

FLUID FLOW IN HIGH MACH NUMBER AXIAL FLOW TURBINES

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Thesis submitted to the University of London for the
degree of Doctor of Philosophy.

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September, 1969

S U M M A R Y

A preliminary study of the combined gas-turbine vapour-turbine engine and a blade-to-blade design method for high Mach number axial flow turbines are presented.

Cycle calculations were carried out and vapour turbines were examined to determine the probable plant performance and to provide an indication of the vapour turbine design problems using a number of different vapour cycle working fluids. It was shown that the combined engine could have an efficiency slightly higher than a regenerative gas-turbine and specific output 30% higher than a simple gas turbine. Vapour turbines using Refrigerant fluids are shown to be comparatively small and to present the design problems of high flare and high Mach number. Refrigerant 21 appears to be the best working fluid.

A blade-to-blade design tool for analysing the subsonic, transonic and supersonic flows in axial turbines was formulated using the streamline curvature method. The convergence of the numerical procedure was analysed and a satisfactory method developed which produced convergence for a digital computer and computed results were compared with results from experiment and other theory. Comparisons for turbine type cascades up to Mach 1.0 show a satisfactory agreement and techniques introduced give improved leading edge results and allow a good approximation to the trailing edge Kutta-Joukowski condition to be applied. Predictions for convergent divergent supersonic wind tunnel type nozzles agree closely with experiment and with other methods. Numerical difficulties are found to be more acute for subsonic-supersonic turbine cascades and experimental agreement is acceptable only up to low supersonic Mach numbers.

ACKNOWLEDGEMENTS

The Author wishes to express his thanks to:-

Dr. A.D. Carmichael for his able, understanding and sacrificial supervision of the research,

Dr. J.A.C. Kentfield for his willing administration of the latter part of the research,

The South African Council for Scientific and Industrial Research, Shell Research Limited, the Science Research Council and the Imperial College Computer Unit for their financial support,

Colleagues P.E.J. Fish, K.L. Kumar, M.O. Khan, N.K. Asamoah, D.J. Hayes and N.C. Remedios for their assistance in computation and discussion,

Mrs. L.W. Roberts who typed this thesis,

his friends of all nations who made his stay in London as happy and beneficial as possible.

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NOMENCLATURE

A	total flow area normal to the streamlines (Sketch 5.1)
a	Stream tube flow area, flow area of one streamline (Sketch 5.1)
C	matrix in c (equation (5.21), chord length
c	convergence constant as in equations 5.14 and 5.22
D	channel width (equation 4.10)
d_f	damping factor for convergence (equation 5.28)
g_o	constant in Newtons second law (Section 2.4c)
h	enthalpy (equation 4.4), streamwise grid spacing (Sketch 5.1)
h_o	total enthalpy (equation 4.4)
h'_m, h'_m	grid spacing constants for subsonic and supersonic flow (equations 5.10 and 5.11)
$h_{1m},$ h_{2m}	grid spacings associated with normal m (Sketch A1.2)
i	subscript counter used in convergence formulation (Appendix 1)
J	energy conversion factor
K	Barkefs constant (Section 2.4a)
K_m	grid constant for convergence (equations 5.8 and 5.9)
k	proportionality constant assumed between velocity and density (Appendix 1)
ℓ	height of fluid element normal to the plane of flow (Sketch 4.3)
M	Mach number, number of normals considered in dummy convergence grid
m	index counter for flow normals (Sketch 5.1)
$\dot{m}$	mass flow (equation 5.2)
N	r.p.m. (Section 2.4c), number of streamlines (Sketch 5.3)
N_s	specific speed (Section 2.4c)
n	distance normal to streamline (Sketch 4.3), index counter for streamlines (Sketch 5.1)
p	pressure (Sketch 5.1)
Q	volumetric turbine exhaust flow rate (Section 2.4c)

R	streamline radius of curvature (Sketch 4.3)
r	radius
s	entropy (equation 4.4), distance along a streamline (Sketch 4.3)
S_{fct}	factor between 0 and 1.0 multiplying radius of curvature in numerical procedure
T	absolute temperature (equation 4.4)
t	profile thickness (Figure 7.1)
V	velocity
X	matrix in x (equation 5.21)
x	coordinate (Figure 6.2), dimensionless velocity change (equation 5.13)
Y	matrix in y (equation 5.21)
y	coordinate (Figure 6.2), dimensionless area change at a normal (equation 5.7)
β_1, β_2	inlet and outlet cascade flow angles (Figure 7.1)
γ	stagger angle (Figure 7.1), perfect gas specific heat ratio
ψ	stream function
ρ	density
θ	angle subtended by streamline segment (Sketch 4.3), angle in direction of rotation
θ_c	blade turning angle (Figure 7.1)

CHAPTER 1 : INTRODUCTION

Economic studies of methods of power generation show that the simple cycle gas turbine, because of its relatively low capital cost, is competitive with other types of power plants in situations when the fuel cost is low, such as at natural-gas fields, or when the operational times are low, as in peak-load power production. If the level of thermal efficiency of gas turbines could be raised, they would become competitive with other types of power plants in other applications, provided the cost of the additional components necessary for the improvement in efficiency does not result in a large increase in the capital cost of the plant. Regeneration is one method of improving the thermal efficiency of gas turbine engines by transferring heat from the engine exhaust to the air entering the combustion chamber. Unfortunately the cost and complexity of the regenerator increases the capital cost of the engine without increasing the power. An alternative* method of improving the thermal efficiency of the gas turbine cycle is to provide a vapour turbine cycle to utilize some of the available waste heat in the gas turbine exhaust gases. This arrangement, termed the combined gas-turbine and vapour-turbine engine, provides the possibility of obtaining a high efficiency power plant at approximately the same low capital cost of the gas turbine. This is because the increase in thermal efficiency is coupled with an increase in power output. Whether this potential can ever be realized by a combined cycle power plant is difficult to assess until detailed design, cost and market studies have been made.

Preliminary studies were carried out at Imperial College to investigate the combined gas-turbine vapour-turbine engine. The results of these studies showed that there were potential gains in efficiency with the combined engine. In the present thesis a more thorough investigation of the engine cycles for a range of working fluids is presented followed by an analysis of the geometries of vapour turbines using the most promising working fluids. The design of turbines having high Mach number level high annulus flare is isolated as probably the most important feature of the design of the vapour turbines. The provision of a computer programme to facilitate the design of the high Mach number, highly flared turbines is described in the remainder of the thesis.

* Total energy and process heating have not been considered here because power generation is the main topic of this thesis.

CHAPTER 2 : THE GAS-TURBINE VAPOUR-TURBINE ENGINE

2.1 Introduction.

It is the purpose of this Chapter to establish the level of expected performance of the combined gas-turbine and vapour-turbine engine for a range of design conditions and for several working fluids in the vapour turbine. In addition, details of preliminary designs of the vapour turbines for a wide range of power outputs are presented to assess the turbine design problems and to indicate the optimum fluids for the vapour cycle. The work described in this Chapter has been published as Reference 2.7.

2.2 Survey of previous work.

There have been several papers on combined gas-turbine and vapour-turbine engines. McAnneny (2.1) described the design of a vapour turbine that was coupled to a small gas turbine. Athans and Dixon (2.2) and Aronson (2.3) have discussed some possible working fluids and plant configurations. It is known that the Central Electricity Generating Board have an interest in this topic and have carried out economic studies of the combined engines for peak load operations (see Horn (2.4)). At Imperial College preliminary studies were made of combined engines by Barkef (2.5) and Nedjat (2.6). Barkef carried out preliminary designs of Refrigerant turbines and Nedjat completed preliminary designs of boilers and water cooled condensers. The main conclusions of these studies were that the vapour cycle components were relatively small. The work described in this Chapter is a continuation of the investigations of Barkef and Nedjat.

2.3 Thermodynamic analysis of the combined gas turbine and vapour turbine cycle.

2.3a Description of the cycle.

The combined gas turbine and the vapour turbine engine may be considered as two separate thermodynamic cycles linked at the exhaust system of the gas turbine where the boiler of the vapour cycle is placed. One possible arrangement of the components of the combined engine is shown diagrammatically on Figure 2.1. The gas turbine has the usual compressor, combustion chamber and turbine arrangement of the simple gas turbine. The vapour turbine has the boiler, turbine, condenser, and feed pump arrangement of the simple Rankine

cycle. Other more complicated gas turbine cycles are discussed later in this Chapter. More complicated vapour cycles utilizing "feedback" heat exchangers have not been considered because of their complexity although these could enhance the thermodynamic performance of vapour cycles using very high molecular-weight working fluids.

2.3b Preliminary selection of potential working fluids for the vapour cycle.

From theoretical considerations, the most suitable fluid would be one which best utilises the available heat in the gas turbine exhaust stream. Figure 2.2 shows the available exhaust heat on a temperature entropy plot. Also shown is an arbitrary vapour saturation line with two vapour cycles indicated, one supercritical and one subcritical. It can clearly be seen that of the two cycles shown, the supercritical one entails the lowest heat transfer irreversibilities as it follows the available heat curve more closely. Supercritical cycles would therefore appear to be thermodynamically the better choice and fluids which have the required critical temperatures the most suitable. Other considerations however eliminate the choice of fluid on this basis alone and the preliminary and final selection is made by considering overall characteristics and performances here and in the subsequent sections.

A wide range of working fluids could be used in the vapour cycle but the number of potentially useful fluids can be reduced if it is assumed that inflammable liquids would not be used because they could incur fire and explosion risks. A further reduction in the number of working fluids can be made if it is assumed that for practical reasons of leakage and stress fluids having condenser pressures greater than 300 psia would not be used. The condenser pressures at 80°F, 100°F and 120°F (27, 38 and 49°C) of some fluids are presented in Table 1. Thermodynamic data are available for all the fluids given in this table.

Organic fluids such as the Refrigerants tend to decompose at temperatures greater than approximately 300°F (149°C), however, it will be shown later that there is little gain in cycle performance when the Refrigerants are operated above this temperature.

2.3c Temperature limitations.

There are two gas side temperature limitations which have an influence on the performance of the combined gas turbine and vapour turbine engine. The first limitation arises due to the formation of sulphuric acid in the exhaust gases when fuel containing sulphur is burnt in the gas turbine combustion chamber. Serious corrosion problems occur when the temperature of the gas falls below the sulphuric acid dewpoint causing the acid to condense in the exhaust system and heat exchanger. In steam boilers the acid dewpoint is approximately 250°F . The minimum stack temperature was therefore taken as 300°F (149°C) in the cycle calculations. Sulphuric acid corrosion also occurs on surfaces having temperatures in the region of 180°F regardless of the bulk temperature of the gas. If this problem were found to be serious in practice it would be necessary to eliminate the "economiser" section of the evaporator or boiler in the vapour cycle. This could mean that the stack temperature would be raised to approximately 350°F (167°C) to ensure heat transfer to the boiling fluid.

The second limitation is associated with heat transfer in the boiler where it was assumed that a minimum temperature difference of 100°F (55°C) between the gas and the liquid would be maintained.

2.3d Performance calculations.

The performance of the combined gas turbine and vapour turbine was calculated for a range of gas turbine pressure ratios up to 20:1 and top temperatures of 1500 , 1600 and 1700°F (815 , 870 and 925°C). The performance of simple cycle gas turbine engines and regenerative engines were also estimated using similar levels of component efficiencies. The assumptions and data used in the cycle calculations are summarised in Tables 2 and 3. The assumed values of efficiency and pressure drops used in the cycle calculations are based on good current practice which could be attained using the latest design methods.

The cycle calculations for the combined engine were carried out in two parts. The simple cycle gas turbine performance was first calculated and the enthalpy change in the turbine exhaust from the gas turbine exit temperature to the boiler leaving temperature of 300°F (149°C) was determined. The thermal efficiencies of the vapour cycles were then calculated separately for each working fluid. The thermal efficiencies and the power output of the combined engine were then determined from the separate cycle calculations.

A Mollier diagram illustrating a vapour cycle for Refrigerant 21 is presented on Figure 2.3.

2.3e Results of the cycle calculations.

The predicted values of thermal efficiency and power output for the combined gas turbine and vapour turbine are presented on Figure 2.4. Refrigerant 21 was assumed as the working fluid for the vapour turbine in this figure. These curves show that the maximum efficiency of the combined gas turbine and vapour turbine occurs at high gas turbine pressure ratios and peak power output is provided at a gas turbine pressure ratio of approximately 6 : 1.

The predicted values of thermal efficiency and power output for simple cycle and regenerative gas turbines are also shown on Figure 2.4. This figure indicates that the maximum thermal efficiency of the combined engine is approximately 10 per cent higher than the peak thermal efficiency of the regenerative gas turbine and approximately 30 per cent higher than the thermal efficiency of the simple cycle gas turbine. In addition the power output of the combined engine is about 30 per cent higher than the power output of both the simple cycle and regenerative gas turbines.

The predicted values of thermal efficiency for the combined engine with several different working fluids in the vapour turbine are presented on Figure 2.5 as functions of the boiler pressure. These results show that the best working fluids from thermodynamic considerations are Hexafluorobenzene, H_2O , Refrigerants 11, 21 and 113. In addition it can be seen that there is little to gain by operating the Refrigerant vapour turbines at boiler temperatures above about $300^{\circ}F$ ($149^{\circ}C$).

2.3f Cycle calculations for more complex combined engines.

The results of thermodynamic calculations of more complicated gas turbines combined with vapour turbines are shown on Figure 2.6. The separate and combined effects of reheat and intercooling were considered in this analysis. The results shown on Figure 2.6 indicate that the thermal efficiency can be slightly improved but large gains in specific output can be obtained by the additional features when compared with the performance of the simple cycle combined with a vapour turbine.

2.3g Conclusions from the cycle calculations.

The main conclusions of the cycle study are that the level of thermal efficiency of the combined gas turbine and vapour turbine engine can be higher than the simple cycle and regenerative gas turbine engines. Furthermore, the specific output of the combined engine is approximately 30 per cent higher than the simple cycle and regenerative gas turbines.

Thermodynamic considerations have shown that the most favourable working fluids for the vapour cycle are Hexafluorobenzene, H_2O , Refrigerants 11, 21, and 113.

2.4 Preliminary designs of vapour turbines.

The vapour turbine is an important component of the vapour cycle of the combined engine and preliminary aerodynamic designs have been analysed in order to determine the size and complexity of the turbine unit and to determine the extent and nature of the turbine design problem.

2.4a Turbine R.P.M.

It was decided to limit the investigation to vapour turbines which were directly coupled to the power turbines of the gas turbine engine. The turbine r.p.m. was therefore selected as 3,000 r.p.m. (for a 50 cycle system). It was realized that smaller gas turbines (< 10 MW) would rotate at higher speeds than 3,000 r.p.m. in order to provide operation at optimum turbine efficiency, but a gear box would be placed between the power turbine and the generator. The principle of geometrical similarity can be used to estimate the geometrical arrangement of vapour turbines coupled to these smaller gas turbines. The reasoning is as follows:

If a designer has freedom to select r.p.m. then he would provide optimum geometry for the power turbine regardless of power output and all power turbines would be geometrically similar. The designs of vapour turbines directly connected to the power turbines would therefore also be geometrically similar to each other although the vapour turbine designs would not generally be optimum because the r.p.m. would be selected to suit the power turbine as it develops more power than the vapour turbine. The r.p.m. of geometrically similar power turbines is given by the expression $N = K / \sqrt{MW}$ and Barkef (2.5) has shown that the average value of K for industrial gas turbines is 19,200. This expression suggests that a power turbine developing 41 MW would have an optimum rotational speed of 3,000 r.p.m., and furthermore if allowance is made for the power developed

by the vapour turbine it can be seen that a 50 MW combined engine would ideally rotate at approximately 3,000 r.p.m. In other words the vapour turbines of smaller combined engines (< 10 MW) would be geometrically similar to the turbine for the 50 MW combined engine. The following similarity rules would be used to scale from a 50 MW design:

$$\begin{array}{lll} \text{turbine diameter} & \propto & \sqrt{\text{MW}} \\ \text{r.p.m.} & \propto & 1/\sqrt{\text{MW}} \\ \text{mass flow} & \propto & \text{MW} \end{array}$$

2.4b Design conditions for the vapour turbines.

From figure 2.4 it can be observed that the maximum power output and thermal efficiency occur at gas turbine pressure ratios of 6 : 1 and 15 : 1 respectively. It was concluded that the optimum gas turbine pressure ratio from economic considerations would lie between these two values. A gas turbine pressure ratio of 10 : 1 was selected as the basis for the vapour turbine study; the top temperature of the gas turbine was taken as 1600°F (870°C). It is not proposed to discuss the influence of the gas turbine design point on the vapour turbine design except to note that it has a relatively small effect on the geometry and size of the vapour turbine at the preliminary design stage. Design conditions for the vapour turbines using the selected working fluids are given in Table 4.

2.4c Type of vapour turbine.

Radial inflow and axial-flow turbines were considered for this application. The radial inflow turbine would generally have the lower capital cost as a single stage and could operate efficiently at high pressure ratios. The axial-flow turbine would be preferred when more than one stage is required.

The specific speed can be used as a method of selecting the type and number of stages from purely geometric considerations.

Specific speed is defined as

$$N_s = \frac{N Q^{\frac{1}{2}}}{60(g_o J \Delta h_o)^{\frac{1}{4}}}$$

where

$$N = \text{r.p.m.}$$

$$Q = \text{Volume flow at exit density}$$

Δh_o = Enthalpy drop per stage

J = Energy conversion factor

g_o = Constant in Newton's second law

In this form, specific speed is non-dimensional and any consistent system of units can be used. It is easy to demonstrate that the value of specific speed defines, in general terms, the geometry of the turbine. A low specific-speed machine has short blades and a large disc while a high specific-speed machine has long blades and a small disc. It is generally stated that the radial inflow turbine should be designed for specific speeds between 0.06 and 0.12 while the axial-flow turbine should be designed for specific speeds in the range of 0.05 to 0.20.* It should be remembered that specific speed is related only to turbine geometry and is not used to predict the Mach number levels in the turbine.

The upper and lower limits of specific speed given in the previous paragraph for axial-flow and radial inflow turbines were used to estimate the ranges of power outputs for combined engines using the various working fluids in the vapour turbines. An additional factor was the number of stages and this was decided from consideration of the enthalpy drop across the vapour turbine. The ranges of power - outputs are given in Table 5. It appears from this table that single stage Refrigerant turbines could be designed using Refrigerant working fluids for power outputs between 10 and 100 MW. However, it is uncertain whether these turbines would be acceptable from Mach number considerations. The table also indicates that the power output of Hexaflourobenzine turbines will be very low.

2.4d Axial flow turbine designs.

Preliminary designs of axial-flow turbines were carried out using a simple but representative design procedure. The following assumptions were used to provide the basis for the designs;

1. Free vortex distribution of tangential velocity downstream of the nozzle (or stator) blade row.
2. Zero tangential velocity at exit from each stage.

* It is possible for individual stages to have a much higher specific speed and values between .25 and .4 are used on final stage steam turbines.

3. The flare distributed evenly between nozzle and rotor blades.
4. Equal enthalpy drop across each stage.
5. The reaction specified at the hub of the stage having the smallest radius.
6. 70° nozzle angle at the point where the reaction is specified.

In addition it can be seen from an examination of the properties of Refrigerants, Downing and Long (2.8), that in the regions of interest, the velocities of sound for each Refrigerant are nearly constant. In the design study it was assumed that the velocities of sound for the working fluids were constant at the following values:

1. Refrigerant 21	560 ft/sec
2. Refrigerant 11	463 ft/sec
3. Refrigerant 113	395 ft/sec
4. Hexafluorobenzine	395 ft/sec
5. Steam	1100 ft/sec

Using the assumptions given above together with the design point data provided in Table 4, the vapour-turbine geometries and flow conditions were determined for a range of power outputs (of the combined engine) and the working fluids. Single stage, two stage and three stage vapour turbines using Refrigerants as the working fluid were designed having zero reaction (impulse) conditions at the hub of the last stage. In addition vapour turbines were designed having 30 per cent reaction at the hub of the last stage. Steam and Hexafluorobenzine turbines were also designed using the same assumptions but it was found that at least six stages were required to provide acceptable levels of Mach number.

The geometrical and flow features of the axial-flow turbines are presented in Figures 2.8 - 2.13. On each figure there are numbers which indicate the power output, Mach number, hub-tip ratio and hub reaction. The key for these numbers and their positions on the turbine diagrams are provided on Figure 2.7.

The figures show that the single stage Refrigerant turbines have rather high Mach numbers (about 2.0) at the nozzle hub and rotor tip and these Mach number levels, in conjunction with the high leaving losses would result in rather low turbine efficiencies. Two stage turbines would have Mach numbers slightly above 1.0 while three stage turbines would have entirely conventional levels of Mach number.

Of the Refrigerant working fluids it appears that turbines for Refrigerant 21 are of superior design for the power range 25 - 100 MW at 3,000 r.p.m. For lower power outputs Refrigerants 11, and 113 may be used.

Comparison between the steam and Refrigerant 21 turbine shown on Figure 11 indicates that the steam turbine would have a very much larger diameter and would have more stages than the Refrigerant 21 turbine. The Hexafluorobenzene turbine shown on Figure 2.13 can only be designed for very low power-outputs at 3,000 r.p.m. and it would require several stages. The first few stages of the Steam and Hexafluorobenzene turbines would have partial admission because of the small blade height.

2.4e Radial inflow turbine design.

It was shown in the specific speed analysis that radial inflow turbines could be designed having acceptable geometries for the required range of power outputs using Refrigerant working fluids. It was expected however, that the levels of Mach number would be high. Representative radial inflow turbines of 50 MW were therefore designed to determine the Mach number levels and geometrical features. These preliminary designs for the radial inflow turbines used the following basic design rules:

1. The meridional velocity at rotor inlet was 0.3 of the inlet blade speed.
2. Radial inlet blades were assumed for the rotors.
3. Exit meridional velocity was the same as rotor inlet meridional velocity.
4. Hub diameter of the rotor was one quarter of the tip diameter.

The geometrical and flow features of the Refrigerant turbines are shown on Figure 2.14. This figure shows that the geometries of the Refrigerant 11 and 21 turbines are acceptable while the Refrigerant 113 turbine is a rather higher specific-speed design than is normally used. The nozzle Mach numbers are rather high but there is reason to expect that turbines could be designed with efficiency levels in the region 70 - 80 per cent with these Mach numbers.

2.4f Conclusions from the vapour turbine design study.

The main conclusions of the turbine design study are as follows:

1. Single stage axial-flow and radial inflow turbines could be designed using Refrigerant working fluids

for combined gas turbine and vapour turbine outputs of 10 to 100 MW at 3,000 r.p.m. but the Mach number levels would be high.

2. Two stage axial turbines would have lower Mach numbers and three stage turbines would have conventional Mach numbers.
3. Refrigerant 21 appears to provide the most satisfactory designs of turbines for combined engine outputs from 25 to 100 MW at 3,000 r.p.m.
4. Refrigerants 11 and 113 would be used at lower power outputs.
5. Using steam as the working fluid would result in very large turbines having many more stages than the Refrigerant turbines.
6. Hexafluorobenzene could only be used at very small power outputs and the number of stages would be higher than Refrigerant turbines.

2.5 Conclusions.

The combined gas turbine and vapour turbine engine appears to offer every prospect of being economically competitive with other power plants in some applications. The addition of a vapour cycle to a simple cycle gas turbine would raise the output and thermal efficiency approximately 30 per cent. Refrigerant 21 is the most promising working fluid for the vapour turbine in the power range 25 - 100 MW.

In order to minimize the number of stages in the turbine it would be necessary to develop high efficiency axial-flow turbines operating with high Mach numbers and with highly flared turbine walls. This problem is considered in subsequent chapters.

CHAPTER 3 : THEORETICAL DESIGN METHODS
FOR HIGH MACH NUMBER TURBINES.

3.1 Introduction.

In Chapter 2 it was observed that the vapour turbine in the combined gas-turbine vapour-turbine engine would probably have high wall-flare and it would operate at high Mach numbers. High flare and high Mach numbers also occur in the last few stages of very large steam turbines and, to a lesser extent, in some gas turbines and jet engines. The problems of designing machines having high flare and high Mach number are therefore not completely novel although it is possible with the high molecular weight fluids to design turbines having extremely high flare and high Mach number without approaching the mechanical limitations imposed by disc and blade materials. The restriction to the output per stage of a turbine using high molecular weight fluids is thus likely to be associated with aerodynamic rather than mechanical design problems. In order to understand the aerodynamic features of these turbines it is considered necessary to provide design and analysis methods which enable the designer to calculate or specify the distributions of velocity (outside the boundary layer) on the blade surfaces of the turbine stator and rotor blade rows.

While the flared high Mach number turbine occurs in several applications the majority of turbines operate subsonically. As a consequence of this theoretical methods for this type of flow problem are not as well developed as methods for subsonic flows. In general, theoretical methods are either subsonic or supersonic and no transonic (flow in the region of Mach 1.0) method exists which can be applied to cascade flows. Subsonic methods are widely used which are either incompressible or, if compressible, they are proprietary or contain flow approximations. There is therefore clearly a need for a theoretical design method which is accurate and can be used for subsonic, transonic and supersonic flow. The development of such a method is the main objective of the research described in this thesis.

In the remainder of this chapter theoretical design tools are discussed in greater detail, existing theoretical methods are surveyed and the method considered to be the most suitable for the design of highly flared high Mach number turbines is selected and discussed.

3.2 Theoretical design tools.

3.2a Introduction.

Turbine designers have in the past considered themselves fortunate in that, due to the accelerating nature of the flow which inhibits boundary layer growth, their designs have apparently been successful without the aerodynamic sophistication that was necessary for the success of the axial compressor. The development of theoretical methods did not therefore receive any real impetus until the quest for high efficiencies and overall size reduction became important.

Theoretical methods, as in most fields of fluid flow, are used to assist in an empirically based overall design procedure. The part the theory plays is to provide a basic understanding of the flow process and to reduce the amount of experimental work necessary by indicating to the designer the probable defects of a proposed design.

After using for many years relatively simple hand methods the introduction of the digital computer has enabled the turbine designer to investigate increasingly complicated and sophisticated models of the fluid flow in turbines. As mentioned in Section 3.1 a need clearly exists for an accurate design method that is applicable to flared turbines having subsonic, transonic and supersonic flows.

3.2b The design of axial flow turbines.

The complete three-dimensional viscous model of the flow of compressible fluids in turbines is too complicated to solve with current techniques. Instead the flow problem is simplified by representing the real flow by two approximate models of the flow. First the meridional flow is considered and then the blade element or blade-to-blade flow is analysed.

In order to solve the meridional flow problem it is assumed that the flow is axisymmetric and therefore two-dimensional. Several different forms of the equations of motion for axisymmetric flow appear in the literature and computer programmes have been developed to solve these equations.

The blade-to-blade flow is solved using boundary conditions given by the meridional flow method. As in the case of meridional flow there are many forms of the equations of motion for blade-to-blade flow and also many solutions.

It is not the purpose of this thesis to discuss further the meridional flow problem since it is adequately treated in the literature. Instead the problem of blade-to-blade flow is investigated with particular reference to problems of highly-flared high Mach number turbines.

The use of a blade-to-blade method as a design tool has ideally two parts. First the frictionless flow within the cascade is analysed to determine the pressure or velocity distribution on the blade surfaces. Secondly the pressure distributions obtained are used to determine the behaviour of the boundary layers on the profile surfaces. In some instances it is advisable to modify the blade shape to take account of the boundary layer growth on the blade surfaces. Many methods exist for predicting the boundary layer growth on blade surfaces and this topic will not be considered here. The frictionless blade-to-blade flow is therefore the problem considered in the remainder of the thesis.

The problem of frictionless cascade flow can have two basically different approaches. The first is the direct or analysis method in which the flow equations are solved for a cascade of given geometry. The second is the inverse or design method which provides the geometry of a cascade when the velocity distributions on the blade surfaces are specified.

3.3 Survey of blade-to-blade methods.

A number of blade-to-blade methods have been used for turbine design purposes. These methods are summarised briefly below.

3.3a The method of vortex singularities.

Due to Martensen (3.1), the flow is represented by superimposing a continuous vortex distribution on the surfaces of profiles in a uniform stream. A set of linear equations for the unknown vortex strength distribution results when the profile surface specification of zero internal tangential velocity is made. The equations are readily solved and the vortex distribution gives the surface velocity distribution.

The basic method is incompressible but has been adapted to compressible flows (Reference 3.2). The compressible version of the method is unpublished and proprietary. The incompressible method has also been developed to solve both direct and indirect problems (Wilkinson (3.3)).

3.3b The method of conformal transformation.

A number of transformations have been developed for low camber blades and incompressible flows (Horlock (3.4)). One transformation, into the hodograph plane by Kraft (3.5), can be used for high camber profiles at Mach numbers up to approximately 0.5. The indirect approach is used.

3.3c Stanitz's full solution.

(Stanitz (3.6)). This is a compressible flow ($\gamma = \text{constant}$) or a linearized compressible flow ($\gamma = -1$) solution of the indirect channel flow problem. From specified channel velocity distributions and uniform velocity upstream and downstream, a relaxation solution is found in a transformed stream function, velocity potential plane. From this solution the complete flow pattern including the channel boundary can be found in cartesian coordinates.

3.3d The Green's function solution of Stanitz (3.6, 3.8).

This method is similar to the full solution of Section 3.3c, the difference being that the equation in the transformed plane is solved using a Greens function.

3.3e The rapid approximate method of Stanitz (3.7).

This is a rapid, compressible, indirect method which has been used in turbine design. A variation in flow height can be accommodated. The equations of motion are solved on an average basis for a fluid element spanning the complete channel and the camber line and thickness distribution are successively approximated. The upstream and downstream flows and the profile velocity distributions are specified. Similar methods have been developed to solve the direct or analysis problem.

3.3f The Taylor series solution of Wu and Brown. (3.9).

The two-dimensional compressible flow is approximated by a Taylor series along the mean streamline. The indirect problem is solved when the mean streamline flow is specified. The direct problem is solved by successive approximation after specifying the geometry and the trailing edge Kutta-Joukowski condition.

3.3g Streamline curvature methods.

These methods can be used for compressible flow to solve the direct or semi indirect problem with a flow height variation. In cascade analysis the Mach numbers have been subsonic. In treating aerofoils in free stream the flow has been both subsonic and supersonic and also purely supersonic. (A fuller description with references is give in Section 3.6).

3.4 Transonic flows.

Analytical approaches to transonic flows are hampered by the difference in nature of the flow equations and their respective method of solution in the subsonic and supersonic regimes. Subsonic flow is governed by an elliptical differential equation and supersonic flow by a hyperbolic equation.

Various methods have been applied to the treatment of the equations to eliminate this difficulty but all are approximations only which reduce the two equations to one using a single method of solution. A number of hodograph solutions have been produced by superposition to yield results for a few simple flows (see Shapiro (3.10) Chapter 20). In examining the flow through the throat of a convergent divergent nozzle, many workers have developed various series methods. Examples of these are Sauer (3.11) and Hall (3.12). A finite difference relaxation method has been used by Emmons (3.13 & 3.14) but this also does not differentiate between the flow regimes.

At present the design or analysis of such devices as supersonic wind tunnel nozzles which have subsonic and supersonic flows are treated in three distinct stages, the subsonic flow, the transonic flow and the supersonic flow. An example of this approach is given in Harrop (3.16) in the design of convergent divergent nozzles for supersonic wind tunnel applications. The transonic regime was chosen so as to enable the series method of Sauer (3.11) to be applied. Thereafter, with the flow fully supersonic, the method of characteristics was applied. An even simpler approach than this can be used which makes an assumption regarding the shape and location of the sonic line to enable the method of characteristics to be applied. Examples of this approach are Shapiro (3.10) page 507 and Forster (3.15) who assumed the sonic line to coincide with the geometric throat of a supersonic wind tunnel nozzle and a supersonic turbine cascade respectively.

Transonic methods are thus restricted in application by the limitations of the methods themselves, that is, to flows with small perturbations and, in most cases, to a given symmetrical geometry. As turbine cascades are two

dimensional and have arbitrary unsymmetrical geometries, the present methods cannot be applied. As will be explained more fully in Chapter 4, the streamline curvature method enables a transonic flow in a highly curved geometry to be treated as one flow yet retaining the essential elliptic and hyperbolic differences between the subsonic and supersonic regions of flow.

3.5 Selection of theoretical method.

The direct or analysis approach is selected in preference to the indirect or design approach as it is thought to be the more frequently occurring and better adapted to meeting the all important criteria of strength, vibration resistance and manufacturability.

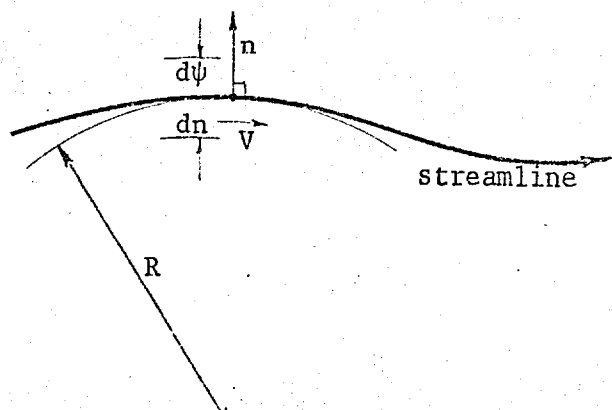
From the survey of two-dimensional methods in Section 3.3 only the method of singularities (3.3a), Wu and Browns series solution (3.3f) and the streamline curvature method (3.3g) are direct methods. At the time the selection was made, the method of singularities had not yet been shown to solve the compressible flow problem and this is apparently restricted to subsonic flows. The streamline curvature method was therefore selected for development as it is the more accurate of the remaining two methods and because the basic simplicity of the method was thought to lead to a simple simultaneous treatment of subsonic and supersonic flows. It is also the only method apart from the rapid approximate method (3.3e) to contain provision for a variation in flow height (flare).

3.6 Survey of the previous use of the streamline curvature method.

3.6a Introduction.

Streamline curvature methods have been used for both the axisymmetric (meridional plane) and the two-dimensional (blade-to-blade) design of turbines. Two dimensional streamline curvature methods are methods which use the equations of motion in a form containing the velocity gradient normal to the streamline, the streamwise velocity and the radius of curvature of the streamline at the point under consideration. The streamline curvature equation of motion and the continuity equation, discussed fully in Chapter 4, are as follows:-

$$-\frac{\partial v}{\partial n} = \frac{v}{R}, \quad \frac{\partial \psi}{\partial n} = \rho v \ell$$



Two basic approaches have been used in the solution of these equations. They are:-

- a) Approximate methods. The streamline curvature equations involve the length n along a normal and the streamline radius of curvature which are closely related to the bounding geometry of the flow field. Accurate assumptions can therefore be made regarding the variation of these quantities which render the equations readily integrable to yield approximate solutions.
- b) Numerical methods. At the cost of additional calculation the equations can be solved numerically using the true values of length n and radius of curvature.

3.6b Approximate Methods.

Greene (3.17) analysed the pressure distribution on an aerofoil in a free stream. Using the attenuation plane to represent both compressible and incompressible flows with a single flow pattern, a circular arc assumption for the radius of curvature variation was introduced. The streamline curvature equations were then formally integrated and the results showed a good agreement with experiment.

Perl (3.18), also analysing the pressure distribution on an aerofoil in a free stream, chose a radius of curvature function in terms of velocities and a constant which was determined from an incompressible solution. The results were compared with exact solutions and a good agreement was obtained, particularly at low Mach numbers.

Alpert (3.19) used the method for the semi indirect design of turbine blades. Two assumptions were made for the variation of radius of curvature along a normal, a vortex assumption and the assumption that the radius of curvature was constant. The curvature variation in the results obtained were examined and compared with the assumptions used. It was found that the vortex assumption was the better one. The method was semi-indirect in that, commencing with a suction surface geometry and velocity distribution, the pressure surface geometry and velocity distribution was calculated.

3.6c Numerical methods.

Stodola (3.20) described the work of Flügel (3.21) who, in applying the method to the design of closely spaced turbine buckets, commenced with an initial pattern of potential lines (normals) and an arbitrary assumption

of the variation in streamline curvature. The streamline curvature equations were then graphically integrated and a new pattern of potential lines and streamlines were produced. If these were found to differ too radically from the initial pattern, the calculation was repeated using the data from the new flow pattern.

Carmichael (3.22) adapted the basic method of Flügel (3.21) to a digital computer. It was found that when the new flow pattern data was used directly in the subsequent calculation, the resulting flow pattern did not converge to a constant pattern but that it diverged to form successively different patterns. A factor, dependant upon the grid spacing, was derived and used to determine an intermediate flow pattern between the new and the old to be used for the next calculation. The flow pattern was then found to converge.

Katsanis (3.23), also using a digital computer, abandoned the use of the orthogonal network of streamlines and potential lines and formulated the streamline curvature equations in terms of an arbitrary network. The method was applied to the analysis of radial inflow turbine blades and the result compared to the result obtained using the approximate method of Stanitz (3.7). A reasonable agreement between the two methods was obtained except in the trailing edge regions. In using this basic method, Wilkinson (3.24) has derived a factor similar to that of Carmichael (3.22) to prevent the divergence of the flow pattern.

CHAPTER 4 : THE STREAMLINE CURVATURE METHOD

4.1 Introduction.

4.1a The channel flow representation of cascade flow.

When two streamlines in the meridional plane (see Sketch 4.1) are rotated to form two closely spaced surfaces of revolution within a turbine cascade, the fluid bounded by these two stream surfaces and the suction and pressure surfaces of two neighbouring blades becomes the basic fluid filament (or element) for consideration in the streamline curvature method. Such a treatment of the flow has been termed the stream filament or blade element treatment. If it is assumed that there is a large number of blades the development of this element on a flat surface can be used instead of the curved one. If it is further assumed that the element does not vary in height, the problem reduces to that of a two dimensional cascade.* When a change in height of the element is present and the velocity gradients caused by the height change normal to the element are small in comparison with the gradients in the plane of the element, the flow is said to be semi three-dimensional.

As will be seen later, the streamline curvature method provides an analysis of the flow between two fixed surfaces or streamlines. In order therefore to apply the streamline curvature method to the basic fluid element, the position and shape of the upstream and downstream stagnation streamlines must be assumed so that the basic element may be extended into the regions well upstream and downstream of the cascade where, in an isolated cascade, the flow may assumed to be uniform. With these streamlines thus assumed the basic fluid element is in the form of a channel bounded by the two blade surfaces and the two stagnation streamlines as shown in Sketch 4.2. Such a representation of the flow is termed a channel flow representation.

Channel flow analyses of cascades have hitherto suffered in accuracy due to the inadequacy of the assumption made regarding the position and form of the stagnation streamlines. Stated differently, channel flow representations have failed to provide a means of satisfying the Kutta-Joukowski condition.

* There is a simple transformation that can be used when the number of blades is small (x, y becomes $x, r\theta$ for axial flow).

It is possible here, using the streamline curvature technique outlined in this chapter, to derive a simple and logical approximation to the Kutta condition. This approximation in combination with a study of the behaviour of it at varying cascade outlet flow angles in Chapter 7, has led to an almost exact channel flow Kutta condition at the trailing edge. A fundamental limitation of the channel flow representation is thus successfully eliminated in the work described in this thesis.

A further inaccuracy is fundamental to a channel flow numerical method. This arises from the high curvature and rapid change of sign of curvature of the channel boundary in the leading and trailing edge regions which introduce practical difficulties into the execution of the numerical procedure. It is therefore necessary to approximate to the true channel boundaries in the regions of difficulty as shown in Sketch 4.2 and to use a coarse finite difference grid. Techniques subsequently developed have considerably improved the accuracy of the streamline curvature method at the leading edge.

4.1b Objectives and layout of the chapter.

The basic streamline curvature equation is derived in Section 4.2. Before proceeding to formulate the numerical procedure, it is necessary to discuss the boundary conditions for the mathematical problem involved, as these influence the formulation of the numerical procedure; this is described in Section 4.3. The fundamental differences between subsonic and supersonic flow using streamline curvature are then considered. As these differences also affect the numerical procedure, they are discussed in Section 4.4 before proceeding to formulate the numerical procedure in Section 4.5.

4.2 Derivation of the basic equations.

The two-dimensional analysis of Shapiro (3.10 page 281) is adapted here to include a fluid filament height variation. Referring to Sketches 4.2 and 4.3, a fluid particle bounded by two streamlines and two normals to the streamlines is considered. The flow is further assumed to be steady and inviscid.

The fluid particle is subject to the pressure forces shown in Sketch 4.3. The nett force towards the center of the streamline radius of curvature R , taking into consideration the components of the pressure forces on the 6 sides, is:-

$$\begin{aligned}
& p \ell R \delta \theta + (p - \frac{1}{2} \frac{\partial p}{\partial s} \delta s + \frac{1}{2} \frac{\partial p}{\partial n} \delta n) (\ell - \frac{1}{2} \frac{\partial \ell}{\partial s} \delta s + \frac{1}{2} \frac{\partial \ell}{\partial n} \delta n) \delta n \frac{\delta \theta}{2} \\
& + (p + \frac{1}{2} \frac{\partial p}{\partial s} \delta s + \frac{1}{2} \frac{\partial p}{\partial n} \delta n) (\ell - \frac{1}{2} \frac{\partial \ell}{\partial s} \delta s + \frac{1}{2} \frac{\partial \ell}{\partial n} \delta n) \delta n \frac{\delta \theta}{2} + 2(p + \frac{1}{2} \frac{\partial p}{\partial n} \delta n) \times \\
& (R + \frac{1}{2} \delta n) \delta \theta \delta n \frac{1}{2} \frac{\partial \ell}{\partial n} - (p + \frac{\partial p}{\partial n} \delta n) (\ell + \frac{\partial \ell}{\partial n} \delta n) (R + \delta n) \delta \theta
\end{aligned}$$

which, after simplifying and neglecting incremental terms of second and higher order becomes:-

$$\begin{aligned}
& (p \ell R + \ell R \frac{\partial p}{\partial n} \delta n + p R \frac{\partial \ell}{\partial n} \delta n + p \ell \delta n - p \ell R - p \ell \delta n - p R \frac{\partial \ell}{\partial n} n) \delta \theta \\
& = \ell R \frac{\partial p}{\partial n} \delta n \delta \theta \quad \text{4.1}
\end{aligned}$$

where:-

- p = pressure
- n = distance measured normal to the streamlines in the plane of the flow
- R = streamline radius of curvature
- ℓ = stream tube height normal to the plane of flow
- s = distance measured along a streamline

The centrifugal force on the fluid particle is:-

$$\begin{aligned}
& \rho \left\{ (\ell + \frac{1}{2} \frac{\partial \ell}{\partial n} \delta n) \delta n (R + \frac{\delta n}{2}) \delta \theta \right\} \frac{V^2}{(R + \frac{\delta n}{2})} \\
& = \rho V^2 \ell \delta n \delta \theta \quad \text{4.2}
\end{aligned}$$

where:-

- ρ = fluid density
- V = streamline velocity

Equating the pressure force and the centrifugal force:-

$$\frac{1}{\rho} \frac{\partial p}{\partial n} = \frac{V^2}{R} \quad \text{4.3}$$

Seeking a substitution for the term $\frac{1}{\rho} \frac{\partial p}{\partial n}$, the thermodynamic equations:-

$$Tds = dh - \frac{1}{\rho} dp$$

$$\text{and } h_0 = h + \frac{V^2}{2}$$

are written as :-

$$\left. \begin{aligned} T \frac{\partial s}{\partial n} &= \frac{\partial h}{\partial n} - \frac{1}{\rho} \frac{\partial p}{\partial n} \\ \text{and } \frac{\partial h_0}{\partial n} &= \frac{\partial h}{\partial n} + V \frac{\partial V}{\partial n} \end{aligned} \right\} \text{----- 4.4'}$$

where:-
 T = absolute temperature
 s = entropy
 h = enthalpy
 h_0 = total enthalpy

Substituting equations 4.4' into 4.3 and simplifying gives:-

$$1 = \frac{R}{V^2} \frac{\partial h_0}{\partial n} - \frac{R}{V} \frac{\partial V}{\partial n} - \frac{TR}{V^2} \frac{\partial s}{\partial n} \text{----- 4.4}$$

The continuity equation in the curvilinear coordinates used is:-

$$\begin{aligned} \frac{\partial \psi}{\partial n} \delta n &= \rho V \ell \delta n \\ \text{or } \frac{\partial \psi}{\partial n} &= \rho V \ell \text{----- 4.5} \end{aligned}$$

where ψ = stream function or mass flow.

Equations 4.4 and 4.5 are the basic streamline curvature equations.

For irrotational flow it can be shown (see Shapiro 3.10 page 283) that:-

$$\frac{R}{V^2} \frac{\partial h_0}{\partial n} - \frac{TR}{V^2} \frac{\partial s}{\partial n} = 0$$

Substituting this into equation 4.4 gives the basic streamline curvature equation for steady, semi three-dimensional, irrotational flow:-

$$1 = - \frac{R}{V} \frac{\partial V}{\partial n} \text{----- 4.6}$$

Equations 4.5 and 4.6 can be written in their finite difference form:-

$$\frac{\Delta\psi}{\Delta n} = \rho V \ell \quad \text{4.7}$$

$$1 = - \frac{R}{V} \frac{\Delta V}{\Delta n} \quad \text{4.8}$$

where $\Delta\psi$, Δn , $\Delta\ell$ and ΔV are finite increments taken normal to the streamlines. Combining equations 4.8 and 4.7 and rearranging:

$$\Delta V = - \frac{\Delta\psi}{\ell \rho R} \quad \text{4.9}$$

This equation, which gives the finite difference velocity increment normal to a streamline in terms of the streamline radius of curvature and other data, is applied using a finite difference grid consisting of streamlines and normals. The radius of curvature at each point in the grid is calculated by passing an approximating curve through the point under consideration and adjacent points on the same streamline.

In applying the two finite difference equations 4.7 and 4.8, two methods of approach were adopted. The first, which is the least accurate of the two, uses property data (V, ρ, R and ℓ) at one streamline to estimate the data at the next streamline. This corresponds to a simple Euler method in initial value problems. The second method is more accurate as it uses average values of V , ρ and ℓ . (The radius of curvature R used remains as above). This corresponds to a Runge-Kutta type treatment of an initial value problem.

4.3 Boundary condition considerations.

4.3a Introduction.

In Section 3.4 it has already been stated that subsonic and supersonic flows are governed by elliptic and hyperbolic differential equations respectively. The two types of equation are very different in the method of solution which must be applied, in the type of solution which results and in the nature of the boundary conditions necessary for a solution. These mathematical differences are physically seen in the differences between subsonic and supersonic flow. In subsonic flow, disturbances at a point are propagated to all parts of the flow field or, alternatively, each point on

the flow field boundary influences the flow at all points within the flow. Supersonic flow is very different. Discontinuities can exist and disturbances at a point are propagated along Mach lines through that point only. Alternatively stated for supersonic flow, a point on the flow field boundary influences only the regime downstream of that point.

4.3b Boundary conditions for elliptic and hyperbolic equations.

The boundary conditions for the two types of differential equation have been variously termed as "closed" and "open", "jury" and "marching". (See Grandall (4.1) page 351). "Closed" and "jury" for elliptic equations indicates a set of boundary conditions completely enclosing the domain and which are similar to an "encircling jury demanding simultaneous satisfaction". "Open" and "marching" for hyperbolic equations indicates a set of boundary conditions which do not completely enclose the domain but has one open boundary in the direction of which a solution is "marched" out, that is, each step in the solution is taken once and for all and is not influenced by subsequent steps in the solution. These boundary condition types are shown in Sketch 4.4. Also shown in this sketch are the boundary conditions used here for completely subsonic and completely supersonic flow.

It is not the intention in this thesis to establish a boundary condition criteria for the uniqueness of the resulting solution, both for these boundary conditions and particularly for the more complex ones of the next section. It is merely stated that in all cases the boundary conditions used produced convergence to a physically realistic solution.

4.3c Boundary conditions for coexistent elliptic and hyperbolic regimes.

A transonic flow contains both subsonic and supersonic regions of flow which implies mathematically the simultaneous existence of elliptic and hyperbolic regions. The boundary between these regimes is the sonic or Mach one line, the position and shape of which is not known at the commencement of the solution. This boundary must therefore be arrived at via other factors which are known. These factors, subsequently explained at length, are:-

- 1) the condition at the "throat",
- 2) definition of the flow type at each section in the channel.

1) The condition at the throat.

The sonic line boundary between the subsonic and the supersonic flow regions in a convergent-divergent one-dimensional channel is known to be a straight line at the section of minimum flow area. In addition it is known that the channel is "choked", that is, the channel is passing the maximum mass flow and that the choking condition occurs at the throat. If the Mach number at all cross sections in the channel were increased by a small amount, the mass flow would increase in the subsonic region, decrease in the supersonic region and remain constant at the throat. The choking flow condition is therefore that the mass flow remains constant when a small velocity (or Mach number) increment is imposed upon the existing velocity distribution at the throat. The sonic line in a two-dimensional channel does not have a simple form and location as in the one-dimensional case. Therefore only the statement regarding choking flow remains to define the throat.

2) Flow type definition at each section in the channel.

Three types of transonic flows can exist in a convergent-divergent channel and each have a simple and obvious statement defining the flow type.

The first is a continuously accelerating flow. The flow defining statement is that the body of the flow (or the average velocity along a normal) is subsonic upstream of the throat and supersonic downstream of it.

The second is an accelerating-decelerating flow with the channel passing the maximum mass flow. The body of the flow is then subsonic everywhere except at the throat which is choked.

The third is an accelerating, decelerating flow with the channel passing less than the maximum mass flow and which contains a small supersonic region. The body of the flow is then subsonic everywhere.

The full boundary conditions and supplementary statements used for transonic flows are shown in Sketch 4.5.

4.4 The elliptic (subsonic) and hyperbolic (supersonic) basic streamline curvature procedures.

4.4a Introduction and formal statement of the two procedures.

It has already been seen that elliptic and hyperbolic equations involve very different mathematical characteristics and procedures. These differences have been expressed in terms of the boundary conditions and must

now be expressed in terms of the actual numerical procedure followed in the two regimes.

Equations 4.9 expresses the velocity increment normal to a streamline in terms of the streamline radius of curvature:

$$\Delta V = - \frac{\Delta \psi}{\rho \Delta R}$$

Stated here and explained at length below, a subsonic (elliptic) or a supersonic (hyperbolic) streamline curvature procedure is determined solely by the manner in which the streamline curvature is calculated. In a subsonic (elliptic) procedure the curvature at a point should be a function of flow data from upstream and downstream while in a supersonic (hyperbolic) procedure it should be a function of upstream flow data only. This is shown in Sketch 4.6.

This statement can be explained or verified as follows:-

- a) by boundary condition considerations,
- b) by comparison with standard finite difference methods,
- c) by simple flow considerations,
- d) by convergence considerations.

4.4b Boundary condition considerations.

In order to meet the "open" or "marching" type of boundary condition for supersonic flow, a solution must be marched out as in Sketch 4.4b & d. Each step taken in the marching process is taken once and for all and must be independent of downstream conditions or subsequent steps. It follows therefore that in order to take a step involving the curvature, that curvature must be a function of upstream conditions only. This is indicated in Sketch 4.7a.

Conversely for subsonic flow, the upstream and downstream boundary conditions can only be met if the curvature at a point involves upstream and downstream data.

4.4c Comparison with standard finite difference methods.

Standard techniques which represent the differential equations as finite differences taken at discreet points in a field, take into account the elliptic and hyperbolic nature of the equations. Elliptic equation finite difference molecules are formulated using differences and gradients taken in all four directions (N, S, E & W) at the grid point under consideration. This is discussed at length in Crandall (4.1) page 243 and illustrated here in Sketch 4.7b. It is obvious that the equivalent streamline curvature finite difference representation can only involve gradients in all four directions by estimating the curvature at a point using upstream and downstream data.

Similarly, finite difference molecules for hyperbolic equations involve differences and gradients in three directions (N, S & W) and march out the next step in the fourth direction (E). This is discussed in Crandall (4.1) page 396 and illustrated here in Sketch 4.7b. The streamline curvature equivalent of this technique must therefore involve the estimation of the curvature at a point using upstream data only as shown in Sketch 4.7b.

4.4d Simple flow considerations.

In this section a simple supersonic flow at a corner is considered. The resulting flow pattern is the well known compression along a straight Mach line. By applying the correct and incorrect streamline curvature procedures to the flow, it is possible to demonstrate that the one result is realistic while the other is absurd. This is indicated below.

Sketch 4.7c shows a simple supersonic flow at a small bend formed by two straight walls. The influence of the bend is propagated along the Mach wave 4, A₅, B₆ and thus the two streamlines shown remain undisturbed until the Mach wave is reached and then bend to flow parallel to the second wall.

The streamline curvature equation is now applied at the points shown to calculate the velocity gradients and streamline spacing between the wall and streamline A. One point upstream and one point downstream is used to evaluate the radius of curvature at each point to show the gross errors involved in such a procedure. The radii at points 1 to 3 are infinite and hence no gradients or stream spacing changes are present and streamline A is as shown up to point A₃. Point 4 is found to have a finite radius

and hence a gradient and stream spacing change is present and streamline A is found to pass through A_4' . This cannot be correct from consideration of the hyperbolic equation since the geometry change at 4 has been propagated upstream. Furthermore, when the same procedure is applied to find streamline B, a finite radius is found at point A_3 and the influence of the change is propagated further upstream to point B_3' . The propagation of a disturbance upstream using this incorrect procedure to estimate the radius at a point is thus a function of the number of stream tubes used in the finite difference approach to a flow of a given width.

Changing the procedure now to use upstream data only to find the radii, points 1 to 4 have infinite radii and hence are correct as shown. Point 5 has a finite curvature and a gradient and a stream spacing change exists which, if correct, estimates point A_5 correctly as shown. Point 6 has once more an infinite radius and the flow downstream of the Mach wave is thus uniform and parallel as shown.

To further illustrate this point, the incorrect procedure is applied to the analysis of the flow in the supersonic region of a convergent divergent nozzle. Sketch 4.8 shows the velocity distributions obtained using 5 and 10 streamlines. The increasing absurdity of the velocity distribution is clearly seen as the degree of upstream propagation of disturbances is increased as the number of streamlines used is increased. The velocity distribution using the correct procedure is also shown for comparison.

4.4e Convergence considerations.

The convergence of the iteration process is described in detail in the next chapter. In this section the correct and the incorrect streamline curvature procedures are discussed in terms of the effect the two approaches have upon convergence.

The problem of convergence is essentially one of ensuring that each successive iteration performed brings the result nearer to the solution that simultaneously satisfies the continuity and momentum equations. When the incorrect elliptic and hyperbolic streamline curvature procedures are used, the convergence process becomes indeterminate for certain combinations of Mach number and orthogonal grid parameters. This is unrealistic as a solution is known to exist.

The details of this indeterminate convergence are not considered to be relevant to this thesis beyond the following brief statement. The matrix

of equation 5.21 becomes singular for the Mach number and grid parameter combination which produces the indeterminate convergence result. This means that changes made between iterations will not influence the next result and convergence is impossible. The origin of the singularities can be traced to the constants of equations 5.8 and 5.9 which, when positive, introduce the possibility of singularities at certain grid parameter combinations. It can be shown that these constants are positive when the "incorrect" elliptic or hyperbolic streamline curvature procedure is used. A similar indeterminate convergence result has been derived for the "incorrect" procedure in supersonic flow by Wilkinson (3.24).

4.5 The streamline curvature numerical procedure.

4.5a Introduction.

Thus far in this chapter the streamline curvature equations have been derived and written in a finite difference form. The boundary conditions for the equations when the flow is subsonic, transonic and supersonic have been formulated and the elliptic (subsonic) and hyperbolic (supersonic) form of the streamline curvature method indicated.

In this section, the numerical procedure used to obtain solutions for the equations is presented. At the outset it appeared that there were two general approaches which could be adopted. The first is the method which has been used in previous work and has been termed here the method of streamline relaxation between channel boundaries. The second is the method used here and termed the method of streamline and boundary collocation between one bounding streamline and one channel boundary. These two methods will be discussed, compared and the reasons given for the choice here of the second method.

Also described in this section are some basic considerations concerning supersonic grid arrangements and the application of an approximate Kutta-Joukowski condition for channel flow treated by the streamline curvature technique.

It should be mentioned that only the general procedure used will be discussed here and the details of the actual numerical methods will be discussed in Chapters 5 and 6.

4.5b Streamline relaxation between channel boundaries.

This is the approach adopted by Stodola (3.20), Carmichael (3.22) and Katsanis (3.23). An approximate mesh of streamlines and normals are installed between the channel boundaries and the curvature of the streamlines calculated at each point in the grid. Using this curvature distribution and length of each normal, the streamline curvature equation and the continuity equation are integrated numerically and solved simultaneously by a process of successive approximation to find the velocity distribution. A new mesh of streamlines and normals is then estimated and used to repeat the above process until the changes between successive results are insignificantly small. This method is termed here the method of streamline relaxation because the process of using the new streamlines to produce convergence is similar to the finite difference relaxation process.

4.5c Boundary collocation between one channel wall and bounding streamline.

Alpert (3.19) has described a semi indirect streamline curvature method whereby, commencing with the suction surface shape and velocity distribution, streamlines are sequentially estimated. The last streamline represents the shape and location of the pressure surface. This basic approach can be adapted to solve the direct problem by iterating the velocity distribution to conform the last streamline to a given channel boundary.

Referring to Sketch 4.9a and proceeding in a manner similar to the above, a velocity distribution is assumed along one channel boundary, termed here the commencing boundary or surface. The channel curvature is calculated at a series of points along the boundary and the streamline curvature equations used to find the stream tube spacing and the velocity at the first streamline. An approximating curve is passed through the points representing the first streamline and the curvature calculated at each point. The equations are then applied in a similar fashion to find the second streamline. By repeating this process sequentially, all the streamlines are produced until the final or flow bounding streamline is reached.

This final streamline will not coincide with the second channel boundary, termed here the terminating surface or boundary, unless the velocity assumption

at the commencing boundary was correct. The deviation between the bounding streamline and the terminating boundary is calculated at each point and used to estimate a new velocity distribution at the commencing boundary. The new velocity distribution is used to restack or regenerate the streamlines and the process repeated until the bounding streamline coincides with the terminating boundary. This method is thus seen to be a process of boundary collocation between one channel wall and the bounding streamline.

4.5d Comparison of the two methods.

The advantages of the boundary collocation method over the relaxation method are:-

1) Kutta-Joukowski condition.

The channel flow representation of cascade flow involves an assumption of the position and shape of the stagnation streamlines. The boundary collocation method deals directly with changes in the bounding streamlines which leads to the simple introduction of an approximate Kutta-Joukowski condition. This will be discussed fully in Section 4.5f. The relaxation method involves fixed channel boundaries and the Kutta-Joukowski condition cannot be applied simply. No attempt appears to have been made to apply the condition in any previous work.

2) Equations of motion satisfied during each iteration.

The boundary collocation method, using true curvature and stream spacing, satisfies the equations of motion at each iteration even though the iteration itself does not satisfy the particular problem boundary conditions (the bounding streamline does not coincide with the channel boundary). The relaxation method, using curvatures and spacings from a previous iteration, satisfies the problem boundary conditions at each iteration but not the equations of motion until the final iteration. This has not been a problem in completely subsonic flow for convergence has been obtained using the method. Some doubt exists that this will also be the case for transonic flows using a finely spaced grid. Stated simply, in a real flow, which satisfies the equations of motion, the flow upstream and downstream of the throat is subsonic or supersonic as outlined under the discussion of the boundary conditions in Section 4.3c(2). When a simulated flow, that is a flow which does not obey the equations of motion, is used, this additional stipulation may render convergence impossible.

The advantages of the relaxation method over the boundary collocation method are:-

1) Importance of initial assumptions.

Experience has shown that the boundary collocation method requires an initial boundary velocity assumption which is sufficiently accurate to enable a streamline stack to be laid at the first iteration. This problem will be fully discussed in Chapter 6 where computational techniques are considered. It will suffice at this point to state that the initial assumption for the boundary collocation method is critical and has led to time consuming methods to achieve solutions using fine grid sizes. No such problem has been reported in this respect using the relaxation process. This factor probably renders the boundary collocation process computationally more time consuming than the relaxation process.

2) Accuracy of the integration used.

The method of integration used in the boundary collocation process is closely linked with the mechanical procedure of streamline stacking and also with the convergence technique developed in the next chapter. Considerable effort is therefore required to introduce more accurate methods of integration and only simple Euler and trapezoidal rule methods are used here. The relaxation method does not have these limitations.

Conclusions.

The boundary collocation procedure was considered more versatile and to provide a better treatment of the critical transonic region. The disadvantages of the critical accuracy required of the initial velocity assumption and the present limitation in accuracy of the integration method used can be offset at the expense of computational time. The boundary collocation method is therefore used here.

4.5e Supersonic grid considerations.

Solutions to hyperbolic equations should be marched out using a grid pattern made up of the characteristics (Crandall (4.1) page 396). This is clearly impossible using the orthogonal streamline curvature network. A rectangular grid may however be used when the characteristic is the diagonal of the rectangular grid. This is seen following the approach of Crandall (4.1) page 397 or, in terms of the hyperbolic streamline curvature approach,

by a similar argument to that which was used to develop the hyperbolic procedure in Section 4.4d. Referring to Sketch 4.10, a flow at a small bend at point 1 is analysed by the streamline curvature grid shown. If the stream tube spacing is such as to place the first streamline at A'' and the same argument is followed as in Section 4.4d, the influence of the bend at 1 has effect upstream of the Mach line or characteristic 1, A₂, 2. This is incorrect. Similarly, if the streamline is chosen at A', the velocity at A₂ will be different from that at 2. This is also incorrect if the wall is straight downstream of the bend.

The streamline must therefore be chosen so that the Mach line becomes the diagonal of the orthogonal grid. When this is so it can be shown that:-

$$h = \frac{D}{N} \sqrt{M^2 - 1} \quad \text{4.10}$$

where:-
 h = streamwise grid spacing,
 $\frac{D}{N}$ = total channel width / number of streamlines
 = spacing normal to streamlines,
 M = Mach number.

The effect of deviating from this spacing criteria is not of importance. Crandall (4.1) page 398 states that the accuracy will be impaired but the instability mentioned when the spacing is incorrect was not encountered in this work. This means that an average Mach number can be used to determine the grid spacing for a curved channel having a Mach number variation normal to the streamlines. This spacing criteria will be further discussed in Chapter 7.

4.5f Kutta-Joukowski approximation for channel flow.

The channel representation of a cascade flow includes an assumption regarding the shape and location of the stagnation streamlines. Unless the assumption made is fortuitously correct or if some other guide is used to determine them, the velocity distribution will be inaccurate in the regions where the stagnation streamline shape is of importance. Depending upon the stagger and turning angles of the cascade, these regions are the leading and trailing edge zones of the suction surface. This is shown in Sketch 4.11. The leading edge stagnation streamline segment BC, if incorrect, will influence the pressure distribution along the suction surface segment bc. This is

mainly due to the error in total flow width available for the flow in that region. Similarly, the trailing edge suction surface segment de is influenced by the stagnation streamline segment DE . The influence of segments JI and HG are not of any great importance to the profile velocity distribution as these are comparatively remote from the regions of flow disturbance and because the major influence of any error would be along segments AB and EF .

In channel flow terms, the Kutta-Joukowski condition implies that the velocities at the stagnation streamlines along tangential lines such as KL and MN must be equal. This condition can be partially met by collocating the upstream and downstream stagnation region terminating surface boundaries to meet the velocity condition instead of the geometric condition. The final streamline shape is therefore such as to satisfy the Kutta condition but not the condition that the two stagnation region channel surfaces must be the same shape and separated tangentially from each other by one pitch length. The Kutta condition is only partially met because the velocity condition is satisfied upstream and downstream of the cascade but not at the leading and trailing edge stagnation points themselves.

It has been shown that the stagnation streamline shape and location are of greatest importance along segments BC and DE of Sketch 4.11. These segments become the regions which are collocated to satisfy the velocity condition when the suction surface is chosen as the commencing surface. The Kutta-Joukowski approximation consists therefore of using the suction surface as the commencing surface and determining the pressure surface stagnation streamline to meet the velocity condition. The inaccuracy arising from the retention of the original suction surface stagnation streamline is shown to be small in Chapter 7 where, in addition, a method is introduced to satisfy the velocity condition at the trailing edge stagnation point as well.

CHAPTER 5 : CONVERGENCE

5.1 Introduction

The method of boundary collocation to provide a solution of the fluid flow in cascades has been described in the previous chapter. The process of convergence in this context is to arrange that the velocity changes at the commencing surface will result in the required changes at the final streamline.

