

ERRATA

p.vii p.111 R radial ordinate, $Re^{0.5} r$ (Fig. 14)
 r non-dimensional radial ordinate (Fig. 14)
 Re reference Reynolds number (Fig. 14)
 p.x insert λ a dimensionless constant, may be regarded as a compression factor i.e. the radius of the circle into which the vorticity in a given portion of the sheet is compressed (section 2.2)

p.xiv line 12 should read: through the vortices at $X = 676.9 \text{ mm}$

p.23 line 8 should read: are supplemented by the equation of state
 $p \propto \rho T$ and the whole

p.51 equation (2.3,2) should read:

$$R_{mn} = e_x(x'_m - x_n) + e_y(s'_m - s_n) + (\Gamma'_m - \Gamma_n)$$

p.72 line 17 should read: an onset of instability; this was

p.144 line 3 should read:

$$\bar{E}_{1,2}^2 = \bar{E}_{0,2}^2 + B_{1,2} \left(\frac{\bar{U} \cos \psi_{1,2} \pm \bar{V} \sin \psi_{1,2}}{\cos \psi_{1,2}} \right)^n$$

line 4 should read:

$$= \bar{E}_{0,2}^2 + B_{1,2} \left(\bar{U} \pm \bar{V} \tan \psi_{1,2} \right)^n$$

Fig. 8 labels on right hand side should read:
 Primary attachment line
 Secondary attachment line

Fig. 9 label on right hand side should read:
 Prandtl uniform vorticity model

Fig. 38 title should read: Turbulence intensity, $\sqrt{u^2}/U_\infty$; a horizontal traverse through the vortices at $X = 676.9 \text{ mm}$ (26.65"): 63" chord delta wing.

IMPERIAL COLLEGE OF SCIENCE AND TECHNOLOGY

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THE INSTABILITY OF AIRCRAFT

TRAILING VORTICES

by

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A B S T R A C T

In this experimental study of the instability of aircraft trailing vortices the flow far behind a delta wing in a low speed wind tunnel was examined. The main emphasis was on searching for a disturbance moving with the outer flow, thus only sample profiles of the mean axial and azimuthal velocities through the core of one of the trailing vortices are given.

A "structured disturbance" in the flow was investigated using a conditional sampling technique based on velocity measurements of the flow in the trailing vortices made using hot wire anemometry. The "structured disturbance" was a random phenomenon and was not linked with the Crow instability. Besides the "structured disturbance" which typically had a wavelength of approximately 1 m, a longer wavelength disturbance (approximately ten times that of the "structured disturbance") was also discovered; this latter disturbance was attributed to vortex meandering and not studied in detail.

A discussion on the formation of the "structured disturbance" as a result of a distortion to the edge of the core of a vortex unstable in the Rayleigh sense is presented. The observed characteristics of the "structured disturbance" indicated that it is a stationary wave; the link between the "structured disturbance" and the solitary wave solution of the Korteweg-de-Vries equation is discussed.

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N O T A T I O N

A	wing planform area
a	core radius
a_c	core radius (section 2.1)
b	wing span (section 2.1 and Table I)
b	separation between the vortices (section 2.3 and ch. 4)
c	wing mean chord (section 2.1 and Table I)
c	vortex core diameter (section 2.3)
c	centreline chord of the wing (ch. 4 and Table VI)
$C_L = L / \frac{1}{2} \rho U_\infty^2 A$	lift coefficient
C_p	specific heat at constant pressure (section 2.2 and appendix A)
d	cut-off distance in self-induction integral, proportional to vortex core diameter (section 2.3, Crow [98])
$E = \bar{E} + e$	sum of the mean, $\bar{E}$, and fluctuating, e , voltages (appendices B, C, D and E)
e	distance for the trailing vortex to roll up (as defined by Spreiter and Sacks [9]) (section 2.1 and Table I)
e_x, e_y, e_z	longitudinal, lateral and vertical unit vectors (section 2.3, Crow [98])
I_0	stagnation enthalpy
K	circulation
k	thermal conductivity (section 2.2 and appendix A)
k	perturbation wavenumber (section 2.3, Crow [98])

L	wing lift
dL	line increment directed along vortex (section 2.3)
M	half the number of point vortices representing the vortex sheet (section 2.1, Moore [36])
N	number of point vortices representing the velocity field of a turn of a spiral (section 2.1, Moore [36])
P	constant boundary, value of pressure (section 2.2 and appendix A)
p	pressure
R	radius of the vortex ring (section 2.3, Widnall and Tsai [126])
R	overheat ratio (appendix B)
R	separation between points on vortices (section 2.3, Crow [98])
$Re_v = \Gamma_\infty / \nu$	vortex Reynolds number
$Re_w = U_\infty c / \nu$	Reynolds number based on centreline chord length
r	vortex radius (section 2.1, Adams [29])
$\underline{r}$	radial vortex displacement, with components (y, z) (section 2.3, Crow [98])
r_c	approximate core radius (section 2.2, Steger and Kutler [64])
r_m	radial location of peak swirl velocity i.e. core radius (section 2.1, Williams [28])
r, θ, z	cylindrical coordinates with positive z along downstream axis
s	position of an unperturbed vortex along $\underline{e}_y$ - axis (section 2.3, Crow [98])
s	wing semi-span at trailing edge (chapters 3 and 4)
s'	vortex separation distance (section 2.1, Spreiter and Sacks [9])

$s' = L/2\rho U_\infty \Gamma_0$	wake semi-span (section 2.1, Williams [28])
T	temperature (section 2.2 and appendix A)
t	time
$t^* = tU/a$	a/U being the natural time scale (section 2.1, Moore [36])
$\bar{U}$	mean level of the oscillation (section 2.4, Leibovitch and Seebass [164])
$\bar{U}$	total induced velocity, $\bar{u} - \bar{e}_z$ ($\Gamma_0/2\pi b$) (section 2.3, Crow [98])
U_∞	free stream velocity
U, V, W	constant, boundary, values of radial, circumferential and axial components of velocity respectively (section 2.2 and appendix A)
U, V, W	total axial, radial and azimuthal velocity components respectively, i.e. the sum of the mean and fluctuating velocities, e.g. $U = \bar{U} + u$ (chapters 3, 4 and 5 and appendices B and E)
$\bar{U}, \bar{V}, \bar{W}$	mean axial, radial and azimuthal components of velocity respectively (chapters 3, 4 and 5 and appendix E)
U_1, U_2, U_3	components of velocity, experiment (iii), U_1 in the axial direction, $U_2 = V \cos 30^\circ - W \sin 30^\circ$, $U_3 = V \sin 30^\circ + W \cos 30^\circ$ (chapters 3, 4 and 5)
u	perturbation velocity at vortex, with components (u,v,w) (section 2.3, Crow [98])

u, v, w	radial, circumferential and axial components of velocity respectively (section 2.2 and appendix A)
u, v, w	fluctuating axial, radial and azimuthal components of velocity respectively (chapters 3, 4 and 5)
u_s, u_n, u_r	components of instantaneous velocity in the streamwise direction, normal to the streamline in the circumferential direction and in the radial direction respectively, experiment (iv) (chapters 3, 4 and 5)
v_θ	transverse velocity relative to the centre of the spiral (section 2.2, Moore and Saffman [571])
W	free stream velocity (section 2.2, Batchelor [12])
w	axial velocity (section 2.2, Moore and Saffman [571])
w_m	peak swirl velocity about vortex centre (occurs at r_m) (section 2.1, Williams [28])
$w^* = \Gamma_0 / \pi s'$	wake reference velocity (section 2.1, Williams [28])
X, Y, Z	Cartesian coordinates X in the downstream direction, Y spanwise and Z vertical (chapters 3, 4 and 5 and Table V)
x	non-dimensional axial ordinate, axial distance/reference length (section 2.2, Newman [55])
x	coordinate along unperturbed vortex (section 2.3, Crow [98])
x, y, z	Cartesian coordinates x vertical, y spanwise and z in the downstream direction (section 2.1)
x, y, z	Cartesian coordinates x in the downstream direction, y spanwise and z vertical (section 2.2)
Y_m	spanwise coordinate based on the mid-point between the centres of the trailing vortices
Y_o	spanwise coordinate based on the centre of a vortex
y_c	displacement of vortex core in spanwise direction (section 2.1 and Table I)
α	temperature coefficient of resistivity (appendix B)
$\alpha = \nu_T / \Gamma$	(section 2.2)

β	dimensionless wavenumber, kb (section 2.3, Crow [98])
Γ	circulation
$\Gamma_m = 2\pi r_m w_m$	core circulation (section 2.1, Williams [28])
Γ_0	bound circulation at wing mid-span (section 2.1)
Γ_0	wing root circulation (Fig. 10)
$\Gamma_0 = \Gamma $	magnitude of circulation (section 2.3, Crow [98])
δ	yaw angle of hot-wire, see Fig. 78 (appendix B)
δ	dimensionless cut-off distance, kd (section 2.3, Crow [98])
$\varepsilon = a/R$	ratio of vortex core radius to ring radius (section 2.3, Widnall and Tsai [126])
ε	half the maximum axial disturbance value (section 2.4, Leibovitch [168])
ζ	independent radial variable (section 2.2, Batchelor [12])
ζ	trailed axial velocity (section 2.1, McCormick et al. [22])
η	transverse coordinate along wing span, origin at wingtip (section 2.1, Adams [29])
θ	temperature (appendix B)
$\theta_c = 360^\circ/N$	(section 2.1, Moore [36])
μ	viscosity
ν	laminar kinematic viscosity of air
ν_T	eddy viscosity
ξ, η, ζ	vorticity components in the r, θ , z directions respectively (appendix A)
ρ	density

$\sigma = \mu C_p / k$	Prandtl number (section 2.2 and appendix A)
ψ	effective angle of a hot wire, see Fig. 75 (appendix B)
ω	vorticity
$\omega_r, \omega_\theta, \omega_z$	components of vorticity vector
Λ	a factor relating to the vortex Reynolds number, $\alpha = \Lambda^2 (\Gamma/\nu)^{-1/2} \quad (\text{section 2.2})$

Subscripts

m	vortex inducing a field (section 2.3, Crow [98])
n	vortex acted upon by that field (section 2.3, Crow [98])
t	differentiation with respect to t (section 2.4)
x	differentiation with respect to x (section 2.4)
1	portside trailing vortex (section 2.3, Crow [98]; Fig. 19)
2	starboard trailing vortex (section 2.3, Crow [98]; Fig. 19)
1	wire 1 (e.g. measuring $U + V$) of a cross-wire anemometer (appendices B, D and E)
2	wire 2 (e.g. measuring $U - V$) of a cross-wire anemometer (appendices B, D and E)

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INTRODUCTION

With the advent of Concorde considerable interest has been focussed on delta wings. The aerodynamic properties of a slender delta wing set at an angle of attack are influenced to a large extent by the vortices associated with boundary-layer separation at the leading edges of the wing. (The boundary layers from the upper and lower surfaces flow outwards separating from the leading edges to yield two shear layers, which in turn roll up into a pair of vortices situated above the wing). These vortices become (when combined with a relatively small contribution of bound vorticity shed from the trailing edges) the trailing vortices, the strength of which is directly related to the lift generated by the wing and inversely related to the free stream velocity, U_∞ ; thus, the trailing vortices from a large aircraft may cause considerable interference to following aircraft, particularly light aircraft. During take-off and landing the relatively low forward speed gives a high circulation for any given lift, and consequently strong trailing vortices are formed with high circumferential velocities. The strength and longevity of these trailing vortices - they may persist for many minutes after their formation - necessitates reduced frequency of take-off and landing at airports. Turbulence in the air helps to break up the vorticity, similarly a cross-wind usually ensures the vorticity is swept away from the path of following aircraft. (If the cross-wind equals the sideways velocity of the vorticity one vortex will remain in

approximately the same position on the runway, the sideways velocity of the other vortex being doubled). Thus, if the break-up decay of trailing vortices could be enhanced, better utilization of airports and air-space would be made by reducing the spacing between aircraft.

The wake-vortex flowfield may be divided into three regions: the roll-up stage as the vortex sheet leaves the wing and rolls up; the intermediate stage, where the vortices are completely rolled-up and may persist for some time, and the final stage when the trailing vortices decay and finally break down. These regions of the flowfield are discussed in further detail in sections 2.1 - 2.3 respectively of chapter 2. Also included in this chapter is a section on solitary waves - these provide the basis to a theory for the phenomenon of vortex breakdown; finally there is a short section on wake vortex minimization.

This study of the trailing vortices formed behind a delta wing concentrates on the instability of the flow in the far field of the wake. Using hot-wire anemometry together with a conditional sampling technique a "structured disturbance" to the flow was established. Detailed measurements of the flow in the vortices were not undertaken as these may be found elsewhere in the literature, see chapter 2 for details. However, a few mean velocities and turbulence measurements are included for the purposes of comparison. The method of detection of the "structured disturbance" together with the instrumentation used is given in chapter 3, the observations are presented and discussed in chapter 4 and conclusions are drawn in chapter 5.

CHAPTER 2 - VORTEX PHENOMENA

2.1 Vortex Roll-Up

When an aerofoil experiences a lift force there must be a circulation of the flow round the aerofoil sections, hence in effect there is a line vortex or set of line vortices running along the span of the aerofoil. These line vortices, which move with the aerofoil, are called the bound vortices of the aerofoil. They are formed by the boundary layer surrounding the surface of the aerofoil and continue in the fluid as free line vortices. As indicated in Figure 1, the free line vortices start at the surface of the aerofoil and pass downstream along the streamlines of the flow. These line vortices are called the trailing vortices of the aerofoil.

A simple type of vortex system occurs when the circulation round the aerofoil sections has a constant value, K , across the span of the aerofoil; the bound vortex system can be thought of as a single line vortex of strength K , and the trailing vortices will be two line vortices of equal strength which come from the tips of the aerofoil and pass downstream in the direction of the streamlines. These line vortices may be assumed with sufficient accuracy to be straight lines parallel to the direction of motion, (in fact they will be curved owing to the variation in the downward component of velocity at different distances behind the model). This gives rise to the simple conception of a "horseshoe" vortex system as shown in Figure 2.

For an aerofoil of finite span a large pressure difference exists between the upper and lower surfaces which decreases towards the tips of the aerofoil. The streamlines passing above the aerofoil tend to flow inwards towards the centre, whilst the streamlines passing below the aerofoil tend to flow outwards. On leaving the trailing edge of

the aerofoil these streamlines form a surface of discontinuity or vortex sheet.

This formation of the vortex sheet was first recognised by Lanchester [1], see Figure 3. The theoretical relation as given by Lanchester that, by integrating over a transverse plane close behind an aerofoil, the total strength of vorticity leaving a semi-span of the aerofoil is equal to the circulation around the median section, was experimentally verified by Fage and Simmons [2]. Their experiments showed very clearly that tip vortex sheets exist in a real flow. Their measurements were made behind a rectangular wing with an aspect ratio of 6.

The free edge of the vortex sheet curls over under the influence of the induced velocity field of the vortex sheet and takes up the form of a spiral with a continually increasing number of turns. The thickness of the vortex sheet is determined by the viscosity of the fluid. Figure 4 shows a sketch of vortex roll-up with the vortex sheet curling up, beginning at the edges, into two vortices, a large portion of the sheet's viscosity being concentrated at their cores.

Kaden [3] gave one of the first solutions of the inviscid problem of roll-up of a sheet of semi-infinite breadth, the initial streamwise vorticity corresponding to steady streaming round a semi-infinite plate. By assuming a two-dimensional self-similar flow and taking the circulation to be a function of radius, he found the equation for the shape of a deforming vortex sheet to be that of a spiral, i.e.

$$\theta = \text{const.} \frac{t}{r^{3/2}} \quad .$$

The success of Kaden's method depended on the absence of any standard of length in the problem. However, Westwater [4] precluded any such treatment by assuming the vortex sheet to be of finite breadth.

An aerofoil may be replaced by a "bound" vortex, which according to Helmholtz' law does not move with the fluid. Thus, Westwater reduced the problem of inviscid roll-up behind a wing to a two-dimensional one by ignoring the bound vortex and treating the trailing vortices as infinite in both directions. Replacing the vortex sheet by a finite number of line vortices of equal strength Westwater used a step-by-step process to calculate the velocity field. The centre of the spirals was found to be given by

$$\begin{aligned} y &= a - 0.426 \left(\frac{U_{\infty} t}{a} \right)^{2/3} \\ x &= - 0.657 \left(\frac{U_{\infty} t}{a} \right)^{2/3} \end{aligned}$$

The result of his calculations (solid lines) together with the purely spiral part of Kaden's solution (dotted lines) are shown in Figure 5. The difference between the two solutions as the motion proceeds becomes very marked with the spiral portion changing from roughly circular to elliptical in section.

Prandtl[5] developed a model describing the inviscid structure of the vortex wake based on the conservation of mechanical energy; the model does not give exact details of the actual roll-up. By equating the kinetic energy per unit length of wake to the induced drag of an elliptically loaded wing, he determined that the vortex tube radius was approximately 8% of the wing span.

Betz[6] model for the inviscid structure of the vortex wake overcame the problem of the somewhat arbitrary choice of swirl velocity distribution in Prandtl's calculations. By progressive application of the Kutta-Joukowski theorem to the relationship between airfoil lift and circulation Betz derived a number of formulae concerning the conduct of vortex systems. Those pertaining to the trailing vortex are as follows :

- i. All the streamwise vorticity shed by each half of the wing is found rolled up in the trailing vortex behind the appropriate half of the wing.
- ii. The centre of gravity of the streamwise vorticity distributions remains at a constant distance from the plane of lateral symmetry.
- iii. The "moment of inertia" of the streamwise vorticity shed by each half of the wing about its centre of gravity is a constant.

For a high aspect ratio wing, having an elliptic spanwise loading,

$$\Gamma = \Gamma_0 \sqrt{1 - (2y/b)^2},$$

the above formulae take the following form:-

$$\int_0^{b/2} \frac{d\Gamma}{dy} dy = \text{const.} = \Gamma_0 = \int_0^{b/2} \frac{d\Gamma}{dr} dr,$$

$$\int_0^{b/2} y \frac{d\Gamma}{dy} dy = \text{const.} = \bar{y} \Gamma_0 = \frac{\pi b}{8} \Gamma_0,$$

$$\int_0^{b/2} (y - \bar{y})^2 \frac{d\Gamma}{dy} dy = \text{const.} = \int_0^{b/2} r^2 \frac{d\Gamma}{dr} dr,$$

where $\bar{y}$ is the spanwise position of the rolled-up trailing vortex and r the radial distance from the centre of the rolled-up vortex. By disregarding the asymmetry stipulated by the mutual interference of the two coiled-up vortices Betz assumed the coiled-up group to be circular, thus presenting the circulation as a pure function of radius i.e. $\Gamma = f(r)$. The vortex group from y to $b/2$ is coiled up into a spiral which fills the circle with radius r , so that the circulation Γ_r must be equal to the circulation of the original vortex

group, $\Gamma_r = \Gamma_y$. Likewise, the inertial moment of the vortices coiled up in this circle must equal the original inertial moment of the vortex group,

$$J_r = \int_0^r \frac{d\Gamma}{dr} r^2 dr = J_y$$

However, when the tip vortices have grown to a large size this assumption of a circular area of vorticity in the cross flow plane is no longer valid.

Measurements made by Verstynen and Dunham [7] behind a CV5 aircraft have been compared with the computed swirl velocity distribution, the Betz model predictions being in far better agreement than those of the Prandtl model, see Donaldson and Bilanin [8] .

The distance, e , for the vortex sheet to become essentially rolled-up was found by Spreiter and Sacks [9] using similarity considerations to be the form of

$$\frac{e}{c} = K \left(\frac{R}{c_L} \right) \left(\frac{b}{c} \right),$$

where K is a constant ($K = 0.28$ for wings having elliptical span loading as given by Kaden).

They deduced that for elliptical span loading the spacing between the vortices, $s' = \frac{\pi}{4} b$, since lift impulse must be preserved throughout the rolling-up process. The rolled-up vortices far behind the wing are not idealized line vortices of potential theory but have cores of finite diameter. The radius of the core may be related to the induced drag by equating the total kinetic energy per unit length to the induced or vortex drag, which for an elliptical span loading gives the core radius to be

$$a_c = 0.197 \frac{s'}{2} = 0.155 \frac{b}{2}$$

Spreiter and Sacks applied the results of Kaden's [3] solution of the rolling up of a vortex sheet of semi-infinite span as approximations to the paths of the vortex cores and to the distance required for the trailing vortices to become essentially rolled-up. Introducing a new coordinate system (η, ζ) , see Figure 6, with the η axis coinciding with the position of the flat vortex sheet and the origin lying at the edge of the sheet, the coordinates of the centres of the vortex cores were found to be

$$\eta_c = 0.34 \left(\frac{\sigma z}{U_\infty} \right)^{2/3}, \quad \zeta = 0.525 \left(\frac{\sigma z}{U_\infty} \right)^{2/3};$$

the circulation distribution in the vicinity of the edge of the flat vortex sheet was assumed to be of the form $\Gamma = 2\sigma\sqrt{\eta}$. From the above equations it can be seen that η_c does not have an asymptote corresponding to the lateral position of the fully rolled-up vortex $(b/2 - s')$ - this is due to the replacing of the finite span with a semi-infinite one. The trailing vortices were considered to be essentially rolled-up at a distance, e , behind the wing, where $\eta_c = (b/2 - s')$. Thus for elliptically loaded wings e is given by

$$\frac{e}{c} = 0.28 \left(\frac{R}{C_L} \right) \left(\frac{b}{c} \right),$$

which compares with $\frac{e}{c} = K \left(\frac{R}{C_L} \right) \left(\frac{b}{c} \right)$ gained by

similarity considerations. Generally, vortices roll-up faster if more of the load is concentrated at the wing tips and vice versa.

The measurements by Dosanjh et al. [10] in a wind tunnel agree with the prediction of Spreiter and Sacks [9].

Chigier and Corsiglia [11] in their measurements behind a rectangular half-wing found that at $(x/c = 4)$ the vortex core radius

was 1.3% of the span, which was an order of magnitude smaller than that predicted by Spreiter and Sacks [9] assuming elliptic span loading. However, Spreiter and Sacks' prediction agrees quite closely with Chigier and Corsiglia's measurements if the actual span loading is used. Chigier and Corsiglia also found that the separation of the vortices centres at roll-up was 94.2% of the span compared to the prediction of $\pi/4$ or 78.5% for elliptic loading. Since tip vortices generate lift in the tip region, the span loading on a rectangular wing differs significantly from that predicted by linear theory; when the increase in lift is accounted for the measured increase in vortex separation and reduced size of core can be explained. Table I shows the distance to roll-up, the inboard displacement of vortex core, y_c/b , and core radius predicted by Spreiter and Sacks [9] for elliptic and actual loading compared with the measurements of Chigier and Corsiglia, as taken from their report.

The effects of viscosity on the trailing vortex have also to be taken into consideration. A flow where the viscous region may be thought of as a thin boundary layer near the wing surface and as a thin wake downstream of the wing is considered. This first implies that skin friction forces act along the surface of the wing resulting in some loss of momentum in the wake far behind the wing. Since on a lifting wing the streamlines in the region of the trailing edge must be inclined to the direction of the mainstream, the wake will be curved as it turns back into the mainstream direction. This curvature of the wake implies that there is a circulation in the viscous wake, i.e. in addition to the streamwise vorticity component there is a vorticity component across the mainstream.

Also, the boundary layer displaces the streamlines in the external inviscid flow outwards; the pressure along the surface of the given wing may be taken to be the same as that in a hypothetical inviscid flow along the displacement surface, obtained by adding the displacement thickness of the boundary layer and of the wake to the wing. This has the effect of changing the pressure distribution over the wing surface everywhere from that calculated for an inviscid flow.

The existence of the boundary layer and wake has two important effects, namely, the drag force which is added to the vortex drag and often referred to as 'profile drag' and the reduction of the lift force and associated changes in the pitching moment. The 'profile drag' is made up of the skin friction drag and the pressure drag. (The skin friction drag may be determined directly from the boundary layer once its development along the surface has been calculated. The pressure drag, or form drag, may be determined by integrating the streamwise component of the pressure around the surface of the wing). The reduction of the lift force by viscosity leads to a reduction of the circulation. Küchemann [18] gives details of the various models which have been proposed which take account of viscous effects.

In his justification for treating the trailing vortex far downstream as axisymmetric Batchelor [12] made use of viscous effects. His argument being that as the effect of viscosity during roll-up is likely to be confined to a diffusive thickening of the vortex sheet shed by the wing, the number of convolutions of the spiral into which the sheet forms increases continually.

Ultimately, the neighbouring turns of the spiral become close enough for viscous spreading to make the distribution of vorticity a smooth one. Viscosity still continues to have an effect on the vortex after the vorticity distribution has been made continuous. Further details of Batchelor's work are given in section 2.2.

A vortex core is also found in the separated flow over the leading edge of a delta wing at incidence, see Figure 7. The flow separates from near the leading edge to form a free shear layer which curves upward and inboard, eventually rolling up into a core of high vorticity, in which the velocity and pressure fields are roughly axially symmetric, and in which very little trace remains of the separate turns of the shear layer. The core, as do most vortex cores, has an appreciable axial component of motion with the fluid spiralling around and along the axis.

Some aspects of separated flow behind a delta wing were investigated by Fink and Taylor [13]. In their work they tested two slender delta wings with sharp leading edges with 80 degrees sweepback. Fink [14] made some preliminary measurements behind a yawed delta wing; the complete set of results using the same wing being made by Harvey [15], who took measurements of surface static pressure, total-head surveys and flow visualisation in order to give the forces acting on the wing and to determine the physical nature of the flow.

The properties of vortex flows over slender wings has been fully discussed by Küchemann [16-19]. In a real flow there is a secondary separation of the boundary layer on the upper surface of the wing underneath the vortex cores. The cores

induce pronounced suction peaks on the wing so that the outflow towards the leading edges subsequently meets an adverse pressure gradient that, in turn, causes separation of the boundary layer. Thus a further singularity of the vortex-sheet type is formed on each half of the wing. The cores of the secondary vortex sheets locally increase the suction on the wing surface, this causes measured pressures to differ from those calculated for a flow with primary vortex sheets only. A typical example is that of an ogee wing with primary separation from the leading edges, see Figure 8. Although the flow is not conical it nevertheless exhibits the features described above. Between the primary attachment lines around the centre line of the wing there is a region of nearly parallel flow. The air drawn into the cores of the primary vortex sheets moves sideways until it meets the secondary separation line, from where a secondary vortex sheet springs. This secondary vortex sheet causes a secondary attachment line on the wing that divides air between that drawn into the secondary vortex cores and that which is not. The position of the secondary separation line is not fixed but depends on the state of the boundary layer and on the Reynolds number.

The problem of vortex wake roll-up has been reviewed by McMahon [20] ; Donaldson [21] has also reviewed the trailing vortex problem focusing on the work of Betz. Figure 9 shows the swirl velocity in the wake of an Army O-1 aircraft as measured by McCormick et al. [22] who made a fairly extensive set of wake data. The load distribution on the wing has been assumed to be elliptic and the swirl velocity has been computed with the Betz model.,

the agreement is quite good. The circulation in the wake as calculated by integrating the measured vorticity distribution sometimes appears not to add up to the wing root circulation, as only 54% of the circulation is found in $r/b < 0.05$. From Figure 10 it can be seen that Betz' model has a rather large spike of vorticity near $r = 0$ and, outward of $r/b = 0.05$ a long tail. This probably contributes to the fact that both Dosanjh et al. [10] and Grow [23] fail to account for the total bound circulation on the wing.

A generalization of Betz' [6] method was made by Mason and Marchman [24, 25] for a circular distribution represented by a Fourier series rather than an elliptical distribution. They concluded that it was not possible to calculate the core radius, a_c , from consideration of the kinetic energy due to swirl without including the effects of viscosity, and calculated the core radius to be

$$a_c = \sqrt{\frac{1.26(4\nu z)}{U_\infty}}$$

The Betz argument assumed that vortex roll-up proceeds monotonically from tip to wing centre, so that the vorticity shed at the trailing edge near the wing tip goes into the centre of the vortex located at the spanwise centroid of vorticity. However, difficulties arise when calculating the roll-up of vortices that trail behind wings with arbitrary span-load distribution. For these cases, Rossow [26] gave two rules for subdividing the vortex sheet into separate segments and for identifying the beginning points of roll-up for each segment.

These are:-

- i. Vortex roll-up sites are located at maxima of sheet strength and at abrupt changes in sheet strength, and
- ii. the edges of the segment of vortex sheet that rolls into a vortex occur where the sheet strength vanishes or changes sign, or where the sheet strength is at a minimum.

The theory is still approximate as interactions of the vortices, viscosity and variations in axial velocity are ignored.

Donaldson et al.[27] extended the Betz method to the case of flapped wings. They found the number of vortices by dividing the vorticity up according to where the minima of $\left| \frac{d\Gamma}{dy} \right|$ occurred, and then calculated each vortex sheet roll-up, assuming that the Betz invariants were preserved in detail, but without allowing for the interaction of vortices on each other.

