

The material in this thesis is entirely the result of my own independent research under the supervision of Dr. G.J. Hampson, Prof. K.L. Gallagher and Dr. R. Labourdette, and is not the outcome of any collaborative work. The research was carried out in the Department of Earth Science and Engineering at Imperial College London. All published or unpublished material used in this thesis has been given full acknowledgment.

No part of this work has been previously submitted to this or any other academic institution for a degree or diploma, or any other qualification.

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Inverse and Forward Modelling of Shallow-Marine Stratigraphy

by

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Abstract

This thesis presents the development and application of a numerical inverse and forward model of stratigraphy applied to shallow-marine wave-dominated sedimentary systems. The approach links a “process-based” forward model of stratigraphy (i.e. BARSIM, developed by J.E.A. Storms, University of Delft) to a fully non-linear stochastic inverse scheme. The inverse problem has been formulated using a Bayesian framework in order to sample the full range of uncertainty and explicitly build in prior knowledge. The methodology combines Reversible Jump Markov chain Monte Carlo and Simulated Tempering algorithms which are able to deal with variable dimensional inverse problems and multi-modal posterior probability distributions, respectively. The numerical scheme requires the construction of a likelihood function to quantify the agreement between simulated and observed data (e.g. sediment ages and thicknesses, grain-size distributions).

Prior to real case study applications, the method has been successfully validated on different scenarios built from synthetic data, in which the impact of data distribution, quantity and quality on the uncertainty of the inferred environmental parameters were investigated. The numerical scheme has then been applied to two case studies: the outcrop-constrained Lower Cretaceous “Standardville” parasequence of the Aberdeen Member of the Blackhawk Formation (Boock Cliffs, Utah, U.S.A.) and the Emsian sub-surface data of South Algeria. The inverse modelling scheme successfully reproduced stratigraphic architecture in both cases, within the constraints of the input data quality. The inferences of the relative sea level, sediment supply and wave regime histories contribute to the understanding of mechanisms that produced the observed stratigraphy. Of equal importance, the inverse results allowed complete characterisation of uncertainties in these forcing parameters and in the stratigraphic architecture developed in between data constraints. These results suggest that the inverse model may ultimately provide a process-based geological complement to standard geostatistical tools for the static characterization of hydrocarbon reservoirs.

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Chapter 1

1 Introduction

1.1 Motivations and Aims

Many analyses of the stratigraphic record aim to infer environmental parameters (such as sediment flux and relative sea level variations) and/or predict facies architecture from sparse observed data. In this context, stratigraphers have to deal with the incompletely preserved record of complex, coupled processes responding to various internal and external forcing parameters through geological time. Simplified conceptual models such as sequence stratigraphy (Vail *et al.*, 1977, 1991) and genetic stratigraphy (Galloway, 1989) are widely used to assess the impact of environmental factors on the stratigraphic record and to predict facies distributions. However, these models assume *a priori* that only certain parameters (e.g. accommodation space or eustasy) are the critical factors controlling stratigraphic architecture. This approach has aroused controversy, as a result of the simplifications needed to produce a tractable formulation and of the conflicting results obtained for the different underlying assumptions (Miall, 1986, 1991, 1992; Miall & Miall, 2001). It is clear that other factors have a strong impact on stratigraphy (e.g. initial physiography of the basin: Thorne, 1991; Thorne & Swift, 1991; Posamentier & Allen, 1993; Gawthorpe *et al.*, 1994; Ulicny *et al.*, 2002; Rasmussen & Dybkjaer, 2005; sediment supply: Schlager, 1993; Leeder *et al.*, 1998; Carroll *et al.*, 2006, and transport efficiency: Burgess *et al.*, 2006) and that many sequence-stratigraphic models do not capture the complexity of real-world processes, parameters and interactions, leading to an underestimation of the different possible combinations of parameters that can account for the same observed stratigraphy (Burgess *et al.*, 2006; Muto *et al.*, 2007).

Furthermore, robust prediction of facies architecture is also a key issue in improving recovery from petroleum reservoirs. Prediction of, and uncertainty in, reservoir facies architecture is typically approached by using geostatistical tools (Deutsch, 2002). However, such techniques do not explicitly incorporate geological processes and thus, are limited in capturing the complexity of sediment distribution and in predicting stratigraphy (although, if used judiciously, they can be successfully used to represent aspects of this record; e.g. Jackson *et al.*, 2005; Labourdette, 2007).

One approach to address many of the issues mentioned above is through the use of numerical stratigraphic models. Such forward models of stratigraphy attempt to predict stratigraphic architecture as a function of input environmental parameters, using calibrated empirical or “process-based” equations to simulate time-dependant sedimentary processes (Eberli *et al.*, 1994; Niedoroda *et al.*, 1995; Niedoroda & Kravits, 1996; Rivanaes, 1992, 1997; Bowman & Vail, 1999; Syvitski *et al.*, 1994, 1999; Carey *et al.*, 1999; Watney *et al.*, 1999; Paola, 2000; Burgess, 2001). In general, such techniques require a large set of input parameters which have to be adjusted to match simulated and observed stratigraphy. However, due to the non-linearity of the interactions between the parameters, their manual adjustment using a trial-and-error approach becomes a non-trivial task.

This modelling process is really an inverse problem in the sense that inverse methods (e.g. Mosegaard & Sambridge, 2002; Mosegaard & Tarantola, 1995; Tarantola, 2005) describe how information (e.g. environmental input parameters) of a parameterized physical system (e.g. forward model of stratigraphy) can be derived from observed data (e.g. wells, seismic data). Lessenger & Cross (1996), Cross & Lessenger (1999), Bornholdt *et al.* (1999) and Wijns *et al.* (2004) conducted the first tests of inverse methods applied to numerical forward models of stratigraphy using both real and synthetic datasets. However to date, the vast majority of stratigraphic forward and inverse applications have been restricted to the optimisation problem, that is finding a single “best” model, so that these procedures do not adequately assess uncertainty (the appraisal aspect of the inverse problem) and do not properly address the issues of eventual non-uniqueness of the inverse problem.

In this thesis, I will address these issues by developing a numerical scheme applied to wave/storm-dominated shallow-marine sedimentary systems. The method combines a “process-response” forward model of stratigraphy (BARSIM; Storms, 2002; 2003) and stochastic non-linear inverse techniques. Throughout this thesis, I will highlight the importance of assessing uncertainty in the inference of model parameters and in all model predictions. I will employ a combination of Bayesian statistics and Markov chain Monte Carlo (MCMC) techniques to achieve these aims.

1.2 Thesis Organisation

I will begin the discussion in Chapter 2 with a concise review of the different approaches for forward modelling of stratigraphy. This is followed by a description of the main characteristics of shallow-marine wave/storm-dominated sedimentary environments that I shall consider when modelling them. The last part of the chapter is dedicated to the presentation of the numerical forward model of stratigraphy (BARSIM, Storms, 2002; 2003) that I will use throughout the thesis.

In the first part of Chapter 3, I will define the general stratigraphic inverse problem and provide an overview of the previous work published in the literature. In the second part of this chapter, I will fully describe the stratigraphic forward and inverse modelling approach that I have developed for this thesis. This part is structured so that, after having provided an introduction to MCMC methodologies and Bayesian statistics, sufficient to allow an understanding of the presented work but leaving detailed proof and complex extensions to the specified references, I will describe the Hybrid Sampler (Denilson et al., 2002), in which adaptations of the Reversible Jump MCMC (Green, 1995) and Simulated Tempering (Marinani & Parisi, 1992; Geyer & Thompson, 1995) algorithms are combined to solve the stratigraphic inverse problem.

Validation tests will be presented in Chapter 4. These applications will aim to infer the sediment supply and relative sea level curves from various numbers of synthetic vertical

successions of grain size (e.g. wells) and synthetic thickness curves (e.g. seismic derived isopachs) extracted at different location from a forward simulation.

In Chapters 5 and 6, I will present the first application of the numerical scheme to geological field data collected in June 2006. Chapter 5 will document a high-resolution stratigraphic and palaeogeographic interpretation of the Aberdeen Member of the Blackhawk Formation, Book Cliffs, Utah, U.S.A., based on these field data. Chapter 6 will focus on the stratigraphic inverse and forward modelling process in which various environmental parameters (relative sea-level, sediment supply, wave height, mud fallout rate) as well as facies architecture along a cross-section oriented along depositional dip will be inferred from eight vertical sections logged from part of the Aberdeen Member.

In Chapter 7, I will present the second application of the numerical scheme to the sub-surface Emsian deposits of South Algeria. In the first part of this chapter, I will give an overview of the geological setting provided by the company Total and describe stratigraphic correlation issues relevant to the Emsian deposits. The second part of Chapter 7 will present the numerical application in which relative sea level, sediment supply, wave-height regime and fallout mud rate parameters were inferred from a dataset of nine wells constraining three parallel, dip-oriented cross-sections of about 75 km in length.

In chapter 8, I will offer a final conclusion and give some recommendations for future work.

1.3 Published Work

Much of the work contained in this thesis has been submitted for publication in various peer reviewed journals. Co-authors on these papers have provided supervisory input as well as editorial and technical discussion. Description of the inverse and forward algorithm (Chapter 3) is found in Charvin et al., 2008a. Additional background information relating to the application of MCMC methods for Earth Science problems can be found in Gallagher *et al.*, 2008. Validation tests of the numerical scheme (Chapter 4) are presented in Charvin *et al.* 2008b.

Stratigraphic description of the Aberdeen Member (Chapter 5) has been submitted as Charvin *et al.*, 2008c. Additionally, the work on the application of the stratigraphic inversion of the “Standardville” parasequence of the Aberdeen Member (Chaptr 6) is found in Charvin *et al.*, 2008d.

Charvin, K., Gallagher, K.L., Hampson, G.J., and Labourdette R., 2008a. A Bayesian approach to inverse modelling of stratigraphy, part 1: Method. Submitted for *Basin Research*.

Charvin K., Hampson, G.J., Gallagher, K.L., and Labourdette, R., 2008b. A Bayesian approach to inverse modelling of stratigraphy, part 2: Validation and sensitivity tests. Submitted for *Basin Research*.

Charvin K., Hampson, G.J., Gallagher, K.L. and Labourdette, R., 2008c. High resolution stratigraphic architecture within an asymmetrical wave-dominated deltaic parasequence. Submitted for *Sedimentology*.

Charvin K., Hampson, G.J., Gallagher, K.L. and Labourdette, R., 2008d. Mechanism driving intra-parasequence shallow-marine wave-dominated stratigraphy: Insights from inverse and forward modelling of stratigraphy. In prep.

Gallagher, K.L., Charvin, K., Nielsen, S., Sambridge, M. and Stephenson, J., 2008. Markov chain Monte Carlo (MCMC) sampling methods to determine optimal models, model resolution and model choice for Earth Science problems. Submitted for *Marine and Petroleum Geology*.

Chapter 2

2 Forward Modelling of Wave/Storm-Dominated Environments

Abstract

In this chapter, I will firstly give an overview of the different types of numerical forward models of stratigraphy that have been published in the literature, with an emphasis on their implementation concepts and computational costs. The second part of the chapter will focus on modelling of shallow-marine wave/storm-dominated sedimentary processes over geological timescale. For that purpose, I will firstly describe the specific features of these sedimentary environments that I need to consider when simulating them. I will secondly describe BARSIM, the “process-response” forward model of stratigraphy which will be used throughout this thesis. I will justify its application to the wave/storm-dominated sedimentary systems previously mentioned.

2.1 Introduction: A Review of Numerical Stratigraphic Forward Modelling Approaches.

Forward numerical models of stratigraphy attempt to predict stratigraphic architecture as a function of environmental parameters (e.g. relative sea level, sediment supply, tectonic subsidence, climate), using calibrated empirical or process-based equations to simulate time-dependant sedimentary processes. Numerical forward models are currently used for a variety of goals from hypothesis testing of environmental parameters at basin scale, to characterizing reservoir-scale stratigraphic architecture by predicting facies distributions and spatial organisation (Eberli *et al.*, 1994; Niedoroda *et al.*, 1995; Niedoroda & Kravits, 1996; Rivanaes, 1992; 1997; Bowman & Vail, 1999; Syvitski *et al.*, 1994, 1999; Carey *et al.*, 1999; Watney *et al.*, 1999; Paola, 2000; Burgess, 2001). Although numerous stratigraphic forward models have been designed to simulate carbonates environments (Burgess, 2001; Warrlich *et al.*, 2002; De Benedictis *et al.*, 2007), I will only consider stratigraphic forward models simulating clastic environments for this thesis.

Such numerical forward models are, generally, divided into two categories, geometrical and dynamic, a distinction based on the detail and complexity of the method applied to simulate erosion and deposition of sediment (Cross, 1990; Watney *et al.*, 1990; Rivaneas, 1992; Slingerland & Harbaugh, 1994; Niedoroda *et al.*, 1995; Niedoroda & Kravits, 1996; Bowman & Vail, 1999; Granjeon & Joseph, 1999; Paola, 2000; Clevis *et al.*, 2004). Geometrical models approximate the result of erosion and sedimentation by moving pre-defined geometrical profiles of different depositional environments in response to changing environmental parameters (e.g Cross *et al.*, 1999; Bowman & Vail, 1999; Houston *et al.*, 2000). These models are generally two-dimensional (focusing on profiles aligned along depositional dip) and are computationally simple and fast, but incorporate few, if any, specific processes. Dynamic models are more focused on the simulation of erosion and sediment transport processes themselves. Three different approaches can be distinguished; hydrodynamic, dynamic slope and “process-response” models. Hydrodynamic models simulate sediment erosion, transport and deposition by running water, via solving simplified forms of the Navier-Stokes equations (Tetzlaff & Harbaugh, 1989; Griffiths *et al.*, 2001). Dynamic slope models (i.e. diffusive

models) simulate long-term large-scale sediment transport as a diffusion process (Moretti & Turcotte, 1985; Rivaneas, 1992; Steckler *et al.*, 1993; Niedoroda *et al.*, 1995; Niedoroda & Kravits, 1996; Fleming & Grotzinger, 1996; Den Bezemer *et al.*, 1999; Granjeon & Joseph, 1999; Paola, 2000). Hydrodynamic and dynamic slope numerical models are computationally complex and generally have high computational costs, which limit their use for multi-iteration inversion processes. “Process-response” models approximate sediment deposition and transport using a simplified set of equations that is generally specific to a single depositional environment (or suite of related depositional environments). They have the combined advantage of incorporating simplified sedimentary processes with relatively low computational overhead (Storms *et al.*, 2002), but should be used carefully regarding the sedimentary environment which needs to be modelled. These properties allow therefore their use in multi-iterative inversion processes.

2.2 Forward Modelling of Wave/Storm-Dominated Environments

2.2.1 Wave/Storm-Dominated Sedimentary Systems

This thesis is focused on wave/storm-dominated systems that are supplied by sediment transported by longshore drift. These systems typically consist of an assemblage of depositional environments comprising coastal plain, strandplain, barrier island and back-barrier lagoon, foreshore (beach), shoreface and shelf (Fig. 2.1; e.g. Walker & Plint, 1992). These environments are not necessarily coeval; for example, strandplains form during shoreline advance, while barrier islands and back-barrier lagoons typically form during shoreline retreat (e.g. Dominguez *et al.*, 1987; Boyd *et al.*, 1992).

In sandy wave-dominated systems, past and modern examples show that the foreshore is characterised by sandstones containing upper-flow-regime planar-parallel laminations caused by the swash and backwash of breaking waves. The foreshore dips generally seaward at 2-3° (Fig. 2.2; Elliot, 1986; Cant, 1991; Walker & Plint, 1992). Associated shoreface deposits typically comprise sandstones characterised by deposits of symmetrical and asymmetrical

ripples and asymmetrical dunes. These structures result from a constant along-shore and cross-shore sediment supply driven by currents caused by shoaling waves under fair-weather conditions. The shoreface dips seaward at approximately $0.1 - 0.3^\circ$ (Fig. 2.2; Davis & Hayes, 1984; Walker & Plint, 1992; Hampson, 2000). The lower shoreface and offshore shelf are characterised by the absence of fair-weather wave action.

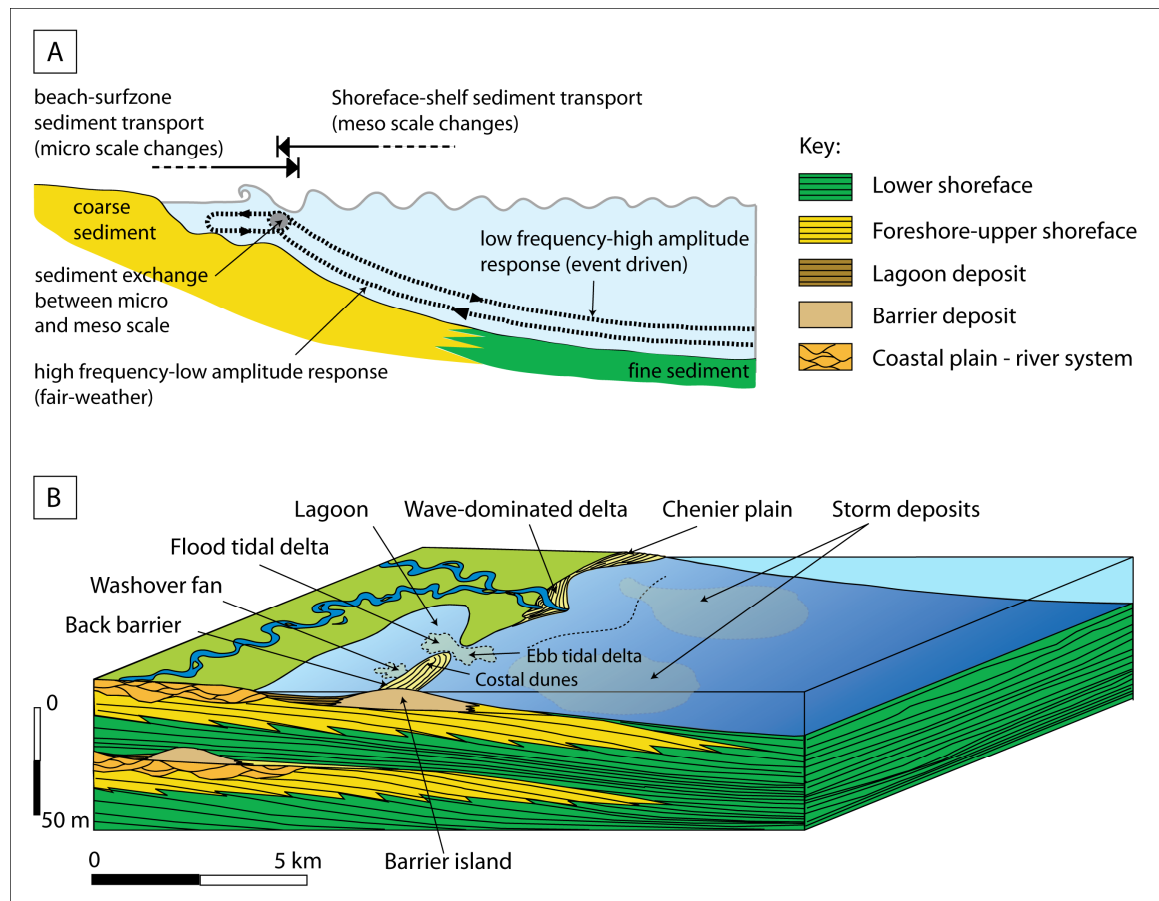


Figure 2.1: Sedimentological architecture of wave-dominated systems. Part (A) highlights the interactions between high frequency – low amplitude processes driving the sedimentary system. Part (B) illustrates the three dimensional architecture of wave-dominated sedimentary environments (figure compiled from Heward, 1981; Kendall, 2003; Storms 2003 and personal data).

Sedimentation is mainly controlled by episodic storm-wave processes that deposit sandstone event beds characterised by hummocky cross-stratification (Dott & Bourgeois, 1982), which alternate with fair-weather shale deposits. The offshore shelf dips at $0.01\text{--}0.003^\circ$ (Fig. 2.2; Cant, 1991; Walker & Plint, 1992). Depositional patterns in wave/storm-dominated systems are the result of complex interactions between high-magnitude, low-frequency storm processes and contrasting low-magnitude, high-frequency fair-weather wave processes (Fig. 2.2A).

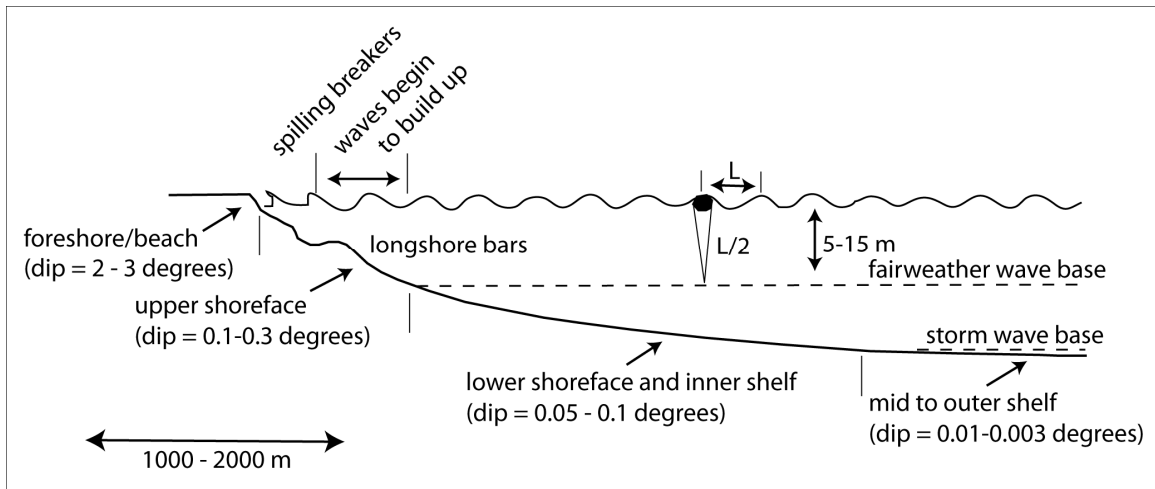


Figure 2.2: Schematic wave-dominated shoreface-shelf profile. The profile illustrates the main elements and processes (from Walker & Plint, 1992 modified by Hampson & Storms, 2003).

During storms, strong winds force the build-up of large waves which create strong water currents that severely scour the shoreface and foreshore areas (Walker & Plint, 1992; Storms, 2002). The remobilised sediments are transported offshore beyond the storm wave-base or into back-shore environments, such as barrier islands and washovers in the back-barrier lagoon. During fair-weather conditions, the shoreface system recovers and steepens as sand is fed onshore by asymmetrical, shoaling waves and associated longshore drift currents (Storms, 2002). In deeper, offshore shelf areas, less energetic fair-weather conditions allow the deposition of silts and mud above older storm beds, because the wave-base becomes shallower at these times. Thus storm-event beds are preserved below the fair-weather wave base (Walker & Plint, 1992).

In the geological record, wave/storm-dominated systems are preserved as facies succession resulting from the combination of very long-term global/regional processes (sea level, sediment supply, subsidence variation) and short-term local processes (wave/storm frequencies and intensities (Heward, 1981; Walker & Plint, 1992; Hampson & Storms, 2003). Short-term local processes determine the sedimentation style, whereas long-term global and regional processes determine accommodation for sedimentation and, thus, transgressive or regressive facies successions. This duality needs to be considered when forward modelling these sedimentary environments.

Because of the process involved, wave-dominated systems are characterized by limited along-strike variability (e.g. Hampson & Howell, 2005).

2.2.2 BARSIM Numerical Forward Model of Wave/Storm-Dominated Shallow-Marine Environments

2.2.2.1 BARSIM: Sediment Erosion/Deposition Implementation

BARSIM (Storms *et al.*, 2002; Storms, 2003; Storms & Swift, 2003) is an event-based “process-response” forward model of stratigraphy that simulates wave/storm-dominated shallow-marine environments over geological timescales (e.g. several hundred thousand years). The model is two-dimensional and simulates erosion, transport and deposition of multiple sediment grain sizes along a two-dimensional cross-shore profile (Fig. 2.3). BARSIM is based on the mass conservation principle:

$$\frac{\partial H}{\partial t} = -\frac{\partial F}{\partial x} + T \quad (2.1)$$

where t is time [T], x is horizontal distance [L], H is topographic elevation relative to a constant reference level [L], F is sediment flux [$L^2 T^{-1}$] and T is rate of subsidence due to vertical movements of the sea floor [$L T^{-1}$]. The spatial derivative of the sediment flux is defined as the difference between the rates of erosion and deposition:

$$\frac{\partial F}{\partial x} = E(x, t) - S(x, t, D) \quad (2.2)$$

where $E(x, t)$ is the rate of erosion [$L T^{-1}$] and $S(x, t, D)$ is the rate of deposition [$L T^{-1}$], which is a function of grain size (D). The shallow-marine environment modelled by BARSIM consists of two spatial domains: the back-barrier and the shoreface-shelf systems (Fig.2.3), which are separated by an internal moving boundary defined by the coastline. Equation 2.2 is solved independently in each domain because they present different sedimentological characteristics. Therefore, instead of calculating fluxes at once in each consecutive cell in the downstream direction over the whole area, erosion and deposition are estimated separately by following three successive phases (Fig. 2.3)

- Phase 1: Calculation of the erosion flux in the shoreface erosion window.
- Phase 2. Grain-size dependent backbarrier deposition.
- Phase 3. Grain-size dependent shoreface-shelf deposition and mud fallout deposition by suspension.

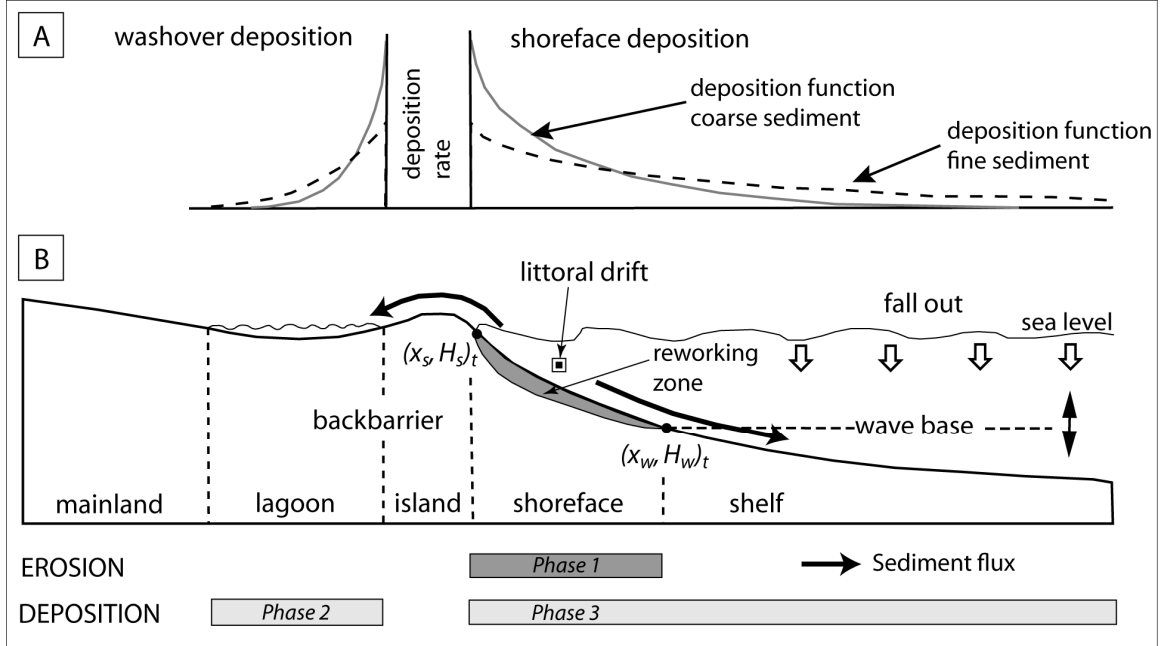


Figure 2.3: BARSIM implementation summary. (A) Deposition functions are used to simulate deposition of each grain-size class. (B) Cross-shore profile showing the geographical domains where equations of sediment erosion and depositions are implemented in BARSIM. Firstly,

BARSIM calculates the erosion in the shoreface area. Secondly, BARSIM calculates sedimentation in the back-barrier area and, thirdly, in the shoreface and shelf area (after Storms, 2003).

Phase 1: Calculation of the erosion flux in the shoreface erosion window.

Erosion rate is defined as:

$$E(x, t) = c_e \cdot c_w(t) \cdot G(x, t) \quad (2.3)$$

where $G(x, t)$ is the local erosion efficiency [-] defined such as it takes values ranging from one in the vicinity of the coastline to zero at the storm wave base:

$$G(x, t) = \left(\frac{H(x, t) - H_w(t)}{H_s(t) - H_w(t)} \right)^m \quad \text{for } x_s(t) < x < x_w(t) \quad (2.4)$$

$$G(x, t) = 0 \quad \text{for } x \leq x_s(t) \text{ and for } x \geq x_w(t)$$

Constant m [-] represents the dependence of the erosion rate to water depth (Table 2.2). Location $x_s(t)$ corresponds to the coastline, which is defined by the intersection of sea level and the topographic profile having an elevation defined as $H_s(t)$ (Fig. 2.3). Erosion is limited to a shoreface erosion window defined by the spatial domain between locations $x_s(t)$ and $x_w(t)$. Outside this domain, erosion $E(x, t)$ is set to be equal to zero. Location $x_w(t)$ is defined by the intersection of the wave base $H_w(t)$ and the topographic profile. The wave base $H_w(t)$ is defined as $H_w(t) = H_s(t) - z_w(t)$, where $z_w(t)$ is the depth of closure [L], which is the maximum water depth for waves to affect the sea bottom. The depth of closure is a function of wave height: $z_w(t) = c_c \cdot w(t)$, in which c_c is a constant (Table 2.1).

The erosion efficiency rate c_e [$L T^{-1}$] is independent of the properties of the unconsolidated substrate (Table 2.2). Parameter $c_w(t)$ [-] corrects for temporal variation in wave energy:

$$c_w(t) = \frac{w(t)}{w} \quad (2.5)$$

where w [L] and $w(t)$ [L] are respectively the fair-weather (Table 2.2) and the current wave heights (i.e. wave height simulated at time t). The current wave height $w(t)$ can either represent the fair-weather wave height w or the event wave height, $w_e(R)$, as discussed in the subsequent paragraph.

The total sediment flux available for deposition, specified for each grain-size class (D), is obtained by adding the volume of sediment eroded during storm events to the sediment supplied via longshore transport (i.e. into and out the modelled 2D cross-section) defined as net sediment input by the model user (Table 2.2):

$$F_t(t, D) = SS(t, D) + \int_{x_s}^{x_w} E(x, t) dx \quad (2.7)$$

where $SS(t)$ is the influx of sediment [$L^2 T^{-1}$] supplied by longshore currents (Table 2.2) which can be specified for different grain sizes (D) and $E(x, t)$ is independent of grain size.

Phase 2 and 3: Grain-size dependent backbarrier and shoreface-shelf depositions.

To simulate washover deposition, back-barrier sediment flux ($F_{in, BB}$) has been defined as a net proportion c_v [-] (Table 2.1) of the total sediment flux $F_t(t, D)$. Thus, sediment fluxes available over the backbarrier and shoreface-shelf domains have been defined as follows:

$$F_{in, BB} = c_v \cdot F_t(t, D) \quad \text{for } 0 \leq c_v \leq 1$$

(2.7)

$$F_{in,SF} = (1 - c_v) \cdot F(t, D)$$

When the backbarrier is completely filled, $F_{out,BB}$ is added to $F_{in,SF}$ prior to the calculation of Phase 3 in order to comply with the mass conservation principle. The deposition function used in the model is given by:

$$S(x, t, D) = \frac{F(x, t, D)}{h(D)} \quad \text{for } x \leq x_s \quad (Phase\ 2) \quad (2.8)$$

$$S(x, t, D) = \frac{F(x, t, D)}{h(D)} + FMR(t, D) \quad \text{for } x > x_s \quad (Phase\ 3) \quad (2.9)$$

where F is the flux of sediment that is in transit and available for deposition [$L^2\ T^{-1}$], h represents the sediment travel distance [L], which depends on the grain size of the sediment and on the environment of deposition. Parameter FMR represents a steady fall-out rate [$L\ T^{-1}$] of fine sediment in calm water at the shoreface-shelf (Table 2.2). The net effects of size-selective transport (i.e. sediment by passing) are simulated by the size dependence of the travel distance h in the deposition function. Cross-shore sediment dispersal is described by the following relationship between the nominal grain diameter D (mm) and h (m) based on data from Terschelling, the Netherlands (Guillén & Hoekstra 1996, 1997):

$$h^*(D) = c_g(t) \cdot c_w(t) \cdot \left[110 + 590 \left(\frac{D_{ref}}{D} \right)^{2.5} \right] \quad \text{for } D > D_{ref} \quad (2.10)$$

$$h^*(D) = c_g(t) \cdot c_w(t) \cdot \left[500 + 200 \left(\frac{D_{ref}}{D} \right)^{0.6} \right] \quad \text{for } D \leq D_{ref}$$

where $D_{ref} = 0.125$ mm. Constant $c_g(t)$ accounts for local variability in sediment transport capacity during storm and fair-weather conditions (Table 2.2). The constant $c_w(t)$ [-] accounts for the temporal variability of wave height.

Barrier geometry (situated between backbarrier and shoreface-shelf depositional area) is modelled via a geometric approach. Geometric parameters need to be specified by the user prior to the simulation (Table 2.2). BARSIM implementation is such that back-barrier and barrier sedimentation are only simulated during transgressive conditions.

2.2.2.2 BARSIM: Event-Driven Implementation

In contrast to algorithms that simulate time-averaged processes, BARSIM distinguishes between storm-event and fair-weather conditions using a stochastic component. At each iteration, BARSIM alternates between storms and fair-weather conditions. Wave-heights during storms and durations of fair-weather periods are stochastically determined from exponential probability distributions (Fig. 2.4) as observed in modern environment at Schiermonnikoog in the Netherlands. Values of the wave height during storm conditions $We(R)$ are determined from a random uniform number $R \langle 0,1 \rangle$ and equation 2.1 (Storms, 2003):

$$w_e(R) = \frac{\ln R}{-a_1} + w \quad \text{for } a_1 > 0 \quad (2.11)$$

where W represents the maximum fair-weather wave height [L] which is assumed to erode smaller previously formed fair-weather wave scours. Constant a_1 [-] describes wave height climate of different coastal settings (Table 2.1). In the standard implementation of BARSIM, the minimum fair-weather wave height W is set to be constant during the simulation period (Fig. 2.4A). However, to better control variations of both fair-weather and storm wave heights during the simulation period and therefore to model effects of long-term changes in climate and/or of minor three-dimensional coastline orientation readjustments, I modified the BARSIM

algorithm and set the minimum fair-weather wave height value W to be time-dependent (Fig. 2.5B).

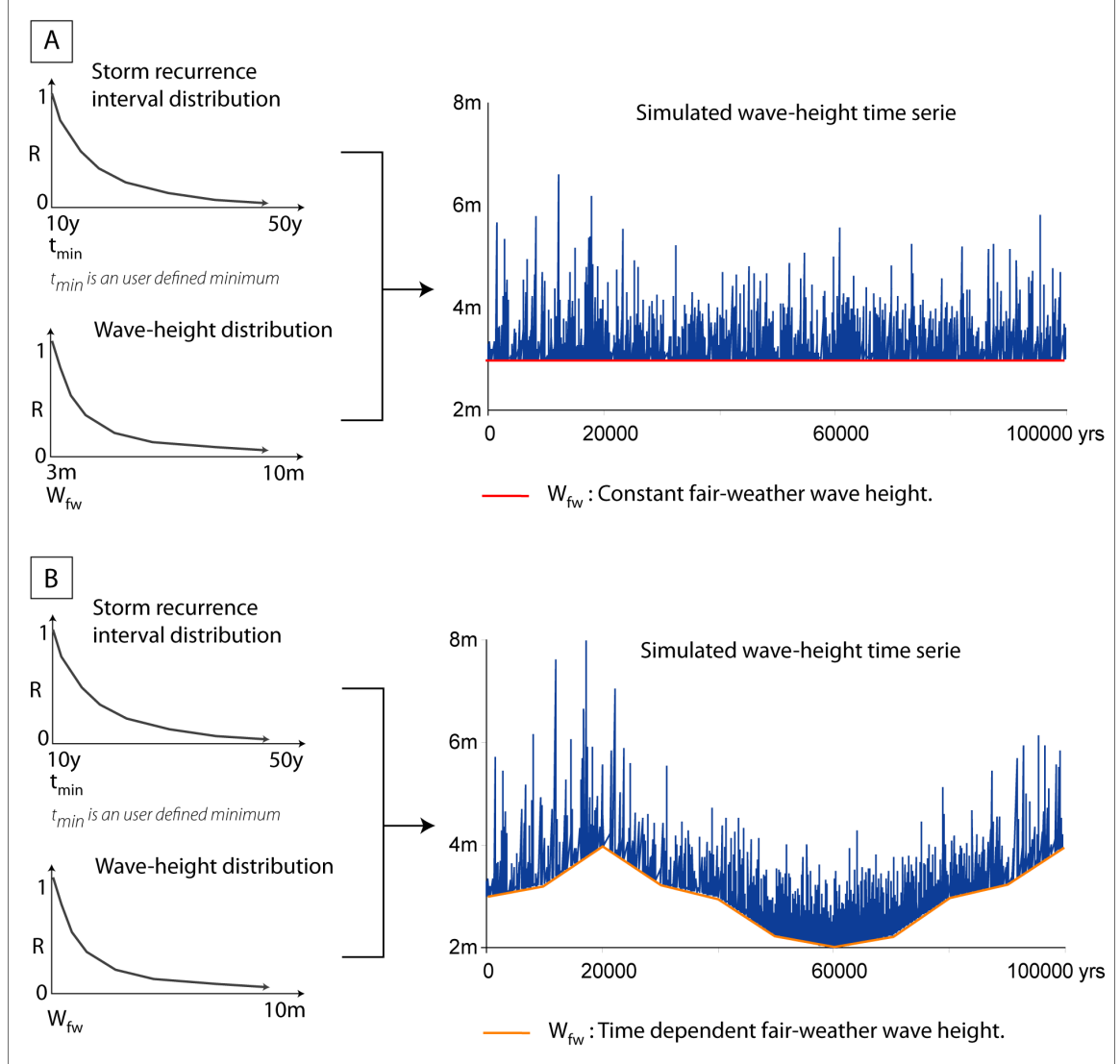


Figure 2.4: BARSIM simulation of wave-height time series based on exponential functions. Independent uniform random numbers between 0 and 1 are used to draw both distributions to create a time series (right), which is used as input for BARSIM (modified from Storms, 2003). (A) Standard implementation of BARSIM in which wave height during storm conditions is drawn from a constant distribution of fair-weather wave height. (B) Modified implementation of BARSIM in which wave height during storm conditions is drawn from a time-dependent distribution of fair-weather wave height.

The duration of a BARSIM time step during fair-weather, Δt_{fw} [T], or storm conditions Δt_e [T], is described as:

$$\Delta t_{fw} = \frac{\ln R}{-a_2} + t_{\min} \quad \text{for } a_2 > 0 \quad (2.12)$$

$$\Delta t_e = 1$$

in which a_2 [-] is a constant that accounts for the duration of fair-weather periods, which is a proxy for wave-height recurrence interval (Table 2.1). To avoid excessive calculation times, $t_{\min}$ can be chosen to be in the order of decades to hundred of years. A long fair-weather period, however, will embrace more small and seasonal storms than a short fair-weather period. Therefore, fair-weather wave height as used in the model is positively correlated with minimum duration of fair-weather periods.

Parameter	Value	Description
c_c	7	Constant calibrating the depth of closure of waves
c_v	0.25	Proportion of sediment flux for available back barrier deposition
a_1	1-5	Constant accounting for wave height climate for different coastal settings
a_2	1-5	Constant accounting for duration of fair-weather periods

Table 2.1: Constant used in BARSIM algorithm.

2.2.2.3 Restrictions on BARSIM Application

Applications of BARSIM are restricted to wave/storm-dominated shallow-marine systems as the model does not include fluvial, tidal and aeolian processes. Furthermore, due to its simplified two-dimensional approach, BARSIM assumes that local 3D variations over short-times scales, such as the behaviour of rip-current channels in the shoreface or the segmentation of the barrier-island system by tidal inlets, have little effect on the preserved stratigraphy.

Thus, like for all numerical models, results should not be over-interpreted beyond these basic assumptions. Moreover, it is important to note that the inversion results will be, at best, only as good as the forward-modelling algorithms in terms of simulating reality.

2.2.2.4 Environmental input parameters in BARSIM

In order to perform one BARSIM forward simulation, two sets of input parameters need to be specified: the internal and external parameters. The internal model parameters (“sediment travel distance correlation event”: $c_g(t)_{event}$, “sediment travel distance correlation shoreface”: $c_g(t)_{fair-weather}$, “power for erosion scaling: shoreline – wave base”: m , “erosion efficiency rate”: c_e , “maximum height and width of the barrier”; Table 2.2) are required by the numerical model formulation and characterise aspects of sediment transport distances, maximum erosion rate and barrier morphology. The erosion efficiency rate c_e is independent of the substrate properties. The parameter labelled “power for erosion scaling: shoreline – wave base” m is a constant that represents the dependence of erosion rate on water depth. The “sediment travel distances correlation event” $c_g(t)_{event}$ and “sediment travel distances correlation shoreface” $c_g(t)_{fair-weather}$ parameters account for local variability in sediment transport during storm and fair-weather conditions, respectively. These two parameters are constant values that are used to match specific local coastal settings for the estimation of sediment travel distance. Higher values of these parameters produce a higher probability for transporting sediment over longer distances.

The external model parameters comprise the initial-condition parameters and the environmental parameters of interests (Table 2.2). The initial-condition parameters are represented by the different grain-size distributions of sediment added by littoral drift, the grain-size distribution of the substrate, and the substrate physiography. The environmental external variables are represented by constant parameters (mud fallout rate, differential subsidence rate) and time-dependent parameters (e.g. relative sea level, sediment supply added by longshore drift current and fair-weather wave height curves). The mud fallout parameter represents the mud deposited by suspension independently from the mud added by longshore

drift currents. Mud fallout is simulated using a uniform rate imposed across the whole shoreface-shelf profile during fair-weather conditions

In this work, both the internal and the external initial-condition parameters will be adjusted manually when initialising the stratigraphic inverse and forward numerical scheme. These parameters will remain constant throughout the inversion process. Initial-condition parameters can easily be estimated from prior analysis of the observed geological data (Chapters 6 and 7). The estimation of the internal parameter is more difficult and depends on observed geological data quantity and quality. Maximum erosion rate, sediment transport and barrier morphology parameters have either been adjusted manually based on preliminary forward simulation tests in cases where the observed data were of sufficient quality (Chapters 6 and 7) or have been set to be equal to default values available in BARSIM (Chapters 6 and 7).

Internal Parameters	External Parameters	
	<i>Initial condition parameters</i>	<i>Environmental parameters</i>
Erosion efficiency rate [m^2/y]	Grain size (shale class) [μm]	Relative sea level curve [m]
Power for erosion scaling: shoreline – wave Base [-]	Grain size (Sand class) [μm]	Sediment supply Curve [m^2/y]
Sediment travel distance correlation event [-]	Supply composition (% of sand) [-]	Fair-weather wave height curve [m]
Sediment travel distance correlation shoreface [-]	Substrate depth [m]	Mud fallout rate shoreface [m^2/y]
Maximum height and width of the barrier [m]	Grain-size distribution of the substrate [μm]	Differential subsidence rate [m/y]

Table 2.2: BARSIM input parameters.

Throughout the thesis, the stratigraphic inverse process will aim to invert the environmental constant and time-dependant parameters. Because prior stratigraphic analyses of the observed data used in case-study applications (Chapter 6 and Chapter 7) did not show any evidence of differential subsidence (i.e. rotation of the substrate when sedimentation occurred), the differential subsidence module has not been used. In BARSIM, the time dependent parameters

consist of three independent matrixes: $SL = \{SL^1, SL^2, \dots, SL^k\}$ of dimension $k \times 2$ for the sea level, $SS = \{SS^1, SS^2, \dots, SS^p\}$ of dimension $p \times 2$ for the sediment supply and $W = \{W^1, W^2, \dots, W^k\}$ of dimension $l \times 2$ for the fair-weather wave height. Each element of SL^i , SS^i and W^i are then given as $SL^i = \{t^i, sl^i\}$, $SS^i = \{t^i, ss^i\}$ and $W^i = \{t^i, w^i\}$ representing sea level (m), sediment supply (m^2/yr) and fair-weather wave height (m) at their respective reference time t^i . These values are interpolated linearly between two successive reference times in order to create a continuous function.

2.2.2.5 BARSIM output format

The output of BARSIM is a cross-section showing depositional patterns along a profile across the shoreline (i.e. oriented along depositional dip). Each node of the BARSIM output grid gives information about its position relative to a fixed reference frame chosen by the model user, the grain-size composition (percentage of each initial grain-size class) and the age of deposition. BARSIM output also provides 1D vertical successions of the grid at specific locations (i.e. equivalent to measured outcrop sections or subsurface wells) and sedimentary interval thickness curves (i.e. equivalent to an isopach derived from outcrop, well and/or seismic data). These are the forms of data that I shall subsequently use for inversion applications.

2.2.2.6 BARSIM adaptation to the inverse algorithm

BARSIM code was available in Visual Basic programming language, with an associated “user friendly” graphic interface. I therefore dedicated the first few months of my PhD to understanding its implementation and to rewriting it in a fully object-oriented C++ version. In this new implementation, graphic interface and non-used modules have been deleted to speed up the code. This simple modification allows me to win up to around 20% of the computational cost. The BARSIM algorithm has been transformed in a C++ class from which the inverse

algorithm object, which I developed subsequently, inherits its public variables. I also modified BARSIM to provide output described in the previous section.

2.3 Summary

BARSIM is an event-based “process-response” forward model of stratigraphy that is able to simulate the dual action of long-term global/regional (sea level, sediment supply, subsidence variation) and short-term local processes (wave/storm frequencies and intensities) over geological timescale that are characteristic of the wave/storm-dominated shallow-marine environments. Its simple formulation allows low computational costs and therefore its use in multi-iterative inverse processes, but its specificity limits strictly its use to wave-dominated stratigraphy. The two-dimensional approach used in BARSIM seems to be justified *a priori* by the limited along-strike variability characteristic of wave-dominated environments. However numerical model results should not be over-interpreted beyond the basic assumptions inherent to the model algorithms.

Chapter 3

3 A Bayesian Approach to Inverse Modelling of Shallow-Marine Wave-Dominated Stratigraphy

Abstract

In this chapter, I will present a new method for the inference of key environmental parameters that control stratigraphy. The approach combines a fully non-linear inversion scheme with a “process-response” forward model of stratigraphy. I formulate the inverse problem using a Bayesian framework in order to sample the full range of possible solutions and explicitly build in prior geological knowledge. The methodology combines Reversible Jump Markov chain Monte Carlo and Simulated Tempering algorithms which are able to deal with variable-dimensional inverse problems and multi-modal posterior probability distributions, respectively. The inverse scheme has been linked to a forward stratigraphic model, BARSIM (developed by Joep Storms, University of Delft), which simulates shallow-marine wave/storm-dominated depositional systems over geological timescales. This link requires the construction of a likelihood function to quantify the agreement between simulated and observed data of different types (e.g. sediment age and thickness, grain size distributions). The work presented in this chapter is partly published in Charvin *et al.* (2008a).

3.1 Introduction

The previous chapter illustrates how numerical forward models of stratigraphy attempt to predict stratigraphic architecture as a function of environmental parameters, using calibrated empirical or process-based equations to simulate time-dependant sedimentary processes. However, despite these attempts to quantify the full range of processes that control stratigraphy, such numerical schemes also suffer from their complexity. In fact, the use of stratigraphic forward models is not as widespread as it should be, at least in part because of the difficulties of calibrating the large number of potentially important input parameters against observed geological data and also of quantifying uncertainty. As a consequence, such numerical models are currently used more as an experimental device (Burgess, 2002), rather than a routine predictive tool. To achieve the latter goal, a robust scheme which combines forward models of stratigraphy and mathematical inversion methods is needed, thus allowing efficient exploration of the parameter space driven by the observed geological data.

In this chapter, I address these issues by developing a numerical scheme combining “process-response” forward models of stratigraphy (BARSIM; Storms *et al.* 2002; Storms, 2003) and non-linear stochastic inversion techniques, which will be applied to shallow-marine wave/storm-dominated sedimentary systems. In doing this, I have formulated a Bayesian approach to quantify the environmental input parameters and their uncertainties, which conditions the parameter inference on observed geological data.

The chapter is structured as follows. Firstly, I will present an overview of the previous work undertaken in inverse and forward modelling of stratigraphy, with an emphasis on the need to create a robust inversion scheme that is able to fully assess the uncertainty inherent to an inverse solution. I will secondly give the background of my inversion approach and thirdly discuss its implementation. Fourthly, a summary will be provided. The following chapter will present validation tests that use synthetic data, with emphasis on computational performance.

3.2 Previous Work

Inverse methods (e.g. Mosegaard & Sambridge, 2002; Mosegaard & Tarantola, 1995; Tarantola, 2005) describe how information (e.g. environmental input parameters) describing a parameterised physical system (e.g. forward model of stratigraphy) can be derived from observed data (e.g. well and seismic data). Moreover, it is philosophically desirable to explicitly incorporate prior information (e.g. knowledge of the geologist, physically plausible parameter values) when constructing inverse models. The applications of such methods to stratigraphic modelling have been limited for two reasons: modelling such geological processes is computationally intensive and time consuming, and quantifying geological knowledge and observations is often difficult. The first reason is becoming less of an issue as computing power increases, and the second, although non-trivial, is not insurmountable, depending on the particular problem under consideration.

Inverse approaches are increasingly used with stratigraphic numerical methods to quantify input environmental parameters. Lessenger & Cross (1996) were the first to apply such sophisticated techniques to stratigraphic modelling (see also Cross & Lessenger, 1999; Lessenger & Lerche, 1999). They presented a successful example of a basin-scale application on the Mesa Verde Group, San Juan Basin (Colorado, USA) by combining a two-dimensional geometrical forward model with “Lerche’s inversion algorithm”, a combination of linear and non-linear search schemes based on gradient descent (Lerche, 1996; Thomsen & Lerche, 1997). However, although their choice of a locally linear inverse technique decreases computational costs, such methods do not overcome the problem of uniqueness of the solution (i.e. the same observed stratigraphy may be explained by different parameters combinations). Moreover, the authors admitted they were not entirely comfortable about their technique for calculating uncertainties (Cross & Lessenger, 1999), also as a consequence of their linearising assumptions.

Bornholdt *et al.* (1999), and more recently, Wijns *et al.* (2004) analysed the application of a fully non-linear global inversion algorithm (Genetic Algorithm, GA) to 3D forward models, respectively FUZZIM (Nordlund, 1999) and SEDSIM (hydrodynamic models) (Tetzlaff, &

Harbaugh, 1989; Griffiths *et al.*, 2001). By formulating their respective inverse schemes as optimization problems and testing them on synthetic data, their work highlighted two points. Firstly, the building of a robust exploration scheme is necessary to sample the full range of possible solutions of the inverse problem. Even by using a GA, Wijns *et al.* (2004) obtained results for some parameters that were far from the known true solution. This emphasizes the importance of assessing uncertainty with a robust method that can efficiently explore the parameter space. Secondly, the use of a computationally fast forward model is desirable to decrease computational cost, thus allowing a complete exploration of the parameter space in a reasonable amount of time. To overcome the problem of computational issues, Lopez *et al.* (2006) used a simplified version of a 3D diffusive forward model (DIONISOS, Granjeon *et al.*, 1999), from which they directly mathematically derived the input parameters (inversion *sensu stricto*). Uncertainty assessment was realised by running several forward realisations calibrated by varying the obtained input parameters values. Although this method is successful in decreasing computational overheads, the technique still does not overcome the problem of uniqueness of the solution and lacks robustness in assessing uncertainty.

Based on this literature review, the philosophy used to design the inverse scheme presented below involves the use of a computationally fast forward model allied to a robust non-linear inversion scheme which has to be able to explore efficiently the parameter space and assess the issues of uniqueness of the solution and of uncertainty estimation.

3.3 A Bayesian Approach to Inverse Stratigraphic Modelling

A forward algorithm can be represented as follows:

$$\mathbf{D}_{sim} = g(\mathbf{X}) \tag{3.1}$$

where $\mathbf{D}_{sim} = \{d_{sim}^1, d_{sim}^2, \dots, d_{sim}^m\}$ represents the simulated data (e.g. wells), m the number of simulated data, $\mathbf{X}$ the total model parameter matrix (in our case, comprised of $\mathbf{SS}$, $\mathbf{SL}$, $\mathbf{W}$ and

FM), and g is the forward model function (e.g. BARSIM). The optimization problem can be formulated as follows, given a set of observed data $\mathbf{D}_{obs} = \{d_{obs}^1, d_{obs}^2, \dots, d_{obs}^m\}$ find values of $\mathbf{X}$ which produce values $\mathbf{D}_{sim}$ that most closely reproduce the observed data $\mathbf{D}_{obs}$. However, the total problem for inversion is to understand the uncertainty on this optimal, or best, model, or distribution of all possible models. Both aspects require some quantitative measure of how well a given model can reproduce the observed data. This is known as the misfit or likelihood function, the latter of which can be thought of as a measure of the probability of reproducing the observed data, given a model with a particular set of parameter values. This can be written as $p(\mathbf{D}_{obs} | \mathbf{X})$, where $a|b$ means a given, or conditional on, b .

In order to incorporate geological information explicitly, we follow the Bayesian approach (Gilks *et al.*, 1996; Denison *et al.*, 2002; Tarantola, 2005). In this, we define a prior probability distribution on the unknown model parameters, $p(\mathbf{X})$, i.e. what we think is reasonable for our particular geological setting, in the absence of any problem specific observations. Then, Bayes rule expresses the relationship between the data and the model as:

$$p(\mathbf{X} | \mathbf{D}_{obs})p(\mathbf{D}_{obs}) = p(\mathbf{D}_{obs} | \mathbf{X})p(\mathbf{X}) \quad (3.2)$$

The term, $p(\mathbf{D}_{obs})$, is often referred to as the evidence or marginal likelihood (Sivia & Skilling, 2006; Sambridge *et al.*, 2006) and is equivalent to the right-hand side integrated over all possible models. In our context, the evidence is not directly a function of the model parameters $\mathbf{X}$, as they are “intergrated out” (Sambridge *et al.*, 2006). Thus the evidence $p(\mathbf{D}_{obs})$ can be regarded as constant (Sivia & Skilling, 2006), so we can write (3.2) as a proportionality relationship

$$p(\mathbf{X} | \mathbf{D}_{obs}) \propto p(\mathbf{D}_{obs} | \mathbf{X})p(\mathbf{X}) \quad (3.3)$$

where $p(\mathbf{X} | \mathbf{D}_{obs})$ is the posterior probability of the model given the observed data $\mathbf{D}_{obs}$. Thus, the posterior distribution can be considered to represent how our prior knowledge of the model parameters is updated once we have some observed data (Fig. 3.1). Therefore, the Bayesian

approach can be thought of in terms of updating information or information retrieval (e.g. Tarantola & Valette, 1982). Clearly, if the prior and the posterior distributions are the same, then we have learnt nothing from the data. To characterize the model space, we want to be able to determine the posterior distribution. In most real problems, this generally involves analytically intractable integrations, and so we need to use numerical methods.

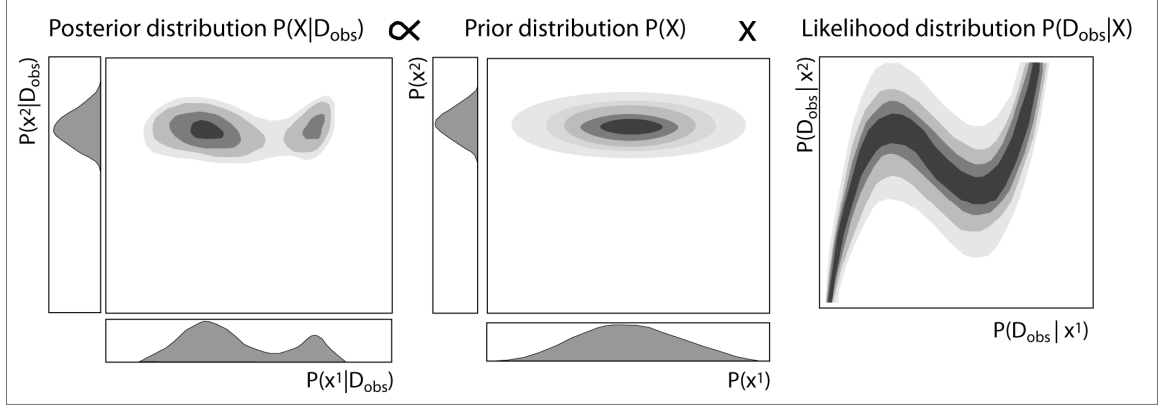


Figure 3.1: Graphic illustration of Bayes law for the distribution of two parameters x^1 and x^2 . The joint posterior probability distribution of the parameters x^1 and x^2 (left) can be considered to represent how our prior knowledge on these parameters (centre) is updated by the likelihood probability distribution (right) once we have some observed data (modified from Tarantola, 2005).

3.4 Determining the Posterior Distribution

The aim of the inversion scheme is to construct a reliable representation of the posterior probability distribution $p(\mathbf{X} | \mathbf{D}_{\text{obs}})$. Given this, it is then straightforward to extract individual models (e.g. the best or expected model), to construct probability distributions for individual model parameters, and to examine correlations between parameters. The best model is the model having the highest posterior probability. Depending on the nature of the prior function, this may or may not be the same as the model that maximizes the data likelihood. One widely used method to achieve these goals is Markov chain Monte Carlo (MCMC), a stochastic

sampling-based approach. Compared to basic Monte Carlo methods that require a lot of samples to be representative of the parameter space, an MCMC algorithm aims to draw samples throughout the range of the posterior probability distribution in appropriate proportions. For that purpose, the MCMC method is intended to draw a sequence of random variables (i.e parameters) $\{\mathbf{X}_0, \mathbf{X}_1, \dots, \mathbf{X}_t\}$ such that the next generation state $\mathbf{X}_{t+1}$ is sampled from a distribution $P(\mathbf{X}_{t+1} | \mathbf{X}_t)$ which depends only on the current state $\mathbf{X}_t$. That is, given $\mathbf{X}_t$, the next step $\mathbf{X}_{t+1}$ does not depend further on the history of the chain $\{\mathbf{X}_0, \mathbf{X}_1, \dots, \mathbf{X}_t\}$. Such a sequence is called a Markov chain and $P(\cdot | \cdot)$ is called the transition Kernel of the chain. In this context, the MCMC algorithm aims to build a Markov chain (i.e. random walk) which will gradually forget its initial state and $P(\cdot | \mathbf{X}_0)$, will explore the parameter space and will eventually converge to a stationary distribution, let's say $\phi(\cdot)$, the posterior probability distribution being its stationary limiting distribution. For the distribution of $\mathbf{X}_t$ to converge to a stationary distribution, the chain needs to satisfy important properties (Gilks *et al.*, 1996):

- It has to be irreducible. From all the starting points, the Markov chain can reach any non-empty step with positive probability in some number of iterations.
- The chain needs to be aperiodic.
- The chain must be positive recurrent. This can be expressed in terms of the existence of a stationary distribution $p(\mathbf{X} | \mathbf{D}_{\text{obs}})$, such that, if the initial value $\mathbf{X}_0$ is sampled for the posterior probability distribution, then all the subsequent iterations will be distributed according to the posterior probability distribution $p(\mathbf{X} | \mathbf{D}_{\text{obs}})$.
- The chain has to be reversible (i.e meet a detailed balance condition), which means that the target posterior probability distribution should be invariant for $P(\cdot | \cdot)$. The probability of jumping from $\mathbf{X}_t$ to $\mathbf{X}_{t+1}$ should be the same as the probability of reversing this jump.

Once the chain reaches stationarity, the distribution of its samples provides a good approximation of the underlying posterior distribution.

Although the theory is complex (useful introductions to this methodology are given in Gilks *et al.*, 1996 and Denison *et al.*, 2002), building such Markov chains is relatively easy by using an MCMC algorithm as defined Metropolis *et al.*, (1953) and later on Hastings, (1970) and Green

(1995). For the stratigraphic inverse problem specification, I designed a Hybrid sampler (Denison *et al.*, 2002) that combines different MCMC-based methods: Fixed Dimension Markov chain Monte Carlo (Metropolis *et al.*, 1953; Hastings, 1970), Reversible Jump Markov chain Monte Carlo (Green, 1995) and Simulated Tempering (Marinani & Parisi, 1992; Geyer & Thompson, 1995) algorithms. The Fixed Dimension Markov chain Monte Carlo sampler will be used in Chapter 6 and 7 to invert the fixed dimension mud fallout rate environmental input parameter. The Reversible Jump Markov chain Monte Carlo sampler will be used in Chapter 4 to invert sediment supply and relative sea level parameters, and in Chapter 6 and 7 to invert sediment supply, relative sea level parameters and fair-weather wave height parameters. The stratigraphic inverse algorithm architecture is illustrated in Figure 3.2.

In this chapter, only the Reversible Jump Markov chain Monte Carlo algorithm will be presented as it represents a generalization of the Fixed Dimension Markov chain Monte Carlo sampler (cf. subsequent paragraph) to variable and non-variable parameter space dimension. For the variable dimension parameters, the target posterior distributions of interest are then, the relative sea level, the sediment supply and the fair-weather wave height curves, their respective uncertainty and possible non-uniqueness (Fig. 3.3). As mentioned earlier, the curves are parameterized by k points, but in the most general case, users do not know the most appropriate value of k . Therefore, I implemented a method where the dimension of the model space is variable. This aspect is relevant for computational efficiency, uncertainty assessment and application to real datasets. Ideally, the inversion procedure has to choose the number of sea level, sediment supply and wave height points without making the curves so simple that they do not fit the observed data well (Fig. 3.3B), or so complex that they effectively over-fit the observed data (Fig. 3.3B). This is a model choice issue which can be solved by the Reversible Jump Markov chain Monte Carlo (RJMCMC) algorithm proposed by Green (1995); a discussion of the application of this algorithm relative to fixed-dimension methods is given in Sambridge *et al.* (2006). The RJMCMC-based sampler is a generalization of the Metropolis-Hastings technique (Metropolis *et al.*, 1953; Hastings, 1970), and is capable of sampling or jumping across spaces of varying dimension (Brooks *et al.*, 2003). A further, and important, advantage of using this method is that Bayesian techniques are naturally parsimonious, avoiding over-complicated models (e.g. Denison *et al.*, 2002; Malinverno, 2002; Sambridge *et al.*, 2006).

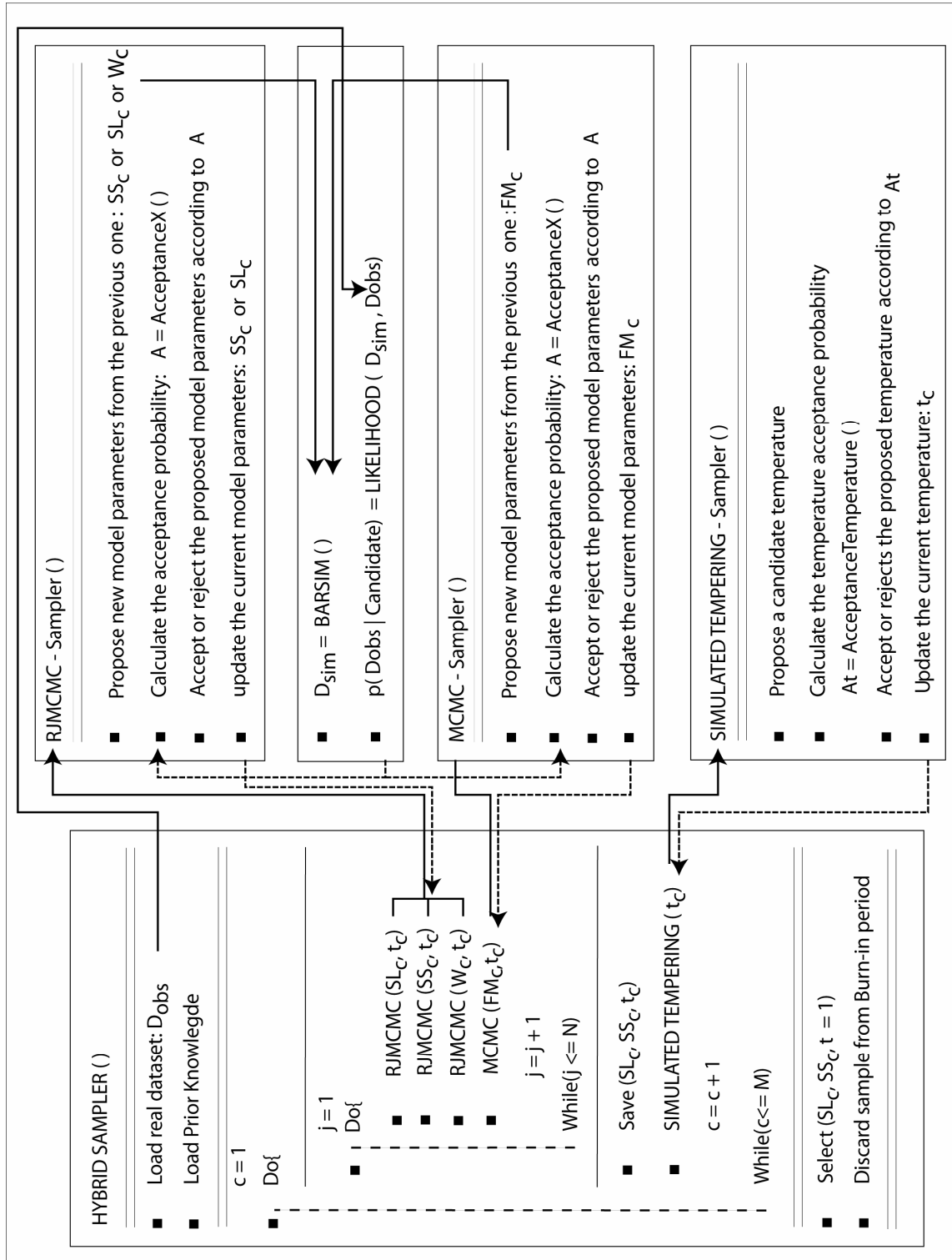


Figure 3.2: Simplified workflow diagram illustrating the inverse method.

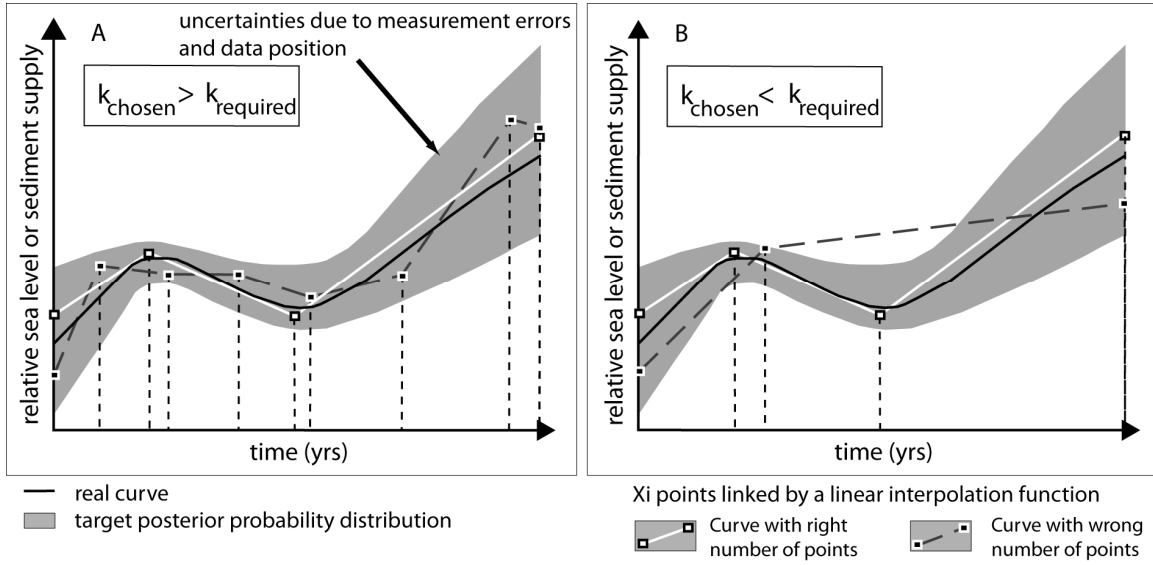


Figure 3.3: Impact of the number of dimensions for MCMC methods. This cartoon illustrates two fixed-dimension MCMC strategies that sample the posterior distribution of the sediment supply or relative sea level curve. In both cases the number of points defining the curves, by linear interpolation, stay constant during the sampling process. (A) In cases where the number of points chosen a priori is larger than the number of points needed to parameterize the curve, computational efficiency is reduced. (B) In cases where the number of points chosen a priori is smaller than the number of point required, it is impossible to correctly sample the posterior target distribution.

3.4.1 Reversible Jump Markov chain Monte Carlo (RJMCMC)

In this thesis, three similar RJMCMC algorithms, sampling independently the relative sea level, the sediment supply and the fair-weather wave height posterior distributions, have been combined in the Hybrid sampler. The method described in the subsequent paragraph refers to a matrix parameter $\mathbf{X}$ (in which each $\mathbf{X}^i = \{t^i, x^i\}$) that corresponds to the matrices $\mathbf{SS}$ or $\mathbf{SL}$ or $\mathbf{W}$ in their respective RJMCMC samplers.

The purpose of the inversion scheme is then to infer the following set of parameters: k (dimension, number of points) and $\mathbf{X}$ (parameter matrix of dimension $k \times 2$). The general algorithm for MCMC can be expressed as follows. The algorithm chooses an initial set of

model parameters at random from the prior distribution of each parameter, and this includes the number of parameters k . It calculates the likelihood for this current model and then perturbs the current model parameters in a variety of ways (explained below) to produce a proposed model. The perturbations are made according to probabilistic rules, coming under the general term of proposal densities. The algorithm calculates the likelihood for the proposed model, and then makes a comparison between the proposed and current models in terms of the likelihood and the prior and proposal probabilities (Fig. 3.4). The aim of this comparison is to decide whether to replace (or update) the current model with the proposed model or not. The algorithm is such that it always accepts a better (data-fitting) model, but it can also accept a less good model, again according to some simple probability-based rules (Fig. 3.4).

