Energy Reports 11 (2024) 5800–5818

Contents lists available at ScienceDirect

Energy Reports

journal homepage: www.elsevier.com/locate/egyr

Research paper

Stochastic optimisation model to determine the optimal contractual capacity of a distributed energy resource offered in a balancing services contract to maximise profit

Ussama Rai [a,*,1], Gbemi Oluleye [b,2], Adam Hawkes [c,3]

[a] Department of Chemical Engineering, Imperial College London, South Kensington Campus, C501, Roderick Hill Building, London SW7 2AZ, UK
[b] Centre for Environmental Policy, Prince's Gardens, Imperial College London, South Kensington Campus, Weeks Building,16-18, London SW7 1NE, UK
[c] The Sargent Centre for Process Systems Engineering, Imperial College London, South Kensington Campus, Roderick Hill Building,, London SW7 2AZ, UK

ARTICLE INFO

Keywords:
Stochastic optimisation
Distributed energy resources
Short-term operating reserve
System operator
Balancing Service
Demand response

ABSTRACT

In the realm of grid balancing services, determining the generation capacity of a distributed energy resource for contractual agreements with the system operator is pivotal. However, prevalent heuristic or deterministic methodologies employed by demand response aggregators often lack risk assessment and may not optimize generation capacity allocation. Consequently, the potential for maximizing utilization profits remains untapped. This paper addresses these limitations and explains the necessity of using the optimal generation capacity of a grid-connected distributed energy resource which is also fulfilling site electricity demand to maximise profit and mitigate penalties both for demand response aggregators and their clients. Demand response aggregators provide these services to the system operator on behalf of their clients whose electrical generation assets they utilize on a profit-sharing basis. The primary challenge investigated in this study lies in effectively managing the uncertainty surrounding both site electricity demand and short-term operating reserve calls by the system operator through a novel two-step approach. Firstly, a demand bin characterization technique is employed to account for site demand uncertainty. Subsequently, a stochastic model utilizing mixed integer nonlinear programming is developed using the General Algebraic Modeling System, incorporating five years of uncertainty regarding the frequency of short-term operating reserve calls which makes it instrumental and novel in determining optimal contractual generation capacity in a balancing service contract, as well as associated profits and penalties, under varying utilization prices. This distinctiveness positions it as an advancement over and distinct from deterministic approaches. Case study results demonstrate the efficacy of the proposed stochastic model in comparison to deterministic methods utilized in prior research. Specifically, the stochastic model yields a realistic profit increase of 12.7% and offers 2.62% higher contractual capacity than the heuristic approach followed by the DR aggregator highlighting its potential to enhance profitability and operational efficiency in grid balancing services.

1. Introduction

The proliferation of distributed energy resources (DER) over the last decade has heralded a departure from traditional centralized power generation models towards decentralized, localized power generation units (Liu et al., 2019). This paradigm shift underscores a global commitment to bolstering the stability and sustainability of electrical grids through the widespread integration of DER, particularly renewable energy sources (Meegahapola et al., 2021). Within the demand response (DR) aggregation sector, the provision of balancing service (BS) contracts to system operators (SO) necessitates a rigorous assessment of the optimal generation capacity of DER (Rai et al., 2022).

There is a lack of understanding of the unit commitment (UC) problem, responsible for finding the optimal generation capacity of a grid-connected generator to be offered in a BS contract e.g. it is a

* Corresponding author.
E-mail address: u.rai17@imperial.ac.uk (U. Rai).
[1] ORCiD: https://orcid.org/my-orcid?orcid=0000-0002-2355-5530
[2] ORCiD: https://orcid.org/0000-0001-7753-0315
[3] ORCiD: https://orcid.org/0000-0001-9720-332X

https://doi.org/10.1016/j.egyr.2024.05.049
Received 12 November 2023; Received in revised form 11 May 2024; Accepted 19 May 2024
Available online 29 May 2024
2352-4847/

Nomenclature

Sets

i ϵ I	Set of the total number of years
j ϵ J	Set of site electricity demand bins. (Site electricity demand bin characterization is the distribution of site electricity demand data in a year into various sections using a histogram i.e., dividing the entire range of annual half-hourly site demand data in a STOR year that runs from 1st of April till 31st of March the following year into a series of interludes and enumerating how many half-hourly site demand values fall into each interlude).

Parameters

$Ah_{(j)}$	Total number of Availability hours in each site electricity demand bin
$AP^{(Gen)}$	Generator availability price in £/Mw/h
$FC^{(Gen)}$	Generator Fuel cost in £/MWh
$Gen^{(Max)}$	Maximum electricity generation capacity of a generator in MW
Gen_m^{AH}	Generator availability hours per month
$Sc_{(i,j)}$	Total number of STOR dispatch calls in each site electricity demand bin each year
$UP^{(Gen)}$	Generator utilization price in £/MWh
Lsc	Duration/Length of STOR call-in hours
$SD_{(j)}$	Electricity demand in each site electricity demand bin

Variables

$P^{(Total)}$	Objective function- Total Annual Profit of the site for all Demand Bins (£s)
P_1^{DB}	Annual Profit for Demand Bin-1 (£s)
P_2^{DB}	Annual Profit for Demand Bin-2 (£s)
P_3^{DB}	Annual Profit for Demand Bin-3 (£s)
P_4^{DB}	Annual Profit for Demand Bin-4 (£s)
P_5^{DB}	Annual Profit for Demand Bin-5 (£s)
$Psc_{(j)}$	Penalty cost for under or no delivery for a single STOR call per site electricity demand bin in each year (£s)
$Pt_{(i,j)}$	Penalty cost for under or no delivery for the total number of STOR calls per site electricity demand bin each year (£s)
$X^{(Gen)}$	Optimal contractual generation capacity to offer in STOR market in MW

Positive Variables

$GFC_{(i,j)}$	Generator fuel cost for the total number of STOR calls per site electricity demand bin each year (£s)
$SAR_{(j)}$	Site availability revenue in each site electricity demand bin (£s)
$TAR^{(Site)}$	Total site availability revenue per year from STOR contract (£s)
$TGFC^{(Site)}$	Total site generator fuel cost per year from STOR contract (£s)
$TPS^{(Site)}$	Total site penalty cost per year from under or no delivery during STOR dispatch calls (£s)
$TUR^{(Site)}$	Total site utilization revenue per year from STOR contract (£s)
$SUR_{(i,j)}$	Site utilization revenue for the total number of STOR calls per site electricity demand bin each year (£s)

Binary Variables

$YP_{(i,j)}$	A binary variable that indicates if there is a penalty or not during a STOR dispatch call

complex optimisation problem with a large-scale, nonconvex, mixed-integer, and nonlinear nature. It is typically considered to be NP-hard problem (Rai et al., 2022). The inclusion of other components like renewable energy sources (RES) and battery energy storage systems (BESS) further amplifies the intricacy of the optimisation dilemma (Anjos and Conejo, 2017). These models remain unpractical due to a lack of comprehensive risk assessment by the DR aggregators (Laur et al., 2020), inadequate consideration of system integration (Park and Baldick, 2020) and the use of the heuristic and conservative approach in finding the optimal generation capacity to offer in the BS contract (Rai et al., 2022).

The motivation behind this study aims and hypothesizes to determine the optimal generation capacity of a grid-connected generator to be offered in a balancing service contract, specifically within the STOR market, by a DR aggregator. There is limited research available on this topic in the body of literature, especially the one focusing on the STOR market. The primary objective is to enhance revenue generation and mitigate penalty costs incurred due to under or non-delivery during grid dispatch calls. By optimizing generation capacity in alignment with market demand and operational constraints, this study seeks to provide actionable insights for improving the profitability and operational efficiency of DR aggregators within the STOR market.

The novelty of this research lies in its innovative approach to addressing the primary challenge of effectively managing uncertainty surrounding both site electricity demand and STOR calls by the SO. This study introduces a novel two-step methodology to tackle this challenge. Firstly, it utilizes a demand bin characterization technique to accurately account for site demand uncertainty. Subsequently, the research develops a stochastic model employing mixed integer nonlinear programming (MINLP) within the General Algebraic Modeling System (GAMS) framework. This model incorporates five years of uncertainty surrounding the frequency of STOR calls, making it unique in its ability to handle complex uncertainties inherent in balancing service contracts. By integrating these novel techniques, the research offers a comprehensive solution for determining optimal contractual generation capacity, as well as associated profits and penalties, across a spectrum of utilization prices within the balancing service contract framework.

1.1. Background

In order to ascertain the contractual generation capacity of a grid-connected generator for inclusion in the BS contract, aggregators commonly employ heuristic or deterministic methods that entail minimal or no risk (Rai et al., 2022). Nevertheless, heuristic approaches fail to produce optimal capacity or maximise profits for both DR aggregators and their clients. In this study, a novel Stochastic Mixed Integer Non-Linear Programming (SMINLP) optimization model is developed to determine the optimal generation capacity of a grid-connected generator to contract with the SO under uncertainty of STOR dispatch calls and uncertain site demand to increase site revenue and mitigate penalties due to reduced or failed delivery of power during a STOR dispatch call. The SMINLP maximizes site profitability by estimating the optimal generation capacity that a DR aggregator can provide for a grid-connected generator in a STOR contract while satisfying the site electricity demand.

1.2. Operational Mechanism of Short-Term Operating Reserve Calls

The purpose of STOR calls issued by the SO is to safeguard the electrical grid's dependability and stability in the face of erratic supply or variable demand. The SO monitors the grid continuously for any imbalances between the supply and demand of electricity (Eicke et al., 2021), (Laugs et al., 2020). A STOR request may be issued by the SO in response to a prospective shortage in generation capacity or an abrupt surge in demand, to rectify the imbalance. Upon identification of the requirement for additional reserve capacity, the SO proceeds to dispatch notifications to registered STOR providers typically DR aggregators. These DR aggregators have clients which consist of combined heat and power (CHP) plant operators, energy storage facilities, and other adaptable assets whose production can be rapidly increased or decreased in accordance with grid demands. Contractual agreements between DR aggregators and the SO generally delineate the conditions and terms under which the providers are obligated to furnish reserve capacity (Langendahl et al., 2019). The SO closely monitors the performance of participating DR aggregators throughout the duration of the STOR event to ensure that sufficient reserve capacity is maintained. The SO compensates DR aggregators for the provision of their reserve capacity and for responding to STOR requests.

If a client rejects a STOR call without a valid reason or without notifying the DR Aggregator in advance, the SO may levy penalties or sanctions under the terms of the DR aggregator's contract which are then imposed on the client by the DR aggregator. Refusing STOR requests repeatedly or neglecting to fulfill contractual commitments can impact the client's ability to secure future contracts from the DR aggregator for revenue generation.

To ensure maximum profitability for both the DR aggregator and the client of the DR aggregator whose DER are being utilized in a BS contract as shown in Fig. 1, it is necessary to strike a balance between various factors. These factors include the running cost of generation, the optimal generation capacity of the generator, the utilization price of the generator offered by the SO, site electricity demand, and the overall reliability of the system.

1.3. Formulation strategy

The formulation in this research explicitly employs scenario-based indexing in the GAMS mathematical model. The specific parameters, such as the total number of availability hours and the site electricity demand in each site demand bin per year are obtained from historical site data. This data inherently considers various scenarios and the formulation incorporates stochastic elements like the uncertainty in the number of STOR dispatch calls in each site demand bin. The uncertainty surrounding STOR dispatch calls can be attributed to the ability of the SO to make predictions and is contingent upon environmental factors, such as temperature, as well as significant events that necessitate a substantial increase in energy consumption. One method employed in this modeling framework for scenario generation is utilizing previous STOR calls data, as seen in Case Study 1 and Case Study 2. In the first case study, STOR maintains a constant dataset for a duration of five years and proceeds to plot the model against different levels of utilization costs. In the second case study, a dataset including diverse STOR calls data over a period of five years is utilized. Subsequently, Elexon data is employed to identify the corresponding utilization pricing associated with the majority of STOR calls observed during the five-year timeframe considered in this study.

1.4. Demand bin characterization

Furthermore, the examination of historical data pertaining to site demand facilitated the detection of trends in site demand. Five distinct scenarios are thereafter produced through the utilization of a sampling approach known as demand bin characterization. The purpose of this analysis is to represent the distribution of site electricity demand data over the course of a year using a histogram. Specifically, this involves dividing the complete range of annual half-hourly site demand data within a STOR year (from April 1st to March 31st of the following year) into a series of intervals. The objective is to determine the frequency of half-hourly site demand values that fall within each interval. In our modeling, we employed five scenarios for characterizing demand bins and the formulation has been specifically intended to encompass and account for uncertainty by utilizing a stochastic optimisation technique.

In addition to this, a binary variable was employed to identify the

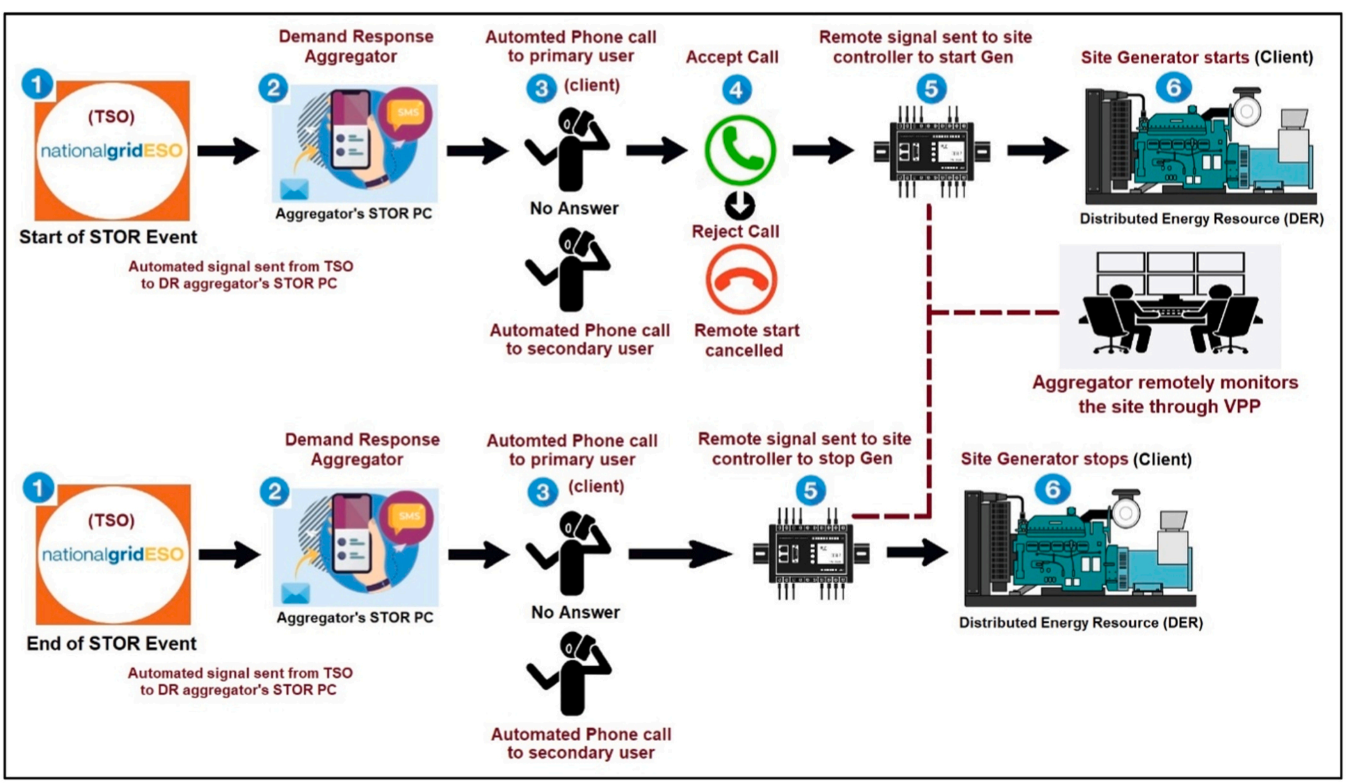


Fig. 1. A detailed depiction of STOR event dispatch through the demand response aggregator's perspective.

presence or absence of a penalty in the context of a STOR dispatch call across the five distinct scenarios. The availability hours for each of the five demand bins are calculated based on the real-time STOR windows data in each season published by the National Grid UK, as described in Appendix A.

