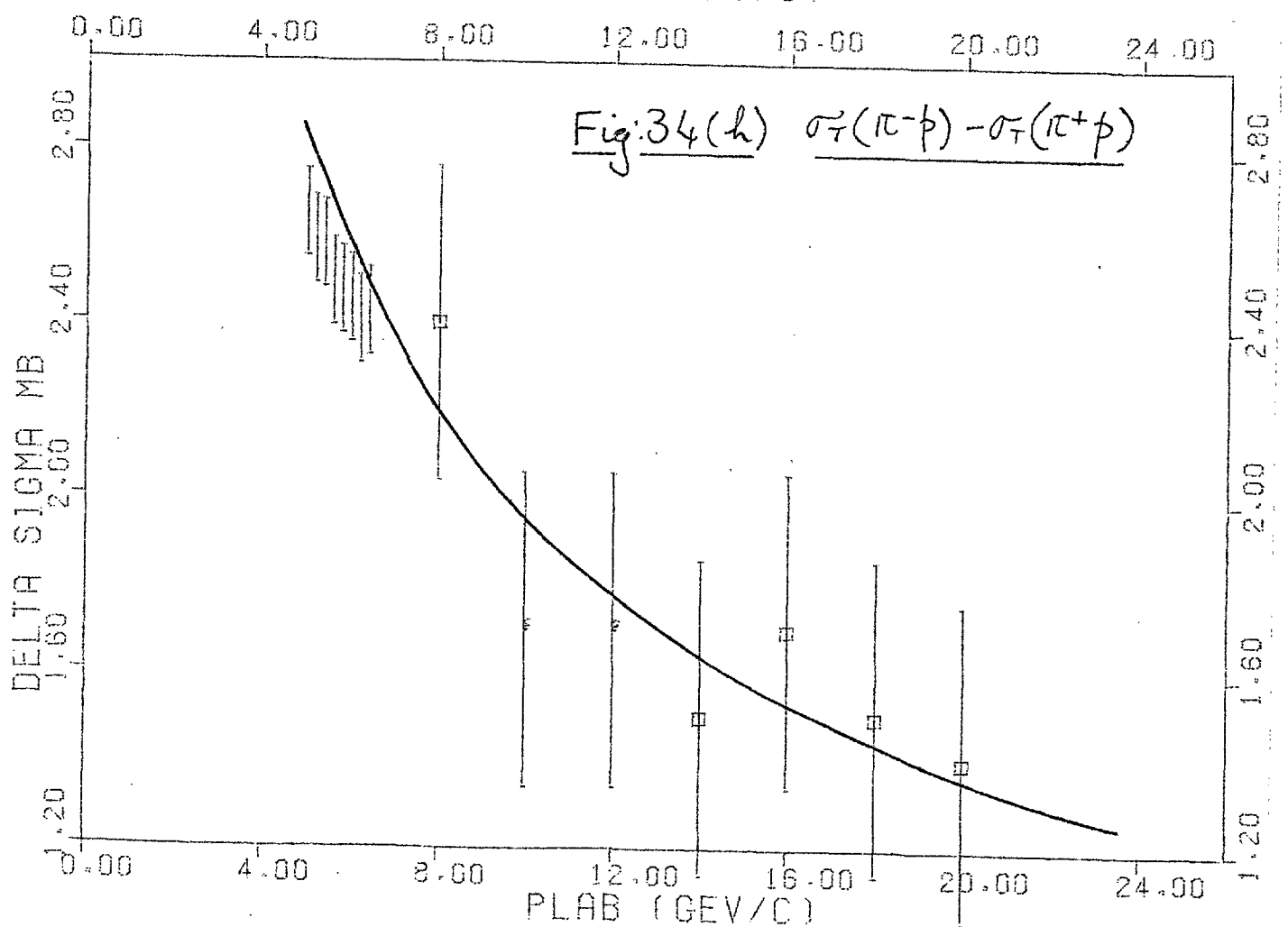
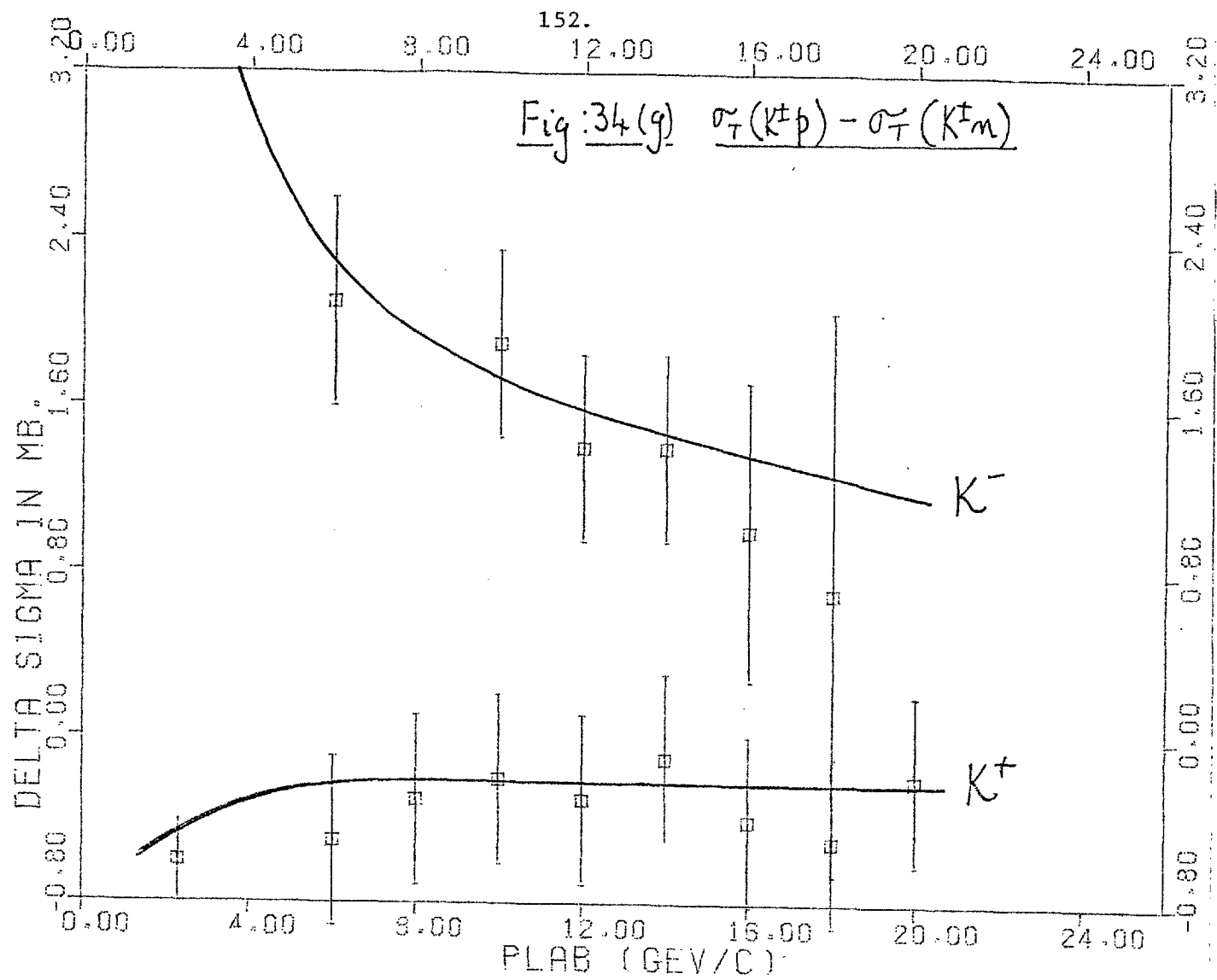












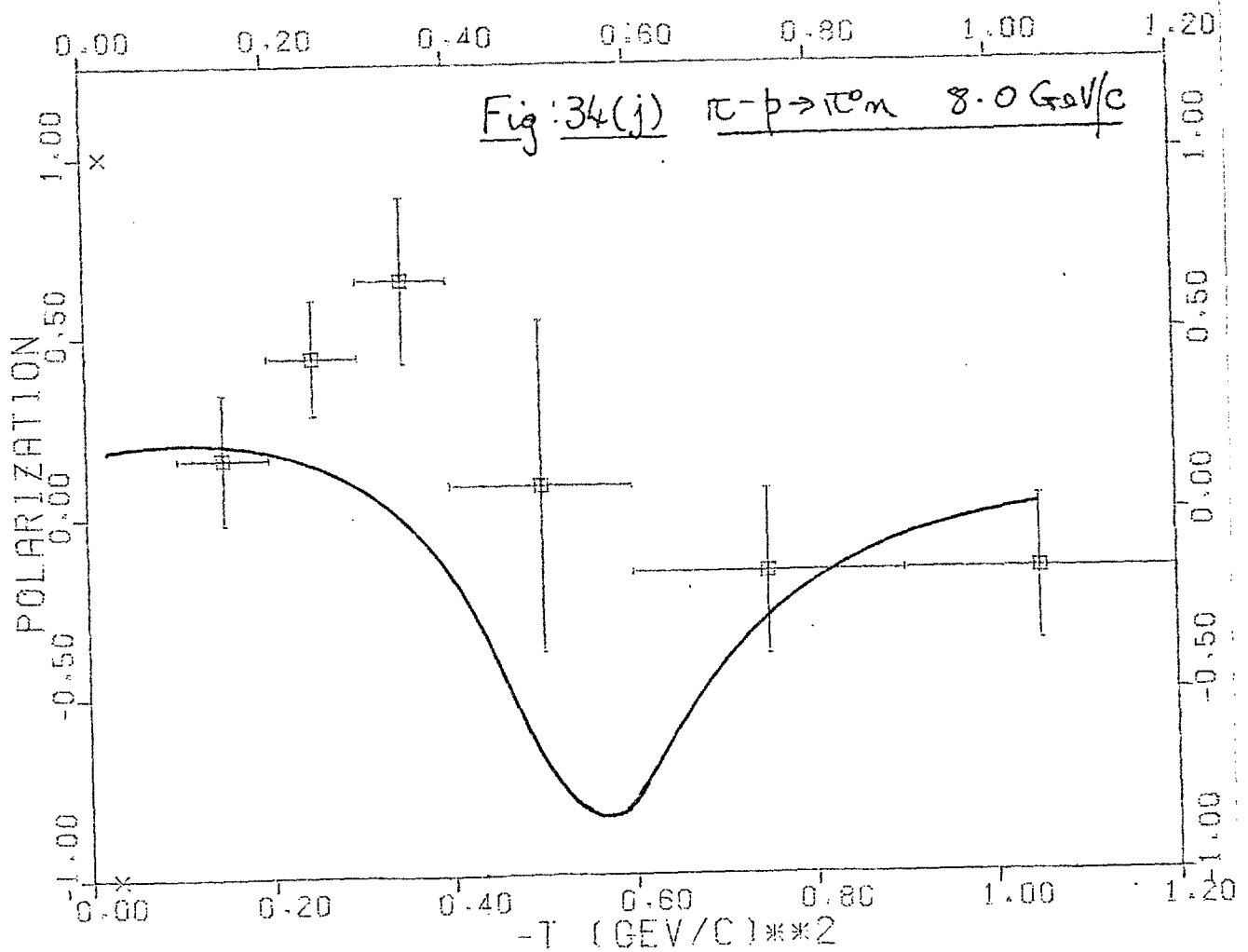
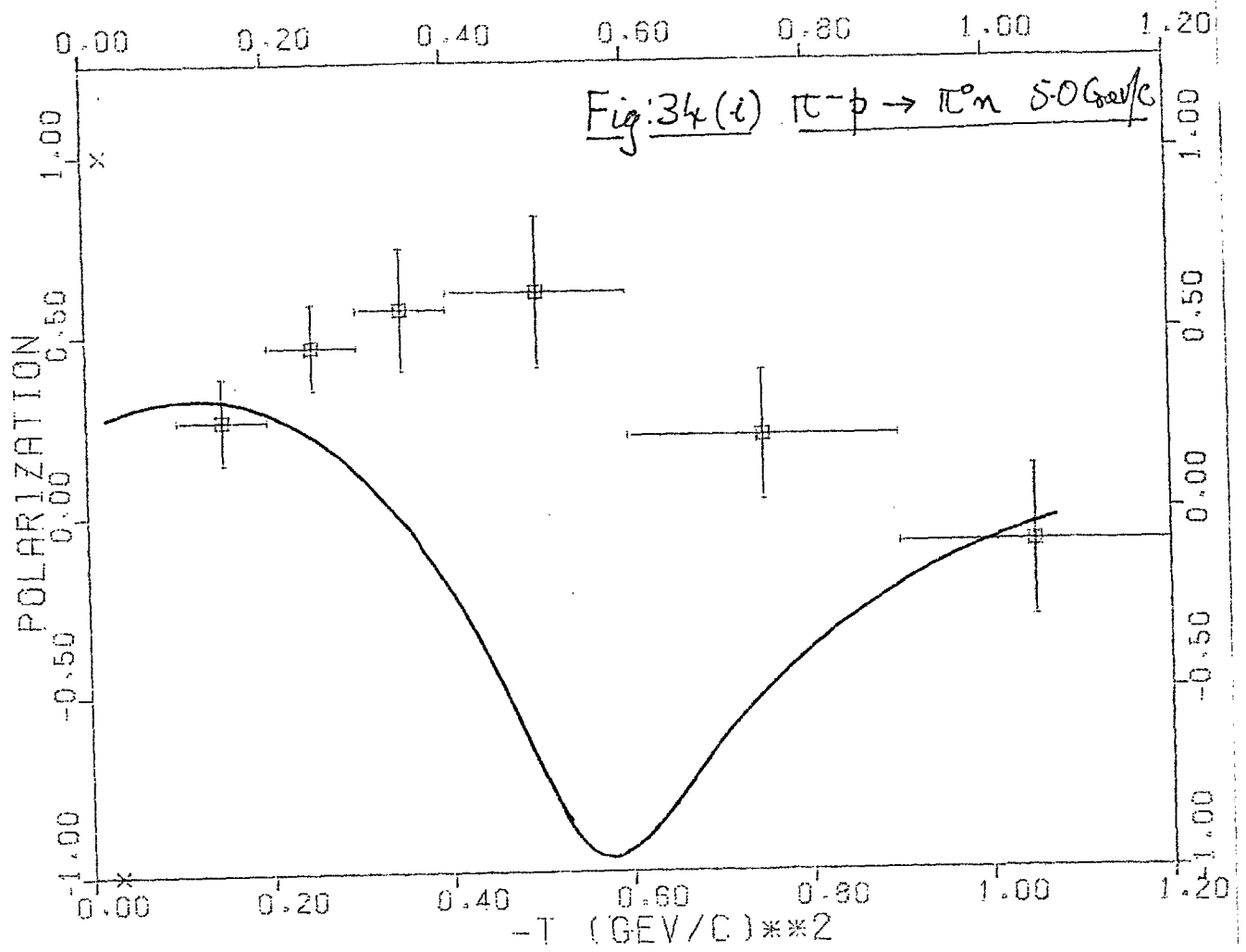
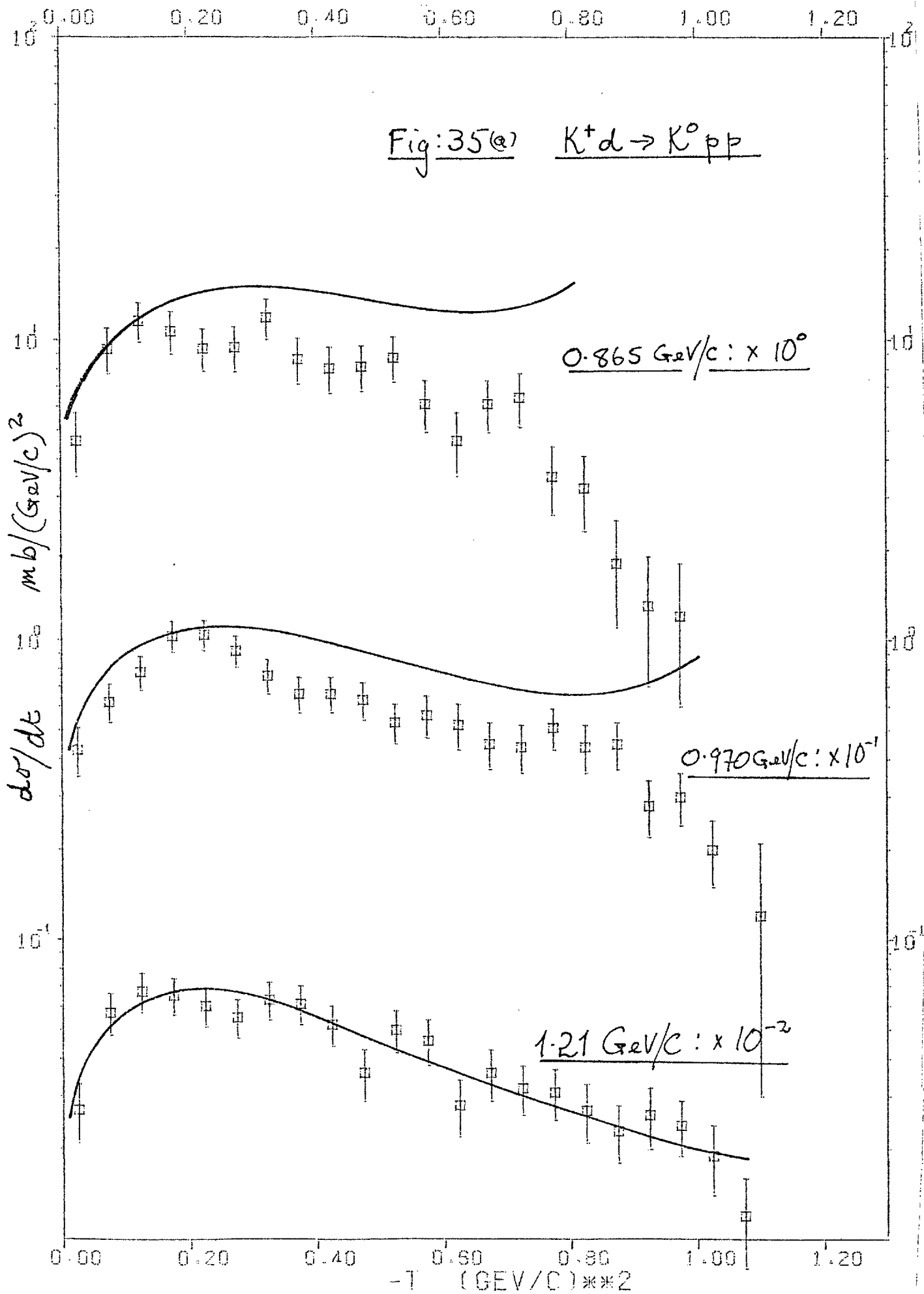
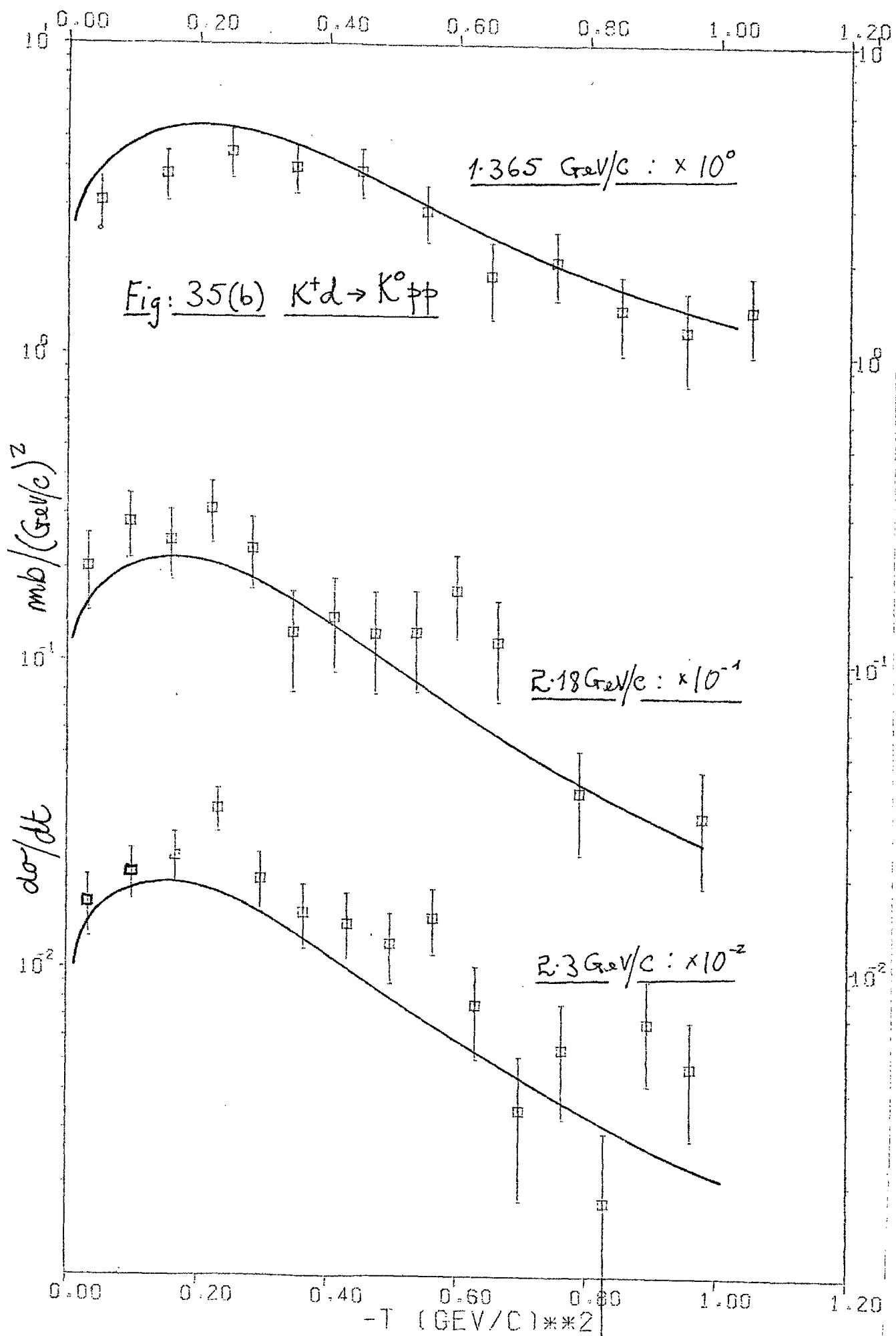


Table 12 $\chi^2/\text{data point} = 2.3.$

$\alpha_0 \rho$	0.59 ± 0.001
$\alpha_0 \rho'$	-0.49 ± 0.002
$\alpha_0 A_2$	0.37 ± 0.002
$\alpha' \rho$	0.92 ± 0.001
$\alpha' \rho'$	1.40 ± 0.005
$\alpha' A_2$	0.39 ± 0.006
$C^0 \rho$	1.38 ± 0.002
$C^1 \rho$	2.76 ± 0.01
$D^0 \rho$	24.4 ± 0.01
$D^1 \rho$	0.30 ± 0.004
$C^0 \rho'$	4.76 ± 0.05
$C^1 \rho'$	7.87 ± 0.1
$D^0 \rho'$	$-246. \pm 2.0$
$D^1 \rho$	3.33 ± 0.03
$C^0 A_2$	5.69 ± 0.02
$C^1 A_2$	0.59 ± 0.019
$D^0 A_2$	$-119. \pm 1.0$
$D^1 A_2$	1.38 ± 0.06

NB "Errors" correspond to 1% changes in χ^2 .