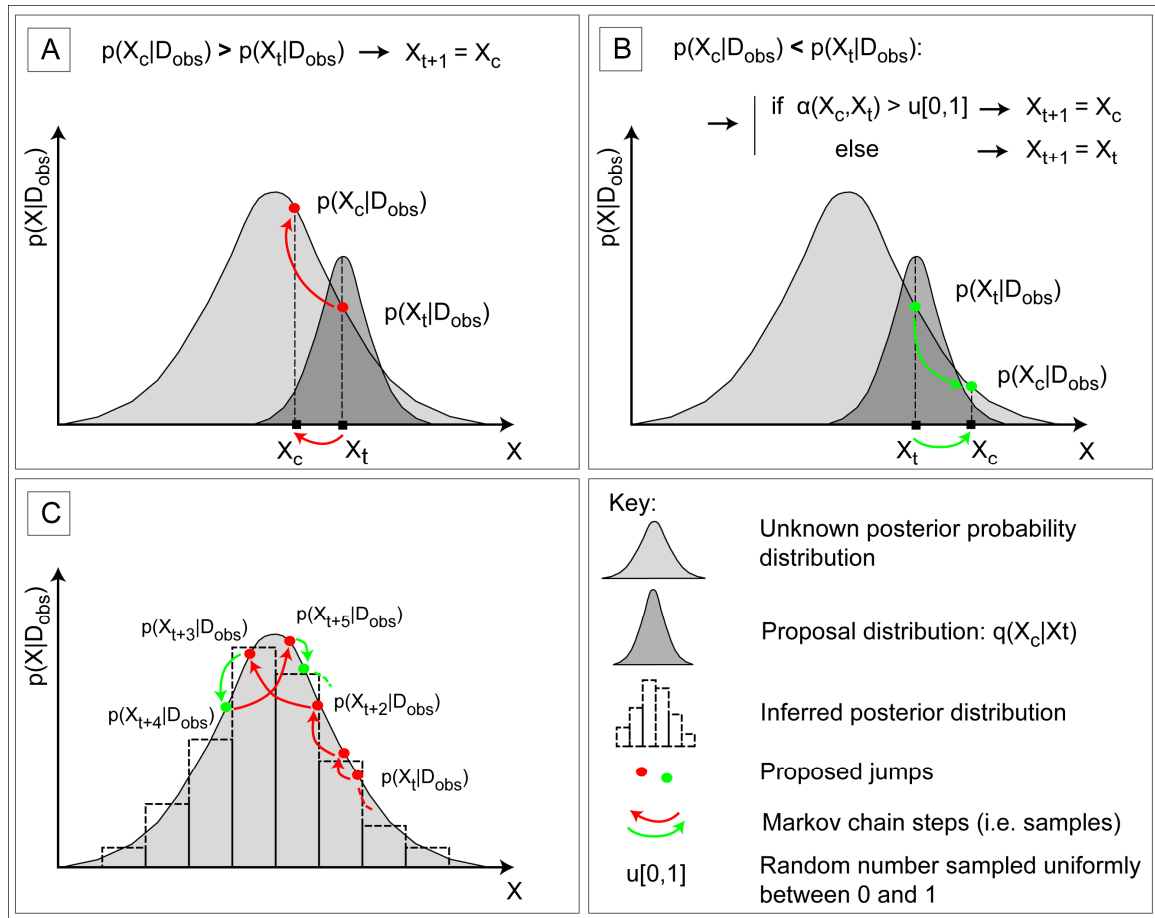


Figure 3.4: Simplified cartoon illustrating MCMC method for a single model parameter X . First, a candidate model parameter X_c is drawn from a proposal density centred on the

current model parameter $\mathbf{X}_t$ (A,B). Second, local calculation of the posterior probability $p(\mathbf{X}_c | D_{obs})$ is estimated via the prior probability and the likelihood function as defined by the Bayes' Rule. Then, (A), If the candidate model parameter $\mathbf{X}_c$ presents an higher posterior probability $p(\mathbf{X}_c | D_{obs})$ than the current model parameter $\mathbf{X}_t$, the acceptance ratio $\alpha(\mathbf{X}_c, \mathbf{X}_t)$ becomes higher than 1 (cf subsequent paragraph): the next step of the Markov chain $\mathbf{X}_{t+1}$ is set to be equal to $\mathbf{X}_c$. If the candidate model parameter $\mathbf{X}_c$ presents lower posterior probability than the current model parameter $\mathbf{X}_t$, the acceptance ratio $\alpha(\mathbf{X}_c, \mathbf{X}_t)$ becomes lower than 1: the next step of the Markov chain $\mathbf{X}_{t+1}$ is set to be equal proposed candidate parameter $\mathbf{X}_c$ if $\alpha(\mathbf{X}_c, \mathbf{X}_t) \geq U$, or to be equal to the current state parameter $\mathbf{X}_t$ if $\alpha(\mathbf{X}_c, \mathbf{X}_t) < U$. U is a random number drawn from a uniform distribution comprised between 0 and 1. The repetition of this method allows the Markov chain to converge to a maximum (i.e. high probability area) and to sample the stationary distribution around it. Once the chain reaches stationarity, the distribution of the samples provides a good approximation of the underlying posterior probability distribution (C).

This simple process not only allows the chain to converge to high probability areas, as for optimisation algorithms, but to also sample the full stationary distribution around the maximum by accepting a less good model. The outcome of this, after many iterations, is an estimate of the posterior distribution for all model parameters. Therefore, following the general framework proposed by Green (1995, 2003), where an initial model is chosen based on the prior distribution, and then at each iteration c :

- The algorithm chooses firstly one type of model dimension update (referred to as a jump), with a certain probability (see Appendix A), to propose a candidate model $\mathbf{X}'$, by perturbing one or more components of the current model $\mathbf{X}_c$ ($\mathbf{X}'$ may or may not have the same dimension of $\mathbf{X}_c$).
- Secondly, this proposed model is accepted or rejected with the following probability, according to the required detailed-balance requirement (Green, 1995):

$$\alpha(\mathbf{X}_c, \mathbf{X}') = \min \left(1, \frac{p(\mathbf{D}_{obs} | \mathbf{X}') p(\mathbf{X}') q(\mathbf{X}_c | \mathbf{X}')}{p(\mathbf{D}_{obs} | \mathbf{X}_c) p(\mathbf{X}_c) q(\mathbf{X}' | \mathbf{X}_c)} \times JR \times |\mathbf{J}| \right) \quad (3.4)$$

where the first ratio $p(\mathbf{D}_{obs} | \mathbf{X}') / p(\mathbf{D}_{obs} | \mathbf{X}_c)$ represents the likelihood ratio, and the second ratio $p(\mathbf{X}') / p(\mathbf{X}_c)$ is the prior ratio. The term $q(\mathbf{X}_c | \mathbf{X}') / q(\mathbf{X}' | \mathbf{X}_c)$ represents the proposal ratio, in which $q(\mathbf{X}_c | \mathbf{X}')$ is an arbitrarily chosen proposal density by which the new candidate is drawn from the current model. JR is the jump ratio (ratio of probability of reversing a proposed jump to the probability of proposing that jump) (Appendix, F.3). $|\mathbf{J}|$ is the Jacobian term which arises due to the change of the model state (Appendix, F.4) and is therefore equal to 1 for fixed dimension jump types. If the candidate model $\mathbf{X}'$ is accepted, this model is set to be the next step of the Markov chain ($\mathbf{X}_{c+1} = \mathbf{X}'$), otherwise the current model is set to be the next step ($\mathbf{X}_{c+1} = \mathbf{X}_c$).

In the algorithm I developed, four jump types have been designed, two of which involve no change in dimension and two of which do:

- The first one, “MoveX” perturbs the x^i coordinate of one of the k $\mathbf{X}^i$ points, randomly chosen, at fixed dimension. The candidate model is accepted with the acceptance probability $\alpha(\mathbf{X}_c, \mathbf{X}')_{MoveX}$ (Appendix B). This jump type is similar to the classical Fixed Dimension MCMC algorithm which will be used in Chapter 6 and 7 for updating the fixed dimension mud fallout rate environmental input parameter in BARSIM.
- The second one, “MoveT” perturbs the t^i coordinate of one of k $\mathbf{X}^i$ points, randomly chosen, at fixed dimension. The candidate model is accepted with the acceptance probability $\alpha(\mathbf{X}_c, \mathbf{X}')_{MoveT}$ (Appendix C).
- The “Birth” jumps across dimensions by adding a new point $\mathbf{X}^i = \{t^i, x^i\}$ to the matrix parameters $\mathbf{X}^{\{k \times 2\}}$. The candidate model is accepted with the acceptance probability $\alpha(\mathbf{X}_c, \mathbf{X}')_{Birth}$ (Appendix D).
- The “Death” is the reverse of the “Birth”, and jumps across dimension by deleting a randomly chosen point $\mathbf{X}^i = \{t^i, x^i\}$ of the matrix $\mathbf{X}^{\{k \times 2\}}$. The model is accepted with the acceptance probability $\alpha(\mathbf{X}_c, \mathbf{X}')_{Death}$ (Appendix E).

It is important to note that the efficiency of such algorithm depends on:

- The number of samples required before the algorithm begins sampling from the target or stationary distribution, known as the burn-in period (burn-in period samples have to be discarded for inference analysis).
- The mixing rate of the chain, which determines how effectively the parameter space is sampled, or explored. In fact, despite the considerable amount of freedom in building such jump types, a key issue in MCMC methods is to design efficient proposal functions in order to maintain a good acceptance rate (around 25%, Gilks *et al.*, 1996; Rotondi, 2002; Al-Awadhi *et al.*, 2002; Brooks *et al.*, 2003).
- The multi-modality and roughness of the posterior distribution in the solution area.

Slow and poor mixing of the chain will lead the inversion process towards an inefficient, often incomplete, sampling of the posterior probability distribution. A highly multi-modal solution area will tend to result in a slow convergence rate for the chain, and will generally increase the risk of becoming trapped at a local sub-optimal mode.

3.4.2 Simulated Tempering (ST)

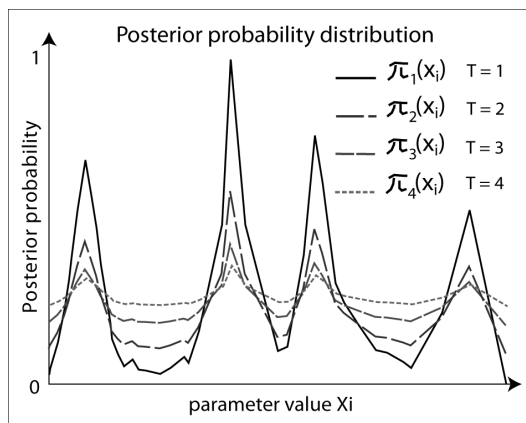


Figure 3.5: Cartoon illustrating the idea of Simulated Tempering. The introduction of a temperature factor in the Markov chain space (here $T = \{2, 3, 4\}$) “flattens” the “cold” posterior probability distribution π_1 of the parameter (X_i), which allows easier jumps between modes for the “warmer” posterior probability distributions π_2, π_3, π_4 .

Initial tests of the RJMCMC samplers applied to the stratigraphic inverse problem showed that they are sometimes inefficient in dealing with a multi-modal and rough solution “landscape”.

This is a well known result with conventional MCMC methods (Brooks *et al.*, 2006). In fact, it is very hard to set up an efficient proposal distribution *a priori* that allows the algorithm to explore both between and within the modes, without becoming “stuck” in local areas. To avoid this problem, a Simulated Tempering (ST) sampler (Marinani & Parisi, 1992; Geyer & Thompson, 1995; Liu & Sabatti, 1999; Brooks *et al.*, 2006) has been incorporated in the general framework of the Hybrid sampler (Fig. 3.2).

Simulated Tempering is a MCMC method which relies on flattening the stationary distribution, allowing easier movement between individual modes (Fig. 3.5). A new term, the temperature τ (which takes values from a finite set of temperature levels, $\tau = \{1, 2, \dots, T\}$ being a finite set of integers) is added as a random variable in the Markov chain state (X, k, τ) . This allows us to use a series of stationary densities $\pi_\tau(X, k)$ whose shape depends on the value of τ :

- The “cold” distribution $\pi_1(X, k)$ is the posterior probability distribution of interest, from which inference is made.
- The “hot” $\pi_T(X, k)$ distribution where rapid mixing takes place.

The method also requires a fixed set of corresponding, arbitrarily chosen, normalising weights ω_τ (with $\omega_\tau > 0$ and $\sum \omega_\tau = 1$; Geyer, 1991). Following Brooks *et al.* (2006), a specific method for “heating up” the target distribution has been used, where $\tilde{\pi}_\tau(X, k) = \pi(X, k)^{1/\tau}$ is taken. Note that, here, π_∞ corresponds to a uniform distribution over the entire parameter space, effectively removing all local modes and allowing easy but not particularly informed movement around the model space.

The Simulated Tempering sampler updates the temperature scheme in a Metropolis-Hasting way, by proposing a new temperature from a proposal probability distribution $q_t(\tau, \tau')$. Following Geyer & Thompson (1995), we set $q_t(\tau, \tau+1) = q_t(\tau, \tau-1) = 1/2$ for $\tau = 2, \dots, T-1$ and $q_t(1, 2) = q_t(T, T-1) = 1$. The candidate temperature is accepted with the probability $\alpha_t(\tau, \tau'; X_c, k)$ defined as follows:

$$\alpha_i(\tau, \tau'; X_c, k) = \min \left(1, \frac{\tilde{\pi}_{\tau'}(X_c, k) \omega_{\tau'} q_i(\tau', \tau)}{\tilde{\pi}_{\tau}(X_c, k) \omega_{\tau} q_i(\tau, \tau')} \right) \quad (3.5)$$

The Hybrid algorithm begins to sample the parameter space at the temperature τ_T by running the RJMCMC and fixed-dimension MCMC samplers during n iterations (n being chosen by the user), then updates the temperature scheme via the Simulated Tempering algorithm. This procedure is repeated until the user estimates that sufficient sampling of the target posterior distribution has occurred.

3.5 Prior Information

The prior information, as specified in equations 3.2 and 3.3, captures both quantitative and qualitative knowledge that the geologist may have *a priori* on the model $\mathbf{X}$. The prior knowledge is expressed as a probability density function which can be described either by a totally uninformative uniform density function, or with more specific prior knowledge like a Gaussian, Poisson, or indeed any other type of probability function. The use of such functions has a direct impact on the efficiency on the inverse scheme, however, because overly precise prior information, if incorrect, can bias the inversion process: the full range of possible solutions may not be properly sampled and the chain may not converge at all to the solution.

The prior information of the x^i value (sl^i , ss^i and w^i respectively for the relative sea level, sediment supply and fair-weather wave height parameters) will be distributed between a user defined global minimum $x_{\min}$ and a global maximum $x_{\max}$ in all the presented applications. The choice of the prior distribution $p(x^i)$, comprised within this interval, will depend on the application presented in the following chapters.

The prior information of the position of the t^i coordinates of the k $\mathbf{X}^i$ points is set to be uniform on $[t_{\min}, t_{\max}]$ with $t^1 < t^2 < \dots < t^k$. For numerical reasons, we set fixed $t^1 = t_{\min}$ and $t^k = t_{\max}$. It is then assumed *a priori* that the time positions are drawn as the order statistic

from a uniform distribution on the forward simulation time interval $U(t_{\min}, t_{\max})$. The joint distribution $p(t^1, \dots, t^k)$ is defined as:

$$p(t^1, t^2, \dots, t^k) = (2k+1)! \left(\frac{1}{t_{\max} - t_{\min}} \right)^{2k+1} \prod_{j=1}^k (t^{j+1} - t^j) \quad (3.6)$$

This joint density provides a gentle preference against two changes occurring too closely in succession (Green, 2001).

The prior information on the number of dimensions k is set to be uniform between $k_{\min}$ and $k_{\max}$ chosen *a priori* by the geologists. The prior density of k is defined as follows:

$$p(k) = \frac{1}{k_{\max} - k_{\min}} \quad (3.7)$$

3.6 Likelihood Function

The likelihood function calculates the probability of obtaining the data $\mathbf{D}_{obs}$ given the model $\mathbf{X}$ by quantifying the difference between the observed data $\mathbf{D}_{obs}$ and the simulated data $\mathbf{D}_{sim}$. As mentioned in chapter 2, I modified BARSIM to extract simulated data which can be compared with available observed data. These simulated data extracted from the numerical BARSIM grid comprise specific 1D vertical successions with associated ages, comparable to well data, and 2D thickness curves for specific stratigraphic intervals, comparable to isopach maps derived from seismic and/or well data.

For practical numerical reasons, we use the log-likelihood function, because maximising the log-likelihood function is the same as maximising the likelihood function (or probability). The log-likelihood function is defined algebraically as:

$$L = L_{th} + L_t + L_{gs} + L_{fs} \quad (3.8)$$

where L represents the total log-likelihood and $L_{th}, L_t, L_{gs}, L_{fs}$ represent respectively the partial log-likelihood accounting for the thickness, the age of deposition at the top and bottom of each wells, the grain-size distribution in each well, and the vertical position of any flooding surfaces in each well. In the formulation of the total log-likelihood, all the terms are assumed to be independent. In order to allow for variations in the number of observed data to which the log-likelihood function is applied, the terms L_t, L_{gs}, L_{fs} have been normalized by the number of wells. Also, the vertical coordinates of the simulated data used to calculate the term L_{gs}, L_{fs} have been normalized to the vertical coordinates of the observed data (Fig. 3.6). This avoids repeating the thickness mismatch effect already evaluated in the first term of the total log-likelihood L_{th} . Equation 3.8 presents the complete formulation of the log-likelihood function. This formulation will be adapted to the available observed data for applications presented in the following chapters.

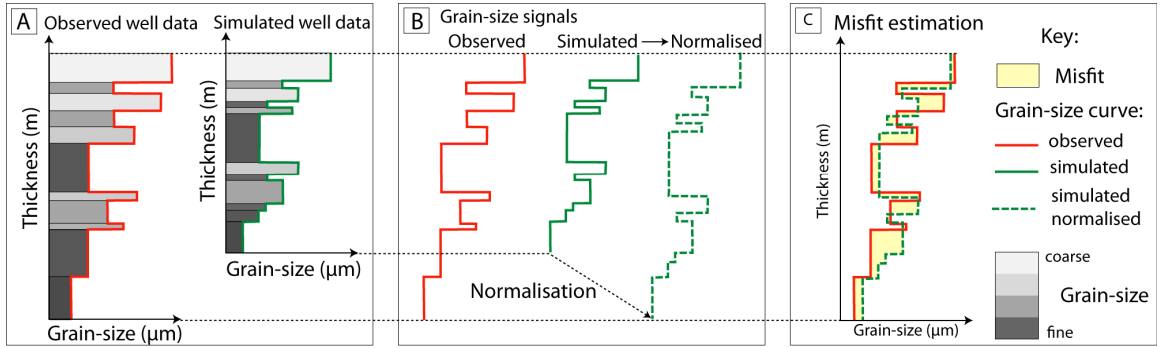


Figure 3.6: Likelihood function design for the stratigraphic inverse problem. This cartoon illustrates how log-likelihood values are evaluated between the simulated and observed grain-size signal at each well. In the first step (A), the simulated and observed grain-size curves are extracted from each well. In a second step (B), the thickness of the simulated grain-size curve is normalised to the thickness of the observed grain-size curve to avoid repeating the thickness mismatch effect already evaluated. In the final step (C), the mismatch between the normalised simulated and observed grain-size curves is evaluated using the least-square distance criterion.

In each partial likelihood function, say L_j , we assumed that measurement errors had a normal distribution with zero mean and were uncorrelated. Using this assumption, the log likelihood is just the well-known least squares criterion technique. This has been used to evaluate the match between the observed data $D_{obs,j}$ and simulated data $D_{sim,j}$ as follows:

$$L_j = - \left[\sum_{i=1}^m \left(\frac{D_{obs,j}^i - D_{sim,j}^i}{\sigma_{i,j}} \right)^2 \right] \times \frac{1}{\tau} \quad (3.9)$$

where m represents the number of observed points, $\sigma_{i,j}$ represents the standard deviation for a given observation, $D_{obs,j}^i$ and τ the temperature.

3.7 Summary

In this chapter, I have presented a formulation for a stochastic inversion scheme linked to a two-dimensional “process-based” forward algorithm that models shallow-marine wave-dominated stratigraphy (BARSIM). The inverse method combines the Reversible Jump Markov chain Monte Carlo and Simulated Tempering algorithms in a hybrid sampler. The multi-parameters inversion approach is driven by the observed geological data and is designed to deal with questions of uniqueness, uncertainty and non-linear interactions. Markov chain Monte Carlo methods achieve this through an iterative sampling strategy. The inverse algorithm proposes candidate model parameters (e.g. BARSIM environmental input parameters) by perturbing the current ones, in order to run one BARSIM realization. The match, or fit, between the resulting simulated and the observed stratigraphy is calculated by the likelihood function. On the basis of the Metropolis-Hasting acceptance-rejection criterion (Metropolis *et al.*, 1953; Hastings, 1970), combining the calculated likelihood and prior probabilities, the candidate model is compared to the current model and therefore accepted or rejected. This loop is repeated until the sampling is judged representative of the target posterior

probability distribution. In this way, it is possible to obtain a large number of samples of the model parameter space, with the distribution of these samples providing a good approximation of the underlying target posterior probability density function.

Chapter 4

4 Validation Tests

Abstract

The stratigraphic inverse technique will be tested in this chapter with synthetic data in which all the parameters are specified to produce a ‘perfect’ forward simulation, although noise is added to simulate measurements errors. I will present a number of examples in which relative sea-level and sediment supply curves will be inferred from synthetic vertical successions of grain size and synthetic thickness curves extracted from the ‘perfect’ forward simulation. These examples represent different scenarios that are designed to test the impact of data distribution, quantity and quality on the uncertainty of the inferred parameters, as well as the sensitivity of the stratigraphic modelling algorithms to the forcing parameters themselves. The inverse modelling approach successfully reproduces the gross stratigraphic architectures and relative sea-level and sediment supply histories of the synthetic forward model simulation, within the constraints of the modelled data quality. The relative importance of the forcing parameters can be evaluated by their sensitivity and impact on the inverted data. Of equal importance, the inverse results allow complete characterisation of the uncertainties inherent to the stratigraphic modelling tool and to the data quality, quantity and distribution. These tests also address convergence and computational-overhead issues and highlight the robustness of the inverse scheme, which successfully deals with the problem of non-uniqueness of the solution of the inverse problem. The work presented in this chapter is partly published in Charvin et al. (2008a, b).

4.1 Introduction

In order to validate the inversion scheme, I considered several synthetic cases, created by BARSIM forward simulations in which all the input environmental parameters are completely specified. In this chapter, I will present the results of tests realised on one of these synthetic cases to provide validation of the method and a basis for discussion of the inverse algorithm efficiency. For these tests, only the relative sea-level and sediment supply parameters will be inferred from synthetic vertical successions of grain size (e.g. wells) and synthetic thickness curves (e.g. seismically derived isopachs) extracted from the ‘perfect’ forward model realisation.

Analysis of the literature shows that Lessenger & Cross (1996), Cross & Lessenger (1999), Bornholdt *et al.* (1999) and Wijns *et al.* (2004) conducted the first tests of their numerical schemes using near-perfect datasets that afforded unusually extensive coverage of the study area by cross-sections and wells. Consequently, in the examples presented, I will consider different scenarios designed to test the impact of data distribution, quantity and quality on the uncertainty of the inferred parameters, as well as the sensitivity of the stratigraphic modelling algorithm to the forcing parameters themselves.

This chapter is structured as follows. I will firstly describe the numerical synthetic data, then I will secondly present and explain the output of the inverse numerical scheme. Thirdly, I will consider the different scenarios relative to the reference synthetic data set in order to assess the sensitivity of the inference of environmental parameters (e.g. relative sea level and sediment supply) and facies architecture prediction (e.g. grain-size and bed-thickness trends) to different measurement errors and observed data locations and numbers. I will conclude with a discussion on computational performance and with suggestions for applications to real datasets.

4.2 Validation Tests on Predefined Synthetic Data, Inference of the Relative Sea Level and Sediment Supply Curves.

4.2.1 Reference Forward Model and Synthetic Data Presentation

The synthetic reference model consists of two aggradationally stacked shallow marine sandstone tongues (Fig. 4.1A), each deposited during a relative sea level fall of 3 m. The sandstone tongues are separated by a flooding surface formed during a short period of sea-level rise of 30 m (Fig. 4.1B).

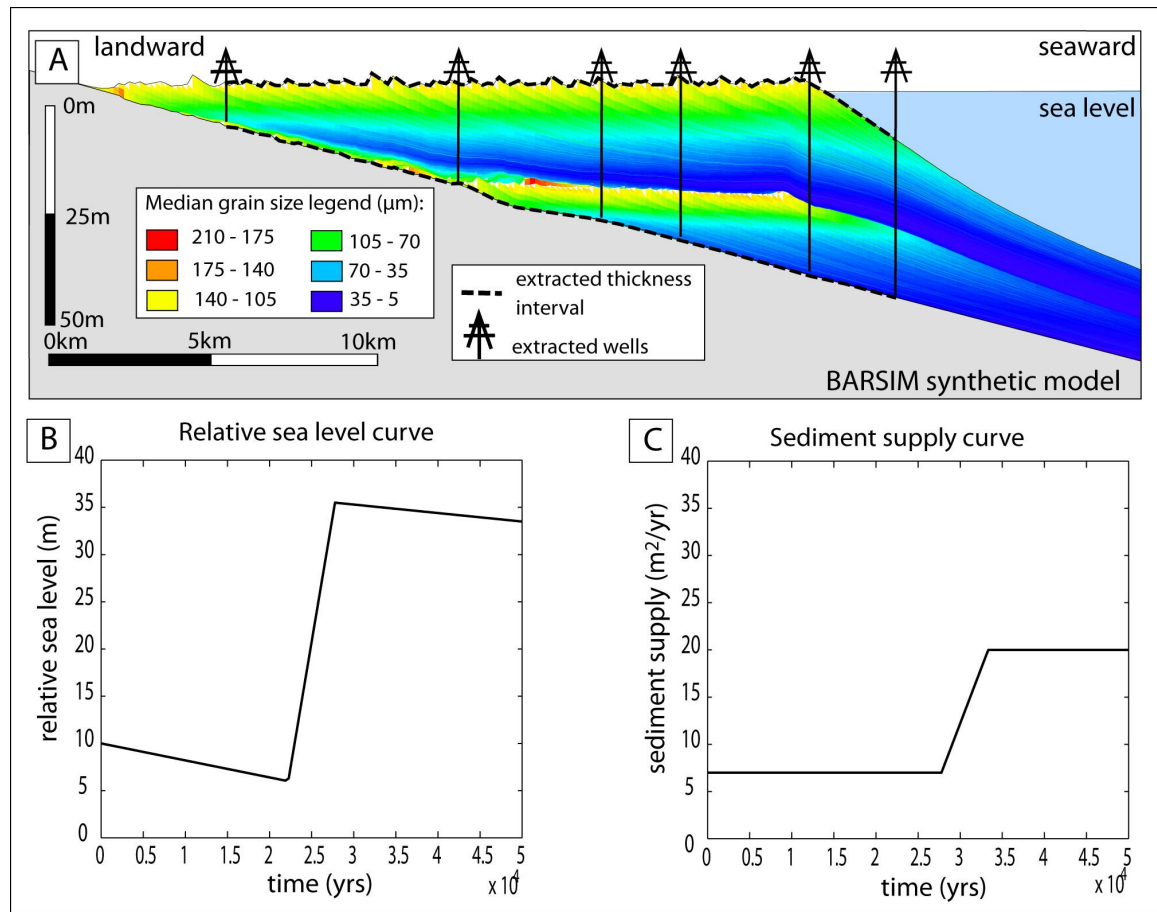


Figure 4.1: BARSIM forward realisation used as a synthetic case to test the inversion scheme. (A) Cross-section resulting from the forward model run showing the two stacked shallow

marine sandstone tongues, their internal stratigraphic architecture and grain-size distribution, and the position of data extracted in order to invert the relative sea level and sediment supply curves (Fig. 6). (B) Relative sea level and (C) sediment supply curves used to create this synthetic model. These curves constitute the target distribution for subsequent inverse modelling.

Sedimentation has been simulated over a period of 50,000 years with a minimum recurrence period of 25 years for major storm events (e.g. minimal timestep). Sediment supply, added by longshore currents, is equal to 7 m²/yr during the first 25,000 years and increases rapidly to reach a constant value of 20 m²/yr until the end of the simulation (Fig. 6C). The horizontal grid comprises 250 cells of 150 m width. The total thickness of simulated strata is around 50 m.

4.2.2 Test Scenarios for Inversion

Vertical successions (wells) and thickness curves of specific stratigraphic intervals between the wells were extracted from the BARSIM synthetic model and used as geological data to infer the relative sea level and sediment supply characteristics. Gaussian random noise has been added to the synthetic data in order to simulate measurement errors. Five different scenarios are presented in this chapter (Fig. 4.2). In the presented scenarios (A-E), all of the synthetic data have been extracted from the same reference model.

Scenario A (Fig. 4.2A) is the reference scenario in which the values extracted effectively represent perfect data. Here, the data, comprising six extracted wells and one thickness curve, cover the full extent of the stratigraphic cross-section and were assigned small measurement errors of 5%. Thus, they provide excellent data control with which to constrain the inversion process.

In scenario B (Fig. 4.2B), the aim was to test the impact of bigger measurement errors on the inferred results. Consequently, I use the same dataset as scenario A, but added Gaussian synthetic noise of 10% (twice as large as scenario A).

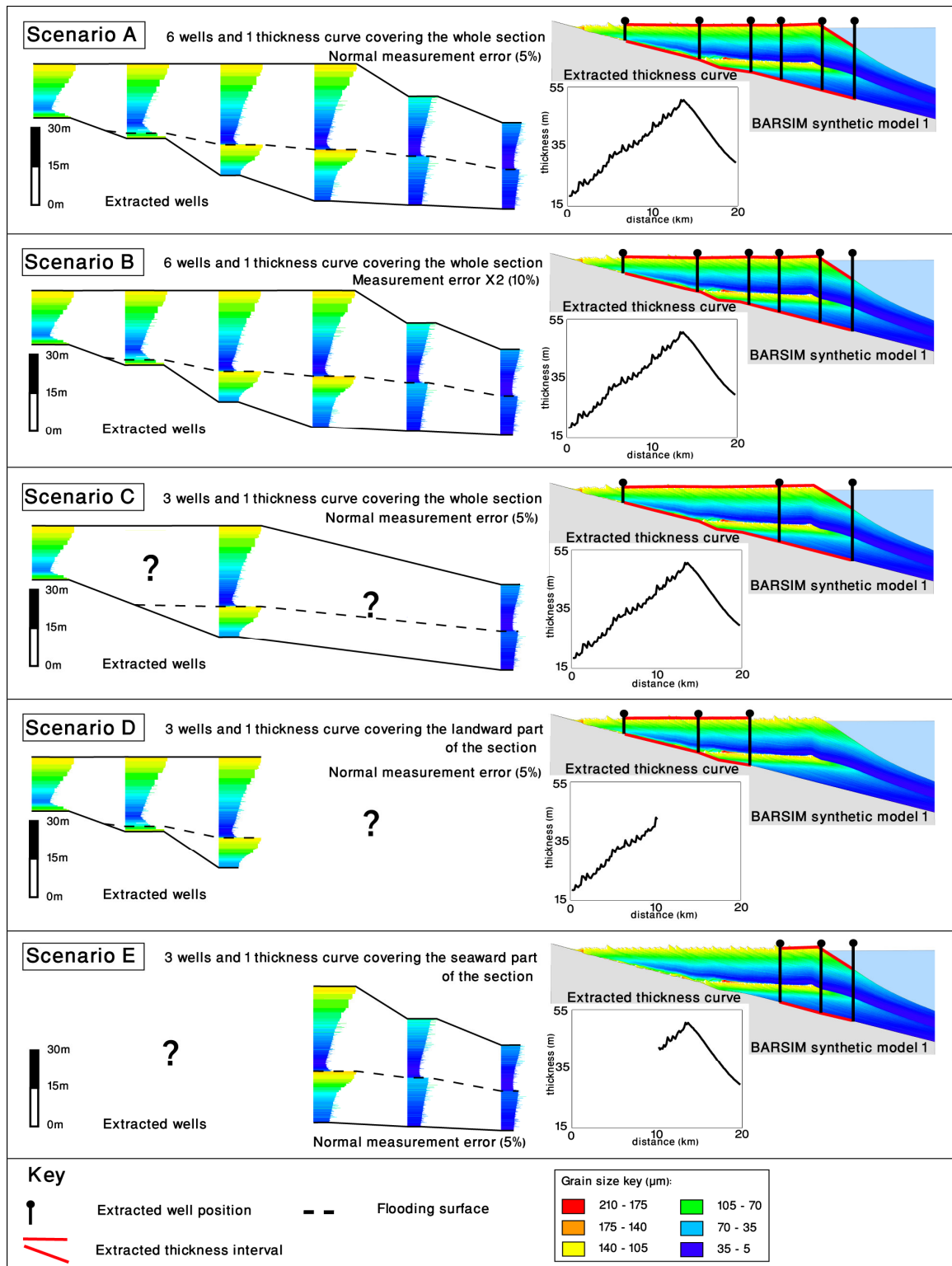


Figure 4.2: Scenarios investigated in order to test the impact of observed data quantity, quality and distribution on the stratigraphic inverse problem. (A) The fullest dataset, comprising 6

wells and 1 thickness curve, which covers the full extent of the stratigraphic cross-section and contains low measurement errors (5%). (B) The same dataset as in (A) has been extracted but larger measurement errors (10%) have been added. (C) Only 3 wells and 1 thickness curve, with low measurement errors (5%), have been extracted across the full extent of the cross-section. (D) 3 wells and 1 thickness curve, with low measurement errors (5%), have been extracted from the landward part of the cross-section. (E) 3 wells and 1 thickness curve, with low measurement errors (5%), have been extracted from the seaward part of the cross-section.

Scenario C (Fig. 4.2C) aims to test the impact of data quantity on the stratigraphic inversion. For that purpose, only three wells and one thickness curve, which cover the full extent of the stratigraphic cross-section, have been extracted from the synthetic model.

Scenario D (Fig. 4.2D) and scenario E (Fig. 4.2E) have been designed to test the impact of observed data (e.g. well location) distribution on the inference of sediment-supply and relative sea-level parameters. Only three synthetic wells and one thickness curve have been extracted from the landward part of the synthetic cross-section for scenario D and from the seaward part for scenario E.

For scenarios C, D and E, the same noise level as scenario A (5%) were used. In this approach, I therefore assume that BARSIM is a perfect simulator and so minimise the impact of forward modelling errors on performance of the inverse scheme.

4.3 Inverse Algorithm Configuration

The five extracted synthetic datasets (scenarios A-E), have been input as observed data in the stratigraphic inversion scheme, in order to run five inversion processes (Fig. 4.3). The inversion of the specific parameters for all runs were identical: the initial random model was the same for every scenario, the Hybrid samplers were run for 65,000 forward simulations each, the prior distributions for relative sea-level and sediment-supply for each scenario (prior

information) were specified to be uniform *a priori* between 0m and 40m, and between $0\text{m}^2/\text{y}$ and $40\text{m}^2/\text{y}$, respectively. For both of these variables, the number $SL^i = \{t^i, sl^i\}$ and $SS^i = \{t^i, ss^i\}$ points were allowed to vary uniformly between 2 and 20. Three levels of temperature $\tau = \{1, 2, 3\}$ and their corresponding weights have been set in order to achieve effective sampling with simulated tempering. The choice of the temperature levels and their respective normalising weights has been carried out arbitrarily by testing different combinations and selecting the one achieving the best temperature sampling. An automatic method to estimate the normalising weights has been proposed by Kerler & Rehberg (1994), but it was not applied here for computational overhead reasons.

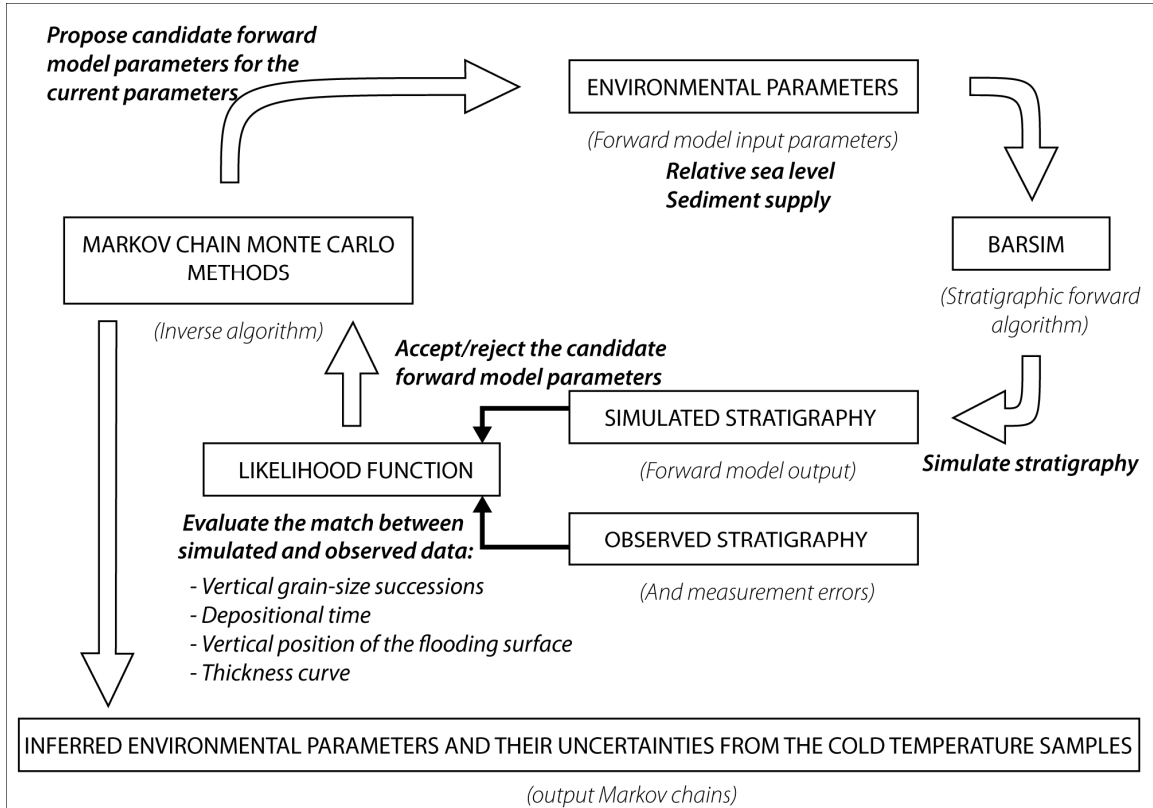


Figure 4.3: Simplified workflow diagram illustrating the inverse method. The inverse scheme has to infer the relative sea level and sediment supply parameters from synthetic data comprising information about vertical grain-size successions, depositional time, vertical position of the flooding surface and thickness.

4.4 Results

In this part, I will first illustrate and explain how the sampling evolves throughout the inversion process with the output of scenario A presented as an example. I will then show the results for the posterior distributions of the inferred parameters for scenarios A to E.

4.4.1 Stratigraphic Inversion Evolution through the Sampling Process

Figure 4.4 illustrates the evolution of the log-likelihood values (i.e. a measure of the agreement between simulated and “real” synthetic data) during the sampling process, as well as three selected models illustrating how the inverse scheme evolved. The three selected models are the first (randomly chosen), intermediate and best-fit BARSIM models. A low log-likelihood value (i.e. large negative number in Fig. 4.4) means poor match between simulated and “real” data, while the maximum possible log-likelihood value (equal to zero in Fig. 4.4) means that the simulated and “real” data are identical.

From the first randomly generated model, the log-likelihood values increase exponentially: the sampling scheme proposes intermediate models that are a progressively better fit to the “real” data. This period of exploration of the model parameter space, which aims to converge to the solution areas, is called the burn-in period. In this case, around 30,000 samples (i.e. 30,000 BARSIM forward iterations), almost half of the samples generated, were required to achieve convergence and then had to be discarded for analysis of the inferred model parameters. This example illustrates the issue of convergence rate impacting directly on computational costs. A decrease in the convergence rate, by adding more prior knowledge and by beginning the sampling process close to the solution area, greatly decreases the computational overhead. However, prior knowledge should not be too restrictive, in order to consider the full range of solutions of the inverse problem.

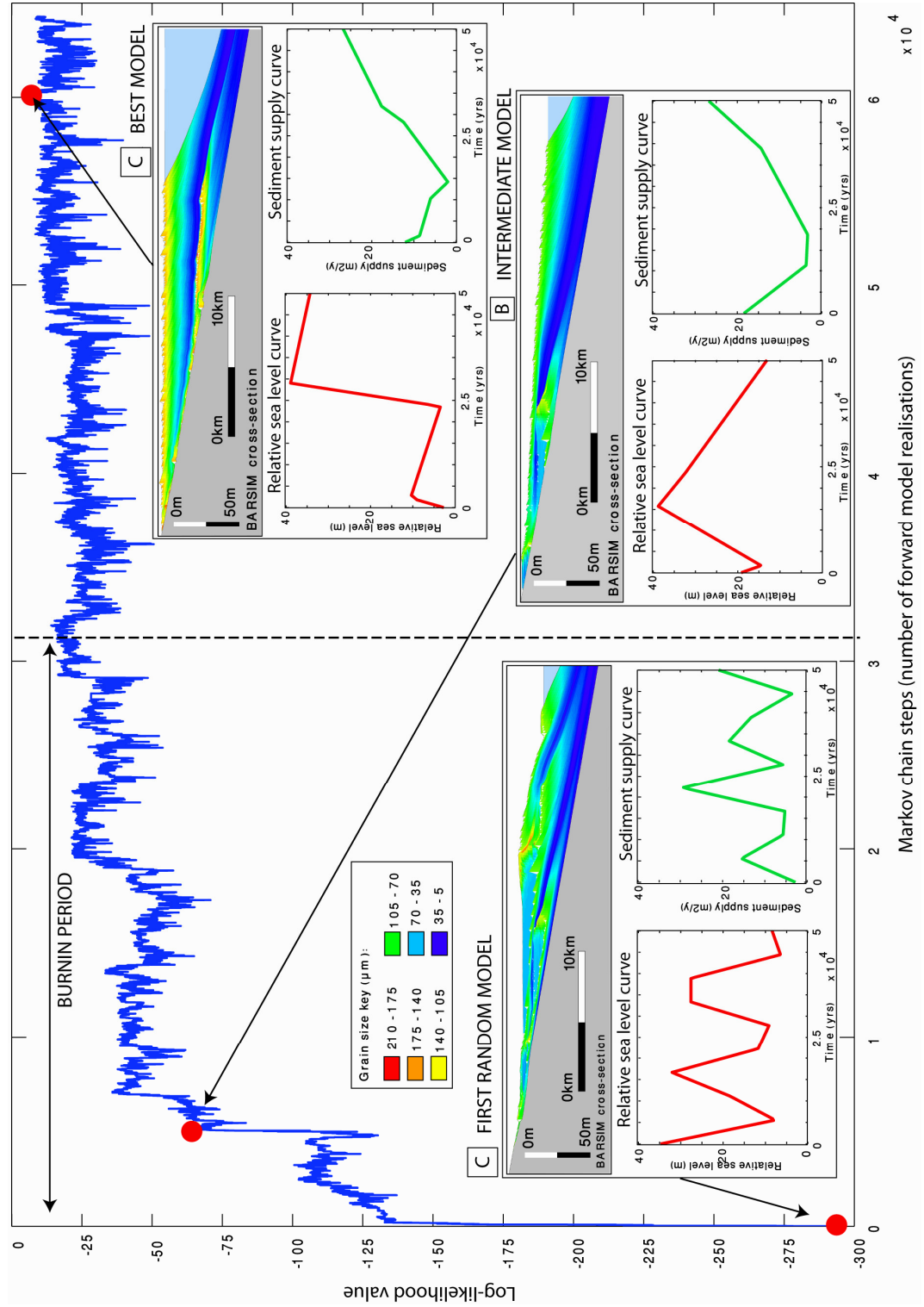


Figure 4.4: Evolution of the log-likelihood value (i.e. match between simulated and “real” data) during the sampling process (blue curve). The figure also shows some selected models

(**A**: first random, **B**: intermediate and **C**: best-fit models) to illustrate how the inversion evolves during the sampling. The length of the burn-in period (convergence to the solution) represents about 30,000 Markov chain steps (i.e. 30,000 stratigraphic forward model iterations).

Once convergence is achieved, the log-likelihood values remain more or less stationary, with values varying between -25 and -4 in an acceptable manner. The numerical values themselves have little intrinsic interest, and clearly will depend on how the likelihood function was designed. The analysis of the best-fit BARSIM model (with the lowest log-likelihood value in Fig. 4.4 C) shows that it is similar to the BARSIM synthetic model (Fig. 4.1) from which the “observed real” data (Fig. 4.2A) have been extracted. This confirms that, during this relatively stationary period of low log-likelihood values, the algorithm is sampling the critical solution area of the inverse problem. Only these post-burn-in samples should be used to infer the posterior probability distribution of the sediment supply and relative sea-level model parameters.

4.4.2 Impact of the Simulated Tempering Sampler on the Inversion Process

Depending on the nature of the prior and proposal distributions in the acceptance criterion (Equ. 3.4), MCMC methods will generally accept better data-fitting (i.e. higher likelihood) models. This leads the inversion procedure to converge to good models. However, it also accepts what are considered worse models (Fig. 4.4). This is required to achieve a representative approximation of the probability distribution of the model parameters. By varying the temperature levels, the Simulated Tempering algorithm allows the acceptance of even poorer data-fitting models (Fig. 4.5). This aims, firstly, to avoid the algorithm becoming “stuck” in local minima, and to focus on potentially higher probability areas when the temperature is again decreased. Secondly, it facilitates the exploration of (*a priori* unknown) multi-modal posterior probability distribution, which allows the algorithm to sample the full range of possible solutions to the inverse problem.

Although, there are clear advantages of using such a robust technique, Simulated Tempering results in the inversion process having a considerably higher computational overhead. In fact, because only the results from the “cold” distribution are used, the number of samples available is roughly inversely proportional to the number of temperature levels chosen *a priori* (Fig. 4.5B). The computational time required for efficient sampling then becomes proportional to the number of temperature levels.

All the results presented in the subsequent paragraphs have been determined only with samples from the “cold” distribution, after having discarded the burn-in samples. Once selected, the sampled coordinate vectors have been used to calculate the posterior probability distributions of the respective curves. From these distributions, the mean inverted curves and the 95% Bayesian credible interval have been calculated and used as statistical estimators. Although credible intervals are often numerically equivalent to the more common confidence intervals, they are philosophically different. The 95% Bayesian credible interval represents the region in which there is a probability of 0.95 of attaining the true parameter value (Bernardo & Smith, 1994).

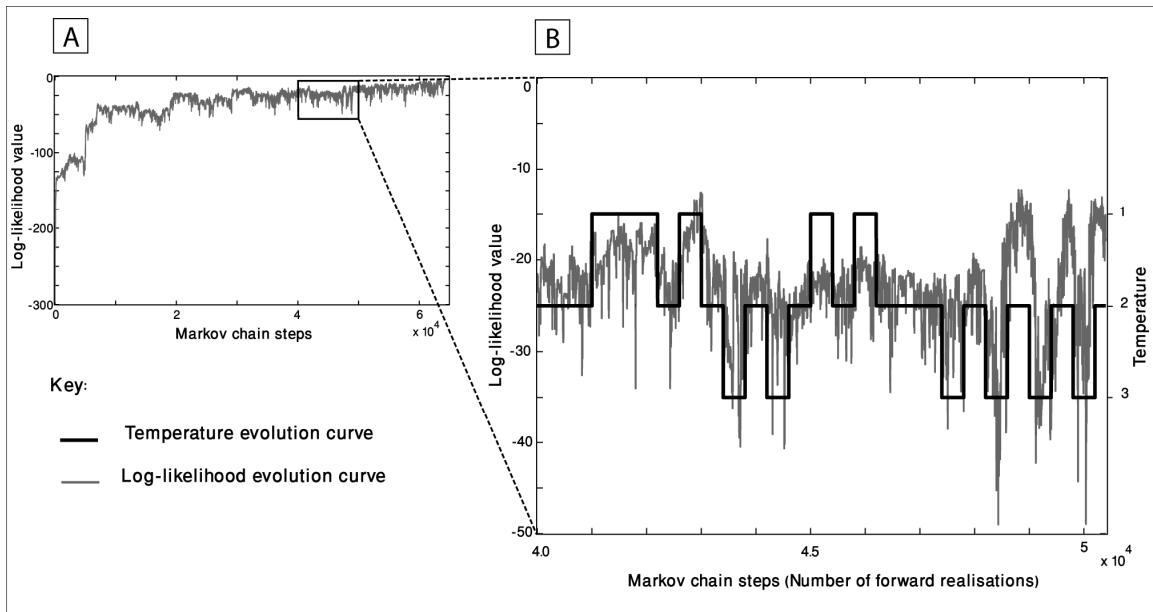


Figure 4.5: Impact of the Simulated Tempering sampler on the log-likelihood evolution. By varying the temperature levels, the Simulated Tempering algorithm allows the acceptance of

even poorer data-fitting models avoiding the algorithm becoming “stuck” in local minima and focusing on potentially higher probability areas when the temperature is again decreased.

4.4.3 Scenario A

4.4.3.1 Posterior Distribution of the Relative Sea Level Curve

Figures 4.6A and 4.6B show that the number k of SL^i points needed to parameterize the relative sea level curve converges toward four, which is the original number of points used to create the input curve (Fig. 4.1B). Figure 4.6A also illustrates that the coordinates of the sampled points tend to be situated around the inflexion area of the real relative sea-level curve, where there is a change from sea-level fall to sea-level rise, or vice versa. It is important to appreciate that the number of points k depends not only on the shape of the curve but also on the spread of the target distribution. A large spread in sample locations, for example, will lead to a smaller number of dimensions by losing the low-amplitude, high-frequency signal of the curve.

Figure 4.7A shows the comparison between the inferred posterior probability distribution and the “real” input relative sea-level curve. It is clear that the high probability area, mapped from the sampling, matches the original relative sea-level curve well. The spread of the Bayesian credible interval is relatively narrow ($\pm 5\text{m}$). However, this spread tends to be larger ($\pm 5\text{m}$ vs. $\pm 2\text{m}$) at the end of each relative sea-level fall, especially the most recent one. This trend can be explained by the lack of constraint offered by the available well data. During the period of highest relative sea level (at c. 25,000 years of simulated time), all the wells record active sedimentation and provide useful information for the inversion. In contrast, during periods of relative sea-level fall, the sedimentary system migrates seaward such that fewer wells record active sedimentation and, consequently, less information is available to constrain the inversion. These results also imply that the inferred relative sea level curve is considerably less well constrained during the relative sea level rise than during periods of relative sea-level fall. This observation is explained in this case study by the scarcity of transgressive deposits preserved in the available well data.

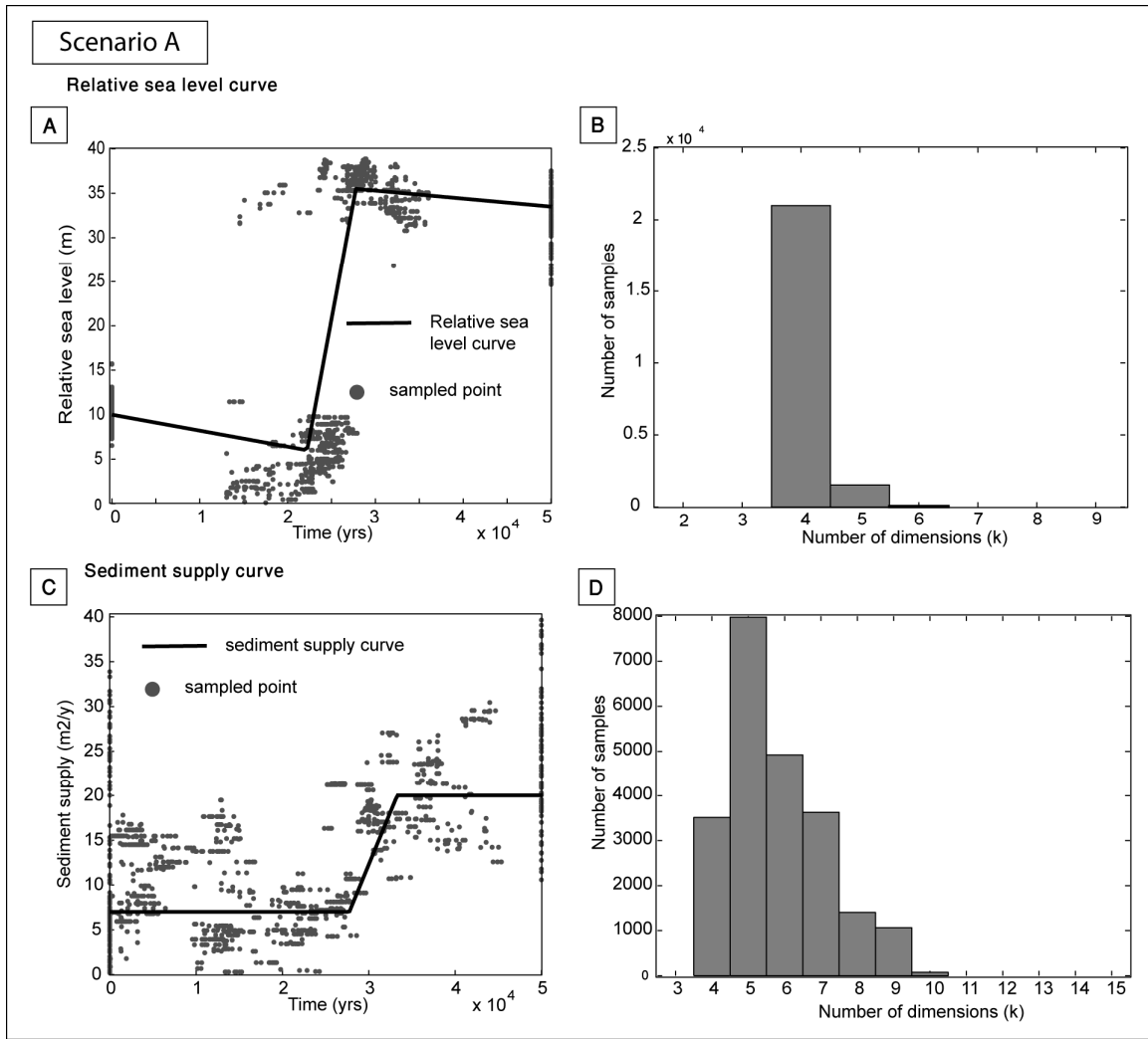


Figure 4.6: Posterior probability distribution of the vectors of coordinates parameterising the relative sea-level (A,B) and sediment supply (C,D) curves. (A) The sampled points tend to cluster around the positions where the input relative sea-level curve (dark line) changes from a rising to falling gradient, or vice versa. (B) The histogram shows the posterior probability distribution of the number of points (i.e. dimensions) required to characterise the relative-sea-level curve. This distribution is centred on the value four. (C) Posterior probability distribution of the vectors of coordinates parameterising the sediment supply curve. The sampled points are distributed widely around the input sediment-supply curve (dark line). (D) The histogram shows the posterior probability distribution of the number of dimensions (i.e. points) required to characterise the sediment-supply curve. The distribution is wider than that for the relative sea-level curve (B), and the most likely number of points is five.

4.4.3.2 Posterior Distribution of the Sediment Supply Curve

The results of the inferred sediment supply curve are more complex than those of the corresponding relative sea level curve. Figures 4.6C and 4.6D show that the number k of SS^i points needed to parameterize the curve do not converge to four as used to define the true input sediment supply curve (Fig. 4.1C), but are sampled at values ranging from four to seven, with five being the most likely value. In contrast to the relative sea-level curve, the positions of the sampled points are more widely and erratically distributed. They do not cluster in an obvious way around the positions where there are changes in the input sediment-supply curve. Figure 4.7B compares the inferred posterior probability distribution and the “real” input data. The inferred target distribution shows the same order of magnitude as the real data and its shape follows roughly the shape of the real curve. However, the mean (Fig. 4.7B) curves do not match the “real” synthetic curve particularly well and, compared to the inversion of the relative sea level parameters, the spread of the 95% Bayesian credible interval is large (± 10 -15 m^2/y). In this case, the inversion scheme is able to detect the change in the sediment supply regime but is unable to quantify this change reliably.

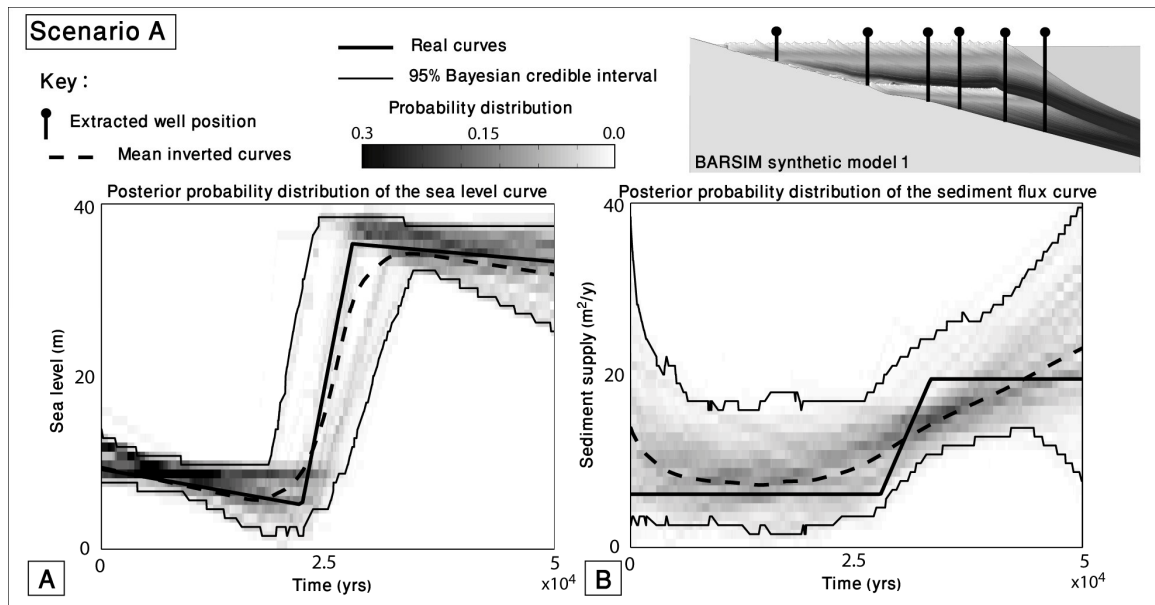


Figure 4.7: Posterior probability distributions of (A) relative sea-level and (B) sediment-supply curves obtained for scenario A. The dark continuous line in each graph is the input curve, the dashed line is the mean inverted curve, and the light grey curves represent the

boundaries of the 95% credible interval. The background shading represents the posterior probability distribution (dark means high probability). Uncertainty tends to be greater at the end of each relative sea-level fall and is even higher during the period of relative sea-level rise.

For this case-study, the results presented above indicate that relative sea-level has a stronger impact on stratigraphic architecture than sediment supply, because the stratigraphic architecture is more sensitive to changes in relative sea-level than in sediment supply. Because of the large uncertainty in the inferred sediment-supply curve (Fig. 4.7B), there is no strong relationship between this parameter and major changes in stratigraphic architecture. Consequently, the model is then less sensitive to sediment supply than to relative sea level, and so the former parameter is harder to extract uniquely from the observed data.

4.4.3.3 Prediction of Grain-Size and Bed Thickness Distribution on Cross-Section

From the posterior probability distributions of the relative sea-level and sediment-supply parameters, grain size and bed thickness can be predicted along a stratigraphic cross-section by running a BARSIM forward simulation that is conditioned to an extracted set of inverted relative sea-level and sediment-flux parameters. To validate the numerical scheme, the best inverted curves have been used to create a single realisation of a BARSIM cross-section (Fig. 4.8), which can be compared with the input synthetic model (Figs. 4.1A, 4.8).

The gross stratigraphic architecture of two stacked shallow marine sandstone tongues separated by a flooding surface is successfully recreated. However, the total grain-size distribution of the inverted cross-section (Figs. 4.8B, 4.8C) is slightly coarser than that of the original BARSIM cross-section (Figs. 4.1A, 4.8A, 4.8C). Two other differences between the two cross-sections are also noted. Firstly, transgressive deposits are slightly thicker and more laterally extensive, implying a higher degree of preservation, in the best inverted cross-section (Fig. 4.8B). The second main difference is related to the stratigraphic architecture of the seaward pinch-outs of the two tongues, which occur in a more landward position in the best inverted cross-section (Fig 4.8B).

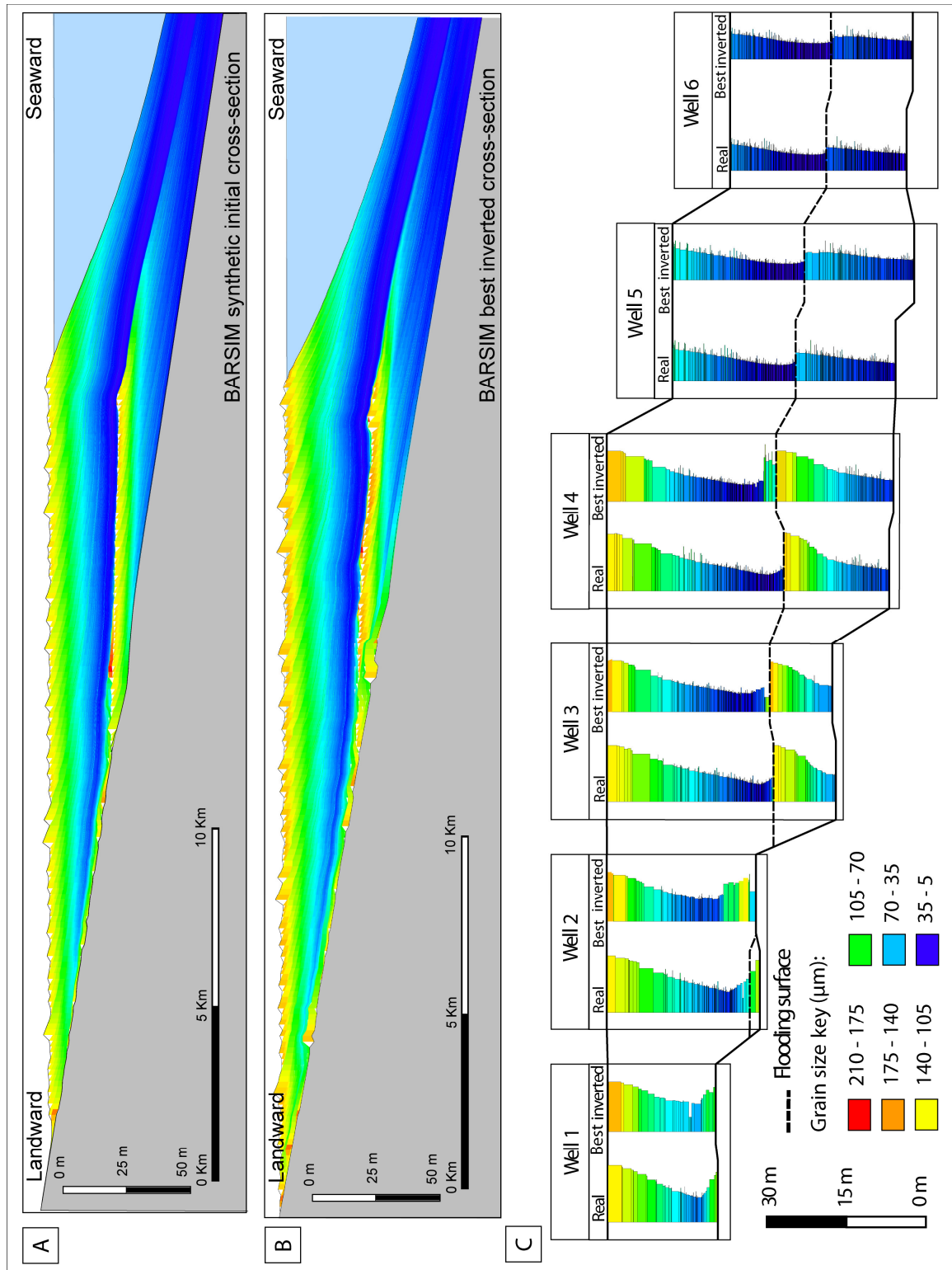


Figure 4.8: Comparison between (A) the input BARSIM cross-section and (B) the inverted BARSIM cross-section, for scenario A. The inverted cross-section is constructed from the best

inferred sediment-supply and relative sea-level curves. The gross stratigraphic architectures in both cross-sections are similar, but differences occur in the thickness and lateral distribution of transgressive deposits and in the position of the seaward pinch-out of each regressive tongue. (C) Comparison between input data (left) and mean inverted data (right) for each well (Fig. 4.2A). The inverse method is able to recreate the grain-size and thickness trends in each well, and the location of flooding surfaces.

This discrepancy is accounted for by the slightly higher value of relative sea level at the end of each period of regression in the inverted model (at c. 20,000 and 50,000 years of simulated time; Fig. 4.4C), which forced regression into more landward locations than in the corresponding initial model.

The grain-size distribution is then predicted according to the geological processes active within wave-dominated systems and conditioned to input data at high-resolution, intra-parasequence scales (Hampson & Storms, 2003; Storms & Hampson, 2005). Grain-size distribution controls porosity and permeability (Beard & Weyl, 1973), and the prediction of these rock properties in inter-well volumes is a principal application of conventional geostatistical techniques in petroleum reservoir characterisation, but high-resolution heterogeneities are generally not incorporated into these approaches. Prediction of uncertainty in grain-size and bed-thickness distributions is therefore also a key feature in petroleum reservoir characterisation and reservoir simulation studies, which often require multiple realisations for flow models.

These distributions can be evaluated by extracting several relative sea level and sediment supply curves from posterior distribution in order to run multiple BARSIM realisations. In principle, various models could be considered, either directly from the MCMC samples, or from the posterior distributions. Figure 4.9B shows the BARSIM realisation calibrated by the mean inverted curves. Figures 4.9C and 4.9D show the two most extreme stratigraphic architectures contained in the 95% Bayesian credible interval envelope, and the figures correspond to the models created respectively by the lower and upper boundaries of the Bayesian credible interval of the sediment flux and relative sea-level distributions. In both cases, the two aggradationally stacked sandstone tongues are recreated. However, in comparison to the input model (Fig. 4.9A), the two cross-sections present significant

differences in (1) the global grain-size distributions, including the volume of sand, (2) the down-dip extent and thickness of each progradational sandbody and (3) the stratigraphic architecture of transgressive deposits. It is important to note that these models have a low probability of existence. They were calculated from the tails of the posterior distributions and do not represent individuals models obtained directly by the inverse algorithm.

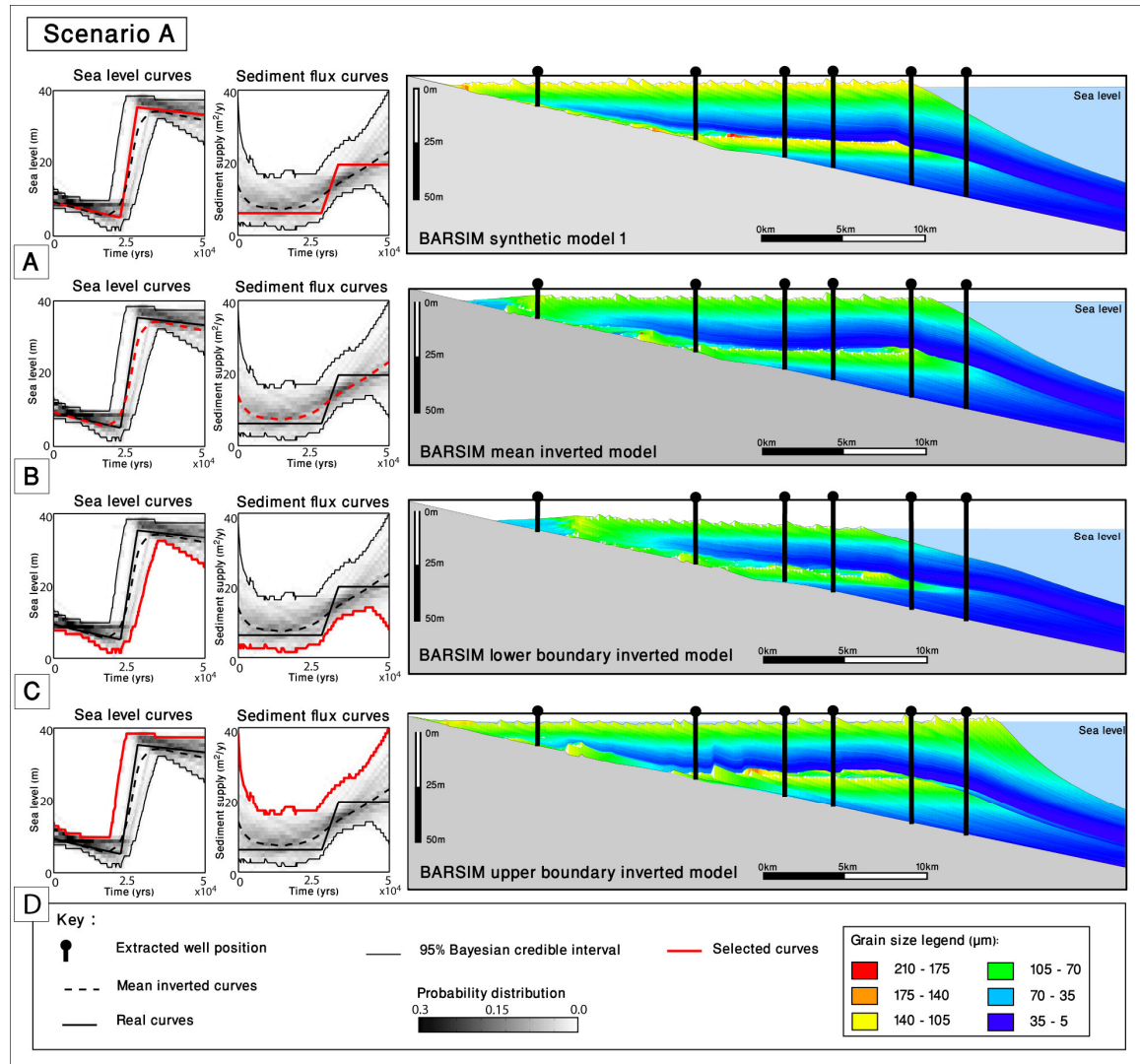


Figure 4.9: Facies distribution assessment for scenario A. Comparison between (A) the original BARSIM cross-section, (B) the inverted BARSIM cross-section constructed from the mean inferred sediment-supply and relative-sea-level curves, (C) the inverted BARSIM cross-section constructed from the lower boundary of Bayesian credible interval of the sediment-

supply and relative sea-level posterior distributions and (D) the inverted BARSIM cross-section constructed from the upper boundary of Bayesian credible interval of the sediment-supply and relative sea-level posterior distributions. The gross stratigraphic architectures in all cross-sections are similar, but differences occur in the volume of sand, the position of the seaward pinch-out of each regressive tongue, and the thickness and lateral distribution of transgressive deposits.

4.4.4 Scenario B

The results of scenario B (Fig. 4.2B) are inferred from the same observed dataset as scenario A, but with synthetic measurement errors that are twice as large.

4.4.4.1 Posterior Distributions of the Relative Sea Level

The high probability area of the relative sea-level posterior distribution, mapped from the sampling, matches the original relative sea-level curve (Fig. 4.10A). As expected intuitively, the spread of the 95% Bayesian credible interval is wider than in scenario A ($\pm 5\text{m}$ to $\pm 10\text{m}$). However, there is no simple quantitative relationship between measurement errors and errors inferred for relative sea level parameters. As a consequence of the increased uncertainty, the mean relative sea-level inverted curve contains a slower rise than the corresponding original curve or that inferred for scenario A.

