

A network based dynamic air traffic flow model for en route airspace system traffic flow optimization

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Abstract: This study proposes a mesoscopic dynamic air traffic model based on a dynamic network for en route airspaces by characterizing the dynamics and distribution of traffic speed. Based on this model, we solve a flow optimization problem for enforcing capacity constraints with the minimum operational cost using a dual decomposition method. A case study of an en route airspace in Shanghai demonstrates the accuracy of the proposed model in successfully capturing the flow dynamics, as well as the effectiveness of the proposed optimization framework to reduce en route delays by balancing the dynamic traffic demand and airspace capacity.

Keywords: air traffic management; en route traffic flow optimization; flight operational cost.

1. Introduction

Global air traffic has grown dramatically during the last few decades, and a recent study from Eurocontrol shows that en route traffic is expected to grow with an annual average rate of 2.5% in the next 7 years [1]. The rapid growth in air traffic with limited airspace resource results in demand-capacity imbalances and flight delays, which may cause potential safety and inefficiency issues in the air traffic control system. To mitigate the imbalance of demand and capacity in air traffic, various automation tools are designed to assist air traffic controllers with decision-making in air traffic flow management (ATFM), with the goal of safe, efficient and cost-effective flow of air traffic. However, most current ATFM tools mainly focus on local regions, e.g. a single airport or sector, while the system-wide decision-making, which takes into account the network effect of those independent decisions, heavily relies on human intuition and past experience of experts [2].

To address the system-wide demand-capacity imbalance in ATFM, this paper presents a novel dynamic network air traffic flow model, based on which an efficient en route optimization method is proposed for air traffic on a particular route and in a network of such routes. Unlike most existing air traffic flow models, the proposed model takes into account the dynamic feature of traffic flow speed, and provides a more practical and flexible mechanism to control air traffic flow at both air route and en route network levels.

The controlled airspaces in China are categorized into (1) tower controlled airspace; (2) approach controlled airspace; and (3) en route controlled airspace, where the aircraft are, respectively, in the landing/take-off phase, the approach phase, and the cruise phase. The en route controlled airspace is further partitioned into area sectors, and controllers guide the aircraft in their assigned sectors along the specified air route according to the flight plan. Aircraft in the en route airspace are all in the cruise phase, which takes up the longest part of most flights. In addition, delay in en route airspace is likely to accumulate and propagate to its upper stream regions and airports (i.e., the regions and airports where aircraft traverse before they fly into the delayed airspace), and therefore responsible for network impact of local delays. A recent study from

Eurocontrol shows that capacity, weather, activity and operation of the en route airspace account for 51.18% of the total flight delays in Europe, which means en route airspace (rather than airport terminal area, surprisingly) is the main reason for ATFM delay in Europe [3]. Advanced and effective ATFM in the en route airspace is thus important to ensure a safe and efficient air transportation system. Motivated by this observation, this paper aims to address the short-term air traffic flow optimization in the en route airspace, by enforcing capacity constraints at the minimum achievable cost. The optimization framework proposed in this paper can be applied to assist air traffic controllers in managing en route traffic flow by providing reliable short-term traffic flow prediction and optimization with the goal of safe, efficient and cost-effective flow of air traffic. Furthermore, the method proposed in this paper can be also used to evaluate airspace performance from the perspective of optimal air traffic operation.

The past few decades have witnessed increased interest in developing ATFM models and tools, which aim to identify and resolve the demand-capacity imbalances through the prediction and optimization of air traffic [4-14]. Trajectory-based models are widely used in the US air traffic control (ATC) system [4][5], as well as the Future Air Traffic Management Concepts Evaluation Tool (FACET, a simulation and analysis tool developed at the NASA Ames Research Center) [6]. The trajectory-based models, which keep track of each individual aircraft, are also known as aircraft-level models. Air traffic demand prediction based on these models is carried out by calculating the total number of aircraft in an airspace by propagating the trajectories of the proposed flights forward in time [7][8]. The aircraft-level models are also adopted for the optimization of traffic to address the imbalanced demand and capacity. The most well-known aircraft-level optimization model is proposed by Bertsimas and Patterson [9]. This model introduces binary variables to characterize whether or not a certain flight enters at a certain sector by a certain time. The objective is to minimize the total operational cost of flights under the capacity constraints of sectors in the en route airspace. Corolli et al. introduce time windows for flight operations such that the minimum delay is achieved under capacity constraints [10]. Kopardekar et al. compared several control strategies for air traffic optimization, including the trade-offs of implementing altitude capping, local rerouting, ground holding, time-based metering, and miles-in-trail restrictions [11]. To predict the corresponding delay with sector capacity constraints, a sector-based traffic flow model using hybrid automata theory is proposed by characterizing the detail of aircraft dynamics under air traffic control commands, such as speed change, heading, holding patterns, etc [12]. More recently, the previous model is further explored by taking into account the dynamic capacity, which is effected by weather conditions, other flights' status, equipment status, etc., to facilitate the optimal operations in various scenarios [13][14].

However, the scales of these models, as well as their computational difficulties, depend on the number of aircraft under consideration. This leads to high computational burdens when modeling or attempting to optimize air traffic flow management problems of large but realistic sizes. Furthermore, recent research reveals that the errors of the aircraft-based flow prediction increase sharply when the forecast time horizon exceeds certain threshold, which is caused by model sensitivity to various uncertain factors related to individual aircraft, such as weather, departure management and human factors, etc [8]. Thus, in practice it is difficult to implement a sound on-line ATFM strategy based on the aircraft-level models for large regions or long planning horizons due to the expensive computational cost and limited forecast horizon.

Efficient air traffic flow management requires reasonable prediction and optimum control of flows of aircraft rather than individual aircraft. Therefore, aggregate air traffic flow models, which focus on the overall distribution of the air traffic flow, are introduced recently. Aircrafts are spatially aggregated to airspace volumes according to their geographical location, which means that the scale of the aggregate model depends solely on the number of airspace volumes rather than the number of aircrafts in the airspace. This aggregate method will, thereby, reduce the computational cost significantly when modeling a large but realistic number of flights. In addition, since the microscopic behavior of the individual aircraft is not explicitly considered in the aggregate models, the prediction results tend to be more robust against the aforementioned uncertain factors related to individual aircraft. Thus, a longer forecast time horizon with less prediction errors can be achieved by the aggregate traffic flow models compared to the aircraft-level ones. The aggregate air traffic flow models support a more macroscopic ATFM management and perform more consistent with the Collaborative Decision-Making (CDM) procedure in ATM (Air Traffic Management), which aims to identify and resolve the demand-capacity imbalances by adjusting the aggregate traffic flow to match the capacity resources of the airspace system [21]. Therefore, the aggregate models are suitable to be used to conduct en route traffic flow management at a system-wide level.

The aggregate models are widely discussed in the recent literatures [15-22]. A stochastic framework with linear dynamic system model is developed by Sridhar et al., where aircraft are aggregated according to their geographic location, e.g., control center. So the scale of this model depends on the number of control centers. Poisson distributions and split parameters, which are estimated from historical traffic data, are introduced to characterize the inflow quantity and the traffic transmission ratio between neighboring control centers, respectively. The established time-invariant model further develops into a time-variant one, where the split parameters are updated each one hour based on mining of historical air traffic operational data [15][16]. Then the stochastic linear dynamic system model is applied to predict air traffic flow at both sector and center levels, which facilitates the identification of capacity-demand imbalances in advance [17][18]. Air traffic optimization can be also implemented based on these stochastic models. Bloem et al. extend the previous model by adapting quadratic cost on cumulative departure delays to analyze pre-departure delays distribution across the National Airspace System [19]. Grabbe et al. and Andreatta et al. propose a linear programming model based on a stochastic aggregate model to make optimal congestion management strategies [20][21][22]. There are mainly three shortcomings in the stochastic framework models. Firstly, these models extensively aggregate aircraft according to their location in control centers, which means the dynamics of the traffic flow inside the control center, e.g. on a certain air route, is not available. Secondly, the same sets of Poisson distributions estimated from historical data are adopted to model the departure flow at each time step for different days, which ignores the fact that the departure traffic flow varies significantly during days, weeks and seasons under different traffic conditions. In addition, the split parameters used in these models bring up the diffusion and dispersion problem, which may result in the inaccurate predictions and optimizations [23].

On the other hand, motivated by the widely used LWR theory (traffic flow theory proposed by Lighthill, Whitham and Richards) [24][25] and the cell transmission model [26][27][28] in highway traffic modeling, Menon et al. derive an aggregate air traffic model by using a baseline Eulerian model, where aircraft are also aggregated into control centers [29]. The derived

one-dimensional model is extended to a two-dimensional one by aggregating aircraft into small grids of surface elements discretized from the realistic airspace with a computer-aided methodology, which is computationally expensive [30]. To resolve the diffusion problem of air traffic flow transmitting between neighboring airspace due to the introduction of the split parameters, Sun et al. use the real origin-destination (OD) information from the flight plan instead of the constant split parameters derived from the historical data [31][32]. On this basis, an Eulerian-Lagrangian model is proposed by aggregating aircraft into discretized air route segments with equal length (1 minute's flight range). Subsequently, this model is further explored for air traffic flow prediction and optimization, where the objective is to minimize the total flight time of all the aircraft in the airspace system under sector capacity constraints. Inspired by the Dantzig-Wolfe decomposition used in the Bertsimas and Patterson model [9], the optimization problem is decomposed into a master problem and a number of independent sub problems by using dual decomposition method. These MILP (Mixed Integer Linear Programming) problems are then relaxed to LP (Linear Programming) problems for the convenience of calculation [33]. However, the discretization method (i.e., divide the air route into 1 minute's equal-length-segments) brings up a huge number of variables, which contribute to an expensive computational cost. Moreover, the dynamics and the uncertainty of air traffic speed are ignored in the discretization and modeling method. Aircraft speed is assumed to be constant in the airspace, that is to say all the aircraft in the upstream 1-minute-segment will fly into the downstream one in 1 minute's time interval.