The approach adopted is to examine the convergence of simple flows using simplifications of the numerical method. This is described in Section 5.2. A basic convergence relationship between velocity changes and streamline changes is derived in Section 5.3. In Section 5.4 a method is devised to apply the simplified analysis to the more complex flows in channels. The procedures used to evaluate the convergence relationships for subsonic and supersonic flow are given in Sections 5.5 and 5.6 respectively. The performance of the convergence theory is discussed in Section 5.7 and the chapter summarised and concluded in Section 5.8. The derivations associated with this chapter are given in Appendix 1.

5.2 Assumptions made

The following flow and numerical assumptions are made:-

- (a) the streamlines are initially straight.*
- (b) the channel shape is such as to produce initially straight streamlines.
- (c) small perturbations of the uniform flow are considered so that incremental terms of higher order than one can be neglected.
- (d) the flow is considered to be either purely subsonic or purely supersonic.
- (e) the flow width, l is assumed constant along a normal.

* This is the converse of the approach adopted in the method of relaxation (Section 4.5b). In that case one streamline is initially perturbed and the iterative process to achieve a flow with all streamlines finally straight is analysed (Carmichael (3.22)). It is more convenient here to commence with straight streamlines everywhere and to study the perturbations in the final streamline.

5.3 Derivation of basic convergence formulae

5.3a Introduction

The basic finite difference equations (4.7 and 4.9) are:-

$$\Delta V = - \frac{\dot{dm}}{\ell \rho R} \quad \text{5.1}$$

and

$$a = \frac{\dot{dm}}{\rho V} \quad \text{5.2}$$

where:-

ΔV = streamline velocity increment, ft/sec

$\dot{dm}$ = streamline mass flow ($d\psi$ in Chapter 4), lbm/sec

ℓ = flow height normal to the plane of flow, ft

ρ = density, lbm/ft³

R = radius of curvature, ft

a = stream tube flow area, ft²

The overall approach of this section is to apply equation 5.1 sequentially to each streamline as outlined in Section 4.5c, before and after an arbitrary velocity change is made at the commencing surface. Subtraction of the overall flow areas in the two cases gives the final streamline area changes at each point. By manipulation of the relevant algebra, a basic convergence relationship between streamline area changes and commencing surface velocity changes is obtained. This is provided for 4 cases, subsonic and supersonic flow and using the two methods of Section 4.2 where the Euler and Runge-Kutta type methods are used in the numerical procedure. There will therefore be four results at each step in the derivation.

5.3b Derivation

Reference to Sketch 5.1 shows the initial and final network of streamlines and normals where, at normals $m-1$, m , $m+1$, etc, velocity changes of dV_{m-1} , dV_m , dV_{m+1} etc, are made at the commencing surface and which produce flow area changes of $da_{m-1,1}$, $da_{m,2}$, $da_{m+1,n}$, etc at streamlines 1, 2, n etc. Using a 3 point parabolic relationship for the streamline curvature at a grid point and following the outline of Section 5.3a, the convergence relation between velocity and streamline area changes is derived in Appendix 1a. The results are, for the four cases:-

Simple Euler type method, subsonic flow:-

$$y_{m,n} = 2y_{m,n-1} - (1 - K_m)y_{m,n-2} - K_m \{h'_m y_{m-1,n-2} + (1 - h'_m)y_{m+1,n-2}\} \quad 5.3$$

Simple Euler type method, supersonic flow:-

$$y_{m,n} = 2y_{m,n-1} - (1 - K_m)y_{m,n-2} - K_m \{(1 + h''_m)y_{m-1,n-2} - h''_m y_{m-2,n-2}\} \quad 5.4$$

Runge-Kutta type method, subsonic flow:-

$$y_{m,n} = (2 + \frac{1}{2}K_m)y_{m,n-1} - (1 - \frac{1}{2}K_m)y_{m,n-2} - \frac{1}{2}K_m \{h'_m(y_{m-1,n-2} + y_{m-1,n-1}) + (1 - h'_m)(y_{m+1,n-2} + y_{m+1,n-1})\} \quad 5.5$$

Runge-Kutta type method, supersonic flow:-

$$y_{m,n} = (2 + \frac{1}{2}K_m)y_{m,n-1} - (1 - \frac{1}{2}K_m)y_{m,n-2} - \frac{1}{2}K_m \{(1 + h''_m)(y_{m-1,n-2} + y_{m-1,n-1}) + h''_m(y_{m-2,n-2} + y_{m-2,n-1})\} \quad 5.6$$

where:-

$y_{m,n}$ = dimensionless flow area change at normal m , streamline n

$$= \frac{da_{m,n}}{Na_m} \quad 5.7$$

N = total number of streamlines

a_m = stream tube flow area at normal m

K_m = dimensionless grid constant

$$= \frac{2 S_{fct} (1 - M_m^2) D_m^2}{N^2 h_{1m} h_{2m}} \quad \text{for subsonic flow} \quad 5.8$$

$$= - \frac{S_{fct} (1 - M_m^2) D_m^2}{N^2 h_{1m} (h_{1m} + h_{2m})} \quad \text{for supersonic flow} \quad 5.9$$

S_{fct} = iteration starting factor, $0 < S_{fct} < 1.0$, required in numerical procedure, described fully in Chapter 6

M_m = Mach number at normal m

D_m = flow width at normal m , ft

h_{1m}, h_{2m} = grid spacings associated with normal m

h'_m = subsonic grid spacing factor at normal m

$$= \frac{h_{2m}}{h_{1m} + h_{2m}} \quad 5.10$$

h''_m = supersonic grid spacing factor at normal m

$$= h_{1m}/h_{2m} \quad 5.11$$

Although appearing to be absent in the convergence relations, the velocity change is actually represented because:-

$$y_{m,1} = -x_m \quad 5.12$$

where x_m = dimensionless velocity change at normal m

$$= \frac{(1-M_m^2)}{N} \frac{dV_m}{V_{m,0}} \quad 5.13$$

A convergence relationship has been derived between the commencing surface velocity changes (x_m, x_{m+1} etc) and the flow area changes $y_{m,n}, y_{m+1,n}$ etc) at the n th streamline in terms of known grid quantities and Mach numbers. When therefore any given iteration is known to deviate by $y_{m,n}, y_{m+1,n}$ etc, at the final streamline ($n=N$), the evaluation of the convergence formulae will determine the velocity changes x_m, x_{m+1} etc, to be made at the commencing surface which will theoretically eliminate the deviations at the next iteration.

Before this can be done two problems must be dealt with. The first is to apply the formulae to a transonic flow (the assumption was made that the flow is either subsonic or supersonic). The second problem is to derive a procedure for evaluating the formulae. These two problems are considered in the next sections.

5.4 General considerations for the application of the simplified analysis to real channel flows.

5.4a Convergence assumption for transonic flow.

The analysis of the previous section assumed that the flow was either subsonic or supersonic. A further assumption is needed for transonic flows. The assumption is that the convergence characteristics of a transonic region is determined by the average Mach number along a normal. Transonic flows for convergence purposes then reduce to subsonic or supersonic flows.

5.4b Subsonic convergence arrangement.

When the average Mach number along a normal is less than one, the subsonic convergence relationship is applied as in Sketches 5.4a, b and c. These sketches represent the purely subsonic flow and the three types of transonic flow discussed in Section 4.3 and Sketch 4.5.

5.4c Supersonic convergence arrangement.

When the average Mach number along a normal is greater than one the supersonic convergence relationship is applied. Section 4.3 has described the boundary conditions for a supersonic region and the "marching" procedure whereby the solution is determined at one normal finally before proceeding to the next downstream normal. This means that in the consideration of the convergence relation for supersonic flow, changes at one normal only need be considered while those upstream remain unchanged. This convergence of one normal at a time in supersonic flow is termed here "marching convergence" to correspond with the numerical procedure used.

The marching convergence procedure cannot however be applied to a transonic region, that is a region where the average Mach number is greater than one and where a subsonic region is present. This is because the subsonic region requires a simultaneous treatment of all the grid points. The region containing the subsonic flow is therefore considered as a whole or as a "block". The simultaneous application of the supersonic convergence relationship to a number of supersonic normals is termed here "block convergence". This is shown in Sketch 5.2d. Also shown in this sketch is the "marching convergence" used once fully supersonic flow is established.

5.5 Calculation procedure for convergence of subsonic flow.

5.5a Introduction.

Equations 5.3 and 5.4 relate area and velocity changes in the streamline curvature grid for subsonic flow. These equations are in the form of a recurrence formulae relating quantities in the two dimensional grid. No method has yet been given to evaluate a given velocity change when a given area change distribution is specified. While the limits of the relationship is obviously between the commencing surface and the final streamline in the normal direction, the formulae in equations 5.3 and 5.4 extends infinitely far upstream and downstream. In this section therefore, the streamwise bounds to the grid are described and then used to devise a simple expression relating a velocity change required at a point to the surrounding flow area change distribution required.

5.5b Description of the dummy grid used.

An examination of equations 5.3 and 5.5 shows that the convergence relation extends one normal to the left ($m-1$) and one normal to the right ($m+1$) of any given normal m . When therefore the conditions along two normals are known (usually that it is fixed and that no changes take place) the convergence formula for the region in between can be evaluated. This is illustrated in Sketch 5.3 which shows a grid of M normals for which it is assumed that normals one and M are fixed. The fixed normal condition is exactly true when the normals are the upstream and downstream grid boundaries. It is approximately true when the normal M is the normal at the throat in a choked flow.

Ideally, the grid of M normals in Sketch 5.3 should represent the whole of the subsonic region. Such an approach would however lead to a large matrix in the final formulation and solution and the following alternative has been adopted. A dummy grid of M normals is used at each point, the size of M being sufficiently large so that the assumption of unchanging normals at the ends does not influence the result at the normal under consideration. The dummy grid is equally spaced on either side of the normal under consideration when possible and this means that M must be an uneven number. When the normal under consideration is near one end of the actual grid, the end of the dummy grid coincides with the end of the actual grid and the location of the normal in the dummy grid can be displaced to one side. These factors are also illustrated in Sketch 5.3.

5.5c Derivation of procedure

It is shown in Appendix 1b that the area changes $y_{m,n}$ may be written for each of the M normals in the dummy grid as:-

$$\begin{array}{l}
 \begin{array}{c} \xrightarrow{M} \\ 1 \end{array} \xrightarrow{i} \\
 y_{1,n} = c_{1,n}x_1 + c_{2,n}x_2 + \dots + c_{i,n}x_i + \dots \\
 \qquad \qquad \qquad + c_{M-1,n}x_{M-1} + c_{M,n}x_M \\
 y_{2,n} = c_{M+1,n}x_1 + c_{M+2,n}x_2 + \dots + c_{M+i,n}x_i + \dots \\
 \qquad \qquad \qquad + c_{M+M-1,n}x_{M-1} + c_{M+M,n}x_M \\
 \begin{array}{c} 1 \quad M \\ \vdots \\ m \end{array} \dots \dots \dots \\
 y_{m,n} = c_{(m-1)M+1,n}x_1 + c_{(m-1)M+2,n}x_2 + \dots + c_{(m-1)M+i,n}x_i + \dots \\
 \qquad \qquad \qquad + c_{(m-1)M+M-1,n}x_{M-1} + c_{(m-1)M+M,n}x_M \\
 \dots \dots \dots \\
 y_{M,n} = c_{(M-1)M+1,n}x_1 + c_{(M-1)M+2,n}x_2 + \dots + c_{(M-1)M+i,n}x_i + \dots \\
 \qquad \qquad \qquad + c_{(M-1)M+M-1,n}x_{M-1} + c_{(M-1)M+M,n}x_M \quad \text{5.14}
 \end{array}$$

where $c_{j,n}$ are constants, n indicating the streamline number and j a counter running from 1 to M^2 as shown. The equation is now in the form of an unknown set of constants $c_{j,n}$ and velocity changes x_m and the known final streamline area changes $y_{m,n}$. The object is now to evaluate the constants. This is done in Appendix 1b where it is shown that a general expression for any constant in equation 5.14 is:-

For the simple Euler type method:-

$$\begin{aligned}
 c_{(m-1)M+i,n} = & 2c_{(m-1)M+i,n-1} - (1 - K_m)c_{(m-1)M+i,n-2} \\
 & - K_m \{ h'_m c_{(m-2)M+i,n-2} + (1 - h'_m)c_{mM+i,n-2} \} \quad \text{5.15}
 \end{aligned}$$

For the Runge-Kutta type method:-

$$\begin{aligned}
 c_{(m-1)M+i,n} = & (2 + \frac{1}{2}K_m)c_{(m-1)M+i,n-1} - (1 - \frac{1}{2}K_m)c_{(m-1)M+i,n-2} - \frac{1}{2}K_m \{ h'_m \\
 & (c_{(m-2)M+i,n-2} + c_{(m-2)M+i,n-1}) + (1 - h'_m)(c_{mM+i,n-2} + c_{mM+i,n-1}) \} \quad \text{5.16}
 \end{aligned}$$

Equations 5.15 and 5.16 can be evaluated for each value of m and i when the coefficients for $n-2$ and $n-1$ are known. The coefficients for subsequent values of n follow sequentially until $n=N$, the total number of streamlines. The calculation sequence is started by finding the coefficients for $n=1$ and $n=2$. From Appendix 1b the starting coefficients are:-

Simple Euler type method

All coefficients zero except:-

$$c_{(m-1)M+m,1} = -1 \quad 5.17$$

$$c_{(m-1)M+m,2} = -2 \quad 5.18$$

Runge Kutta type method

All coefficients zero except:-

$$c_{(m-1)M+m,1} = -1 \quad 5.19$$

for $n=2$ the coefficients are:-

$$c_{(m-1)M+m,2} = -(2 + \frac{1}{2}K_m)$$

$$c_{(m-1)M+m-1,2} = \frac{1}{2}K_m h'_m \text{ for } 3 \leq m$$

$$= 0 \quad \text{for } m=2$$

$$c_{(m-1)M+m+1,2} = \frac{1}{2}K_m (1 - h'_m) \text{ for } m \leq M-2$$

$$= 0 \quad \text{for } m=M-1$$

5.20

With the starting values known the constants for the final streamline ($n=N$) can be found sequentially. Equation 5.14 is then a linear equation in x and y which in matrix form is:-

$$C X = Y$$

and the solution is

$$X = C^{-1}Y \quad 5.21$$

Equation 5.21 gives the velocity change at each point in the dummy grid when the area change distribution is known.

5.6 Calculation procedure for the convergence of supersonic flow.

5.6a Introduction.

As in the case for subsonic flow, the streamwise bounds to the recurrence relationships of equations 5.5 and 5.6 are defined and then used to find a simple relation between velocity and area changes.

5.6b Description of grid used.

Equations 5.5 and 5.6 indicate that the expression for any one normal m contains quantities from two normals to the left ($m-1$ and $m-2$). When therefore the conditions along two adjacent normals are known the convergence formulae become finite and can be evaluated.

In Section 5.4c the "block" and "marching" convergence concepts were described where changes along one or more normals are considered. In both cases the two upstream normals are assumed to be fixed which supplies the condition to evaluate the equation. The fixed normal condition is exactly true for marching convergence, the flow upstream then consisting of supersonic or transonic flow for which convergence has already been established. The condition is approximately true for block convergence for then one normal is the throat normal for which the choking condition renders the changes minimal and the other normal is the last normal in the subsonic region. Changes do take place at this normal but are assumed not to be of consequence.

Sketch 5.4 illustrates the grid of M normals used in the calculation procedure, the first two of which are assumed fixed. The marching convergence case is a special case of the ^{block} convergence where $M=3$.

5.6c Derivation of procedure.

The area changes at each normal are written in terms of an array of constants $c_{j,n}$ in Appendix 1c. Referring to Appendix 1c, the area changes are:-

$$\begin{array}{l}
 y_{1,n} = 0 \quad \xrightarrow[3]{m} i \\
 y_{2,n} = 0 + 0 \\
 y_{3,n} = 0 + 0 + c_{2M+3,n} x_3 \\
 y_{4,n} = 0 + 0 + c_{3M+3,n} x_3 + c_{3M+4,n} x_4 \\
 \dots \dots \dots \\
 y_{m,n} = 0 + 0 + c_{(m-1)M+3,n} x_3 + \dots + c_{(m-1)M+i,n} x_i + \dots + c_{(m-1)M+m,n} x_m \\
 \dots \dots \dots \\
 y_{M,n} = 0 + 0 + c_{(M-1)M+3,n} x_3 + \dots + c_{(M-1)M+i,n} x_i + \dots + c_{(M-1)M+M,n} x_M
 \end{array}$$

The constants are evaluated in Appendix 1c and the general expressions are:-

for the simple Euler type method

$$c_{(m-1)M+i,n} = 2c_{(m-1)M+i,n-1} - (1-K_m)c_{(m-1)M+i,n-2} \\ - K_m \{ (1 + h_m'')c_{(m-2)M+i,n-2} - h_m''c_{(m-3)M+i,n-2} \} \quad 5.23$$

for the Runge-Kutta type method

$$c_{(m-1)M+i,n} = (2 + \frac{1}{2}K_m)c_{(m-1)M+i,n-1} - (1 - \frac{1}{2}K_m)c_{(m-1)M+i,n-2} \\ - \frac{1}{2}K_m \{ (1 + h_m'')(c_{(m-2)M+i,n-2} + c_{(m-2)M+i,n-1}) \\ - h_m''(c_{(m-3)M+i,n-2} + c_{(m-3)M+i,n-1}) \} \quad 5.24$$

As for subsonic flow the coefficients are sequentially evaluated up to $n=N$ commencing with the known starting values for $n=1$ and $n=2$ which are:-

simple Euler type method

all coefficients zero except:-

$$c_{(m-1)M+m,1} = -1 \quad 5.25, \quad c_{(m-1)M+m,2} = -2 \quad 5.26$$

Runge-Kutta type method

for $n=1$, coefficient as in equation 5.25

$$\left. \begin{aligned} c_{(m-1)M+m,2} &= - (2 + \frac{1}{2}K_m) \\ c_{(m-1)M+m-1,2} &= \frac{1}{2}K_m (1 + h_m'') \quad \text{for } m \geq 5 \\ c_{(m-1)M+m-2,2} &= \frac{1}{2}K_m h_m'' \end{aligned} \right\} \begin{aligned} &= 0 \quad \text{for } m=4 \\ &= 0 \quad \text{for } m=3 \end{aligned} \quad 5.27$$

The resulting linear equation in x and y is of the same form as in the subsonic case and the solution is given by equation 5.21.

5.7 Application of the convergence analysis

The convergence analysis of this Chapter is adequate for all the examples treated. Difficulties which were experienced and the steps taken to overcome them are described below. The analysis was derived subject to the assumptions made and channel and flow configurations which do not conform to the assumptions will deviate in behaviour from the ideal predicted. The method does not eliminate all trial and error required for convergence and also for a high rate of convergence. Controlling variables have to be determined by experience in using the method in general and for specific problems in particular. These controlling variables are:-

- 1) a damping factor (d_f) which reduces the theoretical velocity changes and suppresses over corrections, that is,

$$\Delta V \text{ used} = d_f \times \Delta V \text{ theoretical} \quad \text{5.28}$$
- 2) M, the size of the region assumed to influence the change made at a point (see Section 5.5)
- 3) N, the number of points used normal to the flow.

No attempt is made to optimise these variables. The main task has been to obtain solutions and although some indication is given of the effect on the rate of convergence, this was not fully investigated.

The deviations from an ideal convergence behaviour are:-

- 1) Over correction oscillations in final streamline position,
- 2) Gross over corrections in transonic flows near Mach one,
- 3) Divergence and slow convergence through M being too small,
- 4) Divergence in transonic flows through N being too small.

5.7a Over correction oscillations in final streamline position.

When the theoretical velocity changes are too large an oscillation sets in so that the final streamline position alternates above and below the terminating boundary. Convergence fails if the oscillations increase in magnitude or remain constant and convergence is slow if the oscillations become smaller.

Oscillations can be detected by examining the variation of solution error with each iteration. Figure 5.5 gives an example of an oscillation which was suppressed by a reduction in damping factor.

5.7b Gross over corrections in transonic flows near Mach one.

At Mach numbers close to unity, the theoretical velocity changes are large. This is because of the linear assumptions made in the derivations. The best example of the resulting error near Mach one is to consider the theoretical velocity change for a small flow area change at Mach one itself. This is infinite even for the smallest area change ($1-M^2=0$) whereas in reality the smallest velocity increment made would change the Mach number away from one to give a finite velocity increment ($1-M^2$ becomes finite). The same logic applies for Mach numbers near unity and the velocity changes estimated are large because the analysis has made no provision for the change in Mach number introduced by the increment itself.

The resulting gross over correction of velocity using the standard convergence procedure is shown in Figure 5.6a. A special treatment is therefore needed for channels having a particularly slow rate of area convergence and divergence in the throat region.

The treatment devised and found successful was to use a very low damping factor (equation 5.28) in the region of difficulty. The procedure adopted is to use the Mach number far upstream or far downstream of the throat when calculating the actual velocity increment from equation 5.13. The resulting effective damping factor in the throat region is from 15 to 25 times lower than the overall damping factor. The stability of the iteration procedure using this method of damping is shown in Figure 5.6b.

5.7c Divergence and slow convergence through M being too small.

In Section 5.5 the dummy grid of M normals used to find the change required at one normal was described. The quantity M is considerably smaller than the subsonic region itself. The assumption made in the dummy grid analysis is that the normals at the extremities do not change. All other normals in the dummy grid are assumed to undergo the same changes as the actual normals they represent in the real grid. When M is sufficiently large the inaccuracy of the assumptions made at the end of the dummy grid does not influence the center of the grid. If M is too small, the analysis will fail to predict the actual behaviour of the numerical procedure.

Two types of failure occurred. The first is an excessively slow convergence rate as shown in Figure 5.7a. Also shown here is the improvement resulting from an increase in the region of influence. The second

deviation, shown in Figure 5.7b, is a slow divergence which was eliminated by a further increase in the region of influence.

5.7d Divergence in transonic flows through N being too small.

A velocity change in a transonic flow causes opposite effects in the subsonic and the supersonic regions. A velocity increase causes an expansion of the flow in the supersonic region and a contraction in the subsonic region. The result of these effects is the change in final streamline position. It is therefore essential that the finite difference steps normal to the flow be made sufficiently small so that the resulting normal integration adequately represents the effects of the change made. Figure 5.8 shows an example of divergence in a transonic flow with an average Mach number less than one. The divergence indicates that the supersonic region effect predominates. Increasing the number of points on the normal (N) eliminated the divergence.

5.8 Summary and conclusions.

A convergence relationship between velocity changes at the commencing surface and changes in final streamline position for simple flows using approximations of the numerical procedure was developed. These formulae were applied to real channel flows as outlined in Section 5.4 and evaluated by using the calculation procedures of Sections 5.5 and 5.6.

Convergence difficulties encountered in practise were eliminated by a rearrangement of the controlling variables as indicated in Section 5.7. The analysis is thus adequate in providing a rapid and stable convergence. Additional data regarding the choice of the convergence variables is given in Chapter 6 when the computer program input is discussed.

CHAPTER 6 : THE COMPUTER PROGRAMME

6.1 Introduction.

The basic streamline curvature method has been described in Chapter 4 and the convergence techniques in Chapter 5. The remaining numerical procedures and the computer programme itself is described in this chapter. The computer language used is FORTRAN IV and the programme has been used on IBM 7094, 360/65 and CDC 6600 machines.

There are many ways of describing a computer programme and the description must suffice to serve a variety of purposes. The main intention here is to describe the numerical methods used. A secondary aim is to give sufficient detail to enable the computer programme to be used by others. It is felt that the best compromise in presentation is to give an outline of the numerical methods used in each subroutine, a description of the operating techniques, a descriptive programme symbol list, a full programme listing with comment cards and examples of working data decks. Flow charts are used in the description where they are needed to describe the overall flow of calculation.

The chapter and associated appendices are therefore as follows. The main programme and the boundary condition checking subroutine are given in Sections 6.2 and 6.3. Other subroutines excluding those used for convergence are discussed in Section 6.4. All convergence subroutines are described in Section 6.5 and in Section 6.6 the operation and difficulties and limitations of operation are discussed. Section 6.7 outlines a few avenues along which the programme could be developed in future work. Section 6.8 concludes the chapter. Appendix 2a contains an interpreted alphabetical list of the main FORTRAN symbols used in the programme. A full programme listing with comment cards appears in Appendix 2b and Appendix 2c contains three sample working data decks for typical cascades for the three basic channel flow types considered.

6.2 Main programme.

The main programme generates all input data (thermodynamic, geometric and control), and directs the main flow of the calculation. An overall flow diagram is given in Figure 6.1 and the elements of the programme are described in detail below. Three detailed sample data decks are given in Appendix 2c.

6.2a Thermodynamic data.

The thermodynamic data can be supplied either as property values from an expansion line on a chart or as a set of perfect gas constants. The

property chart input enables fluids near or below the saturation line to be represented and the input is specific volume values at equal enthalpy decrements along the assumed (isentropic or otherwise) line of expansion. The specific volume is converted to density and the enthalpy decrease from a specified stagnation value is converted to a velocity. This table of velocity and density is converted to one having equal velocity increments. A similar table is prepared if the fluid is a perfect gas and the fluid density is found at any point in the programme by specifying the velocity only.

The sonic velocity for the expansion process is found by a simple iterative process which converges to the velocity for which the product (velocity x density) is a maximum. This value is used to indicate a subsonic or supersonic flow and to define a choking flow.

6.2b Geometric data.

Referring to Figure 6.2 and Appendix 2c, the geometry of the channel is supplied as two coordinate arrays representing the commencing surface and the terminating surface and a pitch length through which the terminating surface is moved in the positive y direction. The two arrays are usually the pressure and suction profile surfaces respectively. The arrays must have a continuously increasing x coordinate and a flow in the negative x direction cannot be accommodated. For interpolation purposes both arrays must extend beyond the extremities of the grid used for the actual computation. The x coordinates only of the first grid are read in and the corresponding y values found by interpolation.

6.2c Calculation control data.

Data controlling and defining the computational procedure are read in. This data is described in detail below and for purposes of reference are best located via the sample data decks of Appendix 2c and the symbol listing and subroutine reference of Appendix 2a.

6.2d The main calculation and iterative loop.

Prior to the iterative loop an initial velocity distribution is found on a one dimensional basis or read in as input. The iterative loop is then performed a number of times in order to enable a finely spaced grid to be accommodated. It is not possible to iterate directly from a one dimensional velocity estimate towards a solution with a finely spaced grid. This is

explained in Section 6.6b. The fine grid solution is produced gradually in a series of steps, each result providing the initial conditions for the next step.

The boundary conditions are set up upstream and downstream (for a subsonic downstream flow) upon the assumption that the flow is uniform. The two channel surfaces must therefore be straight and parallel in the boundary regions. The boundary conditions are inserted as two straight normals with evenly spaced grid points which do not change during subsequent iterations.

The iterative loop consists of the generation of all the streamlines using the stacking procedure of Chapter 4. The boundary conditions are checked and the changes made to the velocity distribution using the convergence method of Chapter 5. This process is repeated until the boundary conditions are satisfied to within the specified tolerance limits and the iterative loop is terminated. If a supersonic marching procedure is required the relevant subroutine is called. Intermediate output is produced and a subroutine called to generate the data for the next step in producing a fine grid. When all the required steps are executed the solution is complete.

6.3 The boundary condition checking subroutine BCHEK

The boundary conditions are:-

- 1) that the final streamline coincides with the terminating surface,
- 2) that the flow is choked at the throat in choked subsonic flow and in subsonic - supersonic flow,
- 3) that the average Mach number along a normal is subsonic everywhere except at the throat in choked subsonic flow and subsonic upstream of the throat and supersonic downstream of the throat in subsonic - supersonic flow.

The first condition is a relatively simple one and consists of calculating the distance between the end of a normal and the terminating surface. If this distance is within the specified tolerance the boundary condition is considered satisfied. In choked flows this boundary condition is satisfied by iteratively adjusting the channel mass flow.

The second boundary condition is a more complex one as it involves the position of the throat, the choking condition at the throat and is influenced by the other two boundary conditions. It was found by experience that a more stable convergence resulted when all three boundary conditions were simultaneously striven for. When achieved independantly of one another, large gradients arise in the throat region which leads to instabilities when, after one boundary

condition is satisfied, attention is switched to another.

The second boundary condition is achieved as follows. The present location of the throat is checked by determining the deviation at each flow normal from the choking condition.* The throat is assumed to be at the normal having the minimum deviation. When the minimum does not occur at the throat, the location of the throat is changed. The choking condition is achieved by adjusting the throat commencing surface velocity iteratively until the deviation is within the specified tolerance.

The third boundary condition is checked and when not satisfied a velocity increment added at the commencing surface to rectify the situation.

The overall flow chart for the subroutine is shown in Figure 6.3. It should be noted that in choking flows the distances of the final streamline from the terminating surface at each normal are adjusted to allow for the mass flow change made. The final and a very essential step in the subroutine is to print out the boundary condition status. This is necessary to trace the reason for a convergence failure and to provide an indication of the steps to be taken to counter the failure.

6.4. Subroutines excluding those for convergence.

6.4a Subroutine SLCALC.

This subroutine generates (or "stacks") all streamlines sequentially from the first to the last between two specified normals. The overall flow chart is shown in Figure 6.4. The subroutine is executed according to a number of controlling variables most of which are designed to eliminate operational difficulties with the programme. A fuller description of these variables will therefore be found in Section 6.6. The controlling variables are:-

- 1) K2. Controls use of data in streamline curvature calculation. When $K2=1$ the stream tube area and velocity gradient are calculated using the simple Euler type method. When $K>1$ (say 30) an iterative loop is set up whereby the area and gradient are found using the Runge-Kutta type method. The loop is terminated when converged provided the number of iterations does not exceed K2. These two options

* The choking condition used is that the average velocity along a normal is the sonic velocity. No significant difference was seen in the results when the correct condition (Section 4.3c) was applied.

correspond to the two methods already mentioned in Chapters 4 and 5.

- 2) SFCT. Controls the fraction of actual radius of curvature used. (See also Section 6.6). This factor, $0 < \text{SFCT} < 1.0$, multiplies the streamline radius of curvature directly and is designed to ensure a stable and gradual progression between the one dimensional velocity estimate and the final solution ($\text{SFCT}=1.0$).
- 3) IRC. Controls the calculation of streamline curvature and slope. (See also Section 6.6). When $\text{IRC}=1$ the normal procedure of calculating the curvature and slope afresh at each point during the iteration is used. When $\text{IRC}=0$ the calculations are suppressed and slope and curvature values from a previous solution are used.
- 4) KRC. Controls the calculation of streamline curvature and slope. When $\text{KRC}=1$ the curvature and slope are calculated everywhere at the commencement of the iterative process only. A grid must therefore exist from a previous solution. When $\text{KRC}=0$ the normal procedure is used. The first option on this number was not used but is retained for the sake of flexibility as, when used, the programme is similar to the "relaxation" method described in Chapter 4.
- 5) VST. Sonic velocity tolerance. (See also Section 6.6). Experience with transonic flows showed that a grid point near the sonic line could become alternately subsonic and supersonic during the iteration process. The period of these changes is a number of iterations and the solution error introduced by the change between a supersonic and a subsonic flow type at the point prevents convergence. This effect was damped out by introducing the quantity VST and a process whereby a point at which the velocity was initially supersonic is subsequently declared subsonic only after the velocity drops below the sonic velocity by the amount VST. Similarly, a grid point initially subsonic is declared supersonic only when the velocity exceeds the sonic value by VST.
- 6) JRC. Controls flow type declaration at a point. The first of two options, $\text{JRC}=0$, specifies the method of (5) above. The second option, $\text{JRC}=1$, declares the flow type "once for all" at every point at the beginning of the iteration process. This second option, valid only when progressing from a coarse grid to a fine grid, finds the flow type at each point by interpolation from the previous coarse grid. The use of the second option was not fully investigated but is retained for flexibility.

- 7) VMIN, VMAX. Limits the velocity during iteration. (See also Section 6.6). Absurdly high streamline curvature can arise during the streamline stacking procedure and result in correspondingly absurdly high or even negative velocities. When the above limits are set this tendency can be detected and in most situations stably eliminated in subsequent iterations.

Referring to the flow chart in Figure 6.4, the streamline stacking procedure is as follows:-

- 1) All slopes and curvatures are found at the commencing surface,
- 2) all velocity gradients and stream tube spacings are found for the first streamline and the grid points on the first streamline located.
- 3) all slopes and curvatures of the first streamline are found and the process repeated for all streamlines.

6.4b. Subroutine SUPMCH.

This subroutine marches out the supersonic solution. It applies the streamline generating subroutine SLCALC iteratively to satisfy the terminating surface boundary condition at each normal in turn.

6.4c Subroutine DYDXRC.

This subroutine calculates the slope and curvature at a given grid point. A three point parabolic formula is used. A set of dummy axes are drawn through the outer two points which are selected according to the flow type as outlined in Sections 4.4a and 6.4a. If the velocity is subsonic one point upstream and one point downstream are used. Two points upstream are used if the velocity is supersonic. The assumption is made that the grid points are evenly spaced.

6.4d Subroutine NEXTPT.

Given the slope and coordinates at one point on a streamline and the stream tube spacing of the next streamline at that point, the next streamline is located along a normal through the point. The method used is to erect a straight line of length equal to the stream tube spacing at the grid point at 90° to the streamline.

6.4e Subroutine ENDPT.

This subroutine calculates the distance between the end of a normal and the terminating surface. The three terminating surface array points closest to the end of the normal are found and a parabola on transformed axes is passed through the points. The distance, normal to the transformed x axis, between the end of the normal and the parabola is the distance required. The distance is non-dimensionalised by the total flow width.

6.4f Subroutine FINDRO.

Supplied with the velocity this subroutine returns the corresponding density from the velocity-density table generated in the main programme. The tabular values nearest to the required value are determined and used to find the actual density by interpolation.

6.4g Subroutine FINDY.

The commencing surface x coordinates of a grid are supplied and the corresponding y values found by interpolation from the commencing surface array X1, Y1. The interpolation procedure used is the spline fit. (Subroutine EXTRA).

6.4h Subroutine FINDV.

This subroutine performs an iterative "homing" process to find the velocity which gives the value of (mass flow/area) supplied as input. The velocity is specified as being subsonic or supersonic.

6.4i Subroutine VELDIS.

This subroutine generates the mass flow, commencing surface velocity distribution and values of $1-M^2$ (for convergence purposes) either by calculation or as input as specified.* The calculated quantities are based upon a one dimensional estimate of the channel width. The relation $1-M^2 = 1 + \frac{V}{p} \cdot \frac{dp}{dV}$ of Chapter 5 is used to find $1-M^2$.

* The initial velocity distribution may be read in to enable the iterative process to be continuous between computer runs. The option is determined by JREAD and examples are given in Appendix 2c.

6.4j Subroutine NUGRID.

This subroutine reads as input the data for the new step to be taken in the procedure used in the main programme to eventually produce a fine grid solution. When an increase in the number of streamwise grids points (NPS) are specified the distribution of grid points along the commencing surface is arranged in the same form as the original grid. The new velocity, flow area and $1-M^2$ distributions are found by interpolation from the old grid values.

6.4k Subroutine KUTTA.

This subroutine applies the Kutta-Joukowski approximation at the trailing edge of a cascade. (See Section 4.5f). The commencing surface must be the suction surface where the velocities are adjusted until the approximation is satisfied. This approximation is that all terminating surface velocities downstream of the trailing edge must be equal to the corresponding commencing surface velocities at the same distance downstream of the cascade. The required terminating surface velocity change is assumed equivalent to an area change and used as such in the convergence subroutines. This assumption is far from true but convergence was obtained using very low damping factors (0.1 to 0.2). The convergence theory should be extended to relate velocity charges at both walls (see Section 6.7) in future work.

6.4l Subroutine RESULT.

A printout of grid coordinates, velocities and Mach numbers is produced according to various control numbers.

6.4m Subroutine EXTRA.

An interpolation subroutine using the spline method programmed by Fish (ref. 6.1).

6.4n Subroutine PAR.

An interpolation routine using a three point parabolic formula.

6.4o Subroutine EL.

The flow height l normal to the plane of flow is found. The input to the subroutine is the x coordinate at the point at which the height is to be calculated and the fractional overall height increase between two specified

x coordinates. The height is taken to vary linearly between these two points and is taken as constant upstream and downstream of them.

6.5 Convergence subroutines.

This section describes the subroutines associated with convergence. The overall approach is to generate arrays of constants prior to the iterative process which are used to find the new velocities at the end of each iteration. These constants are $1-M^2$ (VCC(N,1)) for each normal, the coefficients of the inverted matrix of equation 5.21 (AI and A) and the numbers of the first normal to be used to calculate the changes at each normal (N1). The constants are generated in subroutine VCCONS and are applied via subroutines VNEW and SUPMCH.

6.5a Subroutine VCCONS.

This subroutine generates the constants as outlined above.

The first step in generating the velocity correction constants (AI and A) is the calculation of the constants K_m (VCC(N,2)) of equations 5.8 and 5.9. This constant involves $1-M^2$ (VCC(N,1)), the flow width D_m (AREA(N)) and the number of streamlines $N(NPN)$.

The next step is to find the first normal (N1(N)) to be used in the calculation of the change at each normal. This involves the size of the region of influence NP (M in Chapter 5) and the location of the normal under consideration within the region.

The final step is to calculate the actual coefficients of the inverted matrix of equation 5.21. This is performed in subroutine ADPT for subsonic flow and SUPCON for supersonic region block convergence. When the region of influence is 3 (that is only changes at one normal are considered as in supersonic marching convergence) a simplified routine CKN is used to generate the constants.

6.5b Subroutine ADPT.

This subroutine performs the calculation procedure outlined in Section 5.5, inverts the resulting matrix and selects and stores the required row in AI(N,J). The description given is for the simple Euler type method. The subroutine for the Runge-Kutta type method is similar and is listed in Appendix 2b.

The array C, using the same subscript rotation as in Chapter 5, is first initialised to zero and the starting coefficients for $n=1$ and 2 found according to equations 5.17 and 5.18. Equation 5.15 is then evaluated

sequentially for $n=3$ to $n=NPN$. The final values of the array C are re-constituted in matrix form and the matrix is inverted. The pertinent row of the matrix is selected and stored in AI.

6.5c Subroutine SUPCON.

This subroutine is similar to ADPT in the overall procedure followed. It generates the constants for the block convergence region and follows the calculation procedure of Section 5.6. Equations 5.23, 5.25 and 5.26 are used and the inverted matrix A retained for use.

6.5d Subroutine CKN.

A simplified version of ADPT and SUPCON valid for subsonic and supersonic regions for $NP=3$. The procedures followed are the same as the above subroutines except that no matrix need be inverted.

6.5e Subroutine VNEW.

This subroutine corrects the velocity distribution everywhere on the commencing surface except at the throat in choking flows. If the average velocity at a normal is incorrect according to the flow type required, a correction is made at the commencing surface to rectify the situation. The solution to the matrix equation 5.21 is then applied at each point. The coefficients of the solution for each normal are in AI and A and the dimensionless velocity changes (x in equation 5.21 and 5.13) are found from the coefficients and from the dimensionless area changes required ($E(N)$ in the programme and y in Chapter 5). The velocity increment is calculated from equation 5.13 and the required correction made using the damping factors DFSUB and DFSUP. It should be noted that if the control index KD is set accordingly, the high throat region damping mentioned in Section 5.7b is invoked. The value of $1-M^2$ far upstream or far downstream is then used in equation 5.13 to find the velocity increment.

6.5f Subroutine SPNIST.

A matrix inversion subroutine of Imperial College. Listed but not described.

6.6 Programme Operation.

6.6a Introduction.

Experience with numerical methods for engineering problems has indicated that the resulting computer programmes do not always work on a wide variety of cases of similar nature. This is because the computer programme only contains techniques which have enabled previously presented cases to be solved. Different cases may require new techniques and in extreme situations may require a radical change in numerical approach.

Experience has also shown that even when the case under consideration is apparently the same as those already successfully handled a result is not always simply obtained. Many controlling variables have to be determined and when judiciously arranged can produce success where none seemed possible. It is therefore necessary to understand the numerical techniques employed and to manipulate the variables according to the difficulties encountered until a result is obtained.

This section outlines the major difficulties of operating the programme and also the limitations of the programme as far as they are understood. These two items are treated in Sections 6.6b and 6.6c.

6.6b Major operating difficulties.

The following two major operating difficulties exist in the programme in its present form:-

- 1) Failure of the grid stacking procedure when the commencing surface velocity distribution is inaccurate.
- 2) The periodic variation of velocity at a grid point above and below Mach one and the effect this has upon convergence.

These two items are discussed at length below.

1) Grid failure

The grid stacking procedure can be thought of as being equivalent to laying bricks upon an uneven curved surface. If the lower layers are not suitably shaped the upper layers become unstable and the final layers tumble when installed. In a similar fashion an error in the velocity assumption at the commencing surface produces a successively more distorted streamline towards the terminating surface until the curvatures and hence the velocity gradients become so large as to produce absurd velocities.

As would be expected this problem is more acute for highly curved channels, for channels with zones of rapidly changing curvature such as at a cascade leading edge and when a finely spaced grid is used. Cascade leading edges presented the most difficulty. The mechanisms to counter grid failure (referred to briefly in the preceding programme description) are:-

Gradual reduction in grid size using the coarser grid result as an initial velocity distribution for the finer grid. (See main programme and subroutine NUGRID).

Suppression of curvature. A grid failure occurs largely due to velocity gradients arising from excessive curvature which, as already explained, is caused by an inaccurate velocity distribution initially. The factor SFCT reduces the value of the actual streamline radius of curvature which is used in the calculation (see SLCALC). By performing a series of "solutions" with gradually increasing values of SFCT, a successful progression from the one dimensional starting estimate (see VELDIS) to the correct solution with SFCT = 1.0 can be achieved without grid failure. This method was found to have a limited application in choked flows with grid aspect ratios (flow width/streamwise spacing) above about 3. Boundary condition problems arose in the throat region which is thought to justify the statement of Section 4.5d(2).

Suppression of curvature calculation. As already stated, grid failure arises from curvature effects and inaccurate velocity distributions. Once a "solution" has been obtained with SFCT < 1 the curvature and slope at all grid points are known and by suppressing curvature and slope calculations during the grid stacking process, SFCT can be increased without grid failure. This method was found particularly useful in applying very fine grids near highly curved leading edges where on one occasion it was found that grid failure persisted beyond SFCT = 0.7. In this case the curvature calculations were terminated and SFCT increased to 1.0. The solution, although not correct, gave a good result in the very difficult leading edge zone. (See Section 7.2d).

Limiting velocities. (See also Section 6.4a(7)). A maximum and minimum is set to the velocity during iteration. In this way the effects of large velocity gradients upon subsequent streamlines is reduced. These limits are usually exceeded during initial iterations but enable grids to be successfully produced and a stable convergence achieved.

Two basic approaches to the problem of producing a fine grid solution were used. The first is to commence with a coarse grid solution (grid aspect ratio between 2 and 3 using if need be the SFCT method) and to

progress gradually towards a fine grid (aspect ratio between 5 and 7). Examples of this approach are given in data decks 2 and 3 in Appendix 2c. The second is to start with a fine grid and to progress towards a final result by gradually increasing SFCT (intermediate curvature calculation suppression may or may not be used). An example of this approach is given in data deck 1 in Appendix 2c. This second method cannot be used for choking flows.

No absolute criterion regarding allowable step sizes in SFCT and NPS is presented beyond the examples in Appendix 2c. It is felt that such criteria are best determined by experience with any given problem as they are highly dependant upon the cascade geometry and the choice of approximate leading edge representations.

2) Periodic variation above and below sonic velocity. This problem and its counter has been described in Section 6.4a(5). The change in flow type at a grid point during the iterative process and the resulting disturbance in the surrounding grid presented the greatest single problem in transonic flows and was the barrier to grid size reduction. Increasing the size of the tolerance band (VST) did in some instances result in convergence. A subsequent reduction in VST was found in one instance to be acceptable but did not change the resulting solution (see Section 7.4b).

6.6c Programme operating limitations.

The operational difficulties discussed in the previous Section imposed limits beyond which the programme would not operate. The limit is difficult to define as it involves largely the expenditure of trial and error in finding the correct combination of variables and techniques. A general guide only is therefore given as to the kind of problem which is likely to present extreme operational difficulties.

No operational limits were encountered in subsonic flows except grid failures in the highly curved leading edge regions which prevented grid aspect ratios beyond about 7 being used. Apart from the leading edge region itself there is little to be gained from using high aspect ratios and the problem of the leading edge is best overcome as outlined in the next chapter, that is by introducing a more accurate leading edge representation and using an intermediate streamline from the previous solution as a channel boundary.

The operational limits in transonic flows were observed to occur due to a combination of grid failure and flow type variations at a point. Narrow

channels with a slow flow acceleration such as a supersonic wind tunnel nozzle or a root section steam turbine profile presented relatively little difficulty. Results for widely spaced channels with rapid flow accelerations such as in the tip region of a steam turbine were extremely difficult to obtain and very coarse grids had to be used. Another source of difficulty were channels having a rapid change in curvature in the transonic or supersonic region. Rapid changes of curvature result when a profile is formed by two arcs of different radii or by the blending of a curved section into a straight one.

6.7 Future development.

It is felt that the following developments of the programme should be undertaken in any future work:-

- 1) The leading edge improvements via the use of an intermediate streamline and a better geometric representation should be incorporated as a standard procedure. Attention should be focussed on the leading edge regions alone and the previous solution immediately upstream and downstream used as boundary conditions to the localised solution.
- 2) The convergence theory should be extended to relate velocity changes at both walls to improve the convergence of the KUTTA subroutine.
- 3) A more flexible channel geometry should be chosen with a particular view to facilitating the application of the Kutta condition at the trailing edge.
- 4) The problem of grid failure should receive attention perhaps in the direction of imposing limits upon the grid curvature during iteration.
- 5) The detection of iterative oscillations and the consequent reduction in damping factor should be made automatic. Oscillations arise locally and a damping factor which varies across the grid should be introduced.
- 6) The problem of flow type oscillations at a point should receive attention. A suggested approach is to avoid grid points near the sonic line by a mechanism which automatically arranges the grid according to the iteratively emerging sonic line.

6.8 Concluding remarks.

The computer programme and operational problems have been described. While these problems have been encountered particularly in transonic flows and require skill and effort to overcome it should be pointed out that few computer programmes exist for which this is not true in some measure. The lengthy

discussion of the operational difficulties should not therefore be interpreted as a warning that the programme is impossible to use. The intention has been to provide a basis of experience and a background to the methods used to guide future users and also to prevent the impression being gained that the programme is foolproof in its operation.

CHAPTER 7 : COMPARISON WITH EXPERIMENTS AND WITH OTHER THEORETICAL METHODS.

7.1 Introduction, objectives and layout.

The objectives of this Chapter are:-

- a) to establish the validity and limitations of the streamline curvature method by comparison with experiment and with other numerical methods,
- b) to determine the changes and additions to the existing techniques necessary to provide accurate theoretical solutions,
- c) to determine the influence of the important numerical parameters,
- d) to consider some unverified cascade results, some with flare.

The need to investigate the validity of the streamline curvature method arises not only because of the extension here of the method to include supersonic flows and to reduce the effects of some of the basic channel flow limitations, but also because only limited investigations have been carried out by previous workers (see Section 3.6). Experimental comparisons and comparisons with other numerical methods are made with published results and with results available from industry. It was felt that there was sufficient data available from these sources to verify the method in its more important aspects and to obviate the need for an experimental programme. The major shortcoming of this approach is that no experimental or theoretical results could be found for channels with a height variation normal to the plane of flow. The ability of the streamline curvature method to predict the flow in flared cascades is therefore not verified.

The first two objectives, the validity of the method and the additions to the present techniques, are approached in three sections. In Section 7.2, subsonic cascades are considered. In Section 7.3 the flow in convergent divergent nozzles is analysed to evaluate the streamline curvature method in subsonic-supersonic flows with small curvatures before proceeding in Section 7.4 to evaluate its use in the curved subsonic-supersonic channels of turbine cascades.

The third objective, to determine the influence of the numerical parameters, is achieved within the discussions of the other three in order to eliminate repetition and because the numerical parameters themselves have an

important bearing on the performance of the streamline curvature method.

The last objective, the discussion of unverified and flared cascade results, is discussed in Section 7.5. The overall conclusions from this Chapter are presented in Chapter 8.

7.2 Subsonic cascades.

7.2a Introduction.

The term subsonic cascade is taken here to mean a cascade having a subsonic outlet velocity. The cascade flow is of the accelerating, decelerating type and transonic flows up to the choking condition are included. This type of blading is important for steam and gas turbines and the method of solution given here provides one of the contributions of this work. The satisfactory prediction of flow in subsonic cascades will provide some confidence in the procedures.

Experimental comparisons are made using the work of Dunavant and Erwin (7.1). A comprehensive set of data is available from this source and a selection is made to provide the broadest and most stringent comparison possible. A comparison is also made with the method of Martensen (ref. 3.1, see Section 3.3a) using the results of Reference (7.2).

7.2b Selection of data from Dunavant and Erwin (7.1) for experimental comparison.

The following aspects were considered when selecting data:-

- 1) Profile geometry. A single profile was chosen and is the most highly cambered profile for which high Mach number test results are given. The profile is from the primary series (reaction type blade) with a 95° camber angle and is shown in Figure 7.1.
- 2) Stagger angle. Test results were selected to cover a wide range of stagger angle. This was undertaken to test the method under varying degrees of overall cascade acceleration and also because the stagger angle determines the influence upon the velocity distributions of the Kutta-Joukowski condition at the trailing edge.
- 3) Mach number. A range of results was selected to cover Mach numbers from incompressible flow up to choking flow.
- 4) Inlet angle. The ability of the theory to predict the influence of changing inlet gas angle was tested by selecting a cascade for which results for two different inlet angles are given.

7.2c Low Mach number, low stagger angle cascade.

This set of results were chosen to test the method at low Mach numbers (incompressible) and for a small degree of overall flow acceleration. The low stagger angle renders the results relatively insensitive to the position of the trailing edge stagnation streamline and a comparison which is virtually independent of one of the basic channel flow limitations (see Sections 4.1a and 4.5f) can be made.

The stagger angle is 16.1° and the Mach number level approximately 0.25. The cascade is shown in Figure 7.1 and the experimental and theoretical results in Figure 7.2a which gives the surface Mach number distributions against chord distance. Additional data, both experimental and numerical, is included in Figure 7.2a.

The comparison between experiment and prediction is satisfactory except close to the leading edge stagnation point and in the leading edge region at the pressure surface.

The difference between the predicted and the experimental results on the pressure surface is due to the inadequacy of the orthogonal grid to represent the flow there. A characteristic of an orthogonal grid is that it fans out towards the pressure surface due to the channel curvature. This is illustrated in Figures 7.2e, 7.17b, 7.18b and 7.19. The grid size at the suction surface is sufficiently small to give a good representation of the flow but has increased by a factor of two at the pressure surface. The minimum grid size is limited as mentioned in Chapter 6. A fine grid can however be achieved at the pressure surface by using the mid streamline of the original grid as the channel boundary for a subsequent solution with a finer grid as shown in Figure 7.2e. In this way the channel width (which largely determines the grid size limit) and the orthogonal spread of the grid is halved. The pressure surface grid size was reduced by a factor of four and the resulting improvement in Mach number distribution is shown in Figure 7.2b.

The remaining difference between theory and experiment near the leading edge stagnation point is due partly to the approximate leading edge representation (see Section 4.1a) and partly to the failure of the grid to represent the small scale flow variations which occur in this highly curved region. The leading and trailing edge representations used are shown in Figure 7.2c and are chosen as shown to produce a smooth transition between the straight stagnation streamline and the curved profile.

The velocity distributions near the leading edge stagnation point can be improved in a similar way to the improvement at the pressure surface discussed above. A major limitation upon the grid size which can be tolerated

is the sharpness of the leading edge representation used (see Section 6.6). A leading edge representation which is closer to the actual stagnation streamline is therefore introduced (see Figure 7.2c) while the channel boundary is taken as an intermediate streamline from original grid solution. Figure 7.2d shows the resulting improvement when this method is applied to the pressure surface leading edge and is the final velocity distribution for this cascade. Figure 7.2e shows the two complete grids used for this cascade.

Although it was not attempted here, further improvements could be made at the leading edge by using streamlines still closer to the profile as channel boundaries thus enabling finer grids to be used and closer representations of the actual leading edge geometries.

The Kutta-Joukowski approximation (see Section 4.5f) was introduced at the trailing edge but did not change the velocity distribution significantly. This is as expected for at small leaving angles the location of the trailing stagnation streamlines has little influence on the velocity distribution within the cascade itself.

7.2d Intermediate Mach number and stagger angle.

This comparison was made to test the method at intermediate Mach numbers and for an intermediate degree of overall flow acceleration. In addition, the influence of the grid parameters (spacing, number of streamlines and numerical method used) remote from the leading edge is discussed.

The Mach number level is approximately 0.4 and the stagger angle is 34.1° . The experimental and predicted results and all relevant data is given in Figure 7.3. The results with and without the Kutta approximation are shown.

The agreement between prediction and experiment for the pressure surface is seen to be good except towards the trailing edge. The suction surface Mach number using the approximate Kutta condition varies above and below the experimental curve by about 4%. The result without the Kutta approximation shows a better agreement.

The disparity between experiment and the Kutta result is largely due to the gas leaving angle being specified incorrectly. The constant deviation of 3.5° used for these cascades is in this case 1.1° higher than the experimental value. This error reveals the limitations of the Kutta approximation which does not take account of the flow leaving angle and simply adjusts one stagnation streamline to satisfy the condition of equal downstream velocities.

The effect of the leaving angle is of greater importance at higher Mach numbers where relatively small inaccuracies in the flow angle cause large errors in the trailing edge region velocity distribution. The question of the flow leaving angle will therefore be considered at greater length in Sections 7.2f and g which is concerned with higher Mach number cascades.

Influence of grid parameters.

The importance of grid size has been discussed in the previous section in connection with the leading edge region velocity distribution. The influence of grid size over the rest of the profile is shown in Figure 7.4a. A fine and a coarse grid result are shown, the coarse grid having a spacing 1.8 times larger than the fine grid. Differences between the two solutions are small and occur mainly in the highly curved leading edge region.

The effect of using the two numerical methods described in Section 4.2, the Euler type method and the Runge-Kutta type method, is shown in Figures 7.4b and c. Figure 7.4b shows the two results using the same overall grid parameters. The two results are seen to be similar, the Runge-Kutta method being approximately 3 and 6% higher on the suction and pressure surfaces respectively. Figure 7.4c compares the two results for a varying normal grid spacing. The two results differ by up to 7% at 5 streamlines but at 24 streamlines the results are closer, differing by only 1 - 2%.

The effect of decreasing the normal spacing is also seen in Figure 7.4c. The result at any point changes rapidly up to 10 streamlines and thereafter changes little indicating that little is to be gained in using more than 15 streamlines.

7.2e Low Mach number, high stagger angle.

These cascade results were selected for comparison to test the method at low Mach numbers and a large overall flow acceleration. At the stagger angle chosen (49.1°), the channel boundary over 50% of the suction surface is the trailing stagnation streamline. The location of this streamline using the Kutta-Joukowski approximation is of prime importance. This comparison is therefore primarily to test the validity of the trailing edge Kutta approximation under extreme conditions.

The predicted results before and after applying the Kutta approximation are shown with the experimental results in Figure 7.5a. The results using the original stagnation streamline show a good agreement except over the latter half of the suction surface where the velocity is up to 12% higher than the

experiment. The agreement using the Kutta approximation shows a considerable improvement and is good except at the centre of the suction surface where it is up to 8% higher than the experimental curve. The need for the Kutta approximation at high gas leaving angles is thus demonstrated. The change in stagnation streamline position due to the Kutta approximation is shown in Figure 7.5b.

In Section 4.5f the assumptions made in arriving at the Kutta approximation are described. The channel boundary formed by the stagnation streamline at the pressure surface (terminating surface) is changed in applying the approximation while the channel boundary at the suction surface (commencing surface) remains unchanged. The effect of this approximation was tested by manually moving the commencing surface stagnation streamline 75% of the distance towards the position estimated by the approximation (see Figure 7.5b) and repeating the calculation. The two Kutta results are shown in Figure 7.5c and the negligible differences seen indicates the validity of the assumptions made.

The error in the centre of the suction surface cannot be explained with any certainty. This error is evident to a lesser degree in the previous cascade (34.1° stagger). As the error increases with the flow leaving angle it is thought to be due to an error in the profile geometry. An error of only 1° in the profile leaving angle would account for the differences of 4 and 8% respectively in the two cascades.

7.2f Cascade at high subsonic Mach number.

This set of cascade results were selected to test the method at high subsonic Mach numbers. The Mach number level is approximately 0.8 and the stagger angle 31.1° . The experimental and theoretical results are shown in Figure 7.6a. A good agreement is seen on the pressure surface and on the leading half of the suction surface. The trailing portion of the suction surface, that is the portion directly influenced by the trailing stagnation streamline, shows a poor agreement with the Kutta result worse than the original result.

As mentioned in Section 7.2d, the inaccuracy of the result in the trailing edge region and the deterioration of the result when the Kutta approximation is applied is due to the incorrect specification of the gas leaving angle. The inaccuracies are seen to be greater at the higher Mach number. No provision is contained in the method to determine the gas leaving angle. An ideal Kutta condition would not only ensure that the velocities

along two stagnation streamlines are the same, as in the approximation used here, but would also ensure that the velocities at the trailing edge stagnation points are equal and that the flow leaves at the correct angle. Reference to Figure 7.6a (Kutta result) shows that the trailing edge velocities on the two surfaces are far from equal.

The disparity of the results in the trailing edge region is shown to be due to the incorrect leaving angle by repeating the result with a better estimate of the angle. The original result was obtained with a specified angle 1.34° lower than the experimental value. The recalculated result therefore uses the experimental flow angle and is shown in Figure 7.6b. This recalculated result indicates that although the velocity is still lower than the experimental curve, it is considerably closer than the original result. In particular, the velocities at the two trailing edge stagnation points are closer to each other. The downstream Mach number is also closer to the experimental value.

The effect of increasing the leaving angle has been to achieve a result closer to the Kutta condition in that the point of equal profile velocity has moved closer to the stagnation point. The possibility is now presented of applying a better approximation to the Kutta condition by adjusting the flow leaving angle until the point of equal velocity occurs at the trailing edge.

To this end a third result is calculated using a leaving angle 1° higher than the experimental value. The new result is included in Figure 7.6b. The point of equal velocity has moved downstream of the trailing edge. For interpolation purposes, the location of the equi-velocity point is plotted against leaving angle in Figure 7.6c. The angle which produces the equi-velocity point at the trailing edge is seen to be only 0.5° higher than the experimental value.

This analysis has shown that, for one particular example, a good approximation to the Kutta-Joukowski condition can be applied at the trailing edge using the Kutta approximation and the effect of the leaving angle upon the equi-velocity point. The linearity of Figure 7.6c indicates that an iterative process to determine the leaving angle would be rapidly convergent. The difference of 0.5° between the experimental and theoretical leaving angle is considered good and could be explained by boundary layer on the blade surfaces. An intermediate velocity distribution for the correct leaving angle is sketched into Figure 7.6b and the maximum difference is only 5% between prediction and experiment.

The solutions without the Kutta approximation for the three leaving angles are shown in Figure 7.6d. The result for the experimental leaving angle appears to agree well with the experimental curve and to locate the equi-velocity point at the trailing edge. In this case the downstream stagnation streamline velocities indicate that the Kutta condition is not satisfied.

7.2g Cascade with small region of transonic flow.

This set of cascade results was selected for comparison with the prediction because small supersonic regions of flow appear towards the suction surface trailing edge and because of the high leaving Mach number of 0.94. The stagger angle is 28.1° . The results using the initial specification of $3\frac{1}{2}^\circ$ deviation appear in Figure 7.7a, which presents the solutions with and without the Kutta approximation.

As in the previous section, the prediction agrees well with the experiment except over the rear portion of the suction surface where velocity differences of up to 25% appear. The downstream velocity is also lower by this amount. These differences are greater than at the slightly lower Mach number of the cascade in the previous section.

The improved Kutta condition developed in the previous section is applied to this cascade and the results repeated using the experimental leaving angle. The new and the original points of equi-velocity are used to determine by extrapolation (Figure 7.7c) the leaving angle required to locate the equi-velocity point at the trailing edge. A third result with this leaving angle is shown in Figure 7.7b with the first two results included for comparison. The three solutions without the Kutta approximation are given in Figure 7.7d.

Figure 7.7b indicates that the equi-velocity point has been successfully located at the trailing edge. This fact, in combination with the Kutta approximation, means that the Kutta-Joukowski condition is satisfied in its most important aspects.* The leaving angle thus determined is 1° higher than the experimental value. The suction surface velocity distribution and the leaving Mach number are 10% lower than the experimental values. This remaining discrepancy in the results are thought to be due to boundary layer blockage effects which causes the real flow to be faster. At this Mach number only a very small

* A repetition of these results using a commencing surface stagnation streamline as determined by the Kutta approximated terminating surface, would indicate, as in Section 7.2e, that the resulting change in velocity distribution is negligible.

displacement thickness is required to cause the differences shown in Figure 7.7b.

Figure 7.7d shows the result without the Kutta approximation. The third result is seen to be close to the experimental curve but the extent to which this solution does not satisfy the Kutta-Joukowski condition is seen in the location of the equi-velocity point and the velocity distributions downstream of the cascade.

7.2h The effect of inlet angle.

The ability of the method to predict the velocity distribution in a given cascade at different gas inlet angles was investigated by selecting results for which two inlet angles are given for the same cascade. The stagger angle chosen is 39.6° and the test results are for low (incompressible) Mach numbers. The two flow inlet angles of 15° and 30° represent respectively angles of incidence of -12° and 5° measured from the profile camber line at inlet.

The experimental and theoretical results for the two inlet angles are given in Figure 7.8. No attempt was made here to improve the results by using very fine grid spacing at the leading edge or by applying the Kutta-Joukowski condition at the trailing edge.

Both predicted velocity distributions show the major differences in the two experimental patterns. The differences in the results on the pressure surface are probably due to the absence of the Kutta condition. The pressure surface differences are thought to be caused by a combination of the coarse leading edge region grid and the inaccuracies involved in reading the small scale presentation of the experimental results in Reference 7.1.

The theoretical method is considered to be adequate to predict the effects of changing inlet angle.

7.2i Comparison with the method of Martensen (3.1).

In this Section a comparison of the streamline curvature method with the method of Martensen (3.1) is made. A comparison of an improved version of the Martensen method with a series of experimental results taken from Dunavant (7.1) has been made by Wilkinson (3.3). This comparison showed a satisfactory overall agreement between the theory and the experiment. The satisfactory overall agreement obtained in the preceding sections between the experimental results of Dunavant and the streamline curvature method leads to

the conclusion that the two theoretical methods give comparable results for the same type of profile.

In addition to the above conclusion a brief comparison is made here using results from industry (Price 7.2). The results, for two profiles, are given in the form of a pressure coefficient with the profiles in Figures 7.9 and 7.10. The agreement between the two methods is satisfactory on the pressure surface and in the centre of the suction surface. The leading regions differ by as much as 50%. The reason for this discrepancy was not investigated and no attempt was made in the streamline curvature method to apply fine grids to improved representations of the leading edges. The Kutta-Joukowski approximation was not attempted at the trailing edges of the two profiles which fact could account for the differences of up to 25% there and in the downstream flow.

7.3 Convergent-divergent nozzles.

7.3a Introduction.

The convergent-divergent, subsonic-supersonic nozzle is used in supersonic wind tunnels and published experimental and theoretical data on the flow in this type of channel are available. As mentioned in Section 3.4, theoretical methods exist for the analysis of the flow in the throat region of these nozzles and the method of characteristics is applied in the supersonic region. The streamline curvature method is compared here in both of these regions to establish the validity of it in flows of this type, that is flows with small curvatures.

The comparisons made here are as follows:-

- 1) with the nozzle of Harrop (3.16). Comparisons are made in the throat region and in the supersonic region with experimental and theoretical results,
- 2) with a hyperbolic throat analysed by the methods of Hall (3.12) and Sauer (3.11). These results are taken from Bird (7.3).

7.3b The nozzle of Harrop (3.16).

This nozzle was designed for wind tunnel purposes to give a uniform Mach number of 2 at the exit of the divergent section. The throat region is parabolic in shape and was analysed using the method of Sauer (3.11). The shape of the divergent section was determined by the method of characteristics commencing from the Mach 1.2 contour.

The nozzle coordinates were adjusted to allow for the boundary layer and the streamline curvature method applied using a grid determined according to the spacing criteria of Section 4.5e for 5 streamlines. The comparisons are presented and discussed below and the effects of the various numerical parameters analysed.

Throat region.

