

Stiffness Modelling and Control for Soft Material Continuum Robotic Manipulators



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Abstract Soft robots, made from compliant materials, offer high flexibility and enable continuous deformation, resulting in inherently safe and adaptive interactions with their environments. Unlike rigid-link robots where compliance is typically localized at discrete joints, soft robots exhibit distributed compliance throughout their structure. This fundamental difference underscores a deep understanding of their intrinsic compliance, or its inverse, stiffness, which is essential for developing other static models such as forward kinematics and contact force models. This work provides a comprehensive review of existing stiffness-related modelling approaches for soft continuum robots, such as techniques based on analytical formulations, numerical methods, and data-driven strategies. Furthermore, it highlights how robot stiffness critically influences the accuracy and effectiveness of static models and contact control methods, which are essential for enabling robust and precise interaction with the environment.

Keywords Soft robots · Compliance and stiffness · Static models

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1 Introduction

The mechanical behaviour of soft robots, in particular, their ability to deform under external loads, depends on their stiffness (or its inverse, i.e., the compliance). This property can be characterised by the Young's modulus, which defines the relationship between the stress and resulting strain. Unlike conventional robots constructed from rigid materials, such as metals or hard polymers, which typically exhibit Young's moduli on the order of $10^9 - 10^{10}$ Pa, soft robots are fabricated from significantly more compliant materials with Young's moduli ranging from approximately $10^4 - 10^6$ Pa [1, 2]. This inherent compliance can enable safer interactions with the environment and living tissues but can also limit the robot's ability to apply large forces when needed [3].

Tuning and analysing stiffness properties is central to the design, modelling, and control of soft robotic systems [4–6]. A variety of stiffening mechanisms have been proposed to modulate compliance, broadly categorised into semi-active and active methods [7]. Semi-active techniques operate by modulating the intrinsic mechanical properties of the materials from which they are made. Examples include the use of field-responsive materials, phase-change alloys [8], and granular jamming systems [9]. In addition, switching between actuation media, such as pneumatic and hydraulic systems, can be employed to alter stiffness [10]. On the other hand, active stiffening strategies often utilise antagonistic configurations, where opposing actuators work against each other. Examples include combined pneumatic and tendon-driven systems [11–13], dual pneumatic actuators [14, 15], and hybrid setups where active components are constrained by passive elements. For instance, pneumatic actuators may be mechanically limited using non-extendable fabric [16–18], locked tendons [19], or by leveraging the inherent elasticity of the material itself [20, 21].

This chapter advances the understanding of stiffness-related control of soft material continuum robotic manipulators by investigating how compliance characteristics influence stiffness response and force generation. Developing mathematical models that capture these relationships is essential for both predictive simulation and real-time control of interaction forces between the robot and the environment, for instance. In this context, the work presented here examines the formulation of models that integrate stiffness properties into the kinematic and force control of soft robotic systems.

Section 2 provides a comprehensive overview of existing approaches to modelling compliance and stiffness in soft robotic structures, while sect. 3 reviews established and emerging frameworks for static force modelling in these highly deformable systems. Section 4 transitions from static, passive compliance and stiffness modelling approaches to a review of active, on-demand stiffness control strategies, with a focus on sensing and controlling interaction forces. Section 5 outlines key open challenges and future research directions in modelling and controlling stiffness and interaction forces in soft material continuum robotic manipulators.

2 Compliance/Stiffness Modelling in Soft Material Robotic Systems

In rigid robots, the compliance is provided by variable stiffness joints [22]. For instance, the 3×3 Cartesian stiffness matrix K_X in the task space is mathematically determined based on the Jacobian-based projection in Eq. 1 [23].

$$K_X = J^{+T} \left(\Phi - \frac{\partial J^T}{\partial \theta} F \right) J^+ \approx J^{+T} \Phi J^+. \quad (1)$$

Φ is the stiffness matrix of the joints with a diagonal form, θ is the displacement of joints, J and J^+ are the Jacobian matrix and pseudo-inverse Jacobian matrix under current robot configuration, F is the external force in the Cartesian space. For the static case of unloaded robot configuration, $\frac{\partial J^T}{\partial \theta} F$ can be approximated as a zero item. The robot compliance is the inverse of the stiffness in Eq. 1. Instead, many soft robots are considered continuum manipulators without physical joints, and the compliance is distributed along the robots [5]. Figure 1 demonstrates the difference of compliance generation between rigid-link and soft continuum robots. Moreover, various static modelling techniques of soft robots usually rely on stiffness properties of robots. For instance, the material properties (e.g., Young's modulus), together with bending, elongation and shear stiffness, are usually considered to calculate curvatures and strains from actuation forces. The resulting curvatures and strains complete the statics models, such as the forward kinematics. However, a thorough stiffness description and analysis for soft robots, e.g., the Cartesian stiffness, is not investigated in these kinematic models.

