



Seismic Waveform Tomography for 3D Impedance Model with Salt Structure

FENGXIA GAO¹ and YANGHUA WANG²

Abstract—Conventional impedance inversion from post-stack zero-offset seismic data is usually based on the convolution model, and wave-equation based inversion is rarely used, although it is capable to precisely describe seismic wave propagation and invert impedance model with higher resolution. That is because there are more than one physical parameters involved in the conventional wave equation, making impedance inversion complicated. In this study, a one-dimensional (1D) wave equation, containing only the impedance parameter, is adopted and applied for the inversion of 1D impedance model by seismic waveform tomography. However, for a three-dimensional (3D) model, direct application of the 1D waveform tomography may lead to lateral discontinuities. Therefore, we propose to utilize a truncated Fourier series to parameterize the 3D impedance model, and then invert for the Fourier coefficients. With this parameterization scheme, the large- and small-scale components of the impedance model can be reconstructed stepwise by gradually increasing the number of Fourier coefficients. To efficiently and effectively invert the coefficients for the 3D model with salt structure, we propose a joint strategy, in which the low-frequency seismic data is used to invert for the Fourier coefficients representing the large-scale components of the model, while the high-frequency seismic data is applied to invert for the Fourier coefficients representing the small-scale components of the model. Tests on a 3D impedance model with salt structure result in models with high resolution and good spatial continuity, proving the feasibility and stability of the impedance inversion procedure.

Keywords: Impedance model, salt structure, Fourier series, waveform inversion, waveform tomography.

1. Introduction

Seismic impedance is an important parameter in reservoir characterization. Conventional impedance

inversion from post-stack zero-offset seismic data is based on the convolution model. Wave-equation based inversion is rare, although it is capable to precisely describe seismic wave propagation and invert impedance model with higher resolution. One of the factors that make the wave-equation based inversion difficult are the multiple physical parameters involved in conventional wave equations. To obtain the impedance model from these wave equations, one can perform multi-parameter inversion (Tarantola, 1986) or assume one parameter known to reduce the unknown parameters or use the empirical formulas for velocity and density. All these methods are possible to cause errors in the inverted impedance model due to the cross-talks in multi-parameter inversion (Köhn et al., 2012; Pan et al., 2016), the inaccurate assumptions and the empirical formulae in single parameter inversion.

An alternative to avoid the multiple parameter problem in impedance inversion is to use a wave equation that contains only the impedance parameter. It is converted from the conventional one-dimensional (1D) wave equation according to a depth to time transformation (Santosa & Schwetlick, 1982), which only works with the 1D wave equation. With this equation, 1D impedance inversion can be performed using waveform tomography, an efficient method for reconstructing subsurface parameters (Tarantola, 1984; Virieux & Operto, 2009; Wang & Rao, 2009). For three-dimensional (3D) models, this 1D impedance inversion procedure will invert the 3D impedance model trace by trace, which may lead to lateral discontinuities, especially for models with salt structure.

To obtain spatially continuous inversion results, the truncated Fourier series parameterization scheme may be applied to the 3D impedance

¹ SinoProbe Center, Chinese Academy of Geological Sciences and China Geological Survey, Beijing 100037, China. E-mail: gaofx@cags.ac.cn

² Centre for Reservoir Geophysics, Resource Geophysics Academy, Imperial College London, South Kensington SW7 2BP, UK. E-mail: yanghua.wang@imperial.ac.uk

inversion. In this scheme, the impedance parameter is parameterized to the Fourier coefficients and then the inverted parameter will be changed from impedance to the Fourier coefficients. After finishing the inversion of Fourier coefficients, the final 3D impedance model will be reconstructed using these coefficients. This parameterization scheme was first applied to seismic traveltime and amplitude inversion for interface geometry and velocity variation (Wang, 1999; Wang & Houseman, 1995; Wang & Pratt, 1997, 2000). It has also been implemented in two-dimensional (2D) impedance inversion (Wang, 2016) and elastic parameter inversion from amplitude versus offset (AVO) data (Liu & Wang, 2020). With this scheme, the large- and small-scale components of the model can be reconstructed stepwise by gradually increasing the number of Fourier coefficients in a model-driven mode.

Combination of the 1D waveform tomography and the model parameterization scheme can be used for the inversion of a 3D impedance model without salt structure using the full-band seismic data (Gao et al., 2023). However, in the inversion of impedance model with salt structure, the salt and subsalt structures cannot be recovered when the full-band data are used for the inversion (Chen et al., 2020). This is because the inversion of the salt structures depends on the low-frequency seismic data. One can apply the data-driven strategy, where the seismic data is inverted from low to high frequency bands, for the Fourier coefficients inversion. In each frequency band, small- to large-valued numbers of Fourier coefficients will be updated. This strategy will increase the computation time. Therefore, the question is how to develop efficient inversion strategies to effectively invert the Fourier coefficients for a 3D impedance model with salt structure.

In this study, we first present the wave equation containing only the impedance parameter, the model parameterization and the theory of seismic waveform tomography. Secondly, we clarify the properties of the model parameterization and propose the joint data-driven and model-driven strategy for efficient inversion of the Fourier coefficients. Finally, we test the inversion method and strategy on a 3D impedance model with salt structure.

2. Theories for Impedance Inversion

2.1. Wave Equation Defined by Impedance

The wave equation defined in terms of impedance is as follows (Wang, 2016).

$$\frac{\partial}{\partial \tau} \left(Z(\tau) \frac{\partial u(\tau, t)}{\partial \tau} \right) - \frac{Z(\tau)}{4} \frac{\partial^2 u(\tau, t)}{\partial t^2} = 0, \quad (1)$$

where u is the displacement, τ is the vertical two-way traveltime, and Z is the impedance, the product of density and velocity. In the subsurface model, we refer to vertical two-way traveltime τ as the depth-time, treated equally as two spatial variables. Because of the use of the depth-time τ , the wave equation contains only a single elastic parameter, the impedance parameter. We solve this equation numerically using a finite difference method (Hall & Wang, 2009).