Figure 7.11a shows the throat region contours of constant Mach number superimposed upon the grid of normals and streamlines from the streamline curvature result. The contours shown are for the experimental, the streamline curvature and for the method of Sauer.

Satisfactory agreement is seen between the three methods. The streamline curvature method shows a slightly better agreement with experiment than does Sauers method particularly near the curved wall below Mach 1.0.

Supersonic region.

Figure 7.11b shows the velocity distribution along the axis and curved wall for the three methods. Agreement between the streamline curvature method and the experimental result in the accelerating region up to the design Mach number is seen to be good. The streamline curvature method is closer to the experimental curve than the design method (characteristics). Both the experimental and the streamline curvature results rise to a Mach number of 2.04 at the end of the nozzle. This value is correct for the area ratio of the nozzle. Although the final streamline curvature velocity is the same as the experimental, it is reached considerably beyond the end of the nozzle. The experimental axis velocity remains constant for some distance before the end of the nozzle and in this respect the method of characteristics is closer to the experimental behaviour.

The reason for the differences between the streamline curvature method and the experiment is thought to result from the failure of the method to represent the wave flow accurately. The acceleration of the streamline curvature flow right up to the end of the divergent section could be evidence of this.

Influence of grid parameters.

The influence of using the two methods of flow area calculation (see Chapters 4 and 5, the Euler and Runge-Kutta type methods) is shown in Figures 7.11c and 7.11d for the two flow regions. Figure 7.11c shows the Mach 2.0 contours for the two methods of calculation and are compared with the result using the method of characteristics. The two streamline curvature methods

are seen to provide reasonably similar results. Figure 7.11d shows the throat region contours and reference to Figure 7.11a indicates that the more accurate streamline curvature method lies closer to the experimental result.

The effect of calculations carried out for grid spacings which deviate from the ideal spacing, (Section 4.5e) are shown in Figures 7.11e and 7.11f. Figure 7.11e shows the throat region contours for the 5 streamline grid and for the arbitrarily spaced grid. The two solutions are the same at the throat but differ slightly as the distance from the throat increases. Figure 7.11f shows the differences at the Mach 2.0 contour resulting from an increase in the number of streamlines from 6 (for which the grid is approximately correct) to 15. The differences seen in these last two figures are not large and indicate that the supersonic grid spacing criterion is not critical.

7.3c Hyperbolic throat analysis.

The flow in a subsonic-supersonic throat of hypobolic shape is analysed with the streamline curvature method and compared with the methods of Hall (3.12) and Sauer (3.11) using the results of Bird (7.3).

Figure 7.12 shows the Mach 1.0 contours for the three theoretical methods. There is satisfactory agreement between the streamline curvature method and the other two methods.

7.4 Subsonic-supersonic cascades.

7.4a Introduction.

The streamline curvature method has been shown to be adequate in high Mach number subsonic cascades and in subsonic-supersonic flows with small curvatures up to low supersonic Mach numbers. In this section the method is applied to curved subsonic-supersonic cascades.

No theoretical methods exist for this type of cascade and only three sources of experimental data could be found. These are Forster (3.15), Rzhiznikov (7.4) and Bird (7.3). These three sets of experimental results are compared below with the computed results.

7.4b The cascades of Forster (3.15).

Forster (3.15) gives the experimental results for three cascades representing the root, mid radius and tip sections of a final stage steam turbine. These three sections are treated separately below.

Root section.

The results for the root section are shown in Figure 7.13a as lines of constant Mach number within the cascade.

The comparison in the subsonic region shows that although the predicted form of the contours are the same as the experimental results, the predicted values are higher particularly near the leading edges. This could be due to the flow restriction of the leading edge representations used in the theoretical case.

In the transonic region the results are similar in that the general form of the contours are the same but with velocity differences of up to 10% near the blade surfaces. The reason for the discrepancy was not ascertained.

This form of cascade is difficult to handle numerically because of the almost constant flow width. The throat location presents the problem as the average Mach numbers along adjacent normals are nearly equal. The greatest similarity in the transonic contours were found at very coarse grid spacings. At high grid aspect ratios the sonic line was wavy as shown in Figure 7.13b. This pattern was not influenced by the number of streamlines or by the velocity tolerance band used to determine a subsonic or supersonic radius of curvature calculation (see Chapter 6). This problem was not encountered in other cascades or in the nozzles of Section 7.3. The wavy sonic line is therefore thought to result from the relatively long boundary between the two flow regions and the large difference between this boundary and the line along which the choking condition is applied (see Chapter 4).

The comparison of results in the supersonic region (Figure 7.13a) shows a reasonable accuracy near the suction surface and an increasing disparity towards the pressure surface. No other experimental contours are available and the theoretical Mach number increases to a value of 1.6 at the suction surface trailing edge. The reason for the increasing discrepancy in the supersonic region was not investigated.

Mid radius section.

The results for the mid radius section are presented in the form of pressure curves for the two profile surfaces in Figure 7.14a and are compared with the predicted results. A contour plot is shown for the cascade in Figure 7.14b to illustrate the theoretical shape of the sonic line and the pattern of flow elsewhere.

Some uncertainty exists regarding the shape of the profile. The two examples of the profile presented in Reference (3.15) are not exactly the same.

A comparison of the two shapes given is included in Figure 7.14b.

A good agreement in the subsonic region is seen apart from near the leading edge. As the Mach numbers approach unity the predicted velocities are higher than the experimental. The agreement between prediction and experiment is considered satisfactory within the limitations imposed by the uncertainty of the profile geometry.

Tip section.

Great computational difficulties were experienced in obtaining results for this tip section profile. The grid used is widely spaced and the comparison shown in Figure 7.15 is not satisfactory.

The unsatisfactory results are thought to be due to the widely spaced grid. The numerical difficulties associated with transonic flows have been discussed in the previous chapter and were extreme for this type of profile which has a rapidly accelerating flow with large pitch to chord and flow width to profile radius ratio.

These difficulties and the unsatisfactory results indicate the limitations of the programme for cascades of this kind.

7.4c Root section of Rzheznikov (7.4).

Rzheznikov (7.4) gives experimental results in the form of velocity contours for a root section of a steam turbine. As with the profiles of Forster (3.15) some uncertainty exists regarding the geometry of the profile. The geometric specification given was found to be inadequate as was the profile taken from the velocity contour plot. The profile used for the predictions was obtained by adjusting the specified radii to give the same overall dimensions as the profile given in the contour plot.

The predicted velocity contours are compared with the experimental in Figure 7.16. The major difference in the results seems to arise from a region of rapid acceleration terminated by a small shock at approximately $M=1.1$ as shown. This does not appear in the predicted result. The Mach numbers remote from the shock (0.7, 0.8, 1.2) show a greater similarity between prediction and experiment.

In view of the geometric uncertainty it is not possible to draw any conclusions from these results beyond that the velocity contours are similar except near the region containing the shock.

7.4d Profile from industry (7.3).

A comparison of the prediction method with the experimental results in the throat region of an industrial gas turbine nozzle (Bird (7.3)) is shown in Figure 7.16b. The nozzle geometry and the streamline curvature representation used is given in Figure 7.16a.

The results, in the form of lines of constant Mach number, show a general similarity between experiment and theory. The differences between the two results are thought to be due to the inadequacy of the finite difference grid to represent the flow in regions of rapidly changing curvature. Reference to Figure 7.16a shows that both the suction and the pressure surfaces undergo step function changes in curvature as the circular arcs upstream of the throat are blended (not tangentially) into the straight walls downstream of the throat.

With such a geometry there are numerical difficulties; the main difficulty arising out of the difference between curvatures in the subsonic and supersonic regions.

The grid points used are shown in Figure 7.16b. A "supersonic" point would have a curvature largely associated with the upstream region (upstream data used) while a "subsonic" point would have a curvature representative of both regions. A change in velocity between iterations from subsonic to supersonic or vice versa will therefore introduce large changes in the surrounding grid and which leads to iterative instability. Because of this difficulty the grid shown was the finest which could be tolerated.

The large change in curvature between grid points in the subsonic and supersonic regions is seen in the change in slope of the contour lines between the two regions along the throat. This rapid change in curvature at the throat is therefore a major cause of the differences between the two results. A reduction in grid size would reduce this effect.

The largest difference between experiment and prediction occurs at the pressure surface where the sharpest change in the geometry occurs. Here the real flow accelerates rapidly round the corner at the throat while the predicted flow accelerates more gradually due to the smoothing effect of the finite difference grid. Once again, a reduction in the grid size would tend to localise the region of curvature change and would cause the predicted flow to accelerate more sharply.

7.5 Cascade results without experimental verification.

7.5a Introduction.

The theoretical method is applied here to the two industrial profiles of Price (7.2) (Figures 7.9 and 7.10, Section 7.2i) for high Mach number subsonic flows, for subsonic-supersonic flows and for flow with a height variation between inlet and outlet for subsonic flow. In addition the root section of Forster (3.15) (Section 7.4b) is analysed with a height variation.

No experimental verification of these results is available and they are presented to demonstrate the application of the method and to give examples of the results for cascades with height variations. No attempt is made in this section to apply the techniques developed in Section 7.2 for the leading and trailing edge regions.

7.5b Profiles of Reference (7.2) at various Mach numbers.

The velocity distributions and contour plots for the two profiles of Reference (7.2) and Section 7.2i at various subsonic outlet Mach numbers are given in Figures 7.17 and 7.18, a & b.

The suction surface of the first profile (Figure 7.17) is formed by two circular arcs which meet slightly before the cascade throat. This step function change in curvature of about 6 causes the velocity decelerations shown. The deceleration at the higher Mach number of 1.4 to 0.8 within one grid spacing is particularly rapid. There were computational difficulties with this geometry and the decelerations obtained indicate that rapid changes of curvature should be avoided to prevent an excessive boundary layer thickening.

The second profile has more evenly curved surfaces and the velocity distributions of Figure 7.18 show lower decelerations nearer to the trailing edge. It should be noted that at the higher Mach number the suction surface velocity distribution is more peaked with greater deceleration at the trailing edge.

Figure 7.19 shows the velocity contour plot and grid for the first profile operating with a supersonic downstream Mach number. The grid shown is the finest which could be used. The contour plot illustrates the capacity of the method to predict a flow pattern for this type of cascade flow.

7.5c Subsonic cascade with flare.

The velocity distributions for the first profile of the previous Section are shown for various degrees of flare (flow height variation) in Figure 7.20. The results shown are for a 10% and a 40% linear variation in flow height between the leading and trailing edges. For comparative purposes the curves for zero height variation are included. The effect of the flare is seen to reduce the overall cascade acceleration and to increase the blade loading towards the leading edge.

7.5d Root section of Forster (3.15) with height variation.

The root section of Forster (3.15) (Section 7.4b) is shown in Figure 7.21 for zero flare and for 10% flare. The profile used was one adjusted to allow for the wake blockage shown in Reference (3.15). The effect of the flare is seen to raise the overall velocity level in the channel and to move the throat towards the leading edge.

CHAPTER 8 : CONCLUSIONS AND RECOMMENDATIONS

The streamline curvature method presented in Chapters 4, 5 and 6 for high Mach number axial turbine profile analysis has been compared with experiments and with other theoretical methods in Chapter 7. The conclusions and recommendations resulting from the comparisons are discussed in this Chapter under the same headings used previously, that is subsonic cascades, convergent-divergent nozzles, subsonic-supersonic cascades and unverified cascade results.

8.1 Subsonic cascades.

Conclusions.

The streamline curvature method as presented predicts the profile velocity distributions with reasonable accuracy in the regions unaffected by the channel flow limitations near the leading and trailing edges. The original inaccuracies seen in the latter two regions can be successfully eliminated using developments of the numerical technique. The method is thus considered to be capable of predicting cascade flows to within (approximately) a 5% accuracy at all subsonic Mach numbers, flow inlet angles and stagger angles. This includes a prediction of the cascade deviation to within a half degree at a Mach number of 0.8. Above this Mach number level, the influence of boundary layer displacement thickness becomes increasingly important and is thought to impair the accuracy of the method. However, boundary layer predictions could be used to improve the high Mach number predictions.

Apart from the necessity of using very fine grid spacings near the leading edge, the numerical parameters are not important provided the grid aspect ratio is in the region of 4 and the number of streamlines used is above 15.

Recommendations.

It is recommended that the streamline curvature method be used as the theoretical design tool for cascades at Mach numbers up to 1.0. The techniques to improve the leading and trailing edge region predictions should be included as standard procedures in the method. It is thought that the inclusion of a boundary layer displacement thickness based upon an initial solution would improve the accuracy at Mach numbers above 0.8 and it is recommended that future work should investigate this possibility.

8.2 Convergent-divergent nozzles.

Conclusions.

The streamline curvature method predicts with reasonable accuracy the low curvature flow in the type of convergent-divergent nozzle used for supersonic wind tunnels. The accuracy is greater in the transonic throat region than in the high Mach number supersonic region.

The important numerical parameter appears to be the accuracy of the computational method (the Runge-Kutta type is more accurate than the Euler type) rather than the grid spacing factors.

The success of the method for this type of flow leads to the further conclusion that the method should provide reliable results for the curved channels of turbine cascades.

Recommendations.

The accuracy of the method for this application leads to the recommendation that it be used to provide an analysis of the subsonic and transonic regions. Existing methods are for transonic regions only with special shapes and the streamline curvature method will thus provide a universal method covering more of the flow. The supersonic region should be treated by the method of characteristics.

In view of the importance of the accuracy of the numerical method employed, it is further recommended that future work should endeavour to introduce more accurate Runge-Kutta methods based upon average streamline curvature values.

8.3 Subsonic-supersonic cascades.

Conclusions.

The flow predictions for subsonic-supersonic cascades were numerically more difficult to obtain than for the cases previously considered and the results generally less accurate. The experimental comparisons in the subsonic and transonic regions showed a reasonable similarity of result but in the supersonic regions the method was not found reliable. The comparisons were however hampered by uncertainty and unsuitability of data and the need exists for more accurate experimental data.

Numerical difficulties were more acute for widely spaced blades with rapidly accelerating flows and for blades with sharp changes in profile shape.

Recommendations.

For design purposes it is recommended that the streamline curvature method be applied in the subsonic and transonic regions and the method of characteristics in the supersonic region.

Future work should endeavour to counter the numerical difficulties of the method as outlined in Chapter 6 and to provide a more reliable experimental comparison for the method.

The use of the method in its present form is not recommended for widely spaced blades as found in the tip section of large low pressure steam turbines.

8.4 Unverified cascade results.

The major flow characteristics of two subsonic cascades were obtained which are thought to be as reliable as were the experimental comparisons for this type of cascade. A rapidly changing blade profile shape was found to affect both the numerical ease of solution and the high Mach number velocity distribution.

The results for flared cascades show the expected tendencies but experimental data are required to establish the validity of the method for height variations.

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TABLE 1 Condenser pressures of vapour cycle working fluids at various condenser temperatures

Working fluid	Pressure, p.s.i.a.		
	80°F (27°C)	100°F (38°C)	120°F (49°C)
H ₂ O	0.5	0.93	1.65
Hexafluorobenzene	1.4	2.55	4.55
Refrigerant 113	6.9	10.6	15.4
Refrigerant 11	16.3	23.8	33.4
Refrigerant 21	28.0	34.5	48.0
Refrigerant 114	33.0	46.5	63.0
SO ₂	59.0	84.0	109.0
Refrigerant 12	99.0	131.0	171.0
Refrigerant 22	160.0	212.0	280.0

TABLE 2 Gas turbine cycle data

Turbine efficiency (polytropic)	0.90
Compressor efficiency (polytropic)	0.90
Total pressure loss:	
simple cycle	0.10
regenerative cycle	0.15
combined cycle	0.11
Combustion efficiency	0.985
Mechanical efficiency	0.99
Generator efficiency	0.99
Regenerator effectiveness	0.75
Additional data for complex engines	
Total pressure loss:	
reheat cycle	0.12
intercooled cycle	0.12
intercooled reheat cycle	0.13
intercooled using intercooler heat	0.12
Intercooler effectiveness	0.85
Reheat temperature = Maximum cycle temperature	
First-stage pressure ratio = Second-stage pressure ratio	
First-stage expansion ratio = Second-stage expansion ratio	

TABLE 3 Vapour turbine cycle data

Turbine efficiency	0.85
Feed pump efficiency	0.70
Condenser temperature	80°F (27°C)
Refrigerant maximum temperature	300°F (149°C)
Steam turbine exhaust dryness	0.85

TABLE 4 Vapour cycle conditions

Quantity	Working fluid								
	R21	R11	R113	H ₂ O	R114	R12	R22	SO ₂	C ₆ F ₆
Turbine inlet pressure, p.s.i.a.	338	300	175	150	400	500	500	500	760
Turbine inlet temperature, °F	300	300	300	515	277	228	201	346	527
Turbine inlet density, lb/ft ³	5.36	7.3	6.2	0.26	16.7	14.3	9.1	—	20.8
Turbine inlet superheat, °F	36	0	0	157	0	13	31	111	—†
Turbine outlet pressure, p.s.i.a.	29.0	16.3	6.9	0.5	33	98.9	158.3	59.7	1.4
Turbine outlet temperature, °F	80	80	80	80	80	80	80	80	80
Turbine outlet density, lb/ft ³	0.65	0.50	0.293	0.002	1.0	2.5	2.86	—	0.042
Turbine outlet dryness, %	100	100	—*	85	—*	100	100	100	—*
Turbine enthalpy drop, Btu/lb	23.3	19.2	18.5	341	12.3	10.2	10.3	33.6	41.6
Thermal efficiency of Rankine cycle	0.1812	0.1841	0.1927	0.2640	0.1509	0.1310	0.1040	0.1560	0.2223
Mass flow of working fluid, lb/sec/MW	9.16	11.45	11.8	0.97	18.3	21.0	16.8	7.4	6.55
Vapour cycle power/total power	0.252	0.247	0.240	0.269	0.185	0.176	0.144	0.203	0.272

†supercritical cycle.

*superheated vapour; low-temperature is not considered.

TABLE 5 Total plant size range for acceptable vapour turbine geometry at 3000 rev/min

Fluid	No. of stages	Plant size range for acceptable specific speed at 3000 rev/min (MW)			
		Axial		Radial	
		Lower limit	Upper limit	Lower limit	Upper limit
R21	1	26	380	38	150
	2	10	146	—	—
R11	1	13	190	19	77
	2	6	90	—	—
R113	1	9	130	13	51
	2	3.4	50	—	—
C ₆ F ₆	4	1.0	18	—	—
	6	0.8	16	—	—
H ₂ O	4	5.4	77	—	—
	6	3.0	43	—	—

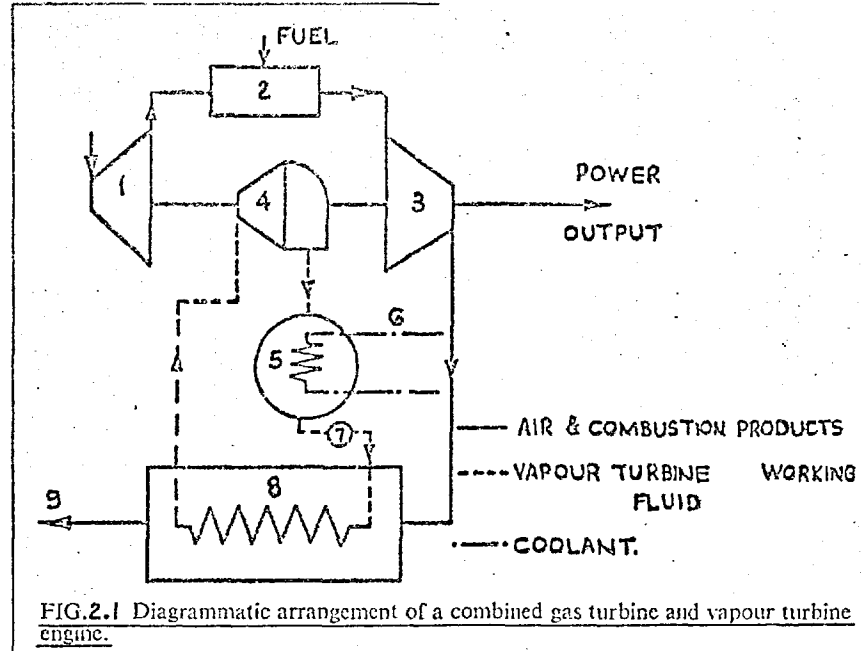


FIG.2.1 Diagrammatic arrangement of a combined gas turbine and vapour turbine engine.

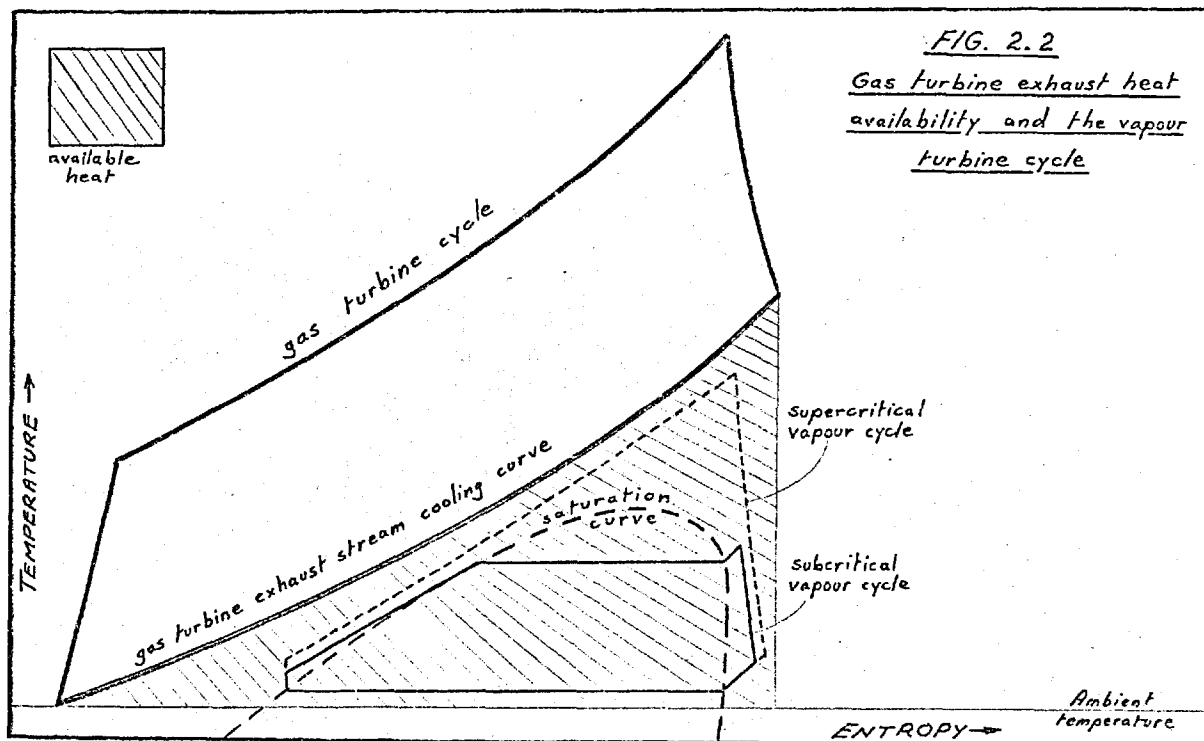


FIG. 2.2

Gas turbine exhaust heat availability and the vapour turbine cycle

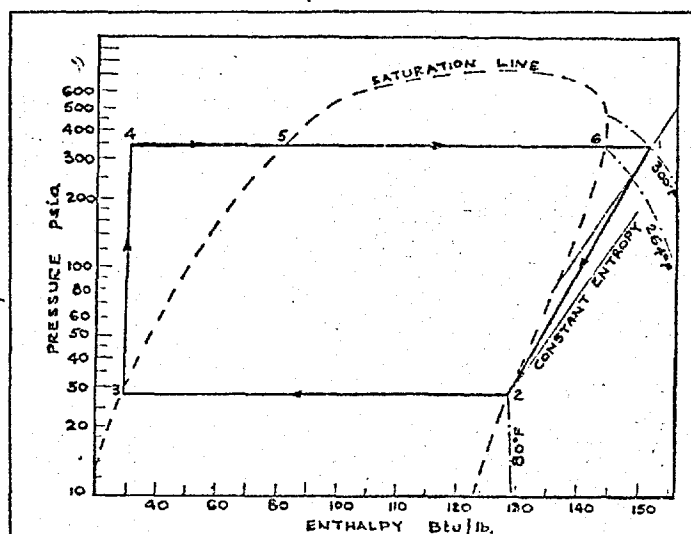


FIG. 2.3 Mollier chart for Refrigerant 21 illustrating the vapour cycle.

Process 1 to 2 Adiabatic expansion in 85% efficient turbine to a saturated exhaust condition.

Process 2 to 3 Condensation at 80°F.

Process 3 to 4 Pressure rise through feed pump.

Process 4 to 5 Temperature rise through 'economizer' section of heat exchanger.

Process 5 to 6 Evaporation at 264°F, 340 p.s.i.a.

Process 6 to 1 Superheating to 300°F.

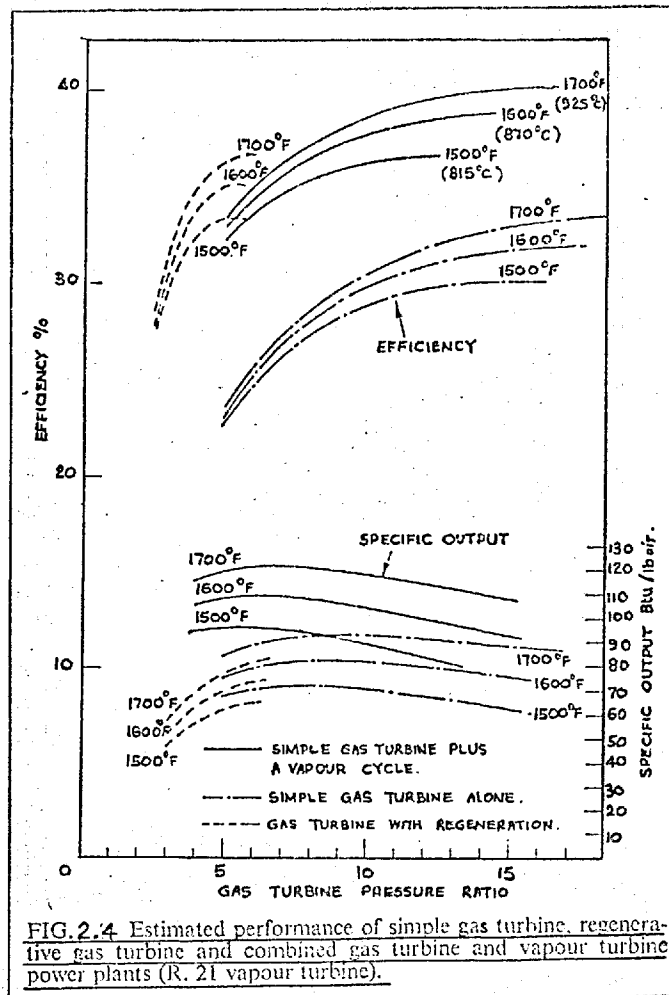


FIG. 2.4 Estimated performance of simple gas turbine, regenerative gas turbine and combined gas turbine and vapour turbine power plants (R. 21 vapour turbine).

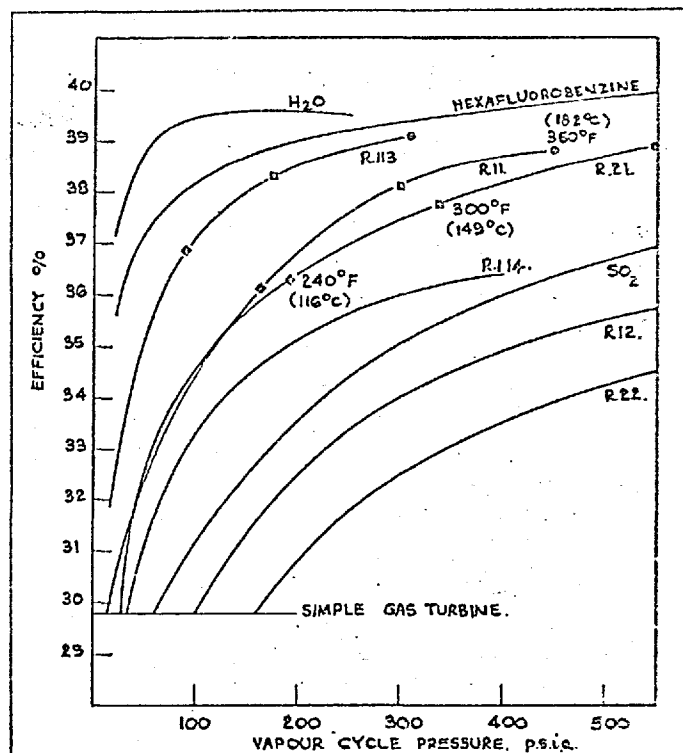


FIG. 2.5 Estimated efficiency of combined gas turbine and vapour turbine engines with various working fluids in the vapour turbine. (Gas turbine pressure ratio = 10:1, turbine top temperature = 1600°F (870°C)).

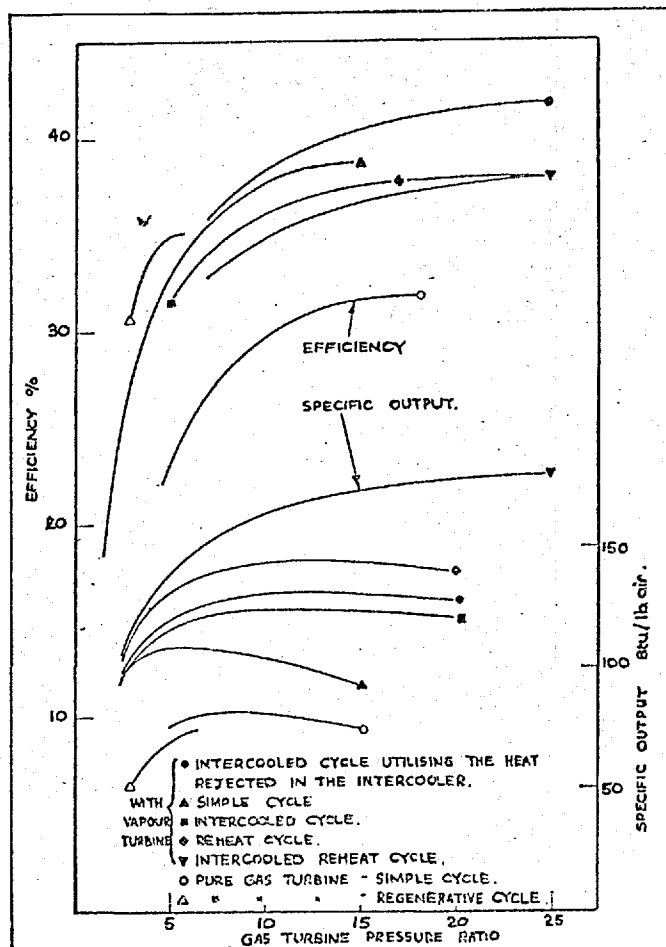


FIG. 2.6 Estimated performance of complex gas turbine cycle.

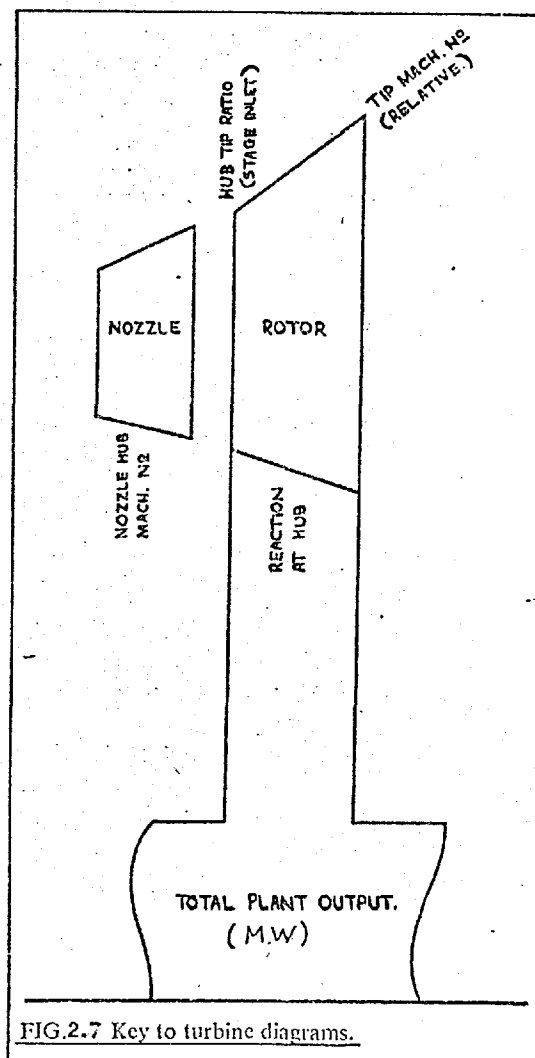


FIG. 2.7 Key to turbine diagrams.

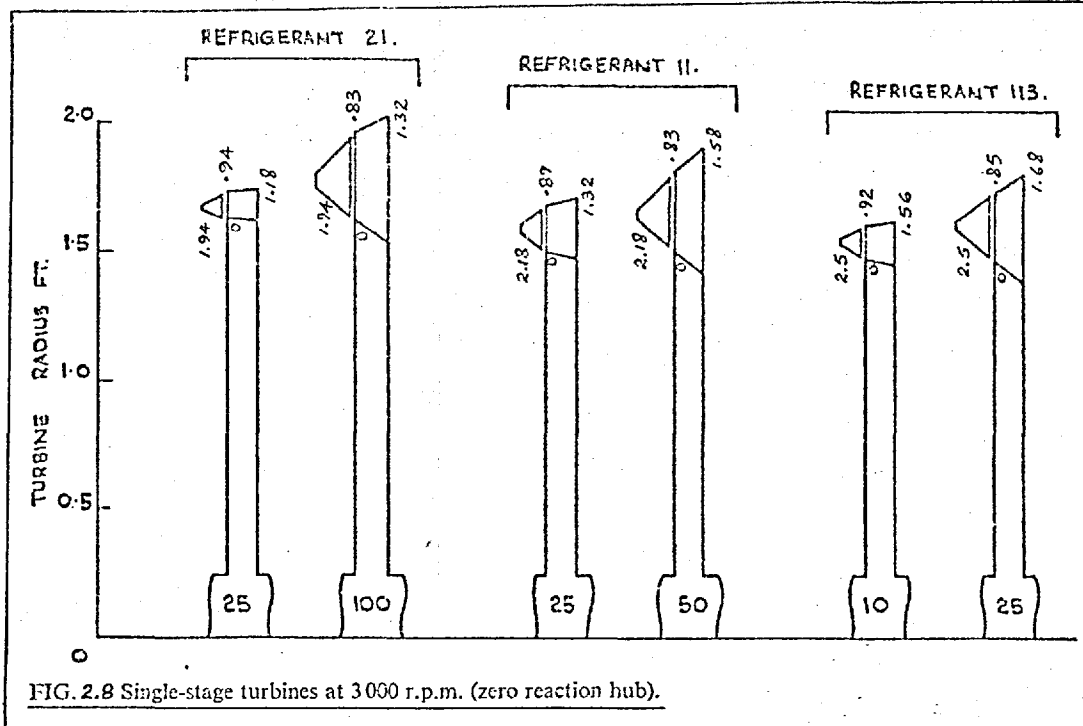


FIG. 2.8 Single-stage turbines at 3000 r.p.m. (zero reaction hub).

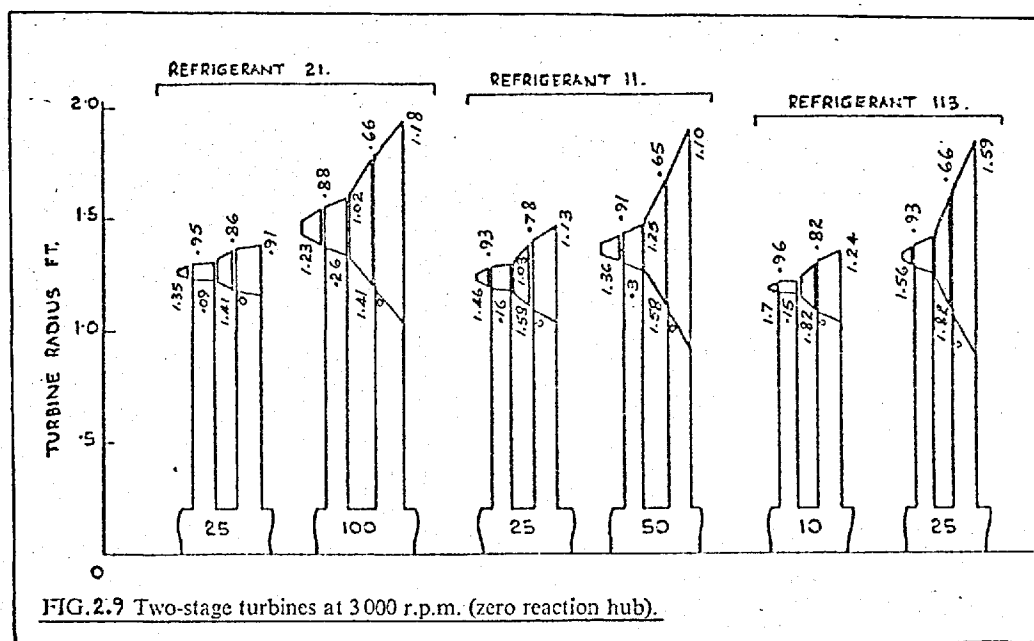


FIG. 2.9 Two-stage turbines at 3000 r.p.m. (zero reaction hub).

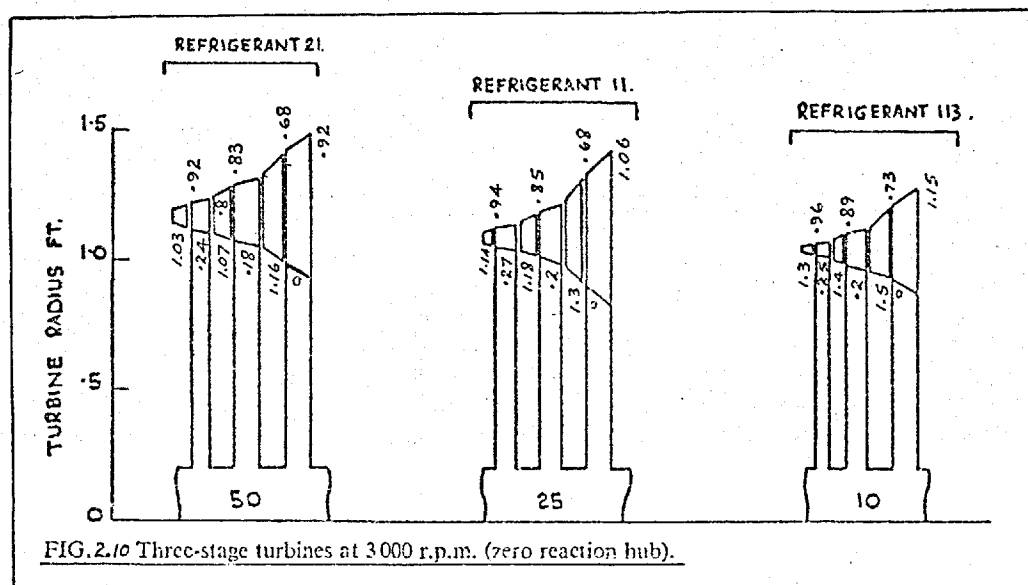
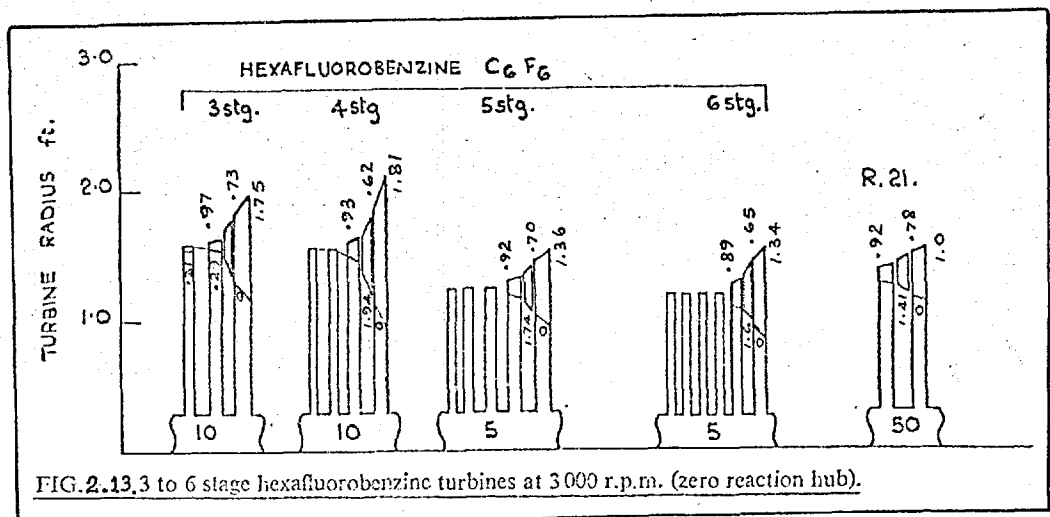
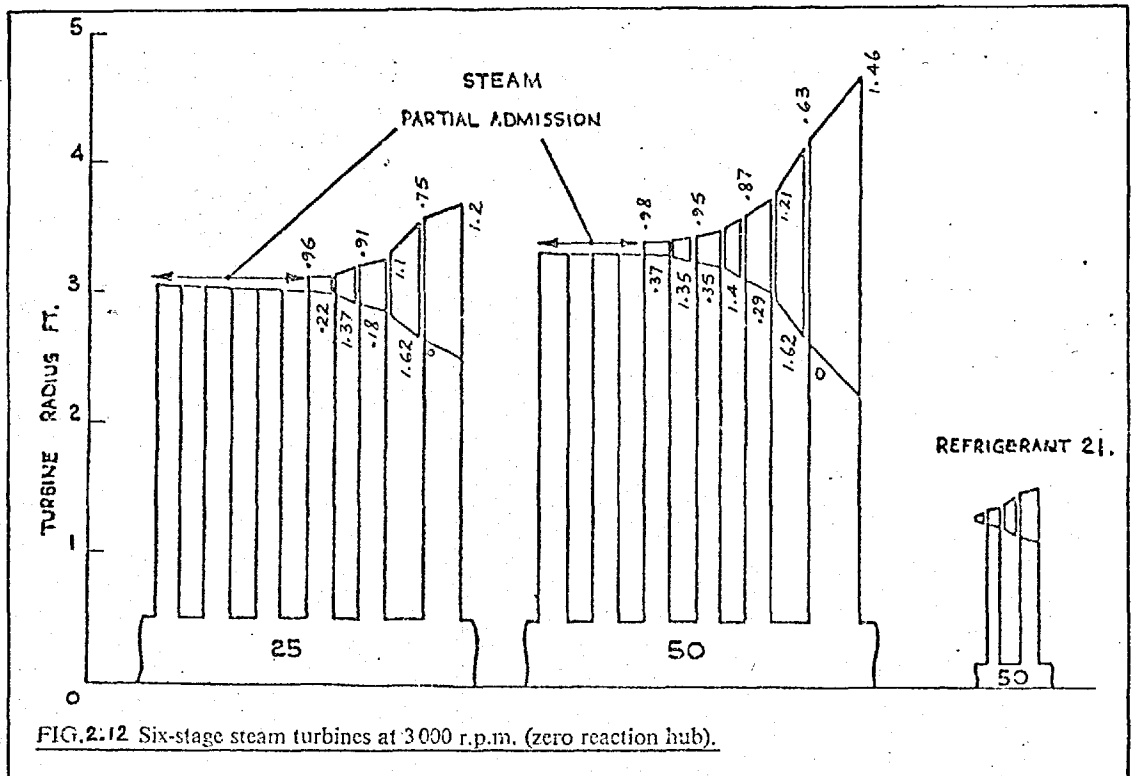
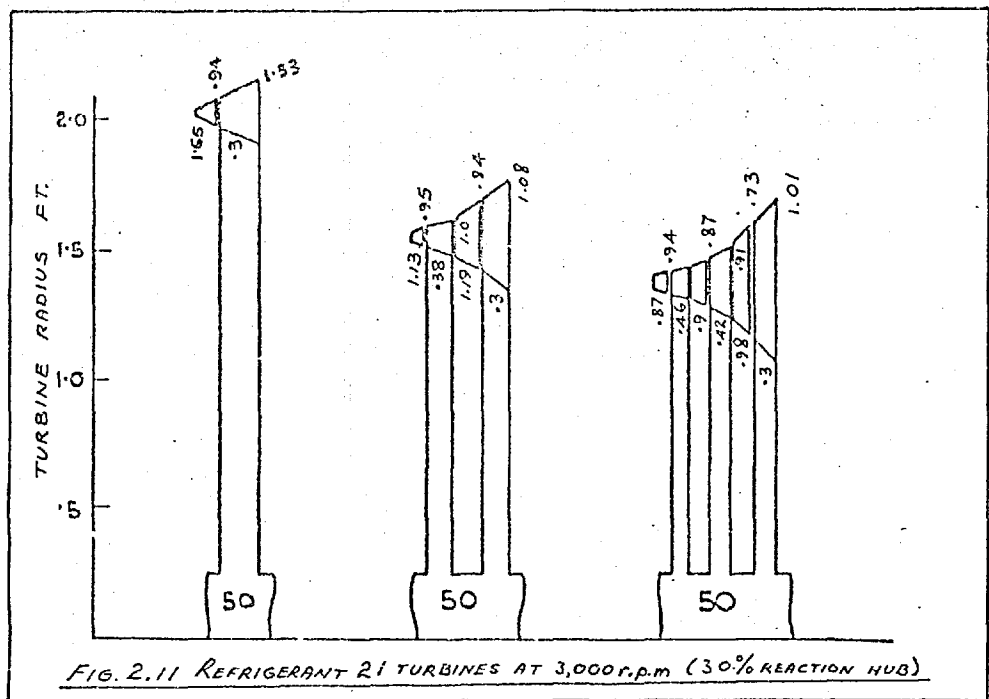
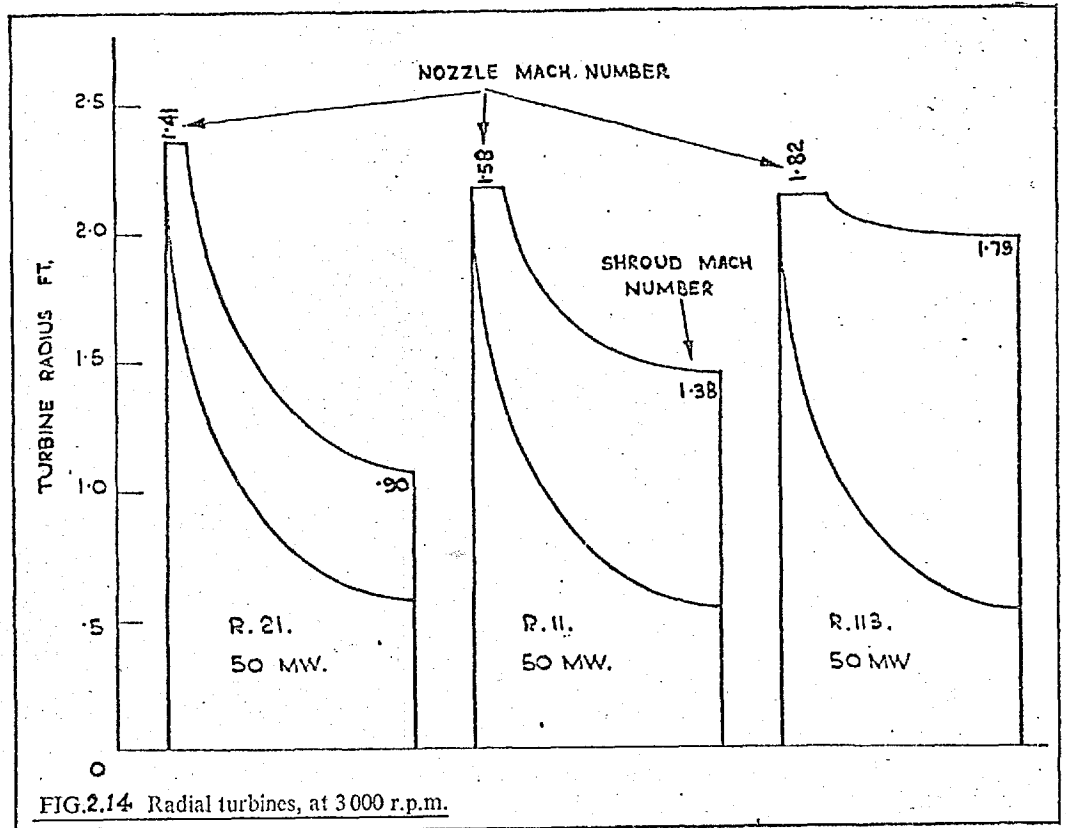


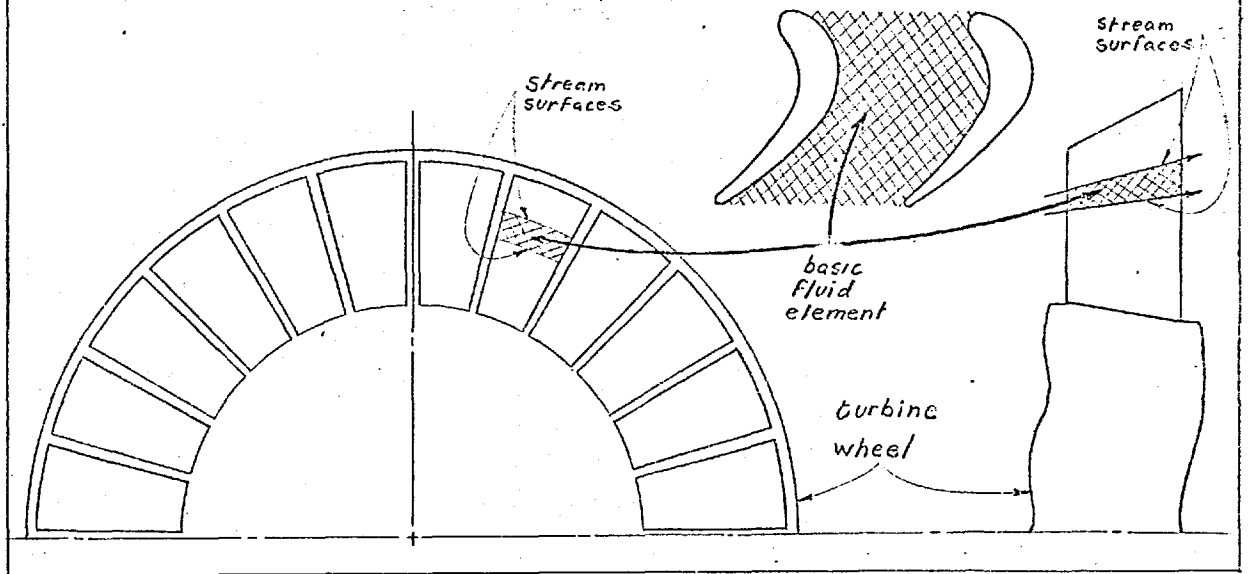
FIG. 2.10 Three-stage turbines at 3000 r.p.m. (zero reaction hub).





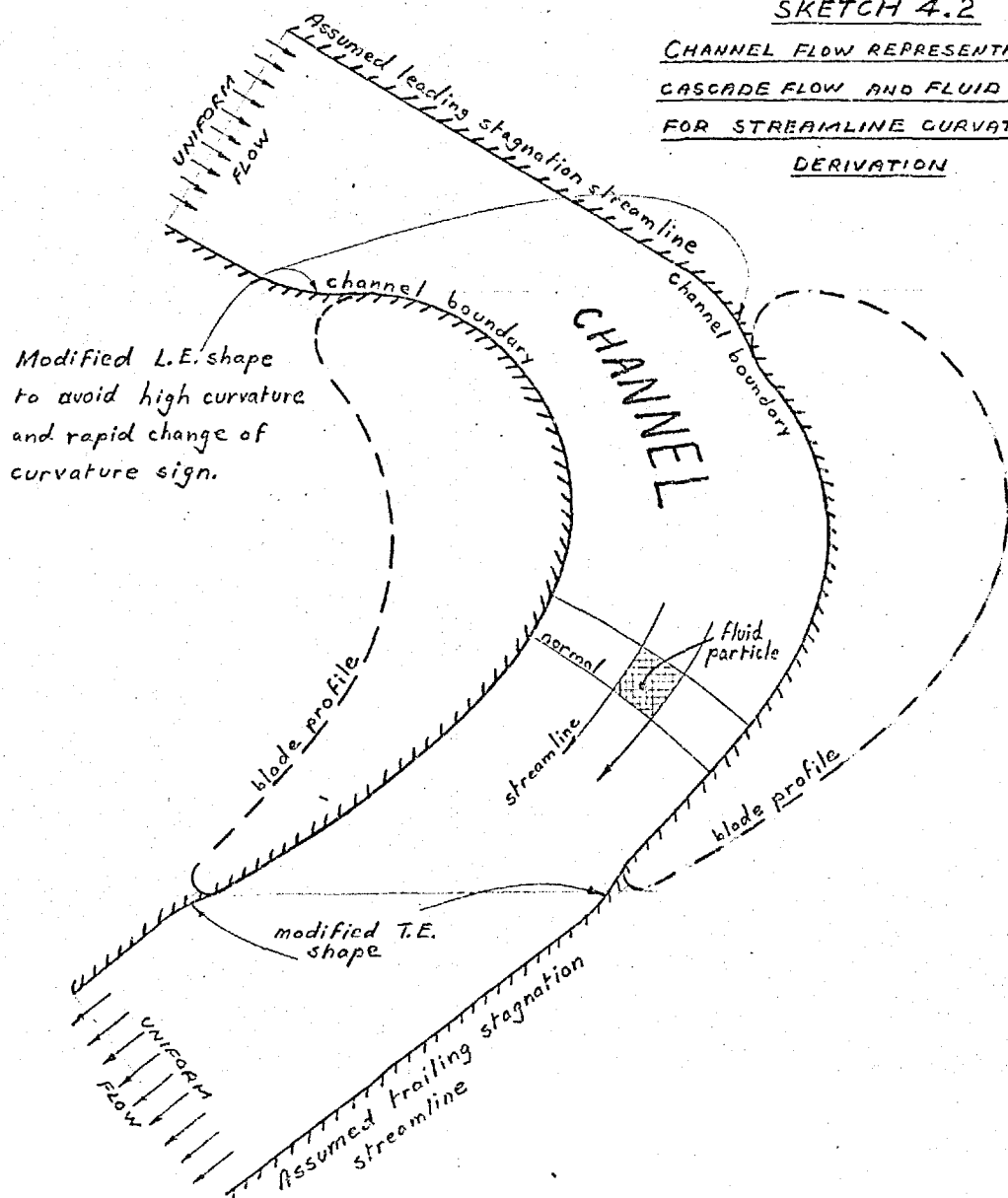
SKETCH 4.1

BASIC FLUID ELEMENT USED IN BLADE TO
BLADE ANALYSIS IN A TURBINE CASCADE



SKETCH 4.2

CHANNEL FLOW REPRESENTATION OF
CASCADE FLOW AND FLUID PARTICLE
FOR STREAMLINE CURVATURE
DERIVATION

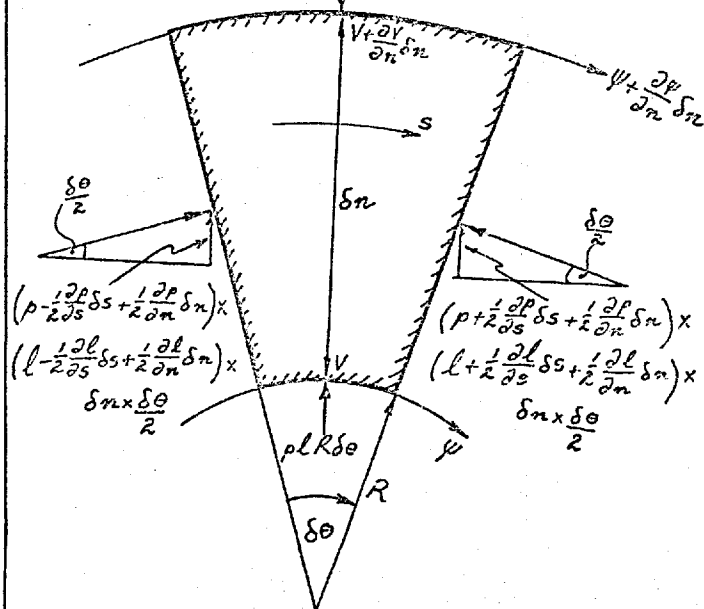


SKETCH 4.3

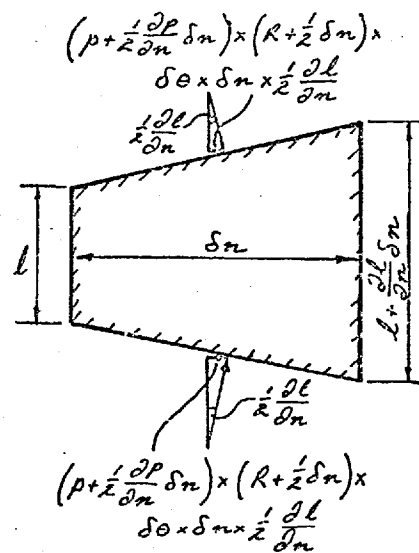
FLUID PARTICLE FOR DERIVATION OF BASIC EQUATION

section through particle center in the plane of flow showing pressure forces acting

pressure $\times$ $(p + \frac{\partial p}{\partial n} \delta n) \times$
 area $(l + \frac{\partial l}{\partial n} \delta n) \times (R + \delta n) \delta \theta$



section through particle center normal to the plane of flow and in the n direction

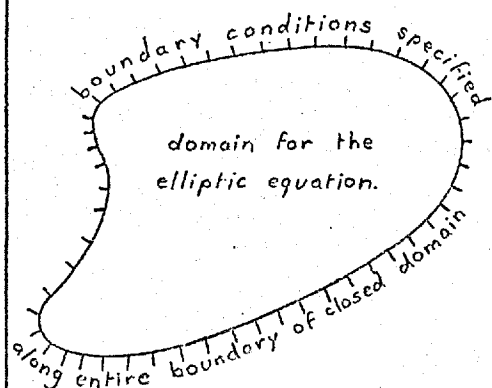


SKETCH 4.4

BOUNDARY CONDITIONS FOR ELLIPTIC AND HYPERBOLIC EQUATIONS AND FOR SUBSONIC AND SUPERSONIC FLOWS

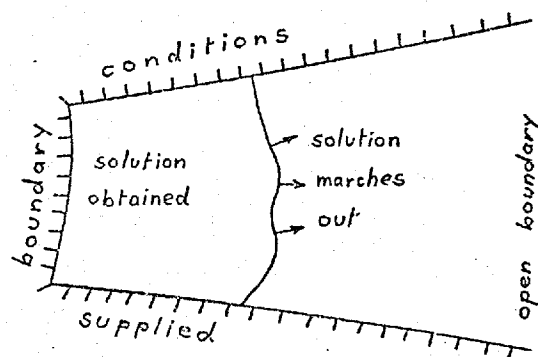
(a)

Elliptic equation



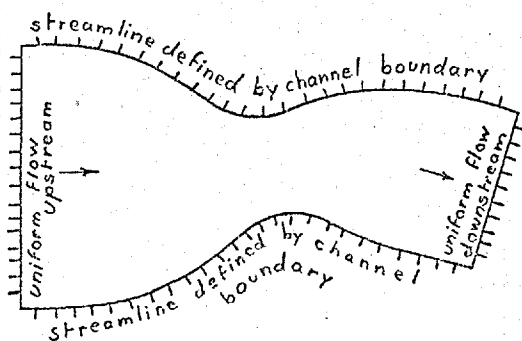
(b)

Hyperbolic equation



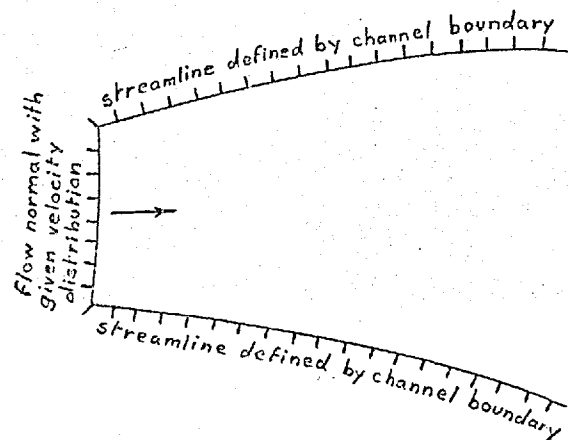
(c)

subsonic flow



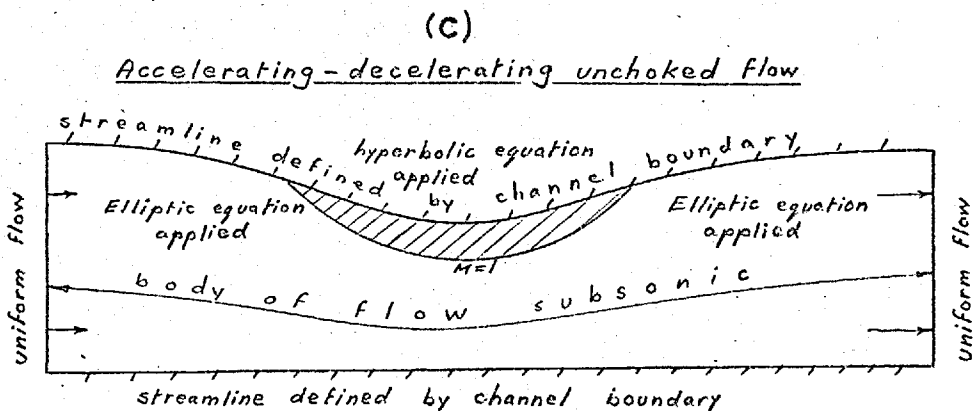
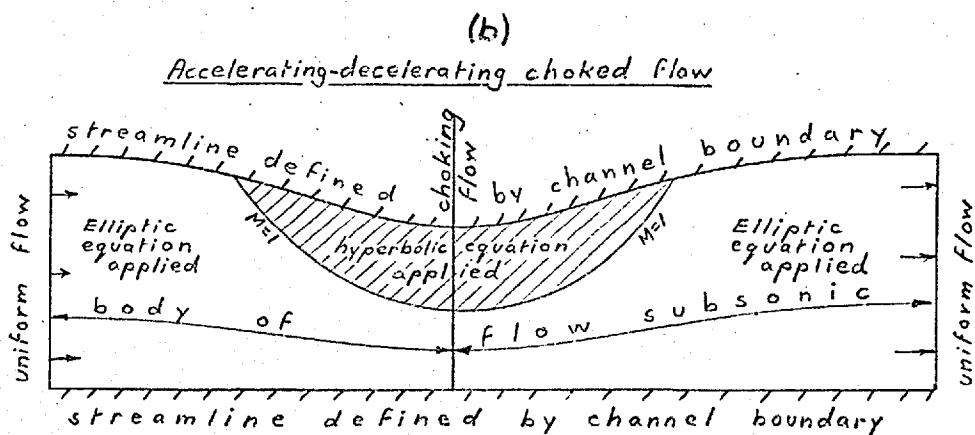
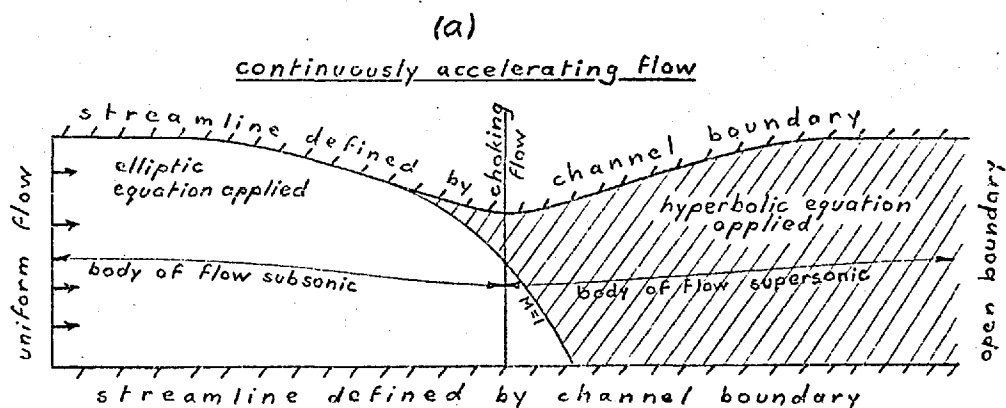
(d)

supersonic flow



SKETCH 4.5

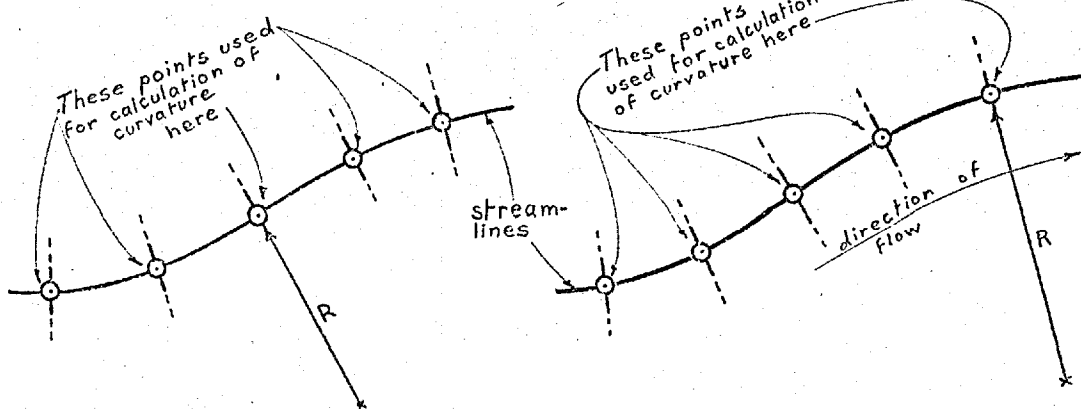
BOUNDARY CONDITIONS AND SUPPLEMENTARY DEFINING STATEMENTS FOR TRANSONIC FLOWS IN CONVERGENT-DIVERGENT CHANNELS