A new viscous vortex model, which is matched in its initial form to the vorticity distribution predicted by the inviscid Betz wake model, for the trailing vortex wake system generated by a lifting wing of finite span is presented by Williams [28]. Figure 11 shows the downwash profile for a typical cross-section of the wake downstream of a lifting wing. Since the modelling techniques are derived from the Betz model, which assumes a wing span load distribution increasing monotonically from the wing tip to the mid-span point, they are inapplicable to wakes generated by highly swept low aspect ratio wing planforms. The analysis demonstrates the important influence of the wing span load distribution on features such as

the peak swirl velocity and the circulation profile in the near and intermediate wake regions. Although the trailing vortex core is shown to be relatively independent of the wing span load distribution, it is considerably smaller than predicted in earlier analytical wake models. Williams shows that these characteristics have also been borne out by experimental evidence.

Adams [29] developed inviscid momentum balance equations to predict initial vortex velocities and pressures from given wing lift and drag distributions. Assuming that the wing circulation distribution is accurately known, he stated that a less complicated and more accurate prediction of an initial vortex circulation profile than the Betz method is given by

$$\Gamma(r=\eta) = \Gamma(\eta),$$

where η is the transverse coordinate along the wing span with the origin at the wing tip.

Nielsen and Schwind's [30] calculation of vortex roll-up using forty equal-strength vortices checks with the experimental results of Fage and Simmons [2]. In their calculations the vortices tended to gravitate to the centre of the spiral becoming very close to each other; when they came closer than a prescribed distance compatible with the accuracy of calculation based on the axial interval size, they were combined at their centre of gravity and the calculation continued downstream. Thus Nielsen and Schwind managed to suppress the chaotic motion experienced by Moore [33] and Takami [32] below.

Jordan [31] disagreed with Westwater's [4] procedure for the calculation of roll-up of a vortex sheet, claiming that he had introduced an artificial disturbance by replacing the continuous

sheet with twenty individual vortices. In Jordan's opinion the deficiency is the external disturbance which starts the rolling up process in Westwater's procedure. He agreed that the vortex sheet would not remain flat in real life, and explained that while ideally it is self-perpetuating, it is unstable with respect to external disturbances. Also, viscous effects of the upwash outside the edges of the sheet would slowly start the rolling up even in the absence of distinct disturbances.

Attempts by Takami [32] and Moore [33] to reproduce Westwater's results were unsuccessful; the vortices moved chaotically, no spiral structure emerged and the rate of roll-up could not be estimated. Westwater's [4] method of replacing the continuous sheet by a finite number of line vortices can lead to unreliable results. Moore [33] concluded that it was the discretization itself which caused the chaotic motion, this being confirmed by showing that increasing the number of vortices made matters worse. Chorin and Bernard [34] suggested that a combination of Euler integration and large time steps, as employed by Westwater, would prevent chaotic motion.

This has been borne out by Clements and Maull [35] who used a time step comparable with the orbital period of the closest vortices. They used a model essentially similar to that described by Westwater to investigate how the vortex sheet rolls up under different loading conditions. They represented the trailing vortex sheet by a series of line vortices of equal strength with their axes parallel to the incident air flow. The vortices were assumed to continue to infinity in the upstream direction and the effect of the bound vortex in the wing was

ignored, thus making the problem two-dimensional in a plane normal to main stream velocity. The circulation distribution along the span was achieved by the spacing of the vortices, and the wing was designed to give a constant lift by keeping the integral of the circulation of the wing constant.

One explanation given by Moore [36] for the inhibition of chaotic motion, as suggested by Chorin and Bernard [34], was that such an integration procedure would quickly increase the separations between the vortices near the ends of the sheet to much greater than their true values, thus tending to suppress the orbiting motion of vortex pairs - this having been a prominent feature of the chaotic motion. Crow [37] pointed out that Euler integration when applied to vortex motion introduces a cumulative error. By studying the finite-difference equations analytically it can be shown that when applied to a pair of vortices of equal strength, Euler integration leads to a vortex separation which continually increases instead of remaining constant. Also, large errors are introduced by a choice of time step comparable with the orbital period. Thus Clemerts and Maull's [35] results are not accurate in the spiral region since they found that a change in time step caused a change in the shape of the spiral comparable with its radius. However, by introducing a tip vortex to represent the tightly rolled portion of the vortex sheet, Moore [36] eliminated the chaotic motion of the vortices and calculated the details of the outer portion of the spiral. The velocity field of the inner portion of the spiral being very nearly axisymmetric is better represented by a single vortex at the

centre of the spiral, than by a finite number of point vortices scattered about on the spiral. In his study of the formation of the leading-edge vortex over a delta wing, Smith [38] also used this representation of a single vortex for the inner portion of a spiral vortex sheet. Figure 12 shows the shape of the spiral as calculated by Moore [36] at time, $t^* = tU/a = 1.0$, where a/U is the natural time scale of the system for a vortex sheet represented by 120 point vortices where the time step, $dt^* = 2 \times 10^{-3}$ with $\theta_c = 90^\circ$ for initially equispaced vortices.

Guiraud and Zeytounian [39] determined the asymptotic behaviour of a rolled-up vortex sheet using a double scale technique which relied on a process of averaging out the saw-tooth-like behaviour of the flow variables, which generates a continuous solution having the structure of a vortex filament. An application to Kaden's [3] problem is worked out with the results being in complete agreement with Moore's [40] asymptotic formulae for the shape of the spiral.

Butter and Hancock [41] and Hackett and Evans [42] were able to extend Westwater's [4] numerical method. Butter and Hancock's extension included the main effect of wing bound vorticity and the effect of the three-dimensional pattern of trailing vorticity. They replaced the wing by a single lifting-line bound vortex representing the circulation distribution around the wing, and approximated the trailing vortex sheet by a number of discrete line vortices. Starting with a nearly planar system (i.e. bound vorticity on wing, trailing streamwise vorticity) and following a step-by-step process, working downstream aft of the wing

trailing edge, the deformation of the trailing vortex sheet is determined.

Using a similar technique to that of Butter and Hancock [41] but truncating the trailing vortices at the upstream and trailing edge Hackett and Evans [42] presented examples to show the effect of sweep, lift coefficient and incidence, the effect of approaching the ground, and the effect of wind-tunnel wall constraint.

Using a "Cloud-In-Cell" technique to calculate the time evolution of the vortex sheet, Baker [43] examined two cases of initial vortex sheet strength; one relating to an elliptically loaded wing and the other simulating a wing with a flap deployed. Many particle simulations in plasma physics have been solved by the "Cloud-In-Cell" technique and results indicate that small scale behaviour plays an important role in vortex roll-up, the small scale perturbations evolving into ever-increasing larger structures by vortex amalgamation.

In order to ascertain the nature and detailed characteristics of the formation and early development of a trailing vortex, Francis [44] made measurements of the incompressible flowfield in the vicinity of a rectangular, untwisted wing having a NACA 64009 laminar flow airfoil cross-section.

2.2 Vortex motion

Vorticity is transported in a viscous fluid by convection and diffusion. At high Reynolds numbers, the large-scale vortex structure is determined primarily by the convective transport mechanism; convection, though essentially a large-scale process also partially determines the small-scale structure. At finite Reynolds numbers vorticity diffuses; the spiral sheet becomes a vortex layer of finite non-zero thickness with successive turns tending to merge into each other. The small-scale structure is determined both by convection and diffusion, the relative effects of which depend on the relative magnitudes of their associated length scales d and $(\nu t)^{1/2}$, where d is the spacing between successive turns of the spiral. Such a vortex has three distinct regions characterised by $d \gg (\nu t)^{1/2}$, $d = O(\nu t)^{1/2}$ and $d \ll (\nu t)^{1/2}$. In the outer region, $d \gg (\nu t)^{1/2}$, discrete turns of the spiral remain evident and diffusion proceeds without constraint. Inside this outer region, $d = O(\nu t)^{1/2}$, the turns of the spiral merge together, the radial gradient of vorticity is reduced causing the diffusion rate to fall. Effectively, a constraint is applied to the diffusion process by the close spacing of successive turns of the spiral layer. Further in, $d \ll (\nu t)^{1/2}$, in the immediate neighbourhood of the axis of the vortex, the diffusive mechanism influences the entire structure. The boundaries between these three regions lie relatively closer to the axis of the vortex the higher the Reynolds number. For further details of the structure within these regions see Maskell [45].

When the vortex sheet has rolled-up the vortex may be considered as one having developed from an infinite line vortex.

Lamb [46] was one of the first to solve the governing equations for the laminar case with no axial velocity perturbations. Considering a flow in which initially the vorticity is zero everywhere, except on the axis $r = 0$, where there is a line vortex of strength C , the equation of motion in two-dimensional polar coordinates is given by

$$\frac{\partial \omega}{\partial t} = \nu \left(\frac{\partial^2 \omega}{\partial r^2} + \frac{1}{r} \frac{\partial \omega}{\partial r} \right) \quad (2.2,1)$$

$$\text{where } \omega = \frac{\partial v}{\partial r} + \frac{v}{r} = \frac{1}{r} \frac{\partial (r^2 \Omega)}{\partial r} ,$$

$\Omega(r, t)$ being the angular velocity of a material cylindrical shell of radius r . $v = C/2\pi r$, as initially the circulation round all circles centred on the axis has the same value as C .

The solution of this problem of a spreading line vortex is given by

$$\omega(r, t) = \frac{C}{4\pi\nu t} \exp \left(-\frac{r^2}{4\nu t} \right) \quad (2.2,2)$$

The velocity distribution is then given by

$$\begin{aligned} v(r, t) &= \frac{1}{r} \int_0^r \omega r dr \\ &= \frac{C}{2\pi r} \left\{ 1 - \exp \left(-\frac{r^2}{4\nu t} \right) \right\} \end{aligned} \quad (2.2,3)$$

which is sketched for several values of t , see Figure 13. At small values of r ($\ll (4\nu t)^{1/2}$) the motion is a rigid-body rotation, with angular velocity $C/(8\pi\nu t)$, while at large values of r ($\gg (4\nu t)^{1/2}$) the motion is irrotational and as it was at the initial instant.

2.2.1 General equations of motion

The equations of motion for an axially symmetric flow of a perfect, viscous, heat-conducting gas in cylindrical polar coordinates, r, θ, z, t with the radial, circumferential and axial components of velocity denoted by u, v and w respectively, are given by Hall [47] as follows:

the continuity equation:-

$$\frac{\partial \rho}{\partial t} + \frac{1}{r} \frac{\partial(r\rho u)}{\partial r} + \frac{\partial(\rho w)}{\partial z} = 0 \quad (2.2,4a)$$

the three momentum equations:-

radial direction:

$$\begin{aligned} \rho \left(\frac{\partial u}{\partial t} + u \frac{\partial u}{\partial r} + w \frac{\partial u}{\partial z} - \frac{v^2}{r} \right) = & - \frac{\partial p}{\partial r} + \mu \left(\frac{\partial^2 u}{\partial r^2} + \frac{1}{r} \frac{\partial u}{\partial r} - \frac{u}{r^2} + \frac{\partial^2 u}{\partial z^2} \right) \\ & + \frac{\mu}{3} \frac{\partial}{\partial r} \left(\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial w}{\partial z} \right) \\ & + \frac{2}{3} \frac{\partial \mu}{\partial r} \left(2 \frac{\partial u}{\partial r} - \frac{u}{r} - \frac{\partial w}{\partial z} \right) \\ & + \frac{\partial \mu}{\partial z} \left(\frac{\partial w}{\partial r} + \frac{\partial u}{\partial z} \right) \end{aligned} \quad (2.2,4b)$$

circumferential direction:

$$\begin{aligned} \rho \left(\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial r} + w \frac{\partial v}{\partial z} + \frac{uv}{r} \right) = & \mu \left(\frac{\partial^2 v}{\partial r^2} + \frac{1}{r} \frac{\partial v}{\partial r} - \frac{v}{r^2} + \frac{\partial^2 v}{\partial z^2} \right) \\ & + \frac{\partial \mu}{\partial r} \left(\frac{\partial v}{\partial r} - \frac{v}{r} \right) + \frac{\partial \mu}{\partial z} \frac{\partial v}{\partial z} \end{aligned} \quad (2.2,4c)$$

axial direction:

$$\begin{aligned} \rho \left(\frac{\partial w}{\partial t} + u \frac{\partial w}{\partial r} + w \frac{\partial w}{\partial z} \right) = & - \frac{\partial p}{\partial z} + \mu \left(\frac{\partial^2 w}{\partial r^2} + \frac{1}{r} \frac{\partial w}{\partial r} + \frac{\partial^2 w}{\partial z^2} \right) \\ & + \frac{\mu}{3} \frac{\partial}{\partial z} \left(\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial w}{\partial z} \right) \\ & + \frac{\partial \mu}{\partial r} \left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial r} \right) \\ & - \frac{2}{3} \frac{\partial \mu}{\partial z} \left(\frac{\partial u}{\partial r} + \frac{u}{r} - 2 \frac{\partial w}{\partial z} \right) \end{aligned} \quad (2.2,4d)$$

and the energy equation:

$$\begin{aligned}
 \rho C_p \left(\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial r} + w \frac{\partial T}{\partial z} \right) - \frac{T}{\rho} \left(- \frac{\partial \rho}{\partial t} \right)_{p=\text{const.}} \left(\frac{\partial p}{\partial t} + u \frac{\partial p}{\partial r} + w \frac{\partial p}{\partial z} \right) \\
 = k \left(\frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} + \frac{\partial^2 T}{\partial z^2} \right) + \frac{\partial k}{\partial r} \frac{\partial T}{\partial r} + \frac{\partial k}{\partial z} \frac{\partial T}{\partial z} \\
 + \mu \left[2 \left(\frac{\partial u}{\partial r} \right)^2 + 2 \left(\frac{u}{r} \right)^2 + 2 \left(\frac{\partial w}{\partial z} \right)^2 + \left(\frac{\partial v}{\partial r} - \frac{v}{r} \right)^2 \right. \\
 \left. + \left(\frac{\partial w}{\partial r} + \frac{\partial u}{\partial z} \right)^2 + \left(\frac{\partial v}{\partial z} \right)^2 - \frac{2}{3} \left(\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial w}{\partial z} \right)^2 \right]
 \end{aligned} \tag{2.2,4e}$$

where C_p , T and k are the specific heat at constant pressure, the temperature and the heat conductivity respectively. The equations are supplemented by the equation of state $p \propto \rho T$ and the whole system is completed when viscosity, μ , conductivity, k , and specific heat, C_p , are specified. Terms involving derivatives of μ and k may be neglected for small temperature variations in the gas, and density, ρ , may be treated as a constant if compressibility effects are small.

The above equations cannot be solved analytically without simplification as they are coupled, elliptic and highly non-linear. Some of the different methods for solving them are as follows:-

- i. Quasi-cylindrical approximation where certain terms are regarded as small and the equations become parabolic,
- ii. Linearizing the equations, which also involves treating some terms as small, and
- iii. Similarity solutions, which allow a transformation of the equations into ordinary differential equations for special boundary conditions.

2.2.2. The Quasi-cylindrical approximation

The quasi-cylindrical approximation assumes that variations in the axial direction may be taken to be small compared

with variations in the radial direction. Since all aircraft vortex cores quickly become turbulent the approximation is applied here to the mean motion of a turbulent flow. A laminar flow can be regarded as a special case in which the turbulent fluctuations are zero.

Starting from the complete equations of motion in cylindrical polar coordinates but without the condition of axial symmetry, Hall [47] derived the equations in the following manner. The flow variables p , ρ , T and u , v , w are each replaced by the sum of a mean value in time and a fluctuating component; for example, u is replaced by $u + u'$, where u is now the mean value and u' the fluctuating component. The mean value of a product of fluctuating quantities is denoted by a bar, for example $\overline{u'w'}$. Fluctuations in time of viscosity, μ , conductivity, k , and specific heat, C_p , are neglected. The mean value of each equation is written down and the following three simplifications made in succession. Firstly, the mean flow is taken to be axially symmetric, thus derivatives of mean values with respect to θ , the circumferential ordinate, are omitted. Secondly, the fluctuations are considered small so that products of fluctuations may be neglected except where the terms they are compared with are also small: for example $\overline{v'^2} \ll v^2$ but $u'v' \not\ll uv$, as the mean value of the radial velocity, u may be small compared with the fluctuation, u' , in quasi-cylindrical flow. Lastly, the quasi-cylindrical approximation assumes that variations in the axial direction of mean values with respect to time are small compared

with the corresponding variations in the radial direction. The equations are reduced to the following forms:-

the continuity equation:-

$$\frac{\partial \rho}{\partial t} + \frac{1}{r} \frac{\partial (r \rho u)}{\partial r} + \frac{1}{r} \frac{\partial (r \rho' \overline{u'})}{\partial r} + \frac{\partial (\rho w)}{\partial z} = 0 \quad (2.2,5a)$$

three momentum equations:-

$$\frac{\rho v^2}{r} = \frac{\partial p}{\partial r}, \quad (2.2,5b)$$

$$\rho \left(\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial r} + w \frac{\partial v}{\partial z} + \frac{uv}{r} \right) = \frac{1}{r^2} \frac{\partial}{\partial r} \left[r \mu \left(\frac{\partial v}{\partial r} - \frac{v}{r} \right) \right] - \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 \rho \overline{u'v'}) - \overline{\rho' u'} \left(\frac{\partial v}{\partial r} + \frac{v}{r} \right), \quad (2.2,5c)$$

$$\rho \left(\frac{\partial w}{\partial t} + u \frac{\partial w}{\partial r} + w \frac{\partial w}{\partial z} \right) = - \frac{\partial p}{\partial z} + \frac{1}{r} \frac{\partial}{\partial r} \left(r \mu \frac{\partial w}{\partial r} \right) - \frac{1}{r} \frac{\partial}{\partial r} (r \rho \overline{u'w'}) - \overline{\rho' u'} \frac{\partial w}{\partial r}, \quad (2.2,5d)$$

and the energy equation written in terms of the mean and fluctuating components of the stagnation enthalpy, I_0 and I'_0 becomes:

$$\begin{aligned} \rho \left(\frac{\partial I_0}{\partial t} + u \frac{\partial I_0}{\partial r} + w \frac{\partial I_0}{\partial z} \right) - \frac{\partial p}{\partial t} \\ = \frac{1}{r} \frac{\partial}{\partial r} \left[\frac{\mu}{\sigma} r \frac{\partial I_0}{\partial r} + \frac{\mu}{\sigma} (\sigma - 1) r \left(v \frac{\partial v}{\partial r} + w \frac{\partial w}{\partial r} \right) - \mu v^2 \right] \\ - \frac{1}{r} \frac{\partial}{\partial r} (r \rho \overline{u'I'}) - \overline{\rho' u'} \frac{\partial I_0}{\partial r} + \frac{5}{3} \frac{1}{r} \frac{\partial}{\partial r} \left[\mu r \left(\frac{v'}{r} \frac{\partial u'}{\partial \theta} + \overline{w' \frac{\partial u'}{\partial z}} \right) \right]. \end{aligned} \quad (2.2,5e)$$

where

$$I_0 = \int C_p dT + \frac{1}{2}(u^2 + v^2 + w^2) + \frac{1}{2}(\overline{u'^2} + \overline{v'^2} + \overline{w'^2}),$$

$$I'_0 = \int C_p dT' + uu' + vv' + ww'$$

and the Prandtl number, $\sigma = \mu C_p/k$.

In the derivation of the above equations for quasi-cylindrical flow, mean values with respect to time have been taken, but time-variations of the mean quantities have been retained to admit "mean" motions which vary slowly with time compared with the fluctuating components.

2.2.3 Boundary conditions for steady motion

For different problems the boundary conditions on the axis of symmetry are the same. However, those conditions on the outer boundaries of a core differ.

On the axis of symmetry, $r = 0$, of a viscous heat-conducting vortex core $u = v = \frac{\partial w}{\partial r} = \frac{\partial T}{\partial r} = 0$. In the absence of sources or sinks, u , the radial velocity must be zero; since there can be no steps or kinks in the profiles the circumferential velocity, v , and the radial gradients of axial velocity and temperature, $\frac{\partial w}{\partial r}$ and $\frac{\partial T}{\partial r}$ are also zero. The above conditions imply for a quasi-cylindrical flow that the strains in an element on the axis are small; an element, therefore, rotates in the manner approaching that of a solid body. As the axis is approached, the radial and circumferential components of vorticity and the radial gradient of the axial component tend to zero, i.e. at $r = 0$,

$$\omega_r = \omega_\theta = \frac{\partial \omega_z}{\partial r} = 0.$$

By equating the viscous terms in the equations of motion to zero the boundary conditions for steady laminar

incompressible flow are:

$$u = -rf'(z); \quad v = \text{constant} \times r; \quad w = 2f(z).$$

There are many forms for the remaining boundary conditions for a core. For a trailing vortex far downstream of a body the flow at the outside edge of the core is irrotational like that due to a potential vortex; whilst for a leading-edge vortex the flow at the edge of the core is rotational and approximately inviscid. As both the above flows are quasi-cylindrical unless breakdown occurs, the only other boundary conditions needed, apart from those on the axis on an outer bounding surface of revolution, are a set on some upstream cross-section. For those cores governed by the general equations of motion, see equations (2.2,4), a further set of boundary conditions is required on some downstream cross-section.

2.2.4 Quasi-Cylindrical Flows

A solution for the growth of a line vortex with time and the spread of a trailing vortex behind a wing due to turbulence was presented by Squire [48]. Using the equation for the vorticity component ζ parallel to the z -axis (equation A,3c), together with the two equations of continuity, see Lamb [46, section 147], and making various assumptions, he obtained the solution for the growth of a vortex in turbulent flow by likening it to a laminar flow with a viscosity $(\nu + \text{constant} \times K)$ instead of ν . His assumptions were that the eddy viscosity was proportional to the circulation, K , of the line vortex, i.e. $\nu_1 = \text{constant} \times K$, that at a sufficient distance downstream the mean longitudinal velocity is large compared with the circumferential component of velocity and

nearly equal to the free stream velocity, U_∞ , and that of the quasi-cylindrical assumption, i.e. mixing in the longitudinal direction is negligible compared with that in the lateral direction.

Owen [49] presented a solution that retained the main features of Squire's [48] solution, the difference being that whereas in Squire's solution apparent viscosity was proportional to the initial circulation about the vortex, Owen proposed that it depended on the Reynolds number defined with respect to the circulation. It might be expected that turbulence in a trailing vortex would be like that in an ordinary wake, modified perhaps by a damping effect of rotation; thus the concepts of self-preservation, mixing length and eddy viscosity would be useful, while the empirical values of the eddy viscosity, for example, would be somewhat less than those for a wake. Measurements have been collected and compared by Owen, see Table 2, of mean-flow quantities with the solution for laminar flow where the laminar kinematic viscosity, ν , has been replaced by an eddy viscosity ν_T ; the above assumption is roughly confirmed; Squire [48] had argued that $\nu_T = cK$, where c is a constant. The measurements show that although c is roughly constant for a given core, at least for the rather limited length examined, it may differ by a factor of ten from one core to the next. Owen's proposed simple model of the turbulent flow postulated self-preservation and argued from the equations of motion that for sufficiently large times the eddy viscosity, ν_T , should be given by

$$\frac{\nu_T}{\nu} = \Lambda \left(\frac{\Gamma}{\nu} \right)^{1/2}, \quad \Lambda = \text{const.}$$

It can be seen from Table II that Λ only varies by a factor of two.

Poppleton [50], from his experiments, obtained a 'first approximation' to the distribution of the Reynolds stresses through the core; this might help in obtaining a more realistic model of the flow than Owen's rough semi-empirical theory. Poppleton also found that the injection of air into the nascent core can have a profound effect on its subsequent development in that it accelerated the decay of the vortex. Kantha et al. [51] have also studied the response of a trailing vortex to axial injection into the core. They demonstrated that it spread out the vorticity concentrated in the core and prematurely aged it.

The existence of axial flow in aircraft trailing vortices has been well established experimentally, see Olsen, Goldburg and Rogers [52]. Wind tunnel tests and free flight observations, see Chigier and Corsiglia [53], show there is usually a deficit of axial velocity in the core of the vortex, although some cases of a velocity increase have been reported. One of the first observations of a deficit of axial velocity in the core was made by Hilton [54]. Chigier and Corsiglia [53] in their wind tunnel measurements found that for a rectangular wing axial velocity changes from a deficit (wake flow) for $\alpha < 9^\circ$ to an excess (jet flow) for $\alpha > 9^\circ$, where α is the angle of attack.

The axial flow in trailing line vortices was accounted for in general terms by Batchelor [12]. When the circumferential velocity, v , is small compared with the free-stream velocity, W , and the axial velocity, w , is nearly equal to W , the equations for quasi-

cylindrical laminar flow (equations A,1) reduce, in the steady incompressible case, to:

$$\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial w}{\partial z} = 0, \quad (2.2,6a)$$

$$\frac{v^2}{r} = \frac{1}{\rho} \frac{\partial p}{\partial r}, \quad (2.2,6b)$$

$$W \frac{\partial v}{\partial z} = \nu \left(\frac{\partial^2 v}{\partial r^2} + \frac{1}{r} \frac{\partial v}{\partial r} - \frac{v}{r^2} \right), \quad (2.2,6c)$$

$$W \frac{\partial w}{\partial z} = - \frac{1}{\rho} \frac{\partial p}{\partial z} + \nu \left(\frac{\partial^2 w}{\partial r^2} + \frac{1}{r} \frac{\partial w}{\partial r} \right) \quad (2.2,6d)$$

A new independent variable, ζ , is introduced instead of r :

$$\zeta = \frac{Wr^2}{4\nu z}$$

The boundary conditions are as follows:

$$\zeta = 0, \quad u = v = \zeta^{1/2} \frac{\partial w}{\partial \zeta} = 0;$$

$$\zeta \rightarrow \infty, \quad v \rightarrow \frac{1}{2} K_c \left(\frac{W}{\nu z} \right)^{1/2} \zeta^{-1/2}, \quad K_c = \text{constant} \quad (2.2,6e)$$

i.e. $rv \rightarrow K_c$ $w \rightarrow W$, $W = \text{constant}$, $\rho \rightarrow P$, $P = \text{constant}$.

Solving equations (2.2,6c), (2.2,6b) and (2.2,6d) successively, for large z the asymptotic form of the solution was found by

Batchelor [12] to be:

$$v = \frac{K_c}{r} (1 - e^{-\zeta}) = \frac{K_c}{2} \left(\frac{W}{\nu z} \right)^{1/2} \zeta^{-1/2} (1 - e^{-\zeta}), \quad (2.2,6f)$$

$$\frac{P - p}{\rho} = \frac{K_c^2 W}{8\nu z} \left[\frac{(1 - e^{-\zeta})^2}{\zeta} + 2\text{ei}(\zeta) - 2\text{ei}(2\zeta) \right] \quad (2.2,6g)$$

$$w = W - \left(\frac{K_c^2}{8\nu z} \log \frac{Wz}{\nu} \right) e^{-\zeta} + \frac{K_c^2}{8\nu^2} f(\zeta) - \frac{aW^2}{8\nu z} e^{-\zeta} \quad (2.2,6h)$$

where

$$f(\zeta) = e^{-\zeta} \{ \log \zeta + \text{ei}(\zeta) - 0.807 \} + 2\text{ei}(\zeta) - 2\text{ei}(2\zeta),$$

$$\text{ei}(\zeta) = \int_{\zeta}^{\infty} \frac{e^{-\xi}}{\xi} d\xi,$$

and 'a' is a constant with the dimensions of area.

From equation (2.2,6h) the axial velocity defect, $W - w$, is small as at sufficiently large z , $w \rightarrow W$ because $\frac{K_c^2}{W\nu z} \log \frac{Wz}{\nu} \rightarrow 0$;

since $K_c^2 \sim v_{\max}^2 \frac{\nu z}{W}$ from equation (2.2,6f) the circumferential

velocity is also small, thus $\left(\frac{v_{\max}}{W} \right)^2 \log \frac{Wz}{\nu} \rightarrow 0$.

In the axial velocity defect $W - w$, (equation 2.2,6h), the

term $\left(\frac{K_c^2}{8\nu z} \log \frac{Wz}{\nu} \right) e^{-\zeta}$ eventually dominates showing the

effect of swirl. With swirl the integral of velocity defect increases as $\log z$, this increase is due to the increase in pressure associated with the decay of the swirl; the velocity defect itself varies as $z^{-1} \log z$. With no swirl, the integral over a cross-section of the axial velocity defect, would be a constant with a sign depending on the upstream conditions, and the velocity defect itself would decrease as z^{-1} by diffusive spreading. The effect of swirl is such that even if the axial velocity defect were negative upstream, with $w > W$, as in a jet, a swirling wake with positive velocity defect ($w < W$) would develop far downstream.

In an earlier solution for the trailing vortex, Newman [55] used the same set of equations as Batchelor [12], with the exception of the term, $-\frac{1}{\rho} \frac{\partial p}{\partial z}$, on the right-hand side of equation (2.2,6d). By excluding this term, there is no means by which the swirl can affect the axial velocity; the solutions for v and p are the same as Batchelor's, see equations (2.2,6f and g), but in the solution for w (equation 2.2,6h) the terms involving K_c are absent. Thus, as Newman assumed, his solution is justifiable only if $K_c^2/(Wyz)$ or $(v_{\max}/W)^2$ is sufficiently small and z is not too large.

