

TO MY FAMILY

"The world is satiated with dogmas,
but people are hungry for life."

Milovan Djilas
(The Unperfect Society, 1969)

ON THE FLOW OF FLUIDIZED SUSPENSIONS

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ABSTRACT

The first part of this work is a study of the flow properties of fluidized beds, which is intended as a contribution to the use of the fluidized technique in the field of solids transport. Experiments were performed on the flow of fluidized beds over inclined planes, under the action of gravity. A method is outlined for the measurement of the point viscosity of a fluidized bed, ^{This avoids} ~~thus avoiding~~ the restricting prevalent assumption of a Newtonian continuum for the fluidized bed. Further, this method has the important advantage over others in that there does not exist any interference with the flow conditions prevailing in the bed during measurement. Further, this method is a general one which is applicable to purely viscous non-Newtonian fluids.

The results of this study are used in the elucidation of the structure of the fluidized state and in the evaluation of criteria for the existence of stable inhomogeneities in the fluidized bed. The measured point viscosity is related to the voidage of the continuous particulate phase of the bed, ^{through the model} and this in turn is related to the presence of inhomogeneities, and to the flow distribution of the fluid through the bed.

The second part of this work comprises an investigation into the flow of fluidized suspensions through orifices in fluidized and pressurized beds. A Newtonian continuum model is assumed for the solids phase, as it was postulated that in this problem, the solids concentration range is sufficiently high for bubbles to be assumed

absent. This continuum model is, in fact, general and it can be applied to various flow configurations.

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PART 1

THE MEASUREMENT OF THE POINT VISCOSITY OF A FLUIDIZED BED

CHAPTER ONE

INTRODUCTION

Fluidization, as a method of contacting granular solids with fluids, has been playing an increasingly important part in a variety of physio-chemical processes, and there is little doubt that this trend will persist in the very near future, provided, as seems almost certain, that academic and industrial interest is maintained if not enhanced. The applications of fluidization can be said to fall into one of two main categories: chemical reactions and catalysis on the one hand, and physical and mechanical processes on the other. The first group has so far tended to occupy the major proportion of processes involving the utilization of the fluidized bed technique. The classical example of this type of application is the use of fluidization in the catalytic cracking of heavy petroleum vapours. In fact, this was the first large-scale application of fluidization, when it was introduced in the United States of America in 1940. Another example of the use of fluidization in chemical operations is the multi-compartment reactor in the calcination of limestone. Surprisingly, the utilization of fluidization in physical and mechanical operations has been far less widespread to date. The most important physical process using fluidization is that of heat transfer between gas or vapour streams. In the field of mechanical applications, fluidization is used in solids drying and coating, the use of stirred beds as blending devices, particle separators and classifiers, and solids transport.

Wen & Simons (1959), for example, have studied the characteristics of dense and solid gas-mixtures in horizontal pipes, at the same time as developing work being carried out by the U.S. Bureau of Mines. It is with this last application that this study is concerned.

Basically, solids can be transported in the fluidized state by one of two ways. The first is the airlide solids conveyer, which is essentially a porous inclined slab that carries a layer of the powder to be conveyed. Air is admitted from below and the solids are either at or above the state of minimum fluidization. Usually, the solids inlet is connected to a hopper. The second type of fluidized transport involves drawing solid material from a conventional - vertical - fluidized bed through an orifice or a pipe in its side. The advantages of fluidized transport lie in the ease of introduction and removal of solids, and in the absence of choking and blocking of the transport lines. Fluidized transport is utilized extensively in fluidized catalytic reactors.

Various studies have been made of the airlide conveyer and these fall into two main groups. One group utilizes an empirical approach and thus has all the inherent limitations of empirical work generally. The other type of investigation is based on the fundamental assumption of the continuum nature of the fluidized bed. This is clearly a much more useful approach, as it may also assist in clarification of the structure of the fluidized bed and throw some light on the differences between "homogeneous" and "heterogeneous" fluidized beds, the distinguishing feature between the two being the presence of finite voids or "bubbles" in the latter. A knowledge of the behaviour of these "bubbles" is crucial in determining the feasibility and mechanics of physico-chemical processes in the fluidized bed.

Despite the nonhomogeneous nature of the aggregatively fluidized bed, it has been found to be both convenient and profitable to approach the system from a continuum point of view by analogy with the particulate fluidized bed. Furukawa and Ohmae (1958) defined and measured such properties of the fluidized bed as viscosity, surface tension, and miscibility in an analogous way with liquid systems. Their experimental work confirmed the liquid-like behaviour of fluidized beds. Wilhelm (1962) discussed these problems and noted that fluidized beds should display non-Newtonian behaviour. The problem is then essentially to relate the viscosity of the fluidized bed to its structure, taking into account the existence of inter-particle forces, and the presence of bubbles in gas-fluidized beds. Schugerl, et al (1961), Siemes and Hellmer (1962), and Turcajová and Neuzil (1969) measured the viscosity of the fluidized system, and other flow properties. However, all these workers treated the fluidized bed as a Newtonian fluid. Saxton, et al (1970) used a statistical-thermodynamic approach to establish an equation of state and a collisional-viscosity relation for particulate fluidized beds. They utilized the smoothed potential cell theory of pure liquids, and they extended their analysis to gas-fluidized systems. Whilst they can obtain a fairly close fit with experimental data for the viscosity of the liquid-fluidized system, the predicted viscosity value for gas-fluidized beds is lower than experimental values of the apparent viscosity by a factor of 2. Davidson and Harrison (1963), Murray (1967), Stewart (1968), and Grace (1970) have considered the effect of viscous-type forces on bubbles rising in fluidized beds.

In this work, the fluidized bed is considered as a purely-viscous non-Newtonian fluid, and its point viscosity is measured

by flow under gravity over an inclined plane. A method is developed to relate the rheological behaviour of the bed to its structure, including especially the existence of bubbles or inhomogeneities in the bed, and the distribution of the fluidizing fluid between the bubble and the particulate phases of the system. Further, the method devised here to measure the point viscosity of the fluidized bed is, in fact, generally applicable to non-Newtonian fluids with viscous but not elastic properties, flowing under low rates of shear.

CHAPTER TWO

LITERATURE SURVEY

A survey of the past work has been conveniently divided into the following two sections:

- 1) Studies of the internal structure of the fluidized bed.
- 2) Studies of the viscosity and the flow properties of the fluidized bed.

1) Internal structure of the fluidized bed

In order to avoid later confusion, it is most useful first to discuss the terminology used in fluidization literature. In the first instance the fluidized state, being one case of fluid-solid systems, is evidently heterogeneous in that these are two phases: fluid and solid particles. However, two main classes of behaviour can be distinguished: homogeneous or particulate fluidization, and heterogeneous or aggregative fluidization. As the names imply, the distinguishing feature between the two classes is the presence of inhomogeneities or "bubbles" in the latter case. It has been found that most gas-fluidized beds belong to the heterogeneous category while liquid-fluidized beds are generally homogeneous in character. It cannot be overemphasized that this classification into homogeneous and heterogeneous fluidized beds is not rigorously defined, but rather is a general one which is justified by the fact that it gives a good qualitative picture of the types of fluidized systems encountered in practise.

Due to the fact that heterogeneous fluidized beds are more

complex in structure than homogeneous beds, it is hardly surprising that the structure and the behaviour of the former are less well understood than the latter. Several models, outlined below, have been proposed to describe heterogeneous fluidized beds.

The first important model proposed to describe heterogeneous fluidized beds was the two-phase model. Toomey and Johnstone (1952) were the first to propose this model of aggregative fluidization. They postulated that the fluidized bed consists of two phases: the continuous phase composed of uniformly distributed solid particles in a supporting gas stream, and the discontinuous phase comprising essentially a particle-free gas in the form of bubbles, channels, or slugs. This is in direct contrast to homogeneous fluidized beds. They concluded that for a given system, the flow of fluid and the density of the continuous phase are independent of the superficial gas velocity, namely, it is always at the point of incipient fluidization. The additional flow of fluidizing gas is carried in the bubble phase. There is a considerable amount of experimental work supporting the two-phase model, which Davidson and Harrison (1963) have surveyed in a concise and systematic manner. It may be noted at this juncture that Lanneau (1960) challenged the assumption that all excess fluidizing fluid passes through the bed as bubbles. However, Davidson, ibid, pointed out that in these experiments, the superficial velocity was of the order of one hundred times the incipient fluidizing velocity, while most other work has been conducted at superficial velocities of the order of ten times the velocity at incipient fluidization.

Davidson and Harrison (1966) studied the relation between the velocity of rise of a single bubble and that of a bubble cloud in a fluidized bed, and the flow distribution of the fluidizing fluid

between the particulate and the bubble phases. Their analysis partly justified the flow distribution assumption of the two-phase model.

Their theory is general in that it holds for multi-size bubbles rising at unsteady velocities through the bed. However, ^{Godard & Richardson} Turner (1968) threw some doubt on the assumption concerning flow distribution.

Lockett, et al (1967) showed that while the flow distribution assumption is not strictly correct, the error it introduces into the analysis is significant only at superficial velocities much higher than that at incipient fluidization.

There is little doubt that the formulation of the two-phase model to describe heterogeneous fluidized beds constituted a turning point in fluidization studies. It certainly provides a more accurate picture of aggregatively-fluidized beds than any single-phase model does. Since then, most efforts to elucidate the structure and the behaviour of heterogeneous fluidized beds have been in essence variations and modifications of the original two-phase model. In this category lie the bubbling bed model, the bubble assemblage model, and further work discussed below.

Kunii and Levenspiel (1968) proposed the bubbling bed model as a basis for fluidized bed reactor design which is valid for bubbles rising more quickly than the interstitial gas. This model views the fluidized bed as composed of three regions: uniformly sized bubbles surrounded by clouds and followed by wakes, rising through an emulsion phase of downward moving solids. In this model, it is assumed that the particulate phase is incipiently fluidized, and that there is an effective bubble size to represent the bubbles in the bed. It is further assumed that the velocity of rise of the solids in the wake of the bubble is that of the bubble itself, and it is postulated that at the bed surface, the wake solids ~~to~~ rejoin the emulsion phase.

and then move downwards at a velocity such that incipient fluidization conditions are maintained in the emulsion phase. Kunii, et al (1967) showed from data on solids circulation, gas flow, flow reversal in the emulsion phase, that this model describes well shallow rather than deep beds. However, the assumption of effective bubble size is more justified in the latter, as in deep beds the maximum stable bubble size may be approached.

Kato and Wen (1969) developed the bubble assemblage model, which is an improvement of the bubbling bed model in that it is based on multi-size bubbles. The fluidized bed is considered as consisting of a series of compartments, each of which is characterized by the bubble size at that height in the bed.

It cannot be overemphasized that despite the extensive use of the terms "bubble" and "bubble phase" in the fluidization literature, bubbles with well-defined boundaries are unlikely to exist in the system. In reality, the system is a mixture of solids and gas in certain regions of which the solids concentration is sufficiently low such that these regions are effectively large voids of gas in an otherwise homogeneous mixture of solids and gas. Bearing this in mind, efforts to investigate the dynamics of these voids in heterogeneous fluidized beds have proved to be extremely useful in the interpretation of the behaviour of such beds. Studies on void or bubble dynamics in fluidized beds have followed a parallel path to that of bubble motion in the gas-liquid system. The first step in this direction, as in the gas-liquid system, has been the study of the motion of a single bubble in an otherwise homogeneous gas-solid suspension which is at the state of incipient fluidization. This is achieved by the injection of a bubble into a fluidized bed maintained at incipient fluidization by a separate supply of gas.

Davidson and Harrison provided extensive experimental data to obtain a relationship between the velocity of a single spherical-cap bubble and its diameter. They obtained a relationship which is analogous to the Davies-Taylor equation for the gas-liquid system; they showed that for a single bubble rising through an incipiently-fluidized bed:

$$U_b = 0.71 g^{1/2} V^{1/6} \quad (1)$$

where U_b = velocity of rise of a single bubble in a fluidized bed.

V = volume of bubble,

and the equation is expressed in c.g.s. units.

By comparing equation (1) with the Davies-Taylor equation, viz:

$$U_b = 0.79 g^{1/2} V^{1/6} \quad (2)$$

they concluded that a single bubble in a gas-fluidized bed rises at a lower velocity than that in an inviscid liquid.

Baumgarten and Pigford (1960) studied the rate of rise of bubbles at conditions ranging from incipient fluidization to a high degree of fluidization. Their results showed that bubbles rise at a slower rate than in the case of an inviscid fluid, and that their motion corresponds to Stokes flow in a fluid of viscosity in the range 10 to 100 poise. Rowe and Partridge (1965) have shown by X-ray photography that the general shape of a single bubble is spherical, and that it contains a wake of solids occupying up to one-third the volume of the bubble. They also showed that

$$U_b = K g^{1/2} d_b^{1/2}$$

where K varies between 0.56 and 0.85 depending on the nature of the fluidized material.

In fluidized beds operating at conditions far removed from incipient fluidization and where single bubble dynamics no longer

represent the motion of the bubble through the bed, such parameters as bubble interactions, voidage distributions, and voidage fluctuations have been studied. Yasuni and Johanson (1958) showed that bubble volume increases with height in the bed, attributing this to bubble coalescence. Toei and Matsumoto (1967) proposed a model for the coalescence of bubbles based on the assumption that the solid particles move around the bubbles as if the particles constituted an inviscid fluid. They studied vertical, horizontal, and oblique coalescence, and their model confirmed that two bubbles side by side on the same horizontal level are unlikely to coalesce.

Rapid bubble movement through the bed causes local variations in the voidage, and this variation can be statistically divided into an average and a fluctuating part. In investigating the average voidage variation, the local fluctuations are ignored. Bakker and Heertjes (1960) carried out extensive experimental work leading to a voidage distribution plot. They showed that the bed can be divided into three major vertical zones. The lower zone is dependent upon the nature of the distributor, and it is dilute in solids. The main central zone comprises most of the bed, and it is essentially at constant voidage. The upper zone is dilute in solids due to the entrainment of solids by the vigorous bubbling at the bed surface.

Teoreanu (1963) carried out an experimental study of the fluctuating part of the voidage. He developed criteria for the existence and stability of inhomogeneities in the fluidized bed, and a model was formulated for the structure of aggregatively-fluidized beds, leading to the conclusion that the structure of the bed is determined by the relation established at any one time between the interparticle force and the hydrodynamic-quasi-Bernoulli forces. He distinguished between two regions in aggregative fluidization:

at lower fluidizing velocities where bubbles are stable and the bed is heterogeneous, and at higher fluidizing velocities where the bubbles "collapse" at the same rate at which they are formed. In this latter region, the bed is said to be quasi-homogeneous. Doichev (1964) derived a relation between the pressure at any point in a fluidized bed and the voidage fraction at that point. He also obtained an expression for the pulsation rate of the particles in the main section of the bed. Vakh^{rushev} and Basov (1968) considered the heterogeneous fluidized bed as a two-phase system with a bubble phase and a particulate phase but ^{relaxed}~~relaxing~~ the assumption of the two-phase theory that the particulate phase is at incipient fluidization. However, the particulate phase was assumed to be at a constant voidage throughout the bed. They obtained expressions for the expansion of homogeneous and heterogeneous fluidized beds, and used the work done for expansion as a criterion of stability. The fluidizing fluid will pass through the bed in such a way that the work done on expansion is minimised.

It is clear from the foregoing that the two-phase model of fluidized beds represents to date the most fundamental and at the same time most useful approach to the structure of aggregatively-fluidized beds. The efforts started by Davidson and Harrison (1959) to apply inviscid flow theory in the study of single bubble motion in fluidized beds has proved to be immensely illuminating, where it is assumed that both fluid and solid particles are in potential flow around the bubble. In actual beds, however, the situation is more complicated due to the presence and motion of numerous interacting bubbles. Vakh^{rushev} and Basov (1968) have tried to relax some of the assumptions of the two-phase model. Vakh^{rushev} and Basov (1968) generalized the two-phase model by avoiding the assumption that the

particulate phase is necessarily at incipient fluidization as in the two-phase model. It is believed that this type of approach will be the most useful in further advances in the understanding of the structure and behaviour of heterogeneous fluidization.

2) Studies of the viscosity and the flow properties of the fluidized bed

Detailed theoretical analysis of multiparticle systems - whether homogeneous or heterogeneous - has been limited by the complexity of the equations of motion. An alternative approach has been to assume that the system behaves as a continuum. It is sufficiently evident that while this approach at least describes the overall structure of homogeneous systems, it is necessarily an approximation in the heterogeneous case.

Matheson, et al (1949) investigated the effect of particle size and density on the density and viscosity of a gas-fluidized bed. They measured the density by pressure drop measurements across the bed, and they found that the bed viscosity decreased with increasing fluidizing velocity, this being due to the existence of bubbles in the bed. The viscosity of the bed was measured using a paddle and cup arrangement by determining the weight required to spin the paddle at 200 revolutions per minute in the bed. This force was corrected for the weight required to spin the paddle at the same rate in air. It should be noted, as Matheson, et al themselves pointed out, that the viscosity measured is an apparent viscosity since the forces required to accelerate the fluidized systems are neglected in calculating the shear viscosity. It was found that the measured viscosity decreased with decreasing particle size and density. They also showed that beds with spherical particles are more viscous than those containing irregularly-shaped particles. This was attributed

to the closer packing obtained with spheres. They linked bed viscosity to bed expansion by postulating that the viscosity is a function of the volume within which the particles are free to move. Thus, the volume occupied by bubbles is excluded as bubbles are assumed to be devoid of solid particles. Furukawa and Ohmae (1958) characterized the fluidized system by analogy with a Newtonian liquid. They proposed that the weight of solid particles corresponds to the attractive forces between liquid molecules, while the forces of friction between the fluid and the particles correspond to the repulsive forces which exist in liquids. They carried out experiments to measure the viscosity of gas-fluidized beds, using a paddle viscometer. The values obtained for the bed viscosity ranged from 0.71 to 22.5 poises, and it was found that viscosity decreased with increasing fluidizing velocity. It was proposed that the particles undergo a harmonic oscillatory motion, from which the mean kinetic energy of the particles can be determined. This last quantity is considered analogous to temperature for a liquid.

Todes (1955) suggested a similar idea. Kramers (1951-2) used an experimental arrangement in which the rotational speed of a dumb-bell was measured as a function of the torque applied to the rotating staff. Measurements were made at several fluidizing velocities, heights, and total expanded heights. It was concluded qualitatively that the viscosity of the bed falls with increasing fluidizing velocity and decreasing bed height. Lin and Orr (1960) devised a new paddle-type spindle to replace the rotating paddle formerly used, as the latter disturbed the flow pattern within the bed. The spindle consisted essentially of two thin-walled brass cylinders located concentrically within one another and mounted axially to the viscometer through a small shaft. As the spindle was suspended

vertically into the fluidized bed, it exposed a relatively small area normal to the direction of gas flow, while retaining a relatively large surface area parallel to the gas stream path, thus ensuring sufficient sensitivity. They found that the gas-fluidized bed behaved as if it were made up of three separate regions. In the lower portion of the bed, the apparent viscosity decreases with increasing gas flowrate. In the middle region, the viscosity can increase or decrease with increasing gas velocity, but the change is small. In the upper zone, the apparent viscosity increases with increasing flowrate. At low flowrates, all regions display pseudoplasticity, while at higher flowrates, Newtonian behaviour is exhibited throughout the bed. The absolute values of the apparent viscosity ranged from 0.1 to 4.6 poise. The results were interpreted as in Matheson's work by relating the viscosity to the voidage fraction at any point or region in the bed. It is worth noting that all the particles used were essentially spherical in shape. Shumster and Haas (1960) used a paddle viscometer to measure the viscosity of beds of spherical gas beads fluidized by air. They observed that the variation of bed viscosity with fluidizing velocity falls into three regions. At low gas flowrates, viscosity decreases rapidly with increasing fluidizing velocity. This is followed at intermediate flowrates by a region in which the viscosity increases and then goes through a maximum. Finally, at high fluidizing flowrates, the viscosity decreases steadily until a constant equilibrium value is attained.

Schugerl, et al (1961) carried out rheological studies on a fluidized bed using a concentric cylinder viscometer. They investigated the effect of shear stresses and centrifugal forces on the structure of the gas-fluidized bed. The bed density was

measured simultaneously using X-ray absorption in order to study the mobility of particles in the bed. They showed that the bed viscosity decreases sharply with increasing gas flowrate initially, and continues to decrease very steadily at higher flowrates. The values obtained for the bed viscosity are in the range 2 to 1000 poise. Haygard and Sacredote (1966) used a torsion pendulum viscometer to measure the apparent viscosity of gas-fluidized beds of spherical particles. The force experienced by a body immersed in the fluidized bed and suspended by a vertical torsionwire was used to determine the bed viscosity. All the particles used were spheres. The viscosity data obtained lie between 2.5 and 250 poise, and the trend of falling viscosity with rising fluidizing velocity is again displayed, with the bed viscosity attaining a constant value at high fluidizing velocities. They concluded that gas-fluidized beds behaved as Newtonian fluids. Siemes and Hellmer (1962) made an extensive study of the flow properties of the fluidized state by the use of a rectangular airslide. In addition to viscosity measurements, they obtained velocity profiles by photographing tracer material in the bed. The values obtained for the viscosity fall between 1 and 12 poise. It was also found that the bed viscosity increases with increasing bed height and bed inclination and with decreasing fluidizing velocity.

Leeden and Bouwhouis (1961) suggested from experiments carried out with air- and water-fluidized beds that the dynamic behaviour of a particulate fluidized bed can best be described with a maximum shear stress or a maximum angle of friction and not with the concept of viscosity. Turcajova, et al (1969) did extensive work on the flow of fluidized beds in an airslide. Their results show that the analogy between the fluidized state and a Newtonian fluid is adequate

to describe the flow characteristics of ^{shallow} fluidized beds. They obtained the following correlation between the friction factor and the Reynolds number:

$$f^* = \frac{24.5}{(Re^*)^{1.35}} \quad (1)$$

Saxton, et al (1970) used a statistical thermodynamic approach based on the smoothed potential cell theory of pure liquids to obtain the collisional viscosity of particulate fluidized beds as a function of the fluid properties and the fluidizing velocity. They extended the analysis to aggregatively-fluidized beds, but the agreement with experimental values of the viscosity was much better in the former than in the latter case. Grace (1970) used an empirical relation based on the shape of spherical-cap bubbles rising in real liquids to predict the viscosity of the fluidized bed. The particulate phase is assumed to be at incipient fluidization. The experimental results obtained by various workers for gas-fluidized beds are summarized below:

<u>Work</u>	<u>Viscometer type</u>	<u>Range of Viscosity</u>	<u>Other Observations</u>
(1) Matheson, <u>et al</u> (1949)	paddle stormer	5-80 g stormer	Viscosity increases due to decreasing U and increasing dp and ds
(2) Furukawa and Ohmae (1958)	paddle stormer	0.71-22.5 poise	Viscosity increases due to decreasing U
(3) Lin and Orr (1960)	paddle-type spindel	0.1-4.6 poise	Newtonian, viscosity decreases with increasing U
(4) Shuster and Haas (1960)	paddle	10-309 stormer	Viscosity decreases with increasing U
(5) Schugerl, <u>et al</u> (1961)	concentric cylinder	2-1000 poise	Viscosity sharply decreases with increasing U at first, but rate of decrease falls to give an equilibrium value

<u>Work</u>	<u>Viscometer type</u>	<u>Range of Viscosity</u>	<u>Other Observations</u>
(6) Haggard and Socredote (1966)	torsion pendulum	2.5-250 poise	Same as in (5), also Newtonian
(7) Siemes and Hellmer (1962)	airslide	1-12 poise	Same as in (5), also viscosity increases with increasing bed height and inclination
(8) Turcajova and Neuzil (1969)	airslide		$f^* = \frac{24.5}{(Re^*)^{1.35}}$

We now turn our attention to the problem of the viscosity of concentrated suspensions of solid particles in fluid media.

Einstein (1906) derived the following theoretical equation for the viscosity of a dilute suspension of spheres.

$$\mu_p = \mu_f (1 + 2.5 \alpha) \quad (2)$$

where

μ_p = viscosity of the suspension

μ_f = viscosity of the fluid

α = volumetric concentration of solid particles
($1 - \epsilon$).

Equation (2) is applicable for values of α up to 2.5%, as it is assumed in the derivation that there is no interaction between the spheres. Vard (1948) and Hawksley (1950) neglected particle collisions and obtained:

$$\mu_p = \mu_f \exp \left(\frac{-2.5 \alpha}{1 - (39/64) \alpha} \right) \quad (3)$$

Equation (3) has been checked experimentally by Hanratty and Bardukwala (1957) for $Re_p < 0.07$, where $Re_p = \frac{d_p d U}{\mu_f}$, and U_r is the superficial relative velocity between the particle and the fluid. Richardson and Zaki (1954) obtained an empirical law from an experimental study of the dynamics of settling in suspensions, thus:

$$\mu_p = \mu_f (1 - \alpha)^{n-2} \quad (4)$$

where n takes one of the following values:

$$\begin{aligned} n &= 4.65, & \text{Re}_p &\leq 0.2 \\ n &= 4.35 \text{Re}_p^{-0.03}, & 0.2 &\leq \text{Re}_p \leq 1 \\ n &= 4.45 \text{Re}_p^{-0.1}, & 1 &\leq \text{Re}_p \leq 500 \\ n &= 2.39, & \text{Re}_p &\leq 500 \end{aligned}$$

Brenner (1958) obtained the viscosity of the suspension by calculating the additional mechanical energy dissipation due to the presence of the particles in a homogeneous fluid. The analysis entails solving the Navier-Stokes equations for creeping flow. Happel (1957) postulated that the disturbance due to each sphere is confined to a frictionless envelope surrounding it. Their final expression for the suspension viscosity is:

$$\mu_p = \mu_f (1 + 5.5 \alpha \psi) \quad (5)$$

$$\text{where } \psi = 1 + \frac{55\gamma^3}{10} \left(\frac{4\gamma^7 + 10\gamma - \frac{84}{11}\gamma^2}{10(1-\gamma^{10}) - 25\gamma^3(1-\gamma^4)} \right) \quad (6)$$

γ being the ratio of the radii of the sphere and its envelope.

Equation (5) gives good agreement with experimental data for values of α in the range 10% to 50%. Frankel and Acrivos (1967) employed a quasi-lubrication analysis to obtain:

$$\mu_p \sim \mu_f C^1 \left(\frac{(\epsilon/\epsilon_s)^{1/3}}{1 - (\epsilon/\epsilon_s)^{1/3}} \right) \quad (7)$$

where C^1 is a constant depending on the particle configuration.