4.4.4.2 Posterior Distribution of the Sediment Supply Curve

The spread in the posterior probability distribution of the sediment supply curve (Fig. 4.10B) is also greater ($\pm 8\text{m}$ to $\pm 20\text{m}$). It is now difficult to recognize the shape of the original curve from the sampled posterior probability distribution. Quantification of the sediment supply regime with low uncertainty also becomes harder.

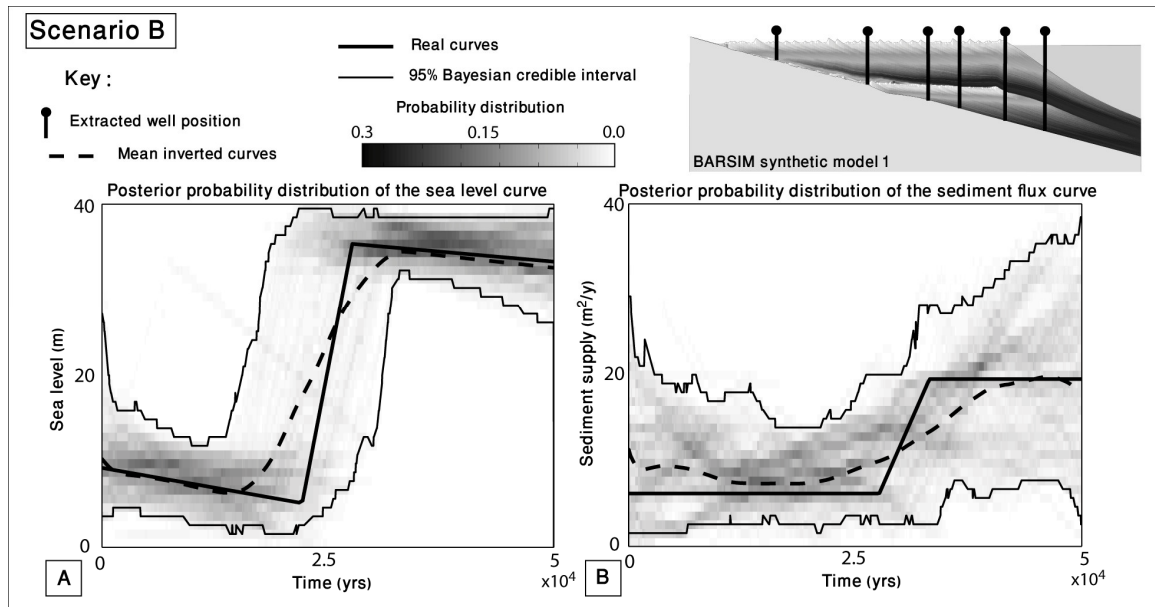


Figure 4.10: Posterior probability distributions of (A) relative sea-level and (B) sediment-supply curves obtained for scenario B. The dark continuous line is the input curve, the dashed line is the mean inverted curve, and the light grey curves represent the boundaries of the 95% credible interval. The background shading represents the posterior probability distribution (dark means high probability). Uncertainty is greater than in scenario A (Fig. 4.7).

Larger measurement errors on observed data increase uncertainty on the inferred environmental parameters by decreasing the sensitivity of the predictions to the model parameters. As a consequence, the low-amplitude, high-frequency signals of the curves tend to not be sampled. This reflects the natural tendency of the Bayesian approach to prefer simple models over complex ones, if the data do not justify the increased complexity.

4.4.5 Scenario C

Scenario C was run to test the impact of the quantity of observed well data on the stratigraphic inversion in the case where the dataset still covers the entire stratigraphic cross-section (Fig. 4.2C).

4.4.5.1 Posterior Distribution of the Relative Sea Level Curve

The result of the inverted relative sea-level history (Fig. 4.11A) provides a close, well-constrained match to the original relative sea level curve despite using a smaller number of observed wells than in scenario A. The Bayesian credible interval shows a relatively narrow spread (from $\pm 2\text{m}$ to $\pm 5\text{m}$) over most of the relative sea-level curve, but is wider ($\pm 10\text{m}$) for the earliest part of the first relative sea-level fall (from 0 to 10,000 yrs of simulated time). This can be explained by the lack of well data in the landward part of the oldest sandstone tongues, which corresponds to the geographical area where the earliest phase of sedimentation occurred (Figs. 4.11A. and 4.2C). The earliest part of the second relative sea-level fall (from 25,000 to 30,000 yrs of simulated time, Fig. 4.11A) has less uncertainty, because of the constraint provided by the most landward well (Fig. 4.2C), which contains the earliest deposits of the second tongue.

4.4.5.2 Posterior Distribution of the Sediment Supply Curve

The results of the inferred sediment supply curve (Fig. 4.11B) are similar to the results for scenario A (Fig. 4.7B): the area of high probability has the same shape and the same spread. However, the uncertainty is bigger (up to $\pm 20\text{m}^2/\text{yr}$) for the first 10,000 years of simulated time because of the lack of well-data containing corresponding deposits.

These results show that the number of observed wells is not crucial in constraining the stratigraphic inversion, as long as they capture a representative cross-section of the stratigraphic architecture. However, these results also show that observed data distribution has an impact on the quality of inversion, because the location of well data is closely related to the time interval(s) that they record.

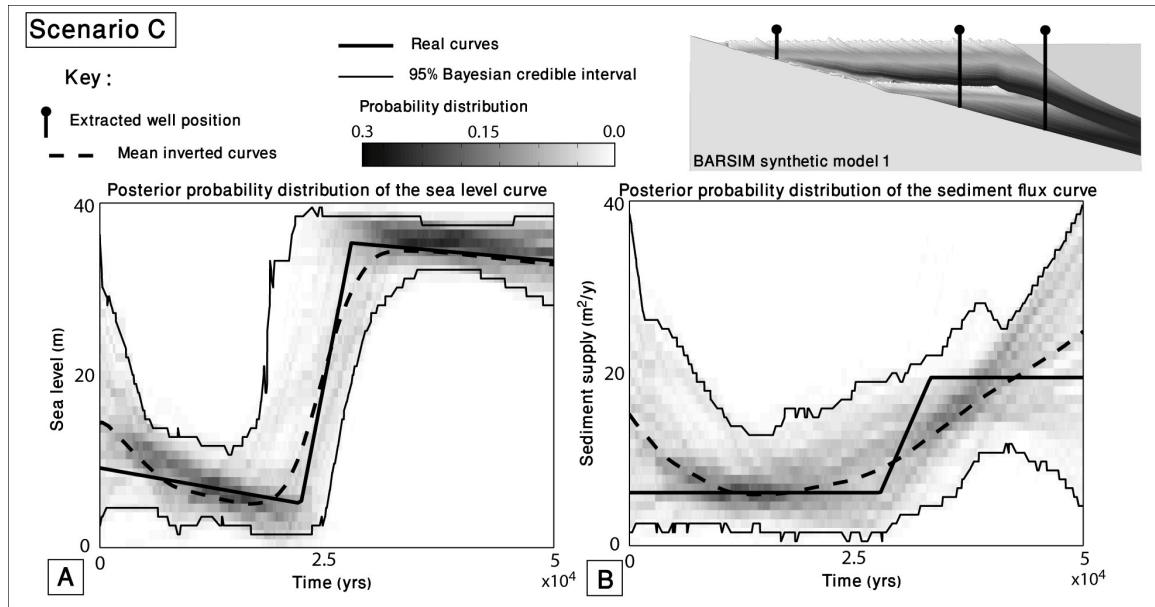


Figure 4.11: Posterior probability distributions of (A) relative sea-level and (B) sediment-supply curves obtained for scenario C. The dark continuous line is the input curve, the dashed line is the mean inverted curve, and the light grey curves represent the boundaries of the 95% credible interval. The background shading represents the posterior probability distribution (dark means high probability). The distributions are similar to scenario A (Fig. 4.7), and only the first 10,000 years of simulated time exhibits greater uncertainty.

4.4.6 Scenario D

Scenario D has been designed to test the impact of observed data location in the case where they are situated only in the landward stratigraphy of the cross-section (Fig. 4.2D) and where no significant prior information was available to constrain the seaward deposits. Figure 4.12 illustrates the posterior probability distribution of the relative sea-level (Fig. 4.12 A) and sediment-supply (Fig. 4.12 B) curves. Both of the sampled distributions display very different patterns to those that result when observed data covering the full extent of the stratigraphic cross-section (scenarios A-C; Figs. 4.7, 4.10, 4.11).

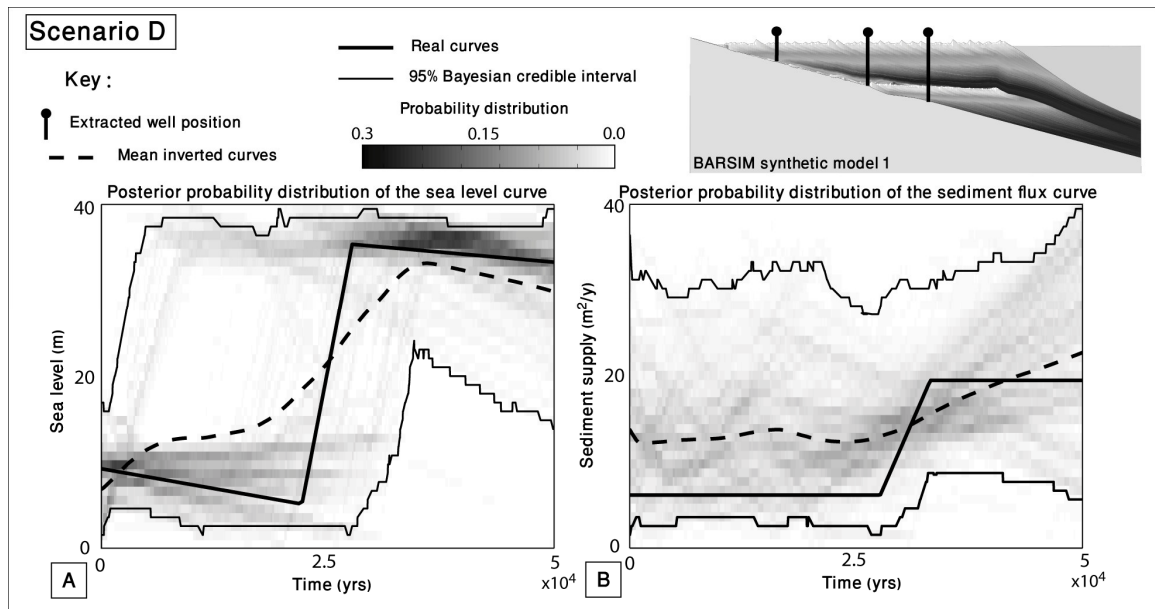


Figure 4.12: Posterior probability distributions of (A) relative sea-level and (B) sediment-supply curves obtained for scenario D. The dark continuous line is the input curve, the dashed line is the mean inverted curve, and the light grey curves represent the boundaries of the 95% credible interval. The background shading represents the posterior probability distribution (dark means high probability). The distributions are very different to those for scenario A, B and C (Figs. 4.7, 4.10, 4.11), and result from the mixing of several local-solution distributions. For scenario D, the solution of the stratigraphic inverse problem is not unique.

4.4.6.1 Posterior Distribution of the Relative Sea Level Curve

The results of the inverted relative sea-level curve (Fig. 4.12A) do not provide a close match with the original relative sea-level history. In detail, the mean inverted curve fits poorly to the original parameters. The spread of the 95% Bayesian credible interval is wide and its shape does not follow the shape of the original curve: the amplitude of the credible interval is about $\pm 35\text{m}$ for the first 30,000 years of simulated time and decreases to about $\pm 20\text{m}$ until the end of the simulation.

Given these reservations, the areas of high probability (darker areas in Fig. 4.12A) within the Bayesian credible interval show three interesting features. Firstly, from 0 to 25,000 years of simulated time, the posterior probability distribution shows an area of high probability following the original relative sea-level curve (i.e. low values of relative sea-level). The probabilities in this area decrease (the area becomes lighter in Fig. 4.12A) and its spread increases with increasing time. Secondly, from 25,000 to 50,000 years of simulated time, the posterior probability distribution contains another area of high probability, corresponding to high values of relative sea level. This area is not restricted to the period of time between 25,000 and 50,000 years of simulated time, but also extends back to 10,000 years, although the probability decreases (the area becomes lighter in Fig. 10A) for progressively earlier times. Thirdly, the posterior probability distribution shows several narrow areas of higher probability that are sub-parallel to the rising limb of original relative sea-level curve, but that occur between 2,500 and 30,000 years of simulated time.

The posterior probability distribution of the relative sea level curve presented in Figure 10A is, then, the result of a mixing of several distributions. Each of these distributions represents a local solution of the stratigraphic inverse problem (i.e. different possibilities explaining the same observed data). This is an example of a multimodal solution space, which the simulated tempering is specifically designed to sample. However, all the local solutions curves have two parts in common: (1) the low values of relative sea level (7m) between 0 and 5,000 years of simulated time, and (2) the high values of relative sea-level (35m) between 30,000 and 40,000 years. These common components of the local solution curves explain the high probability areas described above for these time-periods, and their corresponding geographical positions on the stratigraphic cross-sections correspond to the location of well data that constrain the inversion. The multi-modality of the posterior probability distribution also explains why the mean inverted curve does not fit the real curve. This central estimator is really only useful for a distribution with a single mode. For multimodal solutions, it is possible to use some form of clustering algorithm in high-dimensional space to identify similar models, and then examine the predictions from each cluster. Such a method was proposed for reservoir simulation by Ballester & Carter (2004).

4.4.6.2 Posterior Distribution of the Sediment Supply Curve

The results of the inverted sediment flux history (Fig. 4.12B) do not show, again, a match with the original sediment supply parameters. The spread of the 95% Bayesian credible interval is wide (from $\pm 15 \text{ m}^2/\text{y}$ to $20 \pm \text{m}^2/\text{y}$) and the shape of the mean curve does not follow the shape of the true curve. It is clearly difficult to extract a representative sediment supply history with confidence from the obtained posterior probability distribution. In contrast to the relative sea-level results, this distribution does not exhibit clear areas of high probability, although it is also the result of the mixing of several local solutions. The lack of clear areas of high probability is consistent with the generally lower sensitivity of the inverse model to sediment-supply parameters, which results in wide sample distributions that lack strong peaks.

4.4.6.3 Prediction of Grain-Size and Bed Thickness Distributions on Cross-Section

To examine the hypothesis of non-uniqueness of the inverse problem in scenario D, distinct local solutions have been selected from their respective posterior probability distributions. Relative sea-level and sediment-supply curves for these local solutions have been used to run three BARSIM forward simulations, in order to compare the resulting stratigraphic cross-sections (Figs. 4.13B-D) to the input cross-section (Fig. 4.13A). The three cross-sections for the local solutions show fundamental differences in stratigraphic architecture.

The cross-section for the first local solution (Fig. 4.13B) contains only one sandstone tongue. The lower part of the cross-section instead comprises a transgressive sandbody, which was deposited during an initial phase of relative sea-level rise from 0 to 10,000 years of simulated time. The upper part of the cross-section comprises a thick and laterally extensive tongue that corresponds to a relative sea-level fall from 10,000 to 50,000 years.

The cross-section for the second local solution (Fig. 4.13C) contains two aggradationally stacked sandstone tongues, each deposited during a relative sea-level fall. The two tongues are separated by a flooding surface and a thin, underlying transgressive sandbody that record deposition during a relative sea-level rise. Despite its gross similarity to the input model (Fig. 4.13A), the stratigraphic architecture of the second local solution contains differences in the thickness and extent of the two shallow-marine sandstone tongues, which reflect a relatively high inferred sediment-supply regime.

The cross-section for the third local solution (Fig. 4.13D) also shows two stacked sandstone tongues separated by a flooding surface, reflecting two periods of relative sea-level fall and an intervening relative sea-level rise. The two tongues are retrogradationally stacked, which reflects high sediment supply during the first 25,000 years of simulated time and low sediment supply during the last 25,000 years, relative to the input sediment-supply curve (Fig. 4.13A).

Despite the differences described above, the cross-sections for the three local solutions all contain two aggradationally stacked sandbodies separated by an interval of shale in the area constrained by the observed data (Fig. 4.13). This observation confirms the non-uniqueness of this stratigraphic inverse problem, because all three solutions explain the observed stratigraphic data. This example shows that apparent uniqueness in stratigraphy clearly does not imply uniqueness of solution of the stratigraphic inverse problem. In this case, the non-uniqueness of the inverse problem is due to the lack of constraints coming from the particular locations of the observations, and the non-linearity of the problem. In addition, the example shows that the inversion procedure can be feasibly applied to stratigraphy, even in the case of non-uniqueness, if an appropriate inverse method is used. Scenario D also validates the robustness of our Hybrid sampler: all the possible solutions of the inverse problem have been sampled, and the algorithm did not become “stuck” in a local minimum.

It is important to appreciate that the inverse stratigraphic algorithm in general will not be able to predict uniquely the true solution (in this case, the stratigraphy occurring in the seaward part of the cross-section), particularly without data to constrain it. I believe, however, that it is a powerful tool to assess the full range of possible stratigraphic architectures for a given dataset.

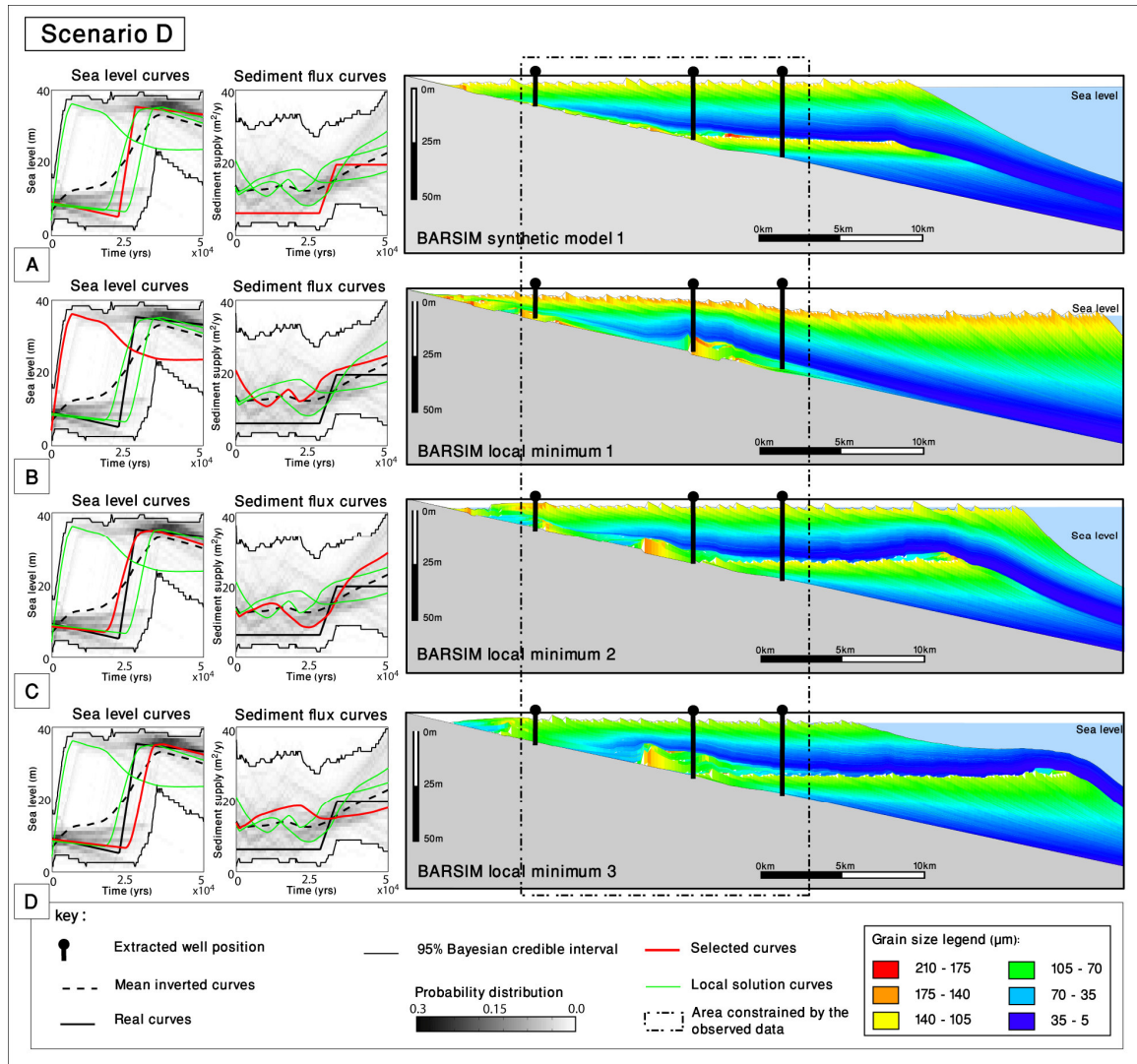


Figure 4.13: Facies distribution assessment for scenario D. Comparison between (A) the original BARSIM cross-section and (B, C, D) three BARSIM cross-sections constructed from local solutions of the relative sea-level and sediment-supply curves. The relative sea-level and sediment-supply curves for each local solution are highlighted in red for the corresponding cross-sections. The gross stratigraphic architectures for each of these local solutions are very different, but contain similar vertical successions in the areas constrained by observed well data.

4.4.7 Scenario E

Scenario E has been designed to test the impact of observed data distribution in the case where they are situated in the seaward part of the cross-section (Fig. 4.2E) and where no significant prior information has been specified to constrain the landward stratigraphy of the cross-section. Figure 4.14 illustrates the results for the posterior probability distributions of the relative sea-level (Fig. 4.14A) and sediment-supply (Fig. 4.14B) curves. Both of the posterior distributions sampled exhibit different patterns to those for scenarios A, B, C and D (Figs. 4.7, 4.10, 4.11, 4.12).

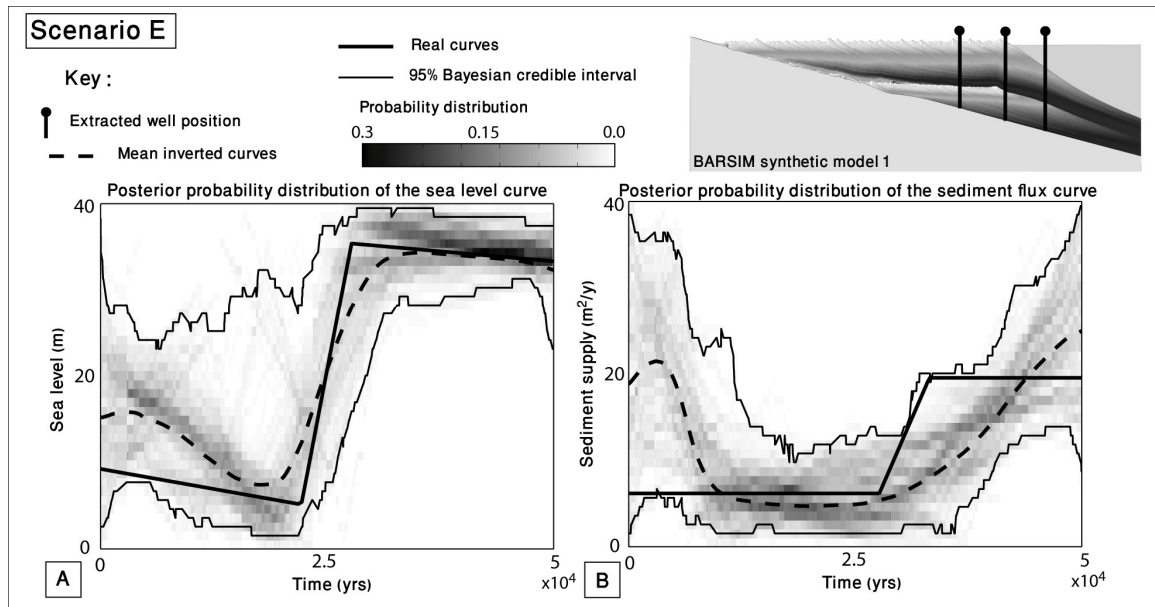


Figure 4.14: Posterior probability distributions of (A) relative sea-level and (B) sediment-supply curves obtained for scenario E. The dark continuous line is the input curve, the dashed line is the mean inverted curve, and the light gray curves represent the boundaries of the 95% credible interval. The background shading represents the posterior probability distribution (dark means high probability). The distributions are different to those for scenarios A, B, C and D (Figs. 4.7, 4.10, 4.11, 4.12), and they reflect mixing of several local-solution distributions. The solution of the stratigraphic inverse problem is not unique, but the seaward position of well data in scenario E implicitly constrains the landward stratigraphy and, thus, limits the range of possible solutions, relative to scenario D (Fig. 4.12).

4.4.7.1 Posterior Distribution of the Relative Sea Level Curve

The posterior probability distribution of the relative sea-level curve (Fig. 4.14A) is complex. The spread of the 95% Bayesian credible interval is wide (± 15 m) for the first 20,000 years of simulated time and it becomes narrower (± 5 m) after 20,000 years, similar to the distribution for scenario A (Fig. 4.2A). The mean inverted curve does not provide a close match to the original relative sea-level curve for the first 20,000 years of simulated time, but it fits relatively well from 20,000 to 50,000 years.

4.4.7.2 Posterior Distribution of the Sediment Supply Curve

The results for the sediment-supply curve (Fig. 4.14) are similar to the results of the relative sea-level curve. For the first 15,000 years of simulated time, the spread of the Bayesian credible interval is wide (up to ± 20 m²/yr) and the mean inverted curve does not match the original curve. However, from 15,000 to 50,000 years of simulated time, the spread of the posterior probability distribution becomes narrower and exhibits a similar pattern to the one obtained in scenario A (Fig. 4.2B), in which inversion was tightly constrained.

As in scenario D, these results illustrate the non-uniqueness of solutions of the stratigraphic inverse problem. In this case, the inference of the relative sea-level and sediment-supply parameters is well constrained for the last 30,000 years of simulated time and multiple acceptable solutions are possible for the first 20,000 years. However, in comparison to scenario D, the range of possible solutions is smaller, as reflected in the narrower spread of the Bayesian credible interval for both relative sea-level and sediment-supply curves (Figs. 4.12, 4.14). This narrower range of possible solutions is explained by the seaward position of the observed data, which provides an implicit constraint on shallow-marine stratigraphy in a landward direction: different stratigraphic architectures can occur in landward locations, provided that they do not greatly impact the seaward stratigraphy, which is constrained by the observed data.

4.4.7.3 Prediction of Grain-Size and Bed Thickness Distributions on Cross-Section

To examine these hypotheses further, the mean inverted curves and those for one of the local solutions have been selected from the posterior probability distributions and then used to run two BARSIM forward simulations, in order to compare the resulting stratigraphic cross-sections (Figs. 4.15B, C) to the original cross-section (Fig. 4.15A). There are significant differences between the three cross-sections.

The mean inverted model (Fig. 4.15B) contains two aggradationally stacked shallow marine sandstone tongues, as does the original model (Fig. 4.15A). However, the lower sandstone tongue differs in thickness and extent relative to the input model, and thin transgressive intervals are preserved at both its landward and seaward pinch-outs.

The local solution model (Fig. 4.15C) also contains two aggradationally stacked shallow marine sandstone tongues, but the lower one has a complex internal architecture. The older, more landward part of the tongue is thick and of limited lateral extent, reflecting deposition under rising relative sea level and high sediment supply during the first 10,000 years of simulated time. The younger, more seaward part of the tongue is thin and laterally extensive with a downstepping, “forced regressive” internal architecture that reflects a rapid and abrupt relative sea-level fall between 10,000 and 15,000 years of simulated time. The upper shoreface tongue overlies a flooding surface that formed during a relative sea-level rise between 20,000 and 30,000 years of simulated time. Despite their differences, the mean inverted and local-solution cross-sections (Figs. 4.15B, C) both contain two aggradationally stacked sandbodies separated by an interval of shale in the area constrained by the observed data.

This last scenario confirms the non-uniqueness of the inverse stratigraphic solution, despite the greater constraint offered by seaward-lying observed data. Different facies architectures can be generated during the earliest phase of deposition, provided that the resulting stratigraphy occurs landward of the observed data. The seaward position of the observed data indirectly constrains the maximum amplitude of relative sea-level and sediment-supply variations. In order to tightly constrain the inverse problem, data should be preferentially collected/sampled (1) in the extremities of the stratigraphic interval, so that they constrain the maximum

amplitude of the environmental parameters (e.g. relative sea-level and sediment supply), and (2) in locations where the data capture the maximum information about the history of the sedimentary system. Once these conditions are satisfied, then the amount of additional observed data is likely to become less critical.

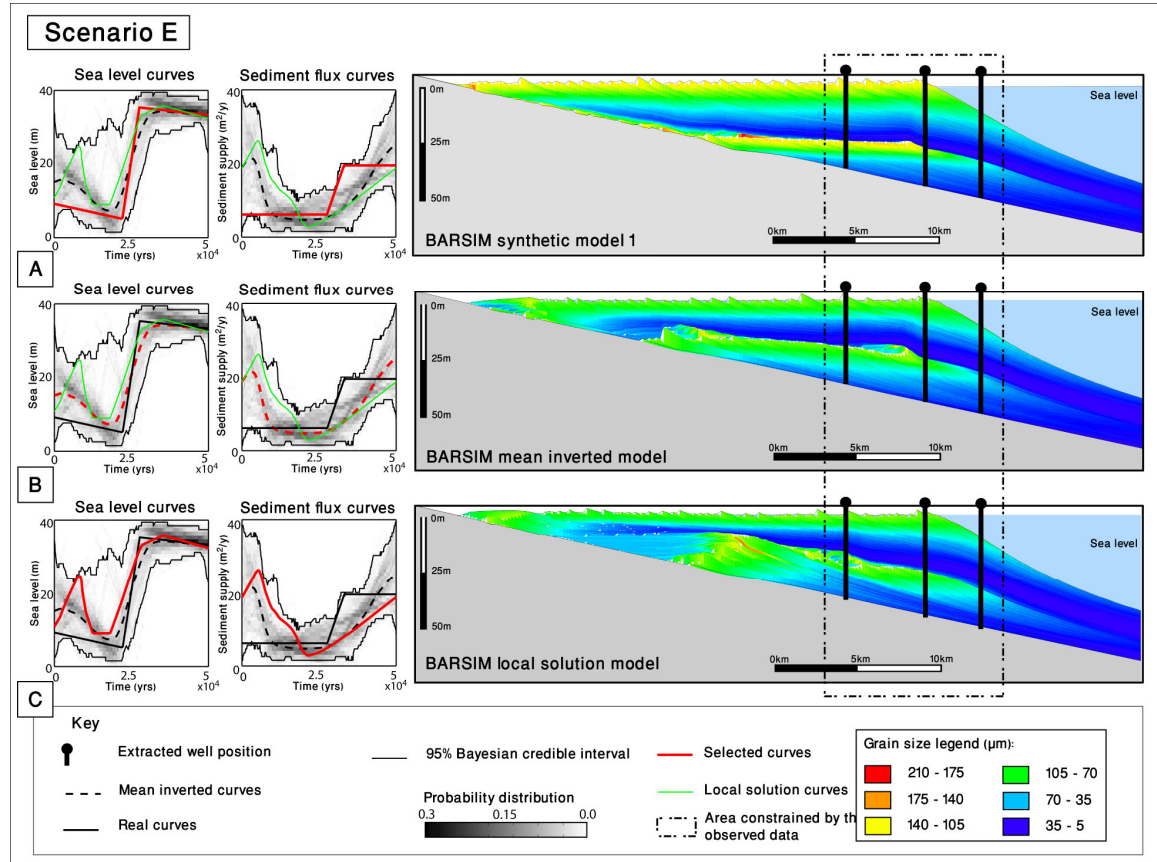


Figure 4.15: Facies distribution assessment for scenario E. Comparison between (A) the original BARSIM cross-section, (B) the BARSIM cross-section constructed from the mean inferred relative sea-level and sediment-supply curves, and (C) a BARSIM cross-section constructed from sediment-supply and relative-sea-level curves for a local solution. The gross stratigraphic architectures differ, but contain similar vertical successions in the areas constrained by observed well data. The seaward position of the wells implicitly constrains the landward stratigraphy, and limits the number of local inverse solutions.

4.5 Discussion

The results of the case study outlined above show that the fully non-linear inverse method combined with a simple 2D “process-response” forward model can be used to infer environmental input parameters (in this case, sediment supply and relative sea level) and stratigraphic architecture from limited observed data. This synthetic case study validates the method and suggests that the inverse scheme may be successfully applied to real datasets, if the latter are selected with due care for the assumptions and simplifications of the underlying forward stratigraphic model. In fact, the utility of stratigraphic inversion is clearly directly dependant on the ability of the forward model to simulate reality. In this chapter, I avoided these issues and assumed that BARSIM was a perfect simulator by creating a “pseudo-reality” from one BARSIM forward simulation in which all the input parameters were known.

The major advantages of this approach are that the inverse scheme is not only able to recreate the known solutions of environmental input parameters, but can also provide full characterisation of uncertainty, an assessment of the model sensitivity to the environmental parameters and an assessment of non-uniqueness of the inverse problem by sampling the full range of possible solutions. Furthermore, the RJMCMC approach lets us consider the issue of model complexity (e.g. the number of free parameters required to model the data) as part of the modelling process.

By adopting a Simulated Tempering sampler, the risk of becoming trapped at a local mode of the posterior distribution is significantly reduced. A small range of temperatures increases the risk of the inversion becoming “stuck” at a local mode, while a big range of temperatures decreases this risk and allows more representative sampling. However, a large number of temperature levels will increase the computational costs and a small number will necessarily increase the temperature difference between levels. In order to achieve smooth temperature transition (i.e. good mixing in the temperature space), the temperatures have to be closely spaced so as to ensure that the distributions of two adjacent temperature levels have sufficient overlap. These computational time and mixing issues can be solved by using a parallel version of Simulated Tempering proposed by Geyer (1991), called Metropolis-Coupled Markov Chain Monte Carlo (MCMCMC). This algorithm runs several Markov chains, with their respective

temperatures corresponding to the number of nodes available on the computer cluster (one chain per node). Mixing between different temperature levels is carried out by proposing a switch of the model parameters between chains of different temperatures after every n iterations and by accepting (or rejecting) each proposal according to the Metropolis rule.

The length of the burn-in period also imposes a computational cost. It is clear that geological prior information is crucial in term of computational cost and inversion quality. In the case study outlined above, prior knowledge was set to be minimal, in order to push the method to its limit, and the first model (i.e. the first step of the Markov chain) was far from the solution area. By defining more prescriptive prior knowledge, it is possible to considerably shorten the burn-in period and to increase the sampling efficiency, as the size of the model space is effectively reduced. However, prior knowledge that is too prescriptive and incorrect can lead the inversion process to sample only partially or not at all in the correct solution area of the model space.

The used of RJMCMC samplers has proved to be efficient in terms of computational costs. This trans-dimensional approach allows the model choice issue to be solved efficiently where the number of dimensions is unknown. Because the Bayesian approach is naturally parsimonious, the inverse scheme will prefer simpler models, focussing the number of parameters towards the minimum acceptable value, and thereby reducing computational costs. As shown by Sambridge *et al.* (2006), it is possible to run a sequence of fixed dimensional MCMC models, and then sample their results. This approach yields results equivalent to the RJMCMC approach but requires more computational effort.

Some other important features emerge from the results presented here. Firstly, the inverse method is able to reproduce the sediment supply and relative sea-level curves from the observed data and provides a direct assessment of uncertainty on these curves. Secondly, the validation tests presented here shows that the inverse method allows the evaluation of model sensitivity to the various environmental input parameters. For example, in this case study, relative sea-level has a stronger impact on stratigraphic architecture than sediment supply, and its signal was then easier to extract from the observed data, with the result that its posterior probability distribution was tightly constrained. Uncertainty in facies architecture (e.g. grain-size and bed-thickness distributions) can also be approached by sampling different models

from the Bayesian credible interval. The numerical experiments demonstrate that uncertainties in the inferred results are directly dependent on the magnitude of measurement errors of the observed data. However, there is no linear relationship linking measurement-error magnitude and resulting uncertainty. High-frequency, low-amplitude signals tend not to be sampled if measurement errors are large; as a result, the quality and precision of the inferred parameters are also directly linked to the quality of the input data, as might be expected intuitively.

By testing different scenarios of data distribution, quantity and quality, our inverse scheme is proved to be robust in dealing with complex multi-modal posterior probability distributions (i.e. in dealing with non-unique solutions of the inverse problem), in contrast to previous work (Cross & Lessenger, 1999; Bornholdt *et al.*, 1999; Wijns *et al.*, 2004). Furthermore, these tests demonstrate that the solution of the inverse problem may not be unique, even though the investigated stratigraphy may be the result of a unique combination of processes and parameters. The non-uniqueness of the inverse problem can result from differences in data quality, quantity and, most importantly, distribution. Observed data distribution is shown to be crucial to the quality of the inversion. In scenarios where data are clustered in only part of the geographical domain (e.g. scenarios D and E above), multiple local solutions of the inverse problem are possible. However, the ranges of possible solutions of the inverse problems differ, reflecting constraints that are implicit in the location of the input data. For example, in the wave-dominated systems studied above, seaward-located data (scenario E) provide a tighter constraint on the maximum amplitude of relative sea-level and sediment-supply variations, and resulting facies architectures, than landward-located data (scenario D). It is also clear that the use of prior knowledge about relative sea-level and sediment-supply parameters can provide important additional constraints on the inversion process, by reducing the number of possible local solutions (e.g. the upper and lower limits of relative sea level and sediment supply values are constrained between 0-40 m and 0-40 m²/yr, respectively, in scenarios A-E). However, once again, prior knowledge should be carefully chosen, because it limits sampling of the posterior distribution in our inversion scheme. If the assumed level of prior knowledge is too restrictive, or if the prior knowledge is inappropriate, then this limits the range of inverse solutions that can be recovered from the input data: thus, a sound knowledge of geological processes is needed to maximise the efficiency of the inversion process.

The validation tests described above also emphasise that data quantity is not the most important criterion in constraining stratigraphic inversion, provided that the available data capture as much information as possible about the complexity and history of the depositional system under investigation. Such information about complexity is necessary, because the Bayesian approach is naturally parsimonious (Malinverno, 2002) and tends to produce simpler inverse solutions that fit only the available data. This last point represents the main limitation of our inversion method, and requires that input data should be selected carefully in order to incorporate the level of complexity (or heterogeneity) required in the inverse solution.

The process-based approach adopted here automatically generates high-resolution features of the facies architecture that are inherent to the depositional system (e.g. clinoforms within regressive sandbodies, transgressive sandbodies of varying thickness and extent), in contrast to conventional geostatistical tools where such features must be recognised and quantified *a priori* by the user. Such features may have an important impact on fluid flow and recovery in some hydrocarbon reservoirs. The approach is therefore a useful tool for both academic research and industrial applications, as it can also help in the design of data acquisition strategies to better constrain the evolution of a stratigraphic system.

4.6 Summary

In this chapter, I have presented validating tests of the stratigraphic inverse and forward modelling scheme described in chapter 2 and 3. These tests consisted of the inference of the sediment supply and relative sea level parameters from synthetic data in which all the environmental parameters are known. These preliminary numerical experiments highlight several important points:

1. The Bayesian formulation of the inverse scheme not only allows the inference of the original relative sea-level and sediment-supply curves from the observed data, but also allows the full assessment of uncertainty relative to these parameters.

2. By providing a full assessment of uncertainty on the input parameters, the inverse method is also able to assess the sensitivity of these parameters to the model formulation. This information can be used to design future data acquisition strategies at low cost that specifically reduce uncertainty in later model predictions.
3. The use of RJMCMC samplers is significant in that it enables a trans-dimensional approach that preferentially avoids over-complicated models, thereby reducing computational costs.
4. The use of the Simulated Tempering sampler considerably improves the robustness of the inverse scheme, because it allows a complete exploration of the parameter space in order to fully sample (1) the posterior probability distribution of the input parameters and (2) all possible solutions of the inverse problem.
5. The numerical experiments demonstrate that uncertainties in the inferred results are directly dependant on the magnitude measurement errors of the observed data, although there is no linear relationship linking measurement-error magnitude and resulting uncertainty.
6. Tests also demonstrate that the solution of the inverse problem may not be unique, even though the investigated stratigraphy may be the result of a unique combination of processes and parameters. The non-uniqueness of the inverse problem can result from differences in data quality, quantity and, most importantly, distribution.
7. Prior knowledge about the environmental input parameters can provide important additional constraints on the inversion process, by reducing the number of possible local solutions and the burn-in period. However prior knowledge should not be chosen to be too restrictive as it can lead the inversion process to sample only partially or not at all in the preferred solution area of the model space.
8. Grain-size and bed-thickness distribution can be predicted with at high-resolution spatial and temporal scales while honouring (1) understanding of the geological

processes active within wave-dominated systems and (2) the input data. Uncertainty assessment on these distributions can be performed by extracting several relative sea-level and sediment-supply curves from the Bayesian 95% credible interval in order to run multiple forward model realisations. These results show that inverse stratigraphic modelling is potentially a powerful tool that can complement and extend the use of conventional geostatistical methods in the building of petroleum reservoir models.

The results of the validation tests suggest that the inverse scheme can be expanded to investigate all the input environmental parameters needed to calibrate BARSIM and also apply it to real datasets, if the latter are selected with due care for the assumptions and simplifications of the underlying forward stratigraphic model. The following chapters will present applications of the stratigraphic inverse and forward numerical scheme to real datasets.

Chapter 5

5 First Application: Sedimentological Documentation of the Aberdeen Member of the Blackhawk Formation, Book Cliffs, Utah, U.S.A.

Abstract

In this chapter, I will document the variability in facies character, palaeo-geomorphology and high-resolution stratigraphic architecture within a single parasequence that is interpreted to record the advance of an ancient asymmetrical wave-dominated delta and from which sedimentary driving processes will be investigated, in the following chapter, by stratigraphic forward and inverse modelling technique. More generally, this chapter will define the second main objective of this PhD thesis, which is the understanding of high resolution environmental parameters variations driving stratigraphy at intra-parasequence scale. The studied parasequence is the Standardville (Ab1) parasequence of the Aberdeen Member, Blackhawk Formation, exposed in the Book Cliffs of central Utah, USA. High-resolution stratigraphic analysis of these deposits reveals that they record a single episode of shoreline progradation characterised by alternating periods of normal regressive and forced regressive shoreline trajectory. The Standardville parasequence contains wave-dominated shoreface-shelf and fluvial-dominated delta-front deposits that are interpreted to represent respectively the updrift and downdrift flanks of a single asymmetrical wave-dominated delta. The work presented in this chapter is partly published in Charvin et al. (2008c).

5.1 Introduction

Modern wave-dominated deltaic edifices are characterised by a complex internal stratigraphy that records spatial and temporal variation in shoreline morphology and physical process regime. Shoreline morphology is shaped by the action of fluvial discharge, which introduces sediment to the system, and sediment reworking/transport by littoral currents induced by fair-weather and storm waves (e.g. Coleman & Wright, 1975; Dominguez, 1996; Bhattacharya & Giosan, 2003). The relative importance of these two processes determines whether the shoreline is linear, cusped or arcuate in plan view. Recent analysis of modern wave-dominated and wave-influenced deltas suggests that shoreline morphology can be defined with a symmetry index, expressed as the ratio of dominance of fluvial and marine influence (Bhattacharya & Giosan, 2003).

Linear shorelines and deflected deltas are formed when net alongshore sediment transport by littoral currents is sufficiently high to anneal the effect of sediment discharge at river mouths. In this context, strandplains are defined by their linear morphology and by the prevalence of reworked sediment supplied by alongshore littoral currents from the wave erosion of older shorelines. Increasing dominance of fluvial processes is expressed by protruding deltaic shorelines with lobate morphologies. Wave-dominated and wave-influenced deltas are characterised by a higher sediment discharge at the river mouth, relative to rates of wave reworking and alongshore sediment supply. Such deltas have been historically been portrayed as symmetrical in plan view (Coleman & Wright, 1975; Reading & Collinson, 1996).

Both fluvial and wave processes can be highly active, thus significant volumes of sand can be supplied by alongshore currents on the updrift flank of the wave-dominated delta. Hence, these deltas may be characterised by a pronounced alongshore variability and a marked asymmetrical morphology with distinctly different sand distributions and facies architectures on their updrift and downdrift flanks (Bhattacharya & Giosan, 2003). Modern examples (Sao Francisco delta, Brazil; Paraibo do Sol delta, Brazil; Danube delta, Romania) show that such deltas resemble strandplains and contain depositional elements like beach ridges and beach-ridge sets on their updrift flank (Bhattacharya & Giosan, 2003; Fig. 5.1). On their downdrift flank, they comprise wave-dominated spits and barriers that enclose sheltered lagoons and embayments, where

fluvial and tidal processes can be preserved from the reworking action of waves in bayhead and flood-tidal deltas (e.g. Elliott, 1986; Dominguez *et al.*, 1987; Boyd *et al.*, 1992; Bhattacharya & Giosan, 2003, Bhattacharya, 2006). Each of these elements may exhibit their own stratigraphic architecture (Fig. 5.1). The morphology of asymmetrical wave-dominated delta typically evolves quickly, over decades to tens of thousands of years, in response to autogenic and allogenic forcing of the relative dominance of fluvial and wave processes (e.g. Dominguez *et al.*, 1987; Rodriguez *et al.*, 2000).

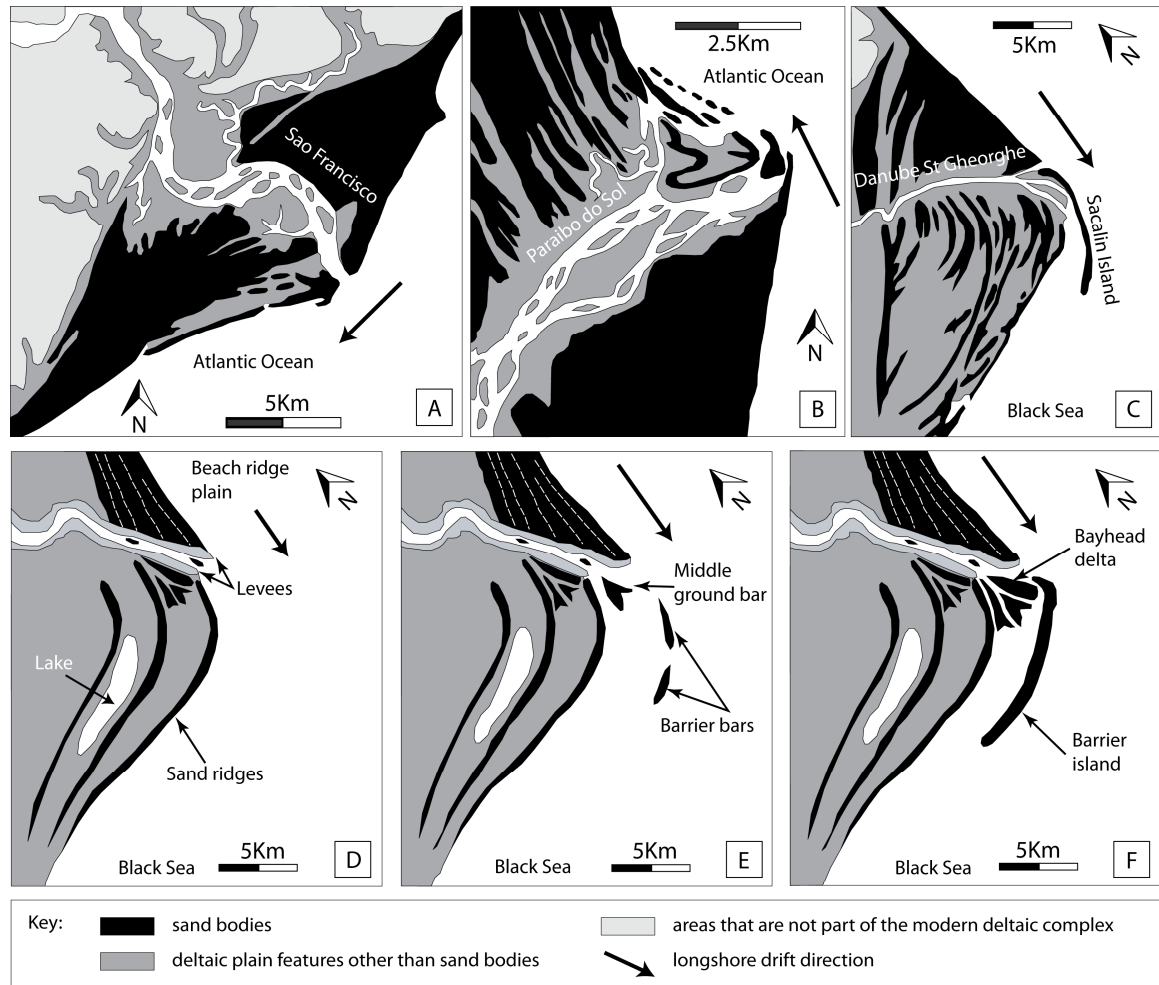


Figure 5.1: Morphology of modern asymmetric deltas/deltaic lobes. (A) Sao Francisco delta, Brazil. (B) Paraibo do Sol, Brazil. (C) Danube St Gheorghe delta, Romania (modified from Bhattacharya & Giosan, 2003). Note the presence of large sand sheets developing updrift of distributary river mouth. (D, E, F) Conceptual evolution model for the modern St Gheorghe deltaic lobe. (D) Subaqueous delta phase – sediment deposition is primarily on the

subaqueous part of the delta: the beach ridge plain on the updrift flank is also advancing. (E) Middle ground bar phase – middle-ground bar forms at the mouth forcing the distributary to bifurcate; linear barrier bars form on the subaqueous delta. (C) Barrier island phase – the linear barrier bars coalesce and become emergent to form a barrier island that rolls over to attach to the mainland; a secondary fluvial-dominated bayhead delta may develop in the sheltered lagoon behind the barrier island. Longshore drift is southward (after Bhattacharya & Giosan, 2003).

Ancient regressive, wave-dominated shallow-marine deposits are characterised by laterally extensive sandstone sheets that reflect shoreline progradation (parasequences *sensu* Van Wagoner *et al.*, 1990; Reynolds, 1999). However, their large extent and sandstone-dominated character often obscure their complex internal stratigraphy. In addition, descriptions of ancient wave-dominated deltas are often limited by sparse data distribution and therefore focus on vertical facies profiles and generalised three-dimensional facies trends. As a result, their interpretations are invariably much simpler than the complex spatial and temporal patterns described in modern systems. In vertical profile, such shorelines are commonly represented by coarsening- and shallowing-upward successions dominated by wave-induced sedimentary structures (e.g. Elliott, 1986; Walker & Plint, 1992; Clifton, 2006; Bhattacharya, 2006) or by successions reflecting a mixture of current- and wave-produced sedimentary structures in areas proximal to river outlets (Walker & Plint, 1992; Reading & Collinson, 1996; Bhattacharya, 2006). Three-dimensional facies trends associated with these vertical profiles define basinward-thinning, progradational tongues (parasequences) that are bounded by marine flooding surfaces and that represent shoreline regression over a relatively long interval (tens to hundreds of thousands of years; Van Wagoner *et al.*, 1990). Geomorphological analysis of these deposits is often conducted at parasequence scale with reference to the end members of Galloway's (1975) tripartite classification of deltas into wave-, tide-, and river-dominated systems. Although this classification has been extended to include relative changes of sea level (Dalrymple *et al.*, 1992) and variability of grain size (Orton & Reading, 1993), it lacks important information about facies variability and related sedimentary processes (e.g. Gani & Bhattacharya, 2007). Moreover, its application at parasequence scale involves considerable averaging of high-resolution (intra-parasequence) palaeogeographic trends, such that the short-term variability observed in modern examples is not resolved. Therefore, subtle variability in

wave-dominated deltaic shorelines in the past, including the occurrence of asymmetrical wave-dominated deltas, are underestimated by this methodology.

Shallow-marine sandstone tongues are commonly regarded as the “building blocks” of sequence stratigraphic interpretations (e.g. Van Wagoner *et al.*, 1990), and their stacking patterns are used for correlation and to interpret the role of various allogenic forcing mechanisms (e.g. eustasy, tectonics, climate). Again, smaller stratigraphic elements, which record shorter timescales of shoreline evolution, are rarely considered when applying sequence stratigraphic methods, although studies of modern deltas highlight high-resolution stratigraphic and geomorphological variability that impacts the larger scale, lower resolution stratigraphic architecture. Hence, the distinction between local and regional stratigraphic signals may be poorly characterised, and interpretations of stratigraphic architecture may lack robustness as a result. Recent studies (Hampson, 2000; Storms & Hampson, 2003; Hampson & Howell, 2005; Hampson *et al.*, 2008; Sømme *et al.*, 2008) show that it is possible to use high-quality datasets to interpret geomorphological and process-regime evolution at intra-parasequence scale, in order to reconstruct high-resolution trends in paleogeography and shoreline trajectory (*sensu* Helland-Hansen & Martinsen, 1996). Such an approach is limited by data quantity and quality, but has the advantage of allowing the investigation of sedimentological, stratigraphic and geomorphological responses to environmental signals at both high and low resolution in order to develop a robust stratigraphic and palaeogeographic framework.

In this chapter, I will present interpretations of facies architecture and physical stratigraphy at intra-parasequence and parasequence scales in several ancient wave-dominated shorelines in the Upper Cretaceous Aberdeen Member (Blackhawk Formation), which is exposed in the Book Cliffs, Utah, USA. This interpretation is based on a dataset that I collected in June-July 2006. These data will be used in the following chapter as observed data from which relative sea level, sediment supply, mud fallout rate and wave-height histories will be inferred by stratigraphic inverse and forward modelling techniques in Chapter 6. Prior to numerical investigation, a conventional sedimentological interpretation is required to assess shoreline orientation, sedimentation style and to qualitatively analyse variations in environmental parameters, which might be used as prior information within the stratigraphic inversion process. The aims of this study are threefold. Firstly, I will document evidence of both wave and river processes along the palaeo-shorelines. Secondly, I will relate high resolution intra-

parasequence facies architecture to palaeogeographic and palaeogeomorphological evolution of the Standardville parasequence shoreline. Thirdly, I will discuss the implication of these high-resolution reconstructions of physical stratigraphy and palaeogeography for previous sequence stratigraphic interpretations of these strata and will define the objectives of the first application of the stratigraphic inverse and forward modelling algorithm.

5.2 Geological Setting and Previous Work

The Campanian (Upper Cretaceous) Blackhawk Formation forms part of a wedge of siliciclastic sediment transported eastward from the Sevier orogenic belt and deposited along the western margin of the Western Interior Seaway (Fouch *et al.*, 1983). The Cretaceous Western Interior Seaway of North America stretched from the present Gulf of Mexico to Arctic Ocean (Fig. 5.2A). This foreland basin was formed by crustal loading and flexural subsidence during thrusting and folding in the Cordilleran Orogen (Burchfield *et al.*, 1992) and thermal subsidence related to dynamic topography of the subducting slab (Liu & Nummedal 2004).

The large scale stratal architecture of the formation records deposition during a long term relative rise in sea level (Taylor & Lovell, 1995; Howell & Flint, 2003). In the Book Cliffs area of western Utah (Fig. 1B), the Blackhawk Formation contains six shallow-marine members (Fig. 5.3) that pass eastward (basinward) into predominantly open-marine Mancos Shale (Young, 1955). Each member comprises several stacked progradational shoreface tongues associated with fluvial and coastal plain deposits. The Blackhawk Formation is unconformably overlain by the Castlegate Sandstone and is separated from the underlying Star Point Formation by a westward-thinning tongue of the Mancos Shale (Young, 1955). Sparse radiometric and palaeontological age data constrain deposition of the Blackhawk Formation to between 82.5 and 79 Ma (Fouch *et al.*, 1983). Each member thus represents an approximate duration of 0.5-0.6 Ma. Outcrop mapping and subsurface correlations of the sequence boundaries and major flooding surfaces that bound shallow marine members (e.g. Kamola & Huntoon, 1992; O'Byrne & Flint, 1995; Taylor & Lovell, 1995; Van Wagoner, 1995; Yoshida, 2000; Hampson *et al.* 2001; Howell & Flint, 2003) define the sequence stratigraphic

framework presented in Figure 5.3 (after Hampson *et al.* 2001; Howell & Flint, 2003; Hampson & Howell, 2005).

The Aberdeen Member is the second oldest shallow-marine member of the Blackhawk Formation. It crops out along the northern part of the Book Cliffs (Fig. 5.2C) and can be traced from near its palaeo-landward termination at Gordon Creek (10 km west of Helper) to its palaeo-seaward termination at Soldier Creek Canyon (15 km of east of Kenilworth).

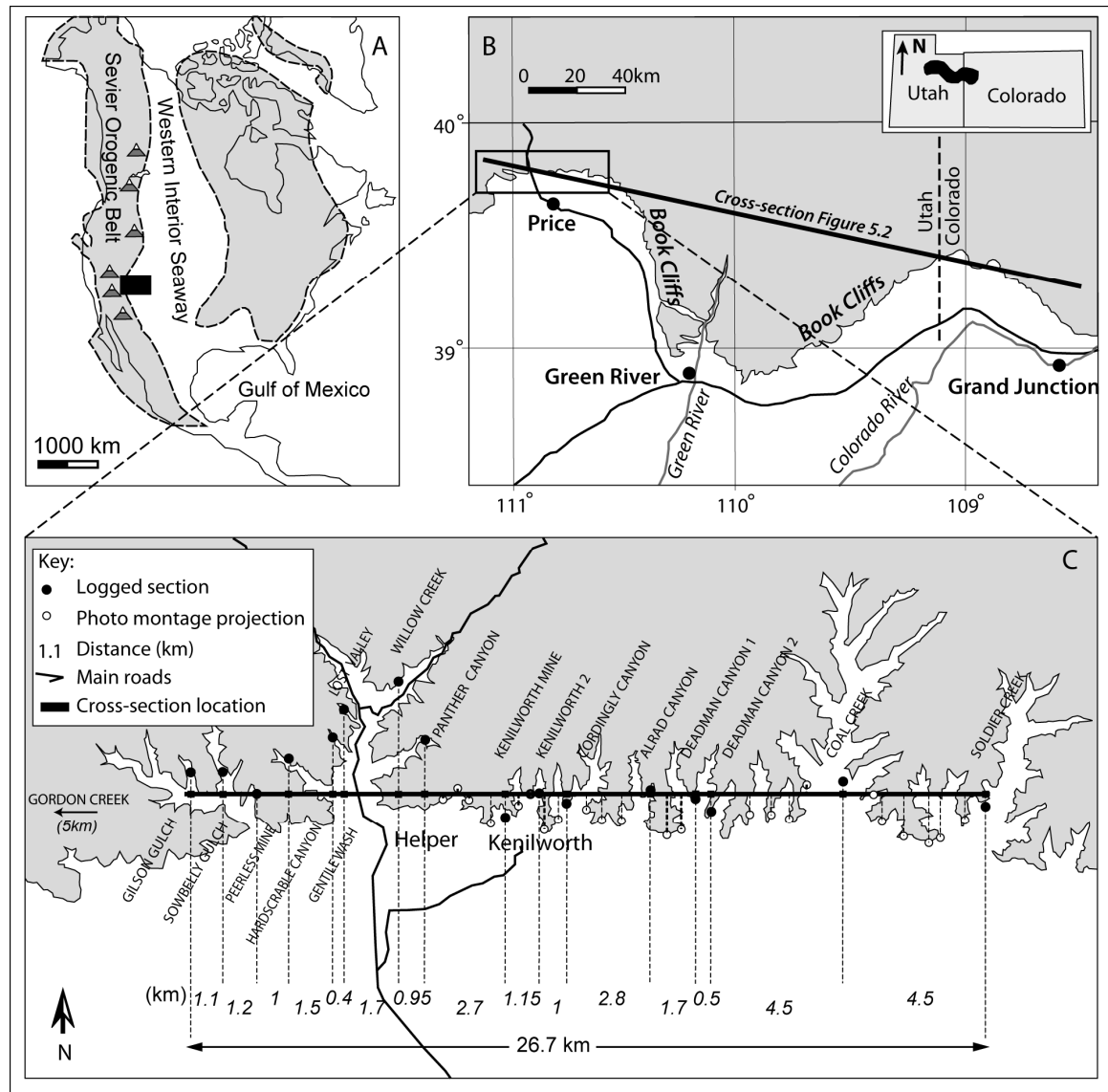


Figure 5.2: Location of the Aberdeen Member of the Blackhawk Formation (Utah, USA). (A) Location of the study area (black square, corresponding to the area shown in Fig. 1B)

superimposed on a palaeogeographic reconstruction of the late Campanian Western Interior Seaway (after Kaufman & Caldwell, 1993). (B) Map of the Book Cliffs area, highlighting the study area and regional stratigraphic cross-section shown in Figure 5.2. (C) Map showing the distribution of cliff-face exposures, measured sections and detailed stratigraphic cross-section shown in Figure 5.12.

Within the study area, the Book Cliffs trend approximately west-east (Fig. 5.2B, C), which is perpendicular to the regional shoreline trend (Balsley, 1980; Howell & Flint, 2003). The distance of overall shoreline progradation during deposition of the Aberdeen Member was about 35 km. Compared to the other well-exposed members which have been widely studied (Kamola & Van Wagoner, 1995; O'Byrne & Flint, 1995; Pattison, 1995; Taylor & Lovell, 1995; Van Wagoner, 1995; Hampson, 2000; Howell & Flint, 2003; Hampson & Howell, 2005; Sømme *et al.*, 2008), the sedimentology and internal stratigraphy of the Aberdeen Member is poorly documented. The parasequence-scale stratigraphic framework of the Aberdeen Member has been outlined by Kamola & Huntoon (1992, 1993, 1995). These workers interpreted five wave-dominated shoreface-shelf parasequences within the Aberdeen Member (labelled Ab0-4 in Fig. 5.3), with the upper two truncated by an incised valley network (Kamola & Huntoon, 1995). The oldest of the five parasequences (Ab0) has not yet been described, but the four younger parasequences (Ab1-4) are documented to contain well-exposed marginal-marine through shallow-marine deposits. Kamola & Huntoon (1995) named these four younger parasequences in ascending stratigraphic order as follows, after their type localities: Standardville (Ab1), Deadman (Ab2), Coal Creek #1 (Ab3) and Coal Creek #2 (Ab4) parasequences. The stacking pattern of these parasequences is strongly progradational for the older parasequences but becomes increasingly aggradational, particularly for the two youngest parasequences (Kamola & Huntoon, 1995). This stratigraphic architecture is attributed to increasing flexural subsidence driven by out-of-sequence uplift and thrust loading, assuming a constant sediment supply (Kamola & Huntoon, 1995; Houston *et al.*, 2000).

The member also contains a unit of fluvial-dominated delta-front deposits that is difficult to place within the parasequence framework outlined above. These deposits lie at the palaeo-seaward limit of wave-dominated shoreface-shelf deposits in the Standardville (Ab1) parasequence, and have been interpreted to represent deposition during a relative sea-level fall

(Kamola & Huntoon, 1992, 1993). In this interpretation, the delta front deposits represent a “stranded lowstand parasequence” (Van Wagoner *et al.*, 1990; Kamola & Huntoon, 1993) with a sequence boundary placed either at their base (Howell & Flint, 2003) or at their top (Kamola & Huntoon, 1992). With the exception of the “stranded” delta front deposits, the various parasequences in the Aberdeen Member are described as consisting of shoreface-shelf deposits (Kamola & Huntoon, 1992, 1995, 1995), implying a strandplain shoreline that was supplied with sediment by alongshore littoral currents.

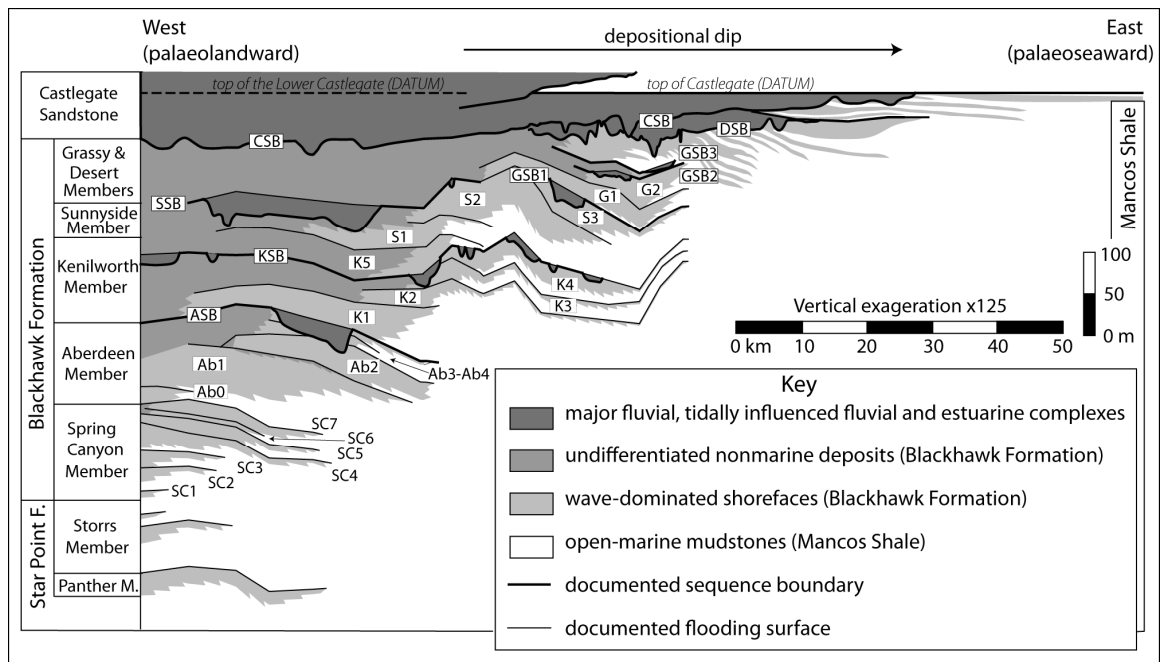


Figure 5.3: Summary stratigraphic cross-section through the late Cretaceous Blackhawk Formation, Book Cliffs, Utah (after Balsley, 1980; Hampson *et al.*, 2001; Hampson & Howell, 2005 and references therein). The cross-section is located in Figure 5.1B. Datum horizons are regionally extensive lithostratigraphic markers. Shoreface-shelf sandstone tongues (parasequences) are numbered within the Spring Canyon (SC1–7), Aberdeen (Ab0–4), Kenilworth (K1–5), Sunnyside (S1–3) and Grassy Members (G1–4). Large-scale stratigraphic architecture is recorded by the stacking patterns of the shoreface-shelf sandstone tongues. Documented sequence boundaries are named after the nomenclature of Hampson & Howell (2005).

5.3 Methodology

Excellent outcrop exposure in the west-east-trending Book Cliffs allows detailed study of sedimentary facies and facies architecture along depositional dip, with some strike control provided by north-south-trending canyons (Fig. 5.2C). The Aberdeen Member was studied using 16 measured sections, combined with detailed photomontages and field sketches covering the entire extent of the outcrop (Fig. 5.2C). No measured sections were collected at the up-dip termination of Aberdeen Member (west of Gilson Gulch; Fig. 5.2C), because these outcrops are both inaccessible and intensively faulted (Hansen, 1996).

The method used to analyse intra-parasequence architecture involved four steps: (1) facies associations and stratigraphic surfaces were described and interpreted in measured sections, (2) based on the interpreted measured sections and the earlier work of Kamola & Huntoon (1992, 1993, 1995), parasequences bounded by transgressive surfaces were correlated over the study area, (3) photomontages and field sketches of the cliff faces were used to trace minor stratigraphic discontinuities and facies association boundaries identified within the parasequences (e.g. in measured sections) throughout the study area at outcrop (after the methods of Hampson, 2000; Hampson & Storms, 2003), and (4) shoreline geomorphology, palaeogeographic evolution and trends in shoreline trajectory were reconstructed from the intra-parasequence stratigraphic architecture (after the methods of Hampson, 2000; Hampson & Storms, 2003). It is important to appreciate that this method is limited by three factors. Firstly, the identification of minor stratigraphic discontinuities within parasequences is limited by photomontage resolution and cliff-face exposure quality. Secondly, accurate thicknesses for the studied succession were obtained only at locations with measured sections. Thirdly, palaeogeographic reconstruction is inherently limited by the largely two-dimensional nature of the cliff-face exposures.

Transgressive surfaces at the base and top of parasequences have been used as local stratigraphic datum surfaces, as they are extensive and clearly identifiable. However, in reality these surfaces do not represent smooth palaeo-horizontal surfaces. Minor, but appreciable, transgressive erosion is often associated with such surfaces (e.g. Swift, 1968; Hwang & Heller, 2002; Storms & Swift, 2003), and they typically dip very gently palaeoseaward to define the

gradient of the resulting shelf (e.g. Storms & Swift, 2003). Consequently, the dips of intra-parasequence stratigraphic discontinuities relative to transgressive datum surfaces are slightly but systematically over-estimated (Hampson, 2000). The main uncertainty in reconstructing the geometry of intra-parasequence stratigraphic discontinuities is the effect of differential compaction, which becomes significant near the palaeo-seaward pinchout of shoreface-shelf sandstones, at the transition from thick sandstone-dominated successions to heterolithic successions containing thinner sandstone beds.

5.4 Facies Associations and Depositional Environments

Ten facies associations have been recognised in the Aberdeen Member. These facies associations are described and interpreted briefly below, and are summarised in Table 5.1.

5.4.1 Wave-Dominated Shallow-Marine Facies Associations

Five facies associations (OT, dLSF, pLSF, USF, FS; Table 5.1) are interpreted to represent wave-dominated shoreface-shelf deposits (Figs. 5.4A, 5.5, 5.6, 5.7). These facies associations are typically arranged in upward-coarsening facies successions, from association OT at the base through to association FS at the top (Fig. 5.4A). Boundaries between the various facies associations in these successions are typically gradational.

Facies association OT (Fig. 5.5A) consists of moderately to intensely bioturbated blue-grey mudstone that is in places interbedded with thin (2-5 cm) siltstone to very fine-grained sandstone beds. These siltstones and sandstones contain parallel lamination, wave and current ripple cross-lamination. Bioturbation is intense and the trace fossil assemblages mainly constitute *Planolites* ichnofacies. This facies association is interpreted to represent offshore transition deposits. Mudstones are interpreted to record deposition from suspension below storm wave base, with bioturbation attributed to subsequent biogenic reworking. Laminated

and cross-laminated siltstone and sandstone beds most likely record deposition from oscillatory and unidirectional currents generated during major storms.