Some local total pressure and flow direction measurements were made by Dosanjh et al. [10] 1 - 8 chord lengths behind a rectangular, symmetrical half aerofoil (NACA 0009) at a Reynolds number, based on the free-stream velocity and chord-length of the aerofoil, of 10,000. The various experimental velocity distributions in the vortex were in good agreement with Newman's analysis; however,

the decay of velocity and geometry parameters was found to be approximately eight to ten times faster than predicted by Newman [55].

In his study of the axial flow in trailing line vortices, Batchelor [12] considered the vortex wake far downstream where it decays under the action of viscosity and its radius is $O(\nu z/U_\infty)^{1/2}$. The viscous effects were found to produce an axial velocity deficit. The kinetic energy of the cross-flow is dissipated by the viscosity and the induced drag effect is transferred from kinetic energy of rotation to a momentum deficit. Batchelor's [12] calculation of the effect of viscosity is not applicable until $(\nu z/U_\infty)^{1/2} \approx a$, where 'a' is a characteristic radius of the inviscid rolled up vortex. A reasonable value of 'a' is one-tenth of the wing-span, thus making the downstream distance $\approx 0.01 c R_c R^2$, where c is the chord, $R_c = U_\infty c/\nu$ is the Reynolds number based on the chord and R the aspect ratio. For Olsen's [56] experiments this distance is several thousand chord lengths.

Moore and Saffman [57] in their work on the axial flow in laminar trailing vortices show that over most of the core viscous effects are negligible with the excess flow in the downstream direction, conforming with Batchelor's inviscid argument. However, there is a viscous controlled inner core in which axial velocities in the opposite direction can develop. A comparable axial velocity deficit is produced by the retardation in the boundary layer over the wing. The total axial flow is obtained by summing the axial velocity perturbation in the viscous core and the axial flow in the viscous core due to boundary layers on the wing. Since the

assumption of inviscid roll-up leads to singularities in the tangential and axial velocities at the centre of the core, these singularities being removed by viscosity, Moore and Saffman assumed that viscous stresses were confined to an inner region, the viscous core, whose radius is $O(\nu t)^{1/2}$. The boundary-layer approximation was used to examine the viscous core, the equations are, in Moore and Saffman's [57] notation:

$$\frac{\partial v_\theta}{\partial t} = \nu \left(\frac{\partial^2 v_\theta}{\partial r^2} + \frac{1}{r} \frac{\partial v_\theta}{\partial r} - \frac{v_\theta}{r^2} \right) \quad (2.2,7a)$$

$$-\frac{v_\theta^2}{r} = -\frac{1}{\rho} \frac{\partial p}{\partial r} \quad (2.2,7b)$$

and

$$\frac{\partial w}{\partial t} = -\frac{1}{\rho U_\infty} \frac{\partial p}{\partial t} + \nu \left(\frac{\partial^2 w}{\partial r^2} + \frac{1}{r} \frac{\partial w}{\partial r} \right) \quad (2.2,7c)$$

where v_θ is the transverse velocity and w the axial velocity. The boundary conditions chosen were such that, in effect, the detailed spiral structure of the inviscid vortex was neglected, and the flow field with discrete jumps replaced by a smoothed out distribution. This procedure is valid; Maskell [45] pointed out that the turns are thickened by viscous diffusion and the spiral structure disappears. The equations are solved by introducing a similarity variable together with initial conditions on v_θ and w which are needed since the boundary-layer equations are parabolic.

The contribution to w from the boundary layers on the wings was treated as follows: if the combined momentum thickness from the upper and lower wing boundary layers at the trailing edge is $\delta_2(x)$, then the initial trailing vortex sheet has a flux deficit $U_\infty \delta_2(x)$ per unit length which is spread out through the core as the sheet rolls up. The circulation distribution was interpreted as a concentration of vorticity initially within a distance x of the tip into a circle of radius x/λ . The vortex lines constituting this distribution move with the fluid, so that the flux at any vortex line is unchanged in the rolling. The same similarity law applies to the flux, and since the flux deficit originally between x and $x + \delta x$ ends up in the core between circles of radii x/λ and $x/\lambda + \delta x/\lambda$, there will be an average axial velocity distribution

$$w_\delta = - \frac{\lambda U_\infty \delta_2(\lambda r)}{2\pi r} \quad (2.2,7d)$$

in the core, which becomes continuous after viscous spreading of the turns between x/λ and $x/\lambda + \delta x/\lambda$ is complete. This value, w_δ , provides the initial and boundary values for the core equation

$$\frac{\partial w}{\partial t} = \nu \left(\frac{\partial^2 w}{\partial r^2} + \frac{1}{r} \frac{\partial w}{\partial r} \right) \quad (2.2,7e)$$

The solution of (2.2,7e) with the condition (2.2,7d) may be added to the w profile already obtained, because the equation for w is linear when $w \ll U_\infty$.

Moore and Saffman's [57] theoretical predictions compared quite reasonably with Olsen's [56] comprehensive set of measurements. Olsen towed a rectangular wing through a water channel and measured the axial velocity by determining the speed of a dye filament. He used various aspect ratios, angles of attack and Reynolds numbers. Profiles of axial velocity have also been reported by Logan [58] , these do not agree so well with the predictions of Moore and Saffman, but $w \ll U_\infty$ is violated. The Reynolds numbers for Logan's measurements are close to those at which transition to turbulence occurs in the boundary layer on the wing, thus it is possibly inappropriate to compare his measurements with a laminar theory.

The roughly axially symmetric cores of leading-edge vortices over slender wings was treated by Hall [59] by assuming that the flow was axially symmetric and rotational, with viscous effects confined to an inner core that was very slender in form. He thus reduced the problem of solving the equations for the core to first obtaining an inviscid outer solution for the outer part whilst ignoring the inner core, and then using this outer solution to specify boundary conditions to obtain a viscous solution for the inner core, i.e. the core was divided into a convective outer part and a diffusive subcore.

The outer solution gave results which agreed fairly well with the experimental results of Earnshaw [60] . The static pressure profiles, outside the subcore, agreed very well and discrepancies were attributed to the theoretical assumption that the

flow field was inviscid, exactly axially symmetric and exactly conical. The predictions of the inner solution were only qualitatively correct giving results that differed appreciably from experimental results. The theoretical velocity and pressure gradients in the subcore were too large and the peaks too pronounced. The difference was explained by Hall to be possibly due to the assumption of laminar flow, and possibly due to the fact that the flow in the subcore of the experiment was significantly turbulent. Hall's [59] argument while shedding considerable light on the mechanism governing the structure of the inner subcore assumes the nature of the outer core region; the assumption is justified by appeal to experiment but is without support of a physical argument. A convincing argument needs to be part of a unified physical interpretation of the observed vortex structure embracing the whole of the vortex from the outer spiral region to the inner diffusive subcore.

Stewartson and Hall [61] dissatisfied with the procedure used by Hall [59] for the inner solution, and in particular with the matching of the inner and outer solutions, presented a solution based on the model of the vortex core previously proposed by Hall [59]. This solution, in special variables, was in the form of an asymptotic expansion containing inverse powers of the logarithm of a Reynolds number. First, the Navier-Stokes equations were simplified by making approximations of boundary-layer type, the appropriate independent variables were defined and the outer solution expressed in terms of these variables; lastly, with this as a guide, an asymptotic

expansion for the inner solution was set down. Substituting the expansion in the equations of motion and equating terms of like magnitude a set of ordinary differential equations was obtained, which, combined with appropriate boundary conditions gave a solution which approached the outer solution with increasing distance from the axis. When applied to a practical problem where $\nu > 0$ there was little difference between Stewartson and Hall's [61] and Hall's [59] solutions. Stewartson and Hall claimed that their solution was easier to apply and more straightforward in the attainment of greater accuracy (by the calculation of extra terms).

2.2.5 Finite-difference solutions

A numerical method of solving the parabolic differential equations (A,1) directly for steady incompressible flow has been developed by Hall [62]. Initial conditions were specified on some upstream cross-section, and boundary conditions on the axis of symmetry and on some bounding surface of revolution. The differential equations were replaced by sets of finite-difference equations, using first-order central differences in an implicit scheme. This method is a development of known finite-difference methods for calculating laminar boundary layers (see, for example, Gadd, Jones and Watson [63]). Explicit techniques have also been used for boundary-layer calculations but they are subject to numerical instability. Hall's [62] calculation used a marching technique that proceeded step-by-step in the axial direction; for each step an iterative plan was followed. The finite-difference equations themselves were solved by straightforward matrix methods. Three examples

of the application of the method were given by Hall [62] ; a trailing vortex, a leading-edge vortex core with non-conical outer flow and a swirling pipe-flow. Figures 14 (a and b) show the profiles of axial and circumferential velocity respectively in a trailing vortex as calculated by Hall [62] . However, at a certain point in the downstream direction he found large gradients in the flow variables which prohibited the computations from proceeding further. This indicated the failure of the quasi-cylindrical approximation, i.e. the axial gradients became large enough to invalidate the quasi-cylindrical approximation. This failure was interpreted by Hall [62] as an indication of the onset of breakdown similar to the failure of the boundary layer equations near the separation point; the analogy may not be strictly valid as the failure of the quasi-cylindrical approximation occurs when the axial velocity is much greater than zero.

Newman's [55] solution for $x = 1.0$, where x is the non-dimensional axial ordinate, axial distance/reference length, is also plotted in Figure 14a; it shows a much more marked increase in axial velocity with increasing distance downstream than is given by the numerical solution. This discrepancy is the net result of an augmentation of velocity in Newman's case due to his neglect of the pressure-gradient term in the equation of motion as mentioned previously, and a reduction of velocity due to Newman's linearization of the equations of motion. From Figure 14b it can be seen that the linear theories of Batchelor [12] and Newman [55] give results for the circumferential velocity that are almost identical to those

obtained numerically.

Finite-difference procedures are potentially more powerful methods for understanding vortex motion than theoretical models, for example, references [8, 42], relying on tracing discrete vortices. Computer programs based on finite-difference methods can ultimately account for flowfield nonlinear effects with few ad hoc assumptions. Implicit finite-difference procedures for the primitive form of the incompressible Navier-Stokes and the compressible Euler equations were used by Steger and Kutler [64] to compute vortex wake flows. Rossow [65] in his survey on the inviscid modelling of aircraft trailing vortices divided the wake-vortex flowfield into three regions, see Figure 15. The roll-up region where the vorticity shed from the wing is rolled-up into two or more vortices is followed by the plateau region where this pattern is then almost stationary and may persist for some time until finally, in the decay region, the vortices begin to diffuse because of viscous and turbulence effects.

Steger and Kutler's [64] primary difficulty in applying finite-difference procedures to solve the wake-vortex plateau and decay regions, was that changes in the gradients of the dependent variables in the crossflow directions are large and need resolving. This requires a refined grid spacing. Changes of the dependent variables in time are frequently small, so that the calculation may have to be computed over a long time interval before the solution of interest is obtained. These difficulties govern the choice of numerical algorithm. So that the time step was restricted only by the desired resolution of the time-varying dependent variables, an implicit

numerical procedure was selected. Fourth-order spatial differencing was used and grid points were clustered in regions containing large changes in the gradients of the dependent variables. The clustering transformation was written as a function of time since vortices can translate. Computational efficiency was achieved by approximate factorization. Steger and Kutler describe both two-dimensional and preliminary three-dimensional calculations.

A series of computations were run to verify the accuracy of the finite-difference procedures for both compressible and incompressible equations. The Lamb vortex - an exact solution of the incompressible Navier-Stokes equations - was used to test the accuracy of their procedures. If the flowfield is initialized using a single Lamb vortex inviscid theory requires that it remain indefinitely. This test serves as a steady-state verification of spatial differencing; a check on the unsteady accuracy is obtained by a solution of the viscous-flow case for which the exact solution is known. Figure 16 shows a check of the accuracy of compressible equations for a single Lamb vortex. The initial vorticity distribution is generated by the super-position of Lamb (or spreading line) vortices given by

$$\begin{aligned} v_\theta &= v/\cos\theta = w/\sin\theta \\ &= \frac{\Gamma}{2\pi r} (1 - e^{-r^2/r_c^2}); \quad r_c^2 = 4\nu t_0 \end{aligned}$$

see Lamb [46 p.592],

where r is the radial distance from the centre of the vortex, r_c is the approximate core radius and t_0 the initial time. A simple viscous decay result is shown in Figure 17, $d_c = 2r_c$. To

demonstrate the capability of the algorithm, a multiple vortex wake representing the flowfield behind a large commercial jet aircraft in a landing configuration was also simulated using the compressible equations.

2.2.6 Solutions of the Navier-Stokes equations

Theoretical work investigating the structure of the flow in the trailing vortex has mostly been done within the framework of a quasi-cylindrical approximation of the Navier-Stokes equations. Jain [69] attempted to understand the structure and decay of a trailing vortex through the numerical solutions of the full unsteady Navier-Stokes equations. The governing equations were recast in terms of circulation, vorticity, and stream function as dependent variables, and a second upwind finite-difference scheme used to integrate them with prescribed initial and boundary conditions. Turbulence was introduced in a simplified way through the concept of eddy viscosity and a fully developed trailing vortex was simulated by a swirling flow coupled with an axial flow. The essential problems lie in prescribing the conditions at the inflow section, either from experimental data or from some other theoretical considerations and computing the flow downstream. Different models of the flow were postulated and solutions obtained describing the development of the flow as integration proceeded in time. In order to understand the various phenomena that may occur in the trailing vortex, Jain [69] carried out a parametric study. Using the Hoffman and Joubert [73] law of circulation at the inflow section, Jain's theory predicted reasonably well the experiment data of Chigier

and Corsiglia [11] on models of the Convair 990 wing and a rectangular wing. Jain did, however, experience difficulty in obtaining convergent solutions with high values of the swirl parameter. Using an exponential law of circulation at the inflow section and an adverse pressure gradient at the outer edge of the trailing vortex led to solutions possibly depicting a vortex bursting through the sudden expansion of the core and through stagnation and subsequent reversal of the flow on the axis.

The motion of a vortex in a two-dimensional incompressible nonuniform stream as solutions of the Navier-Stokes equations was presented by Ting and Tung [70]. Their analysis was capable of being extended to some three-dimensional problems, for example, the motion and decay of a trailing-edge vortex line. Ting [71] also presented solutions of the Navier-Stokes equations constructed as an asymptotic expansion in terms of a small parameter related to the Reynolds number of the vortex. The solution for the inner viscous core of the vortex was matched to the outer inviscid solution. The singularities in the inviscid theory were removed and the velocity of the vortex line was defined by the condition of regularity in the flowfield. Ting applied this scheme to vortices in two-dimensional axially symmetric and in three-dimensional flowfields. The solution of this analysis can be identified with that of classical inviscid theory with the same initial vorticity distribution for the initial instant. After this they disagree as the inviscid theory ignores the diffusion of vorticity in the core.

2.2.7 The solution of Weber et al.

Weber et al. [72] applied a new, general, subsonic potential flow computational technique to solve the three-dimensional flow over wings with leading-edge vortex separation. The method used an inviscid flow model in which the wing, the rolled-up vortex sheets and the wake were represented by piece-wise continuous quadratic doublet sheet distributions. A simpler approach to that used by Smith [38] was adopted for modelling the inner core region. Smith's [38] model when applied to the three-dimensional nonconical flow problem would contain two self-induced infinite forces with no possibility of mutual cancellation. The model of Weber et al. [72] has been verified for predicting the flowfield about swept, sharp-edged wings characterized by the presence of vortex separation at the leading edge.

2.2.8 Similarity solutions

A similarity solution on lines similar to the methods used in turbulent boundary-layer theory for the incompressible, turbulent line vortices was attempted by Hoffman and Joubert [73]. They established that the inner region of fully turbulent vortices was divided into three regions. In the centre or 'eye' of the vortex a region of solid body rotation exists, the circulation $\propto r^2$, the shear stresses are small as there is little slip between concentric layers of fluid. Since there might be rapidly changing tangential velocities as well as changing radial velocities due to wake velocity defects in this region and a small region outside, it might be expected

that tangential inertia forces would dominate. Some form of transition curve may be expected in the transition region. Finally, in the region of the tangential velocity peak where the tangential inertia forces are small but the shear stresses large, the circulation varies logarithmically with radius. The logarithmic distribution of circulation has been confirmed by wind tunnel tests [22, 74] and by the results of smaller scale flight tests [22].

Saffman [75] presented a theory arguing that the turbulent vortex has a triple structure, although his argument was fundamentally different to that of Hoffman and Joubert [73]. In the outer region, $r > r_1$, where r_1 is the radius of maximum tangential velocity, there is a logarithmic distribution of circulation. For $r < r_1$, there is an inner region and viscous core in both of which the motion is close to solid body rotation. He predicted that $r_1 \sim (\nu \Gamma_1 t^2)^{1/4}$, where Γ_1 is the circulation at r_1 ; ν the kinematic viscosity and t the vortex age.

Long [76, 77] obtained a similarity solution for a steady viscous vortex in an infinite compressible fluid. He assumed that the circulation function $K \equiv r v$ tended to a constant K_c at large enough distances from the axis; his solution may be regarded as applying to a swirling jet in an infinite fluid which rotates as a potential vortex.

2.2.9 Overshoot in circulation

An interesting phenomenon that could happen to trailing vortices, and was first analysed by Govindaraju and Saffman [78]

is the overshoot in circulation, i.e. the circulation rises above Γ_∞ , where Γ_∞ is the circulation at ∞ and then falls back to Γ_∞ as $r \rightarrow \infty$. They argued that the vortex must develop an overshoot of circulation if it entrains fluid at a rate greater than that due to molecular diffusion. In their analysis of the effect of turbulent shear on the decay of a single vortex, Donaldson and Sullivan [79] also found a circulation overshoot which results in instability in the Rayleigh sense. Figure 18 shows the circulation profiles, normalised to Γ_∞ from which the circulation overshoot is clear. They suggested that the instability inherent in the overshoot in the circulation well outside the core of a turbulent trailing vortex might be responsible for the "doughnut" shaped rings sometimes observed about the trailing vortices in the wake of a jet aircraft. The overshoot of circulation was also reported in the measurements of Graham et al. [80].

2.2.10 Experimental measurements

The experimental techniques used for studying trailing vortices are usually either a wind-tunnel study, intercepting the vortices shed by one airplane with another airplane, or measuring the vortices shed by a low-flying airplane with ground-based instruments. The conventional wind tunnel is suitable for the study of vortex formation but not the long-time decay process due to the length of working section normally available. Another problem is the simulation of Reynolds number. Simulation of atmospheric conditions such as temperature distribution or the relative scale of turbulence is very difficult and wall effects may be hard to assess.

Kiang [81] used a "moving-wing facility" that consisted of a vortex-generating wing moving along a pair of elevated steel rails so that the vortices shed by the wing remained relatively fixed with respect to ground-based instrumentation, and could be observed and measured throughout their entire life-span. His general conclusions were that the turbulence level was quite high, often exceeding 50%. Neither Squires' [48] or Hoffman and Joubert's [73] theories could correlate the results of his subscale experiments with those of full-scale airplanes; also the trailing vortices generated by the moving wings lasted only a few seconds as compared with several minutes of the full-sized trailing vortices. However, factors affecting his measurements were the effects of starting and stopping the wing and the residual room currents, although these are low they are sufficient to cause irregularities in the path of the vortex centre. Kiang's [81] results were based on the photographing of bubble traces; there is a large scatter in his radial velocity measurements.

Rorke and Moffitt [82] conducted an experimental investigation to determine the important scaling parameters for the flow in the core region of a vortex generated by a rectangular wing tip. They used two geometrically similar wings with chords of 26.0 and 4.25 inches, and by running at different tunnel speeds they obtained Reynolds numbers ranging from 4.4×10^5 to 7.0×10^6 at Mach numbers of 0.2, 0.5 and 0.6. Their results showed that for rectangular planform wings, the measured vortex core diameter to chord ratios, peak tangential velocity ratios and axial velocity ratios are shown

to be functions only of wing lift coefficient and elapsed time from vortex formation, and appear to be independent of both Mach number and Reynolds number. Neither Rorke and Moffitt [82] or Chigier and Corsiglia [11] give reliable quantitative or qualitative information concerning the structure, transport and dissipation of the trailing vortices.

Wind tunnel tests were also performed by Chigier [83] who presented data for tip vortex studies. His results included wing surface pressure distributions, three-dimensional velocity components in the wake and principal vortex characteristics, such as peak tangential velocity and core size distributions. Comparison of his wind tunnel measurements with available flight test data showed that the magnitudes of circumferential velocities, normalised by flight speed and lift coefficient, as well as vortex core radius, normalised by wing span, were in close agreement. For details of other wind tunnel measurements, see El-Ramly [84, 85] .

When probe aircraft are used to study trailing vortices they either follow the generating aircraft, for example, [86, 87], or traverse its vortex system, for example, [66, 88]. Another technique used is that of flying the aircraft past a tower; experiments are performed using ground-mounted instrumentation, for example, [89], ground effects are present in the experiments and the effect of the tower on the vortex system is unknown. The remote measuring of the vortex system which includes using Laser Doppler [90], Doppler Radar [91] and acoustic radar detection techniques [92, 93] are still being developed. McCormick et al. [22] used a technique whereby the aircraft was instrumented to measure its own vortex system.

2.3 Vortex Decay and Disintegration

A trailing vortex system may decay or disintegrate by one of three mechanisms: a) viscous dissipation, b) mutual or hydrodynamic instability and c) vortex breakdown. Atmospheric stability causes relative buoyancy of the wake, with subsequent effects on the vertical transport of the vortices and on their horizontal separation. MacCready [94] concluded that atmospheric turbulence and stability often played a major part in determining the descent, speed, interaction and decay of the vortices.

2.3.1 Decay Due to Viscous Dissipation

Included in viscous dissipation is any dissipation caused by small-scale turbulence that interacts with the velocity gradients in the vortex to produce local Reynolds stresses. Viscous dissipation is a very slow process. The rotating flow motion in the vortex may have a stabilising effect upon the growth of this form of turbulence and act against any significant diffusion of momentum.

An analysis of a trailing vortex pair behind an aircraft in a quiet atmosphere with no buoyancy was made by Nielsen and Schwind [30]. In this situation, viscous dissipation and the interaction between the two vortices, when they grow in size so that their cores start touching and overlapping, are the mechanisms available for decay. Their analysis showed that the rate of circulation decrease due to interaction between the vortices did not lead to rapid decay of the trailing vortices. Kuhn and Nielsen [95]

extended the analysis of Nielsen and Schwind [30] to incorporate insight gained from the experimental work of Chigier and Consiglia [53] who found that the axial velocity defect is confined to the core.

An approximate theory for the decay of the vortex in the presence of a small-increment jet or wake has been developed by Graham, Newman and Phillips [80]. The theory assumes that the jet dominates the flow and provides a scalar eddy viscosity which may be used to predict the growth of the vortex. At large distances downstream the theory becomes invalid, as the rate of strain in the jet-wake falls off more rapidly than that in the outer part of the vortex.

2.3.2 Stability of a Trailing Vortex

Vortices are sometimes seen to undergo a slow, symmetric and nearly sinusoidal instability when the vortex cores recede and draw together in a wavy pattern until eventually they join together at the nearer points to form vortex rings. After the rings are formed the wake rapidly disintegrates into a harmless turbulent state.

Rose and Dee [66] reported the mutual interaction observed between the two vortices during flight tests with a Comet aircraft. Dee and Nicholas [96] confirmed this behaviour of the trailing vortex with flight tests using a Hunter aircraft. Bisgood, Maltby and Dee [97] observed the formation of loops in the vortex wake in their tests behind a Comet aircraft.

The first quantitative analysis of the three-dimensional instability of a vortex pair in an ideal homogenous fluid was given by Crow [98]. He idealized the wake as a pair of nearly parallel

vortex lines interacting in neutrally stable air. The vortices are convected in their own induced field. Gusts or flight-path irregularities displace the two lines slightly in a random fashion, and the displacements amplify under mutual induction.

The analysis began with the kinematic relation (see Batchelor [99] p.509) between vorticity and velocity in an incompressible fluid:

$$U_n = \sum_{m=1}^2 \Gamma_m \int \frac{R_{mn} \times dL_m}{4\pi |R_{mn}|^3} \quad (2.3,1)$$

where n takes on the values 1 and 2. The equation (2.3,1) gives the velocity at a point on the n th vortex in terms of the relative position R_{mn} , length dL_m and strength Γ_m of all the vortex elements in the flow, see Figures 19 and 20 for an illustration of some of the geometrical quantities.

The vector distance from an element of vortex n to another of vortex m satisfies

$$R_{mn} = e_x(x'_m - x_n) + e_y(s_m - s_n) + (L'_m - L_n) \quad (2.3,2);$$

the first two terms on the right involve the locations of the unperturbed vortices, and the third represents radial displacement from their nominal positions. The primes are used to distinguish points lying on the same vortex, in case $m = n$. From Figure 20 dL_n is given by

$$dL_n = (e_x + \partial L_n / \partial x_n) dx_n \quad (2.3,3)$$

The circulations Γ_n and lateral locations s_n are related as follows:-

$$\begin{aligned}\Gamma_1 &= -\Gamma_0, & \Gamma_2 &= +\Gamma_0; \\ s_1 &= -b/2, & s_2 &= +b/2.\end{aligned}$$

The vorticity-transport theorem states that in an inviscid and neutrally buoyant fluid, elements of a vortex line move with fluid particles; in mathematical terms,

$$\frac{\partial \Gamma_n}{\partial t} + u_n \frac{\partial \Gamma_n}{\partial x_n} = \mathcal{E}_y v_n + \mathcal{E}_z w_n, \quad (2.3,4)$$

where (u_n, v_n, w_n) are the components of $\mathcal{U}_n + \mathcal{E}_z(\Gamma_0/2\pi b)$, which is the velocity of convection with respect to the downward moving coordinates. Equations (2.3,1-4) provide a self-contained description of the vortices as they evolve in their induced field. In equation (2.3,1) a difficulty arises with the $m = n$ term on the right-hand side; the line integral diverges logarithmically around $|\mathcal{R}_{nn}| = 0$. This divergence is artificial, as the real viscous vortices have finite core diameter. To solve this problem Crow cut the integral off an arc-length d on either side of the point $|\mathcal{R}_{nn}| = 0$, the cut-off distance, d , was taken to be proportional to the diameter, c , of the vortex cores. To determine (d/c) he applied the cut-off method to two problems whose exact solutions were known a priori, these were Kelvin's [100 pp.152-165] solutions for a displacement wave travelling around a columnar vortex and the speed of a vortex ring [46, p.241]. For vortex displacements that

are small compared with the separation of the vortices and $|c_n|/b \ll 1$ and $|\partial c_n / \partial x| \ll 1$, the equations can be linearised. The equation relating induced velocity to vortex displacement i.e. (2.3,4), gave rise to an eigenvalue problem for the growth rate of sinusoidal perturbations. Stability was found to depend on the products of vortex separation b and cut-off distance d times the perturbation wavenumber. Depending on those products both symmetric and antisymmetric eigenmodes can be unstable, but only the symmetric mode involves strongly interacting long waves. The vortex displacements are symmetric and are confined to fixed planes inclined at 48° to the horizontal. The stability boundaries for the symmetrical, S, and the antisymmetrical, A, modes are shown in Figures 21 and 22, δ/β is used as the abscissa instead of δ , because δ/β equals d/b , a geometrical property of the vortices independent of the wavenumber, k .

The results of Crow's analysis were in good agreement with the observations of [101, 102] of general features of aircraft wake instability and measurements of amplification rates.

The cut-off distance in Crow's [98] analysis removes the fluid mechanics of the vortex core from the problem and the effect of different rotational and axial velocity distribution is not directly apparent. Widnall, Bliss and Zalay [103] presented a general theory for the self-induced motion of a perturbed vortex of finite core size incorporating the effects of an arbitrary distribution of both swirl and axial velocity. The solution is obtained by aid of matched asymptotic expansions, see Van Dyke [104].

Two asymptotic solutions are sought: one approximate solution (the inner solution), valid in the limit $\epsilon \rightarrow 0$, (ϵ is the ratio of the core radius to local radius of curvature) only within the rotational vortex core and its immediate surroundings, and another approximate solution (the outer solution), valid in the limit $\epsilon \rightarrow 0$, in all regions of the flow except the vortex core. They calculated the velocity field induced in the neighbourhood of the vortex core by distant portions of the vortex line for a sinusoidally perturbed vortex filament and for a vortex ring. The effects of axial velocity on the stability of the perturbed vortex pair are: to reduce the most unstable wavelength of the long waves most often observed in flight, to decrease slightly the amplification rate and to cause the merging of the long and short wave branches at lower values of core radius. Experimental results for the distortion and break-up of a perturbed vortex pair are presented and these confirm in a qualitative manner the instability predicted by theory.

Widnall [105] has reviewed the work on trailing vortices and vortex rings.

Another modification to Crow's [98] theory was that of Parks [106], who took account of finite core radii and appropriate distributions of vorticity within these cores, uniform vorticity or one that is a function of radial distance; thus he avoided the difficulty of self-induction encountered by Crow. Parks [106] confirmed the essential features of Crow's theory; Table III, as presented by Parks, compares his results with those of Crow.

