

**Evolution of velocity and scalar gradients in a
spatially developing turbulence**

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Declaration

I hereby declare that this thesis is the product of my own research only and no parts of it has been submitted for another degree. References or quotations from other works are fully acknowledged.

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Immanuvel Paul

Dedicated to the God, my Saviour

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Abstract

This thesis investigates three major topics for a spatially developing turbulence using direct numerical simulations. The subjects of this study are: (i) velocity gradients in fluid turbulence, (ii) temperature gradients in scalar turbulence and (iii) heat transfer from a cylinder due to free-stream turbulence. A square grid-element is used to generate fluid, scalar and free-stream turbulence. The square grid-element is a fundamental building block to both the classical and fractal grids.

The large-scale terms of fluid and scalar turbulence are generated mainly behind the bars of the grid-element due to the action of mean velocity and scalar gradients. On the other hand, the small-scale terms are generated behind the bar predominately by the stretching/compressing of fluctuating velocity and scalar gradient vectors by the turbulent strain-rate. These quantities are then transported to the grid-element centreline through the action of turbulent transport. The best defined $-5/3$ slope of the turbulent velocity and scalar spectra is observed close to the grid-element where the turbulence is highly inhomogeneous and the small-scale terms have just started developing. The universal characteristics of velocity and scalar gradients are observed in the far-downstream where the turbulence is approximately homogeneous and the values of Taylor-length-based Reynolds and Corrsin-length-based Peclet numbers are very low.

The inhomogeneous production region of the grid-element is observed to increase heat transfer from a cylinder substantially although its turbulent intensity is similar to that of the homogeneous decay region. It is noted that the turbulence in the production region is unique with its dominant frequency of fluctuations close to the shedding frequency and with the prevalence of azimuthal vortical structures which start increasing the heat transfer even before they get stretched by the accelerating boundary layer.

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Nomenclature

Greek symbols ---

α	Thermal diffusivity
γ	Intermittency factor
Γ_ϕ	Coefficient in scalar transport equation
$\Delta\phi$	Bandwidth of the selective frequency band
Δn^+	Distance between the surface and the first mesh point along the direction normal to the surface
Δr^+	Distance between the surface and the first mesh point along the radial direction
Δt	Time step of the simulation
Δz^+	Distance between the two mesh points along the spanwise direction
$\Delta_x, \Delta_y, \Delta_z$ and Δ	Local mesh resolutions
$\vec{\Delta}$	Vector linking two centroids on either side of a face
δ	Hydrodynamic boundary layer thickness
δ_T	Thermal boundary layer thickness
ϵ_k	Dissipation of turbulent kinetic energy
ϵ_v	Dissipation of scalar-variance
η	Kolmogorov length scale
η_{OC}	Obukhov-Corrsin length scale
θ	Angle on the surface of the cylinder
κ	Wave number
κ_{tc}	Thermal conductivity
Λ_i	Eigenvalues of the velocity gradient tensor (i=1,2,3)
λ	Taylor length scale
λ_i	Eigenvalues of the strain-rate tensor (i=1,2,3)
λ_T	Corrsin microscale
ν	Kinematic viscosity
ρ	Density of the fluid
ρ_τ	Probability of the timescales

τ	Timescale of the flow
τ_{min}	Smallest timescale of the data-set
τ_{vs}	Timescale of the vortex shedding
τ_2	Upper bound of the mean Nusselt number integral
$\tau_{2(opt)}$	Optimum upper bound of the mean Nusselt number integral
ϕ	A general scalar quantity
ϕ_λ	Normalised dominant frequency of turbulent fluctuations
Ω_i	Mean vorticity components (i=1,2,3)
Ω_{ij}	Anti-symmetric part of the velocity gradient tensor
ω_i	Fluctuating vorticity components (i=1,2,3)
$\omega_r^*, \omega_\theta^*, \omega_z^*$	Instantaneous radial, azimuthal and spanwise vorticity components

Roman symbols

A_{ij}	Velocity gradient tensor
a	Mean straining of the flow
C_D	Drag coefficient
$C_{\epsilon k}$	Kolmogorov constant
$C_{\epsilon T}$	Obukhov-Corrsin constant
C_f	Skin-friction coefficient
C_p	Specific heat
D	Diameter of the cylinder
E_{TT}	Temperature spectrum
E_{uu}	Velocity spectrum
e_i	Eigenvectors of the strain-rate tensor (i=1,2,3)
f_{cyc}	Weighting factor for the cyclic boundary faces
f_x	Weighting factor for the internal fluid faces
f_{xproc}	Weighting factor for the interprocessor boundary faces
f_y	Weighting factor for the outlet boundary faces
f_λ	Dominant frequency of fluctuations
f_{vs}	Frequency of vortex shedding
G_i	Fluctuating scalar-gradient vector components (i=1,2,3)
G_i^*	Instantaneous scalar-gradient vector components (i=1,2,3)
K	Acceleration parameter
k	Turbulent kinetic energy
$\vec{k}$	Non-orthogonal correction vector
L_{TT}	Integral length-scale of scalar turbulence
L_u	Integral length-scale of fluid turbulence

L_x, L_y and L_z	Length of the computational domain in the x, y and z directions
L_0	Length of the grid-element
$\dot{m}$	Mass flow rate
Nu	Instantaneous Nusselt number
$\langle Nu \rangle$	Mean Nusselt number
$\langle Nu \rangle_{max}$	Maximum mean Nusselt number along the circumference of the cylinder
$\langle Nu \rangle_{model}$	Mean Nusselt number obtained using timescale analysis
$\langle Nu \rangle_\tau$	Mean Nusselt number conditioned on the timescale of the flow
Nu_a	Modified Nusselt number
N_P	Number of processors in total
$N_{P_{ref}}$	Number of reference processors in total
N_x, N_y	Number of processors along the x and y directions
$\langle Nu \rangle_0$	Stagnation point mean Nusselt number
p^*	Instantaneous pressure
$\langle p^* \rangle$	Mean pressure
$P, Q,$ and R	First, second and third invariants of the velocity gradient tensor
p	Fluctuating pressure
$Pe_{\lambda T}$	Peclet number based on the Corrsin microscale
Pr	Prandtl number
p^c	Instantaneous correct-pressure
p^g	Instantaneous guessed-pressure
$Q_s,$ and R_s	First and second invariants of the strain-rate tensor
q_w	Prescribed heat applied on the surface
Q_ω	Single invariant of the rotation-rate tensor
r, θ, z	Polar coordinates
Re_D	Reynolds number based on the diameter of the cylinder
Re_{inc}	Reynolds number based on the incident velocity
Re_{L0}	Reynolds number based on the length of the grid-element
Re_{t0}	Reynolds number based on the width of the grid-element
Re_λ	Reynolds number based on the Taylor length scale
St	Strouhal number
St_{L_u}	Strouhal number based on the integral length scale
S_{ij}	Mean strain-rate tensor
s_{ij}	Fluctuating strain-rate tensor
T	Fluctuating temperature
T_{N_P}	Time taken by N_P number of processors
T_{ref}	Time taken by reference number of processors

$\langle T^* \rangle$	Mean temperature
t	Time coordinate
T^*	Instantaneous temperature
Tu	Turbulent intensity
Tu_a	Modified turbulent intensity
Tu_λ	Turbulence parameter
T_∞	Mean temperature at the free-stream
t_0	Width of the grid-element
U_{conv}	Convective velocity at the outlet
U_i	Mean velocity components (i=1,2,3)
U_{inc}	Incident velocity
U_r, U_θ, U_z	Mean radial, azimuthal and spanwise velocity components
U_∞	Mean velocity at the free-stream
u_i	Fluctuating velocity components (i=1,2,3)
u_i^*	Instantaneous velocity components (i=1,2,3)
u_i^c	Instantaneous correct-velocity components (i=1,2,3)
u_i^g	Instantaneous guessed-velocity components (i=1,2,3)
u_r^*, u_θ^*, u_z^*	Instantaneous radial, azimuthal and spanwise velocity components
u_{rms}	Root mean square of the velocity fluctuations
x, y, z	Cartesian coordinates
x_c	Streamwise distance with respect to the cylinder
x_g	Streamwise distance with respect to the grid-element
x^*	Wake-interaction lengthscale
$x^{*'} $	Modified wake-interaction lengthscale

Acronyms ---

AR	Aspect ratio of the grid-element
CFL	Courant Friedrichs Levy number
CMT	Conditional Mean Trajectories
DNS	Direct Numerical Simulation
FVM	Finite Volume Method
HIT	Homogeneous Isotropic Turbulence
JPDF	Joint Probability Density Function
KHMH	Karman Howarth Monin Hill
KOC	Kolmogorov Obukhov Corrsin
LES	Large Eddy Simulation
MPI	Message Passing Interface

PDF Probability Density Function
PETSc Portable Extensive Toolkit for Scientific computation
PISO Pressure Implicit Split Operator algorithm
TNT Turbulent-Non-Turbulent
VGT Velocity Gradient Tensor

Chapter 1

Introduction

1.1 Overview

Funded by the European Commission, the MULTISOLVE project is a combination of 13 PhD topics that investigate various aspects of multi-scale generated turbulence. The current study, which is part of this project, was oriented towards the turbulent flow and associated scalar fields generated by a grid-element, along with an evaluation of the role of grid-element turbulence in heat transfer enhancement.

This thesis considers three major topics of contemporary research for a spatially developing flow through the use of direct numerical simulation (DNS). These topics are: (i) velocity gradients in fluid turbulence, (ii) temperature gradients in scalar turbulence and (iii) the effect of free-stream turbulence on heat transfer from a cylinder. This chapter introduces these topics in sections 1.2, 1.3 and 1.4 respectively along with the relevant literature reviews. The objectives of each topic are also defined. These objectives are defined in accordance with the scope of MULTISOLVE project. In this study, fluid, scalar and free-stream turbulence are generated by a square grid-element which is a fundamental building block of fractal and regular grids. Section 1.5 provides the geometrical details of this grid-element and section 1.6 ends with an outline of the thesis.

Throughout the thesis, the instantaneous, mean and fluctuating velocity fields are denoted as u_i^* , U_i and u_i (where $i = 1, 2, 3$) respectively. The corresponding variables for pressure and temperature are p^* , $\langle p^* \rangle$, p and T^* , $\langle T^* \rangle$, T .

1.2 Fluid turbulence

Turbulence is the last unresolved problem in classical physics according to theoretical physicist Richard Feynman, a Nobel laureate in Physics. Turbulent flows are ubiquitous in nature; they can be seen, for example, in the stirring of water, in a smoking cigar and

numerous other flows. More importantly, most of the engineering applications such as flow around aircraft wings, transportation of oils etc involve turbulent flows. Although turbulent flows are widespread, they are difficult to analyse and understand. The standard textbooks of Tennekes and Lumley (1972) and Tsinober (2009) introduce several characteristics that define a turbulent flow. One of the most important properties of turbulent flow is that it consists of eddying motions of different length and temporal scales. The range of scales increases for increasing Reynolds number, which is defined as the ratio between the inertial forces to the viscous forces acting on the fluid element. The difficulty of studying turbulence arises from the fact that the eddying motions of different scales interact with each other through a non-linear process. In the next two subsections, the well-known phenomenological theory of turbulence and the importance of velocity gradients in turbulence are discussed.

1.2.1 Phenomenology of fluid turbulence

The basis of the phenomenological theory of turbulence is the concept of ‘energy cascade’, which was introduced first by Richardson (1926), and is visualised in figure 1.1. The turbulent kinetic energy ($k = \frac{1}{2} \langle u_i u_i \rangle$), which is produced due to the exchange of energy with the mean flow, enters into the cascade through large eddies which have a characteristic length scale called the integral length scale (L_u).

The large eddies are the energy containing eddies. They are, however, not stable. Due to a nonlinear process, they breakup and give rise to eddies of smaller size. These smaller eddies break up further to produce even smaller eddies. This breakdown process occurs until the size of eddies becomes small enough so that viscosity becomes important. During this breakdown process, the larger eddies transfer energy to the smaller eddies. The energy is finally dissipated at the smallest scale eddies due to viscosity. This process of energy transfer between the eddies of different length scales in turbulent flows is called the energy cascade.

Kolmogorov (1941) advanced the theory of the energy cascade to a phenomenological theory based on three hypotheses. His first hypothesis concerns the isotropy of the small scales. The large scale motions of size L_u are usually anisotropic and are affected by the boundary conditions and the size of the domain. As these large eddies transfer energy to the smaller eddies, the smaller eddies lose their directional basis and become isotropic at high Reynolds numbers. This hypothesis is often called ‘Kolmogorov’s hypothesis of local isotropy’.

Kolmogorov’s second and third hypotheses are based on similarity arguments. For very high Reynolds numbers, turbulent flows have a range of scales from the integral length scale to the smallest length scale. Since the Reynolds number associated with

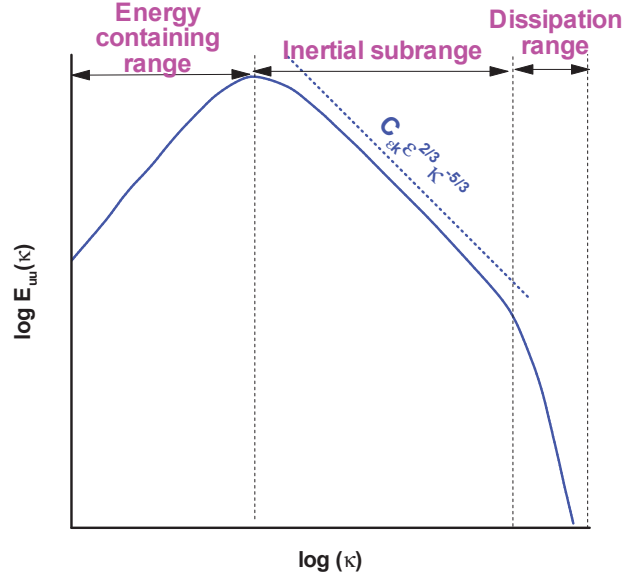


Figure 1.1: The Richardson-Kolmogorov energy cascade. The turbulent kinetic energy enters into the cascade through the small wavenumbers. The energy is transferred to smaller eddies in the inertial subrange until the eddies are so small that viscosity dissipates the energy received.

the smallest length scale is of order one, viscosity becomes important and the turbulent kinetic energy is dissipated. In his second hypothesis, Kolmogorov argued that at sufficiently high Reynolds numbers, the small scales of the flow have a universal form which depends entirely on viscosity (ν) and the dissipation of kinetic energy (ϵ_k). The range of scales where the turbulent kinetic energy is dissipated is known as the dissipation range (see figure 1.1). The turbulent kinetic energy dissipation is defined as

$$\epsilon_k = 2\nu \langle s_{ij}s_{ij} \rangle \quad (1.1)$$

where s_{ij} is the turbulent strain-rate defined as

$$s_{ij} = \frac{1}{2} \left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} \right) \quad (1.2)$$

Based on the second hypothesis, one can deduce, using dimensional arguments, the length scale of the smallest eddy which now bears his name (the Kolmogorov length scale) as,

$$\eta = \left(\frac{\nu^3}{\epsilon_k} \right)^{\frac{1}{4}} \quad (1.3)$$

Having computed the smallest length scale of a turbulent eddy, Kolmogorov's third hypothesis refers to the eddies that are larger than η , but much smaller than the integral length scale L_u . Since these eddies are larger compared to η , their Reynolds number is much higher than unity, therefore viscosity will not play a role. Kolmogorov's third hypothesis states that the eddies in the range of $L_u \gg l \gg \eta$ have a universal form and are dependent on the dissipation (ϵ_k) alone. The range of eddies $L_u \gg l \gg \eta$ is often called the inertial subrange (see figure 1.1). The dissipation range and the inertial subrange are collectively called the equilibrium range. It is important to stress here that all of Kolmogorov's hypotheses are applicable only for high Reynolds number, fully-developed turbulence.

At this point, it would be appropriate to define a length scale which is larger than the Kolmogorov scale but smaller than the integral length scale. Such a length scale is called the *Taylor* length scale (λ) and is defined as,

$$\lambda = \sqrt{\frac{5\nu\langle u_i u_i \rangle}{\epsilon_k}} \quad (1.4)$$

The Reynolds number based on the Taylor length scale is denoted as Re_λ and it is defined as,

$$Re_\lambda = \frac{\lambda}{\nu} \sqrt{\frac{\langle u_i u_i \rangle}{3}} \quad (1.5)$$

Since the inertial subrange depends only on a few parameters, it is possible to derive an energy spectrum that depends on wavenumber (κ) and ϵ_k . Using the dimensional analysis, the energy spectrum in the inertial subrange is shown to have the form:

$$E_{uu}(\kappa) = C_{\epsilon k} \epsilon_k^{2/3} \kappa^{-5/3} \quad (1.6)$$

The above equation is called the Kolmogorov spectrum and $C_{\epsilon k}$ is the Kolmogorov constant.

When written in physical space, the -5/3 power-law of equation (1.6) transforms to the famous Kolmogorov 2/3 scaling. Here, the second order structure function of velocity fluctuations scales as,

$$\langle \delta u^2(\Delta x) \rangle \propto \Delta x^{2/3} \quad (1.7)$$

where $\delta u(\Delta x) = u(x + \Delta x) - u(x)$ and Δx is the distance between two points in stream-wise direction.

1.2.2 Velocity gradients in fluid turbulence

The dynamics of velocity gradient tensor (VGT) $A_{ij} = \frac{\partial u_i}{\partial x_j}$ plays a vital role in fluid turbulence. The VGT dynamics is also believed to exhibit some universal characteristics due to its close association with the small scales of turbulence. The VGT can be decomposed into symmetric and antisymmetric parts. The symmetric part is the strain-rate tensor s_{ij} and the antisymmetric part is the rotation-rate tensor Ω_{ij} (which can be written in terms of vorticity components ω_k as $\Omega_{ij} = -\frac{1}{2}\epsilon_{ijk}\omega_k$, where $\omega_i = \epsilon_{ijk}\frac{\partial u_k}{\partial x_j}$). The interaction between strain and vorticity is considered as an intrinsic process of the very nature of turbulence (Tennekes and Lumley, 1972). Because the governing equations of strain-product ($s_{ij}s_{ij}$) and enstrophy ($\omega_i\omega_i$) involve terms due to production by straining, they intrinsically involve both strain and vorticity:

$$\frac{D\frac{1}{2}\omega_i^2}{Dt} = \omega_i s_{ij} \omega_j + \nu \nabla^2 \omega_i \quad (1.8)$$

$$\frac{D\frac{1}{2}s_{ij}s_{ij}}{Dt} = -s_{ij}s_{jk}s_{ki} - \frac{1}{4}\omega_i s_{ij} \omega_j - s_{ij} \frac{\partial^2 p}{\partial x_i \partial x_j} + \nu s_{ij} \nabla^2 s_{ij} \quad (1.9)$$

In equations (1.8) and (1.9), D/Dt represents the total (substantial) derivative which is given as $D/Dt = \partial/\partial t + u_i \partial/\partial x_i$. The enstrophy production term is $\omega_i s_{ij} \omega_j$. The production of enstrophy is due to the non-linear interaction between strain and vorticity. This term is present also in the governing dynamics of strain-product, but as a sink (refer to (1.9)). The term $-s_{ij}s_{jk}s_{ki}$ is the self-amplification of strain.

The large scale terms of fluid turbulence are characterised by the fluctuating velocity field, and the small-scale terms of fluid turbulence are characterised by the fluctuating velocity gradients (Tsinober (2009)). Therefore, in this thesis, turbulent kinetic energy ($\langle u_i u_i \rangle$) is considered as the large scale term. Mean enstrophy ($\langle \omega_i \omega_i \rangle$) and mean strain-product ($\langle s_{ij} s_{ij} \rangle$) of fluctuations are considered as the small-scale terms of fluid turbulence. The enstrophy production term $\omega_i s_{ij} \omega_j$ is often used as a good measure of non-linearity in turbulence (Tsinober, 2009). Besides, this term represents the direct interaction between strain and vorticity. Note that this term is absent for a two-dimensional flow. The enstrophy production term $\omega_i s_{ij} \omega_j$ can be written in terms of strain and vorticity as,

$$\omega_i \omega_j s_{ij} = \omega^2 \lambda_1 \cos^2(\boldsymbol{\omega}, \mathbf{e}_1) + \omega^2 \lambda_2 \cos^2(\boldsymbol{\omega}, \mathbf{e}_2) + \omega^2 \lambda_3 \cos^2(\boldsymbol{\omega}, \mathbf{e}_3) \quad (1.10)$$

where $\omega^2 = \omega_i \omega_i$, $\boldsymbol{\omega}$ is the vorticity vector, λ_i and $\mathbf{e}_i$ ($i=1,2,3$) are the strain-rate eigenvalues and eigenvectors respectively.

The strain-rate eigenvalues are ordered as $\lambda_1 > \lambda_2 > \lambda_3$. The incompressibility

constraint implies that $\lambda_1 + \lambda_2 + \lambda_3 = 0$. Hence, the largest strain-rate eigenvalue is positive (i.e. $\lambda_1 > 0$), and the third strain-rate eigenvalue is always negative (i.e. $\lambda_3 < 0$). The intermediate strain-rate eigenvalue can either be positive or negative. If λ_2 is positive, it results in two stretching and one compressing directions, which in turn leads to the preference for sheet-like structures. The case of λ_2 negative yields two compression and one stretching directions of strain-rate, and the flow tends to have more tube-like structures in an average sense. For many turbulent flows, the intermediate strain-rate eigenvalue has a positive preference on average, and this property is one of the universal properties of small-scale turbulence (Ashurst et al., 1987; Betchov, 1975; Gomes-Fernandes et al., 2014; Ganapathisubramani et al., 2008).

Another universal property of three-dimensional fluid turbulence is that the average enstrophy production term is always positive (i.e. $\langle \omega_i s_{ij} \omega_j \rangle > 0$) (Betchov, 1975; Taylor, 1938; Tsinober et al., 1992, 1995). From equation (1.10), it is evident that the positiveness of $\langle \omega_i s_{ij} \omega_j \rangle$ should be associated with the positive strain-rate eigenvalues. Since the positive strain-rate eigenvalues represent stretching of the fluid element, the term $\langle \omega_i s_{ij} \omega_j \rangle > 0$ indicates that the vortex stretching is preferred more than the vortex compression in a homogeneous isotropic turbulence (HIT) (Betchov, 1975; Taylor, 1938; Tsinober et al., 1992, 1995).

The self-amplification of strain can be written in terms of the strain-rate eigenvalues as,

$$-s_{ij}s_{jk}s_{ki} = -(\lambda_1^3 + \lambda_2^3 + \lambda_3^3) \quad (1.11)$$

Equation (1.11) shows that the statistics of strain self-amplification is affected by the strain-rate eigenvalues. It also shows that the positive value of $-s_{ij}s_{jk}s_{ki}$ is due to negative strain-rate eigenvalues. This means that the strain self-amplification, which in turn the dissipation of turbulent kinetic energy, is preferentially associated with compressing of the fluid element. Moreover, the average value of $-s_{ij}s_{jk}s_{ki}$ is positive (i.e. $-\langle s_{ij}s_{jk}s_{ki} \rangle > 0$) which is also considered as an universal characteristics of small-scale turbulence (Ashurst et al., 1987; Betchov, 1975; Gomes-Fernandes et al., 2014; Ganapathisubramani et al., 2008).

One of the important parameters that influences the interaction between strain and vorticity is the alignment between the vorticity vector and the strain-rate eigenvectors (refer to equation (1.10)). Kerr (1985) and Ashurst et al. (1987) were the first to compute these alignments for a HIT case. It was noted that the vorticity vector aligns strongly with the intermediate eigenvector ($\mathbf{e}_2$), while the alignment between the extensive strain-rate eigenvector and the vorticity vector is weak. On the other hand, the vorticity vector

aligns perpendicular to the compressive strain-rate eigenvector. These observations have been verified in various studies (Vincent and Meneguzzi, 1994; Tanahashi et al., 2001; Buxton and Ganapathisubramani, 2010; Hamlington et al., 2008b; Jiménez, 1992), and they are considered as one of the universal features of small-scale turbulence.

More insight into the interaction between the strain and vorticity can be gained through the invariants of the VGT. The characteristic equation of the velocity gradient tensor is,

$$\Lambda_i^3 + P\Lambda_i^2 + Q\Lambda_i + R = 0 \quad (1.12)$$

where Λ_i ($i = 1, 2, 3$) are the three eigenvalues and P, Q, R are the first, second and third invariants of the VGT respectively. For an incompressible flow, the first invariant of the VGT is zero ($P=0$). The second and third invariants of the VGT are,

$$Q = \frac{1}{4}(\omega_i\omega_i - 2s_{ij}s_{ij}) \quad (1.13)$$

$$R = -\frac{1}{3}(s_{ij}s_{jk}s_{ki} + \frac{3}{4}\omega_i\omega_j s_{ij}). \quad (1.14)$$

The discriminant of the cubic equation (1.12) for incompressible flow is

$$D = \frac{27}{4}R^2 + Q^3 \quad (1.15)$$

and if it is negative, all three VGT eigenvalues are real and distinct. The invariants of the strain-rate tensor are obtained by setting $\omega_i=0$ in equations (1.13) and (1.14), and they are

$$Q_s = -\frac{1}{2}s_{ij}s_{ij}, \quad (1.16)$$

$$R_s = -\frac{1}{3}s_{ij}s_{jk}s_{ki}. \quad (1.17)$$

Finally, the sole invariant of the rotation-rate tensor is acquired by putting $s_{ij}=0$ in equation (1.13), and it is,

$$Q_w = \frac{1}{4}\omega_i\omega_i. \quad (1.18)$$

For HIT, the space-averaged second and third VGT invariants are zero (i.e. $\langle Q \rangle=0$, $\langle R \rangle=0$). As a result, for homogeneous isotropic turbulence,

$$\langle \omega_i\omega_i \rangle = 2\langle s_{ij}s_{ij} \rangle \quad (1.19)$$

$$\langle \omega_i\omega_j s_{ij} \rangle = \frac{4}{3}\langle s_{ij}s_{jk}s_{ki} \rangle \quad (1.20)$$

The study of the invariants of the velocity gradient, strain- and rotation-rate tensors often reveal information regarding the local topology of the flow field. Furthermore, they also contain information regarding vortex stretching and strain self-amplification. The $Q - R$ and $Q_s - R_s$ diagrams are widely used to study the topology of turbulence, and they are proved to have a universal shape (Chong et al., 1990; Tsinober, 2009).

A typical $Q - R$ diagram has four distinct sectors. To interpret the $Q - R$ diagrams, one needs to rewrite equation (1.14) as

$$3R = -s_{ij}s_{jk}s_{ki} - \frac{1}{4}\omega_i\omega_j s_{ij} - \frac{1}{2}\omega_i\omega_j s_{ij} \quad (1.21)$$

This reveals that $3R$ is the difference between the two production terms: the term $-s_{ij}s_{jk}s_{ki} - \frac{1}{4}\omega_i\omega_j s_{ij}$ (which appears in the equation for $\frac{1}{2}s_{ij}s_{ij}$, equation (1.9)) and the term $\frac{1}{2}\omega_i\omega_j s_{ij}$ (which appears in the equation for $\frac{1}{4}\omega_i\omega_i$, i.e. equation (1.8) multiplied by $\frac{1}{2}$ on both sides). The reason for considering the production of $\frac{1}{4}\omega_i\omega_i$ rather than $\frac{1}{2}\omega_i\omega_i$ lies in the fact that $Q = \frac{1}{4}\omega_i\omega_i - \frac{1}{2}s_{ij}s_{ij}$. Hence in a Q - R plot, the upper left quadrant (where $Q > 0$ and $R < 0$) represents events in which $\frac{1}{4}\omega_i\omega_i$ dominates over $\frac{1}{2}s_{ij}s_{ij}$ and the production of $\frac{1}{4}\omega_i\omega_i$ also dominates over the production of $\frac{1}{2}s_{ij}s_{ij}$. In the upper right quadrant (where $Q > 0$ and $R > 0$) $\frac{1}{4}\omega_i\omega_i$ dominates over $\frac{1}{2}s_{ij}s_{ij}$ but the production of $\frac{1}{2}s_{ij}s_{ij}$ dominates over the production of $\frac{1}{4}\omega_i\omega_i$. The lower left quadrant (where $Q < 0$ and $R < 0$) represents events in which $\frac{1}{2}s_{ij}s_{ij}$ dominates over $\frac{1}{4}\omega_i\omega_i$ and where the production of $\frac{1}{2}s_{ij}s_{ij}$ also dominates over the production of $\frac{1}{4}\omega_i\omega_i$; and finally, the lower right quadrant (where $Q < 0$ and $R > 0$) includes events where $\frac{1}{2}s_{ij}s_{ij}$ dominates over $\frac{1}{4}\omega_i\omega_i$, but the production of $\frac{1}{4}\omega_i\omega_i$ dominates over the production of $\frac{1}{2}s_{ij}s_{ij}$. In many turbulent flows the $Q - R$ diagram has a tear-drop shape as shown in figure 1.2a (Chong et al., 1990; Tsinober, 2009). This shape reflects strong correlations between $Q < 0$ and $R > 0$ on one hand and between $Q > 0$ and $R < 0$ on the other.

Lüthi et al. (2009) extended the two-dimensional $Q - R$ diagram analysis to a three-dimensional joint pdf in $\{Q; -s_{ij}s_{jk}s_{ki}; \omega_i s_{ij} \omega_j\}$ space by decomposing the third invariant of the VGT (R) into strain self-amplification and enstrophy production. They found that the $\{Q; -s_{ij}s_{jk}s_{ki}; \omega_i s_{ij} \omega_j\}$ diagram sheds more light on the dynamics of VGT tensor, in particular with respect to the role of pressure Hessian in small-scale dynamics.

The next VGT-based diagram is the $Q_s - R_s$ diagram. To interpret this diagram, note that R_s can be written in terms of λ_i as $R_s = -\lambda_1\lambda_2\lambda_3$. One might therefore expect some tendency for sheet-like vortex structures if $R_s > 0$ (two extensive strain-rate eigenvalues) and some tendency for tube-like vortex structures if $R_s < 0$ (two compressive strain-rate eigenvalues).

Defining $a = \lambda_2/\lambda_1$, the following equation can be obtained (Chong et al., 1990; Ooi

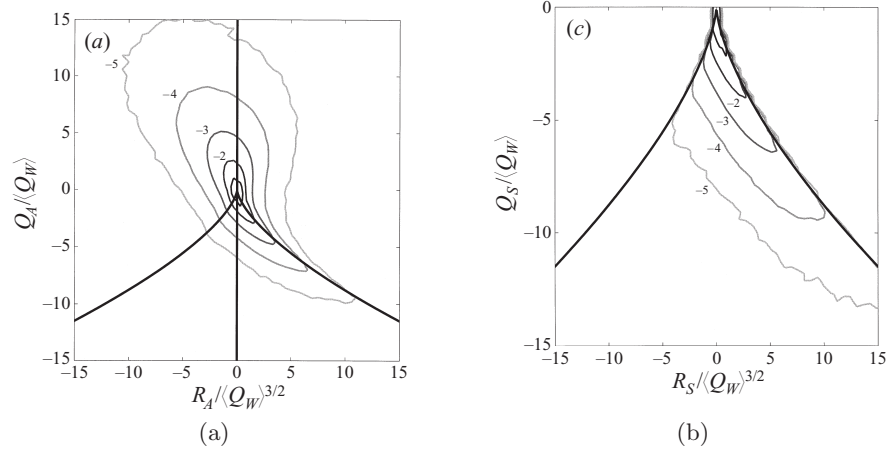


Figure 1.2: (a) Universal shape of the $Q-R$ diagram. (b) Universal shape of the Q_s-R_s diagram. These figures are adopted from Ooi et al. (1999).

et al., 1999):

$$R_s = (-Q_s)^{\frac{3}{2}} a(1+a)(1+a+a^2)^{-\frac{3}{2}} \quad (1.22)$$

The flow topology can be defined in terms of the ratios of the strain-rate eigenvalues. The ratio $\lambda_1 : \lambda_2 : \lambda_3 = 2 : -1 : -1$ ($a = -1/2$) corresponds to axisymmetric contraction, $1 : 0 : -1$ ($a = 0$) is for a two-dimensional flow, $3 : 1 : -4$ ($a = 1/3$) represents biaxial stretching and $1 : 1 : -2$ ($a = 1$) is for axial stretching (da Silva and Pereira, 2008). The universal shape of the Q_s-R_s reported for HIT is presented in figure 1.2b (adopted from the paper of Ooi et al. (1999)). Large probability occurs for $R_s > 0$ which reflects the fact that sheet-like structures are more preferred in HIT.

1.2.3 Literature on velocity gradients in spatially developing flows

The importance of VGT dynamics in turbulent flows has been fully recognised since the works of Taylor (1938) and Betchov (1956). These authors established the importance of vortex stretching and strain self-amplification in relation to the turbulence cascade (see Tsinober (2009)). Betchov (1956) showed that the energy cascade requires vortex stretching and a weakly extensive intermediate strain-rate eigenvalue. Also, the final result of the energy cascade (i.e. energy dissipation) is related to strain (Tsinober, 2009). Although these works were limited to homogeneous isotropic turbulence, they are still relevant for a wider range of turbulent flows that are often only locally homogeneous with isotropic small scales.

Earlier studies on the VGT dynamics, as reviewed by Sreenivasan and Antonia

(1997), predominantly focused on analysing the statistical properties such as flatness and skewness of velocity derivatives. Studies that considered the interaction between strain and vorticity were either fixed point measurements downstream of a grid/cylinder/jet or periodic box simulations (Gulitski et al., 2007a; Tsinober et al., 1992, 1995; Tsinober, 2000; Kholmyansky et al., 2001; Kerr, 1987; Ashurst et al., 1987; Buxton and Ganapathisubramani, 2010; Ganapathisubramani et al., 2008; Vincent and Meneguzzi, 1994; Tanahashi et al., 2001).

Only in the past few years, have a number of authors started addressing the question of the spatial evolution of VGT statistics and dynamics in grid-generated turbulence (Laizet et al., 2013; Gomes-Fernandes et al., 2014; Zhou et al., 2014a, 2015, 2016a,b). The question is, of course, wider and should eventually be addressed in various spatially developing turbulent flows. But grid-generated turbulence (like other turbulent flows) has two regions: a production region nearest to the grid where the turbulence intensity builds up until it reaches a peak value at a certain distance from the grid (Simmons and Salter, 1934; Warhaft and Jayesh, 1992; Hurst and Vassilicos, 2007; Weitemeyer et al., 2013); and an immediate adjacent downstream region where the turbulence decays. The question is to study in which region and where within the region different VGT properties of the turbulence originate from and how they might correlate with statistics traditionally attributed to a turbulence cascade, such as the celebrated $-5/3$ power-law range of the turbulent energy spectrum.

Experimentally, classical regular grids such as those used by Corrsin and his collaborators (Comte-Bellot and Corrsin, 1966, 1971) are very difficult to use for this purpose as they are designed in such a way that the production region is very short. The first and to date the only experimental investigation on the subject (Gomes-Fernandes et al., 2014) was therefore done with space-filling fractal square grids where the production region is greatly magnified and therefore more easily accessible. The first Direct Numerical Simulations (DNS) on the subject (Laizet et al., 2013) was also carried out for space-filling fractal square grids and they showed how the well-known universal tear-drop shape of the Q-R diagram (see Tsinober (2009)) is absent in the grid-centreline region where, however, the $2/3$ power law for the second order structure function of the fluctuating velocity is present.

A series of DNS studies followed from Nagoya University in Japan (Zhou et al., 2014a,b, 2015, 2016a,b) where the fractal square grid was replaced by a single square element of the fractal grid which resembles a hollow square plate. By replacing the fractal grid with its square element, the production region remains magnified, while at the same time isolating the effect of only one of the features of the fractal grid, the largest square pattern. These studies confirmed the appearance of the $2/3$ power law in the second order

structure function in the very near field of the grid-element centreline. Other well-known universal properties of the VGT, such as the tear-drop shape of the Q-R diagram and the alignments between the vorticity vector and the strain-rate eigenvectors (Ashurst et al., 1987; Tsinober, 2009) did also appear until further downstream. They also showed that the pressure Hessian at the centreline is dominant very close to the grid-element, closer than the point where the 2/3 power law first appears as one moves in the streamwise direction.

A particular observation made by Laizet et al. (2013) is that the local Reynolds number based on the Taylor length scale (Re_λ) can be relatively small in the spatial location where the 2/3 power law first appears. The studies of Zhou et al. (2014a,b, 2015, 2016a,b) showed that the well-known universal turbulence properties of the VGT (Q-R diagrams, VGT alignments, etc) are present in the decay region even for values of Re_λ as small as 50 or so. An unrelated study by Schumacher et al. (2014) which was concerned with periodic turbulence, turbulent shear flow between two parallel walls and thermal convection in a closed container, also identified that some universal turbulence properties of velocity gradients are present even at relatively modest Reynolds numbers.

1.2.4 Objectives of velocity gradients analysis

This thesis deals with a spatially developing flow generated by a single square-element. The inlet Reynolds number is kept deliberately low in order to ensure very good DNS resolution. The velocity gradient analysis presented in chapter 3 aims to answer the following questions.

1. The high Reynolds number experimental studies of Laizet et al. (2013, 2015b); Gomes-Fernandes et al. (2015) for turbulence generated by a fractal grid have shown that the -5/3 power-law slope appears even in the near-grid inhomogeneous part of the grid-centreline region. Does this observation also hold true for low Reynolds number turbulence generated by a square grid-element?
2. If there is a -5/3 power-law slope in the energy spectrum for the near grid-element region, how do the small-scale terms like strain and vorticity correlate with the appearance of the -5/3 power-law slope?
3. How are the terms related to the large and small scales of fluid turbulence generated and transported in this spatially developing flow?
4. How does the velocity gradient dynamics of grid-element differ from the VGT dynamics reported for a fractal square grid in Gomes-Fernandes et al. (2014)?.

5. The universal characteristics of VGT dynamics discussed in section 1.2.2 are largely reported for very high values of Re_λ . Do these universal characteristics really require high Reynolds number?

Besides these questions, this thesis also adds to the literature on the subject of VGT dynamics some hitherto undocumented strain-rate and geometrical statistics that are not reported in the grid-element DNS studies of Zhou et al. (2014a,b, 2015, 2016a,b).

1.3 Passive scalar turbulence

The next major topic discussed in this thesis is passive scalar turbulence. A passive scalar is generally transported by the flow but does not react back, i.e. does not influence the carrier flow. Examples include small variations of temperature, pollutant concentration etc. Understanding the statistical behaviour of scalar fluctuations and their gradients is important both from the application as well as the fundamental points of view. Many industrial processes (for instance heat exchange, mixing, combustion) involve the transport of scalar by a turbulent flow. Similarly, understanding of micro-mixing, and thus arriving at an effective mixing model, requires knowledge of the fluctuating scalar gradients (Gulitski et al., 2007a). The study of passive scalar is also fundamental to the understanding of turbulence itself. Numerous studies on various aspects of scalar turbulence have appeared in the literature and are reviewed in Sreenivasan (1991); Sreenivasan and Antonia (1997); Majda and Kramer (1999); Shraiman and Siggia (2000); Warhaft (2000); Falkovich et al. (2001); Dimotakis (2005); Gotoh and Yeung (2013). The phenomenology of passive scalar turbulence and the role of scalar gradients in scalar turbulence is discussed in the next two sub-sections.

1.3.1 Phenomenology of passive scalar turbulence

The basis for the phenomenological theory of scalar turbulence is the scalar cascade. The idea of energy cascade introduced by Richardson (1926) for fluid turbulence can easily be extended to scalar turbulence. In the scalar cascade, as represented in figure 1.3, the scalar variance ($\langle T^2 \rangle$) enters into the cascade through the large eddies. A characteristic size of the large eddy is the scalar integral length scale (L_{TT}). These large eddies are not stable, but they break up into smaller eddies. During the breakup process of larger eddies, the scalar-variance is transferred to smaller and smaller eddies until the thermal diffusivity (α) becomes important. The scalar-variance is dissipated by diffusivity at the smallest scales of passive scalar turbulence. The range of eddy sizes in the scalar turbulence is affected by the Peclet number (besides the Reynolds number). The Peclet number is the ratio between the convective and the diffusive transports of the scalar.

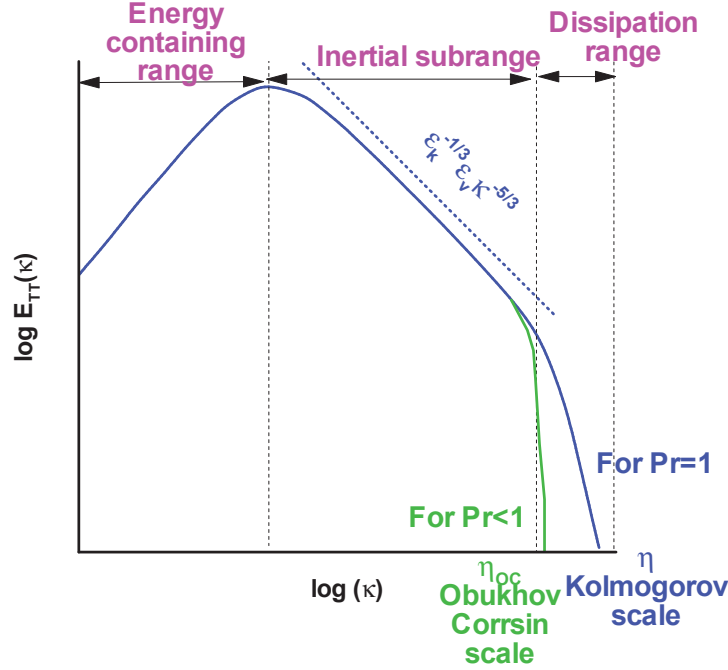


Figure 1.3: A schematic diagram of the spectrum of scalar fluctuations. The scalar variance enters into the scalar field at small wavenumbers, and it is transferred to higher wavenumber before it eventually gets dissipated.

The Peclet number is related to the Reynolds number as $Pe = Re \times Pr$ where Pr is the Prandtl number, defined as the ratio between the kinematic viscosity and the thermal diffusivity (i.e. $Pr = \nu/\alpha$).

The phenomenology of scalar turbulence is based on the similarity theory of Kolmogorov (1941) introduced for the fluid turbulence in section 1.2.1. This theory has been extended to the scalar field by Obukhov (1949) and Corrsin (1951), and it is known as *Kolmogorov – Obukhov – Corrsin* theory (KOC theory). First, the local isotropy hypothesis of Kolmogorov (1941) is applied to the scalar field. It postulates that, at very high Reynolds and Peclet numbers, the small scales of scalar turbulence become isotropic. The second hypothesis of Kolmogorov (1941) is also extended to the scalar. It states that, at very high Reynolds and Peclet numbers, the small scales of scalar have a uniform form which is characterised only by the thermal diffusivity (α) and the dissipation of scalar-variance (ϵ_v). The scalar-variance dissipation is defined as,

$$\epsilon_v = \alpha \left\langle \frac{\partial T}{\partial x_j} \frac{\partial T}{\partial x_j} \right\rangle \quad (1.23)$$

KOC theory obtained the smallest scale of the scalar field from the second hypothesis,

and this length scale is called the Obukhov-Corrsin scale (η_{OC}). It is defined as,

$$\eta_{OC} = \left(\frac{\alpha^3}{\epsilon_k} \right)^{\frac{1}{4}} \quad (1.24)$$

From equation (1.24), the relationship between the smallest scales of fluid and scalar turbulence can be obtained as,

$$\eta_{OC} = \eta Pr^{-3/4} \quad (1.25)$$

This thesis considers the case of $Pr < 1$. From equation (1.25), it is evident that the smallest scale of the scalar field will be larger than that of the flow field when $Pr < 1$. This observation is important while assessing the mesh resolution for scalar gradient analysis.

Extension of Kolmogorov's third similarity hypothesis to the scalar field refers to the length scales that are larger than η_{OC} but smaller than L_{TT} and have a universal form that depends on only ϵ_v and ϵ_k . In this range, neither viscosity nor diffusivity are important, and this range is called the inertial subrange (see figure 1.2).

The equivalent to the Taylor length scale in scalar turbulence is the Corrsin micro-scale (λ_T) which is larger than η_{OC} but smaller than L_{TT} . It is defined as

$$\lambda_T = \sqrt{\frac{6\alpha\langle T^2 \rangle}{\epsilon_v}} \quad (1.26)$$

And the equivalent of Re_λ in scalar turbulence is the Peclet number based on the Corrsin scale ($Pe_{\lambda T}$) which is defined as,

$$Pe_{\lambda T} = \frac{\lambda_T}{\alpha} \sqrt{\frac{\langle u_i u_i \rangle}{3}} \quad (1.27)$$

Since the inertial subrange depends only on ϵ_k and ϵ_v , dimensional analysis can be used to obtain a form of scalar spectrum in the inertial subrange that depends on ϵ_k and ϵ_v . The one-dimensional scalar spectrum (E_{TT}) in the inertial subrange has the form

$$E_{TT}(\kappa) = C_{\epsilon T} \epsilon_k^{-1/3} \epsilon_v \kappa^{-5/3} \quad (1.28)$$

where $C_{\epsilon T}$ is the Obukhov-Corrsin constant. Note that the scalar spectrum given in equation (1.28) is applicable only for $Pr < 1$.

It is important to recall here again that the hypotheses of Kolmogorov and that of Obukhov and Corrsin are applicable only to fully-developed homogeneous isotropic turbulence which has very high values of Re_λ and $Pe_{\lambda T}$.

1.3.2 Scalar gradients in passive scalar turbulence

Scalar gradients play a vital role in the dynamics of passive scalar turbulence, similar to velocity gradients (such as the enstrophy and strain) in fluid turbulence. Moreover, the dissipation of scalar-variance (refer to equation (1.23)) involves scalar gradients. The dynamics of scalar gradients is influenced by the velocity gradients as can be seen from the governing equation of scalar gradient squared $G_i G_i$ where $G_i = \frac{\partial T}{\partial x_i}$.

$$\frac{D \frac{1}{2} G_i G_i}{Dt} = -G_i s_{ij} G_j + \alpha G_i \nabla^2 G_i \quad (1.29)$$

Equation (1.29) shows that only the symmetric part of the VGT (i.e. s_{ij}) has a coupling with the scalar gradients, and this is one way, which means that only the velocity gradients influence the scalar gradients, but not vice-versa.

It is important to examine the one-way coupling between strain and scalar gradients in passive scalar turbulence. The scalar gradient production term can be written as,

$$-G_i s_{ij} G_j = -|\mathbf{G}|^2 \lambda_1 \cos^2(\mathbf{G}, \mathbf{e}_1) - |\mathbf{G}|^2 \lambda_2 \cos^2(\mathbf{G}, \mathbf{e}_2) - |\mathbf{G}|^2 \lambda_3 \cos^2(\mathbf{G}, \mathbf{e}_3) \quad (1.30)$$

where $\mathbf{G}$ is the scalar gradient vector, and $\mathbf{e}_i$ (where $i=1,2,3$) is the strain-rate eigenvector.

Equation (1.30) indicates that the interaction between strain and scalar gradients should be studied through the eigen-contribution of the strain-rate tensor to the scalar gradient production. This includes the alignment of the scalar gradient vector with the strain-rate eigenvector, as well as the relationship between the strain-rate eigenvalues with the scalar gradient production.

The properties of the strain-rate eigenvalues (λ_i , $i=1,2,3$) for HIT have been discussed in section 1.2.2. The other important factor that affects the scalar gradient production is the alignment between the scalar gradient vector and the strain-rate eigenvectors. These alignments were first reported by Ashurst et al. (1987) for a periodic homogeneous isotropic turbulence. It was observed that the fluctuating scalar gradient vector aligns significantly with the compressive strain-rate eigenvector while it aligns perpendicular to the intermediate strain-rate eigenvector. The extensive strain-rate eigenvector aligns 45° to the fluctuating scalar gradient vector. These observations were also verified in the subsequent studies of Vedula et al. (2001); Gulitski et al. (2007a); Abe et al. (2009). These alignment properties are considered as one of the universal characteristics of small-scale scalar turbulence.

Another universal feature of the small-scale scalar turbulence is the positive average scalar gradient production (i.e. $-\langle G_i s_{ij} G_j \rangle > 0$). This was first observed in the numerical

simulation of Ruetsch and Maxey (1991). Later, this observation was verified in various HIT studies (Ruetsch and Maxey, 1992; Vedula et al., 2001; Brethouwer et al., 2003).

1.3.3 Literature on scalar gradients in developing flows

The dynamics of the large and small scales of passive scalar has been explored theoretically (Obukhov, 1949; Corrsin, 1951), experimentally (Warhaft and Lumley, 1978; Sreenivasan and Antonia, 1977; Sreenivasan et al., 1979; Antonia and Van Atta, 1975; Gulitski et al., 2007a), and numerically (Ashurst et al., 1987; Abe et al., 2009; Abe and Antonia, 2011; Pumir, 1994; Holzer and Siggia, 1994; Vedula et al., 2001) for various flow configurations. Some of these configurations have shear (for example jets, wakes, etc) and some are shear-free (grid turbulence). In most shear-free flows, a mean scalar gradient is imposed at the inlet (Sreenivasan et al., 1980; Tong and Warhaft, 1994; Budwig et al., 1985; Mydlarski and Warhaft, 1998; Jayesh and Warhaft, 1994), but there are also studies where the grid was heated (Yeh and Atta, 1973; Warhaft and Lumley, 1978; Antonia et al., 2013). It is important to stress here that almost all of the previous studies on shear-free scalar turbulence are either experiments measured at a far-downstream region of a grid or periodic box direct numerical simulations with an imposed mean scalar gradient. Thus, they have mostly analysed the scalar properties of homogeneous isotropic turbulence (HIT). It is, however, not clear if the high Reynolds and Peclet numbers scalar results obtained for HIT can also be seen in low Reynolds and Peclet numbers scalar turbulence.

As discussed for the literature on VGT dynamics (section 1.2.3), the turbulence generated by fractal grids has an extended turbulent production region followed by a decay region. The production region is characterised by increasing turbulent intensity, which decays further downstream. Even though the decay region is close to homogeneous and Gaussian, the dissipation coefficient is not constant. This peculiar scaling of dissipation is reviewed by Vassilicos (2015). Recently, Valente and Vassilicos (2012) and Zhou et al. (2014a, 2016a) have shown that the turbulence generated by a single square grid-element shares similarities with that generated by a fractal square grid. Previous works on scalar analysis behind a fractal grid (Suzuki et al., 2010a,b; Laizet and Vassilicos, 2015; Ito et al., 2016; Watanabe et al., 2016) have focused on the mechanism that results in increased turbulent fluxes in the cross-streamwise direction observed in the lee of the grid. To the best of the author’s knowledge, there are no comprehensive studies in the literature on the dynamical evolution of scalar-variance and scalar gradient including its alignment with the strain-rate eigenvectors, in turbulence generated by a fractal grid or a single square grid-element. These observations provide the motivation to study scalar fields in relatively low Reynolds and Peclet numbers inhomogeneous developing flows.

1.3.4 Objectives of scalar gradients analysis

The central aim of the scalar gradient analysis is to study the statistical behaviour of a passive scalar injected into a spatially developing turbulence using direct numerical simulation. The turbulence is generated by a single square grid-element which is heated continuously, and the passive scalar is temperature. This study is complementary to the study on the velocity gradients for turbulence generated by the same geometry. The scalar gradient analysis, as presented in chapter 4 of this thesis, aims to answer the following six questions:

1. How are the large scale terms of scalar turbulence (such as the mean scalar, scalar-variance and turbulent heat-fluxes) and the small scale term of scalar turbulence (i.e. scalar-variance dissipation) generated and transported in this spatially developing turbulence?
2. It is sometimes believed that the evolution of scalar-variance dissipation is similar to the mean enstrophy of fluctuations (Corrsin, 1953). Are these two quantities indeed behave similarly in the current case? This study reports the similarities and differences between them along the grid-element and bar centrelines.
3. Do the small scales of scalar turbulence reach isotropy in this developing turbulence? It is reported in the literature that the small scales of scalar rarely reach isotropy (refer to the review of Warhaft (2000) and the references therein). This work explores this issue in the near- and far-downstream regions along the grid-element centerline.
4. Does the $-5/3$ power-law slope of the scalar spectrum appear in the inhomogeneous near grid-element region where the KOC theory is not valid? Previous studies have established that the $-5/3$ slope appears for a wider range of frequencies than that of the velocity spectrum for HIT (refer to the review of Warhaft (2000)). It is not clear if this also holds true when the scalar field is highly inhomogeneous and developing.
5. If indeed there is a power-law in the scalar spectrum in the near grid-element region, how do the small-scale scalar dynamics correlate with the scalar cascade? The non-linear stretching of vorticity vector is believed to be closely associated with the energy cascade in turbulent flows. The equivalent process in the scalar cascade is the stretching/compressing of the passive scalar gradient vector, which is a linear process.

6. How do the scalar spectrum and the scalar gradient dynamics evolve from the near grid-element region to the far-downstream? Since the turbulence is developing, the dynamics of scalar gradients is also expected to vary and this evolution is studied in this thesis.

1.4 Laminar heat transfer due to free-stream turbulence

The final major subject of this thesis is the heat transfer from a cylinder in the presence of a free-stream turbulence. It is well-known from the literature that the free-stream turbulence impinging on a surface makes the developing laminar boundary layer transition earlier. When the surface is exposed to heat, the boundary layer transition is expected to have an impact on heat transfer from the surface due to the presence of turbulent fluctuations. Interestingly, experiments conducted on this subject have revealed that not only on the turbulent boundary layer portion of the surface but a substantial amount of heat transfer increase also occurs even in the places where the boundary layer is still laminar (Dullenkopf and Mayle, 1994, 1995). Such an observation can be seen in the turbine blades of an aircraft engine which are exposed to hot gases. The heat load on the turbine blades is increased due to the free-stream turbulence coming from the combustion chamber as well as the preceding rows of the turbine blades. Although the entire pressure surface and most of the suction surface of the turbine blade remain laminar upon being exposed to free-stream turbulence, those regions also exhibit enhanced heat transfer (Dullenkopf and Mayle, 1994, 1995). The effect of free-stream turbulence on heat transfer through a laminar boundary layer is termed the ‘laminar heat transfer’ which constitutes the third major subject of this thesis. Understanding the characteristics of free-stream turbulence, and their effect on the laminar heat transfer is of paramount importance, for instance, in designing a better turbine blades cooling system. The laminar heat transfer scenario can also be observed at the stagnation point of academic geometries such as the circular cylinder.

1.4.1 Literature on heat transfer due to free-stream turbulence

While there are numerous experimental studies available on the subject of laminar heat transfer, only a few theoretical and numerical studies can be found. These studies can be briefly discussed under three broad categories as: (i) primary factors affecting the laminar heat transfer, (ii) stagnation point heat transfer coefficient correlations, and (iii) heat transfer enhancement mechanism.

Primary factors affecting laminar heat transfer

The literature reveals that the primary factors affecting heat transfer through a laminar boundary layer are: (i) straining of the mean flow (a), (ii) turbulent intensity (Tu , where $Tu = \sqrt{\frac{2k}{3}}/U_\infty \times 100\%$), (iii) Reynolds number, and (iv) turbulence length scale (L_u). Although some secondary factors such as the cylinder blockage, surface roughness, thermal boundary condition, Prandtl number etc can also influence the laminar heat transfer, they are not widely considered in the literature.

Early experiments conducted on flow over a flat plate established the effect of mean flow straining (Kestin et al., 1961; Junkhan and Serovy, 1967). These experiments were carried out at a constant turbulent intensity and for varying pressure gradients. The turbulent intensity did not have an effect on the laminar heat transfer for zero pressure gradient. As a result, these studies concluded that, in order for the free-stream fluctuations to have an effect on the laminar heat transfer, the flow must be accelerating.

Almost all of the existing experiments in the literature report that the laminar heat transfer in an accelerating flow increases for increasing turbulent intensity and Reynolds number (Kestin et al., 1961; Junkhan and Serovy, 1967; Magari and LaGraff, 1994; Paxson and Mayle, 1991; Dullenkopf et al., 1990; Ames, 1995; Liu and Rodi, 1994; Ames et al., 2002; VanFossen and Simoneau, 1987; Kingery and Ames, 2016; Chowdhury and Ames, 2013). Note that the configurations of these experiments are diverse, ranging from flat plates, airfoils, cylinders to rows of airfoils.

The study of Yardi and Sukhatme (1978) provided the first experimental evidence for the importance of turbulence length scales in laminar heat transfer. A turbulence length scale of 4 to 12 times of the laminar boundary layer thickness produced the highest heat transfer increase. The length scales below and above the reported range had a smaller effect on the laminar heat transfer. It was argued that the smallest scales in a turbulent flow get dissipated even before they are stretched by the mean accelerating boundary layer, while the largest scales are generally not stretched resulting in mean variation. Their observation was later verified in various experimental studies (Van Fossen et al., 1995; Lowery and Vachon, 1975; Ames, 1991). A similar result was derived in the theoretical study of Xiong and Lele (2004).

To the best of the author's knowledge, only a handful of numerical studies are available in the literature on the subject of laminar heat transfer (Wissink and Rodi, 2006, 2009; Xiong and Lele, 2007; Bhaskaran and Lele, 2010; Wissink and Rodi, 2011a,b; Venema et al., 2011, 2014). Out of these, the studies of Xiong and Lele (2007) and Bhaskaran and Lele (2010) are the large eddy simulations (LES), while the studies of Venema et al. (2011) and Venema et al. (2014) are the semi-DNS (in the sense that the turbulence generating body is not fully resolved in these simulations). None of these

studies provided a systematic analysis of how the primary factors affect the laminar heat transfer. They have been, however, carried out for different values of these primary factors. A collective conclusion from these studies is that the observations noted by the experiments are indeed valid numerically as well.

Stagnation point heat transfer correlations

The second group of studies focused on arriving at a functional relationship for the stagnation point mean *Nusselt* number, $\langle Nu \rangle_0$, where $\langle Nu \rangle = \frac{q_w D}{(T_w^* - T_\infty^*)}$, q_w is the amount of heat-flux applied on the cylinder, D is the diameter of the cylinder, T_w^* and T_∞^* are the temperatures at the wall and the free-stream respectively. Earlier studies attempted to correlate $\langle Nu \rangle_0$ with the mean flow straining, Reynolds number and turbulent intensity, while omitting the turbulence length scale (Kestin et al., 1961; Lowery and Vachon, 1975). The most successful correlation of such functional form was obtained by Dullenkopf and Mayle (1994), which is given as:

$$Nu_a Pr^{-0.37} = 0.571 + 0.0125 Tu_a \left(1 + \frac{1.8}{1 + (Tu_a/20)^3} \right) \quad (1.31)$$

where Nu_a is the modified Nusselt number $Nu_a = \langle Nu \rangle_0 / \sqrt{a_1 Re_{inc}}$, and Tu_a is a modified turbulent intensity $Tu_a = Tu \sqrt{Re_{inc}/a_1}$. In these definitions, $Re_{inc} = U_{inc} D / \nu$, where U_{inc} is the incident velocity. The normalised free-stream straining is represented as a_1 and it is defined as, $a_1 = a D / U_{inc}$ with ‘ a ’ being the free-stream mean straining of the flow.

Although equation (1.31) was observed to give a better collapse of available heat transfer experimental data at large turbulent intensities, there was still a scatter of data for low turbulent intensity values. This scatter was later mitigated in the study of Dullenkopf and Mayle (1995) where the effect of turbulence length scale was also included. Representing the modified length scale as $L_a = \frac{L_u}{D} \sqrt{a_1 Re_{inc}}$, their correlation is written as:

$$Nu_a Pr^{-0.37} = 0.571 + 0.01 Tu_\lambda \quad (1.32)$$

where, Tu_λ is a turbulence parameter defined as,

$$Tu_\lambda = \frac{Tu_a \sqrt{L_a}}{(1 + 0.004 L_a^2)} \quad (1.33)$$

The constants in equations (1.31) and (1.32) are due to curve fitting. The correlations of the form given in equations (1.31) and (1.32) are presently widely accepted. The

validity of these correlations was assessed in the numerical studies of Wissink and Rodi (2011a,b); Venema et al. (2011, 2014), and was found to be satisfactory.

Mechanism of heat transfer enhancement

The third group of studies predicted the heat transfer enhancement mechanism whereby the laminar boundary layer exchanges heat from the surface. Although it is known from the study of Kestin et al. (1961) that the primary mechanism responsible for heat transfer is the mean straining of the flow, the complete mechanism of heat transfer enhancement is often derived from the rapid distortion theory of Hunt (1973). The reason that the small-scale eddies increase heat transfer upon being stretched by the mean accelerating boundary layer is due to the amplification of their turbulent intensity by stretching. This effect is felt up to three radii from the centre of the cylinder. The large scale eddies are blocked by the boundary layer, and this effect is felt up to one integral length scale from the surface of the cylinder.

Experimental studies provided validation of this mechanism only by checking if the boundary layer is accelerating or not as they were limited to measure just the one point turbulent statistics. The numerical studies painted a better picture of this mechanism. For example, the DNS studies of Wissink and Rodi (2006), Wissink and Rodi (2009), and Wissink and Rodi (2011a) showed that the increase in heat transfer occurs through an upwash of hot fluid away from the cylinder, and a downwash of cold fluid towards the cylinder. The LES studies of Xiong and Lele (2007) and Bhaskaran and Lele (2010) observed longitudinal streamwise vortical structures that increase heat transfer by getting stretched along the surface of the turbine blade.