To reduce the variables and the resulting computational complexity, a more efficient model is proposed, where aircraft are aggregated into a link, i.e., the entry-exit points of sectors for a certain trajectory, instead of 1 minute's flight range [34][35][36]. The variables are reduced, but the dynamics of the air traffic flow inside sectors is missing. Initially, this discretization method (i.e., divides the air route by sectors' boundaries) divides the air route into segments with different length. Secondly, the distribution of en route traffic inside the sector is not taken into account in these link transmission models. The two points above make the model difficult to propagate the trajectories of the flights already airborne at the initial time (whose position information in the sector is not involved in the model) forward in time. Thus the probabilistic component, which is roughly estimated from historical data ignoring its differences in various traffic patterns, is introduced to characterize the transmission of the air traffic flow already airborne at the initial time step. The introduction of the probabilistic component will affect the accuracy of the air traffic flow prediction and add complexity to the optimization. Moreover, air traffic speed is also assumed to be constant in these time-invariant link transmission models.

In summary, there are mainly three discretization methods in regard to the aggregate volumes in the air traffic flow aggregate models as analyzed above, i.e., (1) control center; (2) 1-min air route segment and; (3) link. The first and the third methods are useful for fast but relatively imprecise modeling of air traffic, and they are inefficient to accurately characterize air traffic distribution inside control centers, sectors or air routes. The second method involves detailed information of air traffic distribution in every 1-min segment, but it brings up a huge number of variables which contribute to an expensive computation cost. Therefore, the existing aggregate models have not solved the problem of reducing variables and the resulting computational cost, meanwhile, remaining reasonable modeling accuracy and optimization complexity. Moreover, the main limitation of the existing aggregate models is the assumption that the speed of air traffic is

constant in en route airspace and that all the aircraft flies to the next section of airspace at the initial time of each time intervals. It means that the time-invariant aggregate air traffic model ignores not only the dynamics of air traffic flow both in time and space, but also the uncertain factors like unexpected perturbations, various traffic conditions, etc.

To solve the problems listed above, this paper makes several incremental improves. (1) A new discretization method that divides the airspace by both sector boundaries and key fixes is proposed, based on which a dynamic network for en route traffic flow is established. The variables are reduced by using this method, meanwhile, the traffic distribution inside the sector as well as the dynamic relationship between the traffic flow and the airspace structure are taken into account in this network. (2) The continuity equation in fluid mechanics and the space division concepts in cell transmission model are combined to develop a novel en route traffic flow model, which is able to propagate air traffic in the segments with different lengths. This modeling method need no probabilistic component, thereby, will not affect the accuracy of the air traffic flow prediction and the complexity of the optimization. (3) To the authors' best knowledge, the speed of aircraft on an edge in the network, which is the main parameter in the developed air traffic flow model, is the first to be represented as a random variable. The distributions of these random variables are extracted from a real data set from Shanghai Area Control Center (SACC). (4) Unlike the existing time-invariant model, the model parameters are updated at each time interval by taking into account the dynamics and the uncertainty of air traffic speed in practice. So the results of the developed model are superior over those based on the already exist ones (we refer the reader to Table 2 and Table 4 for the comparison results). (5) A more practical optimization framework based on the improved air traffic flow model is proposed to address the imbalanced demand and capacity from both air route and en route network perspectives, which provides a mechanism to control air traffic flow more efficiently and more flexibly. (6) This paper highlights the importance of the identification of demand-capacity imbalances in ATFM. ATFM is about 1) imbalances' identification and 2) traffic flow optimization. But too many literatures ignore the first part and assume as given the solution to the hardest part of ATFM relying on posteriori knowledge of future traffic situation, which is actual unknown in practical operation. The identification of demand-capacity imbalances is very important in ATFM to know the traffic situation in a short future, based on which optimizations can be made in advance.

Therefore, this paper proposes a novel network-based dynamic air traffic flow model, which aims to assist ATFM planning by modeling, predicting and optimizing air traffic flow in en route airspace with full consideration of the dynamics and uncertainty of the air traffic. The rest of this paper is organized as follows. Section 2 constructs the dynamic network for en route traffic flow, and statistically analyzes the speed distributions both in time and space. Section 3 proposes the dynamic model for the aggregate air traffic flow, based on which the prediction can be easily conducted. Section 4 develops the optimization framework based on the proposed model at both air routes and network levels. Section 5 presents a case study based on a real data set from SACC. Finally, the conclusions are drawn in Section 6.

2. Dynamic network for en route traffic flow

2.1 Definitions

A dynamic network is developed to characterize the static topological structure of airspace [37] as well as the dynamic relationship between the traffic flow and the airspace, which is

illustrated in Figure 1.

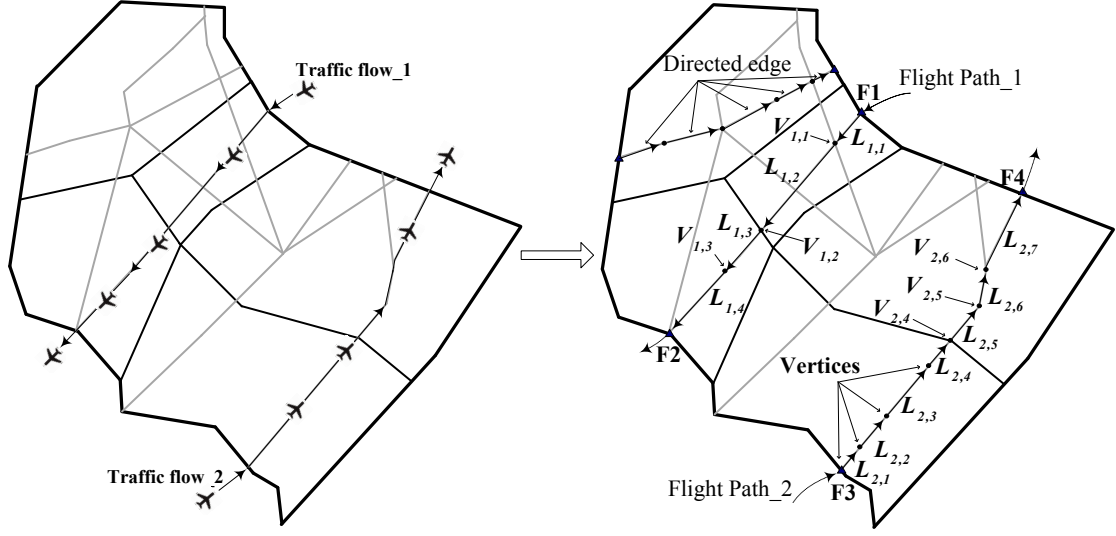


Figure 1. A dynamic network for en route traffic flow.

The left part of Fig.1 shows the actual airspace structure of interest, which consists of five en route sectors. The black bold lines represent the boundary of the airspace system. The dynamic network for the en route traffic flow is developed at the right part of Fig.1, with the definitions as follows.

- (1) Vertices, represent the intersections of air traffic flow and sector boundaries (e.g., vertex $V_{2,5}$) and the key points (e.g., vertex $V_{2,6}$) in the physical structure of the air route, thus the information about the real air route structure is included. There is a special kind of vertices, where air traffic flow transits between the airspace system and the outer airspace. We denote this kind of vertices as interfaces. For example in Fig.1, F1 and F3 represent the interfaces where air traffic flow flying into the airspace system from the outer airspace, while F2 and F4 represent the interfaces where air traffic flow leaving the airspace system to the outer airspace.
- (2) Directed edges, denote the air traffic flow between the interfaces and the nodes, where the direction of the edges is the direction of air traffic, and the length of the edges is proportional to the length of the corresponding segments. For example in Fig.1, $L_{1,1}$ represents the first segment that traffic flow_1 will traverses when entering into the airspace system.
- (3) Edge weight, represents the weight of the directed edge, which characterizes the travel time that the aircraft takes to transit through the corresponding directed edge.

Based on the definitions above, the dynamic network for en route traffic is established to describe an interconnected airspace system with a user-defined size. For example, the airspace system shown in Fig.1 includes 5 sectors, 13 flight paths, 61 directed edges ($L_{1,1}, L_{1,2}, \dots$), 34 vertices ($V_{1,1}, V_{1,2}, \dots, F1, F2, \dots$). And then the traffic flow in the airspace system can be described for simplicity of illustration, e.g., traffic flow_1 in Fig.1 can be simplified as $F1 \rightarrow L_{1,1} \rightarrow V_{1,1} \rightarrow L_{1,2} \rightarrow V_{1,2} \rightarrow L_{1,3} \rightarrow V_{1,3} \rightarrow L_{1,4} \rightarrow F2$.

Note that the weight of a directed edge, i.e., the travel time, varies in different operational conditions. Moreover, if aircraft are prohibited from flying in a specific edge in the network when affected by factors like weather [38], the weight of the directed edge shall be set to infinity or, alternatively, the directed edge shall be deleted from the network.

The proposed network is possible to characterize both the static topological structure of the airspace system and the dynamic relationship between the traffic flow and the airspace structure. Then, aircraft in the airspace system are aggregated into directed edges according to the developed dynamic network, based on which a novel air traffic flow aggregate model can be further developed.

2.2 Analysis of speed variation

The travel time (edge weight) is determined by the length of the segment and the speed of the aircraft traversing it, where the segment length can be derived from NAIP (National Aeronautical Information Publication) and the aircraft speed is analyzed as follows.

The aircraft speed varies in both space and time as suggested by the radar data analysis, which may introduce dynamics and uncertainty to the travel time. To analyze the spatial variation of the speed, the aircraft speeds in each segment are extracted from over 300,000 radar data entries from SACC during one week. The analysis shows that the speed of aircraft on a segment follows a normal distribution and the normal distributions vary for different segments. The distribution of aircraft speed in two segments that randomly selected is illustrated in Fig.2.