To solve the equations of motion Crow [98] linearised them. However, the most unstable long waves are of amplitude comparable to their length when they touch, thus non-linear effects should also be examined. Hackett and Theisen [107] tried to extend Crow's analysis beyond the linear range using a flow modelling, finite element, time-stepping technique. Their computation failed to accommodate the "pinching-off" process by which complete vortex rings are formed in an old contrail thus reducing the flattening tendency of the rings. Downward mutual convection at necking-in points of the vortices are instantaneously replaced in a real flow by upward mutual convection when they break and change partners. Moore [108] also attempted to examine non-linear effects using the same assumptions as in Crow's theory.

The results of the Boeing wake turbulence test program as reported by Condit and Tracy [109] correlated well with Crow's theory; the theory predicted a more rapid break-up than was observed. Both axial flow and mutual interaction instabilities were observed by Olsen [56] in his studies of the trailing vortex in a towing tank. His results were strikingly similar to flight test data, although test Reynolds numbers were far smaller than flight test Reynolds number (10^4 vs. 10^7). The flow in the neighbourhood of the core was destroyed by the instability associated with the axial flow within the core, however the motion far from the core was not destroyed.

Since the velocity of a vortex line depends on its structure i.e. the shape of the cross-section and the detailed

vorticity distribution, Moore and Saffman [110] considered the structure of straight line vortices in a uniform two-dimensional straining field. For irrotational strain, steady exact solutions of the inviscid equations were shown to exist, the boundary of the vortex being an ellipse with principal axes at 45° to the principal axes of strain. Of the two axis ratios possible, provided $e/\omega_0 < 0.15$, where e is the maximum rate of extension and ω_0 the vorticity in the core, the more elongated shape is unstable and the less elongated one is stable to two-dimensional deformations. If $e/\omega_0 > 0.15$ there are no steady solutions of elliptical form. For simple shear, one steady shape of elliptical form exists provided the shear rotation and vorticity are in the same sense and $e' > \omega_0$, where e' is the rate of shear. The major axis is parallel to the streamlines and the shape is stable to two-dimensional deformations. There are two steady elliptical shapes if $e'/\omega_0 < 0.21$ for shear rotation and vorticity in opposite senses, the major axes are then perpendicular to the streamlines; the more elongated form being unstable whilst the less elongated one is stable.

The stability of a line vortex containing axial flow can be treated as a problem in hydrodynamic stability. Widnall and Bliss [111] and Moore and Saffman [112] have considered the stability of a vortex containing axial flow to long-wave disturbances. In their review of their previous work, [103], concerning the effects of axial velocity on the motion of vortex filaments, Widnall and Bliss [111] stated that their results

suggested that a slender body force balance, between the Kutta-Joukowski lift on the vortex cross-section, and the momentum flux within the curved filament, would give some insight into the behaviour of the filament. They investigated the stability of a straight vortex filament containing an axial flow to long wave sinusoidal displacements of its centre-line and obtained the stability boundary. To the lowest order of a/b axial flow reduced the self-induced rotation of a single filament, this could be considered as changing the effective core radius. To the next order of a/b , travelling waves appeared in the instability, the instability mode for the vortex pair became non-planar but the amplification rate of the instability was not affected. Moore and Saffman [112] made a study of infinitesimal waves on a uniform vortex with axial flow by obtaining a general equation for the motion of a vortex filament, valid for arbitrary shape and internal structure and in the presence of an external irrotational flow field. In revising this paper, Moore and Saffman [113] established that a vortex would be unstable in the presence of strain if it could support non-rotating waves; no specific solutions or numerical results were presented.

The characteristics of vortex behaviour and instability have been described by flight tests as reported by Tombach [114], Chevalier [115] and Jones and Chevalier [116]. The vortices were generated by a Cessna 170 for the tests reported by Tombach and a de Havilland Beaver DHC-2 and a Beachcraft T-34B for tests reported by Chevalier. Some of the conclusions were as follows:-

- (i) the existence of vortex wake instability as predicted by Crow [98] was verified;
- (ii) the vortices were destroyed by some form of instability and not observed to decay away due to viscous or turbulent dissipation;
- (iii) two modes of instability were observed; a "localised" bursting of the core of an individual vortex with usually no effect on the adjacent vortex or on distant segments of the same vortex, and the sinuous instability of both vortices resulting in their linking into vortex rings;
- (iv) the majority of the wakes were observed to roll to some degree, and
- (v) the correlation between wake lifetime and atmospheric turbulence, the life of the wake being greatly shortened by small amounts of turbulence.

Singh and Uberoi [117] in their experiments on vortex stability studied the tip vortex of a laminar flow wing and verified the presence of laminar instability modes associated with large axial velocities in the vortex core.

2.3.2.1 Vortex Rings

The vortex core is a region of concentrated vorticity whereas the fluid carried along by the vortex ring may contain little or no vorticity. Some general features of the flow field of a typical vortex ring in nearly steady flow are shown in Figure 23. Maxworthy [118] pointed out that in a real fluid the vorticity may diffuse out of the boundary of the ring and be deposited into a wake;

fluid may also be entrained, increasing the volume carried by the ring, see Figure 24. For slight viscosity these effects only slowly change the character of the flow; at any instant of time the velocities are kinematically related to the distribution of vorticity. Little is known about the process of formation or the resulting properties of the vortex ring, however they are easily created in the laboratory by pulsing fluid through a sharp-edged orifice. Harvey [119] suggested a mechanism for the development of ring vortices, see Figure 25.

When considering vortex rings in steady motion in an ideal fluid, the existence of axisymmetric distributions of vorticity (without swirl) that move without change of shape requires that the velocity field induced by the vorticity distribution be such that the vorticity of a fluid element convected along a stream surface ($\psi = \text{constant}$) changes only in proportion to its length: $\zeta = r f(\psi)$. The governing equation for the stream function of axisymmetric flow is (in cylindrical coordinates r, z):

$$\frac{\partial^2 \psi}{\partial z^2} + \frac{\partial^2 \psi}{\partial r^2} - \frac{1}{r} \frac{\partial \psi}{\partial r} = r^2 f(\psi)$$

The term vortex ring commonly refers to a bounded region of vorticity that moves through a surrounding potential flow so that $f(\psi)$ is nonzero only within some boundary. Two types of solution exist to this problem: general solutions for vortex rings of small cross section but with an arbitrary distribution of vorticity, and solutions for the special case $f(\psi) = \text{constant}$.

Kelvin, in 1867, presented the correct result for the propagation velocity of a ring with 'uniform' vorticity within the

core, $V_0 = (\Gamma/4\pi R)[\ln(8R/a) - 1/2]$, where a is the core size and R the radius of the ring; references to this early work may be found in Lamb [46]. Saffman [120] in his solution for the propagation velocity of a vortex ring of small cross section with an arbitrary distribution of vorticity within the core used the theorems for energy and impulse and a transformation suggested by Lamb [46].

The experimental work of Maxworthy [118] was mostly focussed on the structure of steady rings; he found that they existed in two ranges of initial Reynolds number (Re). For $Re \leq 600$ the ring was stable during the whole of its motion; when $Re \gtrsim 1000$ the ring first became unstable and then out of the disorganised flow so formed a new stable vortex emerged. Rings with very large Re ($\sim 2 \times 10^4$) became unstable immediately after forming. Maxworthy [118] in his study of vortex rings found that when the vorticity was relatively well distributed over the ring there was definitely an exchange of vorticity between the ring and the surrounding medium. When an aircraft sheds two vortices of like sign and magnitude from each side of the fuselage, the vorticity may be soon fairly widely dispersed inside each side of a descending oval; this oval will be similar to the ideal oval shown in Figure 26. Maxworthy [121] also observed the behaviour of rings at higher Reynolds numbers. He found that after the initial instability, which always took place, the vortex ring settled down to turbulent propagation, the radius of the ring increased slowly and the velocity decreased. In a further series of experiments, Maxworthy [122] used flow visualisation and Laser-Doppler techniques to reveal several interesting properties of high Reynolds number vortex rings.

An experimental and theoretical study of the instability of the vortex ring has also been carried out by Widnall and Sullivan [123]. Both theoretical predictions and experimental results indicated that the vortex ring is unstable to wavelengths that are not large in comparison with the size of the vortex core, as required for the application of the cut-off method used in the analysis. The phenomenon of azimuthal wave instabilities which was described qualitatively by Maxworthy [118] was characterized quantitatively in their experiments in which measurements were made of the vorticity distribution, the self-induced velocity and circulation, and the amplification rate for several different rings. Widnall, Bliss and Tsai [124] re-examined the existence of short waves on straight filaments in connection with the study of a short-wave instability of the vortex ring, and concluded that this prediction, (Widnall and Sullivan [123]), of short-wave instability is spurious unless a strong axial velocity exists in the core but that short-wave instabilities having a more complex radial structure can exist.

However, the theoretical work of Widnall and Sullivan [123] was revealing in two important respects. Firstly, the prediction that the vortex ring was unstable at the wavenumber for which the asymptotic analysis of Widnall, Bliss and Zelay [103] spuriously predicts that a sinusoidally perturbed line filament would have zero self-induced rotation ($\omega_0 = 0$), provided a significant clue to the physical mechanism responsible for the instability. Secondly, since this theory was in agreement with the general features of the instability, the amplification rate and the

increase in azimuthal wave numbers with decreasing core size indicated that the instability could likely be predicted by an inviscid analysis of the sinusoidal bending perturbations of a slender vortex ring, without the need to incorporate more complex features of the flow.

A physically plausible, but not mathematically rigorous argument, that bending waves on a vortex filament would be unstable in the presence of a straining flow (such as that of the ring mean velocity field itself) whenever these waves had no self-induced rotation ($\omega_0 = 0$) was presented by Widnall et al. [124]. Generally, the first radial modes of bending of a filament do not have the property that $\omega_0 = 0$ for some value of wavenumber, but $\omega_0 = 0$ does occur for some critical wavenumber $\kappa_2 = \kappa a$ (a is the core radius) in the second radial mode, in which the centre moves in a direction opposite to the boundary of the core; the higher radial modes also have $\omega_0 = 0$ for yet higher values of $\kappa_n = \kappa a$. Since the effects of ring curvature are not considered, and the displaced vortex core is assumed to move with the local free stream at each section of the wave - an invalid assumption for short waves, the analysis is not mathematically rigorous; the model does, however, agree very well with experiment. Tsai and Widnall [125] gave some support for their previous work with Bliss [124] finding that their strained vortex is unstable at wavelengths for which the waves on a corresponding straight filament would not rotate ($\omega_0 = 0$). They analysed the stability of short waves on a vortex filament in a flow field with strain.

A theoretical analysis for the instability of a thin vortex ring, with a core of constant vorticity in an ideal fluid, to short azimuthal bending waves was presented by Widnall and Tsai [126]. Both the mean flow and disturbance flow were found as an asymptotic solution in $\epsilon = a/R$, the ratio of core radius to ring radius. In the stability analysis only terms linear in wave amplitude were retained; however, the solution to $O(\epsilon^2)$ was presented for a special class of bending waves known to be unstable on a line filament in the presence of strain (Tsai and Widnall [125]); Widnall, Bliss and Tsai [124] identified these bending waves as a likely mode of instability for the vortex ring. The ring was always found to be unstable for at least the two lowest critical wavenumbers ($ka = 2.5$ and 4.35); good agreement was found between available experimental results and the theoretical prediction of the amplification rate and wavenumber. Their theory [126] does not cover the persistence of vortex rings in flows at low Reynolds number and with a turbulent core.

There is some similarity between the stability problem for waves on a vortex ring and waves on a straight filament. The waves of interest for the vortex ring instability are short waves such that ka remains constant as $\epsilon \rightarrow 0$ so that ω_0 remains zero. Therefore the wavenumber, k , becomes large in the limit $\epsilon \rightarrow 0$, resulting in the far-field effects from waves around the ring being asymptotically small (as $e^{-k\ell}$) as $\epsilon \rightarrow 0$, so that only local effects of curvature enter into the stability problem. Thus any locally curved filament should exhibit the same instability as the vortex ring.

2.3.2.2. Interactions and Merging of Line Vortices

The merging, pairing, condensing or coalescing of two vortices of like sign results in both the growth of a turbulent shear layer and the rapid ageing of an aircraft vortex wake. The merging vortices ingest or entrain volumes of nearly irrotational fluid which then, through turbulent transport, become mixed with the vortical material; the volume containing vortical material is thereby rapidly increased. From a simple model computation undertaken by Corsiglia et al. [127] a physical description of the merging phenomenon can be obtained, see Figure 27. If a weak vortex, diameter d positioned at $z/d = 1.0$, of vanishingly small strength is placed in the velocity field of a second vortex of strength Γ , positioned at $y = z = 0$, the weak vortex is simply convected about the strong vortex. When two vortices of nearly the same strength merge the axial symmetries of both vortices are destroyed by the strain of vortex upon vortex. The initial phases of merging involve convection; however, the vorticity is redistributed outward by the mechanism of turbulent transport. Interesting tests examining the interaction of a small aircraft with trailing vortices near the ground during approach to landing were reported by Crow and Murman [128].

A theoretical and experimental study of interactions between adjacent line vortices was carried out by Bilanin, Snedeker and Teske [129]. From flow visualisation experiments using smoke as a marker they found that

- (a) vortices of unlike sign do not tend to merge;

- (b) a weak vortex will merge with a strong vortex of the same sign;
- (c) transport in a fluid eventually causes merging of vortices (not predicted by inviscid analysis), and
- (d) vortices of equal strength and opposite sign do not interact strongly even at small separation distances.

Interactions of aircraft wake vortices have also been investigated by Bilanin et al. [130] using both inviscid and viscous models. Viscous wake interactions were predicted using a transport model.

Iversen, Brandt and Pradeep [131] have studied the merging of co-rotational vortices of equal strength using flow-visualization techniques, hot-wire anemometer wind tunnel experiments and by preliminary numerical calculation of vortex merging. Their wind tunnel experiments and numerical calculations indicated that the merging distance was decreased by the effects of turbulence and viscosity from that predicted by inviscid calculations.

2.3.3 Vortex Breakdown

An abrupt and drastic change of structure in the swirling flow of trailing vortices is commonly referred to as 'vortex breakdown' or 'vortex bursting'. Vortex breakdown is usually characterized by the formation of an internal stagnation point on the vortex axis followed by reversed flow in a region of limited axial extent. There are predominately two forms of vortex

breakdown, a "near-axisymmetric" or "bubble-like" and a "spiral" form, for photographs of these see Leibovitch [132, Figure 1] and Figure 28. Introducing a dye filament on the vortex axis, the spiral form is marked by a kink in the filament followed by a corkscrew-shaped twisting of the dye, whilst the bubble form has a nearly axisymmetric envelope resembling a solid body of revolution. Vortex breakdown has been observed only in highly swirling flows; defining the angle of swirl ϕ as $\tan^{-1}(v/w)$ where v and w are the swirl and axial components of velocity respectively, the maximum value ϕ upstream of breakdown is usually greater than 40° .

Another condition for breakdown is the existence of a positive or adverse pressure gradient in the axial direction, this need not appear in the outer part of the core away from the axis. Related to adverse pressure gradient is a divergence of the stream tubes in the vortex core immediately upstream of breakdown. The occurrence of breakdown depends on a balance between the magnitude of the swirl, the external pressure gradient and the degree of divergence of the flow. Variations in Reynolds number appear to have little influence on the position and occurrence of breakdown.

Vortex-breakdown flow fields may be divided into three regions:-

- (a) The approach flow - this consists of a concentrated vortex core embedded in an approximately irrotational flow. The flow is either laminar or has relatively low turbulent intensities.

(b) The breakdown region - where rapid changes in axial direction occur, occupying an axial interval of the order of five vortex-core diameters in length. This interval may be divided into three roughly equal parts as follows:-

- 1) deceleration of the approach flow and formation of stagnation point on the vortex axis,
- 2) the occurrence of flow reversal near the axis; for bubble-like breakdowns the radius is nearly equal to that of the vortex core, and
- 3) the restoration of the original direction of axial flow which is marked by a large increase in turbulent intensity.

(c) The establishment downstream of the breakdown zone of a new vortex structure with an expanded core where axial variations are gradual as in its upstream counterpart.

Vortex breakdown was first discovered by Peckham and Atkinson [133] over wings with highly-swept leading edges when the wings were set at large angles of incidence. Vortex breakdown has been reviewed by Hall [134] and Leibovitch [132]

The explanation of vortex breakdown may be divided into three categories, the basic ideas being as follows:-

- (1) The phenomenon is in some sense like the separation of a two-dimensional boundary layer, for example, Gartshore [135], Hall [136], Bossel [137], Mager [138] and others.
- (2) The phenomenon is a consequence of hydrodynamic instability, for example, Ludwig [139, 140].

- (3) The phenomenon depends in an essential way on the existence of a critical state, for example, Squire [141], Benjamin [142, 143], Bossel [144, 145], Leibovitch [146], Randall and Leibovitch [147] and others.

In the first case it was argued, as for two-dimensional boundary layers, that if at some location in the course of the calculation of a quasi-cylindrical vortex core, the results show appreciable rather than small axial gradients there must also be appreciable axial gradients at that location in the corresponding real vortex core, even though the quasi-cylindrical approximation must fail there. This occurrence of appreciable axial gradients or the failure of the quasi-cylindrical approximation was likened to vortex breakdown. However, this view of vortex breakdown leaves too much unexplained; for example, there is no explanation as to the abruptness of the phenomenon, which is the essence of breakdown; upstream influences other than those that merely modify the boundary conditions along the outside edge of the vortex core are not accounted for, and no description can be given of the flow field at or downstream of breakdown.

Ludwig's [139, 140] proposal was that vortex breakdown, with a local stagnation of the axial flow, is a direct consequence of hydrodynamic instability with respect to spiral disturbances. His suggestion [140] was that after the onset of instability, which is not necessarily breakdown, the spiral disturbances might in suitable circumstances be amplified and induce an asymmetry in the vortex core, which could lead to stagnation when there was a total pressure defect in the core.

Kirkpatrick [148], Hummel [149], and Sarpkaya [150] provided some experimental support for Ludwig's [139, 140] proposal. Some explanation is needed as to how asymmetric stagnation develops from weak spiral disturbances for vortex cores that are otherwise symmetrical and not in such a state that stagnation would occur anyway in the solution of the axisymmetric equations of motion. Associated with breakdown are such a range of velocity gradients that almost inevitably there will be instabilities somewhere thus making Ludwig's explanation difficult to test.

Squire [141] looked for the existence of standing waves in the field of flow, and stated that if they existed disturbances which are generally present downstream will spread forward along the vortex and cause breakdown. For three different forms of swirl distribution he found that standing waves could be supported when the maximum swirl velocity is "rather larger" than the axial velocity, this he proposed as the criterion for vortex breakdown. However, observations of breakdowns in adverse pressure gradients with maximum swirl velocities less than the axial velocity have been made by Sarpkaya [150]. Little explanation was given by Squire [141] as to how an upstream influence could lead to the observed abrupt change in core structure. Vortex breakdown was, according to Benjamin [142, 143], "a transition between two steady states of axisymmetric swirling flow, being much the same in principle as the hydraulic jump in open-channel flow", the transition being from a supercritical flow not able to support standing waves to a subcritical flow which could support them.

Certainly vortex breakdown is like a hydraulic jump, being an abrupt change, the occurrence and position of which is influenced by conditions downstream.

Benjamin's [142, 143] finite-transition theory did not provide a jump condition allowing the (subcritical) wake structure to be inferred from prescribed (supercritical) approach flows. No prediction of the occurrence or position of breakdown is made.

Finite-amplitude waves of a type predicted by Benjamin have been observed propagating in a long cylindrical tube by Pritchard [151], these were solitary waves in which the fluid was displaced inwards slightly, corresponding to a small local acceleration of the flow and not the pronounced retardation of breakdown. A small-perturbation solution for a solitary wave with an outward displacement of the fluid has been obtained by Leibovitch [146].

Vortex breakdown was proposed by Bossel [145] as being a necessary feature of supercritical flows having high swirl close to the critical state, and some axial flow retardation at and near the axis. His own examples [144] showed that breakdown does not necessarily occur in supercritical flows that are close to critical; he does demonstrate, however, that an abrupt change in flow structure is consistent with the critical or near-critical state.

Randall and Leibovitch [147] showed that the propagation of finite-amplitude solitary waves was governed by a modified Korteweg-de-Vries equation, (the K-dV equation with an additional

term that is proportional to wave amplitude), with coefficients that vary slowly with axial location and a term that leads either to wave amplification or damping. Energy is transferred from the basic flow to the wave through deceleration of the basic flow, acceleration causing energy transfer in the opposite direction. A stationary-wave equilibrium in which energy gained from the basic flow is balanced by viscous dissipation exists only between two well-defined axial locations in decelerated portions of the flow (adverse pressure gradient); to persist, these waves must remain "trapped" in this permissible zone. On applying the theory to the flow in which Sarpkaya [150] found a bubble form of breakdown, the trapped-wave amplitude was found to be necessarily large, resulting in formation of a stagnation point and reversed flow; the bubble shape and size were consistent with Sarpkaya's observations. Randall and Leibovitch [147] pointed out some deficiencies in their theory, namely that reversed swirls predicted in the bubble interior are physically impossible, and that although the theory has a rational validity for weakly non-linear disturbances, the disturbances predicted are necessarily large.

A general theory for breakdown was presented by Landahl [152], who used kinematic wave theory to determine under what conditions breakdown of a steady or unsteady laminar flow into high frequency oscillations should occur.

Vortex breakdown behind straight high aspect ratio wings has been observed in towing tank experiments by Olsen [56], Harvey [119], Whithycombe [153] and Lambourne and Bryer [154].

The problems encountered in measuring vortex breakdown flows were described by Harvey [155]; measurements in the vicinity of the vortex breakdown were difficult to make owing to the random variation in the position of the breakdown, and the introduction of measuring probes near the breakdown can completely change the flow as vortex breakdown is highly sensitive to external disturbances. Thus data in the immediate neighbourhood of the breakdown was limited to that obtainable by flow-visualization techniques i.e. swirl-angle distributions in the approach flow - determined by the ratio of azimuthal to axial velocity components (Harvey [155], Sarpkaya [150]), and the centre-line axial-velocity component just upstream of breakdown (Sarpkaya [150]).

The maximum swirl angle obtained by Harvey [155] agreed well with Squire's [141] prediction for the value at which breakdown occurs. Harvey [155] also observed that breakdown is a division between subcritical and supercritical regimes, rather than an onset of stability; this was supported by the theoretical analysis of Benjamin [142]. (The vortex is supercritical when the axial velocity is larger than the swirl, upstream propagation of linear waves is not possible; the vortex is subcritical when the axial velocity is less than the swirl, information can be transmitted in either direction along a vortex). More recently measurements have been made by Falster [156], Falster and Leibovitch [157] and Garg [158]; their results include power spectra and a velocity map of the recirculation region of the bubble type of breakdown.

Flight observations, using an HP115 aircraft, of vortex breakdown by Fennell [159] showed similar characteristics to vortex breakdown observed on models in wind and water tunnels.

2.3.3.1 Vortex Deintensification as a Result of Breakdown

From numerous observations it is clear that breakdown on aircraft trailing vortices can be violently turbulent. Resulting from these observations is the question as to whether the phenomenon aids in lessening the hazard of an aircraft vortex wake. Bilanin, Teske and Hirsch [160] undertook a numerical approach to define the breakdown flowfield. Their calculations showed that the large scale structure of the vortex was not significantly altered by the breakdown phenomenon. The breakdown phenomenon does disrupt the viscous core in a vortex resulting in high levels of turbulence; however, this turbulence tends to destroy the smoke from flow visualization tests, leaving the impression that the vortex has in some way been destroyed. Landahl and Widnall [161] used energy arguments to clarify the phenomenon of vortex bursting and to draw conclusions as to what could be done to prevent or induce bursting.

2.4 Solitary Waves

It is known that small amplitude wave motions can be supported in rapidly rotating fluids, see Greenspan [162, chapter 4], and it is thought by some (see previous section) that solitary waves exist on vortices.

One of the earliest wave theories of vortex breakdown was due to Squire [141] ; his 'criterion' (see section 2.3.3) described flow conditions in the core that are 'critical' in the sense that infinitesimal standing waves of infinite length can be supported by the rotation. Since rotating flows are dispersive media with phase velocity increasing with wave length, the first possible waves to occur for a given axial flow are standing waves of extreme length.

It was shown by Benjamin [143] , in his important paper, that rotating fluids can also support long, axially symmetric, stationary waves of finite amplitude. Further to this, Leibovitch [163] developed a theory of long, non-linear, stationary waves in incompressible swirling flows and extended this, Leibovitch [168], to the case of compressible flows; the non-linear wave theory allows treatment of either axially symmetric or spiral wave modes of vortex breakdown. The Korteweg-de-Vries (K-dV) equation describes the long-time evolution of small but finite amplitude non-linear waves, and it is found that waves are possible with the waveforms of both axially-symmetric and spiral

modes determined by the K-dV equation

$$u_t + \alpha u u_x + \beta u_{xxx} = 0.$$

This nonlinear partial differential equation is one of the simplest model equations in the study of nonlinear dispersive wave phenomena; the nonlinear and dispersive terms are $\alpha u u_x$ and βu_{xxx} respectively.

2.4.1 Derivation of Korteweg-de-Vries equation

Wave motions can survive for long periods and can transmit disturbances over very long distances; they often assume their most distinctive forms after travelling a "large" distance from the region in which they are generated. The equation is derived using the "method of multiple scales"; this method is expected to be asymptotically correct for large times as ϵ , the measure of amplitude, Re^{-1} , dimensionless parameter equal to the inverse Reynolds number, and δ , dispersive parameter, all tend to zero. The method of multiple scales is used in this instance systematically to reduce a system of equations depending upon several small parameters to an equation for a single unknown function.

Systems of nonlinear equations depending upon one or more small parameters that permit nondispersive, nondissipative wave motions when all of the small parameters vanish are considered, where time, t , and the single spatial coordinate, x , are independent

variables. It is assumed that the system may be expressed in the following form :

$$\underline{u}_t + \underline{C}(\underline{u}) \cdot \underline{u}_x = \sum_{k=1}^m \mu_k \mathcal{D}_k(\underline{u}, x, t) \quad (2.4,1)$$

where $\underline{u}$ is the column vector, $\underline{u} = (u_1 \dots u_n)^T$,

$\underline{C}$ is an $N \times N$ matrix that may depend on $\underline{u}$, μ_k are small parameters and $\mathcal{D}_k$ are vector-valued operators on $\underline{u}$ that depend on $\underline{u}$, x and t .

The motion is assumed to be a small disturbance, measured by the parameter ε , to a constant state $\underline{u}_c$ ($\mathcal{D}_k(\underline{u}_c, x, t) = 0$, i.e. $\underline{u}_c$ is assumed to be a solution); an expansion about $\underline{u}_c$ in the small parameters is attempted in the form

$$\underline{u} = \underline{u}_c + \varepsilon \{ \underline{u}_0 + \varepsilon \underline{u}_1 + \dots + \sum_{j=1}^m \mu_j \underline{u}_{j,1} + \dots \} \quad (2.4,2)$$

where omitted terms are proportional to quadratic and higher powers of ε and μ_k .

Substituting equation (2.4,2) into equation (2.4,1) and neglecting higher-order terms

$$\underline{u}_{0,t} + \underline{C}(\underline{u}_c) \cdot \underline{u}_{0,x} = 0 \quad (2.4,3)$$

The matrix $\underline{C}$ is assumed to have the following expansion in order that equation (2.4,1) allows wave motion as $\varepsilon \rightarrow 0$:

$$\underline{C}(\underline{u}_c + \varepsilon \{ \underline{u}_0 + \dots \}) = \underline{C}_0 + \varepsilon \underline{C}_1(\underline{u}_0, \underline{u}_c) + \dots$$

where $\underline{C}_0 = \underline{C}(\underline{u}_c)$ is a matrix with constant coefficients that has N real and distinct eigenvalues

$$\text{i.e. } \underline{C}_0 \cdot \underline{z}_i = \lambda_i \underline{z}_i, \quad (2.4,4)$$

where $\underline{z}_i$ are the right (column) eigenvectors corresponding to the λ_i .

The solution to (2.4,3) is given by

$$u_0 = \gamma_i U(x - \lambda_i t) \quad (2.4,5)$$

where γ_i and λ_i satisfy equation (2.4,4).

In view of equation (2.4,4), equation (2.4,5) is a solution for arbitrary functions $U(X_i)$ and for $N = 2$ is the elementary solution of the linearized wave equation. To proceed with higher-order corrections attention is focussed upon one of the N solutions (2.4,5); assuming one has been selected the subscripts on the selected pair are omitted, thus (λ, v) . By dealing with the N eigenvalues one at a time, attention is confined to the asymptotic behaviour of a single mode for large time or to initial data close to the travelling wave (2.4,5). In the latter case, the initial data should, to lowest order, depend only on $x - \lambda t$; this restriction applies to most applications of multiple scale methods to partial differential equations.