The importance of understanding configuration-dependent stiffness/compliance has been proven for rigid-link and continuum robots [24–27], especially for redundant robotic mechanisms. For instance, the effect of robot configuration on task force capabilities was investigated in [28], force ellipsoid was initially introduced.

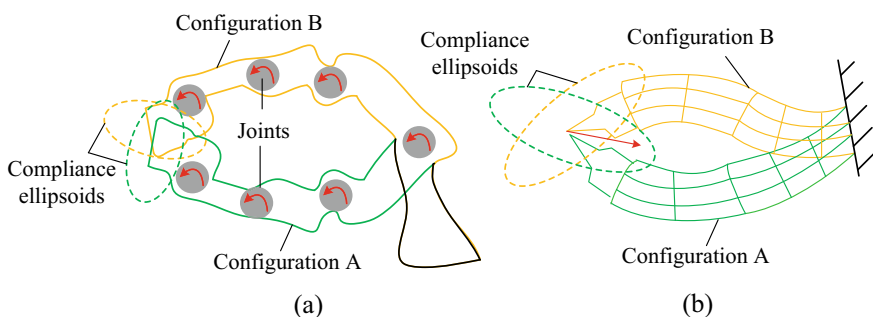


Fig. 1 Illustration of compliance ellipsoids for **a** a rigid-link robot and **b** a soft continuum robot, under two different robot configurations. The compliance of rigid-link robots is typically localized at discrete joints, while soft robots exhibit distributed compliance throughout their structure

To analytically describe the compliance or stiffness properties of soft robots, there are mainly two ways, i.e., Jacobian projection and finite differentiation of forward kinematics. The Jacobian projection approach is motivated by Eq. 1. To this end, the continuum structures of soft robots are discretised into finite elements or joints first to derive the Jacobian matrices, and a pioneer work for achieving compliance modelling of continuum robots using the Jacobian projection approach can be found in [25]. Once the Cartesian compliance matrices are obtained, compliance and force ellipsoids can be employed to achieve analysis, which is analogous to the techniques used in the rigid-link robots. Similarly, the stiffness properties of discretised elements can be written via diagonal stiffness matrices, the Jacobian matrices can be calculated from the Cosserat rod model [29] or piecewise constant curvature (PCC) model [27] for instance. Additionally, the finite differentiation of well-established kinematics models can also deliver the compliance modelling. To achieve this, virtual infinitesimal forces Δf are included in the forward kinematics model to calculate the corresponding variation of robot configuration, the forces and configuration variation are used to derive the compliance or stiffness matrices [30], resulting in Eq. 2.

$$C_X = \left[\frac{p(L) - p(L)^*}{\Delta f_x}, \frac{p(L) - p(L)^*}{\Delta f_y}, \frac{p(L) - p(L)^*}{\Delta f_z} \right] \quad (2)$$

where $p(L)$ and $p(L)^*$ are the 3×1 tip position vectors with and without force increments of Δf_x , Δf_y and Δf_z . For instance, the incremented variables are the tip forces, and the forward kinematics is established based on the Cosserat rod theory [31]. Such a concept was then adopted and developed by [32] to model the stiffness response of a parallel continuum robot under different configurations. Inspired by [31], a new set of compliance differential equations, which was directly coupled to the forward kinematics, to derive the compliance matrix of continuum robots [33]. The stiffness of pneumatic-driven soft parallel robots were analysed with the displacements resulted from applied loading in different directions [34]. However, the robotic manipulators were approximated to the slender Kirchhoff rod, whilst large elongation observed in soft robots were not considered [35, 36]. The stiffness can also be obtained in combination with the Finite Element Method (FEM) and the pseudo-rigid model, as demonstrated in [37]. To account for material nonlinearity resulting from robot elongation, the non-linear matrix structural analysis (NMSA) method presented in [5] can be useful. The NMSA method addresses large deformations in soft bodies by applying load in predefined steps. This method enables the use of a linear strain-stress relation to consider large deformations, where the stiffness matrices update in each step.

