

SOME ASPECTS OF HIGH ENERGY BACKWARD SCATTERING

by

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TO MY PARENTS

ABSTRACT

We study high energy backward scattering in three different approaches.

Firstly, covariant Reggeization techniques are extended to fermion exchange processes including those initiated by photons. The relationship between covariant Regge vertices and s-channel helicity vertices is obtained and tabulated. A consistent way of extending the Carlitz-Kislinger cut model to any backward process is considered and the implication for the dip structure is briefly surveyed.

Secondly, a model with a supermultiplet Reggeon propagator is used to generate polynomial residues for πN backward scattering. The Δ_8 -residue is found to extrapolate very well from the $\Delta(1236)$ pole to the scattering region, where it also exhibits its 'peripheral-like' zero structures. A reasonable good reproduction of the $\pi^- p \rightarrow p \pi^-$ data is obtained. The N_α -residue gives a correct value for the πN -coupling constant, but extrapolates badly to the scattering region presumably due to large symmetry breaking effects.

And finally, the dual absorptive model is extended to the backward direction and applied to πN scattering. All the qualitative features of the observed data are reproduced. The model gives consistent parameter values for the N-amplitude. An inconsistency is found for the Δ -amplitude, which may be attributed to the incorrect form of $\text{Re } \Delta(\Delta\lambda=1)$. Possible mechanisms for a correct form, within the dual absorptive idea, are briefly discussed.

PREFACE

The work presented in this thesis was carried out under the supervision of Professor P.T. Matthews in the Department of Theoretical Physics, Imperial College, London, between 1970 and September 1973. Except where stated the work is original and has not been submitted for a degree of this or any other University.

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CHAPTER I

GENERAL INTRODUCTION

Since the original work by Regge in 1959⁽¹⁾ demonstrating the existence of poles in the complex angular momentum plane for potential scattering, and the subsequent extension of the ideas to relativistic scattering by Chew and Frautschi⁽²⁾ the most useful concept in hadron-physics has been the idea of Regge pole-dominance of high-energy amplitudes. It was then shown by Gribov⁽³⁾ that the baryon Regge poles which dominate the scattering amplitude near the backward direction should occur in pairs, the so called 'parity doublets' if the trajectories are linear, as suggested strongly by Chew-Frautschi plots. However, some of these are certainly not observed and this is usually explained by giving the residue a zero at appropriate points, which is rather unnatural. Lyth⁽⁴⁾ and Jones⁽⁵⁾ suggested a mechanism for changing the form of baryon trajectories at thresholds and thus moving the parity partners away from the expected positions. An alternative was suggested by Carlitz and Kislinger⁽⁶⁾ who introduce a fixed kinematic cut in the scattering amplitude, which is responsible for removing the parity doublets on to the unphysical sheet. This model involves problems concerning the high energy behaviour, the absence of the usual nonsense dips and the most important of all, unitarity in the u-channel. However, Halzen et al.⁽⁷⁾ showed that it is possible to obtain a reasonable fit to the data, albeit with an un-

reasonably large pion-nucleon coupling constant. A modified Carlitz-Kislinger model which respects unitarity has also been proposed by Lyth⁽⁸⁾. In chapter II, the model is discussed briefly in the covariant formalism.

In a simple Regge pole model, dips in the angular distributions are simply explained in terms of nonsense zeros in the Regge residues. However, there are cases where the wrong signature nonsense dips do not appear. This can be explained by introducing either a second trajectory or a Regge cut, but then the simple dip-mechanism is lost and the theory has no predictive power. In connection with the fixed cut model in the covariant formalism, we give a qualitative dip mechanism in chapter II.

A long standing problem in the backward scattering has been the incorrect extrapolation of the Δ_8 -residue from the scattering region to the $\Delta(1236)$ pole. The extrapolated residues are too small to account for the observed width. This indicates the need for a more complicated parametrization of the Δ_8 -residues, which is clearly outside the standard Regge regime, including various types of Regge cut models. This is shown in a recent review by Storrow and Winbow⁽⁹⁾. In chapter III we consider a Reggeized supermultiplet model which generates a polynomial Δ_8 -residue which extrapolates extremely well and also possesses desirable zero structures in the scattering region.

Recent measurements of polarization in $\pi^\pm p$ backward scattering have resulted in new puzzles for backward phenomenology. None of the conventional Regge pole or cut

fits came close to these observations. In view of this situation, Barger and Olssen⁽¹⁰⁾ made a remarkable model-independent analysis of the existing πN backward scattering data, in which the explicit contributions of the $N(I_\alpha = \frac{1}{2})$ and $\Delta(I_\alpha = \frac{3}{2})$ exchanges to the differential cross-section and polarization were deduced. Their essential conclusions are: the N -exchange amplitudes are consistent with the usual N_α -amplitudes with nonsense zeros at $\alpha = -\frac{1}{2}$, but with a completely different phase from that of a pure N_α Regge pole; the imaginary part of the non-flip and the flip (s -channel helicity) $\Delta(I_\alpha = \frac{3}{2})$ -amplitudes have linear zeros at $u \approx -0.15$ and $u \approx -0.5$, respectively. We show in chapter IV that a simple dual absorptive model is consistent with all the above observations.

We may therefore conclude that, in addition to the conventional Regge pole, both weak and strong Regge cuts are also necessary for a correct description of high energy backward amplitudes, but only in the appropriate s -channel helicity amplitudes, as required by duality.

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CHAPTER II

COVARIANT REGGEIZATION FOR BACKWARD SCATTERING2.1 Introduction

A method of Reggeizing invariant amplitudes which uses the idea of elementary particle exchange as a guide was first proposed by Frautschi, Gell-Mann and Zachariasen about a decade ago⁽¹⁾. Due to the difficulty of obtaining the general spin J -propagators, this method was superseded by the conventional method of Reggeizing cross-channel (t or u) helicity amplitudes⁽²⁾. However, the latter method has difficulties associated with the fact that the amplitudes are required to satisfy some complicated kinematic constraints in the forward (backward) direction⁽³⁾ and at various thresholds and pseudo-thresholds⁽⁴⁾. In order to overcome this difficulty two methods have been developed.

The first method is to work with s -channel helicity amplitudes (SHA) instead⁽⁵⁾, exploiting the simplicity of the crossing-matrix at high energy and the fact that the kinematic singularities of the SHA and their constraints occur in regions far from that of physical interest. The advantage of this approach is that the cross-sections and polarizations are easy to obtain and that the explicit form of SHA serves as a natural input for high energy calculations⁽⁶⁾. The main difficulty with this method is in keeping track of the nonsense factors, which appear explicitly in t (or u) channel helicity amplitude.

The second method is that of covariant Reggeization, i.e., Reggeizing invariant amplitudes, which is now made feasible by knowledge of the high-spin propagator-technique⁽⁷⁻¹⁰⁾. This method, which makes use of covariant M-functions rather than helicity amplitudes, is therefore free of the kinematic constraints of cross-channel helicity amplitudes. Furthermore, explicit forms of covariant coupling functions provide a clear and tractable view of nonsense factors (or dip mechanism) present in any particular amplitude; a definite advantage over the helicity formalism. In this approach a single Regge pole amplitude automatically factorizes in terms of covariant couplings and it contains the correct threshold and spin factors which are crucial for continuation in energy and in the complex J -plane. Also the method is closer to field theory and is therefore capable of extension to higher symmetries⁽¹¹⁾. However, the disadvantage of this method is that cross-sections⁽¹²⁾ and density-matrices are rather hard to obtain. Gault and Jones⁽¹³⁾ studied the relationship between the two approaches for forward scattering, where they showed that the two are in fact closely related, thus providing a Reggeization scheme which contains the advantages of both methods.

In this chapter we extend the method of covariant Reggeization to backward scattering processes, where unlike the forward case, factorization into s -channel helicity vertices is not automatic, and Gribov parity doubling of fermion trajectories⁽¹⁴⁾ is also involved. In section 2.2, the covariant formalism of Reggeization is briefly reviewed

and the extension of these techniques to backward scattering is made in section 2.3. Section 2.4 is concerned with the gauge invariance of photon vertices. Regularized gauge invariant photon vertices are derived in this section. In section 2.5, the calculation of s-channel helicity amplitudes corresponding to the asymptotic M-function is considered. From the nature of high-energy kinematics and the form of fermion Regge pole exchanged amplitudes in the s-channel, we show that the SHA are in fact factorizable into helicity vertices in asymptopia. Calculations of helicity vertices for all the coupling in Table I are performed⁽¹⁵⁾ and the results tabulated in Tables II and III. Section 2.6 is concerned with the discussion on the kinematic zeros of the covariant residue functions at nonsense points in J; the dip mechanism. The kinematic cuts arising from the Reggeon mass term are also considered in connection with the extension of the Carlitz-Kislinger-cut model to any backward scattering process. Finally, the conclusions follow in section 2.7.

2.2 The Covariant Formalism

(a) Spin-J Projection Operator

The projection operator for an on-mass-shell ($\Delta^2 = m_J^2$) boson of spin-J is defined by⁽⁷⁾

$$\rho_{\beta_1 \dots \beta_J; \alpha_1 \dots \alpha_J}^J(\Delta) = \sum_{\lambda} \Phi_{\beta_1 \dots \beta_J}^{(\lambda)}(\Delta) \bar{\Phi}_{\alpha_1 \dots \alpha_J}^{(\lambda)}(\Delta), \quad (2.1)$$

where the wave function $\Phi_{\alpha_1 \dots \alpha_J}(\Delta)$ is defined in terms of the basic spin-1 wave function $\varepsilon_{\alpha}^{(\lambda)}(\Delta)$, and the

totally contracted form is

$$\mathcal{P}^J(P, Q; \Delta) = P_{\beta_1} \dots P_{\beta_J} \mathcal{P}_{\beta_1 \dots \beta_J; \alpha_1 \dots \alpha_J}^J(\Delta) Q_{\alpha_1} \dots Q_{\alpha_J}, \quad (2.2)$$

where Q and P are some initial and final momenta defined in Fig. (2.1). Clearly $\mathcal{P}_{\beta_1 \dots \beta_J; \alpha_1 \dots \alpha_J}^J(\Delta)$ is traceless and symmetric in both initial and final indices, and satisfies the condition

$$\Delta_{\beta_1} \mathcal{P}_{\beta_1 \dots \beta_J; \alpha_1 \dots \alpha_J}^J(\Delta) = \Delta_{\alpha_1} \mathcal{P}_{\beta_1 \dots \beta_J; \alpha_1 \dots \alpha_J}^J(\Delta) = 0. \quad (2.3)$$

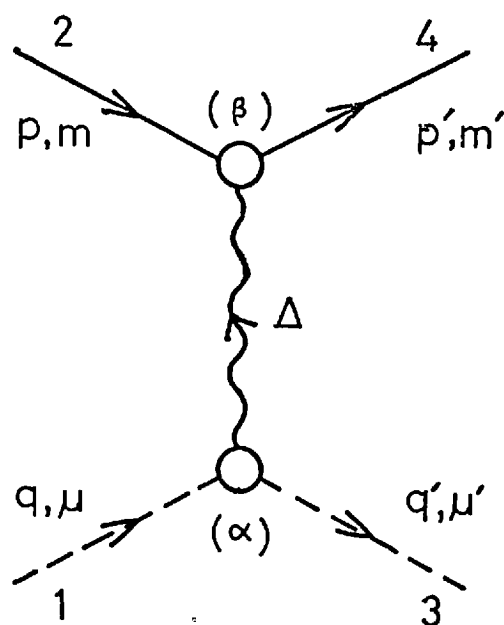
Partially contracted forms, such as $\mathcal{P}_{\beta; \alpha}^J$ or $\mathcal{P}_{\beta; \alpha}^J$, may be obtained from Eq. (2.2) by a differential technique⁽¹⁶⁾ and are given in Ref. 7, e.g.,

$$\begin{aligned} \mathcal{P}_{\beta_1; \alpha}^J(P, Q; \Delta) &\equiv P_{\beta_2} \dots P_{\beta_J} \mathcal{P}_{\beta_1 \beta_2 \dots \beta_J; \alpha_1 \dots \alpha_J}^J(\Delta) Q_{\alpha_1} \dots Q_{\alpha_J} \\ &= \frac{1}{J} \frac{\partial}{\partial P_{\beta_1}} \mathcal{P}^J(P, Q; \Delta), \end{aligned}$$

$$\mathcal{P}_{\beta_1 \beta_2; \alpha}^J(P, Q; \Delta) = \frac{1}{J(J-1)} \frac{\partial}{\partial P_{\beta_1}} \frac{\partial}{\partial P_{\beta_2}} \mathcal{P}^J(P, Q; \Delta), \quad (2.4)$$

etc.

A spin- J boson exchange contribution in the t -channel to a spinless particle amplitude (Fig. 2.1) is proportional to

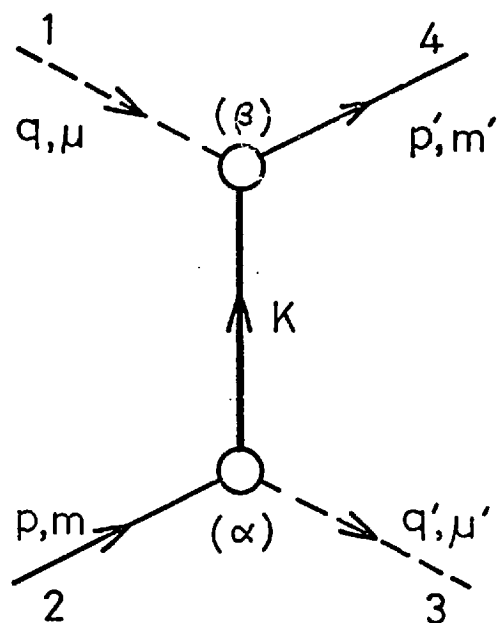


$$P = \frac{1}{2}(p' + p)^2$$

$$Q = \frac{1}{2}(q' + q)^2$$

$$\Delta = (p' - p)^2$$

Fig. (2.1) t-channel kinematics



$$\Lambda' = \frac{1}{2}(p' + q)^2$$

$$\Lambda = \frac{1}{2}(p + q')^2$$

$$K = (p' - q)^2$$

Fig. (2.2) u-channel kinematics

$$\rho^J(P, -Q; \Delta) = c_J \rho_J(\nu), \quad (2.5)$$

$$\text{where } c_J = \frac{\sqrt{\pi}}{2^J} \cdot \frac{\Gamma(J+1)}{\Gamma(J+\frac{1}{2})},$$

and the solid spherical harmonic ρ_J is related to the usual Legendre polynomial P_J by

$$\rho_J(\nu) = |P(\Delta) Q(\Delta)|^J P_J(z_t) \quad (2.6)$$

$$\text{with } \nu \equiv P(\Delta) \cdot Q(\Delta) = |P(\Delta)| |Q(\Delta)| z_t, \quad Q(\Delta) \equiv Q - \frac{Q \cdot \Delta}{m_J^2} \Delta.$$

By differentiation and using the covariant recursion relations for ρ_J , one can easily obtain explicit forms for the partially contracted projection operators, such as

$$\rho_{\beta}^J(P, -Q; \Delta) = \frac{c_J}{J} [Q_{\beta}(\Delta) \rho_J' - Q^2(\Delta) P_{\beta}(\Delta) \rho_{J-1}']. \quad (2.7)$$

In the case of a spin- $J = j + \frac{1}{2}$ fermion (j integral), the projection operator and its totally contracted form are respectively given by

$$\mathcal{G}_{\beta_1 \dots \beta_j; \alpha_1 \dots \alpha_j}^J(K) = \sum_{\lambda} \Psi_{\beta_1 \dots \beta_j}^{(\lambda)}(K) \bar{\Psi}_{\alpha_1 \dots \alpha_j}^{(\lambda)}(K), \quad (2.8)$$

where $\Psi_{\alpha_1 \dots \alpha_j}(K)$ is defined in terms of the basic spin-1 and spin- $\frac{1}{2}$ wave functions, $\varepsilon_{\alpha}^{(\lambda)}(K)$ and $u_{\alpha}^{(\lambda)}(K)$, and

$$\mathcal{G}^J(\Lambda', \Lambda; K) = \Lambda'_{\beta_1} \dots \Lambda'_{\beta_j} \mathcal{G}_{\beta_1 \dots \beta_j; \alpha_1 \dots \alpha_j}^J(K) \Lambda_{\alpha_1} \dots \Lambda_{\alpha_j}, \quad (2.9)$$

where the momenta Λ' , Λ and K are defined in Fig. (2.2).

Again the traceless symmetric tensor $\mathcal{G}_{\beta \dots; \alpha \dots}^J(K)$ satisfies

$$K_{\beta_i} \mathcal{G}_{\beta_1 \dots \beta_j; \alpha_1 \dots \alpha_j}^J(K) = K_{\alpha_i} \mathcal{G}_{\beta_1 \dots \beta_j; \alpha_1 \dots \alpha_j}^J(K) = 0,$$

$$(K - m_J) \mathcal{G}_{\beta_1 \dots \beta_j; \alpha_1 \dots \alpha_j}^J(K) = (K - m_J) \mathcal{G}^J(K) = 0 \quad (2.10)$$

It can be shown⁽⁷⁾, in a similar way as in the boson case, that

$$\mathcal{G}_{\beta_i}^{j+1/2}(\Lambda', \Lambda; K) = \frac{c_{j+1}}{j+1} \left[(K + m_J) \rho'_{j+1} - \Lambda'(K) (K - m_J) \Lambda(K) \rho'_j \right] \quad (2.11)$$

$$\mathcal{G}_{\beta_i}^{j+1/2}(\Lambda', \Lambda; K) = \frac{1}{j} \frac{\partial}{\partial \Lambda'_\beta} \mathcal{G}^{j+1/2}(\Lambda', \Lambda; K) \quad (2.12)$$

etc., where we have used the normalization $\bar{u}u = 2m_J$, and

ρ'_{j+1} denotes the derivative of ρ_{j+1} with respect to the argument τ ;

$$\tau \equiv \Lambda'(K) \cdot \Lambda(K) = |\Lambda'(K)| |\Lambda(K)| z_\alpha. \quad (2.13)$$

(b) Covariant Coupling Functions

The interaction between three on-mass-shell particles of arbitrary spins may be expressed in terms of an effective Lagrangian $\mathcal{L} \sim \bar{\Psi}(3) : \mathcal{C} : \Psi(2) \Psi(1)$ ($1 + 2 \rightarrow 3$) where the covariant coupling function $\mathcal{C}$ depends on the momenta, metric tensors and γ matrices. Parity conservation requires 'normal' and 'abnormal' $\mathcal{C}$ -functions, the normality of the vertex n being defined as the product of the normalities of the particles;

$$n = n_1 n_2 n_3 \quad (2.14)$$

with $n_i = (-)^{s_i}$ x (intrinsic parity) ($s_i = J_i$ for boson, $s_i = J_i - \frac{1}{2}$ for fermion).

Various covariant coupling functions are listed in

Ref. 7, where the 'reduced' coupling functions $\mathcal{C}$ may be introduced by the definition

$$\mathcal{C}_{\beta_1 \dots \beta_J} = \mathcal{C}_{\beta_1 \dots \beta_S} P_{\beta_{S+1}} \dots P_{\beta_J}, \quad (2.15)$$

where as many factors P_β are taken out as possible.

The relevant boson-fermion-fermion reduced coupling functions are listed in Table I.

(c) M-Function and Reggeization

Consider the high energy scattering process $q(s_1) + p(s_2) \rightarrow q'(s_3) + p'(s_4)$ (Fig(2.1) and (2.2)). The M-function is defined from the helicity amplitudes as

$$\langle p' \lambda_4, q' \lambda_3 | T | p \lambda_2, q \lambda_1 \rangle \equiv \bar{\Psi}_{\mu'}^{(\lambda_4)}(p') \bar{\Psi}_{\nu'}^{(\lambda_3)}(q') M_{\mu' \nu'; \mu \nu} \Psi_{\mu}^{(\lambda_2)}(p) \Psi_{\nu}^{(\lambda_1)}(q) \quad (2.16)$$

where the covariant label $\mu, \dots$ stands for $\{\mu_1 \dots \mu_s\}$ for a particle of spin s or $s-\frac{1}{2}$.

The M-function can then be expanded in terms of suitable kinematic covariants $\mathcal{K}^i$ together with kinematic-singularity-free scalar invariant amplitudes $A^i(s, t)$, where s and t (and u) are the usual variables;

$$M_{\mu' \nu'; \mu \nu} = \sum_i A^i(s, t) \mathcal{K}_{\mu' \nu'; \mu \nu}^i \quad (2.17)$$

The crucial step towards the method of covariant Reggeization is the notion that the J^{th} terms in the usual partial wave expansion of A^i 's may be identified with the corresponding contributions to M-function from the exchange of a 'particle' of spin J and mass $\sqrt{t}$ (or $\sqrt{u}$), namely

$$M^{(J)} = c'(s_2, s_4; J) : \frac{\rho^J(P, -Q; \Delta)}{t - m_J^2} : c(s_1, s_3; J), \quad (2.18)$$

near $t = m_J^2$.

For example, in πN scattering (1, 17) we have

$$\begin{aligned} M^{(J)} &= (g_1 P_\beta + g_2 \gamma_\beta) \frac{\rho_\beta^J(P, -Q; \Delta)(g)}{t - m_J^2} \\ &= \frac{g c_J}{t - m_J^2} \left[g_1 \rho_J + \frac{g_2}{J} (\alpha \rho_J' - m^2 \alpha^2 \rho_{J-1}') \right], \end{aligned} \quad (2.19)$$

which, with a suitable identifications of $g g_1$ and $g g_2$, gives the usual partial-wave expansion for the invariant amplitudes A and B⁽¹⁷⁾.

Thus the partial-wave expansion

$$M = \sum_J (2J + 1) M^{(J)} \quad (2.20)$$

can now be converted into a Sommerfeld-Watson integral in a usual way, after the appropriate terms in the expansions have been inverted to give Froissart-Gribov projection for the partial-wave amplitudes. Further details of the technique are given in Ref. 8.

The final result is the Reggeization prescription;

$$\frac{c_J \rho_J(v)}{t - m_J^2} \rightarrow c_\alpha \rho_\alpha(-v) \frac{1 \pm e^{-i\pi\alpha(t)}}{2 \sin \pi\alpha(t)}$$

coupling functions $\rightarrow$ Regge residue; function of t .

$$(2.21)$$

For a single Regge pole exchange, the amplitude is automatically factorized in terms of covariant residue functions. It also contains the correct threshold factor $(PQ)^\alpha$ and the spin factor c_α , so that we have $c_\alpha P_\alpha \sim v^\alpha$ exactly. The factorized residues $g(t)$ are free of kinematic singularities in both $\alpha(t)$ and t , but have kinematic zero corresponding to dip mechanism, etc., which will be discussed in section 2.6.

2.3 Reggeization and Parity Doublets

(a) Fermion Reggeization

It is well known⁽²⁰⁾ that the fermion Regge poles, which govern the asymptotic behaviour of the backward scattering amplitudes, have properties substantially different from those of the boson Regge poles. The amplitudes corresponding to the exchange of a fermion Regge pole with definite parity are not analytic in the usual complex u -plane. This is because of the fact that when we consider the Feynman-pole Reggeization of a fermion, we obtain amplitudes with a kinematic singularity at $u = 0$, owing to the presence of the Reggeon mass term $\sqrt{u}$ of the Reggeon propagator $(K \pm \sqrt{u})$ (Fig. 2.2). The existence of such a singularity is in contradiction with the Mandelstam analyticity properties which are required for any physical amplitude.

There are two distinct methods of dealing with this situation. The first is due to Gribov et al.⁽¹⁴⁾, in which they assumed that the asymptotic behaviour of backward scattering is determined not by individual Regge poles, but by a pair of complex-conjugate Regge poles: the so called

'parity doublets' if one further assumes that the trajectories are linear in u . This relation between the complex-conjugate poles can also be derived via the usual partial wave expansion and is known as the MacDowell symmetry⁽²¹⁾.

An alternative to the above method was given by Carlitz and Kislinger⁽²⁴⁾, who suggested that the amplitude should have a fixed kinematic cut arising from the same Reggeon mass term, $\sqrt{u}$. Therefore, one of the parity partners can be made to move behind this cut on an unphysical J -plane sheet for $\sqrt{u} > 0$, so that no physical particle exists on this trajectory. We will discuss this method further in later sections. For the present, we will follow the argument of Gribov et al., and discuss the problem of parity doubling in our covariant formalism.

(b) Parity Doublets

Consider the high-energy scattering process $q(s_1) + p(s_2) \rightarrow q'(s_3) + p'(s_4)$ near the backward direction, where we have $s = (p + q)^2 \rightarrow \text{large}$, $t = (p' - p)^2$, and $u = (p' - q)^2 \rightarrow \text{fixed value} < u_0$ ($u_0 = \text{threshold}$). The kinematics are defined in Fig. (2.2) for a u -channel exchange where $\Lambda = \frac{1}{2}(p + q')$, $\Lambda' = \frac{1}{2}(p' + q)$ and $K = (p' - q) = (p - q')$.

The contribution to this amplitude of the exchange of a spin $J = j + \frac{1}{2}$ (j integral) fermion of mass m_j and a definite normality ± 1 , in the region $u > u_0$, can be written as

$$M_{\pm}^{(J)} = e^{\pi'}(s_1, s_4; J) : \frac{s^J(\Lambda', \Lambda; K)}{u - m_J^2} : e^{\pi''}(s_2, s_3; J) \quad (2.22)$$

where n' , $n = \pm$ depending on the normalities of the external particles.

The full M-function is given by

$$M^{(J)} = M^{(J)}_+ + M^{(J)}_- , \quad (2.23)$$

which can be written in the form of Eq. (2.17) by using explicit coupling functions and Eq. (2.11), (2.12) for the spin J- projection operator, i.e.,

$$M^{(J)}_{\dots} = \sum_i A^{(J)i}_{(s,u)} K^i_{\dots} \quad (2.24)$$

The M-function can then be rewritten as the sum of definite parity exchange contributions by collecting terms containing $(\not{x} + \sqrt{u})$ and $(\not{x} - \sqrt{u})$;

$$M^{(J)} = A^{(J)}_+ (\not{x} + \sqrt{u}) + A^{(J)}_- (\not{x} - \sqrt{u}) \quad (2.25)$$

where $A^{(J)}_{\pm}$ are the sum of appropriate terms from Eq. (2.24), each corresponding to a definite parity exchange in u-channel.

As an example, we consider πN backward scattering, where we have for the exchange of spin $-J = j + \frac{1}{2}$ fermions of parities $(-1)^{j+\frac{1}{2}}$,

$$\begin{aligned} M^{(J)} &= \frac{c_{j+1}}{j+1} \frac{1}{u-m_\pi^2} \left[g^2 g^{j+\frac{1}{2}}(\Lambda', \Lambda; K) + f^2 \gamma_5 g^{j+\frac{1}{2}}(\Lambda', \Lambda; K) \gamma_5 \right] \\ &= A^{(J)}_+ (\not{x} + \sqrt{u}) + A^{(J)}_- (\not{x} - \sqrt{u}) , \end{aligned}$$

where

$$A_+^{(j)} = \frac{c_{j+1}}{j+1} \frac{1}{u-m_j^2} [g^2 \rho'_{j+1}(\tau) - f^2 (E_u - m)^2 \rho'_j(\tau)],$$

$$A_-^{(j)} = \frac{c_{j+1}}{j+1} \frac{1}{u-m_j^2} [f^2 \rho'_{j+1}(\tau) - g^2 (E_u + m)^2 \rho'_j(\tau)], \quad (2.26)$$

with $E_u \equiv \frac{u - m^2 - u^2}{2\sqrt{u}}$

In deriving Eq. (2.26), we have used Eq. (2.11) along with the fact that

$$\begin{aligned} \hat{\kappa}'(\kappa) (\kappa \pm \sqrt{u}) \kappa(\kappa) &= \left(\hat{\kappa}' - \frac{\Lambda \cdot \kappa}{u} \kappa \right) (\kappa \pm \sqrt{u}) \left(\kappa - \frac{\Lambda \cdot \kappa}{u} \kappa \right) \\ &\rightarrow (E_u \mp m) (\kappa \pm \sqrt{u}). \end{aligned}$$

Going back to the M-function in Eq. (2.25), we can rewrite this in terms of the usual invariant amplitudes A and B in the following way:

$$M^{(j)} = A_+^{(j)} (m_+ + \sqrt{u} - Q) + A_-^{(j)} (m_+ - \sqrt{u} - Q)$$

where we have used the relation

$$\begin{aligned} \kappa &= (\not{x}' - \not{x}) \rightarrow (m' - \not{x}) \\ &= (\not{x} - \not{x}') \rightarrow (m - \not{x}') \end{aligned}$$

$$Q \equiv \frac{1}{2} (q + q'), \quad m_+ \equiv \frac{1}{2} (m + m'),$$

and, if

$$M^{(j)} = A^{(j)} + Q B^{(j)}$$

then

$$A^{(j)} = m_+ (A_+^{(j)} + A_-^{(j)}) + \sqrt{u} (A_+^{(j)} - A_-^{(j)})$$

$$B^{(j)} = - (A_+^{(j)} + A_-^{(j)}) \quad (2.27)$$

Thus $A_{\pm}^{(J)}$ will have kinematic singularities in $\sqrt{u}$, so that $A^{(J)}$ and $B^{(J)}$ satisfy the required Mandelstam analyticity. This can be achieved by requiring that

$$A_{+}^{(J)}(\sqrt{u}) = A_{-}^{(J)}(-\sqrt{u}) \quad (2.28)$$

and $A_{\pm}^{(J)}(\sqrt{u}) \sim 1/\sqrt{u}$ as $u \rightarrow 0$.

We can now perform the covariant Reggeization of the definite parity exchange contributions, e.g. Eq. (2.26), by working in the $\sqrt{u}$ - rather than the usual u -plane and by observing the MacDowell symmetry relation (2.28).

The corresponding Reggeization prescription is thus

$$\frac{c_{J+1/2} \rho'_{J+1/2}(\tau)}{u - m_J^2} \rightarrow c_{\alpha+1/2} \rho'_{\alpha+1/2}(-\tau) \frac{1 \pm e^{-i\pi(\alpha-1/2)}}{2 \sin \pi(\alpha-1/2)},$$

$$J \rightarrow \alpha(\sqrt{u}), \quad (2.29)$$

Coupling function $\rightarrow$ Regge residue; function of $\sqrt{u}$.

If the residue and the trajectory functions corresponding to amplitudes $A_{\pm}^{(J)}(\pm\sqrt{u})$ are $\beta_{\pm}(\sqrt{u})$ and $\alpha_{\pm}(\sqrt{u})$, respectively, then the relation (2.28) requires that

$$\beta_{+}(\sqrt{u}) = \beta_{-}(-\sqrt{u}), \quad \alpha_{+}(\sqrt{u}) = \alpha_{-}(-\sqrt{u}),$$

and for $u < u_0$ we have

$$\beta_{+} = \beta_{-}^{*}, \quad \alpha_{+} = \alpha_{-}^{*}. \quad (2.30)$$

Since the amplitudes are now functions of $\sqrt{u}$, we adopt the convention of Gribov et al⁽¹⁴⁾ for continuation below the singularity point at $u = 0$. We take

$$\sqrt{u} = + i \sqrt{-u} \quad (2.31)$$

which corresponds to approaching the point $u = 0$ from above, i.e., $u \rightarrow u + i\epsilon$ ($\epsilon > 0$).

(c) Leading Order Contribution

A simplified form of the covariant Reggeization method has been given by Gault and Jones⁽¹³⁾, where one deals only with asymptotically leading order contributions. In the case of backward scattering involving (for example) single indexed couplings, Eq. (2.12) and (2.22) give for the M-function;

$$M^{(J)} = \frac{1}{u-m_J^2} e'_\beta \frac{1}{j} \frac{\partial}{\partial \Lambda'_\beta} \frac{\partial}{\partial \Lambda_\alpha} \mathcal{S}^{j+1/2}(\Lambda', \Lambda; K) e_\alpha, \quad (2.32)$$

where e'_β and e_α are covariant reduced coupling functions for $(s_1 + s_4 \rightarrow J)$ and $(s_2 + s_3 \rightarrow J)$ vertices (Fig. 2.2), and the totally contracted projection operator $\mathcal{S}^{j+1/2}$ is proportional to the Legendre polynomial $P'_{j+1}(z_u)$.

Therefore, to leading order in τ , we have from Eq. (2.11), (2.12)

$$\begin{aligned} \mathcal{S}^{j+1/2}(\Lambda', \Lambda; K) &\sim \frac{c_{j+1}}{j+1} (K+m_J) P'_{j+1}(\tau), \\ \mathcal{S}^{j+1/2}_{\beta; \alpha}(\Lambda', \Lambda; K) &\sim \frac{c_{j+1}}{j+1} (K+m_J) \frac{1}{j} \frac{\partial}{\partial \Lambda'_\beta} P'_{j+1}(\tau), \end{aligned} \quad (2.33)$$

and thus Eq. (2.32) becomes

$$M^{(J)} \sim \frac{1}{u-m_J^2} e'_\beta (K+m_J) e_\alpha \frac{1}{j^2} \frac{c_{j+1}}{j+1} \frac{\partial}{\partial \Lambda'_\beta} \frac{\partial}{\partial \Lambda_\alpha} P'_{j+1}(\tau) \quad (2.34)$$

The Reggeized form for either parity exchange is thus given to the leading order in τ as

$$M^{\alpha(\sqrt{u})} \sim e'_\beta (k + \sqrt{u}) e_\alpha \frac{\xi_\pm}{(\alpha(\sqrt{u}) - 1/2)^2} \frac{e_{\alpha(\sqrt{u}) + 1/2}}{(\alpha(\sqrt{u}) + 1/2)} \frac{\partial}{\partial \Lambda'_\beta} \frac{\partial}{\partial \Lambda_\alpha} P'_{\alpha(\sqrt{u}) + 1/2}(-\tau) \quad (2.35)$$

where $\xi_\pm \equiv (1 \pm e^{-i\pi\bar{\alpha}})^{1/2} \sin \pi\bar{\alpha}$ is the signature factor with $\bar{\alpha} \equiv \alpha(\sqrt{u}) - 1/2$, and the coupling constants g and f in e' & e are now Regge residue functions of $\sqrt{u}$ obeying the MacDowell symmetry rule.