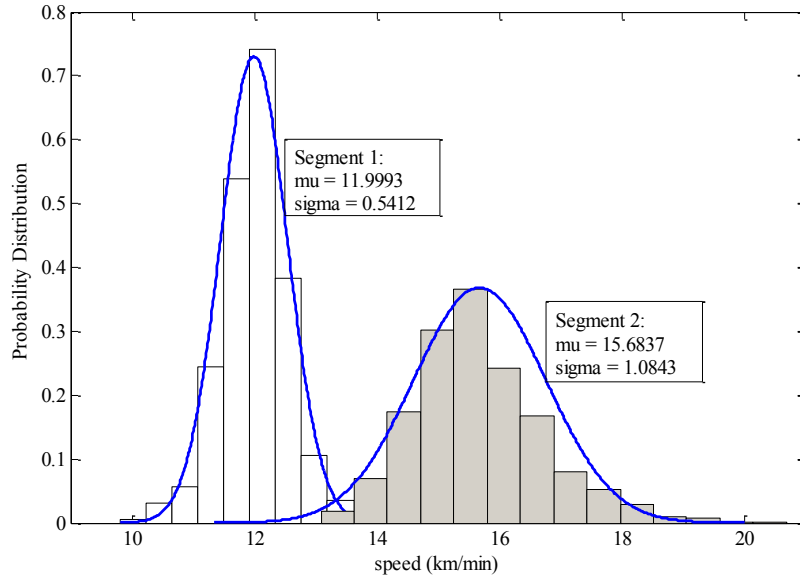


Figure 2. Aircraft speed distribution in different segments.

It can be seen from Fig 2 that both the mean and the standard deviation of the speed in different segment vary significantly. For the aircraft traversing through the same segment, statistical analysis shows that the normal distributions of speed vary with air traffic density (which is time-varying). One segment is randomly selected from the airspace system, and the speed distributions under different conditions of traffic density that extracted from the historical radar data during one week are presented in Fig.3.

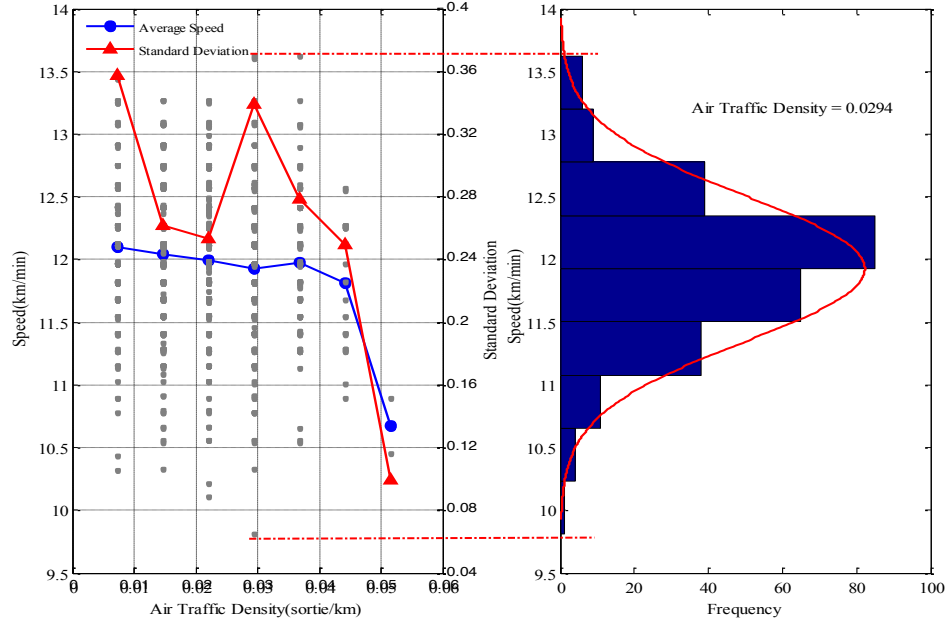


Figure 3. Aircraft speed distributions for a certain segment in different traffic densities.

It can be observed from the left part of Fig.3 that the aircraft speed distributions for the same segment vary under different conditions of traffic density. The right part of Fig.3 also shows that the aircraft speed in the traffic density of 0.0294 sortie/km is distributed normally. The results for the speed distributions under other traffic density conditions are similar.

Analysis above indicates that the distribution of aircraft speed is affected by both spatial position and traffic density which is time-varying. So it can be concluded that the speed varies with both space and time, and then the travel time changes accordingly.

3 Derivation of the dynamic air traffic flow model

Based on the dynamic network developed in Section 2, a novel air traffic flow model is proposed to characterize the dynamic behavior of traffic flow in the en route airspace, which will be further applied to implementing short-term air traffic flow prediction and optimization. Considering the demand of air traffic flow management in China, one hour is selected as the time horizon of the short-term prediction and optimization in this study.

3.1 Modeling the en route traffic flow

This paper focuses on the aggregation of air traffic flow rather than the dynamic behavior of individual aircraft. Therefore, the aircraft are spatially aggregated into ‘packets’ following the notion of cell transmission model [26][27]. According to the dynamic network presented in section 2, several interconnected directed edges make up an air route, where one directed edge, i.e., a segment, is defined as one control volume. Then the air route is divided into interconnected control volumes as illustrated in Fig.4.

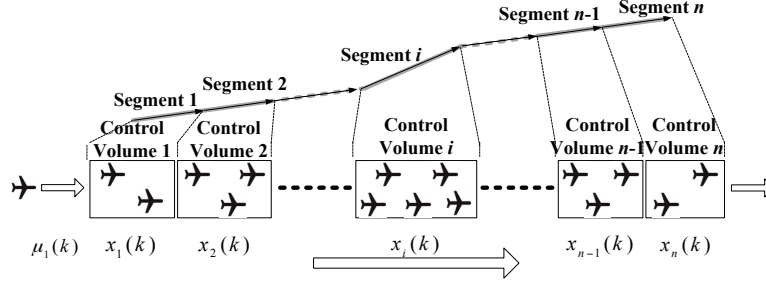


Figure 4. The aggregate en route traffic flow in the airspace system using the control volumes.

As discussed in section 2, the aircraft speed and the travel time vary both in time and space. For computational convenience, the air route is discretized spatially into segment ($i=1,2,3,\dots$) while the time horizon is discretized into intervals ($k=1,2,3,\dots$) of size Δt . The air traffic density, the aircraft speed and the travel time in one specific control volume corresponding to one segment satisfy the following equalities:

$$\rho_i(k) = x_i(k) / L_i \quad (1)$$

$$T_i(k) = L_i / v_i(k) \quad (2)$$

where $\rho_i(k)$ and $T_i(k)$ are the air traffic density and the travel time in the i -th control volume at the k -th time interval, respectively; L_i is the segment length corresponds to the i -th control volume, which can be obtained from NAIP; $v_i(k)$ represents the aircraft speed in the i -th control volume at the k -th time interval.

Based on the analysis in section 2, the aircraft speed $v_i(k)$ in the i -th control volume follows a normal distribution, which is determined by the traffic density in this control volume at the k -th time interval; that is

$$v_i(k) \sim N(\bar{v}_{\rho_i(k)}, \sigma_{\rho_i(k)}^2)$$

where the mean $\bar{v}_{\rho_i(k)}$ and the standard deviation $\sigma_{\rho_i(k)}$ of aircraft speed under the traffic density $\rho_i(k)$ in the i -th control volume at the k -th time interval are derived from the historical radar data.

Considering the dynamic and the continuity of the air traffic flow, the en route traffic flow traversing through the air route is regarded as fluid. According to the continuity equation in fluid mechanics derived from the model of the finite control volume fixed in space, we have

$$\iint_S \rho \mathbf{V} \cdot d\mathbf{S} = -\frac{\partial}{\partial t} \iiint_{\Delta} \rho d\Delta \quad (3)$$

where the left part of the equation denotes the net flow of air traffic out of the entire finite control volume, and the right part denotes the time rate of the decrease of mass flow in it.

Based on Equations (1) and (2), Equation (3) can be further expressed by the discrete-time and discrete-space difference equation

$$\begin{aligned} x_i(k+1) - x_i(k) &= \rho_{i-1}(k)v_{i-1}(k) - \rho_i(k)v_i(k) = x_{i-1}(k)/T_{i-1}(k) - x_i(k)/T_i(k), \text{ i.e.,} \\ x_i(k+1) &= x_i(k) + x_{i-1}(k)/T_{i-1}(k) - x_i(k)/T_i(k), \quad i \in [2, n] \end{aligned} \quad (4)$$

or

$$x_i(k+1) = (1 - 1/T_i(k))x_i(k) + (1/T_{i-1}(k))x_{i-1}(k), \quad i \in [2, n] \quad (5)$$

Now consider the first control volume, i.e., $i=1$, there is

$$x_1(k+1) = (1 - 1/T_1(k))x_1(k) + \mu_1(k) \quad (6)$$

where $\mu_1(k)$ is the system input, which denotes the number of aircraft entering the airspace system through the first control volume from outer airspace during the time interval from k to $k+1$.