DR aggregators have an additional degree of freedom to aggregate these DER and trade them in a single contract with the SO to contribute towards achieving system balancing (Zhang and Domínguez-García, 2018), (Lu et al., 2020), against the influx of renewables in the electricity grid (Radaideh and Ajjarapu, 2015) and counter grid electricity fluctuations (Albadi and El-Saadany, 2007). As a result, the research presented in this article is critical for DR aggregators to contract with the SO at the optimal contractual capacity of a DER that can be delivered during a STOR dispatch call without incurring any penalties due to no or reduced delivery of power. The implementation of this model primarily targets the energy market in the United Kingdom. However, it should be noted that its applicability extends beyond the confines of the UK's electricity market. The model incorporates all yearly short-term operating reserve services offered worldwide, albeit with adjustments to penalty and pricing structures. A taxonomy of the research is given in the taxonomy Table 1.

1.5. Importance of STOR as a balancing service and optimal contractual generation capacity in the current energy framework

Grid technology advancements continue to drive and shape the future of energy grids (Mileva et al., 2016), (Alotaibi et al., 2020) and the role of SO has evolved as a result of the integration of multiple small generators or one large generator in a single contract to participate in BS (Rai et al., 2022) using sophisticated pricing to meet the requirements of transitioning to decarbonized (Ziegler et al., 2019), decentralized (Alstone et al., 2015), and digitalized energy systems (Giordano and Fulli, 2012). The SO procures a substantial amount of its electricity through weather-dependent and variable renewables (Eftekharnejad et al., 2013), necessitating increased grid flexibility to provide an uninterrupted supply of electricity to satisfy demand. To minimize intermittency difficulties (Mazidi et al., 2014), backup generation capacity is provided to a SO in the form of small and medium-sized DER (Bayod-Rújula, 2009), ensuring a reliable balance of electricity generation and consumption (Mengelkamp et al., 2018).

Table 1
Taxonomy table indicating the novelty of the research.

Novelty Category	Description	Method Adopted
Stochastic optimization Modeling	Incorporated stochastic elements to model uncertainty in the number of STOR dispatch calls and site demand.	Using demand bin characterization to resolve site demand uncertainty
Market-Based Optimization	Adapted existing optimization techniques to the specific context of balancing service contracts.	Using Elexon five-year short term operating reserve calls data to incorporate the maximum number of STOR dispatch calls at various utilization prices
Economic Profit Maximization	Focused on maximizing profit as the primary objective within the framework of the balancing service.	Incorporating real revenues, availability, and utilization prices of the contracts and penalty factors into optimization
Balancing services contract	Explored optimization strategies for decentralized energy resources and addressed the optimization of contractual generation capacity offered in contracts for grid balancing.	Balancing supply and demand in energy services contract. Integrating gas/ diesel generators with the electricity grid not only supplying contractual generation capacity to the grid but also fulfills site electricity demand

The relevance of the STOR market has grown due to growing energy demand and the need to address environmental and socioeconomic challenges for long-term growth (Zia et al., 2018). Due to the vulnerability of the grid, which sometimes struggles to match the provision of electricity with demand, there is a proliferation of DER adoption around the world (Wang et al., 2015a), creating a network of decentralized electricity generation assets that require intelligent control and monitoring systems (Hatziargyriou, 2013) to operate effectively with the main power grid (Hatziargyriou et al., 2007). If the actual demand on the system exceeds the predicted demand or if there is unforeseen generation unavailability (Dugmore, 2022), STOR, as one of the BS, is critical to grid balancing.

As illustrated in Fig. 2, DER and DESS are expected to account for a significant portion of the total installed power production capacity by 2028. On-site power generation via various-sized generators and energy storage (Bortolini et al., 2014) has emerged as a way to increase the use of renewables to improve resilience (Fang et al., 2012), reduce carbon emissions (Adefarati and Bansal, 2019), increase stability (Lund et al., 2015), and lower energy costs (Rahbar et al., 2015), which has piqued the interest of both investors and DR aggregators in DER (Rai et al., 2022). When comparing May-July 2020 to the same period in 2019, the cost of auxiliary services jumped thrice from £200 m, emphasizing the importance of supplementary services in low-carbon systems (Badesa et al., 2021).

A general layout of the STOR dispatch call from the SO is shown in Fig. 3. The inability of renewables to match supply with demand reduces the SO ability to match supply with demand (Motevasel and Seifi, 2014). Concurrently, the STOR market is becoming more prevalent inside modern electrical systems (Faisal et al., 2018) because of its diverse generation resources, switch-off loads, and energy storage systems' dispatchable characteristics. A thorough technique for estimating a grid-connected DER contractual generation capacity has the potential to improve the cost-effectiveness and stability of electrical systems (Bell and Gill, 2018) while allowing DR aggregators to dispatch the most cost-effective and reliable assets. As a result, it is critical to assess the appropriate generation capacity of DER participating in the STOR service so that additional generation capacity that these DER can offer to improve contractual revenue is not overlooked. The growing penetration of DER, renewables, and demand response in today's power grids needs a set of advanced strategies and technologies to ensure system dependability and adaptability (Wang et al., 2015b).

SO based on the HH trading time of day predicts whether or not the amount of electricity generated and consumed will differ (Grid, 2022). One of the primary functions SO may play with real-time energy markets in modern power systems is to establish an efficient and effective method for system balancing that employs various types of DER and DR aggregators (Yu and Hong, 2016) to participate in balancing market transactions. In order to participate in system balancing, these DR aggregators require an appropriate contractual generating capacity (Siano, 2014) of each generator, as this defines the revenue stream for both the site and the aggregator.

1.6. Literature Review

Previous research on the scheduling of Distributed Energy Resources (DER) primarily concentrates on optimizing operational expenses, increasing earnings, managing voltage levels, maintaining power equilibrium, shifting loads, reducing peak demands, addressing contractual limitations, and considering regulatory and financial aspects related to demand-side management in grid-connected power plants (Ahmad et al., 2020), (Tumuluru and Tsang, 2018). This study represents a pioneering approach employing stochastic optimization alongside a fixed site demand and variability in the number of STOR calls over a five-year span. It aims to enhance site revenue and mitigate penalty costs for DR aggregators, contrasting with outcomes derived from prior optimization frameworks, to determine the most optimal contractual

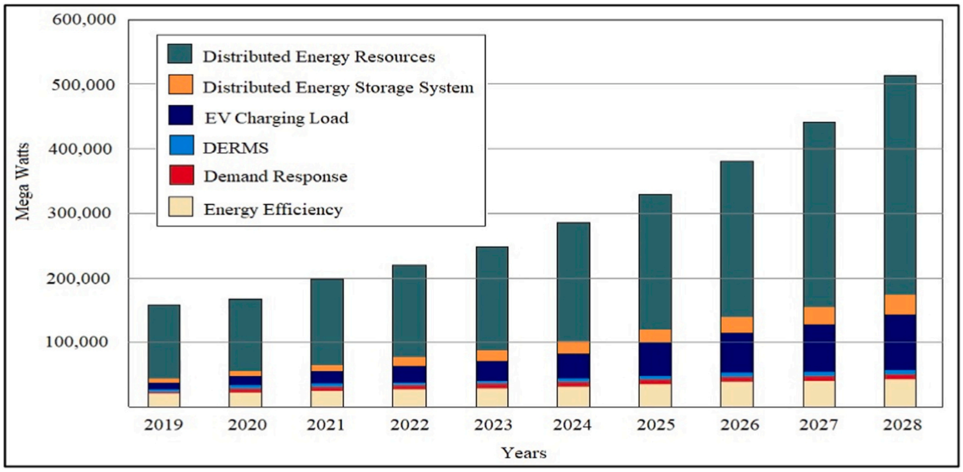


Fig. 2. Annual worldwide installed total DER power capacity by technology (Silverstein, 2022).

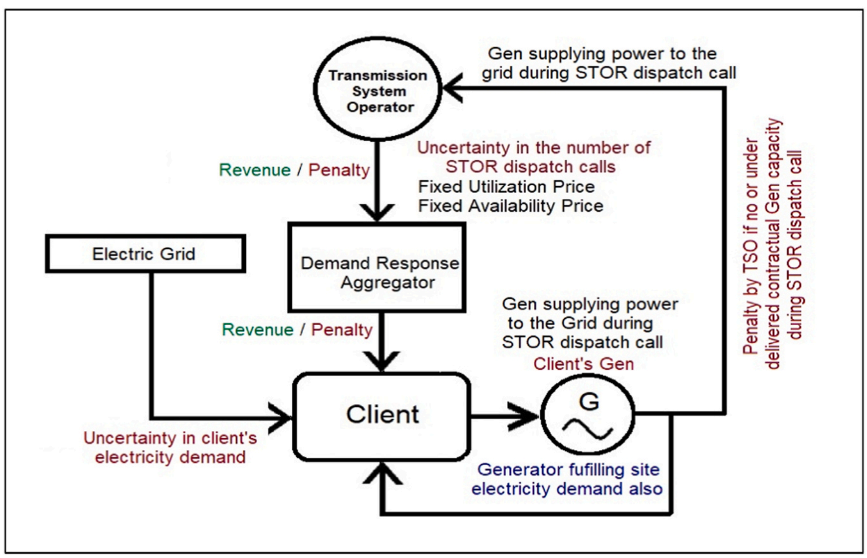


Fig. 3. Layout of the STOR dispatch call mechanism discussed in this research.

generation capacity for a DER to be offered in a BS contract. Stochastic mixed integer optimisation in itself is not new and has been applied to other areas such as solving unit commitment problems (Zheng et al., 2015), stochastic operational scheduling of DER through VPP (Zamani et al., 2016), operational impacts of high wind penetrations in grid (Meibom et al., 2011) and stochastic optimization of renewables-based microgrid incorporating BESS cost for operational scheduling (Nguyen and Crow, 2016), etc. However, in the area of determining the optimal contractual generation capacity of a DER to participate in STOR service, there is a scarcity of studies in literature especially related to grid balancing services.

Research presented by Rai et al (Rai et al., 2022). developed a deterministic methodology for DR aggregators using MINLP to determine the contractual generation capacity of a generator to offer in BS and though their results reveal an increase in revenue from 6.2% to 29.8%, the model due to its deterministic nature lacks the ability to address uncertainty in side demand and the number of STOR calls in a year. Kim et al (Kim and Edgar, 2014). presented a MINLP approach to coordinate the scheduling of a CHP plant within day-ahead electricity markets. This strategy enables the plant to optimize profits by prioritizing the use of more efficient generating units for surplus electricity sales to the grid, while also ensuring local demand is met and adhering to system operational constraints. The models rely on deterministic load and power price forecasts, enabling real-time implementation of the suggested technique by the generator, but do not address uncertainties associated with these parameters. Elsido et al (Elsido et al., 2017). proposed a MINLP model using a two-stage optimisation algorithm for determining the most profitable synthesis and design of a combined heat and power unit (CHP) within a district heating network with heat storage while accounting for optimal unit scheduling throughout the year. The solution quality of the design addressed by the upper-level algorithm cannot be guaranteed due to the algorithm's meta-heuristic nature with no relaxations of the integer variables because they have a low convergence rate, lack of function evaluations, and excess of discrete variables to optimize.

Using a two-stage architecture, Peik-Herfeh et al (Peik-Herfeh and Seifi, 2014). proposed and implemented an additional method for the efficient dispatch of DER in a distribution network using a two-stage architecture. The first stage uses probabilistic price-based UC to let VPP identify how DER should operate in the day-ahead energy market, while the second stage focuses on active distribution network management. This method accounts for both technical and economic network constraints, determining the characteristics of local network points while minimizing hourly operational costs. However, it does not incorporate the utilization of diverse penalty functions. The future significance of selecting an appropriate penalty function to enforce adherence to limitations by the VPP is highlighted as a critical consideration, necessitating careful attention. In another research, Qianfan et al (Qianfan et al., 2013). developed a MILP-based process for ISO that employed a stochastic model for DR aggregators that corresponded to a price-elastic demand curve. In contrast to its deterministic counterpart, employing a stochastic model to represent uncertain DR can result in increased available generation capacity. This is particularly evident when utilizing a two-stage stochastic formulation, where Unit Commitment (UC) decisions are made in the first stage and the Monte Carlo technique is employed to generate scenarios, as well as generation and demand decisions in the second stage. While this approach prioritizes revenue growth for Independent System Operators (ISOs), it diverges from the architecture of the STOR market, which predominantly emphasizes achieving an optimal contractual generation capacity for DER. Liu et al (Liu and Tomsovic, 2015). employed a resilient UC model to minimize the generalized social cost supplied for optimal UC decision-making while accounting for the uncertainty of demand price elasticity. The dualization of the inner maximization problem and the Benders' Decomposition technique were used to solve the multi-stage optimisation problem. The proposed robust model efficiently manages demand response uncertainty when compared to UC with deterministic price elasticity of demand, however, it does not establish the suitable value of generating units to contract with the SO.