The simplification at this stage is due to the fact that $P'_{\alpha(\sqrt{u}) + 1/2}$ may in fact be replaced by its asymptotic form $s^{\alpha(\sqrt{u}) - 1/2}$ (18), thus making the subsequent calculations an easy matter.

2.4 Gauge Invariance; Photon Vertices

(a) Gauge Invariance

Covariant decompositions of photonic amplitudes into invariant amplitudes free of kinematic singularities and zeros have been studied⁽¹⁹⁾. The M-function for processes involving one photon of momentum k , ($k^2 = 0$) can be written as

$$\tilde{M}_\nu = \sum_i \tilde{A}_i \tilde{K}_\nu^i \quad (2.36)$$

where $\tilde{A}_i$ are invariant amplitudes free of kinematic singularities and zeros in s and u (or t), $\tilde{K}_\nu^i$ are linear combinations of covariants like Λ' , Λ and K (Fig. 2.2) and the M-function $\tilde{M}_\nu$ satisfies the gauge condition

$$\tilde{M}_\nu k_\nu = \tilde{K}_\nu^i k^\nu = 0 \quad (2.37)$$

Gauge invariance is guaranteed if we write $\tilde{K}_\nu$ as $\tilde{K}_\nu = \mathcal{G}_{\nu\nu'} K_{\nu'}$, where $K_{\nu'}$ is the corresponding covariant which is in general not gauge invariant, and the gauge projection operator $\mathcal{G}_{\nu\nu'}$ is defined as

$$\mathcal{G}_{\nu\nu'} = g_{\nu\nu'} - \frac{\Lambda_\nu k_{\nu'}}{(k \cdot \Lambda)}, \quad (2.38)$$

which unfortunately introduces kinematic singularities into the invariant amplitudes (i.e. at $k \cdot \Lambda = 0$). A way out of this situation is to regularize the amplitudes by first of all forming all possible less singular combinations and then multiplying by a factor of $(k \cdot \Lambda)$.

In our case the spin-J exchange contribution to the M-function can be written as

$$\tilde{M}_\nu = e'(\Lambda') : \mathcal{S}^J(k) \tilde{e}_\nu(\Lambda) \quad (2.39)$$

where the photon vertex is gauge invariant, that is,

$\tilde{e}_\nu k_\nu = 0$ with $\tilde{e}_\nu = \mathcal{G}_{\nu\nu'} e_{\nu'}$. In the following, we consider the gauge invariant forms of particular photon vertices and the resulting coupling functions are listed in table I.

(b) Photon-Nucleon ($J^P = \frac{1}{2}^+$) Regge Vertex

To obtain the normal vertex $e_{\nu\alpha}^+(\frac{1}{2}, \gamma, j+\frac{1}{2})$, corresponding to normal exchange, the gauge projection operator is applied to the coupling $e_{\nu\alpha}^+(\frac{1}{2}, l, j+\frac{1}{2}) = (g_1 g_{\nu\alpha} + g_2 \gamma_\nu \Lambda_\alpha + g_3 \Lambda_\nu \Lambda_\alpha)$, and the resulting invariants $g'_{\nu\alpha}$ and $\gamma'_\nu \Lambda_\alpha$;

$$g'_{\nu\alpha} = g_{\nu\nu'} g_{\nu'\alpha} = g_{\nu\alpha} - \frac{\Lambda_\nu k_\alpha}{(k \cdot \Lambda)} = g_{\nu\alpha} - \frac{\Lambda_\nu \Lambda_\alpha}{(k \cdot \Lambda)},$$

$$\gamma'_\nu \Lambda_\alpha = g_{\nu\nu'} \gamma_{\nu'} \Lambda_\alpha = \gamma_\nu \Lambda_\alpha - \frac{k \Lambda_\nu \Lambda_\alpha}{(k \cdot \Lambda)} \quad (2.40)$$

are regularized as follows:

$$\text{Since } k \cdot \Lambda = \frac{1}{4} (m^2 - m_J^2) \text{ and } k \rightarrow (m - m_J) \quad (2.41)$$

i.e., $\bar{u}(p') k (k + m_J) \dots = (m - m_J) \bar{u}(p') (k + m_J) \dots$

the linear combinations

$$(m_J + m) \gamma'_\nu \Lambda_\alpha = (m_J + m) \gamma_\nu \Lambda_\alpha - 4 \Lambda_\nu \Lambda_\alpha \equiv \tilde{e}_{\nu\alpha}^{(1)}$$

and

$$(m_J - m) g'_{\nu\alpha} + \gamma'_\nu \Lambda_\alpha = (m_J - m) g_{\nu\alpha} + \gamma_\nu \Lambda_\alpha \equiv \tilde{e}_{\nu\alpha}^{(2)} \quad (2.42)$$

are free of kinematic singularities and zeros in u .

The regularized gauge invariant coupling is thus given by

$$\tilde{e}_{\nu\alpha}^+ (\frac{1}{2}, \gamma, j + \frac{1}{2}) = \tilde{g}_1 \tilde{e}_{\nu\alpha}^{(1)+} + \tilde{g}_2 \tilde{e}_{\nu\alpha}^{(2)+}, \quad (2.43)$$

& comparing with $e_{\nu\alpha}^+ (\frac{1}{2}, 1, j + \frac{1}{2})$, we get

$$\begin{aligned} g_1 &= (m_J - m) \tilde{g}_2 \\ g_2 &= (m_J + m) \tilde{g}_1 + \tilde{g}_2 \\ g_3 &= -4 \tilde{g}_1 \end{aligned} \quad (2.44)$$

Similar calculations can be performed for abnormal vertex and the final result is the replacement:

$$g \rightarrow \gamma_5 f, \quad m_J \rightarrow -m_J.$$

(2.45)

(c) Photon- $N^*(J^P = 3/2^-)$ Regge Vertex

For normal parity exchange, we consider

$$e_{\alpha_1 \alpha_2 \mu \nu}^+ (1, 3/2, j + \frac{1}{2}) \quad \text{as listed in table I. After}$$

the gauge operation, the covariants $g_{\alpha_1\mu} g'_{\alpha_2\nu}$, $g_{\alpha_1\mu} \gamma'_\nu \Lambda_{\alpha_2}$, $g'_{\mu\nu} \Lambda_{\alpha_1} \Lambda_{\alpha_2}$ and $\gamma'_\nu \Lambda_{\alpha_1} \Lambda_{\alpha_2} \Lambda_\mu$ are left, where

$$\begin{aligned} g'_{\alpha_2\nu} &= g_{\alpha_2\nu} - \frac{k_{\alpha_2} \Lambda_\nu}{(k \cdot \Lambda)} = g_{\alpha_2\nu} - \frac{4 \Lambda_{\alpha_2} \Lambda_\nu}{(m^2 - m_J^2)}, \\ \gamma'_\nu &= \gamma_\nu - \frac{k \Lambda_\nu}{(k \cdot \Lambda)} = \gamma_\nu - \frac{4 \Lambda_\nu}{(m + m_J)}, \\ \text{and } g'_{\mu\nu} &= g_{\mu\nu} - \frac{k_\mu \Lambda_\nu}{(k \cdot \Lambda)} = g_{\mu\nu} - \frac{8 \Lambda_\mu \Lambda_\nu}{(m^2 - m_J^2)}, \end{aligned} \quad (2.46)$$

where Eq. (2.41) has been used.

The linear combinations

$$\begin{aligned} (m_J - m) g_{\alpha_1\mu} g_{\alpha_2\nu} + g_{\alpha_1\mu} \gamma_\nu \Lambda_{\alpha_2} &\equiv \tilde{e}_{\mu\nu\alpha_1\alpha_2}^{(1)+}, \\ (m_J + m) g_{\alpha_1\mu} \gamma_\nu \Lambda_{\alpha_2} - 4 g_{\alpha_1\mu} \Lambda_\nu \Lambda_{\alpha_2} &\equiv \tilde{e}_{\mu\nu\alpha_1\alpha_2}^{(2)+}, \\ (m_J - m) g_{\mu\nu} \Lambda_{\alpha_1} \Lambda_{\alpha_2} + 2 \gamma_\nu \Lambda_\mu \Lambda_{\alpha_1} \Lambda_{\alpha_2} &\equiv \tilde{e}_{\mu\nu\alpha_1\alpha_2}^{(3)+}, \\ (m_J + m) \gamma_\nu \Lambda_\mu \Lambda_{\alpha_1} \Lambda_{\alpha_2} - 4 \Lambda_\mu \Lambda_\nu \Lambda_{\alpha_1} \Lambda_{\alpha_2} &\equiv \tilde{e}_{\mu\nu\alpha_1\alpha_2}^{(4)+} \end{aligned} \quad (2.47)$$

are free of kinematic singularities and zeros.

The regularized gauge invariant normal coupling is thus

$$\begin{aligned} \tilde{e}_{\mu\nu\alpha_1\alpha_2}^+ (\gamma, 3/2, j+1/2) &= (\tilde{g}_1 \tilde{e}_{\mu\nu\alpha_1\alpha_2}^{(1)+} + \tilde{g}_2 \tilde{e}_{\mu\nu\alpha_1\alpha_2}^{(2)+} + \tilde{g}_3 \tilde{e}_{\mu\nu\alpha_1\alpha_2}^{(3)+} \\ &\quad + \tilde{g}_4 \tilde{e}_{\mu\nu\alpha_1\alpha_2}^{(4)+}), \end{aligned} \quad (2.48)$$

and the relation to g in $e_{\mu\nu\alpha_1\alpha_2}^+ (1, 3/2, j+1/2)$ is

$$\begin{aligned} g_1 &= (m_J - m) \tilde{g}_1, \\ g_2 &= \tilde{g}_1 + (m_J + m) \tilde{g}_2, \end{aligned}$$

$$\begin{aligned}
g_3 &= -4 \tilde{g}_2 , \\
g_4 &= (m_J - m) \tilde{g}_3 , \\
g_5 &= 2 \tilde{g}_3 + (m_J + m) \tilde{g}_4 , \\
g_6 &= -4 \tilde{g}_4 .
\end{aligned}
\tag{2.49}$$

The abnormal coupling is similarly obtained by the prescription (2.45).

In this way we can obtain the regularized gauge invariant coupling functions for any other photonic vertices. The results are listed in table I.

2.5 Factorization, s-Channel Helicity Vertices

In order to calculate s-channel helicity amplitudes one simply has to sandwich the asymptotic M-function given in Eq. (2.35) between the appropriate helicity spinors or polarization vectors (see Appendix) i.e.,

$$T_{\lambda_4 \lambda_3; \lambda_1 \lambda_2} \sim e_{\beta}^{(\lambda_1 \lambda_4)} (k + \sqrt{u}) e_{\alpha}^{(\lambda_2 \lambda_3)} \frac{\xi_{\pm}}{\bar{\alpha}^2} \frac{\partial}{\partial \Lambda'_{\beta}} \frac{\partial}{\partial \Lambda_{\alpha}} s^{\bar{\alpha}} .
\tag{2.50}$$

Factorization into helicity vertices is not at all obvious at this stage, for we have in Eq. (2.50) the fermion Regge propagator $(k + \sqrt{u})$ connecting the two external fermions through the coupling functions e 's. However, we will show in the following that the amplitude in fact factorizes into helicity vertices in asymptopia. Since every differentiation of $s^{\bar{\alpha}}$ amounts to lowering the power of s it is clear that only those parts of the helicity

vertices which are effectively proportional to Λ'_β , Λ_α respectively give leading order contributions. Thus for the above case we may write

$$e'^{(\lambda_1 \lambda_4)}_\beta (K + \sqrt{u}) e^{(\lambda_2 \lambda_3)}_\alpha \sim \left[(-u)^{\frac{1}{2}|\lambda_1 - \lambda_4|} \Lambda'_\beta V'^{(n')}_{\lambda_1 \lambda_4} \right] \times \left[(-u)^{\frac{1}{2}|\lambda_2 - \lambda_3|} \Lambda_\alpha V^{(n)}_{\lambda_2 \lambda_3} \right] s^{\frac{1}{2}}_{\pm} \quad (2.51)$$

so that

$$T_{\lambda_4 \lambda_3; \lambda_1 \lambda_2} \sim (-u)^{\frac{1}{2}|\lambda_1 - \lambda_4| + \frac{1}{2}|\lambda_2 - \lambda_3|} V'^{(n')}_{\lambda_1 \lambda_4} V^{(n)}_{\lambda_2 \lambda_3} \sum_{\pm} s^{\alpha}_{\pm}, \quad (2.52)$$

where $n, n' = \pm$, depending on the normality of the particles at each vertex. The kinematic factors like $\sqrt{-u}$ always emerge naturally from the covariant vertices. However, in Eq. (2.52) above, the factor $(-u)^{\frac{1}{2}|\lambda_1 - \lambda_4| + \frac{1}{2}|\lambda_2 - \lambda_3|}$ contains both the kinematic and the Reggeon mass factor $\sqrt{u} = i\sqrt{-u}$. Thus it seems that all the SHA are zero at $u = 0$, but this is not the case since the residue functions are assumed to behave like $\frac{1}{\sqrt{u}}$ as $u \rightarrow 0$, and the genuine $\sqrt{u}$ terms will be cancelled by the corresponding Gribov-MacDowell partners.

We will first list a number of asymptotic relations between 'basic' helicity vertices involving only basic spinors $u^{(\lambda)}$ and vectors $\epsilon^{(\lambda)}_\mu$, which will be useful in the subsequent calculations. Using the explicit spinors as given in the Appendix, and noting the fact that the Reggeon mass $\sqrt{u} \rightarrow +i\sqrt{-u}$ in the scattering region, we have

$$\begin{aligned}
\bar{u}^{(1/2)} (\not{k} + \sqrt{u}) u^{(1/2)} &\sim -\sqrt{u} s^{1/2} \\
\bar{u}^{(1/2)} (\not{k} + \sqrt{u}) u^{(-1/2)} &\sim i\sqrt{u} s^{1/2} , \\
\bar{u}^{(1/2)} \gamma_5 (\not{k} + \sqrt{u}) u^{(1/2)} &\sim -i\sqrt{u} s^{1/2} \\
\bar{u}^{(1/2)} \gamma_5 (\not{k} + \sqrt{u}) u^{(-1/2)} &\sim -\sqrt{u} s^{1/2} , \\
\bar{u}^{(-1/2)} (\not{k} + \sqrt{u}) u^{(1/2)} &\sim -i\sqrt{u} s^{1/2} \\
\bar{u}^{(-1/2)} (\not{k} + \sqrt{u}) u^{(-1/2)} &\sim -\sqrt{u} s^{1/2} , \\
\bar{u}^{(-1/2)} \gamma_5 (\not{k} + \sqrt{u}) u^{(1/2)} &\sim -\sqrt{u} s^{1/2} \\
\bar{u}^{(-1/2)} \gamma_5 (\not{k} + \sqrt{u}) u^{(-1/2)} &\sim i\sqrt{u} s^{1/2} ,
\end{aligned} \tag{2.53}$$

from which we obtain the relation

$$\dots (\not{k} + \sqrt{u}) u^{(-1/2)} \sim \dots (\not{k} + \sqrt{u}) (-i) u^{(1/2)} \tag{2.54}$$

for the initial vertex, independent of the nature of the other (final) vertex.

In a similar way, we obtain the relations:

$$\bar{u}^{(-1/2)} (\not{k} + \sqrt{u}) \dots \sim (i) \bar{u}^{(1/2)} (\not{k} + \sqrt{u}) \dots , \tag{2.55}$$

the corresponding abnormal vertex relations (with γ_5) to (2.54) and (2.55) being given by $i \rightarrow -i$, and for the relation between normal and abnormal vertices:

$$\dots (\not{k} + \sqrt{u}) \gamma_5 u^{(1/2)} \sim \dots (\not{k} + \sqrt{u}) (-i) u^{(1/2)} , \tag{2.56}$$

$$\bar{u}^{(1/2)} \gamma_5 (\not{k} + \sqrt{u}) \dots \sim (i) \bar{u}^{(1/2)} (\not{k} + \sqrt{u}) \dots \quad (2.57)$$

Next we consider the vertex $\dots (\not{k} + \sqrt{u}) \not{\epsilon}^{(\lambda')}(q') u^{(\lambda)}(p)$. Using the explicit form of the spin-1 polarization vector (see Appendix), we have

$$\not{\epsilon}^{(0)}(q') \sim \frac{\not{q}'}{\mu'} = \frac{\not{p} - \not{k}}{\mu'} \rightarrow \frac{m - \sqrt{u}}{\mu'} \quad (2.58)$$

and for $\lambda' = \pm 1$

$$\begin{aligned} \not{\epsilon}^{*(\pm)}(q') &= -\underline{\epsilon}^{*(\pm)} \cdot \underline{\gamma} = -\gamma_0 (i\gamma_5) \underline{\epsilon}^{*(\pm)} \underline{\sigma} \\ &= \pm \gamma_0 (i\gamma_5) \frac{1}{\sqrt{2}} \sigma_{\mp} \end{aligned} \quad (2.59)$$

where $\sigma_{\pm} \equiv \sigma_1 \pm i \sigma_2$.

Since we have

$$\begin{aligned} \not{\epsilon}^{*(\pm)} u^{(\lambda)}(p) &= \not{\epsilon}^{*(\pm)} \sqrt{E+m} \left(1 + \frac{i\gamma_5 \not{p} h}{E+m} \right) \phi^{(\lambda)}(\hat{p}) \\ &\sim \not{\epsilon}^{*(\pm)} \sqrt{E} \left(1 + i\gamma_5 h \right) \phi^{(\lambda)}(\hat{p}), \end{aligned} \quad (2.60)$$

substitution of Eq. (2.59) in (2.60) along with the relation (2.54) gives

$$\begin{aligned} \not{\epsilon}^{*(+)} u^{(1/2)} &\sim \sqrt{2} u^{(-1/2)} \sim -i\sqrt{2} u^{(1/2)}, \\ \not{\epsilon}^{*(-)} u^{(1/2)} &\sim 0, \\ \not{\epsilon}^{*(+)} u^{(-1/2)} &\sim 0, \\ \not{\epsilon}^{*(-)} u^{(-1/2)} &\sim \sqrt{2} u^{(1/2)} \sim i\sqrt{2} u^{(-1/2)}. \end{aligned} \quad (2.61)$$

For an abnormal vertex involving γ_5 , we simply have to change $\sqrt{u} \rightarrow -\sqrt{u}$ and $i \rightarrow -i$ in Eq. (2.58) and (2.61), and the expressions for the vertex $\bar{u}^{(\lambda')}\not{x}^{(\lambda)}(\not{k} + \sqrt{u})\dots$ can be obtained in a similar way.

With the various asymptotic relations, (2.54) - (2.57) and (2.61) at our disposal, it is quite obvious now that we can always write any helicity amplitude in a factorized form (2.52). This will be made clear by the following examples.

(a) For the πN -Reggeon normal parity vertex, we have

$$e'^{(\lambda_1 \lambda_4)}(\not{k} + \sqrt{u})\dots = \bar{u}^{(\lambda_4)}(p') g(\not{k} + \sqrt{u})\dots,$$

thus

$$e'^{(0 \frac{1}{2})}(\not{k} + \sqrt{u})\dots = g \bar{u}^{(\frac{1}{2})}(\not{k} + \sqrt{u})\dots$$

Since any form of initial helicity vertex, i.e. $(\not{k} + \sqrt{u}) u \dots \varepsilon^* \dots$ etc., can be reduced, by means of the asymptotic relations, to the form $\sim (\not{k} + \sqrt{u}) u^{(\frac{1}{2})}$, one always ends up with the common factor:

$$\bar{u}^{(\frac{1}{2})}(\not{k} + \sqrt{u}) u^{(\frac{1}{2})} \sim -\sqrt{u} s^{\frac{1}{2}} = i(-u)^{\frac{1}{4}} \times i(-u)^{\frac{1}{4}} s^{\frac{1}{2}}. \quad (2.62)$$

Thus $e'^{(0 \frac{1}{2})} \rightarrow ig(-u)^{\frac{1}{4}} \dots$,

so that $V_{0 \frac{1}{2}}^{(+)(0 \frac{1}{2})} = ig$, (2.63)

and for the initial vertex,

$$V_{0 \frac{1}{2}}^{(+)}(0 \frac{1}{2}) = ig \quad (2.64)$$

Corresponding abnormal vertices are simply obtained by complex conjugation: $i \rightarrow -i$, $g(\sqrt{u}) \rightarrow f(\sqrt{u}) = g(-\sqrt{u})$.

(b) In the case of $N\rho$ -Reggeon normal parity vertex, we have

$$C_{\alpha}^{(\lambda_2 \lambda_3)} = (g_1 g_{\alpha\mu} + g_2 \Lambda_{\alpha} \gamma_{\mu} + g_3 \Lambda_{\alpha} \Lambda_{\mu}) \varepsilon_{\mu}^{*(\lambda_3)}(q') u^{(\lambda_2)}(p). \quad (2.65)$$

Since the polarization vector $\varepsilon^{(0)}(q')$ at high energy behaves like $\sim \frac{q'}{\mu}$, while $\varepsilon^{(\pm)}$ have no large components, we have

$$\begin{aligned} \varepsilon_{\alpha}^{*(0)}(q') &\sim \frac{q'_{\alpha}}{\mu'} \rightarrow \frac{\Lambda_{\alpha}}{\mu'}, \\ \not{\varepsilon}^{*(0)}(q') &\sim \frac{\not{q}'}{\mu'} = \frac{m - \sqrt{u}}{\mu'}, \\ \varepsilon^{*(0)}(q') \cdot \Lambda &\sim \frac{1}{4\mu'} (m^2 - \mu'^2 - u), \end{aligned} \quad (2.66)$$

and

$$\begin{aligned} \varepsilon_{\alpha}^{*(\pm)}(q') &\sim 0, \\ \not{\varepsilon}^{*(\pm)}(q') u^{(\frac{1}{2})}(p) &\sim \begin{cases} -i\sqrt{2} u^{(\frac{1}{2})}(p) \\ 0 \end{cases}, \\ \not{\varepsilon}^{*(\pm)}(q') u^{(-\frac{1}{2})}(p) &\sim \begin{cases} 0 \\ \sqrt{2} u^{(\frac{1}{2})}(p) \end{cases}, \\ \varepsilon^{*(\pm)}(q') \cdot \Lambda &\sim \pm \frac{\sqrt{u}}{2\sqrt{2}}, \end{aligned} \quad (2.67)$$

where the relations (2.61) have been appropriately used.

Thus using Eq. (2.65) - (2.67), we obtain

$$e_{\alpha}^{(\frac{1}{2}0)} \sim \frac{1}{\mu'} \left[g_1 + (m - i\sqrt{u}) g_2 + \frac{1}{4} (m^2 - \mu'^2 - u) g_3 \right] \Lambda_{\alpha} u^{(\frac{1}{2})}(p)$$

so that

$$V_{\frac{1}{2}0}^{(+)}(\frac{1}{2}1) = \frac{i}{\mu'} \left[g_1 + (m - i\sqrt{u}) g_2 + \frac{1}{4} (m^2 - \mu'^2 - u) g_3 \right]. \quad (2.68)$$

Similarly we obtain

$$V_{\frac{1}{2}1}^{(+)}(\frac{1}{2}1) = i \left(-i\sqrt{2} g_2 + \frac{\sqrt{u}}{2\sqrt{2}} g_3 \right); \quad (2.69)$$

$$V_{\frac{1}{2}-1}^{(+)}(\frac{1}{2}1) = -i \frac{g_3}{2\sqrt{2}}. \quad (2.70)$$

The corresponding abnormal parity vertex can be obtained from above by simply taking complex conjugation: $i \rightarrow -i$, $g(\sqrt{u}) \rightarrow g(-\sqrt{u})$, and all the rest are given by the parity relation, which will be defined in the following way.

(c) Parity

We have the parity relation for the SHA as

$$T_{\lambda_3 \lambda_4; \lambda_1 \lambda_2}(\theta, \varphi) = n (-)^{\lambda - \mu} T_{-\lambda_3 - \lambda_4; -\lambda_1 - \lambda_2}(\theta, -\varphi) \quad (2.71)$$

where $n = n_1 n_2 n_3 n_4$; n_i = normality of the i^{th} particle ($i = 1, 2, 3, 4$), $\lambda = \lambda_1 - \lambda_2$ and $\mu = \lambda_3 - \lambda_4$.

In terms of our helicity vertices defined in Eq. (2.52), we have for backward scattering,

$$V_{\lambda_1 \lambda_4}^{(n')} V_{\lambda_2 \lambda_3}^{(n)} = n_1 n_2 n_3 n_4 (-)^{(\lambda_1 - \lambda_2) - (\lambda_3 - \lambda_4)} V_{-\lambda_1 - \lambda_4}^{(n')} V_{-\lambda_2 - \lambda_3}^{(n)}$$

$$= [n_1 n_4 (\pm n_J)] [n_2 n_3 (\pm n_J)] (-)^{(\lambda_1 + \lambda_4) - (\lambda_2 + \lambda_3)} \\ \times V_{-\lambda_1 - \lambda_4}^{(\pm)} V_{-\lambda_2 - \lambda_3}^{(\pm)},$$

since $(\pm n_J)^2 = 1$.

Noting that $n_1 n_4 n_J = n'$, $n_2 n_3 n_J = n$, we obtain the following relation for the helicity vertices:

$$V_{-\lambda_1 - \lambda_4}^{(\pm)} = \pm (-)^{\lambda_1 + \lambda_4} V_{\lambda_1 \lambda_4}^{(\pm)}, \quad (2.72)$$

$$V_{-\lambda_2 - \lambda_3}^{(\pm)} = \mp (-)^{\lambda_2 + \lambda_3} V_{\lambda_2 \lambda_3}^{(\pm)}. \quad (2.73)$$

Calculations of helicity vertices for the couplings listed in Table I are performed in similar ways and the results are displayed in Table II and III. One should note in using our results that the physical SHA are obtained by adding the two parity doublet contributions which are related by the MacDowell symmetry (2.28) or (2.30).

2.6 Kinematic Zeros in J; Kinematic Cuts and Dip Mechanisms

(a) Nonsense Zeros

Although our factorized covariant residue functions $g(\sqrt{u})$ are free of kinematic singularities in J (i.e. poles), they may have kinematic zeros at particular values of J . The reason is based on the fact that the couplings listed in Table I are written for integral $j \geq s - \frac{1}{2}$; s being the sum of spins of the external particles. Therefore for integral $j < s - \frac{1}{2}$, some of the couplings must vanish.

For example, the residue $g_1(\sqrt{u})$ of the ρN -Reggeon vertex (Table I) will vanish at $j = 0$ due to the presence of the 'irreducible' α -label in $g_{\alpha\nu}$. The couplings g_2 and g_3 are not constrained to vanish at this point since $g_2 \Lambda_\alpha \gamma_\nu$ and $g_3 \Lambda_\alpha \Lambda_\nu$ can be reduced further to $g_2 \gamma_\nu$ and $g_3 \Lambda_\nu$ respectively, involving no α -labels.

Therefore, in general, we have the rule: Those couplings which involve more labels of the projection operator than are available at a nonsense point vanish there.

Since no coupling can exist at integral $j < 0$, all of our covariant residues functions will have zeros at negative integer j values, thus eliminating ghosts at right-signature points and giving dips in the differential cross section at wrong-signature points. The first nonsense point, in the scattering region, for the N_α and Δ_δ trajectories occurs at $\alpha = -\frac{1}{2}$ (see Fig. 2.3), which is a wrong-signature point for the former while a right-signature point for the latter.

As in Ref. (13), the relation between our covariant couplings g 's (or f 's) and the usual u -channel factorized couplings β 's⁽²³⁾, near the point $\alpha = -\frac{1}{2}$ is given by

$$\begin{aligned}\beta_s &\sim (g_s) + (g_n) \\ \beta_n &\sim \frac{(g_n)}{(\alpha + \frac{1}{2})^{\frac{1}{2}}},\end{aligned}\tag{2.74}$$

where (g_s) and (g_n) denote simple linear combinations of covariant sense and nonsense couplings respectively.

Table IV shows the different behaviour of g 's and the conventional β 's near the point $\alpha = -\frac{1}{2}$.

The dip, for example, in πN backward scattering is usually associated with the wrong-signature nonsense zero of N_α trajectory at $\alpha_N = -\frac{1}{2}$, corresponding to the "choosing-nonsense" mechanism. This is contrary to the forward case where choosing-sense mechanism is usually employed⁽⁹⁾. In the present case of Regge-pole dominance of the backward scattering amplitudes, the remaining mechanisms, except ^{the} Λ SFP-mechanism, which is irrelevant for pure hadronic processes, would result in higher order zeros in the N_α and Δ_δ exchange-amplitudes at the above point, which would clearly contradict the experimental observations. However, in the presence of cuts all the above mechanisms are possible.

It should be noted that in choosing the non-sense mechanism, the factorized covariant residue functions $g(\alpha)$ will have a branch point at $\alpha = -\frac{1}{2}$.

(b) Kinematic Cuts

As noted in Chapter I, this simple dip mechanism is not entirely satisfactory in explaining the observed dip structures. What is more embarrassing is that the parity partners are not observed at the expected positions. Carlitz and Kislinger⁽²⁴⁾ (CK) had suggested a way of solving these problems by introducing a kinematic cut in J with the branch point at α_0 , corresponding to a fermion trajectory $\alpha(u) = \alpha_0 + \alpha' u$. Since their model (for πN scattering) evolved out of the Feynman-pole Reggeization,

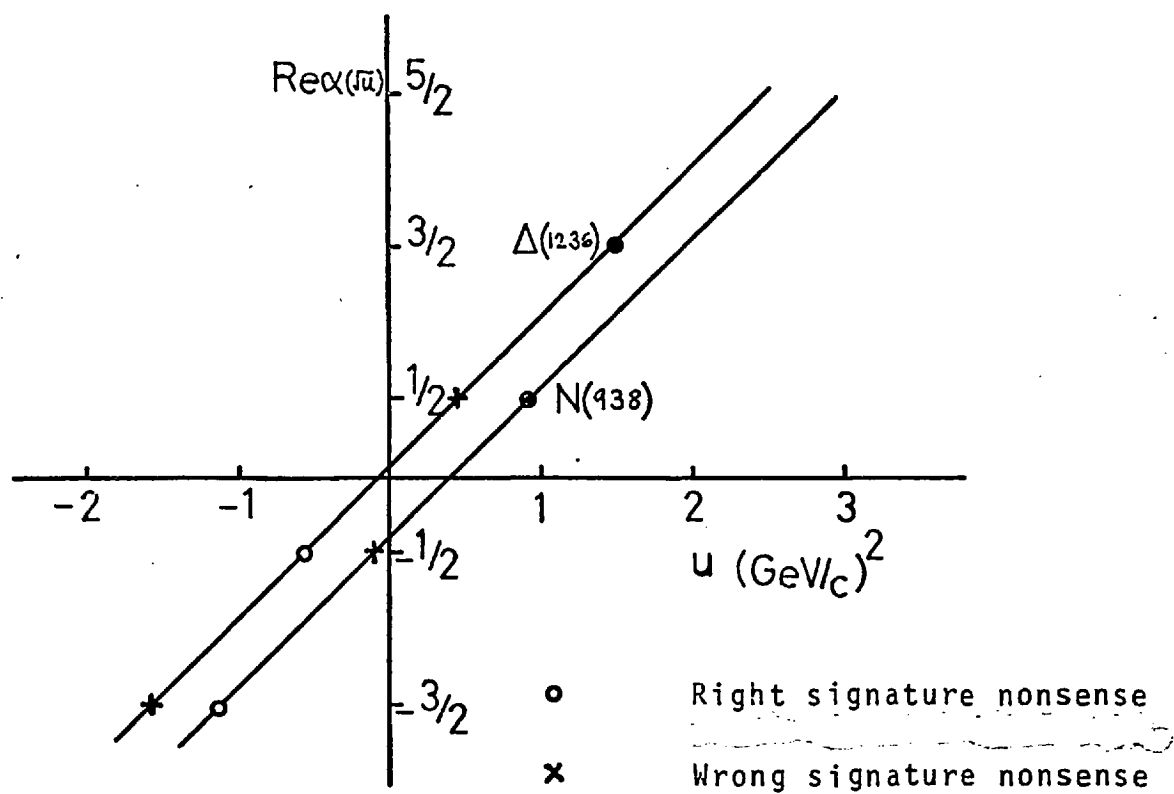
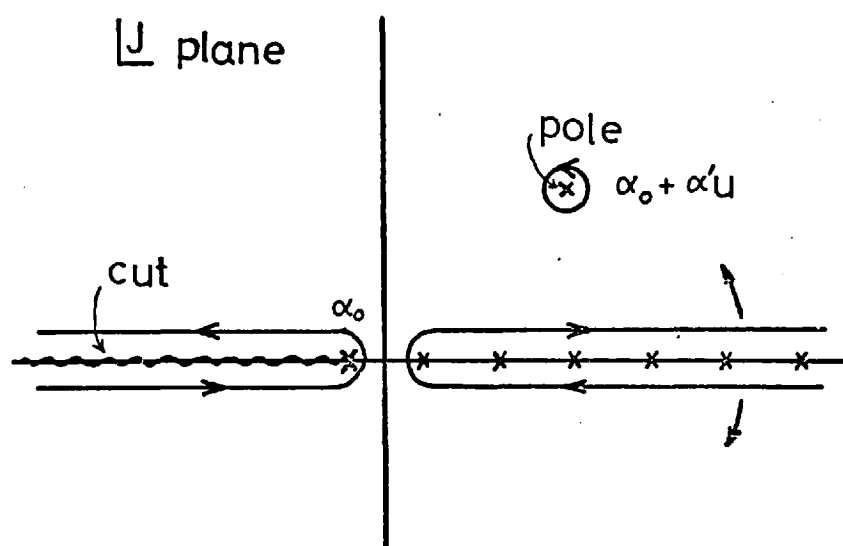
Fig. (2.3) N_α and Δ_δ trajectories.

