

THE MECHANISM OF FRICTIONAL OSCILLATIONS

by

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A Thesis submitted for the degree of

DOCTOR OF PHILOSOPHY

of the University of London

and also for the

DIPLOMA OF IMPERIAL COLLEGE

November 1971

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ABSTRACT

Frictional oscillations, considered as an engineering problem, are of great importance because they produce increased wear rate, inaccurate conditions of operation in machine tools or servomechanisms, noise and similar unwanted phenomena. The function $\mu=\mu(v)$ which governs frictional oscillations is extremely difficult to determine accurately during the frictional oscillation cycle and that is the main reason why simple models with a hypothetical $\mu=\mu(v)$ have previously been employed.

The combination of a new mathematical model for frictional oscillations along with a topological solution to the equation of motion, enables the characteristics of frictional oscillation to be predicted in practice and the function $\mu=\mu(v)$ to be derived experimentally. The model meets the requirement for a generalized explanation of several different forms of frictional oscillations, such as the "reversed stick-slip", the "frictional microvibrations" and the like.

Experimental application of the method to several combinations of specimens and lubricants, most commonly used in tribological practice, showed that successful results can easily be obtained, and revealed the existence of a twin frictional mechanism which explains readily some of the peculiarities of frictional oscillations.

The function $\mu=\mu(v)$ is obtained experimentally within very short periods of time (in some cases in less than 0.01 sec), and this is one of the main advantages of the technique, because time variables which affect the frictional process (e.g. wear and environmental changes) are excluded, thereby eliminating a source of major errors.

ACKNOWLEDGEMENTS

The author acknowledges gratefully the encouragement and advice his supervisor Dr. A. Cameron has given to him throughout this project.

He thanks Mr. P. MacPherson for his generously given help and advice.

He also wishes to thank Mr. R. Dobson of the Lubrication Laboratory and the Workshop Staff of the College for their valuable assistance.

Finally he expresses his gratitude to the Greek Ministry of National Economy for providing a NATO Scholarship which made this study possible.

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NOTATION

A :	Area of contact or amplitude in general
A_r :	Real area of contact
A_c :	Contour area of contact
A_a :	Apparent area of contact
A_{ss} :	Stick-slip amplitude
A_{tro} :	Triggering oscillation amplitude
c :	Constant
C, D :	Damping factor
E :	Energy
E_h :	Frictional heat
F :	Friction in general or function
F_k, F_{k_0} :	Kinetic friction and kinetic friction for $v \rightarrow 0$
$F_s, F_{s_0}, F_{s_t}, F_{s_\infty}$:	Static friction, static friction after zero, t , or ∞ idle time
F_{s_m} :	Minimum value of static friction
h :	Roughness
h :	Separation of the surfaces
$\bar{h}$:	Film thickness
H :	Hardness
k :	Stiffness
$L; N$:	Force, load
m :	Mass
p, p_m :	Pressure, mean pressure
s :	Sliding distance
T :	Absolute temperature
t, t_s :	Time, idle time
v :	Velocity
v_o :	Mean driving velocity
v_r, v_c :	Relative velocity, critical velocity
v_h :	Limiting velocity between boundary and hydrodynamic

	conditions
X, Y, Z, x, y, z :	Displacement, distance
δ_f, δ_n :	Displacements as appear in the experimental traces, in horizontal or vertical direction
ΔF :	Frictional force difference in general and particularly $F_s - F_{k_0}$
$[\frac{\Delta F}{N}]_t$:	The value of the nondimensional ratio $\Delta F/N$ in time t
$\Delta \mu$:	Coefficient of friction difference and particularly $\mu_s - \mu_{k_0}$
ζ, η, ξ :	Parameters in directions x, y, z
η :	Viscosity
θ :	temperature or angular displacement
μ :	Coefficient of friction in general
μ_k, μ_{k_0} :	Kinetic coefficient of friction and kinetic coefficient of friction for $v \rightarrow 0$
$\mu_s, \mu_{s_0}, \mu_{s_t}, \mu_{s_\infty}$:	Static coefficient of friction, static coefficient of friction for 0, t or ∞ idle time
$\{\mu_{k_d}\}_v$:	"Dynamic" kinetic coefficient of friction for sliding velocity v
μ_{k_s} :	"Static" kinetic coefficient of friction (for $\frac{dv}{dt} \rightarrow 0$)
ϕ, φ :	Functions
ω :	Frequency or angular velocity
$\omega_{ss}, \omega_{tro}$:	Frequency of stick-slip and triggering oscillation
ω_n :	Natural frequency

CHAPTER 1 : REVIEW OF THE LITERATURE

1.1. INTRODUCTION

Although numerous investigations into the nature of friction have been made during the last three centuries, it is only in the last thirty years that any real advance has been made towards some slight understanding of frictional self-excited oscillations and related phenomena. This is attributed to two main causes, namely:

a. The three classical "laws of friction" (Amontons' and Coulomb's) predict generally linear behaviour for all frictional pairs, independently of the conditions under which the two bodies, constituting the frictional pair, are rubbed, and consequently energy storage in the system is theoretically impossible and frictional self-excited oscillations cannot occur.

b. The experimental techniques in use for frictional studies some decades ago were incapable of recording fast dynamic phenomena accurately. Attempts at comparative studies of theory with experimental results, did not correlate well.

It is the breakdown of the third "law of friction" (frictional force independent of velocity) that led to the correct methodology for studying frictional self-excited oscillations in theory and practice. As early as 1835 A. Morin [1] had proposed that since the frictional force resisting the start of sliding of two bodies at rest was obviously greater than the resistance after they were in motion, there should be two coefficients of friction, a static one, for surfaces at rest and a kinetic one, for surfaces in motion. Later, observations of Kimball in 1877, Kaufmann in 1910 and Jacobs in 1912 [2] showed more clearly that the validity of Coulomb's law was not universal, and produced an increase of the already strong scepticism about the "laws of friction".

After that initial stimulation of scientific interest in the breakdown

of the classical "laws of friction" some pioneering work followed about the frictional behaviour of solid bodies, under dry or lubricated conditions of sliding. Rankin in 1926 [3] studied the strain of the surfaces in contact, with the applied tangential force, before commencement of sliding (the elastic range of friction), work later revised by Rabinowicz [4] and Mason and White [5].

In 1929 Wells [6] observed self-excited oscillations while attempting to measure the kinetic coefficient of friction at low sliding speeds and concluded that the motion could occur only if the static coefficient of friction were larger than the kinetic one. In 1930 Thomas [7] proved analytically, and showed experimentally, that Wells' conclusion was correct. He noticed that vibrations initiated in any manner between two bodies sliding the one on the other under conditions of dry friction, tend to persist within a certain maximum amplitude without any impressed disturbing force other than that provided by the relative motion of the bodies. He suggested that the continued vibration maintained might be responsible, in some measure, for the production of sound in rubbing contacts. Kaidanowski and Haykin in 1933 [8] studied the relaxation oscillations as applied to mechanical systems having friction varying with velocity. According to their theory mechanical relaxation oscillations occur in an elastic friction system when the curve relating the friction force to the slip velocity has a decreasing character i.e. the basis of this theory is formed by the same assumption as used by Rayleigh who, when studying the transverse vibrations of a violin string, assumed that the force of dry friction between the string and the bow is not constant, but varies [9]. The use of systems where the frictional force was measured by the deformation of an elastic member, which carried one friction surface and pressed it against another (moving) surface led, when an attempt was made to increase the sensitivity of the arrangement by decreasing the

stiffness of the measuring member, to self-excited oscillation. Bowden and Ridler [10] were the first to note regular variations in friction while performing experiments on unlubricated surfaces and concluded that the kinetic coefficient of friction may not be constant. A similar conclusion was reached by Papenhuysen in 1938 [11] who observed the phenomenon while experimenting on the sliding of rubber on glass and other surfaces in order to study the laws governing the skidding of automobile tyres. In 1939 Bowden and Leben [12] attempted to investigate in more detail the physical processes that occur during sliding and the nature of the frictional force that opposes the motion. They are unquestionably the first who used "stick-slip" as a self-sufficient term, to define the most important form of frictional self-excited oscillations, and aroused much interest in their rather revolutionary ideas about the origin of that phenomenon. It seems that although the term "stick-slip" does not define very accurately the phenomenon and many research workers have produced serious objections about its use [13,14], it has prevailed through lack of another more successful one. Deraguin, Push and Tolstoi [15] have proposed the term "self-oscillation of the first kind" or "self-oscillation with stopping" to discriminate it from quasi-sinusoidal or sinusoidal self-excited frictional oscillations which they call "self-oscillations of the second kind" or "self-oscillations without stopping"; terms now in use only among the Russian tribologists.

The first systematic study of stick-slip appears in 1940 [16], when Blok presented a correct method for establishing a quantitative criterion for the appearance of stick-slip. This was shortly followed by the very important contributions of Sampson et. al. [17] and Morgan et.al. [18]

It is not very clear, and many contradictory opinions have been expressed about the date which must be considered as a starting point for the history of the study of frictional self-excited oscillations. Some

put that date as early as 1929 (Wells' paper on boundary friction [6]) and others accept 1930 (Bowden-Ridler [10]) or 1938 (Papehuysen [11]) or even 1939 (Bowden-Leben [12]). In fact it strongly depends on the criteria one uses to estimate the importance of the contribution of a paper on the subject under study, and consequently nothing can be said definitely. Since World War II many researchers have attempted to analyse frictional self-excited oscillations in order to obtain a better understanding and control of them because of their wide and usually undesirable occurrence. Clutch jerking, brake squealling, machining chatter, brush vibration on a slip ring, positioning errors in servomechanisms [19,20] and numerically controlled machine-tools [21,22] all are ascribed to frictional self-excited oscillations or more specifically to the stick-slip phenomenon. Increased wear and non-uniformities of machined surfaces [23,24], periodic thickness variations in the drafted material when drafting fibrous textile materials [25], galloping of electric transmission lines in strong winds [26] and shallow focus earthquakes [27] are some of the quite well known unavoidable consequences of self-excited frictional oscillations. Their influence extends into very different fields of engineering and industrial practice. Obviously such a universal phenomenon attracted considerable interest and a very wide range of investigations has been made into it. Unfortunately, although the attention which is paid to its complexity increases with time, mainly due to the extensive industrial interest for its practical implications, its nature is very differently explained by the research workers involved in its study. The only point of coincidence among them is the fact that generally self-excited frictional oscillations processes can only occur in non-linear systems.

The literature in the field, although extensive, offers very little information which can be applied to a particular problem and many substantial

inconsistencies in the results and the interpretation of frictional tests have been noticed [23,28]. It appears that the results depend almost as much on the test method as on the material being tested.

The reviewed literature has been divided into three main parts:

- a. A survey of the existing theories about self-excited frictional oscillations and the mathematical models employed.
- b. A brief review of the more important experimental techniques used in the past to study frictional oscillations and
- c. A survey of the available literature on frictional oscillations as they are met in practice, in several applications, or phenomena referred to in the literature as being closely related to them.

1.2. THEORETICAL

1.2.1. Conventional theories

1.2.1.1. Theories based on very simple models

The friction-velocity characteristic is the criterion by which the following types of models can be distinguished:

- a. Continuous or non-continuous Coulomb models:

$$F_K = F_S = \text{constant}$$

$$\{(F_K, F_S) = \text{constant}, F_K \neq F_S\}$$

- b. Models with linear characteristic:

$$\left\{ F_K = F_{K_0} + c \cdot v \cdot F_{K_0}, F_S = \text{constant} \geq F_{K_0} \right\}$$

- c. Other types of models.

The first satisfactory theory about frictional self-excited oscillations was produced by Thomas in 1930 [7] who in order to explain the sticking of the bodies, accepted that $F_S > F_K$ in general, and proved that the increase of frictional resistance under static conditions provides a stimulus sufficient to maintain vibrations in spite of other damping influences, if

these are not enough to reduce the oscillation amplitude below a certain limiting value. The equation of motion of the slider is of the general

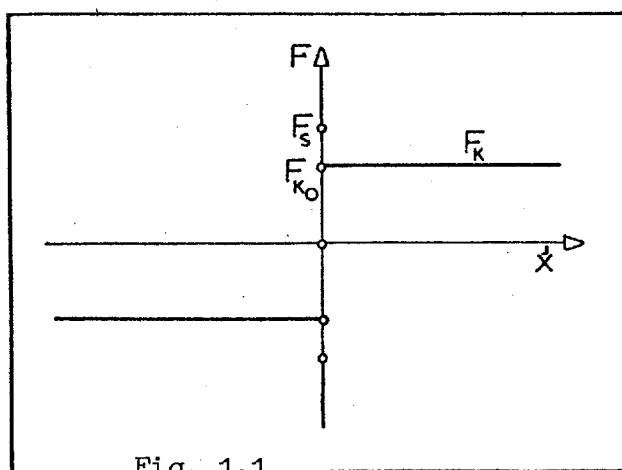


Fig. 1.1

form:

$$m\ddot{x} + c\dot{x} + kx - F_K = 0 \dots (1.1)$$

for $c = 0$ i.e. no additional damping exists in the system (fig. 1.2,a).

The same basic model has been used by Merchant [29], Sinclair [30], Bristow [2], Broadbent [31], Fleischer [32] and Niemann and Ehrlenspiel [32], while Blok [16], Singh and Push [33, 34], Kemper [26], Moisan [35], Matsuzaki [36] used the general form of equation 1.1

($c \neq 0$ fig. 1.2,b). This equation is valid only when $\dot{x} \neq v_0$ and therefore it does not describe the motion of the slider during stick (in case of stick-slip sliding of the bodies). It can be solved analytically and the obtained solutions are harmonic functions of time, which means that the motion of the slider during slip is simple harmonic.

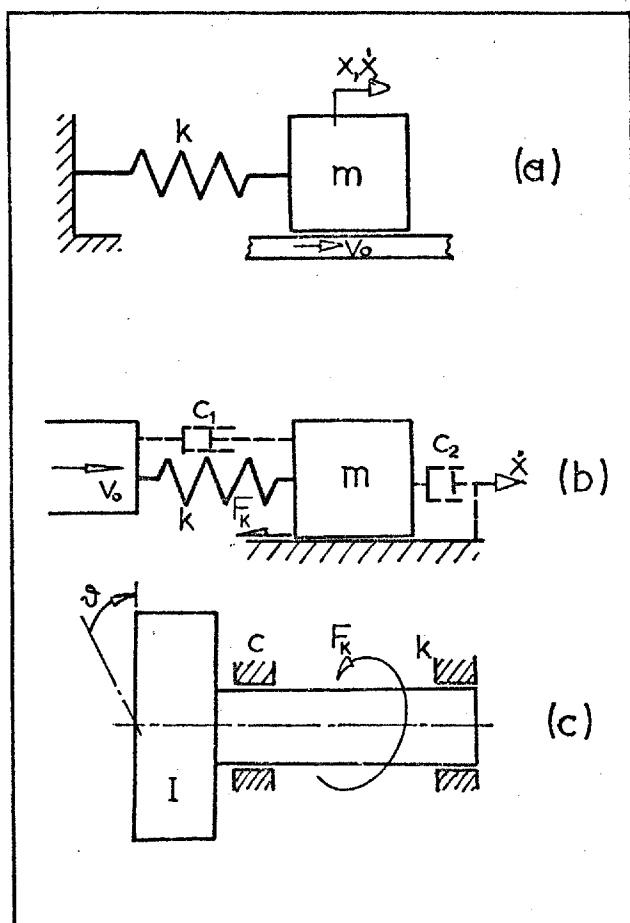


Fig. 1.2

Bristow [2] emphasised the fact that this mathematical representation is oversimplified and more realistic models could give far better results. Also Sinclair [30] concluded that frictional self-excited oscillations can only be produced when an inverse variation of the coefficient of friction with velocity occurs. In Broadbent's model (where $F_s = F_K$) frictional oscillations cannot be maintained in the system (Thomas [7]) and a geometrical theory was employed, explaining the oscillation occurrence in the system as caused by large clearances in the joints of the loading mechanism. The friction-velocity characteristic was used only in a qualitative form in order to explain how an oscillation can be excited in a stable system at rest.

The variation of the static coefficient of friction with time or the sliding distance was studied [32], although such a variation does not agree with the assumed mathematical model, and the importance of the surface roughness and hardness were underlined.

Bowden, Leben and Tabor [37] tried to analyse frictional oscillations in terms of the physical processes involved. Starting from the basic idea that the exact behaviour of sliding bodies depends on the relative physical properties of them, and particularly on the melting point, they classified frictional oscillations into three major categories based on the criteria of hardness and melting point of the materials comprising the frictional pair. Thus suggested that the surface temperature of sliding metals during the slip phase of stick-slip is surprisingly high and may easily reach the melting point at the surfaces, although the mass of the metal remains quite cool. In fact although temperature flashes do exist on the surfaces during slip, their level is much lower than the melting point. Therefore welding of the surface asperities cannot be, (or at least in itself cannot be alone) the cause for initiating frictional oscillations [16].

The stability conditions for the system were studied in detail [16,26,33,34,35,36] and, based on Rankin's theory of the elastic range of friction [3], the idea of frictional microvibrations was conceived [16]. These result from small scale stick-slip due to the material elasticity and rigidity, depending on the internal damping of the material.

Analog computer simulation of the model [34,36] showed good agreement with experimental results.

The simplest forms of models with linear friction-velocity characteristic were proposed by Gemant [38], Michel and Porter [19] and Jania [39]. Self-excited oscillations were attributed to a negative slope of the characteristic

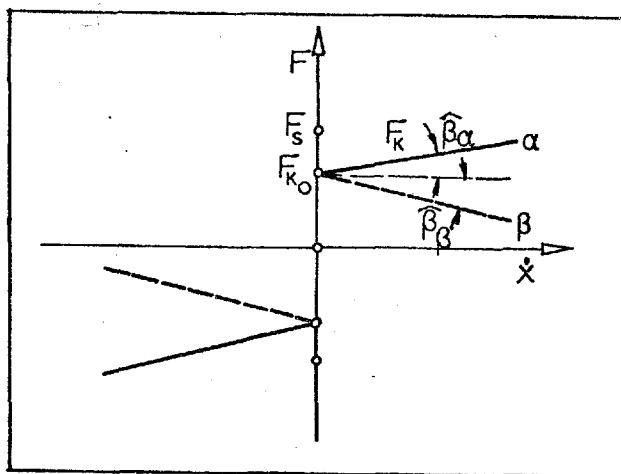


Fig. 1.3

rather (fig. 1.3, negative angle β) than to a positive difference $\Delta F = F_s - F_k > 0$. The abrupt drop in the coefficient of friction as soon as motion starts was considered by Gemant as a secondary phenomenon due to abrasion of the tips of the surface asperities. Michel and Porter admitted that the form of characteristic assumed

(fig. 1.3, α) is only a crude approximation to the real nature of the friction-velocity characteristic. This is not readily definable but, nevertheless, it serves to put the problem on a quantitative basis. The equation of motion was transformed to the form of equ. 1.1. and solved analytically and by means of a differential analyser. A similar mathematical technique was used also by Lauer [40], who obtained the same results, and presented them in the form of phase-plane diagrams. Basford and Twiss [42] developed a similar theory for brake oscillation based on a model having a characteristic

of the form of fig. 1.3.β. The problem is treated statistically and supposed that the frequencies of frictional oscillations are distributed following a Gaussian curve. A relation was derived between the probability of noise and the physical characteristics of the system. In a similar study Jarvis and Mills [43] presented a theory in which the geometry of the system is the predominant factor. They asserted that variation of the coefficient of friction with velocity alone is insufficient to cause oscillation, and the stability of the system is dependent on the manner in which the motions of the components are coupled in that particular system. The friction-velocity characteristic employed was simple ($F_s = F_{K_0}$ fig. 1.3,β) but the final form of the obtained equation of motion was very complicated due to the coupled modes of oscillation. It was solved approximately by the method of slowly varying amplitude and phase, developed by Kryloff and Bogoliuboff [44]. According to this theory it can be shown that even with constant coefficient of friction ($F_s = F_K = \text{constant}$) frictional oscillations can exist depending on the geometry of the system. This is due to nonlinearities introduced in the equation of motion by the geometrical characteristics of the system. Thus the equation became nonlinear independently of the friction-velocity characteristic.

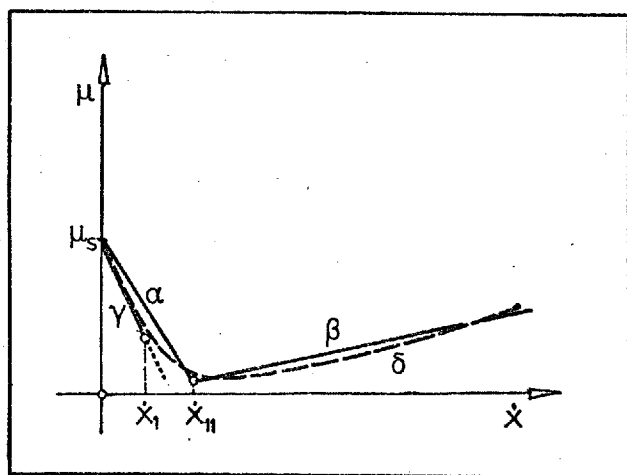


Fig. 1.4

Pavelesku [45] and Häussler and Wonka [46] approximated the realistic friction-velocity characteristic δ (fig. 1.4) by a linear one γ (for $\dot{x} < \dot{x}_I$, Pavelesku) or by the linear in parts $\alpha\beta$ (α for $\dot{x} < \dot{x}_{II}$, Häussler-Wonka). The equation of motion in case of an $\alpha\beta$

characteristic can be treated only numerically (point-by-point solution). Results showed quite good agreement with the experimental traces.

Matsuzaki and Hashimoto [47] studied the stick-slip phenomenon as it is met in hydraulic driving mechanisms. Providing the mechanical construction of the apparatus is such that there is high stiffness and no clearances then slider oscillations produce pressure fluctuations in the hydraulic cylinder instead of spring force fluctuations. The system is represented in fig. 1.2.b with the spring removed and dashpots c_1, c_2 considered not as dampers but as hydraulic pistons for forward and reverse movement. The equation of motion is equ. 1.1 with $k = 0$ and F_K equal to the force produced by the hydraulic mechanism. The characteristic of fig. 1.3.α was used and the solutions of the equation were analysed on a topological plane $\{v, p_m\}$ where p_m is the mean pressure difference in pistons, c_1 and c_2 .

A detailed analysis of stick-slip giving complete information for the prediction of the motion of frictional pairs in practical systems was presented by Derjaguin, Push and Tolstoi [15,23]. The necessary condition for sticking; the dependence of static friction on the duration of stick and the critical velocity were discussed exhaustively. Simple stability criteria and relations for quick practical calculations were formulated for the designer. What is missing in this work is the realistic examination of actual stick-slip as it is met in practice. This was attempted by Sampson, Morgan, Reed and Muscat [17] who tried to solve the equation of motion using a friction-velocity characteristic which was derived experimentally. They were the first to notice a bifurcated $\mu = \mu(v)$ characteristic and concluded that the friction does not return to its static value instantaneously after the motion ceases, and also that the friction is not determined by the velocity alone, but rather by the velocity and the past history of the motion (memory behaviour).

1.2.1.2. More Sophisticated Theories

Three kinds of friction-velocity characteristics are met in the following theories:

a. Models with static friction variable with the time at rest, or kinetic friction variable with the acceleration:

$$\{\mu_s = \mu_s(t_s) \dots\} \text{ or/and } \{\mu_K = \mu_K(\ddot{x} \dots)\}$$

b. Models with bifurcated and/or nonlinear characteristic $\mu = \mu(v)$.

Štěpánek [24] and Elyasberg [41] studied the frictional self-excited oscillations as they appear in machine tool slideway practice. In an effort to explain the difference between steady state and transient friction-velocity characteristic they used a model sensitive to velocity, acceleration and duration of stick:

$$\{\mu_s = \mu_s(t_s) \text{ , } \mu_K = \mu_K(\dot{x}, \ddot{x})\}$$

The effect of acceleration is introduced into the equation of motion as a correction factor (fig. 1.5 from $\{\tau, \epsilon, \alpha\}$ the corrected $\mu = \mu(t_s, \dot{x}, \ddot{x})$ is obtained). This means that the kinetic coefficient of friction is a linear function of acceleration. The derived equations of motion were solved analytically but the form of the solutions is very complicated and in practice simplified versions only can be treated. Juričić [48] and Brockley, Cameron

and Potter [49] had slightly simpler models (insensitive to acceleration). In both cases the models are represented by curves τ, α (fig. 1.5) where τ is an exponential function of t_s of the form:

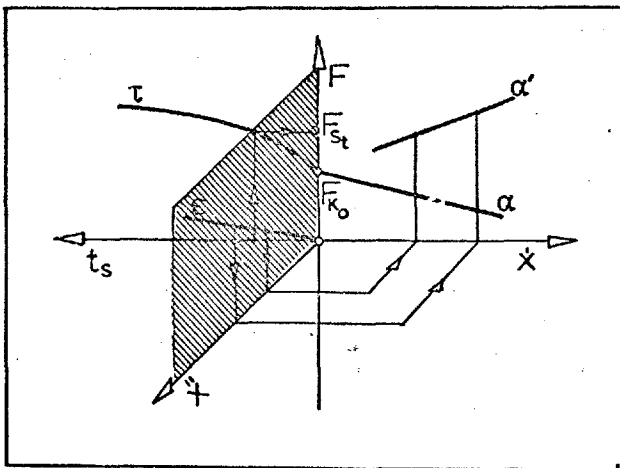


Fig. 1.5

$$F_{s_t} = F_{s_m} + (F_{s_\infty} - F_{s_m})(1 - e^{-c_m t_s}) \dots \dots \dots (1.2)$$

and α the kinetic friction as a linear function of velocity:

$$F_K = F_{K_0} + c \cdot \dot{x} \quad \dots\dots\dots (1.3)$$

Theoretical and experimental results showed good agreement. A fact which readily explains why these two theories are so popular.

In 1949 the first attempt was made by Dudley and Swift [50] to examine the dynamics of frictional relaxation oscillations with a frictional coefficient varying with the sliding speed in a nonlinear way. They tried to ascertain to what extent observations regarding such oscillations can be explained in terms of mechanics as applied to the accepted conditions of operation. The non-linear differential equation of the general form of equ. 1.1 was obtained but with F_K regarded as an empirical function of velocity $\dot{x}$. It was solved by means of the Liénard's graphical construction and some cases were discussed qualitatively. The same topological technique was also used by Hunt, Torbe and Spencer [51] who drew experimental phase-plane trajectories after an experimental derivation of the $\mu = \mu(v)$ curve. It was found that no unique curve of this kind could be obtained.

The tentative hypothesis was made that μ depends in some way, not clearly understood, on acceleration as well as velocity (see also [24,41]). The grapho-analytical technique used constitutes a real improvement because it permits the derivation of experimental trajectories and the study of the phenomenon, independently of the complexity of the function $\mu = \mu(v)$. Banerjee [52] trying to prove that the value of the static friction is without significance for frictional self-excited oscillations, gave an entirely kinetic friction-dependent analysis of stick-slip motion. The friction velocity characteristic assumed as (fig. 1.6.a):

$$F_K = F_{K_0} - a\dot{x} + b\dot{x}^2 \quad \dots\dots\dots (1.4)$$

A characteristic length L was introduced in the equation of motion which

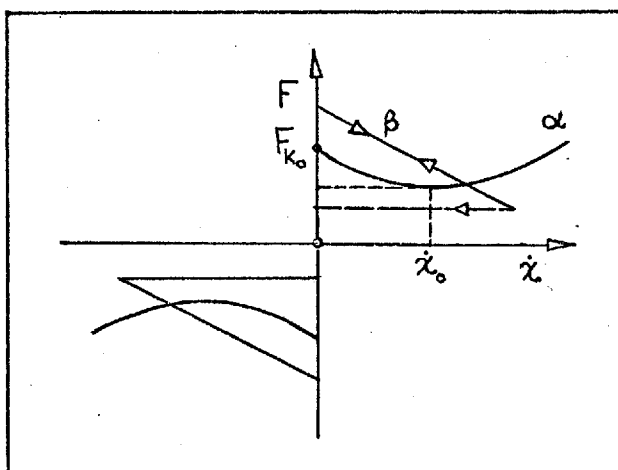


Fig. 1.6

represents the critical slip distance as conceived by Rabinowicz [53]. The equation was solved by the method of slowly varying amplitude and phase [44], although the non-linearity of the system is not weak (see §2.3.1.2.). Stewart and Hunt [54], on the other hand, attempted to use dimensional analysis techniques

to obtain generalised results but they did not derive satisfactory relations except for a very narrow range of sliding velocities. They found experimentally that during stick, a definite very low relative sliding speed existed ($5 \div 15 \mu\text{m/s}$) and suggested that if the feed speed were to fall below these values, there would be no stick-slip.

Ziembra [55] introduced the elasticity and visco-elasticity properties of the bodies in frictional contact into the stick-slip study in order to explain the effect of the mechanics of the system. The function $\mu = \mu(v)$ was derived theoretically as a hysteresis loop attributed to the internal friction hysteresis of the materials. Similar traces were found also by Kato, Yamagushi and Matsubayashi [56,57] who studied the stick-slip phenomenon as it appears in machine-tool slideways. Another bifurcated model (fig. 1.6.β) was proposed by Bell and Burdekin [28], but, for the sake of simplicity in the solutions, the model was linearised by using only the upper straight part of the characteristic fig. 1.6.β. The equation was treated analytically and dynamic friction force-velocity characteristics were obtained experimentally. They observed that although μ_s is affected by

the stick time t_s , the amount $\Delta\mu = \mu_s - \mu_K$ is not.

1.2.1.3. Realistic theories

Under this heading, works either containing models with friction-velocity characteristic derived experimentally, or studying the form of frictional oscillations as they appear in practice, are classified.

Kosterin and Kragelskii [9] tried to complete an older theoretical work by Ishlinskii and Kragelskii, in which frictional oscillations were assumed to be caused solely by variable static friction with stick time, independent of the value or variation of the kinetic coefficient of friction with sliding velocity. The friction-time-velocity characteristic used is that of fig.

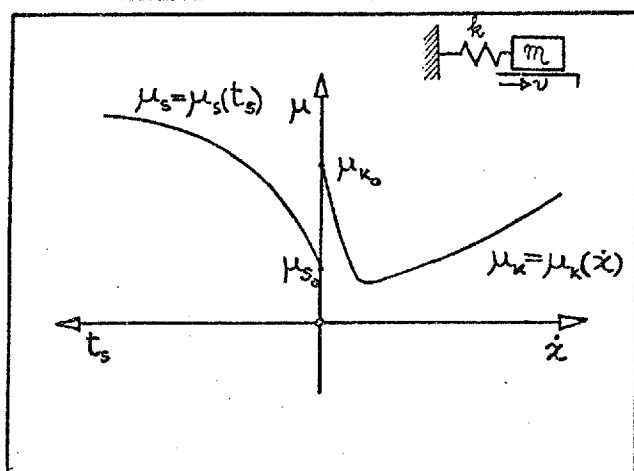


Fig. 1.7

1.7 and the equation of motion is:

$$m\ddot{x} + \varphi(\dot{x}) + kx = 0 \quad \dots (1.5)$$

where $\varphi(\dot{x})$ is the nonlinear factor of the equation. The equation was solved topologically and the effect of several parameters was studied. The same model and similar methodology was also used by Raizada [58] and Watari and Sugimoto [59]. Raizada obtained

experimentally a friction-velocity characteristic $\mu_K = \mu_K(\dot{x})$ and solved topologically and numerically equ. 1.5 where $\varphi(\dot{x})$ was substituted by a linear function $\varphi = \varphi \{ \mu_K(\dot{x}) \}$. The effect of stick time t_s was omitted, because it was correctly supposed that for medium range velocities v and stiffnesses k , the order of magnitude of t_s is such that it can be assumed that $\mu_s(t_s) \approx$ constant. Exactly the same technique and basic assumptions were used by Watari and Sugimoto who emphasised the importance of the topological

characteristics of the problem. Doubtlessly Raizada, Watari and Sugimoto produced the most important contributions in the study of stick-slip phenomenon.

Rabinowicz [53] produced the hypothesis based on previous experimental work, that the friction force depends on the previous history of the experiment.

When the velocity changes abruptly, a sliding distance of 10^{-3} cm is necessary before the coefficient of friction reaches another value. Thus in cases where the velocity is changing rapidly it seems reasonable to express the coefficient of friction as a function of the average sliding velocity of the previous 10^{-3} cm rather than as a function of the instantaneous velocity. Accordingly it is impossible for the stick-slip phenomenon to exist in case where the slip distance (jump) is less than 10^{-3} cm.

Simkins [60] examined the relation $\mu_s = \mu_s(t_s)$ and his experiments revealed that such a relation does not in fact exist. It was also stated that the difference $\Delta\mu = \mu_s - \mu_K$ is not greater than zero and the results of previous investigators where $\Delta\mu > 0$ were ascribed to instrumental errors. It seems that the creep mechanism of contact, explained to some extent by Voorhes [61], played a significant role in that case and produced these contradictory results. The observed "microslip" mechanism of sliding (jumps $0.1 \div 0.8 \mu\text{m}$) fits quite well with the order of magnitude of Rabinowicz's criterion of critical distance ($2 \mu\text{m}$).

Finally Lenkiewicz [62], Anonymous [63], Theyse [64], and Rabinowicz [1] studied and described the morphology of frictional self-excited oscillations under dry or lubricated conditions and generally accepted the dropping friction-velocity characteristic as the main cause of the phenomenon. Schindler [65] gave a general theoretical interpretation of

stick-slip motion based on previous work (Štěpánek [24], Elyasberg [41].) Two types of friction-velocity characteristics were found in existence, one for steady-state conditions (static characteristic) and another depending on the acceleration (dynamic characteristic).

1.2.2. Unconventional theories

1.2.2.1. Electrical theories or analogies

Schnurmann and Warlow-Davies [66,67] produced a most unusual theory for frictional self-excited oscillations. It was observed that the falling friction-velocity characteristic resembles the characteristic of electrical discharges in devices with practically infinite resistance below a definite breakdown potential (voltage-current characteristic). It was thus suggested that due to contact electrification, an electrostatic component of the frictional force appears when the boundary layer has dielectric properties. Cycles of charging and discharging produce cycles of sticking and slipping, for sufficiently low sliding velocity. If the electrostatic interpretation of the friction characteristic is correct, no jerking can be expected when two naked metal surfaces are in frictional contact. This is not in agreement with experimental evidence produced by Bowden and Young [68] who studied the behaviour of clean metal surfaces under high vacuum and found that stick-slip increases as vacuum increases (i.e. contamination of surfaces decreases). Schnurmann's theory although not verified experimentally and rather abolished nowadays, made much sense and the frictional electrification of surfaces is called after Schnurmann, Schnurmann's effect.

Ristow [69] studied self-excited frictional oscillations using electrical analogies. It was observed that the friction-velocity characteristic can be analysed in three components a) the Coulomb, independent of velocity; b) the hydrodynamic, proportional to velocity and c) the boundary, falling according to a hyperbolic law. The same happens in electricity with ohmic,

inductive and capacitive loads in respect of voltage. Thus the following analogies were established

$$F_K \{F, v, m, k, c\} \triangleq E \{V, I, L, R, 1/c\}$$

or

$$F_K \{F, v, m, k, c\} \triangleq E \{I, V, c, 1/R, 1/L\}$$

From the results previously obtained good agreement between the actual case and the electrical analogy was obvious.

1.2.2.2. Multi-degree of freedom systems

Lisitsyn and Kudinov [70,71] produced an analysis based mainly on the theoretical facts that the actual mechanical systems are systems with an infinite number of degrees of freedom. For simplicity of mathematical analysis, they can be considered as systems with a finite number of degrees of freedom, where motion in one of the vibration domains of the system inevitably causes motion in the other domains. They produced, in order

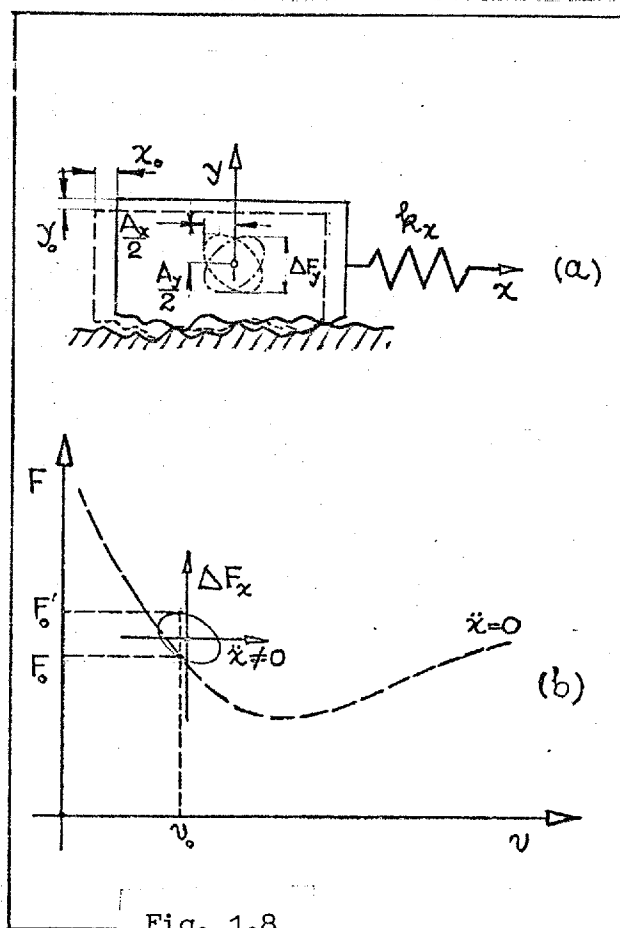


Fig. 1.8

to study frictional self-excited oscillations, a model having two degrees of freedom (fig. 1.8.a) The vertical movement of the slider was assumed to be produced by the surface irregularities of the two bodies in contact.

The motion of the slider is described in that case by the system:

$$\begin{aligned} m\ddot{x} + c_x\dot{x} + k_x x &= \xi y \\ m\ddot{y} + c_y\dot{y} + k_y y &= \xi x \end{aligned} \quad \dots (1.6)$$

Due to the inertia of the system

the forces are out of phase and the calculated amplitude ratio is $A_x/A_y = 20 \div 60$, which explains quite reasonably why oscillations in the vertical direction have never before been taken into consideration. The motion of the slider is the resultant of coupled oscillations of identical frequency but out of phase with each other. The trajectory of such a movement is an ellipse, while another ellipse represents the additional force between the bodies. The fluctuating force ΔF_x produces considerable errors in calculations of the static characteristic of the system, as it is indicated in fig. 1.8.b because for each value of velocity (e.g. v_0), two values $F_0 \neq F'_0$ of frictional force correspond. Therefore the use of static characteristics for analysing transition and self-excited oscillation processes cannot be justified.

Tolstoi and Grigorova [72,73] investigated the effect of self-excited or externally induced high frequency oscillations or impulsive normal forces on sliding in general and, particularly, the stick-slip process. The molecular forces which were assumed to produce the frictional nonlinearities, were measured experimentally and found to decrease with the separation of the surfaces. It was also shown that both the negative slope of the friction-velocity characteristic and the frictional self-excited oscillations are closely associated with the freedom of normal displacement of the slider. Whenever the latter is absent the force of contact friction becomes practically independent of the relative velocity of sliding and stick-slip disappears. A first critical velocity exists, below which no self-excited oscillations can be maintained, as was also found by Burwell and Rabinowicz [74]. Similarly Dolbey [75] evaluated and assessed the effects of normal characteristics of plain slideways and found that friction changes with the separation of surfaces, becoming approximately zero for separation of the order of $1\mu\text{m}$. Experimental traces showed clearly the existence of frictional microvibrations

as they were predicted by Tolstoi. The influence of the stiffness in the vertical direction on the frictional oscillations was studied by Elder and Eiss [76] who found that an increase of the normal stiffness produces a decrease of the stick-slip amplitude, but the phenomenon is less sensitive to stiffness changes than to damping changes in the vertical direction (Tolstoi [72]).

1.2.3. The reverse phenomenon: Externally induced vibrations

The behaviour of a frictional pair under externally induced vibrations is of equal interest as the self-excited oscillation. In many cases a parallel study produced remarkable results.

The existing literature can be divided in two main parts: Works in which the frictional pair was considered as:

- a: Rigid body system
- b: Elastic bodies in contact under tangential forces.

Den Hartog [77] and Nishimura, Timbo and Takano [78] studied forced vibrations of a single degree of freedom system affected by purely Coulomb friction and solved the equations of motion either analytically or numerically. Singh and Mohanti [79], using analog computer simulation techniques on a single degree coulomb model, showed that the critical velocity of the frictional oscillations was considerably increased when the frequency of fluctuations, or that of impressed force, resonates with the natural frequency of the elastic system.

The contact problem of elastic bodies under tangential force was at first solved theoretically by Mindlin and Deresiewicz [80,81]. Mason and White [5, 82, 83] found that no wear is produced due to normal force fluctuation and that all the wear observed is due to tangential sliding. It was also found that as the length of slide is reduced there is a threshold

of motion for which there is no gross slip and very little wear (region $F_T < N \cdot \mu_s$). Investigation of the gross slip region based on Mindlin's theory showed that the specimens reciprocate over the same asperities many millions of times until the material finally becomes fatigued and breaks off.

Johnson [84,85] investigated, mainly experimentally, the microdisplacements between two bodies in contact under steady or oscillating tangential forces. The quantitative results provided considerable support for Mindlin's elastic theory. Measurements of the energy loss revealed that for small amplitudes of oscillating force, the theoretically predicted infinite stress is accommodated by predominantly elastic distortion of surface asperities.

Goodman and Bowie [86], Klint [87] and Halaunbrenner and Sukiennik [88] studied the damping at elastic contacts of spherical or cylindrical specimens. It was shown that within the no-gross-slip region there is a well defined region at the onset of tangential displacement, within which a primary elastic deformation is indicated. Energy dissipation studies indicated that in this region the behaviour is essentially viscoelastic. At amplitudes below this limit no discernable wear was observed, and the damping arises from internal damping of the material.

1.2.4. Effect of externally induced oscillations on self-excited ones

Significant work in this direction has been done by Fridman and Levesgue [89], Lenkiewicz [90] and Lehfeldt [91]. It was found that low or acoustic frequency vibrations reduce the coefficient of friction considerably. The decrease in the coefficient of friction was explained [89] by the breaking of the welded junctions caused by the force exerted on them by the acceleration of the wave form amplitude. This view however does not explain the low

power necessary to reduce static friction completely. The effectiveness of the induced vibration depends on the value of the parameter $A\omega$ [90] and the mean sliding velocity. Similarly Godfrey [92] found that the apparent kinetic friction decreased rapidly after the acceleration of vibration approached and exceeded the acceleration due to gravity. The measured reductions of electrical conductivity showed that the kinetic coefficient of friction is reduced apparently because load is reduced. In such cases fretting corrosion, metal fatigue and cavitation damage are very common.

Basu [93] and Gaylord and Shu [94] studied the influence of oscillating normal force on sliding in general or frictional self-excited oscillations, while Seireg and Weiter [95,96] and Banerjee [97] investigated the effect on frictional behaviour of forces acting in the direction of sliding, of an oscillating or impulsive nature. For impulsive tangential forces the coefficients of friction were found to be independent of the normal load and considerably increased. The coefficient of friction for gross slip under impulsive load, was found to be more than three times higher than the coefficient obtained under vibratory load. Analysis of the effect of forced vibrations on a stick-slip system showed [97] that high frequency oscillations are very effective in eliminating stick-slip and inducing steady sliding.

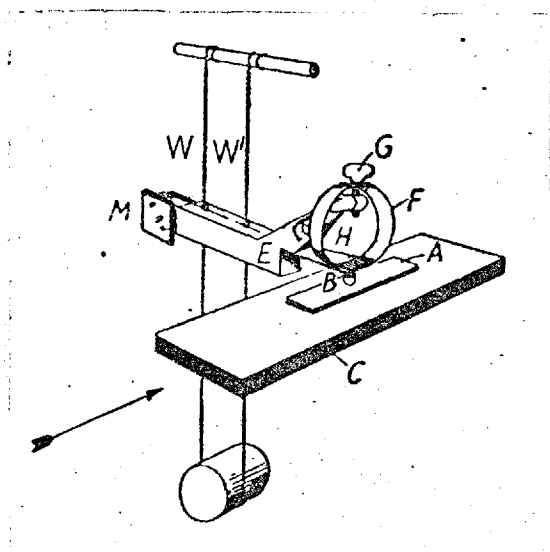
1.3. EXPERIMENTAL

In this section the most important apparatus designed for frictional oscillations studies are presented including those having some historic significance. For classification purposes they are divided in four groups, i.e. apparatus equipped with linear, rotational or reciprocating motion and apparatus designed for some special applied work.

1.3.1. Linear motion

The first successful apparatus was used by Bowden and Leben [12,37]

and it is known as "Bowden-Leben machine" (fig. 1.9). Bowers and Clinton [98] increased the sensitivity of the system considerably by replacing the



A,B: Specimens (ball on flat)

C: Base plate moving on rails in the direction of the arrow, by means of hydraulic pressure.

E: Stiff dynamometer support.

F: Ring spring (normal force application and measurement).

G: Loading screw

H: Horizontal spring, to keep B in plane MEG.

M: Mirror (displacement measurement).

W,W': Piano wires suspension of E.

Fig. 1.9

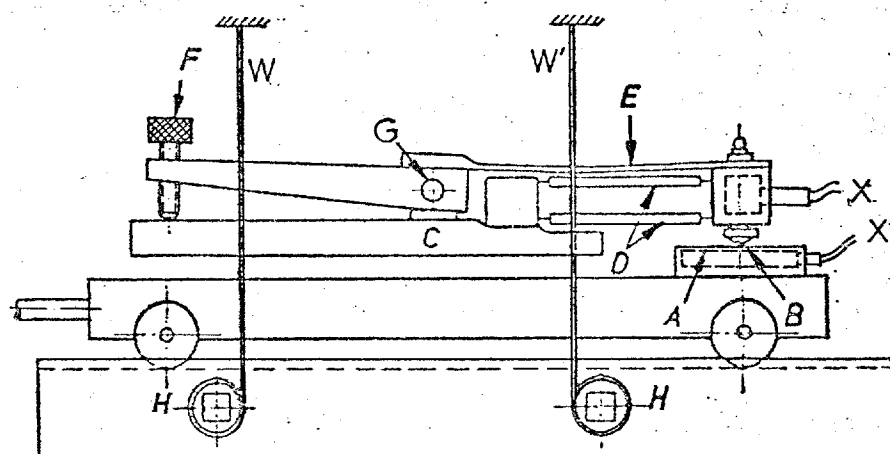


Fig. 1.10

A,B: Specimens (ball on flat)

C: Rigid frame

D: Strain gauges (measurement of load)

E: Loading leaf spring

F: Loading screw

G: Pivot

W,W': Piano wire support of frame C

X,X': Heating-cooling devices (for localized heating of the specimens).

H: Piano wire tensing spindles.

optical deflection measuring device (light-mirror-scale) by an electromechanical transducer. A similar apparatus (fig. 1.10) was designed and used by Bristow [2,13,99] for determination of boundary friction, as a function of velocity, at low speeds. This arrangement was adopted in order to avoid twisting of the specimens, because it was thought that twisting of the surfaces relative to each other might produce increased adhesion between the surfaces. The system is heavily damped by piston-cylinder dampers, and deflections of the slider are measured optically (light-mirror-scale).

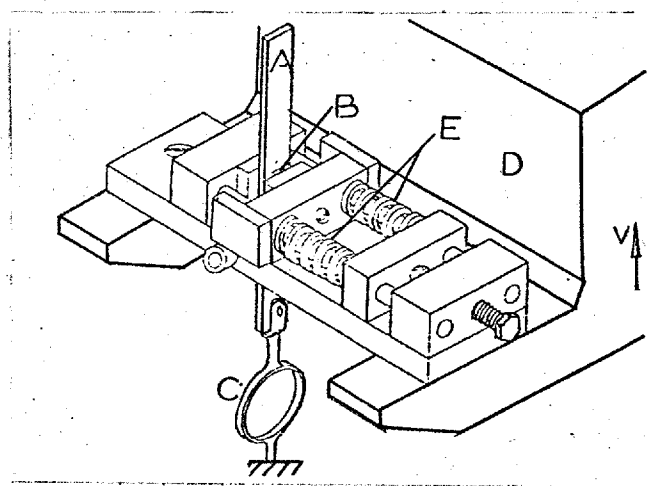


Fig. 1.11

A,B: Specimens (flat on flat)

C: Friction force dynamometer ring.

D: Tinius Olsen tensile machine movable head.

E: Normal force springs

and Rightmire [101].

Basford and Twiss [42,100], on the contrary, used a very simple system for their study (fig. 1.11) which gave reliable results at the very low sliding speeds for which it was designed. The frictional force was applied by means of a Tinius Olsen tensile machine, to the fixed head of which the other end of the strain ring C was fixed.

A special apparatus to operate at extremely low sliding speeds was designed by Heymann, Rabinowicz

The driving velocity of this apparatus is controlled by means of the weight G and the container I of viscous fluid in which the keel K of the carriage moves. Obviously for constant weight G the velocity depends on the viscosity of the fluid in the container I. With this apparatus extremely low velocities have been obtained easily and successfully. A similar

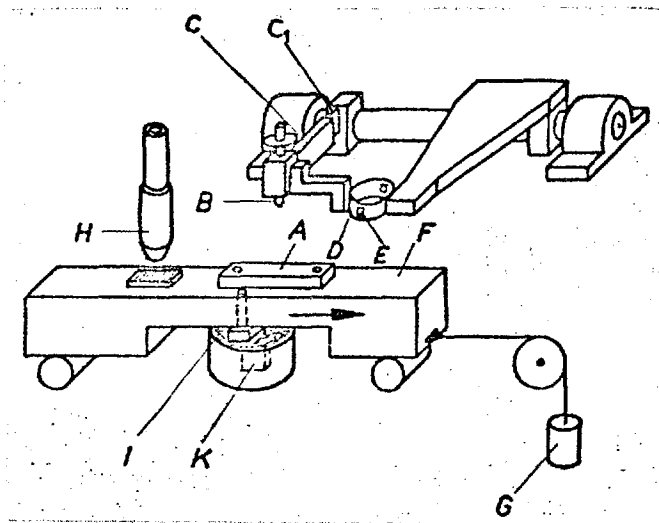


Fig. 1.12

- A,B: Specimens (ball on flat)
- C: Stiff arm carrying specimen B.
- C₁: Flexible plate (axis of rotation of arm C)
- D: Ring spring
- E: Strain gauges
- F: Lower specimen carriage
- G: Pulling weight
- H: Microscope (velocity measurement)
- I: Viscous fluid container
- K: Carriage keel

apparatus was the one used by Simkins [60] in which the driving force is produced by increasing slowly the amount of water contained in a cord suspended container, connected with the movable specimen (fig. 1.13). This technique of continuously increasing driving force presents radical differences from the usual constant velocity technique. Displacements were measured very accurately by an electro-optical displacement transducer.

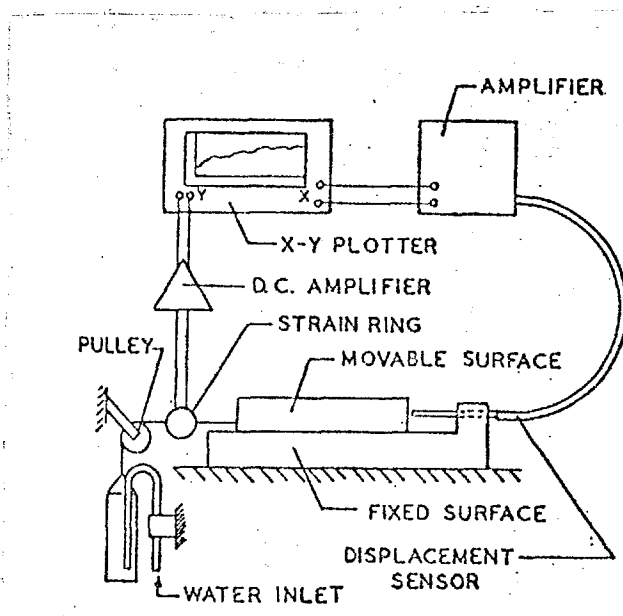


Fig. 1.13

Brockley, Cameron and Potter [49] made an apparatus of which the main details are shown in fig. 1.14. The drive was obtained by a recirculating ball-bearing unit which operated on a screw D. The stiffness of the cantilever loading beam and the magnetic damping were adjustable. Deflections of the slider from the equilibrium

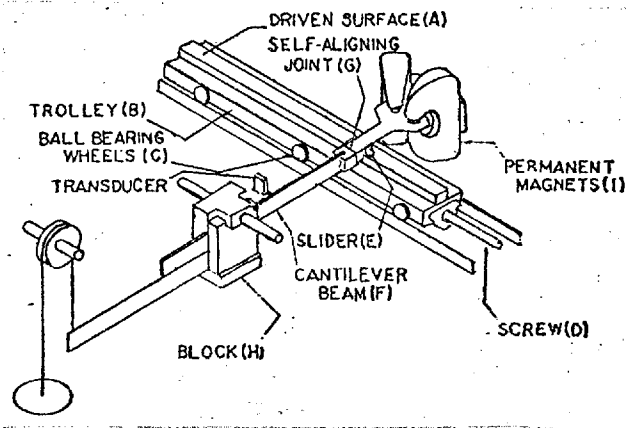


Fig. 1.14

stiffness on stick-slip phenomenon.

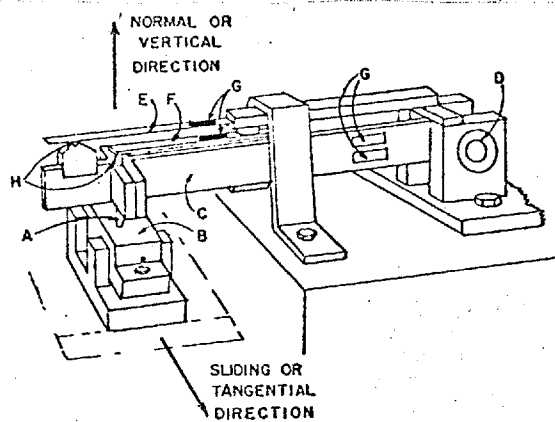


Fig. 1.15

position were measured by a differential transformer transducer.

Finally Elder and Eiss [76] developed an experimental apparatus in which the tangential and normal stiffnesses of the slider supporting mechanism were not coupled (fig. 1.15) in order to study the influence of normal

A,B: Specimens

C: Cantilever beam

D: Bearings

E,F: Upper and lower leaf springs

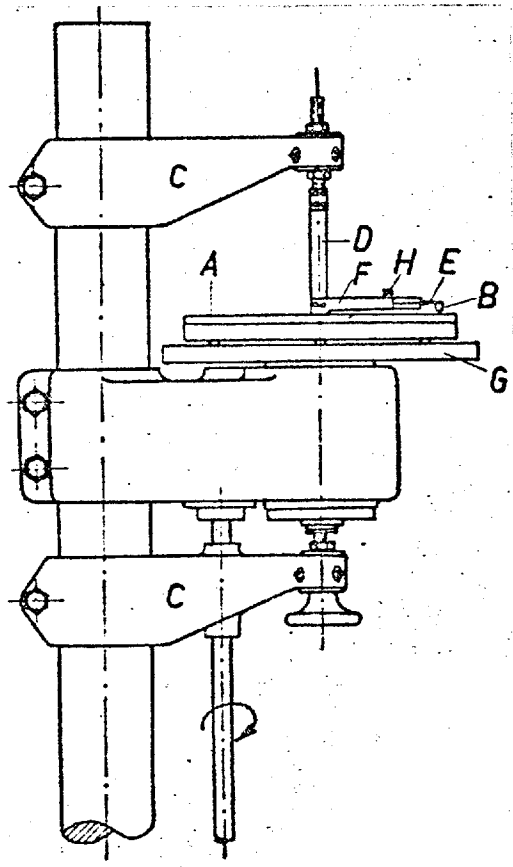
G: Strain gauges

H: Teflon pads.

1.3.2. Rotational motion

The first apparatus equipped with rotational motion and designed for frictional oscillations study is that of Morgan, Muscat, Reed and Sampson [17,18], which is basically a pin and disc friction machine providing low stiffness elastic support D for the pin and low speed rotation for the disc A.

A basically similar arrangement was also used by Dokos [102] and Kaidanowski [103]. Kaidanowski's was equipped with an electromagnet for introduction of controlled damping. Displacements were measured through lamp-mirror-camera optical system HGI (fig. 1.17).



- A,B: Specimens (pin or disc)
- C: Stiff support
- D: Friction measuring spring
- E: Loading leaf spring
- F: Rigid member connecting springs D,E
- G: Base plate
- H: Mirror (for displacement measurement).

Fig. 1.16

Watari and Sugimoto [59], had a system of the pin and ring type, subjected to rotational vibrations (fig. 1.18), and Tolstoi [104] used a friction apparatus especially designed (Tolstoi and Kaplan) for investigations of low speed friction with or without frictional self-excited oscillations (fig. 1.19). Vertical displacements were measured interferometrically, by noting the displacement of Newton rings formed between lens 12 and black-glass plate 13, through a microscope. The improved design of that apparatus permitted extremely successful work to be done and very interesting results were obtained.

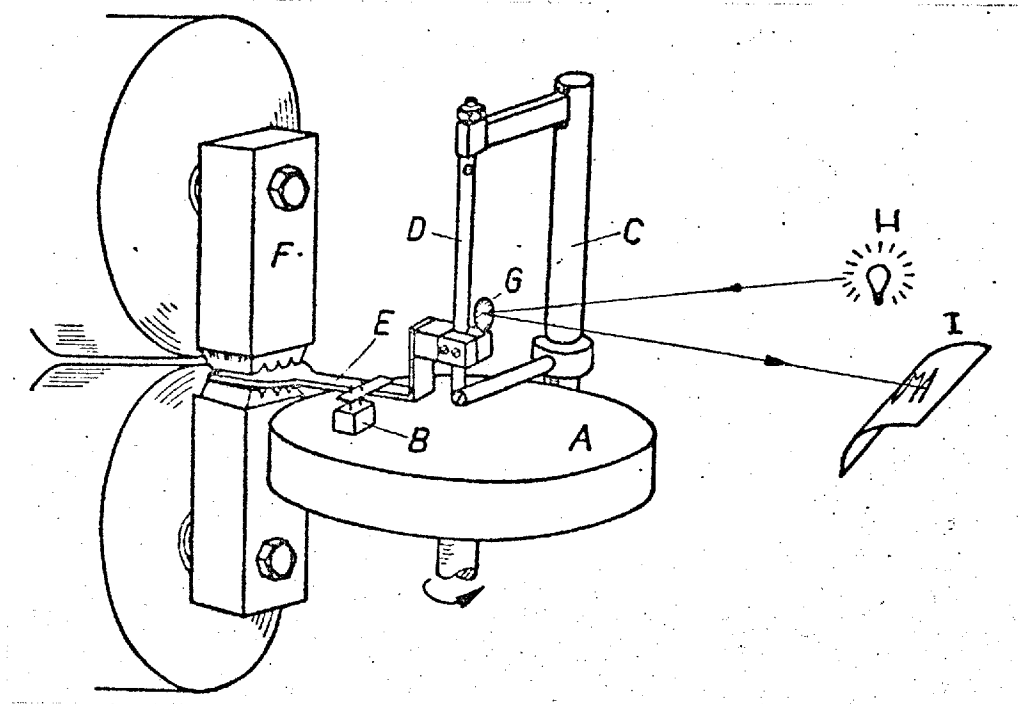
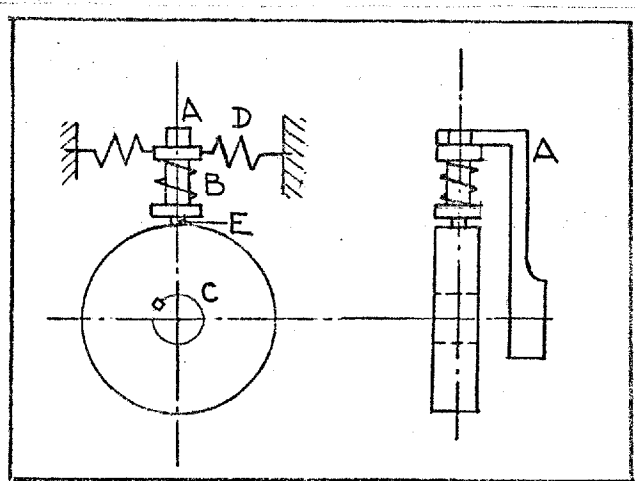


Fig. 1.17

- A,B: Specimens (pin on disc)
- C: Frame
- D: Torsional leaf spring (friction measurement)
- E: Damping plate and upper specimen (pin) support
- F: Electromagnet
- H,G,I: Light-mirror-camera displacement recording system



- A,B: Loading spring and base
- C,E: Specimens (pin on ring)
- D: Torsion springs

Fig. 1.18

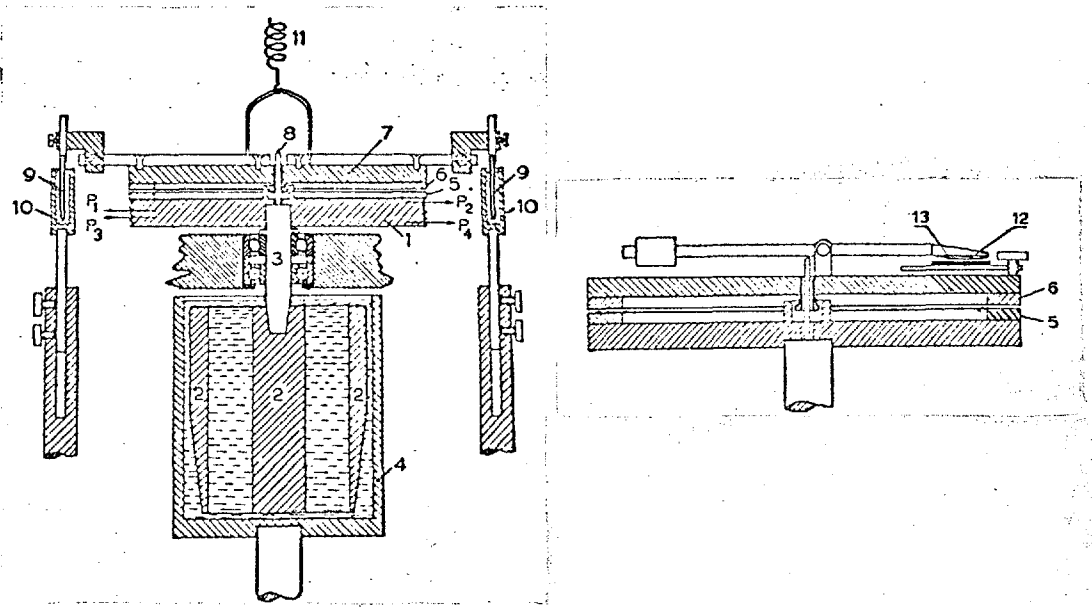


Fig. 1.19

P_1P_2 or P_3P_4 : Cou ple for clockwise or counter clockwise rotation

1 : Base plate (rotating)

2 : Damper for speed control

3 : Shaft

4 : Dashpot

5,6 : Specimens (ring on ring)

7 : Upper plate

8 : Vertical movement styl us

9,10 : Dampers for vertical movement

11 : Supporting-loading spring

12,13 : Interferometric measurement of vertical movements

1.3.3. Reciprocating motion

Sinclair [30] designed the apparatus of fig. 1.20 in order to study the friction of brake lining in reciprocating motion. Quite similar apparatus were also used by Lenkiewicz [90] and Pavelesku [105]. Niemann and Ehrlenspiel [32,106] described two apparatus (fig. 1.21 and 1.22) of the cylinder on flat type of friction machines (linear contact of surfaces). The frictional force in both cases is recorded by means of a special mechanical indicator, constructed by E. Tannert. The reciprocating motion is obtained by means of an eccentric drive.

Finally Simkins [60] used a reciprocating motion apparatus of very different type (fig. 1.23) where the displacement of the slider was measured by an improved optical system.

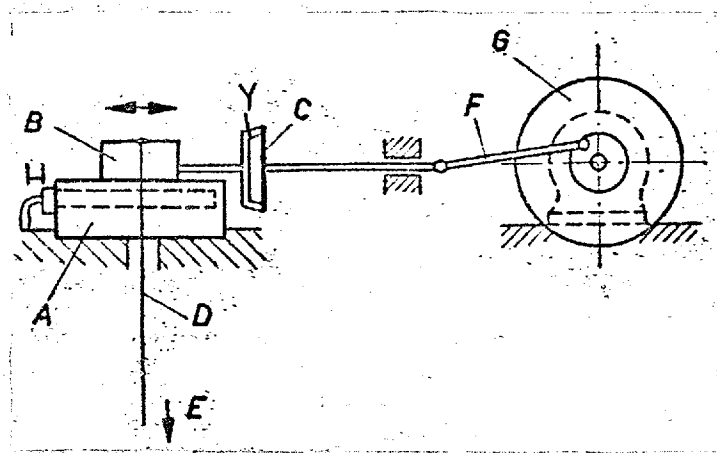


Fig. 1.20

- A,B: Specimens (flat on flat)
- C: Leaf spring (friction measurement)
- D,E: Normal load suspended on piano wire
- F: Connecting rod
- G: Motor flywheel
- Y: Stiff yoke on which spring C is mounted
- H: Heater-cooler

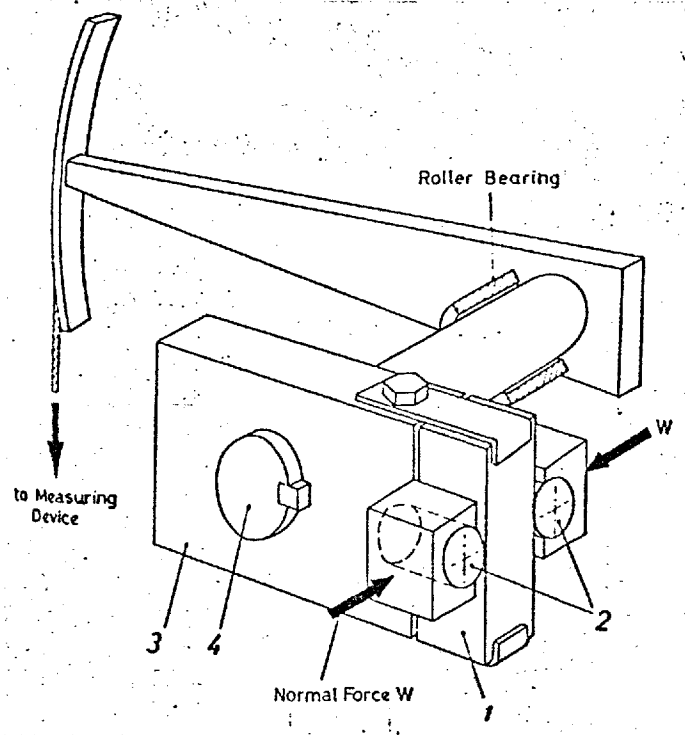


Fig. 1.21.

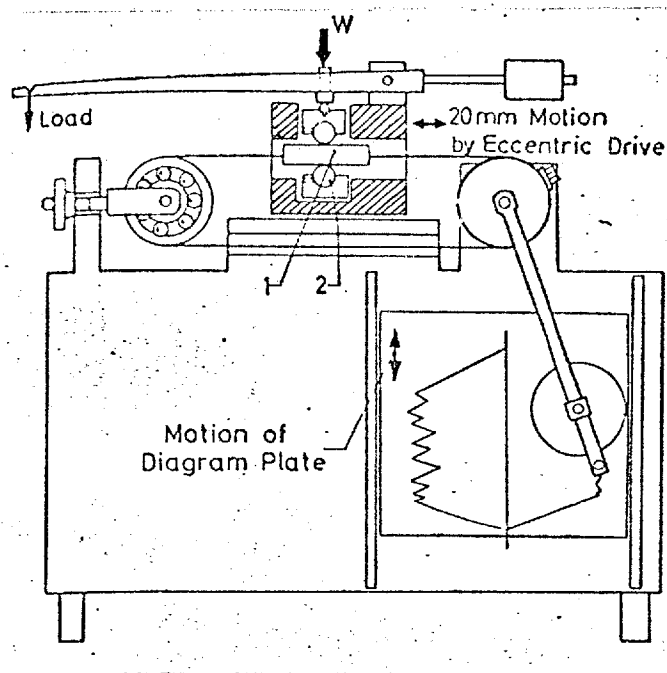


Fig. 1.22

1,2: Specimens (cylinders on flat).

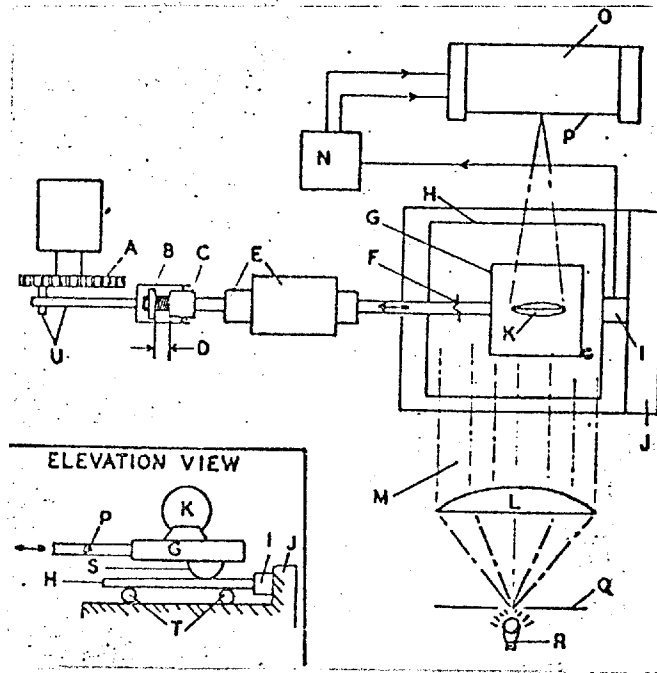


Fig. 1.23.

- A,U,B,C,D: Motor driven wheel, clearance adjustment and cross-head sliding member
- E: Linear ball bearing
- F,G: Oscillating block
- H: Lower specimen
- I: Force transducer
- J: Fixed support
- R,Q,L,M,K: Optical system
- O,P: Visicorder Light oscillograph
- S: Upper specimen
- T: Steel balls

1.3.4. Apparatus designed for applied work

With very rare exceptions these apparatus were designed for the study of frictional oscillations on machine tool feed drives, under realistic conditions of operation.

Elyasberg [41] carried out experimental work on machine tool tables using

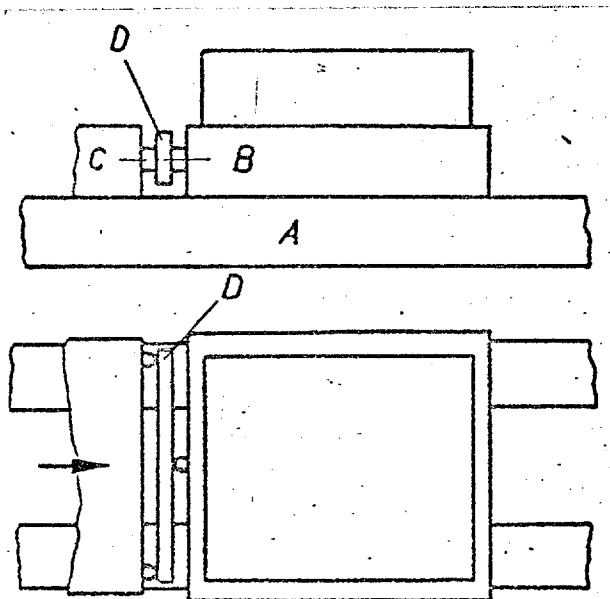


Fig. 1.24

- A,B: Specimens
- C: Driving slide
- D: Elastic link (leaf spring) between driving slide C and driven slide B.