Lithofacies Association	Lithology and Sedimentary Structures	Granulometric Facies	Bioturbation
Foreshore (FS)	Upper fine-grained sandstone. Planar parallel lamination.	Amalgamated sand	Absent
Upper shoreface (USF)	Upper fine- to medium-grained sandstone. Trough and tabular cross-beds, minor planar lamination, and swaley cross-stratification.	Amalgamated sand	Sparse to moderate (<i>Ophiomorpha</i> , <i>Skolithos</i>)
Proximal lower shoreface (pLSF)	Upper fine-grained sandstone. Hummocky cross-stratification and minor wavy lamination.	Amalgamated sand	Sparse to moderate (<i>Ophiomorpha</i> , <i>Thalassinoides</i>)
Distal lower shoreface (dLSF)	Non-amalgamated beds of upper fine-grained sandstone with mudstone and siltstone interbeds. Hummocky cross-stratification and minor wavy lamination.	Interstratified sand and mud	Moderate to intense (<i>Ophiomorpha</i> , <i>Thalassinoides</i>)
Offshore transition shelf/ramp (OT)	Mudstone and siltstone with beds of very fine- to fine-grained sandstone. Parallel lamination, wave and current ripple cross-laminations.	Laminated mud.	Moderate to intense (<i>Planolites</i>)
Proximal delta front (pDF)	Fine- to coarse-grained sandstone. Planar laminations, current ripple laminations, trough and tabular cross-beds, channelised bodies.	Amalgamated sand	Sparse to moderate (<i>Ophiomorpha</i>)
Distal delta front (dDF)	Non-amalgamated beds of fine- to coarse-grained sandstone with mudstone and siltstone interbeds. Planar lamination at bed bases grading into current ripple laminations at bed tops.	Interstratified sand and mud	Moderate to important (<i>Cruziana</i> , <i>Thalassinoides</i>)
Slumps (SL)	Disorganised beds of fine- to coarse-grained sandstone with mudstone. Collapse structures and soft sediment folds	Heterolithic	Absent.
Coastal plain (CP)	Lithologically variable succession of mudstone, siltstone, sandstone and coal	Interstratified sand and mud	Moderate to important (roots)
Channel fill (CF)	Variably fine- to coarse-grained sandstones and interbedded siltstone and mudstones. Erosionally based channel-fill bodies. Cross-beds and current ripple laminations.	Heterolithic	Moderate

Table 5.1. Summary of facies associations.

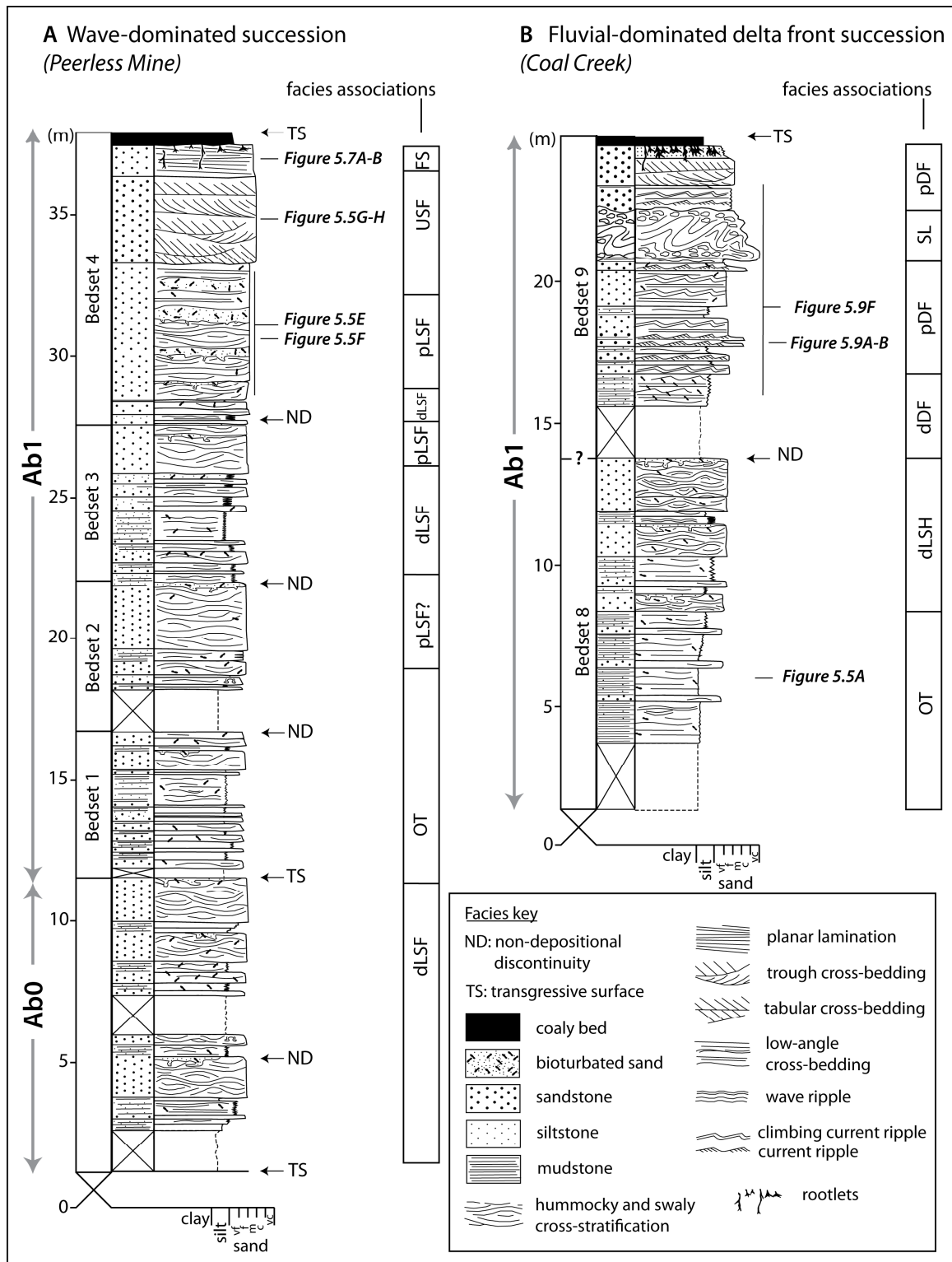


Figure 5.4: Detailed sedimentary logs. Each log shows sedimentology, facies associations, interpreted depositional environment, and stratigraphic surfaces and units. (A) Peerless Mine

measured section, representing a typical wave-dominated shoreface-shelf facies succession from an un-named parasequence (Ab0) and the overlying Standardville parasequence (Ab1). In detail, both parasequences contain several upward-coarsening bedsets bounded by non-depositional discontinuities. (B) Coal Creek measured section, representing a distal wave-dominated shoreface-shelf facies succession (0-14 m in log) overlain by a fluvial-dominated delta front succession (14-25 m in log) near the palaeo-seaward pinchout of the Standardville parasequence (Ab1). Abbreviations for facies associations and depositional environments are given in Table 5.1.

Facies association dLSF (Fig. 5.5B-D) is composed of thin to thick (5-50 cm) beds of very fine-grained, hummocky cross-stratified sandstone with mudstone and siltstone interbeds. Bioturbation in the mudstones and siltstones is moderate to intense. Trace fossil assemblages constitute *Ophiomorpha* and *Thalassinoides* ichnofacies. Facies association dLSF is interpreted to represent distal lower shoreface and inner shelf deposits deposited above storm wave base and below fair-weather wave base. Hummocky cross-stratification records deposition from waning oscillatory flows (e.g. Dumas *et al.*, 2005). Hummocky cross-stratified sandstone beds are interpreted to represent major storm events (e.g. Dott & Bourgeois, 1982; Duke, 1985; Clifton, 2006). Interbedded mudstones and siltstones record deposition from suspension during fair-weather periods between the storms.

Facies association pLSF (Fig. 5.5E-F) comprises amalgamated beds of very fine- to medium-grained sandstone that contain hummocky cross-stratification. Bioturbation is low to moderate in intensity. *Ophiomorpha* and *Thalassinoides* are the most prominent trace fossils. Facies association pLSF is interpreted as proximal lower shoreface to middle shoreface deposits. The pervasive, hummocky cross-stratified character of the sandstones records deposition by storm wave processes (e.g. Dott & Bourgeois, 1982; Duke, 1985), while their preservation implies limited reworking by fair-weather wave processes.

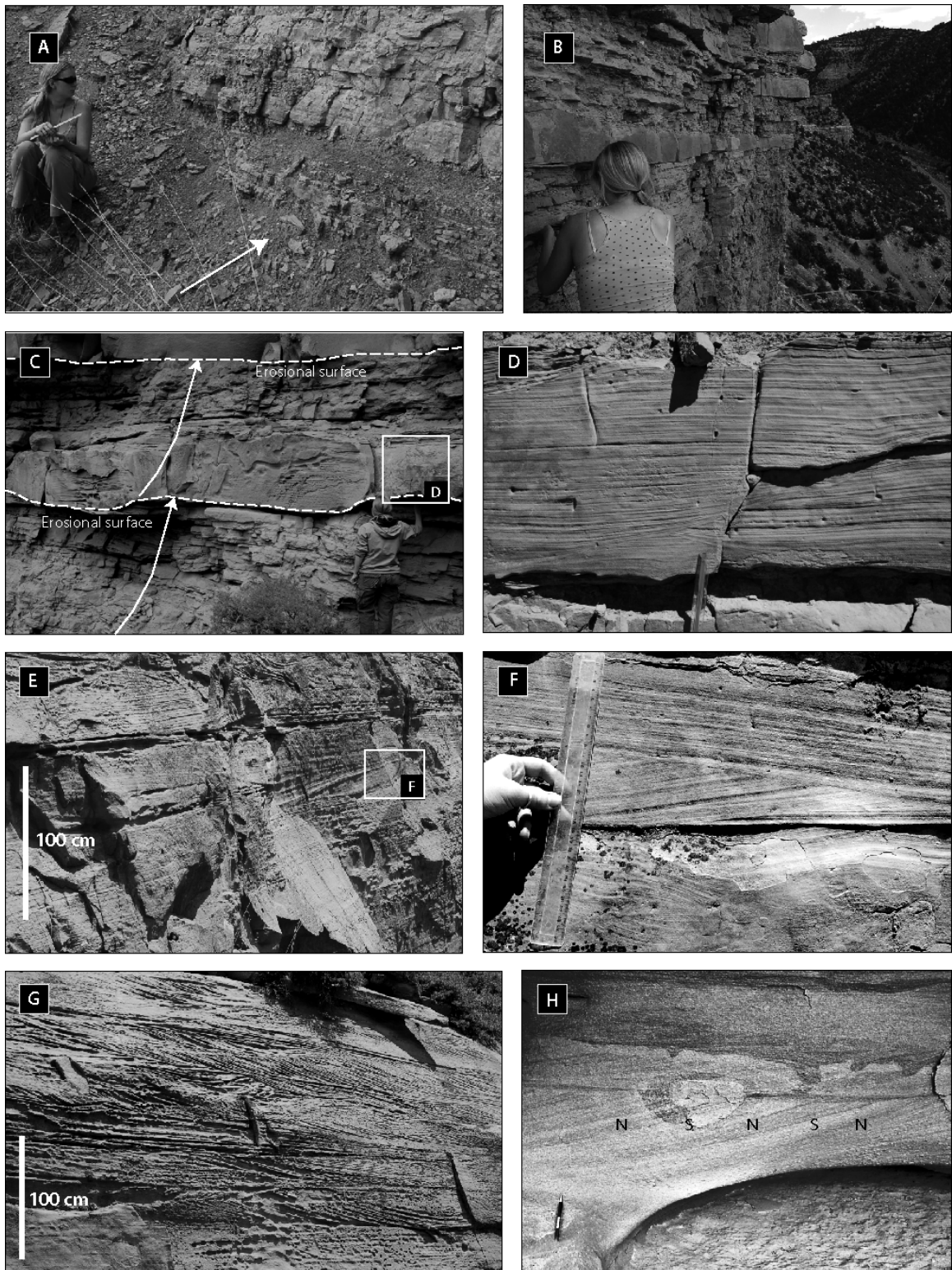


Figure 5.5: Photographs illustrating features of wave-dominated shallow-marine facies associations (OT, dLSF, pLSF, USF; Table 5.1). (A) Blue-grey mudstone interbedded with thin

beds of siltstone to very fine-grained sandstone (white arrow) that contain wave and current ripple cross-lamination. (B) Non-amalgamated, hummocky cross-stratified sandstone beds with siltstone interbeds, which characterise distal lower shoreface deposits (facies association dLSF). (C, D) Upward-fining bedset bounded by erosional surfaces, which are overlain by amalgamated, hummocky cross-stratified sandstone beds. This succession consists of distal lower shoreface deposits (facies association dLSF). (E, F) Amalgamated, hummocky cross-stratified sandstone beds characteristic of proximal lower shoreface deposits (facies association pLSF). (G) Trough and tabular cross-bedding within upper shoreface deposits (facies association USF). The cross-bed has carbonaceous siltstone drapes along its foresets (H), which are arranged into interpreted tidal bundles in which closely spaced drapes represent neap tide conditions (labelled N) and widely spaced drapes represent spring tide conditions (labelled S). Photographs A-B were taken from the Standardville parasequence at Coal Creek (Figs. 5.2C, 5.4B), photographs C-D were taken from the same parasequence at Panther Canyon (Fig. 5.2C), whereas photographs E-H were taken from the same parasequence at the Peerless Mine (Figs. 5.2C, 5.4A).

Facies association USF (Fig. 5.5G-H) consists of upper fine- to medium-grained sandstone containing trough and tabular cross-beds, planar lamination and swaley cross-stratification. Foresets within some cross-beds are lined by drapes of carbonaceous siltstone and mudstone that are arranged in a regular, repeated pattern (Fig. 5.5H). Palaeocurrents measured from cross-beds are predominantly oriented to the southwest or south-southwest (Fig. 5.6A), which is oblique to sub-parallel to the regional shoreline trend (Kamola & Huntoon, 1995). Bioturbation intensity is low, and the dominant trace fossils are *Ophiomorpha* and *Skolithos*. Facies association USF is interpreted as upper shoreface deposits that record deposition above the fair-weather wave base in the breaker zone. Cross-beds record the migration of dunes in response to onshore and/or alongshore currents generated by fair-weather waves (Elliot, 1986; Boyd *et al.*, 1992; Walker & Plint, 1992; Clifton, 2006). Siltstone and mudstone drapes along the foresets of the cross-beds record periods of low flow velocity, and their regular, repeated distribution most probably records velocity variations over several tidal cycles (e.g. spring-neap cycles; Fig. 5.5H). Swaley cross-stratification records an unstable condition of bedform

migration in response to strong oscillatory currents, which may have a minor unidirectional component (Dumas *et al.*, 2005). These structures most likely record storm-wave processes.

Facies association FS (Fig. 5.7A-B) is composed of fine-grained sandstone with gently wedging to sub-parallel planar lamination, and is commonly penetrated by roots (Fig. 5.7B). Bioturbation is sparse to absent. Facies association FS is interpreted as foreshore deposits. Wedging and sub-parallel planar lamination are interpreted to record swash-backwash processes caused by breaking waves (Clifton, 2006).

At a gross scale, upward-coarsening successions of facies association OT through facies association FS record progressive shallowing in water depth (e.g. Fig. 5.4A), and can be regarded as parasequences (Kamola & Van Wagoner, 1995). The vertical facies successions are typical of regressive, barred, sandy shoreface-shelf successions (Elliot, 1986; Walker & Plint, 1992; Clifton, 2006). The trace fossil assemblages in these successions imply deposition in progressively shallower shelfal to nearshore environments (Pemberton *et al.*, 1992). The upper part of these successions (facies associations USF, FS) commonly have a white color, due to early diagenetic leaching by acidic groundwater from coals in the overlying coastal plain deposits (Taylor *et al.*, 2004).

The Standardville (Ab1) parasequence contains a complete shallowing upward succession (facies associations OT, dLSF, pLSF, USF, FS). Offshore transition deposits (facies association OT) are poorly developed and/or missing in palaeolandward areas, but become thicker and more fully developed in palaeoseaward areas. In contrast, foreshore, upper shoreface and proximal lower shoreface deposits (facies associations pLSF, USF, FS) thin and pinch out in a palaeoseaward direction. The Deadman parasequence (Ab2) comprises lower shoreface to foreshore deposits (facies associations dLSF, pLSF, USF, FS). The distal lower shoreface deposits (facies association dLSF) of this parasequence also contain climbing current ripples recording deposition from unidirectional flows. Coal Creek #1 and Coal Creek #2 parasequences (Ab3 and Ab4, respectively) contain only successions from lower shoreface to upper shoreface deposits (facies associations dLSF, pLSF, USF). Foreshore deposits (facies association FS) are locally absent.

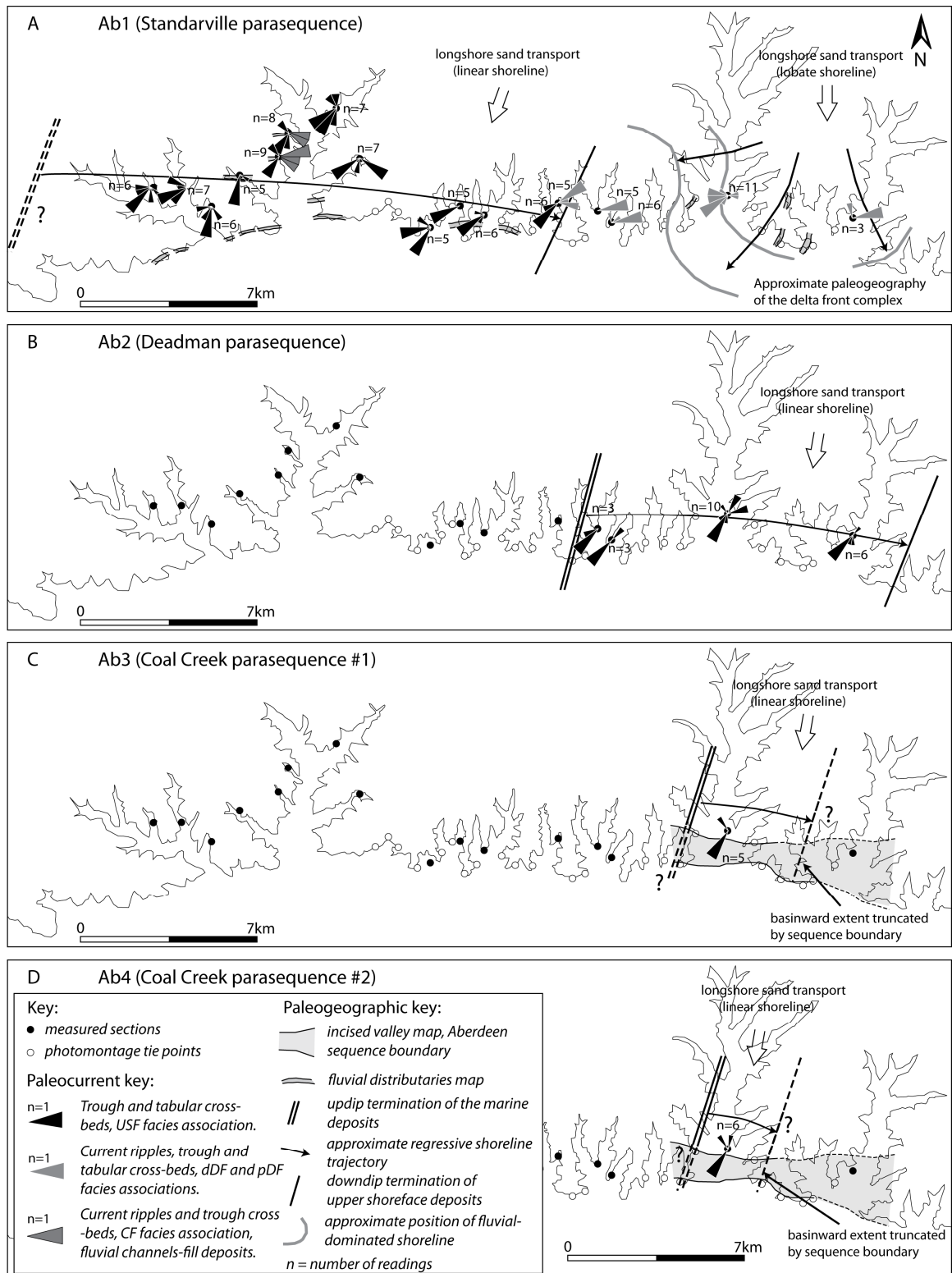


Figure 5.6: Maps showing the extent of shoreline progradation, palaeocurrent data, palaeogeography and plan-view shoreline trajectory for the upper four parasequences of the

Aberdeen Member (Ab1-Ab4, Fig. 5.3). (A) Standardville (Ab1) parasequence map. The palaeo-landward part of the parasequence comprises the deposits of a linear, wave-dominated shoreline with southwest-directed palaeocurrents in upper-shoreface deposits. The palaeo-seaward part of the parasequence comprises fluvial-dominated delta front deposits that define a lobate shoreline, which palaeocurrent data and clinoform geometries indicate built out towards the south and southwest. (B) Deadman (Ab2) parasequence map. The parasequence comprises the deposits of a linear, wave-dominated shoreline with southwest-directed palaeocurrents in upper-shoreface deposits. (C) Coal Creek #1 (Ab3) parasequence map and (D) Coal Creek #2 (Ab4) parasequence map. Both parasequences comprise the deposits of linear, wave-dominated shorelines that are truncated by a sequence boundary near their palaeo-seaward limits.

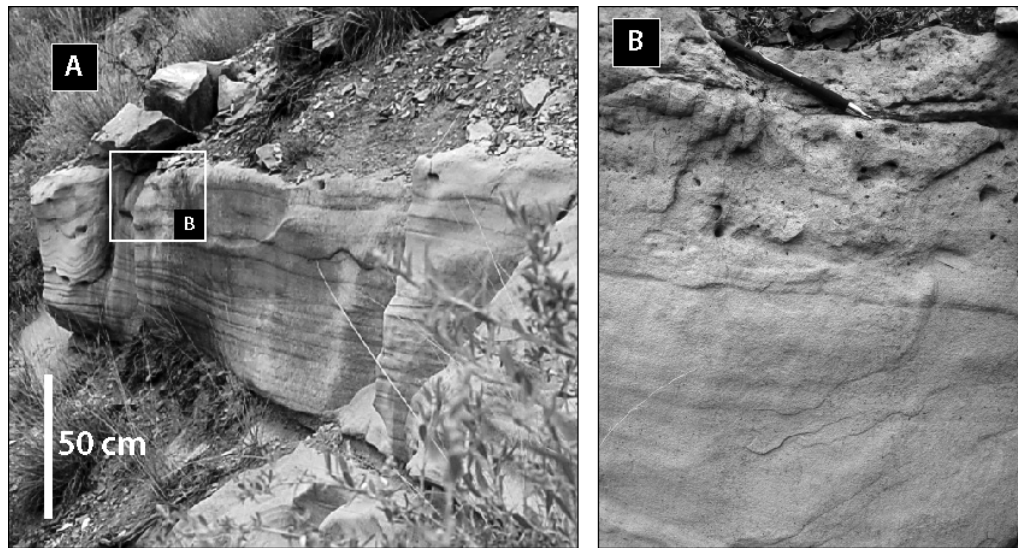


Figure 5.7: Photographs illustrating features of wave-dominated shallow-marine facies associations (FS, Table 5.1). (A) Gently wedging to sub-parallel planar lamination characteristic of foreshore deposits (facies association FS), penetrated by roots (B). Photographs A-B were taken from the Standardville parasequence at Peerless Mine (Fig. 5.2C).

5.4.2 Fluvial-Dominated Delta Front Facies Associations

Three further facies associations are interpreted to represent shallow-marine deposition within fluvial-dominated delta front successions (Figs. 5.4B, 5.8, 5.9, Table 5.1). These facies associations are typically arranged in upward-coarsening facies successions, with association dDF at the base grading upwards into association pDF at the top (Fig. 5.4B). These successions gradationally overlie proximal lower shoreface to offshore transition deposits (facies associations pLSF to OT). Units of facies association SL are interspersed throughout the successions. The fluvial-dominated delta front successions contain a large number of easily recognisable clinoforms (Figs. 5.8). The clinoforms have a concave-up shape and present steep dips of (3-7°), such that each clinoform has an extent down depositional dip of only 200 to 500 m. In the western part of the succession, clinoforms dip towards the south and southwest, which is almost perpendicular to the regional depositional dip. In the eastern part of the succession, clinoforms dip more gently towards the south and southeast. Clinoform distribution is complex; their spacing varies from one metre to several tens of metres, and may be uniform or non-uniform at a particular outcrop location.

Facies association dDF (Fig. 5.9A-B) consists of non-amalgamated fine- to coarse-grained sandstone beds interbedded with bioturbated mudstones and siltstones. Sandstone beds are thin (5-20 cm), and each exhibits a vertical succession of planar lamination at the base grading into current ripple lamination and capped by bioturbated siltstone. Bioturbation is moderate to intense. Prominent trace fossils include *Cruziana* and *Thalassinoides*. This facies association is interpreted to represent distal delta front deposits, because of the abundance of current ripples, which record deposition from unidirectional currents, and the absence of wave- and storm-generated sedimentary structures. The vertical succession within each sandstone bed comprises a partial Bouma sequence (T_{bc}), which records waning of a turbulent flow. Successions of this facies association therefore record repeated deposition of sandstone beds by episodic unidirectional currents, with intervening periods represented by deposition of mudstone and siltstone from suspension. Within a delta front context, the partial Bouma sequences in the sandstone beds are interpreted as the product of hyperpycnal flows caused by episodic discharge of dense, sediment-laden river waters.

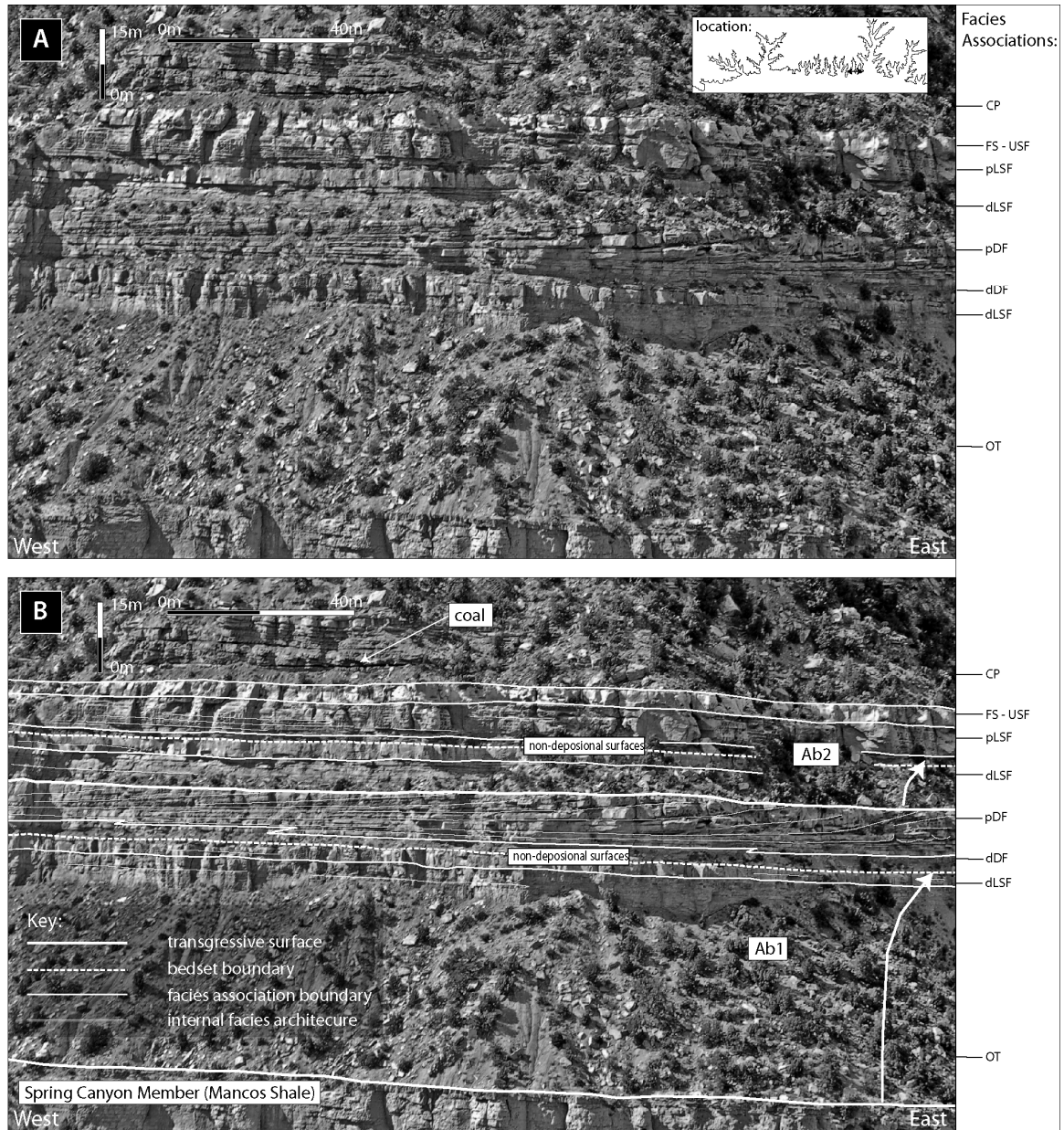


Figure 5.8: Cliff face interpretation of fluvial delta front deposits. (A) Uninterpreted and (B) interpreted aerial photograph of the west face of Coal Creek showing in ascending stratigraphic order: (1) wave-dominated shoreface-shelf deposits of the lower Standardville (Ab1) parasequence, (2) fluvial-dominated delta front deposits of the upper Standardville (Ab1) parasequence, (3) wave-dominated shoreface-shelf deposits of the Deadman (Ab2) parasequence, and (4) coal-bearing coastal plain deposits. Note the very gentle easterly dip of clinoforms in the wave-dominated deposits of the lower Standardville (Ab1) parasequence,

highlighted by bedset boundaries (non-depositional discontinuity surfaces). In contrast, the overlying fluvial-dominated delta front succession contains steeper, westerly dipping clinorforms that downlap the wave-dominated shoreface-shelf deposits. The tops of the delta-front clinoforms are erosionally truncated by a transgressive surface.

Facies association pDF is composed of amalgamated fine- to coarse-grained sandstone beds, which thicken upwards within successions of the facies association. Each sandstone bed in the lower part of the succession has the same internal character as the sandstone beds described in facies association dDF. Sandstone beds in the upper part of the succession are characterised by unidirectional trough and tabular cross-beds (Fig. 5.9C-D) and contain bioturbation of moderate to low intensity; the most prominent trace fossil is *Ophiomorpha*. The upper part of successions of this facies association is also characterised by coarse-grained sandstone beds with erosional U-shaped bases, indicating channelised flow running down the clinoforms toward the south (Fig. 5.9C-D). Facies association pDF is interpreted to represent proximal delta front deposits. Both the partial Bouma sequences (T_{bc}) in the lower part of the succession and trough and tabular cross-beds in the upper part of the succession record deposition by unidirectional currents with little or no wave influence. The increase in sandstone proportion, bed thickness and cross-set scale relative to facies association dDF indicates deposition in a location more proximal to the deltaic distributary. In this context, trough and tabular cross-beds are typically found in mouth bar successions that occur directly at the mouths of terminal distributary channels (e.g. Bhattacharya, 2006; Olariu & Bhattacharya, 2006).

Both distal and proximal delta front deposits (facies associations dDF, pDF) contain numerous collapse structures and beds that contain soft-sediment folds (Fig. 5.9E-G). The latter can be up to several metres thick, and they are assigned to facies association SL. Small-scale raft structures and fold axes are consistently oriented to the south and southwest, down the slope of clinoforms in the successions that contain them. As a result, beds containing these features are interpreted as slumps. The presence of such slumps can be explained by the relatively steep slopes of the prograding clinoforms. Other collapse structures may also be explained by loading and water escape processes.

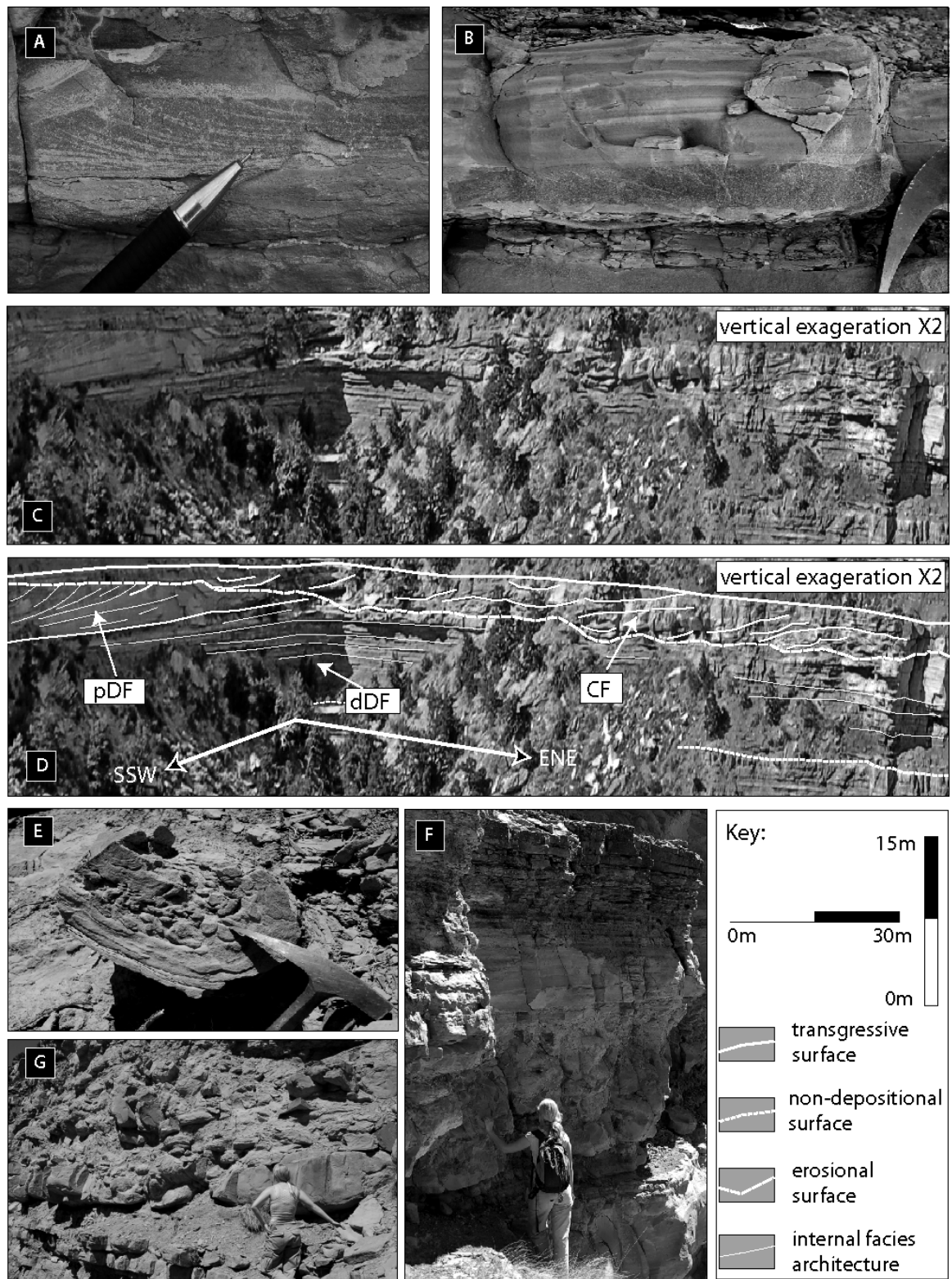


Figure 5.9: Photographs illustrating features of fluvial-dominated delta front facies associations (dDF, pDF, SL; Table 5.1). (A, B) Sandstone beds containing partial Bouma

sequences (T_{bc}) that comprise planar-parallel lamination grading upward into current ripple cross-lamination and capped by bioturbated siltstone. These sandstone beds are interbedded with mudstones and siltstones in distal delta front deposits (facies association dDF). (C) Uninterpreted and (D) interpreted aerial photographs showing proximal delta front deposits (facies association pDF, interpreted as mouth bars) and channel-fill sandstones (facies association CF, interpreted as terminal distributary channel fills) overlying distal delta front deposits (facies association dDF). (E, F, G), Slumps and load structures that occur throughout the delta front succession (facies association SL). All photographs were taken from the Standardville parasequence: photographs C and D were taken between Coal Creek and Soldier Creek (Fig. 5.2C), photographs A, B and F were taken at Coal Creek (Figs. 5.2C), and photographs E and G were taken at Soldier Creek (Figs. 5.2C, 5.4B).

Upward-coarsening successions of facies association dDF through facies association pDF record progressive shallowing in water depth (e.g. Fig. 5.4B), and the successions are typical of regressive fluvial-dominated deltas (e.g. those described in the Spring Canyon Member by Kamola & Van Wagoner, 1995; Hampson & Howell, 2005). The steep, concave-upward clinoforms within these deposits are also typical of fluvial-dominated delta front deposits (e.g. Bhattacharya, 2006). Fluvial-dominated delta front deposits lie at the palaeo-seaward limit of wave-dominated shoreface-shelf deposits in the Standardville (Ab1) parasequence (Fig. 5.4B, 5.6A). Clinoform geometries and palaeocurrent data indicate deposition of at least one delta lobe that was oriented towards the south or southwest. Rapid and localised sedimentation at the mouths of deltaic distributary channels is interpreted to have driven rapid progradation and steep, unstable depositional slopes, which generated slumps and collapse structures. The western part of the lobate delta front comprises facies associations dDf, pDF and SL only, which implies that it was protected from wave action. On the eastern part of the delta front, facies associations dDf and pDF grade into hummocky cross-stratified sandstone beds (facies association dLSF), which indicates reworking by storm waves. This part of the delta front was therefore open to storms that tracked through the Western Interior Seaway.

5.4.3 Coastal Plain and Channel Fill Facies Associations

Two facies associations (CF, CP; Table 5.1) are interpreted to represent non-marine and marginal-marine deposits. These deposits are generally poorly exposed and have not been mapped in detail. As a result, the facies associations described below probably comprise the deposits of multiple depositional environments that have not been differentiated.

Facies association CF is composed of variably sorted, fine- to coarse-grained sandstones and interbedded siltstones and mudstones that occur within erosionally based, channel-fill bodies of single-storey and multi-storey architecture. The sandstones are typically cross-bedded and current ripple cross-laminated. In places, sandstone beds and heterolithic units are arranged into larger scale architectural elements that are inclined parallel to channel margins, thus recording lateral accretion of the channel margin (Figs. 5.9C-D, 5.10, 5.11). This facies association is interpreted to comprise fluvial and distributary channel-fill deposits. Channel-fill deposits are observed in four different contexts: (1) many cut down into proximal, fluvial-dominated delta front deposits (facies association pDF) (Fig. 5.9C-D), (2) several locally erode into foreshore and upper shoreface deposits (facies associations FS, USF) at the top of the Standardville (Ab1) parasequence (Figs. 5.6A, 5.10), (3) channel-fill deposits are stacked into a multistorey and multilateral complex that infills the incised valley network (Fig. 5.6C-D, 5.11) described by Kamola & Huntoon (1992, 1993, 1995), and (4) they occur in coastal plain deposits (facies association CP). The first category of channel-fill deposits is interpreted as infill terminal distributary channels (*sensu* Olariu & Bhattacharya, 2006), based on their context. Internally these channel-fill deposits contain lateral accretion surfaces. They form single-storey sandbodies reaching 2-3m in thickness and up to several tens of metres in lateral extent due to lateral migration. The second category of channel-fill deposits may represent fluvial and/or terminal distributary channels, because their context is ambiguous. The third and fourth categories are interpreted on the basis of their context to be fluvial in origin.

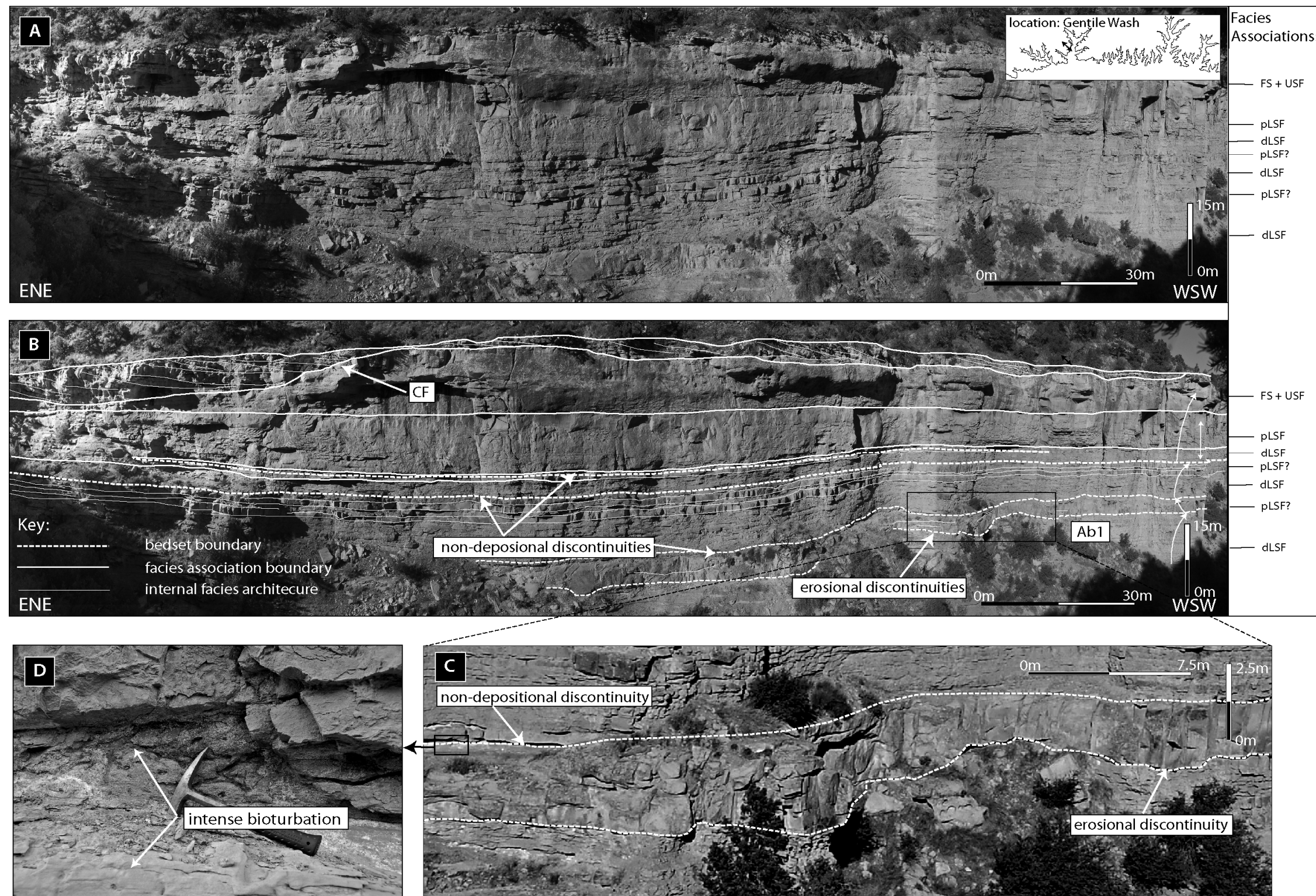


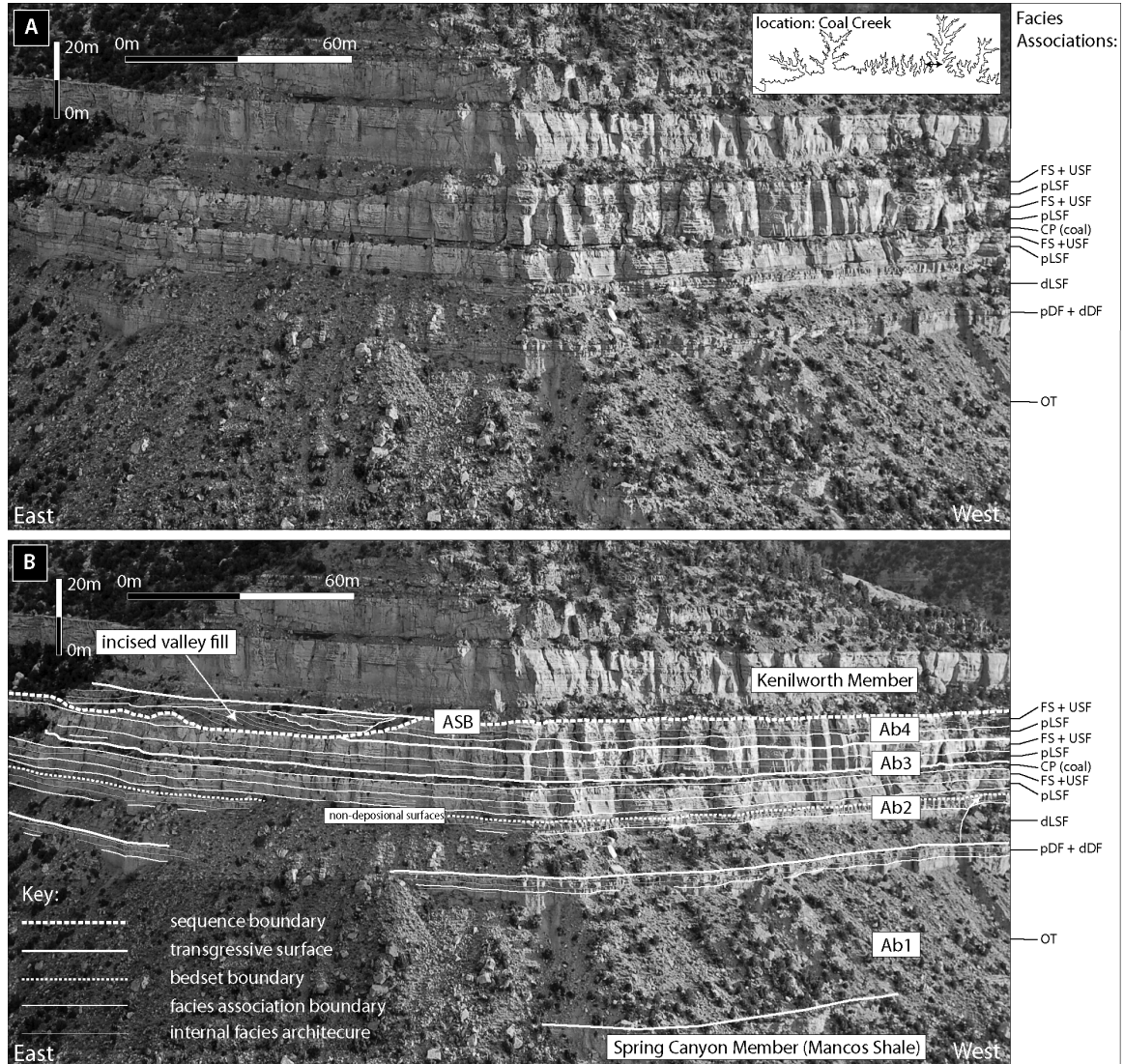
Figure 5.10: (A) Uninterpreted and (B) interpreted photomontage of the Standardville (Ab1) parasequence at Gentile Wash (Fig. 5.2C). The photomontage shows that foreshore and upper shoreface deposits (facies associations FS, USF) near the top of the parasequence are locally incised by channel-fill sandstones (facies association CF, interpreted as fluvial and/or terminal distributary channel fills). Bedsets bounded by non-depositional and erosional discontinuity surfaces are identified in proximal and distal lower shoreface deposits (facies associations pLSF, dLSF) in the lower part of the parasequence. (C) Erosional discontinuity surfaces are characterised by truncation of underlying beds, whereas (C, E) non-depositional discontinuity surfaces are characterised by intense bioturbation.

Facies association CP comprises lithologically variable successions of mudstones, siltstones, sandstones and coals that do not occur as channel-fill bodies. These successions sharply overlie foreshore and proximal delta front deposits (facies associations FS, pDF) at the top of shallow-marine parasequences. The successions are typically carbonaceous and are variably penetrated by roots. This facies association is interpreted to represent undifferentiated coastal plain deposits.

5.5 Stratigraphic Architecture

The stratigraphic architecture of the Aberdeen Member has been interpreted by identifying and correlating various types of stratigraphic surface. The most extensive of these surfaces are transgressive surfaces that bound parasequences. These transgressive surfaces are characterised locally by vertical juxtapositions of lower shoreface deposits (facies associations dLSF, pLSF) above proximal delta front, foreshore and coastal plain deposits (facies associations pDF, FS, CP) (Figs. 5.8, 5.11), which corresponds to a significant increase in water depth and lateral facies dislocations of several kilometres (Van Wagoner *et al.*, 1990; Kamola & Van Wagoner, 1995; Hampson *et al.*, 2008). Sequence boundaries are characterised locally by deep erosion at the base of incised valleys (Fig. 5.11), although they are difficult to interpret in interfluvial areas that lie outside of the valley margins (Van Wagoner *et al.*, 1990). Within parasequences, minor erosional and non-depositional discontinuity surfaces are marked by erosional truncation and intense bioturbation, respectively (Figs. 5.8, 5.10, 5.11), combined with minor vertical and lateral (<1 km) facies dislocations (Hampson, 2000; Hampson & Storms, 2003; Hampson *et al.*, 2008). These intra-parasequence discontinuity surfaces are significantly less extensive than the parasequences that contain them. The facies architecture defined by these various stratigraphic surfaces is summarised in a cross-section (Fig. 5.12) compiled from measured sections, cliff-face photomontages (e.g. Figs. 5.8, 5.10, 5.11) and detailed field maps. The results of our stratigraphic analysis confirm the parasequence-scale stratigraphy of Kamola & Huntton (1995); the Aberdeen Member comprises five wave-dominated shoreface-shelf parasequences and is truncated by an incised valley network that erodes from the top of the member (Fig. 5.11). Additional detail is documented at the intra-parasequence scale, which

helps to elucidate the relationship between the wave-dominated shoreface-shelf deposits of the Standardville (Ab1) parasequence and the fluvial-dominated delta front deposits that lie directly palaeo-seaward of them. Therefore we focus on the intra-parasequence architecture of this interval below.



parasequence (Ab4). The sequence boundary is overlain by stacked channel-fill deposits (facies association FC) that infill an incised valley (Kamola & Huntoon, 1992, 1993).

The Standardville (Ab1) parasequence is unusually thick and laterally extensive due to its development in a high-accommodation setting (Kamola & Huntoon 1995), which allows optimal preservation of intra-parasequence architecture. The stratigraphic and palaeogeographic relationships between wave-dominated and fluvial-dominated shallow-marine deposits in the interval that contains the Standardville (Ab1) parasequence are also poorly characterised in parasequence-scale interpretations (Kamola & Huntoon, 1992, 1993, 1995; Howell & Flint, 2003).

5.5.1 Intra-Parasequence Architecture of Wave-Dominated Shoreface/Shelf Deposits

The wave-dominated shoreface-shelf deposits of the Standardville (Ab1) parasequence contain multiple easterly dipping clinoform surfaces that are marked by minor facies discontinuities (Fig. 5.12). Individual clinoform surfaces have a concave-up shape similar to modern shoreface-shelf profiles (e.g. Cant, 1991; Walker & Flint, 1992) and can be traced from distal lower shoreface deposits into proximal lower shoreface and upper shoreface deposits. Easterly, palaeo-seaward dips of 0.1-0.15° has been calculated in the lower shoreface environment, where these surfaces are most easily identified, relative to the top-parasequence datum surface (Fig. 5.11). These values are relatively similar to those calculated by Hampson (2000), Hampson & Storms (2003) and Sømme *et al.* (2008) in other parasequences exposed in the Book Cliffs succession. As the depositional environment shallows, the clinoform dips become gradually steeper. The gentle clinoform dips documented in the lower shoreface deposits of the Standardville parasequence are consistent with wave-reworking of the shoreface and sediment supply via alongshore littoral currents, and they may represent equilibrium shoreface-shelf profiles for the mean wave climate that prevailed during shoreface progradation.

The clinoform surfaces described above are marked by minor facies discontinuities, and they bound successions of genetically related beds that are here referred to as bedsets (Van

Wagoner, 1990). Two types of discontinuity surface are observed, and these correspond to the non-depositional and erosional discontinuities of Hampson (2000).

Non-depositional discontinuity surfaces are marked by an abrupt decrease in the thickness and amalgamation of storm-generated event beds within proximal and distal lower shoreface deposits and an increase in the intensity of bioturbation (Fig. 5.10B-D). These surfaces are associated with a palaeo-landward dislocation of facies (Figs. 5.4, 5.10C), marked by the sharp base of a palaeo-landward-thinning tongue of more distal deposits (e.g. facies association dLSF) juxtaposed above more proximal deposits (e.g. facies association pLSF) (Fig. 5.12). Within lower shoreface deposits, bedsets bounded by these surfaces are characterised by an upward-coarsening trend that reflects a gradual upward increase in the amalgamation of hummocky cross-stratified sandstone beds. Non-depositional discontinuity surfaces can be traced along depositional dip, from west to east, for 3-8 km in the Standardville (Ab1) parasequence to define clinoform surfaces (Fig. 5.12). Non-depositional discontinuity surfaces are characterised by a decrease in sedimentation which may be associated with a relative rise in sea level level and/or a decrease in storm wave intensity (Hampson, 2000; Hampson & Storms, 2003; Storms & Hampson, 2005). Such surfaces have their strongest expression when there is a concomitant decrease in sediment supply (Sømme *et al.*, 2008). Non-depositional discontinuity surfaces can be forced by both autogenic and allogenic processes that result in a rearrangement of the shoreline geometry and thus local sediment supply and wave/storm influence.

Erosional discontinuity surfaces are marked in vertical sections through lower shoreface deposits by erosion, an abrupt increase in the amalgamation of storm-generated event beds, and associated abrupt increases in grain size and sand content (Figs. 5.5C-D, 5.10C). The surfaces are often associated with steep-walled gutter casts and are sometimes located at the base of “sharp-based” shoreface successions (*sensu* Plint, 1988). Erosional discontinuity surfaces are associated with a palaeo-seaward dislocation of facies (Fig. 5.10B), marked by the sharp base of a palaeo-seaward-thinning tongue of more proximal deposits (e.g. facies association pLSF) juxtaposed above more distal deposits (e.g. facies association dLSF) (Fig. 5.12). Within lower shoreface deposits, bedsets bounded by these surfaces are characterised by an upward-fining trend that reflects a gradual upward decrease in the amalgamation of hummocky cross-stratified sandstone beds (Fig. 5.4C). Erosional discontinuity surfaces can be traced along

depositional dip, from west to east, for 0.5-3 km in the Standardville (Ab1) parasequence to define clinoform surfaces (Fig. 5.12). Erosional discontinuity surfaces are characterised by erosion, a discrete increase in sand supply and an increase in storm wave energy that enhanced event-bed amalgamation. Such surfaces are interpreted to record a lowering of storm wave base in response to: (1) a more energetic wave/storm climate, and or (2) a minor relative fall in sea level (Hampson, 2000; Hampson & Storms, 2003; Storms & Hampson, 2005).

Bedsets bounded by the surfaces described above are well developed in the Standardville (Ab1) parasequence (Fig. 5.12). They have been traced for 4-10 km in down depositional dip (west-east along the main cliff line; Fig. 5.12) and for at least 3 km along depositional strike (north-south along side canyons, Fig. 5.2C). The bedsets have their clearest expression within lower shoreface deposits, where they are defined by variations in the amalgamation of storm-generated event beds. They become less distinct in a palaeo-seaward direction as the event beds thin and become less sand prone, and also in a palaeo-landward direction as shales between the event beds thin and pinch out. None of the bedsets can be traced into upper shoreface deposits (Fig. 5.12). A maximum of four bedsets are observed in any vertical section through the parasequence. Bedset thickness varies from 5 to 20 m, and each bedset records progradation of the lower-shoreface facies belt by 0.5-6 km (Fig. 5.12). These dimensions suggest that bedsets are comparable in scale to beach-ridge sets, as proposed by Sømme *et al.* (2008) using outcrop data from the Sunnyside Member (Fig. 5.3). Modern beach-ridge sets are bounded by seaward-dipping clinoform surfaces across which there is a distinct change in shoreline trajectory (e.g. Hampson *et al.*, 2008).

The bedsets and discontinuity surfaces that bound them are not evenly distributed along the cross-section down depositional dip (Fig. 5.12). Instead, their distribution can be used to define three distinct geographical zones (labelled zones 1-3 in Fig. 5.12): (1) the area west of Panther Canyon predominantly contains non-depositional discontinuity surfaces that bound bedsets which are aggradationally stacked, (2) the area between Panther Canyon and Cordingly Canyon is characterised by an abundance of erosional discontinuity surfaces bounding progradationally stacked bedsets, and (3) the area east of Cordingly contains both non-depositional and erosional discontinuity surfaces that bound aggradationally stacked bedsets.

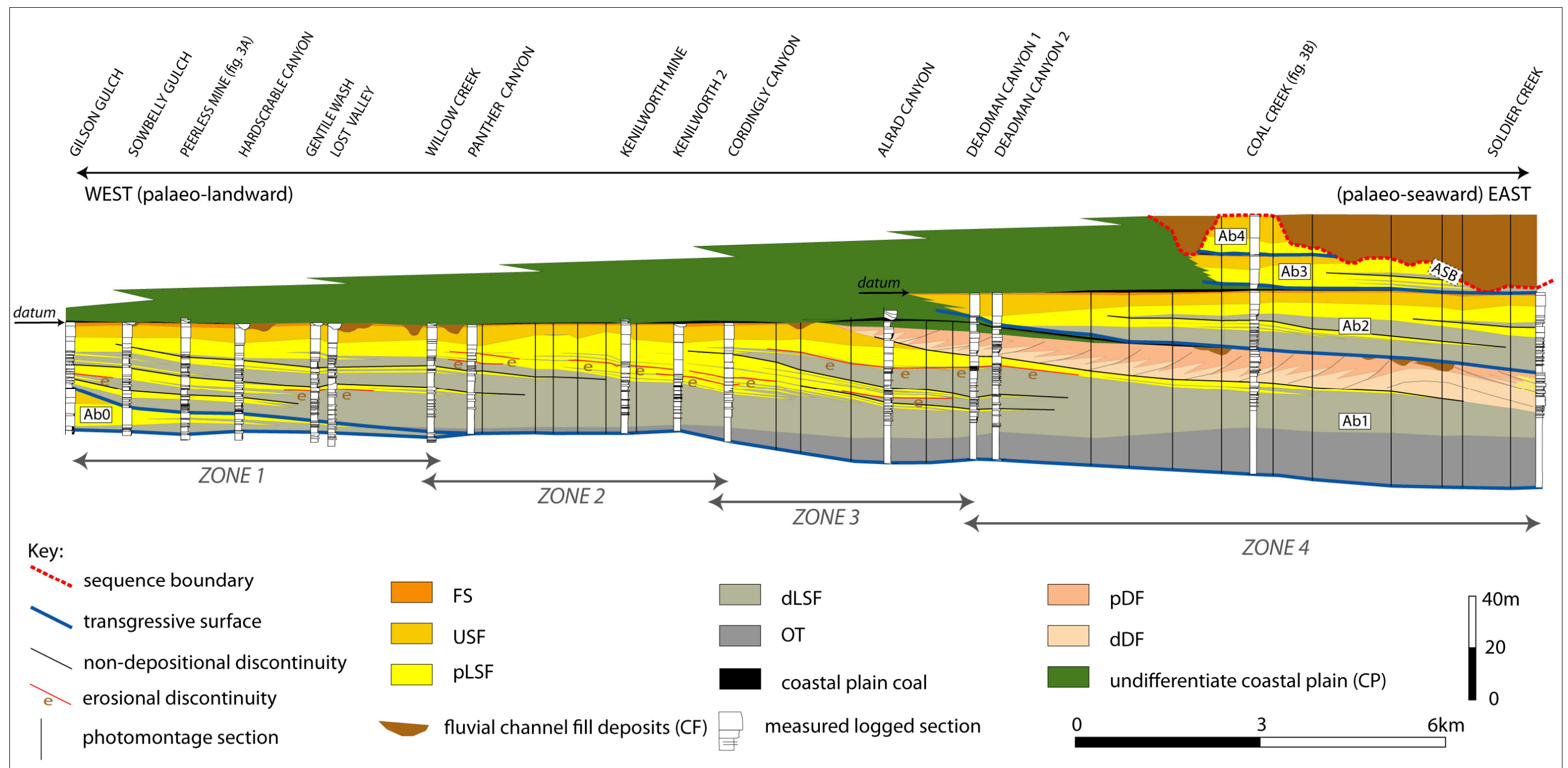


Figure 5.12: Correlation panel aligned parallel to regional depositional dip through the Aberdeen Member. The location of the panel is shown in Figure 5.2C. Datum surfaces for the section are (1) the base of a coal seam that caps the Standardville (Ab1) parasequence, between Gilson Gulch and Deadman Canyon, (2) the top of the Deadman (Ab2) parasequence, marked by a coal seam over much of its extent, between Deadman Canyon and Soldier Creek. Stratigraphic architecture at both parasequence and intra-parasequence scales are shown. Linear, wave-dominated shorelines in each parasequence prograded from west to east, while lobate, fluvial-dominated deltaic shorelines in the upper Standardville (Ab1) parasequence prograded to the south and southwest. The distribution of non-depositional and erosional discontinuity surfaces that define easterly dipping clinoforms within the wave-dominated shoreface-shelf deposits of the lower Standardville (Ab1) parasequence can be used to define four geographical zones, each interpreted to have a distinctive shoreline trajectory. Facies associations are presented in table 5.1. See text for discussion.

The distribution of discontinuity surfaces and associated bedsets can be used to define aspects of shoreline evolution and shoreline trajectory (e.g. Hampson, 2000; Hampson & Storms, 2003; Storms & Hampson, 2005; Sømme *et al.*, 2008). Aggradationally stacked bedsets bounded by non-depositional discontinuities, such as those observed in zones 1 and 3, are interpreted to represent normal regressive (*sensu* Posamentier *et al.*, 1992) shoreline trajectories developed during rising relative sea-level (Hampson, 2000; Hampson & Storms, 2003). Under these conditions, accommodation space was created on the coastal plain, which caused aggradation. As a result, fluvial channels may have become superelevated above the coastal plain and thus prone to avulsion (e.g. Jones & Schumm, 1999), which would have driven switching of deltaic distributary channels in downstream locations. Such repositioning of distributary mouths changes the morphology of a wave-dominated deltaic shoreline, which can then lead to local changes in wave regime, truncation and erosional reworking of old beach-ridge sets, and formation of a non-depositional discontinuity surface (Sømme *et al.*, 2008). Bedsets bounded by erosional discontinuities which show strongly progradational or “downstepping” stacking patterns, such as those observed in zone 2 above, are interpreted to represent forced regressive (*sensu* Posamentier *et al.*, 1992) shoreline trajectories developed during falling relative sea-level (Hampson, 2000; Hampson & Storms, 2003). Forced regressive shoreline trajectories are associated with enhanced scour at wave base, such as that interpreted at each erosional discontinuity surface, which may locally produce a “sharp-based shoreface” as observed in the eastern part of zone 3 above (Plint, 1988; Pattison, 1995; Plint & Nummedal, 2000; Hampson, 2000). Under these conditions, little or no accommodation space was created on the coastal plain, such that fluvial and deltaic distributary channels may have incised the coastal plain and become fixed in their pre-existing positions (Schumm & Etheridge, 1994). The discussion above suggests that a complex history of shoreline trajectory can be interpreted within the wave-dominated shoreface-shelf deposits of the Standardville (Ab1) parasequence. Although the overall trajectory trend was normal regressive, two episodes of forced regressive trajectory can be interpreted within the parasequence (corresponding to zone 2 and the western part of zone 3 in Fig. 5.12; Kamola & Huntoon, 1993). The shoreface-shelf succession deposited during the first interval of negative shoreline trajectory (zone 2) can be correlated palaeo-landward into coastal plain deposits that contain numerous fluvial channels that shallowly incised into foreshore and upper shoreface deposits near the top of the parasequence (above zone 1 in Fig. 5.12).

5.5.2 Internal Architecture of Fluvial-Dominated Delta Front Deposits

An upward-shallowing succession of fluvial-dominated delta front deposits overlies and downlaps the distal toe of wave-dominated shoreface-shelf deposits near the palaeo-seaward limit of the Standardville (Ab1) parasequence (Fig. 5.8, zone 4 in Fig. 5.12). Upper shoreface deposits in the wave-dominated shoreface-shelf succession and proximal delta-front deposits in the deltaic succession are both overlain by the same coal seam, suggesting that the two successions were deposited in lateral continuity within the same parasequence (Fig. 5.12). The fluvial-dominated delta front succession shows a large number of easily recognisable clinoforms (Fig. 5.8). The clinoforms have steep dips ($3-7^\circ$), and an extent down depositional dip of only 200 to 500 m. The difference in clinoform dip and orientation between the fluvial-dominated delta front succession and the underlying wave-dominated shoreface-shelf deposits has formed a downlap surface (Figs. 5.8, 5.12). The thickness of the delta-front succession is 15-20 m, which corresponds to the water depth into which the delta prograded. This estimate of water depth suggests that there was no significant deepening across the downlap surface that separates distal lower shoreface deposits below from distal delta front deposits above. As a result, this downlap surface is interpreted as a non-depositional discontinuity within the Standardville (Ab1) parasequence, rather than a flooding surface that separates two parasequences (Figs. 5.8, 5.12).

In the western part of the succession, deltaic clinoforms dip towards the south and southwest, which is almost perpendicular to the regional depositional dip (Figs. 5.8, 5.12). The absence of wave-generated structures in this part of the succession implies deposition in a sheltered, confined location. In the eastern part of the succession, deltaic clinoforms dip more gently towards the south and southeast (Fig. 5.12). These more gentle slopes are associated with the occurrence of hummocky cross-stratification, which partially replaces fluvial-generated sedimentary structures within the delta-front succession. This decrease in dip may reflect storm-wave reworking of the eastern part of the delta front. Clinoform orientations indicate that the fluvial-dominated delta front had a lobate morphology, and that the delta front prograded towards the south or southwest (Fig. 5.6A). Delta-front deposits are locally eroded by fluvial distributary channels oriented towards the south (Figs. 5.6A, 5.7C-D, 5.12), which supports this interpretation of shoreline migration. The delta-front deposits and the clinoforms that they

contain are truncated by erosion at the transgressive surface that defines the top of the Standardville (Ab1) parasequence. Clinoforms are marked by minor non-depositional surfaces, characterised by an increase in bioturbation and mud content. Non-depositional surfaces are observed in both the proximal and distal parts of the delta front succession. Clinoforms bound bedsets formed by the coalescence of individual mouth bars in the proximal part of the delta front and by an upward-coarsening trend and a gradual upward increase in the amalgamation of turbidite sandstone beds in the distal part of the delta front. Bedset thicknesses vary from 1 to 10 metres. The incremental growth of mouth bars and the presence of non-depositional surfaces suggest that bedset formation reflects variations in fluvial discharge and therefore the generation of sandy hyperpycnal flows on the delta front.

Trends in shoreline trajectory are difficult to interpret within the fluvial-dominated delta-front succession, because the shoreline was lobate and local delta-front progradation occurred at a steep angle relative to the outcrop belt. The occurrence of fluvial channel-fill deposits cutting down into the delta front deposits may support the “forced-regressive” shoreline trajectory interpreted by Kamola & Huntoon (1992, 1993), who placed a sequence boundary at the base of the channels. Alternatively, the fluvial channel-fills may more likely represent terminal deltaic distributaries that eroded into older delta-front deposits as the delta continued to prograde under a normal regressive shoreline trajectory (Olariu & Bhattacharya, 2006).

5.6 Palaeogeographic Reconstructions

Figure 5.13 shows a series of interpreted palaeogeographic reconstructions for the prograding Standardville (Ab1) shoreline. The reconstructions are not unique, but they honour all of the outcrop data and they are based on geomorphological and sedimentary processes observed in modern wave-dominated deltas. Each reconstruction shows a specific stage of the shoreline progradation history, as indicated by a change in cross-sectional shoreline trajectory.

The palaeogeographic reconstructions portray a shoreline that was supplied with sediment by both fluvial discharge and alongshore littoral currents, as suggested by evidence detailed in the

facies descriptions. River outlets changed their positions due to avulsion, which may have been forced by autogenic processes and/or by rising relative sea-level. Reconstruction of the fluvial drainage network, and thus the sediment supply, during shoreline progradation is not straightforward because the three-dimensional geometry of the channel network(s) is sparsely sampled by the outcrop dataset. Indeed, poor exposure of coastal plain deposits, including peats that later formed coal seams, presents the greatest uncertainty in these palaeogeographic reconstructions. It should also be borne in mind that the palaeogeographic reconstructions portray only a small (c. 30 km) stretch of the shoreline, which may form part of a much larger wave-dominated deltaic system (Hampson & Howell, 2005).

Figure 5.13A shows the earliest increment of progradation, with the shoreline portrayed as a strandplain. This progradational increment corresponds to facies architecture zone 1 (Fig. 5.12), which is interpreted to record a normal-regressive shoreline trajectory. Multiple beach-ridge sets are inferred, reflecting local rearrangement of the shoreline morphology in response to fluvial channel avulsion outside of the study area. Palaeocurrent data suggest strong southerly transport of sand by longshore currents in the upper shoreface (Fig. 5.6A). Figure 5.13B illustrates a possible rearrangement of the shoreline caused by the first period of forced-regressive shoreline trajectory, interpreted within facies architecture zone 2 (Fig. 5.12). Minor fluvial incision associated with the forced regression may have helped to create a weakly lobate shoreline, because distributary channels were less able to avulse to new locations. Some unidirectional current indicators are observed within the lower shoreface directly offshore of the distributary channels (e.g. at Cordingly Canyon, Fig. 5.2C). Southerly directed longshore currents remained important, as indicated by palaeocurrent data from upper shoreface deposits (Fig. 5.6A). The next increment of shoreline progradation (Fig. 5.13C) is interpreted to record re-establishment of a strandplain within the study area, and corresponds to the lower part of facies architecture zone 3 (Fig. 5.12), which records a normal-regressive shoreline trajectory. Avulsion of the fluvial channel and deltaic distributaries allowed erosion of the abandoned delta lobe, and reworking of the shoreline into a more linear shape. Avulsion may have been promoted by the rising relative sea-level that is recorded by the normal-regressive shoreline trajectory of this zone.

