

To

My Dearest MEHRI

~~BEHAVIOUR~~
CREEP AND FLOW / OF STEELS
UNDER VARYING BI-AXIAL STRESSES

by

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A Thesis presented for the Degree of
Doctor of Philosophy in the
Faculty of Engineering
at
the University of London
and the Diploma of Membership of the Imperial College
(DIC)

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August 1971

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Abstract

The primary object of this research has been to investigate the yield and subsequent plastic flow behaviour of En 24 steel in both the annealed and pre-strained states.

Following a careful investigation of the heat-treatment necessary to achieve isotropy of the thin-walled tubular test-pieces, an analysis of the room temperature tensile creep behaviour of this steel was carried out. This led to a method of allowing for the time-dependence of the permanent strains in post-yield behaviour.

The initial yield and subsequent plastic flow behaviour of annealed En 24 steel was now investigated, under combinations of axial tension-torsion, axial tension-hoop tension and hoop tension-torsion. These results confirmed the basic laws of plasticity regarding convexity of the yield locus, normality of the plastic strain-increment vectors and the Levy-Mises flow rule shown by the equality of the Lode parameters μ and ν .

The isotropic yield locus obeyed the Maxwell-Mises yield criterion in the axial tension - hoop tension stress space but not in the two tension-torsion stress spaces. The latter two regions exhibited a locus lying midway between the criteria of Maxwell-Mises and Tresca. It is believed that these differences are due to a combination of the different modes of

testing and differences in the size of test-pieces used.

The concepts of equivalent stresses and strains have been examined and the equivalent plastic work hypothesis shown to be invalid for this work. Equivalence of the plastic strains was established empirically and shown to apply over the whole of each stress space. The analysis of the tensile creep behaviour was also extended over the whole of each stress space and for each region an equation was obtained which describes completely the stress-strain-time behaviour under any combination of stress in that region.

Finally the effect of two axial tensile pre-strains one hoop pre-strain and one torsion pre-strain upon the yield and flow of this steel in the same three regions of combined stress as previously examined, was undertaken. The new anisotropic yield loci were established and compared with a recent theory due to Williams and Svensson, and the rotation of the plastic strain increment vectors under subsequent plastic flow was also examined.

Acknowledgements

The author would like to express his deep thanks to Dr. A. Shelton, Ph.D., D.I.C., M.I.Mech.E., for his continued interest and encouragement throughout this investigation.

The advice and assistance of Mr. G. Holloway, B.Sc. (Eng.), C.Eng., F.I.Mech.E., M.I.C.E., M.I. Prod. E. of Enfield College of Technology is also acknowledged.

The financial help of the Science Research Council in getting the specimens made is appreciated.

The assistance of Mr. M. C. Layton and Mr. Williams of M.C. Layton Ltd. Birmingham in manufacturing the specimens is also appreciated.

The author thanks all of the technical staff of the workshops and laboratories of the Mechanical Engineering Department, Imperial College of Science and Technology who helped with the making the preliminary specimens and other parts used in this work.

The author held a scholarship from the Overseas Development Authority through the British Council during the period of this research for which he would like to thank the Directors and Organisers of both Departments.

Notation

σ	direct stress based on initial dimensions.
τ	shear stress based on initial dimensions.
ϵ	direct strain, engineering definition.
γ	pure shear strain, engineering definition.
$\epsilon_{xy} = \frac{\gamma}{2}$	shear strain, tensor definition.
z, θ, r	axial, hoop and radial directions in a thin-walled tube.
$\sigma_1, \sigma_2, \sigma_3$	principal stresses.
$\epsilon_1, \epsilon_2, \epsilon_3$	principal strains.
ϵ^p, γ^p	plastic components of ϵ and γ .
$\bar{\sigma}$	equivalent stress.
$\bar{\epsilon}$	equivalent strain.
$\tau_{oct.}$	octahedral shear stress.
$\gamma_{oct.}$	octahedral shear strain.
t	time in seconds.
$\dot{\epsilon}, \dot{\gamma}$	time derivatives of the strains.
$\dot{\epsilon}t)$ $\dot{\gamma}t)$	creep coefficients obtained from the logarithmic creep law.
$\epsilon_1, \epsilon_{1000})$ $\gamma_1, \gamma_{1000})$	values of strains at time $t = 1, 1000$ secs. Called the creep constant, ϵ_{1000} and γ_{1000} also called "long-time" strains.
$A', B', C',)$ $h, i, j)$	parameters associated with the type of applied stress.
α	angle between the stress vector $\frac{\sigma}{\tau}$ and the strain vector $\frac{\gamma^p}{\epsilon^p}$

All other symbols are defined in the text at the place where they are first used.

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Chapter 1

INTRODUCTION AND SURVEY OF PREVIOUS WORK

1.1 Introduction

The principal objective of the present work is to establish the yield and subsequent plastic flow behaviour of pre-strained En 24 steel over one-half of the bi-axial principal stress space, at room temperature. Subsidiary objectives include an analysis of the creep behaviour under the test conditions and an investigation of whether or not the plastic work hypothesis of plasticity theory describes the relationship between equivalent stress and equivalent plastic strain for all tests on the annealed material.

No attempt has been made to include tests involving compression owing to the inherent friction difficulties entailed in such tests. Thus the tests employed were: (i) torsion-axial tension; (ii) axial tension-hoop tension; (iii) hoop tension-torsion, and these tests are sufficient to cover the above mentioned stress space. The behaviour of the metal in the annealed state is first described and analysed and this is followed by the tests on pre-strained metal. Pre-strain was introduced by either tension, torsion or internal pressure, sufficient to introduce a permanent deformation prior to performing further complex stress tests over the same range as for the annealed material.

In all tests, one test-piece was used for a single point on a given yield locus and then discarded. This has

entailed the manufacture of a considerable number of test-pieces but this was considered preferable to using a single test-piece to establish a number of points on successive yield loci. The strain-history of each test-piece was therefore never in doubt.

1.2 Definition of Yield Stress

As shown by Hsu (1966D) the definition of yield stress used by different authors can give rise to discrepancies between results due to different possible definitions of yielding. Although different materials were used for example by Naghdi (1960), Ivey (1961), Mair and Pugh (1964), and Taylor and Quinney (1931), they also used different definitions of yield stress and reached different conclusions because of this. Their definitions of yielding can be summarized as follows:

- a) departure from linearity of the stress-strain curve as used by both Naghdi and Ivey;
- b) backward extrapolation to intersect the extension of the elastic line of the stress-strain curve i.e. zero plastic strain, as used by Taylor and Quinney.
- c) proof strain.

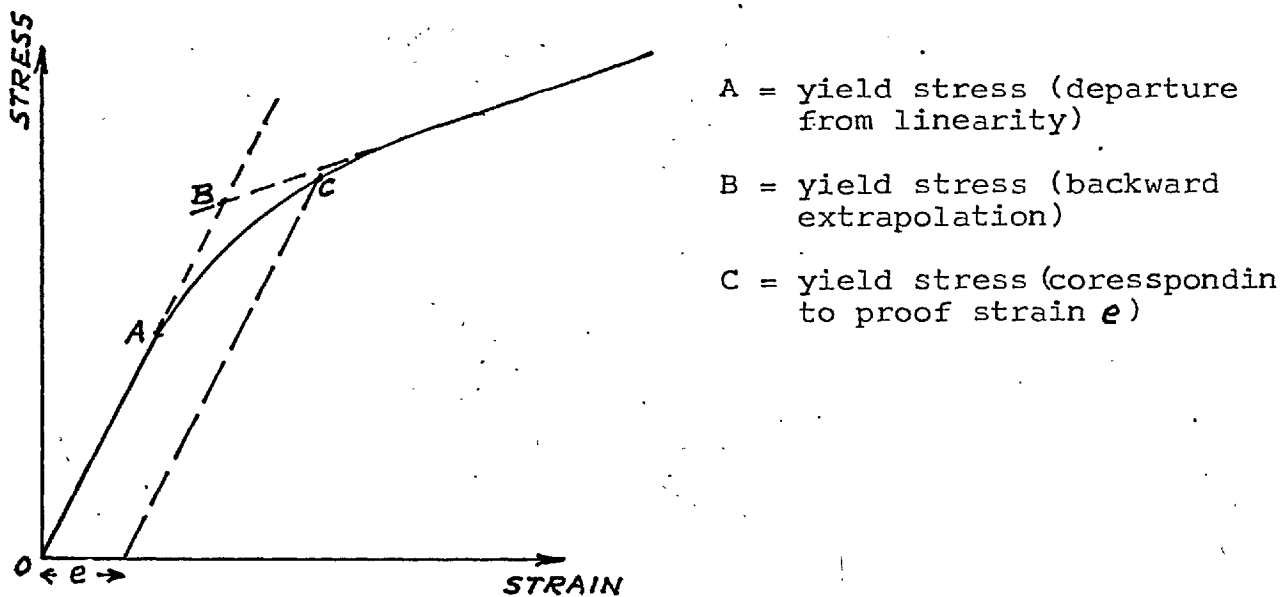
Method (a) is essentially a limit of proportionality criterion. It can be obtained after a small plastic strain has taken place, sufficient to establish that "yield point", but it is clearly unrealistic since a material would not "flow" at that stress.

Method (b) used by Taylor and Quinney (1931) and also used in this work, would appear the most satisfactory in principle, because the value of stress obtained is identical to the "flow" stress in continuous loading and hence shows the change from elastic to plastic strain behaviour. The demerit of this method is the practical one that a considerable inroad into the plastic state is necessary to define a single point on the yield locus. The test-piece is therefore considerably work-hardened and it is not convenient to establish other points on the same locus, using the same test-piece. A large number of test-pieces is therefore necessary to establish the initial and subsequent yield loci required in the present investigation.

Method (c) is an alternative to method (b). A proof strain can be chosen such that the yield stress is similar to that obtained by method (b), but the yield stress obtained will vary with the definition of proof strain used, e.g. 0.1%, 0.2% etc. There is thus no virtue in using method (c) if method (b) can be used directly.

Possible errors in extrapolating the stress-strain curve have been minimised in the present work by making use of the creep results. As shown later, annealed En 24 exhibited logarithmic creep under all stress systems and it has therefore also been possible to extrapolate curves of creep coefficient versus stress in order to obtain the yield stress. The agreement between the two methods was very good in all cases.

The figure below shows the three methods of yield definition discussed and it is clear that yield stress "c" is very much dependent upon the chosen value of the proof strain e .



1.3 Brief Survey of Theories of Yield and Flow

Nadai (1950) gives an excellent review of the theories which have sought to predict the initial yield behaviour of metals under complex stress and then discusses those which experiment has shown to be of most value.

1.3.1 The maximum shear stress or Tresca theory

As a result of extrusion tests on the flow of soft metals through orifices, Tresca (1870) concluded that a metal will yield when the greatest algebraic^a difference between the three principal stresses reaches a constant critical value. Taking $\sigma_1 > \sigma_2 > \sigma_3$, the theory may be expressed as:

$$\frac{\sigma_1 - \sigma_3}{2} = \tau_{\max} = \frac{\bar{\sigma}}{2} \quad \therefore \quad (1)$$

or

$$\sigma_1 - \sigma_3 = \bar{\sigma} \quad \therefore \quad (2)$$

In this theory it is clear that neither the intermediate principal stress σ_2 nor the magnitude of the three principal stresses have any influence upon yielding. Only the maximum stress difference is of importance.

1.3.2 The strength theory of Mohr

In this theory a material is assumed to fail either through plastic slip or by fracture when either the shear stress in the slip planes has increased to a certain value which depends upon the normal stress acting across those planes, or when the largest tensile normal stress has reached a limiting value. Again taking $\sigma_1 > \sigma_2 > \sigma_3$, the theory may be written:

$$\frac{\sigma_1 - \sigma_3}{2} = f\left(\frac{\sigma_1 + \sigma_3}{2}\right) \quad \therefore \quad (3)$$

where $f()$ means function. Again the intermediate principal stress σ_2 has no influence upon yielding.

1.3.3 The shear strain-energy or Maxwell-Mises theory

This theory is variously known as the von Mises, Hencky-Mises or Huber-Hencky theory but was originated in strain form by Clerk Maxwell (see Sopwith and Morrison (1956)). This theory allows for the effect of the intermediate principal stress and states that a material will yield when the shear strain-energy stored in the body reaches a critical value. It is unnecessary to stipulate $\sigma_1 > \sigma_2 > \sigma_3$ as all three principal stress differences are involved and not just the maximum difference as in the Tresca theory. This theory may be written:

$$(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 = 2\bar{\sigma}^2 \quad \dots (4)$$

and is equivalent to the expression:

$$\bar{\sigma}^2 = 3J_2' \quad \dots (5)$$

where J_2' is the second invariant of the deviatoric stresses, given by:

$$J_2' = \frac{1}{2} \sigma'_{ij} \sigma'_{ij} \quad \dots (6)$$

Nadai has shown that this combination of principal stresses is also proportional to the shear stress on the octahedral planes i.e. those four pairs of parallel planes which have their normals defined by the direction cosines $l = m = n = 1/\sqrt{3}$. Thus the onset of yield may also be defined by:

$$\tau_{\text{oct.}} = \frac{1}{3} \{(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2\}^{\frac{1}{2}} = \frac{\sqrt{2}}{3} \bar{\sigma} \quad \dots (7)$$

1.3.4 Other theories

The maximum principal stress theory of Rankine:

$$\sigma_1 = \bar{\sigma} \quad \dots \quad (8)$$

the maximum strain theory of St. Venant:

$$\{\sigma_1 - \nu(\sigma_2 + \sigma_3)\} = \bar{\sigma} \quad \dots \quad (9)$$

and the total strain energy theory of Beltrami:

$$\{(\sigma_1^2 + \sigma_2^2 + \sigma_3^2) - 2\nu(\sigma_1\sigma_2 + \sigma_2\sigma_3 + \sigma_3\sigma_1)\} = \bar{\sigma}^2 \quad (10)$$

have all been suggested in turn as criteria for predicting the initial yield of ductile metals under complex stress.

There is no experimental evidence to support any of these criteria and a simple example illustrates why this is so.

It is well known, e.g. Bridgman (1923), that a metal under hydrostatic stress ($\sigma_1 = \sigma_2 = \sigma_3$) will not yield but only deform elastically, even under enormously high pressure.

All three theories however predict yielding to occur under such a stress system and must therefore be rejected.

1.3.5 Subsequent Flow Behaviour

To describe continued loading past the initial yield locus, two quantities are required. Firstly there must be a single function of the stresses which makes equivalent all different possible states of stress and a single function of the plastic strains which makes equivalent the strains resulting when particular values of the equivalent stress are applied to the material.

The simplest way of expressing the equivalent stress function, or loading function, is obtained when it is said that the form of the initial yield locus will continue to describe the equivalent stress after yielding. Thus $\bar{\sigma}$, which made equivalent the stresses required to initiate yielding continues to make the post-yield stresses equivalent, when loading increases. On radial loading paths this is usually found to express adequately the behaviour of metals, because the directions of the resulting strain vector remain constant during such loading. Therefore, since the strain-increment vector remains normal to the equivalent stress surface, this surface must grow isotropically in the region of each radial stress path, retaining the same shape as the initial yield surface.

It is worthwhile to mention that this does not imply that the post yield surface found subsequent to a prestress will be an isotropic expansion of the initial yield locus, i.e. isotropic hardening.

In fig. 1(a), isotropic hardening, implies that the subsequent yield surface found after pre-straining the material to point 2* would be a proportional expansion of the initial locus I. The statement that the form of the initial yield locus continues to describe the equivalence of post-yield stresses in radial loading tests upon an isotropic material, merely means that as in fig. 1(b) the portion of the initial yield locus at each stress ratio 0 - 1 retains its original shape as each probe moves outwards 0 - 1 - 2. Thus the equivalent-stress locus,

curve II in fig. 1(b) is a proportional expansion of the initial curve. But if instead of an annealed (isotropic) material, these same tests were performed after the material had been pre-strained to 2^* then the subsequent yield surface under each stress ratio could have any shape, e.g. curve II in fig. 1(a).

Fig. 2 taken from Mair and Pugh (1964) shows graphically four theories which seek to explain the shape of the yield locus subsequent to a pre-stress or pre-strain. Fig. 2(a) shows simple isotropic hardening, fig. 2(b) depicts the slip theory of Batdorf and Budiansky (1949) which is not really satisfactory but they suggested later (1954) may be modified to give closer agreement with experiment. Prager (1956) proposed a kinematic hardening law shown in fig. 2(c) which Hodge (1957) combined with isotropic hardening to give fig. 2(d). This combination of kinematic hardening with isotropic hardening is the most promising theory, especially if distortion of the shape of the subsequent locus is incorporated.

Drucker (1950) outlines several loading functions which would combine the features of translation, expansion and distortion but the complexity of the analytic function which will adequately describe the results of the simple loading paths used in the present investigation would be enormous. Briefly, the procedure explained by Drucker is to add an arbitrary constant for every feature that has to be described; thus for example, if ten experiments are performed upon a pre-strained material and in each experiment

there are ten features requiring correlation (such as ten changes in the strain-increment vector direction during each subsequent test) then if the loading function contains one hundred constants, suitable values assigned to these are bound to give perfect correlation of theory with experiment. Drucker examined eight loading functions, the most general of which is of the form:

$$f = \frac{1}{2} D_{ijkl} (\sigma'_{ij} - m \epsilon_{ij}) (\sigma'_{kl} - m \epsilon_{kl}) \quad \dots \quad (11)$$

where m and D are constants, σ' is deviatoric stress and ϵ is the plastic pre-strain. This type of loading function is suitable for small initial anisotropy followed by a marked change in anisotropy in subsequent experimental deformation. It has been used by Svensson (1966) to describe the behaviour of cold rolled aluminium and a related criterion has been used by Hsu (1966A) to describe some results of earlier investigators.

For materials which are initially highly anisotropic, the theories of Hill (1948) and Dorn (1949) have been proposed. Hill's theory has been found several times to describe accurately the behaviour of heavily rolled metals. These quadratic isotropic functions, the most general of which is:

$$f = \frac{1}{2} D_{ijkl} \sigma_{ij} \sigma_{kl} \quad \dots \quad (12)$$

show roughly the same behaviour as, say $f = J_2$, in that subsequent yield loci are expansions of the initial locus. This theory describes the behaviour of a material whose initial anisotropy is so great as to be unaffected by the deformation produced during a pre-stress test.

1.4 Survey of Previous Experimental Work

Some of the more important experiments upon the yield and flow behaviour of metals are reviewed in this section. More detailed comparison with the results of the present investigation is made later in Chapter 7.

1.4.1 Experiments on the initial yield locus

Tresca (1870) was one of the pioneers on investigating the initial yield locus. He established his criterion while drawing steel wires and later others were concerned with establishing the superiority of the Tresca criterion over the principal stress and principal strain criteria.

Guest (1900) experimented with steel, copper, and brass tubes under combined tension-torsion and tension-internal pressure. He also mentions the work done on yielding by McFarlans for Lord Kelvin's article on Elasticity in the Encyclopedia Britannica. This showed that a simultaneous tension lowered the yield point in torsion, but numerical data are not given. Lord Kelvin concluded that as tension lowers the torsional yield point under combined stress, compression would raise it, but his argument does not appear to be rigorous and we now know that either tension or compression would in fact lower it.

Guest used each tube (1.25 inches bore, 0.025 inch wall thickness) six to twenty times. It was first loaded to yield, unloaded, then reloaded in another stress state to

yield, and repeated as before. He measured the yield point carefully but the strain increment vectors were not carefully and accurately measured. He found a different elastic limit after each stress cycle and showed that the Tresca criterion agreed with his results fairly well. He then suggested that a linear relation existed between the critical shear stress τ_{crit} and the normal stresses σ_1 and σ_3 in the form:

$$\tau_{crit} = \frac{1}{2} [(\sigma_1 - \sigma_3) + k (\sigma_1 + \sigma_3)] \quad \dots \quad (13)$$

which is a special form of Mohr's theory.

In Fig. 3 the results obtained by Guest for his tube No. VIII are plotted for tension-torsion and for tension-internal pressure stress space. It is noticeable that the Tresca ellipse $\bar{\sigma} = \sqrt{\sigma^2 + 4\tau^2}$ is nearly followed in tension-torsion, but in tension-internal pressure a rounded yield locus is produced which is far from the square shape predicted by Tresca.

Smith (1909) worked with solid steel bars of different specifications and all about one inch diameter. His tests were in compression, tension and combined compression-torsion or tension-torsion, and showed that for mild steel the Tresca yield law $\bar{\sigma} = \sqrt{\sigma^2 + 4\tau^2}$ was obeyed almost exactly. Two further steels did not have a sudden yield and the limit of proportionality was used instead. One of these - a nickel steel - also favoured the Tresca law but the other - described as a "self-hard" steel - did not; in fact it favoured the Maxwell-Mises law expressed in direct and shear stresses.

Mason (1909) worked on thin walled steel tubes in the as-received and annealed states, under combinations of compression, tension and internal pressure. Since he only applied principal stresses, this author plotted his results in principal stress space and showed that the yield stress in pure shear i.e. equal and opposite tension and compression, was slightly greater than that predicted by Tresca - in fact it lay between the Tresca and Maxwell-Mises criterion postulated at a later date. Clearly, the heat-treatment and test-piece manufacture were at fault in these tests since the simple tension and compression yield points were not in agreement and a variation of as much as 0.007 inch in wall thicknesses of 0.128 inch and 0.08 inch, was reported.

Cook and Robertson (1911) tested thick-walled tubes of cast-iron and mild-steel under internal pressure. They used various ratios of external to internal diameter for both materials and concluded that for cast-iron the maximum principal stress theory of Rankine is obeyed. In the case of mild steel however, initial yield occurred when the internal pressure was about 20 per cent greater than that required by the shear stress theory.

Cook (1932) tested mild steel to determine the relation between the stress at yield point in simple tension and in the non-uniform distribution of stress produced by torsion, flexure and internal pressure in a hollow cylinder, the ratio of external to internal diameters lying between 1.4 - 7.00. He concluded that the maximum shear stress was

the determining factor in elastic failure, and that the ratio of maximum shear stress at yield under non-uniform stress to maximum shear stress in uniaxial tension, was greater than unity.

Lode (1925) carried out tests under combined internal pressure and axial tension. Although his results were interpreted to favour the Maxwell-Mises criterion they are in fact unreliable. This criticism is made because Lode used a single tube for twenty tests and whilst he appreciated that work-hardening would be introduced during the course of each test the empirical law used to allow for work-hardening is of doubtful validity. Lode's principal contribution in this work was to introduce the stress and plastic strain-increment parameters which now bear his name and these are discussed later in Section 1.4.3.

Schmidt (1932) tested rather thick-walled tubes under combined tension-torsion. The four tests reported showed initial yield to favour the Maxwell-Mises criterion.

Morrison (1940) paid particular attention to these previous investigations and the methods of test and decided that they were not accurate enough to determine the yield criterion. His experiments on mild-steel in tension, compression, torsion and combined tension-torsion, however, were carried out without considering the direction of the strain-increment vectors. He went to considerable trouble to ensure accuracy of loading and to account for any sources of error. He showed that the shear yield stress in torsion on solid specimens depends upon the size of the specimen,

i.e. decreasing the diameter increases the stress gradient and increases the apparent shear yield stress. He explained this size effect in torsion as being due to a thin layer at the outer skin of the specimen withstanding the yield stress owing to constraint by the elastic core. Only when the shear yield stress is reached in the whole layer will the material yield. The same effect was observed in a thick-walled tube under torsion when the outside diameter was small. Brown and Rosenbaum (1959) explained this effect later when they found that the average shear stress sustained by a single crystal tested in torsion was 1.12 to 1.25 times the critical resolved shear stress. Morrison had pointed out the importance of the ratio of wall thickness: crystal diameter and showed that very thin-walled tubes with few crystals across the wall, tended to single crystal behaviour.

The main conclusions from this work by Morrison was that the term "yield of a material" was meaningless and one should refer to "yield of a specimen or test-piece". To a first approximation mild steel was shown to yield when a shear stress of constant magnitude was applied i.e. the Tresca criterion.

Lessels and MacGregor (1940) worked on thin-walled tubes of nickel-chrome-molybdenum steel under combined axial tension-internal compression, and combined axial compression-internal pressure; they also carried out torsion tests on the same material using solid bars. They did not indicate the accuracy of dimensions of the test-pieces,

but the testing set-up for combined axial compression-internal pressure - which consisted of a blind hole in the tubes plus copper packing under the compression platens was certain to give eccentric loading and barrelling of the test-piece. A proof stress using a proof strain based upon the octahedral shear strain was used to define yield, and their experimental results agreed most favourably with the Maxwell-Mises or Octahedral criterion.

Davis (1945) carried out tests on steel tubes under combinations of tension-internal pressure. The initial-yield results would give a rounded yield locus when plotted in principal stress space.

Morrison (1948), again using a very careful experimental technique, investigated the criterion of yield of several gun steels. Hardened and tempered En 25 steel in bar form showed an upper and lower yield point when tested in tension, compression, flexure, torsion and internal pressure. The upper yield points favoured the Tresca criterion but the lower yield points favoured the Maxwell-Mises. The same steel in the form of a gun-barrel forging and seven other steels in the same form did not exhibit a sharp yield-point and all correlated with the Maxwell-Mises criterion.

Siebel (1953) performed experiments on thin walled tubes of quenched and tempered low-alloy steel under combined bending and twisting. The results for the initial yield locus agreed with the Maxwell-Mises criterion as did those of Lianis and Ford (1957) when testing a specially prepared

notched aluminium strip in tension. This latter material is of special interest as it had been pickled and rolled at the temperature of liquid nitrogen and in consequence revealed the non-work-hardening curve of "ideal" plasticity.

Naghdi, Essenburg and Koff, (1958) subjected twenty-five specimens of aluminium alloy to combined tension-torsion as did Ivey (1961) who worked with five materials of that type. They both used the limit of proportionality definition of initial yield since a sharp yield point was absent in all tests. The Naghdi et al results obeyed the Maxwell-Mises criterion but Ivey's results lay outside that criterion and remote from the Tresca. Indeed, Ivey found the ratio shear yield stress: tensile yield stress to be 0.63 instead of the 0.577 given by Maxwell-Mises.

Rogan and Shelton (1969A) carried out combined tension-torsion tests on thin-walled tubes of En 24 and En 25 steels. Their primary object was to investigate the creep and subsequent plastic flow behaviour of these steels at room temperature and this is discussed later. Using constant stress ratio paths and a new test-piece for each yield point they obtained initial yield loci which lay between the criteria of Tresca and Maxwell-Mises. The En25 locus was very close to the Tresca criterion and the En24 exactly midway between the two. Both metals were very carefully annealed before testing and the yield point was never in doubt because both metals exhibited a sudden yield under all tests.

1.4.2 Loading beyond initial yield.

Stress-strain relations in the plastic state

Ludwik and Scheu (1925) worked on copper under tension, torsion, and compression. Their results in tension and compression coincided when they plotted true stress v. logarithmic strain. Torsion results using maximum shear stress $\tau_{\max}$ v. maximum logarithmic shear strain $\bar{\gamma}_{\max}$ did not correlate with the former, these curves fell below the other curves. Nadai (1950) pointed out that their definition of maximum logarithmic shear strain was incorrect.

Siebel and Pomp (1927) performed experiments on different materials under tension and compression and found that the end effect in compression due to friction between the material and platens was considerable. They made the ends of their specimens and the platens in cup and cone form to eliminate the friction effect using a cone angle equal to the angle of friction between the two materials. They plotted curves of true stress v. true strain and found that in all materials the compression curves were higher than the tension ones - as much as 20 per cent for steel but much less for copper and aluminium.

Ros and Eichinger (1926) experimented with mild steel tubes under tension, compression, and combined tension-tension. Their material was sharp-yielding and obeyed the Maxwell-Mises yield stress criterion. They later (1930) applied the strain hardening hypothesis:

$$\bar{\sigma} = H(d\bar{\epsilon}^P) \quad \dots \quad (14)$$

where
$$d\bar{\epsilon}^P = \sqrt{\frac{2}{3}} (d\epsilon^P_{ij} d\epsilon^P_{ij})^{\frac{1}{2}} \quad \dots \quad (15)$$

to the post-yield results, but did not obtain correlation in the form $\bar{\sigma}$ v. $\bar{\epsilon}^P$.

Taylor and Quinney (1931) carried out experiments on annealed copper tubes under simple tension and pure torsion and tried to relate the two results by suggesting the hypothesis $\bar{\sigma} = F(W^P)$ i.e. the equivalent stress is a unique function of the total plastic work. They considered the criteria of Tresca and Maxwell-Mises and plotted the two torsion curves obtained from their experiments in conjunction with these criteria. Their experimental tension results were then shown to lie in between these two curves. They noted that if the equivalent stress was defined in accordance with the Maxwell-Mises criterion and the relation between ϵ and γ was defined according to Nadai (1930) a better correlation was obtained.

Schmidt (1932) worked on tubes of copper and mild-steel. His test-pieces were not thin-walled however, since the ratio outside diameter:inside diameter was 1.53. Tests in simple tension and pure torsion were carried out, together with two further tests at different constant stress ratios σ/τ . The resulting strain ratios ϵ^P/γ^P were found to remain constant up to about 30% strain. Schmidt plotted the results using the following stress-strain relations (i) maximum principal stress v. maximum principal strain; (ii) maximum shear stress v. maximum shear strain; (iii) second stress invariant

v. second strain-invariant (Maxwell-Mises). He found the Maxwell-Mises criterion to correlate the results for both steel and copper. Schmidt then proposed a work hypothesis of the form:

$$F = \sqrt{\int_0^{\epsilon} \sigma d\epsilon + \int_0^{\gamma} \tau d\gamma} \quad \dots \quad (16)$$

and a graph of $\bar{\sigma}$ v. F produced very good correlation. This latter correlation would in fact follow since the Maxwell-Mises definition of equivalent stress and equivalent strain had already been shown to fit the results. There is however some doubt over the validity of these results since, as pointed out by Nadai (1950), Schmidt used the incompressibility equation $\epsilon_1 + \epsilon_2 + \epsilon_3 = 0$ which is only applicable to logarithmic strains at the large values used in these tests. Schmidt did not give the values of the stress ratios used in his tests, only the resulting strain ratios. It is therefore impossible to compare the quoted strain ratios with those predicted from the normal to the Maxwell-Mises yield locus, since the points on that locus are not known.

Davis (1937) carried out experiments on solid specimens of annealed pure copper. He plotted the curves of octahedral shear stress against octahedral shear strain for tension, torsion and compression and found fairly good correlation. Up to 10 per cent strain the torsion curve was higher than the tension and compression, but at higher strains it was below the other two. The compression curve became progressively higher than that for tension as the test proceeded. His later work on copper (1943) and on carbon

steel (1945) showed similar results to each other. These tests were under combined tension-internal pressure on thin-walled tubes and maximum shear stress v. maximum shear strain and τ_{oct} v. γ_{oct} gave about the same degree of correlation i.e. either definition showing an experimental difference of about ± 5 per cent.

Bridgman (1943) tested double-grooved tubes to study the effect of compressive stress on the shear stress-strain curve. He discussed the correlation of tension with torsion tests at very large strains up to 600 per cent! He plotted τ_{oct} v. γ_{oct} and found an enormous difference between the two curves with the torsion curve falling increasingly below that for tension.

Osgood (1947) using thin-walled tubes of aluminium alloy under combined tension-internal pressure tests plotted the curves τ_{oct} v. γ_{oct} and obtained similar results to Davis (1945).

Davis and Parker (1948) tested mild steel under combined tension-internal pressure. Four stress ratios were used and the results gave only a fair correlation of equivalent stress v. equivalent strain using the Maxwell-Mises definition.

Morrison (1948) and Morrison and Shepherd (1950) found the Maxwell-Mises definition to give very good correlation of tension tests with torsion tests on several steels and also on aluminium alloy. In the latter paper the authors also showed that in the region where plastic strains are of similar order to the elastic, the

"incremental" flow laws i.e. stress-strain relations of plasticity theory are more exact than "total" strain theories and that the latter could lead to large errors.

Davis (1955) performed a series of combined tension-torsion-internal pressure tests on thin-walled steel tubes. The specimens were tested in such a manner that the principal directions of the increments of natural plastic strain were kept constant, the hoop strain being kept at zero by suitable adjustment of the internal pressure. The octahedral shearing-stress versus octahedral shearing-strain diagrams for the four specimens tested coincided quite closely. The principal directions of stress were also calculated and found to agree well with those of the corresponding increments of strain.

Rogan and Shelton (1969A) performed constant stress-ratio tests on En 24 and En 25 steels under combined tension-torsion. As stated in Section 1.4.1 they first found that neither steel obeyed the Tresca or Maxwell-Mises initial yield criterion exactly and then continued the tests into the plastic strain range. They established a correlation of equivalent stress v. equivalent plastic strain for the simple tension and pure torsion results by utilizing their initial yield law:

$$\bar{\sigma} = \sqrt{\sigma^2 + \alpha \tau^2} \quad \dots \quad (17)$$

as their definition of equivalent stress for post-yield behaviour and then divided the torsion strains by an empirical factor to fit the tensile strains. For the intermediate combined-stress it was only necessary to multiply

the equivalent plastic strain:

$$\bar{\epsilon}^p = \sqrt{\epsilon^p{}^2 + (\gamma^p/b)^2} \quad \dots \quad (18)$$

by the factor $\cos \alpha$ (where α is the angle between the stress vector and plastic-strain vector) to obtain complete correlation of all tests. This empirical equivalence also provided good correlation of the creep-strain rates - measured in these tests and examined for the first time - and these results finally led to the complete correlation of the stress-strain-time behaviour of these metals at room temperature. There is no physical basis whatever for a metal to obey the theoretical yield laws of plasticity theory and much of the earlier work on stress-strain relations under various complex stresses could no doubt have been correlated as in this work.

1.4.3 Experimental evidence of the flow rule using the Lode parameters

Hill (1950) gives an excellent survey of the development of the basic rules of plastic flow. Saint-Venant (1870) used Tresca's yield criterion in his work on plane strain and supposed the work-hardening to be zero. He speculated about the relation between stress and strain and proposed that the principal axes of strain increment (and not the total strain) coincided with the axes of principal stress. Lévy (1871) suggested a relationship between the ratios of the components of the strain increment and the stress ratios in the form:

$$\frac{d\epsilon_{11} - d\epsilon_{22}}{\sigma_{11} - \sigma_{22}} = \frac{d\epsilon_{22} - d\epsilon_{33}}{\sigma_{22} - \sigma_{33}} = \frac{d\epsilon_{33} - d\epsilon_{11}}{\sigma_{33} - \sigma_{11}} = \frac{d\epsilon_{12}}{\sigma_{12}} = \frac{d\epsilon_{23}}{\sigma_{23}} = \frac{d\epsilon_{31}}{\sigma_{31}} = \delta\lambda$$

.. .. (19)

where $\delta\lambda$ is a scalar factor of proportionality. His work remained largely unknown outside his own country until von Mises (1913) suggested the same equations independently. The Lévy-Mises equations, or flow rule as they are known, were expressed by von Mises in the more convenient form:

$$d\epsilon_{11} = \frac{2}{3} \delta\lambda \left\{ \sigma_{11} - \frac{1}{2} (\sigma_{22} + \sigma_{33}) \right\} \text{ etc.}; \quad .. \quad .. \quad (20)$$

$$\text{and } d\epsilon_{12} = \delta\lambda \sigma_{12} \text{ etc.} \quad .. \quad .. \quad .. \quad .. \quad .. \quad (21)$$

In deviatoric stress form and using the tensor notation these equations become:

$$d\epsilon_{ij} = \delta\lambda \sigma'_{ij} \quad .. \quad .. \quad .. \quad .. \quad .. \quad (22)$$

Prandtl (1924) for the plane strain problem and later Reuss (1930) for the general case, allowed for the elastic strain component which is of importance at a plastic-elastic interface or in residual stress problems. The Prandtl-Reuss equations may be written in several ways depending upon whether the material is considered to be work-hardening or not, but the form corresponding to (21) and (22) is:

$$d\epsilon_{11} = d\epsilon_{11}^p + d\epsilon_{11}^e = \frac{2}{3} \delta\lambda \left\{ \sigma_{11} - \frac{1}{2} (\sigma_{22} + \sigma_{33}) \right\} + \frac{1}{E} \{ d\sigma_{11} - \nu (d\sigma_{22} + d\sigma_{33}) \}$$

etc. .. (23)

$$\text{and } d\epsilon_{12} = d\epsilon_{12}^p + d\epsilon_{12}^e = \delta\lambda\sigma_{12} + \frac{d\sigma_{12}}{2G} \text{ etc. } \dots \dots (24)$$

Lode (1926) introduced two parameters in the following form:

$$\mu = \frac{2\sigma_2 - \sigma_1 - \sigma_3}{\sigma_1 - \sigma_3} \dots \dots (25)$$

$$\text{and } \nu = \frac{2d\epsilon_2^p - d\epsilon_1^p - d\epsilon_3^p}{d\epsilon_1^p - d\epsilon_3^p} \dots \dots (26)$$

where $\sigma_1 > \sigma_2 > \sigma_3$ and $d\epsilon_1^p$ $d\epsilon_2^p$ $d\epsilon_3^p$ are the principal plastic strain increments. These Lode parameters provide a useful check on the flow (Lévy-Mises) rule since the latter requires:

$$\mu = \frac{2\sigma_2 - \sigma_1 - \sigma_3}{\sigma_1 - \sigma_3} = \frac{\delta\lambda(2\sigma_2 - \sigma_1 - \sigma_3)}{\delta\lambda(\sigma_1 - \sigma_3)} = \frac{2d\epsilon_1^p - d\epsilon_1^p - d\epsilon_3^p}{d\epsilon_1^p - d\epsilon_3^p} = \nu \dots \dots (27)$$

Equality of μ and ν means that the plastic potential drawn in the deviatoric or π plane is a circle and that the Mohr circles of stress and plastic strain-increment are geometrically similar. Inequality does not necessarily mean that the principal directions of plastic strain-increment and stress do not coincide since the parameters are only a geometrical relationship and do not refer to the actual shape of the yield locus.

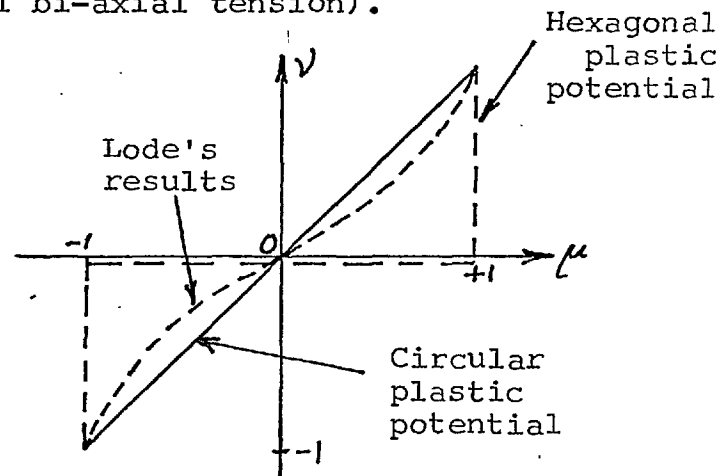
As shown by Hill (1950) for example, an isotropic body without a Bauschinger effect, will have a symmetrical yield locus and plastic potential, and that these are completely specified by a 30° segment on the deviatoric plane. By simple geometry it can be shown for the stresses:

$$\mu = -\sqrt{3} \tan \theta \quad \dots \quad (28)$$

and similarly for the strains, where θ is the direction of the stress vector relative to some origin on the plane.

Thus to cover the $\pm 30^\circ$ segments μ (and ν) will vary between ± 1 . One method of achieving this is to subject an open-ended thin-walled tube to combined tension and internal pressure and varying the ratio axial stress: hoop stress from ∞ to 1. μ and ν will then lie between -1 (simple tension) and +1 (equal bi-axial tension).

If the plastic potential is circular in the π plane, $\mu = \nu$. If it is a regular hexagon, the variation of μ with ν is as shown in the diagram.



Lode's own experiments were on drawn tubes of mild steel using the above method and showed $\mu \neq \nu$ but $|\nu| < |\mu|$, as indicated in the diagram. There was some scatter in the results which Lode attributed to anisotropy in the drawn tubes but later work by the same author (1929) using tubes machined from solid bars supported the deviation between

μ and ν .

Taylor and Quinney (1931) carried out tests under combined tension-torsion on aluminium, copper, and pre-strained mild steel. For these tests μ varied between -1 (simple tension) and 0 (pure torsion), and the stress ratios were varied by maintaining a constant axial load and increasing the torque. The results confirmed Lode's findings.

Schmidt (1932), working on mild steel and copper tubes in combined tension-torsion, disagreed with the Taylor and Quinney findings and obtained results which confirmed the Levy-Mises relation within experimental error. These tests were conducted at constant stress-ratio.

Davis (1943) carried out combined tension-internal pressure tests on annealed copper and again confirmed Lode's results. The agreement between μ and ν this time, however, was much closer.

Fraenkel (1948) also performed combined tension-internal pressure tests. These were on thin-walled tubes of mild steel. He found that for the annealed material $\mu = \nu$ at certain stress ratios, having computed μ and ν from true stresses and natural strains respectively. In the case of rolled material however, Fraenkel obtained similar results to the previous investigations. He believed that the scatter was due principally to inaccuracies in strain measurements which must, for this purpose, apparently be of much higher accuracy than for the determination of stress-strain curves. This was shown to be particularly true for small strains where ν was most sensitive to errors.

Siebel (1953) working on hardened stainless steel under combined bending and torsion, found values lying equally distributed about the line $\mu = \nu$. Similar results were also found by Hundy and Green (1954) Lianis and Ford (1957) and Baraya and Parker (1963) using the suggestion of Hill (1950) in his theory of localised necking that thin strips with asymmetrical notches when pulled in tension provide a means of producing arbitrary uniform states of combined stress. Hundy and Green worked on strips of rolled copper, zinc and stainless steel, Lianis and Ford on the specially treated aluminium referred to earlier, and Baraya and Parker on rolled aluminium with notches and also on further test-pieces with grooves.

Similarly grooved specimens - following Hill (1953) - were used by Ellington (1958) in testing strips of annealed brass and copper. The brass showed $\mu = \nu$ but copper gave $|\nu| > |\mu|$ which is in opposition to any previous results.

Finally Mair and Pugh (1963) working on tubes of high conductivity copper in combined tension-torsion following a small torsional pre-strain found the more usual variation $|\nu| < |\mu|$.

1.4.4 Literature Survey of previous work on Pre-straining and its Effect on Subsequent Yield and Flow Behaviour

Taylor and Quinney (1931) were the pioneers in this field. They tested fairly thick-walled tubes of copper, aluminium, and mild steel in combined tension-torsion. The loading path was initially tension into the plastic.

state followed by a partial unloading which was then held constant whilst sufficient torque was added to induce yielding. The stress ratios were therefore not held constant. The subsequent flow stress was determined by the backward extrapolation of the torque v. angle of twist curve or the curve of torque v. additional axial extension as already referred to in Section 1.2 of this thesis. Hoop strains were not measured, but by carefully finding the changes in the internal volume of the tubes the ratios of the axial strain ϵ_x to the hoop strain ϵ_y and radial strain ϵ_z were determined.

These tests showed that the subsequent yield locus for copper and aluminium was close to the Maxwell-Mises ellipse given by:

$$\bar{\sigma} = \sqrt{\sigma^2 + 3\tau^2} \quad \dots \quad (29)$$

but that for mild steel in various forms was outside the ellipse and in the opposite sense to the Tresca ellipse given by:

$$\bar{\sigma} = \sqrt{\sigma^2 + 4\tau^2} \quad \dots \quad (30)$$

The results are shown in fig. 4.

It is however of interest to note, as pointed out by Rogan (1966), that the yield locus can also be determined from the directions of the strain vector γ^p/ϵ^p . Rogan did this from the data given in the paper by Taylor and Quinney and showed that the vectors γ^p/ϵ^p were in fact normal to the curve $\bar{\sigma} = \sqrt{\sigma^2 + 3.5\tau^2}$ which lies midway between

(29) and (30) and in fact agrees with the results of Rogan and Shelton (1969A) when working with annealed En 24 steel. Clearly there must be doubt about the validity of the Taylor and Quinney results on mild steel and indeed Taylor (1947) later constructed the yield locus for the μ v. v curve in the 1931 paper and showed that it did in fact lie between the two ellipses. The yield stresses given in fig. 4 for mild steel appear to be too high and are probably partly due to the size effect noticed by Morrison (1940). The ratio of outside diameter: inside diameter was 1.44 for the tubes used and the outside diameter was also only 0.25 in. These two factors would tend to increase the apparent shear yield stress, the first because the actual shear stress in a thick tube yielding under torsion is much smaller than the apparent shear stress derived from elasticity considerations (see Nadai's method of finding the shear stress v. shear strain curve from the torque v. twist diagram for a solid bar - a possible extension to the case of torsion of a thick tube having been outlined by Crossland (1965)) and the second because Morrison (1940) showed clearly that the apparent shear yield stress increased as the stress gradient increased i.e. as the outside diameter of the test-piece decreased.

Another result in the Taylor and Quinney paper which was pointed out by Rogan (1966) is that the strain ratio γ^p/ϵ^p was stated to remain constant during the subsequent non-radial loading. Thus γ^p/ϵ^p remained constant although τ/σ was changing. This is in direct contradiction

to the results on glass given in the later part of the 1931 paper and also to all the subsequent work which has been done on the effect of changes in stress vector direction upon the direction of the resulting strain vector.

Rogan's final conclusion on this work was that it was essentially pioneer work of excellence. It was unfortunate that the size of the test-pieces had a serious effect upon the results since up to the time of his own work it was the only work referring to the effect of a pre-stress upon the subsequent yield strength of steels. It should also perhaps be observed that it is surprising that the Taylor and Quinney results showed yield loci to be so close to the Maxwell-Mises criterion - they were, after all, dealing with pre-strained materials.

Naghdi and Rowley (1954) together with Beadle (1955) tested an aluminium alloy possessing severe initial anisotropy in combined tension-torsion. The experimental details were of little value and were largely disclaimed in a later paper by Naghdi, Essenburg and Koff (1958). Here the limit of proportionality was used to define the initial yield locus and also two subsequent loci following shear pre-strains. They reported translation of the yield locus in the direction of the pre-strain with some distortion of shape but no cross-effect. These results are shown in fig. 5. Ivey (1961) pointed out serious errors in the experimental procedure used in these tests and this is discussed later in conjunction with Ivey's paper.

Hu and Marin (1955) tested tubular specimens of 24S-T aluminium-alloy under combined tension - internal pressure. They reported that neither the Maxwell-Mises, Tresca, nor Prager criteria are adequate as a loading function even when these functions are modified for initial anisotropy of the material. Furthermore, the test results definitely disprove the concept of an isotropic expansion of the loading function during plastic deformation. The pre-stress probe carried with it a portion of the initial yield locus whilst the locus behind the probe contracted. The test results also supported, in a qualitative way, the concept of the formation of a yield corner during loading in the plastic range. Very similar results were obtained by Gill (1956) testing brass in tension-torsion and also by Hu and Bratt (1958) testing aluminium-alloy under tension-internal pressure.

It should be observed that in reporting "qualitative existence of a corner" on the yield locus in the direction of the pre-stress, these authors have not been consistent. They have loaded test-pieces into the plastic range to some point on a subsequent yield locus and then on unloading and reloading they have defined their yield point as being the limit of proportionality, which of course is not a point on that yield locus. Hu and Bratt's results are shown in fig. 6.

Philips and various co-workers have published a number of papers concerned with distinguishing between "incremental" and "total" stress-strain relationships and

also with the possible existence of corners on subsequent yield loci. Philips and Gray (1958) tested an aluminium-alloy in combined tension-torsion using the limit of proportionality for defining yield stress. It was shown that the pre-stress probe carried the yield locus along with it, accompanied by a contraction and distortion of the locus behind the probe i.e. a combination of kinematic translation plus distortion. These tests were of a qualitative nature however since apart from the yield definition some of the test-pieces had very serious variations in wall thickness and often only very few grains across the wall.

Parker, with co-workers, has also published work in this field. Using closed-end thin-walled tubes of alpha brass, Gill and Parker (1959) carried out tests under various combinations of torsion-internal pressure in order to investigate the effect of using various loading paths as a test of the state of isotropy of their test-pieces and also to investigate the initial yield criterion. This work showed the material to be isotropic in the plane of the tube surface but anisotropic in the radial direction and that the initial yield locus was fairly close to the Maxwell-Mises. There was no evidence of a corner on this locus. Similar tests by Parker and Kettlewell (1961) again on alpha-brass but pre-strained in torsion, suggested a very strong cross-effect, yielding being defined by the Taylor and Quinney backward extrapolation. Again there was an absence of corners on the yield locus and in these tests the marked

creep was allowed to persist until constant readings were obtained on the strain-measuring instruments. Further tests by Parker and Bassett (1964) on similar pre-strained material but now using open-ended tubes again showed the subsequent yield locus to possess a strong cross-effect together with marked translation and distortion. These authors concluded that the Bauschinger effect did not seem to increase progressively with increasing pre-strain and also that the axes of the yield locus appeared to rotate slightly. Again there was an absence of corners on the subsequent locus.

Ivey (1961), whose results are shown in fig. 7, reverted to the proportional limit definition of yield stress as used by Naghdi et al (1958). Both authors carried out tests in combined tension-torsion on aluminium alloys firstly in the annealed state and then in the torsion pre-stress state. Ivey agreed with the Naghdi findings regarding convexity of the yield surface, absence of a cross-effect (i.e. a constant tensile yield stress which is irrespective of the torsion pre-stress) and a pronounced Bauschinger effect. He disagreed with the finding that the Bauschinger effect gradually vanishes as the tensile stress axis is approached, that the effect of work-hardening rapidly diminishes as the tensile stress axis is approached and that increasing the torsion pre-stress creates a region of high curvature (reminiscent of a corner) about the shear stress axis. Ivey further criticised the Naghdi work regarding the continued use of a single test-piece for

establishing subsequent yield loci. He pointed out that while obtaining a yield point on the first subsequent yield locus by noting when plastic strain first appeared, Naghdi actually stressed his test-piece well beyond the bounds given for the second subsequent locus. From fig. 7 it is clear that Ivey found a marked translation of the yield locus towards the direction of the pre-strain together with a large distortion. Indeed the stress origin is no longer enclosed by the second and third yield loci which implies that the material deforms plastically upon complete unloading. Although the magnitude of the pre-strains were quite different in these two series of tests on aluminium alloys the general conclusions should nevertheless be expected to be the same. Figs. 5 and 7 are however quite different and the work of both Naghdi and Ivey can be criticised in two respects: (i) they should not have used the same test-piece to obtain points on a succession of yield loci - particularly over different stress paths; (ii) they should not have used the limit of proportionality stress to define a yield point since the material does not yield at that stress and indeed there is evidence that the value of the proportional limit stress changes according to the degree of unloading. This latter phenomenon is discussed later in relation to the effect of creep on plasticity experiments.

Bertsch and Findley (1962) determined the initial yield and seven subsequent yield curves from a sequence of loading on the same thin-walled tube test-piece of aluminium alloy,

under combined tension-torsion. The axial and shear strain components agreed well with the normal to the current locus and there was no evidence of a corner.

Pugh and co-workers at the National Engineering Laboratory carried out combined tension-torsion tests on pre-strained copper tubes. Pugh, Mair and Rapier (1962) described the testing machine built for these tests and then reported experiments on the effect of tension pre-strain upon the subsequent yield locus. Using a test procedure based on that of Taylor and Quinney (1931), these authors reduced the tensile stress to some fraction of the pre-stress value and then added sufficient torque to induce yielding. Plastic flow under the combined stresses was continued until sufficient strain was reached to permit a backward extrapolation of the yield stress. Each test-piece was used to determine one point on each of three subsequent yield loci but, as with the earlier papers by other authors already discussed, it is not clear what effect the quite large plastic shear strains given to a test-piece while finding each yield point had upon the results for the next locus. The main conclusions from this work were firstly that there is very little distortion of a subsequent yield locus - thus implying isotropic hardening and a definite cross-effect, unlike the work of Naghdi et al and Ivey - and secondly that although copper gave a yield criterion outside the Maxwell-Mises ellipse when annealed, it tended towards that criterion with increasing tensile pre-strain.

Mair and Pugh (1963) extended the investigation to the effect of a torsion pre-strain and for extrapolation purposes changed the loading paths to constant stress ratio. It was found that to establish subsequent points in the reverse-torsion quadrant, very large excursions into the plastic strain range were necessary owing to the very rounded shape of the stress-strain curve and the need to perform a backward extrapolation. This would entail using a new tube for each point, which the authors were reluctant to do, and they therefore changed their definition of yield to a proof stress on the argument that if this were judiciously chosen it would give a proof stress equal to the flow stress obtained by backward extrapolation. Mair and Pugh used the Maxwell-Mises definition of equivalent plastic strain, given by:

$$d\bar{\epsilon}^p = \frac{2}{\sqrt{3}} \{ d\epsilon_x^p{}^2 + d\epsilon_y^p{}^2 + d\epsilon_x^p d\epsilon_y^p + d\epsilon_{xy}^p{}^2 \}^{\frac{1}{2}} \quad \dots (31)$$

with $\bar{\epsilon}^p$ arbitrarily fixed at 0.1 per cent proof strain. Clearly this was an incorrect procedure since this form of the equivalent plastic strain will not even suit the behaviour of the copper tubes when isotropic let alone when pre-strained. Fig. 8 shows a tensile test from the 1962 paper a torsion test from the 1963 paper and a combined tension-torsion test from the 1966 paper, all performed on identical specimens of the same annealed pure copper and plotted as $\bar{\sigma}$ v. $\bar{\epsilon}^p$ defined by the Maxwell-Mises criterion. It is clear that very poor correlation exists between these tests. Even more serious errors arise when it is considered there is really a whole

set of subsequent yield loci following a pre-strain, as the present work will show, and for each one the corresponding definition of equivalent strain is different. Thus, even if an equivalent strain could be found to correlate the isotropic behaviour of the material this would not be suitable to describe any of the subsequent yield loci.

The conclusions reached by Mair and Pugh were that the initial yield locus conformed closely to the Maxwell-Mises criterion and that subsequent loci were inflations of that locus accompanied by a translation in the direction of the pre-stress and some distortion of shape. The definition of yield stress was found to be of importance particularly in relation to the presence or absence of a cross-effect and all loci were shown to be quite smooth - see fig. 9.

Mair (1966) investigated the possibility of the existence of corners on the yield locus of three metals by subjecting them to a zig-zag stress path in the plastic strain region. The argument used (Drucker and Stockton (1953)) was that if the plastic strain-increment vector followed the stress vector then corners on the yield locus would be present, since by reason of the normality rule the strain vector at a corner would be free to take up any position between the two normals at that point. The results showed that the direction of the plastic strain-increment vectors only deviated slightly from the normal to the locus, defined by the Maxwell-Mises criterion, which implied that there was local distortion of the yield locus but that corners did not exist i.e. the locus

was smooth rather than one with plane segments.

Shiratori and Ikegami (1968) used cross-shaped test-pieces of brass plate, pre-strained in tension along various loading paths or by cold-rolling, to determine the subsequent yield loci. They showed that for bi-axial tension a corner existed on the subsequent yield locus in the direction of any constant stress ratio path, but not elsewhere on that locus, except for the case of uniaxial pre-strain. Test-pieces pre-strained by cold rolling did not exhibit a corner and experimental results were in qualitative agreement with Dorn's theory referred to earlier in this thesis. Zig-zag loading paths showed results similar to those of Mair (1966) and the shape of the subsequent yield loci was an inflation in the direction of the pre-stress axis together with a cross-effect dependent upon the direction of that axis. The latter effect was most marked along the axes of the test-piece where a tensile pre-strain along one axis gave rise to a reduction in the yield stress along the other axis i.e. work-softening. These authors used (i) a proof-stress definition of yield; (ii) a cruciform test-piece of doubtful uniform stress and strain area which was reinforced at the arms, and (iii) measured their strains by the use of wire-wound resistance strain gauges. The first two factors at least, must cause doubt regarding the accuracy of their work.

Rogan and Shelton (1969B) performed combined tension-torsion tests on En 25 steel which had been initially

annealed and then subjected to either tensile or torsion pre-strains. They were careful to use a single test-piece for any given point on a successive yield locus and then discard it. Constant stress-ratios were used in all loading paths and they examined two cases of tensile pre-strain and one torsion pre-strain. Unlike their earlier work on annealed En 24 and En 25 steels (1969A) the pre-strained materials did not exhibit a sudden yield because of the work-hardening introduced. They established each yield point by fulfilling two requirements: (i) each yield locus must pass through the stress space normal to all the plastic strain-increment vectors it encounters; (ii) each yield stress should be an extrapolation from the curves of creep coefficient v. stress and stress v. "long-time" strain. Creep at room temperature was most marked and loads were maintained constant for about 15 minutes at each increment on any given load path, thus permitting the logarithmic creep coefficient to be calculated. The plastic strain-increment vectors were shown to lie in a quite different direction immediately following a pre-strain, in comparison to those for the annealed material, but rotated towards the latter direction with increase in stress. For example, it was shown that a test-piece pre-strained in simple tension to about 3.5 per cent and then twisted in torsion would first show small axial contractions in addition to transverse rotation but further contractions would no longer occur when the shear strain was of the order 3 per cent, i.e. about the same as the tensile pre-strain.

By using the normality rule and the backward extrapolation of both the stress v. strain curves and creep coefficient v. stress curves, these authors were able to overcome the difficulty experienced by earlier workers in defining the yield point on rounded stress v. strain diagrams. Their general conclusions were that a pre-strain hardens a material over that half of bi-axial stress space bisected by the pre-stress axis and hardens it less or even softens it in the reverse half of that space. The cross-effect of a tensile pre-strain is greater than that of a torsion pre-strain and increasing the pre-strain has a gradually reducing cross-effect. There was no evidence of corners on any of the loci.

Some of the results of the work by Rogan and Shelton are shown in fig. 10. Fig. 10A shows the loading paths in combined tension-torsion and the resulting rotation of the plastic strain-increment vectors at different stress levels along each path, for a tensile pre-stress of 39.4 tonf/in^2 i.e. 5.9 per cent strain. The yield locus has been drawn in by the methods described earlier and is repeated in fig. 10B where all three subsequent loci are shown and compared with the initial locus.

Finally, Williams and Svensson (1970) performed tension-torsion-compression tests on commercially pure aluminium tubes, pre-strained in tension to a range of values up to 14 per cent strain. They used a hydraulic testing machine of original design but gave little detail

of their experimental technique, particularly regarding axiality of loading, friction in compression testing, effect of creep etc. Only four points (i.e. tension, compression, forward and reverse torsion) were considered necessary to define the initial yield locus and two further points when defining a subsequent locus. Yield loci for each pre-strain were sketched between these few points for equivalent proof strains lying between 20-200 μ in/in. These curves showed that the definition of yield point had a marked effect upon their shape. A corner developed on the locus in the direction of the stress probe, but this was apparently only really sharp for the smallest proof strain, and this was accompanied by a marked flattening of the locus in the reverse direction. This work has the merit of pointing out the importance of the definition of yield point but the lack of experimental details referred to earlier, the use of the Maxwell-Mises definition of equivalent plastic strain which might suit the isotropic material but certainly not the pre-strained material (see earlier remarks on the work of Mair and Pugh) and also the lack of sufficient experimental points on the yield loci, are serious omissions.

CHAPTER 2

THEORETICAL ANALYSIS OF THE BEHAVIOUR OF METALS UNDER COMPLEX STRESS STATES

2.1 Introduction

This long chapter commences with a review of the basic theories of plasticity for isotropic materials. It commences with a section on stress-strain relations in "total" and "incremental" forms and then leads to a discussion of equivalent stress and equivalent strain. In the latter section the strain-hardening and work-hardening hypotheses are introduced and this is followed by sections on equivalent stress v. equivalent strain based on those hypotheses for both the Maxwell-Mises and the Tresca criteria of yielding.

The anisotropic yield criteria of Hill, Dorn, Drucker Hsu and Williams-Svensson are next considered and the chapter concludes with a section on stress-strain relations for anisotropic flow.

2.2 Review of the Basic Theory of Plasticity for Isotropic Flow

2.2.1 Theoretical stress-strain relations

Various theories have been developed which make it possible to predict the plastic stress-strain behaviour for combined states of stress from the uniaxial tension test. It is desirable for engineers to predict combined plastic stress-strain relations since these relations are used for example in the design of machines and structures and in the mechanics of metal forming.

There are two main types of plasticity theories, referred to as deformation or total and flow or incremental theories. Total type theories may be considered as a special case of the incremental type theories. In the total type theories the ratio of the principal stresses must be maintained essentially constant during loading while in incremental type theories this restriction is not required. It has been shown theoretically that small departures from constancy of the principal stress ratios are nevertheless permissible in the total type theories. A comprehensive review on plastic stress-strain relations was prepared by Drucker (1950).

There is only a limited amount of experimental information on combined plastic stress-strain relations. Some of these results, however, are in approximate agreement with the so-called simple deformation theory developed below, Marin (1962).

(i) Simple total theory

To derive mathematical expressions for defining plastic stress-strain relations by the simple total theory, consider that the true principal stresses are σ_1 , σ_2 , and σ_3 and the corresponding true principal strains are ϵ_1 , ϵ_2 , and ϵ_3 . The theory is based on the following assumptions:

1. The direction of the true principal strains ϵ_1 , ϵ_2 , and ϵ_3 coincide with the directions of the true principal stresses σ_1 , σ_2 and σ_3 ;
2. Plastic deformation causes no volume change; for a cube of dimensions $a \times b \times c$ the initial volume is $V_0 = abc$ and the volume corresponding to nominal strains e_1 , e_2 and e_3 is

$$V = a(1 + e_1) \cdot b(1 + e_2) \cdot c(1 + e_3) \quad (32)$$

then $\frac{V}{V_0} = (1 + e_1)(1 + e_2)(1 + e_3) \quad (33)$

and $\ln \left(\frac{V}{V_0} \right) = \ln(1 + e_1) + \ln(1 + e_2) + \ln(1 + e_3)$

or $\ln \left(\frac{V}{V_0} \right) = \epsilon_1 + \epsilon_2 + \epsilon_3 \quad (34)$

for no change in volume, $V = V_0$ or $\ln \left(\frac{V}{V_0} \right) = 0$ and then by (34)

$$\epsilon_1 + \epsilon_2 + \epsilon_3 = 0 \quad (35)$$

it follows from this that a hydrostatic state of stress will not produce any plastic deformation; experiments show that even for large strains (35) is a good approximation, there is a small elastic change in volume for small strains but as the strains increase in the plastic range this volume change becomes negligible;

3. the ratios of the three maximum shear strains to the maximum shear stresses are equal.

This latter assumption has been confirmed experimentally for certain principal stress combinations and in order to state this mathematically it is noted that the maximum shear stresses are $(\sigma_1 - \sigma_2)/2$, $(\sigma_2 - \sigma_3)/2$, $(\sigma_3 - \sigma_1)/2$, and the corresponding maximum shear strains are $(\epsilon_1 - \epsilon_2)$, $(\epsilon_2 - \epsilon_3)$ and $(\epsilon_3 - \epsilon_1)$ respectively.* Then by this hypothesis

$$\frac{\epsilon_1 - \epsilon_2}{\sigma_1 - \sigma_2} = \frac{\epsilon_2 - \epsilon_3}{\sigma_2 - \sigma_3} = \frac{\epsilon_3 - \epsilon_1}{\sigma_3 - \sigma_1} = \lambda \quad (36)$$

* See M.M. Frocht, Photoelasticity, John Wiley and Sons, Inc. (1941) 1, 31.

Equation (35) and (36) represent four equations that can be solved simultaneously to obtain the principal strains in terms of principal stresses. These solutions give

$$\begin{aligned}\epsilon_1 &= \frac{2\lambda}{3} \left(\sigma_1 - \frac{\sigma_2 + \sigma_3}{2} \right) \\ \epsilon_2 &= \frac{2\lambda}{3} \left(\sigma_2 - \frac{\sigma_3 + \sigma_1}{2} \right) \\ \epsilon_3 &= \frac{2\lambda}{3} \left(\sigma_3 - \frac{\sigma_1 + \sigma_2}{2} \right)\end{aligned}\tag{37}$$

Equations (37) define the true principal strains in terms of the true principal stresses and a quantity λ . A comparison of these equations with the elastic strain equations

$$\begin{aligned}\epsilon_1 &= \frac{1}{E} [\sigma_1 - \nu(\sigma_2 + \sigma_3)] \\ \epsilon_2 &= \frac{1}{E} [\sigma_2 - \nu(\sigma_3 + \sigma_1)] \\ \epsilon_3 &= \frac{1}{E} [\sigma_3 - \nu(\sigma_1 + \sigma_2)]\end{aligned}\tag{38}$$

shows that the two sets of equations are similar. The modulus of elasticity E corresponds to a plasticity modulus $\frac{3}{2\lambda}$, and Poisson's ratio ν becomes equal to 0.5 in the plastic range. An examination of (37) also shows that the strains would be completely defined in terms of the stresses if the value of λ were known. Various total theories are obtained, depending upon the basis used for finding λ . The value of λ will be determined from the stress-strain relation used. Since the distortion energy theory or the octahedral shear stress is considered to be a good measure for the onset of yielding, it

is assumed that for all combined stress cases there is a unique relation between the octahedral shear stress and octahedral shear strain. That is,

$$\tau_{oct} = f(\gamma_{oct}) \quad (39)$$

where

$$\tau_{oct} = \frac{1}{3} [(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2]^{\frac{1}{2}} \quad (40)$$

$$\gamma_{oct} = \frac{2}{3} [(\epsilon_1 - \epsilon_2)^2 + (\epsilon_2 - \epsilon_3)^2 + (\epsilon_3 - \epsilon_1)^2]^{\frac{1}{2}} \quad (41)$$

In order to express completely the values of the principal strains it will be necessary to assume the simple tension stress-strain relation. Using the true stress-strain relation $\sigma = k\epsilon^n$ for simple tension, and noting from (40) and (41) that in simple tension $\tau_{oct} = (\sqrt{2}/3)\sigma$ and $\gamma_{oct} = \sqrt{2}\epsilon$, then $\tau_{oct} - \gamma_{oct}$ relation corresponding to $\sigma = k\epsilon^n$ is obtained by placing $\sigma = (3/\sqrt{2})\tau_{oct}$ and $\epsilon = \gamma_{oct}/\sqrt{2}$ in the equation $\sigma = k\epsilon^n$, or

$$\tau_{oct} = \left(\frac{\sqrt{2}k}{3} \right) \left(\frac{\gamma_{oct}}{\sqrt{2}} \right)^n \quad (42)$$

Equation (42) expresses specifically the $\tau_{oct} - \gamma_{oct}$ relation corresponding to the general form given in (39). Referring to the first of (37) for simple tension $\sigma_2 = \sigma_3 = 0$ and $\sigma_1 = \sigma$, or

$$\epsilon = \frac{2}{3}\lambda\sigma \quad \text{or} \quad \lambda = \frac{3}{2} \left(\frac{\epsilon}{\sigma} \right) \quad \dots \quad (43)$$

Placing values of $\epsilon = \gamma_{oct}/\sqrt{2}$ and $\sigma = (3/\sqrt{2})\tau_{oct}$ in (43)

$$\lambda = \frac{\gamma_{oct}}{2\tau_{oct}} \quad (44)$$

The value of λ is now determined by placing the value of γ_{oct} from (42) into (44) and using the value of τ_{oct} from (40). Thus

$$\lambda = 3(2)^{\frac{-(1+n)}{2n}} \left(\frac{1}{k}\right)^{\frac{1}{n}} [(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2]^{\frac{1-n}{2n}} \quad (45)$$

The principal strains can now be completely defined if the value of λ from (45) is substituted in (37). This leads to

$$\left. \begin{aligned} \epsilon_1 &= \left(\frac{\sigma_1}{k}\right)^{\frac{1}{n}} \left[\alpha^2 + \beta^2 + 1 - \alpha\beta - \alpha - \beta \right]^{\frac{1-n}{2n}} \left(1 - \frac{\alpha}{2} - \frac{\beta}{2} \right) \\ \epsilon_2 &= \left(\frac{\sigma_1}{k}\right)^{\frac{1}{n}} \left[\alpha^2 + \beta^2 + 1 - \alpha\beta - \alpha - \beta \right]^{\frac{1-n}{2n}} \left(\alpha - \frac{\beta}{2} - \frac{1}{2} \right) \\ \epsilon_3 &= \left(\frac{\sigma_1}{k}\right)^{\frac{1}{n}} \left[\alpha^2 + \beta^2 + 1 - \alpha\beta - \alpha - \beta \right]^{\frac{1-n}{2n}} \left(\beta - \frac{\alpha}{2} - \frac{1}{2} \right) \end{aligned} \right\} \quad (46)$$

where $\alpha = \frac{\sigma_2}{\sigma_1}$ and $\beta = \frac{\sigma_3}{\sigma_1}$

Equations(46) represent the principal strains in terms of the principal stresses and the material constants k and n obtained from the simple tension stress-strain relation. It should be emphasized that the above theory applies only for cases in which the principal stress ratios $\alpha = \sigma_2/\sigma_1$ and $\beta = \sigma_3/\sigma_1$ remain constant during loading. Using (46), stress-strain relations for selected values of the stress ratios α and β and a specific tension material constant n can be plotted as

shown in fig. (11).

(ii) Incremental theory

For conditions where the ratios of the principal stresses do not remain constant, a so-called flow or incremental-type theory is commonly used. Flow type theories can be developed in a manner similar to that used in the foregoing for the deformation theories. The difference in the development consists in considering differential increments of strain rather than finite quantities. Then (35) is replaced by

$$d\epsilon_1 + d\epsilon_2 + d\epsilon_3 = 0 \quad (47)$$

Equation (47) can be expressed in terms of the strain rate by writing it as follows:

$$\frac{d\epsilon_1}{dx} + \frac{d\epsilon_2}{dx} + \frac{d\epsilon_3}{dx} = 0 \quad (48)$$

where x is a monotonically increasing parameter such as time or load.

Equation (48) may be written

$$\dot{\epsilon}_1 + \dot{\epsilon}_2 + \dot{\epsilon}_3 = 0 \quad (49)$$

where $\dot{\epsilon}_1 = d\epsilon_1/dx$ etc. are strain rates. Similarly (36) becomes

$$\frac{\dot{\epsilon}_1 - \dot{\epsilon}_2}{\sigma_1 - \sigma_2} = \frac{\dot{\epsilon}_2 - \dot{\epsilon}_3}{\sigma_2 - \sigma_3} = \frac{\dot{\epsilon}_3 - \dot{\epsilon}_1}{\sigma_3 - \sigma_1} = \lambda \quad (50)$$

Solving (49) and (50) for the strain increments, and writing $d\lambda$ instead of λdx , their values in term of the stresses become

$$\left. \begin{aligned} d\epsilon_1 &= \frac{2}{3} d\lambda \left(\sigma_1 - \frac{\sigma_2 + \sigma_3}{2} \right) \\ d\epsilon_2 &= \frac{2}{3} d\lambda \left(\sigma_2 - \frac{\sigma_3 + \sigma_1}{2} \right) \\ d\epsilon_3 &= \frac{2}{3} d\lambda \left(\sigma_3 - \frac{\sigma_1 + \sigma_2}{2} \right) \end{aligned} \right\} \quad (51)$$

Equations (51) can be used to develop the plastic strain-stress relations in a manner similar to that developed for the simple deformation theory. A more complete treatment of flow-type theories is given in Dorn (1948), Drucker (1950), Nadai (1950), Hill (1950), Hoffman and Sacks (1953) in which the Lévy-Mises and Prandtl-Reuss equations have been discussed.

Lévy in 1871 and, independently, von Mises in 1913, introduced the following equations known as the Lévy-Mises equations:

$$\frac{d\epsilon_x}{\sigma'_x} = \frac{d\epsilon_y}{\sigma'_y} = \frac{d\epsilon_z}{\sigma'_z} = \frac{d\epsilon_{yz}}{\tau_{yz}} = \frac{d\epsilon_{zx}}{\tau_{zx}} = \frac{d\epsilon_{xy}}{\tau_{xy}} = d\lambda$$

$$\text{or} \quad d\epsilon_{ij} = \sigma'_{ij} d\lambda \quad (52)$$

where σ'_{ij} is the stress deviator tensor and $d\lambda$ is a non-negative constant which may vary during the loading history. In these equations the total strain increments are assumed to be equal to the plastic strain increments, the elastic strains being ignored. Thus these equations can only be applied to

problems of large plastic flow and cannot be used in the elastoplastic range. The generalization of equations (52) to include both elastic and plastic components of strain is due to Prandtl and Reuss and are known as the Prandtl-Reuss equations.

Reuss assumed that the plastic strain increment is at any instant of loading proportional to the instantaneous stress deviator; i.e.

$$\frac{d\epsilon_x^p}{d\sigma_x} = \frac{d\epsilon_y^p}{d\sigma_y} = \frac{d\epsilon_z^p}{d\sigma_z} = \frac{d\epsilon_{xy}^p}{\tau_{xy}} = \frac{d\epsilon_{yz}^p}{\tau_{yz}} = \frac{d\epsilon_{zx}^p}{\tau_{zx}} = d\lambda$$

or

$$d\epsilon_{ij}^p = \sigma_{ij}^{\cdot} d\lambda \quad (53)$$

Equations (53) can then be considered as a special case of (52) where the elastic strain components are neglected. Considering both elastic and plastic strains

$$\begin{aligned} d\epsilon_x^p &= \frac{2}{3} d\lambda \left[\sigma_x - \frac{1}{2} (\sigma_y + \sigma_z) \right] \\ d\epsilon_y^p &= \frac{2}{3} d\lambda \left[\sigma_y - \frac{1}{2} (\sigma_x + \sigma_z) \right] \\ d\epsilon_z^p &= \frac{2}{3} d\lambda \left[\sigma_z - \frac{1}{2} (\sigma_y + \sigma_x) \right] \end{aligned} \quad (54)$$

To obtain the total strains $d\epsilon_x$, $d\epsilon_y$, and $d\epsilon_z$, the elastic components of strain must be added to (54).

$$\left. \begin{aligned} d\epsilon_x &= \frac{2}{3} d\lambda \left[\bar{\sigma}_x - \frac{1}{2} (\bar{\sigma}_y + \bar{\sigma}_z) \right] + \frac{1}{E} \left[d\bar{\sigma}_x - \nu (d\bar{\sigma}_y + d\bar{\sigma}_z) \right] \\ d\epsilon_y &= \frac{2}{3} d\lambda \left[\bar{\sigma}_y - \frac{1}{2} (\bar{\sigma}_z + \bar{\sigma}_x) \right] + \frac{1}{E} \left[d\bar{\sigma}_y - \nu (d\bar{\sigma}_z + d\bar{\sigma}_x) \right] \\ d\epsilon_z &= \frac{2}{3} d\lambda \left[\bar{\sigma}_z - \frac{1}{2} (\bar{\sigma}_x + \bar{\sigma}_y) \right] + \frac{1}{E} \left[d\bar{\sigma}_z - \nu (d\bar{\sigma}_x + d\bar{\sigma}_y) \right] \end{aligned} \right\} \quad (55)$$

2.2.2 Equivalent stress

Stresses may be said to be equivalent when they produce a specified effect upon a material. For example, in a sharp yielding material all combinations of stresses which produce the same transition in strains from the elastic to the plastic state may be called equivalent to each other. In the case of a material which is already plastic, the equivalent stress is that which induces further plastic strain. The two definitions of equivalent stress are identical if the material work hardens isotropically, i.e. if the equivalent stress $\bar{\sigma}$ necessary to produce plastic flow is a positively increasing function of the applied stress and $\bar{\sigma}$ in the stress space grows isotropically with increase of applied stress. For the type of test in which a radial loading path is applied and no unloading takes place, as in the present work on the annealed material, for $\bar{\sigma}$ to show the behaviour of the material in different loading paths, the locus should hold its original shape at the beginning of each stress path and increase isotropically as the applied stresses grow. In the case of the pre-stressed material, the subsequent $\bar{\sigma}$ will be governed by stress and strain history and the definition of equivalent stress becomes more difficult.

In the case of isotropic materials, three of the most useful criteria have already been discussed in Chapter 1 i.e. those of Maxwell-Mises, Tresca and Mohr, and Drucker (1950) has explained many possible mathematical combinations of stress which may be considered to fit experiments.

2.2.3 Equivalent strain

To plot a unique curve of equivalent stresses $\bar{\sigma}$ v. equivalent strains $\bar{\epsilon}$ so that all different combinations of applied stresses and strains produced can be considered in the same relation, it is necessary to define equivalent strains. The two common ways of defining the relationship between $\bar{\sigma}$ and $\bar{\epsilon}$ are the strain hardening hypothesis and the work hardening hypothesis.

(i) The strain-hardening hypothesis states that $\bar{\sigma}$ is a function of the total plastic strain when all components of strain are considered. Hill (1950) has excellently reviewed the case, and shows that

$$\bar{\sigma} = H \left(\int d\bar{\epsilon}^P \right) \quad (56)$$

where $d\bar{\epsilon}^P$ is some function of $d\epsilon_{ij}^P$, say, for example $d\bar{\epsilon}^P = +\sqrt{\frac{2}{3}} \times (d\epsilon_{ij}^P d\epsilon_{ij}^P)^{\frac{1}{2}}$ or is the same invariant function of the components of the plastic strain-increment tensor (remembering that $d\epsilon_{ii}^P = d\epsilon_{ii}^P$ since $d\epsilon_{ii}^P = 0$) and $\bar{\sigma}$ is a function of the components of the deviatoric stress tensor. The equivalent strain $\int d\bar{\epsilon}^P$, integrated over the strain path provides the measure of the plastic deformation. The function $H()$ may be chosen to be identical with the relation between stress and

strain in a simple tension test. Thus when the equivalent stress $\bar{\sigma}$ is defined then the sum of all the increments of all the plastic strains $d\bar{\epsilon}^P$ is a unique function of $\bar{\sigma}$.

(ii) The work-hardening hypothesis suggests that the degree of hardening is a function only of the total plastic work, and is otherwise independent of the strain-path. This does not imply that the strain energy performed on the lattice to distort it is equal to the external plastic work; in fact, only a small fraction of the external work appears to be used in distorting the lattice permanently, Hill (1950).

Let $d\epsilon_{ij}$ represent an infinitesimal increment of strain during the continued loading, this increment of strain can be shown as:

$$d\epsilon_{ij} = \frac{1}{2} \left[\frac{\partial}{\partial x_j} (du_i) + \frac{\partial}{\partial x_i} (du_j) \right] \quad (57)$$

where du_i ($i = 1, 2, 3$) is the vector representing the incremental displacement of a point whose current position-vector is x_i in Cartesian co-ordinates fixed in the element. The elastic part of this strain that is ~~reversible~~ ^{recoverable} is defined by:

$$d\epsilon_{ij}^e = \frac{d\sigma'_{ij}}{2G} + (1 - 2\nu) \delta_{ij} \frac{d\sigma}{E} \quad (58)$$

where $d\sigma'_{ij} = d\sigma_{ij} - \sigma \delta_{ij}$, $\sigma = \frac{1}{3} \sigma_{ii}$, and E , G , and ν are Young's modulus of elasticity, modulus of rigidity, and Poisson's ratio, respectively. The total external work per unit volume done on the element is equal to

$$dW = dW^e + dW^p = \sigma_{ij} d\epsilon_{ij} \quad (59)$$

where $dW^e = \sigma_{ij} d\epsilon_{ij}^e$ which is recoverable elastic energy and $dW^p = \sigma_{ij} d\epsilon_{ij}^p$ is called the plastic work per unit volume. $d\epsilon_{ij}^p = d\epsilon_{ij} - d\epsilon_{ij}^e$ is the plastic strain increment. dW^p is essentially positive since plastic distortion is an irreversible process, in the thermodynamic sense.

From (59) and the plastic work hypothesis:

$$\bar{\sigma} = F(W^p) = \int \sigma_{ij} d\epsilon_{ij}^p \quad (60)$$

where the function $F(\)$ may be chosen to suit the relation between $\bar{\sigma}$ and W^p in a simple tension test.

To check either of the two hypotheses with the behaviour of materials being used, the equivalent stress $\bar{\sigma}$ must be well defined, especially for the case of a material which does not show ^asharp yield in the annealed state, such as annealed copper. In such cases the only way of defining $\bar{\sigma}$ is by determination of the direction of $d\epsilon^p$ (just the same as used in this work for pre-stressed material), using the normality rule to establish $\bar{\sigma}$. This is one of the merits of this work for the case of pre-stressed material. If for the case of an isotropic material the normality rule or other helpful ways such as creep coefficient (which is dealt with in this work) are not used to establish $\bar{\sigma}$ then defining $\bar{\sigma}$ by some arbitrary function of stresses and then trying to see which of the two hypotheses is correct is not really proving anything, Rogan (1966).

For an isotropic material both theories predict the same $\bar{\sigma} = \sigma_x$ in the x direction under tension or compression. Although these tests will show the same results up to yield,

the plastic work hypothesis, for example, would predict that the two curves of σ vs. ϵ (nominal stress against engineering strain) diverge after a few per cent strain. When using "true" stresses vs. logarithmic strains experiment has shown the divergence to start at higher strains. In the case of annealed copper the tension and compression curves of Ludwik and Scheu (1925) coincided for quite high strains. In the case of steels and soft copper the compression curve of Siebel and Pomp (1927) was higher than the tension one, and in this case the difference between the two curves was greatest for steels and least for soft copper.

Other investigators have also used the work-hardening hypothesis and have shown that this hypothesis correlated well with steels in combined tension and torsion in most cases but not so for copper. Schmidt (1932) found that the plastic work hypothesis worked well with his four tests in tension-torsion as also did Morrison (1948), and Morrison and Shepherd (1950) for hardened steels; but Taylor and Quinney (1931) reached the conclusion that the work-hardening hypothesis was not working well for annealed copper. Jackson et al (1948), Dorn (1949), and Hill (1950) support the work-hardening hypothesis.

Ros and Eichinger (1930) found that ^{the} strain-hardening hypothesis did not agree with their experiments, but Chakrabarty (1970) using the second invariant of the strain-increment tensor to define the strain measure and relating the amount of hardening to this measure of the total plastic strain, has proposed an alternative strain-hardening hypothesis.

The main difference between the two hypotheses is that the strain-hardening hypothesis relates the stresses to all the strains whereas the work-hardening hypothesis considers the components of strains occurring in the direction of the applied stresses.

The two hypotheses for the different theoretical isotropic materials are discussed in the following section and the difference between them is discussed.

(iii) a) Maxwell-Mises criterion examined by the plastic work hypothesis

Nadai (1930), Hill (1950), Drucker (1950), and Rogan (1966) have discussed the matter in detail, and the latter author proceeded as follows:

From the work hardening hypothesis

$$\bar{\sigma} = F \left(\int \sigma_{ij} d\epsilon_{ij}^p \right) \quad (61)$$

Using simple tension, in the x direction $\sigma_{ij} = \sigma_x$, nominal stress, and $d\epsilon_{ij}^p = d\epsilon_x^p$, engineering strain then:

$$\bar{\sigma} = F \left(\int_0^{\epsilon_x^p} \sigma_x d\epsilon_x^p \right) \quad (62)$$

In this case $\bar{\sigma} = \sigma_x$ and a graph of σ_x against $\int_0^{\epsilon_x^p} d\epsilon_x^p$ will give the argument of F as the area under the curve.

In the case of a pure torsion test,

$$\bar{\sigma} = F \int_0^{\gamma_{xy}^p} \tau_{xy} d\gamma_{xy}^p \quad (63)$$

and, using the Maxwell-Mises definition of $\bar{\sigma}$ we have $\bar{\sigma} = \sqrt{3} \tau_{xy}$; thus the curve of $\sqrt{3} \tau$ against $\frac{1}{\sqrt{3}} \int_0^{\gamma^p} d\gamma_{xy}^p$ would give the same argument of F. Thus, the two curves of σ_x vs $\int_0^{\epsilon_x^p} d\epsilon_x^p$ and $\sqrt{3} \tau_{xy}$ vs $\int_0^{\gamma^p} \frac{1}{\sqrt{3}} d\gamma_{xy}^p$ would coincide.

The unique functions which make the two curves coincide are called the equivalent stress v. the equivalent strain.

For the case of combined tension-torsion of a material obeying the Maxwell-Mises criterion, and noting that the increment of plastic work dW^p for any plastic strain increment is

$$dW^p = \bar{\sigma} d\bar{\epsilon}^p = \sigma_x d\epsilon_x^p + \sqrt{3} \tau_{xy} \frac{d\gamma_{xy}^p}{\sqrt{3}} \quad (64)$$

$$\text{and } \bar{\sigma} = \sqrt{\sigma_x^2 + 3\tau_{xy}^2} \quad (65)$$

$$\begin{aligned} \text{We have } \bar{\sigma} d\bar{\epsilon}^p &= \sqrt{\sigma_x^2 + 3\tau_{xy}^2} d\bar{\epsilon}^p \\ &= \sigma_x d\epsilon_x^p + \sqrt{3} \tau_{xy} \frac{1}{\sqrt{3}} d\gamma_{xy}^p \\ &= \sigma_x d\epsilon_x^p \left(1 + \frac{\tau_{xy} d\gamma_{xy}^p}{\sigma_x d\epsilon_x^p} \right) \\ &= \sigma_x d\epsilon_x^p \sqrt{\left(1 + \frac{\tau_{xy} d\gamma_{xy}^p}{\sigma_x d\epsilon_x^p} \right)} \sqrt{\left(1 + \frac{\tau_{xy} d\gamma_{xy}^p}{\sigma_x d\epsilon_x^p} \right)} \quad (66) \end{aligned}$$

Now substituting $\frac{d\gamma_{xy}^p}{d\epsilon_x^p} = 3 \frac{\tau_{xy}}{\sigma_x}$ we have:

$$\begin{aligned}\bar{\sigma} d\bar{\epsilon}^p &= \sigma_x d\epsilon_x^p \left(1 + 3 \frac{\tau_{xy}^2}{\sigma_x^2}\right)^{\frac{1}{2}} \left(1 + \frac{1}{3} \frac{d\gamma_{xy}^2}{d\epsilon_x^2}\right)^{\frac{1}{2}} \\ &= d\epsilon_x^p (\sigma_x^2 + 3\tau_{xy}^2)^{\frac{1}{2}} \left(1 + \frac{1}{3} \frac{d\gamma_{xy}^2}{d\epsilon_x^2}\right)^{\frac{1}{2}} \\ &= (\sigma_x^2 + 3\tau_{xy}^2)^{\frac{1}{2}} \left(d\epsilon_x^p + \frac{1}{3} d\gamma_{xy}^2\right)^{\frac{1}{2}}\end{aligned}\quad (67)$$

As $\bar{\sigma}$ for a Maxwell-Mises material is equal to

$$\bar{\sigma} = \sqrt{\sigma_x^2 + 3\tau_{xy}^2}$$

$$\text{then, } \bar{\sigma} d\epsilon_x^p = (\sigma_x^2 + 3\tau_{xy}^2)^{\frac{1}{2}} d\epsilon_x^p$$

But from (67)

$$\begin{aligned}\bar{\sigma} d\bar{\epsilon}^p &= (\sigma_x^2 + 3\tau_{xy}^2)^{\frac{1}{2}} (d\epsilon_x^p + \frac{1}{3} d\gamma_{xy}^2)^{\frac{1}{2}} \\ \therefore d\bar{\epsilon}^p &= \sqrt{d\epsilon_x^p + \frac{1}{3} d\gamma_{xy}^2}\end{aligned}\quad (68)$$

It should be added that σ_x and τ_{xy} are plane stresses applied to the material and $\bar{\epsilon}^p$ is based on a work hypothesis (planar) equivalence.

(iii) b) Maxwell-Mises criterion examined by the strain-hardening hypothesis

The equivalent plastic strain increment for a material obeying the Maxwell-Mises criterion from the strain-hardening hypothesis is

$$d\bar{\epsilon}^p = \sqrt{\frac{2}{3}} (d\epsilon_{ij}^p d\epsilon_{ij}^p)^{\frac{1}{2}} \quad (69)$$

and for combined tension and torsion this becomes:

$$\begin{aligned} d\bar{\epsilon}^p &= \sqrt{\frac{2}{3}} (d\epsilon_x^2 + d\epsilon_y^2 + d\epsilon_z^2 + 2d\epsilon_{xy}^2)^{\frac{1}{2}} \\ &= \frac{\sqrt{2}}{3} \left[(d\epsilon_x^p - d\epsilon_y^p)^2 + (d\epsilon_y^p - d\epsilon_z^p)^2 + (d\epsilon_z^p - d\epsilon_x^p)^2 \right]^{\frac{1}{2}} \quad (70) \end{aligned}$$

where $d\bar{\epsilon}^p$ is not bound to be in the plane of the stresses.

Considering the constancy of volume $d\epsilon_x^p + d\epsilon_y^p + d\epsilon_z^p = 0$ and taking $d\epsilon_y^p = kd\epsilon_x^p$ and therefore $d\epsilon_z^p = -d\epsilon_x^p (1+k)$ we find:

$$d\bar{\epsilon}^p = \frac{\sqrt{2}}{3} [d\epsilon_x^{p2} \{(1-k)^2 + (1+2k)^2 + (2+k)^2\} + 6d\epsilon_{xy}^{p2}]^{\frac{1}{2}} \quad (71)$$

Using the engineering definition of shear strain

where $d\gamma_{xy}^{p2} = \frac{1}{4} d\epsilon_{xy}^{p2}$ this becomes:

$$d\bar{\epsilon}^p = \sqrt{\frac{4}{3}} d\epsilon_x^{p2} (1+k+k^2) + \frac{1}{3} d\gamma_{xy}^{p2} \quad (72)$$

If, under all combinations of tension-torsion, $k = -\frac{1}{2}$, then (72) is numerically equal to (68), and there would not be any difference between the two hypotheses. If $k \neq -\frac{1}{2}$, e.g. Taylor and Quinney (1931), $d\bar{\epsilon}^p$ found from (72) will be different from the work-hardening hypothesis (68) even for a material obeying the Maxwell-Mises criterion.

(iv) a) Tresca criterion examined by the work-hardening hypothesis

In the case of simple tension σ_x v. $\int d\epsilon_x^p$, the work-hardening hypothesis predicts that the tension curve coincides with the torsion case of $2\tau_{xy}$ v. $\int_0^{\gamma_p} \frac{1}{2} d\gamma_{xy}^p$.

For combined tension-torsion, using the Tresca criterion,

$\bar{\sigma} = (\sigma_x^2 + 4\tau_{xy}^2)^{\frac{1}{2}}$ and $d\gamma_{xy}^p/d\epsilon_x^p = 4\tau_{xy}/\sigma_x$ the work-hardening hypothesis gives:

$$d\bar{\epsilon}^p = \sqrt{d\epsilon_x^{p2} + \frac{1}{4} d\gamma_{xy}^{p2}} \quad (73)$$

(iv) b) Tresca criterion examined by the strain-hardening hypothesis

For the combined tension-torsion case the combination of strains is usually taken to be $\frac{1}{2}$ of the maximum shearing strain, so that for σ_x and τ_{xy} this would be $\frac{1}{2}(d\epsilon_1^p - d\epsilon_2^p)$ where ϵ_1^p and ϵ_2^p lie in the xy plane and

$$\frac{1}{2}(d\epsilon_1^p - d\epsilon_2^p) = \left[\left(\frac{d\epsilon_x^p - d\epsilon_y^p}{2} \right)^2 + \frac{d\gamma_{xy}^{p2}}{4} \right]^{\frac{1}{2}} \quad (74)$$

In the case of annealed material where the direction of principal stresses coincide with the direction of principal strains, and taking $dy_{xy}^p/de_x^p = 4\tau_{xy}/\sigma_x$, we have

$$d\bar{\epsilon}^p = \sqrt{de_x^{p2} + \frac{1}{4} dy_{xy}^{p2}} \quad (75)$$

Equation (75) is the same as (73). If the directions of the principal stresses and principal strains do not coincide or if $dy_{xy}^p/de_x^p \neq 4\tau_{xy}/\sigma_x$ then the two hypotheses will give different results for the Tresca criterion, Rogan (1966).

Rogan and Shelton (1969A) using En24 and En25 steels under combined tension-torsion found that the work-hardening hypothesis was not obeyed by either metal. They therefore resorted to an empirical relationship of the form:

$$\text{En 24} \quad \bar{\sigma} = \sqrt{\sigma^2 + 3.5\tau^2} \quad \text{and} \quad \bar{\epsilon} = \sqrt{\epsilon_x^2 + (\gamma/1.36)^2} \quad (76)$$

$$\text{En 25} \quad \bar{\sigma} = \sqrt{\sigma^2 + 3.8\tau^2} \quad \text{and} \quad \bar{\epsilon} = \sqrt{\epsilon_x^2 + (\gamma/1.4)^2} \quad (77)$$

which resulted in agreement between the stress-strain curves for tension and torsion. For the combined stress tests the equivalent strain was multiplied by $\cos \alpha$ where α was the angle between the stress path and the resultant plastic strain vector of the components de_x^p and dy_{xy}^p . In this way a complete correlation of results was obtained.

As the normal to the yield locus shows the direction of the plastic strain increment vector, so $\int_0^{\epsilon} \bar{\sigma} \cos \alpha d\bar{\epsilon}$

represents the plastic work during the deformation.

Using the above equivalent stresses and equivalent strains, they also analysed the creep behaviour of the annealed steels and the present work is in agreement with their findings.

2.3 Plasticity Theory for Anisotropic Flow

2.3.1 Anisotropic yield criteria

(i) Hill's theory

Hill (1948), (1950), established his anisotropic yield criterion from a general quadratic loading function of the applied stresses. This function is:-

$$f = \frac{1}{2} C_{ijkl} S_{ij} S_{kl} \quad (78)$$

where S is the deviatoric stress and C_{ijkl} is a set of 81 independent constants. The limitations already in the function f are:- (i) the hydrostatic stress $(\sigma_1 + \sigma_2 + \sigma_3)/3$ has no influence upon yielding; (ii) cubic powers of stress also have no effect; (iii) f is independent of any change in sign of S , i.e. tension and compression yield strengths are equal to each other in any particular direction (no Bauschinger effect), but of course not necessarily equal to those taken in another direction. As the product of direct stress times shear stress has no physical significance this type of term is to be avoided in the expansion of the function and f can be written as

$$f = \frac{1}{2} C_{ijij} S_{ij} S_{ij} \quad (79)$$

(N.B. The repeated suffix on C does not imply summation).

$$\text{thus } f = \frac{1}{2} \left\{ \begin{aligned} &C_{1111} S_{11} S_{11} + C_{2121} S_{21} S_{21} + C_{3131} S_{31} S_{31} \\ &C_{1212} S_{12} S_{12} + C_{2222} S_{22} S_{22} + C_{3232} S_{32} S_{32} \\ &C_{1313} S_{13} S_{13} + C_{2323} S_{23} S_{23} + C_{3333} S_{33} S_{33} \end{aligned} \right\} \quad (80)$$

if symmetry is assumed i.e. $S_{ij} = S_{ji}$ etc.

$$f = \frac{1}{2} (C_{1111} S_{11}^2 + C_{2222} S_{22}^2 + C_{3333} S_{33}^2 + 2C_{1212} S_{12}^2 + 2C_{2323} S_{23}^2 + 2C_{3131} S_{31}^2) \quad (81)$$

writing a, b, c, h, i, j for $C_{1111}, \dots, C_{3131}$, and x, y, and z instead of 11, 22, and 33 respectively f becomes:

$$f = \frac{1}{2} (aS_x^2 + bS_y^2 + cS_z^2 + 2hS_{xy}^2 + 2iS_{yz}^2 + 2jS_{zx}^2) \quad (82)$$

From the definition of deviatoric stress:

$$\begin{aligned} S_x &= \frac{1}{3} [2\sigma_x - (\sigma_y + \sigma_z)] \\ S_y &= \frac{1}{3} [2\sigma_y - (\sigma_z + \sigma_x)] \\ S_z &= \frac{1}{3} [2\sigma_z - (\sigma_x + \sigma_y)] \\ S_{xy} &= \sigma_{xy} \text{ etc.} \end{aligned} \quad (83)$$

f leads to:

$$f = \frac{1}{2} \left\{ \frac{1}{9} [A(\sigma_x - \sigma_y)^2 + B(\sigma_y - \sigma_z)^2 + C(\sigma_z - \sigma_x)^2] + 2h \sigma_{xy}^2 + 2i \sigma_{yz}^2 + 2j \sigma_{zx}^2 \right\} = \frac{\bar{\sigma}^2}{3} \quad (\text{say}) \dots \quad (84)$$

$$\left. \begin{aligned} \text{in which } A &= 2a + 2b - c \\ B &= 2b + 2c - a \\ C &= 2c + 2a - b \end{aligned} \right\} \quad (85)$$

and $\bar{\sigma}$ is a constant of the material taken say for the case of simple tension in the direction of x and producing yield when $\sigma_x = X$

$$\text{Thus from (84): } f = \frac{1}{18} (A + C) X^2 = \frac{\bar{\sigma}^2}{3}$$

$$\text{i.e. } \frac{A + C}{6} = \frac{\bar{\sigma}^2}{X^2}$$

Similarly $\frac{B + A}{6} = \frac{\bar{\sigma}^2}{Y^2}$ where Y is the yield stress in simple tension in the y direction

$\frac{C + B}{6} = \frac{\bar{\sigma}^2}{Z^2}$ where Z is the yield stress in simple tension in the z direction

$$\left. \begin{aligned} \text{thus } A &= 3\bar{\sigma}^2 \left(\frac{1}{X^2} + \frac{1}{Y^2} - \frac{1}{Z^2} \right) \\ B &= 3\bar{\sigma}^2 \left(\frac{1}{Y^2} + \frac{1}{Z^2} - \frac{1}{X^2} \right) \\ C &= 3\bar{\sigma}^2 \left(\frac{1}{Z^2} + \frac{1}{X^2} - \frac{1}{Y^2} \right) \end{aligned} \right\} \quad (86)$$

In the case of pure shear, if the shear stress

$\sigma_{xy} = T$ at yield, we have:

$$f = hT^2 = \frac{\bar{\sigma}^2}{3}$$

$$\therefore h = \frac{(\bar{\sigma}/\sqrt{3})^2}{T^2}$$

and $i = \frac{(\bar{\sigma}/\sqrt{3})^2}{R^2}$ where $R = \sigma_{yz}$ at the pure shear yield point

$j = \frac{(\bar{\sigma}/\sqrt{3})^2}{P^2}$ where $P = \sigma_{zx}$ at the pure shear yield point

From (84), $f = \bar{\sigma}^2/3$ and if $f(\sigma)$ is defined that

$$f(\sigma) \equiv \frac{1}{2} \equiv \frac{f}{\frac{2\bar{\sigma}^2}{3}}$$

$$\text{or } 2f(\sigma) \equiv 1 \equiv \frac{A}{6\bar{\sigma}^2} (\sigma_x - \sigma_y)^2 + \frac{B}{6\bar{\sigma}^2} (\sigma_y - \sigma_z)^2 + \frac{C}{6\bar{\sigma}^2} (\sigma_z - \sigma_x)^2 + \frac{3h}{\bar{\sigma}^2} \sigma_{xy}^2 + \frac{3i}{\bar{\sigma}^2} \sigma_{yz}^2 + \frac{3j}{\bar{\sigma}^2} \sigma_{zx}^2 \quad (87)$$

and if $H \equiv \frac{A}{6\bar{\sigma}^2}$, $F \equiv \frac{B}{6\bar{\sigma}^2}$, $G \equiv \frac{C}{6\bar{\sigma}^2}$

$$2N \equiv \frac{3h}{\bar{\sigma}^2}, \quad 2L \equiv \frac{3i}{\bar{\sigma}^2}, \quad 2M \equiv \frac{3j}{\bar{\sigma}^2}$$

then (87) becomes

$$2f(\sigma) \equiv 1 \equiv H(\sigma_x - \sigma_y)^2 + F(\sigma_y - \sigma_z)^2 + G(\sigma_z - \sigma_x)^2 + 2N\sigma_{xy}^2 + 2L\sigma_{yz}^2 + 2M\sigma_{zx}^2 \quad (88)$$

This is Hill's anisotropic theory for yield -
Rogan (1966). As mentioned earlier, this theory does not

however predict a Bauschinger effect while there is ample experimental evidence both in the present work and in that of many earlier authors to show that such an effect does exist in reality. In relation to the present work, a pronounced Bauschinger effect was exhibited in the tests in reversed torsion and a strong cross-effect was also shown in the tests under combined axial tension and internal pressure following a tensile pre-strain.

(ii) Dorn's theory

Dorn (1948) assumes that:

a) the plastic strain rates are functions of the stresses:

b) to a first approximation the components of the strain rates are linear functions of the stresses. The six plastic strain rate-stress equations may therefore be written:

$$\begin{aligned} \dot{\epsilon}_{xx} &= \dot{A}_{11} \sigma_{xx} + \dot{A}_{12} \sigma_{yy} + \dot{A}_{13} \sigma_{zz} + \dot{A}_{14} \sigma_{xy} + \dot{A}_{15} \sigma_{yz} + \dot{A}_{16} \sigma_{zx}, \\ &\text{etc.} \\ \dot{\epsilon}_{xy} &= \dot{A}_{41} \sigma_{xx} + \dot{A}_{42} \sigma_{yy} + \dot{A}_{43} \sigma_{zz} + \dot{A}_{44} \sigma_{xy} + \dot{A}_{45} \sigma_{yz} + \dot{A}_{46} \sigma_{zx} \\ &\text{etc.} \end{aligned} \quad \left. \begin{array}{l} \\ \\ \end{array} \right\} (89)$$

where $\dot{\epsilon}_{xx}$ is the strain rate on the fibre instantaneously in the x coordinate direction, $\dot{\epsilon}_{xy}$ is the shear strain rate between the fibres instantaneously in the x and y directions, σ_{xx} , etc. are the instantaneous components of the normal stresses, σ_{xy} , etc. are the instantaneous components of the shear stresses,

and $\dot{A}_{k\ell}$ ($k, \ell = 1$ to 6) are coefficients of plasticity having dimensions of a strain rate divided by a stress.

The coefficients of plasticity, as introduced here, unlike the coefficient of viscosity or the moduli of elasticity, are not constants; they depend upon the history of flow and deformation; therefore they are not symmetrical (in general $\dot{A}_{k\ell} \neq \dot{A}_{\ell k}$) and the 36 coefficients of plasticity may differ from each other.

In the case of plane plastic flow (89) reduce to:

$$\left. \begin{aligned} \dot{\epsilon}_{xx} &= \dot{A}_{11} \sigma_{xx} + \dot{A}_{12} \sigma_{yy} + \dot{A}_{14} \sigma_{xy} \\ \dot{\epsilon}_{yy} &= \dot{A}_{21} \sigma_{xx} + \dot{A}_{22} \sigma_{yy} + \dot{A}_{24} \sigma_{xy} \\ \dot{\epsilon}_{xy} &= \dot{A}_{41} \sigma_{xx} + \dot{A}_{42} \sigma_{yy} + \dot{A}_{44} \sigma_{xy} \end{aligned} \right\} \quad (90)$$

As with Hill's theory the above relations do not allow for the Bauschinger effect upon reversal of stress and this makes the theory inadequate for comprehensive test conditions. Dorn introduced his theory for heavily rolled sheet metal which exhibits symmetry of the yield locus about the direction of rolling.

(iii) Drucker's theory

Drucker (1950) discussed particular loading functions and stress-strain laws and introduced a comprehensive stress function given by:

$$f(\sigma) = \frac{1}{2} D_{ijkl} (S_{ij} - m\epsilon_{ij}^p)(S_{kl} - m\epsilon_{kl}^p) \quad (91)$$

where S_{ij} is deviatoric stress.

In this function, any initial anisotropy - which will develop still further during subsequent plastic flow - is shown by the pre-strain ϵ_{ij}^p and the Bauschinger effect is also considered by the constant m . This is a direct extension of $f = J_2 - m s_{ij} \epsilon_{ij}^p$. Whereas $f = J_2$ and $f = \frac{1}{2} D_{ijkl} \sigma_{ij} \sigma_{kl}$ shown in fig. 12, display essentially the same features (apart from initial anisotropy) this is not the case with the two corresponding strain theories. In practice, as in the case of the present work, tests on anisotropic materials (with varying small initial anisotropy) will not produce constant plastic strain ratios during radial loading. Therefore it seems reasonable that this assumption should be abandoned. Drucker recognised that this type of anisotropic theory does not lead to constant strain ratios for all but possibly a few isolated loading paths, depending on the manner in which the initial anisotropy was produced. Differentiation of f yields:

$$\frac{\partial f}{\partial \sigma_{ij}} = g_{ijkl} (s_{kl} - m \epsilon_{kl}^p) \quad (92)$$

where $g_{ijkl} = D_{ijkl} - \frac{1}{3} D_{pqkl} \sigma_{ij}$

The ratios
$$\frac{d\epsilon_{ij}^p}{d\epsilon_{kl}^p} = \frac{\partial f}{\partial \sigma_{ij}} / \frac{\partial f}{\partial \sigma_{kl}} = \frac{g_{ijpq} (s_{pq} - m \epsilon_{pq}^p)}{g_{klmn} (s_{mn} - m \epsilon_{mn}^p)}$$

will not, in general, be constant as K increases in $\sigma_{ij} = K \sigma_{ij}^0$ unless $D_{ijpq} = \delta_{ip} \delta_{jq}$. This reduces the theory to the isotropic form of

$$f = \frac{1}{2} (s_{ij} - m \epsilon_{ij}^p)(s_{ij} - m \epsilon_{ij}^p) = J_2 - m s_{ij} \epsilon_{ij}^p + m^2 I_2 \quad (93)$$

In fig. 13 the solid curve is the initial yield curve, identical to the uniform expansion of the locus, but in this case a translation along the σ_x axis is included; in addition it rotates so that tangents at successive points B and B' along a radius vector are no longer parallel. This means that strain ratios alter and that the original anisotropy is not preserved during deformation. There may however exist some direction along which radial loading will produce zero rotation of the ellipse.

(iv) Hsu's theory

Hsu (1966A) derived a general yield criterion for anisotropic materials from the linear relationship between stress and strain components. This yield criterion is discussed for two dimensional stress cases and cases with certain types of symmetry. In the paper it is suggested that a yield criteria for anisotropic materials must imply that: (a) the strain components are linear functions of stress components, and (b) the yield criterion is also the plastic potential. Thus,

$$\frac{d\epsilon_x^P}{d\lambda} = A_{11} (\sigma_x - \delta_{11}) + A_{12} (\sigma_y - \delta_{12}) + A_{13} (\sigma_z - \delta_{13}) + A_{14} (\tau_{xy} - \delta_{14}) + A_{15} (\tau_{yz} - \delta_{15}) + A_{16} (\tau_{zx} - \delta_{16}) \text{ etc.}$$

$$\frac{d\gamma_{xy}^P}{d\lambda} = A_{41} (\sigma_x - \delta_{41}) + A_{42} (\sigma_y - \delta_{42}) + A_{43} (\sigma_z - \delta_{43}) + A_{44} (\tau_{xy} - \delta_{44}) + A_{45} (\tau_{yz} - \delta_{45}) + A_{46} (\tau_{zx} - \delta_{46})$$

where A and δ are parameters representing anisotropy, λ is a parameter representing the progress of the deformation, and ϵ^p , γ^p , σ and τ are plastic direct strain, plastic shear strain, direct stress, and shear stress respectively. Hsu explains that δ_{ii} represents the Bauschinger effect, because reversed stress does not produce a reversed strain increment of the same magnitude, and δ_{ij} ($i \neq j$) are the corresponding constants for the "cross effects".