Equation (7) is valid for values of ϵ/ϵ_s close to unity, ϵ_s being the saturation voidage. Equation (7) has been found to represent experimental data very well in the range of (ϵ/ϵ_s) from 0.8 to 1.0. Murray (1965) presented Carrier and Cashwell's analysis (1956) and suggested that this should be appropriate for fluidized beds. They

considered the relative motion of two layers of particles and obtained from hydrodynamic considerations:

$$\mu_p = \mu_f A (1-\epsilon)/(\epsilon-\epsilon_s) \quad (8)$$

where A is a constant that depends upon the geometry of the particle configuration of O(1). Considering a fluidized bed at incipient fluidization, $\epsilon = \epsilon_m \approx 0.5$, we find that $\mu_p \approx 3\mu_f$. For air at normal atmospheric temperature, $\mu_f = 0.0001827$ poise, and hence $\mu_p \approx 0.0006$ poise. The bulk viscosity of the suspension was also derived theoretically, thus:

$$K_p = \mu_f B \left((1-\epsilon)/(\epsilon-\epsilon_s) \right)^3 \quad (9)$$

This expression was obtained by considering the relative separation of two cylinders, B is a constant which depends on the geometry of the particle configuration, of O(1). For an air-fluidized bed at incipient fluidization, we find that $K_p \approx 0.006$ poise. Both equations (8) and (9) are applicable in the range ϵ_s to 0.7. Bagnold (1954, 1956) studied experimentally the effect of neutrally buoyant particles in a shear flow between two concentric cylinders, one of which is rotated, in the range of ϵ encountered in fluidization. He concluded that interparticle collisions are the predominant factor in modifying the stress on the cylinders.

It can be seen from the experimental measurements which have been reported and from the theoretical expressions obtained by several workers to determine the viscosity of the fluidized bed that the viscosity of the fluidized state is by no means yet understood. In the first instance, apart from the airslide experiments and the concentric cylinder arrangement, all other experimental methods suffer from the fact that the measuring technique involves disturbing the flow conditions in the bed. In addition, all the

experimental work has involved the assumption that the fluidized bed behaves as a Newtonian fluid. As for theoretical expressions for the viscosity, the most extensive work uniquely concerned with fluidization in contradistinction to suspensions in general is that of Murray. His expressions for the shear and bulk viscosities are considered adequate to represent the viscosities of the particulate phase in heterogeneous fluidized beds.

CHAPTER THREE

FLOW OF A PURELY VISCOUS NON-NEWTONIAN FLUID OVER AN INCLINED CHANNEL

The most rheologically interesting type of flows are laminar flows that occur in sufficiently simple geometries to allow exact solutions to be obtained. These are the viscometric flows. The flow is analyzed so that the basic shear diagram, independent of the apparatus used, can be obtained. Consideration has to be given to such sources of error as end effects, slip at the wall, heating effects, and instability. The standard viscometric flows for non-Newtonian fluids are capillary flow and rotational flow. In both types of flows, if the slip velocity is not assumed equal to zero, then measurements must be made at several capillary and cylinder sizes respectively. Astarita, et al (1964) developed a method for the determination of the basic shear diagram of a purely viscous non-Newtonian fluid utilizing flow under gravity along an inclined two-dimensional channel - negligible channel width in comparison with height of fluid in the channel. It cannot be overemphasized that what is being sought is the determination of the basic shear diagram, where the shear stress in the fluid is plotted against the shear rate at the same point in the fluid. Thus, studies in which a mean shear rate is used do not give the basic shear diagram, but these are only useful in obtaining flow correlations between the Reynolds number and friction factor, these two being defined in an analogous manner to Newtonian fluid flow.

In this chapter we give the theory of steady, laminar, uni-directional gravity flow of a purely viscous non-Newtonian fluid over an inclined channel, the fluid exhibiting slip at the wall and unspecified rheological properties. The inclined channel is then used to determine the rheology of the fluid at low shear stresses. In the case of the flow of fluidized beds over inclined channels, we aim to obtain a series of shear diagrams for several fluidizing velocities. The rheological behaviour of the system is best represented in a shear diagram in which the shear stress is plotted as a function of the shear rate (velocity gradient) at the same point. It is noteworthy that as no assumption whatever is made as to the dependence of either the shear rate or the slip velocity on the shear stress, the theory described below is a general one, as shown in figure 1. In deriving a model for the flow, we make the following assumptions:

- (a) The flow is steady, laminar, and uni-directional. Therefore, viscoelasticity has no effect whatsoever on the pattern of flow, and the shear stress is a unique unknown function of the shear rate.
- (b) The velocity at the solid surface is either zero or some finite positive value U_s , the latter being a unique unspecified function of the shear stress at the wall.
- (c) The fluid velocity gradient in the vertical plane at the free surface of the fluid is zero; that is, no shear occurs between the free surface and the layer underneath it.
- (d) There are no waves formed at the free surface.

In the first instance, we consider ~~reducing~~ the question of reducing the geometry of the problem from three to two dimensions. This may be achieved without any assumption or approximation

LEGEND:

A - channel

B - fluid free surface

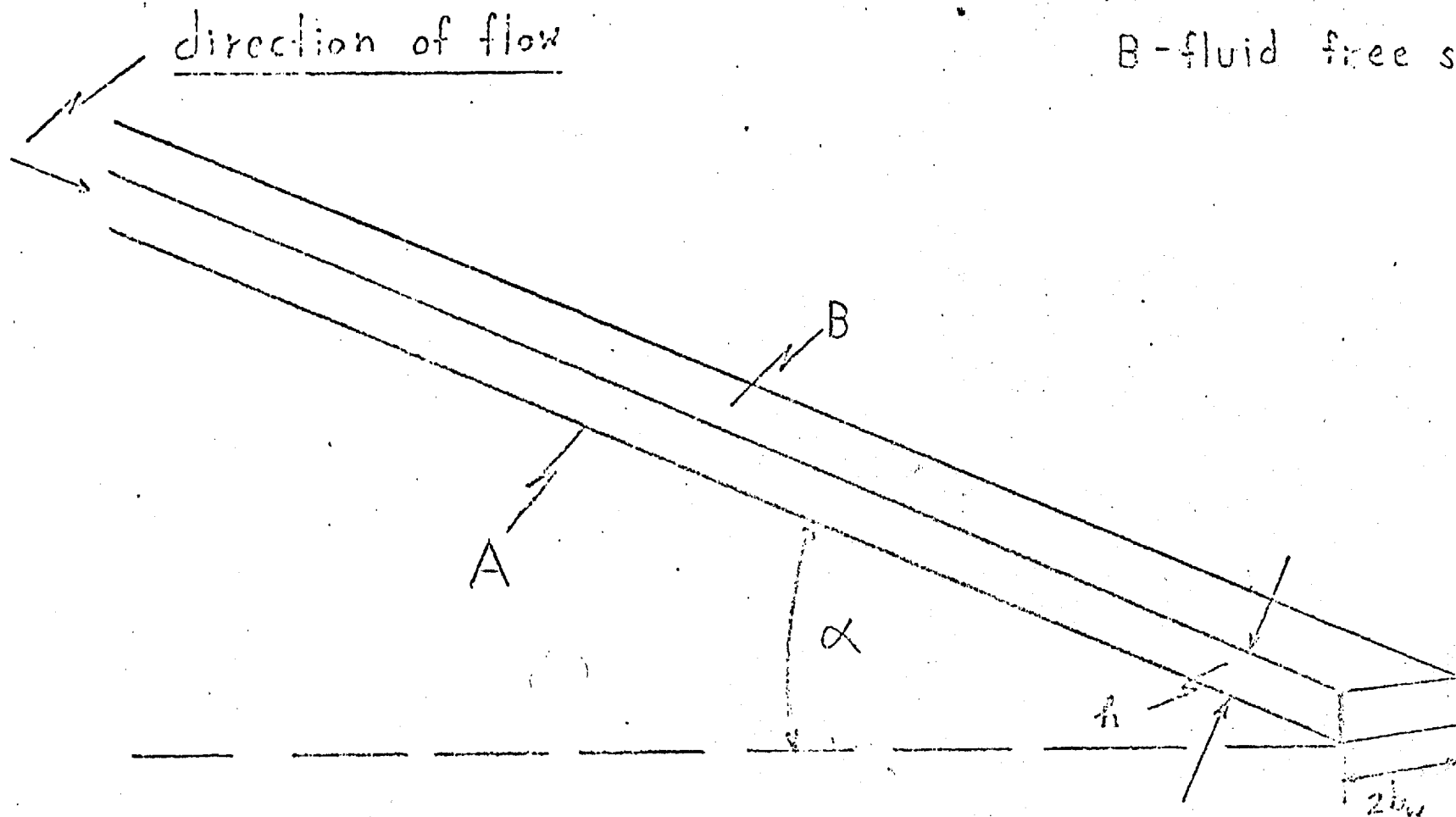


FIGURE 1: EXPERIMENTAL METHOD

concerning the ratio of the width of the channel and the fluid height; namely, the aspect ratio of the open channel.

The flowrate, Q , is given by:

$$\begin{aligned}
 Q &= \int_0^h \left(\int_{-b_w}^{b_w} u dy \right) dz \\
 &= \int_0^h q_z dz
 \end{aligned} \tag{1}$$

where q_z is the flowrate per unit height at any height z .

Differentiating w.r.t. h , we obtain:

$$\left(\frac{dQ}{dh} \right)_h = (q_z)_{z=h} = q$$

where q is the fluid flowrate per unit height at the free surface, i.e. at $z=h$. Thus, from a series of values of Q and h , we can obtain the corresponding values of q , which has the effect of reducing the problem from three to two dimensions.

The next question is that of the shear stress at the wall at the free surface. An averaged value - averaged over the "wetted" perimeter in the channel - of the wall shear stress at the free surface is obtained by an overall momentum balance, thus:

$$\begin{aligned}
 2(b_w + h) \tau_w &= 2db_w h g \sin \alpha \\
 \tau_w &= \frac{db_w h g \sin \alpha}{(b_w + h)} \\
 &= \kappa \beta \lambda
 \end{aligned} \tag{2}$$

where

$$\kappa = db_w g$$

$$\beta = \sin \alpha$$

$$\lambda = \frac{h}{(b_w + h)}$$

d is the fluid density

and α is the angle of inclination of the channel.

It should be borne in mind that the value of τ_w so obtained is the mean over the whole area of contact between the fluid and the wall and not just at the free surface. It is clear that as the wall shear stress at any point decreases with height, then this mean value will be higher than the actual value at the free surface.

Next, we determine a relation between q and τ_w . The flowrate per unit height at the free surface, q , can be expressed as follows:

$$q = 2 \int_0^{b_w} u dy \quad (3)$$

where u is the fluid velocity at any point in the free surface, a distance y from the mid-point of the free surface. Considering the free surface, the shear stress τ at any point is proportional to the distance from the mid-point of the free surface, thus:

$$\tau = \frac{y}{b_w} \tau_w \quad (4)$$

Hence, we can write equation (3) as follows:

$$q = \int_0^{\tau_w} u \frac{b_w}{\tau} d\tau \quad (5)$$

Now:

$$u = \int_y^{b_w} (-du/dy) dy + u_s \quad (6)$$

where $(-du/dy)$ is the unknown velocity gradient at any point in the free surface, and it is some function of the shear stress τ at that point in the free surface. Substituting for u from equation (6) into equation (3), we have:

$$q = 2 \int_0^{\tau_w} \int_{\tau}^{\tau_w} (-du/dy) \frac{b_w^2}{\tau} d\tau + 2b_w u_s \quad (7)$$

Integrating the first term on the right-hand side of equation (7) by parts, we write:

$$q = \frac{2b_w^2}{\tau_w^2} \int_0^{\tau_w} \tau (-du/dy) d\tau + 2b_w u_s \quad (8)$$

The problem is that we now have to solve the integral equation (8) with two unknowns: $(-du/dy)$ being an unknown function of τ , and u_s being an unknown function of τ_w . Instead of solving equation (8) directly, we differentiate it w.r.t. β and λ respectively and obtain two equations. These two equations are used to obtain a linear partial differential equation in u_s in the independent variables β and λ . This equation is solved for u_s , from which $(-du/dy)_w$ is obtained as a function of β and λ . In the first place, λ is varied while β is kept constant. We put $\tau = \beta s$ where s is a dummy variable for λ .

Differentiating w.r.t. s , we have:

$$\frac{d\tau}{ds} = \beta \quad (9)$$

Substituting into equation (8), we have:

$$q = \frac{2b_w^2}{\lambda^2} \int_0^\lambda s (-du/dy) ds + 2b_w u_s \quad (10)$$

Differentiating equation (10) w.r.t. λ , we have:

$$\frac{\partial q}{\partial \lambda} = \frac{2b_w^2}{\lambda} (-du/dy)_w - \frac{2}{\lambda} (q - 2b_w u_s) + 2b_w \frac{\partial u_s}{\partial \lambda} \quad (11)$$

where $(-du/dy)_w$ is the value of $(-du/dy)$ at the wall. We now vary β while keeping λ constant. We put $\tau = \lambda t$, where t is a dummy variable for λ . Differentiating w.r.t. t , we obtain:

$$\frac{d\tau}{dt} = \lambda \quad (12)$$

Substituting into equation (8), we have:

$$q = \frac{2b_w^2}{\beta^2} \int_0^\beta t (-du/dy) dt + 2b_w u_s \quad (13)$$

Differentiating equation (13) w.r.t. β , we have:

$$\frac{\partial q}{\partial \beta} = \frac{2b_w^2}{\beta} (-du/dy)_w - \frac{2}{\beta} (q - 2b_w u_s) + 2b_w \frac{u_s}{\partial \beta} \quad (14)$$

Eliminating $(-du/dy)_w$ between equation (11) and (14), we have:

$$2b_w \beta \frac{\partial u_s}{\partial \beta} - 2b_w h \frac{\partial u_s}{\partial \lambda} = \beta \frac{\partial q}{\partial \beta} - \lambda \frac{\partial q}{\partial \lambda} \quad (15)$$

Equation (15) is a linear partial ^{differential equation} in u_s , β and λ being the independent variables. Equation (15) can be solved analytically provided q is expressed analytically in terms of β and λ . The solution of equation (15) is obtained via solving the auxilliary ordinary differential equations:

$$\frac{d\beta}{2b_w \beta} = \frac{d\lambda}{2b_w \lambda} = \frac{du_s}{\beta \frac{\partial q}{\partial \beta} - \lambda \frac{\partial q}{\partial \lambda}} \quad (16)$$

The equations (16) are solved to give:

$$u_s = \int \frac{(\beta \frac{\partial q}{\partial \beta} - \lambda \frac{\partial q}{\partial \lambda})}{2b_w \beta} d\beta + C_1(\lambda) \quad (17)$$

and

$$u_s = \int \frac{(\beta \frac{\partial q}{\partial \beta} - \lambda \frac{\partial q}{\partial \lambda})}{2b_w \lambda} d\lambda + C_2(\beta) \quad (18)$$

i.e.

$$u_s = \frac{q}{2b_w} - \frac{\lambda}{2b_w} \int \frac{1}{\beta} \frac{\partial q}{\partial \lambda} d\beta + C_1(\lambda) \quad (19)$$

and

$$u_s = \frac{q}{2b_w} - \frac{\beta}{2b_w} \int \frac{1}{\lambda} \frac{\partial q}{\partial \beta} d\lambda + C_2(\beta) \quad (20)$$

where $C_1(\lambda)$ and $C_2(\beta)$ are arbitrary functions determined by the boundary conditions:

$$\beta = 0, \quad u_s = 0, \quad \text{all } \lambda$$

and

$$\lambda = 0, \quad u_s = 0, \quad \text{all } \beta$$

Having obtained u_s as a function of β and λ , $\frac{\partial u_s}{\partial \beta}$ and $\frac{\partial u_s}{\partial \lambda}$ can be immediately obtained. Then, substituting into equation (8) we obtain:

$$(-du/dy)_w = \frac{\lambda}{2b_w^2} \left[\frac{\partial q}{\partial \lambda} - 2b_w \frac{\partial u_s}{\partial \lambda} + \frac{2}{\lambda} (q - 2b_w u_s) \right] \quad (21)$$

Hence, the shear rate - velocity gradient - is obtained from equation (21) as a function of β and λ .

In conclusion, for a given set of values of β and λ , we obtain the corresponding values of τ_w and $(-du/dy)_w$. Since τ_w and $(-du/dy)_w$ are the shear stress and shear rate at the wall; that is, at the same point. Consequently, the shear diagram of the fluid is obtained by plotting τ_w against $(-du/dy)_w$.

CHAPTER FOUR

EXPERIMENTAL APPARATUS

The experimental arrangement consisted of three parts - depicted in figure 2:

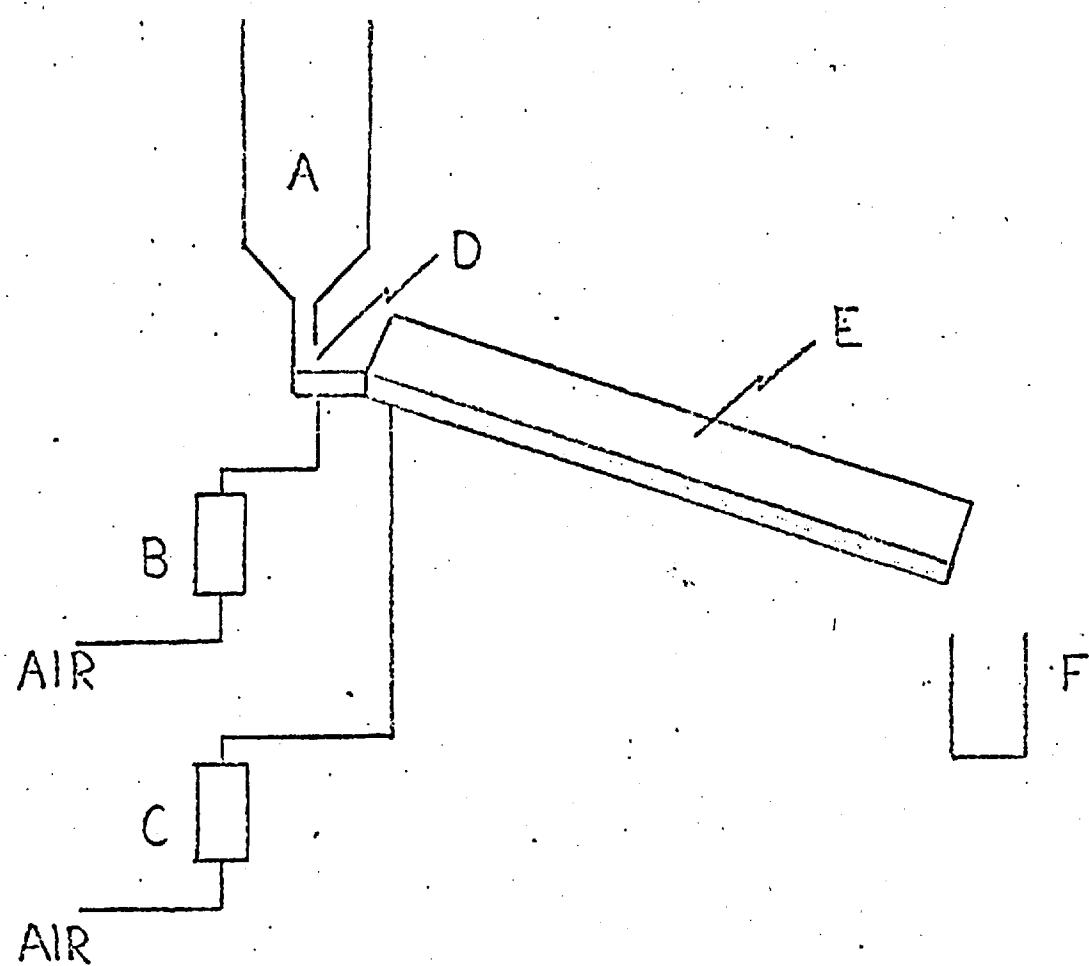
- 1) Inlet hopper
- 2) Flow Channel - the airslide
- 3) Measuring apparatus

1) Inlet hopper

In order to facilitate continuous flow through the test section - part of the flow channel - a specially designed hopper was used as the inlet device from which the fluidized material passes to the flow channel. The hopper consisted of three sections. The lower part starting from the base of the hopper was 23 cm. square and 13 cm. high. In this narrow section of the hopper, the solid material was fluidized by air blown through the porous plastic base of the hopper. The fluidized material flowed from the hopper through a rectangular opening of variable height the side of this narrow part of the hopper into the flow channel. The middle section of the hopper increased gradually in cross section over a height of 63 cm. starting from the lower fluidizing section. The upper part of the hopper was 62 cm. square and 91 cm. high.

2) Flow channel

The fluidized material flowed from the hopper to the flow channel which was an inclined plane with perspex sides 7 cm. high



LEGEND :

- A - hopper
- B - hopper rotameter
- C - channel rotameter
- D - hopper slot
- E - channel
- F - outflow vessel

FIGURE 2: SCHEMATIC DIAGRAM OF
EXPERIMENTAL APPARATUS

and with a base of porous plastic through which air was blown to fluidize the solid material in the channel underneath the fluidizing tile. The channel was 180 cm. long and 8.4 cm. wide. The fluidizing air was distributed longitudinally by a 1-inch diameter pipe with $\frac{1}{8}$ inch diameter holes equally spaced along its length. The inclination of the channel could be varied from 0° to 10° to the horizontal.

The particles were collected in a truck (normally used in mining work) and returned to the hopper in batches. This method was preferred to a solids pumping apparatus largely for economic reasons.

3) Measuring apparatus

In the experiments carried out, three sets of measurements were made

- (a) Flowrate of solid material
- (b) Flowrate of fluidizing air
- (c) Bed height in the channel

(a) Flowrate of solid material

The weight of fluidized material flowing through the system was measured by collecting the particles from the end of the air-slide in a container. A bulk flow balance to measure the collected solids weight was used, reading in increments of 50 g., at times indicated by a stop-clock reading in increments of 5 seconds.

(b) Flowrate of fluidizing material

The two fluidizing air flows - for the channel and for the hopper - were measured by a pair of rotameters - Metric 24 and Metric 65 respectively for the hopper and for the channel -, although it was only the fluidizing air flow through the channel that was required in the subsequent analysis of the experimental results.

The flow ranges of the two rotameters are as follows:

Metric 24: 20-200 $\text{cm}^3/\text{min.}$

Metric 65: 200-2000 $\text{cm}^3/\text{min.}$

(c) Bed height in the channel

The bed height was measured using a centimeter scale calibrated at increments of 0.1 cm. in conjunction with a micrometer which enabled measurements to be made at increments of 0.01 cm. The bed height was measured at several points along the channel and as a necessary condition to satisfy the uni-directional flow assumptions, only those readings which gave an absolute variation of bed height of 0.1 cm. across the channel were accepted for analysis.

CHAPTER FIVE

THEORETICAL MODEL OF HETEROGENEOUS FLUIDIZED BEDS

It was pointed out in chapter 2 that the two-phase theory of fluidization represents a turning point in the attempts to understand heterogeneous fluidization. Almost all of the subsequent theoretical work used the theory as a starting point in order to investigate the two main phenomena in the bed; namely, bubble motion and the flow of the fluidizing fluid through the particulate phase. The two-phase theory assumes that above incipient fluidization, the excess fluid - over that required to incipiently fluidize the bed - passes through the bed in the form of bubbles, while the particulate phase remains at incipient fluidization conditions. This can be represented as follows:

$$U = U_{m_f} + U_b \quad (1)$$

where U_b = the superficial velocity of the bubble phase

$$= \frac{\text{the observed bubble flow}}{\text{cross-sectional area of the bed}}$$

It is this question of the distribution of the fluid flow between the bubble and the particulate phases which lies at the heart of the problem of heterogeneous fluidization. If this distribution can be determined, then the behaviour of heterogeneous fluidized bed is consequently understood in various physico-chemical processes. As for bubble motion, the two-phase theory assumes that the velocity of a bubble in a swarm of bubbles is the same as that of a single bubble, i.e.:

$$U_b = U_{bo} \quad (2)$$

where U_b = actual velocity of a single bubble in a fluidizing bed. Although the term "bubble" is used generously in fluidization nomenclature, there are of course no voids with well-defined boundaries as is the case of gas bubbles in liquids. The term "bubble" suggested itself from the earliest observations of aggregatively-fluidized beds. Its use was extended when it was realized that the gas-liquid analogy could assist in explaining the behaviour and characteristics of aggregatively-fluidized beds. This assumption was made by Davidson and Harrison (1966).

The model proposed here constitutes an interpretation of the measured viscosity of a fluidized bed in terms of its structure and the flow conditions in the bed. While it appears justifiable to treat homogeneous fluidized beds as Newtonian beds, the assumption of Newtonian behaviour needs to be relaxed for heterogeneous fluidized systems. The conclusion arrived at is that if a fluidized bed exhibits Newtonian behaviour, then it is homogeneous or quasi-homogeneous in that the bubbles are sufficiently unstable that their rate of formation equals their rate of dissolution into the particulate phase. On the other hand, non-Newtonian behaviour indicates heterogeneous fluidization, with the presence of bubbles.

1) Assumptions of the model

- phase
- (a) The fundamental assumption of the two^{phase} model is employed, that is, the aggregatively-fluidized bed is composed of two phases: the continuous particulate phase and the discontinuous bubble phase, the two being differentiated by the total absence of solid particles in the bubble phase.
- (b) The continuous particulate phase consists of solid particles

uniformly distributed in the fluid stream. This phase has a uniform voidage ϵ_c , and the fluid is moving through the phase at a uniform actual velocity u_c . This assumption is essentially a generalization of the two-phase theory assumption that the particulate phase is at incipient fluidization. In this model, it is only assumed that the voidage and flow conditions in the particulate phase are constant throughout, but do not necessarily attain incipient fluidization conditions. Clearly, voidage variations in the particulate phase - with height and with distance from bubbles, for example, are neglected. This assumption is likely to represent the conditions in a fluidized bed with numerous bubbles; that is, when the fraction occupied by bubbles is significant.

(c) The motion of the solid particles in the particulate phase is entirely due to bubble motion in the bed. This assumption is best studied by looking at solids motion in a homogeneous bed. Here, the particles undergo vibratory or pulsatory motion about fixed positions. Reverting to the heterogeneous bed, in addition to this pulsation, the particles undergo translatory motion due to the rise of bubbles through the bed. It is clear that the translation of the solid particle in the bubble cloud and wake is of a much greater magnitude than the distance the particles migrate by virtue of their oscillation about fixed positions.