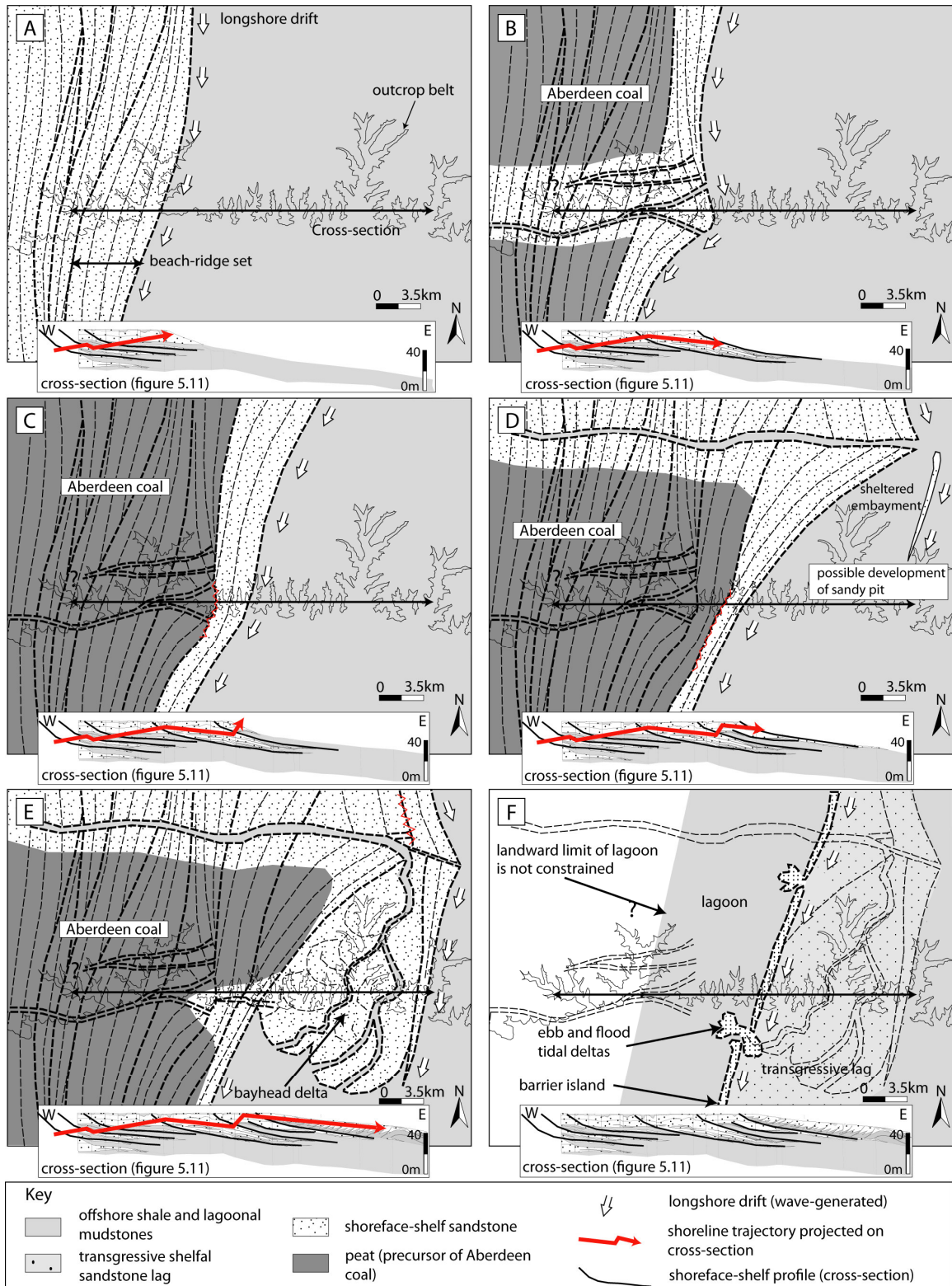


Figure 5.13: Sequential cartoons illustrating possible palaeogeographic development of the Standardville (Ab1) parasequence within the study area. Each palaeogeographic map is linked

to a cross-section along regional depositional dip (inset) that shows an interpretation of shoreline trajectory. (A) Normal regression of a linear strandplain, corresponding to facies architectural zone 1 in Figure 5.12. (B) Forced regression of a near-symmetrical wave-dominated delta lobe, corresponding to facies architectural zone 2 in Figure 5.12. (C) Avulsion of the river channel during a second period of normal regression caused abandonment of the delta lobe and re-establishment of a strandplain, prior to (D) forced regression of a new, asymmetrical wave-dominated delta lobe (facies architectural zone 3, Fig. 5.12). (E) As the asymmetrical wave-dominated delta lobe developed, a fluviially-dominated bayhead delta prograded southward into the sheltered embayment (facies architectural zone 4, Fig. 5.12). (F) A retreating barrier shoreline was created during abandonment and subsequent transgression of the wave-dominated delta system.

Figure 5.13D shows a possible palaeogeography for a second period of forced-regressive shoreline trajectory, corresponding to the upper part of facies architecture zone 3 (Fig. 5.12). The reconstruction shows a strongly asymmetrical wave-dominated delta lobe, which formed due to a combination of southerly directed longshore currents and establishment of a fluvial distributary channel to the north of the outcrop belt. Kamola & Huntoon (1993) suggested that this re-organisation of the shoreline morphology was forced by falling relative sea-level. Fluvial discharge was then deflected southward into a sheltered embayment, possibly as a result of channel branching or avulsion (Fig. 5.13E), implying the establishment of a wave-dominated, sandy spit on the southeastern (downdrift) margin of the deltaic shoreline. Fluvial-dominated delta front deposits are interpreted to result from rapid southerly progradation of a lobate bayhead delta into the sheltered embayment and correspond to facies architecture zone 4 (Fig. 5.12). Although this scenario is consistent with the recent evolution of modern asymmetrical deltas (e.g. the Danube delta; Bhattacharya & Giosan, 2003), it is not unique. An alternative would involve the sheltered embayment having formed behind previously flooded palaeo-topography in the northern part of the study area, perhaps in response to the rapid relative fall of sea level interpreted from the fluvial-dominated delta-front architecture by Kamola & Huntoon (1992, 1993). Subsequently, the entire prograding wave-dominated deltaic edifice was abandoned, and its top was transgressively eroded by a retreating barrier shoreline (Fig. 5.13F).

5.7 Discussion: Intra-Parasequence Architecture as a Tool to Refine Sequence Stratigraphic Interpretations

Shoreface-shelf stratigraphy at intra-parasequence scale is more complex than is represented in current, widely applied facies and sequence stratigraphic models. Conventional sequence stratigraphic analysis relies on the robust identification of sequence boundaries and flooding surfaces, combined with an interpretation of parasequence stacking patterns. This approach implicitly assumes that parasequences have a relatively uniform internal facies architecture with limited lateral variability, such that they record relatively constant environmental conditions over tens to hundreds of thousands of years. However, modern wave-dominated shorelines show important geomorphological and stratigraphic variability due to more frequent changes in environmental conditions over tens to thousands of years. Thus questions arise concerning the robustness of sequence stratigraphic interpretations that do not consider intra-parasequence variability and associated variations in environmental controls at short time-scales. Stratigraphic analysis at intra-parasequence scale can help to improve the robustness of sequence stratigraphic interpretations, by more clearly distinguishing short-term and long-term stratigraphic variability in response to environmental controls. The analysis presented above of the Standardville (Ab1) wave-dominated shoreface-shelf parasequence and “stranded” fluvial-dominated delta-front parasequence provides a good example of how an intra-parasequence-scale approach can significantly improve a pre-existing stratigraphic framework at parasequence scale, as explained below.

The parasequence-scale sequence stratigraphic interpretation of Kamola & Huntoon (1992, 1993) suggested that (1) the wave-dominated shoreface-shelf deposits of the Standardville (Ab1) parasequence and fluvial-dominated delta-front deposits of their “stranded lowstand” parasequence are genetically unrelated, (2) a sequence boundary can be positioned at the base of channel-fill bodies that erosively overlie the “stranded” delta-front deposits, and (3) the fluvial-dominated delta-front deposits were deposited during a rapid relative fall of sea level. Our analysis of intra-parasequence stratigraphic architecture allows more detailed reconstruction of temporal variations in shoreline trajectory and palaeogeography, which provide greater insight into these three aspects of their interpretation.

(1) The analysis presented here shows that there was no major change in water depth between the wave-dominated shoreface-shelf deposits and the fluvial-dominated delta-front deposits, such that they are not separated by a flooding surface but instead belong to the same parasequence. The downlap surface observed between the two successions can be more simply attributed to spatial differences in sedimentary processes and associated geomorphology (e.g. clinoform dip, orientation and plan-view geometry) along the same prograding shoreline. The shoreline re-orientation that occurred across the downlap surface most likely reflects the avulsion of fluvial and distributary channels. However, similar channel avulsions and associated shoreline re-orientations can be interpreted within the wave-dominated shoreface-shelf deposits across bedset boundaries, especially those associated with a change in shoreline trajectory, which likely correspond to beach-ridge-set boundaries (Hampson *et al.*, 2008, Sømme *et al.*, 2008). The different stratigraphic expressions of the shoreline re-orientation arise because the fluvial-dominated deltaic shoreline was deposited in sheltered embayment protected from wave influence (e.g. on the downdrift flank of an asymmetrical wave-dominated delta), whereas the wave-dominated shoreline was subject to wave erosion and sediment reworking once abandoned (e.g. on the updrift flank of an asymmetrical wave-dominated delta). Thus, the entire fluvial-dominated delta-front succession (zone 4 in Fig. 5.12) can be considered as equivalent to a bedset within the wave-dominated shoreface-shelf deposits.

(2) The stratigraphic architecture of the fluvial-dominated delta-front deposits may reflect deposition during a relative sea-level fall, but similar forced regressive shoreline trajectories are interpreted within parts of the wave-dominated shoreface-shelf deposits (e.g. within facies architecture zone 2, Fig. 5.12). Consequently, the erosional bases of channels cut into the fluvial-dominated delta-front succession do not have greater stratigraphic significance than those cut into wave-dominated shoreface-shelf deposits, and the former do not represent a sequence boundary that can be correlated regionally.

(3) The rapid southward progradation of the fluvial-dominated delta front does not require a rapid relative fall of sea level as a driving mechanism, but more likely resulted from large volumes of fluvially derived sediment being deposited over a short timescale within a confined

embayment lacking significant wave influence. Along a wave-dominated shoreline, sediment supplied via a river outlet is dispersed by alongshore littoral currents, resulting in less localised and slower shoreline progradation. This contrast illustrates how the response of different depositional systems can distort the underlying environmental signal, in this case sediment flux, and therefore how a sequence stratigraphic interpretation can become biased when high-resolution palaeogeographic and process-regime evolution are not considered.

5.8 Summary

Quantitative intra-parasequence stratigraphic interpretation and geomorphological analysis applied to the Standardville (Ab1) parasequence illustrates that internal variability and geomorphological structure in an ancient asymmetrical wave-dominated deltaic parasequence is comparably complex to those of analogous modern deltas. Such complexity may need to be considered explicitly when interpreting the stratigraphy and/or rock property distributions within petroleum reservoirs, for example. This study has enabled the revision of an existing parasequence-scale sequence stratigraphic framework in three important respects: (1) wave-dominated shoreface-shelf and fluvial-dominated delta front deposits have been reinterpreted as part of the same asymmetrical wave-dominated deltaic parasequence, rather than as two independent parasequences, (2) a sequence boundary previously interpreted at the top of the delta front deposits has been reinterpreted to have only local significance, and (3) the Standardville parasequence records overall shoreline progradation characterised by alternating periods of normal regressive and forced regressive shoreline trajectory. The approach used in this study is complementary to conventional sequence stratigraphic analysis, and presents the advantage of considering short-term and long-term variations in environmental controls, and the effect of sedimentation style on preservation of these controls. In the next chapter, the stratigraphic inverse and forward modelling approach described in Chapters 2 to 4 will be used to assess the robustness of this interpretation and to aid in distinguishing the role of these temporal variations in environmental parameters. However, following this study, it is now clear that the numerical scheme cannot be applied to the whole wave-dominated deltaic succession of the Standardville parasequence. The two-dimensional, cross-sectional approach of the

numerical scheme cannot simulate the effects of shoreline re-orientation and of changes in progradation direction that occurred during deposition of the fluvial delta front successions. Furthermore, the processes of fluvial-dominated deltaic sedimentation cannot be modelled in the specific “process-based” forward model of BARSIM, which is designed to simulate fair-weather and storm wave processes approach (Chapter 2). Therefore, in the following chapter, I will apply the stratigraphic inverse and forward algorithm to the wave-dominated part of the Standardville parasequence (zones 1-3 in Fig. 5.12, Fig 5.13A, B, C and D), which contains the required limited variability in shoreline orientation and processes. This first application of the numerical modelling scheme to a real dataset will aim to understand the mechanisms driving cross-sectional changes in shoreline trajectory and to predict intra-parasequence wave-dominated stratigraphic architecture from the dataset described above.

Chapter 6

6 First Application: Stratigraphic Inverse and Forward Modelling of the Wave-Dominated “Standardville” Parasequence of the Aberdeen Member.

Abstract

In this chapter, I will present the first application of the stratigraphic inverse and forward modelling algorithm to a real dataset: the first parasequence (Ab0) and the wave-dominated part of the second parasequence (Ab1) of the Aberdeen Member of the Blackhawk Formation (Book Cliffs, Utah, USA). In this first case-study, the aim of the inversion is to infer the relative sea level, sediment supply, wave height and mud fallout histories from the thicknesses and grain-size signals of eight vertical sections previously logged in the field. The inverse modelling approach successfully reproduces the gross stratigraphic architecture of the two progradationally stacked parasequences as well as the gross internal architecture of the second parasequence (Standardville, Ab1 parasequence), within the constrained location. Of equal importance, the inverse results confirm and quantify the qualitative stratigraphic interpretations presented in chapter 5, but with less constraining data. This work can be partly found in Charvin et al. (2008d).

6.1 Introduction

Modern regressive, wave-dominated linear shorelines are characterised by the prevalence of reworked sediment supplied by alongshore littoral currents from wave erosion of older shorelines. In these systems, sedimentation is mainly controlled by the dual action of fair-weather and storm wave processes. During storm periods, the shoreface is eroded and the remobilised sediments are either transported offshore or into back-shore environments. During fair-weather periods, the shoreface system recovers and steepens as sand is fed back to it by asymmetrical shoaling waves and associated longshore drift currents.

Ancient regressive wave-dominated deposits are commonly preserved as a coarsening-up facies succession which records progressive shallowing of water depth (i.e. the parasequence of Van Wagoner et al., 1990). Each parasequence is capped by a condensed marine shale that records flooding of the sedimentary environment (i.e. the flooding surface of Van Wagoner et al., 1990) and is generally interpreted to result from a regional transgressive and regressive cycle with a period of approximately 10^5 - 10^6 years. The sandstone-dominated character of these parasequences often obscures complex internal stratigraphy which reflects shoreline evolution over shorter timescales (10^2 - 10^4 years). Much of this internal variability is expressed by multiple clinoforms surfaces that are interpreted to represent remnants of the shoreface/shelf profile. Clinoforms are marked by minor stratigraphic discontinuities which bound successions of thin (10^0 - 10^1 m) genetically related beds referred to as bedsets (Van Wagoner et al., 1990). Hampson (2000) and more recently Hampson & Storms (2003), Storms & Hampson (2005), Sømme et al. (2008) and Charvin et al. (2008c) documented two types of clinoformal discontinuity surfaces: non-depositional and erosional discontinuities.

Non-depositional discontinuity surfaces bound upward-coarsening bedsets within lower shoreface and shelf deposits. They are interpreted to be characteristic of a decrease in sedimentation rate which may be associated with a relative rise in sea level and/or a decrease in storm energy. Their formation is related to the rearrangements of shoreline geometry due to erosion/deposition of beach ridges and beach ridge sets (Hampson, 2000; Hampson & Storms, 2003; Storms & Hampson, 2005; Sømme et al., 2008; Hampson et al., 2008).

Erosional discontinuity surfaces bound upward-fining bedsets within lower shoreface and shelf deposits. They are characterised by erosion, and an increase in storm event bed amalgamation. They are interpreted to record a lowering of the storm wave base in response to a more energetic storm climate and/or a minor relative fall in sea level (Hampson, 2000; Hampson & Storms, 2003; Storms & Hampson, 2005).

The distributions of the clinoform surfaces and of the facies association belts that they bound record the detailed migration of the shoreface-shelf profile during progradation (shoreline trajectory *sensu* Helland-Hansen & Martinsen, 1996). In fact, zones of strong progradation within the parasequence (associated with the “downstepping” of facies belts and amalgamation of successive erosional discontinuity surfaces) have been interpreted to record episodes of “forced” regression (*sensu* Posamentier et al., 1992). In contrast, zones of strong aggradation within the parasequence (characterised by vertical stacking of bedset bounded by non-depositional discontinuity surfaces) have been interpreted to record episodes of “normal” regression (*sensu* Posamentier et al., 1992). Clinoform distribution is interpreted to be related to local variations in basin parameters (e.g. relative sea level, sediment supply and wave regime) superimposed on regional regression (Storms & Hampson, 2005). Thus, the analysis of their spatial distribution provides a tool to assess explicitly the short- and long-term variation in these environmental parameters.

To date, intra-parasequence variability in wave-dominated shoreface-shelf deposits has been reported by several workers (Campbell, 1979; Valasek, 1990; O’Byrne & Flint, 1995; Pattison, 1995; Jennette & Riley, 1996; Olsen et al., 1999; Hampson 2000; Seidler & Steel, 2001; Hampson & Storms, 2003; Hampson & Howell, 2005; Storms & Hampson, 2005; Sømme et al., 2008; Hampson et al, 2008, Charvin et al., 2008c), but such variability is still rarely considered when applying sequence stratigraphic methods. In fact, parasequences are still commonly regarded as the “building blocks” of sequence stratigraphic interpretation (Van Wagoner et al., 1990; Kamola & Van Wagoner, 1995), although complementary analysis of intra-parasequence facies architecture provides more robustness by considering the entire environmental signal over a wider range of time scales. However, it is clear that this technique is limited by data quantity and quality. Moreover, it relies on understanding of the driving mechanisms creating the discontinuity surfaces, which cannot be determined directly from

modern analogues because of the temporal scales involved. Although their formation has been successfully investigated from several field examples and from numerical forward modelling experiments using BARSIM (Storms & Hampson, 2005; Sømme et al., 2008), the range of interactions between the causative mechanisms and quantification of their relative influence have not been fully investigated. Hence, mathematical inversion represents a means to reconcile the theoretical forward model with observed geological data and thus to infer driving mechanisms at intra-parasequence scales.

Therefore, in this chapter, I will present an application of the stratigraphic inversion scheme to the first and second parasequences (respectively Ab0 and Ab1) of the Aberdeen Member (Blackhawk Formation, Book Cliffs, Utah) that were described in the previous chapter, with a special focus on the well-constrained internal architecture of the wave-dominated part of the Standardville Parasequence (Ab1). The aims of this work are threefold: (1) to test the applicability of the numerical scheme to a real dataset, (2) to infer environmental parameters (e.g. relative sea level, sediment supply, wave regime and mud fallout rate) and their respective uncertainty from the observed data, and (3) to predict facies distributions. The chapter is structured so that after having presented the geological setting and dataset in parts 6.2 and 6.3, the modelling workflow is described in part 6.4 and data preparation in part 6.5. Stratigraphic inverse and forward modelling parameterisation is presented in part 6.6 and the numerical results and interpretations in parts 6.7 and 6.8. A discussion and summary are provided in parts 6.9 and 6.10.

6.2 Geological Setting

The Upper Cretaceous Aberdeen Member of the Blackhawk Formation is exposed in the northern part of the Book Cliffs of Central Utah (USA). The Aberdeen Member comprises five wave-dominated shoreface-shelf parasequences with the upper two truncated by an incised valley network (Kamola & Huntoon, 1995). This study focuses on the two oldest parasequences of the Aberdeen Member, respectively labelled Ab0 and Ab1 (or “Standardville” parasequence), and especially emphasises the internal stratigraphic architecture

of the Standarville parasequence. The overall distance of shoreline progradation for these two parasequences is about 35 km (Fig. 6.1).

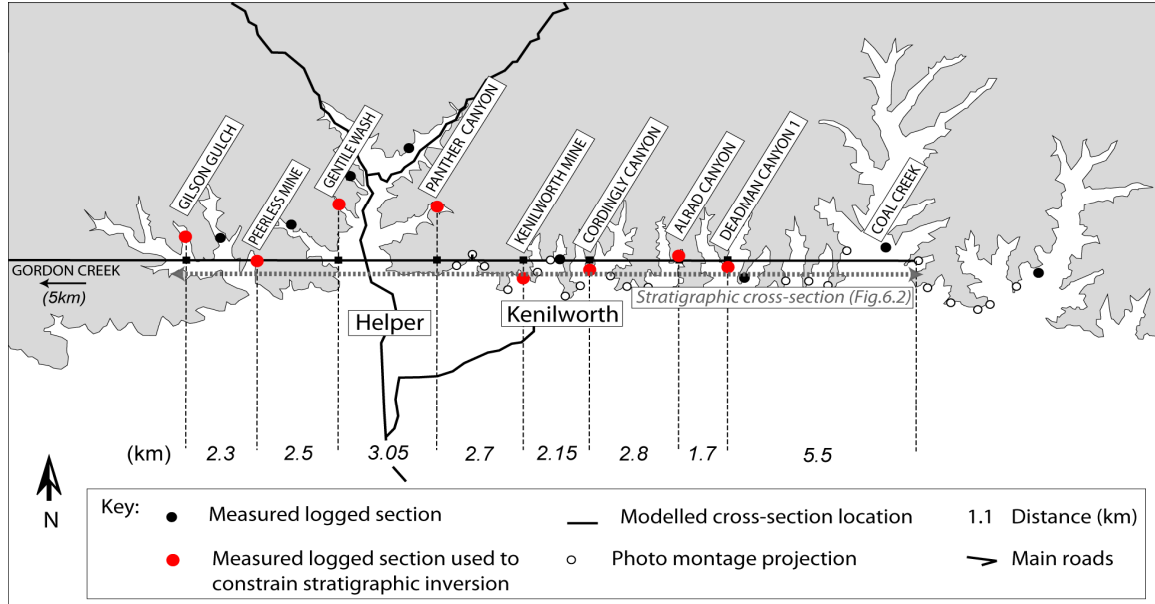


Figure 6.1: Dataset location. Map showing the distribution of cliff-face exposure, measured sections collected in the field, a subset of these measured sections that was used to constrain the stratigraphic inversion, and locations of the detailed cross-section shown in Figure 6.2 and investigated by stratigraphic inversion.

The Standarville parasequence contains wave-dominated shoreface-shelf and fluvial-dominated delta front deposits. The wave-dominated part of the Standarville parasequence can be traced from near its palaeo-landward termination at Gordon Creek to its palaeo-seaward termination at Alrad Canyon (Figs. 6.1, 6.2). It records a single episode of easterly progradation of a generally north-south trending shoreline fed by a strong southerly transport of sand by longshore currents. Its distal part is overlain across a downlap surface by southerly prograding fluvial-dominated delta-front deposits. High-resolution stratigraphic analysis of these deposits reveals that they record a single episode of shoreline progradation characterised by alternating periods of normal regressive and forced regressive shoreline trajectory and that four geographical zones can be recognised as a result (Fig. 6.2): (1) west of Panther Canyon, (2) between Panther Canyon and Cordingly Canyon, (3) between Cordingly Canyon and Deadman Canyon and (4) east of Deadman Canyon. Normal regressive shoreline trajectories

within the wave-dominated shoreface-shelf deposits are marked by aggradational stacking of bedsets bounded by non-depositional discontinuity surfaces, interpreted to record the deposition of beach ridge sets (Zone 1 and lower part of Zone 3 in Fig. 6.2; Fig. 6.3). Forced regressive shoreline trajectories in the same deposits are characterised by shallow incision of fluvial distributary channels and strongly progradational stacking of bedsets mainly bounded by erosional discontinuity surfaces that record enhanced wave-base scour (Zone 2 and upper part of Zone 3 in Fig. 6.2; Fig. 6.3). Fluvial-dominated delta-front deposits record the regression of a bayhead delta into a sheltered embayment lacking wave influence (Zone 4 in Fig. 6.2; Fig. 6.3). The wave-dominated shoreface-shelf and fluvial-dominated delta-front deposits have been interpreted to represent respectively the updrift and downdrift flanks of a single asymmetrical wave-dominated delta that is characterised by highly active wave and fluvial processes (Fig. 6.3). On its updrift flank the delta resembles a strandplain as wave processes are dominant, whereas on its downdrift flank, the fluvial sediments are protected from wave reworking and thus preserved (e.g. Elliott, 1986; Dominguez *et al.*, 1987; Boyd *et al.*, 1992; Bhattacharya & Giosan, 2003, Bhattacharya, 2006).

6.3 Dataset

The inversion scheme has been applied to the first and second parasequences of the Aberdeen Member (Fig. 6.2), with the aim of quantifying the parameters that represent the causative mechanisms responsible for the complex internal stratigraphic architecture of the wave-dominated part of the Standardville parasequence (Ab1). Previous stratigraphic analysis demonstrates that the limited along-strike variability of such shoreface-shelf deposits and their wave-dominated character satisfy the requirements to be investigated by numerical scheme. However, the fluvial-dominated deltaic succession representing the downdrift flank of the asymmetric wave-dominated delta will not be investigated by the modelling technique. This choice has been motivated by two reasons. Firstly, the BARSIM algorithm was not designed to model fluvially driven shallow-marine deltaic processes. Secondly, the two-dimensional approach of the numerical scheme can not account for the drastic changes in shoreline orientation and progradation direction that occurred during the deposition of this fluvial-dominated deltaic succession (Fig. 6.3).

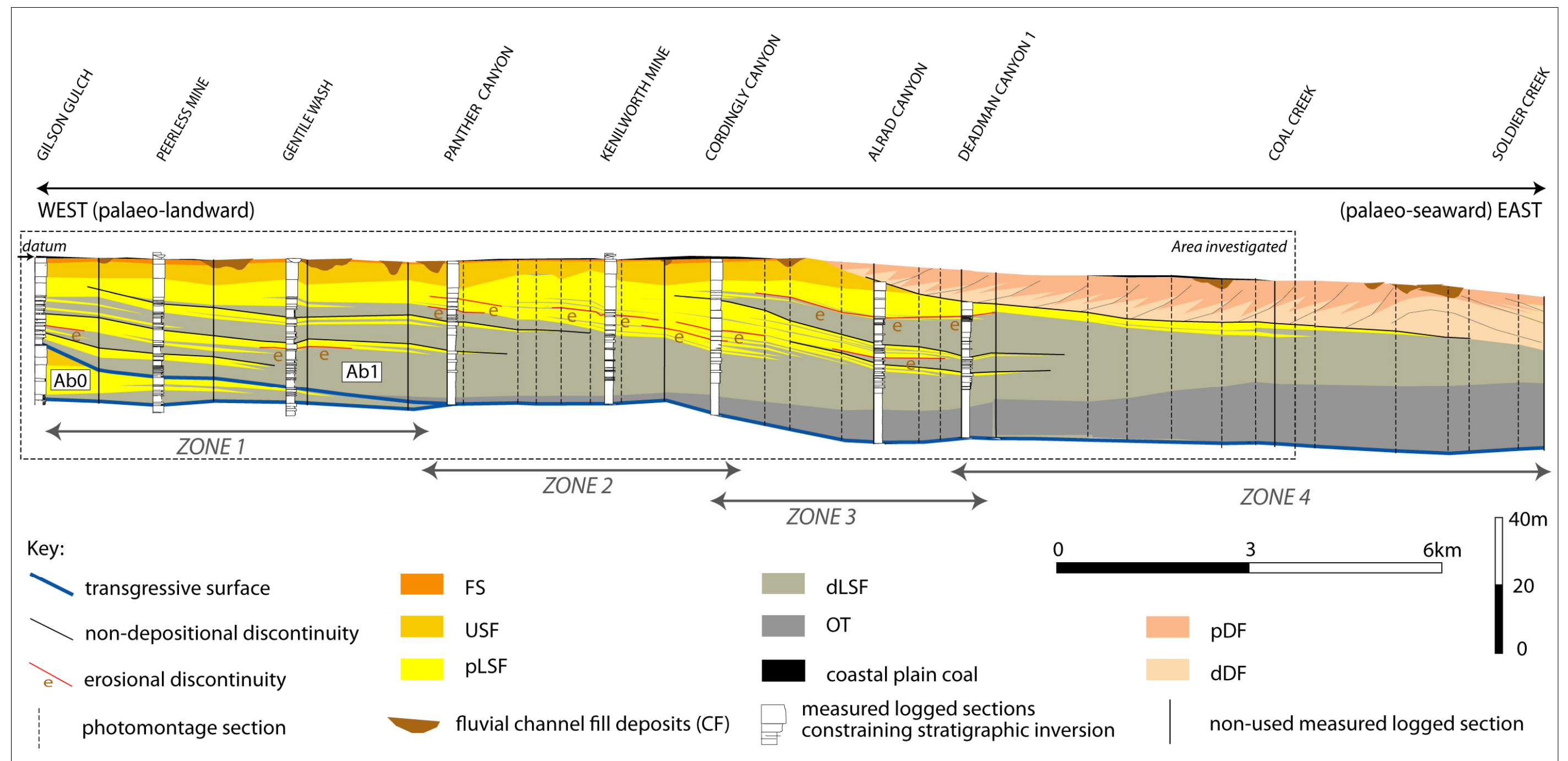


Figure 6.2: Correlation panel of the first (Ab0) and second (Standardville, Ab1) parasequences of the Aberdeen Member. The panel is aligned parallel to regional depositional dip and shows measured cross-sections used to constrain the stratigraphic inversion. The location of the panel is shown in Figure 6.2. The datum surface for the section is the base of the coal seam that caps the Standardville (Ab1) parasequence. Stratigraphic architecture at both parasequence and intra-parasequence scales is shown. Linear wave-dominated shorelines in each parasequence prograded from west to east. The distribution of non-depositional and erosional discontinuity surfaces that define easterly dipping clinoforms within the wave-dominated shoreface-shelf deposits of the lower Standardville (Ab1) parasequence has been used to define four distinct geographical zones, each interpreted to have a distinctive shoreline trajectory. The area investigated by the inverse and forward modelling approach is highlighted by the dashed box and comprises the wave-dominated shoreface-shelf succession.

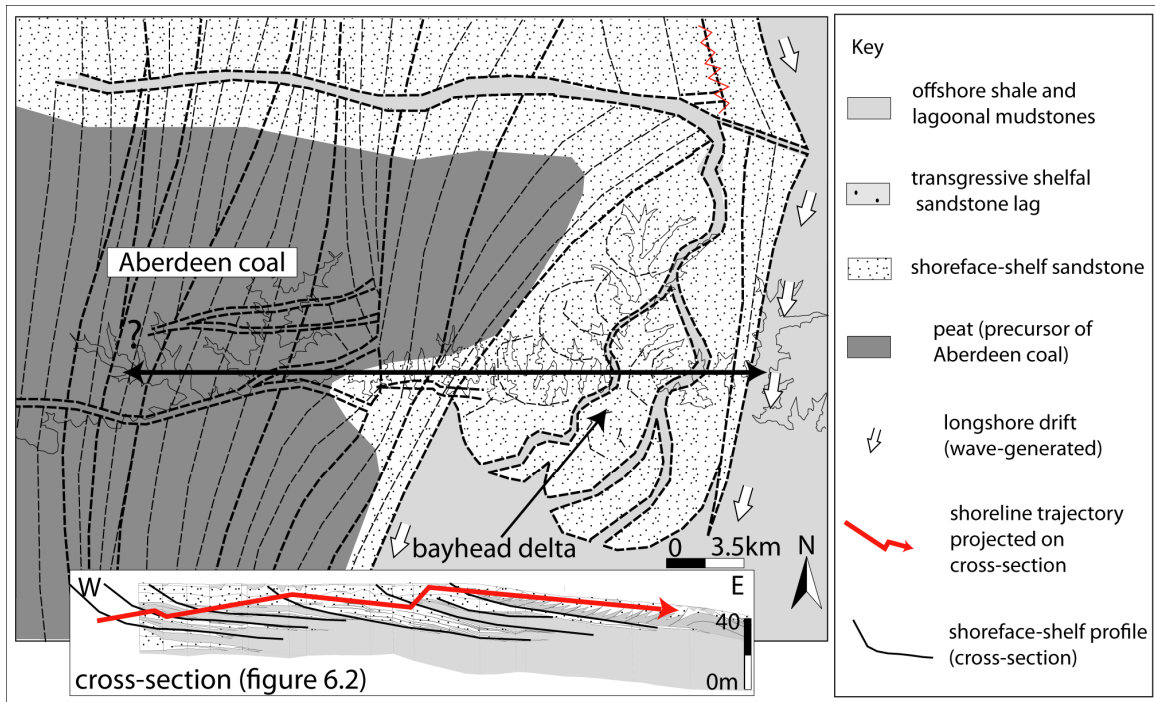


Figure 6.3: Cartoon illustrating possible palaeogeographic development of the Standardville (Ab1) parasequence within the study area. The palaeogeographic map is linked to the cross-section along regional depositional dip (inset) that shows an interpretation of shoreline trajectory. The Standardville parasequence comprises wave-dominated shoreface-shelf and fluvial-dominated delta-front deposits that have been interpreted to represent respectively the updrift and downdrift flanks of a single asymmetrical wave-dominated delta. The shoreface-shelf deposits prograded eastward whereas the fluvial delta front deposits prograded southward.

The stratigraphic analysis and correlation panel presented in Figure 6.2 have been realised from a complete dataset comprising 16 measured logged sections and a complete set of interpreted cliff face photomontages (Chapter 5). However, for this application, the numerical scheme will be constrained only by 8 measured logged sections (Figs. 6.1, 6.2). The choice of conditioning data has been motivated by two reasons. Firstly, as demonstrated in Chapter 4, the observed data quantity is not the most important criterion in constraining stratigraphic inversion, provided that the available data captures as much information as possible about the complexity and history of the depositional system, which is the case for the selected data. Secondly, increasing the amount of data will not always lead to better constraint of the inversion. On the

contrary, such data may add more noise, because of the effect of projecting these additional data into the cross-section. As a result, the additional data may complicate the convergence process by perturbing the main signal information. The eight selected logged-sections cover 17 km (from Gentile Wash to Deadman Canyon) along a cross-section oriented along regional depositional dip and extending from Gordon Creek to Coal Creek (Fig. 6.1). In this cross-section, no data are available to constrain the palaeo-landward stratigraphy cropping out from Gordon Creek to Gentile Wash (Fig. 6.1). The stratigraphic inversion will not be constrained by thickness curves calculated for a specific stratigraphic interval, but only by the thicknesses of each logged section. These sections are distributed relatively uniformly and with close spacing (1-3 km) over the studied area.

6.4 Modelling Workflow

The workflow adopted in this first application consisted of the following steps:

- (1) Attributing numerical grain-size values to the sections logged in the field.
- (2) Decompacting the sedimentary succession in each measured logged section.
- (3) Designing the initial physiographic cross-section profile on which sedimentation occurred.
- (4) Parameterisation of the stratigraphic inverse and forward numerical method by adding geological prior knowledge.
- (5) Running the stratigraphic inverse and forward algorithm.
- (6) Post-processing the output data before interpretation.

6.5 Observed Data Preparation and Errors Estimation

6.5.1 Grain Size Estimation

Grain sizes of the different facies associations described in Chapter 5 (Table 5.1) have been determined qualitatively in the field by using Wentworth's (1922) nomenclature. The first step

of data preparation consisted of transforming qualitative field descriptions into numerical values and then estimating errors on these values. This process has been conducted for the qualitative field description of each facies association (Chapter 5, Table 5.1) by taking the mid point between the minimum and maximum values defining the corresponding grain-size class in Wentworth's (1922) nomenclature. Errors on these values have been estimated by using the range of variation between the minimum and maximum values for a particular grain-size class. Assuming that error distributions are Gaussian, centred on the estimated values, and that 100% of the possible numerical values are contained within this range (i.e. the grain-size range is more or less equal to the interval $[\text{Mean} - 3 \times \text{STD}; \text{Mean} + 3 \times \text{STD}]$), the standard deviations (STD) of these error distributions have been estimated by dividing the ranges by six. Results are summarised in Table 6.1.

Facies association	Qualitative grain-size description	Grain diameter (Wentworth, 1922)		Modelled grain diameter	Estimated errors (STD)
		Min. (mm)	Max. (mm)		
Foreshore (FS)	Upper fine-grained sandstone	0.125	0.170	0.147	0.0075
Upper shoreface (USF)	Upper fine- to medium-grained sandstone	0.125	0.210	0.167	0.014
Proximal lower shoreface (pLSF)	Upper fine-grained sandstone	0.125	0.170	0.147	0.0075
Distal lower shoreface (dLSF)	fine-grained sandstone	0.060	0.125	0.092	0.01
	Siltstone and mudstone	0.002	0.030	0.016	0.0046
Offshore transition (OT)	Siltstone to very fine-grained sandstone	0.010	0.060	0.035	0.0083
	Mudstone	0.001	0.005	0.003	0.0006

Table 6.1: Grain size numerical values and estimated errors.

6.5.2 Decompaction

The BARSIM forward model of stratigraphy does not include a dynamic compaction algorithm, therefore the measured sections have to be decompacted prior to the stratigraphic inversion process. Compaction effects during sedimentation, due to sediment's own load, also cannot be simulated. As a consequence, the inference of the environmental parameters will be systematically but slightly overestimated. To decompact the investigated strata, the maximum burial depth had to be estimated. Nuccio & Roberts (2003) used vitrinite reflectance data to estimate a maximum burial depth for the top of the Mancos Shale (Fig. 5.2, Chapter 5) equal to 11000 feet (3700 m). Therefore, it is reasonable to infer that the maximum burial depth for the Aberdeen Member was 4000 m. Based on this value, the studied stratigraphic interval has been decompacted using the porosity/depth relation of Athy (1930) calibrated by exponents derived from Sclater & Christie (1980) (Table 6.2).

Lithology	Surface porosity ϕ_0	Porosity-depth coefficient $c(km^{-1})$
Shale	0.63	0.51
Sandstone	0.49	0.27
Shaly sandstone	0.56	0.39

Table 6.2: Compaction exponents derived by Sclater & Christie (1980) for different lithologies from North Sea sub-surface data.

To assess uncertainty on the decompaction process, several maximum burial depths (3700, 4000 and 4300m) as well as several exponent values (obtained by perturbing Sclater & Christie's, 1980 original exponents) have been tested for each measured section. The decompacted thickness of the entire cross-section, used as observed data in the likelihood function, has been set to be equal to the average of the values obtained from the different tests. The standard deviations calculated from these values have been used as measurement errors in the likelihood function. Results are summarised in Table 7.3.

Logged sections	Total measured thickness (m)	Averaged decompacted thickness (m)	Standard deviations (m)
Gilson Gulch	42.4	71.5	1.5
Peerless Mine	41.9	70.5	1.5
Gentile Wash	42.1	71.5	1.5
Panther Canyon	38.9	72.5	2.0
Kenilworth Mine	43.3	75.5	2.0
Cordingly Canyon	45.4	77.5	1.5
Alrad Canyon	46.7	83.0	2.5
Deadman Canyon	37.9	73.0	2.0

Table 6.3 Initial measured and averaged decompacted total thickness of the Standardville parasequence at each logged section.

6.5.3 Initial Topographic Profile Design

Specifying the initial physiography of the cross-section on which sedimentation occurred (initial condition parameters of BARSIM, Chapter 2; §2.2.2.3) is an important stage in the inverse and forward modelling workflow. This stage consisted of two steps.

- (1) Reconstruction of the initial topographic profile. This step has been achieved by setting the initial palaeo-water depths at each logged section location to be equal to the corresponding total decompacted thickness. Then values have been assigned to the numerical forward modelling grid by using linear interpolation between the values at each logged section location. This method assumes that the initial topography was completely infilled during progradation of the shoreline, which is reasonable in the palaeo-landward part of the cross-section where the Standardville parasequence is capped by roots and a coal that indicate subaerial exposure (Fig. 6.2). In the palaeo-seaward part of the cross-section that lacks evidence of such subaerial exposure (e.g. at Alrad Canyon and Deadman Canyon, Fig. 6.2), the palaeo-water depths have been determined by adding correction values to the total decompacted thicknesses in order to account for the lack of upper shoreface and foreshore deposits. In the most palaeo-

landward part of the cross-section (i.e. west of Gilson Gulch, Fig. 6.2), where no data are available to constrain the palaeo-topography reconstruction, palaeo-water depth values have been estimated using a slope gradient.

- (2) Evaluation of the underlying grain-size distribution of the substrate. This step has been achieved by using the grain-size values of the top of the underlying Spring Canyon Member at the logged section locations. Then, values have been linearly interpolated into the numerical forward modelling grid from measured section to measured section. This simple approach is justified by the high density of data coverage in the study area, and also supports the decision to model the underlying parasequence (Ab0) in addition to the wave dominated part of the Standardville parasequence (Ab1).

6.6 Stratigraphic Inverse and Forward Model Parameterisation

6.6.1 Stratigraphic Forward Model Parameterisation

The BARSIM forward model grid comprises 300 cells of 100 m each in the horizontal direction. Vertical grid resolution is self-generated according to the thickness of sediment deposited. Each member of the Blackhawk Formation represents an approximate duration of 0.5-0.6 Ma (Fouch et al., 1983). Therefore, sedimentation was simulated with BARSIM over a period of 125,000 years (comprising the estimated duration of the Ab0 parasequence and of the wave-dominated part of the Ab1 parasequence), with a minimum recurrence period of 100 years for major storm events.

Mud fallout rate, relative sea level, sediment supply and wave regime will be inferred by the stratigraphic inverse scheme. The mud fallout parameter represents the mud deposited by suspension independently from the mud added by longshore drift currents and directly subject to offshore transport. Mud fallout is simulated using a uniform rate imposed across the whole shoreface-shelf profile during fair-weather conditions. Prior stratigraphic study of the Aberdeen Member (Chapter 5) showed that no major structural events occurred during the

deposition of the “Standardville” parasequence. Therefore, the differential subsidence rate module of BARSIM has been switched off. The initial composition of sediment supplied by longshore drift currents is one of the internal condition parameters of BARSIM (Chapter 2; §2.2.2.3), and was determined as explained below. Analysis of the grain-size distribution within all the logged sections shows that the lithological composition of the Standardville parasequence is mainly an association of upper-fine-grained sandstones and shales, with overall estimated values of grain diameter of around 160µm and 3µm, respectively (Table 6.1). This analysis, combined with the decompacted thicknesses of sandstones and shales (cf. Table 6.2), was used to set the initial composition of the sediment supplied by longshore drift currents as a mixture of 75% shale (3µm grain diameter) and 25% upper fine sand (160µm grain diameter).

Internal Parameters		External Parameters	
		<i>Initial condition parameters</i>	
Maximum erosion rate [m/y]	2.5		
Power for erosion scaling: shoreline – wave base [-]	1.0	Grain size (shale class) [µm]	3
Sediment travel distance correlation event [-]	3.0	Grain size (Sand class) [µm]	160
Sediment travel distance correlation shoreface [-]	1.0	Supply composition (% of sand)	25
Maximum height and width of the barrier [m]	1.5 -1500		

Table 6.4: BARSIM parameterisation.

Other internal parameters of BARSIM (Chapter 2, §2.2.2.3) have been determined through preliminary tests (“sediment travel distance correlation event”, “sediment travel distance correlation shoreface”) or by using default values (“power for erosion scaling: shoreline – wave base”, “maximum erosion rate”, “maximum height and width of the barrier”). The maximum erosion rate is independent of the substrate properties. The parameter labelled “power for erosion scaling: shoreline – wave base” is a constant that represents the dependence of erosion rate on water depth. The “sediment travel distances correlation event” and “sediment travel distances correlation shoreface” parameters account for local variability in sediment transport during storm and fair-weather conditions, respectively. These two parameters are

constant values that are used to match specific local coastal settings for the estimation of sediment travel distance to the characteristic sediment deposition curves of Guillén & Hoekstra (1996). Higher values of these parameters result in sediment transport over longer distances. Both parameters have been chosen on the basis of preliminary tests, such that convergence of the inverse numerical solutions was achieved most rapidly and such that the obtained geometries were sedimentologically realistic and roughly concordant with the observed stratigraphy. Values are summarised in Table 6.4.

6.6.2 Inverse Method Parameterisation

The strategy developed to solve this stratigraphic inverse problem combines three Reversible Jump samples, one fixed dimension MCMC sampler and one simulated Tempering sampler linked in a Hybrid sampler (Chapter 3; Mariani & Parisi, 1992; Geyer & Thompson, 1995; Green, 1995; Gilks et al., 1996; Denison et al., 2002, Brooks et al., 2003; Charvin et al., 2008a). Details are compiled in Table 6.5.

MCMC-based sampler	BarSim environmental input parameters
Reversible Jump MCMC	Relative sea level curve
Reversible Jump MCMC	Sediment supply curve
Reversible Jump MCMC	Wave height curve
Fixed Dimension MCMC	Mud fallout rate (shoreface)
Simulated Tempering	“Temperature” scheme

Table 6.5: MCMC-based samplers and their application to BarSim input environmental parameters. The fixed dimension MCMC sampler aims to deal with the time-independent mud fallout rate input parameter of BARSIM, whereas the RJMCMC samplers are applied to time dependant curves that described the other input parameters of BARSIM. The Simulated Tempering sampler is designed to deal with multi-modal posterior probability distribution (i.e. non-uniqueness of the inverse problem). The inversion scheme was run for 200,000 forward modelling iterations in order to fully explore the model parameter space.

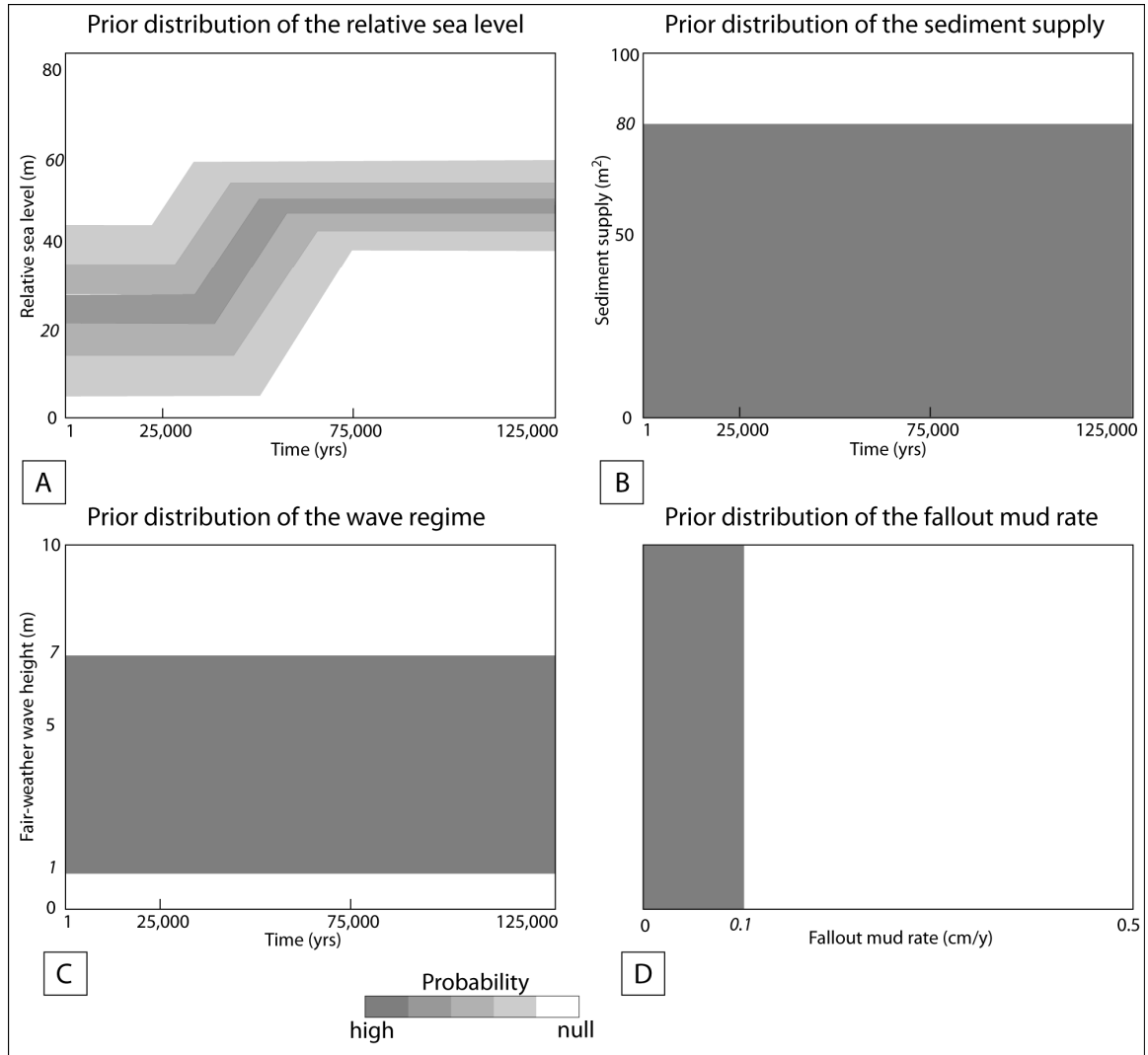


Figure 6.4: Environmental parameter prior probability distributions for the Aberdeen Member case study. (A) Cartoon illustrating the time-dependent Gaussian prior distribution of relative sea level history. (B, C and D) Uniform prior distributions respectively for sediment supply, wave regime and mud fallout rate.

Based on the stratigraphic analysis presented in Chapter 5, it was decided to use a time-dependent Gaussian distribution as prior information for the relative sea level history. This distribution, in which both the mean value and standard deviation vary with simulated time of sedimentation, is presented in Figure 6.4. The general shape of this prior distribution and especially the steep rise in relative sea level are based on the interpretation of a flooding

surface that separates the two parasequences Ab0 and Ab1. For the sediment supplied by longshore drift currents, the wave regime and the mud fallout rate, minimal prior information was used, such that values of these parameters were allocated uniform probabilities *a priori* between 0–80 m²/y, 1–7 m and 0–0.1 cm/y, respectively. The ranges of possible variation in sediment supply and mud fallout rate have been chosen to be sufficiently wide that prior knowledge does not over-influence the inversion in terms of the quantity of sediment preserved in the cross-section. In fact, constant sediment supply and mud fallout rates of 80 m²/y and 0.1 cm/y would provide much more sediment that is present in the cross-section, and thus these values are reasonable upper limits for the two parameters. The range of possible variation in fair-weather wave height has been chosen arbitrarily, but is again broad enough that the assumed prior knowledge does not dominate the inversion. Although 7 m is the upper limit of fair-weather wave height considered by the inversion, storm wave heights which are drawn from fair-weather wave height by the stochastic component described in Chapter 2 are not limited by any prior knowledge and thus can be greater than 7 m. Generally, higher storm wave heights correspond to higher fair-weather wave heights. During the inversion process, environmental model parameters have been perturbed by Gaussian proposal distributions centred on their current values. Details are presented in Table 6.6.

Parameters		Prior distribution	Proposal variance Fixed dim. jump	Proposal variance Trans-dim. jump
Relative sea level	- Sea level values	time-dependent Gaussian	1.5 m	2m
	- Time values	1 – 125,000 yrs	200 yrs	100 yrs
Sediment supply	- Supply values	0 - 80 m ² /y	1.5 m ² /y	5 m ² /y
	- Time values	1 – 125,000 yrs	200 yrs	100 yrs
Wave height regime	- Wave height values	1 - 7 m	0.5 m	1m
	- Time values	1 – 125,000 yrs	200 yrs	100 yrs
Mud fallout	- Fallout rate values	0 – 0.1 cm/y	0.001 cm/y	NA
Temperature level	3 levels: T={1,2,3}			

Table 6.6: Prior and proposal distributions for relative sea level, sediment supply, wave height regime and mud fallout parameters. Sediment supply, wave height regime and mud

fallout parameters are allowed to vary uniformly between minimum and maximum values summarized in the second column. Candidate model parameters are drawn from Gaussian distributions centred on the current parameters. Variances defining the Gaussian proposal distributions for fixed dimensional and trans-dimensional perturbations are presented in the third and fourth columns, respectively.

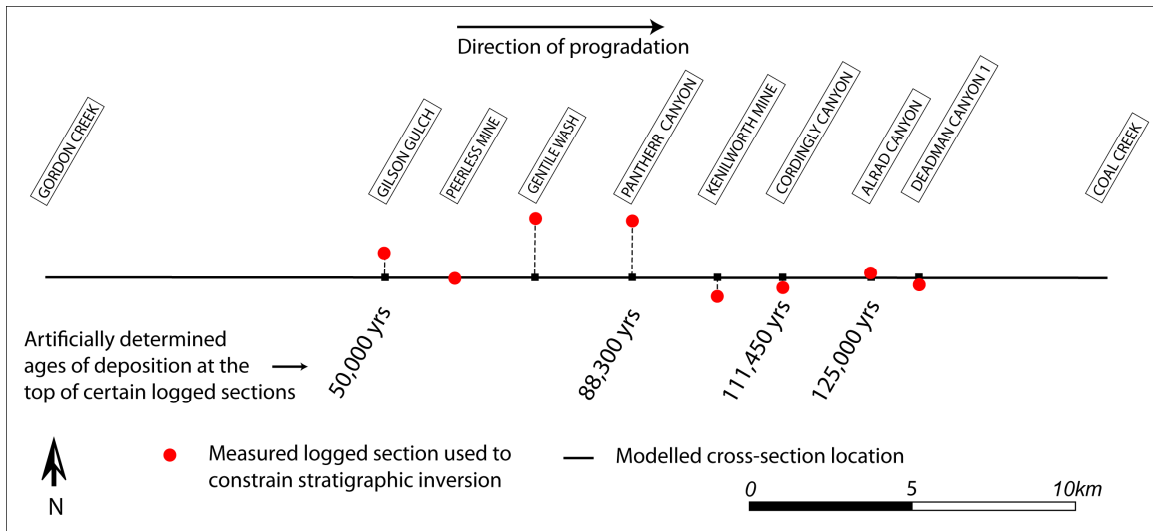


Figure 6.5: Estimated depositional ages attributed to the top of Gilson Gulch, Panther Canyon, Cordingly Canyon and Alrad Canyon logged sections. Each age has been estimated assuming a constant progradation rate and direction over the simulation time period, such that they increase linearly in a palaeo-seaward direction.

Models parameters for initialising the Markov chain were not chosen randomly but manually, sufficiently close to the solution area to decrease the burn-in period and thus computational costs, as suggested in Chapter 4. Values for the first model parameters are presented with the resulting posterior probability distribution in the subsequent analysis (Fig. 6.6). The stratigraphic inversion was constrained by the grain-size signal and the total decompacted thickness of the eight measured logged sections. In this formulation, back-barrier deposits were not incorporated into the likelihood function. No data were available to constrain the timing during of the different stages of shoreline progradation for the Ab0 and Ab1 parasequences. Thus estimated depositional ages were attributed to the top of some sections (i.e. Gilson Gulch,

Panther Canyon, Cordingly Canyon and Alrad Canyon) as required by the likelihood function formulation (Fig. 6.5). These ages have been determined by assuming a constant progradation rate parallel to the cross-section for the two parasequences over the simulated time interval. This approach was necessary to constrain the inversion of the time-dependant parameters (i.e. relative sea level, sediment supply and wave regime), but relatively errors in these age estimates (standard deviation of 2500 years) have been assumed to allow some flexibility in the inversion process.

6.7 Results

The results presented in the subsequent paragraphs have been determined with the post burn-in period samples from the “cold” distribution (Chapters 3 and 4). Therefore, only about 50,000 samples, among the 200,000 available, have been used to assemble the subsequent posterior probability distributions. This inversion requires about one week of computational cost on a standard single-processor PC.

6.7.1 Inferred Posterior Probability Distributions for Environmental Parameters

The inferred posterior probability distributions for the four environmental parameters considered in this case study are shown in Figure 6.6. Each posterior probability distribution is shown with the boundaries of the 95% Bayesian credible interval, the maximum likelihood estimates and the parameters chosen as the initial step of the Markov chain. Averaged values of the parameters are not presented in this figure. This choice has been motivated by the results of Chapter 4, which show that such central estimators are not representative when the posterior probability distributions are complex (i.e. multi-modal distributions).

6.7.1.1 Posterior Probability Distribution of Relative Sea Level

Figure 6.6A shows the inferred posterior probability distribution of the relative sea level curve. The distribution clearly shows two distinct periods, from approximately 0 to 35,000 years and

40,000 to 125,000 years of simulated time. These two periods are separated by a short interval of rapid relative rise of sea level of about 30–40 m, between 35,000 and 40,000 years of simulated time. The first period represents deposition of the first parasequence (Ab0) whereas the second period represents of the Standarville parasequence (Ab1).

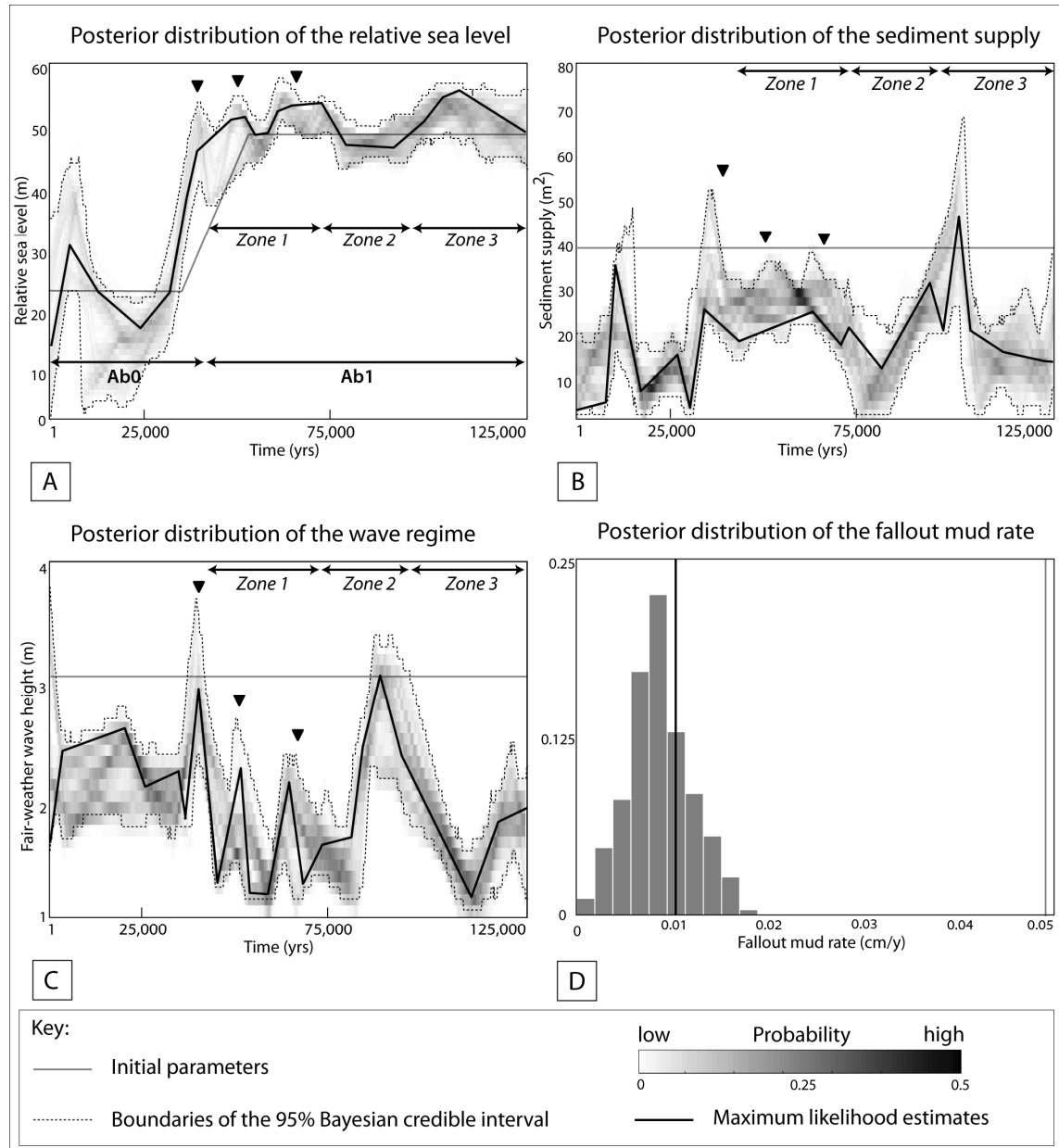


Figure 6.6: Posterior probability distributions for the modeled environmental parameters: (A) relative sea level curve, (B) sediment supply curve, (C) fair-weather wave height curve, and

(D) mud fallout rate. The dark continuous line in each graph is the maximum likelihood estimate; the light grey line represents the initial input curve and the dash lines represent the boundaries of the 95% Bayesian credible interval. The background shading represents the posterior probability distribution (dark means higher probability). Zones 1, 2 and 3 are time equivalent to the geographical zones 1, 2 and 3 of distinctive facies architecture within the Standardville parasequence (Fig. 6.2).

In the first period, the inference of relative sea level is not well resolved, such that the spread of the 95% Bayesian credible interval is wide (± 20 m). This poor resolution can be explained by the lack of constraint offered by the available observed data. In fact, shoreface deposits of the first parasequence (Ab0) are only observed at the bottom part of the two logged sections situated near its palaeo-seaward pinchout (Fig. 6.2). Moreover, no data are available to constrain the palaeo-landward part of the modelled cross-section (Fig. 6.1) where parasequence Ab0 is mostly preserved.

In the second period, from 40,000 to 125,000 years of simulated time, the inference of relative sea level is better constrained. The posterior probability distribution shows a relatively precise high-frequency sea level history. The spread of the Bayesian credible interval becomes narrower (± 2.5 to ± 5 m), but still remains significant. This can be partly related to the measurement errors on the decompacted thicknesses data. In fact, the standard deviations (STD) characterising the Gaussian distributions of these measurement errors have been estimated to vary between 1.5 and 2.5 m, which corresponds more or less to the spread of the credible interval if we consider that 90% of values present in a Gaussian distribution are contained within the interval [mean - 2STD; mean + 2STD]. Detailed analysis shows that this second period corresponds to an overall relative rise of sea level of about 5 to 10 m. The period can be subdivided into three distinct shorter intervals that are respectively time-equivalent to the deposition of geographical zones 1, 2, and 3 within the Standardville parasequence (Fig. 6.2).

The first of these shorter intervals (labelled Zone 1 in Fig. 6.6A), between 40,000 and 70,000 years of simulated time, is characterised by an overall relative rise of sea level of about 5 to 10 m. The spread of the 95% Bayesian credible interval decreases from ± 5 to ± 2.5 m with

increasing simulated time: the inference of the relative sea level curve becomes more tightly constrained as the shoreline prograded towards the area delimited by the observed data. Earlier stages of this interval show clearly that the posterior probability distribution contains two zones of higher probabilities. These features reveal a bi-modal posterior probability distribution but do not indicate non-uniqueness of the stratigraphic inverse problem, as discussed later. This first time interval is also apparently marked by high-frequency relative sea level variations (10^4 years) highlighted by the shape of the Bayesian credible interval boundaries (e.g. black arrowheads at relative sea-level highstands in Fig. 6.6A). These variations can be linked to time-equivalent changes in the posterior probability distribution of the wave height regime (Fig. 6.6C). The second shorter interval (labelled Zone 2 in Fig. 6.6A), between 70,000 and 90,000 years of simulated time, is characterised by a pronounced relative fall of sea level of about 5 m. The spread of the Bayesian credible interval remains narrow (± 2.5 m): the relative sea level inference for this period is tightly constrained. The third shorter interval (labelled Zone 3 in Fig. 6.6A), between 90,000 to 125,000 years of simulated time, begins with a clear and tightly constrained period of relative rise of sea level (of 5-10 m), followed by a poorly constrained period of relative fall of sea level (of up to 5 m) characterised by a large spread of the Bayesian credible interval (± 7.5 m).

Although the inferred relative sea level history has a relatively large uncertainty, the stratigraphic inversion process is successful in describing high resolution variations of sea level that may have occurred during the deposition of the Standardville parasequence. These inferred variations are similar to the qualitative results obtained from the intra-parasequence stratigraphic analysis carried out in Chapter 5.

6.7.1.2 Posterior Probability Distributions of Sediment Supply and Fallout Mud Rate

Figures 6.6B and 6.6D show respectively the posterior probability distributions of sediment supply and mud fallout rate in the shoreface-shelf area. The posterior probability distribution of the fallout mud rate (Fig. 6.6D) is unimodal and has a narrow spread centred on values of

around 0.01cm/y. The volume of mud added by suspension fallout is very tightly constrained, and is relatively low for this sedimentary system in the area of the cross-section.

The inference of the sediment supplied by longshore currents (Fig. 6.6B) is less tightly constrained. In fact, the overall spread of the 95% Bayesian credible interval is large and varies from ± 5 to ± 20 m²/y. This spread is such that overall trends in the original signal can be determined with confidence, but not high-frequency, low-amplitude variations. During deposition of parasequence Ab0 (0-35,000 years of simulated time), sediment supply was generally relatively low (0-20 m²/y) with a peak value of 40 m²/y at around 15,000 years of simulated time. Uncertainty for this period is large, as the spread of the Bayesian credible interval varies from ± 10 to ± 15 m²/y. The major transgressive period, occurring between 35,000 and 40,000 years of simulated time, is characterised by the second major peak of sediment supply, with maximum values of 50 m²/y.

During the deposition of the Standardville parasequence (Ab1), the supply is higher with values situated around 20 to 30 m²/y in average. The intervals corresponding to the deposition of zones 1, 2 and 3 are labelled on the sediment posterior probability distribution (Fig. 6.6B). The first interval (labelled Zone 1 in Fig. 6.6B) contains relatively steady sediment supply conditions with values of around 30 m²/y on average. However, the spread of the Bayesian credible interval is wide (± 10 to ± 20 m²/y) and the relatively uniform distribution may hide some poorly-constrained short-term variability. The second interval (labelled Zone 2 in Fig. 6.6B) is characterised by a drastic reduction in sediment supply occurring from 70,000 to 80,000 years of simulated time, which is followed by a large and rapid rise, which is relatively tightly constrained. This rise in sediment supply continues during the beginning of the third interval (labelled Zone 3 in Fig. 6.6B) to reach maximum values of between 30 and 70m²/y at around 95,000-100,000 years of simulated time. This represents a major peak in sediment supply. The final part of the third interval is characterised by a sharp fall in sediment supply followed by constant low values. Although the spread of the distribution seems to indicate steady conditions during the latter, the detailed probability distribution within the Bayesian credible interval seems to indicate a complex history marked by poorly-constrained high-frequency variations in the sediment supply signal.

6.7.1.3 Posterior Probability Distribution of Fair-Weather Wave Height

Figure 6.6C shows the posterior probability distribution of the fair-weather wave height through time. This distribution is well constrained as its Bayesian credible interval presents an overall spread of only 0.5 m on average. The results indicate that the wave height regime was low to moderate, with fair-weather wave height values generally under 3 m.

During deposition of parasequence Ab0 (0-35,000 years of simulated time), the inference of fair-weather wave height is poorly constrained comparing to the later signal because of the lack of observed data in the palaeo-landward part of the modelled cross-section. Although, the spread of the Bayesian credible interval is still relatively low (just under 0.5 m), the precise wave height signal is not easily extractable from the distribution which has a complex internal structure. The period of transgression, from 35,000 to 40,000 years of simulated time, is characterised by an abrupt rise of fair-weather wave height up to 3-4 m. This peak is synchronous with the first peak of sediment supply shown in Figure 6.6B.

During the deposition of the Standardville parasequence (Ab1), the posterior probability distribution of the fair-weather wave height is extremely well constrained as the spread of its Bayesian credible interval is under 0.5 m. The time intervals corresponding to deposition of zones 1, 2 and 3 (Fig. 6.2) have been reported in Figure 6.6C, to facilitate analysis. The first interval (labelled Zone 1 in Fig. 6.6C) is characterised by an overall decrease of the fair-weather wave-height from 2 to 1.5 m on average. More importantly, the posterior distribution indicates the presence of high-frequency variations expressed by peaks of higher wave-height values (black arrowheads in Fig. 6.6C) alternating with short periods of lower wave-height values situated at around 1m. These high frequency signal variations have a period of around 10^4 years of simulated time, and can be directly correlated to coeval high-frequency cycles of relative sea level (Fig. 6.6A): periods of high relative sea level correspond to periods of large wave height, as shown by the black arrowheads in Figures 6.6A and 6.6C. Furthermore, the peak relative sea level seems to occur slightly before the peak fair-weather wave height in each cycle. The second time interval (labelled Zone 1 in Fig. 6.6C) is characterised by a pronounced rise in wave height (up to 3 m), which is synchronous with a relative sea level fall (Fig. 6.6A) and sediment supply rise (Fig. 6.6B). Maximum wave height coincides with minimum relative

sea level, whereas maximum sediment supply occurs later, at the beginning of the third time interval (labelled Zone 3 in Fig. 6.6C). During this third interval, wave height values decrease to 1 m at around 110,000 years of simulated time and then rise again to 2 m. During this time interval, wave height decreases as relative sea level rises and then increases as relative sea level falls.

The posterior probability distribution of the fair-weather wave-height is extremely well constrained. The model is very sensitive to the wave parameters. The posterior probability distribution shows very high-frequency variations in fair-weather wave-height, which appear to have a complex interplay with the relative sea level history and, to a lesser degree, with sediment supply history.

6.7.2 Prediction of Grain-Size and Bed Thickness Distribution on Cross-Section

6.7.2.1 Maximum Likelihood BARSIM Forward Realisation.

From the inferred posterior probability distributions presented in Figure 6.6, the set of parameters maximising the likelihood function (best fit parameters) have been extracted and used to condition one BARSIM forward realisation in order to predict grain-size and bed thickness distribution along a cross-section. Figure 6.7 presents the resulting forward realisation (Fig. 6.7B) which can be compared to the stratigraphic correlation panel (Fig. 6.7A). In this figure, grain-size and uncompacted bed thickness distributions are compared to compacted facies associations, which introduces some bias in direct visual comparison. For locations situated between the eight logged sections, the gross stratigraphic architecture of the two progradationally stacked parasequences is successfully recreated. The overall internal architecture of the Standardville parasequence (Ab1) is also successfully recreated from the dataset of only eight logged sections (Fig. 6.7B), which represents half of the data used to draw the correlation panel (Fig. 6.7A). The three geographic zones defined in the Standardville parasequence (Fig. 6.2) and their distinctive internal stratigraphic architectures are also reproduced in the forward simulation result.

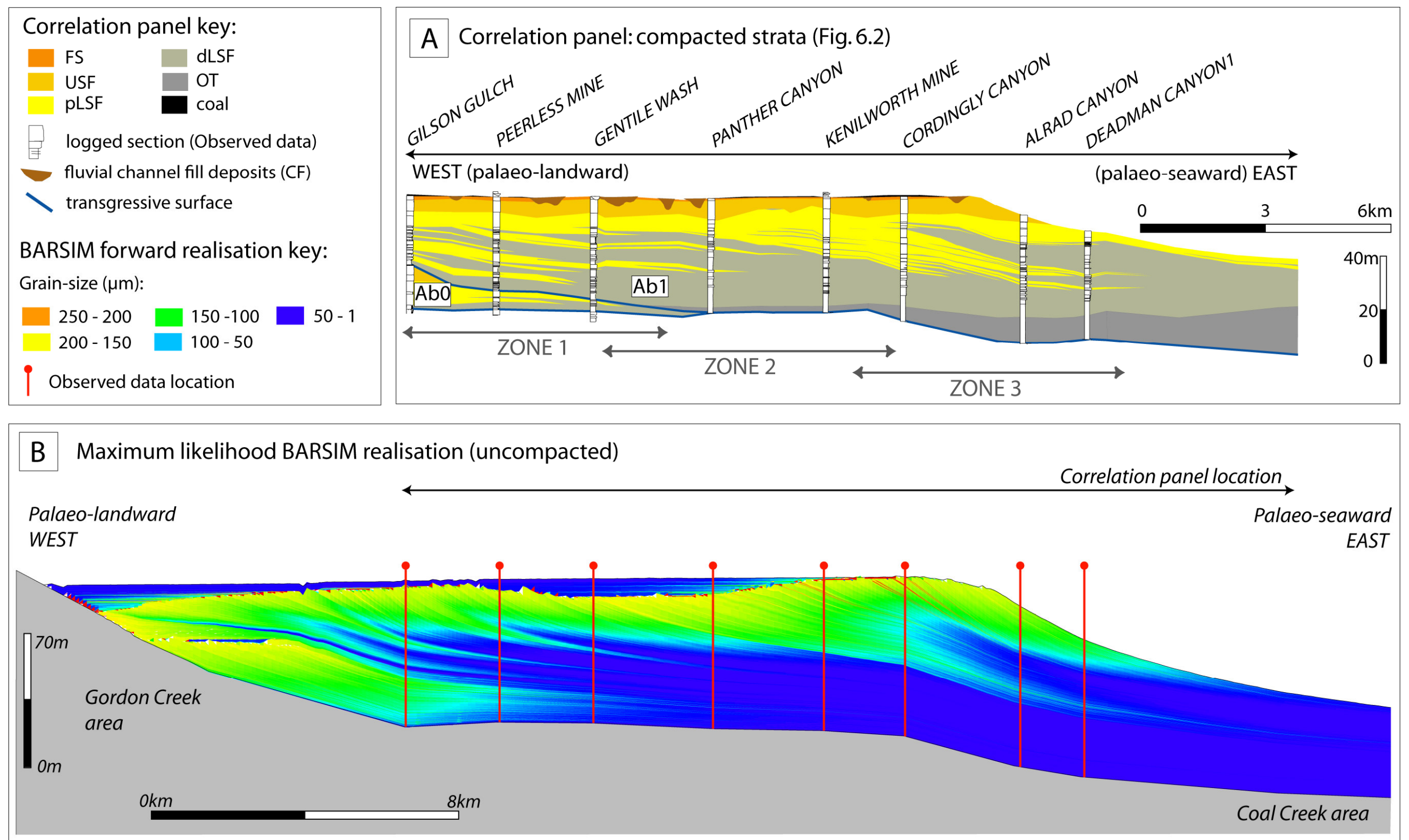


Figure 6.7: Comparison between (A) the stratigraphic correlation panel constructed from the full outcrop dataset and (B) the inverted BARSIM cross-section constructed from the set of parameters that maximise the likelihood function (Fig. 6.6). The gross stratigraphic architecture of the two parasequences (Ab0 and Ab1) and the internal architecture of the second parasequence (Ab1) are similar, but differences occur in the lengths and thicknesses of the three aggradationally stacked sandstone tongues of zone 1 in parasequence Ab1.