In the case of plane strain under plane stress

$$\begin{aligned} \overline{\sigma_z} &= 0 & \tau_{yz} &= 0 & \tau_{zx} &= 0 \\ \frac{d\epsilon_x^p}{d\lambda} &= A_{11}(\overline{\sigma_x} - \delta_{11}) + A_{12}(\overline{\sigma_y} - \delta_{12}) - A_{13}\delta_{13} + A_{14}(\tau_{xy} - \delta_{14}) - \\ & & & & & A_{15}\delta_{15} - A_{16}\delta_{16} \\ \frac{d\epsilon_y^p}{d\lambda} &= A_{21}(\overline{\sigma_x} - \delta_{21}) + A_{22}(\overline{\sigma_y} - \delta_{22}) - A_{23}\delta_{23} + A_{24}(\tau_{xy} - \delta_{24}) - A_{25}\delta_{25} - A_{26}\delta_{26} \\ \text{and} \quad \frac{d\gamma^p}{d\lambda} &= A_{41}(\overline{\sigma_x} - \delta_{41}) + A_{42}(\overline{\sigma_y} - \delta_{42}) - A_{43}\delta_{43} + A_{44}(\tau_{xy} - \delta_{44}) - \\ & & & & & A_{45}\delta_{45} - A_{46}\delta_{46} \end{aligned} \quad (95)$$

For the case of tensile pre-strain, where there is symmetry with respect to the x-axis (in σ_x and τ_{xy} stress field) δ_{15} and δ_{25} vanish. By symmetry with other planes for the same stress field he shows that

$$\left. \begin{aligned} \frac{d\epsilon_x^p}{d\lambda} &= A_{11}(\sigma_x - \delta_{11}) + A_{12}(\sigma_y - \delta_{12}) - A_{13}\delta_{13} \\ \frac{d\epsilon_y^p}{d\lambda} &= A_{21}(\sigma_x - \delta_{21}) + A_{22}(\sigma_y - \delta_{22}) - A_{23}\delta_{23} \\ \frac{d\gamma_{xy}^p}{d\lambda} &= A_{44}\tau_{xy} \end{aligned} \right\} \quad (96)$$

Equations (96) are similar to Dorn's derivation for anisotropic flow, except that there is no δ in Dorn's formulae and therefore the Bauschinger effect is excluded. Considering the second assumption that the yield criterion is the plastic potential and for the strain vector $d\epsilon_x^p/d\epsilon_y^p$ to have a potential at all, A_{12} and A_{21} in (96) must be equal. The yield criterion or plastic potential then take the form :

$$\begin{aligned} A_{11}\sigma_x^2 - 2(A_{11}\delta_{11} + A_{12}\delta_{12} + A_{13}\delta_{13})\sigma_x + 2A_{12}\sigma_x\sigma_y + A_{22}\sigma_y^2 - 2(A_{22}\delta_{22} \\ + A_{12}\delta_{21} + A_{23}\delta_{23})\sigma_y + A_{44}\tau_{xy}^2 = k \end{aligned} \quad (97)$$

where k is the constant of integration. If δ is zero, (97) reduces to Hill's theory, and if, furthermore,

$A_{11} = A_{22} = 2A_{12} = \frac{1}{3} A_{44}$, (97) then becomes the Maxwell-Mises yield criterion for isotropic materials.

v) Williams-Svensson theory

This is the most recent of anisotropic yield criteria and has been used later in this thesis as a basis of comparison with some of the experimental work.

Williams (1969) showed that current phenomenological theories of yield behaviour based on the assumption~~X~~ that a polycrystalline aggregate is a homogeneous continuum, are inapplicable to material behaviour at moderately high degrees of prestrain. He employed a physical model, consisting of spherical anisotropic inclusions embedded in a matrix of elastic-plastic material, and, by virtue of a statistical argument, obtained a yield criterion of the form:

$$f = [I_{ijkl} + A_o \epsilon_{ij}^p \epsilon_{kl}^p] [\sigma_{ij} + \epsilon_{ij}^p (L_{pqrs} \sigma_{pq} \sigma_{rs} - m)] \times$$
$$[\sigma_{kl} + \epsilon_{kl}^p (L_{pqrs} \sigma_{pq} \sigma_{rs} - m)] = k^2 \quad (98)$$

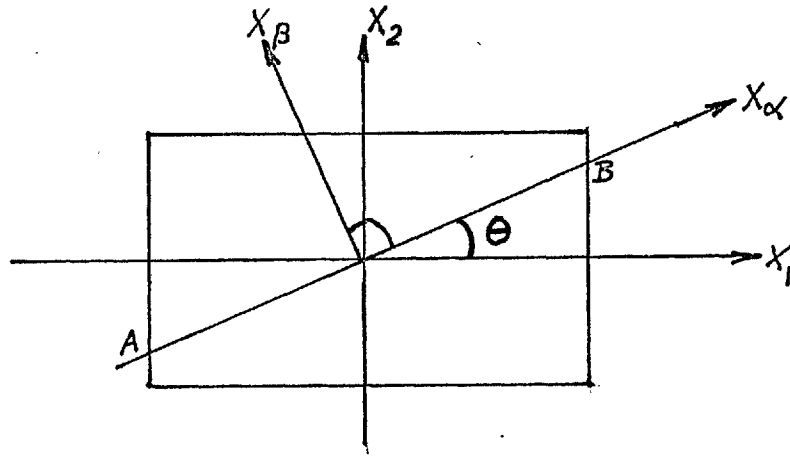
where I_{ijkl} is a spherically isotropic fourth rank tensor representing the isotropic condition, A_o is an anisotropy parameter, ϵ_{ij}^p is the aggregate plastic strain i.e. pre-strain; m is a Bauschinger parameter, L_{pqrs} a distortion parameter and σ_{ij} the applied aggregate stress.

Equation (98) is constructed upon three main considerations, a) arguments involving the local value of shear energy density, b) Eshelby's solution for the local stress acting upon a spherical inclusion in an infinite matrix and c) the statistics of extreme values. Furthermore it is based

upon the fundamental assumption that it is the most probable value of the least value of local plastic strain existing in any one of these inclusions, which determines the yield behaviour of the aggregate.

The development of the theory proceeded as follows.

For a single crystal with a single slip plane the



resolved shear stress on the slip plane AB in direction X_α due to an applied stress X_{ij} is

$$\tau_{\alpha\beta} = l_{\alpha i} l_{\beta j} X_{ij} \quad (99)$$

where the direction cosines $l_{\alpha i}$ are a $f(\theta)$. Thus if simple tension be applied to the crystal in the X_1 direction so that $X_{ij} = X_{11}$ and all other $X_{ij} = 0$ then:

$$\tau_{\alpha\beta} = \cos \theta \sin \theta X_{11}$$

$$\text{and} \quad X_{11} = \frac{\tau_{\alpha\beta}}{\cos \theta \sin \theta} \quad (100)$$

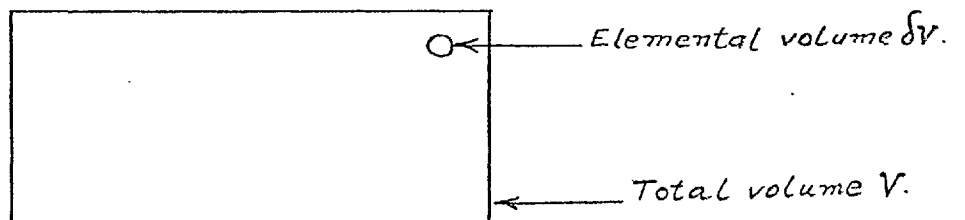
Since the shear strain energy density U_D is proportional to the square of the deviatoric stress tensor:

$$U_D \propto X'_{ij} X'_{ij} \propto \frac{\tau_c^2}{\cos^2 \theta \sin^2 \theta} \quad (101)$$

i.e. there will exist a particular value of θ for which the resolved shear stress $\tau_{\alpha\beta} = \tau_c$ at a minimum value of shear strain energy density. If the orientation of the single slip plane is purely random then the probability of it being in the most favourable position for a given X_{ij} is very small. Thus the shear strain energy density necessary to cause yield will vary with respect to orientation.

For a polycrystal containing a large number of crystals with their slip planes ~~or~~ orientated at random the probability of there being a slip plane in the most favourable position ($\theta = 45^\circ$ gives $(U_D)_{\text{MIN}}$ for simple tension) for a given X_{ij} approaches unity as the number of crystals approaches infinity. Williams (1969) then observes that there is close agreement between the yield criterion for a polycrystal based on the concept of slip and the shear strain energy criterion.

Now consider an elemental volume δV as part of a larger volume V .



Let the material properties of δV obey a stress-strain relationship of the form:

$$e_{ij} = H_{ijkl} X_{kl} + A_{ijkl} X_{kl} \quad (102)$$

and let there be an inverse relationship such that

$$X_{ij} = h_{ijkl} e_{kl} + a_{ijkl} e_{kl} \quad (103)$$

where H_{ijkl} and h_{ijkl} are the Hookean tensors governing elastic response for an isotropic solid. A_{ijkl} is an array of coefficients which represent when $A_{ijkl} \neq 0$ a deviation of the material properties from their original isotropic state. Thus from (102)

$$e_{ij} = \frac{1}{E} [(1 + \nu)X_{ij} - \nu X_{pq} \delta_{pq} \delta_{ij} + A_{ijkl} X_{kl}] \quad (104)$$

where X_{ij} is the local stress acting on the elemental volume δV leading to a local strain e_{ij} . It follows that

$$e_{ii} = \frac{1}{E} [(1 + \nu)X_{ij} \delta_{ij} - 3\nu X_{pq} \delta_{pq} + \delta_{ij} A_{ijkl} X_{kl}] \quad (105)$$

and $X_{ii} = \delta_{mn} X_{mn}$.

As previously outlined, the condition under which this elemental volume will finally yield as X_{ij} are increased has been chosen to be one of critical shear strain energy density. Therefore, assuming that plastic flow is independent

of hydrostatic stress we have:

$$U_s = U - U_v \quad (107)$$

where U_s = shear strain energy density, U = total strain energy density and U_v = volumetric strain energy density.

$$\text{Now } U_v = \frac{1}{2} \Delta p \text{ where } \Delta = e_{ii} \text{ and } p = \frac{1}{3} X_{ii}$$

$$\therefore U_v = \frac{1}{6} e_{ii} X_{mm} \quad (108)$$

$$\text{Also } U = \frac{1}{2} X_{ij} e_{ij} \quad (109)$$

Substituting the appropriate term into (107) leads to

$$\begin{aligned} U_s = \frac{1+\nu}{2E} [\delta_{im}\delta_{jn} X_{ij}X_{mn} - \frac{1}{3} \delta_{ij}\delta_{mn} X_{ij}X_{mn}] \\ + \frac{1}{2E} [A_{ijkl} X_{kl} X_{ij} - \frac{1}{3} A_{ijkl} \delta_{ij}\delta_{mn} X_{mn}X_{kl}] \end{aligned} \quad (110)$$

Since the yield condition is to be a function of U_s it was postulated that:

$$\begin{aligned} f = (\delta_{ik}\delta_{jl} - \frac{1}{3} \delta_{ij}\delta_{kl}) X_{ij}X_{kl} + A'_{ijkl} X_{ij} X_{kl} \\ - \frac{1}{3} A'_{ijkl} \delta_{ij}\delta_{mn} X_{mn}X_{kl} = k^2 \end{aligned} \quad (111)$$

where $A'_{ijkl} = A_{ijkl}(1+\nu)$, would be an appropriate yield criterion for the elemental anisotropic volume.

A'_{ijkl} was described as being an anisotropy tensor related to the strain history of the elemental volume δV by:

$$A'_{ijkl} = A e_{ij}^p e_{kl}^p \quad (112)$$

where e_{ij}^p ^{is the} ~~are~~ plastic strain history tensor and A a scalar function of the strain invariants $I_2 = e_{ij}^p e_{ij}^p$, $I_3 = e_{ij}^p e_{jk}^p e_{ki}^p$ etc. such that:

$$A = A_0 + A_1 \cdot I_2 + B_1 \cdot I_3 + A_2 \cdot I_2^2 + \dots \quad (113)$$

As a first approximation to the state of anisotropy let:

$$A'_{ijkl} = A_0 e_{ij}^p e_{kl}^p \quad (114)$$

Under these conditions and noting that:

$$e_{ij}^p \delta_{ij} = 0 \quad (115)$$

the yield criterion (111) becomes:

$$f = (\delta_{ik} \delta_{jl} - \frac{1}{3} \delta_{ij} \delta_{kl} + A_0 e_{ij}^p e_{kl}^p) x_{ij} x_{kl} = k^2 \quad (116)$$

In the absence of anisotropy the tensor $\delta_{ik} \delta_{jl} - \frac{1}{3} \delta_{ij} \delta_{kl}$ will characterise the isotropic state and can be equated to the isotropic tensor I_{ijkl} so that (116) may be written:

$$f = (I_{ijkl} + A_0 e_{ij}^p e_{kl}^p) x_{ij} x_{kl} = k^2 \quad (117)$$

This is the yield criterion for the elemental volume and Williams noted that for isotropic bodies where $A'_{ijkl} = 0$, (117) may be written

$$f = I_{ijkl} X_{ij} X_{kl} = k^2 \quad (118)$$

which is in fact the most general form of the Maxwell-Mises yield criterion.

It was next assumed that the elemental volume δV undergoes linear translational or kinematic hardening in response to plastic strain with an effect similar to a back stress. Defining C as the strain hardening parameter, (117) was then modified in the form:

$$f = (I_{ijkl} + A_o e_{ij}^p e_{kl}^p)(X_{ij} - c e_{ij}^p)(X_{kl} - c e_{kl}^p) = k^2 \quad (119)$$

This then, is the yield criterion established by Williams (1969) for a small volume element δV of a potentially anisotropic linear strain hardening material subjected to the locally applied stresses X_{ij} . He remarked that if the aggregate volume V is considered to be homogeneous then δV would be representative of V so that the local strain history e_{ij}^p would equal the aggregate strain history ϵ_{ij}^p and the local stress X_{ij} would equal the aggregate applied stress σ_{ij} . Equation (119) could then be written for the aggregate as:

$$f = (I_{ijkl} + A_o \epsilon_{ij}^p \epsilon_{kl}^p)(\sigma_{ij} - c \epsilon_{ij}^p)(\sigma_{kl} - c \epsilon_{kl}^p) = k^2 \quad (120)$$

which corresponds to the continuum theories of Edelman and Drucker (1951) and Svensson (1966). However, Williams does not assume the aggregate to be homogeneous and postulates more generally that both e_{ij}^p and X_{ij} will vary as a function of position within the material, and notes therefore that the aggregate stress σ_{ij} and plastic strain e_{ij}^p are simply statistical averages of the local stresses X_{ij} and plastic strains e_{ij}^p . He next considers that the aggregate at a given stage of deformation is currently composed of n elemental volumes δV embedded in an elastic-plastic matrix and of such size that any one region δV can be considered as a region embedded in an infinite matrix.

Eshelby (1957) has shown that if an elliptical isotropic inclusion undergoes a local stress free strain of e_{ij}^T then, because of the constraint of the matrix, the total strain in the inclusion is:

$$e_{ij}^c = s_{ijkl} e_{kl}^T \quad (121)$$

For a spherical grain:

$$e_{ij}^c = b e_{ij}^T \quad (122)$$

$$\text{where } b = \frac{2(4 - 5\nu)}{15(1 - \nu)} \quad (123)$$

In the absence of external stresses this leads to a local stress:

$$X_{ij} = \lambda (e_{mn}^c \delta_{mn} - e_{mn}^T \delta_{mn}) \delta_{ij} + 2G (e_{ij}^c - e_{ij}^T) \quad (124)$$

Budiansky and Wu (1962) drew a parallel between e_{ij}^T and the local plastic deformation of the spherical inclusion e_{ij}^p . Under these conditions (124) using (122) becomes:

$$X_{ij} = 2G (b - 1) e_{ij}^p \quad (125)$$

However as these authors pointed out, this solution is valid only for an elastic matrix and ignores grain interaction.

For plastic deformation of the matrix and neighbouring grain interaction Williams suggested that the stress free strain in the spherical region be denoted by e_{ij}^p . If many such grains are deforming they will produce an overall plastic strain ϵ_{ij}^p where:

$$\epsilon_{ij}^p = (e_{ij}^p)_{\text{average}} \quad (126)$$

Therefore at any region the local "misfit" strain is:

$$e_{ij}^{p*} = e_{ij}^p - \epsilon_{ij}^p \quad (127)$$

Hence the constrained strain becomes:

$$e_{ij}^c = b(e_{ij}^p - \epsilon_{ij}^p) = b e_{ij}^{p*} \quad (128)$$

The corresponding stresses will then be:

$$X_{ij} = \lambda (e_{mn}^c - e_{mn}^{p*}) \delta_{mn} \delta_{ij} + 2G (e_{ij}^c - e_{ij}^{p*}) \quad (129)$$

which on using (128) becomes:

$$X_{ij} = 2G (b-1) (e_{ij}^p - \epsilon_{ij}^p) \quad (130)$$

If in addition to this there exists an overall aggregate stress σ_{ij} then the resultant stress acting locally on an isotropic spherical inclusion is given by:

$$\sigma_{ij} = \sigma_{ij} + 2G(b - 1)(e_{ij}^p - \epsilon_{ij}^p) \quad (131)$$

Williams (1969) attempted to solve Eshelby's problem for the case of an anisotropic inclusion embedded in an infinite matrix but this proved to be mathematically intractable. Thus, the first order approximation to the local yield criterion for a spherical inclusion in terms of the aggregate stress σ_{ij} , the aggregate plastic strain ϵ_{ij}^p and the local plastic strain e_{ij}^p becomes from (120) and (131):

$$f = [I_{ijkl} + A_{ij} e_{ij}^p e_{kl}^p] [\sigma_{ij} + 2G(b-1)(e_{ij}^p - \epsilon_{ij}^p) - C e_{ij}^p] \times \\ [\sigma_{kl} + 2G(b-1)(e_{kl}^p - \epsilon_{kl}^p) - C e_{kl}^p] = k^2 \quad (132)$$

It now remains for the local strain e_{ij}^p to be related to the known aggregate quantities σ_{ij} and ϵ_{ij}^p and this was done by Williams by using the theory of extreme value statistics - Epstein (1948). If n is the number of defects in the model, it is assumed to vary in the form

$$n = K \exp (C\sigma^2) \quad (133)$$

If the number of defects (inclusion) in the model remains constant during deformation then $C = 0$, but if there is a variation and $C \neq 0$ then n will increase under both tensile and compressive stresses.

To allow for a combination of stress components (133) was then expressed in the more general form:

$$n = K \exp (C_{pqrs} \sigma_{pq} \sigma_{rs}) \quad (134)$$

Because the number of defects will either stay constant or increase it follows that the coefficients C_{pqrs} are non-negative and further that n is an even function of the stress components. This is based on the premise that both positive and negative stress increments will produce an increase in n . Therefore only the following components of C_{pqrs} have non-zero values, i.e.

$$\begin{array}{cccccccc} C_{1111} & C_{2222} & C_{3333} & C_{1212} & C_{1221} & C_{2112} & C_{2323} & C_{2332} & C_{2121} \\ C_{3223} & C_{3232} & C_{1313} & C_{1331} & C_{3131} & C_{3113} & & & \end{array}$$

If $e_{ij}^{p*} n$ is the critical or least probable value of e_{ij}^p it was shown that:

$$\text{If } e_{ij}^{p*} n = e_{ij}^p - \lambda_{ij} \ln \frac{n}{2} \quad (135)$$

where λ_{ij} and e_{ij}^p are parameters describing the Laplacian distribution of n inclusions with local strain e_{ij}^p .

Thus from (134) and (135) Williams showed that

$$\begin{aligned} e_{ij}^{p*} n &= e_{ij}^p - k e_{ij}^p \left(\ln \frac{K}{2} + C_{pqrs} \sigma_{pq} \sigma_{rs} \right) \\ &= e_{ij}^p \left(1 - k C_{pqrs} \sigma_{pq} \sigma_{rs} - k \ln \frac{K}{2} \right) \end{aligned} \quad (136)$$

and putting $D_{pqrs} = k C_{pqrs}$ and $p = K \ln \frac{K}{2}$ we have:

$$e_{ijn}^{p*} = e_{ij}^p \left(1 - D_{pqrs} \sigma_{pq} \sigma_{rs} - p \right) \quad (137)$$

noting that $e_{iin}^{p*} = 0$.

Assuming that the yield criterion of the aggregate is dependent on the most probable value of the least value of e_{ij}^p sustained during prior deformation, substitution of e_{ijn}^{p*} for e_{ij}^p in the yield criterion (132) gives the required yield criterion of the model.

$$\text{Hence } f = [I_{ijkl} + A_0 e_{ij}^p e_{kl}^p] [\sigma_{ij} + 2G(b-1)(-\epsilon_{ij}^p D_{pqrs} \sigma_{pq} \sigma_{rs} - p \epsilon_{ij}^p) - c \epsilon_{ij}^p + c \epsilon_{ij}^p D_{pqrs} \sigma_{pq} \sigma_{rs} + p c \epsilon_{ij}^p] [\sigma_{kl} + 2G(b-1)(-\epsilon_{kl}^p D_{pqrs} \sigma_{pq} \sigma_{rs} - p \epsilon_{kl}^p) - c \epsilon_{kl}^p + c \epsilon_{kl}^p D_{pqrs} \sigma_{pq} \sigma_{rs} + p c \epsilon_{kl}^p] = k^2 \quad (138)$$

$$\text{or } f = [I_{ijkl} + A_0 e_{ij}^p e_{kl}^p] [\sigma_{ij} + \epsilon_{ij}^p (L_{pqrs} \sigma_{pq} \sigma_{rs} - m)] [\sigma_{kl} + \epsilon_{kl}^p (L_{pqrs} \sigma_{pq} \sigma_{rs} - m)] = k^2 \quad (139)$$

$$\begin{aligned} \text{where } L_{pqrs} &= (c - F) D_{pqrs} \\ F &= 2G(b - 1) \\ m &= Fp + c - pc \end{aligned}$$

So far we have not substituted for e_{ij}^p in the tensor of anisotropic deviation. Using (137) we see:

$$e_{ijn}^{p*} e_{kln}^{p*} = \epsilon_{ij}^p \epsilon_{kl}^p (1 - D_{pqrs} \sigma_{pq} \sigma_{rs} - p)(1 - D_{pqrs} \sigma_{pq} \sigma_{rs} - p) \quad (140)$$

Using the approximation that:

$$e_{ijn}^{p*} e_{kln}^{p*} = w \epsilon_{ij}^p \epsilon_{kl}^p \quad (141)$$

we finally arrive at the yield criterion:

$$f = [I_{ijkl} + A_0 \epsilon_{ij}^p \epsilon_{kl}^p] [\bar{\sigma}_{ij} + \epsilon_{ij}^p (L_{pqrs} \bar{\sigma}_{pq} \bar{\sigma}_{rs} - m)] \times \\ [\bar{\sigma}_{kl} + \epsilon_{kl}^p (L_{pqrs} \bar{\sigma}_{pq} \bar{\sigma}_{rs} - m)] = k^2 \quad (142)$$

The form of (142) may be simplified in two ways:

$$\text{Let } I_{ijkl} + A_0 \epsilon_{ij}^p \epsilon_{kl}^p = G_{ijkl} \quad (143)$$

$$\text{and } \epsilon_{ij}^p (L_{pqrs} \bar{\sigma}_{pq} \bar{\sigma}_{rs} - m) = \alpha_{ij} \quad (144)$$

Thus we have

$$f = G_{ijkl} [\bar{\sigma}_{ij} + \alpha_{ij}] [\bar{\sigma}_{kl} + \alpha_{kl}] = k^2 \quad (145)$$

2.3.2 Anisotropic stress-strain relations

After obtaining the loading function which describes the onset of plastic deformation, the next step is to determine the strain increments associated with it.

Stress-strain relationships have been studied by different authors such as Nadai (1931), Dorn (1948), Hill (1950) Drucker (1950), Hsu (1966), Rogan and Shelton (1969) and Williams and Svensson (1970). Following Drucker, the basic concepts for ideal or non-work hardening plasticity are:

(i) the requirement of positive plastic work,

$$\sigma_{ij} d \epsilon_{ij}^p > 0$$

(ii) the requirement of positive plastic work by the stress increments $d\sigma_{ij} d \epsilon_{ij}^p > 0$

(iii) the existence of a loading surface at each stage of plastic deformation.

For work-hardening materials the above concepts are insufficient to determine the stress-strain relation. This indeterminacy can only be overcome by the assumption of a linear relation between $d\epsilon_{ij}^p$ and $d\sigma_{ij}$ of the form:

$$d\epsilon_{ij}^p = H_{ijkl} d\sigma_{kl} \quad (146)$$

Equation (146) is similar to the generalized Hook's law

$$d\epsilon_{ij} = C_{ijkl} d\sigma_{kl} \quad (147)$$

C_{ijkl} for the elastic deformations are constants while H_{ijkl} are functions of stress, strain and history of loading. The assumption of linearity lies in taking the H_{ijkl} as independent of the $d\sigma_{kl}$.

The concepts (i) and (ii) are the well-known normality rule given in Appendix A of this thesis.

The linear assumption (146) implies that the sum of the increments of plastic deformation obtained separately from each of two sets of infinitesimal stress increments $d\sigma_{kl}$, neither of which constitutes unloading, is equal to the plastic deformation obtained from $d\sigma_{kl}$ where

$$d\sigma_{kl} = d\sigma'_{kl} + d\sigma''_{kl} \quad (148)$$

It is clear that when the plastic strains ϵ_{ij}^p were determined by total stresses on the material σ_{ij} , then the function $d\epsilon_{ij}^p = H_{ijkl} d\sigma_{ij}$ would necessarily be correct; this is so if the strain state is completely specified by the current stress state, the Hencky type of total deformation theory, Hencky (1924). The non-incremental theory is not generally true, but for the case of annealed material as in the present work where no unloading is performed on the material throughout the radial loading path the total theory leads to the same results as incremental theory.

From Drucker (1950) and Appendix A using the basic concepts of ideal plasticity and the assumptions of a loading function $f(\sigma_{ij}) > K$ and linearity between the infinitesimals of stress and strain leads to:-

$$d\epsilon_{ij}^p = \left(G \frac{\partial f}{\partial \sigma_{kl}} \cdot d\sigma_{kl} \right) \frac{\partial f}{\partial \sigma_{ij}} \quad (149)$$

in which $\frac{\partial f}{\partial \sigma_{kl}}$ and $\frac{\partial f}{\partial \sigma_{ij}}$ are the gradient of f ; G is a scalar which depends upon stress, strain and history; $d\sigma_{kl}$ is the stress increment and $d\epsilon_{ij}^p$ is the resulting plastic strain increment.

Almost the same equation was obtained by Hill (1950) who used a slightly different approach and obtained

$$d\epsilon_{ij}^p = h \cdot \frac{\partial g}{\partial \sigma_{ij}} df \quad (150)$$

where $df = \frac{\partial f}{\partial \sigma_{ij}} d\sigma_{ij}$, h and g are scalar functions which may depend upon the stress, strain, and history.

The function $g(\sigma_{ij})$ in (150) is called the plastic potential which is related in some way to the yield locus $f(\sigma_{ij})$, and for the case of the plastic potential having the same shape ^{as the} ~~of~~ yield locus, i.e. $f = g$, (150) becomes:

$$d\epsilon_{ij}^p = h \frac{\partial f}{\partial \sigma_{ij}} \cdot df \quad (151)$$

This equation is the same as (149) in which:

$$h df = G \frac{\partial f}{\partial \sigma_{kl}} d\sigma_{kl} = d\lambda, \text{ say, so that}$$

$$d\epsilon_{ij}^p = d\lambda \frac{\partial f}{\partial \sigma_{ij}} \quad (152)$$

The term $\frac{\partial f}{\partial \sigma_{ij}}$ is the differentiation of the yield criterion

with respect to the stresses, e.g. if $f = \frac{1}{2} \dot{\sigma}_{ij} \cdot \dot{\sigma}_{ij}$ then

$\frac{\partial f}{\partial \sigma_{ij}} = \dot{\sigma}_{ij}$ and (152) becomes $d\epsilon_{ij}^p = d\lambda \dot{\sigma}_{ij}$ which is the

Prandtl-Reuss equations dealt with earlier.

Rogan (1966) discussed the subject very well and pointed out that the normality rule (Appendix A) which comes from the basic concepts of plasticity is completely retained in (151) as is the linearity of the differentials which was assumed to find that equation.

It should be noted here that (151) will govern the ratios of the plastic strain increments, not their absolute magnitude; it means that at two different points on the same yield locus $f = \sigma_{ij}$ such as $(\sigma_{ij})_I$ and $(\sigma_{ij})_{II}$ the absolute

magnitude of $(d\epsilon_{ij}^P)_I$ may be different from the magnitude of $(d\epsilon_{ij}^P)_{II}$ for an equivalent increment of stress such as $d\sigma_{ij}$. Thus h in (151) or g in (150) may be functions of stress which express the change of magnitude of $d\epsilon_{ij}^P$ on the different points of the one yield locus $f(\sigma_{ij})$ for a given increment of $d\sigma_{ij}$. Taking $h \cdot df = G^*$, where G^* can vary from point to point on the same yield locus; one should note that $d\lambda$ has a unique value on the same yield surface and is a proportionality factor. The effect of variation of G^* is shown in fig. 14. geometrically; if the yield locus for $(\sigma_{ij})_I$ is line I and that for $(\sigma_{ij})_{II}$ is line II provided $(\sigma_{ij})_{II} > (\sigma_{ij})_I$, and even if locus II is an isotropic expansion of locus I, it is not necessary for locus $(\epsilon_{ij}^P)_I$ to have the same shape as $(\epsilon_{ij}^P)_{II}$ nor should it be expected to be an expansion of $(\epsilon_{ij}^P)_I$. From (151) it is clearly understood that when the stress point moves from a particular point A on the yield locus I to A', by application of a stress increment $d\sigma_{ij}$, then from the normality rule the direction of the resulting plastic strain vector is normal to the yield locus at point A or in other words the ratios of the plastic strain increment components are dictated by the shape of the locus $(\sigma_{ij})_I$. The resultant plastic strain increment vector $d\epsilon_{ij}^P$ will therefore move outward, corresponding to point A, in a direction normal to the yield locus at A.

The magnitude of $d\epsilon_{ij}^P$, in this work, is a function of the stress σ_{ij} in such a way that $|d\epsilon_{ij}^P|$ results from a given stress increment of $d\sigma_{ij}$ and varies according to the

position of σ_{ij} on the yield locus. Rogan and Shelton (1969A) reached the same conclusion and reported that En25 steel under combined tension-torsion produced different $/d\epsilon_{ij}^p/$ for different positions of σ_{ij} .

In the case of a sharp-yielding material such as the annealed En24, used in this work, or En25 steel as used by Rogan and Shelton (1969A), the shape of the yield locus (shape of f) may be readily determined, while measurements of the direction of $d\epsilon_{ij}^p$ at different stress level on the yield surface will show the truth of the $f = g$ hypothesis. The present work showed excellent agreement between f and g .

Turning to the yield criterion of Williams and Svensson as an example, we have by using (152)

$$d\epsilon_{ij}^p = \frac{\partial f}{\partial \sigma_{mn}} \cdot \frac{\partial \sigma_{mn}}{\partial \sigma_{ij}} \cdot d\lambda$$

$$d\epsilon_{ij}^p = \left[\frac{\partial f}{\partial (\sigma_{mn} + \alpha_{mn})} \cdot \frac{\partial (\sigma_{mn} + \alpha_{mn})}{\partial \sigma_{ij}} \right] d\lambda \quad (153)$$

$$= \left[\frac{\partial f}{\partial (\sigma_{mn} + \alpha_{mn})} \cdot \frac{\partial \sigma_{mn}}{\partial \sigma_{ij}} + \frac{\partial f}{\partial (\sigma_{mn} + \alpha_{mn})} \cdot \frac{\partial \alpha_{mn}}{\partial \sigma_{ij}} \right] d\lambda$$

(154)

Thus, using (145) this leads to the stress-strain relation

$$d\epsilon_{ij}^p = \left[2G_{ijkl}(\sigma_{kl} + \alpha_{kl}) + 2(1 + \delta_{ij}) G_{mnkl}(\sigma_{kl} + \alpha_{kl}) \epsilon_{mn}^p L_{ijrs} \sigma_{rs} \right] d\lambda \quad (155)$$

CHAPTER 3

EXPERIMENTAL EQUIPMENT

3.1 Material and Heat Treatment

The chemical composition per cent of the En 24 steel used in this work was as follows:

C	Ni	Cr	Mo	Mn	Si
0.39	1.70	1.30	0.27	0.66	0.26

together with traces of sulphur and phosphorus. The analysis was carried out in the Metallurgy Department of Imperial College and the composition lies within the limits set out in B.S.S. 970 (1955) for this class of steel.

The material was supplied in 1 in. diameter hot rolled bar form, in the as-rolled condition. It was first cut into $9\frac{1}{2}$ in. lengths and then given a low-temperature anneal at 650°C for two hours and furnace cooled for ease of machining. This reduced the Vickers Pyramid Number from 360 to 260 and rough-machined tubes were then prepared.

Some sections taken from the solid bar in the rolling direction and at 45° and 90° to that direction were now given an isothermal anneal. This was derived from the International Nickel Company (Mond) diagram shown in fig. 15. i.e. austenitise at 835°C for one hour, transform at 650°C for 2 hours and then furnace cool. The sections were then polished, etched and

photographed and this revealed marked banding of the ferrite and pearlite structure as shown in fig. 16. The banding in the direction of rolling is seen to be more marked than in the other two directions and it is clear that this recommended heat-treatment would not in fact give the required isotropic grain structure. This conclusion was confirmed by subjecting miniature test-pieces to the same heat-treatment and testing them in an Instron machine. The tensile load-extension curves obtained were not identical as required for isotropy and clearly further investigation of the heat-treatment was necessary before any experimental work could be started.

Rogan (1966) had met a similar phenomenon when testing En 25 steel. His first possible explanation was non-homogeneous dispersion of the phosphorus impurity. Regions of the cast containing high phosphorus content would be regions of low carbon content and on cooling would solidify first. This would form a dendritic structure and produce rods of ferrite and pearlite upon subsequent rolling. He heated miniature test-pieces from the axial and two transverse-directions in a highly evacuated quartz tube to 1250°C and soaked them for 5 hours to give the phosphorus time to diffuse evenly through the test-pieces. However, subsequent examination after an $850^{\circ}\text{--}650^{\circ}\text{C}$ anneal showed ~~no~~^{no} improvement either in the structure or in the shape of the load-extension curves.

Rogan's second possible explanation can be seen in relation to the En24 isothermal transformation diagram given in fig. 15. He argued that banding could be a result of separation

of pre-eutectoid ferrite during the isothermal anneal. That figure shows the time-temperature-transformation diagram and the carbide line indicates when the transforming solution has eutectoid composition. Before the eutectoid point, ferrite is precipitated and after that point ferrite and pearlite are precipitated together. The 650°C isotherm used is the optimum in respect of the time required for transformation but it is seen to give about 30 per cent transformation before any pearlite is laid down. This would clearly give a banded structure in a rolled material.

The austenitising temperature (Rogan 1966) also has a profound effect upon the structure formed during the subsequent isothermal transformation. Low austenitising temperatures, just above the A_3 point, promote spheroidal carbides whereas high austenitising temperatures give lamellar carbides. Thus, low austenitising temperatures combined with low transformation temperatures should therefore produce fine pearlite evenly dispersed in the ferrite with a minimum of banding. This explanation by Rogan was followed in the present work.

After many attempts at varying both the austenitising and transformation temperatures and the soaking times at both temperatures, the following heat-treatment was found to be satisfactory in eliminating the banded structure:

austenitise at 840°C for one hour, cool rapidly to 620°C and soak for one hour then reheat to 650°C and soak for a further one hour and finally cool. Three salt baths pre-heated to the above temperatures were used and this permitted the rapid changes in temperature required. Typical photo-micrographs are

shown in fig. 17, where it is seen that banding has been eliminated and the grain size is very small indeed. Load-extension diagrams from miniature test-pieces taken at 0° , 45° and 90° to the bar axis are shown in fig. 18, where it is seen that the curves are mainly coincident up to the formation of a neck.

In this type of experimental work it is highly desirable not to introduce residual machining stresses into the test-pieces. The annealing just referred to was therefore carried out on all test-pieces after final machining. To avoid oxidation and decarburisation of the surface special containers of the form shown in fig. 19 were made. Drillings and filings of En24, from the previous machining^{ing}, were packed round each test-piece after insertion into the container and the screwed end was tightened on to a copper spacer to ensure an adequate seal. Sealing was most important as the salt-baths solution would have been highly deleterious to the test-piece surfaces. For final cooling of the test-pieces from the 650°C transformation temperature, removal of the container from the salt-bath and air cooling was found sufficient - it was ^{un}necessary to cool the salt bath since each test-piece was not in contact with the surrounding air.

3.2 Test-pieces

Because of the need to use one test-piece for one point on the yield locus, a large number were required. It was also vitally important that high dimensional accuracy be maintained and also a high degree of concentricity. Fig. 20 shows the form of the test-pieces which were all of the same length and bore diameter but three different outside diameters were necessary depending upon the type of test. It was

desirable to avoid buckling in torsion yet under internal pressure the wall thickness could not be too thick owing to the limiting pressure of the pump. After a number of preliminary tests the sizes shown in the figure were decided upon.

Regarding the surface finish and tolerances the whole length of the bore was bored to a finish of 8 μ in. or less with a tolerance on the bore diameter of $+ 0.0005$. This close
- 0.0000
tolerance was necessary since the bore was located on a spigot - undersize would prevent location and oversize would cause oil leakage in the internal pressure tests. The outside test lengths ~~were~~
~~were~~ ground to a finish of 16 μ in or less and both finishes were checked on a Taylor-Hobson machine. The outside diameter was measured on a Societe Genevoise measuring machine and the wall thickness on the micrometer fixture shown in fig. 21, which was made in the departmental workshops at Imperial College. It was therefore unnecessary to measure the bore diameter and the permitted tolerance on the wall thickness was $+ 0.0005$ in.
- 0.0000

Clearly, variation in wall thickness on a given test-piece would give rise to an eccentricity in an axial load and result in bending of the test-piece. This has been investigated for a typical test piece in Appendix E and it is shown there that an eccentricity of 0.0001 in. causes upto 6.15% error in the tensile stress due to bending. In Appendix F the same eccentricity produces upto 8.28% error due to internal pressure.

A total of 140 test-pieces was made, 95 being required for the experimental points and the remainder for possible repeat tests. Many of the latter were in fact used both in repeated work and in pilot test runs.

3.3 Apparatus for Tension-Torsion

The testing machine shown in fig. 22 was a duplicate of that designed by Shelton (1959) and some modifications were described by Rogan and Shelton (1969a). It was built in the workshop of the Mechanical Engineering Department of Imperial College.

The machine is essentially a constant-load device. The tension unit is composed of a single lever with a mechanical advantage of 45:1 and two shackles incorporating knife edges in two orthogonal directions to give axially of loading. The lower shackle~~x~~ is anchored to the machine frame and the upper shackle is connected to the lever to which weights may be added. Calibrated strain ^{au}gauge load cells incorporated in the top shackle suspension are used to measure the tensile load. The torsion unit consists of a system of pulleys which rotate the ends of the test piece in opposite directions and are actuated by another single lever to which further weights may be added. The applied torque is measured by calibrated strain gauge cells in series with the test-piece. Between the torque cells and the shackles, thrust bearings are provided to reduce the frictional restraint to twisting imposed by the tensile load. By reason of this system of knife edges and thrust bearings the apparatus is self-aligning and this was evidenced by the high degree of axial-loading experienced in all tests. The tensile load capacity of the machine is 6 tonf and the maximum torque 2 tonf - ins.

Owing to the change in the shape of the test-pieces - compared with those used by Rogan and Shelton (1969a) - a modification to the method of gripping was required. The new grips* are shown in fig. 23 and consist of a split collet (1) which is actuated by the hexagonal-ended taper unit (2) and both are located in the housing (3) . The housing fits into the existing chuck of the machine and ^{is} also provided with the thrust bearing (4) to assist the tightening of the collet.

A further modification applies to the mandrel located inside the test-piece. This was in solid form when used by Rogan and Shelton (1969a) and was shown as Fig. 2 in ~~this~~ that paper. It assisted in eliminating bending in torsion but now required modification in the form shown in fig. 24 because of the extra length and further extension of the test-pieces in these tests. The new mandrel is in split form and can therefore extend with the extension of the test piece.

The torsion meter used in this work* is shown in figs 25 & 25A. It consists of a pair of annular rings separated by three distance pieces and is provided with six spring-loaded locating pins and three U-shaped locating pieces. The instrument is first located concentrically on the test-piece with the distance pieces in position by using the U-shaped pieces, and the locating pins are then pushed into contact with the test piece and locked. The U-shaped locators and distance pieces are then removed, thereby ensuring that the instrument is located concentrically on the test-piece and that a definite

* Designed by Mr. G. Holloway.

gauge length is established. Shear strains were measured by a spring loaded inductance transducer located in one annular ring and registering against a stop-plate fixed to the other ring. The moving core has a spherical cap and the conversion of transducer displacement into angle of twist requires an allowance for the cap radius. With the arrangement used, the measured angle of twist θ' is given by:

$$\theta' = \frac{\text{transducer displacement}}{R} = \tan \theta - \frac{r}{R} \left(\frac{1}{\cos \theta} - 1 \right)$$

(156)

where θ = actual angle of twist; r = cap radius;

R = distance between transducer centreline and axis of test-piece.

The axial extensometer is shown in fig. 26A and utilises three matched inductance transducers in measuring the relative displacement of two sets of spring loaded knife-edge units. There are three units in a given set and each unit has a knife-edge on one end and narrow positioning feet on the other. The knife-edge locates on a very fine line previously scribed around the circumference of the test-piece on a Societe Genevoise Universal Measuring machine and the positioning feet rest against a very thin piece of p.t.f.e. which minimises friction between instrument and test-piece.

The output from the three transducers is fed into an averaging meter and the mean output obtained from that meter and also the output from the torque transducer are fed through

Boulton-Paul Null-Balance Transducer meters and then on to Honeywell Potentionmetric Recorders. The specially modified meters have three ranges of sensitivity, full scale deflection corresponding to 0.001", 0.005" or 0.025" of transducer displacement. The meter output, recorded on a 12 in. chart recorder was backed off after each creep increment using nulling decade switches on the transducer meter. The strain measuring system can be seen in figs. ^{25 and 26} ~~27~~ and a portion of a typical chart record is enclosed in the pocket at the back of this thesis.

The transducers were all of the same type, with a linear range of 0.2 in. and a manufacturers sensitivity of approximately 1.0 mV/V/0.001 in. The final axial plastic strains were checked by re-measuring the distance between the scribed lines on the test-piece from the Societe Genevoise machine.

3.4 Apparatus for Tension-Internal Pressure

The internal pressure equipment was added to the tension-torsion machine by the arrangement shown in fig. 27. Oil was pumped into the test-piece by a hand operated diesel injection pump with a Budenberg dead-weight tester in the circuit to maintain a constant pressure. Any oil leakage was replaced by turning a screw press on the tester in order to keep the spinning weights floating.

The type A test-pieces of fig. 20 were used in these tests. They had a wall thickness of 0.018 in. and this gave a maximum hoop stress of about 58 ~~ton~~ f/in² for the design pressure of 6,000 lb f/in².

Two sealing methods were used. For stress ratios hoop strain: axial stress greater than 2:1 the closed end method of fig. 28 was used, which eliminated all axial thrust caused by the internal pressure. For stress ratios 2:1 and below the open-ended method of fig. 29 was used which included the axial thrust due to the internal pressure - additional axial tensile stresses being applied by the tensile mechanism of the testing machine. A test pressure of 60,000 lbf/in² was used to check the effectiveness of the seals and this was shown to be satisfactory.

The hoop extensometer is shown in fig. 26B. It consists essentially of a split sleeve which slides over the upper grip of the testing machine and is subsequently clamped in position. This arrangement ensures concentricity with the test piece and the strains are measured with three matching inductance transducers located in the extensometer and in contact with the outside surface of the test-piece. Changes in diameter are thus measured at 120° intervals around the circumference and the transducer outputs are fed into an averaging meter and recorded as previously described for simple tension.

Axial strains were also recorded exactly as described previously, using the same equipment and again the final plastic strains were checked using the Societe Genevoise measuring machine. Changes in diameter were also checked on the same machine at intervals around the circumference and in all cases very close agreement between extensometer and direct readings, existed.

3.5 Apparatus for Torsion-Internal Pressure

The apparatus used in these tests was similar to the arrangements described previously for ~~tension~~ and internal pressure. The sealing method was that used previously for stress ratios hoop stress: axial stress greater than 2:1 and the hoop extensometer was provided with a new sleeve. The latter modification provided three prongs, each of which contained a transducer and was necessary in order for the hoop extensometer to straddle the torsion meter. The arrangement is shown in fig. 26B.

3.6 Accuracies

The tensile load and torque measuring cells in the testing machine were calibrated on a 10 ton Buckton single lever machine and an Avery 15,000 lbf-in torsion machine, respectively. The overall accuracy of stress measurement would be within $\pm 1.6\%$ ($\pm 0.1\%$ on load; $\pm 0.5\%$ on axiality and $\pm 1\%$ on wall thickness) in the elastic strain range and much closer than this when the tubes were fully plastic.

The accuracies for the strain measuring equipment were: transducers 0.1%; transducer meters 0.01% and recorders 0.25%. The overall accuracy was checked in the manner described previously i.e. by measuring the displacements of lines, scribed on the test-pieces in the axial and circumferential direction on a Societe Genevoise Universal Measuring machine and comparing them with the transducer output. In all cases agreement was found to be practically identical.

CHAPTER 4

TENSILE CREEP BEHAVIOUR AT ROOM TEMPERATURE

4.1 Introduction

During preliminary testing of En 24 steel it was found that the metal showed marked creep at stresses in excess of the initial yield. This was shown under both simple and complex stress states and was to be expected in view of the findings of Rogan and Shelton (1969A) and the analysis of Rogan and Alexander (1966).

Literature on the creep of metals at room temperature is sparse and as pointed out by the above authors only three papers on this subject had been published on the behaviour of steel prior to their work, although many other papers on topics in plasticity had mentioned the phenomenon. Papsdorf and Schwier (1958) studied the stress relaxation of steel wire at slightly elevated temperatures; Aepfelbacher (1963) investigated the anelasticity of steel below the initial yield stress, and Hoffman and Vogt (1964) tested high-strength steels in high-pressure atmospheres of air and hydrogen between 20-100°C.

A valuable paper by Wyatt (1953) on the tensile creep behaviour of copper at room temperature formed the initial basis of the investigation by Rogan et al and the same procedure was used in the present tests. In section 4.2 the tests and results are described, they are then analysed by the method of Rogan and Alexander (1966) in section 4.3 and the implications in relation to plasticity experiments are discussed in section 4.4.

4.2 Experimental Results

The testing techniques employed was to load a test-piece of annealed En 24 steel incrementally in tension until initial yield took place. Thereafter creep was allowed at each load increment for about 15 minutes before the next large load increment was applied. Experience showed that relatively large increments were necessary because the use of small increments of load brings in the question of delay times or incubation periods which made the analysis of the creep time-law very difficult. Too large an increment led to comparative instability in that rapid creep would still be observed even after 15 minutes.

Fig. 30 shows the creep behaviour in tension and from this figure it is clear that at all stress levels the material obeyed the logarithmic law:

$$\epsilon = a \ln t + c \quad (157)$$

The slope of the lines is the creep coefficient $\dot{\epsilon}t$ as discussed in the next section and this increases rapidly as the stress level is increased. At high stresses near to the ultimate for this material the logarithmic creep law was no longer strictly obeyed, the curves becoming concave upwards as the creep rate decelerated and eventually became constant.

To check creep recovery most of these test-pieces were unloaded incrementally, allowing about 15 minutes to elapse at each stress level. It was observed that at all times the recovery was very small and never exceeded 0.1%.

The unloading line plotted on a stress-strain diagram was at first parallel to the initial elastic line and then curved slightly towards the origin as the slight recovery occurred at the lower stresses.

The effect of load-unload cycles was investigated and is referred to later in section 4.4 to explain some effects experienced by previous workers who had defined their subsequent yield points by the limit of proportionality. Throughout these tests it was seen that unloading from a stress level of 40 ton f/in^2 say to 37 ton f/in^2 , reduced the creep rate. Further unloading to 34 ton f/in^2 reduced that rate almost to zero. Reloading to 37 ton f/in^2 however resulted in an initially higher creep rate than had been experienced before. If the second unloading had been to a lower stress, say 30 ton f/in^2 instead of 34 ton f/in^2 the newer initial creep rate on reloading to 37 ton f/in^2 would be higher still.

4.3 Analysis of Tensile Creep

The analysis of tensile creep behaviour of metals at room temperature was carried out by Rogan and Alexander (1966) and the theory which follows is summarised from that paper.

Previous analyses of the stress dependence of creep behaviour in metals have been restricted to the secondary phase of creep where the creep rate for each stress level is constant with time. The Rogan-Alexander analysis however is concerned with the primary creep which Rogan's experimental work and the work reported by the present author both show to be of logarithmic form.

Kauzmann (1941) applied Eyring's chemical rate processes theory to the flow of metals and obtained the relation:

$$\dot{\epsilon} = \emptyset \exp (-\Delta H/RT) \sinh(\lambda \tau/RT) \quad (158)$$

where $\dot{\epsilon}$ is the rate of strain with time; $\emptyset$ is a factor composed of lattice dimensions and a frequency term; ΔH is the activation energy (neglecting entropy); τ is the shear stress on the shear plane; λ is the change of activation energy produced by unit shear stress; R is the universal gas constant and T the absolute temperature.

In simple tension the shear stress $\tau = \sigma/2$ where σ is the nominal tensile stress applied at the start of the creep test and (158) therefore leads to:

$$\dot{\epsilon} = \emptyset \exp (-\Delta H/RT) \sinh (\lambda \sigma/2RT) \quad (159)$$

Equations of this type had been shown by previous workers (Kauzmann (1941), Dushman et al (1944), Nadai and McVetty (1943) and Harper and Dorn (1957)) to describe the stress dependence of the secondary creep rate in a range of materials at elevated temperatures, but had never been used previously to describe primary creep behaviour.

A study of the theories of primary or transient creep by Rogan and Alexander showed that such theories account for the time dependence of the creep rate by an exhaustion of the number of activated elements (Wyatt 1953) or by ΔH being

an increasing function of an activation stress (Cottrell 1953). This means that the time dependence of the creep rate $\dot{\epsilon}$ comes from the $\emptyset \exp(-\Delta H/RT)$ term in (159) and not from the hyperbolic sine term.

Cottrell (1953) showed the primary creep at a given stress level to be given by:

$$\dot{\epsilon} = A/t^n \quad (160)$$

where A is a constant at a given stress level, t is time and n a constant.

In logarithmic creep such as that observed in these tests:

$$\dot{\epsilon} = a/t \quad (161)$$

i.e. $A = a$ and $n = 1$ so that $a = \dot{\epsilon}t$ is the slope of the straight lines on the graph of ϵ v. $\ln t$.

Thus re-writing (159) gives:

$$\begin{aligned} \dot{\epsilon} &= \left[\frac{\emptyset \exp(-\Delta H/RT)}{t} \right] \sinh(\lambda \sigma / 2RT) \\ &= f(t) \sinh(\lambda \sigma / 2RT) \end{aligned}$$

where the time dependence is given by the exponential term.

Cross-multiplication therefore gives:

$$\dot{\epsilon}t = [\emptyset \exp(-\Delta H/RT)] \sinh(\lambda \sigma / 2RT) \quad (163)$$

Clearly, (163) shows that a graph of $\dot{\epsilon}t$ (given by the straight line slopes of ϵ v. $\ln t$) when plotted against σ would give a hyperbolic sine relationship.

The graph of creep coefficient $\dot{\epsilon}t$ v. nominal stress σ for En 24 steel is shown in fig. 31 where the line drawn is a fitted hyperbolic sine function and is shown to give a good fit to the experimental results over a wide range of stresses. The creep coefficient is very small below the initial yield stress and then changes discontinuously at yield to a value of 0.00015. After yielding $\dot{\epsilon}t$ increases smoothly as the hyperbolic sine of the stress.

The stress dependence of the primary creep coefficient $\dot{\epsilon}t$ is given by:

$$\dot{\epsilon}t = 3.73 \times 10^{-9} \sinh(\sigma/3.20) + 0.00015 \quad (164)$$

where $\dot{\epsilon}$ is in inch/inch/sec, t is time in secs. and σ is nominal stress in tonf/in².

From these results it is possible to obtain the activation energy ΔH and the change in activation energy produced by unit shear stress. Re-writing (163) in terms of Kauzmann's derivation:

$$\dot{\epsilon}t = \frac{4}{3} \frac{\delta}{L} \frac{kT}{h} \exp(-\Delta H/RT) \sinh(\lambda\sigma/2RT) \quad (165)$$

where k = Boltzmann's constant = 1.38×10^{-16} erg per molecule per degree C; h = Planck's constant = 6.626×10^{-27} erg - second and δ/L is the shear strain resulting from a unit molecular shear process. δ/L depends upon the density of molecules per plane and upon the distance between the shear planes which differs

for metals with different packing. For a body centred cubic metal like En 24 steel, $\delta/L \approx 0.6$. Putting these values into (165) leads to:

$$\dot{\epsilon}t = 5.2 \times 10^{12} \exp(-\Delta H/RT) \sinh(\lambda\sigma/2RT) \quad (166)$$

Because of the discontinuity at yield, comparison with this theoretical exponential term can only be made at high stresses where the + 0.00015 term of equation (164) can be neglected. Thus, equating the appropriate term in (164) and (166):

$$3.73 \times 10^{-9} = 5.2 \times 10^{12} \exp(-\Delta H/RT) \quad (167)$$

$$\sinh 1/3.20 = \sinh \lambda/2RT$$

$$\text{i.e. } \lambda/2RT = 1/3.20 \quad (168)$$

Taking the gas constant $R = 1.9857 \text{ cal/mol } ^\circ\text{K}$ and room temperature as 20°C the following results were obtained for En 24 steel at a nominal tensile stress of 42 ton f/in^2 :

$$\Delta H = 28326 \text{ cal/mol.} \quad (169)$$

$$\lambda = 363.6 \text{ cal/mole/unit shear stress}$$

These values compare with Rogan and Alexander's (1966) values of $\Delta H = 26,000$ and $28,000$ for mild steel and En 25 steel respectively and $\lambda = 410$ and 367 respectively.

To complete the analysis it is necessary to obtain the stress dependence of the constant term in the logarithmic creep equation (157). For convenience the constant 'c' in that

equation is taken to be ϵ_1 which is the strain after an interval of 1 second and was obtained from the backward extrapolation of the strain v. log time curves. Thus (157) may be written:

$$\epsilon = a \ln t + \epsilon_1 \quad (170)$$

The variation of ϵ_1 with σ is shown in fig. 32 and may be put in the form:

$$\sigma = A \epsilon_1^n \quad (171)$$

For those tests the resulting relationship was:

$$\sigma = 88 \epsilon_1^{0.206} \quad (172)$$

The total strain at time t secs after the application of a nominal tensile stress σ tonf /in² may therefore be obtained by substitution of equations (164) and (172) into equation (157):

$$\begin{aligned} \epsilon &= a \ln t + c \\ &= \dot{\epsilon} t \ln t + \epsilon_1 \\ &= \left\{ 3.73 \times 10^{-9} \sinh (\sigma/3.20) + 0.00015 \right\} \ln t + (\sigma/88)^{4.8} \end{aligned} \quad (173)$$

4.4 Effect of Creep upon Plasticity Experiments

Most metals exhibit creep in the plastic strain range at room temperatures and it is not surprising that attempts to "eliminate" or allow for creep in plasticity experiments ~~have~~ ^{have} been inconsistent. Some examples taken at random, illustrate this point.

Parker and Bassett (1964) took readings every 3 minutes after the load increment until two successive readings were equal and then reported this reading as the plastic strain.

Ivey (1961) using five materials, all aluminium-based alloys, took plastic strain readings 3 minutes after the application of the load.

Phillips and Gray ¹⁹⁶¹ (~~1958~~) testing aluminium measured strains every two minutes until the creep strain rate became less than 0.3 per cent per hour.

Bertsch and Findley (1962) using aluminium alloy noted the strains immediately after the application of a load and again two minutes later. The plastic strain-increment vectors for each loading path were given in their paper; the strain-increment resulting from the two-minute readings were not only much greater in magnitude but had quite different vectors compared with the instantaneous readings.

Naghdi, Essenburg and Koff (1958) used a straining machine for their investigation of the yielding of pre-strained aluminium-alloy; their plastic strains were therefore instantaneous values.

In some investigations the time intervals between the application of the load and the recording of strain-increments have not been reported.

There have been cases where workers reported that they used very small increments of load to eliminate the effect of creep - but this is far from correct. Application of very small load increments would increase the creep rate for the next step of loading instead of eliminating it, so the use of very small increments of load makes the entire work an investigation of creep properties! In other cases mentioned above, investigators have just combined creep strains with sudden strains.

Now, as it is clear that creep cannot be eliminated in the plastic range, the question arises, how much creep strain should be included in stress-strain curves?

As explained before the time dependent strain was in all cases a logarithmic function; the graphs of strain v. log time being straight lines as shown in fig. 33(a). To draw the stress-strain graph it is necessary to find the strain at the end of say 100 seconds after the introduction of the load increment and calculate the stress at that stage. The effect of choosing different times however, is shown in fig. 33(b), in which the curve A is for $t = 10$ seconds and the curve B for $t = 1000$ sec.

Inconsistencies will clearly arise when existing definitions of the yield point (or flow stress) are applied to such different stress-strain curves. In the section for the definition of yield it was shown that taking the proof

stress definition would give a yield stress for curve A higher than that for B; also departure from linearity would give a greater value for A than for B. But the backward extrapolation which was introduced by Taylor and Quinney could give a yield stress for curve A either greater or less than that for curve B, depending on the point from where the extrapolation is taken. In this work and in investigations made by Rogan and Shelton (1969A) and from an engineering point of view the most practical results seem to be obtained when the greatest amount of creep is allowed at each stress level coupled with a backward-extrapolation definition of flow stress. The time elapsed at each stress level in the plastic stage of these tests was 15 minutes and it was found that for most cases the stress-strain curve beyond the flow stress was close to a straight line, so that the backward extrapolation was quite clear to follow. In all cases, however, whether the stress-strain curves were straight lines or somewhat curved the curves of $\dot{\epsilon}t$ vs. σ were accepted. These curves are independent of time, as seen in fig. 33(c) and they allow a unique value for flow stress to be determined and permit a check to be made on the yield point from the stress v. strain curves.

Two other points remain to be taken into account which affect plasticity experiments. (1) the effect of unload-reload cycles and (2) the effect of recovery of creep strain.

The definition of yield used in obtaining subsequent yield surfaces has been greatly affected by loading-reloading

cycles; for example, when the departure from linearity has been used for the definition of yield stress, the level of unloading affects the yield point, i.e. the greater the unloading the specimen undergoes the lower will be the yield stress found. This is shown in fig. 34. The results mean that the greater the unloading from a pre-stress level, the faster will be the creep rate upon reloading; but using the backward extrapolation definition for the yield stress eliminates the matter. This is the advantage of using the backward extrapolation definition in this work, since it gives a unique result which is independent of the unloading-reloading cycles. This explains why investigators using departure from linearity have obtained widely different results, particularly in the case of work on pre-strained metals. Loading to obtain the pre-strain, partially unloading and then reloading in another stress direction to obtain the yield point is very dependent upon the degree of unloading.

When unloading from a pre-stress level, it is found in most cases, e.g. Ivey (1961), Williams and Svensson (1969), that the unloading stress-strain line is straight at first but the very small recovery of the forward creep strain experienced during further unloading means that the Bauschinger effect on the material is such that the unloading pierces the yield surface based on the proportional limit definition i.e. the subsequent yield locus could lie outside the zero stress axis.

The use of the backward extrapolation method would give completely different results and be much more realistic from a practical viewpoint in these circumstances.

To build up yield surfaces and introduce theoretical formulation upon the time-independence of plastic strains it is necessary to recall two basic assumptions in plasticity theory:

a) a loading function, $f(\sigma_{ij}) = k$ say, exists such that for every stage of plastic deformation further strain appears only for $f > k$.

b) the infinitesimal stress-strain relation is linear, i.e. if all infinitesimal stresses are increased in ratio, all the infinitesimal strains are increased in the same ratio.

For the material used in this work the creep strain rates are inversely proportional to time if the time considered for creep is fairly large, so the strains taken in this work after elapsing 900 seconds can be accepted as being time-independent for the analysis of the work.

From the experimental results it was found that the strain-increment ratio and the creep strain-rate ratio were identical, so condition (b) above can be applied in this investigation. Also any strain-ageing effects were absent.

In conclusion, it should be noted that if the yield stress is defined by the departure from linearity or by a small proof strain the effect of room temperature creep can be considerable but ~~increasing~~ ^{by using} the backward extrapolation definition the effect is much less; also a graph ^{of} creep coefficient v. stress permits checking of the yield stress and leads to

an unambiguous value of "flow" stress. The stress-strain-time relation for annealed En 24 in simple tension was shown to be of the form:

$$\begin{aligned}\epsilon &= a \ln t + c = \dot{\epsilon} t \ln t + \epsilon_1 \\ &= \left\{ 3.73 \times 10^{-9} \sinh (\sigma/3.20) + 0.00015 \right\} \ln t + (\sigma/88)^{4.85}\end{aligned}$$

and indicates that a similar relationship should hold for this material under complex stress by using appropriate definitions of equivalent stress and equivalent strain.

After this investigation of the creep behaviour of En 24 steel in simple tension and at room temperature, it was possible to go forward to investigate the behaviour under complex stress with some confidence.

CHAPTER 5

EXPERIMENTAL RESULTS FROM ANNEALED EN 24 STEEL

5.1 Introduction

In this chapter, the experimental results for the yield locus and also the subsequent plastic flow behaviour are discussed. The latter includes the creep behaviour under combined stresses and each combined stress field is taken in turn. A later section compares and discusses possible reasons for the differences in yield loci obtained in the different stress regions of these tests and the chapter concludes with a general discussion of previous work on the yield behaviour under different complex stress systems.

5.2 Analysis of the Yield and Flow Behaviour Under Complex Stresses

In this section the yield and subsequent plastic flow behaviour of the annealed material under three different combined stress-states, is analysed.

5.2.1 Behaviour under combined axial tension and torsion

The same results as those of Rogan and Shelton (1969A) were taken for annealed En 24 steel.

In chapter 4.3, the creep behaviour of this material at room temperature under simple tension was analysed and it was shown that the logarithmic creep law, $\epsilon^P = a \ln t + c$, was obeyed. In this function two constants should be found to

completely specify the stress-strain-time behaviour of the material:

a) the creep coefficient a , (which is equal to $\dot{\epsilon}t$), expressed with its stress dependence, and

b) the creep constant C , also expressed with its stress dependence.

The following points were observed for annealed En 24 under combined axial tension and torsion using radial loading paths throughout the whole work.

(1) Logarithmic creep was obeyed by both plastic strain components, i.e. ϵ^p and γ^p ;

(2) The resultant plastic strain increment vectors during a given radial loading path had an almost constant direction with respect to time, therefore for each stress-path of σ/τ , the ratio $\dot{\gamma}/\dot{\epsilon}$ was constant for particular values of $|\sigma|$ and $|\tau|$;

(3) the resultant plastic strain vectors also had a constant direction with respect to increasing stress levels, therefore for any stress probe of σ/τ , $\dot{\gamma}/\dot{\epsilon}$ was constant for any value of $|\sigma|$ and $|\tau|$;

(4) the strain-rate ratio $\dot{\gamma}/\dot{\epsilon}$ was equal to the strain-increment ratio $\Delta\gamma^p/\Delta\epsilon^p$ and to the strain ratio γ^p/ϵ^p ; this ratio was always equal to the gradient of normal to the yield locus;

(5) in this stress field, the equivalence of the stress components could be represented by the function

$$\bar{\sigma} = (\sigma^2 + 3.5\tau^2)^{\frac{1}{2}};$$

(6) the strains corresponding to this equivalent stress could be shown by an equivalent strain function

$\bar{\epsilon}^p = \cos \alpha [\epsilon^{p2} + (\gamma^p/1.36)^2]^{\frac{1}{2}}$, where ϵ^p and γ^p are "long-time" values.

From the above, it is possible to define an equivalent creep-strain-rate coefficient or equivalent creep coefficient $\dot{\bar{\epsilon}}t$ which shows the equivalence of the component creep coefficient $\dot{\gamma}t$ and $\dot{\epsilon}t$ with respect to the equivalent stress $\bar{\sigma}$.

$$\left. \begin{aligned} \dot{\bar{\epsilon}}t &= \cos \alpha [(\dot{\epsilon}t)^2 + (\dot{\gamma}t/1.36)^2] \\ \text{and } \bar{\sigma} &= (\sigma^2 + 3.5 \tau^2)^{\frac{1}{2}} \end{aligned} \right\} \quad (174)$$

Also, the stress-strain-time behaviour of annealed En 24 under combined axial tension and torsion can be analysed by the single equation

$$\bar{\epsilon}^p = a' \ln t + c' \quad (175)$$

Equation (175) is similar to $\epsilon^p = a \ln t + c$ of the simple tension creep.

The curve of $\bar{\sigma}$ v. $\dot{\bar{\epsilon}}t$ for axial tension-torsion of annealed En 24 almost coincides with the curve of σ v. $\dot{\epsilon}t$ in simple axial tension, just the same as reported by Rogan and Shelton (1969A),

The curve of $\bar{\sigma}$ v. $[(\dot{\epsilon}t)^2 + (\dot{\gamma}t/1.36)^2]^{\frac{1}{2}}$ does not correlate well with σ v. $\dot{\epsilon}t$ of the simple axial tension; but $\bar{\sigma}$ v. $\cos \alpha [(\dot{\epsilon}t)^2 + (\dot{\gamma}t/1.36)^2]^{\frac{1}{2}}$ agrees fairly well with it.

To determine the function for $\bar{\epsilon}^p$ in this stress field, two parameters should be evaluated:

(i) the stress dependence of the equivalent creep coefficient ($a' \equiv \frac{\dot{\epsilon}}{\epsilon t}$) and

(ii) the equivalent creep constant c' .

From the curves of $\bar{\epsilon}^p$ and $\log t$ the gradients of the lines are equal to a' , and the amount of $\bar{\epsilon}^p$ in 1 sec taken from ~~back~~ ^(the backward) extrapolation of $\bar{\epsilon}^p$ v. $\log t$ reads c' at each stress level. It was (rather impossible to read c' directly because of the time taken to put a new load on the machine smoothly.

Rogan (1966) in his discussion of the parameters of the creep functions of the materials under investigation, introduced a different time scale i.e. 1000 sec. and recommended measuring c' for that time; his expression for creep is then:

$$\bar{\epsilon}^p = a'' \ln \frac{t}{1000} + c'' \quad (176)$$

where t is in sec., $c'' = \bar{\epsilon}_{1000}^p$ (equivalent plastic strain after 1000 sec); and $a'' = \frac{\dot{\epsilon}}{\epsilon t}$ is the equivalent creep coefficient.

Experimental results for this stress system of annealed material are shown in figs. 35 to 38* and the yield locus is plotted in fig. 39.* *Where a representative number of plots of plastic strain v. time are shown*

5.2.2 Behaviour under combined hoop tension and torsion

The results in this stress field for annealed En 24 were similar to the results taken from combined axial tension and torsion. Uniform expansion of the tubes over a length

of $3\frac{1}{2}$ ins. made it possible to carry out testing in this region. All comments made on combined axial tension and torsion can be applied to this region also. Results are drawn in figs. 40 to 43.

5.2.3 Behaviour under combined axial and hoop tension

Using the cases of tension and torsion combined, the equivalent stress was found to be $\bar{\sigma} = \sqrt{\sigma^2 + 3.5\tau^2}$; transforming this expression into principal stress form gives

$$\bar{\sigma}^2 = \sigma_1^2 - 1.5\sigma_1\sigma_2 + \sigma_2^2 \quad (177)$$

where σ_1 and σ_2 are nominal principal stresses.

In the case of $\sigma_1 = \sigma_2$:-

$$\bar{\sigma}^2 = 0.5\sigma_1^2$$

$$\text{or} \quad \sigma_1 = 1.41 \bar{\sigma} \quad (178)$$

This means that equally applied stresses must each be 1.41 times the simple tension value to yield the material. From the experiment - and repeating a number of times with this stress-ratio - the result $\sigma_1 = \sigma_2 = \bar{\sigma}$ was obtained. Clearly this implies a different yield locus for these tests compared with those for tension-torsion.

The plastic strains of the material under tension and internal pressure are shown in figs. 44 to 48 where ϵ_{θ}^p v. ϵ_z^p has been drawn for different stress-levels on each stress path.

In this stress field, the logarithmic creep law, $\epsilon = a \ln t + c$ was clearly obeyed, so that the curves of ϵ^p_θ v. $\log t$ and ϵ^p_z v. $\log t$ are straight lines. The creep coefficient at each stress-level was found and the plots of $\dot{\epsilon} t$ v. σ for both the axial and hoop directions were also drawn. It can be seen that for each case a sinh curve is derived.

From the experimental results for the annealed En 24 steel under combined tension-internal pressure, the following points were observed:

- a) logarithmic creep was followed by both strain components ϵ^p_θ and ϵ^p_z ;
- b) using a radial loading path, the resultant plastic strain-increment vectors, having components $\dot{\epsilon}_\theta$ and $\dot{\epsilon}_z$, had a fixed direction with respect to time, therefore for any particular σ_θ/σ_z probe, $\dot{\epsilon}_\theta/\dot{\epsilon}_z$ was constant for particular values of $|\sigma_\theta|$ and $|\sigma_z|$;
- c) the resultant strain-increment vectors also had a constant direction with respect to increasing stress levels, therefore for any particular σ_θ/σ_z stress path, $d\epsilon^p_\theta/d\epsilon^p_z$ was constant for any values of $|\sigma_\theta|$ and $|\sigma_z|$;
- d) the ratio of the strain-increment $\Delta\epsilon^p_\theta/\Delta\epsilon^p_z$ was equal to the strain-rate ratio $\dot{\epsilon}_\theta/\dot{\epsilon}_z$ and to the strain ratio $\epsilon^p_\theta/\epsilon^p_z$ which were equal to the gradient of the normal to the yield locus, ^{ie. the} "normality" rule;

e) the equivalent stress could be represented by the function:

$$\bar{\sigma}^2 = \sigma_{\theta}^2 - \sigma_{\theta}\sigma_z + \sigma_z^2 \quad (179)$$

where σ_{θ} and σ_z are hoop tension and axial tension respectively;

f) the equivalent plastic strain function to fit $\bar{\sigma}$ v. $\bar{\epsilon}^p$ with $\sigma_{\theta}/\sigma_z = 1$ stress probe may be shown as:

$$\bar{\epsilon}^p = \cos \alpha [\epsilon_z^p + \epsilon_{\theta}^p]^{\frac{1}{2}} \quad (180)$$

in all cases, even for simple tension. The components of ϵ_z^p and ϵ_{θ}^p are taken into account to compute $\bar{\epsilon}^p$, ϵ_z^p and ϵ_{θ}^p being "long-time" values in the expression, and α the angle between the stress path and the resulting strain.

Now, it is possible to define an equivalent creep-strain-rate coefficient or equivalent creep coefficient $\dot{\bar{\epsilon}}t$ which shows the equivalence of the two creep coefficients $\dot{\epsilon}_z t$ and $\dot{\epsilon}_{\theta} t$ with respect to the equivalent stress $\bar{\sigma}$.

$$\left. \begin{aligned} \bar{\sigma} \text{ is given by } \bar{\sigma} &= \sqrt{\sigma_z^2 - \sigma_{\theta}\sigma_z + \sigma_{\theta}^2} \\ \dot{\bar{\epsilon}}t \text{ is given by } \dot{\bar{\epsilon}}t &= \cos \alpha \sqrt{(\dot{\epsilon}_{\theta}t)^2 + (\dot{\epsilon}_z t)^2} \end{aligned} \right\} \quad (181)$$

The creep behaviour of annealed En 24 under combined axial tension-internal pressure is completely defined by the single equation

$$\bar{\epsilon}^p = A \ln t + C \quad (182)$$

similar to $\epsilon = a \ln t + c$ of the tensile creep.

It is quite clear that to correlate $\dot{\epsilon}_t$ with simple axial tension or simple hoop tension the angle α plays an important role. For the stress path $\sigma_\theta/\sigma_z = 2$ for example where the total plastic strain in the axial direction is almost zero $|\epsilon_z^p| \neq 0$, $\alpha = 26^\circ - 30'$ and $\cos \alpha = 0.895$. Using (181) the experimental results were very similar to those for the case $\sigma_\theta/\sigma_z = 1$ where $\alpha = 0$ and $\cos \alpha = 1$. These curves would clearly not have coincided if the coefficient $\cos \alpha$ had not been introduced.

The yield locus for $\sigma_z - \sigma_\theta$ stress system is shown in fig. 49.

5.3 Comparison Between Tension-Tension and Tension-Torsion Results

As mentioned in the previous section, the values of equivalent stresses for the tension-tension and tension-torsion stress fields do not agree with each other. If the former had to correlate with the latter, each of the tensile stresses σ_θ and σ_z in the case of $\sigma_\theta/\sigma_z = 1$ should have been 1.4 times the simple tensile value whereas the experiments show that in this case $\sigma_\theta = \sigma_z = \bar{\sigma}$.

Possible causes for the discrepancy in yield loci in different stress fields will now be discussed.

1. The size effect

There were three different sizes of specimens used in this work; the bore and length of the specimens were always the same but the wall thicknesses were different. Type "A" was

used for the case of high hoop tension plus low axial tension or shear, type "B" for equal hoop stress and shear stress or equal axial stress and shear, and type "C" which was the thickest of them all was used for high shear stress and low hoop or axial tensile stress. The use of different wall thicknesses was due to (i) the limit of ^{the}oil pump, (ii) the limit of ^{the}axial load in the machine, (iii) ^{the avoidance of}to ~~avoid~~ any plastic buckling from the torsion tests. All tests in the tension-tension stress field were carried out on the very thin specimens.

2. The rotation effect

It is possible that in torsion testing the crystals are freer to rotate than in the case of biaxial normal stresses; this will be dealt in detail later.

3. The definition of yield point

The definition of the yield point has a great effect on the results. This was checked and discussed by many investigators, the most recent being Williams and Svensson (1970) who found quite different loci for different off-set yield definitions. Although a material with sharp yielding is not affected by the definition of yield point nevertheless such a material behaves differently in different stress fields. This is well discussed by Rogan (1966).

5.4 Discussion of Yield Behaviour Under Different Complex Stress Systems

Sachs (1928), Cook (1931), Cox and Sopwith (1937), Morrison (1940) and other investigators have dealt with the yield behaviour of metals which was clearly discussed by Rogan (1966).

Polycrystalline materials are divided into two distinct groups, those which have a sharp yield followed by the formation of flow layers, and those in which the start of plastic deformation is not accompanied by flow layers and do not show a distinct sharp yielding.

The effect of the mode of yielding - i.e. the formation of flow layers or not - upon the required combination of stresses was studied by Sachs (1928) and Cox and Sopwith (1937). From statistical considerations they found that the shear stress to produce yield in a simple shear test, such as a torsion test on a thin walled tube would be 0.5 times the yield stress in a simple tension test for a sharp yielding material. In the case of the material in which the onset of plastic deformation is not accompanied by flow layers and yield takes place in every crystallite of the material the ratio of the two stresses is 0.57.

In fig. 51 the graphs for the two materials have been drawn; graph A is for a material with sharp yielding, such as the annealed En 24 steel used in this work, and graph B is for a material without discrete shear bands both having an equal shear yield stress k . Material A follows the Tresca

ellipse and material B the Maxwell-Mises ellipse.

In the present work it is clear that the yield behaviour of the material obeys neither the Tresca nor the Maxwell-Mises criteria; but lies somewhere between the two:

$$\bar{\sigma}^2 = \sigma^2 + 3.5 \tau^2 \quad (183)$$

The physical explanation of the behaviour of materials to obey Tresca, where the ratio shear yield: tensile yield stress = 0.5 or Maxwell-Mises where the ratio is 0.57 does not explain why a sharp yielding material sometimes does not obey either the Tresca criterion or the Maxwell-Mises.

Now the question that arises is that if the Tresca criterion is fundamental for a material which exhibits sharp yielding, why should engineers consider the J_2 or Maxwell-Mises criterion as a true mathematical model of yielding and introduce the material as an "ideal" material of the model?

The Tresca criterion divides the stress space into different regions and in each region the yield locus deforms sharply and has corners with other regions. The Maxwell-Mises criterion however shows a smooth surface in space without corners; the yield function is quite simple and considers all of the stresses applied to the material in the form of stress differences which in principal stress form are proportional to the maximum shear stress. Hencky (1924) without considering the behaviour of single crystal deformation, showed that the J_2 law was the same as the elastic energy of distortion, reaching a maximum value which may be considered constant for a material;

this law of the elastic energy of distortion does not govern the single crystal deformation and so cannot be considered as a general law of yielding for isotropic material. Nadai (1931) showed that the J_2 law was equivalent to the critical shear stress on the octahedral planes where $l = m = n = \frac{1}{\sqrt{3}}$, i.e. the inclination of the planes is equal to $\frac{1}{\sqrt{3}}$ with all three co-ordinate planes in the space; the criticism about this postulate is that the shear stress on 45° planes in simple tension is $\frac{1}{2}\sigma$, where σ is the tensile stress, and on octahedral plane is 0.422σ ; how is it possible that yield should initiate on a plane where the resolved shear stress is 16% greater than on the $57\frac{1}{2}^\circ$ octahedral planes - Rogan (1966)?

From the point of view of physical interpretation it should be noted that the J_2 law or Maxwell-Mises so-called criterion has no physical significance; it is merely a simple criterion based on the three maximum shear stresses in the form of $(\sigma_1 - \sigma_2)^2 + (\sigma_2 - \sigma_3)^2 + (\sigma_3 - \sigma_1)^2 = \bar{\sigma}^2$ so that it avoids the choosing of different regions and gives a rounded smooth yield surface.

Mohr (1900), Guest (1900) and Nadai (1931) explained that for a sharp yielding material to have the ratio of shear yield stress: tensile yield stress greater than 0.5, the effect of the normal stresses acting upon the shear planes must be considered. If the reasoning of Mohr and Guest on the effect of the influence of the normal stress on the shear plane were accepted then the normal yield stress in compression must be up to 15% greater than the tensile yield stress while this is not

true; also in the case of a tension-tension stress field the Tresca law of square segments should be accepted in principal stress space. Cook (1931) postulated a critical volume of yielded material which failed to interpret the yielding of a thin walled-tube not obeying the Tresca law.

As was explained in the previous section, the mode of stresses influences the behaviour of a material so that the Tresca criterion or indeed any other is not always obeyed by the same material under different stress systems.

It is necessary to observe that none of the sharp yielding materials appear to obey the Tresca criterion in the tension-tension stress field. No work has been reported on such a material having a hexagonal yield surface; the locus is always smooth and rounded and in some cases it has exhibited the Maxwell-Mises locus of $\sigma_1^2 - \sigma_1\sigma_2 + \sigma_2^2 = \bar{\sigma}^2$ just as in the experiments carried out in this work. For example the results of Lode (1925) on machined tubular specimens of annealed mild steel under tension and internal pressure showed close agreement with the Maxwell-Mises criterion.

In the case of normal stress - shear stress tests on a sharp yielding material, it has been observed that such a material obeys the Tresca law fairly well; for example Guest (1900) Smith (1909), Morrison (1948) showed that several steels give good agreement with the Tresca criterion and obtained $\bar{\sigma}^2 = \sigma^2 + 4\tau^2$. Rogan and Shelton (1969a) observed close agreement with Tresca for En 25 steel, namely $\bar{\sigma}^2 = \sigma^2 + 3.8\tau^2$ but for the case of annealed En 24 used by the same investigators and

also in this work the result was $\bar{\sigma}^2 = \sigma^2 + 3.5\tau^2$. For the other stress field of combined axial tension-internal pressure as mentioned before, the present work shows that the Maxwell-Mises law is well obeyed.

As this work is the only investigation on the whole stress space from positive shear to tension and continued to negative shear as far as this author is aware, there is no other experimental evidence to show whether or not an annealed material will obey the same law throughout different principal stress fields. Rogan (1966) attempted to explain these differences of behaviour of sharp yielding annealed materials. He discussed the matter on the basis of a few experimental points of his own, assuming the material follows the Tresca law in all stress fields. According to the normality rule the plastic strain increment vectors must be normal to the yield locus and in the corners he introduced different directions for the plastic strain increment vectors as shown in fig. 52.

The normal to the locus at point A, where $\sigma_1 = \bar{\sigma}$ ($\sigma_2 = \sigma_3 = 0$) and $d\epsilon^P_3 = d\epsilon^P_2 = -\frac{1}{2}d\epsilon^P_1$, is not unique because of the corner at this point; the criterion predicts the yield stress $\sigma_1 = \bar{\sigma}$ and $\tau_{\max} = \frac{\bar{\sigma}}{2}$. If the stress path were changed to OB, i.e. a very small + ve σ_2 added to A, $\sigma_3 = 0$, the Tresca criterion again predicts the yield stress $\sigma_1 = \bar{\sigma}$ and the plastic strain vector in the direction of the σ_1 axis so that $d\epsilon^P_2 = 0$, $d\epsilon^P_3 = -d\epsilon^P_1$.

For the stress path OC where a very small negative σ_2 component is added to the stresses applied to the material

at point A, the material would yield before σ_1 reached $\bar{\sigma}$, the plastic strain vector would be $d\epsilon^p_3 = 0$ and $d\epsilon^p_2 = -d\epsilon^p_1$ which is completely different from the plastic strain vectors at points A and B.

These abrupt changes of the flow of the material that happens in every corner is far from experimental observation which for a sharp yielding material predicts a gradual change of the plastic strain vector from B to A and C, i.e. the locus must be smooth; and as the point A is common on the locus, then the smooth locus must lie outside the Tresca hexagon while the sudden yield of the material causes it to deform by layers and thus the shear stress is the factor to distort it. Rogan then introduced the number of possible shear planes upon which the deformation might occur and divided the mode of deformation into a simple shear where the material is constrained to deform on one plane during the whole deformation and pure shear in which the complexity of deformation and number of possible shear planes are twice as much as the previous one. He believed that one of the important factors affecting the experimental results to make them different in the two regions of σ - τ and σ - σ stress fields is the difference between the mode of deformation.

CHAPTER 6

THE LEVY-MISES FLOW RULE AND THE LODE PARAMETERS

6.1 Introduction

In the first chapter the historical development of the basic rules of plastic flow was outlined. In this chapter the objective is to put forward the results taken from ~~these~~ ^{the present} experiments.

Saint-Venant (1870) was the first investigator to propose that the principal axes of strain increment coincided with the axes of principal stress.

Levy (1870) and then von Mises (1913) independently studied this and proposed the following relations between stresses and strain-increments:

$$\frac{d\epsilon_x - d\epsilon_y}{\sigma_x - \sigma_y} = \frac{d\epsilon_y - d\epsilon_z}{\sigma_y - \sigma_z} = \frac{d\epsilon_z - d\epsilon_x}{\sigma_z - \sigma_x} = \frac{d\epsilon_{xy}}{\tau_{xy}} = \frac{d\epsilon_{yz}}{\tau_{yz}} = \frac{d\epsilon_{zx}}{\tau_{zx}} = d\lambda \quad (184)$$

~~The~~ Prandtl-Reuss equations replaced the total incremental strains by plastic incremental strains in the above equations, thereby allowing for elastic components where applicable.

Two parameters μ and ν were later introduced by Lode (1925 - 1928) such that

$$\mu = \frac{2\sigma_2 - \sigma_1 - \sigma_3}{\sigma_1 - \sigma_3} \quad (185)$$

where σ_1 , σ_2 and σ_3 are principal stresses and $\sigma_1 > \sigma_2 > \sigma_3$ and

$$\nu = \frac{2d\epsilon_2^p - d\epsilon_1^p - d\epsilon_3^p}{d\epsilon_1^p - d\epsilon_3^p} \quad (186)$$

where $d\epsilon_1^p$, $d\epsilon_2^p$, and $d\epsilon_3^p$ are increments of the principal plastic strains.

In the first chapter it was explained that when $\mu = \nu$ the plastic potential drawn in the deviatoric or π -plane is a circle, or in other words the Mohr's circles of stress and plastic strain-increments are geometrically similar. Any inequality of μ and ν does not necessarily imply that the principal directions of plastic strain increment do not coincide with the axes of principal stress nor does their equality determine the yield locus of the material. As mentioned in Chapter 1 the variables μ and ν are helpful to check the Prandtl-Reuss equations for the flow-rule which can be written:

$$\nu = \frac{2d\epsilon_2^p - d\epsilon_1^p - d\epsilon_3^p}{d\epsilon_1^p - d\epsilon_3^p} = \frac{d\lambda(2\sigma_2 - \sigma_1 - \sigma_3)}{d\lambda(\sigma_1 - \sigma_3)} = \mu \quad (187)$$

Lode (1925-1928), Taylor and Quinney (1931), Davis (1943), Fraenkel (1948), Siebel (1953), Hundy and Green (1954), Lianis and Ford (1957), Ellington (1958), ^aBraya and Parker (1963), Mair and Pugh (1963) and some other investigators checked the ν/μ and μ/ν relationship.

In the following sections the experimental results from annealed En 24 and also the pre-strained material will be discussed.

6.2 Relation Between μ and ν For The Annealed Material

From the σ_1 v. σ_2 graphs of the yield locus for En 24 the direction and magnitude of the plastic strain-increments and stress ratios were calculated and graphs of μ v. ν for a number of points were plotted. It was noticed in the experimental results for axial tension-internal pressure that when the ratio of axial stress: hoop stress changed from ∞ to 1, all values for μ and ν such as -1 (for hoop tension) and +1 (for equal tension and hoop stress) were found to lie exactly on the $\mu=\nu$ line and when the stress ratio changed from 1 to 0 the curve was repeated almost exactly as shown in fig. 53.

The graphs for axial tension-torsion and hoop tension - torsion (figs. 54 and 55 respectively) also coincided with the $\mu=\nu$ curve. This therefore shows a most satisfactory state of isotropy for the annealed En 24 and confirms the Levy-Mises flow rule.

6.3 Relation Between μ and ν For the Pre-strained Material

Due to Bauschinger and allied effects on the pre-strained material, to determine the amounts of $d\epsilon^p_1$, $d\epsilon^p_2$ and $d\epsilon^p_3$ at least three plastic strain increments such as $d\epsilon^p_z$, $d\epsilon^p_\theta$, and $d\gamma^p$ were needed. However, due to the difficulty of using three axial and three hoop transducers to measure the average of strain increments in each case, there was no room to set the shear transducer meter, or when the torsion meter was set on the specimen only one more set of transducer devices could be used. Because of these problems it was difficult to determine

the direction of the principal strain axes in space relative to the principal stress axes. The method used in this work to calculate the plastic strain increments for each point on the yield locus was as follows:

1) the principal stresses were calculated from the stress components applied to the material and the yield loci for different pre-stresses were drawn;

2) using the normality rule the plastic strain-increment ratios were determined from the normal to the locus at different points, and the ratios were then put in the equation

$$v = \frac{2d\epsilon_2^p - d\epsilon_1^p - d\epsilon_3^p}{d\epsilon_1^p - d\epsilon_3^p} \quad \text{where } d\epsilon_1^p > d\epsilon_2^p > d\epsilon_3^p \text{ and use}$$

was also made of the constant volume relationship.

$d\epsilon_1^p + d\epsilon_2^p + d\epsilon_3^p = 0$. In Appendix C the method of calculating v for each stress system is explained.

(i) $\mu v.v$ for axial pre-stress of 38.5 T/IN^2

This pre-stress which is almost 4 T/IN^2 above the yield stress of annealed En 24 ^a gave the material a plastic strain of 2.61 per cent. The results of the calculations of μ and v from $\sigma\theta/\sigma_z = \infty$ to $\sigma\theta/\sigma_z = 1$ is shown in fig. 56. From this graph it is obvious that the principal stress and principal strain axes do not coincide, the $\mu v.v$ curve is shifted to the right as a line almost parallel to the $\mu = v$ line.

(ii) μ v. v for circumferential pre-stress of 38.5 T/IN²

For stress ratios of $0 \leq \sigma_{\theta}/\sigma_z \leq 1$, the results of μ v. v are identical to the results for a pre-stress in the axial direction, i.e. the graph shows a translation of $\mu = v$ to the right.

The results for this material pre-stressed in either the axial or the hoop direction followed by combined axial tension-torsion or hoop tension-torsion is very scattered because of the rotation of the principal strain axes during different stress ratios in the subsequent tests. This is one of the important investigations which should be considered for further work. It is necessary to determine the direction of the principal axes of strains for measurement of the principal plastic strain increments and also to calculate $d\epsilon_1^p$, $d\epsilon_2^p$ and $d\epsilon_3^p$ in order that $d\epsilon_1^p > d\epsilon_2^p > d\epsilon_3^p$ for the determination of v .

(iii) μ v. v for axial and hoop pre-stresses of 40.5 T/IN

This pre-stress caused a plastic strain of almost 3.45% in the relative direction. Again, the ratios of $\epsilon_{\theta}^p/\epsilon_z^p$ for the points on the yield loci were determined and the curves plotted. Results show that the curves of μ v. v for the pre-strain in the axial direction when $1 \leq \sigma_{\theta}/\sigma_z \leq \infty$ coincides with the results for test-pieces pre-strained in the hoop direction followed by tests in $0 \leq \sigma_{\theta}/\sigma_z \leq 1$, i.e. the yield loci coincide if the position of the axes are interchanged.

Comparing the results of μ v. ν for the 40.5 T/IN^2 pre-stress with those for 38.5 T/IN^2 it is clear that the higher the pre-stress the greater is the translation of the curve to the right or in other words, the higher the pre-stress the greater is the rotation of the principal axes of strain from the principal axes of stress.

iv) μ v. ν for torsion pre-stress of 19.8 T/IN^2

This pre-stress produced a plastic pre-strain of about 3.2%. In fig. 57 the graph for the two stress fields $0 \leq \sigma_\theta/\sigma_z \leq 1$ and $1 \leq \sigma_\theta/\sigma_z \leq \infty$ shows that the results are almost identical, i.e. the subsequent yield locus in ^{the} σ_1, σ_2 stress field after a pre-stress in shear is symmetrical with ~~the~~ $\sigma_1/\sigma_2 = 1$ line.

Points of μ v. ν for forward and reverse torsions were scattered, with the latter case showing the greater scatter.

CHAPTER 7

THE EFFECT OF PRE-STRAIN ON En 24 STEEL

7.1 Introduction

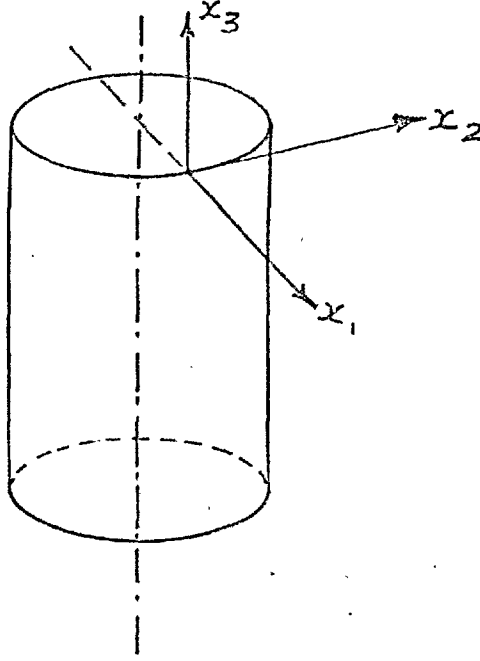
This chapter commences with a theoretical section relating to the anisotropic yield criterion of Svensson and Williams (1970). The general form of this criterion was derived earlier - in chapter 2 - and in this section the specific form of the criterion in relation to tests under combined tension-torsion is now derived. The section concludes with a summary of the coefficients used in calculating the theoretical points on the various yield loci.

Experimental details form the next section of this chapter and the pre-stress and subsequent test procedure is described, as well as the general form of the graphical presentation. The test results from each stress field are then discussed in conjunction with the graphs in the next section and comparisons between theory and experiment made where appropriate.

A general discussion of the influence of the various pre-strains upon the yield and subsequent flow behaviour of En 24 steel, concludes the chapter.

7.2 Application of the Williams-Svensson Anisotropic Theory (Williams (1969)).

For thin-walled tubes the co-ordinate system is as shown.



Noting that the possible direct stresses are σ_{11} σ_{22} and σ_{33} and that the only shear stresses are $\sigma_{23} = \sigma_{32}$ we have from the expansion of (145):-

$$\begin{aligned}
 f = & G_{1111}(\sigma_{11} + \alpha_{11})(\sigma_{11} + \alpha_{11}) + G_{1122}(\sigma_{11} + \alpha_{11})(\sigma_{22} + \alpha_{22}) + G_{1133}(\sigma_{11} + \alpha_{11})(\sigma_{33} + \alpha_{33}) \\
 & + G_{1123}(\sigma_{11} + \alpha_{11})(\sigma_{23} + \alpha_{23}) + G_{1132}(\sigma_{11} + \alpha_{11})(\sigma_{32} + \alpha_{32}) \\
 & + G_{2211}(\sigma_{22} + \alpha_{22})(\sigma_{11} + \alpha_{11}) + G_{2222}(\sigma_{22} + \alpha_{22})(\sigma_{22} + \alpha_{22}) + G_{2233}(\sigma_{22} + \alpha_{22})(\sigma_{33} + \alpha_{33}) \\
 & + G_{2223}(\sigma_{22} + \alpha_{22})(\sigma_{23} + \alpha_{23}) + G_{2232}(\sigma_{22} + \alpha_{22})(\sigma_{32} + \alpha_{32}) \\
 & + G_{2311}(\sigma_{23} + \alpha_{23})(\sigma_{11} + \alpha_{11}) + G_{2322}(\sigma_{23} + \alpha_{23})(\sigma_{22} + \alpha_{22}) + G_{2333}(\sigma_{23} + \alpha_{23})(\sigma_{33} + \alpha_{33}) \\
 & + G_{2323}(\sigma_{23} + \alpha_{23})(\sigma_{23} + \alpha_{23}) + G_{2332}(\sigma_{23} + \alpha_{23})(\sigma_{32} + \alpha_{32}) \\
 & + G_{3211}(\sigma_{32} + \alpha_{32})(\sigma_{11} + \alpha_{11}) + G_{3222}(\sigma_{32} + \alpha_{32})(\sigma_{22} + \alpha_{22}) + G_{3233}(\sigma_{32} + \alpha_{32})(\sigma_{33} + \alpha_{33}) \\
 & + G_{3223}(\sigma_{32} + \alpha_{32})(\sigma_{23} + \alpha_{23}) + G_{3232}(\sigma_{32} + \alpha_{32})(\sigma_{32} + \alpha_{32}) \\
 & + G_{3311}(\sigma_{33} + \alpha_{33})(\sigma_{11} + \alpha_{11}) + G_{3322}(\sigma_{33} + \alpha_{33})(\sigma_{22} + \alpha_{22}) + G_{3333}(\sigma_{33} + \alpha_{33})(\sigma_{33} + \alpha_{33}) \\
 & + G_{3323}(\sigma_{33} + \alpha_{33})(\sigma_{23} + \alpha_{23}) + G_{3332}(\sigma_{33} + \alpha_{33})(\sigma_{32} + \alpha_{32}) = k^2
 \end{aligned} \tag{188}$$

The plastic pre-strains involved are ϵ_{11}^p ϵ_{22}^p ϵ_{33}^p and $\epsilon_{23}^p = \epsilon_{32}^p$ so that taking into account the value of the coefficients G_{ijkl} we have from (188):-

$$\begin{aligned}
 f^p = & \left(\frac{2}{3} + A_o \epsilon_{11}^p \right) (\sigma_{11} + \alpha_{11})^2 + \left(-\frac{1}{3} + A_o \epsilon_{11}^p \epsilon_{22}^p \right) (\sigma_{11} + \alpha_{11}) (\sigma_{22} + \alpha_{22}) \\
 & + \left(-\frac{1}{3} A_o \epsilon_{11}^p \epsilon_{33}^p \right) (\sigma_{11} + \alpha_{11}) (\sigma_{33} + \alpha_{33}) + (A_o \epsilon_{11}^p \epsilon_{23}^p) (\sigma_{11} + \alpha_{11}) (\sigma_{23} + \alpha_{23}) \\
 & + (A_o \epsilon_{11}^p \epsilon_{23}^p) (\sigma_{11} + \alpha_{11}) (\sigma_{23} + \alpha_{23}) + \left(-\frac{1}{3} + A_o \epsilon_{22}^p \epsilon_{11}^p \right) (\sigma_{22} + \alpha_{22}) (\sigma_{11} + \alpha_{11}) \\
 & + \left(\frac{2}{3} + A_o \epsilon_{22}^p \right) (\sigma_{22} + \alpha_{22})^2 + \left(-\frac{1}{3} + A_o \epsilon_{22}^p \epsilon_{33}^p \right) (\sigma_{22} + \alpha_{22}) (\sigma_{33} + \alpha_{33}) \\
 & + (A_o \epsilon_{22}^p \epsilon_{23}^p) (\sigma_{22} + \alpha_{22}) (\sigma_{23} + \alpha_{23}) + (A_o \epsilon_{22}^p \epsilon_{23}^p) (\sigma_{22} + \alpha_{22}) (\sigma_{23} + \alpha_{23}) \\
 & + (A_o \epsilon_{23}^p \epsilon_{11}^p) (\sigma_{23} + \alpha_{23}) (\sigma_{11} + \alpha_{11}) + (A_o \epsilon_{23}^p \epsilon_{22}^p) (\sigma_{23} + \alpha_{23}) (\sigma_{22} + \alpha_{22}) \\
 & + (A_o \epsilon_{23}^p \epsilon_{33}^p) (\sigma_{23} + \alpha_{23}) (\sigma_{33} + \alpha_{33}) + (1 + A_o \epsilon_{23}^p) (\sigma_{23} + \alpha_{23})^2 \\
 & + (A_o \epsilon_{23}^p) (\sigma_{23} + \alpha_{23})^2 + (A_o \epsilon_{23}^p \epsilon_{11}^p) (\sigma_{23} + \alpha_{23}) (\sigma_{11} + \alpha_{11}) \\
 & + (A_o \epsilon_{23}^p \epsilon_{22}^p) (\sigma_{23} + \alpha_{23}) (\sigma_{22} + \alpha_{22}) + (A_o \epsilon_{23}^p \epsilon_{33}^p) (\sigma_{23} + \alpha_{23}) (\sigma_{33} + \alpha_{33}) \\
 & + (A_o \epsilon_{23}^p) (\sigma_{23} + \alpha_{23})^2 + (1 + A_o \epsilon_{23}^p) (\sigma_{23} + \alpha_{23})^2 \\
 & + \left(-\frac{1}{3} + A_o \epsilon_{33}^p \epsilon_{11}^p \right) (\sigma_{33} + \alpha_{33}) (\sigma_{11} + \alpha_{11}) + \left(-\frac{1}{3} + A_o \epsilon_{22}^p \epsilon_{33}^p \right) (\sigma_{22} + \alpha_{22}) (\sigma_{33} + \alpha_{33}) \\
 & + \left(\frac{2}{3} + A_o \epsilon_{33}^p \right) (\sigma_{33} + \alpha_{33})^2 + (A_o \epsilon_{33}^p \epsilon_{23}^p) (\sigma_{33} + \alpha_{33}) (\sigma_{23} + \alpha_{23}) \\
 & + (A_o \epsilon_{33}^p \epsilon_{23}^p) (\sigma_{33} + \alpha_{33}) (\sigma_{23} + \alpha_{23}) = k^2
 \end{aligned} \tag{189}$$

7.2 a) Pre-strain in axial tension followed by combined axial tension-torsion.

(i) Pre-strain conditions: $\sigma_{33} = \sigma_o$; all other $\sigma_{ij} = 0$.

$$\begin{aligned}
 \epsilon_{33}^p &= \epsilon_o^p; \quad \epsilon_{11}^p = \epsilon_{22}^p = -\frac{1}{2} \epsilon_o^p; \quad \text{all other} \\
 \epsilon_{ij}^p &= 0
 \end{aligned}$$

Putting $\alpha_{ij} = \epsilon_{ij}^p X_o$ where $X_o = L_{pqrs} \sigma_{pq} \sigma_{rs} - m$ we see:

$$\alpha_{33} = \epsilon_o^p (L_{3333} \sigma_o^2 - m) = \epsilon_o^p X_o \text{ and } \alpha_{11} = \alpha_{22} = -\frac{1}{2} \epsilon_o^p X_o.$$

(ii) Testing conditions:

$$\sigma_{33} = \sigma; \sigma_{23} = \sigma_{32} = \tau, \text{ all other } \sigma_{ij} = 0$$

$$\epsilon_{33}^p = \epsilon_o^p; \epsilon_{11}^p = \epsilon_{22}^p = -\frac{1}{2} \epsilon_o^p, \text{ all other } \epsilon_{ij}^p = 0$$

$$\therefore \alpha_{33} = \epsilon_o^p (L_{3333} \sigma^2 + 2L_{2323} \tau^2 - m) = \epsilon_o^p X \text{ and } \alpha_{11} = \alpha_{22} = -\frac{1}{2} \epsilon_o^p X$$

Substituting the testing conditions into (189):-

$$\begin{aligned} & \left(\frac{2}{3} + \frac{1}{4} A_o \epsilon_o^{p2}\right) \left(\frac{1}{4} \epsilon_o^{p2} X^2\right) + \left(-\frac{1}{3} + \frac{1}{4} A_o \epsilon_o^{p2}\right) \left(\frac{1}{4} \epsilon_o^{p2} X^2\right) + \left(-\frac{1}{3} - \frac{1}{2} A_o \epsilon_o^{p2}\right) \left(-\frac{1}{2} \epsilon_o^p X\right) (\sigma + \epsilon_o^p X) \\ & + \left(-\frac{1}{3} + \frac{1}{4} A_o \epsilon_o^{p2}\right) \left(\frac{1}{4} \epsilon_o^{p2} X^2\right) + \left(\frac{2}{3} + \frac{1}{4} A_o \epsilon_o^{p2}\right) \left(\frac{1}{4} \epsilon_o^{p2} X^2\right) + \left(-\frac{1}{3} - \frac{1}{2} A_o \epsilon_o^{p2}\right) \left(-\frac{1}{2} \epsilon_o^p X\right) (\sigma + \epsilon_o^p X) \\ & + 2\tau^2 + \left(-\frac{1}{3} - \frac{1}{2} A_o \epsilon_o^{p2}\right) (\sigma + \epsilon_o^p X) \left(-\frac{1}{2} \epsilon_o^p X\right) + \left(-\frac{1}{3} - \frac{1}{2} A_o \epsilon_o^{p2}\right) (\sigma + \epsilon_o^p X) \left(-\frac{1}{2} \epsilon_o^p X\right) \\ & + \left(\frac{2}{3} + A_o \epsilon_o^{p2}\right) (\sigma + \epsilon_o^p X)^2 = K^2 \end{aligned}$$

which gives:

$$\left(\frac{1}{4} \epsilon_o^{p2} X^2\right) \left(1 + \frac{3}{2} A_o \epsilon_o^{p2}\right) + 2 \left(1 + \frac{3}{2} A_o \epsilon_o^{p2}\right) (\sigma + \epsilon_o^p X) \left(\frac{1}{2} \epsilon_o^p X\right)$$

$$+ \left(1 + \frac{3}{2} A_o \epsilon_o^{p2}\right) (\sigma + \epsilon_o^p X)^2 + 3\tau^2 = \frac{3}{2} K^2$$

$$\text{i.e. } \left(1 + \frac{3}{2} A_o \epsilon_o^{p2}\right) \left[\sigma + \frac{3}{2} \epsilon_o^p X\right]^2 + 3\tau^2 = \frac{3}{2} K^2 \quad (190)$$

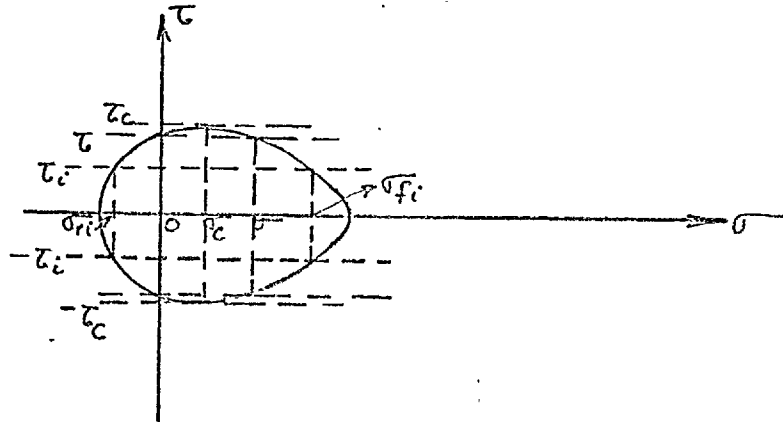
Similarly, by substituting the pre-strain conditions into (189) we have:

$$\left(1 + \frac{3}{2} A_o \epsilon_o^{p2}\right) \left[\sigma_o + \frac{3}{2} \epsilon_o^p X_o\right]^2 = \frac{3}{2} K^2 \quad (191)$$

Thus by equating (190) and (191) and substituting the values of X and X_0 we have the yield criterion:

$$\begin{aligned} & \left[\sigma + \frac{3}{2} \epsilon_0^p (L_{3333} \sigma^2 + 2L_{2323} \tau^2 - m) \right]^2 + \frac{3\tau^2}{\left[1 + \frac{3}{2} A_0 \epsilon_0^p \right]} = \\ & = \left[\sigma_0 + \frac{3}{2} \epsilon_0^p (L_{3333} \sigma_0^2 - m) \right]^2 \end{aligned} \quad (192)$$

Experiment showed the anisotropic yield locus to be symmetrical about the pre-stress axis. The coefficients in (192) can therefore be determined from a relatively few points on the locus which, in general, would be of the form shown.