Bertsimas et al (Bertsimas et al., 2013). proposed a technique using a two-stage adaptive robust UC model for sizable savings on average dispatch and total costs. A significant reduction in the volatility of dispatch cost improves the real-time reliability of the power system operation. The development of a methodology using the Benders decomposition algorithm makes UC solutions of the model robust against various possible realizations of the modeled uncertainties but still fails to determine the optimal contractual generation capacity to offer in the market. Another approach by Ghazvini et al (Fotouhi Ghazvini et al., 2015). proposed a short-term DR scheduling of DER and ESS for incentive-based DR programs, taking into account short-term profits and reduced capacity charges, where the energy retailer can boost the short-term payback by utilizing the DER and ESS in a DR program. Employing a non-dominated sorting genetic algorithm addresses the multi-objective optimization challenge of scheduling various resources available to the electricity retailer. This facilitates the reduction of financial losses, peak demand, and future capacity charges through trade-off decisions. However, this approach does not account for the uncertainty surrounding electricity prices, demand, and grid dispatch calls, nor does it optimize the capacity of each DER to offer in the DR program. Calvillo et al (Calvillo et al., 2016). proposed a stochastic and deterministic approach similar to the one presented in this article to optimally plan and manage the DER systems incorporating energy transitions for the aggregators in the wholesale day-ahead energy market based on the size of the aggregated resources but unable to identify the optimal size of the DER to participate in BS. Hakimi et al (Hakimi et al., 2019). formulated a levy flight-based particle swarm optimisation approach to find the optimal size and location of the DER to be placed in a smart grid to minimize costs and improve reliability but this approach does not accommodate the demand on the site level where the DER are located. Sadeghian et al (Sadeghian and Ardehali, 2016). applied the Benders decomposition algorithm to solve the economic dispatch scheduling of a grid-connected CHP to maximise profit and minimize emissions. This approach regardless of maximizing profits doesn't identify the capacity of a CHP to be contracted with the grid. Ali et al (Zamani et al., 2016). derived a probabilistic model for the integration and optimal scheduling of multiple DER using modified scenario-based decision-making for the optimal day-ahead scheduling of DER in a DR market considering prices, demand, and intermittent renewable power generation. In this model, energy consumption and energy reserve is simultaneously scheduled using BESS and DER rather than identifying any fixed capacity of the assets. Shams et al (Shams et al., 2018). presented an economic dispatch model of a CHP in a microgrid using two-stage MILP stochastic optimisation with an aim to maximize revenue and minimize expected operation costs under operational constraints and uncertainties like renewable generation by investigating the effectiveness of DR to reduce operational costs and enhance security measures. The study investigates the efficacy of DR in reducing operational costs and enhancing security measures. Although the method focuses on short-term operational planning of the microgrid, it overlooked the inclusion of small and medium end-users aggregated as part of the STOR market contract.

Previous research in this field as mentioned in the literature review in comparison with the proposed research addresses the problem from a different perspective primarily the operational schedule of the DER in terms of their operational (Olivella-Rosell et al., 2018), environmental and economic (Ren et al., 2010), security (Fu et al., 2013) and financial constraints to increase revenue and decrease operational costs but rarely there has been a focus on finding the optimal contractual generation capacity of a grid-connected utility-scale generator to offer in the balancing service like STOR to maximise site revenue and mitigate penalty costs due to reduced or no delivery of electricity under uncertainty of STOR dispatch calls in a yearly contract with the SO from DR aggregators perspective and that is the novelty of this research.

1.7. Strengths, novelties and contributions of the Research

To the best of the authors knowledge, this paper is the first to contribute a SMINLP optimisation-based approach under the uncertainty of the number and timing of STOR calls to determine the optimal contractual generation capacity of a utility-scale DER from the DR aggregator's perspective whereas previous research has used SMINLP to compute the production schedule of DER to meet their transmission, distribution, environmental risk, and system-wide constraints to increase site revenue (Nemati et al., 2018). It confirms that the trade-off between the site revenue and the penalties depends on the number of STOR dispatch calls in each site's electricity demand bin. A further innovation in establishing the appropriate selection of the electricity generation capacity of a DER to be contracted with the SO by the DR aggregator takes into consideration varied utilization pricing based on five years of STOR calls data.

The use of active STOR site case studies enhanced the value and strength of this research and supported the concept that the stochastic optimisation model is more robust than the deterministic MINLP model because it includes the following systematic approach that the deterministic model does not;

a) The segmentation and mathematical categorization of the site electricity demand data for the aggregator to select the optimal generation capacity of a DER to contract with the SO.
b) Impact of STOR calls on revenue generation and STOR penalties during asset dispatch under uncertainty of the number and timing of the STOR calls in a particular demand bin.
c) Clarification of the impact of the site temporal electricity demand on the selection of the optimal contractual generation capacity with the grid.
d) Optimal selection of the generation capacity of utility-scale grid-connected DER to be offered in STOR contract based on various utilization prices offered by the SO and five years of STOR calls data.

2. Methodology

2.1. Solution strategy

This research proposes a stochastic MINLP optimization model to determine the optimal generation capacity an aggregator can offer for a grid-connected DER to participate in STOR service to increase revenue and mitigate or reduce penalty costs incurred as a result of no or reduced electricity delivery during a STOR dispatch call. The model addresses a trade-off between the uncertainty of the number of STOR calls in each site's demand bin and the penalty costs associated with no or decreased electricity delivery and the increase in revenue generated by high utilization and availability prices in a STOR contract.

In lieu of a difficult-to-obtain probability distribution on the uncertain data, the model utilized uncertainty of STOR calls and a deterministic site demand over a period of five STOR market years from April 2016 to March 2021, with a set yearly site demand of the site. Classification of the total number of STOR calls dispatched by the grid over five STOR years at various utilization prices from £25 to £500/MWh under five deterministic site demand bins using demand bin characterization technique and use of a separate set of STOR calls against each utilization price in the stochastic model which helps to determine the optimal generation capacity of a generator is used to account for uncertainty in the number and timing of STOR calls over a period of five years.

2.2. Problem Statement

A more detailed and specific description of the problem is explained by these parameters characterized by the UK short term operating reserve market.

- **Given**
- Maximum Generation capacity: The maximum generation capacity of a grid-connected generator at a site participating in the STOR service offered by the National Grid (SO) UK.
- The electricity demand of the site: HH site electricity demand data for the whole year. In both case studies, this demand is kept the same for all five STOR years.
- Generator utilization price: The contractual price in £/MWh paid by the SO to the DR aggregator for the utilization of the DER.
- Generator availability price: The contractual price in £/MW/h paid by the SO to the DR aggregator for committing a generation capacity of DER to be offered in the STOR contract.
- Utilization revenue: The revenue obtained by exporting the contractual electricity on the grid during a STOR dispatch call from the SO.
- Availability revenue: The revenue obtained by committing the grid-connected DER by DR aggregator for a specific contracted generation capacity with the SO for a STOR contract.
- Fuel cost: The cost of fossil fuel i.e. gas or diesel for electricity generation used by a DER during STOR utilization calls.
- Availability hours:This is the total number of hours in a STOR year. There could be three or four seasons of the STOR year.
- Length of STOR call: The average duration of a STOR call.
- Penalties: This is the penalty cost incurred during a STOR dispatch call due to under or no delivery of electricity by the DER. This could be due to a late start or complete failure of the DER to start or due to high site demand at the time of STOR call that DER is unable to deliver the contractual generation capacity.
- No of STOR calls: This is the total number of STOR dispatch calls during a STOR year.

- **Determine**
- To find the optimal generation capacity a DR aggregator can offer in a STOR contract to the SO for a grid-connected DER to maximise site revenue and minimize penalty costs incurred due to under or no delivery of electricity during STOR dispatch calls.

- **Subject to (Constraints)**
- Limitation of the maximum available electricity generation capacity of the grid-connected DER to be contracted in STOR contract with the SO.
- Limitation of the contractual generation capacity of the DER to be offered in the STOR contract due to uncertain site demand.
- **In order to**
- Maximise the total site revenue by offering the optimal electricity generation capacity of a grid-connected DER in the STOR market.
- Mitigate or minimize the total penalty costs incurred by the DER either by no or reduced delivery of electricity due to failing or late start respectively during a STOR dispatch call.

2.3. Optimisation formulation

Given the uncertainty in the time and frequency of STOR calls, the mathematical model is formulated as a deterministic stochastic optimisation model with a trade-off between site revenue and penalties paid owing to no or reduced delivery of electricity during STOR dispatch calls.

A simple graphical representation of the model indicates a variable site demand SD and a variable maximum generation to offer $X^{(Gen)}$ which cannot exceed the maximum generation capacity $Gen^{(Max)}$ of the DER running all day long and fulfilling site demand as shown in Fig. 4.

2.3.1. Constraints

The optimisation model is subject to the constraints shown in Eqs. (1) and (2). The maximum generation capacity of a generator $Gen^{(Max)}$ is constant and the sum of variable site demand SD and optimal generation capacity of the generator $X^{(Gen)}$ that the site can offer cannot exceed $Gen^{(Max)}$.

$$X^{(Gen)} + SD_{(j)} \leq Gen^{(Max)} \tag{1}$$

Furthermore, the optimal generation capacity of a generator offered in grid BS cannot be greater than the maximum generation to offer as there is always a site demand that the generator has to fulfill. The only condition when $X^{(Gen)} = Gen^{(Max)}$ is when there is no site demand.

$$X^{(Gen)} \leq Gen^{(Max)} \tag{2}$$

Site Availability Revenue is the revenue the site generates in an availability window during which the aggregator is required to make their DER available to operate at contracted generation capacity. Even if the site assets are not dispatched during a STOR call, aggregators are still paid for making the assets available. Yearly each contract requires 85% availability as per the STOR contract. Eq. (3) represents the site availability revenue as a product of the number of availability hours $Ah_{(j)}$ in each site demand bin, the contracted availability price $AP^{(Gen)}$ in £/MW/h between the aggregator and the SO and optimal contracted generation to offer $X^{(Gen)}$ by the aggregator in a STOR contract.

$$SAR_{(j)} = Ah_{(j)} \quad AP^{(Gen)} \quad X^{(Gen)} \tag{3}$$

Total site annual availability revenue $TAR^{(Site)}$ is the sum of all availability revenues in each of the five demand bins used in this model and is given in Eq. (6).

$$TAR^{(Site)} = \sum_{j=1}^{m} [SAR_{(j)}] \tag{4}$$

Site utilization revenue $SUR_{(i,j)}$ is generated when the site assets are actually dispatched during a STOR dispatch call. A site is required to deliver the contractual generation capacity during each of the STOR dispatch calls or at least 95% of the total contracted value (Grid, 2022). Site utilization revenue is represented by Eq. (5) and it is the product of optimal generation capacity to offer $X^{(Gen)}$ times generator utilization price $UP^{(Gen)}$, duration of STOR call Lsc and number of STOR calls $Sc_{(i,j)}$ in each site demand bin. The uncertainty is in the number of STOR calls in each demand bin, hence the uncertainty set of STOR calls over a period of five STOR market years from April 2016 to March 2021 is used.

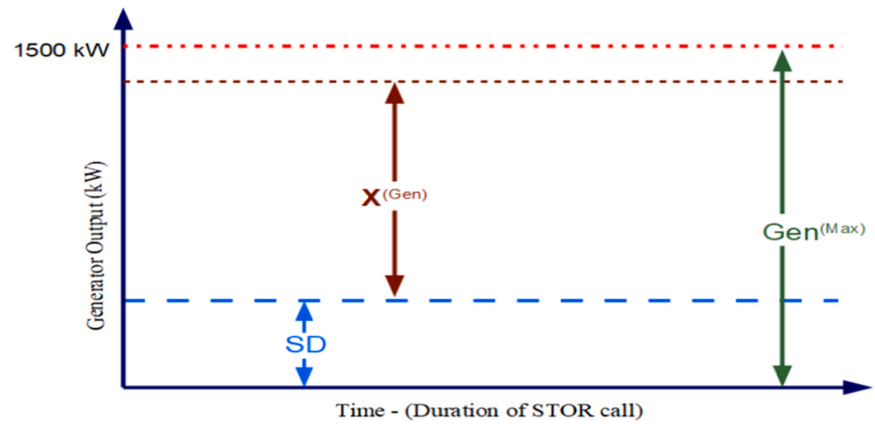


Fig. 4. STOR call with variable site demand (SD) and optimal capacity to offer $X^{(Gen)}$ in STOR contract.

$$SUR_{(i,j)} = X^{(Gen)} \;\; UP^{(Gen)} Lsc \;\; Sc_{(i,j)} \tag{5}$$

$$TUR^{(Site)} = \sum_{j=1}^{m} [SUR_{(i,j)}] \tag{6}$$

Eq. (6) shows the total annual site utilization revenue in each of the five demand bins. Utilization prices are contractual fixed amount that is set at the start of a contract between the aggregator and an SO through a bidding process. Generally, low utilization prices in a bidding process have more chances of winning, and those contracts with low utilization prices are more often called by the SO during the grid dispatch call.

Generator fuel cost is the site expense and it is the amount required to operate the assets during STOR dispatch calls. Generator fuel cost $GFC_{(i,j)}$ in each site demand bin and the total annual generator fuel cost of the site $TGFC^{(Site)}$ in all five demand bins used in the model are given in Eqs. (7) and (9) respectively. Fuel cost is considered as a constant in this model but in real-time it can vary from time to time and depends on the type of fuel as well. Fuel cost is only incurred by the site during a STOR dispatch call. In order to further simplify Eq. (7), a binary variable $YP_{(i,j)}$ is introduced which satisfies the following two conditions.

If there is a penalty $YP_{(i,j)} = 1$

If there is no penalty $YP_{(i,j)} = 0$

$$GFC_{(i,j)} = \left[\left(1 - YP_{(i,j)}\right) \; X^{(Gen)} \; FC^{(Gen)} \; Lsc \; Sc_{(i,j)} \right] +$$

$$\left[YP_{(i,j)} \; \left(Gen^{(Max)} - SD_{(j)}\right) \; FC^{(Gen)} \; Lsc \; Sc_{(i,j)} \right] \tag{7}$$

Generator fuel cost in terms of binary variable is given in Eq. (8).

$$\begin{aligned} GFC_{(i,j)} = {} & \left(X^{(Gen)} \; FC^{(Gen)} \; Lsc \; Sc_{(i,j)}\right) - \left(YP_{(i,j)} \; X^{(Gen)} \; FC^{(Gen)} \; Lsc \;\; Sc_{(i,j)}\right) \\ & + \left(YP_{(i,j)} \;\; Gen^{(Max)} \; FC^{(Gen)} \; Lsc \; Sc_{(i,j)}\right) \\ & - \left(YP_{(i,j)} \; SD_{(j)} \; FC^{(Gen)} \; Lsc \;\; Sc_{(i,j)}\right) \end{aligned} \tag{8}$$

$$TGFC^{(Site)} = \sum_{j=1}^{m} [GFC_{(i,j)}] \tag{9}$$

2.3.2. STOR penalty structure

The SO assesses the delivered MWh energy across all grid utilization calls within each season for each contracted site. If there is a shortfall in total delivered capacity against STOR-instructed contractual capacity, the SO applies a reconciliation against availability revenue made by the aggregator for that site in respect of that Season (Grid, 2022). If the total delivered capacity meets a threshold of 95% or greater as instructed in the STOR contract then no seasonal reconciliation applies and the aggregator is paid in full but if it is less than 95%, this incurs a penalty which is given in the Eq. (10).