As for the efficient mathematical tool for the compliance modelling in robotic field, Lie theory [43] and, in particular, screw theory have been utilised in robotics for it concisely interprets rigid-body motions, such as kinematics and statics modelling [44], parameter calibration [45, 46] and identification [47], and compliance analysis [48–50]. In addition, an analytical model was presented in [40] to model and analyse the stiffness ellipsoids for a tendon-driven soft-rigid gripper. It is worth mentioning that recent study in [41] demonstrated that the Cartesian compliance

Table 1 Examples of modelling and stiffness/compliance analysis for various soft continuum robots

References	Model	Material behaviour	Cartesian compliance modelling	Compliance analysis	Compliance distribution
[31] ⁱ , [33] ⁱ	Cosserat rod	Linear	Finite differentiation	✗	✗
[32] ⁱ	Cosserat rod	Linear	Finite differentiation	Ellipsoid	✗
[38] ⁱ	Cosserat rod	Linear	✗	✗	✓
[39] ⁱ	PCC	Linear	Jacobian projection	✗	✗
[40] ⁱ	Pseudo-rigid model	Linear	Jacobian projection	Ellipsoid	✗
[41, 42]	PCC and Cosserat rod	(Non)linear	Screw theory	Ellipsoid	✓
[37] ⁱ	FEM	Linear	Finite differentiation	✗	✗

✗ and ✓ denotes that the corresponding modelling or analysis is not considered or considered, respectively; The superscript ⁱ denotes the elongation of the backbone is negligible or inextensible.

along the soft continuum robots can be directly integrated with the forward kinematics using the Lie theory. This approach does not require Jacobian transformations or multiple calculations of forward kinematics, which provides a concise approach to model the configuration-dependent compliance at any positions of the continuum robot. Specifically, the general form of the compliance calculation yields in Eq. 3.

$$C(s)^o = \int_0^L \text{Ad}_{ob}^{-T}(s)c(s)^b\text{Ad}_{ob}^{-1}(s)\text{d}(s) \tag{3}$$

$s \in [0, L]$, which is the arc length along the backbone of the robot. $C(s)^o$ is the Cartesian compliance matrix. $\text{Ad}_{ob}(s)$ is the adjoint matrix calculated from the forward kinematics. $c(s)^b$ is the element compliance matrix.

The compliance properties of soft robots exhibit two characteristics. First, the compliance distributes along the robot structure and depends on both the soft robots' configuration and the choice of coordinates (see Fig. 1b). Secondly, modelling of compliance usually involves the calculation of forward kinematics. Specifically, the Jacobian matrices come from the forward kinematics, while the finite differentiation approach needs to calculate the forward kinematics for a few times. The key advantage of Eq. 3 is that it only requires a single forward kinematics computation. It is worth mentioning that these two compliance characteristics are applied for various continuum robots (e.g., tendon-driven robots), as exemplified in Table 1. It can be observed that the compliance/stiffness models are less investigated compared to the kinematics or dynamics models of soft robots. It is also worth mentioning that the

robot compliance relates the loads and deformations, so it is also pivotal for understanding the interaction between the robots and environments, such as the contact force.

3 Modelling Interaction Forces in Soft Continuum Robots

For rigid-link soft robots, the interaction force in the task space results from the torque at revolute or prismatic joints. In contrast, the force generation of soft compliant robots is usually produced from the deformation and determined by the compliance properties of robots. Although soft robots are inherently compliant when contacting with environments, the modelling of the interaction force of soft robots is of paramount importance to understand their force capability and achieve active force control [51].

FEM might be the most straightforward approach to achieve the force prediction [52]. For example, constraints need to be applied to one end of soft robots when predicting the tip contact force. After applying actuation pressures on the internal surfaces of robots, the bending deformation and the tip contact force can be obtained. In [53], Abaqus was used to predict the tip force for soft pneumatic actuators with pleated structures. Similarly, the study in [54] used FEM to evaluate the force generation of a pneumatic-driven soft gripper with two pneumatic networks. A toolbox was presented in [55] to expedite the design of bellow soft robots, where the motions and output forces of soft robots with different parameters are assessed using the FEM. FEM is easy to be implemented; however, it lacks an analytical description of the force generation and often involves intensive computation costs.