1.4.2 Objectives of laminar heat transfer analysis

Almost all of the experimental studies found in the literature on laminar heat transfer used a classical grid to generate the oncoming turbulence, although there were a few studies that had turbulence generator in the form of a circular cylinder. Yet, all these studies are similar in one particular aspect. The oncoming turbulence was always observed to be homogeneous, *Gaussian*, and fully-turbulent in all of the existing studies on laminar heat transfer.

As discussed in the velocity and scalar gradients literature, there has been an increased interest in recent times to analyse non-canonical turbulent flows, such as the turbulence generated by fractal grids, due to their novel intriguing characteristics. As noted earlier, the recent DNS studies have revealed that the fractal grid turbulence characteristics can also be observed in the square grid-element i.e. the largest scale of a fractal grid without fractal iterations (Zhou et al., 2014a, 2016a). In particular, the pro-

duction region behind the grid-element is found to be inhomogeneous and non-Gaussian. The effect of an inhomogeneous developing turbulence on the laminar heat transfer has never been considered in the literature.

The following questions are raised in this thesis for the analysis of laminar heat transfer:

1. What is the effect of different regions such as the production, peak and decay regions of grid-element turbulence on the heat transfer from a cylinder?.
2. When the turbulent intensities of the inhomogeneous production and the homogeneous decay regions are similar, do they also increase heat transfer by the same amount?
3. Are the existing correlations for the stagnation point heat transfer coefficient valid for this developing turbulence?
4. How does heat transfer enhancement happen when the oncoming turbulence characteristics are different from the one reported in the literature?

1.5 Details of the grid-element

The present DNS deals with the flow past a single square grid-element of a fractal square grid (or simply a square grid-element). A fractal square grid is constructed by replicating this element at various length scales. On the other hand, replicating this element at the same length scale forms a classical grid. Therefore, this is a fundamental building block for both the classical and fractal grids. A sketch of the grid-element with the basic dimensions is shown in figure 1.4. The lateral thickness of the grid-element is $t_0=43\text{mm}$, while the length is $L_0 = 5.3t_0 = 229\text{mm}$. The thickness of the grid bar in the streamwise direction is 6mm. These dimensions follow the experimental set up of Laizet et al. (2015a).

The aspect ratio (AR) of the bar, defined as the ratio of the streamwise to the lateral thickness, is 0.14, and not 1.0 as in the case of classical grids. This is because the present thesis examines the single square pattern of the fractal square grid whose streamwise thickness is equal to that of the smallest square element. Therefore, this study considers the streamwise thickness of the fractal square grid used in the studies of Gomes-Fernandes et al. (2012, 2014); Laizet et al. (2015a,b).

Throughout the thesis, the streamwise distance behind the grid-element is normalised with the characteristic wake-interaction length scale, x^* , defined by Mazellier and Vassilicos (2010). In order to derive this length scale, it is assumed that the turbulent

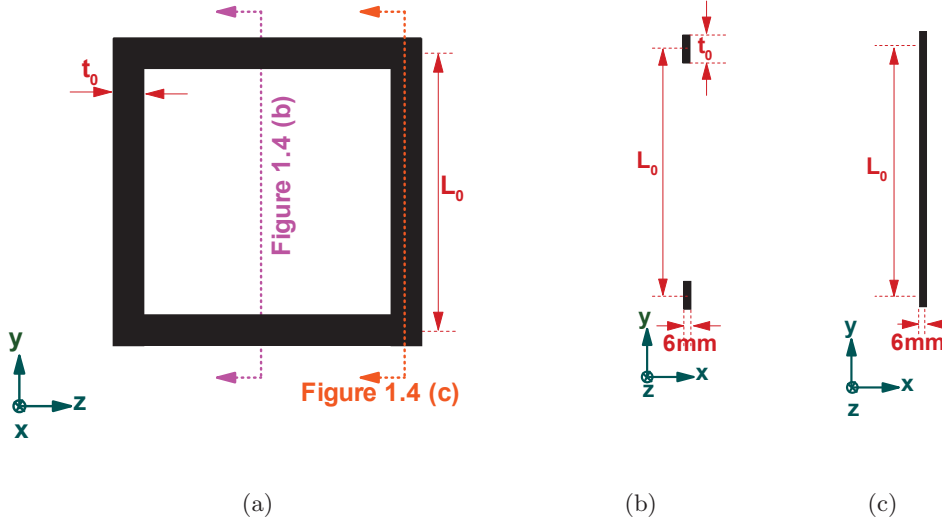


Figure 1.4: (a) Sketch of the single square grid-element. (b,c) Sectional views of the grid-element for the dotted cut sections shown in (a)

wake starts immediately downstream of the bar, and thus the laminar and transitional parts of the wake are neglected. It is known that the wake width of a turbulence generating body of size t_0 grows as $\sqrt{t_0 x}$ in the downstream direction (Townsend (1980)). Therefore, the wakes are expected to meet at a downstream streamwise location which scales with x^* where $L_0 \approx \sqrt{t_0 x^*}$, and L_0 is the distance between the parallel bars. From this, one can obtain the wake-interaction length scale based on the grid-element parameters as $x^* = L_0^2/t_0$.

1.6 Thesis outline

The rest of this thesis is structured as follows. Since the present work is a DNS study, the introduction to the governing equations and the methodologies adopted to solve them numerically are presented in chapter 2. The analysis of velocity and scalar gradients for a developing flow generated by the square grid-element is carried out in chapters 3 and 4 respectively. The effect of free-stream turbulence on heat transfer is analysed in chapter 5. The main conclusions of this thesis and the scope for future work are summarised in the final chapter of this thesis (chapter 6).

Chapter 2

Numerical Methodology

2.1 Overview

Numerical analysis of turbulent flows involves solving the governing equations of velocity and scalar. These governing equations are known as mass, momentum, and scalar conservation equations. In this chapter, a brief discussion on the governing equations followed by the numerical methodology adopted to solve them is presented. Much of the content of this chapter is based on the text books of Patankar (1980); Hirsch (2007); Versteeg and Malalasekera (2007); Ferziger and Peric (2012), and the theses of Muzaferija (1994); Jasak (1996); Croft (1998); Liang (2005).

2.2 Governing Equations

The continuity, momentum and scalar conservation equations for incompressible flow are written as:

$$\frac{\partial u_i^*}{\partial x_i} = 0 \quad (2.1)$$

$$\frac{\partial u_i^*}{\partial t} + u_j^* \frac{\partial u_i^*}{\partial x_j} = -\frac{1}{\rho} \frac{\partial p^*}{\partial x_i} + \nu \frac{\partial^2 u_i^*}{\partial x_j \partial x_j} \quad (2.2)$$

$$\frac{\partial T^*}{\partial t} + u_j^* \frac{\partial T^*}{\partial x_j} = \alpha \frac{\partial^2 T^*}{\partial x_j \partial x_j} \quad (2.3)$$

where ρ, ν, α are the density, kinematic viscosity and thermal diffusivity of the fluid respectively. In this study, these properties are constant and thus the scalar equation is decoupled from the momentum equation. The continuity and momentum equations

(equations (2.4) and (2.2)) are collectively called the Navier-Stokes equations.

The numerical solution of these equations involves two steps: (i) discretisation of the solution domain and (ii) discretisation of the equations. The discretisation of the domain converts the continuous physical space into a finite number of volumes. These volumes form the numerical mesh. The discretisation of an equation transforms the equation into a system of algebraic equations. The *Finite volume method* (FVM) is used in this study for both space and equation discretisations. For space discretisation, the FVM makes use of an arbitrarily unstructured mesh and therefore is more suitable for problems involving complex geometries. The integral form of the conservation equations is considered in the FVM discretisation, which ensures that the basic quantities like mass, momentum, and scalar will also be conserved at the discrete level.

The rest of this chapter is organised as follows. In section 2.3, the computational mesh is defined. The general conservation equation is discretised on this mesh in the unstructured finite volume approach, and the details of the discretisation are described in section 2.4 for every term of the general transport equation. Since the use of an unstructured mesh requires a special treatment of computing the face velocities, section 2.5 is dedicated to the discussion of Rhie-Chow interpolation method. Section 2.6 derives the equation of pressure from the continuity equation for an unstructured mesh. The pressure-velocity coupling in the Navier-Stokes equation and the solution procedure of PISO algorithm are presented in section 2.7. The numerical treatment of boundary conditions is presented in section 2.8. Finally, the parallelisation strategy adopted for unstructured finite volume method is discussed in section 2.9, followed by the closing remarks in section 2.10.

2.3 Discretisation of the solution domain

The space discretisation transforms the continuous space enclosed by the fixed boundaries into a number of control volumes or cells. These control volumes do not overlap, and they fill the continuous space completely. These control volumes are collectively called a numerical mesh. In this study, hexahedral shaped control volumes are used. A typical control volume is shown in figure 2.1. Each control volume has a centroid (shown as P in figure 2.1) defined according by,

$$\int_{V_P} (\mathbf{x} - \mathbf{x}_P) dV = \mathbf{0} \quad (2.4)$$

The value of all dependent variables is stored at the cell centroids, i.e., collocated variable arrangement. As shown in figure 2.1, each control volume consists of vertices

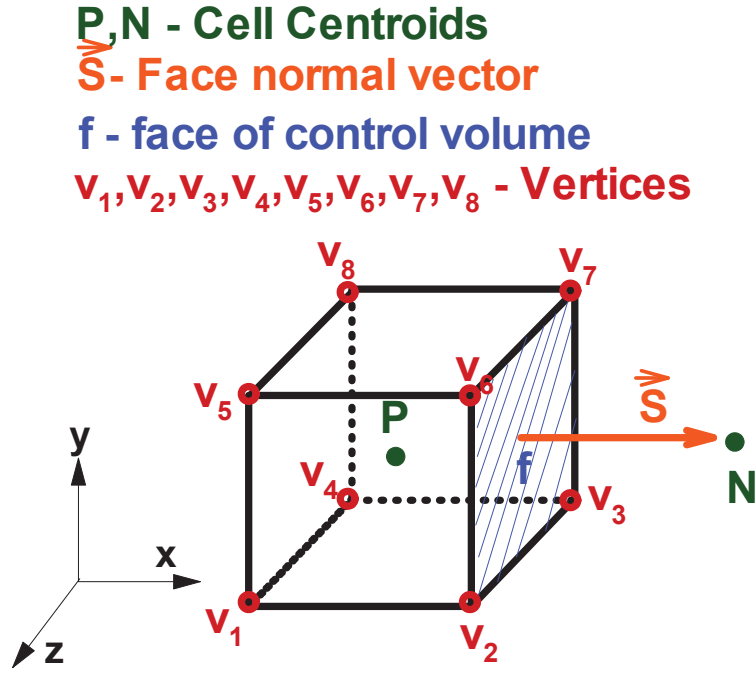


Figure 2.1: A typical hexahedral control volume.

and faces. For a hexahedral control volume, there is one centroid (marked as ‘ P ’ in figure 2.1), six faces (marked as ‘ f ’ in figure 2.1), and 8 vertices (marked as ‘ $v_1, \dots, v_8$ ’ in figure 2.1).

The cell faces are of two types: (i) internal and (ii) boundary faces. The internal faces are shared between two control volumes. The faces coinciding with the boundaries of the continuous space are called boundary faces. For each face, an area vector $\vec{S}$ is defined in such a way that it points outwards from the owner cell towards the neighbouring cell. This vector is normal to the face and has the magnitude equal to the area of the face. For the shaded face in figure 2.1, the owner and neighbour cell centres are marked as P and N respectively, and the face area vector $\vec{S}$ points outwards from the P control volume. Throughout this study, the mesh generation is carried out using ICEM CFD software.

Just like the continuous space, the time domain also has to be discretised. However, time is a parabolic coordinate (Patankar, 1980) and the solution is marched in time from the prescribed initial condition.

2.4 Discretisation of a general transport equation

Having discussed the domain discretisation, this section introduces the discretisation of the general transport equation. The standard form of the transport equation for a scalar

variable ϕ is:

$$\underbrace{\frac{\partial \phi}{\partial t}}_{\text{transient term}} + \underbrace{\nabla \cdot (\rho \vec{U} \phi)}_{\text{convection term}} = \underbrace{\nabla \cdot (\rho \Gamma_\phi \nabla \phi)}_{\text{diffusion term}} + \underbrace{S_\phi(\phi)}_{\text{source term}} \quad (2.5)$$

where Γ_ϕ is a coefficient equal to the viscosity in the momentum equation, and to the thermal diffusivity in the temperature transport equation. The dependent variable ϕ can be u_1^* , u_2^* , u_3^* , T^* etc.

Equation 2.5 is discretised using the finite-volume method. For the case of unsteady problems, the governing equation should also be integrated over time. The integral form of the transport equation over the control volume V_P (see figure 2.1) is:

$$\begin{aligned} \int_t^{t+\Delta t} \int_{V_P} \left(\frac{\partial \rho \phi}{\partial t} \right)_P dV dt = & - \int_t^{t+\Delta t} \int_{V_P} \nabla \cdot (\rho \vec{U} \phi) dV dt + \\ & \int_t^{t+\Delta t} \int_{V_P} \nabla \cdot (\rho \Gamma_\phi \nabla \phi) dV dt + \int_t^{t+\Delta t} \int_{V_P} S_\phi(\phi) dV dt \end{aligned} \quad (2.6)$$

The discretisation practice adopted in this study is second-order accurate in space and time and will be presented in the rest of this section. The individual terms of the transport equation will be treated separately in the following subsections.

2.4.1 Transient term

The time derivative term is approximated using the second order backward differencing scheme as,

$$\left(\frac{\partial \phi}{\partial t} \right)_P^{m+1} = \frac{\frac{3}{2} \phi_P^{m+1} - 2 \phi_P^m + \frac{1}{2} \phi_P^{m-1}}{\Delta t} \quad (2.7)$$

where,

$$\phi^m = \phi(t) \quad (2.8)$$

$$\phi^{m+1} = \phi(t + \Delta t) \quad (2.9)$$

$$\phi^{m-1} = \phi(t - \Delta t) \quad (2.10)$$

Therefore, the final form of the discretised equation with backward differencing in time is:

$$\left[\frac{\frac{3}{2}\phi_P^{m+1} - 2\phi_P^m + \frac{1}{2}\phi_P^{m-1}}{\Delta t} \right] V_P = - \int_{V_P} \left[\nabla \cdot (\rho \vec{U} \phi) \right]^{m+1} dV + \int_{V_P} \left[\nabla \cdot (\rho \Gamma_\phi \nabla \phi) \right]^{m+1} dV + \int_{V_P} \left[S_\phi(\phi) \right]^{m+1} dV \quad (2.11)$$

The right hand side of the above equation is computed at $m+1$ time-level, therefore the temporal discretisation adopted in this study is fully implicit.

2.4.2 Convective term

The Gauss divergence theorem is used to write the convection term in equation (2.11) as:

$$\int_{V_P} \nabla \cdot (\rho \vec{U} \phi) dV = \oint_{\partial V} (\rho \vec{U} \phi) \cdot d\vec{S} \quad (2.12)$$

where ∂V is the closed surface bounding the control volume P , and $d\vec{S}$ represents an infinitesimal surface element with associated normal. This closed surface integral can be split into a sum of integrals over each of the faces surrounding the control volume. Therefore, the surface integral equation can be written as:

$$\oint_S (\rho \vec{U} \phi) \cdot d\vec{s} = \sum_f (\rho \vec{U} \phi)_f \cdot \vec{S} \quad (2.13)$$

Here, the subscript f indicates the value of the variable (in this case $\rho \vec{U} \phi$) at the face, and $\vec{S}$ is the area vector normal to the face of the control volume and directed outwards in relation to the volume (see figure 2.1). Representing the mass flow through the face of control volume as,

$$F_f = (\rho \vec{U})_f \cdot \vec{S} \quad (2.14)$$

The equation (2.13) can be written as,

$$\int_S (\rho \vec{U} \phi) \cdot d\vec{S} = \sum_f F_f \phi_f \quad (2.15)$$

The calculation of face fluxes (F_f) will be discussed separately in this chapter (see section 2.5). For now it is assumed that the flux is calculated from the interpolated values of ρ and $\vec{U}$.

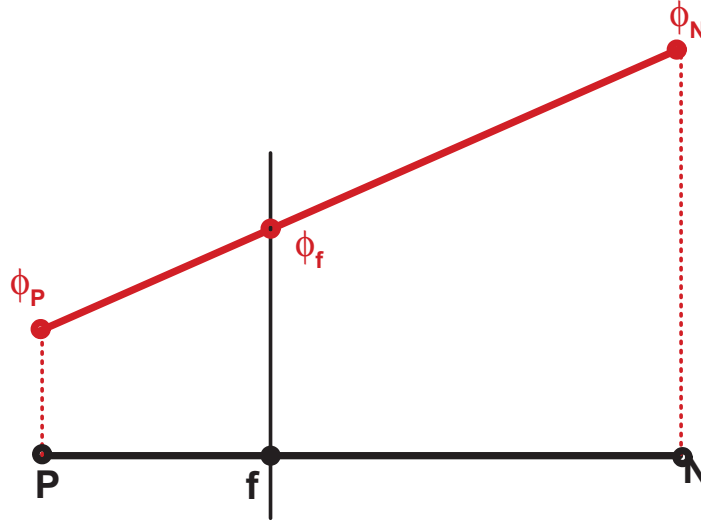


Figure 2.2: Interpolation of the face value.

The next step in discretising the convection equation is to compute the value of ϕ at the cell faces from the values of cell centroids.

Assuming a linear variation of ϕ between P and N as shown in figure 2.2, the face value is calculated according to:

$$\phi_f = f_x \phi_P + (1 - f_x) \phi_N \quad (2.16)$$

Here, the weighting factor f_x is defined as the ratio between distances $\overline{fN}$ and $\overline{PN}$ (see figure 2.2):

$$f_x = \frac{\overline{fN}}{\overline{PN}} \quad (2.17)$$

The differencing scheme used in equation (2.16) to determine the face value of ϕ is called the Central Differencing Scheme (CDS). Ferziger and Peric (2012) has shown that CDS is second order accurate. This is consistent with the overall accuracy of the method. However, CDS produces unphysical oscillations in the solution for poorly resolved simulations (Patankar, 1980; Hirsch, 2007).

2.4.3 Diffusion term

The diffusion term can also be approximated using the Gauss theorem as follows:

$$\int_{V_P} \nabla \cdot (\rho \Gamma_\phi \nabla \phi) dV = \oint_{\partial V} (\rho \Gamma_\phi \nabla \phi) \cdot d\vec{S} \quad (2.18)$$

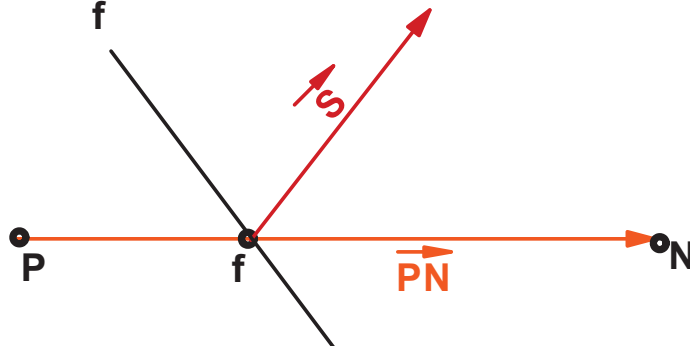


Figure 2.3: An example of non-orthogonality in cell faces.

Again, the closed surface integral can be written as the sum of surface integrals over each of the faces of the control volume. Therefore,

$$\oint_{\partial V} (\rho \Gamma_\phi \nabla \phi) \cdot d\vec{S} = \sum_f (\rho \Gamma_\phi \nabla \phi)_f \cdot \vec{S} = \sum_f (\rho \Gamma_\phi)_f \vec{S} \cdot \nabla \phi_f \quad (2.19)$$

If the mesh is orthogonal, *i.e.* vectors $P\vec{N}$ and $\vec{S}$, in figure 2.3, are parallel, then it is possible to write,

$$\vec{S} \cdot \nabla \phi_f = |\vec{S}| \frac{\phi_N - \phi_P}{|P\vec{N}|} \quad (2.20)$$

Using this equation, the face gradient of ϕ can be calculated from the two values on either side of a face. For the case of non-orthogonal meshes, the product $\vec{S} \cdot \nabla \phi_f$ is split into two parts:

$$\vec{S} \cdot \nabla \phi_f = \underbrace{\vec{\Delta} \cdot \nabla \phi_f}_{\text{orthogonal contribution}} + \underbrace{\vec{k} \cdot \nabla \phi_f}_{\text{non-orthogonal correction}} \quad (2.21)$$

The two vectors introduced in equation (2.21), $\vec{\Delta}$ and $\vec{k}$ are constructed in such a way that:

$$\vec{S} = \vec{\Delta} + \vec{k} \quad (2.22)$$

Vector $\vec{\Delta}$ is chosen to be parallel with $P\vec{N}$ (see figure 2.4). The non-orthogonal contribution is generally computed from the previous time-step. Considering that the gradient of ϕ varies linearly between the cell centroids, the non-orthogonal correction can be

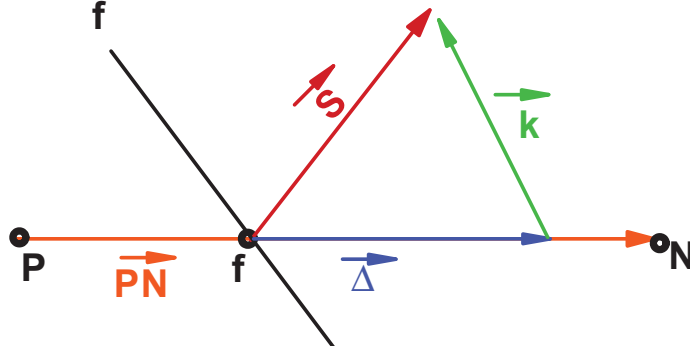


Figure 2.4: Treatment of non-orthogonality used in the current study.

computed as:

$$\vec{k} \cdot \nabla \phi_f = \vec{k} \cdot (f_x \vec{\Delta} \phi_P + (1 - f_x) \vec{\Delta} \phi_N) \quad (2.23)$$

where $\vec{\Delta} \phi_P$ and $\vec{\Delta} \phi_N$ are the values of gradients of ϕ at the cell centroids. Note that the vector $\vec{k}$ in equation (2.23) is constructed within the constraint in equation (2.22).

Therefore, the final form of the difference equation for the diffusion term for the face considered in figure 2.1 with respect to the control volume P is:

$$\begin{aligned} \int_{V_P} \nabla \cdot (\rho \Gamma_\phi \nabla \phi) dV &= (\rho \Gamma_\phi)_f \left(|\vec{S}| \frac{\phi_N - \phi_P}{|P\vec{N}|} \right)^{m+1} + \\ &\quad \left((\rho \Gamma_\phi)_f \vec{k} \cdot (f_x \vec{\Delta} \phi_P + (1 - f_x) \vec{\Delta} \phi_N) |\vec{S}| \right)^m \end{aligned} \quad (2.24)$$

2.4.4 Source term

All other terms that cannot be written as convection, diffusion or temporal terms are treated as sources. The source term, $S_\phi(\phi)$, can be a general function of ϕ . When deciding on the form of the discretisation for the source, its interaction with other terms in the equation and its influence on boundedness and accuracy should be examined. Some general comments on the treatment of source terms are given in the book of Patankar (1980). A simple procedure is explained here.

Before the actual discretisation, the source term is linearised as:

$$S_\phi(\phi) = S_u + S_p \phi \quad (2.25)$$

where S_u and S_p can also depend on ϕ . The volume integral of the source term is then

written as:

$$\int_{V_P} S_\phi(\phi) dV = S_u V_p + S_p V_p \phi_P \quad (2.26)$$

2.4.5 Final form of the discretised equations

Considering the face f with respect to the control volume P as shown in figure 2.1, and substituting equations (2.15), (2.24) and (2.26) in (2.11), the discretised form of the general transport equation is obtained as,

$$a_P \phi_P^{m+1} + \sum_{nb} a_{nb} \phi_{nb}^{m+1} = SU \quad (2.27)$$

where, a_P is the coefficient matrix for the central node, a_{nb} is the coefficient matrix of the neighbouring nodes, and SU is the source term. They are given as,

$$a_P = S_P - \sum_{nb} a_{nb} - (1 - f_x) \quad (2.28)$$

$$S_P = \frac{3}{2} \frac{\rho_f V_P}{\Delta t} + \sum_f F_f \quad (2.29)$$

$$\sum_{nb} a_{nb} = -D_f + (1 - f_x) \quad (2.30)$$

$$D_f = \frac{(\rho \Gamma_\phi)_f |\vec{\Delta}|}{|P\vec{N}|} \quad (2.31)$$

and

$$\begin{aligned} SU = & \frac{4}{2} \frac{\rho_f V_P}{\Delta t} \phi_P^m + \frac{1}{2} \frac{\rho_f V_P}{\Delta t} \phi_P^m + \sum_f F_f \left\{ [f_x \phi_P + (1 - f_x) \phi_N] \right. \\ & \left. \right\}^m + \left((\rho \Gamma_\phi)_f |\vec{k}| (f_x \Delta \phi_P + (1 - f_x) \Delta \phi_N) \right)^m + S_u V_p + S_p V_p \phi_P^m \end{aligned} \quad (2.32)$$

Note that the source term contains the corrections for the convection and diffusion terms. The variable S_P is kept negative and included in the diagonal coefficient of the final coefficient matrix. In this way, the diagonal dominance of the coefficient matrix is improved.

2.5 Rhie-Chow method

The discussion presented in the previous section on equation discretisation is incomplete as the procedure to compute the face velocities and thus the mass fluxes in (2.14) are

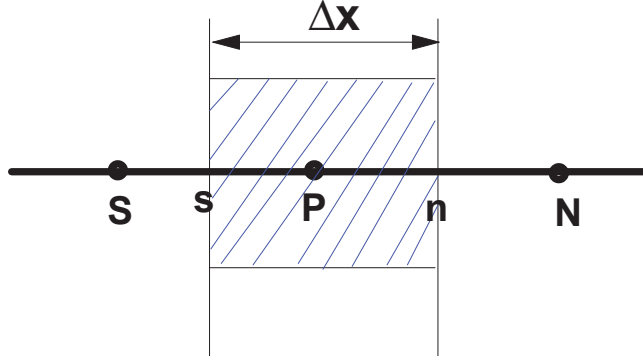


Figure 2.5: One dimensional control volume

not known. This section first introduces the need for a special interpolation method for the computation of face velocities. This study uses the Rhie-Chow interpolation method (Rhie and Chow, 1983), and the details of this method are explained in the rest of this section.

Consider the one-dimensional x -momentum equation

$$\frac{\partial u_1^*}{\partial t} + u_1^* \frac{\partial u_1^*}{\partial x} = -\frac{1}{\rho} \frac{\partial p^*}{\partial x} + \nu \frac{\partial^2 u_1^*}{\partial x^2} \quad (2.33)$$

where u_1^* is the x component of velocity. The momentum equation has an extra term, the pressure gradient term, when compared with the general transport equation (2.5). When discretised over an one-dimensional control volume ‘P’ shown in figure 2.5, the pressure gradient term is transformed into $p_n^* - p_s^* / \Delta x$, where n and s represent the faces of the control volume P (see figure 2.5). Assuming uniform meshing, using the linear interpolation for the face values of pressure, one can obtain:

$$\frac{p_n^* - p_s^*}{\Delta x} = \left[\frac{p_N^* + p_P^*}{2} - \frac{p_P^* + p_S^*}{2} \right] \frac{1}{\Delta x} = \frac{p_N^* - p_S^*}{2\Delta x} \quad (2.34)$$

From equation (2.34), it is evident that the discretisation of the pressure gradient term involves pressure at alternate centroids rather than the adjacent values. This means that the pressure gradient term is discretised over a coarser mesh than the mesh used for all other variables. As a result, an oscillatory pressure field would be seen as a uniform pressure field. For instance, if the pressure distribution in nodes S , P , and N are 1, 100, 1 respectively, then this distribution would appear as uniform as the pressure gradient term is approximated using the centroid values at S and N . This leads to the checkerboard pressure field. This checkerboard field can be added to any given pressure field, and the actual pressure gradient gets affected. Thus if p^* is the correct pressure field and u_1^* the correct velocity field which together satisfy the discretised equations,

then so does $p^* + p_{cb}^*$ and u_1^* where p_{cb}^* is the checkerboard pressure field. This means that any pressure obtained as a result of solving the momentum and continuity equations may predict the correct velocity whilst having an error in the pressure equal to p_{cb}^* .

In order to alleviate the problem of checkerboard effect, a staggered mesh, as suggested by Harlow and Welch (1965), is used in most of the structured mesh simulations. In a staggered mesh, the velocity components are stored at the cell faces, while the pressure is stored at the cell centroids. Due to this staggered arrangement, the fluxes at the faces are stored where they are needed and no interpolation is necessary. The staggered mesh also results in a pressure gradient that involves adjacent nodes as they now lie in between the locations where the velocity components are stored.

Unfortunately, extending the staggered variable arrangement to unstructured mesh is not straightforward. This study, as discussed earlier, uses a collocated method where all the variables are stored at the cell centroid. Therefore, the face values should be computed from the centroids which leads to the problem of checkerboard effect. This brings the need for an alternative to compute the face velocities but does not suffer from the checkerboard effect. Rhie and Chow (1983) introduced such a method for a collocated mesh which now bears their name. The details of this method are discussed in the rest of this section.

The u_1^* momentum equation is

$$\frac{\partial u_1^*}{\partial t} + \nabla \cdot (\vec{u}^* u_1^*) = -\rho \frac{\partial p^*}{\partial x} + \nabla \cdot (\nu \nabla u_1^*) \quad (2.35)$$

This equation can be discretised using the techniques described earlier in this chapter, over the control volume about a node P to produce an equation which can be written in the form

$$a_P u_{1P}^* + \left(\sum_{nb} a_{nb} u_{1nb}^* \right)_P = SU_P + (\nabla_x p^*)_P \quad (2.36)$$

where $\nabla_x p^*$ is the discretised contribution from the pressure gradient term. Similarly for the adjacent node N ,

$$a_N u_{1N}^* + \left(\sum_{nb} a_{nb} u_{1nb}^* \right)_N = SU_N + (\nabla_x p^*)_N \quad (2.37)$$

The u^* velocity at the face between the nodes must also have a discretised momentum

equation of the form

$$a_f u_{1f}^* + \left(\sum_{nb} a_{nb} u_{1nb}^* \right)_f = SU_f + (\nabla_x p^*)_f \quad (2.38)$$

The Rhie - Chow interpolation method uses the equations (2.36) and (2.37) to approximate a solution of equation (2.38). It is assumed that (as done earlier for the variable ϕ) the right hand side of (2.38) may be approximated by using a weighted linear interpolation of the corresponding terms in equations (2.36) and (2.37). Thus

$$\begin{aligned} a_f u_{1f}^* &= - \overline{\left(\sum_{nb} a_{nb} u_{1nb}^* \right)_f} + SU_f + (\nabla_x p^*)_f = \\ &= - \overline{\left(\sum_{nb} a_{nb} u_{1nb}^* \right)_f} + SU_f + \overline{(\nabla_x p^*)_f} = \overline{a_f u_{1f}^*} + \overline{(\nabla_x p^*)_f} \end{aligned} \quad (2.39)$$

Assuming $a_f \approx \overline{a_f}$, then

$$u_{1f}^* = \overline{u_{1f}^*} + \overline{d_f} (\overline{\nabla_x p_f^*} - \nabla_x p_f^*) \quad (2.40)$$

Using the weighting factor f_x in equation (2.16), then

$$\overline{u_{1f}^*} = f_x u_{1P}^* + (1 - f_x) u_{1N}^* \quad (2.41)$$

$$\overline{\nabla_x p_f} = f_x \nabla_x p_P + (1 - f_x) \nabla_x p_N \quad (2.42)$$

$$\nabla_x p_f = \frac{p_N^* - p_P^*}{\overline{PN}} \quad (2.43)$$

$$a_f = f_x a_P + (1 - f_x) a_N \quad (2.44)$$

$$\overline{d_f} = a_f^{-1} \quad (2.45)$$

Equation (2.40) is the final approximation of the face velocities using the Rhie-Chow interpolation method. The difference $(\overline{\nabla_x p_f^*} - \nabla_x p_f^*)$ will detect a checkerboard pressure field, and results in a source term in the pressure equation, that will eventually lead to correct pressure field. The pressure equation is derived in the next section.

2.6 Derivation of the pressure equation

The general transport equation is used to solve for the velocity components (u_1^*, u_2^*, u_3^*) . The Navier-Stokes equation, however, also contains the pressure term, but there is no separate equation for the pressure. Indeed, the pressure is derived from the continuity

equation

$$\nabla \cdot \vec{u}^* = 0 \quad (2.46)$$

In the discretised form, it is written as,

$$\sum_f \vec{u}_f^* \cdot \vec{S} = 0 \quad (2.47)$$

As shown in the previous section, the Rhie-Chow interpolation gives the equation for a face velocity component as,

$$u_{1f}^* = \overline{u_{1f}^*} + \overline{d_f} (\overline{\nabla_x p_f^*} - \nabla_x p_f^*) \quad (2.48)$$

At any solution stage, given the guessed cell centre values for pressure, p^g , and for u_1^* velocity, u_1^g , the guessed face value of velocity component is given as,

$$u_{1f}^g = \overline{u_{1f}^g} + \overline{d_f} (\overline{\nabla_x p_f^g} - \nabla_x p_f^g) \quad (2.49)$$

The other guessed velocity components can also be obtained similar to equation (2.49). The aim is to improve the guessed pressure p^g in order that the guessed velocity components get progressively closer to satisfying the continuity equation. If p^* is the correct pressure, then

$$p^* = p^g + p^c \quad (2.50)$$

where p^c is the pressure correction. Similarly, the velocity components obtained through guessed pressure are related to their correct values by

$$u_1^* = u_1^g + u_1^c \quad (2.51)$$

$$u_2^* = u_2^g + u_2^c \quad (2.52)$$

$$u_3^* = u_3^g + u_3^c \quad (2.53)$$

The correction velocity at cell face is approximated as,

$$u_{1f}^c = -\frac{1}{a_P} \nabla_x p_f^c = \frac{1}{a_P} \frac{p_P^c - p_N^c}{P\vec{N}} \quad (2.54)$$

Therefore, the velocity component at face is,

$$u_{1f}^* = u_{1f}^g - \frac{1}{a_P} \nabla_x p_f^c = u_{1f}^g - \frac{1}{a_P} \frac{p_P^c - p_N^c}{P\vec{N}} \quad (2.55)$$

Similarly, the other corrected velocity components can also be computed. Now the continuity equation (2.47) becomes,

$$\sum_{f,i} \frac{S_f}{a_i} (p_P^c - p_N^c)_f = - \sum_f \vec{u}_f^g \cdot \vec{S} \quad (2.56)$$

where a_i ($i=1,2,3$) is the nodal coefficient for the three coordinate directions. The above equation can be written in the form:

$$a_P p_P^c + \sum_{nb} a_{nb} p_{nb}^c = SU \quad (2.57)$$

where the summation in equation (2.57) is over all elements sharing a face with element P, and

$$a_{nb} = -S_f \sum_{i=1}^3 \frac{1}{a_i} \quad (2.58)$$

$$a_P = - \sum_{nb} a_{nb} \quad (2.59)$$

$$SU = - \sum_f \vec{u}_f^g \cdot \vec{S} \quad (2.60)$$

2.7 Solution procedure for the coupled equation set

The solution procedure adopted to solve the coupled set of Navier-Stokes equations is discussed. This study uses the pressure implicit split operator (PISO) algorithm of Issa (1986). The algorithm involves the following steps:

1. Initialise the values of velocity components and pressure.
2. Solve the momentum equations for velocities using the pressure values at the previous time-step (i.e. p^{*m}). The velocities will not satisfy the continuity equation. This step is often called Predictor step.
3. Solve the pressure correction equation to get a new pressure.
4. Solve the momentum equation with the improved pressure values obtained from the previous step. This step is called the first predictor step. The velocity field will satisfy the continuity equation, but due to approximation involved in equation (2.54), not the momentum equations.
5. Obtain new velocities explicitly from equation (2.36) and solve a new pressure equation. This step yields the more accurate values of velocity and pressure. This step is also known as the second corrector step.

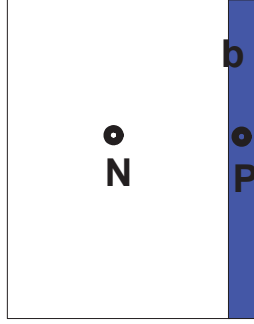


Figure 2.6: Cell with a boundary face

6. Update the previous time-step values.
7. Go to step 2 and proceed to the next time step.

2.8 Boundary conditions and their implementation

The computational mesh consists of a number of faces which coincide with the boundaries of the continuous physical domain. This section discusses how the boundary conditions are implemented on the boundary faces. In the approach presented here, the boundaries are assigned a cell number. These are the fictitious two-dimensional cells, as shown in figure 2.6, where the boundary values are stored.

The boundary conditions are discussed under two broad categories: (i) numerical and (ii) physical boundary conditions.

2.8.1 Numerical Boundary Conditions

Numerical boundary conditions are of two types. Application of a fixed value to the boundary is called the Dirichlet boundary condition. Prescribing a fixed gradient value at the boundary is known as the Von Neumann boundary condition. In the unstructured finite volume method, these boundary conditions are included in the system of algebraic equations (i.e. equation (2.27)).

Fixed Value Boundary Condition

The Dirichlet boundary condition specifies the value of ϕ at the boundary face b (i.e. ϕ_b), and this value is used into the discretised equation of the transport equation.

The convection term according to equations (2.14) and (2.15) is,

$$\int_{V_P} \nabla \cdot (\rho \vec{U} \phi) = \sum_f F_f \phi_f \quad (2.61)$$

It is known that the value of ϕ on the boundary face is ϕ_b . Therefore, the term for the boundary face is:

$$F_b \phi_b \quad (2.62)$$

where F_b is the boundary face flux.

The diffusion term is discretised according to (2.19) as,

$$\int_{V_P} \nabla \cdot (\rho \Gamma_\phi \nabla \phi) dV = \sum_f (\rho \Gamma_\phi)_f \vec{S} \cdot \nabla \phi_f \quad (2.63)$$

The face gradient at b is calculated from the known face value and the cell centre value:

$$\vec{S} \cdot \nabla \phi_b = |\vec{S}| \frac{\phi_b - \phi_N}{|PN|} \quad (2.64)$$

Fixed Gradient Boundary Condition

For the Von-Neumann boundary condition, a fixed gradient value is applied on the boundary face. More specifically,

$$\left(\frac{\vec{S}}{|\vec{S}|} \cdot \nabla \phi \right)_b = g_b \quad (2.65)$$

For **the convection term**, the face value of ϕ is calculated from the value in the cell centre and the prescribed gradient:

$$\phi_b = \phi_N + P \vec{N} \cdot (\nabla \phi)_b = \phi_P + |PN| g_b \quad (2.66)$$

For **the diffusion term**, the dot product between the face area vector and $(\nabla \phi)_b$ becomes,

$$\vec{S} \cdot \Delta \phi = |\vec{S}| g_b \quad (2.67)$$

and the resulting term is:

$$(\rho \Gamma_\phi)_b |\vec{S}| g_b \quad (2.68)$$

2.8.2 Physical Boundary Conditions

The boundary conditions used in fluid mechanics such as the symmetry planes, walls, inlet, outlet and cyclic as well as the boundary conditions used for heat transfer problems like the adiabatic or fixed temperature are called physical boundary conditions.

Inlet boundary

The velocity field and the value of temperature at the inlet are prescribed. In order to implement these conditions, the procedures discussed for the Dirichlet boundary condition in the previous section are used.

Outlet boundary

A convective boundary condition is used at the outlet. It is written as,

$$\frac{\partial \phi}{\partial t} + U_{conv} \frac{\partial \phi}{\partial n} = 0 \quad (2.69)$$

where U_{conv} is the plane-averaged convective velocity normal to the outlet boundary. The value of ϕ at the outlet boundary face is obtained by discretising equation (2.69). Using the first order upwind scheme, the discretised equation for the boundary face in figure 2.6 becomes,

$$\frac{\phi_P^{m+1} - \phi_P^m}{\Delta t} + \frac{\phi_N^{m+1} - \phi_P^{m+1}}{|PN|} U_{conv} = 0 \quad (2.70)$$

From this, the value of ϕ at the boundary can be computed as,

$$\phi_P^{m+1} = \frac{\phi_P^m + (f_y \phi_N^{m+1})}{1 + f_y} \quad (2.71)$$

where f_y is a factor defined as,

$$f_y = \frac{\Delta t U_{conv}}{|P\vec{N}|} \quad (2.72)$$

The computed value in equation (2.71) is then incorporated into the governing equation through the method described earlier for the fixed value boundary condition.

Impermeable no-slip walls

The velocity components at the solid surface are set to zero. Therefore, the Dirichlet boundary condition is used to incorporate this value into the system of algebraic equa-

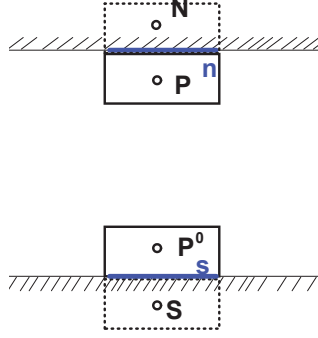


Figure 2.7: Cyclic boundary interpolation.

tions. For the case of heat transfer, a uniform heat-flux is applied on the wall. This is done through utilising the procedure of the Von-Neumann boundary condition.

Cyclic boundary condition

All the lateral boundaries considered in this thesis are cyclic boundaries where velocities and pressure are identical for the cyclic pairs. One such cyclic pair constituting of faces n and s is shown in figure 2.7. A linear interpolation can be easily implemented for the cyclic boundaries using the idea of fictitious cells as introduced earlier. Denoting the weighting factor for the cyclic boundary as, $f_{cyc} = \frac{|\vec{P}_n|}{|\vec{P}_S|}$, the following formula can be used to calculate the variables at the faces n and s .

$$\phi_n = f_{cyc}\phi_P + (1 - f_{cyc})\phi_{P^0} = \phi_s \quad (2.73)$$

and

$$\phi_S = \phi_P \quad (2.74)$$

$$\phi_N = \phi_{P^0} \quad (2.75)$$

2.9 Parallelisation Strategy

The code is parallelised using the PETSc library (a portable extensive toolkit for scientific computation) to run on distributed memory parallel computer architectures (Balay et al., 2014). PETSc is a collection of functions that can be called by an algorithm to perform parallelisation tasks. The domain decomposition method is used to decompose the computational domain into many individual blocks. Each block is assigned to a processor. Message Passing Interface (MPI) is used to exchange communication between the processors.

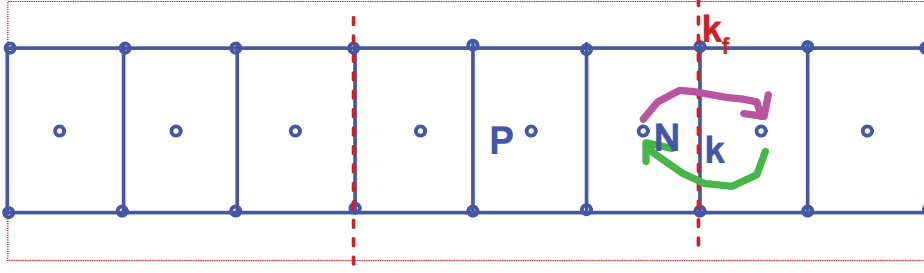


Figure 2.8: Parallelisation explained for one-dimensional domain.

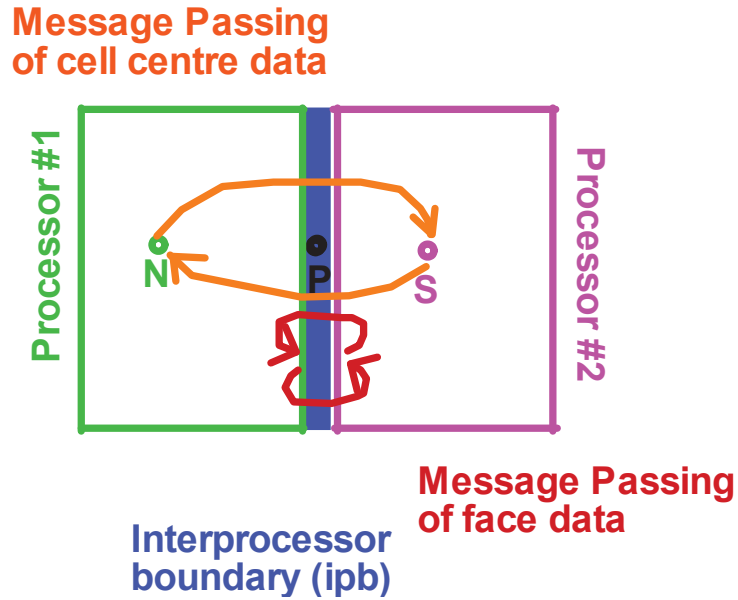


Figure 2.9: Two types of data exchange between the processors.

The domain decomposition is explained for a one-dimensional domain in figure 2.8. The domain decomposition technique splits the computational domain into $N_x \times N_y$ blocks where N_x and N_y are the number of processors in x and y directions respectively, and $N_P = N_x \times N_y$ where N_P is the total number of processors. In figure 2.8, the one-dimensional domain is divided into three processors. Therefore, $N_x = 3$ and $N_y = 1$ for this case. The domain decomposition is performed in a pre-processing stage. It is imperative to keep equal load balancing over the processors by carefully partitioning the mesh with approximately similar size blocks to all the processors. In this work, the unstructured mesh is partitioned manually into almost equal sized sub-domains. Each local control volume is renumbered locally. The vertices are also renumbered locally based on the numbering assigned to the control volumes.

Domain decomposition adds additional faces into the domain in the form of interprocessor faces. These faces are treated like the actual control volume boundary faces. The interprocessor faces share two cell centroids belonging to different processors as shown in figure 2.9. In this figure, the interprocessor face is between two cell centroids N and S where the centroid N is in the processor 1, and the centroid S is in the processor 2. The discretised form of governing equations is written for each of the interprocessor faces. The value of ϕ at the interprocessor face is obtained using the same interpolation method discussed for the internal faces. The computation of the interprocessor face value, and the implementation of Rhie-Chow method requires information about both the cell centroids on either side of the interprocessor boundary. Therefore, the processors have to communicate or exchange data between them. To this end, the MPI protocol is used. There are two types of data that are exchanged between the processes. They are:

1. data exchange of cell centroid values on either side of the interprocessor boundary
2. data exchange of face values of the interprocessor boundaries

In the cell centroids data exchange, the neighbouring cells that lie on either side of the interprocessor faces are identified first. Then the data of these cells are exchanged between the processors. For example in figure 2.9, the data of P is exchanged, therefore, this value can be easily accessible for the computation of boundary value, and subsequently for the interpolation. Although the cell centroids data exchange is sufficient for most operations, for some cases this data exchange is not enough. For instance, the implementation of Rhie-Chow method on the interprocessor boundary involves data exchange of velocity components, pressure, ϕ , pressure gradient, and a_f . These can be simplified by exchanging the face data instead of exchanging centroid data. Therefore, in the second type of data exchange, all the interprocessor faces share their information with the other processors. In Rhie-Chow method, the face velocity is first computed with the pressure gradient of that particular cell for the interprocessor face, and this data is exchanged with other processors. Therefore, the other processors that share the same face get this value, and add the face velocity with the other half of the pressure gradient for that processor, and thus the implementation of Rhie-Chow method is implemented with minimum of data exchange.

In figure 2.9, the boundary face of the interprocessor face is shown with shaded blue colour. The value of the variable in the interprocessor boundary is computed using an interpolation formula similar to the one used for the internal fluid cell elements. Considering the processor 1 in figure 2.9, the value at ϕ at the interprocessor boundary

N_P	Time for 100 time steps (s)	Speedup	Efficiency (%)
384	1015.3	1	100
768	569.7	1.78	89%
1152	481.8	2.11	70%

Table 2.1: Scalability details of the parallel solver.

(P), can be obtained as,

$$\phi_P = f_{xproc}\phi_N + (1 - f_{xproc})(\phi_S)_{exch} \quad (2.76)$$

In the above equation, $(\phi_S)_{exch}$ represents the fact that the value of ϕ_S has been informed to the processor 1 through data exchange. f_{xproc} is the weighting factor of the interprocessor faces which is computed at a pre-processing stage. It is computed using the formula:

$$f_{xproc} = \frac{\overline{PS}}{\overline{NS}} \quad (2.77)$$

Finally, the scalability of the parallel code is assessed by measuring the time taken to complete one iteration of the algorithm listed in section 2.7. In order to include both the calculation of algorithm and the communication between the processors, wall time is considered instead of CPU time. The speedup and efficiency of the parallel algorithm are defined as,

$$\text{Speedup} = \frac{T_{ref}}{T_{N_P}} \quad (2.78)$$

$$\text{efficiency} = \frac{N_{P_{ref}} \times T_{ref}}{N_P \times T_{N_P}} \% \quad (2.79)$$

where T_{ref} is the reference time, and $N_{P_{ref}}$ is the number of reference processors and T_{N_P} is the time taken when the code is run in N_P parallel processors.

Scaling and profiling tests using CRAYPAT, a performance analysis tool, were performed by the Scientific Computing Department of Daresbury Lab. Scaling information from Archer simulations is shown in table 2.1.

The case considered is the 3D flow around an airfoil at Reynolds 50,000 using about 30million cells. The computations were performed using the PISO algorithm. The profiling tests showed that most of the time (about 75%) is spent on Hypre multi grid pre-conditioner, a highly optimised library.

The code performance and scalability were recently improved by replacing the PISO algorithm with a fractional step (FS) method in which the coefficient matrices for the velocity and pressure equations are computed only once at the beginning of the simulation and remain the same at every time step. This means that the set up phase of the Hydre algebraic multi-grid pre-conditioner BoomerAMG (that builds the coarsening and interpolation operators based entirely on the elements of the matrix) is performed only once (at the beginning of the simulation) while in table 2.1 it is performed at every time step. This is an intensive and costly phase which does not scale well, and explain the loss of efficiency at larger processor counts. All simulations reported in the thesis were performed with a number of processors in which scaling is of the order of 90%.

2.10 Summary

A Finite Volume discretisation technique for arbitrarily unstructured meshes has been described. It allows the use of control volumes of arbitrary topology and shape, simplifying the problem of mesh generation for complex geometries. The discretisation approaches for the general transport equation were discussed. The proposed discretisation treatment is second order accurate in space and time. The Rhie and Chow interpolation method for the collocated variable arrangement and the procedure to obtain the equation for pressure were described. The adopted treatment of the pressure-velocity coupling is based on the PISO algorithm. Details on the implementation of boundary conditions were also presented. Finally, information regarding the parallel implementation of the algorithm along with the scalability of the parallel code was discussed.

Chapter 3

Genesis and spatial evolution of velocity gradients

3.1 Overview

The main objective of this chapter is to analyse the VGT dynamics of developing turbulence behind the single square grid-element. The nature of the flow behind the grid-element is studied first through the budgets of turbulent kinetic energy, mean enstrophy and mean strain-product. Then the relationship between the small-scale terms and the energy spectra along the grid-element centreline is analysed. Finally, the invariants of the velocity gradient tensor and the geometrical statistics of small-scale turbulence are presented and discussed.

3.2 Computational parameters

The square grid-element is used to generate the developing flow. A sketch of the grid-element with its basic dimensions and the coordinate system is shown in figure 3.1a. As noted earlier, the lateral thickness of the grid-element is $t_0=43\text{mm}$, while the length is $L_0 = 5.3t_0 = 229\text{mm}$. The thickness of the grid bar in the streamwise direction is 6mm.

The size of the computational domain is $L_x \times L_y \times L_z = 7L_0 \times 2L_0 \times 2L_0 = 1.4x/x^* \times 0.4x/x^* \times 0.4x/x^*$. The blockage ratio of the single square grid-element is 20%, which is the same as in the experiments reported in Laizet et al. (2015a). The grid-element is placed at a distance $1.75L_0$ from the inlet of the computational domain, which is sufficiently large compared to the previous DNS studies (Laizet et al., 2013, 2015a).

The work presented in this chapter has been published in **J. Fluid. Mech.** (Paul et al., 2017) with a title “Genesis and evolution of velocity gradients in a spatially developing turbulence”.

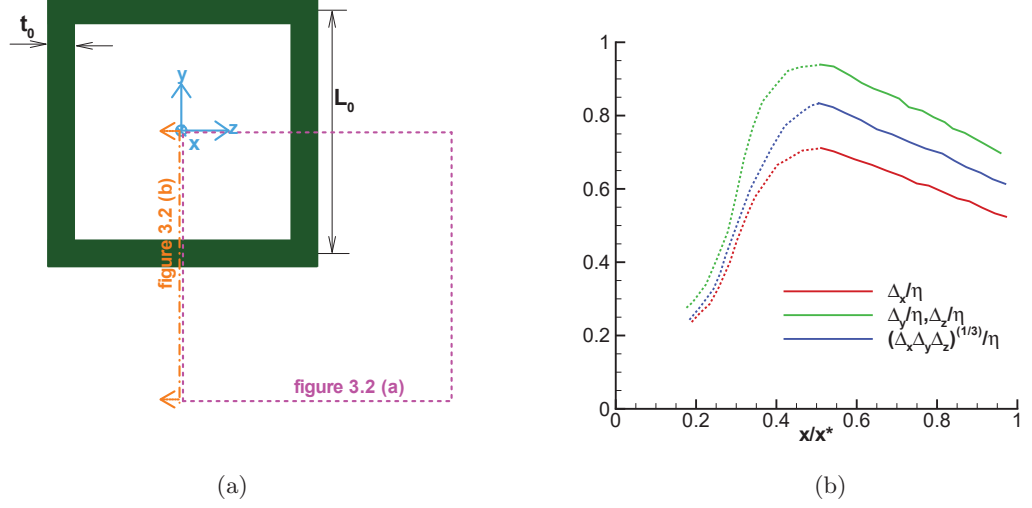


Figure 3.1: (a) Sketch of the single square grid-element and the coordinate system. The dotted box and the dot-dashed cut section are the locations where the computational mesh is shown in figure 3.2, (b) Mesh resolution along the grid-element centreline normalised with the local length scale, η , as a function of the normalised streamwise distance from the grid-element.

The Reynolds number based on the free-stream velocity U_∞ and the bar length L_0 is $Re_{L_0} = 2650$, while based on the lateral thickness t_0 is $Re_{t_0} = 500$. The time-step of the simulation is kept constant at $0.015U_\infty/t_0$, resulting in a maximum CFL number of 0.47. Here, the CFL number is defined as $CFL = \frac{|\mathbf{u}^*| \Delta t}{\Delta}$, where $|\mathbf{u}^*|$ is the velocity magnitude, Δt is the time step of the simulation and Δ is the local mesh resolution. Uniform velocity is prescribed at the inlet, while a convective outlet boundary condition is used at the exit. No-slip boundary conditions are applied on the walls, while all lateral boundaries are considered to be periodic.

The total number of computational cells are 20.4×10^6 . The mesh is body fitted and the cell faces align exactly with the bar walls. The flexibility offered by the unstructured meshing allows for optimal placement of control volumes. Figure 3.2 shows the computational mesh for the locations marked in figure 3.1a. The distance between the bar surface and the closest mesh point in the wall-normal direction is less than 0.15 wall units (i.e., $\Delta n^+ < 0.15$). This shows that the boundary layers developing on the bar surfaces are well resolved. The mesh is refined near the walls to resolve the shear layers that develop after separation from the front sharp corners. There are approximately 17 cells inside the shear-layer closer to the grid-element (around $x/x^* \approx 0.02$), while there are approximately 24 cells inside the shear-layer thickness around $x/x^* \approx 0.5$.

In the region where the wakes have met, the resolution of the present mesh is assessed

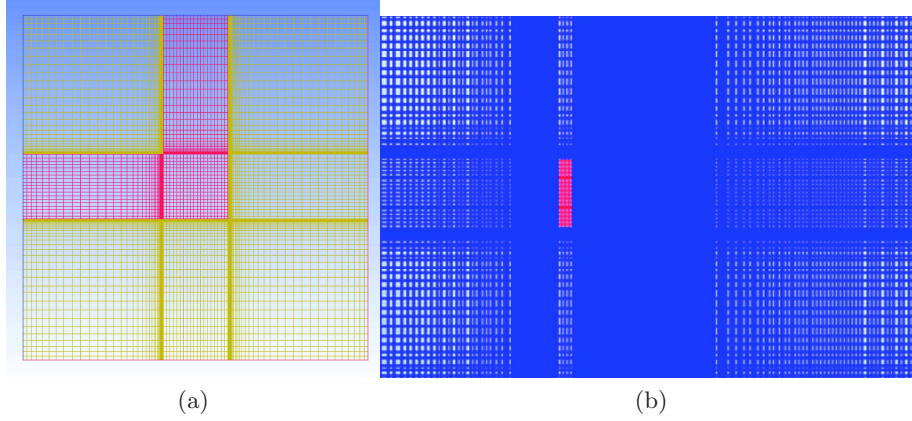


Figure 3.2: (a) Computational mesh in the $y - z$ plane around the lower, right corner of the grid-element (outline shown by dotted line in figure 3.1a), (b) Computational mesh at $x - y$ plane at the centre of the grid-element, through the lower bar (plane shown in figure 3.1a).

in terms of a local length scale (η) defined as the square root of the ratio between the kinematic viscosity and the local standard deviation of fluctuating strain-rate, i.e. $\eta = \sqrt{\nu / \sqrt{\langle s_{ij}s_{ij} \rangle}}$. This length scale can be used to characterise the spatial extent over which the local velocity fluctuations are smoothed out by viscosity and is, as such, of general applicability. It actually turns out that this length-scale is the same as the Kolmogorov microscale ($\eta = (\nu^3/\epsilon)^{1/4}$) in the case of fully developed turbulent flows. One can, therefore, use this length-scale to assess mesh resolution wherever there is fluctuating strain, including in the decaying region where the entire field is fully-turbulent. Figure 3.1b depicts the variation of ratios between the local mesh resolutions Δ_x , Δ_y , Δ_z , Δ (defined as $\Delta = (\Delta_x\Delta_y\Delta_z)^{1/3}$) and the local length scale, η for $x/x^* > 0.2$. The dotted lines denote the region where the turbulence is intermittent, and the solid lines represent the fully-turbulent decay region. The figure shows that the resolution in all directions is always less than one η along the grid-element centreline. Resolution requirements for a proper DNS have been reported in Moin and Mahesh (1998); Donzis et al. (2008). The latter authors mention that a standard resolution for the second order quantities (such as the rms velocities or the energy spectrum which takes very small values and falls off rapidly at wavenumbers $\kappa > 1/\eta$) is $\Delta x/\eta \approx 2$. For higher order moments of velocity derivatives and velocity increments (for example fourth order moments) resolution down to the Kolmogorov scale is required. It is expected, therefore, that the present mesh is sufficiently fine for the computation of all the terms in the balance equations for enstrophy and strain-product. Evidence for this is shown in section 3.4.

The flow is first allowed to develop for 1 flow-through time and then data are collected for 10 flow-through times (or 144,000 time steps) to ensure satisfactory statistical

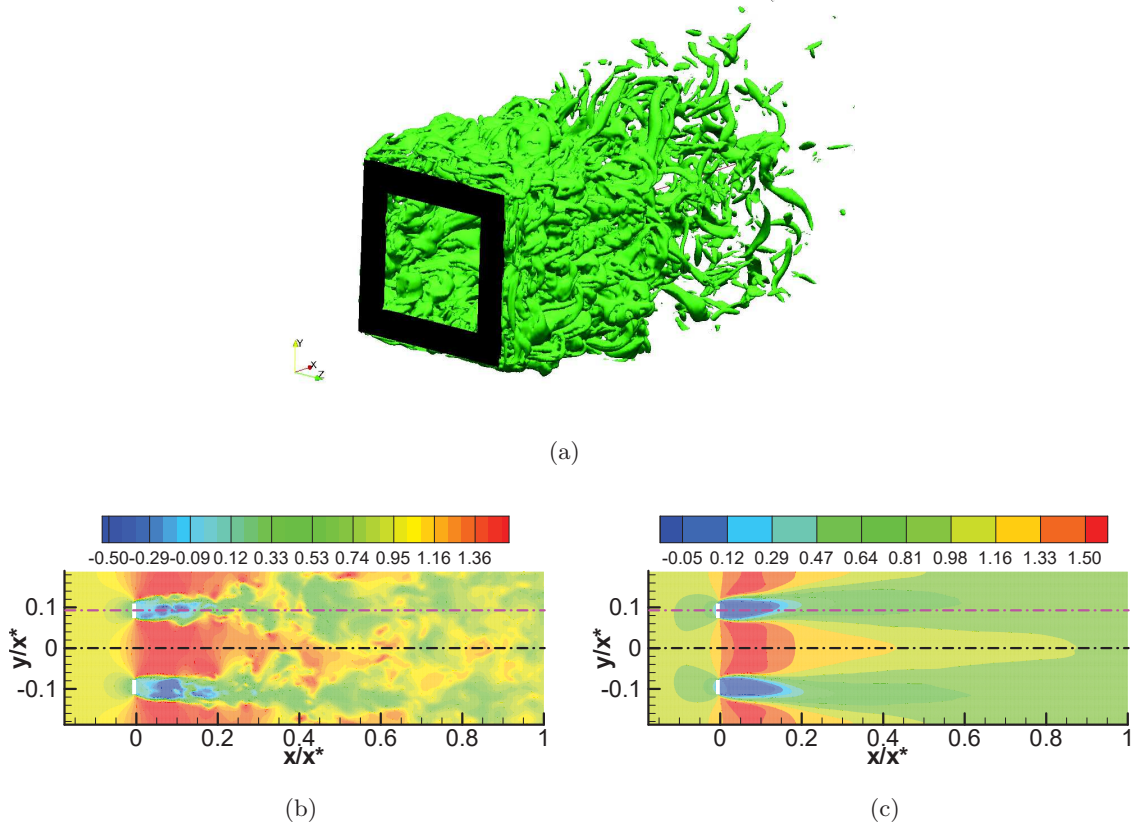


Figure 3.3: (a) Visualisation of turbulent coherent structures through isosurfaces of the fluctuating vorticity vector magnitude of $|\omega|=0.1$ (b) Instantaneous streamwise velocity contour at $z/x^* = 0$ (c) Mean streamwise velocity contour at $z/x^* = 0$. The bar and grid-element centrelines in (b) and (c) are marked by magenta and black dotted-dash lines respectively.

convergence. The simulation was carried out at the United Kingdom national computing facility, Archer, using 448 processors for approximately 1500 clock hours. A total of 2.2TB three-dimensional data were stored for every 4 time steps of the simulation.