Fig. (2.4)

our covariant formalism should serve as a natural way of extension to all other backward scattering processes.

In our formalism, the contribution to the M-function of the exchange of a fermion of spin- $J = j + \frac{1}{2}$, parity $(-)^{(J - \frac{1}{2})}$ and mass m_J is given by Eq. (2.34). The M-function can then be Reggeized by summing over the integer j and transforming the sum into a Sommerfeld-Watson integral (here, we use the normalization $\bar{u}u = 2$), i.e.

$$\begin{aligned}
 M &= \sum_J M^{(J)} \\
 &\sim \frac{i}{2} \int_C \frac{dJ}{\sin \pi (J - \frac{1}{2})} \cdot \frac{1}{u - m^2(J)} \cdot e'_\beta(J) \left(\frac{K}{m(J)} + 1 \right) e_\alpha(J) \\
 &\quad \cdot \frac{1}{(J - \frac{1}{2})^2} \cdot \frac{c_{J+\frac{1}{2}}}{J + \frac{1}{2}} \cdot \frac{\partial}{\partial \Lambda'_\beta} \frac{\partial}{\partial \Lambda_\alpha} \phi'_{J+\frac{1}{2}}(-\tau).
 \end{aligned} \tag{2.75}$$

Assuming that $m^2(J)$ is linear with J , i.e.

$$m^2(J) = \frac{(J - \alpha_0)}{\alpha'}, \tag{2.76}$$

and that $e'_\beta(J)$ and $e_\alpha(J)$ are meromorphic functions of $m(J)$, the contour integral can be opened up in the J -plane to obtain contributions from the pole at $m^2(J) = u$ and the cut with branch point at $J = \alpha_0$ (Fig. 2.4). When the helicity amplitudes are eventually calculated, only those terms proportional to Λ'_β and Λ_α will contribute to the leading order and we obtain

$$\begin{aligned}
M \sim & \frac{\pi \alpha'}{\sin \pi(\alpha - 1/2)} e'(\alpha(u)) \left(\frac{\kappa}{\sqrt{u}} + 1 \right) e(\alpha(u)) s^{\alpha(u) - 1/2} \\
& + \int_{-\infty}^{\alpha_0} \frac{dJ}{\sin \pi(J - 1/2)} \frac{s^{J - 1/2}}{u - m^2(J)} \text{Disc}_J \left[e'(J) \left(\frac{\kappa}{m(J)} + 1 \right) e(J) \right],
\end{aligned}
\tag{2.77}$$

where, for example in the $\pi N \rightarrow N\pi$ process, $e'(J) = e(J) = f(J) \gamma_5$.

The signature factor can easily be incorporated by the replacement $M(\tau) \rightarrow \frac{1}{2} |M(\tau) \pm M(-\tau)|$. However, for clarity, we will just use Eq. (2.77) for the subsequent discussions.

The contributions of the pole term to the SHA can be calculated as described in section 2.5. However, in this case the $\sqrt{u}$ singularities will be cancelled out by the corresponding cut terms, rather than by the Gribov-MacDowell partner, which in fact has moved through the cut onto the unphysical sheet of J -plane.

The main difference between our model and the previous (CK) cut models⁽²⁴⁻²⁷⁾ is that the residue function is now factorized into covariant couplings e' and e , each of which is assumed to be meromorphic in $m(J)$. If we make the simplest assumption that our covariant coupling functions g 's and f 's are analytic in J (i.e. in $m^2(J)$), leaving explicit factors of $m(J)$ to be obtained only from the kinematics, we will have a simple prescription for the presence or absence of fixed cuts in various parts of the SHA depending on whether or not odd powers of $m(J)$ are present in them.

Therefore for a particular SHA, each term of the expression $e'(J) \left(\frac{\kappa}{m(J)} + 1\right) e(J)$ can be written as

$$g'(J) g(J) [m(J)]^n \left(\frac{\kappa}{m(J)} + 1\right), \quad (2.78)$$

where $[m(J)]^n$ is the explicit kinematic factor of $m(J)$ and $g'(J)$ and $g(J)$ are both analytic in J with the usual non-sense zeros at $J = -\frac{1}{2}, -\frac{3}{2}, \dots$. This in fact corresponds to the choosing-sense mechanism of Table IV. This should be contrasted to the choosing-nonsense mechanism usually employed in either Regge pole or CK-cut models. Thus near $J = -\frac{1}{2}, -\frac{3}{2}, \dots$, $g'(J), g(J) \sim [\Gamma(J + \frac{1}{2})]^{-1} \sim (J + \frac{1}{2})$ for small negative u and we may write

$$\begin{aligned} \frac{\alpha' g'(J) g(J)}{\sin \pi (J - \frac{1}{2})} &\simeq g'_0 g_0 (J + \frac{1}{2}) \\ &\simeq g'_0 g_0 (\alpha_0 + \frac{1}{2} + \alpha' m^2(J)), \end{aligned} \quad (2.79)$$

where g'_0 and g_0 are constants.

Therefore the $g'(J) g(J)$ part of the SHA corresponding to (2.77) becomes

$$\begin{aligned} M_{(g'g)} &\sim \pi g'_0 g_0 (\alpha + \frac{1}{2}) (\sqrt{u})^n \left(\frac{\kappa}{\sqrt{u}} + 1\right) s^{\alpha - \frac{1}{2}} \\ &+ \frac{g'_0 g_0}{\alpha'} \int_{-\infty}^{\alpha_0} dJ \frac{[\alpha_0 + \frac{1}{2} + \alpha' m^2(J)]}{u - m^2(J)} s^{J - \frac{1}{2}} \text{Disc}_J \left\{ [m(J)]^n \left(\frac{\kappa}{m(J)} + 1\right) \right\}. \end{aligned} \quad (2.80)$$

With the explicit $[m(J)]^n$ factor, the cut integral

above can be performed using the formula⁽²⁸⁾

$$\int_{-\infty}^{\alpha_0} dJ s^{J-1/2} \frac{[-m^2(J)]^\nu}{u - m^2(J)} = s^{-1/2} (\alpha' u)^\nu \Gamma(\nu+1) \Gamma(-\nu, \alpha' u \ln s), \quad (2.81)$$

where $(\nu > -1)$,

$m(J)^2 = (J - \alpha_0)/\alpha'$, and $\Gamma(\nu, x) = \int_x^\infty e^{-t} t^{\nu-1} dt$, is an incomplete Gamma function⁽²⁸⁾.

Thus we obtain for

$$\begin{aligned} n = 0; \quad M_{(g'g)} &\sim \pi g'_0 g_0 (\alpha(u) + 1/2) \left(\frac{\kappa}{\sqrt{u}} + 1 \right) s^{\bar{\alpha}(u)} \\ &\quad - \pi g'_0 g_0 \kappa \left\{ (\alpha(u) + 1/2) \frac{1}{\sqrt{u}} [1 - \phi(\sqrt{\alpha' u \ln s})] s^{\bar{\alpha}(u)} \right. \\ &\quad \left. - \frac{\alpha' s^{\bar{\alpha}_0}}{\sqrt{\pi \alpha' \ln s}} \right\} \end{aligned} \quad (2.82)$$

and for

$$\begin{aligned} n = 1; \quad M_{(g'g)} &\sim \pi g'_0 g_0 (\alpha(u) + 1/2) (\kappa + \sqrt{u}) s^{\bar{\alpha}(u)} \\ &\quad + \pi g'_0 g_0 \left\{ (\alpha(u) + 1/2) \left[\frac{s^{\bar{\alpha}_0}}{\sqrt{\pi \alpha' \ln s}} - \sqrt{u} (1 - \phi(\sqrt{\alpha' u \ln s})) s^{\bar{\alpha}} \right] \right. \\ &\quad \left. - \frac{\alpha' s^{\bar{\alpha}_0}}{2\sqrt{\kappa} (\alpha' \ln s)^{3/2}} \right\}, \end{aligned} \quad (2.83)$$

where $\bar{\alpha} = \alpha - 1/2$ and $\phi(y) \equiv \frac{2}{\sqrt{\pi}} \int_0^y e^{-t^2} dt$ is an Error-function.

The first and the second terms in (2.82) and (2.83) are, respectively, the pole and the cut contributions, in which the $\sqrt{u}$ singularities cancel explicitly. The remaining expressions are thus free of $\sqrt{u}$ singularities since the Error function $\phi(y)$ is odd in y .

$\pi N \rightarrow N\pi$:

In the case of πN backward scattering, we have $e'(J) = e(J) = f(J)\gamma_5$ which corresponds to ^{the} $\alpha_n = 0$ case above and we essentially get back the original CK-model expressions⁽²⁴⁾, but with an explicit $(\alpha(u) + \frac{1}{2})$ factor which vanishes at $\alpha(u) = -\frac{1}{2}$; the wrong-signature nonsense point for the N_α trajectory and the right-signature-nonsense point for the Δ_δ trajectory. With the trajectories $\alpha_N \approx -0.4 + 0.9u$ and $\alpha_\Delta \approx 0.0 + 0.9u$, these points will occur at $u \approx -0.2$ for N_α and $u \approx -0.5$ for Δ_δ respectively. In the previous application of the CK-model to this reaction by Halzen et al.⁽²⁵⁾ and by Berger and Fox⁽²⁶⁾, such a factor was not present originally in their amplitude, since the usual choosing-nonsense residue was employed. However, by parameterizing their residue as $(a + bu)$, with a & b being free parameters, the authors obtained from their fits to the experimental data the values for a and b for which the residue $(a + bu)$ vanishes at the above points. In particular, Halzen et al. noted that the zeros occur in both the N_α and Δ_δ residues at $u \approx -0.2$ and $u \approx -0.5$, respectively, in complete agreement with our version of the model. The automatic occurrence of the factor $(\alpha(u) + \frac{1}{2})$ in our residue is the direct consequence of our covariant method; the factorization of residues into covariant coupling functions, which have simple analytic property in J .

With a proper signature factor, such a factor above will produce a double zero in the pole term but a single zero in part of the cut term (the term containing $(\alpha + \frac{1}{2})$)

factor in (2.82)) and no zero for the remaining constant cut term (the last term of the cut term), at $u \approx -0.2$ for the N_α amplitude.

For the Δ_0 amplitude, we will have a single zero in both the pole and the first part of the cut term at $u \approx -0.5$, which, however, will not show up as a dip in the differential cross section, for the constant cut term will dominate over the much reduced (varying) pole and cut terms at that point.

It should be noted from (2.82) that we will have no cut in the non-flip ($\Delta\lambda = 0$) amplitude while a fixed cut will be present in the flip ($\Delta\lambda = 1$) amplitude for $\pi N \rightarrow N\pi$ reaction. All the previous conclusions^(25,26) about the detailed comparison with experiments should also be true in our case. The main problem with this kind of model for a pure hadronic reaction seems to come from the fact that the cut discontinuity diverges at the branch point (for $n = 0$), in contradiction to the requirement of u -channel unitarity⁽²⁹⁾.

$\pi N \rightarrow N\rho$:

In this case, the spin-J contribution to the non-flip ($\Delta\lambda = 0$) SHA are

$$T_{\frac{1}{2}0;0-\frac{1}{2}}^{(J)} \sim \frac{f(J)}{\mu'} \left[g_1(J) + g_2(J)(m-m(J)) + g_3(J) \frac{1}{4}(m^2 - \mu'^2 - u) \right]$$

and

$$\times \bar{u}^{(\frac{1}{2})} \gamma_5 \left(\frac{k}{m(J)} + 1 \right) u^{(-\frac{1}{2})}$$

$$T_{\frac{1}{2}1;0\frac{1}{2}}^{(J)} \sim \sqrt{2} f(J) g_2(J) \bar{u}^{(\frac{1}{2})} \gamma_5 \left(\frac{k}{m(J)} + 1 \right) u^{(-\frac{1}{2})}$$

$$+ f(J) g_3(J) \frac{\sqrt{-u}}{2\sqrt{2}} \bar{u}^{(\frac{1}{2})} \gamma_5 \left(\frac{k}{m(J)} + 1 \right) u^{(\frac{1}{2})} \quad (2.84)$$

since we have

$$\bar{u}^{(1/2)} \gamma_5 \left(\frac{K}{m(J)} + 1 \right) u^{(\pm 1/2)} \sim \begin{cases} -i \frac{\sqrt{-u}}{m(J)} s^{1/2} \\ i s^{1/2} \end{cases},$$

explicit $m(J)$ factors corresponding to $n = 0, 1$, will be present in both the nonflip amplitudes.

$\gamma N \rightarrow N\pi$:

Similarly for the $\gamma N \rightarrow N\pi$ process, the only $\Delta\lambda = 0$ amplitude can be written as

$$T_{\frac{1}{2}, \rho; 1, 1/2}^{(J)} \sim -\sqrt{2} \left\{ \frac{u}{m(J)} \tilde{g}_1(J) - [(m(J) + m) \tilde{g}_1(J) + \tilde{g}_2(J)] \right\} (is^J). \quad (2.85)$$

Therefore in both processes, $\pi N \rightarrow N\rho$ and $\gamma N \rightarrow N\pi$, we will have fixed cuts in all the nonflip ($\Delta\lambda = 0$) SHA. It is important to note that the $m(J)$ factor in the expression (2.84) and (2.85) appears as $\frac{\sqrt{-u}}{m(J)}$, $\frac{u}{m(J)}$ or $m(J)$ only, in which $\frac{\sqrt{-u}}{m(J)} \rightarrow \text{constant}$ and $\frac{u}{m(J)}$, $m(J) \rightarrow 0$ like $\sqrt{u}$ as $u \rightarrow 0$. Hence the presence of such factors in the nonflip amplitude could only result in the reduction of the backward peak, and at the same time filling up the dip with the constant cut contributions $s^{\bar{\alpha}_0} / \sqrt{\pi \alpha' l m s}$ and $s^{\bar{\alpha}_0} / 2\sqrt{\pi} (\alpha' l m s)^{3/2}$. Therefore, any form of fixed cuts present in the non-flip SHA will easily destroy the dip structure in the differential cross section. Proceeding in the same way we find that some contributions of kinematic cuts are always present in the non-flip amplitudes for all high-spin backward scattering processes.

In Table V, we list various backward scattering processes where N_α and Δ_δ exchange are assumed to dominate. Dips in differential cross sections at $u \approx -0.2$ are predicted only when there are no kinematic cuts in non-flip ($\Delta\lambda = 0$) amplitudes, i.e. absence of $m(J)$ factors in them. These predictions agree with experiments for all the reactions listed⁽³⁰⁾.

2.7 Conclusions

In this chapter we showed how the techniques of covariant Reggeization may be extended to backward scattering in a consistent way. The kinematic singularities arising from the Reggeon mass term are removed by Gribov parity doubling of fermion trajectories. This requires the individual covariant coupling functions to satisfy the Gribov-MacDowell symmetry relation.

Assuming the dominance of a Gribov parity doublet, and exploiting the specific nature of high energy kinematics in the s-channel, we showed that the SHA for backward scattering processes can be written in terms of factorized helicity vertices. Calculations of helicity vertices for the essential vertices of Table I are performed and the results are displayed in Table II for hadronic and in Table III for photonic vertices. It is therefore possible to construct many backward amplitudes from these few results. Dips in the differential cross-sections are due to the rather natural zeros of the covariant coupling function, which in fact correspond to

the usual wrong-signature-nonsense zeros. The usual dip mechanisms in terms of the covariant residues are displayed in Table IV.

Other methods of dealing with the parity doubling problem may also be employed in the covariant formalism in a straight forward manner. We show that the Carlitz-Kislinger-cut model can be extended in a natural way to any backward scattering process, where the presence or absence of dips is associated with the absence or presence of genuine kinematic $m(J)$ factors in the non-flip SHA. Previous results on the parameters of the model are naturally reproduced.

The results obtained here, or at least the covariant methods we have employed in reaching these results, show how the formalism can be a practical tool for high energy calculations such as absorption or eikonal type calculations for Regge cuts⁽⁶⁾, and various other bootstrap type calculations. One can also generalize the covariant formalism to theories with higher symmetry⁽¹¹⁾, a version of which is considered in the next chapter.

Table I - Covariant Fermion Couplings

$e^+(0\frac{1}{2}j+\frac{1}{2})$	g
$e_{\alpha\nu}^+(1\frac{1}{2}j+\frac{1}{2})$	$g_1 g_{\alpha\nu} + g_2 \Lambda_\alpha \gamma_\nu + g_3 \Lambda_\alpha \Lambda_\nu$
$e_{\alpha\mu}^+(0\frac{3}{2}j+\frac{1}{2})$	$g_1 g_{\alpha\mu} + g_2 \Lambda_\alpha \Lambda_\mu$
$e_{\alpha_1\alpha_2\mu\nu}^+(1\frac{3}{2}j+\frac{1}{2})$	$g_1 g_{\alpha_1\mu} g_{\alpha_2\nu} + g_2 g_{\alpha_1\mu} \gamma_\nu \Lambda_{\alpha_2} + g_3 g_{\alpha_1\mu} \Lambda_{\alpha_2} \Lambda_\nu$ $+ g_4 g_{\mu\nu} \Lambda_{\alpha_1} \Lambda_{\alpha_2} + g_5 \gamma_\nu \Lambda_{\alpha_1} \Lambda_{\alpha_2} \Lambda_\mu + g_6 \Lambda_{\alpha_1} \Lambda_{\alpha_2} \Lambda_\mu \Lambda_\nu$
$\tilde{e}_{\nu\alpha}^+(\gamma\frac{1}{2}j+\frac{1}{2})$	$\tilde{g}_1 (\sqrt{u}+m) \gamma'_\nu \Lambda_\alpha + \tilde{g}_2 [(\sqrt{u}-m) g'_{\nu\alpha} + \gamma'_\nu \Lambda_\alpha]$
$\tilde{e}_{\alpha_1\alpha_2\mu\nu}^+(\gamma\frac{3}{2}j+\frac{1}{2})$	$\tilde{g}_1 [(\sqrt{u}-m) g_{\alpha_1\mu} g'_{\alpha_2\nu} + g_{\alpha_1\mu} \gamma'_\nu \Lambda_{\alpha_2}]$ $+ \tilde{g}_2 (\sqrt{u}+m) g_{\alpha_1\mu} \gamma'_\nu \Lambda_{\alpha_2}$ $+ \tilde{g}_3 [2 \gamma'_\nu \Lambda_{\alpha_1} \Lambda_{\alpha_2} \Lambda_\mu + (\sqrt{u}-m) g'_{\mu\nu} \Lambda_{\alpha_1} \Lambda_{\alpha_2}]$ $+ \tilde{g}_4 (\sqrt{u}+m) \gamma'_\nu \Lambda_{\alpha_1} \Lambda_{\alpha_2} \Lambda_\mu$
Abnormal coupling	$g \rightarrow \gamma_5 f$, $\sqrt{u} \rightarrow -\sqrt{u}$ (for photon couplings)

Table II - Helicity Vertices

$V_{0\frac{1}{2}}^+ (0\frac{1}{2})$	ig
$V_{0\frac{1}{2}}^+ (0\frac{1}{2})$	ig
$V_{\frac{1}{2}1}^+ (\frac{1}{2}1)$	$i(-i\sqrt{2}g_2 + \frac{\sqrt{-u}}{2\sqrt{2}}g_3)$
$V_{\frac{1}{2}0}^+ (\frac{1}{2}1)$	$\frac{i}{\mu'} [g_1 + (m - i\sqrt{-u})g_2 + \frac{1}{4}(m^2 - \mu'^2 - u)g_3]$
$V_{\frac{1}{2}-1}^+ (\frac{1}{2}1)$	$\frac{-ig_3}{2\sqrt{2}}$
$V_{0\frac{3}{2}}^+ (0\frac{3}{2})$	$\frac{ig_2}{2\sqrt{2}}$
$V_{0\frac{1}{2}}^+ (0\frac{3}{2})$	$\frac{i}{2\sqrt{6}} \left\{ i\sqrt{-u}g_2 + \frac{1}{\mu'} [4g_1 + (\mu^2 - m'^2 + u)g_2] \right\}$
Abnormal	$i \rightarrow -i$
vertices	$g(\sqrt{u}) \rightarrow g(-\sqrt{u})$ (Complex conjugation)

Table III - Helicity Vertices (Photons)

$V_{1/2, 1}^+ (1/2 r)$	$\sqrt{2} (m \tilde{g}_1 + \tilde{g}_2)$
$V_{1/2, -1}^+ (1/2 r)$	$-\sqrt{2} \tilde{g}_1$
$V_{1/2, 1}^{' +} (r 1/2)$	$-\sqrt{2} (m \tilde{g}_1 + \tilde{g}_2)$
$V_{-1/2, 1}^{' +} (r 1/2)$	$-i\sqrt{2} \tilde{g}_1$
$V_{1/2, 3/2}^{' +} (r 3/2)$	$i m' (\tilde{g}_3 - i \frac{\sqrt{-u}}{2} \tilde{g}_4)$
$V_{-1/2, 3/2}^{' +} (r 3/2)$	$\frac{i}{2} \tilde{g}_4$
$V_{1/2, 1}^{' +} (r 3/2)$	$-\frac{2}{\sqrt{3}} \left[\frac{1}{m'} \tilde{g}_1 + \tilde{g}_2 + \frac{1}{m'} (u - i m' \sqrt{-u}) \tilde{g}_3 - \frac{m'^2}{4} \tilde{g}_4 \right]$
$V_{-1/2, 1}^{' +} (r 3/2)$	$\frac{i}{\sqrt{3} m'} \left[-2 \tilde{g}_2 + i \sqrt{-u} \tilde{g}_3 + \frac{1}{2 m'} (2 m'^2 - u) \tilde{g}_4 \right]$
Abnormal	$i \rightarrow -i$
vertices	$g(\sqrt{u}) \rightarrow g(-\sqrt{u})$ (Complex conjugation)

Parity: $V_{-\lambda_1, -\lambda_4}^{(\pm)} = \pm (-)^{\lambda_1 + \lambda_4} V_{\lambda_1, \lambda_4}^{(\pm)}$,

$V_{-\lambda_2, -\lambda_4}^{(\pm)} = \mp (-)^{\lambda_2 + \lambda_3} V_{\lambda_2, \lambda_3}^{(\pm)}$.

Table IV - Dip Mechanism at $\alpha = -\frac{1}{2}$

	Choosing sense	Choosing nonsense	NC	Chew	SFP
β_s	1	$\sqrt{\alpha + \frac{1}{2}}$	$\alpha + \frac{1}{2}$	$\sqrt{\alpha + \frac{1}{2}}$	1
β_n	$\sqrt{\alpha + \frac{1}{2}}$	1	$\sqrt{\alpha + \frac{1}{2}}$	$\alpha + \frac{1}{2}$	$\frac{1}{\sqrt{\alpha + \frac{1}{2}}}$
g_s	1	$\sqrt{\alpha + \frac{1}{2}}$	$\alpha + \frac{1}{2}$	$\sqrt{\alpha + \frac{1}{2}}$	1
g_n	$\alpha + \frac{1}{2}$	$\sqrt{\alpha + \frac{1}{2}}$	$\alpha + \frac{1}{2}$	$(\alpha + \frac{1}{2})^{\frac{3}{2}}$	1

GM, NC and SFP stand for Gell-Mann, non-compensating and strong fixed-pole mechanisms respectively.

Table V - Dips or no dips at $u \approx -0.2$

Process	Dominant exchange	m(J) factor absent in $\Delta\lambda=0$ amplitude?	Dips?
$\pi^+p \rightarrow p\pi^+$	N_α	✓	✓
$\pi^-p \rightarrow p\pi^-$	Δ_8	✓	$\times^\dagger$
$\pi^-p \rightarrow n\pi^0$	N_α	✓	✓
$\pi^+p \rightarrow p\rho^+$	N_α	$\times$	$\times$
$\pi^-p \rightarrow n\rho^0$	N_α	$\times$	$\times$
$\pi^-p \rightarrow p\rho^-$	Δ_8	$\times$	$\times^\dagger$
$p\rho \rightarrow d\rho^+$	N_α	$\times$	$\times$
$\bar{K}n \rightarrow \Lambda\pi^-$	N_α	✓	✓
$\bar{K}n \rightarrow \Sigma^0\pi^-$	N_α	✓	✓
$\bar{K}n \rightarrow \Sigma^-\pi^0$	N_α	✓	✓
$\bar{K}p \rightarrow \Sigma^+\pi^-$	Δ_8	✓	$\times^\dagger$
$\gamma p \rightarrow p\pi^0$	N_α	$\times$	$\times$
$\gamma p \rightarrow n\pi^+$	N_α	$\times$	$\times$
$\gamma p \rightarrow p\rho$	N_α	$\times$	$\times$
$\gamma d \rightarrow pn$	N_α	$\times$	$\times$
$\gamma p \rightarrow \Delta^{++}\pi^-$	Δ_8	✓	$\times^\dagger$

† Not relevant at $u \approx -0.2$.

✓ Yes

 $\times$ No

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CHAPTER III

BARYON RESIDUES AND SUPERMULTIPLETS3.1 Introduction

The Regge theory and the supermultiplet theory have been the two most popular theories of the last decade and have in their own way, enjoyed many successes. The most desirable property of the Regge theory is the existence of Regge trajectories which correlate the high energy scattering amplitudes with the low energy resonances, while the supermultiplet theory organizes and correlates the participating hadrons, of which most are resonances. In this sense, the two theories complement one another.

It is becoming increasingly evident that the $SU(6)$ -symmetric-quark-model⁽¹⁻⁵⁾ might in fact be an approximately valid description of the nature of hadrons in general. Feynman et al.⁽¹⁾ have given a consistent description of the data on high energy resonance decays in their relativistic harmonic-oscillator-quark model. However, the problem of the extension of their model to high-energy scattering remains unsolved. From a pure symmetry point of view, Delbourgo, Salam and Strathee, and Arnold, and Freund⁽⁶⁾ have developed a number of Reggeized supermultiplet models with varying resulting hadronic spectra. Although their applications to high-energy scattering have been rather successful, in the sense of having essentially no parameters, one has to introduce parity doubling of quarks in order to eliminate $\sqrt{t}$ or $\sqrt{u}$ kinematic singularities⁽⁷⁾, or to work

only with the on-mass-shell couplings⁽⁸⁾. Also, since the exact symmetries like $U(6) \times U(6)$ or $SU(6)_W$ are at best only approximate symmetries in reality, one has some arbitrariness in the way of breaking them.

On the other hand high energy theoretical and phenomenological studies on two-body scattering processes have indicated that the dominance of simply parametrized Regge poles alone is not sufficient to explain the observed behaviours like cross-over zeros and the irregular appearance of dips in the differential cross sections at $t \simeq -0.6$ and $u \simeq -0.2$, etc^(9,10). Although the introduction of Regge cuts might solve these problems, the essential conclusion is the need for a more complicated parametrization of the residue functions. Additional evidence for such a behaviour comes from the drastic failure of the conventionally parametrized Δ_8 -residue to extrapolate correctly from the scattering region to the $\Delta(1236)$ pole⁽¹¹⁾. From the s-channel point of view, all the above observations can be simply (but qualitatively) explained in the so called 'geometrical' or 'dual absorptive' models⁽¹²⁾, where the cross-over and the dips at $t \simeq -0.2$ and $u \simeq -0.2$, $t = -0.6$, respectively are associated with the Bessel-function-like 'residues'. Therefore it seems to suggest that a correct Regge model should possess polynomial residue functions which guarantee the required polynomial character of their flip-nonflip ratio.

Recently^(2,4) it was noticed that the utilization of the explicit quark spin structure in obtaining the desired hadron spectrum generates, as a result, polynomial Regge

residues possessing many interesting features. Although such features may be produced from a more general case, Delbourgo and Rotelli⁽²⁾ showed explicitly in a recent version of the Reggeized supermultiplet model that the polynomial residues extrapolate very well from the hadron pole to the scattering region (i.e. for Δ_s) and also provide cross-over points for forward scattering at $t \simeq -0.2$. In fact, the residues for the SHA have the same zero structure as those of the corresponding Bessel functions $J_{\Delta\lambda}(r\sqrt{-t})$ ($r \simeq 1$ fermi, $\Delta\lambda =$ total SH-flip).

In this chapter, we shall present the results of the explicit calculations for meson-baryon backward scattering in the above model, giving particular attention to the zero structure of the polynomial baryon-residues⁽¹³⁾. By considering the $I_U = \frac{3}{2}$ and $I_U = \frac{1}{2}$ channels of πN scattering, the residues for the exchange degenerate trajectories (Δ_s, N_s) and (Δ', N') will be studied in detail in connection with their extrapolation and their implications for the differential cross section and polarization.

In section 3.2 a brief description of the covariant Reggeization procedures for the supermultiplet (rest) symmetry group $[U(6) \times U(6)] \times SU(3)$ is given. The techniques of the M-function formalism for writing amplitudes using multispinors are explained. Section 3.3 contains the model for meson-baryon backward scattering, where specific assumptions about the method of Reggeization and the particle spectrum are made. The model is then applied to $\pi N \rightarrow \pi N$

scattering and the results are given. In section 3.4 some detailed studies of the residue functions are made in connection with their particle pole-scattering region extrapolation and their zero structures. Due to a large amount of symmetry breaking involved in the N_α -residue, we are only able to discuss the Δ_s -amplitudes in backward π^-p scattering. Comparison with experimental cross sections at different energies and the polarization at $p_{lab} = 6 \text{ GeV}/c$ are made. We conclude the chapter in section 3.5.

3.2 Supermultiplets and Reggeization

Assuming that the fundamental entity for strong interactions is a 12-component (Dirac) quark of the group $\tilde{U}(12)$, the baryons and mesons can be viewed as being constructed from three quarks and quark-antiquark states. We also assume that all mesons belong to a mass-degenerate $(6, \bar{6}; L)$ representation and all baryons to $(56, \bar{1}; L)$, $(70, \bar{1}; L)$ and $(20, \bar{1}; L)$ representations of the rest group $[U(6) \times U(6)]_{I,S} \times O_L(3)$. (I , S and L denote the unitary spin, intrinsic spin and orbital momentum, respectively, and $J = L + S$). Empirically it is found that all known baryons fall into the $(56, \bar{1}; L = \text{even integer})$ and the $(70, \bar{1}; L = \text{odd integer})$ representations (states belonging to 20 have not yet been identified⁽¹⁴⁾). This can be readily explained in the symmetric quark model, where extra orbital degrees of freedom are associated with a harmonic-oscillator-like potential^(1,7), thereby enlarging $O_L(3)$ to the three-dimensional harmonic oscillator group $SU_N(3)$. Reggeization is then

associated with the 'radial' quantum number N ($L = N, N - 2, \dots, 0$ or 1) of the group, and hence the name 'Reggeized Oscillator model'.

(a) Wave Functions

The wave functions of particle multiplets are defined as follows:

For the orbital part, $SU_N(3)$ multiplets belonging to the representation (N) , $N = 0, 1, 2, \dots$, are represented by the symmetric but not traceless tensor fields $\phi(p)$,

$\phi_\mu(p)$, $\phi_{\mu_1\mu_2}(p)$, $\dots$, satisfying

$$p_{\mu_1} \phi_{(\mu_1\mu_2\dots\mu_N)} = 0,$$

$$p^2 \phi_{(\mu_1\mu_2\dots\mu_N)} = m_N^2 \phi_{(\mu_1\mu_2\dots\mu_N)}. \quad (3.1)$$

For the unitary and intrinsic spin part, the multiplets are represented by

$$\Phi_B^A(p), \quad A, B = 1, \dots, 12; \text{ i.e. } A \equiv a, \alpha \quad \begin{cases} a = 1, \dots, 3 \\ \alpha = 1, \dots, 4 \end{cases}$$

for mesons $(6, \bar{6})$ belonging to the rest symmetry $U(6) \times U(6)$ of $\tilde{U}(12)$, and

$\Psi_{(ABC)}(p)$, $A, B, C = 1, \dots, 12$ (symmetric in A, B, C),
for Baryons $(56, \bar{1})$ and

$$\Psi_{[AB]C} \quad (\text{antisymmetric in } A, B)$$

and satisfying

$$\Psi_{[AB]C} + \Psi_{[BC]A} + \Psi_{[CA]B} = 0,$$

for Baryons $(70, \bar{1})$ belonging to $U(6) \times U(6)$ of $\tilde{U}(12)$.

They satisfy the Bargmann-Wigner equations⁽¹⁵⁾

$$(\not{p}-\mu)_{\mathcal{D}}^A \bar{\Phi}_A^B = \bar{\Phi}_A^B (\not{p}+\mu)_{\mathcal{B}}^D = 0,$$

$$(\not{p}-m)_{\mathcal{D}}^A \bar{\Psi}_{ABC} = (\not{p}-m)_{\mathcal{D}}^A \bar{\Psi}_{[AB]C} = 0, \text{ etc.}$$

(3.2)

where μ and m are the supermultiplet meson and baryon masses, respectively.