Re-arranging the locus (192) we have:

$$\begin{aligned} \sigma + \frac{3}{2} \epsilon_0^p L_{3333} \sigma^2 = & \pm \sqrt{\left[\sigma_0 + \frac{3}{2} \epsilon_0^p (L_{3333} \sigma_0^2 - m) \right]^2 - \frac{3\tau^2}{\left[1 + \frac{3}{2} A_0 \epsilon_0^p \right]} } \\ & - \frac{3}{2} \epsilon_0^p [2L_{2323} \tau^2 - m] \end{aligned} \quad (193)$$

For a given value τ_i say, we have in general two possible values of σ i.e. σ_{fi} and σ_{ri} and substituting in the above and adding:

$$\sigma_{fi} + \sigma_{ri} + \frac{3}{2} \epsilon_o^p L_{3333} (\sigma_{fi}^2 + \sigma_{ri}^2) = -3\epsilon_o^p (2L_{2323} \tau_i^2 - m) \quad (194)$$

In the present tests no compressive stresses were applied and σ_{ri} can conveniently be the value $\sigma_{ri} = 0$, $\sigma_{fi} = \sigma$ say and $\tau_i = \tau$. Substituting in (194) we have:

$$\sigma + \frac{3}{2} \epsilon_o^p L_{3333} \sigma^2 = -3\epsilon_o^p (2L_{2323} \tau^2 - m) \quad (195)$$

Similarly for the co-ords (σ_c, τ_c) we have from (194):-

$$2\sigma_c + 3\epsilon_o^p L_{3333} \sigma_c^2 = 3m \epsilon_o^p - 6\epsilon_o^p L_{2323} \tau_c^2 \quad (196)$$

From (195) and (196) we have:-

$$m = \frac{\sigma}{3\epsilon_o^p} + L_{3333} \frac{\sigma^2}{2} + 2L_{2323} \tau^2 \quad (197)$$

and

$$L_{2323} = \frac{\sigma - 2\sigma_c + 3\epsilon_o^p L_{3333} (\frac{1}{2}\sigma^2 - \sigma_c^2)}{6\epsilon_o^p (\tau_c^2 - \tau^2)} \quad (198)$$

Also by differentiating (193) we have:

$$\frac{d\sigma}{d\tau} = \frac{\frac{+3\tau}{[1+\frac{3}{2} A_o \epsilon_o^p p^2]} \left[[\sigma_o + \frac{3}{2} \epsilon_o^p (L_{3333} \sigma_o^2 - m)]^2 - \frac{3\tau^2}{[1+\frac{3}{2} A_o \epsilon_o^p p^2]} \right]^{\frac{1}{2}} - 6\epsilon_o^p L_{2323}}{1 + 3\epsilon_o^p L_{3333} \sigma} \quad (199)$$

For the forward and reverse yield points where $\tau = 0$ it is clear that $\frac{d\sigma}{d\tau} = 0$. However at (σ_c, σ_c) $\frac{d\sigma}{d\tau} = \infty$ and this will be satisfied if:

$$a) \left[\sigma_o + \frac{3}{2} \epsilon_o^p (L_{3333} \sigma_o^{2-m}) \right]^2 = \frac{3\tau_c^2}{\left[1 + \frac{3}{2} A_o \epsilon_o^p \right]}$$

$$i.e. A_o = \frac{2\tau_c^2}{\epsilon_o^{p2} \left[\sigma_o + \frac{3}{2} \epsilon_o^p (L_{3333} \sigma_o^{2-m}) \right]^2} - \frac{2}{3\epsilon_o^{p2}} \quad (200)$$

$$or b) L_{3333} = - \frac{1}{3\epsilon_o^p \sigma_c} \quad (201)$$

7.2b) Pre-strain in circumferential tension followed by combined circumferential tension-torsion

The procedure in this case is as before excepting that for the pre-strain $\sigma_{22} = \sigma_o$, $\epsilon_{22}^p = \epsilon_o^p$ and $\epsilon_{11}^p = \epsilon_{33}^p = -\frac{1}{2}\epsilon_o^p$ and in the testing condition $\sigma_{22} = \sigma$ and $\sigma_{33} = 0$. For the coefficients, L_{2222} is now written for L_{3333} .

7.2c) Pre-strain in torsion followed by combined axial tension-torsion

- (i) Pre-strain conditions: $\sigma_{23} = \sigma_{32} = \tau_o$, all other $\sigma_{ij} = 0$
 $\epsilon_{23}^p = \frac{1}{2}\gamma_o^p$, all other $\epsilon_{ij}^p = 0$

Putting $\alpha_{ij} = \frac{1}{2}\gamma_{ij}^p Y$ where $Y = L_{pqrs} \sigma_{pq} \sigma_{rs}^{-m}$ we see:

$$\alpha_{23} = \frac{1}{2}\gamma_o^p (2L_{2323} \tau_o^2 - m) = \frac{1}{2}\gamma_o^p Y_o$$

(ii) Testing conditions:

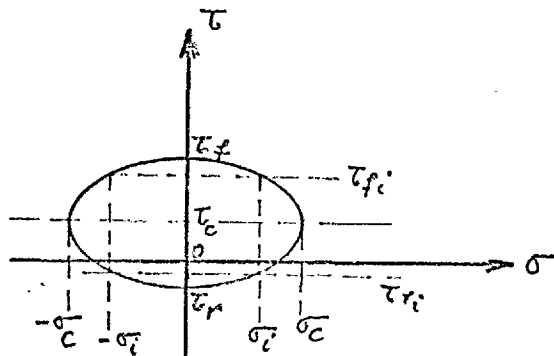
$$\sigma_{33} = \sigma; \sigma_{23} = \sigma_{32} = \tau, \text{ all other } \sigma_{ij} = 0$$

$$\epsilon_{23}^p = \frac{1}{2} \gamma_o^p, \text{ all other } \epsilon_{ij}^p = 0.$$

Substituting the testing conditions and pre-strain conditions into (189) we obtain the yield criterion:

$$\begin{aligned} & \left[\tau + \frac{1}{2} \gamma_o^p (L_{3333} \sigma^2 + 2L_{2323} \tau^2 - m) \right]^2 + \frac{\sigma^2}{3[1 + \frac{1}{2} A_o \gamma_o^p]} = \\ & = \left[\tau_o + \frac{1}{2} \gamma_o^p (2L_{2323} \tau_o^2 - m) \right]^2 \end{aligned} \quad (202)$$

Experiment would again show symmetry of the anisotropic yield locus about the pre-stress axis i.e. the τ axis although in these tests no compressive stresses were applied.



Re-arranging (202) and substituting for some of the principal solutions on the experimental curve, as previously, we obtain:

$$m = \frac{\tau_f + \tau_r}{\gamma_o p} + L_{2323} (\tau_f^2 + \tau_r^2) \quad (203)$$

$$\text{and } L_{3333} = \frac{\tau_f + \tau_r - 2\tau_c + \gamma_o p L_{2323} (\tau_f^2 + \tau_r^2 - 2\tau_c^2)}{\gamma_o p \sigma_c^2} \quad (204)$$

Further, by differentiating the re-arranged form of (202) i.e. $\frac{d\tau}{d\sigma}$ and noting that at (σ_c, τ_c) $\frac{d\tau}{d\sigma} = \infty$ we obtain:

$$A_o = \frac{2\sigma_c^2}{3\gamma_o p^2 [\tau_o + \frac{1}{2}\gamma_o p (2L_{2323}\tau_o^2 - m)]^2} - \frac{2}{\gamma_o p^2} \quad (205)$$

$$\text{and } L_{2323} = \frac{-1}{2\gamma_o p \tau_c} \quad (206)$$

7.2d) Pre-strain in torsion followed by combined circumferential tension-torsion

The procedure in this case is as in 7.2c) excepting that for the testing conditions $\sigma_{22} = \sigma$ and $\sigma_{33} = 0$. For the coefficients L_{2222} is now written for L_{3333} .

7.2 e) Summary of coefficients

Case	Pre-strain %	m	L_{2323}	L_{2222}	L_{3333}	A_o
7.2(a)	2.61	-435.7	-0.9163	0	-0.8514	+25,588
7.2(a)	3.45	-395.9	-0.8697	0	-0.6528	+16,370
7.2(b)	2.34	-395.7	-0.7826	-1.0175	0	+4,647,3
7.2(c)	3.18	-3,256	-6.289	0	-3.298	+5,160
7.2(d)	3.18	-2,981	-5.823	-3.0319	0	+20,872

The calculated theoretical yield points are shown together with the experimental curves in figs. 67, 87, 93, 116 and 126.

7.3 Experimental Details

(i) Pre-stress procedure

The pre-stressing of the thin-walled tubes was carried out in the same way as the tests on the annealed material, using the same testing machine and other apparatus as previously described.

In the axial direction there were two pre-stresses of 38.5 T/IN^2 and 40.5 T/IN^2 and the corresponding plastic strains were 2.61 per cent and 3.45 per cent respectively. Each pre-stress was followed by a complete unloading and then reloading by combined stresses in three stress fields i.e. axial tension - hoop tension, axial tension-torsion, and hoop tension-torsion, using a new test-piece for each test.

A hoop pre-stress of 38.5 T/IN^2 was carried out on a further number of annealed test-pieces and these in turn were subjected to the same combined stresses as previously. A further hoop pre-stress of 40.5 T/IN^2 was next performed on other annealed test pieces and followed by axial tension - hoop tension only. With this latter pre-stressing the whole stress field was not studied as previously because of the number of test-pieces used to repeat many of the tests. For every test a new annealed specimen was used, most tests were duplicated, and in some cases tests were repeated more than twice because during the first, or sometimes the second tests in each subsequent stress-ratio, there was a matter of not knowing what would happen at the next stress level and more useful information was often derived from the duplicate and triplicate tests.

One torsion pre-stress of 19.8 T/IN^2 was carried out and followed by tests in the three subsequent stress fields as previously; this was the case where the Bauschinger effect was very prominent.

To give the same stress histories to all pre-stressed specimens the procedure was identical. From the starting point of the pre-stress, the stress increments were rather small upto the yield stress of the material; this was a good check on the yielding of the material as to whether it was sharp or not, and was also helpful in measuring the deformation of the $\frac{1}{2}$ inch gauge lengths in the axial direction or whatever was the pre-stressing direction. A fixed number of large increments were then applied until the required pre-stresses was reached, the

whole procedure for pre-stressing of the specimens being almost the same as that employed by Rogan (1966). Fifteen minutes, instead of the twenty minutes considered by Rogan, were allowed at each post-yield stress level for creep to take place, then the tubes were unloaded incrementally and the recovery of forward creep measured. This recovery was very small for all types of pre-stresses.

Although strain-ageing was checked and found to be quite negligible, subsequent testing in a particular stress path started almost immediately after the pre-stressing and unloading operation, the subsequent loads being applied simultaneously in rather small increments at first to see if there was a proportional stage of stresses. Larger increments were applied during the later parts of the subsequent loading to analyse the creep behaviour of the material in that probe and this subsequent part of the testing was found to be rather difficult to carry out because of the stress levels to be applied each time. Sometimes the increment of stress was too small so that the analysis of creep was difficult and at other times the stress increments were too large so that the creep strains were too high. Also, the movements of the shear or the axial load pans were up to the end of the stroke and/or the specimens had buckled plastically and in these cases the tests had to be repeated. There were some tests that continued without any bulging or buckling of the specimens.

The merit of this unique procedure for the pre-stress tests is that the pre-stress-strain graph shows up any test-

pieces which have different work-hardening behaviour; in this work nearly every tubular specimen showed a unique pre-stress-strain graph, only two specimens showing rather different behaviour and this was due to slip in the chucks under 40.5T/IN^2 axial pre-stress.

(ii) Strain-ageing

As mentioned earlier, although there was no strain-ageing observed in the material, all of the subsequent tests were carried out immediately following the pre-stress operation which made the work rather prolonged. Rogan (1966) tested En 24 steel for strain-ageing effects and did this by first pre-stressing specimens in shear and then ageing for 30 minutes at 100°C , 150°C , and 200°C . He found that this ageing treatment had no effect upon the reverse-torsion behaviour which was identical to that obtained when a pre-stressed tube was immediately retested in reverse torsion. The isothermal anneal and slow cool of the specimens inside the containers caused any active nitrogen in the material to be precipitated as nitrides and thus there was no active nitrogen left in the material to precipitate onto the slip planes during deformation.

iii) Definitions of stress and strain

For test-pieces with an initial plastic deformation it is possible to define stresses and strains for specimens under combined stress tests in many alternative ways. One difficulty for example is that during the pre-stressing in axial tension or hoop tension, the dimensions of the specimen

change, whereas no change of dimensions appears in the shear pre-stress operation.

The different methods of defining the stresses and strains can be summarized in the following way:

- a) define all stresses and strains with respect to the original, unstrained specimen, even for the subsequent tests;
- b) consider a pre-stressed tube as a "new" specimen, and determine stresses and strains based on the "new" dimensions so that this would give the same definition as case (a) for shear pre-stress;
- c) use "true" stresses and "logarithmic" strains throughout the whole test, although shear strain cannot be logically defined on a logarithmic basis.

In this investigation method (a) was used throughout every test; firstly because initial measurement of the test-piece was needed in calculating the pre-stress and pre-strain; secondly because the results of definition (a) can be quite easily transformed to the other two, and thirdly because final stresses and strains relate to the whole strain history of the deformed test-piece and not to some intermediate state.

According to the above explanation, stresses refer to the initial dimensions of the unstrained specimen and the strains are referred to unit length of unstrained tube. The plastic shear strain γ_o^P refers to the angle of twist at the end of a shear pre-stress; γ^P to the additional plastic angle of twist during any subsequent deformation; γ to the total elastic and plastic shear strains; ϵ_θ^P , ϵ_z^P are the plastic

hoop and axial strains during a test; $\epsilon_{\theta\theta}^p$, ϵ_{oz}^p are the plastic strains after the pre-stress operation in the hoop or axial directions respectively; ϵ_θ , ϵ_z are the total hoop and axial strains during any deformation.

Due to the small pre-strain of the material under any pre-stressing operation, and the small strains in the subsequent tests, there is very little difference between the foregoing three methods of definition. For example, in the case of pre-stressing in the axial direction upto 38.5 T/IN^2 where the plastic pre-strain is 2.6% if method (b) were used, the slopes of all the curves of γ^p/ϵ_z^p would be increased by 2.6%, which would be hardly noticed on the graphs.

(iv) The plastic strain-increments and yield loci

In all of the pre-stress tests logarithmic creep continued to be obeyed by both strain components over almost the whole range of the subsequent bi-axial stress tests. Although the stress increment vector on each loading path was fixed in direction, the plastic strain increment vector continued to rotate during the whole subsequent stress space; this rotation, in all cases, was towards the isotropic direction. For example, if the pre-strain were in the axial direction and this were followed by the stress ratio $\sigma_\theta/\sigma_z = 1$, the ratio of $d\epsilon_\theta^p/d\epsilon_z^p$ was very high at the beginning but rotated towards the isotropic behaviour of $\epsilon_\theta^p/\epsilon_z^p = 1$ for increasing stress. This rotation was more prominent if the direction of the pre-stress were greatly different from the direction of the subsequent

stress probe; for example a specimen pre-stressed axially and followed by a hoop tension.

At the end of each group of tests such as pre-stress in the axial direction ($\sigma_{oz} = 38.5 \text{ T/IN}^2$) followed by $\sigma_\theta - \sigma_z$ stress probes the flow locus has been shown. This locus - which is always checked and confirmed by the three methods, a) back-extrapolation of ^{the} stress-strain curve, b) normality rule, and c) back-extrapolation of stress-creep coefficient curve - represents the onset of plastic deformation. For the combined tension-torsion tests the Williams and Svensson's theory (1970) has been used as a basis of comparison. The general form of this anisotropic yield criterion was derived in Chapter 2 of this thesis and the application to tension-torsion tests has been shown in section 2 of the present chapter.

Rogan and Shelton (1969b) have shown a number of $\bar{\sigma}$ loci as well as the unique flow locus for each pre-stress; as they point out these $\bar{\sigma}$ loci are equivalent stress loci and not flow loci. They were traced out from the normality rule before or after the onset of the plastic flow of the material, without checking by the back-extrapolation of stress-strain and/or stress-creep coefficient curves which give the unique flow locus. The directions of the plastic strain increment vectors mostly depend on the degree of pre-strain of the test-pieces and these vectors would apparently completely rotate to the isotropic direction if the subsequent stress increases sufficiently. This author believes that the equivalent stress

loci are not accurate if the only way of plotting them is the use of the normality rule. To draw an experimental curve there is ^aneed to check the locus by some other methods. On the flow locus of each stress system the direction and magnitude of each stress ratio is shown. The direction of the plastic strain increment vectors and their rotation are shown on the stress plaths. All plastic strain increment vectors are the principal plastic strain directions; for the cases where the direction of the principal stress axes did not coincide with the principal strain axes the plastic strain increment vectors are not shown on the principal stress planes.

It should be noted here that on all of the graphs the axial stress σ_z is shown on the horizontal or x axis which is also that for the σ_1 principal stress; the hoop stress σ_θ is shown on the vertical or Yaxis which is also that for the σ_2 principal stress; shear stress τ is always on the y axis and for $\tau - \sigma_\theta$ bi-axial stress case, σ_θ is on the x axis.

7.4 ^{Subsequent} Effect Of ~~Applied~~ Pre-stress Upon ~~Final~~ Tests

Two tensile pre-stresses were performed, one of 38.5 T/IN² which produced a plastic pre-strain in the axial direction of $\epsilon_z^P = 2.6\%$, and one of 40.5 T/IN² giving an axial plastic pre-strain $\epsilon_z^P = 3.45\%$. A new annealed specimen was used for each experiment and most of the tests were duplicated and a few of them triplicated.

7.4.1 Axial pre-stress of 38.5 T/IN²

The three stress systems for the subsequent tests on these test-pieces were: axial tension - hoop tension, axial tension-torsion, and hoop tension-torsion. Owing to the design of the testing machine and also because of the friction difficulties involved, no compression tests were performed.

a) - Axial tension - hoop tension

The five subsequent σ_{θ}/σ_z ratios were ∞ , 2, 1, $\frac{1}{2}$, and 0 and most of the tests were duplicated. The experimental results are shown graphically in figs. 58 to 61; these results were analysed in the same way as for the annealed material. In all of the figures the value of pre-stress is shown and it should be noted that the whole load was removed from the test-piece after pre-stressing and the test-piece then reloaded in the subsequent stress ratio.

From the graphs of ϵ_{θ}^P v. ϵ_z^P the directions of the plastic strain increment vectors were obtained; the development of the plastic strain increment vector with respect to time and with respect to increasing stress-level is also shown on the graphs.

The yield or flow locus for this stress system is shown in fig. 62. In the case of $\sigma_{\theta}/\sigma_z = 0$ the creep strain started at high stresses which means work-hardening of the material in the pre-stress direction, but it started from a very low stress level at $\sigma_{\theta}/\sigma_z = \infty$ which means work-softening in the direction normal to the pre-stress.

One remarkable point which supports this work strongly

is the subsequent test on $\sigma_\theta/\sigma_z = \infty$ stress probe preceded by axial pre-stress. The σ_θ v. ϵ_θ and σ_θ v. $\dot{\epsilon}_\theta t$ plots are so curved that there was only a very small linear relationship or even none as shown in a repeat of the test; it means that using the limit of proportionality gives a zero yield stress in that direction which is not supported by the normality rule. In this pre-stress the distortion of the plastic strain increment vector was greatest when the stress probe was $\sigma_\theta/\sigma_z = \infty$ and smallest for $\sigma_\theta/\sigma_z = 0$. In all of the stress probes the effect of pre-stress decreased when the stress level increased and plastic strains approached the annealed or non-pre-stress specimen conditions causing the $\Delta\epsilon_\theta^p/\Delta\epsilon_z^p$ vector to rotate towards the isotropic direction.

The subsequent yield stresses are shown on the stress-strain and stress-creep-coefficient curves by back-extrapolation. The stress v. creep-coefficient graphs were plotted as follows: from the stress-strain graphs of σ_θ v. ϵ_θ and σ_z v. ϵ_z the graph of ϵ_θ^p v. ϵ_z^p was drawn by taking off the elastic components from total strains; the development of plastic strain increment vectors with time was determined in the intervals of 10, 30, 60, 120, 240, 480, and 900 seconds and the graphs of ϵ_θ^p v. $\log t$ and ϵ_z^p v. $\log t$ at each stress level was drawn; from this the creep-coefficients related to the stresses were calculated, and finally the stress v. creep-coefficient graphs were plotted.

In fig. 62 the yield locus and the directions of the plastic strain increment vectors are shown. This direction is very helpful for the determination of the tangent to the yield

locus. It can also be seen that the flow stress in a test of $\sigma_\theta/\sigma_z = \infty$ is lower than the one for the pre-stress direction, the whole locus showing that an axial pre-stress has moved the annealed yield locus to the pre-stress direction and rotated it slightly.

b) Axial tension-torsion

Five stress probes were used in this system i.e. $\tau/\sigma_z = \infty, 2, 1, \frac{1}{2}$, and 0, as shown in figs. 63 to 66. In this stress system similar results to those in the $\sigma_\theta - \sigma_z$ stress field were taken. From the yield locus of fig. 67 the cross effect due to the axial tension pre-stress can be seen clearly. The yield locus is obviously decreasing in magnitude when the stress ratio approaches the shear stress axis and this differs from the results of Rogan and Shelton (1969b) in that the value of the yield stress in shear diminishes slightly from the annealed state as the axial pre-stress increases. This might possibly be due to the slight difference between the compositions of En 24 as used in the two investigations and also because of the translation of the yield locus towards the pre-stress direction. Considering the function $\bar{\sigma} = \sqrt{\sigma_z^2 + 3.5\tau^2}$ for the annealed material, a small translation of the locus towards σ_z pre-stress decreases the yield stress on the τ axis and this is the same effect observed in the $\sigma_\theta - \sigma_z$ stress system.

Theoretical values for the yield locus, based on the Williams-Svensson theory, have been plotted. Remarkably close agreement has clearly been established.

c) Hoop tension-torsion

The same five stress ratios of $\tau/\sigma_\theta = \infty, 2, 1, \frac{1}{2}$ and 0 have been used on the pre-stressed specimens figs. 68 to 70, while at least two of them have always been repeated. The two necessary ones are the stress ratios which are common to the other stress systems, i.e. $\tau/\sigma_\theta = \infty$ and $\tau/\sigma_z = \infty$. These are in fact the same test and the only difference was the procedure of the test to measure ϵ_θ in the one case and ϵ_z in the other for the plastic shear strain v. plastic tensile strain (axial or hoop) graphs. One interesting point which can be extrapolated from the yield locus fig. 71 is that due to the transverse plastic pre-strain ϵ_θ^p due to the Poisson's ratio effect. The hoop stress is decreased from 34.5 T/IN^2 for the annealed material to about 32.5 T/IN^2 and if the locus could be extended into the hoop compression-torsion region the reverse hoop yield stress would be less than the forward one because of the shape of the locus. This observation is a logical result from the cross effect of the transverse plastic strain.

d) - The whole yield locus for the first axial pre-stress

The yield locus for the three stress systems is shown on the principal stress axes, fig. 72. The σ_θ - σ_z stress field is the same as the σ_2 - σ_1 section but the τ - σ_θ and τ - σ_z stress fields are completely different since they have to be converted to principal stress form. The shear yield stresses for the two pure shear tests in the 2nd and 4th quadrants are equal (16.5 T/IN^2) in the 2nd quadrant the curve goes towards 32.5 T/IN^2 but in the

4th quadrant the locus goes to 38.5 T/IN^2 , i.e. the pre-stress. This comparison between these two quadrants shows the deviation between the principal stress and strain axes; for subsequent tests on the $\sigma_2/\sigma_1 = 0$ stress path the two sets of axes coincide while on the $\sigma_2/\sigma_1 = \infty$ stress probe they do not.

7.4.2 Axial pre-stress of 40.5 T/IN^2

Two axial pre-stressed tests were chosen to be performed on the specimens in order to investigate the effect of further pre-straining on the material. Again the three stress systems of $\tau - \sigma_\theta$, $\sigma_\theta - \sigma_z$, and $\tau - \sigma_z$ were considered and the yield loci for the three systems are drawn. The rotation of the plastic strain increment vector is shown on each graph and the detailed graphs to show the behaviour of this pre-stress are plotted in the same way as the first pre-stress. Each of the stress systems have been dealt with separately and the three loci after being converted to the principal stress basis have been put together and explained at the end of the subsection.

a) - Axial tension - hoop tension

The graphs for this stress system are shown in figs. 73 to 77; five stress probes $\sigma_\theta/\sigma_z = \sigma_2/\sigma_1$ of 0, 0.5, 1, 2, and ∞ for the subsequent tests were accomplished. From the rotation of the plastic strain increment vector it can be seen

that the material compared with the first axial pre-stress of 38.5 T/IN^2 is now further work-hardened in the direction of pre-stress and further work-softened in the direction normal to it. With the $\sigma_2/\sigma_1 = \infty$ stress probe the plastic strain starts from a very low hoop stress while in the case of the stress probes approaching the pre-stress direction the plastic deformation starts at much higher stresses. The yield locus for this stress system fig. 78 shows that the locus is further translated towards the pre-stress direction and that further rotation about the origin and towards the pre-stress direction has occurred.

b) Hoop tension-torsion

In this series of tests three of the tests were repeated in order to compare the results with the axial tension-torsion tests. All of the specimens were fully annealed and pre-stressed in the axial direction according to the diagrams plotted for each pre-stress operation and then the subsequent tests were performed. Results are plotted in figs. 79 to 81. The plastic strains of hoop and shear v. $\log t$ show a good agreement with logarithmic creep; but all of the three repeated tests were in fact due to the creep-coefficient not being satisfactory in the first experiments. From the yield locus fig. 82 it is clear that the hoop stress has decreased more than that in the first axial pre-stress; the effect of the locus in turning down when coming near to the τ axis shows that the reverse or compressive hoop stress would be less than the

tensile hoop stress and this means that the pre-stress in the axial direction has caused a Bauschinger effect in the hoop direction. This confirms the results for the first axial pre-stress, but in this case the transverse hoop stress is even lower than in the first axial pre-stress. The maximum shear yield stress on the locus is in the tensile hoop stress quadrant and the whole locus has reduced in size especially along the tensile hoop axis.

c) - Axial tension - torsion

The experimental results for the stress probes of $\tau/\sigma_z = \infty, 2, 1, 0.5$, and 0 are given in figs. 83 to 86 and the locus for this system is plotted in fig. 87. Logarithmic plastic strain was obeyed by both shear and axial tension plastic strains and in all of the tests the backward extrapolation resulting from stress v. strain and stress v. creep-coefficient were in agreement with the normality rule. The yield stress for the pre-stress direction was known and following the normality rule the whole curve could be plotted and the graph checked by the other two methods. From the yield locus fig. 87 it is clear that the higher the pre-stress in the axial direction the lower the shear stress; this means that the whole locus for the annealed material has been translated towards the pre-stress and it has been rotated so that the maximum shear yield stress on the locus is smaller than the one for annealed material. This maximum shear stress has been translated into the tensile stress quadrant which means that the axial compressive stress - although not measured - would be lower than the axial pre-stress

i.e. the Bauschinger effect.

Very good agreement between experiment and the anisotropic yield theory of Williams and Svensson is shown from the plotting of the theoretical points.

d) The whole yield locus for the second axial pre-stress

The three yield loci for the hoop tension-torsion axial tension-hoop tension, and axial tension-torsion stress systems have been converted to the principal stress field as σ_1 in the direction of σ_z and σ_2 in the direction of σ_θ , where possible. The whole locus fig. 88 again confirms the translation of the locus in the direction of the pre-stress and rotation towards the same direction. The shear stresses in the positive or negative directions are the same and this is for the same reason as mentioned earlier for the first pre-stress. From the graph it is almost clear that the effect of translation towards the pre-stress is more marked than the rotation about the origin. The total plastic pre-strain is given ^{at} the top left-hand side as $\epsilon_{oz}^p = 3.45\%$ and the pre-stress by $\sigma_{oz} = 40.5 \text{ T/IN}^2$. The direction of the plastic strain increment vectors for each stress level on the stress probe are shown and as previously the direction of the vectors for the negative ratios of $\sigma_2:\sigma_1$ have not been drawn. The expansions of the Maxwell-Mises, Tresca, and initial yield loci are traced on the same axes to show how far removed are the isotropic expansions compared with experiment. The effect of rotation of the experimental locus is also shown quite clearly.

7.4.3 Hoop pre-stress of 38.5 T/IN²

In order to compare the results of axial tension with hoop tension it was decided to use the same pre-stress for both directions. Although there is a small difference between the two plastic pre-strains, $\epsilon_{oz}^p = 2.61$ per cent while $\epsilon_{o\theta}^p = 2.34$ per cent, the results are almost the same for the two pre-stresses. The same three subsequent combined stress systems were observed in the two cases. In the case of the subsequent test in the axial direction the method of back extrapolation is much more inexact because the curves of σ_z vs. ϵ_z and σ_z vs. $\dot{\epsilon}_z t$ are rounded. But by measuring and making use of the plastic strain increment vectors, and by following through the stress space from those results which, given an unequivocal flow stress, a realistic subsequent flow locus could be drawn.

a) Hoop tension-torsion

The results of this stress system are shown in figs. 89 to 92 and the yield locus in fig. 93. Five stress ratios were followed using the same values as in the axial pre-stress tests. The results show that the shear stress is decreased to almost the same amount as in that test, and the locus is translated and rotated towards the pre-stress. In all of the tests logarithmic creep has been followed throughout. Using the three theories of Maxwell-Mises, Tresca, and the isotropic expansion of the initial yield locus it is quite clear that none of them are valid for the pre-stressed material, especially that

part of the locus which is rotated about the pre-stress and resulting in a decrease of the shear stress. The point which is rather interesting in fig. 93 is that the maximum shear stress on the experimental locus is equal to the maximum shear stress on the isotropic expansion of the initial yield locus but is translated in the direction of the pre-stress. The shape of the locus predicts a lower hoop stress for compression, or negative tension, than in tension which illustrates the Bauschinger effect due to pre-stressing.

The direction of the plastic strain increment vector for the subsequent test in pure torsion shows that the material has been work-hardened in the direction of the pre-stress and this effect decreases as the stress level increases. This shows that when the stress becomes large the behaviour of the material tends towards the isotropic state.

Again very close agreement has been shown to exist between experiment and the Williams-Svensson anisotropic yield criterion.

b) Axial tension - hoop tension

The results for five stress probes of $\sigma_\theta/\sigma_z = \infty, 2, 0.98, 0.49, \text{ and } 0$ are shown in figs. 94 to 96. These results are very similar to those for the axial pre-stress tests followed by the same subsequent loading provided that the σ_1 and σ_2 axes are interchanged. Again logarithmic creep strains for the axial and hoop directions are followed so that ϵ_z^p and ϵ_θ^p v. $\log t$ are straight lines for different stress levels and the creep-coefficients are calculated from these graphs. On the yield

locus fig. 97 it is clear that the material has been work-softened in the direction normal to the pre-stress, i.e. σ_1 or σ_z , but work-hardened in the pre-stress direction, so that the material started to deform plastically at a very low σ_1 or axial stress when $\sigma_\theta/\sigma_z = 0$ but at a high stress when the subsequent test was in the σ_2 or hoop direction.

The yield locus also shows that if the yield locus were extended to the negative side of the $\sigma_2 = \sigma_\theta$ axis, the compressive stress would be less than the tensile pre-stresses i.e. the Bauschinger effect would be shown.

c) - Axial tension-torsion

The results of this stress system are shown in figs. 98 to 101, the yield locus being in fig. 102. Results are rather similar to those for the material pre-stressed in the axial direction. No attempt has been made to carry out tests in the compression stress systems but from the yield locus it is evident that the compressive yield stress in the axial direction would be less than that for tension. This point has been discussed for the axial pre-stress followed by hoop tension and torsion but the other point which is of interest here is that the pre-stress direction is normal to the plane of fig. 102, and it is clear that the pre-stress has decreased all the stresses normal to itself - both shear and tensile - on the plane shown.

d) - The whole yield locus for 38.5 T/IN² hoop pre-stress

The yield locus for this pre-stress on the principal axes of stress is shown in fig. 103, where the direction of the plastic strain increment vectors and their rotation on that plane is also shown. In this figure, the torsion shear yield stresses for the 2nd and 4th quadrants are the same while the tensile hoop yield stress is higher than the tensile yield stress in the axial direction. This is due to the rotation of the principal plastic strain axes with respect to the principal stress axes for the case of tension and torsion combined.

Also, when the stress path moves away from the axis of pre-stress the plastic deformation appears increasing for the same

$\bar{\sigma} = \sqrt{\sigma^2 + 3.5\tau^2}$ and this effect would be the greatest when the subsequent test is in the reverse direction to the pre-stress i.e. the Bauschinger effect would then be shown.

7.4.4 Hoop pre-stress of 40.5 T/IN²

This pre-stress was performed to compare the results with the axial pre-stress of the same value, but in this case only the axial tension-hoop tension tests were carried out, due to the limit of the number of test-pieces and the scope of the work.

Results for tension-internal pressure tests are shown in figs. 104 to 107 and the yield locus in fig. 108. In all of the tests the logarithmic creep behaviour was obeyed by both the axial and the hoop plastic strains. In all of the cases the rotation of the plastic strain increment vector shows

work-hardening of the material in the direction of pre-stress and work-softening in the other principal stress directions.

The yield locus is similar to that for the second axial pre-stress, provided the axes are interchanged.

7.4.5 Pre-stress in torsion upto 19.8 T/IN²

The pre-stress operation has been plotted for every single test to show that all of the stresses were taken off before any subsequent test. Three subsequent sets of tests were fulfilled following this pre-stress:

- a) hoop stress - shear stress;
- b) hoop tension - axial tension;
- c) axial tension - shear stress.

In the first and third stress systems the negative or reverse shear stress was also introduced in order to show the Bauschinger effect on the specimens.

a) Hoop tension-torsion

There were nine tests to do in this stress system to study the creep and flow behaviour for this pre-stress. The results are shown in figs. 109 to 115 and the yield locus in fig. 116. The effect of pre-stress in the τ direction is to decrease the yield stress ~~in~~ⁱⁿ hoop tension and also to show a strong Bauschinger effect in reverse torsion. The isotropic locus has been translated towards the pre-stress direction and has contracted in the hoop direction; this is different from Ivey's (1961) results in which he reported no cross effect in

shear pre-strain. From the subsequent test in the hoop direction and rotation of the plastic strain increment vector the decrease of the hoop yield stress is evident; the maximum hoop yield at point c on the yield locus also shows that there is a cross effect. There is no rotation of the plastic strain increment vector during the tests in forward or reverse subsequent torsion.

The Bauschinger effect in the reverse torsion is more prominent than if it were done in simple tension followed by compression; this is due to the effect of the principal stresses during the pre-stressing operation and reversing so that both of the tension and compression principal stresses in the subsequent test are reversed.

The theoretical yield points from the Williams-Svensson theory are shown and again good agreement has been established. This shows that the theory is capable of predicting the Bauschinger effect.

b) - Axial tension-torsion

A total of nine tests are reported for this stress system and are shown in figs. ~~117~~¹¹⁸ to 125. These tests are in positive and negative shear for the subsequent tests and the results are very similar to the hoop tension-torsion stress system. In fig. 126 the yield locus for the forward and reverse shear stress is presented and it can be seen that the gradient of the locus near the τ -axis is normal to it. This shows that the locus is symmetrical with this axis i.e. the shear pre-stress does not introduce a discrepancy between axial

tension and compression. Yet again very good agreement is shown between the Williams-Svensson theory and the experimental results.

c) Axial tension-hoop tension

There were five stress probes to be carried out here and these are shown in figs. 127 to 129 and the yield locus in fig. 130. It should be noted again that logarithmic creep was observed in all of the tests and the creep-coefficient v. stress graph could therefore be drawn. The plane of pre-stress was different from the plane of axial v. hoop stress axes, so it was necessary to show the pre-stress direction on a separate diagram on the graphs. If considering the τ -axis normal to the plane of σ_1 - σ_2 , σ_1 is shown on the principal stress axes instead of σ_z , and σ_2 instead of σ_θ where possible. The yield locus shows that pre-stress in the τ -direction expands the locus in that direction and contracts it in the directions normal to it. This result is again contradicting the isotropic hardening of the material and moreover reveals work-softening instead. On the $\sigma_2/\sigma_1 = \sigma_\theta/\sigma_z = 1$ stress probe the direction of the plastic strain increment vector at low stress levels is rather towards the σ_2 , or σ_θ , axis and gradually rotates till it coincides with the stress path and lies at a normal to the yield locus. Almost the same behaviour occurs with the $\sigma_2/\sigma_1 = 2$ probe, in which case the vector lies initially towards the σ_2 axis and then rotates slowly with increasing stress until it becomes parallel to the σ_2 , or hoop axis; while in the case of $\sigma_2/\sigma_1 = 0.5$ the vector is initially

parallel to the σ_1 stress axis and remains almost parallel throughout the test.

For the same reason as explained earlier, the direction of the plastic strain increment vector is not shown on ~~the~~ figure¹³¹ in the 2nd and 4th quadrants. In each of the two quadrants two curves are plotted, one being related to forward or +ve. shear and the other one to -ve shear. Thus the yield points marked $\odot$ belong to one locus, the ones marked + belong to the other, and the points shown $\oplus$ are common to the two loci.

7.5 Discussion on The Effects of A Pre-Stress Upon Subsequent Behaviour

The results from experiment regarding the effect of pre-stress upon the subsequent flow loci are shown in figs. 132 and 133 where the initial yield locus is also plotted for comparison. The isotropic expansion of the initial experimental yield locus and also the Maxwell-Mises, and Tresca criteria were shown in figs. 88, 93, 108 and 132. where the results of pre-stress were discussed briefly.

In fig. 132 the initial yield locus, the loci for pre-stresses in shear upto 19.8 T/IN^2 , in axial tension upto 38.5 and 40.5 T/IN^2 , and in hoop tension upto 38.5 and 40.5 T/IN^2 are plotted on the $\sigma_z - \sigma_\theta$ axes where these two are also $\sigma_1 - \sigma_2$, the principal stress axes. In this stress field a pre-stress in shear produces a contraction of the initial yield locus in the $\sigma_\theta - \sigma_z$ plane. A pre-stress in the axial direction translates the initial yield locus in the direction

of pre-stress and rotates it so that the yield stress in the direction normal to the pre-stress is decreased. Furthermore, the higher the pre-stress the greater is the effect of work-softening in the transverse direction.

The same effects happen when the pre-stress is in the σ_2 or " σ_θ " direction. The results are very similar to each other.

Ivey (1961) reported no cross effect under pre-stress in torsion; Mair and Pugh (1964), and Rogan and Shelton (1969) said that: "A pre-stress causes both hardening and softening of the materials, the hardening extends over those two quadrants of stress space which include the pre-stress axis and slightly into the two quadrants which do not contain the pre-stress: increasing the pre-stress produces correspondingly smaller increases in hardening, except of course in the immediate neighbourhood of actual pre-stress direction."

In this work the effect of the pre-stress, even in the quadrants which include the pre-stress axis, is to lower the yield stress except in the actual direction of the pre-stress.

For the initial ^{tension-torsion} yield locus $\bar{\sigma} = (\sigma^2 + 3.5\tau^2)^{\frac{1}{2}}$, where σ can be σ_z or σ_θ , $\bar{\sigma}$ will be a circle when plotted in the axes 3.5τ vs. σ .

This type of diagram is shown in fig. 133 with circles corresponding to:-

- a) initial yield locus;
- b) isotropic expansion to axial pre-stress in tension upto 38.5 T/IN^2 .

c) isotropic expansion to axial pre-stress in tension upto 40.5 T/IN^2 .

d) isotropic expansion of shear pre-stress upto 19.8 T/IN^2 .

Also, the experimental flow loci for each of the above pre-stresses are drawn.

Comparison between the experimental results and the isotropic expansion of the initial yield locus for different types of pre-stress show that a pre-stress causes both hardening and softening of the material. The hardening extends over the stress field near to the axis of pre-stress. Increasing the pre-stress produces a further increase in the stress field adjacent to the pre-stress direction and increases the softening effect in the directions normal to the axis of pre-strain.

The anisotropic yield criterion of Williams and Svensson has been shown to give remarkably close agreement with experiment under combined tension-torsion for both the circumferential and axial tensile pre-strains and also for torsion pre-strain. Other cases were examined such as circumferential tensile pre-strain followed by combined axial tension-torsion and axial tensile pre-strain followed by combined circumferential tension-torsion and also all the combined axial tension - internal pressure tests for all pre-strains. The same close agreement was not established for these latter cases and further investigation is therefore ~~required.~~
~~required.~~

CHAPTER 8

GENERAL DISCUSSION OF RESULTS, CONCLUSIONS AND SUGGESTIONS FOR FURTHER WORK

8.1 Introduction

In this chapter the initial yield locus under axial tension-torsion, hoop tension-torsion, and axial tension - hoop tension has been discussed separately for each stress field and then in general when plotted on the principal stress plane. The behaviour of the annealed material beyond the initial yield is then considered and also the experimental check on the basic laws of plasticity.

The main part of this chapter is related to the effect of pre-stress upon the subsequent yield and flow. The gradual translation and rotation of the initial yield locus with increasing pre-stress, to form a new yield locus is discussed and this locus is then compared with that predicted by the anisotropic yield criterion of Williams and Svensson.

The chapter ends with a summary of conclusions and suggestions for possible future work.

8.2 The Initial Yield Locus

The initial yield locus for the annealed En 24 steel under combined tension-torsion is given by $\bar{\sigma} = (\sigma^2 + 3.5\tau^2)^{\frac{1}{2}}$, which shows the yield stress and the direction of the resulting strain-increment vectors almost exactly. The experimental

locus thus lies exactly mid-way between the Maxwell-Mises and Tresca criteria in this stress field.

The results under combined tension-internal pressure however show close agreement with the Maxwell-Mises criterion by $\bar{\sigma} = (\sigma_z^2 + \sigma_\theta^2 - \sigma_z \sigma_\theta)^{\frac{1}{2}}$ where σ_z is the axial stress and σ_θ the hoop stress. These are principal stresses and the criterion could therefore be written $\bar{\sigma} = (\sigma_1^2 + \sigma_2^2 - \sigma_1 \sigma_2)^{\frac{1}{2}}$. Transforming the combined tension-torsion results into principal stresses gives $\bar{\sigma} = (\sigma_1^2 + \sigma_2^2 - 1.5 \sigma_1 \sigma_2)^{\frac{1}{2}}$.

The two equivalent stress functions show that the two loci are not the same, and the discrepancy, in the view of this author, is probably due to a size effect. The ^{testing} ~~tests~~ of a material in any combination of stresses as many other authors have stated, is partly a test of the test-piece and partly of the stress state. In this work the thinnest tubular test-pieces - type "A" of 0.018 inch thick - were used for the $\sigma_z - \sigma_\theta$ tests and the medium thick type "B" of 0.027 inch or the thickest ones "type C" of 0.037 inch were used for the $\sigma_\theta - \tau$ ^{and} ~~or~~ $\sigma_z - \tau$ tests.

Results for the $\sigma_\theta - \tau$ and $\sigma_z - \tau$ stress fields on the annealed material show that the two yield loci are consistent, i.e. the effect of σ_θ on the specimen is the same as the effect of σ_z . It was also noticed that the yield locus for the $\sigma_\theta - \sigma_z$ stress system was symmetrical with respect to the $\sigma_\theta = \sigma_z$ line.

8.3 Post-Yield Behaviour of the Annealed Steel

As far as this author is aware this work is the first comprehensive investigation upon a material under continued loading past the initial yield surface in 180 degrees of principal stress plane.

For the behaviour of the material beyond yielding, the work hypothesis did not produce the same plastic strain in tension as in torsion. It was found that the function of stress and strain which made equivalent the stress-strain curves obtained from an axial tension and a torsion test were not sufficient to correlate the component stress-strain curves resulting from tests in combined axial tension-torsion. This is the effect of the interaction of the combined stresses so that they affect the magnitudes of the resulting plastic strain vectors, but have no influence upon the directions of the vectors. Rogan and Shelton (1969A) introduce a $\cos \alpha$ factor where α is the angle between the stress vector and the strain vector and this enabled axial tension, torsion and combined axial tension-torsion tests to be correlated over the whole axial tension-torsion stress system in their results and also in these. This result was also in agreement with the present hoop tension-torsion stress results. Rogan (1966) explains that the "vector-angle" effect means that a given increment of equivalent stress produces a larger increment of strain when the stress and strain increment vectors are not coincident than is produced when the vectors are coincident or, in other words, the work done in producing a given value of strain is less when the vectors are

not coincident than ^{when} they are. The equivalent plastic strain was found to be $\bar{\epsilon}^p = \sqrt{\epsilon_z^{p2} + (\gamma^p/1.36)^2}$, where ϵ^p may be ϵ_z^p or ϵ_θ^p and the $\bar{\epsilon}^p$ v. $\bar{\sigma}$ curves for tension-torsion agreed well when the ~~the~~ two vectors coincided, but not when the angle between them increased. When the equivalent strain in this stress system is multiplied by the cosine of the angle between the stress and strain vectors, the equivalent stress v. equivalent strain curves coincided for the whole range of both sets of tension-torsion results, i.e. both axial and hoop tension.

The same results can be seen when the combined tension and internal pressure stress system is under consideration and the angle between stress and strain vectors again plays an important role. Thus, in order to coincide $\bar{\sigma} = (\sigma_z^2 + \sigma_\theta^2 - \sigma_1\sigma_2)^{1/2}$ for $\sigma_\theta/\sigma_z = 1$ - where the plastic strain vector and the stress path coincide - with other stress probes, multiplying the equivalent strains by $\cos \alpha$ gives a good correlation.

It has been noticed that within the range of this work the direction of the plastic strain increment vector was always normal to the ~~equivalent stress-strain curves~~ ^{initial yield locus} and the room temperature creep for simple tension, torsion, and combined tension-torsion tests was a log. time function.

Davis and Parker (1948) and Fraenkel (1948) tested tubular specimens of mild steel in combined axial tension - hoop tension and obtained similar results to this work. By multiplying $\bar{\epsilon}$ by a $\cos \alpha$ factor to bring the other curves of $\bar{\sigma}$ vs. $\bar{\epsilon}$ into coincidence with those from the $\sigma_\theta/\sigma_z = 1$ stress probe, they would also have obtained complete correlation.

Mellor (1956) in his report on the study of stretch - forming of copper and a 70:30 brass under fluid pressure compared the balanced biaxial tension results, where $\sigma_x/\sigma_y = 1$, with the simple tension case, where $\sigma_x/\sigma_y = \infty$, on the basis of a Maxwell-Mises $\bar{\sigma}$ and $\bar{\epsilon}$; for the two materials, the curve for $\sigma_x/\sigma_y = 1$, where $\cos \alpha = 1$, was considerably higher than that for $\sigma_x/\sigma_y = \infty$, where $\cos \alpha = 0.89$.

Finally, as mentioned before, Rogan and Shelton (1969A) testing En 24 and En 25, obtained different values of $\bar{\sigma}$ and $\bar{\epsilon}$ for $\tau/\sigma = \infty$ and $\tau/\sigma = 0$ for the two metals and concluded that to correlate tension-torsion results with tension or torsion they needed to introduce $\cos \alpha$ and multiply the equivalent strain by this factor.

Before introducing any parameter to correlate $\bar{\sigma}$ v. $\bar{\epsilon}$ for different stress systems it should be noted that equivalent stress must be well defined. For annealed materials having a sharp yield it is not difficult to define the equivalent stress $\bar{\sigma}$ but in the case of a non-sharp yielding material the best way of defining $\bar{\sigma}$ is that which was used in this work for subsequent tests following a pre-stress: this method was to calculate the plastic strain increments in two directions and determine the direction of the plastic strain increment vector which is normal to the $\bar{\sigma}$ curve i.e. to use the normality rule.

The creep behaviour of the material under tension-torsion was found to be slightly different from Rogan and Shelton (1969A) and this is due to the small difference in the percentage composition of the two En 24 steels used in the two

investigations. The present results gave $\dot{\epsilon}_t = 3.73 \times 10^{-9} \sinh(\sigma/3.20 + 0.00015)$

whilst Rogan and Shelton found

$$\dot{\epsilon}_t = 3.09 \times 10^{-9} \sinh(\bar{\sigma}/3.55) + 0.00025.$$

8.4 Experimental Check on the Laws Of Plasticity

Using the three methods of backward extrapolation of stress v. strain and creep coefficient v. stress and also the normality rule to obtain the yield locus for the material, whether annealed or pre-stressed, was very satisfactory. For the annealed material the results agreed very well with the basic laws of plasticity.

The normality rule in all of the stress systems on the annealed material was obeyed clearly. The convexity of the yield locus and a perfectly smooth curve concave to the origin meant that the linearity of differentials assumption is fully justified so that the direction of the plastic strain increment vector was fixed when performing radial loading tests, i.e. radial stress paths produce radial plastic strain paths.

In the range of the tests the creep strain obeyed the laws of plasticity perfectly; so that $\dot{\gamma}^p/\dot{\epsilon}^p$, $\Delta\gamma^p/\Delta\epsilon^p$, $\dot{\gamma}/\dot{\epsilon}$ in the tension-torsion stress system, whether the tension was axial or hoop, were equal to $\epsilon_\theta^p/\epsilon_z^p$, $\Delta\epsilon_\theta^p/\Delta\epsilon_z^p$, and $\dot{\epsilon}_\theta/\dot{\epsilon}_z$ were also equal, provided that large creep was occurring and this creep has to be analysed at the same time as the "long-time" strains.

For the annealed material in all of the stress systems the relation $\mu = \nu$ was also seen to be obeyed and thus the plastic

potential is smooth with no corners.

When principal stresses such as σ_θ or σ_2 and σ_z or σ_1 were applied on the annealed material the principal strains were in the direction of the stresses applied, but for the case of combined tension-torsion where shear stresses were applied as well as tension the directions of the principal strains were turning slightly during the test. This slight rotation was due to the strains of the specimen upto that stress level. The results were plotted both in tension-torsion and principal stress fields.

The yield locus of the material under investigation for the two stress systems of tension-torsion and tension-tension did not agree. As stated before this was mainly due to the size effect of the specimens.

8.5 The Effect Of Pre-stress Upon The Subsequent Creep And Flow Behaviour

The effect of pre-stress on a material, which yields sharply in the annealed state, is firstly that it eliminates the yield point so the work changes towards the subsequent flow behaviour, which is of course a function of time.

The second effect on the results lies in the definition of yield point which should be defined so that the material could stay deforming plastically if the load is maintained.

The limit of proportionality definition of flow stress which is used very little in practice depends upon: 1) the sensitivity of the strain measuring equipment; 2) the type of unloading after the pre-stress operation, and 3) the subjective judgement of the investigator who analyses the results Hsu (1966D), Szczepinski (1963), and Rogan and Shelton (1969B) have discussed the difficulty of using this definition.

Proof stresses based upon a proof plastic strain which is entirely dependent on the magnitude of the chosen proof strain is another definition, c.f. Iagn and Shishmarev (1958), Szczepinski (1963), and Williams and Svensson (1970). The drawback of using proof strain is in introducing an equivalent strain which must be found from the flow locus not something which is assumed in order to find the flow locus. This demerits the definition of proof strain.

The most satisfactory definition of the yield stress which is therefore used in this work is the backward extrapolation from the graph of stress v. strain obtained in the subsequent test. The point with this definition which should be considered when plotting the graph of stress v. strain is the creep of the material under investigation. This is due to the strain time-dependence of the flow which affects other definitions of yield much more. Fortunately, the analysis of the creep showed that the curves of creep-coefficient v. stress are independent of time and so cancel out this problem. Using the two other theories to fulfil the requirements to plot the yield locus eliminates the subjective judgement of the person

performing the back-extrapolation; these two are 1) the normality rule that defines the yield locus as normal to the resulting plastic strain increment vector at that point; 2) the yield stress must be a reasonable back-extrapolation of the creep-coefficient v. stress curves and of the stress v. strain "long-time" curves.

In addition to the above theories there is a third point which almost eliminates the effect of subjectiveness of the results. This is the consistency of the yield stress with those on neighbouring stress probes, i.e. the yield locus must pass through the pre-stress where the axis of the pre-stress is in the stress-plane of the subsequent tests. The back-extrapolation definition was always consistent and in the few cases where the stress v. "long-time" strain curves were very rounded the normality rule gave the exact direction of the locus.

The shape of the flow loci found for different types of pre-stress have been discussed earlier in detail and show both hardening and softening of the material. Rogan and Shelton (1969B) reported that the effect of pre-stress in one direction is to harden the material over those two quadrants of stress space which include the pre-stress axis and that increasing the pre-stress causes smaller increases in hardening. The results of the present work show that there is work-hardening in the direction of the pre-stress and also in the directions in the neighbourhood of it, but work-softening in the directions approaching the transverse directions whether shear or tensile.

In summarising it should be noted that the effect of work-hardening or work-softening of the material in the transverse direction depends upon the behaviour of the material under investigation. If the material shows a Bauschinger effect and non-work-hardening then there must be a work-softening in the transverse direction of the pre-stress. For a material which shows a Bauschinger effect and work-hardening in the direction of pre-stress, the effect of the pre-stress in the transverse direction depends upon the pre-strain and the rate of the work-hardening.

In figs. 88, 93, 108, 132, and 133 the effects of pre-stress in any direction have been compared with the isotropic expansions of the initial yield, Tresca and Maxwell-Mises loci. It is quite clear from the graphs that any pre-stress translates the initial yield locus in the direction of that stress and rotates the locus about the pre-stress point so that it decreases the transverse yield stress.

The anisotropic yield criterion of Williams and Svensson has been used in relation to some of the tests on pre-strained material. This theory is based on fundamental material properties and is more general than earlier theories in that it makes provision for the Bauschinger effect and does not assume an anisotropic material to be homogeneous. In this last respect, the theory arrives at a solution for the stress state in the element of a body and then uses advanced statistical theory (as explained in Chapter 2) to obtain the most probable value of the least value of local plastic strain existing in an

element (i.e. an inclusion) to determine the yield behaviour of the aggregate.

In application, points on the yield locus must be determined by experiment but this is not surprising since even isotropic theories require at least one point (i.e. the yield stress) to be determined and the theory is more than just a case of curve fitting. The results show remarkably close agreement with experiment over the whole stress range for the specific cases shown. It is not clear why this close agreement was not obtained for all stress fields but it may be that the axial tension-internal pressure results - which disagreed the most strongly - were influenced by the size effect of the test-pieces. Further work on the influence of size of test-pieces on the yield locus must be undertaken before definite conclusions can be drawn.

8.6 Conclusions

The main conclusions for this work may be summarised as follows:

- (1) Under combined axial tension-torsion and also hoop tension-torsion, the annealed En 24 steel obeyed the yield criteria $\bar{\sigma} = \sqrt{\sigma^2 + 3.5\tau^2}$ which is midway between the Maxwell-Mises ($\sqrt{\sigma^2 + 3\tau^2}$) and Tresca ($\sqrt{\sigma^2 + 4\tau^2}$) criteria. Under combined axial tension - hoop tension the material obeyed the Maxwell-Mises criterion $\bar{\sigma} = \sqrt{\sigma_\theta^2 - \sigma_\theta\sigma_2 + \sigma_z^2}$. It is believed that the two different yield criteria experienced was partly due to the difference in the shearing process involved in the

different tests and partly due to the different size of test-piece used.

- (2) The post-yield behaviour showed marked room temperature creep of the logarithmic form $\epsilon = a \ln t + C$, by all strain components under the different stress systems. The plastic strain and strain rate ratios were normal to the appropriate yield locus and remained so ~~for~~^{for} increasing stress.
- (3) The equivalent plastic work hypothesis did not produce satisfactory correlation of the combined stress results on the annealed material. Correlation in the form of a single curve of equivalent stress v. equivalent plastic strain was established empirically for all stress ratio paths in a given field by first establishing the equivalent plastic strain for the cases where the stress and strain paths coincided and then using a factor $\cos \alpha$ for the remaining cases - i.e. where the two paths were not coincident, α being the angle between them.
- (4) Good correlation was established for the creep strain rates in a similar way to that for the plastic strain. This finally led to ^{a correlation of} the complete stress-strain-time behaviour at room temperature.
- (5) The basic laws of plasticity theory have been verified. The yield locus was convex, smooth and the plastic strain increment vectors (and also the strain rate vectors) were normal to it. Lode's relation $\mu = \nu$ was satisfied, thereby

confirming the Levy-Mises stress-strain relations.

- (6) The effect of a pre-stress was firstly to eliminate the initial sudden yield exhibited by the annealed material. It was considered that the most satisfactory definition of yield stress was that obtained from the backward extrapolation method. Three methods were used to define each yield point, i.e. backward extrapolation of stress v. strain and creep coefficient v. stress and also the normality rule. This procedure was found to be very helpful in defining the yield stress for well-rounded curves such as was obtained from reversed torsion tests following a torsion pre-strain.
- (7) Creep was exhibited by all strain components following pre-strain and in general this was again of logarithmic form. The plastic strain increment vectors were initially in a different direction to that shown by the corresponding test on the annealed material and remained constant in direction under time for a given stress. An increase in stress caused an instant rotation of the plastic strain increment vector and this effect continued for further increases, the rotation being towards the isotropic direction.
- (8) The effect of a pre-strain on the initial yield locus is to translate it towards - and to incorporate - the pre-stress value. The material is thus rendered anisotropic and is hardened in the pre-strain direction. For tension-torsion

tests plotted in tension-torsion stress space the new yield locus is symmetrical about the pre-strain axis and no rotation occurs. The transverse stress shows marked work-softening and by implication from the tension pre-stresses and by measurement from the torsion pre-stress, a marked Bauschinger effect is present. The same results when plotted on principal stress space also show a marked rotation of the flow locus. Similar remarks apply to the combined axial tension-internal pressure results which were also plotted on principal stress space.

- (9) The Williams-Svensson anisotropic yield criterion was used for a theoretical comparison with the experimental results. Remarkably close agreement existed for those tension-torsion results where the pre-stress was in the same plane as the yield locus. For other cases such as a circumferential pre-stress followed by combined axial stress-torsion and also the axial tension - internal pressure tests the agreement was erratic and unconfirmable. Further work is recommended to establish the cause of the disagreement in the latter tests.

8.7 Further Work

In a research of this scope it is inevitable that a ~~single~~ ^{single} investigation will not cover all aspects of the problem. A number of queries arose during the course of the investigation and in the writing of this thesis which could lead to further work, and some suggestions are now offered.

- 1) Repeat the investigation on the compression side of the principal stress plane, in order to establish whether or not a unique function covers the whole principal stress field in the compression side. At the same time determine the Bauschinger and allied anisotropic effects by pre-stressing the material in another direction - say axial tension - followed by subsequent testing in the new stress field. ~~Finally~~ ^{Also}, the time-dependence of the flow should be correlated in a way similar to this work.
- 2) Using single sized specimens throughout the whole stress space, find out the correlation between $\sigma_\theta - \sigma_z$ and $\sigma_z - \tau$ and $\sigma_\theta - \tau$ stress systems both for tensile and compressive σ_z and σ_θ . The present work showed different yield loci for tension-torsion compared with tension-internal pressure and such tests should show if there has been a size effect.

Throughout the whole investigation use three sets of transducers to measure ϵ_z , ϵ_θ , and γ in order to calculate the principal plastic strains and their directions relative to the principal stress axes. Then calculate μ and ν , and analyse their relations for different kinds of pre-stress.

- (3) Perform the same investigation as (1) and (2) but use a material which strain ages; pre-stress the material, then age it before subsequent combined-stress testing. Compare the results with the previous investigations.

- (4) Carry out an investigation of the mechanisms of yielding, specially on materials that exhibit sharp yielding. Investigate the difference between the yield in simple tension, compression and pure shear and compare the results to see whether they are consistent or not. Different sized test-pieces should also be considered.
- (5) Investigate the limit of logarithmic creep behaviour. All of the pre-stress tests exhibited creep of this form on subsequent testing ^{as} ~~or~~ did the tests on the annealed material. The pre-strains were never more than about $3\frac{1}{2}$ per cent however and this work has not established the limiting strain beyond which subsequent strains will cease to be logarithmic.
- (6) Investigate the Williams-Svensson anisotropic yield theory further. It has not been established why this theory should agree remarkably closely with the experimental results in some stress fields ^{but} ~~and~~ fail to do so in others.

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APPENDIX A

NORMALITY RULE FOR A WORK-HARDENING MATERIAL*

Before discussing this, it is necessary to discuss the matter for an ideal plastic material.

A) From the definition for ideal plasticity, the property can be formulated as:

$$d\sigma_{ij} d\epsilon_{ij}^p = 0$$

and $f(\sigma_{ij}) \leq k$ with $d\epsilon_{ij}^p \neq 0$ only if $f(\sigma_{ij}) = k$ and $df = 0$.

$$\text{Also } d\epsilon_{ij}^p = 0 \text{ if } f(\sigma_{ij}) < k \text{ or } df < 0 \quad (1)$$

Comparing

$$df = \frac{\partial f}{\partial \sigma_{ij}} d\sigma_{ij} = 0$$

$$\text{and } d\sigma_{ij} d\epsilon_{ij}^p = 0. \quad (2)$$

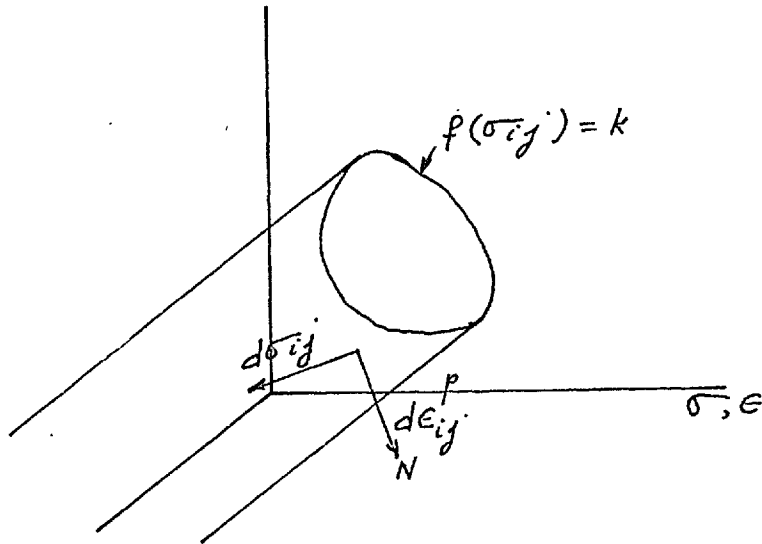
we see that

$$d\epsilon_{ij}^p = \lambda \frac{\partial f}{\partial \sigma_{ij}} \quad (3)$$

where λ is a scalar. The form of f may be as anisotropic as desired and of any degree in the components of stress. In this case f must be symmetric in the stress components.

Equation (3) was proposed originally by von Mises and extended by Prager. In the figure below the loading or yield surface is shown by $f(\sigma_{ij}) = k$ which is fixed in stress space for an ideally plastic material. If the strain axes ϵ_{ij} are

* Drucker, D.C. (1950) - *Marin, J. (1962).



superimposed on the corresponding stress axes, so that vectors also represent strain increment, the normal to the loading surface $\partial f / \partial \sigma_{ij}$ gives the direction of the plastic strain increment vector $d\epsilon_{ij}^p$. The scalar product of $d\epsilon_{ij}^p$ and any $d\sigma_{ij}$, which must lie on the surface, $d\sigma_{ij} d\epsilon_{ij}^p$, is zero, i.e. $d\epsilon_{ij}^p$ must be normal to any line in the yield surface.

B) Two assumptions were made for a work-hardening material:

1. A loading function such as $f(\sigma_{ij})$ exists such that further plastic deformation takes place only for $f(\sigma_{ij}) > k$.

2. The relation between the infinitesimals of stress and strain is linear. The sum of the increments in plastic deformation obtained separately from each of two sets of infinitesimal stress increments $d\sigma'_{kl}$ and $d\sigma''_{kl}$, neither of which constitutes unloading, is the same as the plastic deformation resulting from $d\sigma_{kl} = d\sigma'_{kl} + d\sigma''_{kl}$

$$\frac{\partial f}{\partial \sigma_{kl}} d\sigma'_{kl} \geq 0, \quad -\frac{\partial f}{\partial \sigma_{kl}} d\sigma''_{kl} \geq 0$$

The first-infinitesimal part $d\sigma'_{kl}$ can be arbitrarily chosen so that it produces no plastic strain, and $d\sigma''_{kl}$ is proportional

to the gradient f

$$\frac{\partial f}{\partial \sigma_{kl}} d\sigma'_{kl} = 0 \quad (4)$$

$$\frac{\partial f}{\partial \sigma_{kl}} d\sigma''_{kl} = \frac{\partial f}{\partial \sigma_{kl}} d\sigma_{kl} \quad d\sigma''_{kl} = r \frac{\partial f}{\partial \sigma_{kl}} \quad r > 0 \quad (5)$$

This can always be done as

$$\frac{\partial f}{\partial \sigma_{kl}} d\sigma''_{kl} = r \frac{\partial f}{\partial \sigma_{kl}} \frac{\partial f}{\partial \sigma_{kl}} > 0 \text{ and can be made equal to } \frac{\partial f}{\partial \sigma_{kl}}$$

by a proper choice of r .

$$r = \frac{\frac{\partial f}{\partial \sigma_{kl}} d\sigma_{kl}}{\frac{\partial f}{\partial \sigma_{mn}} \frac{\partial f}{\partial \sigma_{mn}}}$$

From the linearity assumption it is understood that if all $d\sigma_{kl}$ are increased in ratio, all $d\epsilon^p_{ij}$ are increased in the same proportion, so that $d\epsilon^p_{ij} \propto r$ or $d\epsilon^p_{ij} = g_{ij} \frac{\partial f}{\partial \sigma_{kl}} d\sigma_{kl}$ (6)

where the g_{ij} may depend upon stress, strain, and their history but do not involve $d\sigma_{kl}$.

From the work hardening condition:

$$d\sigma_{ij} d\epsilon^p_{ij} = d\sigma'_{ij} d\epsilon^p_{ij} + d\sigma'' d\epsilon^p_{ij}$$

If $d\sigma'_{ij}$ is chosen so that it produces no plastic strain then

$$\frac{\partial f}{\partial \sigma_{ij}} d\sigma'_{ij} = 0 \text{ and } d\sigma''_{ij} \text{ produces } d\epsilon^p_{ij}. \text{ As } d\sigma_{ij} = C d\sigma'_{ij} + d\sigma''_{ij}$$

also constitutes loading for all values of C whether positive or negative and produces the same $d\epsilon_{ij}^P$, the work hardening condition becomes

$$C d\sigma'_{ij} d\epsilon_{ij}^P + d\sigma''_{ij} d\epsilon_{ij}^P \geq 0 \quad (7)$$

$C d\sigma'_{ij} d\epsilon_{ij}^P$ is the work done on the plastic strain increments by a set of stress increments which produces no plastic strain. The magnitude of this term must be zero, for if it were not, a value of C could be chosen, of proper sign and magnitude to violate (7).

$$\text{Therefore, } d\sigma'_{ij} d\epsilon_{ij}^P = d\sigma'_{ij} g_{ij} \frac{\partial f}{\partial \sigma_{kl}} d\sigma_{kl} = 0$$

$$\text{or, } d\sigma'_{ij} g_{ij} = 0 \quad (8)$$

$$\text{where } \frac{\partial f}{\partial \sigma_{ij}} d\sigma'_{ij} = 0 \quad (9)$$

Conditions (8) must be satisfied for all $d\sigma'_{ij}$ satisfying (1) so that $g_{ij} = G \frac{\partial f}{\partial \sigma_{ij}}$ (10)

where G is a scalar which may depend upon stress, strain, and history.

Equation (9) is similar to (3) for ideal plasticity but the λ is now of definite magnitude $G \frac{\partial f}{\partial \sigma_{kl}} d\sigma_{kl}$. Here too the plastic strain increment is normal to the yield surface in stress space. Now, however, the yield surface moves about in stress space, usually changing in size, shape, and position as the plastic deformation proceeds, i.e. $f = f(\sigma_{ij}, \epsilon_{ij}^P, \text{history})$.

APPENDIX B

CONVEXITY OF YIELD SURFACE*

In figure 1, consider some state of stress σ_{ij}^* inside the yielding surface. Let

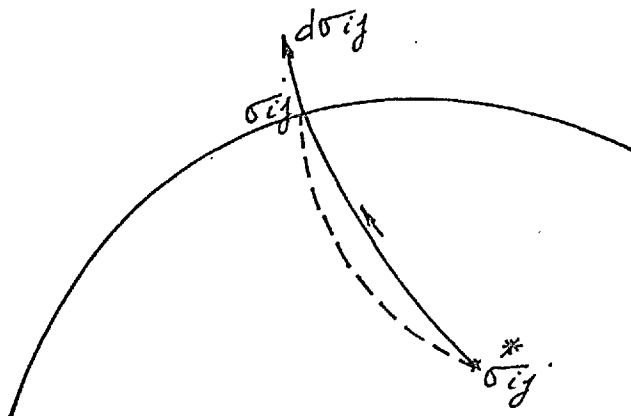


Figure 1.

some external agency add stresses along some arbitrary path inside the surface until a state of stress σ_{ij} is reached which is on the surface. Now suppose the external agency to add a very small outward pointing stress increment $d\sigma_{ij}$ which produces small plastic strain increments $d\epsilon_{ij}^p$, as well as elastic increments. The external agency then releases the $d\sigma_{ij}$ and the state of stress is returned to σ_{ij}^* along an elastic path given by the dotted line. The work done by the external agency over the cycle is

$$\delta W = (\sigma_{ij} - \sigma_{ij}^*) d\epsilon_{ij}^p + d\sigma_{ij} d\epsilon_{ij}^p \quad (1)$$

If the plastic strain co-ordinates are superimposed on the stress coordinates, as in figure 2, δW may be interpreted as the scalar product of the vector $(\sigma_{ij} - \sigma_{ij}^*)$

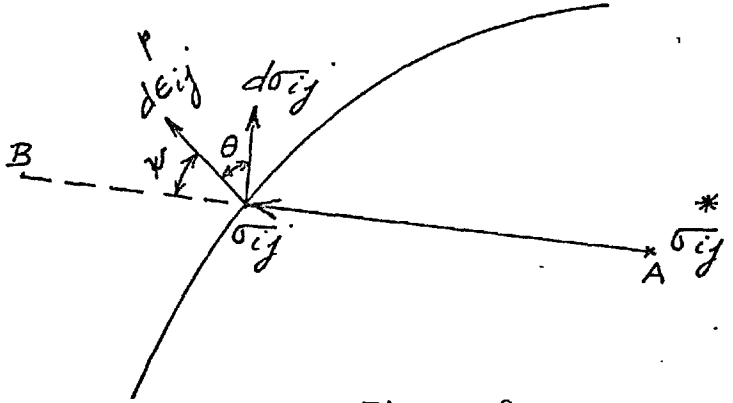


Figure 2

and the vector $d\epsilon_{ij}^p$ plus the scalar product of $d\sigma_{ij}$ and $d\epsilon_{ij}$. From the work-hardening definition quoted in Appendix A:

$$d\sigma_{ij} d\epsilon_{ij}^p \geq 0$$

$$\text{or } /d\sigma_{ij} // d\epsilon_{ij}^p / \cos \theta \geq 0 \quad (2)$$

$$\text{Hence } -\frac{\pi}{2} \leq \theta \leq \frac{\pi}{2}$$

That is, the vectors $d\sigma_{ij}$ and $d\epsilon_{ij}^p$ make an acute angle with each other. In a similar way, since the magnitude of $(\sigma_{ij} - \sigma_{ij}^*)$ can always be made larger than the magnitude of $d\sigma_{ij}$, it follows that

$$(\sigma_{ij} - \sigma_{ij}^*) d\epsilon_{ij}^p \geq 0$$

$$\text{or } /(\sigma_{ij} - \sigma_{ij}^*) // d\epsilon_{ij}^p / \cos \psi \geq 0$$

$$\text{Hence } -\frac{\pi}{2} \leq \psi \leq \frac{\pi}{2} \quad (3)$$

Thus AB, or the direction $(\sigma_{ij} - \sigma_{ij}^*)$ makes an acute angle with the vector $d\epsilon_{ij}^p$ for all choice of σ_{ij}^* . Therefore, all points

σ_{ij}^* must lie on one side of the plane normal to $d\epsilon_{ij}^p$, and since the infinitesimal plastic strain is normal to the yield surface, this plane will be tangent to the yield surface. This must be true for all points σ_{ij} on the yield surface, so that no vector $(\sigma_{ij} - \sigma_{ij}^*)$ can pass outside the surface intersecting the surface twice, as shown in Figure 3. The surface must therefore be convex

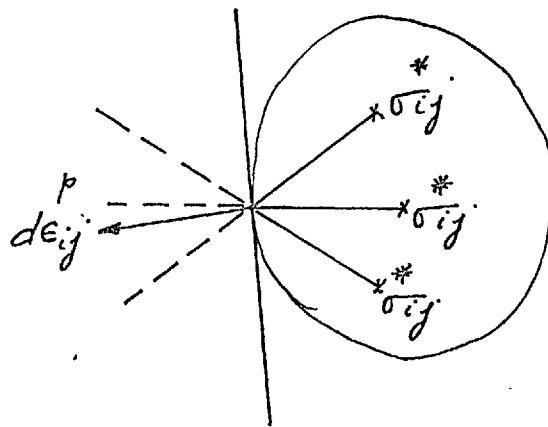


Figure 3

On the other hand, if the surface is not convex, there exist some points σ_{ij} and σ_{ij}^* such that the vector $(\sigma_{ij} - \sigma_{ij}^*)$ forms an obtuse angle with the vector $d\sigma_{ij}$, as shown in figure 4, which contradicts work-hardening theory.

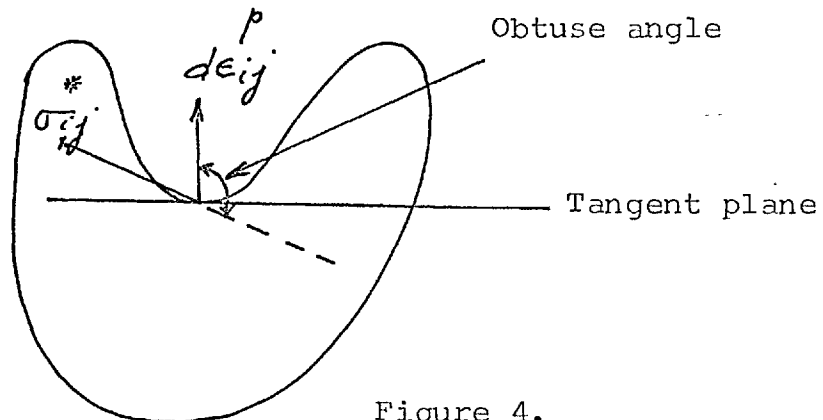


Figure 4.

* Drucker, D.C. (1950) - *Marin, J. (1962).

APPENDIX C

LODE'S PARAMETERS

Lode used the variables μ and ν to represent the stress ratio and the plastic strain-increment ratio respectively, and defined them in terms of the principal stresses and the principal incremental plastic strains as follows:

$$\mu = \frac{2\sigma_2 - \sigma_1 - \sigma_3}{\sigma_1 - \sigma_3} \quad \text{where} \quad \sigma_1 > \sigma_2 > \sigma_3$$

$$\nu = \frac{2d\epsilon_2^p - d\epsilon_1^p - d\epsilon_3^p}{d\epsilon_1^p - d\epsilon_3^p} \quad \text{where} \quad d\epsilon_1^p > d\epsilon_2^p > d\epsilon_3^p$$

a) Lode's Parameters For Combined Axial Tension And Torsion

In this case σ_{33} = axial stress $\tau_{\theta z}$ = shear stress

$$\sigma_1 \text{ and } \sigma_3 = \frac{1}{2}(\sigma_{zz} \pm \sqrt{\sigma_{zz}^2 + 4\tau_{\theta z}^2}) \quad \text{and } \sigma_2 = 0$$

$$\therefore \mu = \frac{0 - \sigma_1 - \sigma_3}{\sigma_1 - \sigma_3}$$

$$= -\frac{\sigma_{zz}}{\sqrt{\sigma_{zz}^2 + 4\tau_{\theta z}^2}}$$

$$\nu = \frac{2d\epsilon_2^p - d\epsilon_1^p - d\epsilon_3^p}{d\epsilon_1^p - d\epsilon_3^p} \quad \text{where } d\epsilon_1^p + d\epsilon_2^p + d\epsilon_3^p = 0$$

$$= \frac{-2d\epsilon_1^p - 2d\epsilon_3^p - d\epsilon_1^p - d\epsilon_1^p - d\epsilon_3^p}{d\epsilon_1^p - d\epsilon_3^p}$$

$$= -3 \times \frac{(d\epsilon_1^p + d\epsilon_3^p)}{(d\epsilon_1^p - d\epsilon_3^p)}$$

$$\text{Now } d\epsilon_1^p \text{ and } d\epsilon_3^p = \frac{1}{2} \left\{ (d\epsilon_{zz}^p + d\epsilon_{\theta\theta}^p) \pm \sqrt{(d\epsilon_{zz}^p - d\epsilon_{\theta\theta}^p)^2 + d\gamma_{\theta z}^{p^2}} \right\}$$

$$\therefore = -3 \times \frac{(d\epsilon_{zz}^p + d\epsilon_{\theta\theta}^p)}{\sqrt{(d\epsilon_{zz}^p - d\epsilon_{\theta\theta}^p)^2 + d\gamma_{\theta z}^{p^2}}}$$

b) Combined Circumferential Tension and Torsion

Proceed exactly as above but interchange suffixes θ and z where necessary.

$$\text{Thus } \mu = -\frac{\sigma_{\theta\theta}}{\sqrt{\sigma_{\theta\theta}^2 + 4\tau_{\theta z}^2}} \text{ and } \nu = \frac{-3(d\epsilon_{\theta\theta}^p + d\epsilon_{zz}^p)}{\sqrt{(d\epsilon_{\theta\theta}^p - d\epsilon_{zz}^p)^2 + d\gamma_{\theta z}^{p^2}}}$$

c) Combined Internal Pressure And Axial Tension

From thin cylinder theory $\sigma_{\theta\theta} = \frac{pd}{2t}$; $\sigma_{zz} = \frac{pd}{4t} \pm \frac{W}{A}$
if pressure is acting along axis of test-piece; $\sigma_{rr} = 0$.

Since $\sigma_{\theta\theta}$ and σ_{zz} are both tensile at all times one will be σ_1 and the other σ_2 (i.e. the intermediate stress) depending upon which is the greater, $\sigma_{rr} = \sigma_3 = 0$ at all times.

When $\sigma_{zz} \geq \sigma_{\theta\theta} > \sigma_{rr}$ (i.e. $\sigma_{zz} = \sigma_1$ and $\sigma_{\theta\theta} = \sigma_2$)

$$\mu = \frac{2\sigma_2 - \sigma_1 - \sigma_3}{\sigma_1 - \sigma_3} = \frac{2\sigma_{\theta\theta} - \sigma_{zz}}{\sigma_{zz}} = 2 \frac{\sigma_{\theta\theta}}{\sigma_{zz}} - 1$$

$$\nu = \frac{2d\epsilon_2^p - d\epsilon_1^p - d\epsilon_3^p}{d\epsilon_1^p - d\epsilon_3^p} \text{ where } d\epsilon_1^p + d\epsilon_2^p + d\epsilon_3^p = 0 \text{ and}$$

$$d\epsilon_1^p > d\epsilon_2^p > d\epsilon_3^p \quad d\epsilon_3^p = -(d\epsilon_1^p + d\epsilon_2^p)$$

$$\nu = \frac{2d\epsilon_2^p - d\epsilon_1^p + d\epsilon_1^p + d\epsilon_2^p}{d\epsilon_1^p + d\epsilon_1^p + d\epsilon_2^p}$$

$$= \frac{3d\epsilon_2^p}{2d\epsilon_1^p + d\epsilon_2^p}$$

$$\nu = \frac{3}{2\left(\frac{d\epsilon_1^p}{d\epsilon_2^p}\right) + 1}$$

APPENDIX D

EFFECT OF THE SIZE OF SPECIMEN ON THE GRADIENT OF HOOP
STRESS AND BUCKLING

The most important point in the design of the specimen is the choice of ratio $\frac{d}{t}$ where d = mean specimen diameter and t =wall thickness. This choice is affected by several conflicting factors.

(1) Circumferential stress variation

From Lamé's analysis on thick cylinders carrying internal pressure p only, the ratio of the hoop stress at the bore to that at the outside diameter in terms of d and t is given by

$$\frac{\sigma_i}{\sigma_o} = \frac{(d/t)^2 + 1}{(d/t - 1)^2}$$

d/t must therefore be large if σ_i/σ_o is to be near unity as required in thin walled tubes.

(2) Buckling

A restriction is imposed on the maximum value of $\frac{d}{t}$ by the onset of buckling when one of the principal stresses is negative, e.g. in the case of pure torsion. As far as buckling is concerned d/t should be a minimum.

On balancing the above factors it was found that for the worst case, type "C" specimen where $\frac{d}{t} \approx 22$, the variation of the hoop stress is slightly above ± 4 per cent of the mean

value and the onset of buckling is at a sufficiently large shear strain. The ratio $\frac{d}{t} \approx 30$ for type "B" and 44.4 for type "A". The latter type was used mostly for the cases where hoop stress was most important and high.

Having chosen $\frac{d}{t}$ it is necessary to decide on the absolute size of the specimen. Again there are conflicting requirements, such as:

a - the accuracy of strain measurements increases with increasing size;

b - the difficulty in manufacturing thin-walled tubes with the necessary concentricity increases as the thickness decreases,

c - for a given $\frac{d}{t}$ the tensile load required is proportional to d^2 while the torque required is proportional to d^3 ,

d - it is difficult to obtain uniform material in large diameter sizes.

Factors a and b favour a large diameter while c and d favour a small one. The choice of d is however more arbitrary than $\frac{d}{t}$

To calculate position of centroid

$$\bar{X} = OG$$

$$A_1 = \frac{\pi \times \overline{OE}^2}{4} = \text{area of the outside circle}$$

$$A_2 = \frac{\pi \times \overline{CD}^2}{4} = \text{area of the inside circle}$$

$$A = A_1 - A_2 = \text{area of the annulus}$$

Taking moment of the areas about origin

$$A \times \bar{X} = A_1 \times OA - A_2 \times OB$$

$$A_1 = 0.54884 \text{ inch}^2$$

$$A_2 = 0.50265 \text{ inch}^2$$

$$A = 0.04619 \text{ inch}^2$$

$$\bar{X} = \frac{0.54884 \times 0.418 - 0.50265 \times 0.419}{0.04619}$$

$$= \frac{0.22941 - 0.21061}{0.04619} = \frac{0.01880}{0.04619}$$

$$= 0.40701 \text{ inch.}$$

Axis of the whole body = GG OG = 0.40701 inch.

Axis of the inside circles and also axis of the pull = BB

$$OB = 0.41900 \text{ inch.}$$

Axis of the outside circle = AA OA = 0.41800 inch

Eccentricity = GB = OB - OG = 0.419 - 0.40701 = 0.01199 inch

$$GA = GB - AB = 0.01199 - 0.001 = 0.01099 \text{ inch}$$

W = axial load, concentric with the internal circle

$$A = \text{sectional area} = \frac{\pi}{4} (\overline{OE}^2 - \overline{CD}^2) = \frac{\pi}{4} (OE + CD)(OE - CD)$$

$$A = 0.04619 \text{ inch}^2$$

$$\text{Axial stress} = \frac{W}{A}$$

$$\text{Bending stress} = \pm \frac{My}{I} = \pm \frac{W \times GB \times y}{I}$$

y = OG for + sing, and GE for - sing

or

y = 0.40701 inch for + sign, and 0.42899 inch for - sing.

$$I_{GG} = I + \text{Area} \times (\text{distance between G and centre of gravity})^2$$

$$\begin{aligned} I_{GG} \text{ for outside circle} &= \frac{\pi \times \overline{OE}^4}{64} + \frac{\pi \times \overline{OE}^2}{4} \times \overline{GA}^2 \\ &= \frac{\pi \times \overline{OE}^2}{4} \left(\frac{\overline{OE}^2}{16} + \overline{GA}^2 \right) \\ &= 0.54884 (0.04368 + 0.00012) \\ &= 0.02404 \text{ inch}^4 \end{aligned}$$

$$\begin{aligned} I_{GG} \text{ for inside circle} &= \frac{\pi \times \overline{CD}^2}{4} \left(\frac{\overline{CD}^2}{16} + \overline{BG}^2 \right) \\ &= 0.50265 (0.04000 + 0.000144) \\ &= 0.02018 \text{ inch}^4 \end{aligned}$$

$$I_{GG} \text{ for the annulus} = 0.02404 - 0.02018 = 0.00386 \text{ inch}^4$$

$$\text{Maximum tensile stress occurring at } \sigma = \frac{W}{A} + \frac{My}{I}$$

$$\text{Mean tensile stress} = \frac{W}{A}$$

$$\text{Minimum tensile stress occurring at E} = \frac{W}{A} - \frac{My}{I}$$

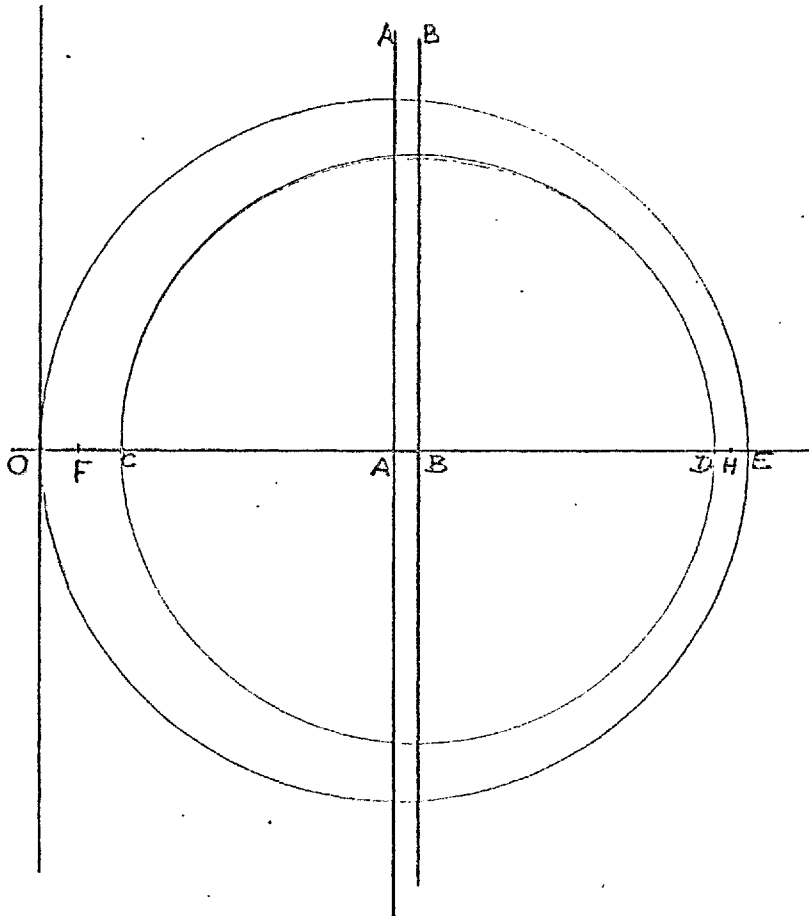
$$\begin{aligned}\text{Min \% Error} &= \frac{M_y}{I} \cdot \frac{W}{A} \times 100 = \frac{W \times G_B \times O_G \times A \times 100}{I \times W} = \frac{0.01199 \times 0.40701 \times 0.04619 \times 100}{0.00386} \\ &= 5.84\%\end{aligned}$$

$$\begin{aligned}\text{Max \% Error} &= \frac{M_y}{I} \cdot \frac{W}{A} \times 100 = \frac{W \times G_B \times G_E \times A \times 100}{I \times W} = \frac{0.01199 \times 0.42899 \times 0.04619 \times 100}{0.00386} \\ &= 6.15\%\end{aligned}$$

This means that ± 0.001 inch difference from the mean wall thickness produces upto 6.15% error in the tensile stress.

APPENDIX F

EFFECT OF ECCENTRICITY OF A THIN-WALLED TUBE ON
THE HOOP STRESS GRADIENT UNDER INTERNAL PRESSURE



Let internal pressure = p lbf/in²

A = centre of outside circle, B = the centre of inside
circle and δl = length of the cylinder .

The component force in the direction of B-B due to internal pressure p_{lb}/in^2 is

$$p \times CD \times \delta l$$

Let the tensile hoop stress in OC = σ_1 lb/in^2

Let the tensile hoop stress in DE = σ_2 lb/in^2

$$p \times CD \times \delta l (\sigma_1 \times OC \times \sigma_2 \times DE) \delta l \quad (1)$$

$\sigma_1 \times OC$ acts on F, midway between O and C

$\sigma_2 \times DE$ acts on H, midway between D and E

$$\sigma_1 \times OC \times FB = \sigma_2 \times DE \times BH$$

$$\frac{\sigma_2}{\sigma_1} = \frac{OC \times FB}{DE \times BH} \quad (2)$$

σ_1 and σ_2 can be calculated from (1) and (2)

If AB = the eccentricity of the annulus = 0.001 inch say

and OE = D = 0.836 inch

$$CD = d = 0.800 \text{ inch}$$

then, Mean thickness = $t = 0.018$ inch

$$\text{Max. thick.} = OC = 0.019 \text{ inch}$$

$$\text{Min. thick.} = DE = 0.017 \text{ inch}$$

$$\text{Mean hoop stress} = \sigma_m = \frac{p \times d}{2t} = \frac{p \times 0.8}{2 \times 0.018} = \frac{0.8p}{0.036} \text{ lb}/in^2 = 22.22 p \text{ lb}/in^2$$

$$\text{from (2)} \quad \frac{\sigma_2}{\sigma_1} = \frac{OC \times FB}{DE \times BH} = \frac{0.019(0.4 + \frac{0.019}{2})}{0.017(0.4 - \frac{0.017}{2})} = \frac{0.019 \times 0.4095}{0.017 \times 0.3915}$$

$$= 1.169$$

from (1) $p \times CD = \sigma_1 \times OC + 1.169\sigma_1 \times DE = \sigma_1(0.019 + 1.169 \times 0.17)$

$$\sigma_1 = \frac{p \times 0.800}{0.019 + 0.01987} = 20.58p \text{ lbf/in}^2$$

$$\sigma_2 = 20.58 \times 1.169 = 24.06p \text{ lbf/in}^2$$

$$\text{Max. \% Error} = \frac{\sigma_2 - \sigma_m}{\sigma_m} \times 100 = \frac{24.06 - 22.22}{22.22} \times 100 = \frac{1.84 \times 100}{22.22}$$

$$= 8.28\%$$

$$\text{Min. \% Error} = \frac{\sigma_1 - \sigma_m}{\sigma_m} \times 100 = \frac{22.22 - 20.58}{22.22} \times 100 = \frac{1.64 \times 100}{22.22}$$

$$= 7.38\%$$

In this case the error is more serious than the effect of the eccentricity of the axial load.

FIG. 1.

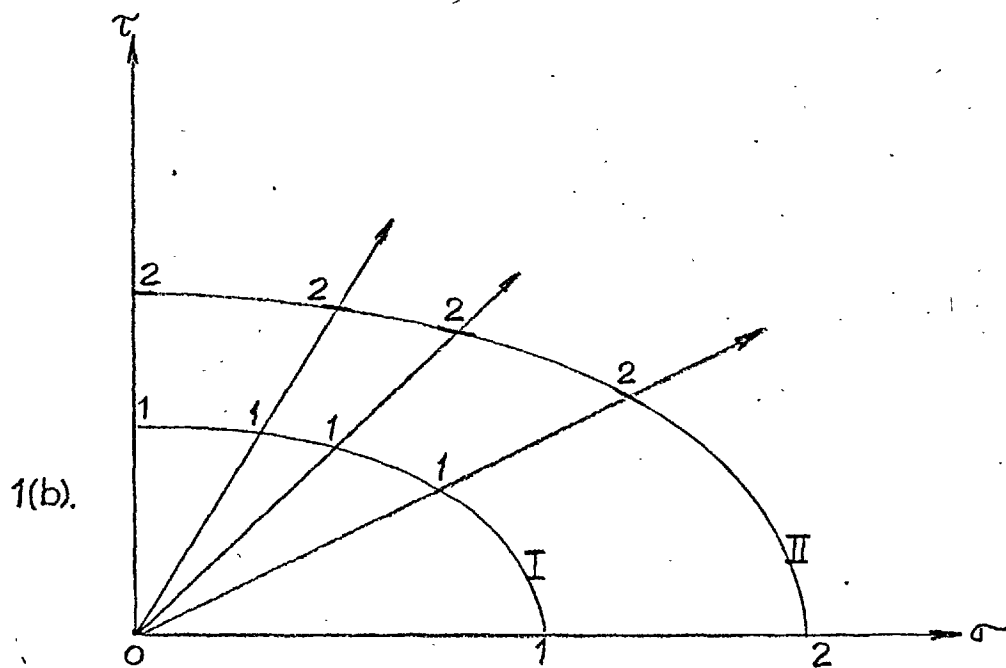
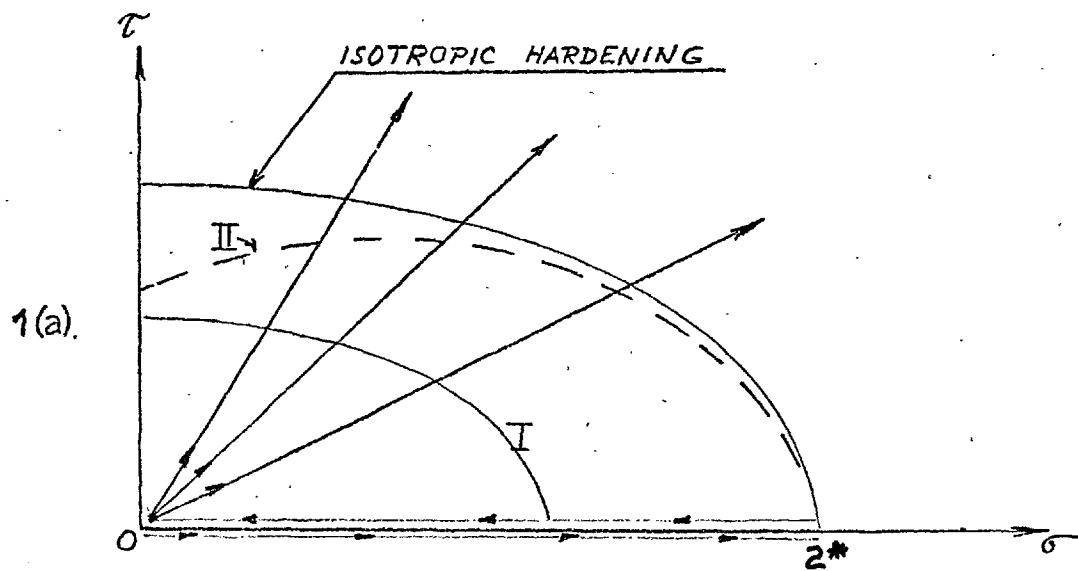


FIG. 1. ISOTROPIC HARDENING RESULTING FROM
RADIAL LOADING ON ISOTROPIC MATERIAL.

FIG. 2.

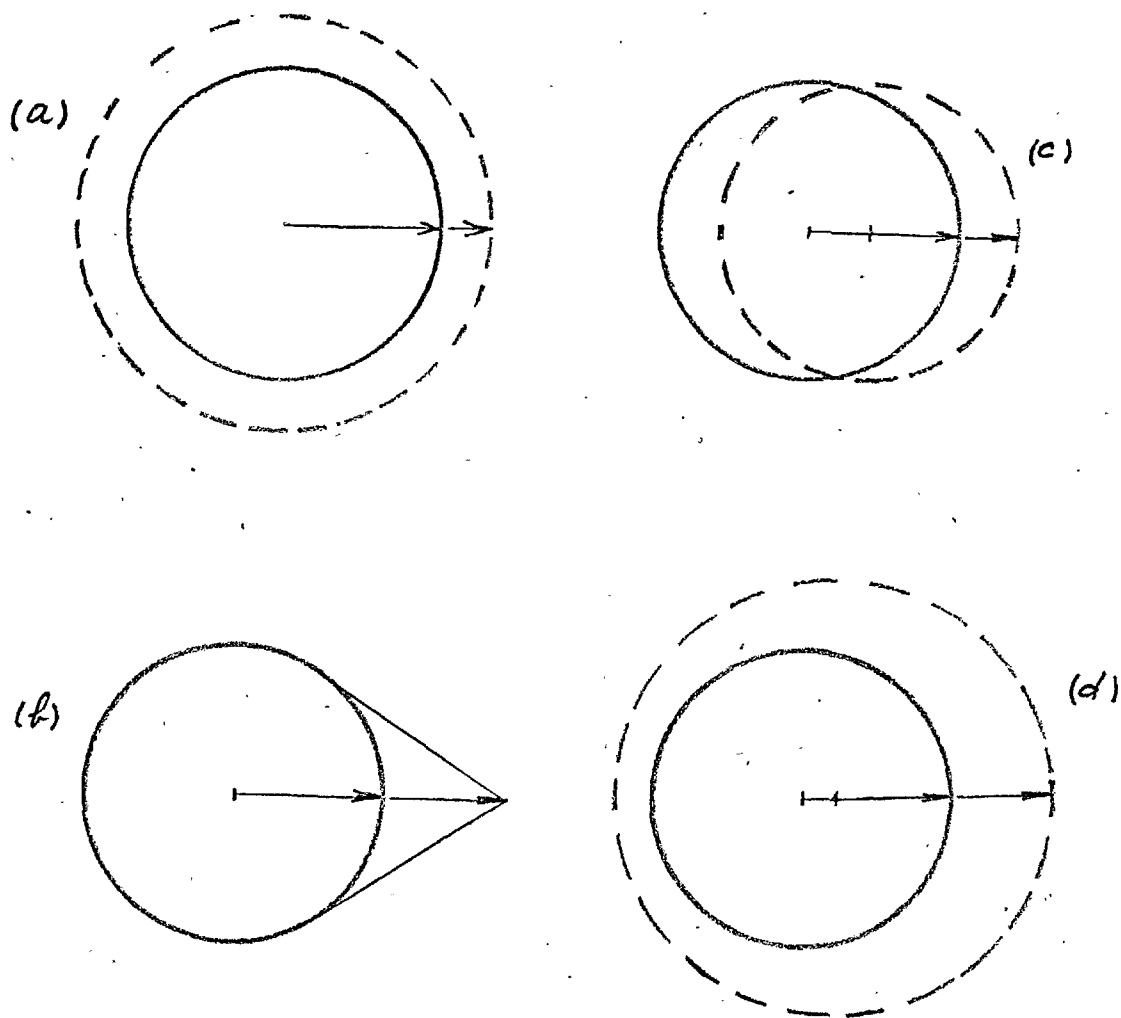


FIG. 2. COMPARISON OF HARDENING LAWS.
MAIR AND PUGH (1964).

FIG. 3

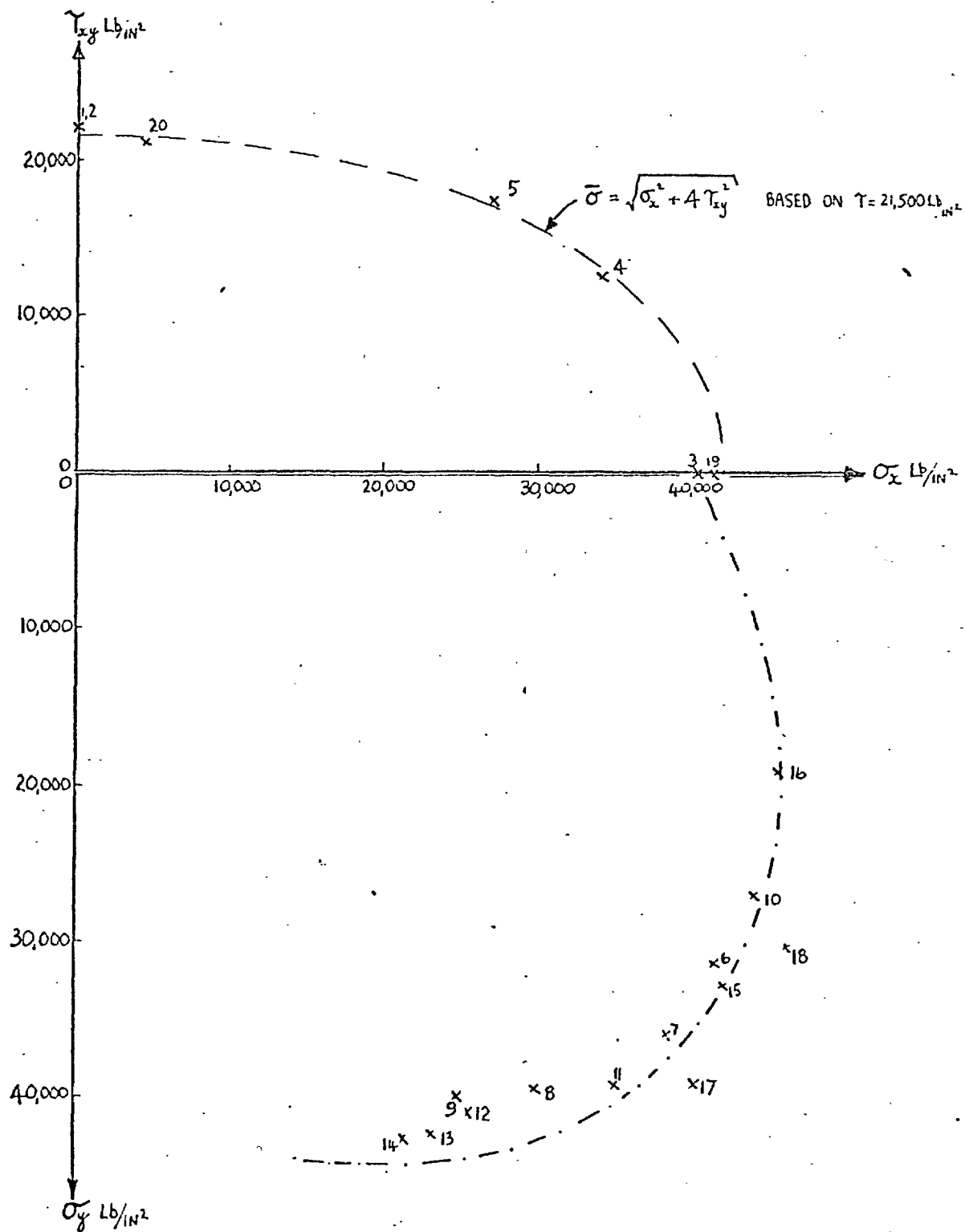


FIG. 3. Results for tube VIII, (J.J. Guest, 1900.)

FIG. 4.

G. I. TAYLOR AND H. QUINNEY ON THE PLASTIC DISTORTION OF METALS

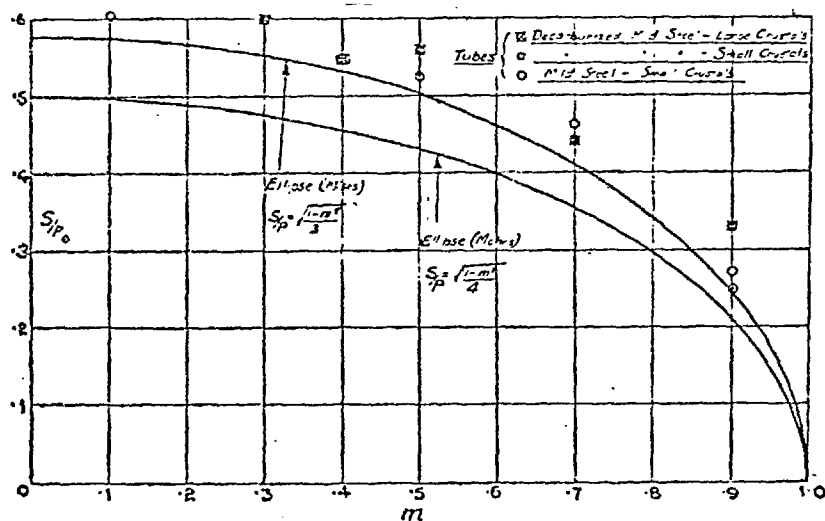
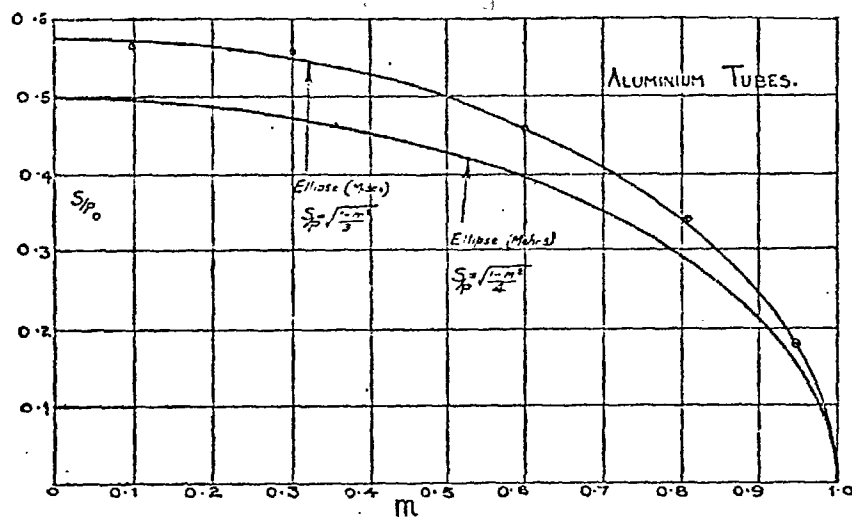
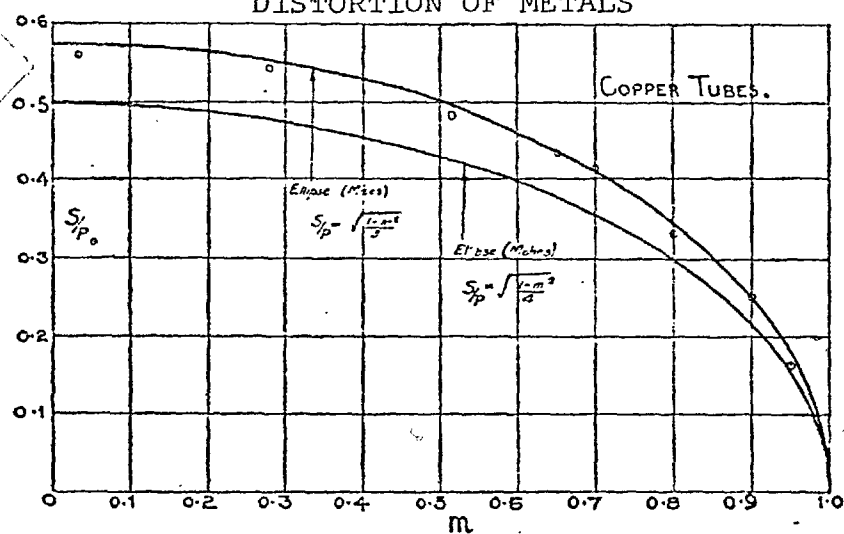


Fig. 4. Results of Taylor and Quinney (1931)

FIG. 5.