2.2. Impedance Parameterization

We parameterize a 3D impedance model using a truncated Fourier series:

$$\begin{aligned} \hat{Z}(p, q, r) = & \sum_{(l,m,n)=(0,0,0)}^{(L-1,M-1,N-1)} [a_{lmn} \cos(l\Delta k_x p) \cos(m\Delta k_y q) \cos(n\Delta k_\tau r) \\ & + b_{lmn} \sin(l\Delta k_x p) \cos(m\Delta k_y q) \cos(n\Delta k_\tau r) \\ & + c_{lmn} \cos(l\Delta k_x p) \sin(m\Delta k_y q) \cos(n\Delta k_\tau r) \\ & + d_{lmn} \sin(l\Delta k_x p) \sin(m\Delta k_y q) \cos(n\Delta k_\tau r) \\ & + e_{lmn} \cos(l\Delta k_x p) \cos(m\Delta k_y q) \sin(n\Delta k_\tau r) \\ & + f_{lmn} \sin(l\Delta k_x p) \cos(m\Delta k_y q) \sin(n\Delta k_\tau r) \\ & + g_{lmn} \cos(l\Delta k_x p) \sin(m\Delta k_y q) \sin(n\Delta k_\tau r) \\ & + h_{lmn} \sin(l\Delta k_x p) \sin(m\Delta k_y q) \sin(n\Delta k_\tau r)], \end{aligned} \quad (2)$$

where $\hat{Z}(p, q, r) \equiv \hat{Z}(p\Delta x, q\Delta y, r\Delta \tau)$ is an approximation to the true impedance model, $(\Delta x, \Delta y)$ are the space intervals, $\Delta \tau$ is the time interval, $\{a, b, \dots, h\}$ with subscripts (l, m, n) are the Fourier coefficients, (L, M, N) are the numbers of Fourier coefficients in each dimension, $(\Delta k_x, \Delta k_y, \Delta k_\tau)$ are the fundamental wavenumbers, set as $\Delta k_x = 2\pi/N_x$, $\Delta k_y = 2\pi/N_y$, and $\Delta k_\tau = 2\pi/N_\tau$, where (N_x, N_y, N_τ) are the number of

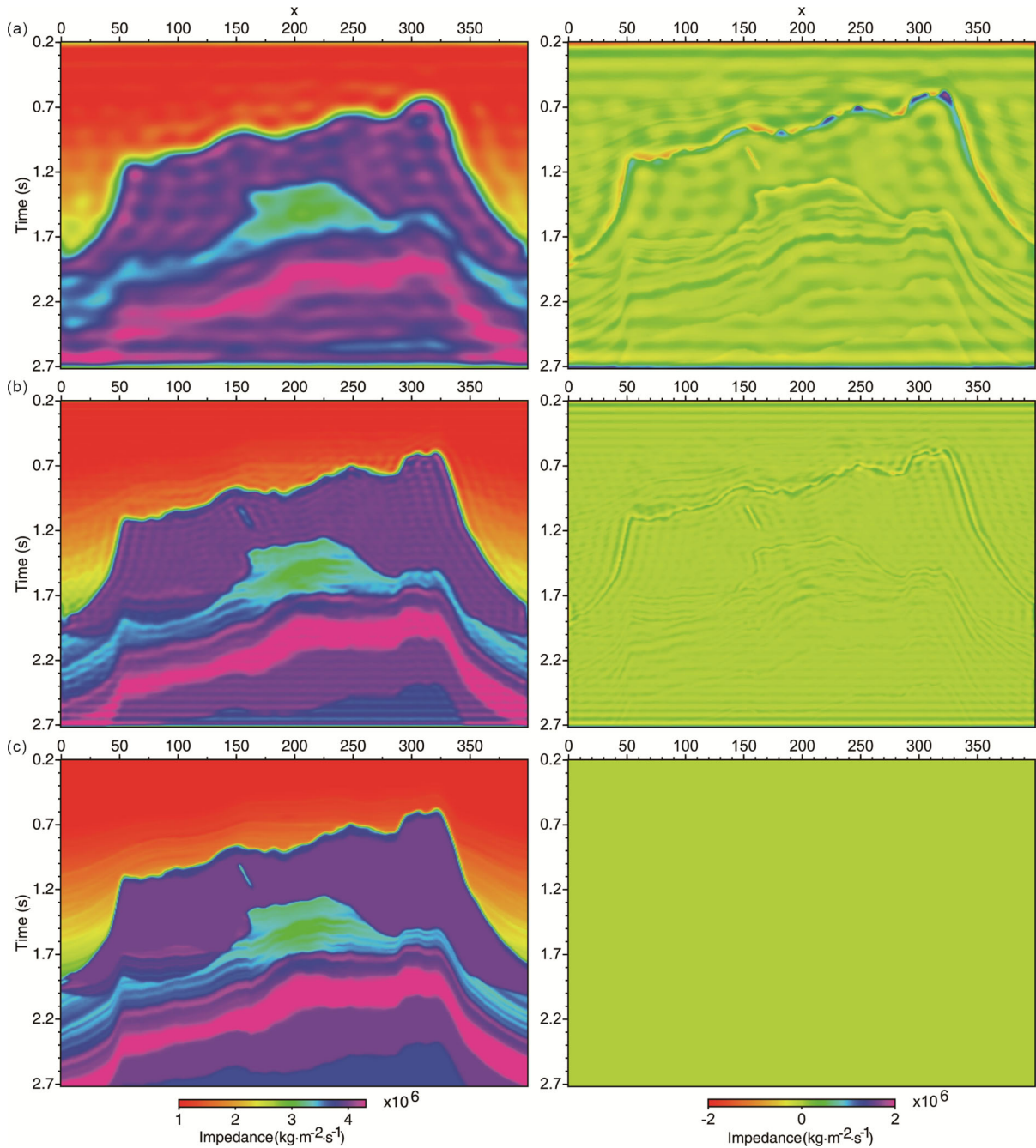


Figure 1

Impedance profiles at $x = 200$ reconstructed by the Fourier coefficients (left panel) and the differences between the reconstructed and true models (right panel) when **a** $(L, M, N) = (15, 15, 15)$, **b** $(L, M, N) = (40, 40, 40)$, **c** $(L, M, N) = (201, 201, 211)$

samples for the model in (x, y, τ) direction, respectively.

After the model parameterization, the parameters inverted will be changed from impedance to Fourier

coefficients. When the inverted Fourier coefficients are obtained, the impedance model will be reconstructed by Eq. 2.