Altogether, the total wave functions are given by

$$\bar{\Phi}_B^A(\mu_1 \dots \mu_N) \quad \text{for } (6, \bar{6}; N),$$

$$\bar{\Psi}_{(ABC)}(\mu_1 \dots \mu_N) \quad \text{for } (56, \bar{1}; N),$$

$$\bar{\Psi}_{[AB]C}(\mu_1 \dots \mu_N) \quad \text{for } (70, \bar{1}; N)$$

satisfying Eq. (2.1) and (2.2).

The explicit decomposition gives⁽¹⁵⁾ $((\mu_N) \equiv (\mu_1 \dots \mu_N))$

$$\bar{\Phi}_{A(\mu_N)}^B(\not{p}) = \frac{1}{2\mu} [(\not{p}+\mu)\gamma_5]_{\alpha}^{\beta} P_{a(\mu_N)}^b - \frac{1}{2\mu} [(\not{p}+\mu)\gamma_{\mu}]_{\alpha}^{\beta} V_{\mu a(\mu_N)}^b \quad (3.3)$$

$$\begin{aligned} \bar{\Psi}_{(ABC)(\mu_N)}(\not{p}) = & \frac{1}{2\sqrt{6}m} \left\{ [(\not{p}+m)\gamma_5 C]_{\alpha\beta} \epsilon_{abd} N_{cr(\mu_N)}^d \right. \\ & \left. + \text{cyclic permutation of } (a\alpha, b\beta, cr) \right\} \\ & + \frac{\sqrt{3}}{2\sqrt{2}m} [(\not{p}+m)\gamma_{\mu} C]_{\alpha\beta} D_{\mu(abc)r(\mu_N)} \end{aligned} \quad (3.4)$$

where D_{μ} satisfies, in addition to the Dirac equation,

$(\gamma_\mu)_\alpha^\beta D_{\mu(abc)}_\beta = 0$, & the matrix operator C is defined by

$$C^\top = C^\dagger = C^{-1}, \quad C^{-1} \gamma_\mu C = -(\gamma_\mu)^\top \quad \text{and} \quad \bar{C} = C.$$

In the above equations, the orbital excitation of the basic quark spin fields $P(0^-)$, $V(1^-)$, $N(\frac{1}{2}^+)$ and $D(\frac{3}{2}^+)$.. are explicitly shown. Further decomposition of the (N) excitation can be done, and by combining with the intrinsic spin component the usual total spin- J components are obtained.

(b) Vertex Functions

In order to conserve the generalized helicity⁽⁷⁾, we assume that the vertices are $U_W(6) \times U(2)$ invariant. They are easily constructed by forming index invariants between fields and momenta; $p_B^A \equiv (\gamma)_\alpha^\beta \delta_a^b$ and p_μ . The number of different ways of doing this gives different types of couplings.

For example, the quark-antiquark coupling to the meson $(6, \bar{6}; N)$ multiplet is given by

$$\begin{aligned} \mathcal{L}_{\text{eff.}} = & \bar{v}^A(\frac{1}{2}K+Q) u_B(-\frac{1}{2}K+Q) [G_0 \delta_A^B Q_\mu + G_1 \delta_A^B \partial_\mu] \\ & \times Q_{\mu_1} \dots Q_{\mu_N} \bar{\Phi}_{C(\mu_1 \dots \mu_N)}^B(K), \end{aligned}$$

which can be rewritten as

$$\mathcal{L}_{\text{eff.}} = \bar{v}^A(\frac{1}{2}K+Q) u_B(-\frac{1}{2}K+Q) [g_0 \delta_A^B + m g_1 \frac{\partial}{\partial Q_B^A}] \bar{\Phi}_{(N)}(K, Q),$$

where $\bar{\Phi}_{(N)} \equiv \frac{1}{\mu^{N+1}} Q_B^A Q_{\mu_1} \dots Q_{\mu_N} \bar{\Phi}_{A(\mu_1 \dots \mu_N)}^B(K)$

Diagrammatically

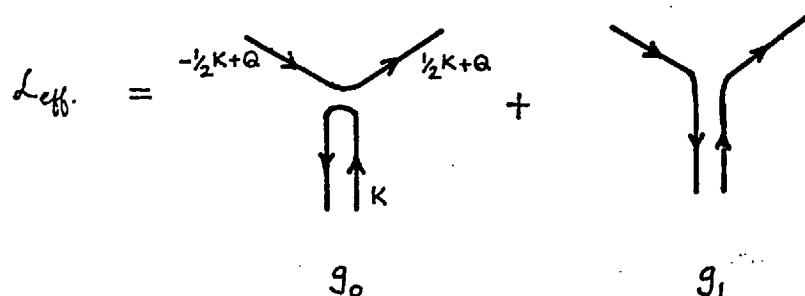


Fig. (3.1)

For our purpose the $(6, \bar{6}; 0) - (\bar{5}\bar{6}, 1; 0) - (56, 1; N)$ coupling is given by

$$\mathcal{L}_{\text{eff}} = g \bar{\Psi}^{(ABD)}(\frac{1}{2}K+Q) \Phi_D^C(-\frac{1}{2}K+Q) \delta_A^E \delta_B^F \delta_C^G \Psi_{EFG(N)}(K) \quad (3.5)$$

where $\bar{\Psi}_{EFG(N)} \equiv \frac{1}{\bar{m}^N} Q_{\mu_1} \dots Q_{\mu_N} \bar{\Psi}_{EFG}(\mu_1 \dots \mu_N)$, with $\bar{m} \equiv \frac{1}{2}(m + \mu)$, the mean mass at the meson-baryon vertex.

Diagrammatically,

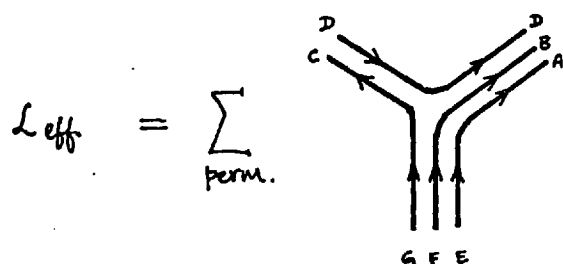


Fig. (3.2).

Similarly, the $(6, \bar{6}; 0) - (7\bar{0}, 1; 1) - (56, \bar{1}; N)$ coupling is

$$\mathcal{L}_{\text{eff}} = \frac{1}{\bar{m}^N} \bar{\Psi}_{\mu}^{[AB]D}(\frac{1}{2}K+Q) \Phi_D^C(-\frac{1}{2}K+Q) \delta_A^E \delta_B^F \delta_C^G (g_0 Q_{\mu} + m g_1 \frac{\partial}{\partial Q_{\mu}}) \times \Psi_{EFG(N)}(K, Q) \quad (3.6)$$

etc.

(c) Propagators

In chapter II, we have noted that the conventional partial wave expansion of the invariant amplitudes can be obtained in an equivalent way by considering the contribution of a spin-J particle exchange to the M-function, thus providing one with the Legendre functions $P^J(\cos \theta)$ for immediate Reggeization. In an exactly analogous way, the generalized Legendre functions are obtained by exchanging a supermultiplet of quantum number N, using the covariant couplings like (3.5) or (3.6). In the present model, the meson $(6, \bar{6}, N)$ and the baryon $(56, \bar{1}; N)$ propagators are respectively given by⁽⁷⁾

$$\begin{aligned} \langle \Phi_{A(\mu_N)}^B(K), \Phi_{B'(\mu_N)}^{A'}(-K) \rangle = & \frac{1}{4\mu^2(K^2 - \mu^2)} (K + \mu)_A^{A'} (K + \mu)_{B'}^B \\ & \times \sum_{\mu_i} \prod_{i=1}^N (-g_{\mu_i \mu_i'}(K)) \end{aligned} \quad (3.7)$$

and

$$\begin{aligned} \langle \Psi_{(ABC)(\mu_N)}(K), \Psi_{(\mu_N)}^{(A'B'C')}(K) \rangle = & \frac{1}{48m^3(K^2 - m^2)} \\ & \times \sum_{\substack{\mu_i \in \mu_N \\ ABC}} (K + m)_A^{A'} (K + m)_B^{B'} (K + m)_C^{C'} \\ & \times \sum_{\mu_i} \prod_{i=1}^N (-g_{\mu_i \mu_i'}(K)) \end{aligned} \quad (3.8)$$

where $g_{\mu\nu}(K) \equiv g_{\mu\nu} - \frac{K_\mu K_\nu}{m^2}$.

The basic rotation functions are now obtained by contracting (3.7) or (3.8) with momenta. For our purpose,

the totally contracted baryon (56) propagator is

$$\begin{aligned}
 \langle \Psi_{(ABC)(N)}(K, Q), \Psi_{(N)}^{(A'B'C')}(K, Q) \rangle &= \frac{1}{48 m^3 (K^2 - m^2)} \\
 &\times \sum_{\substack{\text{perm.} \\ ABC}} (K+m)_A^{A'} (K+m)_B^{B'} (K+m)_C^{C'} \\
 &\times \left[-\frac{Q'(K) \cdot Q(K)}{m^2} \right]^N, \quad (3.9)
 \end{aligned}$$

where $Q_\mu(K) = Q_\mu - \frac{Q \cdot K}{m^2} K_\mu$, (kinematics are defined in Fig. (3.3)) and $-Q'(K) \cdot Q(K) \equiv |Q'| |Q| \cos \vartheta_u$.

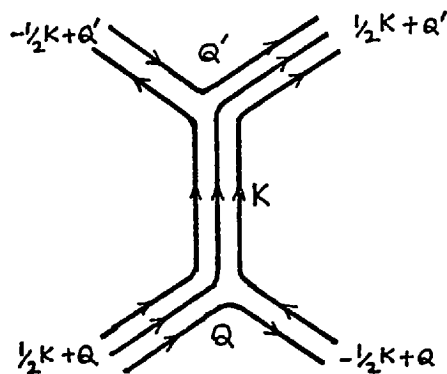


Fig. (3.3)

The expression (3.9) is sufficient for couplings like (3.5). For other couplings with a free orbital label ($\mu \dots$), differential operators $\frac{\partial}{\partial Q_\mu} \dots$ are applied to (3.9) to 'free' the required number of labels. The M-function is then written as

$$M^{(N)} = C'(Q') : \langle \Psi_{(ABC)(N)}(K, Q), \Psi_{(N)}^{(A'B'C')}(K, Q) \rangle : C(Q), \quad (3.10)$$

which is Reggeized through the replacement:

$$\frac{1}{k^2 - m^2} \left(\frac{-Q'(k) \cdot Q(k)}{m^2} \right)^N \longrightarrow \frac{\pi \alpha'_\psi \xi_\psi}{\sin \pi (\alpha_\psi - 3/2)} \left(\frac{s}{2m^2} \right)^{\alpha_\psi - 3/2}$$

$$N \longrightarrow \alpha_\psi - 3/2$$

$$m \longrightarrow \sqrt{k^2} \quad (3.11)$$

where $\xi_\psi \equiv \frac{1}{2} [1 + \exp(-i\pi(\alpha_\psi - 3/2))]$. With the usual nonsense zeros in the supermultiplet residues, we have

$$\frac{\pi}{\sin \pi (\alpha_\psi - 3/2)} \longrightarrow \Gamma(3/2 - \alpha_\psi)$$

in Eq. (3.11).

The final Reggeized M-function will contain kinematic singularities coming from $m \rightarrow \sqrt{u}$, and they are usually eliminated either by assuming Gribov-parity doublet quarks⁽⁷⁾ or by introducing kinematic cuts⁽¹⁶⁾, as described in chapter II. However, such mechanisms will not be necessary in our present model which will be described in the following section.

3.3 The Model

In this model we assume that any two-body hadronic scattering amplitude is composed of three parts; the two particle-particle-Reggeon supermultiplet vertex functions of the last section, which are obtained by taking U(12) traces between the Regge-propagator and the external particles, and the Reggeon supermultiplet propagator.

(a) Baryon Propagator

The baryon propagator in general contains explicit

quark spin-projection operators like $(\frac{K}{m} + 1)_x (\frac{K}{m} + 1)_x (\frac{K}{m} + 1)$ at the pole $K^2 = m^2$ with appropriate indices attached to them. However, the Reggeized propagator is expected to be more complicated than in the case of the simple pair of exchange degenerate 36 meson trajectories, because of the presence of both the 56 and 70 multiplets with completely different symmetry properties. Although the Reggeized Oscillator model of the previous section should provide, in principle, a pure 56 at $L = 0$ and a pure 70 at $L = 1$, both the multiplets (probably including the 20 as well) are present at all higher L , and the simple form of signature for the baryon spectrum is not expected to appear in the usual way. Thus for a general supermultiplet Reggeon propagator, a more general theory like, for example the dual model⁽⁵⁾ is clearly needed. In the absence of such a theory where one can do simple calculations, we simply assume that the supermultiplet scattering amplitude near the backward direction is dominated by the exchange of the Reggeized tri-quark propagator, written explicitly as

$$\begin{aligned} \langle \Psi_{ABC}(K, \alpha), \Psi^{A'B'C'}(K, \alpha) \rangle \sim \frac{\beta_\psi}{(2m)^3 3!} \sum_{\substack{\text{perm.} \\ ABC}} (K+m)_A^{A'} (K+m)_B^{B'} (K+m)_C^{C'} \\ \times \xi_\psi \Gamma\left(\frac{3}{2} - \alpha_\psi(K^2)\right) \left(\frac{s}{s_0}\right)^{\alpha_\psi(K^2) - \frac{3}{2}}, \end{aligned} \quad (3.12)$$

where, as before, $\xi_\psi = \frac{1}{2} [1 + \exp(-i\pi(\alpha_\psi - \frac{3}{2}))]$, the signature factor, $s_0 \approx \frac{1}{\alpha'}$, and β_ψ is the universal baryon-

supermultiplet residue function. We have introduced a trajectory function $\alpha_\psi(k^2)$ with the simple signature factor ξ_ψ in analogy to Eq. (3.11). When exchanged in any backward process the above propagator will provide amplitudes with acceptable analyticity properties and the requisite high energy behaviour.

The difference from the previous supermultiplet-Regge models^(6,7,8) lies in the fact that the mass m (or μ in meson case) is not substituted by $\sqrt{k^2}$, or vice versa, in the Regge propagators, and therefore at the first pole

$\alpha_\psi = \frac{3}{2}$, where $k^2 = m^2$, we only have pure $(56, \bar{1})$ exchanges with no parity partners. However, at all higher masses, Gribov parity partners and $(70, \bar{1})$ should also be present. Thus the validity of our model is perhaps limited to a small momentum transfer region,

Now by assuming the supermultiplet residue function to be a constant, the result of exchanging the propagator (3.12) in any backward amplitude is to produce polynomial baryon residues $\beta(k^2)$ in the conventional Regge amplitudes. The main feature of the data which the polynomial residues of our model hold some hope of explaining is the rapid variation of the Δ_8 -residue⁽¹²⁾. Indeed, we have in our model $\beta(k=m) = 8 \beta(k=0)$ and thus the baryon residues are depressed by a factor of 8 (roughly) when extrapolated from the pole to the scattering region.

Before proceeding with the detailed calculations, we adopt the following rule, which however is rather ad hoc, in descending from the ideal SU(6) symmetry limit to the actual world of broken symmetry: - the trace calculations

are performed in the mass-degenerate-exact symmetry limit, employing the additive quark rule $3\mu = 2m$, with $\mu^2 \simeq 0.6$ $(\text{GeV}/c^2)^2$; then the physical amplitudes are obtained by replacing the momenta and dimensionless wave functions of the external particles by their physical values. The alternative way of replacing all supermultiplet masses by their physical values is to some extent ambiguous when all the hadrons are different and in effect will produce large amounts of symmetry breaking. The ambiguity concerning the symmetry breaking effect on the coupling constant β is reduced by comparing the pole residue to the dimensionless pion-nucleon and pion- Δ coupling constants.

(b) Meson-Baryon Backward Scattering

In this model, the scattering amplitude for meson $(6, \bar{6}) + \text{Baryon } (56, \bar{1}) \rightarrow \text{Baryon } (56, \bar{1}) + \text{Meson } (6, \bar{6})$ process is given by

$$T(s, u) = \Psi^{BCD}(p') \Phi_D^A(q) \langle \Psi_{ABC}(K, \alpha), \bar{\Psi}^{A'B'C'}(K, \alpha) \rangle \Psi_{B'C'D'}(p) \Phi_{A'}^{D'}(-q') \quad (3.13)$$

which by using (3.12) gives

$$T(s, u) = \frac{\beta_\psi}{48 m^3} \xi_\psi \Gamma(3/2 - \alpha_\psi) \left(\frac{s}{s_0} \right)^{\alpha_\psi - 3/2} \bar{\Psi}^{BCD}(p') \Phi_D^A(q) \\ \times \left(\sum_{\substack{\text{perm.} \\ ABC}} (K+m)_A^{A'} (K+m)_B^{B'} (K+m)_C^{C'} \right) \Psi_{B'C'D'}(p) \Phi_{A'}^{D'}(-q'). \quad (3.14)$$

In the exact mass degenerate limit, the $P(\sigma^-)N(\frac{1}{2}^+)$ -
 $\rightarrow N(\frac{1}{2}^+)P(\sigma^-)$ piece of (3.14) is obtained by using
 (3.3) and (3.4) as

$$\begin{aligned}
 T(s,u) = & \left(\frac{1}{2\sqrt{6}m}\right)^2 \left(\frac{1}{2\mu}\right)^2 \left(\frac{1}{2m}\right)^3 \frac{\beta_4}{6} \xi_4 \Gamma(\frac{3}{2}-\alpha_4) \left(\frac{s}{s_0}\right)^{\alpha_4-3/2} \\
 & \times \left\{ \epsilon^{bcs} \overline{N}_s^{d,\delta}(p') [C^{-1} \gamma_5 (\not{p}' + m)]^{\beta r} + \text{cyclic perm. of } (bp, cr, d\delta) \right\} \\
 & \times \left[(\not{q} + \mu) \gamma_5 \right]_s^\alpha P_d^\alpha(q) \\
 & \times \left\{ \sum_{\text{perm.}} \left[(\not{k} + m)_{\alpha}^{\alpha'} \delta_a^{\alpha'} (\not{k} + m)_{\beta}^{\beta'} \delta_b^{\beta'} (\not{k} + m)_{\gamma}^{\gamma'} \delta_c^{\gamma'} \right] \right\} \\
 & \quad (a\alpha, b\beta, c\gamma) \\
 & \times \left[(-\not{q}' + \mu) \gamma_5 \right]_{\alpha'}^{\delta'} P_{\alpha'}^{\delta'}(-q') \\
 & \times \left\{ [(\not{p} + m) \gamma_5 C]_{\beta' \gamma'} \epsilon_{b'c'x} N_{d',\delta'}^x(p) + \text{cyclic perm. } (b'\beta', c'\gamma', d'\delta') \right\}.
 \end{aligned}
 \tag{3.15}$$

The expressions for other processes, like $PN \rightarrow NV$, DP , DV can also be written down in a similar manner.

We then further decompose (3.15) into $SU(3)$ and $U(2,2)$ parts by writing

$$N_{a,\alpha}^b(p) = N_a^b \times u_\alpha(p)$$

$$P_a^b(q) = P_a^b \times 1 \quad ,$$

(3.16)

where N_a^b and P_a^b are the usual SU(3) matrices for baryon ($\frac{1}{2}^+$) and meson (0^-) octets:

$$N_a^b = \begin{pmatrix} \frac{1}{\sqrt{2}} \Sigma^0 + \frac{1}{\sqrt{6}} \Lambda^0 & \Sigma^+ & p \\ \Sigma^- & -\frac{1}{\sqrt{2}} \Sigma^0 + \frac{1}{\sqrt{6}} \Lambda^0 & n \\ -\Xi^- & \Xi^0 & -\frac{2}{\sqrt{6}} \Lambda^0 \end{pmatrix}$$

$$P_a^b = \begin{pmatrix} \frac{1}{\sqrt{2}} \pi^0 + \frac{1}{\sqrt{6}} \eta^0 & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}} \pi^0 + \frac{1}{\sqrt{6}} \eta^0 & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}} \eta^0 \end{pmatrix}$$

(3.17)

As it stands, the expression (3.15) contains 54 terms, which, however, reduce to 27 when the symmetry property of the external and the exchanged 56 baryon is taken into account.

By using (3.16) the amplitude (3.15) can be written

as

$$T = (T_{U(2,2)} \cdot T_{SU(3)}) R$$

$$\text{where } R = \frac{\beta_\psi}{18} \frac{1}{(2m)^5} \frac{1}{(2\mu)^2} \xi_\psi \Gamma(\frac{3}{2} - \alpha_\psi) \left(\frac{s}{s_0}\right)^{\alpha_\psi - 3/2},$$

$$\begin{aligned} T_{U(2,2)} = & \left\{ \bar{u}^\delta(p') [C^\dagger \gamma_5 (\not{p}' + m)]^{\beta\gamma} + \text{cyclic}(\beta\gamma\delta) \right\} \\ & \times [(\not{q} + \mu) \gamma_5]^\alpha_\delta \\ & \times \left\{ (K+m)^\alpha_{\alpha'} (K+m)^{\beta'}_\beta (K+m)^{\gamma'}_\gamma + \text{cyclic}(\alpha\beta\gamma) \right\} \\ & \times [(-\not{q}' + \mu) \gamma_5]^{\delta'}_{\alpha'} \\ & \times \left\{ [(\not{p} + m) \gamma_5 C]_{\beta'\gamma'} u_{\delta'}(p) + \text{cyclic}(\beta'\gamma'\delta') \right\}, \end{aligned}$$

and

$$T_{SU(3)} = \left\{ \epsilon^{bcd} \bar{N}_s^d + \text{cyclic}(bcd) \right\} P_d^a$$

$$\times \left\{ \delta_a^{a'} \delta_b^{b'} \delta_c^{c'} + \text{cyclic}(abc) \right\}$$

$$\times P_{a'}^{d'} \left\{ \epsilon_{b'c'} N_{d'}^{x'} + \text{cyclic}(b'c'd') \right\}.$$

(3.18)

Again, individual terms in the expression for $T_{U(2,2)}$ can be related to each other by their $U(2,2)$ -

algebraic properties and the final expression involves only 14 obviously distinct traces like, for example;

$$\begin{aligned}
 (i) \quad & \bar{u}(p') (q+\mu) (k-m) (q'+\mu) u(p) \\
 & \times \text{Tr} [(p+m) (k+m) (p'+m) (k+m)] , \\
 (ii) \quad & \bar{u}(p') (k+m) u(p) \times \text{Tr} [(p+m) (q'+\mu) (k-m) \\
 & \times (q+\mu) (p'+m) (k+m)] , \\
 (iii) \quad & \bar{u}(p') (k+m) (p+m) (q'+\mu) (-k+m) (q+\mu) (p'+m) \\
 & \times (k+m) u(p) ,
 \end{aligned}$$

(3.19)

where we have used the relations: $\bar{c}^{-1} \gamma_\mu c = -(\gamma_\mu)^T$,
 $\bar{c}^{-1} \gamma_5 c = \gamma_5^T$ etc.

All the remaining terms can be related to the above expressions in (3.19) by commuting the momenta around. The explicit trace calculations are straightforward but rather tedious to perform, and the results obtained can be applied to any $\sigma^- \frac{1}{2}^+ \rightarrow \frac{1}{2}^+ \sigma^-$ reaction. We will give the appropriate results for πN backward scattering in the following section. We used the definitions of kinematics and the Dirac matrices and spinors given in the appendix A.

The SU(3) traces in (3.18) can also be computed in a similar fashion and the combination of the two traces

uniquely determines the K^2 - dependent spin-unitary spin content of the various amplitudes.

3.4 Pion-Nucleon Backward Scattering

(a) N and Δ Amplitudes

In the last section, we showed how to calculate the amplitudes for any backward scattering $P(o^-)N(\frac{1}{2}^-) \rightarrow N(\frac{1}{2}^+)P(o^-)$ in our model. In the following, we shall give the results of the explicit calculations made for πN backward scattering. As given in the Appendix B the SHA are defined by

$$T_{\lambda\lambda} = \bar{u}^{(\lambda')}(\mathbf{p}') (A + B\alpha) u^{(\lambda)}(\mathbf{p}) \quad (3.20)$$

where A and B are given in our model as

$$\begin{aligned} A(s,u) &= \frac{\beta_\psi}{18} \frac{1}{(2m)^5} \frac{1}{(2\mu)^2} \gamma_A(s,u) \xi_\psi \Gamma(3/2 - \alpha_\psi) \left(\frac{s}{s_0}\right)^{\alpha_\psi - 3/2} \\ B(s,u) &= \frac{\beta_\psi}{18} \frac{1}{(2m)^5} \frac{1}{(2\mu)^2} \gamma_B(s,u) \xi_\psi \Gamma(3/2 - \alpha_\psi) \left(\frac{s}{s_0}\right)^{\alpha_\psi - 3/2} \end{aligned} \quad (3.21)$$

Here γ_A and γ_B denotes the polynomial residues of the model.

First, the amplitudes for $\pi^\pm p \rightarrow p \pi^\pm$ and $\pi^- p \rightarrow n \pi^0$ are calculated directly from the expression

(3.18) by picking out the appropriate isospin elements. Then the amplitudes are decomposed back into $N(I_u = \frac{1}{2})$ and $\Delta(I_u = \frac{3}{2})$ exchange amplitudes by use of the isospin relations:

$$\pi^- p \rightarrow p \pi^- : \quad T^- = \Delta$$

$$\pi^+ p \rightarrow p \pi^+ : \quad T^+ = \frac{1}{3}(2N + \Delta)$$

$$\pi^- p \rightarrow n \pi^0 : \quad T^0 = \frac{\sqrt{2}}{3}(N - \Delta)$$

(3.22)

The polynomial residues thus obtained contain a leading term $\sim 0(s)$ and a non-leading term $\sim 0(1)$ and we write for N and Δ amplitudes:

$$\gamma_{\Delta}(s, u) = s \gamma_{\Delta_s}(u) + \gamma_{\Delta'}(u)$$

$$\gamma_N(s, u) = s \gamma_{N_s}(u) + \gamma_{N'}(u)$$

(3.23)

where the leading term is interpreted as that of the highest trajectory (Δ_s, N_s) contribution⁽¹⁷⁾, with a linear trajectory function

$$\alpha(u) = 0.1 + 0.9 u$$

(3.24)

This automatically assigns a trajectory with one unit of angular-momentum lower, i.e. $\alpha(u) = -0.9 + 0.9u$, to

the non leading contribution (Δ' , N'), which contains both the secondary trajectory contribution, i.e. (Δ_α , N_α), as well as the 'tail' or the non leading part of the (Δ_δ , N_δ) contribution. To see this, we write the explicit forms for the Δ and N residues as

$$\begin{aligned}\Delta_\delta: \quad \gamma_A(u) &= -16 \left[m^3(4m^2 + 7m\mu + 3\mu^2) + m(16m^2 + 16m\mu + 3\mu^2)u \right. \\ &\quad \left. + (4m + \mu)u^2 \right] \\ \gamma_B(u) &= 16 \left[m^2(3m^2 + 5m\mu + 2\mu^2) + (8m^2 + 7m\mu + \mu^2)u + u^2 \right] \\ &\quad (3.25)\end{aligned}$$

$$\begin{aligned}N_\delta: \quad \gamma_A(u) &= -50(u - m^2)^2(m + \mu) \\ \gamma_B(u) &= -25(u - m^2) \left[(u - m^2) + (2m + \mu)^2 \right] \\ &\quad (3.26)\end{aligned}$$

for the leading (Δ_δ , N_δ) contribution and

$$\begin{aligned}\Delta': \quad \gamma_A(u) &= -4 \left\{ (u - m^2) \left[(13m^5 + 20m^4\mu + 14m^3\mu^2 - 12m^2\mu^3 - 3m\mu^4) \right. \right. \\ &\quad \left. \left. - 2(13m^3 + 4m^2\mu + 5m\mu^2)u + 5mu^2 \right] \right. \\ &\quad \left. - 4(2m + \mu) \left[(3m^6 + m^5\mu - 8m^4\mu^2 + 5m^3\mu^4 + 3m\mu^5) \right. \right. \\ &\quad \left. \left. + (11m^4 + 5m^3\mu + 8m^2\mu^2 + \mu^4)u - 2m^2u^2 \right] \right\},\end{aligned}$$

$$\begin{aligned}
\gamma_B(u) = 2 \Big\{ (u-m^2) \Big[(13m^4 + 20m^3\mu + 14m^2\mu^2 - 12m\mu^3 - 3\mu^4) \\
- 2(13m^2 - 4m\mu + 5\mu^2)u + 5u^2 \Big] \\
+ 4(2m+\mu) \Big[2(2m^5 + 3m^4\mu - 4m^3\mu^2 - 2m\mu^4 - \mu^5) \\
+ (5m^3 + 4m^2\mu - 16m\mu^2)u + 11m\mu^2 \Big] \Big\}, \\
(3.27)
\end{aligned}$$

$$\begin{aligned}
N': \quad \gamma_A(u) = -2 \Big\{ (u-m^2) \Big[(161m^5 + 235m^4\mu - 92m^3\mu^2 - 144m^2\mu^3 \\
+ 27m\mu^4 + 21\mu^5) \\
+ (149m^3 + 157m^2\mu - 20m\mu^2)u + 10m\mu^2 \Big] \\
+ 4(2m+\mu) \Big[(-3m^6 + 4m^5\mu + 4m^4\mu^2 - m^2\mu^4) \\
- (4m^4 + 4m^3\mu + 4m^2\mu^2 - \mu^4)u + 7m^2u^2 \Big] \Big\},
\end{aligned}$$

$$\begin{aligned}
\gamma_B(u) = - \Big\{ (u-m^2) \Big[(109m^4 - 76m^3\mu - 208m^2\mu^2 - 12m\mu^3 + 15\mu^4) \\
+ 5(41m^2 + 4m\mu + 4\mu^2)u - 10u^2 \Big] \\
+ (2m+\mu) \Big[(322m^5 + 171m^4\mu - 372m^3\mu^2 - 200m^2\mu^3 \\
+ 50m\mu^4 + 25\mu^5) \\
+ (458m^3 + 299m^2\mu - 28m\mu^2)u + 20m\mu^2 \Big] \Big\}, \\
(3.28)
\end{aligned}$$

for the (Δ', N') contribution.

The corresponding expressions at the first pole,
 $u = m^2$, are:

$$\begin{aligned}\Delta_g: \quad \gamma_A(u=m^2) &= -96 m^3 (2m+\mu)^2 \\ \gamma_B(u=m^2) &= 48 m^2 (2m+\mu)^2\end{aligned}\tag{3.29}$$

$$\begin{aligned}N_g: \quad \gamma_A(u=m^2) &= 0 \\ \gamma_B(u=m^2) &= 0\end{aligned}\tag{3.30}$$

$$\begin{aligned}\Delta': \quad \gamma_A(u=m^2) &= 48 m (2m^4 + \mu^4) (2m+\mu)^2 \\ \gamma_B(u=m^2) &= 16 (5m^4 - 4m^2\mu^2 - \mu^4) (2m+\mu)^2\end{aligned}\tag{3.31}$$

$$\begin{aligned}N': \quad \gamma_A(u=m^2) &= 0 \\ \gamma_B(u=m^2) &= -25 (4m^2 - \mu^2)^2 (2m+\mu)^2\end{aligned}\tag{3.32}$$

The expression for $\Delta_g(u=m^2)$ in (3.29) agrees with that of our previous on-mass-shell calculation for a pure spin $\frac{3}{2}^+$ exchange⁽⁸⁾, thus justifying our assignment of the trajectory (3.24) to this. Again (3.30) shows that the N_g residue vanishes at the first pole, as required,

and with the signature zero at the second pole (at $\alpha = 5/2$), the particle spectrum for this trajectory will probably start from a higher value of α (e.g. $\alpha = 7/2$).

Now the secondary contribution $\Delta'(u=m^2)$ in (3.31) is interpreted as that of the 'tail' of $\Delta_S(u=m^2)$ in (3.29). We believe that the residue of the secondary trajectory, i.e. Δ_α which probably exists for arbitrary u , should vanish, as we have for N_S in (3.30), at this point ($\alpha = 1/2$) so that the corresponding particle spectrum will again start from a higher value of α (e.g. $\alpha = 5/2$).

The situation is rather clearer for the N' residue, since the tail of N_S must also vanish along with the vanishing of N_S itself in (3.30). Therefore, what is left in (3.32) is a pure N_α residue, for which the residue γ_A vanishes at the pole, which is required, since the elementary nucleon pole contributes only to the B-amplitude. However at arbitrary u values, we will have both the N_α and the tail of N_S contributions in our N' residue.

Since we have $\alpha_{N_\alpha} = -0.9 + 0.9u$ for the N trajectory in the exact symmetry limit, while the actually observed trajectory is $\alpha_{N_\alpha} \simeq -0.4 + 0.9u$, it is clear that the symmetry is badly broken for our N_α residue.