3.3 Comparison with experiments

The three-dimensional instantaneous flow structure behind the grid-element is visualised using the fluctuating vorticity magnitude $|\omega|$ in figure 3.3a. The flow contains vortical structures of various scales. Figure 3.3b shows instantaneous streamwise velocity contours at plane $z/x^* = 0$. The black and pink dotted-dash lines in the figure represent the grid-element centreline and the bar centreline respectively. The two wakes behind the bars are clearly seen. The wakes are initially separated by a jet that emerges from the gap between the bars, but as the wakes grow downstream, they meet intermittently and begin to interact. This process starts at about $x/x^* \approx 0.2$. Contours of the time-averaged

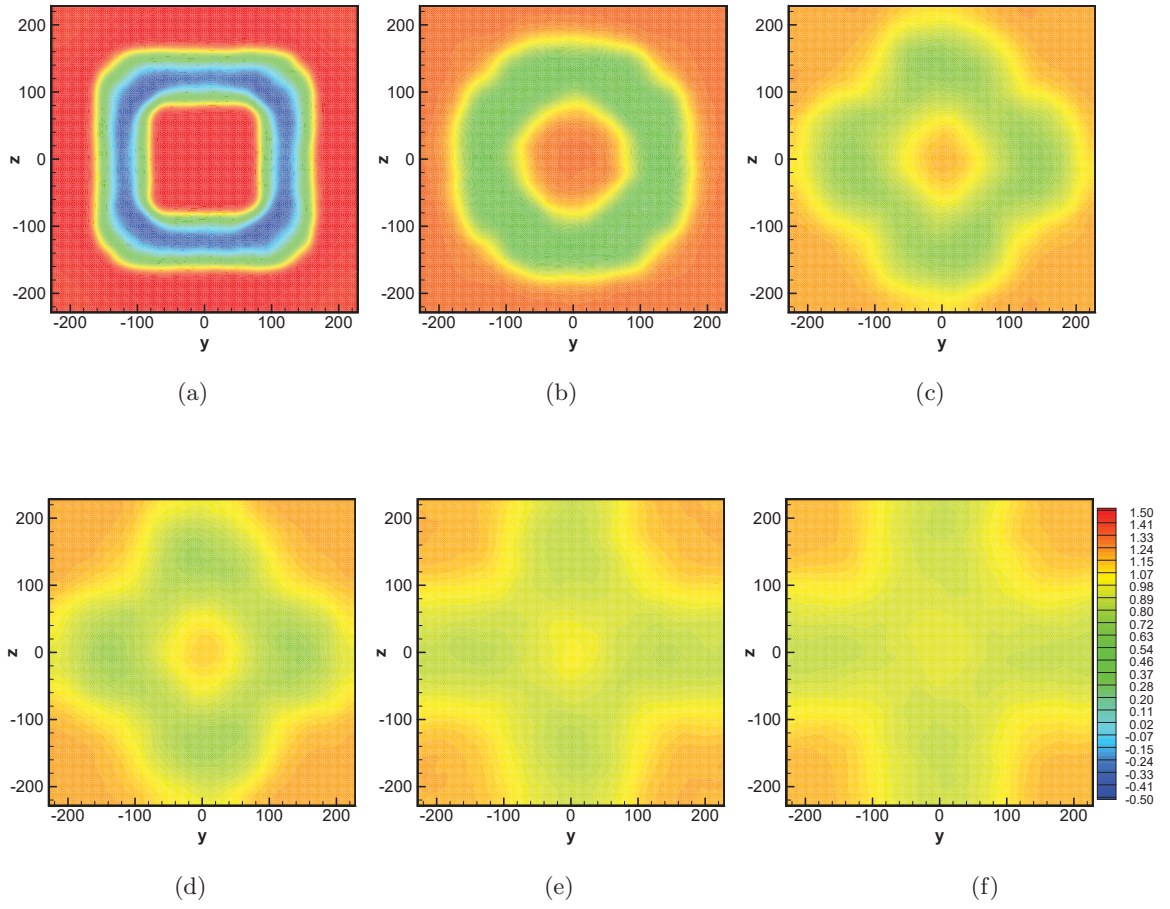


Figure 3.4: Contours of mean velocity in the yz plane at different x/x^* locations: (a) 0.1 (b) 0.2 (c) 0.4 (d) 0.5 (e) 0.75 (f) 0.9

streamwise velocity are shown in figure 3.3c. The flow is highly inhomogeneous in the lee of the grid-element, but as the wakes interact dispersing momentum in the cross-stream direction, it becomes more homogeneous further downstream and is already quite homogeneous at $x \approx x^*$, in agreement with the experimental observations of Seoud and Vassilicos (2007). Evolution of inhomogeneous flow to a near-homogeneous state along the downstream can be better visualised in figure 3.4 which shows the contours of mean streamwise velocity in the $y-z$ plane at different x/x^* locations. Around $x/x^*=0.9$, the flow is approximately homogeneous (see figure 3.4f) while in the near-field it is highly inhomogeneous (see figure 3.4a).

In order to validate the present DNS, one-point velocity statistics are compared with the experiments reported in Laizet et al. (2015a). Figure 3.5a shows the evolution of the time-averaged streamwise velocity (U_1) normalised with the free-stream velocity (U_∞) along the grid-element centreline. The velocity profile obtained from the DNS solver agrees well with the experimental measurements carried out for Reynolds number at least ten times larger than that of the present study. The velocity profile exhibits a jet-like behaviour close to the grid-element, followed by a monotonic decay in the downstream. Note the strong acceleration (by almost 60%) of the streamwise velocity compared to U_∞ in the lee of the grid-element. The velocity profile peaks around $x/x^* \approx 0.1$ (i.e. $x/L_0 \approx 0.5$), and the velocity peak-magnitude is expected to depend on the grid-element blockage and the mesh resolution. For example in the simulations of Laizet et al. (2015a), this peak is underestimated by about 10% when the coarsest mesh is employed. The effect of blockage on the acceleration is strong: the square grid-element of Zhou et al. (2014a, 2016a) has a blockage of 11% (nearly half compared to the blockage of this study) and their acceleration is less than 20%.

Note also that for $x/x^* > 0.4$ the simulations underpredict the experiments by about 5%. This can be explained by the difference between the experimental and computational set ups. As mentioned earlier, all lateral boundaries are periodic in the simulations, while they are solid walls in the experiments. As the wakes expand, they tilt slightly towards the top and bottom boundaries, as seen in figure 3.3c. Previous simulations (Zhou et al., 2014a) have reported the same finding. In the experiments, the boundary layers that form along the lateral boundaries act as a blockage to the flow. As a result, the mean velocity increases in order to maintain a constant mass flow rate. This leads to a slight underprediction of the experiments. Valente and Vassilicos (2014) compared the mean and rms profiles of the wakes interacting with each other as well as with the lateral wall versus the wakes interacting each other in a quasi-periodic arrangement. They reported that the lateral wall surfaces have a meaningful effect on mean and rms velocity profiles only in the far downstream region (i.e., after $x/x^* > 1.25$ as far as

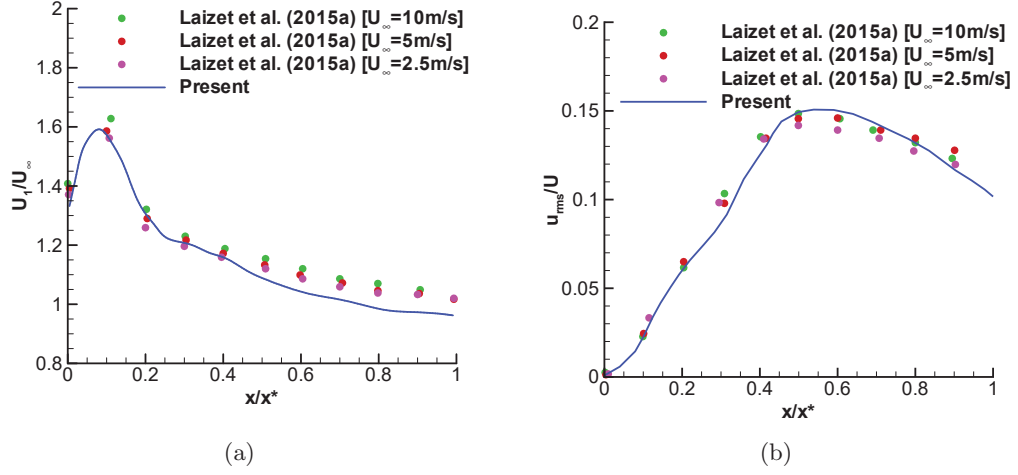


Figure 3.5: Comparison of (a) mean velocity and (b) streamwise turbulent intensity along the grid-element centreline with measurements.

the current simulation is concerned). They also found that the location where the bar wakes meet is not altered by differences in the lateral boundaries. Therefore, periodic boundary conditions are imposed on the lateral boundaries in order to avoid the large computational cost associated with the resolution of the near wall region. Note that the previous numerical studies on grid turbulence also have periodic lateral boundaries (Zhou et al., 2014a,b, 2015, 2016a,b; Laizet et al., 2013, 2015a).

Figure 3.5b depicts the evolution of the streamwise rms velocity (u_{rms}) normalised by the local mean velocity (not U_∞) along the grid-element centreline. The turbulent intensity increases from the lee of the grid-element until $x/x^* \approx 0.5$. The region between $x/x^*=0$ and $x/x^*=0.5$ is termed the production region, and the location where the turbulent intensity attains its maximum value is called the peak location. After the peak, u_{rms} decreases in the decay region. Experiments and predictions match reasonably well. The aforementioned effect of boundary layer growth has been compensated by the normalisation with the local velocity. Note that at $x/x^* = 0$, the predicted rms value of fluctuating velocity matches well with the experiment (both values are close to 0). This means that no artificial fluctuations are produced by the discretization method upstream of the grid-element. The growth of rms velocity from $x=0$ to $x \approx 0.5x^*$ is also well predicted. Simulating this region is particularly challenging because the flow transitions from laminar to turbulent as the turbulent/non-turbulent interfaces from the bar wakes sweep this region. The success in correctly simulating this region further demonstrates that the employed mesh resolution is adequate.

Figure 3.6 shows the p.d.f.s of the streamwise velocity fluctuation at three different locations. The p.d.f.s in the production and peak region show negative skewness and

x/x^*	0.1	0.25	0.35	0.5	0.75	0.95
Re_λ	10.12	22.01	34.49	37.58	39.12	33.12
u_{rms}/U_1	0.02	0.05	0.11	0.15	0.13	0.11
$S(\partial u_1/\partial x)$	-0.11	-0.24	-0.32	-0.34	-0.42	-0.46

Table 3.1: Basic turbulence parameters at the six locations considered grid-element centreline. Here Re_λ is the Taylor length scale based Reynolds number and $S(\partial u_1/\partial x)$ is the skewness of the fluctuating streamwise velocity derivative.

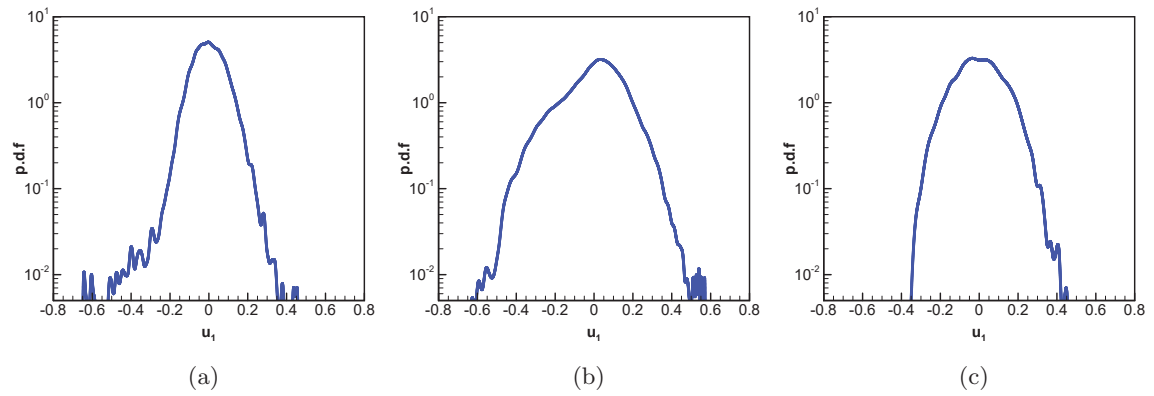


Figure 3.6: Pdf of velocity fluctuation at: (a) $x/x^*=0.25$ (b) $x/x^*=0.5$ (c) $x/x^*=0.9$.

large flatness with a non-Gaussian profile (figures 3.6a and 3.6b). The Gaussian nature of the p.d.f can be seen only in figure 3.6c for the decay region. These trends can also be noted in figures 3.7a and 3.7b which represent the streamwise evolution of skewness and flatness of velocity fluctuation along the grid-element centreline respectively. The values corresponding to Gaussian fluctuating velocities are shown as dotted lines in figures 3.7a and 3.7b. The skewness (S) and Flatness (F) are defined as, $S(u_1) = \frac{\langle u_1^3 \rangle}{\langle u_1^2 \rangle^{3/2}}$, $F(u_1) = \frac{\langle u_1^4 \rangle}{\langle u_1^2 \rangle^2}$ respectively. It can be seen from the figures that the turbulence is highly non-Gaussian in the production region, and then slowly returns to Gaussianity further downstream (it is reminded that the skewness and flatness of a Gaussian random variable are 0 and 3 respectively). The comparison with the experiments is (perhaps surprisingly) good, especially for the flatness. The profiles are smooth which indicates good convergence of the statistics. Reynolds number effects are the strongest in the location closest to the lee of the grid-element ($x/x^* = 0.1$) and this is the region with the largest deviation from the experiments. Further downstream, there is still some dependence on Reynolds number, especially for the skewness, but for flatness this dependency is weak and the matching with the experiments is significantly better. The skewness of fluctuating velocity $S(u_1)$ is negative and decreases in the production region till $x/x^* \approx 0.4$. The value of $S(u_1)$ remains non-zero but increases continuously towards zero further downstream in agreement with the simple closure calculation of Maxey (1987) for a decaying homogeneous turbulence. In the work of Zhou et al. (2014a), the largest negative skewness and positive flatness occur around $x/x^* \approx 0.2$, while in the present case they occur later, at $x/x^* \approx 0.4$. This difference is attributed to the locations where the wakes start to meet. In their low blockage ratio grid-element, the wakes meet at $x/x^* \approx 0.1$ while the wakes meet at $x/x^* \approx 0.2$ in the present case.

All three regions (production, peak and decay) are probed by choosing six different spatial locations along the grid-element centreline. These x/x^* coordinates along with their basic turbulence parameters are provided in table 3.1. The first three points (with $x/x^*=0.1, 0.25$, and 0.35) lie inside the production region and have the following characteristics (in the ascending order of x/x^*): the point closest to the grid-element; the point where a slope close to $-5/3$ in the energy spectrum is first observed; the point in the production region with similar Re_λ as the last point in the decay region. The fourth point ($x/x^*=0.5$) is located at the turbulence peak. The final two locations ($x/x^*=0.75$ and 0.95) correspond, respectively, to: the point which has the maximum Re_λ ; and the point in the decay region with about the same Re_λ as the point 3.

Table 3.1 also records the values of skewness of fluctuating streamwise velocity derivative, $S(\partial u_1/\partial x)$, at all stations. This skewness is very small near the grid-element but in the decay region, it attains values which are closer to those reported in the lit-

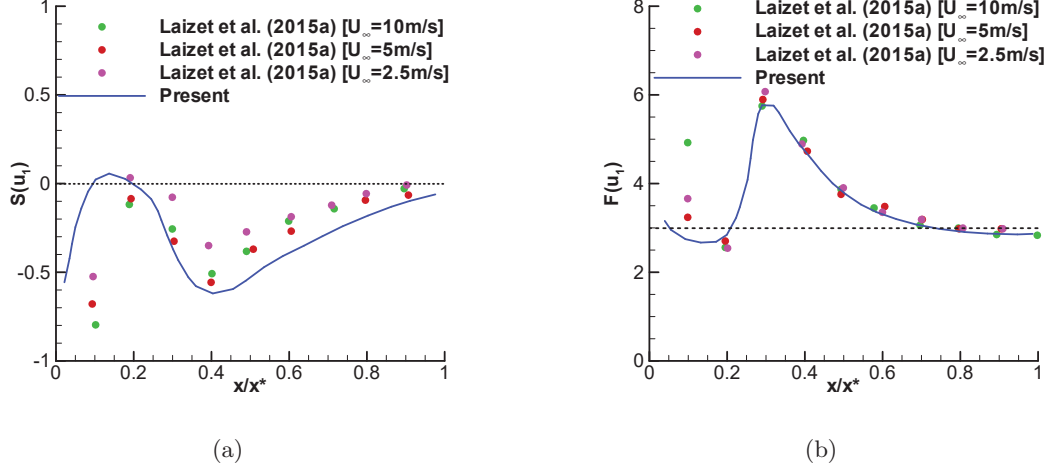


Figure 3.7: Comparison of (a) skewness and (b) flatness of streamwise fluctuating velocity along the grid-element centreline with experiments.

erature for homogeneous turbulence. Indeed, Tavoularis et al. (1978) reported that the value of $S(\partial u_1/\partial x)$ lies in the range -0.35 to -0.45 for relatively low Reynolds number turbulence, a fact also recorded in the review of Sreenivasan and Antonia (1997). The present results are in line with these values in the decay region where the turbulence is most homogeneous.

3.4 Budgets of Turbulent Kinetic Energy, Enstrophy, and Strain-rate

The symmetric (strain-rate) and antisymmetric (vorticity) parts of VGT are analysed through the budgets of mean strain-product and mean enstrophy respectively. As a precursor, the budget of mean kinetic energy is discussed first.

3.4.1 Turbulent kinetic energy balance

The transport equation of the turbulent kinetic energy is (refer to Tennekes and Lumley (1972)),

$$\begin{aligned}
 \frac{\partial}{\partial t} \left(\frac{1}{2} \langle u_i u_i \rangle \right) = & \underbrace{-U_j \frac{\partial}{\partial x_j} \left(\frac{1}{2} \langle u_i u_i \rangle \right)}_{C_k} \underbrace{- \frac{\partial}{\partial x_j} \left(\frac{1}{\rho} \langle u_j p \rangle \right)}_{\pi_k} \\
 & \underbrace{- \frac{\partial}{\partial x_j} \left(\frac{1}{2} \langle u_i u_i u_j \rangle \right)}_{T_k} \underbrace{+ \frac{\partial}{\partial x_j} (2\nu \langle u_i s_{ij} \rangle)}_{D_k} \underbrace{- \langle u_i u_j \rangle S_{ij}}_{P_k} \underbrace{- 2\nu \langle s_{ij} s_{ij} \rangle}_{\epsilon_k}
 \end{aligned} \tag{3.1}$$

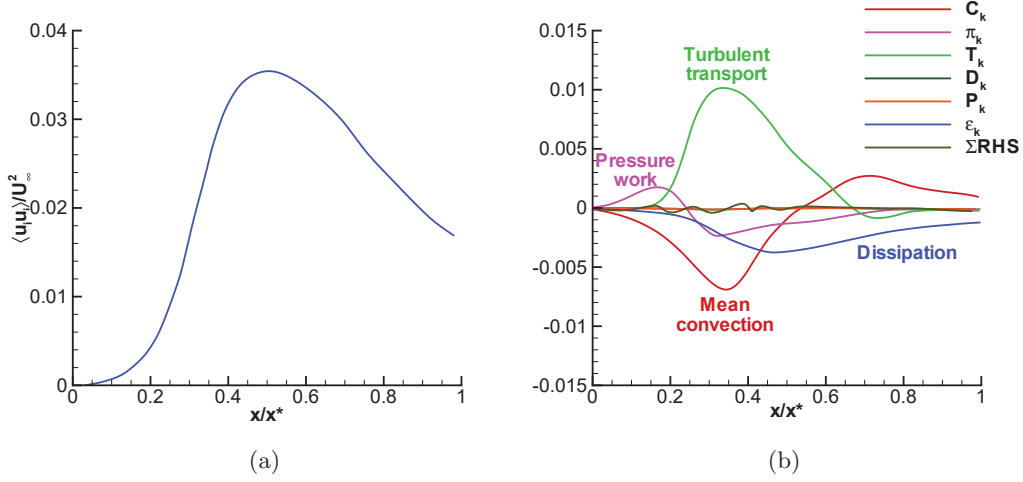


Figure 3.8: (a) Evolution of the turbulent kinetic energy along the grid-element centreline (b) Turbulent kinetic energy budget along the grid-element centreline (all terms are normalised by U_∞^3/t_0).

where S_{ij} is the mean strain-rate which is given as,

$$S_{ij} = \frac{1}{2} \left(\frac{\partial U_i}{\partial x_j} + \frac{\partial U_j}{\partial x_i} \right). \quad (3.2)$$

In equation (3.1), C_k denotes the convection due to mean flow, π_k represents the pressure work, T_k corresponds to the transport by fluctuations, D_k stands for viscous diffusion, the production of turbulent kinetic energy by mean flow is P_k , and the dissipation of turbulent kinetic energy is denoted by ϵ_k .

The streamwise evolution of the turbulent kinetic energy along the grid-element centreline is shown in figure 3.8a. In the production region, the turbulent kinetic energy increases from zero to a peak value. This increase is gradual for $x/x^* < 0.2$ where the flow is mostly irrotational, and steeper for $0.2 \leq x/x^* \leq 0.4$ where the turbulence is developing. The kinetic energy decreases in the decay region (i.e., for $x/x^* > 0.5$).

The budget of turbulent kinetic energy along the grid-element centreline is depicted in figure 3.8b where all the terms in equation (3.1) are normalised by U_∞^3/t_0 . For statistically stationary flows, as the one considered in this study, the transient term on the left-hand side is zero. Figure 3.8b shows that the grid-element centreline turbulent kinetic energy budget is markedly different from some previously reported budgets for boundary layer (Mansour et al., 1988) and backward facing step (Le et al., 1997). The distinguishing feature here is the absence of production term, P_k . The turbulent kinetic energy along the grid-element centreline is not the result of mean-flow gradients producing turbulent fluctuations, as in wall-bounded shear flows. In the region $0 \leq x/x^* < 0.2$,

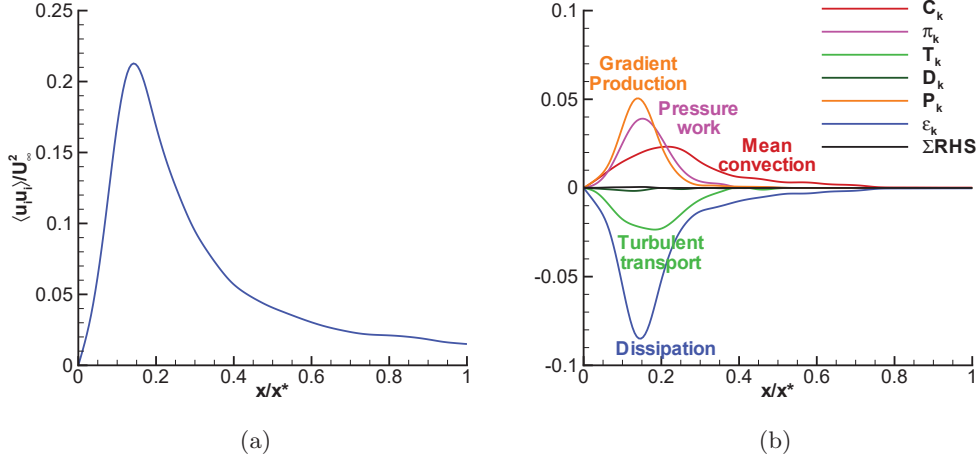


Figure 3.9: (a) Evolution of turbulent kinetic energy along the bar centreline (b) Turbulent kinetic energy budget along the bar centreline (all terms are normalised by U_∞^3/t_0)

the rise of energy is due mainly to pressure work. As shown in the next section, this part of the grid-element centreline is dominated by the irrotational flow. Around $x/x^* \approx 0.2$, the transport by turbulent fluctuations becomes the term responsible for the turbulent kinetic energy growth. This is the direct result of bar wakes starting to reach the grid-element centreline and meet around that location and thereby transporting turbulent kinetic energy towards the grid-element centreline. From $x/x^* \approx 0.2$ to the end of the production region, transport by fluctuations is counter-acted by viscous dissipation, pressure work, and mean advection. At the start of the decay region, the mean flow advection term changes its sign and is mainly balanced by viscous dissipation in most of the decay region. The budget analysis reveals that the flow along the grid-element centreline is initially irrotational and becomes turbulent due to lateral infiltration of turbulent wakes from the grid-element bars.

The turbulent kinetic energy profile along the bar centreline is given in figure 3.9a. The turbulent intensity increases up to $x/x^* \approx 0.13$ along the bar centreline due to the deformation of the Reynolds stress tensor by the mean velocity gradients (production term, P_k) and the pressure work, π_k (refer to figure 3.9b). Notice that P_k is now the dominant term, but π_k is still important. The contribution of P_k would have been even stronger if the budget analysis was performed along the core of the shear layer emanating from the top (or the bottom) face of the bar. This region has the strongest strain-rate, refer to figure 3.3. Near the bar, P_k and π_k are balanced by the sum of turbulent transport and dissipation. Further downstream, the mean turbulent kinetic energy starts to decay as the contribution of pressure-work and mean velocity gradients

decreases. In the far downstream (i.e., after $x/x^* > 0.4$), the mean flow carries the turbulent kinetic energy, and it is balanced by dissipation. Figures 3.8b and 3.9b also show that the mean turbulent kinetic energy equation is properly balanced in the present simulation.

3.4.2 Mean enstrophy balance

The transport equation for mean enstrophy $\langle \omega_i \omega_i \rangle$ is given by (refer to Tennekes and Lumley (1972)),

$$\begin{aligned}
 \frac{\partial}{\partial t} \left(\frac{1}{2} \langle \omega_i \omega_i \rangle \right) = & \underbrace{-U_j \frac{\partial}{\partial x_j} \left(\frac{1}{2} \langle \omega_i \omega_i \rangle \right)}_{C_\omega} - \underbrace{\langle u_j \omega_i \rangle \frac{\partial \Omega_i}{\partial x_j}}_{G_\omega} \\
 & - \underbrace{\frac{1}{2} \frac{\partial}{\partial x_j} \langle u_j \omega_i \omega_i \rangle}_{T_\omega} + \underbrace{\langle \omega_i \omega_j s_{ij} \rangle}_{P_\omega} + \underbrace{\langle \omega_i \omega_j \rangle S_{ij}}_{\alpha_\omega} + \underbrace{\langle \omega_i s_{ij} \rangle \Omega_j}_{\beta_\omega} \\
 & + \underbrace{\nu \frac{\partial^2}{\partial x_j \partial x_j} \left(\frac{1}{2} \langle \omega_i \omega_i \rangle \right)}_{\nu_\omega} - \underbrace{\nu \langle \frac{\partial \omega_i}{\partial x_j} \frac{\partial \omega_i}{\partial x_j} \rangle}_{\epsilon_\omega}
 \end{aligned} \tag{3.3}$$

where Ω_i is the components of the time-averaged vorticity vector and is given by

$$\Omega_i = \epsilon_{ijk} \frac{\partial U_k}{\partial x_j} \tag{3.4}$$

where ϵ_{ijk} is the Levi-Civita symbol. In equation (3.3) C_ω is the advection by mean flow, G_ω represents the production by mean field, T_ω is the transport by turbulent fluctuations, P_ω is the enstrophy production by stretching of fluctuating strain-rate, α_ω corresponds to the production (or removal) of enstrophy by stretching (or squeezing) due to mean strain-rate, β_ω is the mixed production term, while ν_ω and ϵ_ω represent viscous diffusion and the dissipation of enstrophy respectively.

The profile of mean enstrophy along the grid-element centreline is depicted in figure 3.10a. In the lee of the grid-element ($x/x^* < 0.2$) the enstrophy is almost zero. Therefore, the flow in this part of the grid-element centreline may be considered irrotational, which also implies non-turbulent. The mean enstrophy increases slowly after $x/x^* > 0.2$ and it continues to increase until the turbulence peak at $x/x^* = 0.5$. Further downstream in the decay region, the enstrophy continuously decreases.

Further physical insight on the observed variation of enstrophy can be gained from the budget of enstrophy (figure 3.10b). Every term in equation (3.3) is normalised by $(U_\infty/t_0)^3$. The budget demonstrates that all the terms are nearly zero for the irrotational flow part of the grid-element centreline (i.e., for $x/x^* < 0.2$). Moving further from this

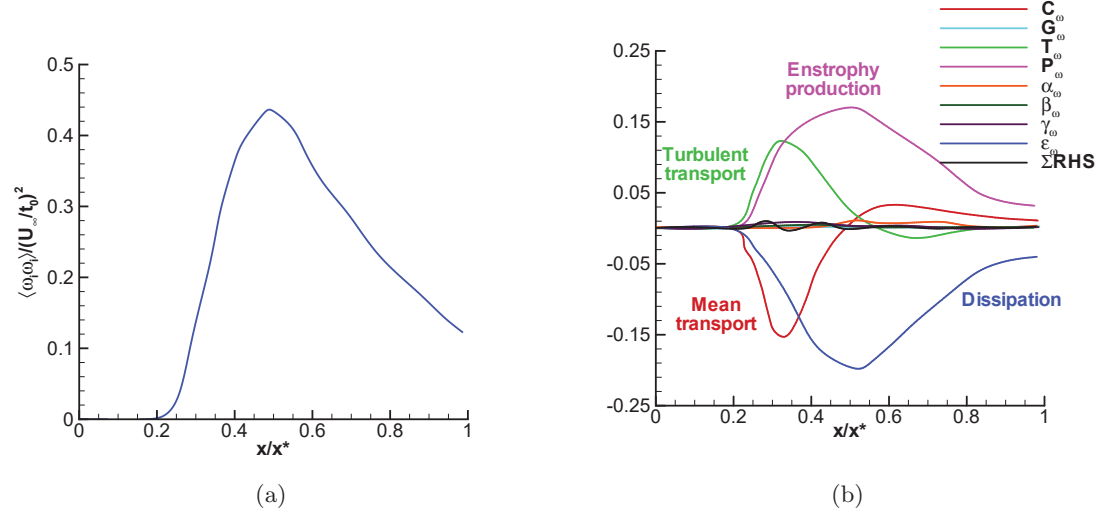


Figure 3.10: (a) Evolution of mean enstrophy along the grid-element centreline. (b) Budget of mean enstrophy along the grid-element centreline (all terms are normalised by $(U_\infty/t_0)^3$).

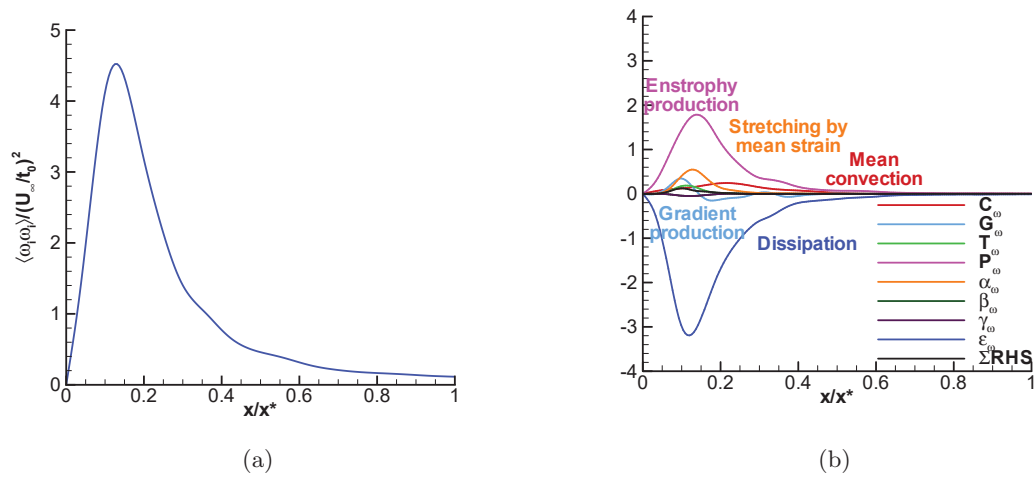


Figure 3.11: (a) Evolution of mean enstrophy along the bar centreline. (b) Budget of mean enstrophy along the bar centreline (all terms are normalised by $(U_\infty/t_0)^3$).

location, the first term in space to contribute to the generation of mean enstrophy is the turbulent transport term, T_ω . The physical mechanism, therefore, is clear: vorticity, which is produced at the grid-element surface due to the no-slip boundary condition, is first shed into bar wakes and then transported towards the grid-element centreline through the bar wakes that disperse vorticity in the lateral directions. The vorticity reaches the grid-element centreline at $x/x^* \approx 0.2$. Interestingly, the skewness and flatness (shown in figure 3.7) are close to their Gaussian values of 0 and 3 respectively exactly at this point and deviate from them further downstream in the production region. In particular, the skewness becomes negative which indicates the presence of strong negative fluctuations. These decelerations correlate with the intermittent interaction between the two wakes in the part of the production region where $x/x^* \approx 0.2$ (see Melina et al. (2016a) for further information on this point).

Shortly after the action of T_ω , the other terms in the transport equation start to become active. For example, after $x/x^* = 0.25$, the fluctuating strain field stretches the (small amount of) existing transported vorticity causing the further production of enstrophy (P_ω term). Both T_ω, P_ω are positive leading to the very rapid growth of $\langle \omega_i \omega_i \rangle$ (figure 3.10a). These two terms are balanced by advection due to mean flow and viscous dissipation.

Turbulent and mean transport terms grow in the production region but are reduced to small values at the peak location. Further downstream in the decay region, P_ω and ϵ_ω become the dominant terms that balance each other. It is worth mentioning here that the terms involving mean velocity gradients do not contribute to the transport of enstrophy anywhere on the grid-element centreline even in the production region, although this region is highly inhomogeneous.

Figure 3.11 illustrates the dynamics of fluctuating enstrophy along the bar centreline. The mean enstrophy profile (figure 3.11a) appears to be qualitatively similar to that of the bar-centreline turbulent kinetic energy with values peaking at the same location $x/x^* \approx 0.13$. Figure 3.11b reveals that once the vorticity is shed into the bar wake, it is continuously stretched by the fluctuating strain-rate throughout the bar centreline. Close to the bar, the increase in mean enstrophy is mostly because of the vortex stretching (P_ω term). Mechanisms such as the stretching of vorticity by mean strain (α_ω), and the production due to mean vorticity gradients (G_ω) also contribute to the growth of enstrophy in the near-bar region although their contribution is small compared to that of the vortex stretching. Further downstream, the vortex stretching term is in balance with the dissipation of enstrophy, and all other terms are negligible (figure 3.11b). Although the mean and turbulent transport terms are the significant budget terms along the grid-element centreline, their contribution to the mean enstrophy budget is minimum along

the bar centreline. This budget analysis also shows that the mixed production term (β_ω) does not contribute to the transport of mean enstrophy along both the grid-element centreline and the bar centreline.

Tennekes and Lumley (1972) predicted that P_ω and ϵ_ω are the dominant terms in equation (3.3). Their prediction appears to be valid on the bar centreline. On the grid-element centreline, however, the current DNS reveals that the turbulent and mean transport terms are also equally important, in particular in the production region. All other terms are predicted to be significantly smaller by Tennekes and Lumley (1972) and this study confirms this.

The balancing of the mean enstrophy equation (figures 3.10b and 3.11b), while not absolutely perfect, the order of magnitude of ΣRHS is very small compared with all other terms, which makes these results acceptable. The choice of low Reynolds number has been essential in achieving this and it is the first time such a point balance is reported in a study of a spatially developing flow. For channel flow, it has been reported by Sandham and Tsinober (2000).

3.4.3 Mean strain-product balance

The transport equation for mean strain-product $\langle s_{ij}s_{ij} \rangle$ is (refer to Tsinober (2009)),

$$\begin{aligned} \frac{\partial}{\partial t} \left(\frac{1}{2} \langle s_{ij}s_{ij} \rangle \right) = & \underbrace{-U_k \frac{\partial}{\partial x_k} \left(\frac{1}{2} \langle s_{ij}s_{ij} \rangle \right)}_{C_s} - \underbrace{\langle u_k s_{ij} \rangle \frac{\partial S_{ij}}{\partial x_k}}_{\alpha_s} \\ & - \underbrace{\frac{1}{2} \frac{\partial}{\partial x_k} \langle u_k s_{ij}s_{ij} \rangle}_{T_s} - \underbrace{2 \langle s_{ij}s_{ik} \rangle S_{kj}}_{\beta_s} - \underbrace{\frac{1}{2} \langle \omega_i s_{ij} \rangle \Omega_j}_{\nu_s} - \underbrace{\langle s_{ij}s_{jk}s_{ki} \rangle}_{P_s} \\ & - \underbrace{\frac{1}{4} \langle \omega_i \omega_j s_{ij} \rangle}_{\omega_s} - \underbrace{\langle s_{ij} \frac{\partial^2 p}{\partial x_i \partial x_j} \rangle}_{\pi_s} + \underbrace{\nu \langle s_{ij} \nabla^2 s_{ij} \rangle}_{\epsilon_s} \end{aligned} \quad (3.5)$$

where C_s is the mean flow advection, α_s is the production by mean strain field, T_s is the turbulent transport, β_s is the production due to stretching by mean strain-rate, ν_s is the mixed production term, P_s is the production due to stretching by fluctuating strain-field (also known as the strain self-amplification term). The enstrophy production term (ω_s) also appears in the transport equation of mean strain-product but with a negative sign and premultiplied by $(-1/4)$, π_s is the strain pressure-Hessian correlation term and ϵ_s is the viscous dissipation term.

Figure 3.12a depicts the profile of mean strain-product along the grid-element centreline. The profile of $0.5 \langle \omega_i \omega_i \rangle$ is also plotted in the same figure. Although the flow in the vicinity of grid-element ($x/x^* < 0.2$) does not contain any significant enstrophy, it does

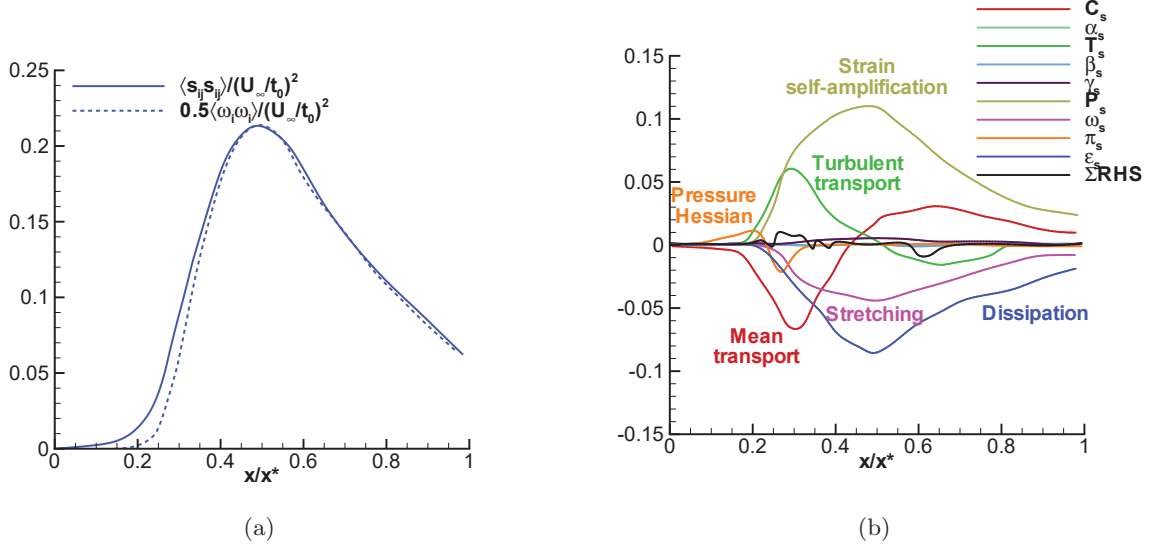


Figure 3.12: (a) Evolution of mean strain-product along the grid-element centreline (b). Budget of mean strain-product along the grid-element centreline (all terms are normalised by $(U_\infty/t_0)^3$).

possess strain (figure 3.12a). The dissipation of turbulent kinetic energy is directly proportional to the strain product, $\epsilon_k = 2\nu\langle s_{ij}s_{ij} \rangle$, so this is a region of mostly irrotational dissipation. The strain increases rapidly after $x/x^* > 0.2$ until the peak region, and then decreases in the decay region. Note that $0.5\langle \omega_i\omega_i \rangle \approx \langle s_{ij}s_{ij} \rangle$ is not valid in the production region (see also table 3.3). Recall that in homogeneous turbulence it is straightforward to prove analytically that $0.5\langle \omega_i\omega_i \rangle = \langle s_{ij}s_{ij} \rangle$ (refer to section 1.2.2). The dominance of strain-product over enstrophy in the production region is also observed in the low-blockage grid-element simulations of Zhou et al. (2016b).

Figure 3.12b shows the budget of mean strain-product along the grid-element centreline. Each term is normalised by $(U_\infty/t_0)^3$. Unlike the budget of enstrophy, some of the terms are active in the region of $x/x^* < 0.2$. It is of interest to magnify this region to see what really contributes to the generation of small-scale strain where there is no vorticity. A magnified view of figure 3.12b for $x/x^* < 0.2$ is shown in figure 3.13. The first term to become active along the grid-element centreline is the strain pressure-Hessian. This term is balanced by nothing else than the convection due to mean flow for $x/x^* < 0.14$. The pressure-Hessian acts on the fluid elements to produce strain in the region where enstrophy is absent. Therefore, the mean strain-product equation for $x/x^* < 0.14$ can be simplified as,

$$U_k \frac{\partial}{\partial x_k} \left(\frac{1}{2} \langle s_{ij}s_{ij} \rangle \right) \approx - \langle s_{ij} \frac{\partial^2 p}{\partial x_i \partial x_j} \rangle. \quad (3.6)$$

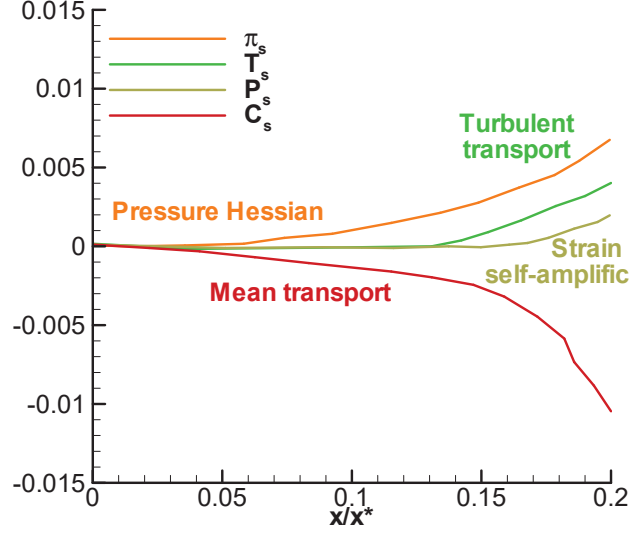


Figure 3.13: Budget of mean strain-product for $x/x^* < 0.2$ along the grid-element centreline.

Note that the pressure Hessian contains isotropic and deviatoric parts. It is only the deviatoric part causes non-local effects (Cantwell, 1992; Ohkitani and Kishiba, 1995). Therefore, in order to better understand the role of pressure Hessian in the near-field, the budget equation of (3.6) should be considered with the decomposed pressure Hessian parts in the future studies. The importance of the pressure Hessian term in the vicinity of the grid-element was observed by Zhou et al. (2015) when the conditional mean trajectories of the Q, R invariants in the $Q - R$ plane were computed. It was not however directly identified as the dominant generating mechanism for strain in the lee of the grid-element, as shown in the present study.

Once the strain is produced by the pressure-Hessian, it is then transported by fluctuations as can be seen from figure 3.13, where the T_s term becomes active after $x/x^* \approx 0.14$, acting to increase $\langle s_{ij}s_{ij} \rangle$. This transport, however, is not due to the lateral fluctuations, but due to the streamwise fluctuations which are generated by the pressure-work mechanism as discussed in section 3.4.1. The increase of small-scale strain leads to turning on of the strain self-amplification which starts being very significant from $x/x^* \approx 0.2$. Therefore, equation (3.5) can be simplified for $0.14 < x/x^* < 0.2$ as,

$$U_k \frac{\partial}{\partial x_k} \left(\frac{1}{2} \langle s_{ij}s_{ij} \rangle \right) \approx -\frac{1}{2} \frac{\partial}{\partial x_k} \langle u_k s_{ij}s_{ij} \rangle - \langle s_{ij}s_{jk}s_{ki} \rangle - \langle s_{ij} \frac{\partial^2 p}{\partial x_i \partial x_j} \rangle \quad (3.7)$$

Looking now at figure 3.12b, the strain pressure-Hessian term eventually becomes a

sink after $x/x^* \approx 0.25$. Around that location, enstrophy has been brought towards the grid-element centreline as discussed earlier and the presence of small-scale strain brought about by the dynamics in equations (3.6) and (3.7) initiates the vortex stretching term as seen in figures 3.12b and 3.10b. From then on, vortex stretching and dissipation by viscosity become sinks of $\langle s_{ij}s_{ij} \rangle$. Therefore, for $0.25 < x/x^* < 0.5$, the transport of mean strain-product is due to transport by fluctuations and strain self-amplification which are balanced by the sum of strain pressure-Hessian, dissipation by viscosity, vortex stretching and convection due to mean flow. This can be written in equation form as,

$$U_k \frac{\partial}{\partial x_k} \left(\frac{1}{2} \langle s_{ij}s_{ij} \rangle \right) \approx -\frac{1}{2} \frac{\partial}{\partial x_k} \langle u_k s_{ij}s_{ij} \rangle - \langle s_{ij}s_{jk}s_{ki} \rangle - \frac{1}{4} \langle \omega_i \omega_j s_{ij} \rangle \quad (3.8)$$

$$- \langle s_{ij} \frac{\partial^2 p}{\partial x_i \partial x_j} \rangle + \nu \langle s_{ij} \nabla^2 s_{ij} \rangle$$

This analysis of mean strain-product suggests that there are two sub-zones in the production region. One is the “strain-dominant” zone where only strain is effectively present, and the other one is located adjacent to it further downstream where both strain and vorticity co-exist.

In the decay region, the pressure-Hessian term becomes zero and transport due to fluctuations becomes a homogenising consuming term. The mean convection term and the strain self-amplification term are balanced by enstrophy production (stretching), transport by fluctuations and dissipation. Therefore, the transport equation of mean strain-product in the decay region can be written as,

$$U_k \frac{\partial}{\partial x_k} \left(\frac{1}{2} \langle s_{ij}s_{ij} \rangle \right) \approx -\frac{1}{2} \frac{\partial}{\partial x_k} \langle u_k s_{ij}s_{ij} \rangle - \langle s_{ij}s_{jk}s_{ki} \rangle - \frac{1}{4} \langle \omega_i \omega_j s_{ij} \rangle \quad (3.9)$$

$$+ \nu \langle s_{ij} \nabla^2 s_{ij} \rangle$$

The mean velocity gradient terms such as α_s , β_s and ν_s do not contribute to the transport of mean strain-product even in the inhomogeneous production region. The same was observed in the transport equation for enstrophy.

Unlike the grid-element centreline, strain and vorticity coexist throughout the bar centreline (see figure 3.14a). The dynamics of strain-product and its budget along the bar centreline, as shown in figure 3.14b, are similar to that of the bar-centreline mean enstrophy discussed in the previous section. Here also, the turbulent stretching of fluctuating strain is the most dominant term, while the gradient production and stretching by mean strain-rate contribute only marginally to the production of mean strain-product. Compared to the enstrophy budget, the only difference is the presence of strain-pressure Hessian correlation term that acts as a source in a near-bar region, and eventually be-

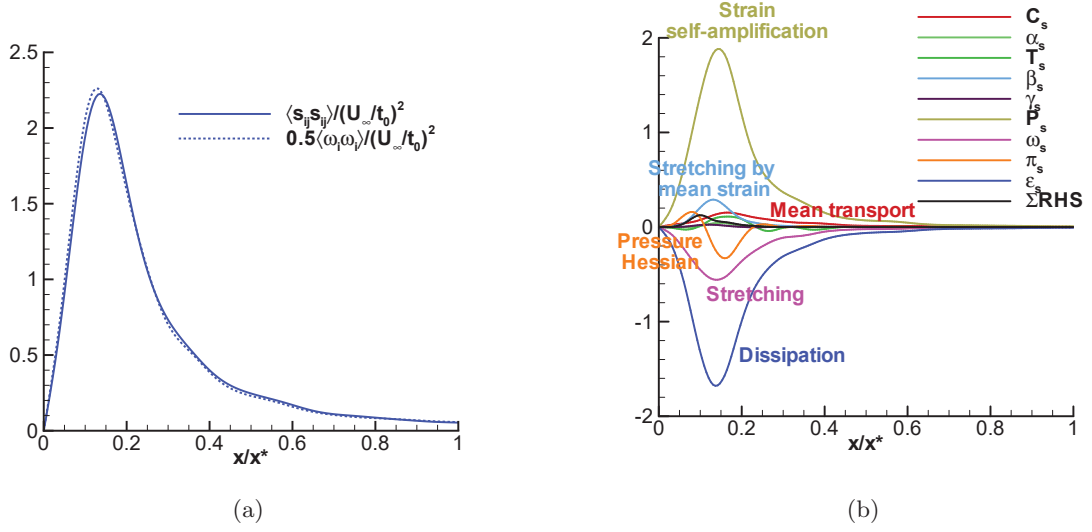


Figure 3.14: (a) Evolution of mean strain-product along the bar centreline (b). Budget of mean strain-product along the bar centreline (all terms are normalised by $(U_\infty / t_0)^3$).

comes a sink further downstream. Far downstream, strain self-amplification is balanced by viscous dissipation.

Here also, the balancing of budgets in the mean strain-product is not perfect, yet the percentage error is minimal which makes these results acceptable. It is repeated here that this is the first time that such numerically well-balanced equations are presented in a study of this kind.

This section is closed by discussing the effect of aspect ratio (AR) of the bar on the near field. As mentioned in section 1.5, the AR of the examined grid-element bar is 0.14, while in classical grids it is equal to 1. It is intuitive to expect that the AR of the bar will influence the near-field, both in terms of the size of the production region as well as the maximum rms velocity in the wake. To estimate the size of the production region, one can employ a similarity argument. Gomes-Fernandes et al. (2012) defined a modified wake-interaction length scale, denoted as $x^{*'}$, that was found to collapse the spatial variation of the normalised turbulent intensities for various grids. The definition of $x^{*'}$ uses the drag coefficient (C_D) of the bar, which is sensitive to AR. Bearman and Trueman (1972) measured the C_D values of rectangular cylinders of different AR's. C_D increases with AR, reaches a maximum value of 3 around $AR \approx 0.6$, and then is reduced. This trend is found to be true also for low Reynolds number transitional flows (refer to Norberg (1993)). Interestingly, for $AR=1.0$ and 0.14, the C_D values are found to be very similar, $C_D \approx 2$. Consequently, the modified wake-interaction length scale $x^{*'}$ of the classical grid-element with $AR = 1.0$ will be very close to one for the current grid-

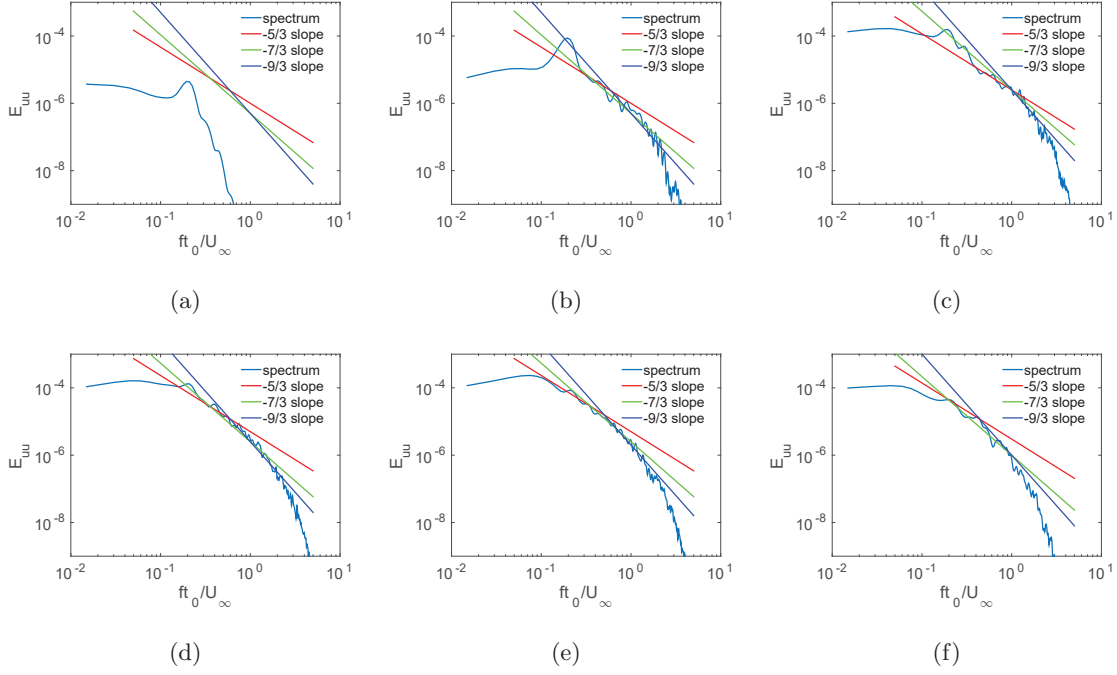


Figure 3.15: Energy spectra at different locations along the grid-element centreline: (a) $x/x^* = 0.1$, (b) $x/x^* = 0.25$, (c) $x/x^* = 0.35$, (d) $x/x^* = 0.5$, (e) $x/x^* = 0.75$, (f) $x/x^* = 0.95$. The frequencies f have been normalised with the ratio of the free-stream velocity U_∞ and the thickness t_0 .

element. Based on this similarity argument, it is expected that the size of the production region for the two ARs to be close to each other. To the best of the author's knowledge, there are no similarity variables to predict the maximum rms values. Therefore one needs to perform additional DNS simulations in order to confirm the prediction of the above similarity argument for the size of the production region as well as to compute the maximum turbulent intensity.

3.5 Energy Spectra

Having understood the mechanism whereby the small-scale terms of turbulence are generated in this developing flow, this section investigates the relation between the small-scale terms and the energy spectra along the grid-element centreline. The frequency spectra is obtained using the time series of velocity fluctuations collected at a specific location along the grid-element centreline. Every velocity signal contains 144,000 data points corresponding to the total number of time steps simulated for this study. Before taking Fast Fourier transform of the data, the velocity signal is divided into several sequences with an overlap of 50% using the Hanning window.

Laizet et al. (2015b) and Gomes-Fernandes et al. (2015) obtained the energy spectra for turbulence generated by a fractal grid. Both studies confirmed that the $-5/3$ power law slope exists for the energy spectrum, even in the production region where the turbulence is only developing, non-Gaussian, and inhomogeneous. In other words, the hypotheses of Kolmogorov presented in section 1.2.1 is not applicable in the production, yet the energy spectrum clearly had a $-5/3$ power-law slope in those studies. In fact, Laizet et al. (2015b) noted that, on the centreline, the $-5/3$ power-law first appears halfway between $x = 0$ and the turbulence peak. The DNS results of Laizet et al. (2013) for fractal grid turbulence and of Zhou et al. (2016b) for a single square grid-element turbulence confirm this finding for the $2/3$ power-law exponent of the second order structure function. Clearly, this power law behaviour in the energy spectrum is not related to the Kolmogorov theory (Kolmogorov, 1941) as none of the assumptions made in the Kolmogorov theory is valid in this region.

Figure 3.15 depicts the energy spectra obtained at all locations considered in the present study. Very close to the grid-element, where the flow is irrotational, the energy spectrum shows a pronounced vortex shedding signature with a normalised frequency (Strouhal number) $ft_0/U_\infty = 0.21$ (figure 3.15a). It is interesting that this shedding frequency is already found at a location in the grid-element centreline where the wakes have not yet met and the flow is irrotational. This indicates that it is probably the result of a global 3D instability of the flow. As can be seen in figure 3.15a, the energy content of this frequency is low (as expected since this point is not directly immersed in the wake) and there is no power law with a $-5/3$ slope. On the other hand, as depicted in figure 3.15b, a power law in the energy spectrum with a slope close to $-5/3$ is first observed at $x/x^* = 0.25$, for at least half a decade of frequency. The point $x/x^* = 0.25$ is exactly half way between $x = 0$ and the turbulence peak which is where Laizet et al. (2015b); Gomes-Fernandes et al. (2015) found the best $-5/3$ power-law slope defined for a wide range of frequency. From the previous section, it is known that $x/x^*=0.25$ is the location where the turbulence is developing and inhomogeneous. The limited frequency range of $-5/3$ in the spectrum is consistent with the fact that the Reynolds number is particularly low. In fact, the local Re_λ is only 22.01 at $x/x^* = 0.25$ (see table 3.1) and it can be surprising that a $-5/3$ power law exists at all, even in such a narrow frequency range. This result is in agreement with the observations made by Laizet et al. (2015b); Gomes-Fernandes et al. (2015) albeit at much higher Reynolds numbers, which explains why their $-5/3$ power law at a location midway between the grid-element and turbulence peak was defined over at least one decade of frequency.

The energy spectra in the further downstream locations of the developing region (i.e $x/x^* < 0.5$) also exhibit a restricted $-5/3$ power law and a less pronounced vortex

shedding signature as seen in figures 3.15c and 3.15d. However, the $-5/3$ slope is progressively eroded in these locations (see figures 3.15c-3.15d) compared to the spectra at $x/x^* = 0.25$ (figure 3.15b). The erosion of the $2/3$ power law from the near-field to the downstream in the second order structure function, directly related to the $-5/3$ law of the spectra, was also observed by Zhou et al. (2016b). It should be reminded that the Kolmogorov theory is not applicable in this developing turbulence region.

The energy spectra at the fully developed turbulent region are represented in figures 3.15e and 3.15f. At these locations, the Kolomgorov theory is applicable as flow in the region of $x/x^* > 0.5$ is fully-turbulent and homogeneous (refer to section 3.4). Note that the local Re_λ of this region is considerably higher than that of $x/x^* = 0.25$ (see table 3.1). Yet, the frequency range of $-5/3$ power-law slope is shorter compared to the frequency range observed at $x/x^*=0.25$.

The energy spectra in 3.15b-3.15e also show existence of a frequency range with a $-9/3$ power-law behaviour prior to the dissipation range. These results serve as a first evidence of the nonequilibrium state imposed on the spectra by the fluctuations of dissipation rate as suggested by Horiuti et al. (2016). However, the evolution of these nonequilibrium state from the near grid-element to the downstream is still not clear.

The energy spectrum in figure 3.15b appears to acquire a shape which includes a short range reminiscent of a $-5/3$ power law at the start of the subregion where equation (3.8) holds and where strain self-amplification *and* vortex stretching have only just started being both significantly present. This is also the subregion where the pressure Hessian acts like a sink. The spectral range which is emulating a short $-5/3$ power law does not grow but in fact, diminishes as vortex stretching and strain self-amplification increase to their maximum values at $x = 0.5x^*$ and this range continues to diminish further downstream in the decay region. These conclusions echo those of Laizet et al. (2013), Gomes-Fernandes et al. (2015) and Laizet et al. (2015b) who obtained energy spectra at much higher Reynolds numbers but were not able to have as much concurrent detail on the small-scale strain and vorticity dynamics as this study presented here.

3.6 Enstrophy production and strain self-amplification

Having identified the mechanism whereby small-scale strain-rate and vorticity fluctuations are amplified, as well as their association with the energy spectra, attention is now turned to the statistical study of two of the most important velocity gradient terms which contribute to the generation of mean enstrophy and strain-product along the grid-element centreline: mean enstrophy production $\langle \omega_i \omega_j s_{ij} \rangle$ and mean strain self-amplification $\langle s_{ij} s_{jk} s_{ki} \rangle$. Their statistics are discussed in this sub-section.