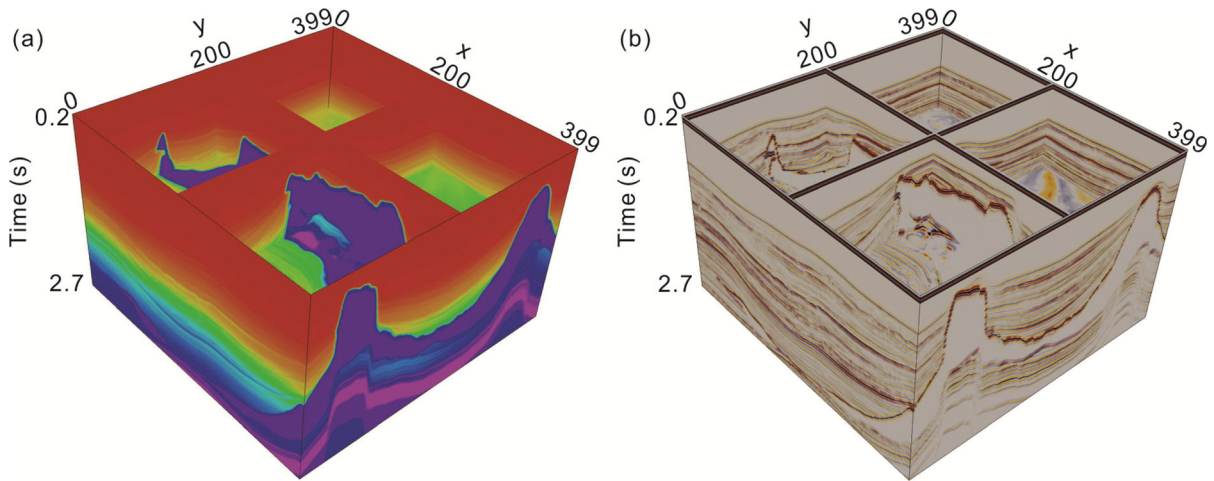


Figure 2
a The 3D impedance model. **b** The synthetic seismic cube

2.3. The Inversion Method for Fourier Coefficients

In the waveform tomography for the Fourier coefficients, we define the objective function in terms of data misfit,

$$\phi(\mathbf{m}) = \frac{1}{2} \sum_p \sum_q \left\| \mathbf{u}_{obs}^{(pq)} - \mathbf{u}^{(pq)}(\mathbf{m}) \right\|^2, \quad (3)$$

where $\mathbf{u}_{obs}^{(pq)} \equiv \mathbf{u}_{obs}(p\Delta x, q\Delta y)$ represents a single observed seismic trace at $(p\Delta x, q\Delta y)$, $\mathbf{m}$ is the “model” consisting of the Fourier coefficients, and $\mathbf{u}^{(pq)}(\mathbf{m})$ is the synthetic seismic trace simulated by Eq. 1.

The gradient vector, which is the first-order derivative of the objective function $\phi(\mathbf{m})$ with respect to the model parameters, taking coefficients a_{lmn} as an example, is

$$\frac{\partial \phi(\mathbf{m})}{\partial a_{lmn}} = - \sum_p \sum_q (\mathbf{u}_{obs}^{(pq)} - \mathbf{u}^{(pq)}(\mathbf{m})) \frac{\partial \mathbf{u}^{(pq)}(\mathbf{m})}{\partial a_{lmn}}. \quad (4)$$

The derivative $\partial \mathbf{u}^{(pq)}(\mathbf{m}) / \partial a_{lmn}$, according to the chain rule, can be explicitly expressed as

$$\frac{\partial \mathbf{u}^{(pq)}(\mathbf{m})}{\partial a_{lmn}} = \sum_r \frac{\partial \mathbf{u}^{(pq)}(\mathbf{m})}{\partial Z_r^{(pq)}} \frac{\partial Z_r^{(pq)}}{\partial a_{lmn}}, \quad (5)$$

where $Z_r^{(pq)} = Z(p\Delta x, q\Delta y, r\Delta \tau)$ is the 1D impedance variable associated with the specific p th in x and q th

in y trace. The first-order derivative $\partial \mathbf{u}^{(pq)}(\mathbf{m}) / \partial Z_r^{(pq)}$ is the derivative of the (p, q) th seismic trace with respect to the r th impedance in the same trace, and the first-order derivative $\partial Z_r^{(pq)} / \partial a_{lmn}$ can be explicitly written as

$$\frac{\partial Z_r^{(pq)}}{\partial a_{lmn}} = \cos(l\Delta k_x p) \cos(m\Delta k_y q) \cos(n\Delta k_\tau r). \quad (6)$$

Combining Eqs. 4–6, the first order derivative of the objective function with respect to a_{lmn} can be expressed as

$$\begin{aligned} \frac{\partial \phi(\mathbf{m})}{\partial a_{lmn}} = & - \sum_p \sum_q \sum_r \left[\frac{\partial \mathbf{u}^{(pq)}(\mathbf{m})}{\partial Z_r^{(pq)}} (\mathbf{u}_{obs}^{(pq)} - \mathbf{u}^{(pq)}(\mathbf{m})) \right] \frac{\partial Z_r^{(pq)}}{\partial a_{lmn}}. \end{aligned} \quad (7)$$

The derivatives to other Fourier coefficients are derived in the same way.

The “model” $\mathbf{m}$ (i.e., Fourier coefficients) are updated along the negative gradient direction, and the step-length is determined by a linear search method (Vigh et al., 2009).

3. The Inversion Strategy

In seismic waveform tomography, a commonly used hierarchical strategy is to invert the seismic data