FEM falls into the category of numerical approaches. Moreover, analytical models have been developed to achieve the force modelling. An analytical contact force model was proposed for PneuNets actuators, where the tip force was analysed considering the equilibrium moment of the PneuNets at the applied pressure [56]. The force model was validated via both FEM simulations and experiments. Overall, the analytical model predicts the tip force with errors of less than 10%. In [57], an analytical model was proposed for PneuNets actuators based on the Euler-Bernoulli finite strain hyperelastic thin cantilever beam theory. The deformation of the air chambers used finite strain membrane theory. The proposed analytical model was applicable to two states of forward kinematics modelling: free-space motion and tip-contact conditions. The model has similar accuracy with the FEM model, but the computational time is only 1% of the FEM model. Building on the PCC assumption and Euler-Bernoulli beam theory, an analytical blocking force model was proposed in [58]. The model reveals that the tip blocking force is impacted by the change of air volume and bending angle, meanwhile, the generated force is highly proportional to the actuation pressure.

In addition to the force models for aforementioned PneuNets actuators, analytical models for fibre-reinforced actuators were investigated. In [59], an analytical model was developed that captures the explicit relationship between the input pressure, the

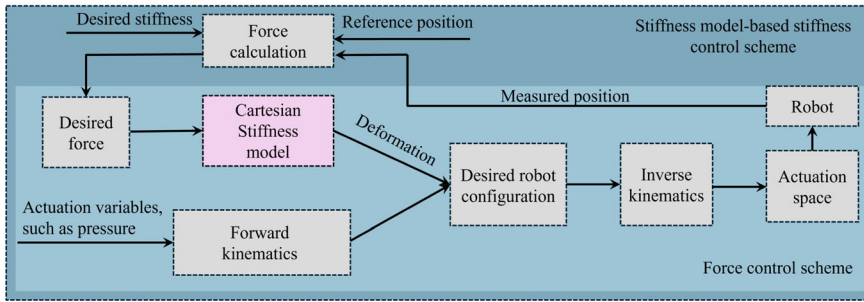


Fig. 2 Diagram of stiffness model-based stiffness and force control, where the stiffness model calculates the robot deformation subject to the desired force

bending angle, and the tip force by considering both the hyperelastic material property of silicone rubber and the geometry of the actuator. The model was built on the Neo-Hookean model and was evaluated on soft robots with different cross-section dimensions and lengths. Results demonstrated that the robot length has minimal influences on the force generation, while the force is primarily impacted by cross-sectional dimensions. The extended work of [59] was reported in [60], where the sensed bending angles of the fibre-reinforced actuator were incorporated to the analytical model. In [61], a comprehensive analytical model was developed for fibre-reinforced bending actuators, the model was also based on the beam mechanics, which can mathematically depict the relation of free-space bending, block force, and deflection shape with respect to the input pressure. Specifically, the block force modelling in [61] was based on the moment equilibrium, and the ODEs of the deflection curve are established. The ODEs were solved using the boundary conditions and resulted in the new force model.

Above force models are usually established on numerical (e.g., FEM) or analytical models, with known robot compliance properties. In particular, the forward kinematics models are often involved to derive the force models. It is worth mentioning that the interaction forces of soft robots can also be predicted via model-free, data-driven approaches, i.e., using neural networks [62]. One prominent example is the soft tactile sensors. The inputs of the neural network come from embedded sensor information, e.g., from camera [20], strain sensors [63], pressure [64]. [65] reviews the recent progress of the flexible tactile sensors. However, these data-driven approaches often overlook the underlying compliance characteristics inherent to soft robots. Section 4.2 further summarizes research on interaction force sensing and control for soft robots.

4 Interaction Compliance and Force Control

Soft robots exhibit passive compliant behaviour resulting from the soft materials. To make the most of their passive compliance properties during robot-environment interaction, active interaction control must be designed. To this end, the robot deformation has to be considered when developing the controllers. Two types of interaction controllers will be discussed in the following sections, including the compliance/stiffness control and the force control. A typical control flow for stiffness model-based interaction control is illustrated in Fig. 2. In this scheme, the stiffness model computes the desired deformation based on the target force. The desired force can be either explicitly specified, as in force control, or derived from the desired stiffness matrix in the case of stiffness control.