Quantity / x/x^*	0.1	0.25	0.35	0.5	0.75	0.95	Field	Grid
$\frac{\langle \omega_i \omega_j s_{ij} \rangle}{\langle s_{ij} s_{ij} \rangle^{3/2}}$	1.04×10^{-6}	0.39	0.46	0.53	0.45	0.38	0.16	0.27
$\frac{-\langle s_{ij} s_{jk} s_{ki} \rangle}{\langle s_{ij} s_{ij} \rangle^{3/2}}$	0.05	0.86	0.42	0.35	0.32	0.29	0.23	0.22
$\frac{-\langle \omega_i \omega_j s_{ij} \rangle}{\langle s_{ij} s_{jk} s_{ki} \rangle}$	1.5×10^{-5}	0.45	1.09	1.51	1.45	1.31	0.7	1.2
Re_λ	10.12	22.01	34.49	37.58	39.12	33.12	10^4	257

Table 3.2: Values of normalised $\langle \omega_i \omega_j s_{ij} \rangle$, $\langle s_{ij} s_{jk} s_{ki} \rangle$ and their ratios at different stations along the grid-element centreline. The values of field experiment are taken from Gulitski et al. (2007b) and the grid experiment is taken from the decay region of Gomes-Fernandes et al. (2014).

It was already shown in figures 3.10a and 3.12a that the mean enstrophy production and strain self-amplification increase in the production region, and then in the decay region their values decrease. Table 3.2 records the values of these quantities normalised by $\langle s_{ij} s_{ij} \rangle^{3/2}$. The normalised values of $\langle \omega_i \omega_j s_{ij} \rangle$ and $\langle s_{ij} s_{jk} s_{ki} \rangle$ decrease from the turbulence peak onwards. These quantities are compared against the reference values taken from the grid and field experiments. It can be seen that the normalised values decay in the region $x/x^* \geq 0.5$ allowing for the possibility that they might tend towards values comparable to the reference values given in the table. However, there is, of course, no guarantee that this is indeed so, and the simulation is not long enough in the streamwise direction for us to check. The ratio of $\langle -\omega_i \omega_j s_{ij} \rangle / \langle s_{ij} s_{jk} s_{ki} \rangle$ is also given in the table and compared with the reference value taken from the classical grid experiment. The reference value is close to the ratio $4/3 (=1.33)$ for homogeneous turbulence (Tsinober, 2009). The value of $(4/3)$ is also closely predicted by the current simulations at the downstream end of the computational domain. Besides, the variation of $\langle -\omega_i \omega_j s_{ij} \rangle / \langle s_{ij} s_{jk} s_{ki} \rangle$ in the simulation qualitatively agrees with the low-blockage grid-element simulations of Zhou et al. (2016a).

Comparison of the normalised enstrophy production and self-amplification values with the values reported by Gomes-Fernandes et al. (2014) (table 6 in their paper) reveals some differences. In their study, the ratios of $\langle \omega_i \omega_j s_{ij} \rangle$ and $\langle s_{ij} s_{jk} s_{ki} \rangle$ to $\langle s_{ij} s_{ij} \rangle^{3/2}$ are 12% and 10% respectively in the middle of the production region and then they increase to 21% and 26% in the decay region. In the present study, however, these ratios are much larger, as can be seen from table 3.2. The reason for this difference is that the denominator $\langle s_{ij} s_{ij} \rangle^{3/2}$ is small in the production region, but increases very rapidly further downstream as shown in figure 3.12a. In fact, it increases by a factor of 7 from the middle of the production region to the peak. On the other hand in figure 10 of Gomes-

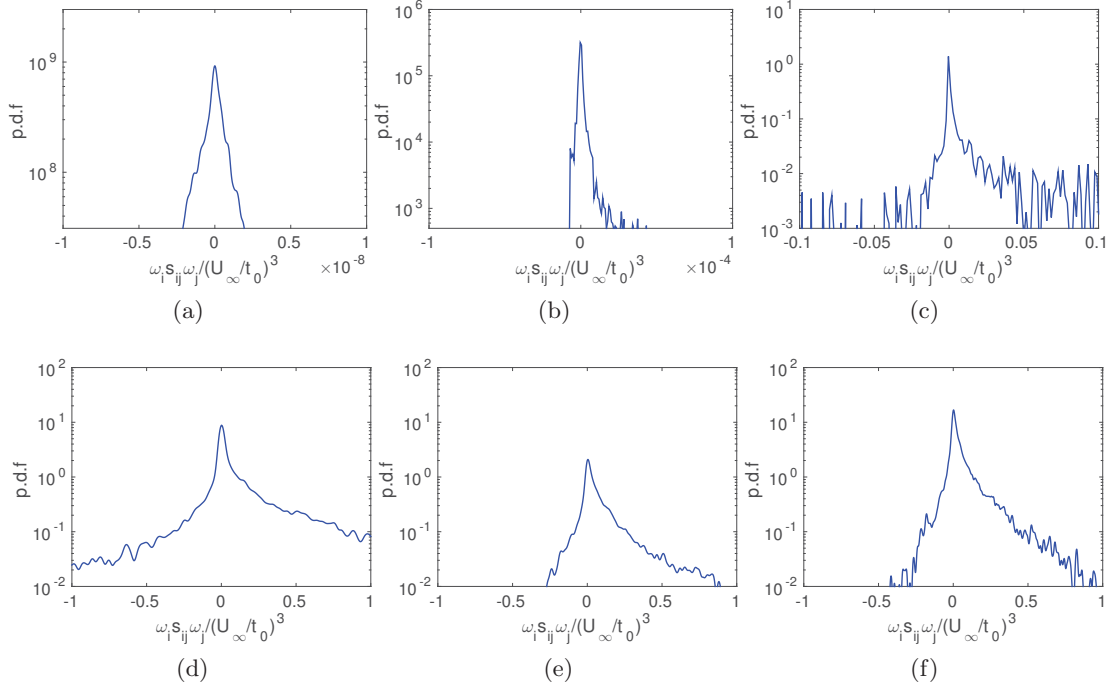


Figure 3.16: PDF of enstrophy production at different locations along the grid-element centreline: (a) $x/x^*=0.1$, (b) $x/x^*=0.25$, (c) $x/x^*=0.35$, (d) $x/x^*=0.5$, (e) $x/x^*=0.75$, (f) $x/x^*=0.95$.

Fernandes et al. (2014), $\langle s_{ij}s_{ij} \rangle$ increases by about a factor of 2 only. This suggests that the fractal grid produces a more homogeneous strain field compared to the single square grid-element. This may not be only due to the multi-scale space filling properties of the grid (i.e. the presence of bars of different sizes that homogenise the flow), but also to the fact that their incoming flow carries significant turbulence intensity, and therefore strain.

It is interesting to note that at $x/x^*=0.25$, where the wakes have already met, the turbulence is developing and the energy spectrum has a $-5/3$ power-law slope (from the current simulation but perhaps more importantly from the higher Reynolds number measurements and simulations of Laizet et al. (2013), Gomes-Fernandes et al. (2015) and Laizet et al. (2015b)), the mean strain self-amplification dominates the mean enstrophy production at least by a factor of two. This observation correlates the emergence of $-5/3$ energy spectra with the dominance of strain self-amplification process rather than of vortex stretching. Downstream of $x/x^*=0.25$, the mean enstrophy production soon dominates over mean strain self-amplification, but the $-5/3$ slope in the energy spectrum gradually weakens. It seems that the strain self-amplification is more essential for the energy spectrum's $-5/3$ power law shape than vortex stretching even though $\langle \omega_i \omega_j s_{ij} \rangle > 0$ (as is, in fact, the case) signifies prevalence of vortex stretching over vortex compression.

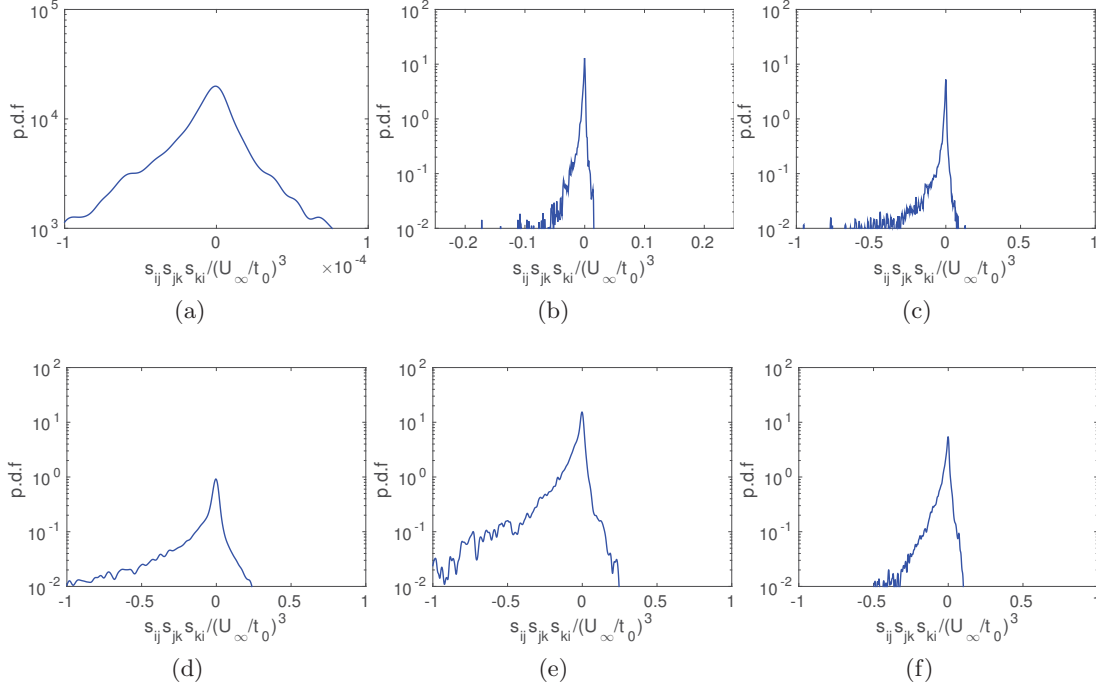


Figure 3.17: PDF of strain self-amplification at different locations along the grid-element centreline: (a) $x/x^*=0.1$, (b) $x/x^*=0.25$, (c) $x/x^*=0.35$, (d) $x/x^*=0.5$, (e) $x/x^*=0.75$, (f) $x/x^*=0.95$.

The PDFs of enstrophy production are shown in figure 3.16 for all the locations considered in this study. The PDF looks almost symmetric in the irrotational flow grid-element centreline region with the negligible values of $\omega_i s_{ij} \omega_j$ (figure 3.16a). At $x/x^* = 0.25$, where the slope of the energy spectrum is first observed to be $-5/3$, the PDF of $\omega_i s_{ij} \omega_j$ is clearly skewed towards positive, as seen in figure 3.16b. This indicates that vortex stretching is preferred over vortex compression at the onset point of $-5/3$ slope in the energy spectrum where the turbulence is still developing and inhomogeneous. This is an interesting result as the experimental study of fractal grid turbulence by Gomes-Fernandes et al. (2014) reported that the tendency for vortex stretching dominance is not evident in the inhomogeneous production region of the fractal grid (see section 4.2 in pp.268 of Gomes-Fernandes et al. (2014)). A plausible reason for this discrepancy is discussed in section 3.8.2 with the help of the intermediate strain-rate eigenvalue λ_2 . In all other locations beyond $x/x^* = 0.25$, the PDF of $\omega_i s_{ij} \omega_j$ is skewed towards positive, signifying the prevalence of vortex stretching over vortex compression, which is a universal characteristic of small scales in turbulent flows (Betchov, 1975; Taylor, 1938; Tsinober et al., 1992, 1995). It is interesting that this universal result is obtained for Re_λ as low as 33 in the decay region where the turbulence is fully-developed and

homogeneous. Although differences are noticed in the near grid-element PDF of $\omega_i s_{ij} \omega_j$ when compared against the experimental result of Gomes-Fernandes et al. (2014), the current results agree with the DNS result of Zhou et al. (2016a).

The PDFs of strain self-amplification are depicted in figure 3.17. At the irrotational flow grid-element centreline, the PDF of $s_{ij} s_{jk} s_{ki}$ is nearly symmetric with very small values of $s_{ij} s_{jk} s_{ki}$ (figure 3.17a). From figures 3.17b-3.17f, it can be observed that the PDFs are skewed towards a negative average in the regions where the wakes have met, which is also considered to be a universal characteristic of turbulent flows (Tsinober, 2000, 2009). Again the universal statistical behaviour of strain self-amplification is noted for very low Re_λ values of homogeneous fully-developed turbulence (see figures 3.17e and 3.17f). Gomes-Fernandes et al. (2014) and Zhou et al. (2016a) also reported negative average values of strain self-amplification in the production region.

3.7 Invariants of the velocity gradient tensor

The next objective is to test the validity of other potentially universal characteristics of the invariants of VGT for the low Reynolds number turbulence considered here. A brief review of equations and terms used in this study has already been presented in chapter 1.

Laizet et al. (2013) were the first to analyse the spatial evolution of $Q-R$ diagrams in a spatially evolving turbulent flow behind a fractal grid. They found that these diagrams obtain their usual tear-drop shape well downstream of the point where the second order structure function acquires its $2/3$ power law in the production region. Gomes-Fernandes et al. (2014) reported from their PIV study that the $Q-R$ diagram clearly gets its usual shape only in the decay region. Zhou et al. (2016b) also presented the $Q-R$ diagrams in the production region of a single square grid-element and confirmed that they are qualitatively different from the well-known tear-drop shape which appears in the decay region. Information on the $Q_s - R_s$ diagram for a single square grid-element turbulence is still missing.

3.7.1 $Q-R$ diagrams

The values of normalised $\langle Q \rangle$ and $\langle R \rangle$ are shown in table 3.3. It can be seen from the table that the values of $\langle Q \rangle$ are not zero in the region $x/x^* \leq 0.35$, making it absolutely clear that this region is not homogeneous, even for small-scale statistics. The table shows that, on average, strain-product dominates over enstrophy in this region as already seen in section 3.4. The strain is the first small-scale velocity gradient quantity generated along the grid-element centreline by the mechanism discussed in section 3.4. Note,

x/x^*	0.1	0.25	0.35	0.5	0.75	0.95
$\langle Q \rangle / \langle s_{ij}s_{ij} \rangle$	-0.5	-0.32	-0.08	0	0	0
$\langle R \rangle / \langle s_{ij}s_{ij} \rangle^{3/2}$	0.02	0.14	0.01	0	0	0

Table 3.3: Values of the normalised mean $\langle Q \rangle$ and $\langle R \rangle$ at different locations along the grid-element centreline.

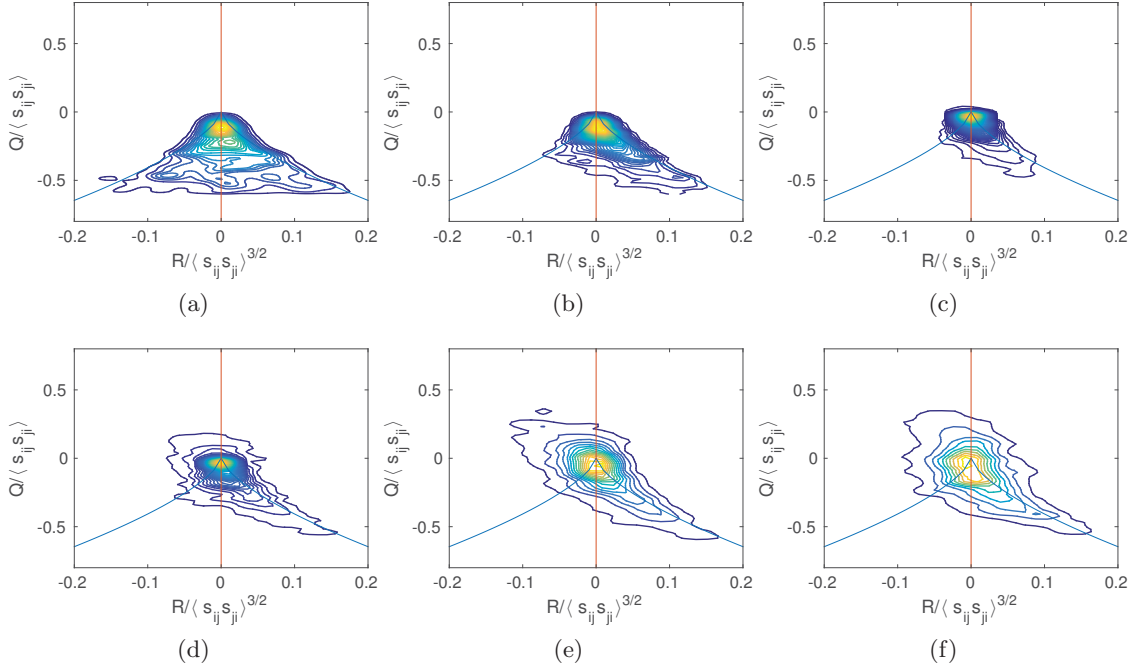


Figure 3.18: Joint PDF of Q and R at different locations along the grid-element centreline: (a) $x/x^*=0.1$, (b) $x/x^*=0.25$, (c) $x/x^*=0.35$, (d) $x/x^*=0.5$, (e) $x/x^*=0.75$, (f) $x/x^*=0.95$. The isocontours range from 10^1 to 10^{-2} .

however, that this strain dominance progressively disappears once vorticity is brought to the grid-element centreline. By the end of the production region, the average of both invariants is 0, which is the expected result for homogeneous turbulence.

Figure 3.18 shows the JPDF of Q and R at six different locations. The blue line in this figure is the locus of points where $D = 0$ (equation (1.15)). The invariants are normalised by the mean strain-product at that location. In the region where the wakes have not met and the flow is irrotational ($x/x^*=0.1$), the shape of the $Q - R$ diagram is very different from the usual tear-drop shape and contains no positive Q values (figure 3.18a). This observation is consistent with and strengthens the previous observation that the flow in the initial part of the grid-element centreline is mostly irrotational, and $Q < 0$ at all $x/x^* < 0.2$. In this region, the VGT is almost equal to the strain-rate

tensor, which is a symmetric tensor, and therefore all the eigenvalues are real. This explains why all isolines are located below the $D = 0$ curve, in an area where $D < 0$. The shape of JPDF at $x/x^* < 0.1$ is similar to the one reported for the non-turbulent side of turbulent/non-turbulent interfaces (da Silva and Pereira, 2008). The shape of figure 3.18a also ascertains that the flow in the initial part of the grid-element centreline is indeed non-turbulent.

Although the wakes have already met by $x/x^* = 0.25$ and the energy spectrum exhibits a short-range approximate $-5/3$ power law exponent, the $Q - R$ diagram, as shown in figure 3.18b, does not show its usual shape. However, it is interesting to note that $Q < 0$ already correlates with $R > 0$ with a visible tail in the $Q - R$ diagram at $x/x^* = 0.25$. This is also the point on the grid-element centreline where strain self-amplification and vortex stretching just started being significant, and the turbulence is still developing, non-Gaussian and inhomogeneous. The undeveloped shape in the $Q - R$ diagram in figure 3.18b for $x/x^* = 0.25$ is due to the fact that the strain still dominates enstrophy. It should be noted that the undeveloped $Q - R$ diagram in the production region reported by Gomes-Fernandes et al. (2014) (see their figure 17a) correlates with an indistinguishable or slight dominance of vortex stretching over vortex compression while the present results show a clear dominance of vortex stretching over compression.

From $x/x^* = 0.35$ onwards the $Q - R$ diagrams evolve towards their usual tear-drop shape as Q starts to take both positive and negative values, and the correlation between $Q > 0$ and $R < 0$ starts to appear as seen in figure 3.18c. This observation suggests that the preference for vortex stretching increases from the production to the decay region.

In this study, the $Q - R$ diagram is observed to attain its usual shape around $x/x^* = 0.5$ i.e. at the end of the production region, as seen in figure 3.18d. The current results agree with the numerical results of Laizet et al. (2013) that the $Q - R$ diagram obtains its tear-drop shape by the end of the production region and well downstream of the place where the $-5/3$ spectra and the $2/3$ structure functions first appear. Zhou et al. (2014a, 2015) obtained the tear-drop shape closer to the grid-element, at $x/x^* = 0.16$. This discrepancy could be due to the difference noticed in the behaviour of skewness and flatness which is the result of different blockage ratios in these two studies (0.11 as opposed to 0.2 in the present study). Also, this is the first study that reports the appearance of a tear-drop shape in the $Q - R$ diagram for a very low Re_λ homogeneous fully-developed turbulence (see figures 3.18e and 3.18f).

3.7.2 $Q_s - R_s$ diagrams and strain-rate eigenvalue ratios

Figure 3.19 shows the JPDF of Q_s and R_s in the production and decay regions. There is no real preference for positive or negative R_s in the irrotational flow region as shown in

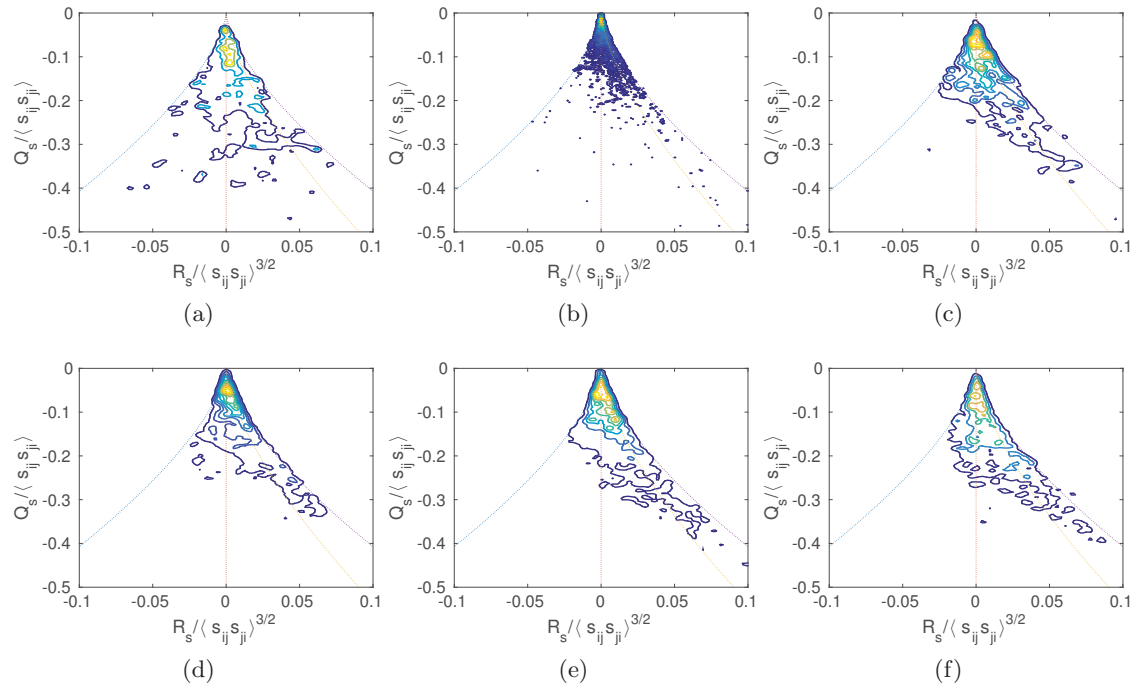


Figure 3.19: Joint PDF of Q_s and R_s at different locations along the grid-element centreline: (a) $x/x^*=0.1$, (b) $x/x^*=0.25$, (c) $x/x^*=0.35$, (d) $x/x^*=0.5$, (e) $x/x^*=0.75$, (f) $x/x^*=0.95$. The isocontours range from 10^1 to 10^{-2} . The four lines from the left to the right in each sub-figure correspond to $\lambda_1:\lambda_2:\lambda_3 = 2:-1:-1$, $1:0:-1$, $3:1:-4$ and $1:1:-2$.

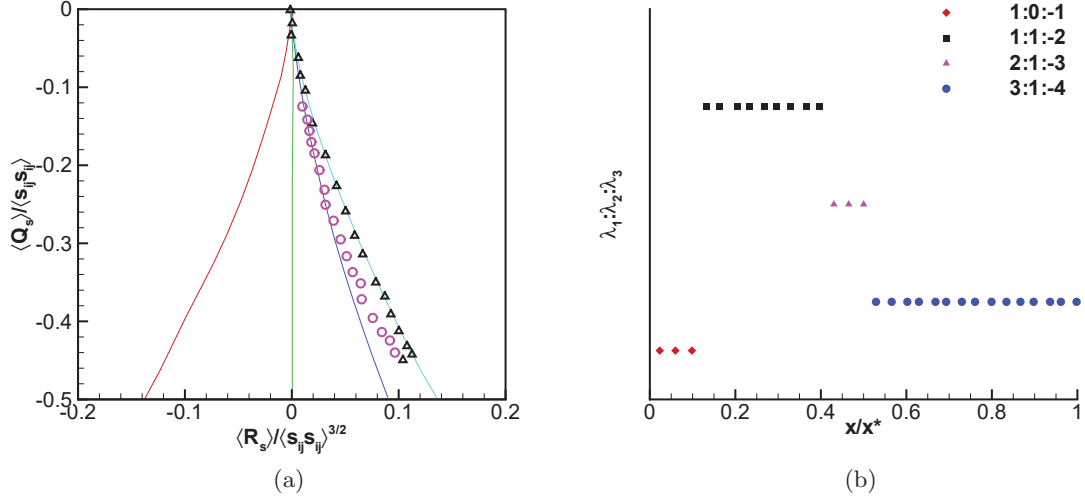


Figure 3.20: (a) Mean trajectory of Q_s and R_s in the phase map. Triangles represent the production region while circles the decay region. (b) Eigenvalue ratios $\lambda_1 : \lambda_2 : \lambda_3$ along the grid-element centreline.

figure 3.19a. The contour lines align with the ratio of 1:0:-1 at $x/x^* = 0.1$ which shows that the irrotational flow is largely two-dimensional. From $x/x^* = 0.25$ onwards the JPDF starts showing a tendency towards positive R_s though the preference is very small at $x/x^* = 0.25$ (figure 3.19b). From $x/x^* = 0.35$ onwards the JPDF looks similar to what is observed for homogeneous isotropic turbulence with larger probabilities occurring between $\lambda_1 : \lambda_2 : \lambda_3 = 3 : 1 : -4$ and $1 : 1 : -2$. Here also, the universal characteristics of Q_s - R_s diagram is obtained for a very low Re_λ homogeneous turbulence of the decay region (see figures 3.19e and 3.19f).

Figure 3.20 shows the trajectory of $\langle Q_s \rangle$ and $\langle R_s \rangle$ in their associated phase map. In figure 3.20a, the triangles represent locations in the production region as one moves downstream along the grid-element centreline. The value of $\langle R_s \rangle$ grows from being negligibly small in the near grid-element region to being slightly above $0.1 \langle s_{ij} s_{ij} \rangle$ at $x/x^* \approx 0.4$ then dropping again as the turbulence peak location is approached. The circles represent locations in the decay region. This phase map can also be plotted in terms of x/x^* as shown in figure 3.20b. The first observation from these figures is that the average invariant values are in the quadrant of $Q_s < 0$ and $R_s > 0$, and that they are mostly on the line $a = 1$ for $0 \leq x/x^* < 0.4$, and mostly on or near the line $a = 1/3$ for the decay region. The transition between axial stretching to biaxial stretching occurs in the region of $0.4 < x/x^* < 0.5$. The eigenvalue ratio in this transitional region is close to 2:1:-3. Note that the most probable eigenvalue ratios obtained in most previous studies are 3:1:-4 and 2:1:-3 (da Silva and Pereira, 2008).

Quantity / x/x^*	0.1	0.25	0.35	0.5	0.75	0.95	Field expt.	HIT
$\frac{-\langle\lambda_1^3\rangle}{\langle s_{ij}s_{jk}s_{ki}\rangle}$	0.45	0.53	0.91	1.24	0.89	0.87	1.62	0.9
$\frac{-\langle\lambda_2^3\rangle}{\langle s_{ij}s_{jk}s_{ki}\rangle}$	0.05	0.07	0.06	0.06	0.06	0.06	0.05	0.06
$\frac{-\langle\lambda_3^3\rangle}{\langle s_{ij}s_{jk}s_{ki}\rangle}$	-1.50	-1.60	-1.97	-2.30	-1.95	-1.93	-2.67	-1.96

Table 3.4: Values of $\langle\lambda_i^3\rangle$ ($i = 1, 2, 3$) normalised by the mean strain self-amplification at different stations along the grid-element centreline. The field experiment data are taken from Kholmyansky et al. (2001), and the homogeneous isotropic turbulence data are taken from the periodic DNS of Onishi et al. (2011) for $Re_\lambda=80$.

3.8 Factors affecting enstrophy production and strain self-amplification

In this section, the factors that affect the important velocity gradient terms are analysed. From equations (1.10) and (1.11), it can be observed that the enstrophy production and strain self-amplification depend on the strain-rate eigenvalues, while enstrophy production also depends on the alignments between vorticity and strain-rate eigenvectors.

3.8.1 Eigenvalues of strain-rate tensor and their contribution to enstrophy production

The statistics of the strain-rate eigenvalues are analysed in this subsection to shed more light onto the mechanisms of enstrophy production and strain self-amplification. This statistical analysis can also help to explain the discrepancy observed in the PDF of $\omega_i\omega_j s_{ij}$ with respect to the PIV study of Gomes-Fernandes et al. (2014).

Table 3.4 gives the values of $\langle\lambda_i^3\rangle$ ($i = 1, 2, 3$) normalised by $-\langle s_{ij}s_{jk}s_{ki}\rangle$ which is the positive source term in the strain product equation as shown in figure 3.12b. From table 3.4 it can be seen that both $\langle\lambda_1^3\rangle$ and $\langle\lambda_2^3\rangle$ are positive i.e., there are two extensive strain-rate eigenvalues, a universal feature of 3D turbulent flows. The tendency of the intermediate eigenvalue to be statistically more positive than negative, i.e., $\langle\lambda_2^3\rangle > 0$ and $\langle\lambda_2\rangle > 0$, was predicted for homogeneous isotropic turbulence by Betchov (1956), first observed in the DNS of Ashurst et al. (1987) and then confirmed by many other studies (e.g. She et al. (1991); Su and Dahm (1996); Tsinober et al. (1992)). This universal turbulence result is found to be valid even for this very low Re_λ turbulence. Since $-\langle s_{ij}s_{jk}s_{ki}\rangle = -(\langle\lambda_1^3\rangle + \langle\lambda_2^3\rangle + \langle\lambda_3^3\rangle)$, the positive sign of $-\langle s_{ij}s_{jk}s_{ki}\rangle$ originates from $-\langle\lambda_3^3\rangle$. Therefore, the strain self-amplification process is associated, in a time-averaged sense, with a fluid mechanical picture involving stretching and compressing

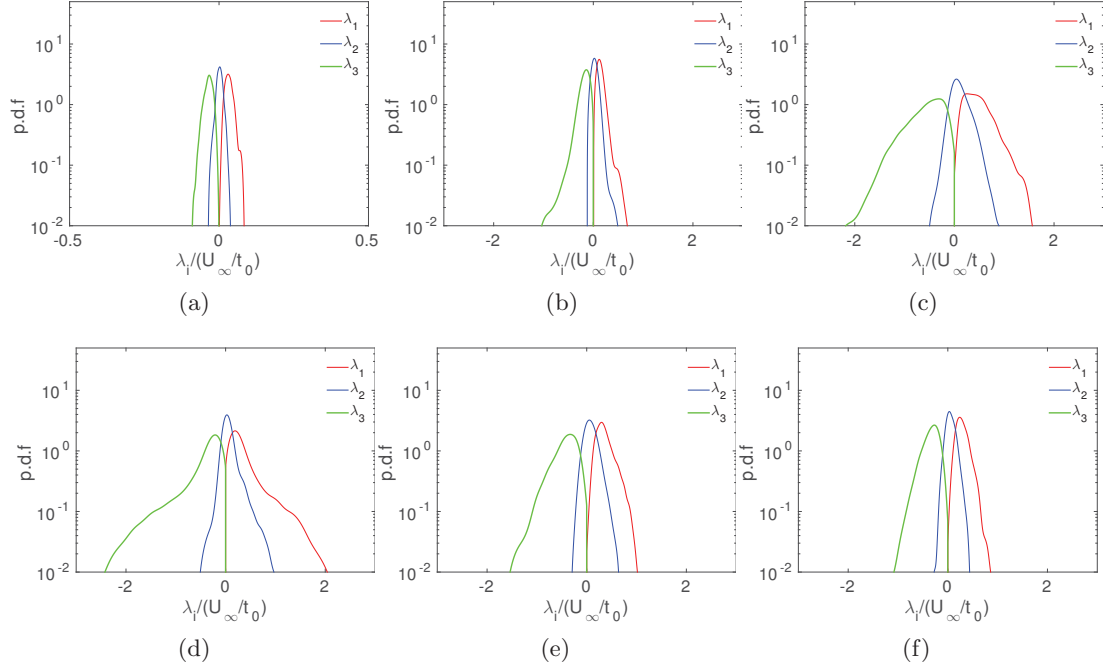


Figure 3.21: PDF of strain-rate eigenvalues at different locations along the grid-element centreline: (a) $x/x^*=0.1$, (b) $x/x^*=0.25$, (c) $x/x^*=0.35$, (d) $x/x^*=0.5$, (e) $x/x^*=0.75$, (f) $x/x^*=0.95$.

velocity gradients into sheets. The values reported in Table 3.4 approximate those of homogeneous isotropic turbulence in the decay region, although there seems to be a deviation from the measurements taken in very high Reynolds number field experiments. In the production region, the values of $\langle \lambda_i^3 \rangle$ normalised by $-\langle s_{ij}s_{jk}s_{ki} \rangle$ are similar to those reported by Gomes-Fernandes et al. (2014) for $i = 2$ but up to an order of magnitude smaller for $i = 1, 3$. The main difference between the production region of this study and that of Gomes-Fernandes et al. (2014) is the Reynolds number, which is lower in this case, and the presence of irrotational flow, which is prominent in the current case given the turbulence generated by all the extra fractal iterations on the fractal grid of Gomes-Fernandes et al. (2014). These fractal iterations seem to greatly increase the intensity of the small-scale stretching and compressing actions. Also, the downstream evolution of the values given in table 3.4 qualitatively agrees with that of Zhou et al. (2016a).

Attention is now turned to the PDFs of the strain-rate eigenvalues. As observed in figure 3.21a, in the irrotational part of the grid-element centreline, the PDFs of λ_1 and λ_3 are near mirror images of each other, and the PDF of λ_2 is symmetric around 0. As the enstrophy production and strain self-amplification depend on the strain-rate eigenvalues linearly (equation (1.10)) and to the power of three (equation (1.11)) respectively,

Quantity/ x/x^*	0.1	0.25	0.35	0.5	0.75	0.95	Field expt.	Grid	HIT
$\frac{\langle \omega^2 \lambda_1 \cos^2(\omega, \mathbf{e}_1) \rangle}{\langle \omega_i \omega_j s_{ij} \rangle}$	–	1.29	0.94	0.86	0.87	0.86	1.44	1.17	0.74
$\frac{\langle \omega^2 \lambda_2 \cos^2(\omega, \mathbf{e}_2) \rangle}{\langle \omega_i \omega_j s_{ij} \rangle}$	–	0.51	0.58	0.61	0.58	0.58	0.47	0.39	0.62
$\frac{\langle \omega^2 \lambda_3 \cos^2(\omega, \mathbf{e}_3) \rangle}{\langle \omega_i \omega_j s_{ij} \rangle}$	–	-0.80	-0.52	-0.47	-0.45	-0.44	-0.97	-0.56	-0.36

Table 3.5: Contribution of strain-rate eigenvalues to enstrophy production along the grid-element centreline. The values are normalised by mean enstrophy production. The field experiment data is taken from Kholmyansky et al. (2001), grid experiment data is from Tsinober (2009), and the homogeneous isotropic turbulence data is from Onishi et al. (2011).

any small symmetry breaking in the PDFs of the strain-rate eigenvalues will be more pronounced on the PDF of strain self-amplification than on the PDF of enstrophy production. This is the reason why the PDF of enstrophy production is perfectly symmetric, and the PDF of strain self-amplification is close to (but not perfectly) symmetric in the irrotational flow region (figures 3.16a and 3.17a).

The PDF of λ_2 becomes positively skewed at $x/x^* = 0.25$ (figure 3.21b) where the turbulence is inhomogeneous and the small-scale terms have just started developing, which is also a characteristic feature of small-scale turbulence (Ashurst et al., 1987; Betchov, 1975; Gomes-Fernandes et al., 2014; Ganapathisubramani et al., 2008). In order to satisfy the incompressibility constraint i.e., $\lambda_1 + \lambda_2 + \lambda_3 = 0$, the PDF of λ_3 becomes negatively skewed thus breaking the mirror image symmetry with λ_1 . Qualitatively similar behaviour is observed in all other locations as seen in figures 3.21c-3.21f. It should be noted here that the study of Gomes-Fernandes et al. (2014) observed symmetry in the PDF of λ_2 in the inhomogeneous production region. The current study, however, clearly shows that λ_2 is positively skewed. This is related to the discrepancy observed in the skewness of the PDF of enstrophy production which depends on the skewness of the PDF of λ_2 . The statistics of λ_2 are further analysed at the end of this section. The tendency for the intermediate strain-rate eigenvalue to skew towards positive is found to be true for the homogeneous turbulence of Re_λ as low as 33 (figure 3.21f).

The contribution of each strain-rate eigenvalue and eigenvector to the enstrophy production is shown in Table 3.5. Note that $\langle \omega^2 \lambda_1 \cos^2(\omega, \mathbf{e}_1) \rangle > \langle \omega^2 \lambda_2 \cos^2(\omega, \mathbf{e}_2) \rangle > 0$ whereas $\langle \omega^2 \lambda_3 \cos^2(\omega, \mathbf{e}_3) \rangle < 0$. This observation suggests that the most extensive eigenvector of the strain-rate tensor is the most responsible for the positive enstrophy production. The ratio $\frac{\langle \omega^2 \lambda_1 \cos^2(\omega, \mathbf{e}_1) \rangle}{\langle \omega^2 \lambda_2 \cos^2(\omega, \mathbf{e}_2) \rangle}$ decreases from its value near the grid-element to its value at the position of the turbulence peak. The ratio $\frac{-\langle \omega^2 \lambda_1 \cos^2(\omega, \mathbf{e}_1) \rangle}{\langle \omega^2 \lambda_3 \cos^2(\omega, \mathbf{e}_3) \rangle}$ in the production

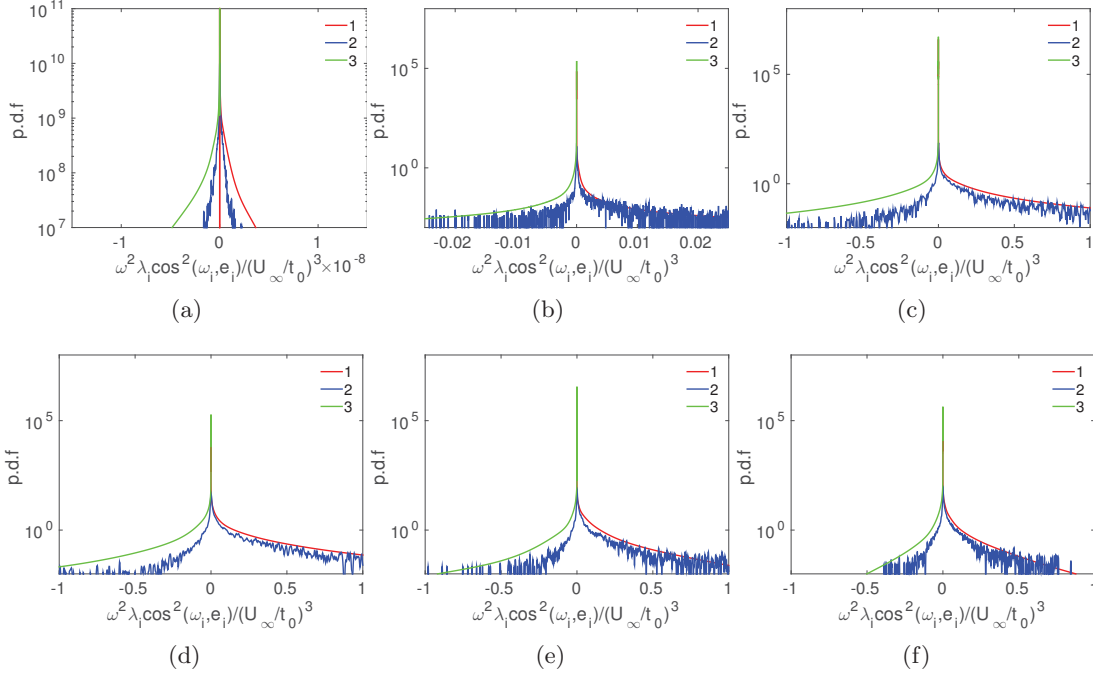


Figure 3.22: PDF of strain-rate eigenvalues contribution to enstrophy production at different locations along the grid-element centreline: (a) $x/x^*=0.1$, (b) $x/x^*=0.25$, (c) $x/x^*=0.35$, (d) $x/x^*=0.5$, (e) $x/x^*=0.75$, (f) $x/x^*=0.95$.

and decay regions is 1.61 and 1.95 respectively. This indicates that the strength of vortex stretching over compression is marginally weaker in the production region compared to the decay region. All three time-averaged quantities in Table 3.5 appear to have a tendency towards approaching the reference HIT values in the restricted decay region, even though they do not quite reach them. Also, this tendency towards reaching a universal value is faster in the present simulation compared to the values reported in Zhou et al. (2016a). Although the value of $\langle \omega^2 \lambda_2 \cos^2(\omega, \mathbf{e}_2) \rangle$ in the production region is closer to the value reported in Gomes-Fernandes et al. (2014), the other two components are smaller. The reason for this discrepancy is the pronounced irrotational region in the current flow configuration which results in weaker stretching/compressing events compared to Gomes-Fernandes et al. (2014) as noted earlier.

The PDFs of the individual components of enstrophy production are depicted in figure 3.22. All the components are small close to the grid-element (figures 3.22a and 3.22b) because of the small values of vorticity. The PDF of $\omega^2 \lambda_2 \cos^2(\omega, \mathbf{e}_2)$ is nearly symmetric only at $x/x^* = 0.1$ as seen in figure 3.22a. The positively skewed PDF of λ_2 in all other locations (figures 3.21b-3.21f) is expected to result into positively skewed PDFs of $\omega^2 \lambda_2 \cos^2(\omega, \mathbf{e}_2)$ as indeed observed in figures 3.22b-3.22f. The tails of the PDFs of $\omega^2 \lambda_1 \cos^2(\omega, \mathbf{e}_1)$ and $\omega^2 \lambda_3 \cos^2(\omega, \mathbf{e}_3)$ are wider in the production region but

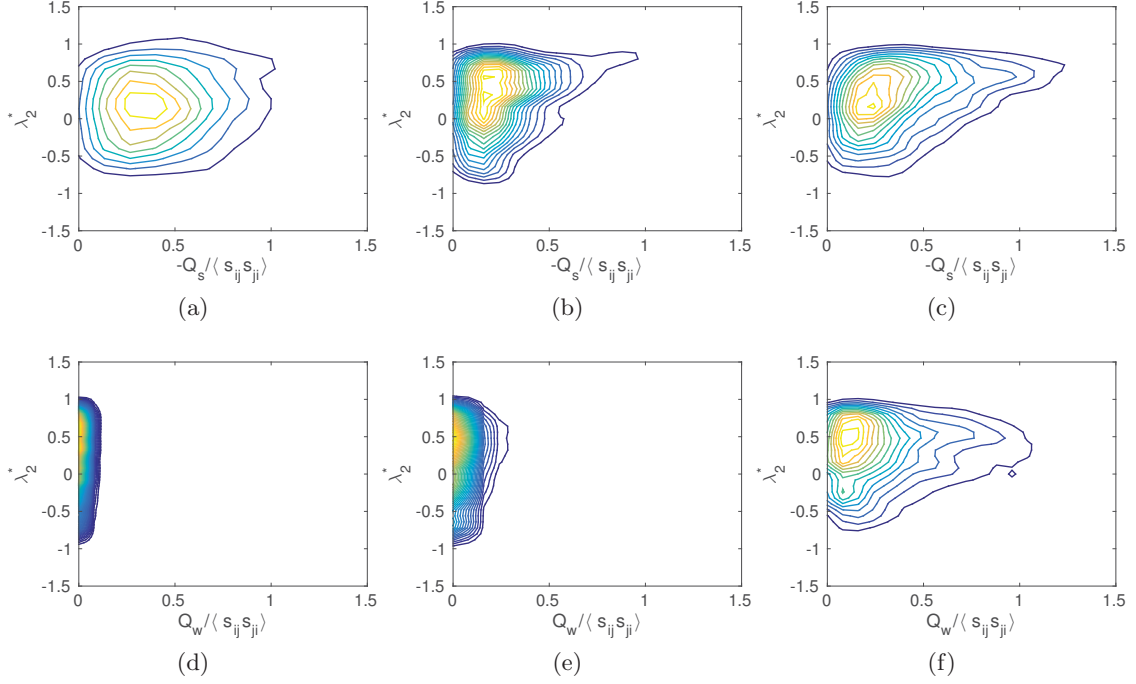


Figure 3.23: Joint PDF of the normalised intermediate strain-rate eigenvalue with Q_s and Q_w at $x/x^*=0.1, 0.25$ and 0.95 along the grid-element centreline. The isocontours range from 10^{-1} to 10^{-3} .

they become more narrow in the decay region. This indicates that strong and rare vortex stretching and compressing events are more prevalent in the production region than in the decay region.

3.8.2 Analysis of the intermediate eigenvalue

As observed in table 3.4, the normalised extensive (λ_1) and compressive (λ_3) strain-rate eigenvalues vary significantly along the grid-element centreline, while the intermediate one (λ_2) remains almost constant. Moreover, the statistical behaviour of this particular eigenvalue is observed to be different in the production region when compared to the PIV study of Gomes-Fernandes et al. (2014). This also causes a discrepancy in the PDF of enstrophy production as the statistical characteristics of $\omega_i s_{ij} \omega_j$ depend on λ_2 . These factors prompt further analysis of λ_2 .

Although various normalisations are available (Ganapathisubramani et al., 2008), the normalisation provided by Ashurst et al. (1987) is used in this study, therefore λ_2 is written as

$$\lambda_2^* = \frac{\sqrt{6}\lambda_2}{\sqrt{\lambda_1^2 + \lambda_2^2 + \lambda_3^2}} \quad (3.10)$$

With this normalisation, λ_2^* is in the interval $[-1, +1]$ owing to the incompressibility constraint. The extreme values $+1, -1$ represent axisymmetric expansion and contraction respectively.

The joint PDFs of λ_2^* with $Q_s (= \frac{1}{2}s_{ij}s_{ij})$ and $Q_w (= \frac{1}{4}\omega_i\omega_i)$ are shown in figure 3.23 for three locations along the grid-element centreline: close to the grid-element, location where the $-5/3$ slope in energy spectrum is first observed for the inhomogeneous turbulence, and in the decay region. As can be seen, λ_2^* is indeed bounded by ± 1 , and this gives further confidence in the simulation data and the procedure followed in normalising and computing the joint PDF. Close to the grid-element (where $x/x^* = 0.1$, see figure 3.23a), the JPDFs of $\lambda_2^* - Q_s$ and $\lambda_2^* - Q_w$ are slightly skewed towards positive values of λ_2^* . This indicates that although $\langle \lambda_2 \rangle \approx 0$, the normalised eigenvalue $\langle \lambda_2^* \rangle > 0$. Further downstream, the JPDFs become more positively skewed (figures 3.23b and 3.23c). Notice that the asymmetry is more pronounced in the $\lambda_2^* - Q_s$ plane compared to $\lambda_2^* - Q_w$ at $x/x^* = 0.25$. This could be perhaps due to the fact that at this location strain strongly dominates enstrophy (see Table 3.3).

It is interesting to notice that in the decay region where the turbulence is most homogeneous, the JPDF peaks at λ_2^* close to 0.5, in very good agreement with previous studies including some which concerned periodic turbulence (Ashurst et al., 1987; Betchov, 1975; Gomes-Fernandes et al., 2014; Ganapathisubramani et al., 2008).

3.8.3 Alignment of vorticity vector and strain-rate eigenvectors

Finally, the geometric alignments between the vorticity vector and the eigenvectors of the strain-rate tensor are discussed as they influence the enstrophy production term (see equation (1.10)).

Figure 3.24 shows the PDFs of the absolute value of the cosine of the angle between the vorticity vector and the strain-rate eigenvectors. These alignments are considered at all locations except at $x/x^*=0.1$ where the magnitude of vorticity is very small. At $x/x^* = 0.25$, where the turbulence is inhomogeneous and strain dominates enstrophy, both the extensive and intermediate strain-rate eigenvectors align with the vorticity vector, although the alignment is weak. This behaviour differs from the universal behaviour of strong alignment between the vorticity vector with the intermediate strain-rate eigenvector in turbulent flows (Kerr, 1987; Ashurst et al., 1987). Further downstream along the grid-element centreline, the alignment of both the extensive strain-rate eigenvectors with vorticity becomes stronger within the inhomogeneous production region (figures 3.24b and 3.24c). This evidence suggests that both the positive strain-rate eigenvectors act effectively to produce enstrophy in the production region, and can explain the rapid growth of the vortex stretching term from $x/x^* = 0.25$ to the turbulence peak location

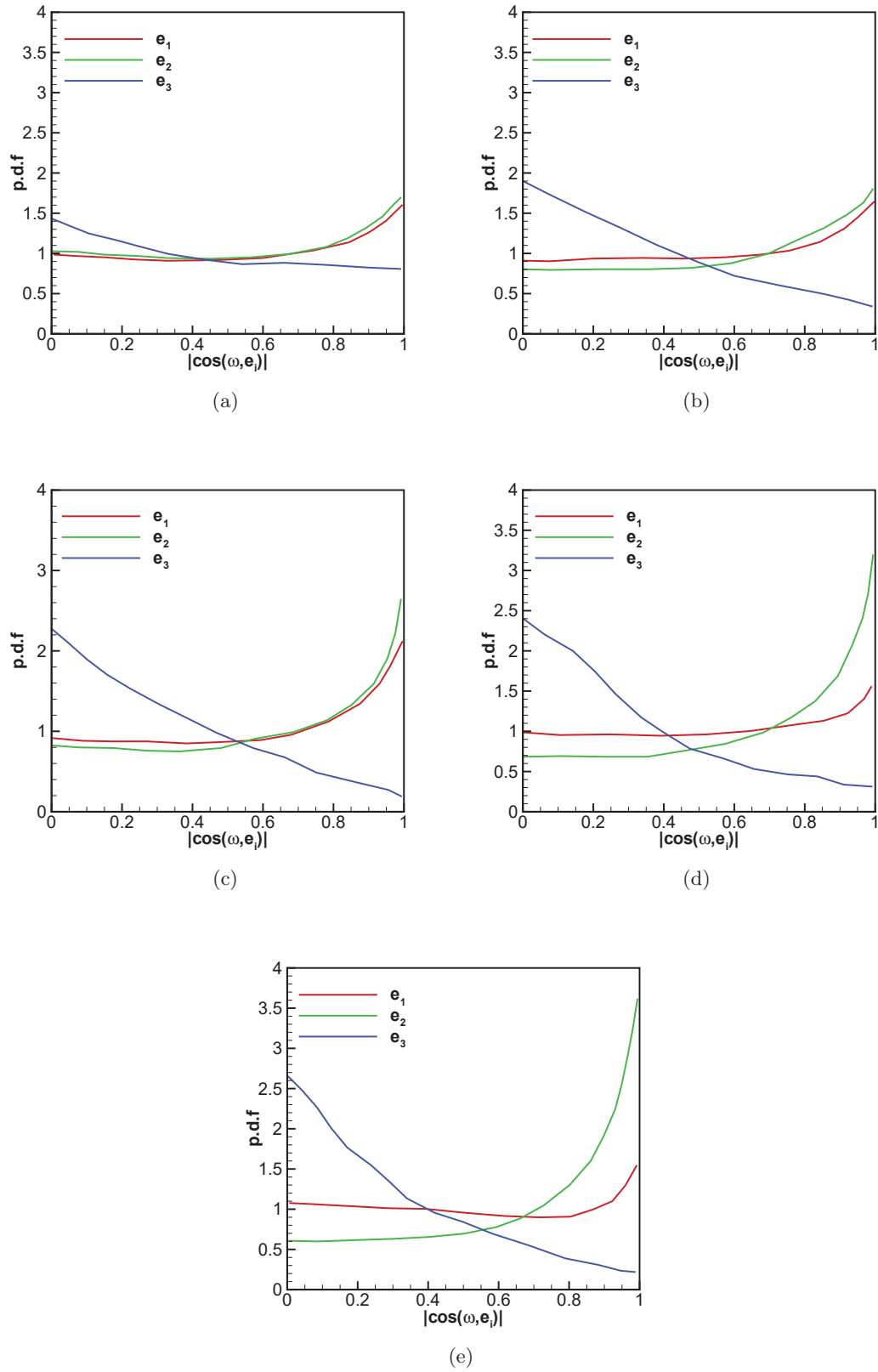


Figure 3.24: PDF of absolute value of the cosine of the angle between the vorticity vector and the eigenvectors of the strain-rate tensor at different locations along the grid-element centreline: (a) $x/x^*=0.25$, (b) $x/x^*=0.35$, (c) $x/x^*=0.5$, (d) $x/x^*=0.75$, (e) $x/x^*=0.95$.

(see figure 3.10a). The universal alignment behaviour is encountered only in the decay region (figures 3.24d and 3.24e) even though the local Re_λ there is only between 30 and 40. In this region, the compressive strain-rate eigenvector aligns perpendicularly to vorticity and the extensive strain-rate eigenvector is indifferent to vorticity. Zhou et al. (2016a) also find that the alignment with the intermediate eigenvector is gradual.

Why is the alignment behaviour different in the production region? As explained in section 3.4.2, vorticity is produced at the grid-element bars, is shed into the bar wake, and then it is transported towards the grid-element centreline by turbulent fluctuations. There it finds itself in an already developed fluctuating strain field which has been produced due to the action of the pressure-Hessian. In other words, most of the strain field at about $x/x^* = 0.2$ or so on the grid-element centreline has not been created by the vorticity, and the two fields are uncoupled. Further downstream, vorticity and strain start to interact but this interaction is gradual and is completed only in the decay region, where the alignments show the universal behaviour.

This scenario is consistent with the findings of Hamlington et al. (2008a). Following Jiménez (1992), these authors decomposed the fluctuating strain field at every point in the domain into two components: one (local) component produced by the local vorticity field around the point in question and another (non-local or background) component produced by vorticity further away. Their numerical simulations in forced, homogeneous isotropic turbulence at $Re_\lambda = 107$ demonstrated that while the intermediate strain-rate eigenvector of the combined strain aligns with the vorticity vector, it is the most extensive eigenvector of the background strain component that aligns with vorticity. Therefore, when most of the strain is not produced by the vorticity, as is the case in much of the production region, it acts as background strain and the alignment differs from the well known universal behaviour. As a result, the extensional strain-rate eigenvector is more strongly aligned with vorticity.

In the present study, fluctuating vorticity is absent for $x/x^* < 0.2$, so the alignment cannot be computed. Once vorticity is present, the results show an equal tendency for alignment between the vorticity vector and the two positive strain-rate eigenvectors. The above analysis suggests that the already existing and dominant background strain (which is the result of non-local pressure) is responsible for the alignment of the most extensive strain-rate eigenvector with vorticity and that this effect coexists in this region with the alignment of the vorticity with its own strain field's intermediate eigenvector. Indeed, Hamlington et al. (2008b) found that the intermediate eigenvector of the vorticity-induced strain tensor aligns very strongly with vorticity. As fluid elements move downstream towards the peak of turbulence intensity and into the decay region, the effect of the vorticity on the strain field becomes stronger and eventually dominates

over the background strain, resulting in the universal alignment behaviour.

This section is closed by comparing the alignment results with those of the experimental study of Gomes-Fernandes et al. (2014). They found the universal alignment behaviour even in the middle of the production region. It has to be borne in mind, however, that there are two significant differences compared to the setup of this study. Firstly, they studied a space filling fractal grid and not a single square grid-element. The fractal grid has small and thin bars located close and around the grid-element centreline that shed wakes which meet much closer to the grid-element compared to the large-scale wakes in the present case. Secondly, the free-stream turbulence intensity was 2.8% for the streamwise velocity component and 4.4% for the spanwise velocity component in their experiment. These two differences indicate that vorticity was already present much closer to the grid-element in their case compared to the current case, and therefore strain and vorticity were very early much more closely coupled. This explains why they see the universal alignment even in the first location they studied, which was in the middle of the production region.

3.9 Summary

This chapter presented the dynamics of velocity gradients for a spatially developing flow generated by a single square element of a fractal square grid at low inlet Reynolds number through direct numerical simulation. This square grid-element is also the fundamental block of a classical grid. The flow along the grid-element centreline is initially irrotational and becomes turbulent further downstream due to the lateral excursions of vortical turbulent wakes from the grid-element bars. The generation and evolution of the symmetric and anti-symmetric parts of the velocity gradient tensor for this spatially developing flow have been studied using the transport equations of mean strain-product and mean enstrophy respectively. The choice of low inlet Reynolds number allows for fine spatial resolution and long simulations, both of which are conducive in balancing the budget equations of the above quantities. The budget analysis is carried out along the grid-element centreline and the bar centreline. The former is observed to consist of two subregions: one in the immediate lee of the grid-element which is dominated by irrotational strain, and one further downstream where both strain and vorticity coexists. In the demarcation area between these two subregions, where the turbulence is inhomogeneous and developing, the energy spectrum exhibits the best $-5/3$ power law slope. This is the same location where the experiments at much higher inlet Reynolds number show a well defined $-5/3$ spectrum over more than a decade of frequencies. Yet, the Q-R diagram remains undeveloped in the near grid-element region, and both the inter-

mediate and extensive strain-rate eigenvectors align with the vorticity vector. Along the grid-element centreline, the strain is the first velocity gradient quantity generated by the action of pressure Hessian. This strain is then transported downstream by fluctuations and strain self-amplification is activated a little later. Further downstream, vorticity from the bar wakes is brought towards the grid-element centreline, and, through the interaction with strain, leads to the production of enstrophy. The strain-rate tensor has a statistically axial stretching form in the production region, but a statistically biaxial stretching form in the decay region. The usual signatures of velocity gradients such as the shape of Q-R diagrams and the alignment of vorticity vector with the intermediate eigenvector are detected only in the decay region even though the local Reynolds number (based on the Taylor length scale) is only between 30 and 40.

Chapter 4

Genesis and spatial evolution of scalar gradients

4.1 Overview

Having analysed the VGT dynamics in the previous chapter, the present chapter focuses on the fluctuating scalar gradient dynamics for the same developing flow generated by a square grid-element. A passive scalar in the form of temperature is injected into the developing flow by means of heating the grid-element. This chapter is complementary to the previous one. The main goal in the scalar gradient analysis is not only to study the dynamics of scalar gradients, but also to understand the interaction between the velocity and scalar gradients. The nature of scalar turbulence behind the grid-element is studied first in terms of the budgets of large-scale terms (mean temperature, turbulent heat-fluxes, scalar-variance) and the budget of small-scale terms (scalar-gradient-variance). Then the behaviour of the small-scale terms of scalar turbulence is studied in relation to the $-5/3$ slope of the scalar spectrum. The scalar gradients are also analysed further in terms of their skewness and flatness. Finally, the interaction between the velocity and scalar gradients is probed through the eigen-contributions of strain to the scalar gradient production.

4.2 Computational parameters

The cross-sectional front view of the computational domain is depicted in figure 4.1a. The geometrical details of the grid-element are already given in chapter 2. A side-view of the computational domain at plane $z/t_0 = 0$, with details on the size of the domain and the description of boundaries, is presented in figure 4.1b. The grid-element and bar centrelines are also marked in figure 4.1b. Note that this study considers the bar

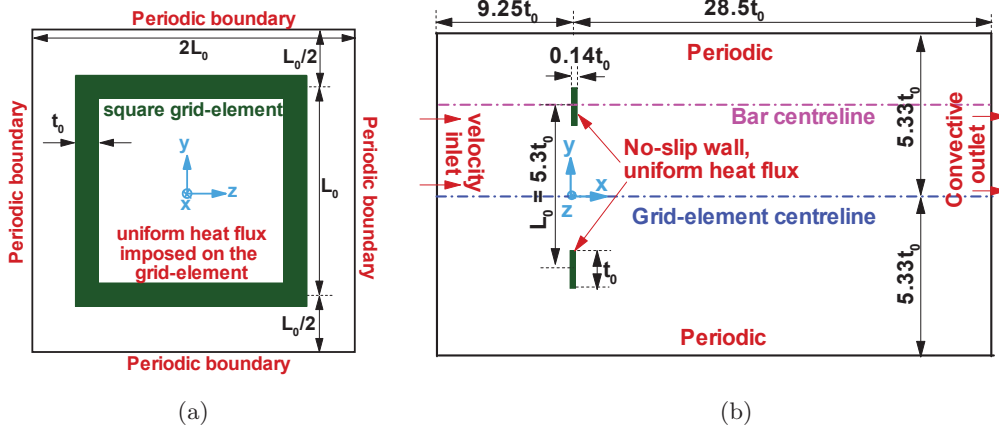


Figure 4.1: Sketch of the computational domain: (a) front view, (b) side view at plane $z/t_0=0$ (the sketches are not to scale).

centreline of the horizontal bars in the $x - y$ plane.