Substituting the series (2.4,2) into (2.4,1) and equating coefficients of $\varepsilon, \mu_1, \dots, \mu_m$ the higher-order terms are found. With $D_k(u_c, x, t) = 0$, D_k is assumed to be such that $D_k(u_c + \varepsilon u_0, x, t) = \varepsilon D_k(u_c, u_0, x, t) + \dots$.

Thus the equations are

$$u_{1t} + \zeta_0 \cdot u_{1x} = -\zeta_1(u_0, u_c) \cdot u_{0x} \quad (2.4,6a)$$

$$\text{and} \quad u_{jt} + \zeta_0 \cdot u_{jx} = D_{j-1}(u_c, u_0, x, t), \quad j = 2, \dots, m+1 \quad (2.4,6b)$$

Each of the vector equations (2.4,6) may represent a solvable problem for the N associated functions. Since (2.4,5) was not generalised to account for multiple time-scaling, the solutions are not valid unless $t \ll \mu^{-1}$ where $\mu = \max\{\varepsilon, \mu_1, \dots, \mu_m\}$,

for details see Leibovitch and Seebass [164, p.109].

For each small paramter a slow time is defined

$$\left. \begin{aligned} \tau_1 &= \varepsilon t \\ \tau_{k+1} &= \mu_k t \quad k = 1, \dots, m \end{aligned} \right\} \quad (2.4,7a)$$

and $\tau_0 = t$ is written for the fast time. (The essence of the method of multiple scales is the formal introduction of this notion with u assumed to be a function of three independent variables, x , t and $\tau = \varepsilon t$, that is, x and two time variables :

$$u(x, t; \varepsilon) = U(x, t, \tau; \varepsilon)$$

The method "works" when a regular perturbation series in ε for U , valid to times of order ε^{-1} may be found by appropriate choices of the τ dependence of U).

After applying the transformation $X = x - \lambda t$

$$\frac{\partial}{\partial t} = \frac{\partial}{\partial \tau_0} + \varepsilon \frac{\partial}{\partial \tau_1} + \sum_{k=1}^m \mu_k \frac{\partial}{\partial \tau_{k+1}} \quad (2.4,7b)$$

The arbitrary function U is explicitly independent of τ_0 but may now depend upon the τ_j , $j \geq 1$. Equations (2.4,6) transformed to coordinates $(X, \tau_0, \tau_1, \tau_{k+1})$, $k = 1, \dots, m$, are

$$u_{1\tau_0} + (\underline{C}_0 - \lambda \underline{I}) \cdot u_{1X} = -\underline{C}_1 \cdot u_{0X} - u_{0\tau_1}$$

and

$$u_{(k+1)\tau_0} + (\underline{C}_0 - \lambda \underline{I}) \cdot u_{(k+1)X} = \underline{D}_k(u_c, u_0, x, t).$$

Since $U(X, \tau_1, \dots, \tau_m)$ is arbitrary, U is chosen so that it is governed by

$$\left. \begin{aligned} \underline{L} \cdot \underline{C}_1 \cdot \underline{U}_X + \underline{L} \cdot \underline{U}_{\tau_1} &= 0 \\ \underline{L} \cdot \underline{D}_k - \underline{L} \cdot \underline{U}_{\tau_{k+1}} &= 0, \quad k = 1, \dots, m \end{aligned} \right\}, \quad (2.4,8)$$

substituting for u_0 from equation (2.4,5).

Since the τ_k are related to t , a single equation governing the time evolution of U may be "reconstituted". Using equation (2.4,7b) the reconstituted equation is

$$\underline{\ell} \cdot \underline{x} U_t + \varepsilon \underline{\ell} \cdot \underline{C}_1 \cdot \underline{x} U_x = \underline{\ell} \cdot \left[\sum_{k=1}^m \mu_k \underline{D}_k(\underline{x} U, X, t) \right]$$

whose solution renders u_0 valid through time $O\left(\frac{1}{\varepsilon} + \frac{1}{\mu_1} + \dots + \frac{1}{\mu_m}\right)$ (assuming that all linear quantities in ε or μ_i are larger than any quadratic combination of these parameters).

Shallow water waves are governed by a set of two equations (equivalent to the approximation of Boussinesq) for which

$$\underline{C}_0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \underline{C}_1(\underline{u}) = \begin{pmatrix} u_2 & u_1 \\ 0 & u_2 \end{pmatrix}.$$

The equations depend upon ε and a single small parameter μ_1 , representing the circumstances of the propagation and the operator $\underline{D}_1$ is

$$\underline{D}_1(\underline{u}) = \begin{pmatrix} \frac{1}{3} u_{2xxx} \\ u_{2xxt} \end{pmatrix}$$

The eigenvalues of $\underline{C}_0$ are $\lambda = 1$, $\lambda = -1$ (since $|\underline{C}_0 - \lambda I| = 0 \Rightarrow (\lambda^2 - 1) = 0$), corresponding to the right column vectors $(1, 1)^T$ and $(1, -1)^T$ and to the left (row) eigenvectors $(1, 1)$ and $(1, -1)$. Thus, selecting $\lambda = 1$ with $\underline{\ell} \cdot \underline{x} = 2$, $\underline{\ell} \cdot \underline{D} = -\frac{2}{3} U_{xxx}$ and $\underline{\ell} \cdot \underline{C} \cdot \underline{x} = 3U$, the appropriate form of equations (2.4,8) are

$$U_{\tau_1} + \frac{3}{2} U U_x = 0$$

$$\text{and} \quad U_{\tau_2} + \frac{1}{3} U_{xxx} = 0$$

and the reconstituted equation is the Korteweg-de-Vries equation

$$U_t + \frac{3}{2} \varepsilon U U_x + \frac{1}{3} \mu_1 U_{xxx} = 0 \quad (2.4,9)$$

2.4.2. Stationary Wave Solutions of the Korteweg-de-Vries Equation

The Korteweg-de-Vries equation (2.4,9) may be re-written (see Leibovitch and Seebass (164, chapter IV)) as

$$U_T + UU_X = \delta U_{XXX} \quad (2.4,10)$$

where $\delta = -\frac{1}{9} \frac{\mu^2}{\epsilon}$, $\mu = \frac{h}{l}$, and $T = \frac{3}{2} \epsilon \tau$

for the case of shallow water waves, h = depth and l is the length of the wave.

Stationary waves - waves that propagate without change of form at a constant speed - may be found by putting

$$\tilde{X} = X - \mathcal{L}T \quad (2.4,11)$$

where $\mathcal{L}$ is the unknown wavespeed.

Substituting equation (2.4,11) into equation (2.4,10) gives

$$\delta U''' = (U - \mathcal{L})U' \quad (2.4,12)$$

an ordinary differential equation where the prime indicates differentiation with respect to $\tilde{X}$.

An integration of equation (2.4,12) yields

$$\delta U'' = \frac{1}{2} U^2 - \mathcal{L}U + \frac{1}{2} c_1 \quad (2.4,13)$$

where c_1 is an arbitrary constant of integration.

Multiplying equation (2.4,13) by U' and integrating again gives

$$(U')^2 = \delta^{-1} \left[\frac{1}{3} U^3 - \mathcal{L}U^2 + c_1 U + c_2 \right] \quad (2.4,14)$$

Real solutions exist only for U such that

$$\delta^{-1}P(U) = \delta^{-1}\left[\frac{1}{3}U^3 - \frac{2}{3}U^2 + c_1U + c_2\right] \geq 0 \quad (2.4,15)$$

The polynomial (2.4,15) may have either one or three real roots, however, no bounded solutions exist for the case of a single real root; thus the polynomial is assumed to have three real roots $U_1 \leq U_2 \leq U_3$ and $P = \frac{1}{3}(U - U_1)(U - U_2)(U - U_3)$. Figure 29 shows three possibilities for the roots of the polynomial: Case A - three distinct roots: Case B - roots U_2 and U_3 coincident and Case C - roots U_1 and U_2 coincident. Solutions are possible for those portions of the curves A, B, C, for which P is non-negative assuming $\delta > 0$. For the case of double roots of the polynomial, which in Figure 29 correspond to values of U at either 'initial' or 'terminal' points of an integral curve of equation (2.4,14), $U' = 0$ and $U'' = 0$ at those points and from equation (2.4,12) all derivatives vanish. Therefore, the constant value of U at a double root is an exact integral of equation (2.4,12). A non-constant solution thus tends towards a double root only asymptotically as $\tilde{X} \rightarrow \infty$ or $\tilde{X} \rightarrow -\infty$.

Consider a point, U_p , near a single zero of a curve such as A, suppose $U'(U_p) > 0$ (see Figure 29); since $U' > 0$ in (U_p, U_2) , U increases as $\tilde{X}$ increases; however, P has a finite negative slope in this portion of the curve, thus $U'' < 0$ and so U' must decrease. At U_2 , U' has decreased to zero, but U'' remains negative; U' therefore becomes negative as $\tilde{X}$ increases and the solution "reflects" from this point retracing its path on

curve A. Thus curve A, with a positive portion lying between two points where P vanishes with a non-vanishing slope, represents a solution of $U(\tilde{X})$ that oscillates between U_1 and U_2 .

Explicitly, the oscillation in terms of the Jacobian elliptic function cn is

$$U = U_2 - (U_2 - U_1)cn^2 \left[\left(\frac{U_3 - U_1}{12\delta} \right)^{1/2} \tilde{X} | m \right] \quad (2.4,16)$$

where $m = \frac{U_2 - U_1}{U_3 - U_1}$; $cn(u|m)$ is defined in Abramowitz and Stegun [165, sections 6.1 and 7.2], $cn^2(u|m)$ oscillates between zero and unity with a period of $2K(m)$, $K(m)$ being the complete elliptic integral of the first kind. (For $m = 0$, $cn(u|0) = \cos u$, for $m = 1$, $cn(u|1) = \text{sech } u$).

The infinite wavetrain equation (2.4,16) is called a cnoidal wave, having a wavelength $2\sqrt{12\delta/(U_3 - U_1)}K(m)$; the amplitude $\tilde{a}$, of the oscillation about the mean value $\bar{U} = (U_1 + U_2)/2$ is $\tilde{a} = (U_2 - U_1)/2$. For waves of small amplitude, sinusoidal oscillations are recovered, since as $m \rightarrow 0$

$$U \sim \bar{U} - \tilde{a} \cos \left(2\tilde{X} \left(\frac{U_3 - U_1}{12\delta} \right)^{1/2} \right)$$

Consider case B of Figure 29 and a solution approaching U_3 as $\tilde{X} \rightarrow -\infty$. As the solution traces curve B from U_3 to U_1 , it decreases to U_1 , where it undergoes a "reflection" returning along B to U_3 as $\tilde{X} \rightarrow \infty$. Plotted against $\tilde{X}$ the solution

resembles a single, symmetrical valley (or if $\delta < 0$, a peak); the wave is of infinite length and is called a solitary wave. It is given by

$$U = U_3 - (U_3 - U_1) \operatorname{sech}^2 \left(\tilde{X} \left(\frac{a}{12\delta} \right)^{1/2} \right) \quad (2.4, 17)$$

The amplitude of the solitary wave peak (or dip) above (or below) U_0 is given by $a = (U_3 - U_1)$.

Case C has sometimes been said to lead to the hydraulic jump - a discontinuous disturbance consisting of two joined semi-infinite regions in which U takes different piecewise constant values. The only solution possible in case C is the single constant value $U = U_1$.

Only two nonconstant waves of permanent form exist, the cnoidal waves of case A and the solitary wave of case B. The wave speed $\mathcal{L}$ is given by $\mathcal{L} = \frac{1}{3}(U_1 + U_2 + U_3)$.

Substituting for $\tilde{X}$ and generalizing for both positive and negative values of δ the final formulae for cnoidal and solitary waves are

$$U = \bar{U} + \frac{\delta}{|\delta|} a_1 \left\{ 1 - \operatorname{cn}^2 \left[\left(\frac{a_1}{6m|\delta|} \right)^{1/2} \left(x - \left[\bar{U} + \frac{\delta}{|\delta|} \frac{2-m}{3m} a_1 \right] \tau \right) | m \right] \right\} \quad (2.4, 18)$$

and

$$U = U_0 - \frac{a\delta}{|\delta|} \operatorname{sech}^2 \left[\left(\frac{a}{12|\delta|} \right)^{1/2} \left(X + \left[\frac{1}{3} a \frac{\delta}{|\delta|} - U_0 \right] \tau \right) \right] \quad (2.4, 19)$$

respectively. In equation (2.4,18) U_1 , U_2 and U_3 are replaced by $\bar{U}$ (the mean level of the oscillation), a_1 (the mean-to-peak and mean-to-trough amplitude) and m (taking values from zero to unity). In equation (2.4,19) U_1 and U_2 are similarly replaced by the undisturbed value, U_0 .

Both the wave speeds and wave shapes depend upon the amplitude of the disturbance and the background state upon which the wave propagates. The appearance of the amplitude is a feature due to nonlinearity. Generally, nonlinear waves cannot be superposed.

2.4.3. Solitary Wave Solutions of the Korteweg-de-Vries Equation

The Korteweg-de-Vries equation is of the form

$$u_t + \alpha u u_x + \beta u_{xxx} = 0$$

Choosing $\alpha = -6$ and $\beta = 1$ it becomes

$$u_t - 6u u_x + u_{xxx} = 0 \tag{2.4,20}$$

and the solitary waves have the form

$$u(x, t) = -\frac{1}{2} a_0^2 \operatorname{sech}^2 \left[\frac{1}{2} a_0 (x - a_0^2 t) \right] \tag{2.4,21}$$

Here the solitary waves propagate to the right at a speed a_0^2 which is proportional to the amplitude and have a width that is inversely proportional to the square root of the amplitude. Solitary wave solutions have a 'linear-like' behaviour; Zabusky and Kruskal [167] observed that two distinct solitary waves, i.e. with distinct amplitudes, interact nonlinearly but emerge from the interaction unchanged.

This particle-like behaviour led to the use of the word 'solitons' for solitary waves. These (nonlinear) solitons may be distinguished from linear solutions in that they are shifted in their positions relative to where they would have been if no interaction had occurred.

It has been shown numerically, Zabusky [167], that in addition to (or instead of) solitons propagating to the right, some solutions will have a dispersing oscillatory state propagating to the left with solitons propagating to the right. The propagation of the oscillatory state to the left is due to the negative group velocity of the linear waves.

Leibovitch [168] argued that the solitary wave provided the basis for a very adequate theory for the phenomenon of vortex breakdown. His proposal, in brief, was as follows:-

- 1) Vortex breakdown is a solitary wave which may carry axial velocity disturbances sufficiently large to cause stagnation of the flow; it is localized in space for long periods of time.
- 2) The wave is caused by a downstream disturbance e.g. the trailing edge of a wing.
- 3) Since breakdown is, by definition, localized in space, the lowest order axial wave speed must vanish, and
- 4) A breakdown is the largest amplitude solitary wave that can be caused by the downstream source of disturbance.

His reasons for this wave theory were as follows:-

- (a) Breakdowns seem to be closely related to downstream conditions, and move up from downstream positions as the flow is started.

- (b) The occurrence of breakdowns is very sensitive to the condition of the primary flow, and occurs when it is nearly critical.
- (c) Viscous effects appear to be only secondary.
- (d) A second breakdown, definitely smaller than the main breakdown and downstream from it was observed in some of Harvey's [155] photographs. From wave theory it would be expected that the main and secondary breakdowns would slowly part company. These secondary breakdowns may be explained as smaller solitons.
- (e) Frequently, unsteady regions of flow appear immediately in the rear of the main breakdown. This feature may possibly be explained as the oscillating tail in the Korteweg-de-Vries solution.

Figures 30-32 show the flow patterns for the decaying vortex as calculated by Leibovitch [168] for values of the (single) free parameter ϵ of 0.8 to 1.0. (ϵ represents one-half of the maximum axial disturbance value). These particular choices of ϵ were made to facilitate a comparison with the breakdown regions observed by Harvey [155], they are too large to expect any quantitative accuracy from the theory for ϵ is not small compared to unity. Nevertheless, the results seem to be in accord with the few available experimental results. The value $\epsilon = 1.0$ agrees well with Harvey's [155] observations, coming close to the observations of both length and maximum radial extent of the eddy. The general features of increasing ϵ are shown in Figures 30a and 30b, as ϵ increases the closed eddy grows and becomes relatively fatter. Figure 31 shows the total axial velocity at the midplane, again for $\epsilon = 0.8$ and 1.0. Figure 32 shows the base flow and

perturbed swirl distributions in a plane through the midsection of the eddy; the streamline dividing the eddy from the outer flow is a surface of zero swirl, a consequence of the conservation of angular momentum.

Further detailed measurements are needed of the actual flow velocities that occur within the region of breakdown, not only to test existing theories of axisymmetric breakdowns but to lead to a better understanding in particular of spiral breakdowns.

2.5 Wake Vortex Minimization

Many proposals have been put forward for "getting rid" of the vortex hazard. The severity of the hazard is dependent on the size, geometry and operating conditions of the generator and following aircraft, the distance between the two aircraft, the angle and altitude of encounter, the atmospheric conditions existing at the time and possibly other unknown factors. A fairly comprehensive review may be found in the proceedings of the symposium on wake vortex minimization [170]. Some of the schemes for reducing the hazard are as follows:-

- 1) The addition of axial velocity to the core of the vortex - for example, mass injection at wing-tips: see references [170-173, 24, 50],
- 2) The depletion of axial velocity in the core of the vortex - for example, vortex dissipator: see references [174-178],
- 3) The addition of vorticity opposing that of the original vortex or tip-load - see, for example, references [179-180],
- 4) The introduction or increase of turbulence into the vortex - this includes the use of spoilers at locations other than the tip: see, for example, references [181-182], and
- 5) Span loading variable with time: see, for example, references [115, 183].

With the first two methods when an inherent destabilizing feature, a jet or wake, is introduced into the vortex, turbulence levels are enhanced; thus method 4 is also contained

in these methods. The spoiler, in method 4, besides introducing turbulence, may have a major effect in changing the wing load distribution, so altering the mean structure of the wake more than the introduction of "turbulence". The time-varying span load is used in the last method to excite the Crow [98] instability and can be effective in causing vortex linking and subsequently very rapid diffusion and/or annihilation of vorticity for the case of clean wings. In the case of flapped wings it is not yet clear whether the linking mechanism is effective. However, in Chevalier's flight tests [115] in which the angle of attack of a light aircraft (a Beaver DHC-2) was periodically oscillated in order to sinusoidally perturb the vertical positions of the trailing vortices, thus inducing the Crow instability, vortex breakdown, which was made visible by smoke, was observed to occur regularly along the trailing vortices. The quantitative deintensification of the wake as a result of breakdown was not known, the pilot in the chase aircraft did report that the wake was reduced to a mild random atmospheric turbulence. An alternative method of provoking instability suggested by Crow [184] and tested by Bilanin and Widnall [183] in a towing tank, using a model wing, was the differential oscillation of two pairs of flaps such that lift was held constant while the centroid of trailed vorticity moved periodically inboard and outboard. This was also seen to excite the Crow instability and the occurrence of vortex breakdown was also observed.

Numerical studies by Bilanin, Teske and Hirsh [185] of two-dimensional aircraft trailing vortex pairs descending in a

neutral atmosphere have shown that dispersion is enhanced when the atmosphere is turbulent, when crosswind shears are present and when the vortices interact with the ground.

The detailed behaviour of the wake of a wing is very sensitive to the lift and drag distributions on the wing, and to any secondary vortices that might be shed by mounting struts or other surfaces generating lift or drag in the vicinity of the wing. The wake behaviour observed behind an aircraft may be classified on the basis of inviscid considerations i.e. whether the vortices remain together and interact strongly or whether they move apart and interact less strongly. However, for precise details it is important to consider the viscous interactions in the wake.

Further details of wake vortex minimization may be found in Donaldson and Bilanin [9].

CHAPTER 3 - THE EXPERIMENT

Vortices have been studied for many years, however much of their behaviour is still poorly understood. Recently, since the advent of the wide-bodied heavy airliner, much attention has been focussed on the formation and decay of aircraft trailing vortices. As these vortices can be strong and persistent enough to pose a hazard to following aircraft, a knowledge of the structure and mechanisms by which vortex wakes are dissipated is clearly desirable. It is thought that structural instabilities, either in the form of periodic variations in the separation of the cores, i.e. the Crow instability [98] discussed previously in section 2.3, or in the appearance of "disc-shaped" disturbances which are thought to be a manifestation of "vortex breakdown", are mechanisms by which the decay of these vortices is accelerated.

These trailing vortices are produced by the rolling-up of the vortex sheets shed from the wing. The early stage in the development of the vortices may be investigated by means of a wind tunnel study; however, the long term behaviour is much less amenable to investigation. Firstly, since the vortices are swiftly carried downstream, a wind tunnel with a long test section is needed in order to study anything but the youngest of vortices. Secondly, any phenomena associated with the vortex near the wing is likely to be stationary with respect to the wing, whereas in the far field developing features may move with a velocity characterised by the fluid surrounding the vortex. Thus, in a wind tunnel this feature will be rapidly swept past the measuring instruments, resulting in problems of interpretation.

By using a water towing tank Harvey and Fackrell [186] were able to avoid these difficulties as the vortices could be studied in a frame of reference stationary with respect to the undisturbed fluid, thus enabling the total history of the vortices to be observed. In their experiment a rectangular planform wing of 2' span and 4" chord was supported rigidly from a towing trolley and towed at two speeds (2' and 4'/sec); any fluctuation in speed was considered to be no more than the equivalent unsteadiness of the flow in a good wind tunnel. Using flow visualisation (a saturated solution of potassium permanganate mixed with acetone until the mixture had the same density as that of the water in the towing tank was used), observations were made of an instability of the flow which led to a rapid destruction of the vortex. This breakdown phenomenon was unlike the usual "vortex breakdown" as it did not emanate from the vortex axis but was confined to the flow outside the core. The breakdown resulted in the formation of toroidal vortices; the association between toroidal vortices and the Rayleigh instability criterion [199] is well established but the mechanism leading to their formation has remained obscure. From the observations made, for details see Harvey [119], some insight into the phenomenon was obtained and a possible explanation is offered in Figure 25.

The study of such a phenomenon would first require a method of detection which could then be used to establish a conditional sampling technique to determine the structure of any phenomenon. As this phenomenon would, in a wind tunnel context, be convected past the observer at the speed, U_∞ , of the surrounding fluid, a wind tunnel with a long working section was needed in order to examine the structure of the phenomenon.

3.1 The Wind Tunnel

The measurements were made in the 4.5' x 4' low speed wind tunnel at Imperial College; the rectangular working section may be arranged either as a short one of 9.78' or a longer one of 28', see Figure 33. To create a longer working section three sections of the diffuser immediately following the test area are removed, the test area is moved to the position of the third section of the diffuser and three rectangular sections, having the same cross-sectional dimensions as that of the test area, are placed in front of the test area. These three rectangular sections are thus placed between the contraction and the test area giving a total working section length of 28'.

So far a detailed calibration of the longer working section has not been carried out, however preliminary measurements show the variation of pressure and velocity across the test area to be acceptable. For the longer working section the calibration factor $\frac{p_0 - p_s}{\Delta p_{ref}}$ across the width of the test area, and the relationship between the pitot-static and the reference pressures at a point 20.4' from the start of the working section are shown in Figure 34 (a and b) respectively. For details of the calibration of the shorter 9.78' working section, see Bearman, Harvey and Gardner [187]. The wind tunnel has a contraction ratio of 4.92.

A three-dimensional traverse mechanism has been installed in the test area. The traverse mechanism has been designed with a low frontal area so as to minimise the disturbance to the flowfield, it is especially suitable for the study of trailing vortices as they are very susceptible to disturbances in the flowfield. The main support arm passing across the test section is 1371 mm long, and the

length and height of the probe carriage are 95.25 and 12.7 mm respectively. This traverse has a resolution of 0.1 mm, the absolute accuracy being less than this due to deflection of the mechanism through air loads which vary from one application to another. It is estimated that probe positions quoted in this thesis are accurate to better than 0.5 mm.

3.2 Delta Wing Models

A more accurate simulation of the flow behind an aircraft is gained by the use of delta wing models rather than rectangular half-wing models; with the former a pair of trailing vortices is formed, whilst from the latter only a single vortex is formed. The delta wing models used had a chord/span ratio of 2, which modelled the centre-line chord/span ratio of a Handley Page 115 aircraft, see Table IV for the principal dimensions of an HP 115. The Handley Page 115 research aircraft was built to enable investigation of the in-flight characteristics of a slender-wing configuration at low airspeeds. The flowfield around the aircraft is generally representative of the flow over a slender wing as described in section 2.1. Flight tests investigating the far field flow behind the HP 115, see Fennell [159] and Bisgood, Maltby and Dee [97], and wind tunnel tests on a model of the HP 115, see Engler and Moss [188] have been carried out. Earnshaw [60] has made detailed measurements of the flow behind a delta wing in a low speed wind tunnel; measurements of the flow behind wings having a general resemblance to the HP 115 wing planform (but not wing section shape) have been made by Earnshaw and Lawford [189] and Elle [190].

In total, three delta wing models were used in my tests. The first delta wing, which was designed to be placed in the contraction

of the wind tunnel was made of 1" plywood with a chord of 63" and a span of 31.5" and was suspended by wires. The delta wing was placed in the contraction of the wind tunnel in order to artificially age the vortices as the longer working section had not, at that time, been constructed. The ageing of vortices in this way was only an interim measure pending the completion of the long test section and it cannot be rigorously defended. There is thus an element of uncertainty in the data gained from tests using this arrangement. The other two delta wings were designed to be used with the longer working section, one was made of 0.25" aluminium with a chord of 12.8" and a span of 6.4", whilst the other was made of 0.5" plywood having a chord of 28" and a span of 14". Both these models were mounted in a similar manner on the floor of the working section, see Figures 35 and 36. All the delta wings had a chamfer of approximately 15° on the edges of the pressure side of the model to aid the formation of the vortices.

3.3 Identification of the Signal to be used as a Trigger for Conditional Sampling

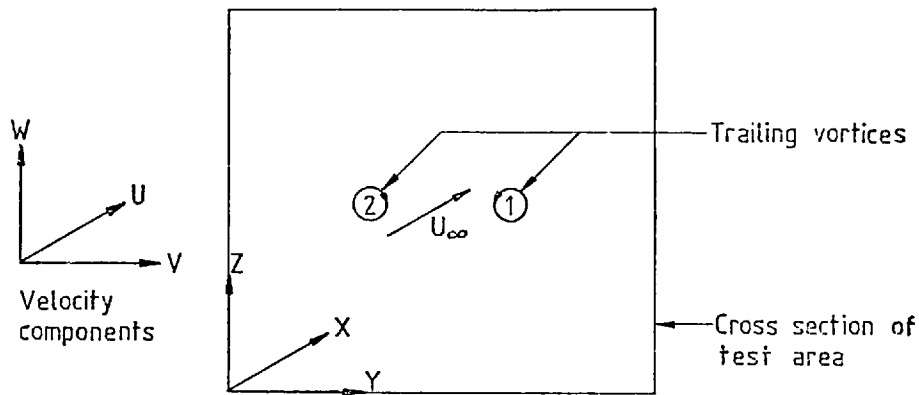
The wind tunnel was set up with the 9.78' working section; the longer, 28', working section being under construction. The 63" chord delta wing was rigidly suspended at an incidence of 15° in the contraction by means of wires. The position of the vortices in the test area of the wind tunnel was first checked by introducing paraffin smoke in front of the delta wing, and the model was adjusted until the vortices were, as near as was possible, equally spaced about the centre of the test area. Thus, all the walls of the working section had an equal effect on the flow. The centres of the vortices were found using a single hot wire anemometer (a gold-plated DISA 55 P01 wire),

which was traversed vertically and horizontally through the flow. The hot wire was mounted on a DISA two-dimensional traverse gear, for details of this traverse gear see Bradshaw [191, section 2.3-21], the measurements were made 189" (3 chord lengths) from the back edge of the delta wing.

The hot wire anemometer was connected to a DISA anemometer (type 55 D01) by a 5m cable. The mean voltage output of the DISA anemometer was measured using a Solartron DVM set on the 0-20V range, the DVM is accurate to within $\pm 0.05\%$ of the range; the rms output of the anemometer was measured with a DATRON rms meter. The centre of a vortex occurred at the highest value of $\frac{\sqrt{U^2}}{U_\infty}$, details of the calculation of $\frac{\sqrt{U^2}}{U_\infty}$ are given in Appendix E. A Kiel tube, see Figure 37, connected to a multi-tube manometer had been used prior to the hot wire to try to locate the centres of the vortices, however, as a consequence of mounting the wing ahead of the contraction, the drop in total pressure at the centres of the vortices was found to be so small that it was difficult to ascertain the exact position of the centre. The depression in total head found in the core of trailing vortices results from the action of viscosity chiefly in the formation process. Placing the delta wing ahead of the contraction of the wind tunnel means that the vortex formation occurs at much reduced velocities with commensurately small losses in total head, which in absolute terms are not significantly affected by the acceleration through the contraction. The centres of the vortices for a free stream speed of 33 msec^{-1} as taken from the static pressure tappings at either end of the contraction were as follows:-

vortex 1:	$Y = 31.1" (790 \text{ mm}),$	$Z = 26.55" (674 \text{ mm})$
vortex 2:	$Y = 21.5" (541 \text{ mm}),$	$Z = 26.9" (683 \text{ mm}).$

The co-ordinate system is sketched below.