4.1 Compliance and Stiffness Control

Models presented in Sect. 2 describe passive compliance/stiffness modelling approaches of soft robots in the Cartesian space. These stiffness properties are inherently determined by the robot design and material selection. Similar to the compliance regulation for rigid-link robots [23], on-demand and active stiffness/compliance control has been investigated for soft robots.

The common approach to regulate robot stiffness is using stiffening mechanisms elaborated in [7]. For instance, once the robots are controlled to desired configurations, the vacuum pressure will be applied to actively increase the rigidity of soft robots, e.g., via layer jamming [66, 67] or granular jamming [68]. To avoid introducing stiffening mechanisms, the antagonistic actuation approach has been developed [69]. For instance, a variable stiffness joint was proposed in [70], where two soft air bellows acted antagonistically on a single joint. In this actuator, various pressure combinations can lead to the same joint angle and allow to additionally adjust the joint stiffness. The same antagonistic stiffening approach was also applied in [71] to vary the stiffness of soft robots made of pneumatic artificial muscles. Thanks to the introduced stiffening mechanisms, the robot configuration and compliance can be controlled independently. However, embedding stiffening mechanisms or applying antagonistic actuation approach in space-restricted applications, for instance, minimally invasive surgery, can be challenging.

As demonstrated in Sect. 2, the robot passive compliance is configuration-dependent and can be analytically or numerically described. In principle, it is possible to control robot configurations to regulate compliance/stiffness of soft robots. The first stiffness control paradigm for soft robots without embedded stiffening mechanisms can be found in [4]. To achieve a desired tip stiffness, the controller actuated the robot to a deflected configuration which produced a required tip force for a measured tip position. And the desired force was calculated from a desired stiffness matrix. The robot kinematics was described using the Cosserat rod model. Similarly, the

study in [72] achieved the stiffness control for a parallel continuum robot based on estimated force and robot pose, using the Cosserat rod theory. Results demonstrated that the stiffness controller can vary the natural stiffness of soft robots by about a factor of two along the x and y -axes. Building on the PCC assumption, a Cartesian stiffness control approach was proposed for a multi-segment soft robot [27]. The Cartesian stiffness was described using Eq. 1. A desired compliance ellipsoid was defined to formulate optimization objective functions, and the robot configuration was then derived from solving the optimization problems. The compliance of the robot was regulated by changing the internal pressure of the tendon-driven robot. A tendon-tensioning method was proposed to control the stiffness of a dual-segment tendon-driven soft robot based on depth vision. A closed-loop controller was designed for stiffness compensation [73]. In line with Fig. 2, a compliance model-based control approach was presented in [74] to regulate the Cartesian compliance in desired directions for pneumatically driven soft robots. It is noteworthy that the compliance regulation in [74] exclusively relies on varying the actuation pressure without additional structures.

4.2 Interaction Force Sensing and Control

To regulate the Cartesian forces of soft continuum robots, force control techniques from rigid-link robots could be referred to. A prominent force control approach is discretising soft robots into finite elements and regulating virtual joint forces from each element using Jacobian projection [75], and the Cartesian stiffness matrix is tuned as a control gain. For instance, an intrinsic force sensing approach was proposed to estimate the wrench exerted at a tendon-driven continuum robot tip by monitoring the actuation forces [76]. Consequently, this estimated wrench is further advanced as feedback information to implement closed-loop force control [77]. In particular, a hybrid motion and force control approach was proposed for tendon-driven continuum robots to achieve force regulation or contact surface estimation from sensed forces in the task space. In aforementioned cases, actuation forces usually can be obtained from force transducers in tendon-driven or parallel continuum robots [32, 78]. In contrast, additional consideration is required to convert fluidic pressure or volume to generalised actuation forces or torques for fluidic-driven robots [79]. For hydraulic-driven soft robots, the combination of fluid volume and pressure can be utilised for intrinsic force sensing and control of soft parallel robotic systems [80], where the Cartesian stiffness matrix was estimated by solving the linear least-squares problem. Likewise, a compliance model-based contact force control approach for soft robots was presented in [81], where the compliance model was analytically derived.