(b) Pole Extrapolation

The normalization of the residue can now be made by considering the $\pi^- p \rightarrow p \pi^-$ process, where only Δ -exchange is possible, and continuing the residue to the

Δ_S -pole where $u = m^2$. From Eq. (3.20), (3.21), (3.22)

and (3.29) we have for $u \rightarrow m^2$

$$\Delta_s \longrightarrow \frac{1}{3m} \frac{\beta_\psi \cdot s}{\alpha' (u - m^2)} \left(\frac{2m + \mu}{4m\mu} \right)^2 \bar{u} (-2m + \mu) u$$

which, when compared to the elementary Δ_s - pole amplitude, given by Eq. (B.9) of Appendix B,

$$\Delta_s = \left(\frac{g_\Delta}{m_N} \right)^2 \frac{s}{4m(u - m^2)} \bar{u} (-2m + \mu) u$$

gives

$$\left(\frac{g_\Delta}{m_N} \right)^2 = \frac{4\beta_\psi}{3\alpha'} \left(\frac{2m + \mu}{4m\mu} \right)^2$$

(3.33)

The pion-nucleon- Δ_s coupling constant g_Δ is calculated from the $\Delta(1236)$ decay width ($\Gamma_\Delta \simeq 120$ MeV) and has a value of $\simeq 14.5$. On the other hand the Δ_s -amplitude in the backward direction ($u \simeq 0$) is

$$\Delta_s^{+-}(u \simeq 0) \simeq \frac{\beta_\psi}{q} \left(\frac{m + \mu}{4m\mu} \right)^2 \xi_\psi \Gamma\left(\frac{3}{2} - \alpha_\psi(0)\right) \left(\frac{s}{s_0}\right)^{\alpha_\psi(0)}$$

Taking $\alpha_\psi(0) \simeq 0.1$, $s_0 \simeq 1$, $\mu/m \simeq 2/3$ and substituting β_ψ from (3.33), we obtain for $p_{\text{Lab}} = 13.7$ GeV/c (i.e. $s \simeq 26.6$ GeV²)

$$\frac{d\sigma}{du}(u \approx 0) \simeq 1.1 \mu b / (\text{GeV}/c)^2.$$

The experimental value⁽²⁰⁾ of $\frac{d\sigma}{du}(u \approx 0)$ for this energy is about $\sim 1 \mu b / (\text{GeV}/c)^2$, which agrees with our prediction to within 10%. Thus the Δ_f - residue extrapolates extremely well from the pole to the scattering region. This is better than we had any right to expect since the exact mass degenerate symmetry must be broken in order to get the physical masses of the particles on the trajectory. However, it should be noted that the physical mass of $\Delta(1236)$ is nearly the same as our baryon supermultiplet mass m ($\simeq 1.16$), so that the symmetry breaking effect on the Δ_f - residue is expected to be minimal.

For the non leading N_α - contribution, a considerable amount of symmetry breaking effect will be present in the residue, since the mass of the nucleon is considerably different from the supermultiplet mass m . To see this, consider the N amplitude given by Eq. (3.20), (3.21) and (3.32) as $u \rightarrow m^2$;

$$N_\alpha \rightarrow - \frac{1}{(2m)^5} \frac{1}{(2\mu)^2} \frac{25 \beta_\psi}{18 \alpha'} \left[(2m + \mu) (4m^2 - \mu^2) \right]^2 \frac{\bar{u} \not{\alpha} u}{(u - m^2)}.$$

Using Eq. (3.33) for β_ψ (with $g_\Delta \simeq 14.5$) and comparing the resulting expression with the elementary N_α - pole amplitude (Eq.(B.6) of Appendix B)

$$N_{\alpha} = -g_{\pi NN}^2 \frac{1}{2m} \frac{\bar{u} \not{x} u}{(u - m^2)}$$

we obtain

$$\frac{g_{\pi NN}^2}{4\pi} = \frac{1}{8\pi} \left[\frac{5}{3} g_{\Delta} \left(1 - \frac{\mu^2}{4m^2} \right) \right]^2 \quad (3.34)$$

in the exact symmetry limit.

Taking $\mu/m \simeq 2/3$, $\mu^2 \simeq 0.6$, we obtain a value of 18 for $g_{\pi NN}^2/4\pi$, which is not at all very different from the experimental value of $\simeq 15$. Indeed this is due to the symmetry breaking effect in the N_{α} - residue, as is seen by replacing $m \rightarrow m_N$ in Eq. (3.34), giving $g_{\pi NN}^2/4\pi \rightarrow 15.5$. This agrees with the experimental value to $\sim 3\%$. From the above procedure we estimate the amount of symmetry breaking in the N_{α} - residue at least near the pole, to be about 20%.

Therefore, we expect a considerable amount of symmetry breaking effect to be present also in the extrapolated N_{α} - residue. In fact we found that the N_{α} - residue extrapolates to a value too low to account for the observed sharp backward peak of the $\pi^+ p \rightarrow p \pi^+$ differential cross section. The inclusion of other contributions coming from N_{δ} , Δ_{δ} and Δ' was found to help raise the peak, but to a value still about an order of magnitude lower than the observed peak. A closer look at the N' -

residues reveals that the non-flip N' -residue has a zero near the extreme backward direction (here, $\mu/m \simeq 2/3$, $\mu^2 \simeq 0.6$). Therefore one can hope that, with a correct way of symmetry breaking, the above zero may be shifted to the dip position and the backward peak may be regained. Another possible reason for this discrepancy may be due to the absence of the N_ρ -contribution coming from the 70 multiplet, in our model. Due to the above problems and uncertainties in our N' -residue, we are forced to abandon the detailed analysis of our model for $\pi^+ p \rightarrow p \pi^+$ and $\pi^- p \rightarrow n \pi^0$ processes.

(c) Δ_δ - Residue

Before proceeding to the detailed comparison between the model prediction and experimental values of differential cross section and polarization for $\pi^- p \rightarrow p \pi^-$, we will look more closely at the polynomial character of the

Δ_δ - residue function in the scattering region. The SHA for Δ - exchange in $\pi^- p \rightarrow p \pi^-$ reaction are written in our model as

$$\Delta^{\lambda\lambda'}(s, u) = \frac{\beta_\psi}{18} \frac{1}{(2m)^5} \frac{1}{(2\mu)^2} \left[s \gamma_{\Delta_\delta}^{\lambda'\lambda}(u) + \gamma_{\Delta'}^{\lambda\lambda'}(u) \right] \zeta_\psi \times \Gamma(3/2 - \alpha_\Delta) \left(\frac{s}{s_0} \right)^{\alpha_\Delta - 3/2} \quad (3.35)$$

where β_ψ is normalized by Eq. (3.33) and $\alpha_\Delta = 0.1 + 0.9u$ ($\lambda', \lambda = \pm 1/2$).

The residue functions $\gamma_\Delta(u)$ for the s-channel helicity flip ($\Delta\lambda = 1$) and nonflip ($\Delta\lambda = 0$) amplitudes

are given in terms of the conventional residues γ_A and γ_B of Eq. (3.25 - 3.28) in the large s limit, as

$$\begin{aligned} (\Delta\lambda=1) \quad \frac{\gamma_{\Delta}^{++}(u)}{\sqrt{s}} &\simeq \sqrt{-u} \gamma_B(u) \\ (\Delta\lambda=0) \quad \frac{\gamma_{\Delta}^{+-}(u)}{\sqrt{s}} &\simeq \gamma_A(u) + m_N \gamma_B(u) \end{aligned} \quad (3.36)$$

where $\Delta = \Delta_S, \Delta'$.

Consider the leading Δ_S contribution in (3.35). Substituting (3.25) in (3.36) by taking $m_N = m$, and $\mu/m \simeq 2/3$, we obtain

$$\begin{aligned} \frac{\gamma_{\Delta_S}^{++}(u)}{\sqrt{s}} &\simeq \sqrt{-u} \times 16 (7.22 m^4 + 13.11 m^2 u + u^2) \\ \frac{\gamma_{\Delta_S}^{+-}(u)}{\sqrt{s}} &\simeq -16 m (2.78 m^4 + 14.89 m^2 u + 3.66 u^2) \end{aligned} \quad (3.37)$$

Now if we take $\mu^2 \simeq 0.6$ which corresponds to $m \simeq 1.16$ ⁽¹⁸⁾ and solve (3.37) for zeros, we find that

$$\begin{aligned} \frac{\gamma_{\Delta_S}^{++}}{\sqrt{-u}} &= 0 \quad \text{at} \quad u \simeq -0.7 \\ \text{and} \quad \gamma_{\Delta_S}^{+-} &= 0 \quad \text{at} \quad u \simeq -0.3, \end{aligned} \quad (3.38)$$

while a slight change in m to $m \simeq 1.01$ gives

$$\frac{\gamma_{\Delta\delta}^{+-}}{\sqrt{-u}} = 0 \quad \text{at} \quad u \simeq -0.6$$

and

$$\gamma_{\Delta\delta}^{+-} = 0 \quad \text{at} \quad u \simeq -0.2 .$$

(3.39)

The second zeros of the residues in (3.37) occur at large $|u|$ values outside the region of interest ($|u| < 1.0 \text{ GeV}/c$).

Therefore we see that our residues possess exactly the same zero structure as those found in the polynomial residues generated in the same model for the forward scattering amplitudes⁽²⁾. The above zero structure in the residues in fact resembles that of the Bessel-function-like 'residues', $\tilde{J}_{\Delta\lambda}(r\sqrt{-u})$ of the dual absorptive model. In Fig. (3.4) the polynomial behaviour of our

Δ_δ - residues is illustrated in comparison with that of the Bessel functions $\tilde{J}_0(r\sqrt{-u})$ and $\tilde{J}_1(r\sqrt{-u})$ (19).

(d) Δ_δ - Amplitude

The dual absorptive model⁽¹²⁾ is based on the assumption that the imaginary parts of the SHA are dominated by the most peripheral resonances (i.e. resonances with $\ell \sim \sqrt{s}$), and consequently one gets $\text{Im } \Delta^{+-} \sim \tilde{J}_0(r\sqrt{-u})$ and

$\text{Im } \Delta^{++} \sim \tilde{J}_1(r\sqrt{-u})$, where the interaction radius $r \simeq 1$ fermi. As will be shown in chapter IV, the DAM

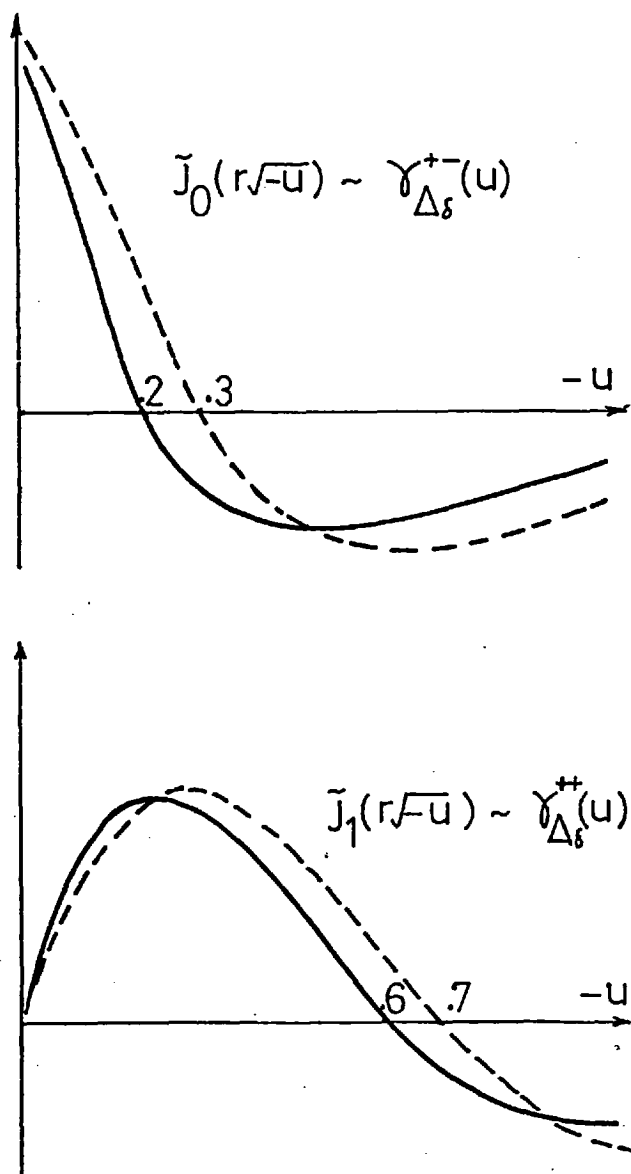


Fig. (3.4) $r \sim 1$ fermi; u in $(\text{GeV}/c)^2$; arbitrary unit in y-axis. — $m \approx 1.01$, --- $m \approx 1.16$, $\mu/m \approx 2/3$.

does not provide the corresponding real parts in any specific form, i.e., the real parts may or may not behave like the Bessel functions. In our case, the zeros of the polynomial residues will produce zeros in both the real and imaginary parts of the flip and non flip amplitudes at $u \simeq -0.6$ and $u \simeq -0.2$ respectively, in addition to the usual signature zero at $u \simeq -0.6$. To see this we write, by using the identity

$$\Gamma(p) \Gamma(1-p) = \frac{\pi}{\sin \pi p},$$

$$\Gamma(3/2 - \alpha_\Delta) [1 + e^{-i\pi(\alpha_\Delta - 3/2)}] = \pi [\Gamma(\alpha_\Delta - 1/2)]^{-1}$$

which gives

$$\times [i + \tan \frac{\pi}{2} (\alpha_\Delta - 1/2)]$$

$$\text{Im } \Delta_\delta \sim \gamma_{\Delta_\delta} [\Gamma(\alpha_\Delta - 1/2)]^{-1}$$

$$\text{Re } \Delta_\delta \sim \gamma_{\Delta_\delta} [\Gamma(\alpha_\Delta - 1/2)]^{-1} \tan \frac{\pi}{2} (\alpha_\Delta - 1/2) \quad (3.40)$$

Noting that $[\Gamma(\alpha_\Delta - 1/2)]^{-1} = 0$ while $[\Gamma(\alpha_\Delta - 1/2)]^{-1} \times \tan \frac{\pi}{2} (\alpha_\Delta - 1/2) \neq 0$ at $\alpha_\Delta = -1/2$, corresponding to $u \simeq -0.6$, we have the following behaviour for our SHA for Δ_δ - exchange:

$$u \simeq -0.2; \quad \Delta_\lambda = 0 \left\{ \begin{array}{l} \text{Im } \Delta_\delta^{+-} \sim [0] \\ \text{Re } \Delta_\delta^{+-} \sim [0] \end{array} \right.$$

$$\Delta\lambda=1 \left\{ \begin{array}{l} \text{Im } \Delta_{\delta}^{++} \\ \text{Re } \Delta_{\delta}^{++} \end{array} \right\} \neq [0]$$

$$u \simeq -0.6 ;$$

$$\Delta\lambda=0 \left\{ \begin{array}{l} \text{Im } \Delta_{\delta}^{+-} \sim [0] \\ \text{Re } \Delta_{\delta}^{+-} \neq [0] \end{array} \right.$$

$$\Delta\lambda=1 \left\{ \begin{array}{l} \text{Im } \Delta_{\delta}^{++} \sim [0]^2 \\ \text{Re } \Delta_{\delta}^{++} \sim [0] \end{array} \right. \quad (3.41)$$

We also have a negative relative sign between the flip and nonflip residues near $u = 0$.

It is clear from (3.41) that the form of our $\text{Im } \Delta_{\delta}$ agrees with those of the DAM for $|u| \lesssim 0.6$; we have an extra single zero coming from $[\Gamma(\alpha_1 - 1/2)]^{-1}$ at $u \simeq -0.6$ for our imaginary parts. It is also worth noting that in a recent DAM-based amplitude analysis for πN backward scattering⁽²¹⁾, the authors concluded that $\text{Re } \Delta^{+-} \sim 0$ near $u \simeq -0.2$ and $\text{Re } \Delta^{++} \sim 0$ near $u \simeq -0.6$; exactly what we have in (3.41), but with an opposite relative sign between the flip and nonflip amplitudes. However, we ignore this disagreement of the relative flip-nonflip sign on account of the fact that it is rather dependent on the detailed assumptions about their input imaginary parts. In fact, Barger and Olssen⁽¹⁰⁾ have pointed out, from their model independent analysis of πN backward scattering data, that $\text{Im } \Delta^{+-} \sim 0$ near $u \simeq -0.2$ and that either Δ^{++}

or Δ^{+-} must have a zero at $u \simeq -0.6$ in both real and imaginary parts. These are all met by our amplitude in (3.14).

(e) Comparison with Experiments

Using the amplitudes (3.35) with the residues (3.25) for Δ_S -contribution we found that the Δ_S -contribution alone is inadequate to explain the experimental $\pi^- p \rightarrow p \pi^-$ differential cross section. For the case of $m \simeq 1.01$ corresponding to (3.39), the predicted differential cross section shows a shallow dip around $u \simeq -0.2$, while for $m \simeq 1.16$, corresponding to (3.38), a broad dip is observed around $u \simeq -0.5$. The prediction for the latter case is shown in comparison with the experimental data in Fig. (3.5). The broad dip is clearly due to the accumulation of zeros of the Δ_S -amplitudes in (3.41), around these u -values. It should be noted that the theoretical curves are not fitted to the data since we have essentially no free parameters.

Although our model in the present form seems to suffer from the defect of having higher order zeros in some of the Δ_S -amplitudes (possibly $\text{Im } \Delta^{++}$ and $\text{Im } \Delta^{+-}$), we find that an acceptable behaviour of the amplitudes is however achieved when the secondary Δ' contribution is also considered. Taking

$$\begin{aligned}\alpha_{\Delta_S}(u) &= 0.15 + 0.9u \\ \alpha_{\Delta'}(u) &= -0.65 + 0.9u\end{aligned}\tag{3.42}$$

and with no other free parameters⁽²²⁾ we are able to reproduce the general behaviour of the differential cross section and polarization⁽²⁰⁾ as shown in Fig. (3.5) and (3.6) respectively. Here we have chosen the trajectory parameter of Δ' arbitrarily (but reasonably) without worrying too much about the presence or absence of particles on the corresponding Δ_α -trajectory. A more serious consideration of this fact would probably be an unnecessary exercise since a large symmetry breaking effect has been shown to present in the N' -residue. In fact we find that the qualitative features of the results are rather insensitive to the parameters of Δ' amplitude.

In Fig. (3.5) we give the comparison between the theoretical and experimental differential cross section for $\pi^- p \rightarrow p \pi^-$ reaction at four different energies. We note that the agreement between theory and experiment is much improved as the energy is increased. The effect of the accumulation of zeros in the Δ_8 -amplitude at about $u \simeq -0.6$ can be seen clearly in the 6.0 GeV/c-curve. The polarization at 6.0 GeV/c is found to be well reproduced by our model as shown in Fig. (3.6). The zero of the polarization at $u \simeq -0.5$ is clearly due to the zeros of the amplitudes around this point. Fig. (3.7) shows the momentum transfer behaviour of the real and imaginary parts of the Δ^{+-} and Δ^{++} amplitudes. We find that the exact zeros of the Δ_8 -amplitudes at $u \simeq -0.3$ and $u \simeq -0.7$ (corresponding to taking $m \simeq 1.16$), are shifted by the presence of the Δ' contribution.

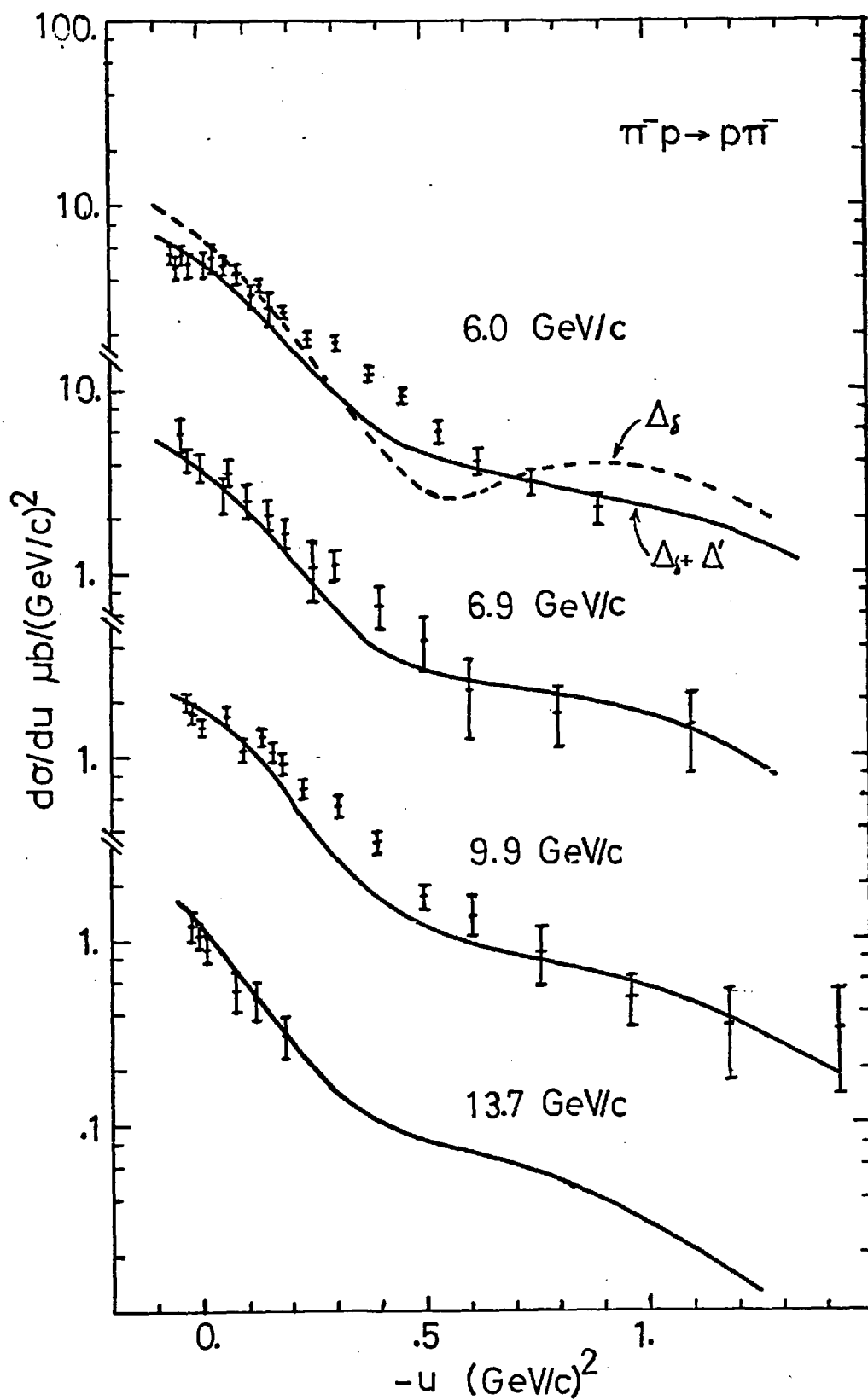


Fig. (3.5)

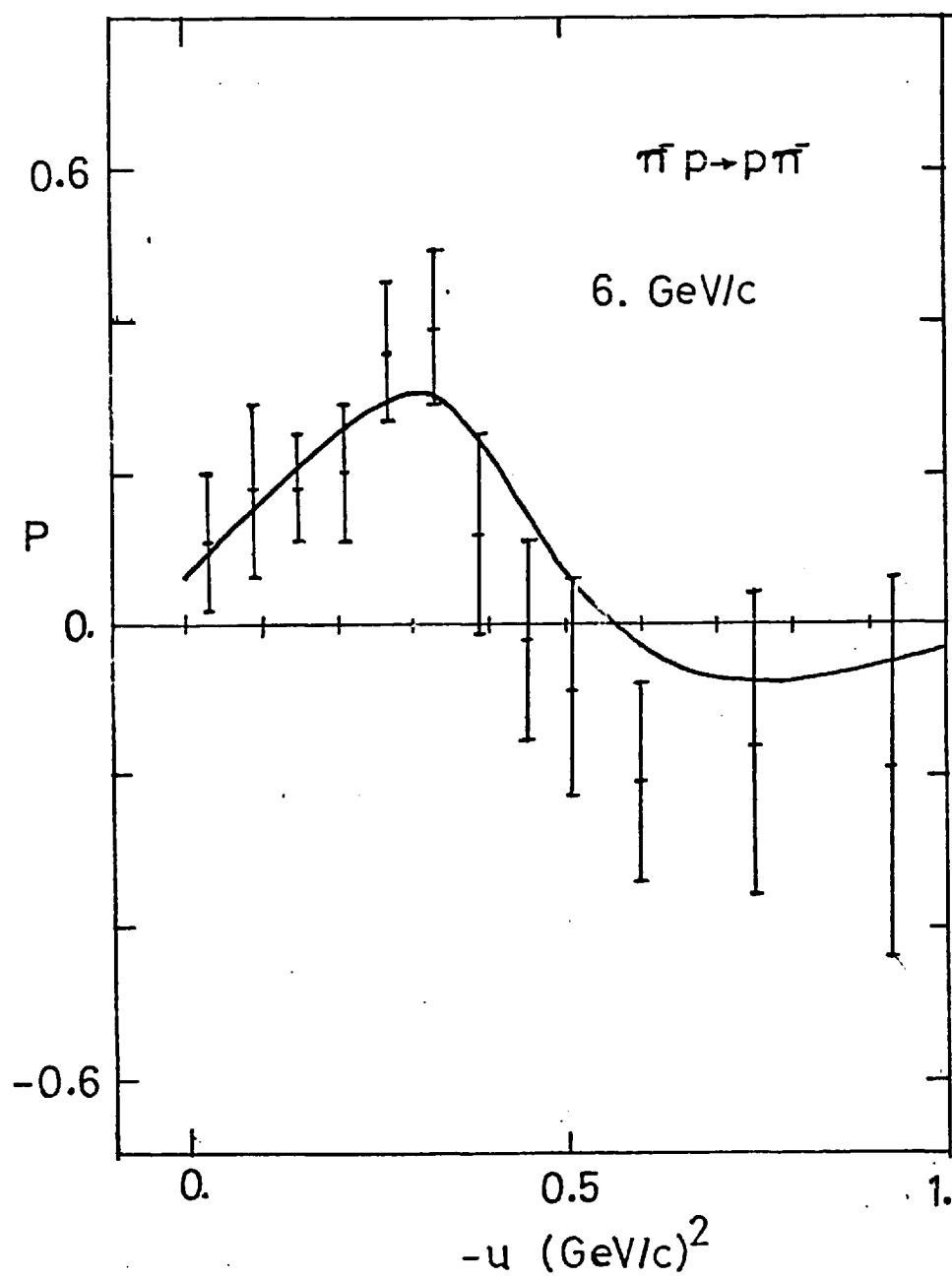


Fig. (3.6)

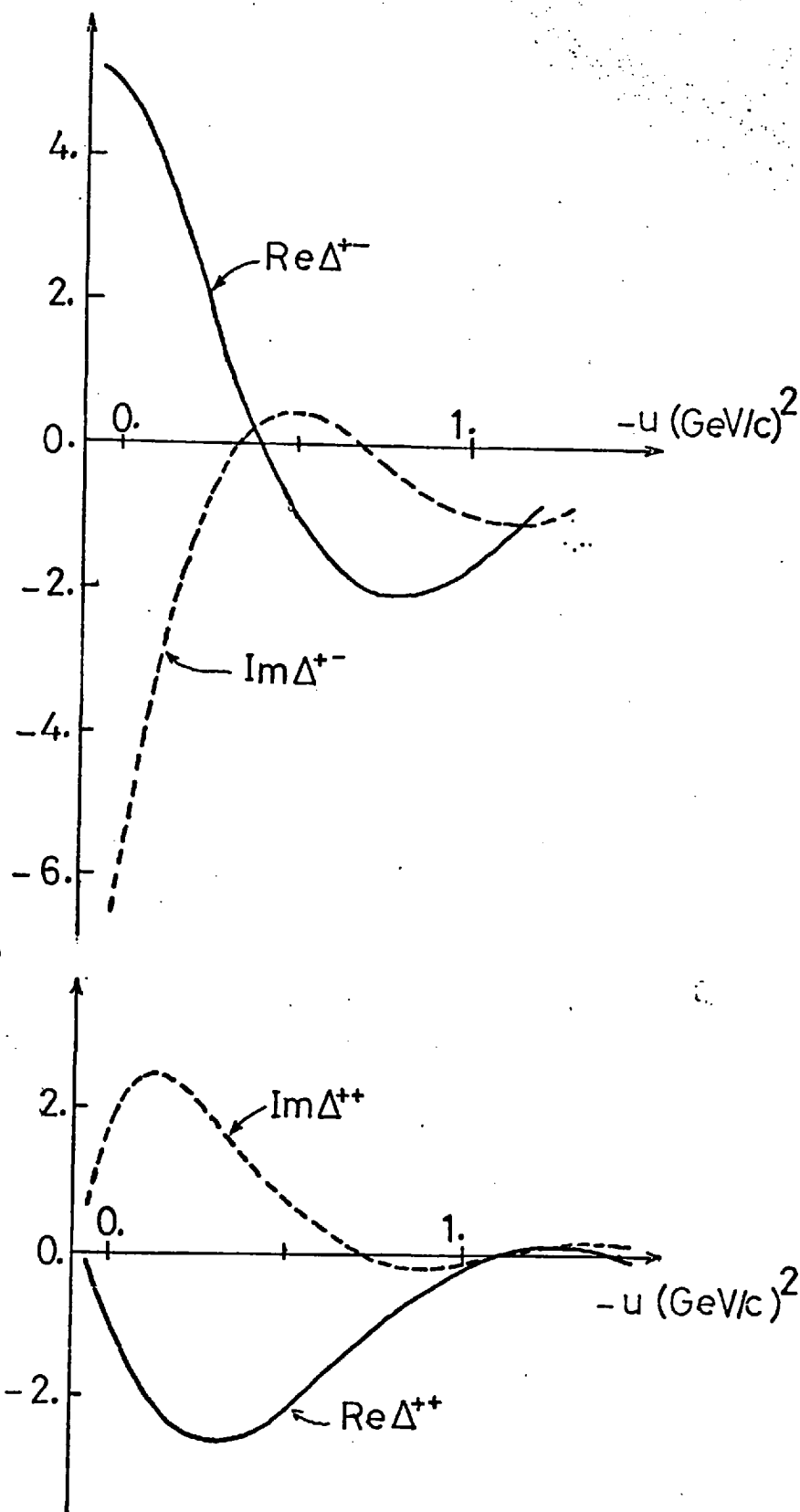


Fig. (3.7)

3.5 Conclusion

We have seen that by using a Reggeized supermultiplet model with explicit quark spin projection operators, which insure the required resonance structure on the Regge trajectory, i.e., presence of a pure $\underline{56}$ without its parity partner at the first pole, we are led to polynomial Regge residues with many desirable features. In particular, the

Δ_8 -residue is found to extrapolate correctly from the pole to the scattering region, where it continues to exhibit the zero structure which is very suggestive of what is observed. Although our explicit calculation does not constitute a proof, the general feature of having a zero at small u values in the non-flip amplitude, while at a larger u value in the flip amplitude, and not vice versa, seems to be rather basic to this tri-quark propagator picture. Similar structures are also observed for forward scattering. Also the N_α -residue provides a correct value for the pion-nucleon coupling constant $g_{\pi NN}^2/4\pi \simeq 15.5$, which however extrapolates badly to the scattering region and fails to account for the observed πN -backward peak. This we interpret as due to the symmetry breaking effect present in the N' -residue, and also possibly due to the absence of ${}^{\text{on}}N_P$ -contribution in our model.

From the above analysis and comparison with the experimental data, we find that the s -channel helicity amplitude for Δ_8 -exchange seems to have undesirable extra zeros around $u \simeq -0.6$. This could well be due to the simple form of signature factor which we assume in our

supermultiple Regge propagator. A way of breaking the conventional signature for the baryon trajectory has been suggested by Mandelstam⁽⁵⁾ in his symmetric-quark model with the Veneziano representation. Here we seem to need such a more general theory to achieve a correct signature.

The partial correctness of our model is however reflected in its ability to reproduce the differential cross-section and polarization for $\pi^- p \rightarrow p \pi^-$ scattering, simultaneously, with essentially no free parameter. Here the lower order Δ' contribution is responsible for the breaking of the signature in the right direction, and consequently the correct non-zero polarization.

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17. The usual notation for (parity, signature) of a trajectory is used: $\alpha = (+,+)$, $\beta = (-,+)$, $\delta = (+,-)$ and $\gamma = (-,-)$.
18. We take these values for m and μ to be consistent with Ref. (2). However, actual values suitable for our case may vary around these values.
19. $\tilde{J}_0, \tilde{J}_1$ denotes the function possessing the general characteristics of the Bessel function $J_0(r\sqrt{u_0-u})$, $J_1(r\sqrt{u_0-u})$ for $|u| \lesssim 1 \text{ GeV}/c^2$. Here $r \sim 1 \text{ fermi}$ ($\sim 5 \text{ GeV}/c^{-1}$) and $u_0 \equiv (m_N^2 - m_\pi^2)^2/s$.
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made to get the present theoretical curves.

CHAPTER IV

A DUAL ABSORPTIVE MODEL4.1 Introduction

The idea of duality⁽¹⁾ between the Regge pole and resonance descriptions of strong interaction processes was first proposed at a time when Regge poles alone were thought to give an adequate description of high energy amplitudes. It has since become clear that Regge cuts, such as those arising from absorptive corrections⁽²⁾ are also important. Therefore, it is desirable to extend the idea of duality to include both Regge poles and Regge cuts in a consistent manner. This led Harari to formulate his dual absorptive model⁽³⁾ (DAM).