The penalty charged by the DR aggregator to the site for a single STOR call due to under-delivery is 0.5% of the monthly availability revenue for every 1% of under-delivered capacity. In addition to this 20% of monthly availability revenue is also charged for that site in case of under-delivery. In case of no delivery due to generator failure or late start, 20% of the monthly availability revenue will be deducted by the aggregator (Rai et al., 2022). This penalty structure is in line with the National Grid UK guidelines. The derivation of Eq. (10) is given in Appendix-A.

$$Psc_{(j)} = \left[\left(224 \;\; AP^{(Gen)} \;\; X^{(Gen)}\right) \; - \left(160 \;\; AP^{(Gen)} \;\; \left(Gen^{(Max)} - \;\; SD_{(j)}\right)\right) \;\; \right] \tag{10}$$

$$Psc_{(j)} = \left[\left(224 \, AP^{(Gen)} \; X^{(Gen)}\right) - \left(160 \, AP^{(Gen)} \; Gen^{(Max)}\right) + \left(160 \, AP^{(Gen)} \; SD_{(j)}\right) \; \right] \tag{11}$$

Total Penalty due to any under-delivery of a contracted generation capacity $X^{(Gen)}$ for all STOR Calls in a single demand bin is represented by Eq. (12). Where $Sc_{(i,j)}$ is the total number of STOR calls and $YP_{(i,j)}$ is the binary variable identifying if there was a penalty or not for the STOR call dispatched by the SO in that demand bin.

$$\begin{aligned} Pt_{(i,j)} = {} & \left[Sc_{(i,j)} \;\;\; YP_{(i,j)}\right] \;\; \left[\left(224 \;\;\; AP^{(Gen)} \;\;\; X^{(Gen)}\right)\right. \\ & \left. - \left(160 \;\;\; AP^{(Gen)} \;\;\; Gen^{(Max)}\right) + \left(160 \;\;\; AP^{(Gen)} \;\;\; SD_{(j)}\right) \;\;\; \right] \end{aligned} \tag{12}$$

$$\begin{aligned} Pt_{(i,j)} = {} & \left[\left(224 \;\;\; AP^{(Gen)} \;\;\; X^{(Gen)} \;\;\; Sc_{(i,j)} \;\;\; YP_{(i,j)}\right)\right. \\ & - \left(160 \;\;\; AP^{(Gen)} \;\;\; Gen^{(Max)} \;\;\; Sc_{(i,j)} \;\;\; YP_{(i,j)}\right) \\ & \left. + \left(160 \;\;\; AP^{(Gen)} \;\;\; SD_{(j)} \;\;\; Sc_{(i,j)} \;\;\; YP_{(i,j)}\right)\right] \end{aligned} \tag{13}$$

The total annual penalty cost $TPS^{(Site)}$ from all STOR calls in a STOR year is given by Eq. (14).

$$TPS^{(Site)} = \sum_{j=1}^{m} \; [Pt_{(i,j)}] \tag{14}$$

The equation for equating the Profit P_X^{DB} for a single demand Bin is given by;

$$P_X^{DB} = \;\; SAR_{(j)} + \;\; SUR_{(i,j)} - \;\; GFC_{(i,j)} \;\; - \;\; Pt_{(i,j)} \tag{15}$$

2.3.3. Demand Bin-1 equation

Profit for the Demand Bin-1 is given by the Eq. (16).

$$P_1^{DB} = \;\; SAR_{(j)} + \;\; SUR_{(i,j)} - \;\; GFC_{(i,j)} \;\; - \;\; Pt_{(i,j)} \tag{16}$$

Substituting the value of Site annual revenue $SAR_{(j)}$, site annual utilization revenue $SUR_{(i,j)}$, site annual generation fuel cost $GFC_{(i,j)}$, and site annual penalty costs $Pt_{(i,j)}$ in Demand Bin-1 and expanding the

Eq. (16) gives.

$$
\begin{aligned}
P_1^{DB} = \;& (Ah_{(j)} \quad AP^{(Gen)} \quad X^{(Gen)}) + \; (X^{(Gen)} \quad UP^{(Gen)} LscSc_{(i,j)}) \\
& - (X^{(Gen)} FC^{(Gen)} LscSc_{(i,j)}) + \left(Yp_{(i,j)} \quad X^{(Gen)} \quad FC^{(Gen)} LscSc_{(i,j)}\right) \\
& - \; (YP_{(i,j)} \quad Gen^{(Max)} \quad FC^{(Gen)} LscSc_{(i,j)}) \\
& + \; (YP_{(i,j)} \quad SD_{(j)} \quad FC^{(Gen)} LscSc_{(i,j)}) \\
& - \; (224 \quad AP^{(Gen)} \quad X^{(Gen)} \quad Sc_{(i,j)} \quad YP_{(i,j)}) \\
& + \; (160 \quad AP^{(Gen)} \quad Gen^{(Max)} \quad Sc_{(i,j)} \quad YP_{(i,j)}) \\
& - \; (160 \quad AP^{(Gen)} \quad SD_{(j)} \quad Sc_{(i,j)} \quad YP_{(i,j)})
\end{aligned}
\tag{17}
$$

Substituting the value of all constants; $AP^{(Gen)}, LscandFC^{(Gen)}$ and availability hours $Ah_{(j)}$ in Eq. (17) gives the profit P_1^{DB} for Demand Bin-1 as.

$$
\begin{aligned}
P_1^{DB} = \;& 6.57X^{(Gen)} + (1.66X^{(Gen)} \quad UP^{(Gen)} \quad Sc_{(i,j)}) - \; (0.1328X^{(Gen)} Sc_{(i,j)}) \\
& - \; (0.8752X^{(Gen)} \quad YP_{(i,j)} \quad Sc_{(i,j)}) \\
& + \; (0.5872 \quad YP_{(i,j)} \quad Sc_{(i,j)}) \quad (Gen^{(Max)} - SD_{(j)} \quad)
\end{aligned}
\tag{18}
$$

2.3.4. Equations for Demand Bins 2–5

Substituting the value of $Ah_{(j)}$, $AP^{(Gen)}, LscandFC^{(Gen)}$ in Eq. (15) for Demand Bin-2, Bin-3, Bin-4 and Bin-5 gives the following equations;

$$
\begin{aligned}
P_2^{DB} = \;& 7.031X^{(Gen)} + (1.66X^{(Gen)} \quad UP^{(Gen)} \quad Sc_{(i,j)}) \\
& - \; (0.1328X^{(Gen)} \quad Sc_{(i,j)}) - \; (0.8752X^{(Gen)} \quad YP_{(i,j)} \quad Sc_{(i,j)}) \\
& + \; (0.5872 \quad YP_{(i,j)} \quad Sc_{(i,j)}) \quad (Gen^{(Max)} - SD_{(j)})
\end{aligned}
\tag{19}
$$

$$
\begin{aligned}
P_3^{DB} = \;& 2.027X^{(Gen)} + (1.66X^{(Gen)} \quad UP^{(Gen)} \quad Sc_{(i,j)}) \\
& - \; (0.1328X^{(Gen)} \quad Sc_{(i,j)}) - \; (0.8752X^{(Gen)} \quad YP_{(i,j)} \quad Sc_{(i,j)}) \\
& + \; (0.5872 \quad YP_{(i,j)} \quad Sc_{(i,j)}) \quad (Gen^{(Max)} - SD_{(j)})
\end{aligned}
\tag{20}
$$

$$
\begin{aligned}
P_4^{DB} = \;& 0.654X^{(Gen)} + (1.66X^{(Gen)} \quad UP^{(Gen)} \quad Sc_{(i,j)}) \\
& - \; (0.1328X^{(Gen)} \quad Sc_{(i,j)}) - \; (0.8752X^{(Gen)} \quad YP_{(i,j)} \quad Sc_{(i,j)}) \\
& + \; (0.5872 \quad YP_{(i,j)} \quad Sc_{(i,j)}) \quad (Gen^{(Max)} - SD_{(j)})
\end{aligned}
\tag{21}
$$

$$
\begin{aligned}
P_5^{DB} = \;& 0.963X^{(Gen)} + (1.66X^{(Gen)} \quad UP^{(Gen)} \quad Sc_{(i,j)}) \\
& - \; (0.1328X^{(Gen)} \quad Sc_{(i,j)}) - \; (0.8752X^{(Gen)} \quad YP_{(i,j)} \quad Sc_{(i,j)}) \\
& + (0.5872 \quad YP_{(i,j)} \quad Sc_{(i,j)}) \quad (Gen^{(Max)} - SD_{(j)})
\end{aligned}
\tag{22}
$$

2.3.5. Objective Function

Adding Eqs. 18 to 22 gives the total annual profit of the site as an objective function of the model.

$$
P^{(Total)} = \; P_1^{DB} + \; P_2^{DB} + \; P_3^{DB} + \; P_4^{DB} + \; P_5^{DB}
\tag{23}
$$

Substituting the values of profits obtained from each demand bin in Eq. 23 gives;

$$
\begin{aligned}
\boldsymbol{P}^{(Total)} = \;& 17.24575X^{(Gen)} + (8.3X^{(Gen)} \quad UP^{(Gen)} \quad Sc_{(i,j)}) \\
& - \; (0.664X^{(Gen)} \quad Sc_{(i,j)}) - \; (4.376X^{(Gen)} \quad YP_{(i,j)} \quad Sc_{(i,j)}) \\
& + \; (2.936 \quad YP_{(i,j)} \quad Sc_{(i,j)}) \quad (Gen^{(Max)} - SD_{(j)})
\end{aligned}
\tag{24}
$$

The objective function of the model $\boldsymbol{P}^{(Total)}$ is to maximize the profit and minimize penalty costs by determining the optimal generation capacity $X^{(Gen)}$ of the DER which in this case is the grid-connected generator participating in STOR service as well as fulfilling site electrical demand. Compared to the deterministic MINLP, the stochastic model can capture this type of variability and can predict the values for optimal results to be assigned to the variables in the objective function during the optimisation process.

3. Application of derived mathematical model

Two distinct case studies are examined using stochastic MINLP. The case study results were compared to the results of the deterministic MINLP model to estimate the possible profitability and to confirm the optimal and realistic generation capacity to be offered in the STOR contract for a grid-connected DER. The site demand data and STOR call data acquired for the case studies are from an actual site using the STOR service. The site HH demand information included in both case studies is depicted in Fig. 5. Applying five years of STOR call data received from Elexon (Elexon, 2022) to the same site demand reveals the profit and penalty outcomes, as well as the effect on the selection of optimal generation to offer. The HH site demand is from the STOR year starting 1st April 2018–31 st March 2019.

Demand bin characterization is the distribution of annual HH site electrical demand data into several divisions, followed by the histogram-based enumeration of the number of values that fall within each division or interval. The annual demand for this stochastic model has been divided into five demand bins. Fig. 6 depicts the demand bin characterization of the site data. Infrequently, the site demand exceeds 1300 kW, according to the data, so the model must compromise between site demand and ideal generation capacity to offer to reduce fines. If the contracted generation to offer is too high and the site receives a STOR call during peak demand, there is a potential that the site will miss the

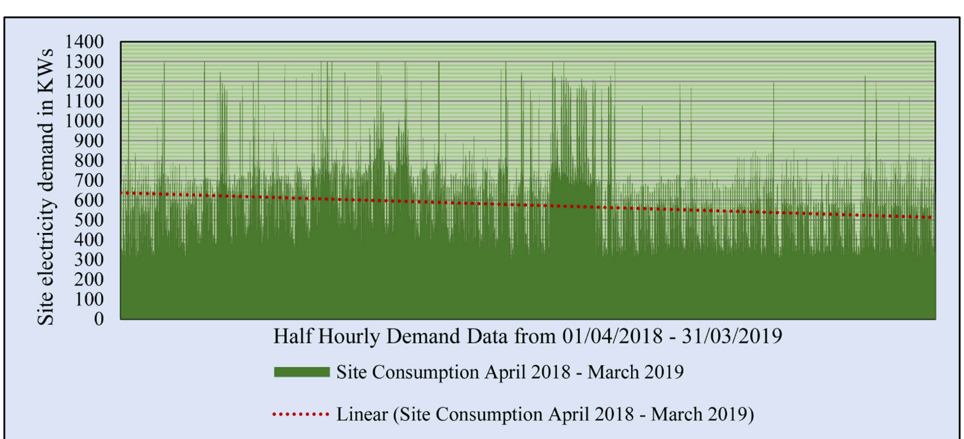


Fig. 5. Annual Half Hourly site-1 demand data.

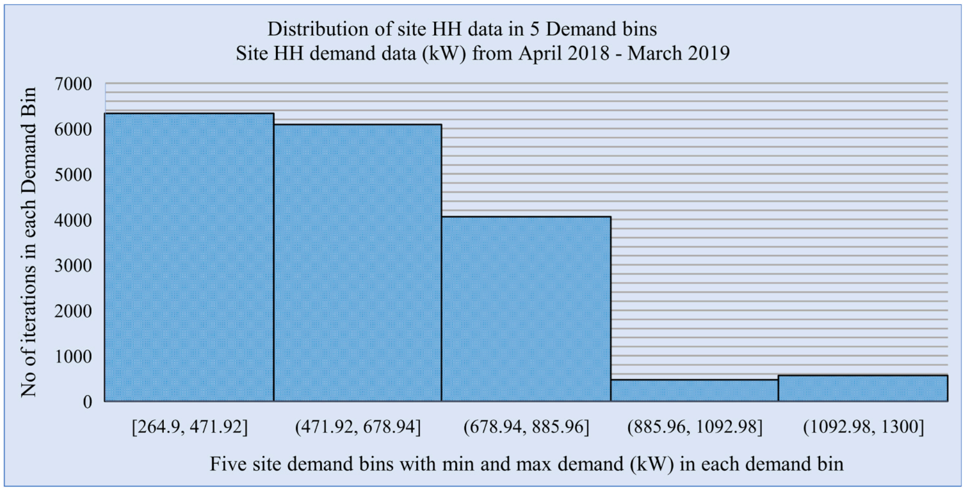


Fig. 6. Demand bin characterization of site HH demand data in 5 demand bins.

Table 2
Utilization prices against the capacity to offer, utilization revenue, availability revenue, generator fuel cost, STOR penalties, and total profit for a deterministic model with 5 demand-bins (Rai et al., 2022).