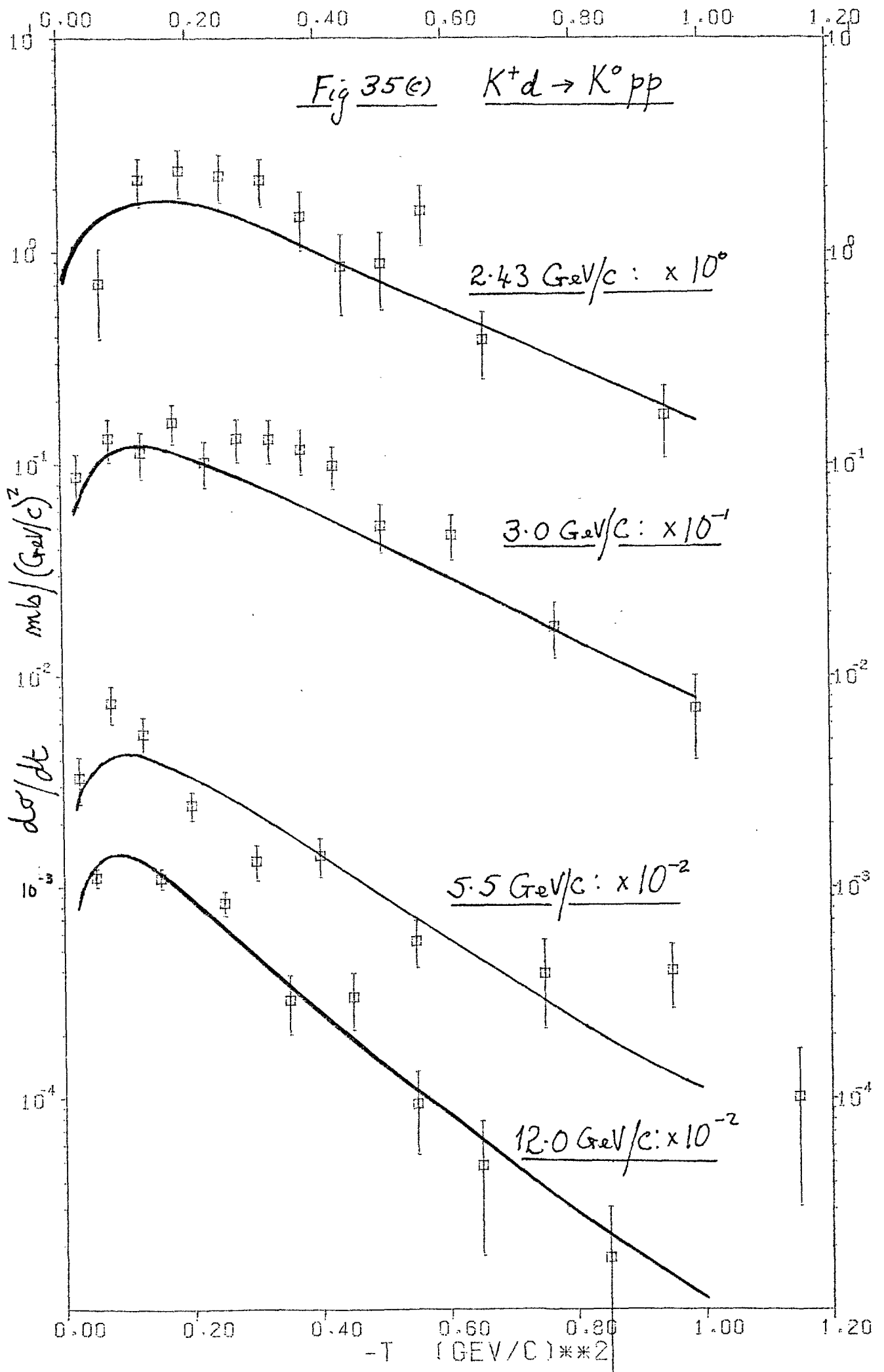


Table 13 $\chi^2/\text{data point} = 1.2.$

$\alpha^0 \rho$	0.58 *
$\alpha^0 \rho'$	$- 0.02 \pm 0.005$
$\alpha^0_{A_2}$	0.37 *
$\alpha^1 \rho$	0.92 *
$\alpha^1 \rho'$	0.45 ± 0.005
$\alpha^1_{A_2}$	0.41 *
$C^0 \rho$	$- 0.02 \pm 0.001$
$C^1 \rho$	2.19 ± 0.08
$D^0 \rho$	30.0 ± 0.07
$D^1 \rho$	1.42 ± 0.08
$C^0 \rho'$	-13.17 ± 0.4
$C^1 \rho'$	0.42 ± 0.06
$D^0 \rho'$	$-533. \pm 2.0$
$D^1 \rho'$	3.48 ± 0.03
$C^0_{A_2}$	$- 0.43 \pm 0.005$
$C^1_{A_2}$	$- 1.46 \pm 0.008$
$D^0_{A_2}$	9.48 ± 0.02
$D^1_{A_2}$	$- 0.004 \pm 0.001$
* fixed parameter.	

Before going on to using a large amount of computer time on an exact fit to the full data set, it was thought reasonable to perform a fit using "semi-exact" amplitudes.

By "semi-exact" is meant that the approximation;

$$P_{\alpha}(\cos \theta_t) \sim \left(\frac{E}{E_0} \right)^{\alpha}$$

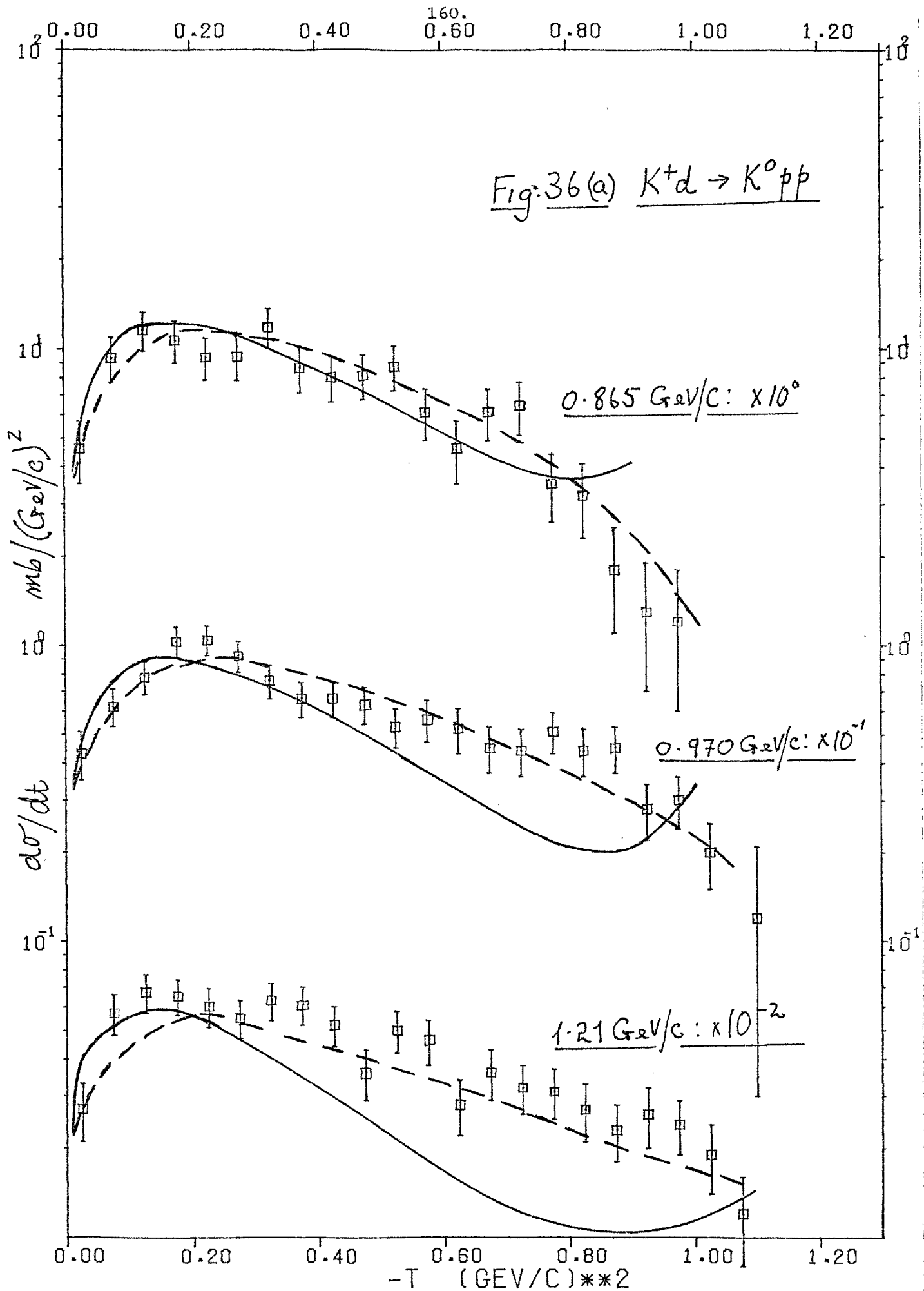
in the asymptotic expressions (5.7), (5.8) is replaced by;

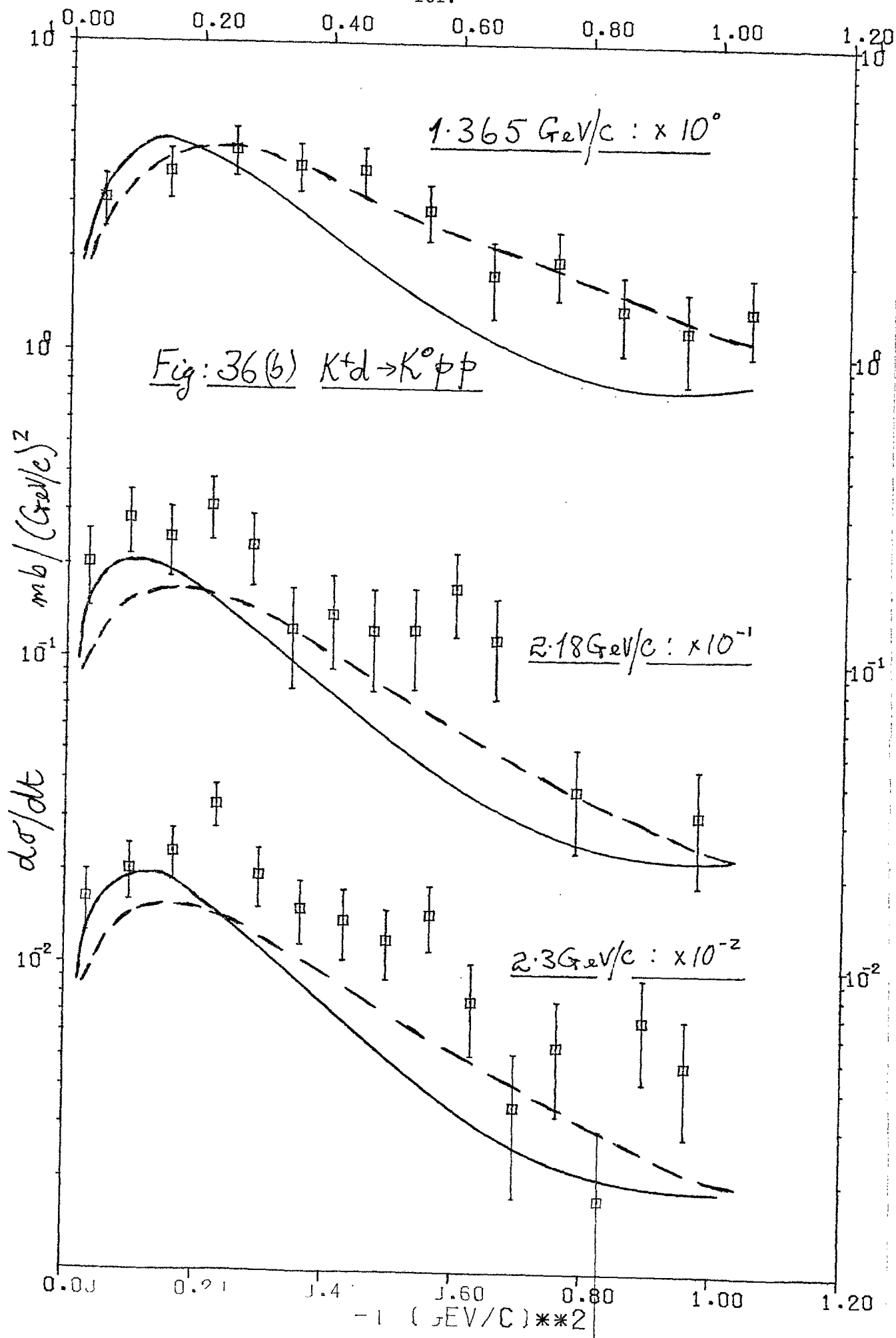
$$P_{\alpha}(\cos \theta_t) \sim (\cos \theta)^{\alpha} = \left(\frac{s - u}{s_0} \right)^{\alpha}$$

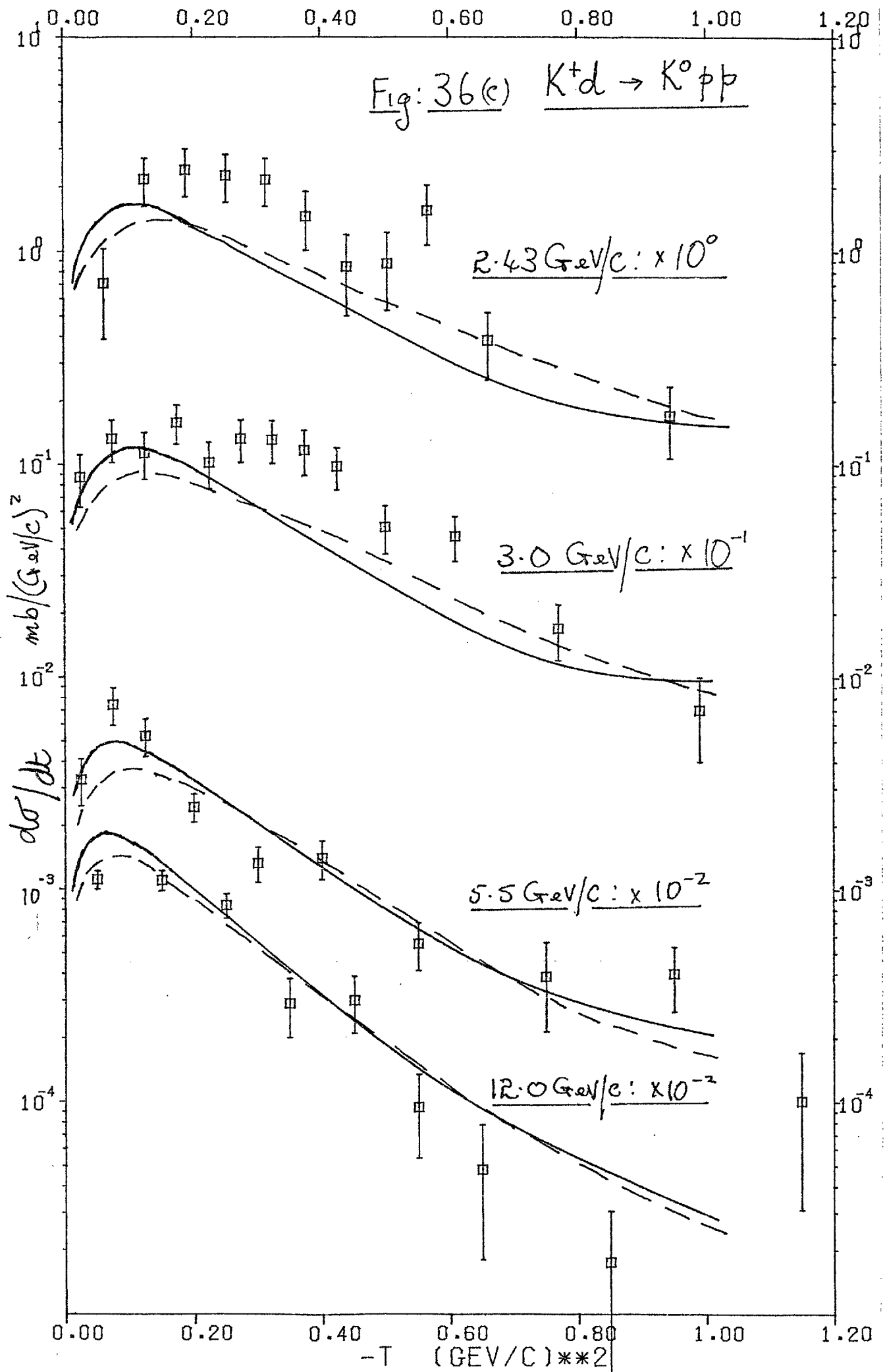
where $\left(\frac{s - u}{s_0} \right) = \left(\frac{s + t/2 - \frac{1}{2} \sum_i m_i^2}{s_0} \right)$.

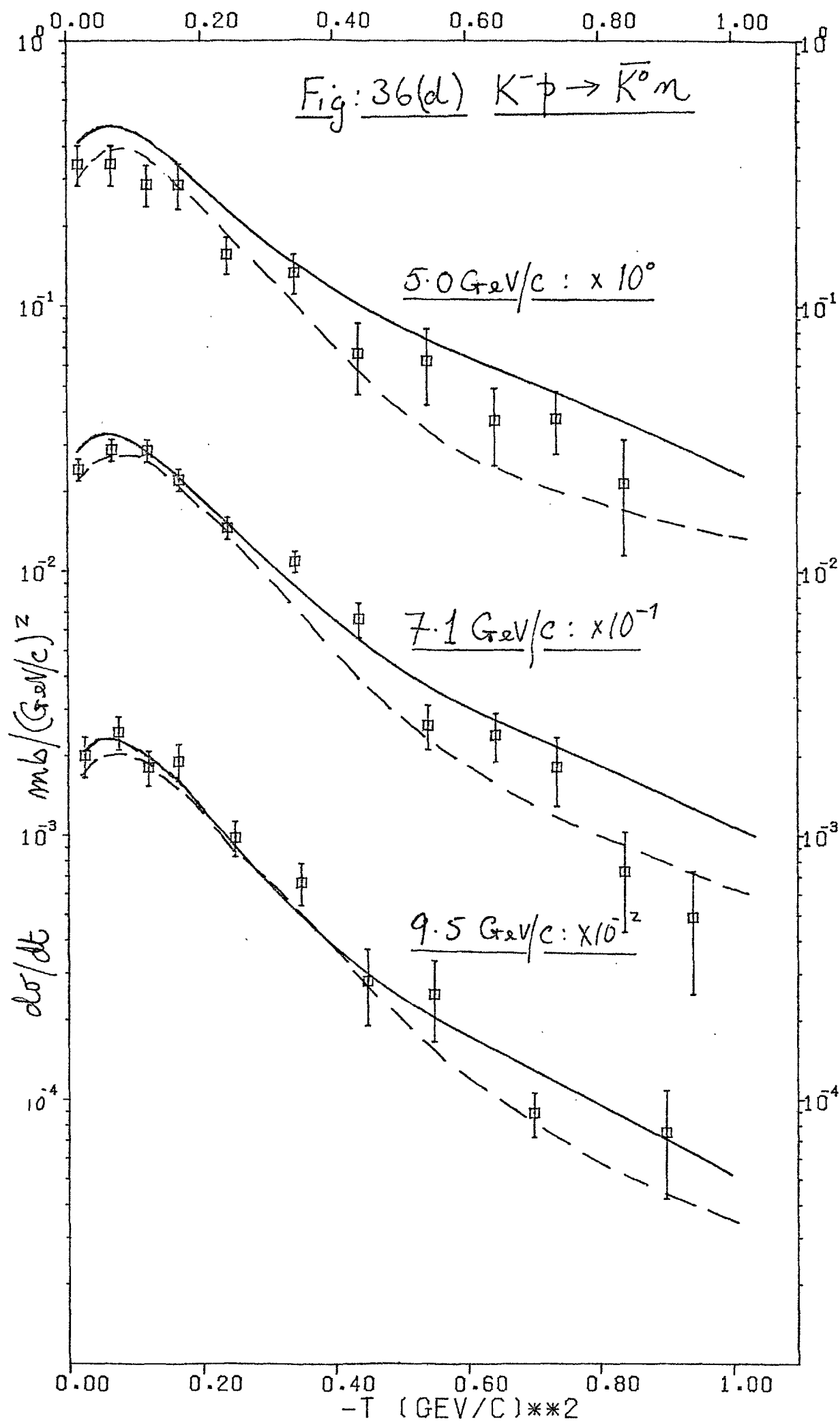
This is a somewhat better approximation, but does not involve large amounts of computer time.

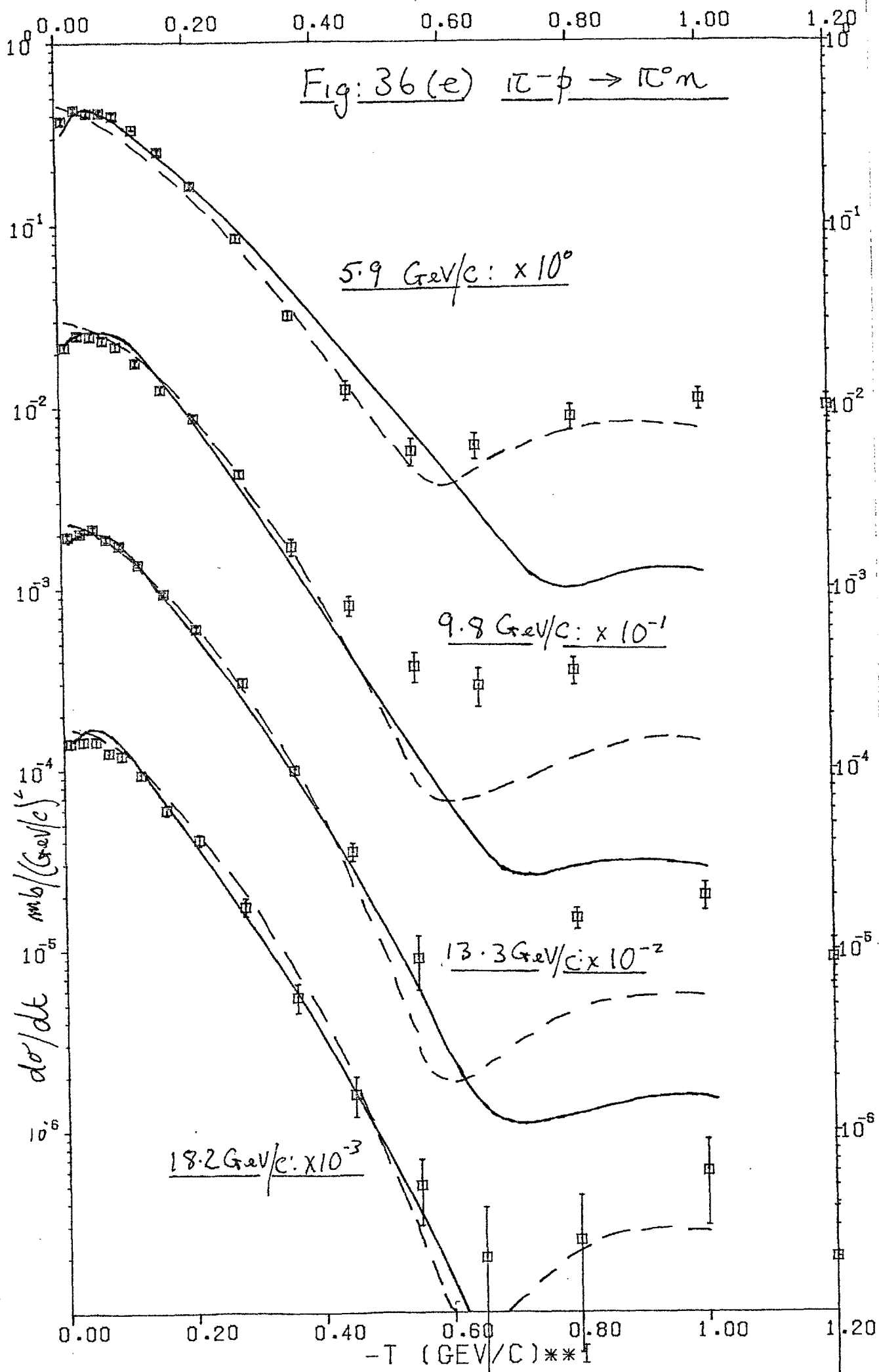
The results obtained are shown dotted in Fig. 36 and Table 14. The ρ and A_2 trajectory functions were fixed parameters and up to 25% SU_3 breaking was allowed. This is different from the original fit of ref. 1 in that the ρ' is much shallower and its non flip residue has changed sign. These differences lead to a more correct representation of the $\pi^- p \rightarrow \pi^0 n$ polarisation, which is now in agreement with the data. These results made it seem useful to perform an exact Legendre fit to the full data set. The results are shown by the full lines in Fig. 36 and by Table 15.