In a simple version of the DAM⁽⁴⁾, which shares many common features with the usual Regge pole model, the real part may be related to the imaginary part by the familiar signature factor provided that absorptive corrections (cuts) are suspected to be negligible and the asymptotic behaviour assumed at an early stage. Otherwise, the real parts are unknown, and for this reason, the DAM is only a qualitative model with little predictive power. In any case, the peripheral nature of the imaginary part is, by itself, a very important characteristic of high energy amplitudes which has been tested in various meson-baryon scattering and photo-production forward processes^(3,4). It would, therefore, be interesting to see whether the model could also account for the systematic features of the

backward scattering data. Such a straightforward extension to the backward direction is not obvious since a large number of cancellations among the various partial wave amplitudes occur and the magnitude of the amplitude becomes small in the backward region. There is, however, some evidence^(5,6) of direct channel effects which suggests the peripheral nature of the imaginary part of backward amplitudes. Such a behaviour for the $\Delta_s(I_u = \frac{3}{2})$ exchanged amplitude has also been obtained in a model⁽⁷⁾ with polynomial Regge residues.

In recent years, a number of phenomenological models⁽⁸⁾ have been proposed to account for the observed features of high energy πN backward differential cross-sections and polarizations^(9,10) but none of the conventional Regge pole and Regge pole plus various cut-models (except possibly the one we mentioned in chapter III) was able to explain the new data on backward $\pi^- p$ polarization at 6 GeV/c⁽¹¹⁾. With the hope of giving some consistent explanation to the above observations we extend the DAM to the backward direction.

In section 4.2 we show how the ideas of duality, peripheral resonances and absorption lead to the prediction of a simple form for the imaginary part of any hadronic two-body amplitude. The extension to the backward direction is made for the imaginary part in section 4.3. Possible ways of determining the real part are also discussed. In section 4.4, a simple version of the dual absorptive model is constructed for two-body backward processes where $N(I_u = \frac{1}{2})$ and $\Delta(I_u = \frac{3}{2})$ exchanges are possible. The model is then

applied to πN backward scattering. Through detailed fits to the available data the momentum transfer behaviour of the N and Δ exchange amplitudes are studied. The conclusions are given in section 4.5.

4.2 Duality, Peripheral Resonances and Absorption

(a) Phenomenological Duality

Duality is based on the empirical fact that certain simple features of high energy amplitudes persist, in a local average sense, down to low energies⁽¹⁾. The most important features are a fixed t or u zero structure, reality of the amplitude and an approximate power-law energy behaviour. There are several versions of duality which differ in their precise definitions. For the present purpose, the two-component duality of Harari⁽¹²⁾ and Freund⁽¹³⁾ is assumed, which, according to Harari⁽¹⁴⁾, states that the imaginary part of any hadronic two-body amplitude involves two components;

$$\text{Im } A(s,t) = R(s,t) + P(s,t) \quad (4.1)$$

From the s -channel point of view, $R(s,t)$ represents the contributions of s -channel resonances, while $P(s,t)$ is given by the non-resonant background. It is important to note that resonance dominance in a local sense applies only to the imaginary part of the amplitude (i.e. the Breit-Wigner formula becomes pure imaginary at the resonance energy), while for the real part, distant resonances may be

important. In the backward direction $P(s,t)$ is absent from (4.1) due to the cancellation of background contributions and we have $R(s,t) \rightarrow R(s,u)$.

From the t -channel point of view, $R(s,t)$ (or $R(s,u)$) represents the contributions of the t (or u)-channel quantum number exchanges which include Regge poles as well as Regge pole-Pomeron cuts. $P(s,t)$ represents the Pomeron exchange (diffractive) part of the amplitude. The above statements are represented diagrammatically in Fig. (4.1).

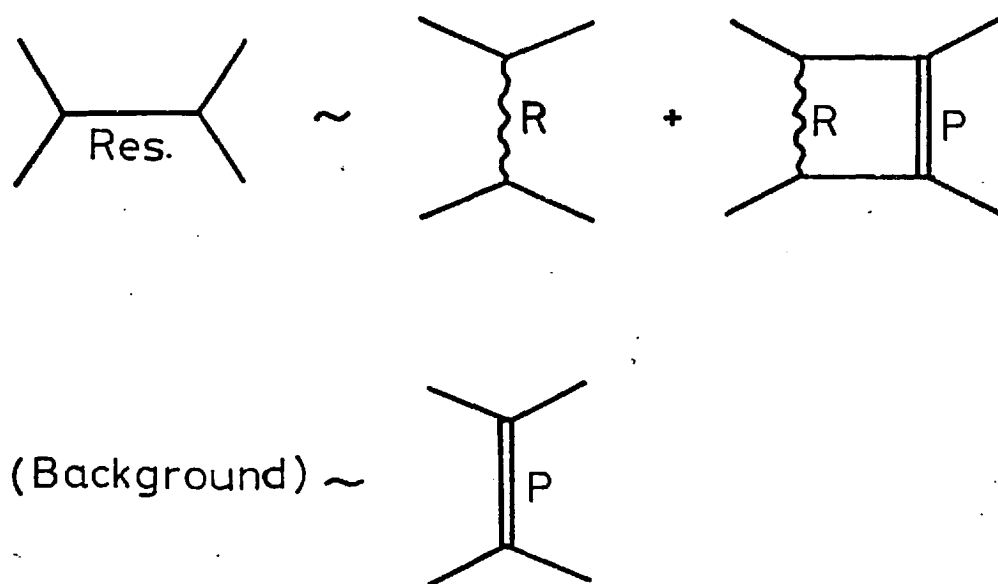


Fig. (4.1)

Many aspects of two-component duality have been successfully tested against experiments⁽¹⁵⁾ with one

exception in the case of baryon-antibaryon scattering.

(b) Peripheral Resonances

The first component $R(s,t)$ has been assumed to have the dual character of being represented as a sum of s -channel resonances or as a combination of cross-channel exchanges. In all simple t -channel models, the observed approximately-fixed structures in the angular-momentum distribution (dips, bumps in t and u distribution) are associated with certain characteristic properties of the singularities in these channels. For example, in the Regge pole model, dips are due to the wrong-signature-nonsense zeros of the Regge residue functions. From the s -channel point of view, such structures at fixed t or u -values at all energies can be reproduced, in the simplest way, if we assume that every single prominent resonance produces such a structure by itself. This imposes strong constraints on the quantum numbers of the prominent s -channel resonances.

To see this, consider the case of spinless particle scattering, where the contribution of a resonance of spin ℓ and mass m_ℓ is proportional to

$$P_\ell(\cos \theta) \sim J_0((\ell + \frac{1}{2}) \sin \theta / 2)$$

where a well known identity between the Legendre polynomial and the Bessel function for large ℓ has been used. Since $\sin \theta / 2 \sim \sqrt{-t}/m_\ell$, a fixed- t structure at all energies imposes a relation of the form

$$\ell \propto m_\ell = \sqrt{s} \quad (4.2)$$

between the spins and masses of the prominent resonances. The observed dips in the t or u -distribution suggest that the proportionality constant is about 1 Fermi ($\approx 5 \text{ GeV}^{-1}$), which puts the dominant resonances on the parabola

$$(\ell + \frac{1}{2}) / \sqrt{s} \approx 5 \text{ GeV}^{-1} \approx 1 \text{ Fermi}$$

These are called peripheral resonances, since they corresponds to a semi-classical picture of scattering from the periphery of a disc of constant radius ~ 1 Fermi (Fig. 4.2).

(c) Absorption

From the above discussions, it is clear that there is a fundamental difference between the above assumptions and those of the absorption model. The basic idea which is common to all versions of absorption models is that inelastic scattering amplitudes are dominated by the most peripheral impact parameter within the interaction radius, which is about 1 Fermi. The contributions from the lower partial waves for this particular channel are 'absorbed' away by the opening of many other competing channels. Therefore, a similar relation to (4.2) is also obtained in all the absorption models. The only difference from the assumption above is that the absorption models assume the dominance of $\ell \propto \sqrt{s}$ partial waves in both the imaginary as well as the real part of the amplitude, while, in our case, a similar assumption is made only for the imaginary part.

Therefore we assume that the non-diffractive component of the imaginary part of any hadronic two body amplitude is

dominated by resonances and that the prominent resonances are peripheral ($\ell \approx \sqrt{s}$). On the other hand, the real part is not dominated by resonances in a local way and we do not expect it *a priori* to be dominated by the peripheral partial waves only.

4.3 Backward Amplitudes

(a) The Imaginary Part

Near the backward direction the Pomeron (or diffractive) component $P(s,t)$ will be absent and Eq. (4.1) takes the form

$$-\text{Im} G(s,u) = R(s,u) \quad (4.3)$$

where $R(s,u)$ represents either the contributions of peripheral ($\ell \approx \sqrt{s}$) s-channel resonances or the contribution of ordinary u-channel pole and pole-Pomeron cut exchanges.

In the following we shall adopt an s-channel impact parameter description of the amplitude, which is more relevant for studying the energy dependence of the imaginary and the real parts. In this picture our basic assumption is that $\text{Im} G(s,u)$ is dominated by the most peripheral impact parameters within an interaction radius r .

Consider the s-channel partial wave expansion of the backward SHA (refer to Fig. (2.2) of chapter II for kinematics and helicity labelling):

$$G_{H_S}(s,u) = \frac{1}{K^2} \sum_J (J+\frac{1}{2}) d_{\lambda\mu}^J(\theta) G_{H_S}^J(s) \quad (4.4)$$

where $\lambda = \lambda_1 - \lambda_2$, $\mu = \lambda_3 - \lambda_4$, $H_S \equiv (\lambda_4 \lambda_3; \lambda_1 \lambda_2)$ and $k =$ s-channel c.m.-momentum. The normalization is given by

$$\frac{d\sigma}{du} = \sum_{H_S} |G_{H_S}(s, u)|^2. \quad (4.5)$$

Here $\theta \approx \pi$ near the backward direction and we have the relation⁽¹⁶⁾

$$d_{\lambda\mu}^J(\theta) = (-)^{J+\lambda} d_{\lambda, -\mu}^J(\pi - \theta) \quad (4.6)$$

At high energy and small $\theta' \equiv (\pi - \theta)$, the rotation function may be approximated by the Bessel function as

$$d_{\lambda\mu}^J(\theta') \approx J_{|\lambda-\mu|}((2J+1)\sin\frac{\theta'}{2}) \quad (4.7)$$

Therefore, (4.6) becomes

$$d_{\lambda\mu}^J(\theta) \underset{\theta \rightarrow \pi}{\approx} (-)^{J+\lambda} J_{|\lambda+\mu|}((2J+1)\cos\frac{\theta}{2}) \quad (4.8)$$

We introduce the impact parameter b by the expression

$$J + \frac{1}{2} = kb \quad (4.9)$$

which corresponds, in classical physics, to the closeness of approach of a particle with angular momentum J to the

target centre (Fig. 4.2). We then make the replacement

$$\sum_J \rightarrow \int_0^\infty k db$$

$$(-)^{J+\lambda} G_{H_s}^J(s) \rightarrow \mathcal{G}_{H_s}(s, b)$$

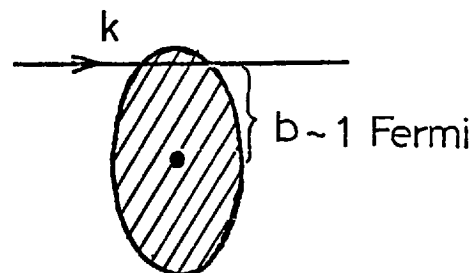


Fig. (4.2)

and (4.4) becomes

$$G_{H_s}(s, u) = \int_0^\infty b db \mathcal{G}_{H_s}(s, b) J_{\Delta\lambda}(b\sqrt{u_0-u}) \quad (4.10)$$

where $\Delta\lambda = |\lambda + \mu|$, $u_0 \equiv (m^2 - \mu^2)^2/s$. We have used $\cos \theta_{1/2} = \sqrt{u_0 - u}/2k$. By using the orthogonality relation between the Bessel functions, we may invert (4.10) to give

$$\mathcal{G}_{H_s}(s, b) = \int_0^\infty (\sqrt{u_0-u}) d(\sqrt{u_0-u}) J_{\Delta\lambda}(b\sqrt{u_0-u}) G_{H_s}(s, u). \quad (4.11)$$

Since $\Delta\lambda = |\lambda + \mu| = |(\lambda_1 - \lambda_4) - (\lambda_2 - \lambda_3)|$, it represents the total s-channel helicity change. We will drop the subscript H_s for clarity and define $u' \equiv u - u_0$.

The extreme case of the peripheral assumption is to

take

$$\text{Im } \mathcal{G}(s, b) \propto \delta(b-r) \quad (4.12)$$

which gives

$$\text{Im } G(s, u) \propto J_{\Delta\lambda}(r\sqrt{-u'}) \quad (4.13)$$

A more realistic impact parameter distribution is however obtained by having a smoothing factor in (4.13) which does not destroy the zero structure of $J_{\Delta\lambda}$ i.e., we assume

$$\text{Im } G(s, u) = A e^{Bu'} J_{\Delta\lambda}(r\sqrt{-u'}) \quad (4.14)$$

which, by using (4.11), gives⁽¹⁷⁾

$$\text{Im } \mathcal{G}(s, b) = \frac{A}{2B} \exp\left(-\frac{r^2+b^2}{4B}\right) I_{\Delta\lambda}\left(\frac{rb}{2B}\right) \quad (4.15)$$

where $I_{\Delta\lambda}(y) = (i)^{\Delta\lambda} J_{\Delta\lambda}(iy)$, which for large y becomes⁽¹⁷⁾

$$I_{\Delta\lambda}(y) = \frac{e^y}{\sqrt{2\pi y}} (1 + O(1/y))$$

Then (4.15) becomes

$$\text{Im } \mathcal{G}(s, b) = \frac{A}{2\sqrt{\pi B}rb} \exp\left(-\frac{(r-b)^2}{4B}\right) \quad (4.16)$$

which is peaked around $b \approx r$, with the width Δb given by

$$\Delta b \sim 2\sqrt{B} \quad (4.17)$$

A typical form of (4.16) is shown in Fig. (4.3). In the limit $\Delta b \rightarrow 0$, i.e. $B \rightarrow 0$ in (4.14), Eq. (4.16) $\rightarrow$ Eq. (4.12) as required.

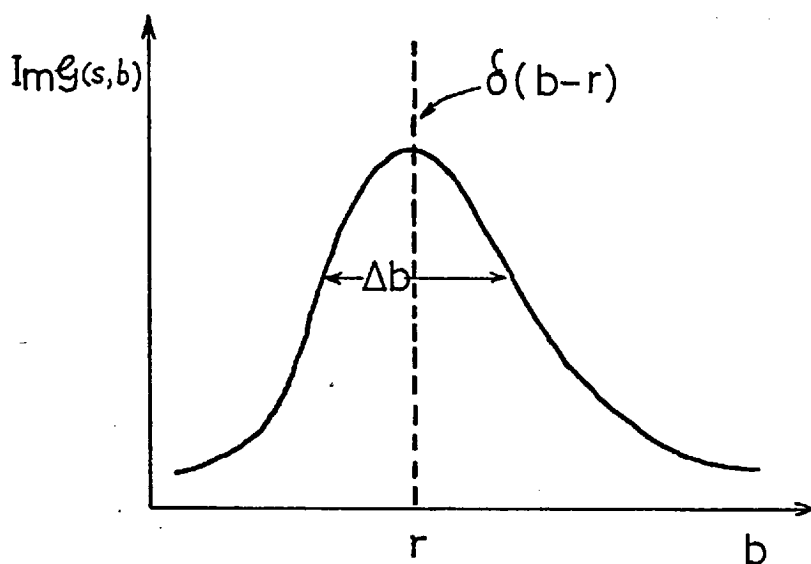


Fig. (4.3)

The three parameters A , B and r are each a function of energy, which may be quite complicated in general. But we expect them to be simple when the cut contribution is negligible. This will become clearer when we discuss the real part. Therefore we assume for $s \rightarrow \infty$

$$\begin{aligned}
A(s) &\sim A s^{\bar{\alpha}_0} \\
B(s) &\sim B + B' \ln s \\
r(s) &\sim r + r' \ln s
\end{aligned}
\tag{4.18}$$

where A , $\bar{\alpha}_0 \equiv \alpha_0 - \frac{1}{2}$, B, B', r and r' are constants. It should be noted from (4.17) that the condition for peripheral dominance is

$$r(s) > \sqrt{B(s)} \tag{4.19}$$

Thus (4.14) becomes

$$\text{Im } G(s, u) = A e^{B u'} s^{\bar{\alpha}_0 + B' u'} J_{\Delta\lambda}((r + r' \ln s) \sqrt{-u'}) \tag{4.20}$$

which gives the usual form of DAM parametrization when $r' = 0$.

(b) The Real Part

Since the real part of the amplitude is not dominated by the resonances in a local sense, it is not restricted by duality to have any specific form.

One may determine the real part by using a fixed- u (or $-t$ for the forward case⁽¹⁸⁾) dispersion relation in which one has to ignore any s -dependence in subtraction constants and assume the knowledge of the energy behaviour of the imaginary part for $s \rightarrow \pm \infty$. Therefore, a definite signature, i.e. a definite connection between the two limits $s \rightarrow \pm \infty$, has to be assumed. However, we know that cut-contributions are also present and there is no reason a priori to expect the pole + cut contribution to have a simple

crossing property (or a definite signature).

Another way of guessing the behaviour of $\text{Re } G(s, u)$ is to make use of the Phragmén-Lindelöf theorem concerning the phase of the terms in asymptotic expansions of crossing symmetric amplitudes⁽¹⁹⁾ i.e. if

$$\text{Im } G(s, u) \sim \sum_i r_i(u) \frac{s^{\bar{\alpha}_i(u)}}{(\ln \ln \dots s)^{\beta_i}} \quad (4.21)$$

then

$$\text{Re } G(s, u) \sim \sum_i r_i(u) \frac{s^{\bar{\alpha}_i(u)}}{(\ln \ln \dots s)^{\beta_i}} \times \begin{cases} -\cot \frac{1}{2} \pi \bar{\alpha}_i(u) & (\text{even}) \\ \tan \frac{1}{2} \pi \bar{\alpha}_i(u) & (\text{odd}) \end{cases} \quad (4.22)$$

The sum on the right-hand-side of (4.21) may be replaced by $s^{\bar{\alpha}(u)}$ where $\alpha(u)$ is some effective trajectory describing the overall effect of poles and cuts. Therefore, unless cut contributions are small, $\alpha(u)$ may not be represented by a simple function like $\alpha(u) = \alpha_0 + B' u'$ of Eq. (4.20). Again the combined pole-cut contribution, in general, may not have simple crossing properties. One should also note that cut terms will reach their asymptotic phases only when $(\ln \ln \dots s) \gg 1$, which is rather hard to realize at energies of a few GeV.

Therefore we may safely assume a simple form of $\text{Im } G(s, u)$ in (4.20) and the asymptotic phase (4.21), (4.22)

only when the cut contributions are suspected to be negligible. Otherwise the real parts remain unpredictable.

(c) J-Plane Structure

Consider $\text{Im } G(s, u)$ as given by (4.2) for the following cases:

(A) $B' = 0, r' = 0$; then $\text{Im } G(s, u) \sim s^{\bar{\alpha}_0} J_{\Delta\lambda}(r \sqrt{-u'})$ and the full amplitude is given by the asymptotic phase relations (4.21), (4.22) as

$$G^\pm(s, u) \sim -J_{\Delta\lambda}(r \sqrt{-u'}) s^{\bar{\alpha}_0} \frac{e^{-i\pi\bar{\alpha}_0} \pm 1}{\sin \pi\bar{\alpha}_0} \quad (4.23)$$

which corresponds to a fixed u -channel pole or cut in the J -plane. In this case $\text{Re } G(s, u)$ is also peripheral ($\pm$ refers to even/odd signature)

(B) $r' = 0$; then $\text{Im } G(s, u) \sim s^{\bar{\alpha}(u)} J_{\Delta\lambda}(r \sqrt{-u'})$ and the full amplitude

$$G^\pm(s, u) \sim -J_{\Delta\lambda}(r \sqrt{-u'}) s^{\bar{\alpha}(u)} \frac{e^{-i\pi\bar{\alpha}(u)} \pm 1}{\sin \pi\bar{\alpha}(u)} \quad (4.24)$$

where $\bar{\alpha}(u) = \bar{\alpha}_0 + B'u'$. This corresponds to an ordinary Regge pole (plus cut) model with a residue function $J_{\Delta\lambda}(r \sqrt{-u'})$. In this case poles (ghosts) will develop at negative integer values of $\bar{\alpha}(u)$, unless $J_{\Delta\lambda}(r \sqrt{-u'})$ vanishes exactly at t -values corresponding to these poles,

or a large cut contribution is present which changes the asymptotic phase relation (4.21), (4.22) in a significant way. In the latter case, the real part will remain unpredicted.

(C) For $\text{Im } G(s, u)$ fully given by (4.20), we will have a more complicated J-plane structure. To see this we use the integral representation of $J_{\Delta\lambda}(x)$ ⁽¹⁷⁾,

$$J_{\Delta\lambda}(x) = \frac{1}{2\pi} \int_{-\pi}^{+\pi} e^{ix \sin \varphi - i\varphi \cdot \Delta\lambda} d\varphi \quad (4.25)$$

to give

$$\text{Im } G(s, u) \sim \frac{1}{2\pi} \int_{-\pi}^{+\pi} s^{\bar{\alpha}(u) + i r' \sqrt{-u'} \sin \varphi} e^{i(r' \sqrt{-u'} \sin \varphi - \varphi \Delta\lambda)} d\varphi \quad (4.26)$$

which may be viewed as a superposition of complex poles at

$$J = \bar{\alpha}_0 + B' u' + i \sin \varphi r' \sqrt{-u'} \quad (-\pi \leq \varphi \leq \pi) \quad (4.27)$$

According to our asymptotic relation (4.21)-(4.22), $\text{Re } G(s, u)$ is then given by:

$$\text{Re } G(s, u) \sim \frac{1}{2\pi} \int_{-\pi}^{+\pi} \left\{ \begin{array}{l} -\cot \frac{\pi}{2} \bar{\alpha}(u, \varphi) \\ \tan \frac{\pi}{2} \bar{\alpha}(u, \varphi) \end{array} \right\} s^{\bar{\alpha}(u, \varphi)} e^{i(r' \sqrt{-u'} \sin \varphi - \varphi \Delta\lambda)} d\varphi ,$$

where

$$\bar{\alpha}(u, \varphi) = \bar{\alpha}(u) + i r' \sqrt{-u'} \sin \varphi$$

(4.28)

Such a complicated J-plane structure is similar to that of the complex-pole models of Zachariasen⁽²⁰⁾ and Oehme⁽²¹⁾, which are rather interesting in the sense of parity-partner elimination. In this case the problem of ghosts does not arise due to the presence of the $i r' \sqrt{-u'} \sin \varphi$ factor in the trajectory function. A typical J-plane structure is shown in Fig. (4.4).

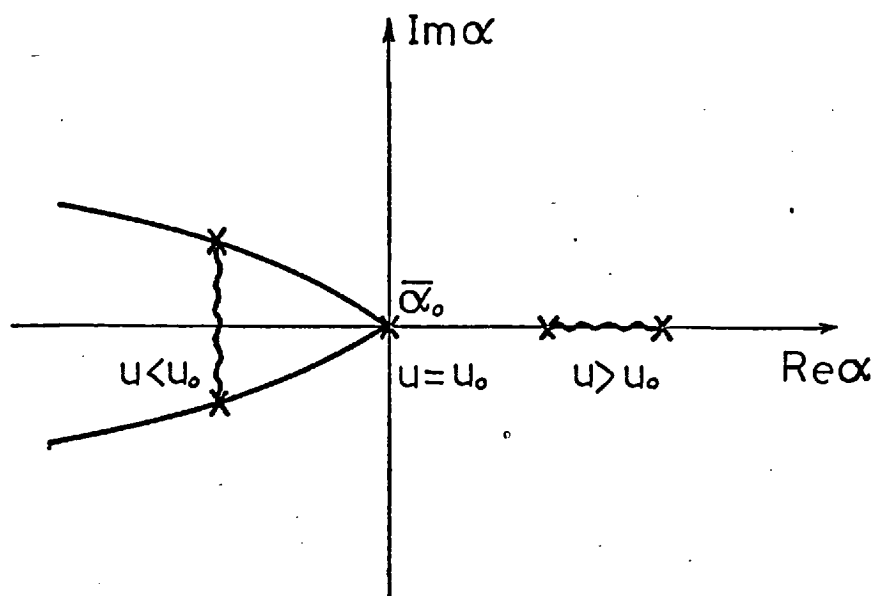


Fig. (4.4)

4.4 A Dual Absorptive Model for Backward πN Scattering

(a) The Model⁽²²⁾

The experimental consequences of the model in case (B) of the previous section are; the Regge-type shrinkage of differential cross-sections, dips in differential cross-sections are fixed in u , and will become less pronounced at large energies as a result of the increased slope which will eventually violate the peripheral condition (4.19).

This type of model is what is usually known as the dual absorptive model (DAM), which has been shown⁽⁴⁾ to account for the qualitative features of experiment data in the forward direction. In this section we apply the model to backward πN scattering, bearing in mind that the other two cases ((A) and (C) in the previous section) are also possible.

We summarize the model as follows:

(i) $\text{Im } G(s, u)$ is dominated by the most peripheral s -channel resonances and is therefore given by a function $\sim A e^{B u'} J_{\Delta\lambda}(\tau\sqrt{-u'})$ where $\Delta\lambda$ is the total s -channel helicity flip.

(ii) $\text{Re } G(s, u)$ is unknown in the s -channel picture. In the u -channel picture, it is expressed in terms of the same pole and cut which describe $\text{Im } G(s, u)$. At asymptotic energies and when the cut contributions are small, it may have the form $\sim A e^{B u'} J_{\Delta\lambda}(\tau\sqrt{-u'}) \cot \frac{1}{2} \pi \bar{\alpha}(u)$ or $\sim A e^{B u'} J_{\Delta\lambda}(\tau\sqrt{-u'}) \tan \frac{1}{2} \pi \bar{\alpha}(u)$ for even or odd signature, respectively. When the cut contributions are strong the

above phase will be reached probably very slowly, leaving $\text{Re } G(s,u)$ undetermined.

The strength of the cuts present in the $N(I_u = \frac{1}{2})$ and $\Delta(I_u = \frac{3}{2})$ amplitudes may be estimated from the form (zero structure) of the corresponding Regge pole amplitudes⁽²³⁾. In the case of the N_α exchange, the usual wrong-signature-nonsense zero of the residue at $\bar{\alpha}_N = -1$ produces a single zero in both $\text{Im } N(\Delta\lambda=0)$ and $\text{Im } N(\Delta\lambda=1)$ at $u \simeq -0.2$. By comparing them with the corresponding Bessel functions $J_0(r\sqrt{-u'})$ and $J_1(r\sqrt{-u'})$ (with $r \simeq 1$ Fermi), we found that a very weak or no cut is required for the $N(\Delta\lambda=0)$ amplitude while a rather strong cut is required for the $N(\Delta\lambda=1)$ amplitude. For Δ_π exchange, a similar analysis shows that a strong cut is needed for the $\Delta(\Delta\lambda=0)$ amplitude while a weak or no cut is required for the $\Delta(\Delta\lambda=1)$ amplitude. The N and Δ amplitudes in our model are summarized in table I, and the corresponding schematic forms are shown in Fig. (4.5).

In table I, $\tilde{J}_{\Delta\lambda} \equiv A_i^{\Delta\lambda} e^{B_i^{\Delta\lambda} u'} J_{\Delta\lambda}$ for $i = N, \Delta$, and $\tilde{C}_i(u) \equiv A e^{B u'} C_i(u)$ where the unknown functions C_i serve as inputs to our analysis. However, they are not arbitrary in the backward direction, since the intercepts of the pole and the pole plus absorptive-cut trajectories are equal at $u' = 0$, irrespective of the strength of the cut. Therefore we normalize them to the corresponding Regge pole phases at $u' = 0$ (with respect to their imaginary parts).

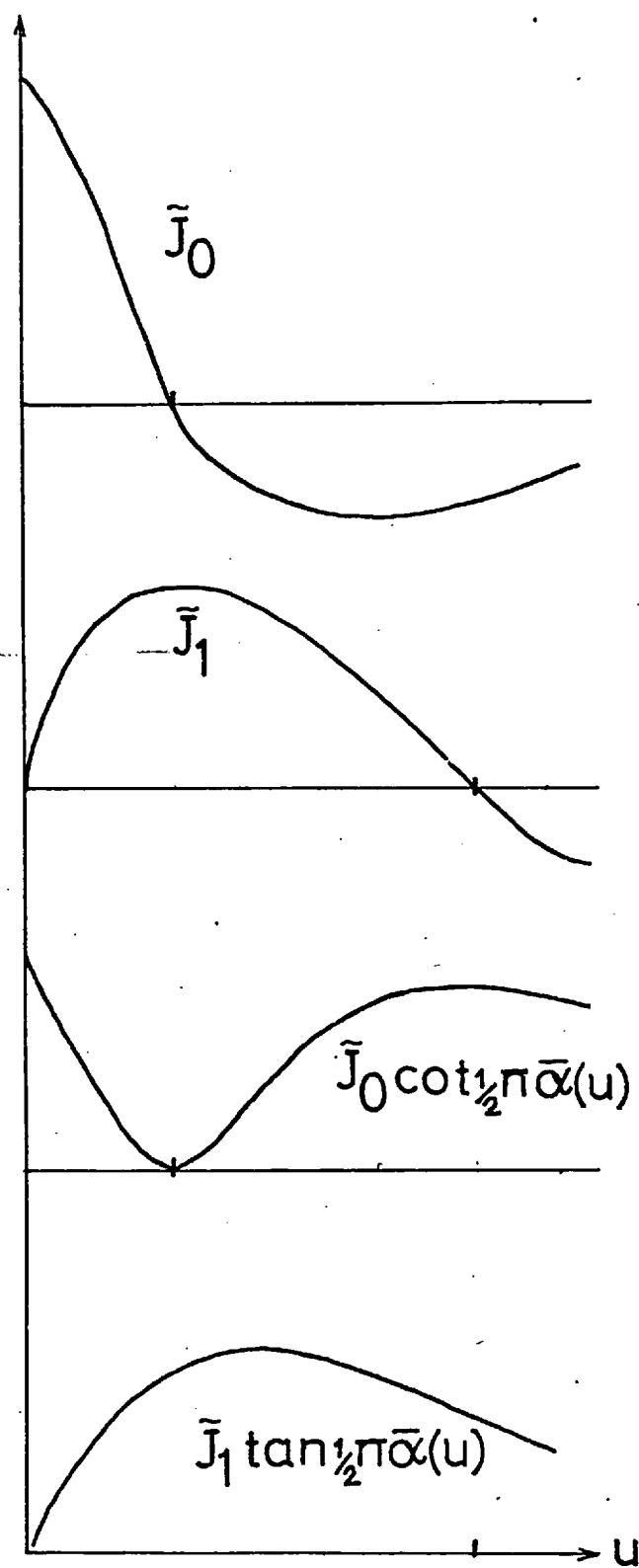


Fig. (4.5)

(b) Experimental Data

The differential cross-section (σ) and the polarization (P) are given by

$$\sigma_i = \sum_{\Delta\lambda} |G_{\Delta\lambda}^i|^2, \quad (\Delta\lambda = 0, 1)$$

$$\sigma_i P_i = 2 \operatorname{Im} (G_{\Delta\lambda=1}^i G_{\Delta\lambda=0}^{i*}), \quad (i = +, -, 0)$$

where for

$$\pi^+ p \rightarrow p \pi^+, \quad G^+ = \frac{1}{3} (2N + \Delta)$$

$$\pi^- p \rightarrow p \pi^-, \quad G^- = \Delta$$

$$\pi^- p \rightarrow n \pi^0, \quad G^0 = \frac{\sqrt{2}}{3} (N - \Delta)$$

(4.29)

Alternatively, the set of observables $(\sigma_+, \sigma_-, \sigma_0)$ can be replaced by a new set $(\sigma_N, \sigma_\Delta, D)$ (5);

$$\sigma_N = \sum_{\Delta\lambda} |N_{\Delta\lambda}|^2$$

$$\sigma_\Delta = \sum_{\Delta\lambda} |\Delta_{\Delta\lambda}|^2$$

$$D = \sum_{\Delta\lambda} \operatorname{Re} (N_{\Delta\lambda}^* \Delta_{\Delta\lambda})$$

(4.30)

and the relations between the two are

$$\sigma_{\Delta} = \sigma_{-}$$

$$\sigma_N = \frac{1}{2} [3(\sigma_+ + \sigma_0) - \sigma_{-}]$$

$$D = \frac{1}{4} [3(\sigma_+ - 2\sigma_0) + \sigma_{-}] \quad (4.31)$$

By using all the common data on $\sigma_{\pm,0}$ for $P_{lab} \geq 4 \text{ GeV}/c$ ⁽⁹⁾, we obtain the experimental σ_{Δ} , σ_N and D from (4.31), which are then used in (4.30), along with the 6 GeV/c data on polarization $P_{\pm}$ ^(10,11), for our fits.

(c) Analyses and Fits

Our model, as given in Table I, involves the parameters $A_N^{\Delta\lambda}$, $B_N^{\Delta\lambda}$, r_N , $A_{\Delta}^{\Delta\lambda}$, $B_{\Delta}^{\Delta\lambda}$ and r_{Δ} which are not all free. The appropriate parameters are fixed in the following way:

We start by looking at the 6 GeV/c data. The parameters A_N^0 and A_{Δ}^0 are fixed by the backward peaks (at $u' = 0$) of σ_N and σ_{Δ} . We obtain

$$A_N^0 \simeq 9.2 ; \quad A_{\Delta}^0 \simeq 1.7$$

The radius r is assumed to be independent of the helicity change $\Delta\lambda$ and the u -channel isospin. We take $r_N^{\Delta\lambda} \simeq r_{\Delta}^{\Delta\lambda} = r \simeq 1 \text{ Fermi}$, and exact values are obtained by requiring $J_0(r_N \sqrt{-u'})$ and $J_1(r_{\Delta} \sqrt{-u'})$ to vanish at u -values where poles develop in $\cot \frac{1}{2}\pi\bar{\alpha}_N$ and $\tan \frac{1}{2}\pi\bar{\alpha}_{\Delta}$, where $\alpha_N(u) \simeq -0.45 + 0.95u$, $\alpha_{\Delta}(u) \simeq 0.0 + 0.95u$. Thus $\text{Re } N_0$ and

and $\text{Re } \Delta_1$ have non-zero finite values at $u \approx -1.0$ and $u \approx -0.5$, respectively.