SKETCH 4.6

Elliptic (subsonic) and hyperbolic (supersonic) streamline curvature procedure determined by the manner in which the curvature is calculated at a point

(a) Elliptic or subsonic

(b) hyperbolic or supersonic

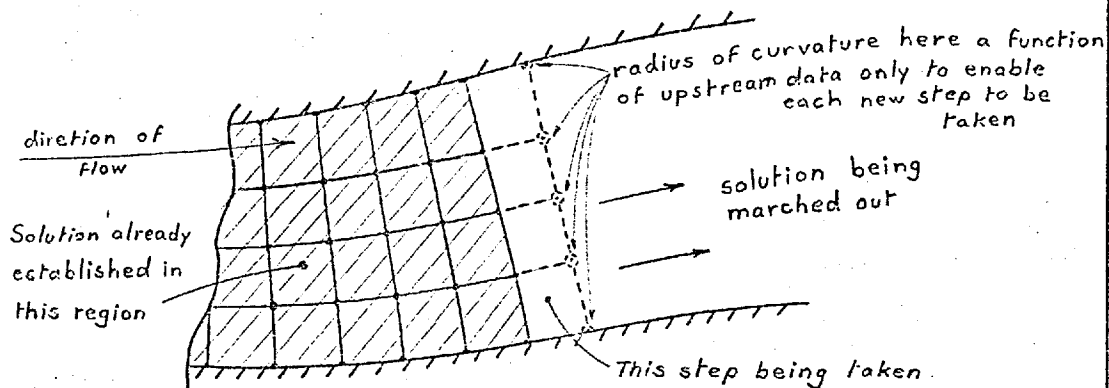


SKETCH 4.7

EXPLANATION OF THE BASIC STREAMLINE CURVATURE PROCEDURES FOR SUBSONIC AND SUPERSONIC FLOW

(a)

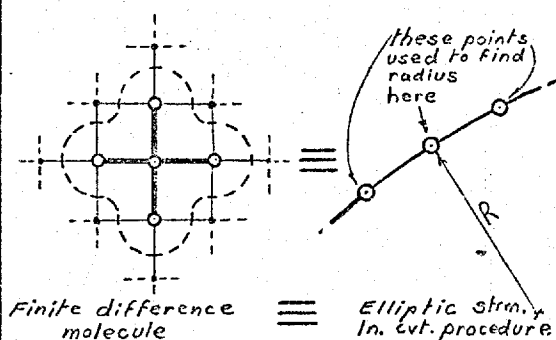
The marching hyperbolic solution and streamline curvature



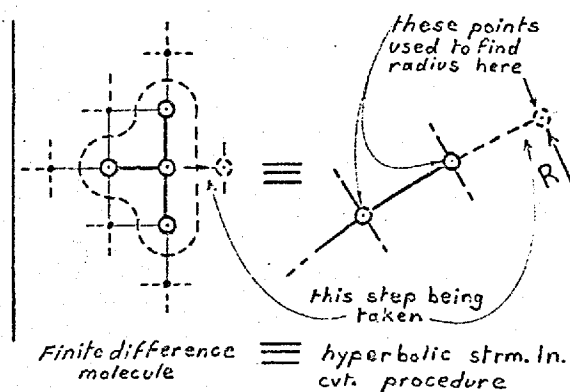
(b)

The two procedures and standard finite difference computational molecules

Elliptic (subsonic) molecule

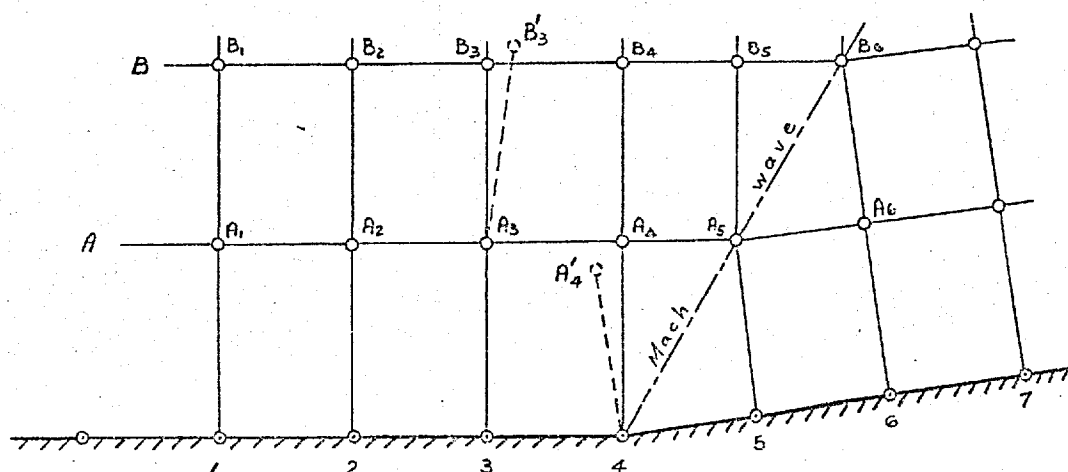


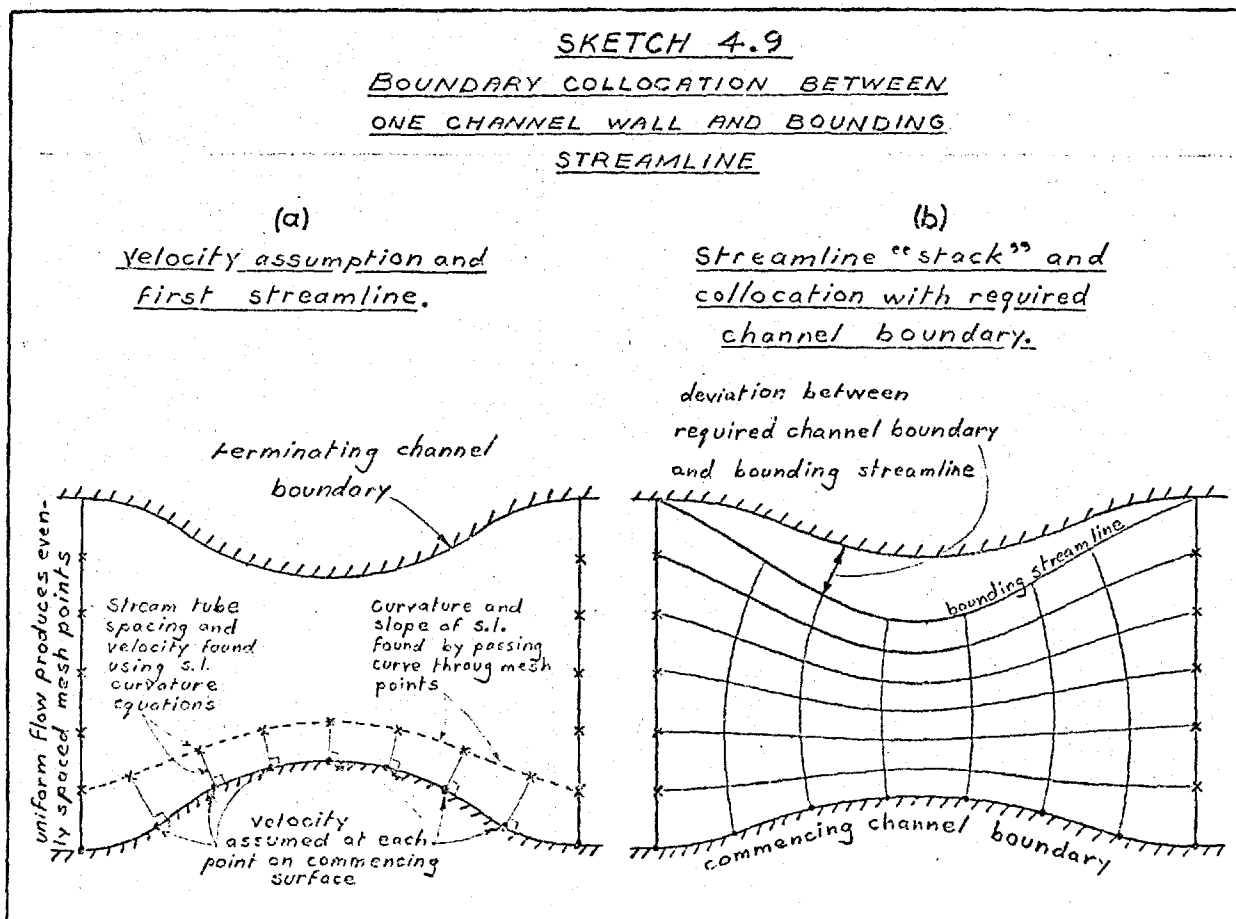
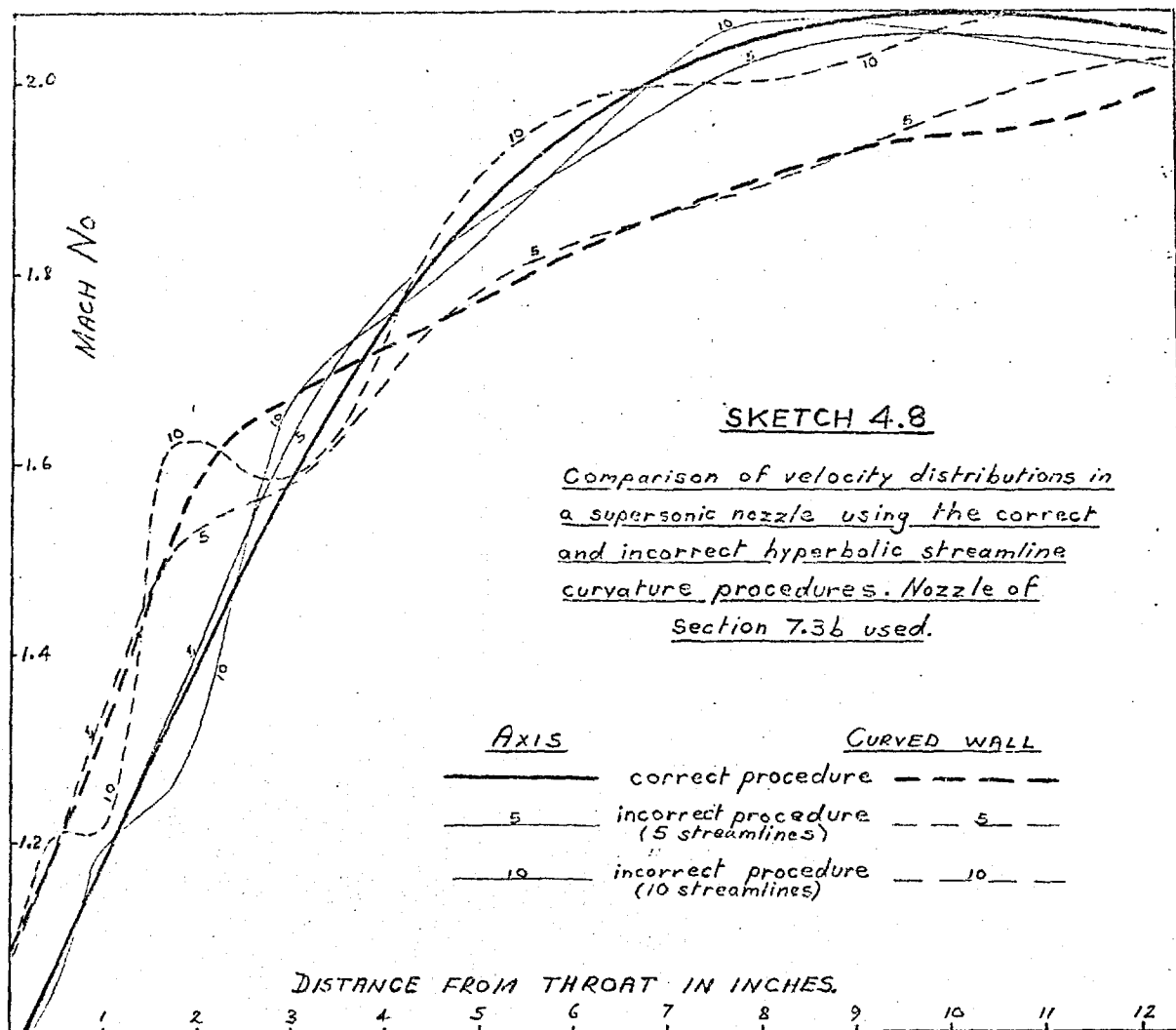
Hyperbolic (supersonic) molecule



(c)

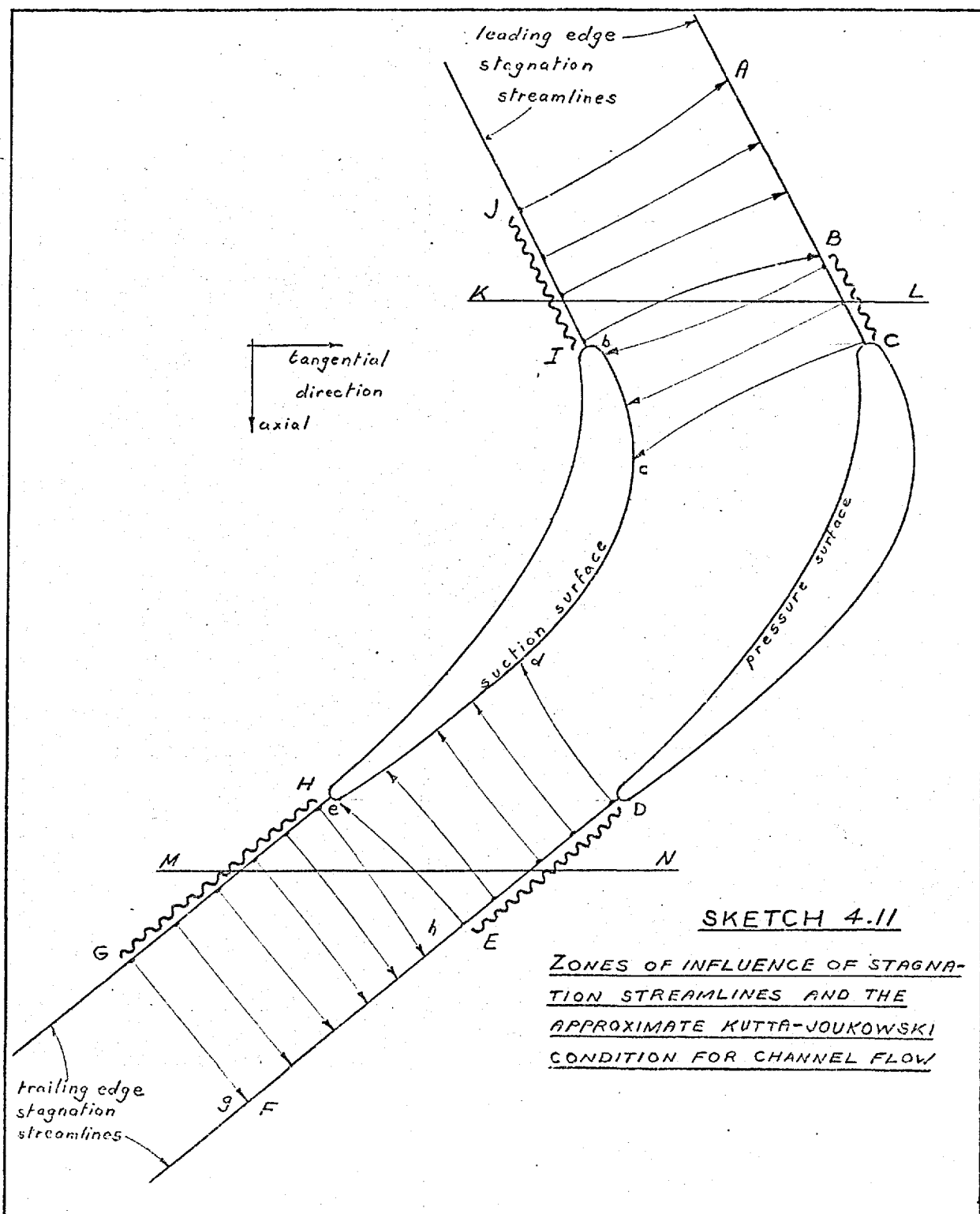
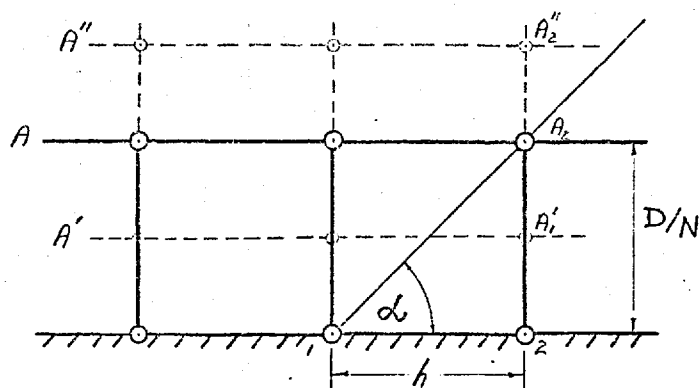
The hyperbolic streamline curvature procedure and a simple wave flow





SKETCH 4.10

Supersonic grid spacing criteria

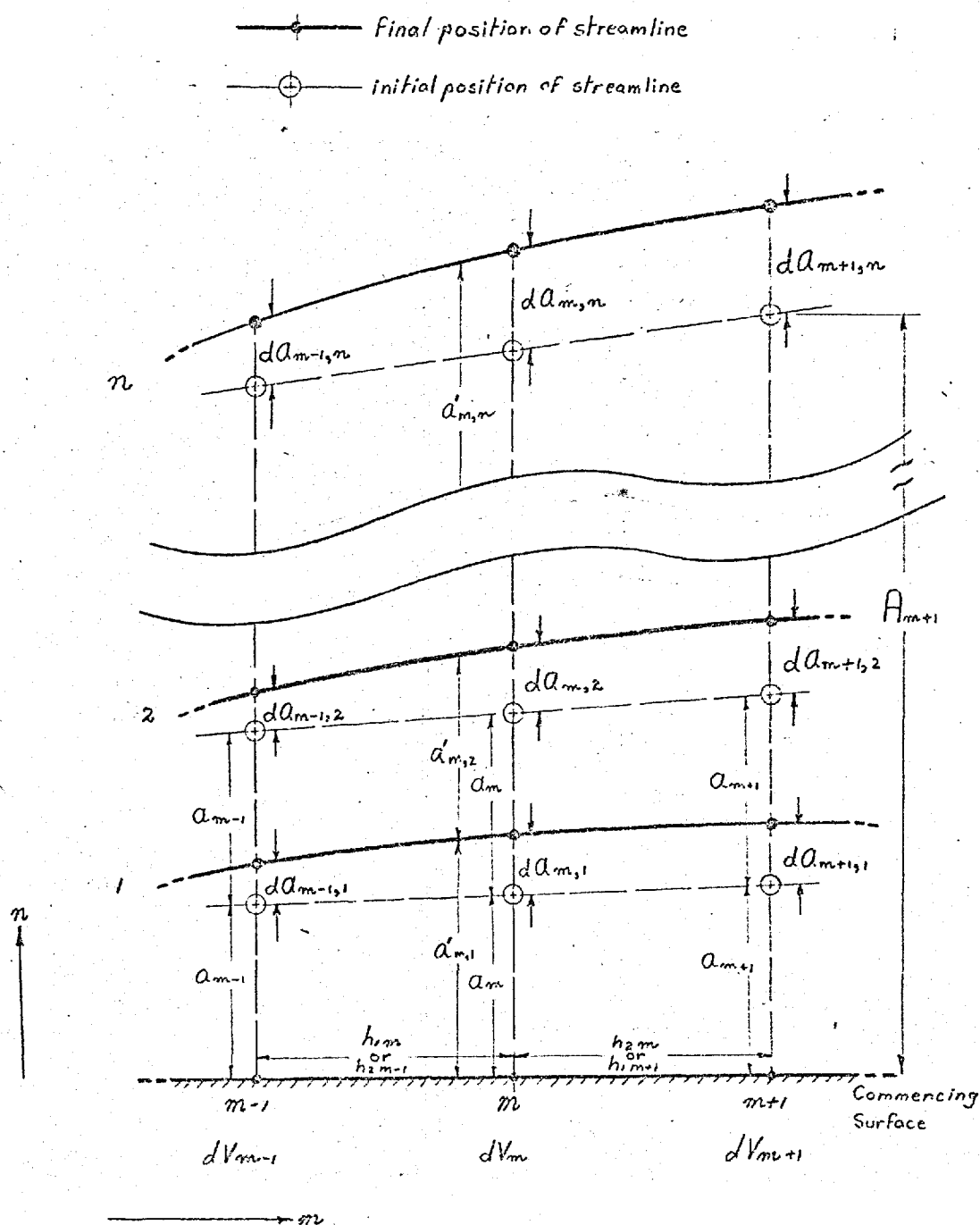


SKETCH 4.11

ZONES OF INFLUENCE OF STAGNA-
TION STREAMLINES AND THE
APPROXIMATE KUTTA-JOUKOWSKI
CONDITION FOR CHANNEL FLOW

SKETCH 5.1

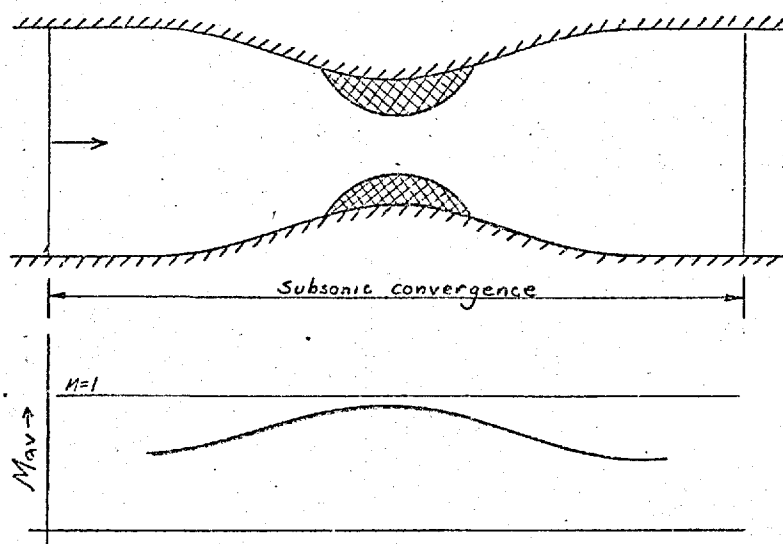
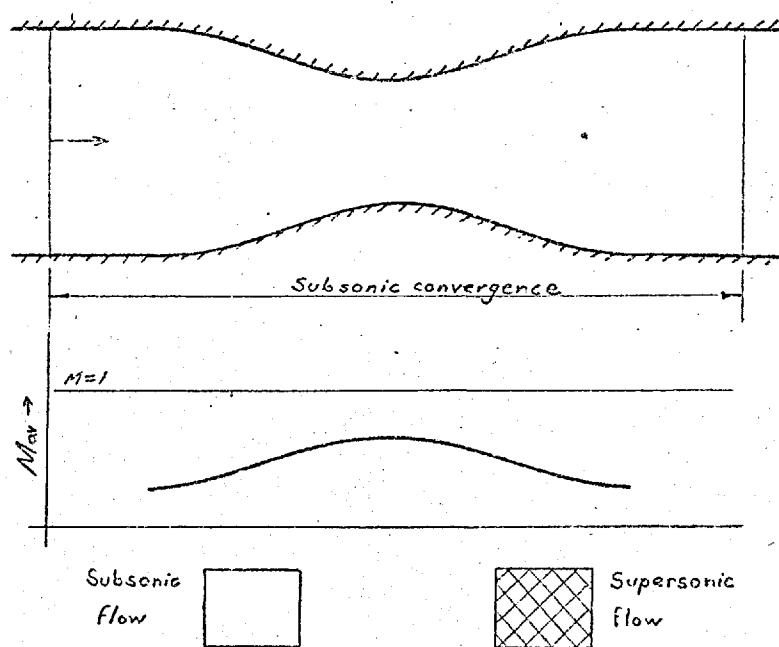
Initial and final orthogonal network of normals
and streamlines with nomenclature for convergence
analysis



SKETCH 5.2 a

Convergence arrangement for low Mach

Number subsonic flow



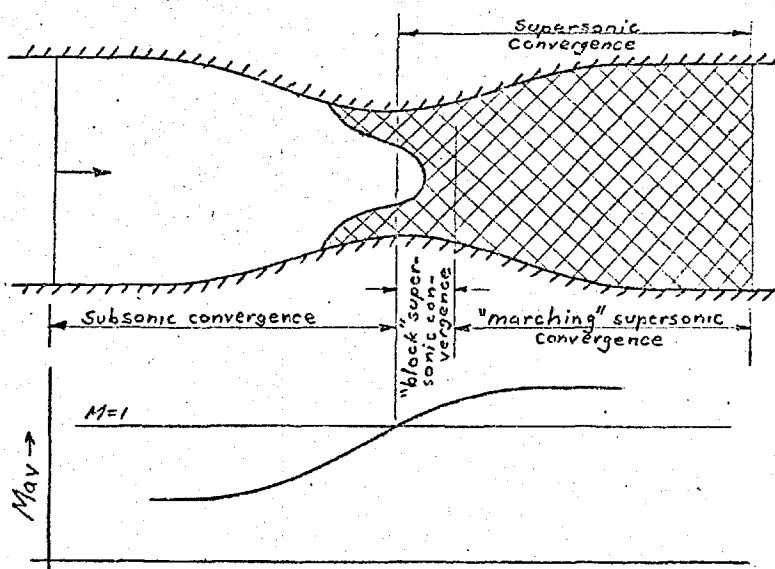
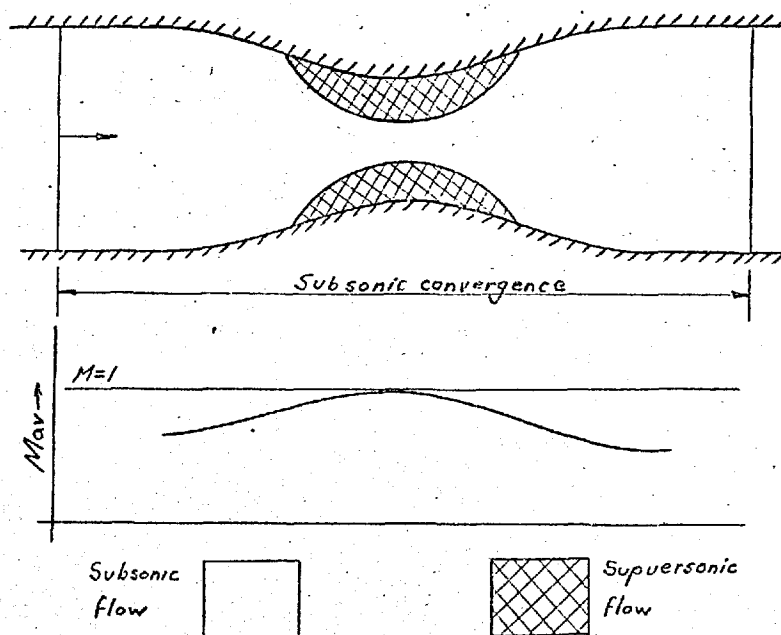
SKETCH 5.2 b

Convergence arrangement for high Mach

Number subsonic flow

SKETCH 5.2c

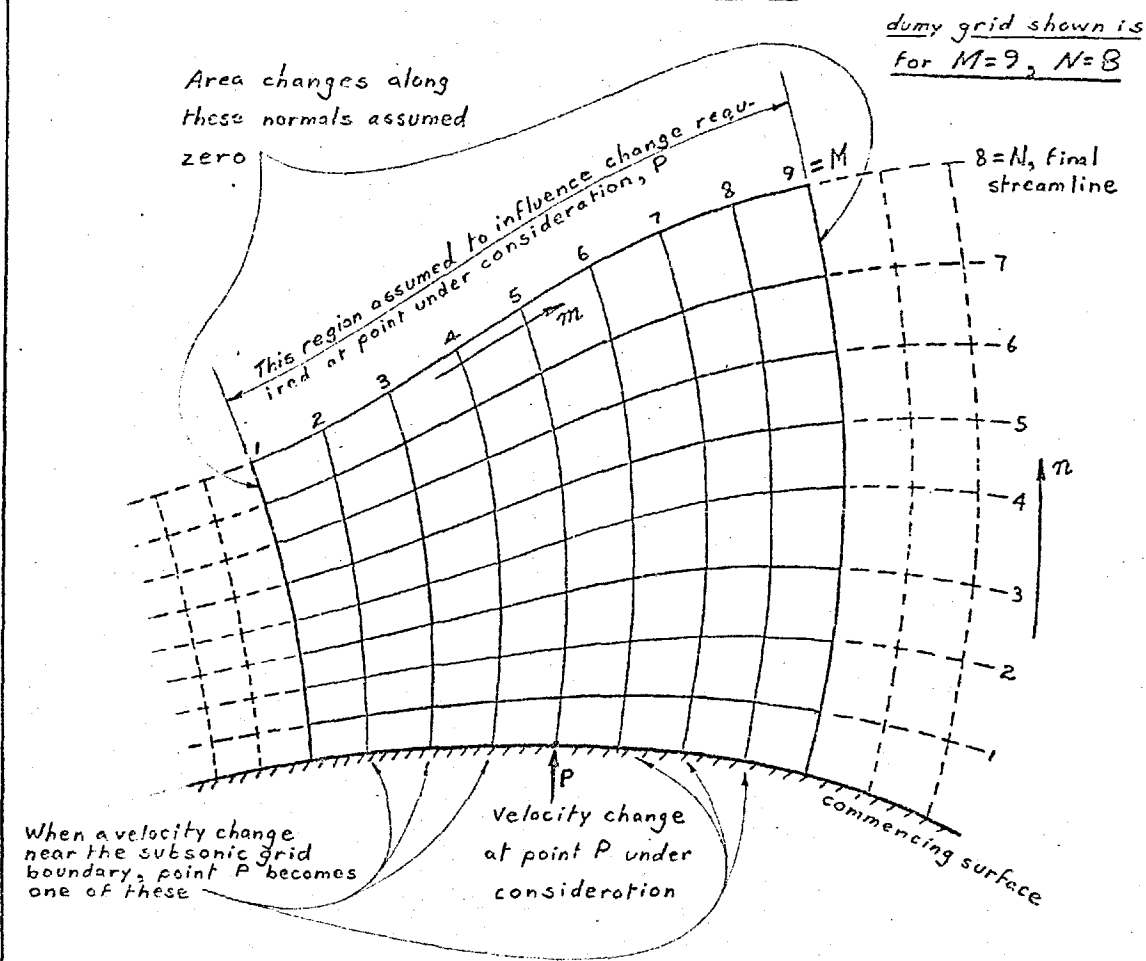
Convergence arrangement for choked subsonic flow



SKETCH 5.2d

Convergence arrangement for Subsonic-Supersonic flow.

SKETCH 5.3
DUMMY GRID ARRANGEMENT FOR SUBSONIC
CONVERGENCE



SKETCH 5.4
EXAMPLE OF GRID USED FOR SUPERSONIC
CONVERGENCE

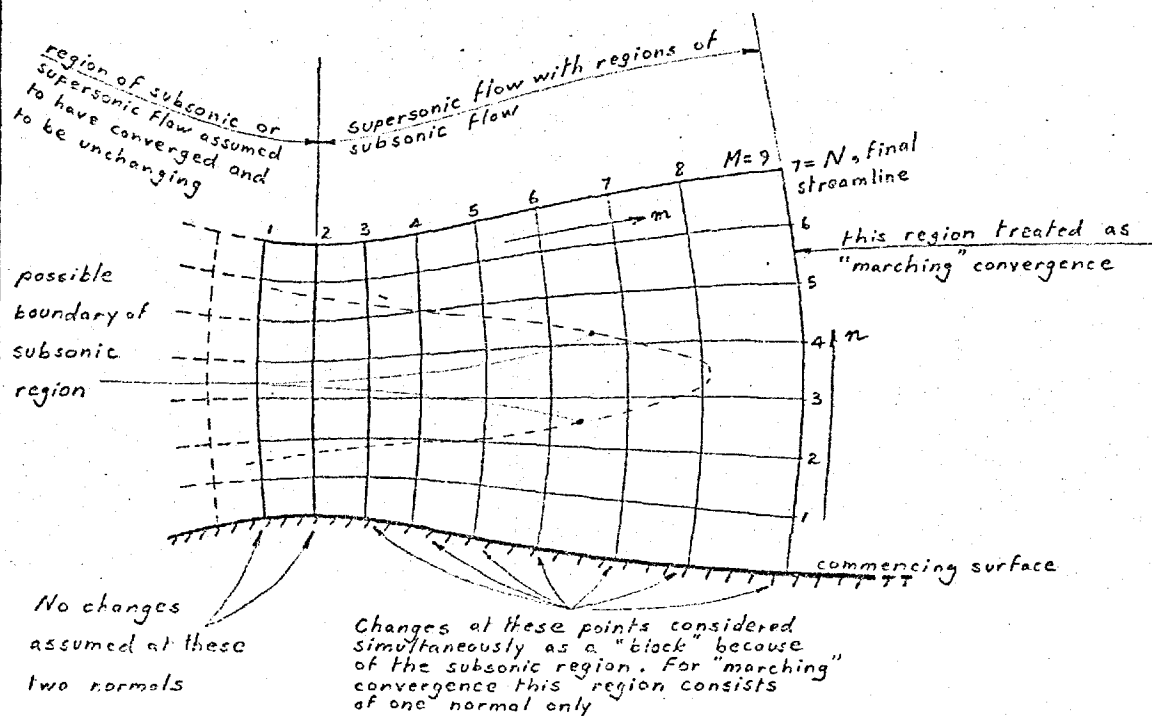
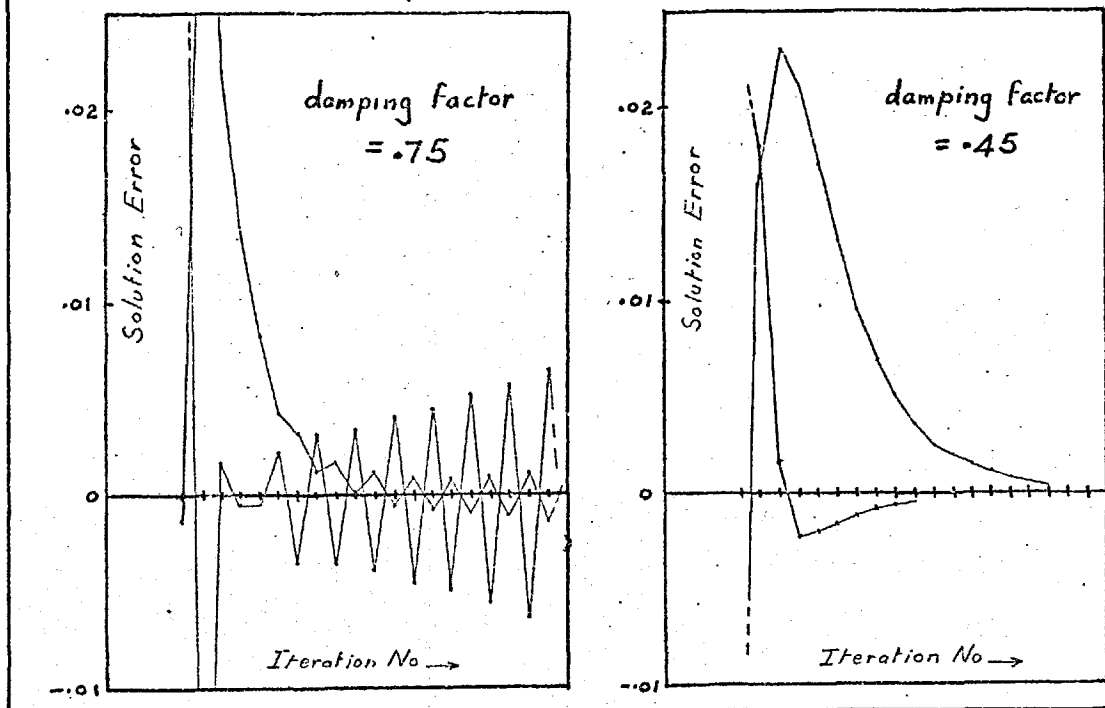


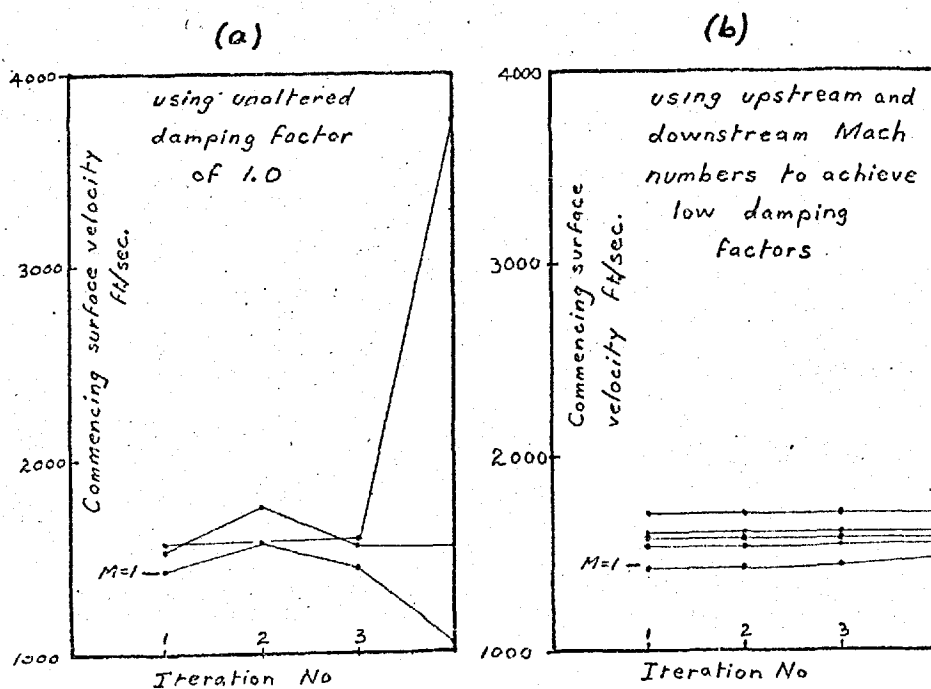
FIGURE 5.5

Oscillations in iteration error
eliminated by a reduction in
damping factor

Example taken from solution of cascade in Fig. 7.2
 Grid points shown are adjacent ones in L.E. region
 Mach No ± 0.2 , Aspect ratio ± 4 , $N_{ps}=30$, $N_{pn}=10$, $M=13$

FIGURE 5.6

Iterative instability in channel without
sharply defined throat



(Example of convergence taken from
 solution of cascade in Fig 7.13)

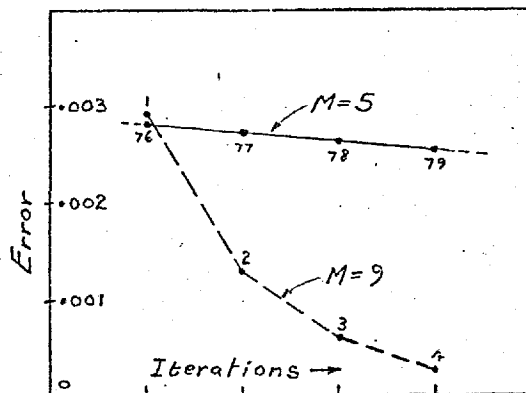
FIGURE 5.7

Slow convergence and divergence through M being too small

Aspect ratio = 7.0 , Mach No. $\doteq$ 0.4

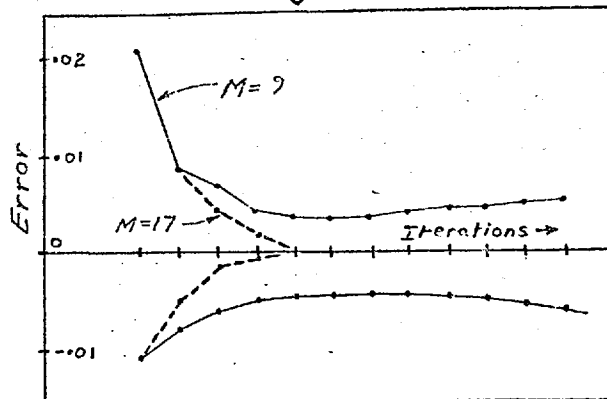
(a)

Slow convergence



(b)

divergence



Example taken from solution of cascade in Figure 7.17

FIGURE 5.8

Divergence in transonic flows through N being too small

Example taken from cascade of Figure 7.17

Aspect ratio = 5 , Mach No (av.) = 0.7

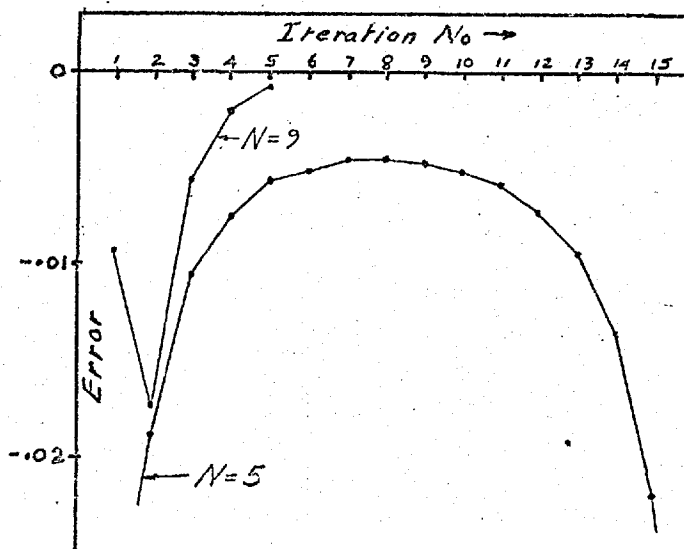
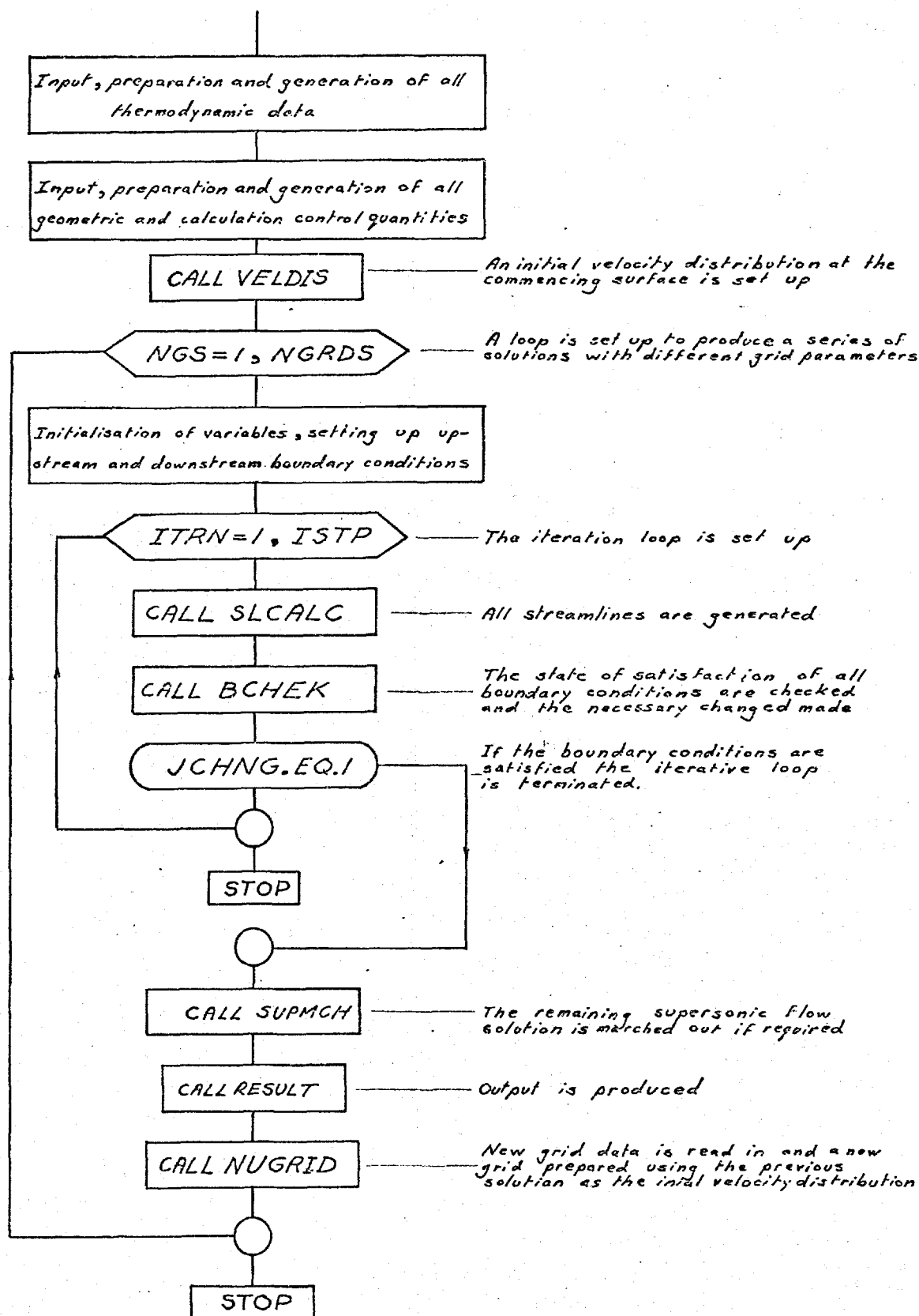


FIGURE 6.1

MAIN PROGRAMME OVERALL FLOW CHART



PROFILE COORDINATE SYSTEM AND DATA SETUP

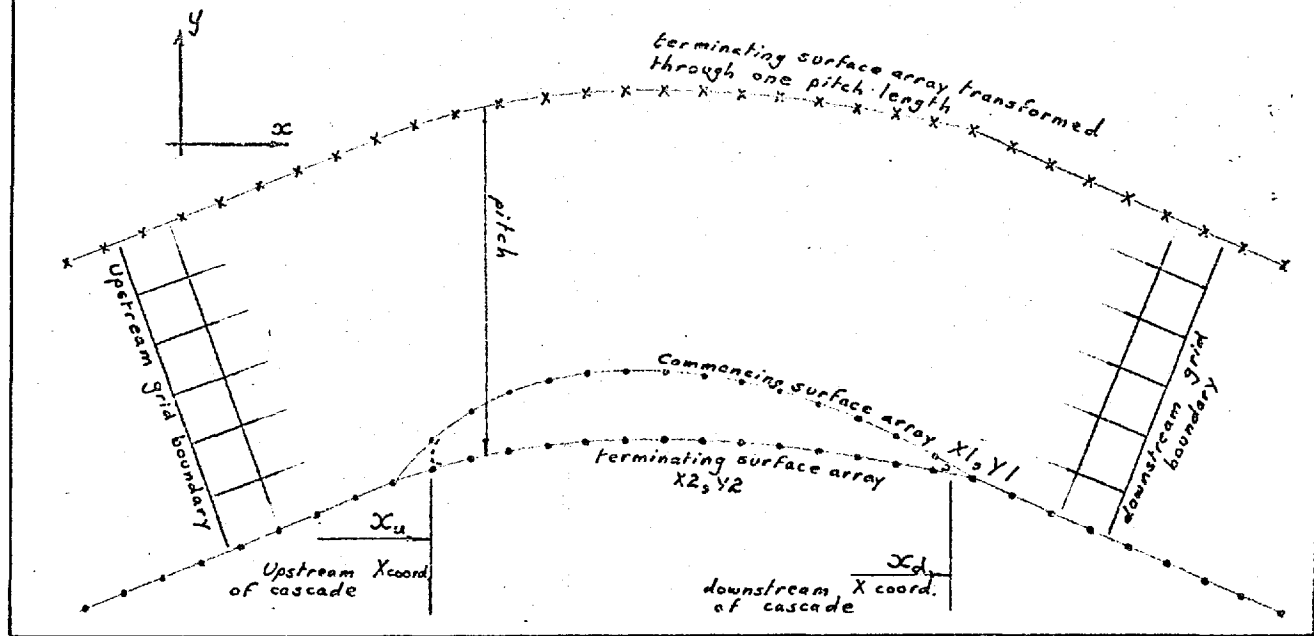


FIGURE 6.3

OVERALL FLOW CHART FOR BOUNDARY CONDITION CHECKING SUBROUTINE

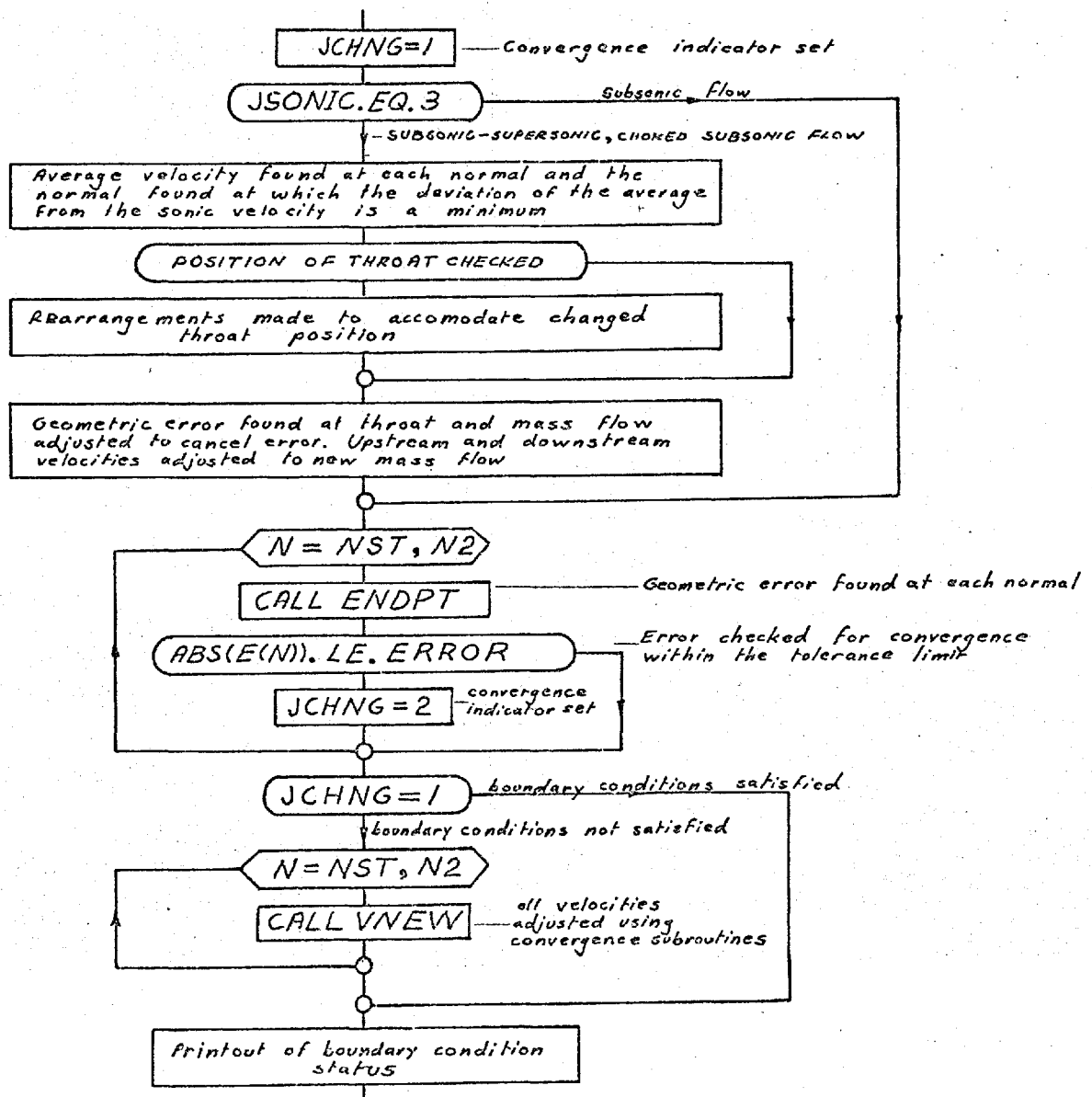
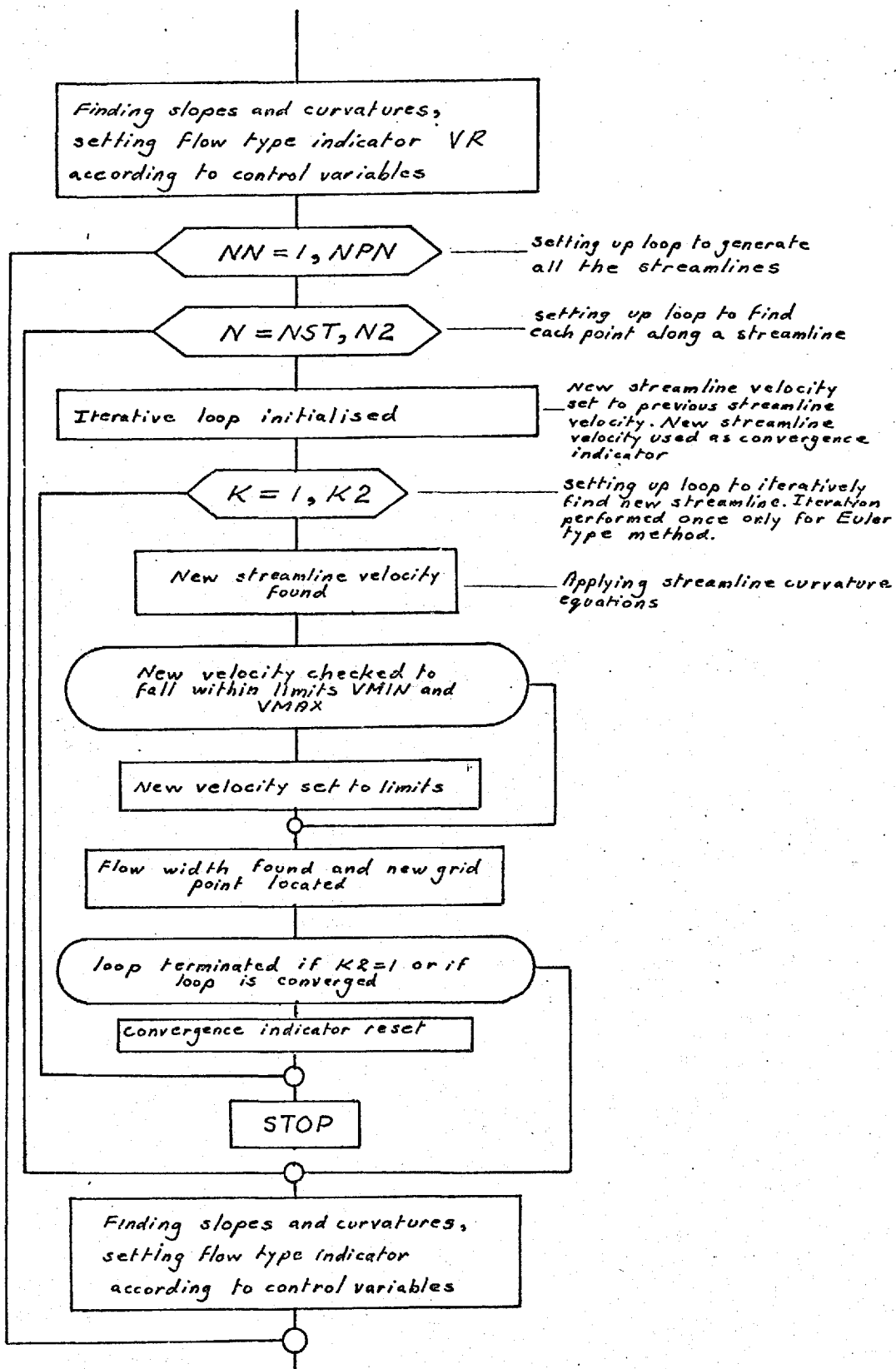


FIGURE 6.4
OVERALL FLOW CHART FOR STREAMLINE
GENERATING SUBROUTINE SLCALC



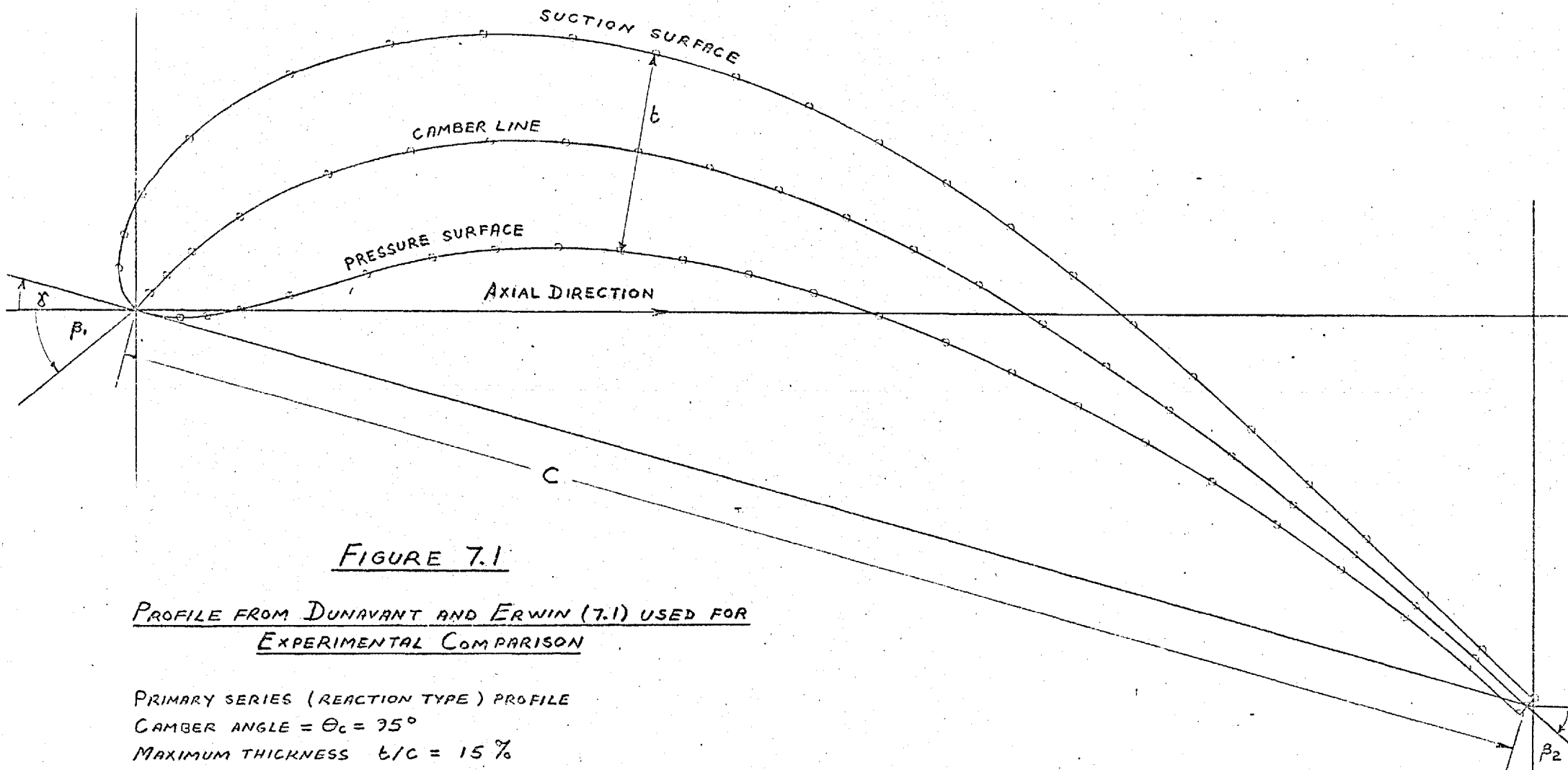


FIGURE 7.1

PROFILE FROM DUNAVANT AND ERWIN (7.1) USED FOR
EXPERIMENTAL COMPARISON

PRIMARY SERIES (REACTION TYPE) PROFILE

CAMBER ANGLE = $\theta_c = 75^\circ$

MAXIMUM THICKNESS $b/c = 15\%$

STAGGER ANGLE SHOWN $\delta = 16.1^\circ$

GAS INLET ANGLE SHOWN $\beta_1 = 40^\circ$

LEAVING STREAMLINE ASSUMED STRAIGHT WITH $3\frac{1}{2}^\circ$ DEVIATION UNLESS OTHERWISE STATED.

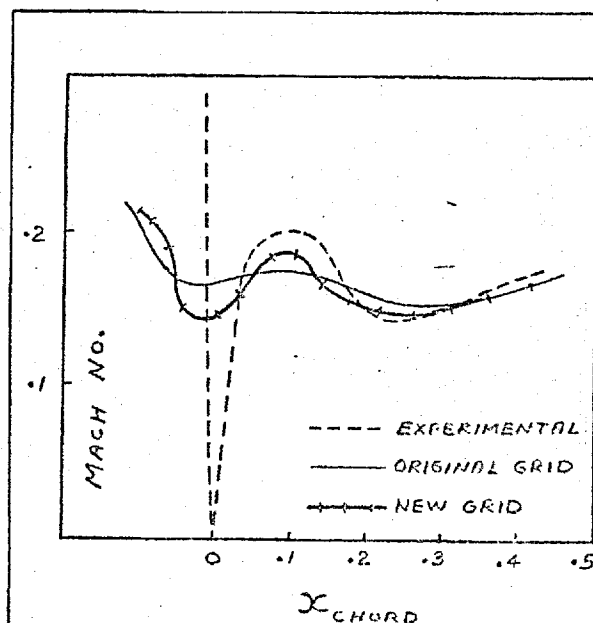
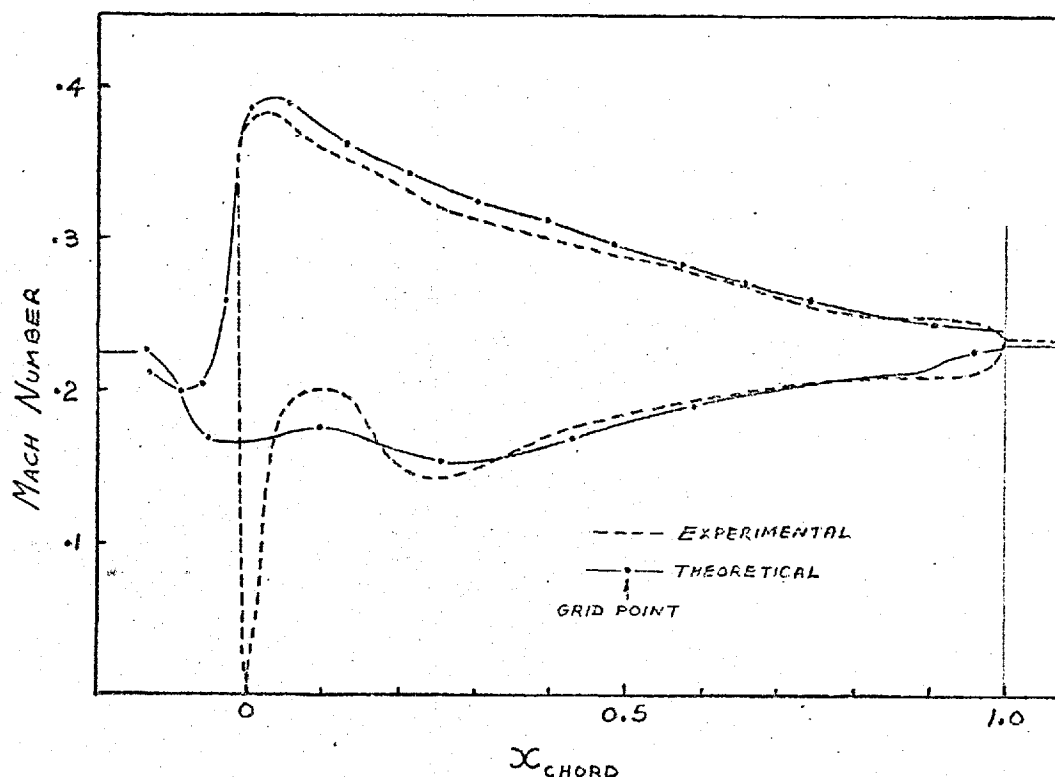
FIGURE 7.2aEXPERIMENTAL COMPARISON FOR LOW MACH NUMBER
AND STAGGER ANGLE CASCADE

STAGGER ANGLE $\delta = 16.1^\circ$, INLET ANGLE $\beta_1 = 40^\circ$
 SOLIDITY CHORD / PITCH = 1.8, PROFILE AS FOR FIGURE 7.1

GRID ASPECT RATIO (SUCTION SURFACE SPACING / CHANNEL WIDTH)
 4.7 AT L.E., 2.2 AT T.E.