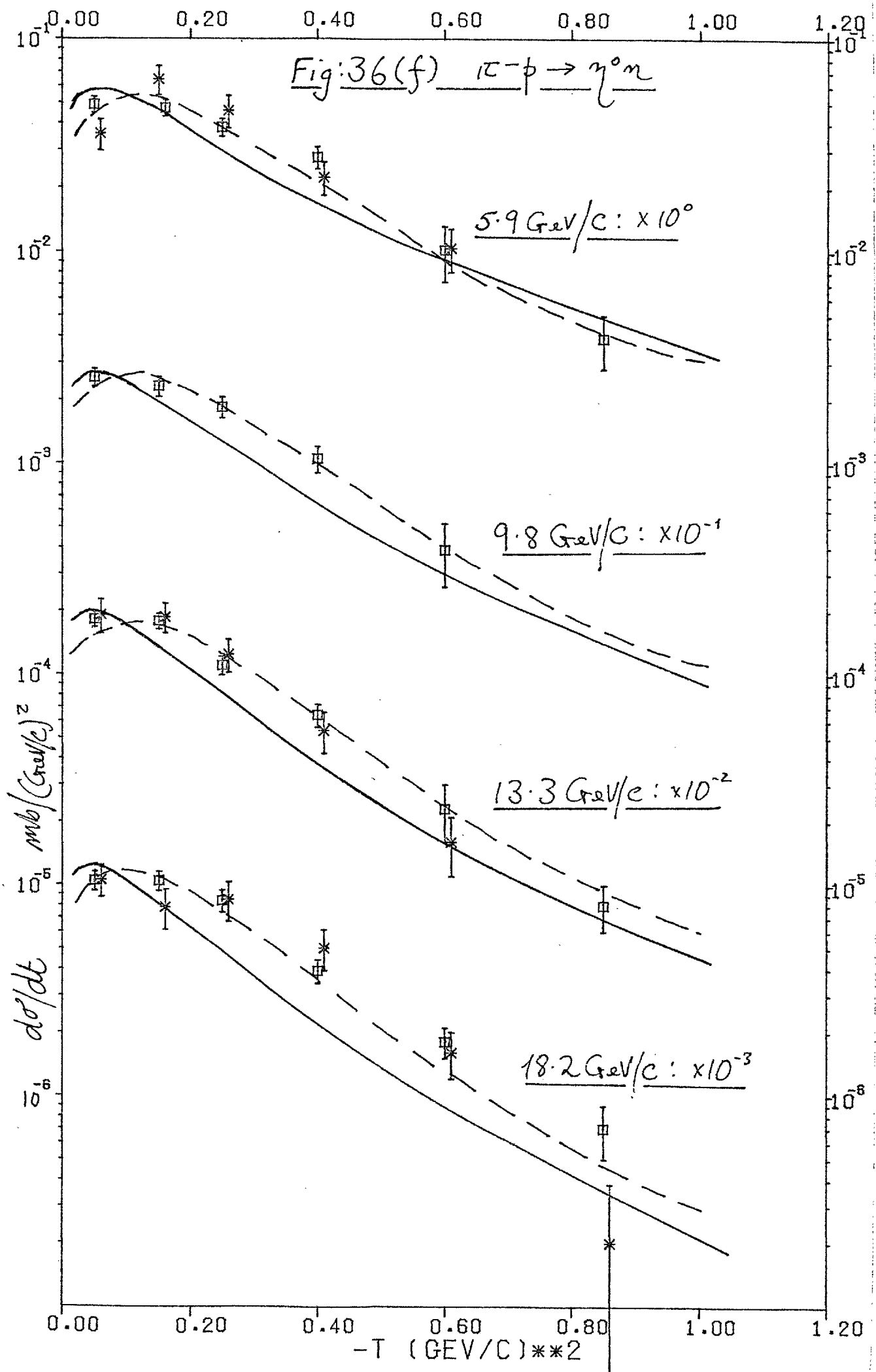


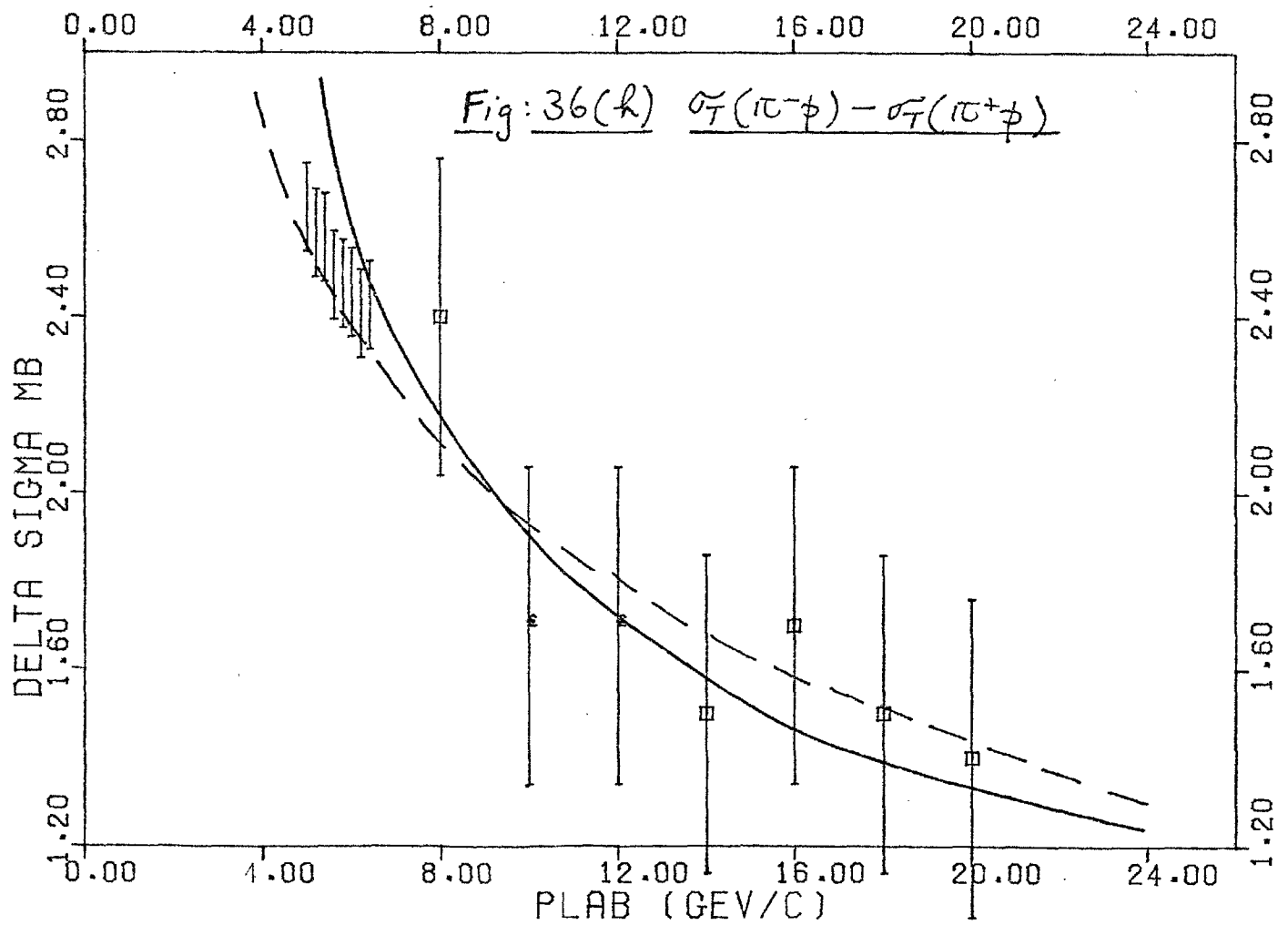
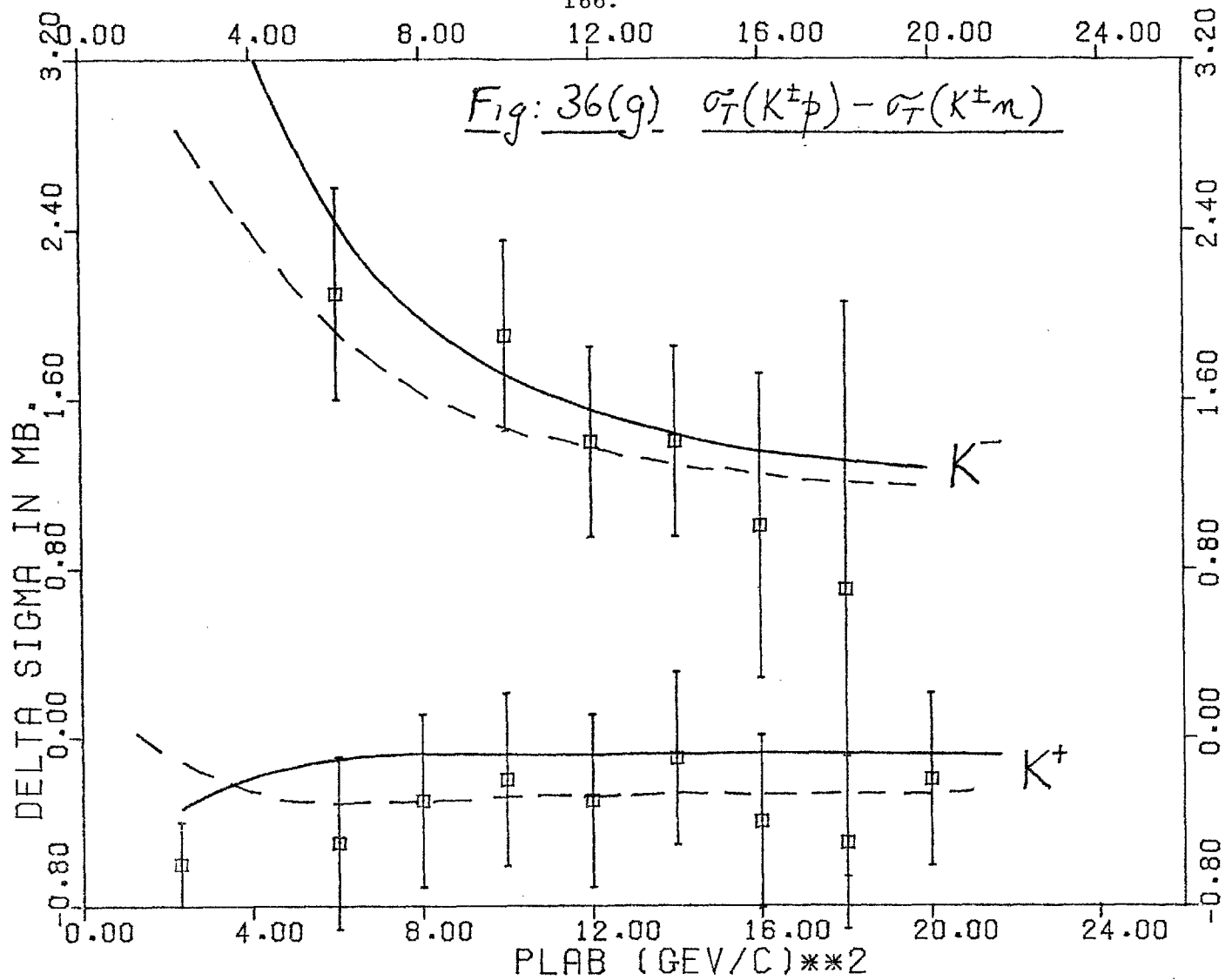












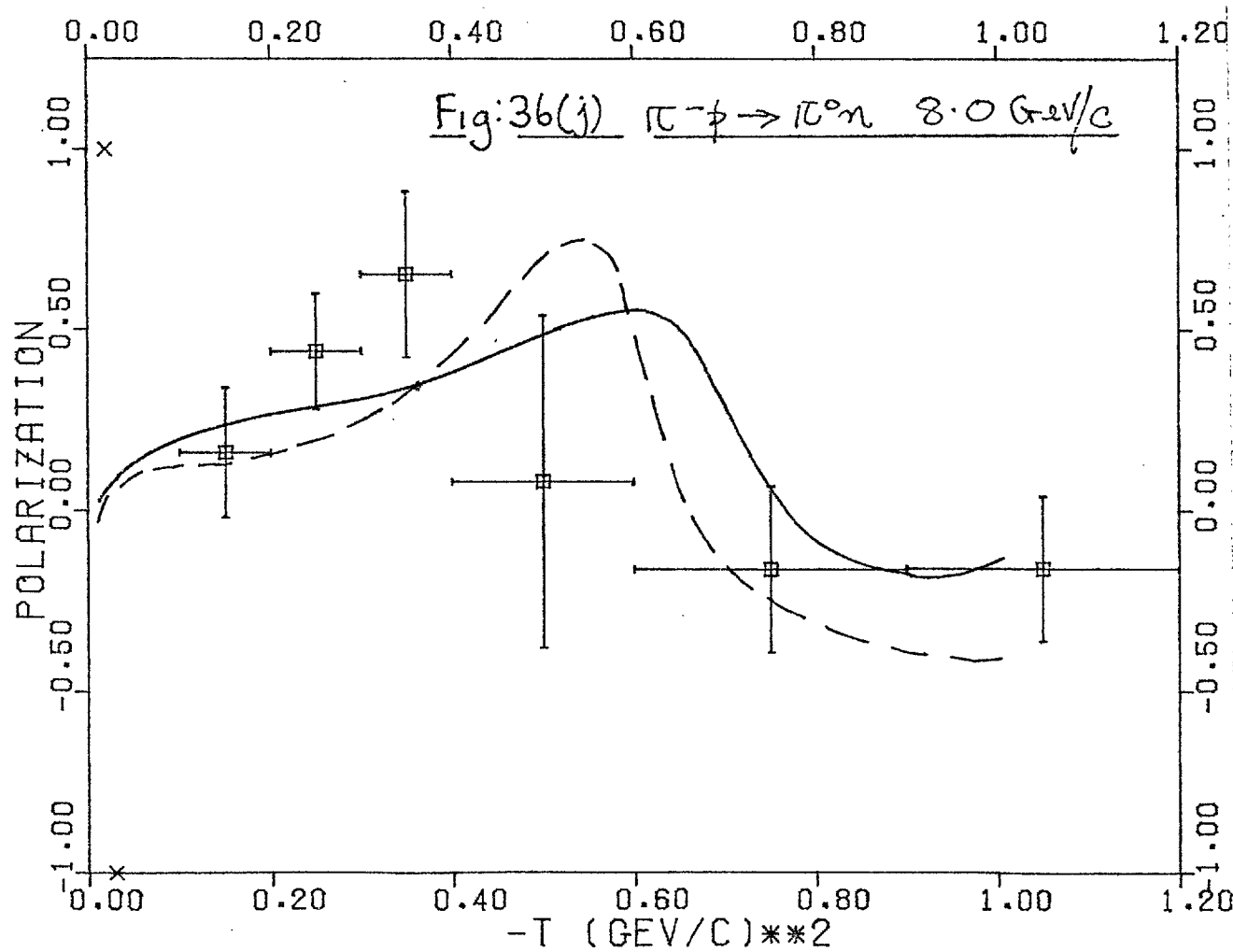
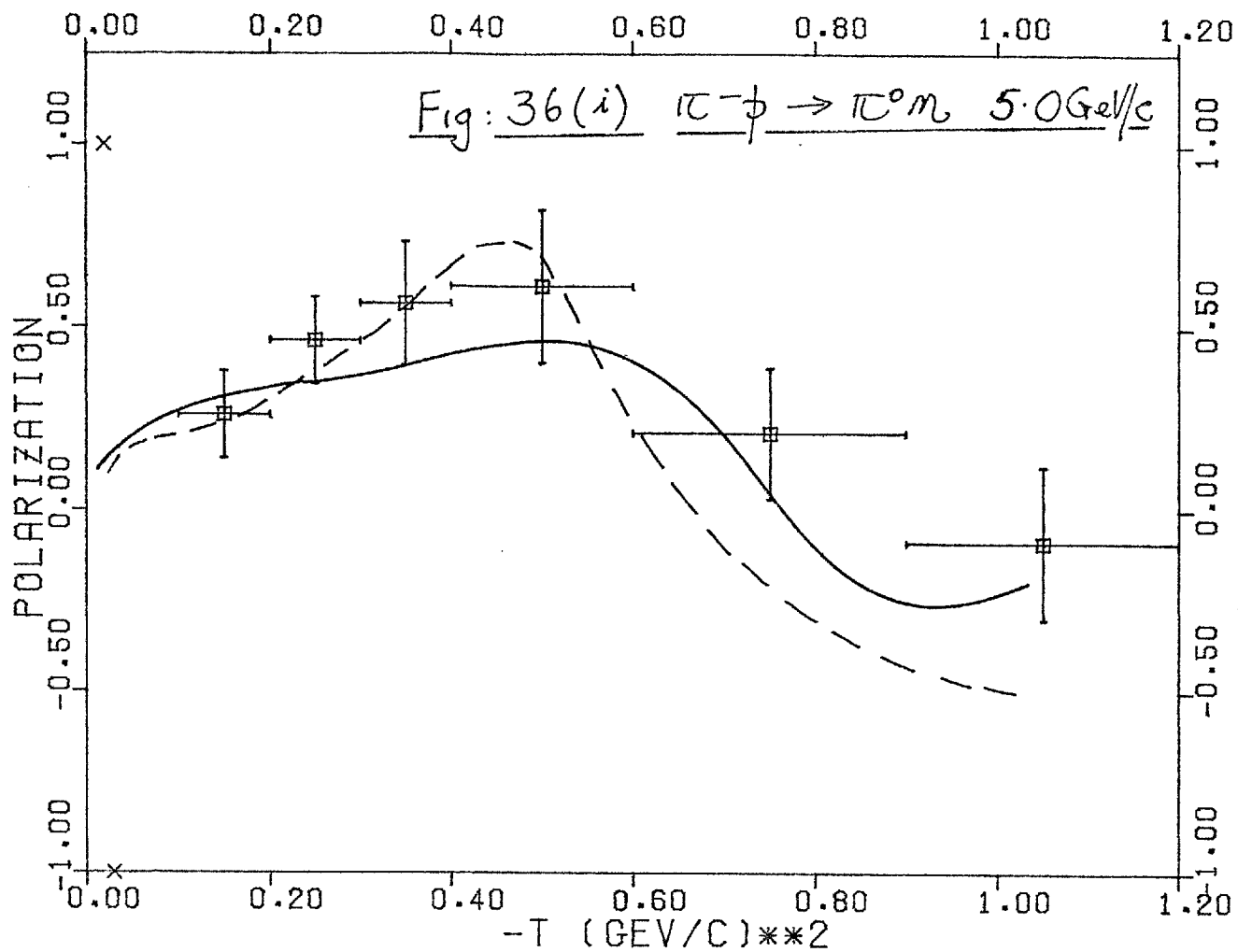


Table 14 $\chi^2/\text{data point} = 2.03.$

$\alpha_0 \rho$	0.58 *
$\alpha_0 A_2$	0.37 *
$\alpha_0 \rho'$	- 0.9 $\pm$ 0.01
$\alpha' \rho$	0.92 *
$\alpha' A_2$	0.41 *
$\alpha' \rho'$	0.41 $\pm$ 0.01
$C^0 \rho$	1.03 $\pm$ 0.008
$C^1 \rho$	1.47 $\pm$ 0.01
$D^0 \rho$	28.2 $\pm$ 0.18
$D^1 \rho$	- 0.27 $\pm$ 0.03
$C^0 \rho'$	- 6.32 $\pm$ 0.4
$C^1 \rho'$	- 0.25 $\pm$ 0.09
$D^0 \rho'$	- 298 $\pm$ 3.4
$D^1 \rho'$	1.28 $\pm$ 0.01
$C^0 A_2$	3.79 $\pm$ 0.01
$C^1 A_2$	- 0.042 $\pm$ 0.020
$D^0 A_2$	- 199 $\pm$ 3.6
$D^1 A_2$	1.24 $\pm$ 0.03
$F^0 \rho$	1.06
$F^1 \rho$	- 0.14
$F^0 \rho'$	0.82
$F^1 \rho'$	- 0.25
$F^0 A_2$	1.04
$F^1 A_2$	- 0.24

constrained

* Fixed Parameter.

Table 15 $\chi^2/\text{data point} = 3.9.$

α_0^{ρ}	$0.60 \pm .001$
$\alpha_0^{A_2}$	$0.35 \pm .002$
$\alpha_0^{\rho'}$	$-0.45 \pm .002$
α^{ρ}	$0.98 \pm .001$
α^{A_2}	$0.62 \pm .005$
$\alpha^{\rho'}$	$0.54 \pm .006$
C^0_{ρ}	$1.28 \pm .002$
C^1_{ρ}	$1.33 \pm .008$
D^0_{ρ}	$22.1 \pm .01$
D^1_{ρ}	$0.73 \pm .004$
$C^0_{\rho'}$	$5.12 \pm .05$
$C^1_{\rho'}$	$17.65 \pm .1$
$D^0_{\rho'}$	-154 ± 2.0
$D^1_{\rho'}$	$3.30 \pm .03$
$C^0_{A_2}$	$7.54 \pm .02$
$C^1_{A_2}$	$-0.20 \pm .02$
$D^0_{A_2}$	$-125 \pm 2.$
$D^1_{A_2}$	$1.16 \pm .05$
Exact SU_3 assumed	

This again is similar to the Rarita Schwartzschild fit, the only major difference being the rather flat ρ' . The major change is that the $\pi^- p \rightarrow \pi^0 n$ polarisation is now correctly described, additionally the fit to $K^+ n \rightarrow K^0 p$ is much improved. Of the two fits shown in Fig. 36 the "semi-exact" amplitudes describe the data much better than the exact Legendre amplitudes. This is not solely due to the SU_3 breaking allowed in the former. The fit to $K^+ n \rightarrow K^0 p$ is much better with the "semi exact" formulation, the exact Legendre showing a tendency to turn up at large t . Also the large t behaviour of $d\sigma/dt(\pi^- p \rightarrow \pi^0 n)$ with the exact formulation is unsatisfactory.

5.2.6 Summary and Criticisms

The above shows that the complete data set requires a much shallower ρ' than found in the original fit.

Fitting to the complete data set, including the low energy $K^+ n$, with the "semi-exact" or exact Legendre amplitudes favourably effects the $\pi^- p \rightarrow \pi^0 n$ polarisation. Without the low energy $K^+ n$ data the Rarita Schwartzschild fit is very stable. Although all the data out to $t = -1.0$ GeV/c is reasonably fitted, the exact amplitudes overestimate the large t behaviour of the $K^+ n$ $d\sigma/dt$ at low energies. This is perhaps to be expected as the Regge model is not crossing symmetric and the large t , low s , region is dominated by u -channel effects.

Apart from the introduction of the ill-established ρ' , and the breaking of Duality, the model has other defects. G.V. Dass⁽¹³⁾ et al. have pointed out that the very large ρ' flip coupling is in disagreement with FESR applied to low energy πN data. FESR also indicate that the model has the wrong sign for the A_2 flip coupling.

These objections apart the model is capable of giving a good description of the

$$0^{-}\frac{1}{2}^{+} \rightarrow 0^{-}\frac{1}{2}^{+}$$

charge exchange class of reactions.

5.3 Exchange Degenerate Model with Veneziano Residues

In the preceding section a model with ~ 20 free parameters was treated. To illustrate how much freedom is removed by making various theoretical assumptions a model incorporating features of the Veneziano amplitude is now examined⁽¹⁴⁾.

5.3.1 Parametrisation of Amplitudes

The most important assumption of this model is that of strong exchange degeneracy. This implies that the ρ and A_2 have identical residue functions, which in turn implies the Gell-Mann mechanism.

The simplest amplitudes incorporating the Gell-Mann mechanism are parametrised as⁽¹⁵⁾

$$A_i^{\pm} = \Gamma(1 - \alpha_i(t)) (1 \pm e^{-i\pi\alpha_i}) s^{\alpha_i} \beta_i^A \quad (5.18)$$

$$B_i^{\pm} = \Gamma(1 - \alpha_i(t)) (1 \pm e^{-i\pi\alpha_i}) s^{\alpha_i} \beta_i^B. \quad (5.19)$$

That this parametrisation is identical to the asymptotic limit of the Veneziano amplitude;

$$A_V = \frac{\beta}{\Gamma(\alpha)} \frac{1 \pm e^{-i\pi\alpha}}{\sin \pi \alpha} (as)^{\alpha}$$

can be seen by using

$$\Gamma(\alpha) \Gamma(1-\alpha) = \frac{\pi}{\sin \pi \alpha}$$

$$\text{so} \quad \Gamma(1-\alpha) = \frac{\pi}{\Gamma(\alpha) \sin \pi\alpha}$$

also the Veneziano model gives

$$a = 1/\alpha' .$$

Taking trajectories to be of unit slope gives the energy dependence s^α .