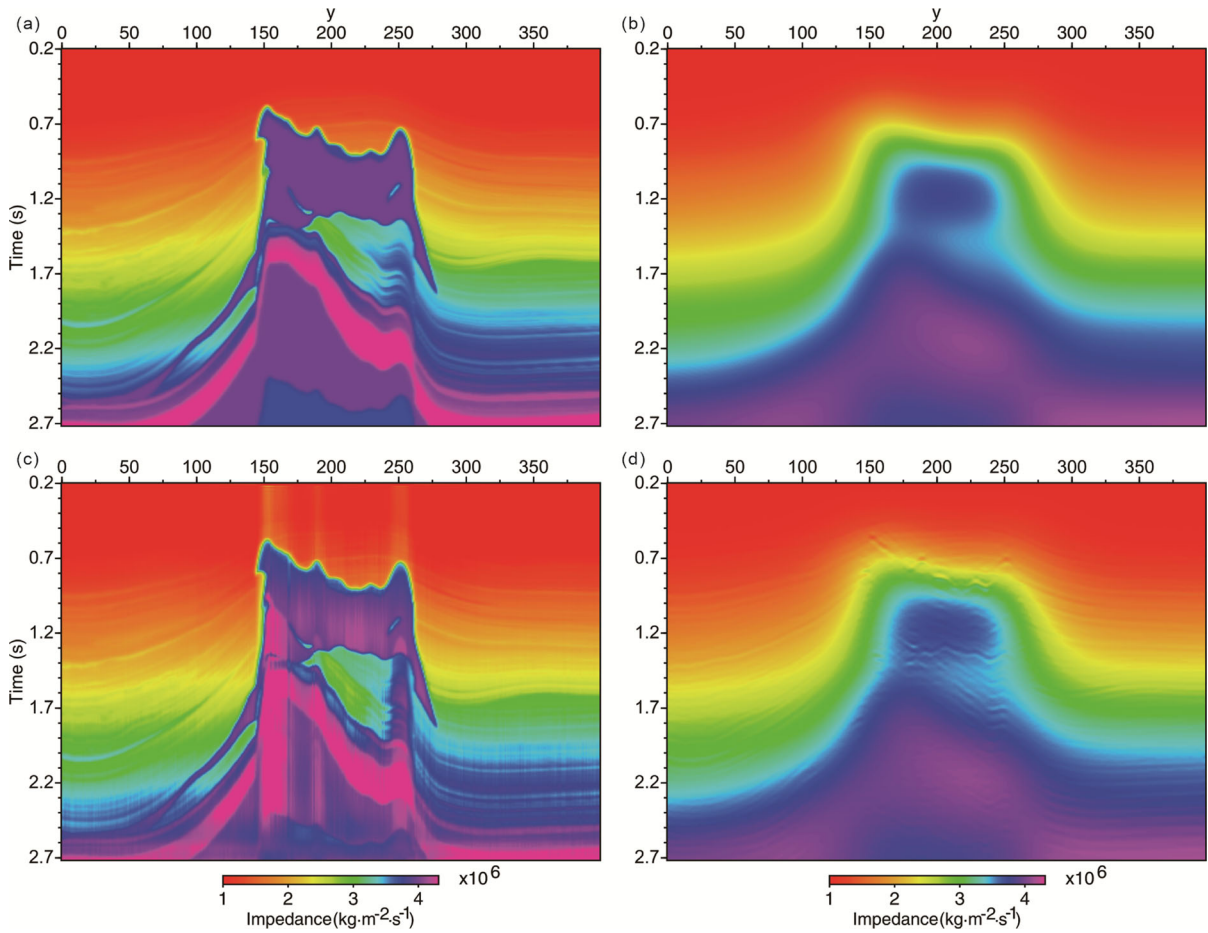


Figure 3

Impedance profiles at $x = 200$ for the **a** true model, **b** starting model, **c** inverted model by the trace-by-trace method, **d** reconstructed model using the inverted Fourier coefficients from the full frequency band seismic data

from low-frequency band to high-frequency band (Bunks et al., 1995; Wang & Rao, 2009, 2020) to get the large-scale and small-scale components of the model gradually. This strategy is referred to as the data-driven mode.

In this study, the model is approximated by the truncated Fourier series in Eq. 2. The values of (L, M, N) in Eq. 2 determine the closeness of the reconstructed model to the true model. Figure 1 presents the reconstructed impedance profiles at $x = 200$ with different (L, M, N) s. It can be seen that with small-valued (L, M, N) , the reconstructed model represents the background trend of the true model (Fig. 1a). With large-valued (L, M, N) , the reconstructed model

is further refined (Fig. 1b). In the right panel of Fig. 1, we also show their differences between the true and the reconstructed models. It can be seen that when the values of (L, M, N) are increased, the differences are smaller. When $(L, M, N) = (201, 201, 211)$, there is no difference between the true and the reconstructed model (Fig. 1c), meaning the reconstructed model is exactly the same to the true impedance model. Note that the true profile in Fig. 1c is extracted from a 3D model at $x = 200$, which will be shown in Fig. 2a.

This characterization of the model parameterization scheme is suitable for designing a model-driven strategy in the inversion. This strategy updates the