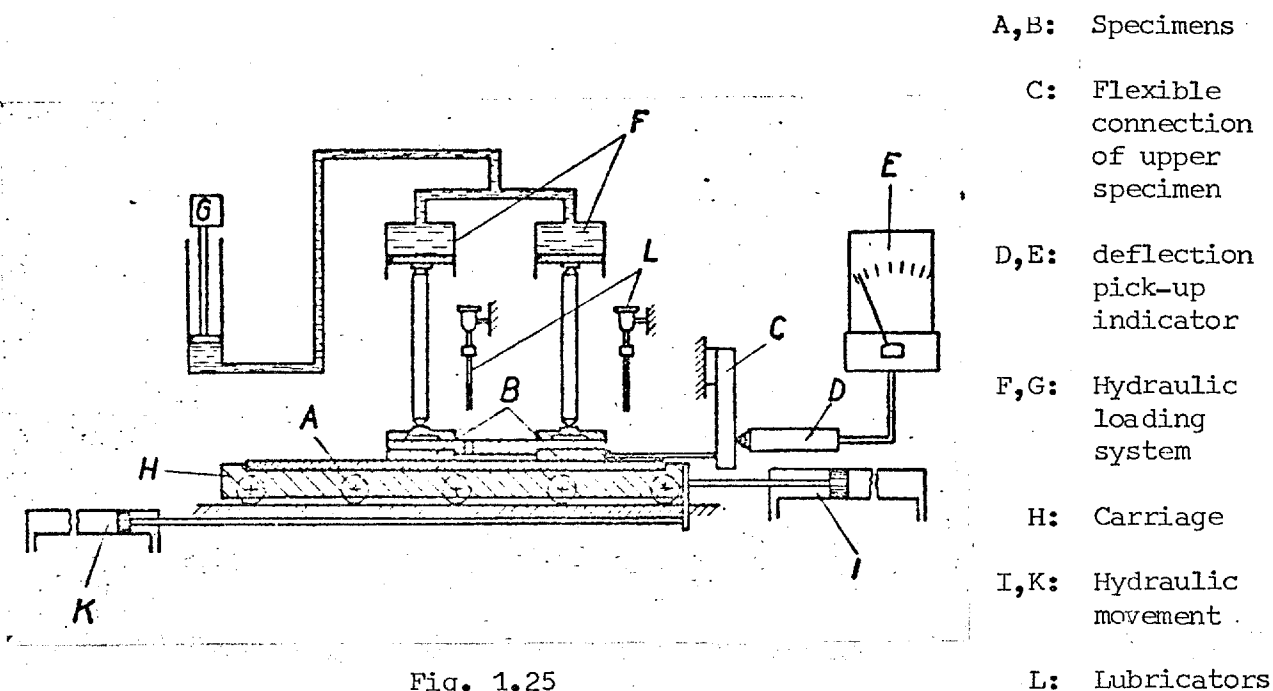


Fig. 1.25

the arrangement of fig. 1.24. The slide B carries pick-ups for displacement, velocity and acceleration measurements. A similar simple system was also used by Moisan [35], while P.E.R.A. proposed a more sophisticated apparatus for general friction studies, based on the same principles [107]. The "P.E.R.A. machine" have been also used by Birchall and Moore [108].

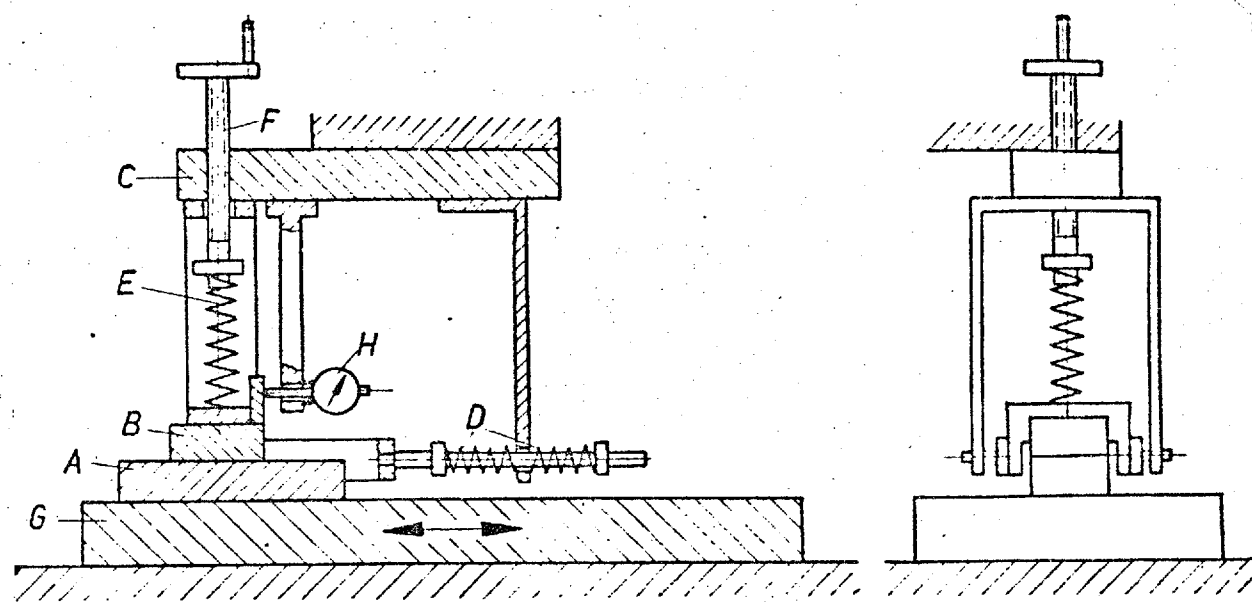


Fig. 1.26

The Shell Research Centre [63] used the apparatus of fig. 1.26 initially designed by Merchant for simulation of real machine - tool conditions conditions. In a later modification, the displacement measuring dial gauge was replaced by a more sensitive optical system. A similar system (fig. 1.27) was used by Fleischer [32] who studied boundary lubrication and related frictional oscillation phenomena.

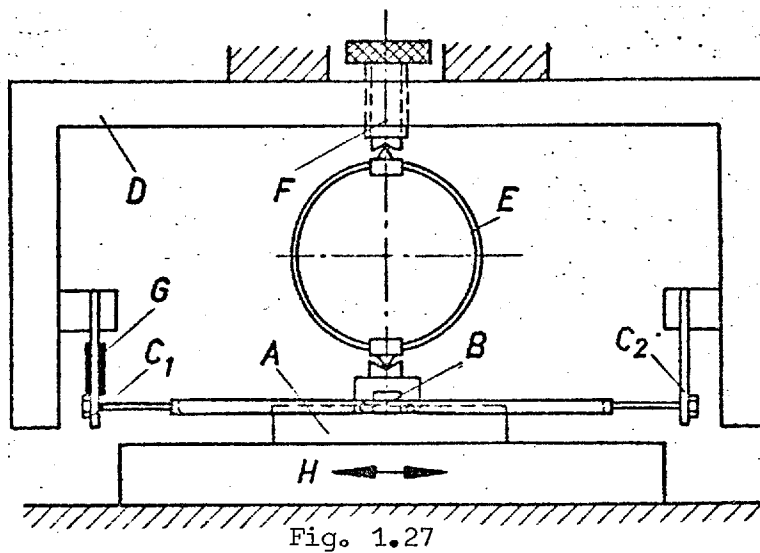


Fig. 1.27

A,B: Specimens

 C_1, C_2 : Flexible support of specimens B.

G: Deflection measuring strain gauges.

F,E: Loading screw, and spring

H: Base plate

D: Rigid frame

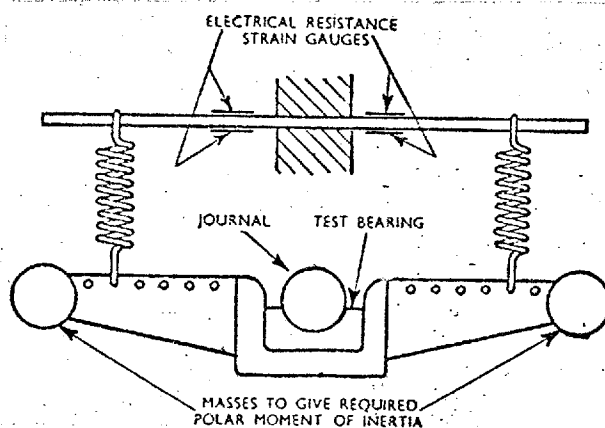


Fig. 1.28

Catling [25] used the arrangement of fig. 1.28 to simulate the operation of a textile drafting machine, while Pavelesku and Dimitrov [21,45] used a device based on the principle of bifilar suspension of the slider, similar to Fleischer's [32].

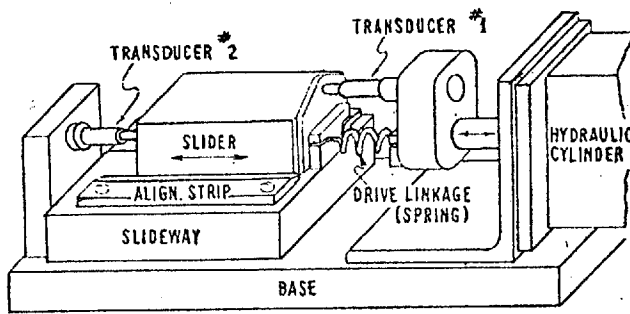


Fig. 1.29.

Voorhes [61] designed an apparatus based on an actual machine tool slideway system (fig. 1.29) and a similar but slightly simpler system was also used by Kato, Matsubayashi and Yamaguchi [56,57]. Similarly Bell and Burdekin [28, 109] constructed a rig

primarily for the study of closed loop drives of machine tool tables employing lead-screws as the transmission device. The rig (fig. 1.30) had the lead-screw replaced by a hydraulic cylinder as this made a more effective transmission element for slideway studies. Piezo-electric load washers measured the thrust

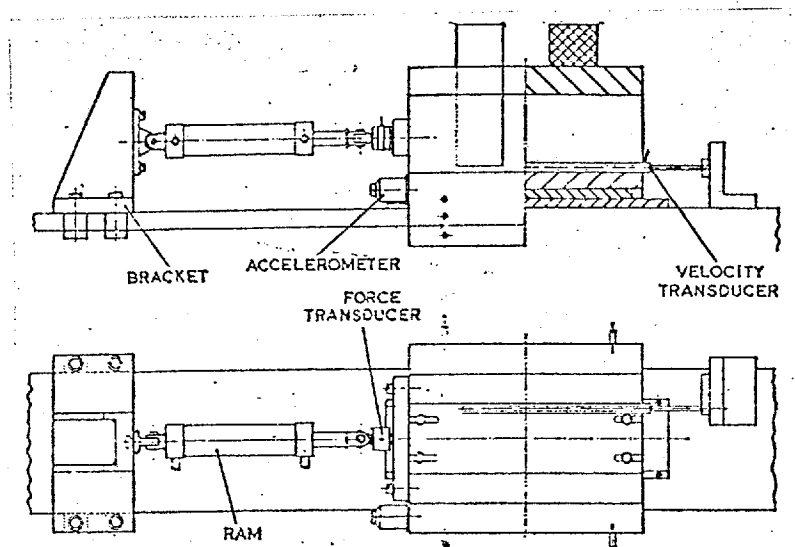


Fig. 1.30

delivered to the table, while the table velocity was monitored by the use of a permanent magnet linear tachometer.

Essentially the same rig was also used by Raizada [58] providing in addition external, shaped, viscous damping.

1.4. FRICTIONAL OSCILLATIONS IN APPLICATIONS

1.4.1. In Machine tool applications

Frictional oscillations as they appear in machine tool practice have been studied rather intensively, especially in recent years. Elyasberg [41] investigated the problem of self-excited oscillations on machine tool slideways and tried to simplify the equations governing the motion of the bodies, in order to establish a practical means of stick-slip calculation. Birchall

and Moore [108] studied in a more general way the friction and lubrication of slideways and the effect of various factors on stick-slip (velocity, load, surface finish, lubricant viscosity). Basu [97] examined the effect of oscillating normal load, and the advantages of using hydraulic preloading to improve the sliding conditions.

Lur'e, Levit and Osher [110, 111, 112, 113] emphasised the fact that there is no practical significance in increasing driving rigidity to decrease the amplitude of stick-slip because, to ensure stable movement over the entire range of speeds, the rigidity of the drive must be increased to such an extent that it is difficult to achieve in practice. It was also found that the standard of machining and assembly of slideway and drive components has a great effect on the uniformity of slow motion, and a method was described for improving slideway lubrication and a technique for the mathematical calculation of slideway features based on friction characteristics. The use of this technique makes possible an estimation of the magnitude of friction in the slideways, and a selection of the optimum slideway parameters. Semi-empirical formulae were used leading to a simple sequence of calculations. This method was extended to cover hydraulic load-relief calculations.

Kudinov and Lisitsyn [71, 114] examined how Coulomb friction damps out externally induced vibrations and, based on the previous work of Lisitsyn [70], studied the setting accuracy and the uniformity of slow movements of machine tool sliding tables under mixed friction conditions.

Wolf [115], Moisan [35] and Poláček and Vavra [116] measured the stick-slip properties of industrial lubricants; studied the dynamic behaviour of slideways and compared several types of slideways (plain, antifriction and hydrostatic). Bell and Burdekin [28, 109, 117, 118] examined the action of polar additives as anti-stick-slip friction modifiers. It was found that in the low velocity region, non-polar oils give more positive damping than the polar oils in a number of conditions.

Emphasis has been placed also on the determination of the difference between steady state and dynamic friction characteristics and their influence on the stability and damping of the sliding motion. The effect of separation of the surfaces on friction and stick-slip was also studied and the results obtained from the dynamic friction characteristic evaluations showed close agreement with Štěpánek's [24] (for non-polar lubricant).

Britton [119] examined the stability of machine tool feed drives and its effect on positional accuracy and stick-slip deterioration of surface finish. An attempt was made to derive straightforward experimental phase-plane traces with fair success. Similarly Schindler [120] and Dolbey [75] examined the effect of materials and the normal dynamic characteristics of slideways, underlining the importance of the separation of the surfaces, and its influence on the frictional force. Steward and Hunt [54] investigated the variation of the coefficient of friction during stick-slip, by using dimensional analysis and introducing new nondimensional parameters. No exact general empirical relations were derived, but some simple methods for estimating the magnitudes of the errors at low sliding speeds, or near the critical speed, were demonstrated.

1.4.2. In servomechanism applications

The effect of nonlinear friction on the stability of servomechanisms was studied by Tustin [121, 122] and Lauer [40]. Step-by-step numerical techniques were used by Tustin to solve the nonlinear differential equation of motion assuming that the frictional force for a range of relative velocities $v_1 < v < v_2$ is given by the exponential relation:

$$F = F_0 - F_c \left(1 - e^{-\frac{v}{v_c}}\right) \dots\dots\dots (1.7)$$

Lauer used a general form of characteristic and solved the equation topologically. It was found that it is important to know over which part of the characteristic the system operates, and the manner (direction and rate) in which the operating point of the system was approached.

On the other hand Haas Jr. [20] and Swamy [123] used simple Coulomb models to study the effect of stiction on servomechanisms and its undesirable consequences such as operating dead zone, low frequency wander and poor dynamic performance for low level signals. Comparative study of experimental and theoretical results has been done using an analog computer.

1.4.3. In other applications

1.4.3.1. Under press-fit conditions

Self-excited frictional oscillations under conditions of high load, low speed and boundary lubrication in press-fit tests were studied by Jones [124, 125]. A modified concept of stick-slip was presented which attempted to take into account the elastic and plastic deformation of welded asperities, prior to slip.

1.4.3.2. In brakes and transmissions

Jania [39] studied the factors influencing the friction clutch transmissions performance. A linear theoretical model was employed and the experimental results obtained showed that stick-slip depends on the steepness of the friction-velocity characteristic. Similar experimental techniques were also used in a publication [126] concerned with automatic transmission shift quality. The effect of fluid friction modifiers (characterized by a high polar activity level) and their concentration was studied as well as the degradation of the oil with the elapsing time.

Broadbent [31] and Spurr [127] examined chatter and squeal of brake blocks. Broadbent's theoretical analysis showed that even for pure Coulomb

friction chatter can be expected, depending on the design of the brake system. However discussion of the equation of motion using a modified more realistic characteristic, showed how stick-slip excites chatter in a brake mechanism. This means that with pure coulomb friction chatter is not to be expected. In favour of stick-slip generation of brake chatter is the noteworthy fact that no chatter occurs when wooden brake blocks are used where the coefficient of friction falls with drop in wheel peripheral speed. A small-scale apparatus was used by Spurr to study the conditions of brake squeal excitation. It was found that squeal might occur independently of the slope of the friction-velocity characteristic. Both these works emphasised the importance of the geometry of the system as a factor controlling the occurrence of brake chatter or squeal.

1.4.3.3. In metal cutting

Arnold [128] made a fundamental investigation of vibration in cutting tools and showed that this phenomenon may be the resultant of both self-excited and forced oscillations. The characteristics of the oscillation were correlated to the cutting parameters and some useful relations were derived. The production of frictional oscillations was explained by means of a dropping friction-velocity characteristic, which was examined in detail.

1.4.3.4. In friction of natural or synthetic fibres and wood

Scheier and Lyons [129, 130] investigated the surface friction of fibres using an electro-mechanical method, in order to determine the nature of stick-slip process in fibre friction and to find the effect of surface finish on this process. In general no correlation was found between frictional parameters and surface geometry, while an attempt to use statistical methods gave very poor results.

The more important variables affecting friction between wood and steel were studied by McKenzie and Karpovich [131], who found that the atmosphere

had pronounced effects on the amplitude of stick-slip oscillation for several species of wood studied under dry or lubricated conditions.

1.4.3.5 In rocks

The friction of rock surfaces was studied by Hoskins, Jaeger and Rosengren [132] by sliding a block with plane parallel surfaces between two others in a special testing machine. With finely ground surfaces, regular stick-slip oscillations occurred whose amplitude was determined by the coefficients of static and kinetic friction and the stiffness of the testing machine. Such oscillations could be produced or inhibited by decreasing or increasing the roughness of the surface.

Brace and Byerlee [27] suggested that shallow focus earthquakes may represent stick-slip during sliding along old or newly formed faults in the earth. Experimental evidence is in favour of this opinion. It was concluded that stick-slip deserves to be considered, in conjunction with the Reid mechanism, as one possible mechanism for shallow focus earthquakes.

1.4.4. Related phenomena

1.4.4.1. Electrical charges on the surfaces

Sohl, Gaynor and Skinner [133] investigated the electrical effects accompanying the stick-slip phenomenon and found a definite time correlation between the mechanical stick-slip and the electrical transients. The characteristics of the electrical discharge appear to favour a charge-discharge mechanism rather than a thermoelectric potential or a dielectric breakdown mechanism, although the situation is not very clear, experimentally. Basis for this work was Schnurmann's theory of stick-slip (electrostatic generation of stick-slip, Schnurmann's effect), but a possibility of existence of Faraday's effect, was not excluded.

1.4.4.2. Wear of the surfaces

Rabinowicz and Tabor [134] studied the metallic transfer between sliding metals using an autoradiographic technique and correlated it with the friction characteristics of the sliding (smooth or intermittent motion etc.) Evdokimov [135] and Pavelesku [105] investigated the wear resistance of a surface subjected to alternating shear loading and found that changes in the values of the elastic-plastic deformations, work hardening and density of dislocation clusters, leads to a difference in the stress of layers subjected to linear or alternating deformations during sliding. On this basis it was assumed that alternating deformation, such as that produced by stick-slip, also affects the wear resistance of the sliding components, which was observed experimentally. Increased wear due to stick-slip affects the value of the static coefficient of friction, which has an immediate effect on the amplitude of stick-slip as it was observed by Wiid and Beezhold [136]. This cyclic effect seemed to be strongly influenced by the surface geometry of the specimens.

Kaminskaya and Kovtun [137] examined the effect of vibrations on the wear of rubbing surfaces, knowing from practice that the life of machines working in conditions of intensive vibrations is greatly reduced. They found that the level of the relative vibrations of two rubbing surfaces has a considerable effect on their wear resistance, and that self-excited vibrations show quite similar action. On the contrary Pavelesku and Dimitrov [45,138] found that there is no definite correlation between stick-slip and wear. The wear rate during stick-slip can be greater equal or smaller, as compared to the case in which by using an elastic system with much greater rigidity coefficient, the stick-slip is practically damped out.

CHAPTER 2 : THEORY

2.1. INTRODUCTION

2.1.1. The problem

Frictional oscillations in general can be divided in three major groups:

- a. - Free frictional oscillations.
- b. - Forced or externally induced frictional oscillations.
- and c. - Self-excited frictional oscillations.

Oscillations of the first and second group do not present any tribological interest because they occur in stationary contacts (in the sense that the mean sliding velocity $v_0 = 0$) while the third group, or a combination of oscillations of the second and third groups, are met in cases where $v_0 \neq 0$ and consequently affect the behaviour of frictional pairs. These oscillations are to be studied.

It has been seen that a mechanical system subjected to frictional oscillations (of group c or combination of groups b and c) can be represented in its simplest form by the system of fig. 2.1. This system consists of a mechanism which can oscillate under the proper conditions of excitation (oscillator - enclosed in contour (a)) and an interface AB on this contour,

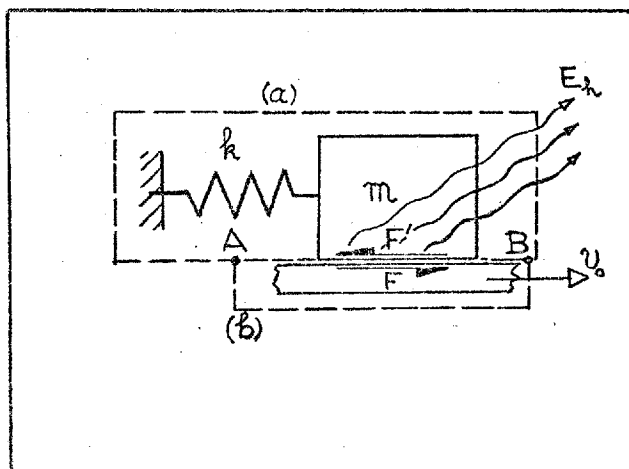


Fig. 2.1.

from which energy exchange with the environment can be achieved. Through the rest of contour (a) only thermal energy exchange can take place (E_h).

The system enclosed in contour (b) acts as a source of available energy from which the

oscillator can draw in synchronism with its own natural oscillations, thus balancing out the unavoidable energy losses caused by damping (heat E_h

dissipated in the environment). This is a typical behaviour of a self-excited system.

Magnus [139] divides self-excited systems into two types according to their design and mode of operation. Systems like fig. 2.2.(a) are called "oscillator" or "vibrator type" systems. A source of energy is provided which

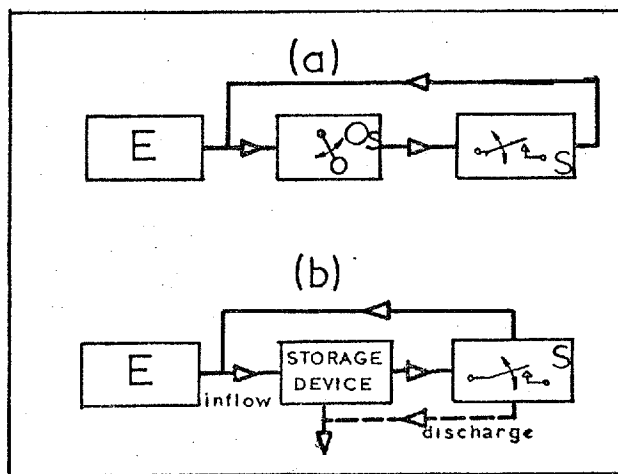


Fig. 2.2.

can supply the system. The supply of energy does not take place at random but is governed by a control mechanism actuated by the vibratory system itself which is indicated by the term "switch" (S on fig. 2.2). This switch reacts upon the connection between energy source and vibratory

system and consequently regulates the supply of energy in rhythm with the natural vibrations of the vibratory system. A significant characteristic of systems like the one portrayed in fig. 2.2.(a) is the back-coupling between vibratory system and power source via the switch. It is only through this back-coupling that self-sustained vibrations become possible.

Fig. 2.2.(b) shows the main elements of a self-excited oscillatory system of the "storage-device" type. Instead of the oscillator there is, here, a storage device through which the energy flows. A switch controlled by the storage device operates either on the inflow or outflow direction of energy, to or from the storage device.

It is not always easy to recognise the individual elements of the block-diagrams of fig. 2.2, and, moreover, the mechanism of energy pick-up for frictional self-excited oscillations is quite complicated. It is a question open to discussion whether frictional self-excited oscillations behave like systems of fig. 2.2.(a) or 2.2.(b). The prevailing opinion is

that they are rather of the type of fig. 2.2.(a) and the role of the "switch" is played by the non-linear friction-velocity characteristic. The system of fig. 2.1. has as energy source, the moving (with constant velocity v_0) lower specimen. The "oscillator" is the upper specimen with its supporting spring, while the function $F = F(\dot{x})$ or $\mu_k = \mu_k(\dot{x})$ acts as the "switch". Writing the differential equation of motion of that system as:

$$m\ddot{x} + \varphi(\dot{x})\dot{x} + kx = 0 \quad \dots\dots\dots (2.1)$$

one can see that the only way in which tribological characteristics could affect the mechanics of the system is through the function $\varphi(\dot{x})$, which in fact is a known function of $\mu_k(\dot{x})$ (fig. 2.3). This is very important because

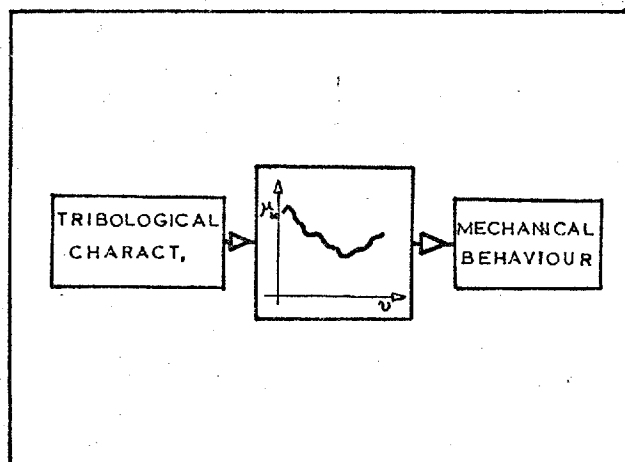


Fig. 2.3

it means, simply speaking, that a parameter affecting the frictional characteristics in some way, could affect the mechanics of the system only by changing the function $\mu_k(v)$. On the other hand it is also obvious that no conclusions can be drawn about the effect of variation of

tribological parameters, in the case where the function $\mu_k(v)$ has a simplified or a theoretically assumed form.. It is thus true that the only correct way to study the tribological aspect of frictional self-excited oscillations is by means of mathematical analysis of the mechanics of the system, based on a real, experimentally derived, friction-velocity characteristic.

2.1.2. Micro- and macro-behaviour and their interaction

Purely tribological factors affecting the friction-velocity characteristic, affect indirectly the motion of the system. All these factors compose the frictional behaviour of the surfaces, which from now on will be called

micro-behaviour of the system to distinct from the mechanics of it, which will be called macro-behaviour. Similarly simplified models of the micro-behaviour will be called micro-models as distinct from macro-models expressing the macro-behaviour (e.g. mass, stiffness, external damping, load, act directly on the macro-behaviour, and they are included in the macro-model).

Based on the fact that micro - and macro-behaviour are related between themselves only through the friction-velocity characteristic, one could study them separately and then find their "interaction" by means of the linking friction-velocity characteristic. In case of complicated micro-behaviour, this characteristic needs a simplification, for the sake of mathematical simplicity. But such a simplification produces inaccuracies and theoretical and experimental results cannot agree any more. To overcome this difficulty

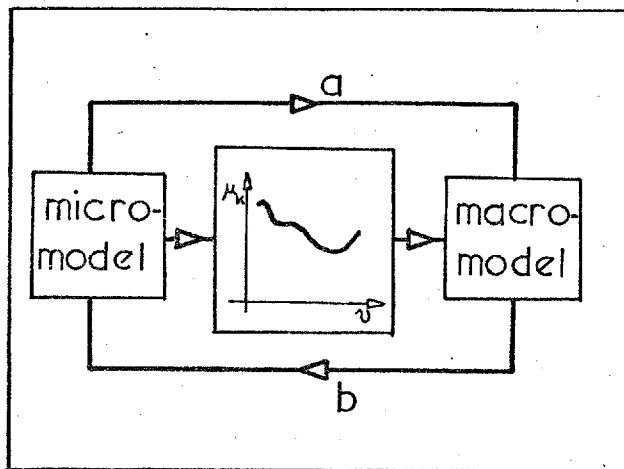


Fig. 2.4

a simple trick can be used, i.e. the friction-velocity characteristic is employed in a simplified form, while two additional links a and b (fig. 2.4) produce the necessary corrections in the macro-behaviour. In that case obviously equ. 2.1. ceases to describe fully the motion of the

system, and corrections are necessary. This technique can give very good results in cases where the friction-velocity characteristic was accepted as a more complicated function, e.g.:

$$\mu = \mu([v]_{v \neq 0}, [t]_{v = 0})$$

or:

$$\mu = \mu([v, \dot{v}]_{v \neq 0})$$

or even:

$$\mu = \mu([v, \dot{v}]_{v \neq 0}, [t]_{v = 0})$$

These characteristics were adopted in order to describe with the highest possible accuracy the macro-behaviour of the system, but the results were not quite satisfactory.

When frictional self-excited and externally induced oscillations co-exist (the second interesting form of frictional oscillations), the problem becomes extremely difficult, because the superposition principle of classical mechanics does not hold for nonlinear systems [140,141]. Furthermore if the amplitude of the forcing function depends upon the frictional characteristics or the effect of the macro-behaviour on the micro-behaviour is to be taken into account, no full analytical solution of the motion of the system must be expected any how.

2.2. THE MICRO-MODEL

2.2.1. Physicochemical factors affecting the micro-model

2.2.1.1. Contact and interaction of surfaces and bulk material

The area of contact of two real surfaces (not ideally smooth) consists of contact spots, the number and area of which increase as the two surfaces A,B (fig. 2.5) approach to one another due to increasing load N . The surface

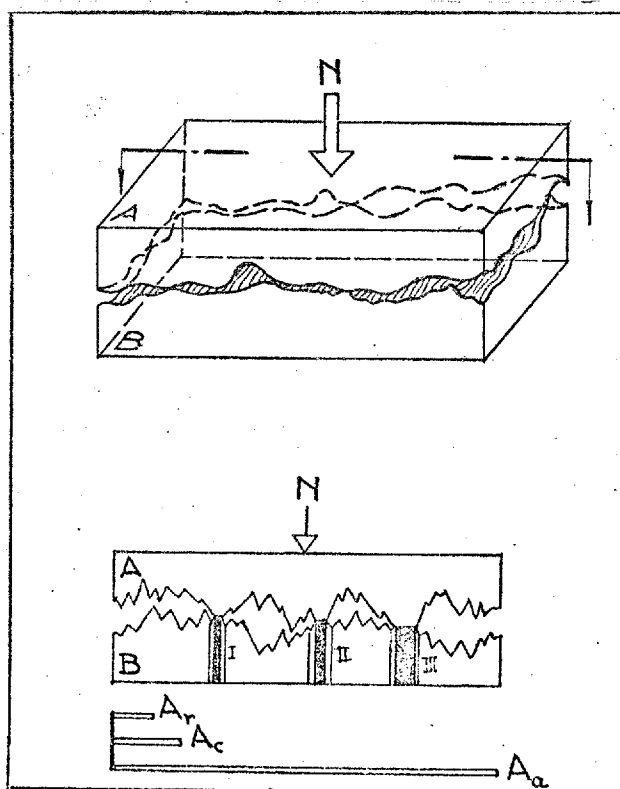


Fig. 2.5

asperities at first present an elastic distortion but as load increases, plastic deformation follows. Removal of the load produces elastic recovery and destruction of some of the contact spots. The number of destroyed contact spots depends on the elastic characteristics of the materials and the microgeometry of the contact. Contact spots which are formed,

exist and disappear under the simultaneous action of the normal and tangential forces are called frictional bonds. Junctions which continue to exist after the removal of the normal load are called adhesional bonds.

The surface layers of the rubbing materials are affected by:

- a. - Frictional heating,
- b. - Physical interaction with the surrounding medium (atmosphere, lubricant, mating surface),
- and c. - Chemical interaction with the surrounding medium (mainly oxidation).

Experiments by McFarlane and Tabor [142,143] showed that with clean hard surfaces in dry air the adhesion is negligibly small. In moist air (above 70% relative humidity) appreciable adhesion may be observed, and it was shown that this is due to the surface tension of thin films of adsorbed water. Adhesion decreases with increasing thickness of surface oxides or other films (metallic interaction is diminished), and with increasing roughness (if the height of the asperities is comparable with the thickness of the adsorbed film). It also increases with idle time and temperature (Mason [83]), and decreases with improved lubrication conditions (Gemant [38]). The adhesion effect on the frictional behaviour of hard elastic surfaces is found to be negligibly small and usually non-measurable.

The action of the lubrication on the interacting surfaces must be seen as comprising:

- 1. - Suppression of the molecular forces (adhesion),
- 2. - Reduction of the surface strength (Kragelskii [144]),
- 3. - Formation of soaps (chemical action),
- 4. - Increase of the separation of surfaces (mechanical action),
- 5. - Decrease of the surface temperature by heat transfer (thermodynamic action).

Apart from the lubricating oils (mineral, synthetic etc.) the following act as lubricants:

- a. - Metal coatings or soft interposed materials,
- and b. - At high speeds the surface layer, which softens under the influence of frictional heating.

2.2.1.2. Sliding friction and coefficient of sliding friction

By friction is understood the necessary force F to introduce sliding between two surfaces kept in contact by a normal load N . Depending on the way the sliding is produced, one can distinguish:

- a. - The static friction F_s for stationary contact.
- b. - The kinetic friction F_k for surfaces sliding with relative velocity $v_r \neq 0$.
- c. - The kinetic friction under zero velocity $F_{k_0} = \lim_{v_r \rightarrow 0} \{F_k\}$.

The above definitions are based upon the two classical frictional laws:

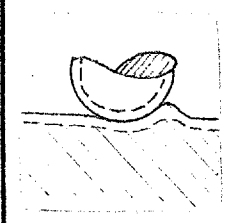
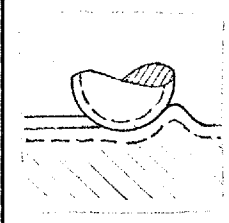
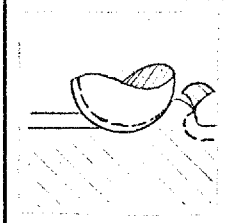
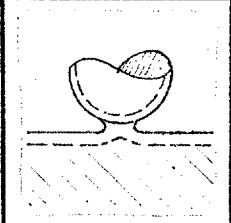
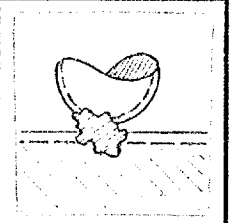
1. That the static coefficient of friction is a function of the idle time t_s : $\mu_s = \mu_s(t_s)$
- and 2. That the kinetic coefficient of friction is a function of the sliding velocity $\mu_k = \mu_k(v_r)$

The kinetic friction under zero velocity might be seen as a boundary state between static and kinetic:

$$F_{k_0} = \lim_{t_s \rightarrow 0} \{F_s\} = \lim_{v_r \rightarrow 0} \{F_k\}$$

The above relationship indicates that both F_s and F_k are manifestations of the same physical entity which behaves according to two different laws, depending on the values of a set of parameters ($t_s, v_r, \dots$), but in the present state of knowledge a generalization is not feasible (some work has been done in that direction by Rubenstein [145,146] and Green [147]).

According to Kragelskii [144] frictional (including adhesional) bonds can be classified as in Table 2.1.

TABLE 2.1: CLASSIFICATION OF FRICTION-ADHESION BONDS acc.to Kragelskii				
MECHANICAL			MOLECULAR	
elastic displacement	plastic displacement	cutting	destruction of surface film	destruction of bulk material
				
I	II	III	IV	V

In this table the friction due to interlocking of surface asperities is not included because it was found that it comprises a negligible percentage of the total frictional force (Strang-Lewis [148]). Electrostatic forces have been omitted likewise (Schnurmann, Warlow-Davies [66,67], Claypoole [149]).

Between two real surfaces in contact, all types of frictional bonds can be met, their distribution being a matter depending on the microtopography of the surfaces. In fig. 2.6, for example, three kinds of frictional bonds

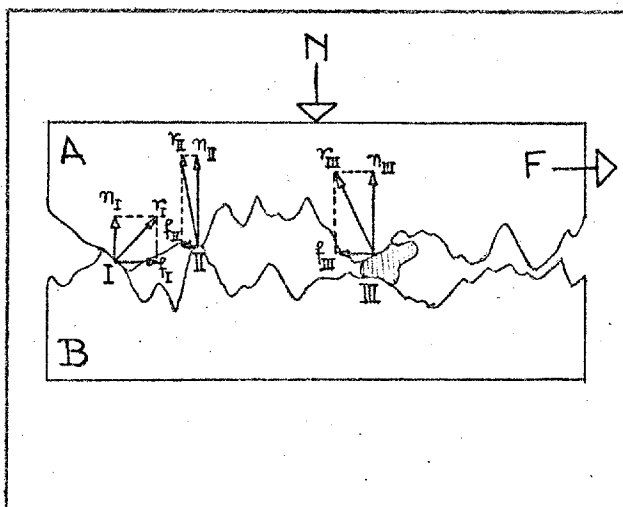


Fig. 2.6

coexist between the two surfaces: Elastic deformation (junction II), Plastic deformation (junction I) and cutting (junction III). According to Newton's law, the system is in equilibrium when (two-dimensional case):

$$(\Sigma F_x = 0, \Sigma F_y = 0, \Sigma M_i = 0) \quad \dots\dots\dots (2.2)$$

Accordingly frictional and normal forces can be expressed, for the general case of co-existing all five types of junctions acc. to Kragelskii, as:

$$F = \sum_{j=1}^k f_k + \sum_{j=1}^l f_l + \sum_{j=1}^m f_m + \sum_{j=1}^n f_n + \sum_{j=1}^p f_p$$

$$N = \sum_{j=1}^k n_k + \sum_{j=1}^l n_l + \sum_{j=1}^m n_m + \sum_{j=1}^n n_n + \sum_{j=1}^p n_p \quad \dots\dots\dots (2.3)$$

and the coefficient of friction is

$$\mu = \frac{\sum_{j=1}^k f_k + \sum_{j=1}^l f_l + \sum_{j=1}^m f_m + \sum_{j=1}^n f_n + \sum_{j=1}^p f_p}{\sum_{j=1}^k n_k + \sum_{j=1}^l n_l + \sum_{j=1}^m n_m + \sum_{j=1}^n n_n + \sum_{j=1}^p n_p} \quad \dots\dots\dots (2.4)$$

Thus, it is obvious that any attempt to express the behaviour of the surfaces by means of simple models cannot give accurate results. On the other hand exact calculations through equ. 2.4. are not feasible. To overcome these difficulties, arising from the nature itself of the coefficient of friction, statistical methods have been proposed (Rabinowicz [150], Saibel [151], Tsukizoe and Hisakado [152], Ling [153], Nagasu [154]) but the stochastic processes used were rather complicated and the obtained formulae were not practically usable.

2.2.1.3. Static friction and coefficient of static friction

Experimental and theoretical works (Jones [155], Wiid and Beezhold [136], Kosterin and Kraghelsky [156], Brockley and Davies [157], Schmidt and Weiter [158]) have shown that static friction increases with idle time, according to some exponential law. Kosterin and Kraghelsky ascribed that to a viscoelastic behaviour of the contacting surfaces while Jones explained it in terms of changes in crystalline structure, changes in shear stresses of the lubricant (if there is one), welding and probably oxidation. The creep mechanism, supported by many investigators (theoretically studied by Arutiunian and Manukian [159]) was denied by others (e.g. Jones). Among the formulae proposed to express the function $\mu_s = \mu_s(t_s)$ the following can be distinguished:

a. - Brockley - Howe - Puddington - Benton:

$$\mu_s = \mu_{k_0} + (\{\mu_s\}_{t \rightarrow 0} - \mu_{k_0})(1 - e^{-ct_s}) \quad \dots\dots\dots (2.5)$$

c = constant depending on the material

b. - Deryaguin - Push - Tolstoi:

$$\mu_s = \mu_{k_0} + \frac{c_1 t_s}{c_2 + t_s} \quad \dots\dots\dots (2.6)$$

$c_1, c_2 = \text{constnats}$

c. - Rabinowicz:

$$\mu_s - \mu_{k_0} = \gamma t_s^\beta \quad \dots\dots (2.7)$$

$\gamma, \beta = \text{constants } (\beta < 1)$

d. - Brockley - Davies:

$$\mu_s - \mu_{k_0} = c_1 \cdot e^{\frac{-c_2 \cdot c_3}{T}} \cdot t_s^{c_3} \quad \dots (2.8)$$

$c_1, c_2, c_3 = \text{constants}$

Equ. (2.8) includes the effect of temperature and predicts a significant increase in friction with time. Fig. 2.7 presents qualitatively the increase of static coefficient of friction with time, and the effect of increasing temperature (θ) according to Brockley and Davies (see also Appendix 1).

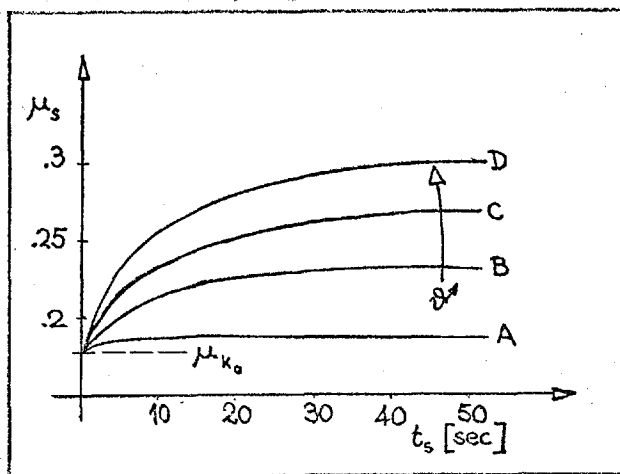


Fig. 2.7

are observed at first (Fig. 2.8). With increasing tangential force, the displacement increases, till gross slip which denotes μ_s according to its

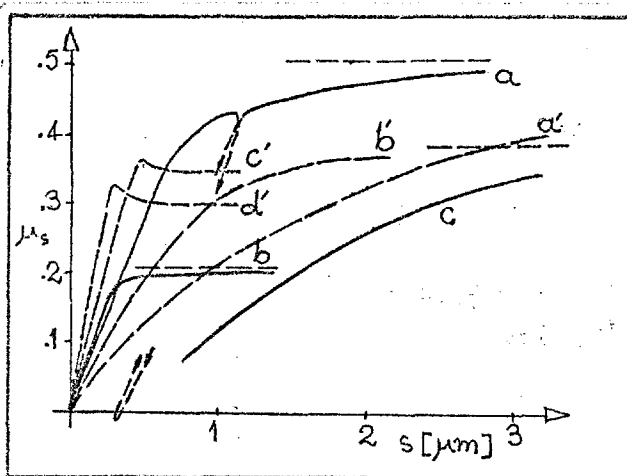


Fig. 2.8

- a: st + st, $N = 920p$, clean
- b: st + st, $N = 920p$, lauric acid + cetane
- c: Pb + glass, $N = 500p$
- a': velocity 0.1 mm/min
- b': velocity 0.3 mm/min
- c': velocity 2.0 mm/min
- d': velocity 5.0 mm/min

— μ_s
 - - - loading - unloading cycles

Repeated experiments over the same track showed decreasing μ_s which means that polishing of the surfaces (decreased surface roughness) produces a decrease in the static coefficient of friction (Jones [155]).

When the tangential force is applied gradually starting from zero, small reversible displacements

definition (Courtney - Pratt, Eisner [160], Parker and Hatch [161]). Obviously the shape of the curves of fig. 2.8 depends on the load and the elastic characteristics of the surfaces. Loading - unloading cycles indicate that the first part of these curves represents mainly elastic deformation of the surfaces, while the rest consists of plastic deformation and relative displacement between the two surfaces. In the elastic region losses are produced exclusively due to internal friction of the materials (hysteretic phenomena) (Greenwood, Minshall,

Tabor [162]. See also Rankin [3]). A technique for definition of the limit of elastic range within the no-gross slip range was produced by Klint [87], based on measurements of dissipated energy, under dynamic loading.

For greater displacements, it seems that μ_s drops with displacement mainly due to wear of the surfaces, which produces reduction of the average height of the surface asperities and consequently lower pressures and less

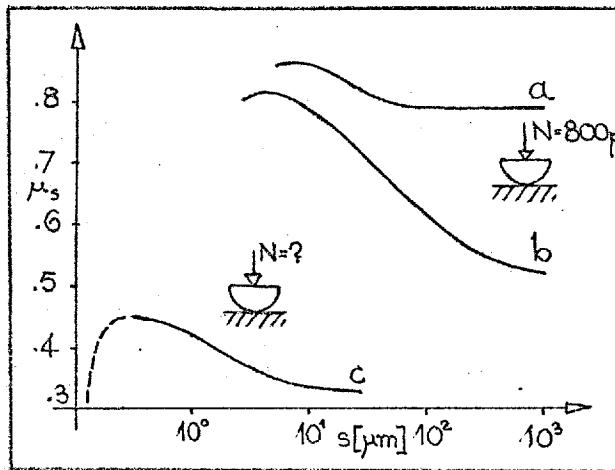


Fig. 2.9

- a: St on Cu
- b: St on St
- c: Cu on St

penetration of the oxide layer, indicated by the increased contact resistance (Rabinowicz [4], Wiid - Beezhold [136]). Repeated passes over the same contact area showed decreasing coefficient of friction. This is not in agreement with previous experimental results (Gaylord-Shu [94], Claypoole [149], Campbell-Summit [163], Bowden-Young [68]) showing that the coefficient of friction increases with sliding distance, probably due to oxide film removal.

A reasonable explanation to that controversy has been given by Bowden and Tabor [164]. They found that friction decreases as thinner films are used because the area of contact becomes smaller. There is however a limit to this and a minimum friction is reached. With thicknesses less than this limit, the film ceases to be effective (Cocks [166]), and the coefficient of friction increases again. Temperature and load was found to have some effect on the surface films and accordingly on the μ_s . The function $\mu_s = \mu_s(s)$ depends also on the time or the rate of application of the frictional force (Burwell - Rabinowicz [74], Claypoole [149], Voorhes [61]) which indicates that a definite creep behaviour exists (see also fig. 2.8 traces a', b', c', d' acc. to Voorhes).

The static coefficient of friction was found to be greatly influenced by externally injected vibrations but rather insensitive to impacts (Fridman-Levesque [89], Parker-Farnworth-Milne [165], Seireg-Weiter [96]). Thus for impacts $0 \div 500 \frac{\text{tn}}{\text{s}}$ a slight increase in μ_s was observed (about 10%) while vibrations of frequencies $6,5 \div 41 \text{ kHz}$ and low power were enough to decrease μ_s by 100%, probably by weakening the junctions between the surfaces.

Finally μ_s decreases with hardness while it seems to increase with load and roughness, but their effect is extremely complicated and even qualitative expressions are still lacking (Whitehead [167], Moore-Tegart [168], Ling-Weiner [169]).

2.2.1.4. Kinetic friction and coefficient of kinetic friction

The most important factor affecting the kinetic friction of metal surfaces is the sliding velocity. Thus a great number of investigators studied the function $\mu_k = \mu_k(v_r)$ and a number of formulae has been proposed, the most representative of which being:

$$1. - \text{Bochet's formula (1851): } \mu_k = \frac{k}{1 + 0.03v_r} \quad \dots\dots\dots (2.9)$$

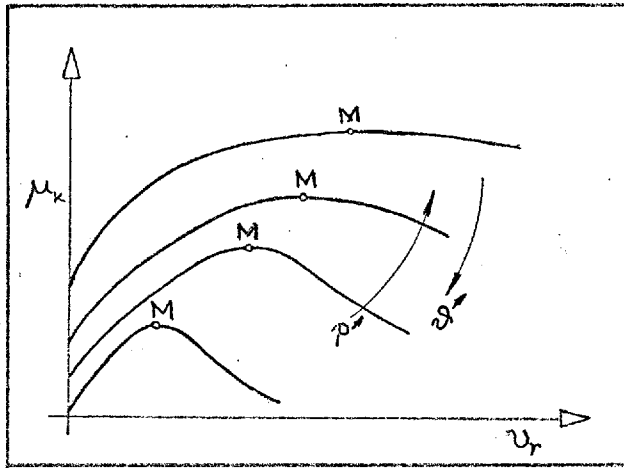
$$2. - \text{Franke's formula (1882): } \left. \begin{array}{l} k = 0.1 \div 0.5 \\ \mu_k = \mu_s e^{-cv_r} \\ c = 0.01 \div 0.1 \end{array} \right\} \quad \dots\dots\dots (2.10)$$

$$3. - \text{Binomial formula (Dobrovolskii)} \quad \left. \begin{array}{l} \mu_k = a + b.v \\ a = 0.1 \div 0.4 \\ b = 0.005 \div 0.02 \end{array} \right\} \quad \dots\dots\dots (2.11)$$

$$4. - \text{Kragelskii's formula: } \left. \begin{array}{l} \mu_k = (a + bv)e^{-cv} + d \\ a, b, c, d = \text{constants} \end{array} \right\} \quad \dots\dots\dots (2.12)$$

Kragelskii's formula 2.12 is in agreement with his theory of viscoelasticity of the metal contacts and predicts a maximum for the function $\mu_k = \mu_k(v_r)$

depending on the contact pressure [144], and the temperature (variation of the mechanical properties of the rubbing materials, the rates of the rheological processes occurring in the region of deformation, change of the surface films). Velocity, according to this theory does not influence μ_k directly,



M the point of $\{\mu_k\}_{\max}$

Fig. 2.10

but only indirectly through the change of the contact properties with temperature. Thus no mathematical formula can express adequately the function

$\mu_k = \mu_k(v_r)$. The typical curves of fig. 2.10 can be qualitatively explained as follows: In practice two independent mechanisms of contact co-exist. That of elastic and that of plastic contact. Since the modulus of

elasticity and the density of the materials vary only slightly with temperature, the frictional force due to elastic contact (hysteresis) is essentially independent of θ and consequently of v_r . During plastic contact, on the contrary, an ascending part of the $\mu_k(v_r)$ curve is observed for low speeds due to viscous effects at the contact and this is replaced by a descending one for higher velocities, due to film formation, easier deformation of heated material etc. (see also Rozeanu-Eliezer [170], Gould [171]). For even higher speeds, μ_k starts increasing anew due to melting of the surfaces and viscous behaviour of the melted material (Bowden-Ridler [10], Bowden-Persson [174], Bailey [175], Earles-Kadhim [176]).

Experiments done with increasing and then decreasing sliding velocity showed an additional complication, i.e. the friction is not a single-valued function of velocity, but takes different values for each cycle of the experiment. (Kato-Matsubayashi [57], Beeck-Givens-Smith [172], Sampson-

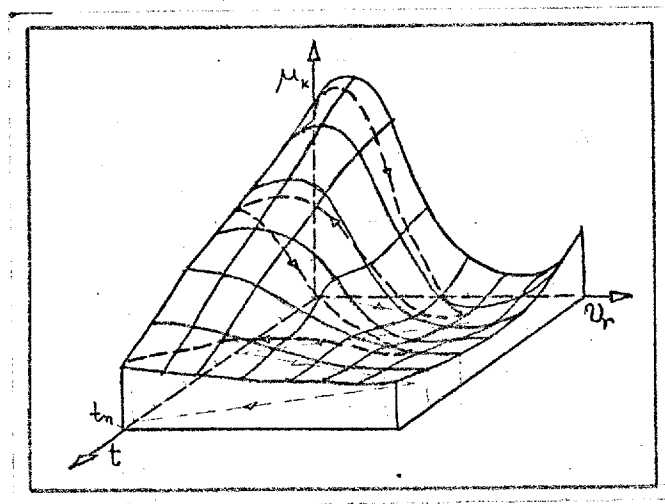


Fig 2.11

Morgan-Reed-Muscat [17]). A simple explanation of this phenomenon, is that if $\frac{dv}{dt}$ is small, then the period of the velocity cycle is long enough to allow an observable change of the surfaces (oxidation, wear etc.) to take place during each cycle. Assuming that the instantaneous $\mu \sim v$ portrait can be obtained, the surface of fig. 2.11 pictures the function $\mu_k(v_r, t)$ for $t_0 < t \leq t_n$. It is obvious that for $\frac{dv}{dt} \neq 0$ the form of the function $\mu_k(v_r, t)$ depends on how the velocity changes with time or in other words depends on $v_r = v_r(t)$.

Although one could expect that for $\frac{dv}{dt}$ large enough bifurcation could diminish, this does not happen (Raizada [58], Bell-Burdekin [28,109,117], Elyasberg [41], Štěpánek [24], Schindler [65,120], Matsuzaki [173]) because the interference of inertia forces ($m\frac{dv}{dt}$) and resonance phenomena (depending on the ω_n of the elastic support of the slider), become predominant. To

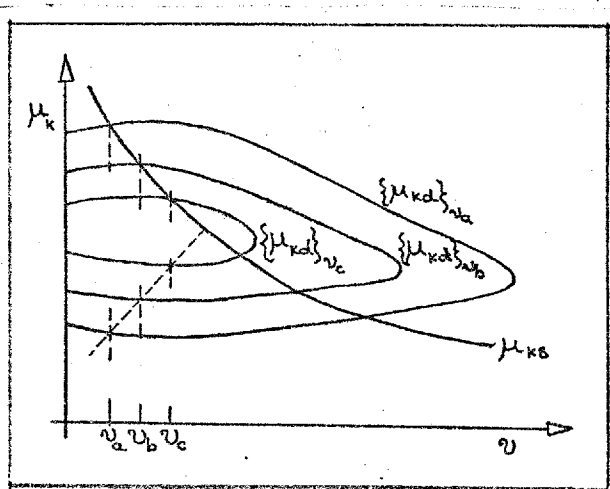


Fig. 2.12

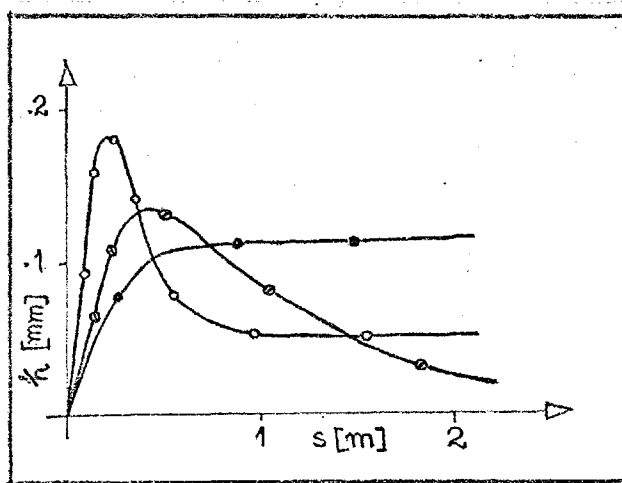
by Hunt, Torbe and Spencer [51].

Experimental results corrected for inertia forces and taken by an apparatus with proper characteristics give μ_{kd} loops of small area depending only on the internal friction of the materials and the irregularity of the frictional characteristics of the surfaces (real $\mu_k = \mu_k(v_r)$ curves). As far as is known there are no available results of that kind in the existing

overcome these difficulties a new entity was fabricated, the "dynamic" kinetic coefficient of friction which depends on velocity (fig. 2.12). As it will be seen this "dynamic" kinetic coefficient of friction has no physical meaning because under dynamic conditions acceleration appears as a latent parameter as correctly suggested

literature.

It was found that the sliding of flat surfaces, produces wear debris which interposes between the two surfaces and increases their separation (Cocks [177], Antler [178]). The separation increases rapidly at the beginning and then a steady-state value is reached (fig. 2.13). The steady state separation is

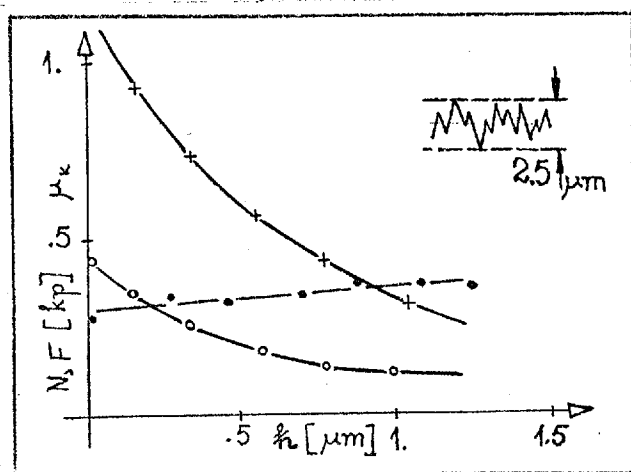


- St on St
- Ni on Ni
- Cu on Cu

Fig. 2.13

usually great enough to diminish the interaction between the surfaces, which is replaced by the triadic system surface-debris-surface. Moreover material transfer from the one surface to the other, work hardening etc. can greatly affect the process. Variations of separation have a direct effect on load and friction, as it was observed by Tolstoi [72,104] (fig. 2.14). The observed drop of coefficient of friction with load agrees with other experimental results (Kragelskii [144] pg. 171, Vinogradov-Korepova-Podolsky [179]).

The surface roughness affects friction according to a number of mechanisms, some of which oppose the others. Consequently the coefficient of friction does not vary monotonically with roughness (Porgess-Wilman [180], Miyakawa [181]). Qualitatively one can say that for smooth surfaces a molecular frictional mechanism prevails and accordingly for decreasing roughness the separation of the surfaces decreases and the friction increases. On the contrary for increasing roughness, a limit is quickly reached after which the molecular forces become negligible and mechanical interaction of the surfaces becomes



- + — Load
- o — Friction
- • — Coefficient of friction

Fig. 2.14

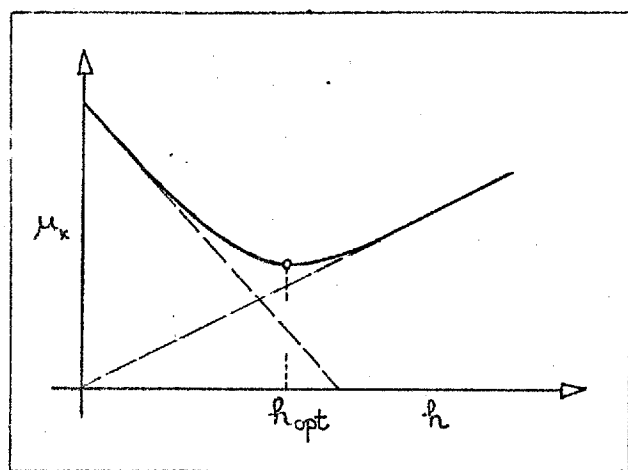


Fig. 2.15

2.2.1.5. The effect of lubrication

When lubrication is employed, the tribological characteristics of the lubricant become predominant, while the characteristics of the metal surfaces are of secondary importance.

the predominant frictional mechanism, which produces increasing friction with roughness. Obviously there is an optimum roughness value h_{opt} for which the combination of the above two mechanisms gives a minimum friction (fig. 2.15).

In the case of low sliding speeds (boundary lubrication) the lubricant acts as adsorbed layer on the surfaces by prohibiting metal to metal contact, while simultaneously affecting the substrate metal which becomes easily deformable (Rebinder effect). The ability of the lubricant to protect the metal surfaces depends on its strength measured from the ease with which it is wiped off the surface (Cameron [182], Kragelskii [144], Dacus-Coleman-Roess [183]).

The function $\mu_s = \mu_s(t_s)$ has, as for the case of dry surfaces, an exponential form but the prevailing parameter is the viscosity of the lubricant which must be squeezed out before metal to metal contact can be observed. The process depends directly on temperature and load (μ_{k_0} increases with increasing temperature and decreasing load according to Kragelskii's experiments [144 pg. 260]).

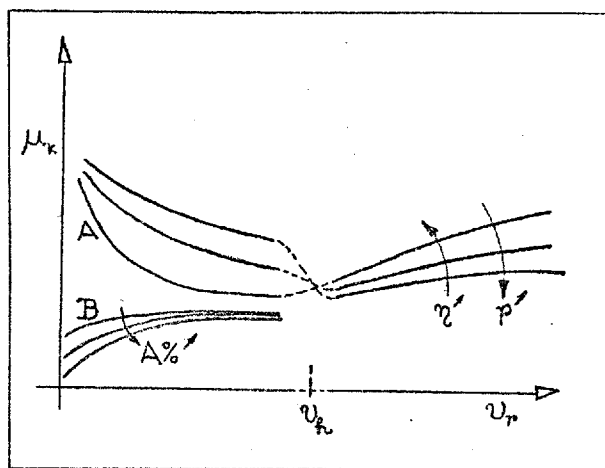


Fig. 2.16

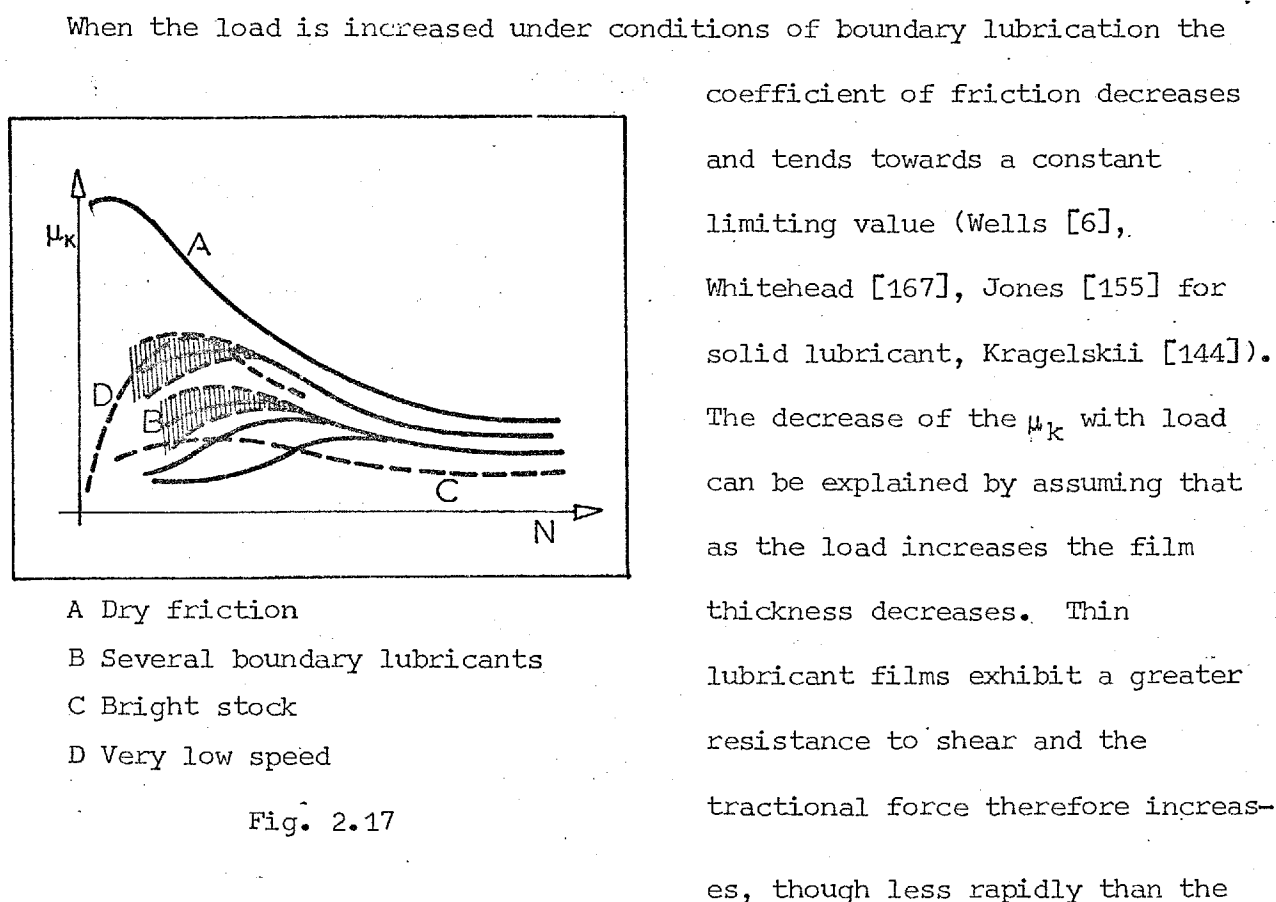
The function $\mu_k = \mu_k(v_r)$ presents two different forms A for pure mineral oils and B (fig. 2.16 Stribeck curves) for fatty acids. Kragelskii explains the behaviour of type A by reduction in the time of localized contact with speed which produces decrease in contact area. Behaviour of type B is met under conditions for which mutual

interpenetration of the surfaces is reduced to a minimum. The viscosity of the lubricant then comes into play and leads to an increase in friction as the rate of frictional force application increases. Thus in this case the rheological properties of the lubricant are eminent (see also Rabinovicz [184]). For speeds higher than a limiting one v_h (depending on the viscosity and the geometry of the surfaces) the conditions of lubrication change from boundary

to hydrodynamic. If the quantity of lubricant present between the surfaces is insufficient to establish a regime of complete hydrodynamic lubrication, mixed conditions of friction appear (Niemann-Banaschek [185], Vogelpohl [186], Chalmers-Forrester-Phelps [187]). In that case (starvation conditions) the separation of the surfaces decreases leading to a less marked dependence of friction on speed. In some other cases friction decreases almost to zero with speed and this is aided by the decrease in the viscosity of the medium with increasing temperature at high speeds.

Modification of the friction-velocity characteristics of a system can be obtained by using oil additives known as "friction modifiers" (or anti-stick-slip or antisquawk additives) (Albertson [188]). The effect of increased concentration of additive in the lubricant A% is shown schematically in fig. 2.16.

Bifurcation of $\mu_k = \mu_k(v)$ was observed for lubricated specimens as for dry friction (Beeck-Givens-Smith [172]).



increase in load. There is some limit over which the decrease in film thickness ceases, and μ_k does not decrease any more. This variation (fig. 2.17) is not absolutely typical because in a number of instances it was observed that coefficient of friction increased with load probably due to partial change from boundary to dry friction. It was also observed that at lower loads there is a much more pronounced trend towards frictional oscillations. At very low sliding speeds Vinogradov et. al. [179] found that coefficient of friction varies according to a law indicated by the curve D (fig. 2.17), in which the ascending part represents progressive development of metal to metal contact, while the descending part is due to plastic deformation of the metal surfaces.

Temperature is another important factor affecting boundary lubrication (Frewing [189]). For temperatures around the melting point of the lubricant or its disorientation temperature, great variations of the value of the coefficient of friction have been observed (Brummage [190]), while Forrester [191] found that the function $\mu_k = \mu_k(\theta)$ is influenced by the sliding speed and the surface roughness (μ_k increases with increasing temperature, roughness and decreasing sliding speed, see fig. 2.18 for bearing alloys lubricated with mineral oil acc. to Forrester).

The kinetic coefficient of friction as a function of the surface roughness

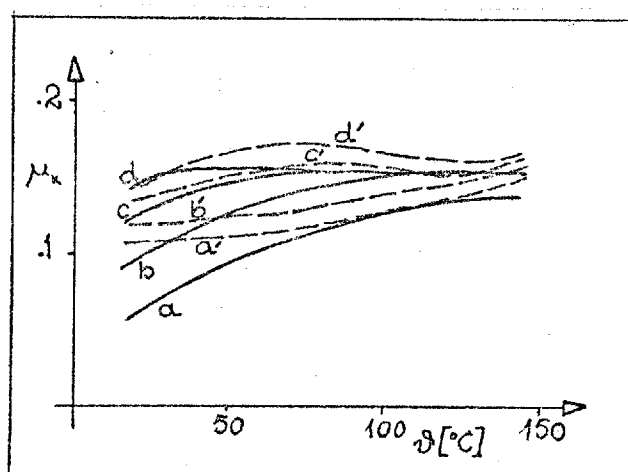


Fig. 2.18

a,b,c,d smooth, incr. velocity
a',b',c',d' rough, incr. velocity

passes through a minimum which is more distinct the thinner the lubricant film applied to the surface. This can be explained by the co-existence of two opposing mechanisms i.e. the real area of contact is proportional to the ratio of the radius of a single asperity

to the maximum height of the asperities. When the roughness increases, this ratio decreases and consequently the real contact area and μ_k decrease as well. On the other hand as roughness increases, thinner films take part in the friction process and this tends to increase the coefficient of friction (Miyakawa [181,192], Forrester [193], Chalmers et. al. [187]).

Finally the viscosity (Lenning [194]) acts on the frictional behaviour indirectly by decreasing the velocity of transition from boundary to hydrodynamic lubrication, opposing load and surface roughness in their action.

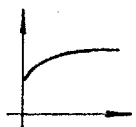
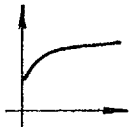
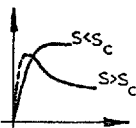
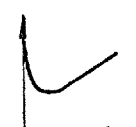
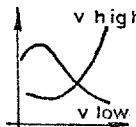
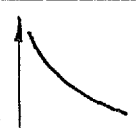
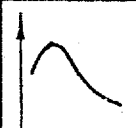
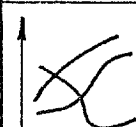
2.2.1.6. Synopsis of the factors affecting the micro-behaviour

Table 2.II presents in a synoptic form the effect of the main tribological factors on the coefficient of friction under dry or lubricated conditions of sliding.

It is clear that the coefficient of friction is in general a very complicated function of a number of parameters, which in many cases act in an undetermined way or oppose one another. A general expression for the coefficient of friction could be:

$$\mu = \begin{cases} \mu_s \{t_s, s, \theta, N, \frac{dF}{dt}, \overset{\circ}{h}, H, h, \} & v_r = 0 \\ \mu_k \{v, \theta, N, A\%, \eta, s, t, \overset{\circ}{h}, H, \check{h}, h\} & v_r \neq 0 \end{cases} \quad \dots\dots (2.13)$$

Obviously such an expression has only theoretical meaning and it cannot be used for friction determination. It is open to discussion the possibility of mathematical formulation of this expression. Previous attempts in that direction failed, as it is shown by the controversial results obtained, and the severe lack of generality. In the present state of tribological knowledge the use, in the mathematical analysis, of expressions derived experimentally seems to fit better than theoretically formed ones as equ.

TABLE 2.II: FACTORS AFFECTING THE COEFFICIENT OF FRICTION						
DRY SLIDING			LUBRICATED SLIDING			
μ_s	t_s		<u>Material</u> $\frac{\partial}{\partial N}$		<u>Lubricant material</u> $\frac{\partial}{\partial N}$	
	s		<u>Material or material and lubricant</u> $v_r, \dot{h}$ $\frac{dF}{dt}, t'$ (<u>viscoelastic behaviour</u>)			
	$\dot{h}$		● <u>*Not so strong effect in case of lubricated surfaces</u>			
	H h	● <u>*Very complicated functions of hardness and roughness especially for dry surfaces</u>				
	∂, N	● <u>indirectly through surface films</u>				
	Vibration impact	● <u>extremely sensitive</u> ● <u>rather insensitive</u>				
μ_k	v_r		<u>Material</u> $\frac{\partial}{\partial N} (p)'$ ● <u>*bifurcation</u>		<u>lubricant additive</u> (A%) η' lubricant $N' (p)$ quantity ● <u>*bifurcation</u>	
	s, t	Two stages: a: <u>running in</u> b: <u>steady-state</u> : approaches a constant value				
	$\dot{h}$	As for μ_s				
	H				$\dot{h}'$ (<u>slight effect</u>)	
	N		<u>material</u>		<u>material</u> <u>lubricant</u>	
	∂	<u>indirectly</u>			<u>lubricant</u> <u>material</u> $v_r, \dot{h}$	
	$\dot{h}$	<u>Slightly increasing</u>				
	h η	<u>Similar to H (there is a minimum)</u>		● <u>indirectly</u>		

2.2.2. The formation of the micro-model

The technique which could lead to realistic yet reproducible and as general as possible results will be examined here, for the case of frictional oscillations experimentation, where the micro-behaviour is of paramount importance.