In fact, the first zone on the modelled cross-section (Zone 1 in Fig. 6.7), situated west of Panther Canyon, is characterised by three aggradationally stacked, sandstone tongues separated by marine shales, and was deposited during a period of relative rise of sea level (Zone 1 in Fig. 6.6). This architecture is similar to that described from the correlation panel, which consists of three aggradationally stacked, upward-coarsening bedsets bounded by non-depositional surfaces, with an overall “normal” regressive shoreline trajectory. The second zone on the modelled cross-section (Zone 2 in Fig. 6.7), situated between Panther Canyon and Cordingly Canyon, is characterised by the presence of strongly progradational stacking of amalgamated sandstone tongues deposited during a period of relative fall of sea level (Zone 2 in Fig. 6.6). This architecture is similar to that described from the correlation panel, which consists of strongly progradational stacked bedsets bounded by erosional and non-depositional discontinuity surfaces, with a “forced” regressive shoreline trajectory. The third zone on the modelled cross-section (Zone 3 in Fig. 6.7), situated east of Cordingly Canyon, is characterised in its palaeo-landward part by facies boundaries that record a “normal” regressive shoreline trajectory, and in its palaeo-seaward part by facies boundaries that record a “forced” regressive shoreline trajectory (Zone 3 in Fig. 6.6).

6.7.2.2 Local Mode BARSIM Forward Realisation.

The analysis of the posterior probability distributions of the environmental parameters revealed the presence of local modes. To test if these local modes represent viable alternative solutions of the stratigraphic inverse problem, environmental parameter sets have been extracted from them in order to condition BARSIM forward realisations. The conclusions for all these tests were similar, thus only one example is presented in this section and in Figure 6.8.

Although similar gross stratigraphic architectures and trends in shoreline trajectories are observed, the modelled cross-section for the local mode (Fig. 6.8E) shows clear differences with the correlation panel (Fig. 6.7A) and the modelled cross-section parameterised by the maximum likelihood estimates (Fig. 6.7B). The principal difference occurs in zone 1, specifically in the area constrained by the Peerless Mine logged section, where only two among

the three observed sandstone tongues have been simulated. This difference is related to differences in the relative sea level and wave-regime signals, which, in this case, lost one of their peaks (Figs. 6.8A, 6.8C). Other differences in environmental parameters create minor disparities in the stratigraphic architecture.

These results show that, as expected, the local mode model fits the observed data less well than the maximum likelihood solution. Moreover, this local mode doesn't match the observed data well enough to be considered as a robust and plausible alternative solution of the stratigraphic inverse problem. This result demonstrates that the presence of local modes in the inferred posterior probability distributions does not prove non-uniqueness of the stratigraphic inverse problem. However, such local modes do complicate the procedure of searching the inverse model space.

6.8 Interpretation

Analysis of the posterior probability distributions for the environmental parameters (Fig. 6.6) and of the grain-size and bed thicknesses predictions along the cross-section of the Standardville parasequence (Fig. 6.7) enables the various mechanisms that drove intra-parasequence architecture to be interpreted. The modelled cross-section (Fig. 6.7B) is not flattened to the top-parasequence datum surface or compacted, as is the case in the outcrop correlation panel (Fig. 6.7A). As a result, it clearly illustrates shoreline trajectories, which facilitates analysis of facies architecture within the three geographical zones that comprise the wave-dominated part of the Standardville parasequence.

Within zone 1 of the Standardville parasequence (Figs. 6.2, 6.7), this analysis indicates that the three aggradationally stacked sandstone tongues were generated by relatively high-frequency variations of environmental parameters (10^4 years of simulated time). In particular, upward-coarsening trends seem to have formed under conditions of relative sea level rise and increasing fair-weather (and associated storm) wave heights (Figs. 6.6A, 6.6C), which increased seaward transport of sand and thus created the tongues.

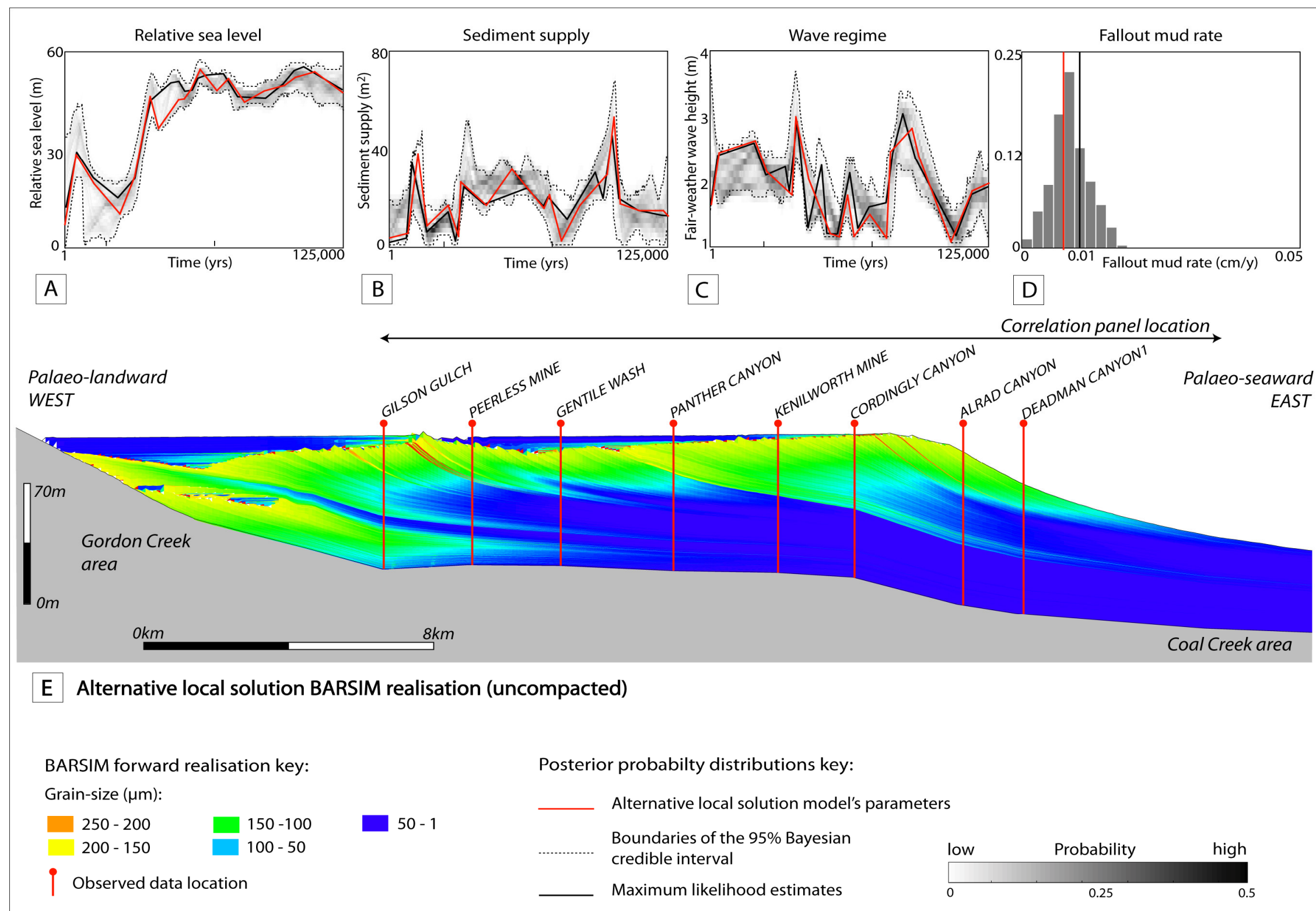


Figure 6.8: BARSIM realization of a local solution. A local solution of the (A) relative sea level, (B) sediment supply, (C) wave height and (D) mud fallout rate parameters results in the construction of (E) a BARSIM cross-section. Red lines in figures A, B, C and D represent local solution parameters, whereas the black lines represent the parameters maximising the likelihood function. Gross stratigraphic architecture and shoreline trajectory trends are similar to the modelled BARSIM cross-section of Figure 6.7B, but major differences occurs at intra-parasequence scale around the Peerless Mine location.

The formation of these upward-coarsening successions may also have been caused, at least in part, by an increase of sediment supply as suggested by the shape of the Bayesian credible interval (Fig. 6.6B). However, uncertainties associated with the posterior distribution are large enough to make the exact combination of driving mechanisms unclear. Marine shale capping the sandstone tongues, which are equivalent to non-depositional surfaces documented at outcrop (Chapter 5), were deposited at the climax of the relative sea level rise and are associated with an abrupt decrease in fair-weather and storm wave heights and a decrease in sediment supply. These results are similar to those obtained from forward numerical modelling experiments by Sømme *et al.* (2008), who related the formation of upward-coarsening bedsets bounded by non-depositional surfaces to the generation of beach ridge sets. During relative rises of sea level, fluvial channels feeding sediment to the shoreline system may aggrade to heights that are super-elevated above the coastal plain and thus prone to avulsion, which results in localised rearrangements of the shoreline geometry and associated wave climate. This mechanism involves the erosion of older beach ridges near the abandoned fluvial channels, forming a beach ridge set boundary, and the redistribution of sand into spits and barriers that protect some stretches of the shoreline from waves and currents. This process can readily explain the formation of non-depositional surfaces and deposition of shale during periods of locally lowered wave height and sediment supply. As erosion of older beach ridges progresses, sand supply via longshore drift currents can drive progradation of these sheltered or abandoned parts of the shoreline, which thus become sandier and more exposed to wave action. At such times, large quantities of sand may be transported seaward to create an upward-coarsening bedset or tongue.

Similar analysis for zone 2 (Figs. 6.2, 6.7) indicates that the formation of progradationally stacked bedsets bounded by non-depositional and erosional discontinuity surfaces within this zone occurred via three sequential steps. Firstly, an initial relative fall of sea level synchronous with an increase of wave height and low sediment supply produced enhanced wave erosion (Figs. 6.6A-C, 6.7). Secondly, as sediment supply increased and the wave regime remained energetic, large volumes of sand were transported seaward and deposited as a series of strongly prograding sandstone tongues (Figs. 6.6B-C, 6.7). Thirdly, relative sea level started to rise and wave heights started to decrease towards the end of deposition of this zone (Figs. 6.6A, C), which caused backstepping of the storm-event beds within an upward-fining succession (Fig.

6.7). Sediment supply remained high at this time, which caused amalgamation of the storm-event beds despite their backstepping stacking pattern (Figs. 6.6B, 6.7). The modelled cross-section shows that zones of fluvial incision recorded independently in the field correspond clearly to zones of negative shoreline trajectory *sensu stricto* (Fig. 6.7). This confirms the interpretations presented in Chapter 5: lowering relative sea level caused incision of fluvial channels and exposure of the shoreline to increased wave action, which in combination drove forced regressive amalgamation of bedsets.

Within zone 3 of the Standardville parasequence (Figs. 6.2, 6.7), deposition of shale above the amalgamated storm-event sandstone beds of zone 2 occurred as a consequence of rising relative sea level, decreased sediment supply and decreased wave height. The palaeo-landward part of this zone records shoreface-shelf deposition during a normal regressive shoreline trajectory (Fig. 6.7), which allowed the accumulation and preservation of coal-bearing coastal plain deposits behind the shoreface (Fig. 6.7B). During deposition of the palaeo-seaward part of zone 3 (Figs. 6.2, 6.7), falling relative sea level combined with increasing wave height and lowered sediment supply (Figs. 6.6A-C) resulted in the formation of a forced regressive, “sharp based” shoreface (*sensu* Plint, 1988) (Alrad Canyon logged section, Figs. 6.7, 6.9).

It is important to note that times of falling relative sea-level (e.g. within the early part of zone 2 and late part of zone 3; Fig. 6.6A) are characterised by falling or lowered sediment supply (Fig. 6.6B). This combination may be, at least in part, an artefact of the way in which depositional ages within the Standardville parasequence were estimated in order to provide some essential constraints on the stratigraphic inversion process. These age estimates assume a constant rate of shoreline progradation (Fig. 6.5). During periods in which relative sea level was lowered, the volume available for sediment to infill was reduced. Thus, sediment supply has to be reduced during such periods in order to maintain the constant progradation rate. Notwithstanding this effect, the model results show considerable variations in sediment supply that are not linked directly to relative sea-level change (Figs. 6.6A, B), and are thus not artefacts of the age estimation. Moreover, major non-uniqueness effects might arise from uncertainty in the estimated time constraints, which has not been fully investigated in this work.

This analysis shows that the stratigraphic forward and inverse algorithm is not only able to infer environmental conditions from observed geological data, but it is also able to predict the gross stratigraphic architecture within the Standarville parasequence. The results from this initial study seem to show that relative sea level and sediment supply define the gross stratigraphic architecture, whereas wave climate is responsible for localised variations in sand and shale content. These results conditioned on observed data also support the forward numerical experimental approach adopted by Storms and Hampson (2005) and Sømme et al. (2008) to investigate intra-parasequence facies architecture.

6.9 Discussion: Application of Stratigraphic Inverse and Forward Modelling to Real Data

The results of this first case study are promising and show clearly that the stratigraphic inverse and forward modelling algorithm can be applied to a real dataset in order (1) to infer environmental parameters (e.g. relative sea level, sediment supplied by longshore drift currents, mud fallout rate and wave height regime) and their respective uncertainties, and (2) to predict stratigraphic architecture from a limited amount of observed data. Furthermore, the stratigraphic inverse and forward model results recover all the key aspects of the architecture recorded in the field: the gross stratigraphic architecture of the two progradationally stacked parasequences has been successfully recreated. The alternating periods of normal regressive and forced regressive shoreline trajectories characterising the internal architecture of the Standarville parasequence (Ab1) and their causative mechanisms have also been successfully investigated. These aspects of the stratigraphic architecture have been recreated as drawn on the outcrop correlation panel (Fig. 6.7A) but from a limited dataset of only eight logged sections (Fig. 6.7B). Moreover, given the available data, the results show unambiguously that there is only one viable solution to explain the architecture at this scale.

The numerical method also has some limitations. High-resolution analysis of the lengths and thicknesses of the bedset-scale sandstone tongues (e.g. within zone 1; Figs. 6.7B, 6.8) reveals differences between observed and simulated data at logged section locations (Fig. 6.9). These bedset- and bed-scale discrepancies show that the BARSIM realisations do not fit the observed

data perfectly at such scales, even if conditioned on the maximum likelihood parameters, and that non-depositional discontinuity surfaces are never properly recreated in the model. Analysis of the posterior probability values indicates that the inversion process seems to find the global solution area of the posterior probability distributions, but did not focus the sampling on the area around the exact solution of the inverse problem, contrary to the validation tests conducted on synthetic data in Chapter 4. In fact, after convergence, the log-likelihood values remain clearly stationary around high values (i.e. higher posterior probabilities) but never reach the highest values close to 0 (i.e. posterior probabilities closed to 1). There are four reasons to explain this behaviour.

1. The BARSIM algorithm, like all the numerical stratigraphic forward modelling approaches, is limited when simulating real sedimentary processes. Moreover, its two dimensional formulation can not assess effects associated with the three dimensional variability of beds and bedsets. Therefore, the exact solution of the inverse problem *sensu stricto* doesn't exist in the sense that the forward model will never be able to completely fit the observed data. Stratigraphic inversion results are, at best, only as good as the capacity of the forward model to simulate reality.
2. Although wave-dominated systems are characterised by limited along-strike variability, the projection of the observed logged sections into the cross-section can modify the true facies architecture geometries and thus interfere with the stratigraphic inversion process.
3. Compaction of the deposits under their own load during sedimentation is not simulated in BARSIM, thus limiting the ability of the forward model to simulate reality. The non-simulation of dynamic compaction leads to slight and systematic misevaluation of the inferred environmental parameters. A compaction algorithm can easily be introduced into the numerical forward algorithm for future work. However, incorporation of such an algorithm might increase considerably computational cost.
4. The user-defined specification of internal and some external parameters within BARSIM (Table 6.4) can also add some bias in the stratigraphic inversion process, even if they are carefully determined by preliminary tests. However, the tests conducted for this case study suggest that model results are not strongly sensitive to these parameters.

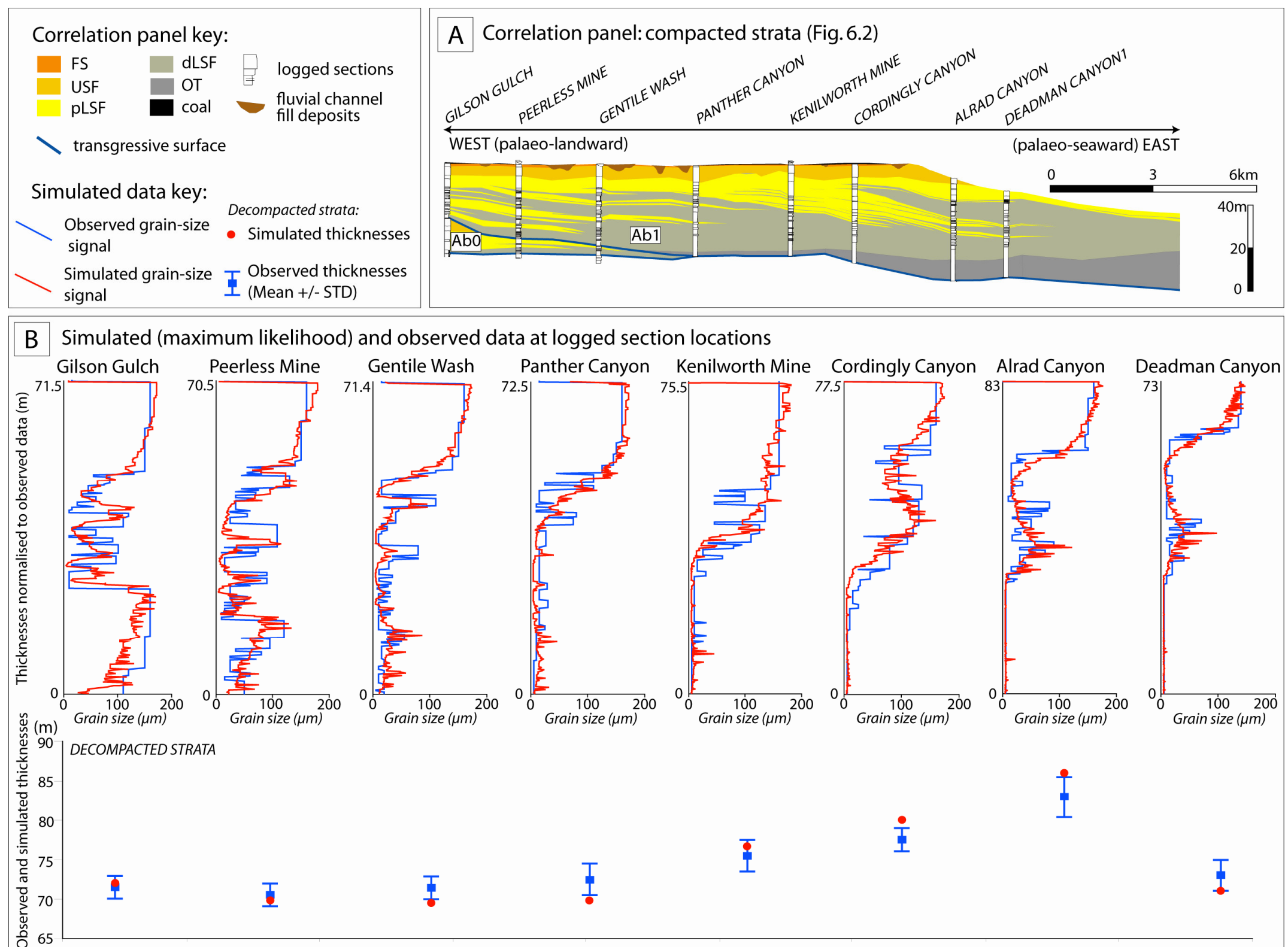


Figure 6.9: (A) Stratigraphic correlation panel showing the location of logged sections used to condition the inverse modelling, and (B) comparison between simulated and observed grain-size and thicknesses at the logged section locations. The simulated grain-size and thicknesses data have been extracted from the inverted BARSIM cross-section constructed from the set of parameters maximising the likelihood function (Figs. 6.6, 6.7B). The overall grain-size trends and thickness distributions are recreated, but differences occur in the detailed distribution of the storm-event sand beds.

These four factors impact directly on the quality of the posterior probability distributions and on their interpretation. By sampling the general solution of the inverse problem, the numerical scheme obtained wider and more complicated posterior probability distributions than it should have done. In fact, because the sampling process never reached the area of very highest posterior probability, the stochastic inversion tended to sample more local modes with lower likelihood probabilities. Therefore, these results demonstrate that the presence of local modes in the inferred posterior probability distributions doesn't necessarily prove the non-uniqueness of the inverse problem. This problem is minor as all the local-mode solutions can be tested visually by running conditioned forward realisations of BARSIM and then discarding *a posteriori* those that are incorrect.

The initial data preparation constitutes another limitation of the method. The preliminary parameterisation for the initial condition parameters (e.g. initial physiographic profile, grain-size classes and supply composition) may add some bias in the stratigraphic inversion process. In fact, misevaluation of grain-size classes and sediment supply composition can prevent convergence, whereas errors in determining the initial physiographic profile can lead to misevaluation of the accommodation infilled by sediment. These effects may bias the inferred posterior probability distributions of sediment supply and mud fallout rate, but they are minor.

The two-dimensional formulation of BARSIM represents another source of errors by assuming that the modelled cross-section was precisely down-dip oriented. In fact, it is clear that its estimated orientation (via the estimation of the palaeo-shoreline orientation) presents errors that directly result from an artificial stretching of the modelled cross-section. As a consequence, the total volume of sediment preserved in the cross-section has been artificially increased implying a bias (over-estimation) in the inferred posterior probability distributions of sediment supply and mud fallout rate. Such issues can be solved in future work by using a three dimensional stratigraphic forward modelling algorithm.

Finally, inversion of time-dependent parameters (e.g. relative sea level, sediment supply, wave regime) is directly dependent on the constraints provided by age data. In this case, no data of sufficient precision were available to constrain the timing of deposition, which then required

the use of assumptions about progradation rate. The absence of major stratigraphic discontinuities within the cross-section supports this approach, although some artefacts in the inferred relative sea level and sediment supply histories have most likely been implicitly introduced by the assumption of a constant progradation rate. This assumption biases the inferred solution toward those having relatively small variations in sediment supply. It is clear that such solutions are not entirely supported by observation of modern systems where shoreline progradation commonly happens at very rapid rates and is generally followed by long periods of non-activity. Therefore, further developments of this work should consider more accurate ways of constraining the timing of deposition by referring to modern analogues.

6.10 Summary: Insights into Intra-Parasequence Stratigraphic Architecture from Stratigraphic Inverse and Forward Modelling

1. The application of the inversion method in this first case study was successful and shows clearly that this method can be applied to real datasets in order to infer the probability distributions of environmental parameters and to predict stratigraphic architecture from a limited amount of observed data.
2. The simulated results not only confirm the stratigraphic interpretations previously proposed in Chapter 5, but also quantify the causative mechanisms and their interactions. Furthermore, the stratigraphic inverse and forward model results recover all the key aspects of the gross, intra-parasequence stratigraphic architecture recorded in the field. The internal stratigraphic architecture of the Standardville parasequence (Ab1) results from complex interplays between relative sea level, wave regime and sediment supply parameters. The results also confirm the presence of alternating periods of normal regressive and forced regressive shoreline trajectories. This last point questions the validity of sequence stratigraphic interpretations that do not consider stratigraphic

architecture at intra-parasequence scale and therefore neglect an important part of the causative environmental signal.

3. The model results show that, overall, the prevailing wave climate had low to medium energy, with values of fair-weather wave height below 3 m in average. Greater fair-weather and storm wave heights are necessary to transport sand seaward to create bedset-scale sandstone tongues, provided that sediment supply is sufficient. Wave climate seems to be mainly responsible for local sand and mud content, whereas relative sea level and sediment supply define the gross internal stratigraphic architecture of the Standarville parasequence.
4. Aggradational stacking of upward-coarsening bedsets bounded by non-depositional surfaces during periods of normal regressive shoreline trajectory was mainly due to a high-frequency increase in wave energy during rising relative sea level rise, provided that sediment supply is sufficient. The generation of these bedsets can be related to the formation of beach reach sets, as proposed by Sømme et al. (2008), and is attributed to autocyclic processes involving fluvial channel avulsion and rearrangement of shoreline morphology, which influences local wave regime.
5. Strongly progradational stacking of bedsets during periods of forced regressive shoreline trajectory was caused by a combination of relative sea level fall, highly energetic wave regime, and low followed by high sediment supply. A time lag between periods of highest wave regime and lowest relative sea level, and periods of higher sediment supply induced firstly the creation of a wave erosion surface and secondly the deposition of amalgamated storm-event beds.
6. The mud fallout rate in the shoreface area was low for this case study.
7. Compared to the stratigraphic correlation panel, which is flattened to a datum surface, BARSIM realisations have the great advantage of showing realistic shoreline trajectories. As a result, comparison of the simulated stratigraphy and the correlation panel shows that incised fluvial channels observed in the field correspond clearly to zones of negative

shoreline trajectory on the modelled cross-section. Furthermore, alternating periods of negative and positive shoreline trajectories created local sections of thickened and potentially better preserved coal-bearing coastal plain deposits.

8. The quality of the results is directly conditioned by the formulation and assumptions of the forward algorithm and therefore its capacity to simulate the real processes. Moreover, the quality of the results is also strongly dependent on the observed data quality, especially for constraining the timing of deposition.

Chapter 7

7 Second Application: Inverse and Forward Stratigraphic Modelling of Sub-Surface Emsian strata of South Algeria

Abstract

In this chapter, I will present the second application of the stratigraphic inverse and forward modelling algorithm to a real dataset: the sub-surface Emsian deposits of South Algeria. In this second case study, the inversion method aims to infer relative sea level, sediment supply, fair-weather wave height and mud fallout rate from a sub-surface dataset of nine wells that define three cross-sections of 75 km length. The results show that the inverse method successfully inferred the environmental parameters from the three cross-sections. The posterior probability distributions for the parameters show coherency in their results across three independent inversions (one for each cross-section). Grain-size and bed thickness predictions between the sparse observed data demonstrate unambiguously that only one stratigraphic correlation hypothesis from two plausible *a priori* scenarios is feasible: the overall architecture is characterised by two progradationally stacked shoreface-shelf sandstone tongues separated by a flooding surface. However, the modelling approach failed to reproduce some features of the stratigraphic architecture, partly because of its two dimensional formulation.

7.1 Introduction

The prediction of sub-surface facies architecture and physical rock-property distribution from sparse well data is challenging in all petroleum reservoirs. Robust building of static reservoir models is depends on (1) reliable stratigraphic interpretations, (2) efficient geostatistical tools chosen appropriately to predict facies architecture within the numerical grid (Deutsch, 2002) and (3) consistent uncertainty assessment.

Stratigraphic interpretation is critical in determining the gross architecture of the static reservoir model. Errors of interpretation can lead to significant misevaluations in volumetric estimation and of reservoir compartmentalisation. Assessment of stratigraphic uncertainty can be realised by proposing and testing different stratigraphic scenarios. However, such an approach is often not completed because of time limitations. Furthermore, due to these restrictions, stratigraphers tend to focus on extreme-case stratigraphic interpretations which have generally low probabilities of existence, in order to capture the full range of uncertainty. Such approaches usually result in misevaluations in the level of heterogeneity and in volumetric estimations. As a result, they may lead to strong biases in the uncertainty range of dynamic reservoir simulation results.

Stratigraphic architecture and facies heterogeneities are generally simulated at reservoir scale (smaller than the case I will illustrate in this chapter) with geostatistical tools conditioned to local well data and an interpretation of larger stratigraphic context. Despite recent improvements (Strebel, 2002), such geostatistical tools do not explicitly incorporate geological processes and are limited in their application to understanding the stratigraphic record. Moreover, their application in areas not constrained by well data remains problematic. Conventional approaches consist of (1) building facies probability cubes derived from stratigraphic interpretation and/or seismic attributes to provide a three-dimensional soft constraint (Harding *et al.*, 2005; Labourdette, 2007; Labourdette *et al.*, 2007) and (2) assessing the variability of small-scale heterogeneity distribution and its impact on dynamic reservoir behaviour by running multiple realisations that populate such heterogeneities within the volume of the facies probability cube. Therefore, uncertainty assessment is directly dependent

on the resolution of the facies probability cube and thus, on seismic data and/or stratigraphic interpretation quality, and may be complicated by problems of non-stationarity (Jenson *et al.*, 2000, Caumon & Journel, 2005). Moreover, in such an approach, the assessment of the impact of small-scale heterogeneities on fluid flow behaviour is also biased by the resolution of cells in a predefined numerical grid.

In this chapter, I will present the second application of the stratigraphic inverse and forward modelling approach to a sub-surface case study: the Emsian reservoir interval of South Algeria in which issues of stratigraphic correlation that control reservoir distribution and quality have to be addressed. The objectives of this application are (1) to assess alternative scenarios of stratigraphic correlation, (2) to predict facies distributions between sparse well data, in this case at a larger scale than conventional reservoir modelling, and (3) to infer variations in the environmental parameters that drove depositional processes (i.e. relative sea level, sediment supply, wave climate and mud fallout rate) and associated uncertainty. This chapter is structured so that, after having briefly described the geological setting of the sub-surface Devonian deposits in southern Algeria, based on Total internal reports (section 7.2), I present the Emsian sub-surface dataset and the specific stratigraphic correlation questions (section 7.3). I then describe the modelling workflow (section 7.4), sub-surface data preparation (section 7.5) and the inverse numerical scheme parameterisation (section 7.6). The results are presented in section 7.7, and discussion of the results and concluding statements in section 7.8.

7.2 Geological Setting and Dataset

7.2.1 Location

The investigated Emsian strata are located in South Algeria at the western part of the Ougarta mountain belt (Fig. 7.1). Recently, several gas fields have been discovered in Lower Devonian sandstones in this area. The basal wave-dominated Emsian unit, which will be investigated throughout this chapter, constitutes one of these potential reservoir units. These deposits are extremely poorly documented in the public literature. Therefore, the geological setting and stratigraphic interpretation presented in this chapter represent a summary of general

information provided by internal reports from the petroleum company Total (Jardine et al., 2007). The ensuing modelling work relies on these previous interpretations.

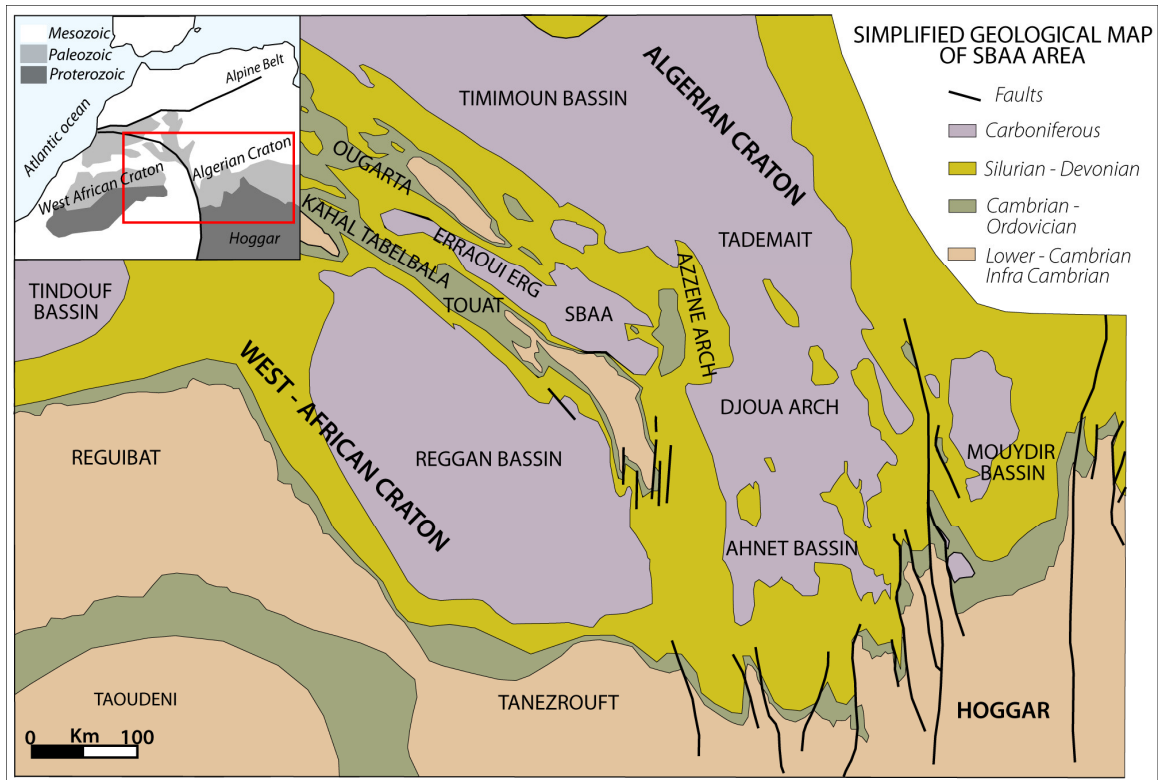


Figure 7.1: Simplified geological map of the Proterozoic and Palaeozoic deposits in the Sbaa area (from Jardine et al., 2007: Total Internal Report).

7.2.2 Tectono-Stratigraphic Setting and Palaeogeographic Reconstruction Hypothesis.

The studied deposits are located in a structurally complex area close to the Pan-African suture between the East and West African blocks. Several major tectonic events occurred during the Paleozoic, but only the Hercynian orogeny affected the Paleozoic units under investigation here. As a result, the Emsian strata are currently structured in a succession of low amplitude and large wavelength anticline-syncline folds oriented N120 – N150 (Figs. 7.1, 7.2).

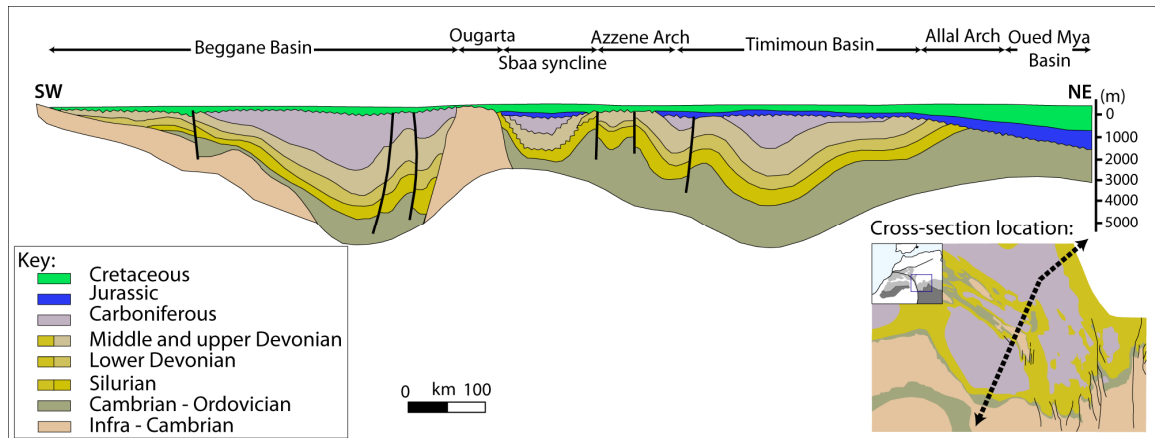


Figure 7.2: SW – NE cross-section over the Sbaa area illustrating the large scale Hercynian folding (from Jardine *et al.*, 2007: Total Internal Report).

At this location, the Palaeozoic sedimentary succession consists of more than 3500 m of siliclastic marine deposits which are discontinuously overlain by Barremian to Turonian deposits (Figs. 7.2, 7.3). No major discontinuities are observed within the Paleozoic succession investigated. The Lower Devonian series consists of shallow-marine strata deposited in tide-dominated environments during the Upper Lochkovian, evolving into wave-dominated environments during the Emsian. The overall vertical facies succession indicates a progressive flooding from the Upper Lochkovian to the Emsian. Lower Devonian units (Lochkovian, Praguian and Emsian) were deposited during a period of relatively calm tectonic activity following the Caledonian orogeny. Their thicknesses are relatively constant over the area investigated (Figs. 7.2, 7.3) and tend to increase northwards where the palaeo-basin was deeper.

Palaeogeographic reconstructions for the Lower-Devonian period, proposed by Total, suggest that the studied area was situated in the western part of a giant estuary in which siliclastic sediments were supplied from the south-south-east. The presence of wave-dominated facies associations, intercalated within the Upper Lochkovian and Praguian deposits, and dominant within the Emsian basal unit, indicates that the estuary was open to the sea.

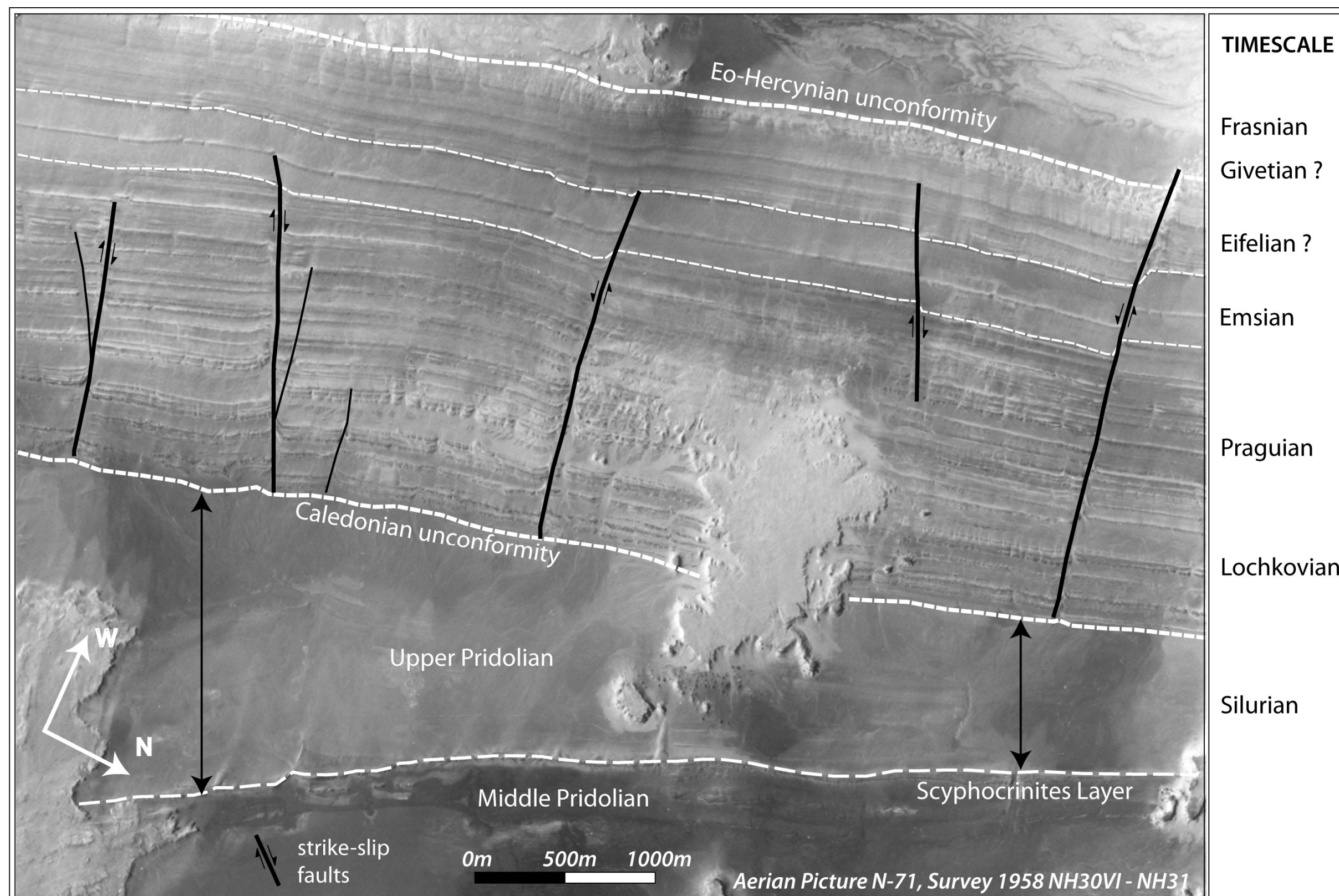


Figure 7.3: Aerial picture over Silurian-Devonian deposits in the studied area. The picture illustrates the constant thickness of the Lower Devonian units and the Caledonian angular unconformity (from Jardine et al., 2007: Total Internal Report).

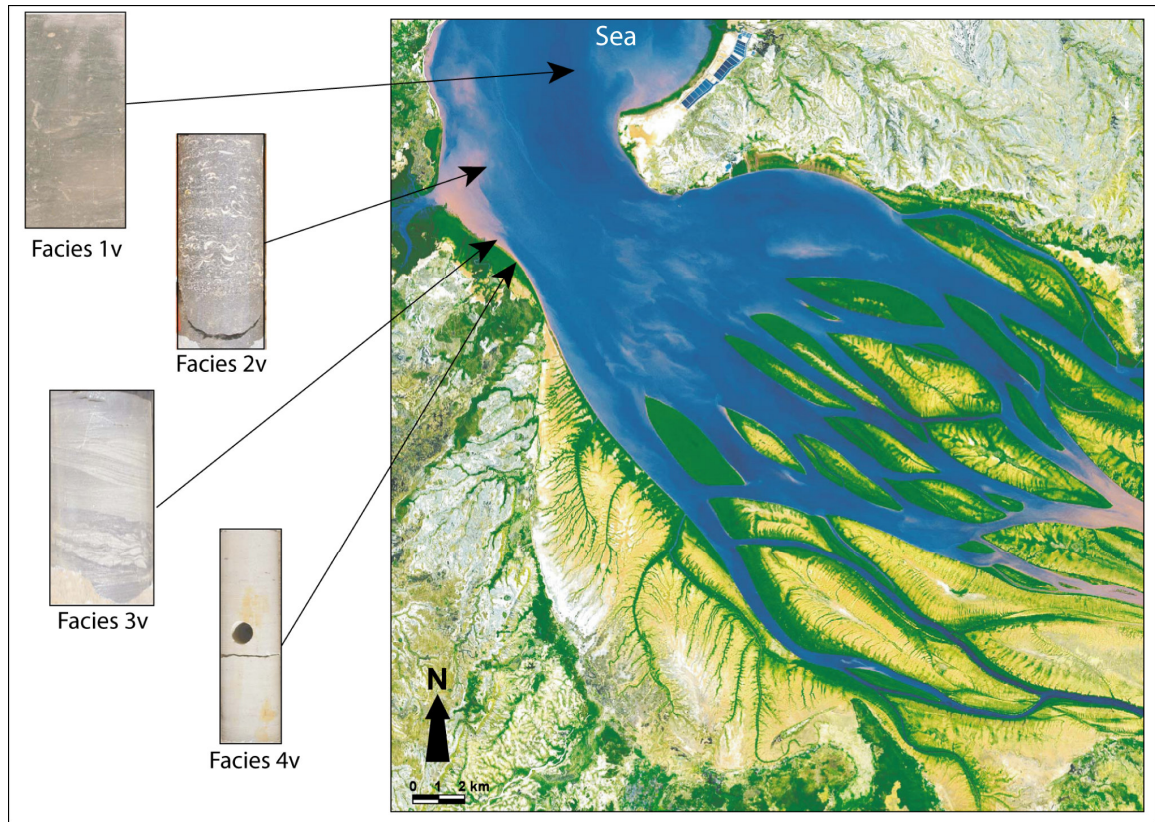


Figure 7.4: Betsiboka / Bombetoka estuary. The estuary (right) has been chosen to be a modern analogue of the Lower Devonian South Algerian estuary in terms of sedimentary processes. On the left, wave-dominated facies associations described in core data are placed in their corresponding geographical context (from Jardine et al., 2007: Total Internal Report).

In such settings, the facies distributions are inherently complex due to interactions between tidal and wave processes, sediment supply and relative sea-level variations. The Betsiboka / Bombetoka estuary (Madagascar) is a modern analogue, chosen by Total, which illustrates well the complexity of these systems (Fig. 7.4). However, it is clear that this modern estuary has numerous differences (i.e. size, geological setting) to its ancient counterpart and should therefore carefully used as an analogue. The Emsian strata are situated a low amplitude anticline, similar to those forming structural traps for the nearby gas fields (Fig. 7.2). The issue of this work is to assess the presence, extent and quality of the Emsian reservoir unit within this structural trap, which depends on stratigraphic correlation and associated facies distributions.

7.2.3 Emsian Deposits

In the investigated area, the Emsian basal unit has been penetrated by 9 wells which are renamed *A, B, C, D, E, F, G, H* and *I* for confidentiality reasons (Fig. 7.5). Their precised geographical location is also hidden for the same reasons. Each well is partially cored (in the reservoir interval) and has a conventional suite of wireline log data available. Gamma Ray and Sonic logs for each well are presented in Figure 7.5. These logs and cores constitute the observed data constraining the stratigraphic inversion process.

Previous analysis of the cored data provided by the company Total showed that four facies associations can be recognized within the Emsian basal unit (Fig. 7.4). Facies association 1v is dominated by mudstone containing lenticular laminations and moderate bioturbation (*Planolites*). This facies association has been interpreted to be deposited in offshore environment. Facies association 2v is characterised by mudstone interbedded with layers of siltstone to very fine-grained sandstone and containing shell fragments. This facies association has been interpreted as offshore transition deposits. Facies association 3v is composed of non-amalgamated to amalgamated beds of siltstone to fined-grained sandstone interbedded with mudstone. Hummocky cross-stratification and wave ripples are the dominant sedimentary structures while bioturbation is low. This facies association has been interpreted as lower shoreface deposits. Facies association 4v is characterised by fine-grained sandstone with minor interbeds of lenticular mudstone. Planar, sub-horizontal, parallel laminations and trough cross-bedding are dominant. This facies association has been interpreted as upper-shoreface to foreshore deposits.

The sedimentary structures described in the four facies associations (1v, 2v, 3v and 4v) are generated by storm and fair-weather wave processes and are therefore characteristic of wave-dominated environments (Elliot, 1986; Walker & Plint, 1992). According to previous interpretations from the company Total, the Emsian basal unit represents a beach system oriented NNW-SSE at studied location (Fig. 7.6). Previous analysis of the well data also showed that the facies succession is dominated by mud derived from the estuarine fluvial outlet and that the reservoir shoreface-sandstone deposits represent thin (5 to 7m) and underdeveloped shoreface facies successions (Figs. 7.4, 7.5, 7.6).

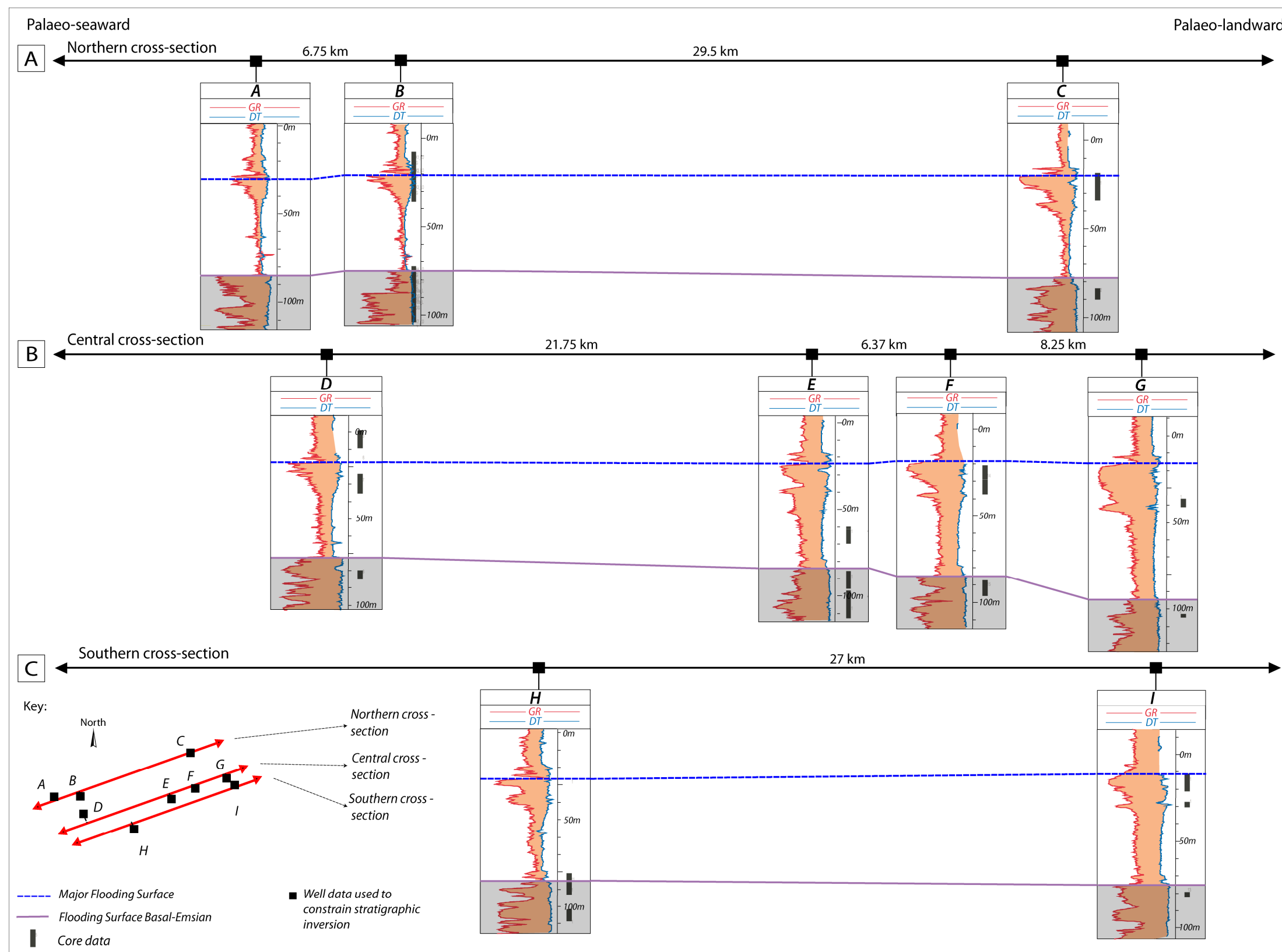


Figure 7.5: Subsurface dataset. The cartoon illustrates the three 75 km long, WSW-ENE-trending cross-sections and the data (Gamma Ray and Sonic logs) used to constrain the stratigraphic inversion. The pink lines represent the bottom of the modelled interval and the dash blue lines represent the top. In combination, the lines delimit the interval for which thickness curves have been calculated.

7.3 Stratigraphic Correlation Issues

From the nine wells, three parallel stratigraphic cross-sections have been defined. Each cross-section presents a length of 75 km and is oriented WSW-ENE (Fig. 7.5), perpendicular to the estimated strike to the palaeo-shoreline orientation (Fig. 7.6). The northern cross-section (Figs. 7.5A) is constrained by 3 wells (A, B and C). The central cross-section (Figs. 7.5 B) is constrained by 4 wells (D, E, F and G). The southern cross-section (Figs. 7.5 C) is constrained by only 2 wells (H and I). One seismic survey is available in this area but, despite its good quality, resolution is too low to visualise the internal stratigraphic architecture of the basal Emsian unit. However, these data indicate that the thickness of the Emsian units increases relatively uniformly toward the north.

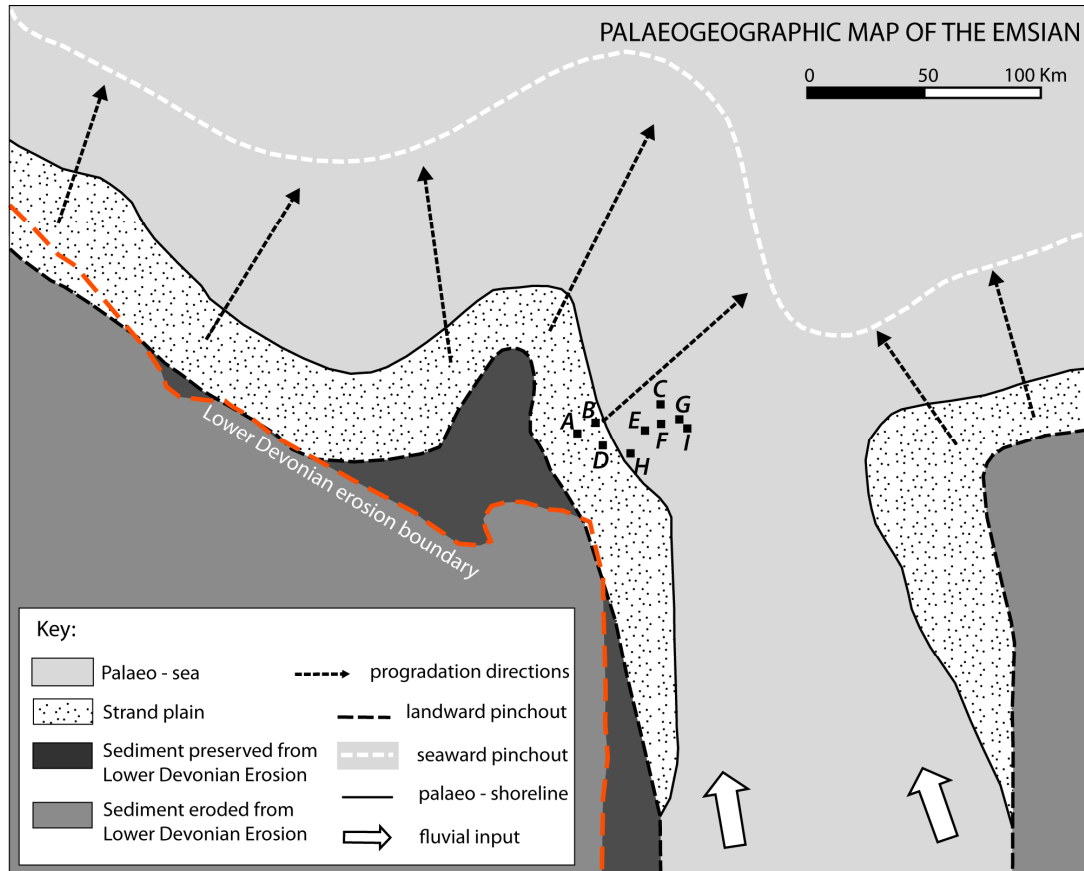


Figure 7.6: Palaeogeographic reconstruction of the Emsian sedimentary environment. The map shows the NNW-SSE orientation of the shoreline near the investigated area and its

proximity to the estuary, which explains the under-development of sandy upper-shoreface deposits due to a high influx of fluvially derived mud (modified from Jardine et al., 2007, Total Internal Report).

Gross well to well correlations have been done for each cross-section on the basis of the Gamma Ray signal and core descriptions (Figs. 7.4, 7.5). The Gamma Ray tool measures clay content. Assuming that sandstone are quartz rich, the Gamma Ray signal has low values for sandstones and higher values for shale. Comparison with core descriptions confirms this tendency and shows that lithology trends, and thus gross grain-size successions, can be qualitatively interpreted from the Gamma Ray signal in this case. However facies boundaries are uncertain in the absence of described core data. On this basis, the Gamma Ray signal of each well shows an overall upward-coarsening trend, whereas core descriptions indicate an overall shallowing-upward facies succession (from facies association 1v at the bottom to facies association 4v at the top). This regressive facies succession (Fig. 7.5) is interpreted to have formed as a consequence of seaward migration of the shoreline and associated progradation of the facies belts toward the ENE at this location. This progradation created laterally extensive sandstone sheets. The top of the shallowing-upward facies succession in the Emsian basal unit is overlain at each well either by facies association 2v followed by facies association 1v, or directly by facies association 1v. This vertical facies juxtaposition represents flooding of the sedimentary system associated with a rapid landward migration of the shoreline.

In palaeo-landward locations (wells A, B, and D), the shallow-marine sandstone tongue is under-developed in thickness (Fig. 7.5). The presence of upper-shoreface and foreshore facies associations at the top of the tongue at these locations indicates the absence of major transgressive erosion of shallow-marine sandstones from the top of the tongue, and thus that the shoreface sandstone was either deposited in a very shallow, restricted area or that the sand proportion supplied by littoral drift was minor compared to the proportion of mud supplied in suspension from the estuary.

Further paleo-seaward, towards ENE, sandstone thickness increases and the Gamma Ray signal becomes more complex, consisting of two upward-coarsening sub-cycles (i.e. two sandstone

explanation). The precise locations of facies boundaries are uncertain in wells where cores are absent.

(1) The upper sub-cycle can be considered as a younger, independent shallow-marine sandstone tongue deposited during a second distinct phase of shoreline regression. The younger tongue is separated from the older one by marine shales deposited during a period of flooding that involved landward migration of the shoreline. In this hypothesis, the two shallow-marine sandstone tongues show a strongly progradational stacking pattern (Fig. 7.7A) which may indicate overall “forced-regressive” conditions (*sensu* Posamentier *et al.*, 1992; Helland-Hansen & Martinsen, 1996).

(2) The upper sub-cycle can be considered as the seaward continuation of the single sandstone tongue described in the landward wells (*A*, *B*, and *D*) and the lower tongue in the paleo-seaward wells can be interpreted as an upward-coarsening bedset (*sensu* Van Wagoner, 1990, Pattison, 1995, Somme *et al.*, 2008, Hampson *et al.*, 2008) of the same tongue. The landward part of this bedset is thus attached to the main prograding tongue. This hypothesis portrays at least one sandstone bedset arranged as part of a low-angle clinoform of limited down-dip extent which is integral to the overall prograding stratigraphic succession (Fig. 7.7B). Thus, in this hypothesis, the shoreline prograded continuously from the palaeo-landward to palaeo-seaward wells without any major flooding of the shoreline and shelf. The bedset may reflect rearrangement of the shoreline morphology due to minor, high-frequency changes in wave regime, sediment supply and/or relative sea level.

These two stratigraphic correlation hypotheses have significant consequences for both exploration and reservoir characterisation purposes. In fact, in the first hypothesis, the recognition of two independent shoreface sandstone tongues having a progradational stacking pattern suggests that the connection between each tongue might be discontinuous at reservoir scale (i.e. they are separated by a laterally continuous flooding surface and shale), thus limiting vertical connectivity of the two tongues. In the second hypothesis, the shallow marine sandstone tongue is continuous and sandbodies are connected at reservoir scale, but the internal stratigraphic architecture is complex, which may affect reservoir sweep and dynamic behaviour. Therefore, a key purpose of this second application of the stratigraphic inverse and

forward numerical scheme will be to assess the feasibility of the correlation hypotheses based on quantification of mechanisms that generated the observed stratigraphy.

7.4 Modelling Workflow

The three stratigraphic cross-sections (Fig. 7.5) have each been inverted using an independent stratigraphic inverse and forward simulation (i.e. one inversion per cross-section). Each inversion has been constrained only by the well data present along the cross-section under consideration and by thickness curves of the stratigraphic interval calculated between the wells in that cross-section (via linear interpolation between the thicknesses measured at each well location).

Early iterations of this work attempted to invert the three cross-sections simultaneously using a specially developed multi-2D inverse and forward numerical scheme, in which the forward simulations were realised for each cross-section and shared the same global environmental input parameters (e.g. relative sea level). However, due to its complex numerical implementation, this stratigraphic inversion process was too CPU intensive and time consuming to obtain enough samples in a realistic time frame. Therefore, this approach was abandoned in favour of the more simplistic, but sufficient, approach presented here.

The method to parameterise and to adapt the simulations to subsurface data consisted of:

- (1) Transforming the Gamma Ray signal of each well to a grain-size signal using a calibration of core and well-log character in cored intervals.
- (2) Decompacting the sedimentary succession of each well.
- (3) Calculating the thickness curves.
- (4) Designing the initial topographic profiles on which sedimentation of the Emsian basal units occurred.
- (5) Parameterisation of the stratigraphic inverse and forward numerical scheme by adding geological prior knowledge.

- (6) Running the stratigraphic inversion.
- (7) Post-processing the output data before interpretation.

7.5 Observed Data Processing

7.5.1 Well data

7.5.1.1 Gamma Ray to Grain Size Transformation

To adapt the sub-surface data to the requirements of the likelihood function, the Gamma Ray signals of the non-cored intervals of the wells have been transformed into grain-size signals by using the core descriptions previously done by the company Total. In these descriptions, the logarithm of the grain-size signal has been normalised between 0 and 5.2 (WellCad format). This format was used throughout the data processing. The workflow is illustrated in Figure 7.8 and consists of the following steps.

- (1) Smoothing the normalised log grain-size curve defined in cored intervals (smoothing window of 25 cm) to extract a low-frequency signal similar to that of the low-resolution Gamma Ray curve (Fig. 7.8A).
- (2) Plotting the resulting smoothed and normalised log grain-size curve against its corresponding Gamma Ray signal in order to calculate linear regressions (Fig. 7.8B). Although the Gamma Ray curve measures clay content, this comparison with the core descriptions supports its use as a proxy to determine lithology trends and thus grain-size signals.
- (3) Transforming the Gamma Ray signal into normalised log grain-size curve assuming that the linear regression calculated in the reservoir cored areas is also valid for non-reservoir mudstone facies (Fig. 7.8C).
- (4) Exchanging the grain-size signal inferred from the Gamma Ray curve for the smoothed version of the observed normalised log grain-size curves in cored intervals (Fig. 7.8D).
- (5) Back-transforming the normalised log grain-size curve into a grain-size curve with standard units of measurement (grain diameter in millimetres) (Fig. 7.8E).

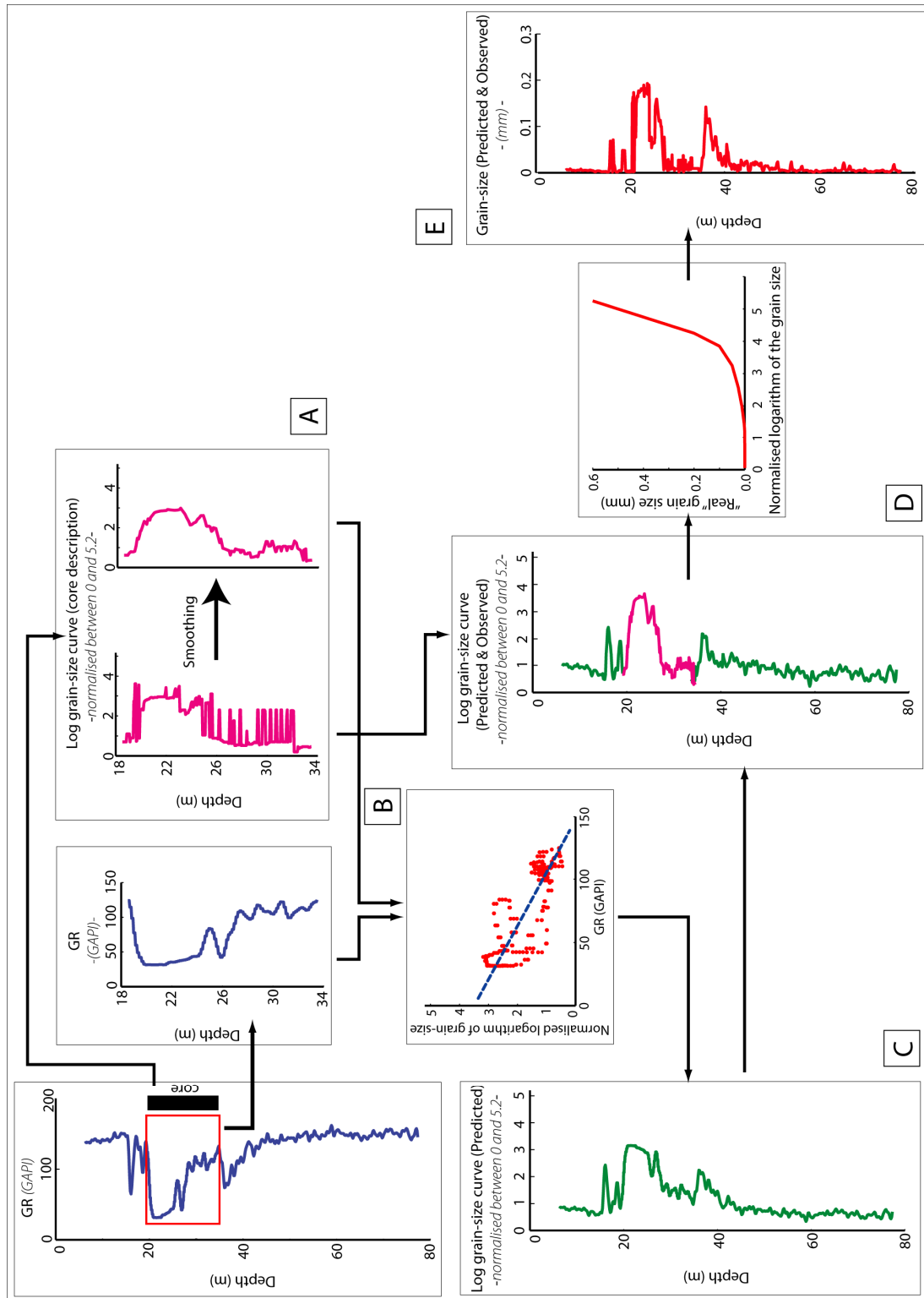


Figure 7.8: Simplified workflow diagram illustrating the Gamma Ray transformation to predict the grain-size signal in well F (see text for explanation).

Data processing has been undertaken for each well independently. Values of calculated grain size in the non-reservoir intervals have been standardised from well to well. Grain-size measurement errors on cored sandstones are low, estimated at around 5 to 10% of the grain-size values. Measurements errors on predicted the shale grain-size signal are higher and have been assessed by varying the slopes of the linear regressions calculated between measured grain size and corresponding Gamma Ray values (steps 2 and 3 above). Errors on the non-reservoir grain-size predictions have been estimated to be around 80 to 100% of the predicted values.

7.5.1.2 Decompaction

The BARSIM algorithm does not include a dynamic compaction module, thus the stratigraphic data available from the wells have to be decompacted prior to the stratigraphic inversion process. To decompact the investigated strata, the maximum burial depth has to be estimated. Vitrinite reflectance is an organic indicator for thermal evolution during burial that can be used to estimate the maximum burial depth using certain assumptions. However, the vitrinite reflectance values in the Emsian well data are extremely high (around 2.5 to 2.8 % Ro) and thus can not be used. In fact, the linear relationship to evaluate the burial depth from the vitrinite reflectance values (Corcorand & Clayton, 2001) is only valid for values between 0.7 and 1.6% of Ro. For example, such a relationship would give a maximum burial depth of 17 km for the Emsian strata. Nevertheless, without quantifying the maximum burial depth precisely, these reflectance values indicate that burial was very deep (over 4-5 km). At such depths, sediments are generally almost fully compacted, which implies that the total sediment thickness, and therefore its associated decompacted value, does not decrease with further burial. Thus, the strategy adopted to decompact Emsian strata was to use an average decompacted value for maximum burial depths between 5 and 7 km. Results are summarised in Table 7.1.

The standard deviations on the results are about 2 to 3m and have been used as measurement error in the likelihood function of the inverse algorithm. Emsian sediments have been

decompacted by using the porosity/depth exponential relationship of Athy (1930), calibrated by exponents derived from Sclater & Christie (1980) (Table 7.2).

Well	Initial thicknesses (m)	Decompacted thicknesses (m)
<i>A</i>	55.75	120.5
<i>B</i>	56.1	121.3
<i>C</i>	57.0	111.9
<i>D</i>	58.5	103.9
<i>E</i>	63.2	112.5
<i>F</i>	65.3	116.3
<i>G</i>	78	138.7
<i>H</i>	53.3	94.96
<i>I</i>	63.25	112.5

Table 7.1: Initial and average decompacted total thickness of the Emsian basal unit in each well.

Lithology	Surface porosity ϕ_0	Porosity-depth coefficient $c(km^{-1})$
Shale	0.63	0.51
Sandstone	0.49	0.27
Shaly sandstone	0.56	0.39

Table 7.2: Exponents derived by Sclater & Christie (1980) for different lithologies from North Sea sub-surface data.

7.5.2 Thickness Curves of Stratigraphic Intervals

Three thickness curves of the Emsian basal unit have been calculated to constrain the inversion process, one thickness curve per cross-section. Thicknesses have been estimated from the most paleo-landward to the most paleo-seaward wells of each cross-section. The curves have been calculated using linear regressions between the total decompacted thickness values of each well, assuming no abrupt lateral variations as suggested by the seismic data. At each well location, the measurement error on the thickness curve has been set to be equal to the error estimated when decompacting the strata at that well (standard deviation of 2 to 3m). At other locations, this small error has been multiplied by a coefficient that increases proportionally with increasing distance from the nearest well, up to a value of 1.5.

7.5.3 Initial Topographic Profile Reconstruction

Specifying the initial physiography of the cross-sections on which Emsian sedimentation occurred (initial condition parameters of BARSIM, Chapter 2; §2.2.2.3) is an important stage in the inverse and forward modelling workflow. This stage consisted of three steps.