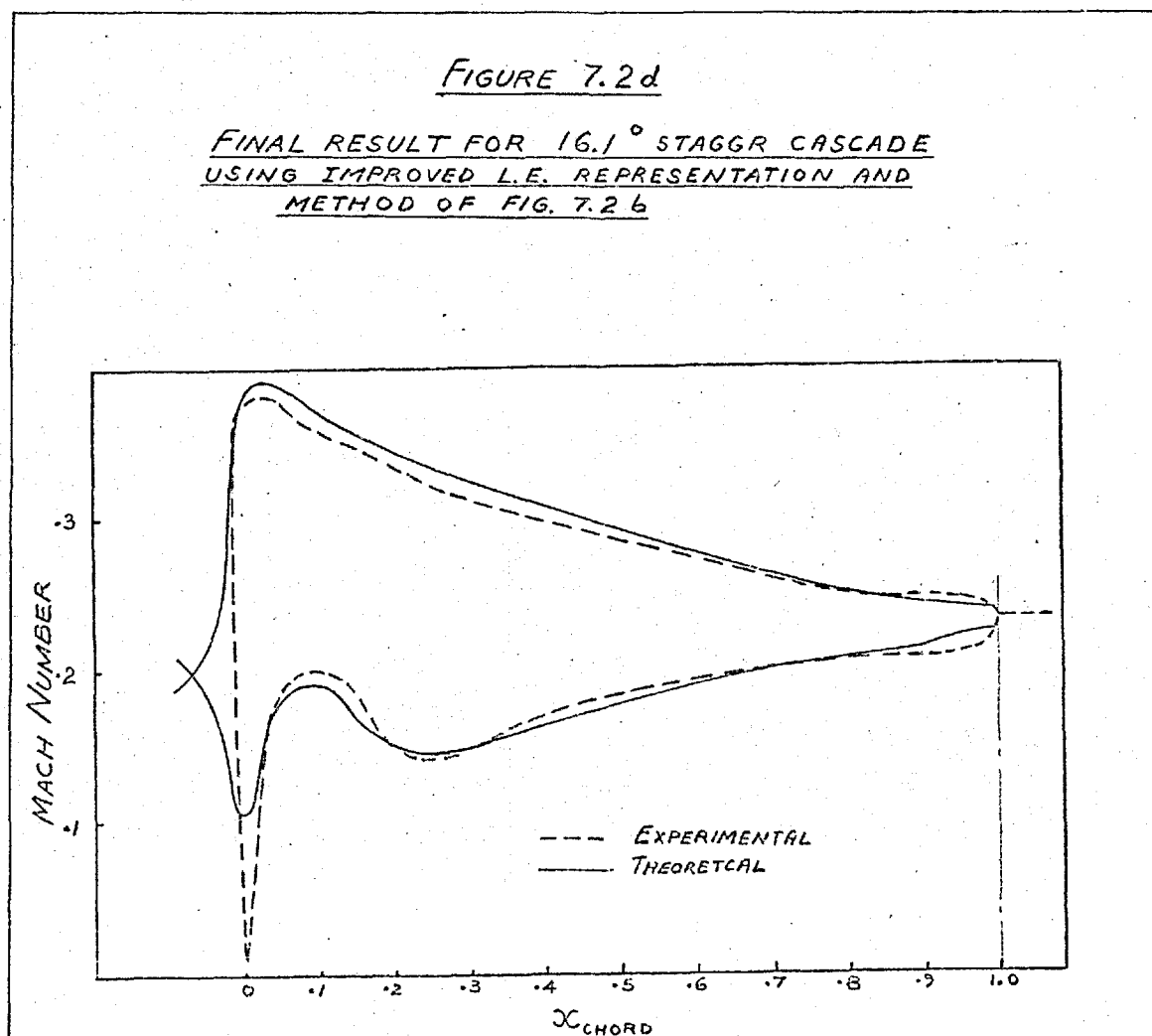
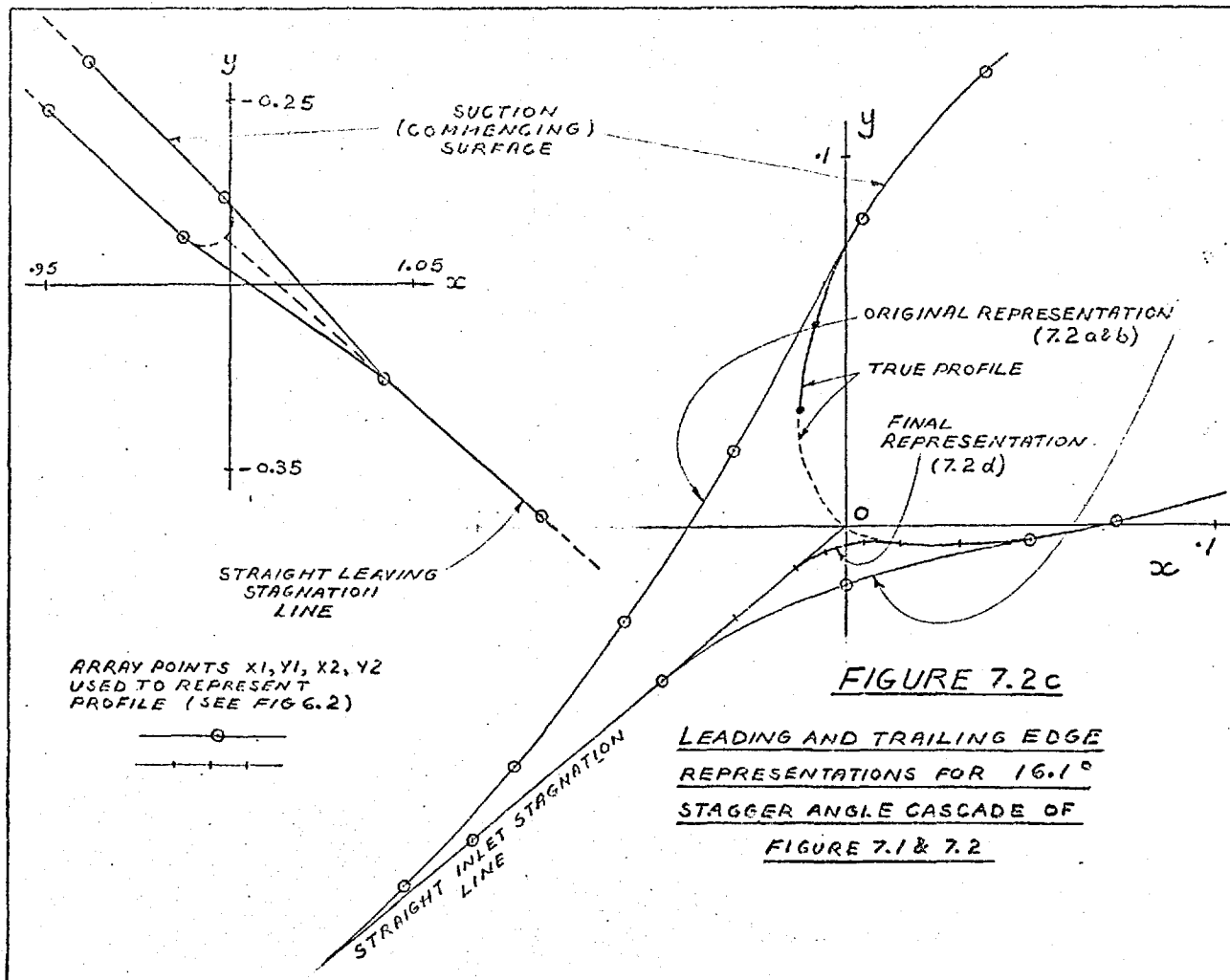
NUMBER OF STREAMLINES = 15

NUMERICAL METHOD - EULER TYPE

FIGURE 7.2bPRESSURE SURFACE LEADING EDGE
IMPROVEMENT USING FINE GRID
AND MID STREAMLINE AS CHANNEL
BOUNDARY

GRID ASPECT RATIO (BASED ON CHANNEL
 FORMED BY MID STREAMLINE) = 6.2

NUMBER OF STREAMLINES = 7



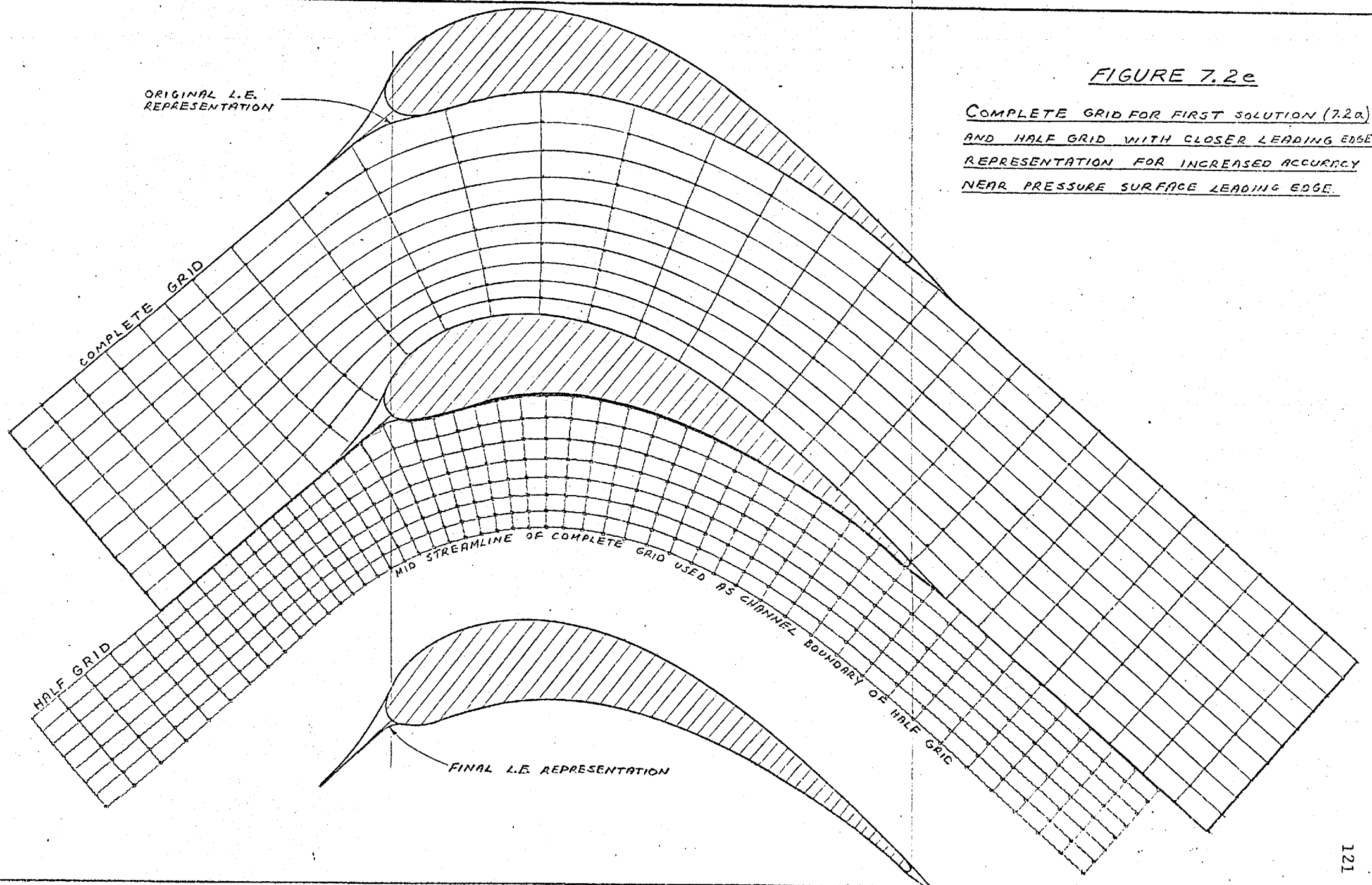


FIGURE 7.2c

COMPLETE GRID FOR FIRST SOLUTION (7.2a)
AND HALF GRID WITH CLOSER LEADING EDGE
REPRESENTATION FOR INCREASED ACCURACY
NEAR PRESSURE SURFACE LEADING EDGE.

EXPERIMENTAL COMPARISON FOR INTERMEDIATE MACH NUMBER
AND STAGGER ANGLE

STAGGER ANGLE $\chi = 34.1^\circ$; INLET ANGLE $\beta_1 = 40^\circ$; SOLIDITY $= 1.8$
 $N_{pr} = 10$, NUMERICAL METHOD - EULER TYPE

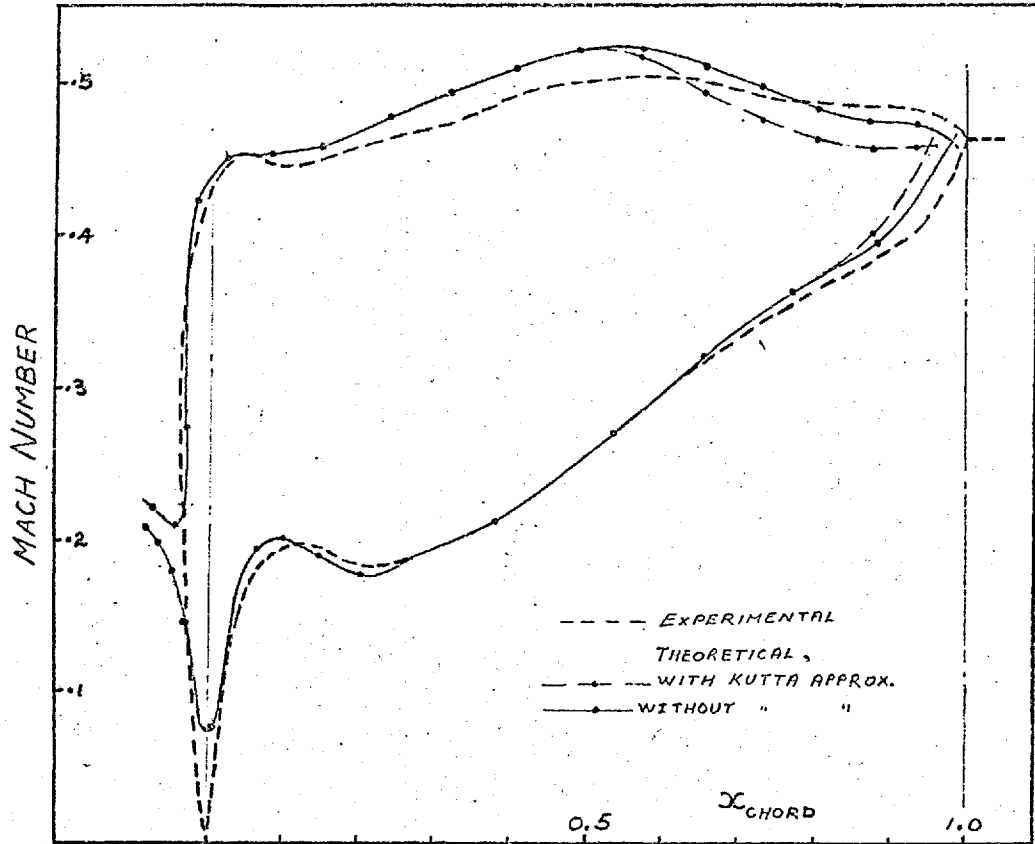


FIGURE 7.4a

INFLUENCE OF GRID SIZE

CASCADE AS FOR FIGURE 7.3
COARSE GRID ASPECT RATIO - 3.5 AT L.E., 2.0 AT T.E

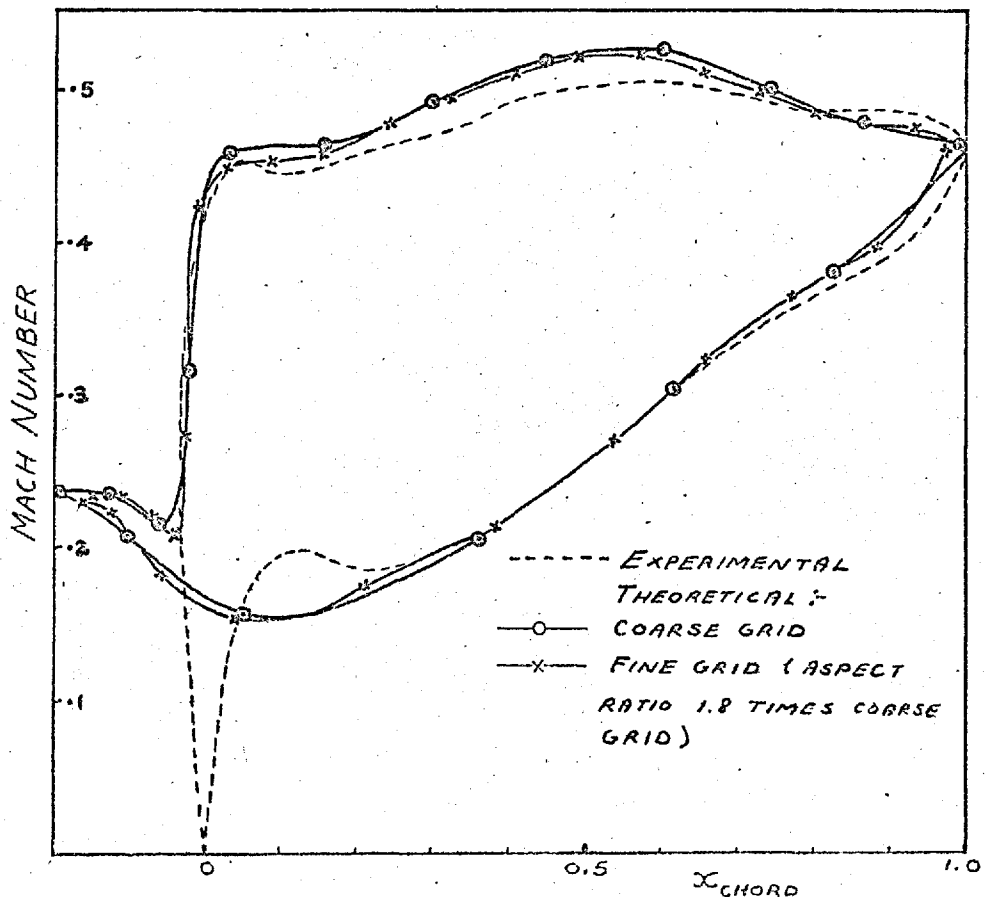


FIGURE 7.4 b

COMPARISON OF THEORETICAL
RESULTS USING TWO CALCULATION
METHODS

NUMBER OF STREAMLINES = 10
CASCADE AS IN FIG. 7.3

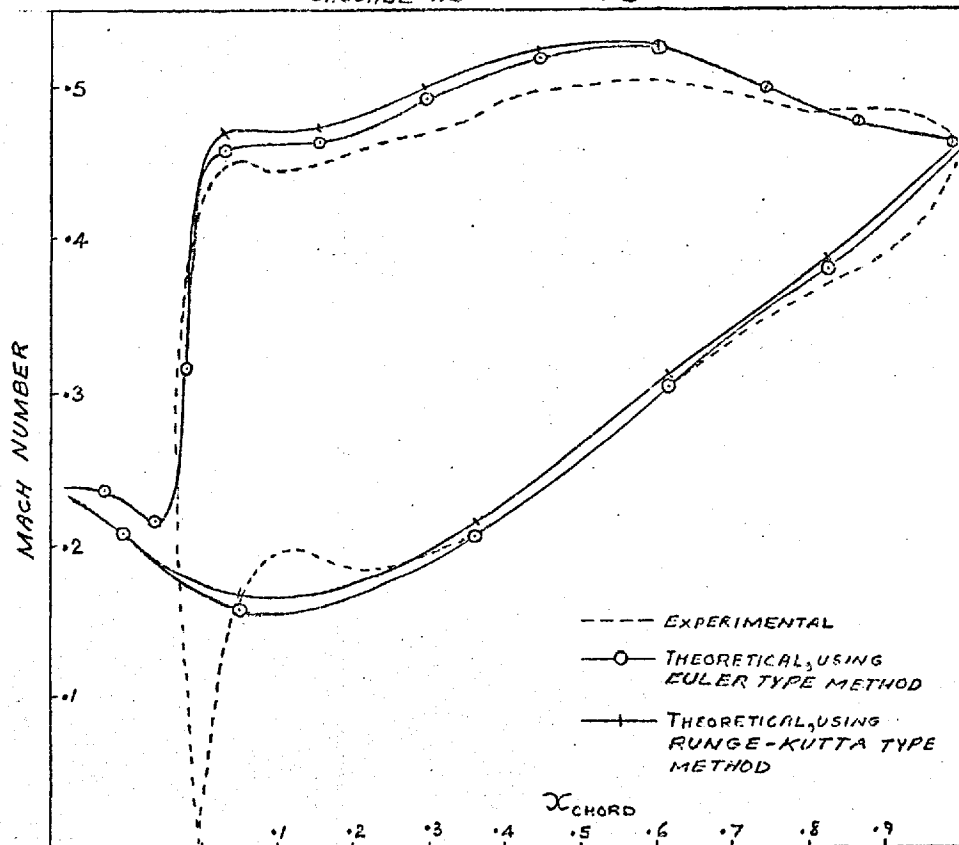


FIGURE 7.4 c

COMPARISON OF THEORETICAL RESULTS
FOR THE TWO CALCULATION METHODS AND
FOR AN INCREASE IN THE NUMBER OF
STREAMLINES

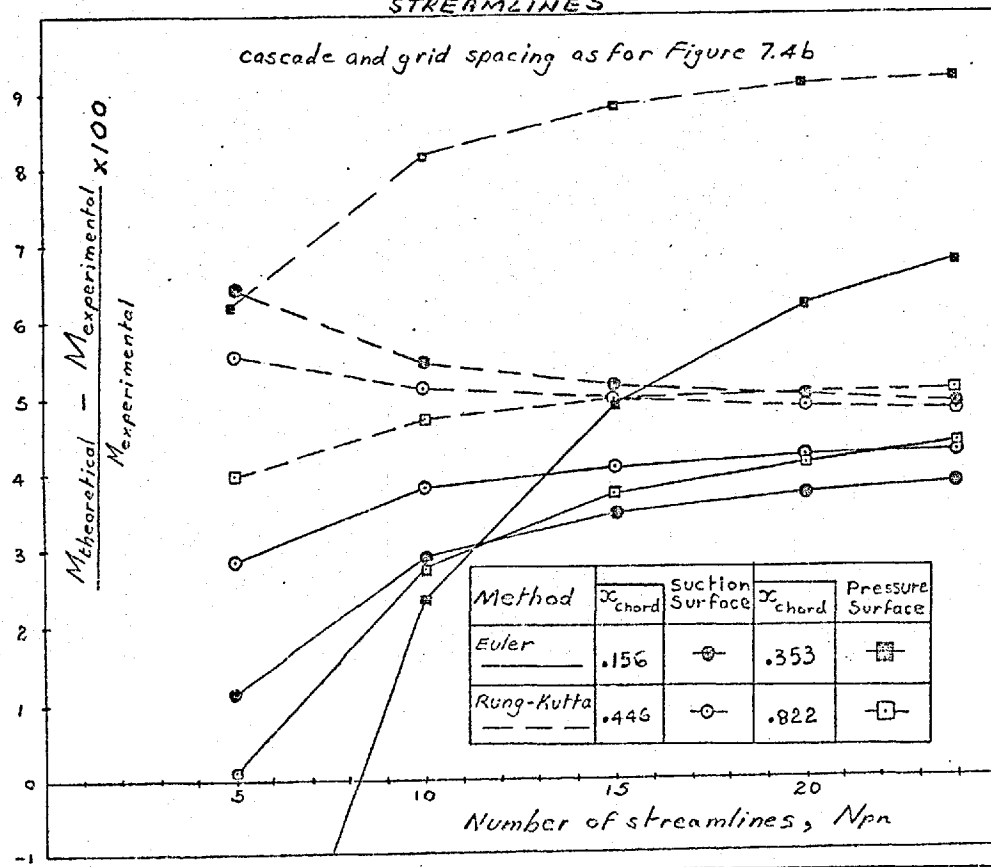


FIGURE 7.5a

EXPERIMENTAL COMPARISON
FOR LOW MACH NUMBER
AND HIGH STAGGER ANGLE

STAGGER ANGLE $\delta = 49.1^\circ$
SOLIDITY, CHORD/PITCH = 1.8
NUMBER OF STREAMLINES = 15
NUMERICAL METHOD - EULER TYPE
MID STREAMLINE, FINE SPACING
TECHNIQUE USED AT L.E.

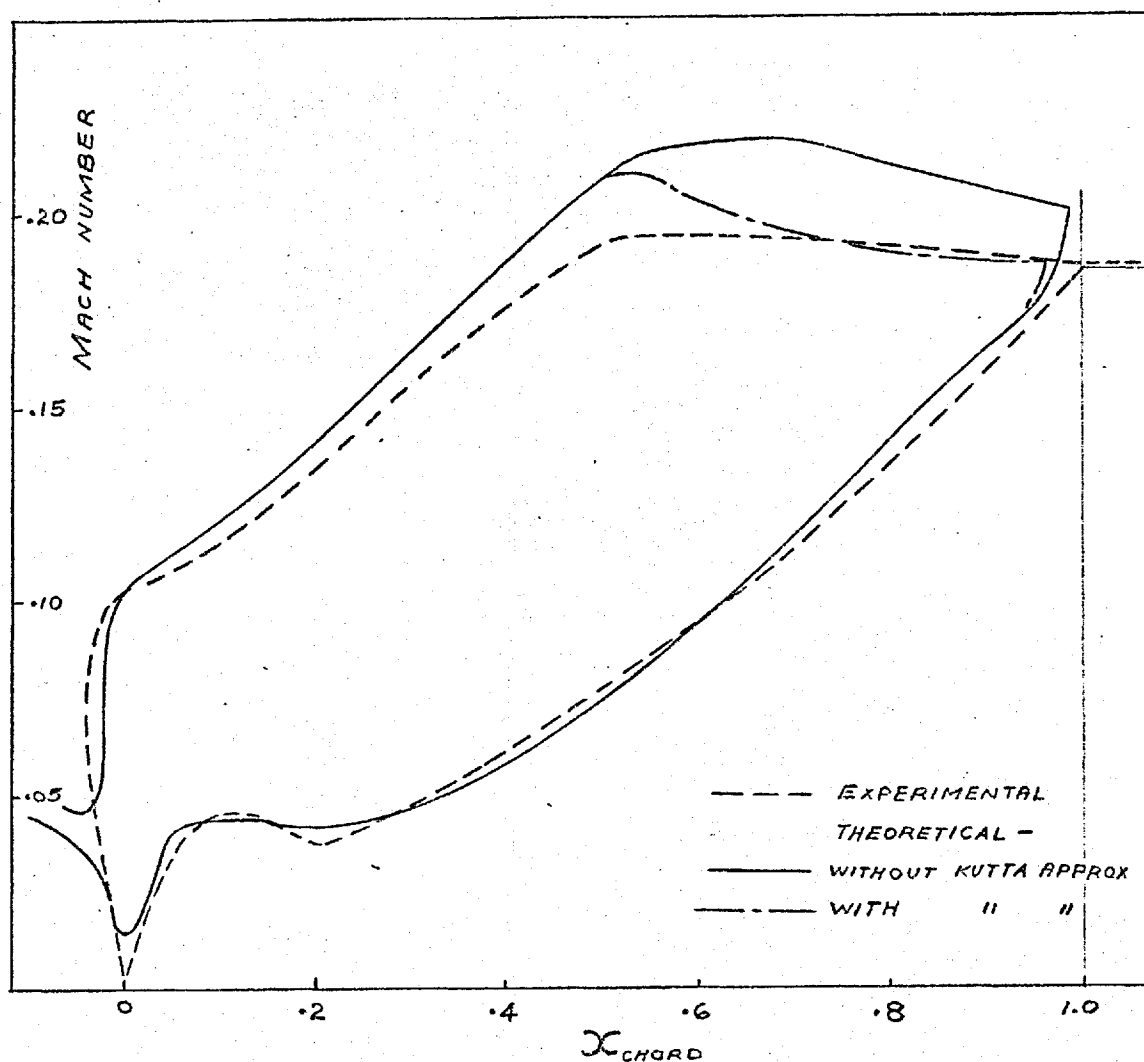


FIGURE 7.5b

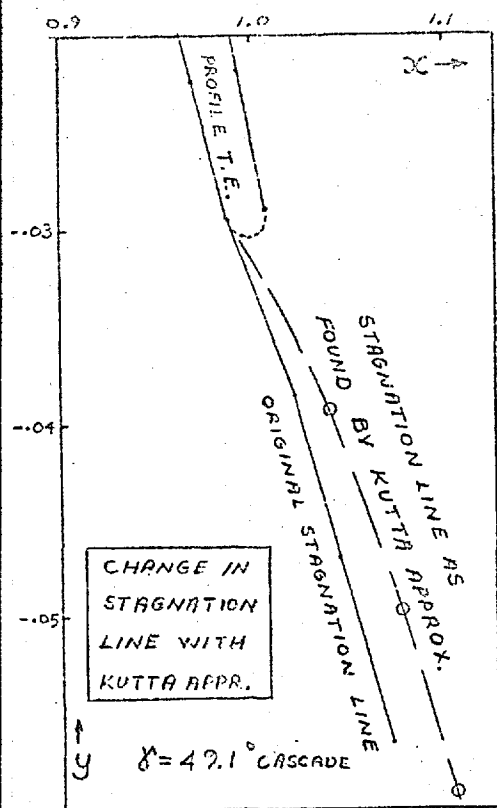


FIGURE 7.5c

EFFECT OF CHANGING COMMENCING SURFACE STAGNATION LINE TOWARD POSITION ESTIMATED BY KUTTA APPROXIMATION.

$\gamma = 42.1^\circ$ CASCADE

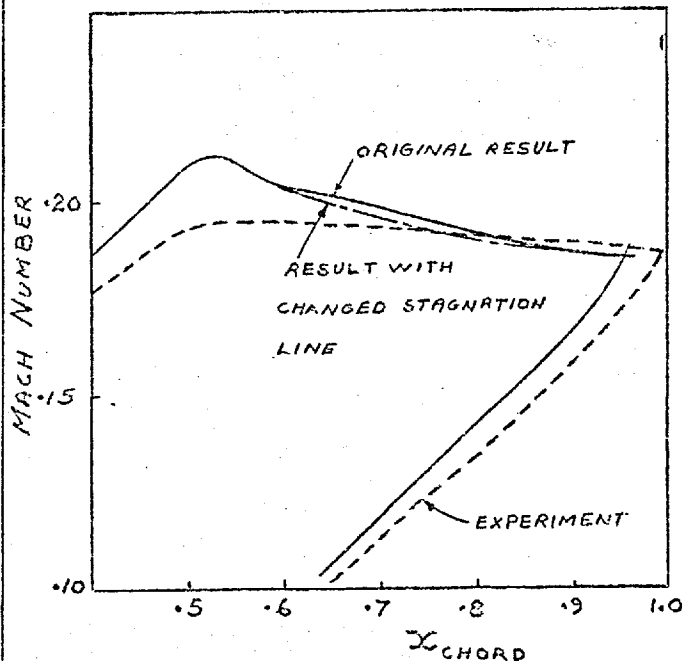
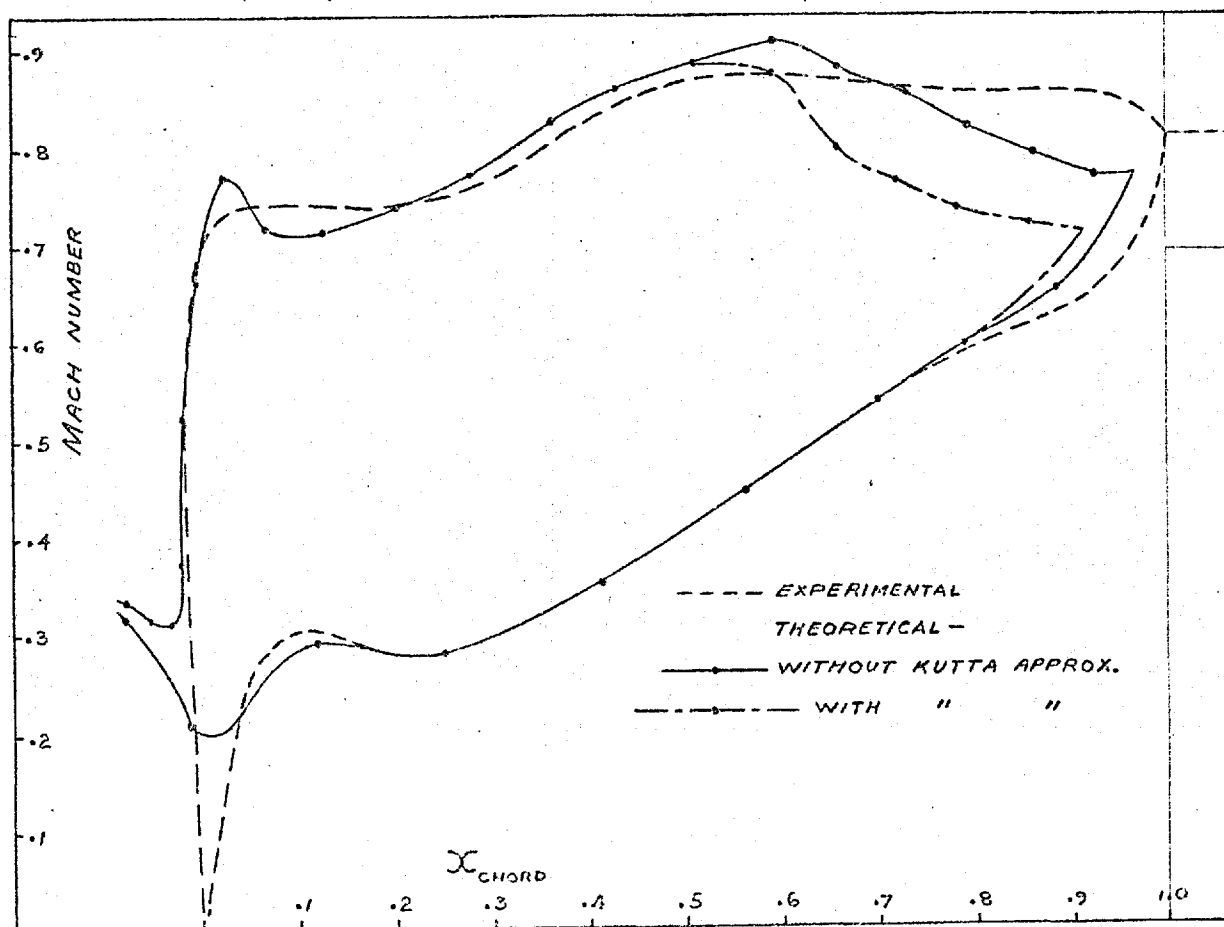


FIGURE 7.6a

EXPERIMENTAL COMPARISON AT HIGH SUBSONIC MACH NUMBER

STAGGER ANGLE $\gamma = 31.1^\circ$, INLET ANGLE, $\beta_i = 30^\circ$
 SOLIDITY = 1.8 , NUMBER OF STREAMLINES = 12
 NUMERICAL METHOD - EULER TYPE
 GRID ASPECT RATIO VARIES FROM 10 AT L.E. TO 4 AT T.E. (MID STREAMLINE TECHNIQUE NOT USED)



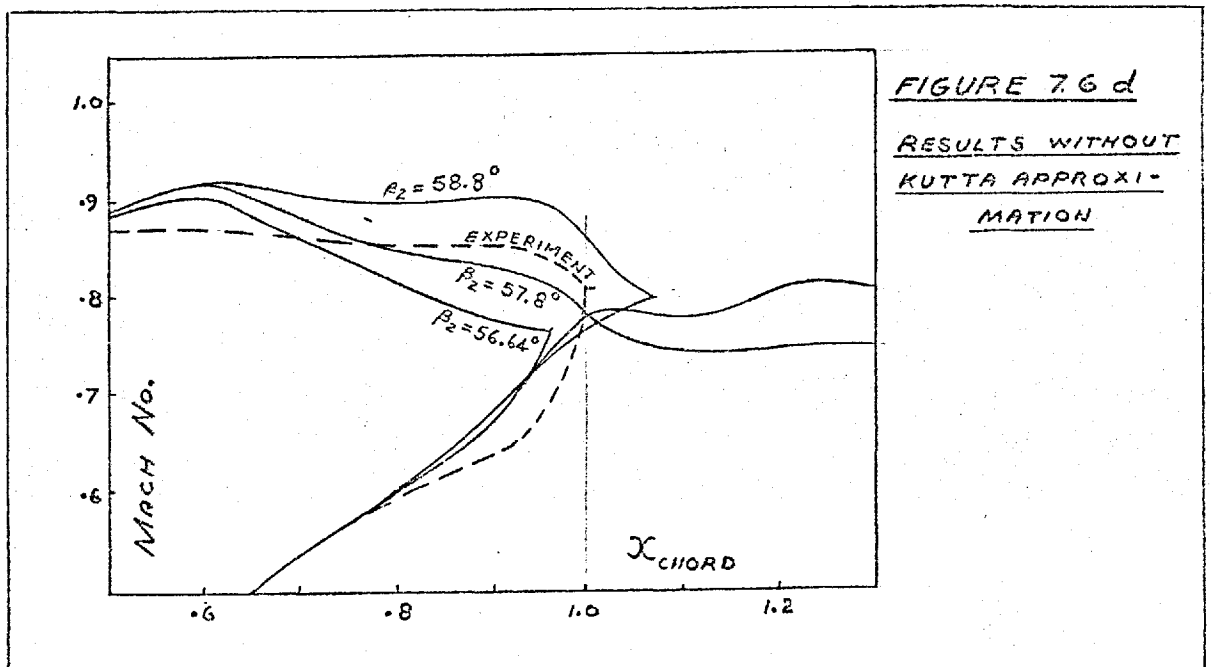
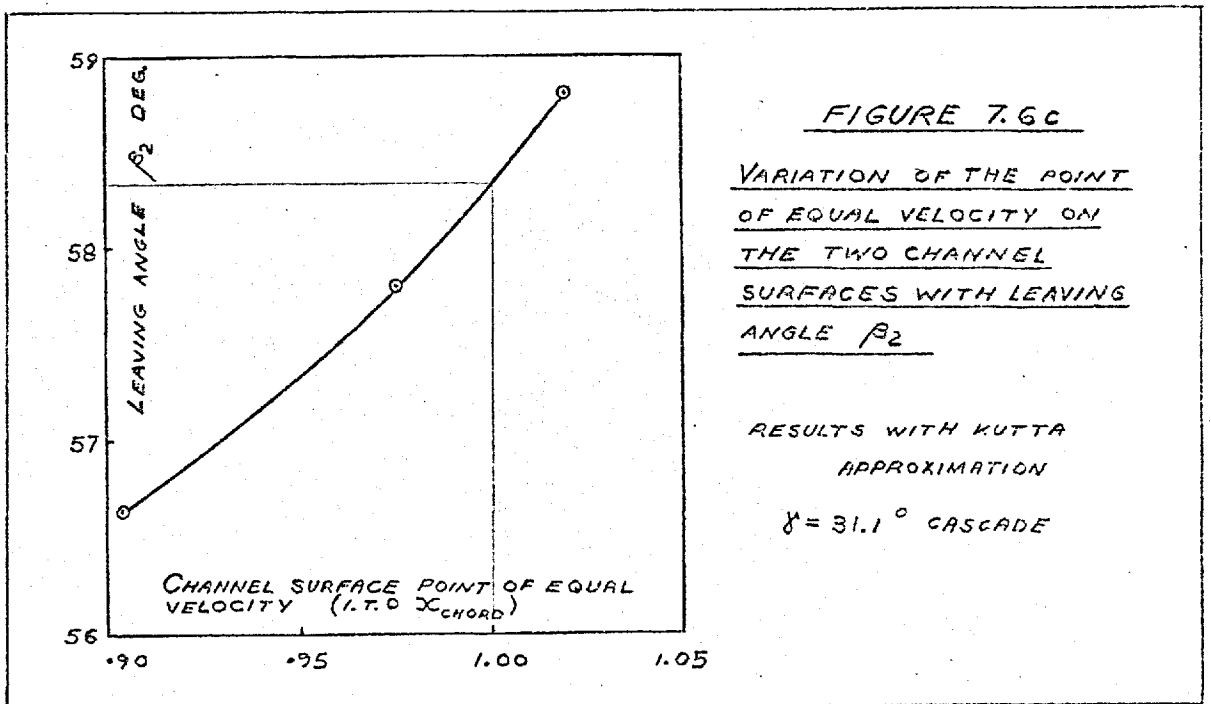
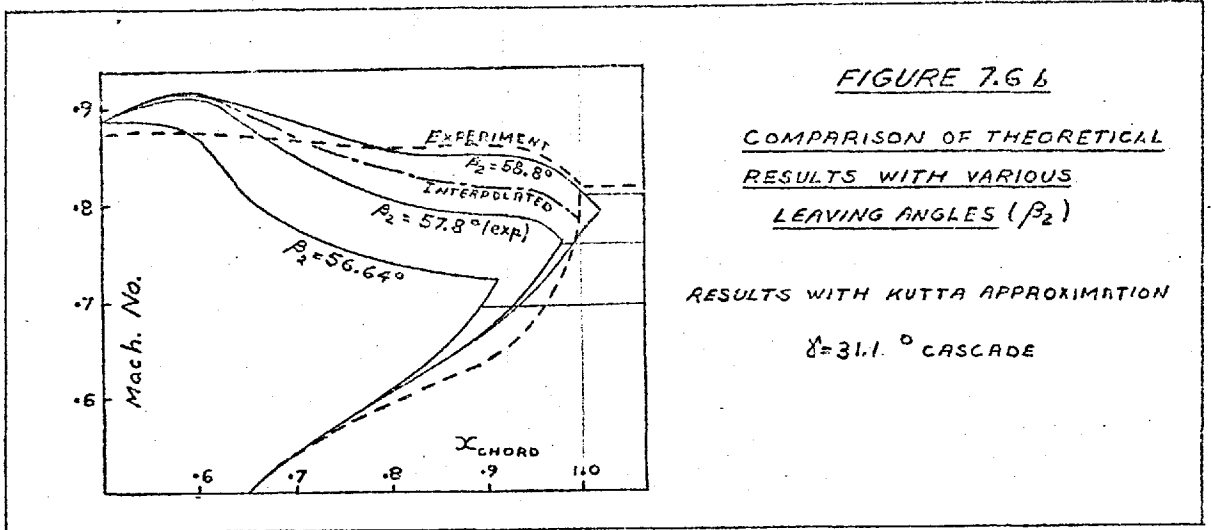


FIGURE 7.7a

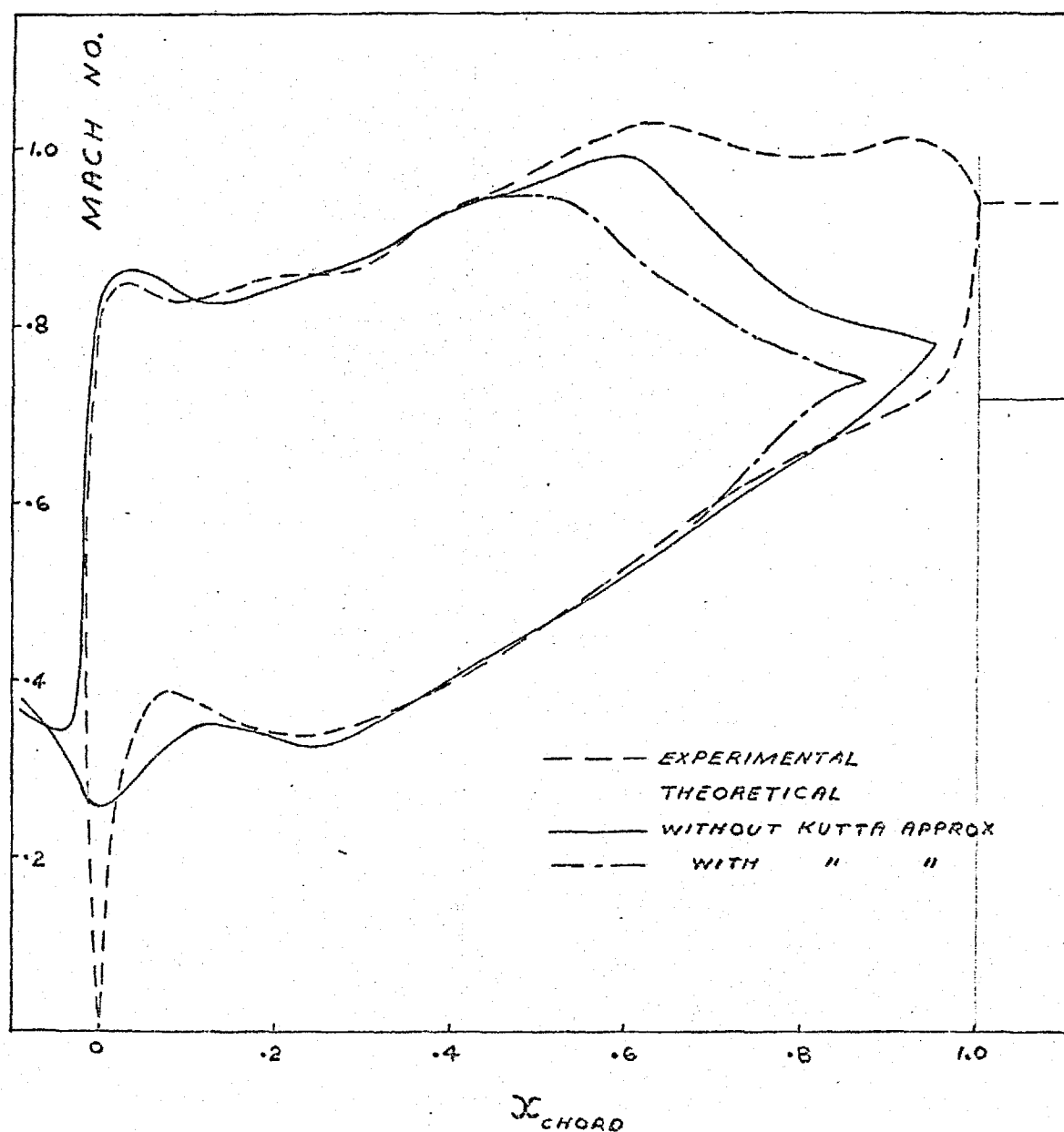
EXPERIMENTAL COMPARISON
FOR CASCADE WITH SMALL
REGIONS OF SUPERSONIC FLOW

STAGGER ANGLE $\delta = 28.1^\circ$

INLET ANGLE $\beta_1 = 30^\circ$

SOLIDITY = 1.8

NUMERICAL METHOD - EULER TYPE



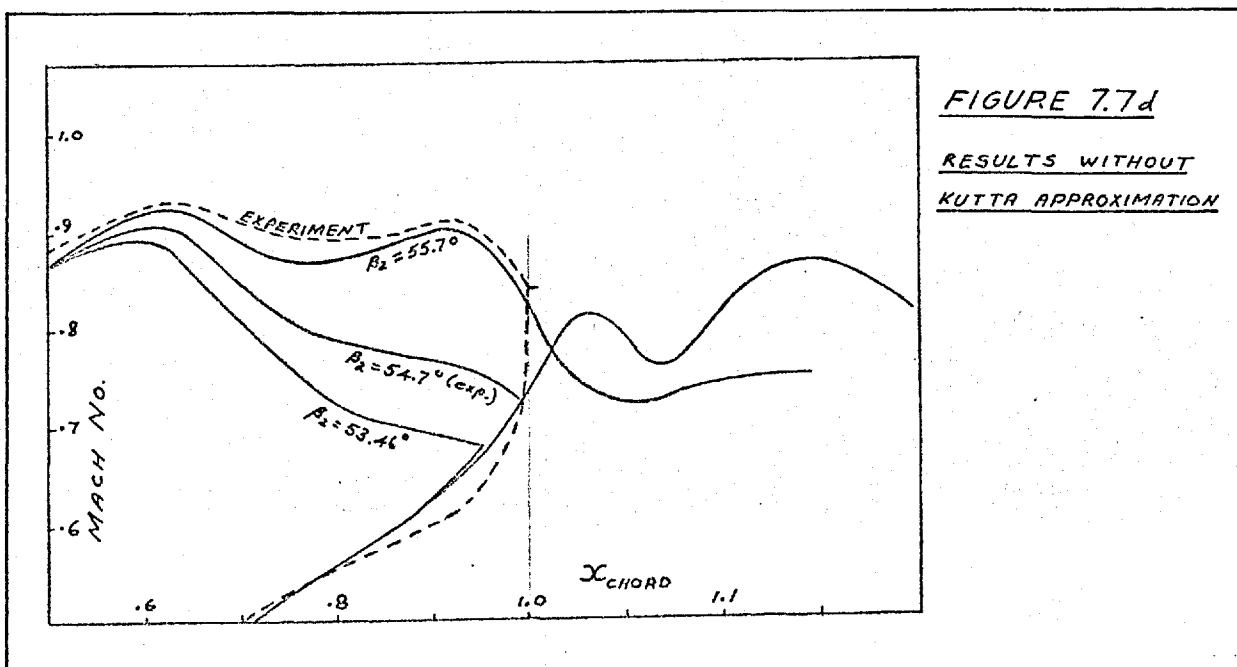
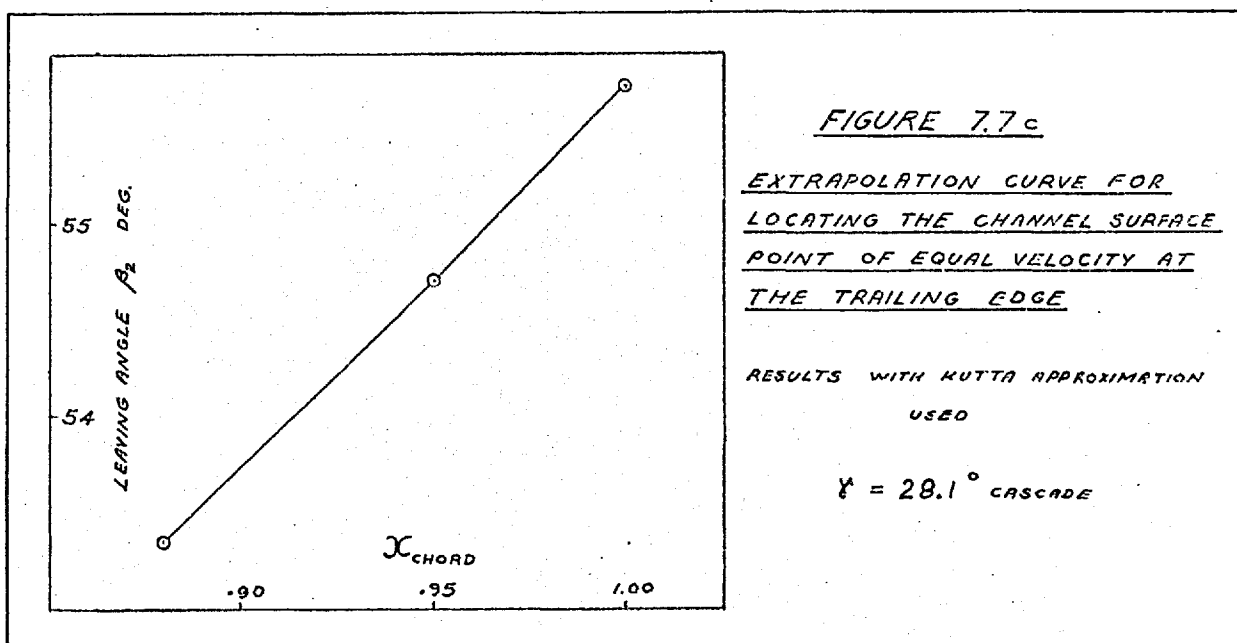
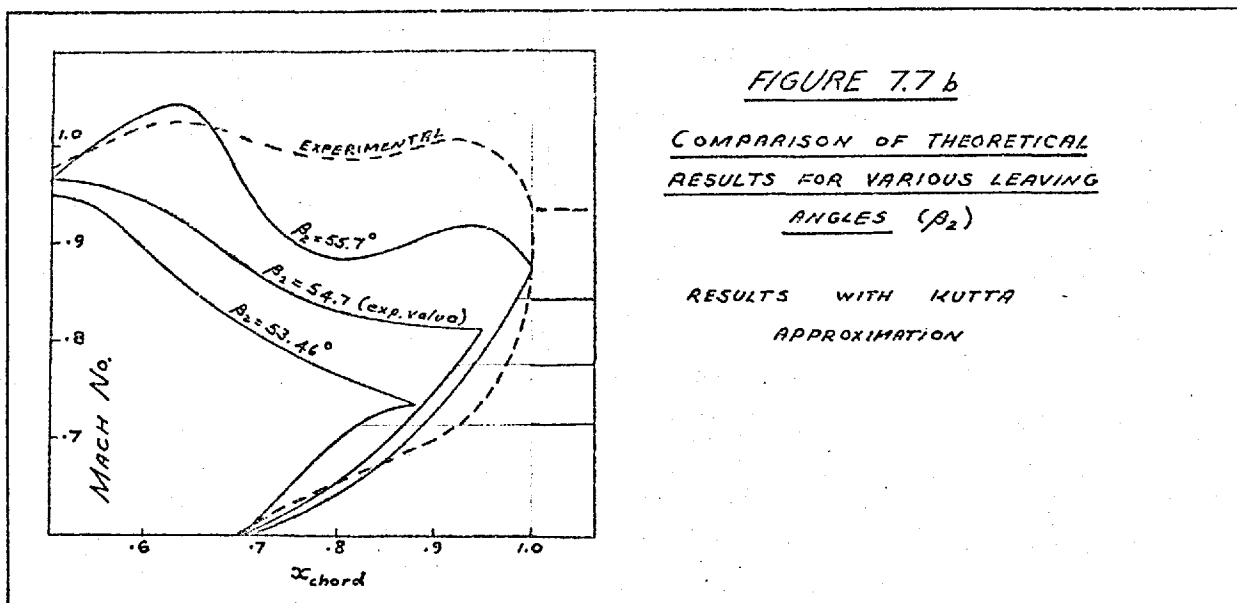
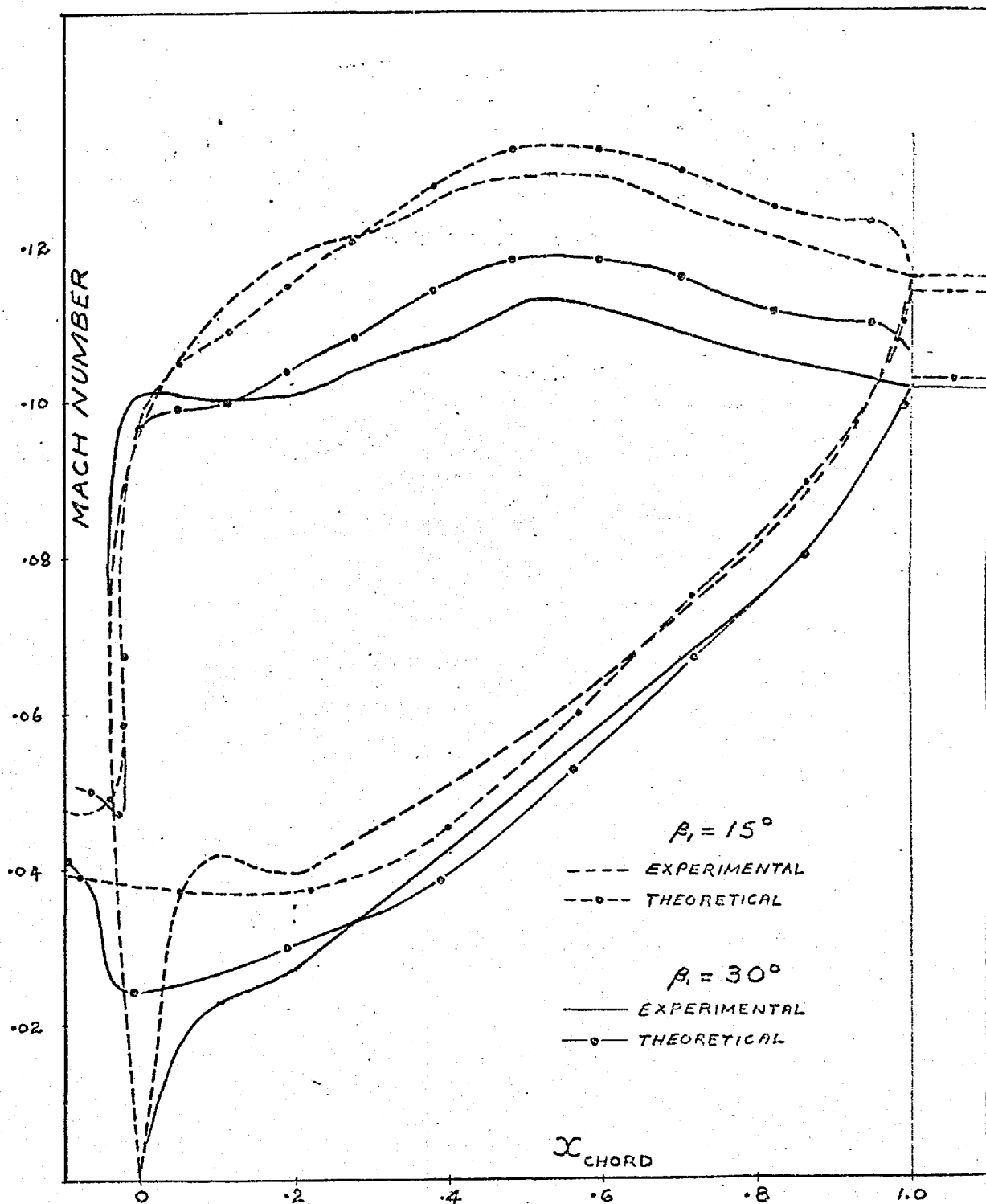


FIGURE 7.8

CASCADE AT TWO FLOW INLET ANGLESSTAGGER ANGLE $\gamma = 39.6^\circ$

SOLIDITY = 1.5

NUMERICAL METHOD - EULER TYPE



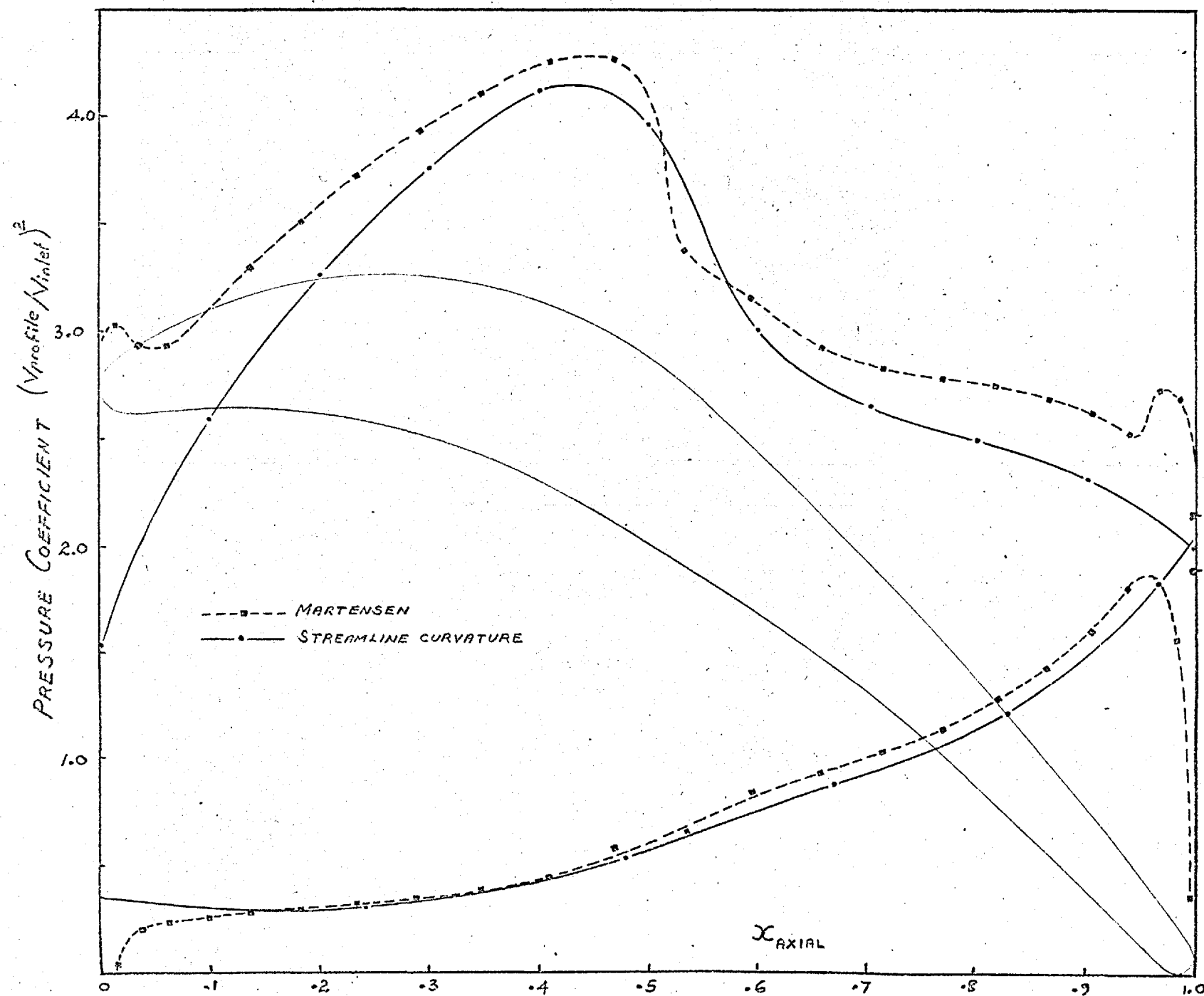


FIGURE 7.9

COMPARISON OF STREAMLINE
CURVATURE METHOD WITH
METHOD OF MARTENSEN

SOLIDITY = 0.976

MACHINUMBER LEVEL -
 INCOMPRESSIBLE (0.1)

INLET ANGLE $\beta_1 = 22.0^\circ$
 LEAVING ANGLE $\beta_2 = 47.5^\circ$

NUMERICAL METHOD - EULER TYPE
 (first profile)