(d) The discontinuous phase consists of non-interacting spherical bubbles of uniform diameter d_b , rising at a uniform velocity u_b , through the particulate phase. In actual fluidized beds, bubbles are multi-size, as the size is a function of height in the bed, and the bubbles interact by coalescence. However, in considering a shallow bed, there is little variation of bubble diameter and velocity with height, but the bubbles maintain their interaction.

(e) There are two contributions to the overall mean voidage of the fluidized bed : one from the bubble phase, and the other from the particulate phase. While the voidage fraction of the particulate phase can take values in the range between the saturation voidage fraction and unity, the voidage fraction within the bubble phase is unity, since the bubbles are assumed to lack solid particles. It may be worth mentioning that the overall mean voidage represents the volumetric fraction of fluid present in the bed and not in any particular phase. Consequently, it does not by itself provide any information about either the bubble or the particulate phase.

(f) The fluid flowing through the fluidized bed distributes itself through the two phases so as to satisfy the equations of motion of both the bubbles and the solid particles in the particulate phase. This assumption stems from the fact that the fluid flow distribution determines the structure of the bubble and the particulate phases, which in turn determines the motion of both the bubbles and the particles.

2) Continuity of fluid flow through the fluidized bed

We consider the continuity of fluid flow through a bed of cross-sectional area S . The fluid flowing through the bed distributes itself between the two phases as follows:

(a) Fluid flow through the particulate phase: This component is independent of bubble motion and is simply given by $Su_c \epsilon_c$.

(b) Fluid flow through the bubble phase: This component consists of two contributions. In the first instance, there is the fluid flowing due to the rise of bubbles, and this is given by $Su_b \epsilon_b$. The other contribution is that of the fluid flowing through the cross-sectional area of the bed occupied by bubbles, at a velocity equal to that of the fluid flowing through the particulate phase. Clearly,

this is independent of bubble velocity, and it is given by

$S(1-\epsilon_b)\epsilon_b u_c \epsilon_c$. This term was referred to by Lockett, et al (1967) as the bubble through flow.

The continuity equation can then be written as follows:

$$\begin{aligned} U &= \epsilon_b(u_c \epsilon_c + u_b) + (1-\epsilon_b) u_c \epsilon_c \\ &= u_b \epsilon_b + u_c \epsilon_c \end{aligned} \quad (3)$$

where by definition:

$$\epsilon_b = \frac{\epsilon_m - \epsilon_c}{1 - \epsilon_c} \quad (4)$$

3) Motion of a single bubble in a fluidized bed

Davidson and Harrison (1963) have correlated the velocity of rise of a single bubble in a fluidized bed with its size by the following expression:

$$u_{bo} = 0.71 g^{\frac{1}{2}} d_b^{\frac{1}{2}} \quad (5)$$

where u_{bo} = velocity of rise of a single bubble in a fluidized bed and d_b = equivalent diameter of the bubble. Equation (5) shows the close analogy of single bubble rise in an inviscid liquid and in a fluidized bed.

We consider a single bubble rising under Stoke's law conditions (small size and or low velocity of rise) ascending the particulate phase of a fluidized bed. Its equation of motion is therefore:

$$3\pi\mu_p d_b (u_{mf} - u_{bo}) \epsilon_{mf} = d_b \frac{\pi}{6} d_b^3 g(1-\epsilon_{mf}) \quad (6)$$

$$\text{where } \mu_p = \mu_f \left(\frac{1-\epsilon_c}{\epsilon_c - \epsilon_s} \right) \quad (7)$$

Here, μ_p denotes the viscosity of the particulate phase, and the Stoke's drag force exerted by the particulate phase on the bubble is balanced by the buoyancy force.

4) Motion of a representative bubble in a bubble swarm in a fluidized bed

In order to simplify the analysis, we first hold the particles stationary by imposing a downward vertical velocity u_b on the system. The resulting situation is one where all the bubbles are stationary, and hence all the particles are fixed in position, whilst the fluid flows through the particulate phase at a velocity $(u_c - u_b)$. Further, we postulate that the bubbles are spherical solid particles of zero density, and we consider a representative bubble in a bubble swarm whose viscosity μ^* is, in fact, the viscosity of the fluidized bed as a whole. Assuming Stoke's law for this representative bubble, we obtain:

$$3\pi \mu^* d_b (u_c - u_b) \epsilon_c = d_s \frac{\pi}{6} d_b^3 g(1 - \epsilon_c) \quad (8)$$

In equation, μ^* rather than μ_p so as to take into account the presence of other bubbles in the swarm and consequently to facilitate the consideration of the equilibrium of a representative bubble. In fact, the presence of other bubbles is incorporated in the viscosity and buoyancy terms of the medium in which the representative bubble is moving, this medium being the swarm of bubbles in the particulate phase, which constitutes the heterogeneous fluidized bed.

5) Motion of a representative particle in the particulate phase

Following the simplifying procedure adopted in section 4) of this chapter, we impose a velocity u_b vertically downwards, thus holding both bubbles and particles stationary. The particles are then in equilibrium under the action of three forces, these being the drag, the buoyancy, and the inter-particle force. The latter is defined in this way, as the difference between the drag and buoyancy forces, and it is zero when the particles are incipiently

fluidized or at any higher degree of fluidization. We consider a representative particle in equilibrium in the particulate phase, and assuming Stoke's law we have:

$$3\pi \mu_p d_b (u_c - u_b) \epsilon_c = d_s \pi \frac{d_p^3}{6} g \epsilon_c - I_p \quad (9)$$

where I_p is the inter-particle force.

6) Shear viscosity of the heterogeneous fluidized bed

In the model formulated here, the heterogeneous fluidized bed is viewed as a swarm of bubble rising through a particulate continuum. Consequently, its rheological behaviour is best studied by analogy to a swarm of bubbles in a liquid. This is a different point of view from that advanced by Grace (1970) that it is the viscosity of the particulate phase which is the relevant one in studying the rheological behaviour of fluidized bed, as this conception ignores the presence of bubbles in the bed.

Batchelor (1967) derived the viscosity of a dilute swarm of gas bubbles in a liquid from the average additional rate of viscous dissipation per unit volume due to the presence of the bubbles in the liquid. He obtained:

$$\frac{\mu^*}{\mu_f} = 1 + \epsilon_b \quad (10)$$

By analogy, the viscosity of the heterogeneous fluidized bed is given by:

$$\frac{\mu^*}{\mu_p} = 1 + \epsilon_b \quad (11)$$

Equation (11) shows the dependence of the viscosity of the fluidized bed on the bubble proportion of the bed. Clearly, for a given superficial fluidizing velocity, a change in the bubble proportion due to changes in the stability of bubbles in the bed leads to a

variation in the viscosity of the bed. This viscosity variation suggests non-Newtonian behaviour. In general, the application of an external shearing force leads to a change in bubble stability in the bed, causing a change in the bubble proportion in the bed. Consequently, variation of bed viscosity with shear stress indicates that the system is heterogeneous with stable bubbles. An increase of viscosity with shear stress suggests that bubbles are becoming less stable, and the reverse is true. The bubble diameter d_b is used as a criterion for bubble stability. Hence, as an increase in μ^* for a given fluidizing velocity U and overall mean voidage ϵ_m , leads to a decrease in d_b , it is deduced that the bubbles tend to be less stable with increasing bed viscosity. This question is discussed fully in Chapter 6. On the other hand, Newtonian behaviour implies the absence of bubbles and that the bed is quasi-homogeneous, where the rate of formation of bubbles equals the rate of their collapse into the particulate phase.

7) Comment on the use of the model

In the model outlined above, the heterogeneous fluidized bed is characterized by equations (3), (4), (5), (6), (7), (8), (9) and (11). By suitable rheological studies, the viscosity of the bed μ^* is determined as a function of shear stress for a given superficial fluidizing velocity U and overall mean voidage ϵ_m . Hence, we have eight algebraic equations in eight unknown variables; namely, u_b , ϵ_b , u_c , ϵ_c , u_p , u_{bo} , d_b and I_p . These eight equations are then solved in order to characterize the bed.

As was pointed out in section 6) of this chapter, Newtonian behaviour of the bed suggests that the bed is quasi-homogeneous, and then the measured bed viscosity μ^* is in fact that of the particulate phase, μ_p , and all variables related to the bubble phase

lose their physical significance as was pointed out by Teoreanu (1962).

Since Stoke's law was assumed in using equations (6), (8) and (9), it is necessary to check at the end of the solution of the eight equations above is made to see how valid that assumption is for each of the equations (6), (8) and (9). It is worth pointing out that if it were thought that Stoke's law was unlikely to hold in the motion of a single bubble, a representative bubble, or a representative particle, then the appropriate equation of motion - (6), (8) and (9) respectively - have to be modified to determine the drag term for flow conditions outside the Stoke's law range. In other words, the model can be modified and used provided suitable modifications are made in equations (6), (8) and (9).

CHAPTER SIX

RESULTS AND DISCUSSION

1) Solid and fluid properties

The solid material used in the flow test experiments was sand with the following properties:

$$\text{density, } d_s = 2.64 \text{ g./cm}^3$$

$$\text{average particle size, } d_p = 266 \text{ } \mu\text{m}$$

Air was the fluidizing medium employed for the experimental work. Its properties, obtained from CRC "Handbook of Chemistry and Physics", 46th Edition (1965-1966), are given below:

$$\text{density, } d_f = 0.001213 \text{ g./cm}^3$$

$$\text{viscosity, } \mu_f = 182.7 \times 10^{-6} \text{ poise}$$

The values were taken at 18°C.

2) Minimum fluidization velocity and velocity-voidage relationship

Separate experiments were performed in a two-dimensional fluidized bed to obtain the minimum fluidization velocity for the system under consideration. For this purpose, the method of Jameson and Mottamedi (1968) was used. The value obtained is 5.06 cm/sec. for minimum fluidization.

Due to the absence of an analytical relationship between the superficial fluidizing velocity and the mean voidage for aggregative fluidization, a graphical correlation based on Leva's work was used. Leva's extensive experimental work on the measurement of the overall mean voidage of aggregatively-fluidized beds was carried out for fluidized beds 10-100 cm. high; namely, deeper beds than the shallow

beds with which our experiments were performed, these being up to 2 cm. high. Furthermore, the beds investigated by Leva were stationary while we considered beds in motion under the action of gravity. Consequently, as there are at present no experimental methods of measuring the overall mean voidage without making numerous assumptions whose validity cannot be ascertained, Leva's graphical method is utilized to relate the mean voidage to the superficial fluidizing velocity. For the fluidizing velocities used in our experiments, the corresponding mean voidage values are listed below, along with the mean voidage values obtained by using the Richardson-Zaki equation for particulate fluidization:

Superficial fluidizing velocity $U(\text{cm/sec.})$	Mean voidage ϵ_m	Voidage from Richardson-Zaki equation
8.38	0.550	0.510
8.82	0.562	0.514
9.33	0.564	0.519
9.73	0.577	0.523
10.23	0.585	0.528
10.68	0.591	0.532
11.22	0.600	0.536
11.70	0.635	0.540
12.22	0.640	0.545
12.76	0.642	0.549

3) Experimental flowrate data

The data of the flow test experiments carried out are tabulated in appendix 1. For each superficial fluidizing velocity, mass flowrates have been measured at different bed heights for several angles of inclination of the flow channel. Experiments have been conducted for degrees of fluidization U/U_{mf} ranging from 1.5 to 2.5. The lower limit of this range is determined by the fact that steady flow under gravity could not be obtained for $U/U_{mf} < 1.5$, as the flow channel

was either blocked at lower degrees of fluidization, or the flow occurred in surges, i.e. it was unsteady. As for the upper limit of the range of U/U_{mf} , it was observed that for $U/U_{mf} > 2.5$, waves appeared on the surface of the flowing bed. Consequently, for $U/U_{mf} < 1.5$ and for $U/U_{mf} > 2.5$, one of the fundamental assumptions of our theoretical model was invalidated, and our experiments were restricted to the range $1.5 < U/U_{mf} < 2.5$.

At the lower fluidizing velocities experiments were performed only at large (5° - 7°) angles of channel inclination, as it was found that for smaller angles flow ceased or became unsteady. On the other hand, at higher fluidizing velocities the whole range (1° - 7°) of angle of channel inclination was covered. Hence, more data points were obtained at higher superficial fluidizing velocities than at the lower velocities.

It was found that the mass flowrate is an increasing function of both bed height and angle of inclination of the channel. This is expected and it was continued by the experimental data. It is worth noting that while bubbles could be observed bursting at the surface of the bed at the low fluidizing velocities studied, the bed takes a more homogeneous appearance with increasing fluidizing velocities and at the highest fluidizing velocities investigated hardly any bubbles could be seen at the surface of the bed.

4) Two-variable surface fitting

In order to solve the linear partial differential equation of chapter 3 for U_s , it is necessary to obtain the volumetric flowrate Q as an analytical expression in terms of β and λ , where

$$\beta = \sin \alpha \quad (1)$$

$$\text{and} \quad \lambda = \frac{h}{h+b_w} \quad (2)$$

The process of two-variable surface fitting is outlined in appendix 2.

The final expressions for the slip velocity and the shear rate are respectively:

$$U_s = \frac{\beta^2 \lambda}{2b_w} \left[a_1(-2n_1 + n_2) + n_2 \beta \left(-4n_1 + \frac{4}{3}n_2 \right) \right] \quad (3)$$

$$\text{and } \left(-\frac{du}{dy} \right)_w = \frac{\beta^2 \lambda}{2b_w} \left[(6 + 4n_1 - 2n_2)a_1 + (6 - 8n_1 - \frac{4}{3}n_2)a_2 \beta + 6a_3 \lambda \right] \quad (4)$$

The significance of the constants a_1 , a_2 , a_3 , n_1 and n_2 is discussed in appendix 2. The results of the surface-fitting procedure and the values of U_s calculated from equation (3), $(-du/dy)_w$ obtained from equation (4), and τ_w obtained from equation () of chapter 3 are tabulated below in table 1. As is shown in appendix 2, Q is fitted as a polynomial in β and λ using the least squares criterion and subject to the following constraints:

$$Q > 0, q > 0, U_s > 0, (-du/dy)_w > 0 \quad (5)$$

It should be noted that the problem of fitting Q in terms of β and λ subject to the constraints (5) is essentially a mathematical programming problem. Unfortunately, the available computer routines did not assist in finding a good surface fit as they were not sufficiently general, and consequently we resorted to a less satisfactory unconstrained surface fit, and by iterative variation of the constants n_1 and n_2 , we found their values which would satisfy the constraints (5) for the highest number of points in any set of data.

5) Shear diagrams

Figure 3-12 display curves representing the basic shear diagrams obtained for the fluidized system. The first point to be made is that of units. τ_w is in dyne/cm^2 , and $(-du/dy)_w$ is in $(\text{sec}^{-1} \times \frac{1}{b_w})$. Further, it was found in sets (2) and (3), very few points satisfied the constraints (5). In fact, for each of sets (2) and (3), only five such points could be obtained. This clearly showed the inadequacy

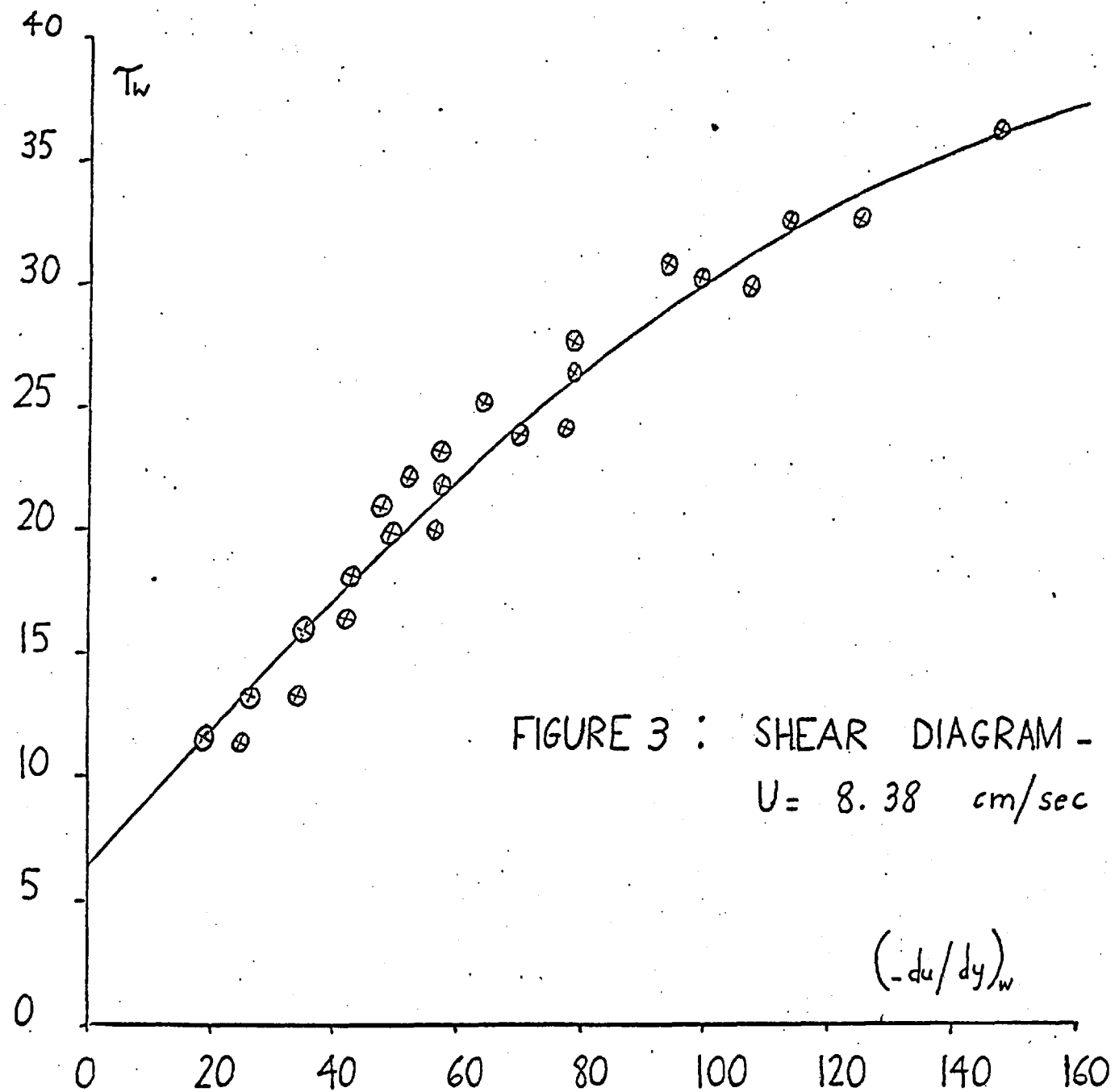


FIGURE 3 : SHEAR DIAGRAM -
 $U = 8.38 \text{ cm/sec}$

FIGURE 4: SHEAR DIAGRAM -

$U = 8.82 \text{ cm/sec}$

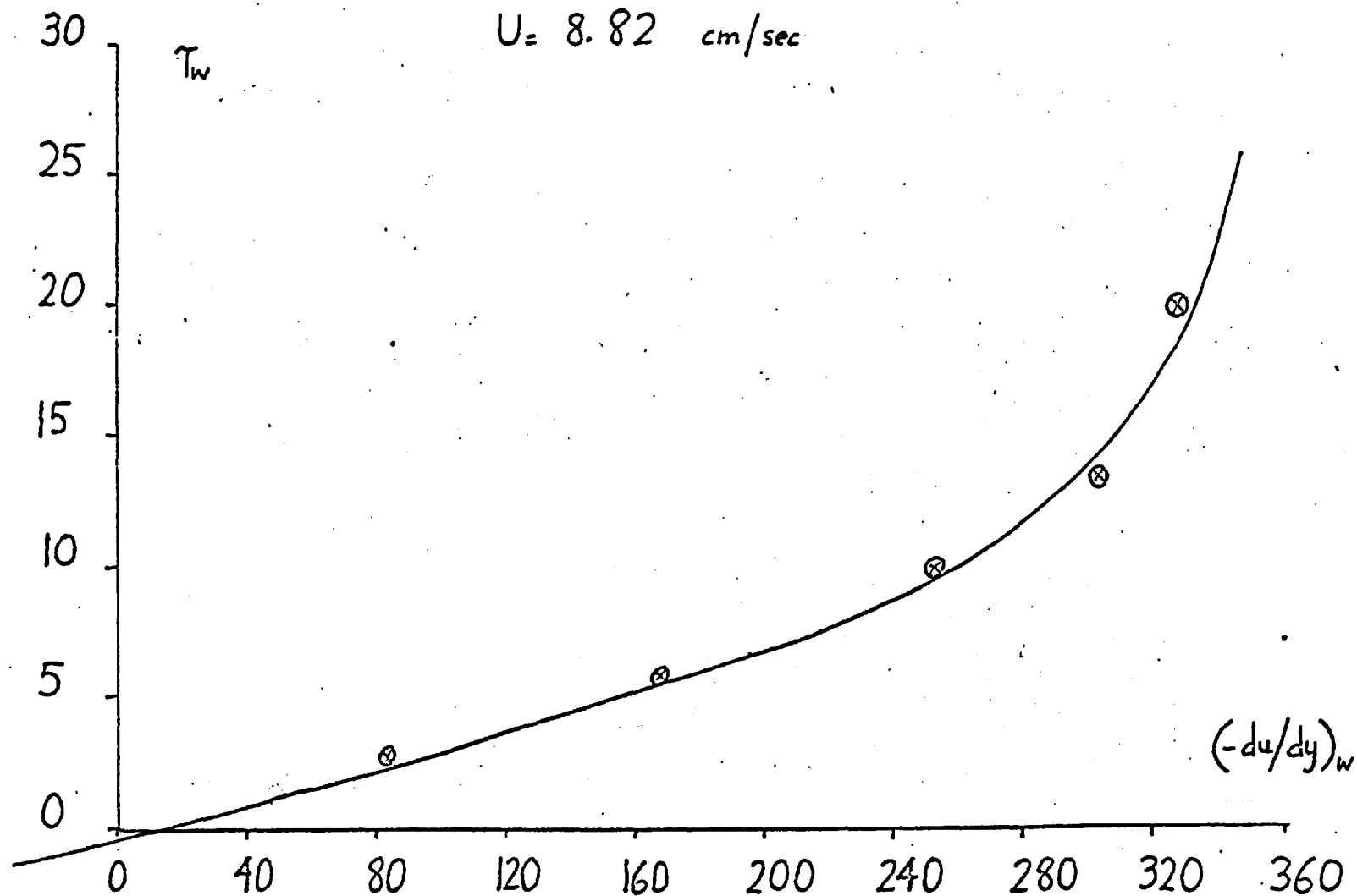
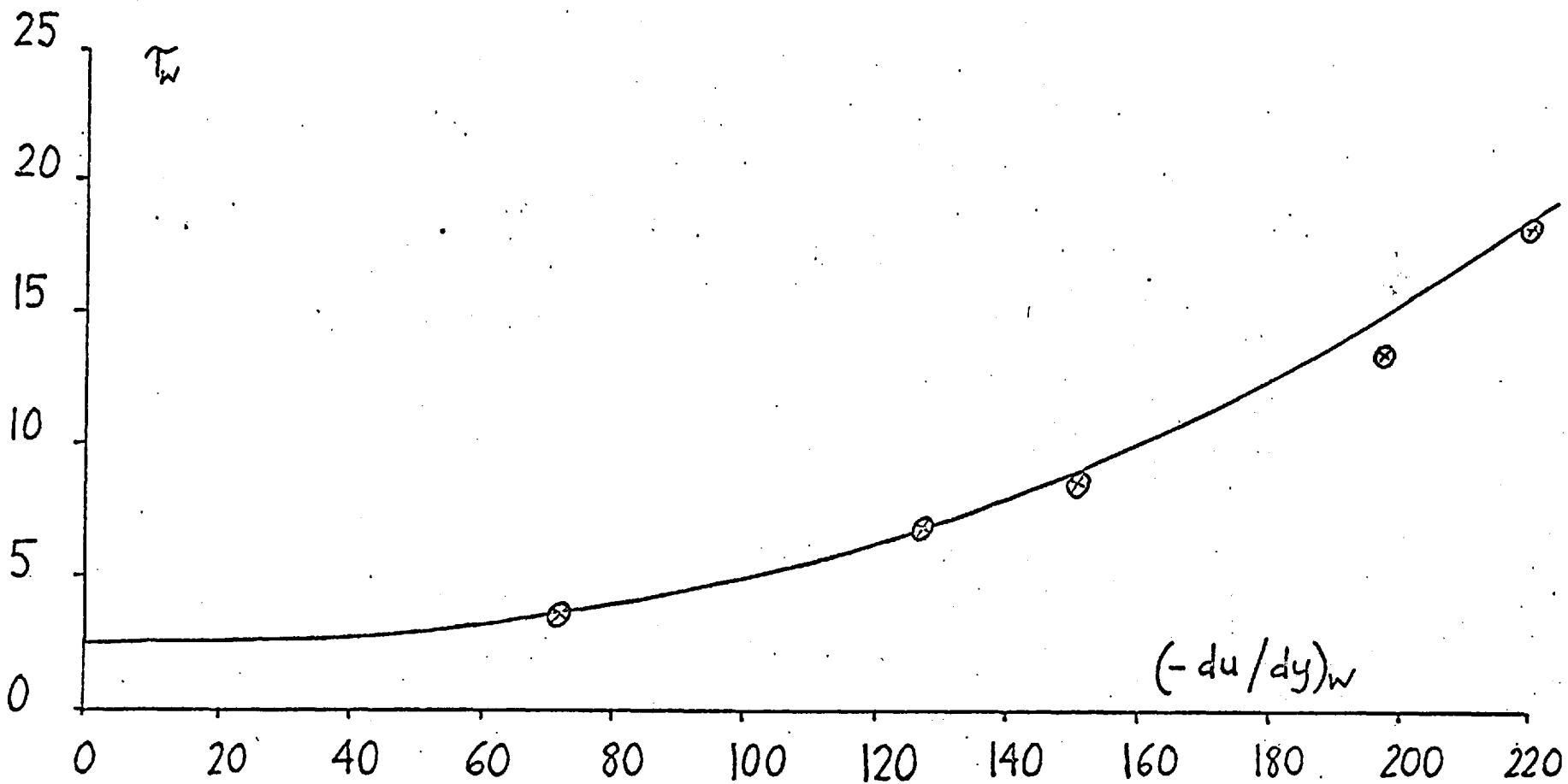