Utilization Prices (£/MWh)	Generation Capacity to Offer in STOR Contract (kW)	Total Availability Revenue (£s)	Total Utilization Revenue (£s)	Total Generation Fuel Cost (£s)	Total Penalties (£s)	Total Profit (£s)
25	821.06	14160.206	0	0	0	14160.206
50	821.06	14160.206	0	0	0	14160.206
75	821.06	14160.206	0	0	0	14160.206
100	821.06	14160.206	3134.807	2507.846	0	14787.167
125	821.06	14160.206	3918.509	2507.846	0	15570.869
150	821.06	14160.206	4702.211	2507.846	0	16354.57
170	821.06	14160.206	5329.172	2507.846	0	16981.532
175	1028.08	17730.525	6327.863	2892.737	4006.273	17159.377
200	1028.08	17730.525	7231.843	2892.737	4006.273	18063.358
225	1028.08	17730.525	8135.824	2892.737	4006.273	18967.338
250	1028.08	17730.525	9039.804	2892.737	4006.273	19871.318
275	1028.08	17730.525	9943.784	2892.737	4006.273	20775.299
300	1028.08	17730.525	10847.765	2892.737	4006.273	21679.279
325	1028.08	17730.525	11751.745	2892.737	4006.273	22583.26
350	1028.08	17730.525	12655.725	2892.737	4006.273	23487.24
375	1028.08	17730.525	13559.706	2892.737	4006.273	24391.22
400	1028.08	17730.525	14463.686	2892.737	4006.273	25295.201
425	1028.08	17730.525	15367.667	2892.737	4006.273	26199.181
450	1028.08	17730.525	16271.647	2892.737	4006.273	27103.162
475	1028.08	17730.525	17175.627	2892.737	4006.273	28007.142
500	1028.08	17730.525	18079.608	2892.737	4006.273	28911.122

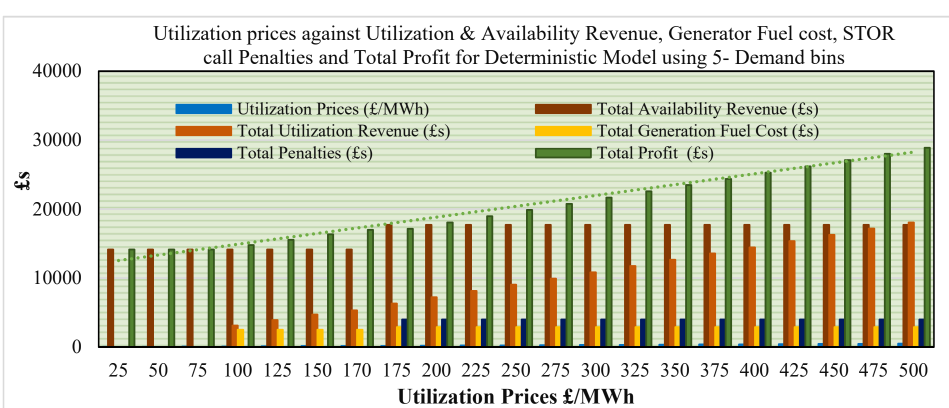


Fig. 7. Utilization prices/ MWh against total profit, penalties, fuel cost, utilization, and availability revenue in the deterministic model using 5 demand-bins (Rai et al., 2022).

committed generation to supply and suffer a penalty.

3.1. Deterministic MINLP model results from previous research

Various utilization prices are plotted in the deterministic MINLP model proposed by Rai et al (Rai et al., 2022). with five demand bins using the GAMS model and the results are indicated in Table 2. In this model researchers used deterministic site demand and one year of STOR calls data. The results show that the generation capacity to offer $X^{(Gen)}$ and penalty costs are not very sensitive to the change in utilization prices. The reason is the model is based on yearly STOR calls data of a single site and most of the STOR calls were dispatched when the site demand was low, hence it offers high contractual generation capacity at high utilization prices to increase revenue.

Logically, this is correct because if the model detects low site demand at the time of STOR dispatch calls over a historical period, it will offer a high contractual capacity to the grid. However, there is a high risk of failing to meet the contractual generation capacity and being penalized by the grid if the STOR call is dispatched during a period of high site demand, the site will be unable to meet the high contractual generation capacity. As a result, the deterministic model clearly has limitations when it comes to hedging against penalty costs and overall profit.

The penalty costs results are not very sensitive to the utilization prices in the deterministic model as shown in Fig. 7. This is because the deterministic model considers only one year of STOR calls data with most of the STOR calls dispatched at the time of low site demand.

3.2. Case Study – 1 (Stochastic formulation with fixed 5 years of STOR calls dispatch data against various utilization prices)

The same site demand as used by Rai et al (Rai et al., 2022). in their deterministic model is used for comparison in the first case study utilizing stochastic formulation with five years of STOR calls data. The actual site electricity demand in each demand bin and the total number of availability hours in each demand bin are shown in Table 3. There could be two or three availability windows each day in a STOR season. Normally there are six STOR seasons in a year.

Table 4. shows the total number of STOR calls dispatched by the SO in five STOR years from 2016 to 2021 against each site demand. According to the STOR call data, the highest concentration of STOR calls occurs when the site demand is around 886 kW, indicating that there is potential for improvement to maximise the contracted generation to offer, which the aggregator chose to be 800 kW (Rai et al., 2022) using a heuristic approach of choosing the contracted generation capacity to offer in the BS.

The case study -1 results are shown in Table 4. By running the data through the GAMS MINLP optimisation model with varied utilization prices, it is determined that the best generation to offer is 821.06 kW between utilization prices of £170/MWh and £450/MWh. This generation to offer $X^{(Gen)}$ is comparable to the deterministic MINLP approach outcome in the model proposed by Rai et al (Rai et al., 2022). between utilization prices of £25/MWh and £170/MWh, as shown in Table 1 by using data from one year of STOR calls.

Results in Table 5. show that the contractual generation capacity to offer $X^{(Gen)}$, STOR call penalties and total profits are sensitive to each

Table 3
Site demand and availability hours in each of the five demand bins.

Site Demand in each demand bin (kW)	Availability hours in each demand bin (Hours)
471.92 kW	1460.00 Hrs
678.94 kW	1562.50 Hrs
885.96 kW	450.50 Hrs
1092.98 kW	145.50 Hrs
1300.00 kW	214.00 Hrs

Table 4
Total number of STOR calls in each demand bin in five STOR years for the Case Study-1 Contract (Elexon, 2022).

Demand →	Year →	STOR calls from April 2016 – March 2017	STOR calls from April 2017 – March 2018	STOR calls from April 2018 – March 2019	STOR calls from April 2019 – March 2020	STOR calls from April 2020 – March 2021
471.92 kW		7	5	2	0	1
678.94 kW		14	9	0	0	0
885.96 kW		15	11	0	1	0
1092.98 kW		4	3	0	0	1
1300.00 kW		4	5	0	0	1

utilization price. The results obtained from the stochastic model are much more realistic as compared to the deterministic model as the model hedges better between profit and penalty costs against each utilization price.

In the GAMS model, utilization prices ranging from £25/ MWh to £500/ MWh are plotted, and Fig. 8. depicts the best generation to offer in STOR service for each utilization price. The trend line represents the available generation capacity. Based on the trade-off between site profit and penalties, the model finds the optimal generation capacity to be 821.06 kW, as opposed to the aggregator's heuristic decision of 800 kW as the capacity to deliver in a STOR contract with the grid (Rai et al., 2022).

Fig. 9. illustrates the overall revenue the site can make at each of the utilization prices if the total number of STOR calls remains the same as shown in Table 3. The trend line in Fig. 9. demonstrates that the maximum revenue may be generated by the site if the utilization price is £300/MWh, but the site selection and utilization price will be determined by the SO during the bidding process. Typically, sites with low utilization rates have greater odds of winning bids. The facility in question participated in the STOR service in real-time, with a utilization price of £170/MWh and a generation to offer of 800 kW.

The results indicated in Table 4. are graphically illustrated in Fig. 10. This demonstrates that higher utilization prices improve revenue, although these prices are rarely supplied by the SO. Furthermore, the prospects of any contract winning the bid at this utilization price are quite slim. When comparing Fig. 7. with Fig. 9, the stochastic model profit results in Fig. 9. vary and they hedge against penalties, as the lower the utilization price, the higher the chances of being called by the grid and missing out on contractual generation capacity and incurring penalties.

3.3. Case Study – 2 (Stochastic formulation with varied 5 years of STOR calls dispatch data against various utilization prices)

In the second case study, another stochastic mathematical model is developed utilizing the identical site demand data as in case study - 1. First, the total number of STOR calls dispatched in the five STOR years from April 2016 to March 2021 at varying utilization prices were taken from the Elexon data (Elexon, 2022) and then employed in this stochastic model one at a time against each of the five demand bins and utilization prices unlike the first case study, where the five years of STOR call data remained constant and only the utilization prices changed. In this case study, only the utilization prices between £25- £500/MWh (with an increment of £25/MWh) were used when there were STOR calls dispatched by the SO as shown in Table 6.

Table 7. shows the predicted generation to offer $X^{(Gen)}$ value by the GAMS model at various utilization prices. It can be noticed that the maximum generation to offer as predicted by the model is 821.06 kW even with five years of STOR calls data against the heuristic value of 800 kW that was offered to the grid by the DR aggregator. This means

Table 5
Utilization prices against the capacity to offer and total profit in case study-1 STOR Contract.

Utilization Prices (£/MWh)	Generation Capacity to Offer in STOR Contract (kW)	Total Availability Revenue (£s)	Total Utilization Revenue (£s)	Total Generation Fuel Cost (£s)	Total Penalties (£s)	Total Profit (£s)
25	734.343	12664.661	2529.444	8219.419	2667.552	10317.150
50	734.343	12664.661	5058.888	8211.714	1455.028	11383.783
75	734.343	12664.661	7588.332	8113.803	242.505	12450.416
100	734.343	12664.661	10117.776	9209.383	908.393	12663.743
125	754.139	13006.076	12988.166	9969.896	3018.269	13005.134
150	795.374	13717.212	16437.987	10113.718	6324.268	13716.218
170	821.06	14160.206	19231.360	10197.946	8101.827	14430.893
175	821.06	14160.206	19796.988	10212.296	7835.329	14669.411
200	821.06	14160.206	22625.129	10187.989	7335.057	15862.000
225	821.06	14160.206	25453.271	10139.072	7111.381	17054.590
250	821.06	14160.206	28281.412	10257.833	9172.829	16495.180
275	821.06	14160.206	31109.553	10257.833	9615.791	17176.659
300	821.06	14160.206	33937.694	10257.833	10058.753	17858.139
325	821.06	14160.206	36765.835	10209.016	12241.781	15621.077
350	821.06	14160.206	39593.976	10231.731	12031.996	16132.187
375	821.06	14160.206	42422.118	10234.348	11995.305	16643.297
400	821.06	14160.206	45250.259	10236.964	11958.615	17154.407
425	821.06	14160.206	48078.400	10239.581	11921.924	17665.516
450	821.06	14160.206	50906.541	10093.289	13378.949	15621.856
475	734.343	12664.660	48059.435	10116.211	5888.490	14657.503
500	734.343	12664.660	50588.879	10131.844	5750.957	14962.255

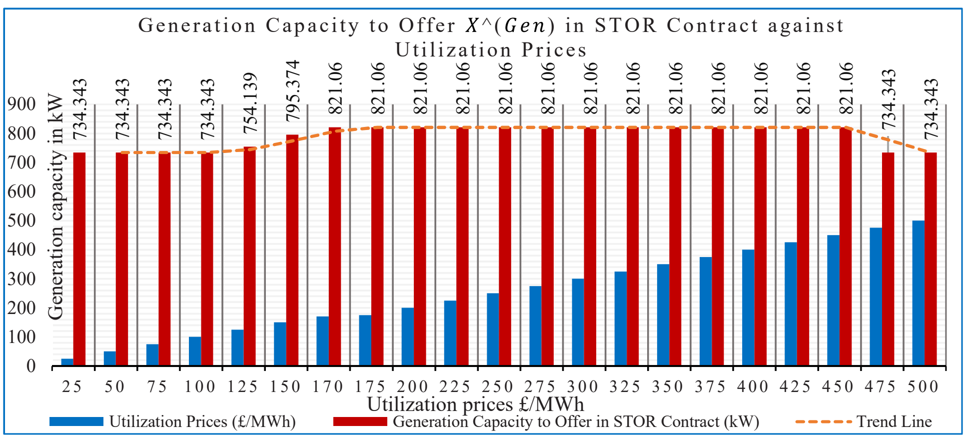


Fig. 8. Case study-1 result: Utilization prices against Capacity to offer in STOR contract.

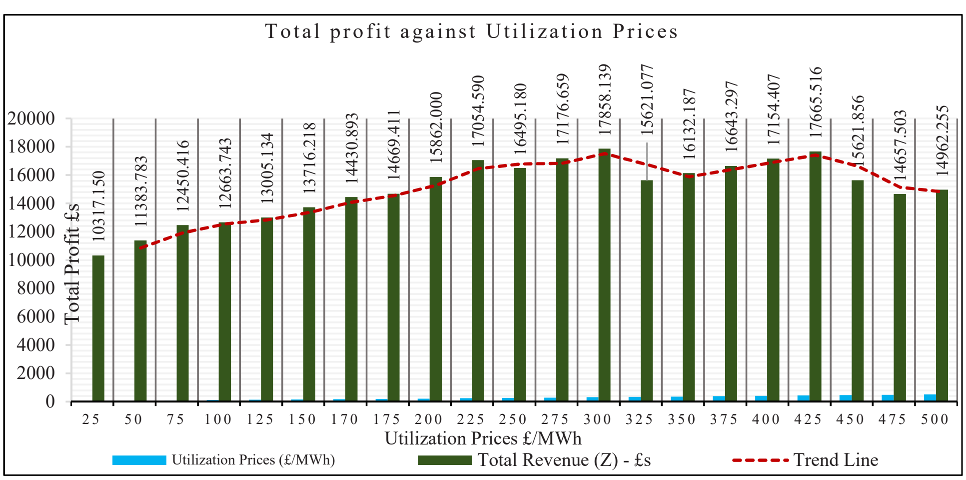


Fig. 9. Case study-1 result: Total Profit against utilization prices for a STOR contract.

that the site can offer slightly more contracted capacity to the SO under the same circumstances to increase revenue.