Sketch of the co-ordinate system

Figure 38 shows $\frac{\sqrt{u'^2}}{U_\infty}$ for a horizontal traverse of the single hot wire through the vortices at $Y = 26.65''$ (676.9 mm).

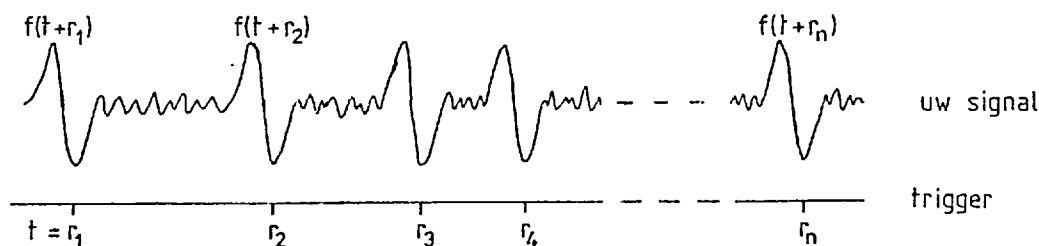
Measurements of the mean and fluctuating velocities of the flow were made with a gold-plated DISA cross-wire (55 P51) which was traversed horizontally through the centre of vortex 2, measurements being taken in both the 'U-V' and 'U-W' planes, the cross-wire was mounted on the DISA traverse gear described above; a schematic diagram of the apparatus used is shown in Figure 39. The two wires of the cross-wire were connected to DISA anemometers by 5m cables. The outputs from the anemometers were input to a switching box connected to a voltmeter, this enabled both mean voltage outputs to be read from the one voltmeter. Errors may be introduced by using two voltmeters which may not give identical readings. The temperature in the test area was recorded for each measurement taken so that variations in the mean voltages due to temperature fluctuations could be calculated using Bearman's formula, see Appendix B. The outputs from the anemometers were also recorded simultaneously on Ampex analogue 1" magnetic tape, for details of the recording technique used see

Appendix C. The data was subsequently digitized and analysed using the program described in Appendix C.

Plots of the mean local velocities $\bar{U}, \bar{W}$ and the turbulence intensities obtained by traversing the cross-wire horizontally through the centre of vortex 2, the data being analysed using the above program, are shown in Figures 40 - 41. From the plot of the mean azimuthal velocity, $\bar{W}$, the size of the core of the vortex may be estimated to be approximately 1.6". Some typical plots of the components of fluctuating velocity and their instantaneous product calculated by the conditional sampling program DIG 2W are shown in Figure 42. It can be seen from these plots, which each represent a timescale of 0.72 sec that the instantaneous velocities remain fairly constant but their product, uw or uv , appears to have large bursts at irregular intervals. The magnitude of these bursts appears to be up to 10 - 12 times that of the rest of the uv or uw signal, and varies in size according to the position of the cross-wire in the vortex. The best position for obtaining regular bursts in the uv or uw signal appeared to be that of the edge of the core of the vortex. In order to study the structure of the vortex at the occurrence of such a burst a conditional sampling technique was established.

3.4 The Conditional Sampling Technique

The 'bursting' phenomenon mentioned in the previous section was more pronounced in the trace of the uw signal. The peak of a burst in this uw trace was used as a triggering device, i.e.



In the above sketch $f(t + r_n)$ is a short repeated solitary wavelike signal, i.e. the 'bursting' phenomenon, which occurs at random times, r_n , superimposed on the 'usual' turbulent signal. A trigger derived from this signal may be used to start a data recording system each time the $f(t + r_n)$ arrives; each $f(t + r_n)$ signal is superimposed on the previous recorded $f(t + r_{n-1})$ signals using an averaging process. This technique eliminates any noise in the resultant $f(t)$ signal. The $f(t + r_n)$ signal was submerged in much noise due to the turbulent nature of the flow. It was hoped that the timing of the start of the data recording system, r_n , was reasonably accurate.

This uw trace was generated, in the wind tunnel set up, by use of electronic PHI modules, see Figure 43. The ac output from the two wires of a cross-wire is $u + w$ and $u - w$ respectively depending on the orientation of the wire. Since $uw = \frac{1}{4} \left[(u + w)^2 - (u - w)^2 \right]$ the instrumentation, as sketched in Figure 43, basically squared and differenced the outputs from the two anemometers. Firstly, to obtain the correct level of input for the PHI multiplier which squared the $u + w$ and $u - w$ signals the output from the anemometers was amplified by a factor of 10 using data amplifier PHI modules. The data amplifiers were ac coupled as only the fluctuating part of the signals was required, since uw is the product of the fluctuating velocities. The gain of a data amplifier is within 5% of its nominal value and is usually closer than this. The amplified $u + w$ and $u - w$ signals were squared; for input signals X, Y not exceeding $\pm 10V$ (peak-to-peak) the multiplier gives an output of $\frac{XY}{10}$ to within 0.05V, greater accuracy is achieved by scaling the input signals to lie within the range

$1V < (X, Y) < 10V$. The resulting signals (proportional to $(u + w)^2$ and $(u - w)^2$) were then differenced using an operational amplifier, the output from which was reduced using an attenuator before proceeding to the voltage comparator. The internal voltage of the comparator was adjusted so that when the peak of a $f(t + r_n)$ signal exceeded a certain pre-set level the data recording system was started. This level was established by monitoring the uw trace on an oscilloscope.

The cross-wire used to obtain the uw trace as outlined above was made from $5\mu m$ tungsten wire. This cross-wire was mounted on a $\frac{3}{8}$ " diameter steel rod, the rod being positioned as far as was possible, at least 4", from the core of the vortex under examination; however, with this set-up it was possible for this cross-wire to be placed anywhere in the test area of the wind tunnel. The cross-wire was always positioned on the edge of the core of the vortex because, as explained in section 3.3, this position gave the most regular 'bursting' phenomenon.

3.5 Examination of the Structure of the Vortex at the Occurrence of this 'Bursting' Phenomenon

A DISA 55 P51 gold-plated cross-wire was mainly used to study the structure of the vortex at the occurrence of the 'bursting' phenomenon, however initially a DISA 55 P01 single gold-plated wire was used. The hot wires were attached to the front of the probe carriage of the traverse mechanism (described in section 3.1); a photograph of a DISA hot wire mounted on the traverse mechanism is shown in Figure 44. The hot wires were mounted so that they were positioned either above or below the arm of the traverse mechanism and approximately 10" in front of it, so as to minimise the interference

to the flow at the measuring position. The holders for the hot wires were also streamlined for minimum interference to the flowfield.

Four experiments were carried out as follows:-

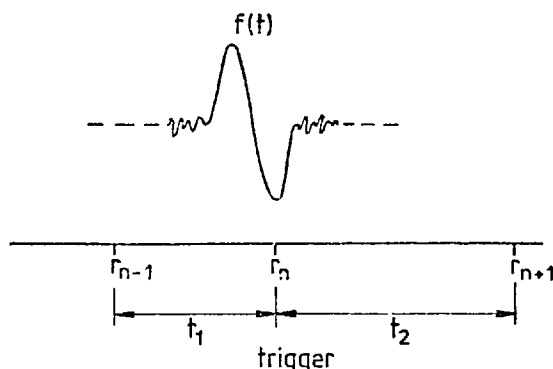
<u>Length of Working Section</u>	<u>Delta Wing</u>	<u>Measuring DISA Hot-wire Anemometer</u>
(i) Short, 9.78'	63" chord mounted in contraction	Single wire
(ii) Short, 9.78'	63" chord mounted in contraction	Cross-wire
(iii) Long, 28'	12.8" chord mounted in working section	Cross-wire
(iv) Long, 28'	28" chord mounted in working section	Cross-wire

The experiments were set up, with the exception of the first, using the same basic instrumentation, see Figure 4.5. In the first two experiments the output from the hot wires and that of the trigger signal was recorded on magnetic tape, see Appendix C, before being analysed. For the first experiment a single hot wire replaced the cross-wire and the sum and difference unit as sketched in Figure 4.5. The output from the cross-wire used for taking velocity measurements was fed to a DISA sum and difference unit, (for details of this unit see Appendix D), which can extract signals proportional to the instantaneous velocity components u, v from the cross-wire outputs e_1, e_2 . The output, e_u and e_v from this DISA sum and difference unit was fed into a Hewlett Packard Model 3721A digital correlator which averaged the signal, e_u or e_v , for a selected timescale

when triggered by the trigger signal. The correlator was used in the signal recovery mode on the ac setting for averaging the signal, the computed function was displayed on the internal cathode ray tube using values computed for 100 discrete delays. The horizontal axis of the display was calibrated in time per mm, this was also the period between successive samples of the analogue input and could be switched from $1\mu\text{s}$ to 1s . The vertical axis was accurately calibrated and the scale factor displayed beside the cathode ray tube. The averaging method used was that of summation, in which the function is computed by summing N samples with the total being divided by N , N was chosen so that the function being computed was a well-defined, fairly smooth curve. The trigger signal had to be a negative-going 6V step on any dc level between $\pm 100\text{V}$. The timescale was selected to give the required display width (time displayed on the cathode ray tube $= 100 \times \text{time/mm}$).

Any instrument with a facility of data storage will contain past information, thus, in this case the trigger signal can be used to instruct the system to retain earlier data, say, back to $-t_1$ and continue to record for t_2 into the future,

i.e.



Thus, data spread either side of the trigger is obtained. The Hewlett Packard correlator is only intended for the acquisition of t_2 type data but due to a modification it retains data of t_1 type. The delay offset (Option Series 01) was set at 50, this enabled the point at which triggering occurred to be that of the centre of the $f(t)$ signal as described above. The output from the correlator, i.e. the averaged $f(t)$ signal, was plotted using a Hewlett Packard plotter.

3.5.1 The methods used for examining the structure of this phenomenon

A summary of the experiments carried out to examine the structure of this phenomenon is given in Table V. The co-ordinate system used for denoting the positions of the vortices and the position of the trigger cross-wire is sketched in section 3.3. The free stream speed quoted for the first two experiments was taken from the static pressure tapings at either end of the contraction. When the model was mounted in the long working section, i.e. for the last two experiments, the free stream speed was measured by a pitot-static tube placed in the test area. The best position for the triggering cross-wire was found by looking at the uw signal on an oscilloscope, this position was just outside the core of the vortex being studied.

A sketch of the set-up in the wind tunnel for the first experiment ((i) in Table V) is shown in Figure 46.

The measuring single hot wire in the first experiment was traversed horizontally through the centre of vortex 2; the single wire was placed both vertically and horizontally. Both the output from the single wire and the trigger, uw , signal was recorded on magnetic tape, the taping speed was 30 ips and the data was recorded for 5 minutes at each position. The gold-plated cross-wire used in the second experiment

was similarly traversed horizontally through the centre of vortex 2, the cross-wire being positioned in both U-V and U-W planes. The outputs from the two wires of the cross-wire were scaled up by a factor of two, using an operational amplifier, and recorded simultaneously with the uw output from the triggering system on magnetic tape at a speed of 15 ips; each position of the horizontal traverse being recorded for 10 minutes.

The output from the measuring cross-wires in the last two experiments was fed directly into the Hewlett Packard correlator and the graphs of $f(t)$ plotted on the Hewlett Packard Plotter. For the third experiment the measuring cross-wire was traversed at an angle of 60° to the horizontal from the centre of the vortex downwards, the cross-wire was set up in the U-V and U-W planes and then rotated clockwise through 30° about the axial axis, the positioning of the cross-wire is shown in Figure 47. The measuring cross probe used in the last experiment was traversed horizontally through the centre of the vortex, the wire being set up in both U-V and U-W planes and then positioned such that it lay parallel to the streamlines of the flow.

The conditional sampling technique, as described in section 3.4, was used to study the "bursting phenomenon" (see section 3.3) of the flow far behind a delta wing. This "phenomenon" will henceforth be termed the "structured disturbance"; this will avoid confusion with "vortex bursting" and "vortex breakdown", both phrases having been used by other authors describing phenomena which are considered to be different from the vortex instability being studied in this thesis. Three delta wing models were used; the experimental set-up in each case is described in section 3.5.1 and outlined in Table V.

4.1 The Structured Disturbance - a "General Picture"

Initially, in order to obtain a "general picture" of the structure of this disturbance a single hot-wire was traversed horizontally through one of the trailing vortices (experiment (i) in Table V). The traces of the fluctuating voltage (which is closely related to fluctuating velocity via King's law) at different positions through the centre of the vortex are shown in Figure 48. The structured disturbance, which was always present throughout the vortex, varied in shape for different positions in the vortex, but at any position the traces were very similar for the single hot-wire placed either horizontally or vertically. This confirmed the existence of the structured disturbance.

By placing the single wire at three downstream (x) positions it was possible to calculate the speed at which the structured disturbance travelled; this was found to be that or

nearly that of the surrounding flow, i.e. $U_{\infty}(1 + \alpha)$, where

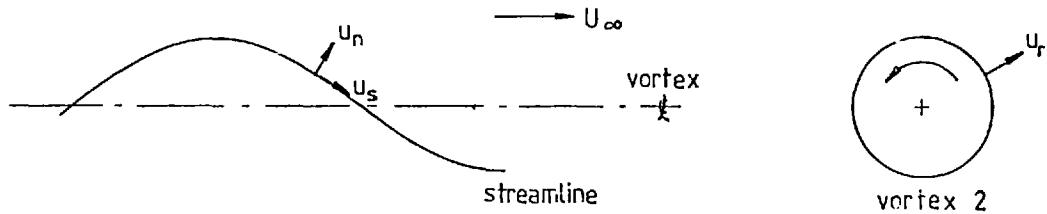
$|\alpha| \ll 1$: from Figure 49 α can be seen to vary between ± 0.25 . Subsequently, a check was also made to ensure that the flow undisturbed by the trailing vortices sustained no structured disturbance, and this was indeed the case. For this check a cross-wire was used, its output signals being processed using exactly the same method as that used to obtain the structured disturbance in the vortex. The resulting output from the Hewlett-Packard correlator was a straight line with a mean of zero, see Figure 50. This is the result to be expected on summing and averaging a turbulent signal.

From the above results it was thus established that a structured disturbance existed which was a phenomenon of the flow-field and which moved with, or nearly with, the speed of the surrounding flow.

4.2 Results from Experiment (iv) - the 28" Chord Delta Wing

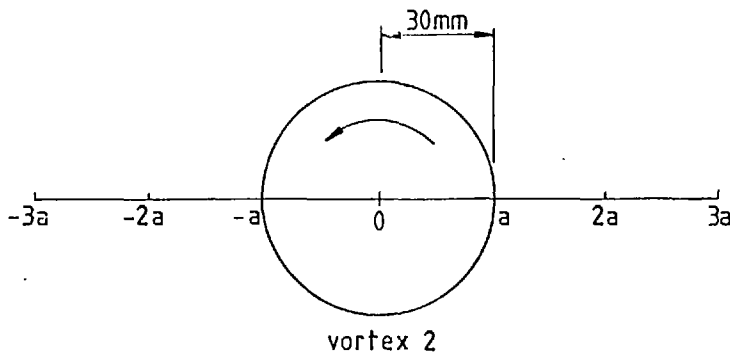
A study of the data from experiment (iv) throws light on that obtained from experiments (ii) and (iii). The 28" chord delta wing was mounted in the long working section, and the measuring cross-wire traversed horizontally through the core of vortex 2 (see the sketch of the co-ordinate system in section 3.3). The cross-wire probe was positioned such that it lay parallel to the local streamlines of the flow. For further details concerning experiment (iv) see Tables V and VI. The notation used for the components of instantaneous velocity relative to a mean streamline is sketched on the next page,

u_s is the component in the streamline direction, u_n the component normal to the streamline in the circumferential direction and u_r the radial direction.



The notation used for the velocity components - experiment (iv)

The mean axial and azimuthal velocity components of the flow in vortex 2 are shown in Figures 51 and 52 respectively; for comparison purposes the experimental data of Graham, Newman and Phillips [80] for the weak jet are also plotted. The diameter of the core of the vortex under study (vortex 2) was approximately 60 mm. The position of the measuring cross-wire is given in terms of the core radius, a , as demonstrated in the following sketch:-



Note:
other vortex
at $a = 8.73$

The positioning of the measuring cross-wire given in terms of the core radius, a .

The wing Reynolds number based on the centre line chord, Re_w , was 1.18×10^6 and the vortex Reynolds number, Re_v , was 6.53×10^4 for experiment (iv). The measuring station was 21 vortex spacings downstream of the trailing edge of the wing.

Velocity measurements were mostly taken using the conditional sampling technique triggered from the negative peak of the uw triggering signal, a few measurements were also taken using the positive peak of the triggering signal. With the measuring cross-wire set up parallel to the streamlines of the flow the $\bar{U}_r$ and $\bar{U}_n$ mean components of velocity should be zero. However, any slight error in angling the probe or changes in flow direction associated with fluctuations in velocity proved especially troublesome in making measurements in the (U_s, U_r) plane. This is because the unresolved and unsteady third component of velocity, U_n , could be large compared with U_r . For this reason the mean radial velocities for the different experiments will not be presented. The turbulence intensities obtained from the measuring cross-wire at the various positions used for studying the structured disturbance are shown in Figure 53.

Using the conditional sampling technique, as described in section 3.4, measurements were taken with the cross-wire for various positions on a horizontal traverse through the centre of the core of vortex 2. Two different disturbances were found to exist; the first, the structured disturbance, occurred in the streamwise component of fluctuating velocity, (the Hewlett-Packard correlator was set-up to admit only the ac signal), and had a wavelength of approximately .96 m, whilst the second occurred in the components of fluctuating velocity normal to the streamline

in the circumferential direction and in the radial direction and had a wavelength approximately ten times longer. These disturbances occurred randomly at a frequency of approximately one per second, a characteristic frequency ($= f d / U_\infty$, where f is the frequency and d the vortex spacing) of 0.011. The uv and uw traces in Figure 42 show that the disturbances occur approximately six times a second, a characteristic frequency of 0.045. This difference in the frequency of occurrence reflects the high triggering level set on the voltage comparator in the signal conditioning system. This high triggering level ensured that data was only recorded at the occurrence of a disturbance and not for any highly turbulent signal.

The structured disturbance is not linked with the Crow [98] instability; the structured disturbance is a random phenomenon whereas the Crow instability has a steady wavelength which, for the case of the maximally unstable long wave (this has a wavelength of $8.4b$), grows by a factor 'e' in a time $9.4 (R / C_L) (b / V_0)$, where R is the aspect ratio, C_L is the lift coefficient, V_0 is the speed of the aircraft and b the separation between the vortices.

Of the three preferred modes of the Crow instability, see Crow [98] for further details, one, i.e. the maximally unstable long wave of wavelength $8.4b$, has been observed; the other two shorter waves of wavelength $0.42b$ have not been observed in practice. From Table V the separation of the vortices for experiment (iv) can be seen to be 262 mm, thus the Crow instability wavelengths would be 2.2 m for the maximally unstable wave and 0.11 m for the short wave; the wavelength for the structured

disturbance of this thesis is, for experiment (iv), $0.96 m_\lambda$. This further demonstrates the dissimilarity between the structured disturbance and the Crow instability.

The structured disturbance, when triggering on the negative peak of the uw trigger signal and using a timescale of 100 msec on the Hewlett-Packard correlator, began to form at -1.93 core radii (a) from the centre of the vortex; at $-3.27a$ there was little trace of a disturbance. This disturbance then increased in amplitude until the $-0.67a$ position; by the next position in the core of the vortex ($-0.33a$) the disturbance had begun to decay and at the centre of the vortex it had died out completely. The structured disturbance had reappeared by $+2.07a$, and at this position in the vortex had the same amplitude as that for the $-1.43a$ position. The perturbations, triggered from the negative peak of the uw triggering signal, in the u_s component of fluctuating velocity are shown as a plot of $u_s(y,t)$, see Figure 55. This gives a helpful picture of the structured disturbance showing its amplitude increasing towards the core of the vortex, and with the disturbance disappearing in the centre of the vortex - the perturbations at $a = 0$ are smaller, having the same order of magnitude compared with those perturbations for the positions between $-1.1a$ and $-0.33a$. A few positions in the vortex were studied using the positive peak of the uw trigger signal in the conditional sampling technique. For the U_s streamwise components of velocity, the positively triggered disturbance, see Figure 56, there appears to be a mirror image of the negatively triggered disturbance; note the $-1.43a$ position

for Figures 54 and 56. In particular, compare position -1.1a in Figure 54 with -1.43a in Figure 56; various points in the structured disturbance have been distinguished by the letters A - D to identify the disturbance and its mirror image. A comparison at the 1.08a position is more difficult since the negatively triggered disturbance is nearly undetectable. This could be due to the set-up in the test area, i.e. interference to the flow from cross-wires and their supports.

The negatively triggered disturbance in the radial, U_r , and normal to the streamline in the circumferential direction, U_n , components of velocity for a timescale of 100 msec is shown in Figures 57 and 58. Unlike the U_s structured disturbance, the disturbance in the U_r and U_n components of velocity increases in amplitude towards the centre of the vortex. This increase in amplitude is more apparent on studying the disturbance for the U_r and U_n components over a time interval of 1 sec, see Figures 59 and 60 respectively. At the -3.27a and -1.93a positions the disturbance does not exist, it is forming for the U_r component at -1.43a, but not in the U_n component; within the core of the vortex the amplitude of the disturbance reaches its maximum. This disturbance with the longer wavelength may represent the wandering of the vortex due to the presence of the measuring probe - the vortex had a tendency to 'jump' either side of the probe.

This data, from experiment (iv), demonstrates the existence of a structured flow with a structured disturbance having a wavelength of approximately .96 m moving with or nearly with the speed of the surrounding flow, i.e. U_∞ ; the

positively triggered disturbance being a mirror image of the negatively triggered one. The other, longer wavelength, disturbance is attributed to the 'meandering' of the vortex. All the measurements taken are restricted to one plane of the vortex. Clearly, more information is needed on the three-dimensional nature of the structured disturbance. Such measurements would involve the use of further cross-wire probes; this would increase the problems of interference to the flow, hence reducing the usefulness of data acquired. Three-dimensional measurements would be extremely difficult to take and the amount of work involved too vast an undertaking. From the available data it is not possible to ascertain whether the structured disturbance is axisymmetric.

Having established the existence of two disturbances it is instructive to study the data from previous experiments, i.e. experiments (ii) and (iii).

4.3 Results from Experiment (ii) - the 63" Chord Delta Wing

For experiment (ii) the 63" chord delta wing was mounted in the contraction of the wind tunnel since only the 9.78' working section was available. This stretching of the trailing vortices will, in a rather arbitrary manner, artificially age the vortices. The mean axial and azimuthal components of velocity for the vortex studied are shown in Figures 61 and 62 respectively. The measuring cross-wire was placed at various positions on a horizontal traverse through the centre of the core of the vortex. The structured disturbance and the longer wavelength disturbance for these positions are shown in Figures 63-65; measurements of the fluctuating velocities were all taken using the negative peak of

the uw trigger signal. The velocity co-ordinate system used is that sketched in section 3.3, i.e. U, V, W , where U is in the same direction as the free stream velocity, U_∞ , V is radial and W is the azimuthal velocity component. The wing Reynolds number Re_w , based on the centre line chord, in this experiment was 1.65×10^6 and the vortex Reynolds number, Re_v , was 9.22×10^4 .

The axial component of velocity, U , for the cross-wire placed in both the horizontal (X, Y) and vertical (X, Z) planes is shown in Figure 63. The structured disturbance, which was present throughout the range $-2.11a$ to $+2.63a$, increased in amplitude in the core of the vortex, and had a wavelength of approximately 1 m. Similarly, the disturbance in the V and W components of velocity, see Figures 64 and 65 respectively, also increased in amplitude towards the centre of the vortex, dying out rapidly towards each end of the range of positions examined; its wavelength was also approximately 1 m. The magnitude of the variation in amplitude of the disturbance may be seen from the vector diagram for the ' $U - V$ ' plane, see Figure 66. The separation between the vortices in this experiment was $2\frac{1}{2}$ mm, thus making the wavelengths of the Crow instability 2 m and .1 m for the maximally unstable long wave and the shorter waves respectively. This again demonstrates that there is no connection between the Crow instability and the 'structured disturbance' of this thesis.

Since the results from experiment (iv) (see Tables V and VI) indicated that the disturbance of most interest, i.e. that with the wavelength of .96 m, appeared along the streamlines of the flow, i.e. the U_s component of velocity, it was hoped that by

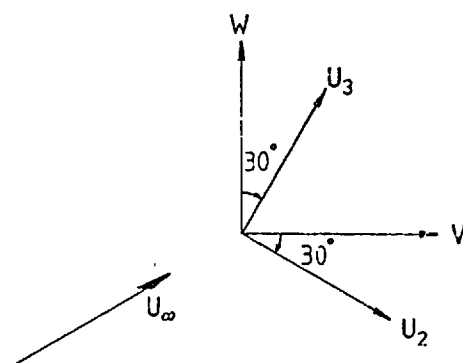
calculating the resultant velocity an indication would be given as to the disturbance along the streamlines of the flow. The resulting disturbance will be different to that obtained by setting-up the cross-wire to lie parallel to the streamlines of the flow. (Setting-up the cross-wire in the U , V , W directions and then calculating the resultant component in any plane will not give the same result as setting-up the cross-wire in the U_s , U_r and U_n directions. In the U_s , U_r , U_n coordinate system the flow will be normal to the wires, whereas in the U , V , W coordinate system there will be considerable flow along the axis of the wires affecting their accuracy, see Appendix D). The resultant disturbance, U_{res} , is shown in Figures 67 and 68; the amplitude again increases towards the centre of the vortex and dies away for positions over 1.5 core radii from the centre of the vortex. In the $U - W$ plane the disturbance inverts in the centre of the vortex and remains inverted for the positions from 0 to $2.63a$.

These results from experiment (ii) are helpful in establishing the existence of the structured disturbance. However, the disturbance obtained by placing the cross-wire in the $U - V$ and $U - W$ planes from readings over a timescale of 100 msec, is probably a combination of both the shorter and longer wavelength disturbances shown in experiment (iv), see Figures 52 - 58.

4.4 Results from Experiment (iii) - the 12.8" Chord Delta Wing

The existence of the structured disturbance was again verified in this experiment; most of the measurements were made of the shorter wavelength disturbance over a timescale of 100 msec; one measurement was made over a timescale of 1 sec which again

resulted in the longer wavelength disturbance. The 12.8" chord delta wing was placed in the long working section for this experiment and the measuring cross-wire was traversed at 30° to the vertical from the centre of the vortex downwards, see Figure 47 for the positioning of the probe in this experiment; the cross-wire was also rotated clockwise (looking in the direction of U_∞) through 30° . By taking measurements at positions further away from the triggering cross-wire it was possible to check that the structured disturbance was not influenced to any large extent by the presence of this cross-wire and its support. The components of velocity for experiment (iii) are U_1 , U_2 and U_3 , where U_1 is in the axial direction and equals U , $U_2 = V \cos 30^\circ - W \sin 30^\circ$ and $U_3 = V \sin 30^\circ + W \cos 30^\circ$: U , V and W are defined in section 3.3; see the following sketch:-



For this experiment the wing Reynolds number was 5.81×10^5 and the vortex Reynolds number 1.47×10^4 . In experiment (iii) the negative peak of the uw trigger signal was used in the conditional sampling technique.

The structured disturbance in the U_1 component of

velocity, see Figure 69, has a wavelength of approximately 1 m; the amplitude of the disturbance dies away for positions further away from the core of the vortex. In the centre of the core a disturbance is shown to exist, this differs from the results for experiment (iv), Figure 54, where the disturbance in the U_s component dies away. The meandering of vortices in a wind tunnel is known to occur and a vortex also tends to 'jump' either side of any measuring probe placed in its core; either or both of these phenomena would tend to show results indicating the existence of a disturbance in the centre of the core of the vortex, particularly as the core of the vortex was relatively small (the core diameter was 27.5 mm) compared with that in experiment (iv) (core diameter of 60 mm), and the measuring cross-wire would represent a relatively larger interference to the flow (the distance between the two wires of the cross-wire is approximately 2.3 mm). The disturbance in the U_1 component of velocity, in the $U_1 - U_2$ plane, in the core of the vortex appears to be 'upside-down' with respect to the similar disturbance for the $U_1 - U_3$ plane; with the cross-wire in the $U_1 - U_2$ plane the large U_3 component of velocity acting along the wires introduces some inaccuracy into the measurements, which could result in the disturbance appearing to be inverted. However, the structured disturbance for the U_1 component, Figure 69, outside the core of the vortex ($a > 1$) does agree fairly well with that obtained for the U_s component (triggered from the negative peak of the uw trigger signal) in experiment (iv), Figure 54.

The disturbance in the U_3 component of velocity, see Figure 70, has a wavelength of approximately .13 m.