- (1) Reconstruction of the palaeo-distance between the well data. The Emsian deposits are structured in large-scale low-amplitude folds such that the maximum dips measured on seismic data are 2-3° degrees. Therefore, in this case study, reconstructing the paleo-distance was unnecessary, as the corrections have been estimated to be less than 100 m, which is smaller than the horizontal grid cell resolution in the models (cf. subsequent paragraph).
- (2) Reconstruction of the initial topographic profile. This has been achieved by setting the palaeo-water depths at each well location to be equal to the corresponding decompacted thickness of the Emsian basal unit. Then values have been assigned to the numerical forward modelling grid by linear interpolation between values of well. This method assumes that the initial topography was completely infilled during progradation of the shoreline, which is justified by the core observations that show fully developed shoreface facies successions from offshore transition to foreshore. In the palaeo-seaward and palaeo-landward parts of the cross-sections, no data were available to constrain the palaeo-topography. Therefore, in the palaeo-seaward parts, palaeo-water depths have been estimated by using the same slope gradients as those in nearby, constrained parts of the cross-sections. In the palaeo-landward parts, palaeo-water depths have been determined by using a steeper gradient, based on data constraints from nearby wells.
- (3) Evaluation of the underlying grain-size distribution of the substrate. This step has been achieved by using the grain-size values of the top of the underlying Praguian deposits available at each well location. Then, values have been linearly interpolated into the numerical forward modelling grid from well to well. The grain-size distribution of the Praguian substrate is very sandy across the cross-sections and contrasts clearly with the mud-dominated succession of the basal Emsian deposits. This observation shows that

Praguian and Emsian sedimentary histories are clearly distinct. Thus, grain-size distributions of the Praguian substrate have not been used to refine the reconstruction of the initial topographic profile on which Emsian sedimentation occurred.

7.6 Model Parameterization

7.6.1 Forward Model

The BARSIM forward model horizontal grid has 200 cells of 375 m each. As in all the forward modelling cases, vertical resolution of the grid is self generated according to the sediment thickness deposited. The duration of deposition of the Emsian basal unit is poorly constrained over this area and has been estimated to be around 1 to 2 My (internal report of the company Total). Regarding the geographical area covered by the cross-sections and the overall progradation direction, the study area has been estimated to represent around 30% of the total progradation distance of the basal Emsian unit (Fig. 7.6). Therefore, it is *a priori* reasonable to state that the studied area represents 30% of the duration of the unit. Thus, it was decided to simulate sedimentation with BARSIM over a period of 300,000 years with a minimum recurrence period of 150 years for major storm events. The choice of using a large minimum recurrence period for major storm events (i.e. time-step) has been motivated by the limited dataset, which does not allow high-resolution analysis of the storm/wave regime, and to reduce computation costs.

For this work, the inversion scheme has to infer the following environmental parameters: mud fallout rate, relative sea level, sediment supply and fair-weather wave height curves. The differential subsidence rate module of BARSIM has been switched off, because the Emsian deposits are interpreted to have been deposited during a period of tectonic quiescence. Analysis of the grain-size distribution within the wells shows that the lithological succession is a composition of 20% upper fine-grained sandstone (estimated mean grain diameter of 100-200 μm) and of 80% shale (estimated mean grain diameter of 6 μm), according to grain-size estimates derived from the Gamma Ray signal. Thus, this analysis was used to set the initial composition of the sediment supplied by longshore drift currents (internal condition parameters

of BARSIM, Chapter 2; §2.2.2.3) to be a mixture of 80% mud (6 μm grain diameter) and 20% upper fine sand (200 μm grain diameter) that is representative of the overall lithological composition observed at well locations.

Internal Parameters		External Parameters	
		<i>Initial condition parameters</i>	
Maximum erosion rate [m^2/y]	2.5		
Power for erosion scaling: shoreline – wave Base [-]	1.0	Grain size (shale class) [μm]	6
Sediment travel distance correlation event [-]	2.0	Grain size (Sand class) [μm]	200
Sediment travel distance correlation shoreface [-]	1.0	Supply composition (% of sand)	20
Maximum height and width of the barrier [m]	1.5 - 1500		

Table 7.3: BARSIM parameterisation.

Other internal parameters of BARSIM (Chapter 2, §2.2.2.3) have been determined through preliminary tests (“sediment travel distance correlation event”, “sediment travel distance correlation shoreface”) or by using default values (“power for erosion scaling: shoreline – wave base”, “maximum erosion rate”, “maximum height and width of the barrier”). The maximum erosion rate is independent of the substrate properties. The “power for erosion scaling: shoreline – wave base parameter” is a constant that represents the dependence of erosion rate on water depth. The “sediment travel distances correlation event” and “sediment travel distances correlation shoreface” parameters are constant values that are used to match specific local coastal settings for the estimation of sediment travel distance to the characteristic sediment deposition curves of Guillén & Hoekstra (1996). Higher values of these parameters produce a higher probability of transporting sediment over longer distances. These two “sediment travel distances correlation event” and “sediment travel distances correlation shoreface” parameters account for local variability in sediment transport during storm and fair-weather conditions, respectively. They have been chosen on the basis of preliminary tests, such that convergence of the inverse numerical solutions was achieved more rapidly and such that the obtained stratal geometries were physically realistic and concordant with the observed stratigraphy. Compared to the case study described in Chapter 6, only the “sediment travel

distance correlation event” parameter is different, with a lower value (2.0, rather than 3.0 in Chapter 6). As consequence, sand is transported over shorter distances during storm conditions. Values are summarized in Table 7.3.

7.6.2 Inverse Method

The strategy developed for this stratigraphic inverse problem combines three Reversible Jump MCMC samplers, one fixed dimension MCMC sampler and one Simulated Tempering sampler linked in a Hybrid sampler (Chapter 3; Mariani & Parisi, 1992; Geyer & Thompson, 1995; Green, 1995; Gilks *et al.*, 1996; Denison *et al.*, 2002, Brooks *et al.*, 2003; Charvin *et al.*, 2008a). Details are compiled in table 7.4.

MCMC-based sampler	BarSim environmental input parameters
Reversible Jump MCMC	Relative sea level curve
Reversible Jump MCMC	Sediment supply curve
Reversible Jump MCMC	Wave height curve
Fixed Dimension MCMC	Mud fallout rate (shoreface)
Simulated Tempering	“Temperature” scheme

Table 7.4: MCMC-based samplers and their application to BarSim input environmental parameters. The fixed dimension MCMC sample aims to deal with the time-independent mud fallout rate input parameter of BARSIM, whereas the RJMCMC samplers are applied to time dependent curves that describe the other input parameters of BARSIM. The Simulated Tempering sampler is designed to deal with multi-modal posterior probability distribution (i.e. non-uniqueness of the inverse problem).

Three independent stratigraphic inversions have been run: one per modelled cross-section (Fig. 7.5). Each inversion was run for 400,000 iterations in order to explore the model parameter space. The northern cross-section has been constrained using the grain-size signals of three wells (A, B and C) and one thickness curve for the stratigraphic interval delimited by these wells (Fig. 7.5A). The central cross-section has been constrained using the grain-size signals of four wells (D, E, F and G) and one associated thickness curve of the stratigraphic interval

(Fig. 7.5B). The northern cross-section has been constrained using the grain-size signals of two wells (*H* and *I*) and one thickness curve (Figs. 7.5C).

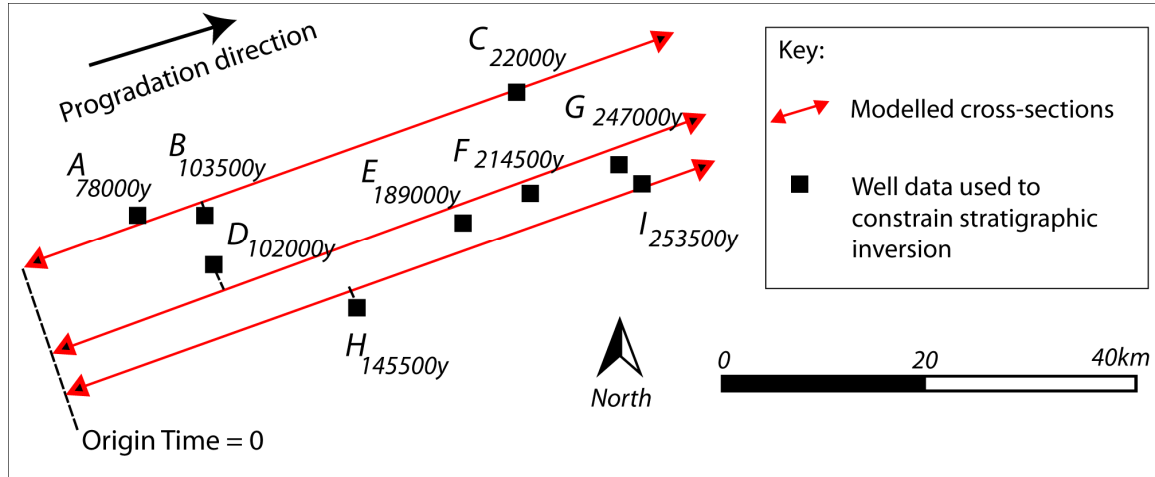


Figure 7.9: Estimated depositional ages attributed at the top of each well, relative to the onset of shoreline progradation into the study area. Each age has been estimated assuming a constant progradation rate and direction (parallel to the cross-sections) over the simulated time period, such that they increase linearly in a palaeo-seaward direction.

No data were available in the wells to constrain precisely the depositional timing of the Emsian basal unit. Thus estimated depositional ages were assigned to the top of each well, as required by the likelihood function. These estimated ages have been evaluated by assuming a constant progradation rate parallel to the cross-sections for the shoreline over the simulated time interval (Fig. 7.9). Such an approach was necessary to provide constraints for inverting the time-dependent BARSIM environmental parameters (i.e. sediment supply, relative sea level and fair-weather wave height regime). Such an approach is *a priori* reasonable in the absence of other data, but might introduce some bias into the inference of these time-dependent parameters. Therefore, measurement errors (standard deviation of 5,000 years) have been assigned to these estimated ages, in order to enable some flexibility to the inversion process.

As mentioned earlier, each cross-section will be inverted independently using data from the restricted subset of wells along it. However, it is clear, for example, that the data contained in the landward wells of the northern cross-section may provide complementary information for

inferring the overall environmental parameters (e.g. relative sea level) that drove the landward stratigraphy of the southern cross-section, where no comparable observed data are available. This information has been integrated via the use of a high level of prior knowledge for the relative sea level parameters, common to each inverted cross-section and defined from the analysis of all nine wells. Estimated ages show that deposition of the shoreface-sandstone tongue in the landward parts occurred between 78,000 and 102,000 years of simulated time (Figs. 7.5, 7.9). This analysis also showed that the upper shoreface-sandstone tongue in the seaward wells was deposited between 145,500 and 253,500 years of simulated time (Figs. 7.5, 7.9). Therefore, based on the decompacted thicknesses of the Emsian basal units, a uniform prior knowledge was used for these two periods in which relative sea level parameters were allowed to vary between 30 and 60 m (Fig. 7.10A). For the three other periods that lack such constraining data (i.e. 0-78,000 years, 102,000-145,500 years and 253,500-300,000 years of simulated time), a relatively uninformative uniform prior in which relative sea level parameters were allowed to vary between 0 and 60 m was used (Fig. 7.10A). This wide range for 102,000-145,500 years of simulated time has been chosen to avoid a bias towards either of the two stratigraphic correlation hypotheses (§7.3); during this period, relative sea level is free to vary in a manner that places no pre-conditioning on the mechanism of formation of the lower sandstone tongue in the palaeo-seaward wells.

For the sediment supplied by longshore drift currents, the wave regime and the mud fallout rate, minimal prior information was used, and values were allowed to vary uniformly *a priori* between 0-40 m²/y, 1-7 m and 0-0.1 cm/y respectively (Fig. 7.10). Ranges of possible variation for the sediment supply and mud fallout values have been chosen to be wide enough that prior knowledge does not over-influence the inversion in terms of the quantity of sediment preserved in the cross-section. The range of possible variation for the fair-weather wave height has been chosen arbitrarily, but again is broad enough that prior knowledge does not dominate the inversion. Although 7 m is the upper limit of fair-weather wave height considered by the inversion, storm wave heights which are drawn from fair-weather wave height by the stochastic component described in Chapter 2 are not limited by any prior knowledge and thus can be greater than 7 m. Generally, higher storm wave heights correspond to higher fair-weather wave heights.

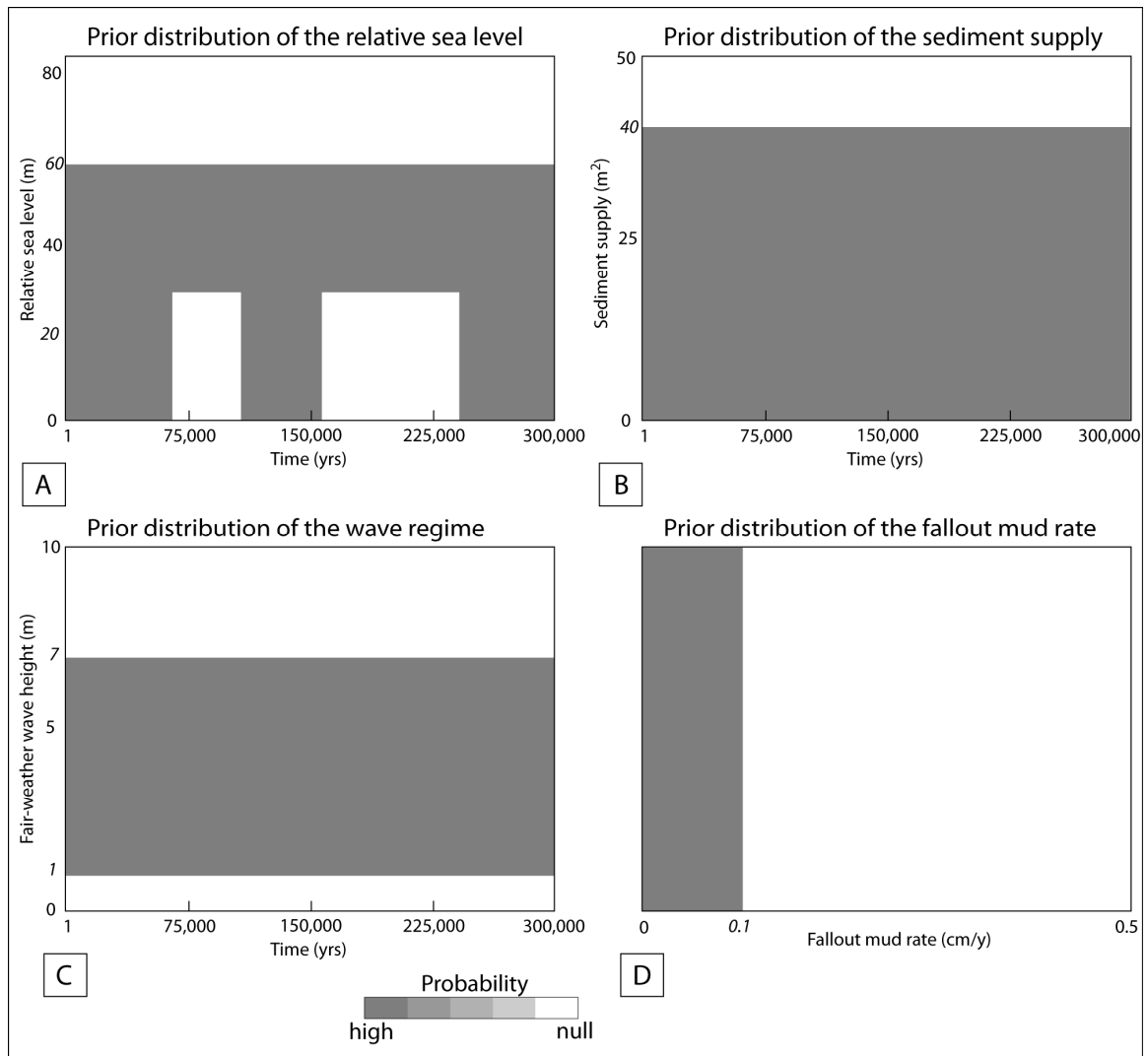


Figure 7.10: Environmental parameter prior probability distributions of the three inverted cross-sections in the South Algerian case study. (A) The probability distribution of the relative sea level parameter has been drawn from the study of the entire dataset in order to give coherency to the three independently inverted cross-sections and to provide a relatively high level of prior information where observed data are lacking. (B, C and D) Uniform prior probability distributions respectively for sediment supply, wave regime and mud fallout rate.

Model parameters have been perturbed using a Gaussian proposal distribution centred on their current values. Details are presented in Table 7.5. The model parameters of the first step of the Markov chain have been selected manually, sufficiently close to the expected solutions of the inverse problem to shorten the “burn-in” (convergence) period of the inversion process. The

model parameters of the first step of the Markov chain are presented with the posterior probability results in the subsequent paragraph (Figs. 7.12, 7.13, 7.14).

Parameters		Prior distribution (Uniform)	Proposal variance Fixed dim. jump	Proposal variance Trans-dim. jump
Relative sea level	- Sea level values	0 – 60, 30-60 m	1 m	4m
	- Time values	1 – 300,000 yrs	1,000 yrs	100 yrs
Sediment supply	- Supply values	0 - 40 m ² /y	1 m ² /y	5 m ² /y
	- Time values	1 – 300,000 yrs	1,000 yrs	100 yrs
Wave height regime	- Wave height value	1 - 7 m	0.1 m	1m
	- Time values	1 – 300,000 yrs	1,000 yrs	100 yrs
Mud fallout	- Fallout rate values	0 – 0.1 cm/y	0.01 cm/y	NA
Temperature level	3 levels: T={1,2,3}			
Observed data	Error	Grain size	Error Thickness	Error Time
Northern cross-section	80 – 100% of the value for Shale		2 - 3 m	5,000 yrs
	5 – 10% of the value for Sand			
Central cross-section	80 – 100% of the value for Shale		2 - 3 m	5,000 yrs
	0 – 10% of the value for Sand			
Southern cross-section	80 – 100% of the value for Shale		2 - 3 m	5,000 yrs
	0 – 10% of the value for Sand			

Table 7.5: Prior and proposal distributions for relative sea level, sediment supply, wave height regime and mud fallout parameters (upper part of the table), and measurement errors (standard deviation) on the observed stratigraphic data constraining the inversion process (lower part of the table). Sediment supply, wave height regime and mud fallout parameters are allowed to vary uniformly between minimum and maximum values summarized in the second column of the upper part of the table. Candidate model parameters are drawn from Gaussian distributions centered on the current parameters. Variances defining the Gaussian proposal distributions for fixed dimensional and trans-dimensional perturbations are presented in the third and fourth columns of the upper part of the table, respectively.

7.7 Results

The results presented in the subsequent paragraphs have been collated from the post burn-in period samples of the “cold” distribution (Chapters 3 and 4). Therefore, only about 100,000 samples, among the 400,000 available for each cross-section, have been used to construct the posterior probability distributions. The inversion of each cross-section requires about two weeks of computational costs on a standard single-processor PC.

7.7.1 Inference of the Environmental Parameters

Each posterior probability distribution is presented with the boundaries of the 95% Bayesian credible interval, the maximum likelihood estimates and the set of parameters chosen as the initial step of the Markov chain.

7.7.1.1 Posterior Probability Distributions of the Relative Sea Level Curve

Figure 7.11 shows the posterior probability distributions of the relative sea level inferred independently for the northern, central and southern cross-sections. Although the relative sea level is a regional parameter, and therefore should be the same for the three inverted cross-sections, the three posterior probability distributions are different. Most of the dissimilarities occur during the first 75,000 years and the last 50,000 years of simulated time. These two periods are conditioned to minimal prior knowledge because sedimentation occurred respectively in the palaeo-landward and palaeo-seaward extremities of the cross-sections, where no observed data constraints are available. During the last period, the differences between the three posterior probability distributions show clearly that no relative sea level history can be extracted with confidence from the available data. These results demonstrate that the method is limited in predicting palaeo-seaward stratigraphy for exploration purposes when no constraining data are available.

The results suggest something different for the first period. In fact, despite their dissimilarities in shape and in the spread of the Bayesian credible intervals, each posterior probability distribution shows a major relative sea level rise of 30-40 m during the first 75,000 years of simulated time. This similarity can be explained by the implicit constraints provided by observed data situated in a more palaeo-seaward position (Chapter 4) and by the prior knowledge used for a later period of simulated time (from 78,000 to 102,000 years). Therefore, the differences between the three cross-sections can be explained by variations in the locations of the most palaeo-landward observed data (i.e. implicit constraints) in each cross-section, combined with the effects of prior knowledge that limits the subsequent relative sea-level history. In fact, the posterior probability distribution inferred for the northern cross-section, which contains the most palaeo-landward observed data (i.e. the earliest implicit constraint), shows the shortest period of sea level rise, whereas the posterior probability distribution for the southern cross-section, characterised by the absence of observed data in its palaeo-landward part, shows the longest period of sea level rise. In this latter case, the relative sea level rise is implicitly constrained mainly by the prior knowledge limits from 78,000 to 102,000 years of simulated time (Fig. 7.10).

Despite these discrepancies, it is clear that the same relative sea level trends and values are observed in the three posterior probability distributions. Following the first period of relative sea level rise (described above), the second period (from around 50,000 to 105,000 years of simulated time) is characterised in all the posterior probability distributions by well constrained steady conditions of high sea level situated at around 40 m (Fig. 7.11). The spread of the Bayesian credible intervals is between 7 and 10 m. These values are relatively high but coherent with the thickness measurement errors estimated when decompacting the strata (STD varying from 2 to 3m; Table 75). The duration of this second period differs between the posterior probability distributions, and it is directly linked to the duration of the preceding sea level rise period and therefore to the palaeo-landward locations of observed data in each cross-section. For example, it is clear that for the southern cross-section, where no observed data are available in the palaeo-landward part, the duration of the relative sea level highstand for this second period is very strongly conditioned to the prior knowledge of sea-level from 78,000 and 102,000 years of simulated time (Fig. 7.10).

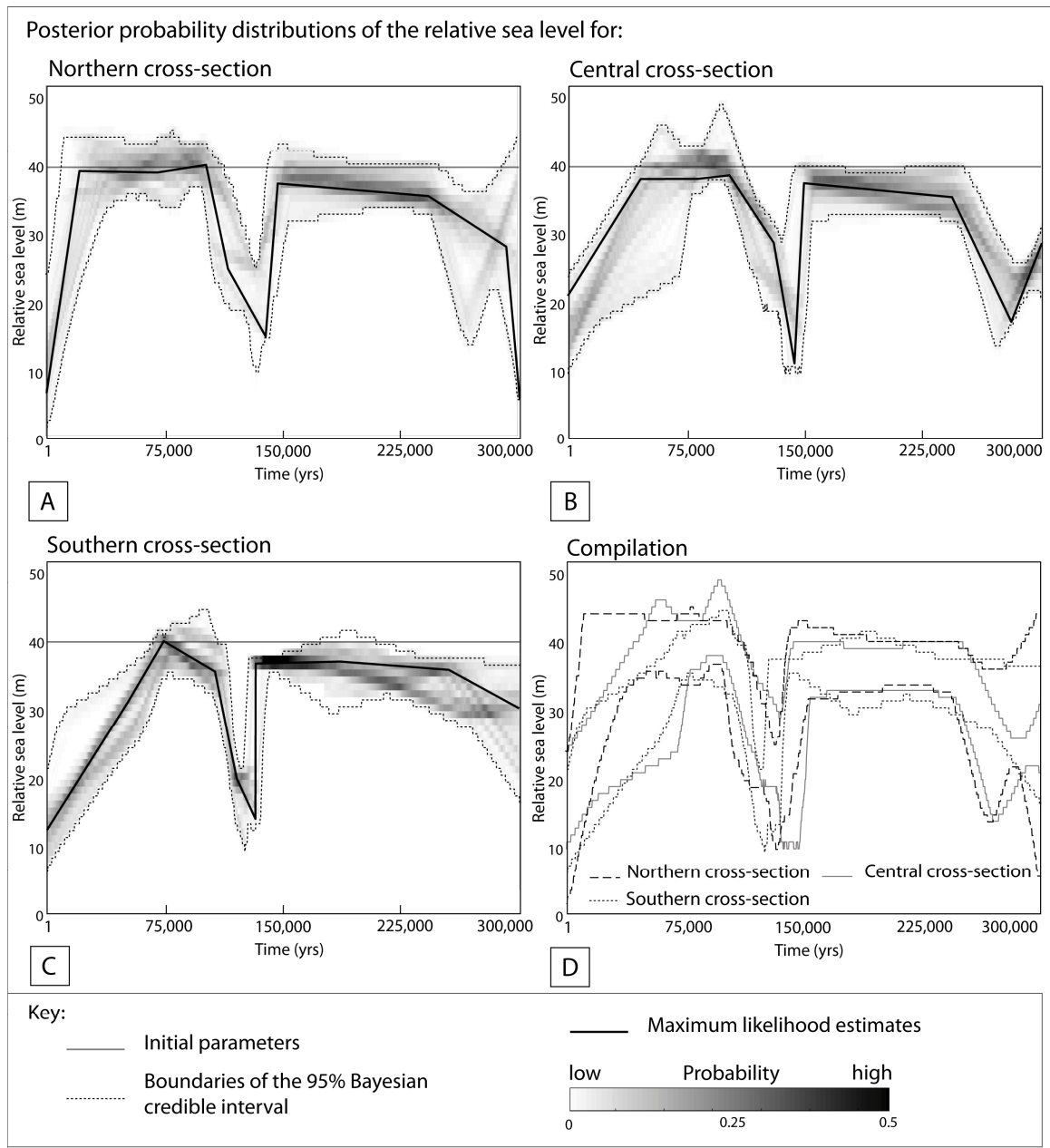


Figure 7.11: Relative sea level posterior probability distributions (A, B and C) respectively inferred from datasets of the northern, central and southern cross-sections. The dark continuous line in each graph is the maximum likelihood estimate; the light grey line represents the initial input curve and the dashed lines represent the boundaries of the 95% Bayesian credible interval. The background shading represents the posterior probability distribution (dark means higher probability). (D) Compilation of the 95% Bayesian credible interval for each modelled cross-section. Although stratigraphic inversions have been run

independently for each cross-section, the inferred distributions show coherency in values and trends.

The third and fourth distinct periods of relative sea level history (Fig. 7.11) are characterised, in all three posterior probability distributions, by a rapid relative sea level fall of 35 m occurring from 105,000 to 135,000-140,000 years of simulated time (third period), followed by a rapid relative sea level rise of 30 m occurring from 135-140,000 to 145,000 years of simulated time (fourth period). These two periods are time equivalent to the second period of limited prior information (Fig. 7.10), that was arbitrarily set up between 102,000 and 145,000 years of simulated time to assess the stratigraphic correlation issues. The spread of the Bayesian credible interval for the three posterior probability distributions is small (around 5 m), shows that the relative sea level signal is well constrained. Between 102,000 and 145,000 years of simulated time, the solution of the inverse problem is clearly unique for the relative sea level parameter: the observed stratigraphy can only be explained by a well defined sea level signal. The inversion therefore clearly supports one of the two stratigraphic correlation hypotheses, as explained later (§7.7.2).

The fifth period of relative sea level history, occurring from 145,000 to 253,000 years of simulated time, is characterised in the three posterior probability distributions by well constrained steady conditions with relative sea level values of around 35m (Fig. 7.11). This period corresponds to the second episode of high prior information, from 145,500 to 253,500 years of simulated time (Fig. 7.10). The spread of the Bayesian credible interval (5 to 7m) is lower than that of the second sea level period (50,000 to 105,000 years of simulated time), which is also conditioned to a high level of prior knowledge. This lower spread reflects the availability of more constraining observed data in the palaeo-seaward parts of the cross-sections, which corresponds to this fifth period of relative sea level history.

Although the three posterior probability distributions contain differences related to the location of geographical areas that are unconstrained by the observed data, the three independent stratigraphic inversion processes were successful in inferring the same trends and values of regional relative sea level history. Of equal importance, the results clearly show a unique solution for the relative sea level parameter, which thus offers a strong basis to evaluate the two alternative hypotheses for stratigraphic correlation (§7.7.2). The posterior probability

distributions of the local environmental parameters (e.g. sediment supply, fallout mud rate and fair-weather wave height) obtained for each cross-section are presented independently below.

7.7.1.2 Posterior Probability Distributions of the Local Environmental Parameters of the Northern Cross-Section

Figure 7.12 shows the posterior probability distributions of the sediment supply (Fig. 7.12B), the fair-weather wave height (Fig. 7.12C) and the mud fallout rate (Fig. 7.12D) obtained for the northern cross-section. The relative sea level posterior probability distribution seen in Figure 7.11A has also been added to facilitate analysis. General analysis of the three time-dependent posterior probability distributions (Fig. 7.12 A, B, and C) demonstrates clearly that there are no time-related relationships between the variations of these parameters. This observation differs from the conclusions drawn from the first application of the numerical scheme in Chapter 6, but it may reflect poor dataset coverage and quantity, which are insufficient to allow inference of high frequency variations in the environmental parameters. In the first case study (Chapter 6), data from 8 logged sections were used to constrain 35 km of cross-section recording 150,000 years of sedimentation; in contrast, here data from only 3 wells were used to constrain 75 km of cross-section recording 300,000 years of sedimentation. Therefore, the results obtained for the time-dependant local environmental parameters should not be over-interpreted. Only their gross trends will be analysed below.

The posterior probability distribution of the sediment supply curve (Fig. 7.12B) shows relatively low values (around 20 m²/y). The spread of the Bayesian credible interval is wide (10 to 15 m²/y), but at gross scale, result show a progressive decrease over the simulation time. The posterior probability distribution of the fair-weather wave height regime (Fig. 7.12C) also contains a very high level of uncertainty, because the Bayesian credible interval is 1 to 1.5 m wide. At gross scale, results demonstrate that the wave regime was relatively low with an averaged fair-weather wave height situated between 1 and 2 m. This trend remains relatively constant over the simulation time. In contrast to the other local environmental parameters, the posterior probability distribution of the mud fallout rate is well constrained and is centred on a relatively high value 0.05cm/y (Fig. 7.12D)

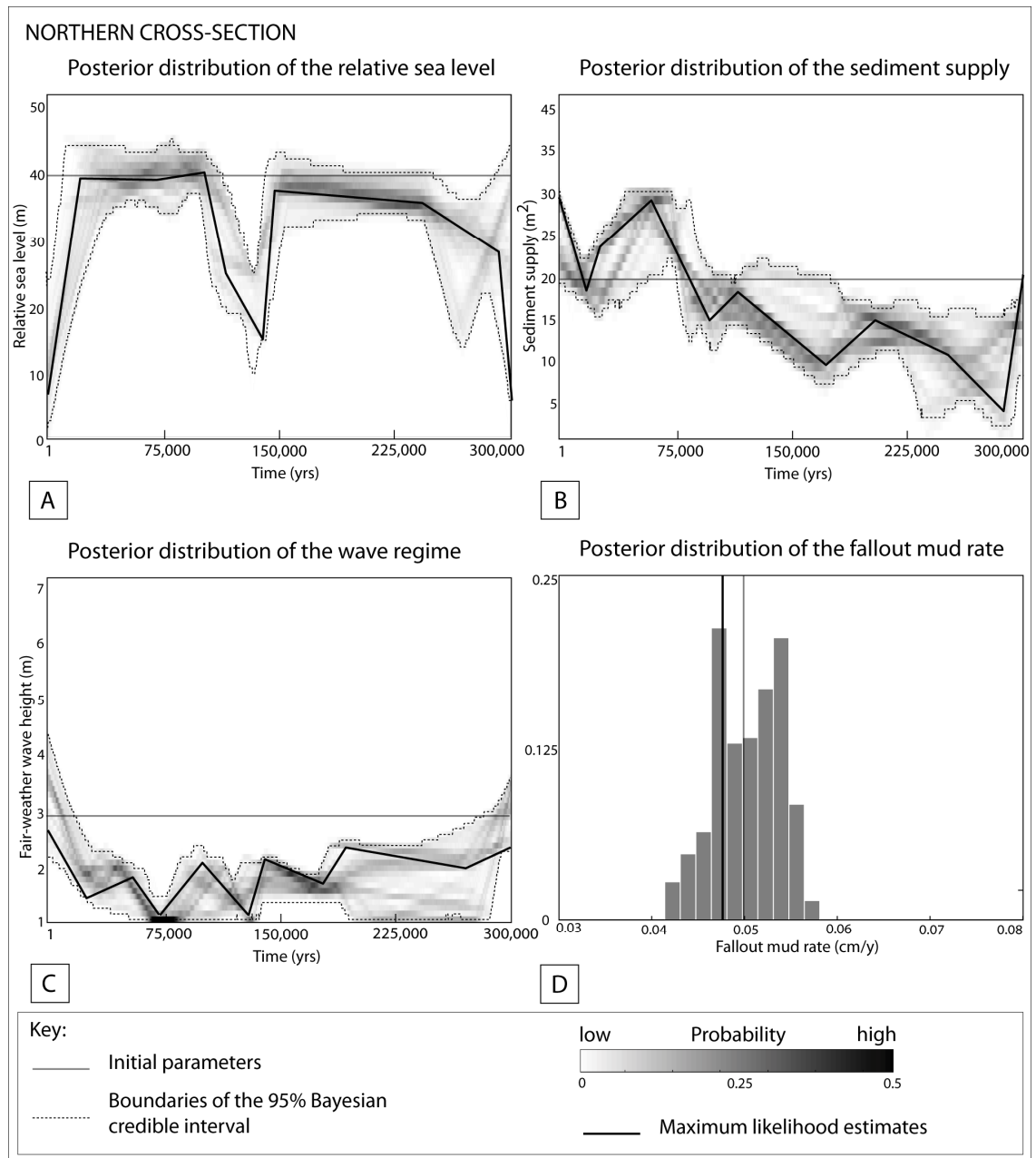


Figure 7.12: Environmental parameter posterior probability distribution inferred from the dataset of the northern cross-section. (A), (B) and (C) show respectively the posterior probabilities distribution of relative sea level (Fig. 7.11A), sediment supply and fair-weather wave height curves. (D) presents the posterior probability distribution of the time-independent mud fallout rate. The dark continuous line in each graph is the maximum likelihood estimate; the light grey line represents the initial input curve and the dashed lines represent the

boundaries of the 95% Bayesian credible interval. The background shading represents the posterior probability distribution (dark means higher probability).

7.7.1.3 Posterior Probability Distributions of the Local Environmental Parameters of the Central Cross-Section

Figure 7.13 shows the posterior probability distribution of the sediment supply (Fig. 7.13B), the fair-weather wave height (Fig. 7.13C) and the mud fallout rate (Fig. 7.13D) obtained for the central cross-section. The relative sea level posterior probability distribution seen in Figure 7.11B has also been added. Again, examination of the three time-dependent posterior probability distributions (Fig. 7.13 A, B, C) demonstrates that there are no obvious relationships between the variations of relative sea level, sediment supply and fair-weather wave height parameters. Even if the central cross-section is better constrained than the northern cross-section, because a dataset of four wells was used to constrain 75km of cross-section recording 300,000 years of sedimentation, data quantity and coverage still do not allow high resolution inference of high frequency variations in the local environmental parameters. Consequently, the results obtained for the time-dependent local environmental parameters should also not be over-interpreted.

The posterior probability distribution of the sediment supply (Fig. 7.13B) shows relatively low values (around 25 to 30 m²/y), which are slightly less than those inferred for the northern cross-section. The spread of the Bayesian credible interval is wide and varies from 5 to 15 m²/y. The trends imply that the supply decreased progressively and slightly during the first 200,000 years of simulated time. Then, it increased up to 35m²/y at around 250,000 years of simulated time, before falling again until the end of the simulation time. This increase in the sediment supply may be related to the increased thickness of the Emsian basal unit and shoreface sandstone tongue in the palaeo-seaward part of the cross-section, as shown in well G. However, transgressive erosion associated with flooding at the top of the basal Emsian unit, which was not simulated in these inversions, may also be invoked to explain the over-thickened shoreface sandstone tongue in well G.

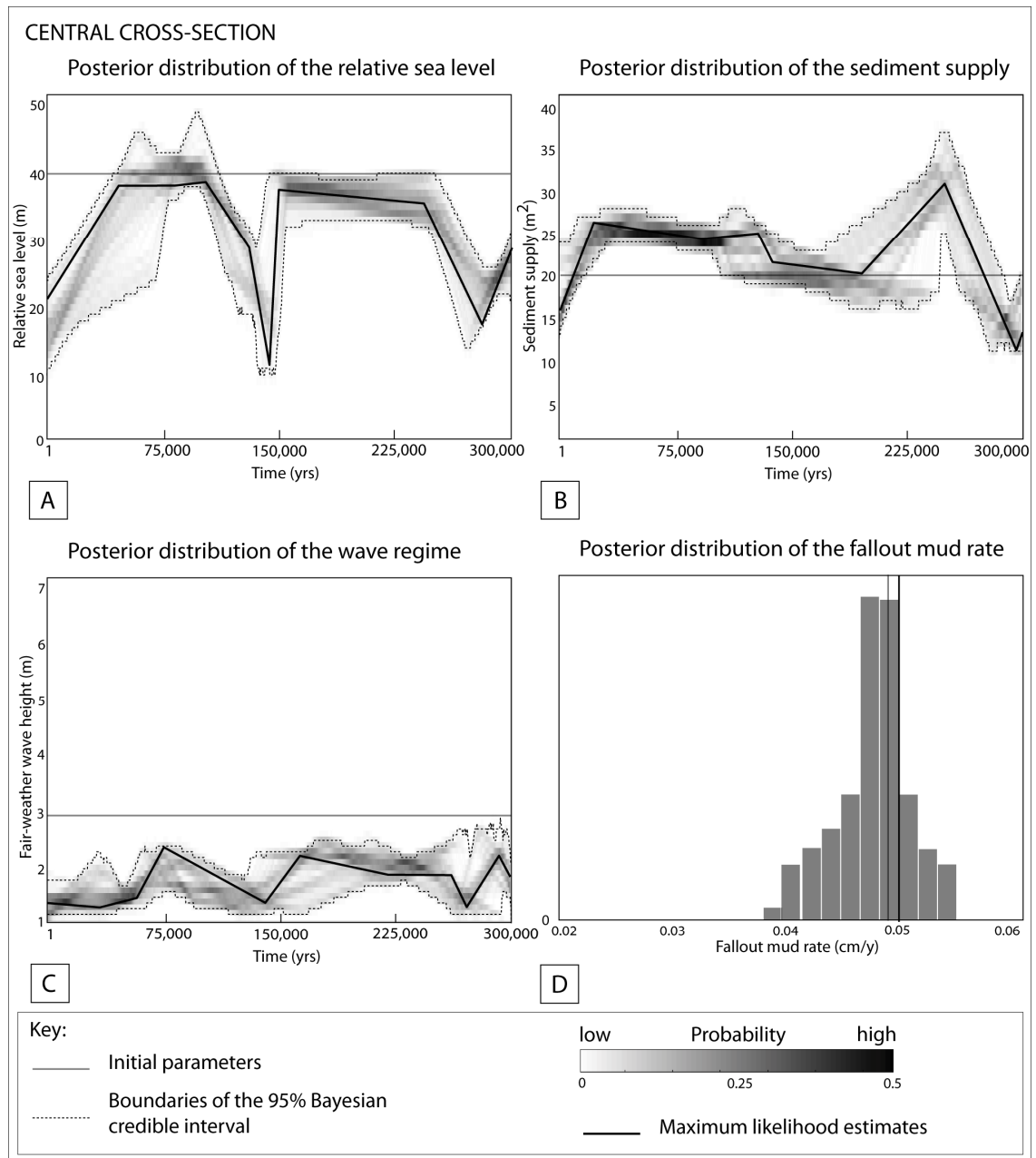


Figure 7.13: Environmental parameter posterior probability distribution inferred from the dataset of the central cross-section. (A), (B) and (C) show, respectively the posterior probabilities distribution of relative sea level (Fig. 7.11B), sediment supply, and fair-weather wave height curves. (D) presents the posterior probability distribution of the time-independent fallout mud rate. The dark continuous line in each graph is the maximum likelihood estimate; the light grey line represents the initial input curve and the dashed lines represent the

boundaries of the 95% Bayesian credible interval. The background shading represents the posterior probability distribution (dark means higher probability).

The posterior probability distribution of the fair-weather wave height regime (Fig. 7.13C) has a large spread: the Bayesian credible interval is 1 to 1.5 m wide. Once again, no high frequency signal can be extracted from this posterior probability distribution. Examination of these results shows that the wave regime was relatively low with an averaged fair-weather wave height of between 1 and 2 m. This trend remains relatively constant throughout the simulation time. The posterior probability distribution of the mud fallout rate is well constrained and is also centred on values of around 0.05 cm/y.

7.7.1.4 Posterior Probability Distributions of the Local Environmental Parameters of the Southern Cross-Section

Figure 7.14 shows the posterior probability distributions of the relative sea level (Fig. 7.14A), the sediment supply (Fig. 7.14B), the fair-weather wave height (Fig. 7.14C) and the mud fallout rate (Fig. 7.14D) parameters obtained for the southern cross-section. Once again, analysis of the three time-dependent posterior probability distributions (Fig. 7.14 A, B, C) demonstrates that there are no obvious relationships between the variations of the relative sea level, sediment supply and fair-weather wave height parameters. High frequency variations in local environmental signals can not be reliably extracted from the posterior probability distributions. This result is even more pronounced in this cross-section, for which a dataset of only two wells in palaeo-seaward locations was used to constrain 75 km of cross-section recording 300,000 years of sedimentation.

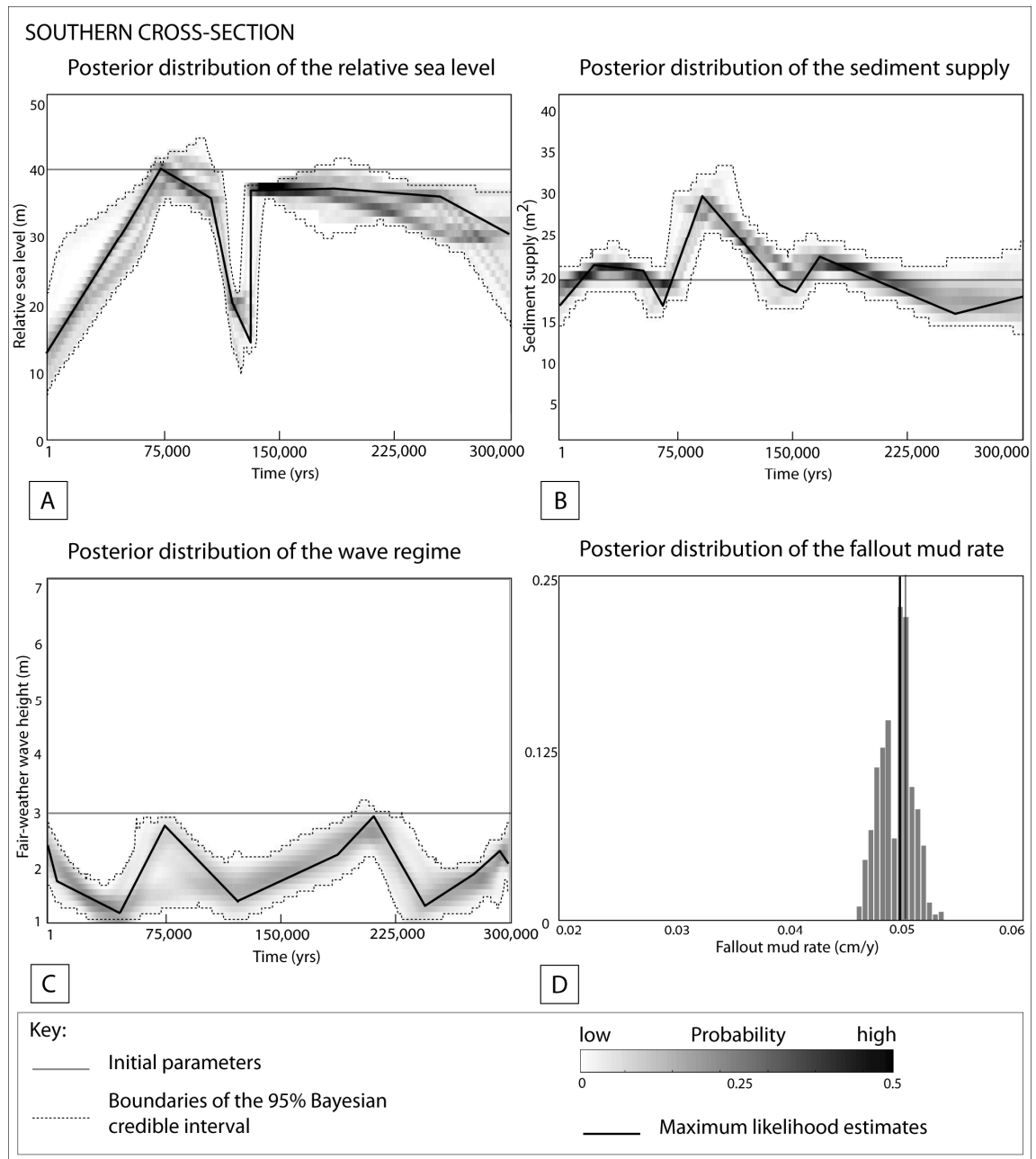


Figure 7.14: Environmental parameter posterior probability distribution inferred from the dataset of the southern cross-section. (A), (B) and (C) show respectively the posterior probabilities distribution of relative sea level (Fig. 7.11C), sediment supply and fair-weather wave height curves. (D) presents the posterior probability distribution of the time-independent fallout mud rate. The dark continuous line in each graph is the maximum likelihood estimate; the light grey line represents the initial input curve and the dashed lines represent the

boundaries of the 95% Bayesian credible interval. The background shading represents the posterior probability distribution (dark means higher probability).

The posterior probability distribution of sediment supply (Fig. 7.14B) shows relatively low values (around 20 m²/y). The Bayesian credible interval is slightly narrower than those of the two first cross-sections (5 to 10 m²/y), but still remains wide. The trend shows that the supply increases up to 30 m²/y during the first 100,000 years of simulated time and then decreases to 15-20 m²/y at the end of the simulation. The posterior probability distribution of the fair-weather wave height regime (Fig. 7.14C) is broad. The Bayesian credible interval is again 1 to 1.5 m wide. Once again, at gross scale, these results demonstrate that the wave regime was relatively low with an averaged fair-weather wave height of between 1 and 2 m during all of the simulated time. The posterior probability distribution of the mud fallout rate is extremely well constrained for this cross-section and is also centred on values of around 0.05 cm/y.

7.7.1.5 Summary of the Posterior Probability Distributions of Environmental Parameters

The inference of the BARSIM environmental parameters from the three cross-sections is relatively successful considering the scarcity of available data to constrain the model parameters. Although the stratigraphic inversions have been independently run for the three cross-sections, the results show coherency for both regional and local parameters. In fact, for all cross-sections, the inferred sediment supply histories are relatively similar, with an average supply value of around 20m²/y and a gently decreasing trend over time. Local variations (e.g. the abrupt rise in sediment supply between 200,000 and 250,000 years of simulated time for the central cross-section) can be related to local configurations, such as the volume of sediment required to infill variations in the initial topographic profile. The fair-weather wave height history is also relatively similar for each cross-section, and is characterised by uniformly low values. Again the mud fallout mud rates for each cross-section are similar and relatively high. The three independent stratigraphic inversion processes were also consistent in inferring the same trends and values for the regional relative sea level history. The results show clearly a unique solution for the sea level parameter: only one stratigraphic correlation hypothesis is

valid, as discussed below (§7.7.2). Of equal importance, the spread of the inferred distributions show that inferences of sediment supply and wave height histories have relatively more uncertainty than inferences of the fallout mud rate and relative sea level history. The inverse models seem to be more sensitive to these two last parameters. However, the number and locations of observed data points are clearly insufficient to obtain better constrained results and to study high-frequency, low-amplitude variations in, and interactions between, the environmental signals.

7.7.2 Prediction of Grain Size, Bed Thickness and Stratigraphic Correlation

In order to predict grain size and bed thickness distributions on the three cross-sections and to assess the two stratigraphic correlation hypotheses, the sets of parameters that maximising the likelihood function in each inversion have been extracted from their respective posterior probability distributions and used to condition three different BARSIM forward simulations, one per investigated cross-section. Figure 7.15 presents the three resulting forward simulations of the northern, central and southern cross-sections. Figure 7.16 shows a high resolution comparison of normalised grain-size signal between simulated and observed data at well locations. As suggested by the coherent results obtained from the three independent inferences of environmental parameters, the three forward simulations of the northern, central and southern cross-sections also present a single, coherent stratigraphic prediction. All of the simulated cross-sections show two progradationally stacked shoreface-shelf sandstone tongues separated by an interval of marine shale. Analysis combining parameter posterior probability distributions and forward simulations shows that the resulting stratigraphic architecture can be separated into five distinct parts.

The first part of the stratigraphic architecture represents the earliest stage of progradation occurring in the palaeo-landward areas of the cross-sections. It is characterised by the proximal segment of the shoreface-shelf sandstone tongue in the palaeo-landward part of the cross-sections. This segment of the tongue has a “normal regressive” shoreline trajectory (*sensu* Posamentier *et al.*, 1992; Helland-Hansen & Martinsen, 1996) (Fig. 7.15), and was deposited during the relative sea level rise occurring from 0 to 25,000-50,000 years of simulated time

(Fig. 7.11). The length of the tongue depends directly on the duration of the period of rising sea level and, therefore, on the palaeo-landward distribution of observed data. Lower initial values of relative sea level are a necessary condition to carry sediment (especially mud) palaeo-seaward and therefore to explain the large thickness of shelf mudstone relative to that of the overlying shoreface-shelf sandstone observed in the palaeo-seaward wells (i.e. the implicit data constraints imposed by these wells). The following period of rising sea level was necessary to explain the occurrence of the shoreface-shelf sandstone tongue in the upper part of the palaeo-landward wells.

The second part of the stratigraphic architecture is characterised by the medial segment of the same shoreface-shelf sandstone tongue, which exhibits a sub-horizontal shoreline trajectory (Fig. 7.15). This segment of the tongue was deposited during the relative sea level highstand occurring from 50,000 to 105,000 years of simulated time (Fig. 7.11), and its architecture is directly constrained by the wells A, B and D. The thickness of shoreface-shelf sandstone is greater in these wells than in the others, due to a combination of higher sediment supply and higher accommodation conditions, the latter characteristic of progradation into an under-filled basin.

The third part of the stratigraphic architecture is characterised by the distal segment of the lower shoreface-shelf sandstone tongue, which contains “down-stepping” facies belts that record a “forced regressive” shoreline trajectory (*sensu* Posamentier *et al.*, 1992; Helland-Hansen & Martinsen, 1996) (Fig. 7.15). This negatively inclined shoreline trajectory records the abrupt fall of relative sea level occurring from 105,000 to 135,000-140,000 years of simulated time (Fig. 7.11), and accounts for the coarsening-upward succession found in the lower part of wells C, E, F, H but not in wells G and I. Despite the high amplitude and rapidity of the sea level fall, this portion of the tongue remains attached to the older shoreface-shelf sandstone deposits. Although the sea level fall occurs during a shorter period (30,000-35,000 years) than the combined duration of the two previous periods of sea level history (105,000 years), the distance of associated progradation of is much greater. This greater progradation distance results from a decrease in accommodation due to the combined actions of lowered relative sea level and high mud fallout rate. In this case, higher sediment supply via longshore drift currents is not a necessary condition to create significant progradation. These lower

accommodation conditions produced a thinned succession in the distal part of the sandstone tongue.

The fourth part of the stratigraphic architecture is characterised by an overall flooding of the sedimentary system (Fig. 7.15) caused by the sea level rise occurring from 135,000-140,000 to 145,000 years of simulated time (Fig. 7.11). The rapidity of the relative sea level rise limited the degree of erosion of the previously deposited tongue, as well as deposition and preservation of transgressive sandstone deposits. The low gradient of the platform constructed by the preceding progradation caused a major landward displacement (15-30 km) of the shoreline and associated facies belts during flooding.

The fifth and final part of the stratigraphic architecture is characterised by a second shoreface-shelf sandstone tongue (Fig. 7.15), which was deposited from 145,000 to 300,000 years of simulated time (Fig. 7.11). The tongue exhibits a sub-horizontal to gently negatively inclined shoreline trajectory and was deposited during steady conditions of high sea level. This tongue corresponds to the upward-coarsening cycle found in the upper part of wells C, E, F, G, H and I (Fig. 7.5). In areas constrained by wells C, E, F and H, the second tongue is underlain by a shale interval and the distal, “forced regressive” segment of the first tongue; generation of this architecture requires a high mud fallout rate and limited accommodation during deposition of the second tongue. Therefore a “sharp-based” shoreface (as shown by the Gamma Ray signal in wells C, E, F and H; Figs. 7.5, 7.15) was generated without a significant fall in sea level. The second shoreface-shelf sandstone tongue does not exhibit this sharp-based character in wells G and I (Figs. 7.5, 7.15) because here the first tongue is absent, and as a result accommodation was higher during progradation of the second tongue.

The results of the stratigraphic inversion processes clearly show that the “bedset” correlation hypothesis (Fig. 7.7B) can be discarded, while the “two progradationally stacked parasequences” correlation hypothesis (Fig. 7.7A) is supported. This outcome is unambiguous, as suggested by the uniqueness of the three posterior probability distributions for sea level history obtained independently for each cross-section, and is successful in allowing the few available data available to constrain the stratigraphic correlation.

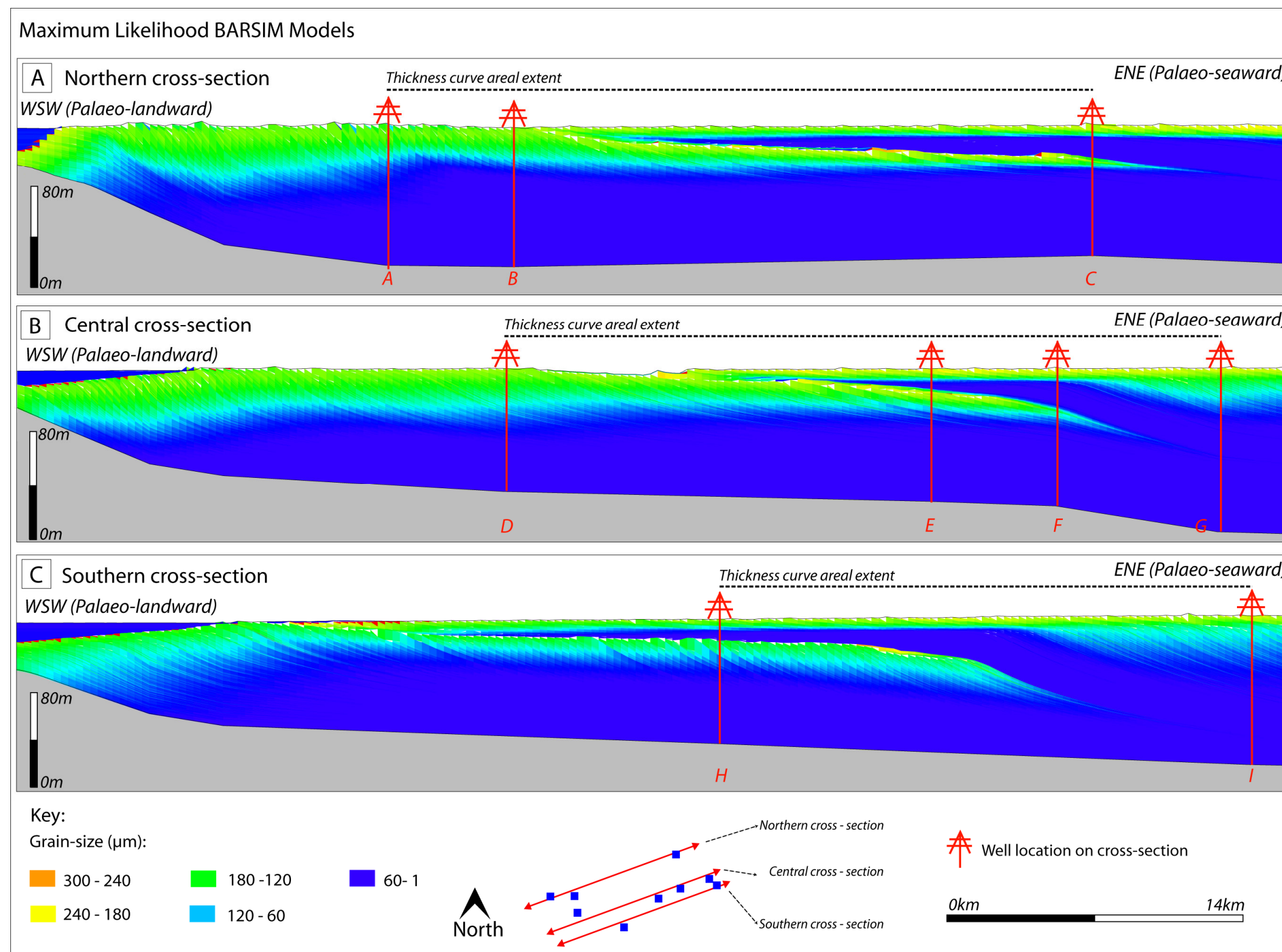


Figure 7.15: Predicted grain size and bed thickness distributions along the northern (A), central (B) and southern (C) cross-sections. These stratigraphic predictions were realised by running three independent BARSIM forward simulations conditions to the sets of parameters maximising the likelihood function of each inversion process. Results show coherency; all three forward simulations contain two progradationally stacked shoreface-shelf sandstone tongues separated by marine shales (i.e. the correlation hypothesis shown in Fig. 7.7A).

The results also show clearly that the overall observed stratigraphy is mainly driven by variations in relative sea level in a context where accommodation decreases progressively due to a high mud fallout rate. Sediment supply by longshore drift currents has a lower impact on the stratigraphic architecture, whereas low wave height regime limited seaward transport of sand and therefore prevented the creation of thicker sandstone tongues. This combination of sediment supply and wave regime conditions is consistent with the estuarine interpretation of the sedimentary environment, in which high concentrations of mud were transported in suspension from the river outlet to the sea. However, the formation of a longshore-oriented subaqueous delta may also explain the high mud fallout rate. Mud wedges, which may or may not be attached to river outlet, are a feature of many such deltas. These features are created by shore parallel hypopycnal plumes formed by the interactions between oceanic circulation and river discharge (Cattaneo *et al.*, 2003). However, these processes imply strongly three-dimensional dispersal of sediment, and they cannot be investigated by the stratigraphic inversion presented above because of two dimensional formulation of the BARSIM forward algorithm.

The results prove that stratigraphic inversion is powerful in predicting gross stratigraphic architecture over a wide area from few observed data. It is clear that conventional geostatistical tools, used to characterised facies architecture at reservoir scale, would fail to recreate realistic facies architectures in such a geographical context without high quality seismic-volume attribute data to condition them. However, high resolution comparison between observed and simulated grain-size successions at some specific well locations (Fig. 7.16) shows that even the maximum likelihood forward simulations do not completely match the observed data (cf Chapter 6). This discrepancy may limit the direct use of this numerical method (and similar three dimensional methodologies) as a tool for the static characterisation of petroleum reservoirs, although stratigraphic inversion can clearly be used as a “process-based” complement to constrain conventional geostatistical realisations.

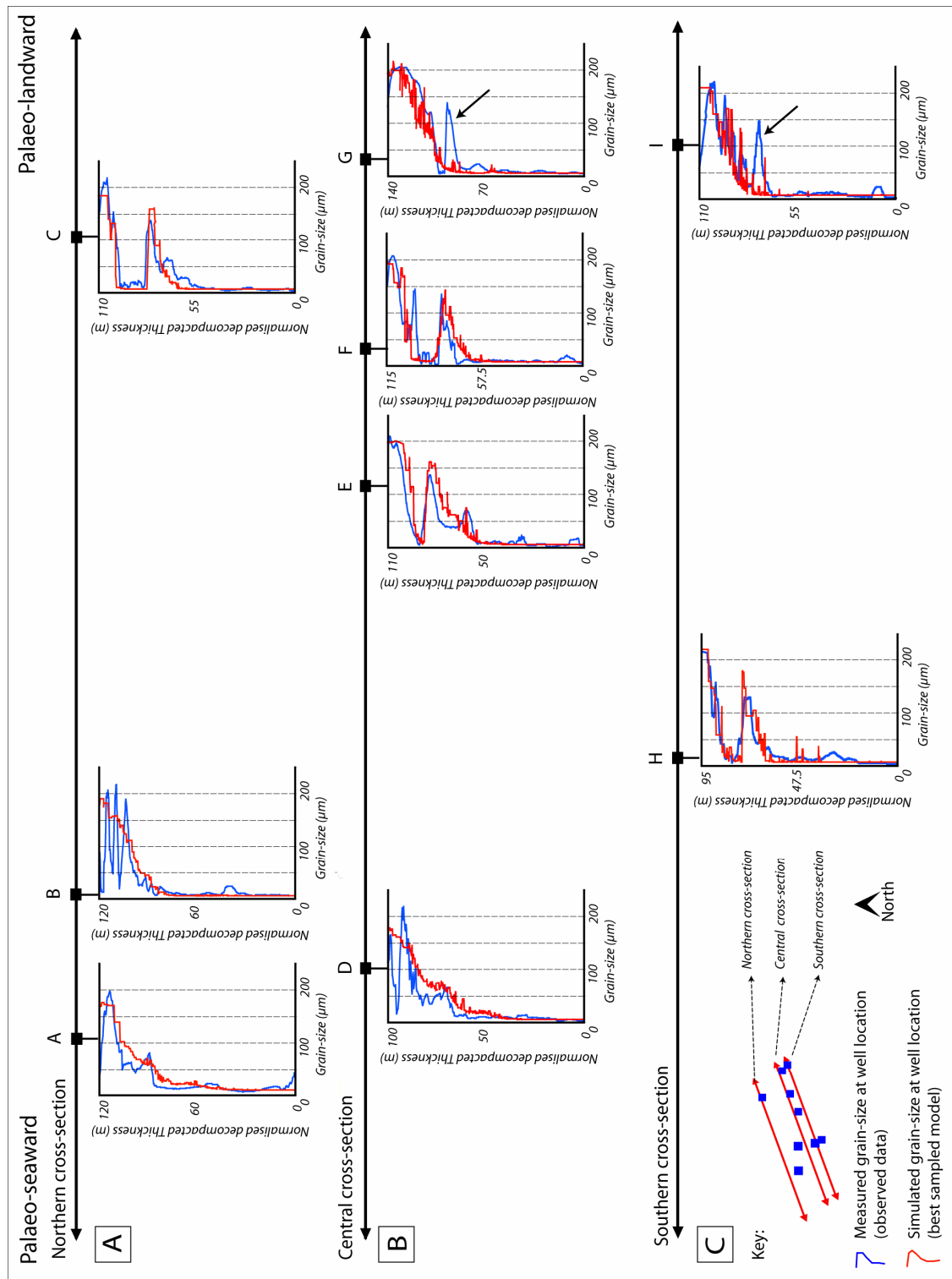


Figure 7.16: Comparison between simulated and observed grain-size signals at well locations for the northern (A), central (B) and southern (C) cross-sections. The simulated grain-size

signals have been extracted from the inverted BARSIM cross-sections conditioned to the parameters maximising the likelihood function of each inversion. The thicknesses of the simulated grain-size signals are normalised to the thicknesses of their observed counterparts. The overall trends of grain size in each well are recreated, but differences occur in the detailed distributions. Simulated data also failed to reproduce the upward coarsening succession in the lower part of well G and I (black arrows).

Detailed analysis of the observed and simulated grain-size successions at well locations (Fig. 7.16) combined with analysis of the length of the “forced-regressive” sandstone tongue and variations in its facies architecture (assessed by sampling different models from the posterior probability distributions) show that the stratigraphic inverse method failed in reproducing the upward-coarsening succession in the lower part of wells G and I (Fig. 7.16). This failure might be explained by three factors.

- (1) Firstly, the forward stratigraphic model failed in simulating the sedimentary processes involved in the formation of this architecture.
- (2) Secondly, the upward coarsening successions found in the lower part of these wells are not related to the upward-coarsening successions observed in the lower part of wells C, E, F and H (Fig. 7.5); they are not part of the same “forced regressive” sandstone tongue. The formation of these anomalous successions can be then explained in terms of clinoformal bedsets that the stratigraphic inverse method failed to reproduce in this case. However, the overall low wave regime and low accommodation context, due to the high mud fallout rate, do not favour this hypothesis of long sandstone-rich bedsets.
- (3) Thirdly, the upward-coarsening successions found in the lower part of these wells are related to the upward coarsening successions observed in the lower part of wells C, E, F and H (Fig. 7.5); they form part of the “forced regressive” sandstone tongue. The stratigraphic inverse methodology failed to reproduce these successions because of its two dimensional formulation. Given the reconstruction of regional palaeogeography (Figs. 7.6), it is plausible that changes in local shoreline orientation from NNW-SSE to WNW-ESE

occurred during progradation (Fig. 7.17). As a direct consequence, the palaeo-seaward position of wells G and I was overestimated in the modelling approach, and these wells may have lain in a more palaeo-landward position consistent with the upward-coarsening trends in their lower parts (Fig. 7.16). Such a change in shoreline orientation, and other effects of three-dimensional basin physiography, cannot be modelled by the two dimensional formulation of the BARSIM algorithm, even by inverting several two-dimensional cross-sections. This mechanism probably represents the most plausible explanation for the discrepancy between observed and simulated data. Future developments of this work should consider the use of a three-dimensional forward model, subject to the constraints of computational time.

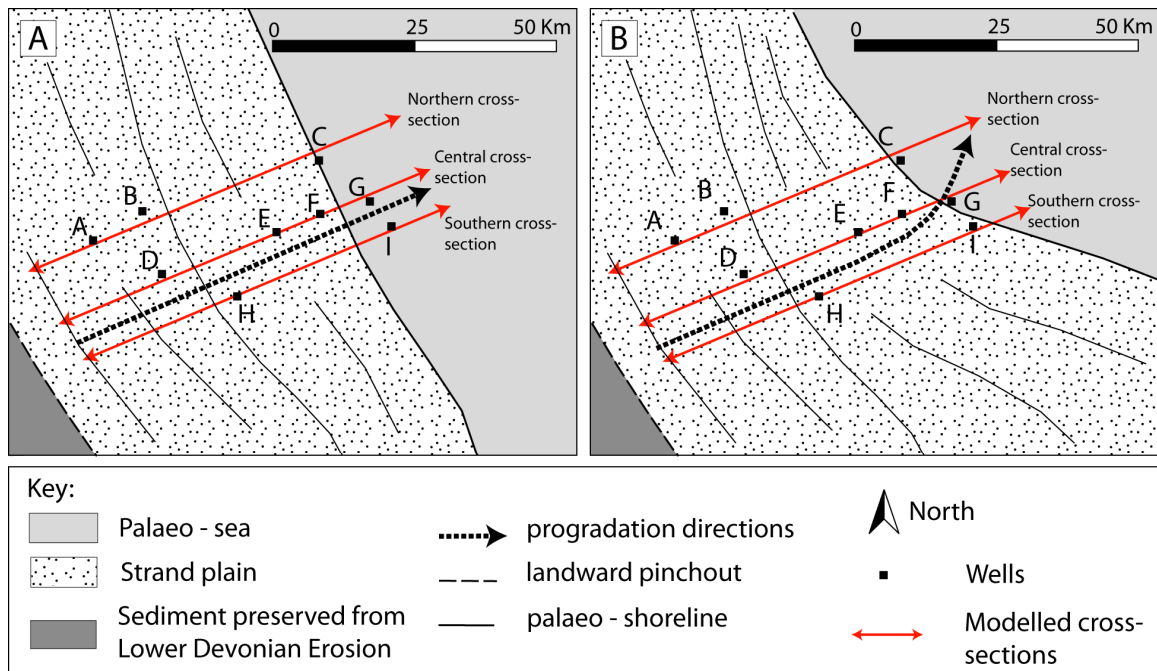


Figure 7.17: Simplified palaeogeographical cartoons illustrating possible changes in shoreline orientation during the progradation of the first “forced-regressive” sandstone tongue. In scenario A, shoreline orientation did not change during progradation. This scenario is implicitly modelled by the inverse stratigraphic numerical scheme because of the two dimensional formulation of BARSIM forward algorithm. In scenario B, shoreline orientation changed from NNW-SSE to WNW-ESE during the progradation because of the three dimensional physiography of the basin. As a consequence, wells G and I are situated in a more

palaeo-landward position than in scenario A. This possible change in shoreline orientation is invoked to explain the occurrence of upward-coarsening successions which have not been reproduced by the inversion scheme in the lower part of wells G and I.

7.8 Discussion and Concluding Statements

The application of the stratigraphic inverse and forward numerical scheme to the Emsian subsurface data was successful, given the observed data quantity, the size of the area investigated and the assumptions of the stratigraphic forward algorithm formulation. In fact, a relatively well resolved stratigraphic history has been inferred for the three cross-sections of 75 km length, which are constrained respectively by only 3, 4 and 2 wells that record 300,000 years of sedimentation. Although three independent stratigraphic inversions have been run for the three modelled cross-sections, the results show coherency in values and gross trends for both local and regional environmental parameters. This coherency shows the robustness of the developed methods. One major finding is that only one correlation strategy is viable across the studied area. This outcome greatly reduces uncertainty in reservoir presence, extent and quality over the investigated area. Such an approach may therefore constitute a robust basis for management decisions to proceed with, or abandon, the development of this prospect.

7.8.1 Inference of Environmental Parameters

The principal results for the inferred environmental parameters are listed below.

1. The relative sea level history is relatively well constrained and can be subdivided into five main periods. The first period was characterised by a relative sea level rise and the second period by steady conditions of high sea level. The third period contained an abrupt fall of sea level, followed by a rapid sea level rise in the fourth period. The last period was characterised by steady conditions of high sea level. Of equal importance,

the relative sea level posterior probability distribution shows a unique, well constrained and unambiguous sea level history for the third and fourth periods.

2. For all three cross-sections, the average sediment supply was approximately $20 \text{ m}^2/\text{y}$ and tended to decrease through simulated time.
3. The fair-weather wave regime was low (1-2m fair-weather wave height on average) and effectively constant.
4. The mud fallout rate is extremely well constrained and was high (around 0.05 cm/y).

The Bayesian formulation of the inverse problem has been shown to have clear advantages over other approaches, with the possibility of adding different levels of prior knowledge for different periods of the sea level history in this case study. In fact, this formulation facilitates the production of coherent independent inversions of the three cross-sections by adding geological prior information, derived from the study of the whole dataset, to parts of individual cross-sections not constrained by well data. Of equal importance, the sediment supply and fair-weather wave heights posterior probability distributions have a greater degree of uncertainty than the relative sea level and mud fallout posterior probability distributions. The inversion process is thus more sensitive to these two last parameters.

However, it is clear that the numerical methods have some weaknesses that might impact subsequently on the inference of the BARSIM environmental parameters.

1. Firstly, as already mentioned in Chapter 6, the BARSIM forward algorithm does not include dynamic compaction processes when simulating sedimentation. In this case study, this absence might be problematic as the studied interval contains a significant proportion of mud. As a result, the widths of the posterior probability distributions are clearly overestimated. This problem is not a fundamental limitation, because a compaction algorithm can be added to BARSIM for future work; however, this addition will diminish the computational efficiency of the BARSIM algorithm.