The results of the Case study-2 GAMS model as shown in Table 6 are plotted in Fig. 11. The estimated revenue is quite high for the utilization price of £75/ MWh. This is because the number of STOR dispatch calls at this utilization price is high too as shown in Table 5. At the time of the

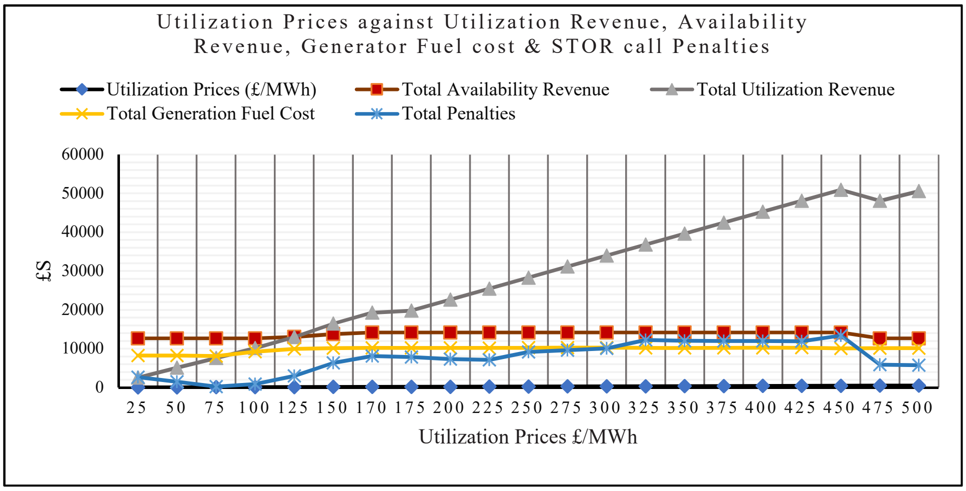


Fig. 10. Case study-1 results: Utilization Prices against Total Utilization Revenue, Total Availability Revenue, Total Generator Fuel cost & Total Penalties.

Table 6
Total number of STOR calls dispatched by the SO in 5 years against various utilization prices and five demand bins (Elexon, 2022).

£25/MWh

Year Price -->	b1	b2	b3	b4	b5
April 2016- March 2017	0	0	0	0	0
April 2017- March 2018	0	0	0	0	0
April 2018- March 2019	18	27	7	2	0
April 2019- March 2020	9	0	12	0	0
April 2020- March 2021	0	0	0	0	0

£50/MWh

Year Price -->	b1	b2	b3	b4	b5
April 2016- March 2017	0	0	0	0	0
April 2017- March 2018	0	0	0	0	0
April 2018- March 2019	2	6	0	0	0
April 2019- March 2020	0	0	0	4	0
April 2020- March 2021	3	2	0	0	0

£75/MWh

Year Price -->	b1	b2	b3	b4	b5
April 2016- March 2017	292	257	107	26	27
April 2017- March 2018	326	186	100	19	39
April 2018- March 2019	0	0	0	0	0
April 2019- March 2020	0	0	0	0	0
April 2020- March 2021	4	0	0	0	0

£100/MWh

Year Price -->	b1	b2	b3	b4	b5
April 2016- March 2017	115	77	66	5	25
April 2017- March 2018	0	0	0	0	0
April 2018- March 2019	1	1	0	0	0
April 2019- March 2020	0	0	0	0	0
April 2020- March 2021	4	0	0	0	0

£125/MWh

Year Price -->	b1	b2	b3	b4	b5
April 2016- March 2017	70	54	14	0	3
April 2017- March 2018	41	31	0	0	1
April 2018- March 2019	9	0	22	1	0
April 2019- March 2020	0	0	0	0	0
April 2020- March 2021	2	1	0	0	0

£150/MWh

Year Price -->	b1	b2	b3	b4	b5
April 2016- March 2017	7	4	4	0	4
April 2017- March 2018	9	32	0	0	0
April 2018- March 2019	0	0	0	0	0
April 2019- March 2020	0	0	0	0	0
April 2020- March 2021	2	0	0	25	5

£175/MWh

Year Price -->	b1	b2	b3	b4	b5
April 2016- March 2017	0	0	0	0	0
April 2017- March 2018	3	0	0	0	0
April 2018- March 2019	14	8	6	0	0
April 2019- March 2020	0	0	0	0	0
April 2020- March 2021	0	0	0	0	0

£200/MWh

Year Price -->	b1	b2	b3	b4	b5
April 2016- March 2017	11	2	0	2	1
April 2017- March 2018	49	23	8	3	4
April 2018- March 2019	0	0	0	0	0
April 2019- March 2020	0	0	0	0	0
April 2020- March 2021	0	0	0	0	0

£225/MWh

Year Price -->	b1	b2	b3	b4	b5
April 2016- March 2017	0	0	0	0	0
April 2017- March 2018	0	0	0	0	0
April 2018- March 2019	0	0	0	0	0
April 2019- March 2020	0	0	0	0	0
April 2020- March 2021	0	0	0	0	0

£250/MWh

Year Price -->	b1	b2	b3	b4	b5
April 2016- March 2017	0	2	0	0	0
April 2017- March 2018	1	0	0	0	0
April 2018- March 2019	0	1	0	0	0
April 2019- March 2020	0	0	0	0	0
April 2020- March 2021	0	0	0	0	0

£275/MWh

Year Price -->	b1	b2	b3	b4	b5
April 2016- March 2017	0	0	0	0	0
April 2017- March 2018	0	0	0	0	0
April 2018- March 2019	0	0	0	0	0
April 2019- March 2020	0	0	0	0	0
April 2020- March 2021	0	0	0	0	0

£300/MWh

Year Price -->	b1	b2	b3	b4	b5
April 2016- March 2017	28	18	3	1	0
April 2017- March 2018	2	2	0	0	0
April 2018- March 2019	0	1	0	0	0
April 2019- March 2020	0	0	0	0	0
April 2020- March 2021	5	0	0	0	0

£325/MWh

Year Price -->	b1	b2	b3	b4	b5
April 2016- March 2017	0	0	0	0	0
April 2017- March 2018	0	0	0	0	0
April 2018- March 2019	0	0	0	0	0
April 2019- March 2020	0	0	0	0	0
April 2020- March 2021	0	0	0	0	0

£350/MWh

Year Price -->	b1	b2	b3	b4	b5
April 2016- March 2017	0	0	0	0	0
April 2017- March 2018	0	0	0	0	0
April 2018- March 2019	0	0	0	0	0
April 2019- March 2020	0	0	0	0	0
April 2020- March 2021	0	0	0	0	0

£375/MWh

Year Price -->	b1	b2	b3	b4	b5
April 2016- March 2017	0	0	0	0	0
April 2017- March 2018	0	0	0	0	0
April 2018- March 2019	0	0	0	0	0
April 2019- March 2020	0	0	0	0	0
April 2020- March 2021	0	0	0	0	0

£400/MWh

Year Price -->	b1	b2	b3	b4	b5
April 2016- March 2017	0	0	0	0	0
April 2017- March 2018	0	0	0	0	0
April 2018- March 2019	0	0	0	0	0
April 2019- March 2020	0	0	0	0	0
April 2020- March 2021	0	0	0	0	0

£425/MWh

Year Price -->	b1	b2	b3	b4	b5
April 2016- March 2017	0	0	0	0	0
April 2017- March 2018	0	0	0	0	0
April 2018- March 2019	0	0	0	0	0
April 2019- March 2020	0	0	0	0	0
April 2020- March 2021	0	0	0	0	0

£450/MWh

Year Price -->	b1	b2	b3	b4	b5
April 2016- March 2017	0	0	0	0	0
April 2017- March 2018	0	0	0	0	0
April 2018- March 2019	0	0	0	0	0
April 2019- March 2020	0	0	0	0	0
April 2020- March 2021	0	0	0	0	0

£475/MWh

Year Price -->	b1	b2	b3	b4	b5
April 2016- March 2017	0	0	0	0	0
April 2017- March 2018	0	0	0	0	0
April 2018- March 2019	0	0	0	0	0
April 2019- March 2020	0	0	0	0	0
April 2020- March 2021	0	0	0	0	0

£500/MWh

Year Price -->	b1	b2	b3	b4	b5
April 2016- March 2017	1	0	0	0	0
April 2017- March 2018	2	1	0	0	0
April 2018- March 2019	0	0	0	0	0
April 2019- March 2020	0	0	0	0	0
April 2020- March 2021	0	2	0	0	0

investigation of site demand and STOR data, the site was participating in the STOR service with a utilization price of £170/ MWh.

The results reported in Fig. 12. reveal that the highest contractual generation capacity the site can offer over a five-year period with uncertainty in the number of STOR dispatch calls and deterministic demand is 821.06 kW. If the DR aggregator offers a higher contractual generation capacity of the site to the grid, the site may be penalized because the STOR dispatch call may occur at a time when the site demand is high and it may be unable to meet the required generation capacity contracted with the grid.

The expected generation to offer and predicted profit against various utilization prices is shown in Fig. 12. and Fig. 13. respectively. The maximum contracted generation to offer in STOR service was calculated to be 821.06 kW based on five years of STOR call data divided among five demand bins and site demand at the time of STOR call occurrence. The stochastic formulation produced more realistic results. While the form of the basic equation is the same in both case studies, the stochastic formulation takes into account the uncertainty in the number of STOR calls in each demand bin, allowing the DR aggregator to better hedge the risk of incurring penalties due to under or no delivery during a STOR dispatch call.

Table 7
Utilization payment against the capacity to offer and total profit in case study-2 STOR Contract.

Utilization Prices (£/MWh)	Generation Capacity to Offer in STOR Contract (kW)	Total Availability Revenue (£s)	Total Utilization Revenue (£s)	Total Generation Fuel Cost (£s)	Total Penalties (£s)	Total Profit - £s
25	734.343	2285.642	12664.661	2098.641	7513.651	6629.647
50	734.343	1036.158	12664.661	5.855	1656.714	12663.743
75	734.343	126441.723	12664.661	10448.455	134178.446	3764.976
100	734.343	35838.869	12664.661	31546.517	4292.352	12663.743
125	754.139	38964.497	13006.076	9509.647	29454.849	13005.134
150	795.374	18220.419	13717.212	8294.434	9925.985	13716.218
175	734.343	6613.125	12664.661	1594.184	3713.382	12663.743
200	821.060	28076.968	14160.206	12583.951	12573.895	14159.180
250	734.343	1219.009	12664.661	447.163	463.639	12663.743
300	821.060	24533.273	14160.206	8120.318	7463.308	16008.659
500	734.343	3657.027	12664.660	334.319	727.979	15028.620

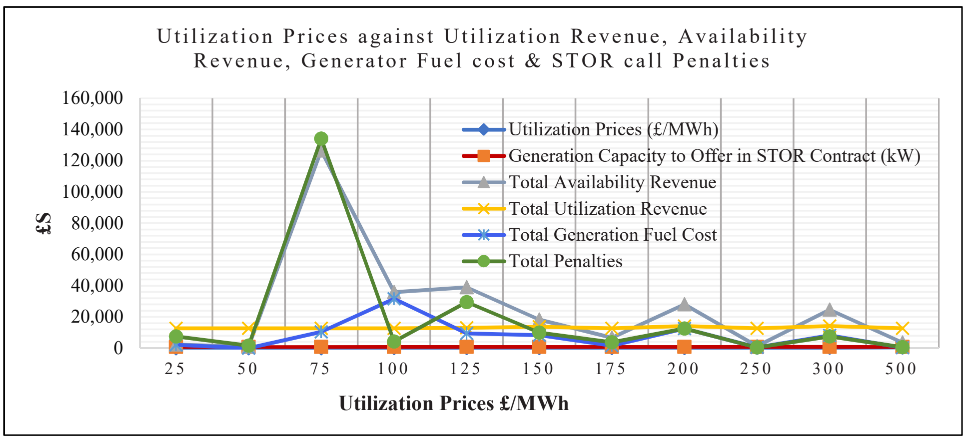


Fig. 11. Case study- 2 results: Utilization Prices against Total Utilization Revenue, Total Availability Revenue, Total Generator Fuel cost & Total Penalties.

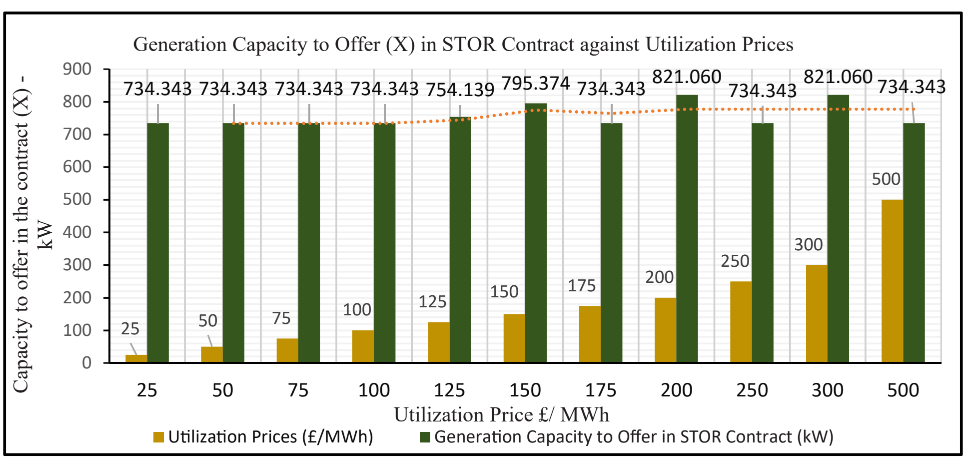


Fig. 12. Case study-2 results: Utilization prices against Capacity to offer in STOR contract.

4. Results: Comparison between deterministic and Stochastic models

The comparison between the optimal generation capacities to offer at various utilization prices is shown in Fig. 14. The model proposed in the previous deterministic approach increases both availability and utilization revenue by mitigating non-delivery penalties during STOR dispatch calls by providing the optimal capacity of a generating unit with deterministic STOR dispatch calls and site demand. Whereas the proposed research provides a more realistic optimal generation capacity by accounting for STOR demand uncertainty. Furthermore, this research takes into account a trade-off between the uncertainty of the number of STOR calls in each site demand bin, the penalty costs associated with reduced or no electricity delivery, and the increase in revenue generated by high utilization and availability prices in a STOR contract based on five years worth of STOR call data, unlike the previous research which only used one year of deterministic STOR call data.