FIGURE 5: SHEAR DIAGRAM - $U = 9.33 \text{ cm/sec}$



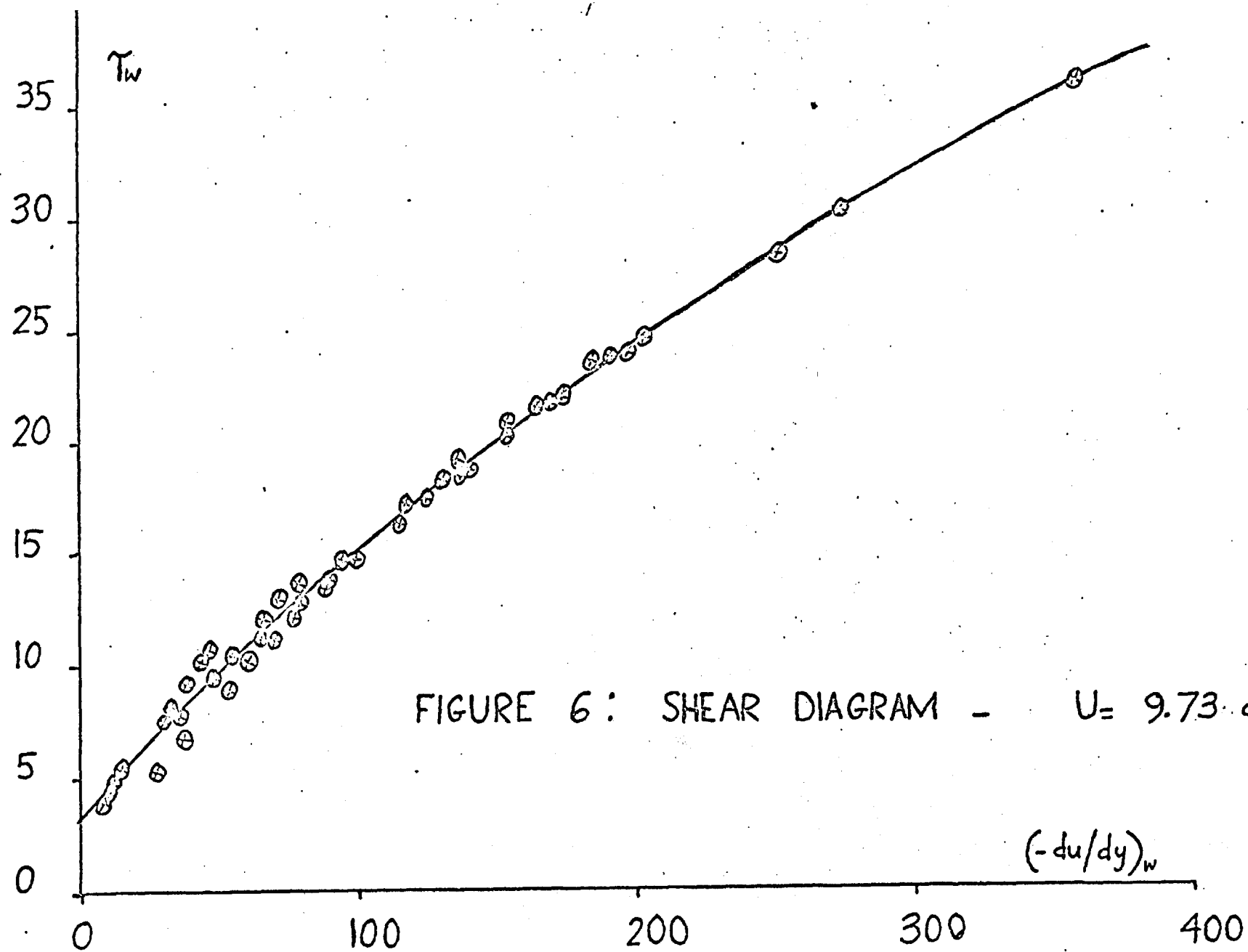
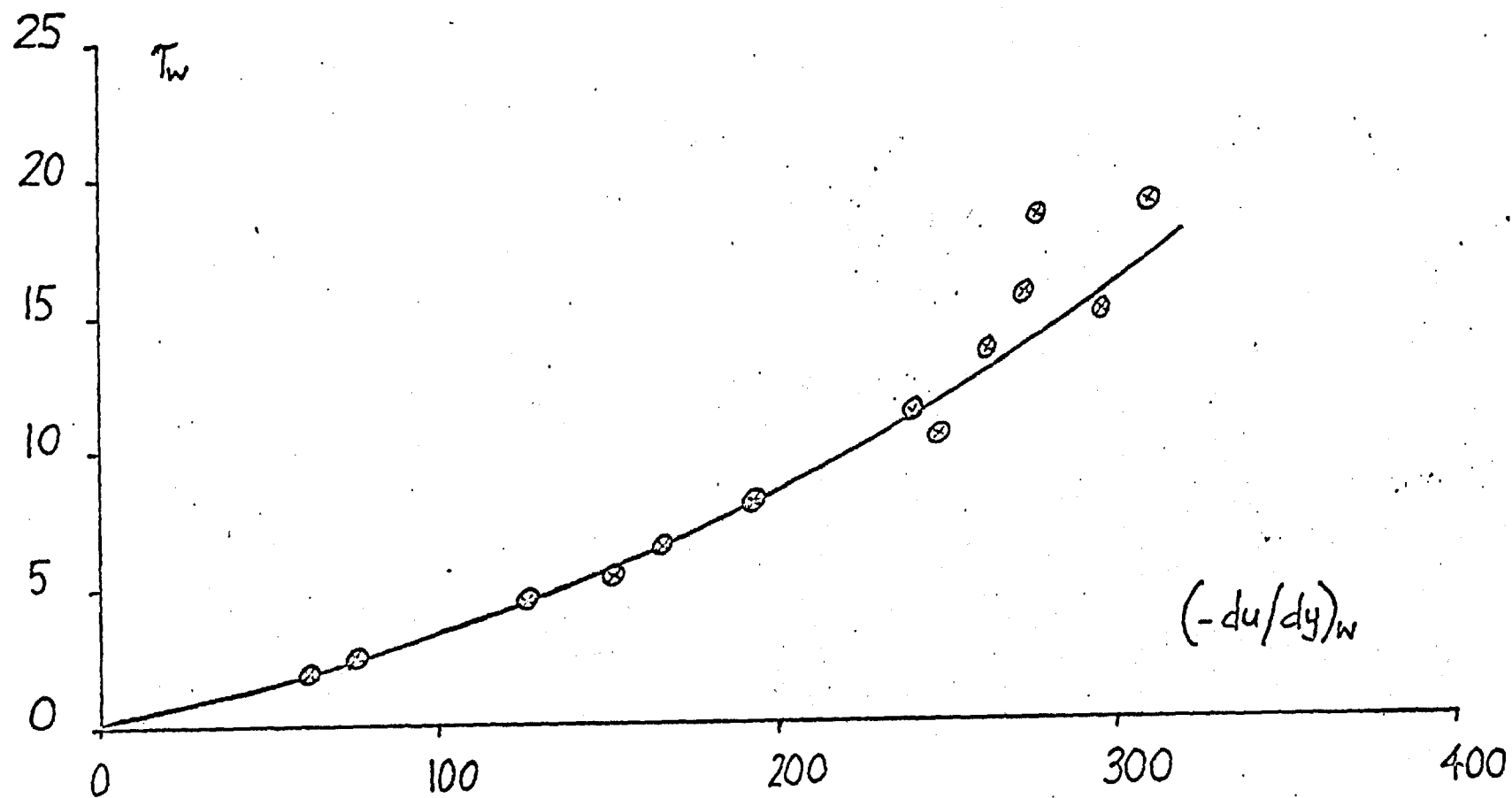
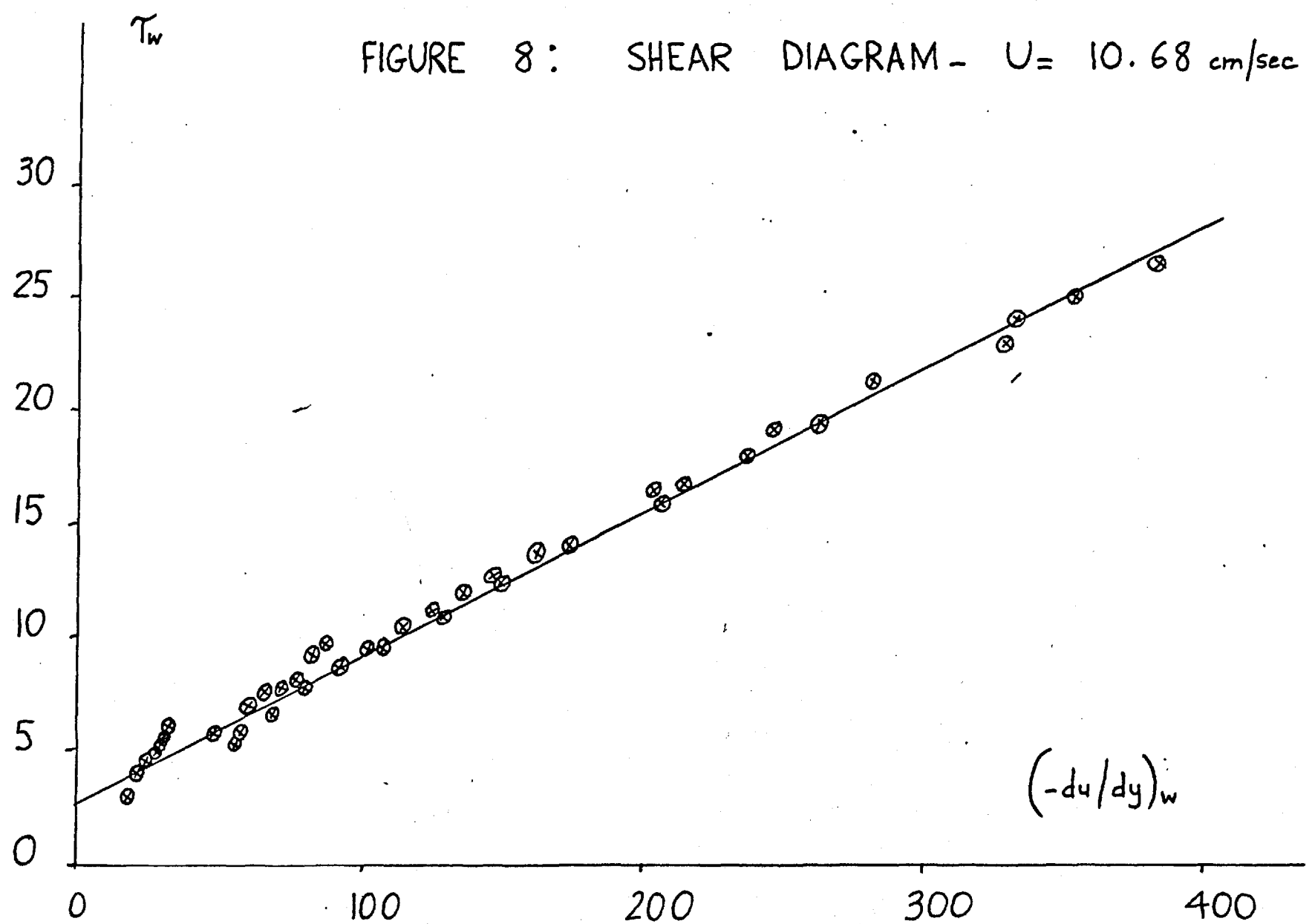
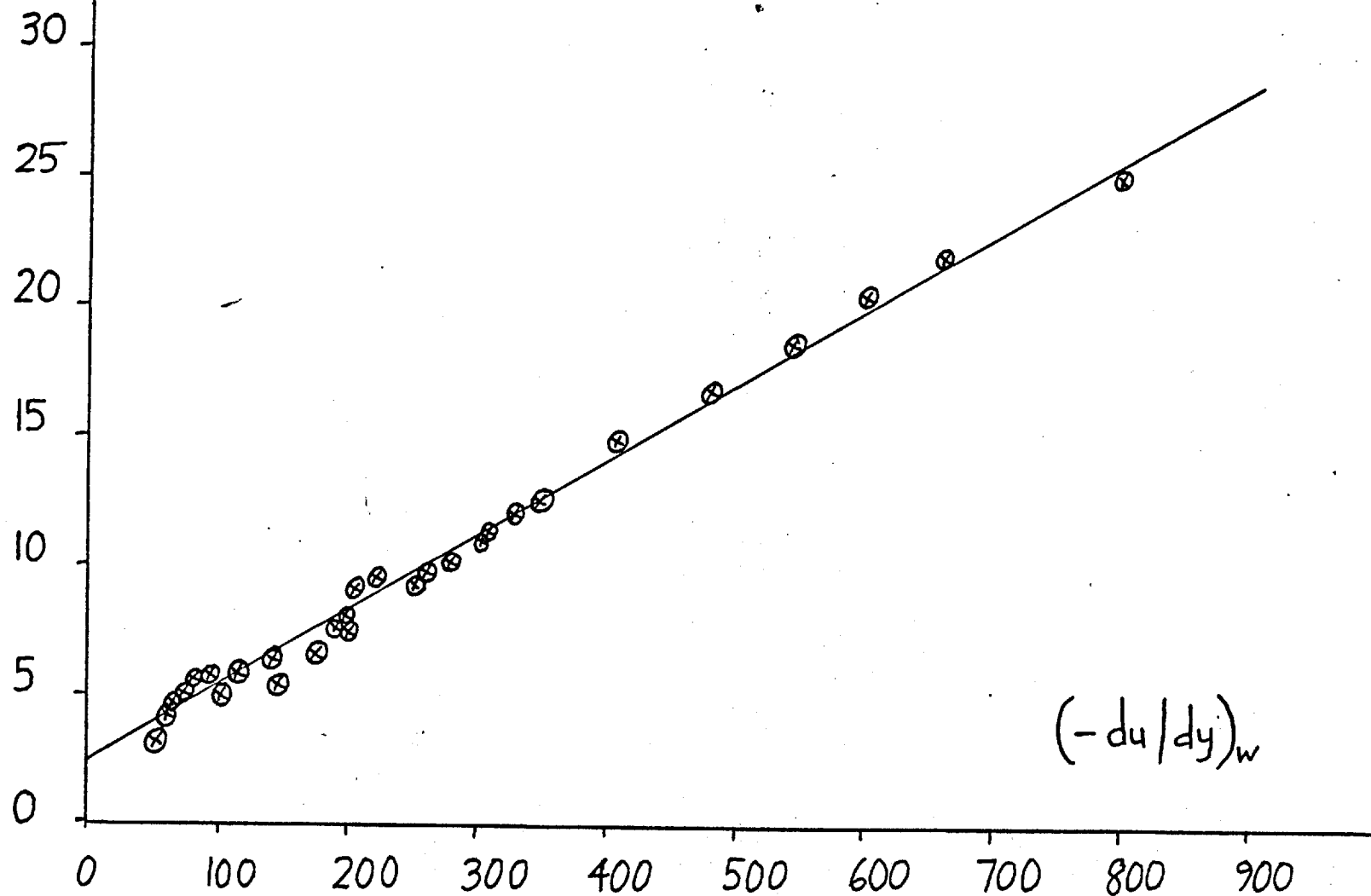


FIGURE 6: SHEAR DIAGRAM - $U = 9.73 \text{ cm/sec}$

FIGURE 7: SHEAR DIAGRAM - $U = 10.23 \text{ cm/sec}$





τ_w FIGURE 9 : SHEAR DIAGRAM - $U = 11.22 \text{ cm/sec}$ 

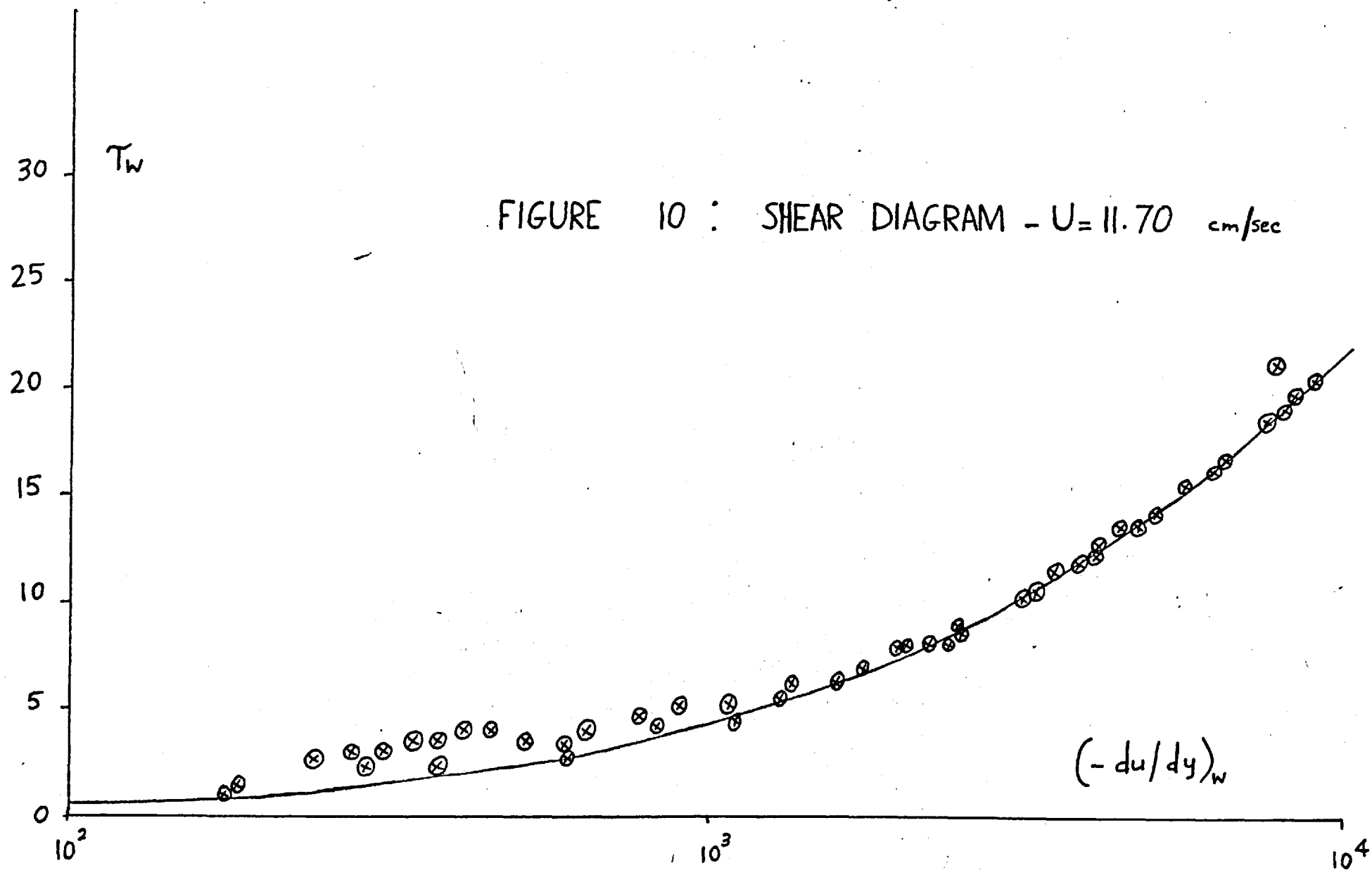


FIGURE 11: SHEAR DIAGRAM - $U = 12.22$ cm/sec

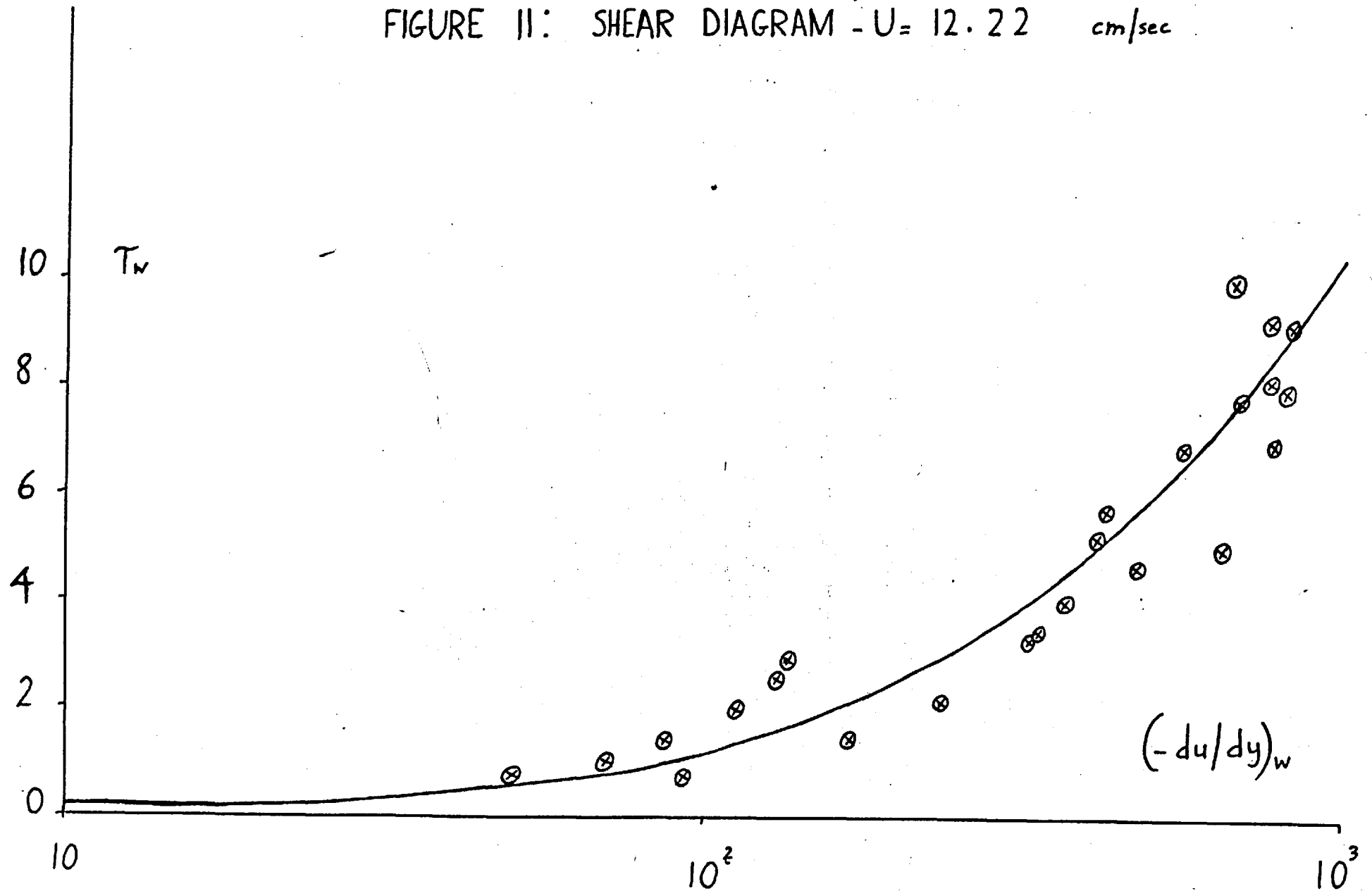
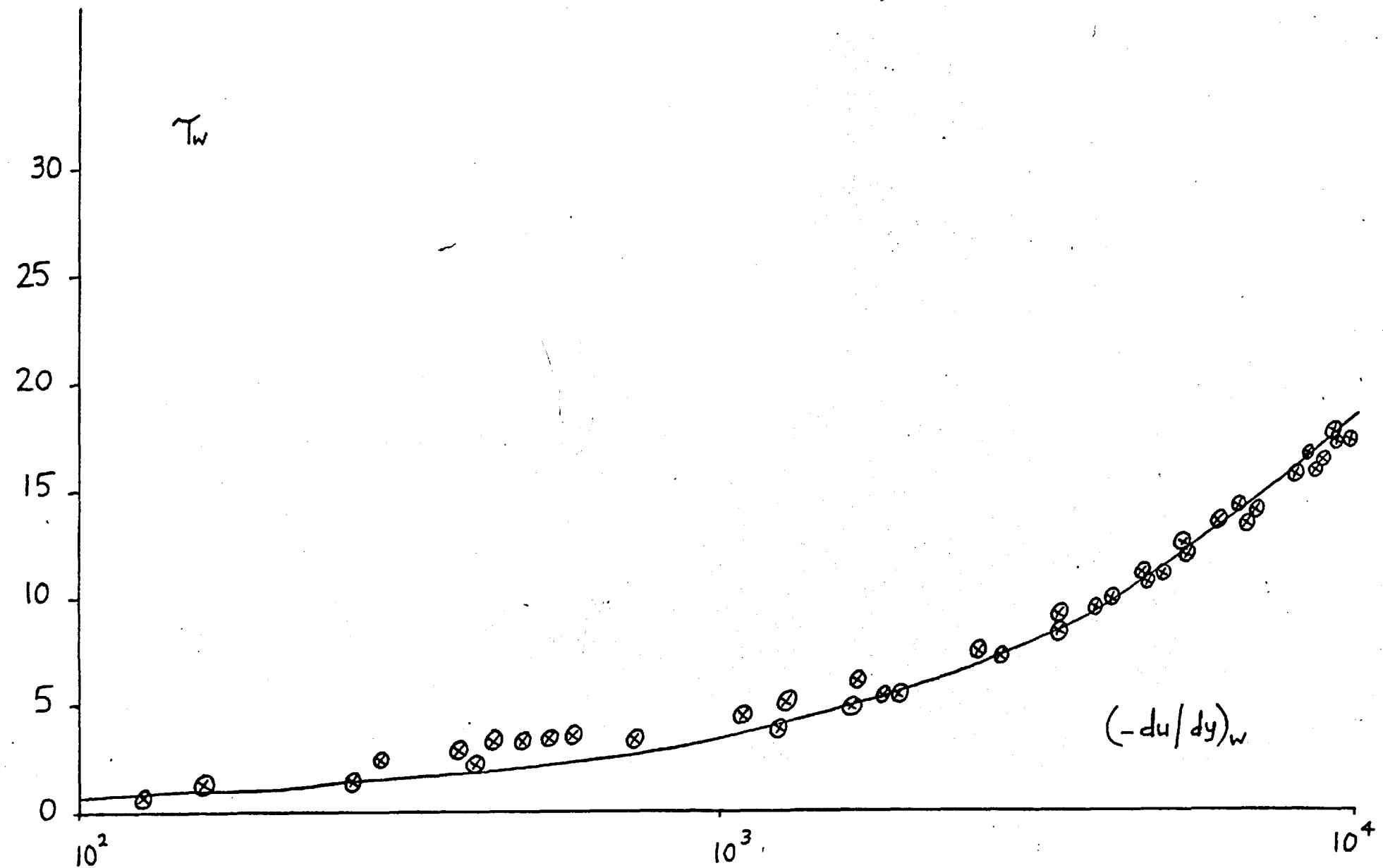


FIGURE 12 : SHEAR DIAGRAM - $U = 12.76 \text{ cm/sec}$



of the surface-fitting routine which was only used because of the unavailability of a sufficiently general constrained surface fitting procedure at our disposal.