In addition to model-based force sensing and control, technological advances enable deployment of force sensing devices on soft robots, e.g., forces exerted on robots can be measured by attached force/torque (F/T) sensors directly [82, 83]. The F/T sensors are usually rigid and limited to its feasibility of being embedded in soft robots. Fibre optics exhibit naturally flexibility [84] and are capable of sensing forces

from measuring strains, e.g., stretchable sensors were developed for prosthetic hand based on optical waveguides [85]. Moreover, material sciences have advanced the development of flexible force sensors, e.g., using piezoelectric polymers [86], liquid metals [87] or pneumatic fluids [88]. In particular, fusion of sensed information and models provides new insights on interaction force estimation and regulation of soft robots [89, 90]. A vision-based external forces sensing approach was proposed in combination with finite element method, where the intensities and the locations of the external forces can be estimated [91]. Similarly, the deformation of soft robots derived from vision or motion tracking information can be utilised to achieve force and motion control [92], estimate or attenuate external force disturbances [73, 93].

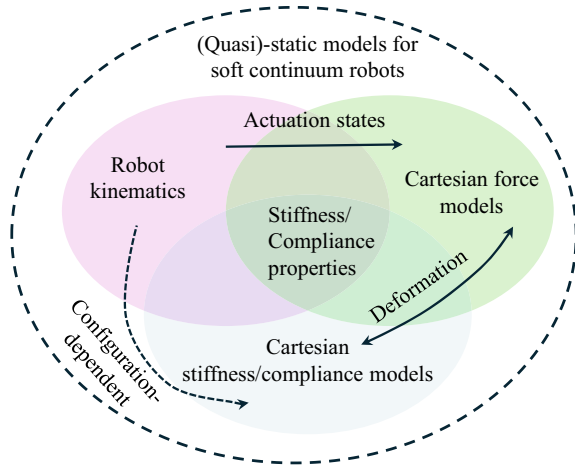
Another force sensing and control paradigm for soft robots is data-driven approach [94]. A sensorless force and displacement estimation approach was introduced by exclusively utilising fluidic pressure and volume in pneumatically powered soft actuators [64]. A learning-based closed-loop force control approach was presented combining embedded soft sensors and recurrent neural networks, which can cope with significant drift and hysteresis in feedback signals [63]. It is worth mentioning that the efficacy of data-driven approaches rely on and is determined by the training set. Please note that robots' stiffness properties may affect the generalizability of data-driven approaches, as high robot compliance can introduce more uncertainties.

5 Conclusions

Robot compliance/stiffness and force capabilities are exhibited characteristics when soft robots interact with environments. The interaction compliance and force behaviour of soft robots result from the continuous deformability of soft materials. The compliance/stiffness properties essentially connect applied wrenches (including forces and moments) and corresponding twists, i.e., linear or angular deformations. Building on various models developed to describe the statics of soft robots, this review reported on the interrelationship between robot kinematics, Cartesian compliance/stiffness, and force generation capability. Understanding these connections is essential for effectively modelling and controlling interactions between soft robots and their environments, as illustrated in Fig. 3. In particular, these static models can be utilised for achieving various model-based control, e.g., the inverse kinematics, compliance and force regulation. Modelling the stiffness of soft robots with hybrid actuation mechanisms remains a challenging task. In such systems, overall stiffness is influenced not only by the material properties but also by factors like actuation pressure and tendon tension. Accurately modelling and controlling the stiffness of these systems is an interesting research direction.

Whereas notable work has been achieved for static cases, modelling the dynamic behaviour of soft robots remains a significant challenge due to the highly non-linear, time-varying, and often distributed nature of their deformation. Unlike rigid-body systems, where mass and inertia are well defined, soft robots require the incorporation

Fig. 3 Summary of the statics models for soft robots. The stiffness properties of soft materials govern the kinematics, as well as Cartesian stiffness and force models of soft robots



of continuum mechanics and viscoelastic properties to accurately capture transient responses. Dynamic modelling is essential for tasks involving high-speed motion, impact mitigation, or interaction with dynamic environments. Coupling dynamic models with stiffness representations enables advanced control strategies, such as variable impedance control, which can regulate both motion and interaction forces in a time-varying manner. However, developing integrated frameworks that jointly model dynamics and compliance remains an challenging problem. Such models must account for actuation principles, non-linear material behaviour, and environmental contact, representing a critical direction for advancing real-time, adaptive control of soft robots.

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