In order to make the incorporation of exact exchange degeneracy convenient a different sign convention is adopted from that used in (5.1) - (5.4), viz.

$$A(K^+ n \rightarrow K^0 p) = -K_\rho^A - K_{A_2}^A \quad (5.20)$$

$$A(K^- p \rightarrow \bar{K}^0 n) = K_\rho^A - K_{A_2}^A \quad (5.21)$$

$$A(\pi^- p \rightarrow \pi^0 n) = \pi_\rho^A \quad (5.22)$$

$$A(\pi^- p \rightarrow \eta^0 n) = \pi_{A_2}^A . \quad (5.23)$$

As far as isospin is concerned this is completely valid. The only thing known about the signs of contributing amplitudes is that even signature trajectories do not change sign in line reversal, while odd ones do. However, this choice of sign convention does have implications for SU_3 .

In terms of the F and D amplitudes⁽¹⁶⁾ of definite G-parity (i.e. $\rho + A_2$ amplitudes in present case) the sign convention chosen by Rarita-Schwartzschild implies;

$$A(K^+ n \rightarrow K^0 p) = -F + D$$

$$A(K^- p \rightarrow \bar{K}^0 n) = +F + D .$$

The present sign convention reverses the sign of the D amplitude; so

$$\begin{aligned}
A(K^+ n \rightarrow K^0 p) &= -F - D \\
A(K^- p \rightarrow \bar{K}^0 n) &= +F - D .
\end{aligned}$$

This has the effect of changing the signs on the F and D SU_3 factors, so

$$A(\pi^- p \rightarrow \pi^0 n) = -\sqrt{2} F = -\sqrt{2} A_\rho$$

now becomes;

$$A(\pi^- p \rightarrow \pi^0 n) = +\sqrt{2} F = +\sqrt{2} A_\rho$$

and,

$$A(\pi^- p \rightarrow \pi^0 n) = +\sqrt{2}/3 D = +\sqrt{2}/3 A_{A_2}$$

now becomes;

$$A(\pi^- p \rightarrow \pi^0 n) = -\sqrt{2}/3 D = -\sqrt{2}/3 A_{A_2} .$$

Considering only (ρ, A_2) contributions to $K^+ n \rightarrow K^0 p$, the non flip invariant amplitude in (5.18) becomes;

$$\begin{aligned}
A_{K^+ n \rightarrow K^0 p} &= -\Gamma(1 - \alpha_\rho(t)) (1 - e^{-i\pi \alpha_\rho}) s^{\alpha_\rho} \beta_\rho^A \\
&\quad - \Gamma(1 - \alpha_{A_2}(t)) (1 + e^{-i\pi \alpha_{A_2}}) s^{\alpha_{A_2}} \beta_{A_2}^A .
\end{aligned}$$

Strong exchange degeneracy requires that;

$$\alpha_\rho(t) = \alpha_{A_2}(t) = \alpha_1(t)$$

and;

$$\beta_\rho^A = \beta_{A_2}^A = \beta_1^A .$$

so;

$$A(K^+ n \rightarrow K^0 p) = -2 \Gamma(1 - \alpha_1(t)) s^{\alpha_1} \beta_1^A . \quad (5.24)$$

Since β and α are real in the Veneziano model, (5.24) gives a purely real amplitude for K^+n charge exchange. A similar expression holds for the B invariant amplitude.

For $K^-p \rightarrow \bar{K}^0n$ the amplitude corresponding to (5.24) is;

$$A(K^-p \rightarrow \bar{K}^0n) = -2\Gamma(1-\alpha_1(t)) e^{-i\pi\alpha_1} \times \beta_1 A_s^{\alpha_1}.$$

Thus these line reversed reactions have amplitudes differing only by a phase. This result, stemming from Duality, gives the well-known line reversal relation;

$$\left(\frac{d\sigma}{dt}\right)_{K^+n \rightarrow K^0p} = \left(\frac{d\sigma}{dt}\right)_{K^-p \rightarrow \bar{K}^0n}.$$

This is experimentally obeyed at 5.5 GeV/c and above, but is strongly broken in the 2-3 GeV/c region⁽¹⁷⁾. This might be attributable to the breakdown of local duality, since there is resonance production in K^-p , but little or none in K^+n in the low energy region. So one might hope that a model such as the one considered here would be satisfactory for low energy K^+n data, if not for low energy K^-p .

In an attempt to make the model more applicable at low energies the more exact form;

$$\left(\frac{s-u}{s_0}\right)^\alpha$$

has been preferred to:

$$\left(s/s_0\right)^\alpha.$$

5.3.2 Trajectories and Fits

The experimentally determined trajectories of the ρ and the A_2 are not exactly degenerate, and in fact vary from process to process⁽¹⁸⁾. Since this model assumes degenerate trajectories there is thus some uncertainty in what values to choose. Since the main purpose here is to demonstrate the general features of this type of model, the degenerate (ρ, A_2) has simply been chosen as;

$$\alpha_{\rho}(t) = 0.5 + 0.9 t.$$

To get non zero $\pi^- p \rightarrow \pi^0 n$ polarisation a degenerate (ρ', A_2') is also introduced, as implied by the Veneziano model. Its parameters are then

$$\alpha_{\rho'}(t) = -0.5 + 0.9 t.$$

The introduction of this daughter breaks the $K^{\pm}N$ line reversal symmetry.

For only one degenerate trajectory;

$$A_{K^- p \rightarrow \overline{K^0} n} = a e^{-i \pi \alpha_1}$$

$$A_{K^+ n \rightarrow K^0 p} = a,$$

$$\text{so } |A^{K-p}|^2 = |A^{K+n}|^2$$

giving the line reversal symmetry.

The introduction of a second trajectory gives

$$A_{K^- p \rightarrow \overline{K^0} p} = a e^{-i \pi \alpha_1} + b e^{-i \pi \alpha_2}$$

$$A_{K^+ n \rightarrow K^0 p} = a + b.$$

Since $\alpha_1(t) \neq \alpha_2(t)$, this results in an interference term in $|A^{K^-p}|^2$ between the two trajectories.

As line reversal symmetry is not experimentally true below 5.0 GeV/c this model is reasonable in that it gives some mechanism for breaking it.

The model then has four free parameters in a fit. Such a fit is shown in Fig. 37 (dotted), Table 16.

Table 16 $\chi^2/\text{data point} = 10.5.$

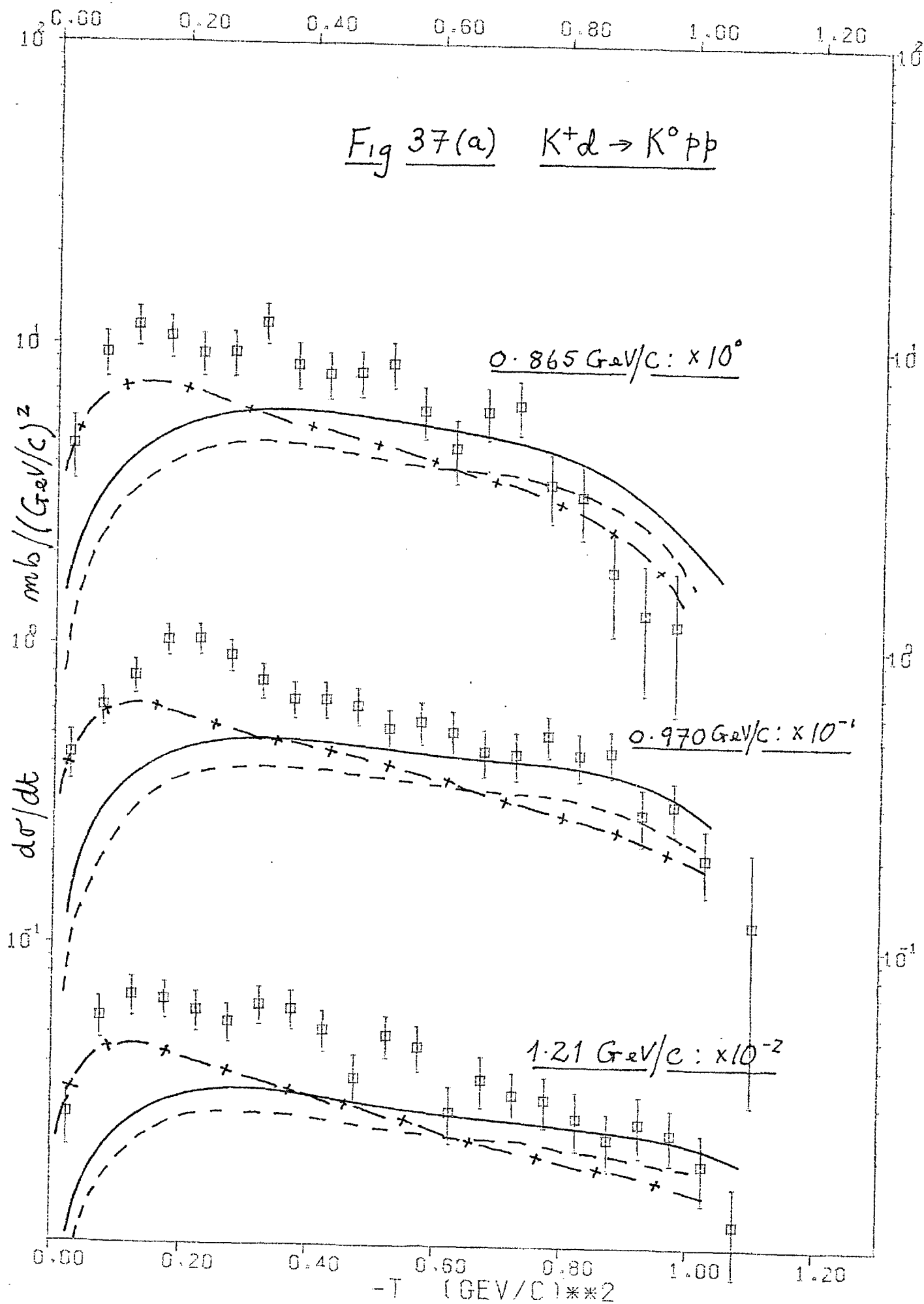
β_1^A	-1.31 ± 0.008	mb×GeV
β_1^B	18.86 ± 0.17	mb
β_2^A	5.33 ± 0.03	
β_2^B	13.08 ± 0.10	

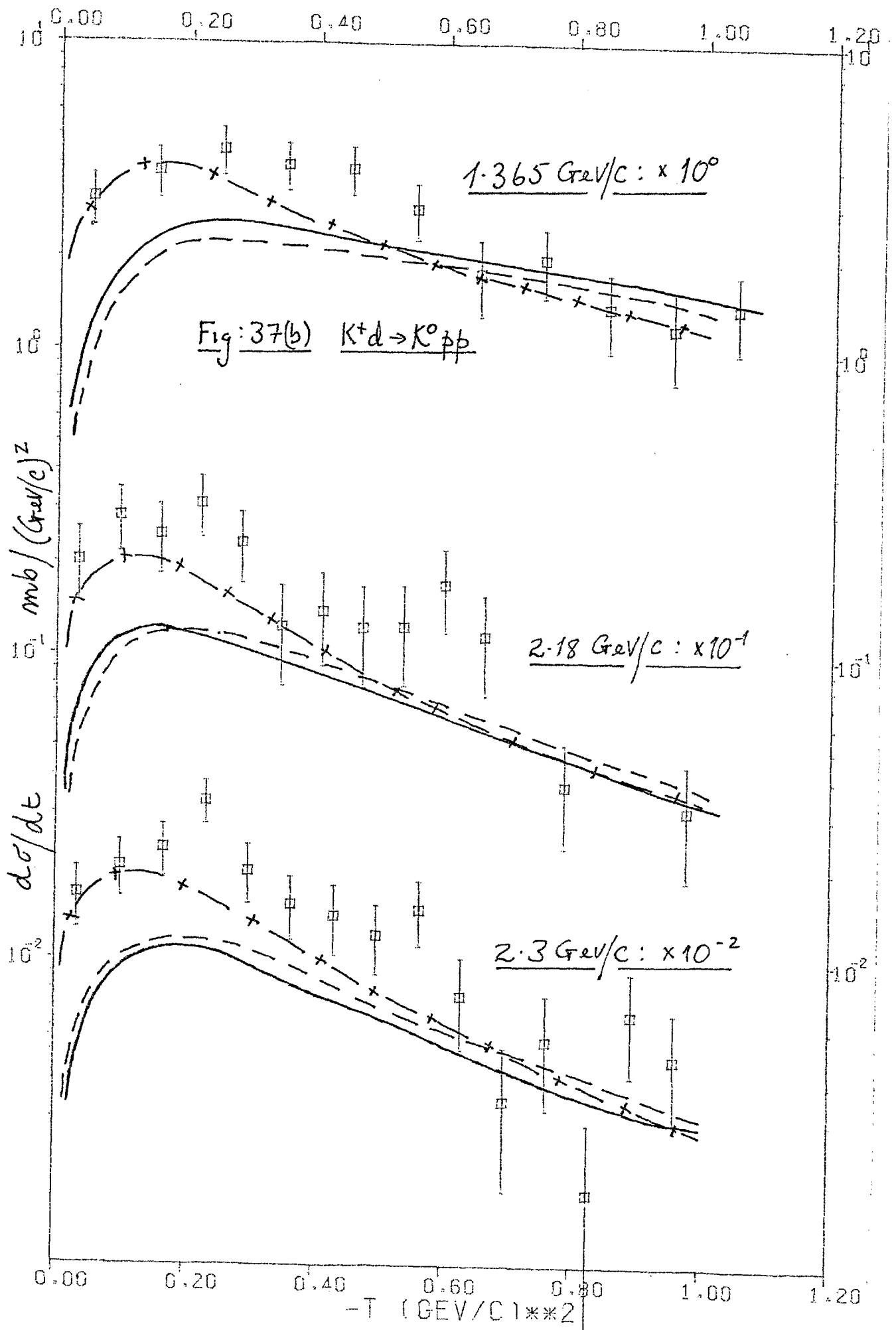
As might be expected for such a highly constrained and simple model it does no more than reproduce the broad outlines of the data. A WSP dip appears in $\pi^-p \rightarrow \pi^0 n$, but none in $\pi^-p \rightarrow \eta^0 n$, in accord with the data, and the form of the $\pi^-p \rightarrow \pi^0 n$ polarisation is correct.

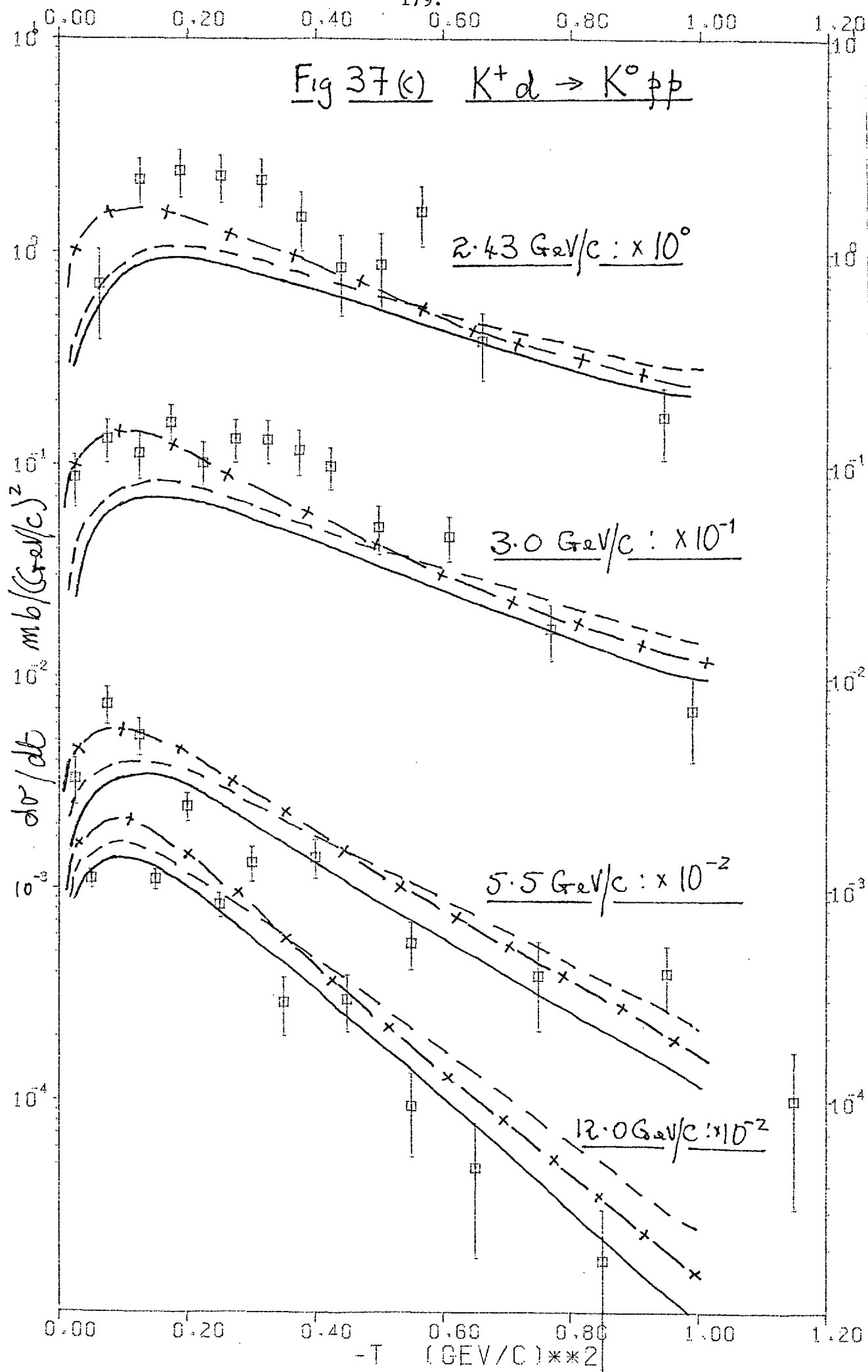
The problem of underestimating the $K^+n \rightarrow K^0p$ $d\sigma/dt$ is very apparent, but this model fares no worse, in this respect, than a typical weak absorptive cut model (ref. 15, e.g.).

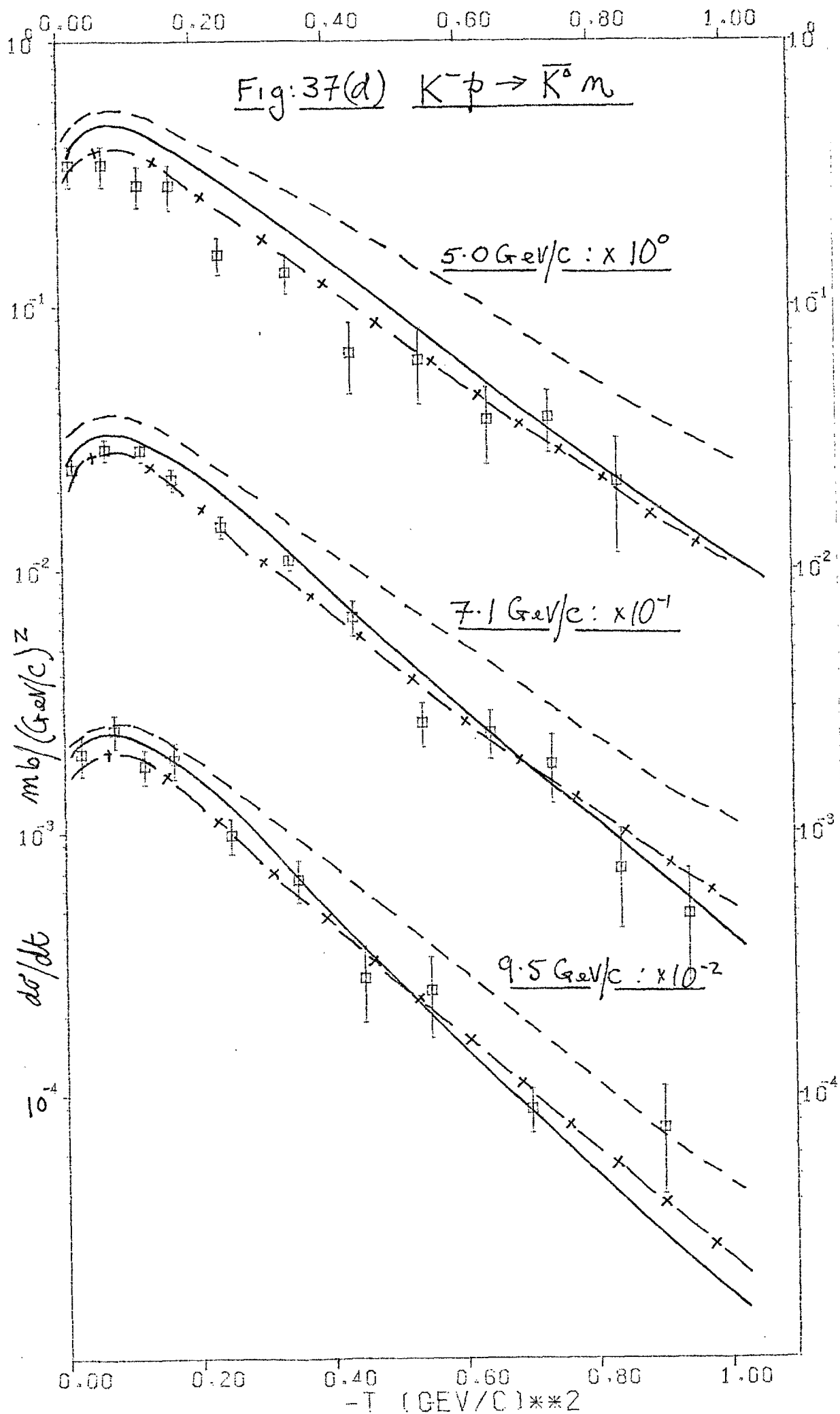
Allowing the trajectories to vary within 10% of the chosen values produces the full line in Fig. 37 with parameters in Table 17.