Following strictly the same technique for the surface preparation of the specimens, made of the same material, a constancy of the initial values of H, h, h^0 is obtained. Also during the stick period of the cycle, N and $\frac{dF}{dt}$ can be assumed constant for $v_0 = \text{constant}$. During slip the temperature rises but it has been proved (Blok [16]) that temperatures higher than 5°C over the ambient temperature must not be expected, because of the heat dissipation in the environment during stick. Thus temperature fluctuation can be ignored and the mean temperature of the surfaces can be accepted as equal to the ambient temperature.

If the experiment lasts only for some cycles of stick-slip, s is very small and consequently its effect on μ_s is negligible.

The effect of idle time seems to be negligible as well, although Kragelskii ascribes stick-slip to that variation of the static coefficient of friction with t_s . In fact using Rabinowicz's formula:

$$\Delta\mu = \alpha \cdot t_s^\beta \quad \dots\dots (2.7.a)$$

with $\alpha = 0.04$, $\beta = 0.2$ taken from Brockley-Davies [157], one can see that even for $t_s = 1$ sec the increase $\Delta\mu$ rises to 4% which is negligible. Moreover if the characteristics of the apparatus employed are chosen in such a way so as to give a stick-slip period much less than unity the effect of t_s really disappears.

Assuming finally that the lubrication conditions are kept constant, follows that μ_k is not varying during the experiment due to $h, \eta, A\%$ variation.

The above elimination of factors acting to the micro-behaviour led to a

modification of equ. 2.13 which became:

$$\left. \begin{aligned} \mu_k &= \mu_k(v) \\ N_s &= \text{const.} \end{aligned} \right\} \dots\dots\dots (2.14)$$

This model is not new. It has been used extensively in the past because of its simplicity (see Chapter I). But its use was not justified in all cases.

To be able to use that model here, without oversimplifying the problem, the following corrections must be made:

a. - Under dynamic conditions and due to coupling of the modes of oscillation in the horizontal and vertical directions, the load fluctuates with the same frequency as the frictional force (but not in phase with it). Thus N is not constant as it was assumed but $N = N(t)$. (see also Lisitsyn [70], Kudinov-Lisitsyn [71]. A secondary fluctuation of other parameters (thickness of the lubricant film etc.) follows.

Is not possible to introduce this load variation in the equation of motion in an explicit form simply because it depends on the frictional oscillation frequency which is not "a priori" known. Thus an iteration numerical technique is advisable for load fluctuation calculation, while all the other factors following the load fluctuation will be assumed, for simplicity, constant.

b. - A close observation of the absolute horizontal and vertical displacements of the slider in typical frictional oscillation experiments (e.g. fig. 2.19) revealed that their variation is not smooth but fluctuates in a more or less irregular way. This can be explained by the fact that the energy is supplied from the lower moving surface to the slider in small finite quantities by some asperity bonding mechanism. Obviously parameters such as the height and strength of surface asperities and their distribution on the contact spot, the load and the mean sliding velocity are expected to affect that fluctuation. Low amplitude (100 ÷ 500 times less than the stick-slip amplitude on which it is superimposed) and rather wide spectrum of frequencies (if this fluctuation is considered as the result of superposition

of a number of harmonic vibrations) make its experimental study extremely difficult.

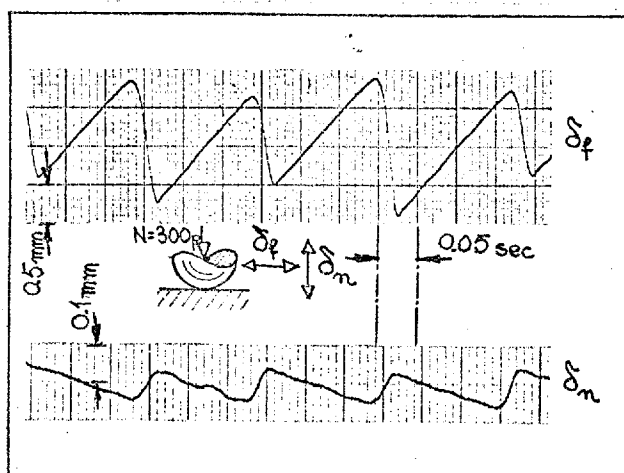


Fig. 2.19

An approximate statistical analysis of some results (see Appendix 2) shows that the characteristic of this fluctuation, which from now on will be called "triggering oscillation", vary mainly with load, mean sliding velocity, and the properties of materials and lubricants in use.

Triggering oscillation will be assumed for simplicity as being a harmonic vibration with amplitude A_{tro} equal to the mean value of amplitudes taken from a big sample of fluctuations and frequency ω_{tro} similarly equal to the mean value of frequencies.

The experiment reveals that the triggering oscillation appears basically in the vertical direction and affects stick-slip due to the coupling of vertical and horizontal modes of vibration. That is why triggering oscillation is hardly observable in the horizontal plane and had never been studied, and once only considered as an independent frictional phenomenon (frictional "microvibrations" of Blok [16]). Its fugitive nature and its probable dependence on the geometry of the apparatus also contributed in that (see also stick-slip theories based on geometry such as Broadbent's [31] and Spurr's [127]).

The frequency spectrum of triggering oscillation contains audio frequencies but their amplitude is so low that they cannot be detected by use of devices made for stick-slip experimentation. Squeal is probably its acoustical effect and some relationship between triggering oscillation and squeal appearance exists but a deeper study of that relationship, under

the present conditions, is rather impossible.

The fact that slight variation of the environmental conditions seriously affects the higher frequencies of the triggering oscillation spectrum is noteworthy (ambient relative humidity higher than 60% makes squeal disappear and decreases the rest spectrum of frequencies drastically).

The above discussion of triggering oscillation properties indicates clearly the second necessary correction which could lead to a realistic micro-behaviour.

Thus the final form of the micro-model is

$$\begin{array}{ll}
 \mu_s = \text{const.} & \} \\
 \mu_k = k(v, N) & \} \\
 N = N(t) & \} \dots\dots\dots (2.14.a) \\
 A_{tro} = A_{tro}(N), \omega_{tro} = \text{const.} & \}
 \end{array}$$

under the condition that all the tribological characteristics mentioned before will be kept constant.

2.3. THE MACRO-MODEL

2.3.1. Macro-behaviour. The mechanics of the system.

2.3.1.1. The equation of motion

The relationship between frictional force and relative velocity is the factor which determines the macro-behaviour of the system. This relationship is supplied by the micro-model. Assuming that the macromodel has only one degree of freedom on the horizontal plane, the system can be represented schematically as in fig. 2.20. The function $F = F(v_r)$ cannot be described in general, mathematically. It is given as an odd function of v_r :

$$\begin{array}{ll}
 F(v_r), v_o > \dot{x} & \} \\
 F(v_r) \equiv -F(-v_r), v_o < \dot{x} & \} \dots\dots\dots (2.15)
 \end{array}$$

or can be expressed by the Kronecker's formula [195]:

$$F = -\mu N \operatorname{sign} \{v_r\} \quad \dots\dots\dots (2.15a)$$

To solve the equation of motion, hypothetical or experimentally derived forms of that function are commonly used.

The self-excitation of oscillations was mainly attributed to the

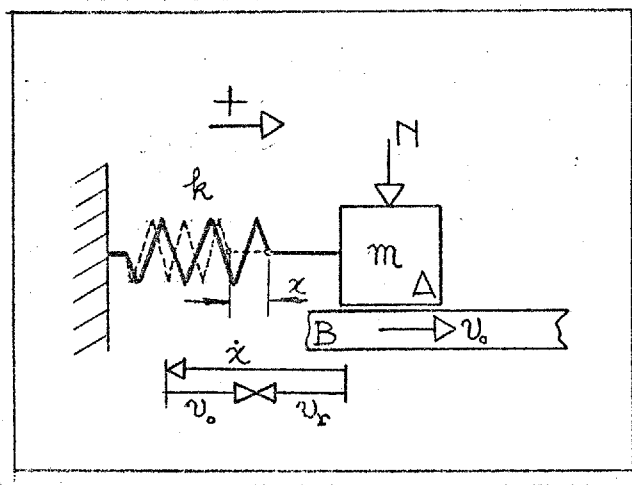


Fig. 2.20

difference between static friction

F_A and minimum kinetic friction

F_B (fig. 2.21) or to the negative

slope of the part AB of the

characteristic (Stoker [141],

Magnus [139], Cunningham [196]).

The position of the slider

A (fig. 2.20) is determined by

its distance x (which is met also

as δ_f on the experimental traces)

from the point at which the spring k supporting the slider is neither stretched nor compressed.

The equation of motion of the slider A is then

$$m\ddot{x} + F_a(\dot{x} - v_0) + kx = 0 \quad \dots\dots\dots (2.16)$$

(see also Watari [59], Hunt, Torbe and Spencer [51]).

By introducing a new variable:

$$X = x + \frac{1}{k} F_a(-v_0) \quad \dots\dots\dots (2.17)$$

which means that the position of the block A is now measured from its

equilibrium position under the combined action of the spring and the friction

forces, since $F_a(-v_0) + kx = 0$, the equ. 2.16 becomes:

$$m\ddot{X} + [F_a(\dot{X} - v_0) - F_a(-v_0)] + kX = 0$$

or:

$$\left. \begin{aligned} m\ddot{X} + F(\dot{X}) + kX &= 0 \\ F(\dot{X}) &= F_a(\dot{X} - v_0) - F_a(-v_0) \end{aligned} \right\} \quad \dots\dots\dots (2.18)$$

where:

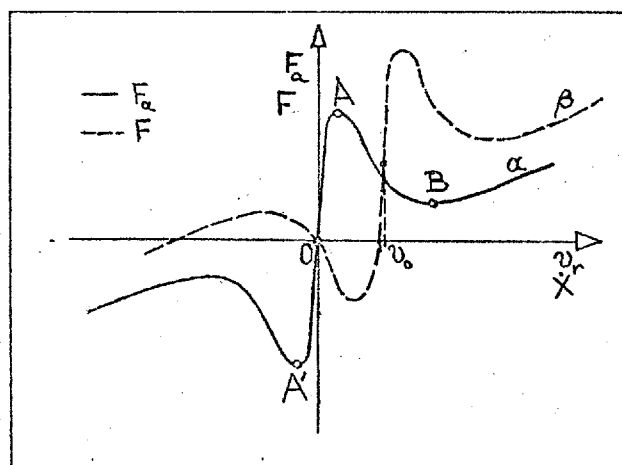


Fig. 2.21

The function $F = F(\dot{X})$ always passes through the origin because:

$$\lim_{\dot{X} \rightarrow 0} \{F(\dot{X})\} = \lim_{X \rightarrow a_0} \{F(\dot{X} - v_0) - F_a(-v_0)\} = 0$$

and appears as in fig. 2.21 (curve β).

The slope of the function $F = F(\dot{X})$ at the origin is very important because theoretically (no triggering oscillation) only for negative slope self-excitation of frictional oscillations can be achieved. This requirement will be fulfilled only if v_0 is such that the friction force decreases numerically with v_0 (negative frictional damping). It is worth noting here that the necessity of negative damping for self-excitation of a vibrational system was noticed by Lord Rayleigh in 1894 [197]. In his equation:

$$m\ddot{x} - (\alpha - \beta\dot{x}^2)\dot{x} + kx = 0$$

there is a predominance of negative damping for small values of the velocity $\dot{x}$.

The nonlinear function contains also all the additional resistances acting on the system as e.g. viscous damping forces and air resistances.

When the system operates under conditions permitting self-sustained oscillation and $\dot{x}$ becomes equal to v_0 , follows $v_r = 0$, $\ddot{x} = 0$ and the movement dies out at equilibrium. This obvious absurdity (stick period always leading to rest) comes from the fact that it was assumed that during stick $\dot{x} = v_0$ which is not correct. In fact numerous investigators (see Chapter 1) noticed slight relative movement during stick and solutions of the equation of motion with characteristic that of fig. 2.21 show also a very low but finite relative velocity during stick. Thus apart from the fact that the characteristic of fig. 2.21 is more realistic (no discontinuities)[58], has also the great advantage that the equation of motion is valid for every value of the relative velocity, which does not happen in case of discontinuous characteristics (slope of AA' tends to infinity, static coefficient of friction "jumps" from $+\mu_s$ to $-\mu_s$ for relative velocity varying from $0 + \Delta v_r$ to t_0).

0 - Δv_r where Δv_r is infinitely small).

2.3.1.2. Solution of the equation of motion

No analytical solutions of equ. 2.18 can be found in general except in the case where $F(\dot{X}) \ll 1$ and consequently the system behaves essentially as linear (weak nonlinearity). When this limitation is not fulfilled, quasi-linearisation techniques (Kryloff, Bogoliuboff [44], Macduff, Curreri [199], Andronow, Chaikin [200], Banerjee [52], etc.) are insufficient and numerical or graphical techniques must be employed.

Purely numerical treatment of the equation has the advantage of simplicity but the physical behaviour of the system cannot be studied adequately. Singularities or critical points cannot be detected easily and special mathematical techniques are necessary when discontinuous variables appear. On the contrary, purely graphical treatment gives a very good qualitative picture of the motion but derivation of quantitative relations is rather laborious and not accurate enough. Improved accuracy is obtained by grapho-analytical or easier by combined graphical-numerical methods which appear accurate and simple being thus the most convenient for application in the present case.

2.3.1.3. Application of Liénard's graphical construction

By introducing the new independent variable:

$$\text{where } \left. \begin{aligned} \tau &= \omega_n t \\ \omega_n &= \sqrt{\frac{k}{m}} \end{aligned} \right\} \dots\dots\dots (2.19)$$

the equation of motion becomes:

$$\frac{d^2 X}{d\tau^2} + \frac{1}{k} F\left(\sqrt{\frac{k}{m}} \frac{dX}{d\tau}\right) + X = 0 \dots\dots\dots (2.20)$$

This transformation, although not essential [196], has been used by many

investigators (e.g. [51], [59]) because it presents several advantages. Since τ is dimensionless, the dimensions of X and $\frac{dX}{d\tau}$ are the same and graphical constructions on the phase-plane (see Appendix 3) are greatly simplified. The solution curves which consist the phase-portrait of the

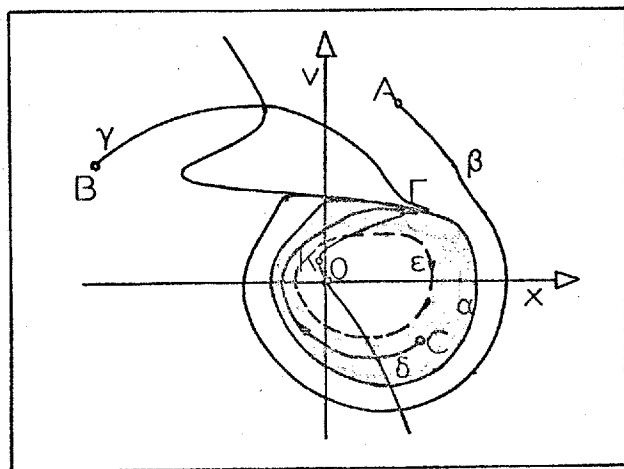


Fig. 2.22

model start from points $A, B, C, \dots$ defined by the initial conditions of each particular solution (Fig. 2.22). The steady state motion is represented by the closed cycle α (assuming that there is only one limiting cycle on the plane), and it can be formed by any ordinary trajectory starting from an arbitrary point $P(x_p, v_p)$, not

singular, because all the trajectories approach with the time cycle α . This is attributed to loss of energy content of external trajectories (like β or γ) or gradual increase of the energy content of internal trajectories (like δ), where the distance of the representative point from the origin, increases with the time (self-excitation). After stabilisation of the trajectories on α , they cannot "escape" due to the stability of the limiting cycle. Thus obviously each trajectory consists of two parts. The first one represents a transient state, while the second one which represents steady-state, coincides in fact with cycle α .

2.3.1.4. Singular points and limiting cycles

According to the theoretical consideration of Liénard's plane (App.3) there is one singular point only, in coincidence with the origin of the plane (equilibrium). The stability of that point O depends solely on the slope of the characteristic line at points very near to O . In fact in the

neighbourhood of 0 the characteristic line is practically straight, except where 0 coincides with K or Γ (fig. 2.22) where the characteristic line appears either as a straight line with slope $\frac{dv}{dx} = \pm \infty$ or as a broken line with two distinct values of slope $(\frac{dv}{dx})_1, (\frac{dv}{dx})_2$.

a. - Stability of ordinary singular point (not coinciding with K or Γ):

The fact that the characteristic line very near to it is essentially straight, means that for low amplitudes of frictional oscillation the system behaves linearly.

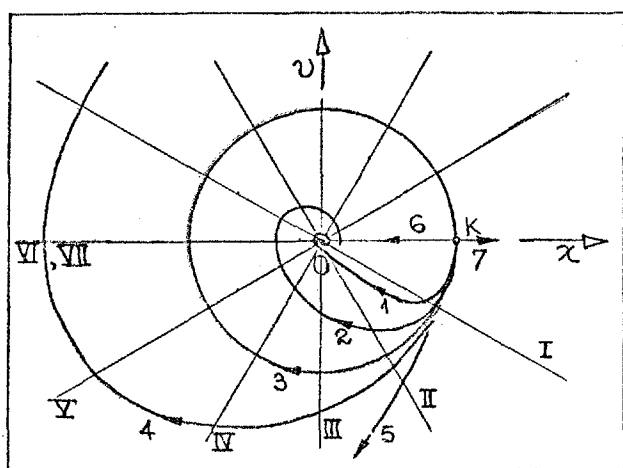


Fig. 2.23

Application of Poincaré's criterion for orbital stability shows that for negative slopes $\frac{dv}{dx}$ (fig. 2.23 I,II) the singular point 0 is stable (trajectories 1,2) while for positive slopes $\frac{dv}{dx}$ (IV,V) is unstable (trajectories 4,5). For infinite slope $\frac{dv}{dx} = \pm \infty$ (III) the trajectory

starting from K ends after 2π revolution of the describing vector to K (trajectory 3) which means that, in that case, point 0 is neither stable nor unstable. This is due to the fact that 0 being the origin and v varying, $\frac{dv}{dx} = \pm \infty$ means that necessarily: $x = 0$, and from the equation of the characteristic line follows $\varphi(v) = 0$. In that case equ. 2.16 ceases to describe the motion around the origin 0 which becomes a critical point.

The system behaves very near 0 in a way characteristic of conservative systems.

For $\frac{dv}{dx}$ approaching zero either from positive or negative values ($\frac{dv}{dx} \rightarrow +0$, $\frac{dv}{dx} \rightarrow -0$) the trajectory comes closer and closer to the characteristic line and finally coincides with the abscissa ($\frac{dv}{dx} = 0$). If that happens for $\frac{dv}{dx} \rightarrow +0$ the trajectory 7 (fig. 2.23) is formed (unstable)

while for $\frac{dv}{dx} \rightarrow -0$ the trajectory 6 is formed (stable). In that case Liénards's construction cannot be applied to relate points of the trajectory and the characteristic line. Without loss of generality it can be assumed for that particular case that, for the common part of trajectory and characteristic, one by one the points of the trajectory correspond to points of the characteristic line and vice-versa, and that the direction of the trajectory is defined by the direction of near lying trajectories.

b. - Stability when 0 coincides with K or Γ : In real systems where the slope of the characteristic is continuous, at the points K and Γ is $\frac{dv}{dx} = \pm \infty$ (fig. 2.24a) and this case has already been examined.

In mathematically simulated characteristics (Monastyrshin [195]) as e.g. in fig. 2.24.b. where the slope presents a discontinuity at K and Γ the

situation is much more complicated.

If K, Γ divide the characteristic in two straight parts such as:

$$\frac{dv}{dx}]_{-} = - \frac{dv}{dx}]_{+}$$

(where the symbols $]_{-},]_{+}$ mean immediately before and after respectively) trajectories starting at a point K' or Γ' very near K, Γ are symmetrical in respect to the abscissa and closed

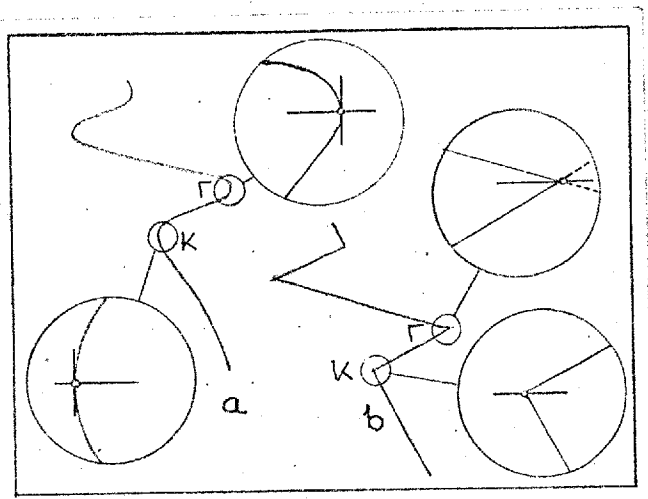


Fig. 2.24

trajectories are produced (degeneration of the equation, behaviour of conservative system). If on the contrary:

$$\frac{dv}{dx}]_{-} \neq - \frac{dv}{dx}]_{+}$$

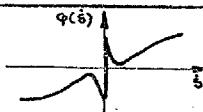
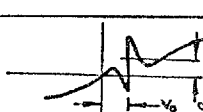
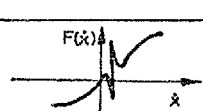
the stability of the singularity 0 depends on the combination of slopes and can be either stable or unstable.

It is obvious that only the case of real characteristics is of some

interest in case of applied work. The stability of the origin is very important because it governs the development of the frictional oscillations. According to Poincaré on the phase-plane of equ. 2.16. there is at least one limit cycle lying around the origin. Application of the criterion for orbital stability shows immediately that this cycle is stable and there is no other cycle around it on the plane. The existence of a second (unstable) limit cycle between the origin and the stable limiting cycle, depends on the stability of the origin. Only if the origin is stable, there is an unstable limiting cycle around it (cycle ϵ fig. 2.22) dividing the area of the stable limiting cycle into two parts, the outer where stabilization is obtained on the cycle α and the inner, where stabilization is obtained on the origin 0.

2.3.1.5. The reverse transformation

It has been seen so far how solution curves can be derived from the equation of motion if the "instrumental" factors (mass, stiffness, damping, mean sliding velocity) and the characteristic line are known. These curves were drawn on Liénard's plane. To transfer information from that plane to the original phase-plane, a reverse transformation is necessary. Table 2.III indicates the transformations used to obtain Liénard's plane:

TABLE 2.III TRANSFORMATION ORIGINAL PHASE-PLANE — LIÉNARD'S PLANE			
Original equation	$m\ddot{x} + F_a(\dot{x} - v_0) + kx = 0$		
Spring-friction equilibrium			$X = x - \frac{1}{k} F_a(-v_0)$
Transformed equation	$m\ddot{X} + F(\dot{X}) + kX = 0$ $F(\dot{X}) = F_a(\dot{X} - v_0) - F_a(-v_0)$		
New time-variable			$\tau = \sqrt{\frac{k}{m}} t$
Final form	$\frac{d^2X}{d\tau^2} + \frac{1}{k} F\left(\sqrt{\frac{k}{m}} \frac{dX}{d\tau}\right) + X = 0$		

It is obvious that to obtain the original phase-plane the following two processes are necessary:

- a. - To multiply the velocity axis by $\sqrt{\frac{k}{m}}$ which gives real velocities instead of non-dimensional ones.
- b. - To subtract v_0 from the v-axis and to position the origin at the center of symmetry of $F(\dot{x})$ by subtracting $a = F_a(-v_0)$.

2.4. MICRO- AND MACRO- MODEL COOPERATION

2.4.1. The final form of the model

The analysis of the micro-behaviour of the system showed that experimental trajectories obtained by a system, assumed as having a single degree of freedom, must be corrected for load $N = N(t)$ and triggering oscillation $\{A_{tro}, \omega_{tro}\}$ fluctuations.

2.4.1.1. Load variation and load correction for real systems

The nonlinear function $F(v_r)$ can be written:

$$F(v_r) = N \mu(v_r) \quad \dots\dots\dots (2.21)$$

where the load N is assumed constant. In fact due to vertical movement of the slider (Lisitsyn [70]) the load is not constant but fluctuates with the same frequency as the frictional force. Instead of solving a system of differential equations of the form proposed by Lisitsyn, it is easier to introduce a correction factor $C_N = C_N(t)$ representing the load fluctuation

$$C_N = \frac{N_{real}}{N_{mean}} \quad \dots\dots\dots (2.22)$$

The movement of the slider in the vertical direction is described by an equation

$$m\ddot{y} + f(\dot{y}) + k_y y = 0 \quad \dots\dots\dots (2.23)$$

where $f(\dot{y})$ is a non-linear factor (due to the influence of the nonlinearity in the horizontal plane).

Differentiation of the vertical displacements $y = y(t)$ gives the velocities $\dot{y} = \dot{y}(t)$. Thus equ. 2.23 can be solved by means of the Liénard's construction precisely as for the movement on the horizontal plane, and the correction factor C_N can be obtained. The real value of the nonlinear function $F(v_r)$ is then:

$$F(v_r)_{\text{real}} = F(v_r) \cdot C_N \quad \text{..... (2.24)}$$

2.4.1.2. Triggering cycle, triggering oscillation correction

Triggering oscillation due to its very low amplitude does not seriously affect the instantaneous phase-portrait of the system. What is affected is the development of the motion in the long run (e.g. it can remove the representative point of an unstable equilibrium and trigger thus a frictional oscillation, which otherwise could only be obtained after infinite time $t \rightarrow \infty$).

Techniques for "smoothing down" the triggering oscillation will be used for the experimental trajectories, because its presence introduces considerable error in the graphical constructions (the determination of the equilibrium point and consequently the precise positioning of the characteristic line cannot be done efficiently even with very low amplitudes of triggering oscillation).

The triggering oscillation appears on the phase-plane as an ellipse γ

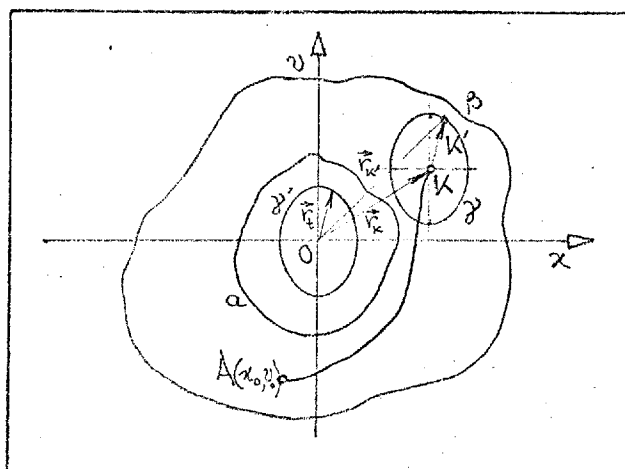


Fig. 2.25

(fig. 2.25) with center the representative point K , and it determines a new representative point K' for an instant t , the position of which obviously depends on $t, A_{\text{tro}}, \omega_{\text{tro}}, x_0, v_0$ and the initial phase difference of the triggering oscillation $\{\omega_{\text{tro}}\}_0$. Transferring γ to γ'

(center at the origin), the following relation is obtained:

$$\vec{r}_{k'} = \vec{r}_t + \vec{r}_k \quad \text{..... (2.25)}$$

and superposition principle can be applied, although this is not correct in general for two superimposed vibrations in a non-linear system.

The ellipse γ' is called the triggering cycle and expresses the idealised form of the triggering oscillation on the phase-plane.

It is noteworthy that due to the existence of the triggering oscillation, the motion on the unstable limiting cycle and the equilibrium at the unstable origin have never been observed experimentally and they were mentioned in the literature only twice [58,59] theoretically. Also convincing is the fact that to have a triggering cycle intersecting limiting cycles and thus affecting the behaviour of the system, it is not necessary to have great triggering oscillation amplitudes. Even assuming the amplitude A to be very small, $A\omega$ (see App.3) could be enough (for ω high) to excite an oscillation. Devices made for mechanical vibration measurements are unable to detect high frequencies at low amplitudes. That is why triggering oscillation very rarely appears in the literature (e.g. Blok's microvibrations [16] explained differently) and never as an independent physical entity.

Although triggering oscillation depends on the load, it will be assumed that small load fluctuations do not affect it (dependence on the mean load N_{mean}).

Obviously triggering oscillation has major importance in the neighbourhood of unstable points or cycles, while it becomes unimportant near stable ones. In case of a stable origin a trajectory approaching it stabilizes itself on the triggering cycle γ' instead of the origin 0, and in that case equilibrium is replaced by quasi-sinusoidal vibration.

2.4.2. Discussion on the theoretical model

2.4.2.1. Effect of the mean driving velocity

In § 2.3.1.3. the importance of the position of the origin 0 was

recognised and found to be a predominant factor affecting the whole phenomenon. This position of 0 is determined by the mean driving velocity v_0 (see also table 2.IV.)

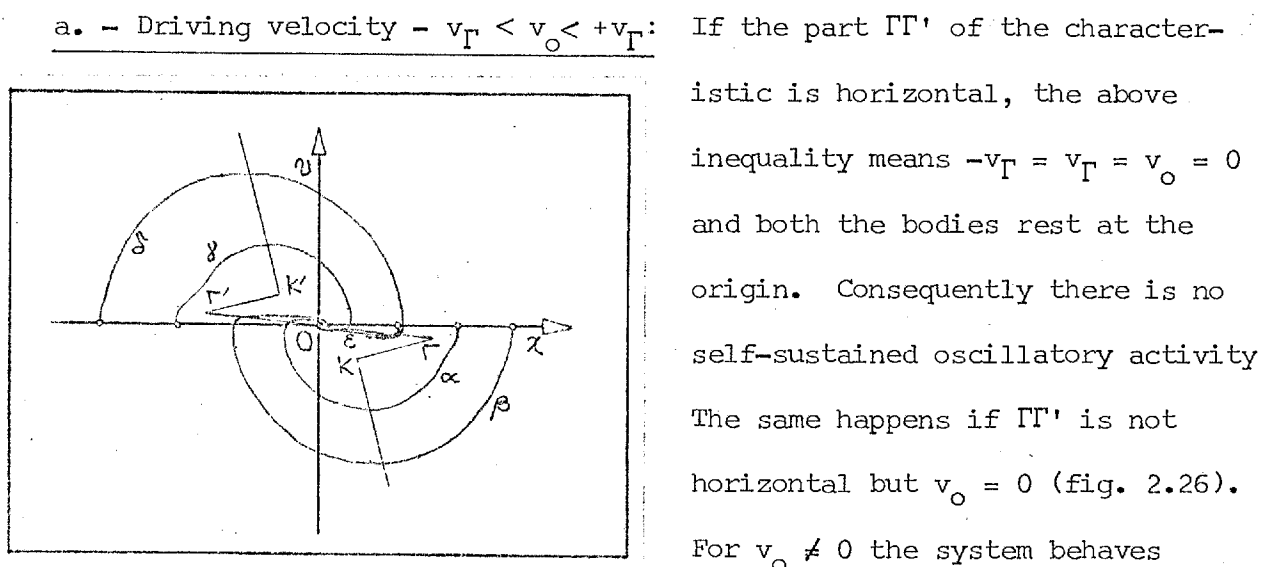


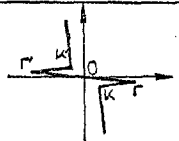
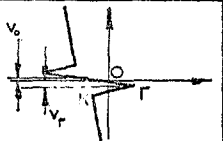
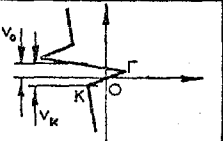
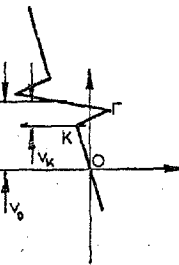
Fig. 2.26

origin is a stable focal point, there are no limiting cycles on the plane and disturbances of the slider equilibrium $x_{\epsilon}, x_{\alpha}, x_{\beta}, x_{\delta}, x_{\gamma}$, produce damped oscillations (trajectories $\epsilon, \alpha, \beta, \delta, \gamma$).

b. Driving velocity: $v_{\Gamma} < v_0 < v_k$ or $-v_k < v_0 < -v_{\Gamma}$: This case almost exclusively attracted the scientific interest because it is the only case where self-excited conditions appear. The positive slope $\frac{dv}{dx} > 0$ at the origin (or the negative slope $\frac{dF(\dot{x})}{dx}$) indicates that the origin is unstable. Accordingly a stable limiting cycle exists on the plane. Equilibrium at the unstable origin (which is an unstable focal point) could be disturbed only by the triggering oscillation.

c. Driving velocity: $v_0 = \pm v_k$ or $v_0 = \pm v_{\Gamma}$: This is a limiting state and the form of the phase-plane trajectories, especially very near the origin, have been seen to depend on the contribution of slopes before and after the origin.

TABLE 2.IV: DRIVING VELOCITY EFFECT ON FRICTIONAL OSCILLATIONS

	DRIVING VELOCITY	CHARACTERISTIC LINE	CLOSED TRAJECTORIES			FRICTIONAL OSCILLATIONS		ORIGIN STABILITY	NOTES
			STABLE L.C.	UNSTABLE L.C.	TRIGGER CYCLE	SELF- SUSTAIN	TRIGGER (QUASI- SIN.)		
1	$v_0=0$							<u>Stable</u>	No motion. The same applies to 2 for horizontal part $\Gamma\Gamma'$.
2	$-v_f < v_0 < v_r$							<u>FIRST CRIT. VEL.</u>	No self-sustained oscillation; equilibrium on the triggering cycle.
3	$v_r < v_0 < v_k$ $-v_k < v_0 < v_r$		+			+		<u>unstable</u>	Self-excited oscillations
4	$v_0 > v_k$ $-v_k < v_0$ but $v_0 < v_{oc}$		$\bar{r}_{SLC} > \bar{r}_{ULC}$		+	Depends on the trigger, oscillation	+	<u>Stable</u>	The behaviour of the system depends on the characteristics of the triggering oscillation
4a	$v_0 = v_{oc}$		$\bar{r}_{SLC} = \bar{r}_{ULC}$			Meta-stable (trig. oscil.)		(SECOND CRIT. VEL.?)	
4b	$v_0 > v_{oc}$							<u>Stable</u>	

d. Driving velocity $v_o > v_k$ or $v_o < -v_k$: The origin becomes stable singular point ($\frac{dv}{dx} < 0$) and an unstable limiting cycle exists between the origin and the stable limit cycle. As v_o increases the stable limiting cycle decreases in size while the unstable limiting cycle increases. Thus for a certain velocity v_{oc} stable and unstable limiting cycles, coincide. For $-v_{oc} < v_o < -v_k$ or $v_k < v_o < v_{oc}$ the existence or not of frictional oscillations of the self-sustained type, depends on the characteristics of the triggering oscillation. In the case where the triggering oscillation is not enough to excite self-sustained oscillation, the system stabilizes itself on the triggering cycle (quasi-sinusoidal behaviour).

For $v_o = \pm v_{oc}$ there is only one limiting cycle, stable from outside and unstable from inside. This "metastable cycle" is obviously affected by the triggering oscillation which can lead to stabilization on the triggering cycle.

Finally for driving velocities $v_o < -v_{oc}$ or $v_o > v_{oc}$ the limiting cycles disappear simultaneously.

Assuming that triggering oscillation increases with the mean driving velocity (§ 2.2.2.) the above discussion is summarized in Table 2.IV, which gives a fair explanation of the very well known three distinguished forms of frictional oscillations.

The rather complicated effect of the mean driving velocity on the frictional oscillation can be studied easily by solving the equation of motion for several values of v_o and then relating the variation of v_o with the variation of the principal geometrical characteristic on the phase-plane. The necessary graphical constructions and calculations for that have been done by means of the computer program LIENG made especially for that purpose (Appendix 4). Thus fig. 2.27 shows the variation of the dimensions of the stable (S.L.C.) and unstable (U.L.C.) limiting cycles with velocity for an assumed simple form of the characteristic line. The magnitudes are expressed in nondimensional form, where v_{om} is the velocity for which the coefficient

of friction has its minimum value, A is the amplitude of frictional oscillation (the distance between two successive points of intersection of the trajectory and the abscissa on the phase-plane) and $A_{o\mu}$ is the amplitude corresponding to driving velocity $v_{o\mu}$. On that diagram one observes that for velocities less than $v_c/v_{o\mu}$ external trajectories^(a) fall on the S.L.C. line and internal^(b,γ) either on the S.L.C. line or on the abscissa depending upon the initial conditions, while for velocities $v_o/v_{o\mu} > v_c/v_{o\mu}$ all the trajectories⁽⁶⁾ move to the abscissa.

The results presented in that diagram agree with Watari and Sugimoto [59] but not with the results of Brockley, Cameron, Potter [49] and Fleischer [32] probably due to their oversimplification of the problem.

2.4.2.2. Effect of the difference $\Delta\mu = \mu_s - \mu_k$

In many of the early works about stick-slip, it was concluded that the difference $\Delta\mu = \mu_s - \mu_k$ is a factor acting a very strong influence on the phenomenon, while μ_s and μ_k affect stick-slip indirectly through variation of $\Delta\mu$. Taking $\Delta\mu_o = \mu_s - \mu_{k_o}$ instead of $\Delta\mu$, where μ_{k_o} is the minimum kinetic coefficient of friction, which means that for μ_k varying $\Delta\mu$ is a variable while $\Delta\mu_o$ is constant and analysing a hypothetical case where all the parameters are kept constant except $\Delta\mu_o$, it was found that in fact the above conclusion is correct.

Speaking in terms of the phase-portrait of the system, variation of $\Delta\mu_o$ produces an increase of the dimensions of the limiting cycles, while variation of μ_s or μ_{k_o} separately under $\Delta\mu_o$ constant produces a change of the limiting cycle position but leaves unaffected its size. Thus the equilibrium position changes but not the amplitude of the oscillation.

Fig. 2.28 shows how the amplitude of self-sustained oscillation varies

Fig. 2.28

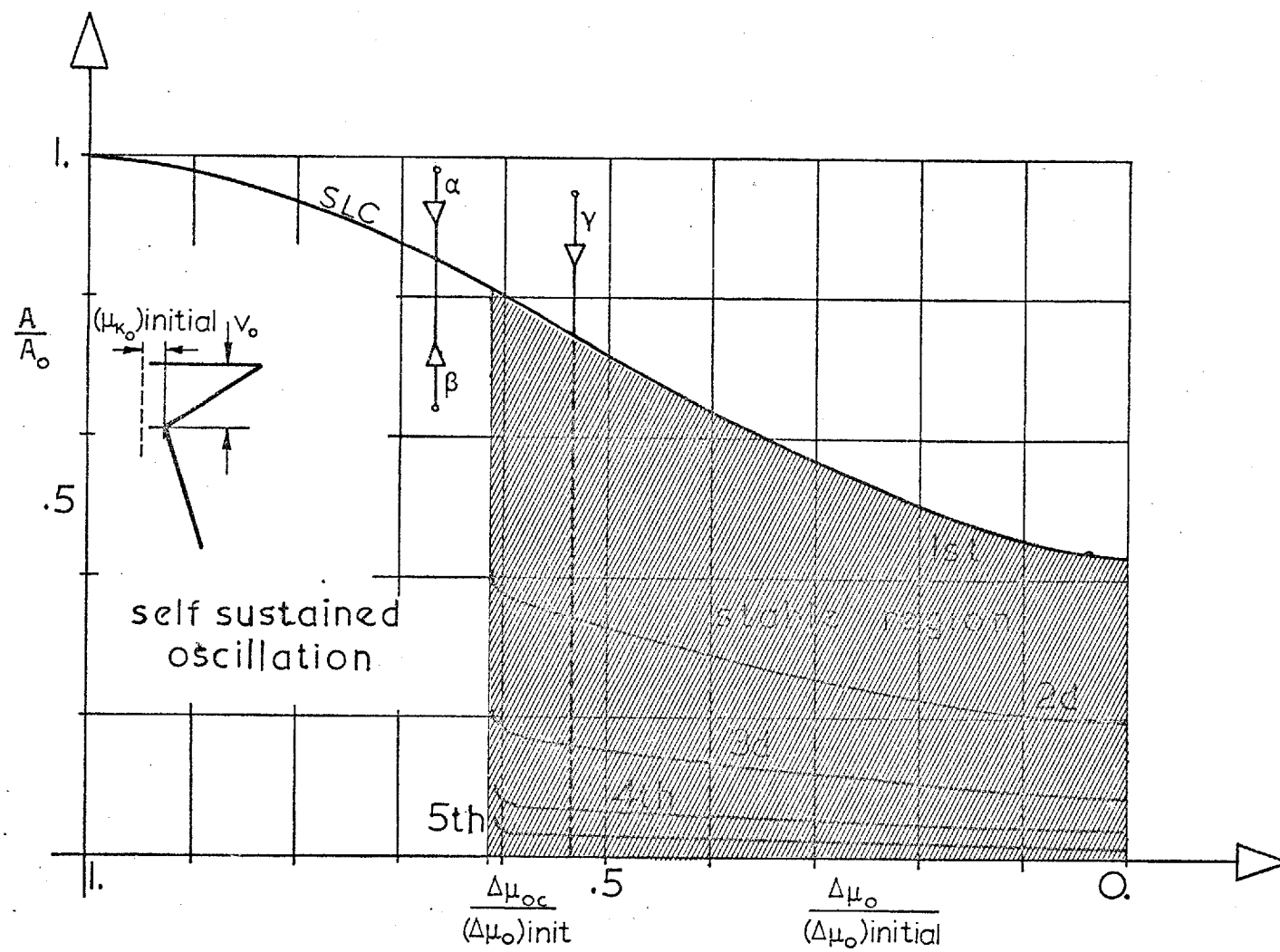
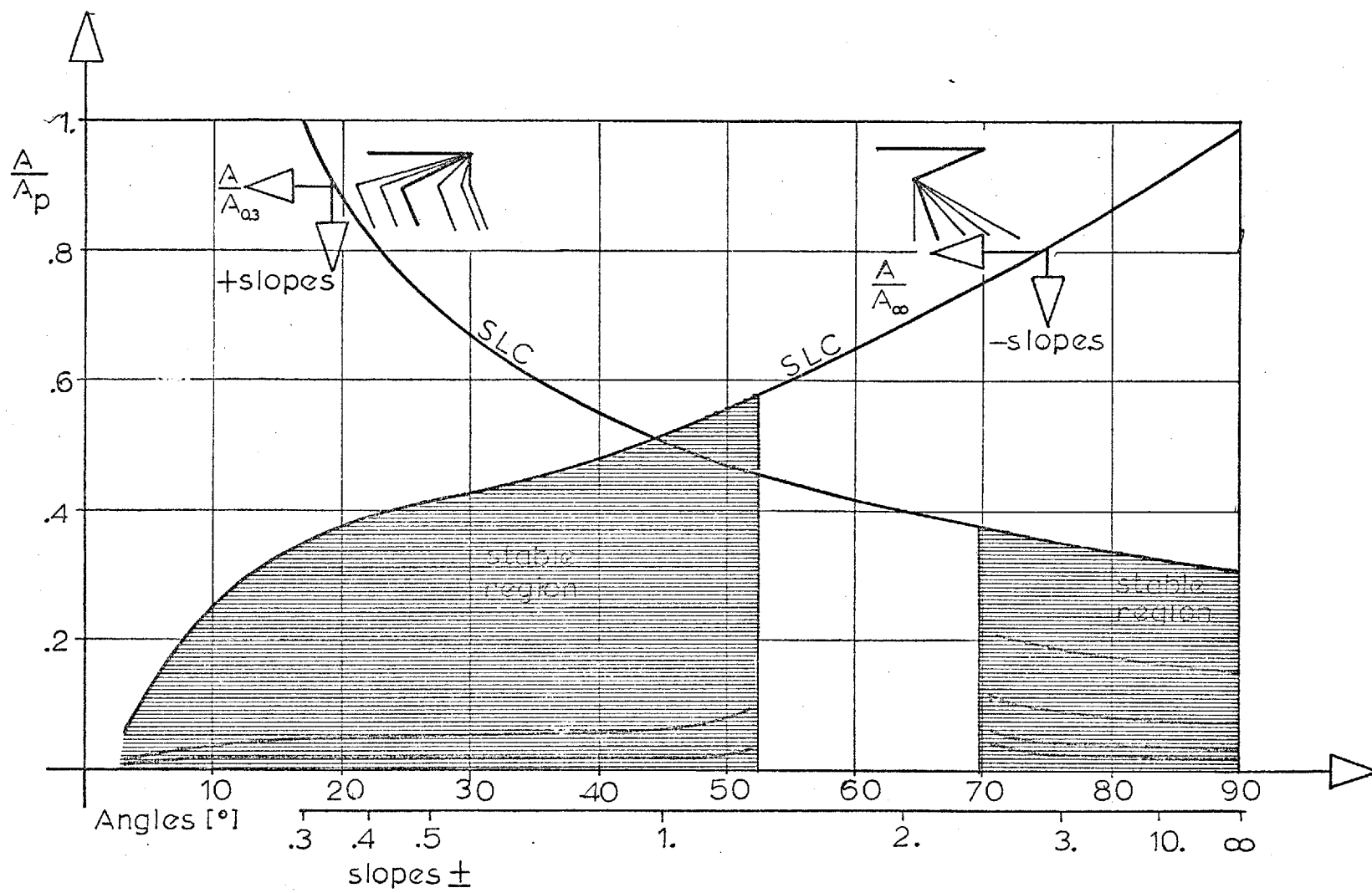


Fig. 2.29.



with $\Delta\mu_0$. What must be emphasized is that for a given mean driving velocity v_0 there is a critical value $\Delta\mu_{oc}$ such that for $\Delta\mu_0 < \Delta\mu_{oc}$ self-sustained frictional oscillations disappear.

Amplitudes of oscillation and $\Delta\mu_0$ are expressed in that diagram in non-dimensional form referred to an arbitrary pair of values $\{A_0, \Delta\mu_{0, \text{initial}}\}$.

2.4.2.3. Effect of the slope of the characteristic

Similarly the effect of varying slope of the characteristic line can be studied. Fig. 2.29 shows how the slope of the two parts of the mathematically simulated characteristic line (for $\mu > \mu_s$) affect the amplitude of oscillation or the stability of the system.

It is obvious that when the slopes of both parts are infinite, the limiting cycle becomes a circle (behaviour of conservative system) and if the first part has zero slope and the second infinite, the limiting cycle consists of a circular arc closed by a straight part and, consequently, the movement is self-sustained oscillation consisting of straight (stick) and harmonic (slip) parts. (see also review of the literature).

CHAPTER 3 : EXPERIMENTAL

3.1 EXPERIMENTAL RIGS

3.1.1. General design principles

Almost exclusively in the existing literature a system exhibiting frictional oscillations is represented by a single-degree of freedom model, like the one of fig. 2.1. For such a simplification to be justified, the fulfilment of certain conditions is necessary:

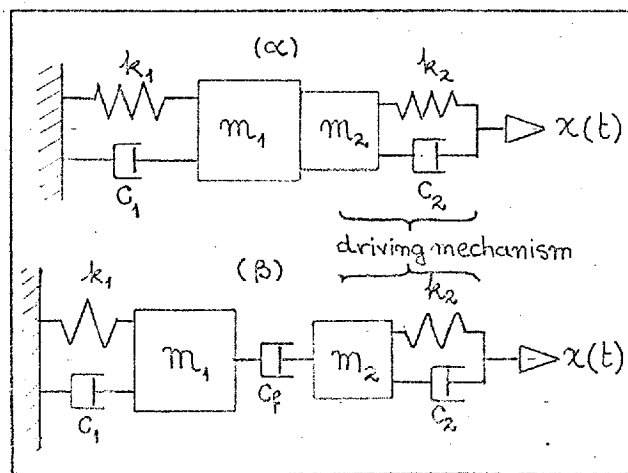


Fig. 3.1.

a. - The system in general consists of the slider and its support (m_1, k_1, c_1 fig. 3.1), the moving specimen and the driving mechanism (m_2, c_2, k_2). During stick the two specimens move as one body (single-degree-of-freedom behaviour, case (α) fig. 3.1) while during slip the two specimens move independently, being coupled only by the dashpot

c_f representing the frictional forces between them. In that case (fig. 3.1.β) the system behaves as a two-degree of freedom system.

Thus a simplification of the system to a single-degree of freedom system is acceptable only if the driving mechanism attached to the moving specimen behaves as a rigid body. That can be obtained by keeping the mass m_2 as low as possible and increasing k_2 in which case (m_2, k_2, c_2) becomes essentially a rigid body, and the system is reduced to the one of fig. 2.1. Due to its high stiffness, no force or displacement measurement can be made by measuring the distortion of the driving mechanism frame.

b. - To avoid errors in force and displacement measurements, no frictional pair must be interposed between the specimens and the force measuring device.

c. - The force measuring device must be as close as possible to the specimens, to avoid errors due to unpredictable distortions of the slider supporting frame.

d. - According to the preceding theory about phase-plane solution of the equation of motion, high natural frequency of the slider system (k_1, c_1, m_1) is desirable. Considering that the stiffness k_1 must be kept low (high sensitivity of displacement measurements) follows that m_1 must be as low as possible.

e. - Finally the coupling between horizontal and vertical modes of oscillation must be weak and the force measuring devices must be sensitive in one direction only. Practically it is not feasible to have full freedom of the horizontal and vertical modes. Thus the measurement error was calculated and subtracted to give the real measurement (normally that error does not exceed 5 - 6%).

3.1.2. Rig Mark I

This is a typical low-speed "pin on disc" friction machine (fig. 3.2) where the disc (A) rotates by means of a low-speed, high stability, servo-controlled turn-table, while the spherically ended pin (B) is fixed on specially designed elastic support, used as the dynamometer. It consists of three rigid frames (C,D of aluminium and E of steel) connected by two pairs of leaf springs (F and G), the horizontal set of springs (F) measuring, by means of strain gauges fixed on the springs, normal forces and displacements, while the vertical set of springs (G) measures frictional forces or displacements in the direction of the friction.

By correct positioning of the pin (B) the contact spot can be located

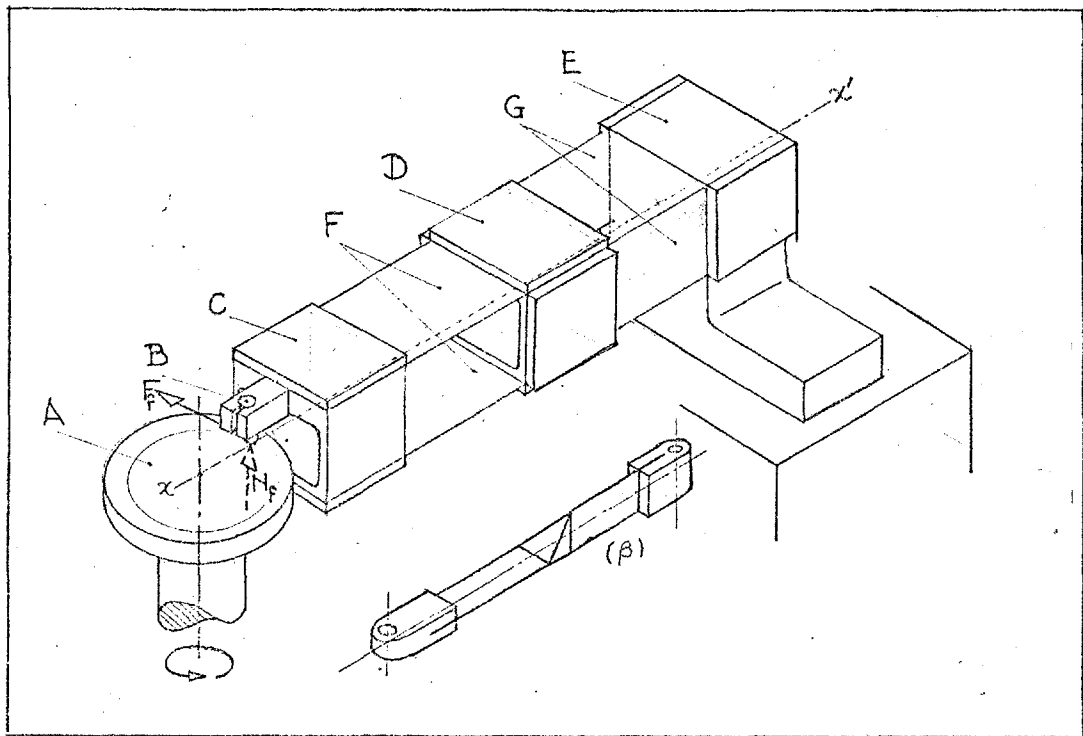


Fig. 3.2.

on the central axis xx' of the frame and consequently the forces N_f, F_f produce, for small displacements, pure bending of springs. For greater displacements the system is loaded eccentrically and torsional couples appear which increase the coupling of the vertical and horizontal modes.

This "twin-leaf-spring" dynamometer is in fact a modified version of the well known "twisted-bar" dynamometer (fig. 3.2. (β)) used several times in tribological applications. The main advantage of the "twin-leaf-spring" dynamometer, compared with the "twisted-bar" is that independently of the displacement magnitude (distortion of the springs) the slider is always kept parallel to itself. Accordingly the frictional conditions between slider and disc remain constant for every value of displacement, provided that the system operates within the elastic range of the springs, because the contact spot remains in the same position on the slider for every displacement, and does not move very far from the wear track on the disc. For the maximum

is obtained and the instrument on the strain gauge apparatus SGA gives the distortion of the springs F which is proportional to the applied load. Then

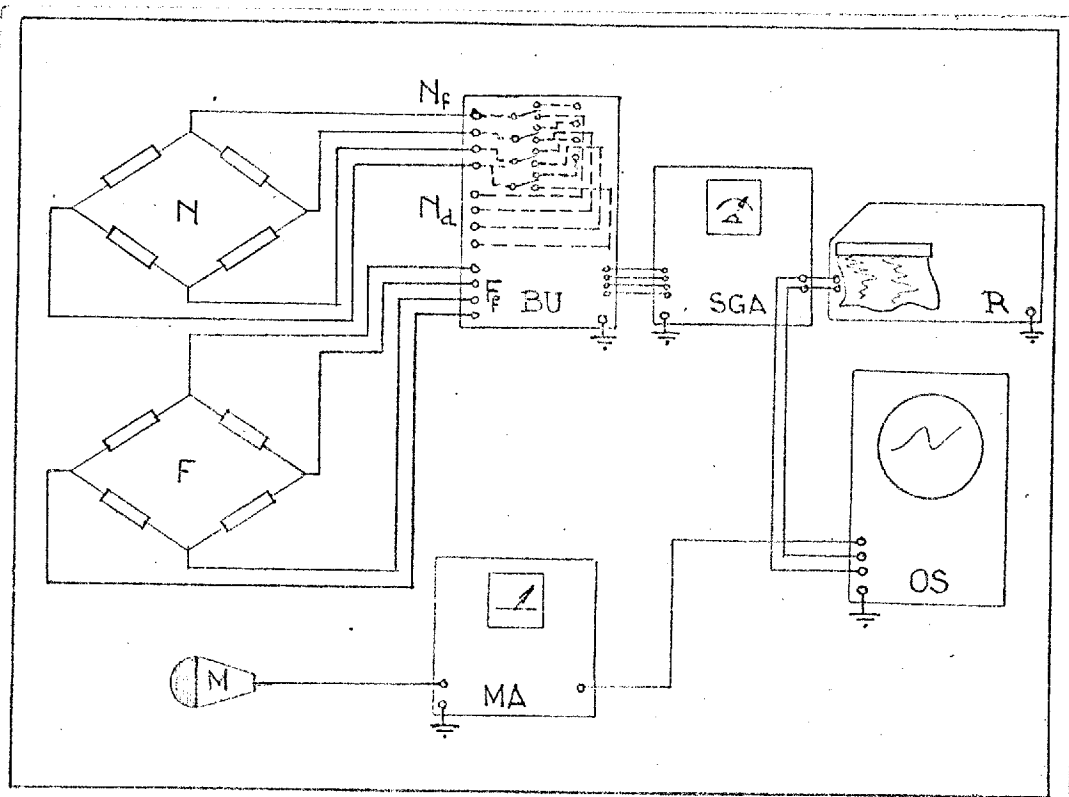


Fig. 3.4.

the disc is fixed and bridge N is connected to the N_d balancing unit and is balanced anew, while bridge F is balanced as well by means of the F_f balancing circuit.

It is obvious from the way N_f -bridge and N_d -bridge are balanced that N_f measures the total distortion of the springs, which under static conditions is proportional to the load, while N_d measures distortion fluctuations, with zero level the distortion under load N . By greatly amplifying the signal coming out of the N_d circuit one can record normal displacement fluctuation while by switching to the N_f circuit, the real load value is monitored.

In this experimental course normal load and normal and frictional displacements were measured or recorded. A modified multi-input Brüel-Kjaer balancing unit a Höttinger strain gauge apparatus and a Höttinger stain



Fig. 3.5

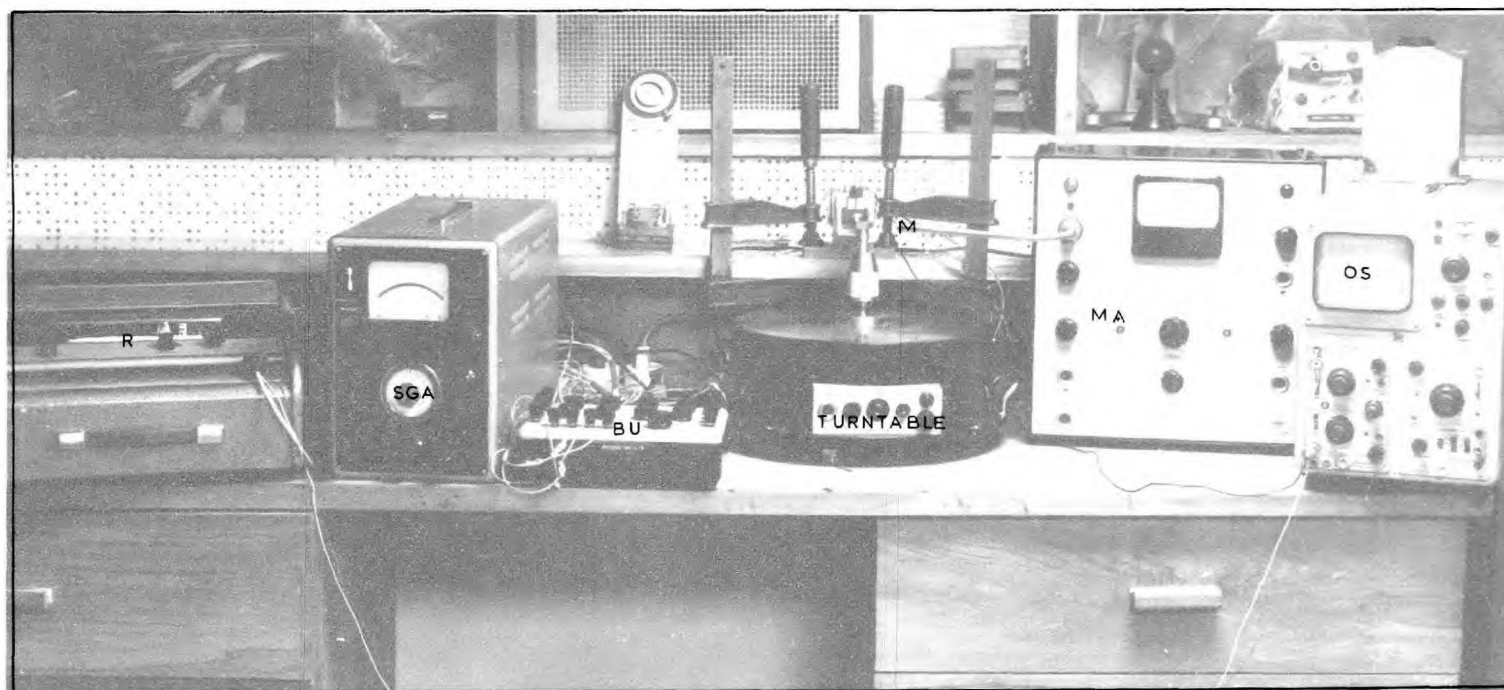


Fig. 3.

recorder were used for recording displacements or measuring the normal load.

A Tectronix Storage oscilloscope connected in parallel with the strain recorder was used to give high resolution samples from the traces recorded on the recorder. Single sweep triggering technique and storing of the picture was used for that purpose. Comparison between the traces recorded on the strain recorder and the ones stored on the oscilloscope screen showed that no measurable differences appear for natural frequencies of the dynamometer less than 60 Hz.

Additionally a high linearity condenser microphone (Brüel-Kjaer) and a microphone amplifier (Brüel-Kjaer) were used for examination of the squeal produced by the frictional pair.

A high stability A.C. power supply was used to feed all the instruments to avoid errors due to fluctuating mains voltage (output voltage stability $\pm 0.1\%$ nominal for input voltage fluctuation $\pm 12\%$ nominal).

As specimens a spherical slider and a disc were used (fig. 3.6).

The general layout of instruments and the apparatus appears in fig. 3.5., while the operational characteristics of Rig Mark I are included in Appendix 5.

3.1.3. Rig Mark II

This is a modified version of a rig used previously in attempts to measure the coefficient of friction at low sliding speeds (Cole [209], Aylward [210], Thorp [211]). It is based on the idea of specimens formed as arc and ring (or ball)(fig. 3.7), where the ring is rotating with constant speed, while the arc oscillates about its axis, executing one oscillatory cycle for each revolution of the ring. The frictional pair consists of the circumference of the ring and the inner surface of the arc.



Fig. 3.6

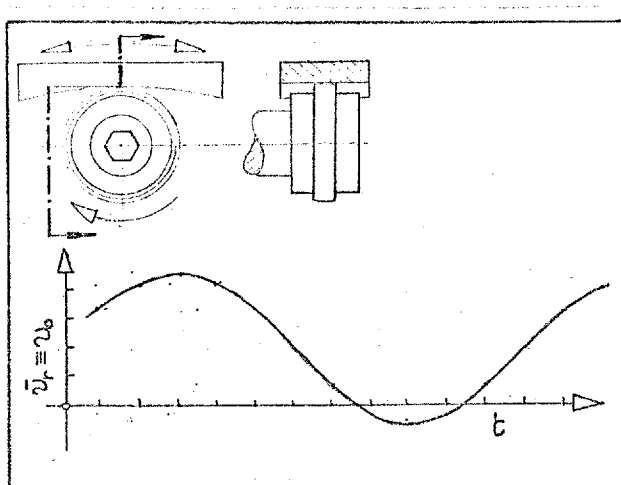


Fig. 3.7.

The mean relative velocity $\bar{v}_r$ (i.e. the driving velocity v_0) is not constant due to the oscillatory motion of the arc but varies periodically with time, according to a law depending on the geometry of the system.

The load is applied on the frictional pair through an octagonal or ring dynamometer, measuring simultaneously horizontal and vertical forces (or displacements). The arc shaped slider is fixed on the lower surface of the dynamometer, thus eliminating errors due to distorted intermediate mechanical parts. By using a series of dynamometers with different geometry, the effect of the stiffness or natural frequency on the frictional oscillations can be studied. The stiffness of the two frames on which the dynamometer-arc and the ring are based is much higher than the stiffness of the dynamometer itself; that means that frictional forces and variation of the normal load produce deflections of the dynamometer arms alone, while the rest of the mechanism does not suffer any distortions.

To optimize the design, a stiffness as high as possible combined with low mass (inertia) is desirable. This is a fundamental rule on which all

the rig modifications have been based.

3.1.3.1. Ring moving mechanism

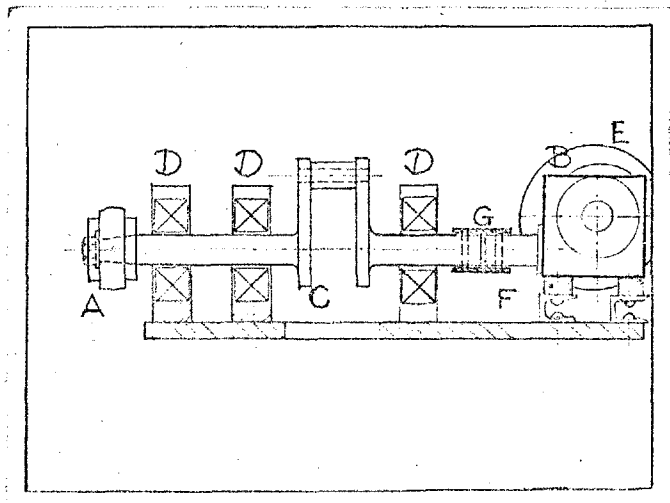


Fig. 3.8.

The ring A is driven by a motor-gearbox system B giving a constant driving speed of 1.1 rpm, through a camshaft C used to drive simultaneously the arc-specimen (oscillatory motion). The camshaft is supported by three self-aligned ball bearings D.

The motor is equipped with a flywheel E for smoother driving, it is properly balanced and it is isolated from the rest of the system by the rubber pads F and the coupling G.

The shaft is designed to stand loads up to 100 kp with less than 5μ deflection at A. The torsional stiffness of the whole driving mechanism is enough to permit the assumption that the driving mechanism behaves as a rigid body.

Additional reduction, if it is necessary can be obtained by means of a separate motor driving the gearbox input shaft through belts and pulleys (final speeds 0.4 rpm or 0.145 rpm).

3.1.3.2. Slider (arc) driving mechanism

The arc B is driven by the crank of the ring moving mechanism A (fig. 3.9) through the driving arm C, which is rigidly fixed onto the frame D, supporting the dynamometer-arc (E,B) system. Frame D is free to rotate around the xx' axis fixed on the frame F which in turn rotates around y axis. The load N is applied at the lower end of frame F.

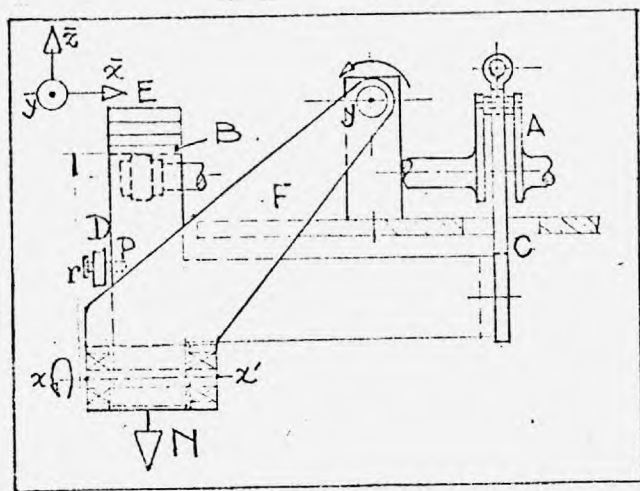


Fig. 3.9

The radius of curvature of arc B is r so that load N does not produce work when the arc rotates around xx' axis. From fig. 3.9 it is obvious that the arc has two degrees of freedom of rotation around the axes xx' and y , which means that practically, for small displacements it can be considered as free to move along $\bar{y}$ and $\bar{z}$ axes (fig. 3.9).

This freedom is necessary for load application and friction-load measurements.

Stiffness measurements under several loading conditions showed that the frames D and F can be considered as perfectly rigid, in which case forces produce deflexions of the dynamometer arms alone.

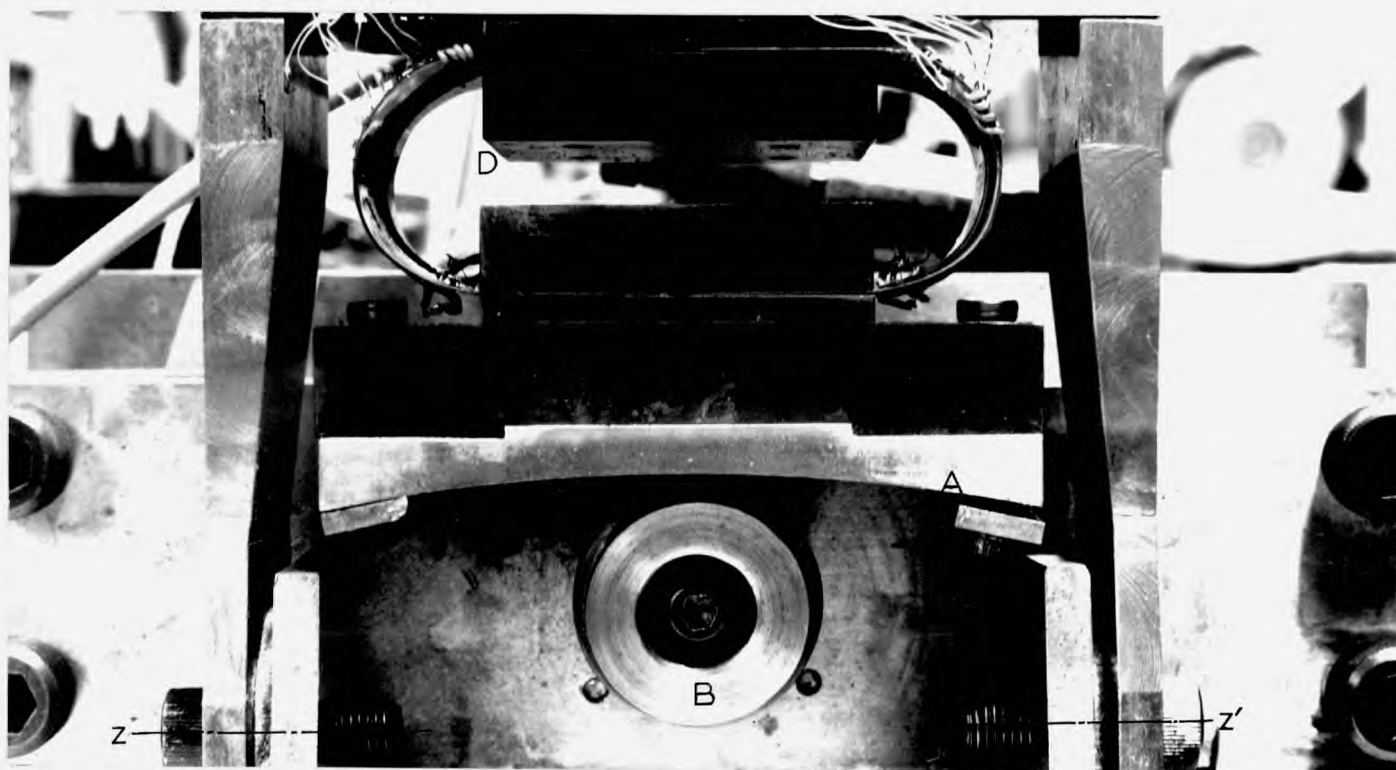


Fig. 3.10.

By using a spherical ring B (fig. 3.10) instead of a cylindrical one the conditions of contact are greatly improved and can be kept constant independently of the load, whereas with the cylindrical ring, increase of the load produces (due to shaft bending) edge-contact effects and alters the position and the size of the contact spot. Additionally this arrangement permits by turning slightly the dynamometer around the axis zz' to achieve a side by side positioning of a number of wear tracks obtained with the same specimens, which makes the comparative microscopic study of the surfaces extremely easy.

The specimens can be fixed on the apparatus easily and accurately which, as it is known, is extremely important for tribological experimentation.

The apparatus has been used in three different ways:

a: As it appears in fig. 3.9 (v_o not constant but a function of time). This case is much more complicated theoretically than the case where $v_o = \text{constant}$ because the characteristic line used for the Liénard's graphical solution of the equation of motion is not fixed but moves on the phase-plane diagram according to a law depending on the variation of v_o . For this case a modified computer program MLIEN (see Appendix 6) has been used, and one can easily see that depending on the initial conditions, self-excited oscillations or stabilization on the origin could appear independently of the value of the mean driving velocity (and its relation to the critical velocity v_c).

b: With the driving arm C removed and frame D fixed by means of the pin P (fig. 3.9) on the apparatus rigid frame. In that case $v_o = \text{constant}$ and the load is applied by changing the vertical position of the dynamometer on the frame D (technique similar to the one applied with rig Mark I).

c: With the apparatus as in (b) but with an eccentrically fixed ring, periodic variation of load is obtained (depending on the eccentricity and the vertical position of the dynamometer on the frame D) and frictional oscillations can be studied for $v_o = \text{constant}$ $N = N(t)$.