The basic shear diagrams obtained indicate that for the lower fluidizing velocities (sets (1)-(5)), the fluidized system is non-Newtonian with an appreciable change in the viscosity with the shear stress. As the fluidizing velocity increases, the systems tends to Newtonian behaviour. The value of the viscosity of the fluidized bed is obtained as the slope of the basic shear diagram at zero shear rate. At this point, the fluidized bed is in a state of mechanical equilibrium in the sense that there is no flow under the action of gravity.

For the sets of data ((1)-(5)) showing non-Newtonian behaviour, the point viscosity μ^* is obtained from the slope of the basic shear diagram and it is inserted in the equations of the theoretical model formulated in chapter 5. However, as was pointed out in that chapter, Newtonian behaviour indicates that the bed is quasi-homogeneous and hence all the bubble variables (U_{bo} , U_b , d_b , ϵ_b) lose their physical significance and the theoretical model of chapter 5 does not describe the fluidized bed anymore. For the sets of data ((6)-(10)) displaying Newtonian behaviour, the viscosity of the bed is assumed to be given by Murray's equation for the viscosity of a concentrated homogeneous suspension, viz. :

$$\mu^* = \frac{\mu_f (1 - \epsilon_c)}{\epsilon_c - \epsilon_s} \quad (6)$$

from which, we obtain:

$$\epsilon_c = \frac{1 + \frac{\mu^*}{\mu_f} \epsilon_s}{1 + \frac{\mu^*}{\mu_f}} \quad (7)$$

6) Results, analysis and observations

In addition to investigating the effect of the superficial fluidizing velocity on the viscosity of the system at zero shear rate, the effect of increasing shear stress on the system at each fluidizing velocity is analyzed. It was explained in chapter 5 that for a given fluidizing velocity, non-Newtonian behaviour indicates a bubbling fluidized bed with stable bubbles. On the other hand, if the bed displays Newtonian characteristics, then bubbles are collapsing into the particulate phase with the same rate at which bubbles are being formed, and the bed attains a quasi-homogeneous structure. Consequently, the first five sets of data ((1)-(5)) exhibiting non-Newtonian behaviour are analyzed with the aid of the equation of the theoretical model formulated in chapter 5. The last five sets of data ((6)-(10)) showing a constant viscosity with shear stress are treated using equations (6) and (7) above for a quasi-homogeneous bed.

We consider first the results obtained for sets (1)-(5). As pointed out in section (5) above, the numbers of data points in sets (2) and (3) that satisfy the constraints (5) above are small, and hence less weight should be attached to results from these two sets than to those obtained from sets (1), (4) and (5). The results of the application of the theoretical model for heterogeneous fluidization are given in table 2 below. We look in the first instance at the effect of the fluidizing velocity on the bed viscosity at zero shear rate $(\mu^*)_0$. It is found that the value of $(\mu^*)_0$ decreases with increasing U and this tendency is shown by all past experimental work. However, the absolute values of $(\mu^*)_0$ were less than the values obtained in past work. Apart from set (2), it is seen that there is a gradual increase in ϵ_c and ϵ_b with increasing U , the increase in the former being much less pronounced than that in the latter. It

is interesting that the values of ϵ_c are just above saturation and very far from the value at incipient fluidization, which is assumed in the formulation of the two-phase theory. Disregarding sets (2) and (3), the value of d_b falls with increasing U , indicating that the bubbles are gradually becoming less stable. We postulate that for bubbles to exist as discrete voids, $d_b \geq 3d_p$. Looking at the inter-particle force, I_p , we observe that its value decreases by a factor of ten with increasing U . It is interesting to note that this trend occurs with increasing ϵ_b appreciably, while there is a very much smaller increase in ϵ_c . Clearly, I_p is much more dependent upon ϵ_b than on ϵ_c , and as the bubble proportion of the bed increases, the inter-particle force decreases. Comparing the absolute values of I_p with the buoyancy of the representative solid particle in the particulate phase, it is seen that I_p is 1-2 orders of magnitude higher than the buoyancy term, and the inter-particle force play an important part in maintaining the equilibrium of the particulate phase and cannot be neglected.

Looking at the variation of the bed viscosity with shear stress at each fluidizing velocity in the first five sets ((1)-(5)), it is concluded from the basic shear diagrams that, disregarding sets (2) and (3) for the moment, sets (1) and (4) display a decrease in bed viscosity with increasing shear stress whilst at the higher fluidizing velocity of set (5), the bed viscosity rises with shear stress. These two trends of viscosity variation with increasing shear stress can be interpreted by investigating the variation of μ^* with the bed variables ϵ_b , ϵ_c , and d_b , for a given fluidizing velocity U and overall mean voidage ϵ_m . If the equation for μ^* presented in the theoretical model of heterogeneous fluidization is expressed solely in terms of ϵ_b , ϵ_c , and d_b , respectively, and each expression is differentiated, we

conclude that:

$$\left(\frac{\partial \mu^*}{\partial \epsilon_b}\right)_{U, \epsilon_m} > 0 \quad (8)$$

$$\left(\frac{\partial \mu^*}{\partial \epsilon_c}\right)_{U, \epsilon_m} < 0 \quad (9)$$

and
$$\left(\frac{\partial \mu^*}{\partial d_b}\right)_{U, \epsilon_m} < 0 \quad (10)$$

Considering sets (1) and (4), we note that as the shear stress is increased for a given fluidizing velocity, the bed viscosity falls, and this leads us to conclude from the inequalities (8), (9) and (10) above that ϵ_b falls while both ϵ_c and d_b rise. The fact that d_b increases with shear stress indicates that the bubbles in the system at this range of fluidizing velocity (sets (1) and (4)) are stable and the application of a shear stress enhances their stability as shown by their growth. Conversely, if we consider set (5), it is seen that an increase in the applied shear stress leads to an increase in the bed viscosity with a consequent rise in ϵ_b and fall in ϵ_c and d_b . Thus the bubbles are destabilized by an increase in the applied shear stress. A trend is therefore clear that in the range of degree of fluidization U/U_{mf} between 1.5 and 2.0, the bubbles are stable at the lower fluidizing velocities, but their instability increases with increasing fluidizing velocity as the decrease in bubble diameter with fluidizing velocity shows.

We now turn to the last five sets of data ((6)-(10)) which exhibit Newtonian behaviour. Their results are shown in table 3 below. As we explained earlier, this indicates that the fluidized bed is quasi-homogeneous with bubble collapsing and forming at equal rates. The first point to note is that the bed viscosity continues to fall with rising fluidizing velocity with a corresponding rise in ϵ_c . The increase of ϵ_c with U is much more appreciable than it is in the

non-Newtonian range where the bed is heterogeneous.

The gradual decrease in the inter-particle force with U is maintained, but the value of I_p is still much higher than the buoyancy term, again indicating that the inter-particle forces should not be neglected in the analysis. It is worth mentioning that strictly speaking for each fluidizing velocity in this range, ϵ_c should equal ϵ_m . The discrepancy is due to the fact that ϵ_m was calculated using a graphical correlation for heterogeneous beds, whilst the bed is in this range essentially quasi-homogeneous. Also, the values obtained for ϵ_c were lower than those predicted by the Richardson-Zaki equation for particulate fluidization.

7) Flow curves for the fluidized bed

Considering the free surface of the fluidized bed as the channel of flow, we can obtain a flow curve using suitably-defined parameters. The Reynolds number, Re^* , is defined with the average velocity defined as: $\frac{q}{2b_w}$. Hence:

$$\begin{aligned} Re^* &= \frac{2b_w d_s (1-\epsilon_m) q}{2b_w} \cdot \frac{1}{\mu^*} \\ &= \frac{d_s (1-\epsilon_m) q}{\mu^*} \end{aligned} \quad (11)$$

The friction factor, f^* , is defined as follows:

$$\begin{aligned} f^* &= \frac{\tau_w}{d_s (1-\epsilon_m) (q/2b_w)^2} \\ &= \frac{4b_w^2 \tau_w}{d_s (1-\epsilon_m) q} \end{aligned} \quad (12)$$

The values of Re^* and f^* are listed in table 4. The points are plotted in sets (1)-(5) for the non-Newtonian range shown in figure 13, and sets (6)-(10) for the Newtonian range shown in figure 14.

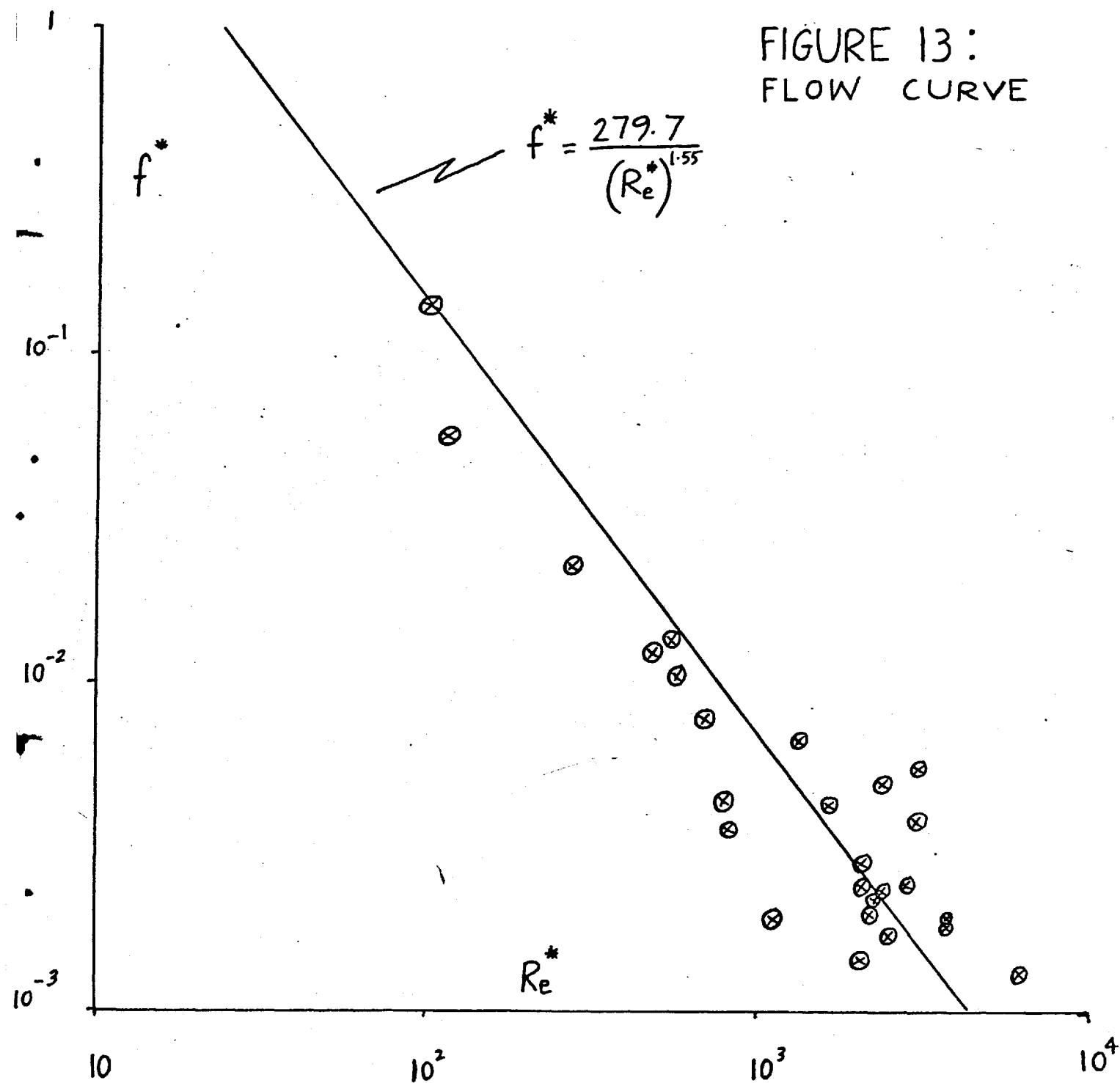
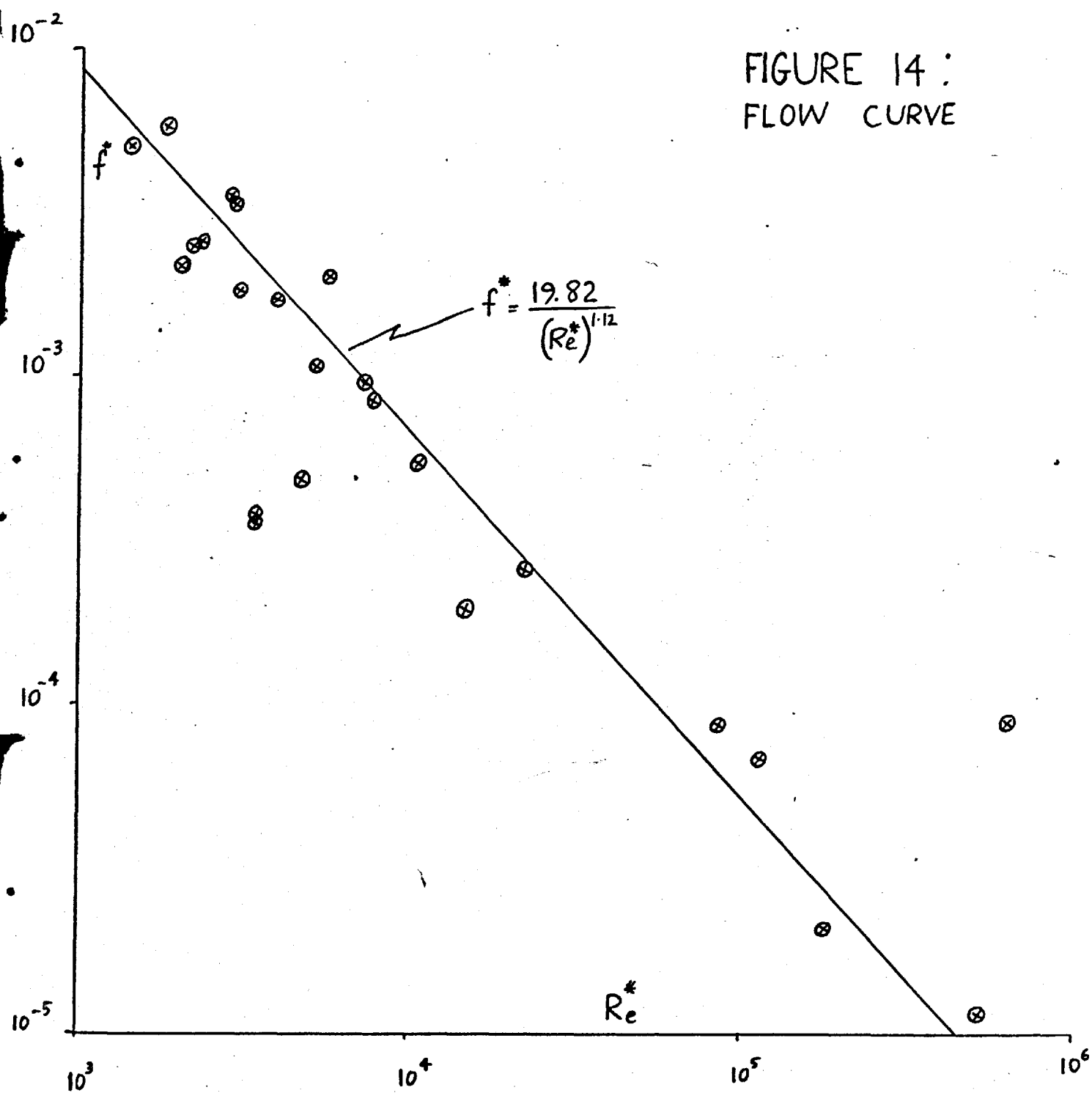
FIGURE 13:
FLOW CURVE

FIGURE 14:
FLOW CURVE

The data are given by the following correlations, using the least squares criterion:

$$f^* = \frac{279.7}{(\text{Re}^*)^{1.55}} \quad , \text{ for sets (1)-(5)} \quad (13)$$

with standard deviation, $s = 3.93$

and:

$$f^* = \frac{19.82}{(\text{Re}^*)^{1.12}} \quad (14)$$

with standard deviation, $s = 4.40$

It is interesting to compare equation (14) with the classical laminar flow correlation for a Newtonian fluid, viz. :

$$f^* = \frac{24}{\text{Re}^*} \quad (15)$$

The similarity between equations (14) and (15) confirm the Newtonian behaviour of the fluidized bed in the range of U/U_{mf} between 2.0 and 2.5.

CHAPTER SEVEN

CONCLUSIONS

The main objective of this work has been the determination of the rheological behaviour of the gas-fluidized bed, and from it the deduction of the structure of the gas-fluidized bed. The system was studied in the range of U/U_{mf} between 1.5 and 2.5. It was found that up to $U/U_{mf} = 2.0$, the system behaved as a non-Newtonian continuum with the equilibrium viscosity decreasing with superficial fluidizing velocity from 1.2 poise to 0.08 poise. There is no ground for supposing that the viscosity reaches the ^{often-quoted} ~~off-gated~~ values of 10.0 or 100.0 poises. In this range, the heterogeneous bed model was found to describe the system, with the bubble proportion of the bed increasing from 0.357, to 0.405. This is to be expected. However, the voidage of the continuous phase is far from that assumed in the two-phase theory; namely, ϵ_{mf} . In fact, ϵ_{cont} is found to be very close to the saturation voidage, ϵ_s . It has also been shown that the inter-particle forces are very high at the lower end of the range, but decrease gradually to less than one-tenth its original value. It was also found that the proportion of gas flowing through the continuous phase increases steadily with increasing superficial fluidizing velocity. This conclusion agrees with the conclusion of Lockett, et al (1966). There is a distinct change in the way the viscosity varies with the shear stress for a given fluidizing velocity. At the lower end, it was found that μ^* decreases with τ . This decrease in μ^* leads to an increase in d_b , suggesting that at the

lower values of U , the bubbles in the bed are quite stable. However, this trend is reversed at the higher values of U , as μ^* increases with τ , resulting in a decrease in d_b . Thus, the bubbles are generally becoming less stable, though their proportion in the bed is on the increase.

In the range of U/U_{mf} between 2.0 and 2.5, it was found that the system is essentially Newtonian, with its viscosity falling from 0.27 to 0.002 poise in this range. It is therefore concluded that the bed is in a quasi-homogeneous state, as suggested by Teoreanu (1962), with the bubbles forming at the same rate at which they are "dissolving" in the particulate phase. As expected in such a case, the inter-particle force is high and decreases with increasing superficial fluidizing velocity. It seems evident that the system will remain Newtonian at higher values of U/U_{mf} .

Finally, it is clear that a more general model is required to take into account the range of bubble size in the fluidized bed. This was not so essential in the case of shallow beds, but once height becomes a variable, bubble expansion plays an important part in defining the state of the bed at any point. This can be done by relating bubble size to bed height. More generally, it has been shown that the bed is heterogeneous in the range of U/U_{mf} 1.0 to 3.0, with stable inhomogeneities. At higher degrees of fluidization, the bed tends to a more homogeneous structure.

PART 2

FLOW OF PARTICLES FROM FLUIDIZED BEDS

1) Introduction

In this section, consideration is given to the flow through orifices of fluidized suspensions from fluidized and pressurized beds. The potential importance of such a study lies in the widespread applicability of the fluidized technique to the addition and withdrawal of particles from various operations and processes. Whatever flow configuration used, the suspension is always allowed to flow from a region of high pressure to one at lower pressure.

All past studies of fluidized suspension flow through orifices have shown that the suspensions display liquid-like behaviour. The first study reported on the flow of solid-gas suspensions through vertical orifices in fluidized beds was by Massimilla, et al (1961). They assumed that the particles flow radially towards the orifice centre and that the effect of the net downward velocity of the particles on the outflow conditions is negligible. It was further assumed by them that the interparticle friction forces on the upstream side of the orifice can be disregarded. From their experimental work, it was concluded that the flowrate of solids depends solely on the orifice diameter and on the efflux pressure, and that the voidage is constant. The ratio of air to solids flowrate was found to depend on the orifice diameter only, and this ratio decreases with increasing orifice diameter. Stockel (1962) considered the flow of fluidized solids

in various channels of constant and changing cross-sectional areas. His theoretical formulation was based on uni-directional, compressible, ideal fluid flow assumptions. The compressibility assumption takes into account the variation of the voidage of the suspension and the compressibility of the continuous gas phase. In extensive experimental work, he found good agreement between theory and experiment. Jones and Davidson (1965) carried out experiments on the flow of particles from fluidized beds through both sharp-edged orifices and shaped nozzles. Their theoretical analysis suggested that, unlike in Massimilla's analysis, the particle inertia and the solid-gas interaction terms are unbalanced in the solids momentum equation, giving rise to an inter-particle friction term. Their theory is similar to that developed for gas bubbles in gas-fluidized beds by Davidson and Harrison (1963). One consequence of their theory, which assumes constant voidage fraction and neglects the gas momentum, is that the amount of air percolating through the solids is given by Darcy's law. Murray (1967) formulated a continuum model for particle-fluid two-phase one-dimensional flow. His theory showed that the voidage fraction could come close to the saturation voidage and consequently lead to the choking of pipelines. In applying his equations of motion to channel flow problems, he neglected the particulate stress altogether. Morris (1969) studied voidage and pressure distributions in the flow of solids from orifices in two- and three-dimensional fluidized beds. Measured values of the voidage variation were open to doubt due to the problems encountered in experimental measurement. He found that the pressure variation agrees with that predicted by Darcy's law.

It is interesting to note that all the workers, who have taken a variety of approaches to the problem of fluidized suspension flow

through orifices, concluded that the mass flow-rate of solids follows a similar trend to that of a liquid of low viscosity; that is, the mass flowrate of solids is directly proportional to the square root of the bed height. Experimental results give rise to a value of 0.5 for the coefficient of discharge for the flow of fluidized particles through beds. In this work, the equations of motion proposed include the particulate stress tensor which was neglected in all past analyses of the orifice flow problem. No restrictions are placed on the possible variation in the voidage fraction except that it should remain higher than the saturation value. It is hoped that the method employed to ensure the existence of physically realizable flows as explained in a later section will prove to be generally applicable to various flow configurations. As an illustration, the problem of orifice flow is investigated and the theoretical solution is compared with experimental results.

2) Theory

The following assumptions about the nature of the flow of fluidized solids are made:

- (a) The flow is uni-directional, steady, and laminar; for the orifice flow problem, the flow is uni-directional in the radial direction towards the centre of the orifice.
- (b) The fluid momentum is negligible in comparison with the momentum of the solid particles; the analysis is thus restricted in applicability to gas-fluidized systems.
- (c) The added mass terms are neglected in the solids momentum equation.
- (d) Gravitational effects are negligible in the flow region.

3) Solids Momentum Equation

While the equations of gas and solids continuity, and the gas momentum equation are easily formulated as will be shown below, it is the solids momentum equation that has proved to be the main stumbling block to a rigorous theoretical analysis of the problems involved in the flow of fluidized solids through various flow configurations. Past theoretical analyses have considered the solids momentum equation in a number of ways. These are summarized below:

(i) Massimilla, et al (1961) equated the solids inertia to the gas solids interaction term. This term arises from the relative motion of gas and solid particles, and it is written as follows:

$$\bar{I} = F(\bar{u} - \bar{v}) \quad (1)$$

where $\bar{I}$ = solids drag force

F = some function, suitably chosen (for example from the Carman-Kozeny equation)

$\bar{u}$ = gas velocity

$\bar{v}$ = particle velocity

Essentially, the function F should include a correction for the presence of other particles on the drag experienced by any one particle at the appropriate flow Reynolds number.

(ii) Stockel (1962) also equated solids inertia to the drag, but he allowed for variations in the voidage fraction. The drag function F is thus a variable parameter; the solids inertia will depend on both velocity and voidage fraction. ⁽ⁱⁱⁱ⁾ Jones and Davidson (1965), assuming constant voidage, balanced solids inertia by the drag plus a term containing the so-called "particle pressure".

(iv) Murray (1967) proposed a continuum model for the solids but he omitted the particulate stress tensor in considering channel

flow. It is worth noting that in his study of fluidized suspension flow over a flat plate, he did include the particulate stress terms.

It is seen that there has been a gradual improvement in the analysis. No solution has so far been obtained which includes the particulate stress tensor in the analysis. We postulate the following:

- (a) There may be continuous variation in the voidage fraction throughout the flow field.
- (b) The fluid pressure drop is an external force acting on the particulate phase, and it is given by the Carman-Kozeny equation;

$$\frac{dp_f}{dr} = K \left(\frac{1-\epsilon}{\epsilon} \right)^2 (\bar{v} - \bar{u}) \quad (2)$$

where $\bar{v}$ = particle velocity, $\bar{u}$ = gas velocity

$$K = \frac{173\mu_f}{d_p^2}, \text{ being an empirical constant.}$$

- (c) The particulate phase behaves like a Newtonian continuum, with two coefficients of viscosity and a static pressure. The normal stress in the particulate stress tensor, which is in the direction of flow, is defined as the inter-particle pressure. Thus the equation of motion of the particulate phase can be written as follows:

$$P_s(1-\epsilon)(D\bar{v}/Dt) = \text{grad } p_f + \cancel{\text{grad}}^{\text{div}} \bar{\tau}_p \quad (3)$$

where (D/Dt) denotes the material derivative, τ_p represents the particulate stress tensor, and p_f acts on the whole flow field.