Equations (5), which characterizes the dynamics of air traffic in one control volume, can be extended to a number of such interconnected control volumes. Define the number of aircraft in each control volume as the state vector of the system at the time step of k , i.e., $\mathbf{X}(k) = [x_1(k) \ x_2(k) \ \dots \ x_n(k)]^T$. The proposed air traffic flow model for the dynamic airspace system composed of multiple interconnected control volumes can be built as follows:

$$\mathbf{X}(k+1) = \mathbf{A}(k)\mathbf{X}(k) + \mu(k) \quad (7)$$

where the $n \times n$ state transition matrix $\mathbf{A}(k)$ is built as follows:

$$\mathbf{A}(k) = \begin{bmatrix} 1 - 1/T_1(k) & 0 & 0 & L & L & 0 \\ 1/T_1(k) & 1 - 1/T_2(k) & 0 & L & L & 0 \\ 0 & 1/T_2(k) & & & & M \\ M & M & 0 & 0 & & M \\ M & M & & 1/T_{n-2}(k) & 1 - 1/T_{n-1}(k) & 0 \\ 0 & 0 & L & 0 & 1/T_{n-1}(k) & 1 - 1/T_n(k) \end{bmatrix}$$

The elements of the state transition matrix can be updated by Equation (2) and the speed distribution at each time step, i.e., $k=1,2,3,\dots$. The time interval is 1 minute in this paper. As to the convergence or divergence cases, where multiple flight paths share an edge or vertex, the common edge is treated as multiple parallel edges. Then the air traffic on the multiple flight paths can be modeled respectively. The state transition matrix is still updated according to the traffic situation in the common edge.

The output of the air traffic flow model $\mathbf{Y}(k)$ is defined as the aircraft count of the user-specified airspace at the k -th time interval, that is

$$\mathbf{Y}(k) = \mathbf{C}\mathbf{X}(k) \quad (8)$$

where $\mathbf{C}$ is a $1 \times n$ vector, and $\mathbf{C} = [1 \ 1 \ L \ 1]$.

3.2 Predicting the en route traffic flow

Based on the dynamic air traffic flow model developed in Section 3.1, an en route traffic flow short-term prediction method is proposed. Select m air routes in the region as the airspace system, where one air route i ($i \in [1, m]$) consists of n_i control volumes. Then the air traffic prediction model for the air route i is built as follows, for each time step $k \geq 1$,

$$\mathbf{Y}_i(k) = \mathbf{C}_i \mathbf{X}_i(k) \quad (9)$$

$$\mathbf{X}_i(k) = \mathbf{A}_i(k-1)\mathbf{X}_i(k-1) + \mu_i(k-1) \quad (10)$$

Assume a multivariate normal distribution for the n_i -dimensional state vector at time $k=0$,

$$\mathbf{X}_i(0) \sim N_{n_i}(\mathbf{m}_0, \mathbf{C}_0) \quad (11)$$

where $\mathbf{X}_i(0)$ ($i \in [1, m]$) is the initial state vector, which denotes the aircraft counts of each control volume on the air route i at the initial time; $\mathbf{m}_0$ and $\mathbf{C}_0$ are the mean value and the variance of $\mathbf{X}_i(0)$, respectively, which are calculated from historical radar data. Given Equation (11), the k -steps-ahead prediction for the state vector $\mathbf{X}_i(k)$ ($X_{i,t+k} | Y_{1:t}$) and the observation $\mathbf{Y}_i(k)$ ($Y_{i,t+k} | Y_{1:t}$), i.e., the traffic flow on the air route i , can be achieved by calculating recursively according to the Equations (9) and (10). The model parameters, i.e., the elements of the state transition matrix $\mathbf{A}_i(k)$ as illustrated in Equation (10), are updated synchronously according to

Equation (2) and the speed distribution. It can be seen in Equation (2) that the element of the state transition matrix is a function of air traffic speed. $v_i(k)$ is a random variable and follows a normal distribution (i.e., $v_i(k) \sim N(\bar{v}_{\rho_i(k)}, \sigma_{\rho_i(k)}^2)$), which varies in different segment under different traffic density as analyzed in section 2. Considering the dynamics and the uncertainty of air traffic speed, it is impossible to update these model parameters $A_i(k)$ in a closed-form, and we have to resort to simulation-based methods, e.g., Monte Carlo methods used in the case study.

This method can be also used to predict the air traffic flow in an en route sector by adding up the prediction results of all the segments (i.e., the control volumes) in the corresponding sector.

4 Optimization of the en route traffic flow

Based on the en route traffic flow prediction method proposed in Section 3, the identification of demand-capacity imbalances in the airspace system can be achieved by comparing the predicted traffic flow (demand) with the airspace capacities. In this section, the en route traffic flow model is also used to propagate traffic flow dynamics in the optimization framework in order to resolve the imbalanced problem from both air route and en route network perspectives.

The en route traffic flow model developed in Section 3.1 is refreshed to facilitate the development of the optimal ATFM control strategies. Two kinds of variables, i.e., the inflow traffic $\mu(k)$ and the traffic flow in the airspace system $X(k)$, are optimally controlled to enforce capacity constraints at the minimum achievable cost. A control variable $d(k)$, which represents the amount of delayed flow that will not advance into the next control volume during the k -th time interval, is introduced to make an optimal strategy for $X(k)$. The Equation (5) and (6) can be rewritten as,

$$x_i(k+1) = (1 - 1/T_i(k))x_i(k) + (1/T_{i-1}(k))x_{i-1}(k) + d_i(k) - d_{i-1}(k), \quad i \in [2, n] \quad (12)$$

$$x_1(k+1) = (1 - 1/T_1(k))x_1(k) + \mu_1(k) + d_1(k), \quad i = 1 \quad (13)$$

Then the en route traffic flow model of Equation (9) can be rewritten accordantly as,

$$X(k+1) = A(k)X(k) + \mu(k) + B(k)d(k) \quad (14)$$

where $B(k)$ is a $n \times n$ control matrix with elements of all zeros except with ones on its diagonal and negative ones on its super diagonal.

4.1 Optimization at the air route level

For the optimization at the air route level, the en route traffic flow on one air route in the studied airspace system is optimally controlled, while those on the other air routes remain unchanged. The optimization ensures that the capacity constraints of the airspace sectors the air route goes through are met to resolve the capacity-demand imbalance problem. The optimization ensures that the capacity constraints of the airspace are met to resolve the capacity-demand imbalance problem. Since the airspace capacity is generally given at the sector level, the capacity constraints are considered as the capacities of the sectors which the air route traverses. The sector capacity, which depends on the controllers' capacity, the geographic location of the sector, the weather conditions, etc., is a restriction on the number of aircraft that may fly in the sector at one time [9].

For one air route i , which traverses s en route sectors and consists of n_i directed edges (i.e., control volumes), the capacity-demand imbalance of the s sectors is resolved by only optimizing the traffic flow on the air route i . The objective is to decide how many flights should advance into the airspace system from the air route i and how the traffic flow on it should be distributed at each time interval in order to enforce the capacity constraints at the minimum achievable cost. The time

interval is also 1 minute as used in the en route traffic flow model, and the optimization problem at the air route level is modeled as follows.

The objective function. The ATFM control strategies contain the airborne traffic flow control (e.g., changing the speed, stretching the flight path, circling, etc.) and the ground-holding. Since it is both safer and less costly to delay flights on the ground than flights that are airborne, the flights coming into the air route i , which should hold outside the airspace system for the capacity-demand imbalance inside it, are considered to hold on the ground prior to their departure. Therefore the objective is to minimize the operational cost of flights flying on the air route i and holding on the ground. The objective function can be expressed as,

$$\min_{x_{i,v}(k), d_{i,v}(k), \mu_i(k)} \theta_a \sum_{k=1}^K \sum_{v=1}^{n_i} x_{i,v}(k) + \theta_g \sum_{k=1}^K \left(\sum_{k'=0}^{k-1} \alpha_i(k') - \sum_{k'=0}^{k-1} \mu_i(k') \right) \quad (15)$$

where $\sum_{k=1}^K \sum_{v=1}^{n_i} x_{i,v}(k)$ is the total flight time of traffic flow on the air route i during the optimization time horizon K ; the variable $x_{i,v}(k)$ is the aircraft number in the v -th control volume ($v \in [1, n_i]$) on the air route i at time step k , where n_i is the number of the control volumes on the air route i ; $\sum_{k=1}^K \left(\sum_{k'=0}^{k-1} \alpha_i(k') - \sum_{k'=0}^{k-1} \mu_i(k') \right)$ is the total ground-holding time of traffic flow coming into the air route i during the optimization time horizon K , where $\alpha_i(k')$ and $\mu_i(k')$ are the planned and actual flights flying into the air route i during the time interval from k' to $k'+1$, respectively; the coefficients θ_a and θ_g are the airborne and ground holding operational cost for 1 minute, respectively.

The constraints. There are some constraints of the ATFM problem including the traffic flow dynamics, the inflow constraints, the control variables constraints and the capacity constraints. The concept of remaining capacity is proposed to characterize the capacity for part of the air route being optimally controlled in a certain sector. The remaining capacity of sector j for the air route i (i.e., $C_{j,i}(k)$) denotes the maximum number of aircraft that can fly on the air route i in sector j at one time, while at the same time the capacity of sector j can be met when the traffic flow on the other air routes in sector j remains unchanged. $C_{j,i}(k)$ is derived from the capacity of sector j minus the traffic flow on the other air routes in the sector at the k -th time step. Remaining capacity is introduced to enforce the capacity constraints of the sectors, which the optimally controlled air route traverses. The constraints of the objective function are listed as follows,

$$X_i(k+1) = A_i(k)X_i(k) + \mu_i(k) + B_i(k)d_i(k), \quad k \in [0, K-1] \quad (16)$$

$$\sum_{k=0}^k \mu_i(k') \leq \sum_{k=0}^k \alpha_i(k'), \quad k \in [0, K-1] \quad (17)$$

$$\sum_0^{K-1} \mu_i(k') = \sum_0^{K-1} \alpha_i(k') \quad (18)$$

$$d_{i,v}(k) \leq 1/T_{i,v}(k) \times x_{i,v}(k), \quad k \in [1, K] \quad (19)$$

$$\sum_{(i,v) \in J_j} x_{i,v}(k) \leq C_{j,i}(k), \quad j \in [1, s], \quad k \in [1, K] \quad (20)$$

$$x_{i,v}(k) \geq 0, \quad d_{i,v}(k) \geq 0, \quad \mu_i(k) \in N \quad (21)$$

Equation (16) characterizes the dynamics of the traffic flow on the air route i . Equation (17) ensures that the total inflow of the air route is not greater than the scheduled inflow at each time step, and Equation (18) ensures that all the scheduled inflow will fly into the air route during the optimization time horizon K . Equation (19) ensures that the control variable of each control volume does not exceed the traffic flow intended to exit if no control measure is taken, where the left part represents the traffic flow that will not enter the next control volume and remain in the current one during the k -th time interval for optimization purpose, and the right part represents the traffic flow which will flow into the next control volume if no optimal strategy implemented. Equation (20), where V_j denotes the set of the control volumes in sector j , takes into account the capacities of the sectors in the airspace system and enforces the traffic flow on a specific part of the air route in a certain sector $j(j \in [1, s])$ not exceed its remaining capacity. Equation (21) ensures that the variables are non-negative, and $\mu_i(k)$ which denotes the number of the aircraft entering the airspace system is integer. Therefore, the traffic flow optimization problem at the air route level is a MILP problem (Mixed Integer Linear Programming), which can be solved with CPLEX or other computation aided tools.