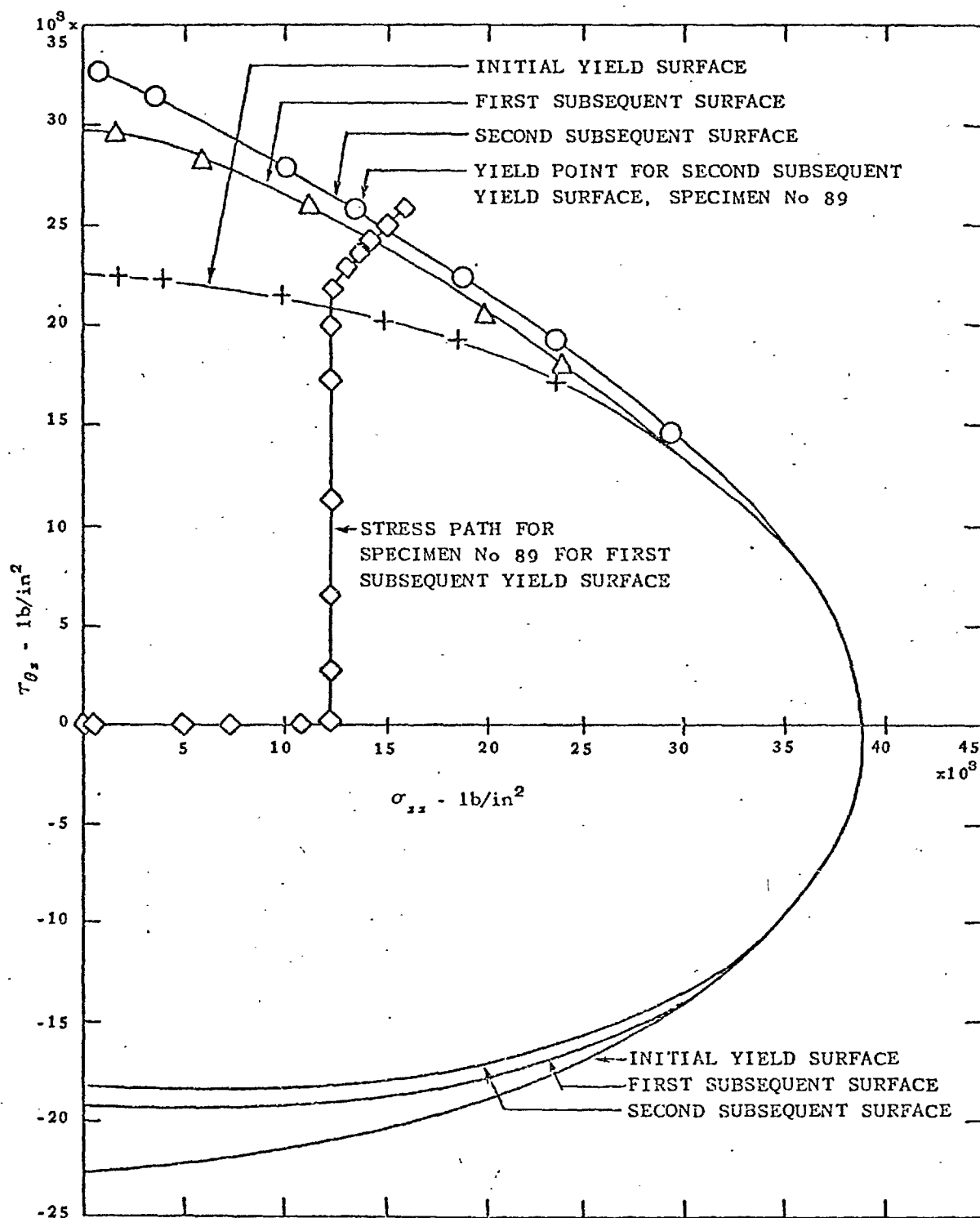
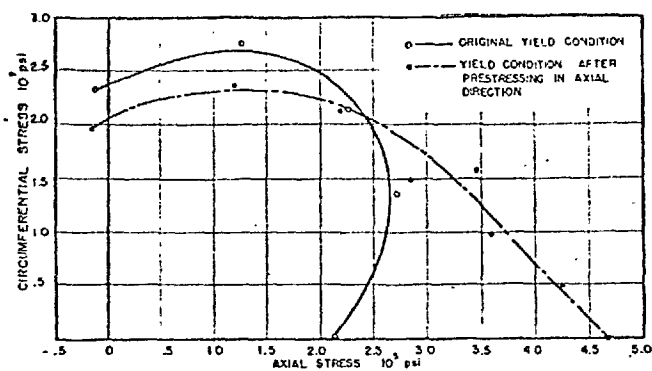


FIG 5 INITIAL AND SUBSEQUENT YIELD SURFACES
(FROM NAGHDI *et al* (1958))

FIG. 6.



Original and subsequent yield conditions

FIG. 6. RESULTS OF HU AND BRATT (1958)

FIG. 7.

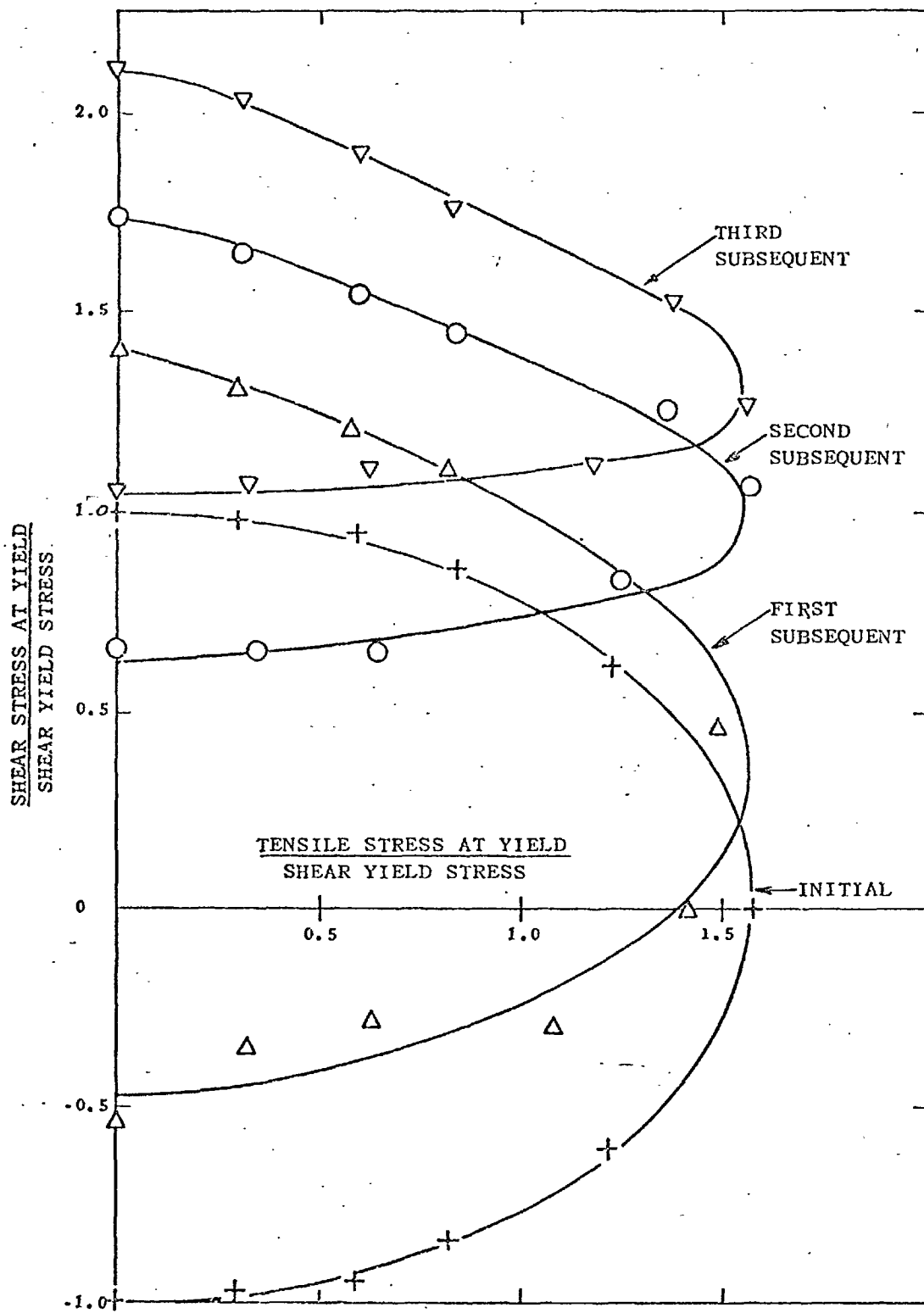


FIG 7 EFFECT OF TORSIONAL OVERSTRAIN ON SUBSEQUENT YIELD SURFACES (FROM IVEY⁽²⁾)

FIG. 8.

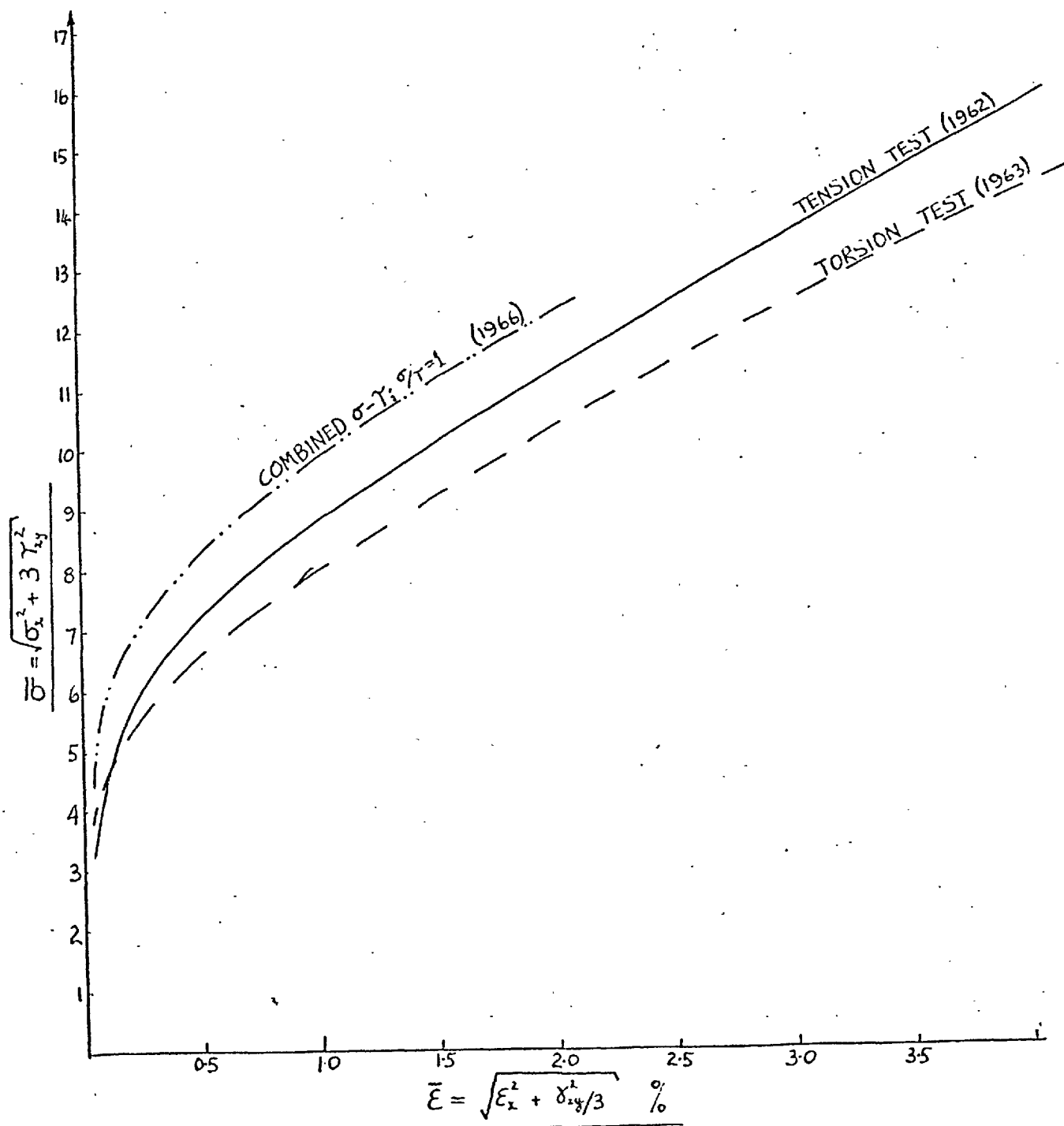


FIG. 8. CORRELATION OF RESULTS BY PUGH ET AL (1962).

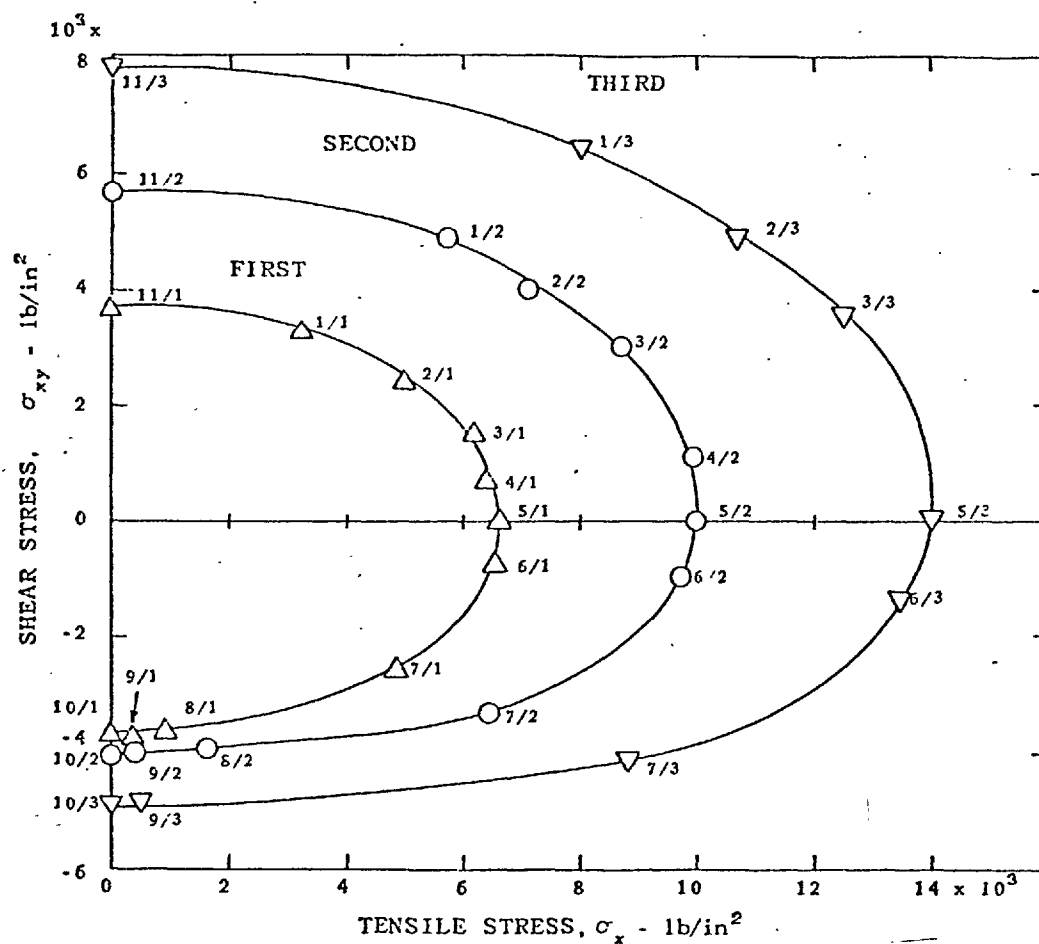
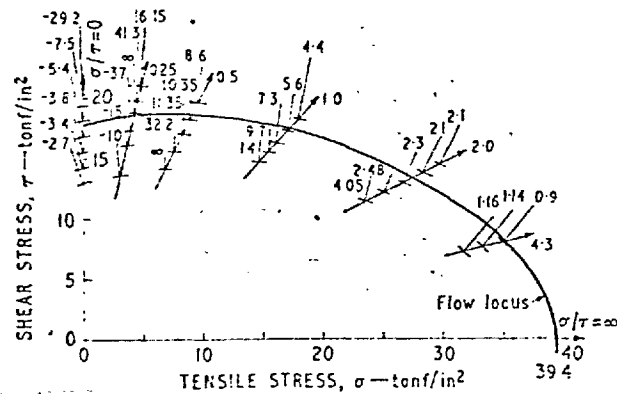


FIG. 9. Initial and Subsequent Yield Loci from Mair and Pugh (1963)

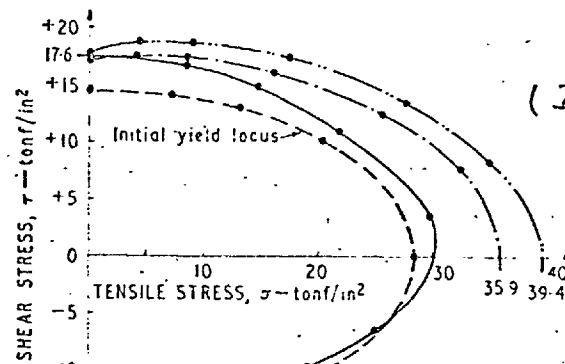
(A)



(A)

Effect of a tensile pre-stress of 39.4 tonf/in² upon direction of plastic-strain-increment vectors

(B)



(B)

Comparison of subsequent flow loci with initial yield locus

Fig. 10 Effect of Tensile Pre-stress on Direction of Strain Vectors and Subsequent Yield Loci from Rogan and Shelton (1969B).

FIG. 11.

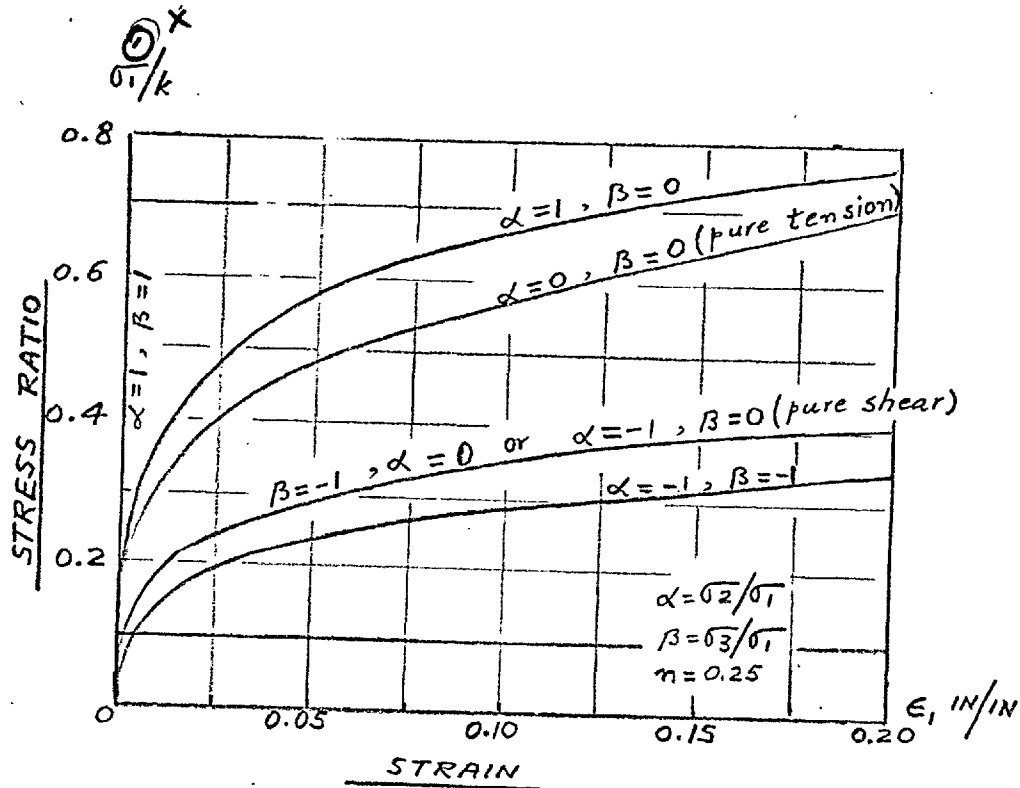


FIG. 11. PLASTIC PRINCIPAL STRESS-STRAIN RELATIONS. (MARIN 1962)

FIG. 12.

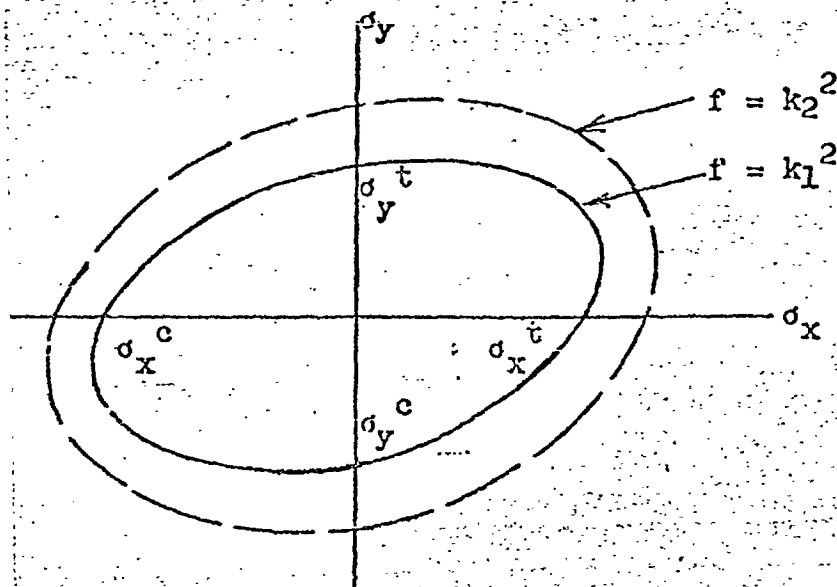


FIG. 12 . SUCCESSIVE YIELD CURVES OF THE ANISOTROPIC QUADRATIC
FORM $f = \frac{1}{2} D_{ijkl} \sigma_{ij} \sigma_{kl}$.

FIG. 13.

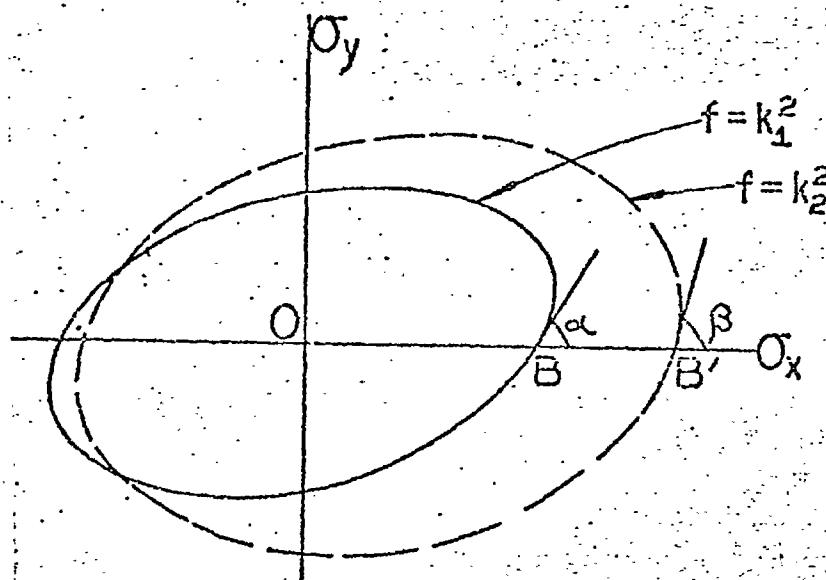


FIG. 13. Successive yield curves of $f = \frac{1}{2} c_{ijkl} (s_{ij} - m_{ij}^p) (s_{kl} - m_{kl}^p)$
in axial (σ_x) tension.

FIG. 14.

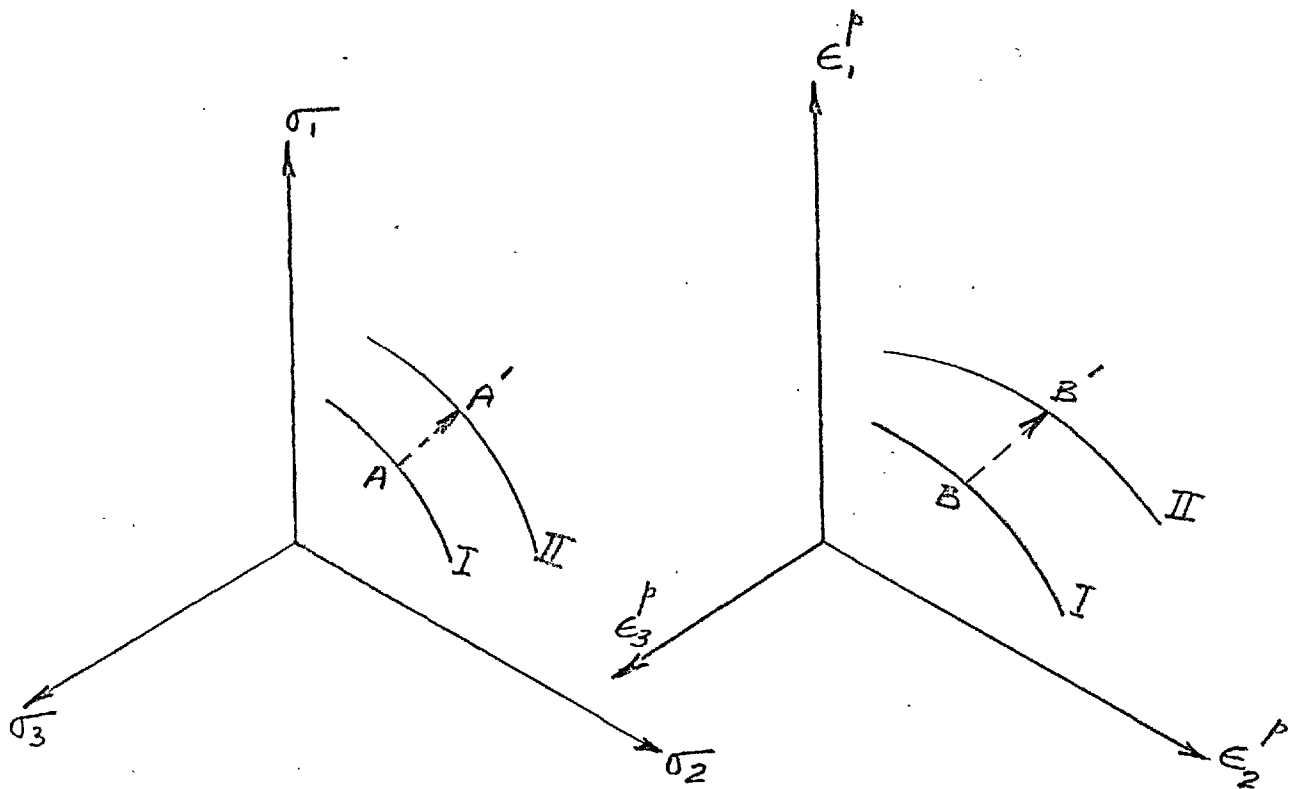


Fig.14. Comparison of an isotropic expansion of yield locus(A) and expansion of equivalent plastic strain locus(B)

EN 24 1½% Ni-Cr-Mo Steel

CHEMICAL COMPOSITION, PER CENT.

FIG. 15.

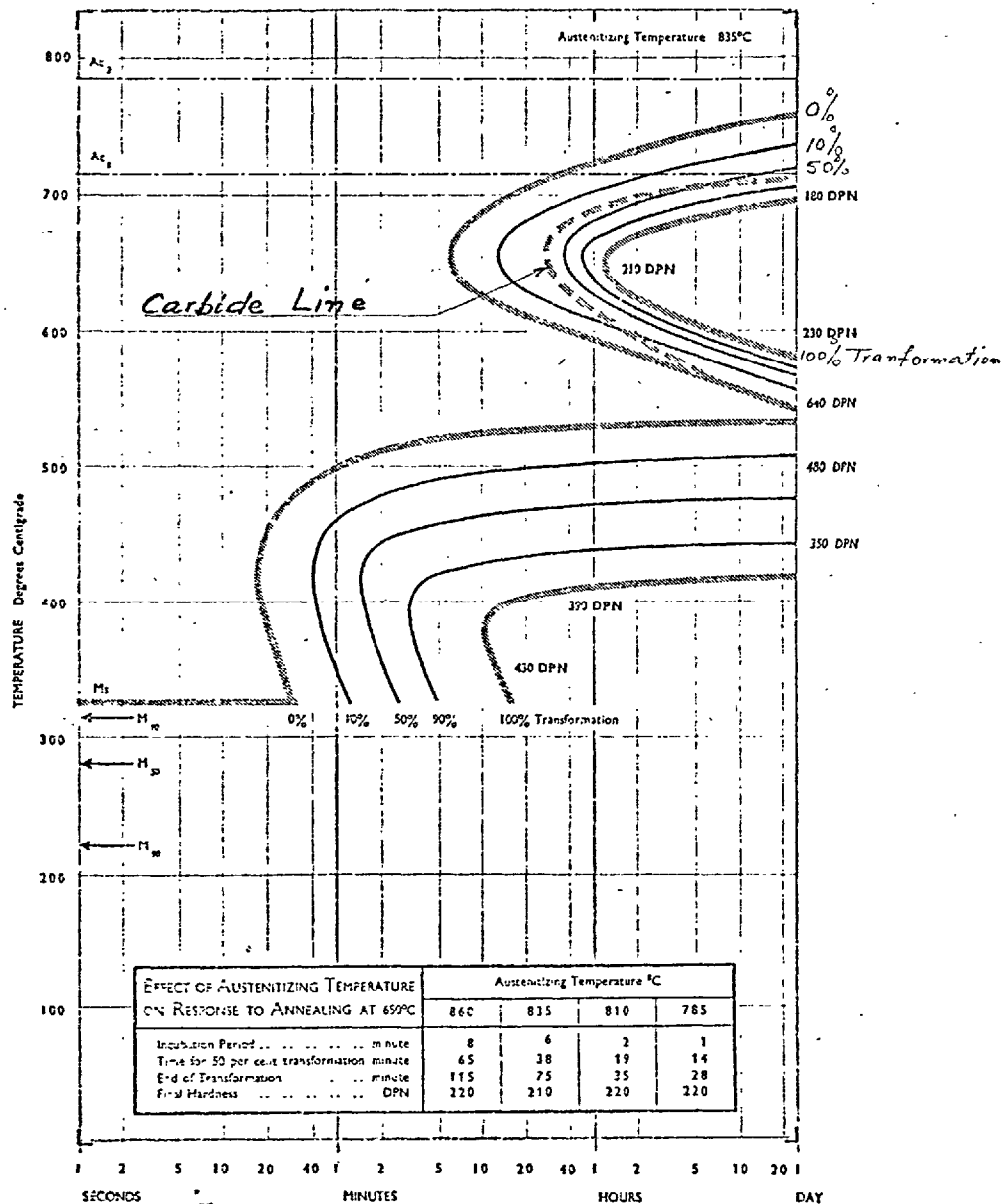
Specification:	C	Si	Mn	S	P	Ni	Cr	Mo
Min.	0.35	0.10	0.45	—	—	1.30	0.90	0.20
Max.	0.45	0.35	0.70	0.05	0.05	1.80	1.40	0.35
Steel Studied	0.36	0.22	0.52	0.005	0.007	1.52	1.17	0.27

METHOD OF MANUFACTURE:— Basic Electric Arc

GRAIN SIZE:— As-Quenched Grain Size, 7 & 8 (A.S.T.M.)

McQuaid-Ehn Grain Size, 4 to 5 (A.S.T.M.)

ISOTHERMAL TRANSFORMATION DIAGRAM

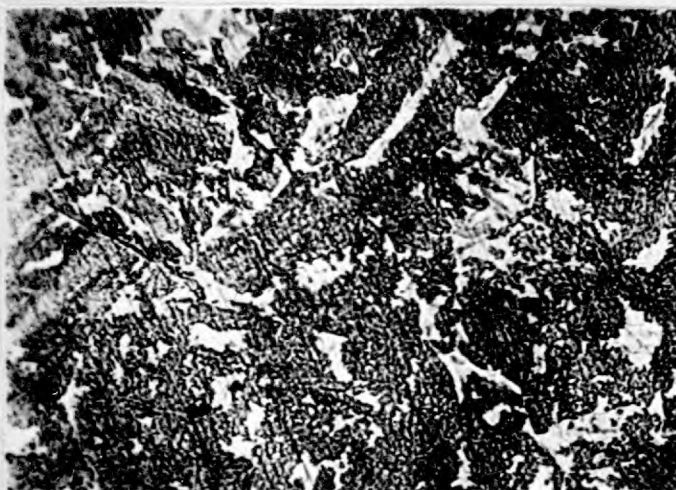


Hardness values in bold figures are those of the fully transformed steel; the values in italics represent structures developed by holding at the selected temperatures for 24 hours and then quenching to room temperature.

FIG. 15. INTERNATIONAL NICKEL COMPANY (MOND) LIMITED.



(1)



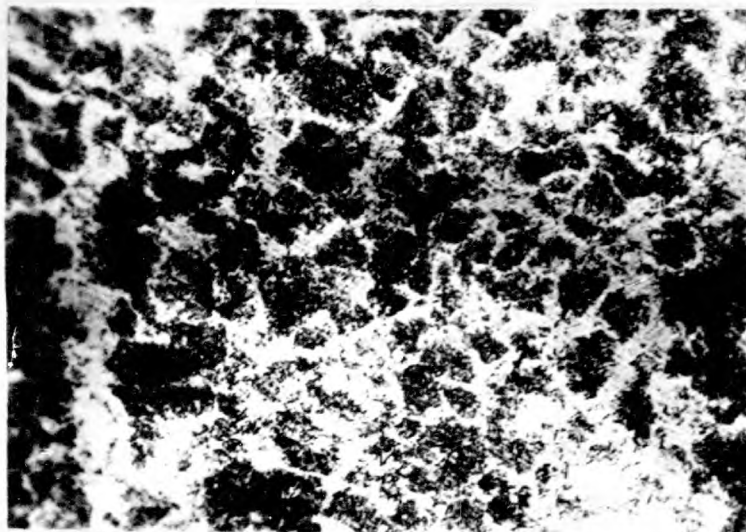
(2)



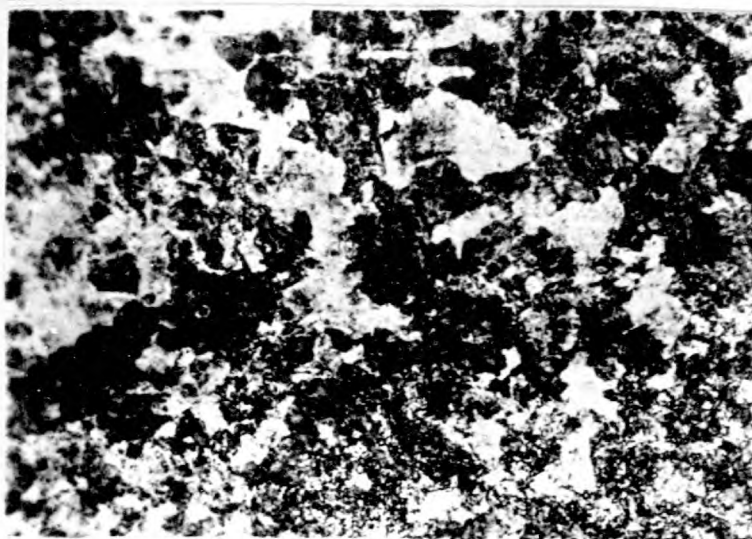
(3)

Fig. 16 Microstructure 600X

- (1) Rolling Direction.
- (2) 45° to Rolling Direction.
- (3) Transverse Direction.



(1)



(2)

Fig. 17 Microstructure after annealing 450X

(1) Rolling direction

(2) 45° to Rolling direction

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8/67

FIG. 18.

FIG. 18. STRESS-STRAIN CURVES FROM ISOTROPY TEST.

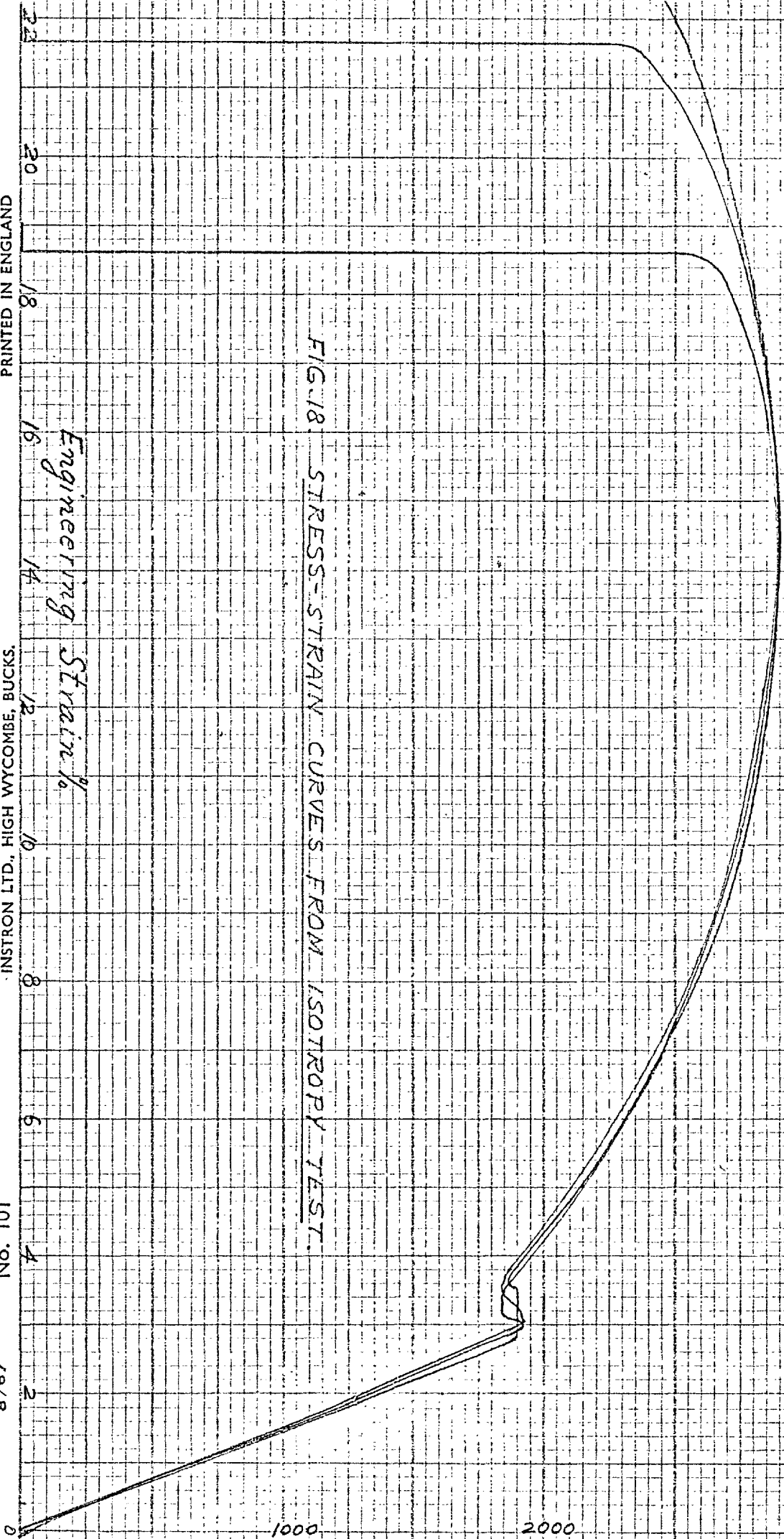
Engineering Strain %

0 2 4 6 8 10 12 14 16 18 20 22

1000

2000

3000 Load lbf.



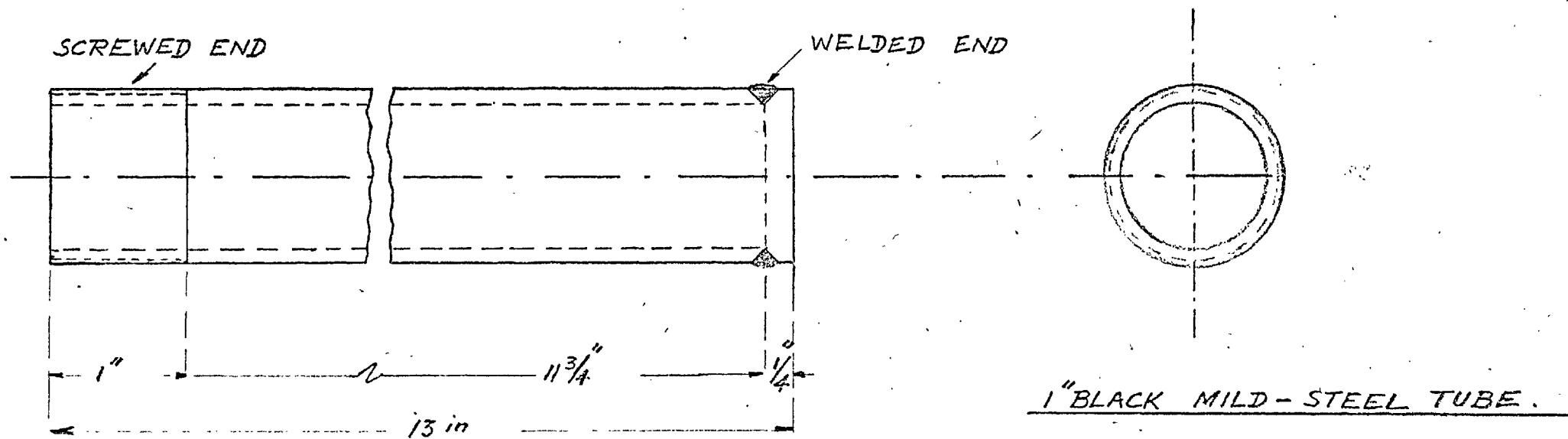


FIG. 19, CYLINDRICAL CONTAINER FOR ANNEALING
OF SPECIMENS

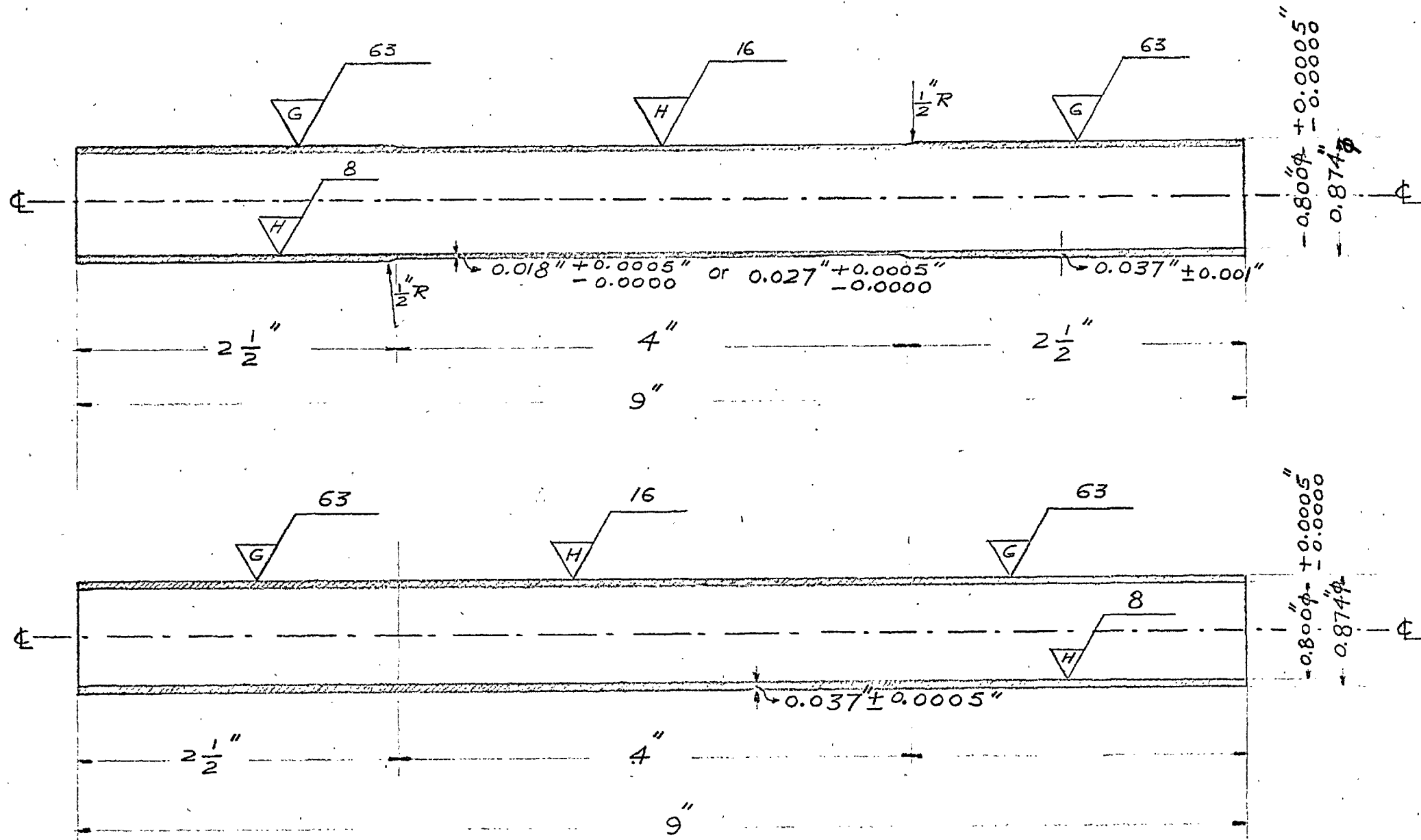


FIG. 20. TEST-PIECES FOR TWO-DIMENSIONAL STRESS TESTS.

FIG. 21.

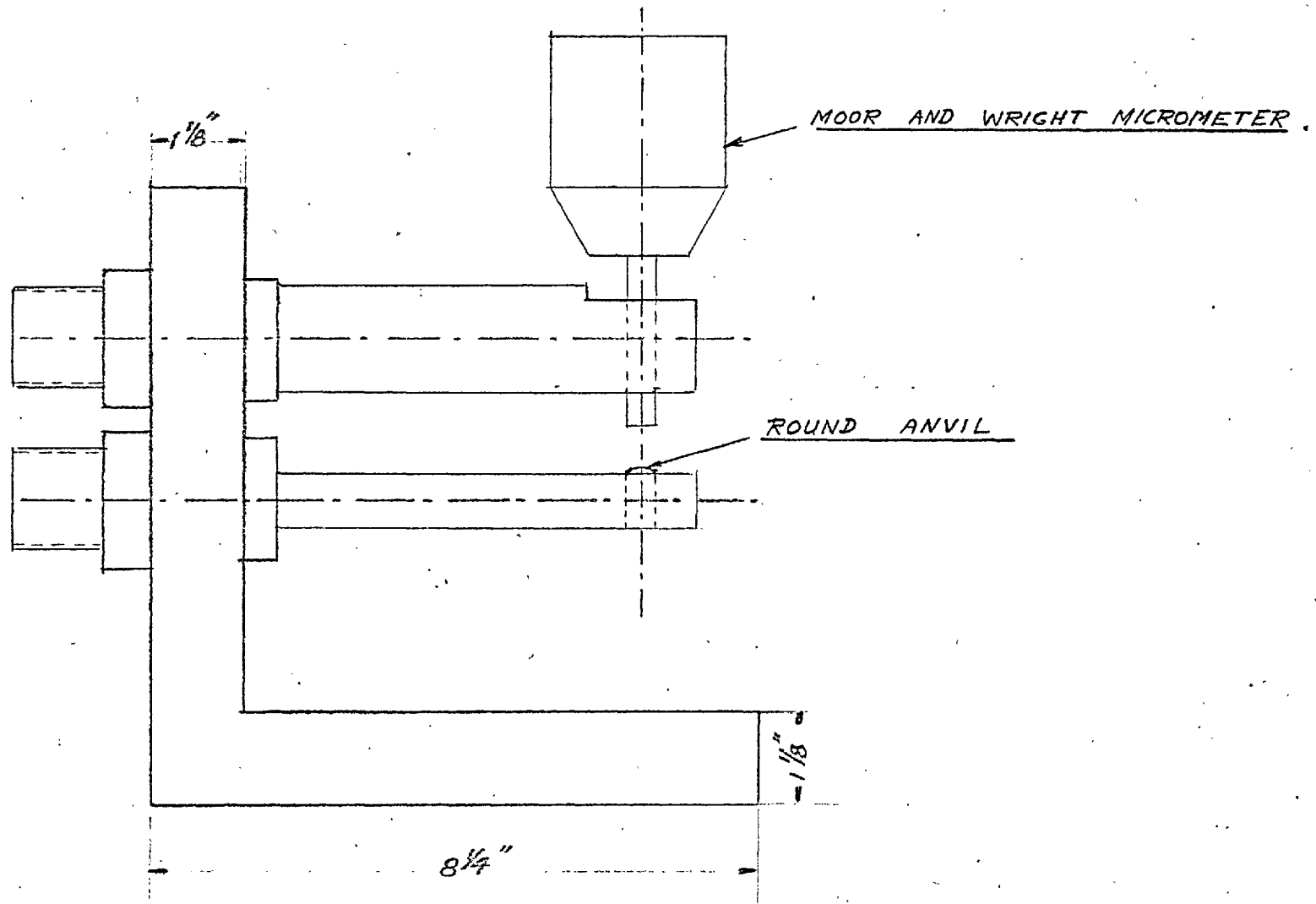


FIG. 21. MICROMETER TO MEASURE THE THICKNESS
OF THE WALL OF THE SPECIMENS.

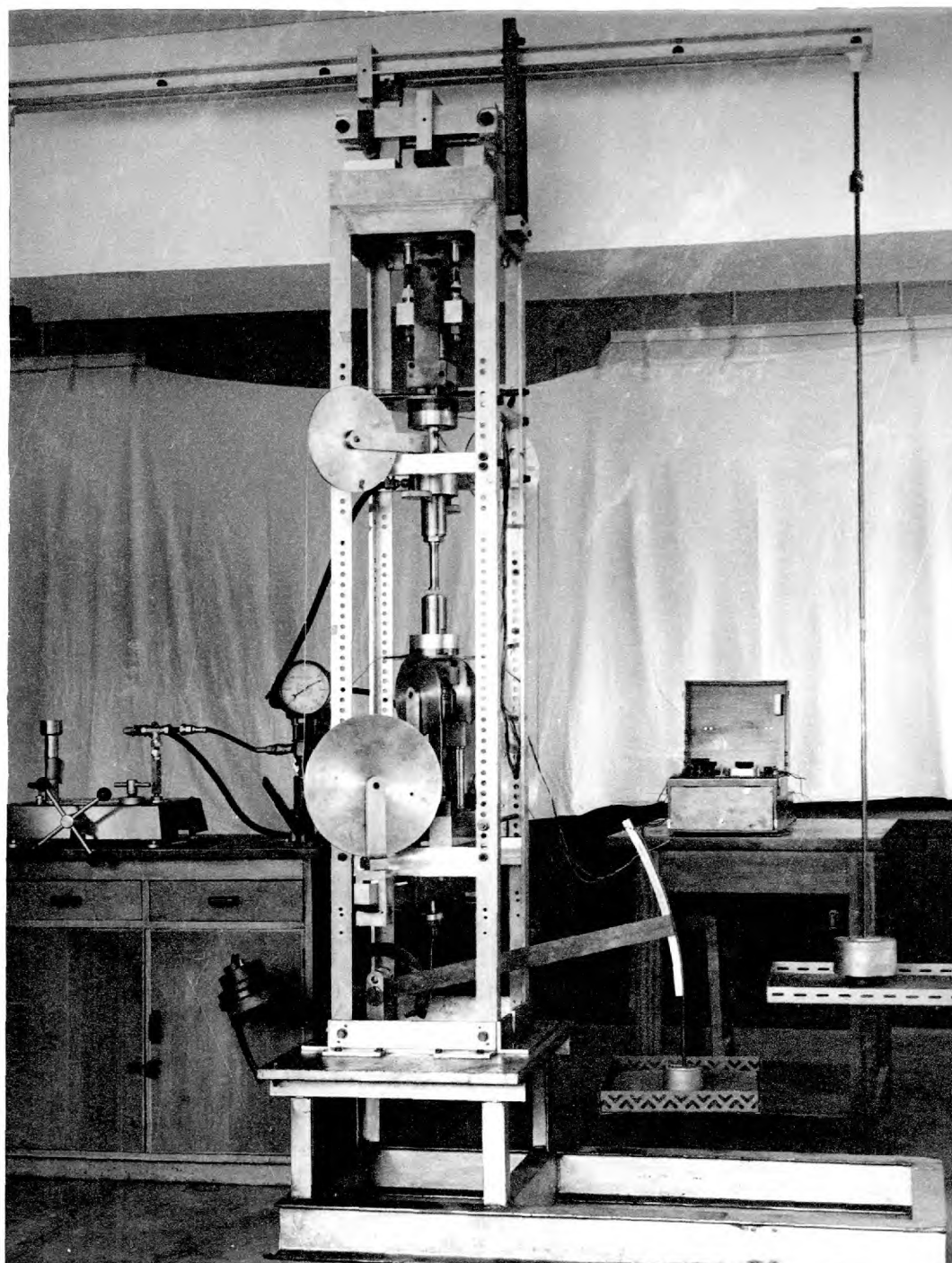


Fig. 22 The Testing Machine Combined Tension-Torsion
in Action

FIG. 23.

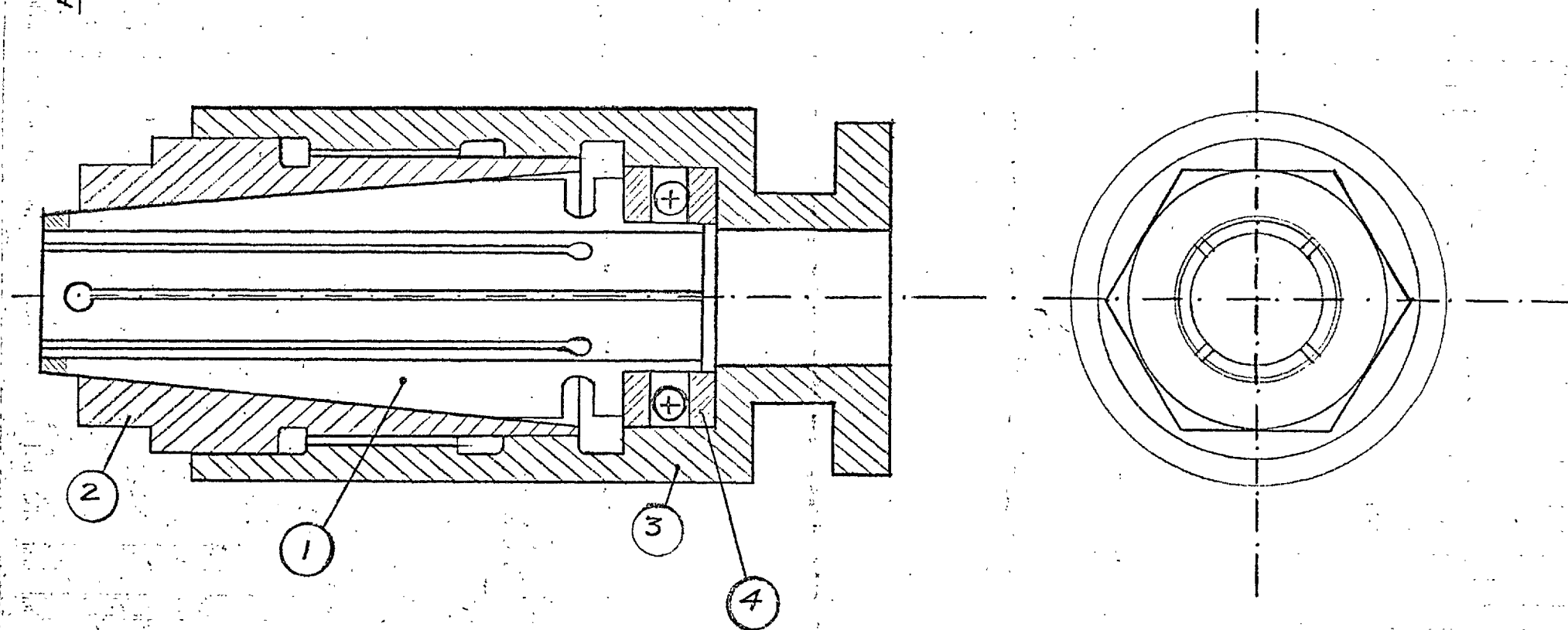


FIG. 23. CHUCK FOR GRIPPING THE SPECIMEN.

FIG. 24.

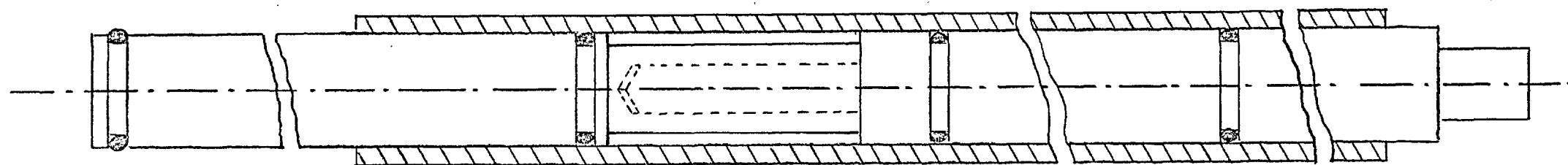


FIG. 24. SPLIT SLIDING MANDREL TO ELIMINATE
BENDING IN TORSION.

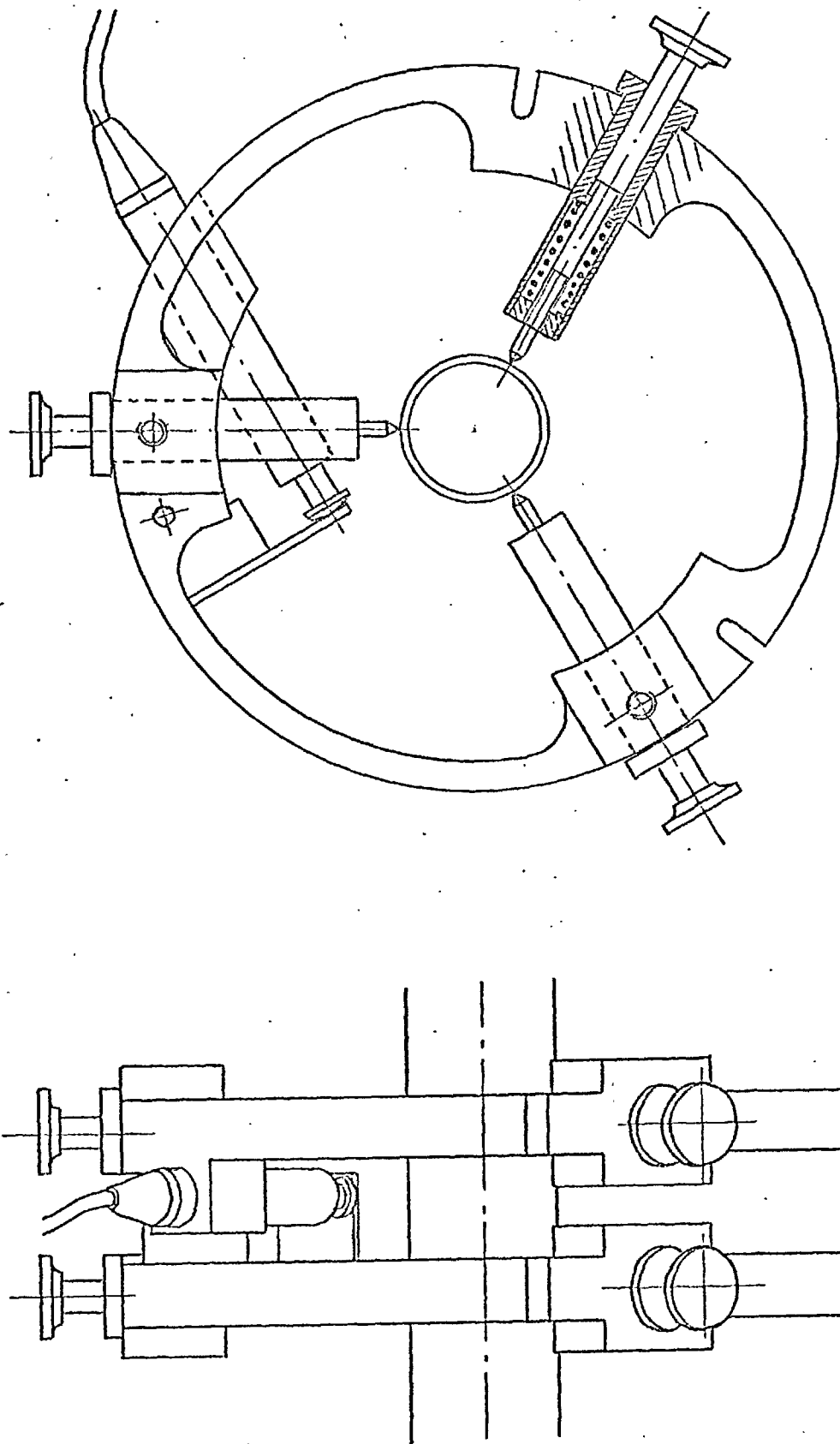
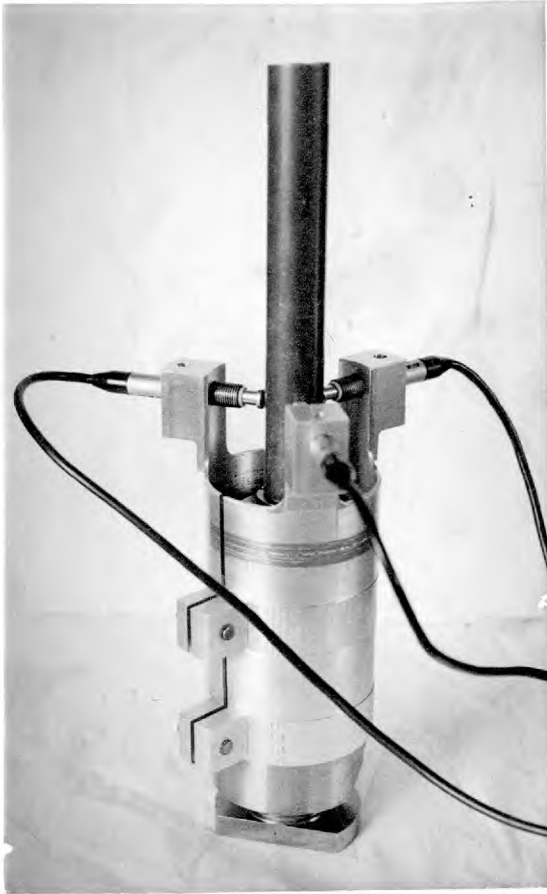


FIG. 25. TORSION METER.



Fig. 25A Torsion meter set on the specimen and connected to the transducer.



26(B)



26(A)

Fig. 26 Extensometers:

(A) Axial

(B) Circumferential or hoop

FIG. 27.

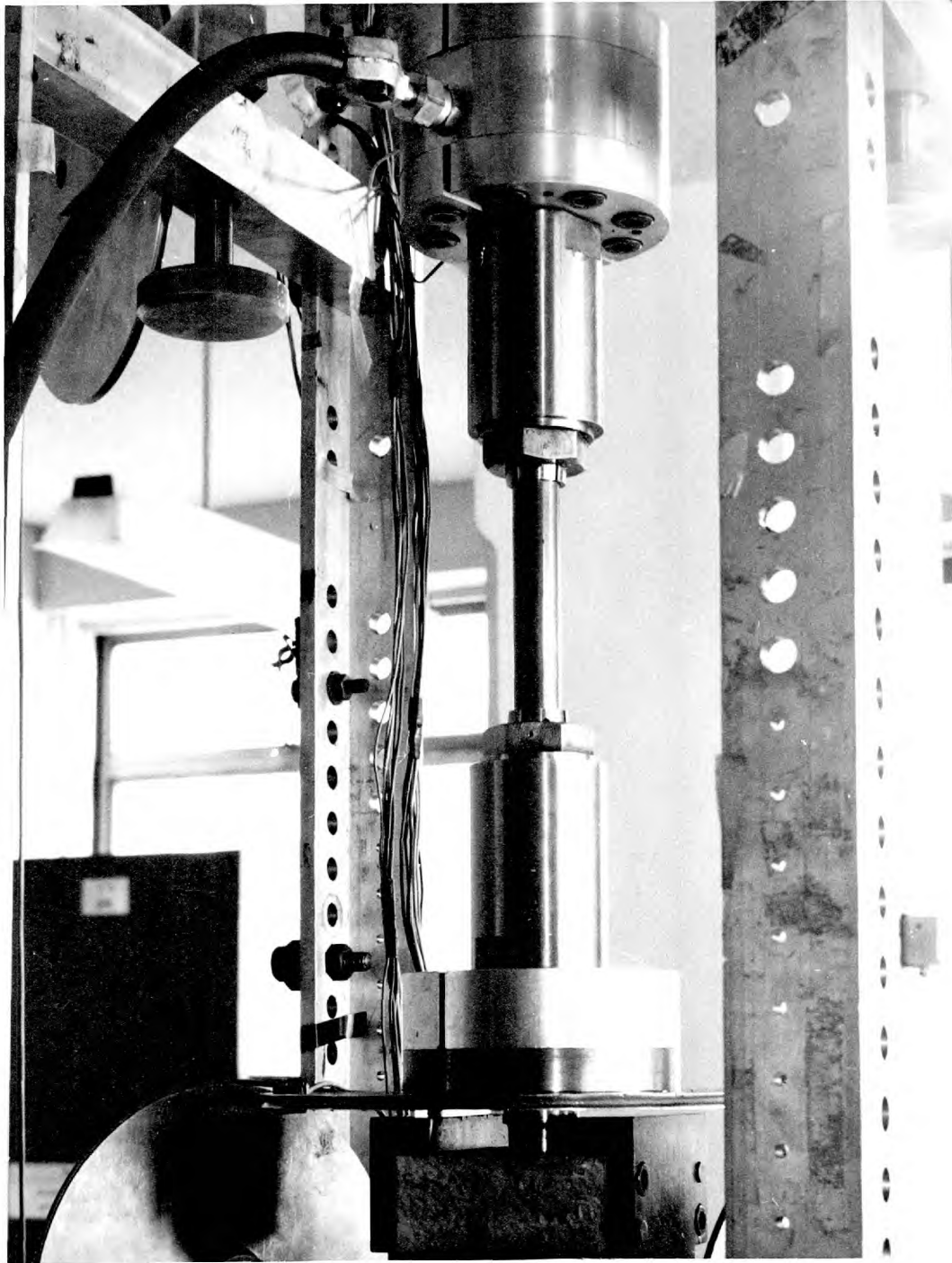


Fig. 27 Apparatus for Tension-Internal Pressure.

FIG. 28.

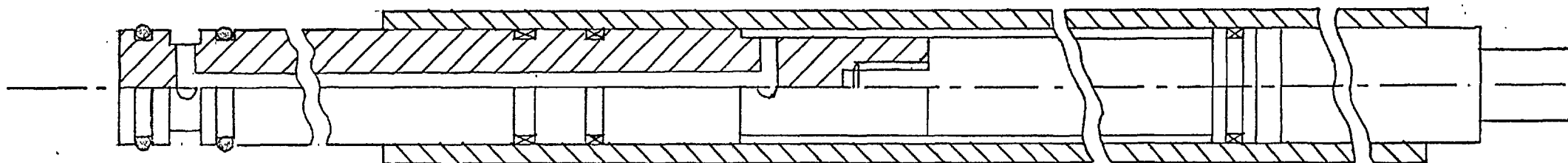


FIG. 28 . MANDREL TO ELIMINATE AXIAL THRUST
DUE TO INTERNAL PRESSURE.

FIG. 29.

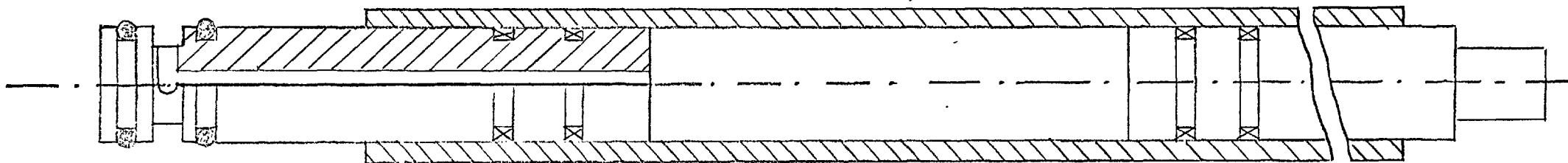


FIG. 29 . SPLIT MANDREL TO OBTAIN AXIAL THRUST
UNDER INTERNAL PRESSURE.

FIG. 30.

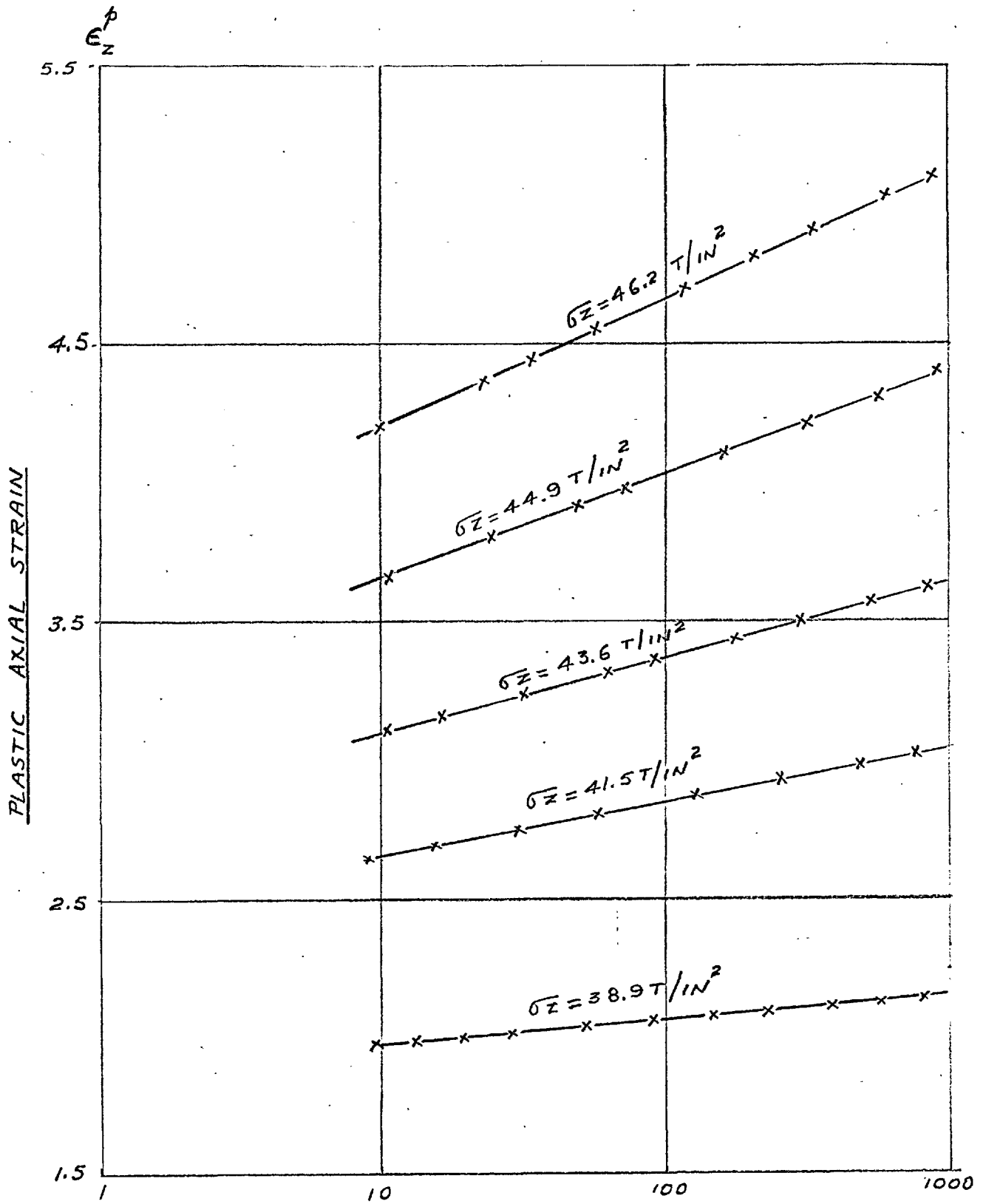


FIG. 30. TIME SEC. LOG. SCALE.

FIG. 31.

TENSILE STRESS

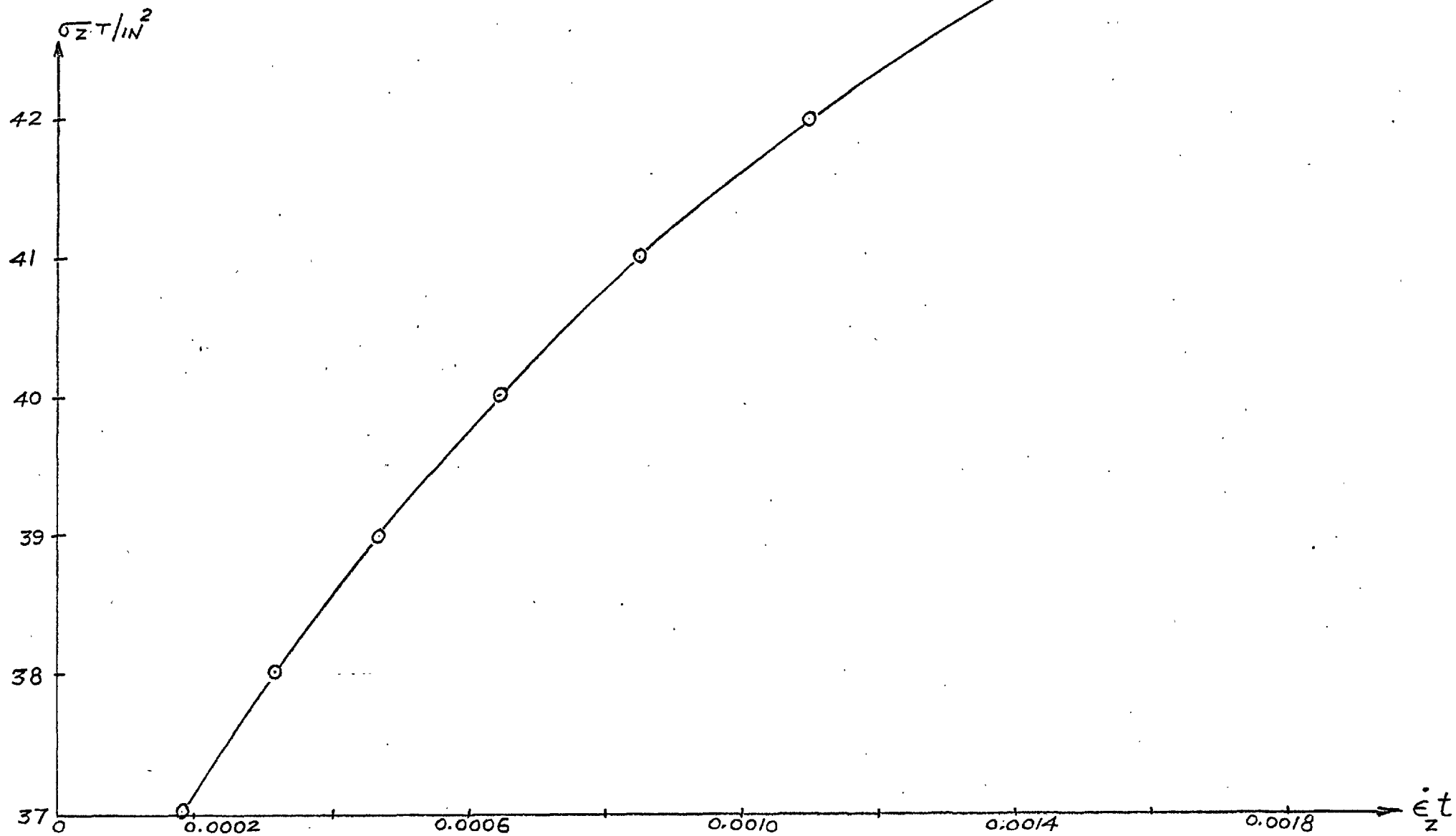


FIG. 31. CREEP COEFFICIENT

FIG. 32.

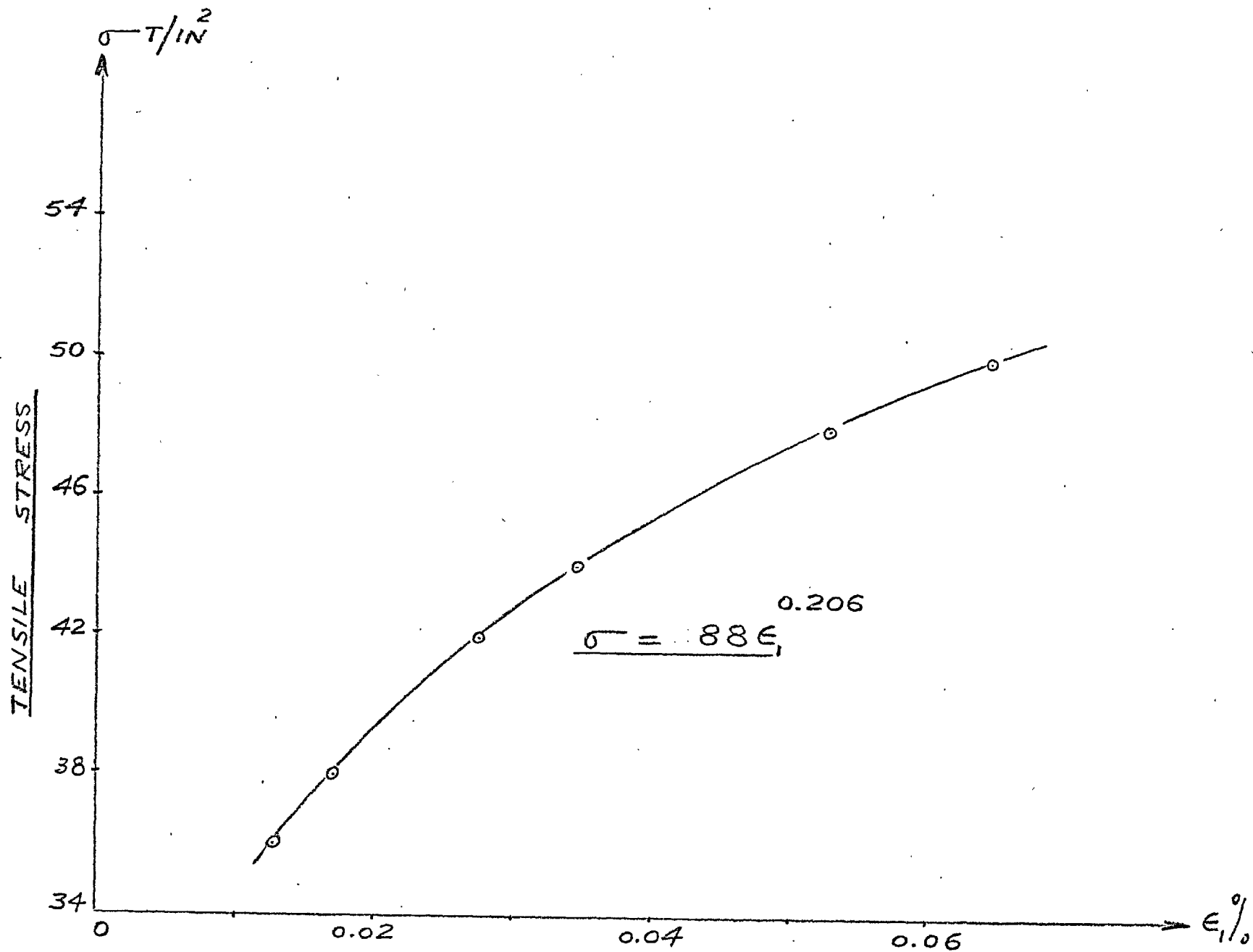
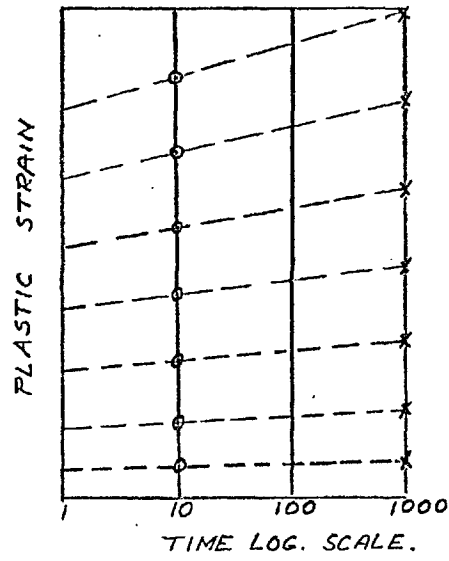
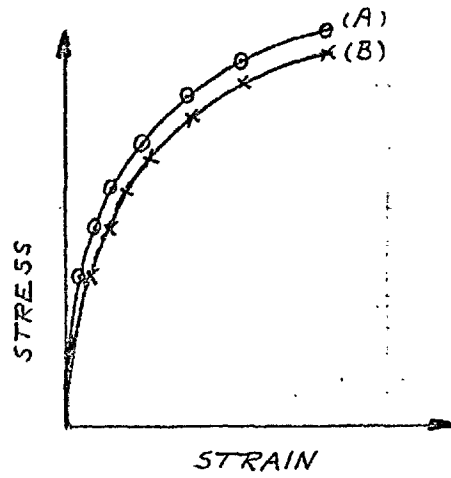


FIG. 32. CREEP CONSTANT

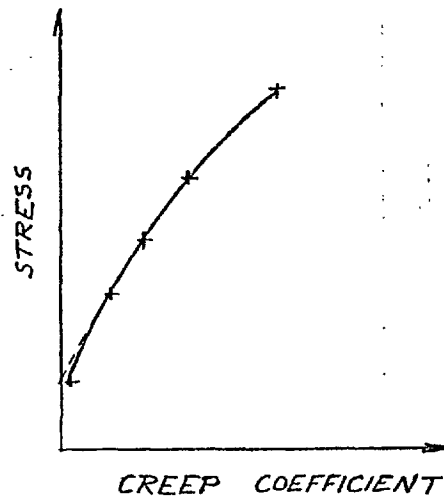
FIG. 33.



(a)



(b)



(c)

FIG. 33.

FIG. 34.

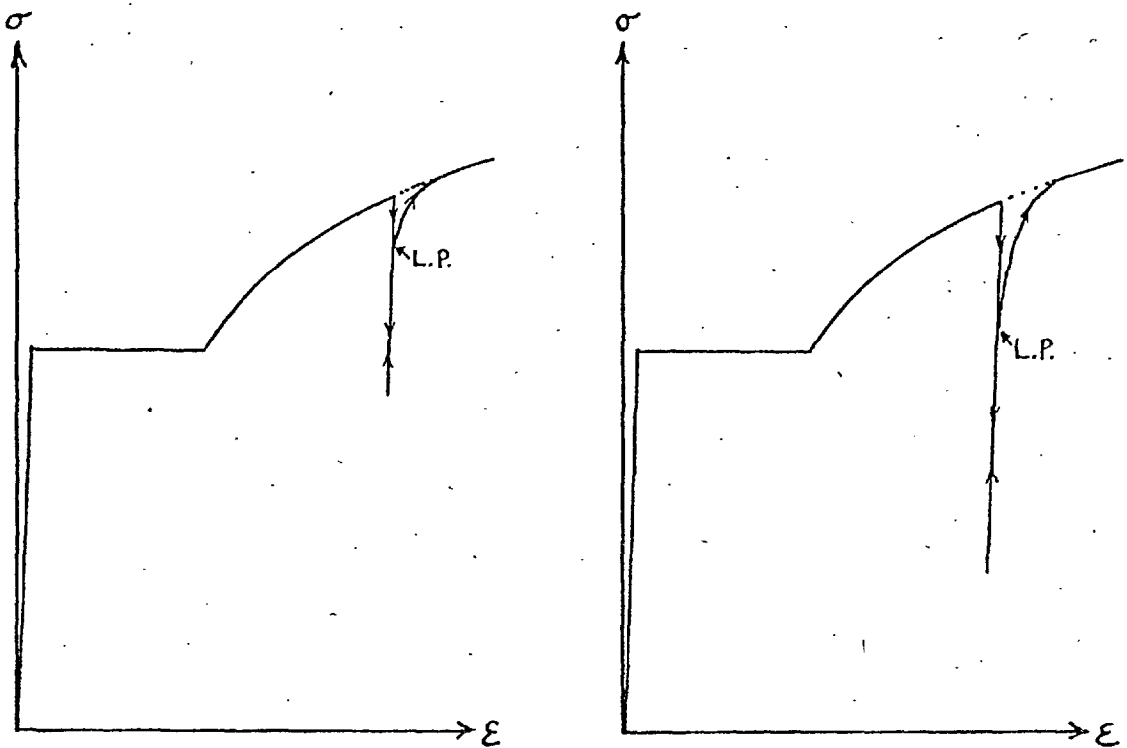


FIG.34. Sketches showing how the amount of unloading affects the subsequent Limit of Proportionality.

FIG. 35.

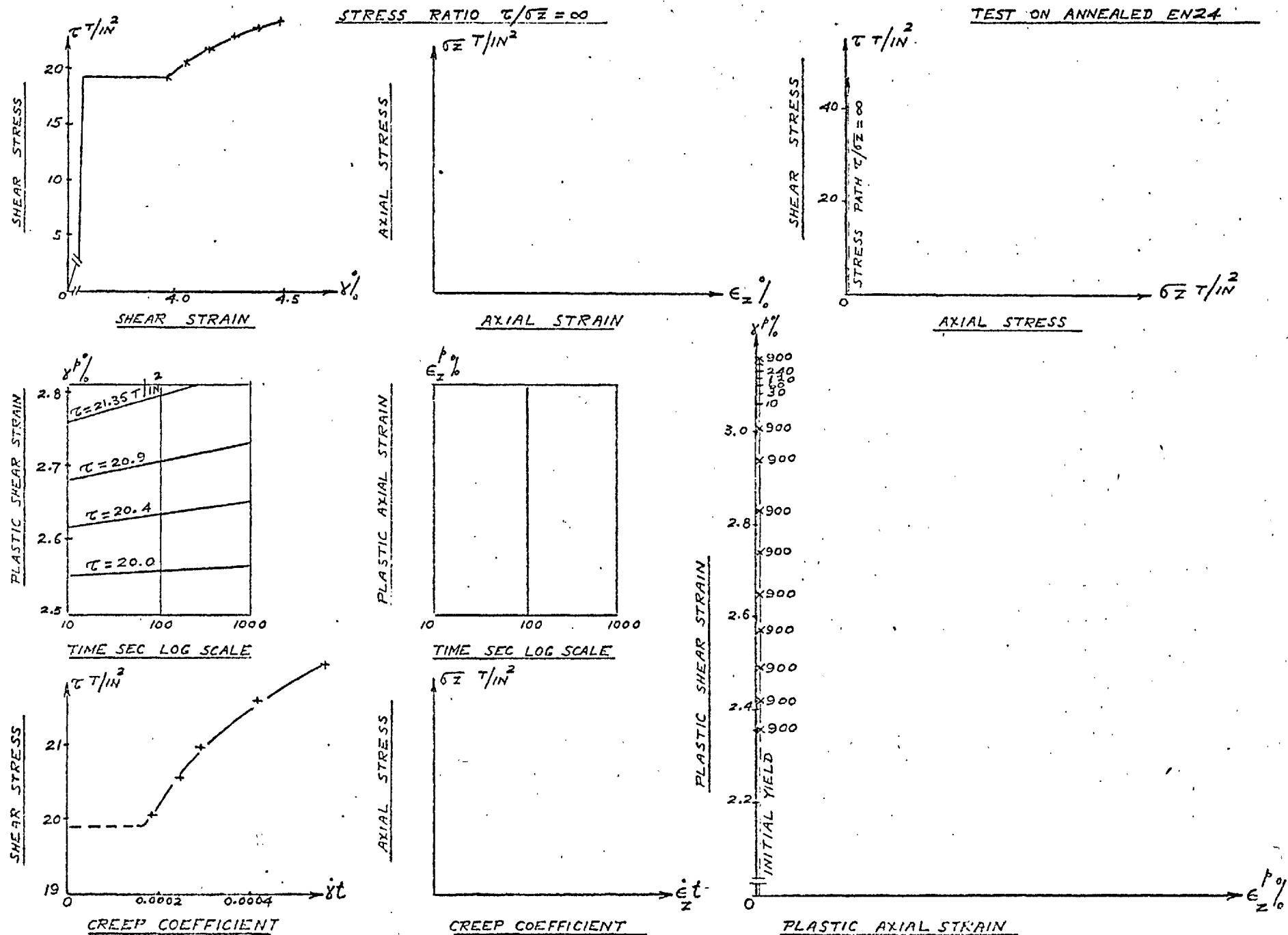


FIG. 35.

FIG. 36.

STRESS RATIO $\tau/\sigma = 2.25$

(ANNEALED EN24)

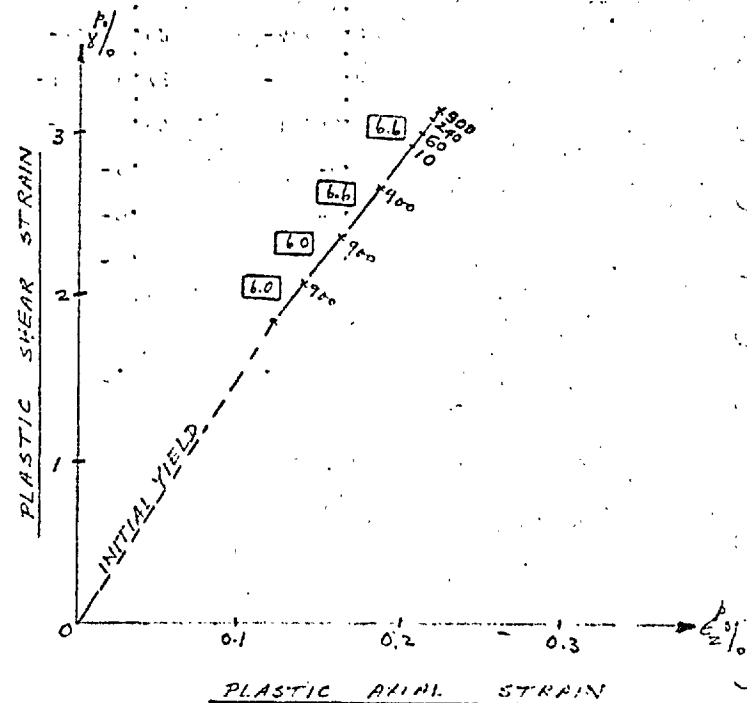
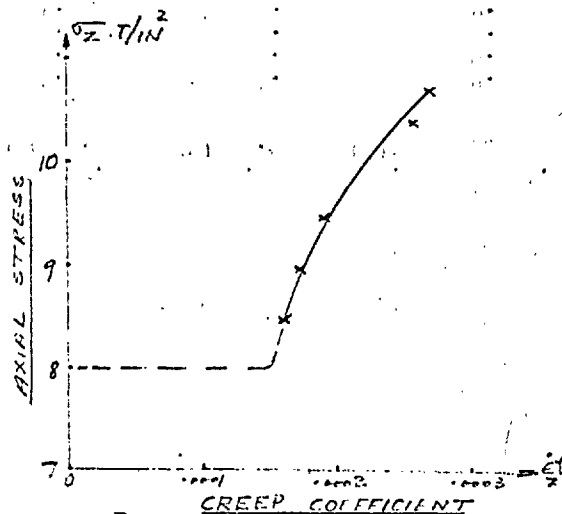
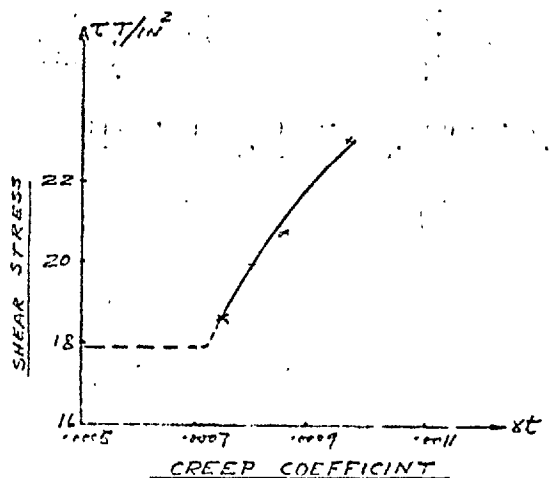
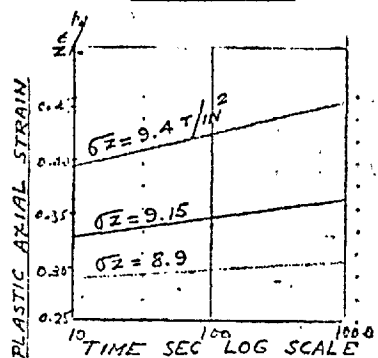
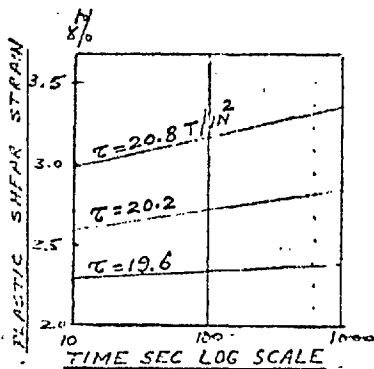
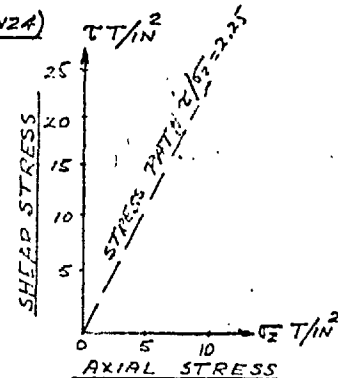
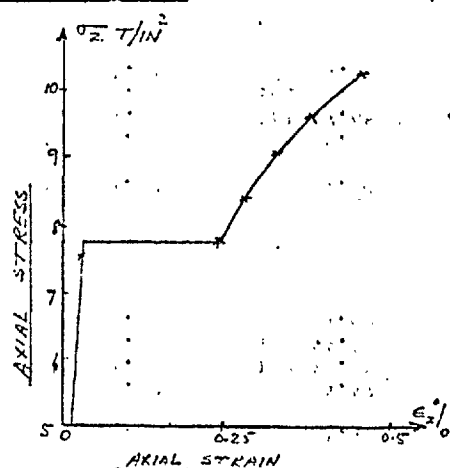
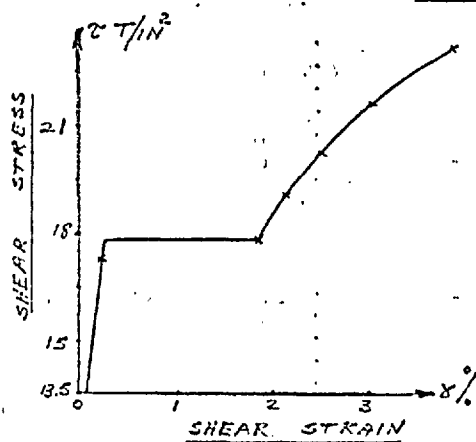


FIG. 37.

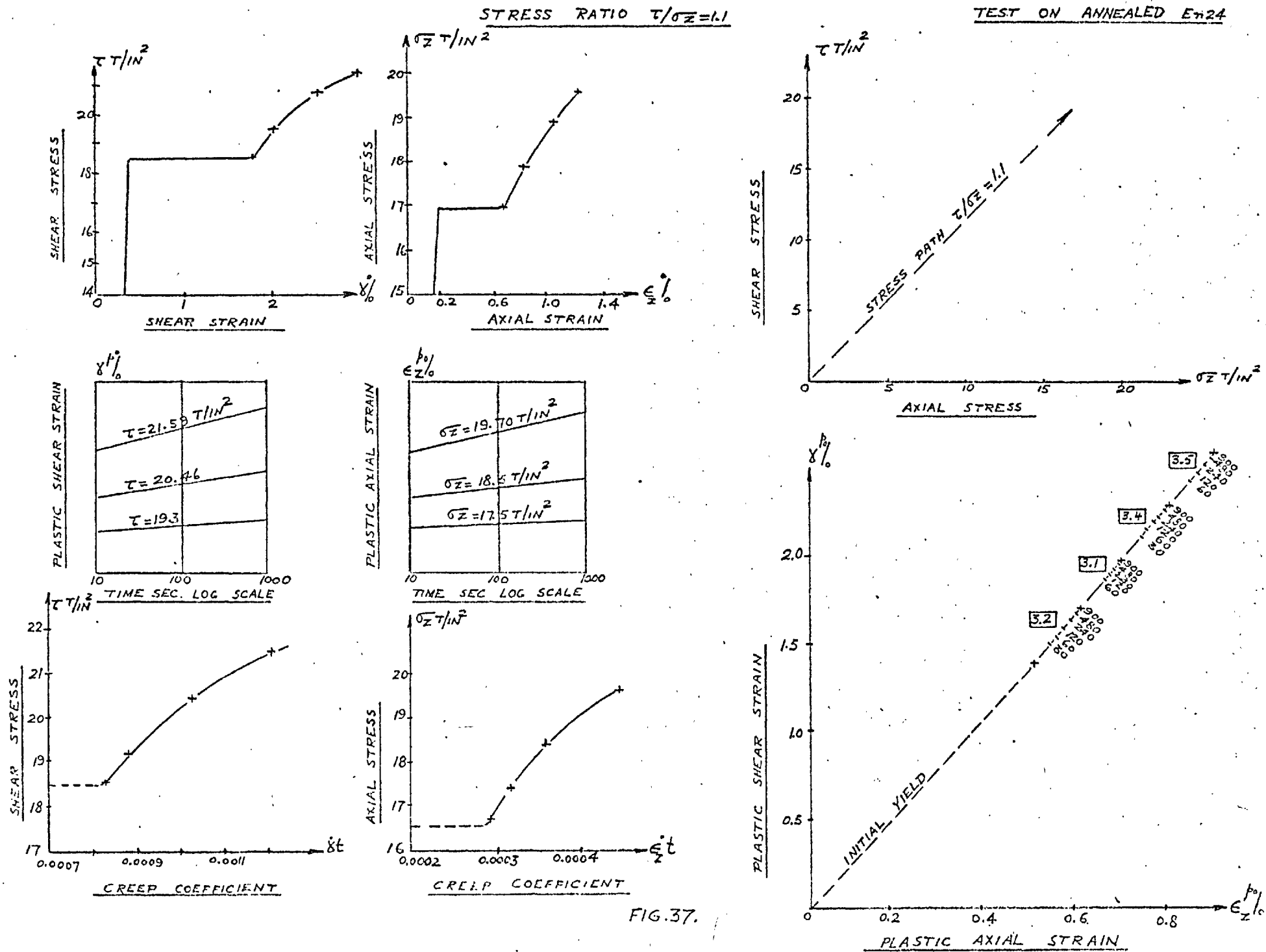
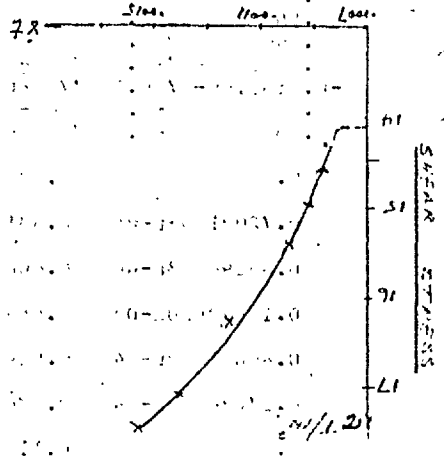
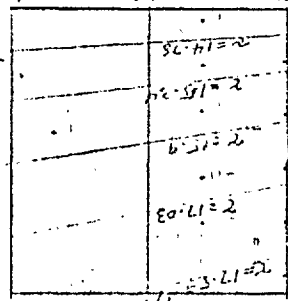


FIG. 37.

CREEP COEFFICIENT

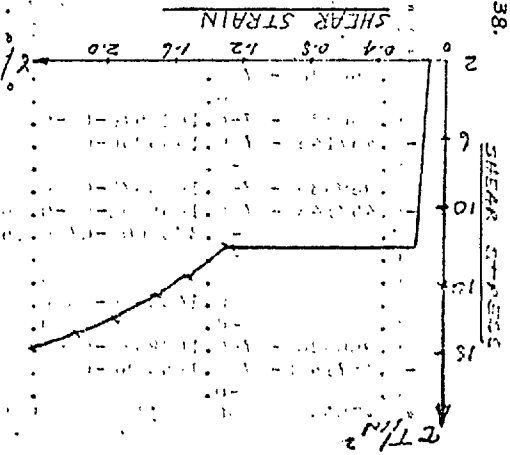


TIME-SEC. LOG SCALE



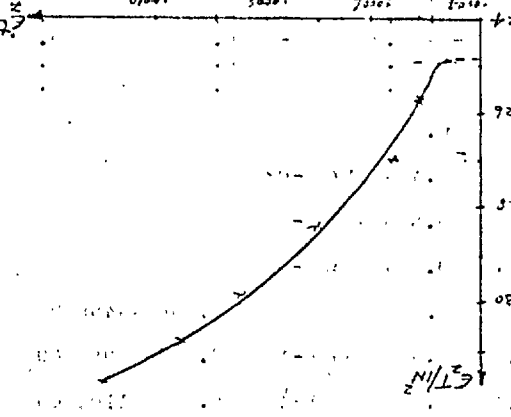
PLASTIC SHEAR STRAIN

FIG. 38.

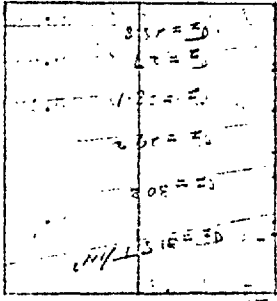


SHEAR STRAIN

CREEP COEFFICIENT

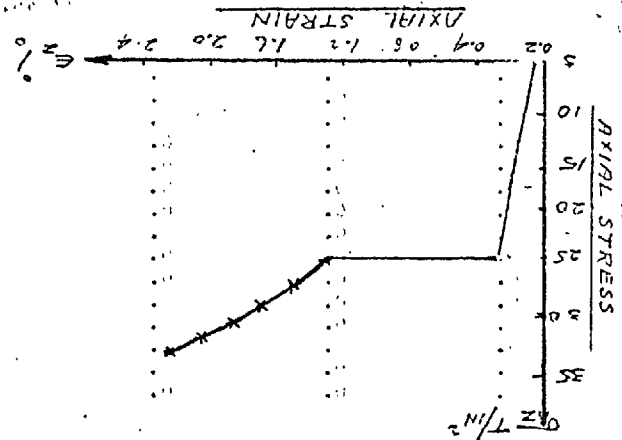


TIME-SEC. LOG SCALE



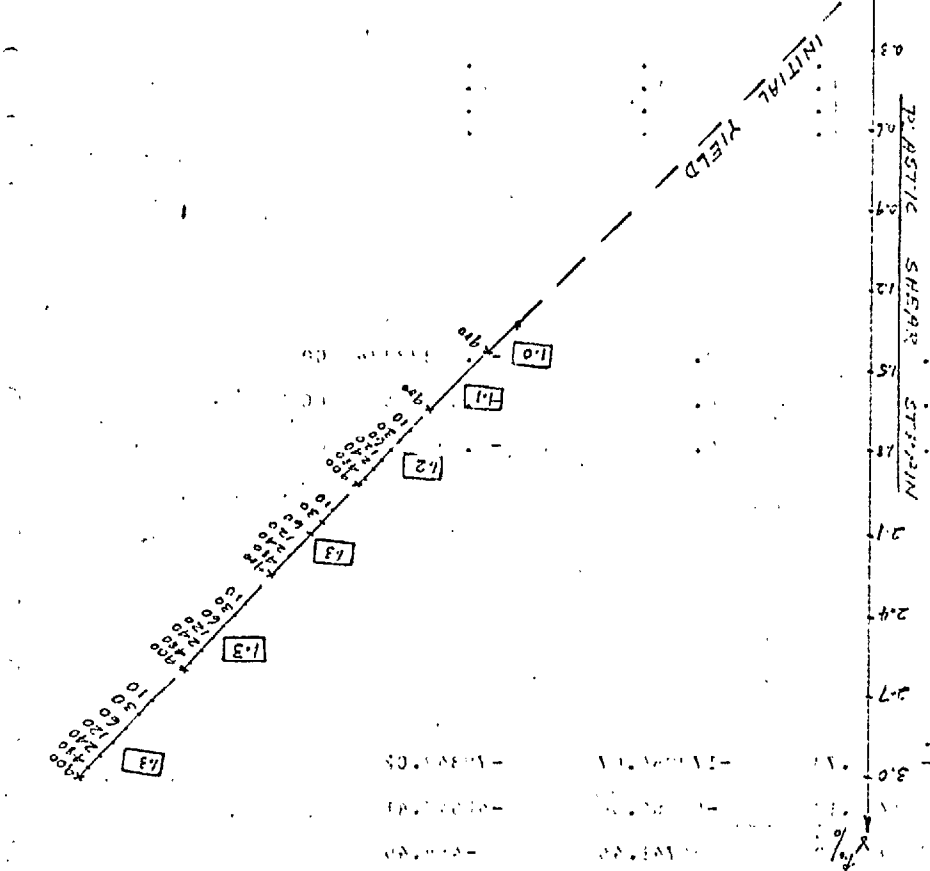
PLASTIC AXIAL STRAIN

STRESS RATIO $\tau/\sigma = 0.57$



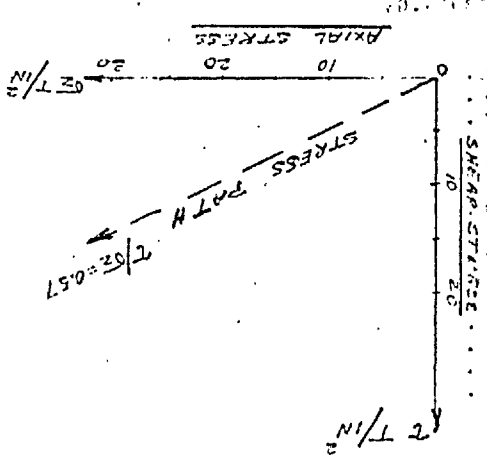
AXIAL STRAIN

INITIAL YIELD



PLASTIC SHEAR STRAIN

TEST ON ANNEALED EN24



AXIAL STRESS

STRESS PATH $\tau/\sigma = 0.57$

ANNEALED EN24

FIG. 39.