3.1.3.3. The dynamometer

The slider (arc A fig. 3.11) is fixed on the lower end of the dynamometer which operates according to the principle of the extended ring (β -type) or the octagonal (α -type) dynamometer. Dynamometers of this kind are commonly used to measure two (normal to each other) components of the cutting forces in machine-tools (Rabinowicz-Cook [212], Loewen-Marshall-Shaw [213], Loewen-Cook [214], Cook-Loewen-Shaw [215], Königsberger-Marwaha-Sabberwall [216]).

The basic idea is that a ring-spring (or a half-ring-spring, fig. 3.11. γ) when loaded with a radial force N suffers maximum strain at the points 1,3 (tension for N compressive) and 2,4 (compression for N compressive) while there are points (5,6,7,8) practically unstrained. Angle φ defining the position of these points is found to be about 45° . Similarly for a tangential force F the maximum strain is observed at 5,7 (tension for F -direction as in fig. 3.11) and 6,8 (compression) while points 1,2,3,4 remain practically unstrained. Thus two four-arm strain gauge bridges fixed at 1,2,3,4 and 5,6,7,8 measure simultaneously frictional forces and loads without the one measurement interfering with the other. Practically there is a small interference (less than 5-6%) which usually is omitted or can be taken into account in the numerical treatment of the results (see Appendix 5). This type of dynamometer is not sensitive to bending moments.

The high-stiffness octagonal dynamometer made for the experiments, was cut from a solid block of mild steel (fig. 3.11. α and 3.12) while the low-stiffness dynamometers consisted of clamping plates B,C holding tightly the rounded leaf-springs D,E. In both cases, the same specimen holder F and dynamometer base-plates G,H were used.

The distance between the center lines of the two cooperating half-rings was kept as large as the rig design would allow because that improves the dynamic stability of the dynamometer.

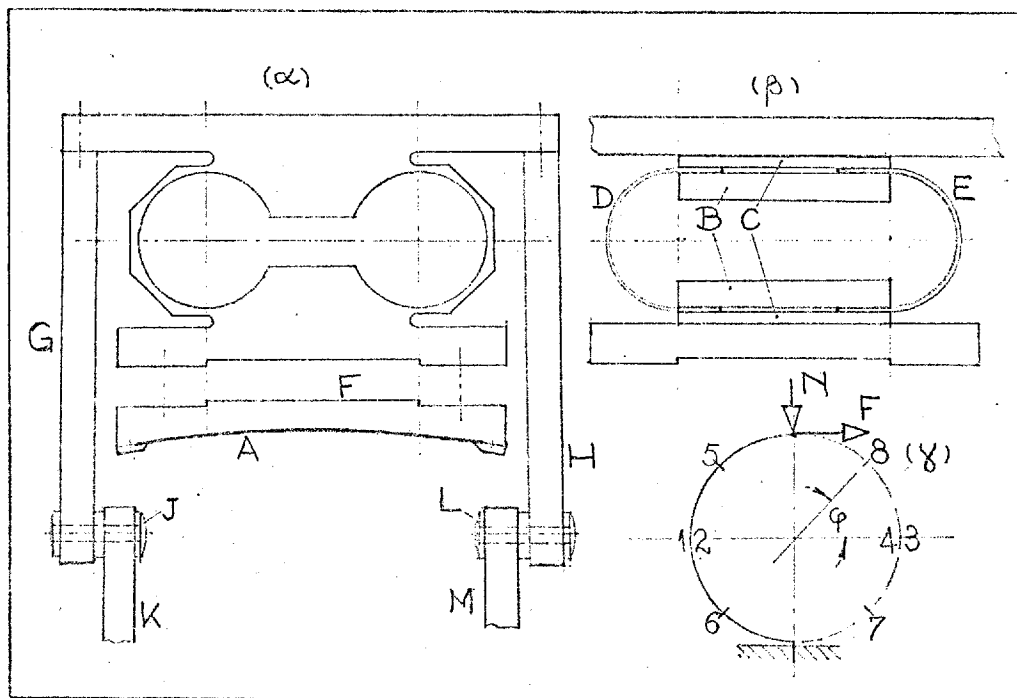


Fig. 3.11



Fig. 3.12.

The base plates were connected to the arc moving frame K,M by means of the adjustable screw connections J,L. Damping pads can be interposed between the plates G,H and K,M, to isolate the dynamometer and the frictional pair from external vibrations. This isolating technique was abolished after the trial runs because it was found that it radically changes the behaviour of the system, which in that case behaves as a two-degree of freedom system (fig. 3.1) due to the additional freedom of movement.

The dynamometers of fig. 3.11 also present the advantage of keeping the tribological conditions almost constant, for small displacements. If the displacement is not small then a load variation is introduced and correction of the numerical values of the results is necessary.

3.1.3.4. Lubrication

The lubricant was applied (as fig. 3.13 shows schematically) by means of a shallow oil container A placed under the ring specimen in such a way

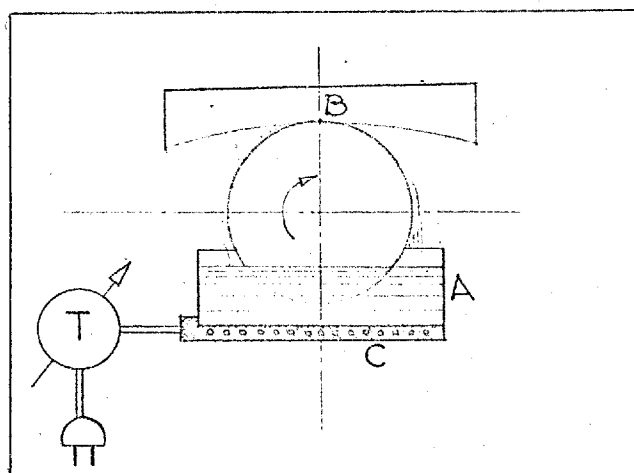


Fig. 3.13.

that the periphery of the ring touches the surface of the oil, and a small amount of it is brought in the contact spot B. A thermostatically controlled heater C permitted experiments at different oil temperatures. The temperature at B was assumed to be the mean temperature

between two points on the surface of the ring, the one before and the other after B at equal small distances from it.

3.1.3.5. Measurements

Two strain gauge bridges measuring normal and frictional forces (N, F in fig. 3.14) are the basic measuring equipment in Mark II apparatus,

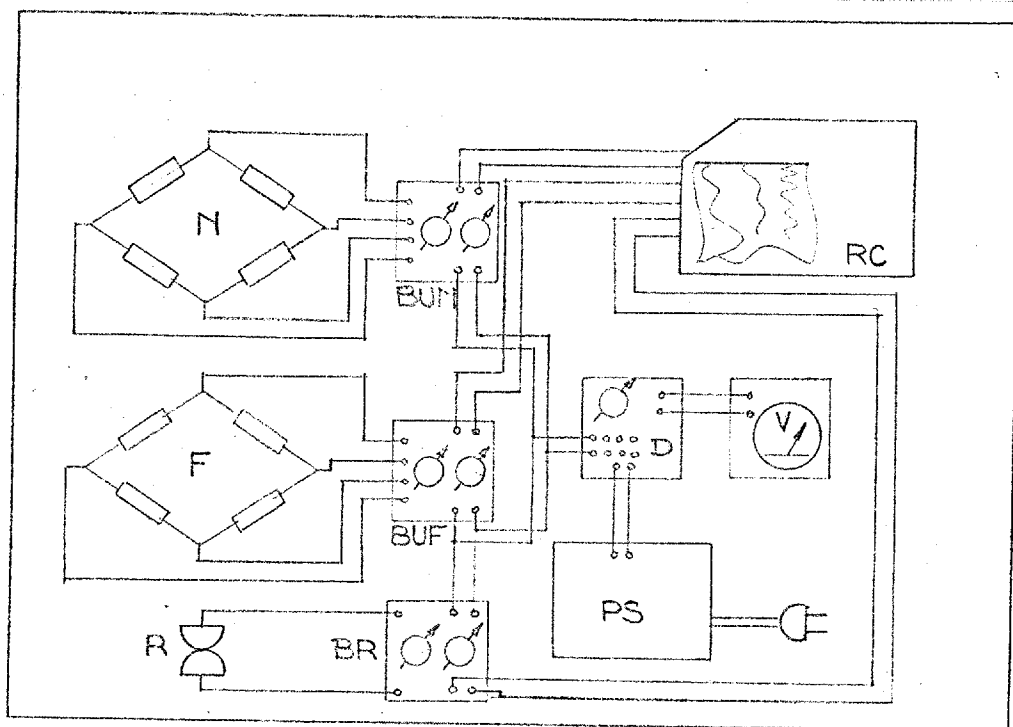


Fig. 3.14.

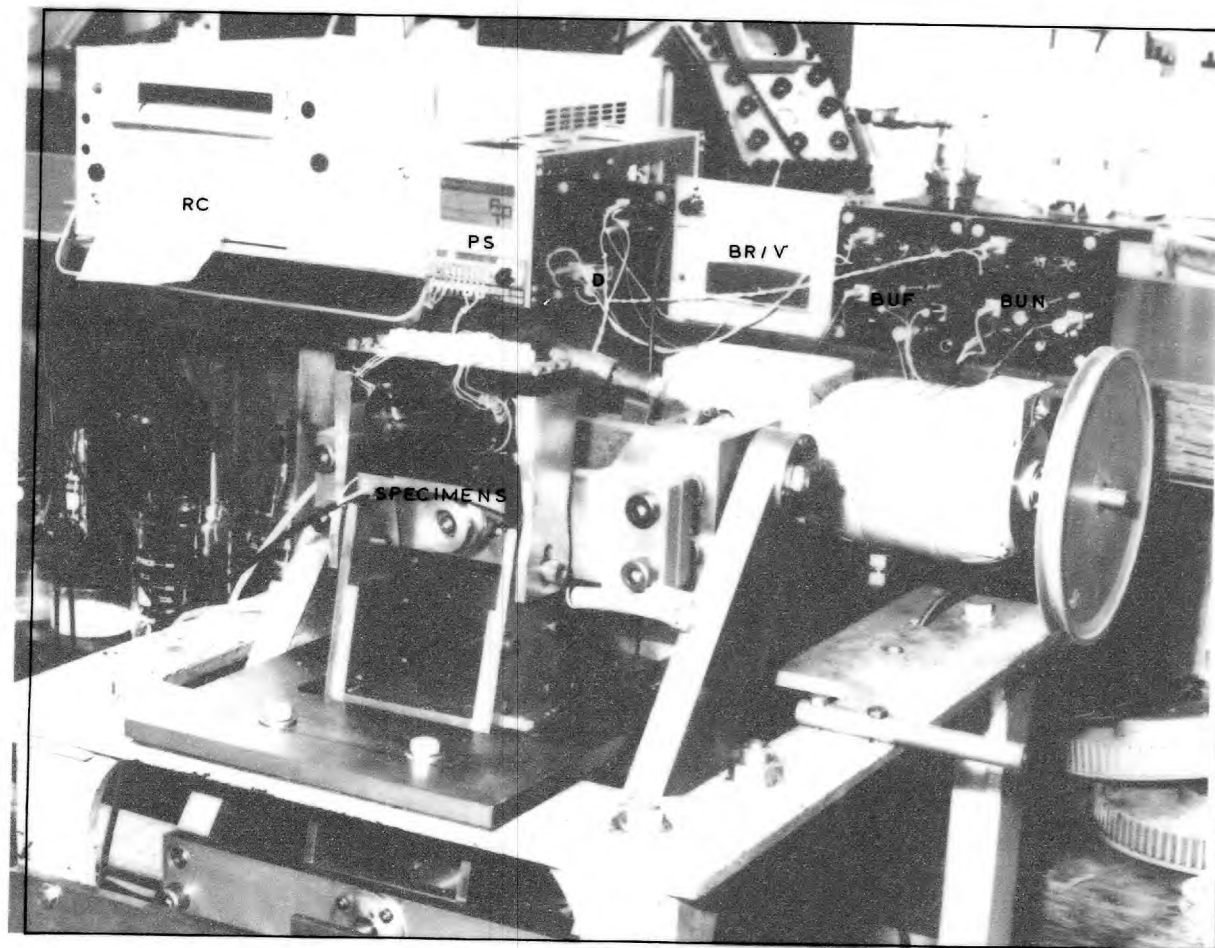


Fig. 3.15

as in Mark I one. The bridges are fed with DC stabilised voltage through the D.C. Power Supply PS, the distributor D and the two independent balancing units BUN, BUF. Due to the high sensitivity of the u/v galvanometric recorder (S.E. Laboratories) used in this experimentation, no amplification of the signals was necessary.

Additionally measurements of the electrical contactivity of the contact spot have been made to give an estimation of the contact of the surfaces during the experiments.

During the preliminary experimentation a dual beam oscilloscope (SOLARTRON) was used as in case of rig Mark I.

Fig. 3.15. presents the general layout of apparatus and instruments, while Appendix 5 gives additional information about their design.

3.2. EXPERIMENTAL TECHNIQUE

3.2.1. Choice of tests

It was made clear in the theory (Chapter 2) that a very small number of frictional oscillation cycles is enough to give the necessary information for a correct analysis of the motion and the study of the coefficient of friction and its variations during frictional oscillations.

Bearing in mind the scope of this work, which is the establishment of an improved general technique for frictional oscillation study, it becomes obvious that only a small number of experiments is necessary to support the preceding theory.

Thus a number of experiments was executed for specimens made of several materials and several lubricants under varying load (Rig Mark I), while the effect of load, surface conditions, velocity, lubricant, temperature (and consequently lubricant viscosity) were also studied (Rig Mark II).

Table 3.1. gives a picture of the experiments done for justification of the

TABLE 3.1 : EXPERIMENTAL PARAMETERS

NUMB. OF EXPER.	SPECIMEN		RIG	LOAD [kp]	LUBRIC.	RUNS RECORDED	TRACES ANALY- SED	NOTES
	LOWER	SLIDER						
1 2 3 4 5 6	St	St	MKI	.600	-	2, 5, 25, 50, 75, 100	+ + + + +	
7 8	St	St	MKI	1.450		10 10	+ +	Anomalous phenomenon. Additional excitation was used (*)
9 10 11	St	St	MKI	1.480	-	10 10	+	Effect of load and environment. Magnif- ied N traces to show triggering oscillation
12 13 14 15	St	St	MKI	1.480	-	50	+ +	Additional excitation. Stop-start cycles. Surface treatment effect.
16 17 18 19	St	St	MKI	1.480	-	10	+ +	Short (quasi-sinusoid- al) stick-slip; effect of surface treatment and oxide films.
20 21 22	St	St	MKI	1.480	-	10		Stop-start cycles. Additional excitation.
23 24 25 26 27 28	St	St	MKI	1.480	-	1, 50, 100, 150	+ + +	Environmental condit- ions stable at $\theta = 20^{\circ}\text{C}$ rel. humidity 56%
29 30 31	St	H-st	MKI	.600	-	10, 50, 100	+	Irregular stick-slip. Effect of the surface contamination. Material

Cont'd.....

NUMB. OF EXPER.	SPECIMEN		RIG	LOAD [kp]	LUBRIC.	RUNS RECORDED	TRACES ANALY- SED	NOTES
	LOWER	SLIDER						
32 33 34 35 36	br	br	MKI	1.400	-	1,2		Effect of material
37 38 39	St	St	MKI	1.400	C e a r b o n	50,100		Continued till starv- ation. Effect of lubricant
40 41 42 43	St	St	MKI	1.400	M S O 2	50,S,S+ 50		Continued till starvation. Effect of lubricant
44 45 46 47	br	br	MKI	1.400	P a r a o i f .1	10	+	Lubrication Effect Additional excitation was used
48 49 50	H-st	H-st	MKII	0.900	-	contin- uous	+	Ion bombardment technique for cleaning the surfaces. Roughness (p-p) $\frac{1}{4}\mu$
51 52 53	H-st	H-st	MKII	0.800	-	contin- uous	+	Ion bombardment technique for cleaning the surfaces. Roughness (p-p) $\frac{1}{4}\mu$
54 55 56	H-st	H-st	MKII	1.800	-	contin- uous	+	Surface roughness 0.25 μ (p-p)
57 58 59	H-st	H-st	MKII	7.200	-	contin- uous	+	Surface roughness 0.25 μ (p-p)

cont'd....

NUMB. OF EXPER.	SPECIMEN		RIG	LOAD [kp]	LUBRIC.	RUNS RECORDED	TRACES ANALYSED	NOTES
	LOWER	SLIDER						
60 61 62 63 64 65	H-st	H-st	MKII	0.900	-	Continuous	+	Surface roughness 0.1 μ p-p.
66 67 68 69	St	St	MKII	0.4 ÷ 7.00	used em	"	+	During each experiment the load was varying. Static coefficient of friction measurements
70 71 72	St	St	MKII	"	-	"	+	"
73 74 75	St	St	MKII	0.4 ÷ 7.00	-	"	+	Ion bombardment cleaning. "
76 77 78	St	St	MKII	0.4 ÷ 7.00	Parallel fil	"	+	"
79 80 81	St	St	MKII	0.4 ÷ 7.0	220- Mob- il	"	+	"
82 83 84 85 86 87	St	St	MKII	3.000	-	"	+	Soft-spring experiments. Additional excitation
88 89 90 91 92 93 94	St	H-st	MKII	3.000	-	"	+	Effect of surface contamination. Additional excitation

* To obtain wide-spectrum phase-portrait.

theory.

Of the total of about 100 experiments, 32 representative ones have been analysed completely to show possible differences in the function $\mu = \mu(v)$ or in phase-portrait of the system due to variation of the values of the parameters.

Due to the great number of parameters, a more systematic study of the phenomenon is not possible, if the extent of the study is to be kept within reasonable limits.

3.2.2. Specimens

Steel and bronze specimens have been used, (Rig Mark I), polished by means of metallographic polishing paper (increasing grades up to 600) under running water. A rather uniform surface roughness was obtained by that technique, not exceeding peak to valley heights of $0.5 \mu\text{m}$.

With rig Mark II steel (mild or hardened) specimens only have been used. The surface was treated with metallographic polishing paper and diamond paste and the polishing was performed strictly in one direction only (along the periphery of the ring and arc). Thus the mating surfaces contact each other with parallel surface roughness grooves and movement in a direction parallel to them. The finally obtained surface (diamond paste $1\mu\text{m}$, $\frac{1}{4}\mu\text{m}$) had a rather uniform roughness with maximum peak to valley height not exceeding $0.4 \mu\text{m}$.

3.2.3. Cleaning of the surfaces

After polishing, the surfaces were degreased in hot toluene and then acetone. Usually this technique is adequate enough, leaving on the surface no films other than thin oxide. Ion bombardment was used in some experiments as the final cleaning process, but it was found that the experimental repeatability is then very poor and ion bombardment was finally abandoned.

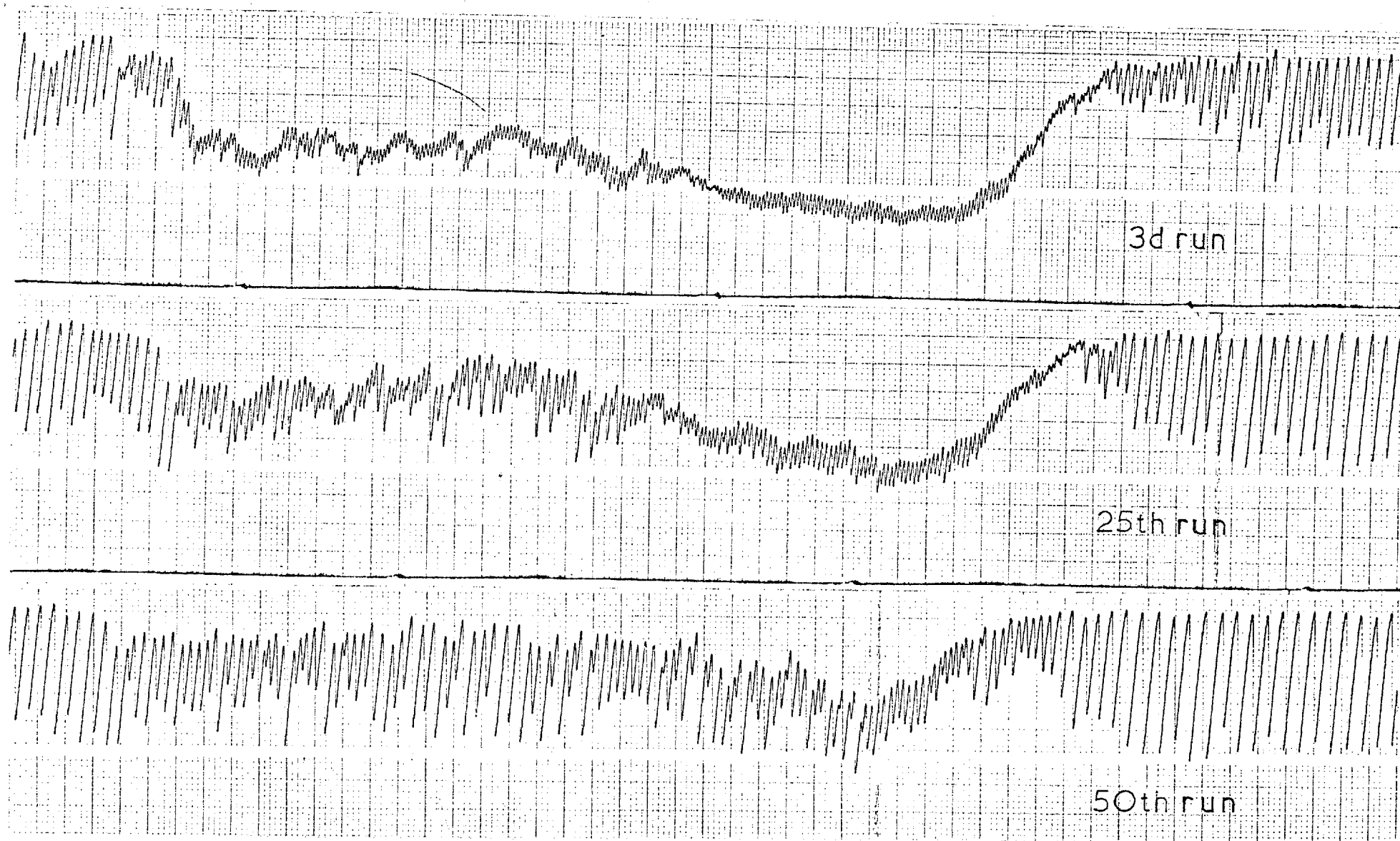


Fig. 3.16

After cleaning, the specimens were stored (for time less than one hour) in acetone.

3.2.4. Positioning of the specimens

In rig Mark I the pin is fixed on the "trunk" of the dynamometer, the N bridge is balanced, the disc is fixed on the chuck of the turntable and then, by lifting the disc, contact between pin and disc is achieved. Further elevation of the disc applies a normal load N between pin and disc. N - and F -bridges are balanced anew and the sensitivity is increased for vertical and horizontal displacement measurements.

After the impression of the driving velocity, frictional oscillations start, usually in an irregular form. A number of revolutions (20 - 100) are necessary (under the loads used) to obtain a regime of regular frictional oscillations (e.g. fig. 3.16). This is due to the fact that as the slider passes repeatedly over the same points of the disc, the thin oxide films are gradually removed and clean metal contact participates in the formation of stronger regular bonds between the two surfaces. Thus the initial oxide-oxide friction is gradually transformed to metal-metal friction.

After the establishment of steady-state conditions (regular, constant amplitude stick-slip which means that the rate of oxide film formation equals the rate of oxide removal and by no means that all the oxide film has been removed) a number of oscillation cycles is recorded. These cycles expressed on the phase-plane, can be analysed by the reversed Liénard's construction (program TRC, Appendix 7) and the variation of the coefficient of friction with velocity, during a single frictional oscillation cycle is obtained.

Additionally some experiments with displacement excitation by external means were performed to complete the phase-portrait of the system (external trajectories) and some experiments with continuously varying driving velocity

v_o for comparative purposes.

In rig Mark II the ring is fixed on the shaft by means of a conical fitting and screw, and a special extractor is used to remove the ring easily from the shaft.

The position of the dynamometer-arc combination is adjustable in the vertical direction and a small inclination (10° to either side) can be given to it, obtaining thus an accurate positioning of the contact spot.

The technique followed for load application is in general the same as for rig Mark I.

In the experimentation with rig Mark II, apart from the "running-in" period, necessary to achieve regular stick-slip conditions, a "running-out" period was observed as well during which stick-slip decreases again and finally an irregular form was established. This can be ascribed to the much higher environmental humidity during these experiments. Thus the initial oxide removal is followed by the formation of more coherent oxide films, formed quickly due to the participation of the water vapour in the atmosphere. The fact that "running-out" phenomena never appeared in the case of lubricated surfaces, intensifies this opinion (fig. 3.17). As representative cycles of frictional oscillation, the cycles after "running-in" and before the initiation of the "running-out" period were accepted.

The main experimental course with rig Mark II comprises the following three types of experiments:

- a. - Under constant $v_o, \bar{N}$ with or without additional excitation.
- b. - Under $\bar{N} = \text{constant}$, $v_o = v_o(t)$ with or without additional excitation.
- c. - Under $v_o = \text{constant}$ $\bar{N} = \bar{N}(t)$ with or without additional excitation.

Additionally some experiments under very low driving speed ($v_o = 10^{-5} - 10^{-2} \text{ mm/s}$) have been performed to enable a more accurate study of the first slip and the effect of the idle time on the static coefficient of

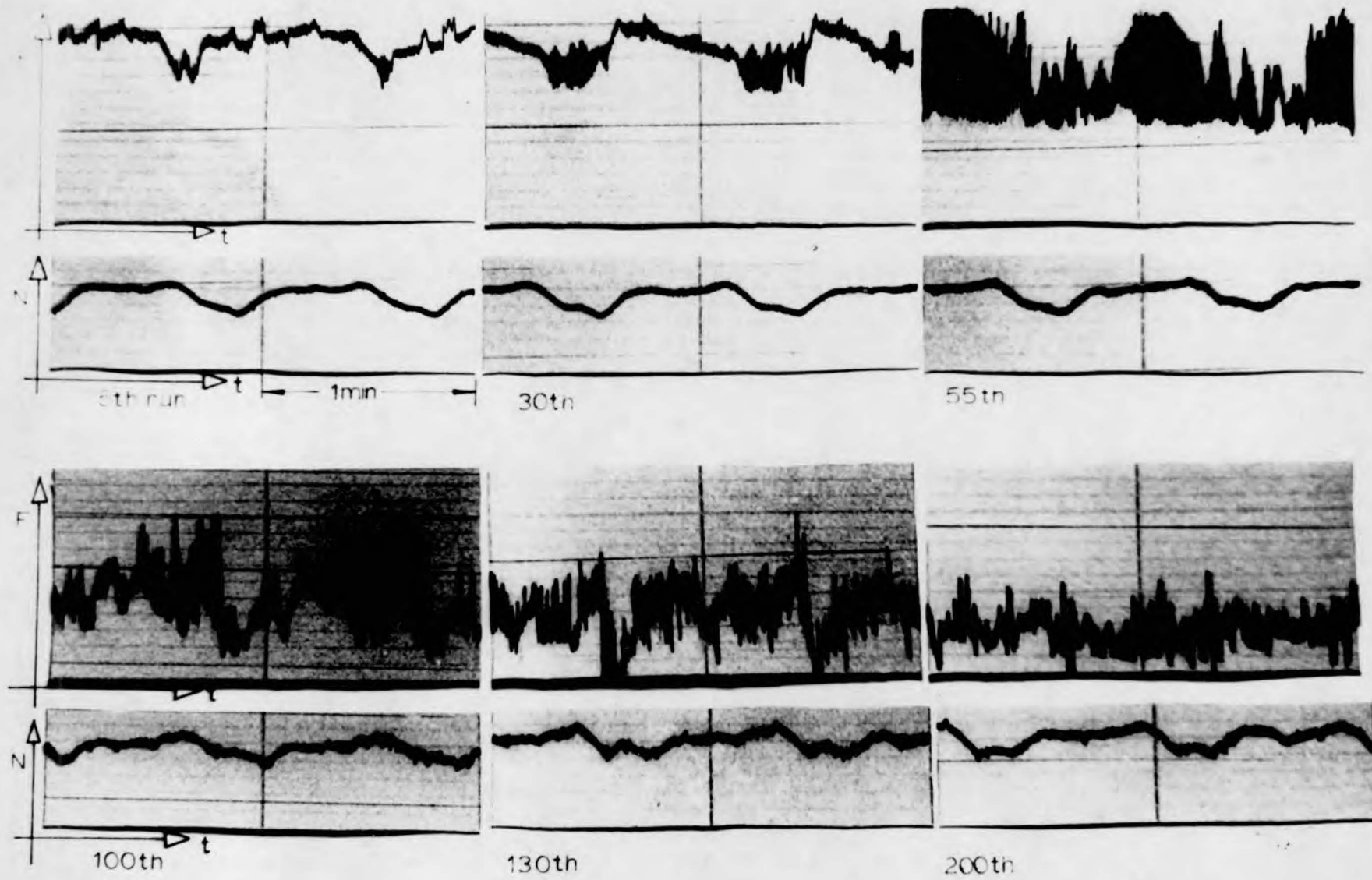


Fig. 3.17

friction (see App.1). For these experiments the load was applied in two different ways:

a. - The first load N_1 (lowest value) was applied and then v_o was impressed. After the first gross-slip, v_o was reversed and when zero frictional force was obtained the load was increased to its new value N_2 and the experiment was repeated.

b. - The first load N_1 (lowest value) was applied and then v_o was impressed. After the first gross-slip, the load was removed completely, the new load N_2 was applied and the experiment repeated.

The essential difference between these two techniques for load application is that in the first one the contact spot is continuously loaded. So if there is any "memory effect" in friction due to viscoelastic behaviour of the combination metal-lubricant-metal, the static coefficient of friction in that case is expected to be sensibly higher than in the second case.

3.2.5. Environment

All the tests with rig Mark I have been performed in controlled environment of temperature $20^{\circ} - 23^{\circ}\text{C}$ and relative humidity 40% - 56%.

It was observed that for relative humidity less than 50 - 53% audio frequency vibration was produced having inadequate amplitude to be detected by the measuring devices used for frictional oscillation measurements, but appearing as a pure harmonic oscillation on the oscilloscope screen when the microphone-microphone amplifier were employed. This oscillation appeared irregularly and no adequate study of it, was possible by any means.

All the tests with rig Mark II have been performed under higher relative humidity (over 70%) and temperatures varying from $18^{\circ} - 22^{\circ}\text{C}$. When the oil-heater was used the temperature range was extended up to about 150°C . No audio frequency oscillation was observed, probably due to the higher environmental humidity (Tingle [217], Bowden-Young [68], McFarlane-Tabor [142]).

3.2.6. Lubrication

Cetane, medicinal paraffin oil and MoS_2 were used with rig Mark I, and the experiments were prolonged till "starvation" conditions were established. Thus the gradual change from lubricated to unlubricated conditions was studied.

Two commercial anti-stick-slip oils were additionally studied with rig Mark II (see App. 5).

3.3. EXPERIMENTAL RESULTS

3.3.1. Necessary information for the analysis

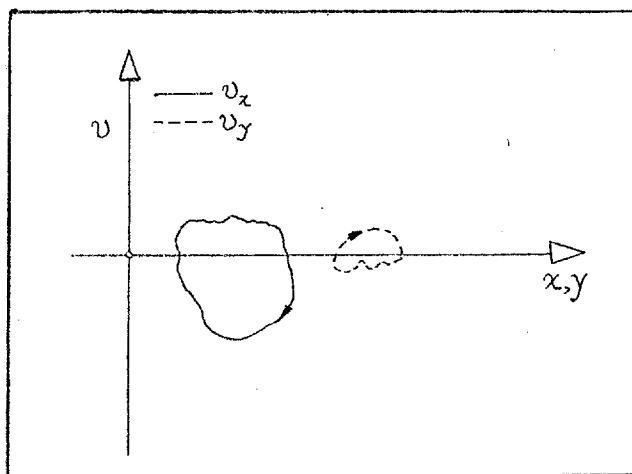


Fig. 3.18

and two phase-plane diagrams can be drawn (fig. 3.18) $\{v_x, x\}$, $\{v_y, y\}$ representing the horizontal and vertical movement of the slider.

The function $y=y(x)$ is represented on a $\{x, y\}$ plane by an ellipse

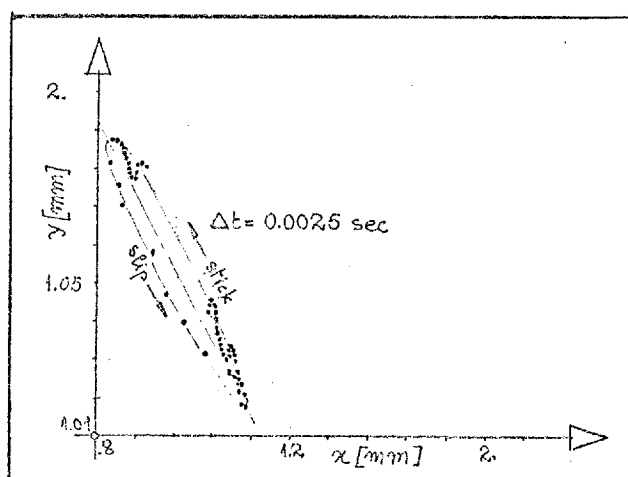


Fig. 3.19

$\Delta t = 0.0025$ sec.

The vertical movement phase-plane trajectory $\{v_y, y\}$ is used to correct the trajectory v_x, x thus avoiding complicated two-degree of freedom phase-

The horizontal and vertical (absolute) displacements $x=x(t)$, $y=y(t)$ comprise the only necessary information for derivation of the experimental phase-portrait of the system. By differentiating these functions in respect of time the velocities $v_x(t) = \frac{dx(t)}{dt}$, $v_y(t) = \frac{dy(t)}{dt}$ are obtained

(fig. 3.19) showing clearly the phase difference between horizontal and vertical modes of oscillation and giving a clear picture of the superimposed triggering oscillation (Lisitsyn's ellipse). The points in fig. 3.19 are taken from an experiment and they are equi-distant in time with

plane (or phase-space) presentations (Ku[202,203,204,205,206]).

3.3.2. Experimental trajectory treatment

Due to the existence of the triggering oscillation it has been seen that the application of the reversed Liénard's construction to obtain the function $\mu = \mu(v)$ is not accurate. Thus a "smoothing" numerical technique must be applied to remove triggering oscillation from the $\{v_x, x\}$, $\{v_y, y\}$ trajectories. It is very important to use the correct "smoothing technique

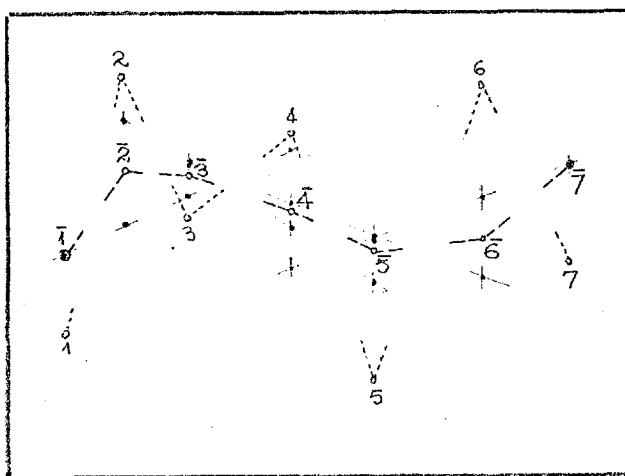


Fig. 3.20

which will not affect seriously the basic form of the trajectory. The "group-fitting" of a polynomial which approximates roughly the trajectory, seems to be the most satisfactory technique.

Assuming that an experimental curve consisted of the points 1,2,3,4,5,6,7 (fig. 3.20)

is to be "smoothed", two decisions must be made at first:

- a. - The degree of polynomial D which is going to be fitted, and
- b. - The number of points n comprising each "fitting-group".

There are no theoretical criteria for choosing these two factors.

Assuming that $\{D=1, n=4\}$ a first degree of polynomial must be fitted over four groups of four points each ($\overline{1234}, \overline{2345}, \overline{3456}, \overline{4567}$). After the fitting of the polynomial each of these groups of points is replaced by a new group as follows:

$$\overline{1234} \rightarrow \overline{11, 21, 31, 41}$$

$$\overline{2345} \rightarrow \overline{22, 32, 42, 52}$$

$$\overline{3456} \rightarrow \overline{33, 43, 53, 63}$$

$$\overline{4567} \rightarrow \overline{44, 54, 64, 74}$$

(where a number nm indicates a point obtained from the initial point n after fitting of the polynomial in the m-th group). The final values of each point (represented by $\bar{n}$) is the arithmetic mean of the above first fitted values. Thus for the above example:

$$\bar{1} = \frac{(11)}{1}$$

$$\bar{2} = \frac{(21)+(22)}{2}$$

$$\bar{3} = \frac{(31)+(32)+(33)}{3}$$

$$\bar{4} = \frac{(41)+(42)+(43)+(44)}{4}$$

$$\bar{5} = \frac{(52)+(53)+(54)}{3}$$

$$\bar{6} = \frac{(63)+(64)}{2}$$

$$\bar{7} = \frac{(74)}{1}$$

The whole "smoothing" procedure can be repeated a number of times N giving each time an improved smoothness. There is for each case an optimum triad of values $\{D, n, N\}$. For the present case that optimum found after a number of trials to be:

$$\{D, n, N\} = \{2, 5, 8\}$$

D and N must be kept as small as possible because increase of D or N produces a disproportional increase of the computation time.

From the "smoothed" $\{v_x, x\}$, $\{v_y, y\}$ phase-plane diagrams, Liénard's diagrams are obtained easily and the $\{v_x, x\}$ diagram is corrected by the introduction of the correction factor c_N (see § 2.4.1.1.) derived from the

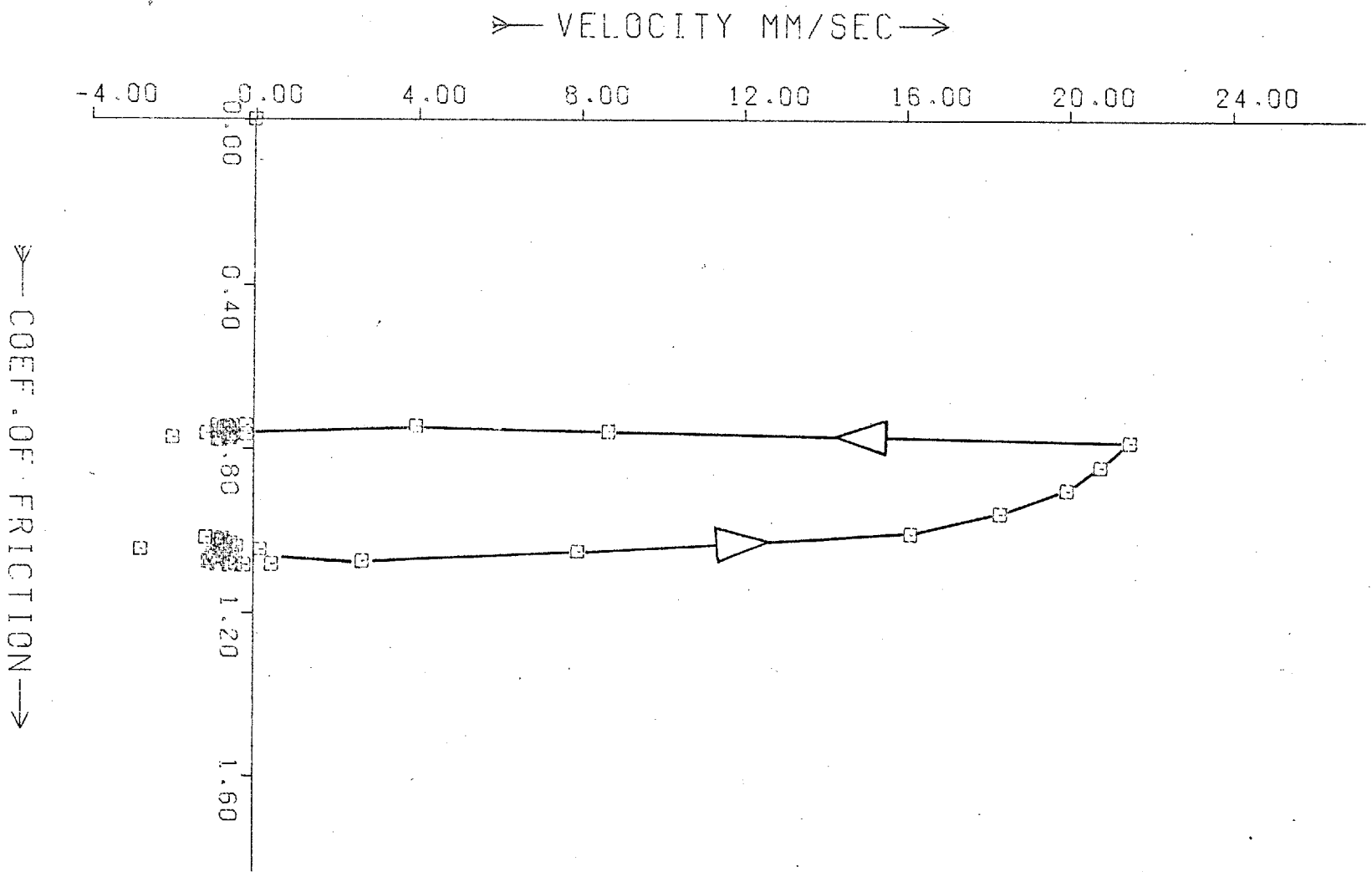


Fig 3.21

$\{v_y, y\}$ diagram. Only this final corrected form of the $\{v_x, x\}$ diagram is susceptible to comparative study with the theoretical phase-portraits.

For comparison with previous experimental results, the apparent coefficient of friction as a function of velocity was plotted (dynamometer indications were accepted as real force N, F indications, under static calibration and inertia forces were neglected). Diagrams like that indicate quite close similarity with previous $\mu = \mu(v)$ experimental traces (Štěpánek [24], Schindler [25,120], Matsuzaki [173], Hunt-Torbe-Spencer [51]). (fig. 3.21).

3.3.3. Experimental $\mu = \mu(v)$ function

One of the basic aims of this work is the experimental derivation of the function $\mu = \mu(v)$ during a frictional oscillation cycle. To do that, the reverse Liénard's graphical construction is used and from the obtained characteristic line the reverse transformation (§2.3.1.5) gives the function $\mu = \mu(v)$.

This technique has the great advantage that a full cycle of oscillation (i.e. a full description of $\mu = \mu(v)$) is completed in very short periods of time, which means that errors due to slowly varying factors (such as wear) disappear.

The graphical construction used fails to determine a point of the characteristic line if the corresponding point of the trajectory (M) lies on the abscissa. For this case two auxillary points (M', M'') lying on the trajectory and in both sides of the intersection of the trajectory with the abscissa were used, and it was assumed that for small $\overline{M'M''}$ it is sensibly $\overline{MM} = \overline{MM''}$ and also that the same relations hold for the corresponding points of the characteristic line. Thus M_c (corresponding to M) is found as the bisecting point of the $\overline{M'_c M''_c}$ (where M'_c, M''_c can be found easily by the reverse Liénard's graphical construction).

If a number of cycles is expressed in the form of phase-plane trajectories,

diagrams like fig. 3.22. (a for $v_0 = \text{const}$ or b for $v_0 = v_0(t)$) are obtained,

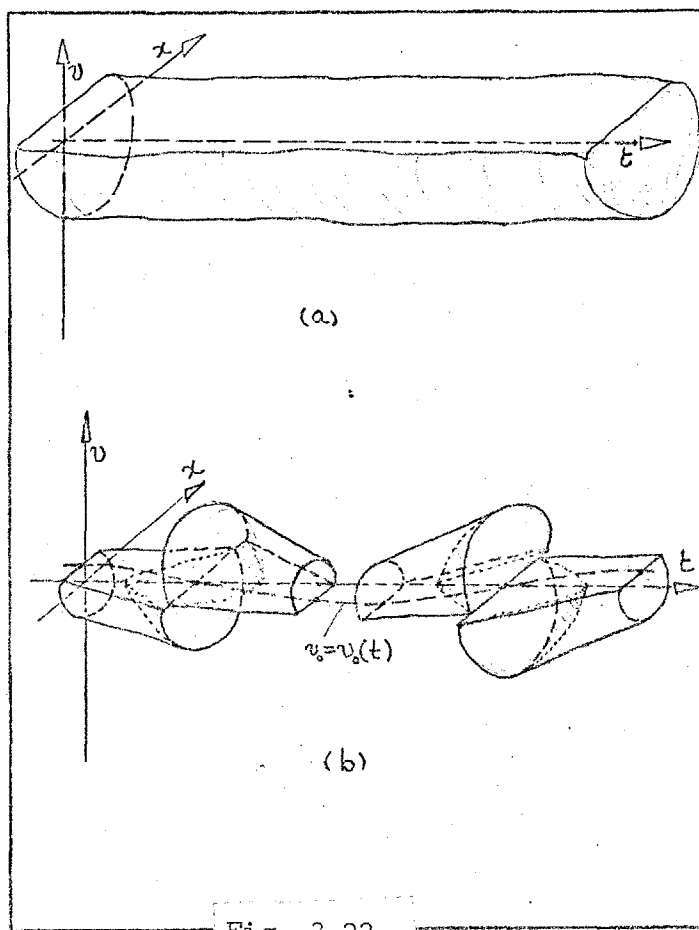


Fig. 3.22.

giving a picture of the variation of the trajectory with time and consequently of the variation of the function $\mu = \mu(v)$ with time. Fig. 3.22 also shows clearly how much more complicated the situation becomes in the case where v_0 is not constant but varies harmonically with time.

CHAPTER 4 : RESULTS AND CONCLUSIONS

4.1. GENERAL DISCUSSION OF THE RESULTS

Analysis of a large number of stick-slip cycles showed that systematically the "experimental characteristic line" appears as a wide loop, which cannot be ascribed to hysteresis phenomena. Comparative examination of characteristics obtained under the same experimental conditions and plotted on the same graph (e.g. fig. 4.1 for dry steel on steel friction, eight pairs of traces $x=x(t)$, $y=y(t)$ as shown in fig. 4.2, β) revealed the following character of the characteristic line, common for all the experiments performed:

Starting at point A which marks the end of stick period, with negative slope $\frac{dx}{d(-\dot{x})}$ (or $\frac{d\mu}{d\dot{x}} < 0$), it continues with gradually decreasing slope and increasing velocity $-\dot{x}$. At a certain point B where velocity $-\dot{x}_b$ is reached the coefficient of friction starts increasing and a "jump" appears from branch α to branch β of the characteristic line. During that "jump" and due to increasing μ_k the slider stops accelerating and the deceleration part of the slip starts. For a much lower velocity $-\dot{x}_c$ an abrupt decrease of

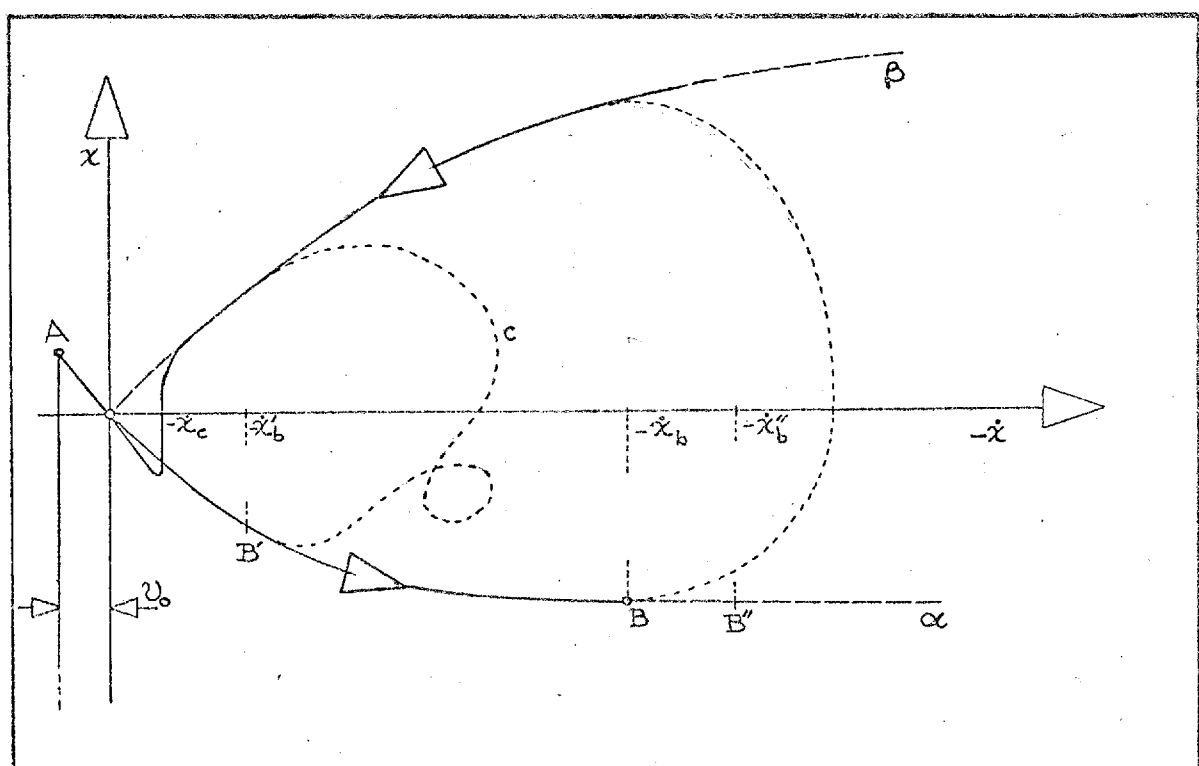


FIG. 4

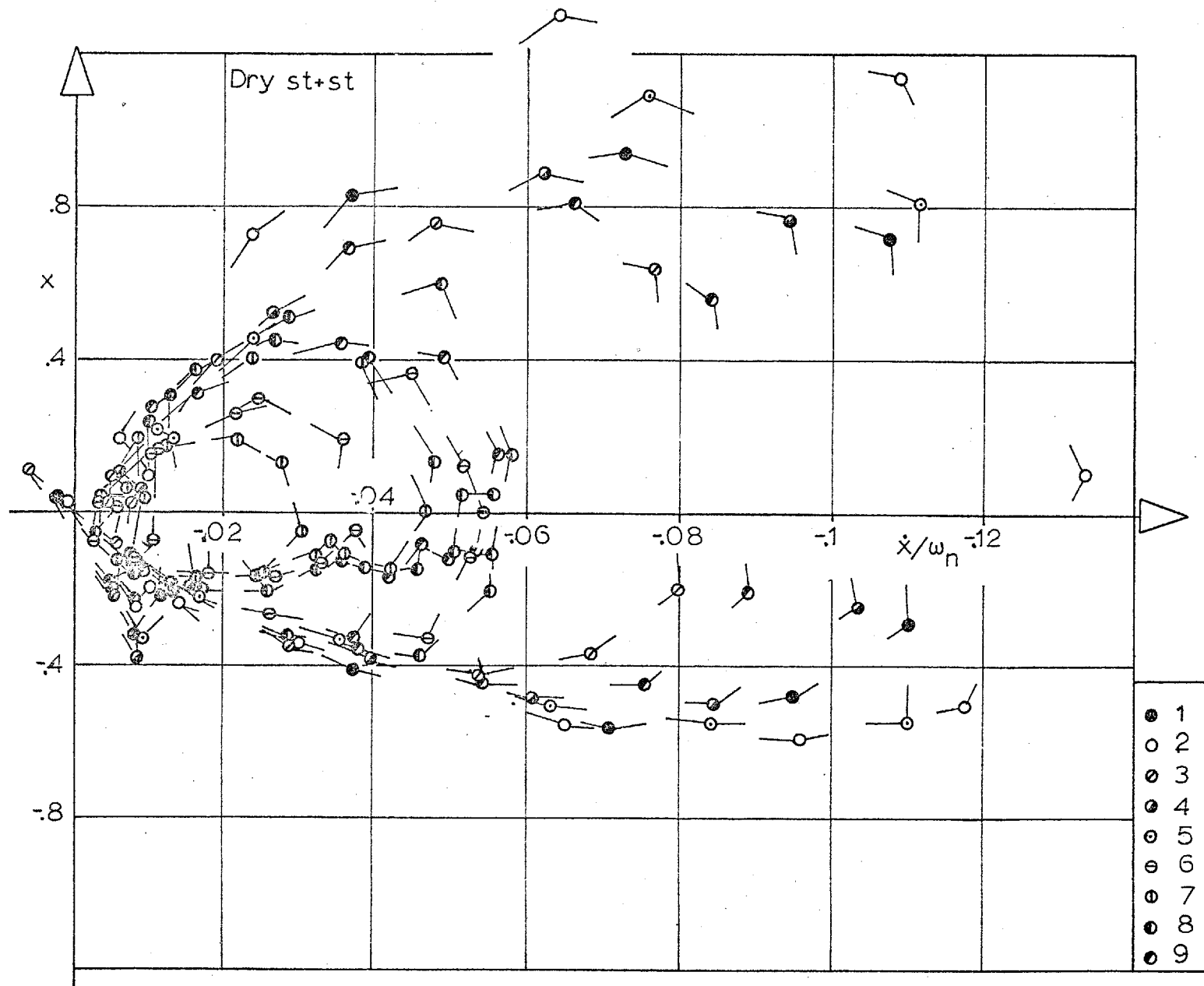


Fig. 4.1.

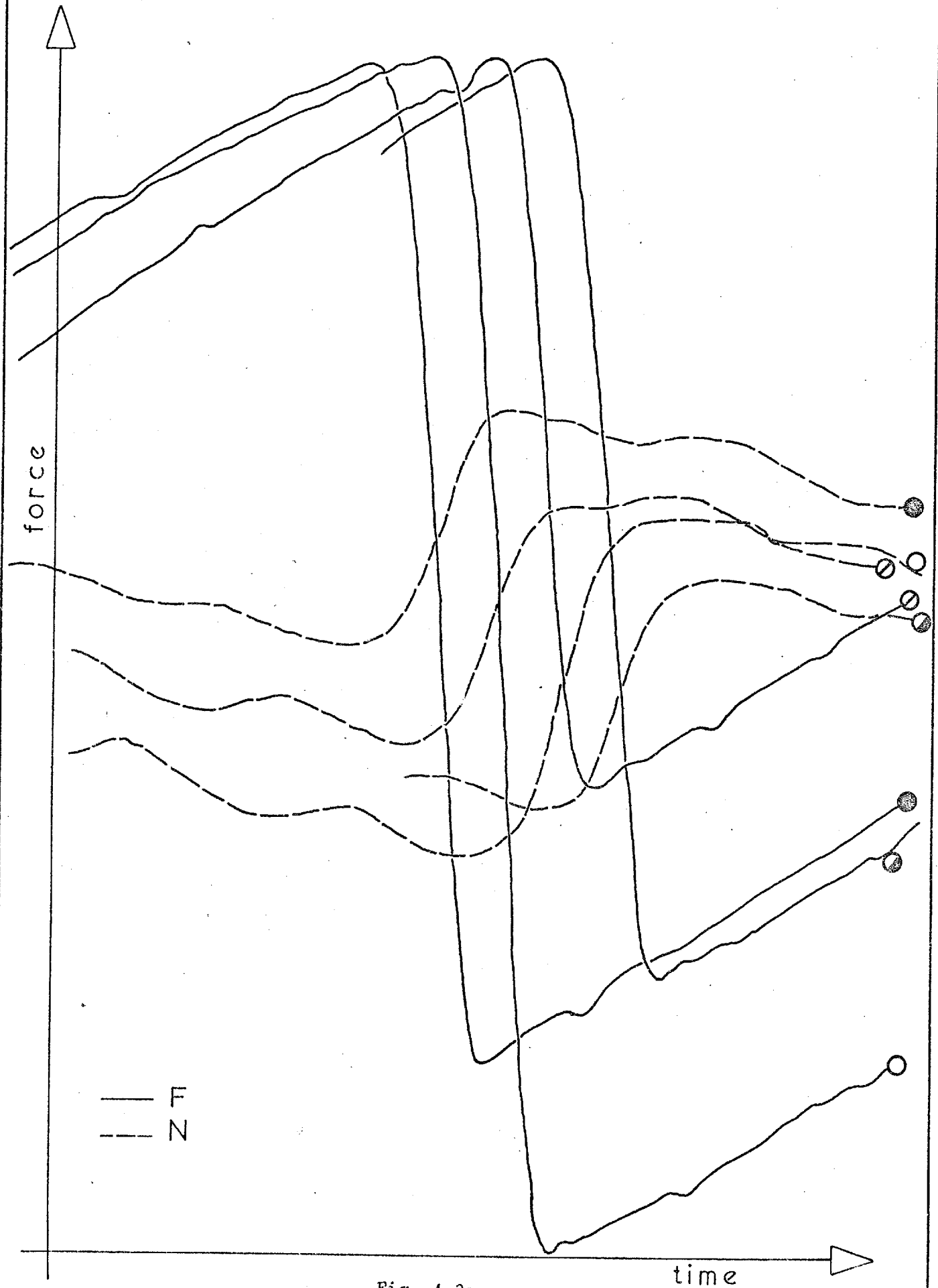


Fig. 4.2a.

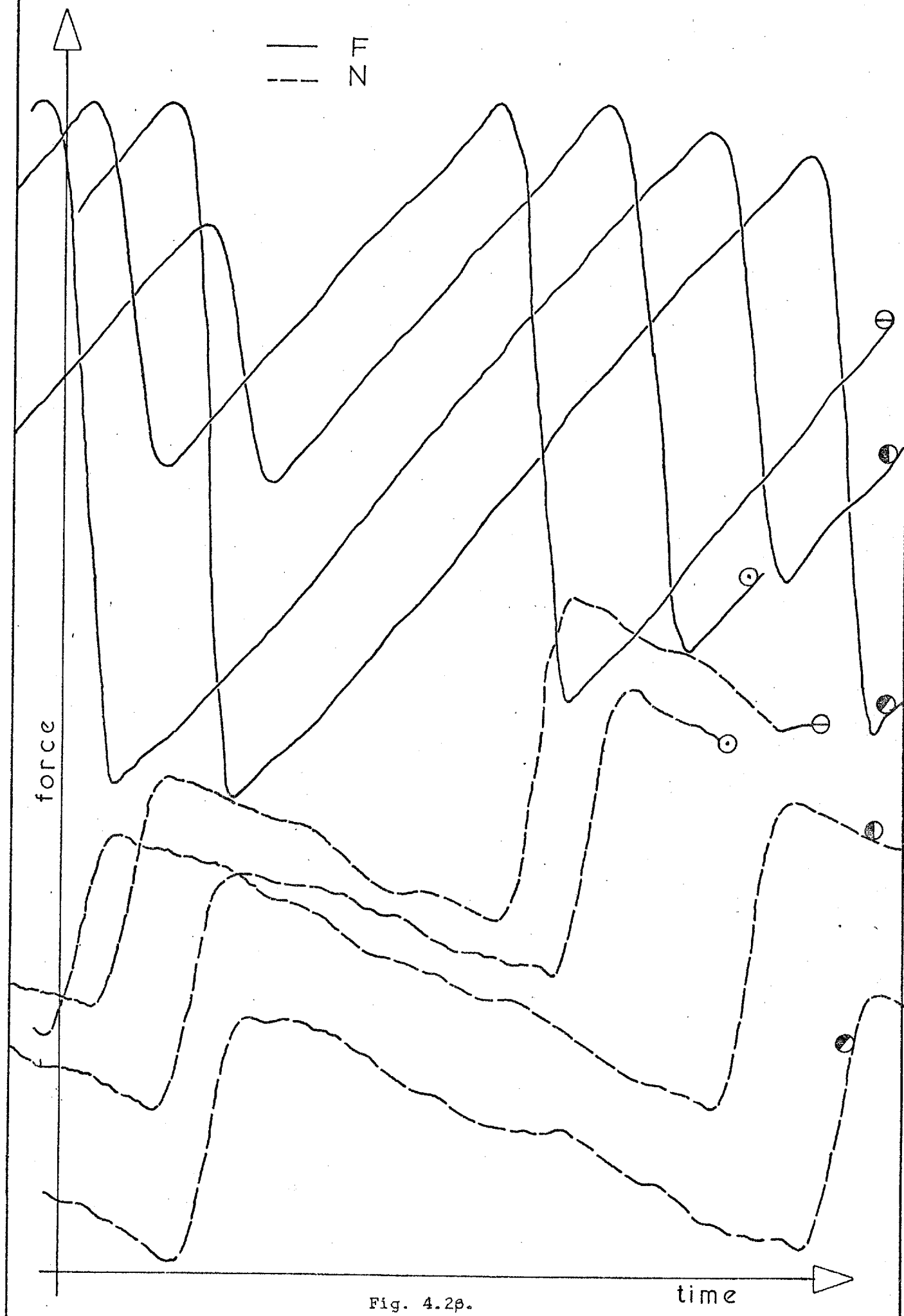


Fig. 4.2p.

time

the ordinate of the characteristic line is observed and the cycle closes at A, following again the α branch of the characteristic. Thus becomes clear that for velocities within the range $-\dot{x}_c \sim -\dot{x}_b$ the sliding is governed by two and not one $\mu=\mu(v)$ characteristic lines. The position of the point B is not definite. It was found that B lies on α between two extreme points B', B'' defining the range of velocities $-\dot{x}'_b \sim -\dot{x}''_b$ in which the "jump" from α to β takes place. No systematic correlation between the position of B and the tribological parameters was found, but it seems reasonable to assume that the topical characteristics of the surfaces and triggering oscillation are the main factors affecting it. As B moves to the left (lower velocities $-\dot{x}$) the stick-slip cycle becomes shorter (higher $\bar{\mu}_k$) but also the "jump" from α to β more irregular (e.g. trace c) indicating an instability in the cause producing the "jump".

The "transition velocity" $-\dot{x}_c$ for which the characteristic line branches α and β meet is clearly defined and its position is fairly constant (see also §4.2.2.).

Obviously only one satisfactory explanation can be given to that form of the characteristic line: that two different and independent mechanisms of friction coexist.

Using the reverse transformation, the bifurcated (in a different sense than the one met in the literature) $\mu=\mu(v)$ function was obtained (fig. 4.4). The mean values of μ_k on the two branches (0.1 and 1.40) help the following hypothesis to be made about the frictional mechanism:

At first the slider moves over surface oxide films and as velocity increases, the separation of the surfaces increases (Tolstoi [104]) and the coefficient of friction drops. With increasing velocity, the kinetic energy of the slider increases and (with the contribution of the triggering oscillation) the slider asperities, instead of jumping over the lower

specimen asperities, break through them. This ploughing mechanism increases the coefficient of friction and also damages the surface oxide films, producing a further increase of the coefficient of friction. This phase lasts for a very short time (0.0025 - 0.01 sec) and atmospheric oxidation cannot counterbalance the oxide film removal. The great scattering of experimental points around the upper branch of the characteristic is in agreement with this hypothesis.

To find if greasy films and other atmospheric contamination contribute to some extent to the low value of the coefficient of friction of branch α , an experiment under variable temperature (19°C - 167°C) was performed. The results of this experiment were negative, and such an effect was considered to be non-existent, at least for specimens cleaned and prepared according to the method described in § 3.2.3.

An idea of the extent to which the existence of surface films can effect frictional oscillations is given by fig. 4.4.a, where the same specimen was treated three successive times before regular stick-slip could be established. (a,b,c represent the friction forces during the 10th run after the first, second and third treatment respectively). No further improvement could be obtained after the third treatment, which indicates that surfaces were virtually clean. Similarly, explanations employing the chemistry of the surface films must be excluded.

When the system operates at the upper part of the characteristic (deceleration period); the coefficient of friction decreases with decreasing velocity and as $-\dot{x}_c$ is reached the kinetic energy of the slider is not enough and the initial frictional mechanism prevails again, as indicated by the abrupt drop of μ_k .

The positive slope of branch β of the characteristic means that if one could force the system to operate on it, no stick-slip phenomenon could

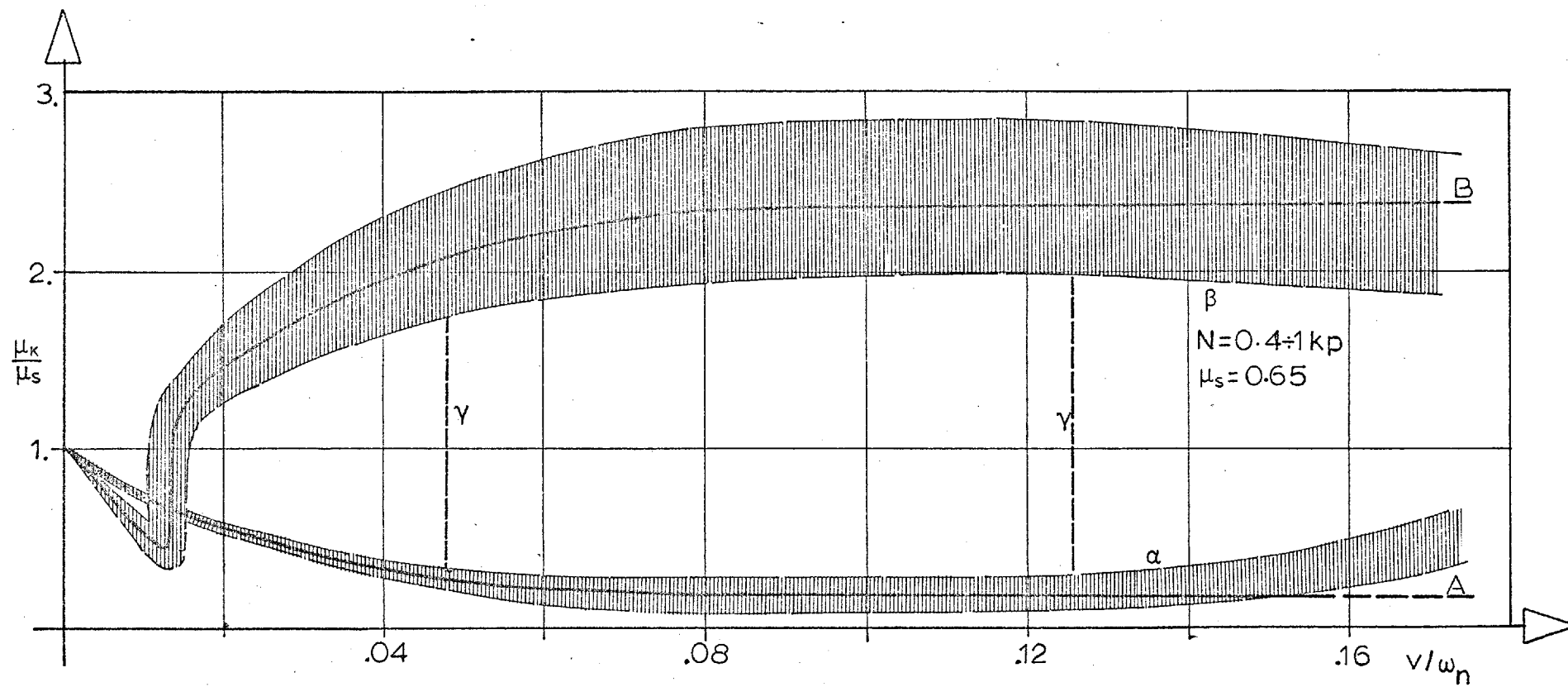


Fig. 4.4.

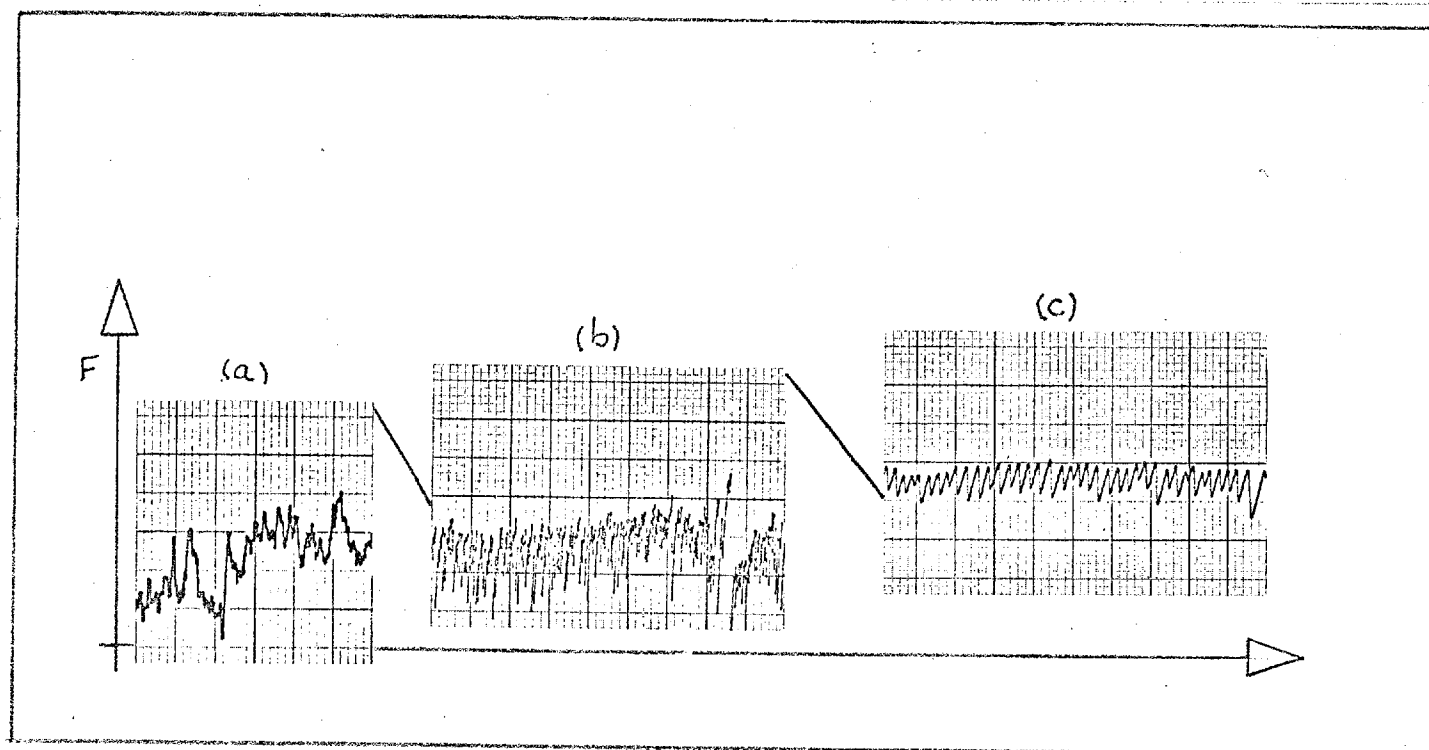


Fig. 4.4.a

appear, but smooth driving would be obtained and higher μ_k values. In fact a simple way to do this is to prohibit the vertical movement of the slider e.g. by heavy damping of the vertical mode. In this case, "... stick-slip disappears and friction markedly rises not only above the mean value observed during self-excited oscillations, but also above the static values corresponding to the maxima at the end of the stick stages in stick-slip sliding. ..." (Tolstoi [104]. See also Elder-Eiss [76]). The explanation given by Tolstoi to that, is however different. The disappearance of stick-slip is ascribed to the damping of the normal vibrations themselves which lowers the slider level. This explains the observed increase of the coefficient of friction but it does not explain why stick-slip disappears. Unfortunately no further comparison between the present results and Tolstoi's can be made because his stick-slip traces are unsuitable for phase-plane trajectories derivation.

Another proof for the twin mechanism existence is the fact that many experimental trajectories (e.g. the trajectories in fig. 4.5, 4.6 for two cycles of frictional oscillation) show very clearly, especially at their lower ends, that they are formed by two independent non-linear phenomena so that their left part is governed by a different differential equation of motion than their right part.