4.2 Optimization at the en route Network level

In this section, traffic flow in the en route network is optimally controlled at the system-wide level, while the sector capacities of the airspace system should be met to resolve the capacity-demand imbalance problem. The optimization at the network level can provide an optimal strategy from a global perspective, which is more efficient and cost-effective for the system-wide ATFM decision-making.

For the airspace system, which consists of I air routes and J en route sectors, the capacity-demand imbalance is resolved by optimizing the en route traffic flow in the dynamic network. In order to minimize the total operational cost, the objective is to decide how many flights should enter the airspace system and how the traffic flow in it should be distributed at each time interval (this is the decision that belong to the ATFM authority), rather than to decide the temporal-spatial distribution or the control strategy for each individual aircraft (this is the decision that belong to the air traffic controllers). The optimization problem at the network traffic flow level is modeled as follows.

The objective function. The objective is to minimize the operational cost of flights flying in the airspace system and holding on the ground. The objective function can be expressed as,

$$\min_{x_{i,v}(k), d_{i,v}(k), \mu_i(k)} \theta_a \sum_{i=1}^I \sum_{k=1}^K \sum_{v=1}^{n_i} x_{i,v}(k) + \theta_g \sum_{i=1}^I \sum_{k=1}^K \left(\sum_{k'=0}^{k-1} \alpha_i(k') - \sum_{k'=0}^{k-1} \mu_i(k') \right) \quad (22)$$

where $\sum_{i=1}^I \sum_{k=1}^K \sum_{v=1}^{n_i} x_{i,v}(k)$ is the total flight time of traffic flow in the airspace system (i.e., on the I air routes and in the J en route sectors) during the optimization time horizon K ; $\sum_{i=1}^I \sum_{k=1}^K \left(\sum_{k'=0}^{k-1} \alpha_i(k') - \sum_{k'=0}^{k-1} \mu_i(k') \right)$ is the total ground-holding time of the traffic flow which enters the airspace system during the optimization time horizon K .

The constraints. The constraints of the objective function (22) are listed as follows,

$$X_i(k+1) = A_i(k)X_i(k) + \mu_i(k) + B_i(k)d_i(k), \quad i \in [1, I], \quad k \in [0, K-1] \quad (23)$$

$$\sum_{k=0}^k \mu_i(k') \leq \sum_{k=0}^k \alpha_i(k'), \quad i \in [1, I], \quad k \in [0, K-1] \quad (24)$$

$$\sum_{k=0}^{K-1} \mu_i(k') = \sum_{k=0}^{K-1} \alpha_i(k'), \quad i \in [1, I] \quad (25)$$

$$d_{i,v}(k) \leq \frac{1}{T_{i,v}(k)} \times x_{i,v}(k), \quad i \in [1, I], \quad k \in [1, K] \quad (26)$$

$$\sum_{(i,v) \in J_j} x_{i,v}(k) \leq C_j(k), \quad i \in [1, I], \quad j \in [1, J], \quad k \in [1, K] \quad (27)$$

$$x_{i,v}(k) \geq 0, \quad d_{i,v}(k) \geq 0, \quad \mu_i(k) \in N \quad (28)$$

The constraints are similar to those in Section 4.1, but the dimensions of the variables above have been extended from one air route to the en route network in the airspace system.

The optimization problem at the air route level is less complex compared to that at the network level. And controllers only make extra strategies for air traffic on one air route according to the optimization at the air route level, which means that the workload of controllers can be reduced compared to making extra strategies for aircraft in the whole network. But optimization at the network level can provide an optimal strategy from a global perspective, which is more efficient and cost-effective for the system-wide ATFM decision-making.

4.3 Optimization algorithm

The ATFM optimization is composed of two steps, 1) air traffic flow prediction, based on which the identification of the demand-capacity imbalances can be achieved; 2) traffic flow optimization, which facilitates the goal of safe, efficient and cost-effective flow of air traffic. The optimization algorithms for the two steps are presented as follows.

A dynamic network for the en route traffic flow in the airspace system is constructed according to Section 2.1. The network traffic flow in the next one hour at each time step of one minute is predicted according to Section 3, where the model parameters, i.e., the elements of the state transition matrix, are updated synchronously. The procedure above is repeated until the air traffic flow prediction results for 24 hours are obtained. Considering the dynamics and the uncertainty of air traffic, it is impossible to update the model parameters (Equations (10)) in a closed-form, and we have to resort to simulation-based methods, e.g., Monte Carlo methods (MC methods) [39]. The short-term prediction algorithm is given by the following procedure:

Procedure 1. En route traffic flow prediction.

For $l=1,2,\dots,10000$ (10000 MC iterations)

For $h = 1,2,\dots,24$ (24 hours)

Initialization, $k = 0$;

For $k = 1,2,\dots, 60$ (short-term with one- minute time step)

Compute $X(k)$ and $Y(k)$ according to Equation (9)(10)

For $i = 1,2,\dots, n_i$

Sample $v_i(k) \sim N(\bar{v}_{\rho_i(k)}, \sigma_{\rho_i(k)}^2)$

Update the state transition matrix $A(k)$

Building on the traffic flow prediction procedure, the identification of the demand-capacity imbalances is conducted once the prediction results exceed the airspace capacities. Then the optimization is implemented to resolve the imbalance problems at both air route and en route network levels. When air traffic mainly concentrates on one air route, the imbalance problem can

be resolved by taking control strategies for the traffic flow on only one air route. While in other cases, the control strategies for the whole network traffic flow should be taken.

The MILP problem of the optimization at the air route level (i.e., Equation (15-21)) can be directly solved by using the mathematical programming solver, CPLEX. The optimization problem at the en route network level can hardly be solved efficiently for the increased dimensions of the model variables. To make the problem tractable, the primary problem is decomposed route by route into some sub-problems by using the dual decomposition method derived from reference [2]. Then the sub-problems can be solved similar to the optimization problem at the air route level. Define the traffic flow on the i -th air route in the j -th sector at time k as $y_i^j(k)$, the Equation (27) can be rewritten as,

$$\sum_{i=1}^I y_i^j(k) \leq C_j(k), \quad j \in [1, J], \quad k \in [1, K] \quad (29)$$

The dual problem of the primary problem (Equation (22)) is derived by introducing the partial Lagrangian for the constraints (Equation (29)), where a dual variable, i.e., Lagrange multiplier $\lambda_j(k)$, is incorporated:

$$\max_{\lambda \geq 0} \min_{x_{i,v}(k), d_{i,v}(k), \mu_i(k)} \sum_{i=1}^I [\theta_a \sum_{k=1}^K \sum_{v=1}^{n_i} x_{i,v}(k) + \theta_g \sum_{k=1}^K (\sum_{k'=0}^{k-1} \alpha_i(k') - \sum_{k'=0}^{k-1} \mu_i(k'))] + \sum_{k=1}^K \sum_{j=1}^J \lambda_j(k) (\sum_{i=1}^I y_i^j(k) - C_j(k)) \quad (30)$$

Subject to: (23-26),(28)

$$\sum_{(i,v) \in E_j} x_{i,v}(k) = y_i^j(k), \quad i \in [1, I], \quad j \in [1, J], \quad k \in [1, K] \quad (31)$$

The objective function of the dual problem (i.e., Equation (30)) is decomposed to form the term route by route as $\max_{\lambda \geq 0} [-\sum_{k=1}^K \sum_{j=1}^J \lambda_j(k) C_j(k) + \sum_{i=1}^I d_i^*(\lambda)]$, where

$$d_i^*(\lambda) = \min_{x_{i,v}(k), d_{i,v}(k), \mu_i(k)} \theta_a \sum_{k=1}^K \sum_{v=1}^{n_i} x_{i,v}(k) + \theta_g \sum_{k=1}^K (\sum_{k'=0}^{k-1} \alpha_i(k') - \sum_{k'=0}^{k-1} \mu_i(k')) + \sum_{k=1}^K \sum_{j=1}^J \lambda_j(k) y_i^j(k) \quad (32)$$

$d_i^*(\lambda)$ is the objective function of the sub-problem for the i -th air route. The constraints of the dual problem (i.e., Equation(23-26),(28),(31)) are also decomposed route by route as shown in Procedure 2. Thereby, the sub-problems can be solved independently similar to the optimization problem at the air route level. Then the Lagrange multiplier $\lambda_j(k)$ is updated synchronously by using the subgradient method based on the solution of the sub-problems. In order to approach the global optimum, the procedures above are iteratively repeated until the master problem converges to an acceptable level or the maximal iteration is reached. The optimization algorithm at the network traffic flow level is given by the following procedure:

Procedure 2. Network traffic flow optimization .