4) Flow through an orifice

In investigating the flow of solid particles from an orifice, it may be assumed that the flow is radial towards the centre of the orifice. Furthermore, if it is assumed that at each radial

distance from the orifice centre, the velocity is uniform, so that shear stresses in the particulate phase are absent, and the flow problem becomes uni-directional. Having made these assumptions about the nature of the flow for a three-dimensional orifice, the continuity equations for the solids and gas phases can be written in their integral form, thus:

$$2\pi r^2 d_s (1-\epsilon) v_r = M_s \quad (4)$$

$$2\pi r^2 d_f \epsilon u_r = M_f \quad (5)$$

where v_r = particle radial velocity at radius r

u_r = fluid radial velocity

M_s = solids mass flowrate

M_f = fluid mass flowrate

and v_r , u_r are positive in the direction of flow, i.e. towards the orifice. In writing equation (5), it is assumed that the fluid flow is also in the radial direction towards the orifice centre.

Equations (4) and (5) can be rewritten as follows:

$$1 - \epsilon = \frac{A}{r^2 v_r} \quad (4a)$$

$$\epsilon = \frac{B}{r^2 u_r} \quad (5a)$$

where $A = M_s / (2\pi d_s)$

and $B = M_f / (2\pi d_f)$

We assume that at a very large distance away from the orifice where the particles are incipiently fluidized, there is no relative motion between the gas and the particulate phases. Although this is not strictly true because of "leakage", it affords a simple means of determining the fluid mass flowrate from a knowledge of the solids mass flowrate. Thus, far away from the orifice:

$$v_r = u_r$$

Hence:

$$\frac{M_s}{d_s(1-\epsilon_{m_f})} = \frac{M_f}{d_f\epsilon_{m_f}} \quad (6)$$

The gas momentum equation can be written as follows:

$$\frac{dp_f}{dr} = K\left(\frac{1-\epsilon}{\epsilon}\right)^2 (v_r - u_r) \quad (7)$$

where K is the same empirical constant as in equation (2). In equation (7), the inertia of the gas phase has been neglected. The interaction term between the two phases has been assumed to be given by the Carman-Kozeny equation.

The momentum equation for the solids phase for this particular geometry becomes from equation (3):

$$d_s(1-\epsilon)V_r \frac{dv_r}{dr} = -\frac{dp_f}{dr} + \frac{1}{r^2} \frac{d[r^2(s_{rr})_d]}{dr} \quad (8)$$

where

$$(s_{rr})_d = -p_p + 2\mu_p \frac{dv_r}{dr} + \frac{1}{r^2} \frac{d(r^2 v_r)}{dr},$$

the assumption here being that p_p is zero, an assumption whose validity has been demonstrated for particulate systems.

In equation (8), $(s_{rr})_p$ denotes the normal stress in the particulate phase, and this is physically represented by the interparticle pressure. For the two coefficients of viscosity of the particulate phase, Murray's expressions, theoretically derived from hydrodynamic considerations, are utilized thus:

$$\mu_p = \mu_f \left(\frac{1-\epsilon}{\epsilon - \epsilon_s} \right) \quad (9)$$

$$\text{and } \kappa_p = \mu_f \left(\frac{1-\epsilon}{\epsilon - \epsilon_s} \right)^2 \quad (10)$$

For most particulate systems, the voidage fraction at minimum fluidization is approximately 0.5, and the saturation voidage

fraction is around 0.3. Hence, at incipient fluidization:

$$\mu_p = 3\mu_f \text{ approximately}$$

$$\text{and } \kappa_p = 27\mu_f \text{ approximately}$$

Thus the bulk viscosity is one order of magnitude higher than the shear viscosity. On the other hand, close to saturation, say at $\phi = 0.31$, the viscosity coefficients become:

$$\mu_p = 70\mu_f \text{ approximately}$$

$$\text{and } \kappa_p = 10^5\mu_f \text{ approximately}$$

Hence, as saturation is reached from incipient fluidization, the shear viscosity loses significance rapidly in comparison with the bulk viscosity. Consequently, in flow problems, most of the work done goes in overcoming viscous resistance to contraction.

On rearrangement, equation (8) becomes:

$$\begin{aligned} (2\mu_p + \kappa_p) \frac{d^2 v_r}{dr^2} = & \left(d_s \frac{A}{r^2} - \frac{4\mu_p}{r} - \frac{4d\mu_p}{dr} - \frac{4\kappa_p}{r} - \frac{d\kappa_p}{dr} \right) \frac{dv_r}{dr} \\ & - \left(\frac{2\kappa_p}{r^2} + \frac{2}{r} \frac{d\kappa_p}{dr} \right) v_r + \frac{dp_f}{dr} \end{aligned} \quad (8a)$$

This equation may be solved numerically, employing suitable initial conditions, as is discussed in the next section.

5) Numerical solution

Equation (8a) is solved numerically with the assistance of equations (4a), (5a), (6), (7), (9) and (10), so that voidage and pressure distributions are obtained for the flow region around the orifice. In the solution, r is taken as the independent variable and v_r the dependent variable. For the purpose of comparing the numerical solution of (8a) with actual situations, experimental results are taken from the work of Howard, et al (1969). These are listed below:

<u>Solid material</u>	<u>dp (μm)</u>	<u>d_s (g./cm³)</u>	<u>r_e (cm)</u>
Sand	330	2.60	0.16
Sand	330	2.60	0.24

The bed used in these experiments was of diameter 14.2 cm and height 75cm. It should be noted that the experimental results of Howard, et al (1969) were used as a basis of comparison with the results obtained from this theory, as the present work model is a part of the theoretical and experimental study of the flow of fluidized particles from pressurized beds conducted at the departments.

The Runge-Kutta method was utilized to solve the second-order ordinary differential equation (8a). The best step length to be used in the integration of (4a) is obtained by carrying out the integration for several orders of magnitude of step length in ascending order, and taking the highest step length for which the solution is not a function of the step length used. For all the solutions obtained, it was found that the most suitable step length is 0.001. For each set of data-material, particle diameter, solids density, and orifice diameter - equation (8a) is solved for several values of the solids mass flowrate, M_s .

The next point concerns the initial conditions. These are formulated as the equilibrium conditions prevailing at some distance - yet to be found - away from the orifice. Thus, the initial conditions can be written:

$$r = nr_e, \quad \epsilon = \epsilon_{m_f}, \quad v_r = \frac{A}{(1-\epsilon_{m_f})2\pi(nr_e)^2}, \quad \frac{dv_r}{dr} = 0$$

In order to find n, the equation is solved for increasing values of n starting from 2 (n always be taken as an intager), and the

value assigned finally to n is the least one which gives a physically feasible solution - namely, ϵ always $> \epsilon_s$ - and at the same time a numerically stable solution.

6) Discussion of results

The voidage variation obtained from the theoretical solution of equation (8a) is shown in figures 15 and 16 for the two orifices considered. Throughout the range of mass flowrates investigated it was found that the voidage fraction drops sharply from ϵ_{mf} to some value just above ϵ_s at the beginning of the integration process, that is at $r = nre$, and the voidage fraction remains approximately constant for the rest of the integration. As the distance over which the initial sharp drop in the voidage fraction takes place is too small to study in detail from the point of view of comparison with experimental voidage profiles, when they become available in the future, a gap is left in the curve in this small distance. In practice, the flow is not uni-directional, particularly at large values of r/r_e . Consequently, the initial fall in voidage fraction is unlikely to be so abrupt.

The pressure distribution obtained from the present model is shown in figure 17, and it is found to coincide with the Jones-Davidson profile (1965), which represents the actual pressure profile extremely well. Also shown in figure 17 is the pressure distribution obtained by the use of Bernoulli's equation for an inviscid incompressible fluid. Morris (1969) showed that the pressure profile is the same whether the fluid is assumed incompressible or compressible provided the particulate stress tensor is omitted in the equations of motion. In figure 17, p_H denotes the fluid pressure inside the bed far away from the orifice,

FIGURE 15 : EFFECT OF MASS FLOW ON
VOIDAGE VARIATION - $r_e = 0.24$ cm

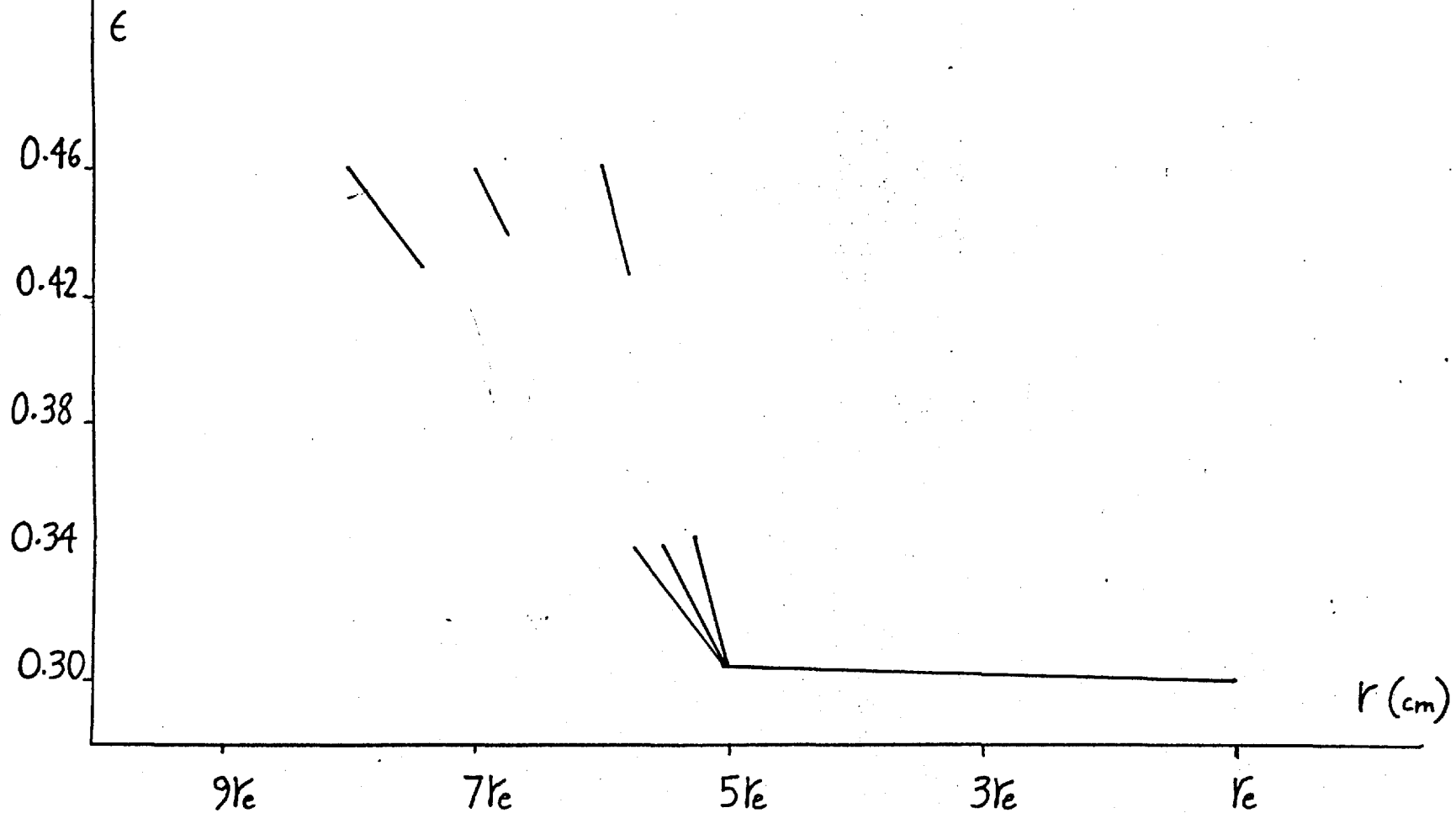


FIGURE 16 : EFFECT OF MASS
FLOW ON VOIDAGE
VARIATION - $k_e = 0.16$ cm

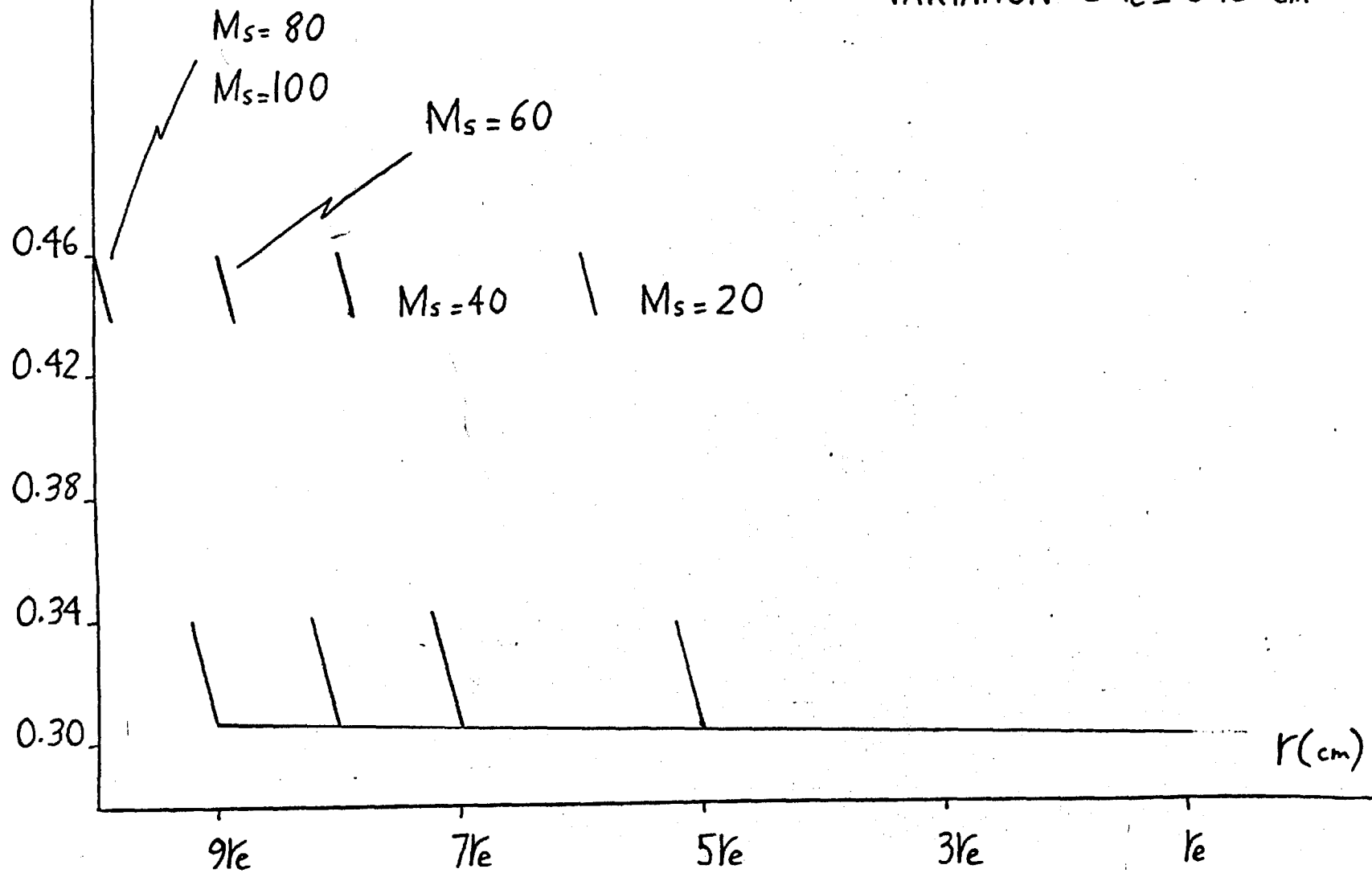
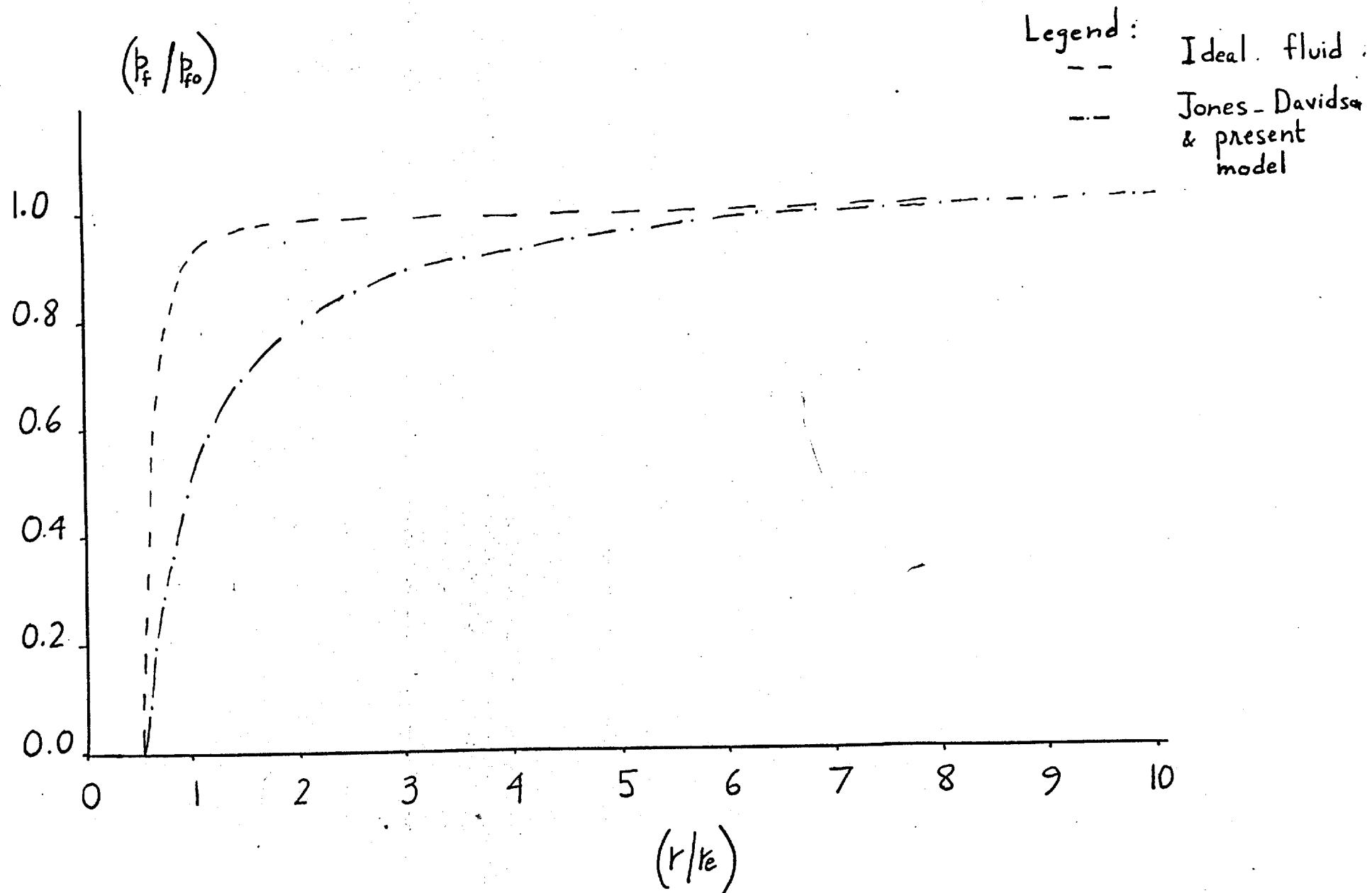


FIGURE 17 : PRESSURE DISTRIBUTION



taken at 10 times the orifice radius. Hence, the pressure profiles in figure 17 have been scaled, and it can be seen that the dimensionless pressure profile obtained from the present model is coincident with the Jones-Davidson profile. However, the absolute values of the pressure at any point is in fact rather higher in our model than that given by the Jones-Davidson analysis, this being due to the fact that while in both models the voidage fraction is constant in the flow region, its value is that at incipient fluidization in the Jones-Davidson theory, while it is just above saturation in our analysis. Another factor which may explain the discrepancy in the absolute pressure values between the two models is that the most significant viscous term in equation (8a) is that due to bulk viscosity, and the expression used for the bulk viscosity of the suspension can only be expected to give the order of magnitude and not the exact value of the coefficient of bulk viscosity of the suspension. Hence, much more significance can be assigned to the relative pressure profile than the absolute pressure variation, as the latter is extremely sensitive to the value of the coefficient of bulk viscosity used. The question of boundary conditions to be used may also explain the rather high absolute pressure values obtained by the present model. It was pointed out in section 5) of this chapter that equation (8a) is solved starting at $r = nr_e$ for a series of integral values of n . The absolute value of the pressure at any point was found to depend on the value of n , but the relative variation of the pressure followed the same trend. Clearly, before much more insight is attained into the question of boundary conditions, absolute values of the pressure must be considered with caution in so far as comparison with experimental absolute pressure profiles is concerned.

7) Conclusions

The problem of flow of fluidized suspensions from pressurized beds is approached from the continuum point of view, assuming the particulate phase to behave as a compressible Newtonian fluid. As an example of this approach, the problem of three-dimensional orifice flow is investigated, and the solution obtained is compared with available experimental results. In the particulate stress tensor, theoretical expressions were employed for the shear and bulk viscosities of the suspension, the latter being the predominant one in the concentration range considered, this being slightly above saturation.

The voidage around the orifice varies in two successive stages. Initially, there is a sharp decrease in the voidage from ϵ_{m_f} to a value slightly above ϵ_s . For the rest of the region of flow until the orifice radius is reached, the voidage remains approximately constant. This trend is due to the viscosity of the particulate phase - in particular, the bulk viscosity. Hence, whilst the bulk of the bed is at incipient fluidization, the region around the orifice is near saturation.

The dimensionless pressure distribution obtained from the model advanced here coincides with the Jones-Davidson profile, and is in excellent agreement with the experimental pressure profile. The absolute value of the fluid pressure is found to be higher than in the Jones-Davidson model, and this may be due to uncertainty in both the theoretical expressions used for the bulk viscosity of the suspension and the formulation of the boundary conditions. The fact that the pressure p_H in this study for a given solids mass flowrate is higher than that predicted by the inviscid fluid theory implies that the coefficient of discharge for fluidized suspension

flow should be less than unity. Due to the fact that the integration cannot be started at an infinite radial distance, but rather at some finite radial distance away from the orifice centre, the value of the coefficient of discharge predicted from this model cannot be reliably used with values obtained by Jones and Davidson (1965), and Howard, et al (1969).

Finally, the continuum approach is applicable to various flow configurations, such as constant and varying cross-section channel flow. The formulation of the initial equilibrium conditions needs further investigation, and the assumption of Newtonian behaviour for the particulate phase has to be relaxed to allow for non-Newtonian flows. Further, reliable point measurements of the voidage in the flow system will be found to be most useful in providing a direct comparison between the physical situation studied and the theoretical model proposed.