Using the experimentally derived characteristic to construct the slope field of the trajectories (for the same series of experiments with dry steel on steel and loads varying between 0.5 to 1.5 kp) one has to face the following problem: During an experiment or series of experiments under precisely the same conditions, the size of the trajectory varies considerably, due to variations of the function $\mu = \mu(v)$ around its mean value (fig. 4.4). This makes difficult the identification of the part of the trajectory representing the "jump" $\alpha \rightarrow \beta$ on the characteristic which as

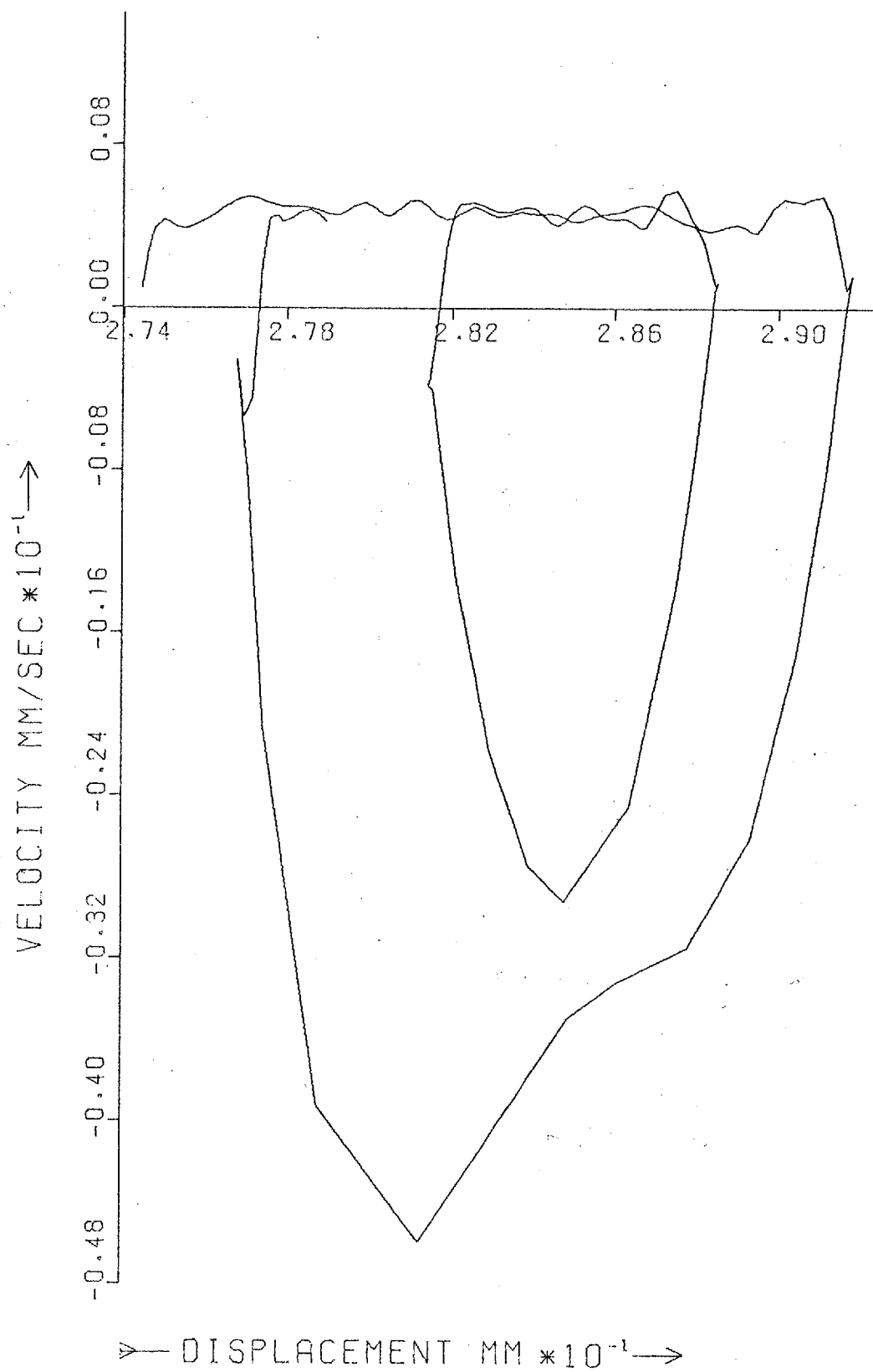


Fig. 4.5

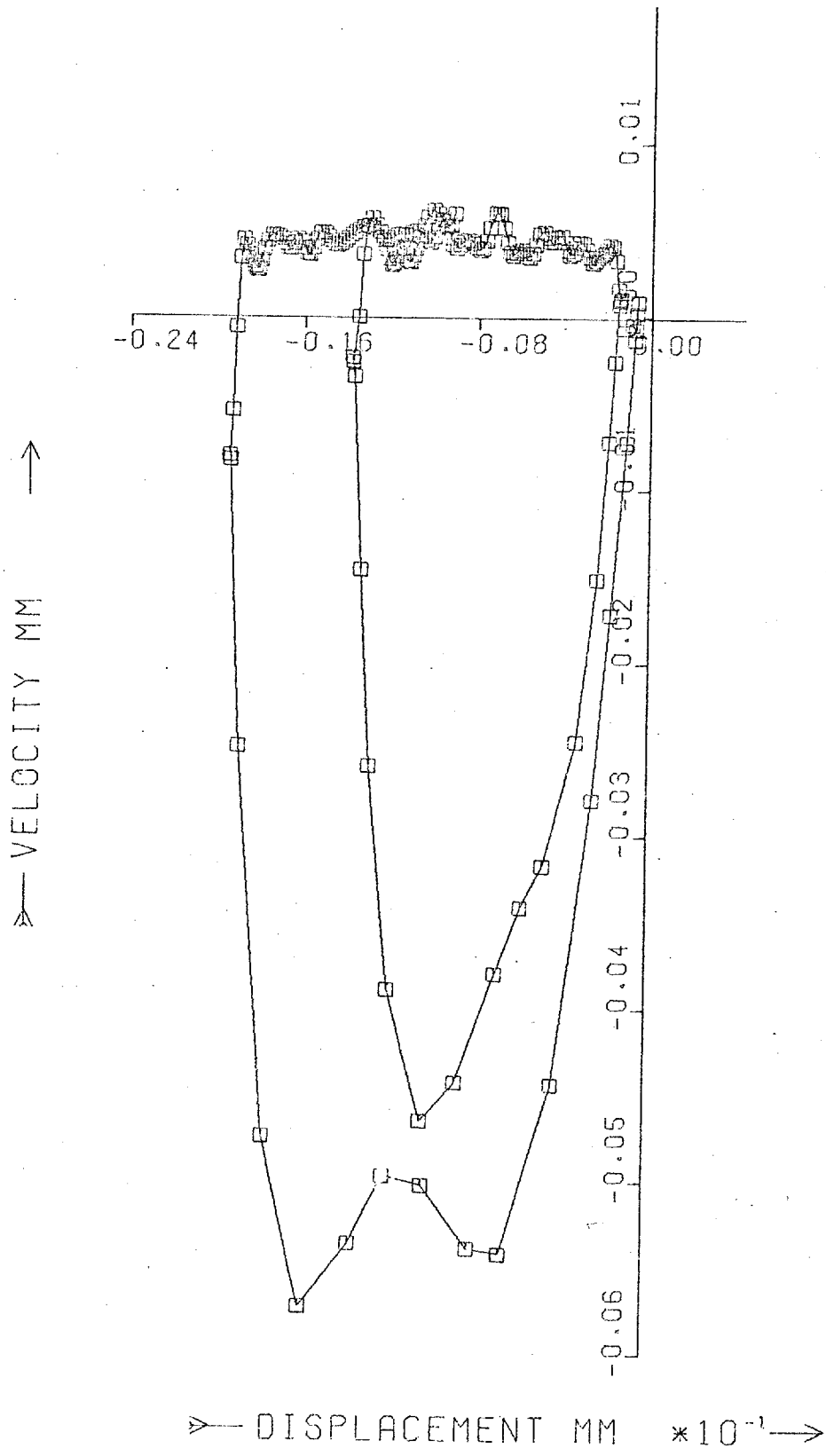


Fig. 4.6

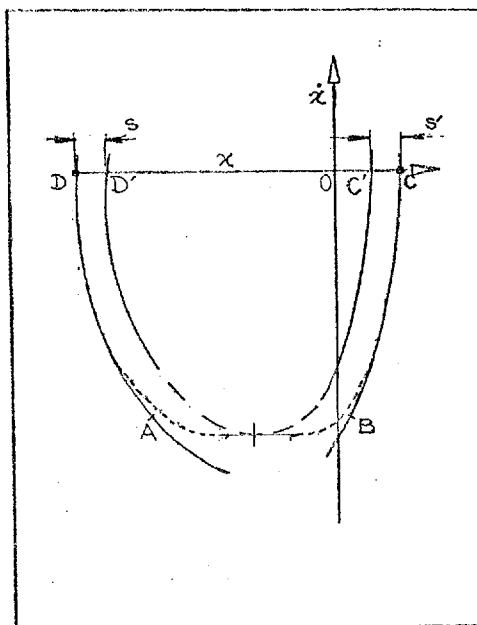


Fig. 4.7.

(fig. 4.7). Thus the points of fig. 4.8 (master curve $\text{---}\bullet\text{---}$) are transformed to the well defined trajectory of fig. 4.9. On the same figure the slope field appears, constructed theoretically after solving the differential equation of motion with characteristic line, the one appearing on top (derived experimentally). The shaded area on the upper diagram shows the exact position of the lower diagram to give an idea of the relative positions of trajectory and characteristic line.

The fact that the coefficient of friction as a function of velocity jumps from α to β at a certain velocity has also been observed by Hunt-Torbe-
Spencer [51], but a different explanation has been given to it. (see also § 4.2.2.).

Comparison of the curve $\mu=\mu(v)$ for dry steel on steel with previous results (fig. 4.10) shows that the lower part of the curve is in agreement with them, but no previous result was found to agree with the upper part of the characteristic. Dis regarding the inertia forces and consequently accepting the trajectory as being proportional to the coefficient of friction

pertaining to neither branch has no interest and must be discarded. To overcome this difficulty one of the trajectories under study was assumed as the "master curve" and then, by increasing x , ($x'=x+s'$) for the right-hand side or decreasing it ($x'=x-s$) for the left-hand side, a fitting of the remaining trajectories on the "master" one was obtained

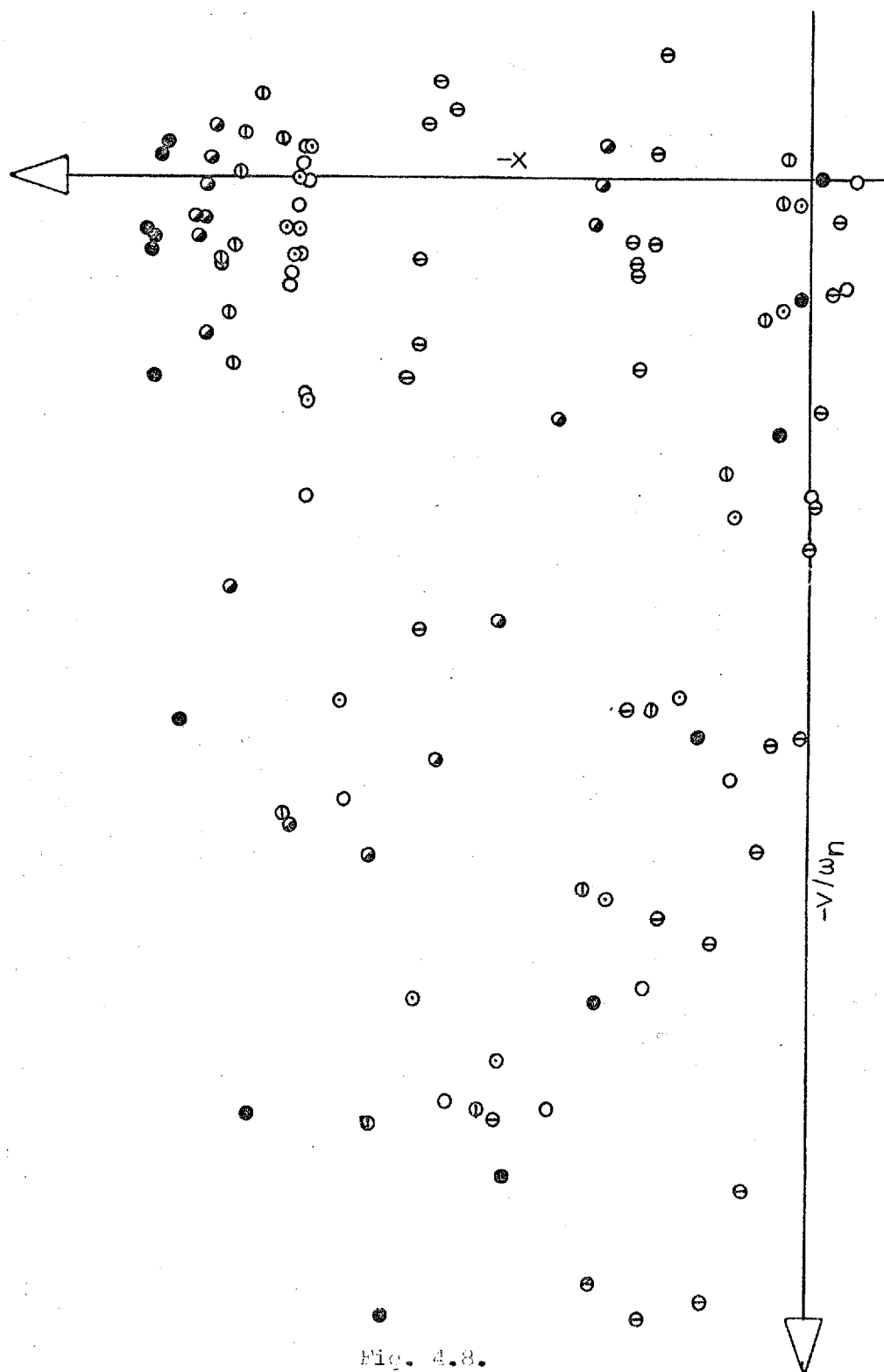
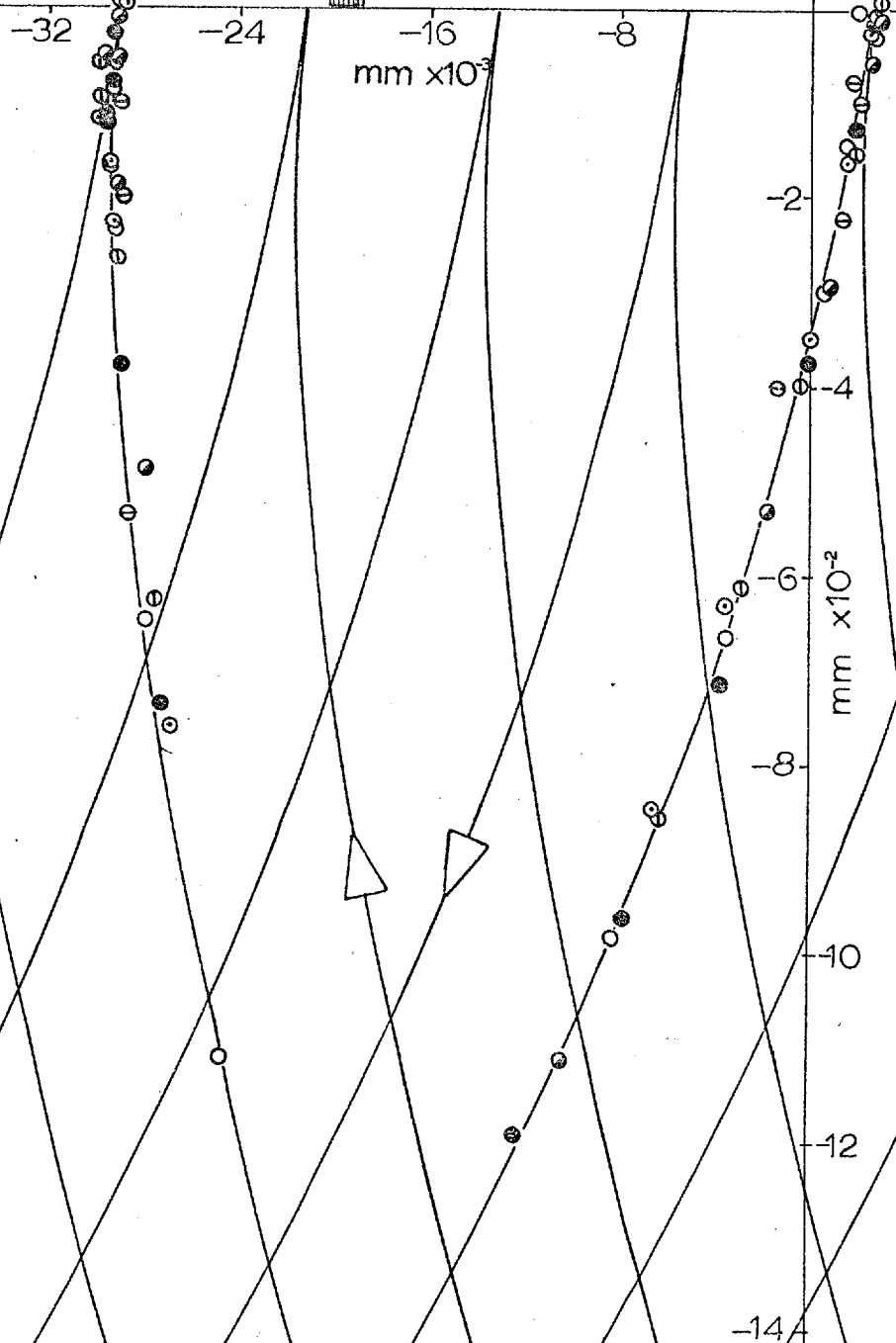
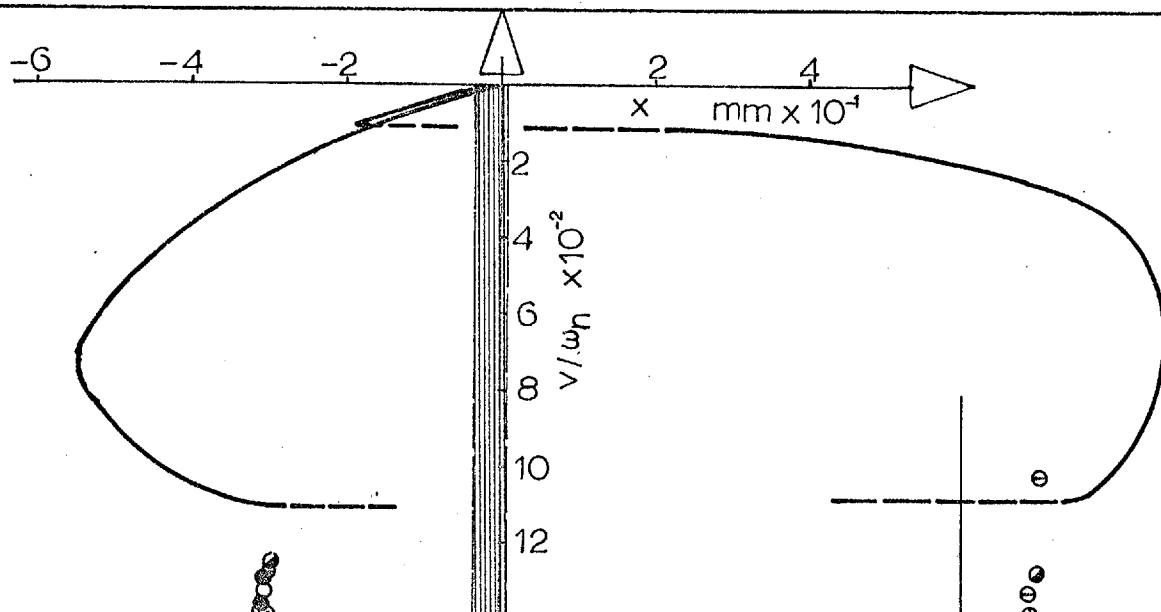


Fig. 4.8.



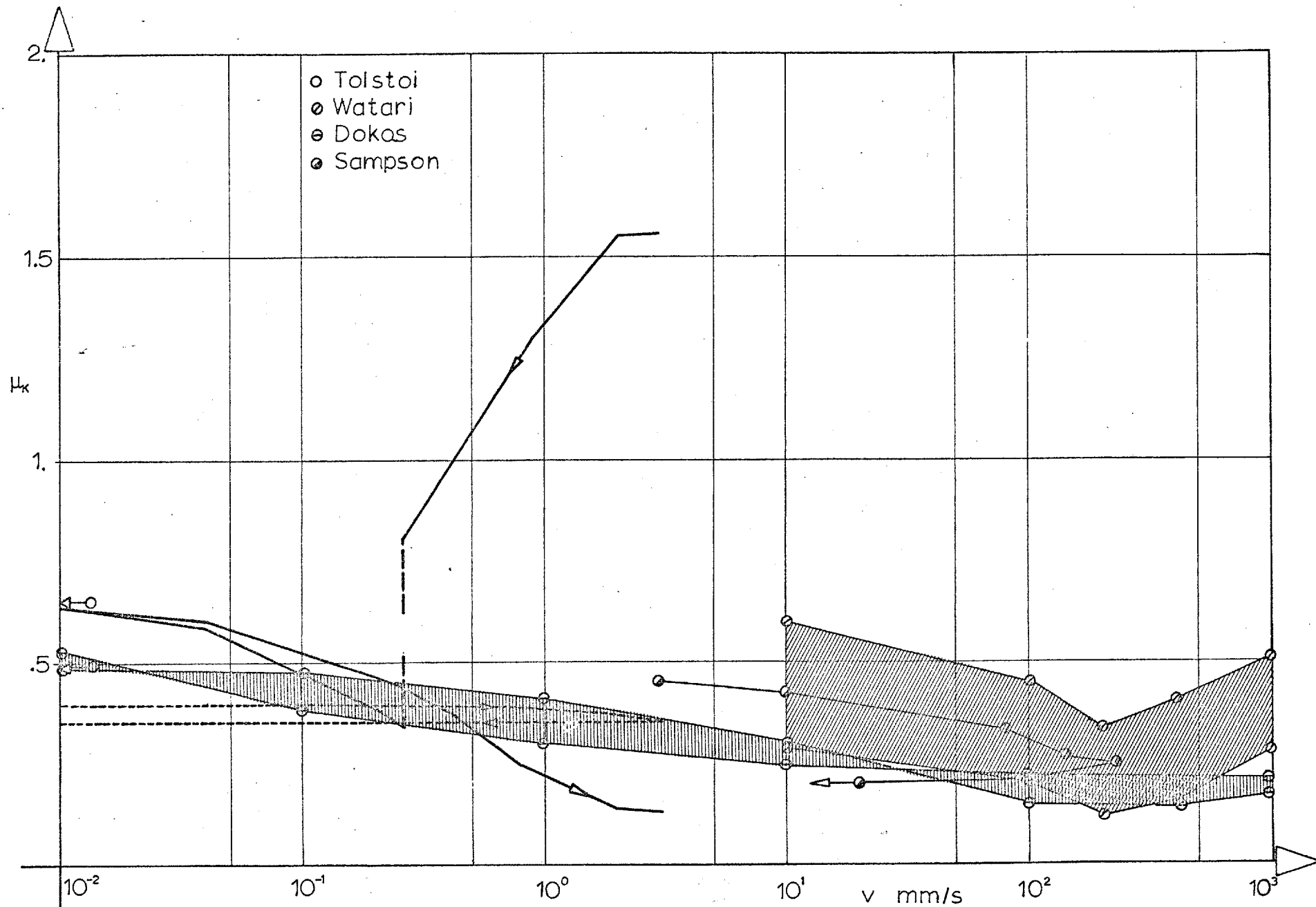


Fig. 4.10

the dotted line characteristic (A) is obtained, fitting much better in the previous results and giving the wrong impression that the coefficient of friction is almost constant with velocity. This is due to the fact that wear does not interfere with these measurements as in the case of other experiments where strong bifurcation was noticed (e.g. Sampson et.al. [17]). One notices also that previous bifurcated results are described in a clockwise direction (agree with the trajectory (A) accepted as proportional to $\mu=\mu(v)$) while the real function $\mu=\mu(v)$ in a counter clockwise direction. Fig. 4.10 explains clearly why as stick-slip commences the wear rate increases considerably although the coefficients of friction do not increase proportionally, or sometimes they remain at about the same levels, as was noticed by almost all the investigators who studied the stick-slip phenomenon.

Finally of importance is the fact that results obtained with both apparatus present remarkable reproducibility. Thus no "instrumental" factors were employed to explain the scattering of the experimental results.

4.2. RESULTS

4.2.1. Dry friction

4.2.1.1. Steel on steel

The experimentally derived function $\mu=\mu(v)$ for unlubricated steel on steel has already been examined in § 4.1. (Fig. 4.4. the characteristic and fig. 4.9. the relative positions of one phase-plane trajectory and characteristic line $\mu=\mu(v)$).

The much greater scattering of experimental points around the upper part of the characteristic results from the obviously irregular variation of the percentage of naked metal to metal contact. It is also probable that atmospheric oxidation processes contribute to that. On the contrary the lower part of the characteristic cannot be affected by these two factors.

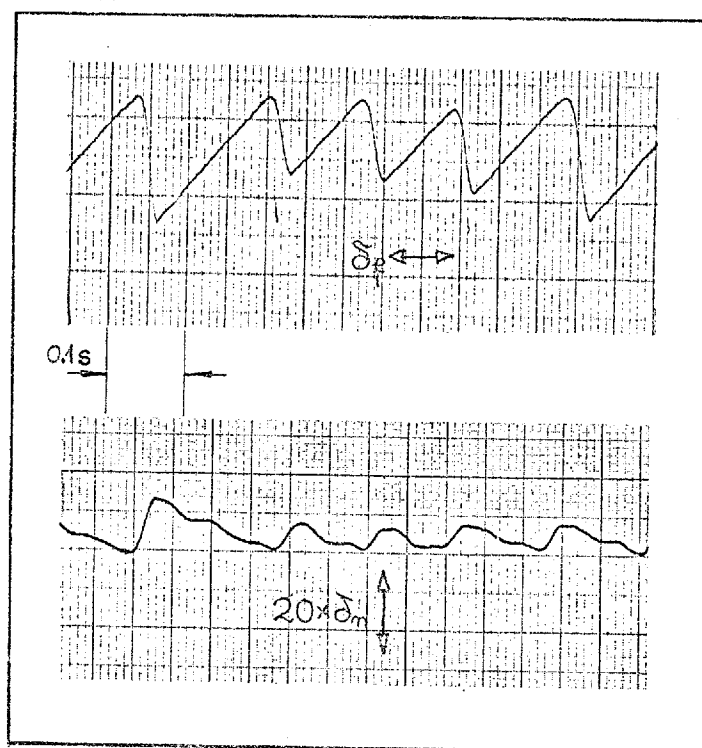


Fig. 4.11

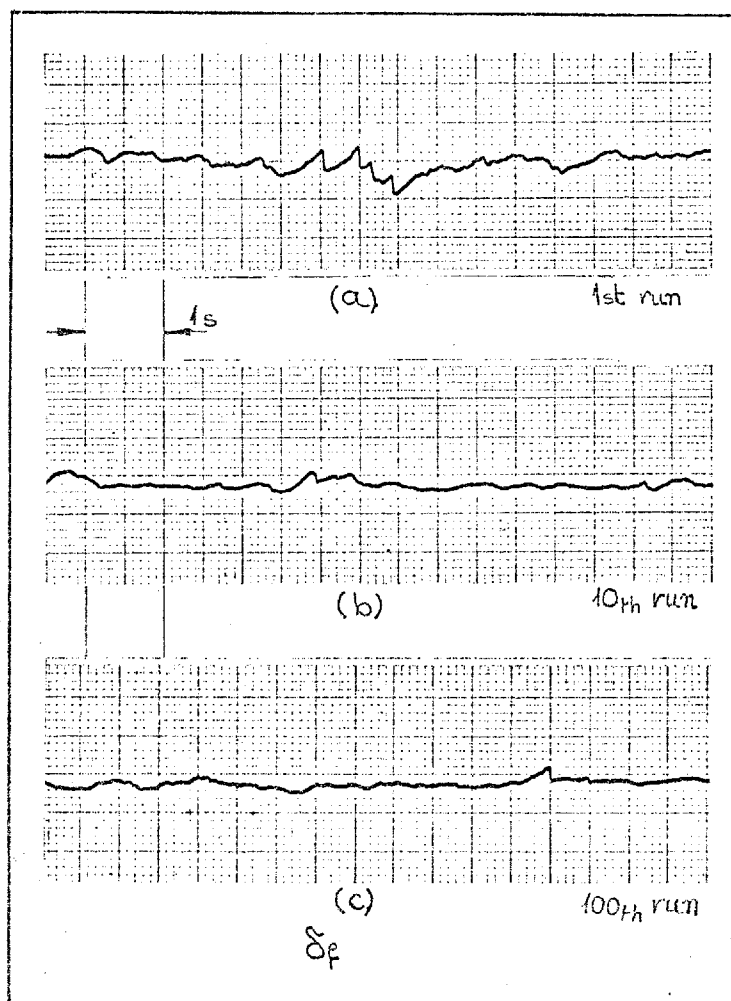


Fig. 4.12

surface films and that justifies their irregular appearance.

Analysis of stick-slip cycles during the first run gave extremely scattered results and no characteristic line could be drawn.

An explanation of this unusual behaviour of hard bearing steel, which is in agreement with the hypothesis of the twin mechanism of friction, is as follows. For the first runs interposed oxide films regulate the frictional behaviour of the system, thus, in dependence upon their properties, bifurcated characteristic can be observed. This is more or less irregular depending

In the case of non-lubricated hardened bearing steel (EN31) sliding on mild steel no regular stick-slip (e.g. as obtained with mild steel on mild steel, fig. 4.11) could be obtained, under the same experimental conditions.

The "running-in" period instead of establishing regular stick-slip as it was observed for mild steel, here establishes a state of almost smooth sliding with but a few "jerks" (fig. 4.12) of very small amplitude.

The friction fluctuations observed during the first few runs are produced by the

on how strongly the surface films are connected with the metal surface. After a few runs the bulk of these films is removed and due to the great difference in hardness, the hardened steel surface asperities plough through the mild steel asperities. This is a typical behaviour for the upper part of the characteristic which has positive slope at the origin and no stick-slip could appear. Thus after the "running-in" period the lower branch of the characteristic disappears because for any speed the ploughing mechanism operates as a result of the hardness difference. Short "jerks" appearing in the friction trace must be treated not as slip parts of stick-slip motion but rather as casual friction fluctuations (a lack of periodic character of these "jerks" supports this, as one can see in fig. 4.13).

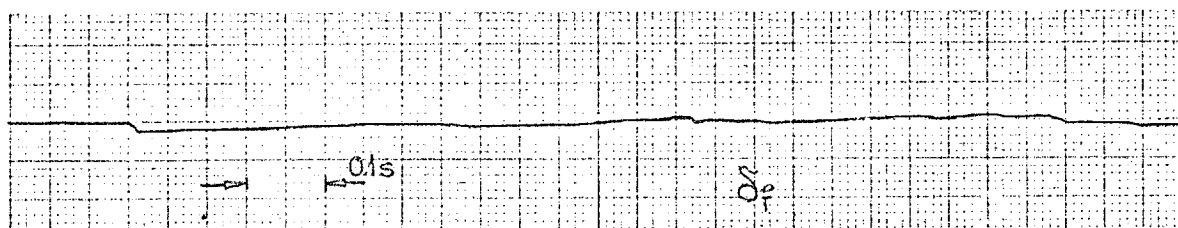


Fig. 4.13.

4.2.1.2. Bronze on bronze

Bronze being a typical bearing alloy has also been used as a specimen (see Chapter 3).

The high level and rather irregular triggering oscillation in many cases made the drawing of the vertical phase-plane diagram problematical. (fig. 4.14 shows a typical trace).

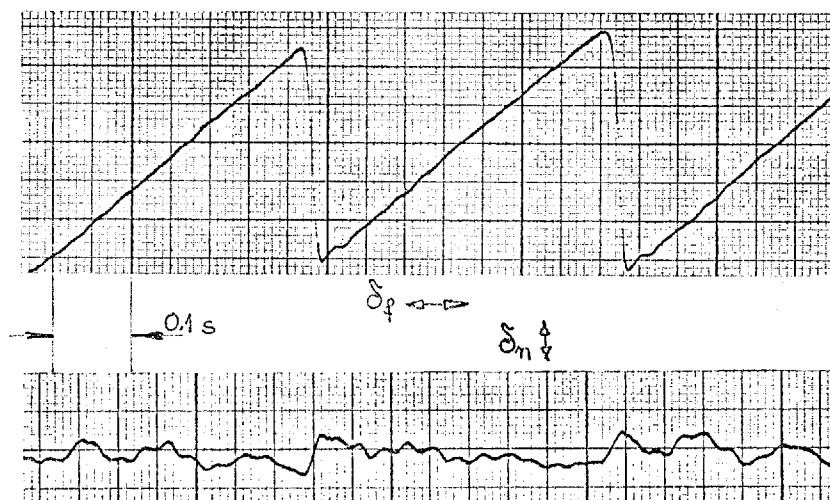


Fig. 4.14.

on the experimental function $\mu = \mu(v)$, fits quite well with the experimental points (fig. 4.16). Unfortunately no previous results of friction as a function of velocity for bronze were found in the literature for comparison.

Fig. 4.15 reveals two basic differences between bronze and steel frictional pairs:

1. The upper part of the characteristic is almost horizontal for a much wider range of velocity (probably because the oxidation process does not interfere so vigorously as for steel on steel).
2. The "transition velocity" is not so clearly defined.

4.2.2. The effect of the lubricant

With the two typical industrial lubricants containing anti-squeal

The amplitude of the triggering oscillation increases with the "running in" time (as was expected) and a decaying oscillation appears following every slip of the slider which can be ascribed to the high triggering oscillation level (see also App.9).

Analysis of stick-slip traces gave the $\mu = \mu(v)$ characteristic appearing in fig. 4.15, while a theoretically drawn field of slopes of the trajectories, based

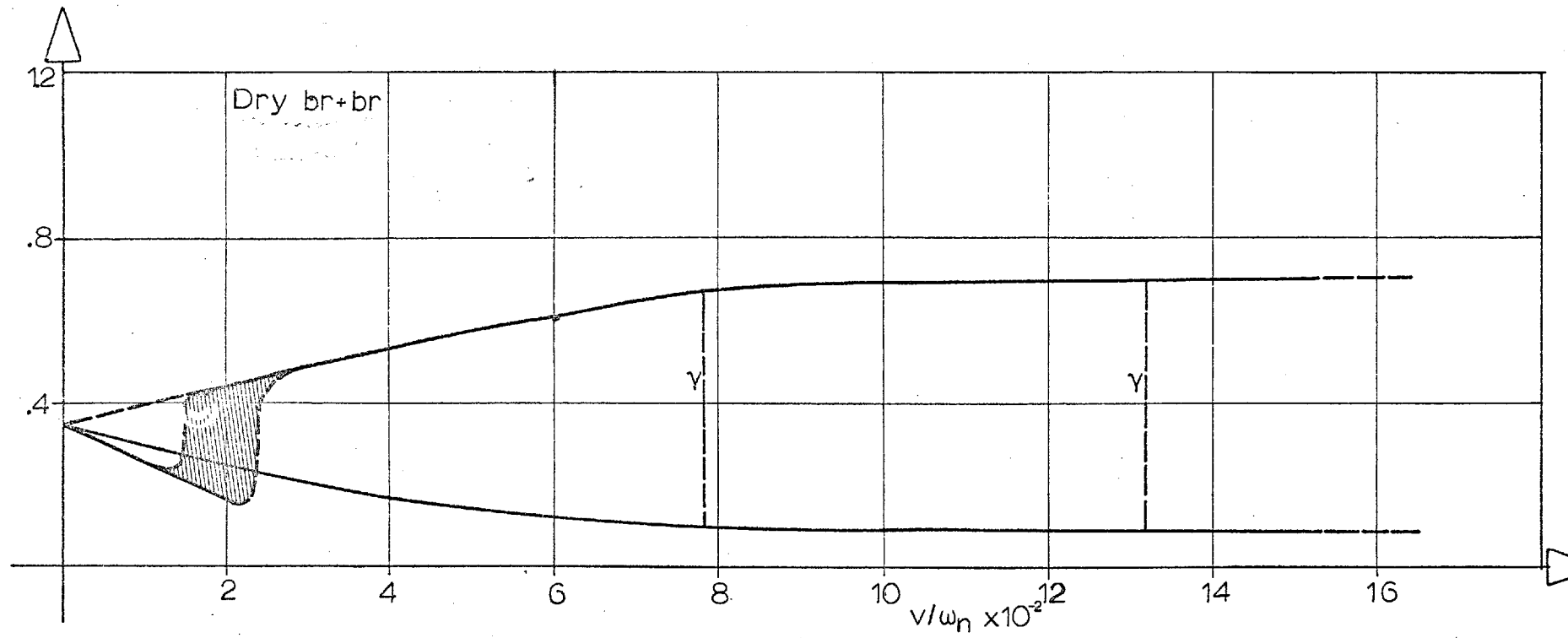
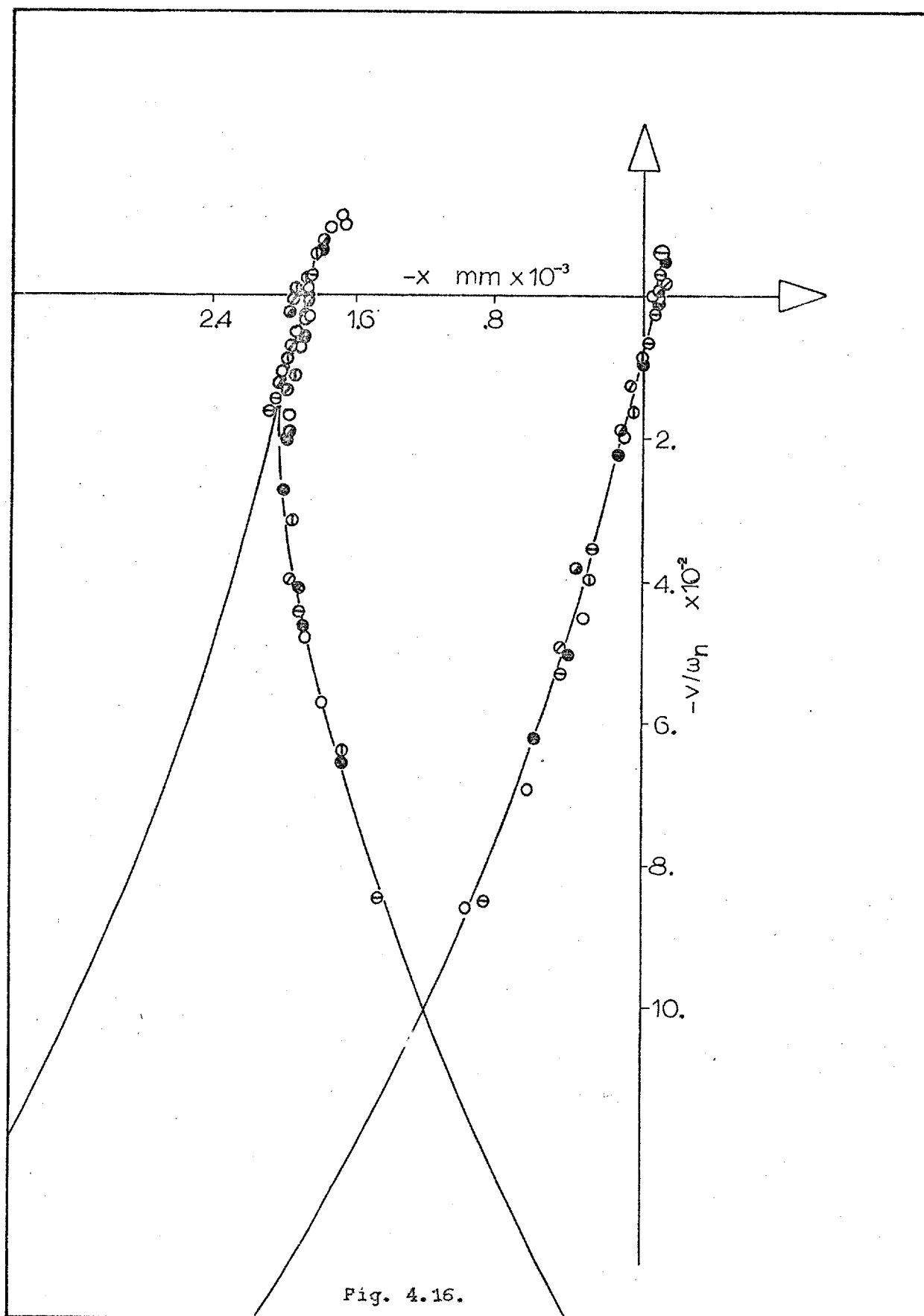


Fig. 4.15.



additives (friction modifiers) no stick-slip was observed within the range of load and velocities used and, consequently, no analysis of $\mu=\mu(v)$ obtained from phase-plane trajectories is feasible.

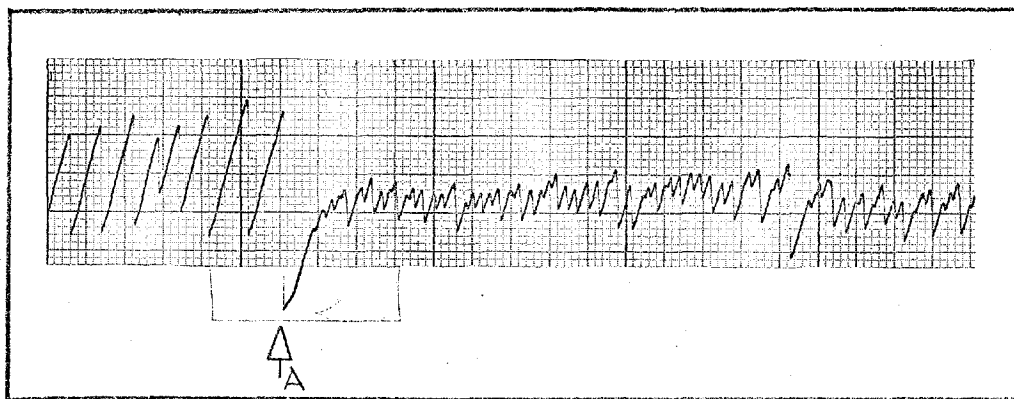


Fig. 4.17

The application of cetane or paraffin oil as lubricant following the establishing of regular stick-slip with dry specimens, produced traces similar to the one in fig. 4.17 (A marks the moment when the lubricant was applied). Thus lubricant application is followed by drastic decrease of the static coefficient of friction (measured in the conventional way from the displacement of the slider and found to be after the lubricant application about 0.1 - 0.25), and also decrease (5 - 20 times) of stick-slip amplitude.

Much more pronounced was the effect of a rubbed-in MoS_2 film as a lubricant. The (conventional) static coefficient of friction dropped to about 0.06 - 0.18, the amplitude of stick-slip decreased about 15 - 40 times and the shape of frictional oscillations changed from stick-slip to quasi-sinusoidal (fig. 4.18). This form of oscillation combined with short slip "jerks" - was described by Lenkiewicz ([220],[221, pg. 52]) as an individual phenomenon (reverse stick-slip). The $\mu=\mu(v)$ function derived from the analysis of stick-slip cycles appears in figures 4.19, 4.20. In these traces it can be observed that although the twin mechanism of friction

remains under lubricated conditions of sliding, the scattering of the experimental points around the upper part of the characteristic is much greater, and the transition from the upper to the lower branch is gradual

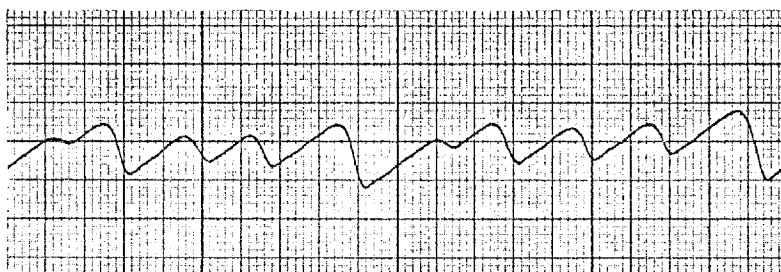


Fig. 4.18

while the "transition velocity" loses its sense.

In Fig. 4.21 a number of results from the literature, obtained under "static" conditions, has been drawn on the same graph with results for lubricated specimens. Also fig. 4.22 presents the only "dynamic" tests in the literature put on the same graph with currently derived similar results to make a comparison easy. The agreement with results obtained by Hunt-Torbe-Spencer [51] or Štěpánek [24] is obvious, and this between Schindler's results and the phase-plane trajectory A from which the characteristic B was obtained is also evident.

4.3. CONCLUSIONS AND RECOMMENDATIONS FOR FUTURE WORK.

a. - The use of the model established in chapter 2 and the topological expression of the differential equation of motion made possible the experimental derivation of the function $\mu = \mu(v)$. This function is strongly nonlinear and bifurcated due to the coexistence of two frictional mechanisms (bifurcation in a different sense than the one met in the literature). Consequently linearization or quasi-linearization of the problem or use of analogies without a nonlinearity of the same kind are not justified.

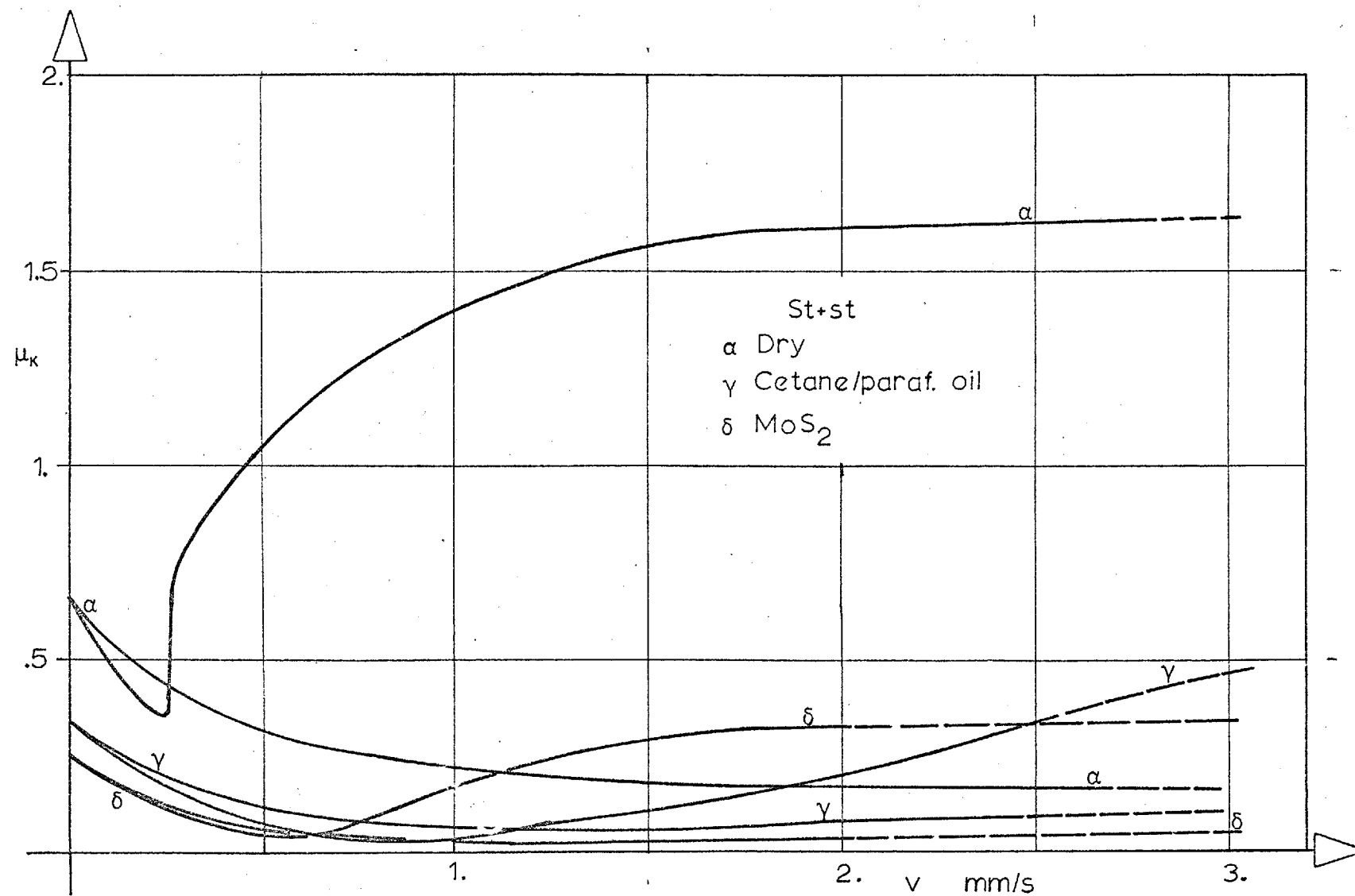


Fig. 4.19.

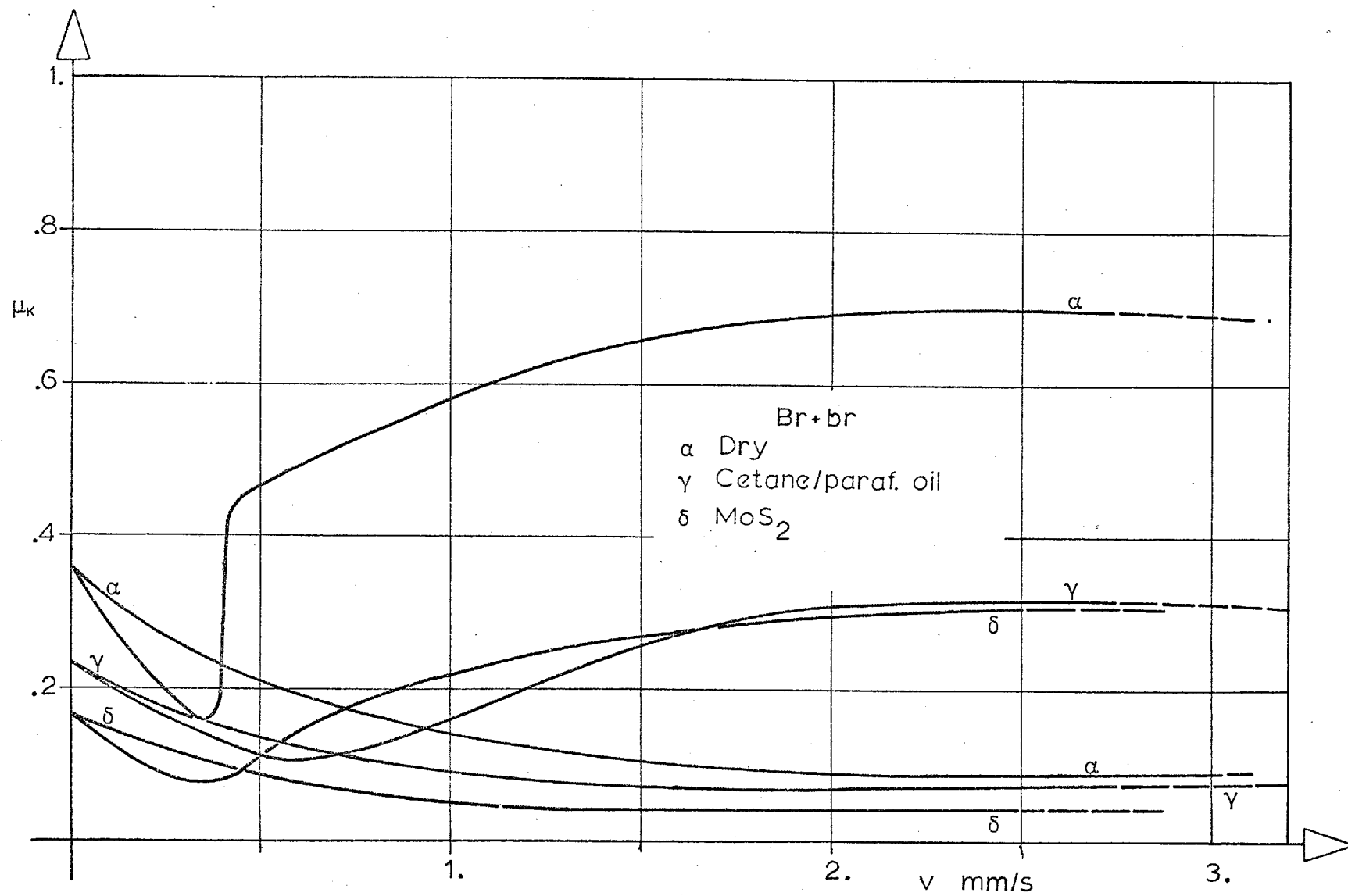


Fig. 4.20

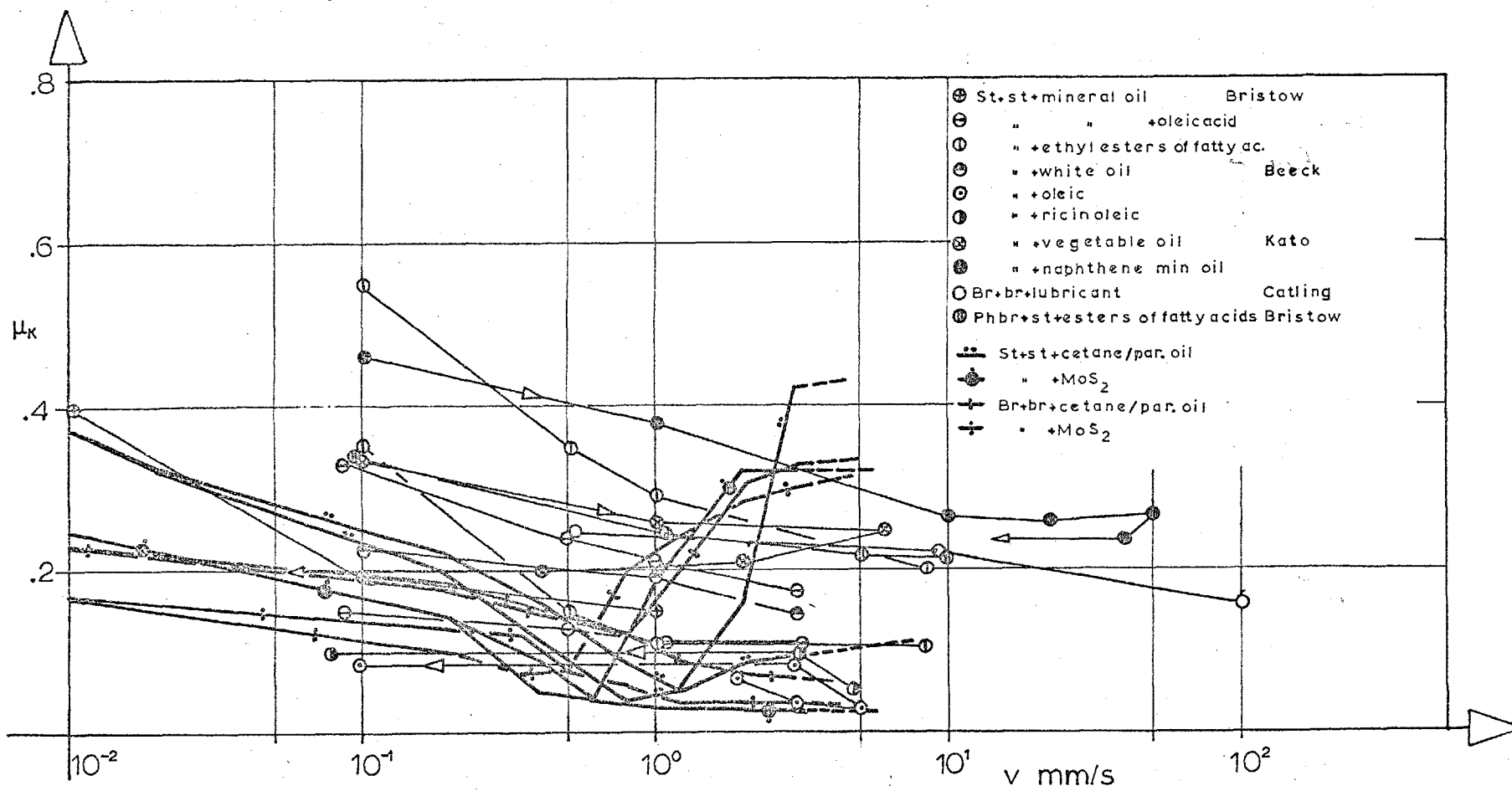


Fig. 4.21.

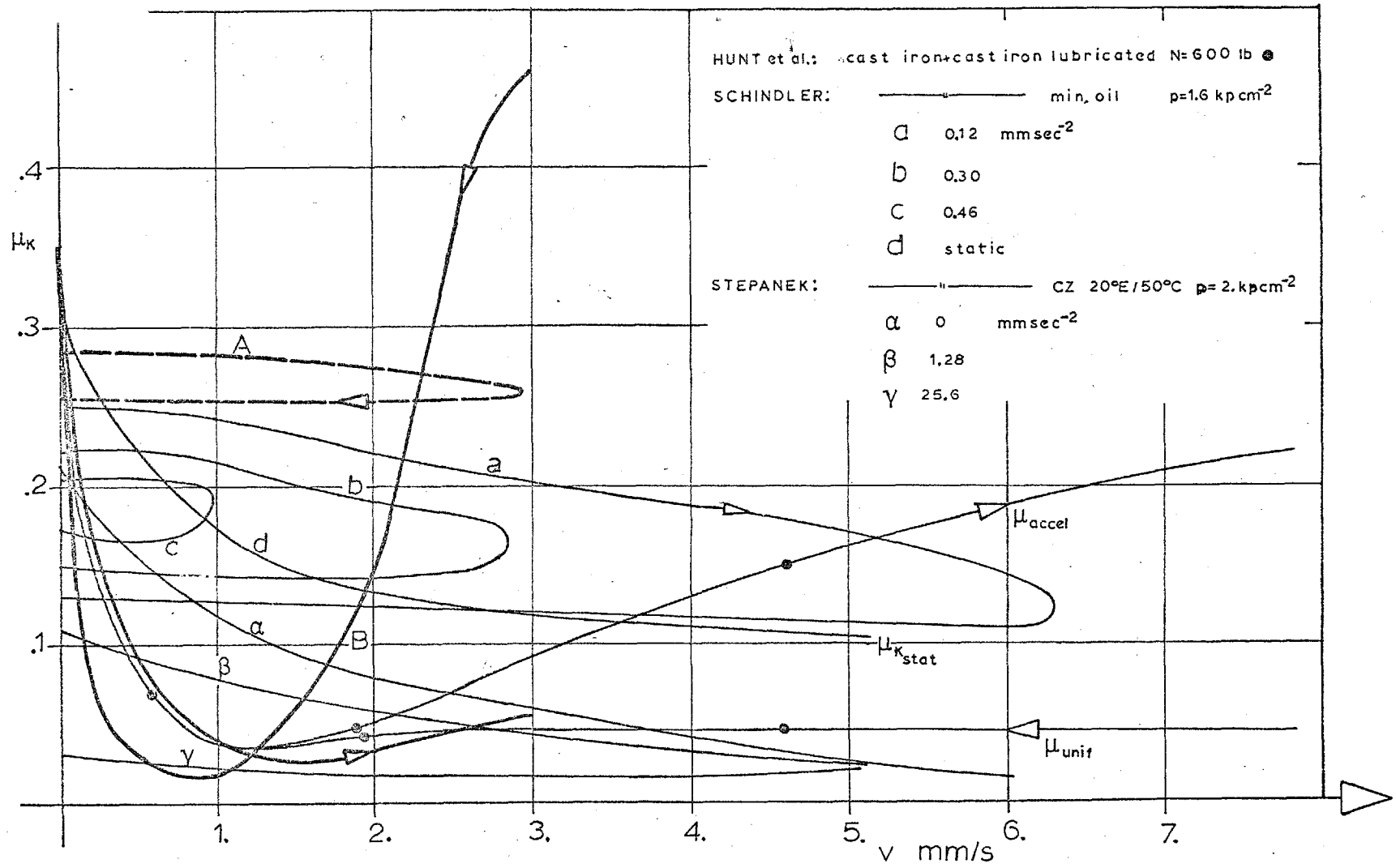


Fig. 4.22.

b. - Characteristics such as the mean driving velocity v_0 , the difference $\Delta\mu = \mu_s - \mu_k$ or the slope of the function $\mu = \mu(v)$ for $v=0$ found to effect friction oscillations in the way predicted by some previous investigators.

c. - Model and mathematical treatment of it present the advantage of generality. Thus it was not necessary to employ any "instrumental" factors to explain discrepancies in the experimental results. The model explains reasonably well the observed irregularities of frictional oscillation (e.g. the three distinct phases of stick-slip according to Bowden and Leben [12]) purely in terms of the mechanics of the system. This model is also susceptible of a further improvement and generalization.

d. - The triggering oscillation (probably the same as the "microvibration of Blok or Tolstoi's "seismic oscillations") appear, not as an externally produced oscillation interfering on the frictional oscillations (Tolstoi) but, as a manifestation of the tribological characteristics of the system under study (micro-model), because a phase-plane analysis of more than one successive cycles of stick-slip always shows that the "slip" phases start or end with triggering oscillation "in-phase" (see e.g. traces of App. 8). Unfortunately apparatus and instrumentation were not designed with triggering oscillation in mind; thus the available information is insufficient for a complete study of it.

e. - Suppression of the freedom of movement in the vertical direction leads to uniform movement, no matter how low the mean driving velocity v_0 is (observed at first experimentally by Tolstoi). This technique for obtaining smooth driving is especially advantageous in cases where the differences between the upper and the lower branch of the characteristic is small.

f. - Smooth driving can also be obtained by adjusting the characteristics of the system in such a way that the "stick" period duration t_s is less than a limiting time t_e that is a characteristic of the surfaces under study (e.g. for dry steel on steel $t_e \approx 0.04s$). This criterion is virtually the same as that of Rabinowicz critical length (see App.1).

g. - For varying $v_0 = v_0(t)$, transformation of stick-slip to quasi-sinusoidal oscillation can be obtained by adjusting the characteristics of variation of the driving velocity v_0 .

h. - The relative acceleration between the specimens was found to have no effect on frictional oscillations as an independent factor.

i. - The load variation during a stick-slip cycle is the main source of inaccuracies in the experimental results.

j. - More work must be done in the direction of making clear the mechanism of triggering oscillation, and its origin. It is true that although the results obtained were satisfactory, the manipulation of triggering oscillation was rather arbitrary to avoid mathematical implications.

k. - Modifications are necessary for the graphonumerical technique used to solve the equation of motion. This, in combination with better methods for determination of the relative position of trajectory and characteristic on the phase-plane and on-line computation techniques, is expected to lead to a more accurate study of the coefficient of friction, and its variation with the tribological characteristics of the system.

APPENDICES

Appendix 1.

Experiments on the variation of the coefficient of friction with time and the effect of temperature.

The function $\mu_s = \mu_s(t_s)$ was examined experimentally for the materials and lubricants used in the main experimentation in order to justify the fact that the dependence of the static coefficient of friction on the idle time is not significant enough (under the conditions of the main experimentation) to be taken into account.

Specimens

Ordinary mild steel (EN1)

Lubricants

Cetane and paraffin oil were used for comparison with a typical industrial anti-stick-slip fluid. (MOBIL)

Preparation of the surfaces

As for the main experimental work (clean surfaces, mean final height of the surface asperities 0.5μ)

Experimental methodology

The static coefficient was measured (rig Mark II) as a function of the idle time t_s for $t_s = 1 \div 100$ sec.

Parameters were the lubricant (none, oxide films on specimens exposed in atmosphere for several days, cetane, paraffin oil, and fluid Mobil 220), the temperature (20 to approx. 150°C), the rate of application of the frictional force and the method of application of the normal load. In parallel with μ_s , the magnitude of jump from the static to the kinetic value ($\mu_s - \mu_{k_0} = \Delta F$) was measured.

Effect of the lubricant

All the lubricated experiments gave a variation $\mu_s(t_s)$ similar to that predicted theoretically (e.g. by the formula of Rabinowicz with properly chosen constants), with only one exception for the paraffin oil, during the

first stages of the experiment (curve 2 fig. A1-1). A reasonable explanation is that paraffin oil acts chemically on the metal surfaces producing an oxide film which lowers the coefficient of friction. This oxide formation counterbalances the increase of the static coefficient of friction with idle time. In the case of fluid 220, $\mu_{s_{\infty}}$ is obtained in about 10 sec and after that the static coefficient of friction is insensitive in idle time changes.

Effect of the temperature

Increased temperature is expected to affect μ_s by:

- a.- Accelerating chemical reactions
- b.- Changing the viscoelastic properties of the specimens
- c.- Changing basic properties of the lubricant (e.g. viscosity, orientation of the molecules etc.)

Fig. A1-1 shows that (for cetane as lubricant) temperature variation of the order of 50°C produces an overall change in friction of approx 16% (see also Brockley-Davies [157]), while fig. A1-2 shows that cycles of heating-cooling have an irreversible effect on the coefficient of friction due to chemical reactions occurring within the cycle (see also Niemann-Ehrlenspiel [32]). The variation of the non-dimensional magnitude $\left[\frac{\Delta F}{N}\right]_{\infty}$ which is proportional to the amplitude of stick-slip, follows precisely the variations of the $\mu_{s_{\infty}}$ with time, and becomes zero for high temperatures, an indication that at these temperatures the static coefficient of friction is less than the kinetic one, even for extremely low velocities.

Effect of load

Experiments with dry surfaces, exposed to the air for some days, or clean, showed that the function $\mu_s = \mu_s(t_s)$ is affected by the load in a very complicated way (fig. A1-3). There are no satisfactory results to be found in the literature and it would seem that only statistical treatment of the problem could lead to some production of relevant formulae.