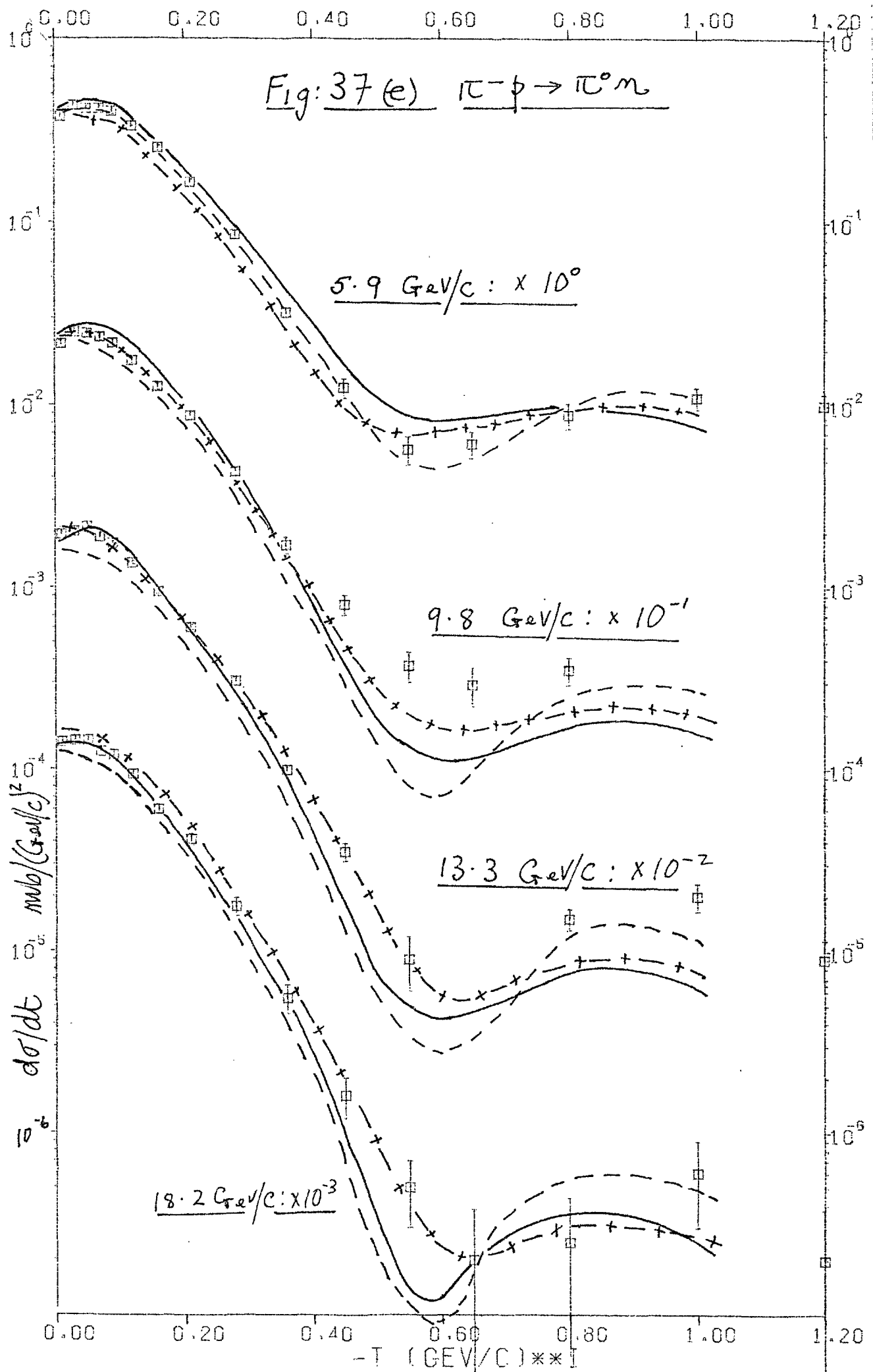
Fig 37(a) $K^+d \rightarrow K^0 pp$

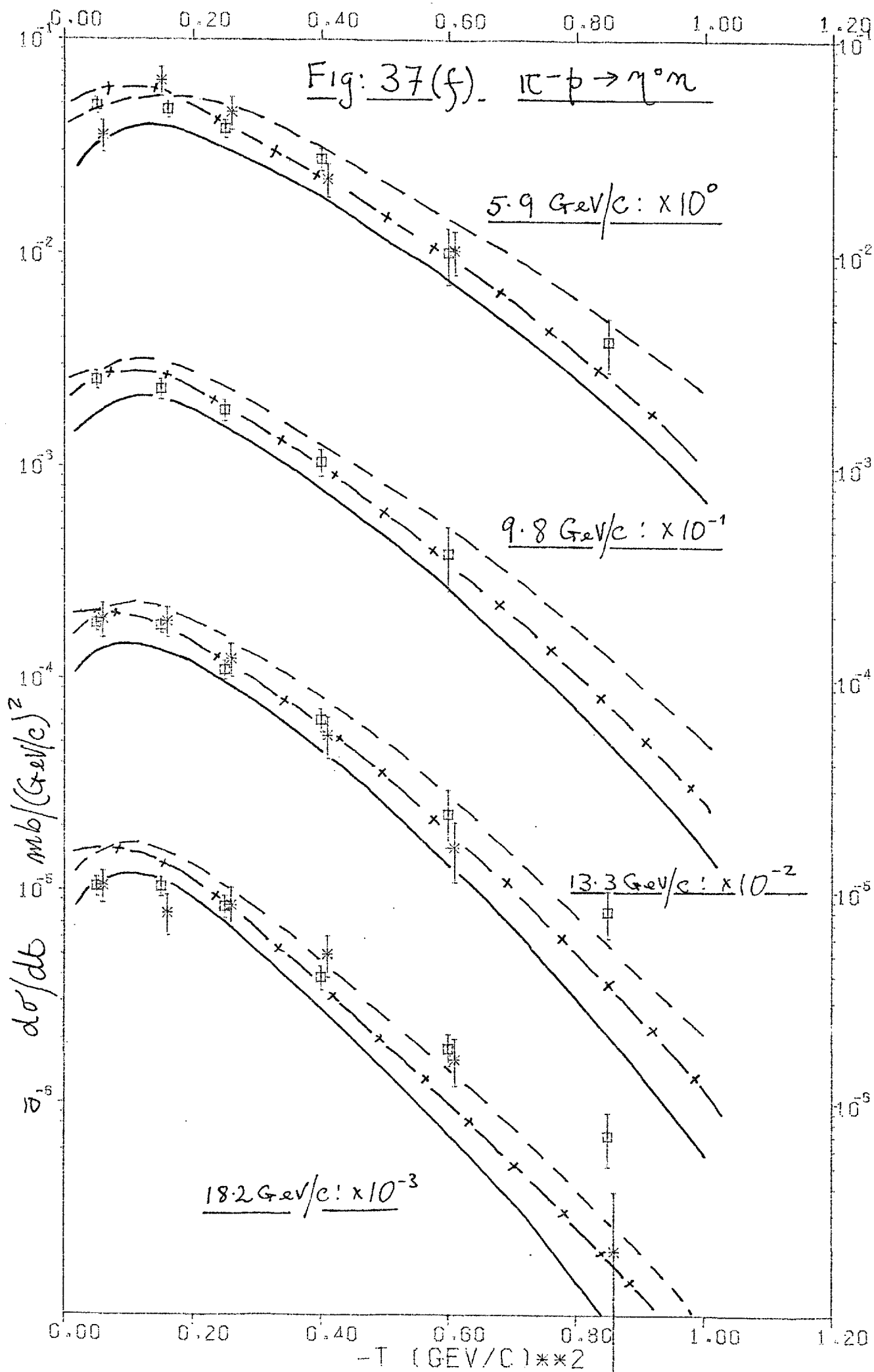


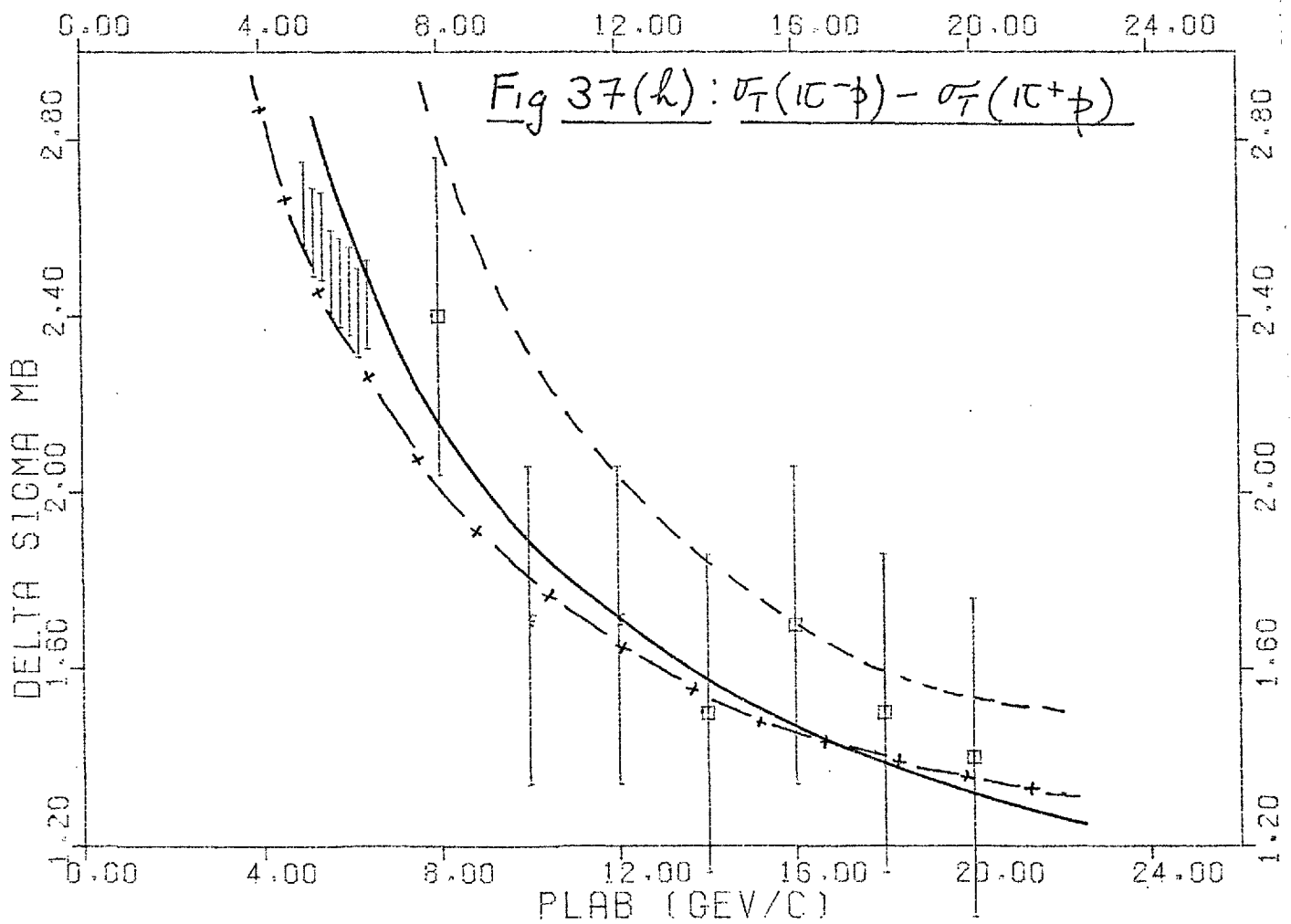
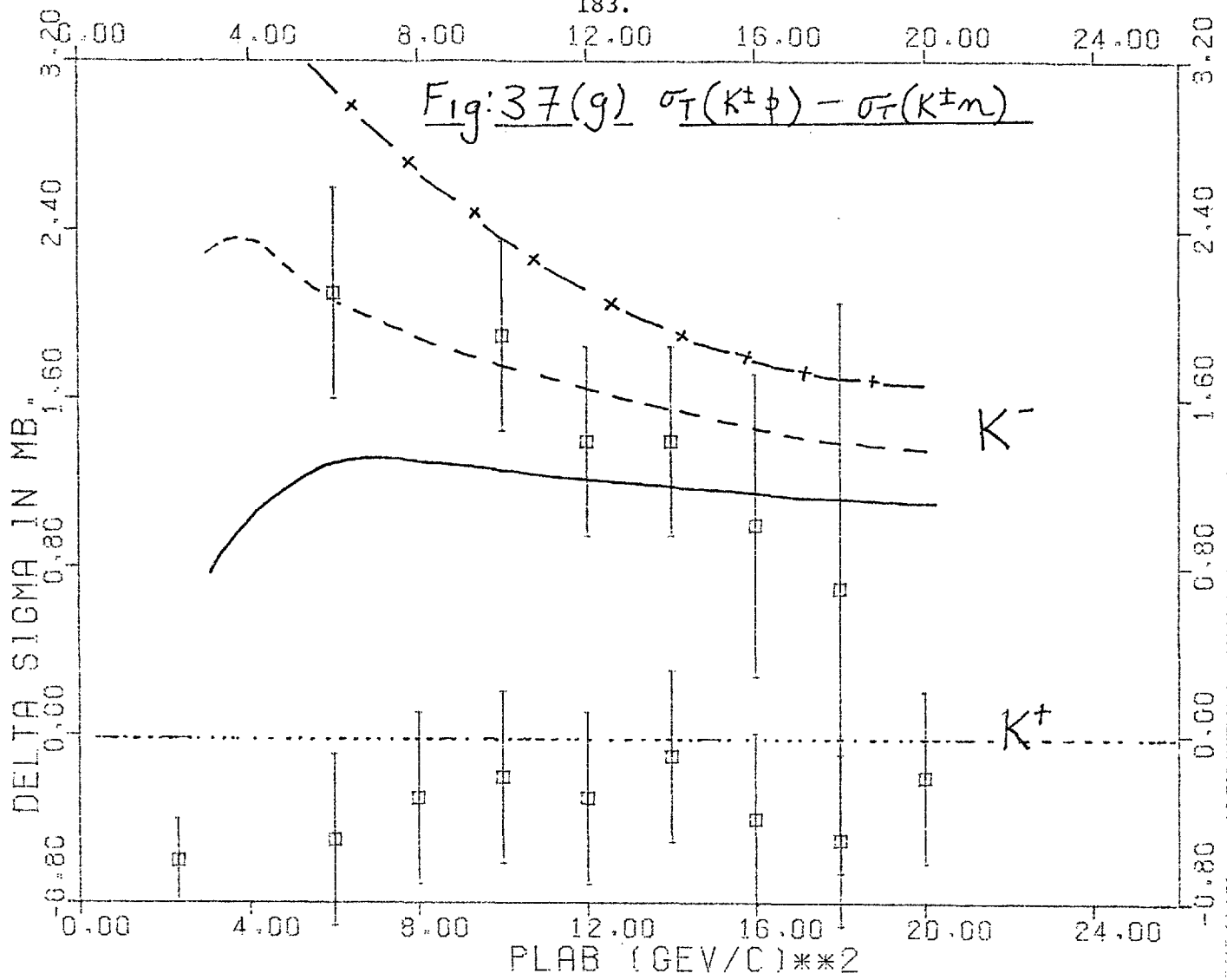












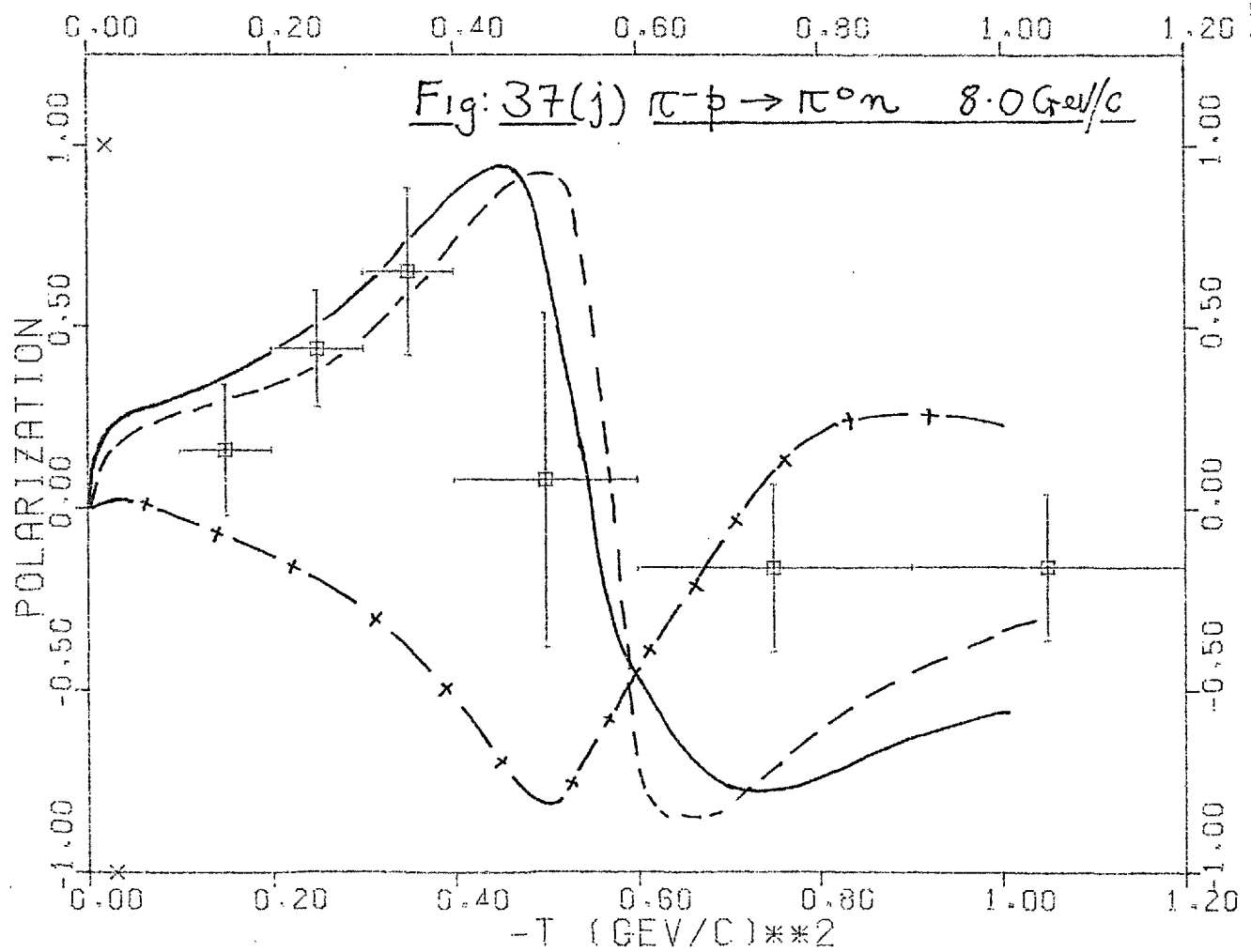
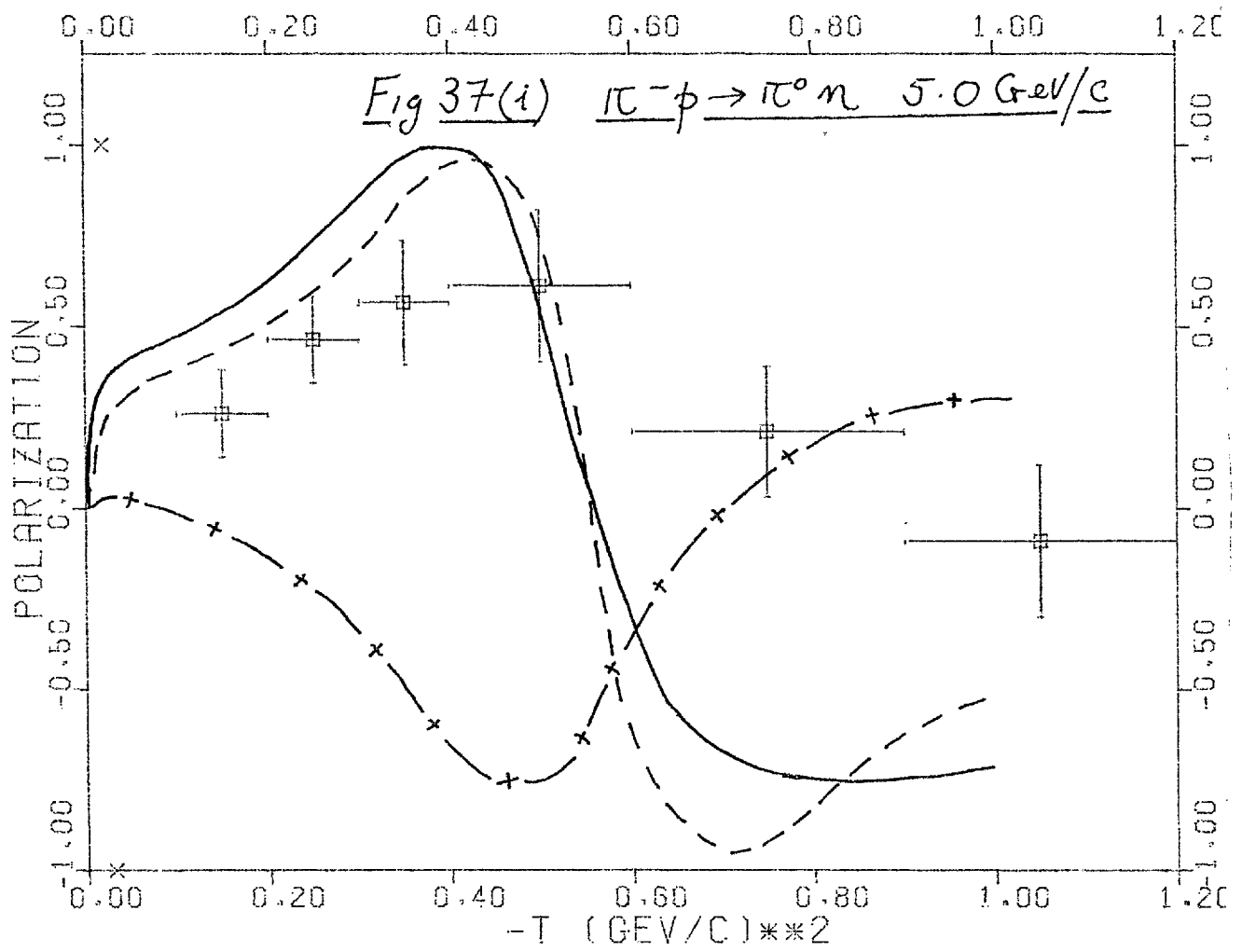


Table 17 $\chi^2/\text{data point} = 4.3.$

α_1^0	0.6 ± 0.007
α_1^1	1.07 ± 0.01
α_2^0	-0.4 ± 0.001
α_2^1	0.88 ± 0.003
β_1^A	-0.63 ± 0.004
β_1^B	13.8 ± 0.01
β_2^A	5.43 ± 0.02
β_2^B	22.35 ± 0.05

Which is something of an improvement, although the $K^+n \rightarrow K^0p$ is still unsatisfactory. By making the trajectories completely free the chain dotted fit in Fig. 37 is obtained. This is a considerable improvement for all the differential cross sections. But the high lying daughter (Table 18) and the incorrect polarisation indicate that the model is seriously lacking.

Table 18 $\chi^2/\text{data point} = 4.5.$

α_1^0	0.63 ± 0.005
α_1^1	0.95 ± 0.02
α_2^0	0.083 ± 0.007
α_2^1	1.03 ± 0.03
β_1^A	-0.54 ± 0.03
β_1^B	9.45 ± 0.32
β_2^A	-3.11 ± 0.06
β_2^B	16.3 ± 0.53

This latter indicates that although the polarisation is not a strong constraint on the fitting, it is a strong constraint on the interpretation of how meaningful a fit is. Without the polarisation information the chain dotted fit would be preferable.

5.3.3 Summary

This model does not produce particularly good fits⁽²³⁾. Nonetheless it shows that exact SU_3 symmetry of residues with the exchange of a small number of degenerate trajectories does reproduce the salient features of the data. This leads to the conclusion that such a picture is a first approximation to reality. A first approximation which is perturbed by effects not included in the model.

It is already known that SU_3 is broken and it appears that even weak exchange degeneracy is also broken. Ref. 17 has hypothesised that the breaking of line reversal might be due to weakly coupled resonances in K^+n . A more likely explanation is the presence of singularities other than Regge Poles in the complex ℓ - plane, such as cuts.

5.4 The ρ'

In the above models the ρ' was introduced on purely phenomenological grounds, to give non zero polarisation.

A ρ' has also been introduced in other processes, e.g. forward pn and $\bar{p}p$, where it was identified with the $\delta(896)$ ⁽¹⁹⁾.

Recently, Leader⁽²⁰⁾ has produced a model for $\pi^-p \rightarrow \pi^0n$ which requires a rather flat ρ' , i.e.

$$\alpha_{\rho'} = -0.15 + 0.3 t.$$

This is tentatively identified with a recently⁽²¹⁾ observed

$$e^+ e^- \rightarrow \pi^+ \pi^- \pi^+ \pi^-$$

spin 1 enhancement at 1968 MeV.

Additionally evidence for the ρ' has been presented by Smadja et al.⁽²⁴⁾ They presented strong evidence for the photoproduction of a vector meson ($\rho'?(1290 \pm 32)$) at 9.5 GeV/c. The ρ' has also been examined using FESR⁽¹³⁾ and Continuous Moment Sum Rules (CMSR)⁽²²⁾. This treatment indicates a weakly coupled, or non-existent, ρ' . Which directly contradicts the original Rarita-Schwartzschild fit. It may be that a ρ' with a flat trajectory, as found here, does not contradict the sum rules. The ρ' trajectories used by various authors are very inconsistent, α^0 ranging from -0.5 to $+0.34$, and α' from 0.3 to 1.4 (see ref. 1b chapter 4).

Chapter 5 - REFERENCES

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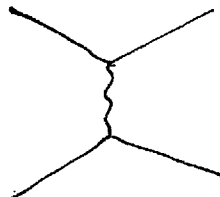
CHAPTER 6
REGGE CUT MODELS

In the previous chapter the deficiencies of models assuming the exchange of Regge trajectories alone were pointed out. In this chapter a different approach, employing absorptive cuts, is examined. After a discussion of the general classes of models proposed a particular model is examined in detail.