We are therefore left with three free parameters (at every s) A , B^0 and B^1 for each N and Δ amplitude. There are two possibilities of having the same or opposite relative sign between the flip and non-flip amplitudes, corresponding to $A^1 > 0$ or < 0 , respectively. In each case, the unknown functions C_N and C_Δ are determined as follows.

C_N : The isospin bounds on the polarization (P_N) due to N -exchange amplitudes⁽⁵⁾;

$$P_{N_{\text{lower}}} \leq P_N \leq P_{N_{\text{upper}}}$$

where

$$P_{N_{\text{lower}}} = \frac{1}{4} \left\{ 3 \left[\frac{\sigma_+}{\sigma_N} (1 + P_+) \right]^{\frac{1}{2}} - \left[\frac{\sigma_-}{\sigma_N} (1 + P_-) \right]^{\frac{1}{2}} \right\}^2 - 1 ,$$

$$P_{N_{\text{upper}}} = 1 - \frac{1}{4} \left\{ 3 \left[\frac{\sigma_+}{\sigma_N} (1 - P_+) \right]^{\frac{1}{2}} - \left[\frac{\sigma_-}{\sigma_N} (1 - P_-) \right]^{\frac{1}{2}} \right\}^2 ,$$

(4.32)

may be converted, using our amplitudes in Table I, to the isospin bounds on C_N , namely

$$C_{N_{\text{lower}}} \leq C_N \leq C_{N_{\text{upper}}} \quad (4.33)$$

The bounds on C_N , of course, depend on the parameters A_N and B_N . In both cases $A_N^1 \gtrless 0$, we normalize C_N at $u' = 0$

according to the Regge-pole phase and thus it is expected to be small near $u = u_d$ ($u_d \approx -0.15$). As a start, we take $C_N = 0$ at $u = u_d$ and estimate A_N^1 from the value $\sigma_N(u_d)$ for $B_N^1 = 1$. With these values for A and B, an estimate of the bounds on C_N are obtained. The inset of Fig.

(4.8b) shows the C_N bounds for $A_N^1 > 0$. We then take an input C_N , as shown in the figure, for the detailed fitting.

C_Δ : Since we have both the σ_- and P_- data at 6 GeV/c for fitting, we parametrize C_Δ , with two additional free

parameters, in several different ways but in each case normalizing to the Regg-pole phase at $u' = 0$. Then we proceed to fit simultaneously the data for σ_N , D , σ_- and P_- all at 6 GeV/c, first by taking $A^1 < 0$, which corresponds to the phase convention used in our previous qualitative analysis⁽²²⁾. The fits obtained are bad in the sense that the parameter values for B's are unreasonably large.

In the case $A^1 > 0$, which corresponds to the phase convention used by Ferro-Fontan⁽²⁴⁾ and by Takahashi and Kohsaka⁽²⁵⁾, we obtain excellent fits for all $u \geq -0.6$, which are shown in Figs. (4.6b), (4.7), (4.10b) and (4.11b). We obtain an overall χ^2/ND ($ND = \text{No. of degrees of freedom}$) of about 1.5. The corresponding N and Δ amplitudes are displayed in Figs. (4.8) and (4.9), and the calculated cross-sections and polarizations are shown in Figs. (4.10) and (4.11) respectively.

(d) The N-Amplitude

From the fits we obtain the values for the parameters

A_N^1 , B_N^0 and B_N^1 as tabulated in Table II. We find that $B_N^0 \approx B_N^1$, and the magnitude is quite consistent with those obtained in the DAM-analysis of forward scattering⁽⁴⁾.

$\text{Im } N_0$ and $\text{Im } N_1$ are clearly peripheral, since the width of the corresponding b-distributions are much smaller than the radius r_N . However, the real parts are clearly not peripheral. Contrary to the conventional Regge pole or weak-cut models our peripheral $\text{Im } N_1 \sim J_1$ is non-zero at $u = u_d$. But we found that this non-zero (but small) background is required in order to obtain the beautiful fits shown in Fig. (4.6).

In view of this encouraging support for the model, we also examine the s-behaviour of the parameters r_N and B_N . The magnitude of backward peaks of σ_N at different s gives

$$A_N^0 \approx (283 \pm 25) s^{-1.4} \quad (4.34)$$

and we assume an exactly similar s-dependence for A_N^1 , the normalization being given by the value at 6 GeV/c. Thus we obtain

$$A_N^1 \approx (77.4 \pm 8.0) s^{-1.4} \quad (4.35)$$

We then proceed to fit the data on σ_N having, at each energy, $B_N^0 = B_N^1$ and r_N as free parameters, and the best fits are shown in Fig. (4.6). The values obtained are shown in Table II. As in the forward case, r_N is found to be nearly constant in s (~ 1 Fermi) and B_N increases like $\ln s$.

corresponding to the usual Regge shrinkage.

(e) The Δ -Amplitude

The parameter values for the Δ -amplitude obtained from the best fits are given in Table II. Although we have $B_{\Delta}^0 \approx B_{\Delta}^1$, the magnitudes are rather large and the result is a poor peripheral nature of the b-distributions for $\text{Im } \Delta_0$ and $\text{Im } \Delta_1$. The fits to D , $\sigma_{\perp}$ and $P_{\perp}$ as shown in Figs. (4.7), (4.10b) and (4.11b) are reasonably good down to $u \approx -0.6$, showing the approximate correctness of the phases of the Δ -amplitudes. The behaviour of the Δ -amplitudes are displayed in Fig. (4.9).

A more detailed look at the Δ -amplitudes seems to suggest that $\text{Re } \Delta \sim J_1(r\sqrt{-u}) \tan \frac{1}{2} \pi \bar{\alpha}_{\Delta}$ should be very small or zero near $u \approx -0.5$, the solution obtained in the DAM-based amplitude analysis of Ref. (25). This form of is rather hard to realize in a simple DAM. However, particular asymptotic energy behaviour of the parameters B and r , corresponding to the case (A) and (C) in the previous section, could lead to non-Regge-pole-type J-plane structure (e.g., fixed poles, a super-position of complex-conjugate pairs of poles etc.). Then the resulting real parts would also have zero structure around u (or t) values where the corresponding imaginary parts vanish. This kind of mechanism seems to operate for both $\text{Re } \Delta_0$ and $\text{Re } \Delta_1$.

4.5 Conclusions

We have applied a simple form of DAM to backward scattering with the intention of explaining the observed

features of the πN differential cross-sections and polarizations. We found that the model works very well for the N -amplitudes for all $u > -1.0$.

The peripheral N - and Δ - amplitudes have an interaction radius of $r \simeq 1$ Fermi. r_N varies very little with energy while B_N increases like $\ln s$, which is compatible with standard Regge behaviour. Also the magnitude of B_N is consistent with the corresponding forward values. Contrary to standard Regge pole or weak-cut models, our $\text{Im } N_1$ is non-zero at $u \simeq u_d$. However, $\text{Re } N_1$ has a zero around this point. In our model σ_N is almost totally dominated by the non-flip (N_0) amplitude; the flip-amplitude (N_1) being very small near the backward direction presumably due to the presence of a strong cut.

The model also works reasonably well for the Δ -amplitude for $u > -0.6$. The model, however, seems to fail to give the correct $\text{Re } \Delta_1$ for regions beyond this value. The slope parameter B_Δ is rather large and the resultant impact parameter distributions $\text{Im } \Delta$ are much less peripheral. This inconsistency is interpreted as due to the incorrect form of $\text{Re } \Delta_1$, as given by the model. In order to obtain a consistent value for B_Δ , $\text{Re } \Delta_1$ must be very small or zero near $u \simeq -0.5$. This type of behaviour for $\text{Re } \Delta_1$ may be achieved in other classes of DAM where both B and r behave like constant (in s) or both increase like $\ln s$. The corresponding consequences for the peripherality, dip positions and shrinkage of the differential cross-section will be different from that of our simple DAM. Strong

absorption models as well as a covariant version of the fixed-cut (Carlitz-Kislinger cut) model of chapter II are both consistent with our conclusion about $\text{Re } \Delta_1$.

Table I - The DAM-amplitudes

s-channel- helicity flip	N_α -exchange		Δ_δ -exchange	
	Im	Re	Im	Re
$\Delta\lambda=0$	$\tilde{J}_0$	$-\tilde{J}_0 \cot \frac{1}{2} \pi \bar{\alpha}_N$	$\tilde{J}_0$	$-\tilde{C}_A (?)$
$\Delta\lambda=1$	$\tilde{J}_1$	$\tilde{C}_N (?)$	$\tilde{J}_1$	$\tilde{J}_1 \tan \frac{1}{2} \pi \bar{\alpha}_\Delta$

Table II - Parameter values

	N			Δ	Units
P_{lab}	4	6	10	6	GeV/c
A_0 (fixed)	15.5	9.2	4.1	1.7	$\mu b / (GeV/c)^2$
A_1 (fixed)	3.95	2.34 [*]	1.28	3.4 [*]	"
B_0	0.73	0.93	1.45	4.0	$(GeV/c)^{-2}$
B_1	0.73	0.9	1.45	3.2	"
R	5.0	5.2 ⁺	5.2	4.8 ⁺	$(GeV/c)^{-1}$

* (+) : Free (fixed) for (σ_N , D, σ_- , P_-) simultaneous fits at 6 GeV/c.

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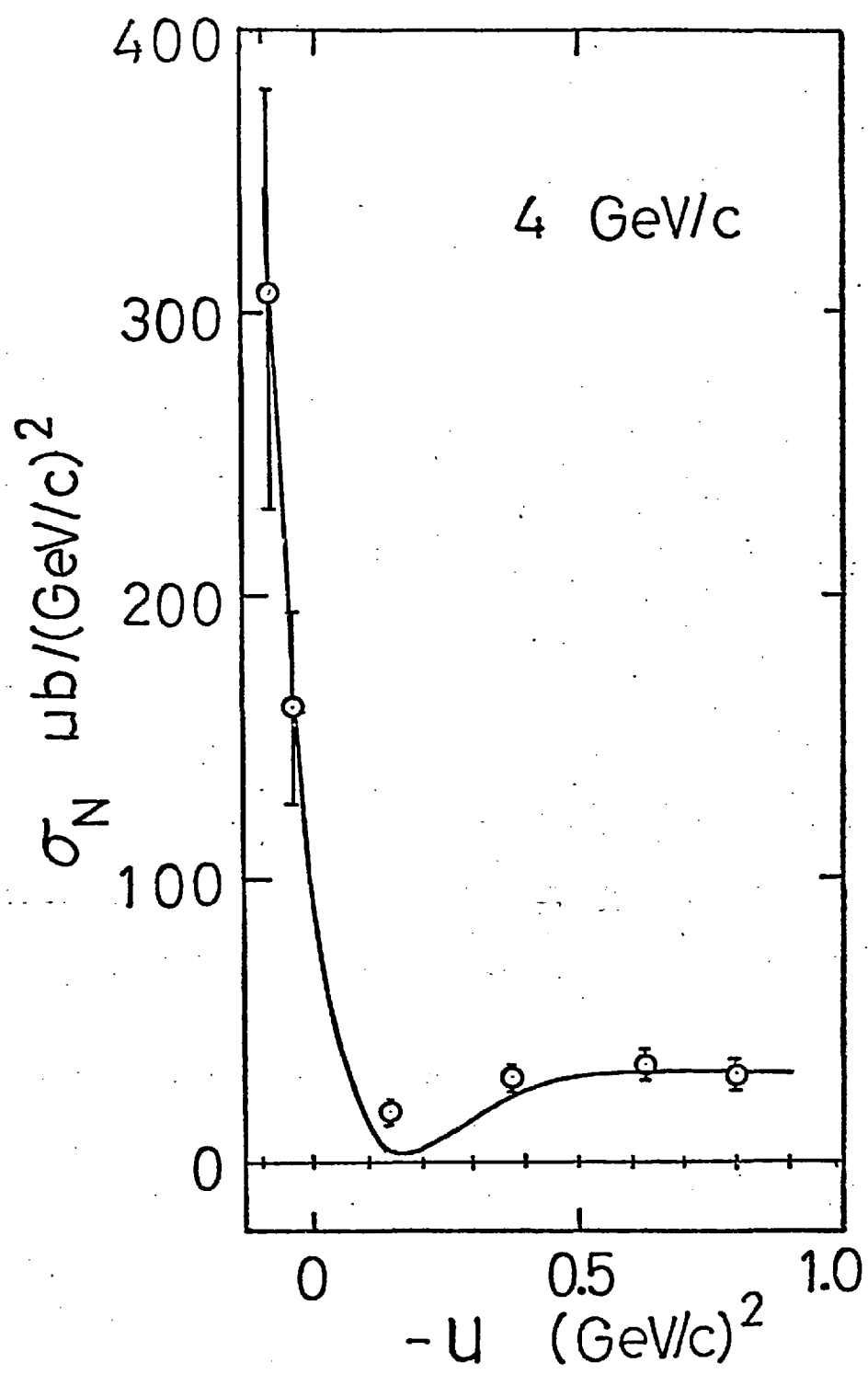


Fig. (4.6a)

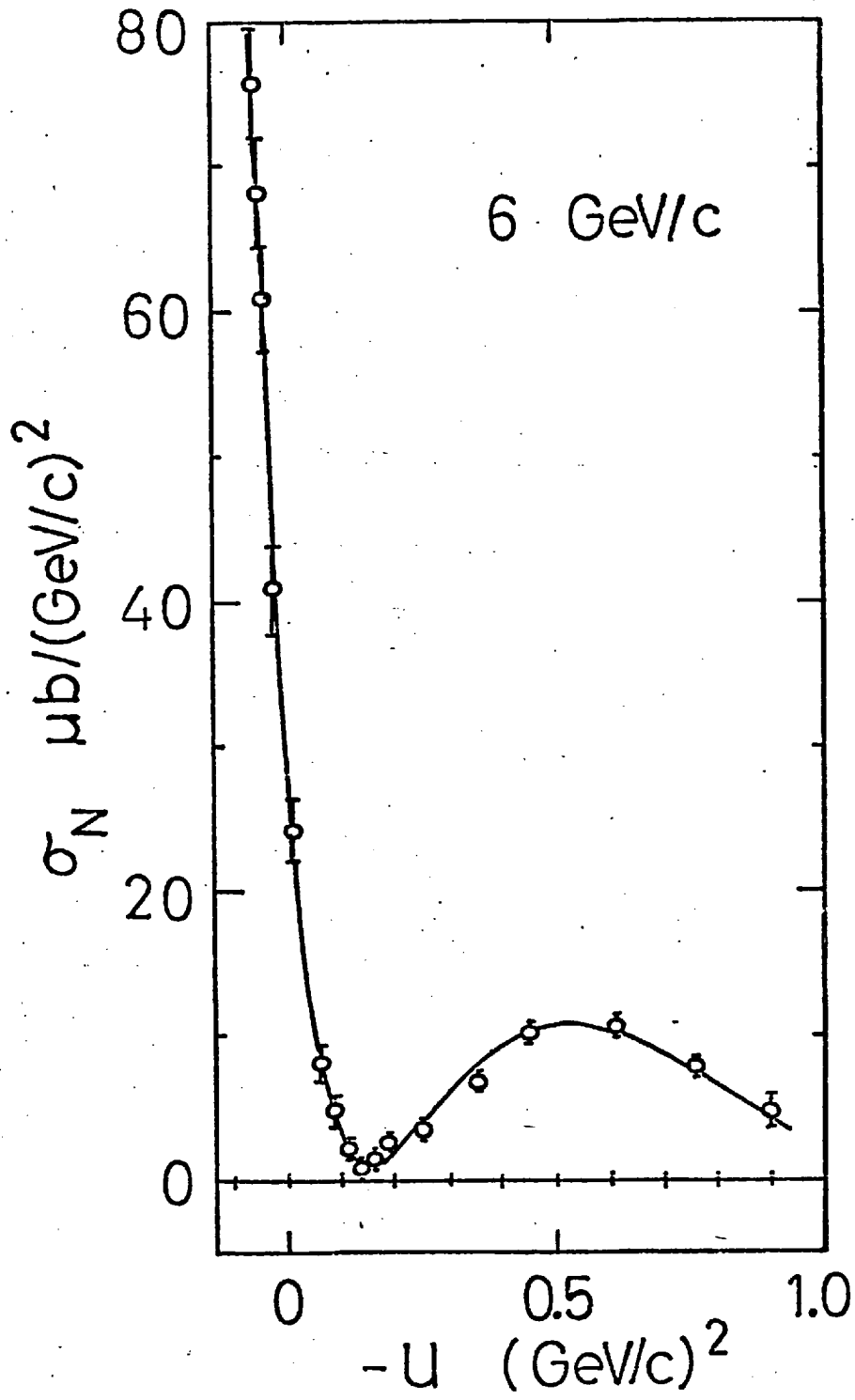


Fig. (4.6b)

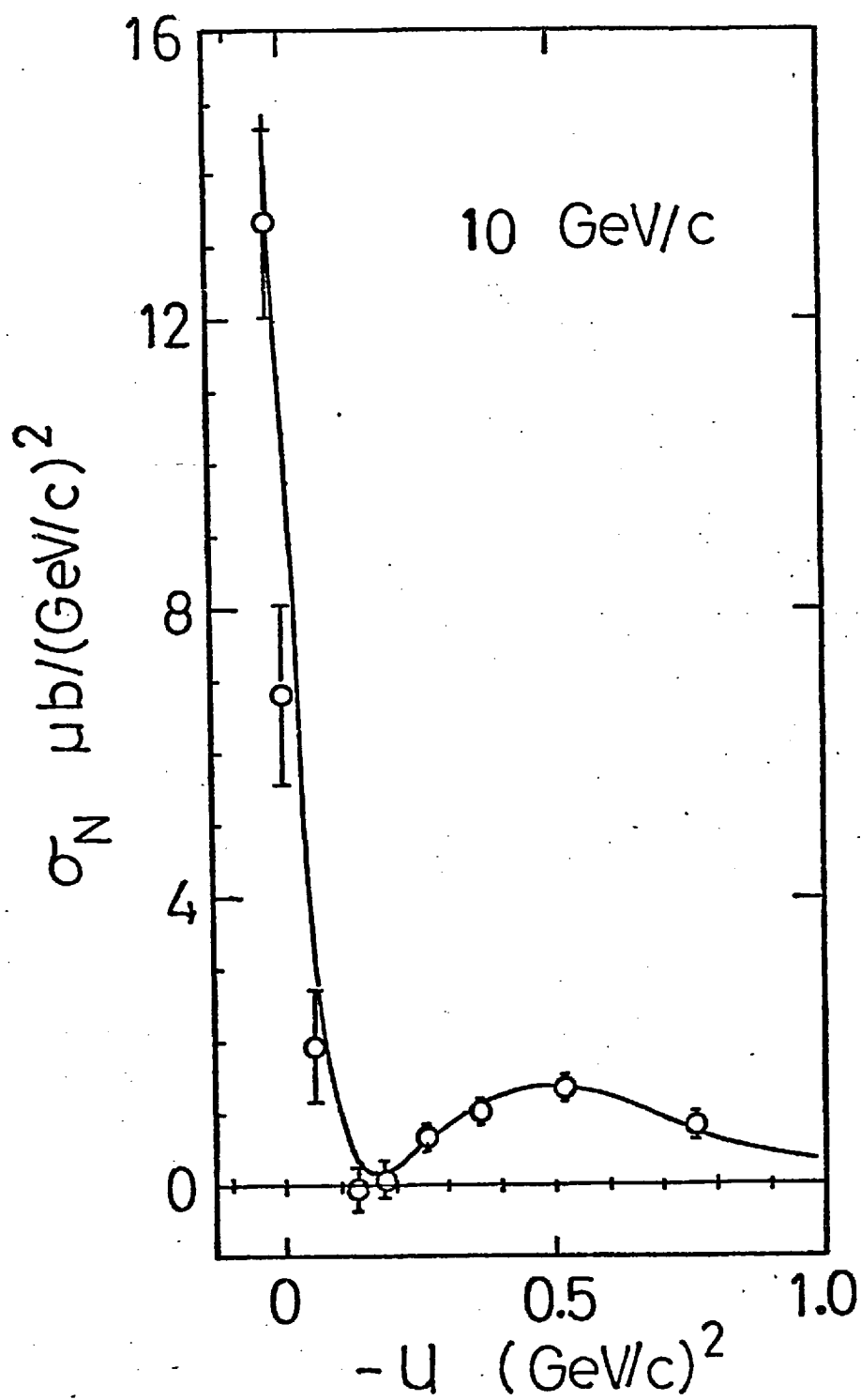


Fig. (4.6c)

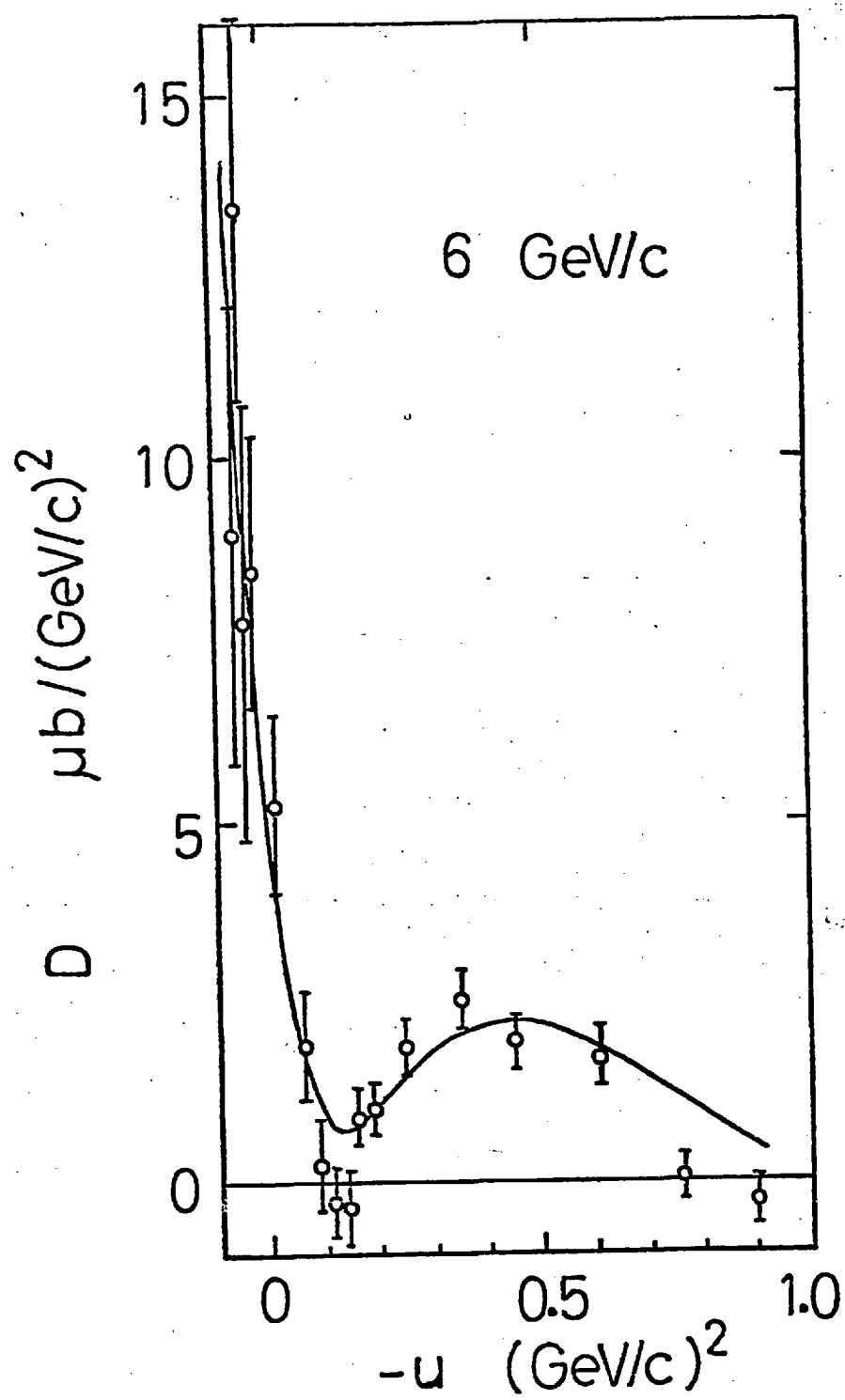


Fig. (4.7)

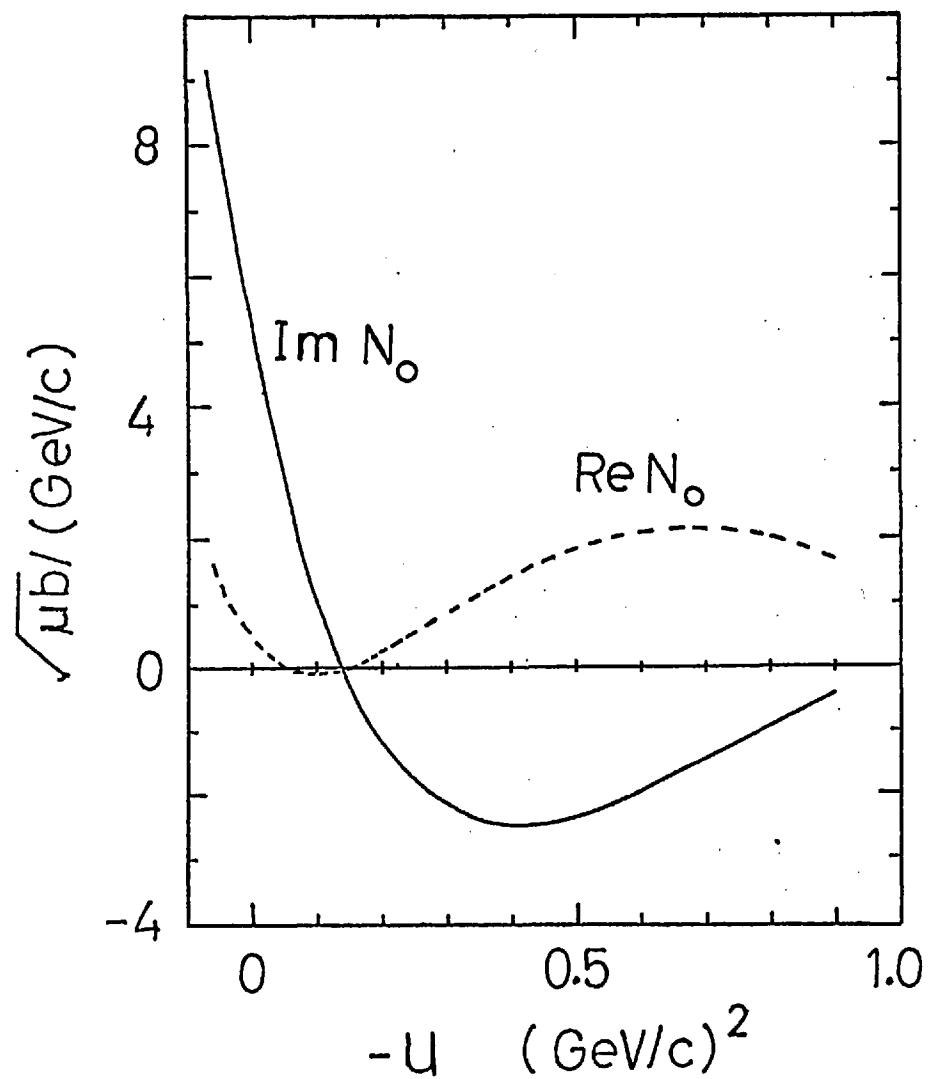


Fig. (4.8a)

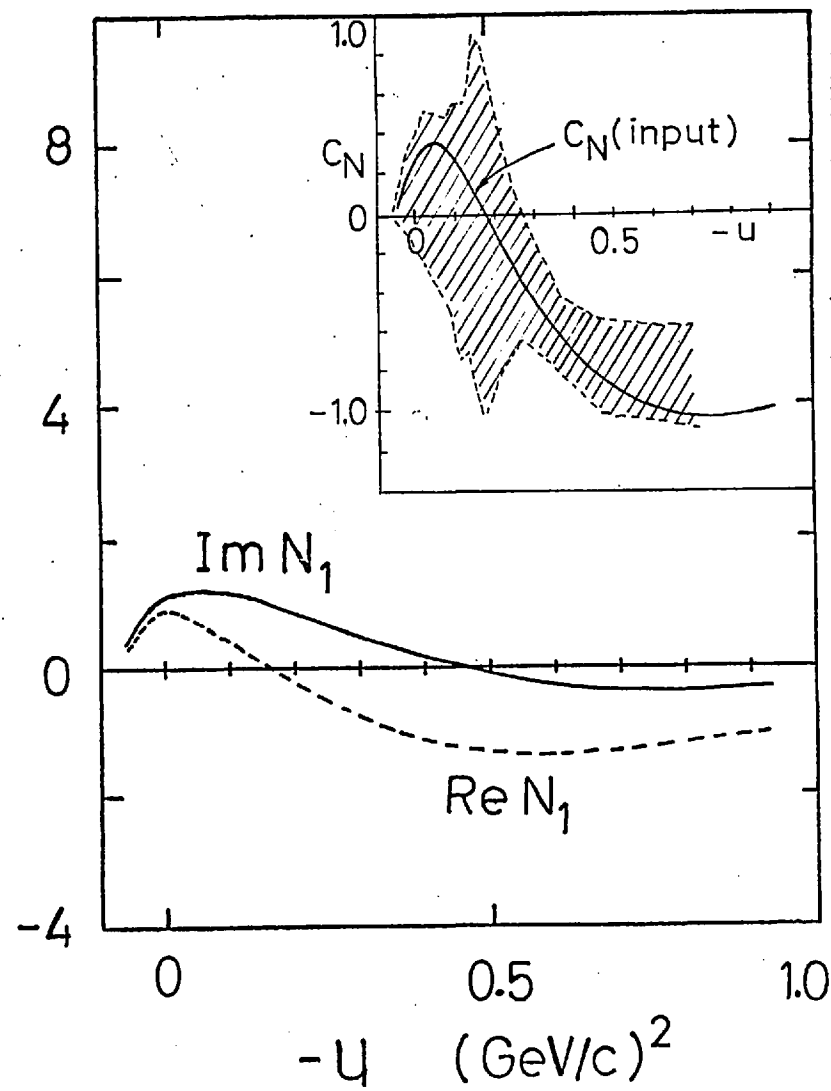


Fig. (4.8b)

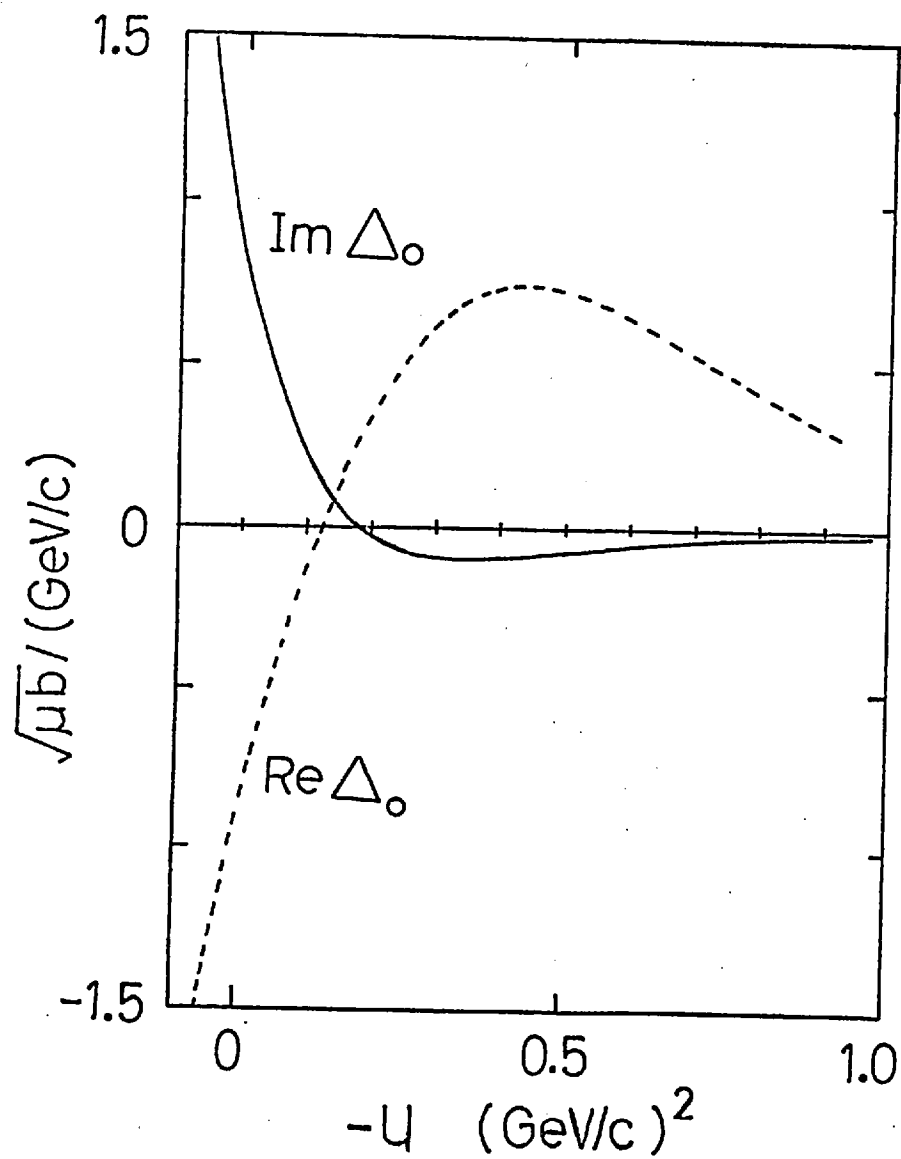


Fig. (4.9a)