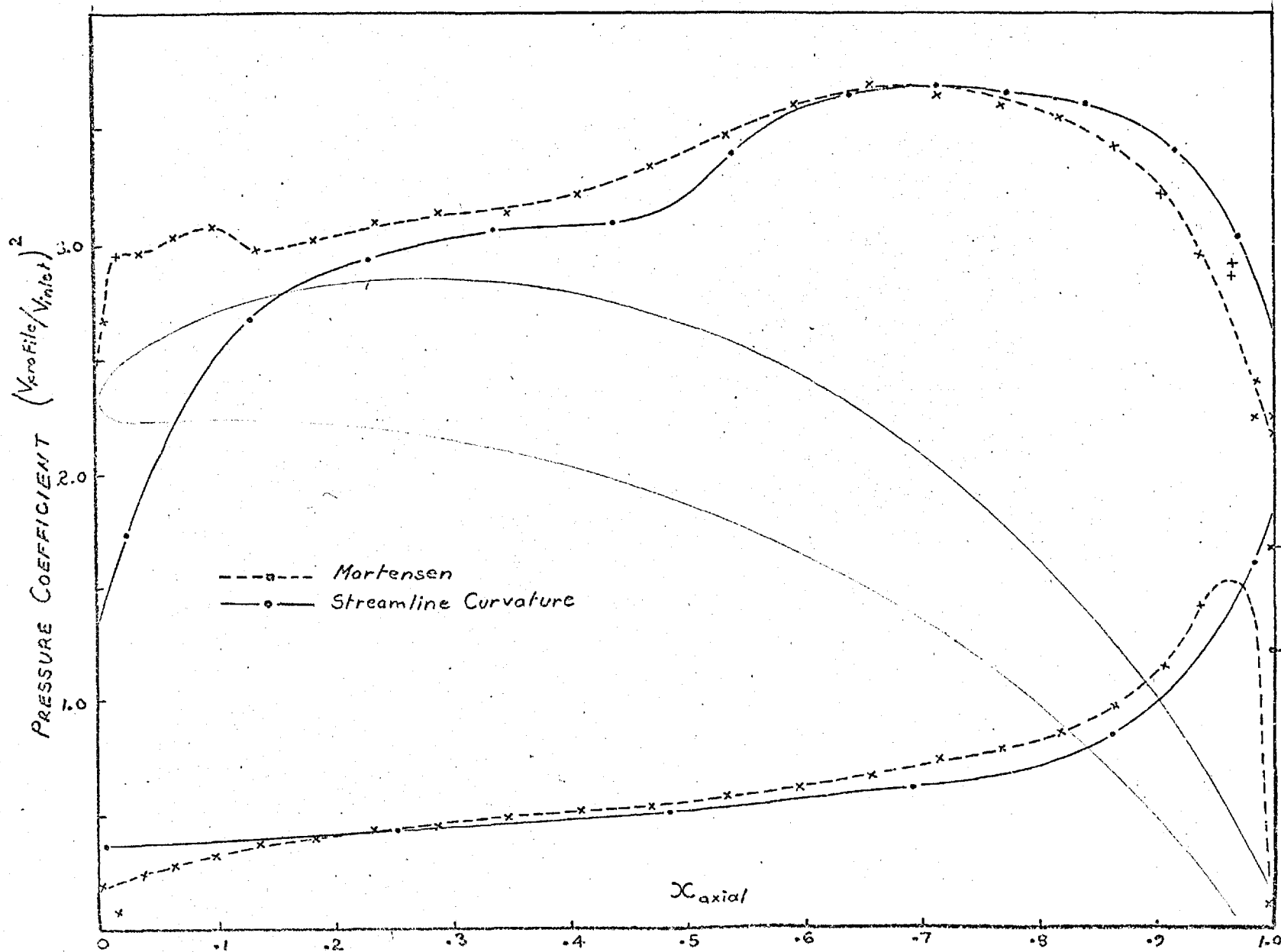


FIGURE 7.10

Comparison of streamline
curvature method with
method of Mortensen

Solidity = 0.943

Mach No. level-
incompressible (0.1)

Inlet angle = 22.0° (β_1)

Leaving angle = 50.8° (β_2)

Numerical method - Euler type
(second profile)

FIGURE 7.11a

COMPARISON OF STREAMLINE CURVATURE, METHOD
OF SAUER AND EXPERIMENTAL RESULTS FOR
THROAT REGION OF CONVERGENT-DIVERGENT
NOZZLE

STREAMLINE CURVATURE METHOD USED - RUNGE-KUTTA TYPE

GRID DATA AS SHOWN

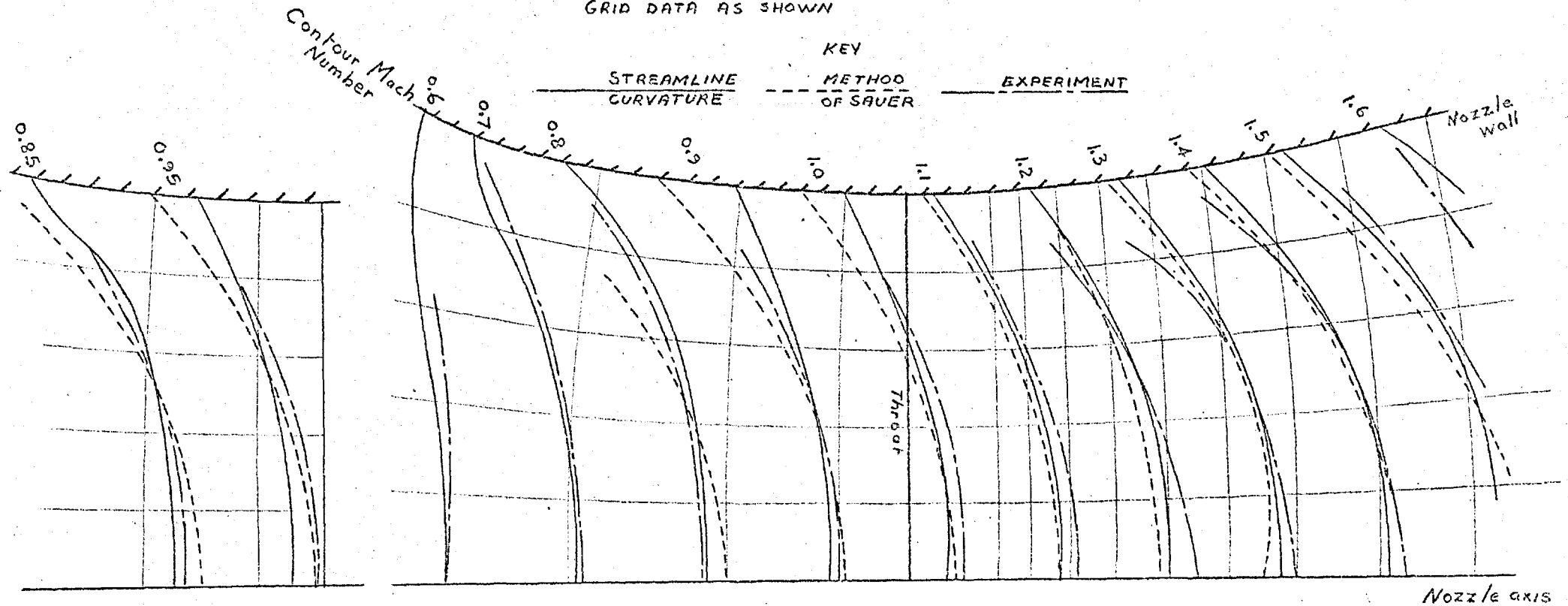


FIGURE 7.11 b

COMPARISON OF STREAMLINE CURVATURE
METHOD WITH METHOD OF CHARACTER-
ISTICS AND EXPERIMENT IN SUPERSONIC
OF CONVERGENT-DIVERGENT NOZZLE

NUMERICAL METHOD USED - RUNG-KUTTA TYPE
GRID AS SHOWN IN FIGURES 7.11 a & c

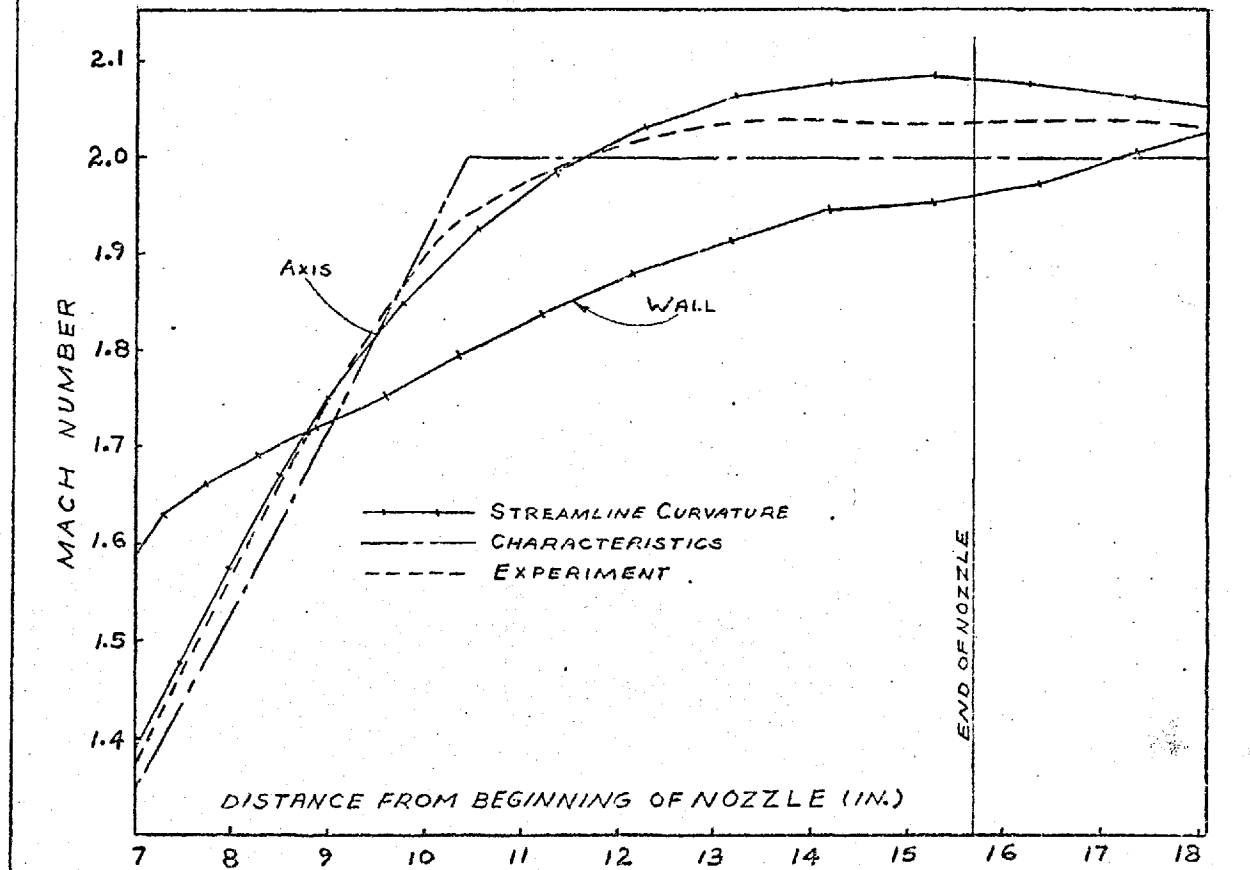


FIGURE 7.11 c

COMPARISON OF STREAMLINE CURVATURE
METHODS

CHARACTERISTICS

RUNGE-KUTTA TYPE

EULER TYPE

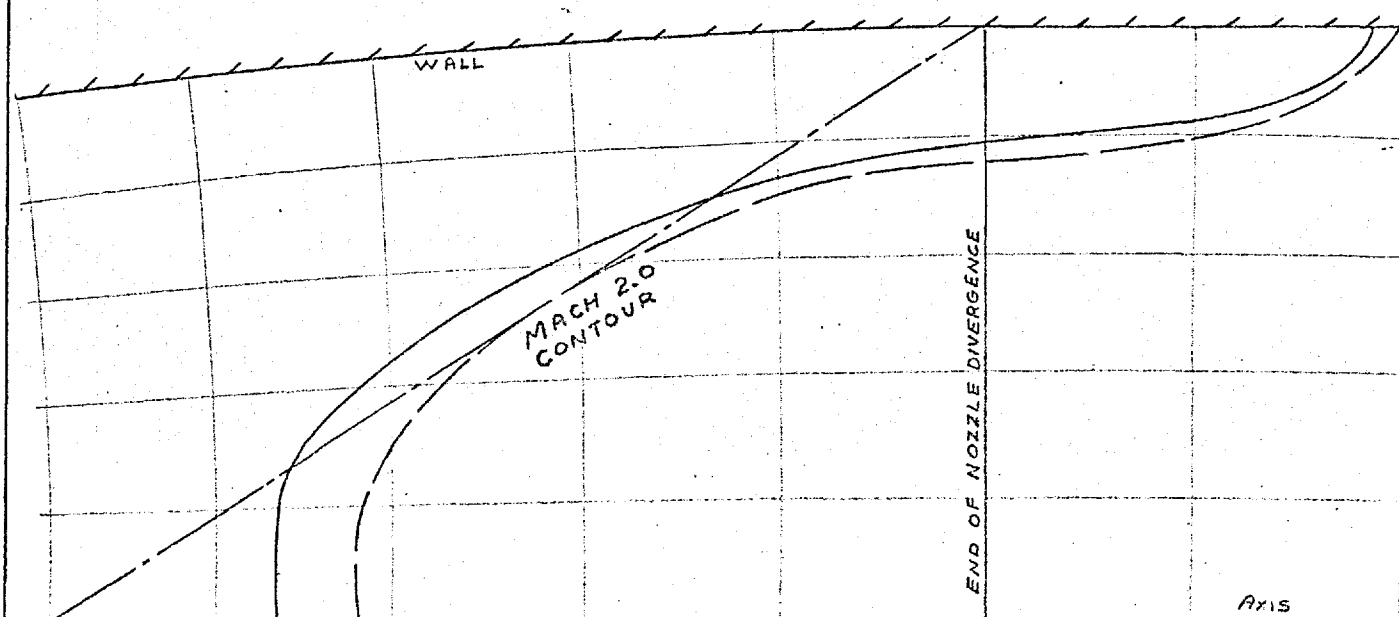
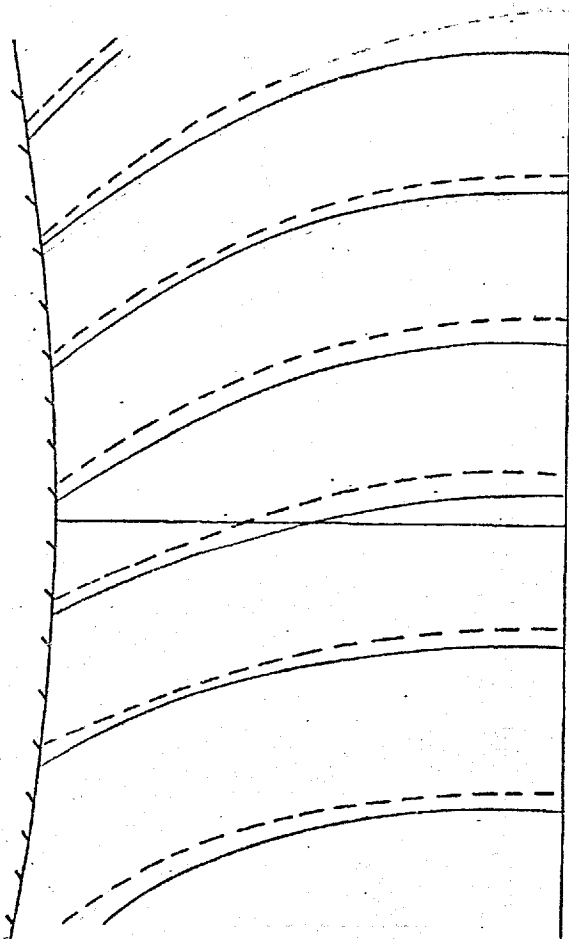
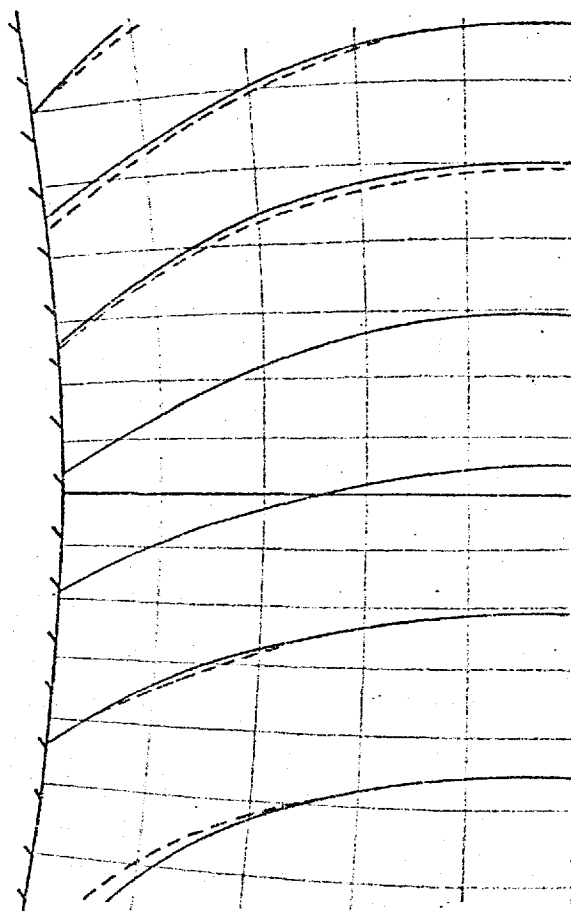


FIGURE 7.11dCOMPARISON OF STREAMLINE
CURVATURE METHODS

RUNGE-KUTTA TYPE -----
 EULER TYPE _____

FIGURE 7.11eCOMPARISON OF RESULTS FOR
DIFFERENT GRIDS

GRID OF FIG. 7.11a -----
 EVENLY SPACED GRID SHOWN -----

FIGURE 7.11fEFFECT OF NOT OBEYING GRID SPACING
CRITERIA

6 STREAMLINES -----
 15 " -----
 CHARACTERISTICS -----

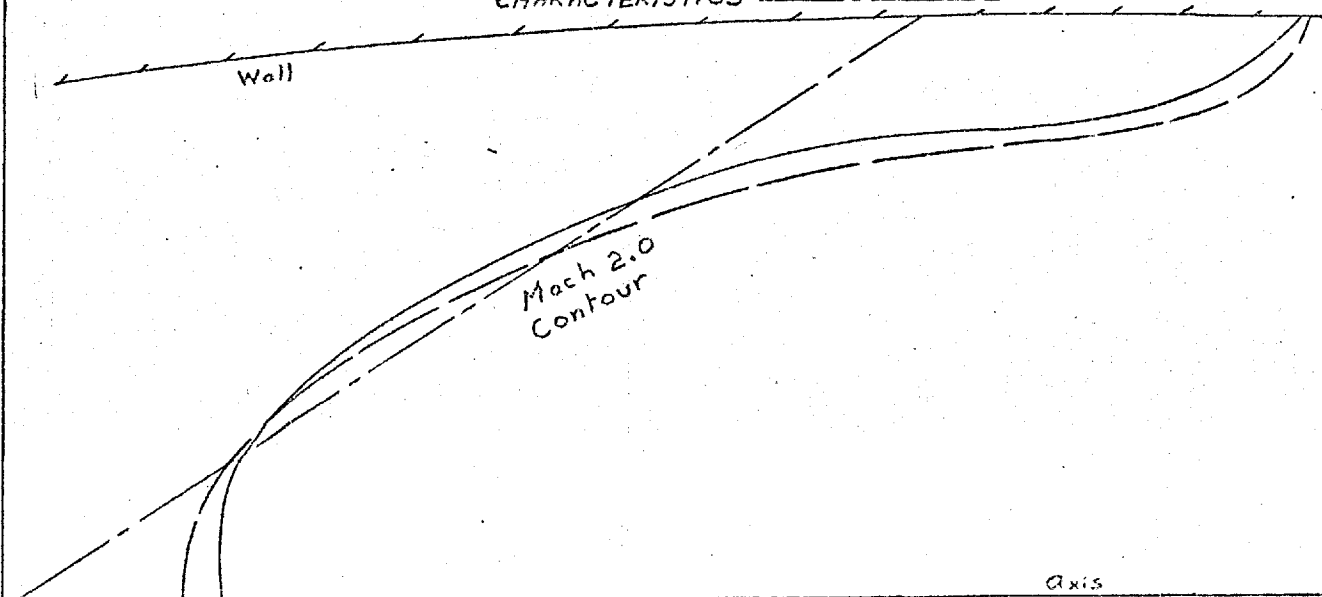


FIGURE 7.12

COMPARISON OF THEORETICAL METHODS
IN THROAT REGION OF CONVERGENT
DIVERGENT HYPERBOLIC NOZZLE

numerical method - Runge-Kutta type

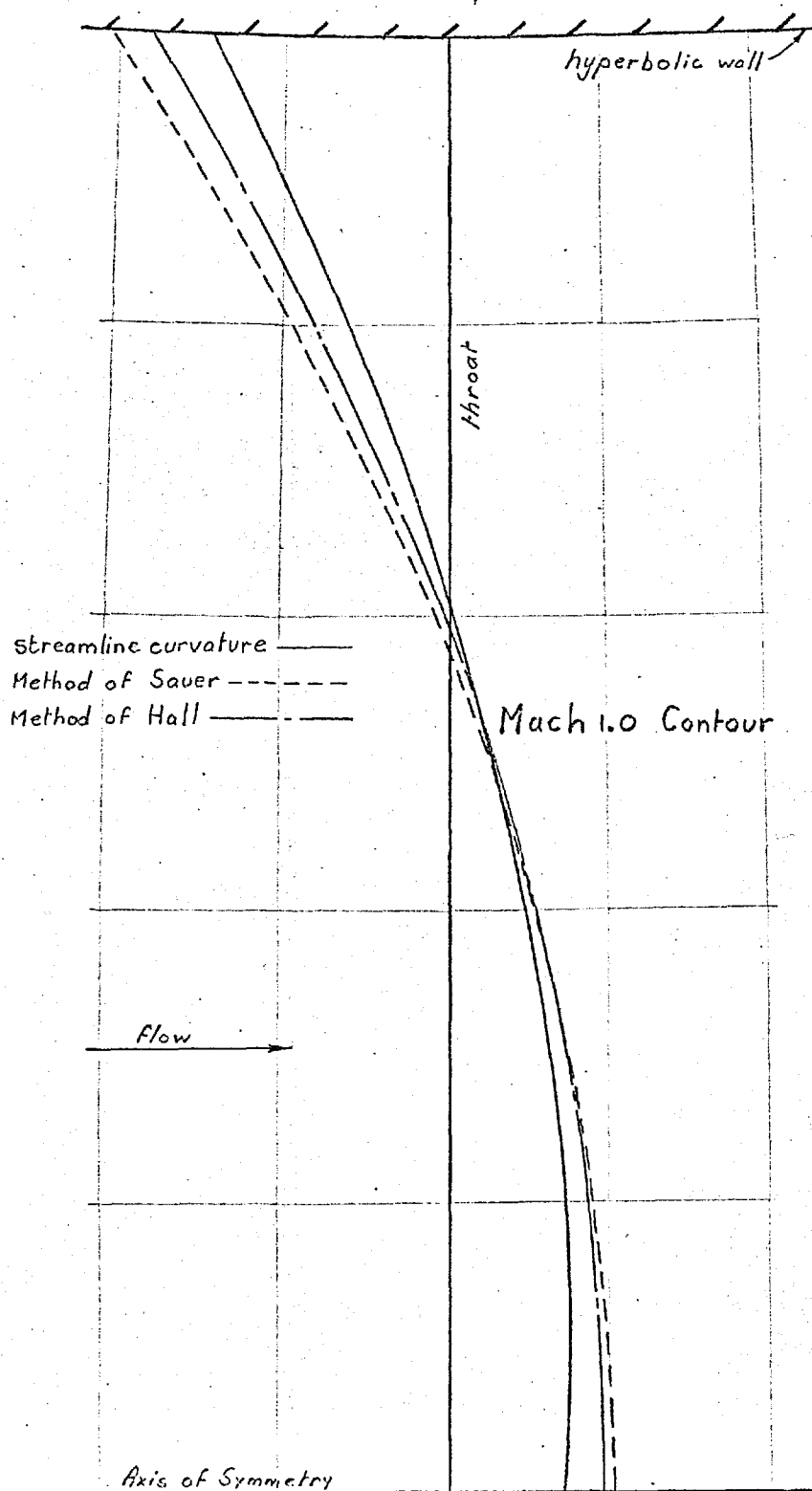


FIGURE 7.13a

Comparison of theoretical and experimental
results for root section steam turbine
profile

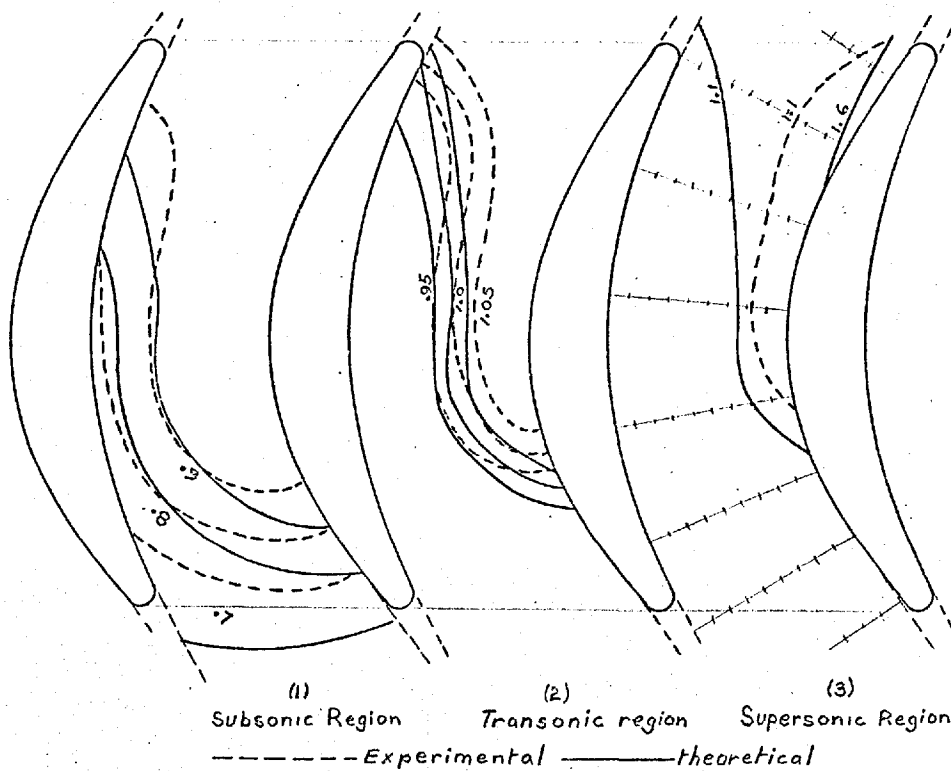
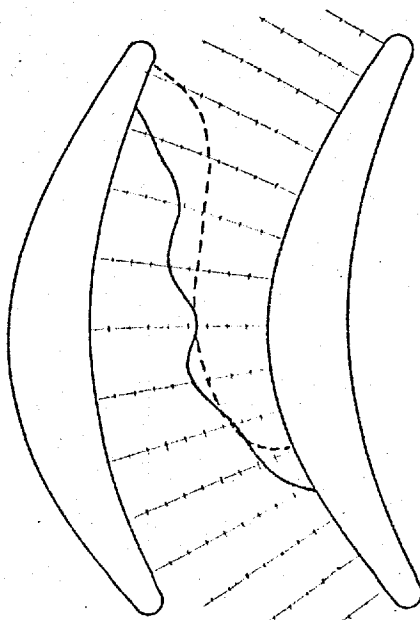


FIGURE 7.13 b

WAVY THEORETICAL SONIC LINE
FOR HIGH ASPECT RATIO
GRID



Experimental -----
Theoretical ——

FIGURE 7.14a

COMPARISON OF EXPERIMENTAL AND
THEORETICAL RESULTS FOR MID-RADIUS
SECTION STEAM TURBINE PROFILE

EXPERIMENTAL - - - - -

STREAMLINE CURVATURE —●—

CHARACTERISTICS TAKING
SONIC LINE AS AT
GEOMETRIC THROAT

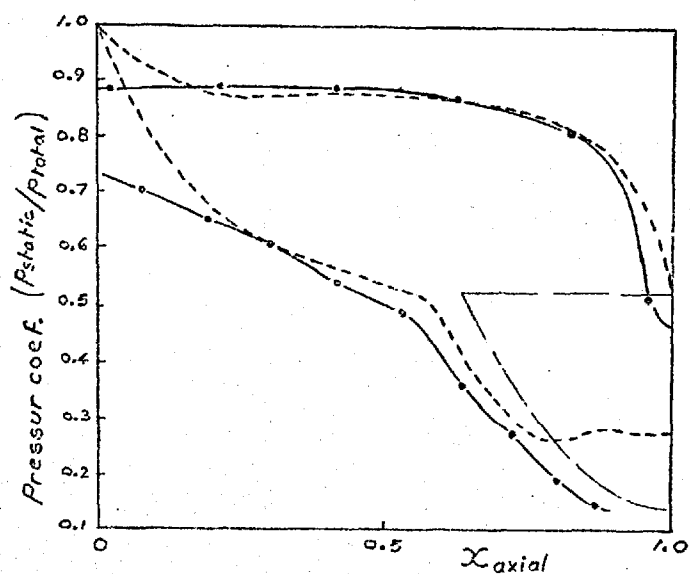


FIGURE 7.14b

CONTOUR PLOT AND GRID FOR
MID RADIUS STEAM TURBINE PROFILE

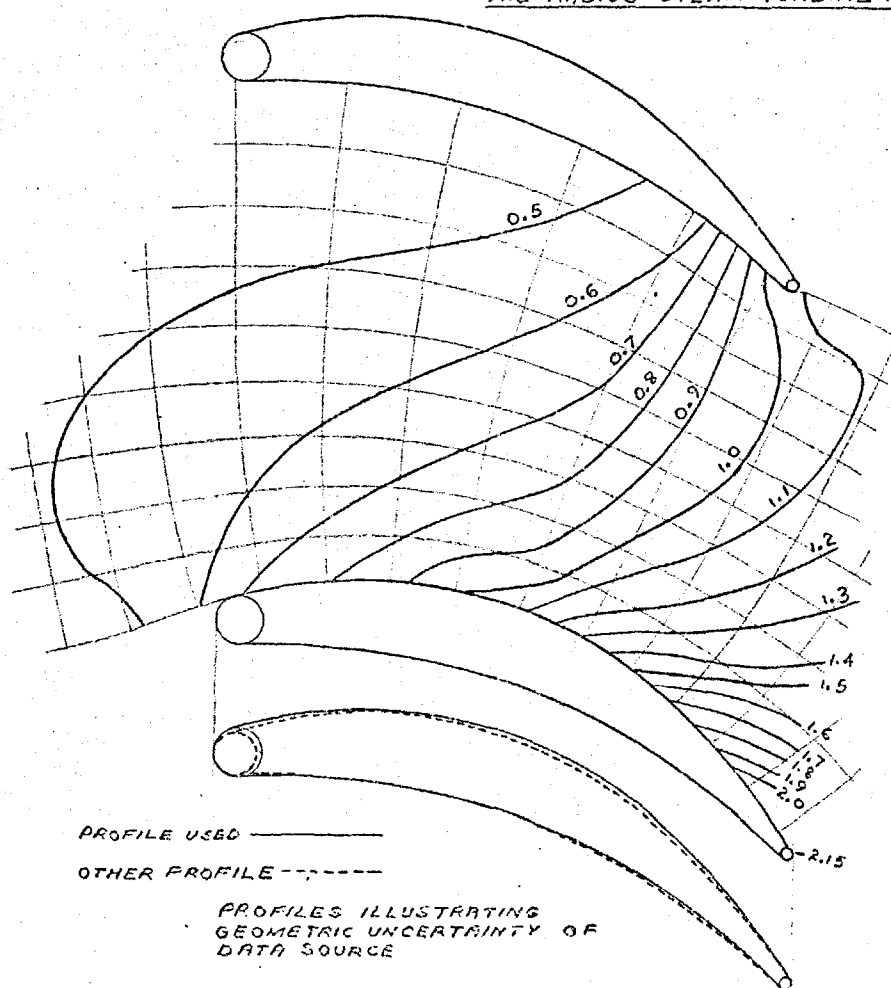


FIGURE 7.15

EXPERIMENTAL AND THEORETICAL
RESULTS FOR TIP SECTION STEAM
TURBINE PROFILE

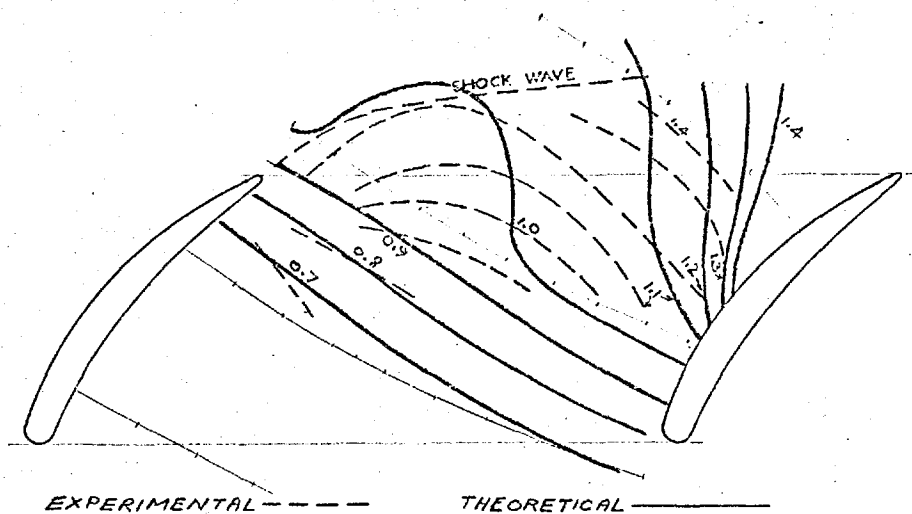


FIGURE 7.16

EXPERIMENTAL AND THEORETICAL
RESULTS FOR PROFILE OF
RZHEZNIKOV (7.4)

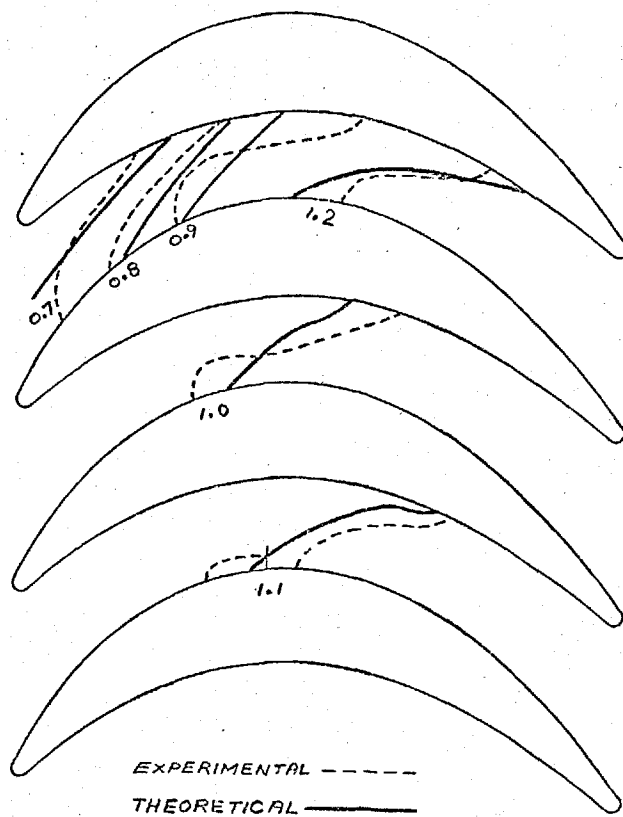


FIGURE 7.16 a

CASCADE AND STREAMLINE
CURVATURE GRID USED
FOR INDUSTRIAL PROFILE

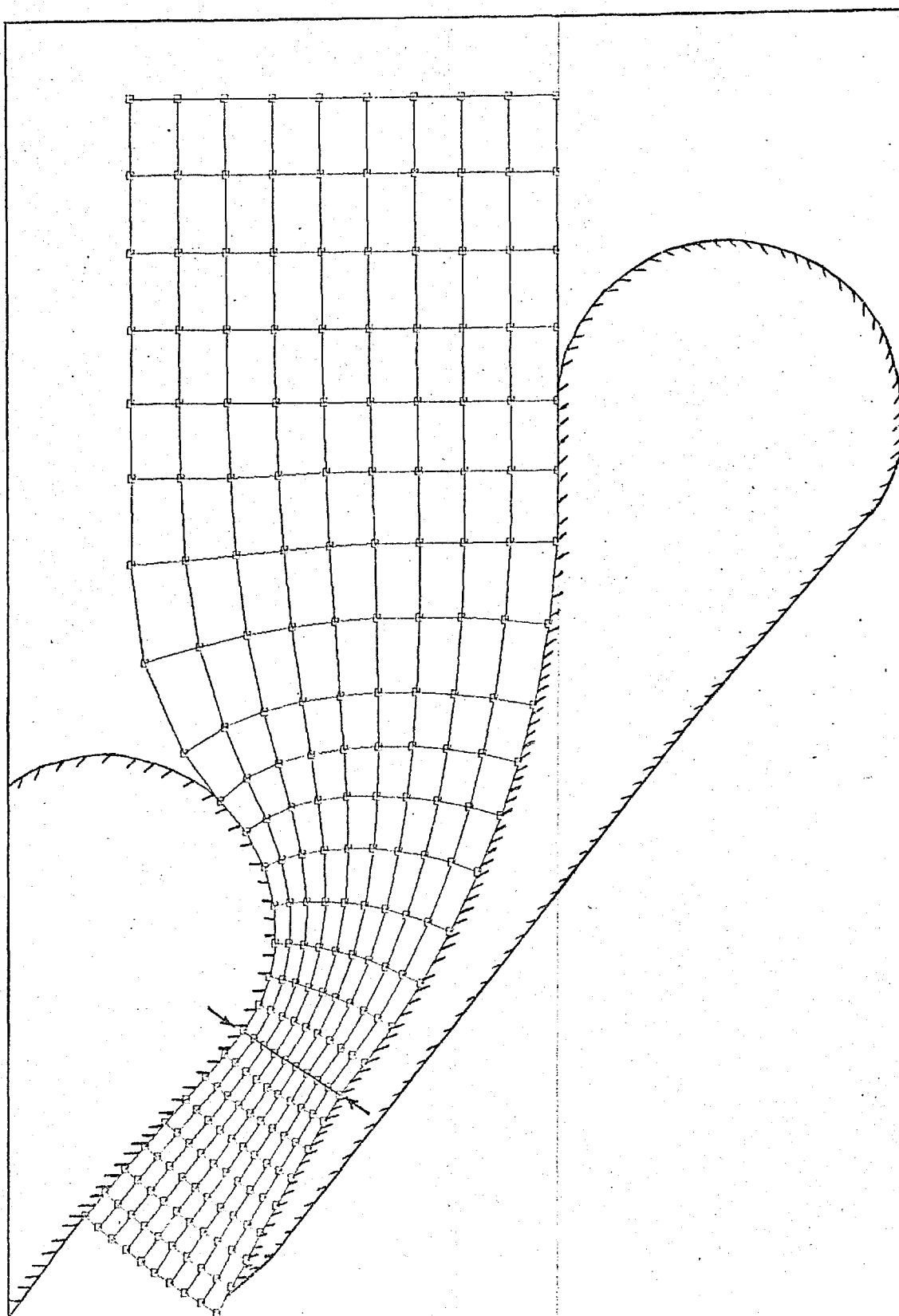


FIGURE 7.16 b
EXPERIMENTAL COMPARISON
FOR THROAT REGION OF INDUS-
TRIAL PROFILE

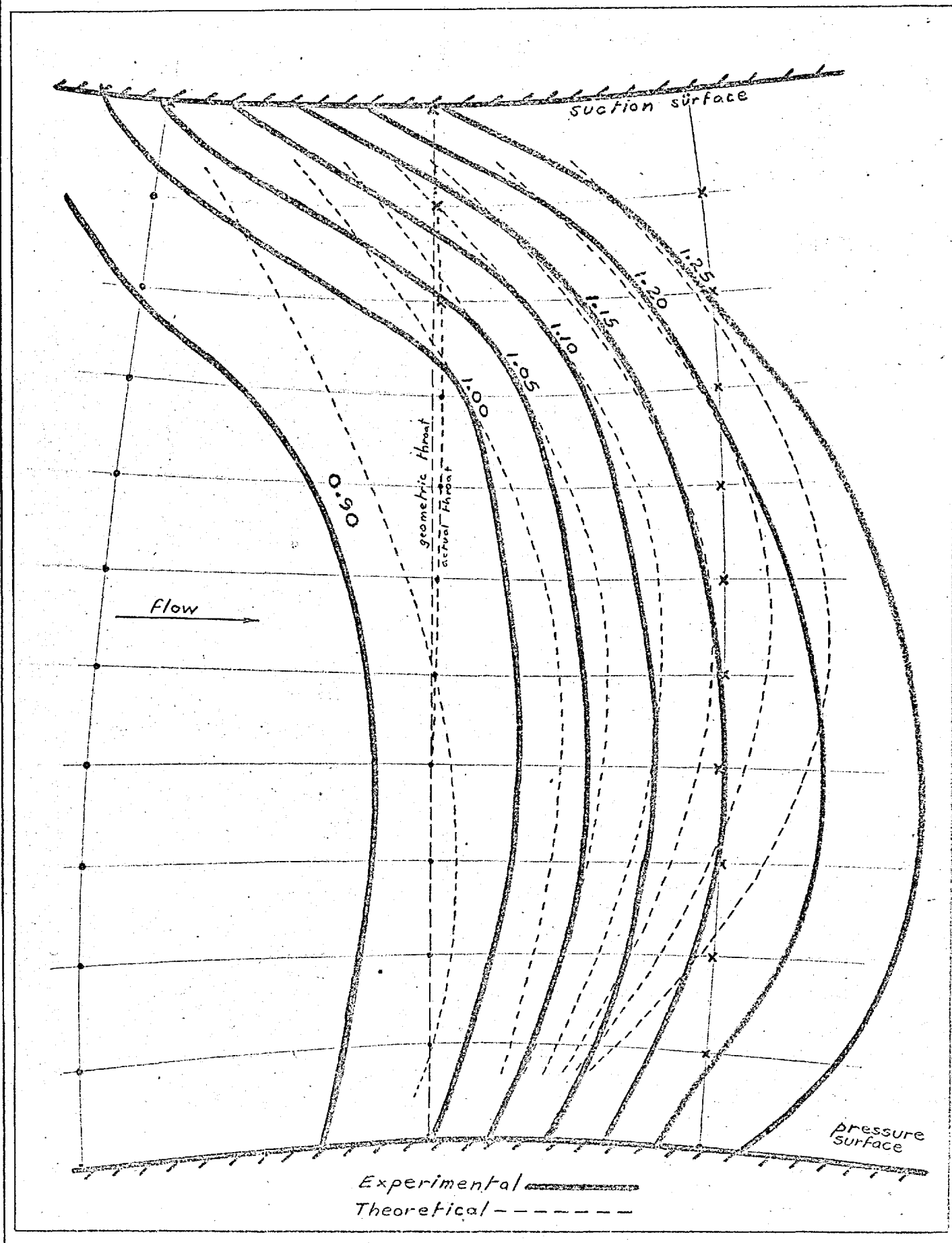
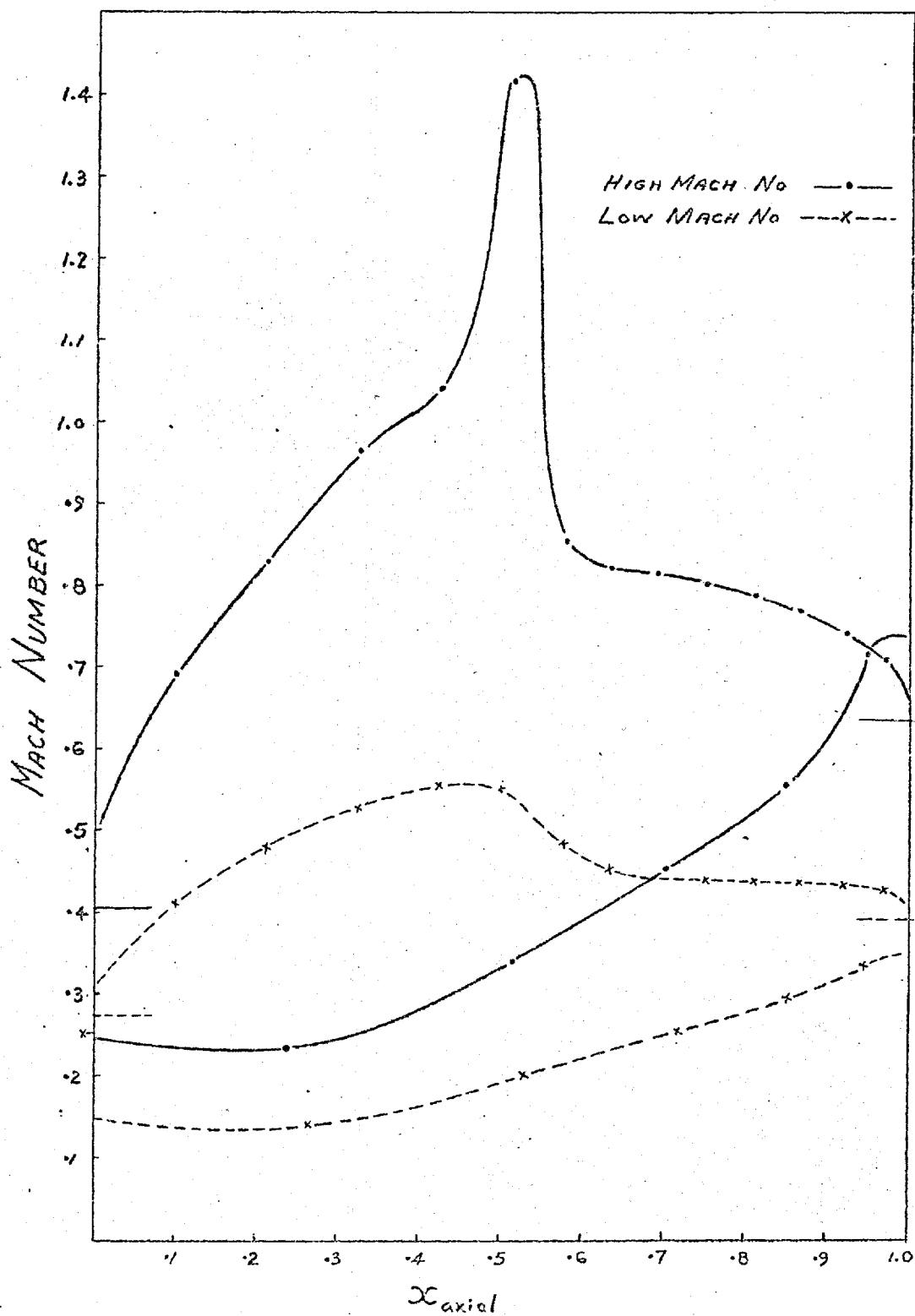


FIGURE 7.17a

PROFILE OF REF 7.2 AT HIGH AND
LOW SUBSONIC LEAVING MACH
NUMBERS

CASCADE DATA AS IN FIGURE 7.9

NUMERICAL METHOD - EULER TYPE



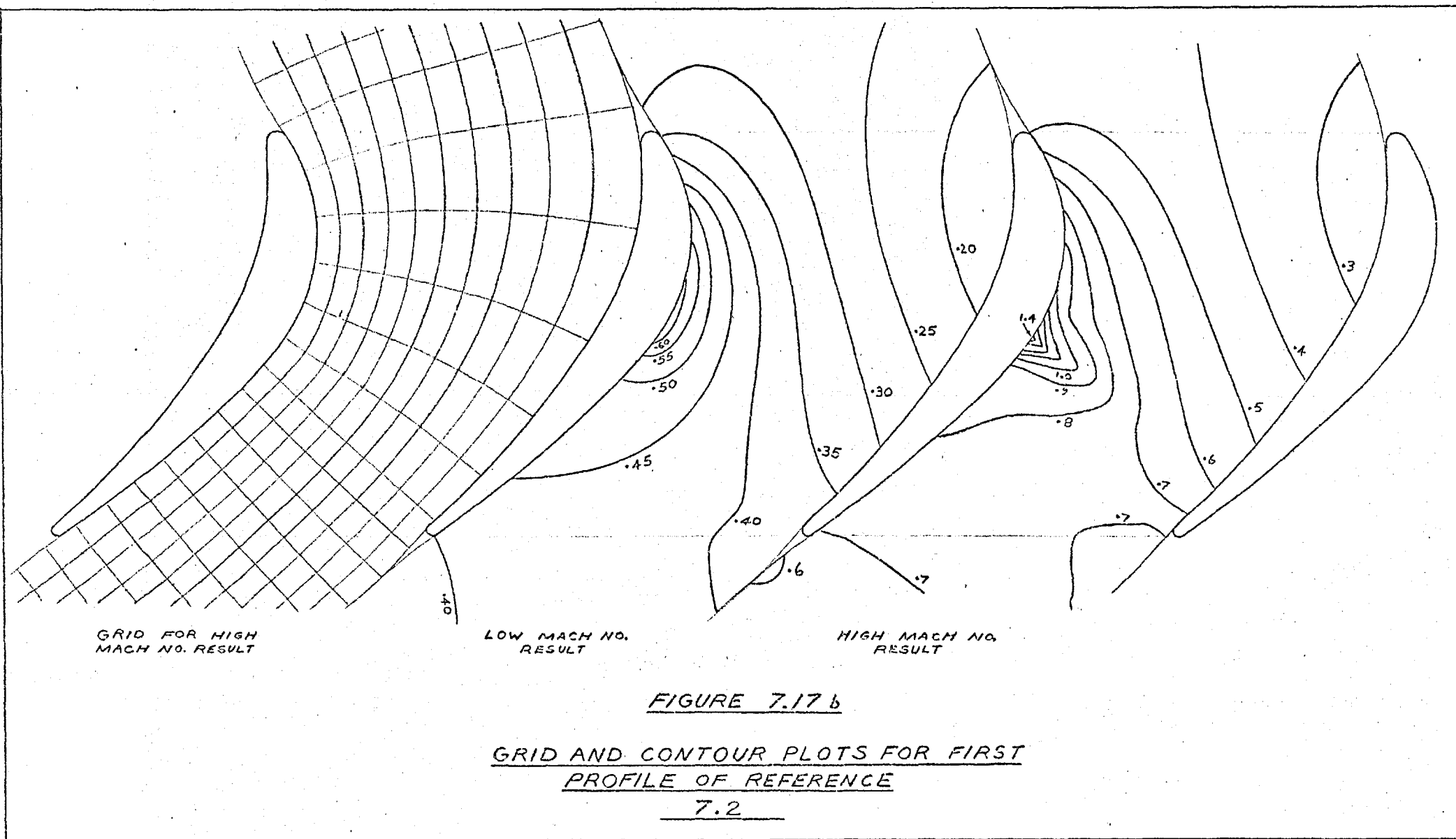
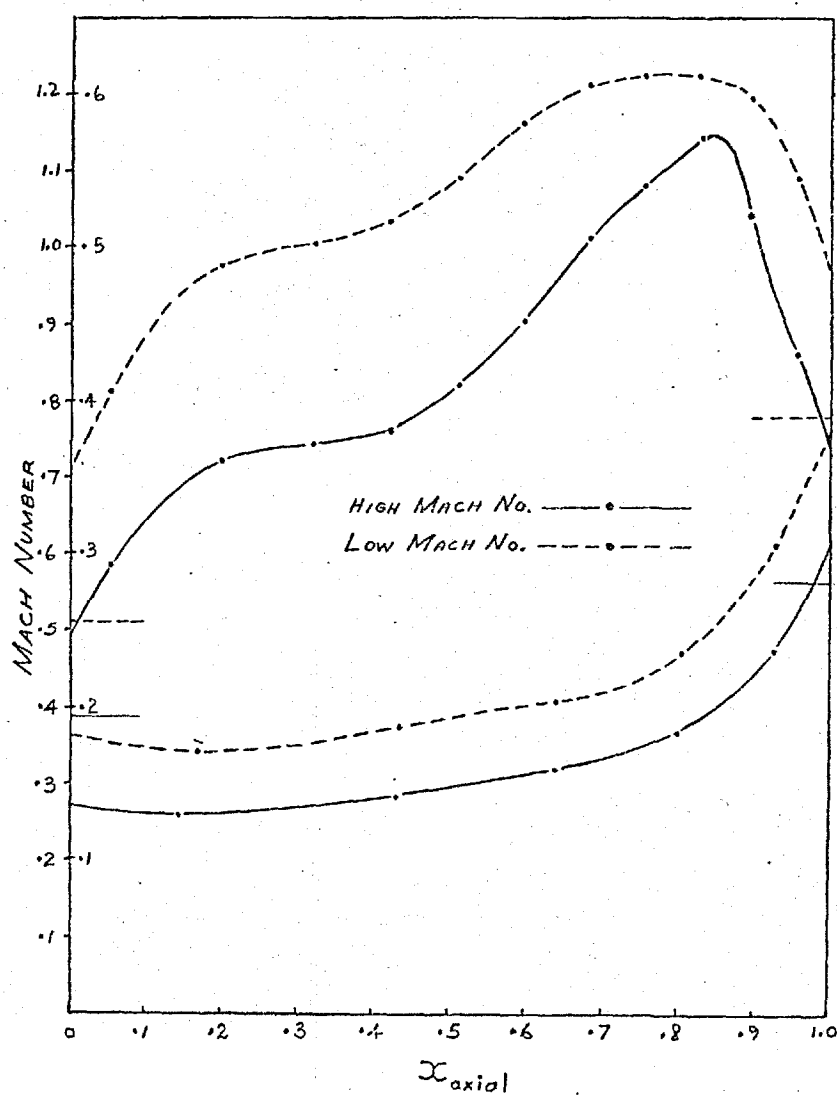


FIGURE 7.18 a

PROFILE OF REFERENCE
7.2 AT HIGH AND LOW
SUBSONIC LEAVING
MACH NUMBERS

CASCADE DATA AS IN FIGURE 7.10

NUMERICAL METHOD - EULER TYPE



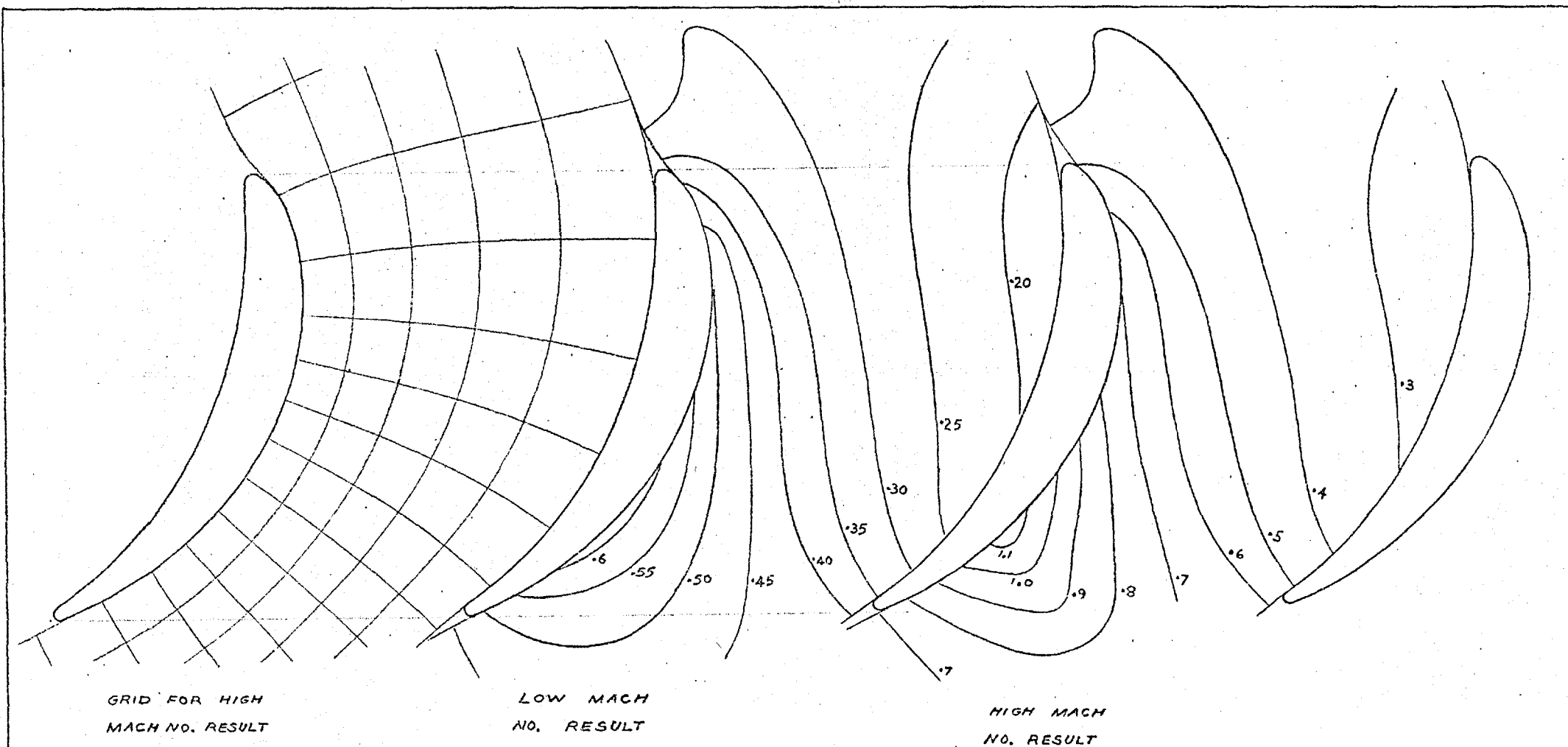
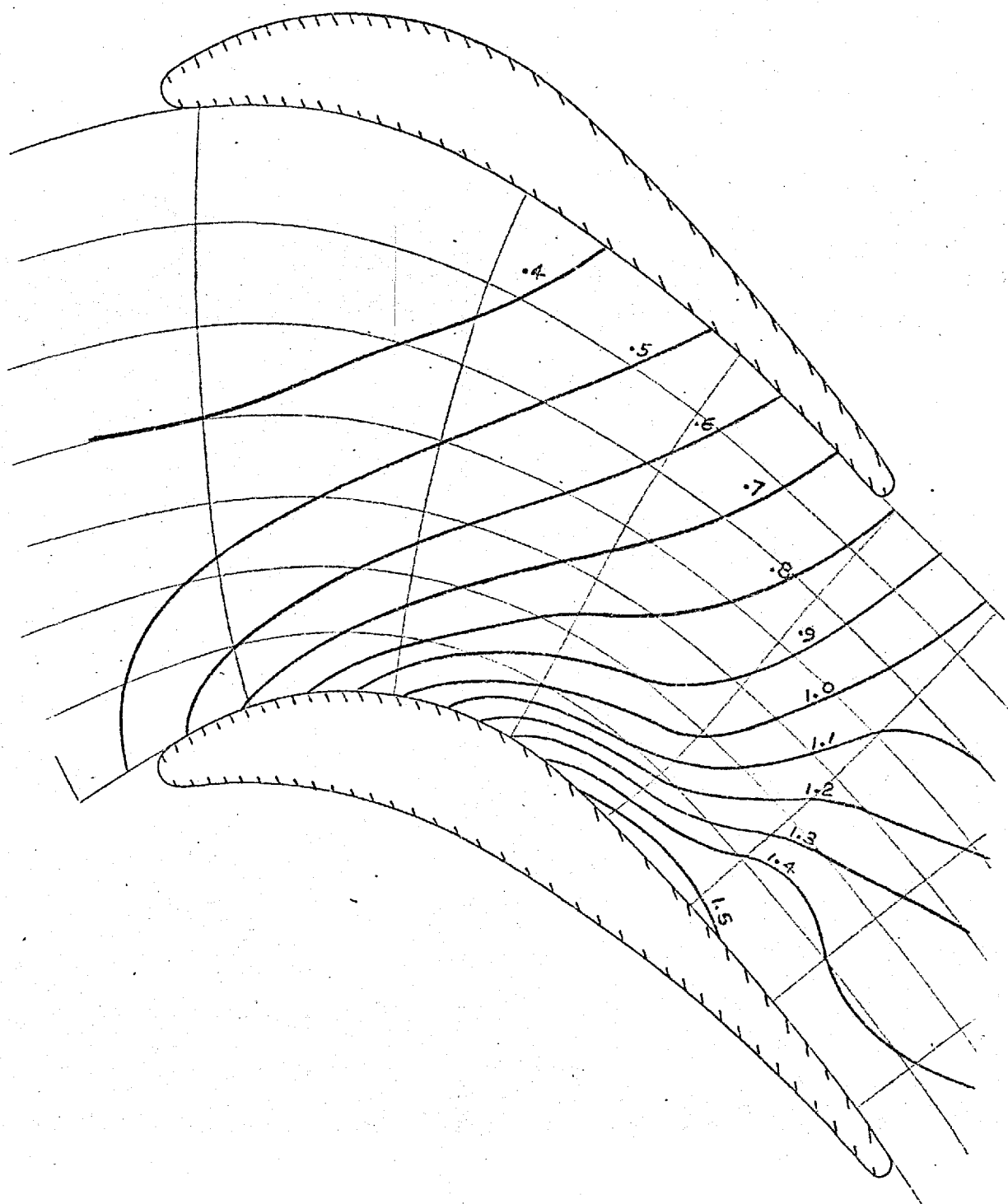


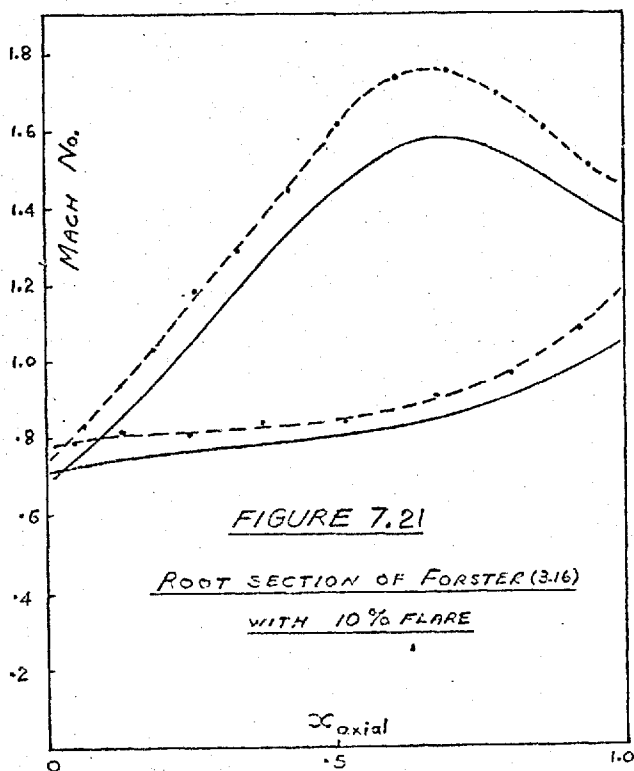
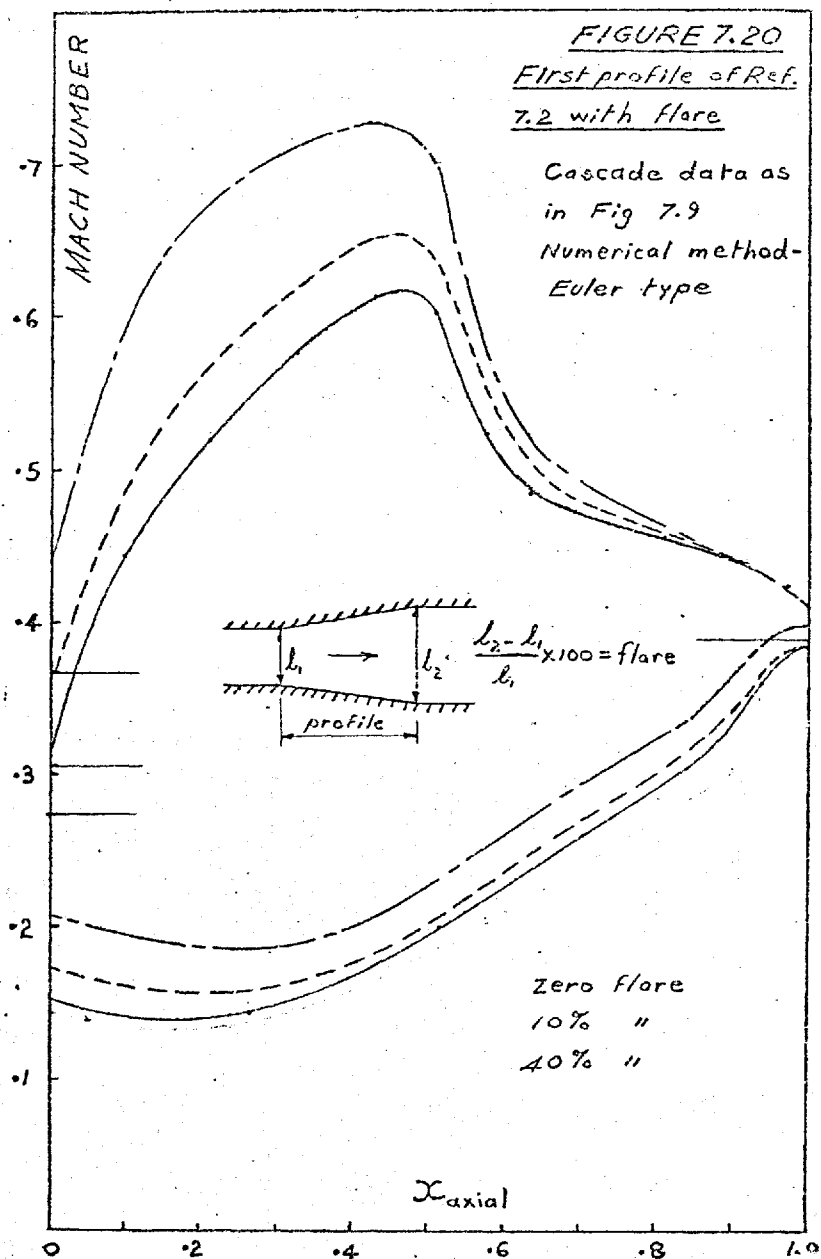
FIGURE 7.18 b
GRID AND CONTOUR PLOTS FOR SECOND
PROFILE OF REFERENCE 7.2

FIGURE 7.19

FIRST PROFILE OF REF 7.2
WITH SUPERSONIC LEAVING
MACH NO.