With dry surfaces, the oxidation mechanism interferes and becomes

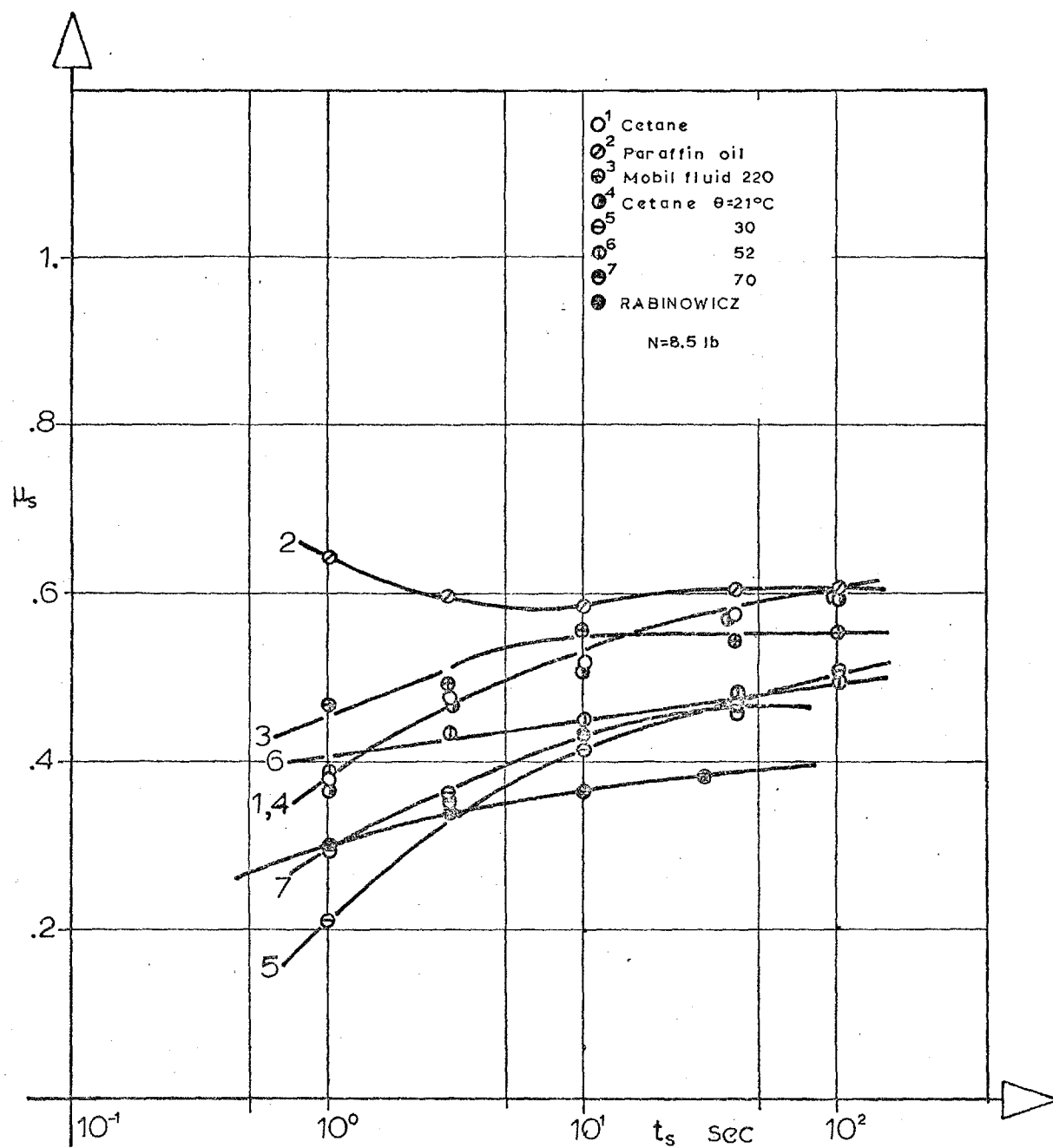


Fig. A1-1

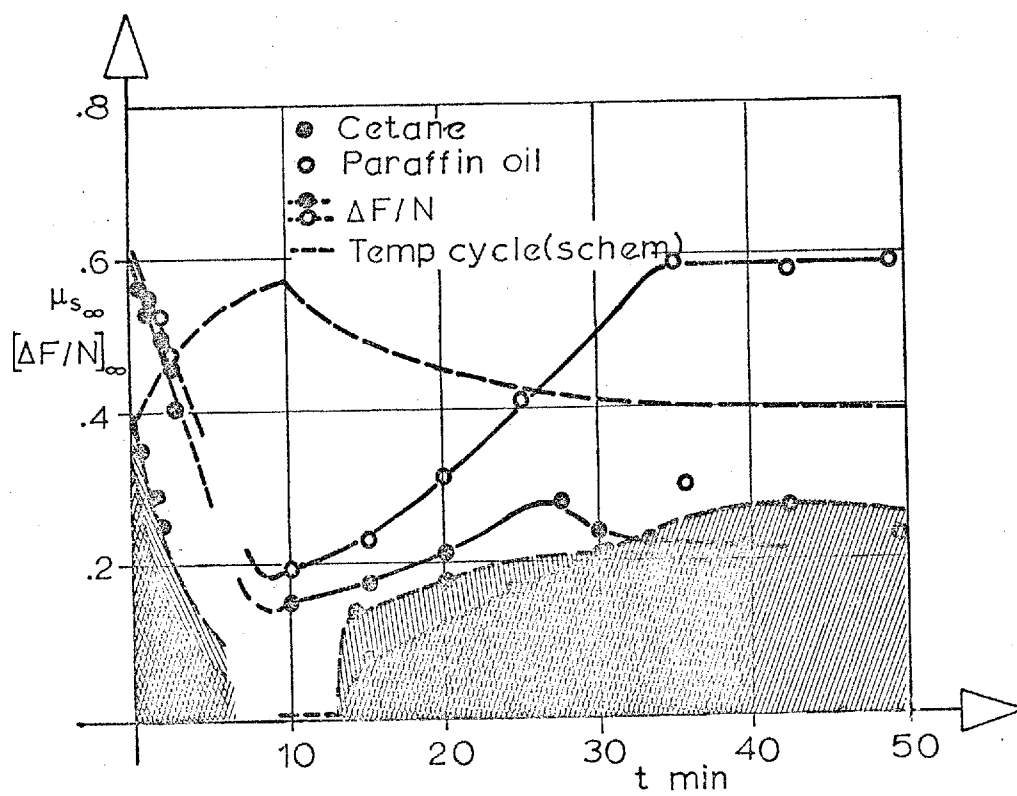


Fig. A1-2

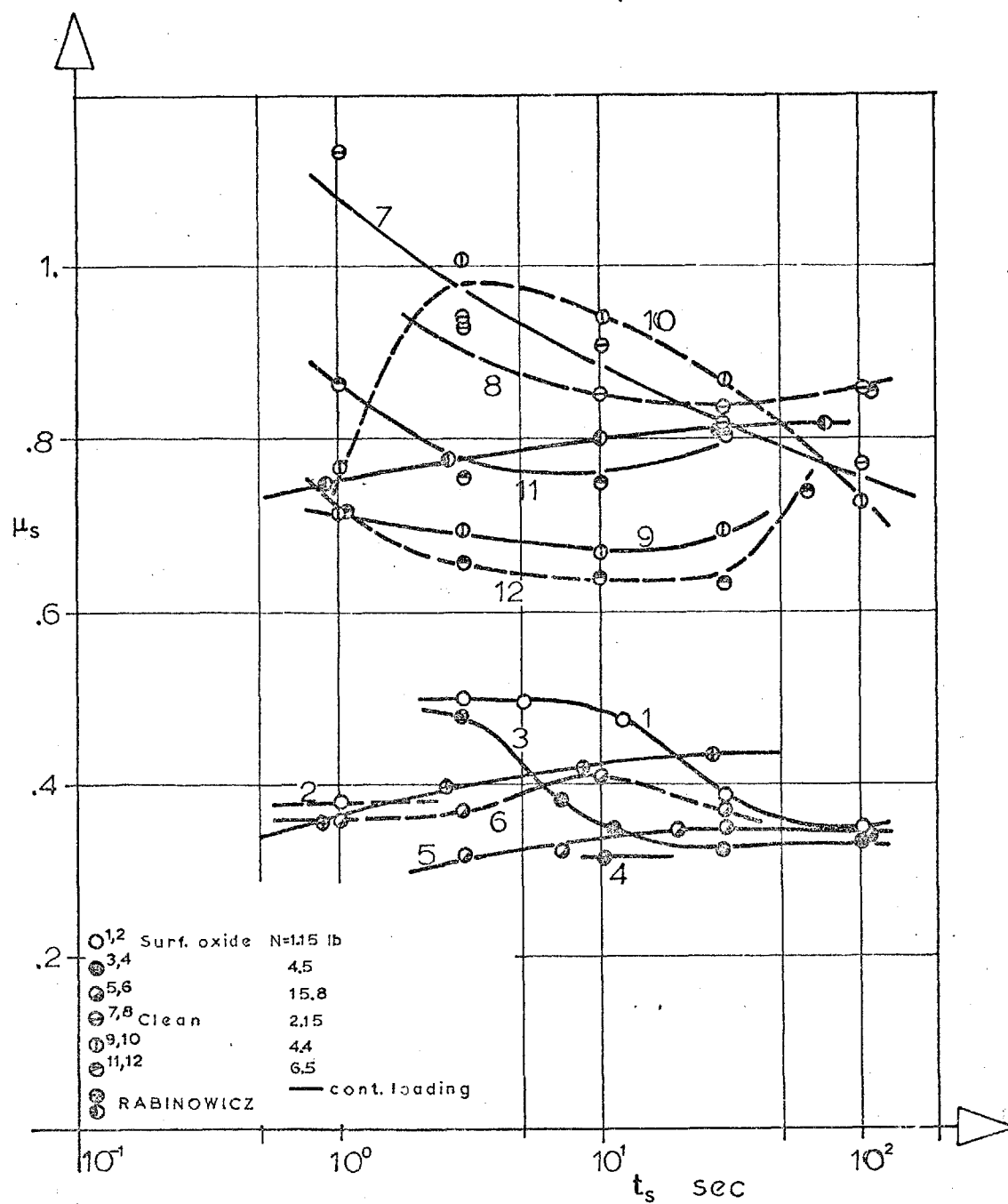


Fig. A1-3

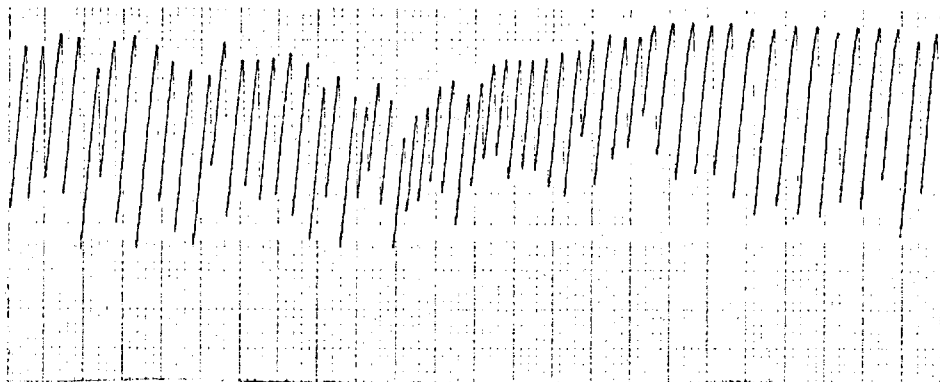


Fig. A1-4

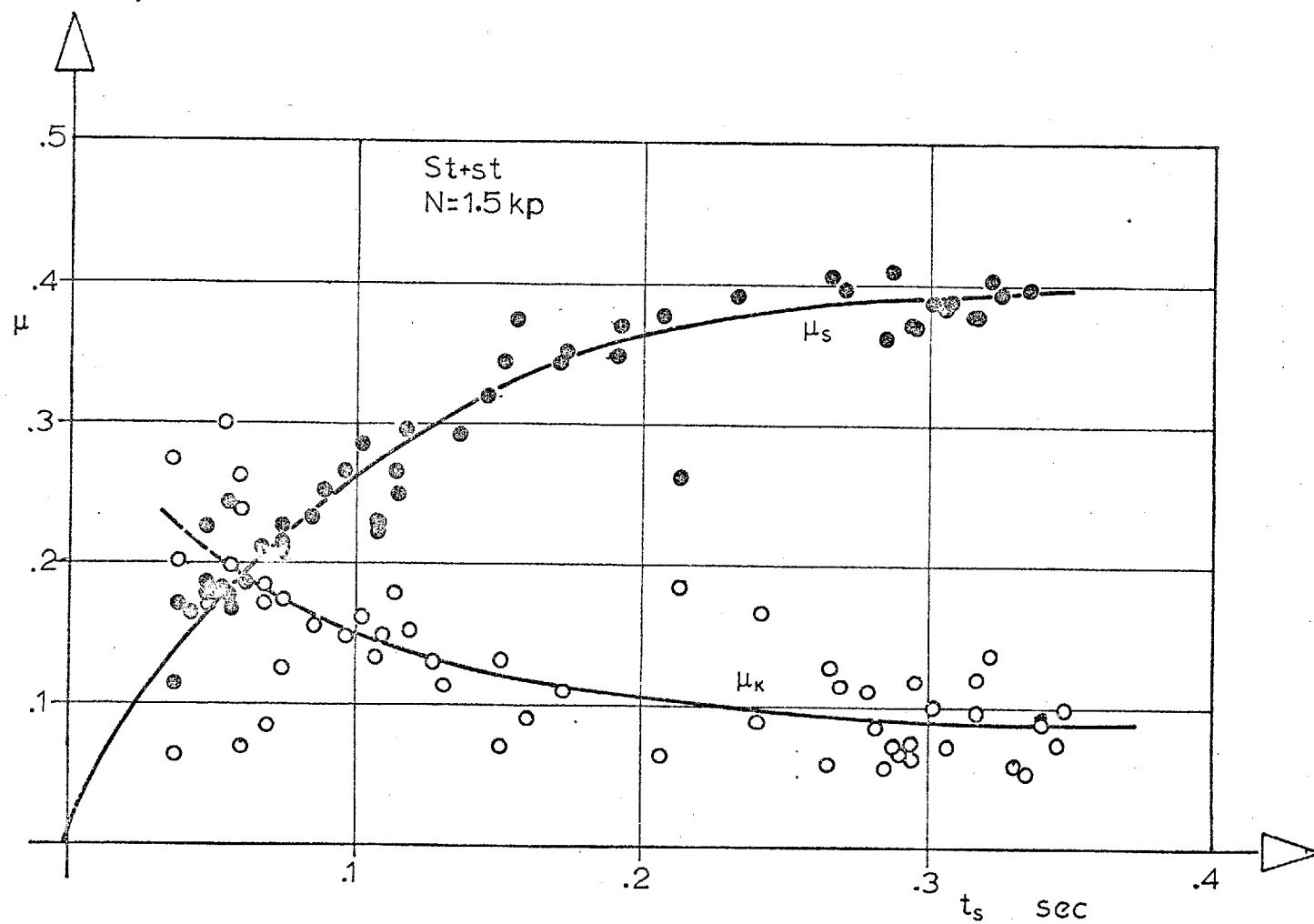


Fig. A1-5

predominant. This explains the stepwise drop in friction with t_s for oxide covered surfaces (traces 1,2,3,4,5,6) and the continuous drop with "fresh" surfaces (traces 7,8,9,10,11,12). The two techniques used for load application (load-unload. cycles and continuous stepwise loading) produced quite similar results.

Effect of friction rate

For rates of frictional force application in the region $2^{lb}/s - 0.02^{lb}/s$ no difference was observed in the function $\mu_s = \mu_s(t_s)$. This an indication that the specimens under the present specific experimental conditions do not exhibit any observable viscoelastic behaviour.

Short-time experiments

For idle times shorter than 1 sec the previously followed technique fails to give good results. For this case indirect measurements from stick-slip showed (fig. A1-5) that μ_s and also μ_k as they are measured on the frictional force traces, seem to be affected by idle time. Important is the fact that for $t_s < 0.04$ sec, $\mu_k > \mu_s$ which means that no stick-slip can exist for these specimens if the stick duration is less than 0.04 sec, or for $v_o = 0.7^{mm}/s$ there is a limiting length of $30 \mu m$ under which no slip distance could appear. This agrees in the order of magnitude with Rabinowicz's critical length of $10 \mu m$.

Appendix 2

Analysis of typical triggering oscillation traces

The effect of load, mean sliding velocity, material and lubricant on the triggering oscillation are to be studied. Amplitudes and frequency spectra of triggering oscillation will be treated statistically.

Specimens:

Ordinary mild steel on mild steel (EN1) and bronze on bronze.

Lubricants:

Cetane, medicinal paraffin oil and one industrial anti-stick-slip fluid.

Preparation of the surfaces:

As for the main experimental work.

Methodology

The triggering oscillation was separated from the stick parts of stick-slip cycles. This increases the accuracy of the method because the slope of the stick parts is known ($v_o = \frac{\Delta\delta_f}{\Delta t}$ and acc. to Lisitsyn $\Delta\delta_n = \zeta\Delta\delta_f$ thus $\frac{\Delta\delta_n}{\Delta t} = \zeta \cdot v_o$). In cases where the amplitude was very low or the wave-form complicated the reliability of the method became very poor.

The results were extracted from statistical samples containing 50 - 100 elements.

The effect of load

Fig. A2-1 shows clearly that increased load produces an almost proportional increase of the amplitude A_{tro} of triggering oscillation while its frequency remains constant, at least for the range of loads tested ($N = 380 - 1530p$).

Similarly for Bronze on Bronze although for low load there is a much wider spectrum of frequencies, the mean frequency remains almost constant. For increasing load (fig. A2-2), the frequency spectrum becomes narrower and

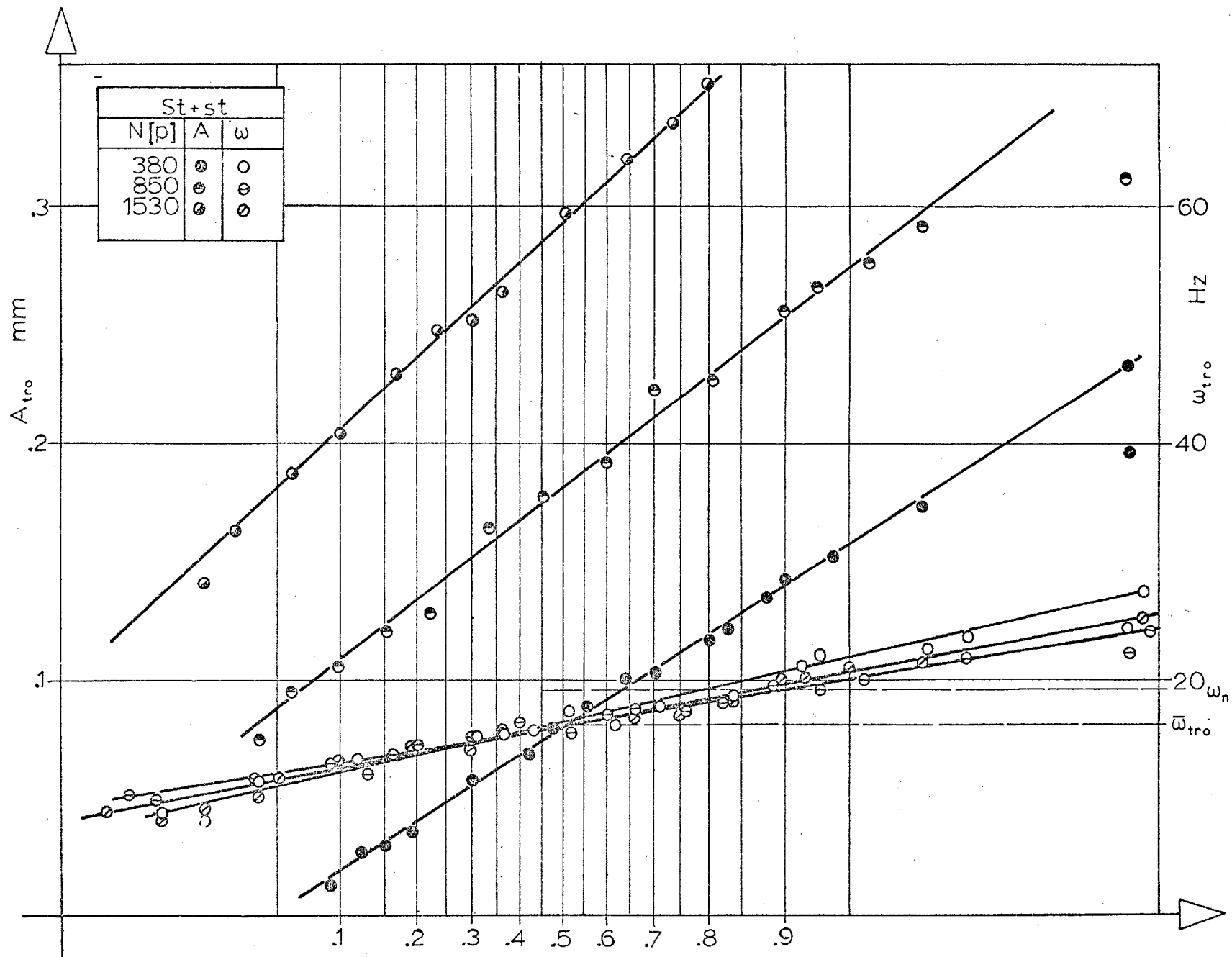


Fig. A2-1

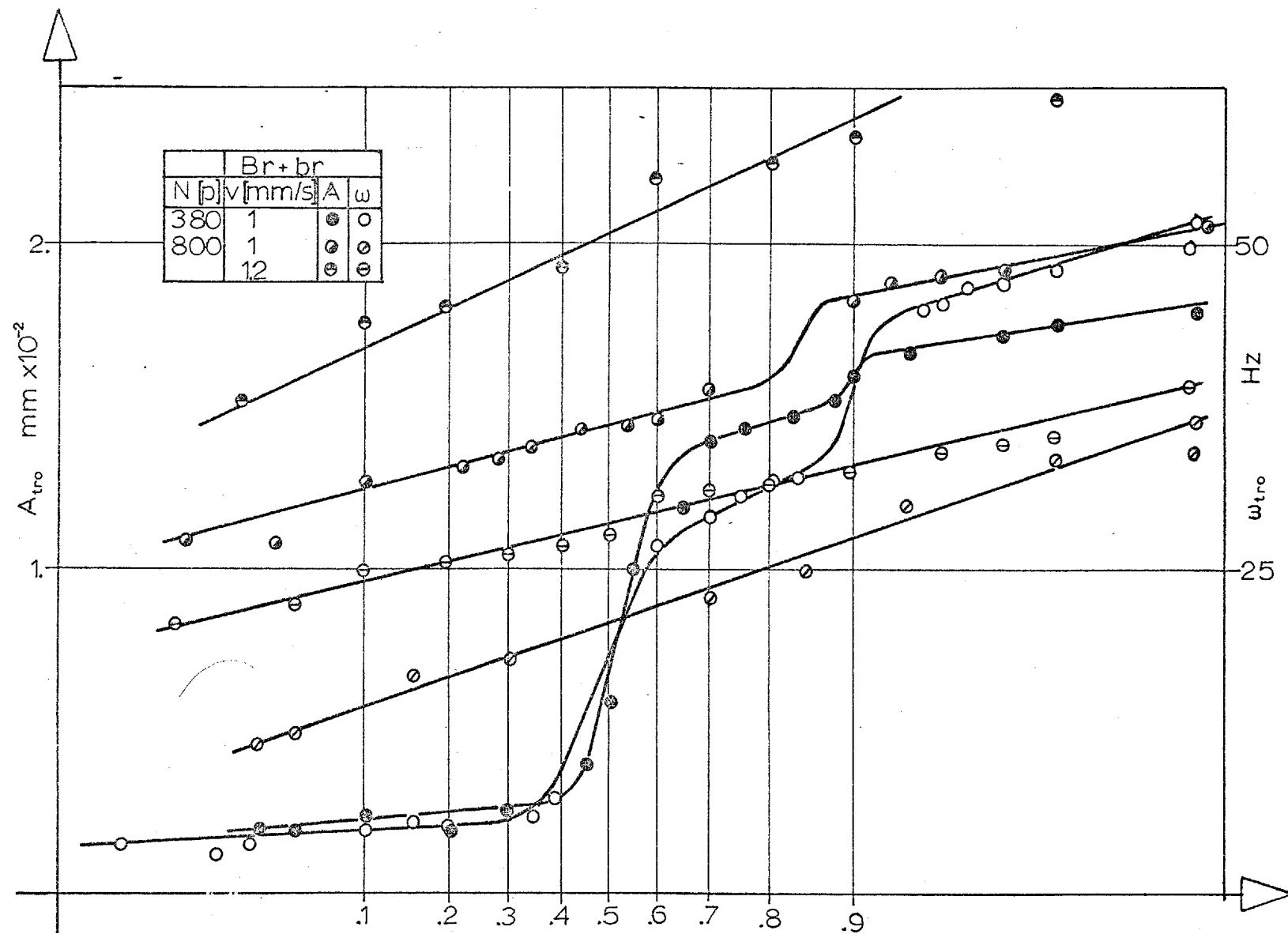


Fig. A2-2

the amplitude (much less than in case of steel on steel) increases almost proportionally with it.

The effect of mean sliding velocity

According to fig. A2-2 amplitude and frequency of triggering oscillation increase with mean sliding velocity, frequency, in proportion with it, and amplitude following a second power law. This seems to be in agreement with the conception that energy is exchanged between lower surfaces and slider in small finite amounts. In that case the mean displacement produced (δ_n or δ_f) is proportional to the mean kinetic energy of the asperities of the lower surface, which is proportional to v_o^2 . The frequency for the same surface topography varies, on the other hand, in proportion to v_o .

The effect of materials and lubricants

An idea about how the material or lubricant characteristics could affect triggering oscillation can be obtained by studying the diagrams of fig. A2 - 1, 2 and 3.

What is noticeable is the drastic decrease in amplitude with changing specimens from St + St to Br + Br (a five-fold drop) and the decrease in amplitude with oils of higher "oiliness" while frequency seems to be not very much affected by it.

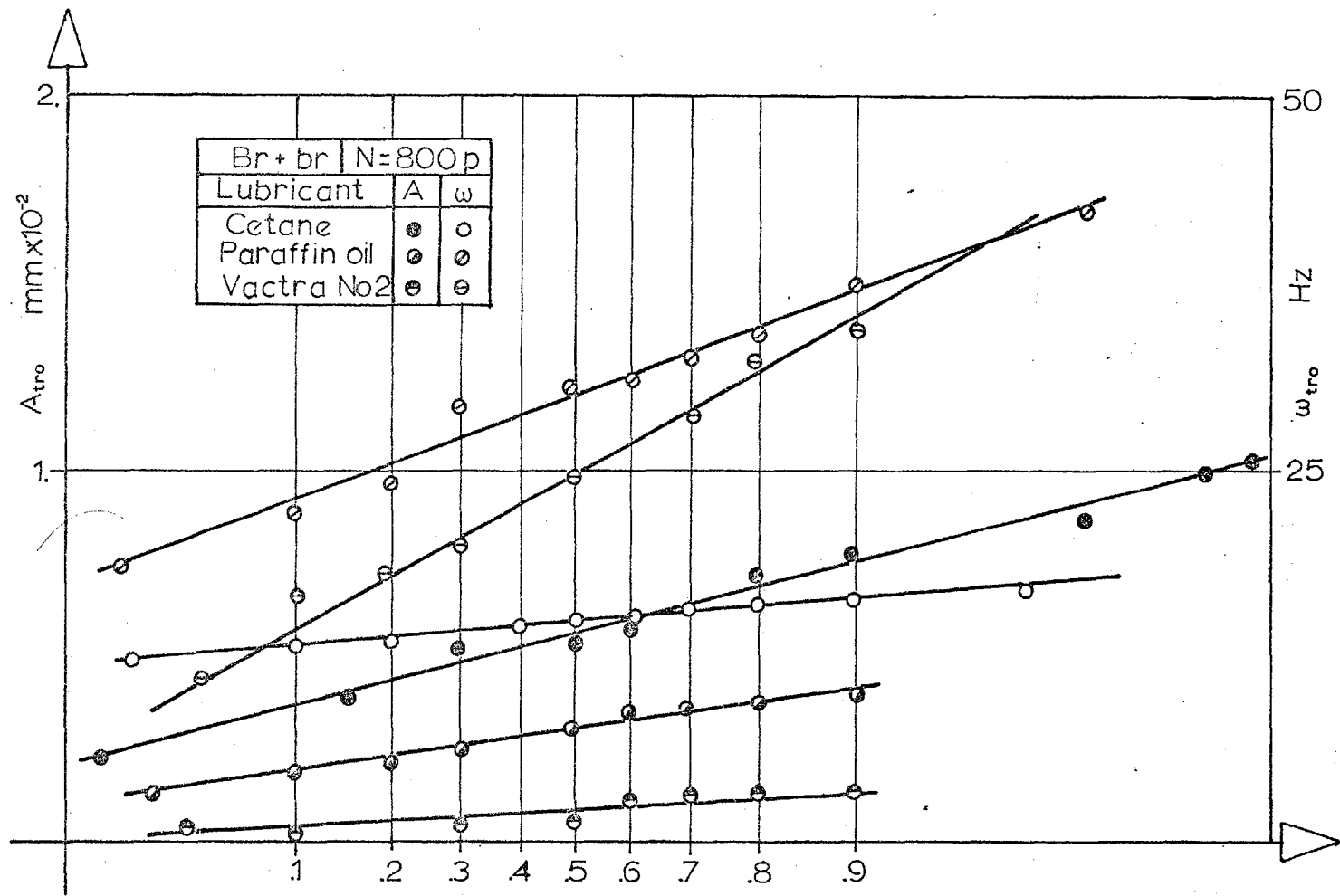


Fig. A2-3

Appendix 3

The phase-plane diagram-Energy curves

An $\{x, v\}$ plane (where $v = \frac{dx}{dt}$) is called in topology "phase-plane".

Taking into account that x, v are functions of time, the curves on the x, v plane may be regarded as given in parametric form with t as parameter. These curves are called "energy" or "integral" curves or "phase-plane trajectories".

A differential equation of the second order:

$$\ddot{x} = Q(x, \dot{x}) \quad \text{..... (A3-1)}$$

can be reduced to the form:

$$\frac{dx}{dt} = P(x, \dot{x}) = y$$

$$\frac{dy}{dt} = Q(x, \dot{x}) = Q(x, y) \quad \text{..... (A3-2a, \beta)}$$

because time t does not appear explicitly so a new variable $v = \dot{x}$ can be used.

More specifically, the equation:

$$\ddot{x} + \varphi(\dot{x}) + f(x) = 0 \quad \text{..... (A3-3)}$$

can be written as:

$$\left\{ \frac{dv}{dt} = -f(x) - \varphi(v), \frac{dx}{dt} = v \right\} \quad \text{..... (A3-4)}$$

or:

$$\frac{dv}{dt} = \frac{-f(x) - \varphi(v)}{v} \quad \text{..... (A3-5)}$$

The existence of the term $\varphi(v)$ in equ. A3-4 or A3-2 does not permit separation of the variables in general to obtain solution curves in x, v plane by explicit integration. In spite of this, the geometric interpretation of equations A3-2, A3-4, A3-5 as equations defining fields of directions in the x, v plane can lead to useful information, even though the solution curves themselves cannot be obtained explicitly. The trajectories can be defined as curves which have everywhere the field direction defined by the

above equations. The vector representing that field:

$$\vec{V}_p = \vec{V}_p \left\{ \frac{dx}{dt}, \frac{dv}{dt} \right\} \quad \dots\dots\dots (A3-5)$$

is always tangential to a solution curve and points along it in the direction of motion of the point $P_t \{x(t), v(t)\}$, which is called "representative" point on the $\{x, v\}$ plane, with increasing t .

The velocity of the representative point on the phase-plane is called "phase velocity" and it is:

$$\frac{ds}{dt} = \left\{ \left(\frac{dx}{dt} \right)^2 + \left(\frac{dv}{dt} \right)^2 \right\}^{\frac{1}{2}} \quad \dots\dots\dots (A3-6)$$

Equation A3-5 ceases to define directions on the phase-plane at points where numerator and denominator of the right-hand side vanish simultaneously (Equ. A3-5 gives $\vec{V}_p = \vec{V}_p(0,0)$ in that case). Such a point is called a singular point of the differential equation. Thus a point $P_0(x_0, v_0)$ is singular when, and only when, the function $P(x_0, y_0) = Q(x_0, y_0) = 0$ simultaneously. On that point the passing trajectory degenerates into a single point, the singular point itself.

Through every ordinary point (not singular) on the phase-plane there passes one and only one trajectory. If $P_0(x_0, v_0)$ is a singular point, a trajectory passing through an ordinary point $P(x, v)$ at a certain instant, will never reach $P_0(x_0, v_0)$ for any finite value of the time parameter t , because the only trajectory passing through $P_0(x_0, v_0)$ is the degenerate trajectory consisting of this point alone. But it may approach it for $t \rightarrow \pm \infty$ which means that:

$$\begin{aligned} \lim_{t \rightarrow \pm \infty} \{ x(t) \} &= x_0 \\ \lim_{t \rightarrow \pm \infty} \{ v(t) \} &= v_0 \end{aligned} \quad \dots\dots\dots (A3-7)$$

Phase-plane trajectories have the following general characteristics:

a. - Can only travel from left to right in the upper half of the phase-plane and from right to left in the lower half plane. The sense of direction of the trajectory is thereby uniquely determined.

b. - All the trajectories cut the abscissa vertically ($v = 0$ when x has a stationary value). No point on the phase-plane (except on the x -axis) can have a vertical trajectory tangent. Only degenerate trajectories do not cut the abscissa normally (the point of intersection in that case is a singular point).

c. - A trajectory represents a specifically determined pattern of motion. An overall picture of all possible motions of a vibratory system is given by a group of trajectories which comprise the "phase portrait" of the system.

d. - Closed trajectories correspond to periodic motions:

$$x(t + T) = x(t)$$

$$v(t + T) = v(t)$$

where T is the period calculated by the line integral:

$$T = \oint \frac{dx}{v}$$

along the closed trajectory in the direction t' . Closed trajectories are called "limit cycles".

e. - A closed trajectory represents a mode of vibration where the energy supplied E_z is spent in damping E_D in such a way that the energy level remains constant ($E_z = E_D$). That can happen for more than one trajectory on the same phase-plane.

f. - Closed trajectories can also exist in the phase portrait of free oscillations (in conservative systems). When a system is conservative however, periodic oscillations of arbitrary amplitudes are possible, that is, the phase portrait consists of closed trajectories surrounding the origin. These trajectories are not limit cycles since adjacent trajectories do not approach each other asymptotically. In other words, a limit cycle is an

isolated closed trajectory. In phase portraits of conservative systems there are neither exciting nor damping zones. On the contrary the phase-plane of self-sustained systems can be divided into exciting and damping zones, whose boundaries are the limit cycles. Periodic motion of self-excited systems is thus only possible under certain specific conditions. Accordingly the simple harmonic vibration:

$$x = a \cos \omega t$$

$$v = \frac{dx}{dt} = -a\omega \sin \omega t$$

is represented on the phase-plane by an ellipse with axes:

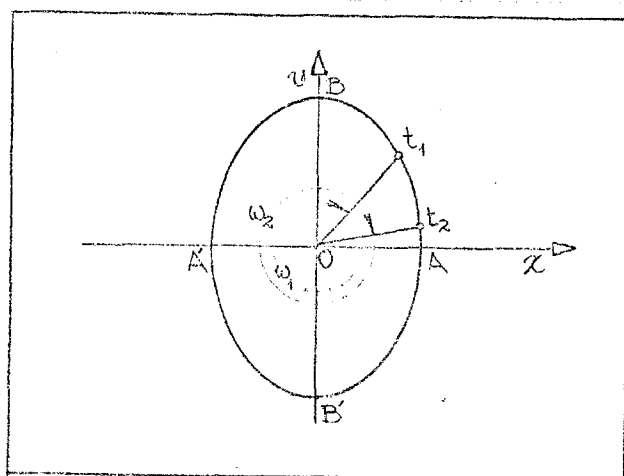


Fig. A3-1

$$\overline{AA'} = 2a$$

$$\overline{BB'} = 2a\omega_0$$

That means that variation of the amplitude of the vibration effects both axes of the ellipse proportionally, while for constant amplitude, variation of the frequency influences the vertical only axis of the ellipse

g. - The phase-plane trajectory involves time implicitly so that a time scale can be set up along a solution curve. This process requires a step-by-step integration and can be performed in several ways.

The simplest and more straight forward way is based on the fact that for small increments Δx and Δt the average velocity is:

$$v_{av} = \frac{\Delta x}{\Delta t} \quad \dots\dots\dots (A3-8)$$

A small increment Δx can be measured on the phase-plane curve and the corresponding v_{av} can be determined. The increment in t needed to traverse

the distance Δx is then:

$$\Delta t = \frac{\Delta x}{v_{av}} \quad \text{..... (A3-9)}$$

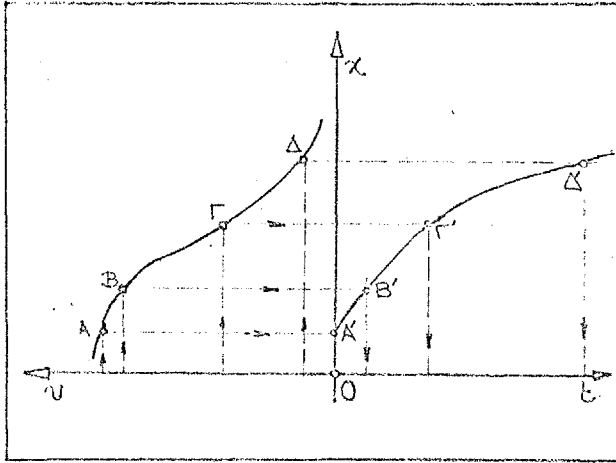


Fig. A3-2

but:

$$v_{av} = \frac{v(B)+v(A)}{2} = v(A) + \frac{\Delta v}{2}$$

and:

$$\Delta v = 2v_{av} - 2v(A)$$

or:

$$\Delta v = \frac{2}{\Delta t} \Delta x - 2v(A) \quad \text{..... (A3-10)}$$

If a fixed value of Δt is chosen equ. A3-10 represents a straight line of slope $\frac{2}{\Delta t}$ and Δv intercepts at $-2v(A)$ where $\Delta v, \Delta x$ are measured from $v(A)$, $x(A)$, which are at the beginning of the increment (fig. A3-3). The intersection of this line with the trajectory, locates the point satisfying

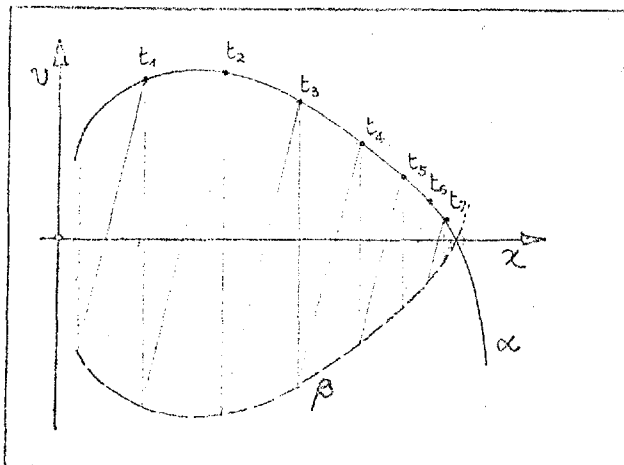


Fig. A3-3

and the semi-graphical construction of fig. A3-2 follows. A purely graphical construction can also be used for locating points equally spaced in time along a trajectory. Equation A3-9 can be written as:

$$\Delta x = v_{av} \cdot \Delta t$$

simultaneously the original differential equation and also equation A3-10. Thus this intersection is the point on the solution curve representing a point reached after an interval Δt (Cunningham [196], Macduff, Curreri [199]).

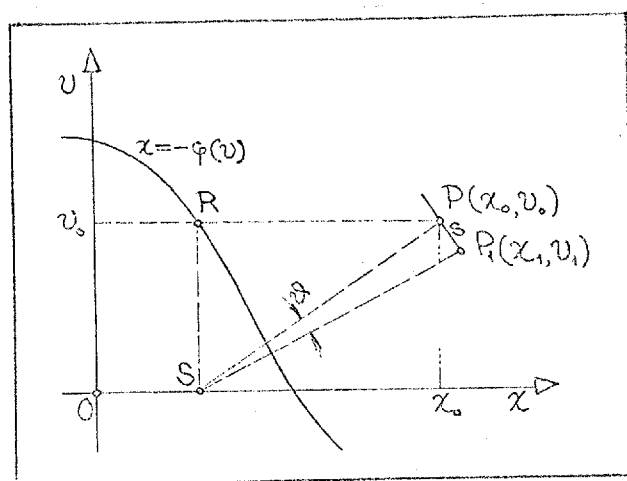


Fig. A3-4

Liénard's construction

Transformations as the ones used in § 2.3.1. lead to formation of the trajectories not on the original, ordinary $\{x, v\}$ phase-plane but on a transformed one obtained from the original by some transformation of the system of coordinates.

The transformations of § 2.3.1. lead to a phase-plane which is called "Liénard's plane" because in that case Liénard's graphical construction can very easily be applied.

Liénard's construction [201] is a simple geometrical technique for definition of the slope field if the nonlinear function is known, even in the case where it is not expressible mathematically. The technique can be applied easily only on second order differential equations without a forcing factor. Later modifications (Ku [202, 203, 204, 205, 206], Jacobsen [207], Ito, Muto, Shinoda [208]) enlarged its field of application and although the method is basically topographic in nature, numerical calculations with reasonable accuracy became possible.

From equ. A3-5 (for $f(x) = x$ due to the transformations made):

$$\frac{dv}{dx} = \frac{-\varphi(v) - x}{v}$$

where: $v = \frac{dx}{dt}$ (A3-5a)

the field direction at any point of the phase-plane is obtained graphically as follows:

The curve $x = -\varphi(v)$ which is called "characteristic line" of the system is first plotted (Fig. A3-4). To determine the field direction at a point $P(x_0, v_0)$ a line is drawn parallel to the x -axis until it cuts the curve $x = -\varphi(v)$ at R . From R a perpendicular is dropped to the x axis at s . The field direction at P is then orthogonal to the line $\overline{SP}$, for $\frac{v}{x + \varphi(v)}$ is the slope of $\overline{SP}$. Thus the position of the next point P_1 of a trajectory starting

at P can be determined. If θ (or s) is small. P_1 can be assumed without great error as lying on the tangent to the trajectory at P. Obviously the error decreases as θ or s decrease. There is an optimum value s or θ giving the highest accuracy, under which truncation errors in the numerical calculations increase very rapidly with decreasing s, θ and produce increased error.

The physical meaning of the characteristic line $x = -\varphi(v)$ is that it represents the trajectory of a degenerated form of the system ($m=0$). In that case the phase-plane degenerates also to a phase line, the characteristic line itself (no inertia forces).

Basic Properties of Liénard's plane

The limiting cycles and their properties have been studied in detail by Liénard, Van der Pol, Levinson, Smith, and others.

Poincaré proved [141] that in the case of a nonlinear differential equation of the form of equ. 2.16 there is at least one limiting cycle. The stability of the limiting cycles can be examined by means of Poincaré's criterion for orbital stability. According to that criterion a closed trajectory A_0 (Fig. A3-5) passing through the point $F\{x_F, 0\}$ is stable when, and only when, another trajectory A_1 starting from a point $K(x_F + \Delta x_0, 0)$

terminates after a 2π rotation of the describing vector $\vec{r}_1$ to a point $L(x_F + \Delta x_1, 0)$ such that:

$$0 \leq \Delta x_1 < \Delta x_0 \quad \dots \dots \dots (A3-11)$$

and another trajectory A_2 starting from a point $K'(x_F - \Delta x_0, 0)$ terminates after 2π rotation of the vector $\vec{r}_2$ to a point $L'(x_F - \Delta x_2, 0)$ such that:

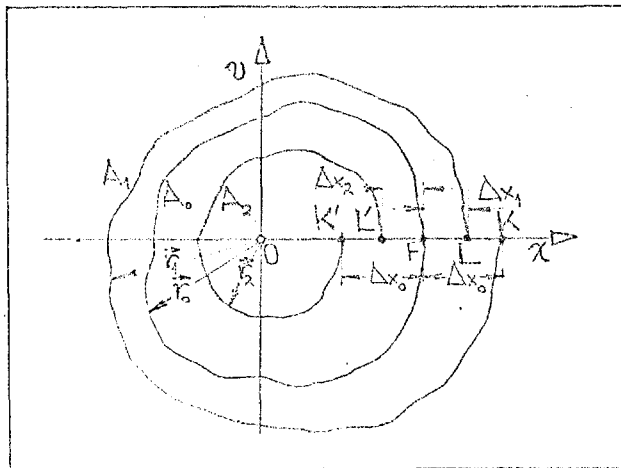


Fig. A3-5

$$0 \leq \Delta x_2 < \Delta x_0 \quad \dots\dots\dots (A3-12)$$

The cycle is metastable if one of the above conditions A3-11,12 is not fulfilled, and particularly is stable from inside and unstable from outside

if: $0 < \Delta x_0 < \Delta x_1$ \dots\dots\dots (A3-13)

$$0 \leq \Delta x_2 < \Delta x_0$$

is stable from outside and unstable from inside if:

$$0 \leq \Delta x_1 < \Delta x_0 \quad \dots\dots\dots (A3-15)$$

$$0 < \Delta x_0 < \Delta x_2$$

and finally the cycle is unstable if:

$$0 < \Delta x_0 < \Delta x_1 \quad \dots\dots\dots (A3-15)$$

$$0 < \Delta x_0 < \Delta x_2$$

Poincaré's criterion can be used in exactly the same way for stability examination of singular points on the phase-plane.

The following theorems (due to Poincaré) given here without proof, complete the basic study of the topology of Liénard's plane. Some of these theorems come from simple geometrical consideration of the phase-plane (Minorski [140])

- a. - A closed trajectory contains in its interior at least one singularity
- b. - This singularity can be a nodal or focal point and not a vortex or saddle point
- c. - Between two stable limit cycles must be an unstable one.
- d. - Between a stable singularity and a stable limit cycle around it there is always an unstable limit cycle.
- e. - Critical points are the points at which the differential equation describing a phenomenon in a certain domain, ceases to describe it.
- f. - Whenever the representative point following a trajectory reaches a critical point, a discontinuity occurs in some variable of the system.

Appendix 4

Program LIENG-1

This computer program produces the phase-portrait of a non-linear system based on Liénard's method. It consists of the main program and two subroutines (PINPUT and POINT), and calls a CALCOMP routine for plotting of the produced phase-portrait.

The main program

The main program starts by calling the subroutine PINPUT by which the input data are fed in and the geometry of the characteristic line is examined. Then starting from an initial point XINP, YINP a trajectory is calculated point by point by means of the POINT subroutine. For points very near the abscissa a correction of the time calculation is introduced and then the arrays X,Y,T containing the calculated coordinates of the points of each trajectory are printed in the output file. Finally the arrays X,Y,T and XCHL, YCHL (characteristic line) are arranged anew and by calling GRAF (CALCOMP) the plotting is made.

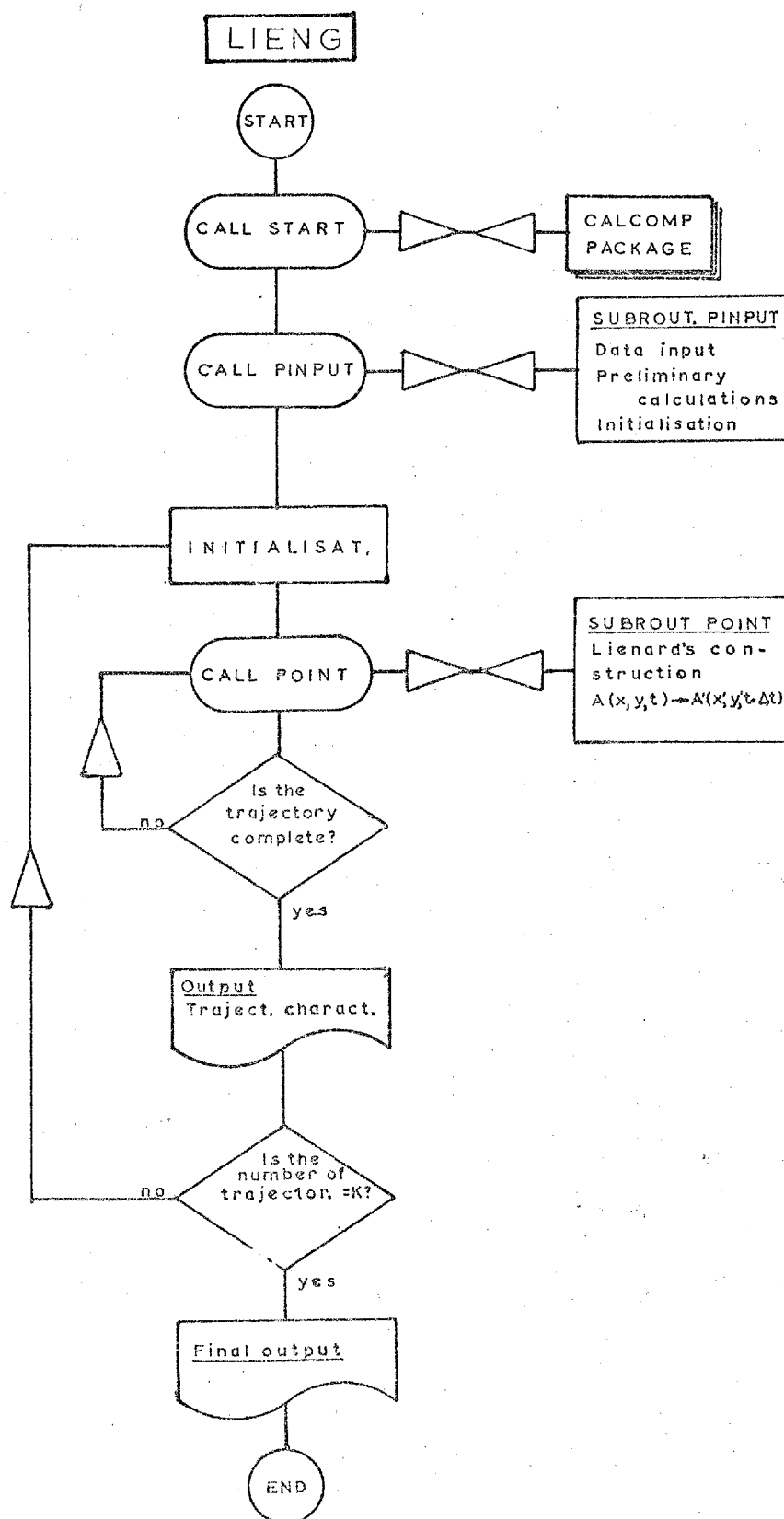
Subroutine PINPUT

This subroutine reads the input data and writes them if KEY=1. Then it locates the centre of symmetry XMDL, YMDL of the characteristic, which gives the mean driving velocity and messages in case the characteristic is not symmetric or has parts where $\frac{dv}{dx} = 0$ in which case the Liénard's construction is not applicable.

Subroutine POINT

This subroutine contains the graphical construction by which, from a known point $P_K\{X(K), Y(K), T(K)\}$ of a trajectory, the next point $P_{K+1}\{X(K+1), Y(K+1), T(K+1)\}$ can be found.

The spacing of the trajectory points (i.e. the distance $\overline{P_K P_{K+1}}$) depends on the parameter DTHETA. There is an optimum DTHETA value for which the error in trajectory determination is a minimum.



```

PROGRAM LIENG(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT,TAPE25,TAPE27)
C   S. ANTONIOU      UMEM109
C MOD CNE-ONE
      DIMENSION XCHL(200),YCHL(200),          X(3000),Y(3000),T(3000),
1XXX(3000),YYY(3000),NUM(10),TTT(3000),NAM(9)
      COMMON XCHL,YCHL,          X,Y,T
C
C
C *****
CPHASE-PLANE SOLUTION OF A NON LINEAR DIFFERENTIAL EQUATION BY THE
C LIENARD'S METHOD.
C
C
C
C INPUT OF DATA AND GENERAL NOTATION AS FOLLOWS**ARRAYS(XCHL,YCHL) THE
C COORDINATES OF THE CHARACTERISTIC LINE POINTS.ARRAYS(DXCHL,DYCHL) THE
C DIFFERENCES OF THE COORDINATES OF THE CHARACTERISTIC LINE POINTS.SLO-
C PE ARRAY IS THE SLOPE AT ANY POINT OF THE CHARACTERISTIC LINE. N THE
C NUMBER OF POINTS WITH WHICH THE CHARACTERISTIC LINE IS DETERMINED.
C N MUST BE .LE.200. OMEGA THE CYCLIC FREQUENCY OF THE DRIVING VELOCITY
C CHANGE. AMPVO THE AMPLITUDE OF THE DRIVING VELOCITY CHANGE. (XINP,
C YINP) THE COORDINATES OF THE INITIAL POINT FOR T=0. DTHETA THE ANGLE
C INCREAMENT OF THE BASIC LIENARD'S CONSTRUCTION. M M THE EXPECTED MA-
C XIMUM CHARACTERISTIC LINE SLOPE MULTIPLIED BY 1000. YCHL1(J) AN ARRAY
C CONTAINING THE Y COORDINATES OF THE PART OF THE CHARACTERISTIC LINE
C HAVING THE LOWEST SLOPE. THE ARRAYS X AND Y ARE THE COORDINATES OF
C THE TRAJECTORY POINTS.T IS THE TIME ARRAY FOR THE TRAJECTORY.THETA IS
C THE ARGUMENT OF EVERY POINT OF THE TRAJECTORY
C
C
C
C PART ONE *** INPUT OF DATA AND CHARACTERISTIC LINE ***
C
C THE CHARACTERISTIC LINE DISPLACEMENT DURING THE TIME IS NOT CONSIDERED
C IN THIS PROGRAM.THE PHYSICAL PICTURE IS THAT OF A TIME INSTANT T(ARC
C STATIONARY).
C
C
C
      CALL START
      CALL PINPUT(N,OMEGA,AMPVO,XINP,YINP,DTHETA,M,KEY,XMDL,YMDL,XINCR,
1YINCR)

```

```

C
C
C
2000 VO=YMDL
      PHI=ARCOS(VO/AMPVO)
      WRITE(6,1009)VO,PHI
1009 FORMAT(2F50.8/
C
C PART THREE *** INTRODUCTION OF THE INITIAL POINT.INIT. POINT MANIPULA-
C TION
C
      MK=0
60 NL=0
      L=1
      T(L)=0.
      X(L)=XINP
      Y(L)=YINP
      WRITE(6,100)X(L),Y(L),T(L)
100 FORMAT(3F10.3)
10 CALL POINT(N,R,KEY,DTHETA,VO,PHI,AMPVO,OMEGA,L,XB,XK,DS,DX,DY,DT)

      IF(L.EQ.3000)GO TO 50
      L=L+1
      GO TO 10
50 CONTINUE
      X(1)=XINP
      Y(1)=YINP
      T(1)=0.
      DO 140 J=5,3000
      A=((T(J-4)-T(J-5))+(T(J-3)-T(J-4))+(T(J-2)-T(J-3))+(T(J-1)-T(J-2))
1) /4.
      AA=1.1 *A
      XCO=T(J)-T(J-1)
      IF(ABS(Y(J-1)).LT..2 .AND.XCO .GT.AA)GO TO 150
      GO TO 145
150 D=XCO -A
      XCO =A
      T(J)=T(J-1)+XCO
      DO 160 K=J,3000
      T(K)=T(K)-D
160 CONTINUE
145 CONTINUE
140 CONTINUE
      DO 70 L=1,3000,60
      L1=L+10
      L2=L+20
      L3=L+30
      L4=L+40
      L5=L+50
      WRITE( 6,80)X(L),Y(L),T(L),L,X(L1),Y(L1),T(L1),L1,X(L2),Y(L2),
1T(L2),L2,X(L3),Y(L3),T(L3),L3,X(L4),Y(L4),T(L4),L4,X(L5),Y(L5),
1T(L5),L5
80 FORMAT(6(F6.1, 2F5.1,15))

```

```

      IF(MK.GT.0)GO TO 200
      DO 210 IA=1,N
      XXX(IA)=XCHL(IA)
      YYY(IA)=YCHL(IA)
      TTT(IA)=0.
210  CONTINUE
200  CONTINUE
      DO 201 IA=10,3000,10
      IB=IA/10
      XXX( N+MK*300+IB)=X(IA)
      YYY( N+MK*300+IB)=Y(IA)
      TTT( N+MK*300+IB)=T(IA)
201  CONTINUE
      NUM(1)=N
      DO 202 I=2,10
      NUM(I)=300
202  CONTINUE
      DO 203 I=1,9
      NAM(I)=300
203  CONTINUE
      70  CONTINUE
      XINP=XINP-XINCR
      YINP=YINP-YINCR
      MK=MK+1
      IF(MK.GT. 8)GO TO 90
      GO TO 60
      90  CONTINUE
C
C
      II=N+MK*300
      DO 330 IC=1,II
      X(IC)=XXX(IC)
      Y(IC)=YYY(IC)
      T(IC)=TTT(IC)
330  CONTINUE
      CALL GRAF(XXX,YYY,NUM,10,0,13HDISPLACEMENTS,13,10HVELOCITIES,10,
14,6,8,2)
      DO 340 IC=1,II
      XXX(IC)=X(IC)
      YYY(IC)=Y(IC)
      TTT(IC)=T(IC)
340  CONTINUE
      DO 400 I=1,2700
      XXX(I)=XXX(I+N)
      TTT(I)=TTT(I+N)
400  CONTINUE
      CALL GRAF(XXX,TTT,NAM,9,0,13HDISPLACEMENTS,13,8HTIME SEC,8, 4,6,
18,2)
      DO 350 IC=1,II
      XXX(IC)=X(IC)
      YYY(IC)=Y(IC)
      TTT(IC)=T(IC)
350  CONTINUE
      DO 440 I=1,2700
      YYY(I)=YYY(I+N)
      TTT(I)=TTT(I+N)
440  CONTINUE
      CALL GRAF(YYY,TTT,NAM,9,0,10HVELOCITIES,10,8HTIME SEC,8, 4,6,8,2)
      DO 360 IC=1,II

```

```
      XXX(IC)=X(IC)
      YYY(IC)=Y(IC)
      TTT(IC)=T(IC)
360  CONTINUE
      DO 320 I=1,3000
      IF(YYY(I).GT.25.)GO TO 300
      IF(YYY(I).LT.15.)GO TO 310
      GO TO 320
300  YYY(I)=25.
      GO TO 320
310  YYY(I)=15.
320  CONTINUE
      CALL GRAF(XXX,YYY,NUM,10.0,13HDISPLACEMENTS,13,10HVELOCITIES,10,
1  4.6,8.2)
      CALL ENPLOT(10.0)
      STOP
      END
```



```

      READ(5,30) YCHL(111),YCHL(112),YCHL(113),YCHL(114),YCHL(115)
      IF(KEY.EQ.0)GO TO 334
      WRITE(6,33)YCHL(111),YCHL(112),YCHL(113),YCHL(114),YCHL(115)
33  FORMAT(5F20.2)
334 CONTINUE
C FORTY CARDS                      FOR YCHL(J)
C
C THE CHARACTERISTIC LINE IS FED IN THE MEMORY
C
C PART TWO *** THE INITIAL DRIVING VELOCITYFOR T=0 IS ASSUMED THAT COIN-
C CIDES WITH THE MIDDLE POINT OF THE CONSTANT NEGATIVE SLOPE PART OF THE
C CHARACTERISTIC LINE
C
      K=N-1
      DO 90 J=1,K
145  DXCHL   =XCHL(J)-XCHL(J+1)
      DYCHL   =YCHL(J)-YCHL(J+1)
C
C THE SLOPE MUST NOT BE INFINITE AT ANY POINT OF THE CHARACTERISTIC LINE
C THUS DXCHL ≠ 0 AT ANY POINT. ***WARNING***CONTROL STATEMENT DOES NOT E-
C XIST.
C
      SLOPE(J)=DYCHL /DXCHL
      IF(KEY.EQ.0)GO TO 1002
      WRITE(6,1000)J,XCHL(J),YCHL(J),DXCHL ,DYCHL ,SLOPE(J)
1000 FORMAT(17,2X,2(F20.4,2X)/55X,3(F20.8,2X))
1002 CONTINUE
      90 CONTINUE
      J=N
      IF(KEY.EQ.0)GO TO 1003
      WRITE(6,1001) J,XCHL(J),YCHL(J)
1001 FORMAT(17,2X,F20.4,2X,F20.4)
1003 CONTINUE
C
C
C LOCATION OF POINTS WITH MINIMUM SLOPE
C
C
      I=1
135 IF(SLOPE(I).LT.0)GO TO 140
      I=I+1
      IF(I.GE.K)GO TO 175
      GO TO 135
140 L=I+1
141 CONTINUE
      IF(SLOPE(L).LT.0)GO TO 150
      L=L+1
      GO TO 141
150 A=SLOPE(I)-SLOPE(L)
      IF(A.LT.0)GO TO 160
      L=L+1
      IF(L.GT.K)GO TO 170
      GO TO 141
175 WRITE(6,1175)
1175 FORMAT(40H THERE IS NOT A NEGATIVE SLOPE POINT )
      GO TO 180
160 I=L

```

```

        GO TO 140
170 IF(KEY.EQ.0)GO TO 1305
    WRITE(6,1005)I,SLOPE(I)
1005 FORMAT(///120,F20.8)
1305 CONTINUE
180 CONTINUE
C
C
C
C SEARCH FOR EQUAL-SLOPE POINTS AROUND THE CENTER OF SYMMETRY
C
C
    LL=I
    LL=LL-1
185 IF(SLOPE(LL).GE.0.) GO TO 190
    LL=LL-1
    GO TO 185
190 LL=LL+1
    LLL=LL
    MM=I
    MM=MM+1
195 IF(SLOPE(MM).GE.0.)GO TO 200
    MM=MM+1
    GO TO 195
200 MM=MM-1
    MMM=MM
    IF(SLOPE(LLL).LT.SLOPE(MMM)) GO TO 210
    IF(SLOPE(LLL).GT.SLOPE(MMM)) GO TO 220
    XMDL=(XCHL(LLL)+XCHL(MMM))/2.
    YMDL=(YCHL(LLL)+YCHL(MMM))/2.
    IF(KEY.EQ.0)GO TO 1506
    WRITE(6,1006)XMDL,YMDL
1006 FORMAT(/////32H FROM SLOPE(LLL).EQ.SLOPE(MMM)      , F20.8,F20.8)
1506 CONTINUE
    GO TO 2001
210 SLOPEX=SLOPE(MMM)
    AA=SLOPEX-SLOPE(LLL)
    BB=SLOPE(LLL+1)-SLOPE(LLL)
    CC=XCHL(LLL+1)-XCHL(LLL)
    XX=(AA/BB)*CC+XCHL(LLL)
    DD=YCHL(LLL+1)-YCHL(LLL)
    YY=(AA/BB)*DD+YCHL(LLL)
    XMDL=(XX+XCHL(MMM))/2.
    YMDL=(YY+YCHL(MMM))/2.
    IF(KEY.EQ.0)GO TO 1507
    WRITE(6,1007) XMDL,YMDL
1007 FORMAT(/////32H FROM SLOPE(LLL).LT.SLOPE(MMM)      , F20.8,F20.8)
1507 CONTINUE
    GO TO 2001
220 SLOPEX=SLOPE(LLL)
    AA=SLOPEX-SLOPE(MMM-1)
    BB=SLOPE(MMM)-SLOPE(MMM-1)
    CC=XCHL(MMM)-XCHL(MMM-1)
    XX=(AA/BB)*CC+XCHL(MMM-1)
    DD=YCHL(MMM)-YCHL(MMM-1)
    YY=(AA/BB)*DD+YCHL(MMM-1)
    XMDL=(XX+XCHL(LLL))/2.
    YMDL=(YY+YCHL(LLL))/2.
    IF(KEY.EQ.0)GO TO 1508

```

```
      WRITE(6,1008)XMDL,YMDL
1008 FORMAT( //32H FROM SLOPE(LLL).GT.SLOPE(MMM)      , F20.8,F20.8 )
1508 CONTINUE
2001 IF(KEY.EQ.0) GO TO 4010
      WRITE(6,4012)
4012 FORMAT( 90H      THE KEY VALUE WAS KEY=1)      *** THE INPUT DATA ARE
      1 STORED AND DISPLAYED
      GO TO 2002
4010 WRITE(6,4011)
4011 FORMAT( 90H      THE KEY VALUE WAS KEY=0 .      THE INPUT DATA ARE
      1 STORED BUT NOT DISPLAYED
2002 RETURN
      END
```

```

SUBROUTINE POINT(N,R,KEY,DTHETA,VO,PHI,AMPVO,OMEGA,L,XB,XK,DS,DX,
1DY,DT)
  DIMENSION XCHL(200),YCHL(200),          X(3000),Y(3000),T(3000)
  COMMON XCHL,YCHL,      X,Y,T

```

C
C
C
C
C
C
C
C
C
C
C

THIS SUBROUTINE CONTAINS THE GRAPHICAL CONSTRUCTION WITH WHICH FROM
THE POINT(X(N),Y(N)),THE (X(N+1),Y(N+1)) POINT CAN BE FOUND

LINEAR INTERPOLATION ON THE CHAR. LINE

```

305 LL=0
  DO 315 JJ=1,N
    C=Y(L)-YCHL(JJ)
    IF(C) 310,320,330
310 CONTINUE
315 CONTINUE
316 GO TO 337
320 LL=1
  XB=XCHL(JJ)
  GO TO 340
330 IF(JJ.EQ.1) GO TO 335
  XB=((XCHL(JJ)-XCHL(JJ-1))*(YCHL(JJ-1)-Y(L))/(YCHL(JJ-1)-YCHL(JJ))
  1+XCHL(JJ-1)
  GO TO 340
335 LL=1
  XB=XCHL(1)
  GO TO 340
337 IF(LL) 340,342,340
342 XB=XCHL(N)
340 XK=X(L)-XB
  R=SQRT(Y(L)**2+XK**2)

```

C
C
C
C
C
C
C
C
C
C
C

INITIAL ANGLE OF THE LIENARD'S CONSTRUCTION

THETA =ATAN(Y(L)/XK)

LOCATION OF THE REST TRAJECTORY FROM THE INITIAL POINT

```

  DS=DTHETA*R
  DX=ABS(DS*SIN(THETA))
  DY=ABS(DS*COS(THETA))
  DT=ABS((2.*DX)/(2.*Y(L)-DY))
  T(L+1)=T(L)+DT
  IF(XK.GE.0..AND.Y(L).GE.0.)GO TO 360
  IF(XK.GE.0..AND.Y(L).LT.0.)GO TO 370
  IF(XK.LT.0..AND.Y(L).LT.0.)GO TO 380
  IF(XK.LT.0..AND.Y(L).GE.0.)GO TO 390
360 X(L+1)=X(L)+DX
  Y(L+1)=Y(L)-DY
  GO TO 400
370 X(L+1)=X(L)-DX
  Y(L+1)=Y(L)-DY
  GO TO 400
380 X(L+1)=X(L)-DX

```

```
      Y(L+1)=Y(L)+DY  
      GO TO 400  
390  X(L+1)=X(L)+DX  
      Y(L+1)=Y(L)+DY  
400  CONTINUE  
C  
C THE STATEMENTS 410,360,370,380,390 CONTROL THE DIRECTION OF THE TRA-  
C JECTORY EVOLUTION  
C  
  
501 RETURN  
    END
```

Appendix 5APPARATUS : DESIGN AND CHARACTERISTICSDetails of rig Mark I dynamometer

All the parts of the dynamometer, except for its base and the springs, were made of aluminium in order to keep its mass as low as possible.

The free length of the springs is adjustable so that their stiffness is adjustable too. Four-arm strain gauge bridges were fixed on the springs and their static calibration is indicated in fig. A5 - 1, A5 - 2 for two different thicknesses of the springs.

Under dynamic conditions, it was found experimentally that for the α pair of springs (fig. A5-1, A5-2) the two natural frequencies are (see fig. A5-3):

$$\omega_{n_N} = 18 \text{ Hz}$$

$$\omega_{n_F} = 19 \text{ Hz}$$

and the internal damping of each pair of springs, by use of the approximate formula:

$$D = \frac{c}{c_c} \approx \frac{\delta}{2\pi} \quad \text{..... (A5-1).}$$

where the logarithmic decrement:

$$\delta = \frac{x_n - x_{n+1}}{x_n} \quad \text{..... (A5-2)}$$

where x_n the n -th maximum amplitude on the trace and x_{n+1} the next maximum, (see Den Hartog [218] pg. 40), is:

$$D_N \approx D_F \approx 3.8 \times 10^{-3}$$

These values justify the assumption that the system is virtually undamped

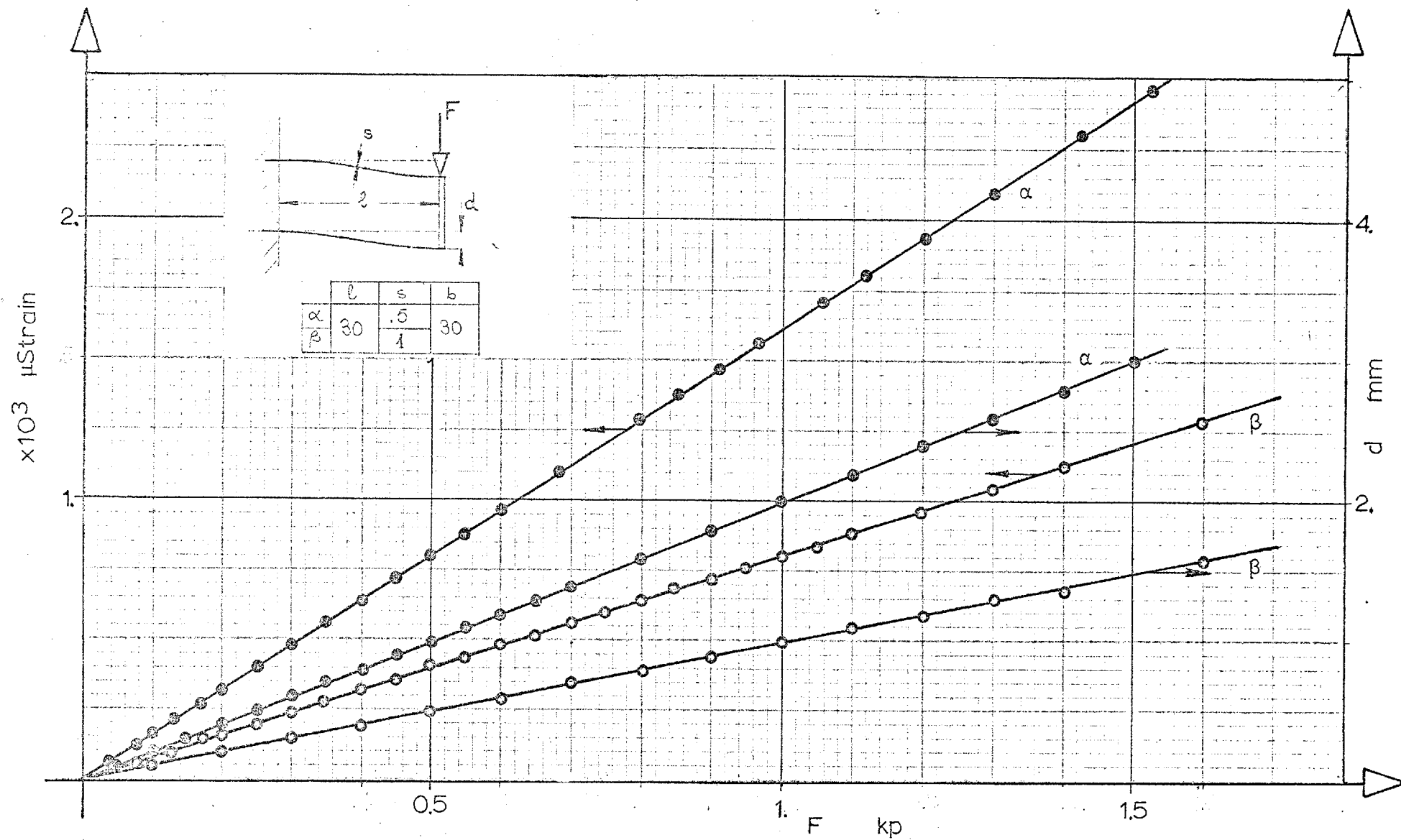


Fig. A5-1

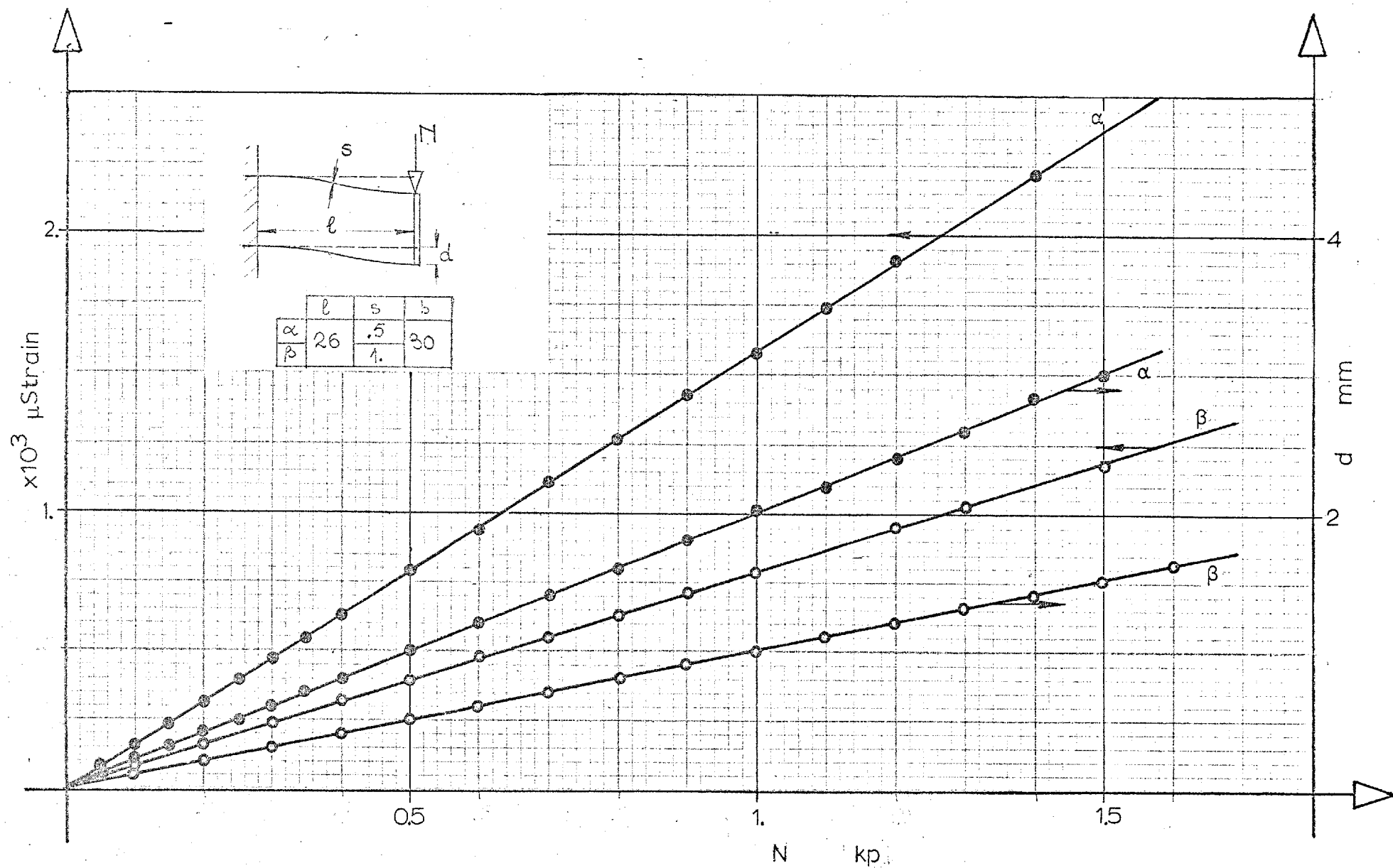


Fig. A5-2

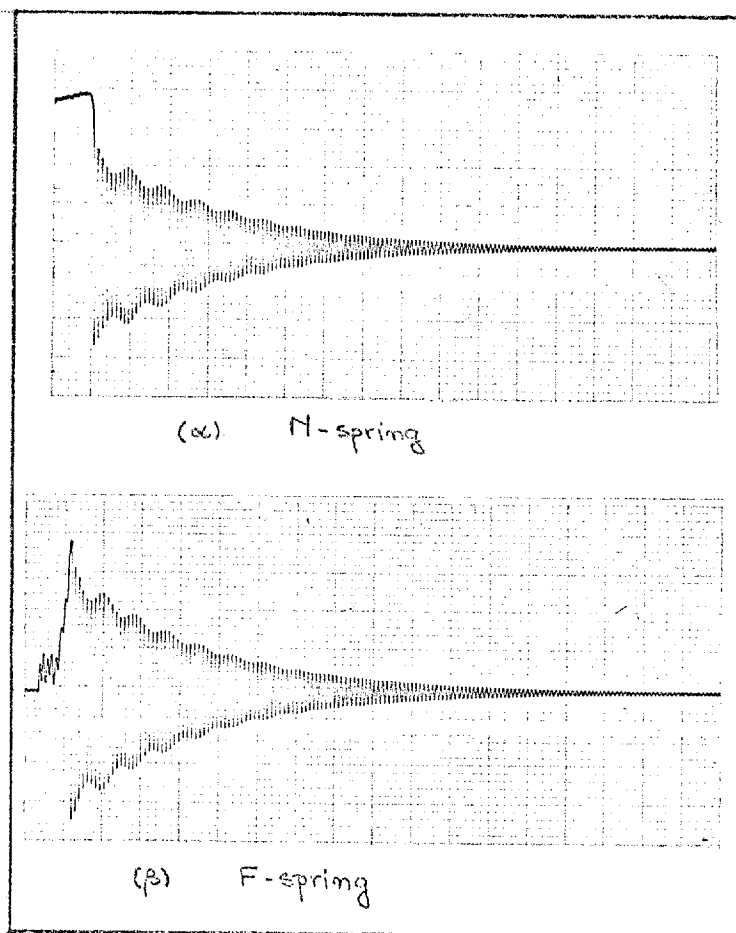


Fig. A5-3

(apart of the friction damping introduced by the frictional pair itself).