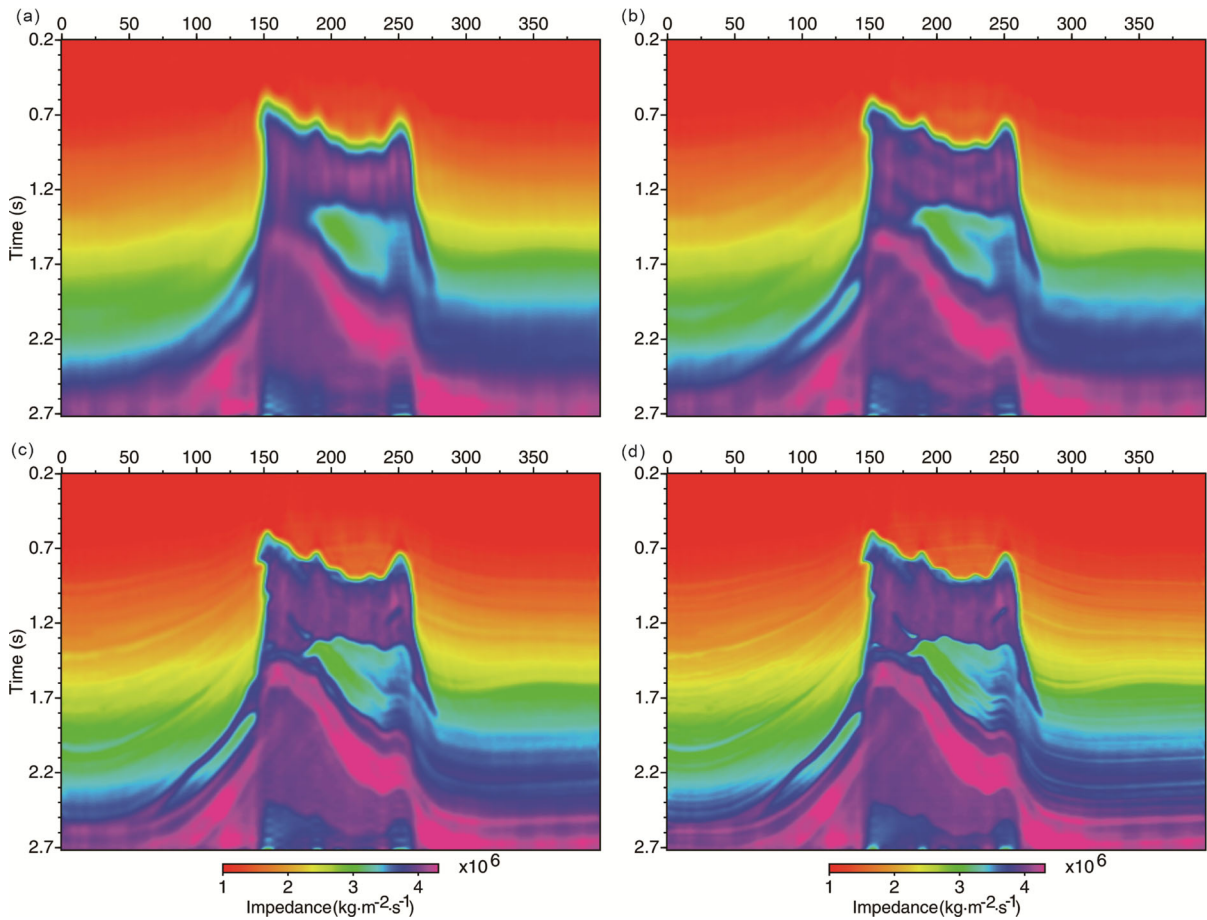


Figure 4

Impedance profiles in different stages at $x = 200$. **a** The reconstructed impedance profile when $(L, M, N) = (40, 40, 40)$ and the seismic data used for inversion is within $[0, 1]$ Hz. **b** The reconstructed impedance profile when $(L, M, N) = (40, 40, 40)$ and the seismic data is within $[0, 3]$ Hz. **c** The reconstructed impedance profile when $(L, M, N) = (65, 65, 65)$ and the seismic data is within $[0, 5]$ Hz. **d** The reconstructed impedance profile when $(L, M, N) = (90, 90, 90)$ and the seismic data is within $[0, 25]$ Hz

Fourier coefficients with small valued (L, M, N) first, which represents the large-scale components of the model, and then updates the Fourier coefficients with large-valued (L, M, N) , which indicates a further refined model.

In this study, we propose to combine the data-driven and model-driven strategies together. Specifically, in this joint strategy, Fourier coefficients with small-valued (L, M, N) will be updated by the low-frequency seismic data, while Fourier coefficients with large-valued (L, M, N) will be inverted using the high-frequency seismic data.

4. Application in Seismic Impedance Inversion

We test the proposed inversion scheme on the inversion for a 3D impedance model with salt structure.

Figure 2a is a 3D impedance model. It has the features of (a) complex structure; (b) high impedance contrast and steep interfaces between the salt and surrounding sedimentary layers; (c) thin layers inside/beneath the salt body; (d) thick salt structure and thin sediment layers. In the model, there are 400 samples in both the x and y directions with space interval of 25 m, and 420 samples in time direction with interval

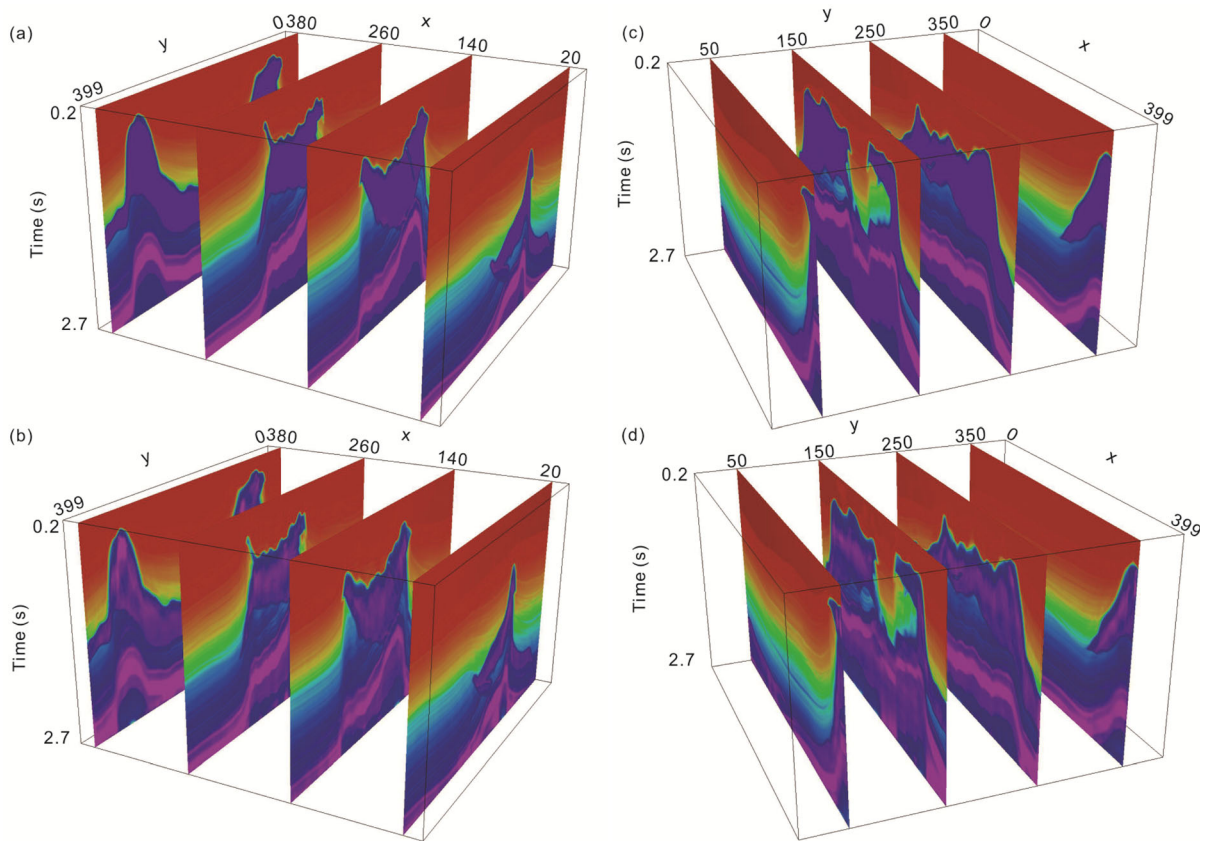


Figure 5

Comparisons between the true and reconstructed impedance profiles. The left panel is the impedance profiles at $x = 20, 140, 260$, and 380 . The right panel is impedance profiles at $y = 50, 150, 250$, and 350 . **a** and **c** are the true ones, while **b** and **d** are the reconstructed counterparts. The colour bar is the same to that in Fig. 3

of 6 ms. Figure 2b is the corresponding seismic cube generated by the 1D wave equation simulation. It is assumed to have full frequency band components that is it has the low, medium and high frequency components. There are 1300 samples in each trace and the time interval is 2 ms. Figure 2b shows only the time range of 0.18–2.7 s. The wavelet used for the simulation is Ricker wavelet with the peak frequency of 25 Hz (Wang, 2015a). For field data test, the wavelet may be estimated by the generalized wavelet estimation method, which is calculated from the observed seismic data and well-independent (Wang, 2015b). With this estimated wavelet, the proposed method is able to invert the impedance model reliably.