CASCADE DATA AS IN FIG 7.9
NUMERICAL METHOD - EULER TYPE





APPENDIX I CONVERGENCE ANALYSIS

This appendix contains the full derivation of the convergence analysis of Chapter 5. The derivation is presented in 4 parts. The basic convergence formulae are derived in part (a) and the subsonic and supersonic calculation procedures for all the convergence formulae in parts (b) and (c) respectively. Part (d) contains a short proof in connection with part (a).

Appendix 1a Derivation of basic convergence formulae.

Section 5.3a has described the overall procedure followed. Equation 5.1 is applied sequentially to each streamline along normal m (Sketch 5.1) to obtain the new velocity distribution. The continuity equation is applied to find the new flow area which, after the original flow area is subtracted, gives the change in flow area at each streamline.

In this analysis only the n th streamline radius, density and flow width will be used to calculate the velocity increment between the n th and $n+1$ th streamline.

Applying equation 5.1 along normal m in Sketch 5.1, the velocity distribution can be written as:-

$$\begin{aligned}
 V_{m,1} &= V_{m,0} + dV_m & (R_{m,0} = \infty) \\
 V_{m,2} &= V_{m,1} - S_{fct} \frac{d\dot{m}}{\rho_{m,0} \ell_m R_{m,1}} = V_{m,0} + dV_m - S_{fct} \frac{d\dot{m}}{\rho_{m,0} \ell_m R_{m,1}} \\
 &\dots\dots\dots \\
 V_{m,n} &= V_{m,n-1} - S_{fct} \frac{d\dot{m}}{\rho_{m,0} \ell_m R_{m,n-1}} \\
 &= V_{m,0} + dV_m - S_{fct} \frac{d\dot{m}}{\rho_{m,0} \ell_m R_{m,1}} - \dots\dots\dots - S_{fct} \frac{d\dot{m}}{\rho_{m,0} \ell_m R_{m,n-1}} \quad \text{A 1.1}
 \end{aligned}$$

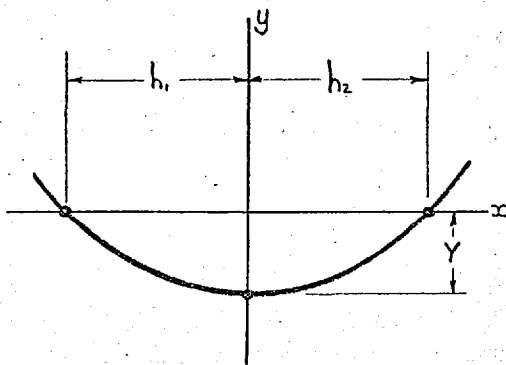
(It should be noted that the effect of using $\rho_{m,0}$ instead of $\rho_{m,n}$ is negligible because, for small increments, $\frac{dV}{\rho+d\rho} = \frac{dV}{\rho}$)

Before introducing the continuity equation, a substitution for the radius of curvature is sought. For a 3 point parabolic formula the required radius is shown in Sketch A 1.1a to be, for subsonic flow:-

$$R_{m,n} = \frac{h_{1m} h_{2m} \ell_m}{2 \Delta a_{m,n}} \quad \text{A 1.2}$$

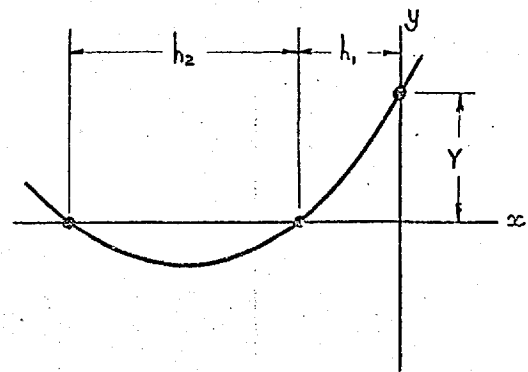
(a)

Subsonic flow



(b)

Supersonic flow

General form of parabola: $y = ax^2 + bx + c$

$$1) 0 = ah_1^2 - bh_1 + c$$

$$2) Y = c$$

$$3) 0 = ah_2^2 + bh_2 + c$$

$$\therefore a = -\frac{Y}{h_1 h_2}$$

$$1) 0 = a(h_1 + h_2)^2 - b(h_1 + h_2) + c$$

$$2) 0 = ah_1^2 - bh_1 + c$$

$$3) Y = c$$

$$\therefore a = \frac{Y}{h_1(h_1 + h_2)}$$

$$\text{assuming: } R \doteq -\frac{1}{d^2y/dx^2} = -\frac{1}{2a}$$

$$R = \frac{h_1 h_2}{2Y}$$

$$R = -\frac{h_1(h_1 + h_2)}{2Y}$$

SKETCH A1.1

Parabolic radius of curvature

for supersonic and subsonic

flow

and in Sketch A 1.1b for supersonic flow:-

$$R_{m,n} = - \frac{h_{1m}(h_{1m} + h_{2m}) \ell_m}{2 \Delta a_{m,n}} \quad \text{A 1.3}$$

where $\Delta a_{m,n}$ = streamline area deviation from a straight line as defined in Sketch A 1.2.

The streamline deviation from a straight line, $\Delta a_{m,n}$, by reference to Sketch A 1.2 can be shown to be, for subsonic flow:-

$$\left. \begin{aligned} \Delta a_{m,n} &= da_{m,n} - h'_m da_{m-1,n} - (1 - h'_m) da_{m+1,n} \\ \text{or } \Delta y_{m,n} &= y_{m,n} - h'_m y_{m-1,n} - (1 - h'_m) y_{m+1,n} \end{aligned} \right\} \quad \text{A 1.4}$$

and similarly for supersonic flow:-

$$\Delta y_{m,n} = y_{m,n} - (1 + h''_m) y_{m-1,n} + h''_m y_{m-2,n} \quad \text{A 1.5}$$

$$\text{where } \Delta y_{m,n} = \frac{\Delta a_{m,n}}{Na_m}$$

Substituting equations A 1.2 and A 1.3 into equation A 1.1 gives:-

$$V_{m,n} = V_{m,o} + dV_m - Z_m \sum_{i=1}^{n-1} \Delta a_{m,i} \quad \text{A 1.6}$$

where:-

$$Z_m = 2 S_{fct} \frac{d\dot{m}}{\rho_{m,o} \ell_m^2 h_{1m} h_{2m}} \quad \text{for subsonic flow (equation A 1.2)}$$

and

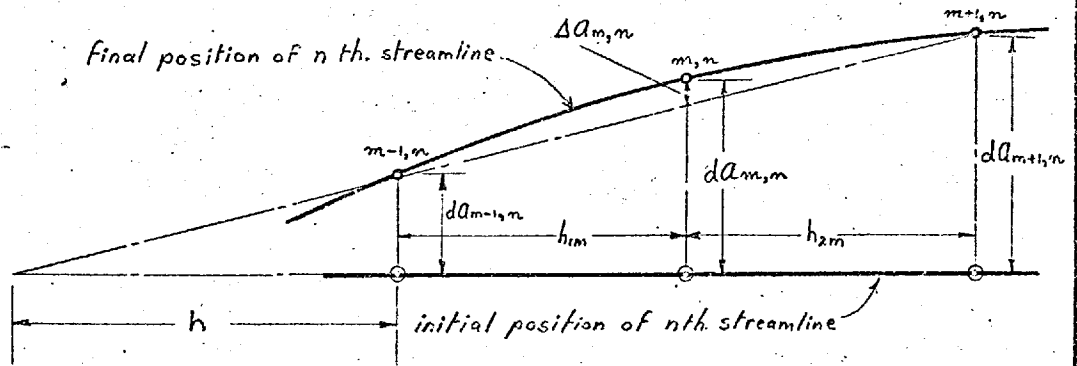
$$Z_m = - 2 S_{fct} \frac{d\dot{m}}{\rho_{m,o} \ell_m^2 h_{1m} (h_{1m} + h_{2m})} \quad \text{for supersonic flow (equation A 1.3)}$$

Proceeding now to apply the continuity equation, a relation between velocity and density changes is required.

For small changes in velocity a linear relationship between velocity and density is assumed so that:-

$$\frac{d\rho}{dV} = k \quad \text{and} \quad \rho = \rho_o + k dV \quad \text{A 1.7}$$

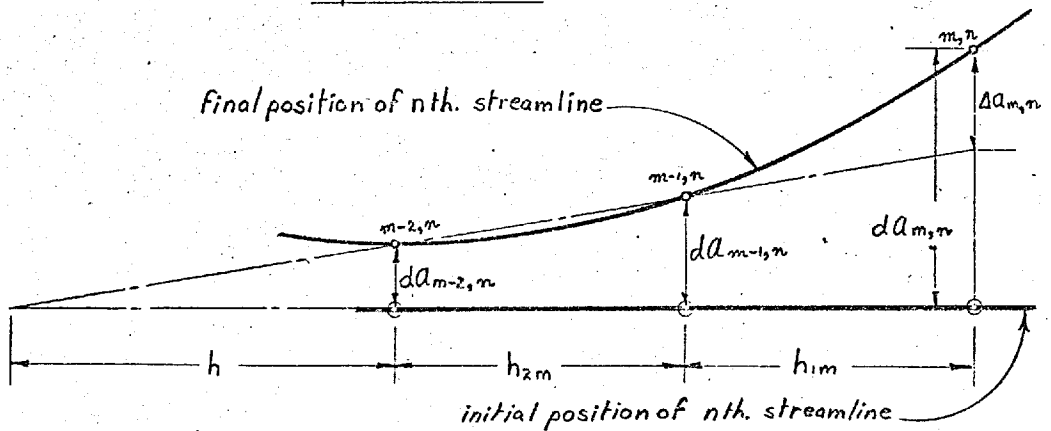
Subsonic Flow



From similar triangles: $\frac{da_{m,n} - \Delta a_{m,n}}{h_{1m} + h} = \frac{da_{m+1,n}}{h_{2m} + h_{1m} + h} = \frac{da_{m-1,n}}{h}$

or: $\Delta a_{m,n} = da_{m,n} - \frac{h_{2m}}{h_{1m} + h_{2m}} da_{m-1,n} - \left(1 - \frac{h_{2m}}{h_{1m} + h_{2m}}\right) da_{m+1,n}$

Supersonic Flow



From similar triangles: $\frac{da_{m,n} - \Delta a_{m,n}}{h_{1m} + h_{2m} + h} = \frac{da_{m-1,n}}{h_{2m} + h} = \frac{da_{m-2,n}}{h}$

or: $\Delta a_{m,n} = da_{m,n} - \left(1 + \frac{h_{1m}}{h_{2m}}\right) da_{m-1,n} + \frac{h_{1m}}{h_{2m}} da_{m-2,n}$

SKETCH A1.2

Derivation of the streamline deviation from a straight line

Combining equations A 1.6 and A 1.7, the density at each streamline is:-

$$\rho_{m,n} = \rho_{m,o} + k \, dV_m - k \, Z_m \sum_{j=1}^{n-1} \Delta a_{m,j} \quad \text{-----A 1.8}$$

The new streamline flow area $a'_{m,n}$ is found in two ways as outlined in Section 4.2. The first, the Euler type method, uses velocity and density data from the previous n th streamline to calculate the next $(n+1)$ th streamline flow area. The second, the more accurate Runge-Kutta type method, uses average data from the two streamlines. These two results, for subsonic and supersonic flow are presented together for compactness.

For the Euler type method the new streamline flow area is, from equation 5.2:-

$$a'_{m,n} = \frac{d\dot{m}}{\rho_{m,n-1} V_{m,n-1}}$$

Substituting equations A 1.6 and A 1.8 and simplyfying:-

$$\begin{aligned} a'_m &= d\dot{m} / (\rho_{m,o} + k dV_m - k Z_m \sum_{j=1}^{n-2} \Delta a_{m,j}) (V_{m,o} + dV_m - Z_m \sum_{j=1}^{n-2} \Delta a_{m,j}) \\ &= d\dot{m} / \{ \rho_{m,o} V_{m,o} + (1 + k \frac{V_{m,o}}{\rho_{m,o}}) \rho_{m,o} dV_m - (1 + k \frac{V_{m,o}}{\rho_{m,o}}) Z_m \sum_{j=1}^{n-2} \Delta a_{m,j} \}. \end{aligned}$$

It is shown in Appendix 1d that:-

$$1 + k \frac{V}{\rho} = 1 - M^2$$

where M is the Mach number.

$$\therefore a'_{m,n} = \frac{d\dot{m}}{\rho_{m,o} V_{m,o}} \{ 1 + (1 - M_m^2) (\frac{dV_m}{V_{m,o}} - \frac{Z_m}{V_{m,o}} \sum_{j=1}^{n-2} \Delta a_{m,j}) \} \quad \text{----- A 1.9}$$

Similary using the Runge- Kutta type method:-

$$\begin{aligned} a'_{m,n} &= \frac{d\dot{m}}{(\frac{\rho_{m,n-1} + \rho_{m,n}}{2}) (\frac{V_{m,n-1} + V_{m,n}}{2})} \\ &= \frac{d\dot{m}}{\rho_{m,o} V_{m,o}} \{ 1 + (1 - M_m^2) (\frac{dV_m}{V_{m,o}} - \frac{Z_m}{V_{m,o}} \sum_{j=1}^{n-2} \Delta a_{m,j} - \frac{1}{2} \frac{Z_m}{V_{m,o}} \Delta a_{m,n-1}) \} \quad \text{----- A 1.10} \end{aligned}$$

The new streamline flow areas have been derived . The area changes at each streamline are now found and are, with reference to Sketch 5.1 as follows:-

$$da_{m,n} = da_{m,n-1} + a'_{m,n} - a_m \quad \text{A 1.11}$$

Substitution of equation A 1.9 into A 1.11 and simplyfying gives the result for the Euler type method:-

$$\begin{aligned} da_{m,n} &= da_{m,n-1} + \frac{\dot{dm}}{\rho_{m,o} V_{m,o} \left\{ 1 + (1 - M_m^2) \left(\frac{dV_m}{V_{m,o}} - \frac{Z_m}{V_{m,o}} \sum_j \Delta a_{m,j} \right) \right\}} - \frac{\dot{dm}}{\rho_{m,o} V_{m,o}} \\ &= da_{m,n-1} + \frac{\dot{dm}}{\rho_{m,o} V_{m,o}} \left[\frac{(1 - M_m^2) \left(\frac{dV_m}{V_{m,o}} - \frac{Z_m}{V_{m,o}} \sum_j \Delta a_{m,j} \right)}{1 + (1 - M_m^2) \left(\frac{dV_m}{V_{m,o}} - \frac{Z_m}{V_{m,o}} \sum_j \Delta a_{m,j} \right)} \right] \\ &= da_{m,n-1} + \frac{\dot{dm}}{\rho_{m,o} V_{m,o}} \left\{ (1 - M_m^2) \left(\frac{dV_m}{V_{m,o}} - \frac{Z_m}{V_{m,o}} \sum_j \Delta a_{m,j} \right) \right\} \end{aligned}$$

Dividing by Na_m , substituting $\frac{\dot{dm}}{\rho_{m,o} V_{m,o}} = a_m$ and equations 5.7 and 5.13 gives

$$y_{m,n} = y_{m,n-1} - x_m + (1 - M_m^2) \frac{Z_m a_m}{V_{m,o}} \sum_j \Delta y_{m,j} \quad \text{A 1.12}$$

the term in equation A 1.12 is simplified for subsonic flow as:-

$$\begin{aligned} K_m &= \frac{(1 - M_m^2) Z_m a_m}{V_{m,o}} = \frac{(1 - M_m^2)}{V_{m,o}} 2S_{fct} \frac{\dot{dm}}{\rho_{m,o} \ell_m^2 h_{1m} h_{2m}} \frac{\dot{dm}}{\rho_{m,o} V_{m,o}} \\ &= 2S_{fct} (1 - M_m^2) \frac{\dot{dm}^2}{\rho_{m,o}^2 V_{m,o}^2 \ell_m^2 h_{1m} h_{2m}} \frac{1}{\ell_m^2 h_{1m} h_{2m}} \\ &= \frac{2 S_{fct} (1 - M_m^2) N^2 a_m^2}{N^2 \ell_m^2 h_{1m} h_{2m}} = \frac{2 S_{fct} (1 - M_m^2) \frac{D_m^2}{h_{1m} h_{2m}}}{N^2} \end{aligned}$$

where $D_m = \frac{N a_m}{\ell_m} = \text{total flow width,}$

and for supersonic flow:-

$$K_m = - \frac{S_{fct} (1 - M_m^2)}{N^2} \frac{D_m^2}{h_{1m} (h_{1m} + h_{2m})} \quad \text{A 1.14}$$

Equation A 1.12 becomes:-

$$y_{m,n} = y_{m,n-1} - x_m + K_m \sum_{j=1}^{n-2} \Delta y_{m,j} \quad \text{A 1.15}$$

Writing equations A 1.15 for the $n-1$ th streamline and subtracting from equation A 1.15:-

$$y_{m,n-1} = y_{m,n-2} - x_m + K_m \sum_{j=1}^{n-2} \Delta y_{m,j}$$

$$y_{m,n} = 2y_{m,n-1} - y_{m,n-2} + K_m \Delta y_{m,n-2}$$

Substituting equation A 1.4 the result for subsonic flow is:-

$$y_{m,n} = 2y_{m,n-1} - y_{m,n-2} + K_m \{y_{m,n-2} - h'_m y_{m-1,n-2} - (1 - h'_m) y_{m+1,n-2}\}$$

or

$$y_{m,n} = 2y_{m,n-1} - (1 - K_m) y_{m,n-2} - K_m \{h'_m y_{m-1,n-2} + (1 - h'_m) y_{m+1,n-2}\} \quad \text{A 1.16}$$

Similarly, substituting equation A 1.5 gives the result for supersonic flow:-

$$y_{m,n} = 2y_{m,n-1} - (1 - K_m) y_{m,n-2} - K_m \{(1 + h''_m) y_{m-1,n-2} - h''_m y_{m-2,n-2}\} \quad \text{A 1.17}$$

Proceeding from equation A 1.11 in a similar way to the above, the use of equation A 1.10 gives the result for the Runge-Kutta type method:-

$$y_{m,n} = y_{m,n-1} - x_m + K_m \left\{ \sum_{j=1}^{n-2} \Delta y_{m,j} + \frac{1}{2} \Delta y_{m,n-1} \right\} \quad \text{A 1.18}$$

$$y_{m,n} = 2y_{m,n-1} - y_{m,n-2} + \frac{1}{2} K_m \{\Delta y_{m,n-2} + \Delta y_{m,n-1}\} \quad \text{A 1.19}$$

Substitution of equation A 1.4 gives the result for subsonic flow:-

$$y_{m,n} = (2 + \frac{1}{2}K_m)y_{m,n-1} - (1 - \frac{1}{2}K_m)y_{m,n-2} - \frac{1}{2}K_m\{h'_m(y_{m-1,n-2} + y_{m-1,n-1}) + (1 - h'_m)(y_{m+1,n-2} + y_{m+1,n-1})\} \quad \text{A 1.20}$$

Substitution of equation A 1.5 into A 1.19 gives the result for supersonic flow:-

$$y_{m,n} = (2 + \frac{1}{2}K_m)y_{m,n-1} - (1 - \frac{1}{2}K_m)y_{m,n-2} - \frac{1}{2}K_m\{(1 + h''_m)(y_{m-1,n-2} + y_{m-1,n-1}) - h''_m(y_{m-2,n-2} + y_{m-2,n-1})\} \quad \text{A 1.21}$$

Equations A 1.16, A 1.17, A 1.20 and A 1.21 are the required convergence formulae relating velocity and area changes in terms of grid parameters. A method of evaluating these formulae is presented in the next two sections.

Appendix 1b

Calculation procedure for the evaluation of the subsonic convergence formulae.

The calculation procedure of Section 5.5 is presented fully in this section.

Inspection of equation A 1.15 for $n=1, 2$ etc. shows that it may be written, for the M normals of Sketch 5.3, as:-

$$y_{m,n} = c_{1,n}x_1 + c_{2,n}x_2 + \dots + c_{M,n}x_M \quad \text{A 1.22}$$

where $c_{j,n}$ are constants.

Introducing a subscript notation for c ranging from 1 to M^2 , equation A 1.22 is written for each of the M normals as in equation 5.14. It is now assumed, as explained in Section 5.4b, that no changes take place along normals 1 and M . With this assumption equation 5.14 becomes:-

$$\begin{array}{c}
 \xrightarrow{M} i \\
 1 \\
 y_{1,n} = 0 \\
 \vdots \\
 \begin{array}{c} 1 \quad M \\ \downarrow \\ m \end{array} y_{m,n} = 0 + c_{(m-1)M+2,n} x_2 + \dots + c_{(m-1)+i,n} x_i + \dots + c_{(m-1)M+M-1,n} x_{M-1}^{+0} \\
 \vdots \\
 y_{M,n} = 0
 \end{array}
 \quad \text{----- A 1.23}$$

Equation A 1.23 (that is equations A 1.23 for $y_{m,n-1}$, $y_{m,n-2}$, $y_{m-1,n-2}$ and $y_{m+1,n-2}$) can now be substituted into the R.H.S. of equation A1.16 to give:-

$$\begin{aligned}
 y_{m,n} = & 2(c_{(m-1)M+2,n-1} x_2 + \dots + c_{(m-1)M+i,n-1} x_i + \dots + c_{(m-1)M+M-1,n-1} x_{M-1}) \\
 & - (1-K_m)(c_{(m-1)M+2,n-2} x_2 + \dots + c_{(m-1)M+i,n-2} x_i + \dots + c_{(m-1)M+M-1,n-2} x_{M-1}) \\
 & - K_m \{ h'_m (c_{(m-2)M+2,n-2} x_2 + \dots + c_{(m-2)M+i,n-2} x_i + \dots + c_{(m-2)M+M-1,n-2} x_{M-1}) \\
 & + (1+h'_m)(c_{mM+2,n-2} x_2 + \dots + c_{mM+i,n-2} x_i + \dots + c_{mM+M-1,n-2} x_{M-1}) \}
 \end{aligned}
 \quad \text{----- A 1.24}$$

Equations A 1.23 and A 1.24 are both expressions for $y_{m,n}$. The i th coefficient of equation A 1.23 must therefore be equal to the i th coefficient of equation A 1.24. Equating these coefficients gives:-

$$c_{(m-1)M+i,n} = 2c_{(m-1)M+i,n-1} - (1-K_m)c_{(m-1)M+i,n-2} - K_m \{ h'_m c_{(m-2)M+i,n-2} + (1-h'_m)c_{mM+i,n-2} \} \quad \text{----- A 1.25}$$

Similarly for the Runge-Kutta type method (substituting equation A 1.23 into equation A 1.20) :-

$$\begin{aligned}
 c_{(m-1)M+i,n} = & (2 + \frac{1}{2}K_m)c_{(m-1)M+i,n-1} - (1 - \frac{1}{2}K_m)c_{(m-1)M+i,n-2} \\
 & - \frac{1}{2}K_m \{ h'_m (c_{(m-2)M+i,n-2} + c_{(m-2)M+i,n-1}) + (1-h'_m)(c_{mM+i,n-2} + c_{mM+i,n-1}) \}
 \end{aligned}
 \quad \text{----- A 1.26}$$

As explained in Section 5.5c, the starting coefficients for the general expressions of A 1.25 and A 1.26 are needed. These are found as follows. From equations A 1.4 and A 1.15, for $n = 1$:-

$$y_{m,1} = 0 - x_m + 0$$

Comparison of this equation with equation A1.23 indicates that all the coefficients are zero except that of x_m which is -1.

Therefore:-

$$c_{(m-1)M+m,1} = -1 \quad \text{A 1.27}$$

Similarly for $n = 2$:-

$$y_{m,2} = -x_m - x_m + 0 = -2x_m$$

and

$$c_{(m-1)M+m,2} = -2 \quad \text{A 1.28}$$

The starting coefficients for the Runge-Kutta type method are found using equations A 1.5 and A 1.18. For $n = 1$:-

$$y_{m,1} = 0 - x_m + 0$$

therefore

$$c_{(m-1)M+m,1} = -1 \quad \text{A 1.29}$$

For $n = 2$:-

$$\begin{aligned} y_{m,2} &= -x_m - x_m + \frac{1}{2}K_m \{-x_m + h'_m x_{m-1} + (1 - h'_m)x_{m+1}\} \\ &= -(2 + \frac{1}{2}K_m)x_m + \frac{1}{2}K_m h'_m x_{m-1} + \frac{1}{2}K_m (1 - h'_m)x_{m+1} \end{aligned}$$

Comparison of this equation with equation A 1.23 indicates that all the coefficients are zero except those of x_{m-1} , x_m and x_{m+1} which are:-

$$\left. \begin{aligned} c_{(m-1)M+m,2} &= -(2 + \frac{1}{2}K_m) \\ c_{(m-1)M+m-1,2} &= \frac{1}{2}K_m h'_m \quad \text{for } 3 \leq m \\ &= 0 \quad \text{for } m = 2 \\ c_{(m-1)M+m+1,2} &= \frac{1}{2}K_m (1 - h'_m) \quad \text{for } m \leq M-2 \\ &= 0 \quad \text{for } m = M-1 \end{aligned} \right\} \quad \text{A 1.30}$$

Appendix 1cCalculation procedure for the evaluation of the
supersonic convergence formula.

The recurrence formulae for the convergence of supersonic flow are given in equations A 1.17 and A 1.21. The method and the grid notation used to apply the formulae to an actual channel has been described in Section 5.6 and Sketch 5.4. Equations A 1.17 and A 1.21 can be evaluated as follows.

An examination of the equations show that $y_{m,n}$ is a function only of x_j where $j \leq m$. When therefore $y_{m,n}$ is written in the form of equation A 1.22, the coefficients of x_j for $j > m$ must be zero. Referring to Sketch 5.4, no changes occur at normals 1 and 2 and the coefficients for $m = 1$ and $m = 2$ are also zero. Adopting the same subscript notation as in equation 5.14, a similar equation is written for the supersonic case in equation 5.22.

Equation 5.22 for $y_{m,n-1}$, $y_{m,n-2}$, $y_{m-1,n-2}$ and $y_{m-2,n-2}$ is substituted into the R.H.S. of equation A 1.17 to give:-

$$y_{m,n} = 2(c_{(m-1)M+3,n-1}x_3 + \dots + c_{(m-1)M+i,n-1}x_i + \dots + c_{(m-1)M+m,n-1}x_m) \\ - (1-K_m)(c_{(m-1)M+3,n-2}x_3 + \dots + c_{(m-1)M+i,n-2}x_i + \dots + c_{(m-1)M+m,n-2}x_m) \\ - K_m\{(1+h_m'')(c_{(m-2)M+3,n-2}x_3 + \dots + c_{(m-2)M+i,n-2}x_i + \dots + c_{(m-2)M+m,n-2}x_m) \\ - h_m''(c_{(m-3)M+3,n-2}x_3 + \dots + c_{(m-3)M+i,n-2}x_i + \dots + c_{(m-3)M+m,n-2}x_m)\}$$

————— A 1.31

Equations 5.22 and A 1.31 are both expressions for $y_{m,n}$. The i th coefficient of equation 5.22 must therefore be equal to the i th coefficient of equation A 1.31. Equating these coefficients:-

$$c_{(m-1)M+i,n} = 2c_{(m-1)M+i,n-1} - (1-K_m)c_{(m-1)M+i,n-2} \\ - K_m\{(1+h_m'')c_{(m-2)M+i,n-2} - h_m''c_{(m-3)M+i,n-2}\} \text{ ————— A 1.32}$$

Similarly, using equations A 1.21, the result for the Runge-Kutta type method is:-

$$c_{(m-1)M+i,n} = (2 + \frac{1}{2}K_m)c_{(m-1)M+i,n-1} - (1 - \frac{1}{2}K_m)c_{(m-1)M+i,n-2} \\ - \frac{1}{2}K_m \{ (1 + h_m'')(c_{(m-2)M+i,n-2} + c_{(m-2)M+i,n-1}) \\ - h_m''(c_{(m-3)M+i,n-2} + c_{(m-3)M+i,n-1}) \} \quad \text{A 1.33}$$

As for subsonic flow, equations A 1.32 and A 1.33 can be evaluated sequentially for increasing values of n once the initial constants for $n = 1$ and $n = 2$ are known. These two starting coefficients for the simple Euler type method are found from equations A 1.15 and A 1.5. For $n = 1$:-

$$y_{m,1} = -x_m$$

Comparing this result with 5.22 indicates that all the coefficients are zero except that of x_m . Therefore:-

$$c_{(m-1)M+m,1} = -1 \quad \text{A 1.34}$$

Similarly for $n = 2$, all coefficients are zero except that of x_m which is:-

$$c_{(m-1)M+m,2} = -2 \quad \text{A 1.35}$$

The starting coefficients for the Runge-Kutta type method are found from equations A 1.18 and A 1.5. For $n = 1$:-

$$y_{m,1} = -x_m \quad \text{and} \quad c_{(m-1)M+m,1} = -1 \quad \text{A 1.36}$$

For $n = 2$:-

$$y_{m,2} = -(2 + \frac{1}{2}K_m)x_m + \frac{1}{2}K_m(1 + h_m'')x_{m-1} - \frac{1}{2}K_m h_m''x_{m-2}$$

Comparing this equation with equation 5.22 indicates that all the coefficients are zero except those of x_m , x_{m-1} and x_{m-2} . Therefore:-

$$\text{for } m \geq 5 \quad c_{(m-1)M+m,2} = -2(2 + \frac{1}{2}K_m), \quad c_{(m-1)M+m-1,2} = \frac{1}{2}K_m(1 + h_m'')$$

$$c_{(m-1)M+m-2,2} = -\frac{1}{2}K_m h_m''$$

$$\text{for } m = 4 \quad c_{(m-1)M+m-2,2} = 0, \quad \text{with the other two coefficients as for } m \geq 5.$$

$$\text{for } m = 3 \quad c_{(m-1)M+m-1,2} = c_{(m-1)M+m-2,2} = 0, \quad \text{with the other coefficient as for } m \geq 5$$

Appendix 1dDerivation of $1 + k \frac{V}{\rho} = 1 - M^2$

It is required to show that :- $K_m = 1 + k \frac{V}{\rho} = 1 - M^2$

From equation A 1.7 :-

$$K_m = 1 + \frac{d\rho}{dV} \frac{V}{\rho}$$

For a perfect gas :-

$$T = T_0 - \frac{V^2}{2g_0 J C_p} \quad \text{and} \quad \frac{T}{T_0} = \left(\frac{\rho}{\rho_0} \right)^{\gamma-1}$$

Combining :-

$$V = (2 g_0 J C_p T_0 \{1 - \left(\frac{\rho}{\rho_0} \right)^{\gamma-1}\})^{\frac{1}{2}}$$

$$\text{Differentiating :- } \frac{dV}{d\rho} = \frac{1}{2} \{2g_0 J C_p T_0 (1 - \left(\frac{\rho}{\rho_0} \right)^{\gamma-1})^{\frac{1}{2}}\} \times \left\{ - \frac{2g_0 J C_p T_0 (\gamma-1) \rho^{\gamma-2}}{\rho_0^{\gamma-1}} \right\}$$

$$= - \frac{g_0 J T_0 \gamma \rho^{\gamma-2}}{V \rho_0^{\gamma-1}} \quad (R\gamma = C_p (\gamma - 1))$$

$$1 + \frac{d\rho}{dV} \frac{V}{\rho} = 1 - \frac{V^2}{g_0 J R \gamma T} = 1 - \frac{V^2}{V_{sonic}^2} = 1 - M^2$$

APPENDIX 2 THE COMPUTER PROGRAMME

2a

FORTTRAN Symbol List

<u>FORTTRAN Symbol</u>	<u>Description of symbol</u>	<u>Algebraic Equivalent/ Source/ Main Areas of Use.</u>
A(N,J)	Inverted matrix array for super-sonic block convergence.	Chapter 5, VCCONS, SUPCON, VNEW.
AI(N,J)	Salient row of matrix inverse at normal N.	Chapter 5, VCCONS, ADPT, VNEW.
AREA(N)	Flow area along normal N.	D.
A2	Flow area required at a normal.	ENDPT, BCHEK.
CC(289,3)	Array used for generating velocity correction constants.	ADPT, SUPCON.
CP	Cp for perfect gas.	Main.
DA(N)	Flow area change required at normal N.	da, ENDPT.
DANPS	Flow area downstream.	BCHEK.
DAI	Flow area upstream.	BCHEK.
DFSUB	Damping factor used for a subsonic flow.	d_f , VNEW, NUGRID.
DFSUP	Damping factor used for a super-sonic flow.	d_f , VNEW, SUPMCH, NUGRID.
DFT	Throat velocity correction damping factor.	BCHEK.
DHDATA	Tabular difference in enthalpy table.	Main.
DL	Fractional flow height variation between XU and XD.	EL.
DLPL1	DL + 1.0, for use in channel height calculation.	EL.
DLDXU	DL/(XD - XU), for use in channel height calculation.	EL.
DM	Streamline mass flow lb/sec.	$\dot{m}$, Chapter 4 & 5, SLCALC.
DN	Stream tube flow width.	dn, Chapter 4, SLCALC, NEXTPT.
DV	Tabular velocity increment.	FINDRO.
DYDX(N,M)	Slope of streamline M at normal N.	SLCALC, DYDXRC, NEXTPT.
E(N)	Non-dimensionalised flow area change required at normal N.	Y, Chapter 5, ENDPT, BCHEK, SUPMCH, VNEW.
EM	Molecular weight of perfect gas.	Main.

ERROR	Terminating surface boundary condition tolerance. For convergence/E(N)/ must be less than ERROR.	BCHEK, SUPMCH.
GAMMA	γ for perfect gas.	Main.
HODATA	Highest tabular enthalpy value.	Main.
HOI	Total enthalpy of fluid.	Main.
IRC	Calculation control number 0 = curvature and slope calculation suppressed during stacking procedure 1 = normal ie. curvature and slope calculated during stacking procedure.	SLCALC.
ISTP	Maximum number of iterations allowed for any one grid.	SUPMCH, KUTTA.
ITRN	Counter for number of iterations.	SLCALC, SUPMCH, KUTTA.
JCHNG	Indicates state of convergence. 1 = convergence obtained, 2 = a change has been made and further iteration required.	BCHEK.
JRC	Calculation control number. 0 = normal procedure used to declare a point "subsonic" or "supersonic". 1 = each point declared once for all at commencement of iteration process by interpolation from a previous solution.	SLCALC, NUGRID.
JREAD	Number controlling generation of initial flow parameters TM, VCC(N,1) V(N,1). All data is calculated one dimensionally and then overwritten by input as follows:- 1 = nothing overwritten 2 = VCC(N,1) overwritten 3 = VCC(N,1), TM, V(N,1) overwritten 4 = TM, V(N,1) overwritten	VELDIS.
JSONIC	Channel flow type control number. 0 = wholly supersonic flow, 1 = subsonic-supersonic flow, 2 = choked subsonic flow, 3 = unchoked subsonic flow.	_____
KD	Calculation control number. 1 = normal damping factor used, 0 = low damping factor used in throat region.	VNEW.
KDK	Output control. 0 = deck of grid coordinates and Mach numbers produced, 1 = no deck produced.	RESULT.
KRC	Calculation control number. 1 = curvature and slope calculated at commencement of iterative process only 0 = normal i.e. curvature and slope calculated during each iteration.	

K2	Calculation control number. 1 = previous streamline data used, 1 < average streamline data used.	SLCALC.
NDATA	Number of expansion line data points supplied or generated.	Main.
NGAS	Data type control number. 1 = perfect gas, - 1 = enthalpy and specific volume expansion line data used.	Main.
NGRDS	Number of grids used to achieve final solution.	Main.
NP	Region of influence affecting convergence at a point.	Chapter 5, VCCONS, ADPT, VNEW.
NPB2	Size of terminating surface array X2, Y2.	ENDPT.
NPN	Number of points on a normal.	N.
NPS	Number of points on a streamline.	_____
NSP	Normal at which velocity is specified.	VELDIS.
NSPI	Number of supersonic points used in "block" convergence.	Chapter 5, VCCONS, SUPCON, VNEW.
NSPIC	Calculation control number. 1 = whole supersonic region treated as a "block", 0 = size of block as specified.	Main.
NST	Number of first normal to be used in streamline stacking procedure.	SLCALC, BCHEK, SUPMCH, KUTTA.
NTAB	Number of tabular values in velocity density table.	FINDRO.
NTD	Increment to NTHRT for new grid.	NUGRID.
NTHRT	Number of throat normal.	_____
NXI	Size of commencing surface array.	FINDY.
NI(N)	Number of first normal to be used in velocity correction calculation.	VCCONS, ADPT, VNEW.
N2	Last normal in streamline stacking procedure.	SLCALC, BCHEK, SUPMCH, KUTTA.
PITCH	Distance through which terminating surface input Y coordinates are moved to form the required cascade channel.	Main.
P1	Total pressure of perfect gas flow.	Main.
RC(N,M)	Radius of curvature of streamline Mat normal N.	R, Chapter 4 & 5, SLCALC, DYDXRC.
RHO(J)	Specific volume array of expansion line input.	Main.
RO(J)	Tabular values of density in lb/ft ³ .	ρ , FINDRO.

SFCT	Radius of curvature "starting factor". A quantity between 0 and 1 to facilitate the erection of a grid. When less than 1 only a fraction of the actual streamline curvature is used to calculate the velocity gradients.	S_{fct} , SLCALC, NUGRID.
TITLE(J)	An alphameric array used for titling the printout.	Main, RESULT.
TM	Total channel mass flow lb/sec.	$\dot{m}$.
T1	Total temperature of perfect gas flow.	Main.
V(N,M)	Velocity of streamline M at normal N.	V.
VAV(N,M)	Average velocity of streamline M at normal M.	_____
VCC(N,3)	Velocity correction constants at normal N.	$1 - M^2$, K_m and h'_m or H''_m . Chapter 5, VCCONS.
VEL(J)	Tabular value of velocity in ft/sec.	V, FINDRO.
VMAX	Maximum allowable velocity at any grid point during iterative process.	SLCALC.
VMIN	Minimum allowable velocity at any grid point during iterative process.	SLCALC.
VR(N,M)	Velocity used to declare a grid point "subsonic" or "supersonic".	SLCALC.
VSONIC	Sonic velocity in ft/sec.	_____
VSPEC	Velocity specification at normal NSP in unchoked subsonic flow.	VELDIS.
VST	Sonic velocity tolerance used as damping during iteration in the declaration of a "subsonic" or "supersonic" grid point.	SLCALC.
VTT	Throat velocity tolerance on the choking boundary condition at the throat.	BCHEK.
X(N,M)	x coordinate of streamline M at normal N.	x
XD	Downstream x coordinate of cascade. Intended to describe channel height changes.	EL, KUTTA.
XU	Upstream x coordinate of cascade. Intended to describe channel height changes.	EL.
X1(J)	x coordinate array of commencing surface.	FINDY.

X2(J)	x coordinate array of terminating surface.	ENDPT.
Y(N,M)	y coordinate of streamline M at normal N.	y.
Y1(J)	y coordinate array of commencing surface.	FINDY.
Y2(J)	y coordinate array of terminating surface.	ENDPT.

APPENDIX 2 B

PROGRAMME LISTING

```

C ** MAIN PROGRAMME. GENERATES ALL DATA AND DIRECTS THE MAIN FLOW OF
C ** CALCULATION
C
COMMON/BLKA/X(50,25),Y(50,25)/BLKB/AREA(50)/BLKC/V(50,25)
1/BLKD/RC(50,25)/BLKE/DYDX(50,25)/BLKF/RO(100),VEL(100),NTAB
2/BLKG/DA(50)/BLKH/VAV(50,25)/BLKI/X2(70),Y2(70),NPB2/BLKJ/E(50)
3/BLKK/A2/BLKL/NPS/BLKM/NPN/BLKN/DM/BLKO/NTHRT/BLKP/JSONIC
4/BLKQ/VSPEC,VSONIC/BLKR/TM/BLKS/DV/BLKT/DN/BLKU/XU,XD,DLPL1,DLXDXU
5/BLKW/SFCT /BLKX/ERROR/BLKY/JCHNG/BLKZ/ITRN,IMF,ICYC,ITV
6 /BLKBB/NSP/BLKCC/N2,NST/BLKDD/VCC(50,3),A1(50,15),N1(50)
7,AM(15,30),NSPI,NP/BLKEE/DA1,DANPS/BLKFF/DFSUB,DFSUP/BLKGG/CC(289,
8 3)/BLKHH/X1(70),Y1(70),NX1/BLKII/VR(50,25),VST/BLKJJ/IRC,KRC,JRC
9/BLKKK/TITLE(90) /BLKLL/VTT,DFT/BLKMM/KD
DIMENSION RHO(100),H(100),RKM(100),RM(100)
C ** INPUT OF THERMODYNAMIC DATA
READ (5,130) NGAS,NDATA,H01,H0DATA,DHDATA,DV,T1,P1,GAMMA,CP,EM
WRITE(6,130) NGAS,NDATA,H01,H0DATA,DHDATA,DV,T1,P1,GAMMA,CP,EM
IF(NGAS.EQ.-1) READ (5,140) (RHO(N),N=1,NDATA)
IF(NGAS.EQ.-1) WRITE(6,140) (RHO(N),N=1,NDATA)
C ** PREPARING THE THERMODYNAMIC DATA
IF(NGAS.EQ.-1) GO TO 23
C ** GENERATING ENTHALPY AND SPECIFIC VOLUME TABLE FROM PERFECT GAS DATA
EXP=1.0/(GAMMA-1.0)
CONST=T1*(1545.43*T1/(EM*P1*144.0))**(GAMMA-1.0)
DT=DHDATA/CP
DO 22 N=1,NDATA
22 RHO(N)=(CONST/(T1-DT*FLOAT(N-1)))*EXP
23 CONTINUE
C ** CONVERTING THE SPEC VOL TO DENSITY AND GENERATING THE ENTHALPY
C ** TABLE
DO 30 N=1,NDATA
RHO(N)=1.0/RHO(N)
30 H(N)=H0DATA-FLOAT(N-1)*DHDATA
C ** GENERATING THE VELOCITY TABLE AND INTERPOLATING FROM THE DENSITY
C ** TABLE TO FIND THE CORRESPONDING DENSITY
GJ2=2.0*32.16*778.16
DO 42 N=1,98
VEL(N)=FLOAT(N-1)*DV
HH=H01-VEL(N)**2/GJ2
IF(HH.LE.H(NDATA-1)) GO TO 43
K=1
DO 40 NN=1,NDATA
IF(HH.LE.H(K).AND.HH.GT.H(K+1)) GO TO 41
40 K=K+1
41 RO(N)=PAR (H(K),H(K+1),H(K+2),HH,RHO(K),RHO(K+1),RHO(K+2))
42 NTAB=N
43 CONTINUE
RKM(1)=1.0
C ** PREPARING A TABLE OF MACH NUMBER AND 1-M**2
DO 44 N=2,NTAB
RKM(N)=1.0+(RO(N)-FINDRO(VEL(N)-5.0))*VEL(N)/RO(N)/5.
44 RM(N)=SQRT(1.0-RKM(N))
C ** CALCULATING THE SONIC VELOCITY
VSUB=1.0
VSUP=VEL(NTAB)

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DO 52 N=1,100
VSONIC=.5*(VSUB+VSUP)
IF (ABS(VSONIC-VSON).LE.1.0) GO TO 53
VSON=VSONIC
DZ=1.0/(FINDRO(VSONIC+.25)*(VSONIC+.25))-
1 1.0/(FINDRO(VSONIC-.25)*(VSONIC-.25))
IF (DZ) 50,53,51
50 VSUB=VSONIC
GO TO 52
51 VSUP=VSONIC
52 CONTINUE
WRITE(6,400)
53 CONTINUE
C ** THERMODYNAMIC DATA PRINTOUT
C WRITE(6,300) NGAS,NDATA,NTAB,H01,H0DATA,DHDATA,DV,T1,P1,GAMMA,
C 1CP,EM,VSONIC
C WRITE(6,210) (N,H(N),RHO(N),N=1,NDATA)
C WRITE(6,320)
C WRITE(6,211) (N,VEL(N),RO(N),RKM(N),RM(N),N=1,NTAB)
C ** GEOMETRIC AND CONTROL DATA INPUT
READ (5,101) TITLE,
1NPB2,NX1,NSPI,ISTP,NGRDS,K2,JSONIC,JREAD,NSPIC,KD,KDK,
2NPS,NTHRT,NPN,NP,IRC,KRC,JRC,ERROR,DFSUB,DFSUP,SFCT,
3PITCH,XU,XD,DL,VSPEC,VMIN,VMAX,VST,VTI,DFT
WRITE(6,101) TITLE,
1NPB2,NX1,NSPI,ISTP,NGRDS,K2,JSONIC,JREAD,NSPIC,KD,KDK,
2NPS,NTHRT,NPN,NP,IRC,KRC,JRC,ERROR,DFSUB,DFSUP,SFCT,
3PITCH,XU,XD,DL,VSPEC,VMIN,VMAX,VST,VTI,DFT
READ (5,120) (X(N),N=1,NPS)
WRITE(6,121) (X(N),N=1,NPS)
READ (5,122) (X2(N),Y2(N),N=1,NPB2)
WRITE(6,123) (X2(N),Y2(N),N=1,NPB2)
READ (5,122) (X1(N),Y1(N),N=1,NX1)
WRITE(6,123) (X1(N),Y1(N),N=1,NX1)
C ** GEOMETRIC DATA PRINTOUT AND PREPARATION
CALL FINDY
DO 10 N=1,NPB2
10 Y2(N)=Y2(N)+PITCH
DLPL1=DL+1.0
DLXDXU=DL/(XD-XU)
DO 13 NN=2,25
DO 13 N=1,50
13 X(N,NN)=X(N,1)
N2=NPS
DO 15 N=1,NPS
15 CALL DYDXRC(N,1,1.0)
WRITE(6,200) NPB2,NX1,NSPI,ISTP,NGRDS,K2,JSONIC,JREAD,NSPIC,KD,
1KDK,NPS,NTHRT,NPN,NP,IRC,KRC,JRC,ERROR,DFSUB,DFSUP,SFCT,
2PITCH,XU,XD,DL,VSPEC,VMIN,VMAX,VST,VTI,DFT
WRITE(6,202) (N,X1(N),Y1(N),N=1,NX1)
WRITE(6,204) (N,X2(N),Y2(N),N=1,NPB2)
WRITE(6,206) (N,X(N,1),Y(N,1),N=1,NPS)
WRITE(6,208) (N,DYDX(N,1),RC(N,1),N=1,NPS)
C ** MAIN CALCULATIONS
CALL VELDISE(JSONIC,JREAD,1)
C ** INITIAL VELOCITY ASSUMPTION AND MASS FLOW FOUND
C ** SETTING UP A LOOP TO PRODUCE SUCCESSIVELY MORE ACCURATE GRID
C ** SOLUTIONS
DO 74 NGS=1,NGRDS

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      DM=TM/FLOAT(NPN)
C **  SETTING UP UPSTREAM AND DOWNSTREAM BOUNDARY CONDITIONS
      DO 71 NN=1,NPN
        V(1,NN+1)=V(1,1)
        VAV(1,NN)=V(1,1)
        DYDX(1,NN)=DYDX(1,1)
        DN=DA1/(FLOAT(NPN)*EL(X(1,1)))
        CALL NEXTPT(1,NN)
        IF(JSONIC.EQ.1) GO TO 71
        V(NPS,NN+1)=V(NPS,1)
        VAV(NPS,NN)=V(NPS,1)
        DYDX(NPS,NN+1)=DYDX(NPS,1)
        DN=DANPS/(FLOAT(NPN)*EL(X(NPS,1)))
        CALL NEXTPT(NPS,NN)
      71 CONTINUE
C **  SETTING VALUES FOR FIRST AND LAST NORMALS IN STREAMLINE STACKING
C **  PROCEDURE
      NST=2
      N2=NPS-1
      IF(NSPIC.EQ.1) NSPI=NPS-NTHRT
      IF(JSONIC.EQ.1) N2=NTHRT+NSPI
C **  GENERATING THE VELOCITY CORRECTION CONSTANTS
      CALL VCCONS(NGS)
C **  SETTING UP THE MAIN ITERATIVE LOOP
      DO 72 ITRN=1,ISTP
C **  GENERATING ALL THE STREAMLINES
        CALL SLCALC(K2,VMIN,VMAX)
C **  CHECKING ALL BOUNDARY CONDITIONS AND MAKING THE CHANGES NECESSARY
        CALL BCHEK
C **  CHECKING FOR CONVERGENCE
        IF(JCHNG.EQ.1) GO TO 73
      72 CONTINUE
        STOP
C **  MARCHING OUT A SUPERSONIC SOLUTION IF REQUIRED
      73 IF(JSONIC.EQ.1.AND.N2.LT.NPS.AND.JCHNG.EQ.1) CALL SUPMCH(K2,ISTP,
      1VMIN,VMAX)
C **  OUTPUT PRODUCED
      CALL RESULT(NGS,NGRDS+KDK,1,NPS)
C **  GENERATING DATA FOR NEXT GRID
      CALL NUGRID(NGRDS-NGS,NGRDS-NGS)
      74 CONTINUE
        STOP
C **  FORMAT STATEMENTS
      101 FORMAT(5(18A4/),6I4,5I2/4I4,3I2,E8,1,3F5,2/4F7,4,6F7,1)
      120 FORMAT(14F5,3)
      121 FORMAT(14F7,3)
      122 FORMAT(10F7,5)
      123 FORMAT(10F9,5)
      130 FORMAT(2I4 / 2F7,1,F4,1,F6,1,F7,1,F6,1,F5,2,F7,4,F7,2)
      140 FORMAT(10F7,4)
      150 FORMAT( 5I4/E10,0,F7,1,2F5,2)
      200 FORMAT(15H1GEOMETRIC DATA//8H NPB2   =I4/8H NX1   =I4/8H NSPI   =I4/
      18H ISTD   =I4/8H NGRDS =I4/8H K2     =I4/8H JSONIC=I4/8H JREAD =I4/
      28H NSPIC  =I4/8H KD     =I4/8H KDK    =I4/8H NPS   =I4/8H NTHRT =I4/
      38H NPN    =I4/8H NP     =I4/8H IRC    =I4/8H KRC    =I4/8H JRC    =I4/
      48H ERROR  =E13.5/8H DFSUB =E13.5/8H DFSUP =E13.5/8H SFCT   =E13.5/
      58H PITCH  =E13.5/8H XU     =E13.5/8H XD     =E13.5/8H DL     =E13.5/
      68H VSPEC  =E13.5/8H VMIN  =E13.5/8H VMAX  =E13.5/8H VST    =E13.5/
      78H VTT    =E13.5/8H DFT    =E13.5)

```

```

202 FORMAT(31H0COMMENCING SURFACE DATA ARRAY //
14(6X,1HN11X,2HX111X,2HY1) / (4(17,2E13.5)))
204 FORMAT(31H0TERMINATING SURFACE DATA ARRAY //
14(6X,1HN11X,2HX211X,2HY2) / (4(17,2E13.5)))
206 FORMAT(25H0INITIAL GRID COORDINATES //
14(6X,1HN12X,1HX12X,1HY) / (4(17,2E13.5)))
208 FORMAT(36H0INITIAL GRID SLOPES AND CURVATURES //
14(6X,1HN9X,4HDYDX11X,2HRC) / (4(17,2E13.5)))
300 FORMAT(19H1THERMODYNAMIC DATA // 8H NGAS =14/8H NDATA =14/
28H NTAB =14/8H H01 =E12.5/8H H0DATA=E12.5/8H DHDATA=E12.5/
38H DV =E12.5/8H T1 =E12.5/8H P1 =E12.5/8H GAMMA =E12.5/
48H CP =E12.5/8H EM =E12.5/8H VSONIC=E12.5//
44(30H N H RHO ))
320 FORMAT(1H02(9X,1HN4X,3HVEL11X,2HRO7X,3HRKMBX,2HRM))
400 FORMAT(34H0VSONIC CALCULATION NOT CONVERGING )
END

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SUBROUTINE BCHEK

```

C ** SUBROUTINE TO CHECK ALL BOUNDARY CONDITIONS AND MAKE THE
C ** ITERATIVE CHANGES
C
COMMON /BLKB/AREA(50)/BLKC/V(50,25)/BLKH/VAV(50,25)/BLKJ/E(50)
1/BLKK/A2/BLKL/NPS/BLKM/NPN/BLKN/DM/BLKO/NTHRT/BLKP/JSONIC
2/BLKQ/VSPEC,VSONIC/BLKR/TM/BLKX/ERROR/BLKY/JCHNG/BLKZ/ITRN,11,12,
313/BLKCC/N2,NST/BLKEE/DA1,DANPS/BLKLL/VTT,DFT
4/BLKDD/VCC(50,3),A1(50,15),N1(50),A(15,30),NSPI,NP
DIMENSION KCHNG(50),C(50)
C ** SETTING THE CONVERGENCE INDICATOR
JCHNG=1
C ** FINDING THE DEVIATION OF THE AVERAGE NORMAL VELOCITY FROM THE
C ** SONIC VELOCITY
DO 10 N=NST,N2
E(N)=0.0
DO 9 M=1,NPN
9 E(N)=E(N)+VAV(N,M)
E(N)=E(N)/FLOAT(NPN)
10 C(N)=VSONIC-E(N)
C ** SEPARATING SUBSONIC FLOWS FROM CHOKING FLOWS
IF(JSONIC.EQ.3) GO TO 30
C ** FINDING THE MINIMUM DEVIATION OF THE AVERAGE VELOCITY ALONG A
C ** NORMAL FROM THE SONIC VELOCITY AND CHECKING THAT IT COINCIDES WITH
C ** PRESENT THROAT POSITION
DV=1.0E10
DO 11 N=NST,N2
VCC(N,1)=1.0+(FINDRO(E(N)+2.5)-FINDRO(E(N)-2.5))
1 *0.2*E(N)/FINDRO(E(N))
IF(ABS(C(N)).GT.ABS(DV)) GO TO 11
DV=C(N)
K=N
11 CONTINUE
IF(K.EQ.NTHRT) GO TO 12
C ** IF THROAT POSITION HAS CHANGED, VELOCITY CORRECTION CONSTANTS
C ** ARE REGENERATED
NTHRT=K
NSPI=N2-NTHRT
CALL VCCONS(2)
WRITE(6,200) NTHRT,(N,C(N),N=NST,N2)

```

```

C ** CHECKING THROAT BOUNDARY CONDITIONS. (CHOKING FLOW AND GEOMETRIC)
C ** AND MAKING CORRECTIONS TO MASS FLOW AND THROAT COMMENCING SURFACE
C ** VELOCITY
12 IF (ABS(C(NTHRT)).GT.VTT) JCHNG=2
   V(NTHRT+1)=V(NTHRT+1)+DFT*C(NTHRT)
   CALL ENDPT(NTHRT)
   IF (ABS(E(NTHRT)).GT.ERROR) JCHNG=2
   F=1.0+DFT*(A2/AREA(NTHRT)-1.0)
   TM=TM*F
   DM=TM/FLOAT(NPN)
C ** RECALCULATING UPSTREAM AND DOWNSTREAM VELOCITIES
   V(1,1)=FINDV(TM/DA1,1)
   IF (JSONIC.GE.2) V(NPS,1)=FINDV(TM/DANPS,NPS)
   DO 20 N=1,NPN
     IF (JSONIC.GE.2) V(NPS,N+1)=V(NPS,1)
20  V(1,N+1)=V(1,1)
C ** CHECKING BOUNDARY CONDITION AT TERMINATING SURFACE
30 DO 50 N=NST,N2
   IF (N.EQ.NTHRT.AND.JSONIC.LE.2) GO TO 40
   CALL ENDPT(N)
   IF (JSONIC.LE.2) E(N)=(A2-AREA(N)*F)/A2
40  KCHNG(N)=0
   IF (ABS(E(N)).LT.ERROR) GO TO 50
   KCHNG(N)=1
   JCHNG=2
50  CONTINUE
C ** CHECKING FOR SATISFACTION OF ALL BOUNDARY CONDITIONS VIA INDICATOR
   IF (JCHNG.EQ.1) GO TO 70
C ** CHANGING VELOCITY DISTRIBUTION AT COMMENCING SURFACE
   DO 60 N=NST,N2
60  CALL VNEW(C,N)
C ** PRINTING OUT STATE OF CONVERGENCE
70  WRITE(6,100) ITRN,DV,F,TM,V(1,1),V(N2,1),(N+KCHNG(N),E(N),V(N,1),
   1N=NST,N2)
   RETURN
100 FORMAT(9H0ITRN NO 13,10X,3H0V=F5.1,3H F=E15.8,4H TM=E15.8,8H V(1,1
   1)=F7.1,9H V(N2,1)=F7.1,/(6(13,12,E10.2,F7.1)))
200 FORMAT(27H0THROAT POSITION CHANGED TO 14/(12(14,F7.1)))
END

```

SUBROUTINE SLCALC(K2,VMIN,VMAX)

C ** SUBROUTINE TO GENERATE (OR STACK) STREAMLINES

C

```

COMMON/BLKA/X(50,25),Y(50,25)/BLKB/AREA(50)/BLKC/V(50,25)
1/BLKD/RC(50,25)/BLKL/NPS/BLKM/NPN/BLKH/VAV(50,25)/BLKN/DM/BLKT/DN
2/BLKCC/N2,NST/BLKF/RO(100),VEL(100),NTAB/BLKW/SFCT
3/BLKZ/ITRN,IMF,ICYC,ITV/BLKQ/VSPEC,VSONIC/BLKI/VR(50,25),VST
4/BLKJJ/IRC,KRC,JRC
ID=1

```

C ** CALCULATING SLOPES AND CURVATURES ONCE FOR ALL IF KRC = 1

```

IF (ITRN.EQ.1.AND.KRC.EQ.1) GO TO 97
GO TO 99

```

```

97 DO 98 N=NST,N2
   DO 98 NN=2,NPN

```

```

98 CALL DYDXRC(N,NN,VR(N,NN))

```

C ** SUPPRESSING SLOPE AND CURVATURE CALCULATIONS IF JRC = 1

```

99 IF (JRC.EQ.1) GO TO 105

```

```

      DO 104 N=NST,N2
      AREA(N)=0.0
C **  SETTING THE GRID FLOW TYPE INDICATOR VR AT COMMENCING SURFACE
      IF(ITRN.GT.1) GO TO 100
      IF(V(N.1).LE.VSONIC)VR(N.1)=VSONIC-VST
      IF(V(N.1).GT.VSONIC)VR(N.1)=VSONIC+VST
      GO TO 104
100  IF(VR(N.1).LT.VSONIC.AND.V(N.1).GT.VSONIC+VST) GO TO 101
      IF(VR(N.1).GT.VSONIC.AND.V(N.1).LT.VSONIC-VST) GO TO 102
      GO TO 104
101  VR(N.1)=VSONIC+VST
      GO TO 103
102  VR(N.1)=VSONIC-VST
103  WRITE(6,13) N
C **  CALCULATING SLOPE AND CURVATURE AT COMMENCING SURFACE
104  CALL DYDXRC(N.1,VR(N.1))
105  CONTINUE
C **  GENERATING ALL STREAMLINES
      DO 6 NN=1,NPN
      M=NN+1
      DO 4 N=NST,N2
      V2=V(N,NN)
      EL1=EL(X(N,NN))
      DO 2 K=1,K2
      ELAV=.5*(EL1+EL(X(N,M)))
      C=.5*(V(N,NN)+V2)
      Z=DM/(FINDRO(C)*ELAV)
      V(N,M)=V(N,NN)-Z*SFCT/RC(N,NN)
C **  CHECKING THAT NEW VELOCITY DOES NOT EXCEED LIMITS SET
      IF(V(N,M).GT.VMIN.AND.V(N,M).LT.VMAX) GO TO 1
      ID=2
      IF(V(N,M).LE.VMIN) V(N,M)=VMIN
      IF(V(N,M).GE.VMAX) V(N,M)=VMAX
1    DN=Z/C
      CALL NEXTPT(N,NN)
C **  PROCEEDING TO NEXT POINT IF NON AVERAGE CALCULATION TYPE IS
C **  SPECIFIED OR IF THE ITERATIVE AVERAGE CALCULATION HAS CONVERGED
      IF(ABS((V2-V(N,M))/V2).LE.1.0E-5.OR.K2.EQ.1) GO TO 3
2    V2=V(N,M)
      WRITE(6,9) NN,N
      STOP
3    AREA(N)=AREA(N)+DN*ELAV
4    VAV(N,NN)=.5*(V(N,NN)+V2)
C **  SETTING GRID FLOW TYPE INDICATOR AND CALCULATING SLOPE AND
C **  CURVATURE ACCORDING TO CONTROL VARIABLES IRC AND JRC
      IF(NN.EQ.NPN.OR.IRC.EQ.0) GO TO 6
      DO 204 N=NST,N2
      IF(JRC.EQ.1) GO TO 204
      IF(ITRN.GT.1) GO TO 200
      IF(V(N,M).LE.VSONIC)VR(N,M)=VSONIC-VST
      IF(V(N,M).GT.VSONIC)VR(N,M)=VSONIC+VST
      GO TO 204
200  IF(VR(N,M).LT.VSONIC.AND.V(N,M).GT.VSONIC+VST) GO TO 201
      IF(VR(N,M).GT.VSONIC.AND.V(N,M).LT.VSONIC-VST) GO TO 202
      GO TO 204
201  VR(N,M)=VSONIC+VST
      GO TO 203
202  VR(N,M)=VSONIC-VST
203  WRITE(6,13) N,M

```

```

204 CALL DYDXRC(N,M,VR(N,M))
6  CONTINUE
   IF(ID.EQ.2) WRITE(6,12)
   RETURN
9  FORMAT(30H SLCALC SUBROUTINE LOOPING NN=13,3H N=13)
12 FORMAT(35H VELOCITY LIMITS EXCEEDED IN SLCALC )
13 FORMAT(38H CURVATURE CALCUTATION TYPE CHANGED N=14,4H M=14)
END

```

```

SUBROUTINE SUPMCH(K2,ISTP,VMIN,VMAX)
C ** SUBROUTINE TO MARCH OUT THE SUPERSONIC REGION SOLUTION
C
COMMON /BLKC/V(50,25)/BLKJ/E(50)/BLKL/NPS/BLKX/ERROR/BLKY/JCHNG
1/BLKDD/VCC(50,3),AI(50,15),N1(50),A(15,30),NSPI,NP/BLKCC/N2,NST
2/BLKFF/DFSUP,DFSUP/BLKZ/ITRN,IMF,ICYC,ITV
DIMENSION ER(50)
WRITE(6,10)
C ** SETTING THE VALUE OF THE FIRST NORMAL AND SETTING UP A LOOP TO DO
C ** EACH NORMAL SUCCESSIVELY, OBTAINING CONVERGENCE AT EACH ONE
N=N2+1
DO 3 NST=N,NPS
N2=NST
DO 2 ITRN=1,ISTP
C ** GENERATING ALL THE GRID POINTS ON ONE NORMAL
CALL SLCALC(K2,VMIN,VMAX)
C ** CHECKING THE BOUNDARY CONDITION AND MAKING THE CHANGE NECESSARY
CALL ENDPT(NST)
ER(ITRN)=E(NST)
JCHNG=1
IF(ABS(E(NST)).GT.ERROR) GO TO 1
WRITE(6,11) NST,ITRN,(ER(I),I=1,ITRN)
GO TO 3
1 V(NST,1)=V(NST,1)*(1+DFSUP*E(NST)*AI(NST,1))
2 JCHNG=2
WRITE(6,11) NST,ISTP,(ER(I),I=1,ISTP)
STOP
3 CONTINUE
RETURN
10 FORMAT(32H0SUPERSONIC REGION ERROR HISTORY )
11 FORMAT(4H0N =13,8H ITRN =13,9E11,3/5(18X,9E11,3/))
END

```