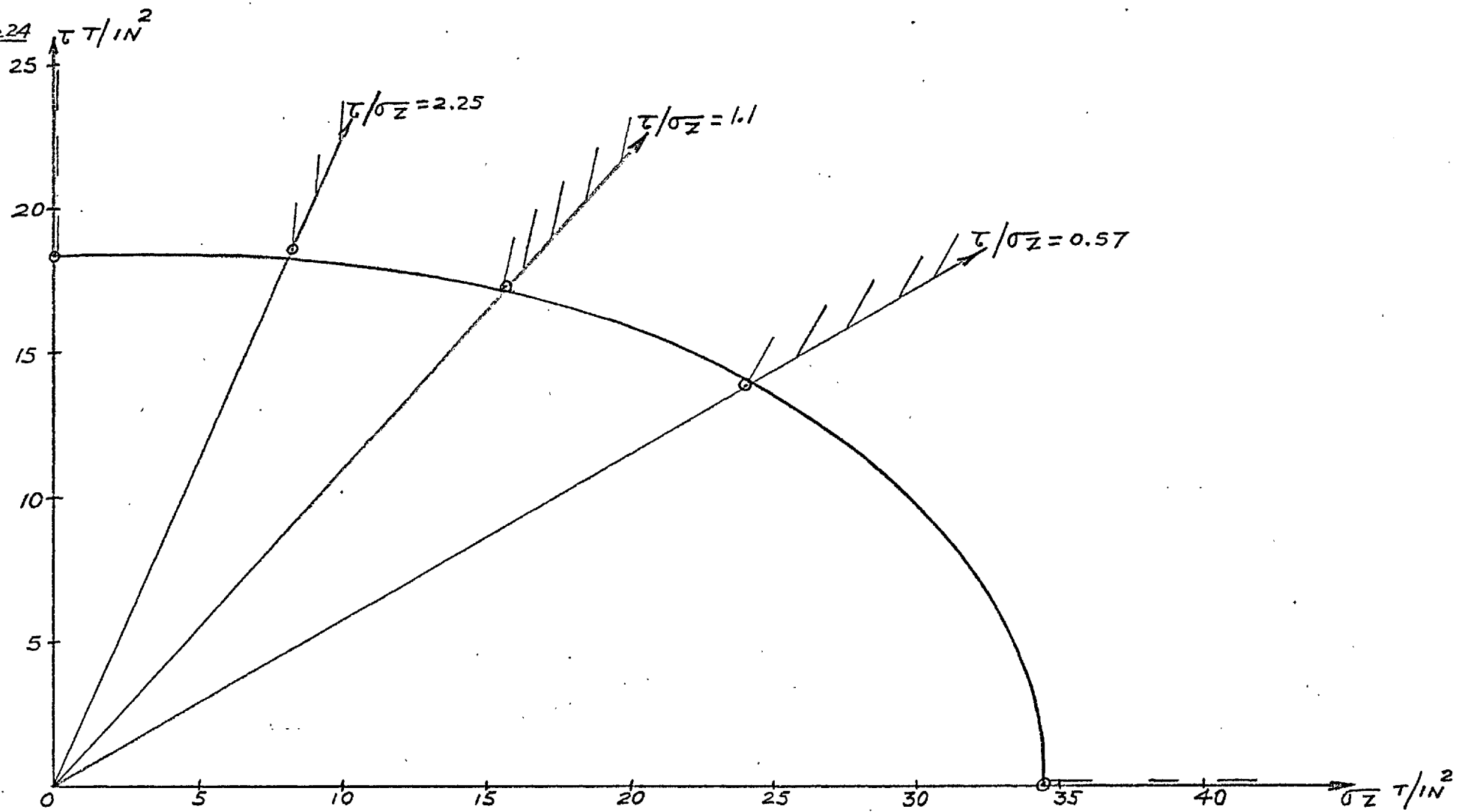
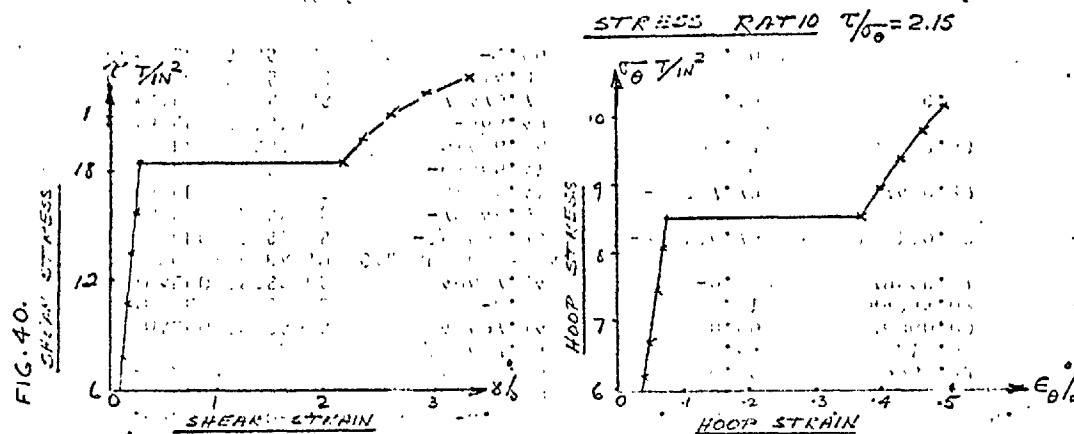


FIG. 39.

YIELD LOCUS FOR ANNEALED MATERIAL UNDER
COMBINED AXIAL TENSION-TORSION.

FIG. 40.



TEST ON ANNEALED EN24

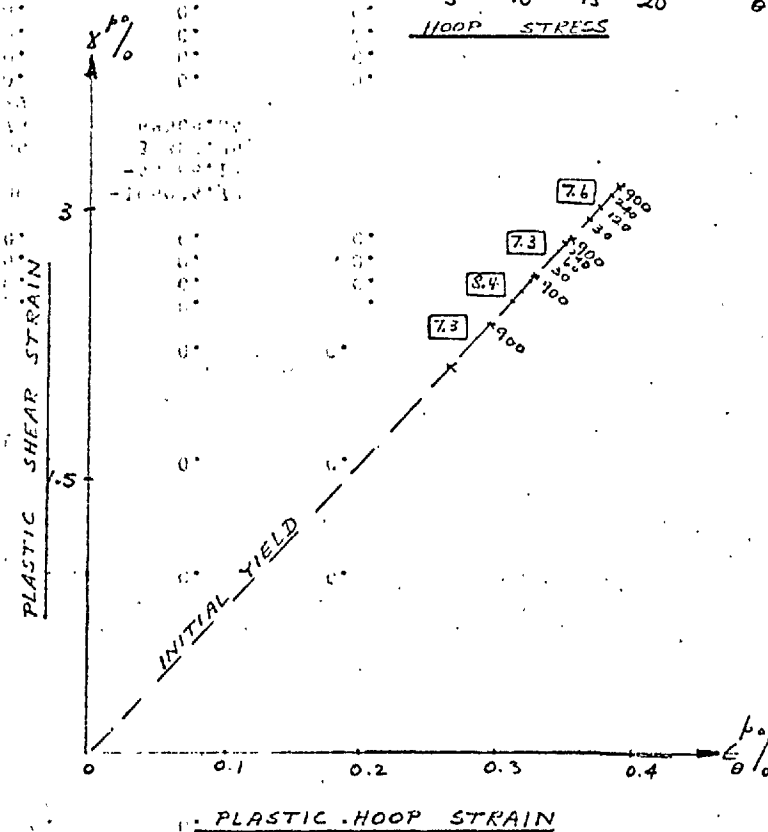
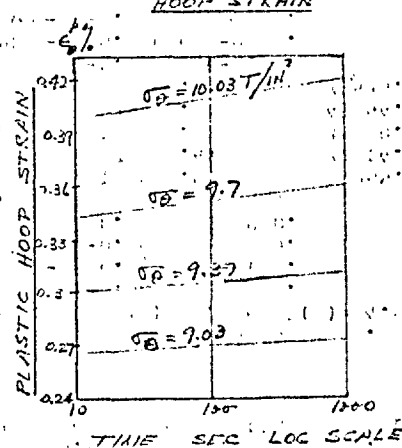
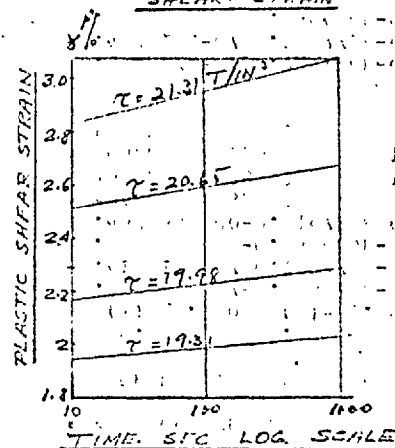
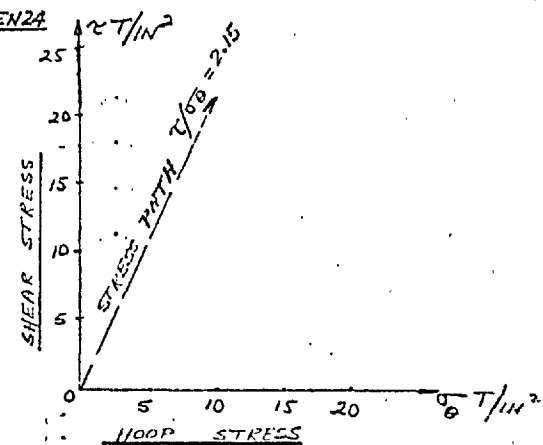


FIG. 40.

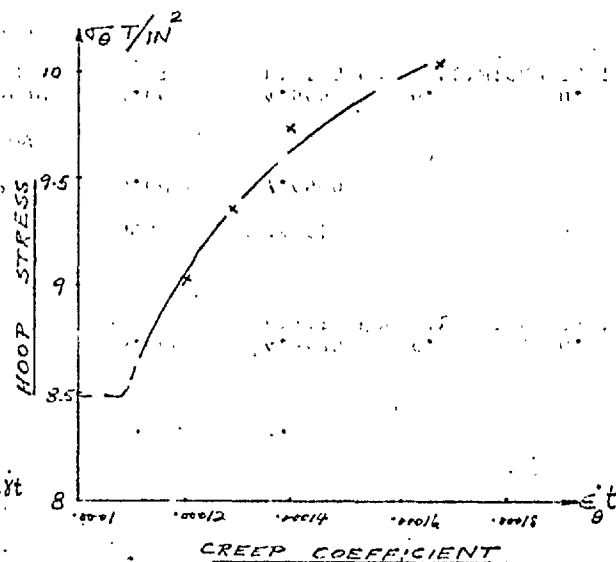
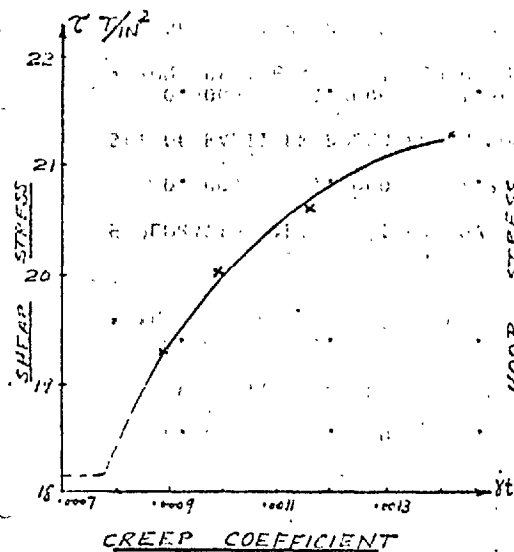
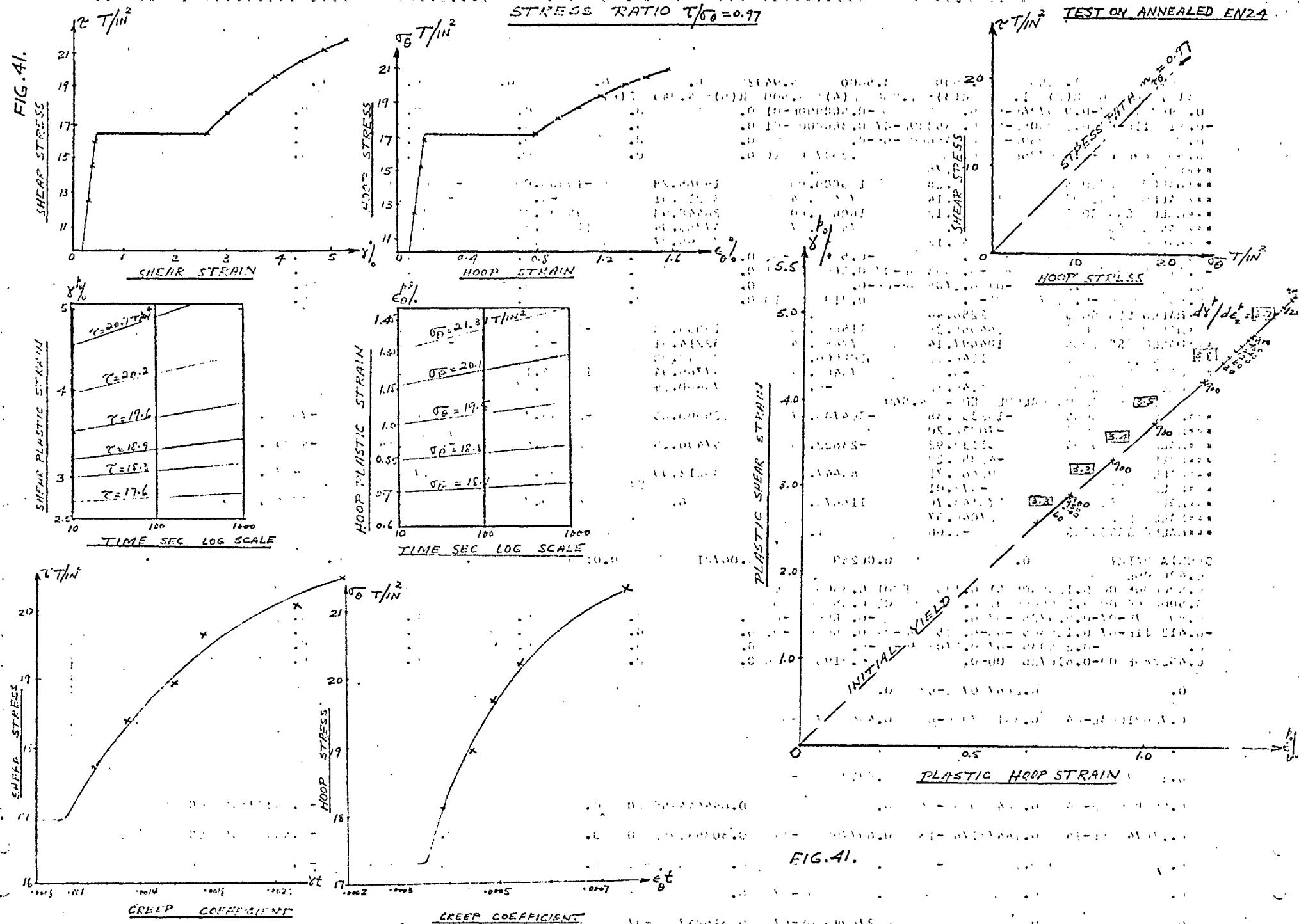


FIG. 41.



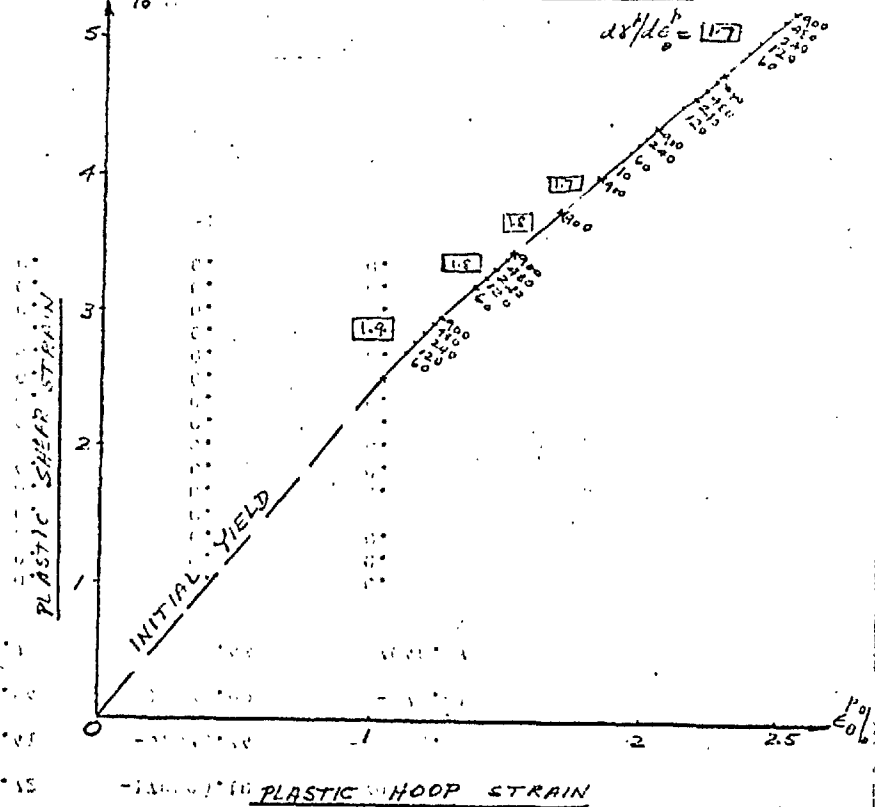
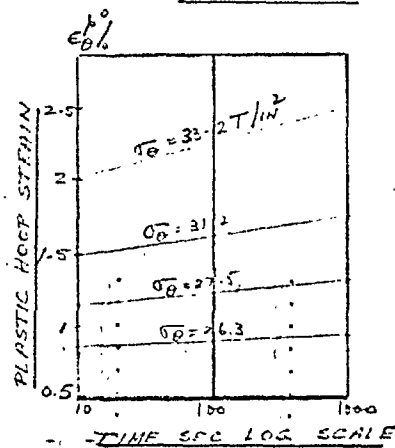
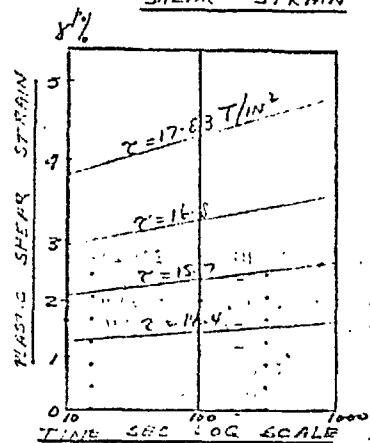
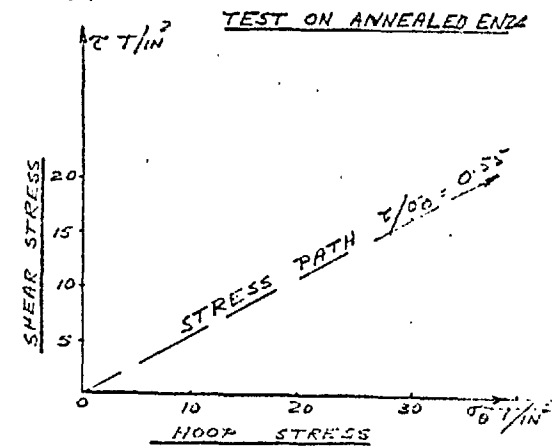
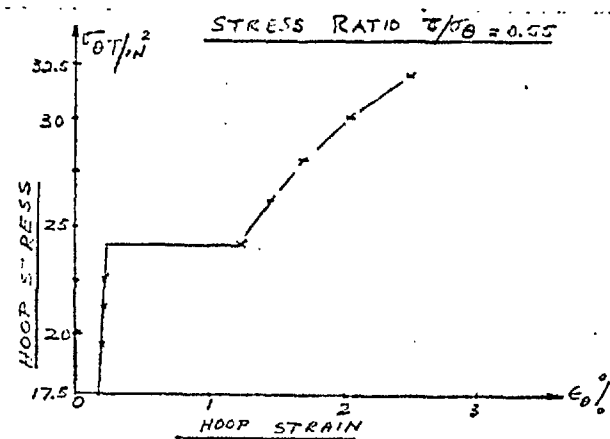
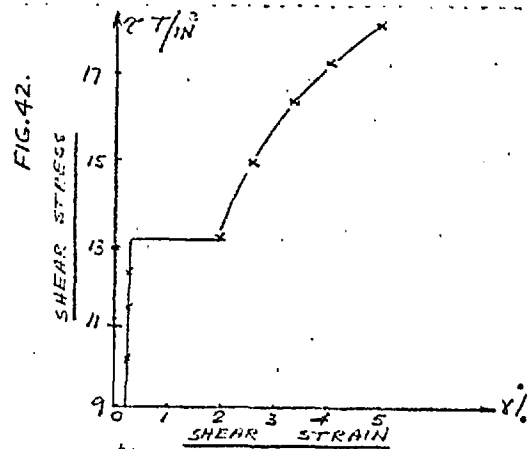


FIG. 42.

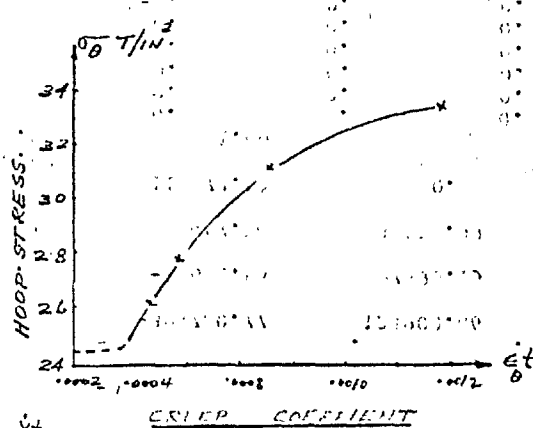
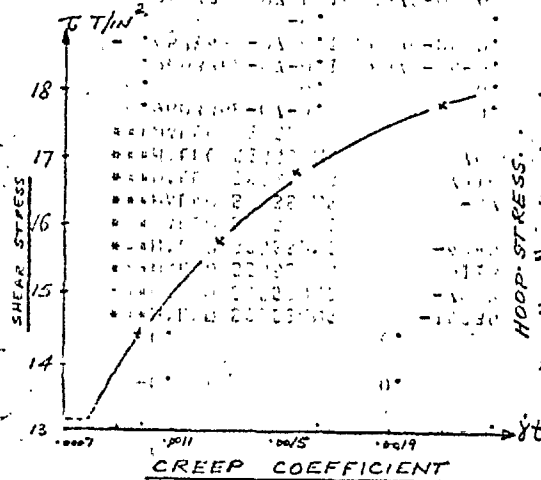


FIG. 43.

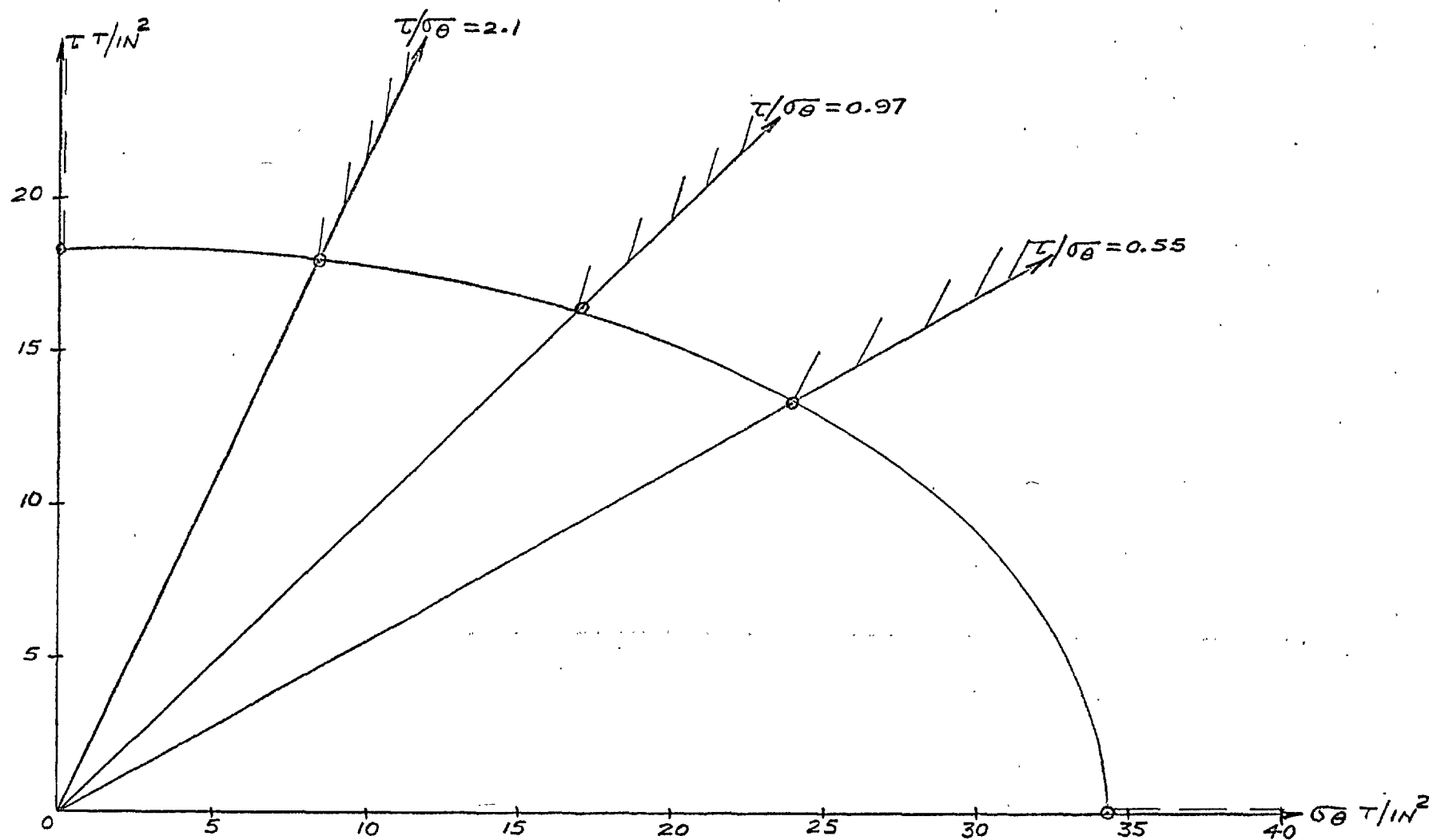
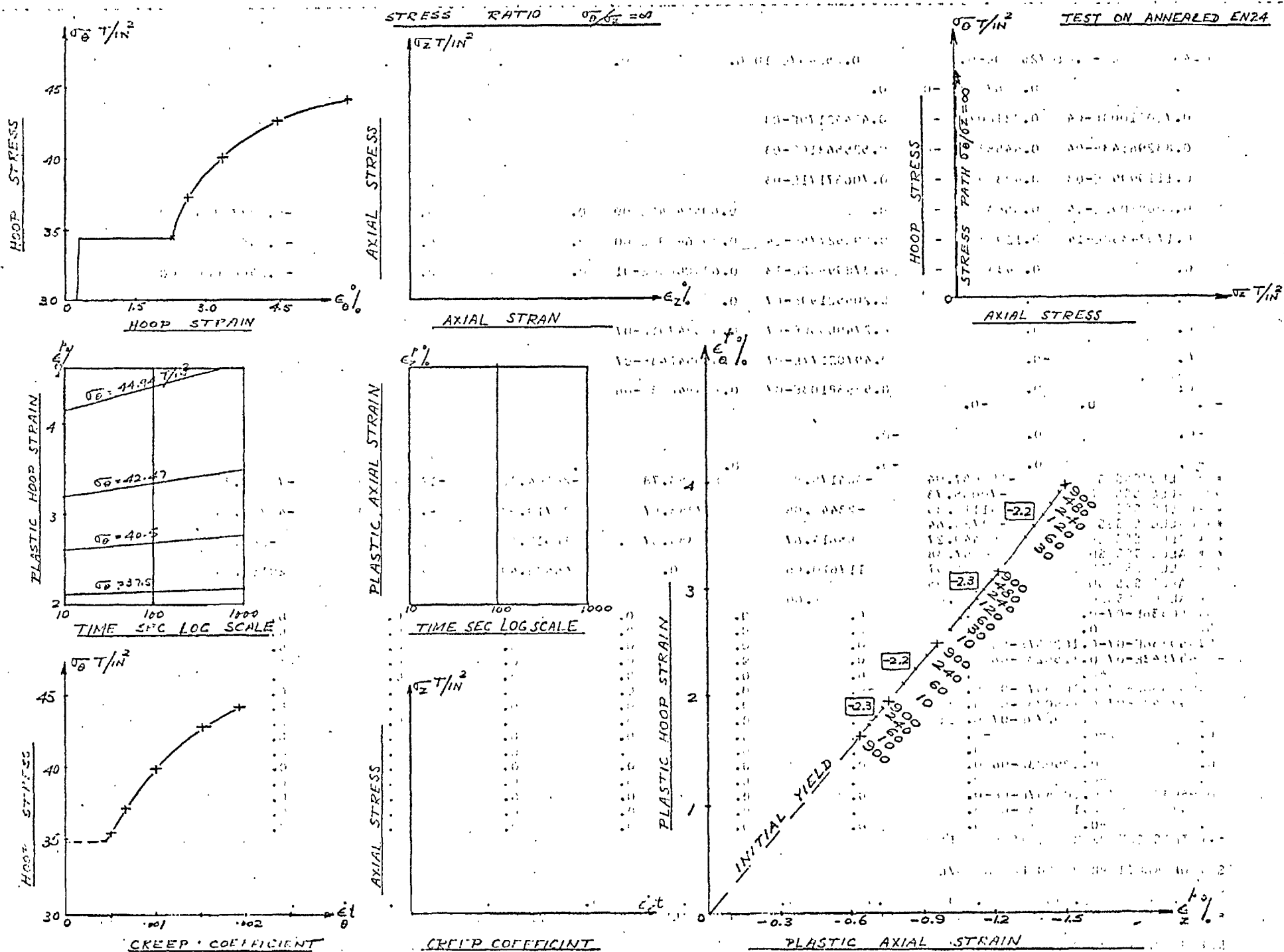


FIG. 43.

ANNEALED MATERIAL UNDER HOOP TENSION - TORSION

FIG. 44.



STRESS RATIO $\sigma_\theta/\sigma_z = 2$

TEST ON ANNEALED EN24

FIG.45.

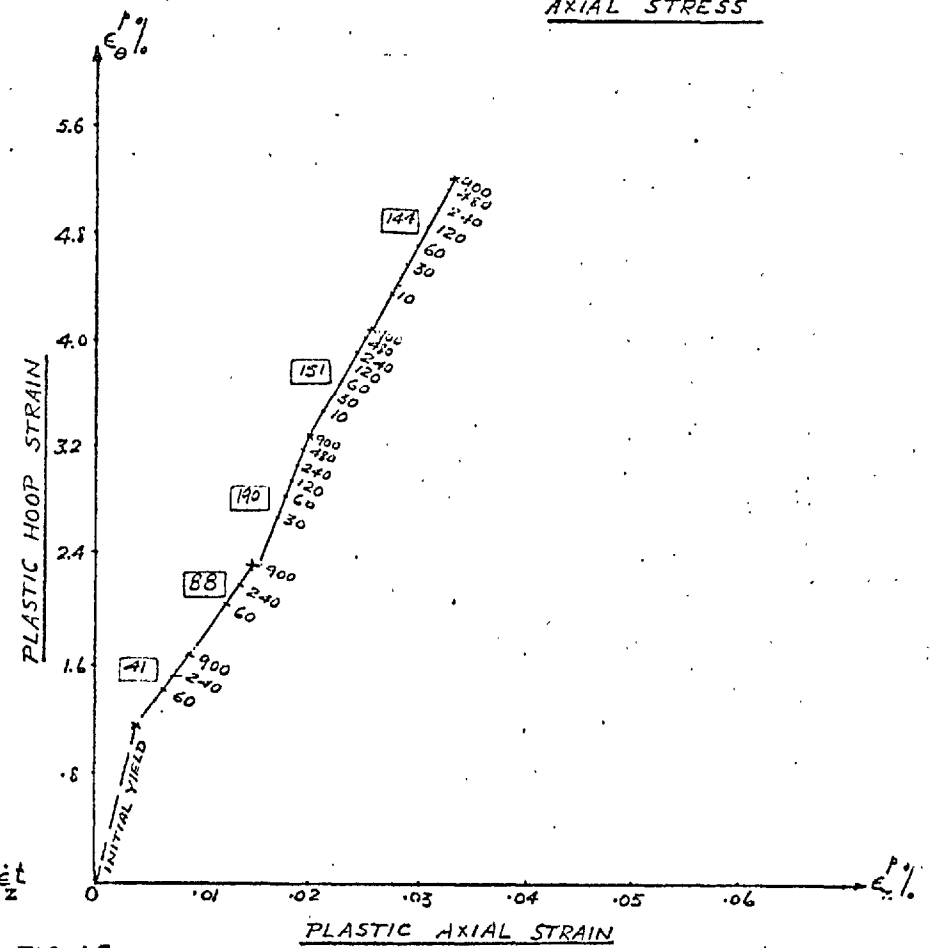
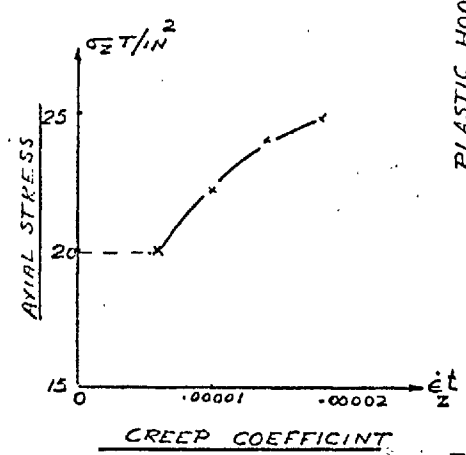
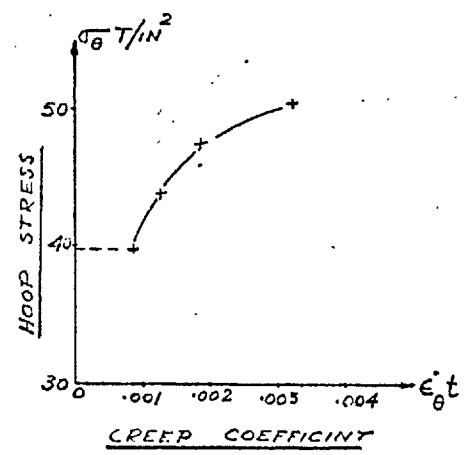
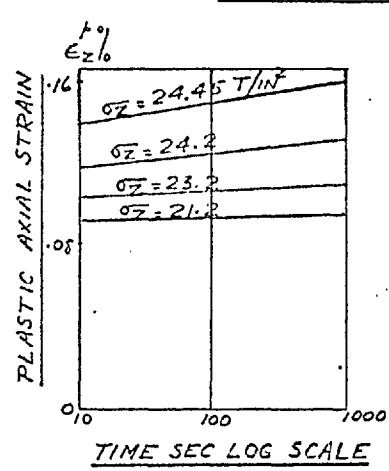
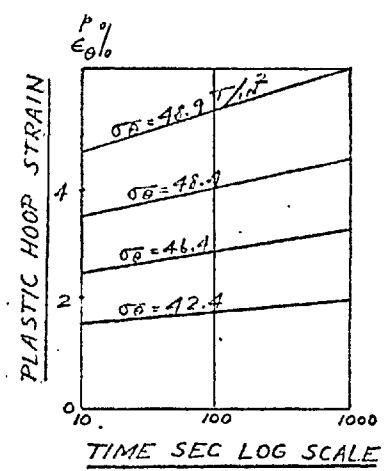
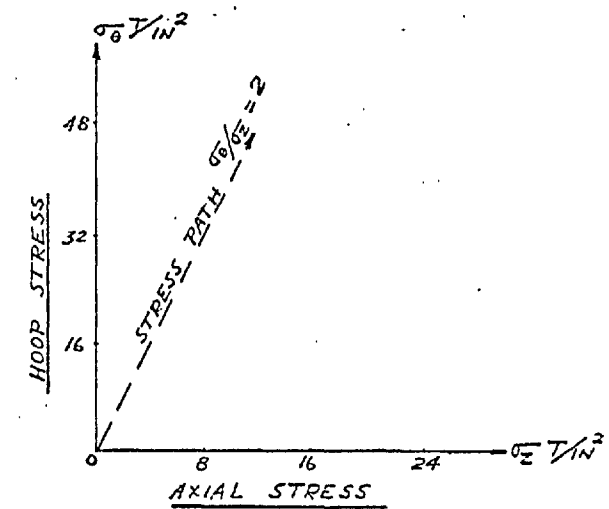
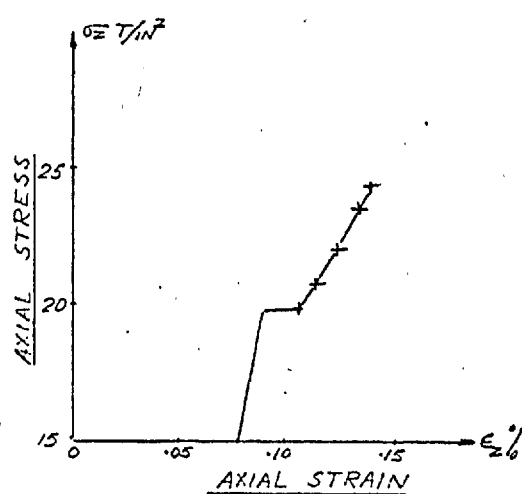
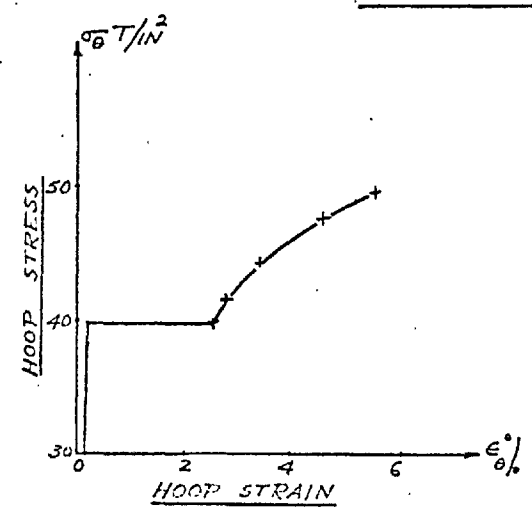


FIG. 46.

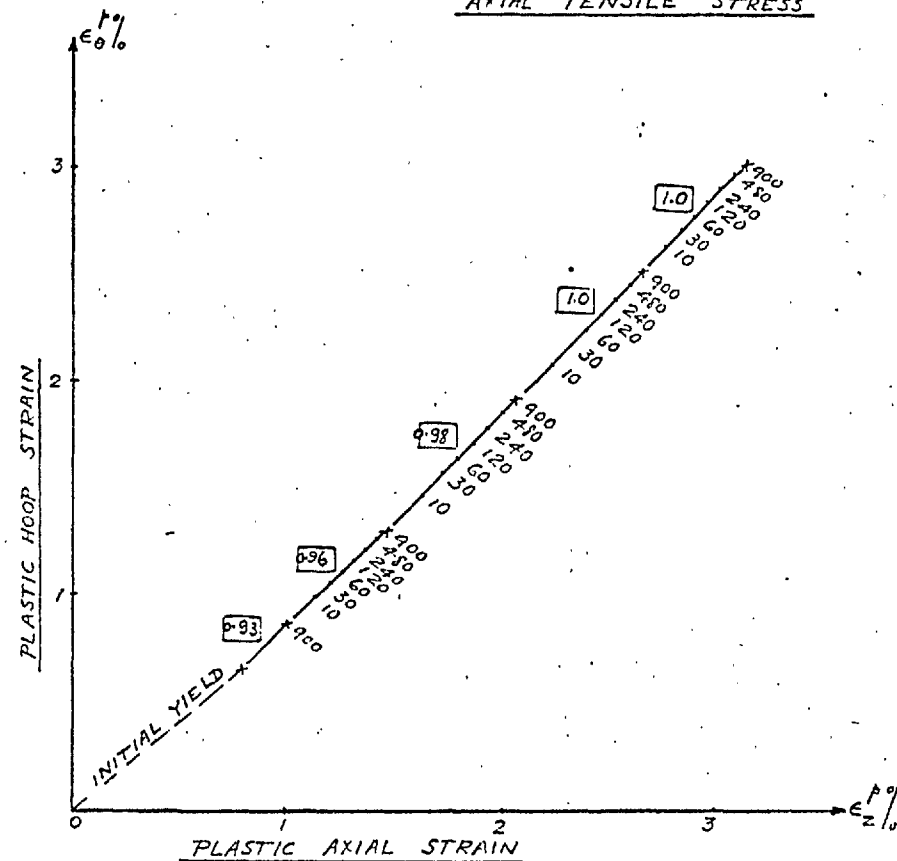
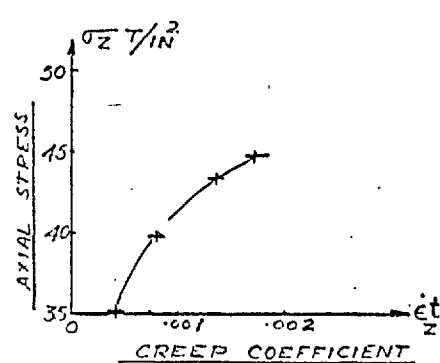
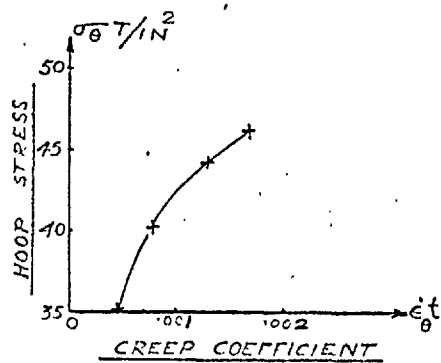
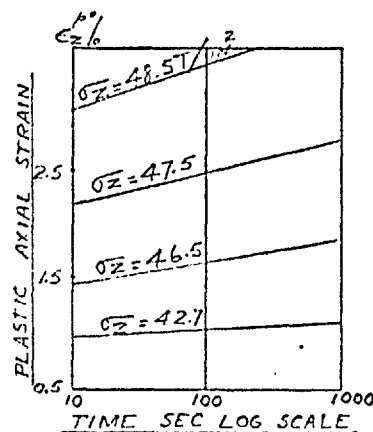
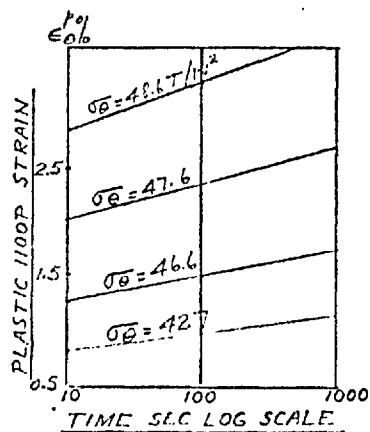
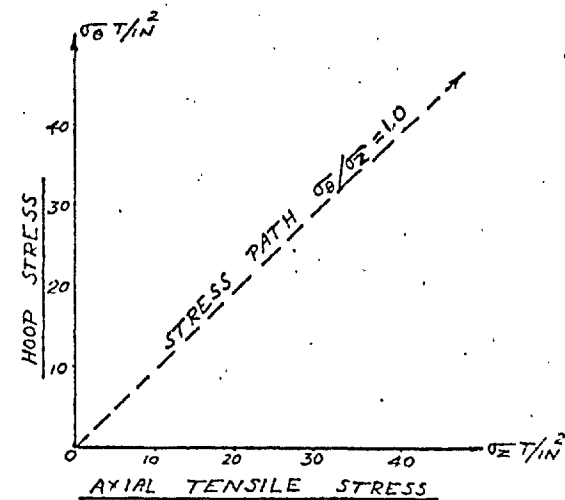
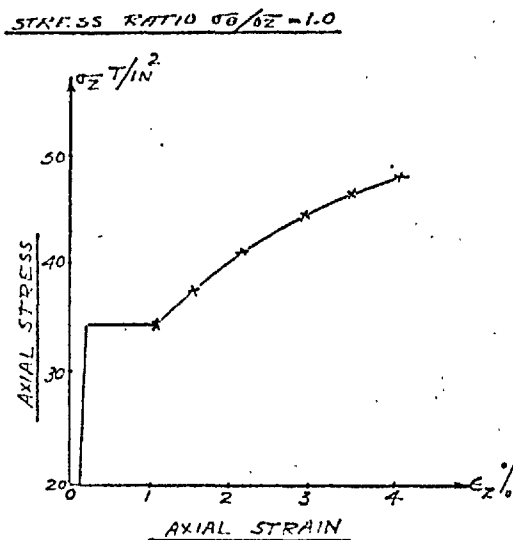
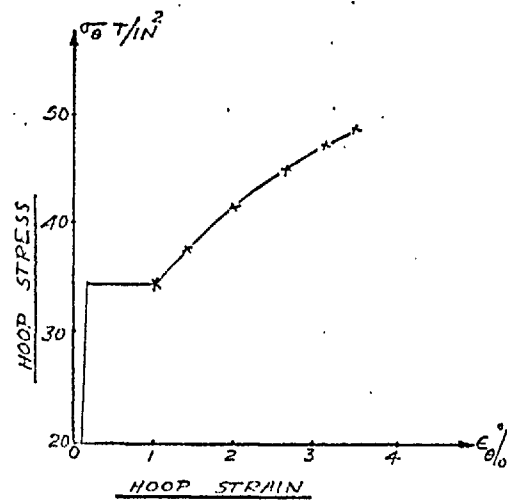


FIG. 46.

FIG. 47.

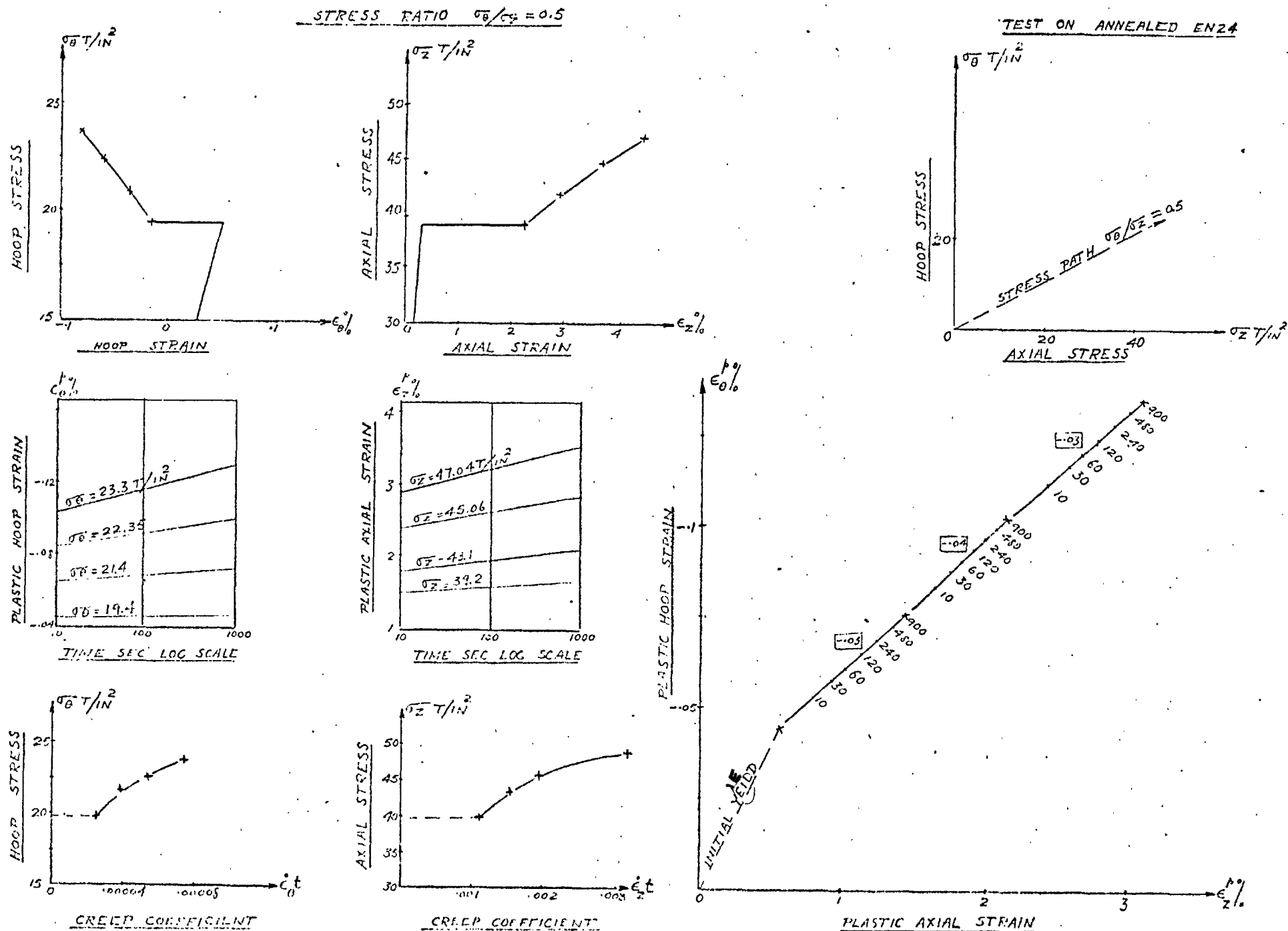


FIG. 47.

FIG. 48.

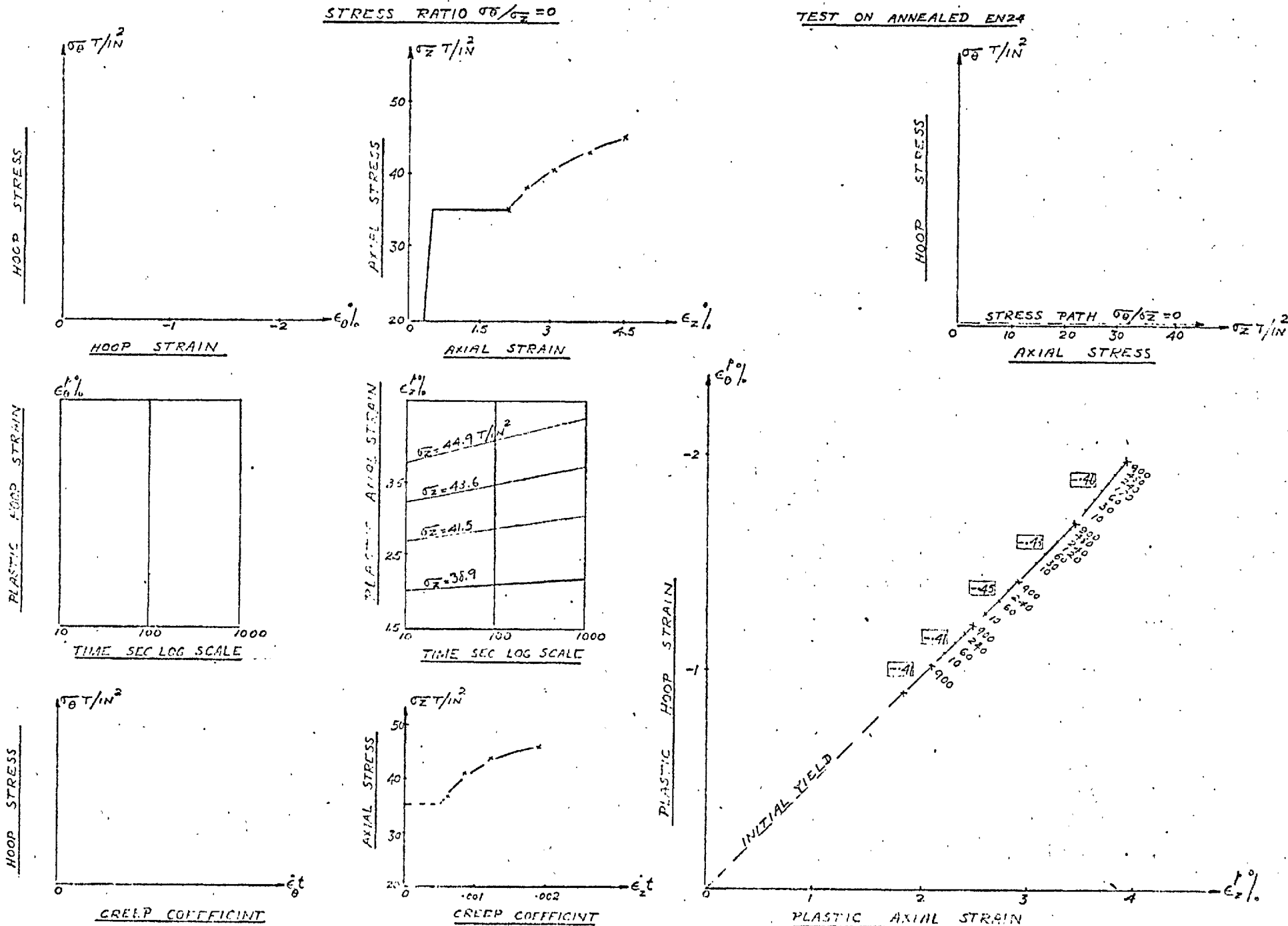


FIG. 48.

FIG. 49.

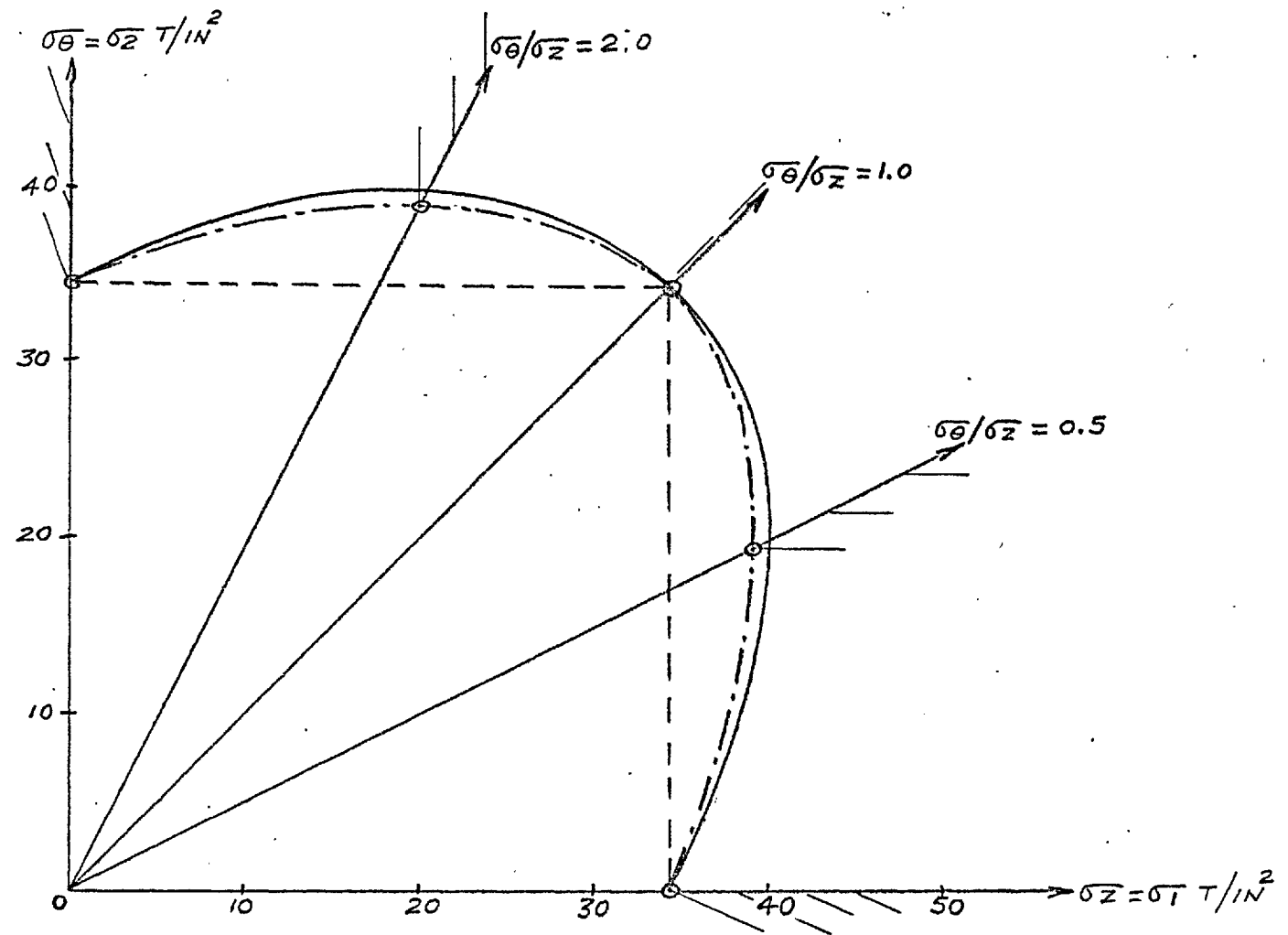


FIG. 49.

YIELD LOCUS FOR ANNEALED MATERIAL
UNDER AXIAL TENSION-HOOP TENSION.

EXPERIMENT — — — — —

MAXWELL-MISES —————

TRESCA — — — — —

ANNEALED EN24

FIG. 50.

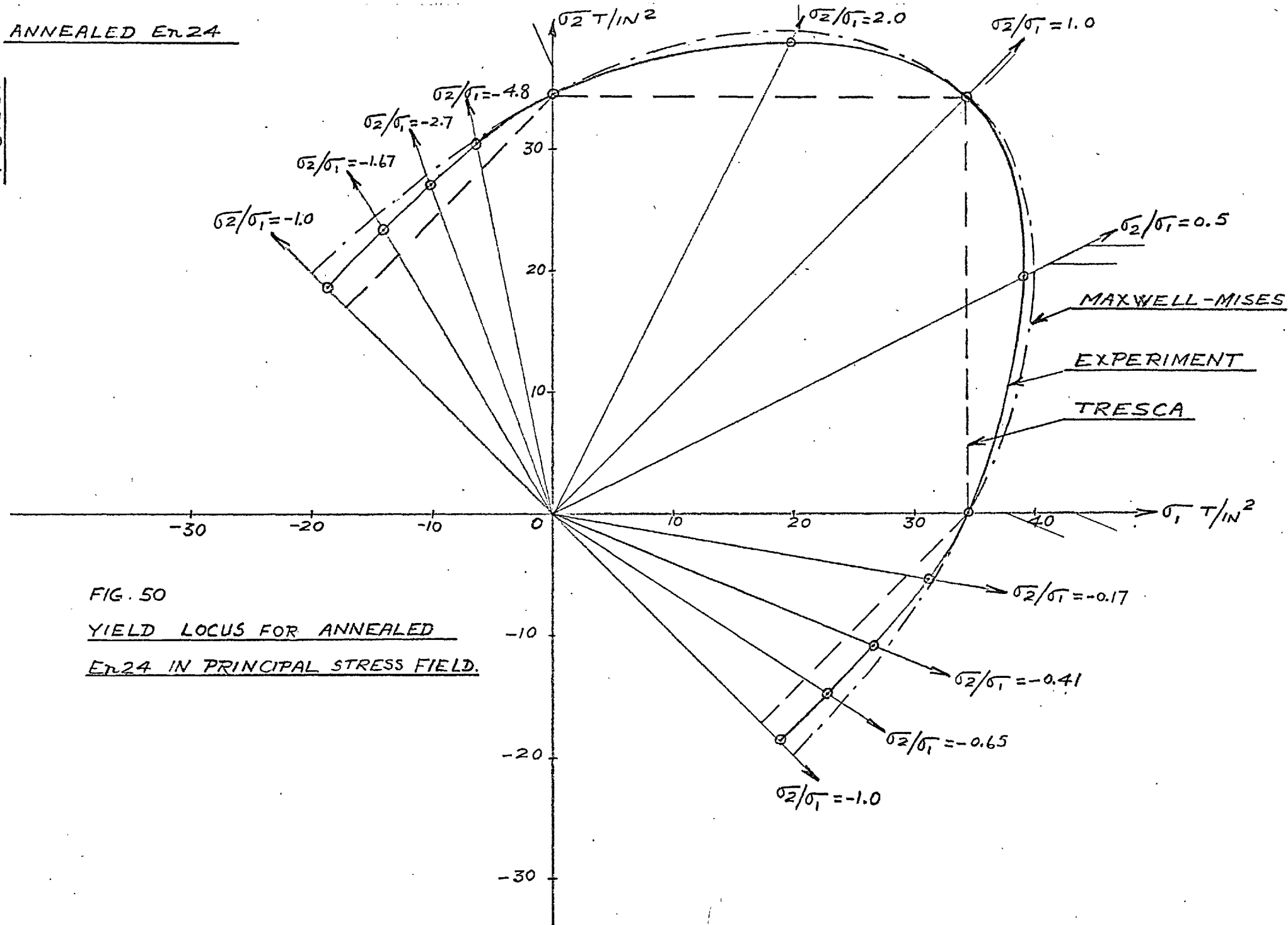


FIG. 50

YIELD LOCUS FOR ANNEALED
EN24 IN PRINCIPAL STRESS FIELD.

FIG. 51.

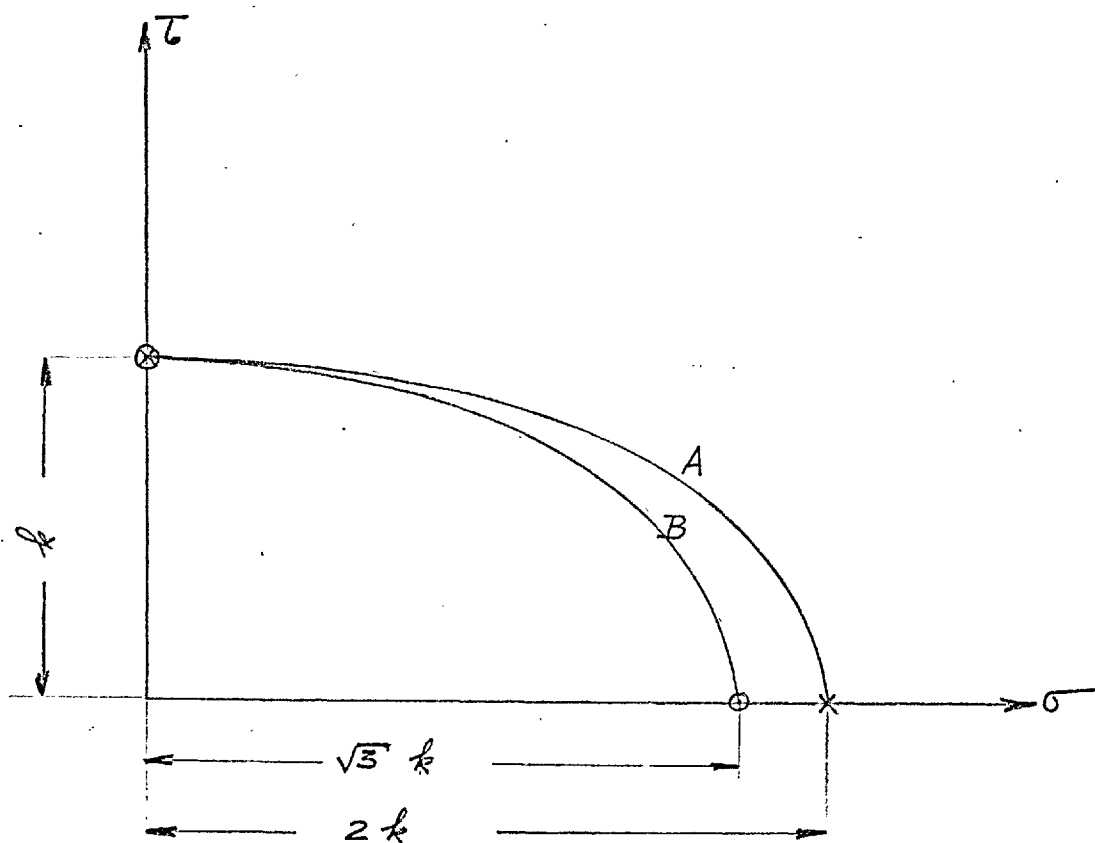


Fig.51. Comparison of Tresca and Maxwell-Mises criteria.

FIG. 52.

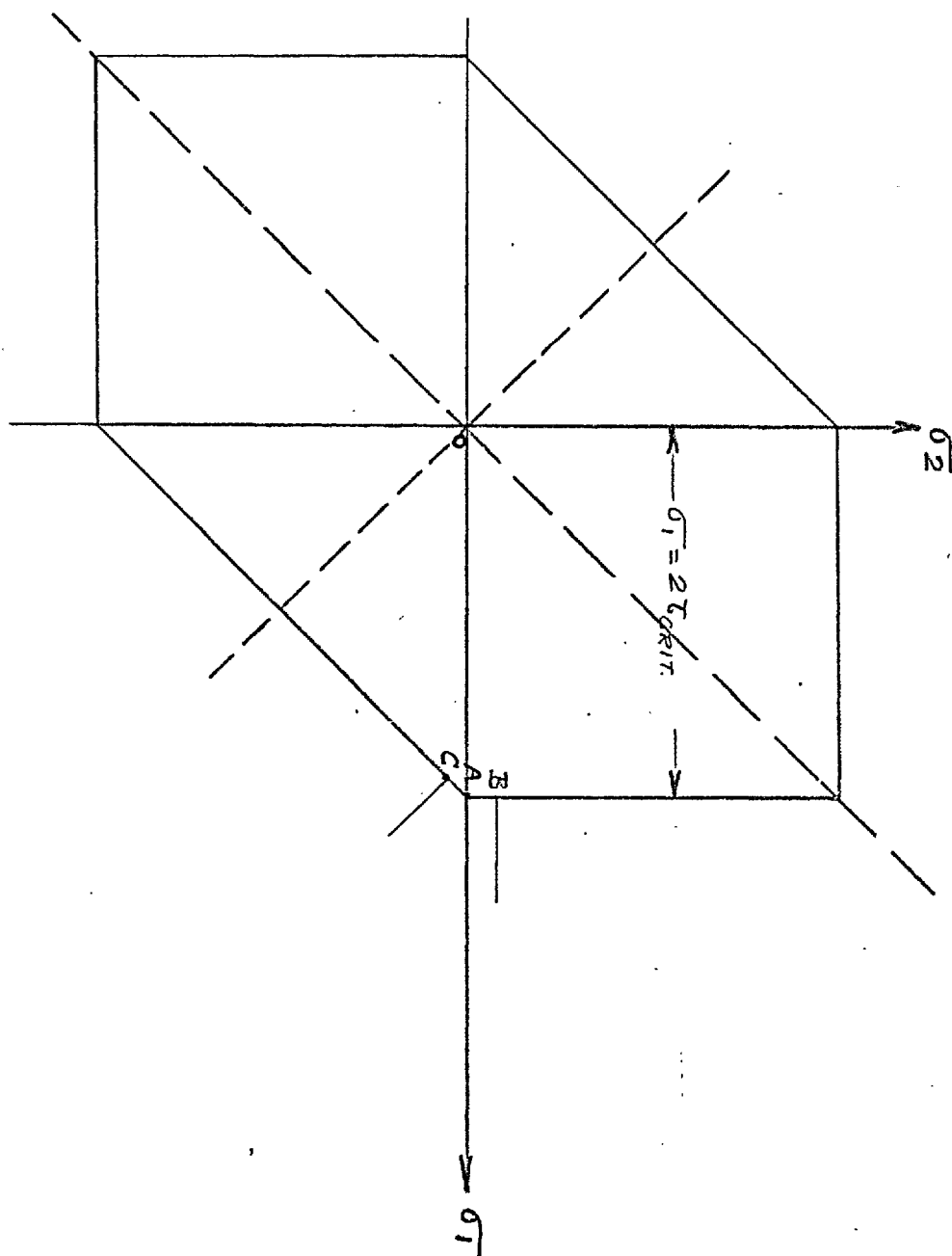


Fig. 52. Yield locus for the Tresca law on $\sigma_1 \sigma_2$ plane ($\sigma_3 = 0$)

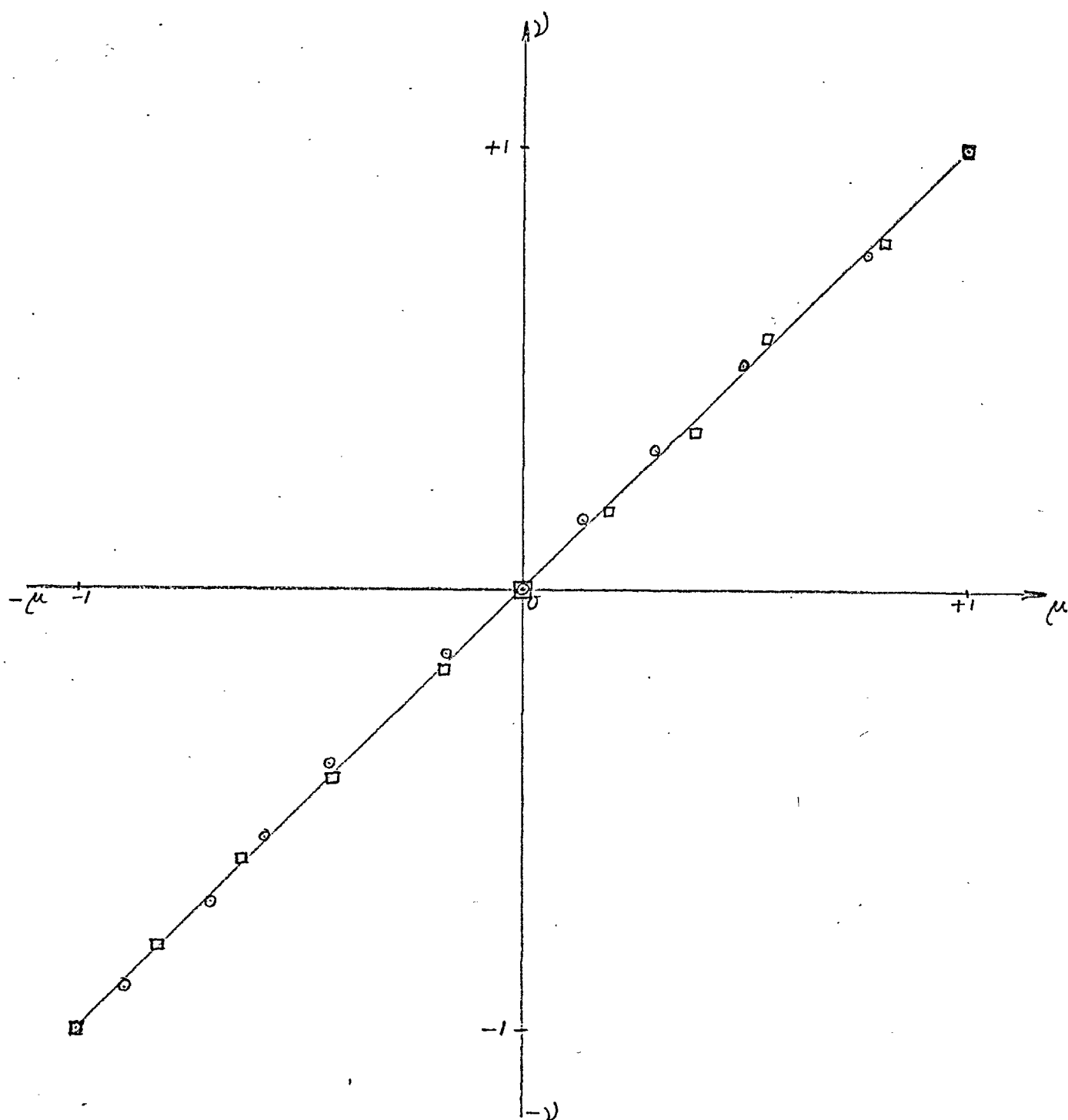


Fig. 53. Relation between μ and ν for annealed En24 under combined axial tension-injunct pressure.

$\odot$ $\sigma_\theta / \sigma_z = 0 \rightarrow 1$

$\square$ $\sigma_\theta / \sigma_z = 1 \rightarrow \infty$

FIG. 54.

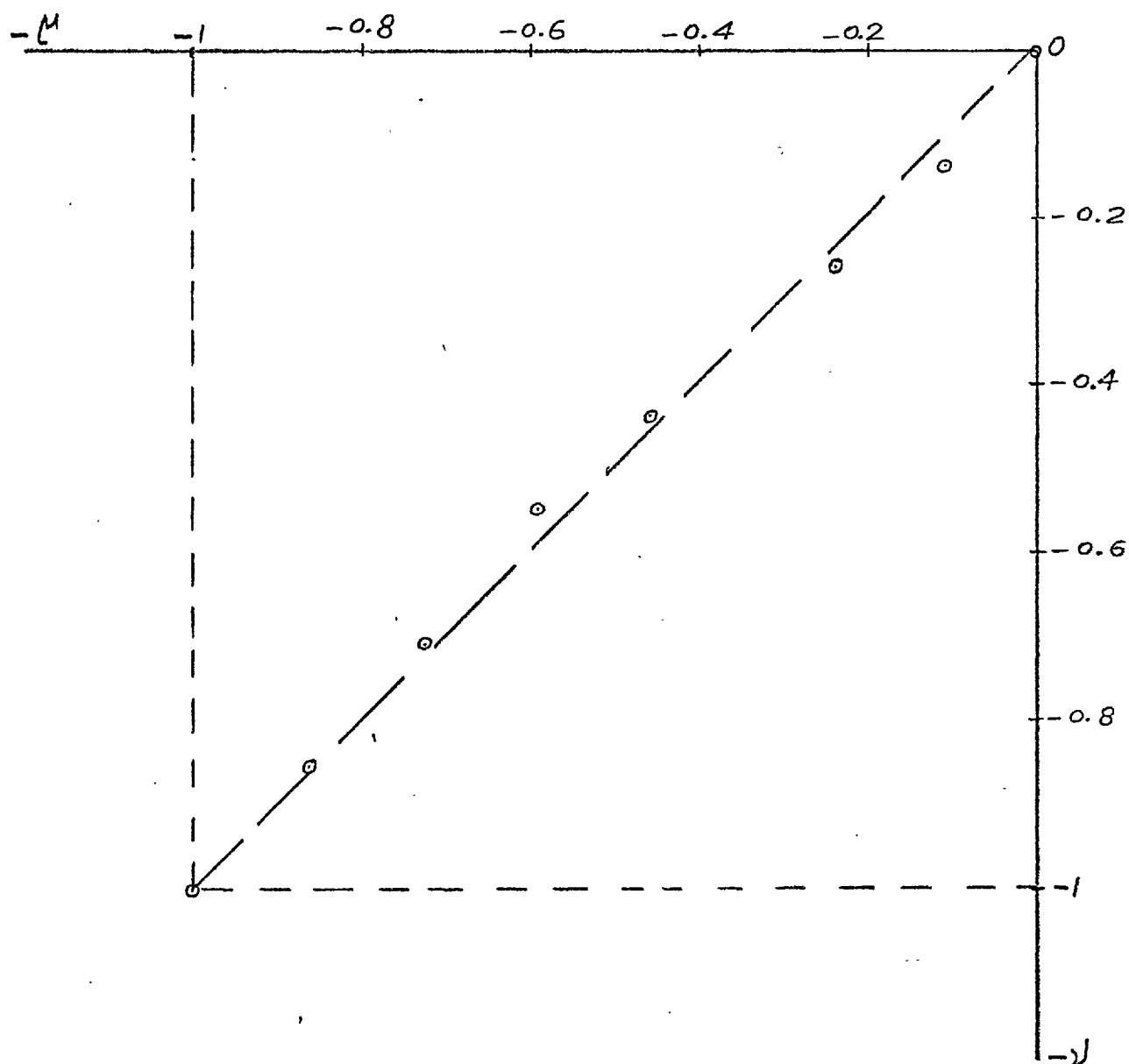


Fig.54. μ and ν for axial tension-torsion combined (annealed material).

FIG. 55.

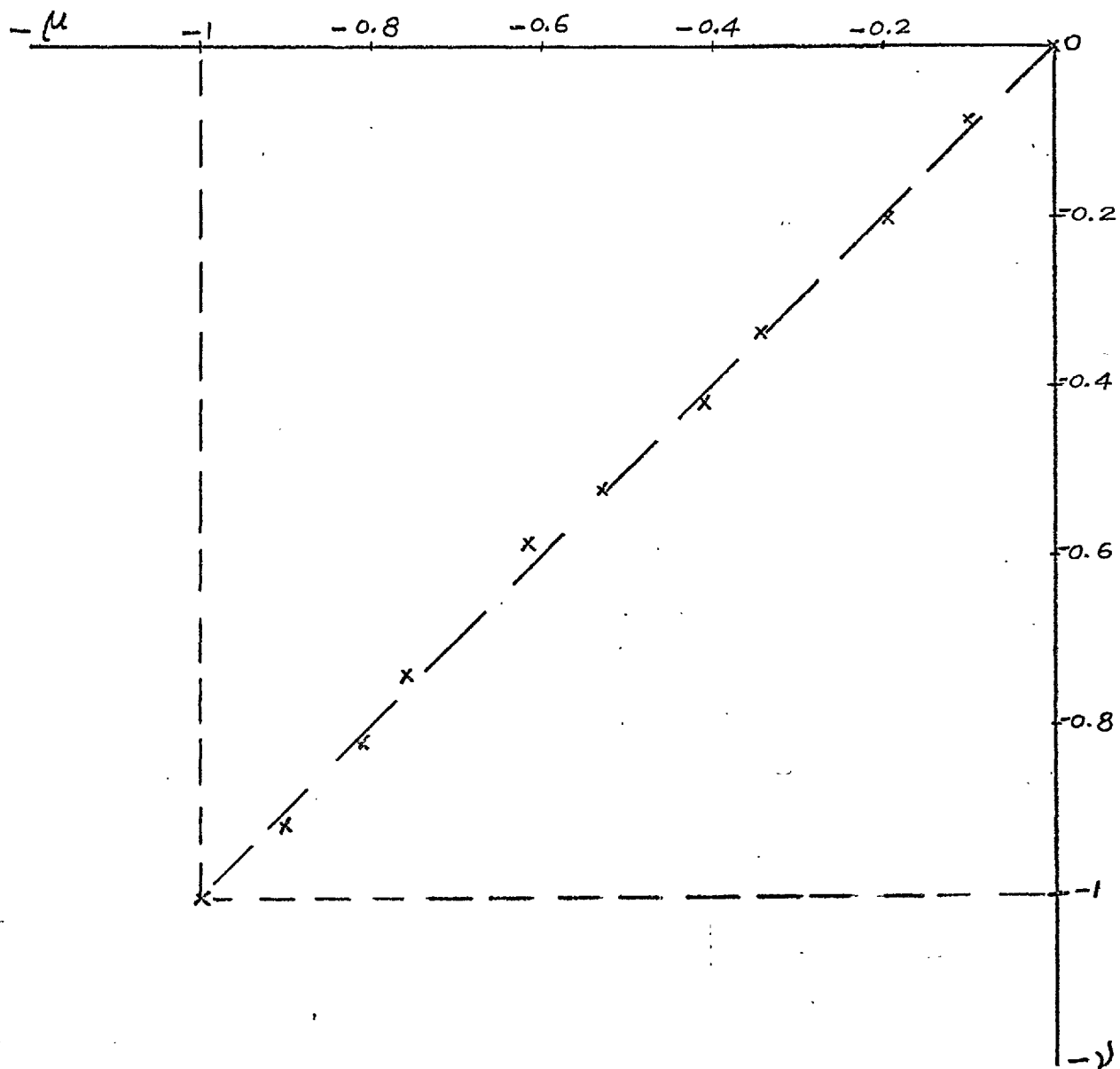


Fig.55. μ and ν for hoop tension-torsion combined (annealed material).

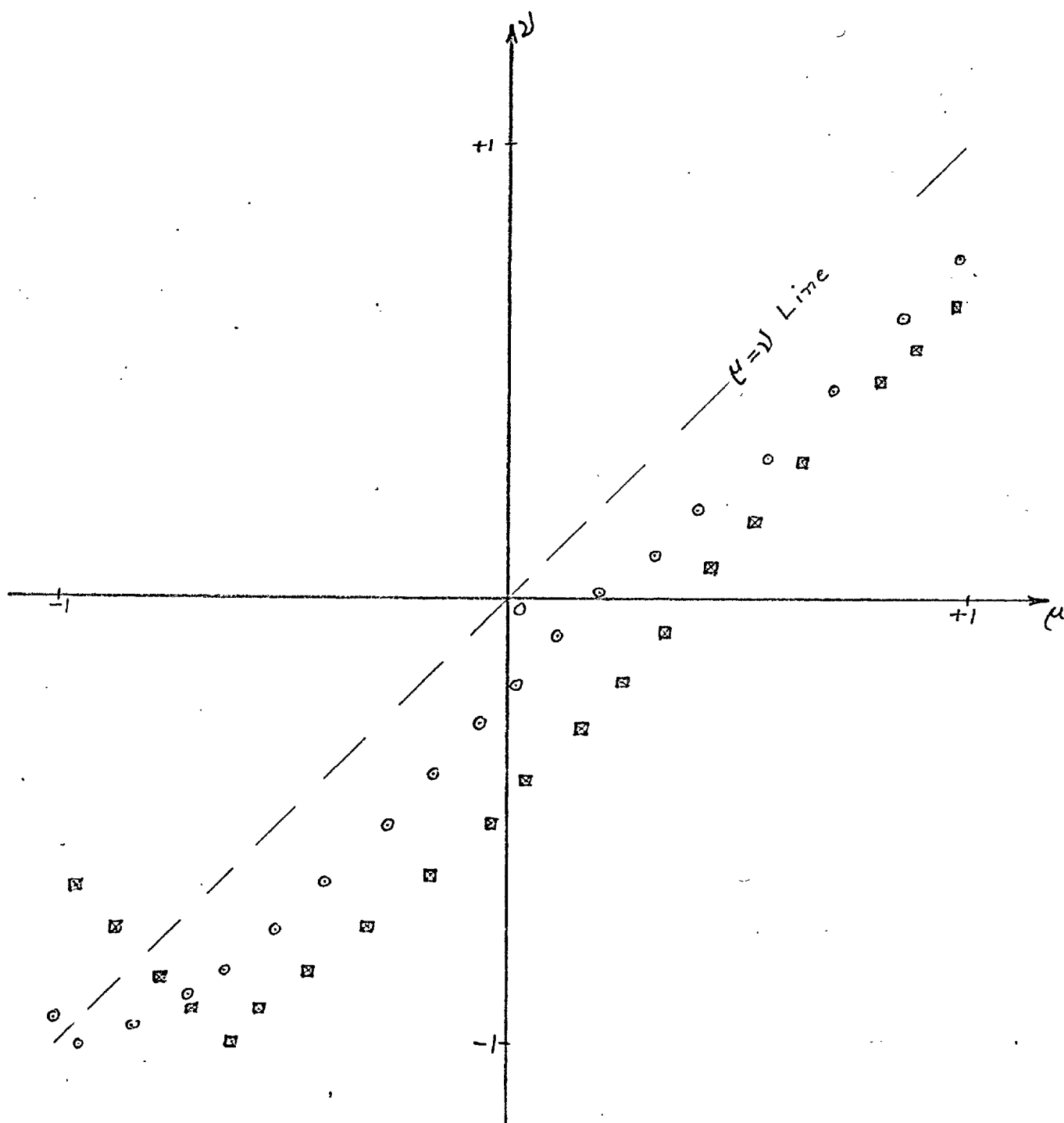


Fig.56.Relation between μ and ν for axial pre-strained material

○ plastic pre-strain=2.61% and $\sigma_\theta / \sigma_z = 1 \longrightarrow \infty$

⊠ plastic pre-strain=3.45% and $\sigma_\theta / \sigma_z = 1 \longrightarrow \infty$

FIG. 57.

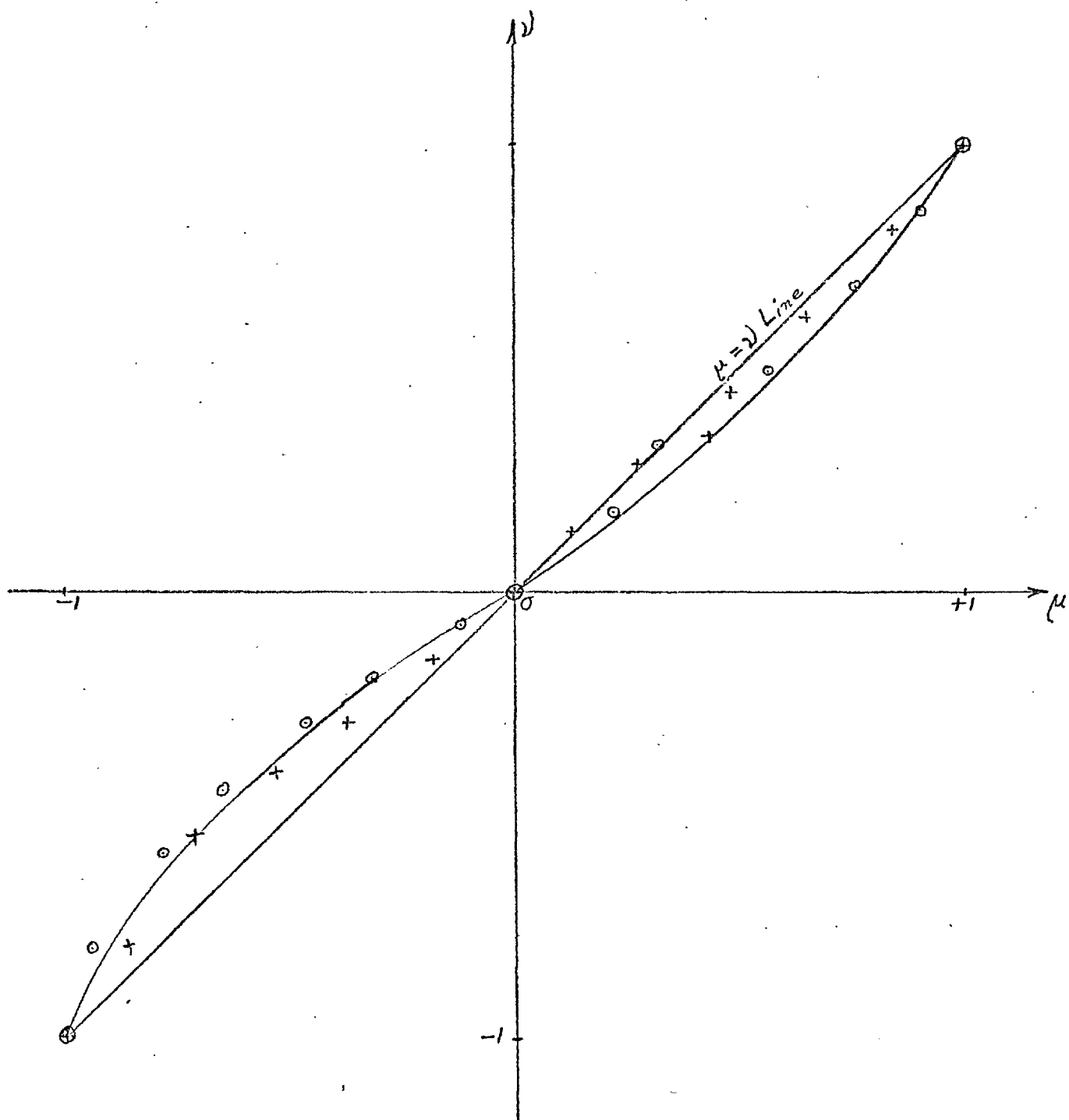


Fig.57. Relation between μ and v for En24 pre-strained in torsion

for $\sigma_\theta / \sigma_z = 0 \longrightarrow 1$

for $\sigma_\theta / \sigma_z = 1 \longrightarrow \infty$

FIG. 58.

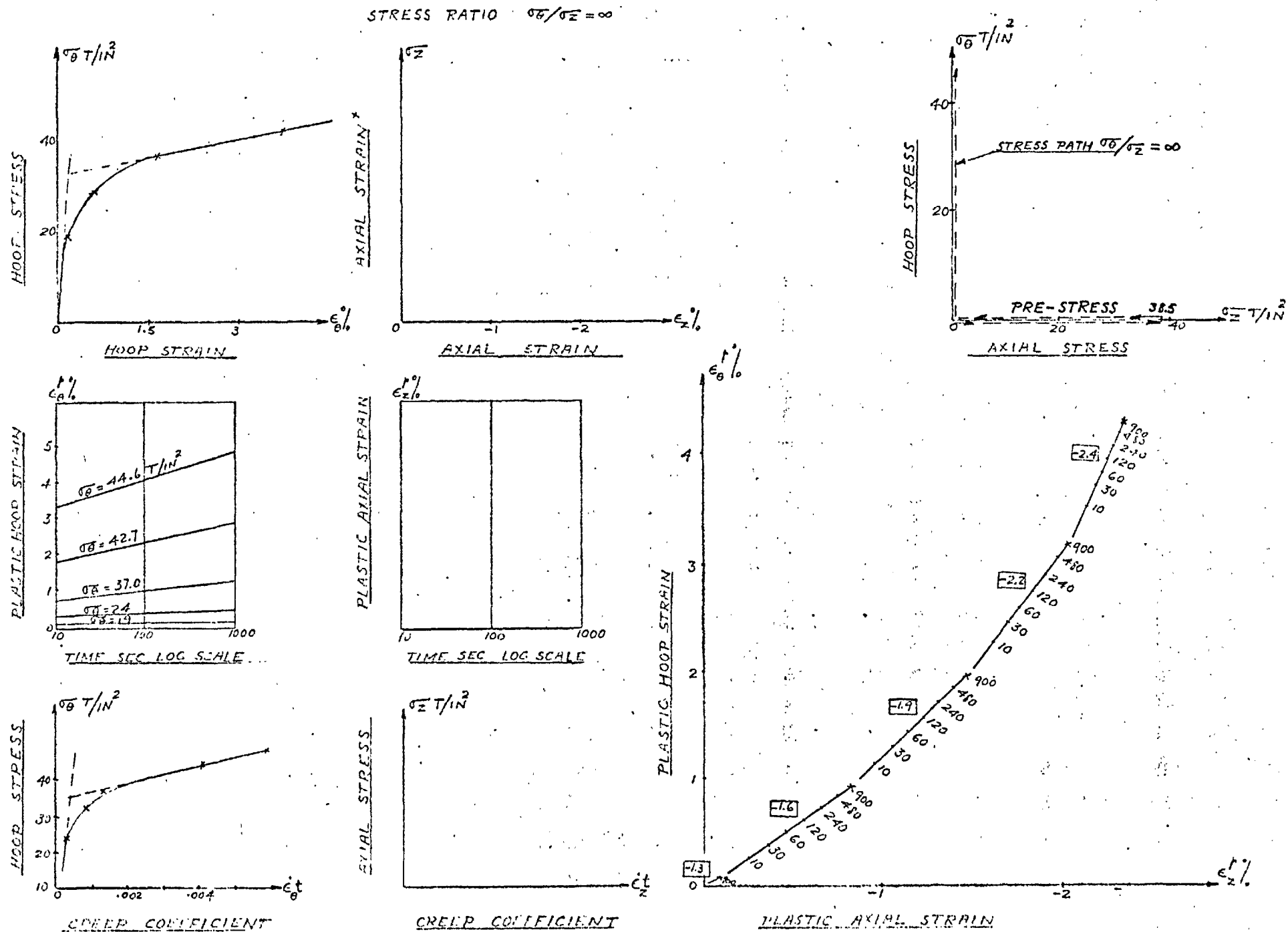


FIG. 59.

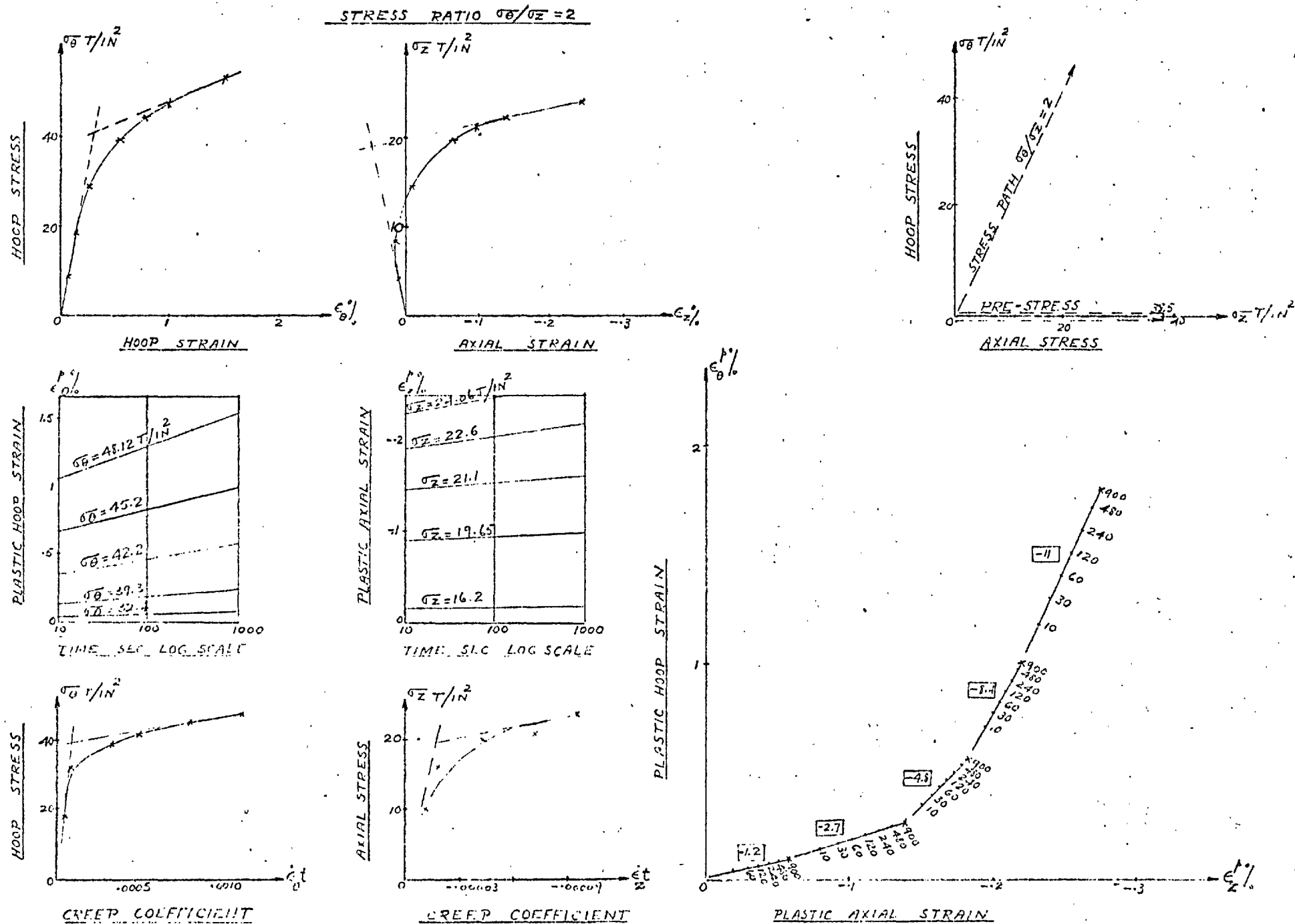


FIG. 59.

FIG. 60.

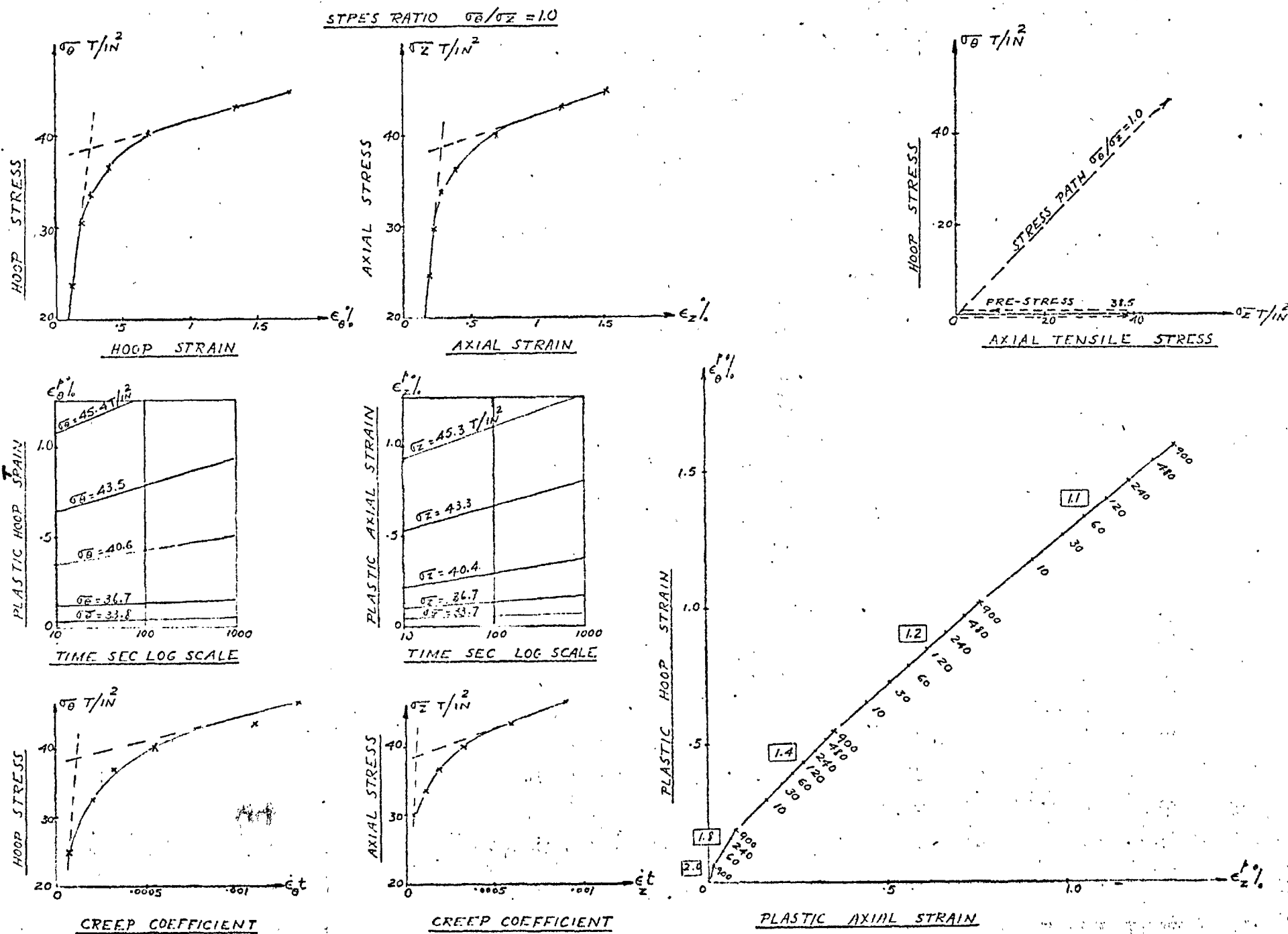
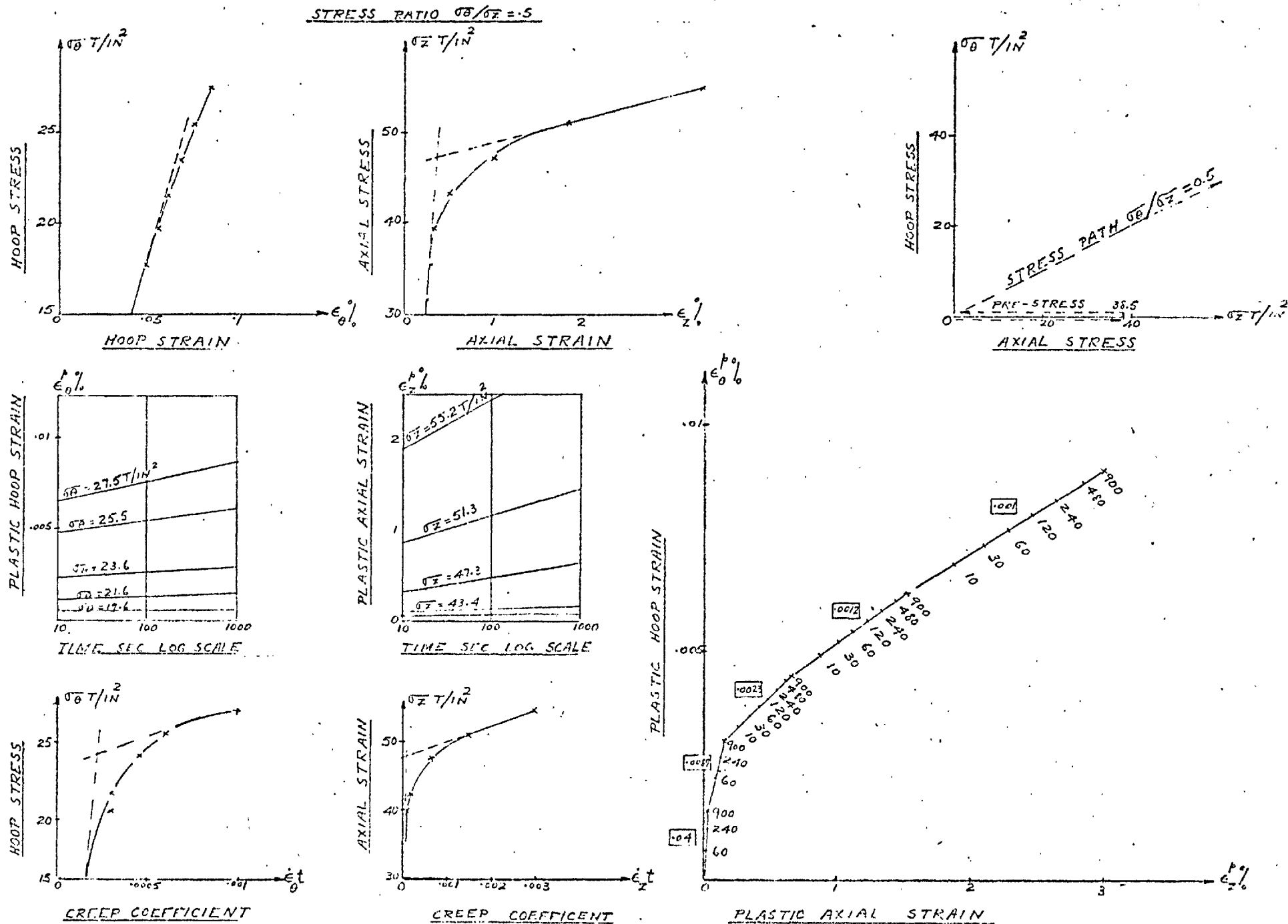


FIG. 60.

FIG. 61.



$$\epsilon_{OZ}^p = 2.61\%$$

$$\sigma_{OZ} = 38.5 \text{ T/IN}^2$$

FIG. 62.

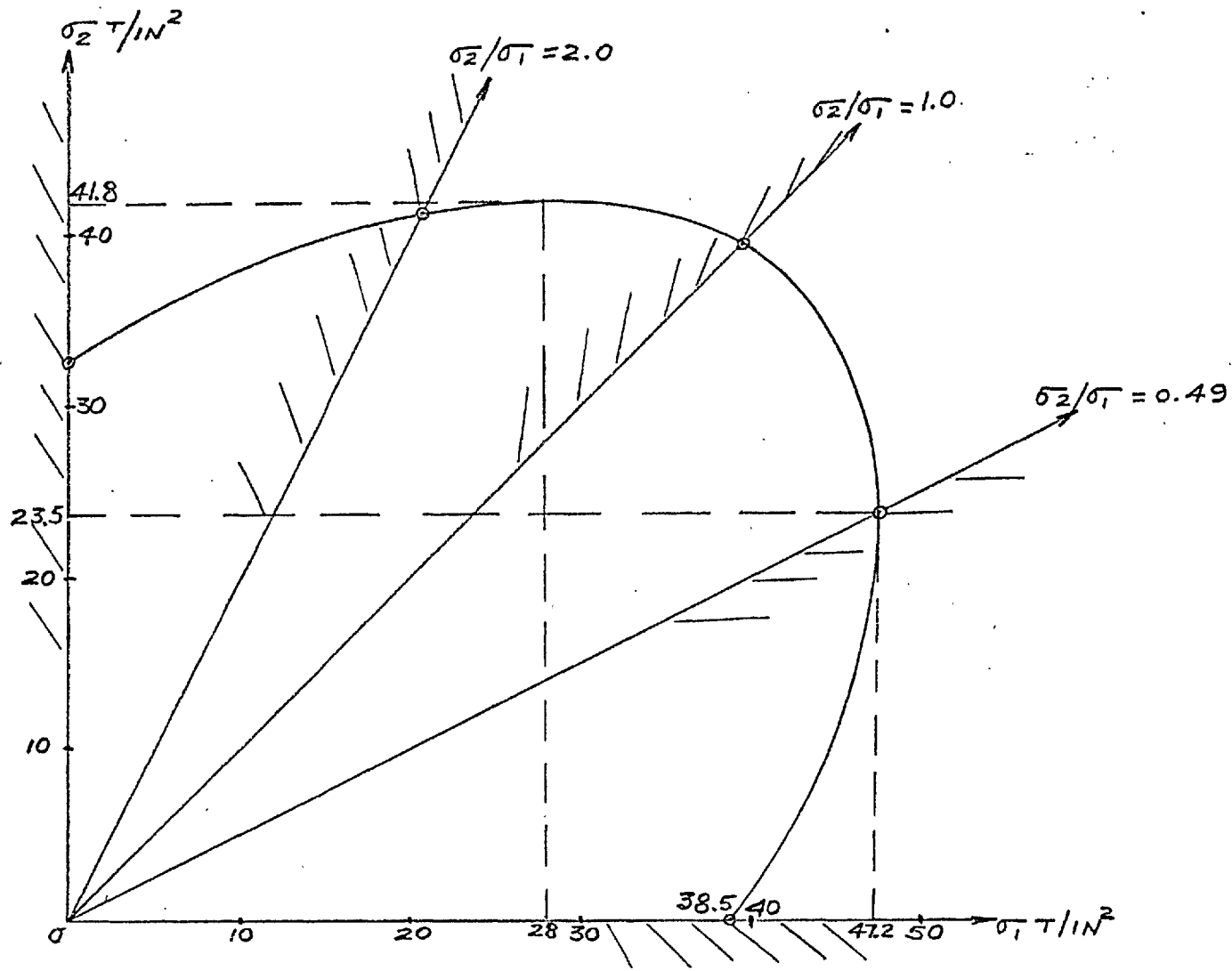


FIG. 62. YIELD LOCUS FOR En24 PRE-STRESSED
IN AXIAL DIRECTION UPTO 38.5 T/IN²

FIG. 63.