A comparative analysis of profitability examining the differences between deterministic and stochastic models is shown in Fig. 15. The profit of the deterministic model indicates a consistent upward trend, without taking into account the potential consequences of failing to meet contractual grid demand during periods of high energy usage at site. The presence of variability in the stochastic model reflects the existence of profits at various utilization prices, while also accounting for

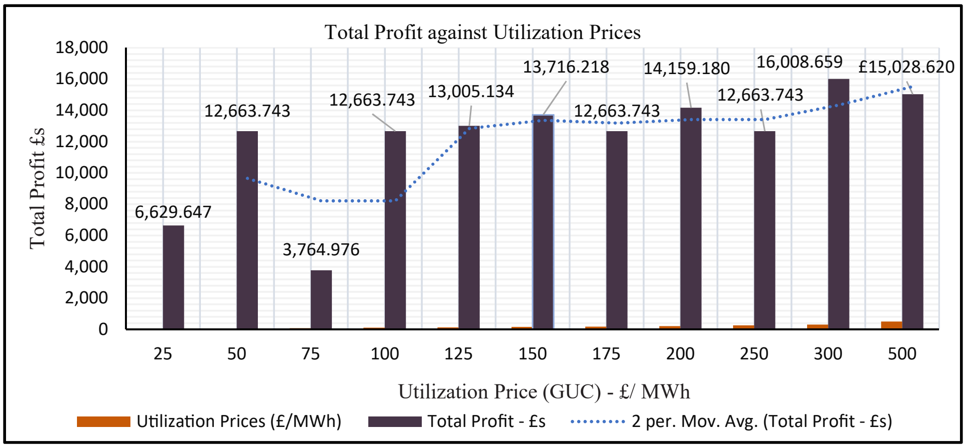
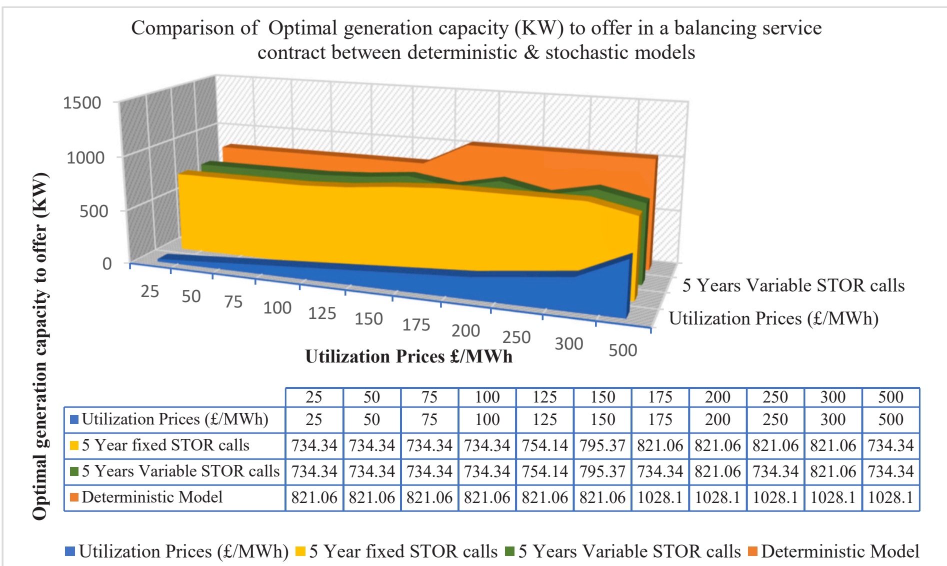


Fig. 13. Case study-2 result: Total Profit against utilization prices for a STOR contract.



	25	50	75	100	125	150	175	200	250	300	500
Utilization Prices (£/MWh)	25	50	75	100	125	150	175	200	250	300	500
5 Year fixed STOR calls	734.34	734.34	734.34	734.34	754.14	795.37	821.06	821.06	821.06	821.06	734.34
5 Years Variable STOR calls	734.34	734.34	734.34	734.34	754.14	795.37	734.34	821.06	734.34	821.06	734.34
Deterministic Model	821.06	821.06	821.06	821.06	821.06	821.06	1028.1	1028.1	1028.1	1028.1	1028.1

Fig. 14. Comparison between optimal generation capacity to offer at various utilization prices.

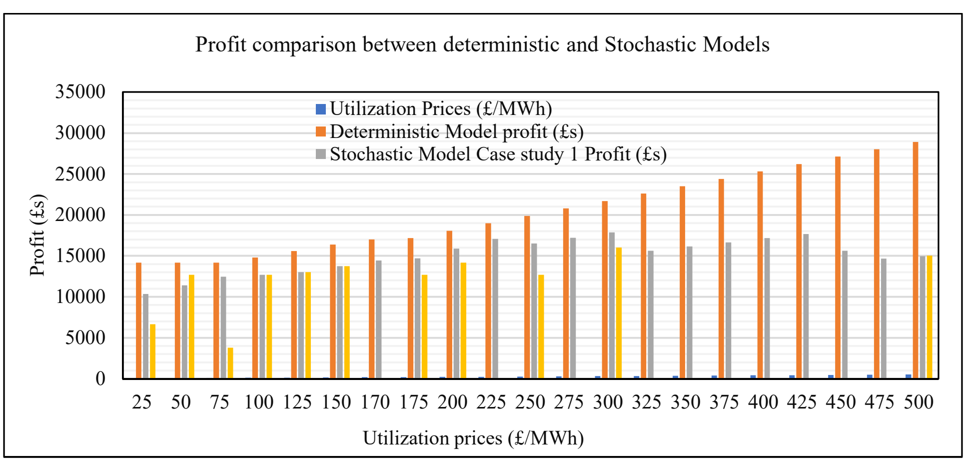


Fig. 15. Comparative analysis of profits between deterministic and stochastic models.

the uncertainty associated with STOR dispatch calls and the fluctuating demand at the site. In practical applications, it is not advisable to utilize higher capacity as suggested in deterministic models, as this may increase the likelihood of incurring penalty risks. An increased contractual capacity of the grid-connected generator, while still satisfying the site demand, raises the probability of its failure to meet the grid requirement as a result of the heightened site demand. Consequently, the aggregator may impose penalties on the client. The values of contractual generation capacities calculated from stochastic models at various utilization prices exhibit a higher degree of similarity to the actual contractual capacities given by the aggregator in real contracts.

5. Conclusions

In this study, we have developed a novel stochastic optimization-based MINLP model to determine the optimal contractual generation capacity of a utility-scale grid-connected DER, while also ensuring it meets site electricity demand for participation in the BS market by DR aggregators. From the case studies carried out, we made the following observations.

- Comparative analysis, bolstered by real-life case studies, demonstrates that our stochastic model significantly enhances profit returns for DR aggregators by 12.7% compared to the deterministic approach adopted in previous studies by offering an optimal contractual capacity of a DER to the SO in a BS contract. Similarly, it offers a 2.62% higher optimal contractual capacity to be offered in the BS contract as compared to the heuristic approach adopted by the DR aggregator.
- These findings underscore the efficacy and practical applicability of our stochastic optimization framework in optimizing generation capacity and maximizing profit in the evolving landscape of BS, especially the STOR market.
- Prior research in this area predominantly utilized deterministic MINLP models to estimate generation capacity in BS. However, our findings reveal that these models exhibit limited sensitivity to varying utilization prices. This limitation arises from deterministic models relying on a single year of STOR call data, with most calls occurring during periods of low site demand.
- The stochastic optimization approach introduced in our study sheds light on the trade-off between site profits, penalty costs, and utilization prices. This trade-off is influenced by not only the uncertainty in the number and occurrence of STOR calls but also by the uncertainty surrounding site electricity demand. Our research further advances by incorporating five years' worth of STOR call data to inform the selection of electricity generation capacity for DERs.

The outcomes derived from the case studies significantly augment both the practical applications and theoretical underpinnings of this research. The techniques elucidated herein hold the potential to assist DR aggregators in discerning the optimal generation capacity of a DER for participation in STOR contracts or similar annual BS agreements. These findings underscore the utility of our approach in addressing the evolving demands of energy markets and advancing the efficacy of DR strategies in optimizing grid operations Through these advancements, our research contributes valuable insights and methodologies for stakeholders navigating the complexities of grid balancing and demand response in contemporary energy systems.

Future development

The findings from our research highlight the efficacy of stochastic models in capturing the inherent uncertainty and variability present in real-world energy market scenarios. The congruence observed between the model outputs and actual contractual agreements suggests that stochastic models adeptly replicate the strategic behavior of aggregators in response to market dynamics. This alignment underscores the capability of stochastic models to accurately simulate capacity offers in reaction to fluctuations in utilization pricing, thus reflecting the statistical properties of decision-making processes. Looking ahead, our future work will focus on modeling uncertainty in site demand and utilization bidding prices to identify the optimal generation capacity for maximizing revenue from a DER in a day-ahead BS contract under robust optimization. Additionally, we plan to explore the integration of DER in the form of both generators and Battery Energy Storage Systems (BESS) to enhance capacity offerings in balancing service contracts and to further mitigate penalties. These endeavors will further advance our understanding and application of stochastic optimization techniques in the evolving landscape of energy markets.

CRediT authorship contribution statement

Ussama Javed Rai: Conceptualization, Data curation, Formal analysis, Funding acquisition, Investigation, Methodology, Project administration, Resources, Software, Writing – original draft, Writing – review & editing. **Gbemi Oluleye:** Investigation, Methodology, Supervision, Validation, Writing – review & editing, Visualization. **Adam Hawkes:** Investigation, Methodology, Supervision, Validation, Visualization, Writing – review & editing.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

Data will be made available on request.

Acknowledgments

The authors present their gratitude to Kiwi Power Ltd subsidiary of Engie, a DR aggregator and a global VPP solution provider based in London. Kiwi Power provided access to the real-time site demand data and portal to download Elexon STOR calls data at various utilization prices to be used in this research.

Appendix A

Derivation of Penalty Function Eq. (10) in the model

Penalty charges for failed start or under delivery are standard for all the sites in the contract as per the aggregator. Only the site that underperforms is charged for penalties in the contract.

$$\text{Generator monthly Availability Revenue} = G_m^{AH} \times AP^{(Gen)} \times X^{(Gen)} \quad (A1)$$

Where G_m^{AH} is generator availability hours per month, $AP^{(Gen)}$ is generator availability price in £/ MW/h and $X^{(Gen)}$ is contractual generation capacity to offer in a STOR contract.

The penalty for under-delivery is;

$$Psc_{(j)} = \begin{bmatrix} 0.005 \quad G_m^{AH} \quad AP^{(Gen)} \quad X^{(Gen)} \left[\dfrac{\left(X^{(Gen)} - X_{ACT(j)}\right)}{X} \right] \quad 100+ \\ 0.2 \quad G_m^{AH} \quad AP^{(Gen)} \quad X^{(Gen)} \end{bmatrix} \tag{A2}$$

Where;

$G_m^{AH} = 320$ hours and XACT $_{(j)}$ is the actual generator capacity delivered in each demand bin during the STOR dispatch call

Substituting the value of G_m^{AH} in equation (b) gives;

$$Psc_{(j)} = \begin{bmatrix} 0.005 \times \quad 320 \times AP^{(Gen)}(X^{(Gen)} - X_{ACT(j)}) \quad \times 100+ \\ 0.2 \times 320 \times gAP^{(Gen)} \quad X^{(Gen)} \end{bmatrix} \tag{A3}$$

$$Psc_{(j)} = \begin{bmatrix} \left(160 \quad AP^{(Gen)} \quad X^{(Gen)}\right) - \\ \left(160 \quad AP^{(Gen)} \quad XACT_{\ (j)}\right) + \\ \left(64 \quad AP^{(Gen)} \quad X^{(Gen)}\right) \end{bmatrix} \tag{A4}$$

(d)

$$Psc_{(j)} = \left[\left(224 \quad AP^{(Gen)} \quad X^{(Gen)}\right) \quad - \left(160 \quad AP^{(Gen)} \quad X_{ACT(j)}\right) \quad \right] \tag{A5}$$

Replacing $X_{ACT(j)}$ with $(Gen^{(Max)} - \quad SD_{(j)})$ in equation (e) gives;

$$Psc_{(j)} = \left[\left(224 \quad AP^{(Gen)} \quad X^{(Gen)}\right) \quad - \left(160 \quad AP^{(Gen)} \quad \right) \quad \left(Gen^{(Max)} - \quad SD_{(j)}\right)\right) \quad \right] \tag{A6}$$

Where$Gen^{(Max)}$ is the total generation capacity of the generator and $SD_{(j)}$ is the site demand per bin.

Season	Dates		Monday - Saturday			Sunday - Holidays		
	Start	End	Start time	End Time	Duration	Start time	End Time	Duration
12.1	01-Apr-18	29-Apr-18	06:00	13:00	07:00	10:00	14:00	04:00
			19:00	21:30	02:30	19:30	21:30	02:00
12.2	30-Apr-18	19-Aug-18	06:30	14:00	07:30	10:30	13:30	03:00
			16:00	18:00	02:00	19:30	22:00	02:30
			19:30	22:00	02:30	-	-	-
12.3	20-Aug-18	23-Sep-18	06:30	13:00	06:30	10:30	12:30	02:00
			16:00	21:00	05:00	19:30	21:30	02:00
12.4	24-Sep-18	28-Oct-18	06:00	13:00	07:00	10:30	13:00	02:30
			17:00	20:30	03:30	17:30	20:00	02:30
12.5	29-Oct-18	27-Jan-19	06:00	13:00	07:00	10:30	13:30	03:00
			16:00	20:30	04:30	16:00	19:30	03:30
12.6	28-Jan-19	31-Mar-19	06:00	13:00	07:00	10:30	13:00	02:30
			16:30	20:30	04:00	16:30	20:00	03:30

STOR calls availability window for the year 2018–2019

Appendix B

Abbreviations

BS	*Balancing Service*
CHP	*Combined Heat and Power*
DER	*Distributed Energy Resource*
DESS	*Distributed Energy Storage System*
DR	*Demand Response*
ESS	*Energy Storage System*
HH	*Half Hourly*
ISO	*Independent System Operator*
MILP	*Mixed Integer Linear Programming*
MINLP	*Mixed Integer Non-Linear Programming*
SMINLP	*Stochastic Mixed Integer Non-Linear Programming*
SO	*System Operator*
STOR	*Short-Term Operating Reserve*
UC	*Unit Commitment*
VPP	*Virtual Power Plant*

References

Liu, W.H., et al., 2019. Development and optimization of an integrated energy network with centralized and decentralized energy systems using mathematical modelling approach. Energy 183, 617–629. https://doi.org/10.1016/j.energy.2019.06.158.

Meegahapola, L., Mancarella, P., Flynn, D., Moreno, R., 2021. Power system stability in the transition to a low carbon grid: a techno-economic perspective on challenges and opportunities. Wiley Interdiscip. Rev. Energy Environ. 10 (5), e399-n/a. https://doi.org/10.1002/wene.399.

Rai, U., Oluleye, G., Hawkes, A., 2022. "An optimisation model to determine the capacity of a distributed energy resource to contract with a balancing services aggregator. Art no. 117984 Appl. Energy 306. https://doi.org/10.1016/j.apenergy.2021.117984.

Anjos, M.F., Conejo, A.J., 2017. Unit commitment in electric energy systems. Found. Trends® Electr. Energy Syst. 1 (4), 220–310. https://doi.org/10.1561/3100000014.

Laur, A., Nieto-Martin, J., Bunn, D.W., Vicente-Pastor, A., 2020. Optimal procurement of flexibility services within electricity distribution networks. Eur. J. Oper. Res. 285 (1), 34–47. https://doi.org/10.1016/j.ejor.2018.11.031.