However, the disturbance in the U_2 component, see Figure 71, has a much longer wavelength, possibly 10 m, judging from the one position at which measurements were taken using a timescale of 1 sec on the Hewlett-Packard correlator, see Figure 72. The disturbance in the U_2 component of velocity compares well with that in the U_r component (figure 57) from experiment (iv); there is close agreement between the wavelengths and the maximum amplitude, which occurs in the core of the vortex, decreases with distance from the centre of the vortex.

The resultant components, U_{res} , of velocity in the $U_1 - U_2$ and $U_1 - U_3$ planes are shown in Figures 73 and 74 respectively. The disturbance in the U_{res} resultant component for the two planes is in closer agreement than the disturbance in the U_1 component for the two planes throughout the positions at which measurements were taken.

4.5 Summary and Discussion

The structured disturbance, which is a random phenomenon, has been shown to move at a speed equal or nearly equal to that of the undisturbed flow, i.e. at $(1 + \alpha)U_\infty$, where $\alpha \ll 1$, see Figure 49. Another, longer wavelength disturbance has also been found, and is attributed to the 'meandering' of the vortex. The longer wavelength disturbance appears to increase in amplitude towards the centre of the vortex, whilst the shorter wavelength structured disturbance increases in amplitude reaching a maximum at the edge of the core before dying out completely in the centre of the vortex. The longer wavelength disturbance is thus regarded as trivial and not studied in detail in this thesis. The shorter wavelength

structured disturbance when triggered from the positive peak of the uw trigger signal is the mirror image of that obtained from the negative peak. A summary of the wavelength of the structured disturbance for each experiment, together with the diameter of the core of the vortex under study and the wing and vortex Reynolds' numbers is given in Table VI. As discussed in section 4.2 the structured disturbance is not linked with the Crow instability as it occurs randomly, the mean frequency not coinciding with the Crow frequency.

Photographs, using smoke visualisation, of the flow behind the 12.8" and 28" chord delta wings in the long working section are shown in Figures 35a, 75 and 76. Although it is realised that the introduction of smoke could provide interference to the flow and show other characteristics which are not normally present, (e.g. if the smoke is introduced unevenly the resulting 'bursts' might be confused with vortex bursting), certain characteristics may be observed from the photographs. The rolling-up of the vortices from the 12.8" chord delta wing is shown in Figure 35a, and the trailing vortices behind this wing are shown in Figure 75. Figure 76a shows the trailing vortices formed behind the 28" chord delta wing, whilst the flow just behind the 12.8" chord delta wing is shown in Figure 76b. The axial flow in the vortices is clearly shown in Figure 76b. Figures 75 and 76a show the trailing vortices further downstream; in Figure 75b a Crow [98] instability could be just starting to form. Some of the characteristics shown in the photographs of the vapour trails behind a wide-bodied jet, Figure 28, may also be seen in the photographs taken in the wind tunnel. The

spiralling in the vortex is clearly shown in Figure 28b together with possibly a Crow instability; one vortex, (the lower in the photograph), does appear to meander. Figure 28a demonstrates the vortices appearing to break-up and then reform, usually the radius of the core grows rapidly in size before the vortex disappears, reappearing again with an enlarged core before continuing as though undisturbed. This phenomenon is similar to that described by Harvey and Fackrell [186] in their towing tank experiments, see chapter 3; it can be seen that this phenomenon does not occur in connection with the Crow instability since the phenomenon does not take place simultaneously in both trailing vortices as does the Crow instability.

A possible explanation as given by Harvey and Fackrell is as follows: if at some instance the vortex becomes unstable in the Rayleigh [199] sense the core will not then maintain its true asymmetry. If, further, its edge is distorted a reduction in its rate of rotation will result. Thus, bundles of vortex filaments from the vortex core will revolve less rapidly about the vortex axis at the site of the instability, and be distorted into a U-shaped disturbance eventually leading, after much stretching, to quasi-ring vortices, see Figure 25.

This distortion to the edge of the core of the vortex might also result in the structured disturbance found in the experiments described in this thesis. Since the disturbance moves with the flow it seems probable that some instability in the flowfield could account for its appearance.

4.5.1 A possible link between the structured disturbance and a stationary wave

From Figure 49 it appears that the structured disturbance travels downstream without change of form. By definition, see Leibovitch and Secbass [164], a stationary wave is one that propagates without change of form at a constant speed. Since the structured disturbance seems to fit this definition, see section 4.1, it is thought it may be a stationary wave.

For long times, wave development is governed by the Korteweg-de-Vries (KdV) equation; only two stationary wave solutions of permanent form exist, cnoidal waves and solitary waves, for details of these see section 2.4. The uniform wave-train solutions, cnoidal waves, are periodic and may be expressed in terms of Jacobi elliptic functions. Benjamin [142, 143] constructed a "finite transition" theory analogous to hydraulic jumps in open channel flow; the term "finite transition" and "large amplitude wave of unknown structure" can be used interchangeably. The mechanism by which a supercritical stream is converted into a subcritical one was denoted a "finite transition". (Vortex flows may be classified, Benjamin [142], as "supercritical" if wave propagation is only possible with phase speed in the downstream direction or "subcritical" if waves can also propagate with phase speeds in the upstream direction. The two categories of flow are separated by "critical flow" on which standing waves of extreme length are possible). Cnoidal waves, switched on by the finite transition, were favoured by

Benjamin as an explanation of vortex breakdown. Benjamin's theory appears to be valid for breakdowns of arbitrary amplitude and to apply more to strong breakdowns. A possible connection between solitary waves and vortex breakdown was mentioned by Benjamin. However, he was principally interested in providing support for the finite transition theory. The suggestion of a periodic character to the flow downstream of a breakdown referred to as secondary breakdowns and sometimes found by Harvey [155] have been suggested by Benjamin to be the observable remnants of a trail of cnoidal waves. Such an explanation is not widely accepted and cannot be argued entirely from the KdV equation or its stationary forms since infinite wave trains are not physically relevant.

The solitary wave solution appears to be more compatible with the structured disturbance, one further reason being that the structured disturbance occurs randomly, whilst cnoidal waves are periodic. A small perturbation solution for a solitary wave with an outward displacement of the fluid has been obtained by Leibovitch [163]. Like the majority of other theories which attempt to explain breakdown the theory is applied to vortex flows confined in tubes, and has the restriction of axially symmetric perturbations. Details of his approach are presented in section 4.5.1; the basic outline of the theory is given in section 2.4. This work has been extended by Leibovitch and Randall [147] and Randall and Leibovitch [192], for details see section 2.4. Several criticisms of the Randall-Leibovitch calculations have been raised; one is that their

model equation is not valid for the large amplitude waves which are computed, whilst another is that the reflection of waves downstream is not possible.

A possible method of analysis for a stationary wave moving at small velocities with respect to the undisturbed fluid, i.e. the structured disturbance, is now presented, although it is realised that many other approaches to the solution of this problem may be possible.

4.5.2 A possible method of analysis for stationary waves moving at small velocities with respect to the undisturbed fluid

The flow far downstream of a delta wing has been shown by means of wind tunnel experiments to support what appear to be stationary waveforms which travel downstream at a speed equal or nearly equal to that of the undisturbed fluid. Similar disturbances have been observed in the condensation trails of a conventional wing. The general theory establishing the existence of solitary waves on trailing vortices is presented in section 2.4. However, workers such as Leibovitch were especially interested in "vortex breakdown" and thus concentrated on waves not moving with respect to the wing. The following is an outline for a possible method of analysis for stationary waves moving at small velocities with respect to the undisturbed flow in an incompressible fluid; the method follows on from that used by Leibovitch [163].

4.5.2.1 Equations of motion

The axially symmetric form of the Euler equations for

a constant density fluid in cylindrical (r, θ, z) coordinates with velocity components (u, v, w) are:-

$$u_t + uu_r + wu_z - r^{-1}v^2 = \rho^{-1}p_r \quad (4.5.2,1a)$$

$$v_t + uv_r + wv_z + r^{-1}uv = 0 \quad (4.5.2,1b)$$

$$w_t + uw_r + ww_z = -\rho^{-1}p_z \quad (4.5.2,1c)$$

$$\text{and} \quad r^{-1}(ru)_r + w_z = 0 \quad (4.5.2,1d)$$

where the subscript denotes differentiation, i.e. $u_t = \frac{\partial u}{\partial t}$.

The fluid can sustain an unperturbed flow with velocity vector,

$$V_0 \{ 0, V(r), W(r) \},$$

and pressure,

$$P(r) = \rho V_0^2 \int r^{-1} V^2(r) dr,$$

where V, W are arbitrary nondimensional functions of the radius, r .

A perturbation and non-dimensional variables may be introduced. Velocities are compared to V_0 , a typical swirl velocity, radial distances to R , and axial disturbances to L , a length characterising the axial scale of the disturbances. Letting unstarred quantities be dimensionless and putting $k = R/L$, then

$$u^* = \varepsilon V_0 k u \quad (4.5.2,2a)$$

$$v^* = V_0 [V(r) + \varepsilon v] \quad (4.5.2,2b)$$

$$w^* = V_0 [W(r) + \varepsilon w] \quad (4.5.2, 2c)$$

$$\text{and } p^* = P(r) + \varepsilon \rho V_0^2 p \quad (4.5.2, 2d)$$

The quantity εV_0 is a measure of the perturbations of the base flow, ε being a Rossby number. (The Rossby number is given by $U/L\Omega$, where Ω is the angular velocity, U is the representative velocity magnitude (relative to rotating axes), and L is the measure of the distance over which u varies appreciably). From the equations of motion the scale of time is L/V_0 in order to retain nontrivial first order dynamic effects in the limit as $\varepsilon \rightarrow 0$. Thus, let $t^* = LV_0^{-1}t$.

The scale factors are chosen so that in the limit $\varepsilon \rightarrow 0$, $k = 1$, and $V = r$ the perturbation equations reduce to the linearized case which admits dispersive waves. A perturbation procedure applicable to the case of small ε and k is used. Small k implies slow axial variations, or in the case of wave motion, (where k is then a "wavenumber"), long waves.

Adopting the above set of scale factors (4.5.2,2) equations (4.5.2,1) assume the following dimensionless form:-

$$2Vr^{-1}v - p_r = -\varepsilon r^{-1}v^2 + k^2(u_t + Wv_z) + \varepsilon k^2(uu_r + ww_z) \quad (4.5.2, 3a)$$

$$v_t + Wv_z + r^{-1}(rV)_r u = -\varepsilon(r^{-1}u(rv)_r + ww_z) \quad (4.5.2, 3b)$$

$$w_t + Ww_z + uW_r + p_z = -\varepsilon(uw_r + ww_z) \quad (4.5.2, 3c)$$

$$(ru)_r + rw_z = 0 \quad (4.5.2, 3d)$$

The equations governing the perturbations when they are small and gradual, i.e. as $\varepsilon \rightarrow 0$ and $k \rightarrow 0$, have now to be derived.

4.5.2.2 The perturbation equations

It has been established, (see Leibovitch [163]) that a uniformly valid (in time) description of finite amplitude waves is possible only when $\varepsilon = O(k^2)$.

For $\varepsilon = 0$, equations (4.5.2,3) admit a separation of variables solution of the form

$$(u, v, w) = (u_0(r)A_z(z, t), v_0(r)A(z, t), w_0(r)A(z, t))$$

$$p = p_0(r)A(z, t)$$

if A satisfies to $O(1)$, the equation

$$A_t + c_0 A_z = 0. \quad (4.5.2,4)$$

In equation (4.5.2,4) c_0 is constant and is the eigenvalue of the equations for (u_0, v_0, w_0, p_0) .

If a separable perturbation solution exists for the above set of equations (4.5.2,3) it must be, in a sense, singular, i.e. this "singular" behaviour is the non-existence for all (z, t) of higher order corrections; i.e. the wave function, A , depends upon ε .

For a stationary wave solution, if the wave function, A , is expressed in coordinates moving with the zeroth order wave speed, c_0 , and if time is suitably scaled, a regular perturbation of the form

$$\psi = A \phi_0(r) + \sum_{n=1}^{\infty} \varepsilon^n \sum_{j=1}^N \phi_n^{(j)}(r) B_n^{(j)}(A), \quad (4.5.2,5)$$

is permissible, where B_n^j is an operator acting upon A.

In equation (4.5.2,5) $\psi(r, z, t) = \phi_0(r)A(z, t) + \dots$,

where $\phi_0(r) = -ru_0(r)$ is a "streamfunction".

Then A is expressed as follows:-

$$\left. \begin{aligned} \xi &= z - c_0 t, \\ \tau &= \epsilon t, \\ A &= A(\xi, \tau). \end{aligned} \right\} \quad (4.5.2,6)$$

For a stationary wave analysis, where the waveform is travelling at a constant speed equal or nearly equal to that of the fluid surrounding the vortex, i.e. at $(1 + \alpha)U_\infty$, where $\alpha \ll 1$, the frame of reference would have to be transformed to one moving with or nearly with the fluid. Assuming that at time, $t = 0$, the two frames of reference coincide and denoting the frame of reference moving with the fluid by a prime, the relationship between the two frames of reference in cylindrical coordinates (r, θ, z) is given by:-

$$\left. \begin{aligned} t &= t' \\ r &= r' \\ \theta &= \theta' \\ z &= z' + |(1 + \alpha)U_\infty| t \end{aligned} \right\} \quad (4.5.2,7)$$

Applying this Galilean transformation (4.5.2,7) to the expression for A given by equation (4.5.2,6) the perturbation equations would be similar in form to those for the stationary

wave of Leibovitch [163], i.e. let

$$\left. \begin{aligned} u &= u_0(r)A(\xi, \tau) + \varepsilon[u_1^{(1)}(r)(\frac{1}{2}A^2)_\xi + u_1^{(2)}(r)A_{\xi\xi\xi}] + \dots \\ v &= v_0(r)A(\xi, \tau) + \varepsilon[v_1^{(1)}(r)\frac{1}{2}A^2 + v_1^{(2)}(r)A_{\xi\xi}] + \dots \\ w &= w_0(r)A(\xi, \tau) + \varepsilon[w_1^{(1)}(r)\frac{1}{2}A^2 + w_1^{(2)}(r)A_{\xi\xi}] + \dots \\ p &= p_0(r)A(\xi, \tau) + \varepsilon[p_1^{(1)}(r)\frac{1}{2}A^2 + p_1^{(2)}(r)A_{\xi\xi}] + \dots \\ \phi_n^j(r) &= -ru_n^{(j)}(r), \quad n, j = 1, 2, \dots \end{aligned} \right\} \quad (4.5.2,8)$$

Substituting equations (4.5.2,8) into equations (4.5.2,3), Leibovitch found that the condition of separability required:-

$$A_\tau = c_1 A A_\xi + c_2 A_{\xi\xi\xi}, \quad (4.5.2,9)$$

where c_1 and c_2 are constants to be determined, (for details of the determination of c_1 and c_2 see Leibovitch [163]).

Equation (4.5.2,9) is the Korteweg-de-Vries equation; for all initial profiles $A(\xi, 0) = f(\xi)$, with $\lim_{|\xi| \rightarrow \infty} f(\xi) = 0$, a

finite number of solitary waves will emerge asymptotically as $\tau \rightarrow \infty$. These solitary waves will be the only surviving nonzero disturbances as $\tau \rightarrow \infty$. The initial distribution, $f(\xi)$, uniquely determines the propagation speeds and the number of solitary waves that emerge. The explicit solution

for a solitary wave is found by putting $A = A(\xi + \beta\tau)$, then if 'a' is the amplitude of the wave,

$$A = a \operatorname{sech}^2 \left[\frac{1}{2} \sqrt{\frac{c_1 a}{3c_2}} (\xi + \frac{1}{3} a c_1 \tau) \right] \quad (4.5.2,10)$$

In equation (4.5.2,10) the ratio c_1/c_2 determines whether the function A represents a wave of elevation ($a > 0$) or depression ($a < 0$). For further details of the solution to the Korteweg-de-Vries equation and of the connection between solitary waves and vortex breakdown, see section 2.4.

The resulting flow pattern in the vicinity of the breakdown for the example of the exponential vortex, (see equation (2.2,3) section 2.2),

$$rV = K(1 - e^{-ar^2})$$

is shown in Figures 30-32 for ϵ , the (single) free parameter equal to 0.8 and 1.0; ϵ has been set at half the maximum axial disturbance. These particular values of ϵ were chosen to facilitate a comparison with the breakdown regions observed by Harvey [155].

In the case of the above suggested analysis for random stationary waves moving at small velocities with respect to the undisturbed flow, it is hoped that a condition of similarity would also emerge which could hopefully be solved to obtain this type of stationary waveform. It may then be possible to link the structured disturbance with the solitary wave solution of Leibovitch.

CHAPTER 5 .. CONCLUSIONS

An experimental study of a structured disturbance in the flow far behind a delta wing in a low speed wind tunnel has been carried out. The main emphasis was on searching for a disturbance moving with the outer flow. A conditional sampling technique was used to study the structure of this disturbance, with velocity measurements of the flow in the trailing vortices being made using hot wire anemometry. Sample profiles of the mean axial and azimuthal velocities through the core of one of the trailing vortices are given.

The occurrence of a disturbance to the flow was marked by a large peak, (its magnitude was ten times that of the rest of the turbulent signal), in the product, uw , of the fluctuating velocities. This peak was used as a triggering device which initiated automatic recording of the fluctuating velocity signals for a preset timescale. Besides the structured disturbance, which typically had a wavelength of approximately 1 m, a longer wavelength disturbance (approximately ten times that of the structured disturbance) was also discovered; this latter disturbance was attributed to vortex meandering and not studied in detail. A summary of the details of the experiments is given in Tables V and VI, further details are given in chapter 4.

For one experiment (experiment (iv)) measurements were also taken with the triggering level based on the positive

peak of the uw signal as well as those measurements taken using the negative peak. A mirror image of the structured disturbance was obtained when triggering from the positive peak as compared with that obtained using the negative peak of the signal. It was found that the structured disturbance moved at a constant speed equal or nearly equal to the free stream velocity, and that it appeared to travel downstream without change of form.

The mechanism leading to the formation of the structured disturbance may be similar to that described by Harvey and Packrell [186] for the "disturbance" studied in their towing tank experiments, for details see section 4.5.

It is proposed that the structured disturbance is a stationary wave and may be linked with the solitary wave solution of the Korteweg-de-Vries equation as given by Leibovitch [163]. A possible method of analysis for stationary waves moving at small velocities with respect to the undisturbed flow in an incompressible fluid has been suggested. This analysis should be pursued in further work to find a solution for the structured disturbance.

Further experimental work is necessary to examine the structured disturbance in greater detail. The use of hot wire anemometry poses many problems, the main one being interference to the flow due to the presence of the measuring probe; this tends to result, when making measurements in the core of the vortex, in the vortex 'jumping' randomly either side of the

probe. Thus in any further work it would seem reasonable to use a measuring technique that does not interfere with the flow; for example, laser anemometry. From the available data it is not possible to ascertain whether the structured disturbance is axisymmetric; further data is needed to establish any such symmetry. It would be helpful to take more measurements of the structured disturbance at two downstream positions to establish in greater detail the speed at which the disturbance travelled downstream and its shape as it progressed downstream. Tests to establish if the structured disturbance is a solitary wave, i.e. examine its properties, would be most valuable.

Appendix A The Equations of Motion - Some Special Cases

A.1 Equations for laminar flow

Equations (2.2,5) take a simpler form for a number of cases. For negligible temperature variations the viscosity and conductivity are constant, and if compressibility effects are negligible the mean density is constant and the fluctuations of density are negligible. If the fluctuations are all zero then the equations take the form appropriate for laminar flow. These equations may be derived from the Navier-Stokes equations, in the manner of boundary-layer theory, by assuming variations in the axial direction to be vanishingly small compared with variations in the radial direction and taking a limit as the viscosity and radial distance from the axis tend to zero.

The equations for laminar flow are as follows:-

$$\frac{\partial \rho}{\partial t} + \frac{1}{r} \frac{\partial(r\rho u)}{\partial r} + \frac{\partial(\rho w)}{\partial z} = 0, \quad (\text{A-1a})$$

$$\frac{\rho v^2}{r} = \frac{\partial p}{\partial r} \quad (\text{A-1b})$$

$$\rho \left(\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial r} + w \frac{\partial v}{\partial z} + \frac{uv}{r} \right) = \frac{1}{r^2} \frac{\partial}{\partial r} \left[\mu r^2 \left(\frac{\partial v}{\partial r} - \frac{v}{r} \right) \right] \quad (\text{A-1c})$$

$$\rho \left(\frac{\partial w}{\partial t} + u \frac{\partial w}{\partial r} + w \frac{\partial w}{\partial z} \right) = - \frac{\partial p}{\partial z} + \frac{1}{r} \frac{\partial}{\partial r} \left(\mu r \frac{\partial w}{\partial r} \right) \quad (\text{A-1d})$$

and

$$\begin{aligned} & \rho \left(\frac{\partial I_o}{\partial t} + u \frac{\partial I_o}{\partial r} + w \frac{\partial I_o}{\partial z} - \frac{\partial p}{\partial t} \right) \\ & = \frac{1}{r} \frac{\partial}{\partial r} \left[\frac{\mu}{\sigma} r \frac{\partial I_o}{\partial r} + \frac{\mu}{\sigma} (\sigma - 1) r \left(v \frac{\partial v}{\partial r} + w \frac{\partial w}{\partial r} \right) - \mu v^2 \right] \quad (\text{A-1e}) \end{aligned}$$

A.2 Turbulent Flow with Quasi-cylindrical Approximation

It is often assumed for turbulent flows that turbulent diffusion dominates the structure such that all terms involving μ or k in equations (2.2,5) may be neglected; these terms being only appreciable close to solid walls. The equations then become:-

$$\frac{\partial \rho}{\partial t} + \frac{1}{r} \frac{\partial (r \rho u)}{\partial r} + \frac{1}{r} \frac{\partial (r \overline{\rho' u'})}{\partial r} + \frac{\partial (\rho w)}{\partial z} = 0, \quad (\text{A-2a})$$

$$\frac{\rho v^2}{r} = \frac{\partial p}{\partial r}, \quad (\text{A-2b})$$

$$\rho \left(\frac{\partial v}{\partial t} + u \frac{\partial v}{\partial r} + w \frac{\partial v}{\partial z} + \frac{uv}{r} \right) = - \frac{1}{r^2} \frac{\partial}{\partial r} (r^2 \rho \overline{u'v'}) - \overline{\rho' u'} \left(\frac{\partial v}{\partial r} + \frac{v}{r} \right), \quad (\text{A-2c})$$

$$\rho \left(\frac{\partial w}{\partial t} + u \frac{\partial w}{\partial r} + w \frac{\partial w}{\partial z} \right) = - \frac{\partial p}{\partial z} - \frac{1}{r} \frac{\partial}{\partial r} (r \rho \overline{u'w'}) - \overline{\rho' u'} \frac{\partial w}{\partial r}, \quad (\text{A-2d})$$

and

$$\rho \left(\frac{\partial I_o}{\partial t} + u \frac{\partial I_o}{\partial r} + w \frac{\partial I_o}{\partial z} \right) - \frac{\partial p}{\partial t} = - \frac{1}{r} \frac{\partial}{\partial r} (r \rho \overline{u'I_o'}) - \overline{\rho' u'} \frac{\partial I_o}{\partial r} \quad (\text{A-2e})$$

A.3 Equation of Motion in Vorticity Form

The three components of vorticity, ξ , η , ζ are defined in cylindrical polar coordinates, as

$$\xi = \left(\frac{1}{r} \frac{\partial w}{\partial \theta} - \frac{\partial v}{\partial z} \right), \quad \eta = \left(\frac{\partial u}{\partial z} - \frac{\partial w}{\partial r} \right), \quad \zeta = \frac{1}{r} \left(\frac{\partial}{\partial r} (r v) - \frac{\partial u}{\partial \theta} \right)$$

For steady incompressible flow the equation of motion may be

written in vorticity form, see Lamb [46], section 328, as follows:-

$$\frac{\partial \xi}{\partial t} + u \frac{\partial \xi}{\partial r} + \frac{v}{r} \frac{\partial \xi}{\partial \theta} + w \frac{\partial \xi}{\partial z} = \xi \frac{\partial u}{\partial r} + \frac{\eta}{r} \frac{\partial u}{\partial \theta} + \zeta \frac{\partial u}{\partial z} + \nu \nabla^2 \xi, \quad (\text{A-3a})$$

$$\frac{\partial \eta}{\partial t} + u \frac{\partial \eta}{\partial r} + \frac{v}{r} \frac{\partial \eta}{\partial \theta} + w \frac{\partial \eta}{\partial z} = \xi \frac{\partial v}{\partial r} + \frac{\eta}{r} \frac{\partial v}{\partial \theta} + \zeta \frac{\partial v}{\partial z} + \nu \nabla^2 \eta$$

(A-3b)

$$\frac{\partial \zeta}{\partial t} + u \frac{\partial \zeta}{\partial r} + \frac{v}{r} \frac{\partial \zeta}{\partial \theta} + w \frac{\partial \zeta}{\partial z} = \xi \frac{\partial w}{\partial r} + \frac{\eta}{r} \frac{\partial w}{\partial \theta} + \zeta \frac{\partial w}{\partial z} + \nu \nabla^2 \zeta$$

(A-3c)

The equation of continuity being - Lamb [46 , section 147]

$$\frac{1}{r} \frac{\partial}{\partial r} (\xi r) + \frac{1}{r} \frac{\partial \eta}{\partial \theta} + \frac{\partial \zeta}{\partial z} = 0$$

(A-3d)

Appendix B Calibration of Hot-Wire Anemometers

The usual method of calibration of hot-wire anemometers operated at constant temperature uses King's law:-

$$E^2 = A + BU^n \quad (B-1)$$

where E is voltage. $A, = E_0^2$, is found by plotting E^2 against U^n (using an 'n' which gives the best straight line - n equal to 0.45 is usually found to give a good straight line in the velocity range $5 - 50 \text{ msec}^{-1}$) and extrapolating back to zero velocity. The value of E_0^2 actually measured at $U = 0$ is therefore not necessarily equal to A . The value of B is given by the slope of the straight line. Calibration of the wire is advantageously made in low levels of turbulence, highly turbulent flows have to be avoided because of the integration time needed to obtain a true mean value and because of the nonlinear errors affecting the measurements (these errors considerably affect the partial derivatives of the calibration curves so changing the sensitivity of the wire, for further details see Comte-Bellot [1931]).

Voltages were corrected for variations in temperature, as the temperature increases the voltage decreases, using Bearman's [194] formula:-

$$\frac{1}{E} \frac{\partial E}{\partial \theta} \bigg|_U = \frac{-\alpha}{2(R-1)} \quad (B-2)$$

where α is the temperature coefficient of resistivity and has the values

$$3.5 \times 10^{-3} \text{ } ^\circ\text{C}^{-1} \text{ for platinum wire}$$

$$\text{and } 5.2 \times 10^{-3} \text{ } ^\circ\text{C}^{-1} \text{ for tungsten wire,}$$

R , being the overheat ratio of the anemometer set-up, is usually equal to 1.8.

For a cross-wire the effective angles of the wires also need to be found. Consider a wire at an angle $(90 - \psi)$ to the X-axis (Figure 77). Assuming that the wire responds only to the component of velocity normal to its axis, (this is the most common assumption for the yaw response), the effective velocity is $U \cos \psi$ if $V = 0$. The effective angle, ψ , is not necessarily exactly equal to the geometrical ψ , thus it is better practice to calibrate the wire for yaw rather than measure the geometric angle ψ . The cross-wire probe is mounted in a free stream of constant velocity, and the anemometer output voltages are measured for various yaw angles; the angles used were, 0° , $\pm 5^\circ$ and $\pm 10^\circ$. Using suffices 1 and 2 for the two wires the equations become, for a yaw angle δ (Figure 78)

$$E_1^2 = E_0^2 + B_1 [U \cos(\psi_1 + \delta)]^n \quad (B-3)$$

and
$$E_2^2 = E_0^2 + B_2 [U \cos(\psi_2 - \delta)]^n \quad (B-4)$$

where ψ_1 and ψ_2 are the two unknown effective angles. From equation (B-3) and considering the result for $\delta = 0$

$$E_1^2 = E_{0_1}^2 + B_1 U^n \cos^n \psi$$

it follows that

$$\frac{\cos(\psi_1 + \delta)}{\cos \psi_1} = \left[\frac{E_1^2 - E_{0_1}^2}{(E_1^2 - E_{0_1}^2)_{\delta=0}} \right]^{1/n}$$

and hence

$$\sin \delta \tan \psi_1 = \cos \delta - \left[\frac{E_1^2 - E_{0_1}^2}{(E_1^2 - E_{0_1}^2)_{\delta=0}} \right]^{1/n} \quad (B-5)$$

Plotting the right hand side of equation (B-5) against $\sin \delta$ gives a straight line through the origin with a slope of $\tan \psi_1$. The second wire is treated similarly. Any deviation from a straight line after allowing for experimental errors is a measure of the error caused by assuming a cosine-law response. Values of δ up to about $\pm 15^\circ$ usually give reasonable straight lines, thus the cosine law is usually expected to give satisfactory results for low intensity turbulence, clearly as ψ increases the errors caused by adopting the simple cosine law increase, if only because there is a higher probability of the flow vector being along or near to the axis of the wire.