```

SUBROUTINE DYDXRC(N,M,V)
C ** SUBROUTINE TO FIND THE SLOPE AND CURVATURE AT A POINT USING A 3
C ** POINT PARABOLIC FORMULA
C
COMMON/BLKA/X(50,25),Y(50,25)/BLKD/R(50,25)/BLKE/S(50,25)/BLKL/NPS
1/BLKQ/VSPEC,VSONIC/BLKCC/N2,NST
C ** SETTING THE CENTER POINT J OF THE 3 TO BE USED ACCORDING TO THE
C ** LOCATION OF N IN THE GRID AND THE VELOCITY V
IF(N.GT.1) GO TO 1
J=2
GO TO 3
1 IF(V.GT.VSONIC.OR.N.EQ.N2) GO TO 2
J=N

```

```

      GO TO 3
      2 J=N-1
C ** SETTING UP DUMMY AXES THROUGH THE OUTER TWO POINTS AND FINDING THE
C ** PARABOLIC CONSTANTS A AND B
      3 THETA=ATAN((Y(J+1,M)-Y(J-1,M))/(X(J+1,M)-X(J-1,M)))
      ST=SIN(THETA)
      CT=COS(THETA)
      X1=(Y(J,M)-Y(J-1,M))*ST+(X(J,M)-X(J-1,M))*CT
      Y1=(Y(J,M)-Y(J-1,M))*CT-(X(J,M)-X(J-1,M))*ST
      X3=(X(J+1,M)-X(J-1,M))/CT
      A=Y1/(X1*(X1-X3))
      B=-A*X3
C ** FINDING THE RADIUS OF CURVATURE  $R=-1/2A$ 
      IF(ABS(A).LT.1.0E-10) A=1.0E-10
      R(N,M)=-1.0/(A+A)
C ** FINDING THE SLOPE AND TRANSFORMING IT BACK INTO THE ORIGINAL
C ** COORDINATE SYSTEM
      IF(N.EQ.1) S(N,M)=TAN(THETA+ATAN(B))
      IF(J.EQ.N) S(N,M)=TAN(THETA+ATAN(2.*A*X1+B))
      IF(J.EQ.N-1) S(N,M)=TAN(THETA+ATAN(2.*A*X3+B))
      RETURN
      END

```

```

      SUBROUTINE NEXTPT(N,NN)
C ** SUBROUTINE TO PLOT THE NEXT POINT ALONG NORMAL N FROM STREAMLINE
C NN
      COMMON/BLKE/DYDX(50,25)/BLKT/DN/BLKA/X(50,25),Y(50,25)
C ** DIFFERENTIATING BETWEEN POSITIVE, NEGATIVE AND ZERO SLOPE
      IF(DYDX(N,NN).GT.1.0E-5) GO TO 2
      IF(DYDX(N,NN).LT.-1.0E-5) GO TO 1
      X(N,NN+1)=X(N,NN)
      Y(N,NN+1)=Y(N,NN)+DN
      RETURN
      1 THETA=ATAN(-1.0/DYDX(N,NN))
      GO TO 3
      2 THETA=ATAN(-1.0/DYDX(N,NN))+3.1416
C ** FINDING THE NEW POINT N,NN+1
      3 X(N,NN+1)=X(N,NN)+DN*COS(THETA)
      Y(N,NN+1)=Y(N,NN)+DN*SIN(THETA)
      RETURN
      END

```

```

      SUBROUTINE ENDPT(N)
C ** SUBROUTINE TO FIND THE DISTANCE BETWEEN THE FINAL STREAMLINE
C ** (NPN+1) AND THE TERMINATING SURFACE AT NORMAL N
C
      COMMON/BLKA/X(50,25),Y(50,25)/BLKB/AREA(50)/BLKG/DA(50)
      1/BLKI/X2(70),Y2(70)/NPB2/BLKJ/E(50)/BLKK/A2/BLKM/NPN
C ** FINDING THE NEAREST 3 TERMINATING SURFACE POINTS
      DMIN=1.0E20
      DO 1 M=1,NPB2
      D=SQRT((X(N,NPN+1)-X2(M))**2+(Y(N,NPN+1)-Y2(M))**2)
      IF(D.LT.DMIN) GO TO 1
      L=M-1
      GO TO 2

```

```

1 DMIN=D
2 IF(L.GT.1.AND.L.LT.NPB2) GO TO 3
  WRITE(6,20) N
  IF(L.EQ.1) L=2
  IF(L.GT.2) L=NPB2-1
C
C ** A PARABOLA ON DUMMY AXES IS DRAWN THROUGH TERMINATING SURFACE
C ** POINTS L-1 AND L+1. POINT L IS TRANSFORMED TO X1,Y1, POINT L+1 TO
C ** X3,0 AND THE STREAMLINE POINT TO XP,YP. THE PARABOLIC CONSTANTS
C ** A AND B ARE FOUND. THE DISTANCE REQUIRED IS THE DISTANCE NORMAL TO
C ** THE X AXIS BETWEEN THE NORMAL POINT AND THE VALUE OF THE PARABOLA
C ** AT XP. THE DISTANCE IS CORRECTED FOR FLOW HEIGHT EL AND NON
C ** DIMENSIONALISED USING THE FLOW AREA
C
3 THETA=ATAN((Y2(L+1)-Y2(L-1))/(X2(L+1)-X2(L-1)))
  ST=SIN(THETA)
  CT=COS(THETA)
  X1=(Y2(L)-Y2(L-1))*ST+(X2(L)-X2(L-1))*CT
  Y1=(Y2(L)-Y2(L-1))*CT-(X2(L)-X2(L-1))*ST
  X3=(X2(L+1)-X2(L-1))/CT
  XP=(Y(N,NPN+1)-Y2(L-1))*ST+(X(N,NPN+1)-X2(L-1))*CT
  YP=(Y(N,NPN+1)-Y2(L-1))*CT-(X(N,NPN+1)-X2(L-1))*ST
  A=Y1/(X1*(X1-X3))
  B=-A*X3
  DA(N)=(XP*(A*XP+B)-YP)*EL(X(N,NPN+1))
  A2=AREA(N)+DA(N)
  E(N)=DA(N)/AREA(N)
  RETURN
20 FORMAT(24H DISORDER IN ENDPT AT N=14)
  END

FUNCTION FINDRO(V)
C ** SUBROUTINE TO FIND THE FLUID DENSITY AT INPUT VELOCITY V
C
COMMON/BLKF/RO(100),VEL(100),NTAB/ BLKS/DV
C ** VELOCITY IS CHECKED TO FALL WITHIN TABULAR LIMITS
IF(V.LT.0.0.OR.V.GE.VEL(NTAB)) GO TO 1
C ** NEAREST TABULAR VELOCITY TO V IS FOUND
N=IFIX(V/DV)+1
IF(V.LT.DV) N=N+1
C ** DENSITY FOUND BY PARABOLIC INTERPOLATION BETWEEN 3 TABULAR VALUES
FINDRO=PAR (VEL(N-1),VEL(N),VEL(N+1),V,RO(N-1),RO(N),RO(N+1))
RETURN
1 WRITE(6,2) V
2 FORMAT(3H0V=E12.5,53HOUTSIDE LIMITS OF TABULATED DATA IN FINDRO SU
  IBROUTINE )
  STOP
  END

SUBROUTINE FINDY
C ** SUBROUTINE TO FIND THE COMMENCING SURFACE Y COORDINATES
C
COMMON /BLKA/X(50,25),Y(50,25)/BLKL/NPS/BLKHH/X1(70),Y1(70),NX1
DIMENSION XP(11),YP(11)
C ** FINDING THE ARRAY POINTS NEAREST THE GRID POINT. GENERATING A

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C ** SYMMETRICAL DUMMY ARRAY XP,YP OF SIZE NPEX FOR INTERPOLATION TO
C ** FIND Y(N,1)
DO 5 N=1,NPS
  DMIN=1.0E20
  NPEX=11
  DO 2 M=1,NX1
    D=ABS(X(N,1)-X1(M))
    IF(D.LT.DMIN) GO TO 2
  DO 1 J=1,4
    L=M-NPEX/2-1
    IF(L.GE.1.AND.L.LE.NX1-NPEX+1) GO TO 3
1  NPEX=NPEX-2
    WRITE(6,10) N,L
    IF(N.LT.NPS/2) L=1
    IF(N.GE.NPS/2) L=NX1-2
    GO TO 3
2  DMIN=D
3  DO 4 M=1,NPEX
    XP(M)=X1(L)
    YP(M)=Y1(L)
4  L=L+1
5  Y(N,1)=EXTRA(YP,XP,X(N,1),NPEX)
  RETURN
10 FORMAT(30H0ARRAY X1 EXCEEDED IN FINDY N=I4,3H L=I4)
END

```

```

FUNCTION FINDV(TMA,N)
C ** SUBROUTINE TO FIND THE VELOCITY WHEN THE SPECIFIC MASS FLOW TMA
C ** (MASS FLOW/AREA) IS SPECIFIED
C
COMMON/BLKF/RO(100),VEL(100),NTAB/BLKP/JSONIC/BLKO/NTHRT
1/BLKQ/VSPEC,VSONIC
C ** SETTING THE INITIAL VELOCITIES FOR THE HOMING PROCEDURE ACCORDING T
C ** WHETHER THE GRID POINT N SHOULD BE SUBSONIC OR SUPERSONIC
V1=VEL(NTAB)
IF(JSONIC.GE.2.OR.(JSONIC.EQ.1.AND.N.LT.NTHRT)) V1=0.0
V3=VSONIC
C ** EXECUTING THE HOMING PROCEDURE
DO 3 K=1,50
  FINDV=.5*(V3+V1)
  IF(ABS(V3-V1).LE.1.0) RETURN
  Y2=FINDV*FINDRO(FINDV)
  IF(TMA-Y2) 1,3,2
1  V3=FINDV
  GO TO 3
2  V1=FINDV
3  CONTINUE
  WRITE(6,4) N
  STOP
4  FORMAT(39H0FINDV SUBROUTINE NOT CONVERGING AT N = I4)
END

```

```

SUBROUTINE VELDIS(JSONIC,JREAD,NSP)
C ** SUBROUTINE TO FIND THE INITIAL VELOCITY DISTRIBUTION, MASS FLOW
C ** AND 1-M**2 BY ONE DIMENSIONAL ESTIMATE OR BY INPUT ACCORDING TO
C ** JREAD SPECIFICATION
C
COMMON/BLKA/X(50,25),Y(50,25)/BLKC/V(50,25)/BLKG/DA(50)/BLKL/NPS
1/BLKM/NPN/BLKN/DM/BLKO/NTHRT/BLKEE/DA1,DANPS/BLKR/TM/BLKB/AREA(50)
2/BLKE/DYDX(50,25)/BLKT/DN/BLKQ/VSPEC,VSONIC
3/BLKDD/VCC(50,3),AI(50,15),NI(50),A(15,30),NSPI,NP/BLKH/VAV(50,25)
C ** FINDING THE ONE DIMENSIONAL FLOW AREAS USING ENDPT
DO 1 N =1,NPS
DYDX(N,NPN+1)=DYDX(N,1)
X(N,NPN+1)=X(N,1)
Y(N,NPN+1)=Y(N,1)
CALL ENDPT(N)
1 AREA(N)=DA(N)
DA1=DA(1)
DANPS=DA(NPS)
C ** FINDING THE MASS FLOW ACCORDING TO THE FLOW TYPE SPECIFICATION
C ** JSONIC. SUBSONIC MASS FLOW IS FOUND ACCORDING TO WIDTH AT NORMAL NSP
IF(JSONIC.LE.2) TM=VSONIC*FINDRO(VSONIC)*DA(NTHRT)
IF(JSONIC.EQ.0) TM=VSPEC*FINDRO(VSPEC)*DA(NTHRT)
IF(JSONIC.EQ.3) TM=VSPEC*FINDRO(VSPEC)*DA(NSP)
C ** FINDING THE ONE DIMENSIONAL VELOCITIES
DO 3 N=1,NPS
IF(N.EQ.NTHRT.AND.JSONIC.LE.2) GO TO 2
V(N,1)=FINDV(TM/DA(N),N)
GO TO 3
2 V(N,1)=VSONIC
IF(JSONIC.EQ.0.AND.N.LE.NTHRT) V(N,1)=VSPEC
3 VCC(N,1)=1.+(FINDRO(V(N,1)+2.5)-FINDRO(V(N,1)-2.5))*2*V(N,1)/
1FINDRO(V(N,1))
C ** OVERWRITING THE CALCULATED VALUES BY INPUT ACCORDING TO JREAD SPEC
IF(JREAD.EQ.1) GO TO 5
IF(JREAD.LE.3) READ(5,18) (VCC(N,1),N=1,NPS)
IF(JREAD.GE.3) READ(5,19) TM,(V(N,1),N=1,NPS)
5 DO 6 N=1,NPS
C ** INITIALISING ARRAYS AND PRINTING RESULTS
DO 6 NN=1,25
VAV(N,NN)=V(N,1)
6 V(N,NN)=V(N,1)
WRITE(6,21) TM,(N,V(N,1),N=1,NPS)
RETURN
18 FORMAT(18F4.2)
19 FORMAT(E15.8/(10F7.1))
20 FORMAT(29H0VELDIS SUBROUTINE LOOPING N=13)
21 FORMAT(34H1INITIAL VELOCITY DISTRIBUTION TM=E15.8/(10(14,F8.1)))
END

```

```

SUBROUTINE NUGRID(NRD,NRR)
C ** SUBROUTINE TO READ IN NEW GRID DATA AND GENERATE THE NEW GRID
C ** INITIAL VELOCITY ASSUMPTION BY INTERPOLATION FROM THE OLD
C
COMMON /BLKA/X(50,25),Y(50,25)/BLKB/AREA(50)/BLKC/V(50,25)
1/BLKD/RC(50,25)/BLKE/DYDX(50,25)/BLKH/VAV(50,25)/BLKL/NPS
2/BLKM/NPN/BLKO/NTHRT/BLKP/JSONIC/BLKBB/NSP/BLKFF/DFSUB,DFSUP
3/BLKDD/VCC(50,3),AI(50,15),ND(50),AM(15,30),NSPI,NP/BLKX/ERROR

```

```

4/BLKW/SFCT/BLKJJ/IRC*KRC/JRC/BLK11/VR(50,25)*VST
  DIMENSION C1(11)*C2(11)*C3(11)*C4(11)*C5(11)
C **  READING NEW GRID DATA
      IF(NRD.NE.0) READ (5,20) NPS2,NTD,NPN,NP,IRC,KRC,JRC,
1ERROR,DFSUB,DFSUP,SFCT
      IF(NRR.NE.0) WRITE(6,21) NPS2,NTD,NPN,NP,IRC,KRC,JRC,
1ERROR,DFSUB,DFSUP,SFCT
      IF(NPS2.EQ.NPS.OR.NRR.EQ.0) RETURN
C **  INCREMENTING THROAT NORMAL BY NTD
      NT2=NTD+NTD
      N1=1
      M1=1
C **  FOR CHOKED FLOWS, POSITION OF THROAT IS RETAINED CONSTANT
      IF(JSONIC.EQ.0.OR.JSONIC.EQ.3) GO TO 1
      NZ=2
      N2=NT2
      M2=NTD
      GO TO 2
1  NZ=1
      N2=NPS2
      M2=NPS
C **  FINDING THE NEW GRID COMMENCING SURFACE X COORD, VELOCITY, 1-M**2
C **  AND FLOW AREA BY INTERPOLATION FROM THE OLD DISTRIBUTION
2  DO 10 K=1,NZ
      NA=N1+1
      NB=N2-1
      DO 8 N=NA,NB
      M=FLOAT((N-N1)*(M2-M1))/FLOAT(N2-N1)+FLOAT(M1)+.5
      NPEX=11
      DO 3 J=1,5
      MS=M-NPEX/2
      IF(MS.GE.1.AND.MS.LE.NPS-NPEX+1) GO TO 4
3  NPEX=NPEX-2
      NPEX=3
      IF(M.EQ.M1) MS=M1
      IF(M.EQ.M2) MS=M2-2
4  DO 5 J=1,NPEX
      C1(J)=X(MS,1)
      C2(J)=FLOAT(MS-M1)/FLOAT(M2-M1)
      C3(J)=V(MS,1)
      C4(J)=VCC(MS,1)
      C5(J)=AREA(MS)
5  MS=MS+1
      X(N,2)=EXTRA(C1,C2,FLOAT(N-N1)/FLOAT(N2-N1)*NPEX)
      Y(N,2)=EXTRA(C3,C1,X(N,2)*NPEX)
      VCC(N,2)=EXTRA(C4,C1,X(N,2)*NPEX)
      VCC(N,3)=EXTRA(C5,C1,X(N,2)*NPEX)
      DO 7 L=2,NPN
      MS=MS-NPEX
C **  FINDING THE ONCE FOR ALL GRID FLOW TYPE INDICATORS VR BY
C **  INTERPOLATION FROM THE OLD VELOCITY DISTRIBUTION
      DO 6 J=1,NPEX
      C3(J)=V(MS,L)
6  MS=MS+1
      VR(N,L)=EXTRA(C3,C1,X(N,2)*NPEX)
8  CONTINUE
      DO 9 M=2,NPN
9  VR(N2,M)=V(M2,M)
      X(N2,2)=X(M2,1)

```

```

      Y(N2*2)=V(M2*1)
      VCC(N2*2)=VCC(M2*1)
      VCC(N2*3)=AREA(M2)
      N2=NPS2
      N1=NT2
      M2=NPS
10  M1=NTHRT
      NPS=NPS2
      NTHRT=NT2
      DO 11 N=2,NPS
        X(N*1)=X(N*2)
        V(N*1)=Y(N*2)
        VR(N*1)=V(N*1)
        VCC(N*1)=VCC(N*2)
11  AREA(N)=VCC(N*3)
C ** FINDING THE NEW GRID COMMENCING SURFACE Y COORDINATE AND
C ** CURVATURES
      CALL FINDY
      DO 12 N=1,NPS
        CALL DYDXRC(N*1,V(N*1))
        DO 12 NN=3,25
          X(N,NN)=X(N*1)
          Y(N,NN)=Y(N*1)
          V(N,NN)=V(N*1)
          VAV(N,NN)=V(N*1)
12  RC(N,NN)=RC(N*1)
      WRITE(6,22) (N,X(N*1),Y(N*1),V(N*1),RC(N*1),DYDX(N*1),AREA(N),
1  VCC(N*1),N=1,NPS)
      IF(JRC.EQ.0) RETURN
      DO 13 M=1,NPN
13  WRITE(6,23) M,(N,VR(N,M),N=2,NPS)
      RETURN
20  FORMAT(4I4,3I2,E8.1,3F5.2)
21  FORMAT(19H1NEW GRID DATA NPS=I4,5H NTD=I4,5H NPN=I4,4H NP=I4,
15H IRC=I2,5H KRC=I2,5H JRC=I2,7H ERROR=E8.1,7H DFSUB=F5.2,
27H DFSUP=F5.2,6H SFCT=F5.2)
22  FORMAT(4H0 N14X,1HX14X,1HY14X,1HV13X,2HRC11X,4HDYDX11X,4HAREA9X,
16H1-M**2/ (I4,7E15.5 ))
23  FORMAT(3H M=I4,3X,12(I3,F7.1)/(6X,12(I3,F7.1)))
      END

      SUBROUTINE KUTTA(ISTP,K2,VMIN,VMAX)
C ** SUBROUTINE TO APPLY THE KUTTA - JOUKOWSKI APPROXIMATION AT THE T E
C
      COMMON/BLKY/JCHNG/BLKCC/N2,NST/BLKC/V(50,25)/BLKL/NPS/BLKM/NPN
1/BLKJ/E(50)/BLKX/ERROR/BLKZ/ITRN,11,12,13/BLKA/X(50,25),Y(50,25)
2/BLKU/XU,XD,DLPL1,DLXDXU
      DIMENSION KCHNG(50),C(50)
      DATA C/50*1.0/
C ** SETTING UP THE ITERATIVE LOOP
      DO 50 ITRN=1,ISTP
        JCHNG=1
        K=NPN+1
C
C ** FINDING THE DEVIATION FROM THE SPLIT BOUNDARY CONDITION I.E
C ** TERMINATING SURFACE POINTS WITHIN THE CASCADE OBEY THE GEOMETRIC
C ** CONDITION AND THOSE DOWNSTREAM THE CONDITION THAT VELOCITIES AT

```

```

C ** THE SAME X COORD MUST BE THE SAME
C
DO 20 N=NST,N2
  IF(X(N,K).GT.XD) GO TO 15
  CALL ENDPT(N)
  GO TO 20
15 DO 16 M=NST,N2
  J=M
  IF(X(J,1).GE.X(N,K)) GO TO 17
16 CONTINUE
  E(N)=1.0-V(NPS,1)/V(N,K)
  GO TO 20
17 E(N)=1.0-PAR(X(J-2,1),X(J-1,1),X(J,1),X(N,K),
1      V(J-2,1),V(J-1,1),V(J,1)) /V(N,K)
20 CONTINUE
C ** CHECKING THE BOUNDARY CONDITION
DO 30 N=NST,N2
  KCHNG(N)=0
  IF(ABS(E(N)).LT.ERROR) GO TO 30
  KCHNG(N)=1
  JCHNG=2
30 CONTINUE
  IF(JCHNG.EQ.1) RETURN
C ** CHANGING THE VELOCITY DISTRIBUTION
DO 40 N=NST,N2
40 CALL VNEW(C,N)
C ** PRINTING OUT THE STATE OF THE BOUNDARY CONDITION AND REGENERATING
C ** THE STREAMLINES
WRITE(6,100) ITRN,(N,KCHNG(N),E(N),V(N,1),V(N,NPN+1),N=NST,N2)
CALL SLCALC(K2,VMIN,VMAX)
50 CONTINUE
WRITE(6,110)
STOP
100 FORMAT(15HOKUTTA ITRN NO I4/(4(I7,I2,E10.2,2F7.1)))
110 FORMAT(35HONO CONVERGENCE IN KUTTA SUBROUTINE)
END

SUBROUTINE RESULT(NOUTP,KDK,N3,N2)
C ** SUBROUTINE FOR OUTPUT OF VELOCITY, MACH NO, AND COORDINATES AT EACH
C ** GRID POINT AND FOR SURFACE PRESSURE DISTRIBUTION
C
COMMON /BLKA/X(50,25),Y(50,25)/BLKB/AREA(50)/BLKC/V(50,25)
1/BLKH/EM(50,25)/BLKL/NPS/BLKM/NPN/BLKR/TM/BLKY/JCHNG/BLKJ/CP(50)
2/BLKZ/ITRN,IMF,ICYC,ITV/BLKDD/VCC(50,3),AI(50,15),NI(50),AM(15,30)
3,NSPI,NP/BLKKK/TITLE(90)/BLKO/NTHRT
  IF(JCHNG.EQ.1) WRITE(6,8) NPS,NPN,TM,ITRN,NTHRT
  IF(JCHNG.EQ.2) WRITE(6,9) NPS,NPN,TM,ITRN,NTHRT
  NPN1=NPN+1
  DO 2 N=1,NPS
  Z=0.0
  DO 1 M=1,NPN1
  EM(N,M)=SQRT(ABS((FINDRO(V(N,M)+5.))-FINDRO(V(N,M)-5.))
1      *.1*V(N,M)/FINDRO(V(N,M))))
  IF(M.NE.1) Z=Z+.5*(EM(N,M)+EM(N,M-1))
1 CONTINUE
  VCC(N,2)=Z/FLOAT(NPN)
2 VCC(N,1)=1.0-VCC(N,2)**2

```

```

NM=0
ND=1
IF(SFCT.GT.0.999) CALL RESULT(NGRDS-NGS,KDK*1,NPS)
IF(NOUTP.GT.0) ND=NPN
DO 3 M=1,NPN1,ND
WRITE(6,10) NM,(N,V(N,M),EM(N,M),X(N,M),Y(N,M),N=1,NPS)
3 NM=NM+ND
WRITE(6,11) (N,VCC(N,1),N=1,NPS)
WRITE(6,14) (N,VCC(N,2),N=1,NPS)
Z=1.429*(1.0+0.2*EM(1,1)**2)**3.5/EM(1,1)**2
CPME1=Z*(1.0-1.0/1.2**3.5)
WRITE(6,15) CPME1
DO 5 M=1,NPN1,NPN
DO 4 N=1,NPS
4 CP(N)=Z*(1.0-1.0/(1.0+0.2*EM(N,M)**2)**3.5)
5 WRITE(6,16) (N,X(N,M),CP(N),N=1,NPS)
IF(KDK.GT.0) RETURN
NPTS=N2-N3+1
WRITE(6,12) TITLE,NPTS,NPN1
WRITE(6,13) ((X(N,M),Y(N,M),N=N3,N2),M=1,NPN1)
WRITE(6,13) ((EM(N,M),N=N3,N2),M=1,NPN1)
RETURN
8 FORMAT(18HOSOLUTION FOR NPS=I4.5H NPN=I4.4H TM=E15.8.6H ITRN=I4.
17H NTHRT=I4)
9 FORMAT(24 HONO CONVERGENCE FOR NPS=I4.5H NPN=I4.4H TM=E15.8.
16H ITRN=I4.7H NTHRT=I4)
10 FORMAT(15HOSTREAMLINE NO I2/4(3H N6X.1HV6X.1HM7X.1HX7X.1HY) /
1(4(I3.F7.1.F7.4.2F8.4)))
11 FORMAT(7H01-M**2// (10(I4.F9.4)))
12 FORMAT(26H CARD OUTPUT FORMAT NEEDED / 5(18A4/).2I4)
13 FORMAT(26H CARD OUTPUT FORMAT NEEDED / (10F7.4))
14 FORMAT(16HOAVERAGE MACH NO // (10(I4.F9.4)))
15 FORMAT(44HOSURFACE PRESSURE DISTRIBUTION: CP FOR M=1 = F6.3 /
17(4X.1HN6X.1HX4X.2HCP))
16 FORMAT(7(I5.F7.3.F6.3))
END

```

FUNCTION EXTRA(YP,XP,X,M)

C ** INTERPOLATION SUBROUTINE USING THE SPLINE FIT METHOD
C

```

DIMENSION XP(11),YP(11),D2Y(11),A(11,11),B(11),C(11),D(11),E(11)
DIMENSION DY(11),F(11)
MM=M-1
N=MM/2+1
DO 101 I=1,MM
F(I+1)=XP(I+1)-XP(I)
IF(ABS(F(I+1)).LT.1.0E-10) F(I+1)=1.0E-10
DY(I)=(YP(I+1)-YP(I))/F(I+1)
101 CONTINUE
DY(M)=DY(MM)
DO 102 J=1,M
B(J)=0.0
DO 102 K=1,M
A(K,J)=0.0
102 CONTINUE
DO 103 J=2,MM
A(J-1,J) =F(J)/6.0

```

```

      A(J,J)=(F(J)+F(J+1))/3.0
      A(J,J-1)=F(J)/6.0
      B(J)=DY(J)-DY(J-1)
103  CONTINUE
      A(1,1)=F(2)/3.0
      A(M,M)=F(M)/3.0
      A(M,MM)=F(M)/6.0
      A(MM,M)=F(M)/6.0
      C(1)=A(1,1)
      D(1)=A(1,2)/C(1)
      E(1)=B(1)/C(1)
      DO 104 J=2,MM
        C(J)=A(J,J)-A(J,J-1)*D(J-1)
        D(J)=A(J,J+1)/C(J)
        E(J)=(B(J)-A(J,J-1)*E(J-1))/C(J)
104  CONTINUE
      C(M)=A(M,M)-A(M,MM)*B(MM)
      D2Y(M)=(B(M)-A(M,MM)*E(MM))/C(M)
      DO 105 J=1,MM
        K=M-J
        D2Y(K)=E(K)-D(K)*D2Y(K+1)
105  CONTINUE
      NP =MM/2+1
      H=X-XP(N-1)
      G=XP(N)-X
      R=G*(D2Y(N-1)*(G*G/F(N)-F(N))/6.0+YP(N-1)/F(N))
      S=H*( D2Y(N) *(H*H/F(N)-F(N))/6.0+ YP(N) /F(N))
      EXTRA=R+S
      RETURN
      END

```

```

      FUNCTIONPAR(X1,X2,X3,X,Y1,Y2,Y3)
C **  INTERPOLATION SUBROUTINE USING A SIMPLE 3 POINT PARABOLIC FORMULA
C
      XX1=X1*X1
      XX2=X2*X2
      XX3=X3*X3
      XX=X*X
      A=((Y2-Y3)*(X2-X1)-(Y2-Y1)*(X2-X3))/((XX1-XX2)*(X2-X3)-(XX3-XX2)*
1 (X2-X1))
      B=(Y2-Y3+A*(XX3-XX2))/(X2-X3)
      C=Y1-A*XX1-B*X1
      PAR=A*XX+B*X+C
      RETURN
      END

```

```

      FUNCTION EL(X)
C A SUPROGRAM FOR FINDING THE FLOW PASSAGE HEIGHT EL USING A LINEAR
C HEIGHT VARIATION BETWEEN XU AND XD
C
      COMMON/BLKU/XU,XD,DLPL1,DLXDXU
      IF(X,GE,XU,AND,X,LE,XD) GO TO 1
      IF(X,LT,XU) GO TO 2
      EL=DLPL1
      RETURN

```

```

1 EL=1.0+DLXDXU*(X-XU)
  RETURN
2 EL=1.0
  RETURN
  END

```

SUBROUTINE VCCONS(NGS)

C ** SUBROUTINE TO GENERATE VELOCITY CORRECTION CONSTANTS USING
C ** CONVERGENCE THEORY

C

```

COMMON /BLKA/X(50,25),Y(50,25)/BLKB/AREA(50)/BLKL/NPS/BLKM/NPN
1/BLKO/NTHRT/BLKP/JSONIC/BLKDD/VCC(50,3),AI(50,15),N1(50),A(15,30)
2,NSPI,NP/BLKW/SFCT/BLKJJ/IRC,KRC,JRC

```

C ** $VCC(N,1) = 1 - M^{**2}$, $VCC(N,2) = K$, $VCC(N,3) = H2/(H1+H2)$ FOR

C ** SUBSONIC FLOW $VCC(N,3) = H1/H2$ FOR SUPERSONIC FLOW

C ** IF CURVATURE CALCULATION IS SUPRESSED IT IS EQUIVALENT TO SFCT = 0
SF=SFCT

```
IF(IRC.EQ.0) SF=0.0
```

C ** CALCULATING THE CONSTANT K AND GRID SPACING CONSTANT H
FN=NPN**2

```
DO 4 N=2,NPS
```

```
H1=SQRT((Y(N,1)-Y(N-1,1))**2+(X(N,1)-X(N-1,1))**2)
```

```
IF(JSONIC.LE.1.AND.N.GT.NTHRT) GO TO 3
```

```
IF(N.EQ.NPS) GO TO 2
```

```
H2=SQRT((Y(N+1,1)-Y(N,1))**2+(X(N+1,1)-X(N,1))**2)
```

```
2 VCC(N,2)=2.0*VCC(N,1)*AREA(N)**2/(H1*H2*FN)*SF
```

```
VCC(N,3)=H2/(H1+H2)
```

```
GO TO 4
```

```
3 H2=SQRT((Y(N-1,1)-Y(N-2,1))**2+(X(N-1,1)-X(N-2,1))**2)
```

```
VCC(N,2)=-2.0*AREA(N)**2*VCC(N,1)/(H1*(H1+H2)*FN)*SF
```

```
VCC(N,3)=H1/H2
```

```
4 CONTINUE
```

```
VCC(1,1)=VCC(2,1)
```

```
IF(NGS.EQ.1) WRITE(6,20) (N,(VCC(N,M),M=1,3),N=2,NPS)
```

```
IF(JSONIC.EQ.0) GO TO 8
```

C ** FINDING THE VALUE N1(N) OF THE FIRST NORMAL TO BE USED FOR

C ** VELOCITY CORRECTION AT NORMAL N

```
NC=(NP-2)/2
```

```
NPS1=NPS-1
```

```
DO 7 N=2,NPS1
```

```
N1(N)=N-NC
```

```
IF(N-1.LE.NC) N1(N)=2
```

```
IF(NPS-N.LE.NC) N1(N)=NPS-2*NC-1
```

```
IF(JSONIC.GE.2) GO TO 6
```

```
IF(NTHRT-N.LE.NC.AND.N.LT.NTHRT) N1(N)=NTHRT-2*NC-1
```

```
IF(N.GE.NTHRT) GO TO 7
```

C ** GENERATING THE SUBSONIC REGION CONSTANTS

```
6 CALL ADPT(N,NGS)
```

```
7 CONTINUE
```

```
IF(NGS.EQ.1) WRITE(6,23) (N,N1(N),N=2,NPS1)
```

```
IF(JSONIC.GE.2) RETURN
```

C ** GENERATING THE SUPERSONIC REGION CONSTANTS

```
IF(NSPI.GE.2) CALL SUPCON(NGS)
```

```
8 M=NTHRT+NSPI
```

```
IF(M.EQ.NPS) RETURN
```

C ** GENERATING THE SUPERSONIC REGION MARCHING CONVERGENCE CONSTANTS

```
DO 9 N=M,NPS
```

```

9 AI(N*1)=CKN(VCC(N*1)*VCC(N*2)*NPN)
  IF(NGS.EQ.1) WRITE(6,24) (N*AI(N*1),N=2,NPS)
  RETURN
20 FORMAT(30H0VELOCITY CORRECTION CONSTANTS //3(4H   N6X,6H1-M**211X,
  11HK4X,8HVCC(N*3))/(3(14,3E12,4)))
23 FORMAT(13H0VALUES OF N1/(17(2I3,1H)))
24 FORMAT(19H0CONSTANTS FROM CKN /(8(13,E12,4)))
  END

```

```

      SUBROUTINE ADPT(NM,NGS)
C ** SUBROUTINE TO GENERATE THE SUBSONIC REGION CONVERGENCE CONSTANTS
C ** FOR THE CALCULATION METHOD USING THE EULER TYPE METHOD
C
      COMMON /BLKM/NPN/BLKDD/V(50,3)*AI(50,15)*N1(50)*A(15,30)*NSP1*NP
      1/BLKGG/C(289,3)
C ** VCC(N*1) = 1-M**2 , VCC(N*2) = K , VCC(N*3) = H2/(H1+H2)
      NP1=NP-1
      NP2=NP-2
      NPNP=NP*NP
C ** INITIALISING ARRAY C TO ZERO
      DO 1 N=1,3
      DO 1 M=1,NPNP
      1 C(M,N)=0.0
C ** SETTING INITIAL VALUES OF ARRAY C FOR N=1 AND N=2
      DO 2 M=2,NP1
      J=NP*(M-1)+M
      C(J,1)=-1.0
      2 C(J,2)=-2.0
C ** EVALUATING THE SUBSONIC RECURRENCE RELATIONSHIP SEQUENTIALLY
C ** FROM N=3 TO N=NPN
      DO 4 N=3,NPN
      DO 3 M=2,NP1
      LL=N1(NM)+M-1
      DO 3 I=2,NP1
      J=NP*(M-1)+I
      K=NP*(M-2)+I
      L=NP*M+I
      3 C(J,3)=2.0*C(J,2)-(1.0-V(LL,2))*C(J,1)-
      1 V(LL,2)*(V(LL,3)*C(K,1)+(1.0-V(LL,3))*C(L,1))
      DO 4 M=1,NPNP
      C(M,1)=C(M,2)
      4 C(M,2)=C(M,3)
C ** CONVERTING ARRAY C INTO A SQUARE MATRIX A
      DO 5 K=1,NP2
      DO 5 J=1,NP2
      L=NP*K+J+1
      5 A(K,J)=C(L,3)
C ** INVERTING MATRIX A
      CALL SPNIST(A,NP2,15,ISIG)
C ** EXTRACTING RELAVANT ROW FOR NORMAL N AND STORING COEFFTS IN AI
      L=NM-N1(NM)+1
      DO 6 N=1,NP2
      6 AI(NM,N)=A(L,N)
      IF(NGS.EQ.1) WRITE(6,7) NM,(AI(NM,N),N=1,NP2)
      7 FORMAT(4H AI(12,6H,N) 1P13E9,2/12X,13E9,2)
      RETURN
      END

```

```

SUBROUTINE ADPT(NM,NGS)
C ** SUBROUTINE TO GENERATE THE SUBSONIC REGION CONVERGENCE CONSTANTS
C ** FOR THE CALCULATION METHOD USING THE RUNGE KUTTA TYPE METHOD
C
COMMON /BLKM/NPN/BLKDD/V(50,3)*A1(50,15)*N1(50)*A(15,30)*NSPI*NP
1/BLKGG/C(289,3)
NP1=NP-1
NP2=NP-2
NPNP=NP*NP
DO 1 N=1,3
DO 1 M=1,NPNP
1 C(M,N)=0.0
DO 2 M=2,NP1
I=NP*(M-1)+M-1
J=NP*(M-1)+M
K=NP*(M-1)+M+1
L=N1(NM)+M-1
C(J,1)=-1.0
IF(M.GT.2) C(I,2)=V(L,2)/2.0*V(L,3)
IF(M.LT.NP1) C(K,2)=V(L,2)/2.0*(1.0-V(L,3))
2 C(J,2)=-2.0-V(L,2)/2.0
DO 4 N=3,NPN
DO 3 M=2,NP1
LL=N1(NM)+M-1
DO 3 I=2,NP1
J=NP*(M-1)+I
K=NP*(M-2)+I
L=NP*M+I
3 C(J,3)=(2.0+V(LL,2)/2.0)*C(J,2)-(1.0-V(LL,2)/2.0)*C(J,1)-V(LL,2)/2.0*
1(V(LL,3)*(C(K,1)+C(K,2))+(1.0-V(LL,3))*(C(L,1)+C(L,2)))
DO 4 M=1,NPNP
C(M,1)=C(M,2)
4 C(M,2)=C(M,3)
DO 5 K=1,NP2
DO 5 J=1,NP2
L=NP*K+J+1
5 A(K,J)=C(L,3)
CALL SPNIST(A,NP2,15,ISIG)
L=NM-N1(NM)+1
DO 6 N=1,NP2
6 A1(NM,N)=A(L,N)
IF(NGS.EQ.1) WRITE(6,7) NM,(A1(NM,N),N=1,NP2)
7 FORMAT(4H A1(12,6H,N) 1P13E9.2/12X,13E9.2)
RETURN
END

```

```

SUBROUTINE SUPCON(NGS)
C ** SUBROUTINE TO GENERATE SUPERSONIC REGION BLOCK CONVERGENCE
3 ** CONSTANTS USING THE EULER TYPE METHOD
C
COMMON /BLKM/NPN/BLK0/NTHRT/BLKDD/V(50,3)*A1(50,15)*N1(50)*A(15,30
1)*NSPI*NDUM/BLKGG/C(289,3)
C ** VCC(N,1) = 1-M**2 * VCC(N,2) = K * VCC(N,3) = H1/H2
NP=NSPI+2

```

```

      NPNP=NP**2
C **  INITIALISING ARRAY C TO ZERO
      DO 1 N=1,3
      DO 1 M=1,NPNP
1 C(M,N)=0.0
C **  SETTING INITIAL VALUES OF ARRAY C FOR N=1 AND N=2
      DO 2 M=3,NP
      J=NP*(M-1)+M
      C(J,1)=-1.0
2 C(J,2)=-2.0
C **  EVALUATING THE SUPERSONIC RECURRENCE RELATIONSHIP SEQUENTIALLY
C **  FROM N=3 TO N=NPN
      DO 4 N=3,NPN
      DO 3 M=3,NP
      NM=NTHRT+M-2
      DO 3 I=3,M
      J=NP*(M-1)+I
      K=NP*(M-2)+I
      L=NP*(M-3)+I
3 C(J,3)=2.0*C(J,2)-((1.0-V(NM,2))*C(J,1)-
1 V(NM,2)*((1.0+V(NM,3))*C(K,1)-V(NM,3)*C(L,1))
      DO 4 M=1,NPNP
      C(M,1)=C(M,2)
4 C(M,2)=C(M,3)
      IF(NGS.EQ.1) WRITE(6,10)
C **  CONVERTING ARRAY C INTO A SQUARE MATRIX A
      DO 6 K=1,NSPI
      DO 5 J=1,NSPI
      L=NP*(K+1)+J+2
5 A(K,J)=C(L,3)
      IF(NGS.EQ.1) WRITE(6,11) (A(K,J),J=1,NSPI)
6 CONTINUE
C **  INVERTING MATRIX A
      CALL SPNIST(A, NSPI,15,1SIG)
      IF(NGS.EQ.1) WRITE(6,12)
      DO 7 K=1,NSPI
      IF(NGS.EQ.1) WRITE(6,11) (A(K,J),J=1,NSPI)
7 CONTINUE
      RETURN
10 FORMAT(19HOMATRIX FROM SUPCON )
11 FORMAT(10E11,3)
12 FORMAT(15HOMATRIX INVERSE )
      END

```

```

      FUNCTION CKN(RKM,RK,NPN)
C **  SUBROUTINE SIMILAR TO SUPCON AND ADPT FOR GENERATING CONVERGENCE
C **  CONSTANTS FOR NP = 3
C
      COMMON/BLKGG/C(867)
      C(1)=-1.0
      C(2)=-2.0
      OMK=1.0-RK
      DO 1 N=3,NPN
1 C(N)=2.0*C(N-1)-OMK*C(N-2)
      CKN=FLOAT(NPN)/(C(NPN)*RKM)
      RETURN
      END

```

```

SUBROUTINE VNEW(C,N)
C ** SUBROUTINE FOR CHANGING COMMENCING SURFACE VELOCITY DISTRIBUTION
C ** ACCORDING TO CONVERGENCE THEORY
C
COMMON /BLKC/V(50,25)/BLKH/VAV(50,25)/BLKJ/E(50)/BLKM/NPN
1/BLKP/JSONIC/BLKQ/VSPEC,VSONIC/BLKDD/VCC(50,3),AI(50,15),N1(50),
1A(15,30),NSPI,NP/BLKFF/DFSUB,DFSUP/BLKCC/N2,NST/BLKO/NTHRT
3/BLKMM/KD
DIMENSION C(50)
IF(N.EQ.NTHRT.AND.JSONIC.LE.2) RETURN
C ** SEPARATING SUBSONIC AND SUPERSONIC FLOWS
IF(N.GT.NTHRT.AND.JSONIC.EQ.1) GO TO 1
C ** CHECKING THE CONDITION THAT NORMALS UPSTREAM OF THE THROAT ARE
C ** SUBSONIC AND DOWNSTREAM OF THE THROAT SUPERSONIC OR SUBSONIC
C ** DEPENDING UPON THE FLOW TYPE
IF(C(N)) 2,11,3
1 IF(C(N)) 3,11,2
2 WRITE(6,20) N
C ** IF THE FLOW TYPE IS WRONG A CORRECTION IS MADE
V(N,1)=V(N,1)+1.1*C(N)
RETURN
C ** USING THE INVERTED MATRIX COEFFICIENTS AI AND AREA CHANGES
C ** REQUIRED AT THE TERMINATING SURFACE TO FIND THE NEW VELOCITY
3 VV=0.0
IF(JSONIC.EQ.1.AND.N.GT.NTHRT) GO TO 5
DF=DFSUB
M=NP-2
DO 4 J=1,M
L=N1(N)+J-1
4 VV=VV+AI(N,J)*E(L)
GO TO 7
5 DF=DFSUP
L=N-NTHRT
DO 6 J=1,NSPI
M=NTHRT+J
6 VV=VV+A(L,J)*E(M)
C ** INTRODUCING A HIGH DAMPING IN THROAT REGION VIA USE OF 1 M**2 FAR
C ** UPSTREAM OR DOWNSTREAM IF KD = 0
7 IF(KD.EQ.1) J=N
IF(KD.EQ.0.AND.N.LT.NTHRT) J=1
IF(KD.EQ.0.AND.N.GT.NTHRT) J=N2
C ** FINDING NEW VELOCITY
V(N,1)=V(N,1)*((1.0+DF*VV*FLOAT(NPN)/VCC(J,1)))
RETURN
RETURN
11 WRITE(6,22) N
RETURN
20 FORMAT(21H0FLOW TYPE CHANGED N=14)
22 FORMAT(18H0DVC FOUND ZERO N=14)
END

```

```

SUBROUTINE SPNIST(A,M,KK,ISIG)
C ** IMPERIAL COLLEGE LIBRARY SUBROUTINE FOR MATRIX INVERSION
C
  DIMENSION A(1)
  ISIG = 0
  N = M
  NN = KK
  N2 = N + N
  DO 10 J=1,N
    NJCOL = (N + J - 1) * NN
    DO 10 I=1,N
      KINJ = NJCOL + I
      IF(I-J)4,6,4
    4  A(KINJ) = 0.
      GO TO 10
    6  A(KINJ) = 1.
  10 CONTINUE
C DETERMINE MAXIMUM ABS OF VARIABLE BEING ELIMINATED. THIS BECOMES PIV
C OTAL ROW
  L = 0
  12 L = L + 1
  LCOL = NN*L-NN
  KLL = LCOL + L
  IF(L - N)13,30,1000
C FIND THE LARGEST ELEMENT IN THE LTH COLUMN.
  13 J1 = L
  C=ABS(A(KLL))
  L1 = L + 1
  DO 20 I = L1,N
    KIL = LCOL + I
    X=ABS(A(KIL))
    IF(C - X)14,20,20
C RECORD THE NUMBER OF THE ROW HAVING THE GREATER ELEMENT.
  14 J1 = I
C C BECOMES THE GREATER.
  C = X
  20 CONTINUE
C INTERCHANGE ROW J1 WITH ROW L. J1 IS THE ROW WITH THE LARGEST ELEMENT
C TEST TO SEE IF INTERCHANGING IS NECESSARY.
  IF(J1 - L)22,30,22
  22 DO 24 J = L,N2
    JCOL = NN*J-NN
    KJIJ = JCOL + J1
    HOLD = A(KJIJ)
    KLJ = JCOL + L
    A(KJIJ) = A(KLJ)
    A(KLJ) = HOLD
  24 CONTINUE
C IF THE LARGEST ABSOLUTE ELEMENT IN A COLUMN IS ZERO WE HAVE A SINGUL
C AR MATRIX
  30 IF (ABS(A(KLL)) - .00000001)33,33,32
  33 WRITE(6,100)
  ISIG = 4
  GO TO 1000
C ZERO ALL THE ELEMENTS IN THE LTH COLUMN BUT THE PIVOTAL ELEMENT.
  32 L1 = 1
  L2 = L - 1
  IF(L2)321,321,323
  321 IF(L-N)322,46,322

```

```

322 L1 = L + 1
    L2 = N
323 DO 324 I = L1,L2
    KIL = LCOL + I
    Z = -A(KIL)/A(KLL)
    DO 324 J = L,N2
    JCOL = NN*J - NN
    KIJ = JCOL + I
    KLJ = JCOL + L
324 A(KIJ) = A(KIJ) + Z*A(KLJ)
    IF(N - L2)12,12,321
C   DIVIDE BY DIAGONAL ELEMENTS.
46 DO 48 I = 1,N
    KKK = NN*I - NN + I
    ZZ = A(KKK)
    DO 48 J = 1,N2
    KKI = NN*J - NN + I
48 A(KKI) = A(KKI)/ZZ
C   RETURN AFTER PUTTING A INVERSE INTO B
49 DO 50 J = 1,N
    JCOL = NN*J - NN
    NJCOL = NN * N + JCOL
    DO 50 I = 1,N
    KIJ = JCOL + I
    KINJ = NJCOL + I
50 A(KIJ) = A(KINJ)
100 FORMAT('//20X,42H MATRIX IS SINGULAR. NO INVERSE OBTAINABLE//')
1000 RETURN
    END

```

APPENDIX 2 C SAMPLE DATA DECKS AND I.B.M. 7094 RUNNING TIMES.

THERMODYNAMIC DATA

AIR AS A PERFECT GAS

NGAS NDATA

1 80

H01	H0DATA	DHDATA	DV	T1	P1	GAMMA	CP	EM
500.0	500.0	2.0	50.0	1000.0	14.7	1.40	0.2400	28.97

EXPANSION LINE DATA FOR FREON 21 AT AN ISENTROPIC EFFICIENCY OF 0.85

1 34

152.0 154.0 1.0 25.0

RHO(N) (1.E SPECIFIC VOLUME)

.2430	.2610	.2820	.3060	.3320	.3620	.3930	.4270	.4630	.5030
.5460	.5940	.6450	.6980	.7580	.8210	.8900	.9630	1.0450	1.1390
1.2430	1.3500	1.4680	1.5900	1.7210	1.8540	1.9950	2.1270	2.2630	2.4000
2.5333	2.6700	2.8050	2.9430						

GEOMETRIC AND CONTROL DATA

LISTED BELOW ARE THREE EXAMPLES OF DATA DECKS FOR TYPICAL CASCADES. THE CONTROL VARIABLES GIVEN PRODUCE SUCCESSFUL OPERATION OF THE PROGRAMME. THE THREE EXAMPLES ARE FOR SUBSONIC FLOW, CHOKED SUBSONIC FLOW AND SUBSONIC-SUPERSONIC FLOW. THE RELEVANT SOURCE AND DESCRIPTION OF EACH DATA DECK IS GIVEN ON THE DATA DECK TITLE CARDS.

EXAMPLE 1 DATA DECK FOR SUBSONIC CASCADE. RUNNING TIME = 2.5 MIN.

CASCADE OF DUNAVANT AND ERWIN (REF 7.1). BLADE TURNING ANGLE = 95 DEG. 1
 STAGGER ANGLE = 16.1 DEG. FLOW INLET ANGLE = 40 DEG. DEVIATION CHOSEN 3. 2
 5 DEG. CASCADE IS SUBSONIC AND IS TAKEN FROM FIGURE 46A PAGE 90 OF REF. 3
 GRID REDUCTION PROCESS. STEP 1 COARSE GRID LOW SFCT. STEP 2 SFCT CONST 4
 GRID REDUCED. STEP 3 SFCT AND NPN GRADUALLY INCREASED 5

NPB2 NX1 NSPI ISTEP NGRDS K2 JSONIC JREAD NSPIC KD KDK
 55 55 0 50 7 1 3 1 0 1 1 6

NPS NTHRT NPN NP IRC KRC JRC ERROR DFSUB DFSUP SFCT
 14 0 5 9 1 0 0 3.0E -3 0.50 0.50 0.10 7

PITCH	XU	XD	DL	VSPEC	VMIN	VMAX	VST	VTT	DFT
.5782	0.0	1.0	0.0	347.0	175.0	700.0	30.0	0.0	0.0

8

EXAMPLE 1 (CONT)

X(N:1) I.E X VALUES OF FIRST GRID

440 -300 -160 -040 100 300 500 680 840 1000 1140 1280 1420 1560 9

X2(N):Y2(N) I.E TERMINATING SURFACE ARRAY (NPB2 VALUES)

1.4000-1.1747-1.3000-1.0908-1.2000-1.0069-1.1000-0.9230-1.0000-0.8391 10
 .9000-0.7552-0.8000-0.6713-0.7000-0.5874-0.6000-0.5035-0.5000-0.4196 11
 .4000-0.3356-0.3000-0.2517-0.2000-0.1678-0.1000-0.0839-0.0500-0.0420 12
 .0000-0.0160 0.0502-0.0038 0.0737 0.0008 0.1092 0.0104 0.1638 0.0267 13
 .2110 0.0384 0.2561 0.0447 0.3008 0.0462 0.3457 0.0441 0.3900 0.0376 14
 .4370 0.0273 0.4836 0.0141 0.5307-0.0020 0.5782-0.0210 0.6258-0.0425 15
 .6736-0.0665 0.7213-0.0926 0.7687-0.1208 0.8157-0.1509 0.8621-0.1829 16
 .9077-0.2169 0.9521-0.2529 0.9887-0.2876 1.0432-0.3268 1.0863-0.3649 17
 1.1295-0.4030 1.1726-0.4412 1.2158-0.4793 1.2589-0.5175 1.3021-0.5556 18
 1.3453-0.5937 1.3884-0.6319 1.4316-0.6700 1.4747-0.7081 1.5179-0.7463 19
 1.5611-0.7844 1.6042-0.8225 1.6474-0.8607 1.6905-0.8988 1.7337-0.9370 20

X1(N):Y1(N) I.E COMMENCING SURFACE ARRAY (NX1 VALUES)

1.4000-1.1747-1.3000-1.0908-1.2000-1.0069-1.1000-0.9230-1.0000-0.8391 21
 .9000-0.7552-0.8000-0.6713-0.7000-0.5874-0.6000-0.5035-0.5000-0.4196 22
 .4000-0.3356-0.3000-0.2517-0.2000-0.1678-0.1200-0.0975-0.0900-0.0655 23
 .0600-0.0255-0.0300 0.0200 0.0046 0.0831 0.0377 0.1231 0.1095 0.1696 24
 .1804 0.1915 0.2476 0.1991 0.3109 0.1966 0.3706 0.1857 0.4280 0.1691 25
 .4802 0.1479 0.5307 0.1222 0.5789 0.0932 0.6252 0.0615 0.6698 0.0276 26
 .7132-0.0080 0.7556-0.0450 0.7974-0.0830 0.8389-0.1218 0.8803-0.1609 27
 .9218-0.2004 0.9638-0.2403 1.0004-0.2772 1.0432-0.3268 1.0863-0.3649 28
 1.1295-0.4030 1.1726-0.4412 1.2158-0.4793 1.2589-0.5175 1.3021-0.5556 29
 1.3453-0.5937 1.3884-0.6319 1.4316-0.6700 1.4747-0.7081 1.5179-0.7463 30
 1.5611-0.7844 1.6042-0.8225 1.6474-0.8607 1.6905-0.8988 1.7337-0.9370 31

CARDS SIMILAR TO NO 7 EACH GIVING NEW GRID DATA

30 0 5 9 1 0 0 3.0E -3 0.50 0.50 0.10 32
 30 0 6 9 1 0 0 3.0E -3 0.50 0.50 0.30 33
 30 0 7 11 1 0 0 3.0E -3 0.50 0.50 0.50 34
 30 0 8 11 1 0 0 3.0E -3 0.50 0.50 0.70 35
 30 0 9 11 1 0 0 3.0E -3 0.50 0.50 0.85 36
 30 0 10 13 1 0 0 0.5E -3 0.50 0.50 1.00 37

EXAMPLE OF READ IN VELOCITY DISTRIBUTION AND MASS FLOW FOR ABOVE
EXAMPLE AND AN INCREASE OF THE NUMBER OF STREAMLINES

55 55 0 50 2 1 3 4 0 1 0 6
 30 7 10 17 1 3 0 0.5E -3 0.45 0.45 1.00 7
 440 -377 -315 -252 -187 -131 -077 -023 035 106 195 286 377 467 9
 552 633 708 780 851 924 995 1059 1121 1183 1246 1309 1372 1435 9A
 1497 1560 9B
 .59467009E 1 (MASS FLOW TM) 31A

V(N:1) I.E VELOCITY AT COMMENCING SURFACE

3472 3459 3438 3409 3295 3118 3198 3913 5605 5609 31B
 5232 4957 4717 4588 4409 4238 4075 3924 3803 3741 31C
 3729 3321 3511 3527 3539 3547 3548 3552 3549 3555 31D
 30 7 15 17 1 3 0 0.5E -3 0.45 0.45 1.00 32

EXAMPLE 2 CHOKED SUBSONIC CASCADE RUNNING TIME = 2.5 MIN.

CASCADE OF DUNAVANT AND ERWIN (REF 7.1); BLADE TURNING ANGLE = 95 DEG. 1
 STAGGER ANGLE = 34.1 DEG. FLOW INLET ANGLE = 30 DEG. DEVIATION CHOSEN 2
 5 DEG. CASCADE IS CHOKED WITH A SUBSONIC OUTLET MACH NO. EXAMPLE FOR 3
 COMPARISON WITH FIG 44B OF REF. GRID REDUCTION PROCESS-INCREASE OF NPN 4
 AND SFCT FOR COARSE GRID AND THEN A GRADUAL DECREASE OF GRID SIZE 5

55 55 0 50 10 1 2 1 0 1 1 6
 17 10 5 9 1 0 0 3.0E -3 0.75 0.75 0.20 7
 .671 0.0 1.0 0.0 0.0 300.0 1700.0 30.0 2.0 0.5 8
 620 -460 -300 -140 000 140 320 480 620 740 840 920 1000 1080 9
 1160 1240 1320 10

1.4000-0.8083-1.3000-0.7506-1.2000-0.6928-1.1000-0.6351-1.0000-0.5774 11
 .9000-0.5196-0.8000-0.4619-0.7000-0.4041-0.6000-0.3464-0.5000-0.2887 12
 .4000-0.2309-0.3000-0.1732-0.2000-0.1155-0.1000-0.0577-0.0500-0.0325 13
 .0000-0.0190 0.0540-0.0222 0.0817-0.0256 0.1242-0.0276 0.1903-0.0293 14
 .2466-0.0332 0.2987-0.0425 0.3485-0.0568 0.3973-0.0753 0.4438-0.0984 15
 .4920-0.1266 0.5387-0.1578 0.5849-0.1925 0.6305-0.2304 0.6754-0.2713 16
 .7195-0.3149 0.7628-0.3608 0.8050-0.4089 0.8460-0.4590 0.8858-0.5109 17
 .9238-0.5648 0.9599-0.6205 0.9879-0.6719 1.0340-0.7346 1.0679-0.7922 18
 1.1019-0.8497 1.1358-0.9073 1.1698-0.9648 1.2037-1.0224 1.2377-1.0800 19
 1.2716-1.1375 1.3056-1.1951 1.3395-1.2526 1.3735-1.3102 1.4074-1.3677 20
 1.4414-1.4253 1.4753-1.4829 1.5093-1.5404 1.5432-1.5980 1.5772-1.6555 21

1.4000-0.8083-1.3000-0.7506-1.2000-0.6928-1.1000-0.6351-1.0000-0.5774 22
 .9000-0.5196-0.8000-0.4619-0.7000-0.4041-0.6000-0.3464-0.5000-0.2887 23
 .4000-0.2309-0.3000-0.1732-0.2000-0.1155-0.1000-0.0530-0.0500-0.0060 24
 .0200 0.0280 0.0099 0.0629 0.0349 0.0901 0.0858 0.1223 0.1817 0.1479 25
 .2677 0.1467 0.3446 0.1309 0.4136 0.1055 0.4755 0.0720 0.5329 0.0332 26
 .5829-0.0090 0.6295-0.0554 0.6723-0.1047 0.7119-0.1563 0.7491-0.2097 27
 .7841-0.2646 0.8177-0.3206 0.8502-0.3775 0.8820-0.4352 0.9137-0.4932 28
 .9454-0.5517 0.9774-0.6108 1.0045-0.6645 1.0340-0.7346 1.0679-0.7922 29
 1.1019-0.8497 1.1358-0.9073 1.1698-0.9648 1.2037-1.0224 1.2377-1.0800 30
 1.2716-1.1375 1.3056-1.1951 1.3395-1.2526 1.3735-1.3102 1.4074-1.3677 31
 1.4414-1.4253 1.4753-1.4829 1.5093-1.5404 1.5432-1.5980 1.5772-1.6555 32

17 10 10 13 1 0 0 3.0E -3 0.75 0.75 0.20 33
 17 10 10 13 1 0 0 3.0E -3 0.75 0.75 0.50 34
 17 10 10 13 1 0 0 3.0E -3 0.75 0.75 0.70 35
 17 10 10 13 1 0 0 0.5E -3 0.75 0.75 1.00 36
 19 1 10 13 1 0 0 0.5E -3 0.75 0.75 1.00 37
 21 1 10 13 1 0 0 0.5E -3 0.75 0.75 1.00 38
 23 1 10 13 1 0 0 0.5E -3 0.75 0.75 1.00 39
 25 1 10 13 1 0 0 0.5E -3 0.75 0.75 1.00 40
 27 1 10 13 1 0 0 0.5E -3 0.75 0.75 1.00 41

EXAMPLE 3 SUBSONIC-SUPERSONIC CASCADE. RUNNING TIME = 1.5 MIN.

CASCADE OF FORSTER (REF 3.15) PROFILE IS BLADE NO 1. FIGURE 20 OF REF. 1
GRID REDUCTION PROCESS IS TO COMMENCE WITH AN ASPECT RATIO OF APPROX2.5 2
AND A LOW VALUE OF SFCT. THE NUMBER OF STREAMLINES IS INCREASED AND 3
THEN SFCT GRADUALLY INCREASED TO 1.0. THE GRID SIZE IS THEN DECREASED 4
IN SMALL STEPS UNTIL NO FURTHER REDUCTION IS POSSIBLE DUE TO FAILURE 5

35 35 2 50 9 1 1 1 1 1 1 6

8 6 5 5 1 0 0 3,0E -3 0,30 0,30 0,30 7

0.99	-0.5	0.52	0.0	0.0	300.0	2700.0	50.0	1.0	0.3	8
------	------	------	-----	-----	-------	--------	------	-----	-----	---

1200 -850 -600 -350 -100 150 350 500 9

1.5000 0.2730-1.4000 0.2900-1.3000 0.3075-1.1000 0.3415-0.9000 0.3750 10

•8000 0,3915-0,7500 0,4000-0,7000 0,4020-0,6200 0,4030-0,5500 0,4030 11

•5000 0,4040-0,4000 0,4070-0,3000 0,4015-0,2000 0,3920-0,1000 0,3725 12

0.0000 0.3480 0.1000 0.3110 0.2000 0.2630 0.2500 0.2350 0.3000 0.2050 13

•3500 0.1705 0.4000 0.1315 0.4500 0.0910 0.4800 0.0620 0.5000 0.0500 14

●5200 0.0400 0.5500 0.0300 0.6000 0.0020 0.7000-0.0460 0.9000-0.1500 15

1.1000-0.2540 1.3000-0.3580 1.5000-0.4620 1.7000-0.5660 1.9000-0.6700 16

1.5000 0.2730-1.4000 0.2900-1.3000 0.3075-1.1000 0.3415-0.9000 0.3750 17

0.8000 0.3915-0.7500 0.4000-0.7000 0.4150-0.6200 0.4400-0.5500 0.4670 18

0.5000 0.4820-0.4000 0.5090-0.3000 0.5200-0.2000 0.5210-0.1000 0.5110 19

0.0000 0.4870 0.1000 0.4470 0.2000 0.3920 0.2500 0.3560 0.3000 0.3130 20

•3500 0.2665 0.4000 0.2090 0.4500 0.1460 0.4800 0.1080 0.5000 0.0800 21

0.5200 0.0600 0.5500 0.0300 0.6000 0.0020 0.7000-0.0460 0.9000-0.1500 22

1.1000-0.2540 1.3000-0.3580 1.5000-0.4620 1.7000-0.5660 1.9000-0.6700 23

8 6 9 5 1 0 0 3.0E -3 0.30 0.30 0.30 24

8	6	9	5	1	0	0	3.0E	-3	0.30	0.30	0.50	25
---	---	---	---	---	---	---	------	----	------	------	------	----

8	6	9	5	1	0	0	3.0E	-3	0.30	0.30	0.70	26
---	---	---	---	---	---	---	------	----	------	------	------	----

8	6	9	5	1	0	0	3.0E	-3	0.30	0.30	0.85	27
---	---	---	---	---	---	---	------	----	------	------	------	----

8	6	9	5	1	0	0	0.5E	-3	0.30	0.30	1.00	28
---	---	---	---	---	---	---	------	----	------	------	------	----

11	2	9	7	1	0	0	0.5E	-3	0.30	0.30	1.00	29
----	---	---	---	---	---	---	------	----	------	------	------	----

14	2	9	9	1	0	0	0.5E	-3	0.30	0.30	1.00	30
----	---	---	---	---	---	---	------	----	------	------	------	----

```
17  2  9 11 1 0 0 0.5E -3 0.30 0.30 1.00
```

EXAMPLE OF READ IN VELOCITY DISTRIBUTION AND MASS FLOW FOR ABOVE
EXAMPLE AND AN INCREASE OF THE NUMBER OF STREAMLINES

35 35 2 50 2 1 1 4 1 1 1 ..6

17 12 9 11 1 0 0 0.5E -3 0.30 0.30 1.00 7

1200-1032 -883 -758 -645 -533 -419 -305 -192 -076 041 150 237 311 9

380 443 500 9A

•25072823E 02 23A

723.79 723.35 714.72 664.89 728.23 890.68 1073.38 1164.18 1260.14 1392.71 23B

1489,591746,521926,322123,992239,552380,852405,27 23C

17 12 15 11 1 0 0 0.5E -3 0.30 0.30 1.00 25