Uniform velocity and temperature profiles are prescribed at the inlet (U_∞ and T_∞ respectively) and a convective boundary condition is imposed at the outlet. All lateral boundaries are periodic. On the grid-element surface, no-slip condition for velocity and a uniform heat-flux (q_w) for temperature are applied.

The Reynolds number based on the the free-stream velocity U_∞ and the bar length L_0 is $Re_{L_0} = 2650$, while based on the lateral thickness t_0 is $Re_{t_0} = 500$. The Prandtl number, Pr , is 0.71.

The same mesh, reported in the previous chapter, is used for this chapter as well. It has been shown in the previous chapter that the ratio of a characteristic mesh size to the Kolmogorov length scale is less than one throughout the computational domain. Since $Pr < 1$ for the current study, the Obukhov-Corrsin scale (η_{OC}) is larger than the Kolmogorov length scale (see section 1.3.1). Therefore, the mesh resolution is finer for the scalar field compared to the velocity field. At worse, the mesh resolution is $0.63\eta_{OC}$ in the whole domain. More details about the mesh, the numerical setup and the high performance computing details of the simulation were presented in the previous chapter.

The validation of the solver against the experiments for one-point velocity and velocity gradient statistics was presented in the previous chapter on the VGT dynamics. The scalar analysis relies upon this validation since there are no experiments available for the scalar configuration of this study.

4.3 Balance of transport equations

The main theme of this section is to investigate the dynamics and transport mechanisms of time-average scalar, scalar-variance, turbulent heat-fluxes and scalar-variance dissipation (or scalar-gradient-variance). The transport equations of these quantities are analysed along the bar and grid-element centrelines because of the different dynamics in these two locations.

4.3.1 Mean scalar balance

The three dimensional fluctuating temperature field is visualised in figure 4.2a. Contours of the instantaneous scalar field are depicted in figure 4.2b. The scalar is being injected into the wake by heating the grid-element. For the time instant shown in the figure, the scalar wakes behind the grid-element bars start meeting at $x/x^* \approx 0.4$. As shown later in this section, the scalar wakes on average meet at $x/x^* \approx 0.2$, which is similar to the location reported in the previous chapter for the meeting of vortical wakes.

The time-averaged scalar field is shown in figure 4.2c. The iso-line of normalised mean vorticity vector magnitude is superimposed in order to compare the spreadings of vorticity and scalar. These two spreadings are comparable as one might expect from the known fact that when a scalar is released from the turbulence generating body, the spreading behaviour of momentum and scalar wakes are similar (Tennekes and Lumley, 1972). A mean recirculation region exists behind the grid-element bars and its length is approximately $3.7t_0$ (in terms of x^* , it is $0.13x^*$). The scalar is trapped inside this region, leading to high values in the centreline (depicted as a small circular red area behind the bars in figure 4.2c).

Figure 4.3 depicts the development of mean scalar field downstream of the grid-element in $y - z$ plane. The scalar field is highly inhomogeneous in the near field (see figures 4.3a and 4.3b) but it becomes approximately homogeneous in the far downstream as seen in figure 4.3f plotted for $x/x^*=0.9$.

Decomposing the instantaneous scalar field as the sum of mean (i.e. time-averaged) and fluctuations, $T^* = \langle T^* \rangle + T$, the equation for the mean scalar field $\langle T^* \rangle$ is obtained after Reynolds averaging equation (2.3):

$$\frac{\partial \langle T^* \rangle}{\partial t} = \underbrace{-U_j \frac{\partial \langle T^* \rangle}{\partial x_j}}_{C_T} + \underbrace{\alpha \frac{\partial}{\partial x_j} \left(\frac{\partial \langle T^* \rangle}{\partial x_j} \right)}_{D_T} - \underbrace{\frac{\partial}{\partial x_j} (\langle u_j T \rangle)}_{Q_T} \quad (4.1)$$

where C_T is convection by the mean flow, D_T is the diffusion and Q_T is the transport due to turbulent fluctuations $\langle u_j T \rangle$ (where $j = 1, 2, 3$). The transient term on the left

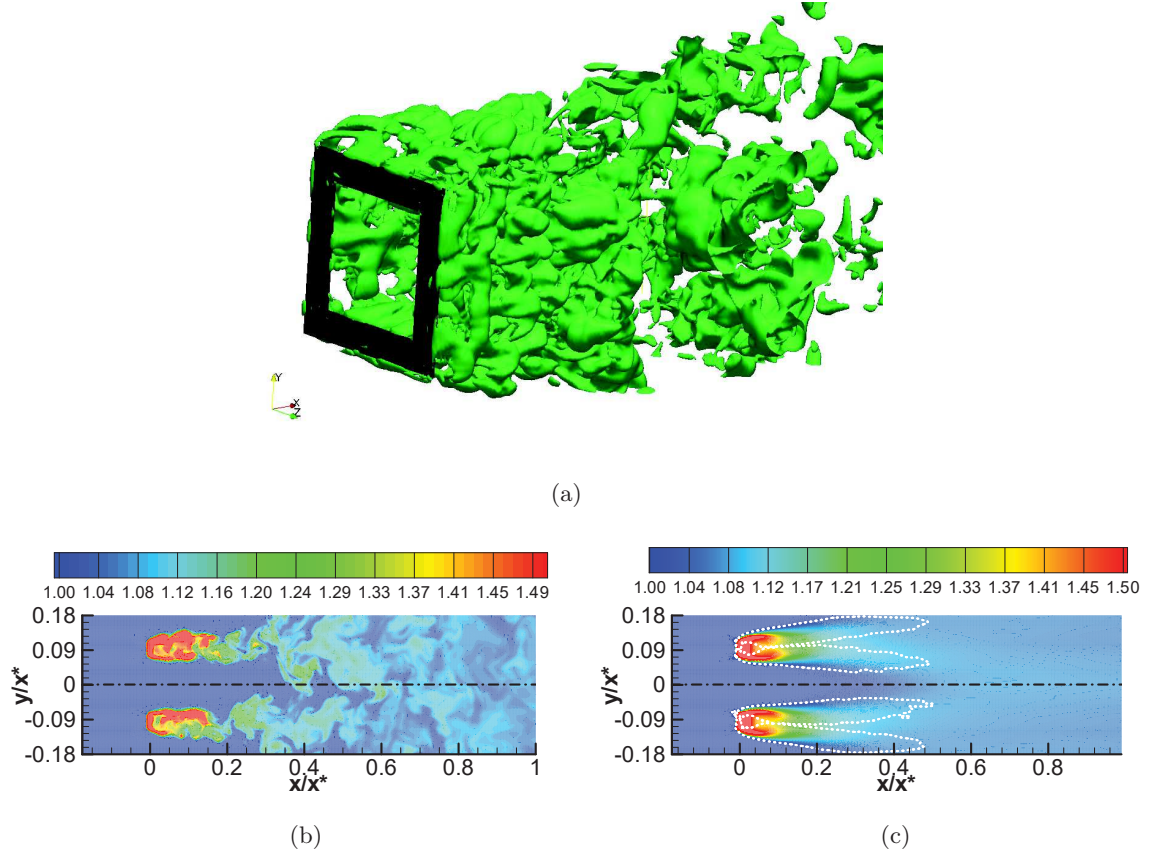


Figure 4.2: (a) Iso-surfaces of fluctuating temperature $T=0.1$. Contours of (b) instantaneous temperature, (c) mean temperature fields at $z/x^*=0$. The isocontours range from 1 to 1.5 of T_∞ . The white coloured dotted line in (c) is the isoline of normalised mean vorticity vector magnitude ($\langle |\boldsymbol{\omega}^*| \rangle t_0 / U_\infty$) of 0.18. The black coloured dotted-dashed-line in (b) and (c) is the centreline of the grid-element, while the bar centreline is represented by a pink dotted-dashed-line.

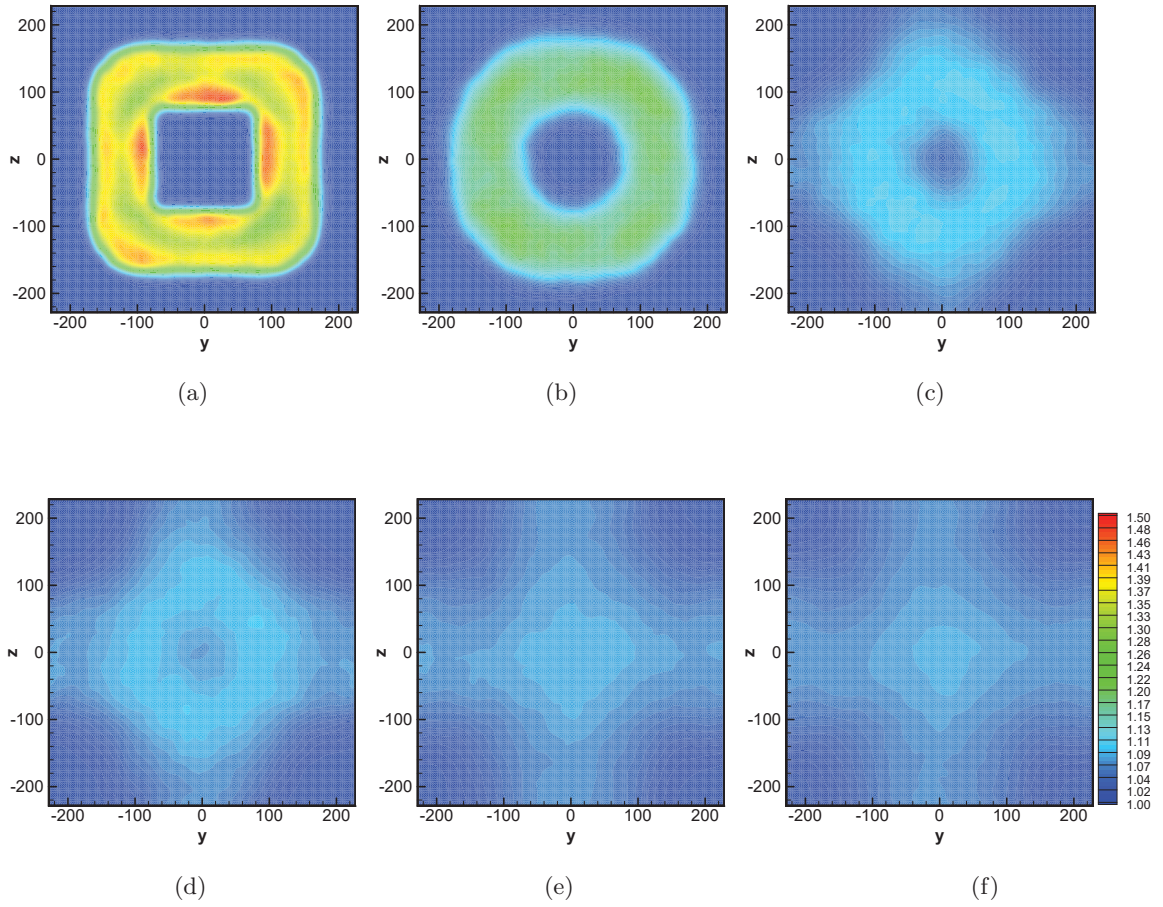


Figure 4.3: Contours of mean temperature in the yz plane at different x/x^* locations: (a) 0.1 (b) 0.2 (c) 0.4 (d) 0.5 (e) 0.75 (f) 0.9

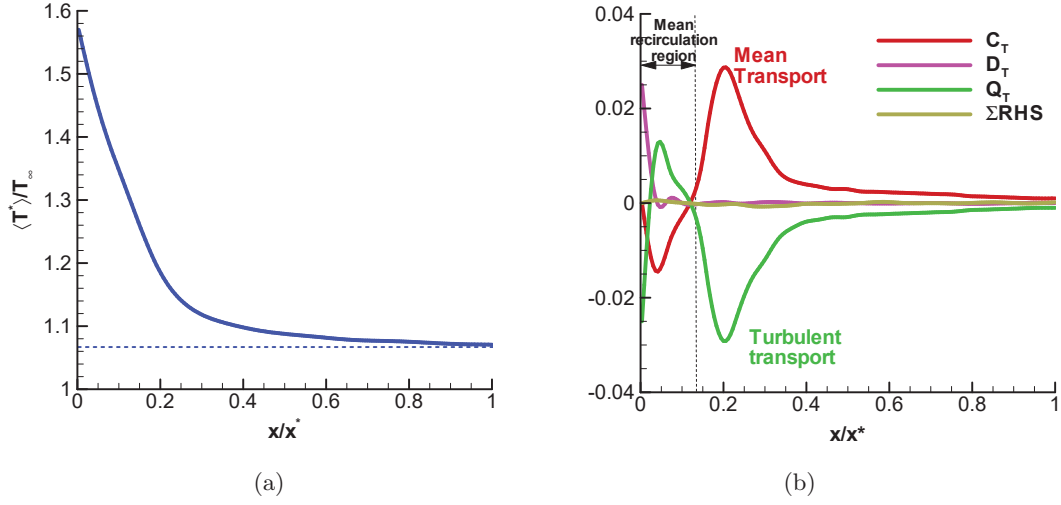


Figure 4.4: (a) Evolution of mean scalar profile along the bar centreline, (b) Evolution of budget terms of mean scalar along the bar centreline (all the terms are normalised by $U_\infty T_\infty / t_0$). The dotted-line in (a) denotes the outlet temperature predicted by global energy balance.

hand side is zero.

Figure 4.4a presents the evolution of mean scalar along the bar centreline; the values are normalised by the free-stream value, T_∞ . The variation is as expected: the highest value occurs at the grid-element due to heating applied on the wall and the scalar decays monotonously downstream. A simple energy balance in the whole domain yields:

$$E_{out} - E_{in} = \Delta E_{system} \quad (4.2)$$

where E_{in} , E_{out} are the convective heat-fluxes entering and leaving the domain respectively and ΔE_{system} is the total heat added to the system (per unit time). The resulting temperature increase $\langle \Delta T^* \rangle$ is computed as,

$$\langle \Delta T^* \rangle = \frac{q_w A_{grid-element}}{\dot{m} C_p} \quad (4.3)$$

where q_w is the imposed heat-flux on the grid-element with surface area $A_{grid-element}$, $\dot{m}$ is the inlet mass flow-rate and C_p is the specific heat (related to the thermal diffusivity with $\alpha = \kappa_{tc} / (\rho C_p)$, where κ_{tc} is the thermal conductivity). $\langle \Delta T^* \rangle$ is computed to be 0.068, therefore the outlet temperature is 1.068, a value asymptotically approached by the numerically computed mean scalar profile in figure 4.4a.

The budget of mean scalar along the bar centreline is plotted in figure 4.4b. Very

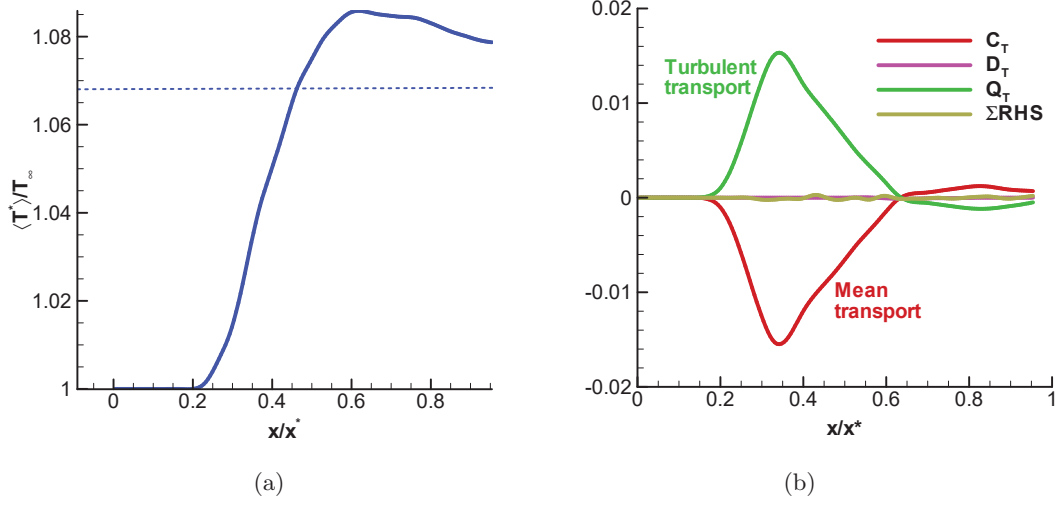


Figure 4.5: (a) Evolution of mean scalar profile along the grid-element centreline, (b) Evolution of budget terms of mean scalar along the grid-element centreline (all the terms are normalised by $U_\infty T_\infty / t_0$). The dotted-line in (a) denotes the outlet temperature predicted by global energy balance.

close to the grid-element, the viscous diffusion process is balanced by the turbulent transport. The figure shows that the heat-flux applied to the grid-element is injected into the wake through a diffusion process. The diffusion term, however, becomes negligible immediately downstream of the grid-element and the turbulent transport is balanced by the mean convection. These two terms change sign at the end of the recirculation zone but continue to balance each other far downstream (figure 4.4b).

The mean scalar profile along the grid-element centreline is depicted in figure 4.5a. The boundary condition at the inlet ($T^* = T_\infty$) persists until $x/x^* \approx 0.2$. This is the same location where the vortical wakes were first observed to meet in the previous chapter. Temperature increases in the region $x/x^* \approx 0.2 - 0.6$. From the budget plot (figure 4.5b) it is clear that this increase is due to the turbulent transport caused by the intermittent meeting of the bar wakes. For $x/x^* > 0.6$, the mean scalar decays. The diffusion term is found to be insignificant throughout the grid-element centreline, and the dominant terms are the convection due to mean flow and the turbulent transport that balance each other. At the end of the domain, there remains a slight inhomogeneity in the cross-streamwise direction; the temperature in the grid-element centreline is slightly higher compared to the bar centreline (compare figures 4.5a and 4.4a).

4.3.2 Turbulent heat-flux balances

Reynolds averaging of the scalar transport equation results in the mean scalar field

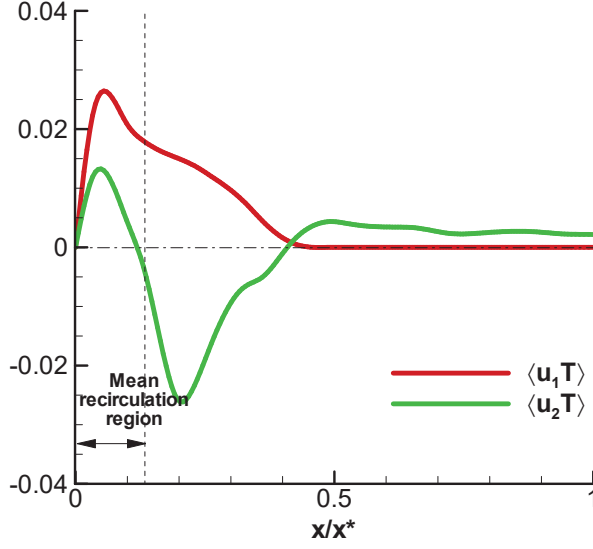


Figure 4.6: Evolution of turbulent heat-fluxes along the bar centreline (fluxes are normalised by $U_\infty T_\infty$.)

equation with additional unknowns in the form of turbulent heat-fluxes $\langle u_i T \rangle$ (where $i = 1, 2, 3$). In this subsection, the budgets of these terms are analysed. The transport equation of turbulent heat-fluxes is given as (refer to Launder (1988)),

$$\begin{aligned} \frac{\partial \langle u_i T \rangle}{\partial t} = & \underbrace{-U_k \frac{\partial \langle u_i T \rangle}{\partial x_k}}_{C_q} - \underbrace{\langle u_i u_k \rangle \frac{\partial \langle T^* \rangle}{\partial x_k}}_{P_q} - \underbrace{\langle T u_k \rangle \frac{\partial U_i}{\partial x_k}}_{G_q} + \underbrace{\left\langle \frac{p}{\rho} \frac{\partial T}{\partial x_i} \right\rangle}_{\pi_q} - \underbrace{(\alpha + \nu) \left\langle \frac{\partial u_i}{\partial x_k} \frac{\partial T}{\partial x_k} \right\rangle}_{\epsilon_q} \\ & - \underbrace{\frac{\partial \langle u_i T u_k \rangle}{\partial x_k}}_{T_q} - \underbrace{\frac{\partial}{\partial x_k} \left\langle \frac{p T}{\rho} \delta_{ik} \right\rangle}_{Pr_q} + \underbrace{\nu \frac{\partial}{\partial x_k} \left\langle T \frac{\partial u_i}{\partial x_k} \right\rangle}_{\alpha_q} + \underbrace{\alpha \frac{\partial}{\partial x_k} \left\langle u_i \frac{\partial T}{\partial x_k} \right\rangle}_{\beta_q} \quad (4.4) \end{aligned}$$

where C_q is the mean convection, P_q and G_q are the production due to mean- scalar and mean-velocity gradients respectively, the correlation of pressure and fluctuating scalar gradient is denoted as π_q , ϵ_q is the dissipation, the turbulent transport due to velocity and pressure are marked as T_q and Pr_q respectively, the viscous and molecular diffusions are denoted as α_q and β_q respectively.

The variation of turbulent heat-fluxes along the bar centreline is depicted in figure 4.6. The spanwise heat-flux ($\langle u_3 T \rangle$) is negligible throughout the bar centreline and is not shown. From figure 4.6, it can be seen that the values of $\langle u_1 T \rangle$ are larger than $\langle u_2 T \rangle$ close to the grid-element, and they peak at the same location inside the recirculation zone. The value of the streamwise heat-flux, however, becomes negligible after $x/x^* \approx 0.5$. Far downstream, only the cross-streamwise turbulent heat-flux is non-zero while the other

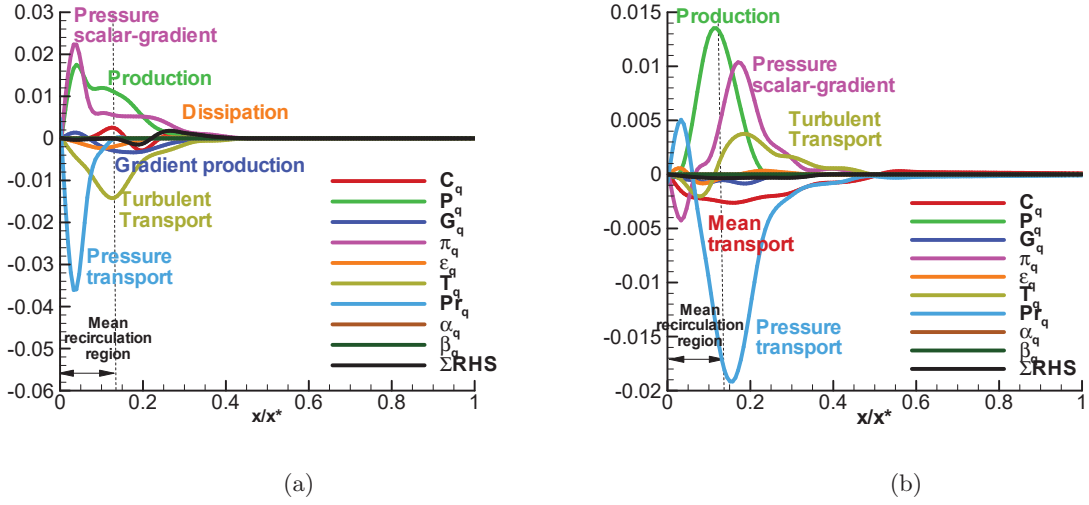


Figure 4.7: Evolution of turbulent heat-flux budgets along the bar centreline: (a) streamwise turbulent heat-flux ($\langle u_1 T \rangle$), (b) cross-streamwise turbulent heat-flux ($\langle u_2 T \rangle$). All the terms in the budget equation are normalised by $U_\infty^2 T_\infty / t_0$.

two heat-fluxes are insignificant.

The budget of the streamwise turbulent heat-flux along the bar centreline is shown in figure 4.7a (all the terms are normalised by $U_\infty^2 T_\infty / t_0$). The growth of the streamwise heat-flux close to the grid-element is mainly due to the pressure-scalar-gradient correlation and the production due to mean scalar gradient, which are balanced by the pressure transport and the turbulent transport terms. The importance of the pressure-scalar-gradient correlation and the pressure transport has been reported in the literature before. It was first established by Kasagi et al. (1992) in a channel flow simulation that these terms are important for the transport of turbulent heat-fluxes, and they often have opposite signs. Similar observations are noted for the present flow configuration. The production due to mean scalar gradient is stronger closer to the grid-element, and it decays slowly to zero around $x/x^* \approx 0.3$. The evolution of the pressure-scalar-gradient correlation is similar to that of the production by mean scalar gradient. It is interesting that the mean transport term is much smaller than the turbulent transport term, and is negligible. The contribution of mean velocity gradients to the production of turbulent heat-flux is much smaller compared to that of mean scalar gradients. For $x/x^* > 0.5$, all the transport terms are approximately zero.

Figure 4.7b depicts the budget of the cross-streamwise turbulent heat-flux $\langle u_2 T \rangle$. Very close to the grid-element, the pressure-scalar-gradient correlation acts as a sink and is balanced by the pressure transport term. The pressure-scalar-gradient and the pressure transport change sign a little further downstream. A similar observation was

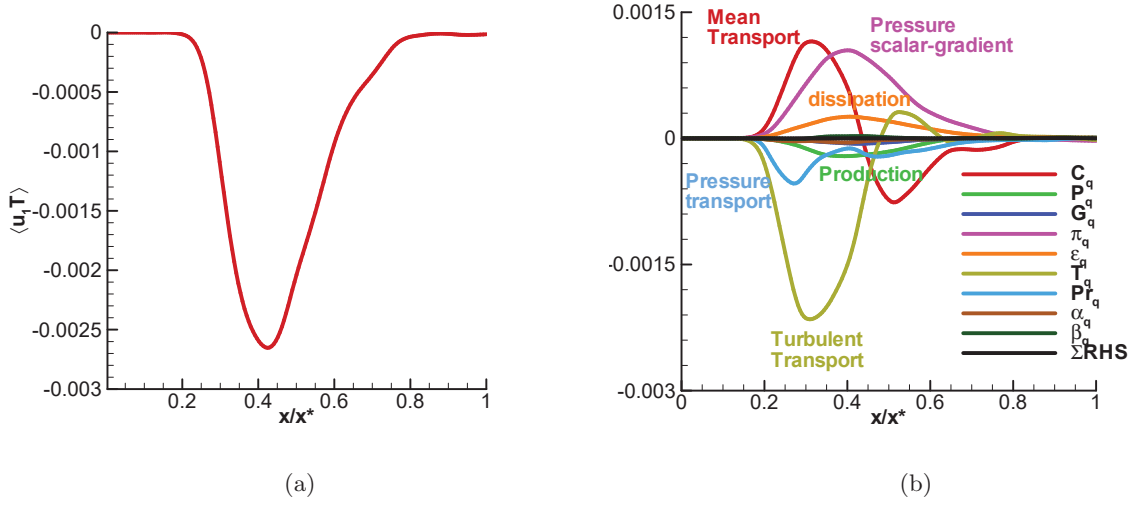


Figure 4.8: (a) Evolution of the streamwise turbulent heat-flux ($\langle u_1 T \rangle$) along the grid-element centreline, (b) Budget terms of the streamwise turbulent heat-flux ($\langle u_1 T \rangle$) along the grid-element centreline. All terms are normalised by $U_\infty^2 T_\infty / t_0$.

noted in the budget of cross-streamwise heat-flux budget in channel flow (Kasagi et al., 1992). After the recirculation zone, the pressure and mean transport terms are balanced by the production due to mean scalar gradient, the turbulent transport and the pressure-scalar-gradient correlation terms. In the far downstream (i.e. $x/x^* > 0.5$), the cross-streamwise heat-flux is carried by the mean flow while being in balance with the turbulent transport.

Along the grid-element centreline, the streamwise turbulent heat-flux $\langle u_1 T \rangle$ is zero in the region of $0 \leq x/x^* < 0.2$ because the bar wakes have not met yet (figure 4.8a). The other two heat-fluxes (i.e. $\langle u_2 T \rangle$ and $\langle u_3 T \rangle$) are very small throughout the grid-element centreline, therefore they are not considered. In figure 4.8a, $\langle u_1 T \rangle$ decreases from $x/x^* \approx 0.2$ to 0.4, and then it increases to zero around $x/x^* \approx 0.8$.

The growth of streamwise heat-flux after $x/x^* \approx 0.2$ along the grid-element centreline is due to the turbulent transport term as seen in figure 4.8b. The components of the turbulent transport term are found to be $\frac{\partial \langle u_1 T u_2 \rangle}{\partial x_2} \approx \frac{\partial \langle u_1 T u_3 \rangle}{\partial x_3} \geq \frac{\partial \langle u_1 T u_1 \rangle}{\partial x_1}$, i.e. the turbulent transport arises from fluctuations in the y and z directions. This demonstrates that the turbulent heat-flux found on the grid-element centreline has been transported laterally from the bar wakes, where it was generated by the action of mean scalar gradient and the pressure-scalar-gradient correlation. Although a streamwise mean scalar gradient exists in the region of $0 \leq x/x^* \leq 0.6$ (see figure 4.5a), its contribution to the production of streamwise heat-flux seems to be small compared to the contribution of turbulent transport term. From $x/x^* \approx 0.2$ to 0.5, the growth of streamwise heat-flux is also due

to the pressure transport. These two terms are balanced by the pressure-scalar-gradient correlation, the mean convection and the dissipation. Further downstream, the turbulent transport becomes a sink, the source is due to the mean convection and the pressure transport, and they are balanced by the dissipation and the pressure-scalar-gradient correlation. Note that the transport equations of turbulent heat-fluxes along the bar and the grid-element centrelines (figures 4.7a, 4.7b and 4.8b) are properly balanced.

4.3.3 Scalar-variance balance

The next quantity of both fundamental and engineering importance is the scalar-variance $\frac{1}{2}\langle T^2 \rangle$. The transport equation of scalar-variance is written as (see Tennekes and Lumley (1972))

$$\begin{aligned} \frac{\partial(\langle \frac{1}{2}T^2 \rangle)}{\partial t} = & \underbrace{-U_j \frac{\partial(\langle \frac{1}{2}T^2 \rangle)}{\partial x_j}}_{C_v} \underbrace{-\langle u_j T \rangle \frac{\partial \langle T^* \rangle}{\partial x_j}}_{G_v} \underbrace{-\alpha \left\langle \frac{\partial T}{\partial x_j} \frac{\partial T}{\partial x_j} \right\rangle}_{\epsilon_v} \underbrace{-\frac{\partial}{\partial x_j} \left\langle \frac{1}{2} u_j T^2 \right\rangle}_{T_v} \\ & + \underbrace{\alpha \frac{\partial}{\partial x_j} \left(\frac{\partial \frac{1}{2} \langle T^2 \rangle}{\partial x_j} \right)}_{D_v} \end{aligned} \quad (4.5)$$

where C_v is the mean flow convection, G_v is the production by mean scalar gradient, ϵ_v is the dissipation of scalar-variance, T_v is the transport by turbulent fluctuations and D_v is the molecular diffusion.

Figure 4.9a presents the evolutions of scalar-variance and turbulent kinetic energy along the bar centreline. The scalar-variance increases from $x = 0$ until the end of the mean recirculation region (i.e. $x = 0.13x^*$) and then decays downstream. This trend is very similar to the evolution of the turbulent kinetic energy.

The dynamics of scalar-variance is further analysed through the evolution of its budget terms (figure 4.9b). Closer to the bar, the mean scalar gradients produce scalar-variance which is further carried by the mean flow. The values of mean scalar production and mean transport terms increase until the end of the mean recirculation region. Here, they are balanced by the turbulent transport and dissipation terms. Downstream of the mean recirculation region, the gradient production and mean transport decrease. After $x/x^* \approx 0.5$, the turbulent transport term becomes negligible and the scalar-variance decays due to the dissipation as it is carried by the mean flow.

The evolutions of scalar-variance and turbulent kinetic energy along the grid-element centreline are depicted in figure 4.10a. Although the variation was almost identical behind the bars as shown in the previous figure, along the grid-element centreline the two variables exhibit intriguing differences. Firstly, the scalar-variance is negligible for

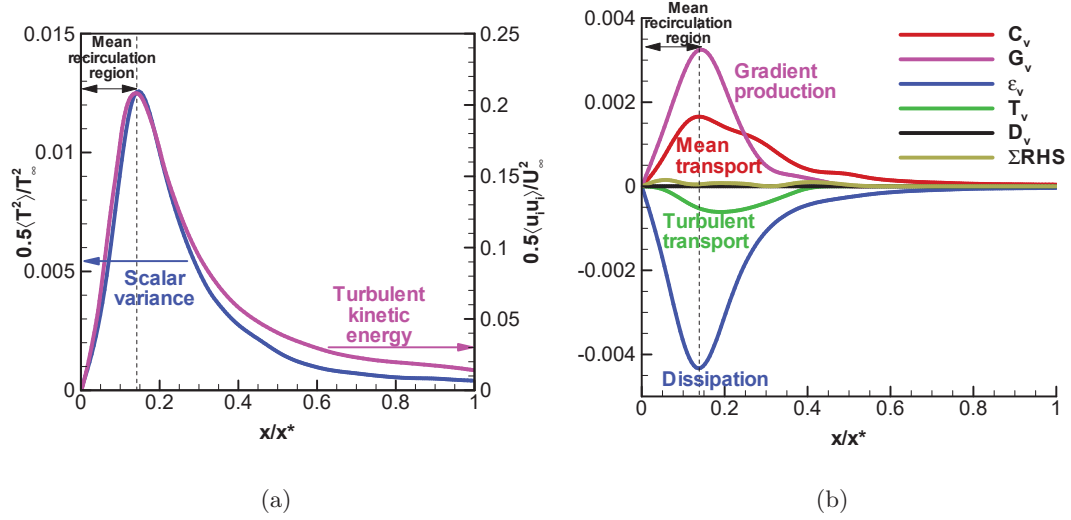


Figure 4.9: (a) Evolution of scalar-variance and turbulent kinetic energy along the bar centreline, (b) Evolution of budget terms of scalar-variance along the bar centreline. All terms are normalised by $U_\infty T_\infty^2 / t_0$.

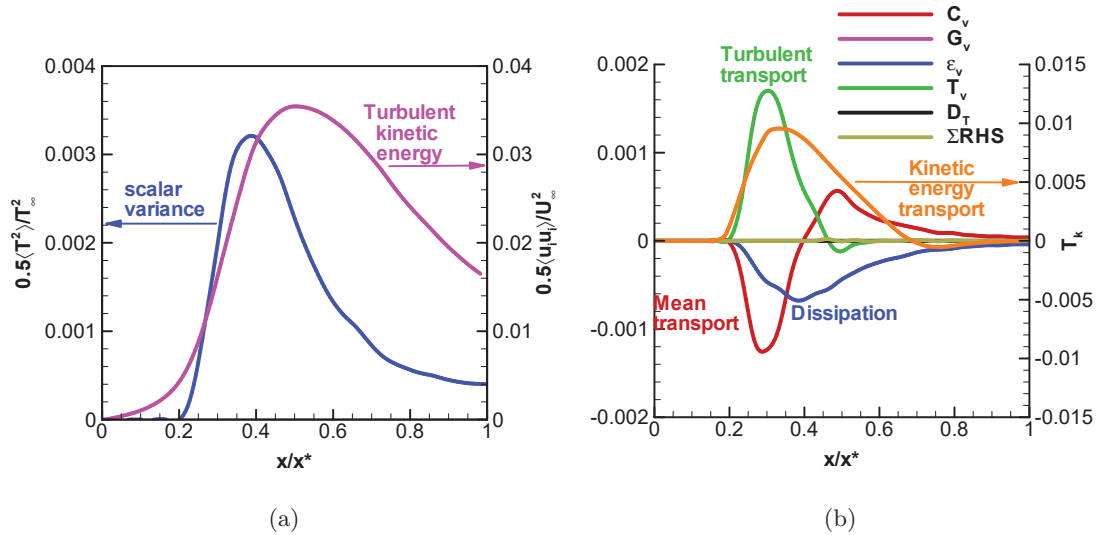


Figure 4.10: (a) Evolution of scalar-variance and turbulent kinetic energy along the grid-element centreline, (b) Evolution of budget terms of scalar-variance along the grid-element centreline. All terms are normalised by $U_\infty T_\infty^2 / t_0$. The transport due to fluctuations of the turbulent kinetic energy is also shown.

$x/x^* < 0.2$ but the kinetic energy is clearly not. This difference is explained by the generating mechanism in the form of pressure-work for turbulent kinetic energy, a mechanism which is absent for the scalar. Secondly, the onset of scalar-variance decay occurs much earlier in space compared to that of the turbulent kinetic energy. While the turbulent kinetic energy starts decaying around $x/x^* \approx 0.5$, the decay of scalar-variance starts around 0.4. Comparing figures 4.9a and 4.10a, it is interesting that the variation of scalar-variance and turbulent kinetic energy is qualitatively similar, with both quantities reaching a peak value at exactly the same location along the bar centreline, but along the grid-element centreline, their behaviour is markedly different.

The budget terms, as shown in figure 4.10b, provide further insight into the evolution of scalar-variance along the grid-element centreline. The first term to become active along the grid-element centreline is the turbulent transport term, $T_v = -\frac{\partial}{\partial x_j} \left\langle \frac{1}{2} u_j T^2 \right\rangle$. The largest contribution to this term is calculated to be coming from the lateral directions, $j = 2, 3$. Therefore the scalar-variance, which was produced by the mean scalar gradients near the bar, is brought to the grid-element centreline through the intermittent meeting of the wakes. This turbulent transport is balanced by the mean convection and dissipation. Eventually, the turbulent transport term becomes negligible around $x/x^* \approx 0.5$. Far downstream, the scalar-variance is carried by the mean flow while it is dissipated by the thermal diffusivity. Again note that the budgets of scalar-variance are properly balanced along the grid-element and the bar centrelines.

The budget analysis reveals that the evolution of scalar-variance along the grid-element centreline is controlled by two terms, the turbulent transport and the dissipation. Although the turbulent transports of turbulent kinetic energy and scalar-variance reach a peak value around $x/x^* \approx 0.3$, their evolution downstream of $x/x^* \approx 0.3$ differs (see figure 4.10b). At the same time, as shown in the next subsection, the dissipation of scalar-variance peaks earlier along the grid-element centreline than that of turbulent kinetic energy. This means that the differences in the evolution of scalar-variance and turbulent kinetic energy observed in figure 4.10a are due to differences in the turbulent transport and dissipation of the two quantities.

4.3.4 Scalar-gradient-variance balance

Having studied the budget of scalar-variance, this section focuses on the dissipation of scalar-variance (i.e. scalar-gradient-variance) $\langle G_i G_i \rangle$, where $G_i = \frac{\partial T}{\partial x_i}$. Denoting the instantaneous scalar gradient as $G_i^* = \frac{\partial T^*}{\partial x_i}$, the transport equation of scalar-gradient-

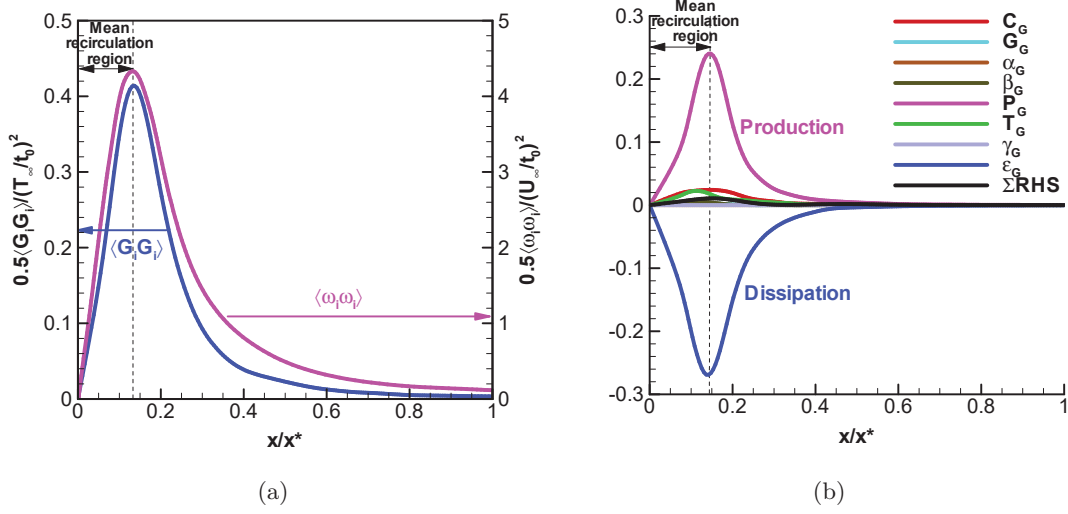


Figure 4.11: (a) Evolution of scalar-gradient-variance and enstrophy along the bar centreline, (b) Evolution of budget terms of scalar-gradient-variance along the bar centreline. All the terms are normalised by $U_\infty T_\infty^2 / t_0^3$.

variance is written as (see Launder (1988)):

$$\begin{aligned}
 \frac{\partial (\langle \frac{1}{2} G_i G_i \rangle)}{\partial t} = & \underbrace{-U_j \frac{\partial \langle G_i G_i \rangle}{\partial x_j}}_{C_G} - \underbrace{\langle G_i u_j \rangle \left\langle \frac{\partial G_i^*}{\partial x_j} \right\rangle}_{G_G} - \underbrace{\left\langle G_i \frac{\partial u_j}{\partial x_i} \right\rangle \left\langle \frac{\partial G_i^*}{\partial x_j} \right\rangle}_{\alpha_G} \\
 & - \underbrace{\langle G_i G_j \rangle \frac{\partial U_j}{\partial x_i}}_{\beta_G} - \underbrace{\left\langle G_i s_{ij} G_j \right\rangle}_{P_G} - \underbrace{\frac{1}{2} \frac{\partial}{\partial x_j} \langle G_i G_j u_j \rangle}_{T_G} \\
 & + \underbrace{\frac{\alpha}{2} \frac{\partial^2}{\partial x_j \partial x_j} (\langle G_i G_i \rangle)}_{D_G} - \underbrace{\alpha \left\langle \frac{\partial G_i}{\partial x_j} \frac{\partial G_i}{\partial x_j} \right\rangle}_{\epsilon_G} \quad (4.6)
 \end{aligned}$$

where C_G is the convection by mean flow, G_G is the production by mean scalar gradient, α_G is the mixed production, β_G is the production by velocity gradient, P_G is the production by turbulent stretching/compression, T_G is transport by turbulent fluctuations, D_G is the diffusion, and ϵ_G is the dissipation.

The evolution of scalar-gradient-variance along the bar centreline is presented in figure 4.11a. This variation is similar to that of the mean enstrophy of fluctuations, also shown in the same figure, with values reaching a peak value by the end of the mean recirculation region, and then a monotonic decay further downstream. The corresponding budget terms, as shown in figure 4.11b, reveal that the growth of scalar-gradient-variance is due to the stretching/compression of the scalar gradient vector by the fluctuating strain

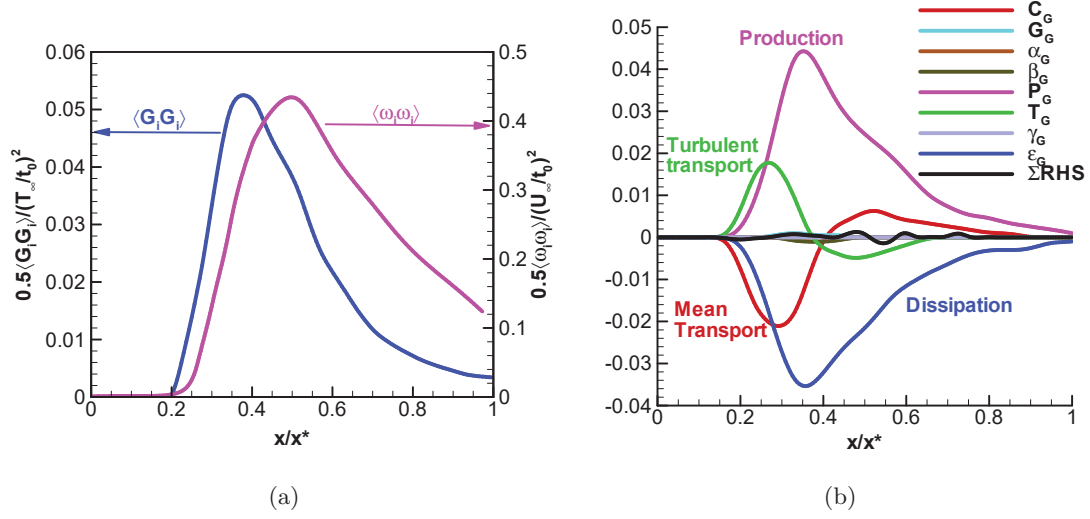


Figure 4.12: (a) Evolution of scalar-gradient-variance and enstrophy along the grid-element centreline, (b) Evolution of budget terms of scalar-gradient-variance along the grid-element centreline. All the terms are normalised by $U_\infty T_\infty^2 / t_0^3$.

field (term P_G), which is balanced by the dissipation throughout the bar centreline. The contribution of all the other terms is negligible. The insignificant contribution of mean fields (both velocity and scalar) to the generation of small scales of scalar was also observed in the field experiments of Gulitski et al. (2007a).

Although the spatial evolution of scalar-gradient-variance is similar to that of the mean enstrophy along the bar centreline, they differ along the grid-element centreline. The scalar-gradient-variance is negligible in the region before the meeting of the wakes, and it starts increasing after $x/x^* \approx 0.2$ until $x/x^* \approx 0.4$ (figure 4.12a). Scalar-gradient-variance starts decaying earlier along the grid-element centreline compared to mean enstrophy. This difference is further discussed in section 4.6.

The budget of scalar-gradient-variance along the grid-element centreline is plotted in figure 4.12b. The very first term in space to contribute for the growth of scalar-gradient-variance is the turbulent transport term. This indicates that the fluctuating scalar-gradients have been brought to the grid-element centreline by the fluctuations, just like the turbulent heat-fluxes and the scalar-variance. Once there, the background strain, which is mainly due to pressure Hessian and turbulent transport (see section 3.4.3 for more details on these strain-generating mechanisms), acts on the scalar gradient in the form of turbulent stretching/compression which results in activation of the production term P_G . These two terms are balanced by the mean convection and the dissipation. After $x/x^* \approx 0.4$, the mean convection and the production are balanced by the turbulent transport and the dissipation. Far downstream, the transport terms (both mean and

x/x^*	0.1	0.25	0.35	0.5	0.75	0.95
$Pe_{\lambda T}$	5.21	12.31	19.45	25.6	24.71	23.89
Re_{λ}	10.21	22.01	34.49	37.58	39.12	33.12

Table 4.1: Values of $Pe_{\lambda T}$ and Re_{λ} at different locations along the grid-element centreline.

turbulent) become negligible, and only the production and dissipation terms remain that balance each other.

As an interim summary, this study has identified the dominant mechanisms responsible for the production and transport of various scalar quantities such as mean scalar, turbulent heat-fluxes, scalar-variance and its dissipation along the bar and the grid-element centrelines. The turbulent heat-fluxes and scalar-variance are mainly produced by the action of mean scalar gradients along the bar centreline, while the growth of these terms along the grid-element centreline is the result of lateral turbulent transport from the bar wakes to the grid-element centreline. On the other hand, the scalar-gradient-variance is predominately generated by the turbulent stretching/compression of the scalar gradient vector along the bar as well as the grid-element centrelines.

4.4 Statistics of the fluctuating scalar and its gradients

The analyses presented in the rest of this chapter have been carried out only along the grid-element centreline for the following reasons. Firstly, as noted in section 4.3.4, the scalar-gradient-variance and the turbulent kinetic energy dissipation evolve with a lag from each other along the grid-element centreline, while they evolve together along the bar centreline. Secondly, the grid-element centreline exhibits both intermittent and fully-turbulent regions while the bar centreline contains only the fully-developed turbulence. Finally, the scalar gradient statistics and dynamics, as will be shown in the rest of this chapter, exhibit intriguing novel results only along the grid-element centreline.

The values of the Taylor-length-based Reynolds number (Re_{λ}) and the Corrsin-length-based Peclet number ($Pe_{\lambda T}$) at different stations along the grid-element centreline are given in table 4.1. Note the very low values of Re_{λ} and $Pe_{\lambda T}$ along the grid-element centreline. These very low values are deliberately chosen so as to check the applicability of high Reynolds and Peclet number scalar turbulence properties for the low Re_{λ} and $Pe_{\lambda T}$ case.

First, the scalar fluctuations and their gradients are characterised using statistical

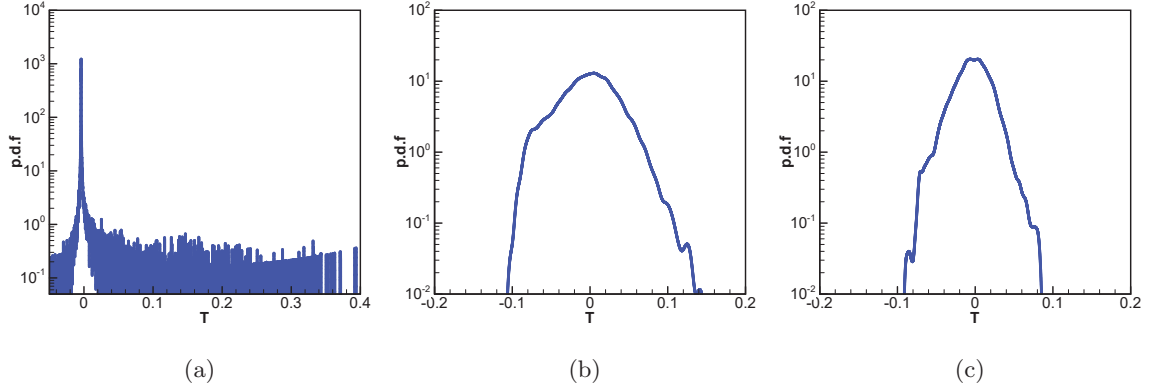


Figure 4.13: Pdf of temperature fluctuation at: (a) $x/x^*=0.25$ (b) $x/x^*=0.5$ (c) $x/x^*=0.9$.

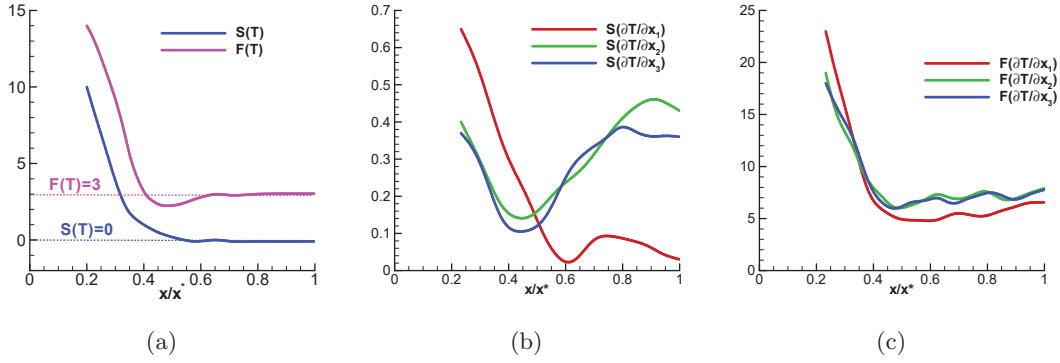


Figure 4.14: Evolution of various statistics along the grid-element centreline: (a) Skewness and flatness of scalar fluctuation, (b) Skewness of fluctuating scalar gradients, (c) Flatness of fluctuating scalar gradients. The dotted lines in (a) denote the values for Gaussian distribution.

measures like skewness and flatness. As this study has chosen very low Re_λ and $Pe_{\lambda T}$ turbulence, it remains to be seen if the scalar spectrum will have a $-5/3$ power-law slope or not. This question is addressed in section 4.5. Finally, the production term P_G of scalar-gradient-variance is analysed in terms of strain-rate eigenvalues and geometrical alignments. The fundamental mechanism to explain why the onset of the dissipation of scalar-variance decay occurs earlier than that of the turbulent kinetic energy is also provided.

4.4.1 Skewness and flatness of scalar fluctuations

The pdf of fluctuating scalar is shown for the production, peak and decay regions in

figures 4.13a, 4.13b and 4.13c respectively. While the production region pdf shows large positive skewness, the level of skewness decreases in the peak region. In the far-downstream decay region, the pdf resembles that of a Gaussian variable. The evolution of skewness of scalar fluctuations along the grid-element centreline is plotted in figure 4.14a. The scalar fluctuations are highly non-Gaussian with a large positive skewness in the region where the scalar-variance is increasing ($0.2 < x/x^* < 0.4$). Zero skewness, as in Gaussian statistics, appears around $x/x^* \approx 0.5$. Comparing the skewness of fluctuating scalar with that of the streamwise velocity fluctuation (see figure 3.7a of the previous chapter), two observations can be made. Firstly, the skewness of velocity fluctuations is negative close to the grid-element, indicating highly decelerating turbulent events, but for the scalar field it is positive due to the transport of heated fluid elements to the grid-element centreline from the bar wakes. Secondly, the scalar field reaches Gaussianity earlier along the grid-element centreline than the fluctuating velocity field.

Figure 4.14a also depicts the downstream evolution of the flatness $F(T)$ of fluctuating scalar along the grid-element centreline. As flatness is a measure of the peakedness of a statistical distribution, very high values of $F(T)$ in the region of $0.2 \leq x/x^* \leq 0.4$ indicates the presence of infrequent intense events. The origin of the high $F(T)$ values is the intermittent meeting of the wakes. Gaussianity is reached in the turbulent decay region, $x/x^* > 0.5$.

4.4.2 Skewness of fluctuating scalar gradients

Attention is now turned to the isotropy of small scales, which is a central assumption in the KOC theory (see section 1.3.1 of chapter 1). Many previous studies have shown that the small scales of scalar field rarely reach isotropy (refer to Warhaft (2000) and the references therein). The validity of these observations is explored for the present developing turbulence of low Re_λ and $Pe_{\lambda T}$ numbers. Local isotropy implies that the odd moments of $\partial u_i / \partial x_j$, $i \neq j$, and $\partial T / \partial x_i$ are zero, and that the derivatives of velocity and scalar fluctuations are related as (Tavoularis and Corrsin, 1981; Budwig et al., 1985; Tong and Warhaft, 1994),

$$\left\langle \left(\frac{\partial u_1}{\partial x_1} \right)^2 \right\rangle = \frac{1}{2} \left\langle \left(\frac{\partial u_2}{\partial x_1} \right)^2 \right\rangle = \frac{1}{2} \left\langle \left(\frac{\partial u_3}{\partial x_1} \right)^2 \right\rangle \quad (4.7)$$

$$\left\langle \left(\frac{\partial T}{\partial x_1} \right)^2 \right\rangle = \left\langle \left(\frac{\partial T}{\partial x_2} \right)^2 \right\rangle = \left\langle \left(\frac{\partial T}{\partial x_3} \right)^2 \right\rangle \quad (4.8)$$

The results from the isotropy test for velocity and scalar gradients are presented in figure 4.15. It is very interesting to notice that the small scales of velocity fluctuations (that dominate the spatial derivatives) are locally isotropic even in the region where

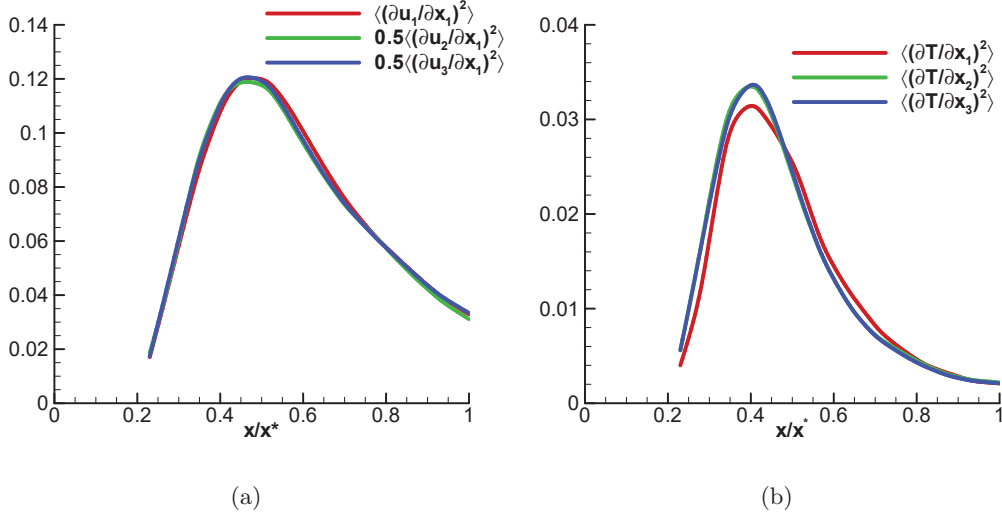


Figure 4.15: Test of isotropy for velocity and scalar gradients along the grid-element centreline: (a) Evolution of velocity derivatives, (b) Evolution of scalar derivatives.

the turbulence is intermittent, i.e. from $x/x^* \approx 0.2$ to 0.5 . The small scales of scalar fluctuations are moderately anisotropic throughout the grid-element centreline (figure 4.15b). Note that the scalar derivatives in the y and z directions must be identical due to symmetry and this is indeed so, indicating a good convergence of second-order derivative statistics.

It should be mentioned here that local isotropy is not expected in the near-grid-element region because: (i) it is very close to the grid-element, and therefore the small scales may feel the boundary effects from where they were generated, (ii) the local Re_λ and $Pe_{\lambda T}$ are low (see table 4.2) and (iii) the turbulence in this region is highly intermittent and developing. Therefore, it is indeed surprising that the velocity gradients are locally isotropic even in the near-grid-element centreline region. For high Reynolds number flows, Laizet et al. (2015b) have shown that the turbulence generated by fractal square grids reaches isotropy faster along the grid centreline than that generated by the classical grid of same blockage.

The isotropy assumption also implies that the skewness of the fluctuating scalar derivative is zero due to reflectional symmetry. It was, however, first reported by Freymuth and Uberoi (1971) that the skewness of the scalar derivative in the direction of mean scalar gradient is non-zero, and it decreases for increasing Re_λ . This observation was initially thought to be associated with the hot-wire velocity probe contaminating the measured temperature (Wyngaard, 1971). Later, in the studies of Mestayer et al. (1976); Gibson et al. (1977) this error was found to be minimal. As a result, it became clear

that the skewness of the derivative of scalar fluctuation in the direction of imposed mean gradient is indeed non-zero.

The skewness of the three fluctuating scalar derivatives along the grid-element centreline is plotted in figure 4.14b. In the region of $0.2 \leq x/x^* < 0.4$, the values are large. Moving further from this intermittent region to the more homogeneous turbulent decay region, $x/x^* > 0.5$, different behaviours can be noticed for the skewness of streamwise and lateral derivatives. The former is reduced to small values but the latter increase downstream. Along the grid-element centreline, $S(\partial T/\partial x_2) = S(\partial T/\partial x_3)$ because of geometric symmetry. It can be seen that $S(\partial T/\partial x_2)$ and $S(\partial T/\partial x_3)$ follow the same trend, are close to each other, but are not identical. This demonstrates the difficulty in getting fully-converged statistics for a third order moment that can take positive and negative values. On the other hand, as shown in figure 4.14c, the flatness of the lateral derivatives (fourth order moment) in the two lateral directions are almost indistinguishable, indicating better convergence for a quantity that is always positive.

In the literature, the skewness of scalar derivatives has been studied almost exclusively for homogeneous isotropic turbulence and was found to be non-zero in the direction of mean scalar gradient (Sreenivasan et al., 1979; Pumir, 1994; Holzer and Siggia, 1994). Therefore, it is very intriguing that a non-zero scalar derivative skewness is observed in the approximately homogeneous decay region where there are no mean scalar gradients. To the best of the author's knowledge, this is the first study that reports almost zero streamwise scalar derivative skewness and non-zero lateral skewness for a heated grid-element scalar turbulence of very low Re_λ and $Pe_{\lambda T}$. Sreenivasan et al. (1979) compiled various experimental data from HIT and noted that the value of scalar derivative skewness is in the range of 0.4 – 0.8 for low Re_λ and $Pe_{\lambda T}$ turbulence. In the present case, a value around 0.5 is found in the far-downstream homogeneous decay region, which falls in the reported range.