TABLE 1

Set no. (1) $U = 8.38$ cm./sec., $\epsilon_m = 0.550$

β	λ	$q(\text{cm}^3/\text{sec})$	$q(\text{cm}^2/\text{sec}) \times \frac{1}{x_{bw}}$	$U_s(\text{cm}/\text{sec}) \times \frac{1}{x_{bw}}$	$(\text{sec}^{-1}) \times \frac{1}{x_{bw}} (-du/dy)_w$	$(\text{dyne}/\text{cm}^2) \times \frac{1}{\tau_w}$
0.087	0.118	23.80	469.82	275.23	18.82	11.95
0.087	0.208	92.31	1092.38	485.54	48.15	21.08
0.087	0.218	104.04	1178.54	509.71	52.35	22.14
0.087	0.229	118.14	1277.83	536.57	57.21	23.30
0.087	0.249	144.48	1453.15	581.72	65.87	25.26
0.087	0.273	183.29	1692.85	639.50	77.82	27.77
0.087	0.303	238.89	2008.55	709.98	93.72	30.83
0.105	0.110	27.68	584.69	287.04	26.48	13.42
0.105	0.132	42.49	761.05	344.52	35.15	16.11
0.105	0.150	57.20	915.61	390.25	42.85	18.25
0.105	0.163	70.46	1043.40	425.59	49.27	19.90
0.105	0.180	88.83	1207.43	468.16	57.58	21.89
0.105	0.202	117.74	1443.53	525.05	69.65	24.55
0.105	0.216	140.51	1615.75	563.86	78.51	26.36
0.105	0.249	199.00	2019.33	647.86	99.43	30.29
0.105	0.268	241.24	2285.80	699.01	113.36	32.68
0.122	0.081	17.29	487.46	203.29	24.83	11.50
0.122	0.101	28.55	660.65	252.71	39.66	14.29
0.122	0.166	39.66	808.52	290.74	41.82	16.45
0.122	0.141	63.54	1083.93	354.30	56.55	20.04
0.122	0.172	102.24	1462.68	430.87	76.97	24.37
0.122	0.211	169.77	2020.84	528.62	107.26	29.90
0.122	0.231	213.91	2344.17	579.45	124.89	32.78
0.122	0.255	276.58	2766.65	641.09	147.99	36.26

Set no. (2) $U = 8.82$ cm./sec., $\epsilon_m = 0.562$

0.122	0.019	9.59	990.09	61.36	83.79	2.58
0.122	0.041	42.44	1890.22	134.91	167.98	5.68
0.122	0.071	110.51	2601.72	232.42	252.27	9.79
0.122	0.099	186.61	2769.76	324.06	303.11	13.65
0.122	0.145	300.88	1991.60	474.72	326.91	19.99

Set no. (3) $U = 9.33 \text{ cm./sec.}, \epsilon_m = 0.564$

β	λ	$Q(\text{cm}^3/\text{sec})$	$q(\text{cm}^2/\text{sec})$ $\times \frac{1}{x_b^w}$	$U_s(\text{cm/sec})$ $\times \frac{1}{x_b^w}$	$(\text{sec}^{-1}) \times \frac{1}{x_b^w}$ $(-du/dy)_w$	(dyne/cm^2) $\times \frac{1}{x_b^w}$ τ_w^w
0.122	0.026	11.16	832.55	40.10	72.08	3.51
0.122	0.050	38.58	1391.48	78.21	127.41	6.85
0.122	0.063	57.65	1594.27	98.20	151.31	8.60
0.122	0.099	121.18	1830.45	155.10	200.00	13.58
0.122	0.132	179.87	1599.50	207.77	219.59	18.20

Set no. (4) $U = 9.73 \text{ cm./sec.}, \epsilon_m = 0.577$

0.017	0.221	14.59	149.05	47.22	8.56	4.22
0.017	0.249	19.10	175.09	53.18	10.05	4.75
0.017	0.262	21.50	187.99	56.01	10.79	5.01
0.017	0.275	23.97	200.79	58.74	11.52	5.25
0.017	0.286	26.27	212.21	61.11	12.17	7.88
0.035	0.206	45.47	500.31	147.45	29.18	7.88
0.035	0.221	53.20	549.62	157.99	32.04	8.44
0.035	0.249	69.88	647.90	177.94	37.72	9.51
0.035	0.275	87.94	745.20	196.51	43.34	10.50
0.035	0.286	96.46	788.55	204.46	45.84	10.92
0.052	0.139	38.14	608.02	180.49	36.34	7.99
0.052	0.172	60.80	800.60	222.26	47.69	9.84
0.052	0.188	74.45	903.51	243.02	53.74	10.76
0.052	0.216	103.27	1100.56	280.31	65.30	12.41
0.052	0.231	119.80	1204.57	298.91	71.39	13.23
0.052	0.245	137.18	1308.48	316.83	77.47	14.02
0.070	0.150	70.20	1061.70	261.20	64.96	11.45
0.070	0.173	97.78	1295.30	302.06	78.95	13.24
0.070	0.197	131.48	1550.32	343.41	94.16	15.05
0.070	0.229	187.78	1929.23	399.93	116.68	17.53
0.070	0.254	239.11	2240.61	442.90	135.11	19.41
0.070	0.277	294.45	2551.06	483.20	153.45	21.18
0.070	0.313	393.95	3602.45	545.09	183.57	23.89
0.087	0.145	87.51	1391.44	267.96	87.87	13.81
0.087	0.158	107.88	1579.80	293.37	99.40	15.12
0.087	0.175	135.96	1819.91	324.02	114.04	16.69

Set no. (4) (continued)

β	λ	$Q(\text{cm}^3/\text{sec})$	$q(\text{cm}^2/\text{sec})$ $\times \frac{1}{b_w}$	$U_s(\text{cm}/\text{sec})$ $\times \frac{1}{b_w}$	$(\text{sec}^{-1}) \times \frac{1}{b_w}$ $(-du/dy)_w$	$(\text{dyne}/\text{cm}^2)$ $\times \frac{1}{\tau_w}$
0.087	0.198	183.24	2187.54	367.79	136.35	18.95
0.087	0.215	221.52	2461.20	398.24	152.88	20.52
0.087	0.231	262.61	2737.10	427.63	169.51	22.03
0.087	0.234	270.37	2787.44	432.84	172.54	22.30
0.087	0.261	352.32	3292.15	482.84	202.83	24.88
0.087	0.300	497.77	4097.75	555.92	250.95	28.64
0.105	0.060	14.71	532.64	85.62	52.68	9.27
0.105	0.081	28.11	776.13	114.76	69.68	11.52
0.105	0.101	45.93	1038.79	142.65	76.72	12.40
0.105	0.108	54.28	1148.53	153.48	98.54	14.94
0.105	0.130	83.45	1491.49	184.88	125.08	17.75
0.105	0.155	125.06	1913.38	219.60	138.70	19.09
0.105	0.167	148.79	2131.43	236.24	196.79	24.28
0.105	0.212	266.14	3070.44	300.50	266.92	54.87
0.122	0.041	65.56	347.64	9.59	26.69	5.49
0.122	0.079	28.19	819.64	18.42	59.88	10.54
0.122	0.106	56.44	1252.71	24.83	89.09	14.20
0.122	0.139	107.67	1874.12	32.52	129.92	18.60
0.122	0.163	158.80	2396.08	38.12	163.59	21.81
0.122	0.181	205.58	2824.42	42.31	190.92	24.20
0.122	0.229	371.87	4133.34	53.53	273.291	30.62
0.122	0.272	576.95	5494.21	63.50	357.70	36.33

Set no. (5) $U = 10.23 \text{ cm./sec.}, \epsilon_m = 0.585$

0.105	0.019	6.98	724.55	32.09	61.73	2.10
0.105	0.041	31.30	1413.59	70.54	125.55	4.62
0.105	0.058	59.19	1809.81	100.07	167.02	6.55
0.105	0.071	83.22	2025.42	121.53	193.05	7.95
0.105	0.101	148.54	2293.89	172.76	241.25	11.31
0.105	0.121	196.12	2276.20	208.29	263.12	13.63
0.105	0.139	235.94	2125.01	239.20	274.44	15.65
0.105	0.165	285.46	1691.05	283.25	278.21	18.54
0.105	0.205	331.17	5202.05	351.12	255.52	22.98
0.122	0.019	8.53	881.76	36.76	75.53	2.45

Set no. (5) (continued)

β	λ	$Q(\text{cm}^3/\text{sec})$	$q(\text{cm}^2/\text{sec})$ x_{bw}^{-1}	$U_s(\text{cm}/\text{sec})$ x_{bw}^{-1}	$(\text{sec}^{-1})x_{bw}^{-1}$ $(-du/dy)_w$	$(\text{dyne}/\text{cm}^2)$ x_{bw}^{-1} τ_w
0.122	0.041	37.89	1694.75	80.81	152.24	5.38
0.122	0.079	118.99	2464.08	155.25	247.57	10.34
0.122	0.114	207.70	2500.18	224.03	297.42	14.92
0.122	0.145	277.98	1998.24	284.36	310.89	18.94

Set no. (6) $U = 10.68 \text{ cm./sec.}, \epsilon_m = 0.591$

0.017	0.191	26.67	295.26	116.98	17.15	3.53
0.017	0.221	36.36	350.27	135.39	20.34	4.08
0.017	0.235	41.52	377.08	144.09	21.89	4.34
0.017	0.249	46.86	403.43	152.49	23.41	4.60
0.017	0.275	57.99	454.71	168.40	26.38	5.08
0.017	0.286	63.16	477.18	175.21	27.67	5.28
0.017	0.309	74.94	525.78	189.62	30.48	5.72
0.017	0.320	80.95	549.39	196.47	31.84	5.92
0.035	0.158	60.56	808.11	291.55	48.57	5.85
0.035	0.191	89.95	1005.52	351.28	60.33	7.05
0.035	0.206	106.07	1102.39	379.45	66.09	7.62
0.035	0.221	123.01	1197.94	406.58	71.76	8.16
0.035	0.235	140.67	1292.12	432.72	77.35	8.69
0.035	0.249	158.99	1284.90	457.92	82.85	9.19
0.035	0.262	177.88	1476.25	482.24	88.25	9.68
0.052	0.103	44.04	899.47	284.01	57.02	5.69
0.052	0.121	62.71	1090.49	336.00	68.96	6.73
0.052	0.139	84.05	1218.36	385.86	80.84	7.72
0.052	0.157	107.83	1471.62	433.17	92.64	8.68
0.052	0.172	131.34	1642.03	475.17	103.18	9.51
0.052	0.188	158.94	1830.17	519.54	114.79	10.40
0.052	0.203	188.59	2016.78	562.23	126.27	11.26
0.052	0.231	249.33	2366.36	639.02	147.72	12.79
0.070	0.129	95.73	1601.85	319.47	108.72	9.50
0.070	0.150	133.08	1930.66	372.04	130.31	11.07
0.070	0.168	171.62	2233.46	418.03	150.06	12.44
0.070	0.191	226.01	2619.18	473.75	175.05	14.09

Set no. (6) (continued)

β	λ	$Q(\text{cm}^3/\text{sec})$	$q(\text{cm}^2/\text{sec})$ $x \frac{1}{b_w}$	$U_s(\text{cm}/\text{sec})$ $x \frac{1}{b_w}$	$(\text{sec}^{-1}) x \frac{1}{b_w}$ $(-du/dy)_w$	$(\text{dyne}/\text{cm}^2)$ $x \frac{1}{b_w} \tau_w$
0.070	0.218	303.69	3113.21	541.12	206.83	16.10
0.070	0.243	388.81	3602.58	604.12	238.10	17.97
0.070	0.263	464.54	4005.32	635.58	263.71	19.44
0.070	0.311	682.93	5050.29	773.59	329.68	23.01
0.070	0.225	326.65	3249.71	559.04	215.57	16.63
0.087	0.060	19.87	696.96	4.80	55.92	5.58
0.087	0.083	39.15	1017.46	6.59	80.22	7.66
0.087	0.103	62.04	1322.29	8.14	102.82	9.47
0.087	0.130	105.41	1798.34	10.36	137.38	12.04
0.087	0.150	143.68	2158.43	11.89	163.05	13.83
0.087	0.180	217.06	2761.58	14.27	205.36	16.59
0.087	0.208	302.43	3375.41	16.48	247.73	19.16
0.087	0.229	381.62	3890.76	18.21	282.87	21.17
0.087	0.259	509.12	4646.92	20.58	333.85	23.93
0.087	0.272	570.96	4989.19	21.60	356.74	25.12
0.087	0.287	647.96	5397.75	22.78	383.93	26.49

Set no. (7) $U = 11.22 \text{ cm./sec.}, \epsilon_m = 0.600$

0.017	0.206	87.61	871.12	375.92	51.03	3.73
0.017	0.235	114.71	1003.36	428.68	58.75	4.25
0.017	0.249	128.88	1066.79	453.65	62.46	4.50
0.017	0.275	158.14	1188.58	501.00	69.57	4.96
0.017	0.286	171.63	1241.29	521.26	72.64	5.17
0.017	0.309	202.12	1354.02	564.12	79.22	5.59
0.017	0.320	217.55	1408.21	584.52	82.38	5.79
0.035	0.141	128.37	1858.44	714.65	114.34	5.10
0.035	0.158	162.44	2101.11	801.82	129.20	5.72
0.035	0.175	199.14	2337.52	885.57	143.65	6.32
0.035	0.221	321.86	3010.92	1118.18	184.75	7.98
0.035	0.235	366.11	3224.03	1190.06	197.74	8.50
0.035	0.249	411.65	3431.76	1259.37	210.39	8.99
0.035	0.262	458.30	3634.27	1326.25	222.71	9.47
0.052	0.103	111.09	2212.50	653.98	147.46	5.56

Set no. (7) (continued)

β	λ	$Q(\text{cm}^3/\text{sec})$	$q(\text{cm}^2/\text{sec})$ $x_{\frac{1}{b_w}}$	$U_s(\text{cm}/\text{sec})$ $x_{\frac{1}{b_w}}$	$(\text{sec}^{-1})x_{\frac{1}{b_w}}$ $(du/dy)_w$	$(\text{dyne}/\text{cm}^2)$ $x_{\frac{1}{b_w}}$ τ_w
0.052	0.121	156.70	2647.54	773.69	176.16	6.58
0.052	0.139	208.19	3073.48	888.49	204.17	7.55
0.052	0.172	319.90	3857.84	1094.15	255.57	9.30
0.052	0.188	384.91	4257.70	1196.32	281.68	10.17
0.052	0.203	453.55	4648.77	1294.61	307.16	11.01
0.052	0.216	518.06	4993.34	1379.93	329.57	11.73
0.070	0.134	221.22	3441.44	325.56	263.78	9.68
0.070	0.155	299.54	4054.20	376.24	308.66	11.20
0.070	0.173	378.77	4607.28	420.79	349.32	12.52
0.070	0.203	529.92	5541.78	493.20	417.46	14.67
0.070	0.231	696.11	6448.61	560.57	482.98	16.68
0.070	0.257	874.21	7326.92	623.40	545.93	18.55
0.070	0.281	1061.63	8176.43	682.15	606.39	20.29
0.070	0.303	1256.17	8997.26	737.20	664.45	21.93
0.070	0.345	1659.56	10554.80	837.50	773.76	24.91

Set no. (8) $U = 11.70 \text{ cm./sec.}, \epsilon_m = 0.635$

0.017	0.160	219.91	4174.22	1248.23	239.56	2.64
0.017	0.176	365.63	4782.85	1376.72	274.37	2.91
0.017	0.192	466.22	5399.57	15002.28	309.62	3.17
0.017	0.208	533.21	6022.18	1619.17	345.19	3.42
0.017	0.222	626.15	6648.75	1733.65	380.98	3.67
0.017	0.236	724.59	7277.63	1843.98	416.89	3.90
0.017	0.250	828.10	7907.37	1950.36	452.89	4.12
0.035	0.023	16.51	1464.10	606.35	86.40	0.77
0.035	0.045	66.83	3110.06	1185.14	182.96	1.50
0.035	0.067	151.50	4909.54	1738.20	288.07	2.20
0.035	0.106	422.65	8874.50	2773.72	518.66	3.51
0.035	0.125	607.39	11000.40	3259.13	641.90	4.12
0.035	0.143	823.23	13199.90	3724.71	769.20	4.71
0.035	0.160	1068.67	15459.10	4171.68	899.78	5.28
0.052	0.023	32.44	2889.89	1095.81	175.30	1.15
0.052	0.045	132.40	6207.03	2141.82	374.30	2.25

Set no. (8) (continued)

β	λ	$Q(\text{cm}^3/\text{sec})$	$q(\text{cm}^2/\text{sec})$ $x_b \frac{1}{w}$	$U_s(\text{cm}/\text{sec})$ $x_b \frac{1}{w}$	$(\text{sec}^{-1}) x_b \frac{1}{w}$ $(du/dy)_w$	$(\text{dyne}/\text{cm}^2)$ $x_b \frac{1}{\tau_w}$
0.052	0.067	302.21	9886.54	3141.33	593.36	3.30
0.052	0.087	542.47	13872.20	4097.39	829.32	4.30
0.052	0.106	852.50	18115.50	5012.77	1079.45	5.26
0.052	0.125	1230.66	22574.00	5890.00	1341.39	6.18
0.052	0.160	2181.66	31995.00	7539.20	1892.73	7.92
0.070	0.085	804.48	21303.00	5376.10	13069.0	5.60
0.070	0.106	1340.58	28193.00	6731.12	1763.37	7.01
0.070	0.121	1816.19	34762.60	7677.42	2112.45	8.00
0.070	0.136	2362.42	40838.0	8592.56	2473.77	8.95
0.070	0.173	4212.15	58468.90	10960.60	3516.98	11.42
0.070	0.192	5422.42	68517.20	12167.80	4108.91	12.68
0.070	0.203	6189.29	74481.90	12846.60	4459.58	13.39
0.087	0.077	863.19	25471.20	5146.80	1619.26	6.34
0.087	0.106	1821.59	40037.80	7117.92	2512.18	8.76
0.087	0.129	2851.54	5280.50	8606.48	3286.62	10.60
0.087	0.148	3996.86	65205.80	9907.33	4033.95	12.20
0.087	0.173	5856.53	82973.60	11590.40	5098.53	14.27
0.087	0.203	8674.28	106543.00	13584.80	6502.58	16.73
0.087	0.227	11413.50	127033.00	15156.10	7717.66	18.66
0.087	0.247	14255.90	146588.00	16547.20	8873.63	20.38
0.087	0.277	19068.90	176926.00	18540.90	10661.40	22.83
0.105	0.048	348.87	16321.70	2403.66	1112.49	4.71
0.105	0.081	1169.38	33712.20	4086.74	2229.45	8.00
0.105	0.103	2043.89	47599.50	5177.11	3101.64	10.13
0.105	0.138	4165.39	74486.50	6944.44	4763.51	13.59
0.105	0.163	6378.12	97734.50	8245.21	6182.74	16.14
0.105	0.194	9827.46	129053.00	9785.33	8078.10	19.16
0.105	0.228	14890.50	168899.00	11505.70	10471.00	22.52
0.105	0.249	18663.10	195626.00	12551.50	12067.60	24.57
0.122	0.026	91.41	7815.01	166.42	602.06	2.94
0.122	0.075	1064.12	34030.20	488.32	2395.79	8.63
0.122	0.104	2394.21	56754.50	681.25	3868.38	12.04
0.122	0.134	4470.64	84688.50	873.88	5636.03	15.44

Set no. (8) (continued)

β	λ	$Q(\text{cm}^3/\text{sec})$	$q(\text{cm}^2/\text{sec})$ $\times \frac{1}{b_w}$	$U_s(\text{cm}/\text{sec})$ $\times \frac{1}{b_w}$	$(\text{sec}^{-1}) \times \frac{1}{b_w}$ $(du/dy)_w$	$(\text{dyne}/\text{cm}^2)$ $\times \frac{1}{\tau_w^w}$
0.122	0.160	7037.35	113583.00	1043.28	7435.92	18.43
0.122	0.172	8436.25	127790.00	1118.90	8313.58	19.77
0.122	0.202	12851.00	168176.00	1314.02	10789.50	23.22
0.122	0.219	16080.10	194765.00	1430.15	12407.80	25.27
0.122	0.246	21833.90	238074.00	1603.79	15028.90	28.34

Set no. (9) $U = 12.22 \text{ cm./sec.}, \epsilon_m = 0.640$

0.017	0.045	13.91	580.94	32.22	50.24	7.40
0.017	0.067	28.50	787.77	47.26	70.04	10.85
0.017	0.087	46.17	947.34	61.65	86.81	14.15
0.017	0.125	86.40	1145.67	88.62	112.53	20.34
0.017	0.160	127.98	1211.95	113.43	129.61	26.03
0.017	0.176	147.92	1204.61	125.10	135.46	28.71
0.017	0.208	184.32	1123.60	147.14	142.70	33.77
0.017	0.222	200.33	1054.83	157.54	144.39	36.16
0.035	0.023	13.10	1094.15	53.41	92.80	0.76
0.035	0.045	47.28	1955.19	104.39	170.99	1.48
0.035	0.067	100.00	2610.61	153.11	236.22	2.17
0.035	0.106	217.21	3397.98	244.33	333.42	3.46
0.035	0.125	282.27	3569.69	287.08	367.79	4.07
0.035	0.160	408.19	3551.46	367.47	413.08	5.21
0.035	0.176	465.48	3388.81	405.29	425.65	5.74
0.052	0.160	685.14	5076.55	632.85	692.50	7.81
0.052	0.203	861.58	2870.77	803.07	676.31	9.91
0.070	0.077	304.80	6492.60	375.37	652.73	5.00
0.070	0.106	502.27	6703.57	519.14	773.71	6.92
0.070	0.121	600.13	6328.36	592.12	807.79	7.89
0.070	0.141	716.32	5334.19	688.57	824.58	9.18
0.087	0.101	468.80	5489.17	429.80	768.18	8.18
0.087	0.114	535.57	4498.52	486.53	772.33	9.26
0.105	0.048	134.34	4409.96	63.60	476.47	4.64
0.105	0.810	269.56	3096.57	108.13	560.60	7.89

Set no. (10) $U = 12.76 \text{ cm./sec.}, \epsilon_m = 0.642$

β	λ	$Q(\text{cm}^3/\text{sec})$	$q(\text{cm}^2/\text{sec})$ $\times \frac{1}{b_w}$	$U_s(\text{cm/sec})$ $\times \frac{1}{b_w}$	$(\text{sec}^{-1}) \times \frac{1}{b_w}$ $(du/dy)_w$	(dyne/cm^2) $\times \frac{1}{b_w}$ τ_w
0.017	0.045	27.41	1263.44	521.93	73.00	0.74
0.017	0.087	109.03	2717.27	998.48	156.67	1.41
0.017	0.143	325.89	5128.48	1640.36	295.04	2.31
0.017	0.176	526.33	6828.69	2026.33	392.44	2.86
0.017	0.192	641.28	7694.90	2208.18	442.02	3.11
0.017	0.208	765.15	8567.83	2383.16	491.97	3.36
0.017	0.222	897.27	9444.95	2551.67	542.14	3.60
0.017	0.236	1037.01	10324.10	2714.05	592.40	3.82
0.035	0.023	23.93	2117.57	871.28	125.66	0.75
0.035	0.045	96.49	4474.58	1702.95	264.63	1.47
0.035	0.067	218.01	7033.18	2497.64	414.80	2.16
0.035	0.106	604.99	12628.80	3985.62	741.60	3.44
0.035	0.143	1173.50	18692.20	5352.12	1094.13	4.62
0.035	0.160	1520.70	21847.70	5994.37	1277.15	5.18
0.035	0.192	2329.34	28326.30	7204.77	1652.22	6.22
0.052	0.155	2833.39	42562.50	10110.60	2538.96	7.52
0.052	0.191	4620.15	57515.80	12448.30	3415.82	9.26
0.052	0.231	7293.49	76491.30	15059.80	4524.10	11.20
0.052	0.257	9445.14	90032.30	16747.90	5312.79	12.45
0.052	0.280	11784.80	103598.00	18326.20	6101.50	13.63
0.052	0.294	13214.00	111429.00	19193.80	6556.24	14.27
0.052	0.345	19659.40	143687.00	22499.60	8426.16	16.73
0.052	0.364	22491.40	156679.00	23730.60	9178.07	17.65
0.070	0.070	886.37	25667.10	6324.72	1603.32	4.98
0.070	0.114	2156.91	43615.90	9366.98	2692.15	7.37
0.070	0.148	3975.51	63410.60	12174.80	3880.12	9.58
0.070	0.168	5390.94	76608.60	13839.20	4667.33	10.89
0.070	0.186	6858.35	89062.30	15297.00	5407.50	12.03
0.070	0.203	8478.43	101780.00	16693.90	6161.21	13.13
0.070	0.242	13039.30	133738.00	19887.50	8047.74	15.64
0.070	0.267	16684.60	156559.00	21954.40	9389.90	17.27
0.070	0.291	20633.90	179436.00	23888.60	10732.20	18.79
0.087	0.069	891.35	29203.00	5209.53	1913.92	5.55

Set no. (10) (continued)

β	λ	$Q(\text{cm}^3/\text{sec})$	$q(\text{cm}^2/\text{sec})$ $\frac{1}{x_{bw}}$	$U_s(\text{cm}/\text{sec})$ $\frac{1}{x_{bw}}$	$(\text{sec}^{-1}) \frac{1}{x_{bw}}$ $(du/dy)_w$	$(\text{dyne}/\text{cm}^2)$ $\frac{1}{x_{bw} \tau_w}$
0.087	0.106	2429.79	53507.30	8062.79	3433.25	8.60
0.087	0.136	4334.74	76593.90	10292.50	4852.62	10.97
0.087	0.168	7295.98	106287.00	12756.80	6658.16	13.60
0.087	0.198	10966.10	137749.00	15042.30	8555.07	16.04
0.087	0.215	13389.50	156537.00	16291.40	9682.10	17.37
0.087	0.247	19093.00	196706.00	18743.80	12080.90	19.98
0.087	0.291	28874.80	257148.00	22020.10	15669.90	23.48
0.105	0.041	316.00	17058.90	1377.74	1233.59	3.98
0.105	0.056	637.91	25849.20	1883.42	1829.74	5.44
0.105	0.103	2611.52	61389.50	3438.46	4145.33	9.94
0.105	0.123	4077.53	81301.10	4129.38	5407.77	11.94
0.105	0.145	6065.54	104694.00	4847.80	6872.88	14.01
0.105	0.170	9108.07	135916.00	5697.92	8807.28	16.47
0.105	0.194	12742.90	168852.00	6499.08	10829.40	18.79
0.105	0.228	19379.20	221725.00	7641.72	14047.80	22.09
0.105	0.270	30112.10	295701.00	9037.16	18511.40	26.12

TABLE 2

Heterogeneous bed results

Set no.	(cm./ sec.) U	ϵ_m	(μ^*) _o (poise)	ϵ_c	ϵ_b	μ_p (poise)	(cm) d_b	(cm./ sec.) u_{bo}	(cm./ sec.) u_c	(cm./ sec.) u_b	(dyne) I_p
(1)	8.38	0.550	1.2054	0.3001	0.357	1.2789	0.14	8.32	15.71	10.28	0.513
(2)	8.82	0.562	0.1424	0.301	0.373	0.1277	0.06	5.45	17.58	9.46	0.070
(3)	9.33	0.564	0.0092	0.318	0.361	0.0069	0.02	3.14	21.18	7.14	-0.004
(4)	9.73	0.577	0.5552	0.3003	0.396	0.4263	0.10	7.03	17.35	11.52	0.179
(5)	10.23	0.585	0.0785	0.302	0.405	0.0638	0.05	4.97	20.07	10.30	0.037

TABLE 3

Quasi-homogeneous bed results

Set no.	U (cm./sec.)	ϵ_m	μ^* (poise)	ϵ_c	I_p (dyne)
(6)	10.68	0.591	0.2680	0.3004	0.707
(7)	11.22	0.600	0.1205	0.301	0.332
(8)	11.70	0.635	0.0202	0.306	0.051
(9)	12.22	0.640	0.0462	0.303	0.134
(10)	12.76	0.642	0.0185	0.307	0.052

TABLE 4

Friction factor - Reynolds number data

Set no. (1) $U = 8.38 \text{ cm./sec.}, \epsilon_m = 0.550$

<u>Re*</u>	<u>f*</u>
112.16	0.57×10^{-1}
268.57	0.23×10^{-1}
470.10	0.12×10^{-1}
552.65	0.11×10^{-1}
1303.49	0.66×10^{-2}
2343.72	0.50×10^{-2}

Set no. (2) $U = 8.82 \text{ cm./sec.}, \epsilon_m = 0.562$

<u>Re*</u>	<u>f*</u>
2015.57	0.28×10^{-2}
2439.09	0.17×10^{-2}
1992.35	0.16×10^{-2}
1083.94	0.19×10^{-2}
4713.48	0.54×10^{-4}

Set no. (3) $U = 9.33 \text{ cm./sec.}, \epsilon_m = 0.564$

<u>Re*</u>	<u>f*</u>
1318.22	0.55×10^{-2}
1291.97	0.38×10^{-2}
797.15	0.36×10^{-2}
768.66	0.44×10^{-2}
671.68	0.77×10^{-2}

Set no. (4) $U = 9.73 \text{ cm./sec.}, \epsilon_m = 0.577$

<u>Re*</u>	<u>f*</u>
98.11	0.15
531.18	0.13×10^{-1}
1283.24	0.65×10^{-2}
2752.60	0.25×10^{-2}

Set no. (4) (continued)

<u>Re*</u>	<u>f*</u>
3557.77	0.18×10^{-2}
3631.97	0.19×10^{-2}
6022.59	0.13×10^{-2}

Set no. (5) $U = 10.23 \text{ cm./sec.}, \epsilon_m = 0.585$

<u>Re*</u>	<u>f*</u>
1610.00	0.43×10^{-2}
2041.02	0.25×10^{-2}
2234.18	0.22×10^{-2}
2114.15	0.21×10^{-2}
21.85	23.15