Park, H., Baldick, R., 2020. Optimal capacity planning of generation system integrating uncertain solar and wind energy with seasonal variability. Electr. Power Syst. Res. 180, 106072 https://doi.org/10.1016/j.epsr.2019.106072.

Eicke, A., Ruhnau, O., Hirth, L., 2021. Electricity balancing as a market equilibrium: an instrument-based estimation of supply and demand for imbalance energy. Energy Econ. 102, 105455 https://doi.org/10.1016/j.eneco.2021.105455.

Laugs, G.A.H., Benders, R.M.J., Moll, H.C., 2020. Balancing responsibilities: effects of growth of variable renewable energy, storage, and undue grid interaction. Energy Policy 139, 111203. https://doi.org/10.1016/j.enpol.2019.111203.

Langendahl, P.-A., Roby, H., Potter, S., Cook, M., 2019. Smoothing peaks and troughs: Intermediary practices to promote demand side response in smart grids. Energy Res. Soc. Sci. 58, 101277 https://doi.org/10.1016/j.erss.2019.101277.

Zhang, J., Domínguez-García, A.D., 2018. Evaluation of demand response resource aggregation system capacity under uncertainty. IEEE Trans. Smart Grid 9 (5), 4577–4586. https://doi.org/10.1109/TSG.2017.2663780.

Lu, X., Li, K., Xu, H., Wang, F., Zhou, Z., Zhang, Y., 2020. Fundamentals and business model for resource aggregator of demand response in electricity markets. Energy 204, 117885. https://doi.org/10.1016/j.energy.2020.117885.

Radaideh, A., Ajjarapu, V., 2015. Demand Response planning in day-ahead market for improving power system flexibility with high wind penetration levels. 2015 North Am. Power Symp. . (NAPS) 4-6 Oct. 2015, 1–6. https://doi.org/10.1109/NAPS.2015.7335137.

M.H. Albadi and E.F. El-Saadany, "Demand response in electricity markets: an overview," ed, 2007, pp. 1-5.

Mileva, A., Johnston, J., Nelson, J.H., Kammen, D.M., 2016. Power system balancing for deep decarbonization of the electricity sector," (in English). Appl. Energy 162, 1001–1009. https://doi.org/10.1016/j.apenergy.2015.10.180.

Alotaibi, I., Abido, M.A., Khalid, M., Savkin, A.V., 2020. A comprehensive review of recent advances in smart grids: a sustainable future with renewable energy resources. Energy 13 (23), 6269. https://doi.org/10.3390/en13236269.

Ziegler, M.S., et al., 2019. Storage requirements and costs of shaping renewable energy toward grid decarbonization. Joule 3 (9), 2134–2153. https://doi.org/10.1016/j.joule.2019.06.012.

Alstone, P., Gershenson, D., Kammen, D.M., 2015. Decentralized energy systems for clean electricity access," (in English). Nat. Clim. Change 5 (4), 305–314. https://doi.org/10.1038/nclimate2512.

Giordano, V., Fulli, G., 2012. "A business case for smart grid technologies: A systemic perspective. Energy Policy 40 (1), 252–259. https://doi.org/10.1016/j.enpol.2011.09.066.

Eftekharnejad, S., Vittal, V., Heydt, G.T., Keel, B., Loehr, J., 2013. Impact of increased penetration of photovoltaic generation on power systems. IEEE Trans. Power Syst. 28 (2), 893–901. https://doi.org/10.1109/TPWRS.2012.2216294.

Mazidi, M., Zakariazadeh, A., Jadid, S., Siano, P., 2014. Integrated scheduling of renewable generation and demand response programs in a microgrid. Energy Convers. Manag. 86, 1118–1127. https://doi.org/10.1016/j.enconman.2014.06.078.

Bayod-Rújula, A.A., 2009. Future development of the electricity systems with distributed generation. Energy 34 (3), 377–383. https://doi.org/10.1016/j.energy.2008.12.008.

Mengelkamp, E., Garttner, J., Rock, K., Kessler, S., Orsini, L., Weinhardt, C., 2018. Designing microgrid energy markets; a case study: the Brooklyn Microgrid. Appl. Energy 210, 870. https://doi.org/10.1016/j.apenergy.2017.06.054.

Zia, M.F., Elbouchikhi, E., Benbouzid, M., 2018. Microgrids energy management systems: a critical review on methods, solutions, and prospects," (in English). Appl. Energy 222, 1033–1055. https://doi.org/10.1016/j.apenergy.2018.04.103.

Wang, T., O'Neill, D., Kamath, H., 2015a. "Dynamic control and optimization of distributed energy resources in a microgrid. IEEE Trans. Smart Grid (6), 2884–2894. https://doi.org/10.1109/TSG.2015.2430286.

Hatziargyriou, N., 2013. Microgrids: Architectures and Control (Microgrids: Architectures and Control). Wiley-IEEE Press, pp. 1–317.

Hatziargyriou, N., Asano, H., Iravani, R., Marnay, C., 2007. Microgrids," (in English). IEEE Power Energy Mag. 5 (4), 78–94. https://doi.org/10.1109/MPAE.2007.376583.

Dugmore, S., 2022. STOR Market Information Report - TR39. National Grid ESO, Warwick, UK. Accessed: 05 April 2022. [Online]. Available: ⟨https://www.nationalgrideso.com/industry-information/balancing-services/reserve-services/short-term-operating-reserve-stor?market-information⟩.

Silverstein, K., 2022. Distributed Energy Resources to Grow 15.9% CAGR." Microgrid Knowledge. https://microgridknowledge.com/distributed-energy-resources-navigant/ (accessed 23 April, 2022)..

Bortolini, M., Gamberi, M., Graziani, A., 2014. Technical and economic design of photovoltaic and battery energy storage system," (in English) (Article). Energy Convers. Manag. vol. 86, 81–92. https://doi.org/10.1016/j.enconman.2014.04.089.

Fang, X., Misra, S., Xue, G., Yang, D., 2012. Smart grid - The new and improved power grid: a survey," (in English). IEEE Commun. Surv. Tutor., Rev. vol. 14 (4), 944–980. https://doi.org/10.1109/SURV.2011.101911.00087.

Adefarati, T., Bansal, R.C., 2019. Reliability, economic and environmental analysis of a microgrid system in the presence of renewable energy resources," (in English). Appl. Energy 236, 1089–1114. https://doi.org/10.1016/j.apenergy.2018.12.050.

Lund, P.D., Lindgren, J., Mikkola, J., Salpakari, J., 2015. Review of energy system flexibility measures to enable high levels of variable renewable electricity. Renew. Sustain. Energy Rev. 45, 785–807. https://doi.org/10.1016/j.rser.2015.01.057.

Rahbar, K., Xu, J., Zhang, R., 2015. "Real-time energy storage management for renewable integration in microgrid: an off-line optimization approach. IEEE Trans. Smart Grid 6 (1), 124–134. https://doi.org/10.1109/TSG.2014.2359004.

Badesa, L., Strbac, G., Magill, M., Stojkovska, B., 2021. "Ancillary services in Great Britain during the COVID-19 lockdown: a glimpse of the carbon-free future. Appl. Energy 285. https://doi.org/10.1016/j.apenergy.2021.116500.

Motevasel, M., Seifi, A.R., 2014. Expert energy management of a micro-grid considering wind energy uncertainty," (in English). Energy Convers. Manag. 83, 58–72. https://doi.org/10.1016/j.enconman.2014.03.022.

Faisal, M., Hannan, M.A., Ker, P.J., Hussain, A., Mansor, M.B., Blaabjerg, F., 2018. Review of energy storage system technologies in microgrid applications: Issues and challenges," (in English). IEEE Access 6, 35143–35164. https://doi.org/10.1109/ACCESS.2018.2841407.

Bell, K., Gill, S., 2018. Delivering a highly distributed electricity system: technical, regulatory and policy challenges. Energy Policy 113, 765–777. https://doi.org/10.1016/j.enpol.2017.11.039.

Wang, Q., Zhang, C., Ding, Y., Xydis, G., Wang, J., Østergaard, J., 2015b. Review of real-time electricity markets for integrating distributed energy resources and demand response," (in English). Appl. Energy 138, 695–706. https://doi.org/10.1016/j.apenergy.2014.10.048.

N. Grid. "Short term operating reserve (STOR) frequently asked questions." Contracts & Settlements. ⟨http://www2.nationalgrid.com/uk/services/balancing⟩- services/reserve-services/short-term-operating- reserve/ (accessed 25 April, 2022).

Yu, M., Hong, S.H., 2016. Supply-demand balancing for power management in smart grid: A Stackelberg game approach," (in English). Appl. Energy 164, 702–710. https://doi.org/10.1016/j.apenergy.2015.12.039.

Siano, P., 2014. Demand response and smart grids—a survey. Renew. Sustain. Energy Rev. 30, 461–478. https://doi.org/10.1016/j.rser.2013.10.022.

Ahmad, S., Naeem, M., Ahmad, A., 2020. Unified optimization model for energy management in sustainable smart power systems. Int. Trans. Electr. Energy Syst. 30 (4), e12144 https://doi.org/10.1002/2050-7038.12144. ⟨https://doi.org/10.1002/2050-7038.12144⟩, 2020/04/01.

Tumuluru, V.K., Tsang, D.H.K., 2018. A two-stage approach for network constrained unit commitment problem with demand response. IEEE Trans. Smart Grid 9 (2), 1175–1183. https://doi.org/10.1109/TSG.2016.2580578.

Zheng, Q.P., Wang, J., Liu, A.L., 2015. Stochastic optimization for unit commitment - a review," (in English). IEEE Trans. Power Syst. 30 (4), 1913–1924. https://doi.org/10.1109/TPWRS.2014.2355204.

Zamani, A.G., Zakariazadeh, A., Jadid, S., Kazemi, A., 2016. Stochastic operational scheduling of distributed energy resources in a large scale virtual power plant," (in English). Int. J. Electr. Power Energy Syst. 82, 608–620. https://doi.org/10.1016/j.ijepes.2016.04.024.

Meibom, P., Barth, R., Hasche, B., Brand, H., Weber, C., O'Malley, M., 2011. Stochastic optimization model to study the operational impacts of high wind penetrations in Ireland," (in English). IEEE Trans. Power Syst. 26 (3), 1367–1379. https://doi.org/10.1109/TPWRS.2010.2070848.

Nguyen, T.A., Crow, M.L., 2016. "Stochastic optimization of renewable-based microgrid operation incorporating battery operating cost," (in English). IEEE Trans. Power Syst. 31 (3), 2289–2296. https://doi.org/10.1109/TPWRS.2015.2455491.

Kim, J.S., Edgar, T.F., 2014. Optimal scheduling of combined heat and power plants using mixed-integer nonlinear programming," (in English). Energy 77, 675–690. https://doi.org/10.1016/j.energy.2014.09.062.

Elsido, C., Bischi, A., Silva, P., Martelli, E., 2017. Two-stage MINLP algorithm for the optimal synthesis and design of networks of CHP units," (in English). Energy 121, 403–426. https://doi.org/10.1016/j.energy.2017.01.014.

Peik-Herfeh, M., Seifi, H., 2014. and M. Kazem Sheikh-El-Eslami, "Two-stage approach for optimal dispatch of distributed energy resources in distribution networks considering virtual power plant concept. Int. Trans. Electr. Energy Syst. 24 (1), 43–63. https://doi.org/10.1002/etep.1694.

Qianfan, W., Jianhui, W., Yongpei, G., 2013. Stochastic unit commitment with uncertain demand response. Power Syst., IEEE Trans. 28 (1), 562–563. https://doi.org/10.1109/TPWRS.2012.2202201.

Liu, G., Tomsovic, K., 2015. Robust unit commitment considering uncertain demand response," (in English). Electr. Power Syst. Res. 119, 126–137. https://doi.org/10.1016/j.epsr.2014.09.006.

Bertsimas, D., Litvinov, E., Sun, X.A., Zhao, J., Zheng, T., 2013. Adaptive robust optimization for the security constrained unit commitment problem," (in English). IEEE Trans. Power Syst. 28 (1), 52–63. https://doi.org/10.1109/TPWRS.2012.2205021.

Fotouhi Ghazvini, M.A., Soares, J., Horta, N., Neves, R., Castro, R., Vale, Z., 2015. A multi-objective model for scheduling of short-term incentive-based demand response programs offered by electricity retailers," (in English). Appl. Energy 151, 102–118. https://doi.org/10.1016/j.apenergy.2015.04.067.

Calvillo, C.F., Sánchez-Miralles, A., Villar, J., Martín, F., 2016. Optimal planning and operation of aggregated distributed energy resources with market participation," (in English). Appl. Energy 182, 340–357. https://doi.org/10.1016/j.apenergy.2016.08.117.

Hakimi, S.M., Hasankhani, A., Shafie-khah, M., Catalão, J.P.S., 2019. "Optimal sizing and siting of smart microgrid components under high renewables penetration considering demand response," (in English). IET Renew. Power Gener. 13 (10), 1809–1822. https://doi.org/10.1049/iet-rpg.2018.6015.

Sadeghian, H.R., Ardehali, M.M., 2016. A novel approach for optimal economic dispatch scheduling of integrated combined heat and power systems for maximum economic profit and minimum environmental emissions based on Benders decomposition," (in English). Energy 102, 10–23. https://doi.org/10.1016/j.energy.2016.02.044.

Shams, M.H., Shahabi, M., Khodayar, M.E., 2018. Stochastic day-ahead scheduling of multiple energy Carrier microgrids with demand response," (in English). Energy 155, 326–338. https://doi.org/10.1016/j.energy.2018.04.190.

Olivella-Rosell, P., et al., 2018. Local flexibility market design for aggregators providing multiple flexibility services at distribution network level. Energy 11 (4), 822. https://doi.org/10.3390/en11040822.

Ren, H., Zhou, W., Nakagami, K. i, Gao, W., Wu, Q., 2010. Multi-objective optimization for the operation of distributed energy systems considering economic and environmental aspects. Appl. Energy 87 (12), 3642–3651. https://doi.org/10.1016/j.apenergy.2010.06.013.

Fu, Y., Li, Z., Wu, L., 2013. Modeling and solution of the large-scale security-constrained unit commitment. IEEE Trans. Power Syst. 28 (4), 3524–3533. https://doi.org/10.1109/TPWRS.2013.2272518.

Nemati, M., Braun, M., Tenbohlen, S., 2018. Optimization of unit commitment and economic dispatch in microgrids based on genetic algorithm and mixed integer linear programming," (in English). Appl. Energy 210, 944–963. https://doi.org/10.1016/j.apenergy.2017.07.007.

Elexon. 2022 "Elexon BMRS." Elexon. ⟨https://www.bmreports.com/bmrs/?q=/balancing/balancingserviceadjdata/2021-01-31/1⟩ (accessed 10 March, 2022).