Initialization: $\lambda_j(k) \geq 0, \quad j \in [1, J], \quad k \in [1, K];$

Initialization: $x_{i,v}(0), \quad i \in [1, I], \quad v \in [1, n_i]$

For iteration $l=1, 2, \dots, L$

For $i=1, 2, \dots, I$ (I air routes in the airspace system)

Solve the sub-problem $d_i^*(\lambda)$, objective function: Equation (32);

s.t. $X_i(k+1) = A_i(k)X_i(k) + \mu_i(k) + B_i(k)d_i(k), \quad k \in [0, K-1]$

$$\sum_{k=0}^k \mu_i(k') \leq \sum_{k=0}^k \alpha_i(k'), \quad k \in [0, K-1]$$

$$\sum_{k=0}^{K-1} \mu_i(k') = \sum_{k=0}^{K-1} \alpha_i(k')$$

$$d_{i,v}(k) \leq \frac{1}{T_{i,v}(k)} \times x_{i,v}(k), \quad k \in [1, K]$$

$$\sum_{(i,v) \in \mathcal{V}_j} x_{i,v}(k) = y_i^j(k), \quad j \in [1, J], \quad k \in [1, K]$$

$$x_{i,v}(k) \geq 0, \quad d_{i,v}(k) \geq 0, \quad \mu_i(k) \in N$$

$$\text{Conduct the master problem: } \max_{\lambda \geq 0} \left[- \sum_{k=1}^K \sum_{j=1}^J \lambda_j(k) C_j(k) + \sum_{i=1}^I d_i^*(\lambda) \right];$$

For $j=1, 2, \dots, J$ (J en route sectors in the airspace system)

$$\text{Compute the subgradient } g_j(k) = \sum_{i=1}^I y_i^j(k) - C_j(k);$$

$$\text{Update the Lagrange multiplier } \lambda_j(k) = (\lambda_j(k) + \beta_l g_j(k))_+;$$

5 Case study

A case study was carried out based on a real data set from SACC to verify the developed en route traffic flow optimization framework. The data used in this study mainly contains the airspace structure data and the operational radar data, which provides the position, speed and altitude of all the airborne aircraft in SACC for every 30 seconds. In the case study, a dynamic network of 5 interconnected en route sectors at the altitude of 7800 meters and above in SACC is constructed according to section 2.1, which consist of 13 air routes (i.e., flight paths, aircraft fly along the applied air route as in the field flight plans) and 61 control volumes. The air traffic flow prediction is carried out at both air route and en route sector levels according to Procedure 1, based on which the identification of the capacity-demand imbalance in the airspace system is achieved. Subsequently, the optimization procedure is executed to resolve the imbalance problem at both air route and en route network levels according to Procedure 2.

5.1 En route traffic flow prediction

An air traffic flow dynamic model is proposed according to Section 3, based on which the en route traffic flow prediction at both air route and en route sector levels are conducted by Procedure 1. The prediction time horizon is one hour. One minute is selected as the time interval for the modeling in order to capture the dynamics and the uncertainty of air traffic as possible. In practice, the predicted values of every minute is too redundant for air traffic controllers to make a sound control strategy, and controllers are mainly concerned about the worst traffic situation in the next 15 minutes. Meanwhile to show the prediction results more clearly, the 15 minutes peak flow is used to trace the traffic situations on the air routes and in the en route sectors.

Take the No.7 air route in the airspace system as an example, which is one of the busiest air routes in China operating the flights between Shanghai and Guangzhou direction. Fig.5 shows the prediction results of 15-minute peak flow on this air route and the actual traffic observation extracted from the real radar data.

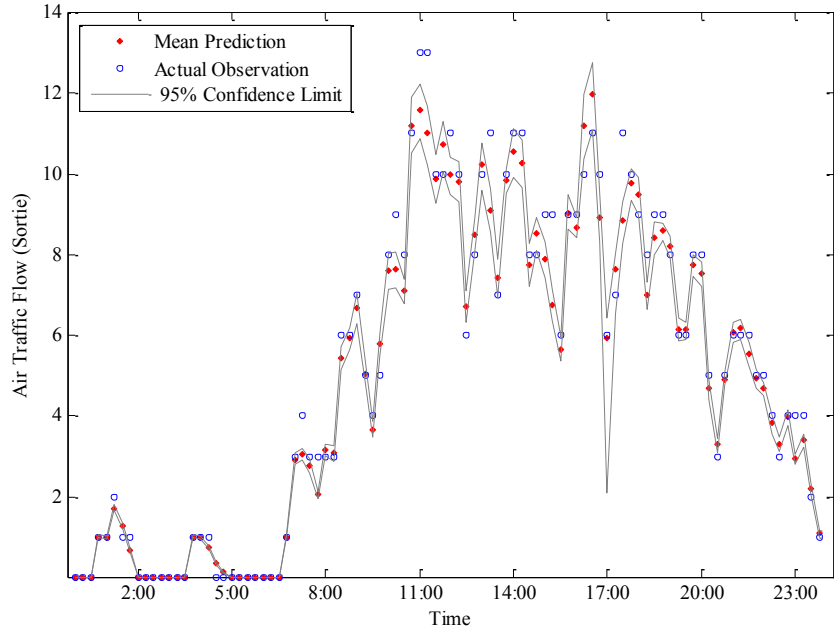


Figure 5. Prediction results of 15-minute peak flow on air route No.7.

It can be seen from Fig.5 that the prediction results of 15 minutes peak flow closely approximate to the actual observations. To further discuss the prediction performance, Fig.6 presents a detailed picture of errors distribution and frequency analysis of the prediction results for the air route No.7 during the 96 time intervals.

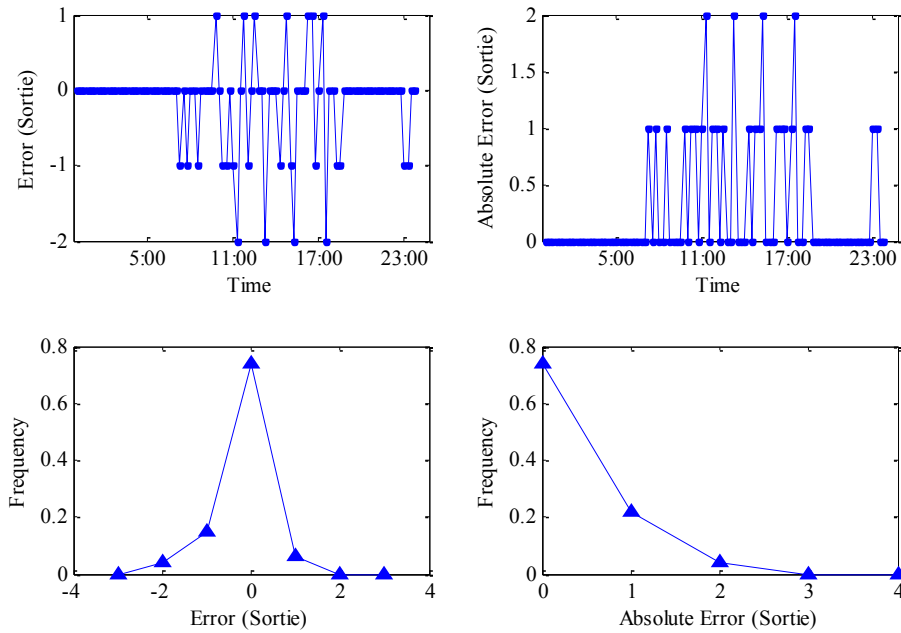


Figure 6. Errors analysis of the prediction results for the air route No.7.

It can be seen from Fig.6 that the errors of the air traffic flow prediction results for the air route No.7 are most frequently 0 and is between -1 and 1 over 95% of the time, and that the absolute errors of over 95% of the time are within 1. The results for the other twelve air routes in the airspace system are similar, and the detailed distributions of absolute errors of the prediction results for 13 air routes are presented in Table 1.

Table 1. Absolute errors distributions of 13 air routes.

Absolute error*	Cumulated Probability												
	No.1	No.2	No.3	No.4	No.5	No.6	No.7	No.8	No.9	No.10	No.11	No.12	No.13
0	0.75	0.74	0.84	0.91	0.85	0.79	0.74	0.74	0.95	0.95	0.78	0.81	0.68
<=1	0.99	1.00	1.00	1.00	1.00	0.99	0.96	1.00	1.00	1.00	0.98	1.00	0.97
<=2	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00	1.00

Note: Absolute Error represents the difference between prediction results and actual aircraft counts, and the unit of Absolute Error is aircraft count.

It can be seen from Table 1 that the absolute errors of 13 air routes fell in the range of 0 to 1 in more than 95% time intervals, and that all the absolute errors are within 2.

To further discuss the advantage of the proposed time-variant dynamic air traffic flow model (denote as TVD model for the purpose of discussion) compared with other aggregate models, an air route traffic flow prediction is also conducted based on the existing models. Since previously developed models assume aircraft speed to be constant, we denote the existing models as time-invariant constant air traffic flow model (TIC model) for the purpose of discussion. A comparative analysis of the prediction accuracy of these two methods is given in Table 2.

Table 2. Comparison of the air route traffic flow prediction results between models.