In the waveform tomography, the starting model is a smoothed version of the true impedance model,

calculated by a 3D smooth function. Figure 3a and b are two profiles at $x = 200$, extracted from the true model and the starting model, respectively. We first invert the impedance directly without using the model parameterization scheme by the Fourier coefficients, which is implemented trace by trace. This method is referred to as trace-by-trace method. In this test, each impedance trace is inverted from its corresponding seismic trace by using the data-driven strategy, which means that the impedance in each trace is inverted by the seismic trace from the low-frequency band to high-frequency band gradually. To recover the high-valued salt body/layers, the inversion starts from an ultra-low frequency band, $[0, 1]$ Hz. In the test, five frequency bands are used for the impedance inversion, which are $[0, 1]$, $[0, 3]$, $[0, 5]$, $[0, 15]$, $[0, 25]$ Hz. In each frequency band, the

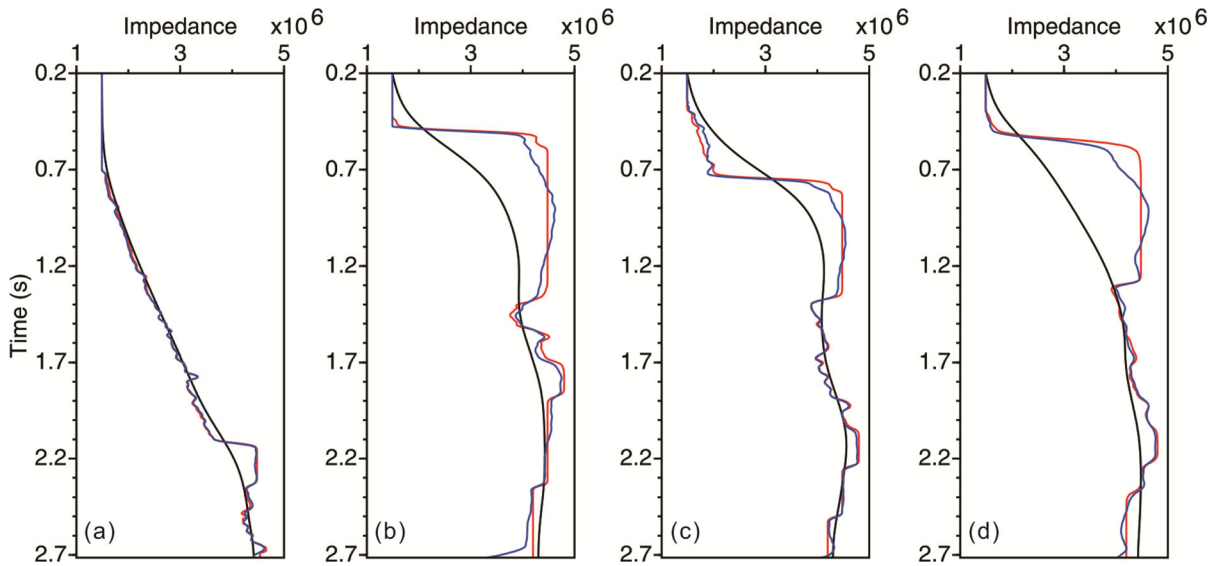


Figure 6

Impedance trace comparisons at **a** $(x, y) = (20, 50)$, **b** $(x, y) = (140, 150)$, **c** $(x, y) = (260, 250)$, and **d** $(x, y) = (380, 350)$. The true, starting and reconstructed impedance traces are presented by the red, black and blue curves, respectively

inversion will terminate when the iteration reaches 10. Figure 3c is the inverted impedance profile at $x = 200$ by the trace-by-trace method. It can be seen that impedance values beneath the salt body in positions around $y = 150$ and $y = 250$ are not well recovered. Besides, there are also some lateral discontinuities between 2.0 and 2.7 s.

We then try to use the model-driven strategy to invert the Fourier coefficients by using the full frequency band seismic data. In the model-driven inversion procedure, considering the computation time and the convenience for sub-groups setting in the inversion, we select approximately twentieth to tenth of $(N_x/2+1, N_y/2+1, N_z/2+1)$ as the starting values of (L, M, N) , where N_x, N_y, N_z are the number of samples of the model in three directions, and the increment of (L, M, N) is usually set as 1–2 times of the starting values of (L, M, N) (Gao et al., 2023). Therefore, in this test, the values of (L, M, N) of the Fourier coefficients start from (15, 15, 15) and terminate at (90, 90, 90). The increments of (L, M, N) is (25, 25, 25). Figure 3d is a profile at $x = 200$ from the reconstructed impedance model using the inverted Fourier coefficients by the model-driven strategy. It is obvious that thick salt body/layers with high impedance values are not recovered. The reason is that the

seismic data is not divided into low to high frequency bands, instead the full frequency band seismic data is directly used to invert for the Fourier coefficients. The full frequency band data is equivalent to the high-frequency bands data in data-driven strategy, which mainly works for the recovery of the small-scale components of the model. Since the starting model in Fig. 3b is far away from the true model, only using the high frequency band seismic data cannot reconstruct the large-scale components of the model, as shown in Fig. 3d.