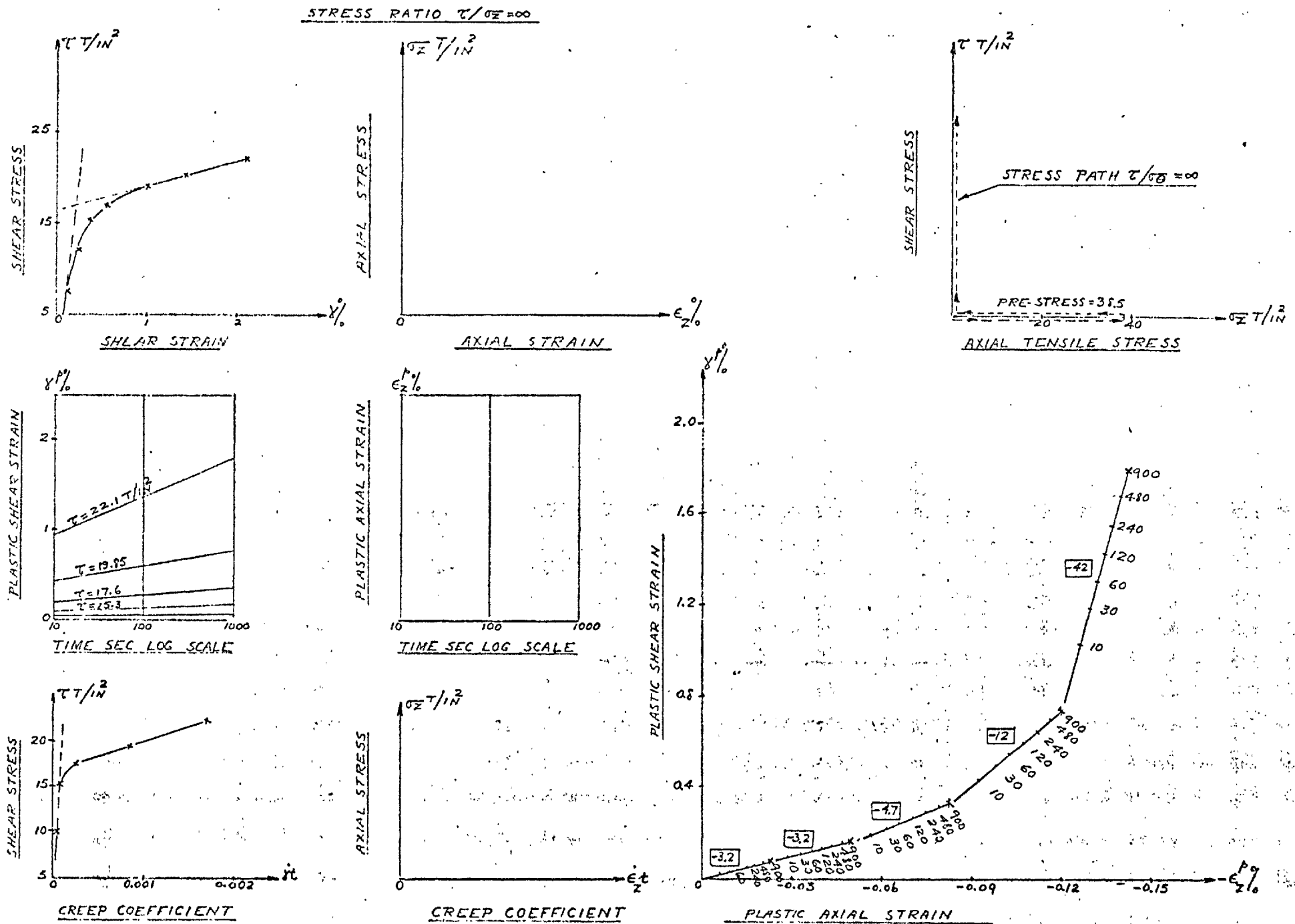


FIG. 63.

FIG. 64.

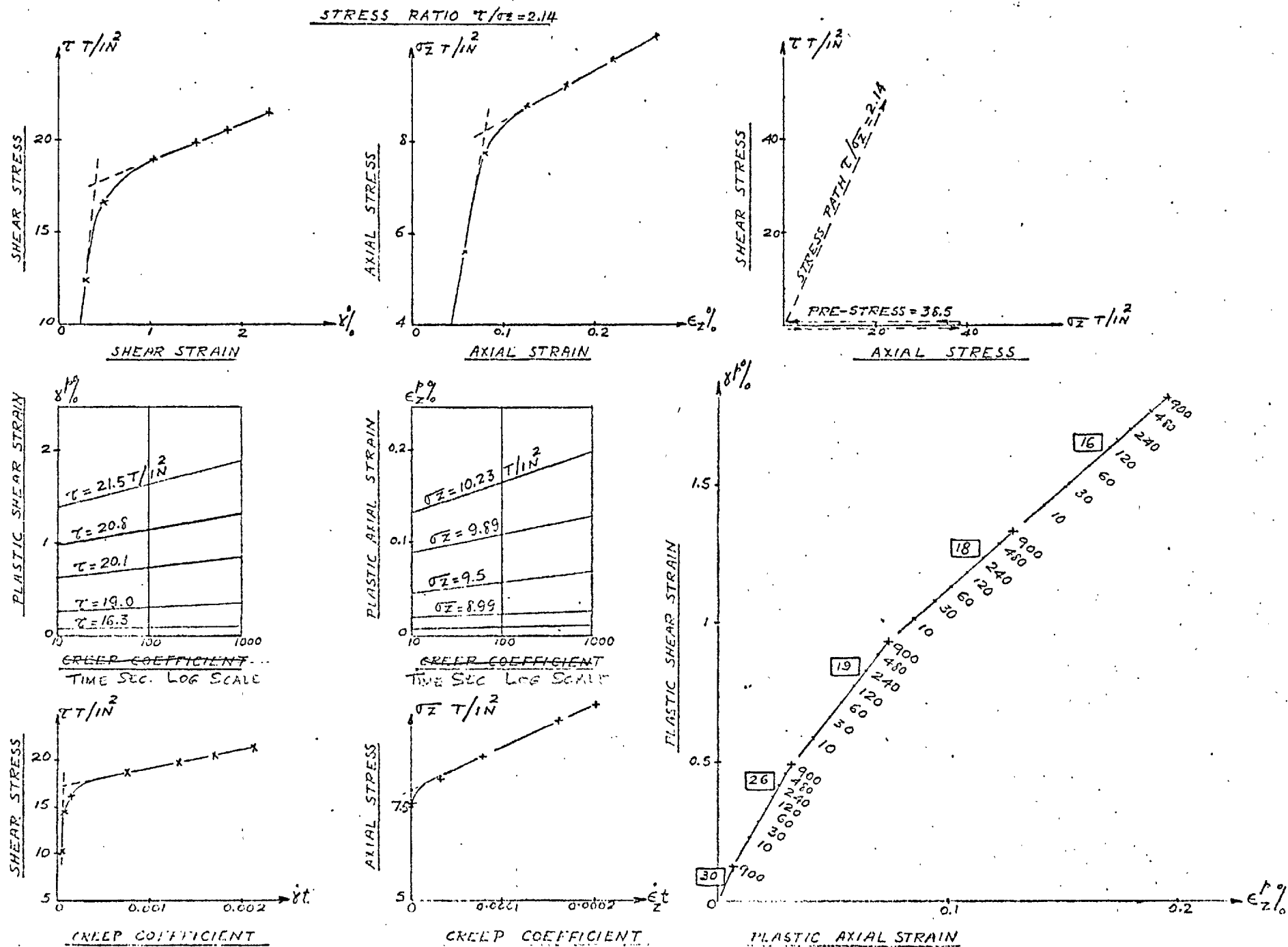


FIG. 64.

FIG. 65.

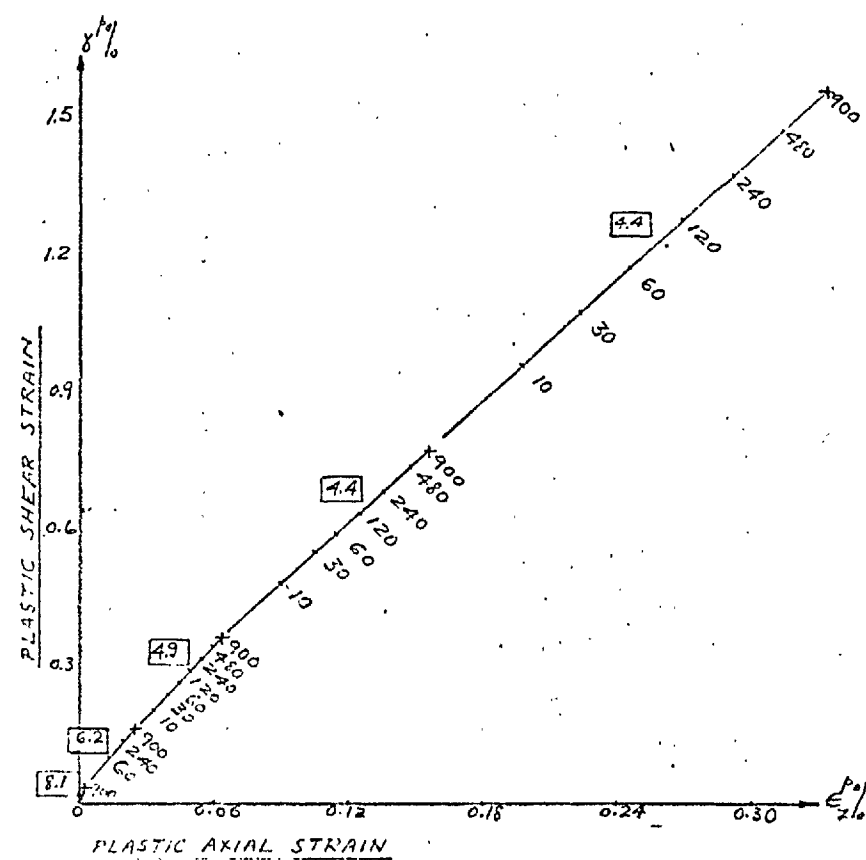
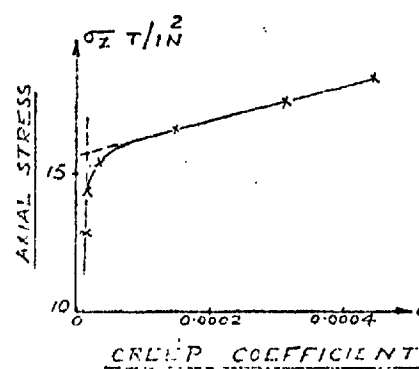
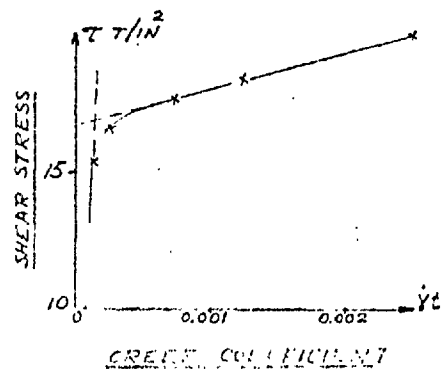
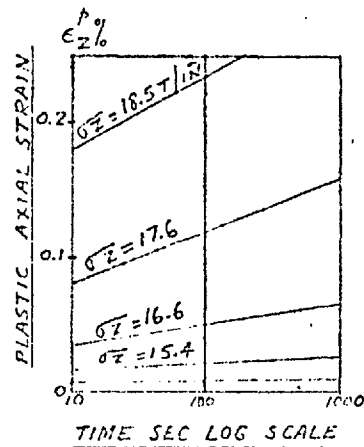
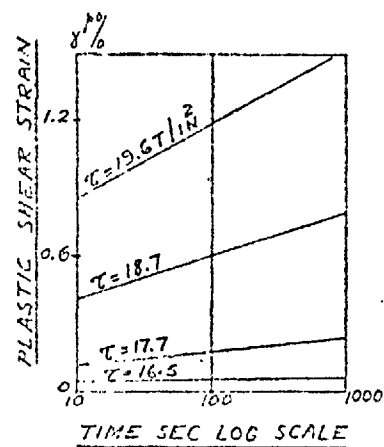
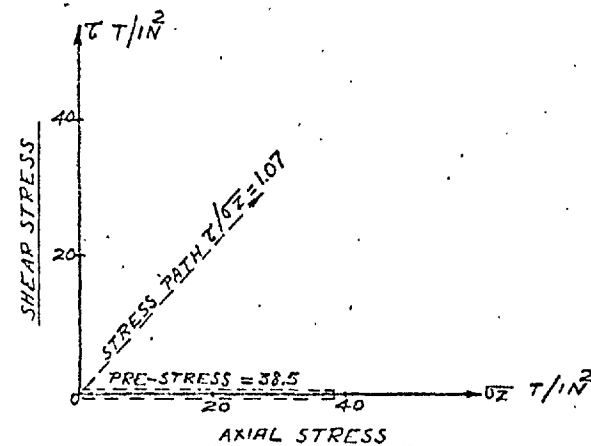
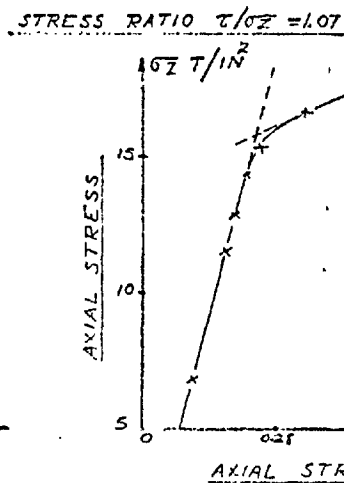
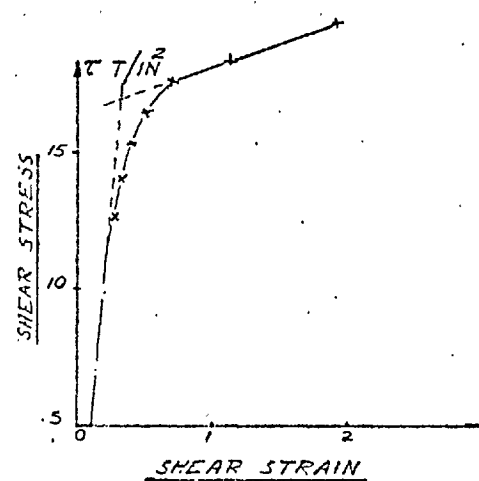


FIG. 66.

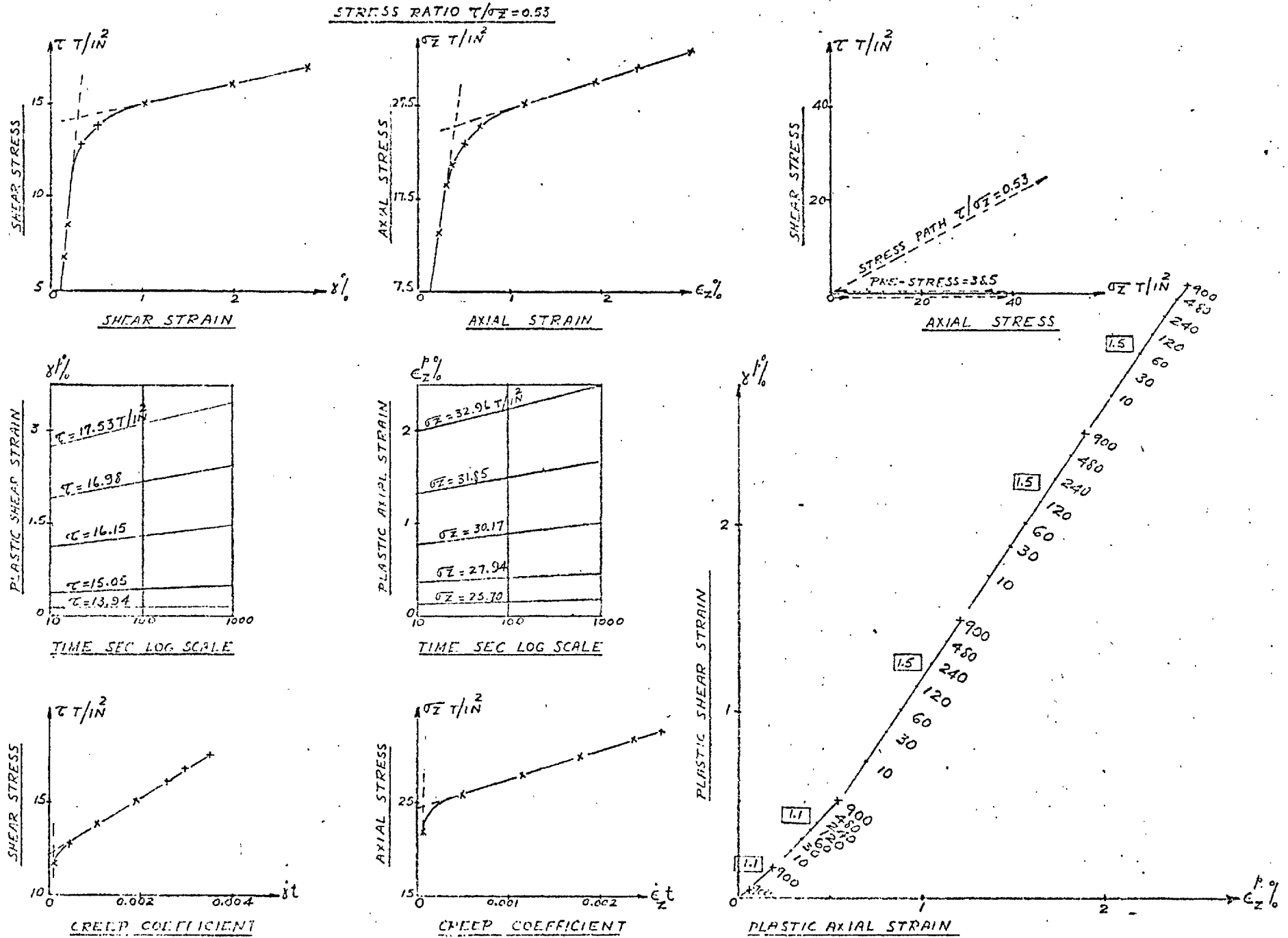


FIG. 66.

$p = 2.61\%$
 $\sigma_z = 38.5 \tau / \text{IN}^2$

FIG. 67.

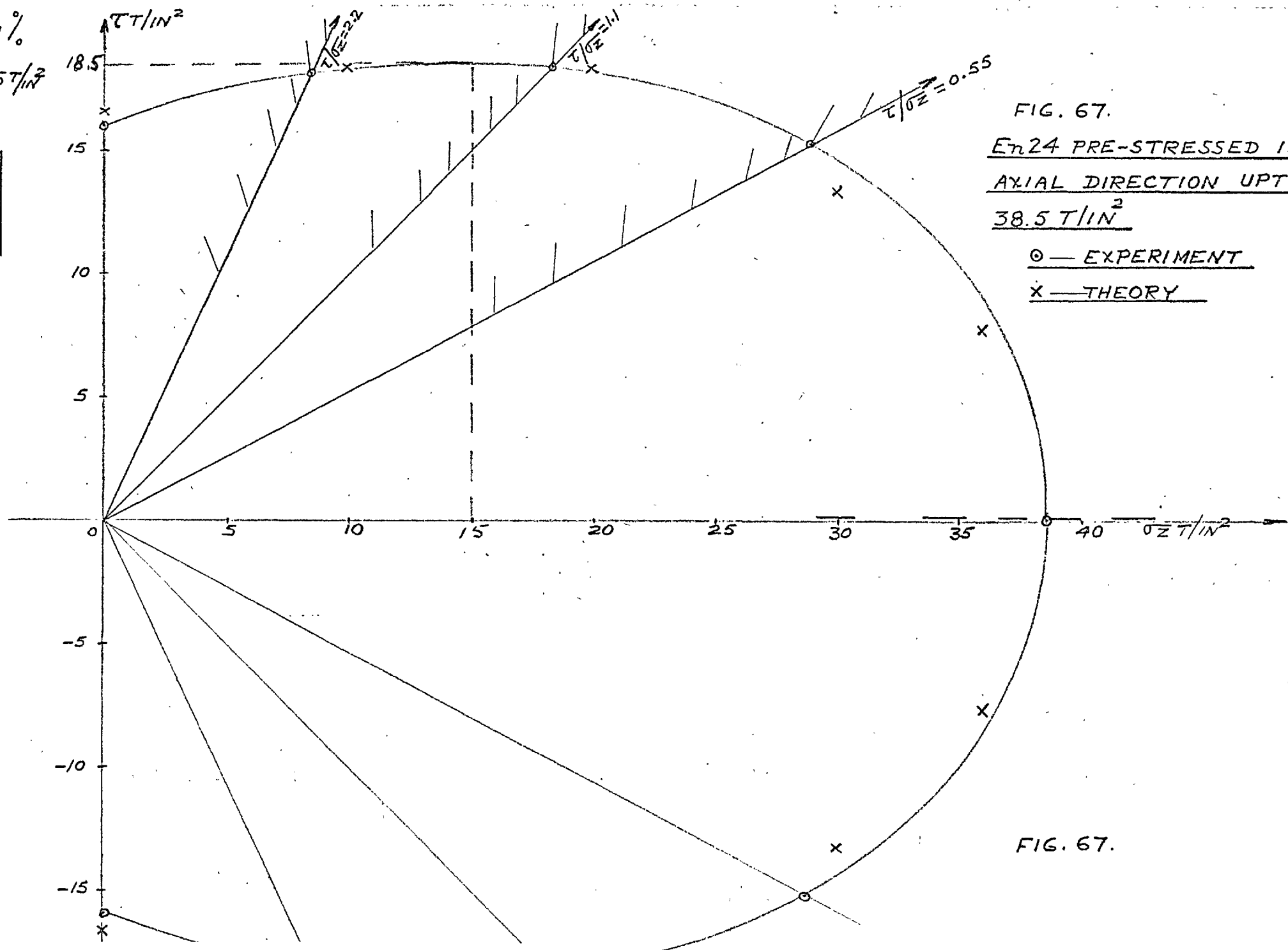


FIG. 67.
 EN24 PRE-STRESSED IN
 AXIAL DIRECTION UPTO
 $38.5 \tau / \text{IN}^2$

FIG. 67.

FIG. 68.

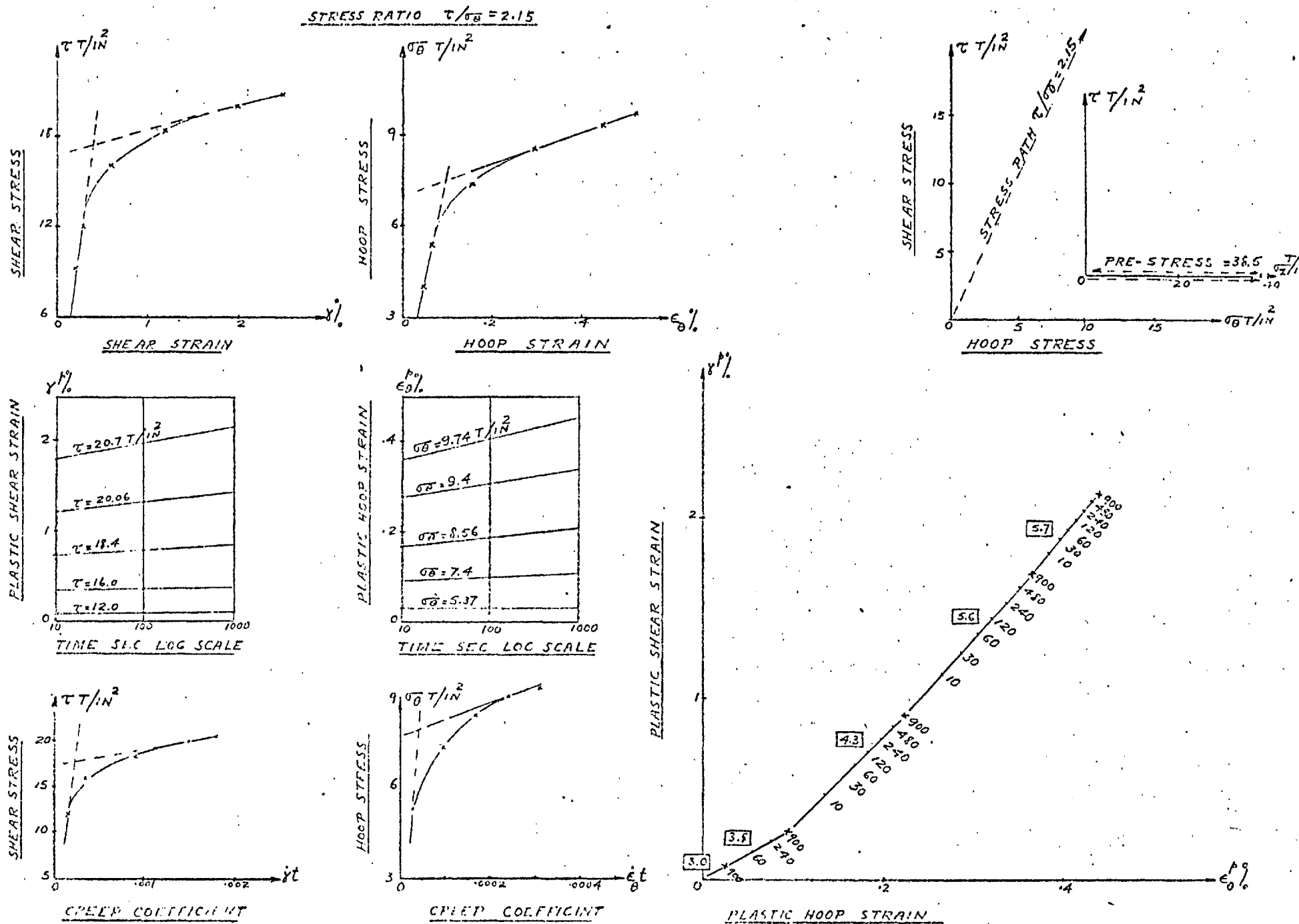


FIG. 68.

FIG. 69.

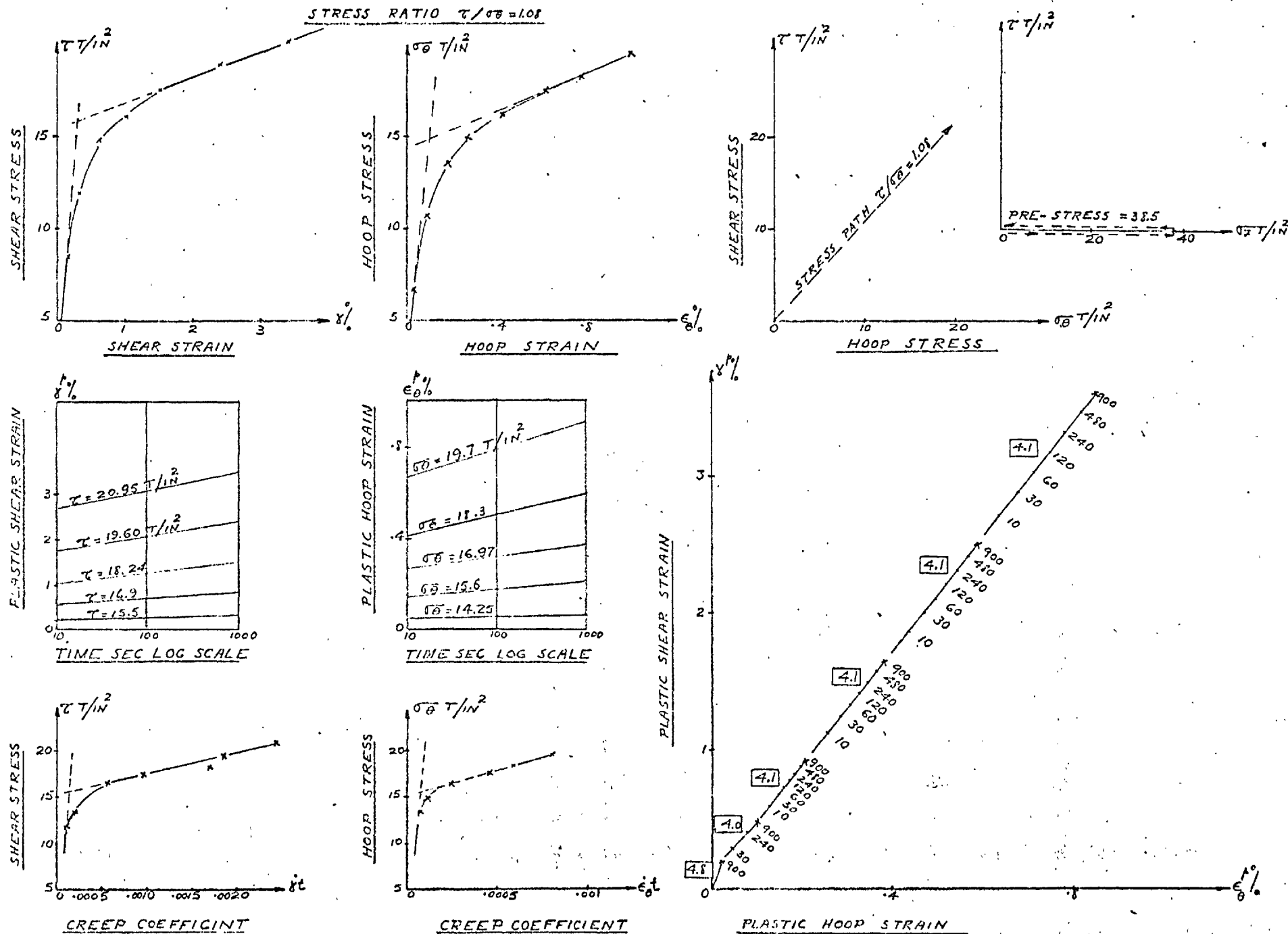


FIG. 69.

The graph shows the relationship between Shear Stress (τ) and Shear Strain for mild steel. The y-axis represents Shear Stress in T/in^2 , with marked values at 2.5, 7.5, and 12.5. The x-axis represents Shear Strain in percent, with marked values at 0, 1, 2, 3, and 4. The curve starts at the origin, rises steeply in a linear fashion, and then gradually curves downwards, indicating a non-linear relationship at higher strains. A dashed line is drawn tangent to the curve at the point where the stress is 12.5 T/in^2 and strain is approximately 1.5%.

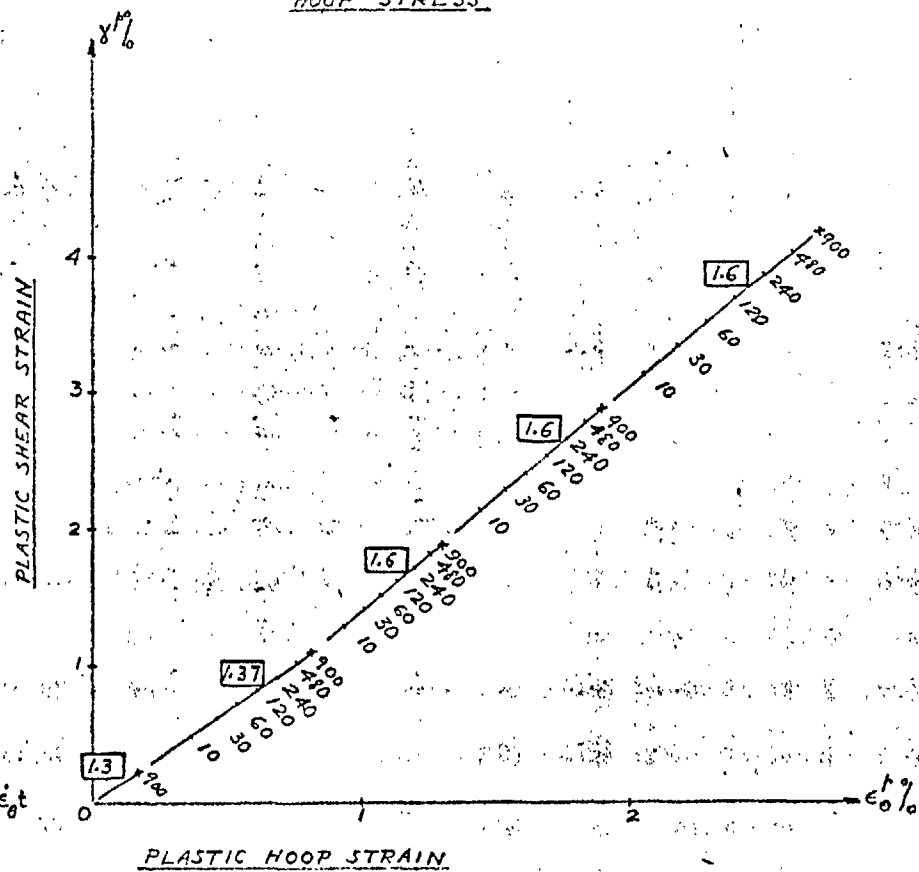
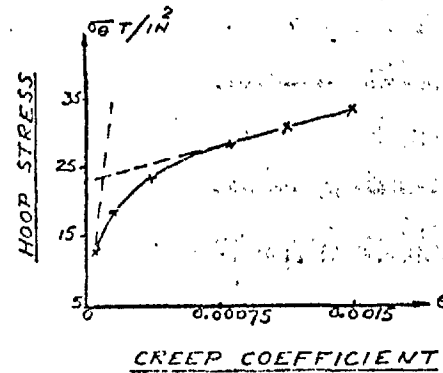
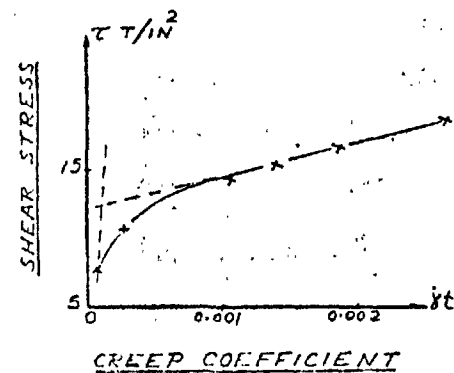
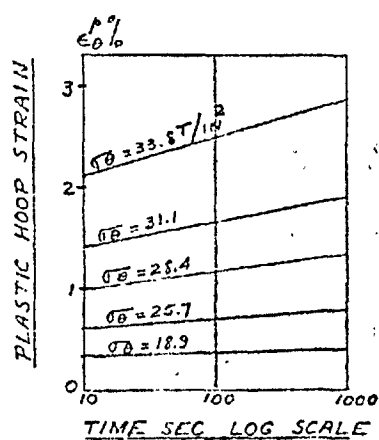
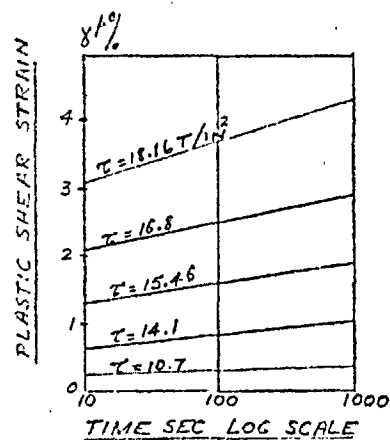
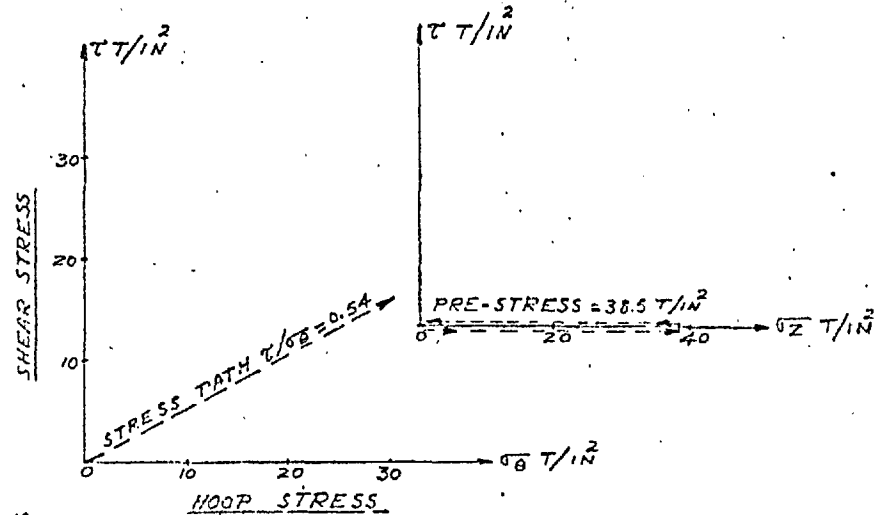
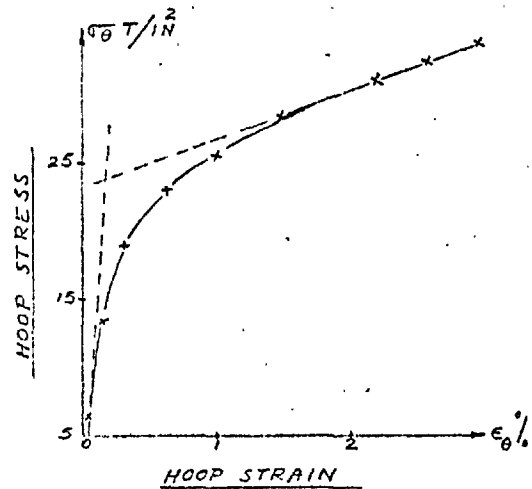
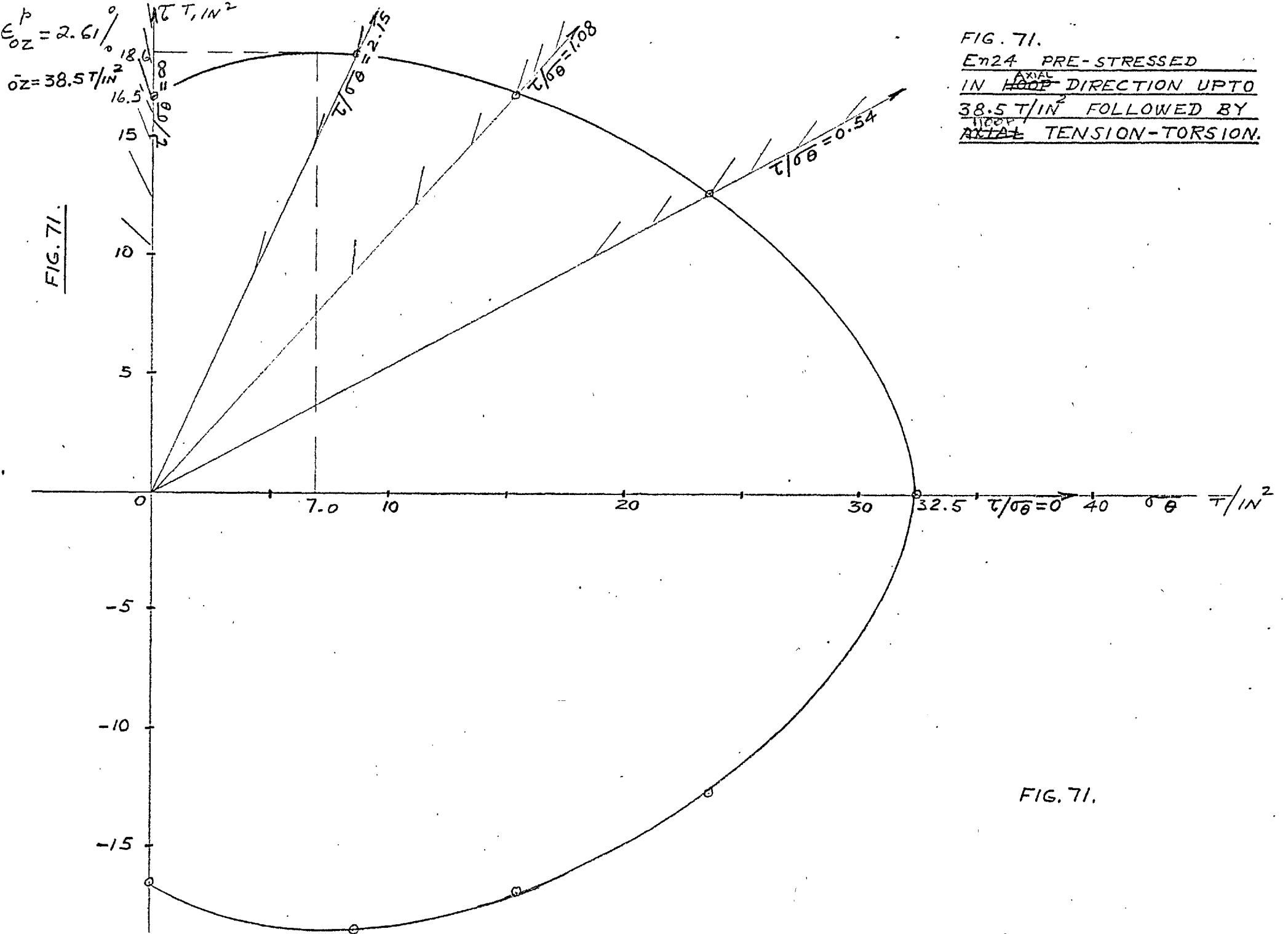


FIG. 70.



$$\epsilon_{oz}^p = 2.61\%$$

$$\sigma_{oz} = 38.5 \tau / \text{IN}^2$$

FIG. 72.

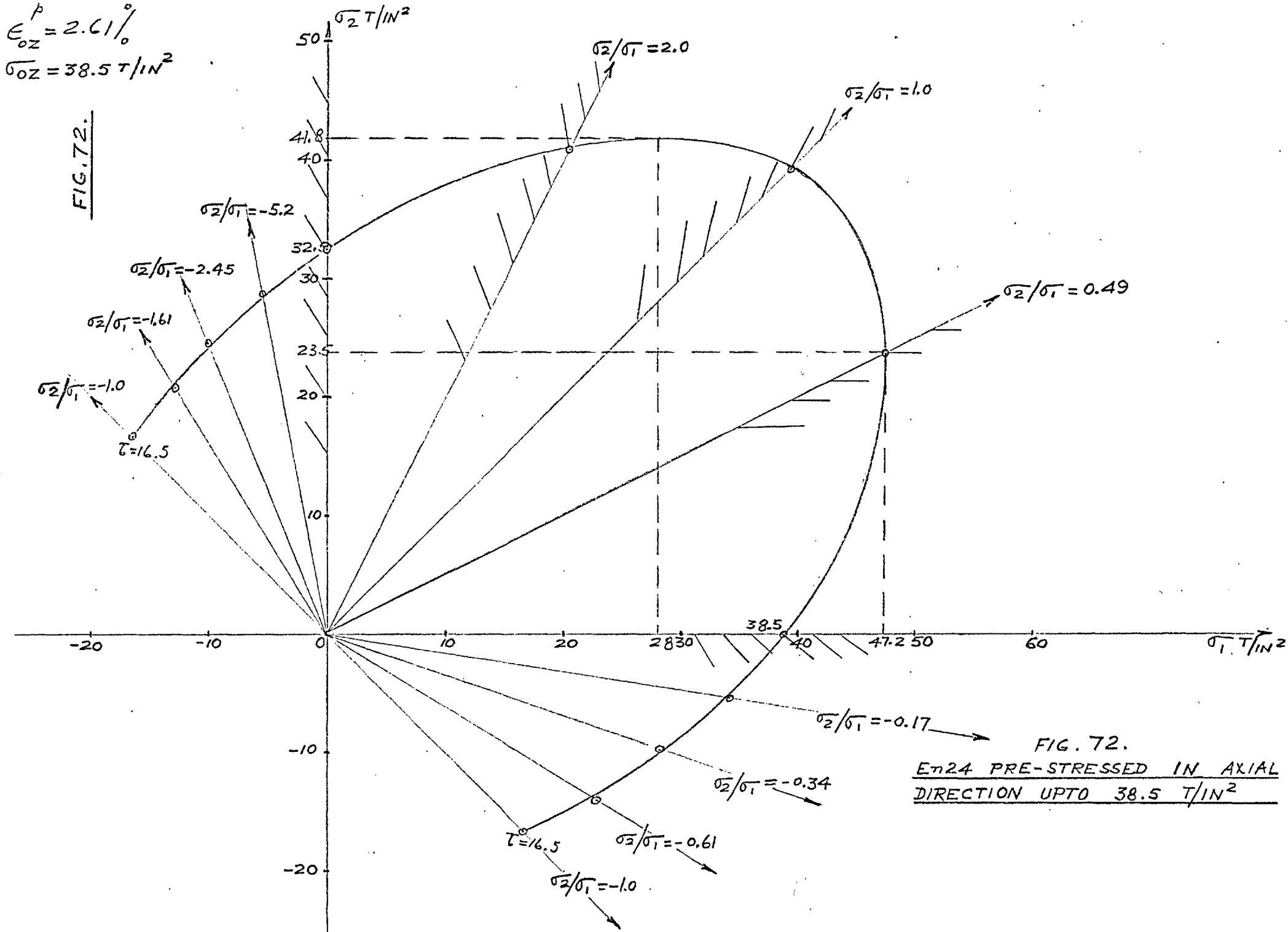


FIG. 73.

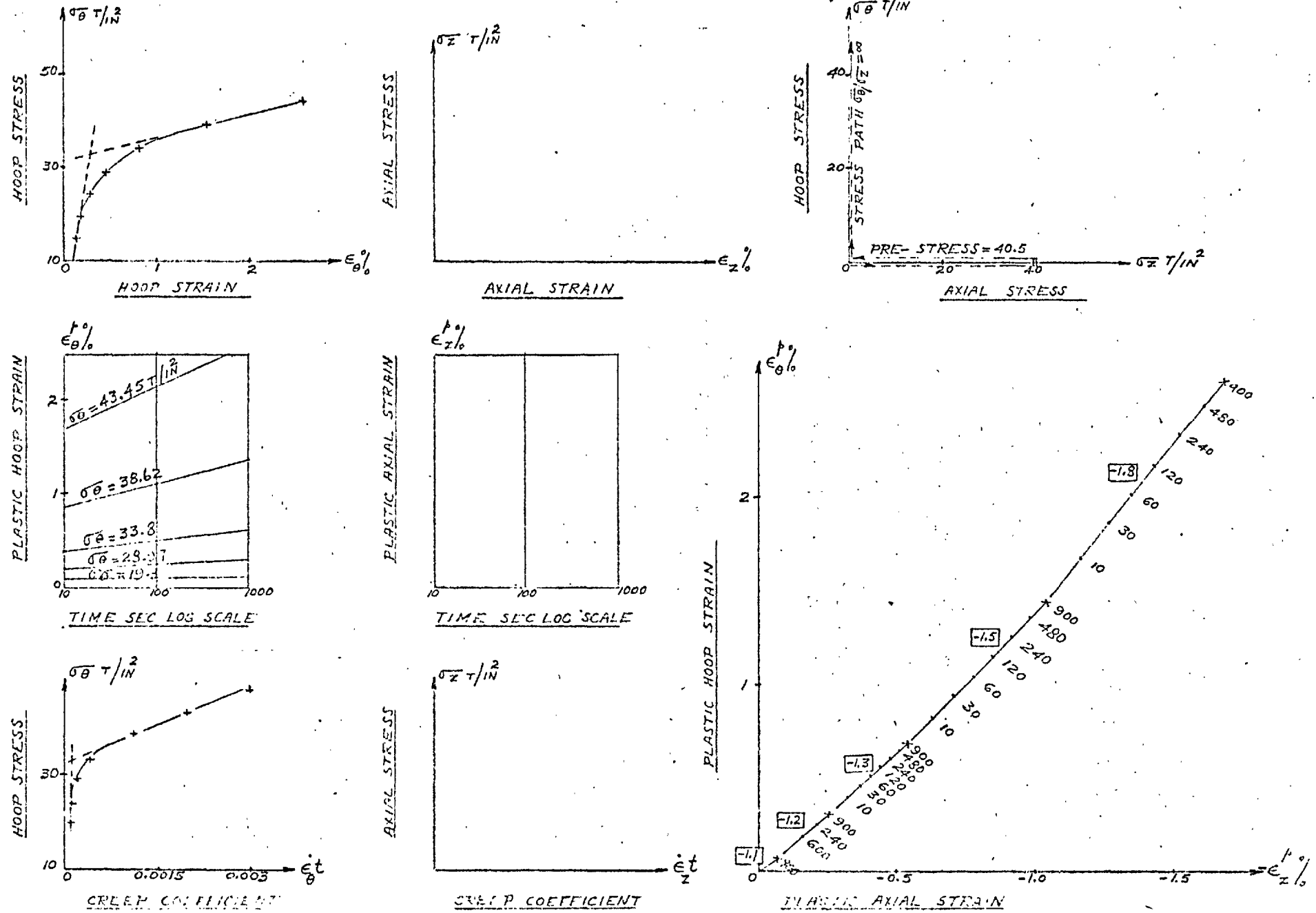


FIG. 73.

FIG. 74.

STRESS RATIO $\sigma_B/\sigma_Z = 2$

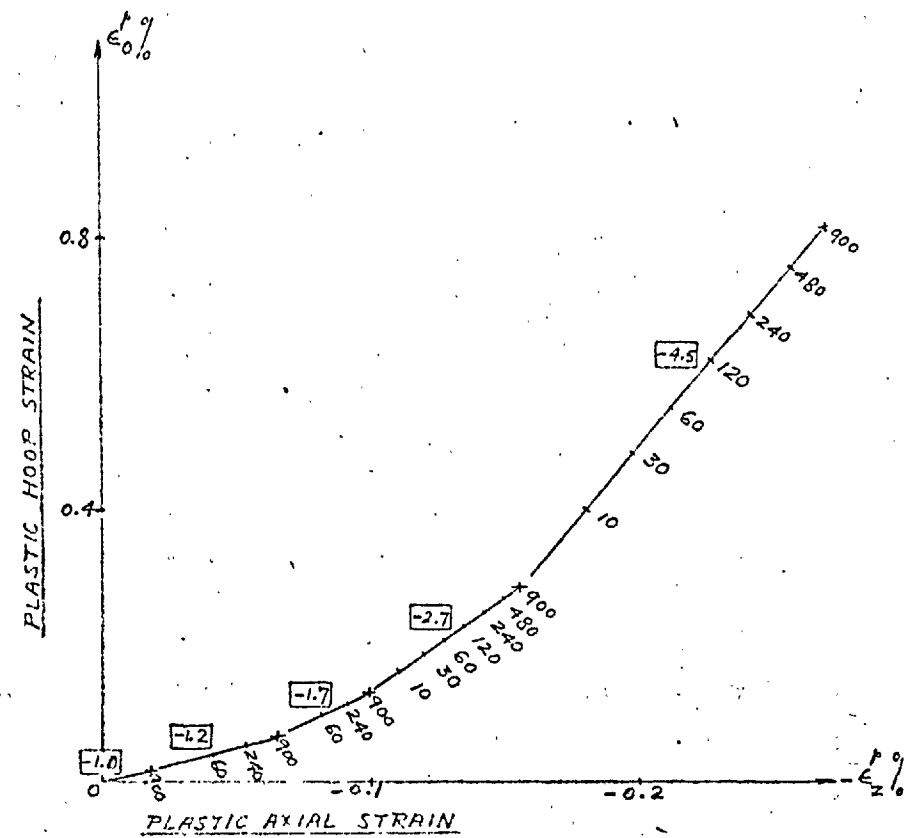
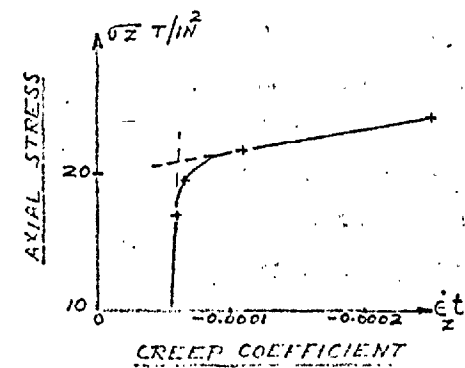
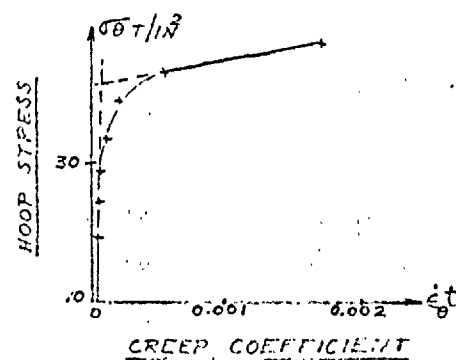
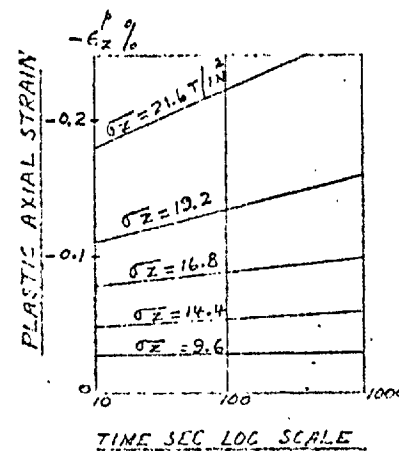
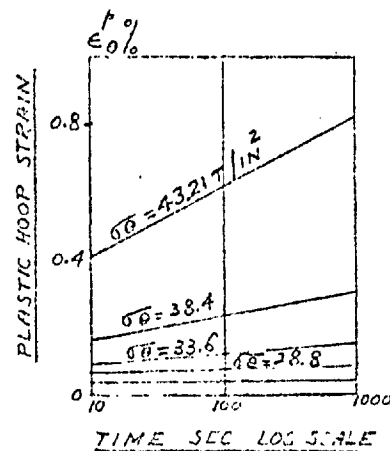
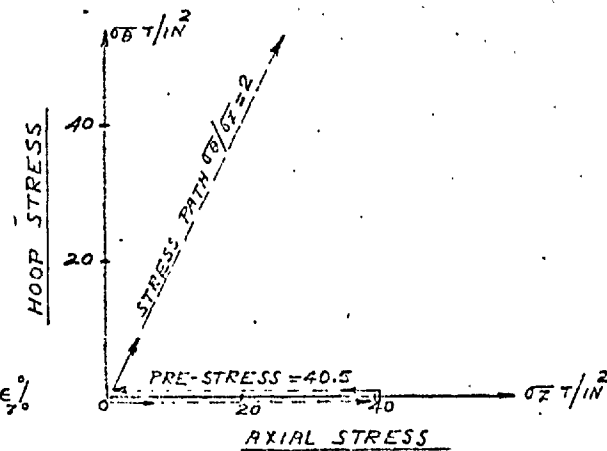
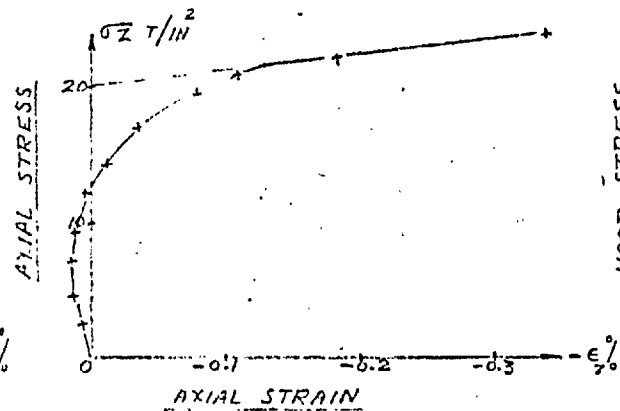
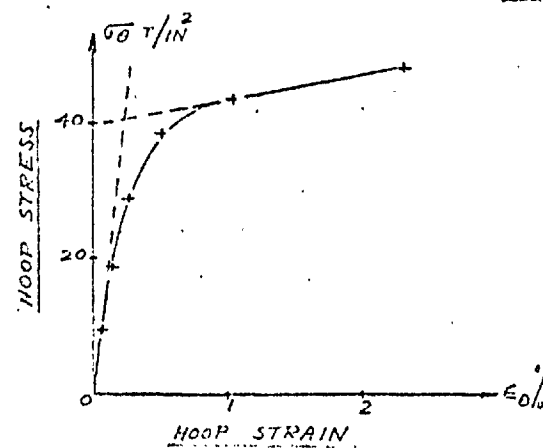


FIG. 75.

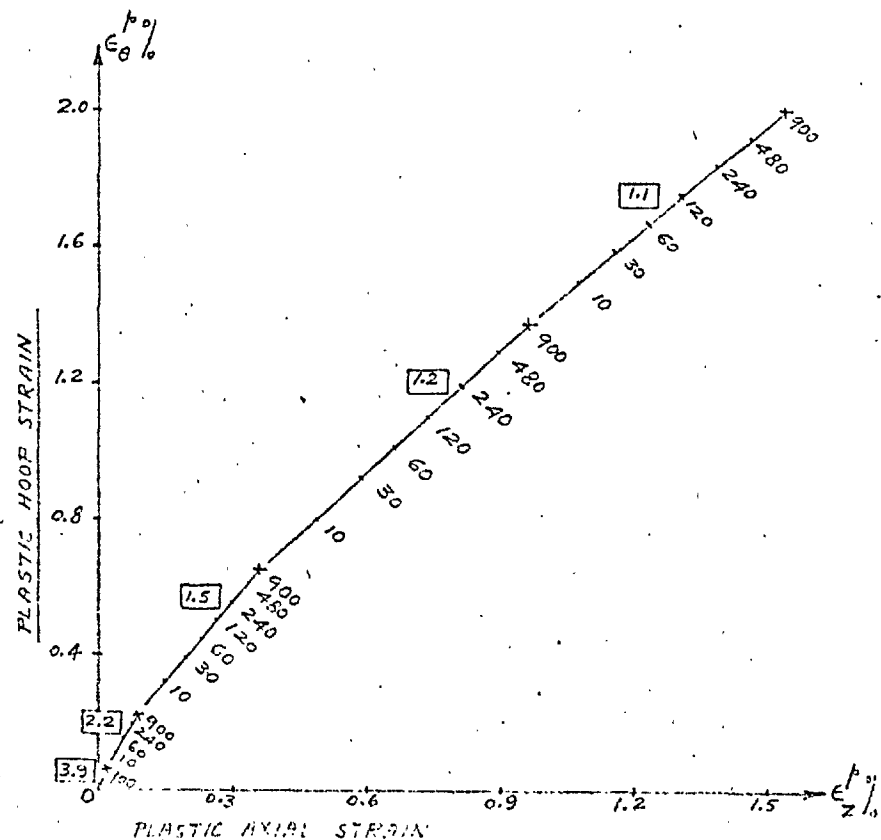
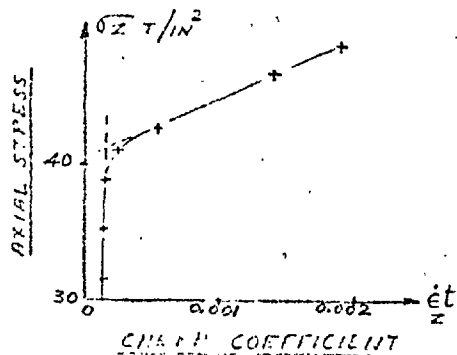
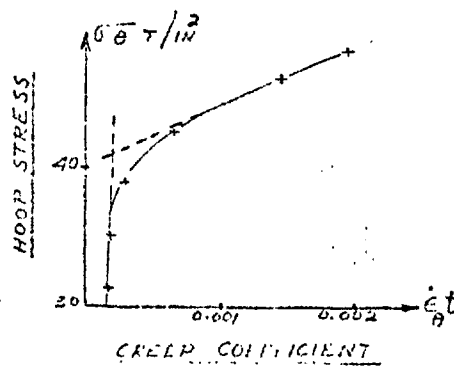
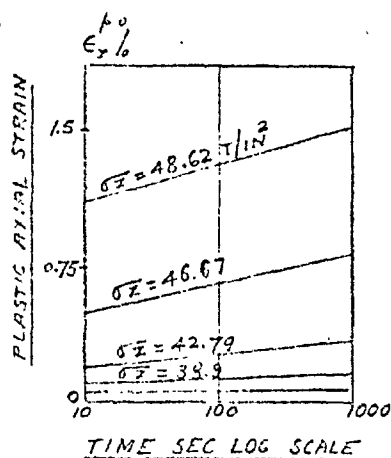
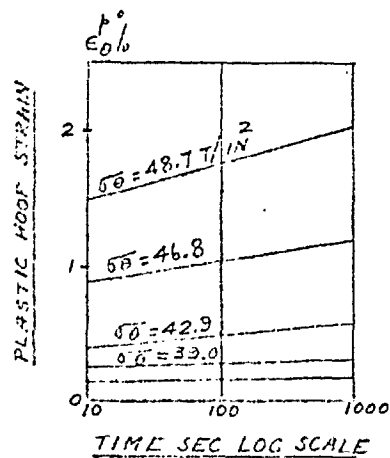
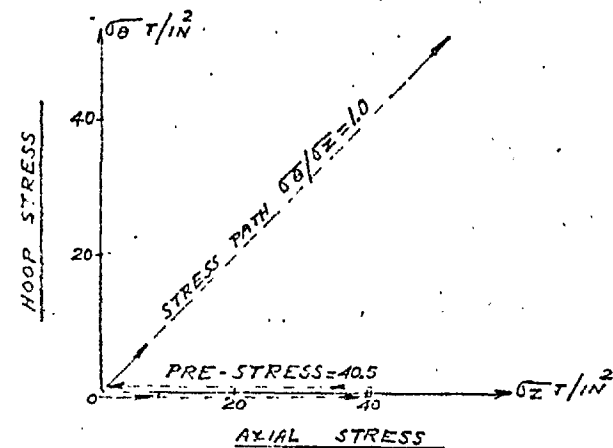
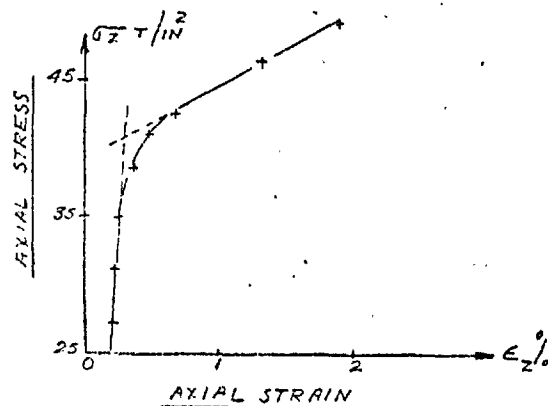
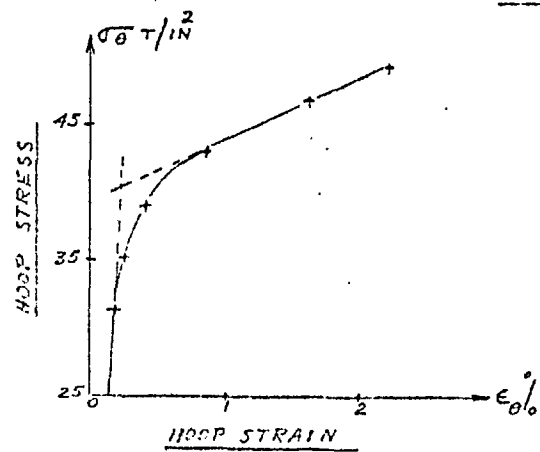


FIG. 75.

FIG. 76.

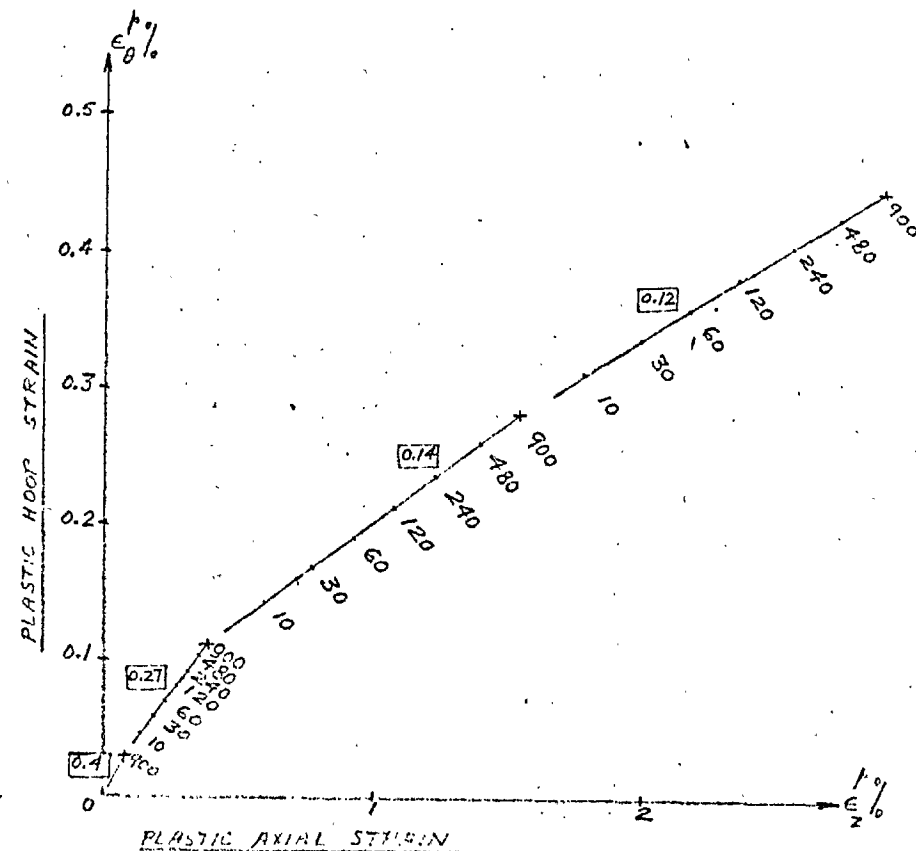
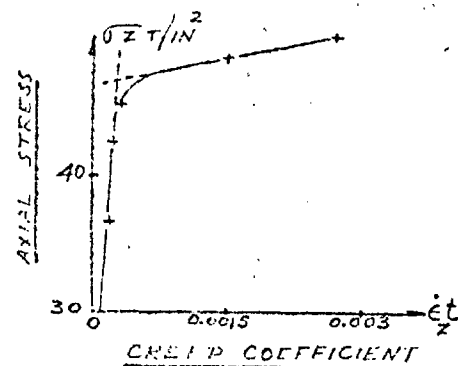
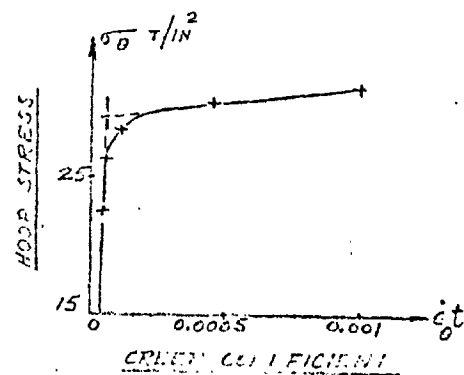
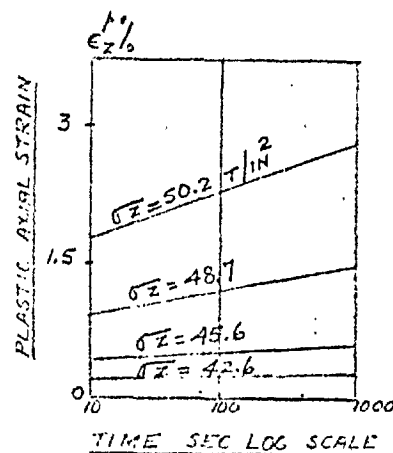
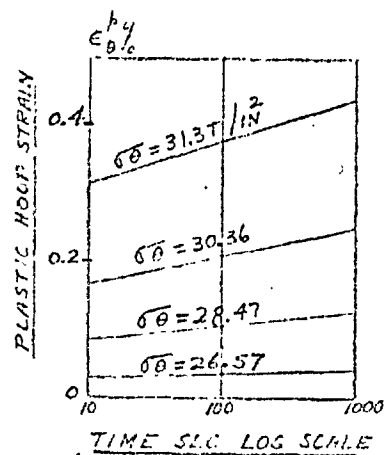
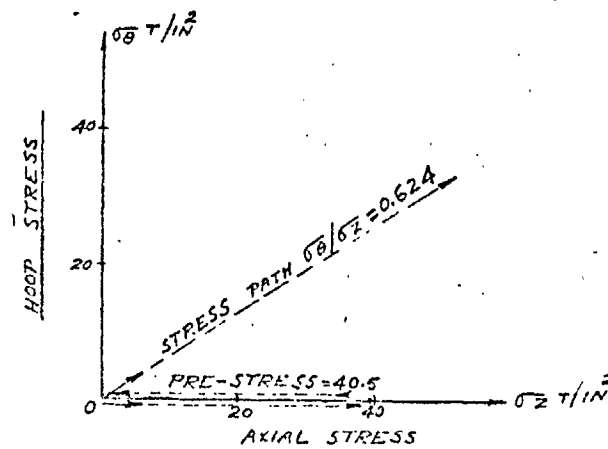
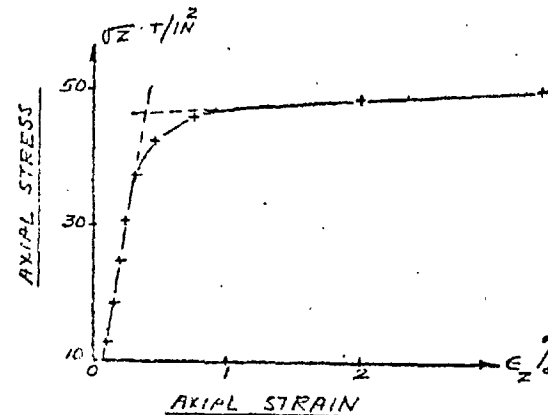
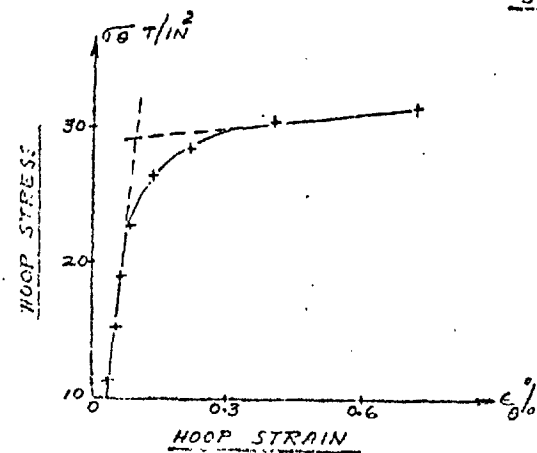


FIG. 76.

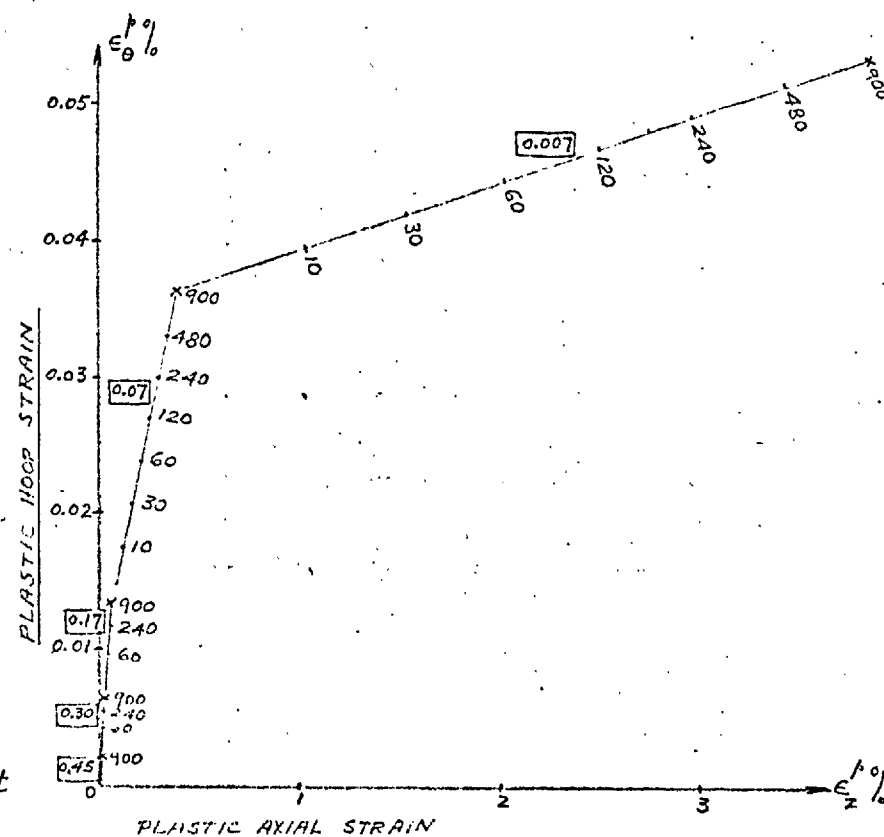
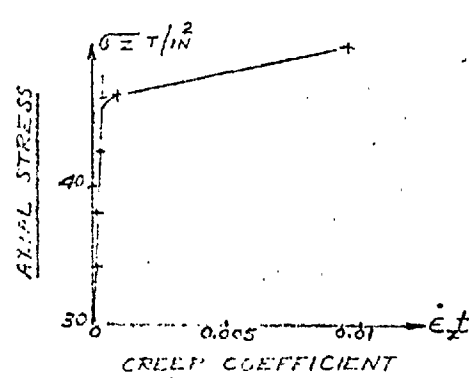
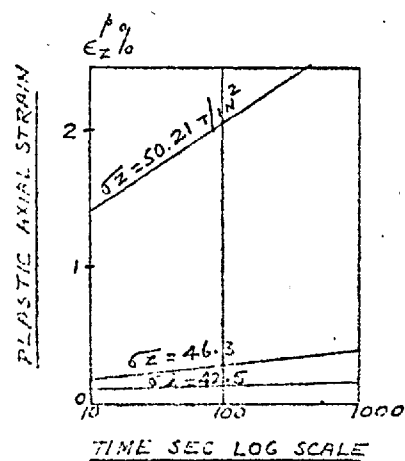
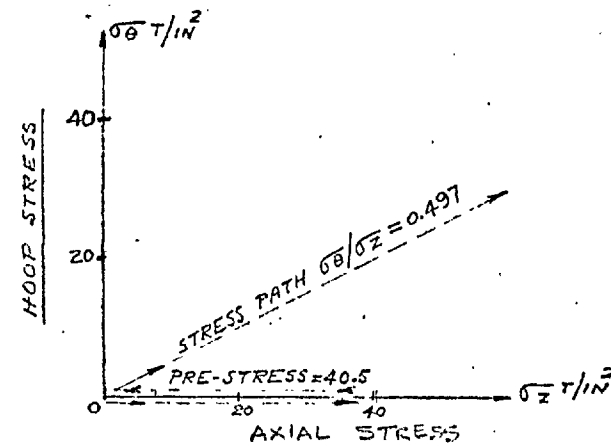


FIG. 77.

$$\epsilon_{0Z}^p = 3.45 \%$$

$$\sigma_{0Z} = 40.5 \text{ T/IN}^2$$

FIG. 78.

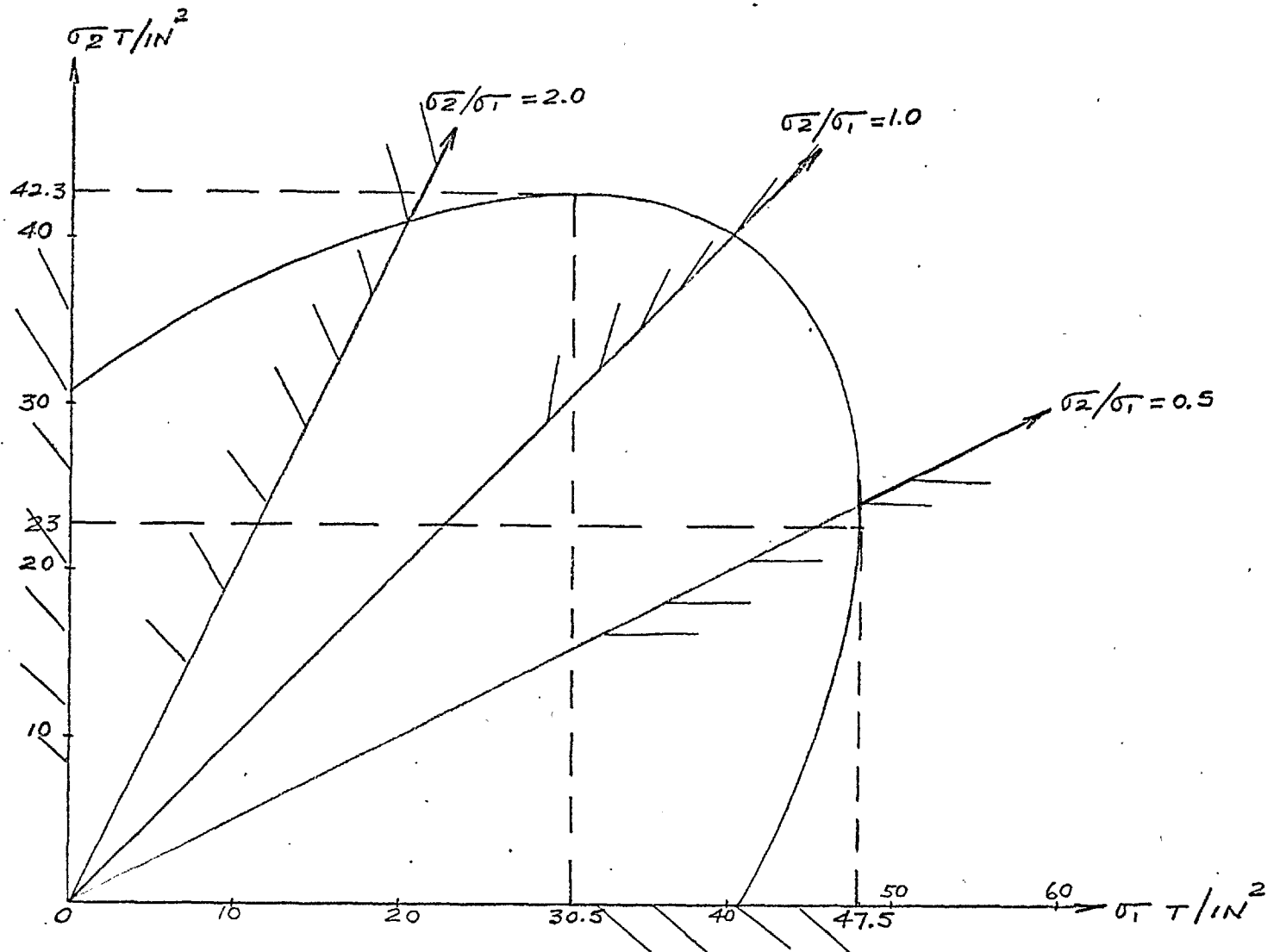


FIG. 78. En24 PRE-STRESSED IN AXIAL DIRECTION UPTO 40.5 T/IN^2

FIG. 79.

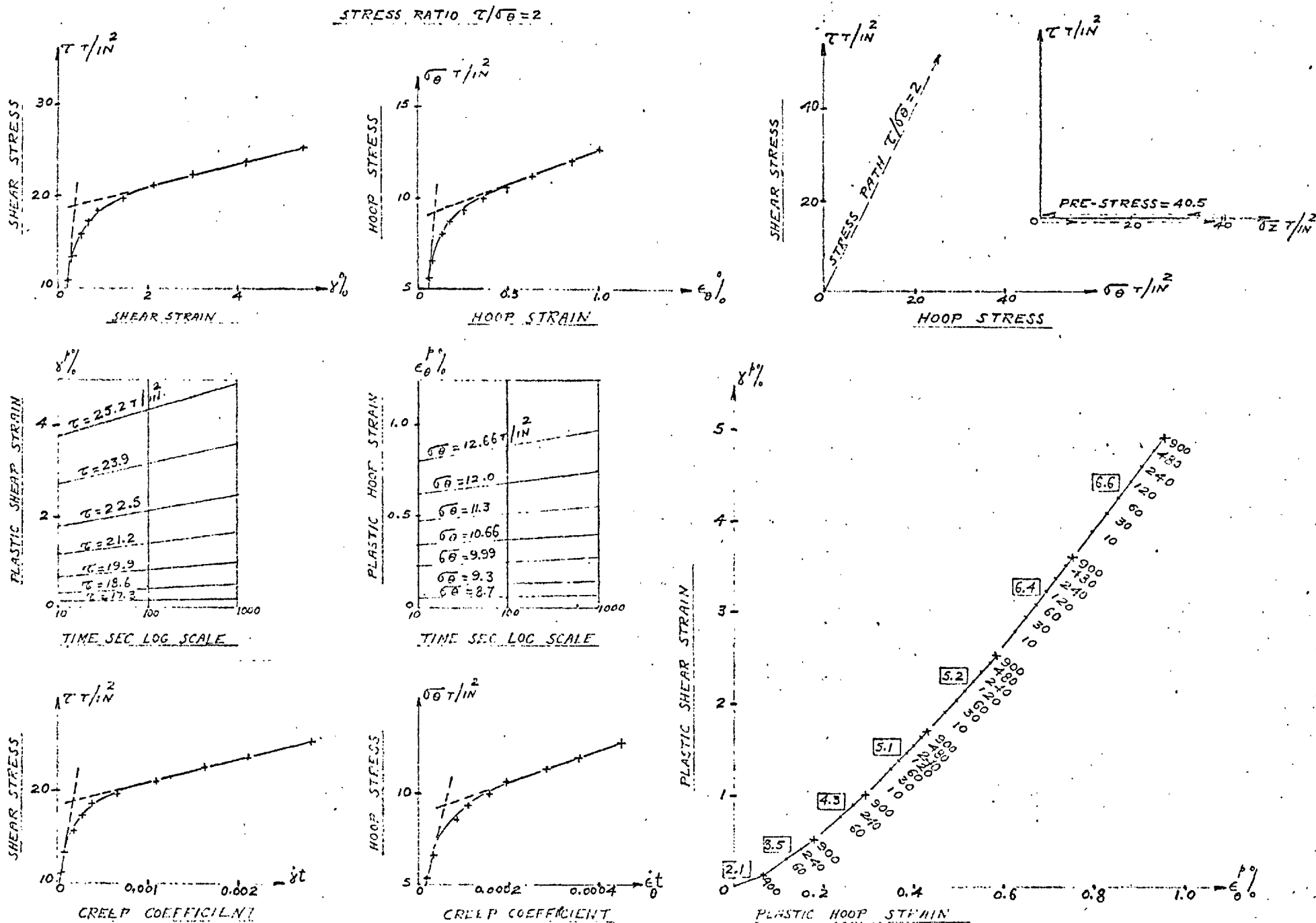


FIG. 79.

FIG. 80.

STRESS RATIO $\tau/\sigma = 1.1$

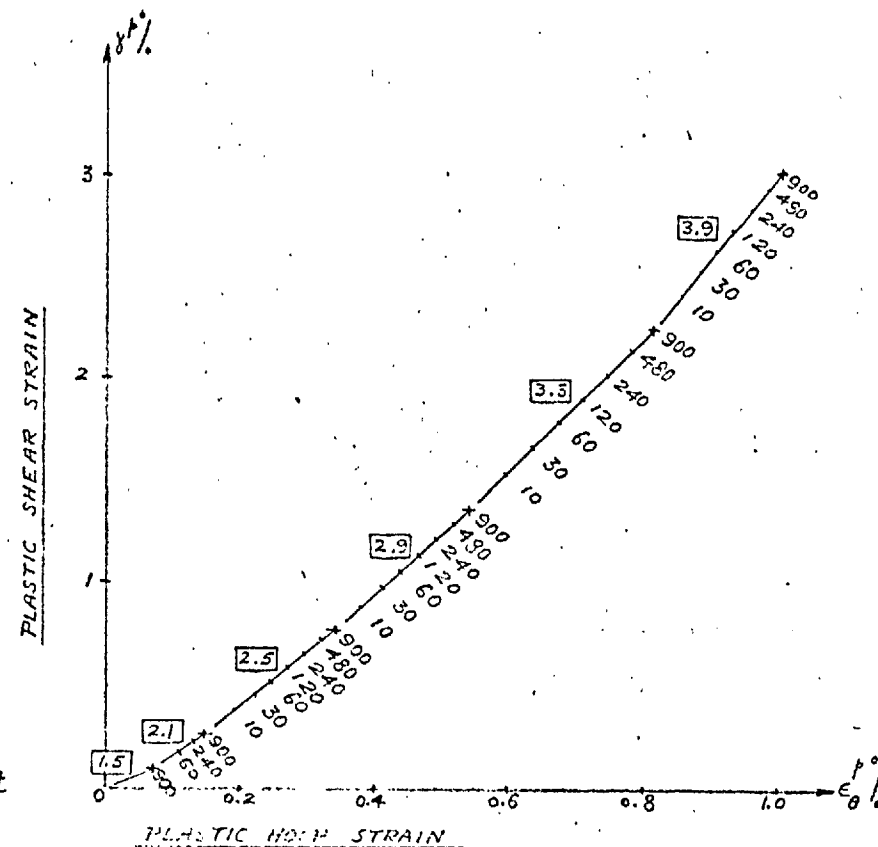
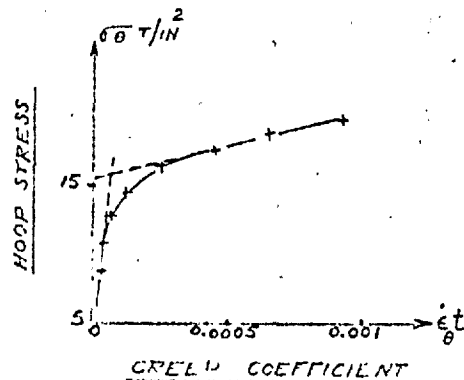
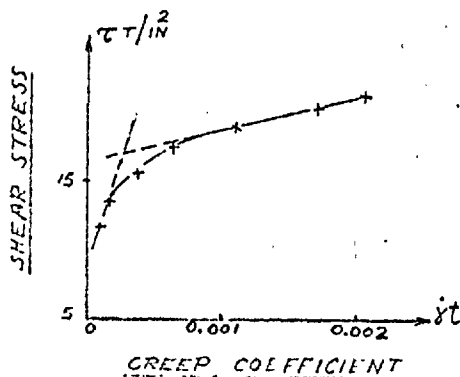
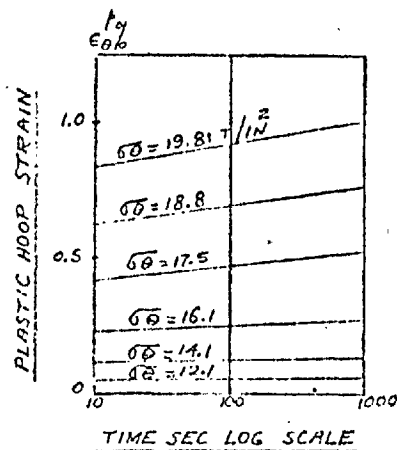
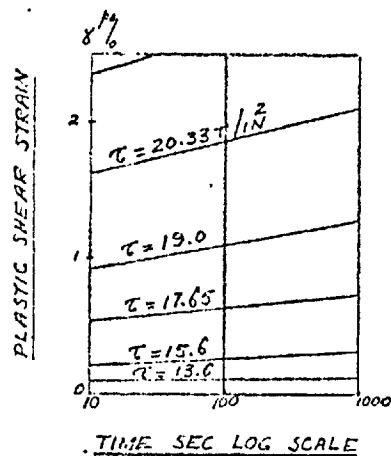
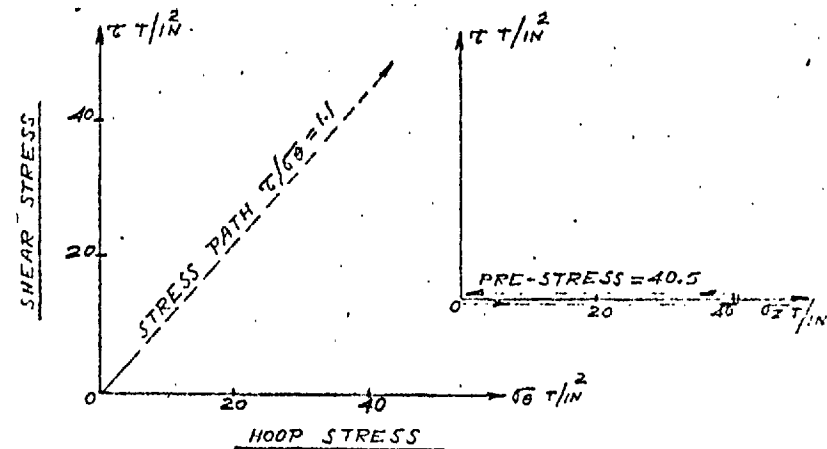
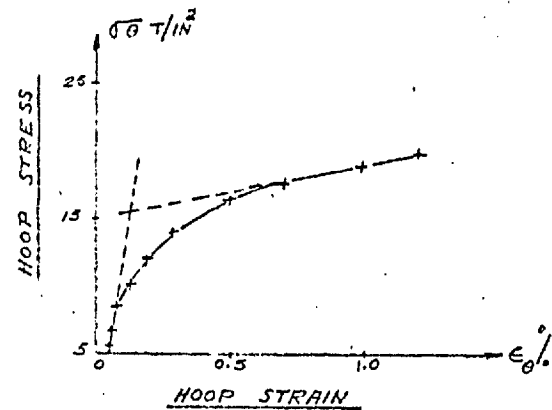
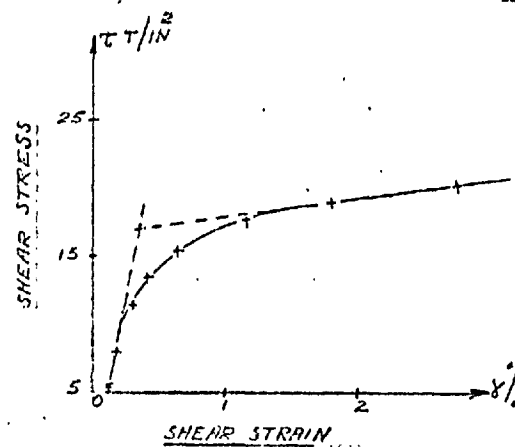


FIG. 80.

FIG. 81.

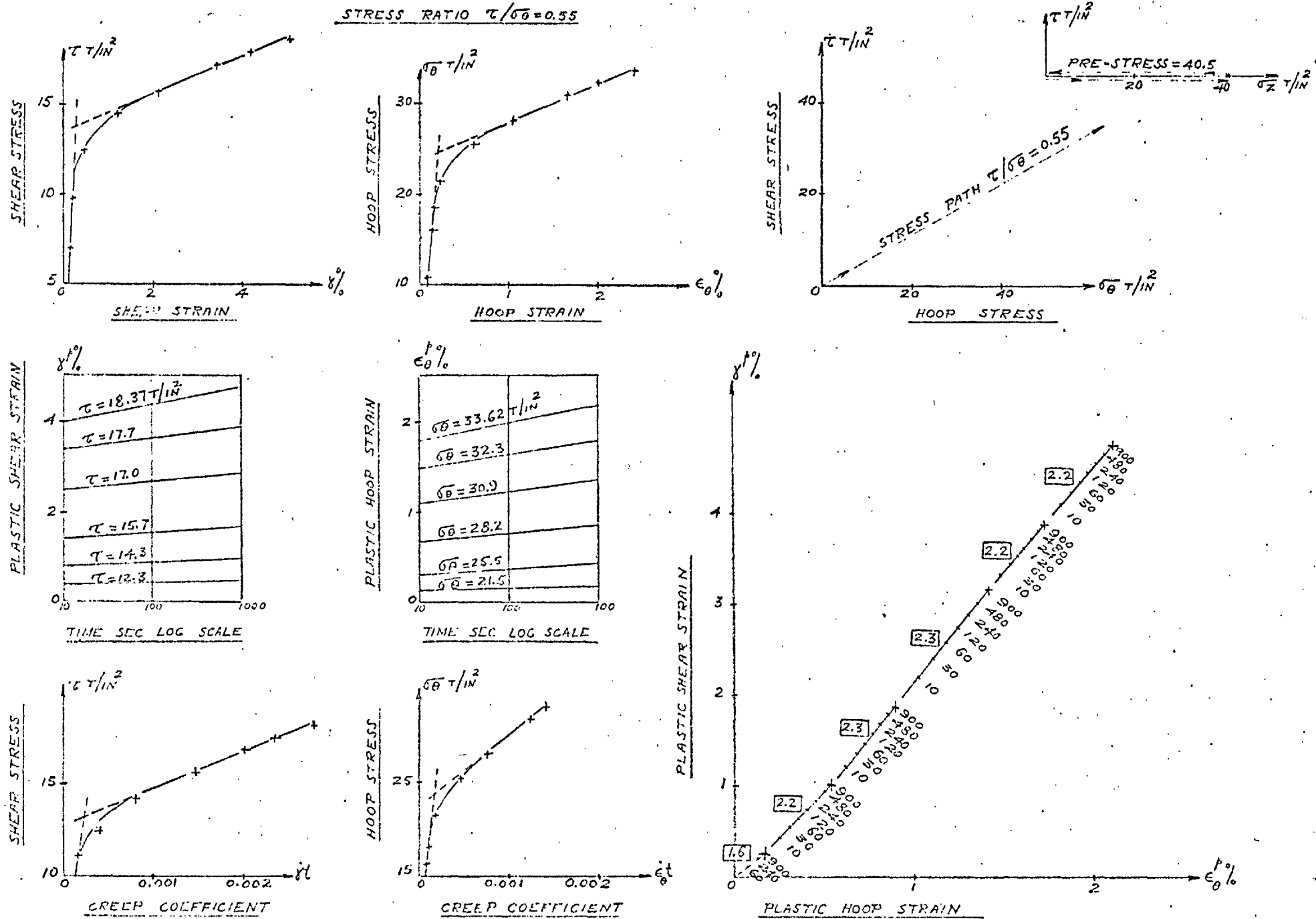
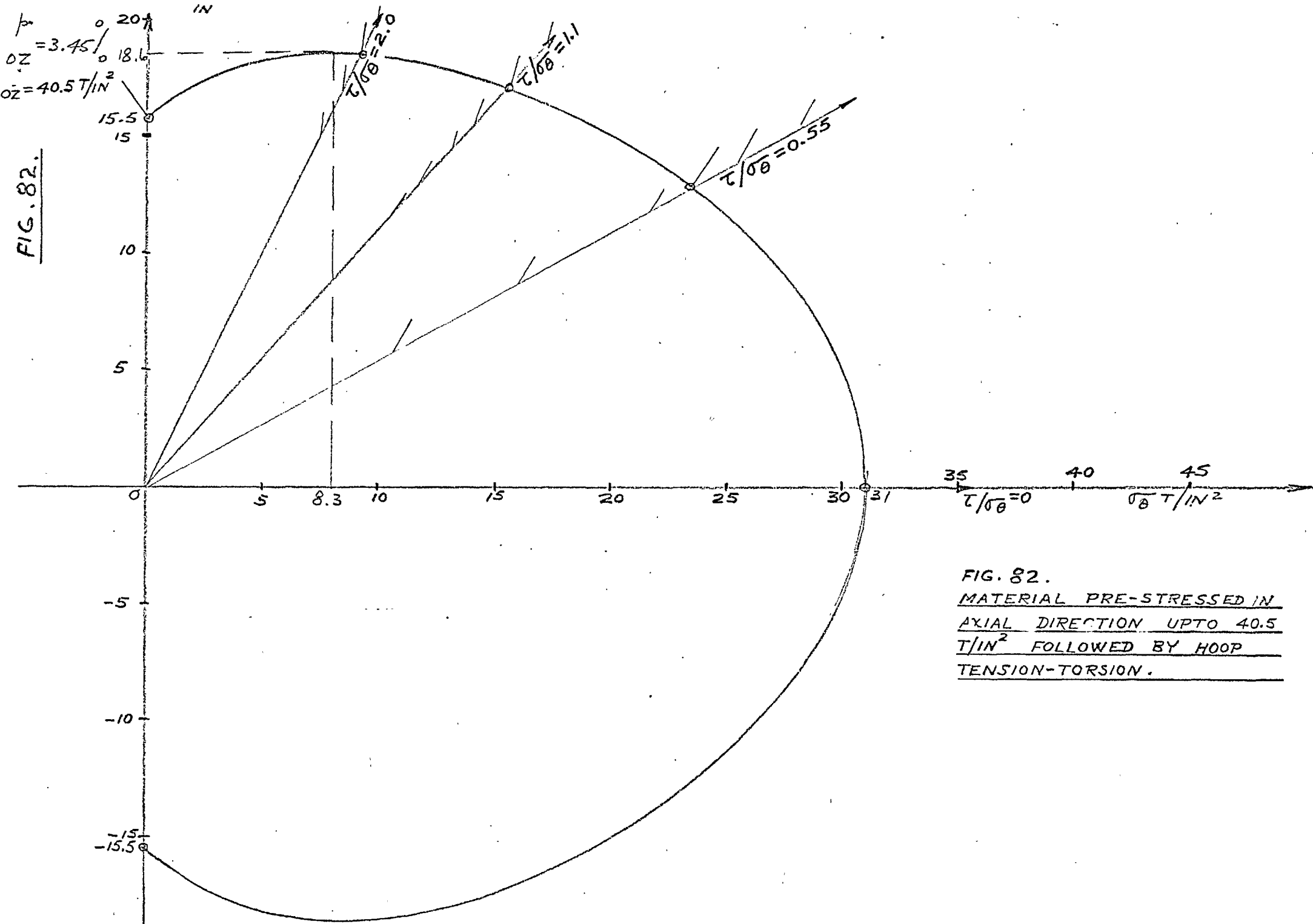


FIG. 81.



STRESS RATIO $\sigma/\sigma_F = \infty$

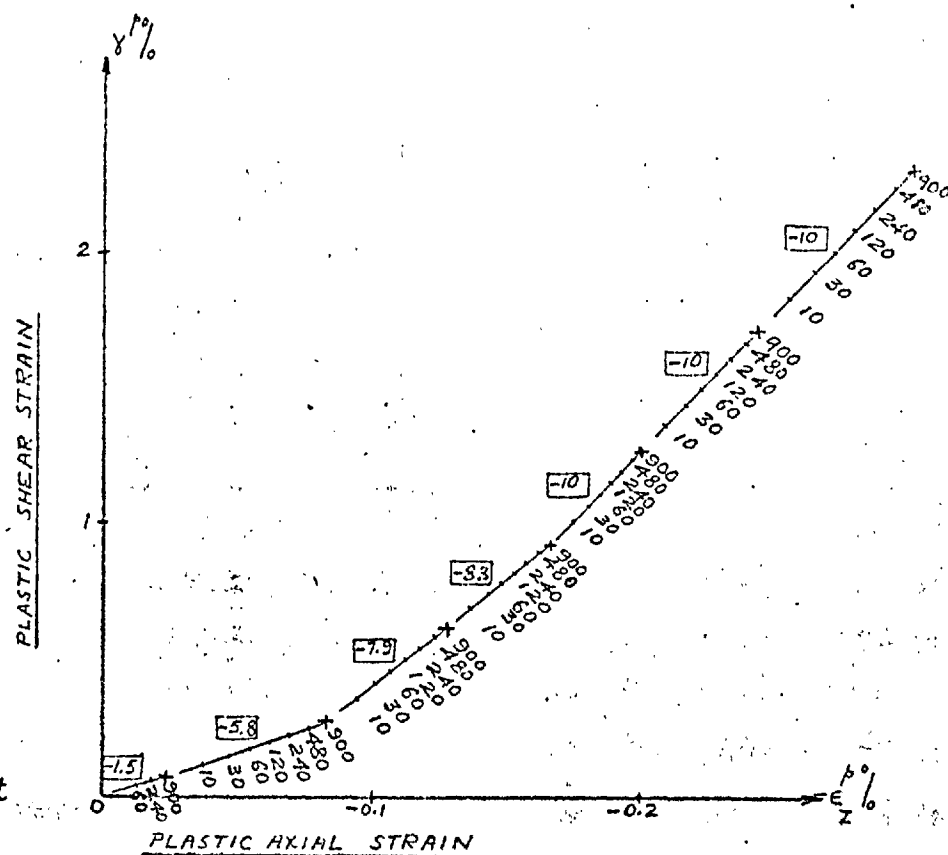
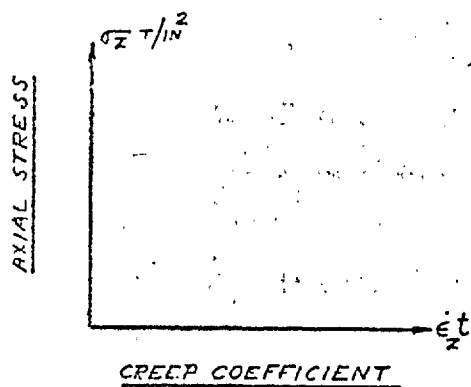
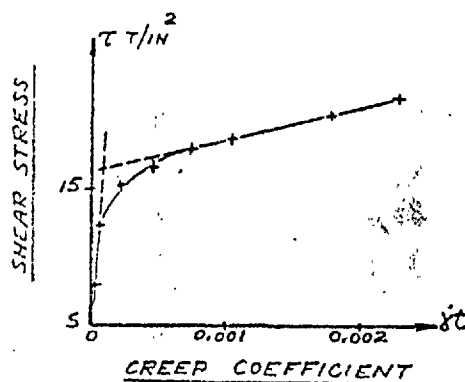
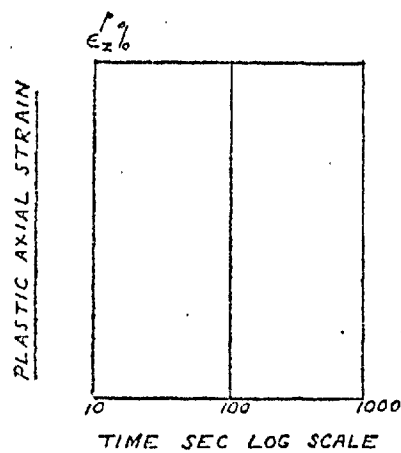
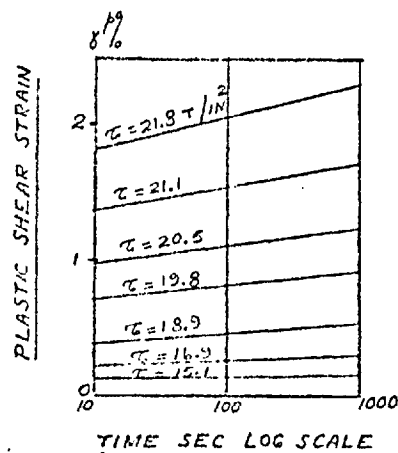
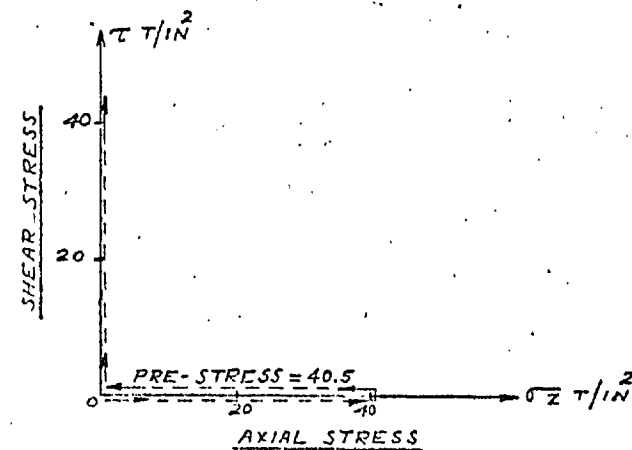
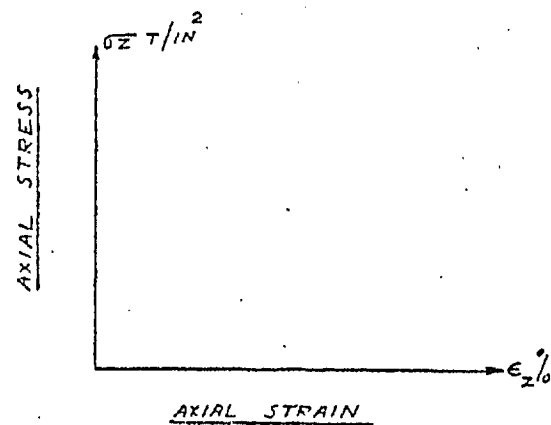
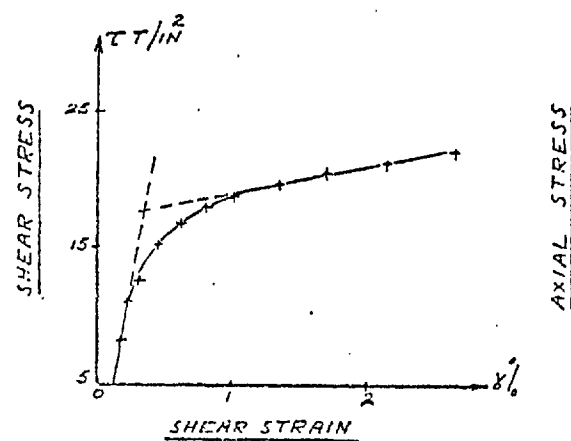


FIG. 83.

FIG. 84.

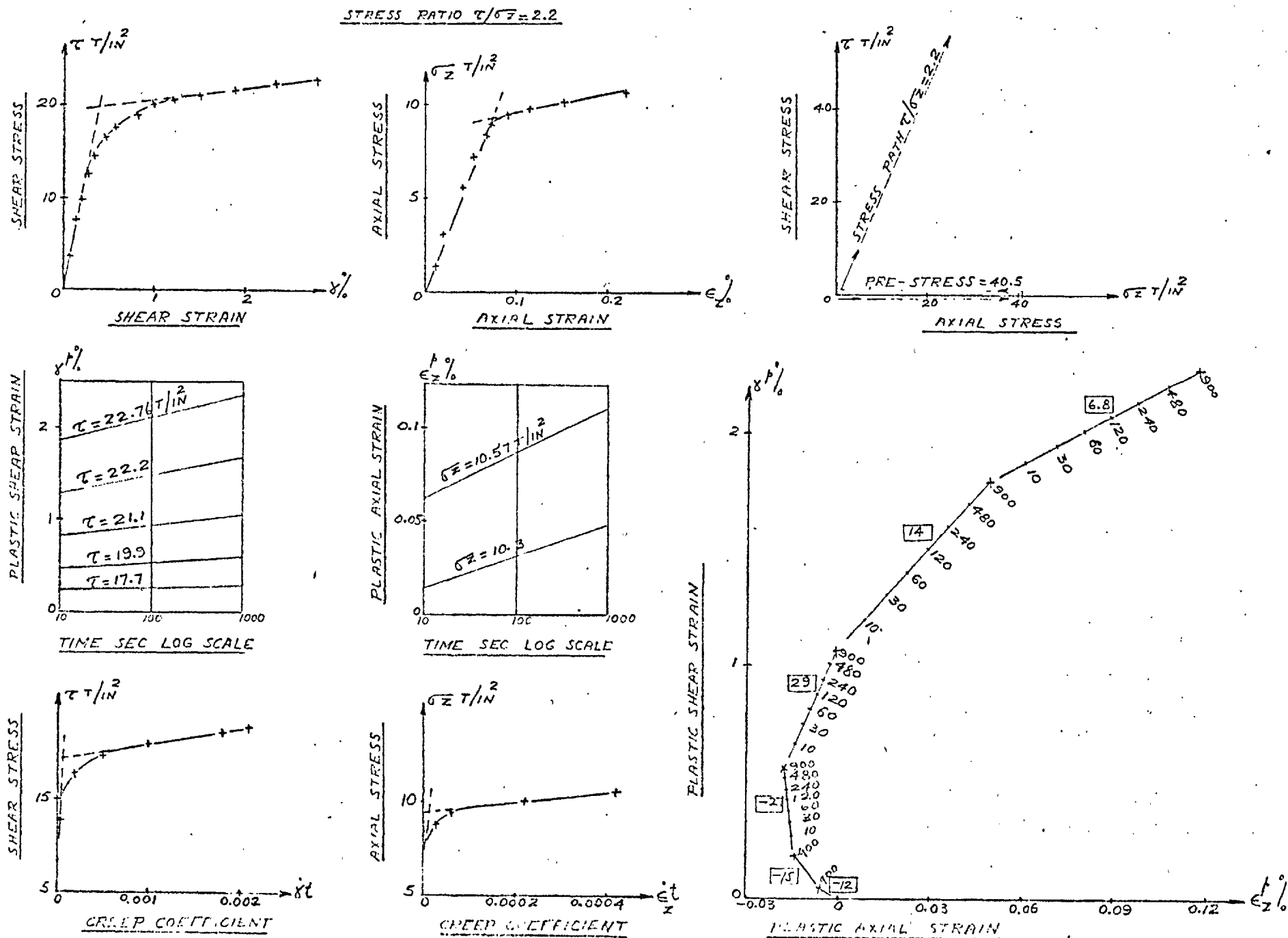


FIG. 84.