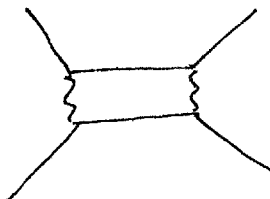
6.1 Theoretical Basis of Regge Cuts

Returning to the discussion of sense-nonsense choosing in section 4.2.6. In terms of Regge trajectory exchange the dip in π -p $\rightarrow \pi^0 n$ $d\sigma/dt$ at $\alpha \approx 0$ is explained in terms of a wrong signature nonsense zero. However the Mandelstam representation of an amplitude contains integrals over three double spectral functions ($\rho_{st}, \rho_{tu}, \rho_{su}$). If $\rho_{su} \neq 0$ then there is a singularity in the amplitude at wrong signature points. Obviously something must prevent such fixed poles from contributing to the physical amplitude. In fact Mandelstam showed that these fixed poles are "shielded" by moving cuts. The presence of these cuts arises from ρ_{su} itself in that

$\int \frac{\rho_t(t') dt'}{(t'-t)}$ corresponds to exchange diagrams of the form



and $\iint \frac{\rho_{su}(u') du' ds'}{(s'-s)(u'-u)}$ to diagrams of the form.



This gives some clue as to the physical interpretation of the moving cuts. The single spectral functions correspond to single particle exchange and the double spectral functions to double particle exchange.

The presence of cuts complicates the interpretation of dips in $d\sigma/dt$ considerably, as these can now be created or destroyed by interference between pole and cut contributions⁽¹⁾.

Since no dynamical theory exists in which cuts might be shown to exist, it is necessary to resort to constructing models which give rise to "cut-like" terms. For $O^{-\frac{1}{2}+} \rightarrow O^{-\frac{1}{2}+}$ process only Pomeron-Reggeon cuts can contribute⁽²⁾ (as opposed to double Reggeon). An intuitively attractive cut picture is then one in which the cuts are generated by "absorption", i.e. elastic scattering, modifying Regge Trajectory exchange, i.e.

$$\text{Amplitude} = \begin{array}{c} \text{---} \\ | \\ \text{---} \end{array} + \begin{array}{c} \text{---} \\ || \\ \text{---} \end{array} + \begin{array}{c} \text{---} \\ | \\ \text{---} \end{array}$$

$\begin{array}{c} || \\ \text{---} \end{array} - \text{Pomeron}$
 $\begin{array}{c} \} \\ \text{---} \end{array} - \text{Reggeon}$

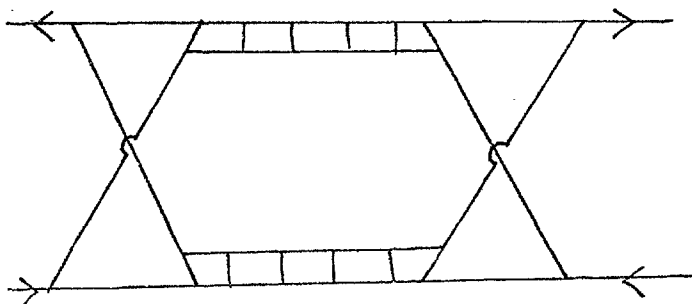
Such a model is open to several theoretical criticisms. The most basic is that box diagrams such as



in field theory do not contribute to the total amplitude. In fact they are cancelled by higher order diagrams. Lovelace has pointed out⁽³⁾ that, within the frame work of Duality, the box diagram corresponds only to a self energy term.

Again in field theory⁽⁴⁾ it happens that the high energy behaviour of diagrams with internal lines is dominated by terms in which the internal lines are off mass-shell. However, in most cut models these intermediate lines are taken to be on the mass-shell.

In fact it can be shown⁽⁵⁾ that planar diagrams (no crossing internal lines) in general, do not give rise to cuts. The sort of diagram which does give rise to cut contributions is



This can be interpreted⁽³⁾ as the virtual break-up of the incident hadrons, and the subsequent scattering and recombination of the constituents.

Thus cuts would appear to be a manifestation of the non-elementary nature of hadrons.

From all this the "Absorptive Cut Model" would seem to be rather dubious. The argument for its use is that it generates cut-like terms in a way which corresponds to a definite physical process.

6.2 Regge Cut Models

The lack of theoretical constraints has lead to an abundance of Regge Cut models. The various models do, however, have in common the fact that cuts are generated from absorptive effects. What they do not have in common is the relative importance assigned to cut and pole terms.

In Weak Cut models the Regge trajectories are assumed to be the dominant factor. All structures in $d\sigma/dt$ are assumed to result from structure in the Regge Pole residues (i.e. dips at WSNP, etc.). This pictures the cuts as being small perturbations on the basic fact of Regge Trajectory exchange (ref.1 for example).

Strong cut models arise from a very different point of view. The absorptive effects are assumed to be dominant⁽⁶⁾. Structure in differential cross sections stems from the Bessel function form of the impact parametrisation of the amplitude. The Regge Pole amplitudes are assumed to be structureless. That is, there are no nonsense dips, no conspiracies, no ghosts, no exchange degeneracy effects, etc; The supporters of this model term this "choosing simplicity".

6.2.1 Weak Cut Models

Typically, weak cut models parametrise the t-channel invariant Regge amplitudes in the manner employed by the exchange degenerate model treated in the last chapter⁽¹⁾.

$$A^t(s,t) = \beta \Gamma(1-\alpha) (1 \pm e^{-i\pi\alpha}) s^\alpha.$$

These t-channel amplitudes are related to the S-channel helicity amplitudes via

$$\text{NON FLIP} \quad F_{++} = \frac{1}{4\pi W} \cos \Theta/2 [m_n A + (E_{\text{LAB}} - m_n^2) B]$$

$$\text{FLIP} \quad F_{+-} = \frac{1}{4\pi W} \sin \Theta/2 [E_{\text{LAB}} A + m_n (W - E_{\text{LAB}}) B].$$

(W is centre of mass energy).

The s-channel amplitudes are employed since absorption is essentially an s-channel process.

From the partial wave (in terms of helicity states) expansion

$$F_{\lambda\mu}(s,t) = \sum_J (2J+1) T_{\lambda\mu}^J(s) d_{\alpha\beta}^J(\theta) .$$

(α, β are initial-final helicity differences) it is possible to obtain the expression for an individual partial wave amplitude

$$T_{\lambda\mu}^J(s) = \frac{1}{2} \int_{-1}^{+1} F_{\lambda\mu}(s,t) d_{\alpha\beta}^J(\theta) d(\cos \theta)$$

The absorptive corrections are then applied to the individual partial waves

$$T_{\lambda\mu}^J(s) \xrightarrow{\text{ABSORBED}} T_{\lambda\mu}'^J(s)$$

which are then resubstituted in the partial wave expansion to yield the total absorbed amplitude

$$F_{\lambda\mu}'(s,t) = \sum_J (2J+1) T_{\lambda\mu}'^J(s) d_{\alpha\beta}^J(\theta)$$

Large numbers (~30) of partial waves are used, and the partial wave amplitudes are calculated by accurate numerical integration. This is then an extremely accurate method of applying absorptive corrections. The accuracy of the method implies that any predictive short-comings are due to the basic physics of the model, rather than to inaccuracies arising from approximations.

The Weak cut model has several general difficulties⁽⁵⁾ :

It is unable to predict very high energy cross sections, gives incorrect positions for elastic differential cross section cross-overs, and predicts $d\sigma/dt$ dips in some processes where none are found experimentally. More relevant from the present point of view is its behaviour for $0^{-}\frac{1}{2}^{+} \rightarrow 0^{-}\frac{1}{2}^{+}$ charge exchange. If only (ρ, A_2) exchange is assumed

the model gives the wrong sign of $\pi^- p \rightarrow \pi^0 n$ polarisation and underestimates the $K^+ n \rightarrow K^0 p$ $d\sigma/dt$ by a factor of 3⁽⁷⁾. Even at 5.5 GeV/c the $K^+ n \rightarrow K^0 p$ forward peak is underestimated. In fact this form of the model is little better than pure pole models.

Rather better fits to $\pi^- p \rightarrow \pi^0 n$ polarisation are obtained by altering the phase relationship between pole and cut terms⁽⁸⁾. What is done is to apply the absorptive corrections only to the signatured part of the Regge Pole amplitude

$$\begin{aligned} A &= \beta \Gamma(1-\alpha) (1 \pm e^{-i\pi\alpha}) s^\alpha \\ \text{Absorbed } A^{\text{sig}} &= \pm \beta \Gamma(1-\alpha) e^{-i\pi\alpha} s^\alpha \\ \text{Unabsorbed } A^{\text{nonsig}} &= \beta \Gamma(1-\alpha) s^\alpha. \end{aligned}$$

Although this gives the $\pi^- p \rightarrow \pi^0 n$ polarisation correctly, there is no improvement with respect to the $K^+ n \rightarrow K^0 p$ $d\sigma/dt$. The model has the form of a simple (ρ, A_2) pole model in this reaction; already known to be unsatisfactory.

A further modification⁽⁹⁾ is to return to absorbing both parts of the amplitude, but rotating the unsignatured part through $\pi/2$.

$$\begin{aligned} A_{\text{sig}}^{\text{absorbed}} &= C(X) A_{\text{sig}}^{\text{pole}} \\ A_{\text{unsig}}^{\text{absorbed}} &= i C(X) A_{\text{unsig}}^{\text{pole}} \end{aligned}$$

This retains the correct sign of the $\pi^- p \rightarrow \pi^0 n$ polarisation and gives a mechanism for improving the $K^+ n \rightarrow K^0 p$ $d\sigma/dt$. In fact the model behaves well at 5.5 GeV/c as $K^+ n \rightarrow K^0 p$ and $K^- p \rightarrow \bar{K}^0 n$ $d\sigma/dt$ have the correct ratio. This is in contrast to early weak cut models^(1,10), which broke line reversal in the wrong direction.

With this final modification the weak cut model performs rather well on $0^{-}\frac{1}{2}^{+} \rightarrow 0^{-}\frac{1}{2}^{+}$. There is, however, no real theoretical justification for the prescription adopted. The authors of ref.9 suggest that some justification arises from Duality. Their prescription corresponds to a normal absorption model for direct channel resonance effects, with a contribution from u-channel effects which is purely real. They state that the prescription for the signed part is correct, from duality, and that duality does not contradict their treatment of the unsigned part.

6.2.2 Strong Cut Models

Just as an object with sharp edges produces a diffraction pattern in optics, the strong cut model produces $d\sigma/dt$ structure by strong absorption of low order partial waves. Because the absorption is very dependent on the order of the partial waves involved, the impact parameter representation is convenient to use. Then the amplitude at a particular impact parameter, b , is

$$\chi(b) = \int_0^{\infty} \frac{d(-t)}{4q^2} A(t) J_0(b\sqrt{-t}) .$$

J_0 is a 0th order Bessel function. The inverse of the above is

$$A(t) = 2q^2 \int_0^{\infty} b db J_0(b\sqrt{-t}) .$$

The absorbed amplitude is written as

$$A(b) = S^{el}(b) A^{Regg}(b)$$

elastic
S-matrix

Regge
Pole amplitude

This is known as the SOPKOVITCH prescription.

In terms of the T-matrix

$$S^{el}(b) = 1 + i T^{el}(b)$$

and since elastic scattering is mainly imaginary

$$S^{el} \approx 1 - |T^{el}(b)|$$

so

$$A(b) = A^{Reg}(b) - |T^{el}(b)| A^{Reg}(b) .$$

From this the momentum transfer amplitude $A(t)$ is proportional to

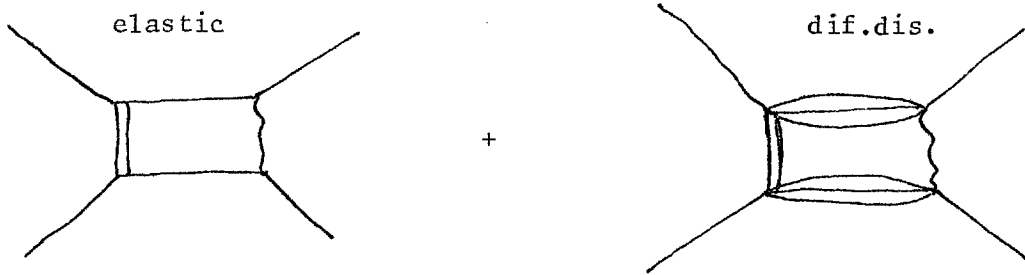
$$\int_0^\infty b db J_0(b \sqrt{-t}) = \frac{J_1(b_0 \sqrt{-t})}{\sqrt{-t}}$$

where J_1 is a first order Bessel function and b_0 is the "edge" of the absorption. This leads to the amplitude having zeros at

$$b_0 \sqrt{-t} = 3.8, 7.0, \text{ etc.}$$

and this is what gives rise to structure in $d\sigma/dt$.

Physically the "potential" is not sharp edged and this wipes out higher order zeros. In fact this simple absorption gives zeros in the wrong places, $t \approx -0.2$ rather than ≈ -0.6 . If the absorption arises from elastic scattering, then this misplacing of zeros leads to the conclusion that the absorption is too weak. Ross et al.⁽⁶⁾ reason that this deficiency is due to neglecting diffractively produced intermediate states i.e. that the correct diagrams are



This correction cannot be calculated, but is allowed for by a multiplication factor λ , such that

$$A(b) = A^{\text{Reg}}(b) - |T^{\text{eff}}(b)| A^{\text{Reg}}(b)$$

where $T^{\text{eff}}(b) = \lambda T^{\text{el}}(b)$.

λ is then a free parameter in fits.

The strong cut model has had some definite successes. π exchange forward peaking is explained and elastic differential cross section cross-overs are correctly predicted, as are high energy total cross sections. For ρ dominated process the $d\sigma/dt$ structure is well reproduced, but for A_2 dominated processes the model predicts dips which are not experimentally observed. This latter seems a serious failing, but a tentative explanation has been given in terms of the short range of A_2 exchange forces⁽¹¹⁾.

This form of the strong cut model also gives the wrong sign for $\pi^- p \rightarrow \pi^0 n$ polarisation. Again this can be corrected by introducing ad hoc complications, in this case complex radii of interactions⁽¹²⁾. The behaviour of the strong cut model is thus even less satisfactory in $0^{-\frac{1}{2}+} \rightarrow 0^{-\frac{1}{2}+}$ than that of the weak.

6.2.3 A Hybrid Cut Model

In an effort to combine the different successes of the two classes of cut models, R.J.N. Phillips has proposed⁽¹³⁾ that there are very strong cuts in the non-flip amplitudes, and weak cuts in the flip. This implies that the $d\sigma/dt$ structure of net helicity flip = 1 processes arises from pole residues. While the net helicity flip = 0 structure arises from very strong absorption.

Collins and Gault⁽¹⁴⁾ have fitted a model along these lines to πN processes, with some success, although a ρ' is introduced to give the $\pi^- p \rightarrow \pi^0 n$ polarisation. In the following section this model is extended to cover all the $0^{-\frac{1}{2}} \rightarrow 0^{-\frac{1}{2}+}$ charge exchange processes.

6.3 The Eikonalised Regge Model

As pointed out in the introduction the absorption prescription is effectively the addition of multiple elastic scatters to the leading, Regge trajectory exchange, term. Most models only take the first order of multiple scattering into account. In the model to be examined multiple scatters of order up to 6 are included, to give an accuracy of 1 part in 10^4 .

6.3.1 The Eikonal Series

The Eikonal method is a way of handling, as a perturbative expansion, a situation in which multiple scattering is involved. It has already been mentioned in the context of a deuterium target (Chapter 3). As pointed out in ref. 3, this is no accident, as both cases deal with scattering from non-elementary objects. The only difference is in the magnitude of the forces involved.

The Eikonal method involves summing the series of ladder diagrams



The impact parameter representation is used, the incoming wave packet being decomposed into partial waves at definite impact parameters. Each impact parameter then corresponds to a definite phase change during the scattering process. In the Eikonalised cut model the first term of the series is identified with Regge trajectory exchange. Higher terms are interpreted as multiple absorptive cuts.

6.3.2 Cuts by Eikonalisation

Initially consider elastic scattering and assume it to be mediated by Pomeron ($\mathbb{P}$) exchange. Obviously multiple as well as single Pomeron exchange can occur. So, in this model, the physical Pomeron is a sum of multiple bare-Pomeron cuts. This is a problem (which is ignored) as the Pomeron input to the model is assumed to be the physical (re-normalised) Pomeron; by summing the Eikonal series double counting is introduced.

The Pomeron exchange amplitude is denoted by $A^{\mathbb{P}}(s, t)$. The Eikonal phase (the phase shift at a given impact parameter) is defined by

$$\chi^{\mathbb{P}}(s, b) = \frac{1}{8\pi s} \int_{-\infty}^0 dt J_0(b \sqrt{-t}) A^{\mathbb{P}}(s, t) . \quad (6.1)$$

The impact parameter, b , is defined by:

$$J = q_s b - \frac{1}{2}$$

$J \rightarrow$ total angular momentum

$q_s \rightarrow$ centre of mass 3-momentum.

The first order of the Eikonal approximation to the total elastic amplitude is

$$\chi^{\text{el}}(s,b) = i \left[1 - e^{i \chi^{\text{P}}(s,b)} \right] \quad (6.2)$$

The full scattering amplitude in the momentum transfer representation is given by the Fourier-Bessel transform of (6.2).

$$A^{\text{el}}(s,t) = 4 \pi s \int_0^\infty b db \, i \left[1 - e^{i \chi^{\text{P}}(s,b)} \right] J_0(b\sqrt{-t}).$$

This can be expanded as

$$A^{\text{el}}(s,t) = 4 \pi s \int_0^\infty b db \left[\chi^{\text{P}} + i \frac{(\chi^{\text{P}})^2}{2!} - \frac{(\chi^{\text{P}})^3}{3!} - i \frac{(i \chi^{\text{P}})^n}{n!} \right] J_0(b\sqrt{-t}) \quad (6.3)$$

The first term of (6.3) is

$$4 \pi s \int_0^\infty b db \, \chi^{\text{P}} J_0(b\sqrt{-t})$$

which is the inverse of (6.1) and so corresponds to single Pomeron exchange.

To introduce spin it is necessary to define an Eikonal phase for each s-channel helicity state.

$$\chi_{H_s}^{\text{P}}(s,b) = \frac{1}{8\pi s} \int_{-\infty}^0 dt \, J_{\frac{1}{2}}(b\sqrt{-t}) A_{H_s}^{\text{P}}(s,t).$$

In (6.4) H_s is the helicity index, and $\chi_{H_s}^{\text{P}}$ is now a matrix of possible helicity changes, i.e.

$$\chi_{H_s}^{\text{P}} = \begin{pmatrix} \chi_{++}^{\text{P}} & \chi_{+-}^{\text{P}} \\ \chi_{-+}^{\text{P}} & \chi_{--}^{\text{P}} \end{pmatrix}$$

As pointed out in chapter 4, parity conservation implies that only the two diagonal elements are independent.

$\bar{\mu}$ is the net helicity flip defined by

$$\bar{\mu} \equiv |(\mu_1 - \mu_2) - (\mu_3 - \mu_4)| \quad (6.5)$$

To generate a total amplitude whose leading term is Regge exchange, and whose subsequent terms are Regge-Pomeron cuts one writes

$$\chi_{H_S}^{\text{TOT}}(s,b) = \chi_{H_S}^{\text{Regge}}(s,b) e^{-i \chi^{\mathbb{P}}(s,b)} \quad (6.6)$$

This is essentially the Sopkovitch prescription for absorption since $e^{i \chi^{\mathbb{P}}(s,b)}$ is the elastic S-matrix, (6.6) expands as

$$\chi_{H_S}^{\text{TOTAL}}(s,b) = \chi_{H_S}^{\text{Regge}} + i \chi^{\mathbb{P}} \chi_{H_S}^R - \frac{(\chi^{\mathbb{P}})^2 \chi_{H_S}^R}{2!} \text{ etc.} \quad (6.7)$$

(6.7) only includes absorption from purely elastic scattering.

As pointed out in the context of the strong cut model it is necessary to enhance this absorption to allow for diffractively produced intermediate states. This is again achieved by means of an overall enhancement factor λ , which is assumed to be independent of s and b .

Then

$$\chi_{H_S}^{\text{TOTAL}}(s,b) = \chi_{H_S}^{\text{Regge}}(s,b) e^{i \lambda \chi^{\mathbb{P}}(s,b)}$$

which expands as

$$\chi_{H_S}^{\text{TOTAL}}(s,b) = \chi_{H_S}^R + i \lambda \chi^{\mathbb{P}} \chi_{H_S}^R - \lambda^2 \frac{(\chi^{\mathbb{P}})^2}{2!} \chi_{H_S}^R + \dots$$

or, in general

$$\chi_{H_S}^{\text{TOTAL}}(s,b) = \sum_{n=1}^{\infty} \frac{\chi_{H_S}^R(s,b) [i \lambda \chi_{H_S}^P(s,b)]^{n-1}}{(n-1)!} \quad (6.8)$$

6.3.3 Introduction of Regge Pole Amplitude

(6.8) is completely general in that no assumptions have been made concerning the form of the exchange amplitudes. To give the expression any content it is necessary to assume some form of the amplitudes giving rise to the Eikonal phases $\chi_{H_S}^R$, and $\chi_{H_S}^P$. In this case it is assumed that they are standard Regge pole amplitudes.

Because of the essentially s -channel nature of absorption (6.8) involves s -channel helicities. This means that it is not possible to employ the simple t -channel Regge Pole amplitudes used in the models in the previous chapters.

The s -channel Regge helicity amplitude is written as

$$A_{H_S}^R(s,t) = \left(\frac{-t}{s_0} \right)^{\frac{\bar{\mu}+x}{2}} e^{-\frac{i\pi\alpha(t)}{2}} \chi_{H_S}(t) F(\alpha) (s/s_0)^\alpha \quad (6.9)$$

This is a helicity amplitude rather than an invariant amplitude, so the s -dependence of both flip and non-flip amplitudes is s^α . The factor $(-t/s_0)^{\frac{\bar{\mu}+x}{2}}$ is included to satisfy angular momentum conservation in that the flip amplitude ($\bar{\mu}=1$) will vanish at $t=0$.

$x \equiv |\mu_1 - \mu_3| + |\mu_2 - \mu_4| - \bar{\mu}$ and $= 0$ for $0^{-\frac{1}{2}+} \rightarrow 0^{-\frac{1}{2}+}$.

$\chi_{H_S}(t)$ is now a residue function factorisable in terms of s -channel helicities.

The normal signature factor

$$\left(\frac{1 \pm e^{-i\pi\alpha}}{\sin \pi\alpha} \right)$$

has been rewritten as

$$F(\alpha) e^{-i\pi\alpha/2}$$

Where $F(\alpha)$ is a "nonsense" factor whose value depends on whether a trajectory chooses sense or nonsense, and also on the signature.

Linear trajectory functions are assumed

$$\alpha(t) = \alpha_0 + \alpha' t$$

and the residues are parametrised in a fashion similar to the Rarita-Schwartzschild model.

$$\gamma_{H_S}(t) = G_{H_S} e^{at}.$$

So

$$A_{H_S}^R(s, t) = \left(\frac{-t}{s_0} \right)^{\frac{\bar{\mu} + x}{2}} e^{-\frac{i\pi\alpha_0}{2}} F(\alpha) G_{H_S} \left(\frac{s}{s_0} \right)^{\alpha_0} e^{Ct} \quad (6.10)$$

where $\left(\frac{s}{s_0} \right)^{\alpha' t} = e^{\alpha' t \ln(s/s_0)}$ has been used.

C is defined by

$$C = a + \alpha' [\ln(s/s_0) - i \frac{\pi}{2}] .$$

In order to carry out the Fourier-Bessel transform, and its inverse, the two following relations are necessary.

$$\int_{-\infty}^0 e^{Ct} (-t)^{\frac{\bar{\mu}}{2} + m} J_{\bar{\mu}}(b\sqrt{-t}) dt = \left(\frac{b}{2} \right)^{\bar{\mu}} \left(\frac{\partial}{\partial C} \right)^m \frac{e^{-b^2/4C}}{C^{\bar{\mu}+1}} \quad (6.11)$$

$$\int_0^\infty e^{-b^2/4C} (b^2)^{\frac{\bar{\mu}}{2} + m} J_{\frac{\bar{\mu}}{2}}(b\sqrt{t}) b db = (-t)^{\frac{\bar{\mu}}{2}} \left(4 C^2 \frac{\partial}{\partial C} \right)^m \times (2C)^{\bar{\mu}+1} e^{Ct}. \quad (6.12)$$

6.3.4 Construction of Eikonalised Amplitudes

Because of the assumption of cuts in the non-flip amplitude only, Eikonalisation is only carried out for the non-flip case.