A number of theories were proposed to explain the observation of non-zero scalar derivative skewness and whether it originates from small or large scales. Using dimensional analysis, Freymuth (1976) proposed two different scaling laws so that, if the non-zero skewness is due to the large scales, then it should scale as $S \propto Pr^{-3/2} Re_\lambda^{-3}$, while if it is due to the small scales, it should be independent of Re_λ . Sreenivasan and Antonia (1977) suggested that the non-zero skewness is due to the large scales. In particular, they reasoned that the ramp-cliff structures present in the fluctuating scalar signal cause non-zero derivative skewness. Their observation was further verified in diverse flow fields such as the shear-flows (Sreenivasan et al., 1979), atmospheric boundary layers (Antonia and Van Atta, 1978), shear-free grid-element turbulence (Tavoularis and Corrsin, 1981; Budwig et al., 1985; Tong and Warhaft, 1994) and numerical simulations (Pumir, 1994;

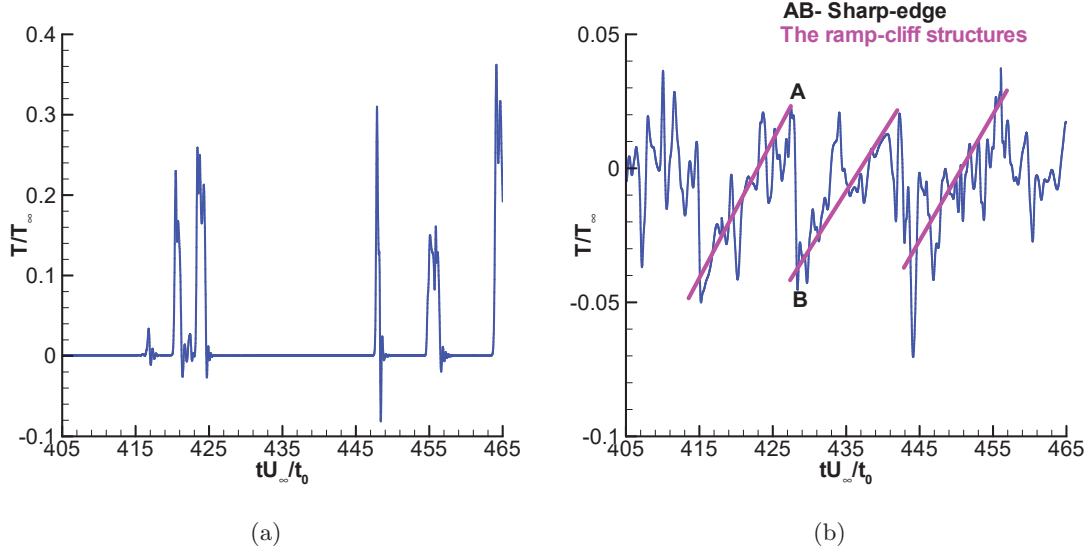


Figure 4.16: A segment of fluctuating scalar signal along the grid-element centreline at: (a) $x/x^* = 0.25$, (b) $x/x^* = 0.95$.

Holzer and Siggia, 1994).

The next question is to understand how the ramp-cliff structures are generated in the turbulent flow field. The quest to find an answer to this question led to various phenomenological theories such as those of Budwig et al. (1985) and Thoroddsen and Van Atta (1992) who hypothesised that the fluid blobs moving against the mean scalar gradients cause ramp-cliff structures to form. These models rely on the correlation between velocity and scalar fluctuations and are not applicable to all flows. The studies of Antonia et al. (1986) and Holzer and Siggia (1994) eventually established the mechanism whereby the ramp-cliff structures are formed. The large-scale eddies expel the scalar field resulting in ramps in the scalar signal, over which the variation of fluctuating scalar is small. These large eddies form the converging and diverging separatrices in the flow. Due to this expulsion, the scalar is concentrated on the diverging separatrix which causes the cliffs where the variation of the scalar is rapid within a thin region. However, the reason for the polarisation of ramps followed by cliffs is not found in the literature.

In order to provide insight on the sources responsible for the non-zero scalar derivative skewness for the present case, figure 4.16 presents the fluctuating scalar signal at two different locations. In the near-grid-element centreline location, $x/x^* \approx 0.25$, intermittent bursts in the signal can be clearly seen in figure 4.16a. These intense bursts are the source of high frequency content of scalar fluctuations and cause positive skewness of scalar fluctuations and, equally, also positive skewness of the scalar derivative in the near-grid-element centreline region.

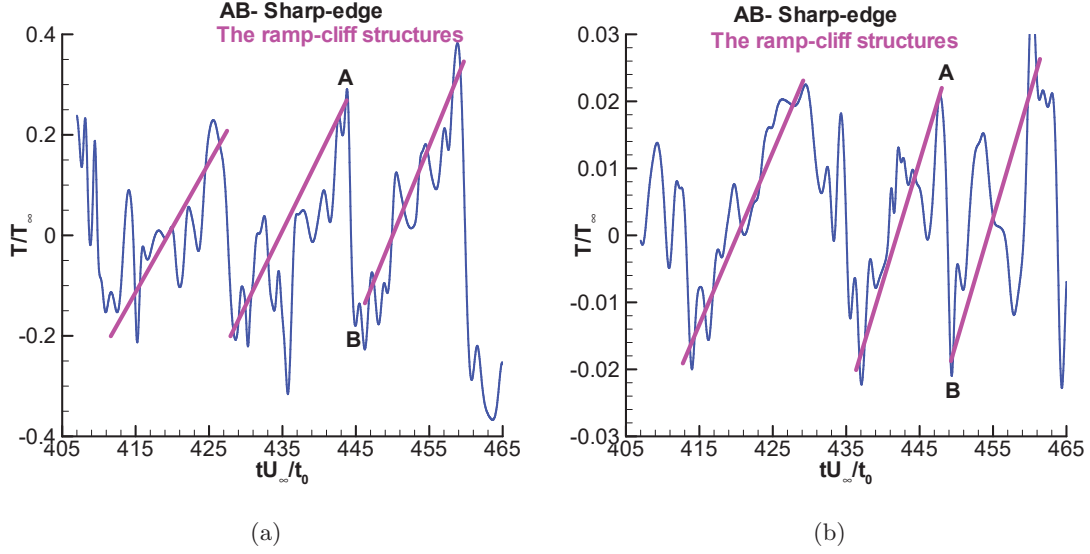


Figure 4.17: A segment of fluctuating scalar signal along the bar centreline at: (a) $x/x^* = 0.2$, (b) $x/x^* = 1.0$.

Concerning the far-downstream region, the ramp-cliff structures result in derivative skewness as discussed earlier. Previous experiments using a heated-grid, however, could not detect the large-scale characteristic ramp-cliff structures (refer to Sreenivasan et al. (1980)). Only the grid turbulence experiments with imposed transverse mean scalar gradients were able to detect these structures (Tavoularis and Corrsin, 1981; Budwig et al., 1985; Tong and Warhaft, 1994). Even the numerical simulations that reported the existence of such structures were also simulated for imposed mean scalar gradient (Pumir, 1994; Holzer and Siggia, 1994).

The decay region of the current study does contain the previously-reported ramp-cliff structures, even though there are no lateral mean scalar gradients as seen in figure 4.2c. Figure 4.16b clearly shows these large-scale ramp-cliff structures with superimposed small scale fluctuations. Sreenivasan et al. (1979) argued that sharp edges such as the one marked as 'AB' in figure 4.16b are the main reason why the scalar derivative skewness is non-zero. It is surmised that the ramp-cliff structures identified in the far-downstream decay region are the result of mean scalar gradients downstream of the grid-element bars. In fact, the ramp-cliff structures are identified even around $x/x^* \approx 0.2$ along the bar centreline (see figure 4.17a). It is known from the literature that the mean scalar gradients cause these structures to form. The ramp-cliff structures survive throughout the bar centreline although the mean scalar gradients become negligible moving downstream from the near-grid-element region. Evidence for this can be seen in figure 4.17b which shows the presence of ramp-cliff structures around $x/x^* \approx 1.0$

along the bar centreline where there are no local mean scalar gradients. Therefore, the ramp-cliff structures are generated in the downstream of the bar due to lateral mean scalar gradients and they are convected downstream along the flow. This is the reason why the ramp-cliff structures are observed in this study even in the places where there are no mean scalar gradients.

As mentioned earlier, the previous experiment on heated-grid scalar turbulence could not observe the large-scale characteristic ramp-cliff structures (Sreenivasan et al., 1980). Since the measurements of their study were taken far-downstream of a classical heated-grid (for $x/x^* > 3.75$), the temperature fluctuations of their experiment become isotropic much like the variation observed in figure 4.15b. Besides, the magnitude of temperature fluctuations continues to become smaller in the downstream as seen in figure 4.17b compared with the near-field plotted in figure 4.17a. As a result, by $x/x^* \approx 3.75$, identification of ramp-cliff structures becomes even more harder. These are the two reasons which could explain why the experiment of Sreenivasan et al. (1980) could not observe the ramp-cliff structures.

4.4.3 Flatness of fluctuating scalar gradients

Finally, the evolution of flatness of scalar derivatives is shown in figure 4.14c. The flatness values of lateral scalar derivatives are closer to each other compared to the skewness. Large values of scalar derivative flatness in $0.2 \leq x/x^* < 0.4$ are due to the high intermittency of this region. The review of Sreenivasan and Antonia (1997) records that the value of flatness for low Re_λ of about 50 is in the range of 4 to 6. In the decay region, flatness values close to 6 are observed, in agreement with the literature.

4.5 Spectrum of the fluctuating scalar

Having realised that the small scales of scalar in this developing turbulence do not become isotropic, this section explores if the scalar spectra exhibit a $-5/3$ power-law slope.

The computed and compensated spectra at different locations along the grid-element centreline are illustrated in figure 4.18. At $x/x^* = 0.25$, where the velocity spectrum was reported to exhibit the best $-5/3$ power-law slope for the first time along the grid-element centreline (refer to Paul et al. (2017)), the scalar spectrum has a well-defined $-5/3$ slope for about a decade of frequencies (see the compensated spectra for $x/x^* = 0.25$ in figure 4.18b). It is indeed surprising that such a well-defined power-law slope appears at a location where not only the local Re_λ and Pe_{λ_T} numbers are too low for an inertial subrange to exist, but also the scalar small-scales are anisotropic as well

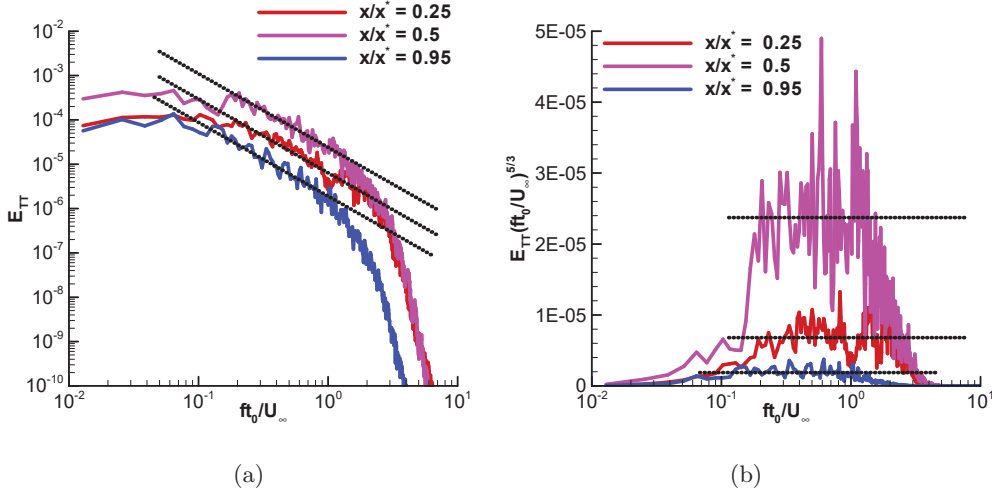


Figure 4.18: (a) Spectra of the fluctuating scalar at different locations along the grid-element centreline, (b) Compensated spectra plotted in linear-logarithmic axes at the same locations. The dotted lines in (a) represent the $-5/3$ slope, while in (b) they represent a constant value of $E_{TT}(ft_0/U_\infty)^{5/3}$.

as inhomogeneous, and the turbulent fluctuations are highly intermittent. In fact, Re_λ is only 22.01 and $Pe_{\lambda T}$ is a mere 12.31 (see table 4.1). Due to these very small values and the developing nature of the turbulence, the scalar spectrum is not at all expected to have any power-law scaling at this location. Yet, the spectrum exhibits a definite $-5/3$ power-law slope for a decade of frequencies. Note also that, $x/x^* = 0.25$ is the location where the fluctuating scalar and its gradients have just been brought to the grid-element centreline by the turbulent transport from the bar wakes (see §4.3).

The power-law behaviour observed in the near-grid-element region is not related to the KOC theory. It appears that the mere existence of large and small scales, irrespective of whether they are isotropic or homogeneous, is sufficient enough to cause a $-5/3$ slope in the scalar spectra. The relationship between the small scales of scalar and the well-defined $-5/3$ power-law slope is further analysed in §4.7. Note that, this is the first time a study reports a well-defined $-5/3$ power-law in scalar spectrum for a highly-intermittent turbulence of very low Re_λ and $Pe_{\lambda T}$ numbers.

The scalar spectra in the more homogeneous decay region are also plotted in figures 4.18a and 4.18b. The frequency range of $-5/3$ slope appears to be decreasing in these locations compared to that of the near-grid-element region. This is consistent with the observations made for the velocity spectra in Paul et al. (2017). At these fully-turbulent locations, Paul et al. (2017) reported that the velocity spectra have power-law slopes defined only for a limited range of frequencies. However, the power-law is clearly defined

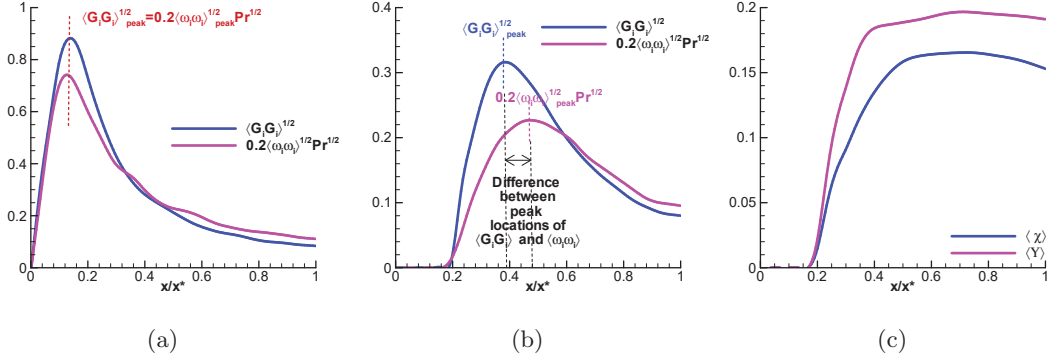


Figure 4.19: Evolution of turbulent kinetic energy and scalar-variance dissipations along: (a) the bar centreline, (b) the grid-element centreline. (c) Evolution of the time-averaged quantities χ and Υ (defined in the text) along the grid-element centreline.

in the scalar spectra for a wider range of frequencies. The observation that scalar spectra have a wider $-5/3$ slope than the velocity spectra has been reported in the literature on heated-grid fully developed homogeneous turbulence but for high Re_λ and $Pe_{\lambda T}$ values (Yeh and Atta, 1973; Warhaft and Lumley, 1978).

4.6 Stretching of vorticity vector versus stretching of a passive vector

The aim of this section is to provide an explanation as to why the dissipation rate of scalar-variance ($\alpha \langle G_i G_i \rangle$) starts decaying earlier along the grid-element centreline than that of the turbulent kinetic energy ($\nu \langle \omega_i \omega_i \rangle$). These two dissipations are sometimes believed to have similar behaviour (Corrsin, 1953) and the relationship is generally given as $\langle G_i G_i \rangle^{1/2} = C \langle \omega_i \omega_i \rangle^{1/2} Pr^{1/2}$, where C is a constant. Pumir (1994) noted that the value of the constant C is of the order of 2 in periodic box simulation.

The spatial evolution of these two dissipation quantities along the bar and the grid-element centrelines is plotted in figures 4.19a and 4.19b respectively. First, it can be noted that the relationship $\langle G_i G_i \rangle^{1/2} = C \langle \omega_i \omega_i \rangle^{1/2} Pr^{1/2}$ is just approximately valid, if at all, only along the bar centreline and for $x/x^* > 0.6$ on the grid-element centreline. Indeed, figure 4.19a reveals that the qualitative behaviour of these two dissipation profiles are similar along the bar centreline, with values peaking at the same position. However, at the grid-element centreline, the onset of scalar-variance dissipation decay occurs earlier than the decay of turbulent kinetic energy dissipation (see figures 4.19b and 4.12a). This observation further ascertains that the grid-element centreline contains more intriguing turbulence properties than the bar centreline. Secondly, the value of the constant C in

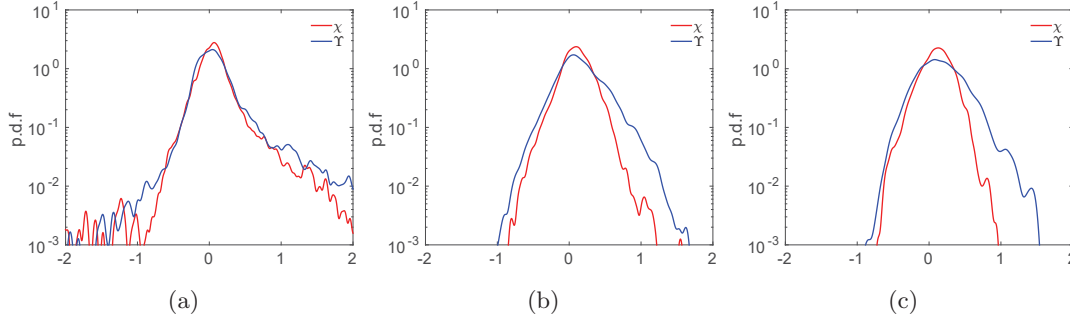


Figure 4.20: PDF of scalar gradient production rate and enstrophy production rate at different locations along the grid-element centreline: (a) $x/x^*=0.25$, (b) $x/x^*=0.5$, (c) $x/x^*=0.95$.

the current simulation is approximately 0.2 although it should be remembered here that the value of C depends on the normalisation parameters.

As the dissipations of scalar-variance and turbulent kinetic energy are determined by turbulent stretching/compression (i.e. $-G_i s_{ij} G_j$ for the scalar-variance dissipation and $\omega_i s_{ij} \omega_j$ for the turbulent kinetic energy dissipation), the explanation for this difference lies in the way the turbulent strain-rate stretches/compresses the velocity gradient based vector and the passive scalar gradient vector.

The processes that determine the stretching/compression of the vorticity and scalar gradient vectors are different. As noted in Tsinober (2009), the vorticity vector has a two-way coupling with the turbulent strain-rate, while the scalar gradient vector has a one-way coupling. It can be seen from the governing equations of enstrophy ($\omega_i \omega_i$) and strain-product ($s_{ij} s_{ij}$) (see equations (1.3) and (1.8)) that the vortex stretching/compression term appears in both equations: as a source term for enstrophy and a sink for strain-product. This means that the growth of enstrophy reduces the strain-rate and thus weakening the stretching/compression which eventually leads to the enstrophy damping.

Following the work of Ohkitani (2002), the normalised stretching/compression rates of vorticity and scalar gradient vector are defined as: $\chi = \frac{\omega_i s_{ij} \omega_j / \omega_i \omega_i}{\sqrt{\langle \omega_i \omega_i \rangle}}$, and $\Upsilon = \frac{-G_i s_{ij} G_j / G_i G_i}{\sqrt{\langle \omega_i \omega_i \rangle}}$. The PDFs of these normalised rates are compared at different grid-element centreline locations in figure 4.20. Clearly, the stretching/compression rate of the passive vector is stronger than that of the vorticity vector throughout the grid-element centreline. The dominance of passive vector stretching/compression over vorticity vector was first reported in Ohkitani (2002). The current results, however, cannot be compared with that study as the spectra of fluctuations of velocity and scalar are significantly different in the present case while the study of Ohkitani (2002) had a similar spectrum for both the velocity and scalar.

The reason for the observed stretching/compression difference lies in the two-way coupling mentioned earlier. When the mean enstrophy increases in the region of $0.2 < x/x^* < 0.5$ (see figure 3.11a), the stretching of vorticity also increases in this region, but this increase weakens the strain-rate magnitude (because the stretching/compression term acts as a sink for the strain) which in turn reduces the strength of vortex stretching/compression that depends on the strain magnitude. This can explain why the stretching/compression rate of the vorticity vector is smaller than that of a passive vector.

Although the comparison of stretching/compression rates between the vorticity vector and the scalar gradient vector did confirm the validity of Ohkitani's observation for the current case, it cannot explain why the onset of scalar-variance dissipation decay occurs much earlier along the grid-element centreline. The evolution of mean stretching/compression rates (χ and Υ) are plotted in figure 4.19c. The curve shows that the growth of these rates is suddenly hampered. This process occurs early for the passive scalar field and late for the vorticity field. Now the question is, what is this weakening process that suddenly impedes the growth of stretching/compression rates? It is intuitive that this process should be related to the stretching/compression term. Therefore, this term is analysed further in the next section, and it will be shown that the onset of scalar-variance dissipation decay happens earlier than the turbulent kinetic energy dissipation due to difference in alignments between the scalar gradient vector and the strain-rate eigenvector.

4.7 Eigen-contributions of strain-rate to the scalar gradient production

In this last section, the production term of scalar-gradient-variance ($-G_i s_{ij} G_j$), which is the source term of fluctuating scalar gradient production, is analysed in terms of the eigenvalues λ_i (where $i=1,2,3$) and eigenvectors e_i (where $i=1,2,3$) of the strain-rate tensor. As shown in chapter 1, $-G_i s_{ij} G_j$ can be written in terms of λ_i and e_i .

Equation (1.30) shows that the scalar gradient production term depends on the strain-rate eigenvalues and the alignment between the scalar gradient vector and the strain-rate eigenvectors. The statistical behaviour of the scalar gradient production term is discussed first, followed by a discussion on the eigen-contributions to the scalar gradient production.

The PDFs of scalar gradient production are plotted for different grid-element centreline locations in figure 4.21. The PDFs are skewed towards positive in all the grid-element centreline stations. This positive skewness, i.e. $-\langle G_i s_{ij} G_j \rangle > 0$, is considered as a univer-

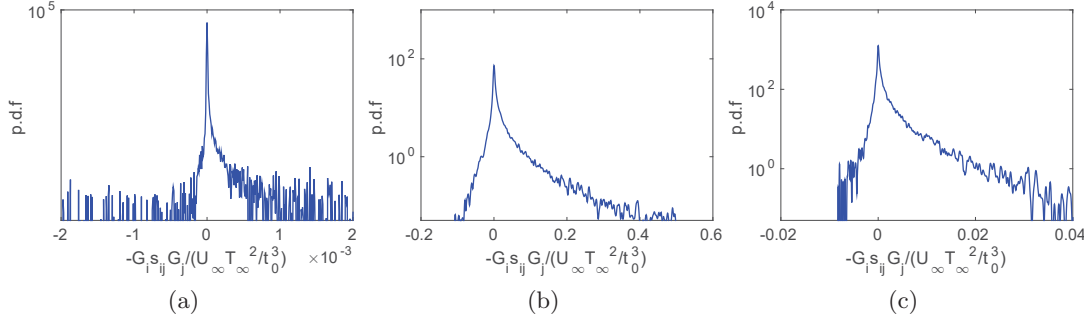


Figure 4.21: PDF of scalar gradient production at different locations along the grid-element centreline: (a) $x/x^*=0.25$, (b) $x/x^*=0.5$, (c) $x/x^*=0.95$.

x/x^*	0.1	0.25	0.35	0.5	0.75	0.95
$-\langle G_i s_{ij} G_j \rangle / \langle G^2 \rangle \langle s^2 \rangle^{1/2}$	—	0.11	0.18	0.04	0.03	0.02

Table 4.2: Values of normalised scalar-gradient-variance production term at different locations along the grid-element centreline.

sal behaviour of small-scale scalar turbulence (Ruetsch and Maxey, 1991, 1992; Vedula et al., 2001; Brethouwer et al., 2003; Gulitski et al., 2007a; Tsinober, 2009). This universal behaviour is observed also here in locations where the local Re_λ is as low as 20 and when the flow is only intermittently turbulent.

Table 4.2 records the values of scalar gradient production normalised by the product of mean strain-product ($\langle s_{ij} s_{ij} \rangle$) and scalar-gradient-variance (G^2 , i.e. $\langle G_i G_j \rangle$). This ratio initially increases rapidly from $x/x^* \approx 0.2$ to 0.35. This increase shows that $-\langle G_i s_{ij} G_j \rangle$ increases faster than the product of $\langle s_{ij} s_{ij} \rangle$ and $\langle G_i G_j \rangle$. Further downstream, the ratio decreases drastically. As $\langle s_{ij} s_{ij} \rangle$ increases only slightly in this region (i.e. from $x = 0.35x^*$ to $0.5x^*$, see figure 3.12a), this drastic decrease is not just related to increase in $\langle s_{ij} s_{ij} \rangle$. Rather, it should only be due to the rapid decay of $-\langle G_i s_{ij} G_j \rangle$ compared to that of $\langle G_i G_i \rangle$. In the far-downstream, the ratio decreases even more, ascertaining again that the decay of $-\langle G_i s_{ij} G_j \rangle$ is faster than that of $\langle G_i G_i \rangle$ and $\langle s_{ij} s_{ij} \rangle$. Therefore, the growth and decay of $-\langle G_i s_{ij} G_j \rangle$ are faster than both $\langle s_{ij} s_{ij} \rangle$ and $\langle G_i G_i \rangle$. The reason for this behaviour will become clearer later in this section.

4.7.1 The role of strain-rate eigenvalues in scalar gradient production

It is known from the previous chapter (see section 3.8.1) that in the grid-element centreline region $0.25 < x/x^* < 1$, the PDF of the intermediate strain-rate eigenvalue is skewed positive resulting in two stretching directions and one compressive direction.

Quantity / x/x^*	0.1	0.25	0.35	0.5	0.75	0.95	Field experiment
$\langle \lambda_1 \rangle / \langle s^2 \rangle^{\frac{1}{2}}$	–	0.40	0.48	0.54	0.55	0.56	0.53
$\langle \lambda_2 \rangle / \langle s^2 \rangle^{\frac{1}{2}}$	–	0.09	0.09	0.12	0.13	0.13	0.09
$\langle \lambda_3 \rangle / \langle s^2 \rangle^{\frac{1}{2}}$	–	-0.49	-0.57	-0.66	-0.68	-0.69	-0.62
$\langle \lambda_1^2 \rangle / \langle s^2 \rangle$	–	0.36	0.38	0.37	0.36	0.36	0.4
$\langle \lambda_2^2 \rangle / \langle s^2 \rangle$	–	0.05	0.05	0.05	0.05	0.05	0.04
$\langle \lambda_3^2 \rangle / \langle s^2 \rangle$	–	0.59	0.57	0.58	0.59	0.59	0.56
$\langle \lambda_1^3 \rangle / \langle s^2 \rangle^{\frac{3}{2}}$	–	0.74	0.43	0.32	0.28	0.28	0.48
$\langle \lambda_2^3 \rangle / \langle s^2 \rangle^{\frac{3}{2}}$	–	0.05	0.02	0.02	0.05	0.05	0.01
$\langle \lambda_3^3 \rangle / \langle s^2 \rangle^{\frac{3}{2}}$	–	-1.57	-0.86	-0.66	-0.60	-0.60	-0.73

Table 4.3: Values of normalised eigenvalues of strain-rate tensor at different locations along the grid-element centreline. The values are compared against the field experiments of Gulitski et al. (2007a).

Due to this, sheet-like structures may be preferred over tube-like structures in some average sense. In this chapter, more statistics of the normalised strain-rate eigenvalues are provided in table 4.3. The normalised values of $\langle \lambda_i^2 \rangle$ (where $i=1,2,3$) do not vary downstream, while the normalised values of $\langle \lambda_1 \rangle$ and $\langle \lambda_3^3 \rangle$ increase and $\langle \lambda_1^3 \rangle$ and $\langle \lambda_3 \rangle$ values decrease. The downstream evolution of $\langle \lambda_2 \rangle$, $\langle \lambda_2^2 \rangle$ and $\langle \lambda_2^3 \rangle$ remains insensitive to any kind of normalisation although the PDF of λ_2 showed remarkable difference from the near-grid-element to the decay regions as reported in section 3.8.1. These normalised values are compared to those from the field experiments carried out for the atmospheric boundary layer (Gulitski et al., 2007b) in table 4.3. Although the Reynolds numbers of the field experiments are too large to permit more than a qualitative comparison, the simulated values of normalised mean strain-rate eigenvalues are surprisingly close, in particular for $\langle \lambda_i \rangle$ and $\langle \lambda_i^2 \rangle$.

The attention is now turned to the effect of the strain-rate eigenvalues on the scalar gradient production. This analysis is carried out for two stations along the grid-element centreline: (i) one in the near-grid-element region (figures 4.22a-4.22c for $x/x^*=0.25$) and (ii) another point in the far-downstream decay region (figures 4.22d-4.22f for $x/x^*=0.95$). Figure 4.22 shows the joint probability distribution (JPDF) between the strain-rate eigenvalues and the scalar gradient production. These JPDFs help to understand the role of the strain-rate eigenvalues in scalar gradient production. It is evident from figure 4.22 that the extensive λ_1 and compressive λ_3 strain-rate eigenvalues are positive and negative respectively, while the tendency for the intermediate strain-rate eigenvalue λ_2 to skew towards positive is conspicuous in both the near-grid-element and far-downstream regions. Since the strain-rate eigenvalues are skewed noticeably, the production term is

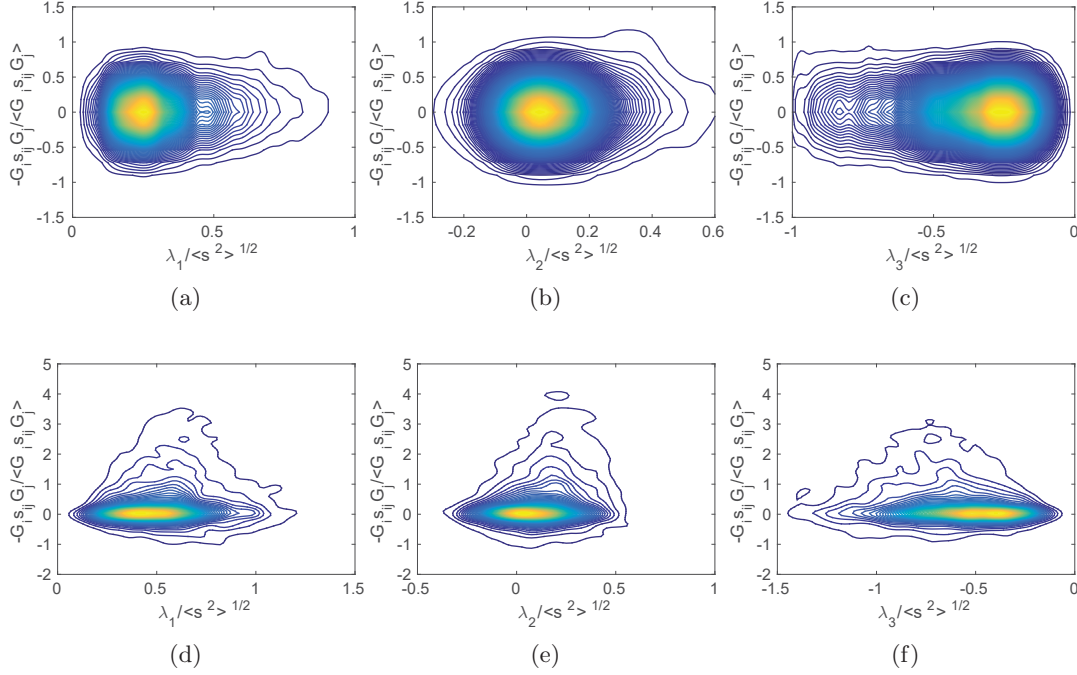


Figure 4.22: JPDF of eigenvalues of strain-rate tensor against the scalar gradient production: (a-c) at $x/x^*=0.25$ (near-grid-element inhomogeneous region), (d-f) at $x/x^*=0.95$ (decay region). The isocontours range from 10^1 to 10^{-1} .

also expected to have a marked skewness if the eigenvalues really influence the statistics of scalar gradient production. Yet, figures 4.22a-4.22c (i.e. in the near-grid-element region) are near-symmetric with respect to the horizontal line of $-G_{is_{ij}}G_j/\langle G_{is_{ij}}G_j \rangle=0$. This observation from this JPDF leads to two important interpretations. Firstly, it appears that there is no correlation between the strain-rate eigenvalues and the positive skewness of scalar gradient production in the inhomogeneous near-grid-element region. Secondly, the preference for the positive scalar gradient production is only marginal at a location where the scalar spectrum exhibits the best -5/3 power-law slope. Since stretching/compression is believed to have a close relationship with the energy cascade, it might be intuitive to expect a stronger preference for the production term, but in reality that does not occur. This observation leads to a statement that the stretching/compression process of scalar gradients is not really the most important process related to the scalar cascade. This is further ascertained in figures 4.22d-4.22f where the production of scalar gradient is strongly preferred (as evident from the asymmetry in the JPDF with respect to $-G_{is_{ij}}G_j/\langle G_{is_{ij}}G_j \rangle=0$), yet the spectrum at this location has a power-law defined for a lesser range of frequencies than that of the near-grid-element region. Figures 4.22d-4.22f also reveal that there is a correlation between the strain-rate eigenvalues and the positive skewness of scalar gradient production in the homogeneous decay region. Fig-

ures 4.22d-4.22f also look similar to those reported in Gulitski et al. (2007b) (see figure 16 of their paper) for the atmospheric boundary layer.

4.7.2 The role of geometrical alignments in scalar gradient production

Having understood the role of the strain-rate eigenvalues in scalar gradient production, the other factor that affects the production of scalar gradient, the geometrical alignments (refer to equation 1.30), is analysed in this subsection.

The PDFs of the absolute cosine angle between the fluctuating scalar gradient vector and the strain-rate eigenvectors are plotted for different grid-element centreline locations in figure 4.23. The alignment behaviour at $x/x^*=0.25$ and 0.35 , as seen in figures 4.23a and 4.23b respectively, is not the universal alignment behaviour reported in the literature. Here, only the compressive strain-rate eigenvector aligns with the scalar gradient vector. Note that these are the locations where the alignment behaviour of the vorticity is also reported to be anomalous (see section 3.8.3) as the vorticity vector aligns with both the extensive and intermediate strain-rate eigenvectors. In the near-grid-element region, the extensive and intermediate eigenvectors align perpendicular to the scalar gradient vector meaning that they, on average, do not act on the fluctuating scalar gradient for the production of scalar gradient. Note also that the alignment of the compressive strain-rate eigenvector with the scalar gradient vector becomes stronger from $x/x^*=0.25$ to 0.4 giving a possible answer as to why the scalar-gradient-variance increase in the region $0.2 < x/x^* < 0.35$. The alignment behaviour for the region $0.2 < x/x^* < 0.35$ agrees with the results of Brethouwer et al. (2003) who reported that strain-dominated regions strengthen the alignment of the scalar gradient vector with the compressive strain-rate eigenvector. The alignment behaviour after $x/x^* \approx 0.4$ starts to exhibit the universal trend noticed in the literature as the PDFs in figures 4.23c and 4.23d, plotted for $x/x^*=0.5$ and 0.95 respectively, are comparable to the results of Vedula et al. (2001). This universal alignment behaviour prevails throughout the decay region although the values of $Pe_{\lambda T}$ and Re_{λ} are significantly low.

4.7.3 The combined effects of strain-rate eigenvalues and eigenvectors to scalar gradient production

The combined effects of strain-rate eigenvalues and eigenvectors on the scalar gradient production is plotted in figure 4.24. At $x/x^*=0.25$, where the scalar-gradient-variance has just started to develop and the scalar spectrum has the best-defined $-5/3$ power-law slope, the eigen-contribution to the production of scalar gradient is predominantly due to the compressive strain-rate eigenvalues and eigenvectors as only the PDF of $-|\mathbf{G}|^2 \lambda_3 \cos^2(\mathbf{G}, \mathbf{e}_3)$ is skewed to the positive. A similar finding was reported in Gulitski

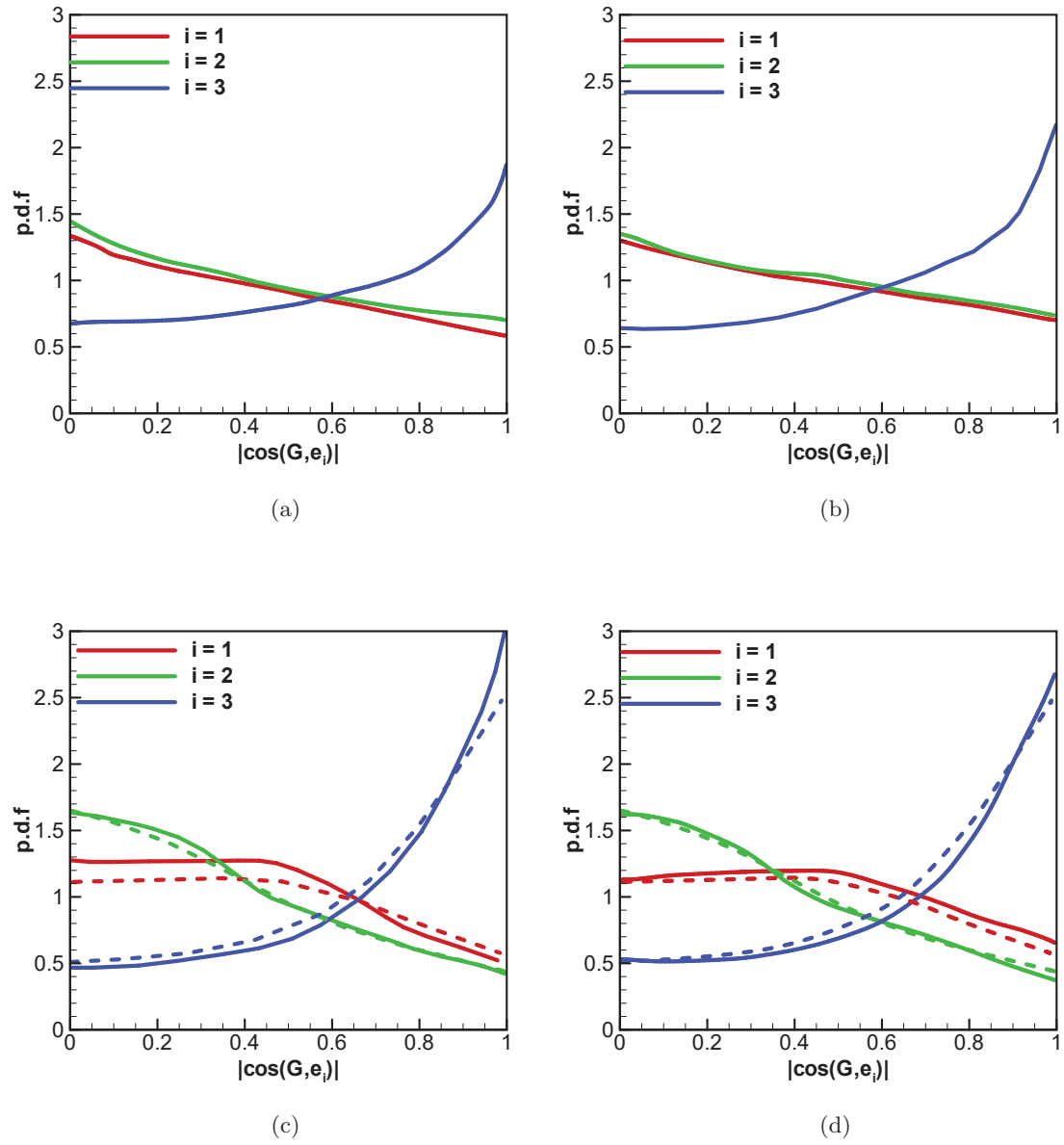


Figure 4.23: PDF of cosine of the angle between scalar gradient vector and strain-rate eigenvectors at different locations along the grid-element centreline: (a) $x/x^*=0.25$, (b) $x/x^*=0.35$, (c) $x/x^*=0.5$, (d) $x/x^*=0.95$. The dotted lines in (c) and (d) are the results from Vedula et al. (2001).

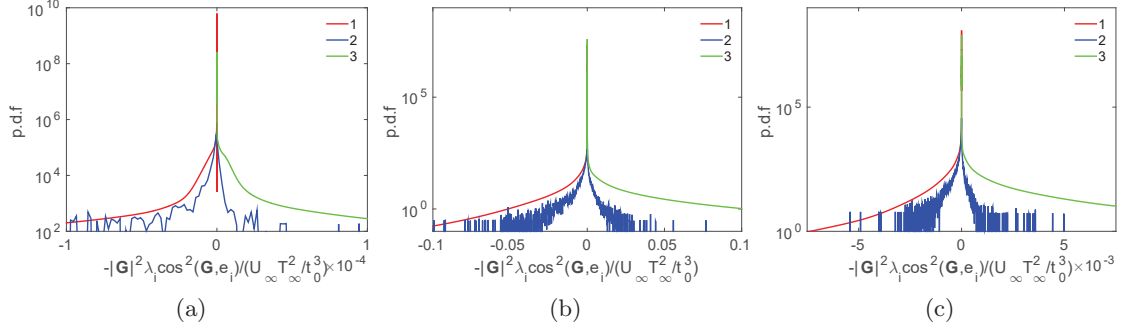


Figure 4.24: PDF of eigen-contribution to scalar gradient production at different locations along the grid-element centreline: (a) $x/x^*=0.25$, (b) $x/x^*=0.5$, (c) $x/x^*=0.95$.

Quantity/ x/x^*	0.1	0.25	0.35	0.5	0.75	0.95	Field exp.
$\langle \mathbf{G} ^2 \lambda_1 \cos^2(\mathbf{G}, \mathbf{e}_1) \rangle / \langle G_i s_{ij} G_j \rangle$	–	-0.18	-0.31	-0.27	-0.25	-0.22	-0.64
$\langle \mathbf{G} ^2 \lambda_2 \cos^2(\mathbf{G}, \mathbf{e}_2) \rangle / \langle G_i s_{ij} G_j \rangle$	–	-0.03	-0.04	-0.04	-0.04	-0.04	-0.05
$\langle \mathbf{G} ^2 \lambda_3 \cos^2(\mathbf{G}, \mathbf{e}_3) \rangle / \langle G_i s_{ij} G_j \rangle$	–	1.21	1.35	1.31	1.29	1.26	1.69

Table 4.4: Contribution of strain-rate eigenvalues to scalar gradient production. The values are compared with the field-experiments of Gulitski et al. (2007a).

et al. (2007a) for a homogeneous isotropic turbulence. Therefore, the small scales of scalar are, on average, due to prominent compressing of fluid elements, although the small scales of velocity fluctuation are indeed due to prominent vortex stretching. As a novelty, this study takes this result one step further and shows that this compressing of fluid elements is mostly associated with the strain-rate eigenvectors for the developing inhomogeneous turbulence. In the decay region, although some portion of the PDF of $-|\mathbf{G}|^2 \lambda_2 \cos^2(\mathbf{G}, \mathbf{e}_2)$ is in the positive x-axis, the overall behaviour is on the destruction of scalar-gradient-variance as the intermediate strain-rate eigenvector, on average, aligns perpendicular to the scalar gradient vector. In this decay region, the production of scalar gradient is due to the combined action of both the compressive strain-rate eigenvalues and eigenvectors.

The observation noted in the PDFs of figure 4.24 is further analysed through the mean values of individual components of the eigen-contribution to the scalar gradient production (see table 4.4). The values of $\langle |\mathbf{G}|^2 \lambda_1 \cos^2(\mathbf{G}, \mathbf{e}_1) \rangle$ and $\langle |\mathbf{G}|^2 \lambda_2 \cos^2(\mathbf{G}, \mathbf{e}_2) \rangle$ are negative and the only positive component is $\langle |\mathbf{G}|^2 \lambda_3 \cos^2(\mathbf{G}, \mathbf{e}_3) \rangle$. This observation ascertains that the production of scalar gradient is due to the compressive action of strain-rate eigenvalues and eigenvectors (Gulitski et al., 2007a). The mean eigen-contribution

values are compared against the very high Reynolds number field experiments of Gulitski et al. (2007a). Although the values of $\langle |\mathbf{G}|^2 \lambda_2 \cos^2(\mathbf{G}, \mathbf{e}_2) \rangle$ and $\langle |\mathbf{G}|^2 \lambda_3 \cos^2(\mathbf{G}, \mathbf{e}_3) \rangle$ are closer to the literature values, the values of $\langle |\mathbf{G}|^2 \lambda_1 \cos^2(\mathbf{G}, \mathbf{e}_1) \rangle$ show some significant discrepancy. Nonetheless, it should be noted that the present simulation is carried out for a very low Reynolds number, and the observed discrepancy could well be a Reynolds number effect.

Concerning the evolution of $\langle |\mathbf{G}|^2 \lambda_3 \cos^2(\mathbf{G}, \mathbf{e}_3) \rangle$ with respect to $-\langle G_i s_{ij} G_j \rangle$, it can be noted that the behaviour of these two terms are similar; both terms increase from $x/x^*=0.2$ to 0.4 , and then they decay. Noting from table 4.3 that the values of $\langle \lambda_3 \rangle$ decrease from $x=0.25x^*$ to $0.4x^*$, the rapid increase of $-\langle G_i s_{ij} G_j \rangle$ compared to $\langle G_i G_i \rangle$ and $\langle s_{ij} s_{ij} \rangle$ is the result of rapid increase in the strong alignment between the compressive strain-rate eigenvector and the scalar gradient vector. The fast decay of $-\langle G_i s_{ij} G_j \rangle$ compared to $\langle G_i G_i \rangle$ and $\langle s_{ij} s_{ij} \rangle$, however, is the result of intermediate eigenvector aligning 45° to the scalar gradient vector and thus weakening the $-\langle G_i s_{ij} G_j \rangle$ (see figures 4.24b and 4.24c). In simple terms, the asymmetric evolution of $-\langle G_i s_{ij} G_j \rangle$ compared to $\langle G_i G_i \rangle$ and $\langle s_{ij} s_{ij} \rangle$ is the result of asymmetric evolution of the alignment behaviour between the strain-rate eigenvector and the scalar gradient vector.

The study on scalar gradients is closed by providing a probable reason as to why the onset of the dissipation of scalar-variance decay occurs much earlier along the grid-element centreline than that of the turbulent kinetic energy. It was noticed that the productions of these dissipations depend on the strain-rate eigenvalues and eigenvectors (see figures 4.10b, 4.12b and equation (1.30)). Since the strain-rate eigenvalues appear the same way in both the production terms (see equation (1.10) for $\omega_i s_{ij} \omega_j$, and equation (1.30) for $-G_i s_{ij} G_j$), the difference in evolution between the production terms should be coming from the geometrical alignments. For the case of kinetic energy dissipation, the extensive and intermediate eigenvectors aid the growth of $\langle \omega_i \omega_i \rangle$ (see figure 3.22). On the other hand, as was noticed in figure 4.24, the growth of $\langle G_i G_i \rangle$ occurs predominantly due to the compressive eigenvector. This is because, as was noticed earlier, the extensive eigenvector has a negative impact on the growth of scalar-gradient-variance, and the intermediate eigenvector aligns perpendicular to the scalar gradient vector. The vorticity vector aligns with both the extensive and intermediate eigenvectors in the region $0.2 < x/x^* \leq 0.5$, and thus acting as a main agent to increase $\langle \omega_i \omega_i \rangle$ in that region (see figure 3.24). If $\langle G_i G_i \rangle$ has to increase in $0.2 < x/x^* \leq 0.5$, then the scalar gradient vector is expected to align only with the compressive eigenvector. Yet, such an alignment behaviour is noticed only in the region $0.2 < x/x^* < 0.4$ where the extensive and intermediate eigenvectors align perpendicular to the scalar gradient vector and thus they do not hamper the growth of scalar-gradient-variance due to the compressive eigenvector.

When the extensive strain-rate eigenvector aligns 45° to the scalar gradient vector (i.e. after $x/x^* < 0.4$), the negative effect due to this alignment on the scalar-gradient-variance is felt in the form of decreasing the scalar-gradient-variance. Comparing the alignment behaviour of the scalar gradient vector with that of the vorticity vector, it is observed that the vorticity vector changes its alignment behaviour later along the grid-element centreline (it occurs only after $x/x^*=0.5$) than that of the scalar gradient vector. Thus, the delayed change in alignment behaviour of vorticity vector causes the scalar-variance dissipation to decay earlier along the grid-element centreline than the turbulent kinetic energy dissipation.

4.8 Summary

This chapter presented the evolution of passive scalar statistics in a spatially developing turbulence using direct numerical simulation. Turbulence is generated by a single-square grid-element which is heated continuously, and the passive scalar is temperature. The dominant mechanisms responsible for the dynamical evolution of scalar-variance, turbulent heat-fluxes, and scalar-gradient-variance are traced along the bar and the grid-element centrelines. The scalar-variance and turbulent heat-fluxes are generated predominantly by the action of mean scalar gradient behind the bar and they are transported laterally by turbulent fluctuations to the grid-element centreline. The contribution of mean velocity and scalar fields to the production of scalar-gradient-variance is found to be insignificant along both the centrelines. The scalar-gradient-variance is produced primarily by the compressing action of the turbulent strain-rate. This study also explored the small-scale scalar statistics along the grid-element centreline. The skewness of lateral fluctuating scalar derivatives is found to be non-zero throughout the grid-element centreline. In the near grid-element inhomogeneous region, the non-zero derivative skewness is associated with external intermittency. In the far-downstream homogeneous region, the fluctuating scalar signal exhibits the presence of ramp-cliff structures, that are known to cause skewness in lateral scalar derivatives. In the grid-element centreline closer-to-the-grid-element region, even though the fluctuating scalar field is highly inhomogeneous as well as anisotropic, and the local Corrsin-microscale-Peclet number is too small, the scalar spectrum exhibits a well-defined $-5/3$ power law in violation to the Kolmogorov-Oboukhov-Corrsin theory. At this location, the PDF of scalar gradient production is only slightly skewed towards positive, the correlation between the strain-rate eigenvalues and the scalar gradient production is not clearly defined, and the fluctuating scalar gradient vector aligns only with the compressive strain-rate eigenvector. The scalar gradient vector is stretched/compressed stronger than the vorticity

vector throughout the grid-element centreline. However, the alignment of the former changes much earlier in space than that of the latter, resulting in scalar-variance dissipation to decay earlier along the grid-element centreline compared to the turbulent kinetic energy dissipation. The universal alignment of the scalar gradient vector is found in the decay region although the local Reynolds (based on the Taylor-length scale) and the Peclet (based on the Corrsin microscale) numbers are very low.

Chapter 5

Effect of free-stream turbulence on heat transfer from a circular cylinder

5.1 Overview

This chapter deals with the final major topic of this thesis, the effect of free-stream turbulence on heat transfer from a circular cylinder. The same square grid-element, as in the previous two chapters, is used to generate the oncoming free-stream turbulence. In order to study the effect of turbulence generated by the grid-element on heat transfer from the cylinder, the grid-element is insulated (i.e. adiabatic) while the circular cylinder is heated. The circular cylinder is placed at three distinct locations in the wake of the grid-element. These locations are carefully chosen in such a way that the production and decay regions have the same turbulent intensity of 11%, while the peak location has the highest turbulent intensity of 15%.

The numerical challenges associated with studying the current configuration are highlighted first before presenting a detailed explanation of the present numerical setup and boundary conditions. The characteristics of oncoming turbulence are discussed in terms of velocity, rms profiles, and statistical measures such as skewness and flatness. The intermittent nature of the oncoming flow is also properly characterised. This is followed by the analysis of the effects of free-stream turbulence on flow and thermal characteristics on the surface of the cylinder. The validity of the existing stagnation point heat transfer coefficient correlations is assessed and found to be invalid in the production and peak regions. The following sections provide explanations as to why the stagnation point heat transfer correlations are not valid. This includes an analysis of the domin-

ant frequency of turbulent fluctuations and a timescale analysis. Finally, the physical mechanism responsible for heat transfer increase is identified and explained for each region.

5.2 Present configuration and numerical challenges

This study presents DNS results of heat transfer from a cylinder immersed in the turbulence generated by a single square grid-element. A study of this kind is numerically challenging due to several factors. Firstly, the surfaces of the turbulence-generating grid-element require a fine resolution in order to capture the correct dynamics of the turbulent flow it generates, while the near-wall flow close to the surface of the cylinder also has to be resolved well in order to predict the correct heat transfer. Such a requirement increases the number of cells significantly, in particular for high Reynolds numbers. To the best of author's knowledge, only the DNS studies of Venema et al. (2011, 2014) included the turbulence-generating body in their computational domain. Nonetheless, the turbulence-generating body was not resolved fully in those studies, and they are, therefore, only semi-DNS studies. In the DNS studies of Wissink and Rodi (2011a,b), the inlet fluctuations are obtained from a separate precursor simulation. Secondly, the spanwise length of the grid-element is larger compared to the standard spanwise size used for flow around a single cylinder. For example, the study of Venema et al. (2011) had a spanwise length of $2.5D$, while Wissink and Rodi (2011b) and Wissink and Rodi (2011a) had $2D$ and $0.6D$ respectively. The presence of the grid-element increases this size by at least 7 times which in turn increases the number of cells correspondingly. Thirdly, the wetted-surface of the grid-element is much larger than that of a small cylinder which means more refinement for the grid-element surface. Fourthly, if the effect of different regions of the grid-element turbulence on heat transfer is to be studied, then the distance between the cylinder and the turbulence-generating body becomes larger than the typical distances used in the previous DNS studies. In the numerical studies of Wissink and Rodi (2011b,a) the distance between the turbulence-generating small cylinder and the heated large cylinder is usually set to less than $2D$. The present study, however, places the cylinder up to a maximum of $27D$ from the turbulence-generating grid-element. This will also increase the number of computational cells substantially. Taking these factors into consideration, simulating heat transfer from a cylinder in the presence of a grid-element even at a moderate Reynolds number will require a large number of cells. These may be the major reason why no such DNS study is available in the literature for the configuration considered in this study.

As shown in chapter 3, the turbulence generated by a single square element shares

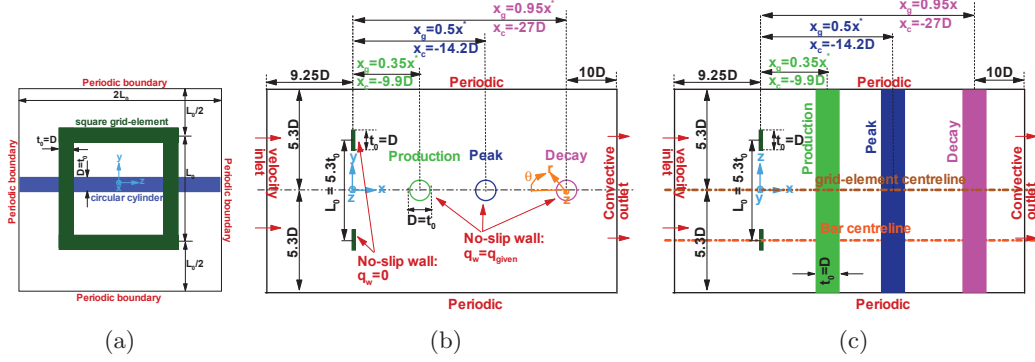


Figure 5.1: Sketch of the computational domain: (a) front view, (b) sectional view at the plane $z/D=0$, (c) sectional view at the plane $y/D=0$. (The sketches are not to scale).

many similarities with the turbulence generated by regular or fractal grids. Moreover, having a regular or fractal grid to generate the oncoming turbulence will only increase the complexities of the simulation as they have a larger wetted-area compared to the grid-element. Due to these reasons, this work explores the effect of turbulence generated by a single square grid-element on laminar heat transfer from a circular cylinder. In particular, the current analysis focuses on the effect of the production, peak and decay regions of grid-element turbulence on heat transfer from a circular cylinder. In order to make meaningful comparisons between these regions, the turbulence intensities of the production and decay regions are kept similar.

5.3 Computational details

5.3.1 Simulation setup

The front view of the computational domain is shown in figure 5.1a where the dimensions of the grid-element and the cylinder are noted. The square grid-element has a lateral thickness of $t_0=43\text{mm}$, a length of $L_0 = 5.3t_0 = 229\text{mm}$, and a streamwise thickness of $0.14t_0 \approx 6\text{mm}$. The diameter of the cylinder (D) is equal to the lateral thickness of the grid-element bar, i.e. $D = t_0$. The side view of the computational domain (figure 5.1b) shows that the square grid-element is placed at a distance $9.25D$ from the inlet. Note that, for this simulation setup, two different variables and their corresponding length scales are used to represent the streamwise distance. With respect to the grid-element, x_g is used to represent the streamwise distance and its origin is at the grid-element as shown in figure 5.1a. On the other hand, x_c , with its origin at the centroid of the cylinder (see the polar coordinates in figure 5.1b), is used to represent the streamwise distance with respect to the cylinder. A wake-interaction length scale, as defined in section 1.5

as $x^* = L_0^2/t_0$, is used to normalise x_g . The diameter of the cylinder, D , is used to normalise x_c .

On the surface of the cylinder the velocity vector is decomposed into radial, azimuthal, and spanwise components which are denoted by the subscripts of r, θ , and z respectively. Therefore, the instantaneous velocities on the cylinder surface are represented as, u_r^* , u_θ^* , and u_z^* . The corresponding mean velocity components are denoted as U_r , U_θ , and U_z . Finally, the fluctuating velocity components on the surface of the cylinder are u_r, u_θ and u_z .

The cylinder is placed at three different locations downstream of the turbulence-generating grid-element. The terminology of these simulations is based on the turbulent kinetic energy evolution along the grid-element centreline as shown in figure 3.8a (chapter 3). The turbulent kinetic energy increases for $x_g/x^* < 0.5$, and this region is called the production region. The turbulent kinetic energy attains its maximum value around $x \approx 0.5x^*$ which is called the peak region. For $x_g/x^* > 0.5$, the value of turbulent kinetic energy decreases, and this region is called the decay region. Accordingly, the simulation where the cylinder is placed at $x_g = 0.35x^*$ ($x_c = -9.9D$) from the grid-element is termed ‘*Production*’. The second simulation is termed ‘*Peak*’ due to the fact that the cylinder for this case is placed at $x_g = 0.5x^*$ ($x_c = -14.2D$) from the grid-element. For the third simulation, the cylinder is placed at $x_g = 0.95x^*$ ($x_c = -27D$) from the grid-element, and this simulation is termed ‘*Decay*’. The locations of the cylinder in these simulations are carefully chosen so as to have similar oncoming turbulent intensity values for the production and decay simulations, while the simulation ‘peak’ has the highest turbulent intensity value. The distance between the cylinder and the outlet is kept constant at $10D$. The Reynolds number based on the inlet velocity and diameter of the cylinder (or the thickness of the grid-element) is $Re_D = U_\infty D / \nu = Re_{t_0} = U_\infty t_0 / \nu = 500$. The blockage for the grid-element is 20%, while for the cylinder it is approximately 10%. The details of these simulations are documented in table 5.1. The Prandtl number of these simulations (defined as $Pr = \nu / \alpha$) is 0.71.

There are also two baseline simulations. The first baseline simulation is the flow past the grid-element without a cylinder. The Reynolds number of this simulation Re_{t_0} is 500. This simulation is termed ‘*grid – element*’, and the details of this simulation were already discussed in chapter 3. The second baseline simulation is the heat transfer from a single cylinder at $Re_D = 500$ without any oncoming turbulence, and this simulation is termed ‘*Cylinder*’. The size of the computational domain in cross-streamwise and spanwise directions is $2L_0 \times 2L_0$.

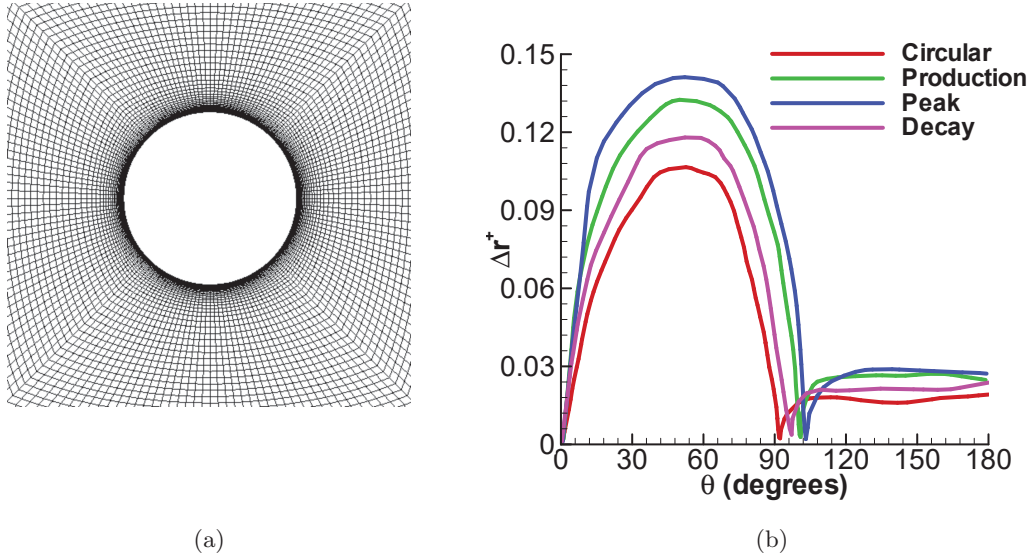


Figure 5.2: (a) Sectional view of an O-type mesh adopted around the cylinder. (b) Variation of Δr^+ of the first grid point in the radial direction.

Simulation name	Maximum $\Delta\theta^+$	Maximum Δz^+	Number of cells	Re_{inc}	Tu
Production	0.12	0.08	18.7×10^6	600	11%
Peak	0.14	0.12	29.31×10^6	550	15%
Decay	0.11	0.09	48.5×10^6	500	11%
Circular	0.08	0.05	8.1×10^6	500	0
Grid-element	0.15 (Δn^+)	0.15 (Δn^+)	20.4×10^6	—	—

Table 5.1: Details of the simulations.