TABLE 5

<u>Set no. (1)</u>	$a_1 = 0.21892 \times 10^6$ $a_2 = -0.750810 \times 10^6$ $a_3 = 0.61982 \times 10^6$	$n_1 = -1.0$ $n_2 = 1.0$ $s = 0.179863 \times 10^2$
<u>Set no. (2)</u>	$a_1 = 0.127837 \times 10^7$ $a_2 = 0.575026 \times 10^7$ $a_3 = -0.698685 \times 10^7$	$n_1 = 0.0$ $n_2 = 0.1$ $s = 0.193555 \times 10^3$
<u>Set no. (3)</u>	$a_1 = 0.188625 \times 10^7$ $a_2 = -0.511039 \times 10^7$ $a_3 = -0.431682 \times 10^7$	$n_1 = 0.0$ $n_2 = 0.1$ $s = 0.996735 \times 10^2$
<u>Set no. (4)</u>	$a_1 = 0.816993 \times 10^6$ $a_2 = -0.494356 \times 10^7$ $a_3 = 0.113991 \times 10^7$	$n_1 = 0.0$ $n_2 = 1.0$ $s = 0.909548 \times 10^2$
<u>Set no. (5)</u>	$a_1 = 0.306031 \times 10^7$ $a_2 = -0.107121 \times 10^8$ $a_3 = -0.594558 \times 10^7$	$n_1 = 0.0$ $n_2 = 0.1$ $s = 0.104510 \times 10^3$
<u>Set no. (6)</u>	$a_1 = 0.251482 \times 10^7$ $a_2 = -0.216049 \times 10^8$ $a_3 = 0.140908 \times 10^7$	$n_1 = 0.0$ $n_2 = 1.0$ $s = 0.121284 \times 10^3$
<u>Set no. (7)</u>	$a_1 = 0.782187 \times 10^7$ $a_2 = -0.789286 \times 10^8$ $a_3 = 0.160415 \times 10^7$	$n_1 = 0.0$ $n_2 = 1.0$ $s = 0.427508 \times 10^3$
<u>Set no. (8)</u>	$a_1 = 0.298211 \times 10^8$ $a_2 = -0.181275 \times 10^9$ $a_3 = 0.673732 \times 10^8$	$n_1 = 0.0$ $n_2 = 1.0$ $s = 0.696761 \times 10^4$
<u>Set no. (9)</u>	$a_1 = 0.276951 \times 10^8$ $a_2 = -0.190421 \times 10^9$ $a_3 = -0.497390 \times 10^8$	$n_1 = 0.0$ $n_2 = 0.1$ $s = 0.170087 \times 10^4$
<u>Set no. (10)</u>	$a_1 = 0.446399 \times 10^8$ $a_2 = -0.299028 \times 10^9$ $a_3 = 0.910495 \times 10^8$	$n_1 = 0.0$ $n_2 = 1.0$ $s = 0.107116 \times 10^5$

APPENDIX 1

EXPERIMENTAL DATA

(1) Set no.1 $U = 8.38 \text{ cm./sec.}$, $\epsilon_m = 0.550$

$\alpha(\circ)$	$h \text{ (cm.)}$	$M_s \text{ (g./sec.)}$
5.0	0.56	22.5
5.0	1.10	90.0
5.0	1.17	148.2
5.0	1.25	157.5
5.0	1.39	202.5
5.0	1.58	252.5
5.0	1.83	292.5
6.0	0.52	25.0
6.0	0.64	42.5
6.0	0.74	61.7
6.0	0.82	73.3
6.0	0.92	102.5
6.0	1.06	135.0
6.0	1.16	160.0
6.0	1.39	230.0
6.0	1.54	235.0
7.0	0.37	30.0
7.0	0.47	40.0
7.0	0.55	43.8
7.0	0.69	87.5
7.0	0.87	137.5
7.0	1.12	207.5
7.0	1.26	247.5
7.0	1.44	267.5

(2) Set no.2 $U = 8.82 \text{ cm./sec.}$, $\epsilon_m = 0.562$

$\alpha(o)$	$h(\text{cm.})$	$M_s (\text{g./sec.})$
4.0	0.70	13.7
4.0	0.81	50.0
4.0	0.92	68.3
4.0	1.10	95.0
4.0	1.21	103.2
5.0	0.90	27.5
5.0	1.08	80.0
5.0	1.20	112.5
5.0	1.30	135.0
5.0	1.45	167.5
5.0	1.59	202.5
5.0	1.81	367.5
5.0	2.03	365.0
6.0	0.64	30.0
6.0	0.74	52.8
6.0	0.82	61.7
6.0	0.92	95.0
6.0	1.03	127.5
6.0	1.10	145.0
6.0	1.28	187.5
6.0	1.38	287.5
7.0	0.08	35.0
7.0	0.18	36.7
7.0	0.32	52.5
7.0	0.46	80.0
7.0	0.71	132.5
7.0	1.01	230.0
7.0	1.43	315.0

(3) Set no.3 $U = 9.33 \text{ cm./sec.}$, $\epsilon_m = 0.564$

3.0	0.87	40.0
3.0	0.97	67.5
3.0	1.07	125.0
3.0	1.16	153.9
3.0	1.26	173.2

Set no.3 (continued)

<u>$\alpha(o)$</u>	<u>$h(cm.)$</u>	<u>$M_s(g./sec.)$</u>
3.0	1.45	215.0
3.0	1.55	267.5
4.0	0.81	30.0
4.0	0.99	48.8
4.0	1.13	78.3
4.0	1.25	117.5
4.0	1.35	160.0
4.0	1.58	217.5
4.0	2.02	347.5
5.0	0.94	33.7
5.0	1.08	71.7
5.0	1.23	106.7
5.0	1.31	132.5
5.0	1.41	162.5
5.0	1.59	245.0
5.0	1.71	255.0
5.0	2.07	365.0
6.0	0.64	30.0
6.0	0.71	42.5
6.0	0.82	75.0
6.0	0.92	105.0
6.0	1.06	142.5
6.0	1.16	167.5
6.0	1.32	207.5
6.0	1.60	275.0
7.0	0.11	35.0
7.0	0.22	61.6
7.0	0.28	70.0
7.0	0.46	100.0
7.0	0.64	155.0
7.0	1.03	252.5
7.0	1.25	302.5
7.0	1.39	365.0

(4) Set no.4 $U = 9.73 \text{ cm./sec.}$, $\epsilon_m = 0.577$

$\alpha(o)$	$h(\text{cm.})$	$M_s (\text{g./sec.})$
1.0	1.19	6.3
1.0	1.39	7.6
1.0	1.49	8.5
1.0	1.59	11.4
1.0	1.68	20.6
2.0	1.09	35.0
2.0	1.19	46.7
2.0	1.39	76.7
2.0	1.59	234.7
2.0	1.68	348.4
3.0	0.68	16.6
3.0	0.87	45.0
3.0	0.97	102.5
3.0	1.16	187.5
3.0	1.26	332.5
3.0	1.36	505.0
4.0	0.74	23.7
4.0	0.88	68.3
4.0	1.03	118.2
4.0	1.25	140.1
4.0	1.43	171.6
4.0	1.61	235.0
4.0	1.91	320.0
5.0	0.71	23.7
5.0	0.79	58.3
5.0	0.89	85.0
5.0	1.04	130.0
5.0	1.15	137.0
5.0	1.26	167.5
5.0	1.28	182.5
5.0	1.48	212.5
5.0	1.80	485.0
6.0	0.27	20.8
6.0	0.37	30.0
6.0	0.47	51.7

Set no.4 (continued)

$\alpha(o)$	$h(cm.)$	$M_s (g./sec.)$
6.0	0.51	54.9
6.0	0.63	87.5
6.0	0.77	115.0
6.0	0.84	130.0
6.0	1.13	180.0
7.0	0.18	35.0
7.0	0.36	65.0
7.0	0.50	97.5
7.0	0.68	152.5
7.0	0.82	192.5
7.0	0.93	232.5
7.0	1.25	302.5
7.0	1.57	375.0

(5) Set no.5 $U = 10.23 \text{ cm./sec.}, \epsilon_m = 0.585$

1.0	1.39	14.2
1.0	1.49	17.7
1.0	1.59	21.6
1.0	1.68	26.5
1.0	1.78	35.4
1.0	1.98	50.8
1.0	2.08	80.1
1.0	2.28	130.0
2.0	0.89	30.9
2.0	0.99	40.0
2.0	1.19	55.5
2.0	1.29	62.5
2.0	1.39	112.6
2.0	1.49	125.9
2.0	1.59	133.7
2.0	1.68	251.8
3.0	0.58	28.3
3.0	0.68	80.0
3.0	0.78	115.0
3.0	1.07	267.5

Set no.5 (continued)

$\alpha(o)$	$h(cm.)$	$M_s(g./sec.)$
3.0	1.16	325.0
4.0	0.84	41.7
4.0	0.99	86.7
4.0	1.17	121.8
4.0	1.32	245.0
4.0	1.72	295.0
4.0	1.98	375.0
4.0	2.53	575.0
5.0	0.51	45.0
5.0	0.73	70.0
5.0	0.87	105.0
5.0	0.98	130.0
5.0	1.09	175.0
5.0	1.24	195.0
5.0	1.42	235.0
5.0	1.60	285.0
5.0	1.89	380.0
6.0	0.08	23.8
6.0	0.18	32.5
6.0	0.26	47.5
6.0	0.32	60.0
6.0	0.47	92.5
6.0	0.58	112.5
6.0	0.68	137.5
6.0	0.83	230.0
6.0	1.08	255.0
7.0	0.08	33.8
7.0	0.18	47.5
7.0	0.36	77.5
7.0	0.54	127.5
7.0	0.71	167.5
7.0	0.93	215.0
7.0	1.07	252.5
7.0	1.29	297.5
7.0	1.43	345.0

(6) Set no.6 $U = 10.68 \text{ cm./sec.}, \epsilon_m = 0.591$

$\alpha(o)$	$h(\text{cm.})$	$M_s (\text{g./sec.})$
1.0	0.99	7.5
1.0	1.19	43.7
1.0	1.29	47.0
1.0	1.39	51.3
1.0	1.59	83.3
1.0	1.68	110.0
1.0	1.88	132.0
1.0	1.98	194.8
2.0	0.79	43.3
2.0	0.99	58.7
2.0	1.09	86.7
2.0	1.19	111.2
2.0	1.29	126.2
2.0	1.39	168.3
2.0	1.49	262.5
3.0	0.48	22.5
3.0	0.58	48.7
3.0	0.68	70.0
3.0	0.78	108.2
3.0	0.87	130.2
3.0	0.97	190.0
3.0	1.07	252.5
3.0	1.26	490.0
4.0	0.62	33.3
4.0	0.74	48.3
4.0	0.85	56.6
4.0	0.99	80.0
4.0	1.17	112.5
4.0	1.35	142.5
4.0	1.50	190.0
4.0	1.90	302.5
4.0	2.22	422.5
5.0	0.27	22.5
5.0	0.38	42.5
5.0	0.48	46.3

Set no.6 (continued)

<u>$\alpha(o)$</u>	<u>$h(cm.)$</u>	<u>$M_s (g./sec.)$</u>
5.0	0.63	70.0
5.0	0.74	95.0
5.0	0.92	137.5
5.0	1.10	180.0
5.0	1.25	215.0
5.0	1.47	270.0
5.0	1.57	305.0
5.0	1.69	395.0
6.0	0.20	21.3
6.0	0.38	46.7
6.0	0.56	95.0
6.0	0.71	142.5
6.0	0.82	150.0
6.0	0.96	195.0
6.0	1.18	267.5
6.0	1.50	335.0
7.0	0.11	31.3
7.0	0.25	45.0
7.0	0.43	70.0
7.0	0.61	115.0
7.0	0.71	167.5
7.0	0.83	177.5
7.0	1.00	202.5
7.0	1.32	305.0

(7) Set no.7 $U = 11.22 \text{ cm./sec.}, c_m = 0.600$

1.0	1.09	211.0
1.0	1.29	218.3
1.0	1.39	219.8
1.0	1.59	229.9
1.0	1.68	251.9
1.0	1.88	265.3
1.0	1.98	456.6
2.0	0.69	48.0
2.0	0.79	71.2

Set no.7 (continued)

<u>$\alpha(o)$</u>	<u>$h(cm.)$</u>	<u>$M_s (g./sec.)$</u>
2.0	0.89	108.9
2.0	1.19	148.8
2.0	1.29	200.0
2.0	1.39	243.8
2.0	1.49	323.4
3.0	0.48	65.0
3.0	0.58	90.0
3.0	0.68	170.0
3.0	0.87	247.5
3.0	0.97	295.0
3.0	1.07	395.0
3.0	1.16	460.0
4.0	0.65	12.5
4.0	0.77	32.5
4.0	0.88	51.7
4.0	1.07	80.0
4.0	1.26	165.0
4.0	1.45	185.0
4.0	1.64	222.5
4.0	1.83	277.5
4.0	2.21	387.5
5.0	0.28	20.0
5.0	0.40	42.5
5.0	0.50	43.3
5.0	0.66	65.0
5.0	0.73	95.0
5.0	0.88	142.5
5.0	1.04	165.0
5.0	1.38	250.0
5.0	1.53	272.5
6.0	0.21	31.3
6.0	0.43	56.7
6.0	0.56	57.3
6.0	0.78	115.0
6.0	0.97	167.5

Set no.7 (continued)

$\alpha(o)$	$h(cm.)$	$M_s(g./sec.)$
6.0	1.20	217.5
6.0	1.32	252.5
6.0	1.58	327.5
7.0	0.11	31.3
7.0	0.23	43.3
7.0	0.46	82.5
7.0	0.76	175.0
7.0	1.14	252.5
7.0	1.25	287.5

(8) Set no.8 $U = 11.70 \text{ cm./sec.}, \epsilon_m = 0.635$

1.0	0.80	95.0
1.0	0.90	115.0
1.0	1.00	125.0
1.0	1.10	135.0
1.0	1.20	198.2
1.0	1.30	219.3
1.0	1.40	231.5
2.0	0.10	40.0
2.0	0.20	67.5
2.0	0.30	125.0
2.0	0.50	153.6
2.0	0.60	173.2
2.0	0.70	215.0
2.0	0.80	267.5
3.0	0.10	62.5
3.0	0.20	95.0
3.0	0.30	145.0
3.0	0.40	172.5
3.0	0.50	202.5
3.0	0.60	222.5
3.0	0.80	615.0
4.0	0.39	18.8
4.0	0.50	32.5
4.0	0.58	53.3

Set no.8 (continued)

$\alpha(o)$	$h(cm.)$	$M_B (g./sec.)$
4.0	0.66	77.5
4.0	0.88	137.5
4.0	1.00	150.0
4.0	1.07	177.5
5.0	0.35	21.7
5.0	0.50	50.0
5.0	0.62	62.5
5.0	0.73	100.0
5.0	0.88	132.5
5.0	1.07	162.5
5.0	1.23	195.0
5.0	1.38	217.5
5.0	1.61	257.5
6.0	0.21	18.8
6.0	0.37	36.3
6.0	0.48	56.6
6.0	0.67	95.0
6.0	0.82	142.5
6.0	1.01	165.0
6.0	1.24	227.5
6.0	1.39	287.5
7.0	0.11	30.0
7.0	0.34	62.5
7.0	0.49	87.5
7.0	0.65	142.5
7.0	0.80	165.0
7.0	0.87	172.5
7.0	1.06	245.0
7.0	1.18	285.0
7.0	1.37	320.0

(9) Set no.9 $U = 12.22 \text{ cm./sec.}, \epsilon_m = 0.640$

1.0	0.20	22.1
1.0	0.30	41.9
1.0	0.40	53.2

Set no.9 (continued)

<u>$\alpha(o)$</u>	<u>$h(\text{cm.})$</u>	<u>$M_s (\text{g./sec.})$</u>
1.0	0.60	73.1
1.0	0.80	99.3
1.0	0.90	143.2
1.0	1.10	185.1
1.0	1.20	266.3
2.0	0.10	18.0
2.0	0.20	62.5
2.0	0.30	116.2
2.0	0.50	157.5
2.0	0.60	193.5
2.0	0.80	307.5
2.0	0.90	432.5
3.0	0.80	52.5
3.0	1.07	102.5
3.0	1.26	140.0
3.0	1.45	185.0
3.0	1.64	232.5
3.0	1.83	262.5
3.0	2.02	340.0
3.0	2.40	480.0
4.0	0.35	22.5
4.0	0.50	33.8
4.0	0.58	42.5
4.0	0.69	67.5
4.0	0.85	97.5
4.0	1.11	150.0
4.0	1.30	177.5
5.0	0.47	46.3
5.0	0.54	60.0
5.0	0.81	100.0
5.0	1.04	177.5
5.0	1.26	222.5
5.0	1.38	240.0
5.0	1.91	425.0
6.0	0.21	22.5

Set no.9 (continued)

<u>$\alpha(o)$</u>	<u>$h(cm.)$</u>	<u>$M_s(g./sec.)$</u>
6.0	0.37	38.8
6.0	0.52	60.0
6.0	0.71	85.0
6.0	0.86	137.5
6.0	0.93	172.5
6.0	1.17	212.5
6.0	1.43	265.0
6.0	1.51	307.5
7.0	0.08	22.5
7.0	0.23	32.5
7.0	0.34	65.0
7.0	0.46	85.0
7.0	0.61	122.5
7.0	0.76	142.5
7.0	0.87	172.5
7.0	1.10	235.0
7.0	1.48	365.0

(10)Set no.10 $U = 12.76 \text{ cm./sec.}, e_m = 0.642$

1.0	0.20	56.3
1.0	0.40	77.5
1.0	0.70	125.8
1.0	0.90	147.6
1.0	1.00	196.2
1.0	1.10	244.0
1.0	1.20	271.2
1.0	1.30	277.5
2.0	0.10	29.1
2.0	0.20	122.5
2.0	0.30	165.0
2.0	0.50	190.0
2.0	0.70	257.5
2.0	0.80	305.0
2.0	1.00	432.5
3.0	0.77	28.3

Set no.10 (continued)

<u>$\alpha(o)$</u>	<u>$h(cm.)$</u>	<u>$M_s(g./sec.)$</u>
3.0	0.99	87.5
3.0	1.26	140.0
3.0	1.45	192.5
3.0	1.64	217.5
3.0	1.75	265.0
3.0	2.21	387.5
3.0	2.40	415.0
4.0	0.35	23.7
4.0	0.54	52.5
4.0	0.73	92.5
4.0	0.85	120.0
4.0	0.96	142.5
4.0	1.07	157.5
4.0	1.34	227.5
4.0	1.54	292.5
4.0	1.72	332.5
5.0	0.31	25.0
5.0	0.50	43.7
5.0	0.66	77.5
5.0	0.85	120.0
5.0	1.04	167.5
5.0	1.15	172.5
5.0	1.38	237.5
5.0	1.72	342.5
6.0	0.18	12.5
6.0	0.25	32.5
6.0	0.48	62.5
6.0	0.59	95.0
6.0	0.71	130.0
6.0	0.86	140.0
6.0	1.01	190.0
6.0	1.24	240.0
6.0	1.55	320.0
7.0	0.08	27.5
7.0	0.19	45.0

Set no.10 (continued)

<u>$\alpha(o)$</u>	<u>$h(\text{cm.})$</u>	<u>$M_{\text{S}} (\text{g./sec.})$</u>
7.0	0.30	65.0
7.0	0.46	90.0
7.0	0.57	122.5
7.0	0.76	147.5
7.0	0.87	175.0
7.0	1.06	232.5
7.0	1.40	350.0

APPENDIX 2

Two-Variable Polynomial Surface Fitting

It was pointed out in section (4) of chapter 6 that the flowrate, Q is expressed analytically as a polynomial in β and λ where $\beta = \sin\alpha$, and $\lambda = h/(h+b_w)$. A computer program SURFIT was used to accomplish this surface fitting (Ref.: M. Grossman). In order to satisfy the boundary conditions:

$$Q = 0, \text{ all } \beta \text{ and all } \lambda$$

$$q = \frac{\partial Q}{\partial h} = 0, \text{ all } \beta \text{ and } \lambda$$

$Q/(\beta^2 \lambda^2)$ was fitted as a function of β and λ . It was found that for the experimental values obtained for Q , the best analytical expression was of the form:

$$Q/(\beta^2 \lambda^2) = a_1 + a_2 \beta + a_3 \lambda$$

$$\text{i.e.} \quad Q = a_1 \beta^2 \lambda^2 + a_2 \beta^3 \lambda + a_3 \beta^2 \lambda^3 \quad (1)$$

Now $q = \frac{\partial Q}{\partial h} = \frac{1}{b_w} \frac{\partial Q}{\partial \lambda}$, assuming that $b_w \gg h$, which is valid. Hence:

$$q = \frac{1}{b_w} (2a_1 \beta^2 \lambda + 2a_2 \beta^3 + 3a_3 \beta^2 \lambda^2) \quad (2)$$

$$\therefore \quad \frac{\partial q}{\partial \lambda} = \frac{1}{b_w} (2a_1 \beta^2 + 2a_2 \beta^3 + 6a_3 \beta^2 \lambda)$$

$$\text{and} \quad \frac{\partial q}{\partial \beta} = \frac{1}{b_w} (4a_1 \beta \lambda + 6a_2 \beta^2 + 6a_3 \beta \lambda^2)$$

$$\therefore \quad I_1(\beta) = \beta \int \frac{1}{\lambda} \frac{\partial q}{\partial \beta} d\lambda = \frac{1}{b_w} (4a_1 \beta^2 \lambda + 6a_2 \beta^3 \lambda + 3a_3 \beta^2 \lambda^2)$$

$$I_2(\lambda) = \lambda \int \frac{1}{\beta} \frac{\partial q}{\partial \lambda} d\beta = \frac{1}{b_w} (a_1 \beta^2 \lambda + \frac{2}{3} a_2 \beta^3 \lambda + 3a_3 \beta^2 \lambda^2)$$

∴ The two solutions for U_s are:

$$U_s = g_1 = \frac{q - I_1(\beta)}{2b_w} + C_1(\beta)$$

and $U_s = g_2 = \frac{q - I_2(\lambda)}{2b_w} + C_2(\lambda)$

Applying the boundary conditions $U_s=0$, all β and all λ , we conclude that:

$$C_1(\beta) = C_2(\lambda) = 0$$

∴ $g_1 = \frac{q - I_1(\beta)}{2b_w} = -\frac{1}{b_w} (2a_1\beta^2\lambda + 4a_2\beta^2\lambda)$

and $g_2 = \frac{q - I_2(\lambda)}{2b_w} = \frac{1}{b_w} (a_1\beta^2\lambda + \frac{4}{3}a_2\beta^3\lambda)$

∴ $\frac{\partial g_1}{\partial \lambda} = -\frac{1}{b_w} (2a_1\beta^2 + 4a_2\beta^3)$

and $\frac{\partial g_2}{\partial \lambda} = \frac{1}{b_w} (a_1\beta^2 + \frac{4}{3}a_2\beta^3)$

We take $U_s = n_1g_1 + n_2g_2$, since the equation for U_s is linear.

Hence:

$$U_s = \frac{1}{b_w} (a_1(-2n_1+n_2)\beta^2\lambda + a_2(-4n_1+\frac{4}{3}n_2)\beta^3\lambda) \quad (3)$$

∴ $\frac{\partial U_s}{\partial \lambda} = \frac{1}{b_w} (a_1(-2n_1+n_2)\beta^2 + a_2(-4n_1+\frac{4}{3}n_2)\beta^3)$

$$\begin{aligned} \therefore (-du/dy)_w &= \frac{1}{2b_w} (a_1(6+4n_1-2n_1)\beta^2\lambda + a_2(6+8n_1-\frac{4}{3}n_2)\beta^3\lambda \\ &\quad + 6a_3\beta^2\lambda^2) \end{aligned} \quad (4)$$

For each set of data, the constants n_1 and n_2 are so chosen that the maximum number of points satisfying the constraints: $Q > 0$, $q > 0$, $U_s > 0$, $(-du/dy)_w > 0$, are obtained.

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NOMENCLATURE

PART 1

CHAPTER TWO

- d_b - bubble diameter
- f^* - friction factor
- g - gravitational acceleration
- Re^* - Reynolds number
- Re_p - particle Reynolds number
- U_{bo} - velocity of a single bubble
- V - volume of a single bubble
- ϕ - volumetric concentration of solid particles
- ϵ - voidage fraction
- ϵ_s - voidage fraction at saturation
- K_p - bulk viscosity coefficient of the particulate phase
- μ_f - shear viscosity coefficient of the fluid
- μ_p - shear viscosity coefficient of the particulate phase

CHAPTER THREE

- b_w - half-width of the channel $\longrightarrow$ d - fluid density
- g - gravitational acceleration
- h - height of fluid in the channel
- Q - fluid flowrate
- q - fluid flowrate per unit height
- u - fluid velocity
- U_s - fluid slip velocity
- α - angle of inclination of the channel

τ - shear stress

τ_w - shear stress at the wall

CHAPTERS FIVE AND SIX

d_b - bubble diameter

d_f fluid density

d_s - solids density

f^* - friction factor

I_p - inter-particle force

Re^* - Reynolds number

U - superficial fluidizing velocity

U_b - superficial velocity of the bubble phase

U_{mf} - superficial fluidizing velocity at incipient fluidization

u_b - actual velocity of the bubble phase

u_{bo} - actual velocity of a single bubble

u_c - actual velocity of the fluid through the particulate phase

ϵ_b - bubble volumetric fraction

ϵ_c - voidage fraction in the particulate phase

ϵ_m - overall mean voidage fraction

ϵ_{mf} - voidage fraction at incipient fluidization

ϵ_s - voidage fraction at saturation

μ^* - shear viscosity coefficient of the bubble swarm and the heterogeneous fluidized bed

μ_f - shear viscosity coefficient of the fluid

μ_p - shear viscosity coefficient of the particulate phase

PART 2

d_f - fluid density

d_p - particle diameter

d_s solids density

$\vec{I}$ - solids drag force vector

M_f - fluid mass flowrate

M_s - solids mass flowrate

p_f - static pressure in the fluid phase

p_p - static pressure in the particulate phase

r_e - orifice radius

s_{rr} - normal stress in the particulate phase in the radial direction

$\vec{u}$ - fluid velocity vector

u_r - fluid velocity component in the radial direction

$\vec{v}$ - solids velocity vector

v_r - solids velocity in the radial direction

ϵ - voidage fraction

ϵ_{m_f} - voidage fraction at incipient fluidization

ϵ_s - voidage fraction at saturation

K_p - bulk viscosity coefficient of the particulate phase

μ_f - shear viscosity of the fluid phase

μ_p - shear viscosity of the particulate phase

τ_p - particulate stress tensor