Air route number	TIC model				TVD model				improvement	
	MAE	Normalized	STD	Normalized	MAE	MAE	STD	STD	Normalized	Normalized
	1*	MAE 2* (%)	STD 3*	STD 4* (%)	MAE	MAE (%)	STD	STD (%)	MAE (%)	STD (%)
1	0.38	13.24	0.60	21.16	0.26	9.19	0.46	16.31	30.56	22.91
2	0.27	11.21	0.47	19.33	0.23	9.48	0.42	17.39	15.40	10.05
3	0.18	10.00	0.38	21.56	0.16	8.82	0.36	20.50	11.81	4.91
4	0.09	5.88	0.29	18.29	0.09	5.88	0.29	18.29	0.00	0.00
5	0.20	10.86	0.40	21.86	0.15	7.89	0.35	19.36	27.32	11.42
6	0.58	20.22	0.77	26.78	0.22	7.58	0.44	15.18	62.50	43.32
7	0.92	17.74	1.04	20.08	0.30	5.85	0.54	10.50	67.03	47.71
8	0.38	14.17	0.53	19.86	0.26	9.84	0.44	16.59	30.57	16.46
9	0.06	10.17	0.24	39.39	0.05	8.47	0.22	36.15	16.71	8.21
10	0.08	11.76	0.31	44.02	0.05	7.35	0.22	31.37	37.50	28.74
11	0.46	14.01	0.54	16.46	0.24	7.32	0.47	14.47	47.76	12.10
12	0.26	8.47	0.46	15.03	0.19	6.10	0.39	12.70	28.02	15.53
13	0.55	8.41	0.76	11.61	0.35	5.40	0.54	8.22	35.81	29.21

Note: 1*. MAE represents mean absolute error, and absolute error is the difference between prediction results and actual aircraft counts.

2*. Normalized MAE is defined as the ratio of MAE to the actual mean observation.

3*. STD represents the standard deviation of absolute errors, which is used to characterize the stability of the prediction.

4*. Normalized STD is defined as the ratio of STD to the actual mean observation.

It can be seen from Table 2 that the TVD model based prediction results have smaller MAE and STD than those based on the TIC model. The average improvement of the normalized MAEs of 13 air routes is 31.62%, and the average improvement of normalized STD is 19.27%, which indicates that the prediction performance of the developed TVD model is more stable with smaller

errors.

The traffic flow prediction for the en route sectors in the airspace system is carried out in a similar way, and the detailed distributions of the absolute errors of the prediction results for the 5 en route sectors are presented in Table 3.

Table 3. Absolute errors distributions of 5 en route sectors.

Absolute error	Cumulated Probability				
	No.1	No.2	No.3	No.4	No.5
0	0.42	0.59	0.45	0.50	0.39
<=1	0.72	0.96	0.80	0.86	0.84
<=2	0.88	1	0.96	0.98	0.95
<=3	0.97	1	0.98	1	0.99
<=4	0.99	1	0.99	1	1
<=5	1	1	1	1	1

It can be seen from Table 3 that the absolute errors of sector No.2 to No.5 fell in the range of 0 to 2 in more than 95% time intervals, and that the absolute errors of sector No.1 are within 2 in more than 85% time intervals and within 3 in 97% time intervals. This is because that the size of sector No.1 is bigger than the other 4 sectors, and that the average flow of sector No.1 is 1.67 times that of sector No.2. A comparative analysis of the prediction accuracy of en route sector traffic flow between models is illustrated in Table 4.

Table 4. Comparison of the en route sector traffic flow prediction results between models.

Sector number	TIC model				TVD model				improvement	
	Normalized		Normalized		Normalized		Normalized		Normalized	Normalized
	MAE	MAE (%)	STD	STD (%)	MAE	MAE (%)	STD	STD (%)	MAE (%)	STD (%)
1	1.05	11.40	1.24	13.49	1.03	11.17	1.14	12.36	1.98	8.36
2	0.56	10.13	0.69	12.42	0.45	8.07	0.57	10.36	20.36	16.59
3	1.15	16.11	1.22	17.21	0.82	11.57	0.96	13.46	28.20	21.82
4	0.96	12.30	1.01	12.96	0.66	8.42	0.76	9.77	31.52	24.57
5	1.08	17.78	1.03	16.86	0.83	13.68	0.85	13.95	23.09	17.28

From the results listed in Table 4, we can see that the TVD model based prediction results have smaller MAE and STD. The average improvement of the normalized MAEs of 5 en route sector is 21.03%, and the average improvement of normalized STD is 17.73%, which indicates that the prediction performance of the developed TVD model is more stable with smaller errors.

From the prediction results for both air route and en route sector granularities, we can see that the prediction accuracy and stability based on the proposed TVD model are superior over those based on the existing TIC model. This can be attributed to the incorporation of the dynamics and the uncertainty of the realistic air traffic for modeling. Therefore we apply the developed TVD model to predict the air traffic flow in SACC, which is further used to characterize the traffic flow dynamics in the optimization process.

5.2 En route traffic flow optimization

Based on the identification of the capacity-demand imbalance in the airspace system by comparing the traffic flow prediction results with the capacities of the en route sectors, the optimization of the en route traffic is carried out at both air route and network traffic flow levels according to the algorithm in Section 4.3. The time horizon of the optimization is also one hour as

that of the prediction in the case study.

5.2.1 Air route traffic flow level optimization

In this section, the capacity-demand imbalance problem is resolved by taking control of the en route traffic flow on an air route. Still take the No.7 air route in the airspace system as an example, which traverses the en route sector No.3 and No.5 and is thus constrained by the capacities of the two sectors. The optimization time horizon is set as 18:30-19:29, since the traffic flow in the sector No.3 approaches or even exceeds its capacity constraint during half of this time horizon. Additionally, the traffic flow on the No.7 air route during the time horizon accounts for over 40% of the total traffic in the two sectors. Therefore, the traffic flow on the No.7 air route is chosen to be optimally controlled to resolve the imbalance problem in the sector No.3 and No.5.

Two scenarios are defined in this paper, i.e., the optimization scenario and the uncontrolled scenario. The optimization scenario represents the scenario where air traffic flow is optimally scheduled by the method proposed in the paper. The uncontrolled scenario represents the scenario where aircraft are flying according to their filed flight plan and controlled by air traffic controllers. In the uncontrolled scenario, controllers manage air traffic flow relying on current ATFM tools, human intuition and past experience of experts. The traffic situation of the optimization scenario is compared to that of the uncontrolled scenario to test whether or not the demand-capacity imbalance problem can be effectively solved by the proposed method. Fig.7 illustrates a comparison of traffic flow on the air route No.7 between two scenarios.

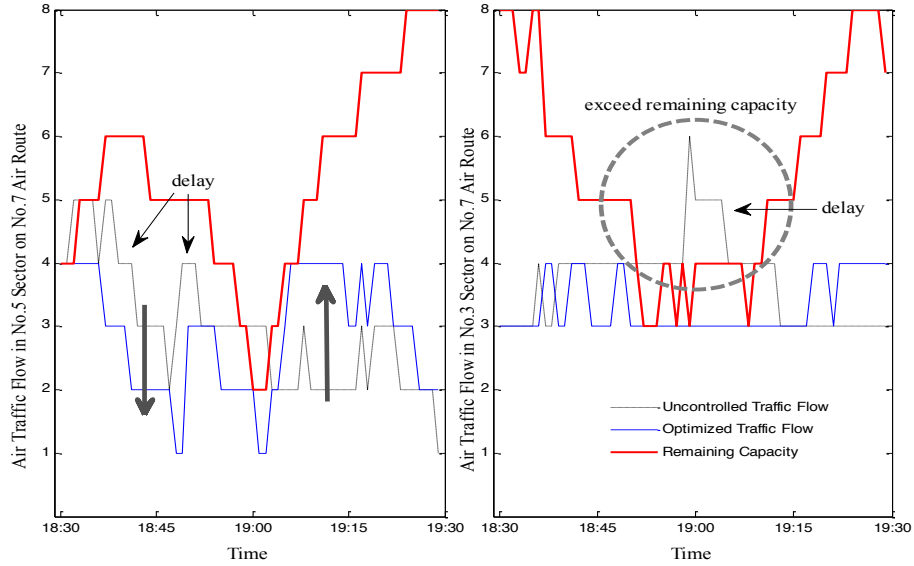


Figure 7. Comparison between traffic flow on the air route No.7 in two scenarios.

The traffic flow on the No.7 air route is divided into two parts according to their spatial distribution, i.e., 1) part 1: the upstream traffic on the No.7 air route in sector No.5 and 2) part 2: the downstream traffic on the No.7 air route in sector No.3. It can be seen in Fig.7 that the traffic flow in the right part exceeds the remaining capacity during 18:50-19:10 if no control strategies are taken. While in the optimization scenario, parts of the upstream traffic are controlled outside of the airspace system. The aircraft are restricted to enter the airspace, and delay may take place. So the optimized upstream traffic flow decreases compared to the uncontrolled flow shown as the down arrow in the left figure. The downstream traffic flow as shown in the right figure decreases accordingly, and the capacity constraints are, thereby, satisfied (the optimized traffic flow is under the remaining capacity limitation). Then the delayed traffic will fly into the airspace when the

capacity constraints have been met and the airspace resource becomes available (shown as the up arrow in the left figure). A comparison between traffic flow of No.5 and No.3 sectors in the two scenarios above is shown in Fig.8.