FIG. 85.

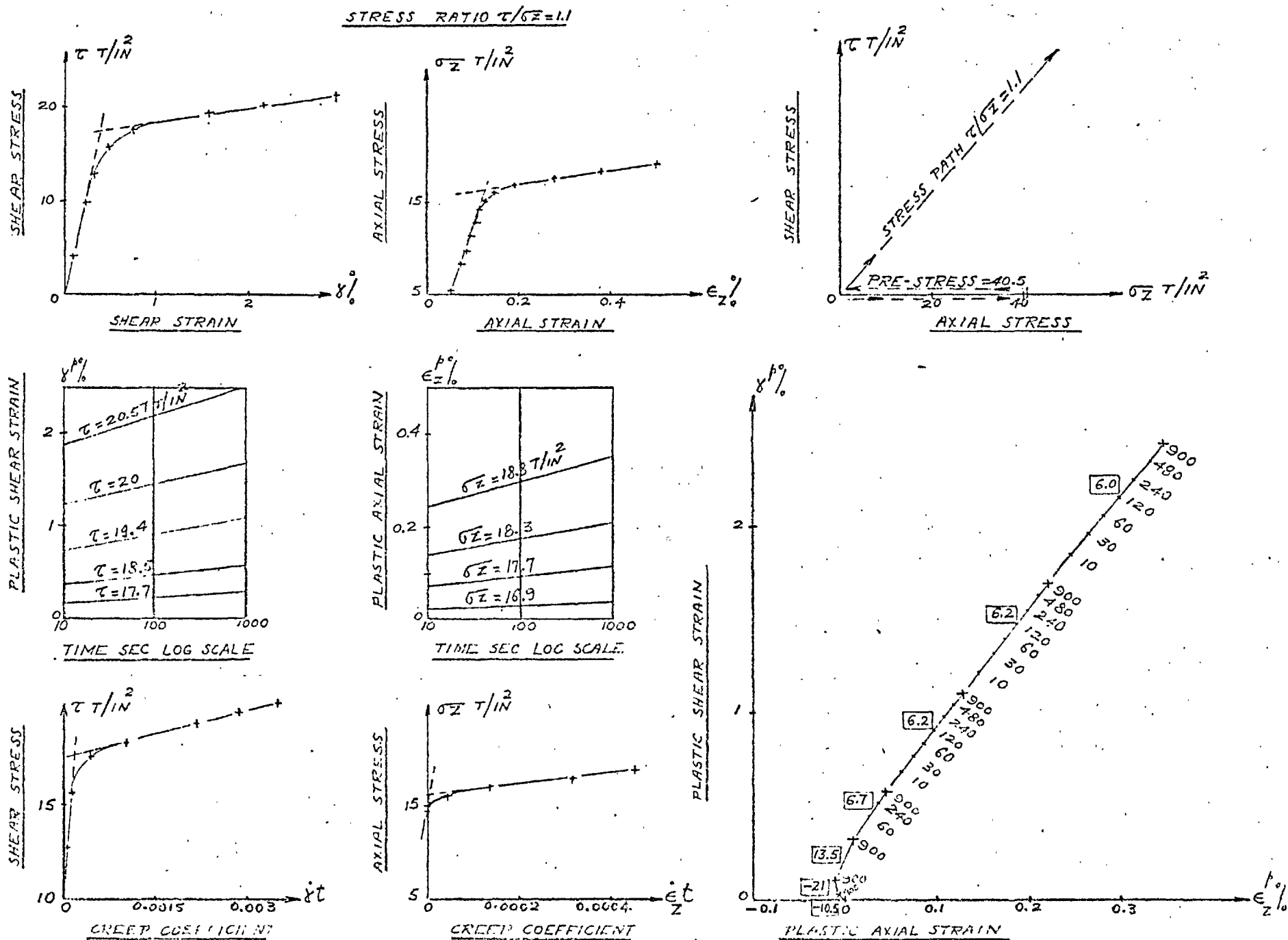


FIG. 86.

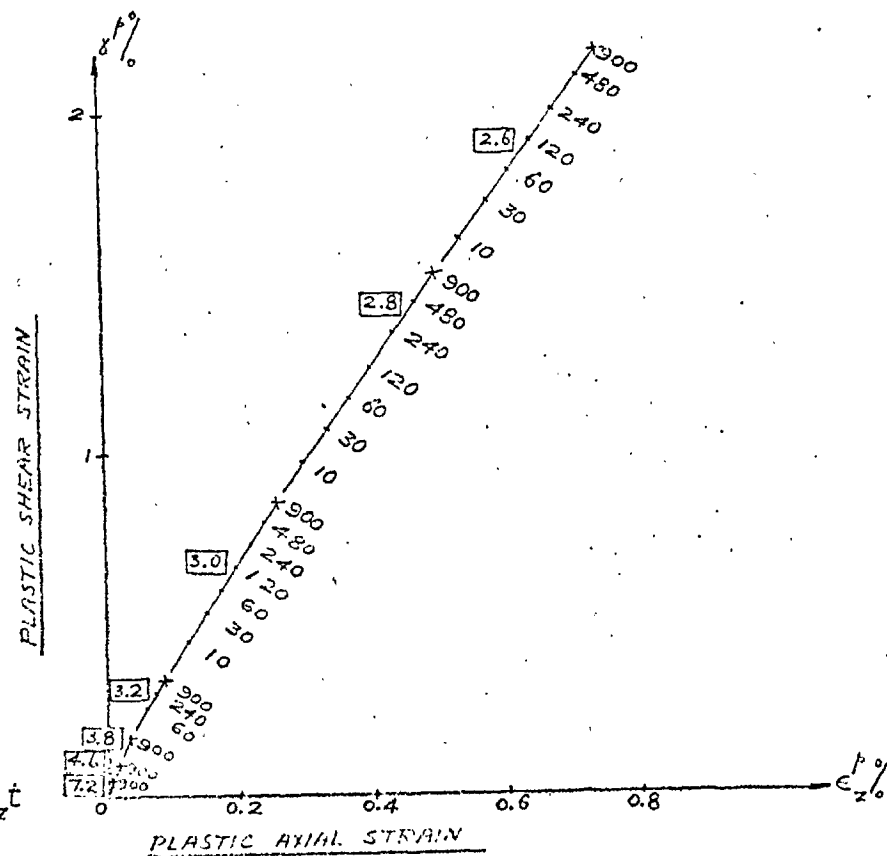
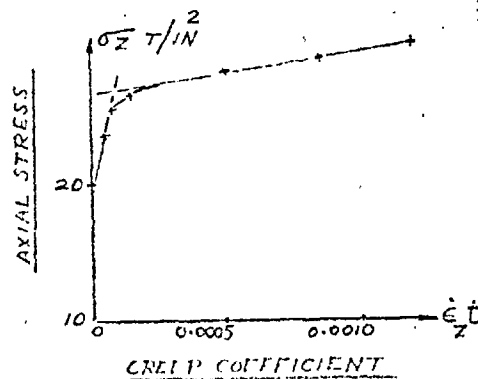
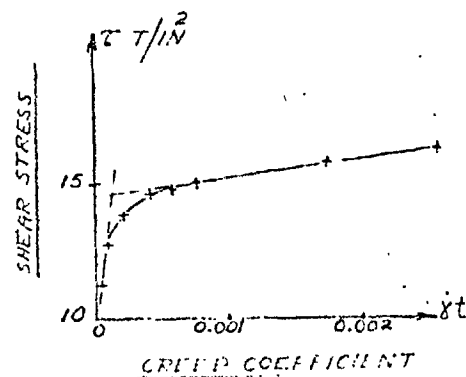
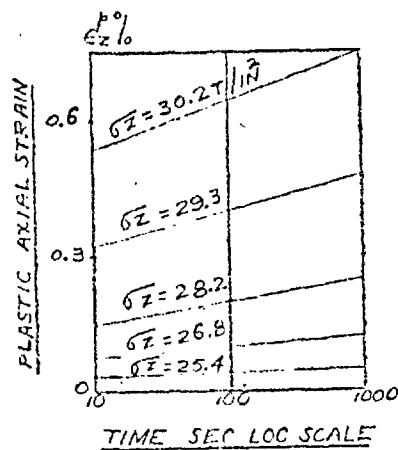
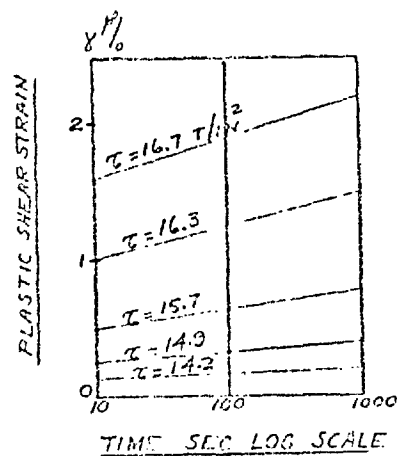
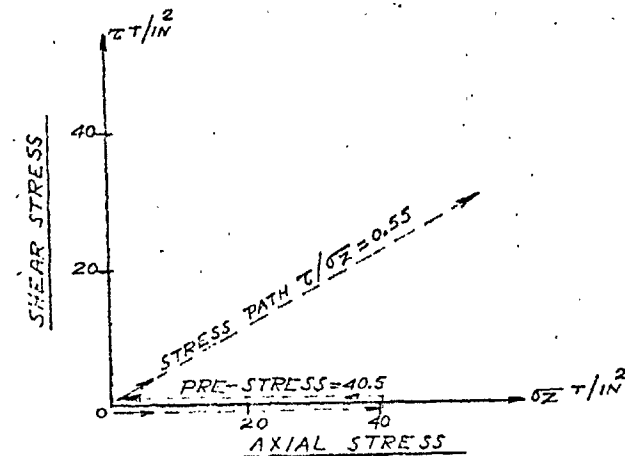
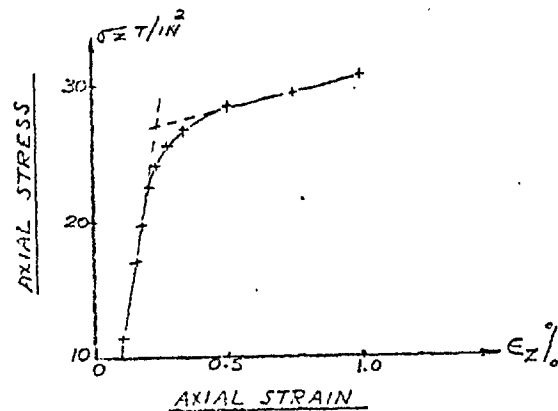
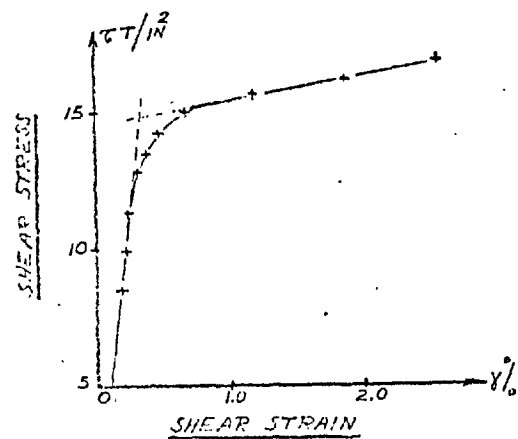
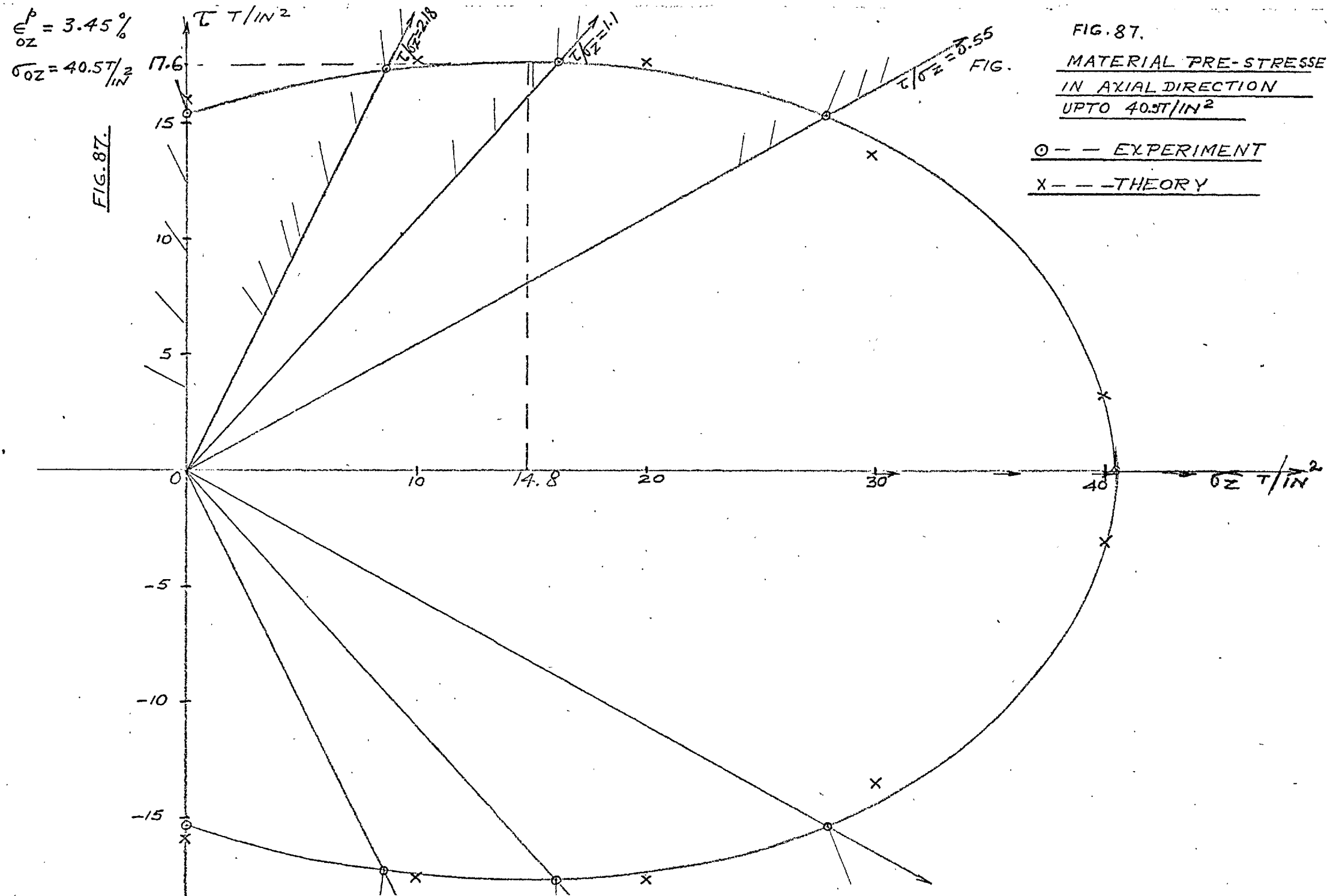


FIG. 86.



$$\epsilon'_{02} = 3.45 \%$$

$$\overline{\sigma}_2 = 40.5 \tau / \text{IN}^2$$

FIG. 88.

FIG. 88.

EN 24 PRE-STRESSED IN AXIAL
DIRECTION UPTO $40.5 \tau / \text{IN}^2$

MAXWELL-MISES ———
TRESCA ———
EXPANSION OF INITIAL ———
EXPERIMENT ———

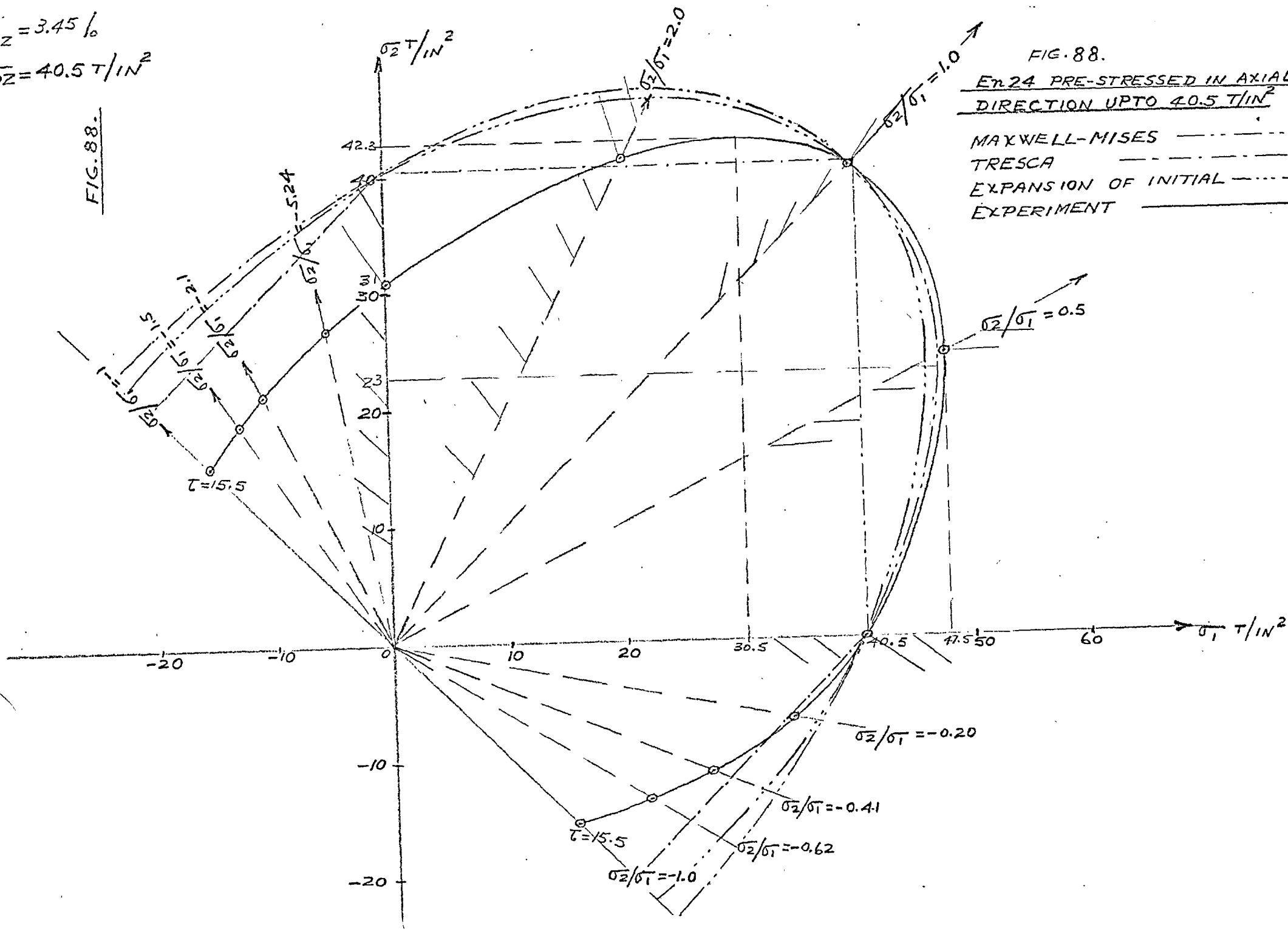
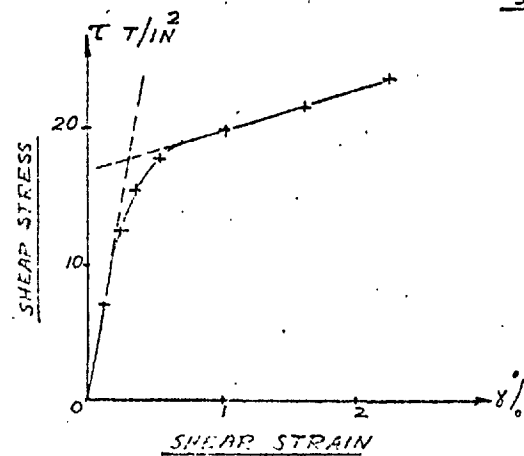
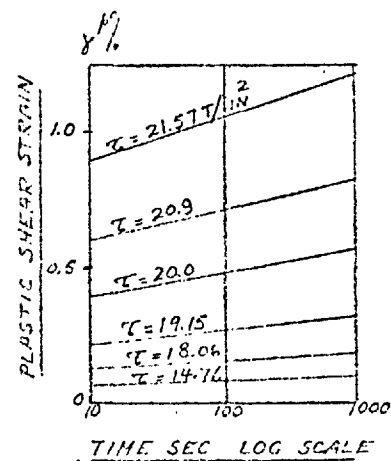
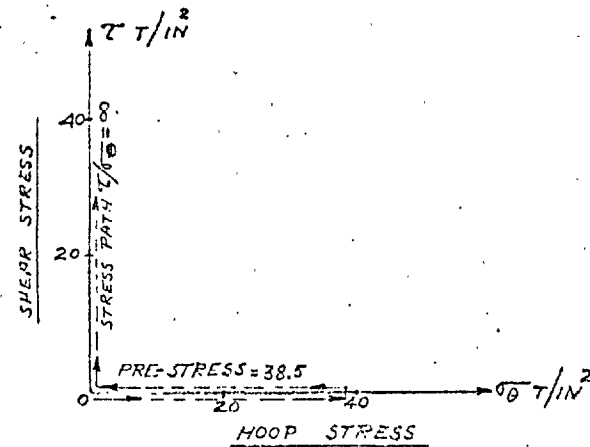
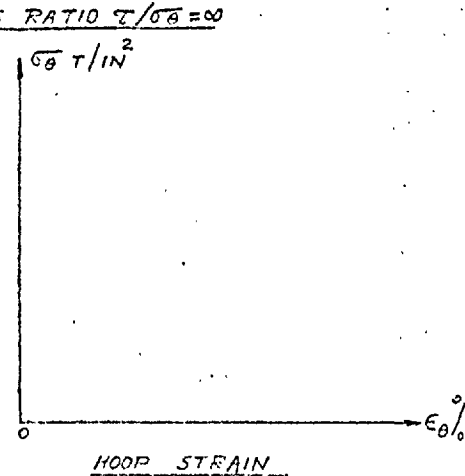


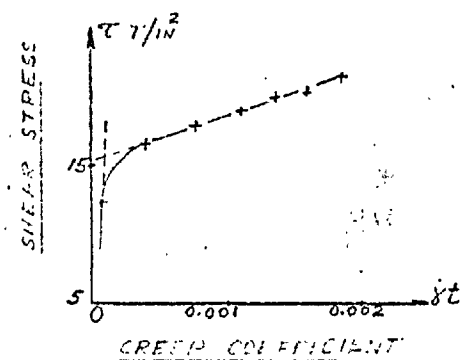
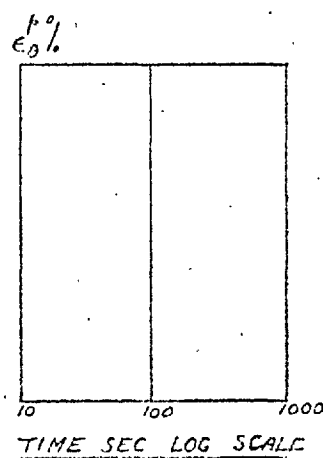
FIG. 89.



HOOP STRESS



PLASTIC HOOP STRAIN



HOOP STRESS

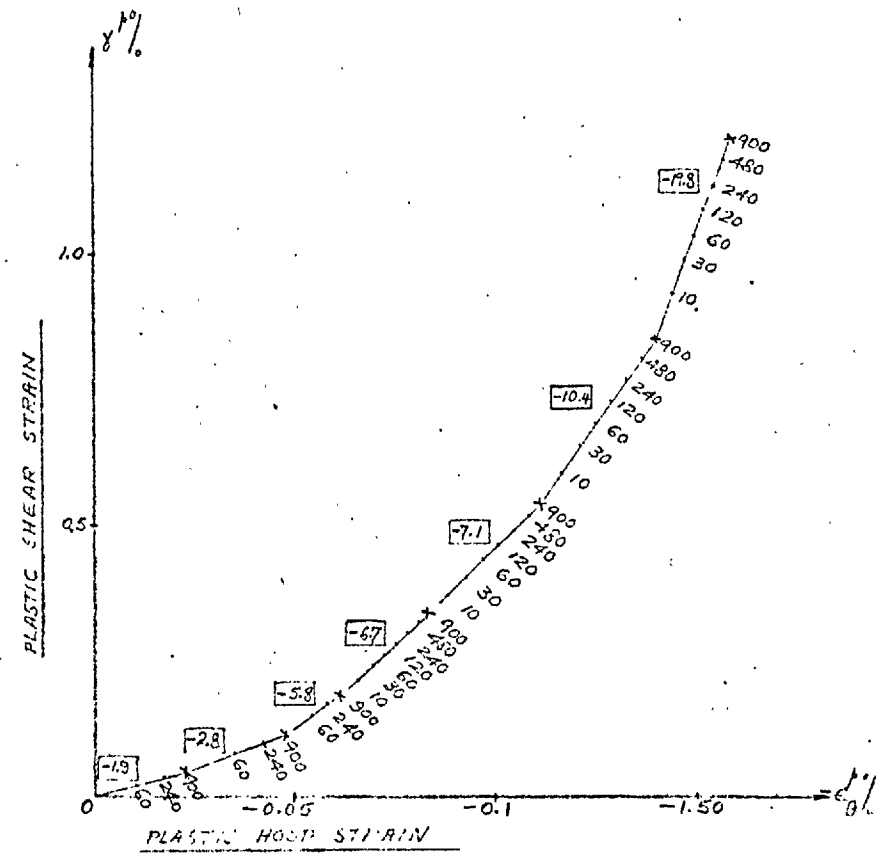
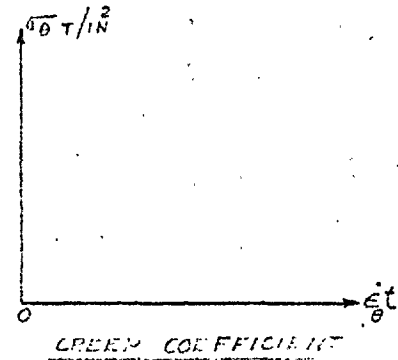


FIG. 89.

FIG. 90.

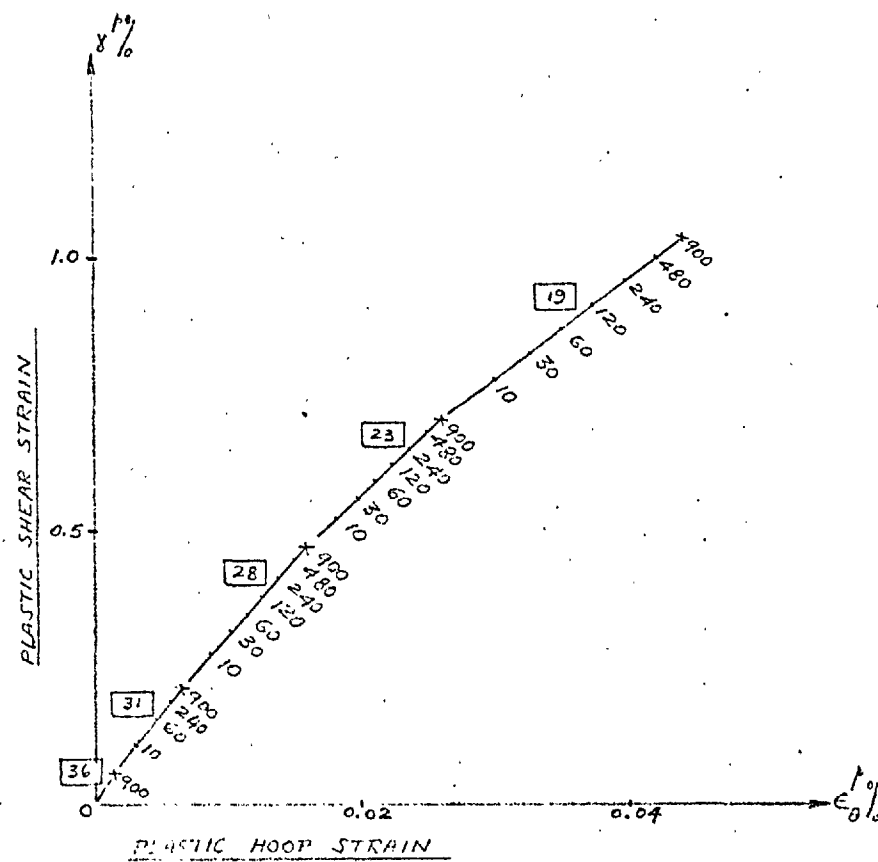
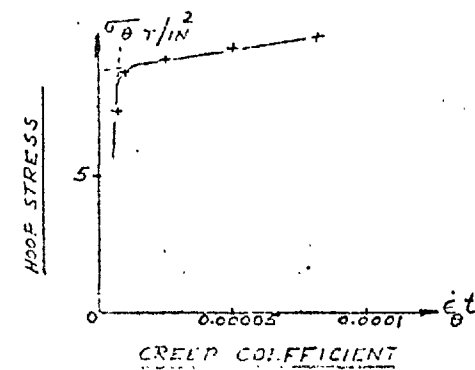
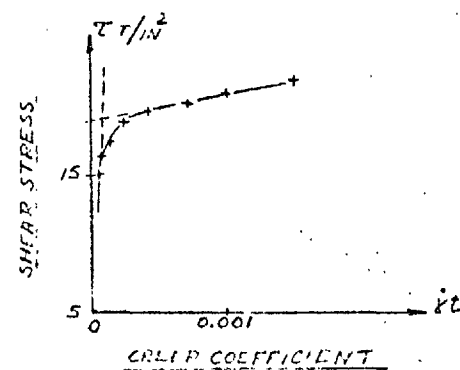
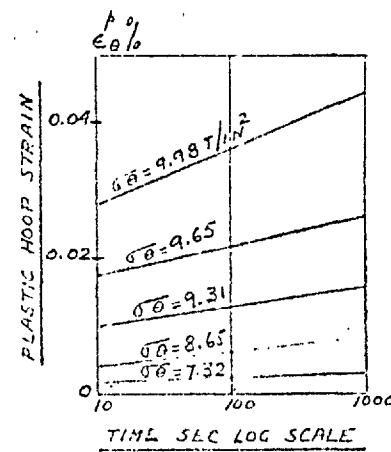
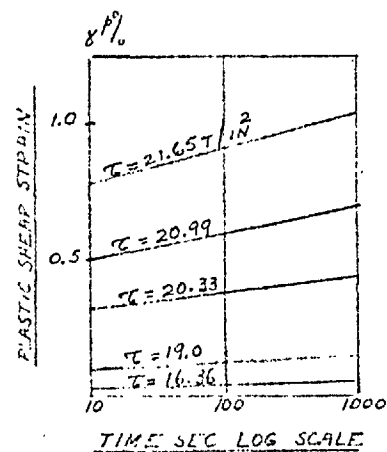
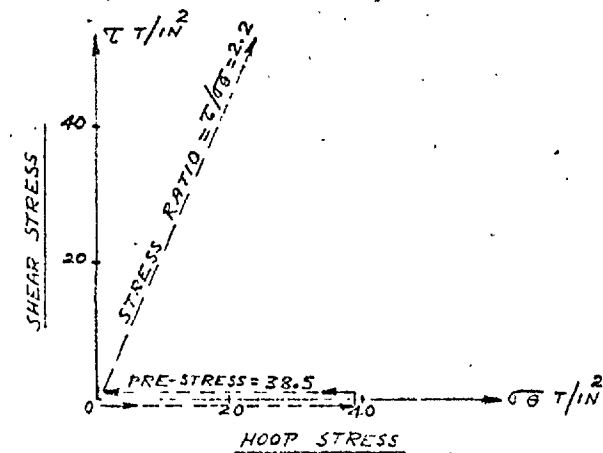
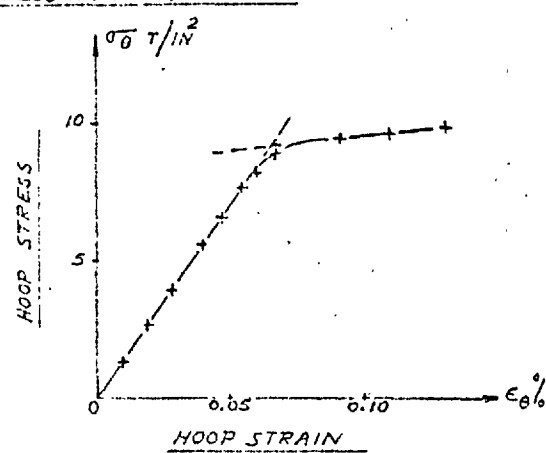
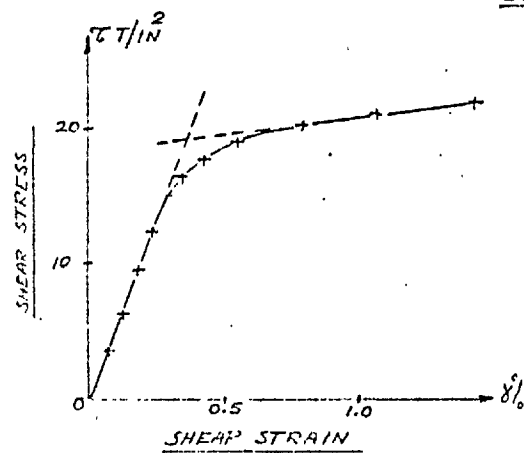
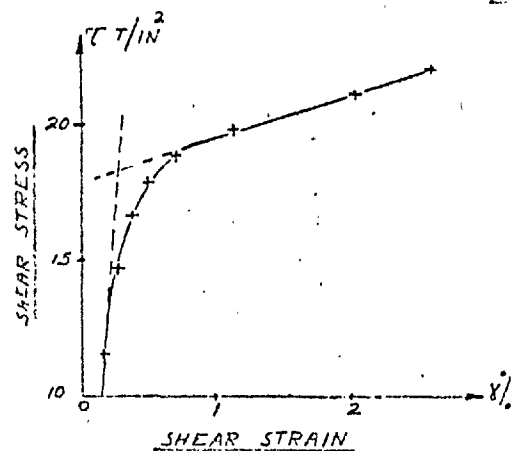


FIG. 90.

FIG. 91.



STRESS RATIO $\tau/\sigma = 1.1$

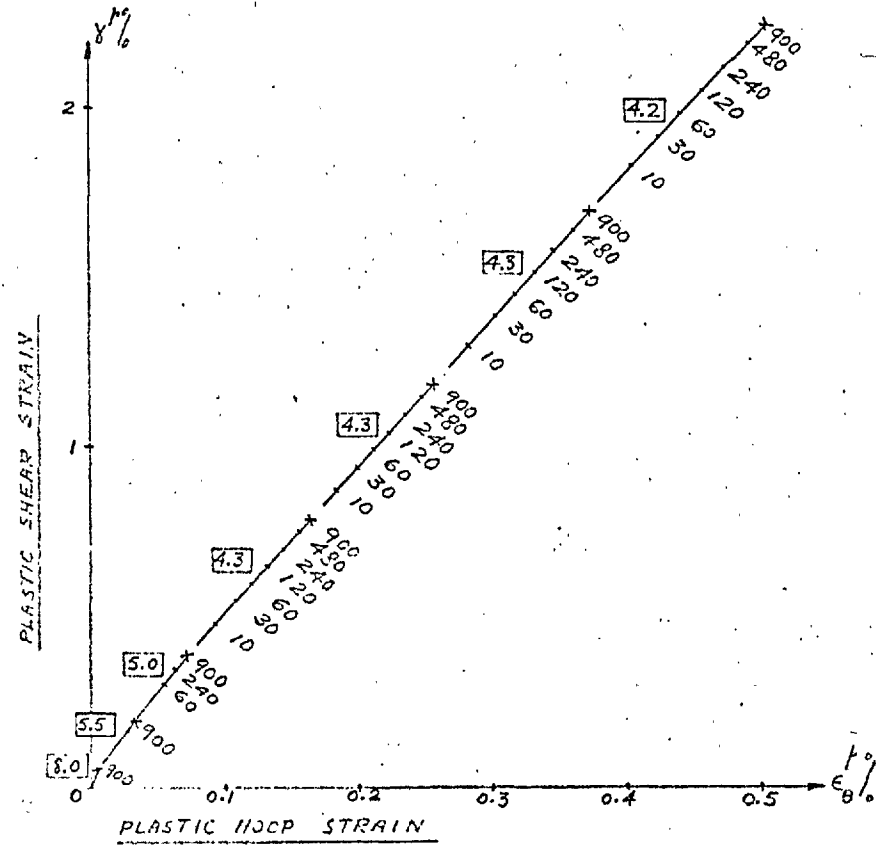
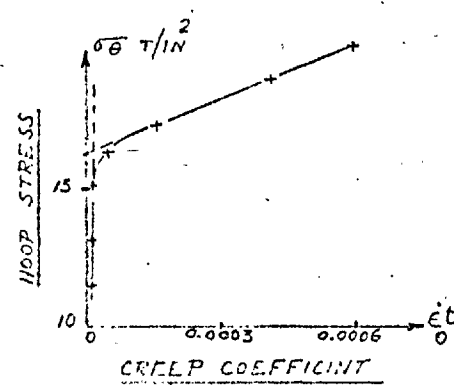
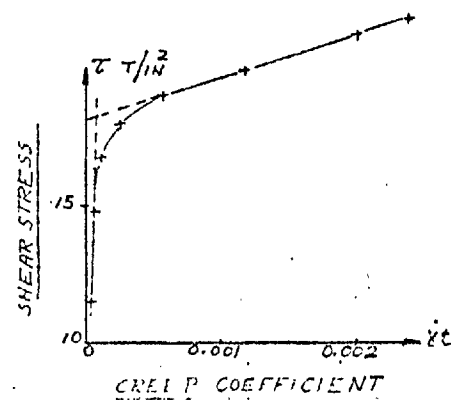
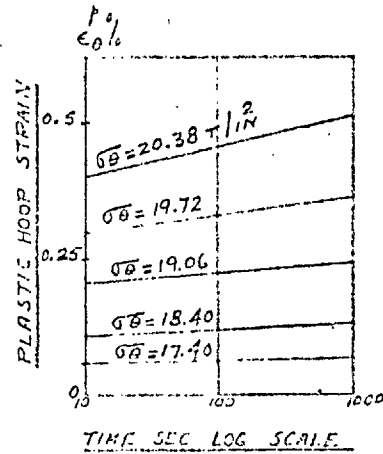
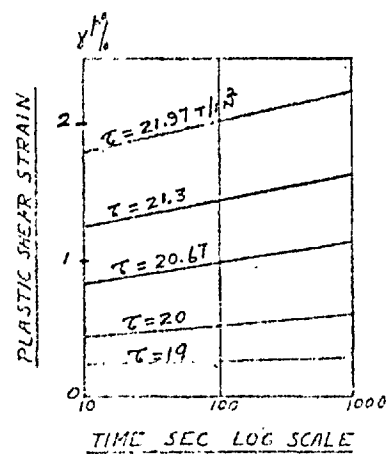
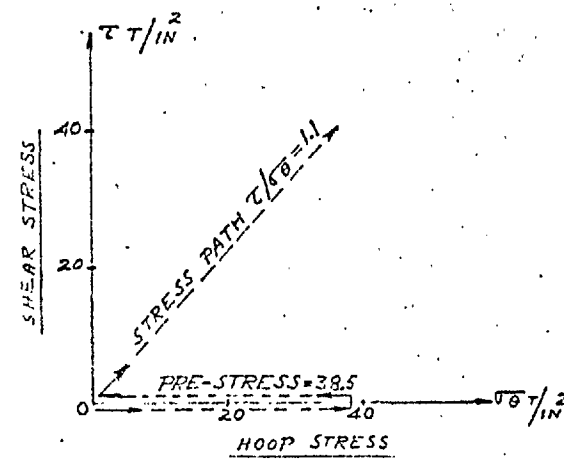
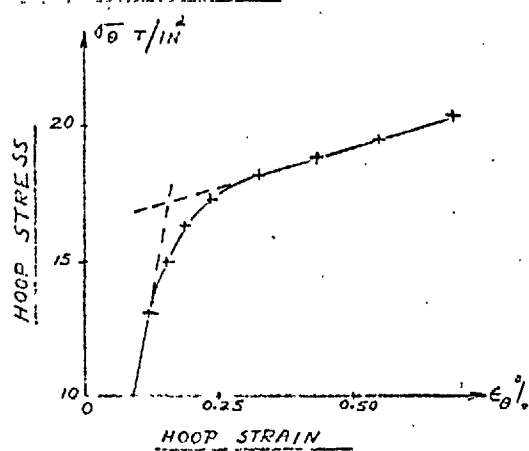


FIG. 91.

STRESS RATIO $\tau/\sigma = 0.55$

FIG. 92.

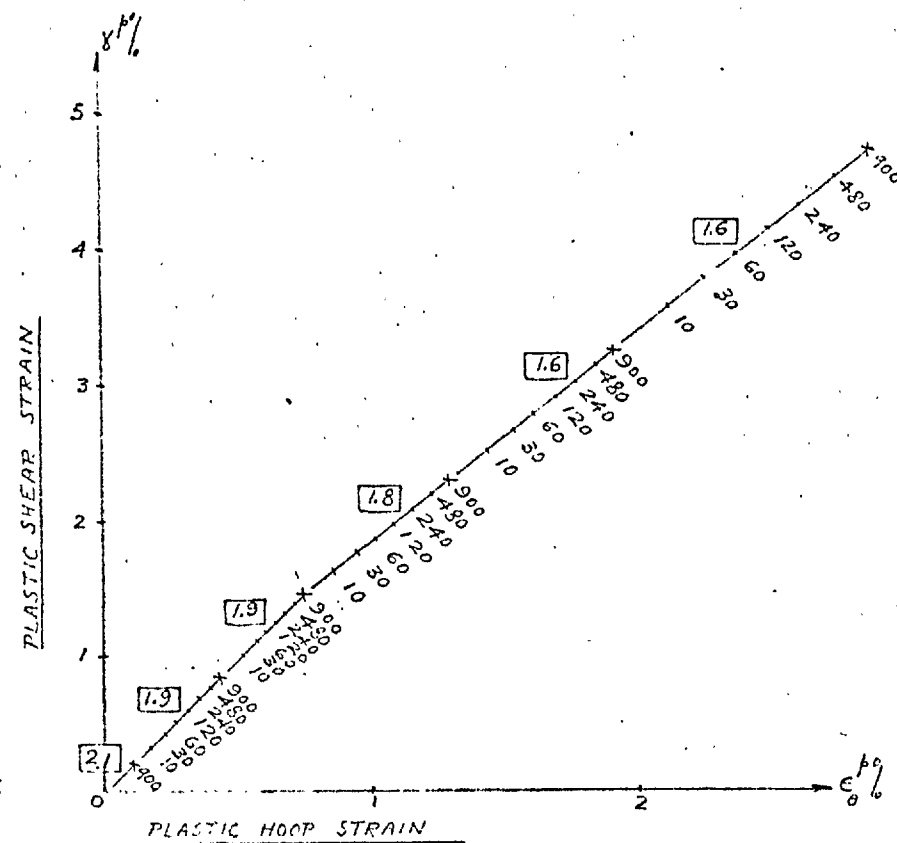
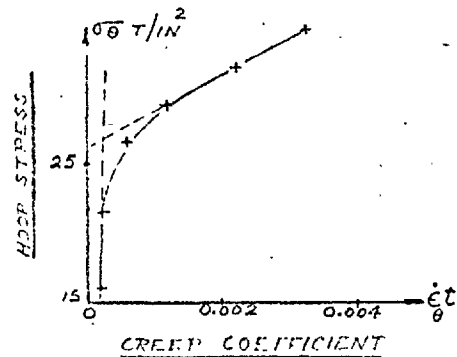
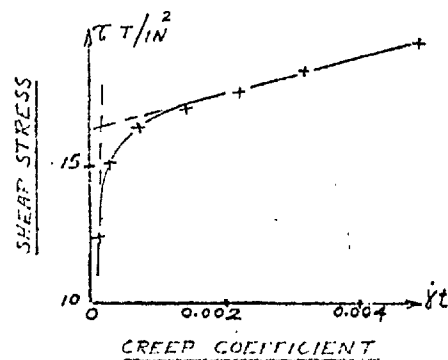
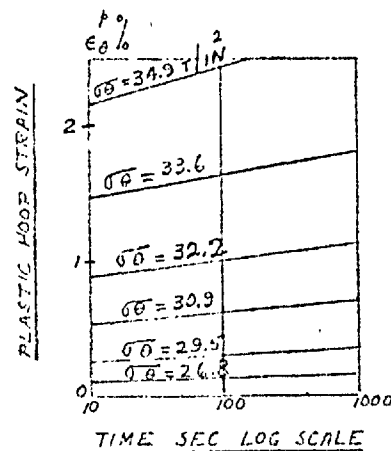
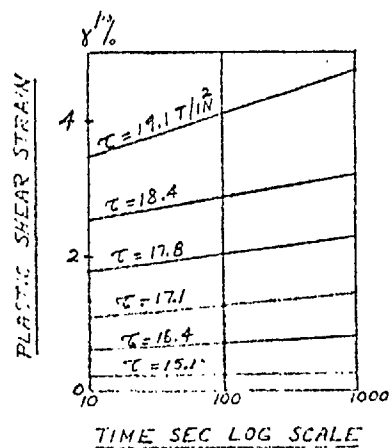
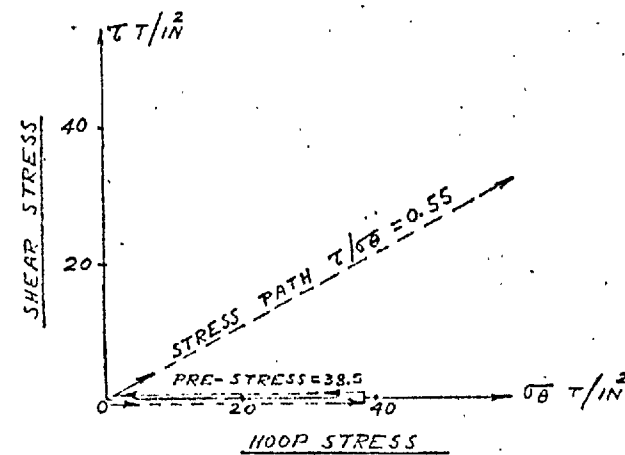
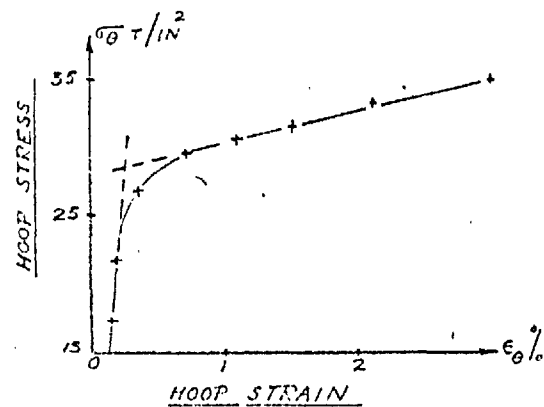
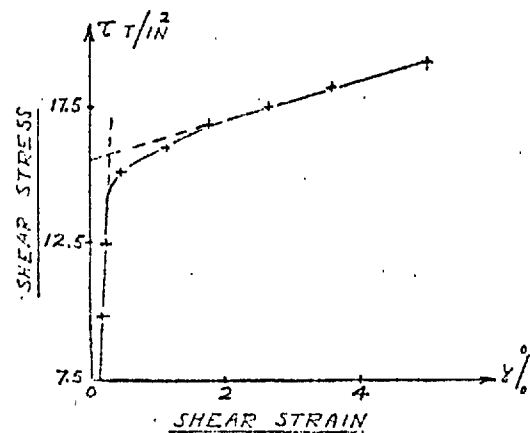


FIG. 92.

ρ
 $\epsilon_{\theta\theta} = 2.34\%$
 $\sigma_{\theta\theta} = 38.5 \tau / \text{IN}^2$

FIG. 93.

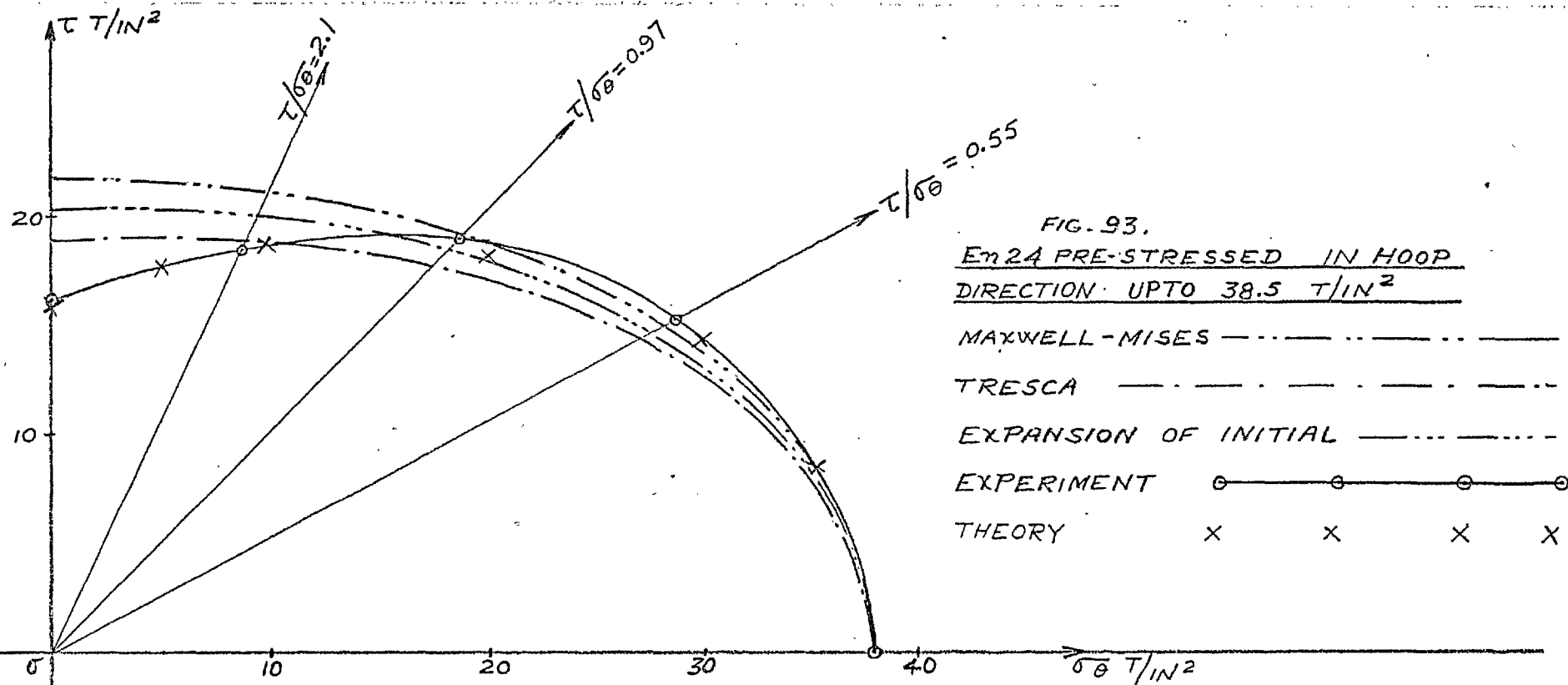
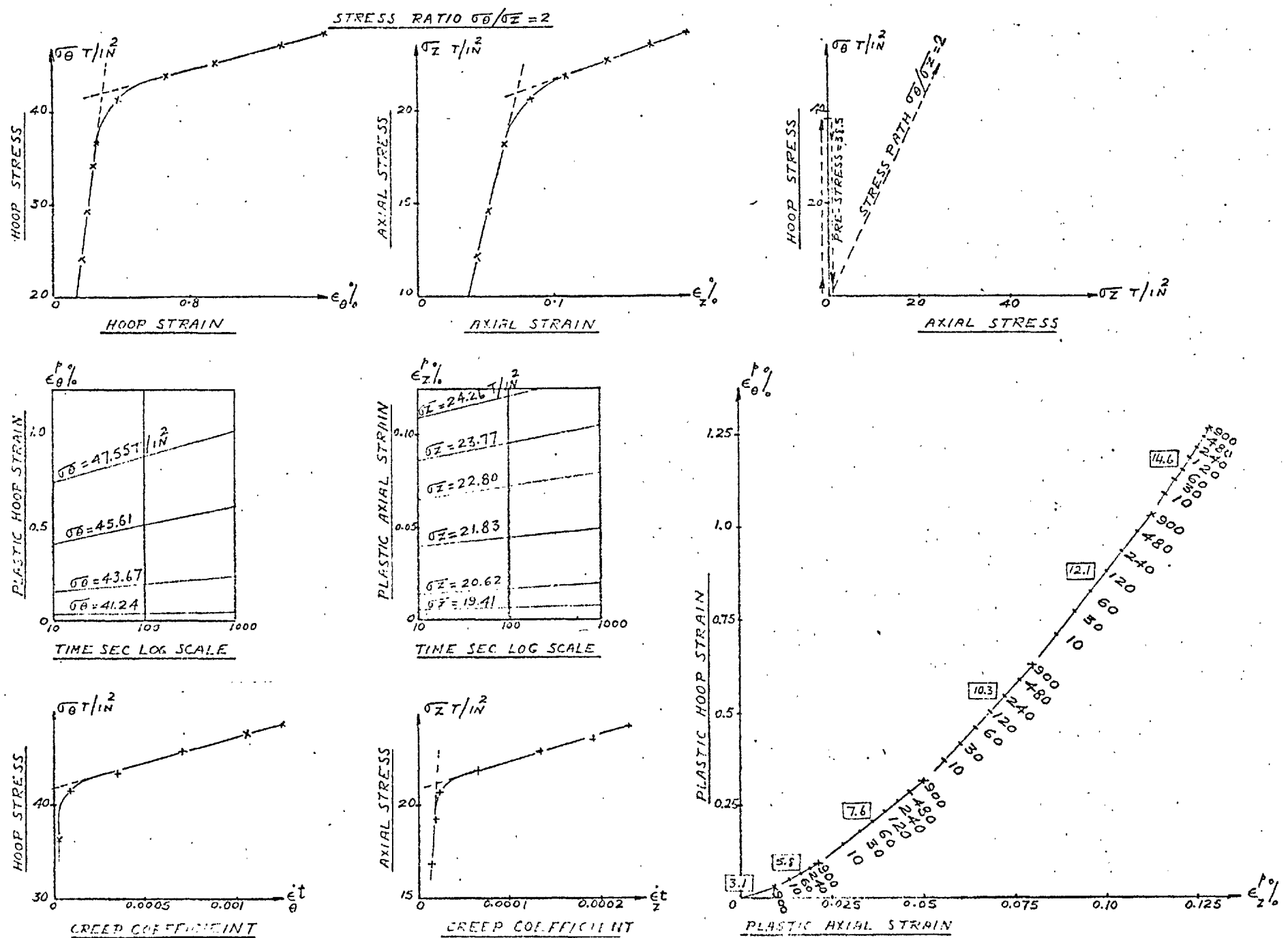


FIG. 94.



The figure consists of three graphs illustrating the relationship between stress and strain for a material under different loading conditions.

Graph 1: Hoop Stress vs. Hoop Strain
 The y-axis is labeled $\sigma_{\theta} T/in^2$ and the x-axis is labeled $\epsilon_{\theta} \%$. The curve shows a non-linear relationship, starting at the origin and increasing with a decreasing slope. A dashed line is drawn tangent to the curve at a strain of approximately 0.2%.

Graph 2: Axial Stress vs. Axial Strain
 The y-axis is labeled $\sigma_z T/in^2$ and the x-axis is labeled $\epsilon_z \%$. The curve shows a non-linear relationship, starting at the origin and increasing with a decreasing slope. A dashed line is drawn tangent to the curve at a strain of approximately 0.2%.

Graph 3: Hoop Stress vs. Axial Stress
 The y-axis is labeled $\sigma_{\theta} T/in^2$ and the x-axis is labeled $\sigma_z T/in^2$. The graph shows a linear relationship between the two stresses, with a slope of $\sigma_{\theta}/\sigma_z = 0.98$. A pre-stress of 38.5 is indicated on the y-axis.

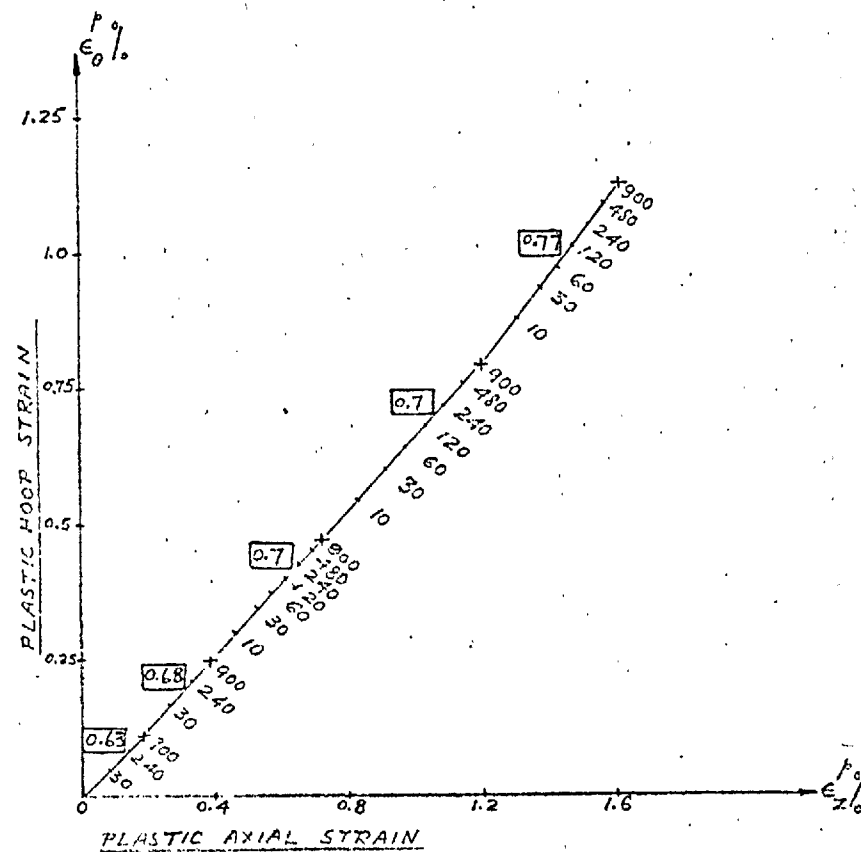
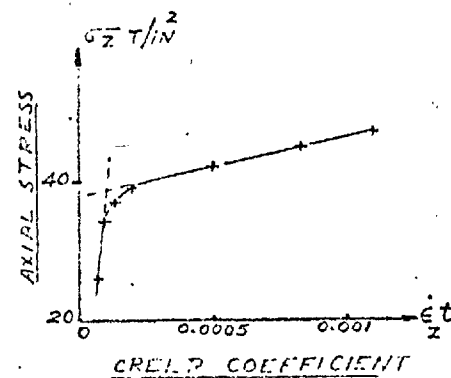
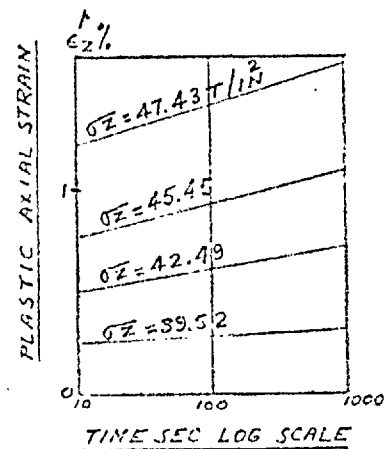


FIG. 95.

FIG. 96.

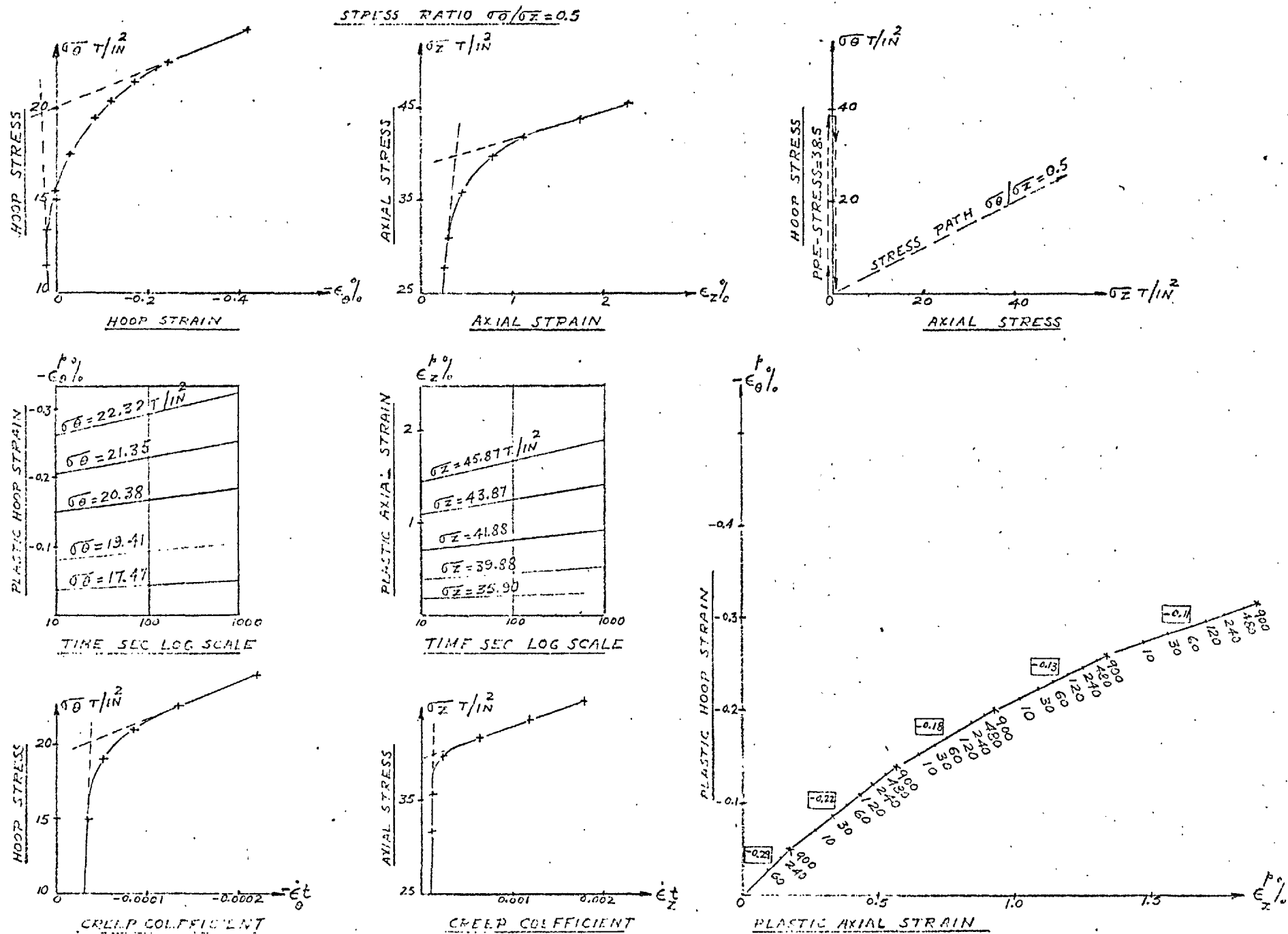


FIG. 96.

FIG. 97.

EN 24 PRE-STRESSED IN
HOOP " σ_2 " DIRECTION UPTO 38.5 T/IN^2

$$\epsilon_{\theta\theta} = 2.34\%$$

$$\sigma_{\theta\theta} = 38.5 \text{ T/IN}^2$$

FIG. 97.

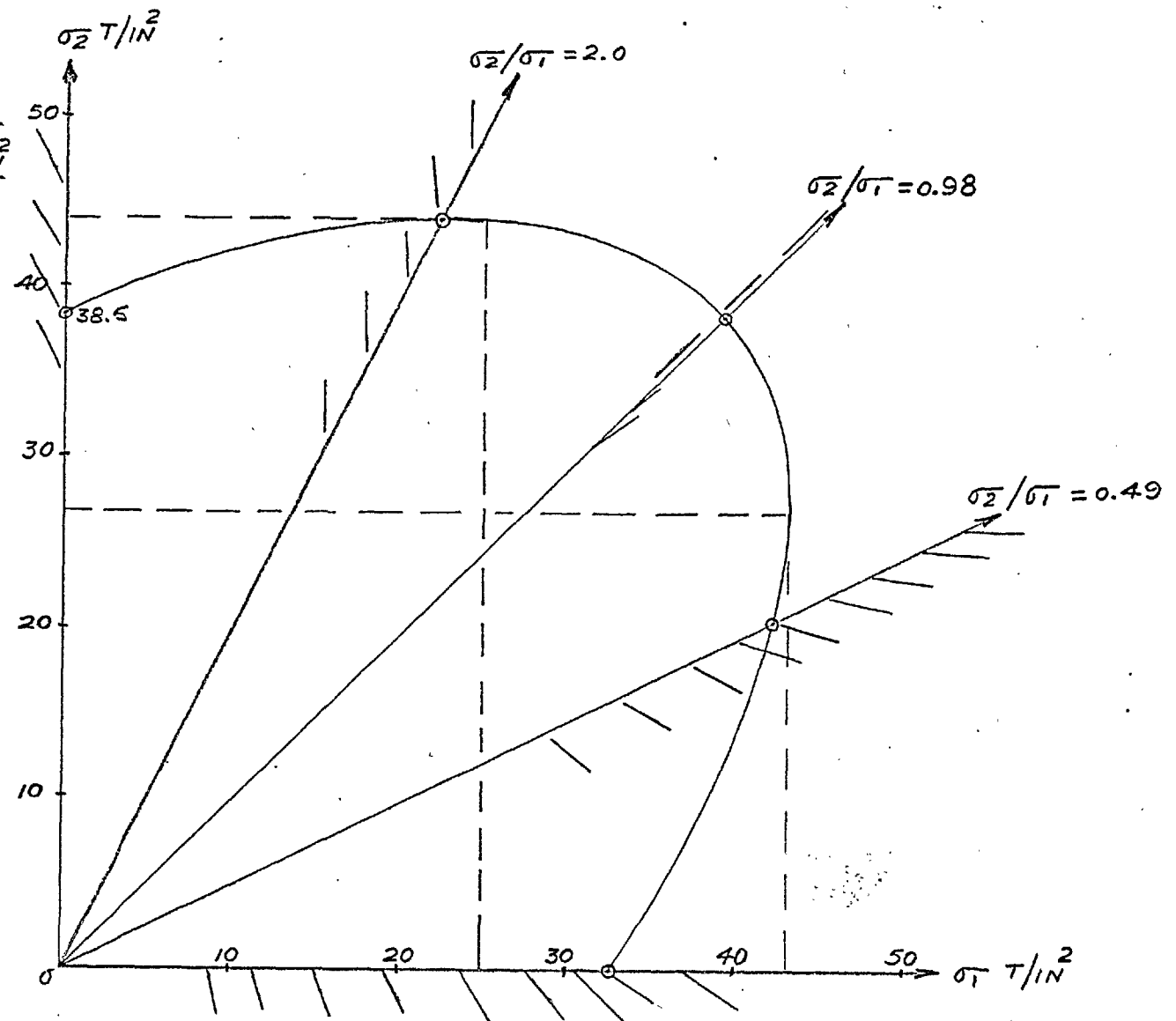


FIG. 97.

STRESS RATIO $\tau/\sigma_z = 2.18$

FIG. 99.

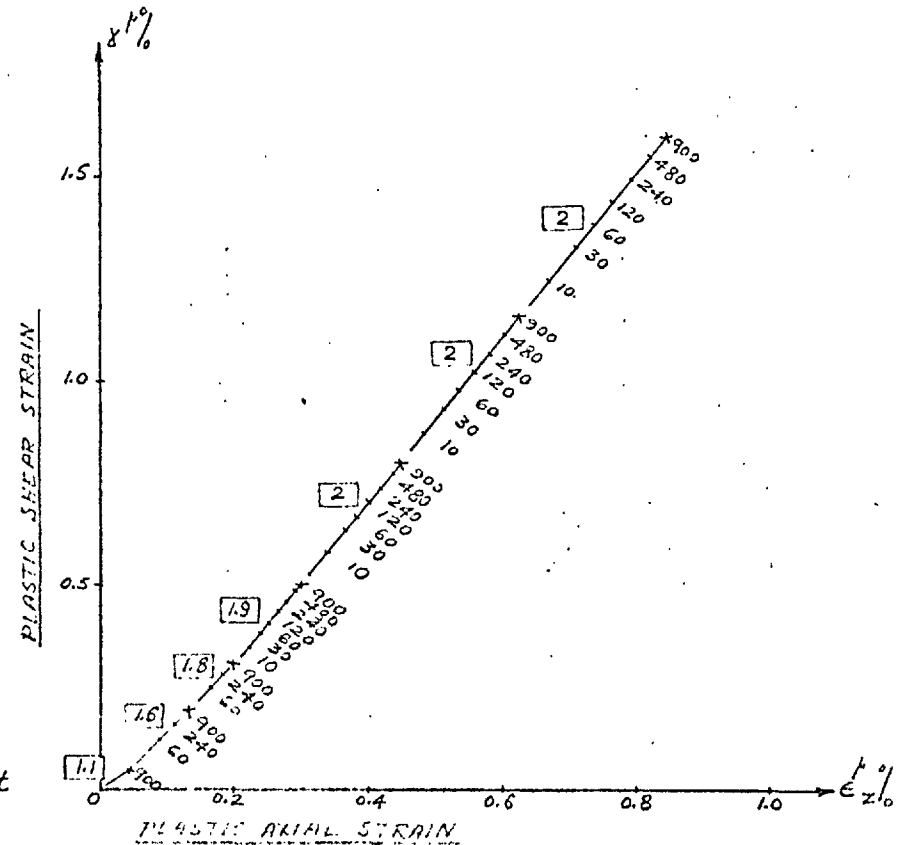
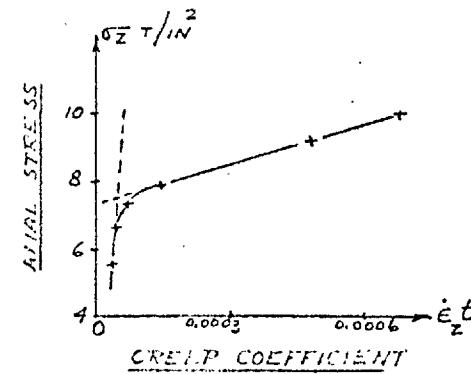
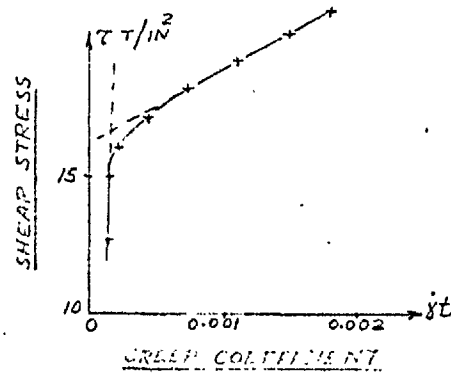
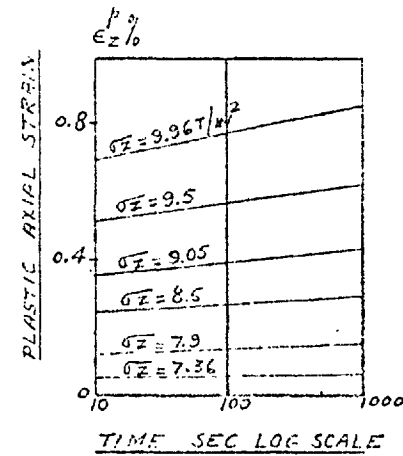
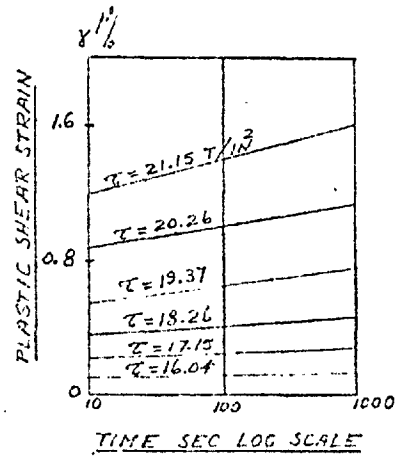
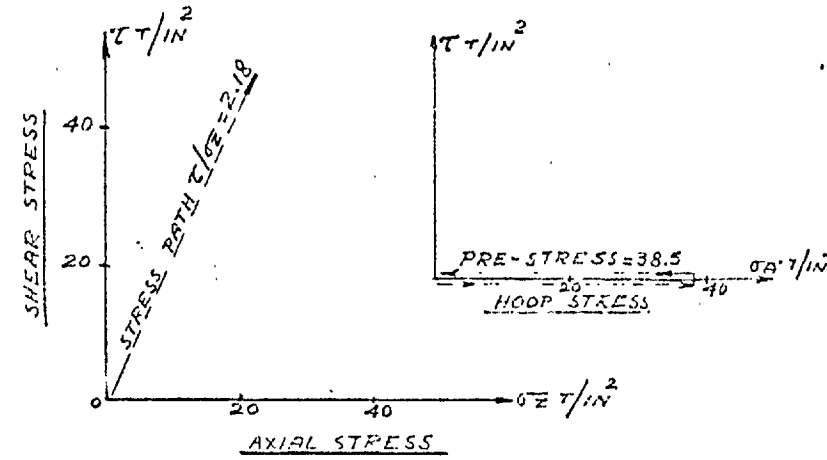
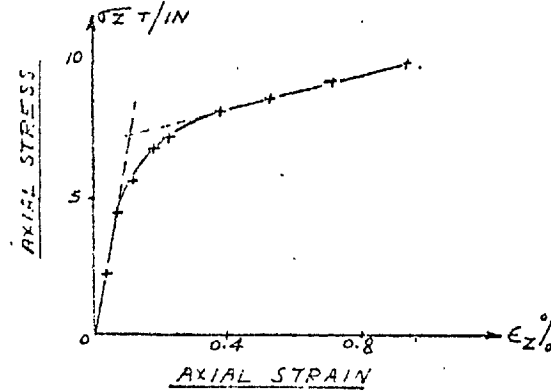
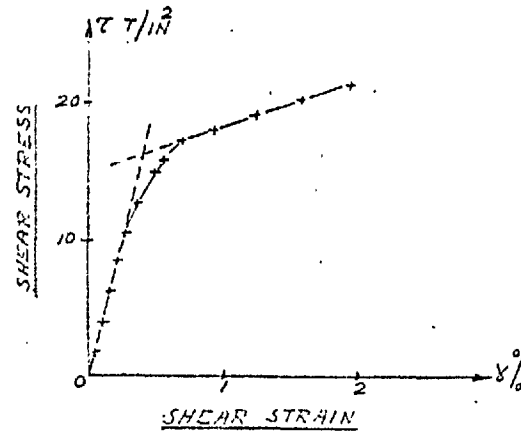


FIG. 99.

FIG. 100.

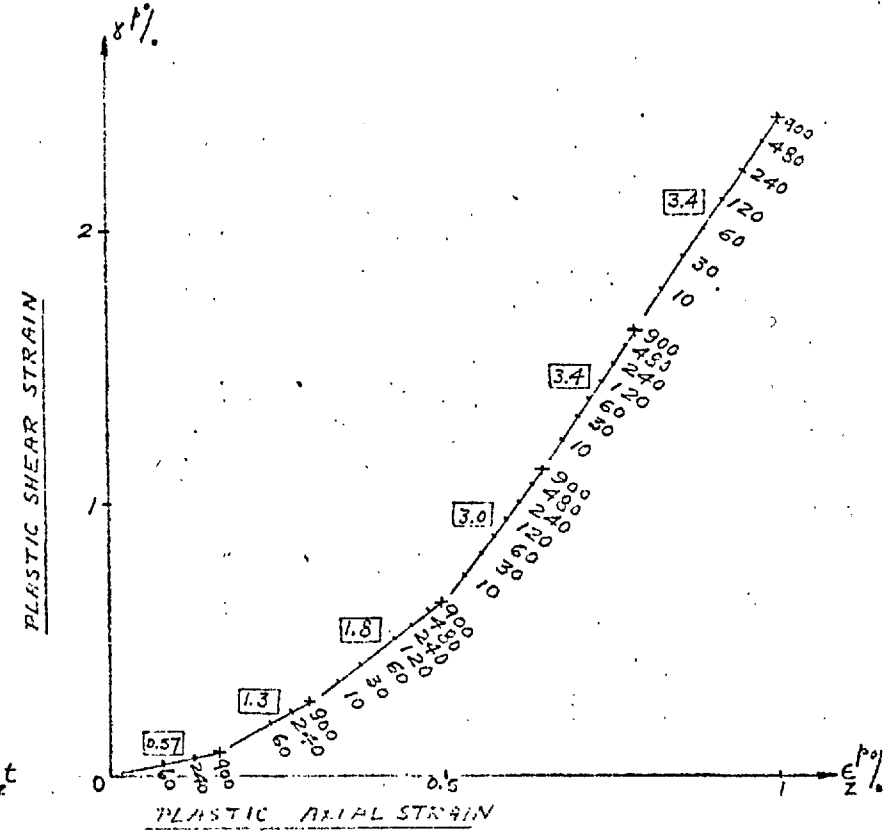
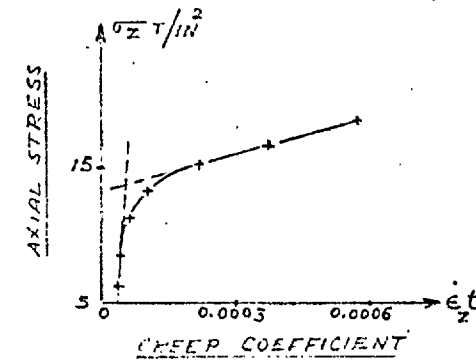
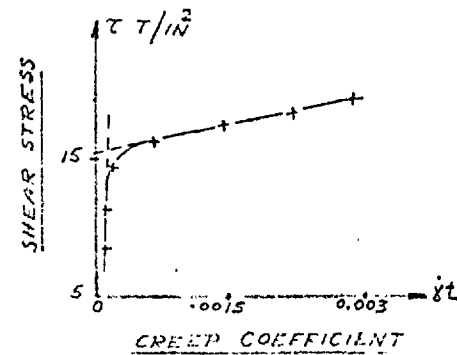
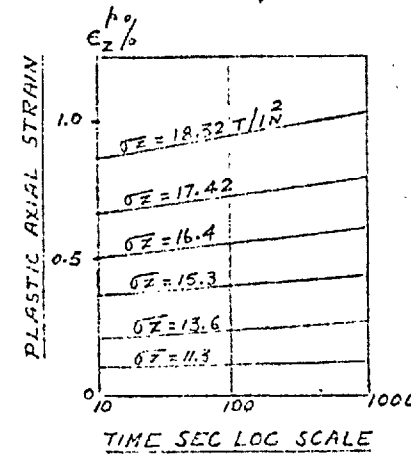
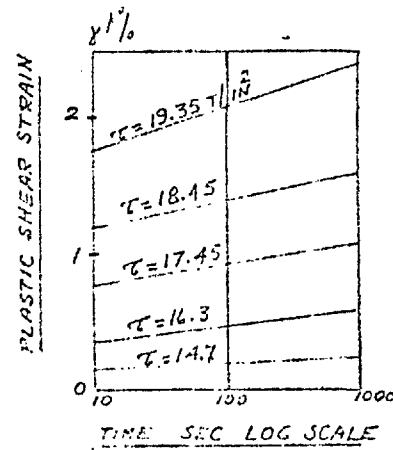
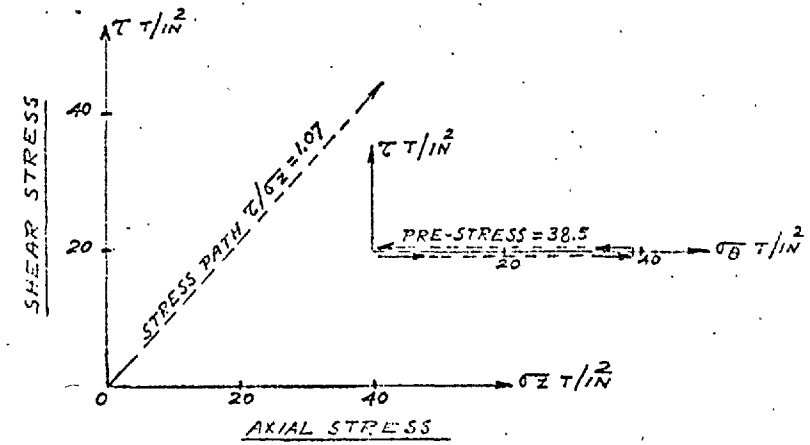
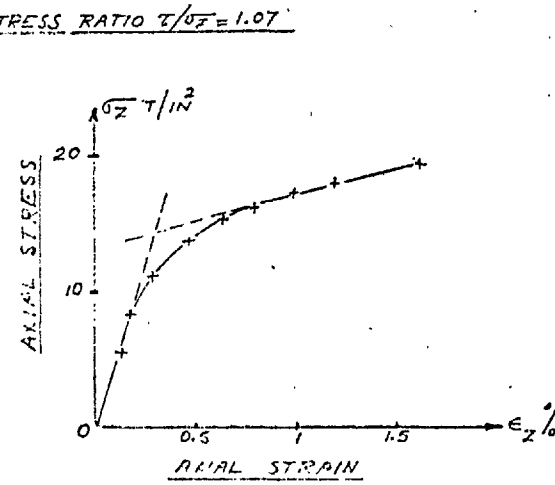
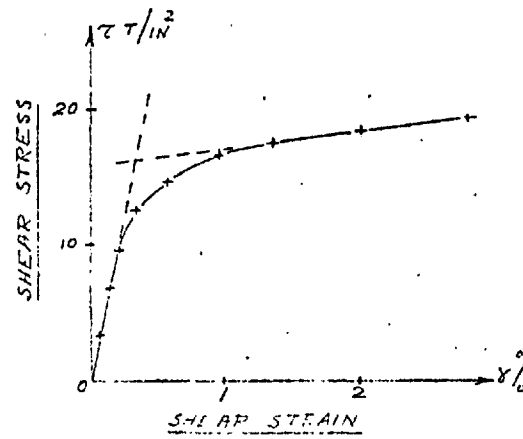


FIG. 100.

FIG. 101.

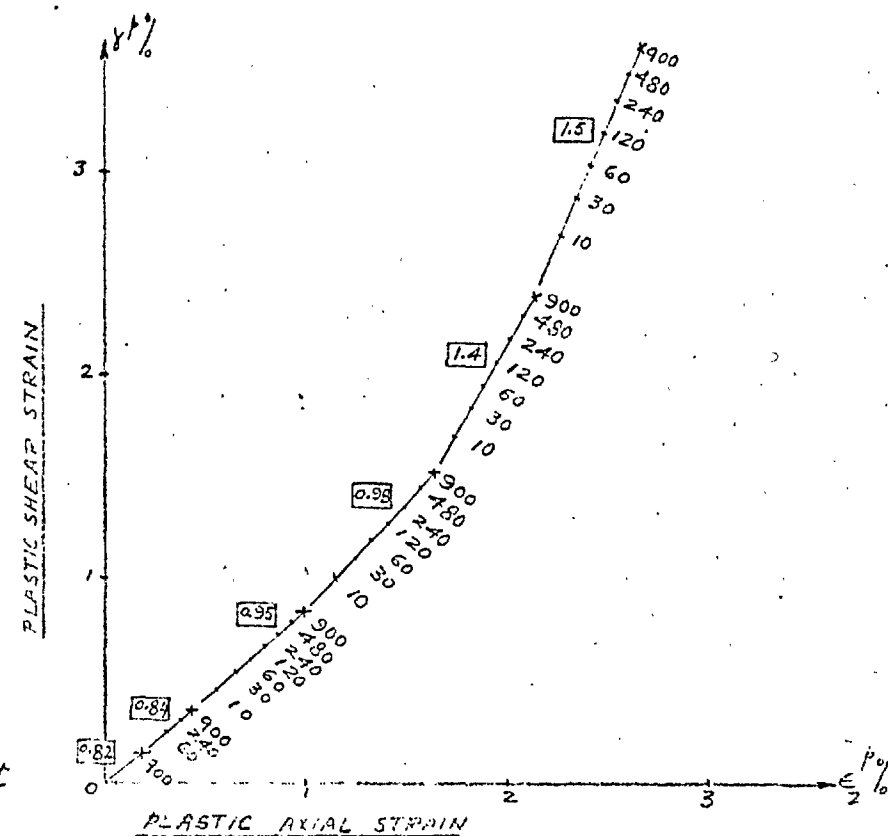
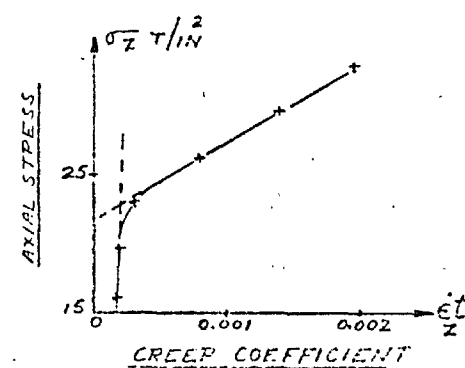
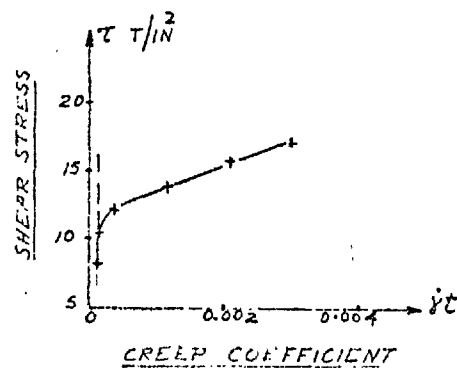
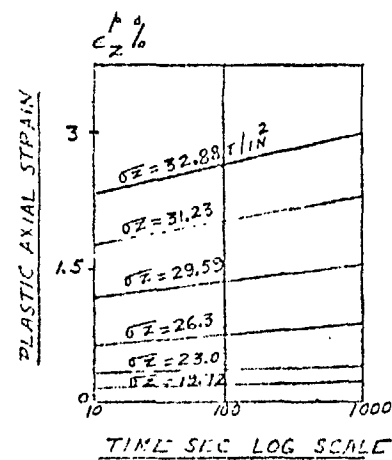
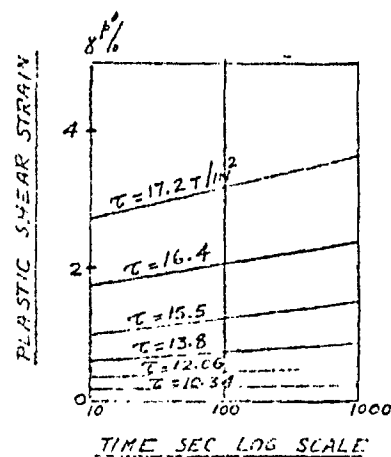
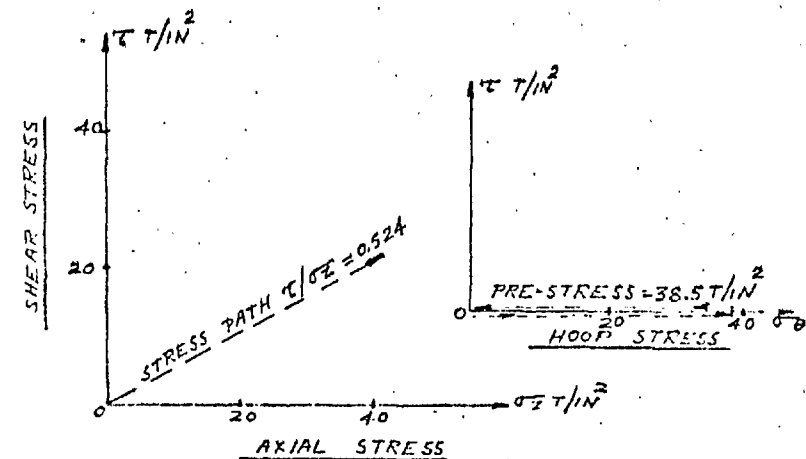
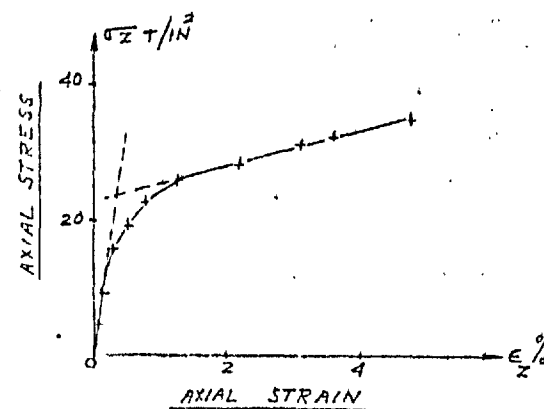
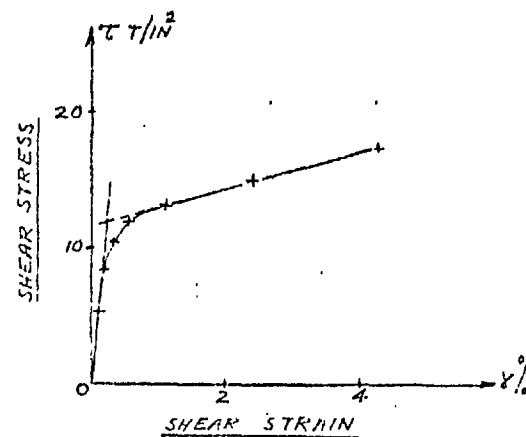


FIG. 101.

$$\epsilon_{\theta\theta}^p = 2.34\%$$

$$\sigma_{\theta\theta} = 38.5 \text{ T/IN}^2$$

FIG. 102.

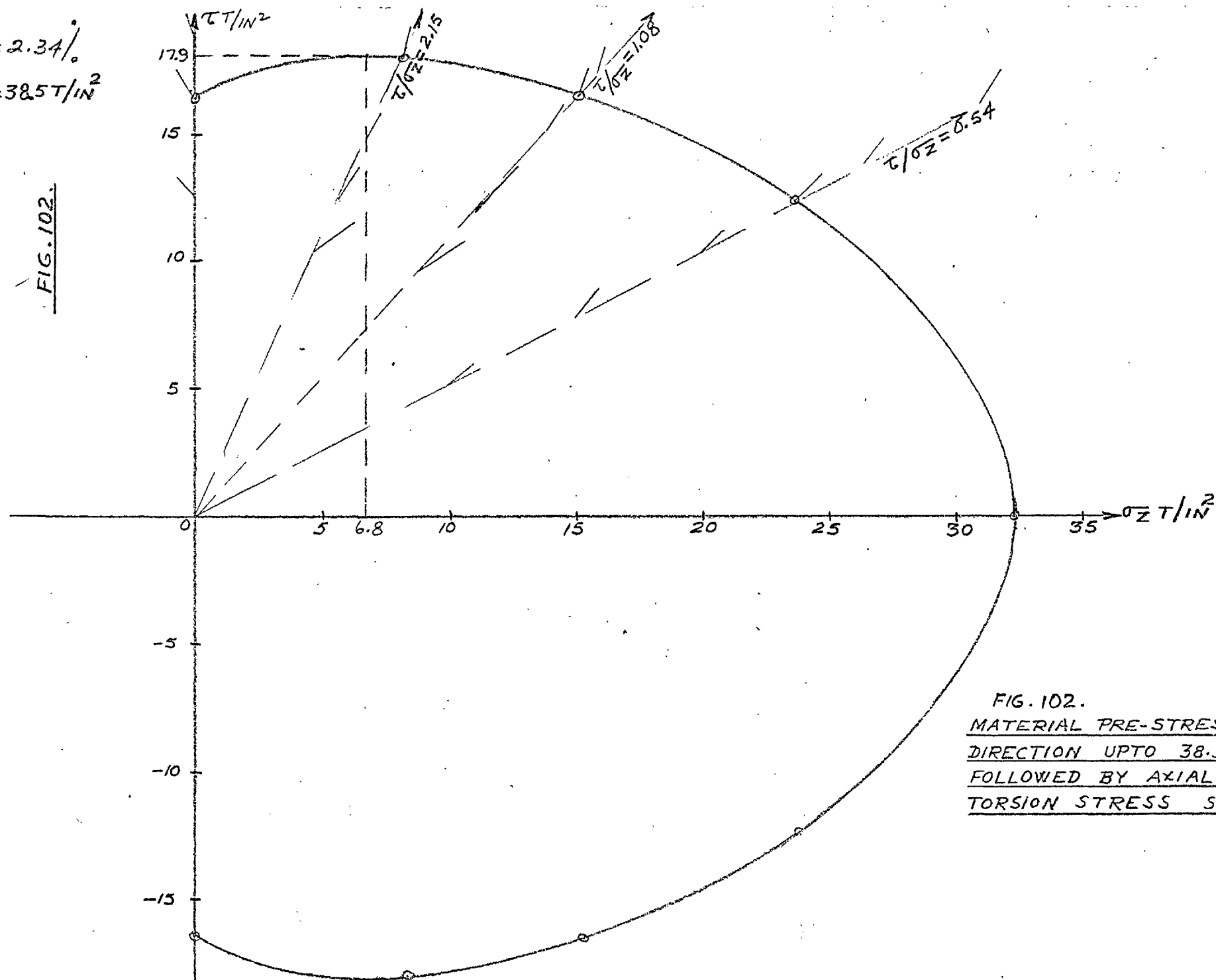


FIG. 102.
MATERIAL PRE-STRESSED IN HOOP
DIRECTION UPTO 38.5 T/IN^2
FOLLOWED BY AXIAL TENSION-
TORSION STRESS SYSTEM.

$$\epsilon_{0\theta} = 2.8\%$$

$$\sigma_{0\theta} = 38.5 \text{ T/IN}^2$$

FIG. 103.

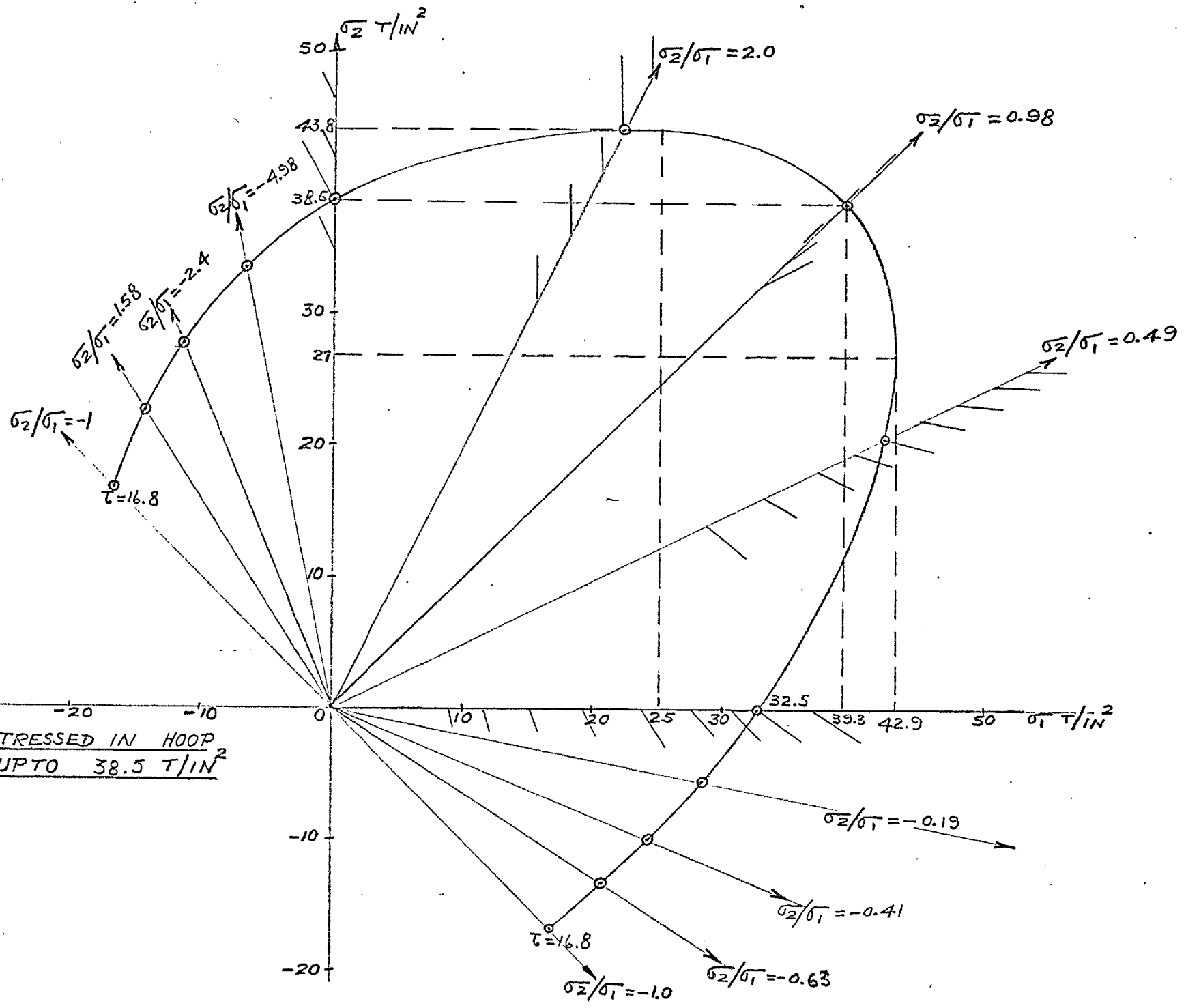


FIG. 103.
E724 PRE-STRESSED IN HOOP
DIRECTION UPTO 38.5 T/IN^2

FIG. 104.

STRESS RATIO $\sigma_\theta/\sigma_z = 2$

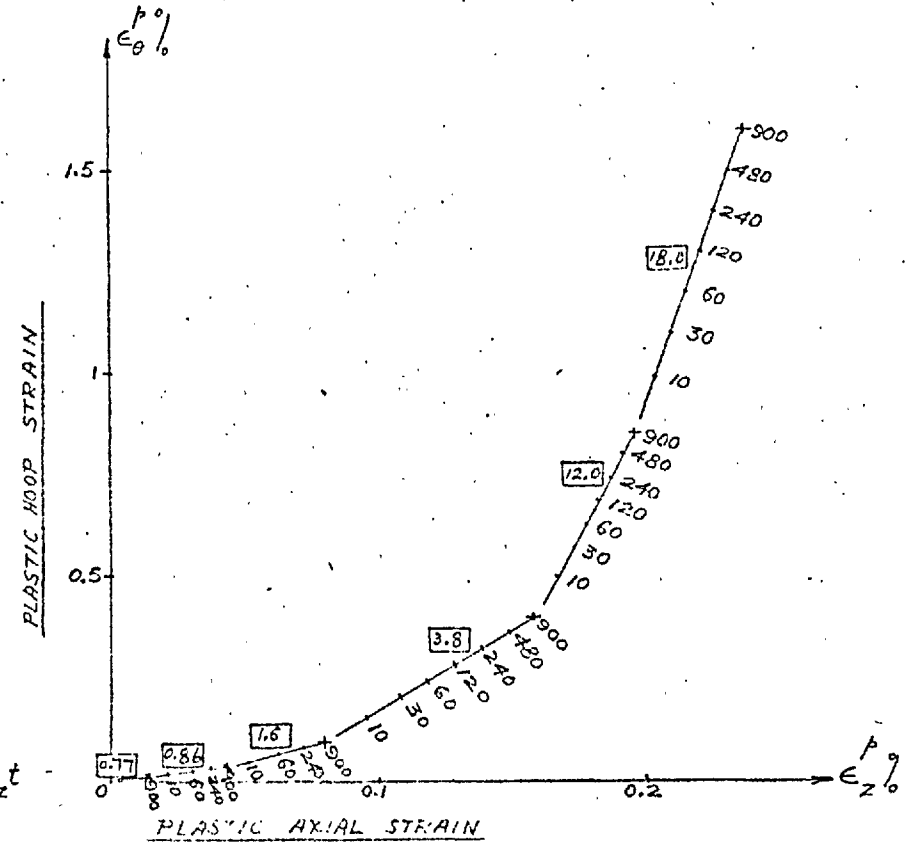
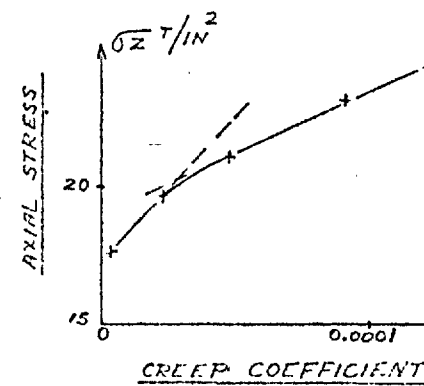
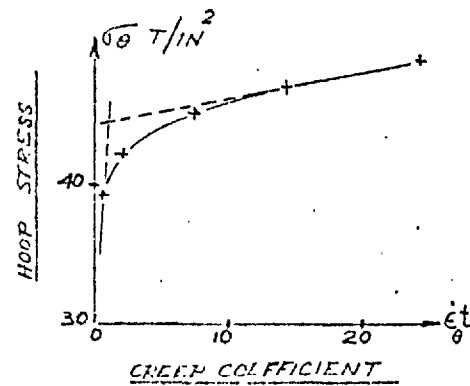
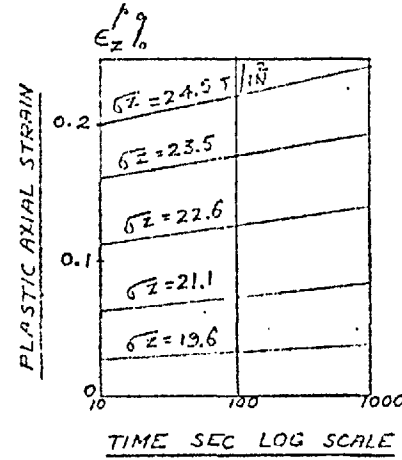
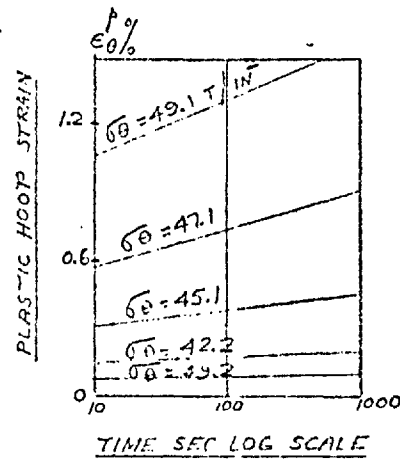
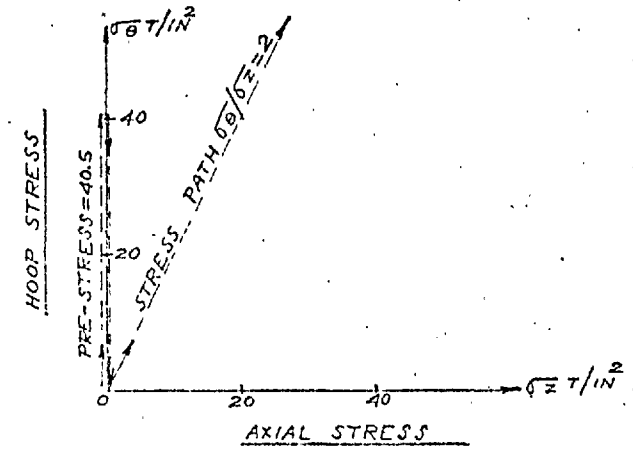
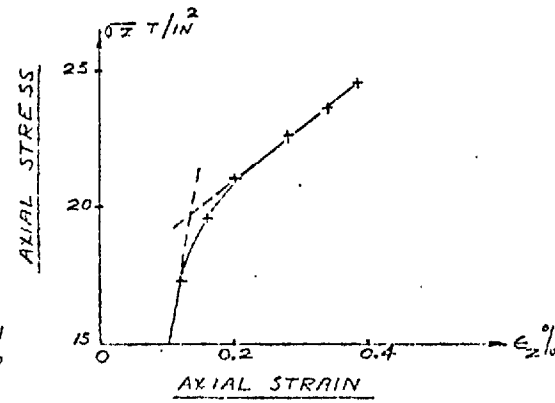
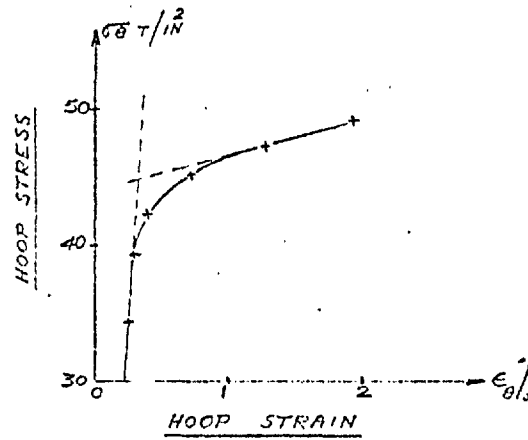


FIG. 104.

STRESS RATIO $\sigma_\theta/\sigma_z = 1.0$

FIG. 105.