2. Secondly, as also already mentioned in Chapter 6, the two-dimensional formulation of BARSIM represents another source of errors by assuming that the modelled cross-sections were precisely down-dip oriented. Errors in their estimated orientation lead to an artificial increase of the total volume of sediment preserved in the cross-sections and therefore to an over-estimation in the inferred posterior probability distribution of the sediment supply and mud fallout rate. However, such issues can easily be solved in future works by using a three dimensional stratigraphic forward modelling algorithm
3. Thirdly, the initial manual design of the physiographic profile in areas unconstrained by well data might add some bias in the inference of environmental parameters by impacting directly on the volume and architecture of sediment in the inverted cross-sections.
4. Fourthly, the application to subsurface data required the transformation of the Gamma Ray signal of each wireline log into a grain-size signal. In this case study, this process has been realised in a very simple manner by using a linear correlation between the two signals in cored intervals, and then extrapolating the relationship into uncored intervals. It is clear that the relationship Gamma Ray character grain size relationship is more complex. In particular, shales are poorly sampled by core and their “grain-size” values might be misevaluated. However, this method yields overall grain-size trends in the non-reservoir intervals, and provide useful constraining data for the reservoir intervals of interest. Moreover, the Bayesian formulation allowed the inversion scheme to deal with measurements errors in the estimated grain-size values when inferring the environmental parameters.
5. Fifthly, the last weakness of this application does not come from the numerical model itself but from uncertainty related to the lack of age data. This lack of data requires depositional ages to be estimated, which in turn requires a number of assumptions that also impact the design of the prior probability distribution for relative sea level. In this case, estimated ages were assigned to observed well data assuming a constant overall progradation rate and shoreline orientation over the whole period of simulated time. This simple approach was chosen because of the large uncertainty inherent to

stratigraphic correlation within the study area. With the benefit of hindsight about the most likely stratigraphic correlation hypothesis, it is clear that the duration of periods of falling and rising of relative sea level has been *a priori* underestimated as a direct result of the assumption of a constant progradation rate. In fact, deposition of the lower tongue and subsequent flooding was forced to take place in a short time interval occurring after the estimated age at well B and before the estimated age at well H, which enhanced the rapid character of the relative sea level fall and rise. The posterior probability distributions of time-dependent environmental parameters should have been “stretched” to encompass longer periods of relative sea-level fall and rise. Although this assumption may have introduced an unconscious bias against the “two parasequences” correlation hypothesis, because it forced rapid relative sea level changes, the stratigraphic inversion shows that this correlation hypothesis remains the only viable solution.

The issue of estimated age constraints needs to be addressed carefully in future developments of this modelling approach, although it is strongly related to the specific case being investigated by stratigraphic inversion. Potential solutions should firstly consider the use of fewer age estimates, and/or of artificially increased errors on these age estimations, in areas where stratigraphy is ambiguous (e.g. in this case, in wells B, I and H). These solutions should secondly compensate the lack of timing constraint via the use of information-rich prior probability distributions for the time-dependent parameters, in this case especially for relative sea level. However, contrary to the prior distribution of relative sea level used in this study, such prior information should also be more flexible in the sense that time boundaries and probability values between periods with different levels of prior knowledge should be progressive and should include *a priori* all the potential solutions of the inverse problem without introducing bias. It is clear that such prior information is difficult to set up, as it requires a clear and objective view of the range of potential stratigraphic correlations and of the parameters having the strongest impact on the observed stratigraphy. Such an approach might also be more CPU intensive.

7.8.2 Grain Size and Bed Thickness Prediction

The stratigraphic inverse and forward modelling approach successfully predicts grain size and bed thickness distributions for the three cross-sections from a very little amount of observed data. The method also shows the possibility of assessing uncertainty in these distributions. More importantly, the results clearly demonstrate that the “bedset” correlation hypothesis is not supported by the data, while the “two progradationally stacked parasequences” correlation hypothesis is favoured. In fact, forward realisations conditioned by parameters extracted from the posterior probability distributions show two progradationally stacked shoreface-sandstone tongues separated by a flooding surface, with alternating periods of “normal regressive” and “forced regressive” shoreline trajectories occurring in each tongue. The uniqueness of the three posterior probability distributions obtained for the relative sea level parameters suggests that this result is unambiguous.

These results also demonstrate that the overall stratigraphic architecture was mainly driven by sea level variations in a context where accommodation decreased because of a high mud fallout rate. Sediment added by longshore drift currents has a lower impact on stratigraphy, whereas the low-energy wave regime limited the seaward transport of sand. These sediment supply characteristics support an estuarine interpretation of the sedimentary environment, in which high concentrations of mud were transported in suspension from the river outlet to the sea.

Although these results are promising, the inversion results again show some weaknesses in their prediction of stratigraphy. Firstly, the inverse approach fails to reproduce and explain - upward-coarsening trends in the lower part of the two most palaeo-seaward wells (G and I). These trends are probably related to a forced regressive sandstone tongue with a similar expression in wells C, E, F and H. However, the method falls down because of its two dimensional formulation, which limits its capacity to deal either with a complex three-dimensional basin physiography or with changes in shoreline orientation. Future developments of this work should consider the use of a three dimensional forward model of stratigraphy, although this will considerably reduce the computational speed of the stratigraphic inverse and forward modelling scheme. Detailed analysis also showed that the numerical method did not

match the observed data at bed- and bedset-scales, which may limit its use as a direct tool for the numerical static characterisation of petroleum reservoirs at these scales.

Overall, however, this approach has many positive features and therefore should be considered as complement to traditional geostatistical tools. First, it can simulate facies distributions using few and widely distributed conditioning data, in contrast to conventional geostatistical tools which would require a larger amount of “soft” conditioning data. Secondly, the approach is based on real-world sedimentary processes and recreates realistic sediment-body geometries and stratigraphic architectures as a result. Thirdly, the method can be used to evaluate stratigraphic correlation hypotheses and to propose different realistic scenarios for uncertainty assessment. Fourthly, vertical grid cell resolution is self-generated according to the thickness of sediment deposited. Consequently, the characterisation of small-scale sedimentary heterogeneities is not limited by a pre-defined grid cell resolution. Thus, the inverse numerical method developed in this work can be used to complement existing reservoir modelling techniques in two ways: (1) to construct facies probability cubes that provide “soft” conditioning constraints on geostatistical tools, and (2) to design complex numerical grids representative of stratigraphic architecture (e.g. clinoforms within shoreface-shelf sandstone tongues).

Chapter 8

8 Conclusions and Future Work

In this thesis, I have discussed the topic of the development and linkage of stochastic numerical inverse methods to a two dimensional “process-based” forward modelling algorithm for shallow-marine wave-dominated sedimentary environments (BARSIM; Storms, 2003), and have presented two applications to real dataset. Throughout this work, I have emphasised the importance of assessing uncertainties in inferred model parameters and in model predictions, as well as addressing the issues of possible non-uniqueness of the stratigraphic inverse problem. For that purpose, I have developed an appropriate inverse method, formulated in a Bayesian framework, using both conventional MCMC, trans-dimensional MCMC and Simulated Tempering algorithms. The link between the inverse strategy and the numerical forward model of stratigraphy required the construction of a likelihood function to quantify the agreement between simulated and observed data of different types (e.g. sediment age and thickness, grain-size distributions). The major contributions and conclusions for this work are outlined below.

On the Efficiency of the Inverse and Forward Modelling Scheme

The detailed implementation of the newly developed stratigraphic inverse and forward modelling scheme was presented in Chapters 3 and 4. The inversion algorithm was tested and validated on synthetic data in which all the parameters were specified to produce a ‘perfectly known’ BARSIM forward simulation. Different scenarios have been investigated in these tests and the main conclusions are:

1. The Bayesian formulation of the inverse problem not only allows the inference of the BARSIM environmental input parameters and their respective uncertainty but also allows assessment of their sensitivity to the model formulation. The numerical experiments demonstrate that uncertainties in the inferred results are directly dependent on the magnitude of measurement errors for the observed data, although there is no linear relationship linking measurement-error magnitude and resulting uncertainty.
2. Because the Bayesian approach is naturally parsimonious, the use of RJMCMC samples is preferred in that it enables a trans-dimensional approach that preferentially avoids over-complicated models for the available data, thereby reducing computational costs. This point also represents a limitation of the inversion method as it requires that input observed data should be selected carefully in order to incorporate the level of complexity (or heterogeneity) required in the inverse solution.
3. The use of the Simulated Tempering sampler considerably improves the robustness of the inverse scheme, because it allows a complete exploration of the parameter space in order to fully sample (1) the posterior probability distribution of the input parameters and (2) all possible solutions of the inverse problem. Tests also demonstrate that the solution of the inverse problem may not be unique, even though the stratigraphy may be the result of a unique combination of processes and parameters. The non-uniqueness of the inverse problem can result from differences in data quality, quantity and, most importantly, distribution.
4. Geological prior information about the environmental input parameters can provide important additional constraints on the inversion process, by reducing the number of possible local solutions and the burn-in period. However prior knowledge should not be chosen to be too restrictive (unless justified from geological data, for example) as it can lead the inversion process to sample only partially or not at all in the preferred solution area of the model space.
5. The multi-iterative inversion process was relatively CPU intensive as it required the obtaining of a large number of samples (i.e. stratigraphic forward realisations), extracted from the “cold ” post-burn-in Markov chain, to construct a reliable representation of the target posterior probability distribution. This intense CPU usage required the use of a quick stratigraphic forward modelling algorithm and of an inversion scheme able to explore efficiently the parameter space.

Following these successful tests, in which BARSIM was assumed to be a perfect simulator, the inverse and forward modelling scheme was applied to two real case studies: the Upper Cretaceous “Standardville” parasequence of the Aberdeen Member of the Blackhawk Formation (Book Cliffs, Utah, USA) in Chapter 6 and the sub-surface Emsian deposits of South Algeria in Chapter 7. In terms of algorithm efficiency, the main conclusions of these applications are:

6. The inverse and forward modelling scheme showed clearly its ability to be successfully applied to real datasets, provided that the latter are selected with due care for the assumptions and simplifications of the underlying forward stratigraphic model. Interactions between environmental parameters were successfully inferred with different levels of resolution according to the density of available observed data. Grain-size and bed-thickness distributions were predicted on cross-sections at high-resolution spatial and temporal scales while honouring (1) understanding of the sedimentological processes active within wave-dominated systems and (2) the input observed data.
7. Inversion results were clearly conditioned on, and only as good as the ability of, the forward modelling algorithm (i.e. model formulation, simplifications and assumptions) to simulate the sedimentological processes responsible for deposition of the studied strata and to consider the full range of environmental parameters that drove the past stratigraphy.
8. Because of the two dimensional formulation and inherent simplifications of BARSIM, predicted stratigraphy will never perfectly match the observed data. As a consequence, the inversion process never reached the highest probability solution areas and sampled more local modes presenting lower probabilities, as shown in Chapter 6: in this case, a multi-modal posterior probability distribution did not mean non-uniqueness of the stratigraphic inverse problem.
9. Robust data preparation, prior to the inversion process, was fundamental to reduce bias in the inferred parameters, as shown in Chapter 7. Moreover, the gathering of age data to constrain the inversion of time-dependent parameters remained an issue (Chapter 6 and Chapter 7) that has been overcome by determining artificial ages based on an assumed constant progradation rate. However such an assumption biases the inferred solutions towards those having smoother transitions between different regimes, which is not entirely

supported by observation of modern systems that exhibit periods of rapid shoreline progradation followed by long periods of non-activity).

10. Again, use of prior knowledge has proved to be a real advantage in constraining the inversion process. For example, the case study described in Chapter 7 shows the value of incorporating contextual geological information provided by analysis of a larger dataset than that used directly to constrain stratigraphic inversion.

On the understanding of Shallow-Marine Wave-Dominated Environments

Applications to real datasets greatly contributed to the understanding and quantification of the past sedimentary processes driving both intra-parasequence and basin-scale stratigraphy within the limits of observed data quality, quantity and distribution. The major contributions of the case study of intra-parasequence facies architecture described in Chapter 6 are:

1. The numerical application (Chapter 6) not only confirmed, but also quantified, all of my stratigraphic interpretations previously proposed on the basis of field observations (Chapter 5). The internal stratigraphic architecture of the studied parasequence is characterised by alternating periods of normal regressive and forced regressive shoreline trajectory, which reflect complex, high-frequency interplays between relative sea level, wave height regime and sediment supply parameters.
2. The inferred presence of high-frequency variations of the relative sea level at such temporal and spatial scales suggests that conventional sequence stratigraphy methodology lacks robustness when intra-parasequence variability is not considered.
3. Large fair-weather and storm wave heights were necessary to transport sand seaward in order to create bedsets, provided that sediment supply was sufficient.
4. Strongly progradational stacking of bedsets reflects periods of falling relative sea level, increased wave-height regime and increased sediment supply.
5. Aggradationnal stacking of upward-coarsening bedsets reflects periods of rising relative sea level and increased wave height, provided that sediment supply was sufficiently large.
6. Mud fallout rate in the shoreface area was low and accommodation was high, thus allowing the development and preservation of the complex internal stratigraphy observed in the studied parasequence.

7. BARSIM simulated cross-sections have the advantages of allowing us to visualise directly shoreline trajectories. Alternating periods of normal regressive and forced regressive shoreline trajectories created conditions under which lenses of back-barrier deposits were developed and potentially preserved. Predicting the location of these lenses might have direct implications for exploration and production of coal seams.

The major contributions of Chapter 7 for sub-surface prediction of stratigraphy are:

8. The numerical application successfully and unambiguously identified a favoured stratigraphic correlation, even with very limited data spread over a large area: the Emsian stratigraphy is characterised over the investigated area by two progradationally stacked shoreface-shelf sandstone tongues (parasequences) that internally contain alternating intervals of normal regressive and forced regressive shoreline trajectory.
9. Stratigraphic architecture in this case study was mostly driven by relative sea level variations associated with a high mud fallout rate which contributed significantly to the progressive infilling of accommodation. A combination of low wave-height regime and low sediment supply via longshore drift currents limited seaward transport of sand and enhanced the development of thin shoreface-shelf sandstone tongues. These results are consistent with the interpreted palaeogeographic setting of a large estuary.
10. The numerical approach can be used as a “process-based” complement to conventional geostatistical tools in the static characterisations of hydrocarbon reservoirs. It can either be used for the building of facies probability cubes for “soft” conditioning of geostatistical realisations or for the construction of complex numerical grids that represent stratigraphic architecture and sedimentary heterogeneities in a geologically realistic and computationally efficient manner. Also, the fact that we characterise uncertainty as probability distributions means we can readily run multiple realisations, drawn from the posterior.

Future Work

Throughout this thesis, the possibilities for future work and improvements of the stratigraphic inverse and forward modelling algorithm have been highlighted. Three main areas of fruitful future work can be identified.

The first main area for future development would be the use of a three-dimensional forward model of stratigraphy. This would allow fuller consideration of stratigraphic architecture associated with (1) the high-resolution stratigraphic and palaeogeographic evolution of the asymmetrical wave-dominated delta described in Chapter 5, (2) the impact of wave approach relative to the shoreline orientation and of fluvial avulsions in the formation of beach ridge sets and associated bedsets (e.g. those described in Chapter 5), (3) the effects of complex basin physiography and changes in shoreline orientation, as hypothesised in Chapter 7. It would also (1) solve the issue in the inferred posterior probability distributions of sediment supply and mud fallout rate introduced by misevaluated down-dip orientation of the cross-sections and (2) allow the comparison between environmental posterior distributions resulting from the application of different stratigraphic forward algorithms (linked to the same inverse strategy) to the same observed data.. A numerical three dimensional forward model of stratigraphy should also be sufficiently fast to enable the inversion process in a reasonable timeframe, and should also include dynamic compaction processes. Computational efficiency can be improved by developing a parallel version of the numerical scheme (Geyer, 1991).

The second area of future work is the development of a more efficient and “intelligent” likelihood function to decrease the burn-in period and improve the sampling strategy of the inverse process. For this work, a very simple likelihood function was designed that evaluated the match between observed and simulated data in term of sediment age, thickness and grain-size distributions. More efficient likelihood functions should be able to assess sediment body geometries and volumes, as well as to determine the “geological plausibility” of a solution by adding prior information directly in the likelihood function (Wijns *et al.*, 2003), although it is difficult to distil this latter assessment into a single value. More “intelligent” likelihood functions should be able to perturb the model parameters during the inversion process, according to evaluations of the degree of fit for its different components.

In the third main area, future work should apply the stratigraphic inverse and forward modelling scheme to numerous case studies of more poorly understood depositional systems and stratigraphic architectures, in order to increase our general understanding of the effects of interactions between environmental parameters. An obvious example would be applications to

transgressive wave-dominated sedimentary systems, which are more poorly preserved than their regressive counterparts, and thus more poorly understood.

Bibliography

- AL-AWADHI, F., HUM., JENNISON, C., (2002) Improving the acceptance rate of reversible jump MCMC proposals, *Stat. Probab. Lett.* 69(2),p 189-198.
- ATHY, L.F. (1930). Density, porosity and compaction of sedimentary rocks. *Bulletin of American Association of Petroleum Geologists*, 14, p.1-24.
- BALLESTER, P.J. & CARTER, J.N. (2004). An Algorithm to identify clusters of solutions in multi-modal optimisation. *Lectures Notes in Computer Science* 3059, Springer, pp. 42 – 56.
- BALSLEY, M.D., (1980). Cretaceous Wave-dominated delta systems, Book Cliffs, East-Central Utah. *American Association of Petroleum Geologists, Continuing Education Course Field Guide*. AAPG, Tulsa, OK.
- BHATTACHARYA, J.P., (2006), Deltas, *in* H.W., Posamentier and R., Walker, ed., *Facies Models Revisited*, SEPM Special Publication No. 84, p. 237 – 292.
- BHATTACHARYA, J.P., & GIOSAN, L., (2003), Wave - influenced deltas: geomorphological implications for facies reconstruction: *Sedimentology*, v. 50, p. 187 - 210.
- BEARD, D.C., & WEYL, P.K., (1973) Influence of texture on porosity and permeability of unconsolidated sand: *AAPG Bulletin*, v. 57, no. 2, p. 349-369.
- BERNARDO, J. & SMITH, A.F.M., (1994), *Bayesian Theory*, John Wiley and Sons, Ltd., Chichester.

- BORNHOLDT, S., NORDLUND, U. & WESTPHAL, H. (1999) Inverse stratigraphic modelling using genetic algorithms. In: Harbaugh, J., Watney, W., Rankey, E., Slingerland, R., Goldstein, R., Franseen, E. (Eds.) 1999. SEPM Special Publication 62: *Numerical Experiments in Stratigraphy: Recent Advances in Stratigraphic and Sedimentologic Computer Simulations*. SEPM (Society for Sedimentary Geology): 85-90.
- BOWMAN, S.A., & VAIL, P.R., (1999) Interpreting the stratigraphy of the Baltimore canyon Section, offshore New Jersey with PHIL, a stratigraphic simulator. In: Harbaugh, J., Watney, W., Rankey, E., Slingerland, R., Goldstein, R., Franseen, E. (Eds.) 1999. SEPM Special Publication 62: *Numerical Experiments in Stratigraphy: Recent Advances in Stratigraphic and Sedimentologic Computer Simulations*. SEPM (Society for Sedimentary Geology): 117-138.
- BOYD, R., DALRYMPLE, R. & ZAITLIN, B.A. (1992) Classification of clastic coastal depositional environments. *Sedimentary Geology*, 80. p 139-150.
- BROOKS, S.P., GIUDICI, P. & ROBERTS, G.O. (2003) Efficient construction of reversible jump Markov chain Monte Carlo proposal distribution, *J. R. Stat. Soc., Ser. B90 J.Stat. Methodol.* 65, p3-39.
- BROOKS, S.P., FAN, Y., & ROSENTHAL, J.S. (2006) Perfect Forward Simulation via Simulated Tempering. *Communications in Statistics-Simulation and Computation*, 35: 683–713.
- BURCHFIELD, B.C, COWMAN D.C., & DAVIS, G.A., (1992), Tectonic over-view of the Cordilleran orogen in the western United States. *In*: Burchfield, B.C. et al. (ed), *The Cordilleran Orogen: Con-terminous, U.S. Geological Survey of America, The Geology of North America*, vol. G-3, pp. 407-480.
- BURGESS, P.M. (2001) Modelling carbonate sequence development without relative sea-level oscillations. *Geology*, v. 29, p. 1127-1130.

- BURGESS, P.M., LAMMERS, H., VAN OOSTERHOUT, C. & GRANJEON, D. (2006) Multivariate sequence stratigraphy: Tackling complexity and uncertainty with stratigraphic forward modelling, multiple scenarios, and conditional frequency maps. AAPG Bulletin, v.90, n.12, pp 1883-1901.
- CAMPBELL, C.V., (1979) Model for beach shoreline in Gallup sandstone (Upper Cretaceous) of northwestern New Mexico: New Mexico Bureau of Mines and Mineral Resources, Circular 164, 32p.
- CANT, D.J. (1991) Geometric modelling of facies migration: theoretical development of facies successions and local unconformities. Basin Research, v. 3, p51-62.
- CAREY, J.S., SWIFT, D.P., STECKLER, M., REED, C.W., & NIEDORODA, A., (1999) High-resolution sequence stratigraphic modeling 2: effects of sedimentation processes. In: Harbaugh, J., Watney, W., Rankey, E., Slingerland, R., Goldstein, R., Franseen, E. (Eds.) 1999. SEPM Special Publication 62: *Numerical Experiments in Stratigraphy: Recent Advances in Stratigraphic and Sedimentologic Computer Simulations*. SEPM (Society for Sedimentary Geology): 151-164.
- CAUMON, G., & JOURNEL, A.G. (2005). Early uncertainty assessment: Application to a hydrocarbon reservoir appraisal. In: Leuangthong, O. & Deutsch, C., (eds): Geostatistics Banff 2004, Quantitative Geology and Geostatistics, vol. 14., p. 551-557.
- CHARVIN, K., GALLAGHER, K.L., HAMPSON, G.L., & LABOURDETTE, R., (2008a). A Bayesian approach to inverse modelling of stratigraphy, part 1: Method. Basin Research, in press.
- CHARVIN, K., GALLAGHER, K.L., HAMPSON, G.L., & LABOURDETTE, R., (2008b). A Bayesian approach to inverse modelling of stratigraphy, part 2: Validation and sensitivity tests. Basin Research, in press.

- CHARVIN, K., HAMPSON, G.J., GALLAGHER, K.L. & LABOURDETTE, R., (2008c), High resolution stratigraphic architecture within an asymmetrical wave-dominated deltaic parasequence. Submitted for *Sedimentology*.
- CHARVIN, K., HAMPSON, G.J., GALLAGHER, K.L. & LABOURDETTE, R., (2008d), Mechanism driving intra-parasequence shallow-marine wave-dominated stratigraphy: Insights from inverse and forward modelling of stratigraphy, in prep.
- CLEVIS, Q.J.W.A., BOER, P.L. de & NIJMAN, W. (2004) Differentiating the effect of episodic tectonism and eustatic sea-level fluctuations in foreland basins filled by alluvial fans and axial deltaic systems: insights from a three-dimensional stratigraphic forward model. *Sedimentology*, 51(4), 809-835.
- CLIFTON, E.H., (2006), A re-examination of facies models for clastic shorelines, in H.W., Posamentier and R., Walker, ed., *Facies Models Revisited*, SEPM Special Publication No. 84, p. 293 – 337.
- CORCORAN, D.V. & CLAYTON, G. (2001) Interpretation of vitrinite reflectance profiles in sedimentary basins, onshore and offshore Ireland. In: Shannon, P.M., Haughton, P.D.W., & Corcoran, D.V. (Eds) 2001. Special Publication of the Geological Society of London, 188: *The Petroleum Exploration of Ireland's Offshore Basins*. p. 61-90.
- CROSS, T. & LESSENGER, M., (1999) Construction and application of stratigraphic inverse model. In: Harbaugh, J., Watney, W., Rankey, E., Slingerland, R., Goldstein, R., Franseen, E. (Eds.) 1999. SEPM Special Publication 62: *Numerical Experiments in Stratigraphy: Recent Advances in Stratigraphic and Sedimentologic Computer Simulations*. SEPM (Society for Sedimentary Geology): 69-83.
- COLEMAN, J.M. & WRIGHT, L.D., (1975), Major river deltas: variability of processes and sandbodies, in Broussard, M.L., ed., *Deltas, Models for Exploration*: Houston Geological Society, p. 99 – 149.

- DALRYMPLE, R.W., ZAITLIN, B.A., & BOYD, R., (1992), Estuarine facies models: conceptual basin and stratigraphic implication: *Journal of Sedimentary Petrology*, v. 62, p. 1130 – 1146.
- DAVIS, Jr.R.A & HAYES, M.O. (1984) What is a wave-dominated coast? In: B. Greenwood and R.A. Davis, Jr (Eds), *Hydrodynamics and Sedimentation in Wave-Dominated Coastal Environments*. Marine Geology, 60: p313-329.
- DE BENEDICTIS, D., BOSENCE, D.W.J, & WALTHAM, D.A., (2007), Tectonic control of peritidal carbonate parasequence formation: an investigation using forward tectono-stratigraphic modelling. *Sedimentology*, 54, 587-605.
- DEN BEZEMER, T., KOOL, H. & CLOETINGH, S., (1999) Numerical modeling of fault-related sedimentation. In: Harbaugh, J., Watney, W., Rankey, E., Slingerland, R., Goldstein, R., Franseen, E. (Eds.) 1999. SEPM Special Publication 62: *Numerical Experiments in Stratigraphy: Recent Advances in Stratigraphic and Sedimentologic Computer Simulations*. SEPM (Society for Sedimentary Geology): 177-196.
- DENISON, D.G.T., HOLMES, C.C., MALLICK, B.K & SMITH, A.F.M., (2002). *Bayesian Methods for Nonlinear Classification and Regression*, Wiley, Chichester, 2002.
- DEUTSCH, C.V., 2002, *Geostatistical reservoir modeling (applied geostatistics)*. Oxford University Press, 384p.
- DOMINGUEZ, J.M.L., (1996). The São Francisco strandplain: a paradigm for wave-dominated deltas? in De Batist, M., and Jacobs, P., eds., *Geology of siliciclastic shelf seas*: Geological Society of London, Special Publication 117, p. 217-231.
- DOMINGUEZ, J.M.L., MARTIN, L. & BITTENCOURT, A.C.S.P. (1987). Sea-level history and Quaternary evolution of river mouth-associated beach-ridge plains along the east-southeast Brazilian Coast: a summary, in Nummedal, D., Pilkey, O.H., Howard, J. D., and Price, W. A., eds., *Sea-Level Fluctuation and Coastal Evolution*: SEPM, Special Publication 41, p. 115-127.

- DOTT, R.H., & BOURGEOIS, J., (1982) Hummocky stratification: significance of its variable bedding sequences: *Geological Society of America Bulletin*, v. 93, p.663-680.
- DUKE, W.L., (1985), Hummocky cross stratification, tropical hurricanes, and intense winter storms: *Sedimentology*, v. 32, p. 167-194.
- EBERLI, G.P., KENDALL, C.G.St.C, MOORE, P., Whittle, G.L., & CANNON, R., (1994) Testing a seismic interpretation of great Bahama Bank with a computer simulation in: *AAPG Bulletin*, v. 98, n.6., p. 981-1004.
- ELLIOTT, T., (1986), Clastic shorelines, in Reading, H.G., (Eds), *Sedimentary Environments and Facies*: Oxford, Blackwell Scientific Publications, p. 143–177.
- FLEMINGS, P.B. & GROTZINGER, J.P. (1996) STRATA: Freeware for analyzing classic stratigraphy problems IN: *GSA Today*, v. 6, n.12, p. 1-7.
- FOUCH, T.D, LAWTON, T.F., NICHOLS, D.J., CASHION, W.B, & COBBAN, W.A., (1983). Patterns and timing of synorogenic sedimentation in Upper Cretaceous rocks of Central and Northeast Utah. In: Reynolds, M.W., Dolly, E.D., (ed), *Mesozoic Paleogeography of the West Central United States*. SEPM Rocky Mountains Section, Rocky Mountain Paleogeography Symposium, vol. 2, pp. 305-336.
- GALLAGHER, K.L., CHARVIN, K., NIELSEN, S., SAMBRIDGE, M., & STEPHENSON, J., (2008), Markov chain Monte Carlo (MCMC) sampling methods to determine optimal models, model resolution and model choice for Earth Science problems. Submitted for *Marine and Petroleum Geology*.
- GALLOWAY, W.E., (1975), Process framework for describing the morphologic and stratigraphic evolution of deltaic depositional systems, in Broussard, M.L., ed., *Deltas; Models for Exploration*: Houston Geological Society: Houston, p. 87-98.

- GALLOWAY, W.E., (1989) Genetic stratigraphy sequences in basin analysis 1: architecture and genesis of flooding-surface bounded depositional units. AAPG Bulletin, v.73: 125–142.
- GANI, M.R. & BHATTACHARYA, J.P., (2007), Basic building blocks and process variability of a Cretaceous delta: Internal facies architecture reveals a more dynamic interaction of river, wave and tidal processes than is indicated by external shape. Journal of Sedimentary Research, v. 77, pp. 284-302.
- GAWTHORPE, R.L., FRASER, A.J., & COLLIER, R.E.L (1994) Sequence stratigraphy in active extensional basins: Implication for the interpretation of ancient basin fills. Marine and Petroleum Geology, v.11, p. 642-658.
- GEYER, C., (1991) Markov Chain Monte Carlo Maximum Likelihood. Computing Science and Statistics, *Proceedings of the 23rd Symposium on the Interface*, 156–163.
- GEYER, C.J. & THOMPSON, E.A. (1995) Annealing Markov chain Monte Carlo with applications to ancestral inference. J. Amer. Statist. Assoc., 90, 909-920.
- GILKS, W.R., RICHARDSON, S. & SPIEGELHALTER, D.J., (1996) Markov Chain Monte Carlo in Practice. London: Chapman & Hall.
- GRANJEON, D. & JOSEPH, P., (1999) Concepts and applications of a 3-D multiple lithology, diffusive model in stratigraphic modeling. In: Harbaugh, J., Watney, W., Rankey, E., Slingerland, R., Goldstein, R., Franseen, E. (Eds.) 1999. SEPM Special Publication 62: *Numerical Experiments in Stratigraphy: Recent Advances in Stratigraphic and Sedimentologic Computer Simulations*. SEPM (Society for Sedimentary Geology):197-210.
- GREEN, P.J. (1995) Reversible jump Markov chain Monte Carlo computation and Bayesian model determination. Biometrika 82 (4): 711-732.
- GREEN, P.J. (2001) A Primer on Markov Chain Monte Carlo. In: Barndorff-Nielsen, O.L., Cox, D.R., Kluppelberg, C. (Eds.): *Complex Stochastic Systems*. Chapman and Hall.

- GREEN, P.J., (2003) Trans-dimensional MCMC, in: P.J. Green, N. Hjort and S. Richardson (Eds.), *Highly Structured Stochastic Systems*, Oxford Statistical Sciences Series, Ch.6, pp. 179-196.
- GRIFFITHS C.M., DYT, C., PARASCHIVOIU, E. & LIU, K., (2001) Sedsim in hydrocarbon exploration, in Merriam, D., Davis, J. C. (Eds.) *Geologic Modeling and Simulation*, Kluwer Academic, New York, p. 71- 97.
- GUILLEN, J., & HOEKSTRA, P. (1996) The "equilibrium" distribution of grain size fractions and its implications for cross-shore sediment transport: a conceptual model. *Marine Geology* 135, pp. 15-33.
- HAMSPON, G.J. (2000). Discontinuity surfaces, clinoforms, and facies architecture in wave dominated, shoreface-shelf parasequence. *Journal of Sedimentary Research* Vol. 70, No. 2. p325-340.
- HAMSPON, G.J., HOWELL, J.A., & FLINT, S.S., (1999). A sedimentological and sequence stratigraphic re-interpretation of the Upper Cretaceous Prairie Canyon Member ("Mancos B") and associated strata, Book Cliffs Area, Utah, U.S.A. *Journal of Sedimentary Research*, Vol. 69., No. 2, March, 1999, p. 414-433.
- HAMPSON, G.J., BURGESS, P.M., & HOWELL, J.A., (2001). Shoreface tongue geometry constrains history of relative sea-level fall: examples from Late Cretaceous strata in the Book Cliffs area, Utah. *Terra Nova*, vol. 13, No. 3, pp. 188-196.
- HAMPSON, G.J. & STORMS, J.E.A., (2003) Geometric and sequence stratigraphic variability in wave-dominated, shoreface-shelf parasequences. *Sedimentology* 50, 667-701.
- HAMPSON, G.J., & HOWELL, J.A., (2005), Sedimentologic and geomorphic characterization of ancient wave-dominated shorelines: examples from the Late Cretaceous Blackhawk

- Formation, Book Cliffs, Utah, *in* Bhattacharya, J.P., and Giosan, L., eds., *Deltas Old and New*: SEPM, Special Publication 83, p. 133-154.
- HAMPSON, G.J., RODRIGUEZ, A.B., STORMS, J.E.A., JOHNSON, H.D., & MEYER, G.T., (2008), Geomorphology and High-Resolution Stratigraphy of Progradational Wave-Dominated Shoreline Deposits: Implication on Reservoir-Scale Facies Architecture. *In* Hampson, G.J., Steel, R.J., Burgess, P.M., and Dalrymple, R.W. (eds.), Recent advances in models of siliciclastic shallow-marine stratigraphy: SEPM, Special Publication 90, in press.
- HANSEN, C.D., (1996). Interim geologic map of the Jump Creek Quadrangle, Carbon County, Utah. Utah Geological Survey Open-File Report, 2pl.,3/96 ref. OFR-334.
- HASTINGS, W.K. (1970) Monte Carlo sampling methods using Markov chains and their applications. *Biometrika* 57, p97-109.
- HELLAND-HANSEN, W., & MARTINSEN, O.J., (1996), Shoreline trajectories and sequences: description of variable depositional - dip scenarios: *Journal of Sedimentary Research*, v. B66, p. 670 - 688.
- HEWARD, P. (1981). A review of wave-dominated clastic shoreline deposits. *Earth Science Reviews*, 17. p 223-276.
- HOUSTON, W.S., HUNTON, J.E., & KAMOLA, D.L. (2000) Modelling of Cretaceous foreland-basin parasequences, Utah, with implications for timing Servier thrusting. *Geology*, 28. p. 267-270.
- HOWELL, J.A., & FLINT, S.S., (2003). Tectonic setting, stratigraphy and sedimentology of the Book Cliffs; Parasequences of the Book Cliffs succession; Sequences and systems tracts in the Book Cliffs; Sequence stratigraphical evolution of the Book Cliffs succession, *in* Coe, A., ed., *The sedimentary record of sea-level change*: Cambridge University Press, p. 135-208.

- HWANG, I-G., & HELLER, P.L., (2002), Anatomy of a transgressive lag : Panther Tongue Sandstone, Star Point Formation, central Utah : *Sedimentology*, v. 49, p. 977-999.
- JACKSON, M.D., YOSHIDA, S., & MUGGERIDGE, A.H., (2005) Three-dimensional reservoir characterization and flow simulation of heterolithic tidal sandstones, *AAPG Bulletin*, v. 89, p 507 – 528.
- JARDINE, Ch; DEHARBES, J.M., SONTOT, Ch., VACHERESSE, C., RUBINO, J.L, GERARD, J., & ROSS, J.B., (2007). Synthèse sédimentaire du Devonien Inferieur du permis de Timimoun, Bassin de Timimoun, Algérie. TOTAL Internal Report, p.201.
- JENNETTE, D.C, & RILEY, C.O, (1996) Influence of relative sea-level on facies and reservoir geometry of the Middle Jurassic lower Brent Group, U.K., North Viking Graben. *In* Aitkin, J.F. & Howell, J.A., (eds), *High Resolution Sequence Stratigraphy: Innovations and Applications: Geological Society of London, Special Publication 104*, p. 97-113.
- JENSON, J.L., LAKE, L.W., CORBETT, P.W. & GOGGIN, D.J., (2000) *Statistics for Petroleum Engineers and Geoscientists*, Elsevier, Amsterdam, 338p.
- JONES, L.S., & SCHUMM, S.A., (1999), Causes of avulsion : an overview, *in* Smith, N.D., and Rogers, J., eds., *Fluvial Sedimentology VI: IAS, Special Publication 28*, p. 171-178.
- KAMOLA, D.L. & HUNTOON, J.E., (1992), Sequence boundary variations within the Aberdeen Member, Cretaceous Blackhawk Formation, Utah (Abs.): American Association of Petroleum Geologists, 1992 Annual Convention Program, p.61-62.
- KAMOLA, D.L. & HUNTOON, J.E., (1993), Sequence boundary development and influence on "stranded" deltaic parasequences, Upper Cretaceous Blackhawk Formation, Utah (Abs.): American Association of Petroleum Geologists Annual Convention Official Program, p. 125-126.

- KAMOLA, D.L. & HUNTOON, J.E., (1995). Repetitive stratal patterns in an foreland basin sandstone and their possible tectonic significance. *Geology* 23, p.177-180.
- KAMOLA, D.L. & VAN WAGONER, J.C., (1995). Parasequence stratigraphy and facies architecture of the Spring Canyon Member, Upper Cretaceous, Blackhawk Formation. *In*: Van Wagoner, J.C., and Bertram, G.T. (eds), *Sequence Stratigraphy of Foreland Basin deposits: Outcrop and Subsurface Examples from the Cretaceous of North America*: American Association of Petroleum Geologists, Memoir 64 p.27-54.
- KERLER, W. & REHBERG, P., (1994) Simulated-tempering procedure for spin-glass simulations. *Phys. Rev., E* 50, p 4220 – 4225.
- LABOURDETTE, R., (2007). Integrated three-dimensional modelling approach of stacked turbidite channels. *AAPG Bulletin*, 91(11), p. 1603-1618.
- LABOURDETTE, R., & JONES, R.R., (2007). Characterization of fluvial architectural elements using a three-dimensional outcrop dataset: Escanilla braided system, South-Central Pyrenees, Spain. *Geosphere*, vol. 3, Iss. 6, p. 422-434.
- LEEDER, M., HARRIS, T., & KIRKBY, M. (1998) Sediment supply and climate change: Implications for basin stratigraphy. *Basin Research*, v. 10, p7-18.
- LERCHE, I., (1996) An inverse method for determining parameters for folded structures: *Quarterly of Applied Mathematics*, v.54, p. 625-636.
- LESSENGER, M. & CROSS, T.A., (1996) Is stratigraphic inversion possible?: *Energy Exploration & Exploitation*, v. 14, n. 6, p. 627-637.
- LESSENGER, M., & LERCHE, I., (1999) White paper in inverse modeling, in J.W. Harbaugh, W.L. Watney, E.C. Rankey, R. Slingerland, R.H. Goldstein, E.K. Franseen, (Eds.), *SEPM Special Publication 62 Numerical Experiments in Stratigraphy: Recent Advances in*

- Stratigraphic and Sedimentologic Computer Simulations*: SEPM (Society for Sedimentary Geology), p. 29-31.
- LIU, J.S., & SABATTI, C., (1999) Simulated sintering: Markov chain Monte Carlo with spaces of varying dimension. In: Bernardo, J. M., Smith, A. F. M., Dawid, A. P. Berger, J. O (Eds.). *Bayesian Statistics 6*, New York: Oxford University Press. p. 389–414.
- LIU, S., & NUMMEDAL, D., (2004). Late Cretaceous subsidence in Wyoming: quantifying the dynamic component: *Geology*, v. 32, p. 397-400.
- MALINVERNO, A., (2002) Parsimonious Bayesian Markov chain Monte Carlo inversion in a non linear geophysical problem, *Geophys. J. Int.* 151, 675-688
- MARINARI & PARISI (1992) Simulated tempering: a new Monte Carlo scheme. *Europhys. Lett.* 19: 451-458.
- METROPOLIS, N., ROSENBLUTH, A.W., ROSENBLUTH, M.N., TELLER, A.H., & TELLER, E. (1953) Equations of state calculations by fast computing machines. *J. Chem. Phys.* 21, 1087-91.
- MIALL, A.D., (1986) Eustatic Sea Level Changes Interpreted from Seismic Stratigraphy: A Critique of the Methodology with Particular Reference to the North Sea Jurassic Record: *AAPG Bulletin*, v. 70, p. 131–137.
- MIALL, A.D., (1991) Stratigraphic Sequences and Their Chronostratigraphic Correlation: *J. Sediment. Petrol.*, v. 61, p. 497–505.
- MIALL, A.D., (1992) Exxon Global Cycle Chart: An Event for Every Occasion? *Geology*, v. 20, p. 787–790.
- MIALL, A.D., & MIAL, C. (2001) Sequence Stratigraphy as a Scientific Enterprise: The Evolution and Persistence of Conflicting Paradigms. *Earth Science Reviews*, 54: 321-348.

- MORETTI, I. & TURCOTTE, D.L. (1985) A model for erosion, sedimentation and flexure with applications to New Caledonia. *Journal of Geodynamics* 3, p. 155-168.
- MOSEGAARD, K. & TARANTOLA, A. (1995) Monte Carlo sampling of solutions to inverse problems: *Journal of Geophysical Research* 100, B7, p. 12431-12447.
- MOSEGAARD, K. & SAMBRIDGE, M. (2002) Monte Carlo analysis of inverse problems. *Inverse Problems*, 18: R29-R54.
- MUTO, T., STEEL, R.J. & SWENSON J.B. (2007) Autostratigraphy: A framework norm for genetic stratigraphy. *Journal of Sedimentary Research*, v. 77, pp 2-12.
- NIEDORODA, A.M., REED, C.W., SWIFT, D.J.P. ARATA, H. & HOYANAGI, K. (1995) Modeling shore-normal large-scale coastal evolution. *Marine Geology*, 126, 181-199.
- NIEDORODA, A. & KRAVITZ, J.H., (1996) STRATAFORM: A program to study the creation and interpretation of sedimentary strata on continental margin. *In Oceanography*, v.9, n. 3, p.146-152.
- NORDLUND, U., (1999) FUZZIM: forward stratigraphic modeling made simple. *Computers and Geosciences*, v. 25, n. 4, p. 449-456(8).
- NUCCIO, V.F. & ROBERTS, L.N.R., (2003) Thermal maturity and oil and gas generation history of petroleum systems in the Uinta-Piceance province, Utah and Colorado. *In* Petroleum Systems and Geologic Assessment of Oil and Gas in the Uinta-Piceance Province, Utah and Colorado: United States Geological Survey, Digital Data Series DDS-69-B, Chapter 4.
- O'BYRNE, C.J., & FLINT, S.S., (1995). Sequence, parasequence and intraparasequence architecture of the Grassy Member, Blackhawk Formation, Book Cliffs, Utah, USA. *In*: Van Wagoner, J.C., and Bertram, G.T. (eds), *Sequence Stratigraphy of Foreland Basin deposits*:

- Outcrop and Subsurface Examples from the Cretaceous of North America: American Association of Petroleum Geologists, Memoir 64 p. 225-255.
- OLARIU, C., & BATTACHARYA, J.P., (2006), Terminal distributary channels and delta front architecture of river-dominated delta systems: *Journal of Sedimentary Research*, v. 76, p. 212 - 233.
- ORTON, G.J. & READING, H.G., (1993). Variability of deltaic processes in terms of sediment supply, with particular emphasis on grain size: *Sedimentology*, v. 40, p. 475 – 512.
- OLSEN, T.R., MELLERE, D., & OLSEN, T. (1999) Facies architecture and geometry of landward stepping shoreface tongue: the Upper Cretaceous Cliff House Sandstone (Mancos Canyon, south-west Colorado): *Sedimentology*, v. 46, p. 603-635.
- PAOLA, C., (2000) Quantitative models of sedimentary basin filling, *Sedimentology*, 47 (suppl. 1), 121-178.
- PATTISON, S.A.J., (1995). Sequence stratigraphic significance of sharp-based lowstand shoreface deposits, Kenilworth Member, Book Cliffs, Utah. *AAPG Bulletin*, v. 79, p. 444-462.
- PEMBERTON, S.G., MAC EACHERN, J.A., & FREY, R.W., (1992), Trace fossil facies models: environmental and allostratigraphic significance, *in* Walker, R.G., and James, N.P., eds., *Facies Models; Response to Sea-Level Change*: Geological Association of Canada, p. 47-72.
- PLINT, A.G., (1988). Sharp-based shoreface sequence and “offshore bars” in the Cardium formation of Alberta: their relationship to relative changes in sea level. *In*: Wilgus, C.K., Hastings, B.S., Kendall, C.G.St.C., Posamentier, H.W., Ross, C.A. and Van Wagoner, J.C., (eds), *Sea Level Changes – An Integrated Approach*: SEPM, Special Publication 42, p.357-370.

- PLINT, A.G. & NUMMEDAL, D., (2000). The falling system tract: recognition and importance in sequence stratigraphic analysis. *In*: Hunt, D. and Gawthorpe, R.L., (eds), *Sedimentary Responses to Forced Regression*: Geological Society of London, Special Publication 172, p1-17.
- POSAMENTIER, H.W., ALLEN, G.P., JAMES, D.P. & TESSON, M. (1992) Forced regressions in a sequence stratigraphic framework: Concepts, examples, and exploration significance: *American Association of Petroleum Geologists Bulletin*, v. 76, p. 1687-1709.
- RASMUSSEN, E.S. & DYBKAER (2005), Sequence stratigraphy of the Upper Oligocene-Lower Miocene of eastern Jylland, Denmark: Role of structural relief and variable sediment supply in controlling sequence development: *Sedimentology*, v. 52, p. 25- 63.
- READING, H.G. & COLLINSON, J.D. (1996), Clastic coasts, in Reading, H.G (ed), *Sedimentary Environments: Processes, Facies and Stratigraphy*. 3rd ed., Blackwell Science, Oxford, 154–231.
- REYNOLDS, A.D. (1999), Dimensions of paralic sandstone bodies: *American Association of Petroleum Geologists, Bulletin*, v. 83, p. 211-229.
- RIVANAES, J.C., (1992), Application of a dual lithology, depth-dependent diffusion equation in stratigraphic simulation: *Basin Research*, v. 4, p. 133-146. Blackwell Sciences Ltd.
- RIVENAES, J.C., (1997), Impact of sediment transport efficiency on large-scale sequence architecture: results from stratigraphic computer simulation. *Basin Research*, v9/2, p. 91-105.
- RODRIGUEZ, A.B., HAMILTON, M.D., & ANDERSON, J.B., (2000), Facies and evolution of the modern Brazos River Delta, Texas: wave versus flood influence: *Journal of Sedimentary Research*, v. 70, p. 283-295.

- ROTONDI, R. (2002) On the influence of the proposal distribution on a reversible jump MCMC algorithm applied to the detection of multiple change-points. *Computational Statistics and Data Analysis* 40, p 633-653.
- SAMBRIDGE, M., GALLAGHER, K., JACKSON, A., & RICKWOOD, P. (2006) Trans-dimensional inverse problems, Model comparison and the Evidence. *Geophy. J. Int.*, 167, 528-542.
- SCHLAGER, W. (1993) Accommodation and supply – A dual control on stratigraphic sequences. *Sedimentary Geology*, v. 86, p.111-136.
- SCLATTER, J.G., & CHRISTIE, P.A.F., (1980). Continental stretching: an explanation of the post Mid-Cretaceous subsidence of the central North Sea basin. *Journal of Geophysical Research*, 85, p. 3711-3739.
- SCHUMM, S.A. & ETHRIDGE, F.G., (1994), Origin, evolution and morphology of fluvial valleys. *In*: Dalrymple, R., Boyd, R., and Zaitlin, B.A., (eds), *Incised-valleys systems; origin and sedimentary sequences*, SEPM Special Publication 51, p. 11-27.
- SEIDLER, L. & STEEL, R. (2001) Pinch-out style and deposition of tidally influenced strata in a regressive-transgressive wave-dominated deltaic sandbody, Twentymile Sandstone, Mesaverde Group, NW Colorado: *Sedimentology*, v. 48, p. 399-414.
- SIVIA, D., & SKILLING, J., (2006) *Data analysis: A Bayesian tutorial* . Oxford University Press, 246p.
- SØMME, T.O., HOWELL, J.A., HAMSPON, G.J. & STORMS, J.E.A., (2008), Architecture and genesis of intra - parasequence discontinuity surfaces in wave - dominated deltaic deposits: Upper Cretaceous Sunnyside Member, Blackhawk Formation, Book Cliffs, Utah, USA. *In* Hampson, G.J., Steel, R.J., Burgess, P.M., and Dalrymple, R.W. (eds.), *Recent advances in models of siliciclastic shallow-marine stratigraphy: SEPM, Special Publication 90*, in press.

- STECKLER, M.S., REYNOLDS, D.J., COAKLEY, B.J., SWIFT, B.A. & JARRAD, R.D., (1993), Modeling passive margin sequence stratigraphy, in Posamentier, H.W., Summerhayes, C.P., Haq, B.U., and Allen, G.P., (Eds.), *Sequence Stratigraphy and Facies Associations*. International Association of Sedimentologists Special Publication, v. 18.
- STREBELLE, S., (2002), Conditional simulation of complex geological structures using multiple-point statistics, *Mathematical Geology*, 34(1), p.1-21.
- STORMS, J.E.A. (2003) Simulating event-based shallow marine deposition over geological timescales. *Marine geology* 199, 83-100.
- STORMS, J.E.A., WELTJE, G.J., van DIJKE, J.J., GEEL, C.R. & KROONENBERG, S.B. (2002) Process-response modeling of wave-dominated coastal systems: simulating evolution and stratigraphy on geological time scales. *Journal of Sedimentary Research*, 72(2), 226-239.
- STORMS, J.E.A. & SWIFT, D.J.P. (2003) Shallow marine sequences as the building blocks of stratigraphy: insights from numerical modelling. *Basin Research* 15, 287-303.
- STORMS, J.E.A. & HAMPSON, G.J. (2005), Mechanisms for Forming Discontinuity Surfaces Within Shoreface–shelf Parasequences: Sea Level, Sediment Supply, or Wave Regime? *Journal of Sedimentary Research*; 75: 67 - 81.
- SWIFT, D.J.P., (1968), Coastal erosion and transgressive stratigraphy: *Journal of Geology*, v. 76, p. 444– 456.
- SYVITSKI, J. P. M. & ANDREWS, J. T. (1994) Climate Change: Numerical modelling of sedimentation and coastal processes, Eastern Canadian Arctic. *Arctic and Alpine Research*, 26, 199-212.
- SYVITSKI, J. P., PRATSON L., & O'GRADY, D., (1999), Stratigraphic predictions of continental margins for the U.S. Navy. . In: Harbaugh, J., Watney, W., Rankey, E.,

- Slingerland, R., Goldstein, R., Franseen, E. (Eds.) 1999. SEPM Special Publication 62: *Numerical Experiments in Stratigraphy: Recent Advances in Stratigraphic and Sedimentologic Computer Simulations*. SEPM (Society for Sedimentary Geology): 219-236.
- TARANTOLA, A. (2005). *Inverse problem theory and methods for models parameter estimation*. SIAM. 358p.
- TAYLOR, D.R. & LOVELL, R.W.W., (1995), High-frequency sequence stratigraphy and paleogeography of the Kenilworth Member, Blackhawk Formation, Book Cliffs, Utah, U.S.A., in Van Wagoner, J.C and Bertram, G.T. (ed), *Sequence Stratigraphy of Foreland Basin deposits: Outcrop and Subsurface Examples from the Cretaceous of North America*: American Association of Petroleum Geologists, Memoir 64 p. 257-275.
- TAYLOR, K.G., GAWTHORPE, R.L., & FANNON-HOWELL, S., (2004). Basin-scale diagenetic alteration of shoreface sandstones in the Upper Cretaceous Spring Canyon and Aberdeen Members, Blackhawk Formation, Book Cliffs, Utah. *Sedimentary Geology* 172, p. 99-115.
- TETZLAFF, D.M. & HARBAUGH, J.W. (1989), *Simulating Clastic sedimentation*. New York: Van Nostrand Reinhold.
- THORNE, J.A., (1992) An analysis of the implicit assumptions of the methodology of seismic sequence stratigraphy. In J.S. Watkins (Eds.), *Geology and Geophysics of Continental Margins*: AAPG Memoir 53, p. 375-394.
- THORNE, J.A. & SWIFT, D.J.P., (1991) Sedimentation on continental margins: II. Application of the regime concept. In D.J.P. Swift, G.F., Oertel, R.W. Tillman, and J.A. Thorne, (Eds.). *Shelf sand and sandstone bodies: Geometry, facies and sequence stratigraphy*: International Association of Sedimentologists Special Publication 14, p33-58.
- THOMSEN, R.O., & LERCHE, I., (1997) Relative contributions to uncertainties in reserve estimates. *Marine and Petroleum Geology*, v. 14, n. 1, pp. 65-74(10).

- ULICNY, D., NICHOLS, G. & WALTHAM, D (2002) Role of initial depth at basin margins in sequence architecture: field examples and computer models. *Basin Research*, 14, 347-360.
- VAIL, P. R., MITCHUM, R. M., Jr. & THOMPSON, S., III, (1977) Seismic Stratigraphy and Global Changes of Sea Level, Part 4: Global Cycles of Relative Changes of Sea Level, in C. E. Payton, ed., *Seismic Stratigraphy-Applications to Hydrocarbon Exploration*: AAPG, Memoir 26, p. 83-97.
- VAIL, P.R., AUDMARD, F., BOWMAN, S.A., EISNER, P.N. & PEREZ-CRUZ, G.,(1991), The Stratigraphic Signature of Tectonics, Eustasy and Sedimentology, In G. Einsele, W. Rickeng, and A. Sielacher (Eds).
- VALASEK, D.W. (1990) Compartmentalisation of shoreface sequences in the Cretaceous Gallup Sandstone, New Mexico: Implication for reservoir quality and continuity: AAPG Bulletin, v.74, p 784.
- VAN WAGONER, J.C., (1995). Sequence stratigraphy and marine to non-marine facies architecture of foreland basin strata, Book Cliffs, Utah, USA. *In*: Van Wagoner, J.C. and Bertram, G.T. (eds). Sequence Stratigraphy of Foreland Basin deposits: Outcrop and Subsurface Examples from the Cretaceous of North America: American Association of Petroleum Geologists, Memoir 64 p. 137-223.
- VAN WAGONER, J.C., MITCHUM, R.M., CAMPION, K.M. & RAHMANIAN, V.D., (1990) "Siliciclastic Sequence Stratigraphy in Well Logs, Cores, and Outcrops: Concepts for High-Resolution Correlation of Time and Facies", AAPG Methods in Exploration, No. 7, pp55.
- WALKER, R.G. & PLINT, A.G. (1992). Wave and storm-dominated shallow marine systems. *In*: Walker, R.G. & James, N.P. (Eds), *Facies Models; Response to Sea-Level Change*. Geological Association of Canada, St. John's, Newfoundland, 219-238.

- WARRLICH, G.M.D., WALTHAM, D.A. & BOSENCE, D.W.J. (2002), Controls on atoll stratigraphies; insights from 3D forward computer modelling. *Basin Research*, 14, 416.
- WATNEY, W. L., RANKEY, E. C., & HARBAUGH, J., (1999), Perspectives on stratigraphic simulation models: current approaches and future opportunities in J.W. Harbaugh, W.L. Watney, E.C. Rankey, R. Slingerland, R.H. Goldstein, E.K. Franseen, (Eds.), *SEPM Special Publication 62 Numerical Experiments in Stratigraphy: Recent Advances in Stratigraphic and Sedimentologic Computer Simulations*: SEPM (Society for Sedimentary Geology): 3-21.
- WENTWORTH, C.K., 1922, A scale of grade and class terms for clastic sediments: *Journal of Geology*, v. 30, p. 377-392.
- WIJNS, C., POULET, T., BOSCHETTI, F., DYT, C. & GRIFFITHS, C.M.,(2003) Interactive inverse methodology applied to stratigraphic forward modelling. In: Curtis, A. and Wood, R. (Eds.) 2004. *Geological Prior Information Informing Science and Engineering*. Geological Society, London, Special Publications, 239:147-156.
- YOSHIDA, S., (2000). Sequence and facies architecture of the Upper Blackhawk Formation and the Lower Castlegate Sandstone (Upper Cretaceous), Book Cliffs, Utah, U.S.A., *Sediment. Geol.* 136, p239-276.
- YOUNG, R.G., (1955), Sedimentary facies and intertonguing in the Upper Cretaceous of the Book Cliffs, Utah-Colo: *Geological Society of America Bulletin*, v. 66, p. 177-202.

Appendix : Algorithm Implementation

A. RJMCMC Jump probabilities

At each iteration of the RJMCMC, one of the four jump types is randomly chosen, with probabilities b_k , d_k , mx_k and mt_k respectively for “Birth”, “Death”, “MoveX” and “MoveT” jumps (ensuring that $b_k + d_k + mx_k + mt_k = 1$).

We set $b_{kmax} = 0$ and $d_{kmin} = 0$, so that the limits of k can not be exceeded.

Hence if $k = kmax$, then $b_{kmax} = 0$, $d_{kmin} = 0.2$, $mx_{kmin} = mt_{kmin} = 0.4$

if $k = kmin$, then $b_{kmin} = 0.2$, $d_{kmin} = 0$; $mx_{kmin} = mt_{kmin} = 0.4$

Otherwise ($kmin < k < kmax$) $b_k = d_k = 0.15$ and $mx_k = mt_k = 0.35$

For a fixed dimension jump, the jump ratio is then always equal to one.

B. RJMCMC “MoveX” jump.

B.1 Proposal

A change to a x^i value is attempted by first choosing one of the $x^1, \dots, x^k$ coordinates at random, obtaining say x^j , and by, secondly, proposing a new value, say x'^j , from a normal proposal distribution $N(x^j, \sigma_x^2)$ centred on the current x^j value with variance σ_x^2 .

The transformation can be summarized as follows:

$$X_c^{\{2k\}} = \{t^{\{k\}}, x^{\{k\}}\} \rightarrow X'^{\{2k\}} = \{t^{\{k\}}, x'^{\{k\}}\}$$

B.2 Acceptance

The acceptance probability $\alpha(X_c, X')_{MoveX}$ for each “MoveX” jump becomes equal to likelihood ratio time the prior ratio of the values x^j and $x^{j'}$:

$$\alpha(X_c, X')_{MoveX} = \min(1, LR) = \min\left(1, \frac{p(D_{obs} | k, X'^{\{2k\}})p(x^{j'})}{p(D_{obs} | k, X_c^{\{2k\}})p(x^j)}\right)$$

C. RJMCMC “MoveT” jump

C.1 Proposal

For a “MoveT” jump type, one the $t^2, \dots, t^{k-2}$, is chosen at random, obtaining, say t^j . The candidate $t^{j'}$ is drawn from a normal distribution $N(t^j, \sigma_t^2)$ centred on the current value with proposal variance σ_t^2 , conditioned on the interval $[t^{j-1}, t^{j+1}]$ ($t^{j'} \in [t^{j-1}, t^{j+1}]$) with respect to the requirement $t^1 < t^2 < \dots < t^k$.

The transformation can be summarized as follows:

$$X_c^{\{2k\}} = \{t^{[k]}, x^{[k]}\} \rightarrow X'^{\{2k\}} = \{t'^{[k]}, x^{[k]}\}$$

C.2 Acceptance

The acceptance probability $\alpha(X_c, X')_{move-t}$ for each “MoveT” jump becomes:

$$\alpha(X_c, X')_{MoveT} = \min\left(1, \frac{p(D_{obs} | k, X'^{\{2k\}})}{p(D_{obs} | k, X_c^{\{2k\}})} \times \frac{(t_{j+1} - t'_j) \times (t'_j - t_{j-1})}{(t_{j+1} - t_j) \times (t_j - t_{j-1})}\right)$$

The prior ratio PR of a “MoveT” jump being equal to:

$$PR = \frac{p(k, X^{\{k \times 2\}})}{p(k, X_c^{\{k \times 2\}})} = \frac{p(k)}{p(k)} \times \frac{p(t^{\{k\}})}{p(t^{\{k\}})} \times \frac{p(x^{\{k\}})}{p(x^{\{k\}})} = \frac{p(t^{\{k\}})}{p(t^{\{k\}})}$$

$$\Leftrightarrow PR = \frac{p(t^1, t^2, \dots, t^{j'}, \dots, t^k)}{p(t^1, t^2, \dots, t^j, \dots, t^k)} = \frac{(t^{j+1} - t^{j'}) \times (t^{j'} - t^{j-1})}{(t^{j+1} - t^j) \times (t^j - t^{j-1})}$$

D. RJMCMC “Birth” jump

D.1 Proposal

To maintain a good acceptance rate, this jump type has been designed to draw “Birth” points close to the target posterior probability distribution.

- To draw the time coordinate t^* of the candidate point X^* :
 - Firstly, one of the pre-existing time coordinate $t_1, t_2, \dots, t_{k-1}$ is chosen at random, say t_j .

(with probability $1/(k-1)$).

- Secondly, t^* is drawn as follows:

$$t^* = t_j + u_1 \times \sigma_{t-birth}$$

Where u_1 is a random number drawn from a uniform distribution $U[0,1]$ ($g_1(u_1) = 1$)

and $\sigma_{t-birth} = \beta(t_{j+1} - t_j)$, with β being a coefficient chosen a priori comprised between 0 and 1.

- The corresponding x^* value of the candidate birth point X^* is drawn as follows:

$$x^* = x^\alpha + u_2 \times \sigma_{x-birth}$$

Where u_2 is a random number drawn from a uniform distribution $U[-1/2, 1/2]$ ($g_2(u_2) = 1$), $\sigma_{x-birth}$ is chosen a priori and x^α is the corresponding coordinate of t^* ,

calculated by linear regression between the points X^{j+1} and X^j :

$$x^\alpha = x_j + \left(\frac{(t^* - t_j)(x_{j+1} - x_j)}{(t_{j+1} - t_j)} \right)$$

The birth move transformation can be summarized as follows:

$$(X_c^{\{2k\}}, k) \rightarrow (X'^{\{2k+2\}}, k+1)$$

with, for the dimension: $k \rightarrow k+1$

where, for the sea level coordinates of the points X , $x \rightarrow x'$ means:

$$x^{i'} = x^i \quad \text{for } i=1, 2, \dots, j-1$$

$$x^{i*'} = x^\alpha + u_2 \times \sigma_{x-birth} \quad \text{for } i = j$$

$$x^{i'} = x^{i-1} \quad \text{for } i = j+1, \dots, k+1$$

and where, for the time coordinates of the points X , $t \rightarrow t'$ means:

$$t^{i'} = t^i \quad \text{for } i=1, 2, \dots, j-1$$

$$t^{i*'} = t_j + u_1 \times \sigma_{t-birth} \quad \text{for } i = j$$

$$t^{i'} = t^{i-1} \quad \text{for } i = j+1, \dots, k+1$$

Note that when there are k X^i points, there are k t^i coordinates and k x^i coordinates, making the dimension of the system d_1 equal to $2k$. A birth proposes a move to a system of dimension d_2 equal to $2k+2$, from a random vector $U=\{u_1, u_2\}$ of dimension $r_1 = 2$.

D.2 Acceptance

The acceptance probability $\alpha(X_c^{\{2k\}}, X'^{\{2k+2\}})_{Birth}$ for each “Birth” jump becomes (Appendix, part 6, sections 6.1, 6.2, 6.3, 6.4):

$$\alpha(X_c, X')_{Birth} = \min \left(1, \frac{p(D_{obs} | X'^{\{2k+2\}}, k+1)}{p(D_{obs} | X_c^{\{2k\}}, k)} \times \frac{p(x'^{\{k+1\}} | k+1)}{p(x^{\{k\}} | k)} \times \frac{(2k+3)(2k+2)}{(t_{\max} - t_{\min})^2} \times \frac{(t^{j+1} - t^*)(t^* - t^{j-1})}{(t^{j+1} - t^j)} \times \sigma_{t-birth} \times \sigma_{x-birth} \times JR \right)$$

Having chosen $\sigma_{t-birth} = \beta(t^{j+1} - t^j)$, the acceptance ratio becomes:

$$\alpha(X_c, X')_{Birth} = \min \left(1, \frac{p(D_{obs} | X'^{\{2k+2\}}, k+1)}{p(D_{obs} | X_c^{\{2k\}}, k)} \times \frac{p(x'^{\{k+1\}} | k+1)}{p(x^{\{k\}} | k)} \times \frac{\beta(2k+3)(2k+2)(t^{j+1} - t^*)(t^* - t^{j-1})}{(t_{\max} - t_{\min})^2} \times \sigma_{x-birth} \times JR \right)$$

E. RJMCMC “Death” jump

E.1 Proposal

Following the dimension balance requirement, the death move consists on deleting one of the k X^k points randomly chosen.

The death move transformation can be summarized as follows:

$$(X_c^{\{2k\}}, k) \rightarrow (X^{\{2k-2\}}, k-1)$$

E.2 Acceptance

The acceptance probability of the “death” move has the same form as the birth acceptance probability with the appropriate change of labeling of the variables, and the ratio terms inverted.

F. Detailed implementation of the acceptance probability of a “Birth” jump.

F.1 Prior Ratio PR

$$PR = \frac{p(X^{\{2k+2\}}, k+1)}{p(X_c^{\{2k\}}, k)} = \frac{p(k+1)}{p(k)} \times \frac{p(\mathbf{x}^{\{k+1\}} | k+1)}{p(\mathbf{x}^{\{k\}} | k)} \times \frac{p(\mathbf{t}^{\{k+1\}} | k+1)}{p(\mathbf{t}^{\{k\}} | k)}$$

With:

$$(1) \quad \frac{p(k+1)}{p(k)} = 1,$$

Any configuration of k are as likely as any others in the interval $[k_{\min}, k_{\max}]$

$$(2) \quad \frac{p(\mathbf{t}^{\{k+1\}} | k+1)}{p(\mathbf{t}^{\{k\}} | k)} = \frac{(2k+3)! \left(\frac{1}{t_{\max} - t_{\min}} \right)^{2k+3} \prod_{j=1}^{2k+3} (t^{j+1} - t^j)}{(2k+1)! \left(\frac{1}{t_{\max} - t_{\min}} \right)^{2k+1} \prod_{j=1}^{2k+1} (t^{j+1} - t^j)}$$

$$\Leftrightarrow \frac{p(\mathbf{t}^{\{k+1\}} | k+1)}{p(\mathbf{t}^{\{k\}} | k)} = \frac{(2k+3)(2k+2)}{(t_{\max} - t_{\min})^2} \times \frac{(t^{j+1} - t^*)(t^* - t^{j-1})}{(t^{j+1} - t^j)}$$

Then the prior ration PR becomes:

$$PR = \frac{p(\mathbf{x}^{\{k+1\}} | k+1)}{p(\mathbf{x}^{\{k\}} | k)} \times \frac{(2k+3)(2k+2)}{(t_{\max} - t_{\min})^2} \times \frac{(t^{j+1} - t^*)(t^* - t^{j-1})}{(t^{j+1} - t^j)}$$

F.2 Proposal ratio PrR

In the case of a Birth Jump, the proposal ratio probability is the ratio of reversing a proposed jump (death, dimension $k+1$) to proposing that jump (Birth, k), then PrR becomes:

$$P_R R = \frac{Death(k+1)}{Birth(k)} = \frac{g'(\mathbf{u}', \mathbf{X}^{\{2k+2\}})}{g(\mathbf{u}, \mathbf{X}^{\{2k\}})} = \frac{\frac{1}{k}}{\frac{1}{(k-1)} g_1(u_1) g_2(u_2)} = \frac{(k-1)}{k}$$

F.3 Jump ratio JR

The jump ratio probability is the ratio of proposing the reverse move of the proposed move.

$$JR = \frac{d_{k+1}}{b_k}$$

Then,

(1) If k is different from $k_{\min}$ and $k_{\max-1}$, then $JR = 1$

(2) If $k = k_{\min}$, then $RJ = \frac{d_{k_{\min}+1}}{b_{k_{\min}}} = \frac{0.15}{0.2}$

(3) If $k = k_{\max-1}$, then $RJ = \frac{d_{k_{\max}}}{b_{k_{\max-1}}} = \frac{0.2}{0.15}$

(There is no birth proposed for $k = k_{\max}$)

F.4 The jacobian | J |

To calculate the Jacobian $\left| \frac{\partial(\mathbf{X}^{\{2k+2\}}, \mathbf{u}')}{\partial(\mathbf{X}^{\{2k\}}, \mathbf{u})} \right|$, an order $2k+2$ determinant is needed, but because many of the vector components are not altered by the transformation, the Jacobian reduce to:

$$|\mathbf{J}| = \left| \frac{\partial(t^*, x^*)}{\partial(u_1, u_2)} \right|$$

$$\Leftrightarrow |\mathbf{J}| = \left| \begin{array}{cc} \sigma_{t-birth} & 0 \\ \sigma_{t-birth} \times \frac{(x^{j+1} - x^j)}{(t^{j+1} - t^j)} & \sigma_{x-birth} \end{array} \right|$$

$$\Leftrightarrow |\mathbf{J}| = \sigma_{t-birth} \times \sigma_{x-birth}$$



TOTAL

Exploration & Production

DGEP/GSR / _ _ _

Autorisation de Communication

1/ Identification

Auteur / R. Labourdette	
De [Div/Dept] : GSR/TG/G&I	Initiative personnelle [] demande du management []
Auteur principal: Karl Charvin	Compagnie / Div Imperial College London
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Co-auteur:	Compagnie /
Co-auteur:	Compagnie /

2/ Communication

Titre : Inverse and forward modelling of shallow marine wave-dominated stratigraphy
Domaine technique principal : <i>Inverse and forward modelling</i>
Mots-clés:

3/ Statut

Cette demande est : le premier résumé [] le suivant ou le document final [X]
Date limite de soumission

4/ Présentation orale, Poster ou Démonstration technique

Intitulé de l'événement :
Lieu et Date
Présentateur ou participant prévu:
Type de présentation: <i>Orale</i> [X] <i>Poster</i> [] <i>Démonstration</i> [] reprise dans proceedings [] journal [] CD-ROM []

5/ Publication écrite (article)

Titre du Périodique ou de l'Ouvrage : Thèse PhD
Date de publication pressentie:

6/ Budget (si nécessaire)

Coût:	Origine (R&D; Consortium; Prestation filiale; Autre): R&D
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7/ Données et contenu

Document JOINT: (Résumé/ Texte complet/ Figures / Diapos/ Poster/ Autre): Texte complet
Intérêt / Raison de communiquer sur ce sujet: Mise en valeur du savoir faire en sédimentologie
Les résultats sont-ils innovants ? O Peuvent-ils être exploités par la concurrence? O
Confidentialité / Concurrence (O; N; A définir; Sans objet) N Cette action est-elle liée à un projet R&D? (O/N) O
Données originales? Approbation des partenaires des données Texte Figure (si oui, approbation formelle à joindre)

8/ Approbations (Commentaires obligatoires si désapprobation)

Auteur Nom	Hierarchie de l'auteur Nom & position	Approbation patrimoniale ¹ Nom & position	Approbation technique Nom & position
Richard Labourdette	<i>PONCET Jérôme</i> Chef de Service G&I		<i>B. PIRONON</i> 13.05.08
Date & Signature 31 Mars 2008	Date & Signature 31 mars 08	Date & Signature Patrim.: GSR/CAE/Coord. Géograph.	Date & Signature Techn.: si Géologie = B. Pironon si Géophysique = J.L. Piazza si Réservoir = F. Daviau si Supports = J. Tachet des Combes
↓	↓	↓	↓
Faxer à J.-F. Raynaud – 85388	GSR/FIT – Geneviève Desbordes : – retour de l'original à l'auteur. CSTJF -D 101 – copie à Bernadette Cazenave pour la base de données E&P – copie à J.F. Raynaud pour enregistrement.		

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