5.3.2 Boundary conditions and computational mesh details

Uniform velocity and temperature profiles are prescribed at the inlet. Convective boundary condition is imposed for velocity and temperature at the outlet. No-slip condition is applied on all solid surfaces for velocity. The grid-element is insulated (i.e. a zero heat-flux boundary condition for temperature), while a uniform heat-flux is imposed on the cylinder. All lateral boundaries are periodic.

The mesh resolution for the grid-element is the same as the one reported in chapter 3. This means the turbulence generated by the grid-element is resolved up to the order of Kolmogorov length scale. The details of mesh resolution for the grid-element can be found in chapter 3.

The cross-sectional view of the mesh around the cylinder is shown in figure 5.2a. An

Name	N_P	Wall-clock hours (approx.)	Data stored (in TB)
Production	448	1000	1.5
Peak	448	1200	1.7
Decay	744	2300	3.1
Circular	240	350	0.65
Grid-element	448	1500	2.2

Table 5.2: High performance computing details of the simulations. (N_P - Number of processors)

O-type mesh is used for the cylinder which allows fine resolution of boundary layers. Figure 5.2b presents the resolution of the mesh in terms of wall units along the circumference of the cylinder. This figure illustrates that the boundary layer is well-resolved as the radial distance between the wall and the nearest mesh point is always less than 0.15. This means that the viscous sub-layer in these simulations is well resolved. Other details regarding the mesh can be found in table 5.1.

The high performance details of the simulations are presented in table 5.2. Due to the numerical difficulties associated with the current setup, simulating even for low Reynolds number requires a large amount of computing time and storage.

5.3.3 Validations

For the simulation setup considered here, it is important to validate the mesh resolution for the grid-element as well as the cylinder. The validation pertaining to the mesh used for the turbulence-generating grid-element will be presented later while characterising the oncoming turbulence. Due to lack of literature on flow around the cylinder at $Re_D=500$, only limited validation is performed for the mesh adopted for the cylinder, and they are given in figure 5.3. Since this study deals with near-wall phenomena, it is imperative to show validations for near wall parameters. The validation for the skin-friction coefficient ($C_f = (\nu \partial U_\theta / \partial r) / U_\infty^2$), and the time-averaged Nusselt number ($\langle Nu \rangle$) are provided for the mesh used for the cylinder.

The variation of the skin-friction coefficient along the circumference of the cylinder is shown in figure 5.3a. The current result is found to be in good agreement with the experimental study of Dimopoulos and Hanratty (1968). Zdravkovich (1997) reports that the mean separation angle for $Re_D = 500$ will be around 90° , and our study also produces a similar result. The time-averaged Nusselt number is compared with the literature in figure 5.3b. Again, the current result is found to be in good agreement with the

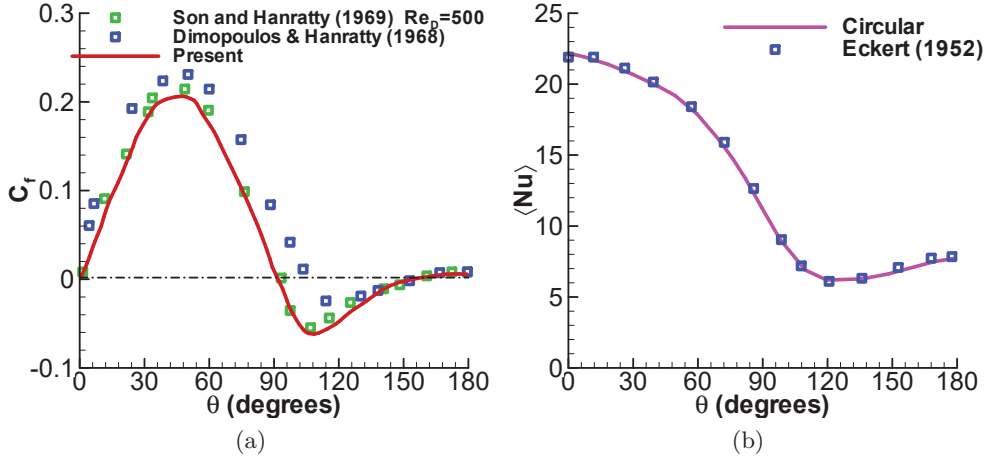


Figure 5.3: (a) Comparison of the skin-friction coefficient with the literature. The data for $Re_D=339$ is taken from Dimopoulos and Hanratty (1968), and for $Re_D=500$ is from Son and Hanratty (1969), (b) Comparison of mean Nusselt number variation with the experiments of Eckert (1952).

experiment of Eckert (1952). Krall and Eckert (1973) proposed correlations for the stagnation point Nusselt number $\langle Nu \rangle_0$, and for the overall Nusselt number $\langle Nu \rangle_{overall}$, which is the circumferential average of $\langle Nu \rangle$. They are given as: $\langle Nu_0 \rangle = 0.95 Re_D^{0.5}$, and $\langle Nu_{overall} \rangle = 0.43 + 0.48 Re_D^{0.5}$. It is found that $\langle Nu_0 \rangle$ and $\langle Nu_{overall} \rangle$ are 22.01 and 11.5 respectively, and their percentage error with respect to the correlations of Krall and Eckert (1973) for $Re_D=500$ are 3.01% and 3.19% respectively. This validation provides evidence that the numerical mesh resolution employed in this study is adequate to compute the correct near-wall parameters of fluid flow and heat transfer.

5.4 Characteristics of oncoming turbulence

This study focuses only on the $x - y$ centreplane at $z/D=0$ (see figure 5.1b) for two important reasons. Firstly, as shown in chapter 3, in this plane, the oncoming flow exhibits different types of turbulence such as the intermittent and fully-developed turbulent states. Secondly, as shown later in this study, the stagnation point heat transfer coefficient correlations are not valid along the grid-element centreline $x - y$ plane, while they are valid in the $x - y$ plane where the bar centreline passes through. For these reasons, the grid-element centreline $x - y$ plane is not only intriguing but also provides the opportunity to study the laminar heat transfer for turbulence types which are not considered in the literature.

Typical instantaneous flow fields of the simulated cases at a particular time instant

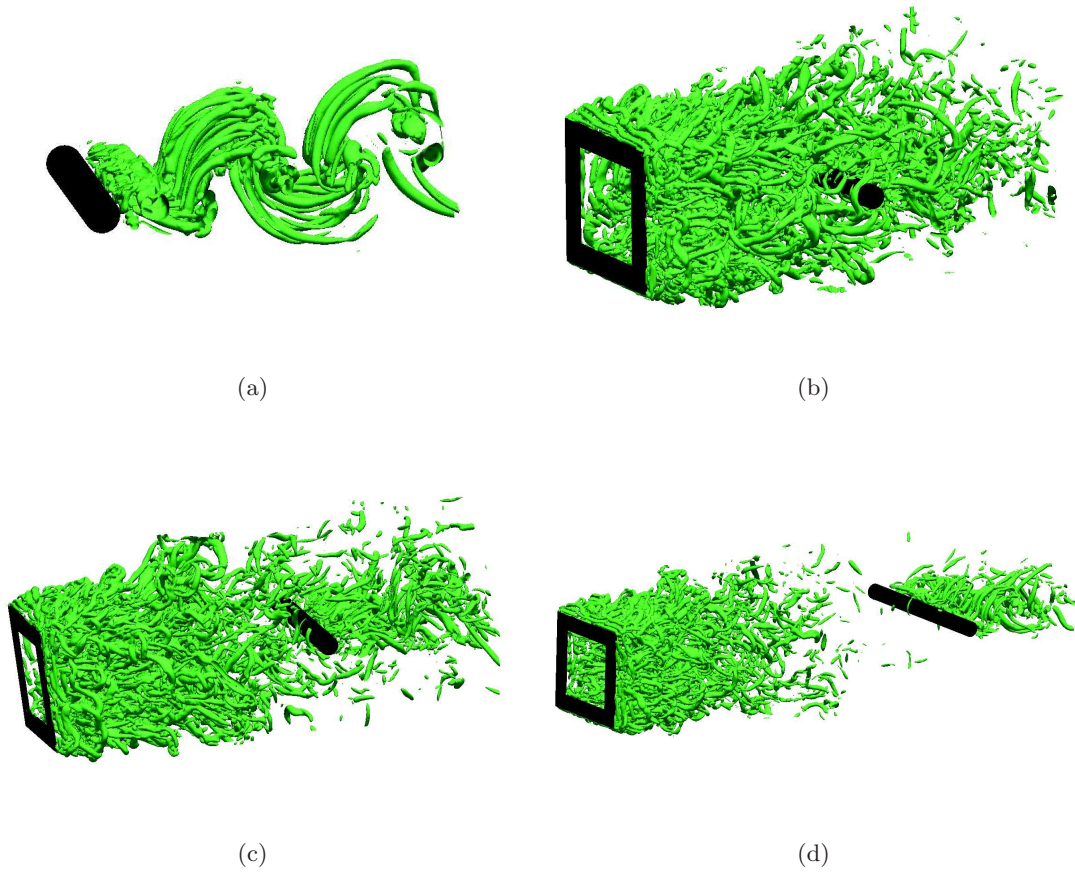


Figure 5.4: Visualisation of turbulent coherent structures through iso-surfaces of the second invariant of the fluctuating velocity gradient tensor ($Q/(U_\infty/D)^2=3$): (a) Circular, (b) Production, (c) Peak, (d) Decay.

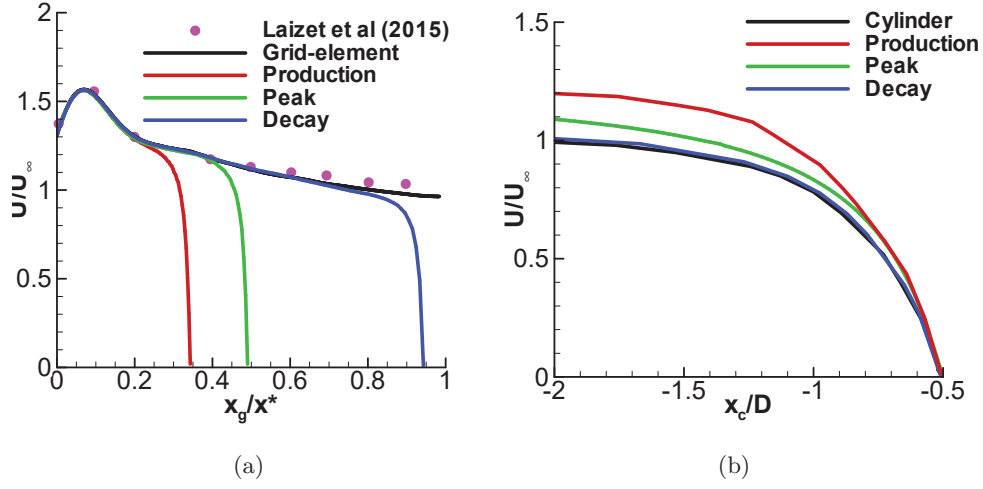


Figure 5.5: (a) Variation of mean velocity along the grid-element centreline, (b) Mean velocity profiles upstream of the cylinder in the plane $z/D=0$.

are shown in terms of the instantaneous iso-surfaces of Q -criterion in figure 5.4. It can be seen that the turbulent flow from the grid-element has a wider range of scales compared to that of flow past the cylinder at this Reynolds number. This section is devoted to the characterisation of the oncoming turbulence generated downstream of the single square grid-element.

5.4.1 Mean and rms velocity profiles

Mean velocity profiles along the centreline of the grid-element are shown in figure 5.5a for all the cases considered in this study. For the production, peak and decay cases, the approaching flow reaches zero at the streamwise location where the cylinder is placed due to the no-slip boundary condition applied on the cylinder surface. The velocity profiles of all the cases are indistinguishable for $x_g/x^* < 0.2$ which is a proof that the resolution downstream of the grid-element in every case is the same. The velocity profile exhibits a jet-like behaviour close to the grid-element, followed by a monotonic decrease further downstream. Such a mean velocity variation implies that the local Reynolds number, which the cylinder “feels”, is not the same as of the inlet Reynolds number. The incident Reynolds number, which is defined in terms of velocity outside the boundary layer that develops on the cylinder surface, is computed. In the cases considered here, the incident velocity (U_{inc}) is measured at $x_c/D = -1.5$ as this location is well outside the boundary layer, and the mean and rms velocities do not vary significantly at this point. Figure 5.5b reveals that the incident mean velocity of the production case is the largest due to the jet-like behaviour of mean velocity profile. On the other hand, the incident velocities

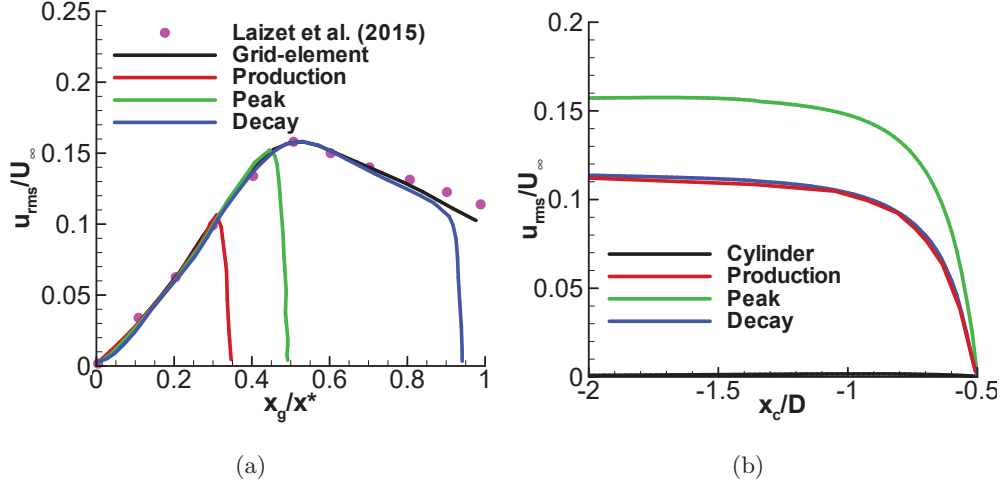


Figure 5.6: (a) Variation of rms velocity along the grid-element centreline, (b) Rms velocity profiles upstream of the cylinder in the plane $z/D=0$.

of the decay and the cylinder cases are almost equal, and they are less than that of the peak case. The computed incident Reynolds number ($Re_{inc}=U_{inc}D/\nu$) values are documented in table 5.1. The mean velocity profiles in figure 5.5a are also compared against the experimental results of Laizet et al. (2015a), and the matching is found to be good. This matching provides the first evidence that the mesh resolution downstream of the grid-element is similar to the mesh used in chapter 3, and the grid-element and its downstream is properly resolved.

The rms-velocity profiles along the grid-element centreline are shown in figure 5.6a and the experimental results of Laizet et al. (2015a) are also included. Again, the comparison is found to be reasonable which further proves that the turbulence generated by the grid-element is indeed resolved properly. Three distinct regions are identified in the rms-velocity profile. The first one is the production region which lies in the lee of the grid-element and identified by increase in rms-velocity. The location where the rms velocity value reaches its maximum is called the peak region. In the decay region, the turbulence decays. The circular cylinders are kept at these three regions. Turbulent intensity is defined in accordance with the previous studies (Wissink and Rodi, 2011b; Venema et al., 2011; Wissink and Rodi, 2011a) as,

$$Tu = \sqrt{\frac{1}{3} \frac{\langle u_i u_i \rangle}{U_\infty^2}} \times 100\% \quad (5.1)$$

This study chose to present rms velocity profiles instead of turbulent intensity profiles solely because the rms velocity profiles can be compared against the experiments. Yet,

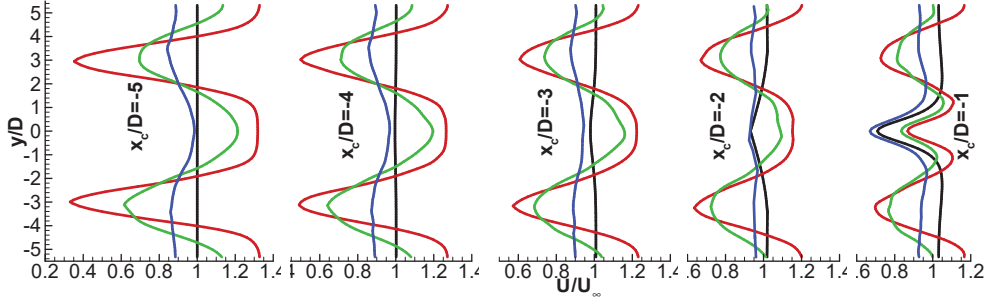


Figure 5.7: Development of mean velocity profiles upstream of the cylinder in the plane $z/D=0$ (red - Production, green-Peak, blue-Decay, black-Cylinder)

the values and variation of Tu are similar to that of u_{rms} , and Tu reaches a plateau around $x_g/x^* \approx -1.5D$ much like the mean velocity profile. This can also be seen in the rms velocity profiles plotted upstream of the cylinder in figure 5.6b. The values of Tu at the edge of boundary layer (i.e. at $x_c/D=-1.5$) for each case is given in the table 5.1. It is important to note in table 5.1 that the oncoming turbulent intensities for the production and decay cases are approximately similar while their incident Reynolds numbers differ.

The profiles of mean velocity along the cross-stream direction at different x_c/D locations upstream of the cylinder are plotted in figure 5.7. The x_c/D locations are chosen to be -5, -4, -3, -2, -1. At $x_c/D=-5$, although the profile of the cylinder case is uniform in cross-stream direction, the profiles of other cases exhibit velocity deficits around $y/D \approx \pm 3$ in them due to the presence of grid-element bars. This velocity deficit is more prominent for the production and peak cases, while it is weaker for the decay case. This is expected as the production and peak regions are closer to the grid-element, while the decay region is further away. Further downstream of $x_c/D=-5$, the wake deficits of the bars decrease in magnitude along with the decrease in grid-element centreline mean velocity. The stagnation starts to have an effect on the cross-stream variation of the velocity profiles around $x_c/D=-2$ where the profiles of the cylinder and decay cases almost match each other. The cross-stream plane velocity profiles at $x_c/D=-1$ clearly feel the presence of the cylinder. At the grid-element centreline, the value of mean velocity is the highest for the production case due to the jet-like behaviour as discussed earlier. For the cases of cylinder and decay, the mean velocity at the grid-element centreline is observed to be similar ascertaining the conclusions made from the mean velocity evolution along the streamwise direction.

The development of u_{rms} profiles upstream of the cylinder is illustrated in figure 5.8. These profiles are normalised by the free-stream velocity (U_∞). The presence

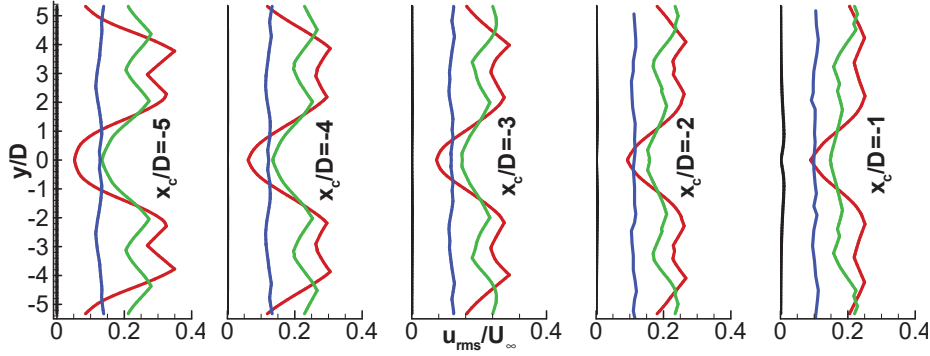


Figure 5.8: Development of rms velocity profiles upstream of the cylinder in the plane $z/D=0$ (red - Production, green-Peak, blue-Decay, black-Cylinder)

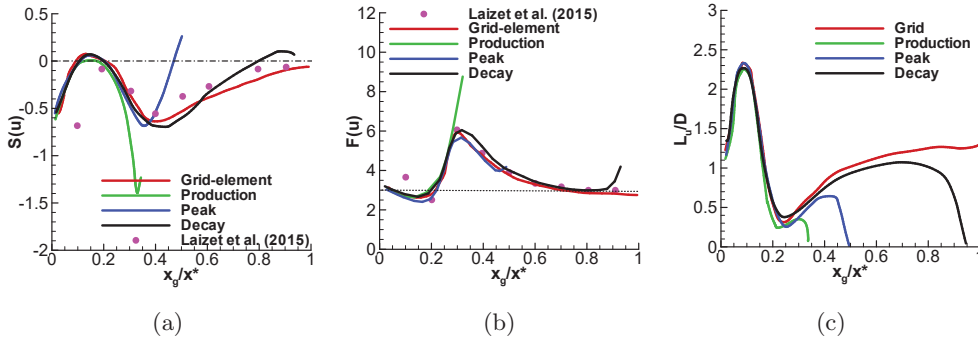


Figure 5.9: Variation of skewness, flatness and length scales of turbulence along the grid-element centreline: (a) Skewness, (b) Flatness, (c) Length scale.

of grid-element bars make the turbulence in the production and peak cases highly inhomogeneous with higher values of rms velocity behind the bars. On the other hand, the turbulence appears to be nearly homogeneous for the decay region. Mazellier and Vassilicos (2010) reported that the turbulence behind a fractal grid becomes approximately homogeneous around $x \approx x^*$. Therefore, it is not surprising that the turbulence upstream of the cylinder in the decay case is nearly homogeneous. The values of u_{rms} at $y/D = 0$ are similar for the production and decay regions at $x_c/D = -2$ and -1 .

5.4.2 Statistical characteristics of the oncoming turbulence

The characteristics of oncoming turbulence is examined next through skewness and flatness of the streamwise fluctuating velocity. The profiles of skewness and flatness are depicted in figures 5.9a and 5.9b respectively. When the turbulence is Gaussian, its skewness and flatness are 0 and 3 respectively, which are shown as dashed lines in fig-

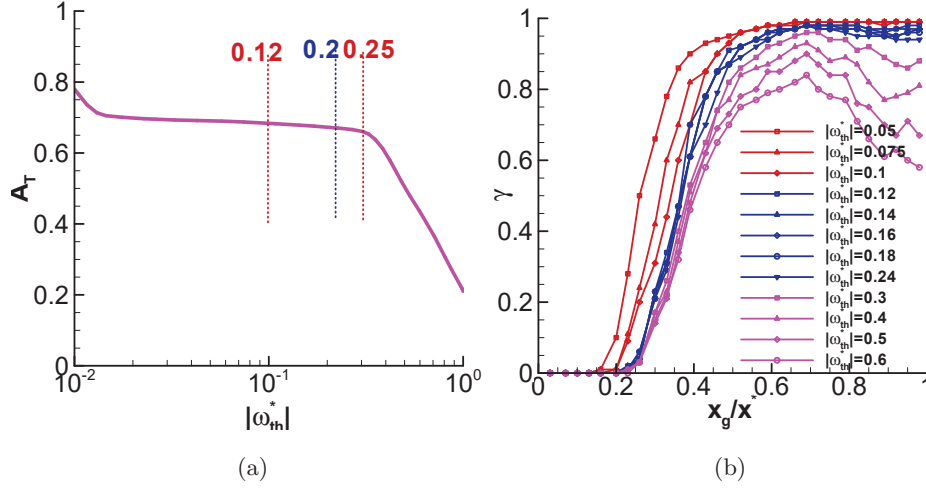


Figure 5.10: (a) Calculation of vorticity magnitude threshold, (b) Sensitivity of vorticity magnitude on intermittency factor.

ures 5.9a and 5.9b. The experimental results of Laizet et al. (2015a) are compared with the current results, and the comparison is found to be satisfactory, especially for the flatness. In the production region, both skewness and flatness values are highly deviating from the Gaussian values. Large negative values of skewness in the production region represent extreme decelerating turbulent events (Mazellier and Vassilicos, 2010), while the large positive values of flatness denote intense rare events. Therefore, the production region is identified as non-Gaussian with intense rare events of decelerating turbulence. Only in the decay region, the values of skewness and flatness reach the Gaussian values which resembles that the turbulence at the decay region is Gaussian. It is interesting to observe that placing a cylinder in the production region significantly decelerates the turbulence further (figure 5.9a), besides aiding the creation of even more intense events (5.9b). However, such an observation is not seen for the peak and decay cases.

The integral length scale (L_u) of the oncoming turbulence is computed through the auto-correlation function of fluctuating streamwise velocity which is defined as, $R_{uu}(t) = \langle u(t)u(t+\Delta t) \rangle / u_{rms}^2$. First, an integral time scale is computed as: $L_{time} = \int_0^{R_\psi} R_{uu}(t) dt$, where R_ψ is the first zero crossing of $R_{uu}(t)$. Then the integral time scale is multiplied by the local mean velocity to obtain the integral length scale: $L_u = U_{local} L_{time}$. The integral length scales of turbulence normalised by the diameter of the cylinder are plotted in figure 5.9c. The integral length scale profile qualitatively resembles that of Valente and Vassilicos (2011) reported for a fractal grid.

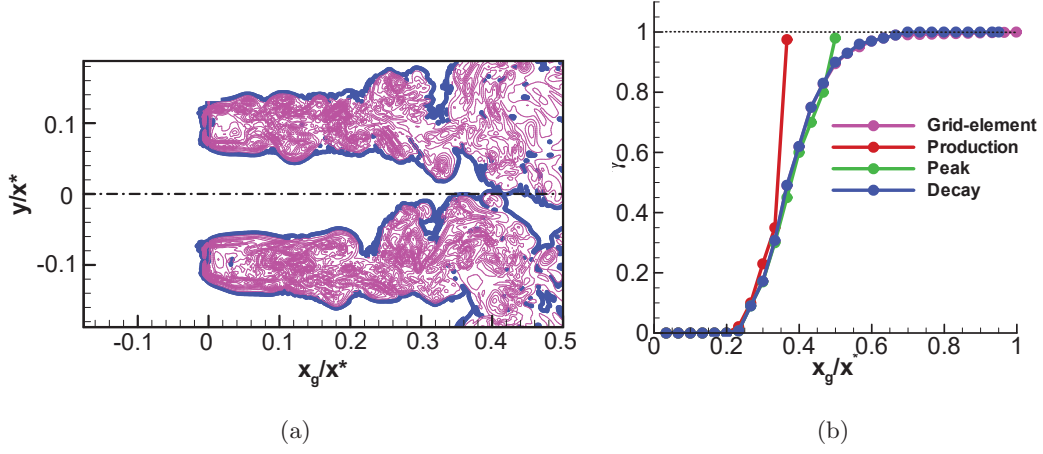


Figure 5.11: (a) Visualisation of the turbulent/non-turbulent (T/NT) interface and vortical structures surrounding the centreline. The 2D region plotted in this figure has the dimensions of $-1 \leq y/t_0 \leq 1$, and $0 \leq x_g/x^* \leq 0.5$ downstream of the grid-element. The blue lines represent the T/NT interfaces with $|\omega^*|_{th}=0.2$ while the pink lines represent the isolines of $0 \leq |\omega^*| \leq 3$. (b) Evolution of intermittency factor along the grid-element centreline

5.4.3 Characterisation of oncoming turbulence intermittency

Finally, the intermittency of the oncoming turbulence is analysed. The study of intermittency is required as the flatness of fluctuating streamwise velocity indicated that the production and peak regions are dominated by rare intense events. The intermittency factor (γ) is determined through the instantaneous magnitude of the vorticity vector ($|\omega^*|$) whose components are given as $\omega_i^* = \epsilon_{ijk} \frac{\partial u_k^*}{\partial x_j}$. Following the studies on turbulent/non-turbulent interfaces (Bisset et al., 2002; Watanabe et al., 2014; Taveira et al., 2013), the flow is defined as turbulent when the vorticity magnitude ($|\omega^*|$) is greater than a certain threshold vorticity magnitude ($|\omega^*|_{th}$). Based on this, the intermittency factor is defined as, $\gamma = \text{probability of } (|\omega^*| \geq |\omega^*|_{th})$. As can be seen from this definition, the vorticity threshold value plays a crucial role in determining the intermittency factor. The method adopted for estimating $|\omega^*|_{th}$ in this study closely follows that of Taveira et al. (2013). Since this study focuses only on the grid-element centreline, a two-dimensional instantaneous vorticity slice of xy plane at $z=0$, whose dimensions are $0 \leq x \leq 0.5x^*$ and $-5.3D \leq y \leq 5.3D$, is considered. A wide range of $|\omega^*|_{th}$ values from 0 to 10 is also considered. For each value of $|\omega^*|_{th}$, the fraction of turbulent area ($A_T = \text{Area of } (|\omega^*| > |\omega^*|_{th}) / \max(A_T)$) is computed. The variation of A_T with $|\omega^*|_{th}$ is depicted in figure 5.10a. Although A_T decreases for increasing $|\omega^*|_{th}$, a plateau can be seen in the plot for a range of $|\omega^*|_{th}$. It is generally believed that the

intermittency factor weakly depends on $|\omega^*|_{th}$ when the threshold is chosen from this plateau (Watanabe et al., 2014; Taveira et al., 2013). This fact is checked in figure 5.10b which shows the variation of intermittency factor for various $|\omega^*|_{th}$. The profile of γ is found to be insensitive to $|\omega^*|_{th}$ for $0.12 \leq |\omega^*|_{th} \leq 0.25$. Therefore, this study chooses $|\omega^*|_{th}$ as 0.2.

The detected TNT interface using this threshold value is visualised in figure 5.11a. The figure shows that the vortical structures are enveloped by the interface. The value of $|\omega^*|_{th}$ is highly sensitive to flow configuration. For example, Bisset et al. (2002) reported a value of 0.7 for the far-wake of a cylinder, while for jet it is found to be 0.3 (Watanabe et al., 2014). The variation of intermittency factor along the grid-element centreline for all the cases is presented in figure 5.11b. As can be seen from the figure, the region $0 \leq x_g/x^* \leq 0.2$ is purely irrotational and the region $0.2 \leq x_g/x^* \leq 0.5$ is intermittent. The rapid increase of γ for the production and peak cases near the cylinder signifies the fact that those points are situated inside the developing boundary layer on the surface of the cylinder. Therefore, the intermittency analysis reveals that the oncoming turbulence in the production is indeed highly intermittent for the production case, while it is less intermittent for the peak case, and it is fully turbulent for the decay case.

As an interim summary, the oncoming turbulence generated by the single square grid-element contains various characteristics downstream of the grid-element. In the production and peak regions, the turbulence is highly non-Gaussian, inhomogeneous, and intermittent. On the other hand, the turbulence at the decay region is Gaussian, homogeneous and fully turbulent.

5.5 Flow characteristics on the surface of the cylinder

Having examined the characteristics of oncoming turbulence, this section provides details of how the impinging turbulence affects the flow characteristics on the surface of the cylinder.

5.5.1 Mean velocity and turbulent stress profiles inside the boundary layer

The boundary layer profiles (i.e. the azimuthal velocity profiles) of all the cases are compared in figure 5.12 at different angles on the surface of the cylinder. Note that the profiles are plotted normal to the cylinder surface at plane $z/D=0$. The value of mean azimuthal velocity at the front stagnation (i.e., at $\theta = 0^\circ$) is zero due to symmetry. Moving further clockwise from this angle, the velocity gradually increases. The boundary layer theory predicts that the thickness of laminar boundary layer (δ) has a power-law

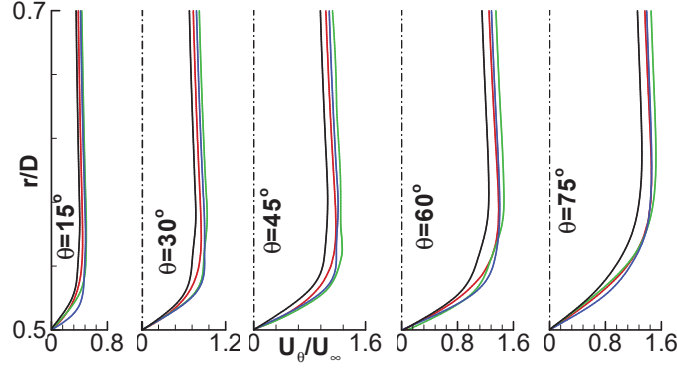


Figure 5.12: (a) Mean azimuthal velocity profiles inside the boundary layer in the plane $z/D=0$ (green-Production, blue-Peak, black-Decay, red-Cylinder)

relationship with Reynolds number as, $\delta/D = Re_D^{-0.5}$. This relationship appears to be satisfied for the cases considered in this study as the simulations indicate boundary layer thickness approximately as $0.05D$, a value closer to the theoretical prediction. The profiles show distinguishable behaviour for $\theta \leq 60$ with larger approaching velocities for the production and peak cases, and lower approaching velocity for the decay case. The profiles, however, become indistinguishable after $\theta = 60^\circ$, with an exception of the decay case whose velocity is low compared to the other cases including the cylinder case. Despite being exposed to turbulent fluctuations, the boundary layer profiles of production, peak and decay cases are comparable to the cylinder case. This ascertains that the boundary layer on the surface of the cylinder in all cases is laminar. Indeed, Jones and Launder (1973) showed that the boundary layer remains laminar for acceleration parameter (which is defined as $K = \frac{\nu}{U_\infty^2} \frac{dU_\theta}{ds}$, where ds is the circumferential distance on the surface of the cylinder) in excess of 2.5×10^{-6} . The value of K at 75° is computed to be around 1.52×10^{-5} , a value much larger than the critical value. Therefore, the boundary layer flow in the range of $0^\circ \leq \theta \leq 75^\circ$ will not undergo transition.

The turbulent stress profiles along the radial direction are plotted for all the cases in figure 5.13. The turbulent stresses considered here are: (i) radial stress ($\langle u_r u_r \rangle$), (ii) azimuthal stress ($\langle u_\theta u_\theta \rangle$), (iii) spanwise stress ($\langle u_z u_z \rangle$) and (iv) Reynolds stress ($\langle u_r u_\theta \rangle$). These stresses are normalised by U_∞^2 . Effect of free-stream turbulence is felt on these profiles as these stress magnitudes are larger near the cylinder surface for the production, peak and decay cases while they are zero for the cylinder case. Within the boundary layer thickness, the azimuthal and spanwise stresses are higher than the radial stress. The magnitude of stresses gradually increases moving clockwise from the front stagnation point. The peak case exhibits the largest stress at all location as the

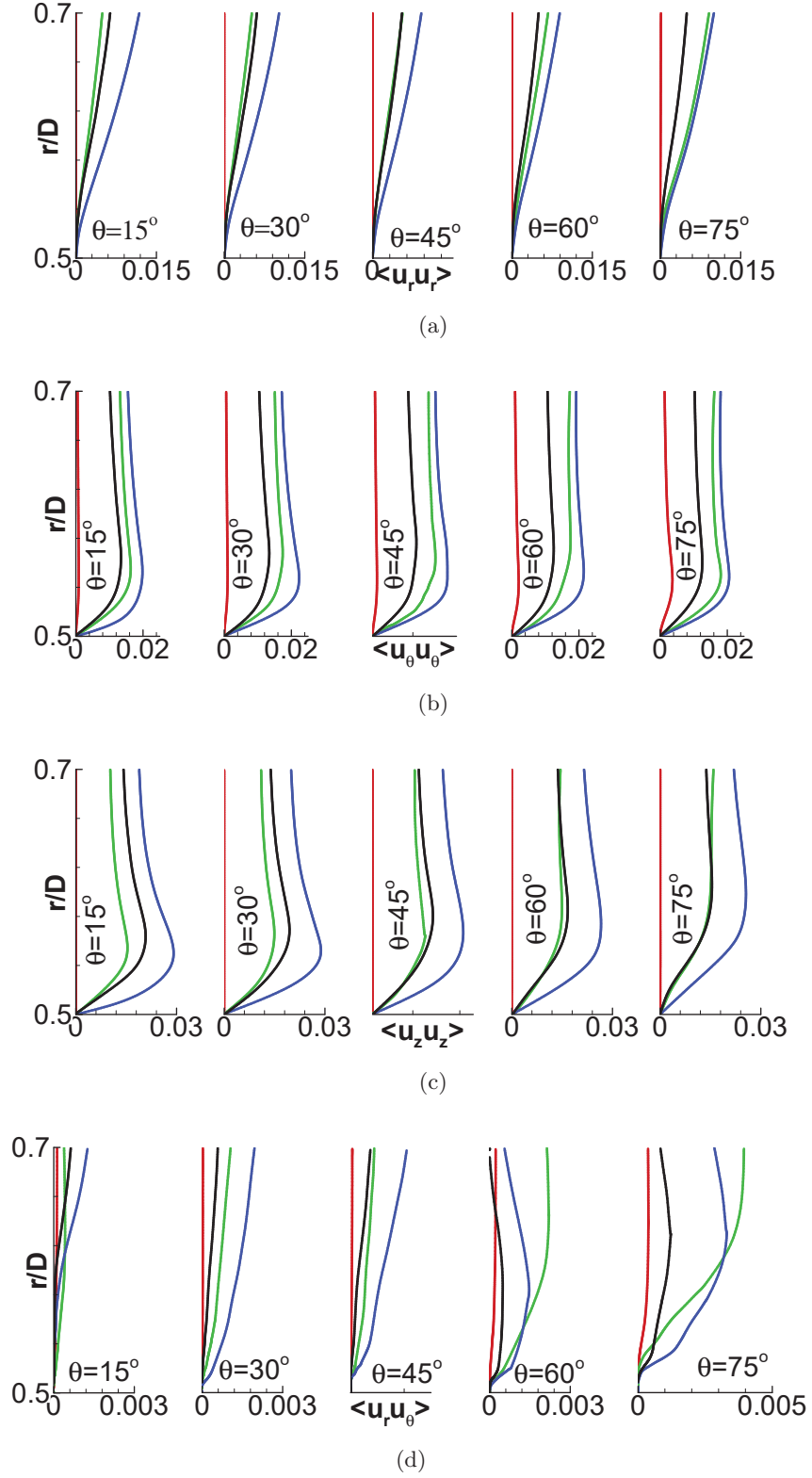


Figure 5.13: Hydrodynamic stress profiles inside the boundary layer in the plane $z/D=0$: (a) Radial stress, (b) Azimuthal stress, (c) Spanwise stress, (d) Reynolds stress. (green-Production, blue-Peak, black-Decay, red-Cylinder).

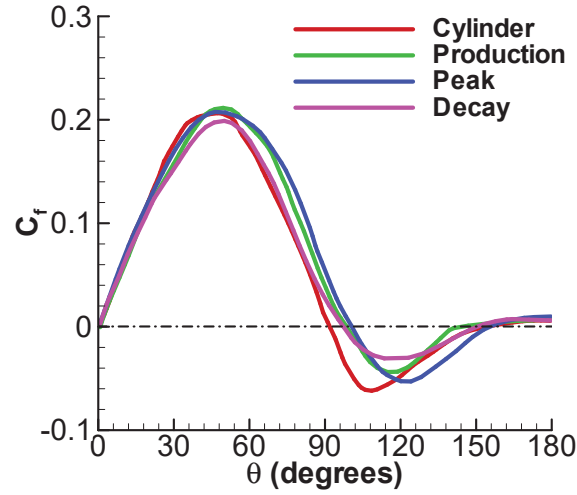


Figure 5.14: Variation of friction coefficient along the circumference of the cylinder in the plane $z/D=0$.

oncoming turbulence intensity is maximum for this case. The radial and spanwise stress profiles of the production and decay cases are self-similar, especially for $\theta \geq 45^\circ$. Such observation, however, is not spotted for the azimuthal as well as the Reynolds stress profiles.

5.5.2 Effect of free-stream turbulence on skin-friction profile

The effect of free-stream turbulence on the flow characteristics can be seen in the skin-friction coefficient (C_f) variation along the circumference of the cylinder (figure 5.14). The mean separation angles for the cylinder, production, peak and the decay cases are computed to be 93.5, 100.2, 103.5, and 97 respectively. The higher values of θ_s for the cases of production, peak and decay compared to that of the cylinder case signify the influence of free-stream turbulence in energising the laminar boundary layer which delays its separation from the cylinder surface.

5.6 Heat transfer characteristics on the surface of the cylinder

This section explores the effect of free-stream turbulence on heat transfer characteristics.

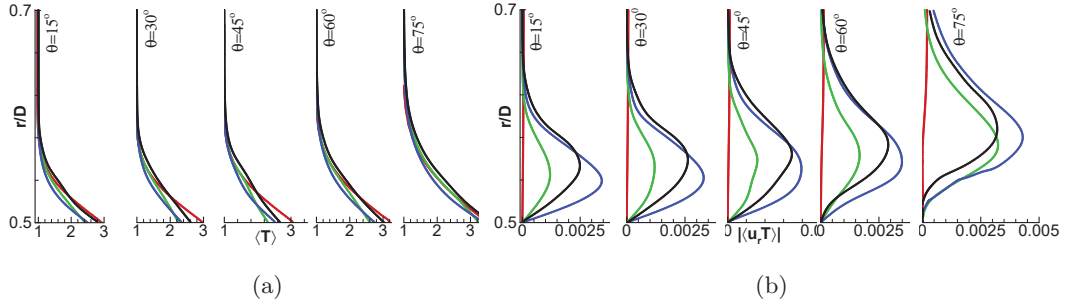


Figure 5.15: (a) Mean temperature profiles inside the thermal boundary layer in the plane $z/D=0$, (b) Turbulent heat-flux ($|\langle u_r T \rangle|$) profiles inside the thermal boundary layer in the plane $z/D=0$ (green-Production, blue-Peak, black-Decay, red-Cylinder).

5.6.1 Mean temperature and turbulent heat-flux profiles inside the boundary layer

Figure 5.15a shows the thermal boundary layer profiles at different angles on the surface of the cylinder. These profiles are normalised by the free-stream temperature (T_∞). The thickness of thermal boundary layer, δ_T , is computed to be larger than that of the velocity boundary layer. This is because the Prandtl number is less than one, and therefore the thermal diffusion is larger than the viscous diffusion, which results in thicker thermal boundary layer than the hydrodynamic boundary layer. Indeed, δ_T is computed approximately to be $0.1D$ in our simulations which is twice of the hydrodynamic boundary layer thickness. The mean temperature profiles of the production, peak and decay cases exhibit larger gradients than that of the cylinder case resembling that the heat transfer is larger than that of the cylinder case. Comparing the profiles of the production and decay cases, the production case clearly has larger gradients, meaning that it will have the highest heat transfer rate. These observations will be further verified in the time averaged Nusselt number ($\langle Nu \rangle$) profiles.

The wall-normal heat-flux profiles ($|\langle u_r T \rangle|$) for all the cases are depicted in figure 5.15b. The presence of the cylinder restricts the wall-normal fluctuation, so the heat transfer near the cylinder is mainly due to molecular diffusion (Wissink and Rodi, 2011a). Away from the cylinder surface, the role of wall-normal fluctuations becomes important. Moving clockwise from $\theta = 0^\circ$, the wall-normal region where turbulent fluctuations are active keeps expanding radially outward from the surface. The maximum turbulent wall-normal heat-flux is observed in the peak case. Comparing the profiles of the production and decay cases, the production case has larger heat-flux than the decay case. This is in-line with the previous observation that the velocity deficit for the case of the production is larger than that of the decay case.

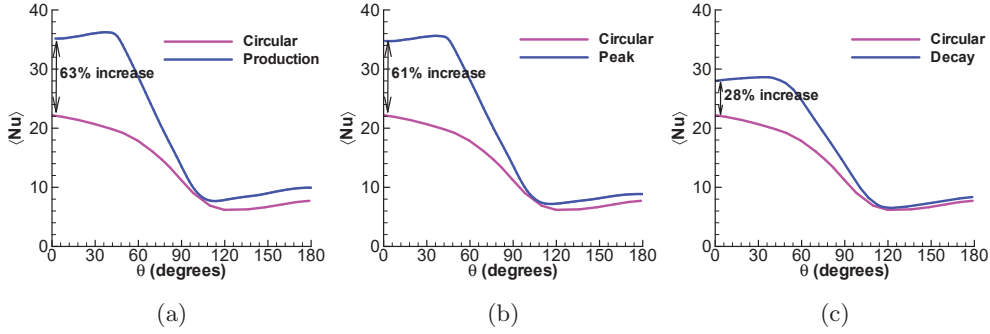


Figure 5.16: Variation of mean Nusselt number along the circumference of the cylinder in the plane $z/D=0$: (a) Production, (b) Peak, (c) Decay.

Simulation name	Re_{inc}	$\langle Nu_0 \rangle$ (theory)	$\langle Nu_0 \rangle$ (present)	% increase
Production	600	24.49	35.12	44%
Peak	550	23.45	34.9	49%
Decay	500	22.0	28.1	28%

Table 5.3: Effect of incident Reynolds number on stagnation point heat transfer coefficient.

5.6.2 Effect of free-stream turbulence on Nusselt number profiles

The variation of mean Nusselt number, $\langle Nu \rangle$, for the production, peak, and decay cases are shown in figures 5.16a, 5.16b and 5.16c respectively. The variation of $\langle Nu \rangle$ for the cylinder case is also plotted. The characteristic observations noted from the mean temperature and turbulent heat-flux profiles are reflected in the $\langle Nu \rangle$ variation. The heat transfer rate of the production, peak and decay cases are higher than that of the cylinder. Interestingly, the maximum percentage increase of heat transfer at the front stagnation point occurs for the production case with a substantial 63% increase compared to the cylinder case. On the other hand, the peak case, which has a maximum oncoming turbulence intensity of 15%, results in 61% of heat transfer increase compared to the cylinder case. The decay case exhibits the lowest percentage of heat transfer increase of 28% although the oncoming turbulent intensities of the production and decay cases are similar.

It can be argued that the effect of local increase in mean velocity for the production and peak cases can explain why the increase in heat transfer is larger. Therefore, it is reasonable to compare the stagnation point mean Nusselt number, $\langle Nu_0 \rangle$, against the equivalent Reynolds numbers, rather than the cylinder case. In order to clarify the

effect of incident Reynolds number on heat transfer, $\langle Nu_0 \rangle$ values are compared against the values predicted by the theory for the same incident Reynolds number. Such a comparison is given in Table 5.3. Consideration of incident Reynolds number reveals that the peak case has the highest increase of stagnation point heat transfer, while the decay case shows the lowest. This observation is intuitive as the peak case has the highest oncoming turbulent intensity of all other cases. It is interesting, however, to note that the production case has much higher heat transfer increase than that of the decay case although both the cases have a similar turbulent intensity value. Clearly, this difference in heat transfer enhancement is not due to the differences in incident Reynolds number. The reasons for the production case to exhibit higher heat transfer rate than that of the decay case will be further discussed later in this study by means of timescale separation analysis and examining the dominant frequency of the turbulent fluctuations.

The next important observation noted in the $\langle Nu \rangle$ profiles (figures 5.16) is the location of the maximum Nusselt number ($\langle Nu \rangle_{max}$). The front stagnation point on the surface of the cylinder is generally expected to have the highest heat transfer rate. This is because the cold oncoming flow makes contact with the cylinder surface first on the front stagnation point, and as a result, the rest of the cylinder surface is exposed to flow which has the temperature higher than the one that has impinged on the front stagnation point. This observation is valid only in the cylinder case where there is no oncoming turbulence. When a turbulence-generating grid-element is introduced, figures 5.16a-5.16c show that the $\langle Nu \rangle_{max}$ is shifted around 45° away from the front stagnation point. Similar observations have been noted in the experimental study of Magari and LaGraff (1994) and the semi-DNS study of Venema et al. (2011). In those studies the cylinder was subject to wake turbulence generated by a small cylinder placed upstream. They concluded that the shift in $\langle Nu \rangle_{max}$ is due to the presence of small cylinder which creates a wake deficit which in turn decreases the convective heat transfer at the front stagnation point. The study of Wissink and Rodi (2011a,b), where the wake generator was simulated in a precursor simulation and the turbulence data was imposed on the inlet of the domain, could not detect such a shift in $\langle Nu \rangle_{max}$.

The characteristic of $\langle Nu \rangle$ behind the grid-element bars as well as the grid-element centreline is now visualised. The $\langle Nu \rangle$ contour on the surface of the cylinder is shown for all the cases in figure 5.17. For the cylinder case, the $\langle Nu \rangle$ contour is uniform in the spanwise direction (figure 5.17a) as expected. This uniformity is lost for the other cases. This is attributed to the fact the values of Reynolds number, turbulent intensity and integral length scales vary drastically along the spanwise direction. As can be seen in figure 5.17b, the value of $\langle Nu \rangle$ attains its maximum around $\theta \approx 45^\circ$ along the grid-element centreline. This feature is more prominent in the production case

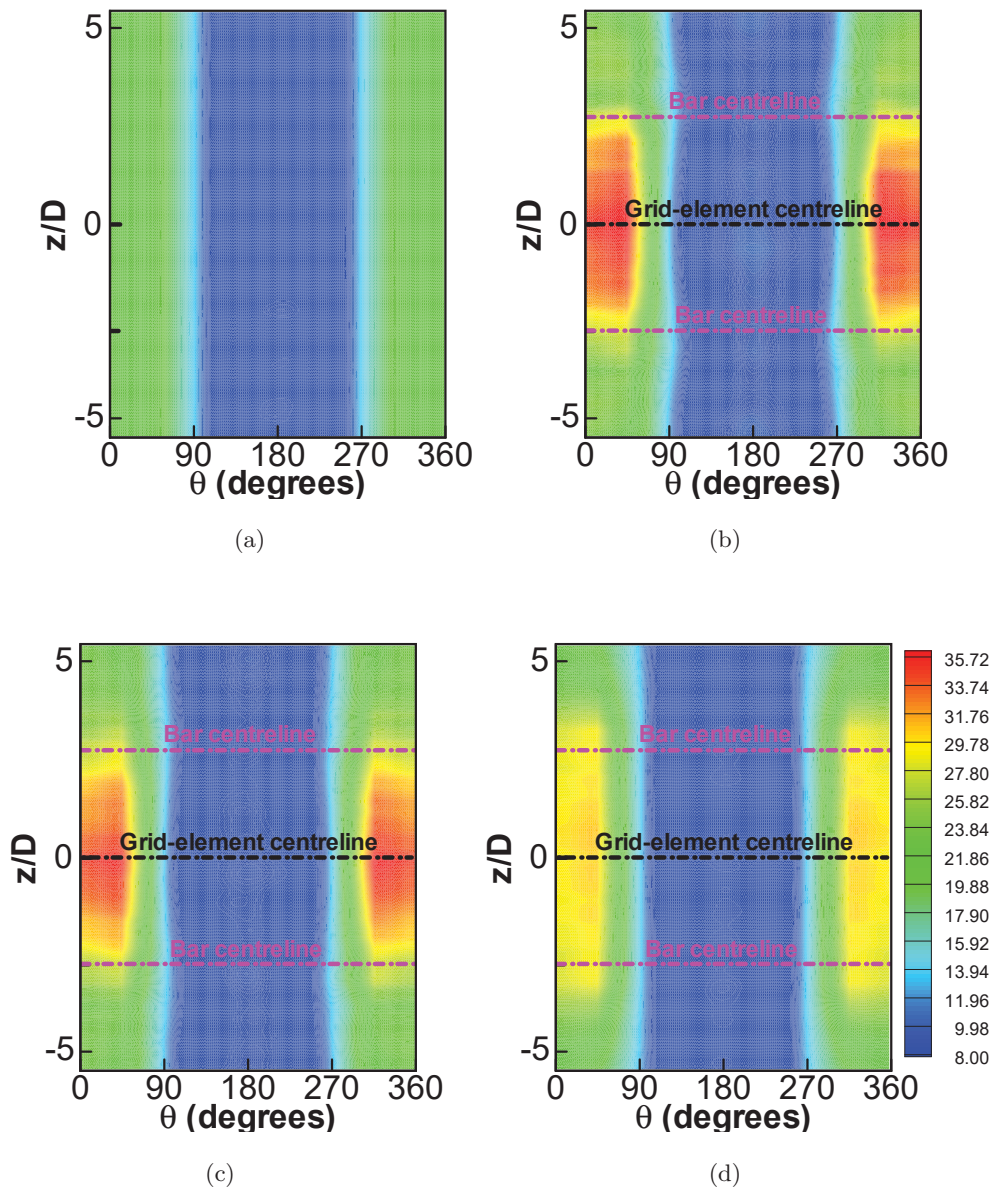


Figure 5.17: Contours of mean Nusselt number on the cylinder surface: (a) cylinder, (b) Production, (c) Peak, (d) Decay. The isocontours range from 8 to 36.

and becomes less significant for the decay case. Since the grid-element centreline is not directly immersed in the wakes of the bars, the presence of wake deficit, as reported by Venema et al. (2011), cannot answer why the $\langle Nu \rangle_{max}$ is shifted from the front stagnation point. Note also that although the turbulent intensity is very high along the bar-centreline, in particular for the production case (see figure 3.9a), the maximum $\langle Nu \rangle$ is not observed along the bar centreline, instead it is felt only on the grid-element centreline. The reason for this observation will become clearer when the dominant frequency of the turbulent fluctuations is discussed in section 5.7.

5.6.3 Factors causing the shift in $\langle Nu \rangle_{max}$

The shift in $\langle Nu \rangle_{max}$ from the front stagnation point is further explored through probability density functions (PDFs) of Nusselt number. The PDFs of Nusselt number at various azimuthal locations for all the cases are plotted in figure 5.18. The PDFs of Nusselt number for the cylinder case are Gaussian with near-perfect symmetry. The other cases exhibit an interesting behaviour. The PDF at the stagnation point appears to be near-symmetric with a Gaussian profile. Yet, for the region of $0^\circ < \theta \leq 45^\circ$, the PDFs lose their symmetry with more skewness towards higher values of Nu . The level of skewness increases from $\theta > 0^\circ$ to $\theta \approx 45^\circ$. After $\theta \approx 45^\circ$, the PDFs again morph to a near-symmetric state for the production, peak and decay cases. Therefore, it appears that additional intense events occur in the region of $0 < \theta \leq 45$ compared to the front stagnation point. Such events can be clearly seen in figure 5.19 where the PDFs of Nusselt number at the front-stagnation point and at $\theta = 45^\circ$ are compared. The green shaded areas in these figures represent the additional intense events occurring at $\theta = 45^\circ$ for the cases of production, peak and decay. Note that the shaded area in these figure is larger for the production case, and continues to decrease for the peak and decay. Therefore, it is now clear that these additional events cause the PDFs of Nusselt number to skew more towards higher values, thus causing higher heat transfer rate at $\theta = 45^\circ$.

Having noted that the intense additional events occur at $\theta=45^\circ$, now the question is that what causes such events. The answer can be found in the oncoming turbulence profiles. It is known that the Nusselt number is a function of Reynolds number, turbulent intensity, and length scales. The length scales are not expected to change drastically in a limited range of $0 \leq \theta < 45^\circ$. The profiles of mean velocity show that there exists a jet-like behaviour as discussed earlier. The effect of this jet behaviour is that it limits the velocity deficit along the grid-element centreline. Therefore, the cylinder sees a much higher velocity at locations $0 < \theta \leq 45$ as seen in figure 5.7 for $x_c/D=-1$. Adding to that, the oncoming turbulence intensity is also highly inhomogeneous in the cross-stream direction with lesser intensity at the stagnation point and higher values away from it (see

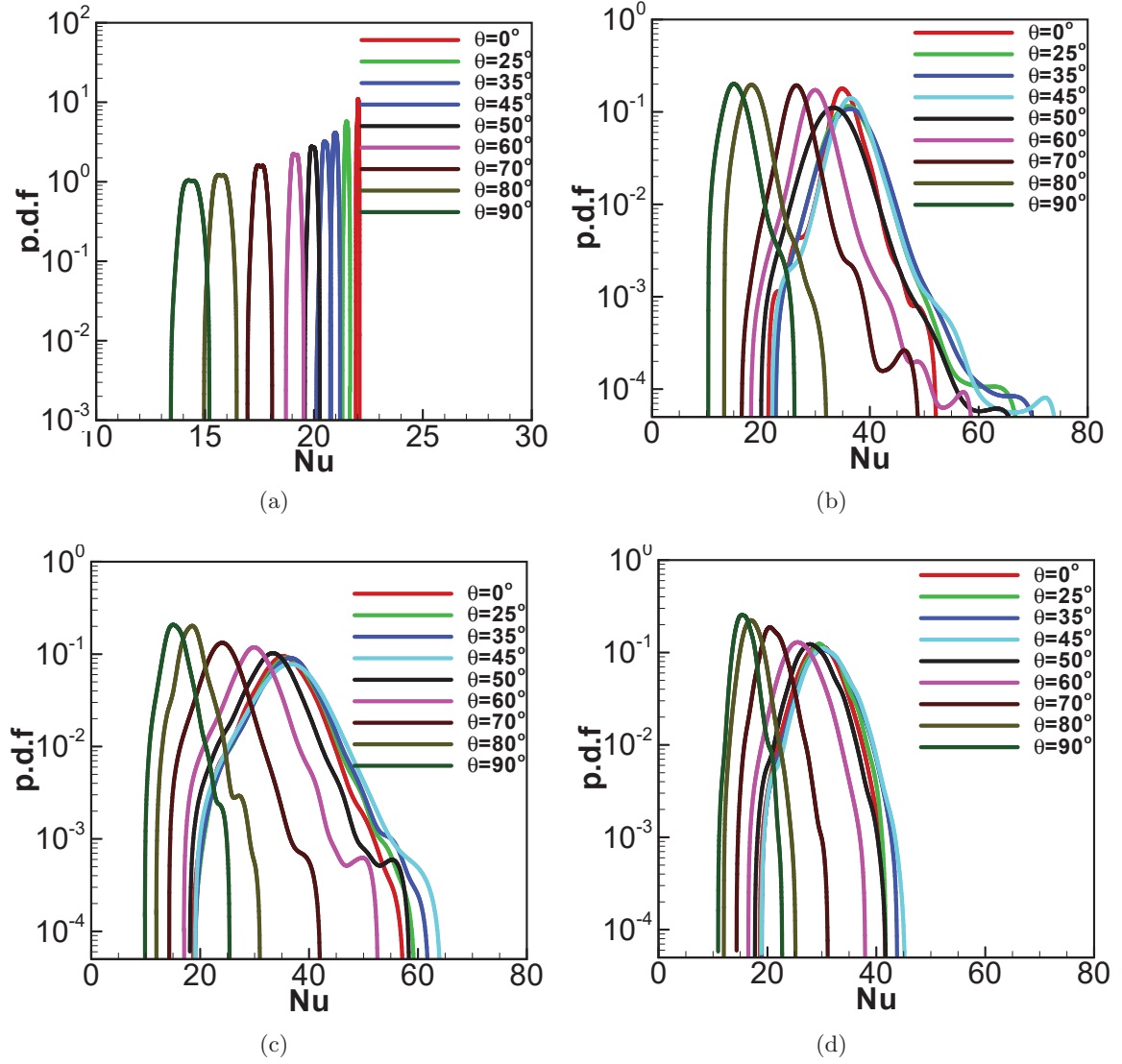


Figure 5.18: PDFs of Nusselt number at different locations along the circumference of the cylinder in the plane $z/D=0$: (a) Cylinder, (b) production, (c) peak, (d) decay

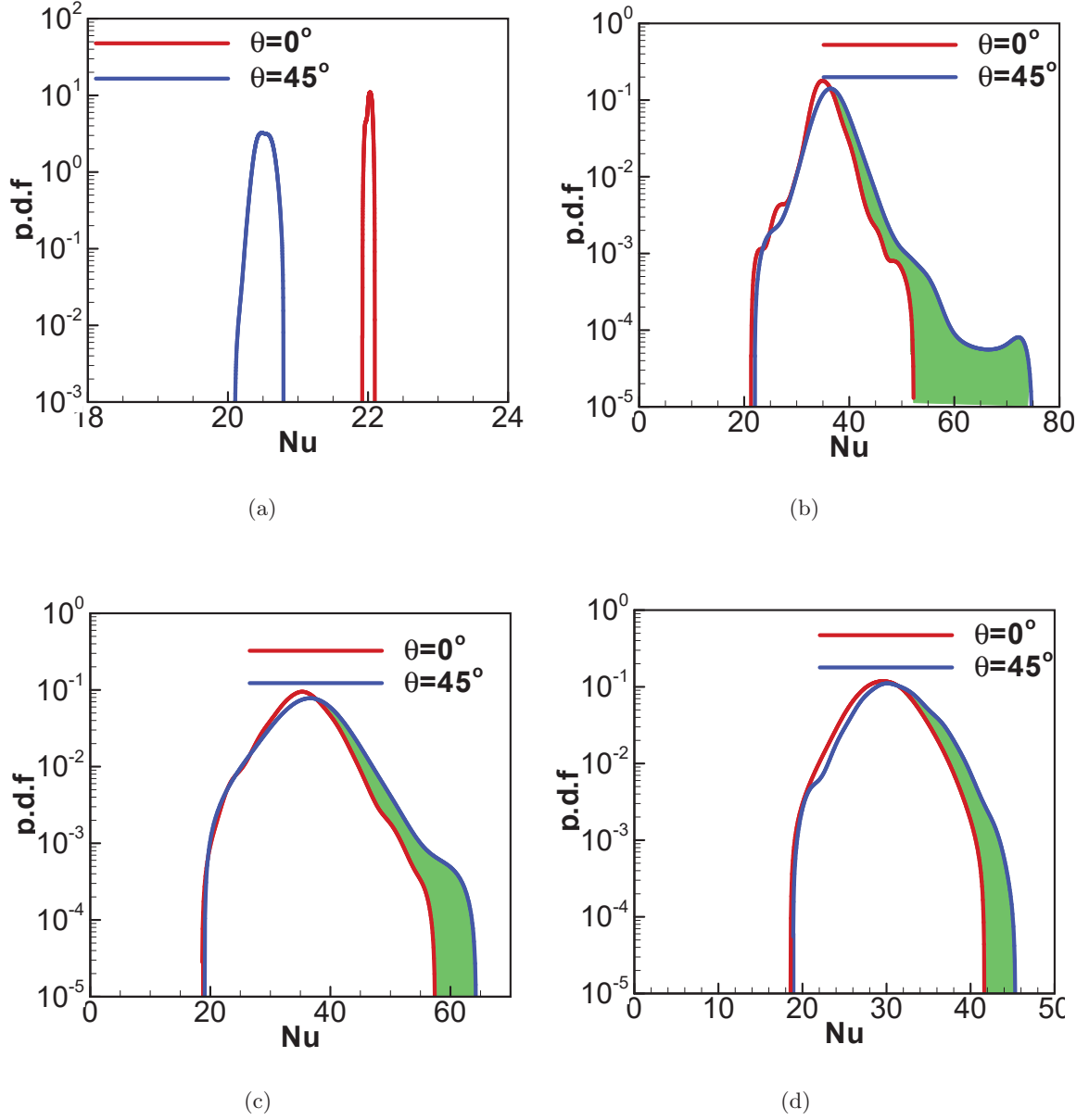


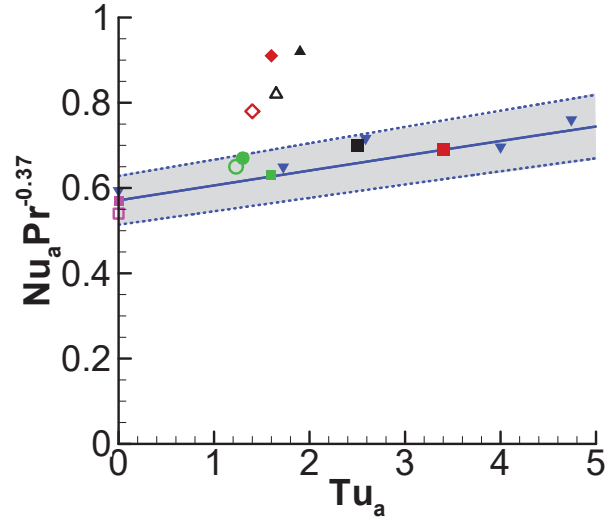
Figure 5.19: Illustration of additional intense events occurring at the $\langle Nu \rangle_{max}$ point in the plane $z/D=0$ for the case of: (a) Circular, (b) Production, (c) Peak, (d) Decay

figure 5.8). Therefore, it is believed that the increase in local mean incident flow and its turbulent intensity at $\theta \approx 45^\circ$ compared to the front stagnation point is related to the additional intense events which in turn shift the $\langle Nu \rangle_{max}$ by 45° from the stagnation point.