Fig. A5-3 shows also that under free-oscillation conditions, the coupling of the vertical and horizontal modes of oscillation is weak and can be disregarded.

Under load conditions the vertical distortion of the dynamometer interferes with the horizontal force measurements and

vice-versa. Thus an error is introduced (usually less than 10%) and a corrective technique is necessary. Assuming that $n_{F \rightarrow N}$, $n_{N \rightarrow F}$ represent the percentage of interference of the horizontal distortion on the vertical measurement and vice-versa respectively, it was found that $n_{N \rightarrow F}$, $n_{F \rightarrow N}$ vary proportionally with the real force values N, F respectively. Thus the recorded values N_r, F_r are:

$$\begin{aligned} N_r &= N + \frac{n_{F \rightarrow N}}{100} F \\ F_r &= F + \frac{n_{N \rightarrow F}}{100} N \end{aligned} \quad \dots\dots\dots (A5-3)$$

And by solving this system in respect of N, F :

$$\begin{aligned} F &= \frac{F_r - \frac{n_{N \rightarrow F}}{100} N_r}{1 - \frac{n_{N \rightarrow F} \cdot n_{F \rightarrow N}}{10,000}} \\ N &= \frac{N_r - \frac{n_{F \rightarrow N}}{100} F_r}{1 - \frac{n_{N \rightarrow F} \cdot n_{F \rightarrow N}}{10,000}} \end{aligned} \quad \dots\dots\dots (A5-4)$$

For $n_{N \rightarrow F}$, $n_{F \rightarrow N}$ less than 10% the ratio $\frac{n_{N \rightarrow F} \cdot n_{F \rightarrow N}}{10,000}$ can be ignored. Thus finally the corrected values of the horizontal and vertical dynamometer readings are:

$$F = F_r - \frac{n_{N \rightarrow F}}{100} N_r$$

$$N = N_r - \frac{n_{F \rightarrow N}}{100} F_r$$

..... (A5-5)

The above correction of the dynamometer reading is included in the numerical treatment of the experimental results (see Appendix 7.).

Details of rig Mark II dynamometer

To design an octagonal or extended-ring dynamometer, the following empirical formulae (Lowen-Cook [214]) were used:

$$\epsilon_o = 0.7 \frac{F_v \cdot R}{E \cdot b \cdot t^2} \cdot 10^6 \frac{\mu m}{m}$$

$$\epsilon_{45^\circ} \simeq 1.4 \frac{F_H \cdot R}{E \cdot b \cdot t^2} \cdot 10^6 \frac{\mu m}{m}$$

..... (A5-6)

$$\delta_v \simeq \frac{F_v \cdot R^3}{E \cdot b \cdot t^3} \cdot 10^4 \mu m$$

$$\delta_H \simeq 3.7 \frac{F_H \cdot R^3}{E \cdot b \cdot t^3} \cdot 10^4 \mu m$$

(Where ϵ_o the strain at the points 1,2,3,4 fig. 3.11, ϵ_{45° the strain at the points 5,6,7,8, F_H, F_v horizontal and vertical force respectively, δ_H , horizontal displacement at 5,6,7,8 due to F_H , δ_v vertical displacement of 1,2,3,4 due to F_v , R the mean radius of the spring in cm, E the modulus of

elasticity of the spring material in kpcm^{-2} , b the width of the spring in cm).

An optimum combination sensitivity-stiffness is obtained when:

$$\left\{ \frac{\epsilon_o}{\delta_v} = 0.7 \frac{t}{R^2}, \quad \frac{\epsilon_{45^\circ}}{\delta_H} = 0.379 \frac{t}{R^2} \right\} \quad \dots\dots\dots (\text{A5-7})$$

According to the above formulae stress and stiffness in the horizontal and vertical directions are given by:

$$\begin{aligned} \sigma_v &= \epsilon_o \cdot E & \text{kp.cm}^{-2} \\ \sigma_H &= \epsilon_{45^\circ} \cdot E & \text{kp.cm}^{-2} \\ k_v &= \frac{F_v}{v} & \text{kp.cm}^{-1} \\ k_H &= \frac{F_H}{H} & \text{kp.cm}^{-1} \end{aligned} \quad \dots\dots\dots (\text{A5-8})$$

According to the above formulae, four dynamometers, the first octagonal and the rest of the extended-ring spring type were designed, with the following characteristics:

		α	β	γ	δ
b	[mm]	50	50	50	50
t	[mm]	4	1	0.5	0.1
R	[mm]	13	13	13	13
ϵ_v	$\left[\frac{\mu\text{m}}{\text{m}}\right]$	114	182	364	1820
ϵ_H	$\left[\frac{\mu\text{m}}{\text{m}}\right]$	228	364	728	3640
δ_v	[mm]	0.0068	0.044	0.0352	4.4
δ_H	[mm]	0.019	0.163	0.130	16.3
k_v	$[10^3 \text{kp.mm}^{-1}]$	29.4	0.456	0.284	0.45×10^{-3}
k_H	$[10^3 \text{kp.mm}^{-1}]$	10.5	0.123	0.077	0.12×10^{-3}
$\frac{t}{R^2}$	$[\text{cm}^{-1}]$	0.23	0.059	0.029	0.0059
σ_v	$[\text{kp.cm}^{-2}]$	251	364	728	3630
σ_H	$[\text{kp.cm}^{-2}]$	508	728	1456	728
$F_v = F_H$	[kp]	200	20	10	2

The values included in this table have only indicative meaning because due to the empirical origin of the formulae used and slight deviations in the geometry of the spring, differences as high as 40 or 50% can easily be observed. The dynamometers γ, δ were finally made of spring steel of thickness 0.25 and 0.125 mm respectively. The static calibration of the springs used in the experimentation appears in fig. A5-4 and A5-5 (for maximum bridge voltage $V_B = 6V$).

Under free vibration excitation it was found that the two natural frequencies of the spring are

$$\omega_{n_F} = 22 \text{ Hz}$$

$$\omega_{n_F} = 48 \text{ Hz}$$

(see fig. A5-6) and the damping of the system (acc. to formulae A5-1,2):

$$D_F \approx 30,3 \times 10^{-3}$$

$$D_N \approx 14,3 \times 10^{-3}$$

Although these damping values are much higher than the ones obtained with the leaf-spring dynamometer, they are still small enough to consider the system as being virtually undamped. The interference factors were found to be:

$$n_{F \rightarrow N} = 2,5\%$$

$$n_{N \rightarrow F} = 2,3\%$$

Except for this error due to the interference of distortions in the one direction on the measurements in the other direction, an additional error is introduced in the readings in the case of variable driving velocity $v_o = v_o(t)$ operation of the apparatus.

Thus, assuming that at instant t the dynamometer is at the position β

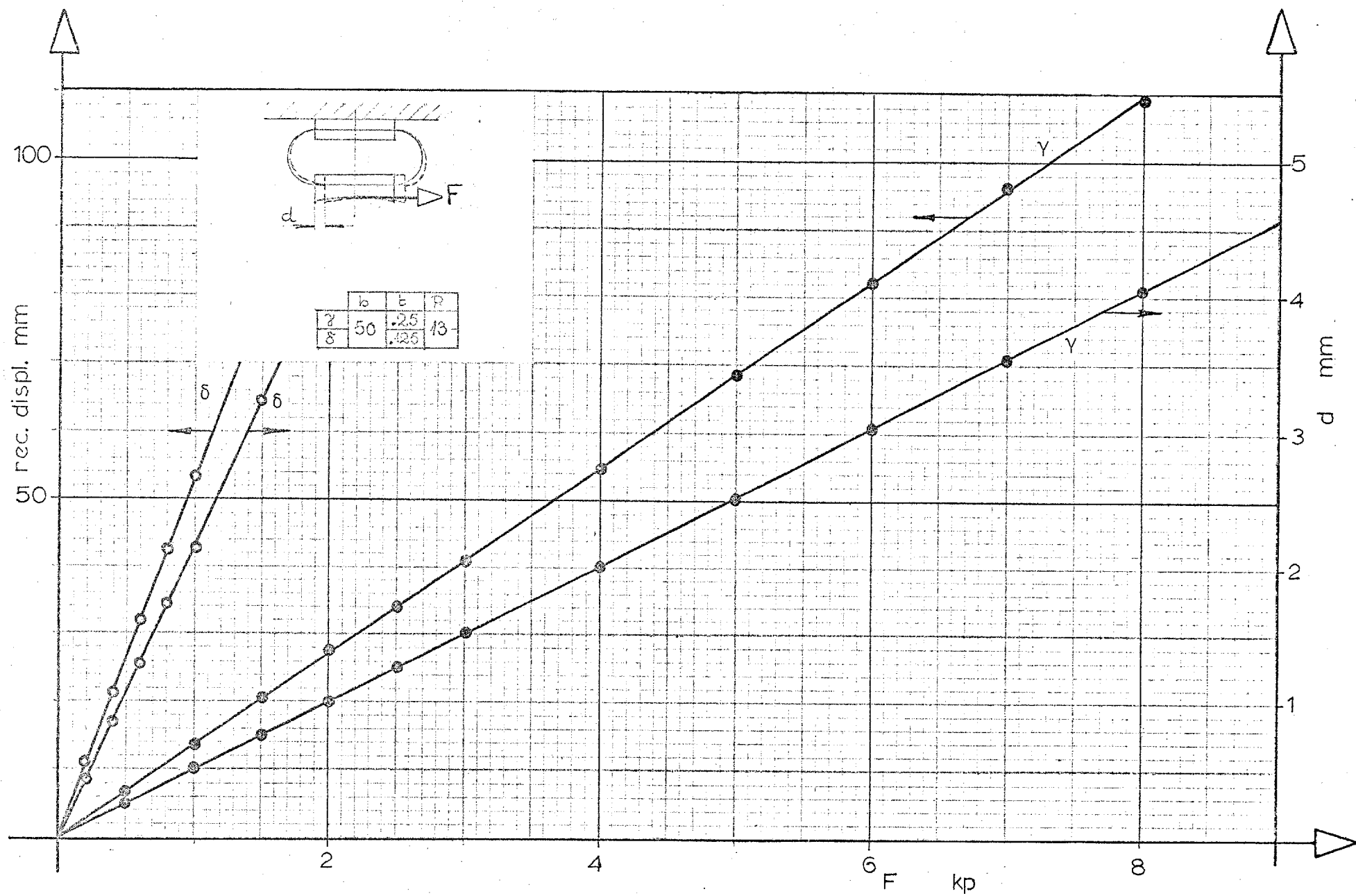


Fig. A5-4

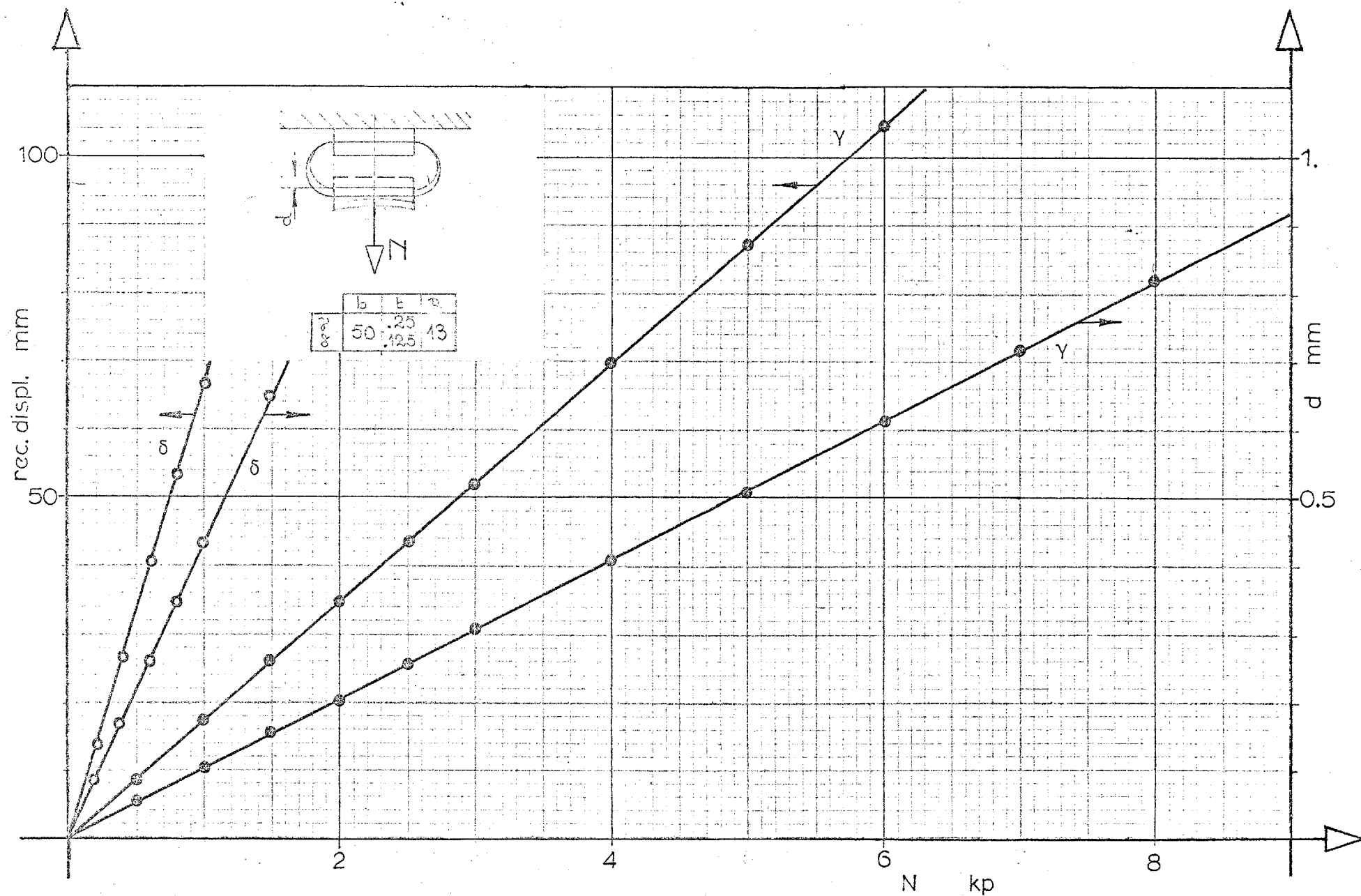
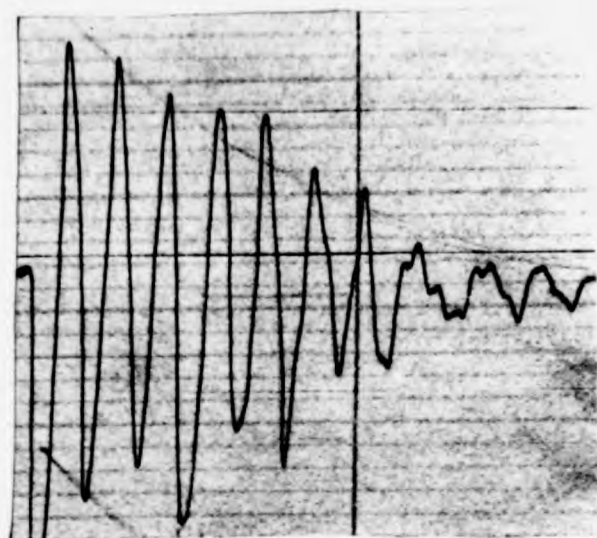
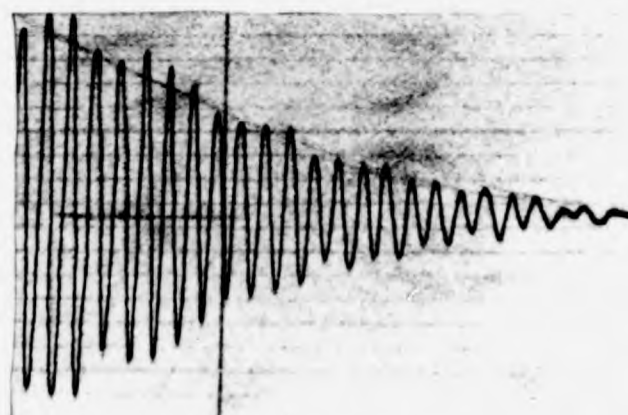


Fig. A5-5

(fig. A5-7), the indicated forces N_r, F_r , the real vertical and horizontal



F-mode γ -spring — 0.1 s —



N-mode γ -spring

Fig. A5-6

forces and the angle φ

are related by the

following relations:

$$N = N_r \cos \varphi + F_r \sin \varphi$$

$$F = F_r \cos \varphi - N_r \sin \varphi$$

Thus, knowing the

law of variation of

$\varphi = \varphi(t)$ and measuring

F_r, N_r the real values

F, N are obtained. This

correction was also

included in the numerical

treatment of the results

(program TRC). For $\varphi = 9^\circ$

the error is about $\Delta F = 14.4\%$

$\Delta N = 16.8\%$.

Relative velocity and acceleration between the specimens

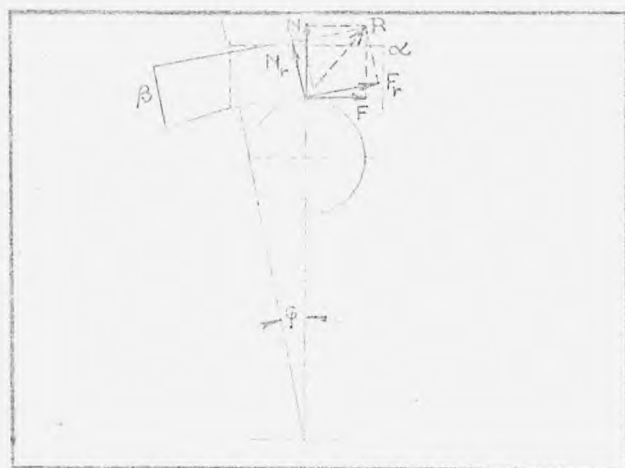


Fig. A5-7

The geometry of the

kinematic mechanism of

the apparatus appears in

fig. A5-8 where a, b are

the arc and ring surfaces

rotating around B, A

respectively, r is the

crank radius and r' the

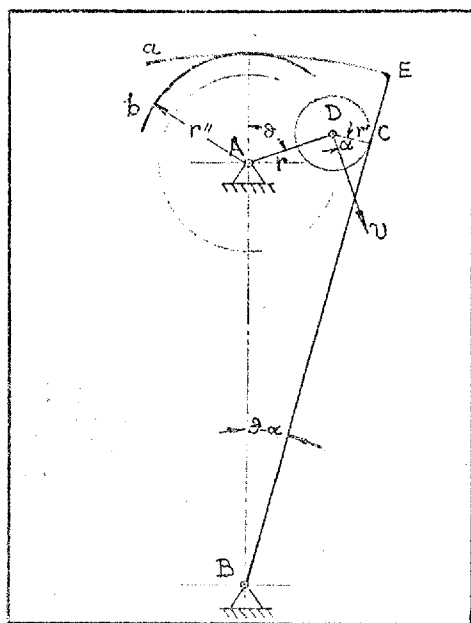


Fig. A5-8

crank pin radius.

The (linear) velocity of the pin centre D is:

$$v_D = r \cdot \omega$$

where ω is the constant angular velocity of the crank. The component of v_D parallel to DC is:

$$\{v_D\}_{DC} = r \cdot \omega \cdot \cos \alpha$$

And if angular velocity of the arm

BCE is Ω then:

$$\Omega = \frac{r \omega \cos \alpha}{BC}$$

$$\text{but } \overline{BC} = \overline{BA} \cos(\theta - \alpha) + r \cos \alpha$$

thus (calling the fixed length $\overline{AB} = L$)

$$\Omega = \frac{r \cdot \omega \cdot \cos \alpha}{L \cos \theta \cos \alpha + L \sin \theta \sin \alpha + r \cos \alpha}$$

or

$$\Omega = \frac{r \cdot \omega}{L \cos \theta + L \sin \theta \tan \alpha + r}$$

The relative velocity between ring and arc is ($\overline{BE} = L_a$):

$$v_r (=v_o) = v_b - v_a =$$

$$= r' \omega - L_a \frac{r \cdot \omega}{L \cos \theta + L \sin \theta \tan \alpha + r} \quad \dots\dots\dots (A5-9)$$

For the determination of angle α , from the geometry of the system:

$$r' + r \sin \alpha = L \sin(\theta - \alpha)$$

$$\text{or: } r' + r \sin \alpha = L \sin \theta \cos \alpha - L \cos \theta \sin \alpha$$

$$\text{or: } \frac{r'}{\cos \alpha} + \tan \alpha = L \sin \theta - L \cos \theta \tan \alpha$$

$$\text{and: } \frac{r'}{\sqrt{1 + \tan^2 \alpha}} = L \sin \theta - L \cos \theta \tan \alpha - r \tan \alpha$$

which reduces to the quadratic equation in $\tan\alpha$:

$$(r'^2 - L^2 \cos^2 \theta - 2rL \cos \theta - r^2) \tan^2 \alpha + (2L^2 \sin \theta \cos \theta + 2rL \sin \theta) \cdot \tan \alpha + (r'^2 - L^2 \sin^2 \theta) = 0 \quad \dots\dots\dots (A5-10)$$

Equations (A5-9) and (A5-10) solved simultaneously give the wanted relative velocity $v_r = v_o = v_o(t)$.

To find the relative acceleration the following formula was used:

$$\gamma_r = \frac{dv_r}{d\theta} = \frac{dv_r}{d\theta} \cdot \frac{d\theta}{dt} = \omega \cdot \frac{dv_r}{d\theta} \quad \dots\dots\dots (A5-11)$$

where $\frac{dv_r}{d\theta}$ is easily found by differentiating equation (A5-9) in respect of θ ($\frac{d(\tan \theta)}{d\theta}$ obtained by differentiating equ. A5-10 in respect of θ).

Fig. A5-9 shows how relative velocity v_r and acceleration γ_r vary with angle θ for ring angular velocity $\omega = 1\text{rpm}$. For other values ω , the curves of fig. A5-9 must be multiplied by ω in rpm for velocities or by ω^2 in rpm^2 for accelerations.

Electrical part of the dynamometers

In both cases four-arm strain gauge bridges were used to detect horizontal or vertical distortions of the dynamometer springs.

In the Mark I apparatus the bridge was fed with 5kHz A.C., voltage e_i giving current through the gauges less than 25mA, the output voltage e_o :

$$e_o = \frac{n}{4} \cdot k \cdot \epsilon \cdot e_i \cdot 10^{-6} \text{ [V]} \quad \dots\dots\dots (A5-12)$$

(where n the number of active gauges in the bridge, $k = \frac{\frac{\Delta R}{R}}{\frac{\Delta l}{l}} = \frac{\frac{\Delta R}{R}}{\epsilon}$

the sensitivity factor of the gauges, ϵ the μ strain along the sensitive axis of the gauge [$\frac{\mu\text{m}}{\text{m}}$]) was amplified, rectified (to remove the 5kHz carrier frequency) and then fed to the pen recorder.

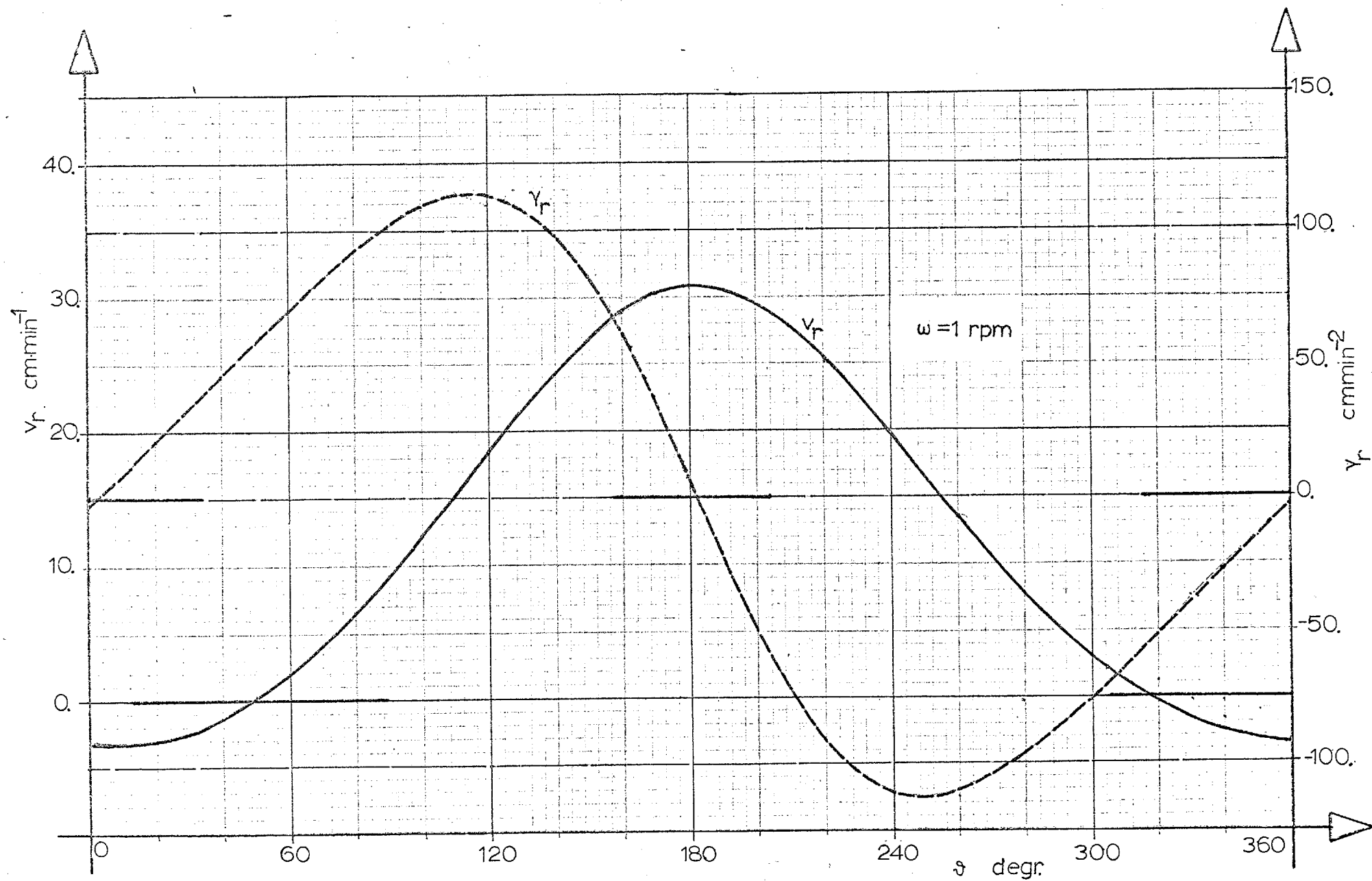


Fig. A5-9

In the Mark II apparatus, due to the high sensitivity of the U/V galvanometric recorder employed, no amplification of the output voltage e_o

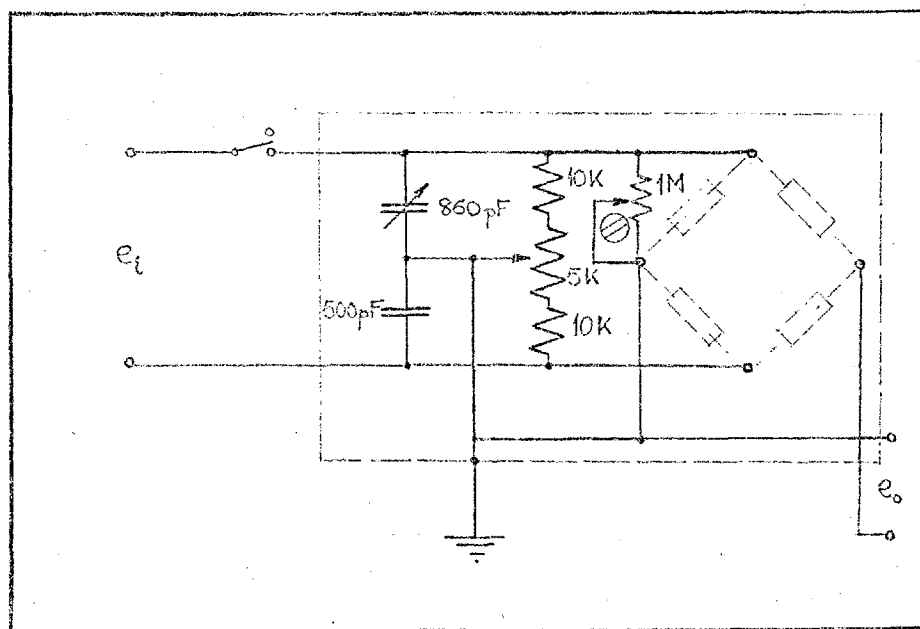


Fig. A5-10

was necessary. Thus a high stability D.C. power supply was used (10,000:1 stabilisation ratio over range $\pm 10\%$ nominal input) with high quality output (ripple and noise output less than $250 \mu\text{V}$ peak-to-peak, high frequency output impedance less than 0.10Ω o - 200 kHz or less than 0.5Ω up to 1MHz, temperature coefficient of voltage, over the range $25^\circ\text{C} \pm 35^\circ\text{C}$ less than $0.005\% \text{ V}/^\circ\text{C}$) feeding two balancing units BUF, BUN (fig. 3.14) constructed according to fig. A5-10), through a voltage regulator operating according to the rheostat principle. The output of the balancing units was fed to the U/V recorder. The frequency response of the galvanometers used (flat 0 - 60 Hz $\pm 5\%$) and their sensitivity ($0.0037 \frac{\text{mA}}{\text{cm}}$ or $0.13 \frac{\text{mV}}{\text{cm}} \pm 10\%$) are quite satisfactory as parallel tests (recorder-oscilloscope) proved.

Electrical conductivity measurement

The simple circuit of fig. A5-11 was used for the measurement of the

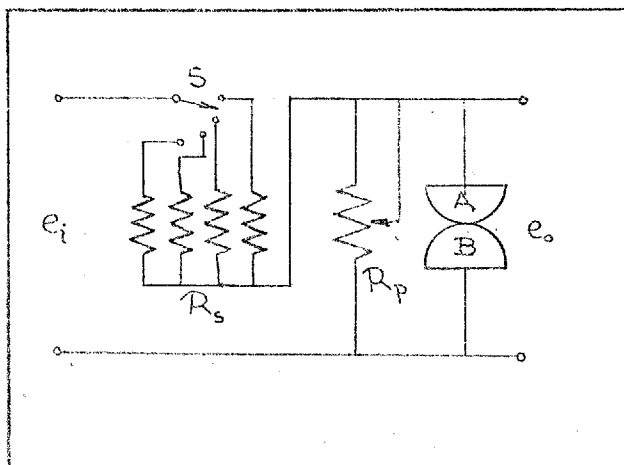


Fig. A5-11

electrical conductivity of the contact spot. By changing the values of R_s and R_p the desired sensitivity can be obtained. The voltage V_{AB} across the contact was kept less than 15 mV which is accepted as a safe limit to avoid electrical pitting. The information obtained by this method was not sufficient and the measurement was finally abandoned.

Commercial Oils used in the experiments

A. Mobil ATF 220

This is a fluid recommended for automatic transmissions and hydraulically operated units. It contains friction modifiers and has the following characteristics:

Specific Gravity	60/60°F	0.890
Pour Point		- 50°F max.
Flash Point	C.O.C.	380°F
Viscosity		
	S.S.U. 100°F	195
	S.S.U. 210°F	50
	Redwood No. 1 70°F	350
	Redwood No. 1 100°F	170
	Redwood No. 1 140°F	82
	Redwood No. 1 200°F	48
Viscosity index (D2270/64)		150
Colour		Red

B. Mobil Vactra Oils

These are solvent refined oils specially recommended for the lubrication of machine-tool slideways. They contain film strength and adhesive additives and also a defoamant. Typical characteristics:

	Specific Gravity	Pour Point °F Max.	Flash Point C.O.C. °F Min.	Viscosity						
				Centistokes		S.S.U.		Redwood No.1		
				100°F	210°F	100°F	210°F	70°F	140°F	200°F
Mobil Vactra Oil No. 1	0.880	-30	320	34	5.8	160	45	305	69	43
Mobil Vactra Oil No. 2	0.895	0	340	70	9	320	56	710	116	54
Mobil Vactra Oil No. 4	0.905	0	380	204	18.3	950	91.5	2440	279	90

Appendix 6

Program MLIEN (LIENG-2)

This computer program is a modified version of LIENG-1 being able to produce phase-portraits of a non-linear system under v_0 variable $v_0 = v_0(t)$. Triggering oscillation is also simulated by this program and the results are plotted by a "calcomp" routine.

The main program

The main program energise the CALCOMP package (CALL START) and then calls subroutine PINPUT by which the input data are fed in. By calling the subroutine CHALIN the geometry and position of the characteristic line are examined and some preliminary calculations for the Liénard's construction are executed.

By means of the POINT subroutine a trajectory is calculated point by point starting from the initial point X_{INP} , Y_{INP} , as for LIENG-1, and finally the arrays $X, Y, T, XCHL, YCHL$ are plotted.

Subroutine PINPUT

Basically the same as subroutine PINPUT of LIENG-1. Minor changes were necessary to accomodate subroutines CHALIN and TIMDIS.

Subroutine CHALIN

This writes the coordinates of the points that form the characteristic line (depending on the value of KEY) and locates the centre of symmetry of the characteristic line. Finally it calculates the mean driving velocity v_0 at time t at which the characteristic line was given ($t \geq 0$).

Subroutine POINT

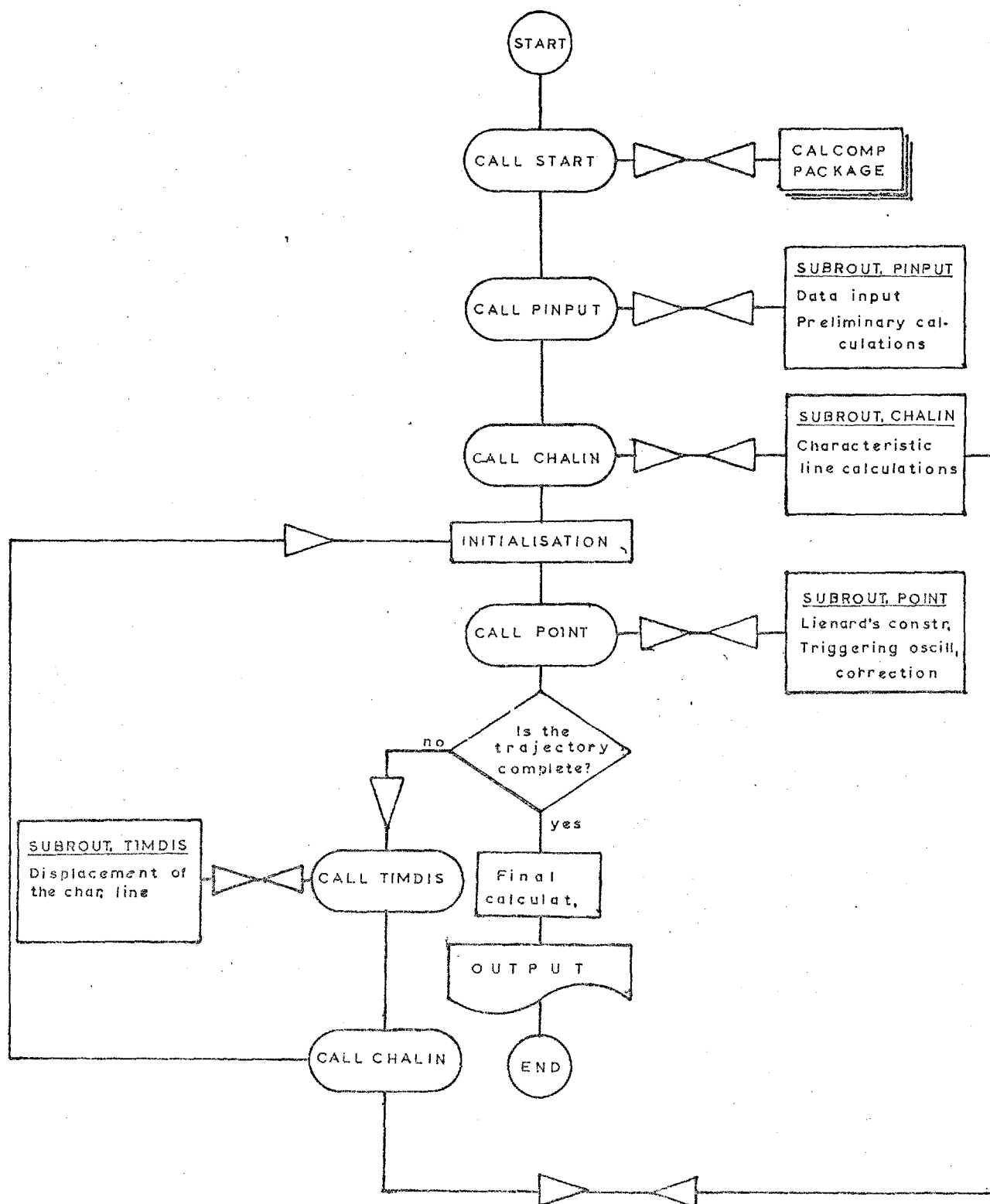
Basically the same as POINT of LIENG-1. From the known point $P_k \{X(K), Y(K), T(K)\}$ gives the next point $P_{k+1} \{X(K+1), Y(K+1), T(K+1)\}$ which is the result of the combination of the basic non-linear oscillation and the

triggering oscillation.

Subroutine TIMDIS

This produces a displacement of the characteristic line with increasing T according to a law $v_o = v_o(t)$. Thus conditions of variable mean driving velocity are simulated.

LIENG2-MLIEN



```

PROGRAM LIENG(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT,TAPE25,TAPE27)
  DIMENSION XCHL(200),YCHL(200),SLOPE(200),X(3000),Y(3000),T(3000),
  1THETA(3000),NL(3000),TTT(3000),NUM(2),YYY(3000),NAM(1)
  COMMON XCHL,YCHL,SLOPE,X,Y,T,THETA,NL,TTT,NUM,YYY

```

```

C
C *****

```

```

C 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2
C 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2
C 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2
C 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2
C 2 2 2 2 2 2 2 2 2 2 2 2 2 2 2

```

```

C *****

```

```

C PHASE-PLANE SOLUTION OF A NON LINEAR DIFFERENTIAL EQUATION BY THE
C LIENARD'S METHOD.

```

```

C INPUT OF DATA AND GENERAL NOTATION AS FOLLOWS**ARRAYS(XCHL,YCHL) THE
C COORDINATES OF THE CHARACTERISTIC LINE POINTS.ARRAYS(DXCHL,DYCHL) THE
C DIFFERENCES OF THE COORDINATES OF THE CHARACTERISTIC LINE POINTS.SLO-
C PE ARRAY IS THE SLOPE AT ANY POINT OF THE CHARACTERISTIC LINE. N THE
C NUMBER OF POINTS WITH WHICH THE CHARACTERISTIC LINE IS DETERMINED.
C N MUST BE .LE.200. OMEGA THE CYCLIC FREQUENCY OF THE DRIVING VELOCITY
C CHANGE. AMPVO THE AMPLITUDE OF THE DRIVING VELOCITY CHANGE. (XINP,
C YINP) THE COORDINATES OF THE INITIAL POINT FOR T=0. DTHETA THE ANGLE
C INCREAMENT OF THE BASIC LIENARD'S CONSTRUCTION. M M THE EXPECTED MA-
C XIMUM CHARACTERISTIC LINE SLOPE MULTIPLIED BY 1000. YCHL1(J) AN ARRAY
C CONTAINING THE Y COORDINATES OF THE PART OF THE CHARACTERISTIC LINE
C HAVING THE LOWEST SLOPE. THE ARRAYS X AND Y ARE THE COORDINATES OF
C THE TRAJECTORY POINTS.T IS THE TIME ARRAY FOR THE TRAJECTORY.THETA IS
C THE ARGUMENT OF EVERY POINT OF THE TRAJECTORY

```

```

C
C CALL START

```

```

C CALL PINPUT(N,OMEGA,AMPVO,XINP,YINP,DTHETA,M,KEY,XMDL,YMDL)

```

```

C CALL CHALIN (N,XMDL,YMDL,VO,PHI,AMPVO,OMEGA,KEY)

```

```

C PART THREE *** INTRODUCTION OF THE INITIAL POINT.INIT. POINT MANIPULA-
C TION

```

```

C
5 NL(1)=0
  L=1
  T(L)=PHI/OMEGA
  X(L)=XINP
  Y(L)=YINP
10 CALL POINT(N,R,KEY,DTHETA,VO,PHI,AMPVO,OMEGA,L,XB,XK,DS,DX,DY,DT)
  IF(L.EQ.3000) GO TO 50
  LL=L+1
  K=10
  NN=K+LL
  NL(LL)=NN/K
  NLN=NL(LL)-NL(LL-1)
  IF(NLN.EQ.0)GO TO 20
C *****WARNING*****ONLY FOR THE TRIAL RUNS*****
  WRITE(6,1010)R,X(L),Y(L),T(L),XMDL,YMDL,L
1010 FORMAT(6F10.3,I8)
C *****WARNING*****ONLY FOR THE TRIAL RUNS*****
20 CONTINUE
  L=L+1
  CALL TIMDIS(VO,AMPVO,OMEGA,N,KEY,L,XMDL,YMDL)
  CALL CHALIN (N,XMDL,YMDL,VO,PHI,AMPVO,OMEGA,KEY)
  GO TO 10
C
50 CONTINUE
C LAST POSITION OF THE CHARACTERISTIC LINE
  DO 210 IA=1,N
    THETA(IA)=XCHL(IA)
    YYY(IA)=YCHL(IA)
    TTT(IA)=0.
210 CONTINUE
  DO 201 IA=5,3000,5
    IB=IA/5
    THETA(N+IB)=X(IA)
    YYY(N+IB)=Y(IA)
    TTT(N+IB)=T(IA)
201 CONTINUE
  NUM(1)=N
  NUM(2)=600
  NAM(1)=600
  CALL GRAF(THETA,YYY,NUM,2,0,13HDISPLACEMENTS,13,10HVELOCITIES,10,
1 4.6,7.8)
  DO 3000 I=1,600
    THETA(I)=X(5*I)
    TTT(I)=T(5*I)
    YYY(I)=Y(5*I)
3000 CONTINUE
  CALL GRAF(THETA,TTT,NAM,1,0,13HDISPLACEMENTS,13,8HTIME SEC,8, 4.6,
17.8)
  CALL GRAF(YYY,TTT,NAM,1,0,10HVELOCITIES,10,8HTIME SEC,8, 4.6,7.8)
C
C
C
C
11 CALL ENPLOT(10.0)
  STOP
  END

```

```

SUBROUTINE PINPUT(N,OMEGA,AMPVO,XINP,YINP,DTHETA,M,KEY,XMDL,YMDL)
DIMENSION XCHL(200),YCHL(200),SLOPE(200),X(3000),Y(3000),T(3000),
1THETA(3000),NL(3000),TTT(3000),NUM(2),YYY(3000)
COMMON XCHL,YCHL,SLOPE,X,Y,T,THETA,NL,TTT,NUM,YYY

```

```

C
C
C
C THIS SUBROUTINE READS THE INPUT DATA AND WRITES THEM(DEPENDING UPON
C THE KEY VALUE).ALSO FINDS THE ADDITIONAL PARAMETERS OF THE CHARACTERI-
C STIC LINE.
C
C
C

```

```

      READ(5,8) KEY
      8 FORMAT(I1)

```

```

C
C
C THE CONSTANT KEY CONTROLS THE DISPLAY OF THE INPUT DATA. IF KEY.EQ.0
C NO INPUT DATA ARE DISPLAYED. IF KEY.EQ.1 ALL THE INPUT DATA ARE DISP-
C LAYED.      *
C
C
C

```

```

      READ(5,10) N,OMEGA,AMPVO,XINP,YINP,DTHETA,M
10  FORMAT(15,5F10.2,I7)
      IF(KEY.EQ.0) GO TO 410
      WRITE(6,41) N,OMEGA,AMPVO,XINP,YINP,DTHETA,M
41  FORMAT(1H1, 42H LIENARD'S GRAPHICAL CONSTRUCTION WITH DATA ,//
144H THE CHARACTERISTIC LINE IS DETERMINED WITH ,15,8H POINTS, /
224H THE CYCLIC FREQUENCY IS ,F10.5,11HRAD PER SEC ,/51H THE AMPLIT
3UDE OF THE DRIVING VELOCITY VARIATION IS ,F10.5,10HMM PER SEC ,/
445H THE COORDINATES OF THE INITIAL POINT ARE X= ,F10.2,5X,2HY= ,
5F10.2,5H MM ,/25H THE ANGLE INCREMENT IS ,F10.2,4HRADS,/35H T
6HE SLOPE COMPARISON FACTOR IS M= ,I7//////////)
410 CONTINUE
      DO 302 J=1,196,5
        JJ1=J
        JJ2=J+1
        JJ3=J+2
        JJ4=J+3
        JJ5=J+4
        READ(5,30) XCHL(JJ1),XCHL(JJ2),XCHL(JJ3),XCHL(JJ4),XCHL(JJ5)
        IF(KEY.EQ.0) GO TO 302
        WRITE(6,30) XCHL(JJ1),XCHL(JJ2),XCHL(JJ3),XCHL(JJ4),XCHL(JJ5)
30  FORMAT(5F10.2)
302 CONTINUE
C FORTY CARDS      FOR XCHL(J)
31 CONTINUE
      DO 334 I=1,196,5
        II1=I
        II2=I+1
        II3=I+2
        II4=I+3
        II5=I+4
        READ(5,30) YCHL(II1),YCHL(II2),YCHL(II3),YCHL(II4),YCHL(II5)
        IF(KEY.EQ.0) GO TO 334
        WRITE(6,33) YCHL(II1),YCHL(II2),YCHL(II3),YCHL(II4),YCHL(II5)
33  FORMAT(5F20.2)
334 CONTINUE

```

```
C FORTY CARDS          FOR YCHL(J)
C
C THE CHARACTERISTIC LINE IS FED IN THE MEMORY
C
C
C PART TWO *** THE INITIAL DRIVING VELOCITYFOR T=0 IS ASSUMED THAT COIN-
C CIDES WITH THE MIDDLE POINT OF THE CONSTANT NEGATIVE SLOPE PART OF THE
C CHARACTERISTIC LINE
      RETURN
      END
```

```

SUBROUTINE CHALIN(N,XMDL,YMDL,VO,PHI,AMPVO,OMEGA,KEY)
  DIMENSION XCHL(200),YCHL(200),SLOPE(200),X(3000),Y(3000),T(3000),
1 THETA(3000),NL(3000),TTT(3000),NUM(2),YYY(3000)
  COMMON XCHL,YCHL,SLOPE,X,Y,T,THETA,NL,TTT,NUM,YYY

```

C

```

  K=N-1

```

```

  DO 90 J=1,K

```

```

45 DXCHL =XCHL(J)-XCHL(J+1)

```

```

  DYCHL =YCHL(J)-YCHL(J+1)

```

C

```

C THE SLOPE MUST NOT BE INFINITE AT ANY POINT OF THE CHARACTERISTIC LINE
C THUS DXCHL=0 AT ANY POINT. ***WARNING***CONTROL STATEMENT DOES NOT E-
C XIST.

```

C

```

  SLOPE(J)=DYCHL /DXCHL

```

```

  IF(KEY.EQ.0)GO TO 1002

```

```

  WRITE(6,1000)J,XCHL(J),YCHL(J),DXCHL ,DYCHL ,SLOPE(J)

```

```

1000 FORMAT(17,2X,2(F20.4,2X)/55X,3(F20.8,2X))

```

```

1002 CONTINUE

```

```

  90 CONTINUE

```

```

  J=N

```

```

  IF(KEY.EQ.0)GO TO 1003

```

```

  WRITE(6,1001) J,XCHL(J),YCHL(J)

```

```

1001 FORMAT(17,2X,F20.4,2X,F20.4)

```

```

1003 CONTINUE

```

C

C

C

```

C LOCATION OF POINTS WITH MINIMUM SLOPE

```

C

C

```

  I=1

```

```

135 IF(SLOPE(I).LT.0.) GO TO 140

```

```

  I=I+1

```

```

  IF(I.GE.K)GO TO 175

```

```

  GO TO 135

```

```

140 L=I+1

```

```

141 CONTINUE

```

```

  IF(SLOPE(L).LT.0)GO TO 150

```

```

  L=L+1

```

```

  GO TO 141

```

```

150 A=SLOPE(I)-SLOPE(L)

```

```

  IF(A.LT.0)GO TO 160

```

```

  L=L+1

```

```

  IF(L.GT.K)GO TO 170

```

```

  GO TO 141

```

```

175 WRITE(6,1175)

```

```

1175 FORMAT(40H THERE IS NOT A NEGATIVE SLOPE POINT )

```

```

  GO TO 180

```

```

160 I=L

```

```

  GO TO 140

```

```

170 IF(KEY.EQ.0)GO TO 1305

```

```

  WRITE(6,1005)I,SLOPE(I)

```

```

1005 FORMAT(///120,F20.8)

```

```

1305 CONTINUE

```

```

180 CONTINUE

```

C

C

C

C SEARCH FOR EQUAL-SLOPE POINTS AROUND THE CENTER OF SYMMETRY

C
C

```

      LL=I
      LL=LL-1
185  IF(SLOPE(LL).GE.0.) GO TO 190
      LL=LL-1
      GO TO 185
190  LL=LL+1
      LLL=LL
      MM=I
      MM=MM+1
195  IF(SLOPE(MM).GE.0.)GO TO 200
      MM=MM+1
      GO TO 195
200  MM=MM-1
      MMM=MM
      IF(SLOPE(LLL).LT.SLOPE(MMM)) GO TO 210
      IF(SLOPE(LLL).GT.SLOPE(MMM)) GO TO 220
      XMDL=(XCHL(LLL)+XCHL(MMM))/2.
      YMDL=(YCHL(LLL)+YCHL(MMM))/2.
      IF(KEY.EQ.0)GO TO 1506
      WRITE(6,1006)XMDL,YMDL
1006 FORMAT(//////32H FROM SLOPE(LLL).EQ.SLOPE(MMM)      , F20.8,F20.8)
1506 CONTINUE
      GO TO 2001
210  SLOPEX=SLOPE(MMM)
      AA=SLOPEX-SLOPE(LLL)
      BB=SLOPE(LLL+1)-SLOPE(LLL)
      CC=XCHL(LLL+1)-XCHL(LLL)
      XX=(AA/BB)*CC+XCHL(LLL)
      DD=YCHL(LLL+1)-YCHL(LLL)
      YY=(AA/BB)*DD+YCHL(LLL)
      XMDL=(XX+XCHL(MMM))/2.
      YMDL=(YY+YCHL(MMM))/2.
      IF(KEY.EQ.0)GO TO 1507
      WRITE(6,1007) XMDL,YMDL
1007 FORMAT(//////32H FROM SLOPE(LLL).LT.SLOPE(MMM)      , F20.8,F20.8)
1507 CONTINUE
      GO TO 2001
220  SLOPEX=SLOPE(LLL)
      AA=SLOPEX-SLOPE(MMM-1)
      BB=SLOPE(MMM)-SLOPE(MMM-1)
      CC=XCHL(MMM)-XCHL(MMM-1)
      XX=(AA/BB)*CC+XCHL(MMM-1)
      DD=YCHL(MMM)-YCHL(MMM-1)
      YY=(AA/BB)*DD+YCHL(MMM-1)
      XMDL=(XX+XCHL(LLL))/2.
      YMDL=(YY+YCHL(LLL))/2.
      IF(KEY.EQ.0)GO TO 1508
      WRITE(6,1008)XMDL,YMDL
1008 FORMAT( //32H FROM SLOPE(LLL).GT.SLOPE(MMM)      , F20.8,F20.8 )
1508 CONTINUE
2001 CONTINUE
      VO=YMDL
      PHI=ARCOS(VO/AMPVO)
2002 RETURN
      END

```



```

380 X(L+1)=X(L)-DX
    Y(L+1)=Y(L)+DY
    GO TO 400
390 X(L+1)=X(L)+DX
    Y(L+1)=Y(L)+DY
400 CONTINUE

C
C THE STATEMENTS 410,360,370,380,390 CONTROL THE DIRECTION OF THE TRA-
C JECTORY EVOLUTION
C
C
C CONTROL OF THE PROCEEDING CONTINUATION.THE NUMBER INDICATES THE TOTAL
C REVOLUTION OF THE CHARACTERISTIC VECTOR IN RADS
C
C *****TRIGERING OSCILLATION INPUT*****
    OMINTR=1.25
C OMINTR THE INITIAL ANGLE OF TRIG. OSCILL. IN RADS
    OMETRI=3000.
C OMETRI THE ANGULAR VELOCITY OF THE TRIG.OSC. IN RAD/SEC
    ATRI=10.
    BTRI=2.
C THE AXIS IN X DIRECTION IN MM AND BTRI IN Y DIRECTION IN MM/SEC
    OMTRIL=OMETRI*T(L+1)+OMINTR
    AKTRI=OMTRIL/2.*3.1415927
    NKTRI=AKTRI
    ANKTRI=AKTRI
    AAKTRI=ANKTRI*2.*3.1415927
    PHITRI=OMTRIL-AAKTRI
    XTRI=ATRI*COS(PHITRI)
    YTRI=BTRI*SIN(PHITRI)
    X(L+1)=X(L+1)+XTRI
    Y(L+1)=Y(L+1)+YTRI
C *****TRIGERING OSCILLATION INPUT*****
501 RETURN
    END

```

```

SUBROUTINE TIMDIS(VO,AMPVO,OMEGA,N,KEY,L,XMDL,YMDL)
  DIMENSION XCHL(200),YCHL(200),SLOPE(200),X(3000),Y(3000),T(3000),
  1THETA(3000),NL(3000),TTT(3000),NUM(2),YYY(3000)
  COMMON XCHL,YCHL,SLOPE,X,Y,T,THETA,NL,TTT,NUM,YYY
C TIME CHANGE OF THE CHARACTERISTIC LINE
C
C
C
C OM=OMEGA*T(L)
C A=OM/2.*3.1415927
C N=A
C AN=N
C ANN=AN*2.*3.1415927
C PHI1=OM-ANN
C
C *****
C ***** ONLY FOR THE TRIAL RUNS ***** WARNING *****
C WRITE(6,100)OMEGA,T(L),OM,A,N,AN,ANN,PHI1
C 100 FORMAT(4F14.7,I8,3F14.7)
C *****
C VON=AMPVO*COS(PHI1)
C VON =AMPVO*COS(OMEGA*T(L))
C DIFF=VO-VON
C DO 315 JJ=1,N
C C=DIFF-YCHL(JJ)
C IF(C) 310,320,330
310 CONTINUE
315 CONTINUE
GO TO 340
320 ADIFF=XCHL(JJ)
GO TO 340
330 ADIFF=((XCHL(JJ)-XCHL(JJ-1))*(YCHL(JJ-1)-DIFF)/(YCHL(JJ-1)-YCHL(
1JJ)),)+XCHL(JJ-1)
GO TO 340
340 CONTINUE
DO 3000 J=1,N
XCHL(J)=XCHL(J)-ADIFF
3000 YCHL(J)=YCHL(J)-DIFF
3100 CONTINUE
C THE NEW POSITION OF THE CHARACTERISTIC LINE IS NOW FULLY DESCRIBED
501 RETURN
END

```

Appendix 7

Program TRC

This program is used for the derivation of the experimental phase-plane trajectories, the correction of the coordinates of the experimentally obtained points of the trajectories and then gives, by means of the reverse Liénard's construction, the function $\mu = \mu(v)$.

By means of CALCOMP routine the results are plotted as well.

The main program

The main program starts by energising the CALCOMP package and initialising the arrays which are to be used. Then it reads the experimental parameters (scales of the recorded traces, paper speed of the recorder, natural frequencies of the dynamometer in use, sensitivities of the two strain gauge bridges, number of points comprising each trace, interference factors $n_{F \rightarrow N}$, $n_{N \rightarrow F}$) and the coordinates of the experimental points (in the form of three arrays $NT(y)$, $NH(y)$, $NHN(y)$).

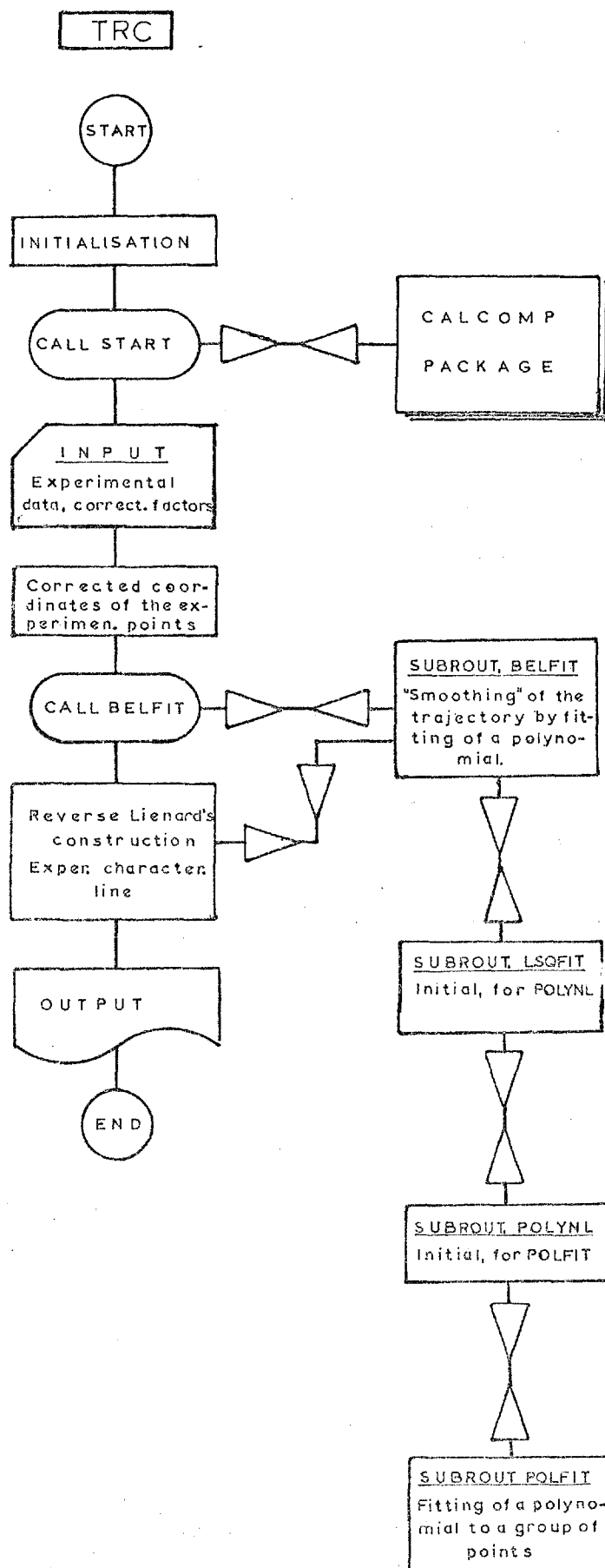
After the correction of the experimental values "smooths" down the trace, to permit correct location of the origin on the Liénard's plane (see Chapter 2) by the use of the BELFIT subroutine. The reverse Liénard's construction follows for the horizontal and vertical mode of oscillation and the results are written in the output file in the form of a table and plotted.

Subroutine BELFIT

This subroutine applies the numerical technique explained in 3.3.2. on the experimental values. It calls repeatedly subroutine LSQFIT for the fitting of a polynomial to the experimental points.

Subroutines LSQFIT, POLYNL, POLFIT

Subroutine LSQFIT (called by BELFIT) is used to initialise POLYNL and connect that to BELFIT, while POLYNL initialises and calls POLFIT which does the fitting of the polynomial.



```

PROGRAM TRC(INPUT,OUTPUT,TAPE5=INPUT,TAPE6=OUTPUT,TAPE25,TAPE27)
C   S. ANTONIOU      UMEM109
C
C   VERS  ONE      TRAJC
C
  DIMENSION N(250),T(250),X(250),V(250),XN(250),VN(250),AM(250),
  1NT(250),NHN(250),H(250),HN(250),TC(250),NH(250),NUM(2),XX(250),
  2YY(250),AMT(250),NAM(1),XLNR(250),XNLNR(250),VLN(250),VNLN(250),
  3YYN(250),XXN(250),AMTN(250),XXX(250),YYY(250),XCX(500),YCY(500)
C VELOCITY-DISPLACEMENT TRAJECTORY CONSTRUCTION FROM DYNAMOMETER READING
C (DYNAMOMETER MARK 1)
C TRACE DESCRIPTION WITH 200 POINTS AT MAXIMUM
C INITIAL ZEROING OF THE ARRAYS
  DO 5 I=1,250
    N(I)=0
    T(I)=0.
    X(I)=0.
    V(I)=0.
    XN(I)=0.
    VN(I)=0.
    AM(I)=0.
    NT(I)=0
    NHN(I)=0
    H(I)=0.
    HN(I)=0.
    TC(I)=0.
    NH(I)=0
    XX(I)=0.
    YY(I)=0.
    AMT(I)=0.
    XLNR(I)=0.
    XNLNR(I)=0.
    VLN(I)=0.
    VNLN(I)=0.
    YYN(I)=0.
    XXN(I)=0.
    AMTN(I)=0.
  5 CONTINUE
  CALL START
  READ(5,10)HO,HON,VEL
  10 FORMAT(3F20.5)
C HO,HON IN MICROSTRAIN THE REFERENCE LEVEL RESPECT. VEL THE PAPER
C VELOCITY IN MM PER SEC.
  HOR=HO/500.
  HONR=HON/568.2
C IN MM DISPLACEMENT
  READ(5,11)OMNH,OMNV
  11 FORMAT(2F20.5)
C OMNH,OMNV THE NATURAL FREQUENCIES OF THE DYNAMOMETER
  READ(5,15)SENSF,SENSN,AN1,AN2
  15 FORMAT(4F10.3)
  READ(5,16)NN
  16 FORMAT(I5)
C SENS- THE SENSITIVITIES OF FRICTIONAL OR NORMAL FORCE RESP.,AN- THE
C PER CENT ERROR DUE TO CROSS-INTERFERENCE BETWEEN THE TRANSDUCERS(1 FOR
C THE FRICTIONAL FORCE).
C NN THE NUMBER OF POINTS CONSISTING EACH TRACE
C SENS- THE SENSITIVITIES OF FRICTIONAL OR NORMAL FORCE RESP.,AN- THE
C PER CENT ERROR DUE TO CROSS-INTERFERENCE BETWEEN THE TRANSDUCERS(1 FOR

```

C THE FRICTIONAL FORCE).

NUM(1)=NN

NUM(2)=NN

NAM(1)=NN

NNN=NN+4

DO 20 J=1,NNN,6

JJ1=J

N(JJ1)=JJ1

JJ2=J+1

N(JJ2)=JJ2

JJ3=J+2

N(JJ3)=JJ3

JJ4=J+3

N(JJ4)=JJ4

JJ5=J+4

N(JJ5)=JJ5

JJ6=J+5

N(JJ6)=JJ6

READ(5,30)NT(JJ1),NH(JJ1),NHN(JJ1),NT(JJ2),NH(JJ2),NHN(JJ2),
2NT(JJ3),NH(JJ3),NHN(JJ3),NT(JJ4),NH(JJ4),NHN(JJ4),NT(JJ5),NH(JJ5),
3NHN(JJ5),NT(JJ6),NH(JJ6),NHN(JJ6)

30 FORMAT(18I4)

20 CONTINUE

DO 40 I=1,NN

TK=NT(I)

TC(I)=TK/100.

HK=NH(I)

H(I)=HK/100.

HNK=NHN(I)

HN(I)=HNK/100.

40 CONTINUE

T(1)=0.

V(1)=0.

VN(1)=0.

X(1)=HOR+H(1)*SENSF/500.

XN(1)=HONR+HN(1)*SENSN/568.2

XREAL=(X(1)-AN2*XN(1))/(1.-AN1*AN2)

XNREAL=(XN(1)-AN1*X(1))/(1.-AN1*AN2)

X(1)=XREAL

XN(1)=XNREAL

C -REAL THE CORRECTED REAL DISPLACEMENT VALUES

DO 50 I=2,NN

T(I)=((T(I-1)*VEL)+TC(I))/VEL

X(I)=HOR+H(I)*SENSF/500.