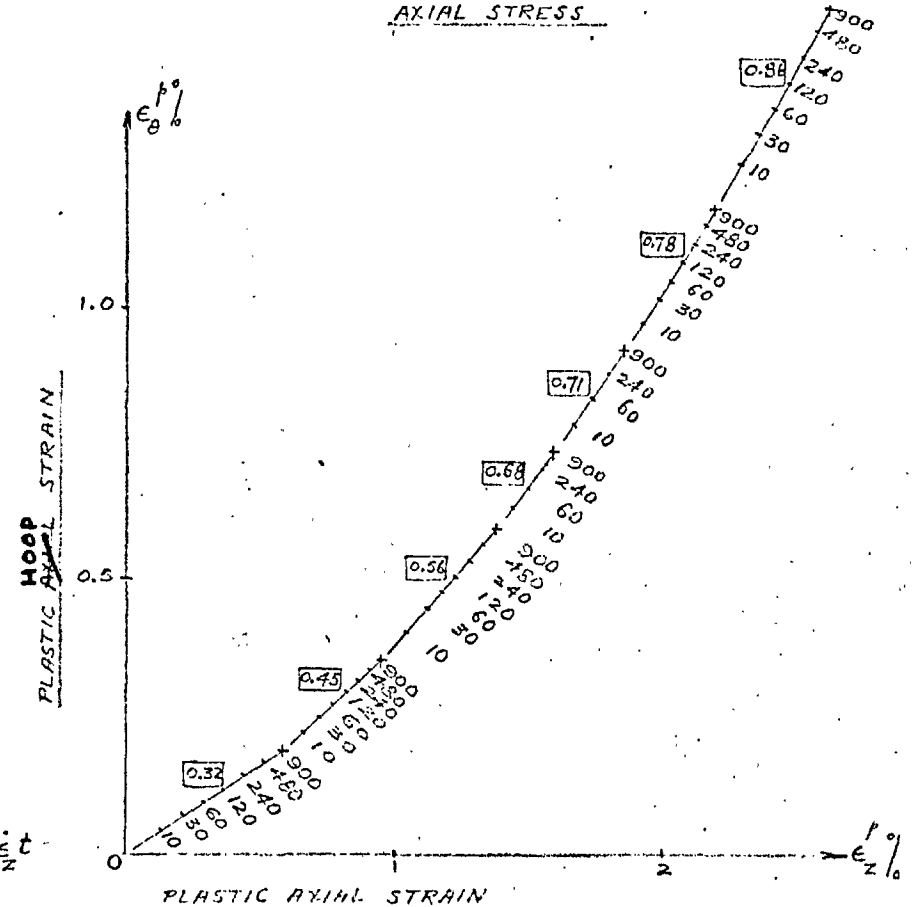
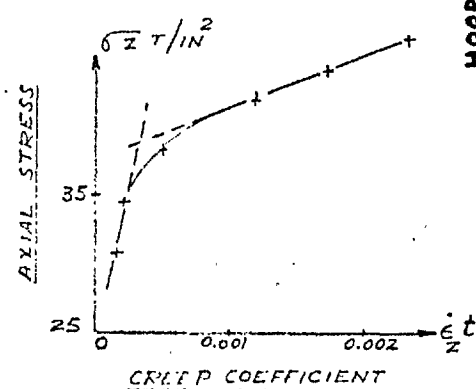
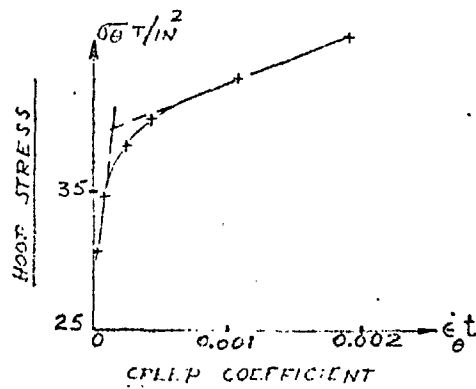
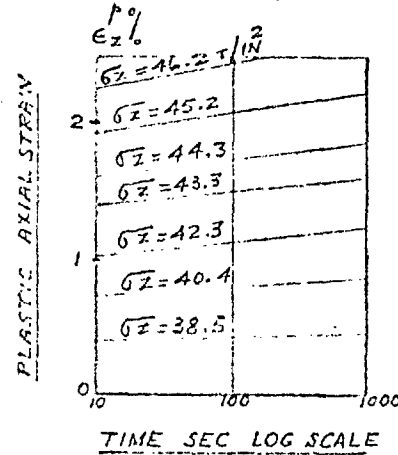
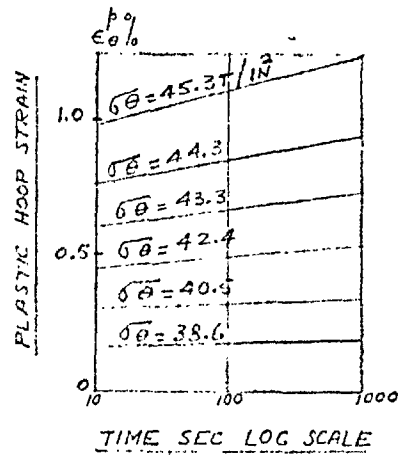
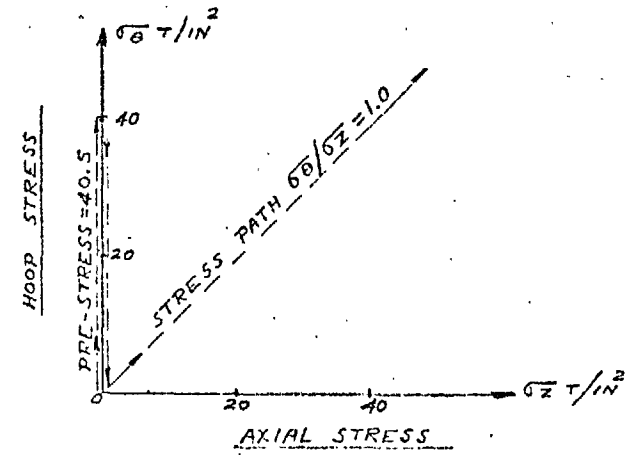
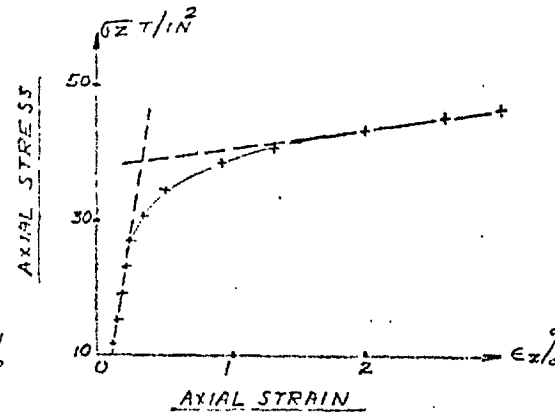
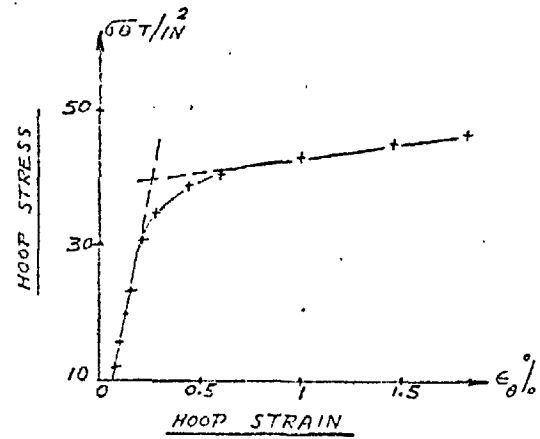


FIG. 105.

FIG. 106.

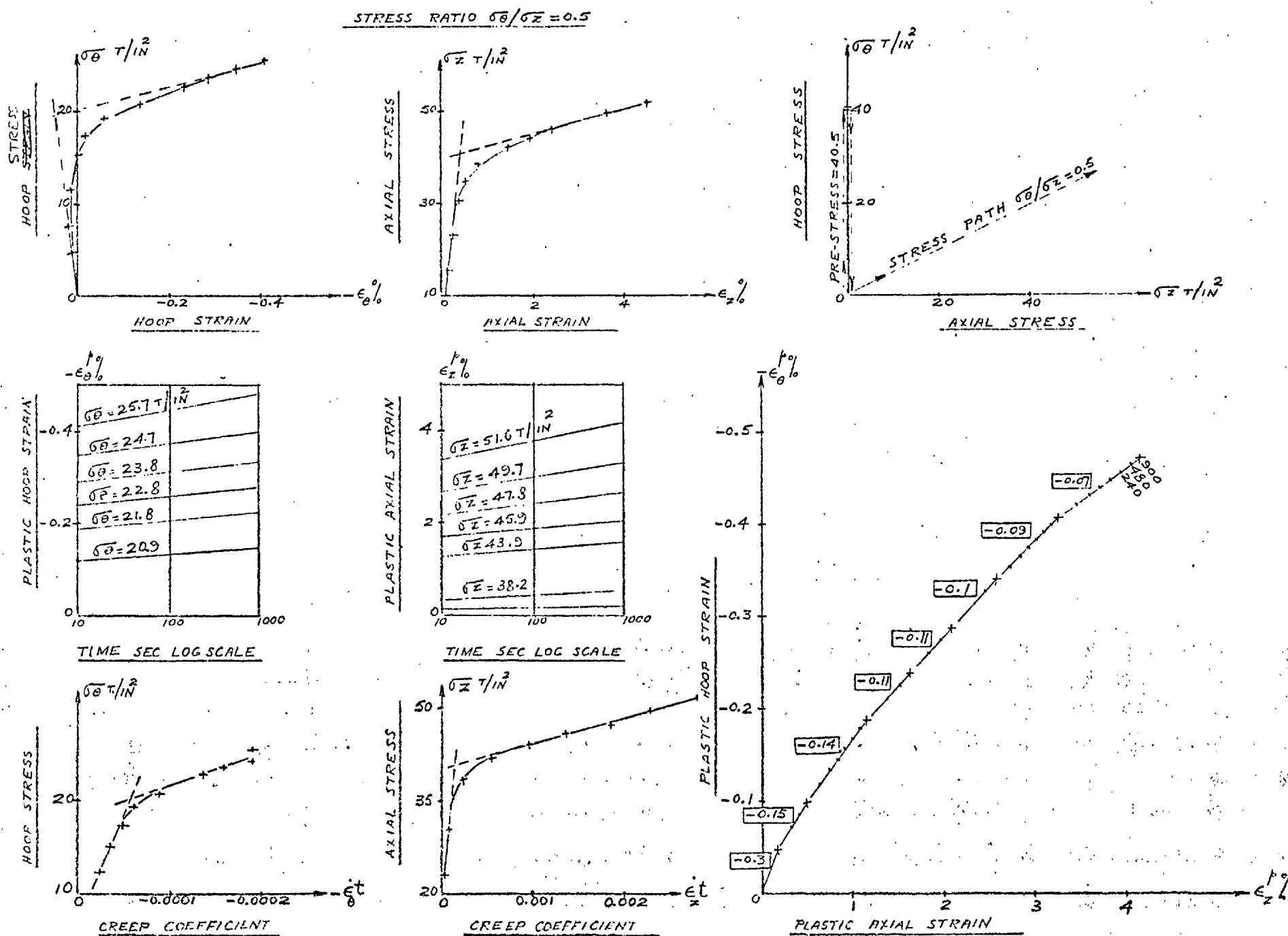
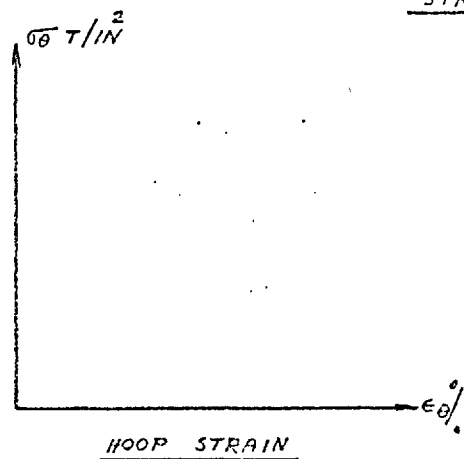


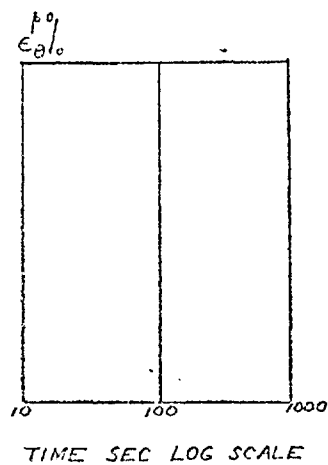
FIG. 106.

FIG. 107.

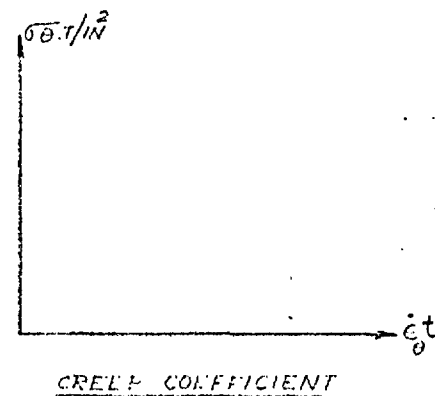
HOOP STRESS



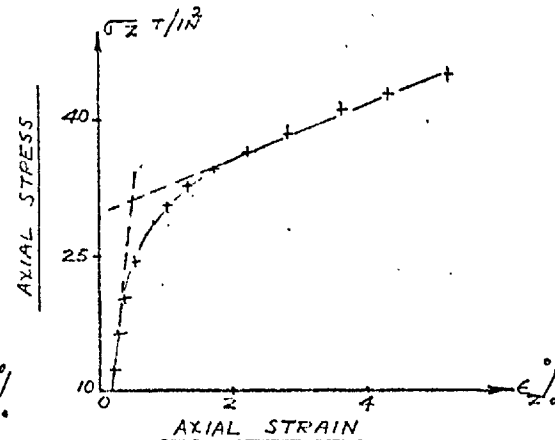
PLASTIC HOOP STRAIN



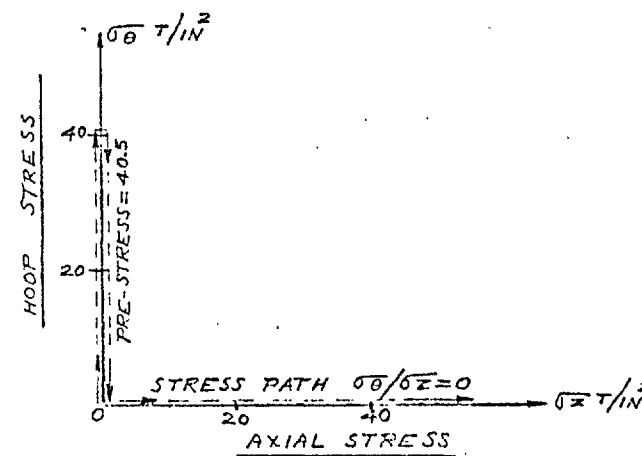
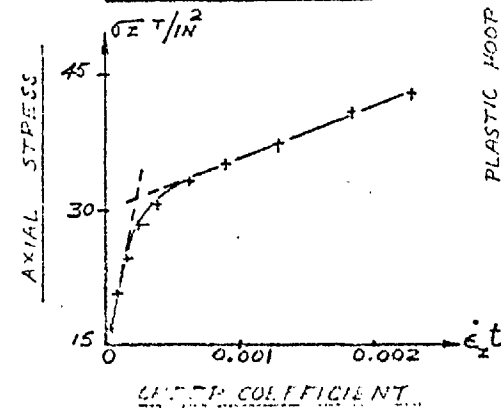
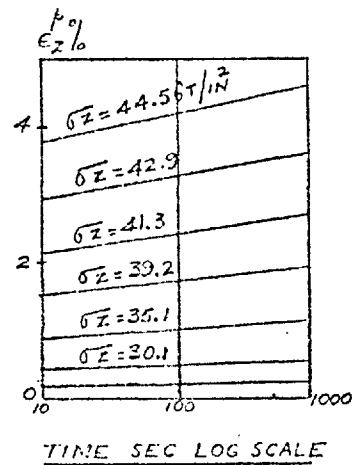
HOOP STRESS



STRESS RATIO $\sigma_{\theta} / \sigma_z = 0$



PLASTIC AXIAL STRAIN



PLASTIC HOOP STRAIN

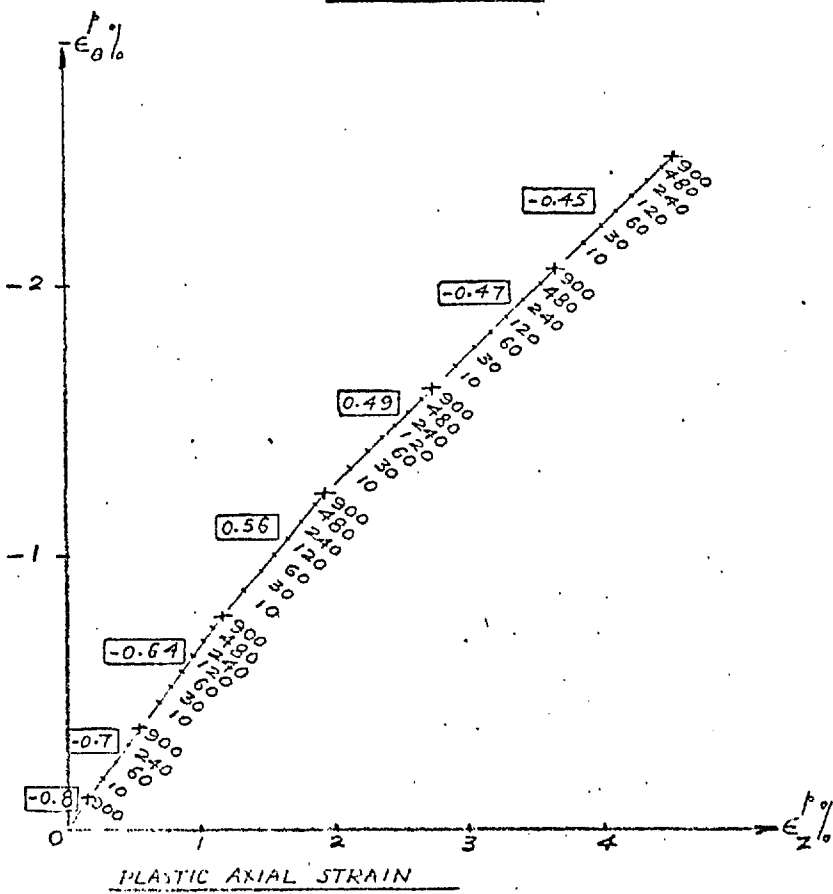


FIG. 107.

$$\epsilon_{00} = 3.34\%$$

$$\overline{\sigma_0} = 40.5 \text{ T/IN}^2$$

FIG.108.

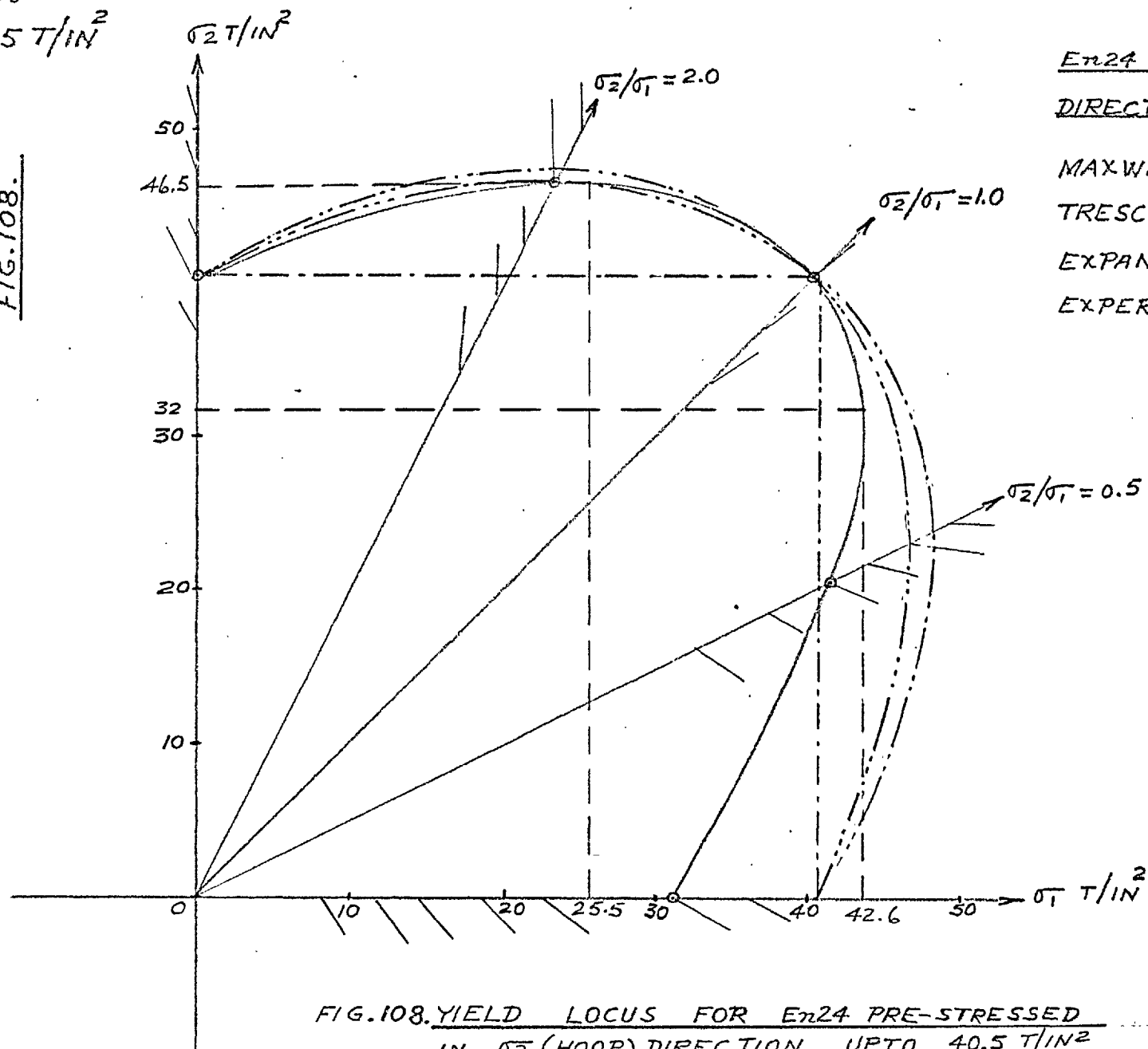


FIG.108.

En24 PRE-STRESSED IN HOOP, σ_2 , DIRECTION UPTO 40.5 T/IN^2

MAXWELL-MISES — · · · · ·
TRESCA — · · · · ·
EXPANSION OF INITIAL — · · · · ·
EXPERIMENT —————

FIG.108. YIELD LOCUS FOR En24 PRE-STRESSED IN σ_2 (HOOP) DIRECTION UPTO 40.5 T/IN^2

FIG. 109.

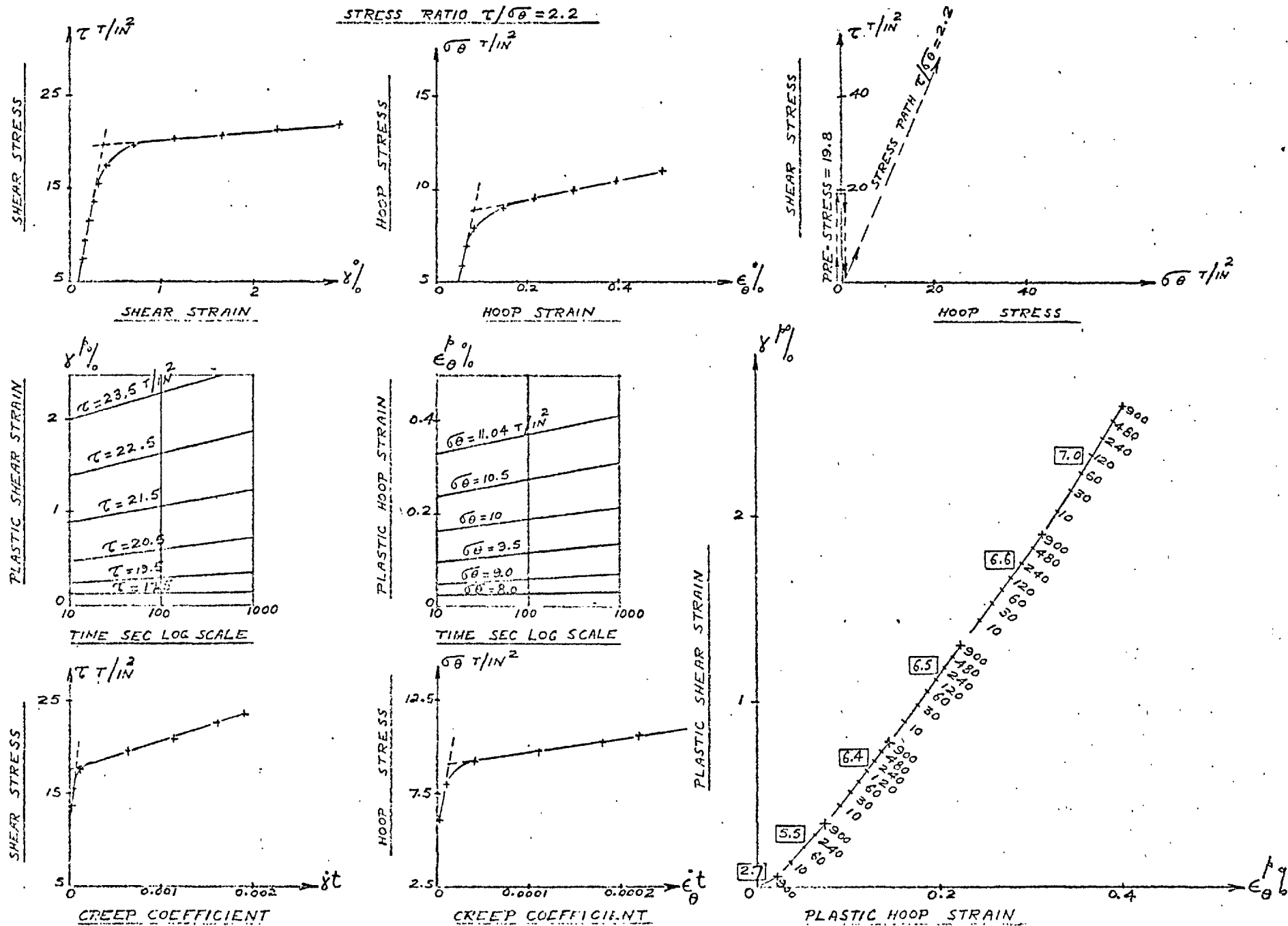


FIG. 109.

FIG. 110.

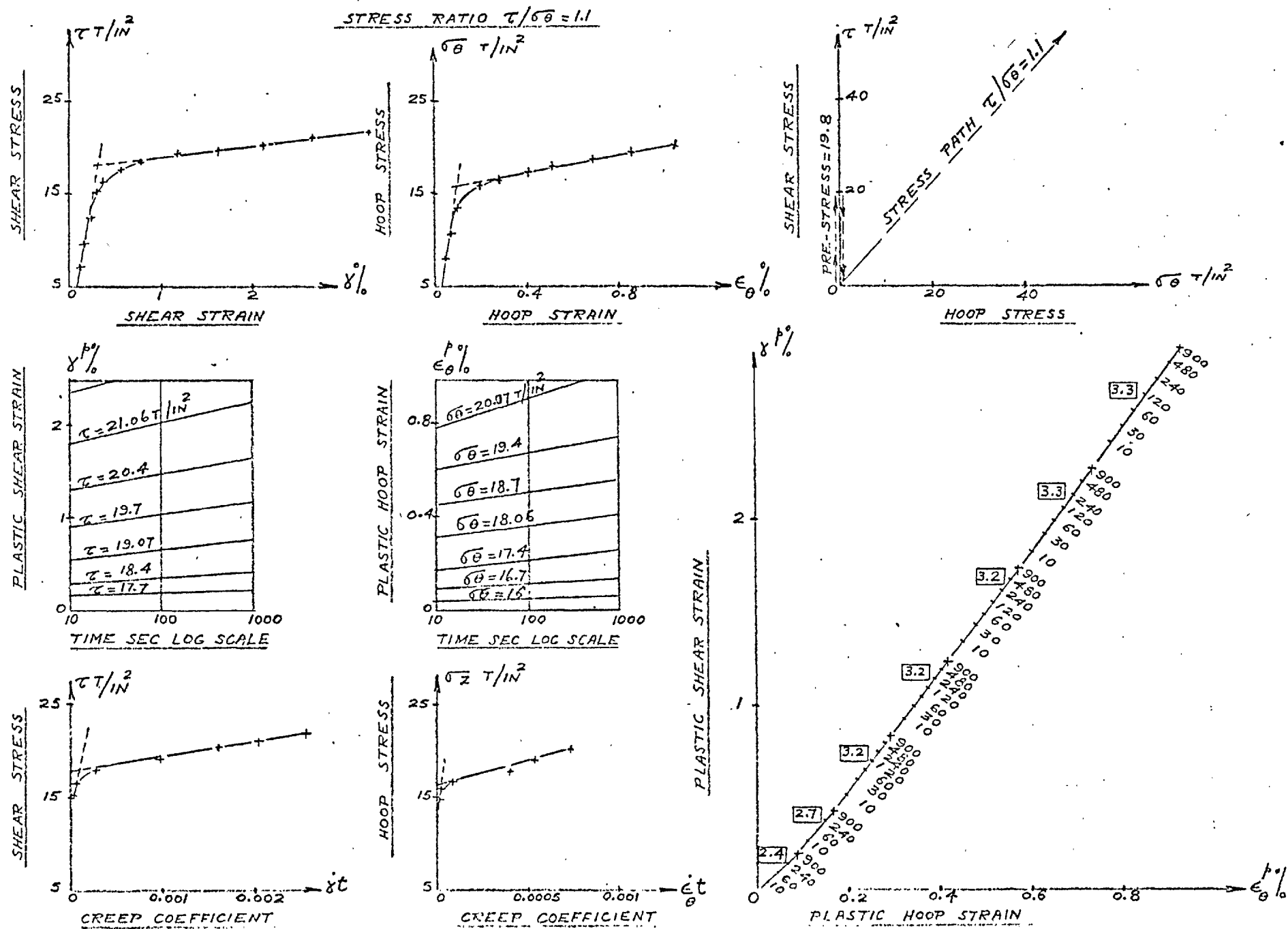


FIG. 110.

FIG. III.

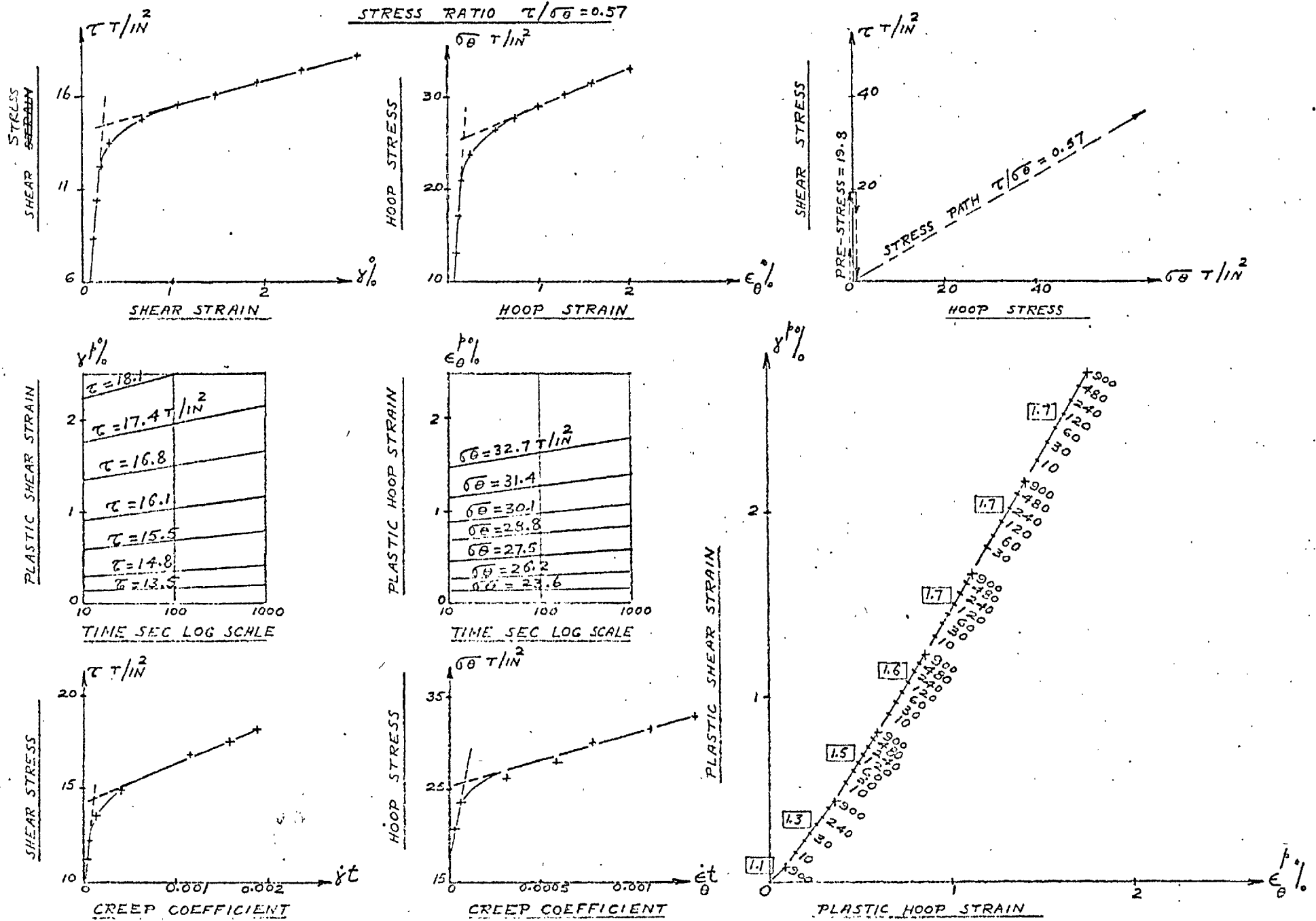


FIG. III.

FIG. 113.

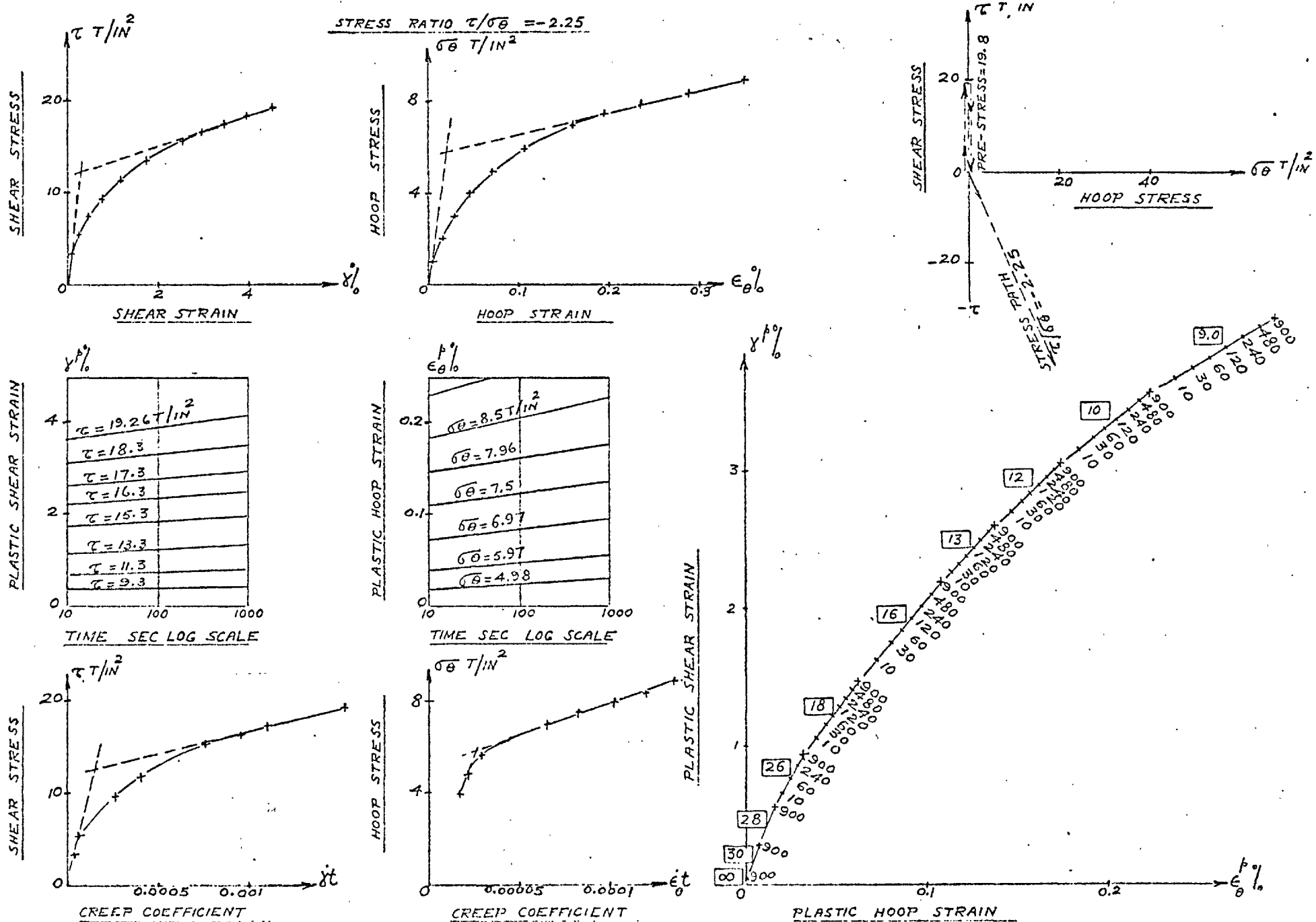


FIG. 113.

FIG. 114.

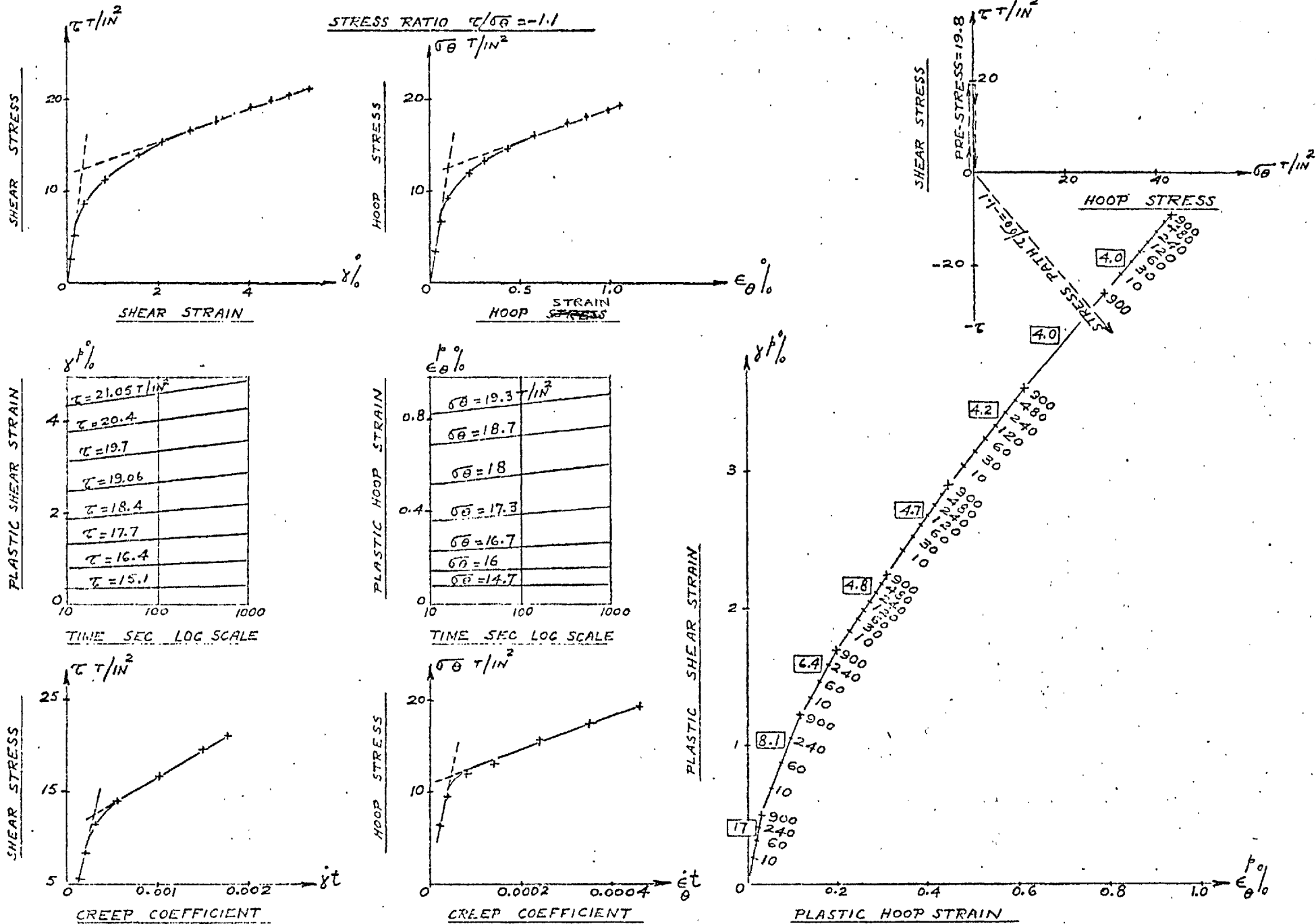


FIG. 115.

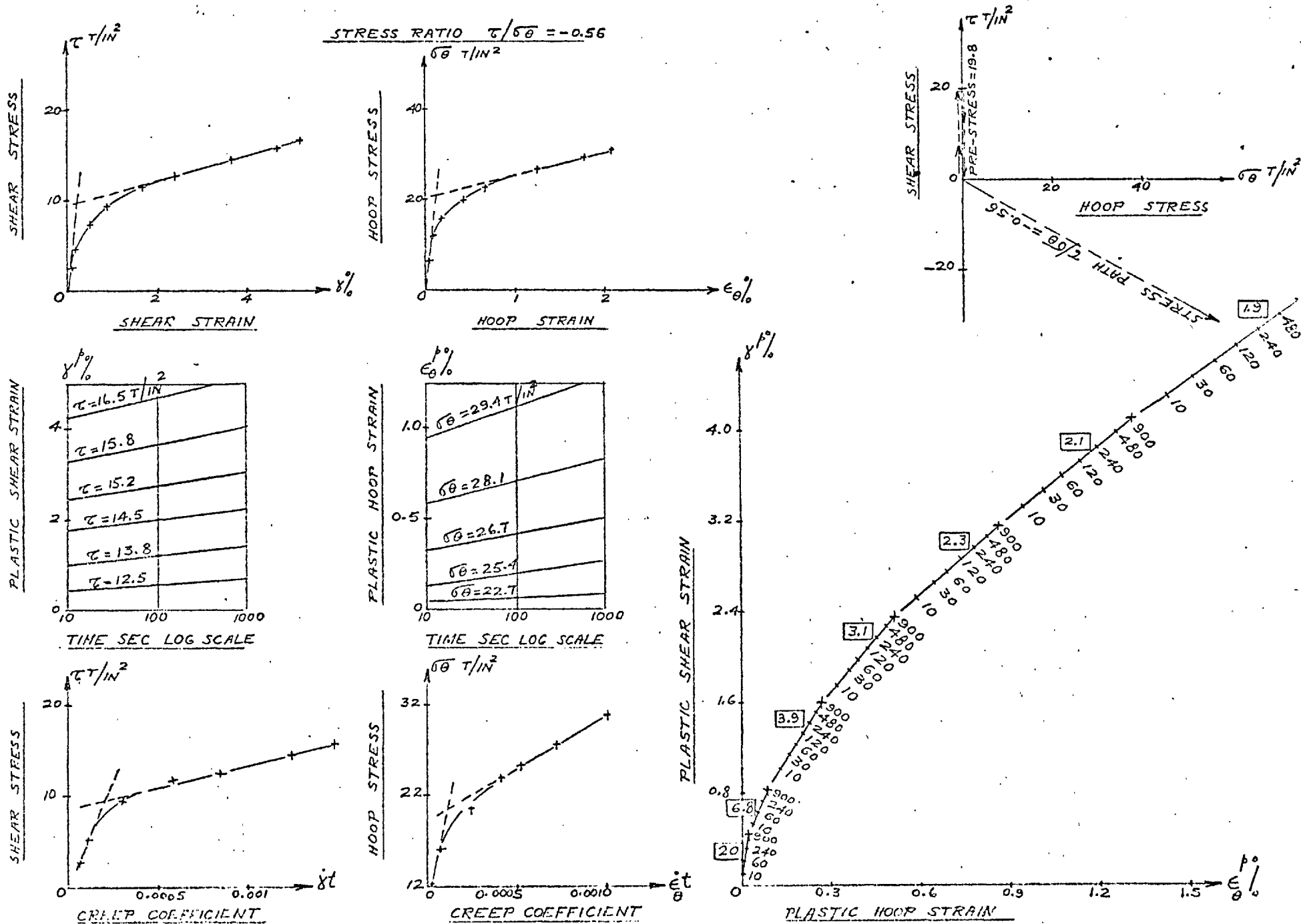


FIG. 115.

$\sigma_c = 3.18\%$
 $\sigma_c = 19.8 \text{ T/IN}^2$
 $c = 2.5 \text{ T/IN}^2$
 $\sigma_c = 31.8 \text{ T/IN}^2$

FIG. 116.

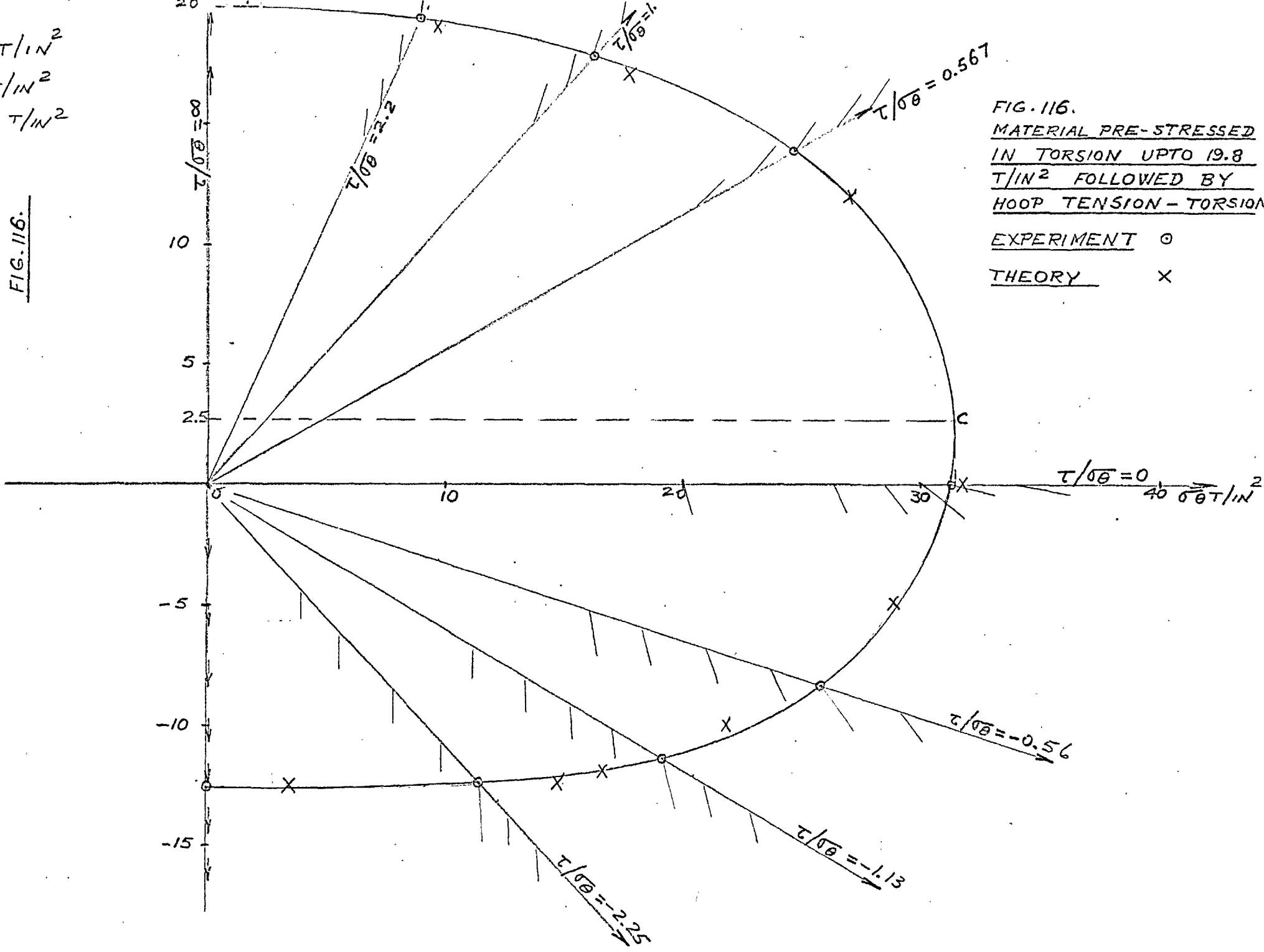


FIG. 116.
 MATERIAL PRE-STRESSED
 IN TORSION UPTO 19.8
 T/IN² FOLLOWED BY
 HOOP TENSION - TORSION.

EXPERIMENT O

THEORY X

$$\gamma_o^p = 3.18\%$$

$$\tau_o = 19.8 \tau / \text{IN}^2$$

FIG. 117.

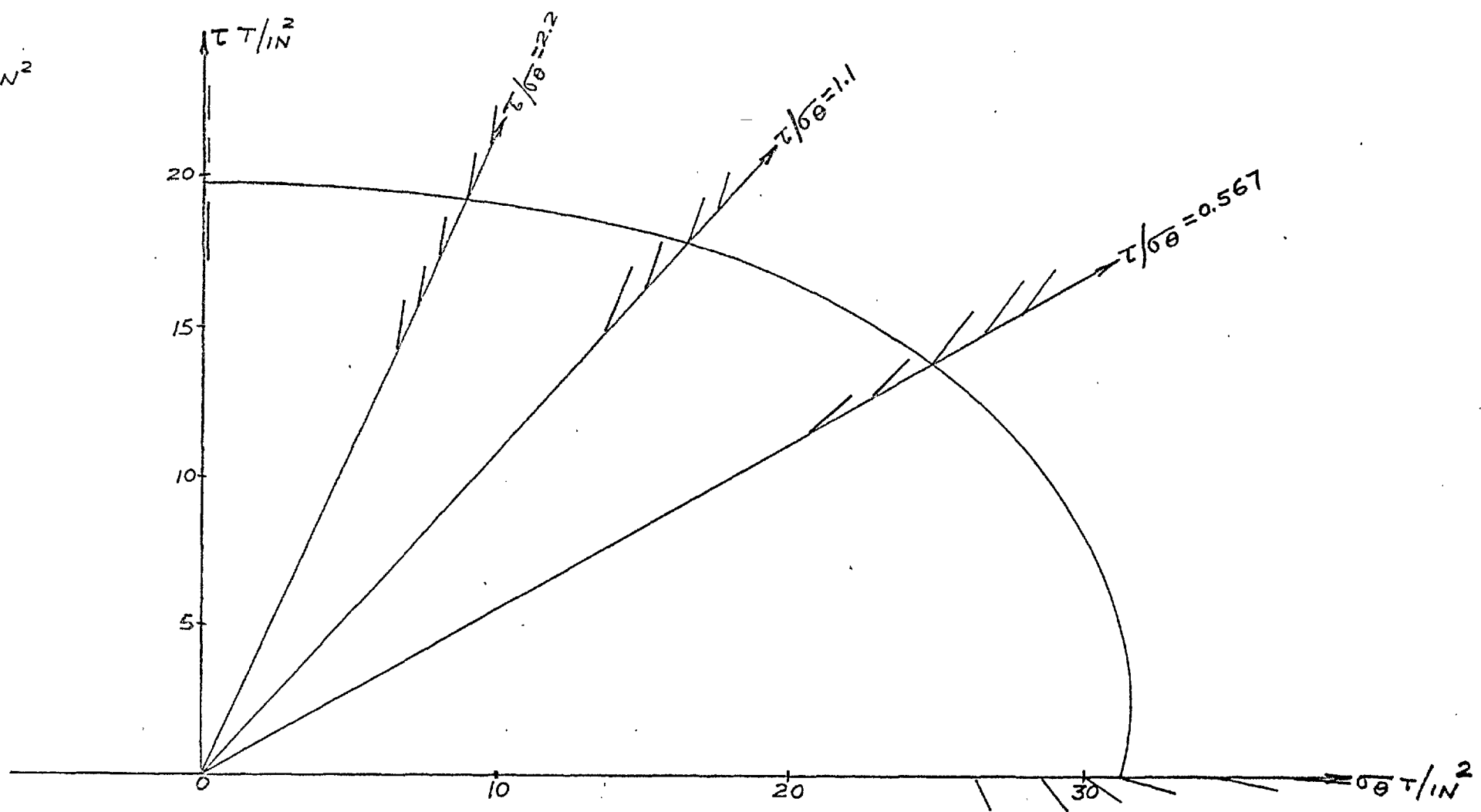


FIG. 117. En24 PRE-STRESSED IN TORSION UPTO $19.8 \tau / \text{IN}^2$ FOLLOWED BY HOOP TENSION-TORSION IN +VE. QUADRANT.

FIG. 118.

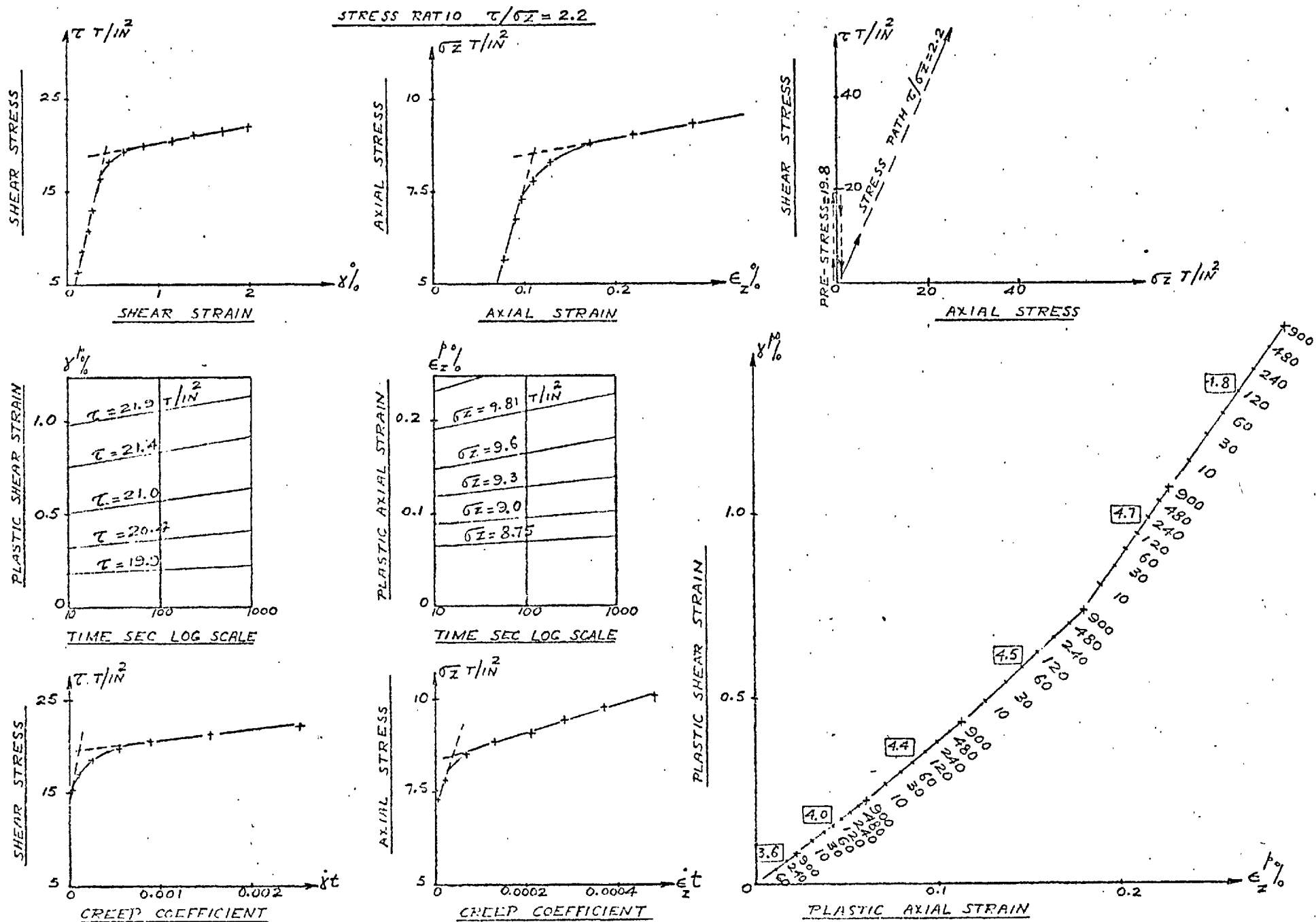


FIG. 118.

FIG. 119.

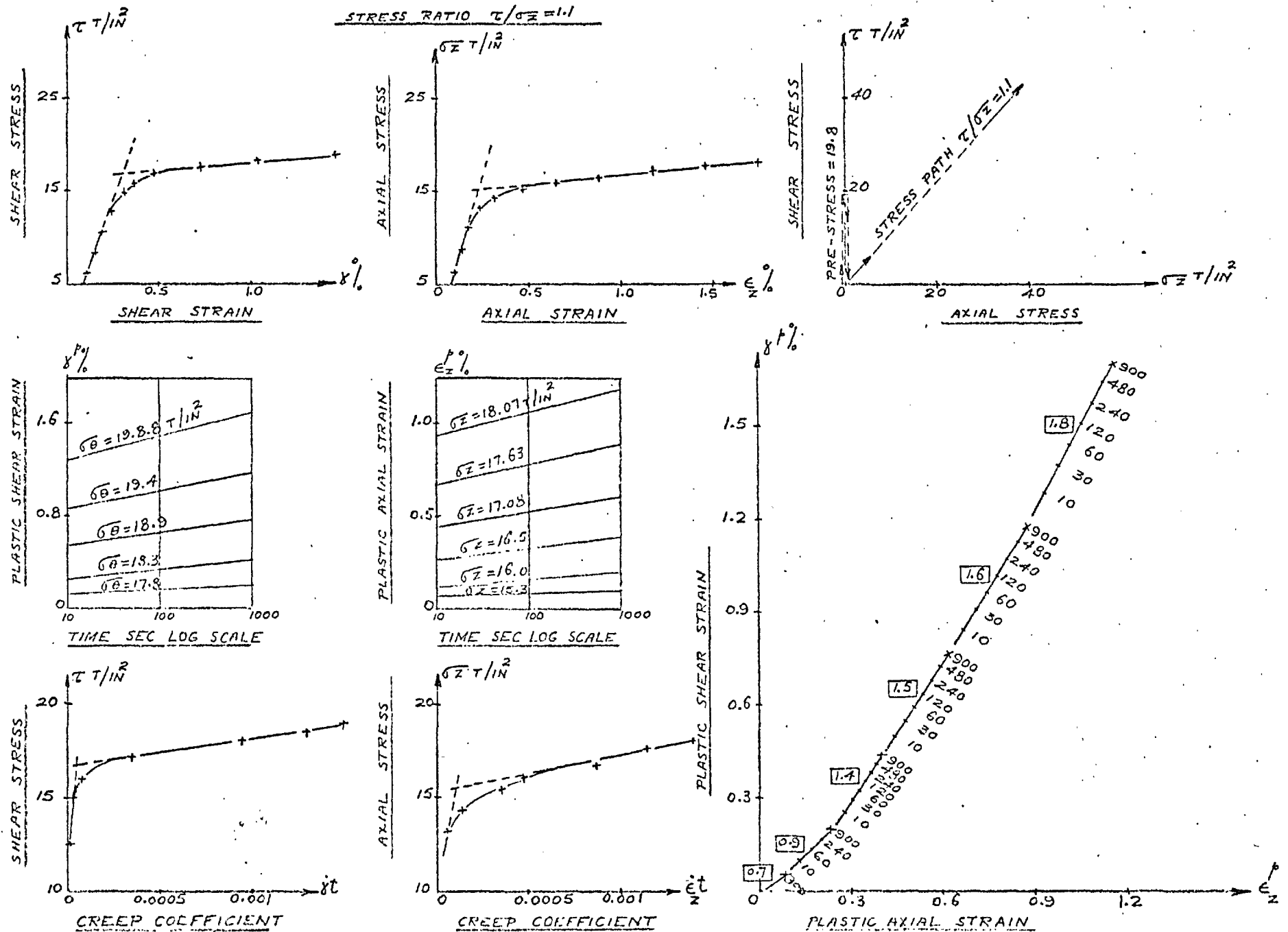


FIG. 119.

FIG. 121.

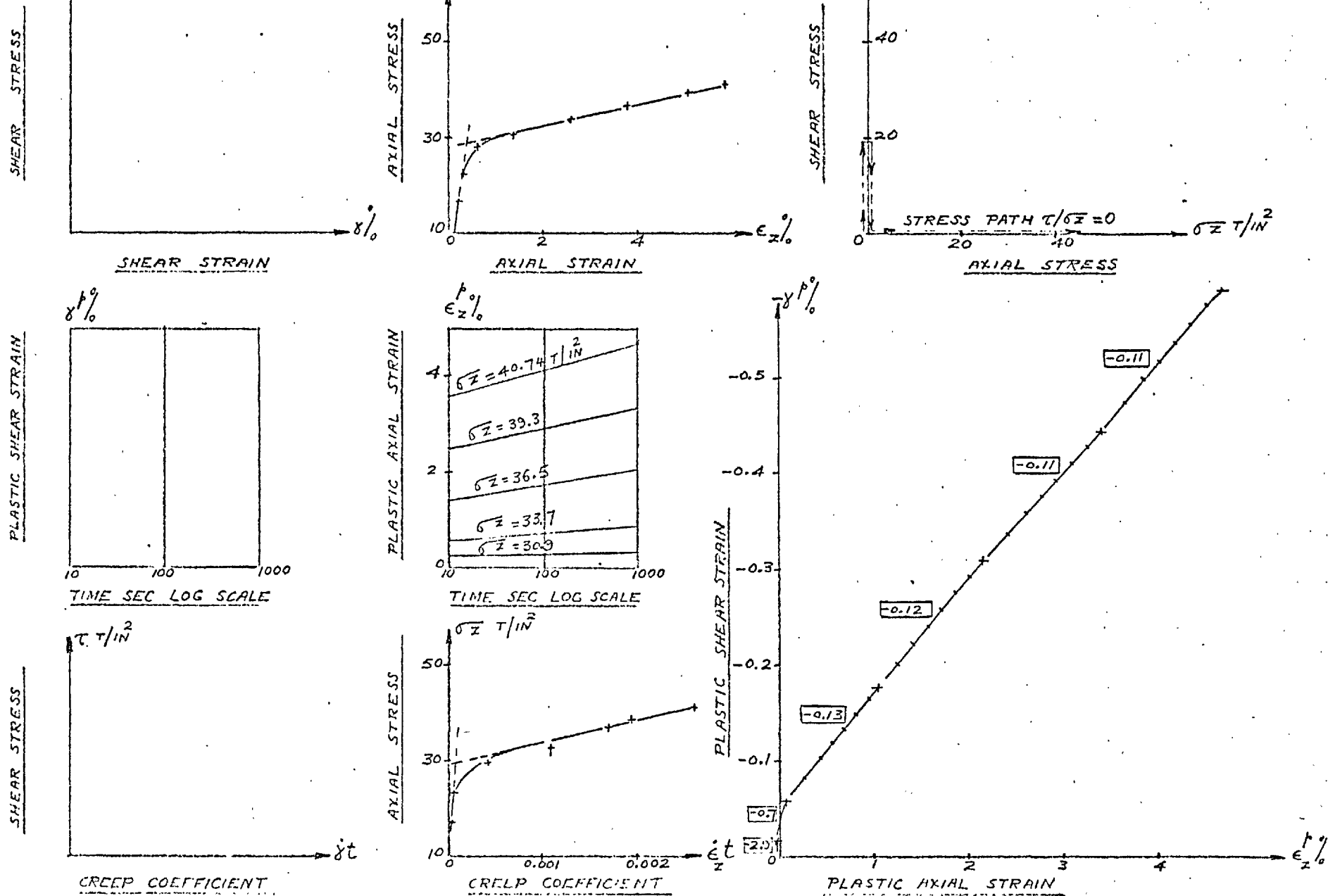


FIG. 121.

FIG.122.

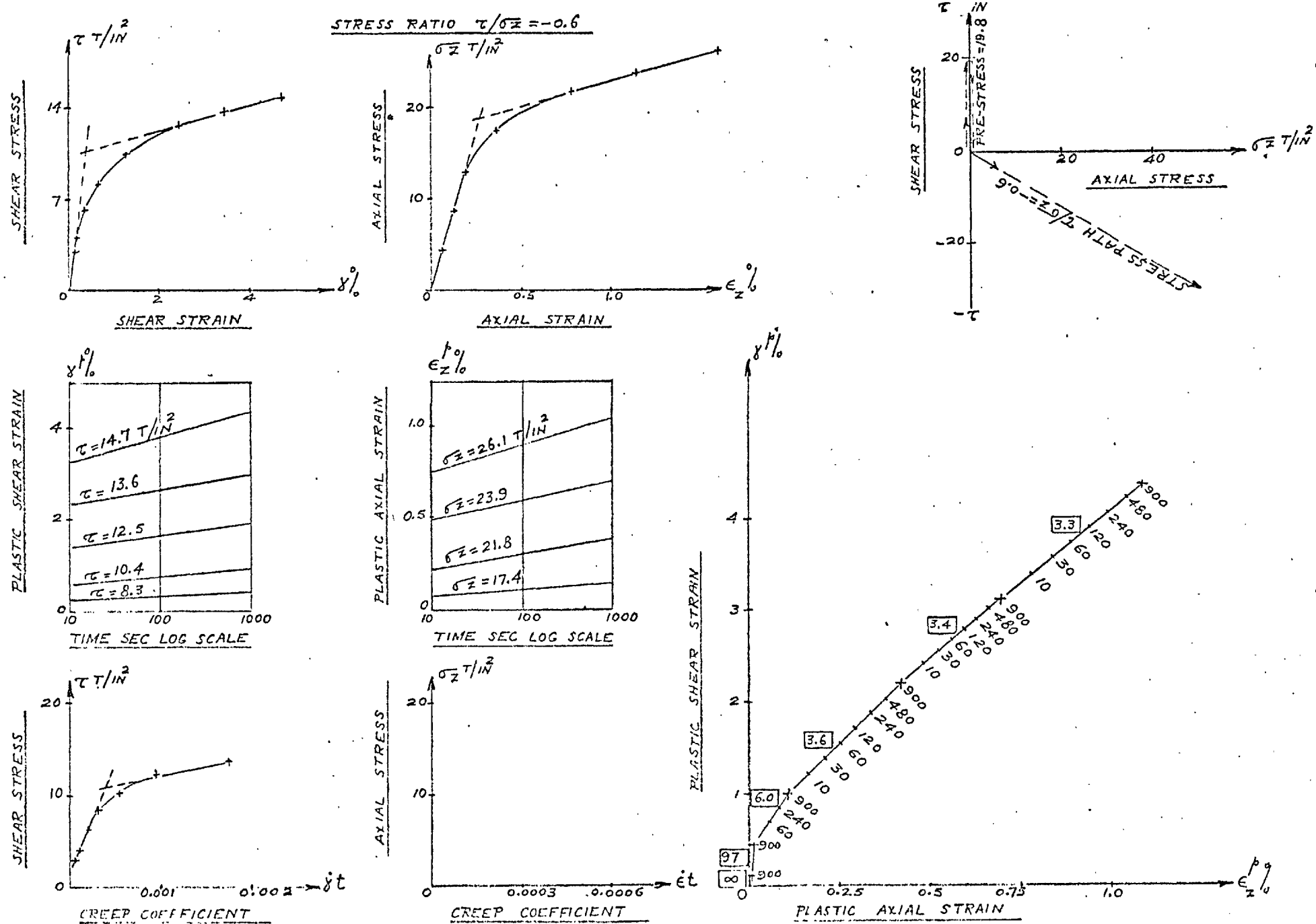


FIG.122.

FIG. 123.

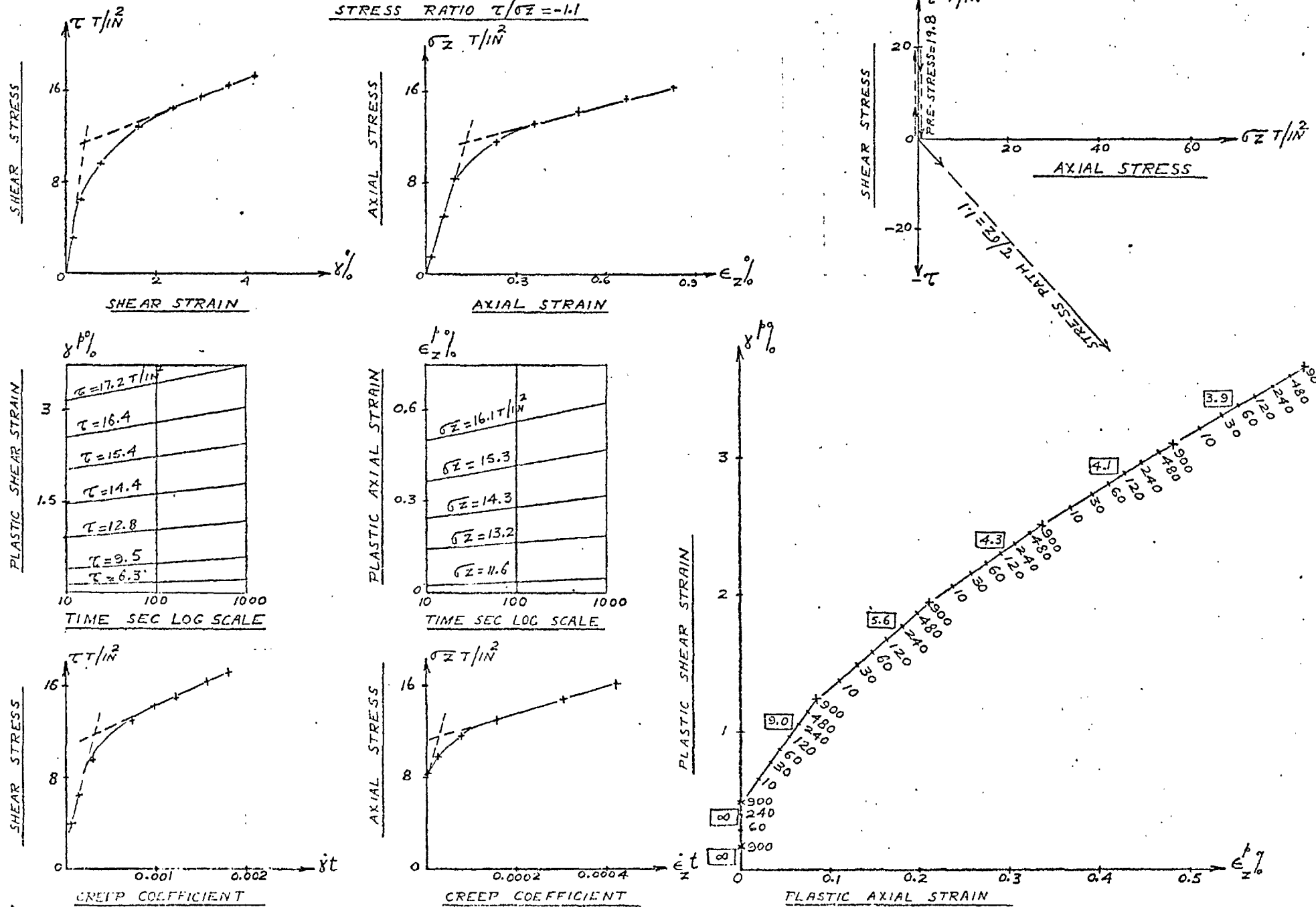


FIG. 123.

FIG. 124.

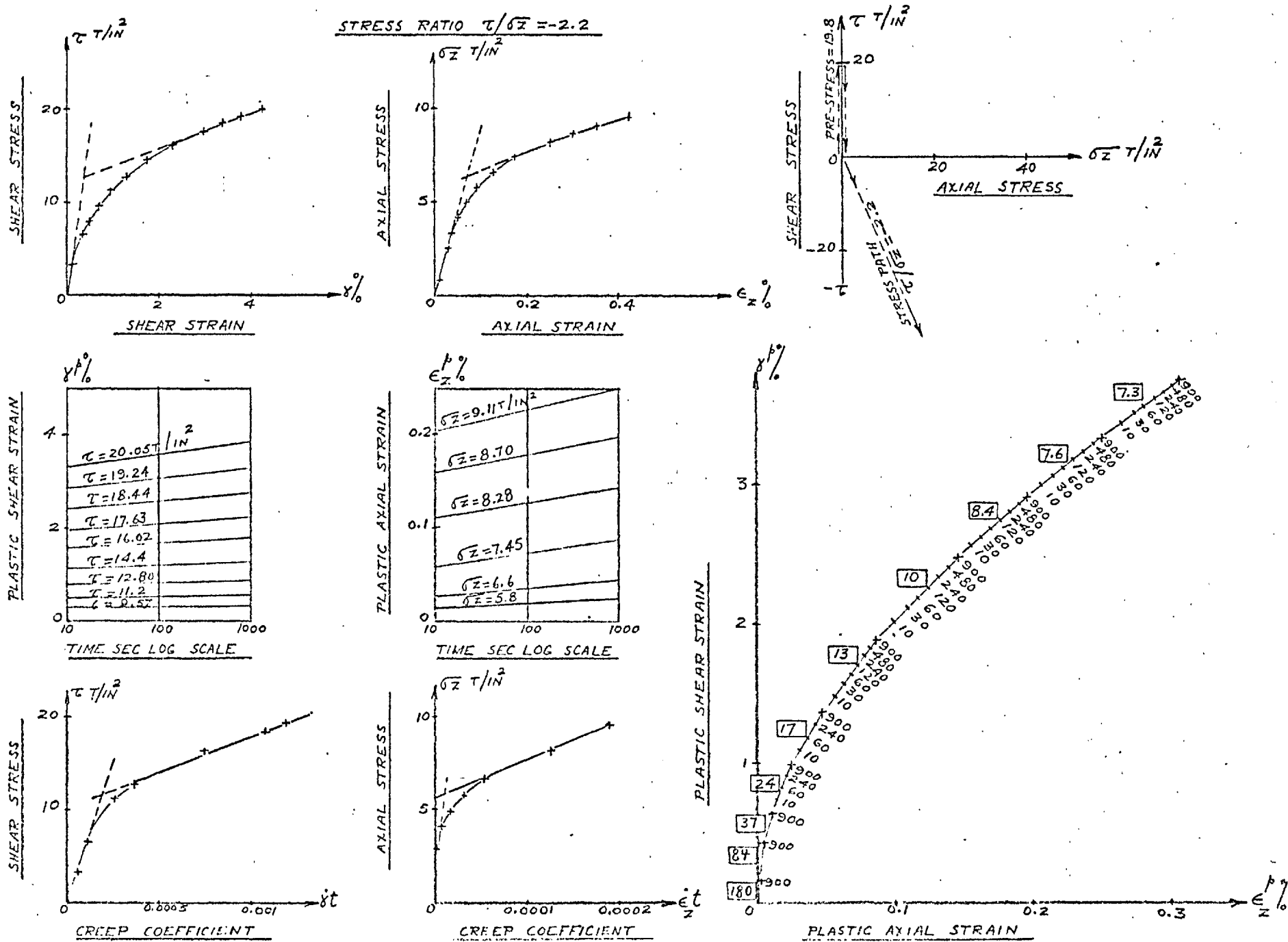


FIG. 124.

FIG. 125.

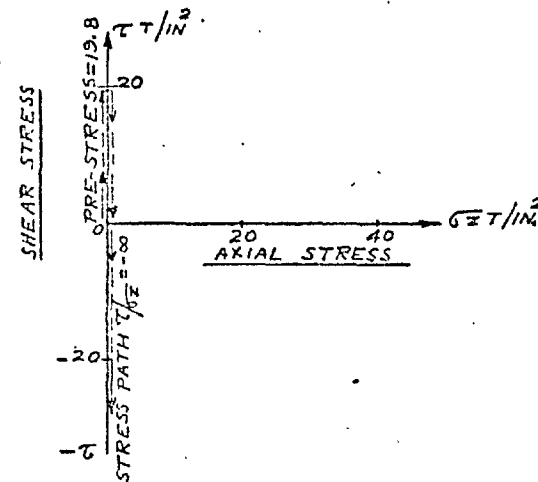
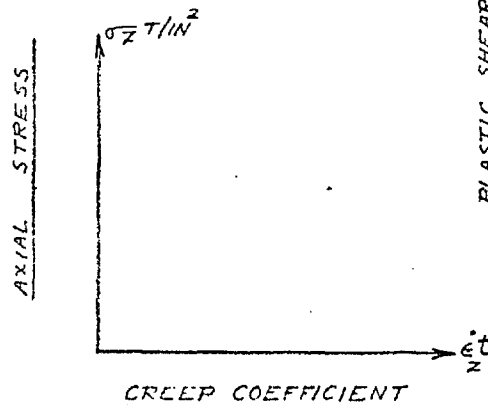
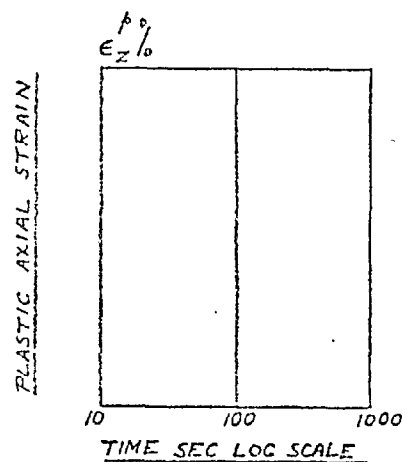
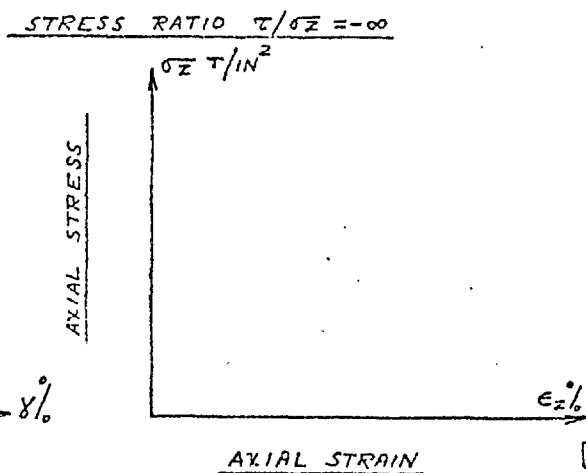
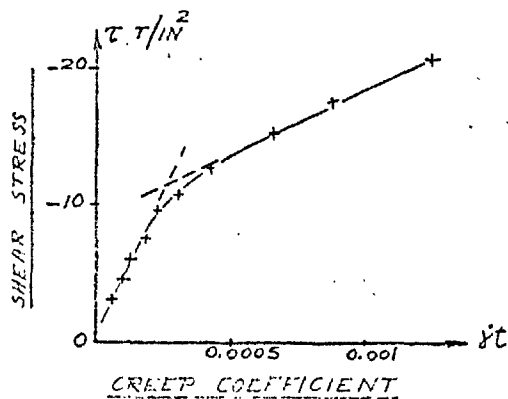
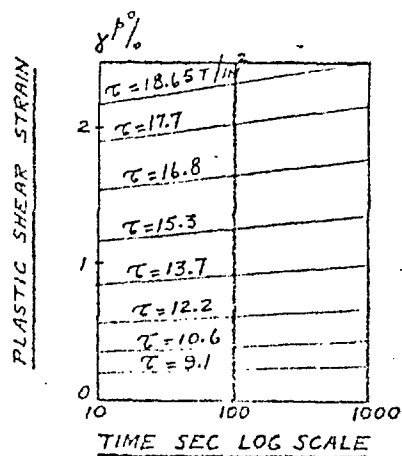
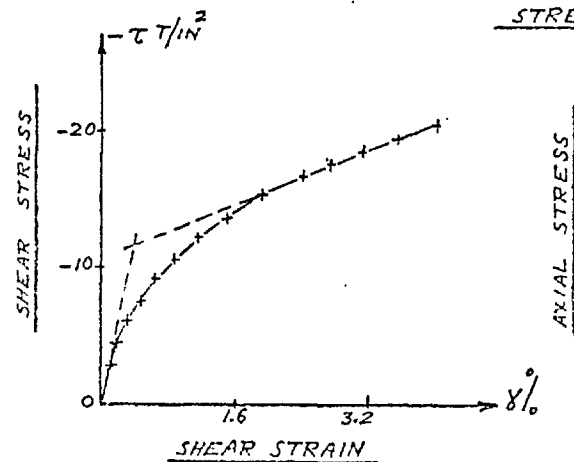


FIG. 125.

$$\gamma_0 = 3.18\%$$

$$\tau_0 = 19.8 \text{ T/IN}^2$$

FIG. 126.

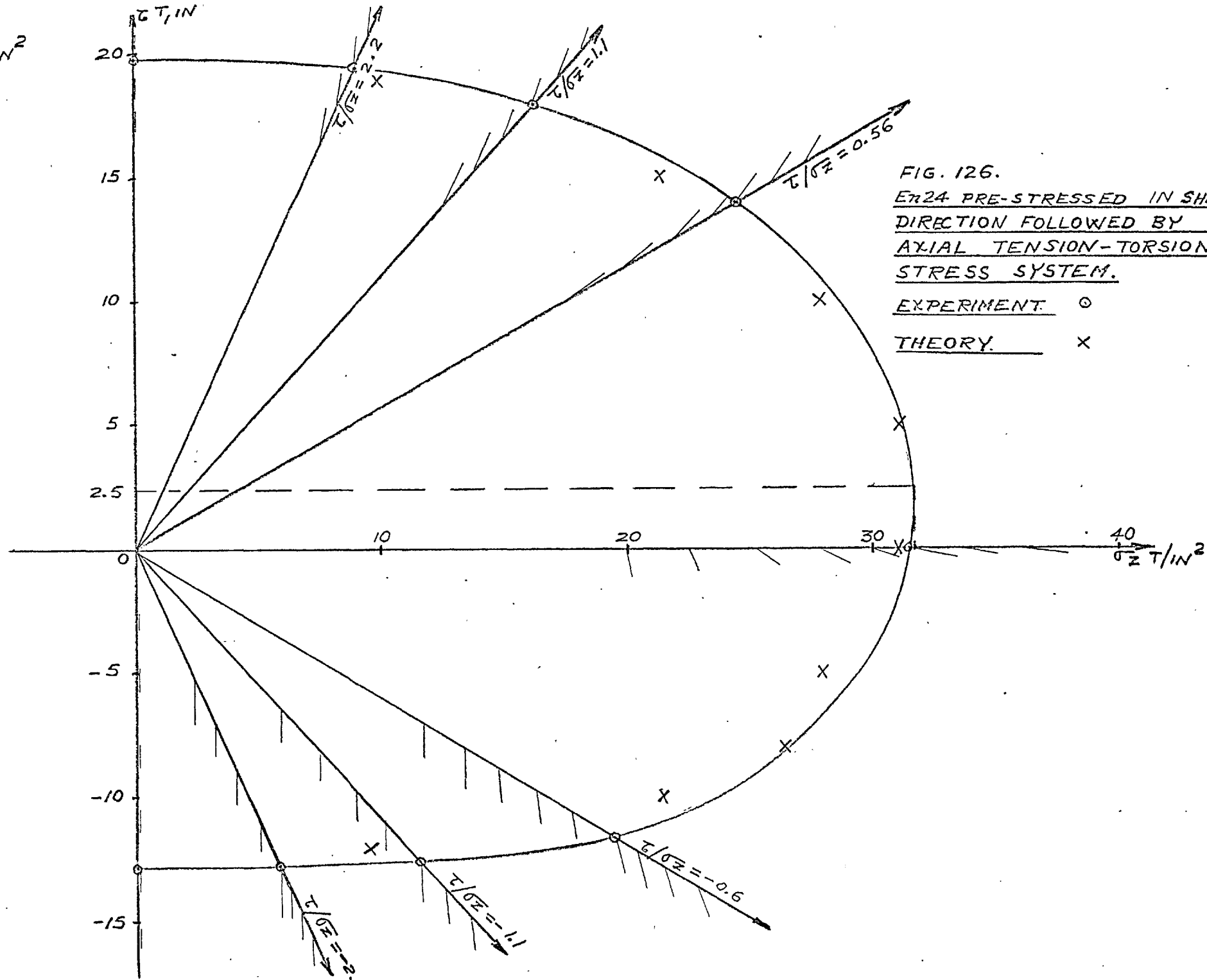


FIG. 128.

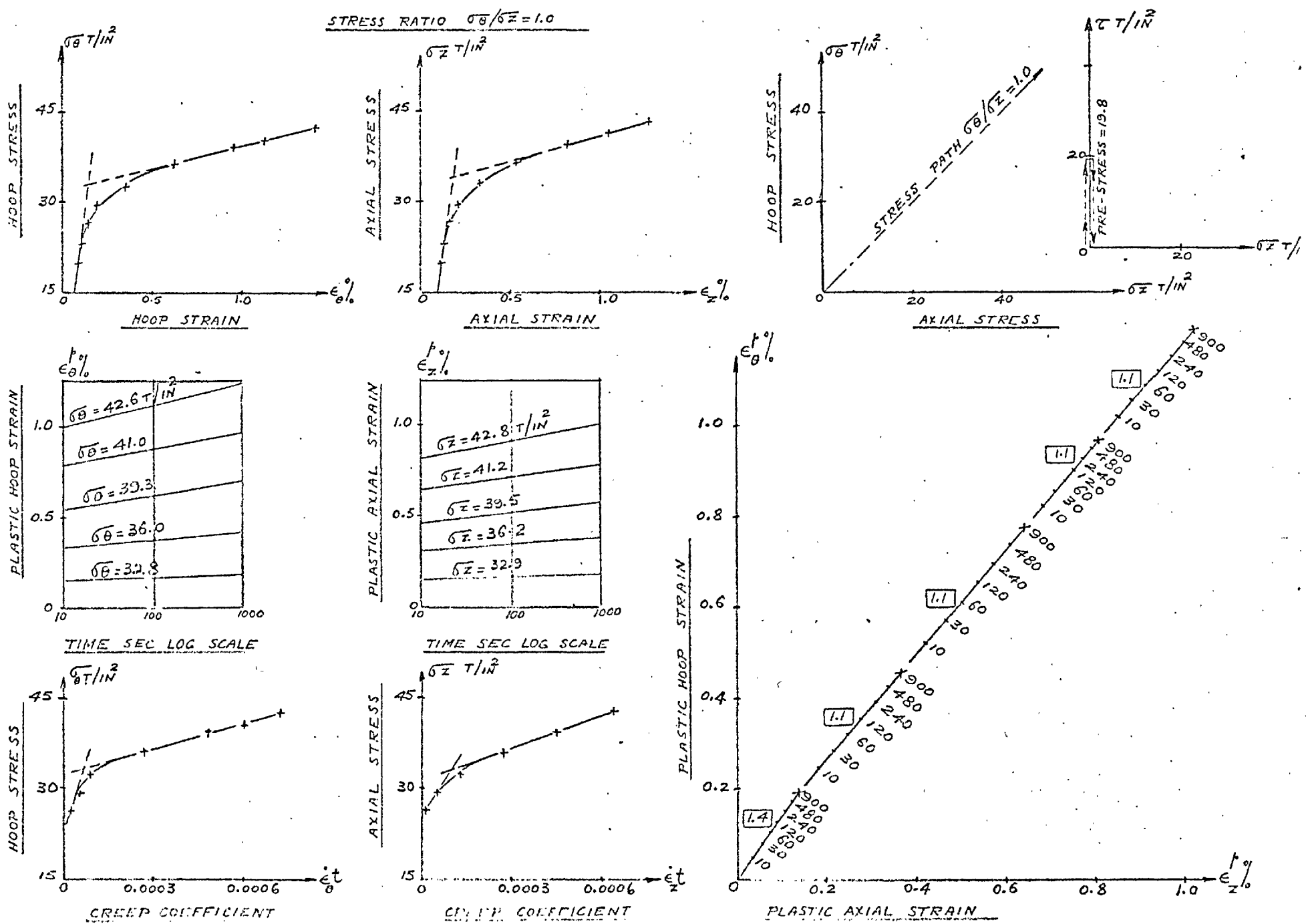


FIG. 128.

FIG. 129.

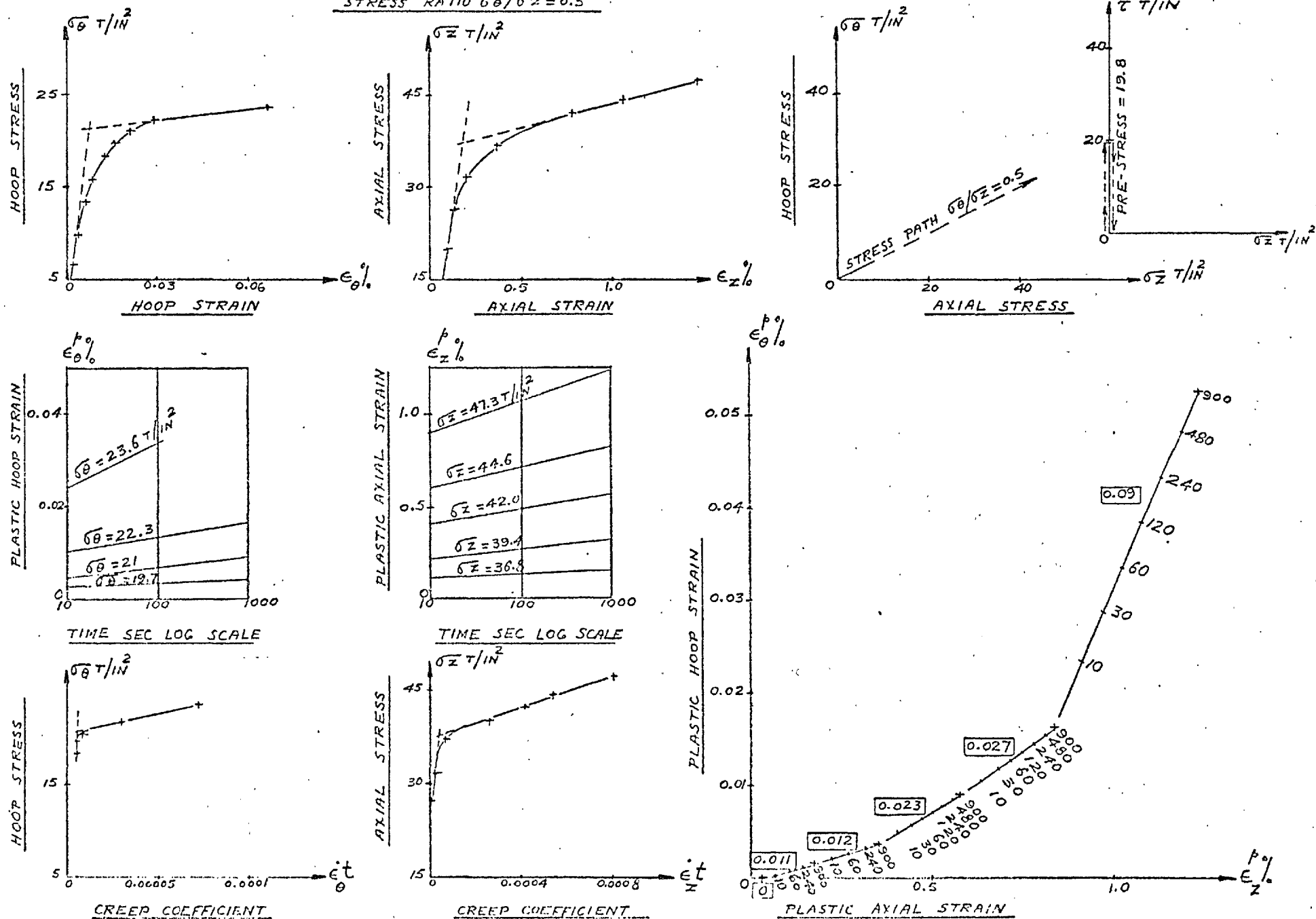


FIG. 129.

$$\chi'_0 = 3.18\%$$

$$\tau_0 = 19.8 \text{ T/IN}^2$$

FIG. 130.

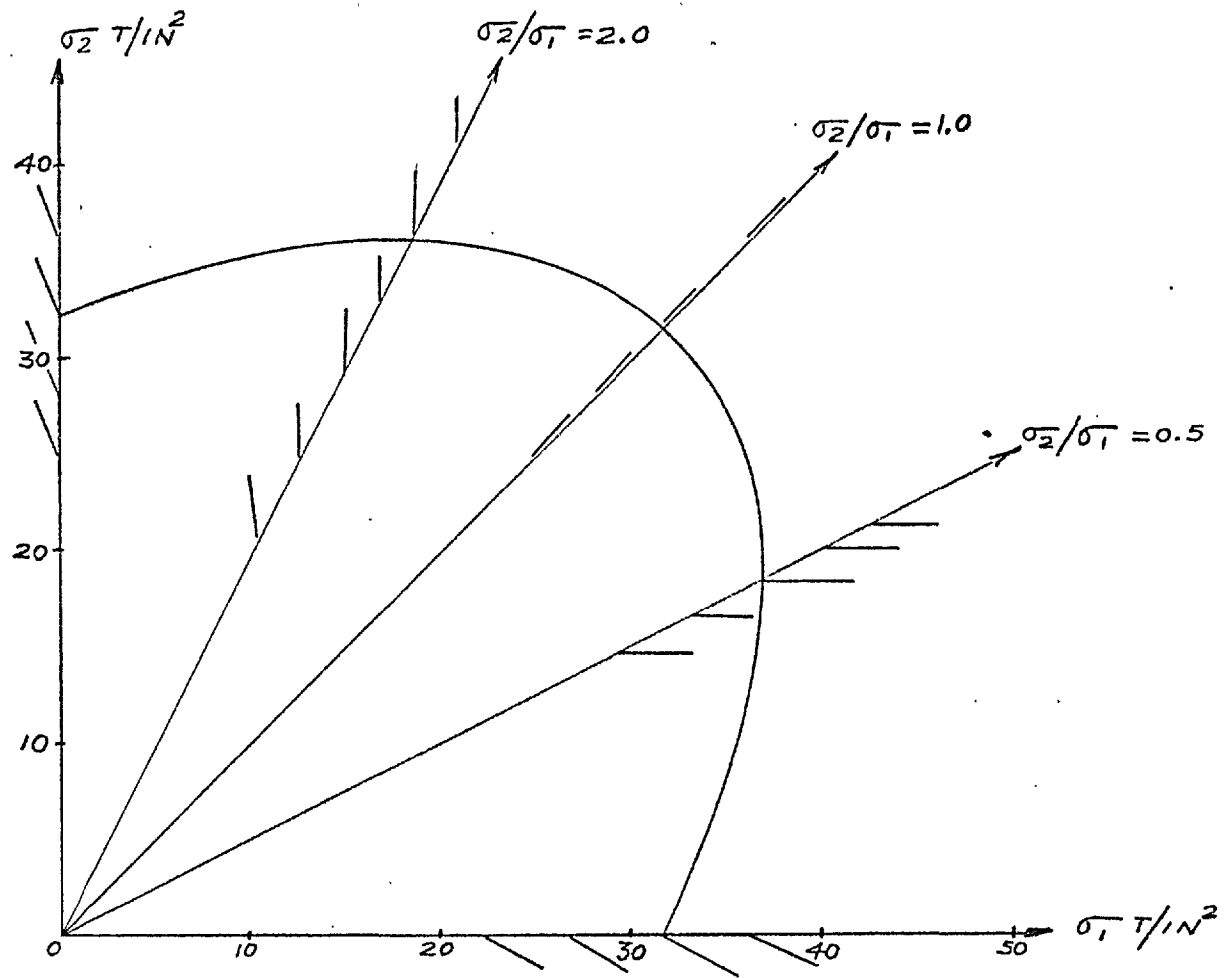


FIG .130 . MATERIAL PRE-STRESSED IN SHEAR
UPTO 19.8 T/IN² FOLLOWED BY AXIAL
TENSION-HOOP TENSION.

$$\delta_o^p = 3.18\%$$

$$\tau_o = 19.8 \text{ T/IN}^2$$

FIG. 131.

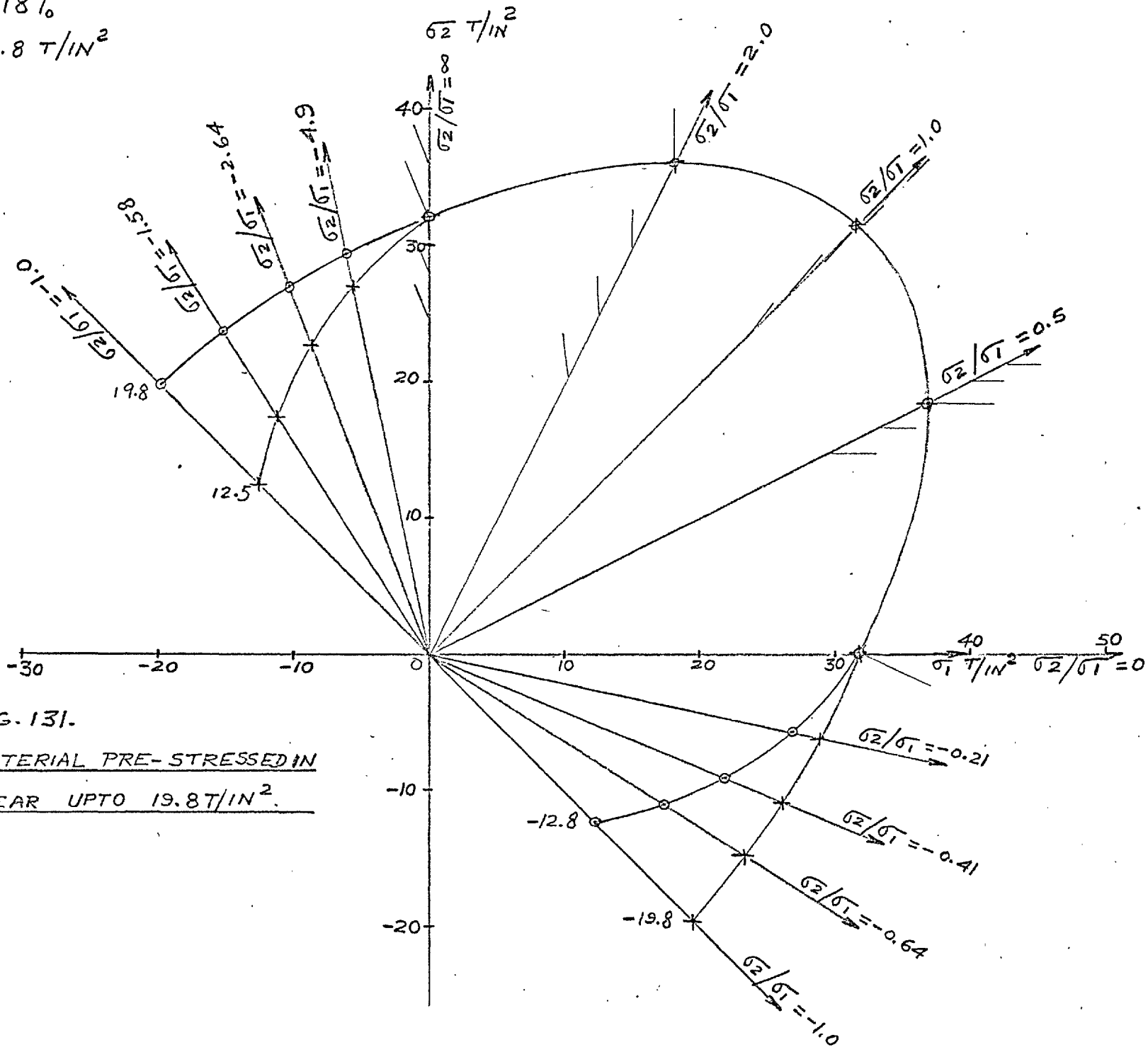


FIG. 131.

MATERIAL PRE-STRESSED IN
SHEAR UPTO 19.8 T/IN^2 .

FIG. 132.

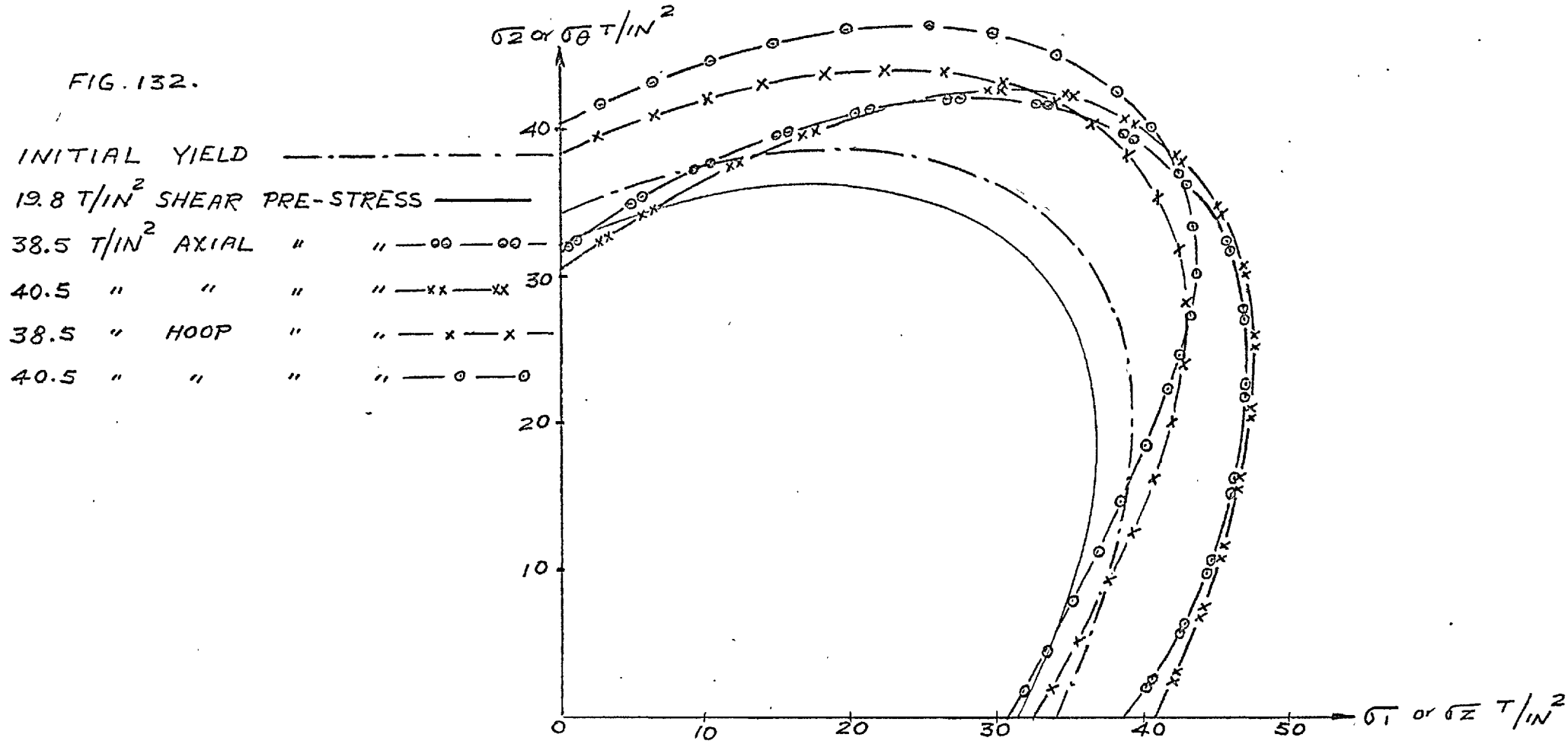


FIG. 132.

COMPARISON OF THE EXPERIMENTAL YIELD LOCI

UNDER VARIOUS PRE-STRESS.

FIG. 133. YIELD LOCUS FOR

MATERIAL PRESTRESSED IN:

TORSION UPTO 19.8 T/IN^2 - - - - -

AXIAL TENSION 38.5 " - - - - -

" " 40.5 " - xx - xx - xx

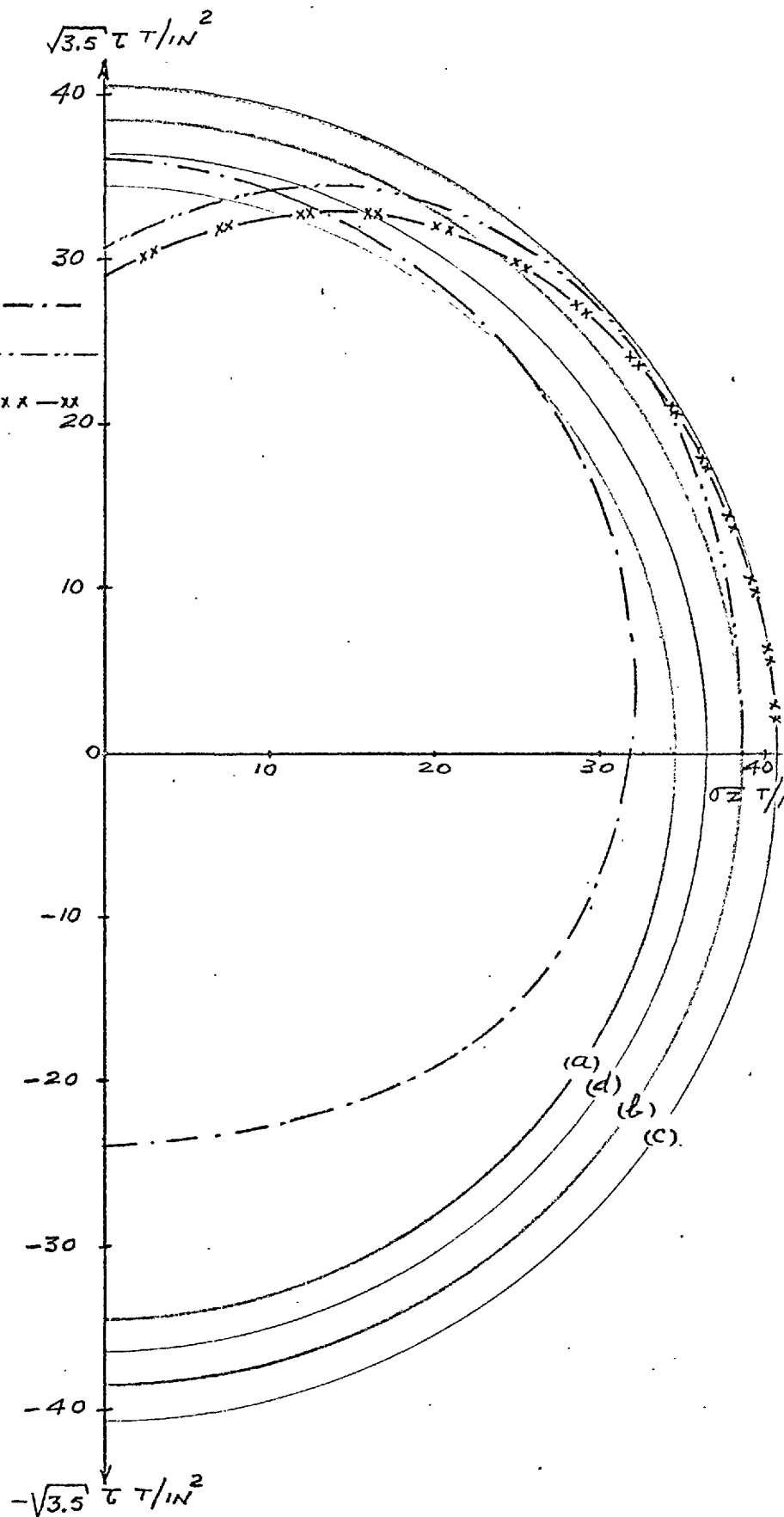


FIG. 133. COMPARISON BETWEEN THE ISOTROPIC EXPANSION OF INITIAL YIELD LOCUS AND EXPERIMENTS.