When the proposed strategy is applied in impedance inversion, the low-frequency seismic data and Fourier coefficients with small-valued (L, M, N) will be applied for the reconstruction of large-scale components of the salt body and boundaries between the salt body and sedimentary layers, while the high-frequency seismic data and Fourier coefficients with large-valued (L, M, N) will be used for the refinement of the boundaries and recovery of the thin layers. In this test, the frequency bands used are the same to that in the trace-by-trace method. For each frequency band, different numbers of Fourier coefficients are set. When the seismic data are filtered with [0, 1] Hz and [0, 3] Hz, the (L, M, N) s start with (15, 15, 15) and terminate at (40, 40, 40). When the seismic data

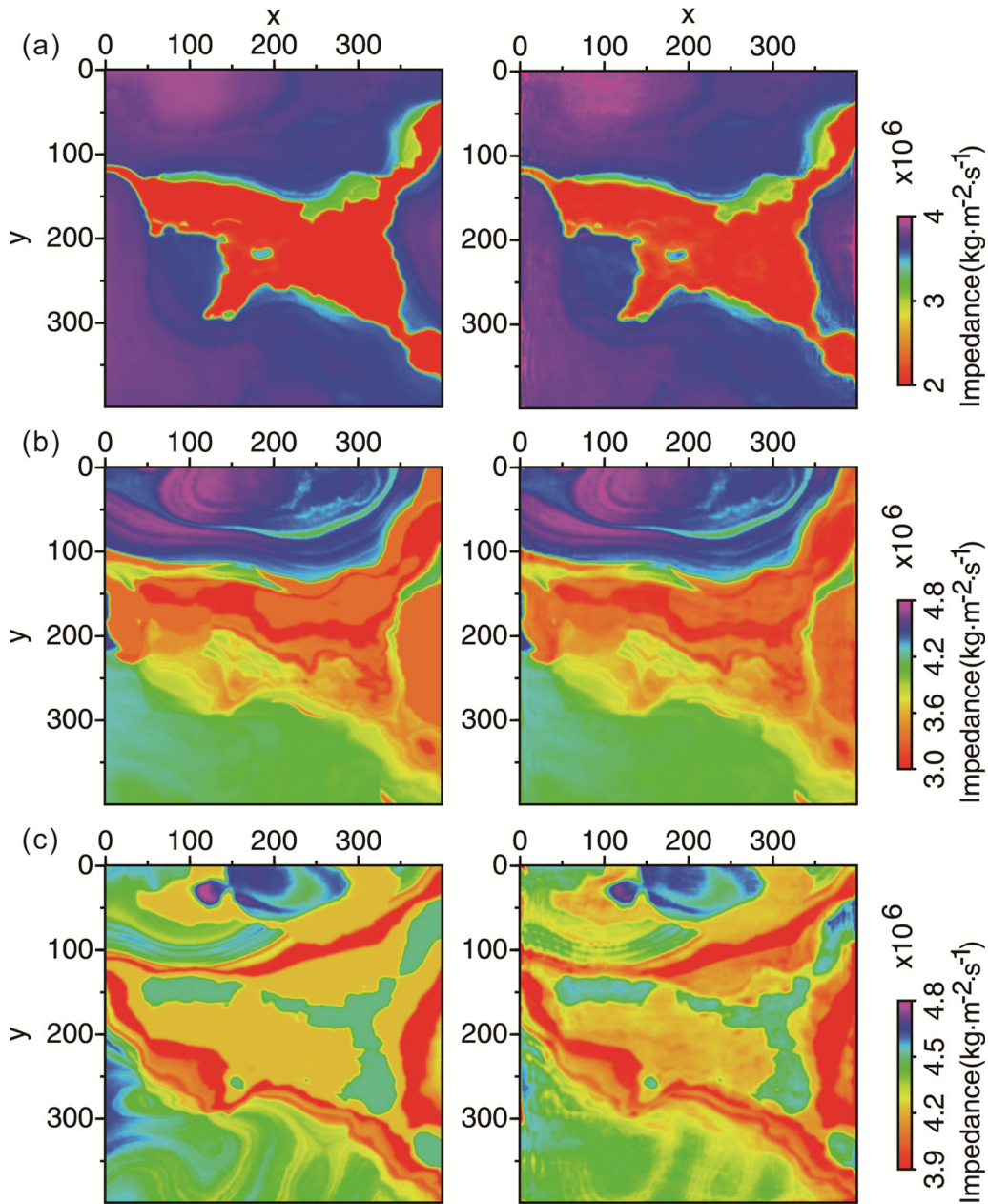


Figure 7

Time slices comparison between the true and reconstructed models. The left panel is for the true impedance model and the right panel is for the reconstructed model. The time slices are at 0.9 s (a), 1.8 s (b) and 2.4 s (c)

are within $[0, 5]$ Hz, the starting and final (L, M, N) s are $(15, 15, 15)$ and $(65, 65, 65)$, respectively. When the seismic data are filtered by high frequency bands, i.e. $[0, 15]$ and $[0, 25]$ Hz, the (L, M, N) s begin at $(40,$

$40, 40)$ and $(65, 65, 65)$, and stop at $(65, 65, 65)$ and $(90, 90, 90)$, respectively.

Figure 4 compares the impedance profiles at $x = 200$ between the true and reconstructed models at different stages. From Fig. 4, it can be seen that even

when the starting impedance profile (Fig. 3b) is far away from the true model (Fig. 3a), with the ultra-low frequency seismic data, the large-scale components of the impedance model can be recovered, as shown in Fig. 4a and b. Based on the large-scale components, with large valued numbers of the Fourier coefficients and high frequency seismic data, the small-scale components of the impedance model are reconstructed and finally a gradually refined model is obtained in Fig. 4c and d. Compared to Figs. 3c and d, Fig. 4d gives a continuous impedance profile with well recovered salt and subsalt structure. However, if the seismic data lacks the low frequency components, which is useful for inverting the large-scale components of the model, the starting model is better to include these large-scale components.

Figure 5 compares 8 pairs of true and reconstructed impedance profiles, which are at $x = 20, 140, 260, 380$ in Fig. 5a and b and at $y = 50, 150, 250, 350$ in Fig. 5c and d, respectively. The reconstructed impedance profiles by the proposed strategy at different positions are close to the true impedance profiles.

Figure 6 shows 4 groups of impedance traces at $(x, y) = (20, 50), (140, 150), (260, 250),$ and $(380, 350)$. In each group, the true, starting and reconstructed impedance traces are presented. The true and reconstructed impedance traces are nearly the same except some small fluctuations around the salt body.

Figure 7 presents the true and reconstructed impedance slices at time 0.9 s, 1.8 s and 2.4 s, respectively. The true and reconstructed slices are also nearly the same, which prove the validity of the proposed strategy in 3D impedance inversion.

5. Conclusion

In this study, the wave equation containing only the impedance parameter is used in seismic waveform tomography for the impedance model. The subsurface impedance model is parameterized by a truncated Fourier series and the inverted parameters are changed to Fourier coefficients. Waveform tomography and model parameterization are combined for the inversion of the salt model. We have proposed a joint data- and model-driven strategy to

invert the Fourier coefficients with small-valued numbers by the low-frequency seismic data and to update these coefficients with large-valued numbers by the high-frequency seismic data. Model tests show that with this inversion strategy the salt and subsalt structures can be well recovered with high resolution and good spatial continuity.