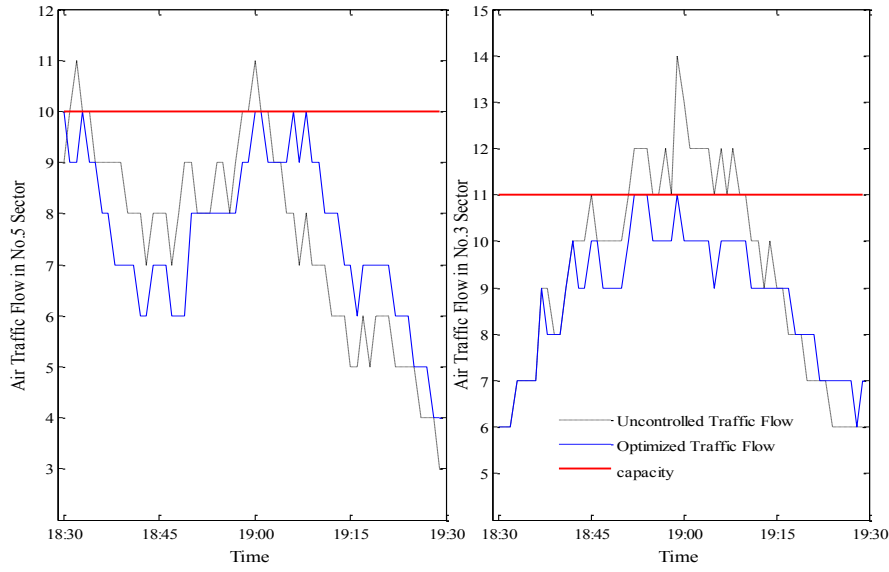


Figure 8. Comparison between traffic flow of sector No.5 and No.3 in two scenarios

It can be seen in Fig.8 that the capacity constraints are satisfied in the both two sectors by only implementing ATFM strategies on the No.7 air route. To characterize the performance of the optimization at the air route level, a capacity-exceeding frequency index (CEFI), which represents the total exceeding part of demand at all the time steps, is introduced. The operational cost can be calculated based on the ground holding time and the flight time, and the operational cost of ground holding and airborne flight per minute are 15 euro/min and 58 euro/min, respectively, according to the European airline delay cost reference values [40]. Note that ground holding may also take place in the uncontrolled scenario under current ATFM restrictions before air traffic fly into the studied airspace system. But this part of delay in the uncontrolled scenario is the same to that in the optimization scenario. Moreover, driven by the need to test the optimization method proposed in this paper, the case study focuses on the operational cost that produced during the optimization process rather than the cost of ground holding delay produced before the optimization. So this part of ground holding delay is not included in the comparison analysis. The detailed analyze of the No.7 air route operation between two scenarios are listed in Table 5.

Table 5. Analysis of the No.7 air route operation between two scenarios.

Scenarios	Ground Holding Time (min)	Total Flight Time (min)	Operational Cost (euro)	CEFI
uncontrolled scenario	0	467	27086	17
optimization scenario	73	458	27659	0

It can be seen in Table 5 that ground holding is implemented in order to meet the capacity constraints (CEFI=0), and that the total flight time is decreased since the capacity-demand imbalance has been resolved in the optimization scenario. The traffic flow optimization at the air route level can balance the demand-capacity in the en route sectors, where the controlled air route traverses, at the expense of ground holding delay and the corresponding increased operational

cost.

5.2.2 Network traffic flow level optimization

In this section, the network traffic flow in the airspace system (consists of 5 en route sectors as defined in the case study) is optimally controlled to resolve the capacity-demand imbalance problem from the system-wide perspective. The optimization time horizon is set as 16:30-17:29, since the airspace system is relatively congesting during this period of time. Fig.9 illustrates a comparison between traffic flow in the neighboring en route sectors No.5 and No.1 in the uncontrolled scenario and the optimization scenario.

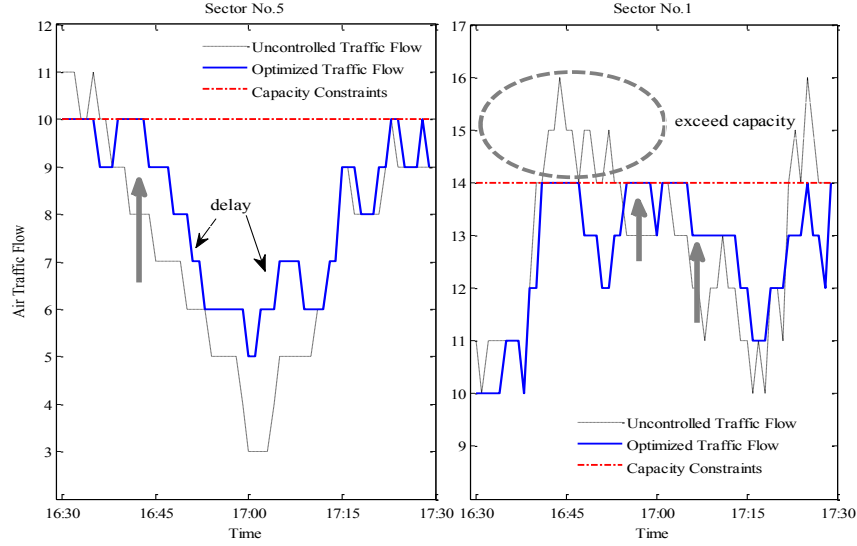


Figure 9. Comparison between traffic flow in sector No.5 and No.1 in two scenarios

Fig. 9 shows that the air traffic in the en route sectors is below the capacity in the optimization scenario. To balance the air traffic in sector No.1 in the uncontrolled scenario during 16:40-16:52, aircraft are delayed by remaining in its neighboring sector No.5 (airborne delay) or keeping outside of the airspace system (delay outside of the airspace system, which is considered to be ground holding delay in this paper) in the network traffic flow level optimization. Then the delayed traffic will fly into the sector No.1 during 16:55-17:21 (shown as the up arrow in the right figure) after the capacity-demand imbalance has been resolved. The capacity-demand imbalance problem has thereby been resolved, and meanwhile the utilization of airspace resource has been improved. The results for the other three sectors in the airspace system are similar. The detailed analyze of the airspace system operation between two scenarios are listed in Table 6.

Table 6. Analysis of the airspace system operation between two scenarios.

Scenarios	Ground Holding Time (min)	Total Flight Time (min)	Operational Cost (euro)	CEFI
uncontrolled scenario	0	3690	214020	70
optimization scenario	54	3571	207928	0

The scale of the number of flights in this airspace system during the optimization period is 148. It can be seen in Table 6 that the traffic is successfully balanced in the system-wide optimization scenario during the 1-h time horizon (CEFI is decreased from 70 to 0), and that the ground holding delay is the tradeoff for satisfying the capacity constraints (induces 54 min ground

holding time). Since the capacity-demand imbalance in the airspace system has been resolved from the system-wide perspective which facilitates the smooth and efficient air traffic, the total flight time is decreased (from 3690 minutes to 3571 minutes), as well as the corresponding operational cost (from 214020 euros to 207928 euros). It can be seen from Table 5 and Table 6 that the optimization at the network level is more efficient and cost-effective to address the imbalanced demand and capacity than that at the air route level. This can be attributed to the network-based dynamic air traffic flow modeling and optimization from a global perspective. To characterize the efficiency of the optimization algorithm, the convergence speed is analyzed in Fig.10.

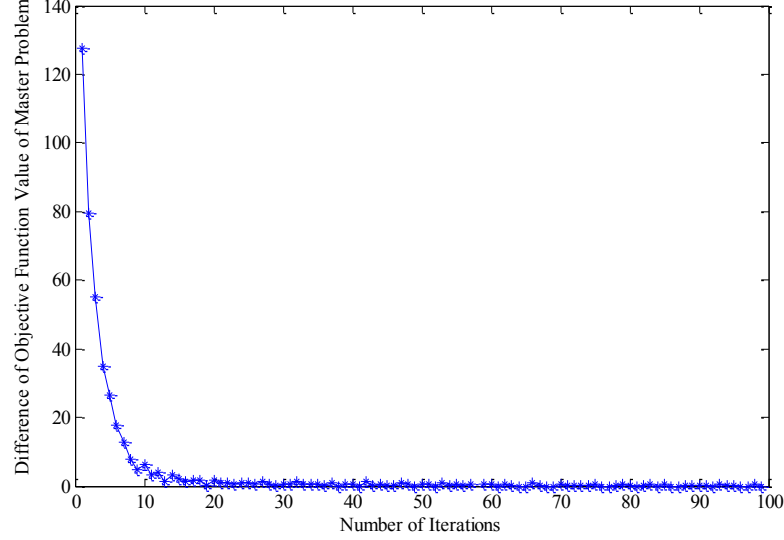


Figure 10. Difference of the objective function value of the master problem during the iterations by using Procedure 2.

It can be seen from Fig.10 that the difference of the objective function value of the master problem between the adjacent iterations converges to 0 in around 30 iterations. The computational time for the 1-h network traffic flow optimization problem is about 104.27 seconds. The air traffic flow optimization problem at the network level is decomposed route by route into some sub-problems by using the dual decomposition method, which is highly suitable for parallel computing in terms of a large scale optimization problem. Therefore when considering the air traffic flow optimization problem for a larger airspace system, e.g., the national airspace system, a parallel computing method can be applied to improve the computational efficiency.

6. Conclusions

In this paper, we focus on optimizing air traffic flow in en route airspaces by balancing the dynamic demand and capacity with the goal of minimizing en route delays. A mesoscopic air traffic flow model is proposed to characterize realistic dynamics and uncertainty of air traffic. Based on this model, an en route traffic flow optimization framework is proposed at both air route and network levels, by dynamically scheduling the flights.

This paper makes three contributions to the field of air traffic flow optimization. Firstly, a dynamic network is constructed (Section 2), which encapsulates the topological structure of the airspace and captures the dynamics of the traffic flow. Secondly, a mesoscopic dynamic air traffic flow model is developed to capture air traffic flow characteristics at an aggregate level in the

proposed dynamic network, where the continuity equation in fluid mechanics and space-time discretization adopted from cell transmission models are introduced to take into account the dynamics and the uncertainty of air traffic flow. Thirdly, at both air route and network levels, en route traffic flow optimization problems are solved with a dual composition method. Thus, a new method is developed, which solves more realistic models of the en route traffic flow optimization problem.

The proposed method is applied to a real-world radar data set of en route airspace in SACC to enforce capacity constraints with a look-ahead-time of one hour. The prediction results show that our method can accurately predict the dynamic air traffic (with the normalized MAE of the traffic flow prediction results below 10% and 14% for air route and en route sector levels, respectively). Meanwhile, the optimization results show that our method can effectively balance the demand and capacity of air traffic system at both air route and en route network levels. This can be attributed to the network-based dynamic air traffic flow modeling and optimization from a global perspective, which facilitates the incorporation of the dynamic characteristics of air traffic at an aggregate level for the system-wide ATFM decision-making.

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