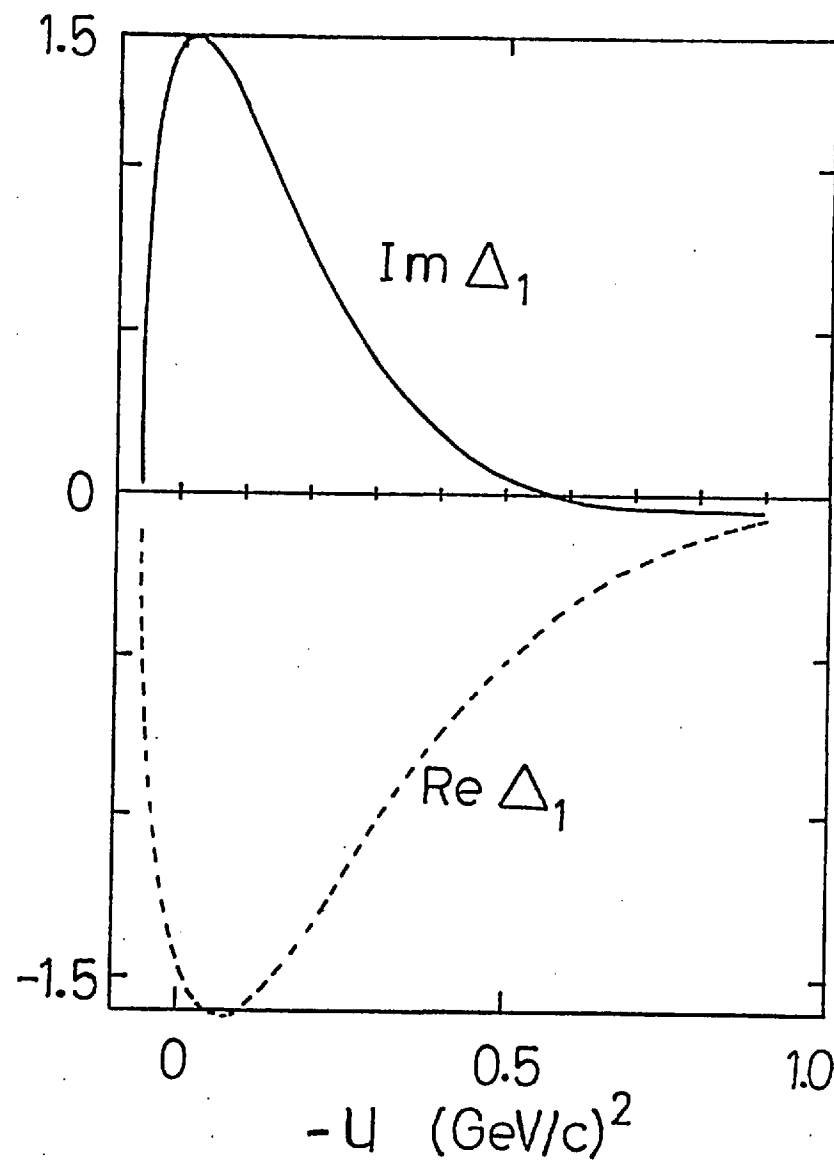


Fig. (4.9b)

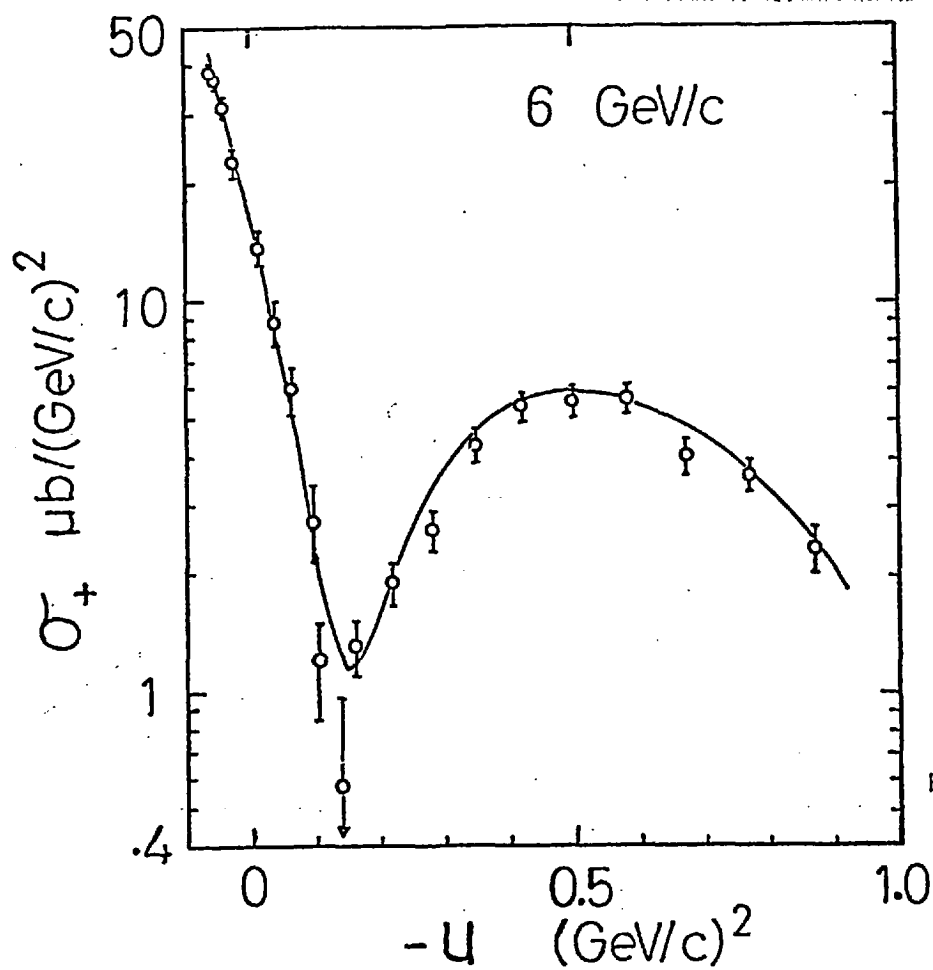


Fig. (4.10a)

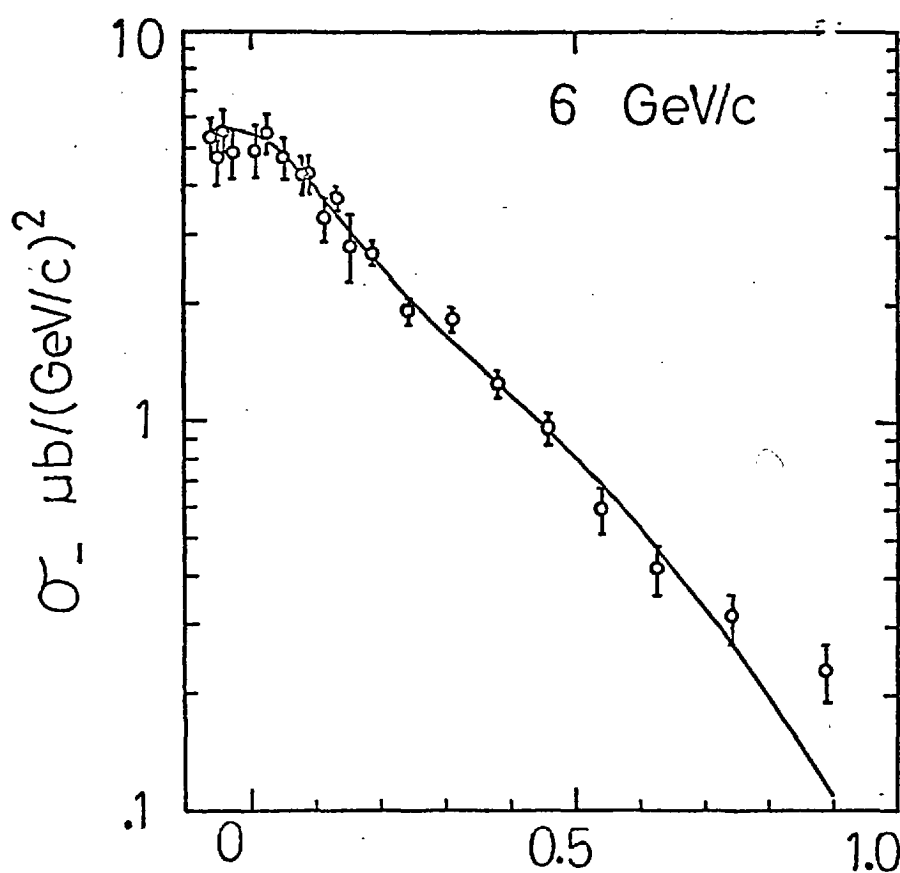


Fig. (4.10b)

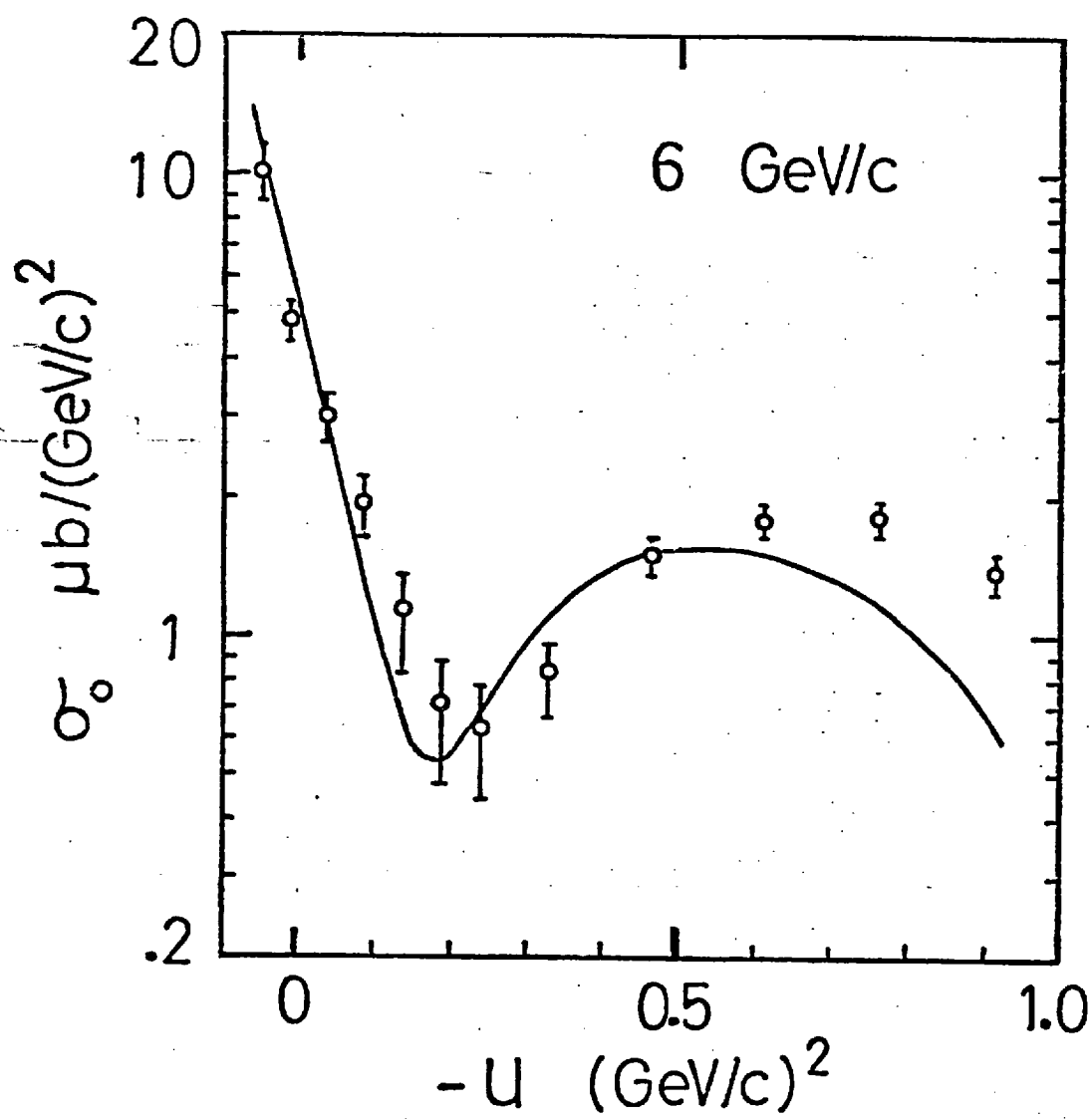


Fig. (4.10c)

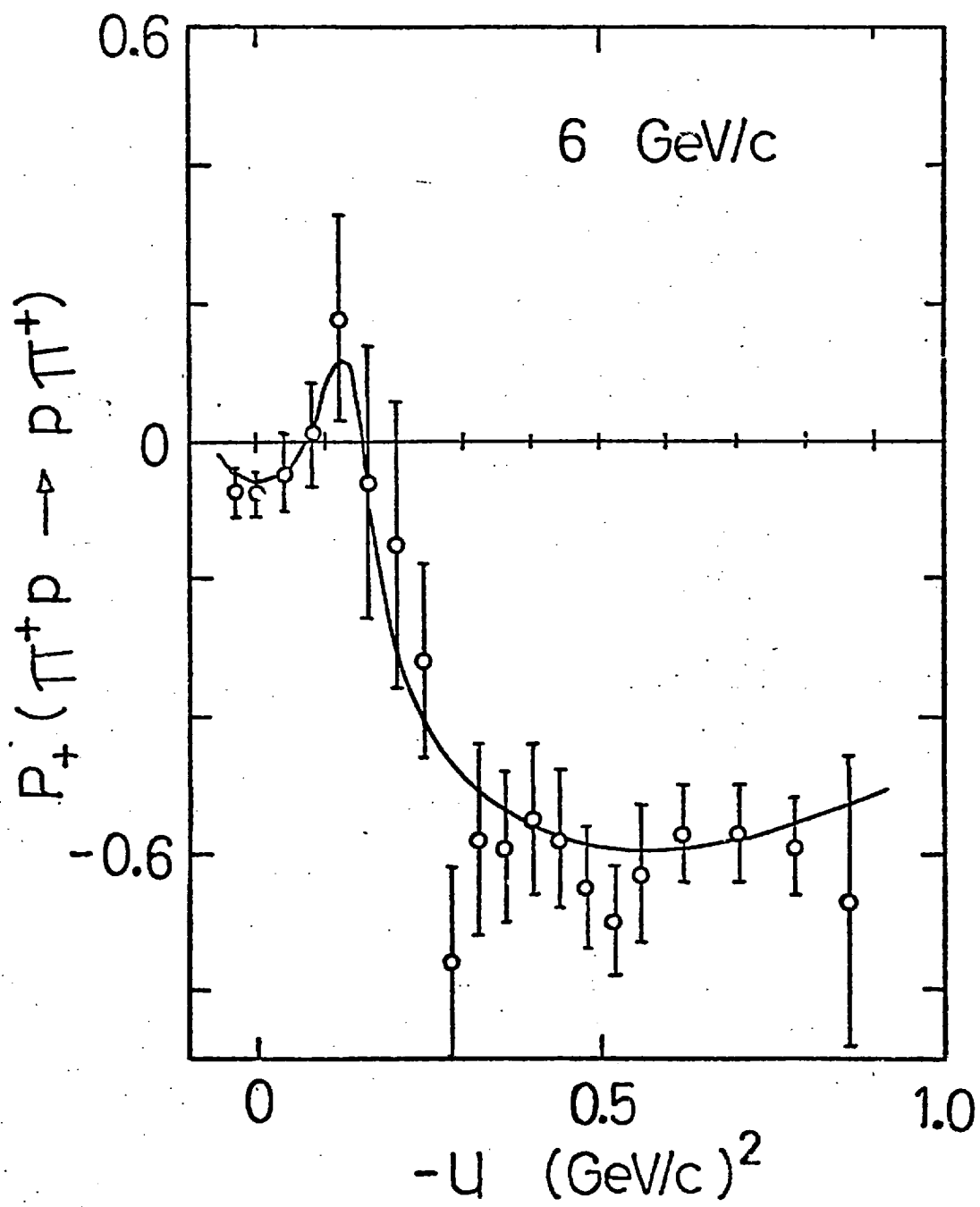


Fig. (4.11a)

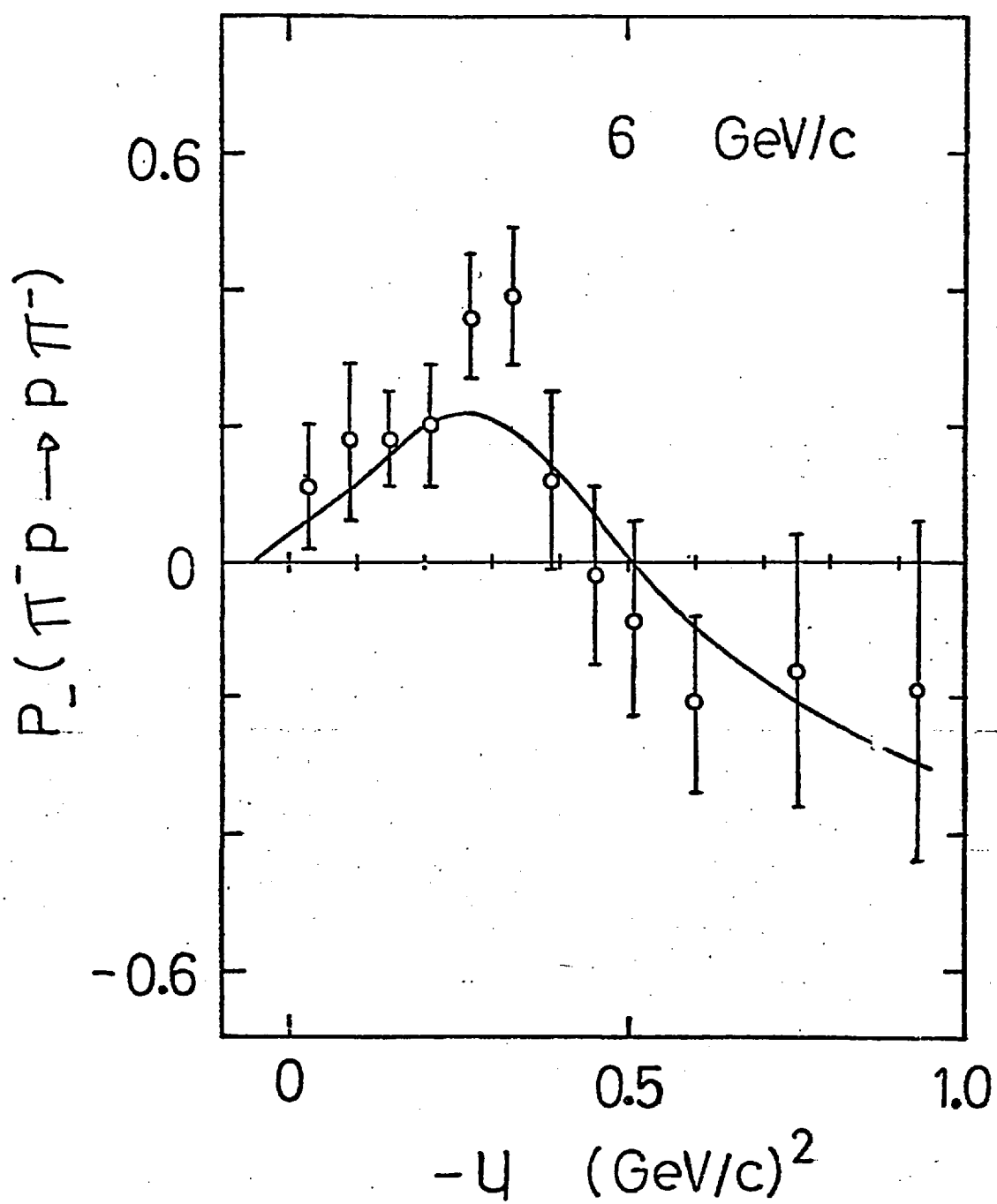


Fig. (4.11b)

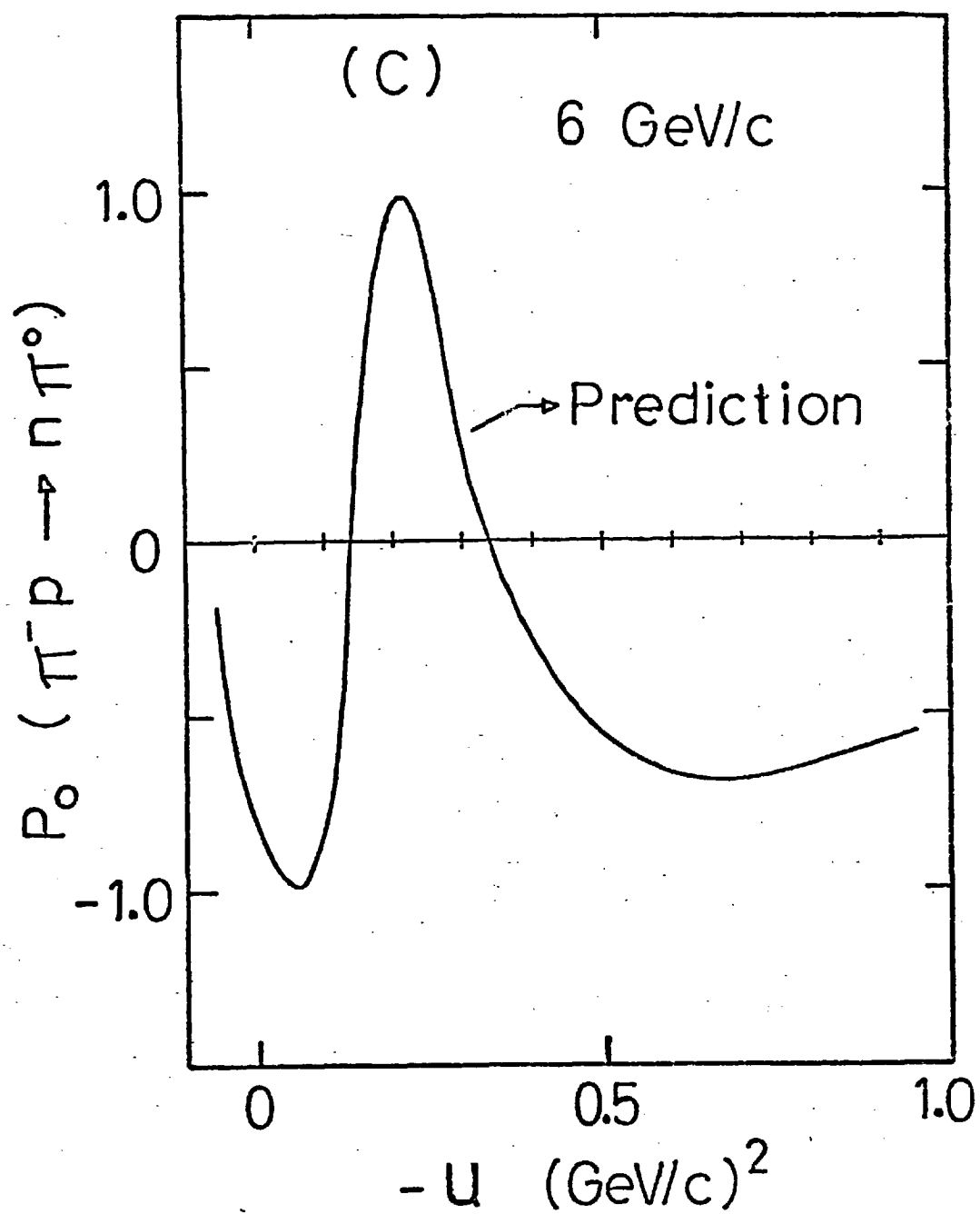


Fig. (4.11c)

Appendix A

(a) Dirac Spinors

The Dirac Spinors for the initial and final spin- $\frac{1}{2}$ fermions with s-channel helicity λ and λ' , respectively, are given by

$$u^{(\lambda)}(p) = \sqrt{E+m} \left(1 + \frac{i\gamma_5 p h}{E+m} \right) \phi^{(\lambda)}(\hat{p}),$$

$$\bar{u}^{(\lambda')}(p') = \sqrt{E'+m'} \phi^{(\lambda')}(\hat{p}') \left(1 - \frac{i\gamma_5 p' h'}{E'+m'} \right),$$

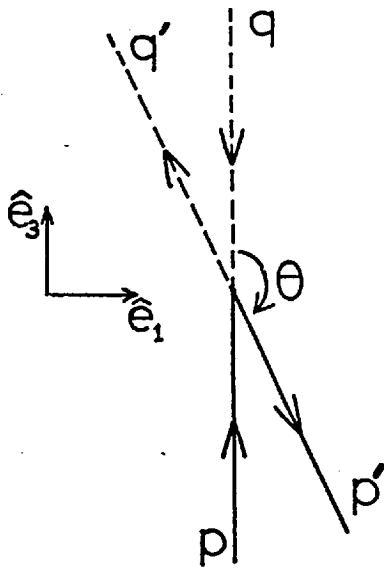


Fig. (A.1)

$$\text{with } \phi^{(\frac{1}{2})}(\hat{p}) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\phi^{(-\frac{1}{2})}(\hat{p}) = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$\phi^{(\frac{1}{2})}(\hat{p}') = \begin{pmatrix} \cos \theta/2 \\ \sin \theta/2 \end{pmatrix}$$

$$\phi^{(-\frac{1}{2})}(\hat{p}') = \begin{pmatrix} -\sin \theta/2 \\ \cos \theta/2 \end{pmatrix},$$

(A.1)

and $p = |p|$, $h \equiv 2\lambda$, where we have taken $\underline{p}$ along $\hat{e}_3$, $\underline{q}$ along $-\hat{e}_3$, $\underline{p}'$ at an angle θ ($\phi = 0$) with respect to $\hat{e}_3$ and $\underline{q}'$ at an angle $(\pi - \theta)$ with respect to $\hat{e}_3$, as shown in Fig. (A.1). The Pauli representation is taken, where

$$\gamma_5 = \gamma_0 \gamma_1 \gamma_2 \gamma_3, \quad \underline{\gamma} = \gamma_0 (i\gamma_5) \underline{\sigma}, \quad \gamma_5^2 = -1, \quad \text{i.e.}$$

$$\gamma_0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad \underline{\gamma} = \begin{pmatrix} 0 & \underline{\sigma} \\ -\underline{\sigma} & 0 \end{pmatrix}, \quad (\text{A.2})$$

and the spinors are normalized according to $\bar{u}u = 2m$, $\underline{\sigma}$ are the usual Pauli matrices.

(b) The polarization vectors for spin-1 particles corresponding to the momentum q' in Fig. (A.1) are given by

$$\begin{aligned} \varepsilon_{\nu}^{(0)}(q') &= \frac{1}{\mu'} (q'; -\omega' \sin \theta, 0, -\omega' \cos \theta), \\ \varepsilon_{\nu}^{(\pm)}(q') &= \frac{1}{\sqrt{2}} (0; \pm \cos \theta, -i, \mp \sin \theta), \end{aligned} \quad (\text{A.3})$$

where $q' = |q'|$, with the normalization $\varepsilon^{*(\lambda)} \varepsilon^{(\lambda)} = -1$.

(c) In the case of a spin- $3/2$ fermion, the spinor-vector wave function is given by

$$u_{\mu}^{(\lambda)}(p) = \sum_{r,s} (1 \frac{1}{2} r s | \frac{3}{2}, \lambda) \varepsilon_{\mu}^{(r)}(p) u^{(s)}(p),$$

where $(1 \frac{1}{2} r s | \frac{3}{2}, \lambda)$ are the Clebsch-Gordon coefficients and ε_{μ} and u are the spin-1 and Dirac Spinor wave functions. Explicitly

$$\begin{aligned} u_{\mu}^{(3/2)}(p) &= \varepsilon_{\mu}^{(+)} u^{(1/2)} \\ u_{\mu}^{(1/2)}(p) &= \frac{1}{\sqrt{3}} (\varepsilon_{\mu}^{(+)} u^{(-1/2)} + \sqrt{2} \varepsilon_{\mu}^{(0)} u^{(1/2)}) \\ u_{\mu}^{(-1/2)}(p) &= \frac{1}{\sqrt{3}} (\varepsilon_{\mu}^{(-)} u^{(1/2)} + \sqrt{2} \varepsilon_{\mu}^{(0)} u^{(-1/2)}) \\ u_{\mu}^{(-3/2)}(p) &= \varepsilon_{\mu}^{(-)} u^{(-1/2)}. \end{aligned}$$

Appendix B

(a) πN Kinematics

As stated earlier, the kinematics for πN backward scattering are defined by taking p, q and p', q' for the in- and out-going nucleon and pion momenta, as shown in Fig. (B.1), so that

$$s = (p + q)^2$$

$$t = (p - p')^2$$

$$u = (p - q')^2$$

$$p + q = p' + q', \quad p'^2 = p^2 = m^2,$$

$$q'^2 = q^2 = \mu^2, \quad Q \equiv \frac{1}{2}(q' + q)$$

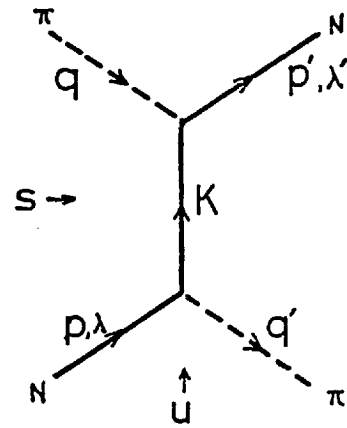


Fig. (B.1)

(B.1)

The s-channel helicity amplitudes are given in terms of the usual invariant amplitudes A and B as

$$T_{\lambda'\lambda} = \bar{u}^{(\lambda')}(p') (A + B \not{Q}) u^{(\lambda)}(p), \quad (B.2)$$

and explicitly,

$$T_{++} = 2 [m A + (E \sqrt{s} - m_N) B] \cos \theta_{1/2}$$

$$T_{+-} = 2 [E A - m_N (E - \sqrt{s}) B] \sin \theta_{1/2} \quad (B.3)$$

where $\pm$ stands for nucleon helicity $\pm \frac{1}{2}$, E and m_N are the c.m. energy and mass of the nucleon and θ is the c.m.

scattering angle.

At fairly large s , we can approximate

$$T_{++} \sim \sqrt{-u} B s^{\frac{1}{2}} \quad (\text{helicity flip})$$

$$T_{+-} \sim (A + m_N B) s^{\frac{1}{2}} \quad (\text{helicity non-flip})$$

(B.4)

The A and B amplitudes are specifically given by the model.

The differential cross section and the polarization are respectively given by the expressions

$$\frac{d\sigma}{du} = \frac{389}{64 \pi k^2 s} (|T_{++}|^2 + |T_{+-}|^2) \quad \mu b / (\text{GeV}/c)^2,$$

and

$$P \frac{d\sigma}{du} = 2 \operatorname{Im} (T_{++} T_{+-}^*)$$

(B.5)

(b) Pole Couplings

We define the usual pion-nucleon coupling constant, $g_{\pi NN}$ by writing the N_α -pole amplitude ($u \approx m^2$) as

$$N_\alpha = g_{\pi NN}^2 \frac{1}{2m} \frac{\bar{u} \gamma_5 (\not{K} + m) \gamma_5 u}{(u - m^2)}$$

$$= -g_{\pi NN}^2 \frac{1}{2m} \frac{\bar{u} \not{Q} u}{(u - m^2)},$$

(B.6)

where we adopt the normalization $\bar{u}u = 1$, to be consistent with the trace calculation in section 3.4.

For the pion-nucleon- Δ coupling, g_Δ , we write the decay amplitude for the process $\Delta \rightarrow N \pi$ as

$$T = \frac{g_\Delta}{m_N} \bar{u}(p') \not{p}'_\mu u_\mu(k) \quad (\text{B.7})$$

and the c.m. decay width as

$$\Gamma_\Delta = \frac{1}{4\pi} \frac{k}{2m_\Delta^2} \frac{1}{4} \sum_{\text{hel.}} |T|^2$$

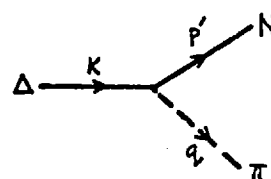


Fig. (B.2)

where k is the c.m. three-momentum of the nucleon.

Explicitly, we have

$$\Gamma_\Delta = g_\Delta^2 \frac{k^3}{24\pi m_N^2 m_\Delta^2} \left[(m_\Delta + m_N)^2 - m_\pi^2 \right] \quad (\text{B.8})$$

which gives, for $\Gamma_\Delta = 120$ MeV, the value $g_\Delta \simeq 14.5$.

Using (B.7), the elementary $\Delta(1236)$ -pole amplitude in πN scattering is given, to leading order as

$$\Delta_s \sim \left(\frac{g_\Delta}{m_N} \right)^2 \frac{s}{4m} \bar{u}(p') \frac{(-2m + \not{q})}{(u - m^2)} u(p). \quad (\text{B.9})$$