The method used is to determine $\chi_{++}^{\mathcal{P}}(s,b)$ and $\chi_{++}^R(s,b)$ from the momentum transfer representation (6.10), using (6.11), these are used in (6.8) to obtain $\chi_{++}^{\text{TOT}}(s,b)$. (6.12) is then used to transform $\chi_{++}^{\text{TOT}}(s,b)$ into the momentum transfer representation. $A_{++}^{\text{TOT}}(s,b)$.

$\chi_{++}^{\mathcal{P}}$

The Pomeron is treated as a conventional Regge trajectory.

So from (6.1)

$$\chi_{++}(s,b) = \frac{1}{8\pi s} \int_{-\infty}^0 dt J_0(b\sqrt{-t}) e^{\frac{-i\pi\alpha_0}{2}} F(\alpha^{\mathcal{P}}) G_{++}^{\mathcal{P}}(s/s_0)^{\alpha_0^{\mathcal{P}}} \times e^{C_{++}^{\mathcal{P}} \cdot t}.$$

Using (6.11) this is:

$$\chi_{++}(s,b) = \frac{F(\alpha^{\mathcal{P}})}{8\pi s} e^{\frac{-i\pi\alpha_0}{2}} G^{\mathcal{P}} \frac{e^{-b^2/4C^{\mathcal{P}}}}{C^{\mathcal{P}}} (s/s_0)^{\alpha_0^{\mathcal{P}}}$$

Assuming the Pomeron to be an even signature trajectory, this becomes $[F(\alpha^P) = -1]$.

$$\chi_{++}^\rho(s, b) = -\frac{e^{-i\frac{\pi\alpha_0^P}{2}}}{8\pi s C^P} \left(\frac{s}{s_0}\right)^{\alpha_0^P} G^P e^{-b^2/4C^P} \quad (6.13)$$

$$\underline{\chi_{++}^R(s, b)}$$

The following holds for both the ρ and A_2 , although

$$F(\alpha^\rho) = i\alpha^\rho; \quad F(\alpha^{A_2}) = -1.$$

$$\chi_{++}^\rho(s, b) = F(\alpha^\rho) e^{-\pi\alpha_0^\rho} \left(\frac{s}{s_0}\right)^{\alpha_0^\rho} G_{++}^\rho \frac{e^{-b^2/4C_{++}^\rho}}{8\pi s C_{++}^\rho} \quad (6.14)$$

using (6.14) and (6.13) in (6.8) gives

$$\chi_{++}^{\text{TOT}}(s, b) = \sum_{n=1}^{\infty} \left\{ F(\alpha^\rho) e^{-i\frac{\pi\alpha_0^\rho}{2}} \left(\frac{s}{s_0}\right)^{\alpha_0^\rho} G_{++}^\rho e^{-\frac{b^2}{4C_{++}^\rho}} \frac{1}{8\pi s C_{++}^\rho} \frac{1}{(n-1)!} \right. \\ \left. \times \left(\frac{-i\lambda e^{-i\frac{\pi\alpha_0^P}{2}} \left(\frac{s}{s_0}\right)^{\alpha_0^P} G^P e^{-b^2/4C^P}}{8\pi s C^P} \right)^{n-1} \right\}$$

Since $i = e^{i\pi/2}$ one can write

$$-i\lambda e^{-i\pi\alpha_0^P/2} = \lambda e^{-i\frac{\pi}{2}(\alpha_0^P - 1)}$$

so

$$\chi_{++}^{\text{TOT}}(s,b) = \sum_{n=1}^{\infty} \left\{ \frac{F(\alpha_0^\rho) e^{-i \frac{\pi}{2} \alpha_0^\rho} (s/s_0)^{\alpha_0^\rho} G_{++}^\rho e^{-b^2/4C_{++}^\rho}}{8\pi s C_{++}^\rho (n-1)!} \right. \\ \left. \times \left(\frac{-\lambda G_{++}^\rho e^{-i \frac{\pi}{2} (\alpha_0^\rho - 1)} (s/s_0)^{\alpha_0^\rho} e^{-b^2/4C_{++}^\rho}}{8\pi s C_{++}^\rho} \right)^{n-1} \right\}$$

The momentum transfer representation $A_{++}^{\text{TOT}}(s,t)$ is obtained by using (6.12)

$$A_{++}^{\text{TOT}}(s,b) = 4\pi s \int_0^\infty b db \chi_{++}^{\text{TOT}}(s,b) J_0(b\sqrt{-t}) \\ = 4\pi s \sum_{n=1}^{\infty} \left\{ \frac{F(\alpha_0^\rho) e^{-i \frac{\pi}{2} \alpha_0^\rho} (s/s_0)^{\alpha_0^\rho} G_{++}^\rho}{8\pi s C_{++}^\rho (n-1)!} \right. \\ \left. \times \left(\frac{-\lambda G_{++}^\rho e^{-i \frac{\pi}{2} (\alpha_0^\rho - 1)} (s/s_0)^{\alpha_0^\rho}}{8\pi s C_{++}^\rho} \right)^{n-1} \int_0^\infty e^{-b^2/4C_{++}^\rho} J_0(b\sqrt{-t}) dt \right\}$$

where $X_n = \frac{C_{++}^\rho}{(n-1) C_{++}^\rho + C_{++}^\rho}$

carrying out the integration

$$= 4\pi s \sum_{n=1}^{\infty} \left\{ \frac{F(\alpha_0^\rho) e^{-i \frac{\pi}{2} \alpha_0^\rho} (s/s_0)^{\alpha_0^\rho} G_{++}^\rho}{8\pi s C_{++}^\rho (n-1)!} \right. \\ \times \left(\frac{-\lambda G_{++}^\rho e^{-i \frac{\pi}{2} (\alpha_0^\rho - 1)} (s/s_0)^{\alpha_0^\rho}}{8\pi s C_{++}^\rho} \right)^{n-1} \\ \left. \times 2 C_{++}^\rho X_n e^{C_{++}^\rho X_n t} \right\}$$

Setting $\alpha_0^\rho = 1$ and using the optical theorem to give $G_{++}^\rho = \sigma^T s_0$ ($s_0 = 1$), σ^T being the asymptotic total cross section.

$$\begin{aligned}
A_{++}^{\text{TOT}}(s, t) &= F(\alpha^\rho) e^{-i \frac{\pi}{2} \alpha_0^\rho} (s)^{\alpha_0^\rho} G_{++}^\rho \\
&\times \sum_{n=1}^{\infty} \left(\frac{-\lambda \sigma^{\text{TOT}}}{8 \pi C^{\mathbb{P}}} \right)^{n-1} \frac{X_n}{(n-1)!} e^{C_{++}^\rho X_n t} \quad (6.15)
\end{aligned}$$

(6.15) is applicable for A_2 exchange by merely setting $F(\alpha^{A_2}) = -1$, and replacing $\alpha^\rho \Rightarrow \alpha^{A_2}$.

$$\begin{aligned}
\text{For } \rho \text{ exchange } F(\alpha^\rho) &= i \alpha^\rho \\
&= i \alpha_0^\rho \left(1 + \frac{\alpha'^\rho}{\alpha_0^\rho} t \right)
\end{aligned}$$

The form of (6.12) implies that the factor t in $F(\alpha^\rho)$ results in a factor $\frac{\partial}{\partial C_{++}^\rho}$ after integration, i.e. in (6.15)

$$F(\alpha^\rho) = i \alpha_0^\rho \left(1 + \frac{\alpha'^\rho}{\alpha_0^\rho} \frac{\partial}{\partial C_{++}^\rho} \right)$$

so

$$\begin{aligned}
A_{++}^{\text{TOT}}(s, t) &= i e^{-i \frac{\pi}{2} \alpha_0^\rho} (s)^{\alpha_0^\rho} G_{++}^\rho \\
&\times \sum_{n=1}^{\infty} \left(\frac{-\lambda \sigma^{\text{TOT}}}{8 \pi C^{\mathbb{P}}} \right)^{n-1} \frac{\alpha_0^\rho}{(n-1)!} \left\{ X_n + \frac{\alpha'^\rho}{\alpha_0^\rho} \left[\frac{(1-n)}{C^{\mathbb{P}}} X_n^2 + X_n^3 t \right] \right\} \\
&\times e^{C_{++}^\rho X_n t} \quad (6.16)
\end{aligned}$$

Putting $n=1$ in either (6.15) or (6.16) gives back the Regge Pole amplitude (6.10). This confirms the interpretation of the leading term of the Eikonal series as Regge pole exchange.

6.3.5 Flip Amplitude

The unabsorbed Regge Pole amplitude is used for the helicity flip contribution.

$$A_{+-}^{\text{TOT}}(s,t) = F(\alpha) e^{-i \frac{\pi}{2} \alpha_0} (s)^{\alpha_0} G_{+-} \sqrt{-t} e^{C_{+-} t}.$$

The $\sqrt{-t}$ factor comes from $(-t/s_0)^{\frac{\bar{\mu}+x}{2}}$.

since $\bar{\mu} = 1$ in the flip case.

6.3.6 SU₃ Factors and Isospin

All trajectories choose nonsense, and again a ρ' is introduced and assumed to behave similarly to the ρ .

The sign conventions for contributions to the total amplitudes are as in (5.1) $\rightarrow$ (5.4); the SU₃ factors are then identical to those used in the Rarita-Schwartzschild model. This is valid since, although the R-S amplitudes are t-channel and the present amplitudes s-channel, the crossing matrix is diagonal⁽¹⁵⁾. The expressions used to evaluate the physically interesting quantities are (4.16) + (4.17).

6.3.7 Results of Fit

This is a model of what is a rather complex situation. Due to the lack of theoretical constraints fitting to the data involves a large number of free parameters.

If no knowledge of the trajectories, residues, or SU₃ behaviour is assumed, there are 28 free parameters. Assuming exact SU₃

immediately reduces this to 22. In the present fit the ρ was also fixed leaving 20 variable parameters.

Assuming (ρ, A_2) weak exchange degeneracy would further reduce the freedom. However the A_2 was merely constrained within acceptable limits. The ρ' was constrained to have +ve slope and -ve intercept. (> -1).

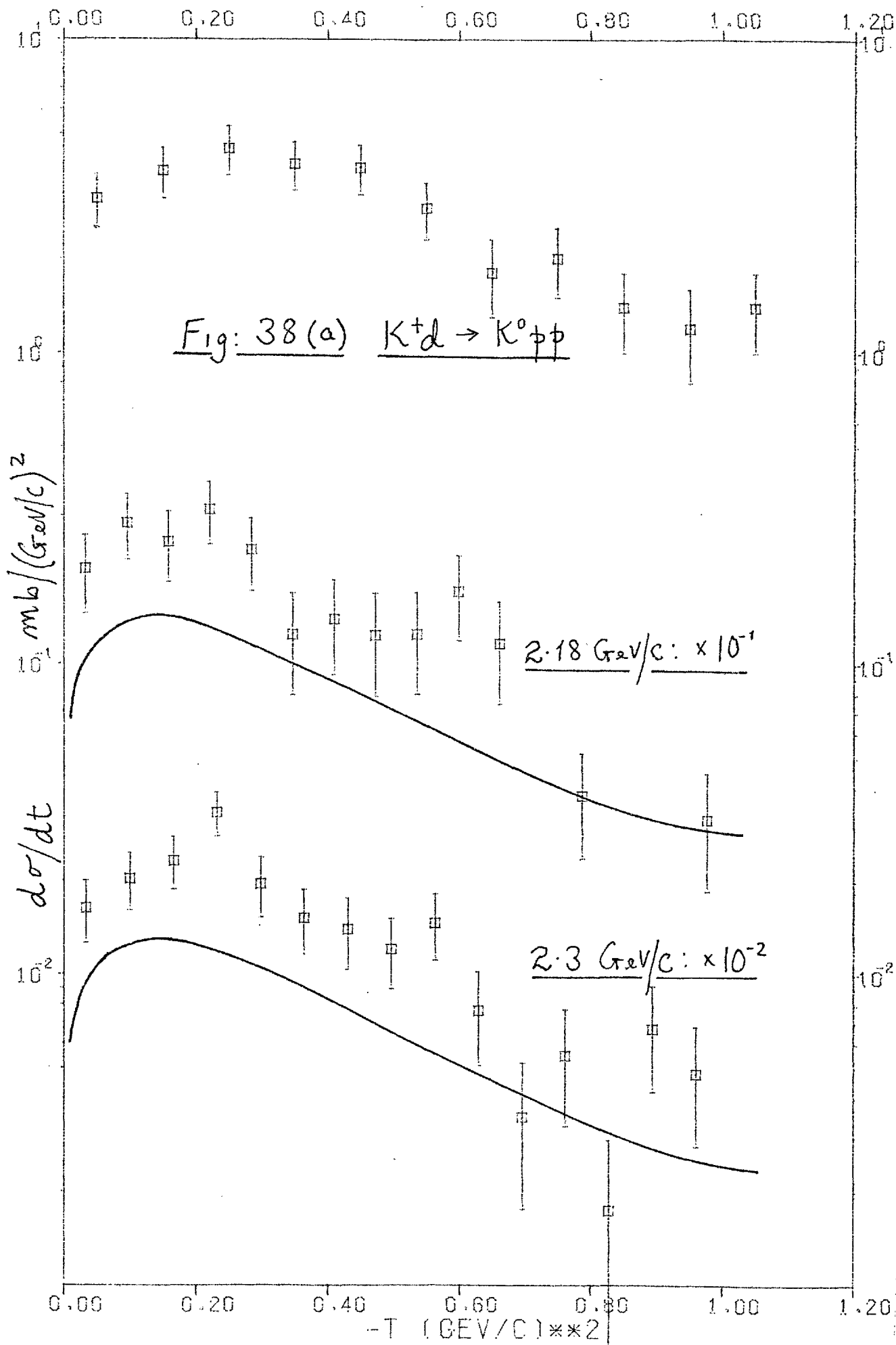
Thus the fit involved 20 free parameters, 4 of which were constrained.

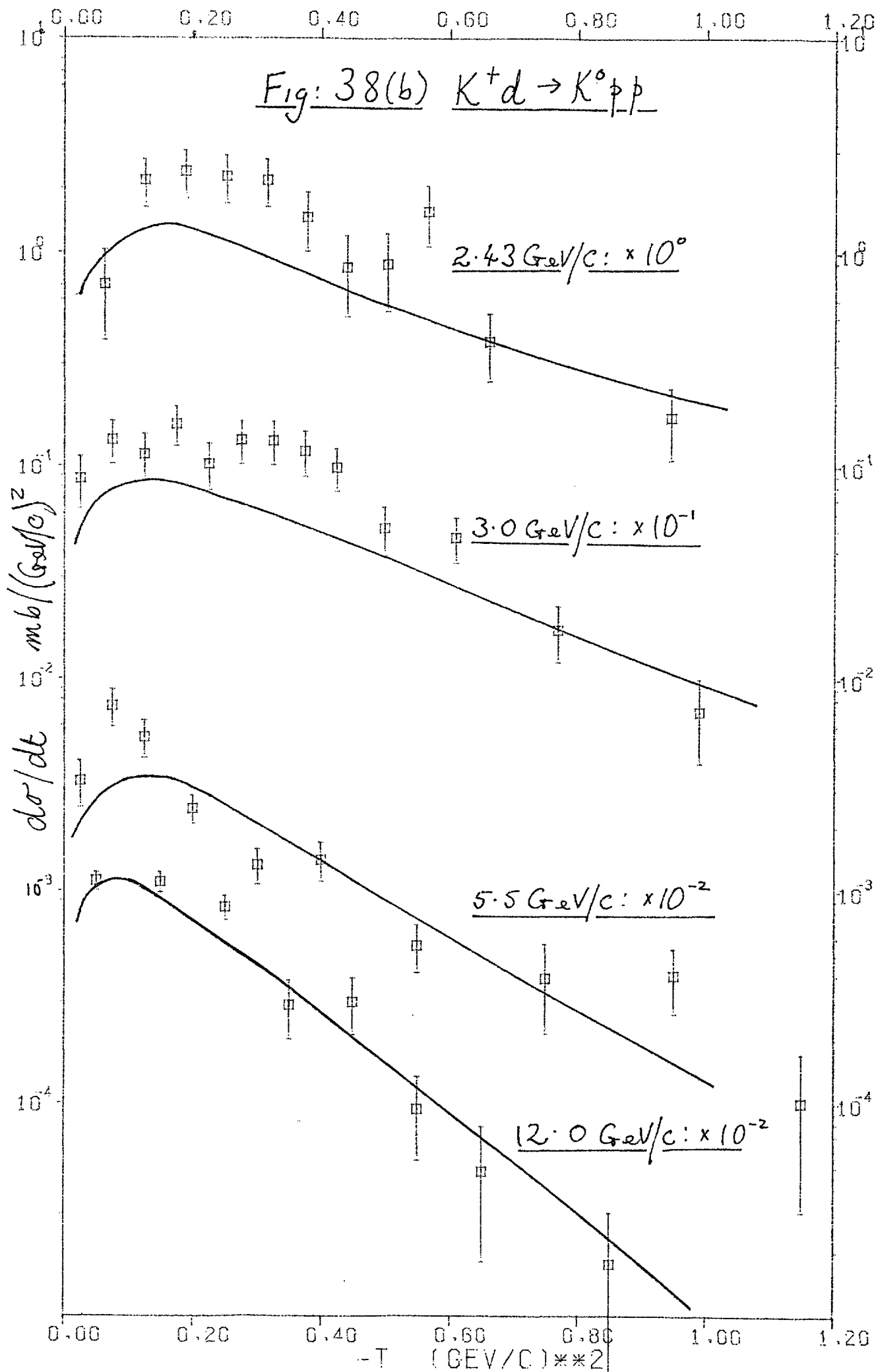
The model was initially fitted to $\pi^- p \rightarrow \pi^0 n$ ($d\sigma/dt$), $|\vec{P}|$). The A_2 parameters were then set at the same values as the ρ parameters and the full data set fitted to.

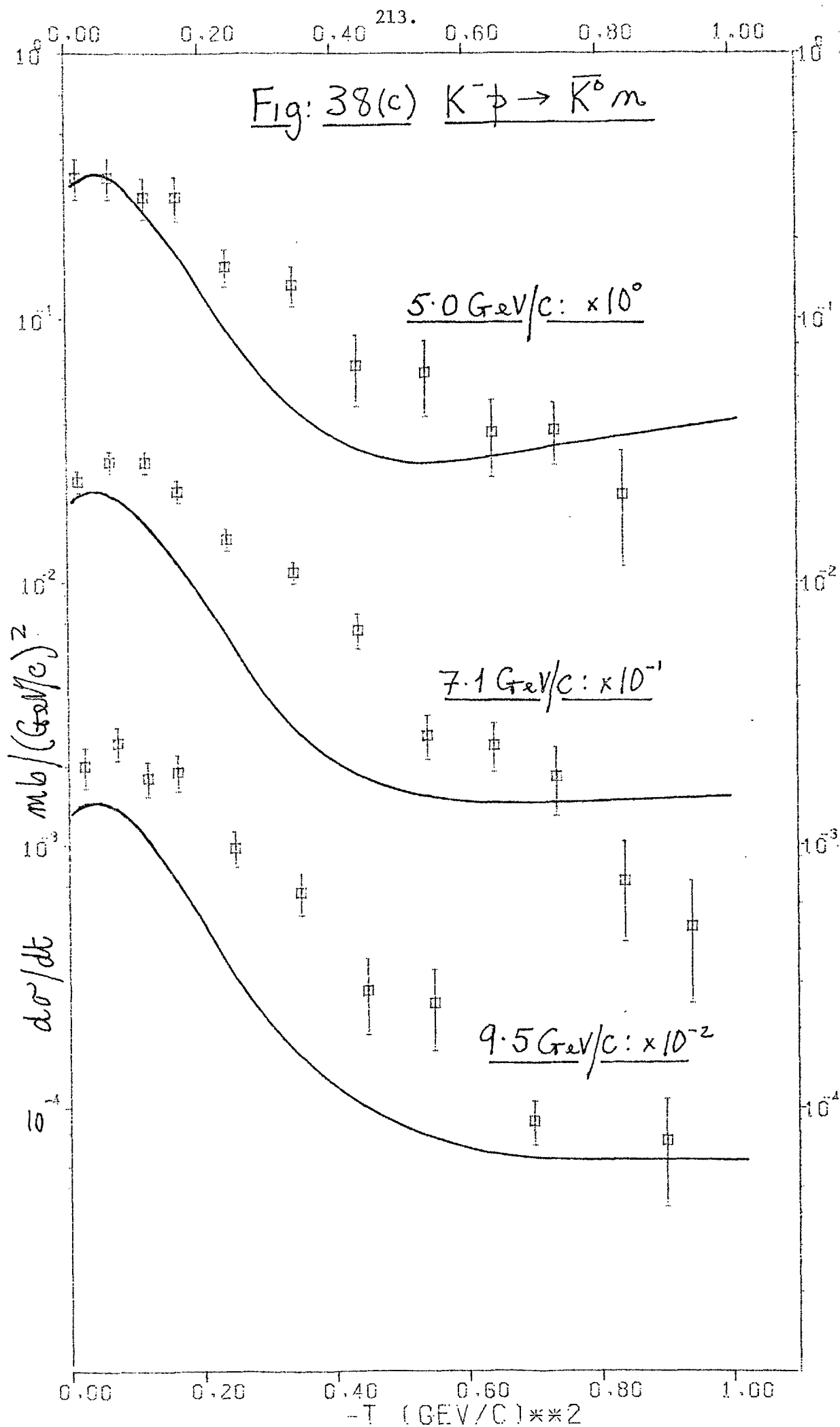
The $K^+ n \rightarrow K^0 p$ data below 2.18 GeV/c was omitted as it was considered to be at too low an energy for the Eikonal method to be valid. Although the data at 2 $\rightarrow$ 3 GeV/c is also perhaps at too low an energy, one would at least expect the model to reproduce the forward peak, even if the large t -behaviour were unsatisfactory.

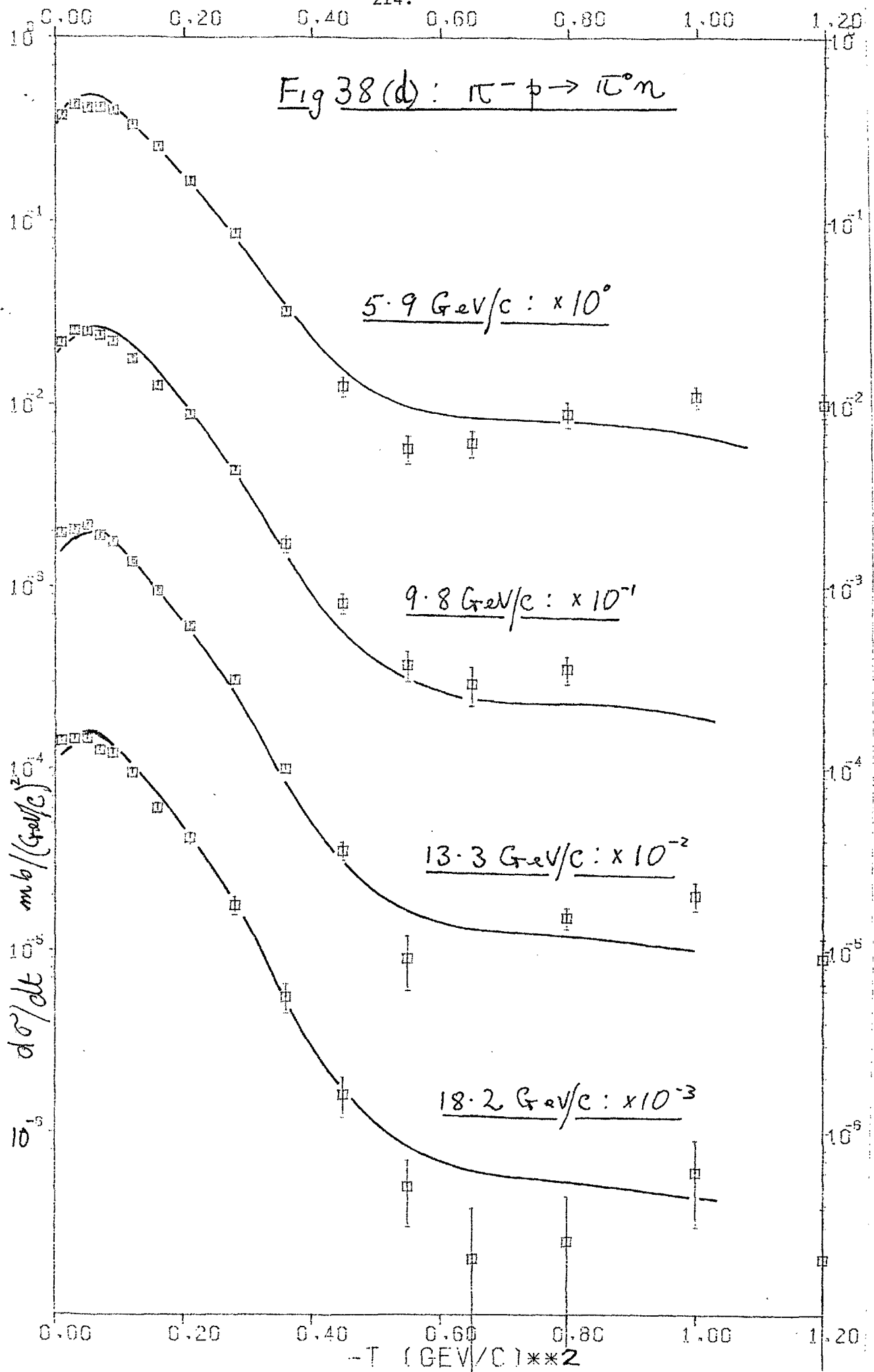
The overall fit is displayed by the full lines in Fig. 38 and in Table 19.