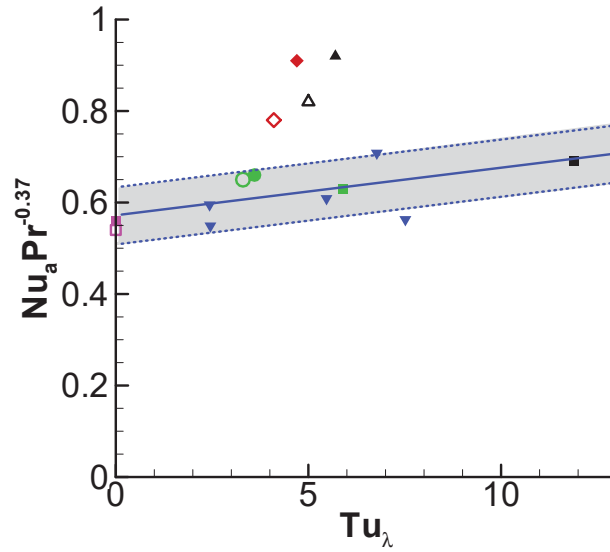
5.6.4 Validity of the existing stagnation-point heat transfer correlations

The validity of existing correlations for the stagnation point heat transfer coefficient is assessed in this subsection. These correlations are given in the introduction chapter of this thesis (refer to equations (1.31) and (1.32)) and they require the calculation of free-stream strain rate, ' a_1 '. The value of ' a_1 ' is difficult to measure experimentally, and most of the experimental studies assume the normalised free-stream straining $a_1=4$ which corresponds to potential flow. The procedure outlined in Venema et al. (2011) is followed in this study to determine the value of ' a_1 '. On the grid-element centreline, a_1 is defined as $a_1 = -\frac{\partial U}{\partial x} \frac{D}{U_{inc}}$. The variation of ' a_1 ' along the centreline reaches a plateau around $x_c/D=-1.5$ (much like the plateau observed for mean and rms velocity profiles), and the value of a_1 is chosen from this plateau to be 3, 3.2, 3.6, and 3.8 for the production, peak, decay, and circular cases respectively. With the help of a_1 , the values of Nu_a and Tu_a are calculated for our cases, and the values are checked against the correlation curve in figure 5.20a. As the computation of a_1 involves uncertainties, the correlation parameters are also computed using the potential flow constant $a_1=4$. The closed symbols represent the computed values of a_1 , while the open symbols represent the potential flow value. The cylinder case, which also forms the origin of the correlation curve, is well predicted. It is interesting, however, to note that the correlation curve is not valid for the production and peak cases, where the computed values are much higher than the correlation error bound. On the other hand, the decay case satisfies the correlation curve. One could presume that the exclusion of the turbulence length scale in equation (1.31) could be a reason why the production and peak cases do not satisfy this correlation. Therefore, the correlation proposed by Dullenkopf and Mayle (1995), which includes the effect of the integral length scale (refer to equation (1.32)), is also assessed.

The two sides of the correlation equation (1.32) are computed and compared against the correlation curve in figure 5.20b. This correlation is also observed to be invalid for the production and peak cases similar to the previous correlation. Therefore, the inclusion of length scale into the correlation equation does not change the scenario. This situation shows that the turbulence in the production and peak cases possess some unique characteristics which produce higher heat transfer compared to previous studies.



(a)



(b)

Figure 5.20: Assessment of (a) the correlation (1.31) of Dullenkopf and Mayle (1994) and (b) the correlation (1.32) of Dullenkopf and Mayle (1995). The square symbols represent data for the bar centreline, and they are differentiated in terms of their colour as Red - Production, Black - Peak, Green - Decay, and Pink - Cylinder. The filled inverted triangles are the literature data from Wissink and Rodi (2011a), Wissink and Rodi (2011b), Venema et al. (2011) and Venema et al. (2014). The other symbols represent the grid-element centreline correlations, and they are identified as, Triangle-Production, Diamond- Peak, Circle- Decay. The open symbols are the results obtained with the potential value of free-stream strain rate $a_1 = 4$, and the filled symbols are from the computed value of a_1 . The dotted lines represent the $\pm 5\%$ error bounds of the correlations.

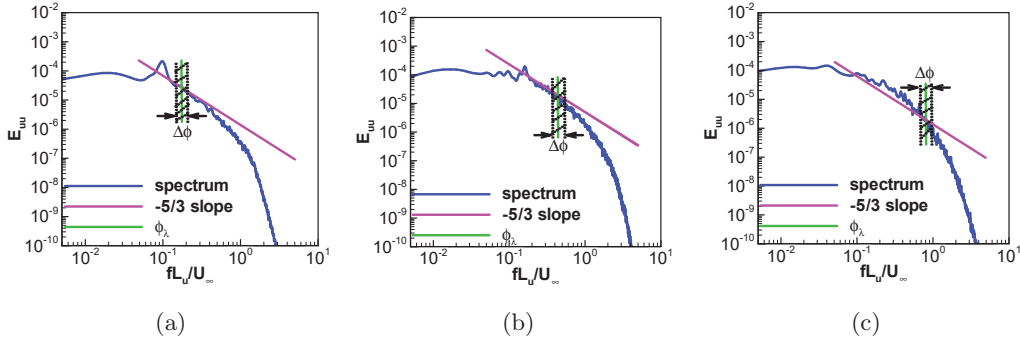


Figure 5.21: Energy Spectrum at $x_c/D=-1.5$ in the plane $z/D=0$: (a) Production, (b) peak, (c) decay

The reasons for this excessive heat transfer rate in the production and peak cases will be discussed in the next two sections. Note also that this is the first DNS study that reports some invalidity of the correlations (1.31) and (1.32). The previous numerical studies (Venema et al., 2011; Wissink and Rodi, 2011a,b) showed that the correlations curves are valid for the case of wake turbulence generated by a small cylinder placed upstream of a larger cylinder. The correlations are found to be valid only along the bar-centreline (see figures 5.20a and 5.20b filled square symbols). It is indeed intriguing to note that the presence of turbulence along the grid-element centreline exhibits unusual heat transfer results, while the bar-centreline exhibits results which are already found in the literature. This is the main reason, as mentioned at the beginning of this chapter, why this study focuses only on the $x - y$ plane which has the grid-element centreline (i.e. $z = 0$ plane).

5.7 Effect of the dominant frequency of fluctuations on heat transfer

This section investigates further why the stagnation point heat transfer correlations are not valid in the production and peak cases. The receptivity theory states that the laminar boundary layer is sensitive (or receptive) to the free-stream disturbances only in a selective band of frequencies. The representative frequency of such selective band is obtained using receptive theory as $f_\lambda = 0.03U_\infty/\delta$. Therefore, it is not the root mean square of the fluctuations, but it is the dominant frequency of the fluctuations that affects the response of the boundary layer to the free-stream turbulence. For stagnation flows, the boundary layer thickness is $\delta \approx 2.4\sqrt{\nu/a}$. Using this formula, Dullenkopf and Mayle (1995) formulated an equivalent of f_λ normalised by the integral length scale,

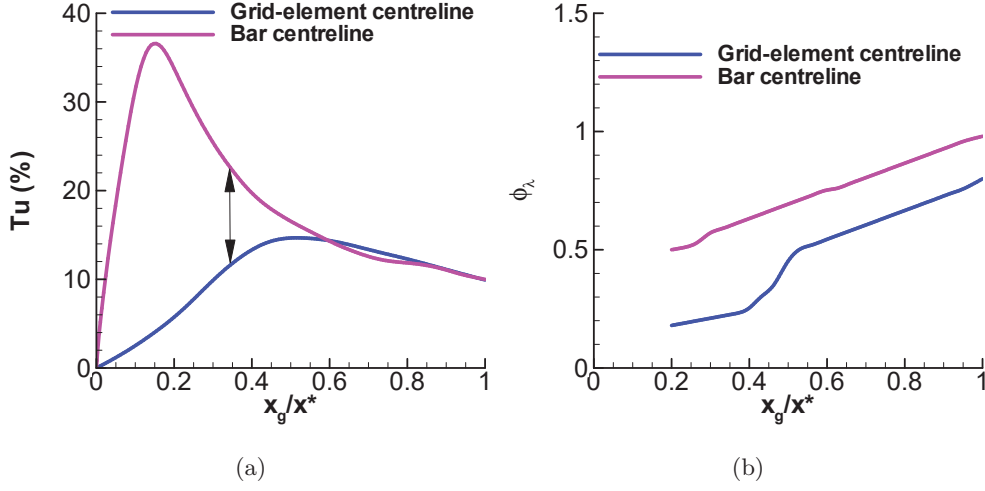


Figure 5.22: Variations of: (a) turbulent intensity and (b) dominant frequency of fluctuations along the grid-element and bar centrelines in the plane $z/D=0$.

called normalised dominant frequency of fluctuations, to which the boundary layer is most receptive, and is given as

$$\phi_\lambda = 0.008 \left(\frac{L_u}{D} \right) \sqrt{Re_{inc} a_1} \quad (5.2)$$

For our simulations ϕ_λ is computed and plotted in figure 5.21 along with the spectra of velocity fluctuations. The selective bandwidth of frequencies ($\Delta\phi$) to which the boundary layer is receptive is also shown indicatively in the figure. This frequency band is constructed with the constraint $\Delta\phi/\phi_\lambda \ll 1$ as proposed by Dullenkopf and Mayle (1995). In this study, the ratio $\Delta\phi/\phi_\lambda$ is taken as 10^{-3} . As seen in figure 5.21, the dominant frequency lies very close to the shedding frequency that has higher energy content for the production case (figure 5.21a) while it lies in higher frequencies that have lower energy (figure 5.21c) for the decay case. This means that although the rms of the velocity fluctuations is similar, the production case disturbances that impinge on the cylinder surface are far more effective in making the boundary layer to react to the disturbances than the oncoming fluctuations in the decay case. As a result, the production region has the largest heat transfer rate compared to the decay case.

The reason behind the appearance of $\langle Nu \rangle_{max}$ along the grid-element centreline rather than the bar centreline in figure 5.17 can be explained in terms of the dominant frequency of fluctuations. The turbulence intensity for the production region bar centreline (i.e at $x_g/x^*=0.35$ along the bar-centreline) is 22%, while it is only 11% along the grid-element centreline, (see figure 5.22a). Yet, figure 5.17b clearly shows that the

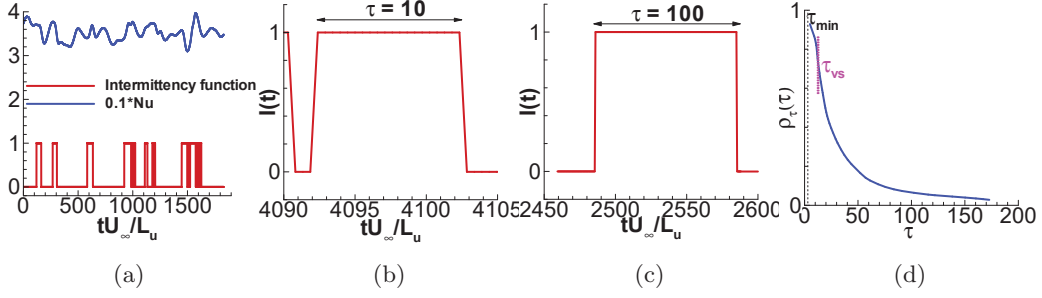


Figure 5.23: (a) A typical segment of intermittency function signal with its corresponding instantaneous Nusselt number. The Nusselt number signal is plotted for $x_c/D=-0.5$ (i.e. at the surface of the cylinder) while the intermittency function signal is plotted for $x_c/D=-1.5$, (b) An example of a shedding event, (c) An example of a non-shedding event, (d) Probability of timescales of turbulent events.

maximum heat transfer occurs only in the grid-element centreline. It is observed that, along the bar centreline, the turbulent intensity of the production region is at least twice to that of the grid-element centreline, yet the dominant frequency of this turbulence is high with low energy, and thus not having any effect on the heat transfer (see figures 5.22a and 5.22b). On the other hand, although the turbulence intensities along the grid-element centreline are similar compared to that of the bar-centreline for the peak and decay cases, their dominant frequency is lower with very high energy content. This evidence proves that the most important parameter for heat transfer increase is the dominant frequency of fluctuations rather than the turbulent intensity.

5.8 Timescale separation analysis and mechanisms of heat transfer enhancement

In this section, a novel approach is used to provide more insight about the nature of the flow that causes higher heat transfer rate for the production case. A timescale analysis is performed at $x_c/D=-1.5$ (i.e at the edge of the laminar boundary layer) as follows. The turbulent flow events are identified at that point through the intermittency function, $I(t)$, which is defined as

$$I(t) = \begin{cases} 0, & \text{if } |\omega^*(t)| < |\omega_{th}^*| \\ 1, & \text{otherwise} \end{cases} \quad (5.3)$$

where $I(t)=0$ means that the flow can be considered irrotational while for $I(t)=1$ as turbulent. We choose $|\omega_{th}^*|=0.2$ as determined in section 5.4.3. A typical intermittency function and its corresponding Nusselt number signal is shown in figure 5.23a for the production case. Note that, while the intermittency function is computed at $x_c/D=-1.5$,

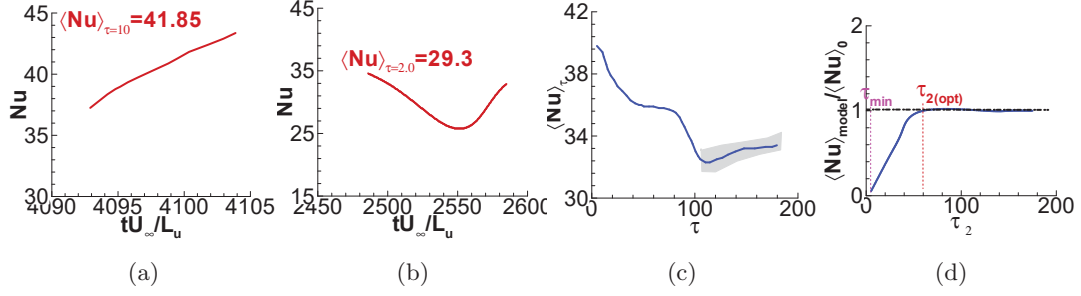


Figure 5.24: (a) Variation of instantaneous Nusselt number in a shedding event, (b) Variation of instantaneous Nusselt number in a non-shedding event, (c) Variation of conditional mean Nusselt number with respect to the time-scales of turbulent events, (d) Calculation of optimum τ_2 .

the Nu signal is computed at the front stagnation point ($x_c/D = -0.5$). Therefore, the time-lag in the Nu signal is appropriately adjusted. As can be seen in figure 5.23a, the flow is highly intermittent, and the turbulent flow has a wide range of timescale events ($\tau = \Delta t U_\infty / L_u$) ranging from short to long. Typical short and long timescale events are shown in figures 5.23b and 5.23c respectively. Computing the probability of the timescales ($\rho_\tau(\tau)$) for the entire signal reveals that the flow is dominated by short timescale events (figure 5.23d). The smallest timescale event observed in our data set is represented as τ_{min} . The value of τ_{min} depends on the data set and the total simulation time. As the current simulation has been run for sufficiently long time (as proven from the well-converged statistics presented earlier), it is expected that the turbulent events of timescale $\tau < \tau_{min}$ could occur very rarely. Therefore, it is surmised that the probability curve in figure 5.23d will have a decay from τ_{min} to 0. The timescale of the short events, as noted in figure 5.23d, is comparable to the timescale of vortex shedding ($\tau_{vs} = 1/St_{L_u}$), where St_{L_u} is the Strouhal number based on the integral length scale ($St_{L_u} = f_{vs} L_u / U_\infty$, see figure 5.21). Therefore, the short timescale events are called shedding events, while the long timescale events are called non-shedding events. In this way, the energetic shedding events are considered as an equivalent to the dominant frequency of fluctuations. It must be stressed here that the Strouhal frequency in figure 5.21 is the frequency of the flow itself, while the frequency in figure 5.23d is only the representative frequency of a turbulent event.

Typical variations of instantaneous Nu for the shedding and non-shedding events are depicted in figures 5.24a and 5.24b respectively. The time averaged Nusselt number conditioned for every timescale event τ , $\langle Nu \rangle_\tau$, is given in figure 5.24c. The shaded area in this figure represents the lack of statistical convergence in $\langle Nu \rangle_\tau$ as only very few non-shedding events occur in the production case. Yet, the main interpretation from

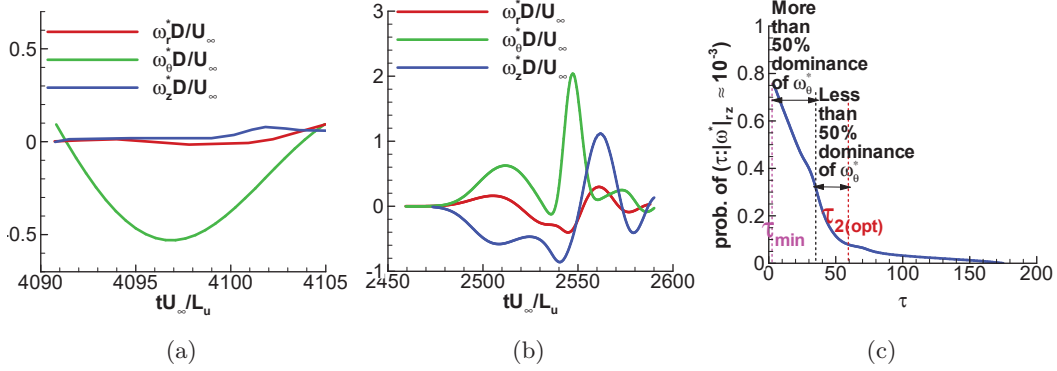


Figure 5.25: (a) Variation of vorticity components in a shedding event, (b) Variation of vorticity components in a non-shedding event, (c) Conditional probability of $|\omega^*|_{rz} \approx 10^{-3}$.

this figure is that the largest heat transfer occurs at the shedding events, and the value of $\langle Nu \rangle_\tau$ decreases for increasing τ . This signifies the importance of shedding events for higher heat transfer rates. These shedding events, as noted earlier, are related to the dominant frequency of turbulent fluctuations (ϕ_λ).

Based on the DNS results of this study, a simple model for the mean Nusselt number is proposed. It is given as,

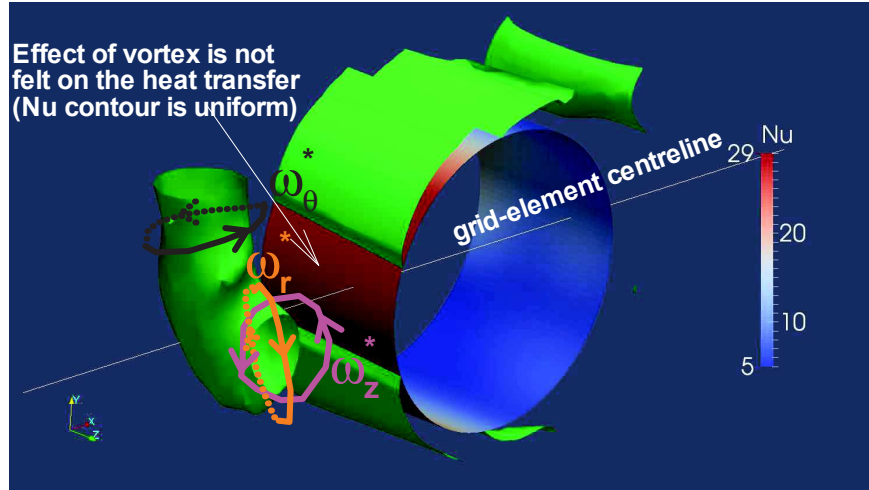
$$\langle Nu \rangle_{model} = \int_{\tau_{min}}^{\tau_2} \rho_\tau(\tau) \langle Nu \rangle_\tau d\tau \quad (5.4)$$

Mathematically, equation (5.4) should be integrated from 0 to infinity. For intermittent flows, however, it is integrated from τ_{min} to τ_2 . The optimum value of τ_2 , $\tau_{2(opt)}$, is determined by integrating equation (5.4) for various values of τ_2 and $\tau_{2(opt)}$ is chosen to be the value of τ_2 for which the $\langle Nu \rangle_{model}$ becomes equal to the value computed by the simulations (see figure 5.24d). The timescale range of $\tau_{min} < \tau < \tau_{2(opt)}$ is termed the effective timescale range for heat transfer. Note also that equation (5.4) is applicable only for production and peak cases as there is no proper way to compute $|\omega_{th}^*|$ for fully turbulent decay region.

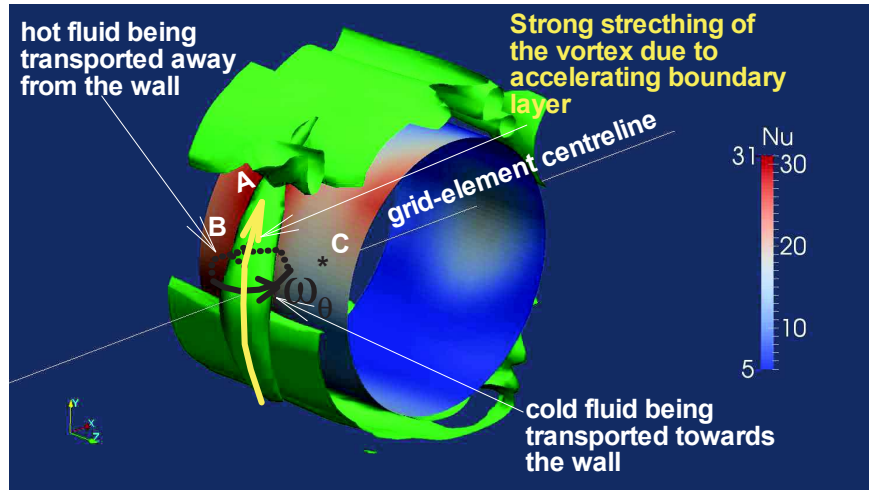
The shedding and non-shedding events are further analysed using the instantaneous vorticity components decomposed with respect to the polar coordinates (much similar to the velocity field). The radial, azimuthal, and spanwise components of instantaneous vorticity are denoted as ω_r^* , ω_θ^* , and ω_z^* respectively. Variations of vorticity components in the shedding and non-shedding events are shown in figures 5.25a and 5.25b respectively. It is interesting to observe that the non-shedding event consists of all three components of vorticity while the shedding event is dominated by the azimuthal vorticity component

(ω_θ^*). This observation is further checked for all the timescale events by computing the conditional probability for the radial and spanwise vorticity components to be zero (i.e., $|\omega^*|_{rz} \approx 10^{-3}$, where, $|\omega^*|_{rz} = \sqrt{\omega_r^{*2} + \omega_z^{*2}}$), and the result is presented in figure 5.25c. This plot clearly shows that the probability of dominating azimuthal component is maximum for the shedding events, and this probability decreases for increasing timescales. It is known that the vortical structures generated due to the vertical bars are predominately dominated by the azimuthal component. Now it is surmised that, for the shedding event, these vortical structures move fast and impinge on the cylinder with the same predominance of azimuthal component. On the other hand, the predominance of azimuthal component is lost for the long non-shedding event. Therefore, we classify the shedding events as the ones that have the timescale which is closer to that of vortex shedding and contain more than 50% dominance of ω_θ^* compared with other two vorticity components. On the other hand, the non-shedding events are classified as the ones having large timescales and have less than 50% dominance of ω_θ^* compared with ω_r^* and ω_z^* . Figure 5.25c also shows that more than 75% of the effective timescale range (i.e. $\tau_{min} < \tau < \tau_{2(opt)}$) is occupied by azimuthally dominated events (which are also happen to be the shedding events). At the same time, there is also some contribution to the effective timescale range from the non-shedding events although the probability of such events is lesser compared with the shedding events (see figure 5.23d).

Although it is now clear that much of the heat transfer in the production case is due to azimuthal vortical flow, it is still not clear as to how exactly the azimuthal vorticity domination correlates to the higher heat transfer rate observed for the production case. Analysing the way the vortical structures transfer heat from the cylinder surface sheds light on this question. As mentioned earlier, the heat transfer in the production case is mostly due to shedding events with small contribution from the non-shedding events. The heat transfer mechanism for the non-shedding events is addressed first as this mechanism is similar to the one found in the literature. It is known that the vortical structures for the non-shedding event will have all three components. A typical vortical structure approaching the cylinder is shown for the non-shedding event in figure 5.26a. This particular vortical structure does not leave any impact on the cylinder surface even when it comes closer to the cylinder (or before it is being stretched by the mean accelerating flow) as the Nu contour on the wall is almost uniform in figure 5.26a. Once this vortical structure is stretched along the cylinder surface by the action of the accelerating boundary layer, the effect is felt on the surface of the cylinder with largest Nu in the location where the vortex lies (denoted as ‘A’ in figure 5.26b), and lesser Nu in other locations (points ‘B’ and ‘C’ in figure 5.26b). Therefore, in the case of non-shedding events, the oncoming vortex does not contribute much to heat transfer unless



(a)



(b)

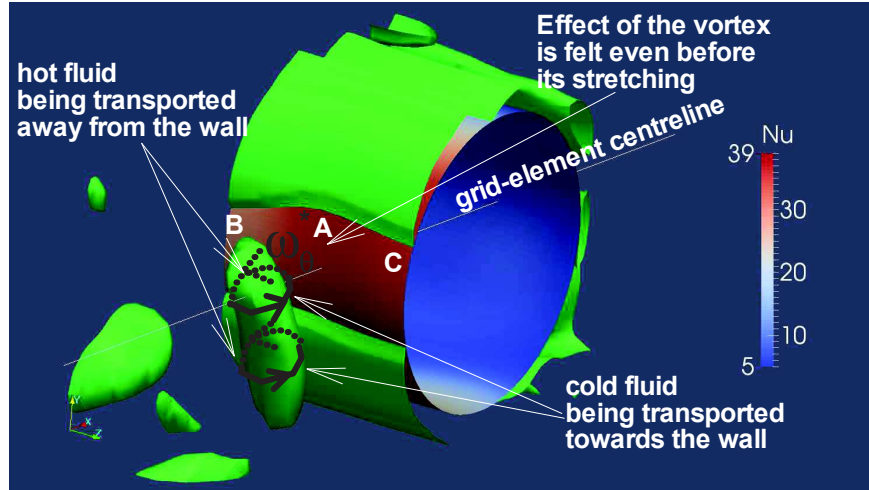
Figure 5.26: Typical vortical structures of a non-shedding event: (a) While approaching the cylinder (b) After being stretched by the mean boundary layer. The 3D region plotted in this figure has the dimensions of $-1 \leq y/D \leq 1$, $-1 \leq z/D \leq 1$ and $-1 \leq x_c/D \leq 1$

it is stretched by the accelerating boundary layer. Note also that the vortex is stretched in the azimuthal direction causing an increase in azimuthal vorticity while reducing the other two components. Due to this, the colder fluid is transported towards the cylinder, and the hot fluid from the cylinder is transported away from the cylinder. The aforementioned stretching mechanism by the boundary layer is similar to the hypothesis proposed by Hunt (1973) based on rapid distortion theory. The observed heat transfer mechanism is similar to the one observed in the simulations of Wissink and Rodi (2006); Xiong and Lele (2007); Bhaskaran and Lele (2010).

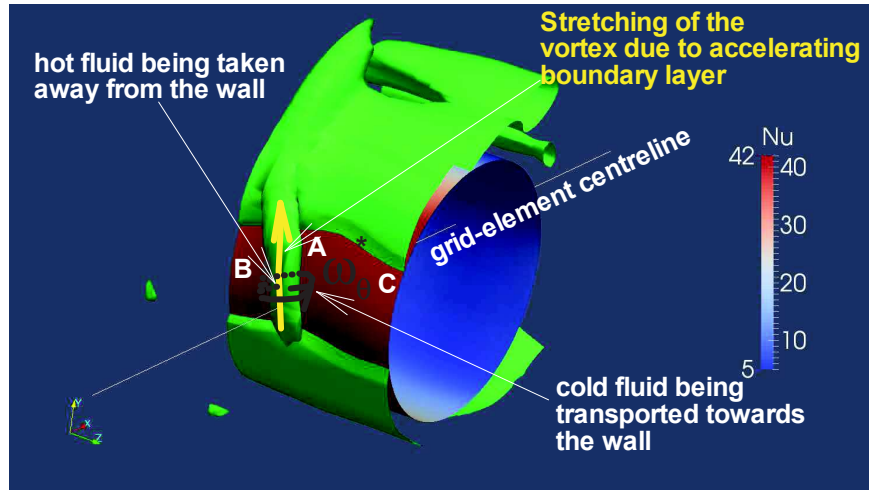
Attention is now turned to the heat transfer mechanism for the shedding event which is mostly responsible for heat transfer in the production case. A typical vortical structure of the shedding event approaching the cylinder is depicted in figure 5.27a. This plot reveals an interesting result. The heat transfer on the cylinder is clearly affected by the vortex even before the oncoming vortex is stretched by the accelerating boundary layer. This can be seen in the Nu contour on the cylinder wall in figure 5.27a as the contour is not uniform with the highest values aligning with the vortex direction (point ‘A’ in figure 5.27a) and lesser values on the sides of the vortex (points ‘B’ and ‘C’ in figure 5.27a). This can be attributed to the fact that the azimuthal vortex transports fluid perpendicular to the cylinder surface causing an effective heat transfer from the cylinder. The effectiveness of this vortex is further increased when the mean accelerating flow acts on it by means of stretching along the cylinder wall surface as seen in figure 5.27a. This stretching increases further the azimuthal vortical strength causing highest heat transfer in the places where the vortex is aligned (point ‘A’ in figure 5.27b). This heat transfer mechanism is different to the previously noted mechanism for the non-shedding event as the approaching vortex contributes to heat transfer even before it gets stretched.

Therefore, the shedding events, which consist of dominant azimuthal vortical component, increase heat transfer even without the help of the mean accelerating flow. In this way, turbulence in the production case is unique: firstly the dominant frequency of turbulence is more energetic, and the flow is dominated by shedding events which contain mostly the azimuthal vorticity components that are very effective in heat transfer. These characteristics are the main reasons as to why the production case has much higher heat transfer rate than the decay case although the oncoming turbulent intensities are similar.

Although the discussion and the conclusions made in this section are based on the production case, it can be easily extended to the peak case. The intermittency level decreases for the peak case, and thus the number of shedding events are less when performing the timescale separation analysis. Therefore, both the heat transfer mechanisms of shedding and non-shedding events play a role in heat transfer enhancement for the



(a)



(b)

Figure 5.27: Typical vortical structures of a shedding event: (a) While approaching the cylinder (b) After being stretched by the mean boundary layer. The 3D region plotted in this figure has the dimensions of $-1 \leq y/D \leq 1$, $-1 \leq z/D \leq 1$ and $-1 \leq x_c/D \leq 1$

peak case. Here, the peak case exhibits higher heat transfer because of the dominance of azimuthal vorticity component associated with the shedding events combined with higher turbulent intensity. On the other hand, the timescale separation analysis is not applicable to the decay case as it is fully turbulent and there is no proper way to determine $|\omega_{th}^*|$. However, the heat transfer mechanism discussed for the non-shedding event is valid for the decay case. This is the same mechanism also reported in the literature.

5.9 Comparison with high Reynolds number experiments

Finally, the observations of this DNS study are compared with an experimental study carried by Melina et al. (2016b), and two significant differences are noted. The first difference is that the decay region of the experiment was observed to have higher heat transfer than the production region although the locations examined in both the regions had approximately similar turbulent intensity. Secondly, the maximum heat transfer coefficient was observed at the front stagnation point unlike the shift of approximately 45° observed in this DNS study. These two differences can be answered through one simple reason as discussed below.

Concerning why the experiments predict higher heat transfer for the decay case than the production case, equation (5.2) predicts that the value of the dominant frequency will be higher for large Re values. It appears that in the high Reynolds numbers experiment (Melina et al., 2016b), the dominant frequency is a higher frequency with lower energy as ϕ_λ is directly proportional to Reynolds number (refer to equation (5.2)). Therefore, the characteristics of turbulence in the production case of the experiments are not as effective as compared to the present low Reynolds number DNS, and that is why the experimental production case exhibits lower heat transfer than the decay case. Such an observation can be noted even in the present simulations as discussed in section 5.7.

The second difference can also be explained based on the dominant frequency concept and figures 5.17. Notice in figure 5.17a that the shift in $\langle Nu \rangle_{max}$ is more prominent only along the grid-element centreline where the dominant frequency of the fluctuations are more energetic. Along the bar-centreline, where the oncoming turbulent intensity is much higher, the shift in $\langle Nu \rangle_{max}$ is less prominent. Turbulent intensity increases along the azimuthal direction from the front stagnation point to a certain angle due to the presence of the grid-element bars for the production case (see figure 5.8). Due to higher values of turbulent intensity along with an energetic dominant frequency, $\langle Nu \rangle_{max}$ is shifted by 45° from the front stagnation point. On the other hand, in the experiments, although the turbulent intensity increases along the azimuthal direction from the front stagnation point, this increase becomes meaningless as the laminar boundary layer does

not react to this increase as the oncoming turbulence having a low energetic content. Due to this, the experiments (as well the bar-centreline of the simulations) will have a near-constant or a slow decay of $\langle Nu \rangle$ from the front stagnation point to 45° .

5.10 Summary

This DNS study, the first of its kind, explored the effect of different grid-element turbulence regions on heat transfer from a circular cylinder. A single square grid-element is used to generate the oncoming turbulence upstream of the cylinder. Due to fine-scale resolution requirements for the grid-element and the cylinder, the simulations are carried out for a transitional Reynolds number. Three distinct regions of grid-element turbulence are considered. The turbulence intensities of the production and the decay regions are almost equal to 11%, while the peak location has the highest turbulent intensity of 15%. A baseline simulation of heat transfer from the cylinder without oncoming turbulence is also simulated. The turbulent flow in the production and peak locations is highly intermittent, non-Gaussian and inhomogeneous, while it is Gaussian, homogeneous, and fully-turbulent in the decay region. Although the oncoming turbulent intensities are similar, the production region increases the stagnation point heat transfer by 63% while the decay region only by 28%. The existing correlations for the stagnation point heat transfer coefficient are found to be invalid for the production and peak locations, while the decay region satisfies the correlations. It is detected that the production and peak regions are dominated by shedding events where the predominant vorticity component is the azimuthal one which increases the heat transfer even before it gets stretched by the accelerating boundary layer. Apart from this, the dominant frequency of turbulent fluctuations in these regions lies closer to the vortex shedding frequency, thus making the laminar boundary layer to react effectively to the impinging disturbances. The highest Nusselt number point along the circumference of the cylinder is shifted by 45° from the front stagnation point. This shift is due to the turbulence-generating grid-element bars that result in a prevalence of intense events at the maximum Nusselt number point compared to the stagnation point.

Chapter 6

Conclusions and future work

6.1 Overview

This work is a part of the MULTISOLVE project that investigated various aspects of multi-scale turbulence through 13 PhD topics. The current work is oriented towards investigating fluid and scalar fields of the grid-element turbulence and its role in increasing heat transfer from a cylinder. Therefore, this thesis considered three major topics for a spatially developing flow. The topics considered were: (i) fluctuating velocity gradients in fluid turbulence, (ii) fluctuating temperature gradients in scalar turbulence and (iii) the effect of free-stream turbulence on laminar heat transfer. A single square grid-element was used to generate the developing flow. The scalar field was generated by heating the grid-element. A series of questions were raised in chapter 1 for each topics. These research questions were formulated in-line with the objectives of the MULTISOLVE project. This thesis used a finite-volume-based DNS in order to answer these questions. The answers to them are presented in this final chapter along with the scope for future for each of these topics.

6.2 Velocity gradients

Chapter 3 investigated the genesis and evolution of velocity gradients for a spatially developing flow generated by a single square grid-element using DNS. The flow along the grid-element centreline is initially irrotational (i.e. non-turbulent), but it becomes turbulent later due to the infiltration of the turbulent vortical wakes from the grid-element bars. The dynamics of antisymmetric and symmetric parts of the velocity gradient tensor have been discussed for this developing flow using the transport equations of mean turbulent enstrophy and strain-product. As these transport equations contain third-order moments of velocity gradients, the simulation has been run for a relatively

low inlet Reynolds number to make sure these equations are well balanced. The objective of VGT analysis is to answer the questions raised in section 1.2.4. The answers are given below.

1. *The high Reynolds number experimental studies of Laizet et al. (2013, 2015b); Gomes-Fernandes et al. (2015) for turbulence generated by a fractal grid have shown that the $-5/3$ power-law slope appears even in the near-grid inhomogeneous part of the grid-centreline region. Does this observation also hold true for relatively low Reynolds number turbulence generated by a square grid-element?*

In agreement with the previous studies on fractal grid turbulence where the Reynolds number was significantly higher (Laizet et al., 2013, 2015b; Gomes-Fernandes et al., 2015), the turbulence energy spectrum of the current study exhibits its best defined $-5/3$ power law shape midway between the grid-element and the turbulence peak location on the grid-element centreline. At this point, the value of Re_λ is just about 20 or so, yet the $-5/3$ slope in the energy spectrum is observed for about half a decade of frequencies. The higher Reynolds number experiments and simulations of Gomes-Fernandes et al. (2015) and Laizet et al. (2015b) report a $-5/3$ power law-shaped energy spectrum over more than a decade of frequency at the same location. This location is at the heart of the production region where the turbulence is highly inhomogeneous and non-Gaussian as seen, for example, from the contours of mean streamwise velocity (figure 3.3c) and the skewness values of fluctuating streamwise velocities (figure 3.7b).

2. *If there is a $-5/3$ power-law slope in the energy spectrum for the near grid-element region, how do the small-scale terms like strain and vorticity correlate to the $-5/3$ power-law slope?*

The best $-5/3$ power-law slope of the energy spectrum in this study is observed at a location where the small-scale terms such as the mean enstrophy and mean strain-product have just started developing. It is also the location where the strain dominates vorticity, and the importance of strain self-amplification process is greater than the vortex stretching. At this very location, the alignment between the vorticity vector and the strain-rate eigenvectors are found to be anomalous with the vorticity vector aligning with both the extensive and intermediate strain-rate eigenvectors. This unusual alignment behaviour is believed to be the effect of dominance of non-local strain which promotes alignment between the vorticity vector and the extensive strain-rate eigenvector. As the small-scale terms started developing, and vortex stretching started to dominate strain self-amplification, the $-5/3$ slope progressively eroded although the alignment between the vorticity vector and the

strain-rate eigenvector starts showing universal characteristics. These observations make this study to conclude that the most important process associated with the energy cascade is the strain self-amplification, not the vortex stretching.

3. *How are the large and small scale terms of fluid turbulence generated and transported in this spatially developing flow?*

The turbulent kinetic energy is produced by the action of mean velocity gradients and the pressure work behind the bar. It is then transported to the grid-element centreline due to fluctuations.

For the small-scale terms, along the grid-element centreline, the fluctuating strain is created initially by actions of pressure-Hessian and is then transported by turbulent fluctuations. Self-production of strain starts very closely after that (figure 3.13). The end of this strain-dominated region is marked by excursions of vorticity from the sides which is brought towards the grid-element centreline from the wakes of the bars by turbulent fluctuations. The already existing fluctuating strain starts to act on this vorticity to cause the production of enstrophy through vortex stretching (figure 3.12b). Note that terms involving interactions between mean and fluctuating quantities are negligible throughout the grid-element centreline even though the turbulence is highly inhomogeneous in the production region. A novel observation, not reported in previous works, is that the strain field has a statistically axial stretching form in the production region which, within a short space slightly upstream of the turbulence peak, transforms into a statistically biaxial stretching form in the decay region.

4. *Since this thesis considers the single square element of a fractal square grid, it is natural to ask how does the velocity-gradient dynamics of grid-element differs from the VGT dynamics reported for a fractal square grid in Gomes-Fernandes et al. (2014).*

The characteristics of small scale turbulence generated by the single square grid-element at relatively low inlet Reynolds number are similar to those of a fractal grid at high Reynolds number only in the decay region. Important differences are detected in the production region. Firstly, the stretching/compressing events are much stronger in the fractal grid due to the presence of small bars close to the grid-element centreline that create small scale turbulence. This leads to differences in the PDFs of the intermediate eigenvalue of the strain-rate tensor and the associated term that quantifies the contribution of this eigenvalue to the enstrophy production. This difference is also reflected in the JPDF between Q and R . Secondly, the alignment characteristics of the vorticity vector with the strain-rate eigenvectors

are different due to the prevalence of irrotational flow strain in the quite extended strain-dominated subregion of the production region, a feature which is absent in the experiments of Gomes-Fernandes et al. (2014). These differences are not just because of the differences in Reynolds number and grid geometry, but also due to the experimental setup which had non-negligible incoming turbulent intensity.

5. *The universal characteristics of VGT dynamics discussed in section 1.2.2 are largely reported for very high values of Re_λ . Do these universal characteristics really require high Reynolds number?*

Some of the universal turbulent flow characteristics, such as: (i) the statistical behaviours of enstrophy production, strain self-amplification, and strain-rate eigenvalues, (ii) the shapes of $Q - R$ and $Q_s - R_s$ diagrams, and (iii) the geometrical alignments, that are recorded for very high Reynolds number classical grid experiments and the other studies of isotropic homogeneous turbulence are also detected in the far-downstream decay region of the square grid-element where the flow is fully turbulent and the value of Re_λ is as low as 33.

6.3 Scalar Gradients

The second major topic of this thesis was the statistical analysis of a passive scalar injected into a spatially developing turbulence. The turbulence was generated by a single square grid-element, and a passive scalar in the form of temperature was injected into the wakes of the grid-element through heating the grid-element continuously. To the best of the author's knowledge, the current analysis of a passive scalar is the first study on fluctuating scalar gradients for turbulence generated by a grid-element. The main objectives of this study were to find answers to the six questions raised in section 1.3.4. The answers to those questions are given below:

1. *How are the large scale quantities of scalar turbulence (such as the mean scalar, scalar-variance and turbulent heat-fluxes) and the small scale quantity of scalar turbulence (i.e. scalar-variance dissipation) generated and transported in this spatially developing turbulence?*

In this developing turbulence, the large scales scalar quantities (such as turbulent heat-fluxes, scalar-variance) are generated mainly behind the bars due to the action of mean scalar gradients. They are then laterally transported to the grid-element centreline through turbulent transport in the form of intermittent meeting of the bar wakes. Both the velocity and scalar mean fields have minimal effect on the production of small scale term (i.e. scalar-gradient-variance), even behind the

grid-element bars. The small scale quantities are produced by a linear stretching/compressing process by turbulent strain rate. This study has shown that the production of scalar small scale quantity is exclusively due to compressive action of strain-rate, while it is the stretching action that produces small scale quantities of velocity fluctuation. In particular, the current work established that the compressive action of strain-rate for the production of scalar gradient is mainly in the form of alignments rather than the strain-rate eigenvalues in the near-grid-element inhomogeneous region.

2. *What is the relationship between enstrophy and scalar-variance dissipation downstream of the grid-element?*

Although the evolution of dissipation of scalar-variance is similar to that of the turbulent kinetic energy along the bar-centreline, these dissipations behave differently along the grid-element centreline. Through a detailed analysis, it is found that the change in alignment behaviour between the scalar gradient vector and the vorticity vector with the strain-rate eigenvectors is the main reason why the scalar-variance dissipation decays much earlier along the grid-element centreline than that of the turbulent kinetic energy dissipation.

3. *Do the small scales of scalar reach isotropy in this developing turbulence?*

The small scales of velocity fluctuation exhibit local isotropy even in the near-grid-element region where the turbulence is highly intermittent. On the other hand, the small scales of scalar are observed to be anisotropic throughout the grid-element centreline. External intermittency in the form of bursts in the scalar signal is believed to cause very high values of scalar derivative skewness for the near-grid-element region. Surprisingly, the large scale characteristic ramp-cliff structures are identified in the homogeneous decay region, and these ramp-cliff structures cause nonzero skewness of scalar derivatives in the far-downstream region.

4. *Does the $-5/3$ power-law slope of the scalar spectrum appear in the inhomogeneous near grid-element region where the KOC theory is not valid?*

The scalar spectrum is observed to exhibit the best $-5/3$ power-law slope in the near-grid-element region. The novelty of this observation lies with the fact that, it is shown for the first time that the $-5/3$ slope can occur even in the highly-intermittent, non-Gaussian and inhomogeneous regions where the local Re_λ and Pe are significantly low.

5. *If indeed there is a power-law in the scalar spectrum in the near grid-element region, how does the small-scale scalar dynamics correlate with the scalar cascade?*

The best $-5/3$ slope in the scalar spectrum is observed in the regions where the small scale terms of scalar have just started developing. The scalar spectrum has a well-defined $-5/3$ power-law slope for at least a decade of frequencies in the locations where the strength of stretching/compressing of scalar gradient by strain-rate is weak. On the other hand, in the locations where the stretching/compressing strength is high, the frequency range on which the $-5/3$ power is defined becomes smaller. This is contrary to the general belief that the linear stretching/compressing process of scalar gradients by strain-rate, which is equivalent to the vortex stretching process in velocity field, is the most important process associated with the scalar cascade.

6. *How do the scalar spectrum and the scalar gradient dynamics evolve from the near grid-element region to the far-downstream?*

Moving from the near-grid-element centreline region to the far-downstream, it is noted that: (a) the $-5/3$ slope in the scalar spectrum slowly erodes, (b) the scalar turbulence becomes more *Gaussian* and homogeneous, (c) the action of strain-rate eigenvalues on the production of scalar gradients becomes more prominent, and (d) the alignment between the scalar gradient vector and the strain-rate eigenvector morphs to the universal alignment behaviour reported in the literature.

6.4 Laminar heat transfer

The final major topic of this thesis was devoted to the investigation of how different regions of a grid-element turbulence affect heat transfer from a circular cylinder. Unlike the previous studies, the current work had a single square grid-element to generate turbulence upstream of the cylinder. Here, due to the fine resolution requirements for the grid-element and the cylinder, the simulations were carried out for a transitional Reynolds number regime. The cylinder was placed at three distinct locations downstream to the grid-element namely the production, peak, and decay regions. In order to make meaningful comparisons between these regions, the turbulent intensity of the production and decay regions were kept similar. The peak has the highest turbulent intensity of 15%, while the production and peak regions have a turbulent intensity of 11%. The oncoming turbulence in the production, and peak regions was identified to have the following characteristics: (i) non-*Gaussian*, (ii) non-homogeneous, and (iii) intermittent. On the other hand, the turbulence in the decay region contained the following characteristics: (i) *Gaussian*, (ii) homogeneous, (iii) fully turbulent. The questions raised for the laminar heat transfer analysis in section 1.4.2 are answered below:

1. *What is the effect of different regions such as the production, peak and decay regions of grid-element turbulence on the heat transfer from a cylinder?*

The ability to increase the heat transfer is the greatest in the close-to-the-grid-element region where the turbulence is inhomogeneous and developing. As the turbulence becomes more homogeneous and developed further downstream, it loses its effectiveness of increasing the heat transfer.

2. *When the turbulent intensities of the inhomogeneous production and the homogeneous decay regions are similar, do they also increase heat transfer by the same amount?*

It was found that the production region has the highest heat transfer increase even though its turbulent intensity is the same as that of the decay region where heat transfer increase was much less compared to the production region. A systematic analysis was carried out to find the reasons as to why the production and peak turbulence cases have indifferent heat transfer increase compared to the previously reported studies. It is found that the turbulence in the production and peak regions is unique, with their dominant frequency of fluctuations lying closer to the vortex shedding frequency. This makes the laminar boundary layer react sharply to the impinging disturbances. Besides this, this turbulence is also observed to be dominated by shedding events, a peculiar feature of which is that they have only the azimuthal component of vorticity, which starts increasing the heat transfer even before it is stretched by the mean accelerating boundary layer. These two unique characteristics results in very high increase of heat transfer.

3. *Are the existing correlations for the stagnation point heat transfer coefficient valid for this developing turbulence?*

It was found that the production and peak cases of the grid-centreline do not satisfy the existing correlations for the stagnation point heat transfer, but the grid-centreline decay case satisfies the correlation. On the other hand, the bar-centreline regions were found to be satisfying the existing correlations. Note that this is the first DNS study that reports some invalidity of such correlations.

4. *How does heat transfer enhancement happen when the oncoming turbulence characteristics is different from the one reported in the literature?*

The heat transfer enhancement mechanism for the inhomogeneous developing free-stream turbulence is found to be different to the one reported for the homogeneous free-stream turbulence in the literature. Here, the approaching vortex is found to increase heat transfer even before being stretched by the accelerating boundary layer. This is because, the inhomogeneous developing turbulence is dominated by

vortical flow where the predominant vorticity component is azimuthal. This azimuthal component does not need any help from the boundary layer to effectively take heat away from the cylinder.

6.5 Recommendations for future work

Although this thesis discussed the topics of velocity and gradient gradients, and the role of free-stream turbulence in heat transfer in context to the aims of the MULTISOLVE project, there are many interesting associated research directions remain unexplored. Of the topics considered in the current thesis, the following would be worthy of analysing further:

6.5.1 Velocity gradients

1. Since intermittency plays a vital role in the production region, the turbulent/non-turbulent (TNT) regions are expected to play an active role in the energy case. Building further upon section 5.4.3, it would be worth to analyse the behaviour of small-scale statistics at the TNT interfaces of the production region.
2. Helicity (which is the dot product of velocity and vorticity vectors) is a good measure of swirling structures. Application of helicity to the current study would help to understand the fluid structure when the flow transitions from potential to vortical along the grid-element centreline.
3. As mentioned in the thesis, the experimental study of Gomes-Fernandes et al. (2014) was conducted with some considerable free-stream turbulence. In order to make a more meaningful comparison with the experiment, it would be essential to have some free-stream fluctuations imposed on the inlet of the domain.
4. The best defined $-5/3$ slope of the energy spectra is observed in a place where the turbulence is highly inhomogeneous and intermittent. The other important phenomena associated with the $-5/3$ power-law is the interscale energy transfer and its association with the energy dissipation rate. In order to understand these phenomena in the near field, the Karman-Howarth-Monin-Hill (KMH) equation can be used as this equation is valid for inhomogeneous, anisotropic and unsteady turbulence.
5. The energy spectra discussed in section 3.5 showed the existence of $-9/3$ slope before the fluctuations are dissipated at small scales. This observation serves as a first evidence of the nonequilibrium state induced by fluctuating dissipation rate

- (Horiuti et al., 2016). However, it is not entirely clear as how they develop from the near grid-element to the downstream. A study similar to Horiuti et al. (2016) could perhaps answer the nonequilibrium dynamics of dissipation rate in energy spectra and their evolution downstream to the grid-element.
6. In order to understand better the reason behind the anomalous alignment of vorticity vector with the strain-rate eigenvector, it would be necessary to study the evolution equation of the alignments (i.e. equations for $\cos(\boldsymbol{\omega}, \mathbf{e}_1)$, $\cos(\boldsymbol{\omega}, \mathbf{e}_2)$ and $\cos(\boldsymbol{\omega}, \mathbf{e}_3)$) as shown in Dresselhaus and Tabor (1992). These transport equations will reveal the physical processes associated with the alignments.
 7. The current work predicted that the vorticity created due to local effects tends to have stronger alignment with the extensive strain-rate eigenvector. In order to shed further light on this result, the vorticity vector, following the works of Jiménez (1992) and Hamlington et al. (2008b), can be decomposed into local and non-local parts, and then alignments conditioned on these two parts can be computed.
 8. As shown in section 3.4.3, the pressure Hessian plays a main role in the near grid-element region. It is still unclear if the isotropic or the deviatoric part of the pressure Hessian is important in the near field. The pressure Hessian term in the transport equation for $\langle s_{ij}s_{ij} \rangle$ should be decomposed into isotropic and deviatoric parts. Such a study will clarify the non-local effects in the production region.
 9. As the domination of strain self-amplification over enstrophy production changes within the production region (see section 3.6), the ideal way to study the relationship between these two production quantities (i.e. enstrophy production and strain self-amplification) is to study the evolution of velocity gradient tensor in three-dimensional $\{Q; -s_{ij}s_{jk}s_{ki}; \omega_i s_{ij} \omega_j\}$ space rather than in two-dimensional $Q - R$ space. Such a study would add more information about the small-scale statistics of strain- and enstrophy-dominated regions of the flow.
 10. The 3-D space of $\{Q; -s_{ij}s_{jk}s_{ki}; \omega_i s_{ij} \omega_j\}$ would also help to analyse the effect of pressure Hessian further. For instance, Lüthi et al. (2009) have shown that projecting the conditional mean trajectories (CMT) of the restricted Euler equation on the $\{Q; -s_{ij}s_{jk}s_{ki}; \omega_i s_{ij} \omega_j\}$ space would help identifying the roles of individual terms in the VGT evolution equation. Such a study, in turn, will lead to the better modelling of the pressure Hessian for the flow field considered in this thesis.
 11. The possible parametric studies in VGT analysis include to analyse the influence of (a) the grid-geometry, (b) the Reynolds number (c) the aspect ratio of the

element and (d) the blockage of the grid-element. Such studies would be useful to determine the dependency of the location where the bar wakes meet, and the size of the potential and production regions along the grid-element centreline on various parameters.

6.5.2 Scalar gradients

1. As shown by Westerweel et al. (2002), the TNT interfaces can also be defined based on the scalar magnitude. Therefore, it would be interesting to detect the threshold magnitude of the scalar for the TNT interface and the subsequent computation of intermittency factor along the grid-element centreline. Such a study not only will complement the TNT study using vorticity magnitude but also will establish the spreading behaviours of vorticity and scalar when they are generated/released from the same point in a flow field.
2. Since the production region is highly intermittent, the small-scale statistics of scalar turbulence could be studied on the TNT interfaces detected using the scalar magnitude. In particular, the budgets of mean scalar, turbulent heat-flux, scalar-variance and scalar-gradient-variance across the TNT interfaces will yield a significant understanding of the scalar turbulence for the current case.
3. Since the best defined $-5/3$ power-law of scalar spectra is observed in the highly intermittent region, the relationship between TNT interfaces and the scalar cascade has to be established. To this end, one can perform conditional statistics of quantities such as $G_i G_i$ and $-G_i s_{ij} G_j$ with respect to the TNT interface. This study will establish the importance of stretching/compressing of scalar gradient vector by the turbulent strain-rate eigenvectors in relation to the TNT interfaces.
4. The interscale scalar-variance transfer and its relationship with scalar-variance dissipation can be studied using an equation similar to KHMH. Such an equation can be derived for the scalar-variance in two-point space, and then applied to the current case.
5. The evolution equations of $\cos(\mathbf{G}, \mathbf{e}_1)$, $\cos(\mathbf{G}, \mathbf{e}_2)$ and $\cos(\mathbf{G}, \mathbf{e}_3)$ are given in Brethouwer et al. (2003). Applying these equations to the current case will reveal why the background strain aids for stronger alignment between the scalar gradient vector and the extensive strain-rate eigenvector.
6. In order to understand the small-scale scalar mixing, the topology of the scalar gradients should be studied. This can be done by, (a) analysing the flow and

scalar fields simultaneously, (b) two-point correlations of fluctuating vorticity and scalar-gradient, and (c) joint probability distributions between quantities involving velocity and scalar gradients (Abe et al., 2009). Note that, although vorticity does not play a role in the production of scalar gradients, it does rotate and thus change the direction of scalar gradient vector (refer to Gulitski et al. (2007a)). Therefore, the topological understanding of scalar gradients also requires consideration of vorticity vector.

7. The observed dominant scalar gradient stretching compared with vortex stretching can be studied in a Lagrangian framework as in the study of Guala et al. (2005). Such a study will reveal the physical processes associated with the growth of scalar gradient and vorticity stretching/compressing.
8. Although this thesis is the first to show that the ramp-cliff structures are present even in places where there are no mean scalar gradients, it does not provide the physical mechanism whereby these structures are generated. All the physical models of ramp-cliff structures available in the literature involve mean scalar gradients. The physical model of ramp-cliff structures for the current study requires identification of whether the mean scalar gradient behind the bar generates these structures or they evolve from the intermittent bursts. To this end, three dimensional data projected on a time axis could help finding the source of these structures.
9. As discussed in section 1.3.3, most of the studies on scalar analysis have a mean scalar gradient in the inlet. Therefore, it would be interesting to see the effect of different boundary conditions (for instance a uniform temperature applied on the grid-element or an insulated grid-element with a uniform mean temperature gradient at the inlet) on the scalar gradient dynamics.
10. There are numerous parametric studies possible for the scalar case. Firstly, with respect to the Peclet number. Secondly, with respect to the grid geometry. Finally with respect to the Prandtl number. It should be mentioned here that studying the dependency of Prandtl number for $Pr > 1$ is numerically challenging as the small scales of scalar becomes smaller than that of the fluid turbulence when $Pr > 1$.

6.5.3 Laminar heat transfer

1. One of the important results of heat transfer analysis is that the heat transfer enhancement depends on the frequency of forcing. This result can be verified by formulating a control theory problem based on the frequency of fluctuations.

Since the equation of frequency to which the boundary layer is more receptive is readily available (Dullenkopf and Mayle, 1995), such a study will yield the optimum frequency at which the free-stream turbulence should be forced for a given set of flow and thermal conditions.

2. The second important result concerning heat transfer is that the azimuthal vortical structures are the most effective in taking heat away from a cylinder. However, the missing link here is how effective they are with respect to other components of the vorticity vector. This question can be answered by formulating a simple numerical problem where only the heat transfer from a cylinder is simulated. By imposing the desired vorticity component at the inlet of such a simulation, it could be possible to estimate the effectiveness of azimuthal vortical structures and to gain a better understanding of their interaction with a cylinder.
3. The other important conclusion from this heat transfer study is that the stagnation point heat transfer coefficient correlations, which are widely considered to be universal, are proved to be invalid in intermittent regions. Therefore, it is conspicuous that the effect of intermittency has to be included in the correlations of (1.31) and (1.32). However, at least to the author's opinion, this is not possible with numerical studies. Experiments at low Reynolds number, such as the one considered in this thesis, have to be carried out extensively to get more data in order to facilitate accurate curve fitting.
4. As shown in the experiments of Melina et al. (2016b), the effect of different regions downstream to the grid-element on heat transfer significantly differs for very high Reynolds number flows. High Reynolds number simulations are required to shed more light onto the different heat transfer enhancement mechanisms. However, at least to the author's point of view, direct numerical simulations are very difficult to carry out for the Reynolds numbers equivalent to the experiments. Either a semi-DNS or large eddy simulations (LES) could well be an alternative choice.
5. The parametric studies in this part include the influence of (a) the grid-geometry, (b) the Reynolds number and (c) the Prandtl number on laminar heat transfer. Such studies are highly expensive numerically and therefore can be carried out only experimentally.

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