XN(I)=HONR+HN(I)*SENSN/568.2

XREAL=(X(I)-AN2*XN(I))/(1.-AN1*AN2)

XNREAL=(XN(I)-AN1*X(I))/(1.-AN1*AN2)

X(I)=XREAL

XN(I)=XNREAL

V(I)=(X(I)-X(I-1))/(T(I)-T(I-1))

VN(I)=(XN(I)-XN(I-1))/(T(I)-T(I-1))

C IN MM/SEC

VLN(I)=V(I)/OMNH

VNLN(I)=VN(I)/OMNV

C IN NON-DIMENTIONAL FORM

AM(I)=X(I)/XN(I)

50 CONTINUE

C *****

CALL BELFIT(X,VLN,NN,XXX,YYY)

```

C *****
DO 701 IAK=1,NN
X(IAK)=XXX(IAK)
VLN(IAK)=YYY(IAK)
701 CONTINUE
NK=NN-1
DO 51 J=2,NK
YY(J)=VLN(J)
XK=X(J)+(VLN(J)*(VLN(J-1)-VLN(J)))/(X(J-1)-X(J))
XL=X(J)+(VLN(J)*(VLN(J)-VLN(J+1)))/(X(J)-X(J+1))
AL1=SQRT((X(J-1)-X(J))**2+(VLN(J-1)-VLN(J))**2)
AL2=SQRT((X(J)-X(J+1))**2+(VLN(J)-VLN(J+1))**2)
XX(J)=(XK*AL1+XL*AL2)/(AL1+AL2)
C NON DIMENSIONAL FRICTION TRACE FOR THE HORIZONTAL PLANE
YYN(J)=VNLN(J)
XKN=XN(J)+(VNLN(J)*(VNLN(J-1)-VNLN(J)))/(XN(J-1)-XN(J))
XLN=XN(J)+(VNLN(J)*(VNLN(J)-VNLN(J+1)))/(XN(J)-XN(J+1))
ANL1=SQRT((XN(J-1)-XN(J))**2+(VNLN(J-1)-VNLN(J))**2)
ANL2=SQRT((XN(J)-XN(J+1))**2+(VNLN(J)-VNLN(J+1))**2)
XXN(J)=(XKN*ANL1+XLN*ANL2)/(ANL1+ANL2)
C NON DIMENSIONAL NORMAL FORCE CHANGES
51 CONTINUE
DO 52 I=2,NN
IF(VLN(I).GT.0..AND.VLN(I+1).LT.0..AND.VLN(I+2).LT.0..AND.
2VLN(I+3).LT.0..AND.VLN(I+4).LT.0.)GO TO 552
52 CONTINUE
GO TO 510
552 EL=(XX(I+1)*YY(I)-XX(I)*YY(I+1))/(YY(I)-YY(I+1))
510 DO 520 I=2,NK
IF(VNLN(I).LT.0..AND.VNLN(I+1).GT.0..AND.VNLN(I+2).GT.0..AND.
2VNLN(I+3).GT.0..AND.VNLN(I+4).GT.0.)GO TO 562
520 CONTINUE
GO TO 530
562 ELN=(XXN(I+1)*YYN(I)-XXN(I)*YYN(I+1))/(YYN(I)-YYN(I+1))
530 DO 540 I=1,NN
XLNR(I)=X(I)-EL
XNLNR(I)=XN(I)-ELN
XX(I)=XX(I)-EL
XXN(I)=XXN(I)-ELN
AMT(I)=(XX(I)*OMNH**2)/9810.
AMTN(I)=(XXN(I)*OMNV**2)/9810.
C -
C NO CORRELATION BETWEEN MT VALUES AND STATIC MEAN VALUES
C -
540 CONTINUE
WRITE(6,70)
70 FORMAT(1H1,
190H *****
2***** ,/
390H * STICK-SLIP CYCLE ANALYSIS
4 * ,/
590H *****
6***** ,/
790H * DATA
8 * ,/
990H * ----
1 * )
WRITE(6,110)
110 FORMAT(

```

```

290H      *      SURFACES      STEEL ST 37.11 ON STEEL ST 37.11
3          *      ,/
490H      *      SURFACE ROUGHNESS      20 MICROIN CLA FOR BOTH
5SPECIMENS *      ,/
690H      *      SURFACE HARDNESS      40-45 RC FOR BOTH SPECIMEN
7S        *      ,/
890H      *      GEOMETRY      1/2 IN BALL ON FLAT DISC
9          *      ,/
190H      *      LUBRICANT      NONE
2          *      ,
WRITE(6,120)
120 FORMAT(
390H      *      AMBIENT TEMPERATURE      21 DEG CENT.
4          *      ,/
590H      *      RELATIVE HUMIDITY      57 PER CENT
6          *      ,/
790H      *      *****
8*****
990H      *      N      T      X      V      XN      VN
1M      MT      *      ,/
290H      *      -      SEC      MM      MM/SEC      MM      MM/SEC
3-      -      *      ,)
WRITE(6,111)
111 FORMAT(
190H      *      *****
2*****
DO 90 I=1,NN
WRITE(6,80)N(I),T(I),X(I),V(I),XN(I),VN(I),AM(I),AMT(I)
80 FORMAT(11H      *,I6,2F8.4,F8.3,F8.4,F8.3,2F8.4,9H      *      )
90 CONTINUE
WRITE(6,100)
100 FORMAT(
390H      *
4          *      ,/
190H      *      *****
2*****
DO 600 I=1,NN
WRITE(6,635)XX(I),YY(I),XLNR(I),XNLNR(I),VLN(I),VNLN(I),YYN(I),
2XXN(I),AMTN(I)
635 FORMAT(9F12.5)
600 CONTINUE
DO 700 I=1,NAM
XCX(I)=X(I)
YCY(I)=XN(I)
700 CONTINUE
CALL GRAF(XCX,YCY,NAM,1,1,15HHOR.DISPLAC. MM,15,15HVER.DISPLAC. MM
1,15,4.6,7.8)
C *****
CALL BELFIT(X,XN,NAM,XXX,YYY)
C *****
DO 710 I=1,NAM
XCX(I)=XLNR(I)
YCY(I)=XNLNR(I)
710 CONTINUE
CALL GRAF(XCX,YCY,NAM,1,1,15HHOR.DISPLAC. MM,15,15HVER.DISPLAC. MM
1,15,4.6,7.8)
C *****
CALL BELFIT(XLNR,XNLNR,NAM,XXX,YYY)
C *****

```

```

      KP=NN+1
      KNP=2*NN
      DO 121 I=KP,KNP
      XCX(I-KP+1)=X(I-KP+1)
      YCY(I-KP+1)=V(I-KP+1)
      XCX(I)=XN(I-KP+1)
      YCY(I)=VN(I-KP+1)
121  CONTINUE
      CALL GRAF(XCX,YCY,NUM,2,1,15HDISPLACEMENT MM,15,15HVELOCITY MM/SEC
1,15,4,6,7,8)
      DO 1210 I=1,KNP
      IF(AM(I).LT.3.)GO TO 1210
      AM(I)=3.
1210  CONTINUE
      DO 720 I=1,NAM
      XCX(I)=V(I)*(-1)
      YCY(I)=AM(I)
720  CONTINUE
      CALL GRAF(YCY,XCX,NAM,1,-1,16HCOEF.OF FRICTION,16,15HVELOCITY MM/S
1EC,15,4,6,7,8)
C *****
      CALL BELFIT(AM,V,NAM,XXX,YYY)
C *****
      DO 122 I=KP,KNP
      XCX(I-KP+1)=XLNR(I-KP+1)
      YCY(I-KP+1)=VLN(I-KP+1)
      XCX(I)=XNLNR(I-KP+1)
      YCY(I)=VNLN(I-KP+1)
122  CONTINUE
      CALL GRAF(XCX,YCY,NUM,2,1,15HDISPLACEMENT MM,15,15HVELOCITY MM/SEC
1,15,4,6,7,8)
      DO 123 I=KP,KNP
      XCX(I-KP+1)=XX(I-KP+1)
      YCY(I-KP+1)=VLN(I-KP+1)
      XCX(I)=XXN(I-KP+1)
      YCY(I)=VNLN(I-KP+1)
123  CONTINUE
      DO 1230 I=1,KNP
      IF(XCX(I).LT.20.)GO TO 1230
      XCX(I)=20.
1230  CONTINUE
      CALL GRAF(XCX,YCY,NUM,2,1,16HNON-DIM.C.OF FR.,16,15HVELOCITY MM/SE
1C,15,4,6,7,8)
      DO 124 I=KP,KNP
      XCX(I-KP+1)=AMT(I-KP+1)
      YCY(I-KP+1)=VLN(I-KP+1)
      XCX(I)=AMTN(I-KP+1)
      YCY(I)=VNLN(I-KP+1)
124  CONTINUE
      DO 1240 I=1,KNP
      IF(XCX(I).LT.3.)GO TO 1240
      XCX(I)=3.
1240  CONTINUE
      CALL GRAF(XCX,YCY,NUM,2,-1,16HNON-DIM.C.OF FR.,16,15HVELOCITY MM/S
1EC,15,4,6,7,8)
      CALL ENPLOT( 4,6)
      STOP
      END

```

```

SUBROUTINE BELFIT(X,Y,N,XXX,YYY)
  DIMENSION X(250),Y(250),PSI(250),CHI(5),PSII(5),C(4),ESTY(5),
1XXX(250),YYY(250)
  M=5
  K=2
  L=8
  DO 10 I=1,L
    NN=N-M+1
    DO 110 LM=1,4
      C(LM)=0.
110 CONTINUE
    DO 15 KAA=1,N
      PSI(KAA)=0.
  15 CONTINUE
    DO 20 KA=1,NN
      CHI(1)=X(KA)
      PSII(1)=Y(KA)
      CHI(2)=X(KA+1)
      PSII(2)=Y(KA+1)
      CHI(3)=X(KA+2)
      PSII(3)=Y(KA+2)
      CHI(4)=X(KA+3)
      PSII(4)=Y(KA+3)
      CHI(5)=X(KA+4)
      PSII(5)=Y(KA+4)
      CALL LSQFIT(5,2,1,0,0,0,2,0,CHI,PSII,C,ESTY)
      DO 40 KK=1,M
        KN=KA+KK-1
        PSI(KN)=C(4)*CHI(KK)**2+C(3)*CHI(KK)+C(2)+PSI(KN)
40 CONTINUE
20 CONTINUE
    IFF=6
    DO 50 KL=1,N
      IF (KL.LE.M) GO TO 60
      IF (KL.GT.M.AND.KL.LT.NN) GO TO 80
      IFF=IFF-1
      ID=IFF
      GO TO 70
60 ID=KL
      GO TO 70
80 ID=M
70 CONTINUE
      Y(KL)=PSI(KL)/FLOAT(ID)
50 CONTINUE
      IF (I.EQ.8) GO TO 9
      GO TO 10
9 CONTINUE
      DO 100 IAK=1,N
        XXX(IAK)=X(IAK)
        YYY(IAK)=Y(IAK)
100 CONTINUE
      CALL GRAF(X,Y,N,1,0,6HBEFIT,6,6HBEFIT,6,4,6,7,8)
10 CONTINUE
      RETURN
      END

```

```

SUBROUTINE LSQFIT(NUMBER,KDEG,AP,EQWT,WRITER,X,Y,FIT POL,ESTY)
C   LEAST SQUARES FIT OF POLYNOMIAL, WITH STATISTICAL ANALYSIS
C
  DIMENSION X(5),Y(5),WT(5),RESID(5),U(5),V(5),VAR(5),
1 FITPOL(4),F(4),ORTPOL(4),H(4),SSQS(4),C(4),NOMAX(4),
2 RESMAX(4),DINV(4),EL(6),COV(6),ESTY(5)
C
  NOUT=6
  NIN=5
  NDIM=5
  KD=4
  KE=6
  DO 7 I=1,NUMBER
7  WT(I)=1.0
3  CALL POLYNL
  1(NUMBER,KDEG,WRITER,KD,NDIM,KE,X,Y,WT,RESID,U,V,VAR,FITPOL,
2 F,ORTPOL,H,SSQS,C,RESMAX,NOMAX,DINV,EL,COV,ESTY)
  RETURN
  END
  SUBROUTINE POLYNL
  1 (NUMBER,KDEG,WRITER,KD,NDIM,KE,X,Y,WT,RESID,U,V,VAR,FITPOL,
2 F,ORTPOL,H,SSQS,C,RESMAX,NOMAX,DINV,EL,COV,ESTY)
  DIMENSION X(NDIM),Y(NDIM),WT(NDIM),RESID(NDIM),U(NDIM),V(NDIM),
1 VAR(NDIM),FITPOL(KD),F(KD),ORTPOL(KD),H(KD),SSQS(KD),C(KD),
2 RESMAX(KD),NOMAX(KD),DINV(KD),EL(KE),COV(KE),ESTY(NDIM)
C
C   ,RESID(I), HOLDS THE CURRENT DIFFERENCE BETWEEN THE DATA Y(I) AND
C   THE FITTED VALUE, AND IS ALTERED AFTER EACH INCREASE OF THE DEGREE
C   OF THE POLYNOMIAL. ,U(I), AND ,V(I), ARE USED IN THE RECURRENCE
C   RELATION TO CALCULATE THE VALUE OF AN ORTHOGONAL POLYNOMIAL IN
C   TERMS OF THE PREVIOUS TWO ORTHOGONAL POLYNOMIALS.
C   ,VAR(I), CONTAINS THE VARIANCE OF A FITTED Y VALUE, AND INCREASES
C   WITH EACH INCREASE OF DEGREE OF THE FITTED POLYNOMIAL.
  NOUT=6
  DO 1 I=1,NUMBER
  RESID(I)=Y(I)
  U(I)=1.0
  V(I)=0.0
1  VAR(I)=0.0
C
C   IN ORDER TO AVOID ZERO SUBSCRIPTS, THE J,TH POLYNOMIAL IS REFERRED
C   TO BY THE INDEX (J+1) OR (J+2). ,FIT POL, AND ,ORT POL, REFER TO
C   THE FITTED AND THE ORTHOGONAL POLYNOMIALS RESPECTIVELY. ,C(J+2),
C   IS THE LEAST SQUARES ESTIMATE OF THE BEST MULTIPLE OF THE J,TH
C   ORTHOGONAL POLYNOMIAL.
  K2=KDEG + 3
  DO 22 J=1,K2
  FIT POL(J) =0.0
  F(J)=0.0
  ORT POL(J) = 0.0
22 H(J)=0.0
C
  ORT POL(2) =1.
  C(1)=0.0
  CALL POLFIT
  1(NUMBER,KDEG,WRITER,KD,NDIM,KE,X,Y,WT,RESID,U,V,VAR,FITPOL,F,
2 ORTPOL,H,SSQS,C,RESMAX,NOMAX,DINV,EL,COV,ESTY)
  RETURN
  END

```

```

SUBROUTINE POLFIT
1 (NUMBER,KDEG,WRITER,KD,NDIM,KE,X,Y,WT,RESID,U,V,VAR,FITPOL,F,
2 ORTPOL,H,SSQS,C,RESMAX,NOMAX,DINV,EL,COV,ESTY)
  DIMENSIONX(NDIM),Y(NDIM),WT(NDIM),RESID(NDIM),U(NDIM),V(NDIM),
  1VAR(NDIM),FITPOL(KD),F(KD),ORTPOL(KD),H(KD),SSQS(KD),C(KD),
  2RESMAX(KD),NOMAX(KD),DINV(KD),EL(KE),COV(KE),ESTY(NDIM)

```

```

C
C   ,DIV, IS THE DIVISOR USED TO CALCULATE VAR(C).
C   NOUT=6
C   DIV=1.0
C   DINV(1)=0.0
C   A=0.0
C   B=-1.0
C   KPLUS=KDEG+1

```

```

C
C   DO 60 J=1,KPLUS
C   EX=0.0
C   EY=0.0
C   Z=0.0
C   BIG RES =0.0
C   NO RES = 0

```

```

C
C   DO 61 I=1,NUMBER
C   IF(J-1)49,49,6
6  VAR(I)=VAR(I)+V(I)*V(I)*DINV(J-1)
  RESID(I)=RESID(I)-C(J)*V(I)
7  IF(WT(I)-0.0)49,49,64
64 ABS RES = ABS (RESID(I))
  IF(ABS RES - BIG RES)49,49,62
62 BIG RES = ABS RES
  NO RES = I
49 W=(X(I)-A)*V(I) - B*U(I)
  U(I)=V(I)
  V(I)=W
  W=WT(I)*V(I)*V(I)
  EX=EX+W
  W=W*X(I)
  EY=EY+W
  W=RESID(I)*WT(I)*V(I)
61 Z=Z+W

```

```

C
C   ,RES MAX(J), AND ,NO MAX(J), CONTAIN THE BIGGEST RESIDUAL AND ITS
C   NUMBER AFTER FITTING THE POLYNOMIAL OF DEGREE (J-1). ,S SQS, IS
C   THE REDUCTION IN THE SUM OF SQUARES OF THE RESIDUALS.

```

```

8  A=EY/EX
  B=EX/DIV
  DIV=EX
  RES MAX(J) = BIG RES
  NO MAX(J) = NO RES
  C(J+1)=Z/EX
  S SQS(J) = Z*C(J+1)
  J POWER = J-1
  DO 66 I=1,J
  NT= I+1
  FIT POL(NT) = FIT POL(NT) + C(J+1)*ORT POL(NT)
  I POWER = I-1
66 CONTINUE
  DINV(J)=1.0/DIV

```

```

      IF(WRITER-3.5)5.5,11
11  IF(J-1)3,10,3

```

```

C
C      THE COEFFICIENTS OF THE ORTHOGONAL POLYNOMIALS ARE STORED IN THE
C      ARRAY ,EL(210),, FOR USE IN SUBROUTINE COVAR.

```

```

10  EL(1)=1.0

```

```

      GO TO 5

```

```

      3  N1=(J*(J-1))/2

```

```

      DO 2 I=1,J

```

```

      L=N1+I

```

```

      2  EL(L)=ORTPOL(I+1)

```

```

      5  DO 63 I=1,NT

```

```

      NS=I+1

```

```

63  H(NS) = ORT POL(I) - A*ORT POL(NS) - B*F(NS)

```

```

      DO 60 I=1,NT

```

```

      NS=I+1

```

```

      F(NS) = ORT POL(NS)

```

```

60  ORT POL(NS) = H(NS)

```

```

C
      DET=DINV(KPLUS)

```

```

      RETURN

```

```

      END

```

Appendix 8EXPERIMENTAL TRAJECTORIES

- Fig. 1.,1a : Regular stick-slip trajectory and Lisitsyn diagram (steel on steel).
- Fig. 2.,2a : Regular stick-slip with strong triggering oscillation (Bronze on bronze).
- Fig. 3.,3a : Irregular stick-slip (Steel EN31 on steel EN1 during the first stages of sliding).

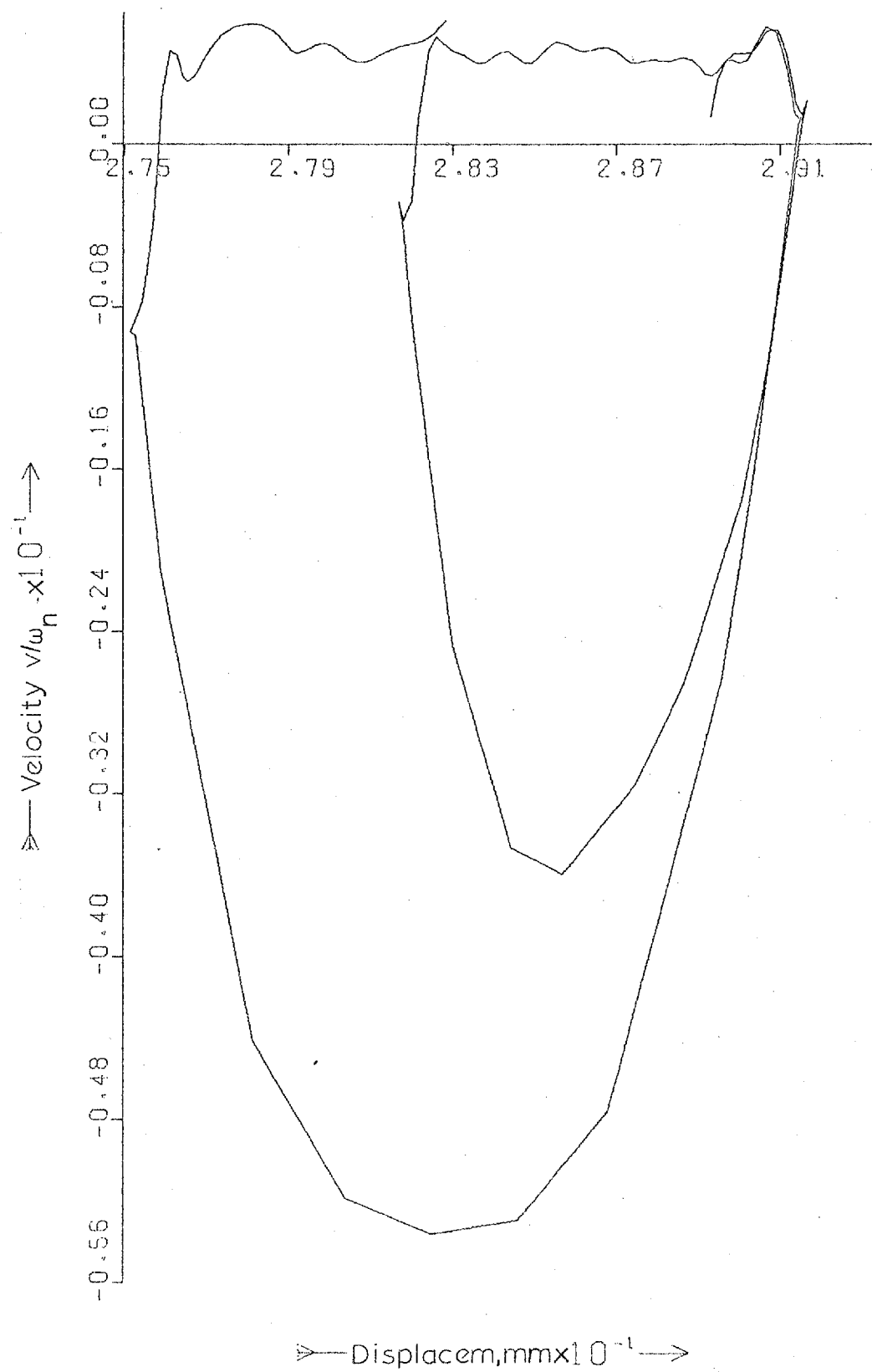


Fig. A8 - 1

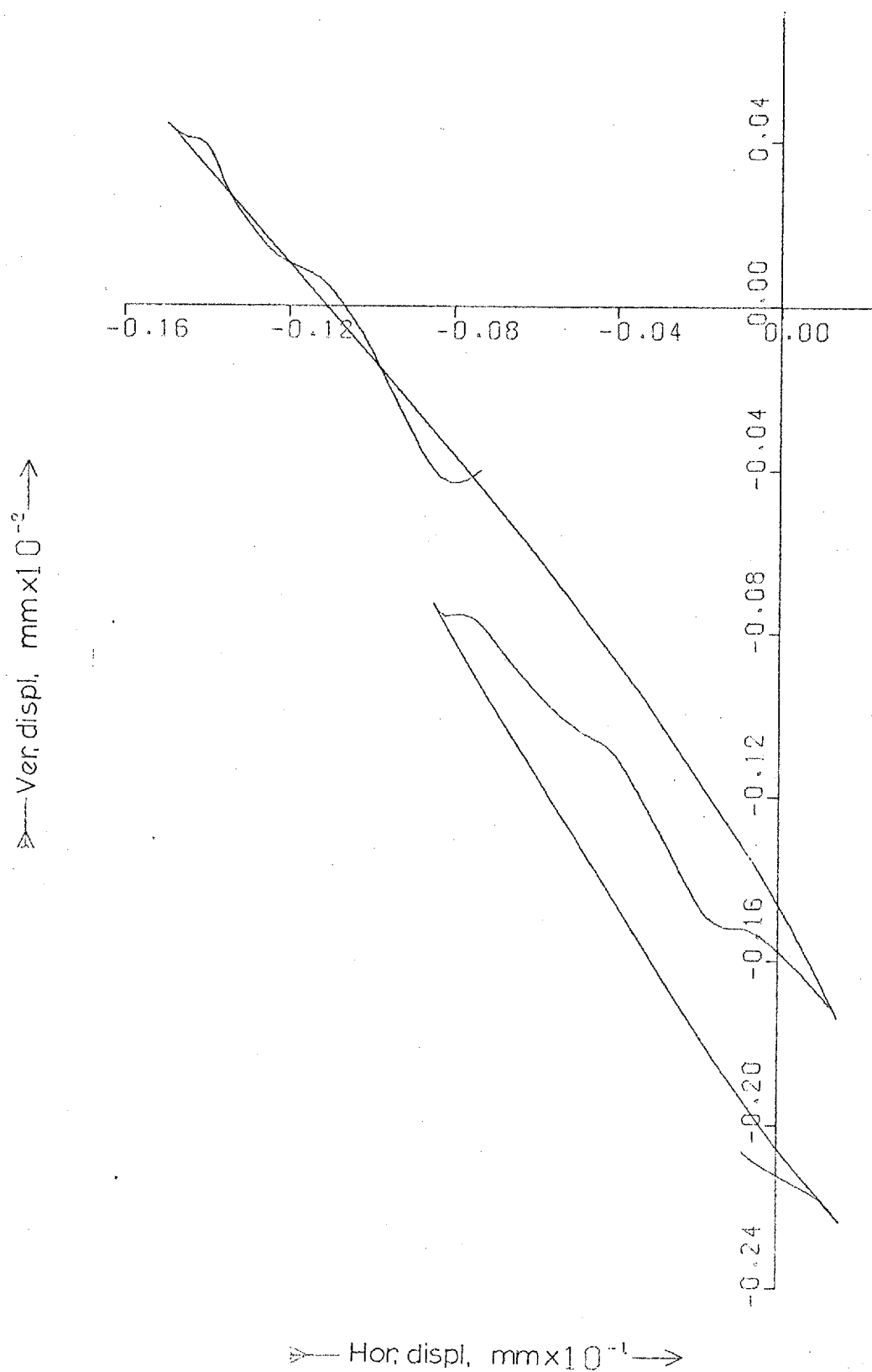


Fig. A8-1a

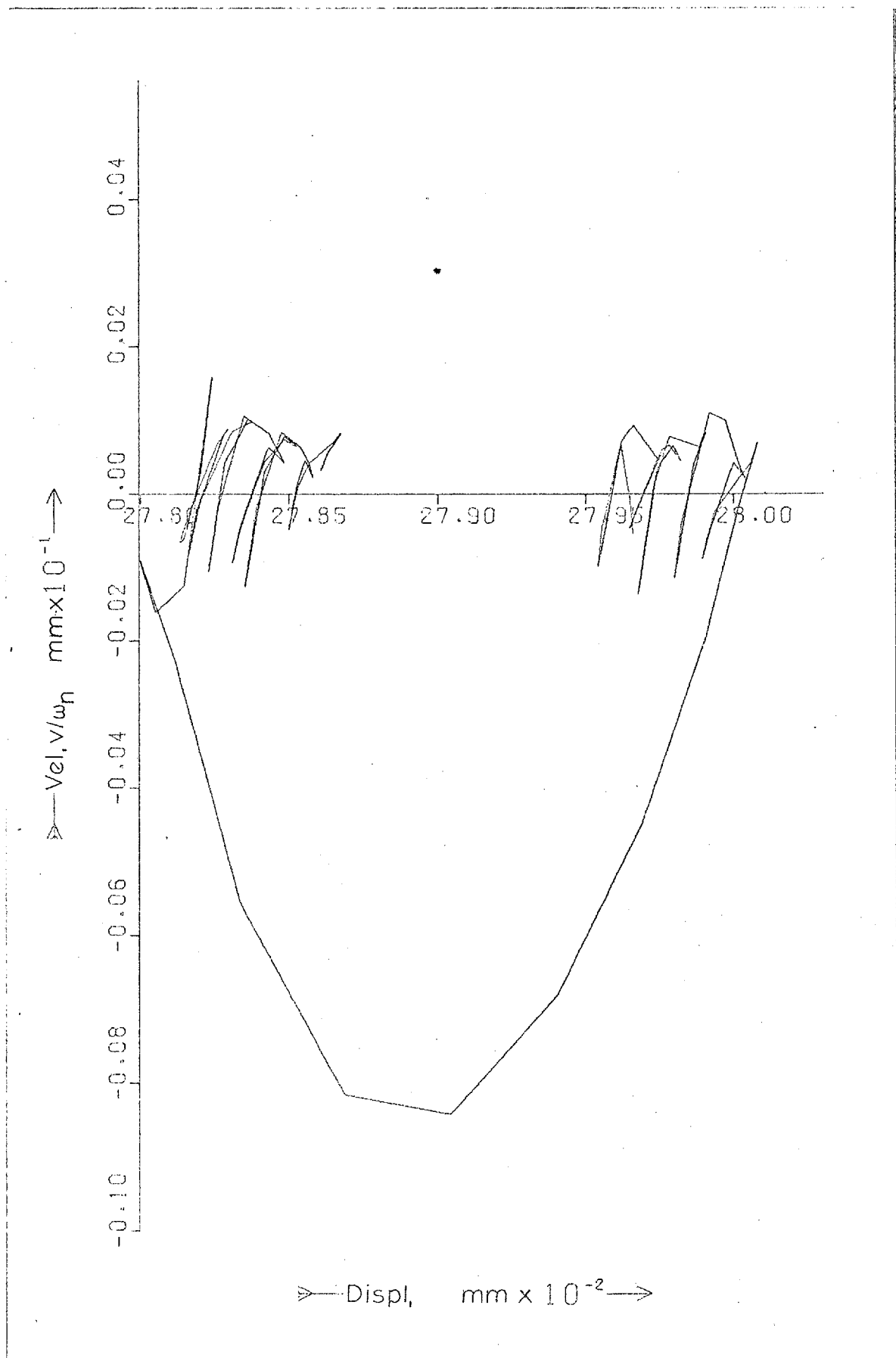


Fig. A8-2

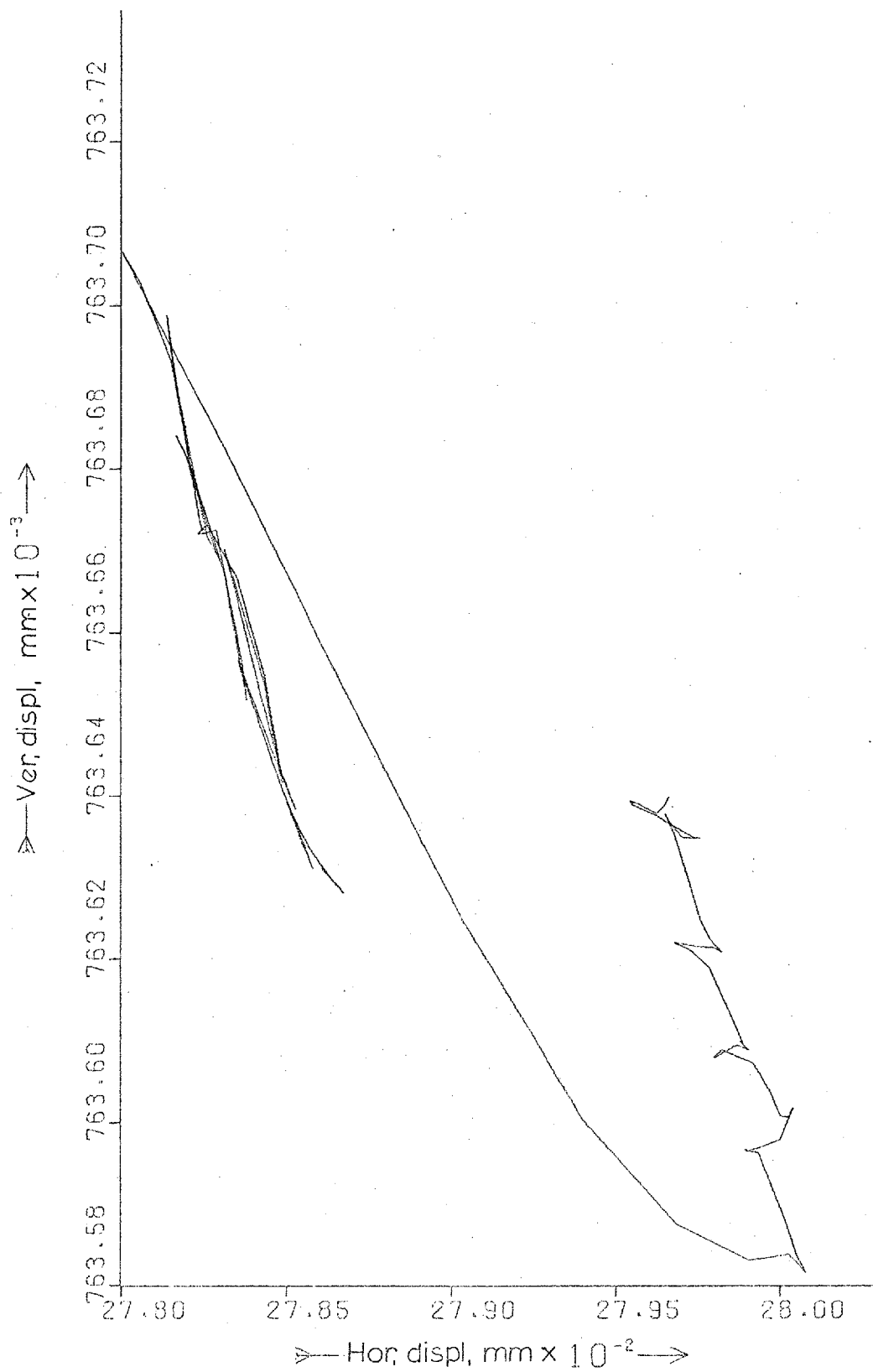


Fig. A8-2a

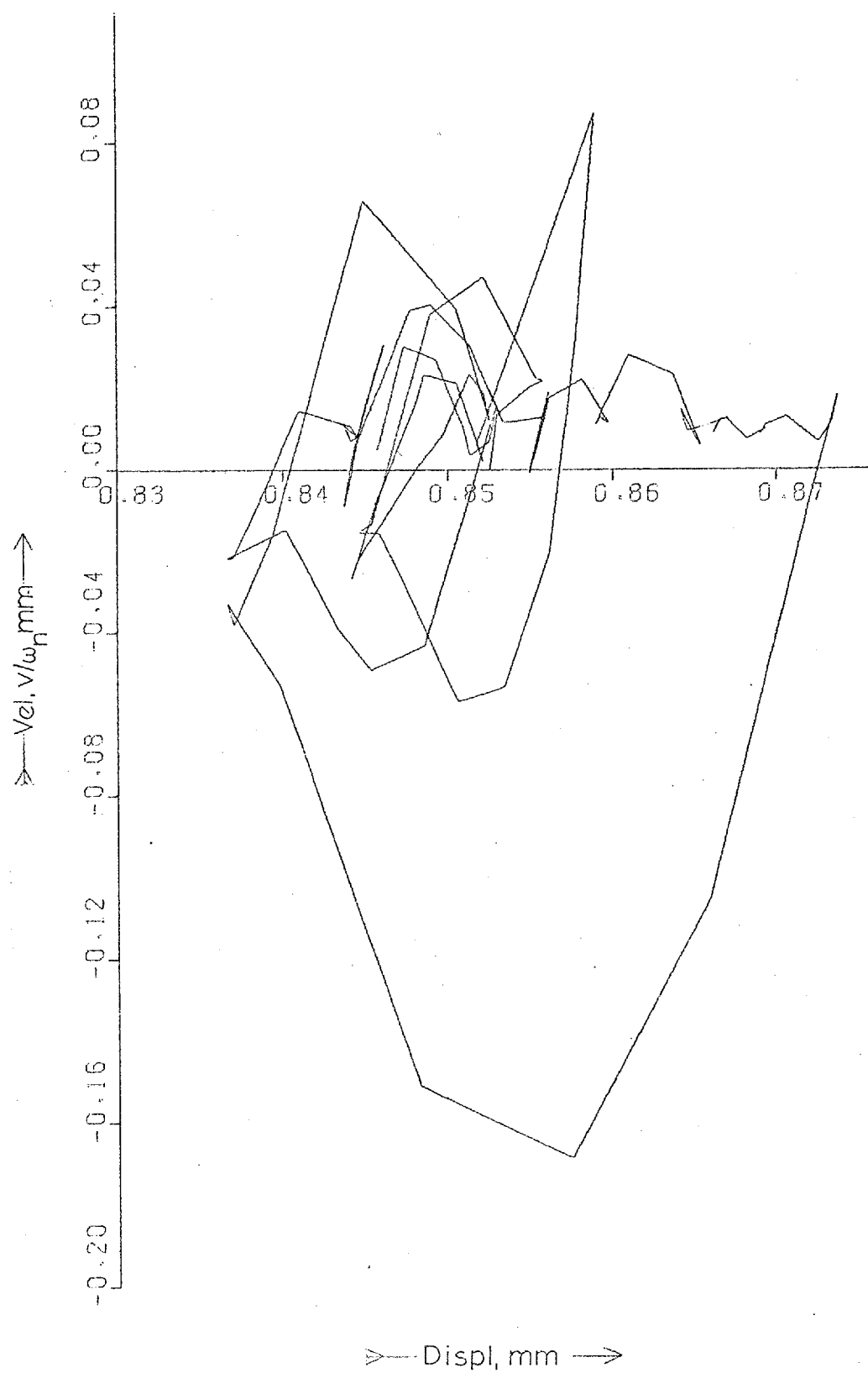


Fig. A8-3

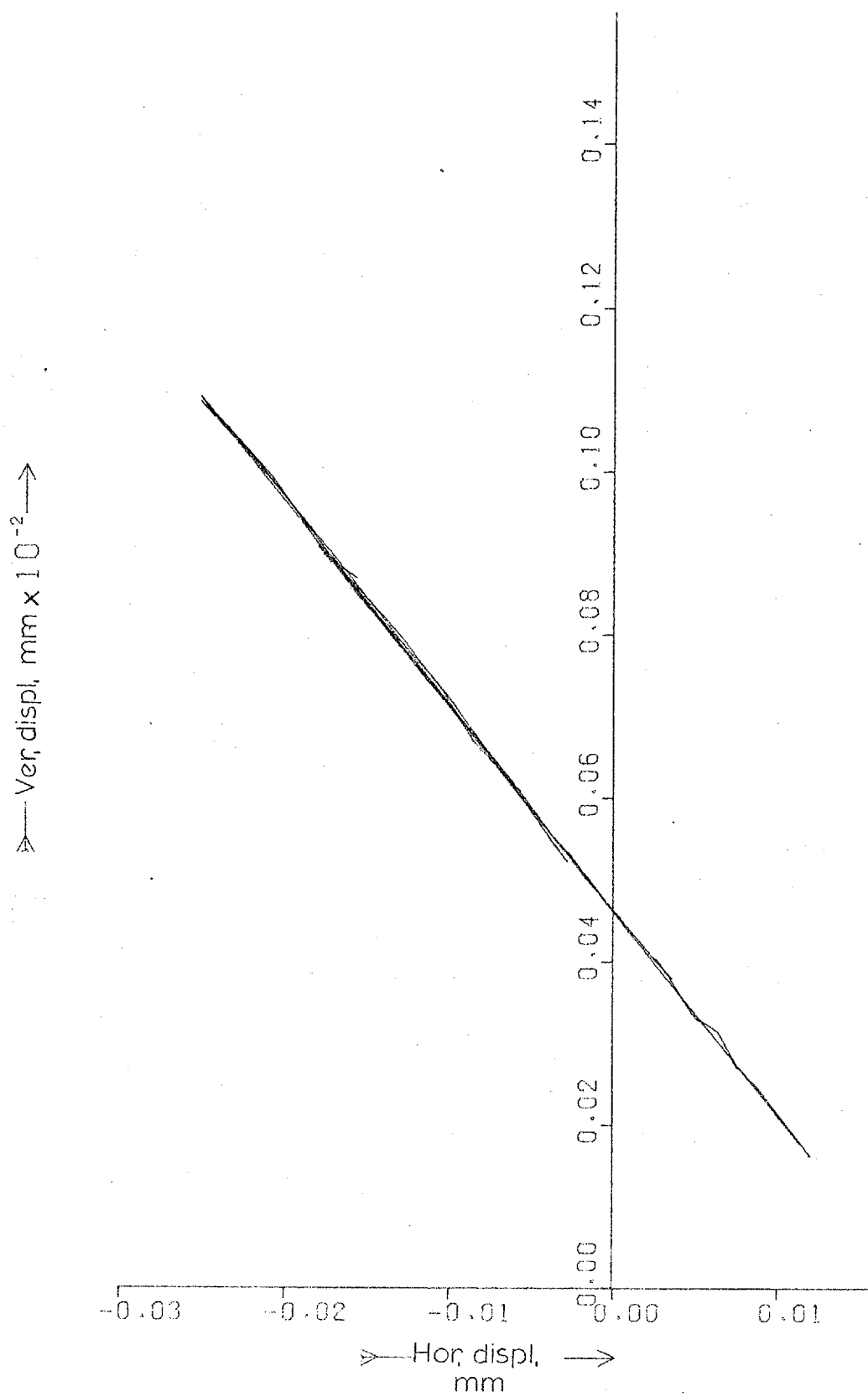


Fig. A8-3a

Appendix 9

THE THEORETICAL MODEL

The theoretical model was used to simulate actual frictional oscillations. Thus fig. A9-1 to A9-6 show how the geometry of the characteristic line affects the trajectories and consequently the stick-slip traces, while fig. A9-7 is simulation in the general case of $v_o = v_o(t)$ and triggering oscillation interferes in the formation of the trajectories. Fig. A9-7 has a particular importance because it shows how the triggering oscillation can produce the observed dying-out oscillation after the slip part of stick-slip (fig. A9-8). According to Eiss [219] this is explained as an oscillation of the dynamometer springs subjected to the effect of the dynamic energy released during slip, which is spent by the internal damping of the springs and consequently no relative motion appears necessarily between the specimens, but with this explanation does not agree the magnitude of damping factor which was found to range between $D = 8.90 \times 10^{-3}$ and $D = 59 \times 10^{-3}$ i.e. about 3 - 20 times greater than the internal damping of the springs.

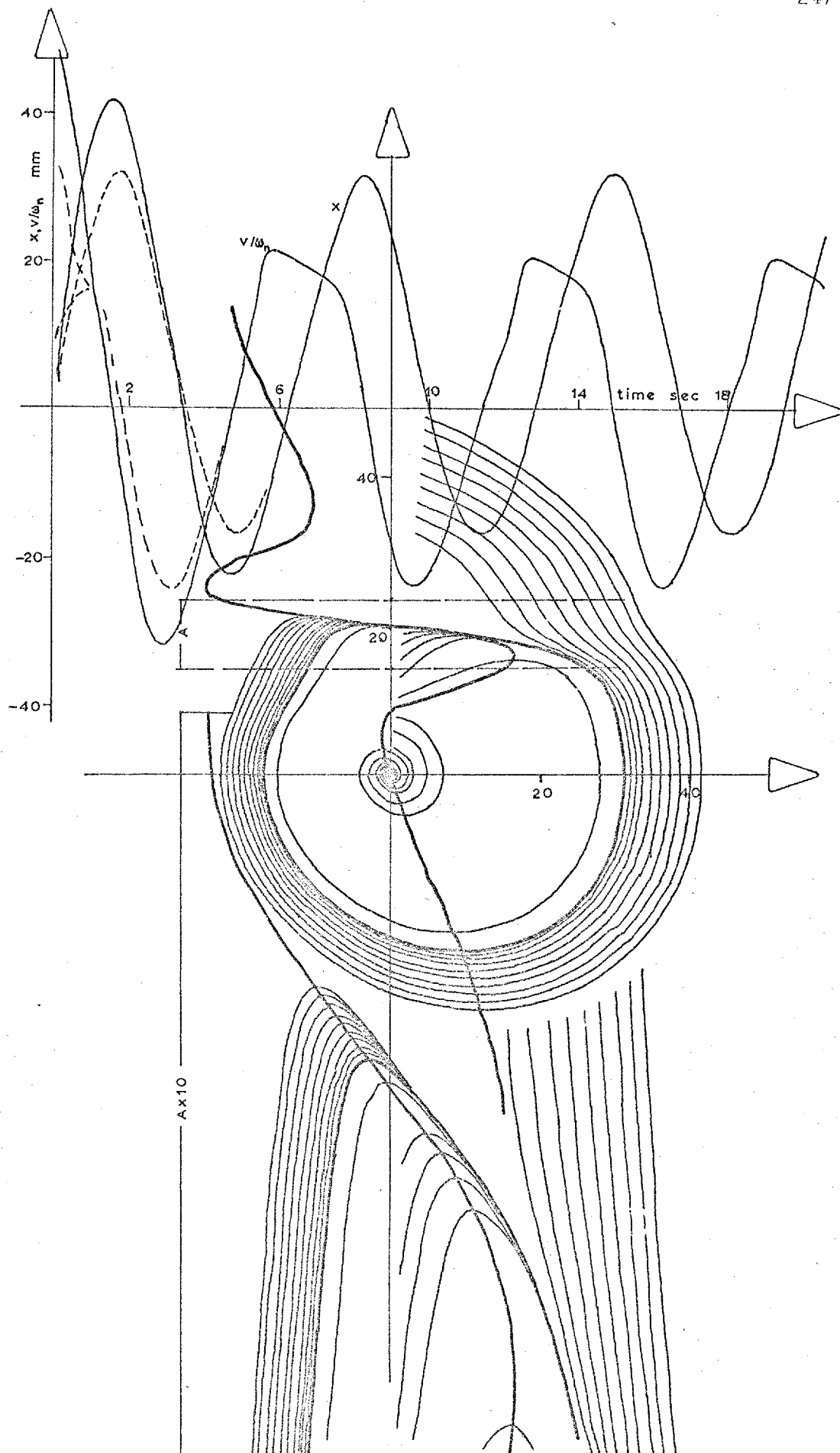


Fig. A9-1,2

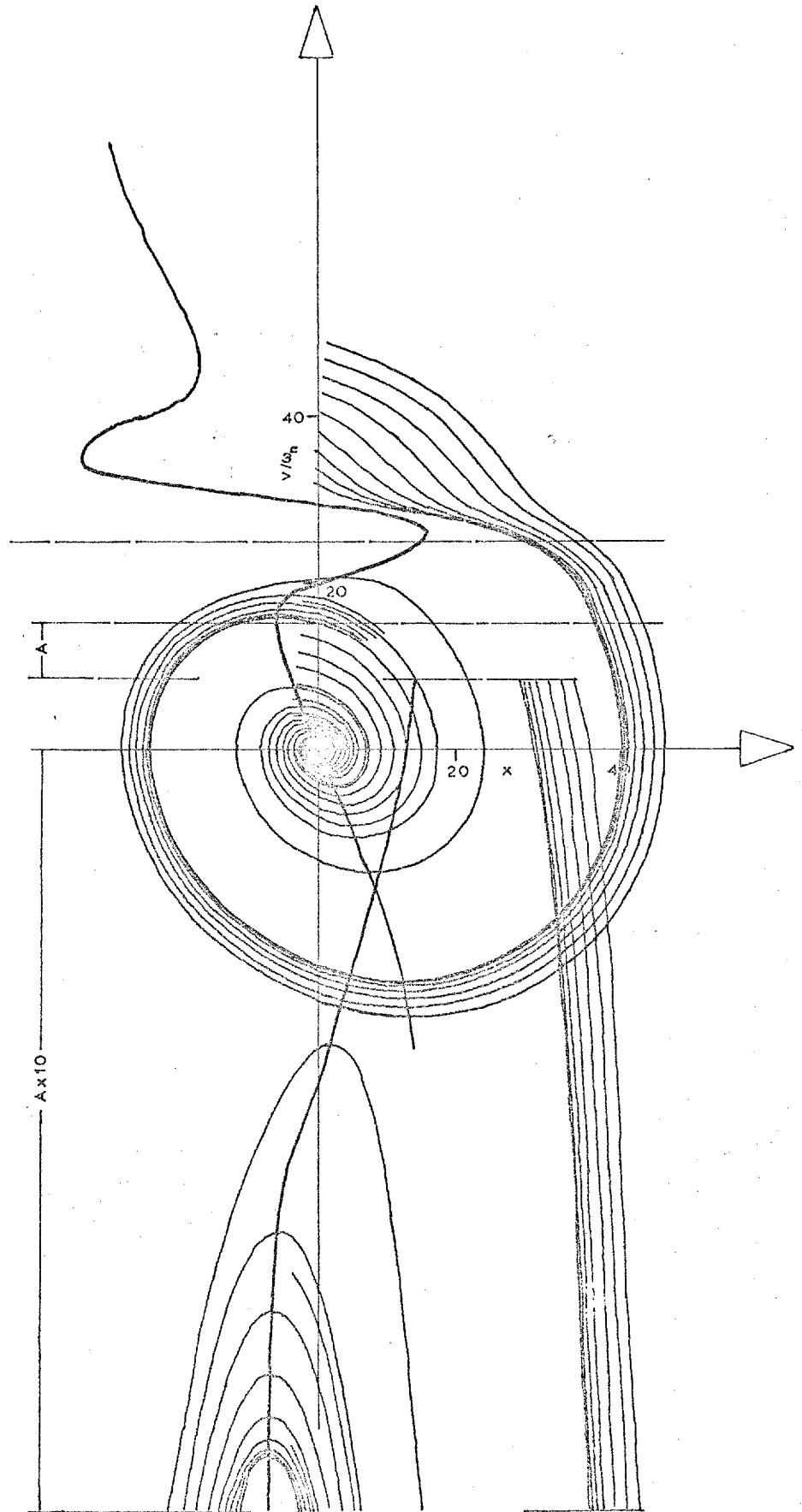


Fig. A9-3

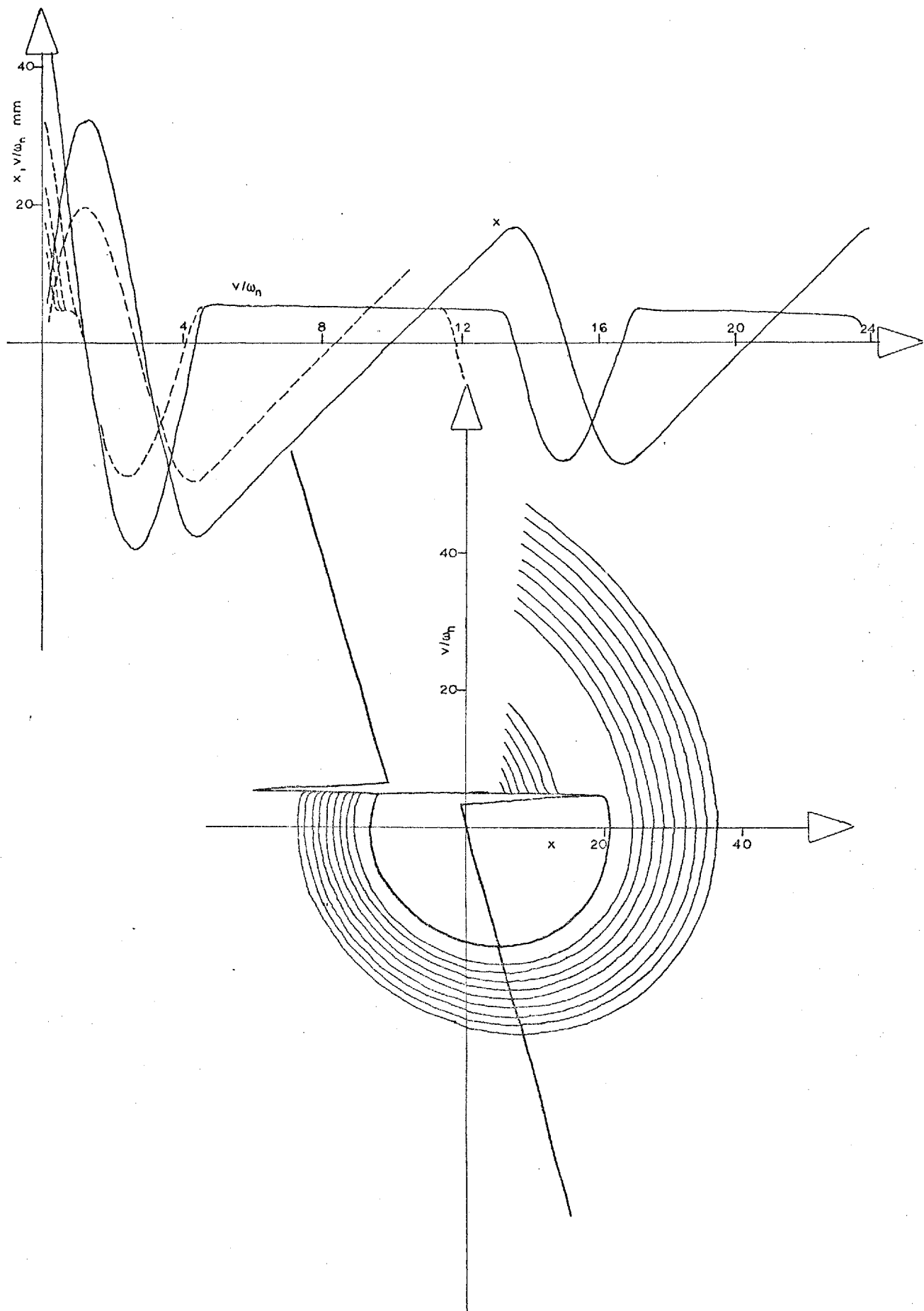


Fig. A9-4

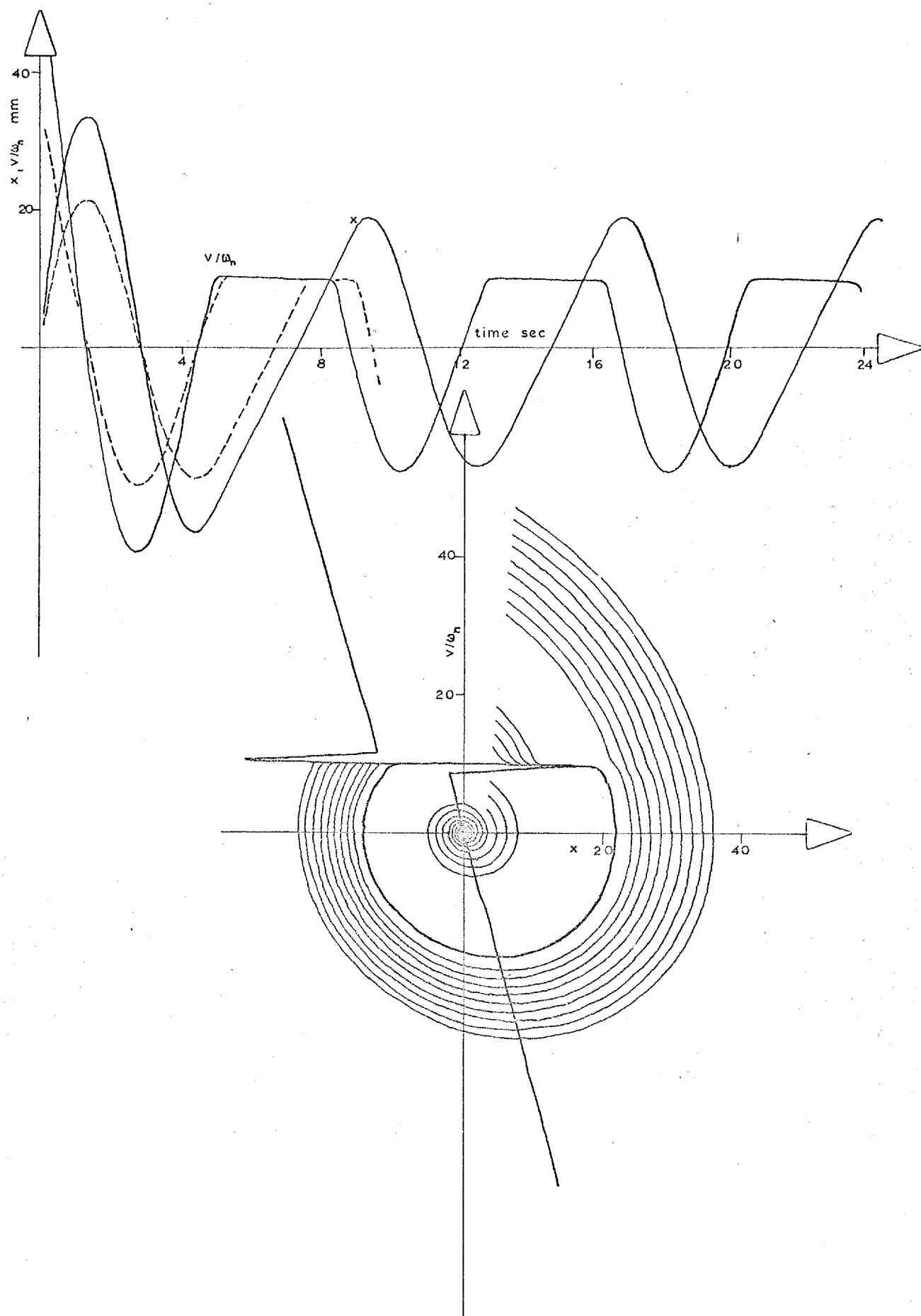


Fig. A9-5

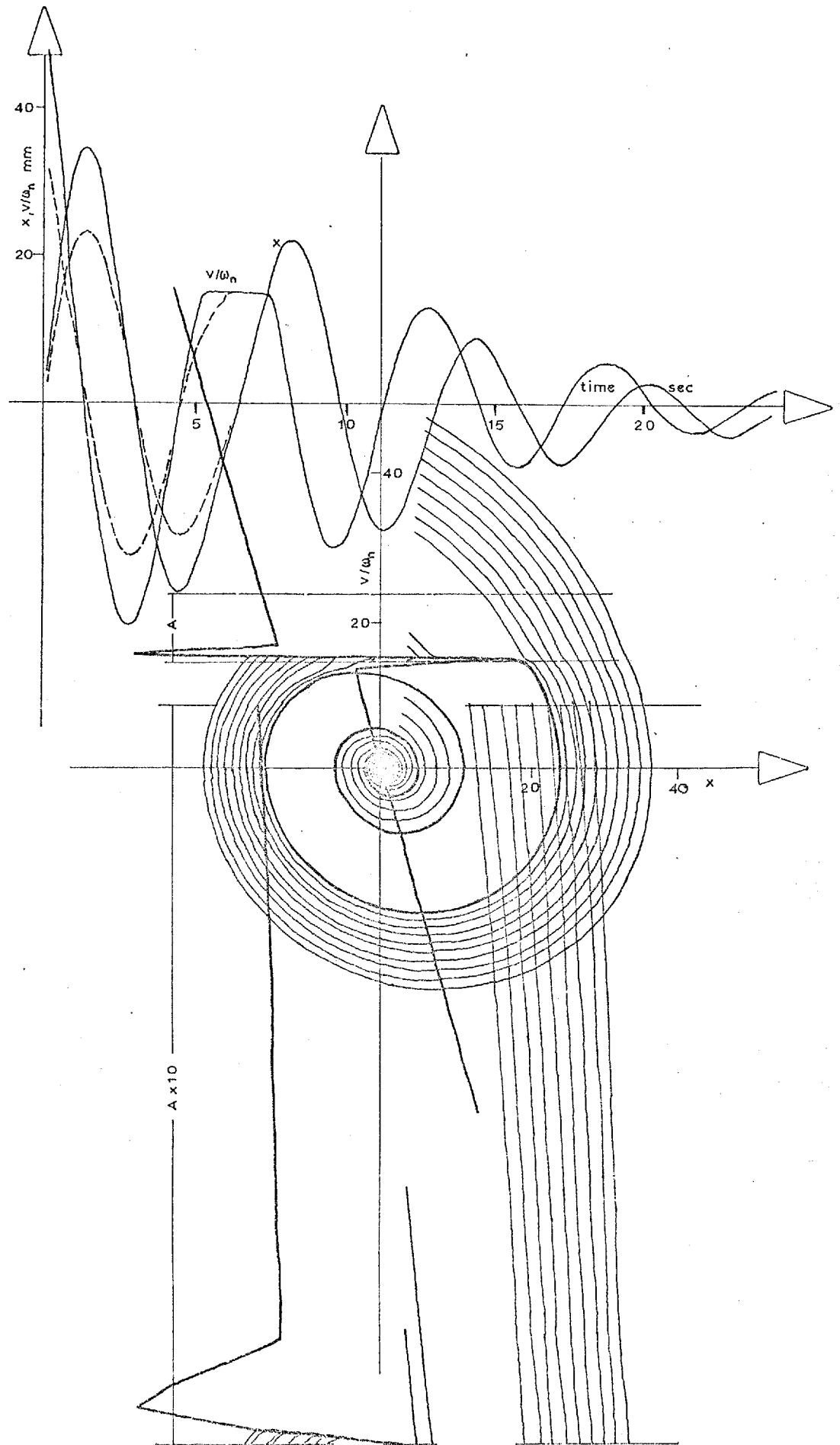


Fig. A9-6

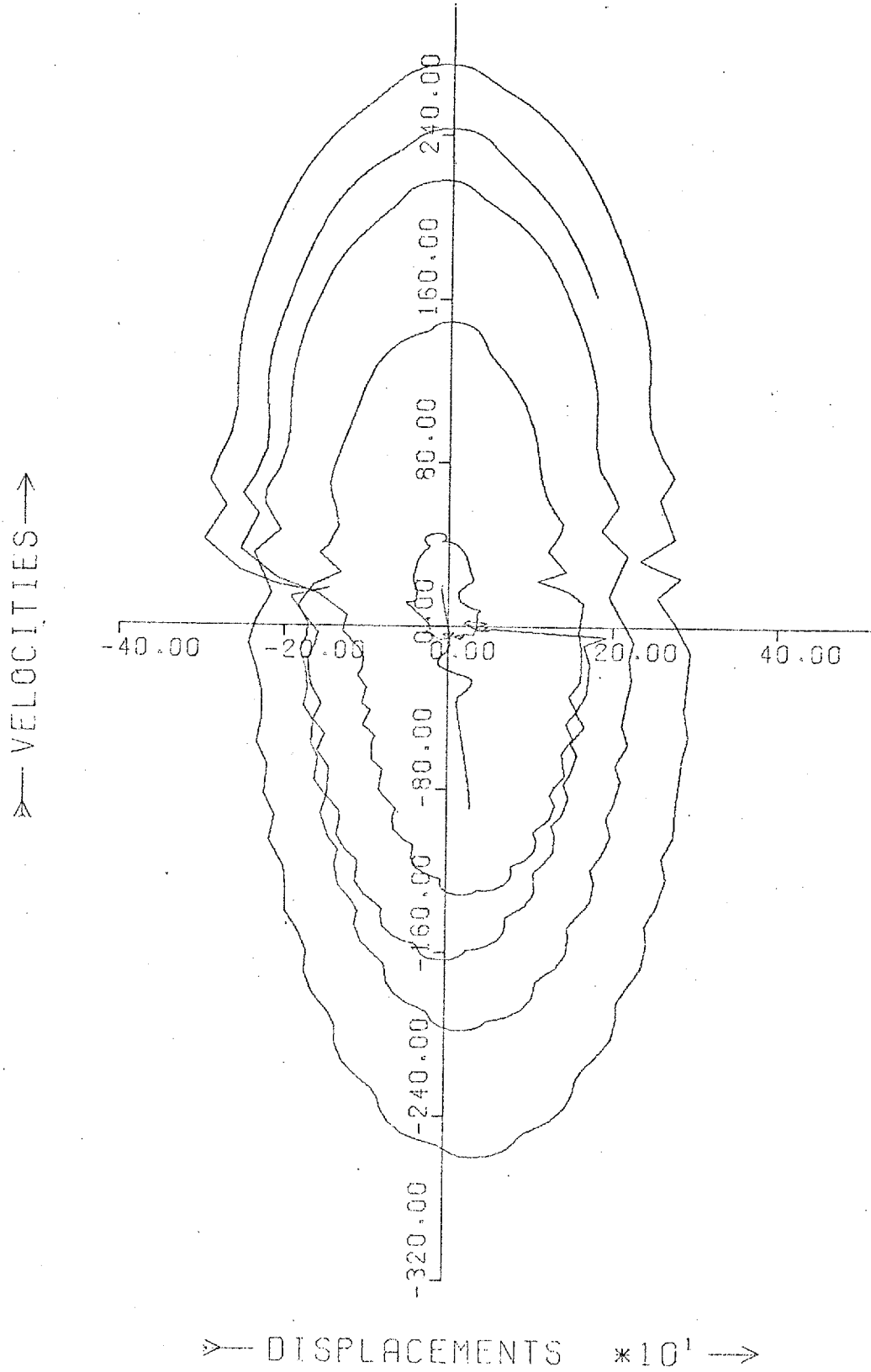


Fig. A9-7a

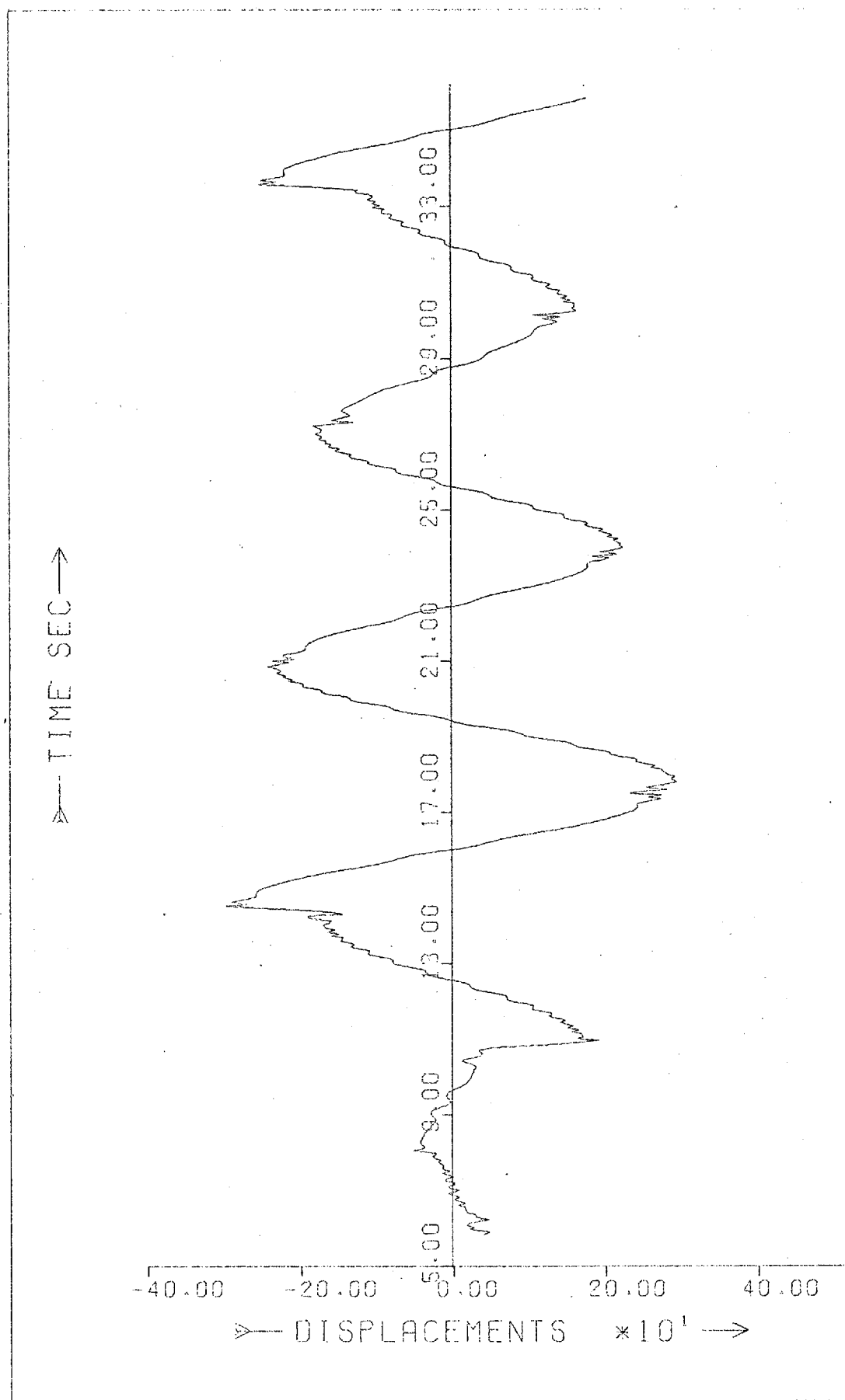


Fig. A9-7b

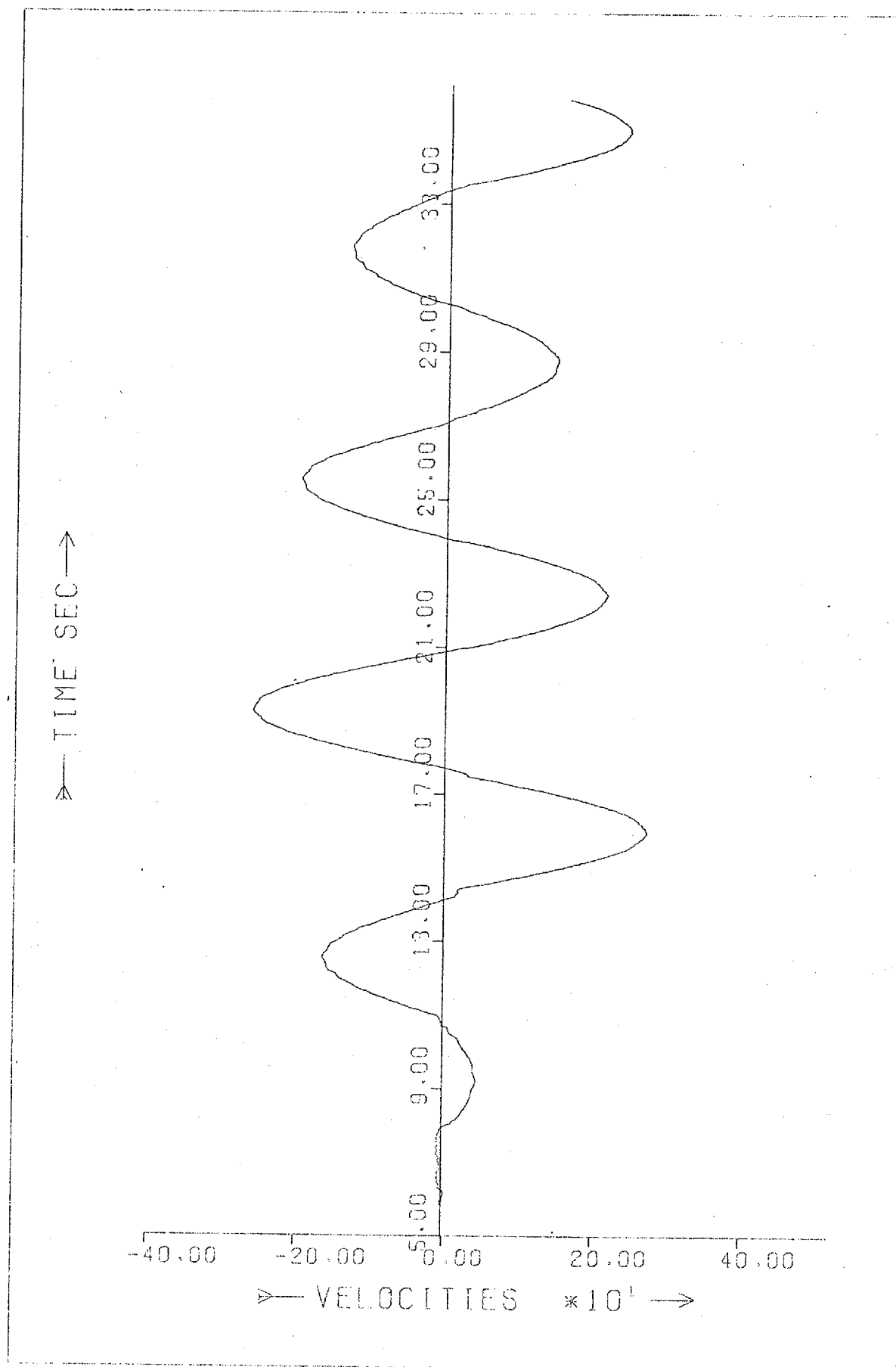


Fig. A9-7c

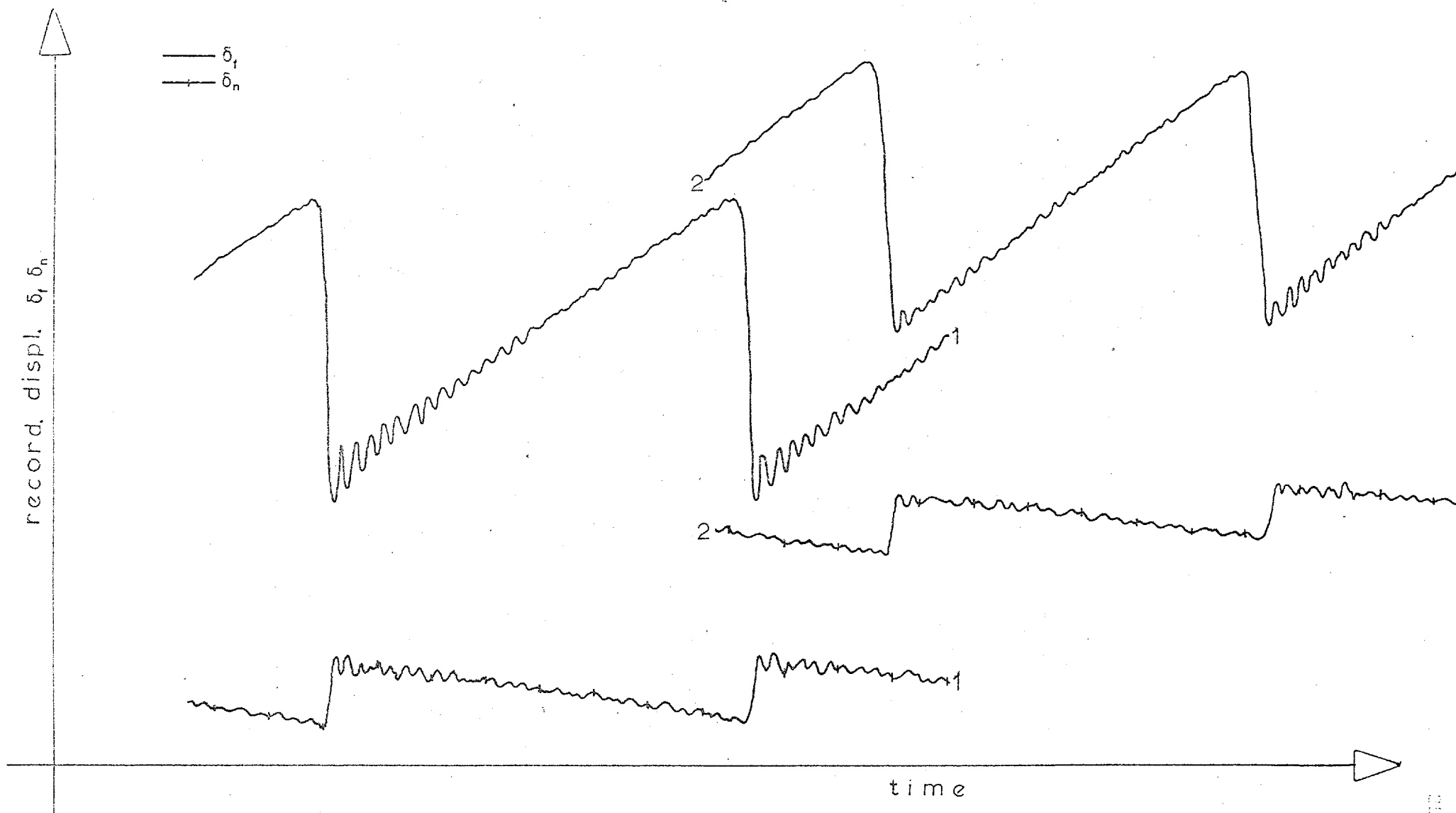


Fig. A9-8a

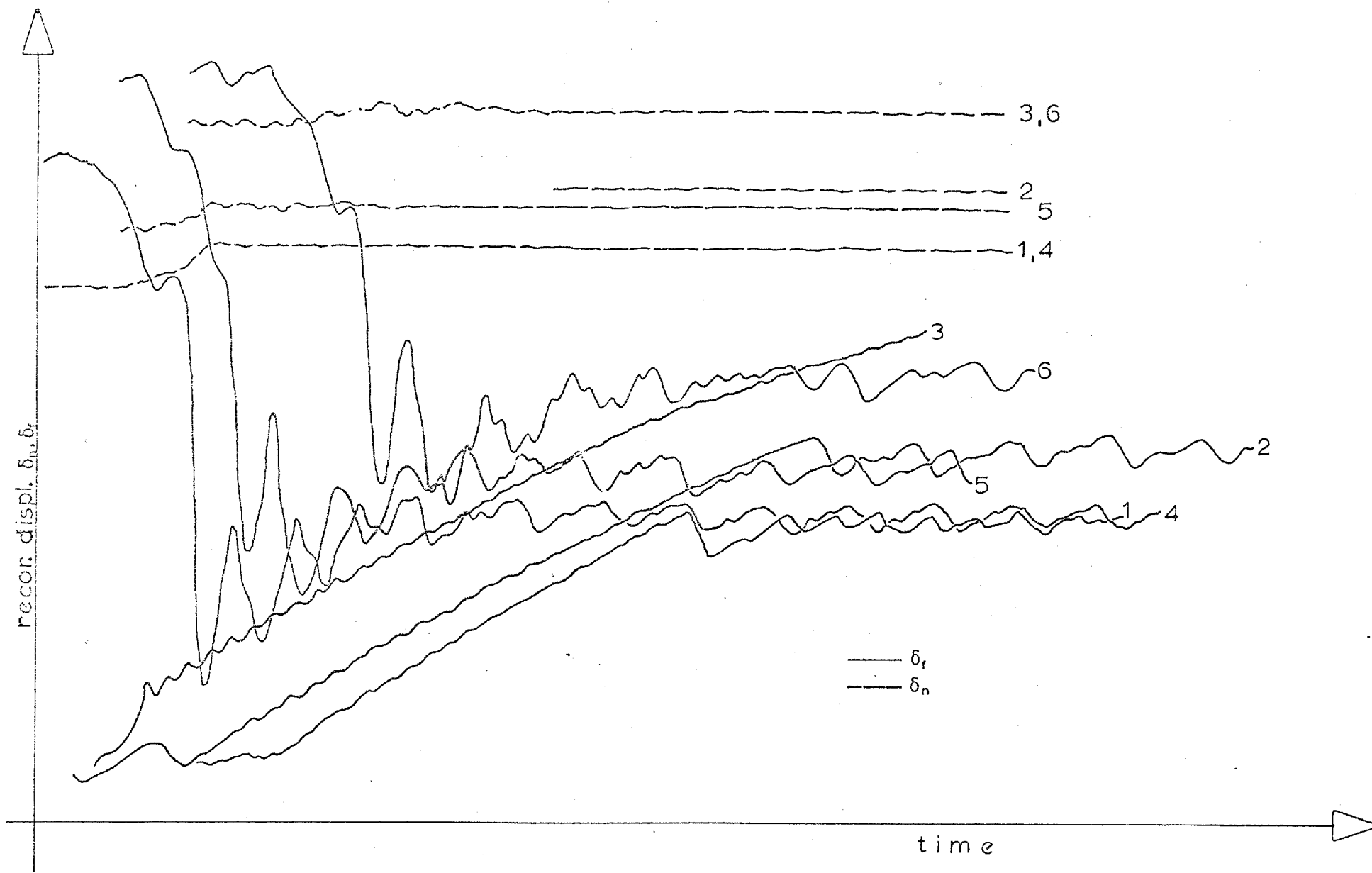


Fig. A9-8b

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