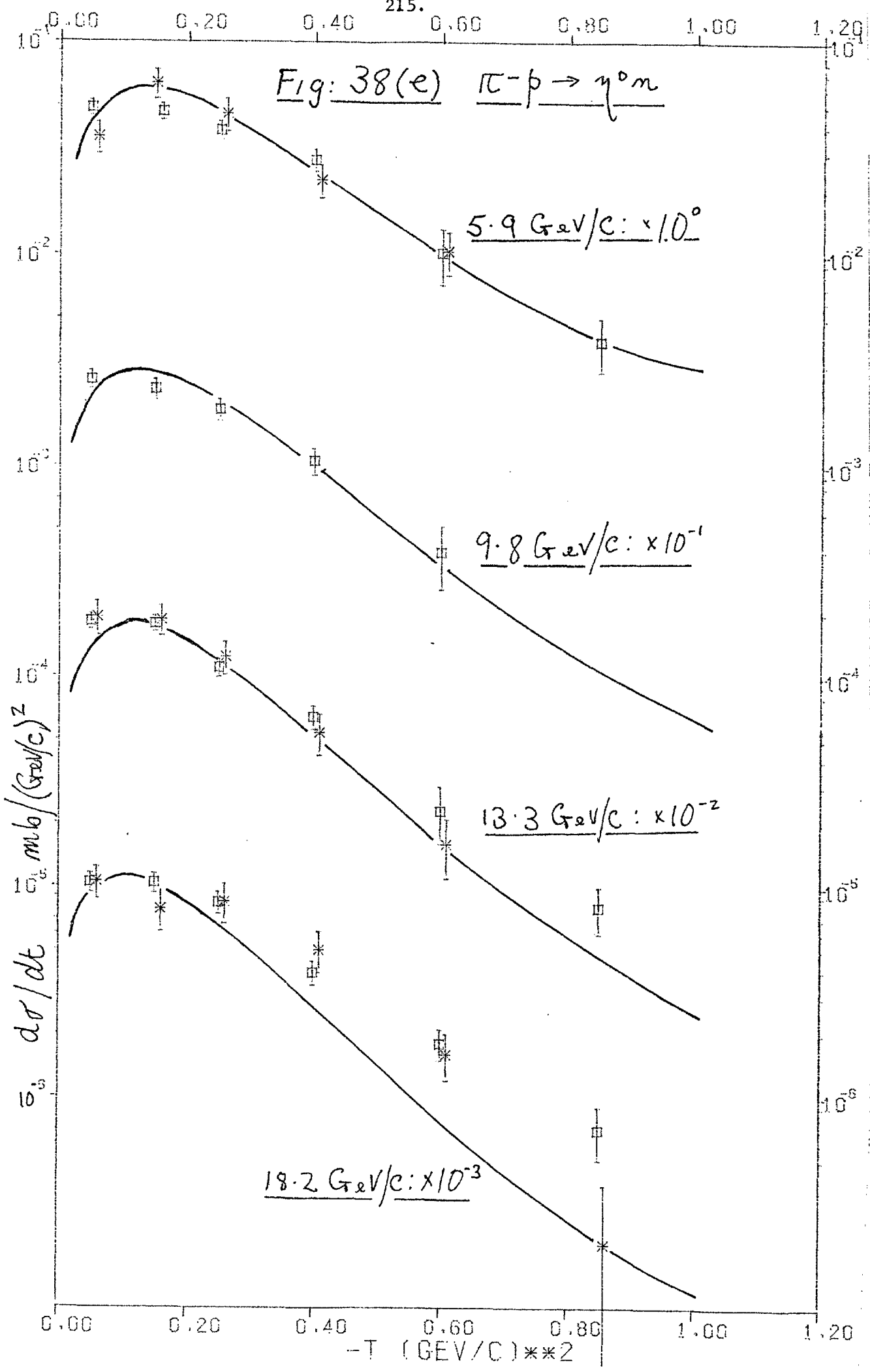
The pion $d\sigma/dt$ are satisfactory, being completely compatible with the data. The large t -behaviour of $d\sigma/dt$ $\pi^- p \rightarrow \pi^0 n$ illustrates the effect of cuts in smearing out the WSN dip due to the pole residue. The $\pi^- p \rightarrow \pi^0 n$ polarisation is well described.

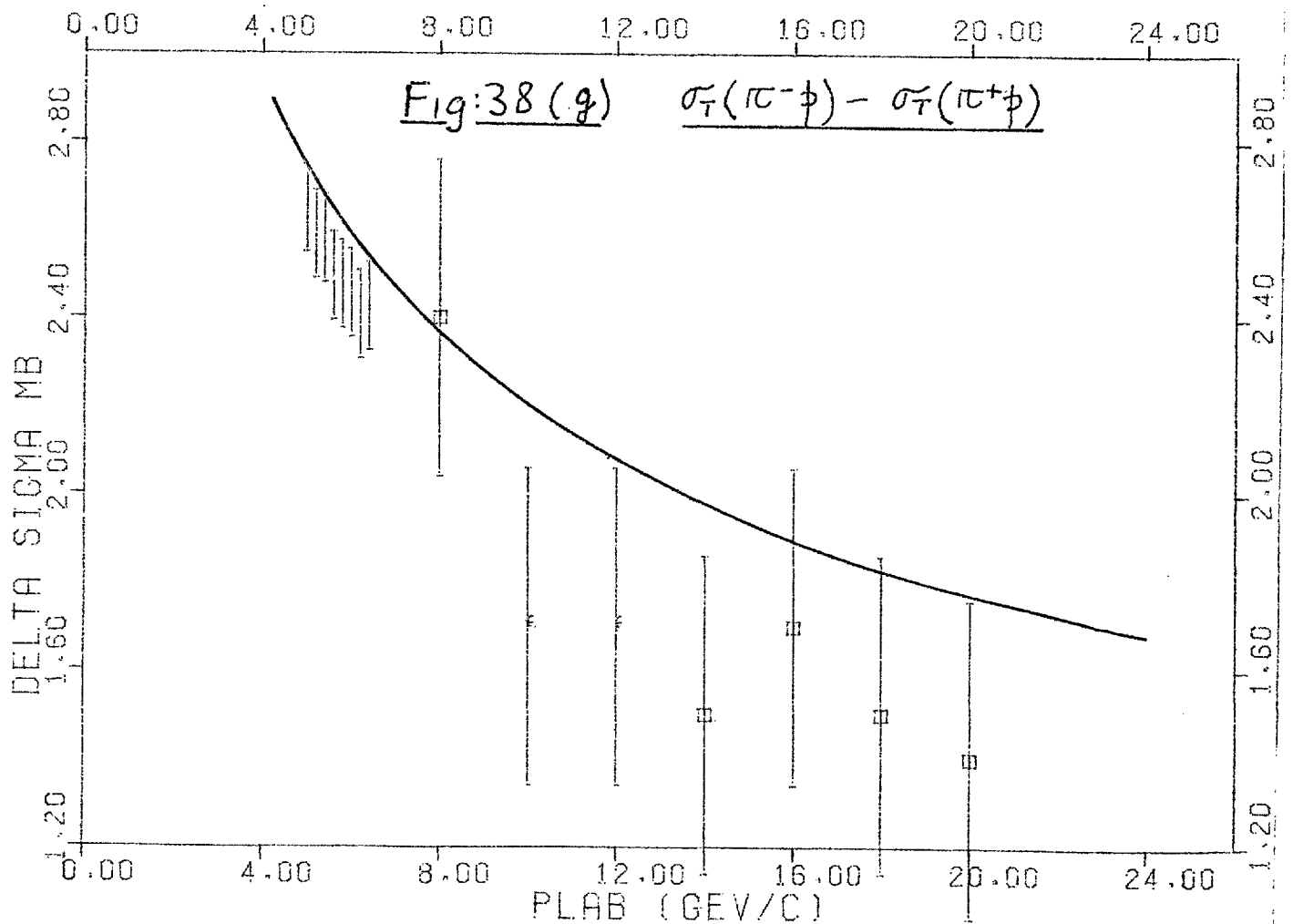
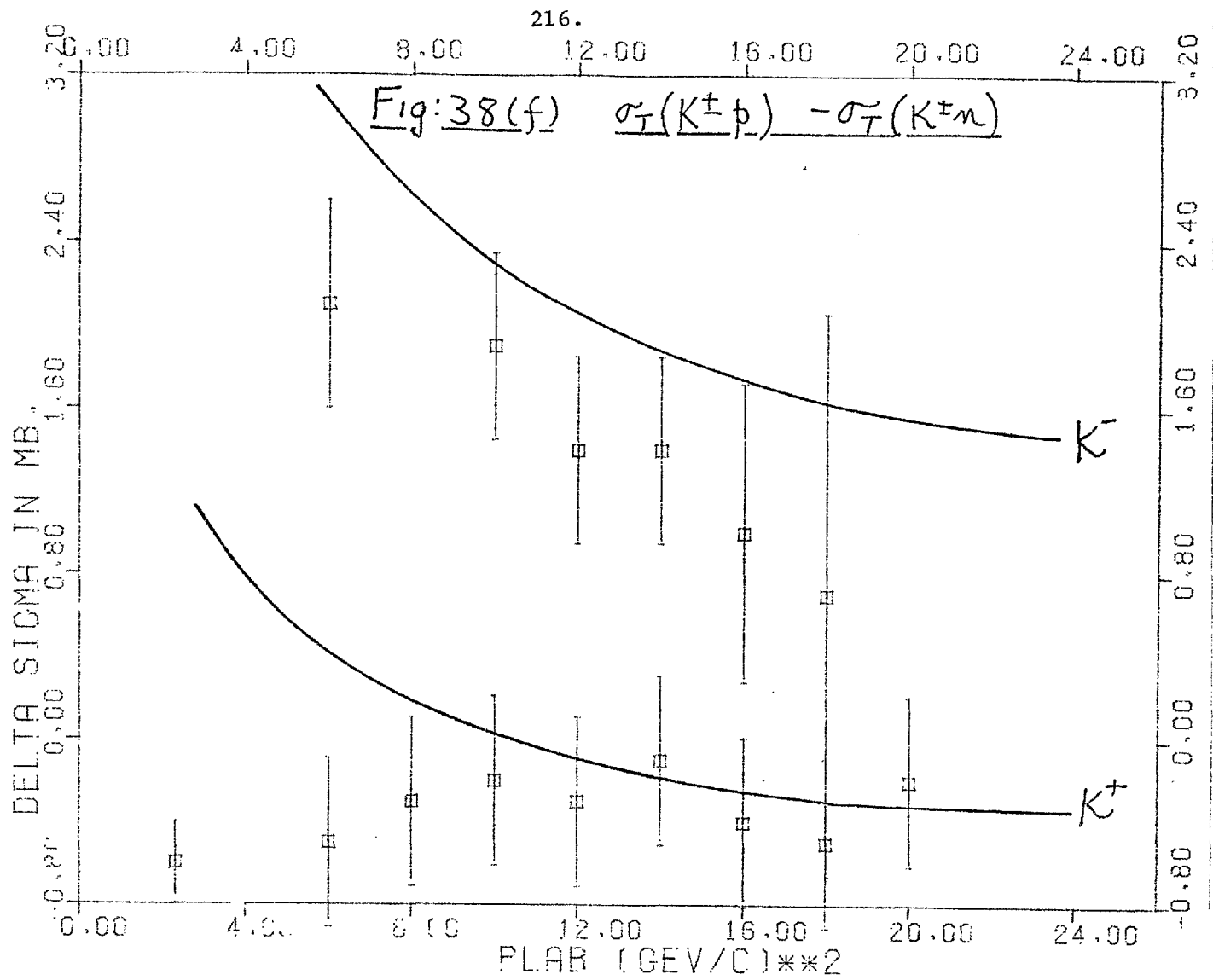












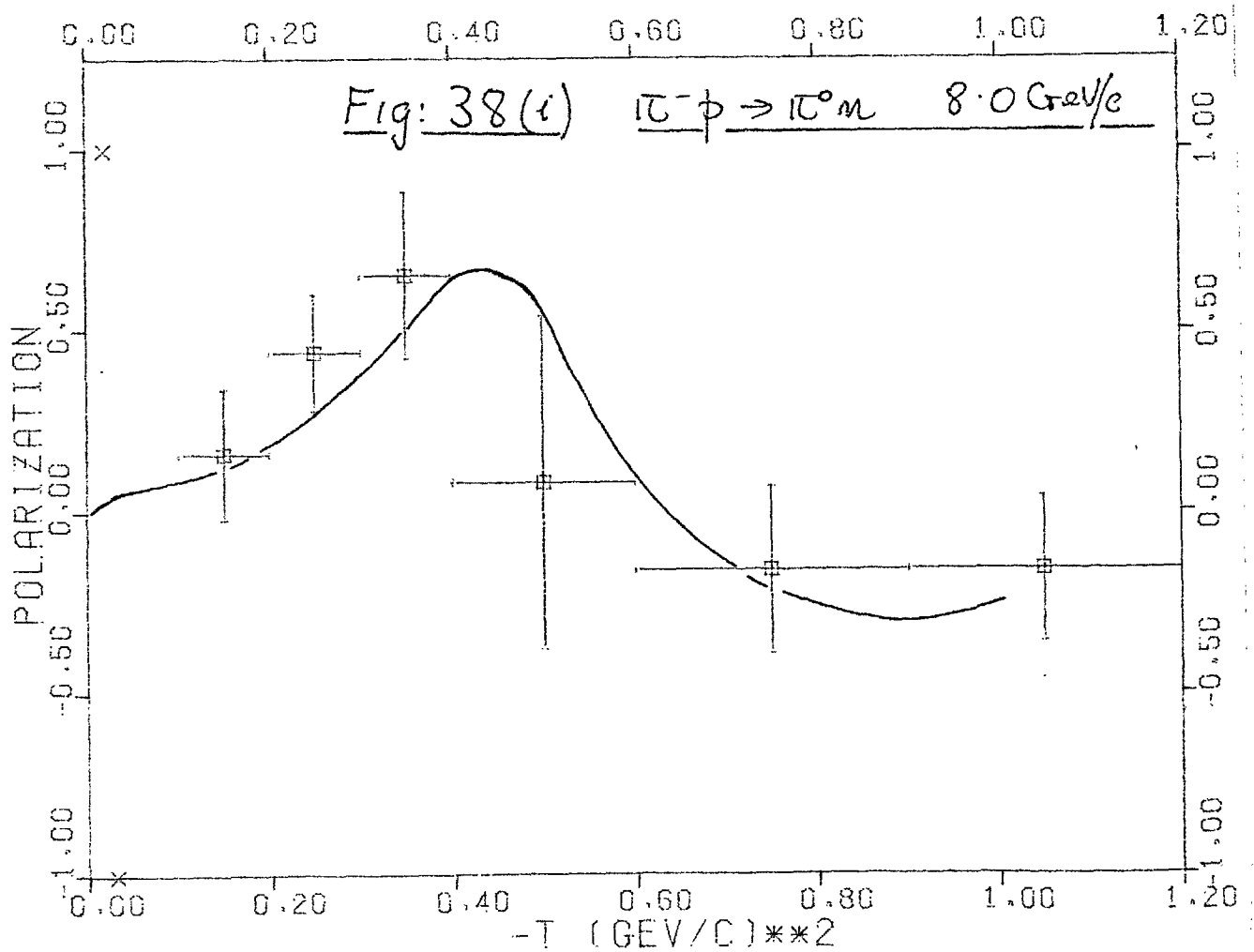
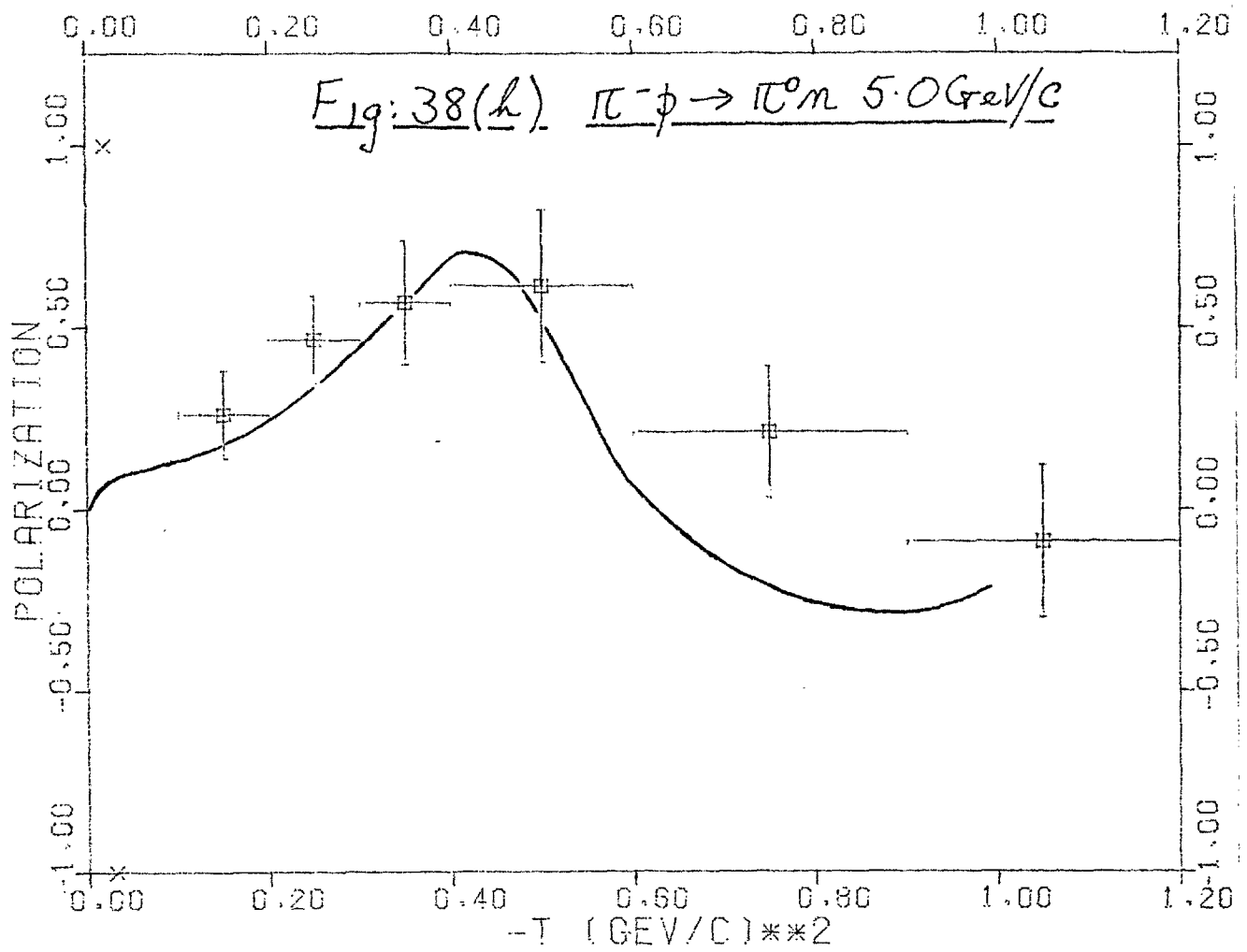


Table 19

 χ^2 / data point = 5.1.

α_0^ρ	0.57 *	} Fixed
α_1^ρ	0.91 *	
$\alpha_0^{\rho'}$	-0.25 ± 0.005	
$\alpha_1^{\rho'}$	0.22 ± 0.01	
$\alpha_0^{A_2}$	0.30 ± 0.005	
$\alpha_1^{A_2}$	0.78 ± 0.01	
$\alpha_1^{\mathbb{P}}$	0.52 ± 0.01	
G_{++}^ρ	11.15 ± 0.3	mbxGeV
G_{+-}^ρ	133.5 ± 1.0	mbxGeV
$G_{++}^{\rho'}$	140.0 ± 1.0	
$G_{+-}^{\rho'}$	-1000. -	
$G_{++}^{A_2}$	19.54 ± 0.2	
$G_{+-}^{A_2}$	82.81 ± 0.2	
λ	2.56 ± 0.05	
σ^{TOT}	31.16 ± 0.14	mb
a_{++}^ρ	4.34 ± 0.1	$(\text{GeV})^{-2}$
a_{+-}^ρ	3.07 ± 0.1	
$a_{++}^{\rho'}$	-0.04 ± 0.01	
$a_{+-}^{\rho'}$	47.4 ± 0.25	
$a_{++}^{A_2}$	-1.49 ± 0.02	
$a_{+-}^{A_2}$	1.86 ± 0.02	
$a_{\mathbb{P}}$	3.34 ± 0.3	

The low energy ($< 5 \text{ GeV}/c$) $d\sigma/dt$ ($K^+n \rightarrow K^0p$) exhibit behaviour common to all cut models, being too small in the region of $t = -0.2 \text{ (GeV}/c)^2$. Also the large t -behaviour of $d\sigma/dt$ ($K^-p \rightarrow \bar{K}^0n$) is unsatisfactory. This would appear to be due to the selective absorption. The ρ' flip residue showed a tendency to increase indefinitely, with very little effect on the form of the $d\sigma/dt$. It was limited, rather arbitrarily, to 10^3 .

The Pomeron and A_2 trajectories are conventional. The A_2 being more or less degenerate with the ρ . The ρ' is again rather flat which serves to confirm the flat ρ' found in the R-S pole model. This flatness of the ρ' trajectory is what gives the correct polarisation behaviour in both pole and pole + cut models.

In summary, the adoption of cuts in the non-flip amplitudes, only, does not improve the absorptive cut model for this class of reactions. Comparing the behaviour of the models treated, the simple pole model is as satisfactory as the cut model. Since there is now some evidence for a ρ' its inclusion in the pole models is not a particularly strong argument against them. This is especially true in view of the theoretical arguments against all present cut prescriptions. The treatment of these three models has been illustrative rather than rigorous. The results illustrate well the fact that widely differing assumptions can all produce reasonable fits to the data. This indicates that the Regge approach is valuable as a phenomenological tool, rather than as a predictive theory.

ACKNOWLEDGEMENTS

I would like to extend my thanks to all members of the Imperial College HENP group for creating such a pleasant environment. In particular I thank the people under whose direction the group has been during my stay here; initially Prof. S.J. Goldsack and latterly Prof. I. Butterworth.

Very many thanks go to Dr. D.B. Miller who has acted as my supervisor and whose patience, good humor, and wide knowledge have been invaluable. Special thanks go to Azarhul Islam for his friendly cooperation throughout our period of mutual suffering.

I also acknowledge the contribution towards my financial support made by the Science Research Council.

Finally, and above all, I thank, Flissy, Becky, and Lucy for their patience and understanding.

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