Acknowledgements

The authors are grateful to Professor Ying Rao, China University of Petroleum (Beijing), for providing the 3D salt model.

Author contributions FG wrote the first draft of the main manuscript text and prepared the figures. YW refined the manuscript and the figures. All authors reviewed the manuscript.

Funding

This research is funded by the Centre for Reservoir Geophysics, Imperial College London.

Data availability

Data associated with this research are confidential and cannot be released.

Declarations

Conflict of interest The authors have not disclosed any competing interests.

Open Access This article is licensed under a Creative Commons Attribution 4.0 International License, which permits use, sharing, adaptation, distribution and reproduction in any medium or format, as long as you give appropriate credit to the original author(s) and the source, provide a link to the Creative Commons licence, and indicate if changes were made. The images or other third party material in this article are included in the article's Creative Commons licence, unless indicated otherwise in a credit line to the material. If material is not included in the article's Creative Commons licence and your intended use is not permitted by statutory regulation or exceeds the permitted use, you will need to obtain permission directly from the copyright holder. To view a copy of this licence, visit <http://creativecommons.org/licenses/by/4.0/>.

Publisher's Note Springer Nature remains neutral with regard to jurisdictional claims in published maps and institutional affiliations.

REFERENCES

- Bunks, C., Saleck, F. M., Zaleski, S., & Chavent, G. (1995). Multiscale seismic waveform inversion. *Geophysics*, 60(5), 1457–1473. <https://doi.org/10.1190/1.1443880>
- Chen, G., Yang, W., Chen, S., Liu, Y., & Gu, Z. (2020). Application of envelope in salt structure velocity building: From objective function construction to the full-band seismic data reconstruction. *IEEE Transactions on Geoscience & Remote Sensing*, 58(9), 6594–6608. <https://doi.org/10.1109/TGRS.2020.2978125>
- Gao, F., Rao, Y., Zhu, T., & Wang, Y. (2023). 3-D seismic inversion by model parameterization with Fourier coefficients. *IEEE Transactions on Geoscience and Remote Sensing*, 61, 1–16. <https://doi.org/10.1109/TGRS.2023.3268410>
- Hall, F., & Wang, Y. (2009). Elastic wave modelling by an integrated finite difference method. *Geophysical Journal International*, 177(1), 104–114. <https://doi.org/10.1111/j.1365-246X.2008.04065.x>
- Köhn, D., De Nil, D., Kurzmann, A., Przebindowska, A., & Bohlen, T. (2012). On the influence of model parameterization in elastic full waveform tomography. *Geophysical Journal International*, 191(1), 325–345. <https://doi.org/10.1111/j.1365-246X.2012.05633.x>
- Liu, J.Y., & Wang, Y. (2020). Seismic simultaneous inversion using a multidamped subspace method. *Geophysics*, 85(1), R1–R10. <https://doi.org/10.1190/geo2018-0470.1>
- Pan, W., Innanen, K. A., Margrave, F. G., Fehler, C. M., Fang, X., & Li, J. (2016). Estimation of elastic constants for HTI media using Gauss–Newton and full-Newton multiparameter full waveform inversion. *Geophysics*, 81(5), R275–R291. <https://doi.org/10.1190/geo2015-0594.1>
- Santosa, F., & Schwetlick, H. (1982). The inversion of acoustical impedance profile by methods of characteristics. *Wave Motion*, 4(1), 99–110. [https://doi.org/10.1016/0165-2125\(82\)90017-8](https://doi.org/10.1016/0165-2125(82)90017-8)
- Tarantola, A. (1984). Inversion of seismic reflection data in the acoustic approximation. *Geophysics*, 49(1), 1259–1266. <https://doi.org/10.1190/1.1441754>
- Tarantola, A. (1986). A strategy for nonlinear elastic inversion of seismic reflection data. *Geophysics*, 51(10), 1893–1903. <https://doi.org/10.1190/1.1442046>
- Vigh, D., Starr, E. W., & Kapoor, J. (2009). Developing earth models with full waveform inversion. *The Leading Edge*, 28(4), 432–435. <https://doi.org/10.1190/1.3112760>
- Virieux, J., & Operto, S. (2009). An overview of full-waveform inversion in exploration geophysics. *Geophysics*, 74(6), WCC1–WCC26. <https://doi.org/10.1190/1.3238367>
- Wang, Y. (1999). Simultaneous inversion for model geometry and elastic parameters. *Geophysics*, 64(1), 182–190. <https://doi.org/10.1190/1.1444514>
- Wang, Y. (2015a). Frequencies of the Ricker wavelet. *Geophysics*, 80(2), A31–A37. <https://doi.org/10.1190/geo2014-0441.1>
- Wang, Y. (2015b). Generalized seismic wavelets. *Geophysical Journal International*, 203(2), 1172–1178. <https://doi.org/10.1093/gji/ggv346>
- Wang, Y. (2016). *Seismic inversion, theory and applications*. Wiley Blackwell.
- Wang, Y., & Houseman, A. G. (1995). Tomographic inversion of reflection seismic amplitude data for velocity variation. *Geophysical Journal International*, 123(2), 355–372. <https://doi.org/10.1111/j.1365-246X.1995.tb06859.x>
- Wang, Y., & Pratt, R. G. (1997). Sensitivities of seismic travel-times and amplitudes in reflection tomography. *Geophysical Journal International*, 131, 618–642. <https://doi.org/10.1111/j.1365-246X.1997.tb06603.x>
- Wang, Y., & Pratt, R. G. (2000). Seismic amplitude inversion for interface geometry of multi-layered structures. *Pure and Applied Geophysics*, 157, 1601–1620. <https://doi.org/10.1007/PL00001052>
- Wang, Y., & Rao, Y. (2009). Reflection seismic waveform tomography. *Journal of Geophysical Research*, 114(B3), B03304. <https://doi.org/10.1029/2008JB005916>
- Wang, Y., & Rao, Y. (2020). *Seismic, Waveform Modeling and Tomography, Encyclopedia of Solid Earth Geophysics* (2nd ed.). Springer. https://doi.org/10.1007/978-3-030-10475-7_211-1

(Received March 21, 2023, revised May 23, 2023, accepted May 28, 2023, Published online June 10, 2023)