More complicated formulae for the yaw response of a hot-wire, most of them including a term involving the velocity component tangential to the wire, may be adopted; however, so many other uncertainties exist in the operation of hot-wires that the increase in accuracy attainable by using one of the tangential cooling methods would most likely be lost in the general 'background scatter' that is observable in the general run of published results.

Appendix C Apparatus and Programs used for Digital Analysis of
Fluctuating Velocities in the Flow

C.1 Recording Data on Magnetic Tape

To record the output from the hot-wire anemometers (DISA 55 D01) on analogue tape it was necessary to amplify the signal by a factor of two to obtain a suitable voltage level for recording purposes. If the voltage level of the signal is too low the noise generated by the recording interferes with the signal, which is then ill-defined. The peak-to-peak voltage of the signal being recorded had necessarily to be less than 2 volts (for low distortion it should be less than 1 volt). The signals were amplified by passing them through an operational amplifier module; the accuracy of this method was checked by passing a sine wave through the module and viewing the output on an oscilloscope.

The tape recorder was an Ampex FR 1300 "Portable" analogue recorder; the taping speed mostly used was 30 ips giving a frequency response of 10 KHz. During one experimental run a taping speed of 15 ips with a frequency response of 5 KHz was used, however, on feeding this data into the Hewlett Packard correlator it became apparent that some of the u_{wz} signal, used for conditional sampling purposes, of higher frequency had possibly been lost. As the signals were recorded they were simultaneously played back and constantly monitored on an oscilloscope. The dc portion of the fluctuating signal was filtered out before recording, this improves the signal/noise ratio. The odd and even channels on the tape recorder are on separate heads, thus the phase relation between them is unknown and when recording more than one channel at a time data was recorded on either all odd or all even channels. A few seconds of known sine wave signal was recorded

on each channel for calibration purposes.

One method of filtering out the dc portion of a fluctuating signal was to pass the signal through a passive filter network. The network used, which was designed by A. J. Smits, provided a high pass filter at approximately 0.8 Hz for each channel, this eliminated the dc component; the two channels were phase matched to better than 1° for frequencies less than 10 KHz. A low pass filter with a time constant of approximately two seconds gave a reasonable average of the dc output of the signal with the filter being switched between the two channels as required. A low pass filter set at 160 Hz reduced the output noise level from the anemometer - the filter box was placed between the anemometer output and the tape recorder, see Figure 39. The inputs to the filter box were designed for positive voltage levels and had to come from an instrument with an output impedance less than 1Ω , e.g. a DISA anemometer. When measuring the dc output from the passive filter network an instrument with input impedance $\geq 10\text{ M}\Omega$ was required, i.e. a Solartron DVM on the 20V scale. Further details of this passive filter network may be found in Bradshaw [195, section 6.1]. A similar three-channel version of this network was also used.

C.2 Digital Analysis of Fluctuating Velocities

To analyse the data on magnetic tape the analogue voltages are first converted into 10-bit binary numbers and recorded in pure binary form on 7 track $\frac{1}{2}$ " digital magnetic tape in "blocked IBM" format compatible with the College CDC computer. The need for this conversion arises as the digital tape format used on computers (a) accepts only one channel per tape and (b) has a gap between every few

thousand binary numbers to allow the tape recorder to be stopped if necessary. The analogue-to-digital converter should preferably sample all the input channels simultaneously and regularly, but due to (a) the digital outputs must be presented to the tape recorder sequentially, i.e. samples from two input channels are recorded 1, 2, 1, 2 etc., and due to (b) the digital outputs that appear while the tape recorder is writing an inter-block gap must be stored until the recorder is ready to use them.

A mini-computer, a PDP 8/L, is used to control the converter and recorder and to store the digital data during an inter-block gap, see Figure 79 for a block diagram of the system. For further details of this system see Bradshaw [196] .

Digitized hot-wire data was processed on the College computer using the conditional sampling program DIG 2W described by Weir and Bradshaw [197] . The program calculates the conventional and conditional statistical averages of two components of velocity and their higher moments; a plot of the two components of the fluctuating velocity, their instantaneous product and the intermittency function, I , may also be obtained.

Appendix D The DISA Sum and Difference Unit

The DISA sum and difference unit operates for input signals e_1, e_2 in the range $1\text{mV} < |e| < 5\text{V}$ and output signals $e_u = A(e_1 + Be_2), e_v = C(e_1 - De_2)$; A, B, C and D are set by screws in the front of the module to be compatible with the cross-wire in use. Usually A and C are set to 1,

$$B = \frac{\tan \psi_1}{\tan \psi_2} \cdot \frac{\bar{E}_2}{\bar{E}_1} \cdot \frac{B_1}{B_2}$$

and
$$D = \frac{B_1}{B_2} \cdot \frac{\bar{E}_1}{\bar{E}_2}; \quad \psi_1, \psi_2, B_1 \text{ and } B_2$$

are gained from the calibration of the cross-wire, see Appendix B.

$\bar{E}_1$ and $\bar{E}_2$ are the mean output voltages from the cross-wire, subscripts 1 and 2 denoting wire 1 = $u + v$ and wire 2 = $u - v$ respectively.

Since for the flowfield under consideration $\bar{E}_1 \approx \bar{E}_2$ it was assumed that $\bar{E}_1 = \bar{E}_2$, thus settings of B and D were unchanged throughout a traverse of the flow. During one experimental run B and D were changed for some positions in the vortex to allow for $\bar{E}_1 \neq \bar{E}_2$, at the same positions data was also collected assuming $\bar{E}_1 = \bar{E}_2$; no difference was found between the two sets of data.

The maximum output of a DISA sum and difference unit is $\pm 10\text{V}$ (i.e. 20V peak-to-peak), the input impedance is $100\text{k}\Omega$, and the general accuracy is about 0.5% .

Appendix E Calculation of Mean and Fluctuating Velocities from Hot-Wire Anemometers

Hot-wire anemometers consist of thin metallic elements heated by an electric current and cooled by the incident flow, which acts by virtue of its mass flux and temperature (predominantly the effects of forced convection). The forced convection loss is proportional to the temperature difference and to the square root of the velocity, see Ser [198] . For a comprehensive review on hot-wire anemometry, see Comte-Bellot [193] .

E.1 Calculation of Mean and Fluctuating Velocities for a Single Hot-Wire Anemometer

E.1.1 Mean velocity

King's law states:-

$$E^2 = A + BU^n, \quad (E-1)$$

where $A \approx E_o^2$ and B and n are obtained from the calibration of the wire, see Appendix B.

Hence, from equation (E-1) the mean velocity is given by:-

$$\bar{U} = \left(\frac{\bar{E}^2 - \bar{E}_o^2}{B} \right)^{\frac{1}{n}} \quad (E-2)$$

E.1.2 Fluctuating velocity

The total velocity, U, equals the sum of the mean and fluctuating velocities, i.e. $U = \bar{U} + u$; similarly $E = \bar{E} + e$.

Substituting for U and E in King's Law (equation (E-1)) gives:-

$$(\bar{E} + e)^2 = \bar{E}_0^2 + B(\bar{U} + u)^n$$

Expanding the term $B(\bar{U} + u)^n$ using the binomial theorem the above equation becomes:-

$$(\bar{E} + e)^2 = \bar{E}_0^2 + B\bar{U}^n \left(1 + \frac{nu}{\bar{U}} + 0 \left(\frac{n^2}{2} \right) \right) \quad (E-3)$$

Combining equations (E-3) and (E-2) the fluctuating velocity, u , is given by:-

$$u = \frac{2\bar{E}\bar{U}e}{n(\bar{E}^2 - \bar{E}_0^2)} \quad (E-4)$$

E.2 Calculation of the Mean and Fluctuating Velocities for the Cross-Wire Anemometer

In Figure 77 U_{n_1} is the velocity normal to wire 1 - it is assumed that the wire is infinitely long so that its heat-loss rate depends only on the velocity component U_n normal to the wire ($U_n = U \cos \psi$).

Calibration of the wire (single or cross-wire) is advantageously made in low levels of turbulence; highly turbulent flows should be avoided due to the integration time needed to obtain a true mean value and the nonlinear errors affecting these measurements, (these errors considerably affect the partial derivatives of the calibration curves so changing the sensitivity of the wire, see Comte-Bellot [1931]). During calibration of the wire $V(t) = 0$, thus $U_n = U \cos \psi$; however, when taking measurements, $V(t) \neq 0$, and thus

$$U_n = U \cos \psi \pm V \sin \psi \quad (E-5)$$

When calibrating the wire King's law (equation (E-1)) becomes:-

$$E^2 = \bar{E}_0^2 + B \left(\frac{U_n}{\cos \psi} \right)^n \quad (E-6)$$

E.2.1 Mean velocity measurements

On taking mean velocity measurements equation (E-6)

becomes:-

$$\begin{aligned}\overline{E}_{1,2}^2 &= \overline{E}_{o,2}^2 + B_{1,2} \left(\frac{\overline{U} \cos \psi \pm \overline{V} \sin \psi}{\cos} \right)^n \\ &= \overline{E}_{o,2}^2 + B_{1,2} (\overline{U} \pm \overline{V} \tan \psi)^n\end{aligned}$$

Thus, $\overline{U} \pm \overline{V} \tan \psi_{1,2} = \left(\frac{\overline{E}_{1,2}^2 - \overline{E}_{o,2}^2}{B_{1,2}} \right)^{\frac{1}{n}}$ (E-7)

where subscripts 1,2 refer to wires 1 (U+V) and 2 (U-V) respectively.

From the two inherent equations (E-7) the mean velocities $\overline{U}$ and $\overline{V}$ are given as follows:-

$$\overline{V} = \frac{1}{(\tan \psi_1 + \tan \psi_2)} \left[\left(\frac{\overline{E}_1^2 - \overline{E}_{o,1}^2}{B_1} \right)^{\frac{1}{n}} - \left(\frac{\overline{E}_2^2 - \overline{E}_{o,2}^2}{B_2} \right)^{\frac{1}{n}} \right] \quad (E-8)$$

$$\text{and } \overline{U} = \frac{1}{(\tan \psi_1 + \tan \psi_2)} \left[\left(\frac{\overline{E}_1^2 - \overline{E}_{o,1}^2}{B_1} \right)^{\frac{1}{n}} \tan \psi_1 + \left(\frac{\overline{E}_2^2 - \overline{E}_{o,2}^2}{B_2} \right)^{\frac{1}{n}} \tan \psi_2 \right] \quad (E-9)$$

These equations apply to a cross-wire positioned in the U-V plane, similar equations exist for $\overline{U}$ and $\overline{W}$ for a cross-wire positioned in the U-W plane.

E.2.2 Fluctuating velocity measurements

From King's law (equation (E-1),

$$E_{1,2}^2 = \overline{E}_{o,2}^2 + B_{1,2} (U \pm V \tan \psi_{1,2})^n, \quad (E-10)$$

where subscripts 1 and 2 refer to wires 1 (U+V) and 2 (U-V) respectively. Now $U = \bar{U} + u$ and $V = \bar{V} + v$, i.e. the total velocity equals the sum of the mean and fluctuating velocities, where $\bar{U}$, $\bar{V}$ denote mean velocities and u, v fluctuating velocities; similarly $E = \bar{E} + e$. Substituting for U, V and E in equation (E-10) gives:-

$$(\bar{E} + e)_{1,2}^2 = \bar{E}_{0,1,2}^2 + B_{1,2} \left((\bar{U} + u) \pm (\bar{V} + v) \tan \psi_{1,2} \right)^n \quad (E-11)$$

For $|x| < 1$ the Binomial Theorem states:-

$$(1 + x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \dots \quad (E-12)$$

From equation (E-10):-

$$\begin{aligned} B_{1,2} \left((\bar{U} + u) \pm (\bar{V} + v) \tan \psi_{1,2} \right)^n \\ = B_{1,2} \left[(\bar{U} \pm \bar{V} \tan \psi_{1,2}) + (u \pm v \tan \psi_{1,2}) \right]^n \\ = B_{1,2} (\bar{U} \pm \bar{V} \tan \psi_{1,2})^n \left[1 + \frac{(u \pm v \tan \psi_{1,2})}{(\bar{U} \pm \bar{V} \tan \psi_{1,2})} \right]^n \end{aligned}$$

using the binomial theorem (equation (E-12)):-

$$= B_{1,2} (\bar{U} \pm \bar{V} \tan \psi_{1,2})^n \left[1 + \frac{n(u \pm v \tan \psi_{1,2})}{\bar{U} \pm \bar{V} \tan \psi_{1,2}} + o\left(\frac{1}{0}\right)^2 \right]$$

Substituting the above expression back into equation (E-11), and making use of equation (E-10) it can be seen that:-

$$2 (\bar{E}e)_{1,2} = B_{1,2} (\bar{U} \pm \bar{V} \tan \psi_{1,2})^n \cdot \frac{n(u \pm v \tan \psi_{1,2})}{(\bar{U} \pm \bar{V} \tan \psi_{1,2})},$$

$$\text{thus } u \pm v \tan \psi_{1,2} = \frac{(\bar{U} \pm \bar{V} \tan \psi_{1,2}) \cdot 2 (\bar{E}e)_{1,2}}{B_{1,2} (\bar{U} \pm \bar{V} \tan \psi_{1,2})^n} \quad (\text{E-13})$$

On separating out the two equations in equation (E-13) the fluctuating velocities are given by:-

$$u = \frac{(\bar{U} - \bar{V} \tan \psi_2) \cdot 2 \bar{E}_2 e_2}{B_2 (\bar{U} - \bar{V} \tan \psi_2)^n} + v \tan \psi_2 ,$$

$$v = \frac{1}{(\tan \psi_1 + \tan \psi_2)} \left[\frac{(\bar{U} + \bar{V} \tan \psi_1) 2 \bar{E}_1 e_1}{B_1 (\bar{U} + \bar{V} \tan \psi_1)^n} - \frac{(\bar{U} - \bar{V} \tan \psi_2) 2 \bar{E}_2 e_2}{B_2 (\bar{U} - \bar{V} \tan \psi_2)^n} \right] .$$

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T A B L E I

		e/c	y_c/b	a_c/b
Spreiter and Sacks [9]	Elliptic loading	11.0	0.107	0.078
	Actual loading	3.6		0.012
Chigier and Corsiglia [11]	Experimental measurements	4.0	0.029	0.017

T A B L E II

	Γ/ν	ν_t/ν	$10^3\alpha$	Λ
Rose and Dee [66]	$\approx 10^7$	2000	0.2	1.22
Dosanjh, Gasparek and Eskinazi [10]	2×10^3	10	5	0.81
Newman [55]	2×10^4	30-50	2	0.69-0.89
Templin [67]	5×10^4	70	1.4	0.84
Mabey [68]	5×10^4	77	1.5	1.01

T A B L E III

<u>Crow [98]</u>		<u>Parks [106]</u>
8.4 S'	Most unstable long wavelength:	7.2 S'
9.4 $\frac{RS'}{C_L U_\infty}$	Grew by a factor e in time:	9.5 $\frac{RS'}{C_L U_\infty}$
0.42 S'	Most unstable short wavelength:	0.57 S'
7.75 $\frac{RS'}{C_L U_\infty}$	Grew by a factor e in time:	7.75 $\frac{RS'}{C_L U_\infty}$

T A B L E IV

Principal Dimensions of the Handley-Page 115 Aircraft

Overall length	15.3m
Gross wing area	40.2m ²
Span	6.1m
Centreline chord	12.2m
Leading edge sweep	74° 42'
Trailing edge sweep	0°
Dihedral	0°
Mass	2280kg

T A B L E V

Summary of the Experiments carried out to examine the Structure
of the 'Bursting' Phenomenon (the Structured Disturbance)

Experiment	Delta Wing, Angle of Incidence, Position of Wing	Length of Working Section	Free Stream Speed, U_{∞}	Measuring DISA hot-wire Anemometer
(i)	63" chord delta wing, 15°, contraction of wind tunnel.	9.78'	33.5 msec ⁻¹	Single wire DISA 55 P01
(ii)	63" chord delta wing, 15°, contraction of wind tunnel.	9.78'	33.6 msec ⁻¹	Cross wire DISA 55 P51
(iii)	12.8" chord delta wing, - 12.5° (nosedown), working section.	28'	26.09 msec ⁻¹	Cross wire DISA 55 P51
(iv)	28" chord delta wing, - 12.5° (nosedown), working section.	28'	24.34 msec ⁻¹	Cross wire DISA 55 P51

Continued/.....

TABLE V (Continued)
Summary of the Experiments carried out to examine the Structure
of the 'Bursting' Phenomenon (the Structured Disturbance)

Experiment	Position of Vortices (in mm)	Vortex under study	Position of Trigger cross- wire (in mm)	Distances between probes and rear edge of model
(i)	Vortex 1: Y = 795 Z = 692.5 Vortex 2: Y = 545 Z = 700.5	Vortex 2	Y = 545 Z = 662.5	15.3' (2.92 chord lengths)
(ii)	Vortex 1: Y = 792 Z = 692 Vortex 2: Y = 550 Z = 692	Vortex 2	Y = 550 Z = 654	15.3' (2.92 chord lengths)
(iii)	Vortex 1: Y = 690 Z = 792.5 Vortex 2: Y = 585 (approx.) Z = 792.5	Vortex 1	Y = 705 Z = 816	18.67' (17.5 chord lengths)
(iv)	Vortex 1: Y = 850 Z = 681 Vortex 2: Y = 588 Z = 699	Vortex 2	Y = 588 Z = 749	18.08' (7.75 chord lengths)

T A B L E VI

Summary of the Characteristic Phenomena in Experiments (i) - (iv)

Experiment	Delta Wing, Angle of Incidence, Position of Wing	Wing Reynolds number, $Re_w = \frac{U_\infty c}{\nu}$	Vortex Reynolds number, $Re_v = \frac{\Gamma_\infty}{\nu}$	Core diameter of the Vortex under study (in mm)	Wavelengths of the Structured Disturbance (approx.)
(i)	63" chord delta wing, 15°, contraction of wind tunnel.	1.65×10^6	9.22×10^4	40	1.4 m
(ii)	63" chord delta wing, 15°, contraction of wind tunnel.	1.65×10^6	9.22×10^4	38	1 m
(iii)	12.8" chord delta wing, - 12.5° (nosedown), working section.	5.81×10^5	1.47×10^4	27.5	1.04 m
(iv)	28" chord delta wing, - 12.5° (nosedown), working section.	1.18×10^6	6.53×10^4	60	.96 m

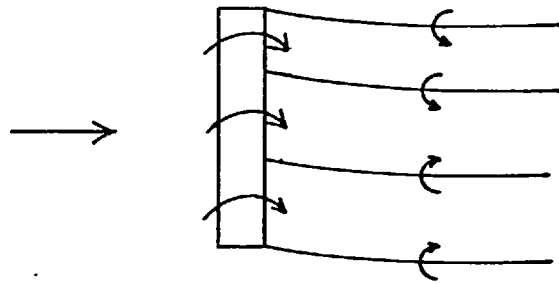


Fig.1 Line vortices starting at the surface of an aerofoil.

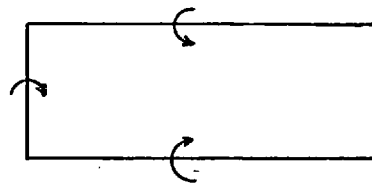


Fig.2 A horseshoe vortex system.

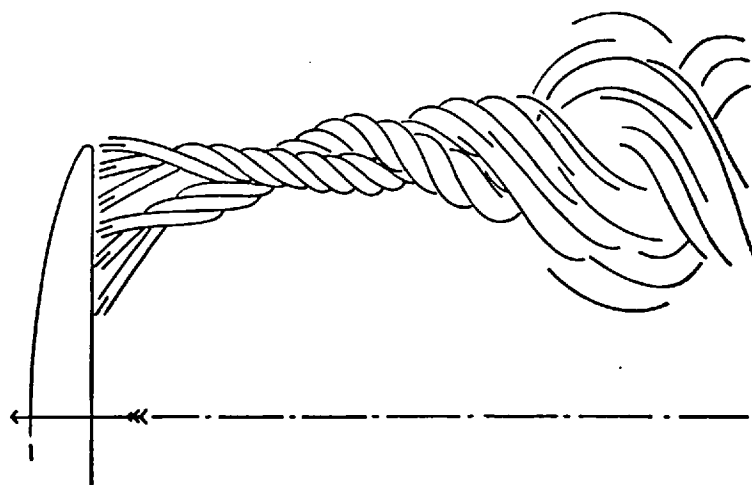


Fig. 3 Lanchester's concept of tip vortex roll-up [1].

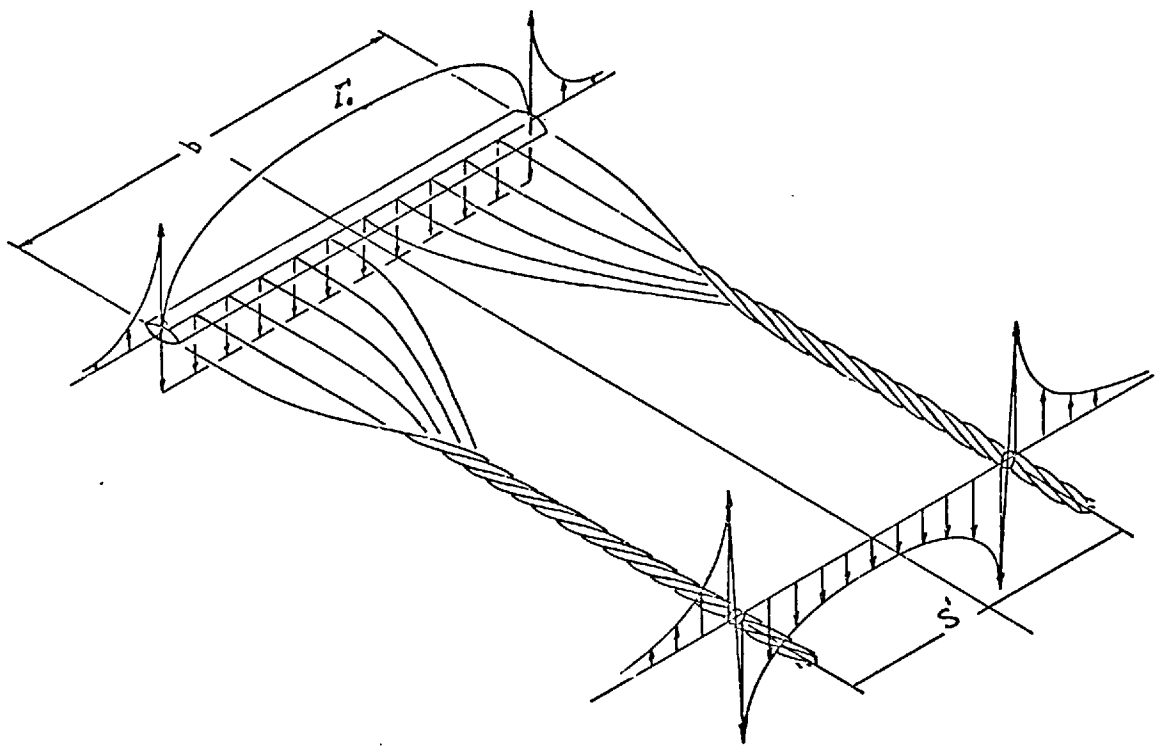


Fig.4 Sketch of vortex roll-up.

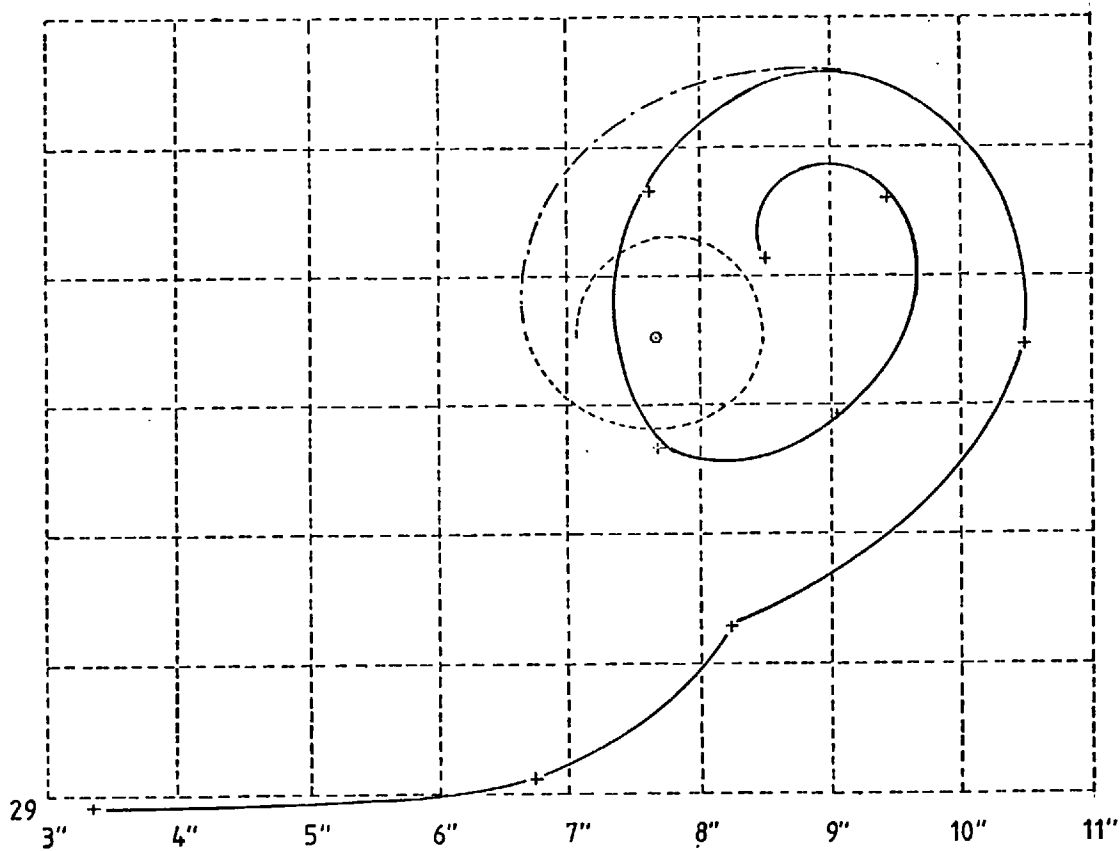
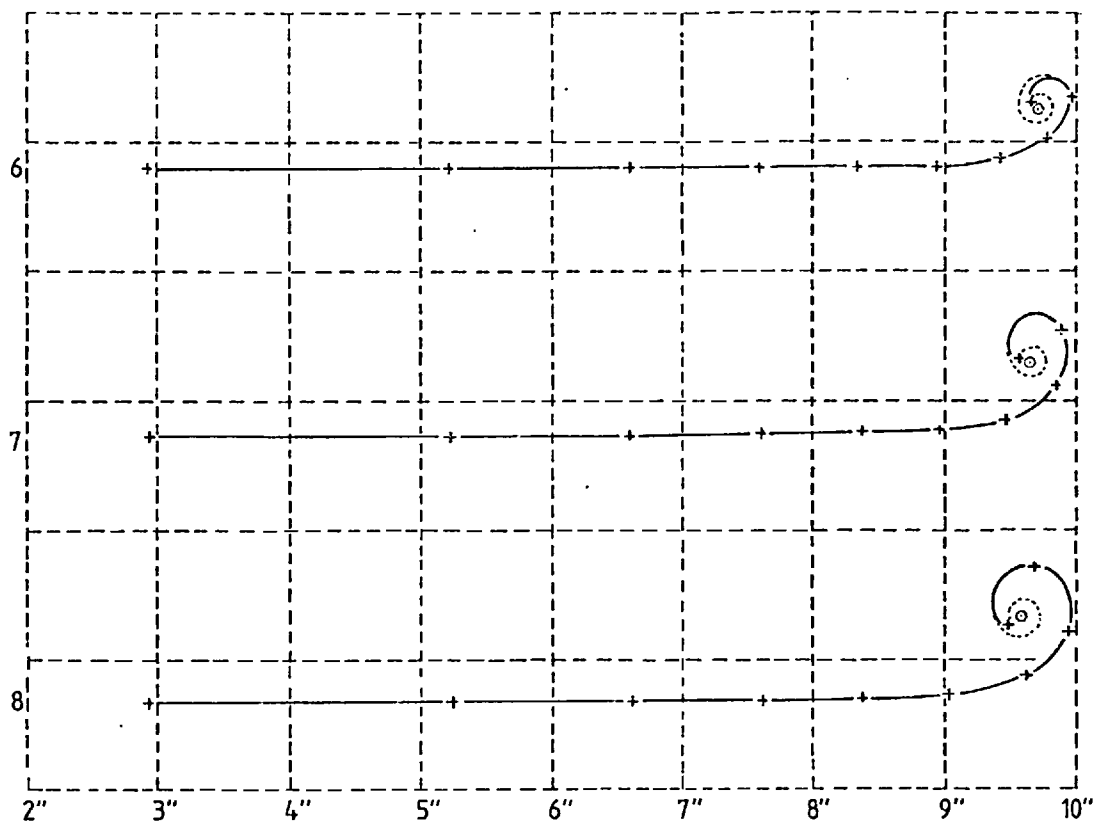


Fig.5 Westwater's calculation of roll-up [4].

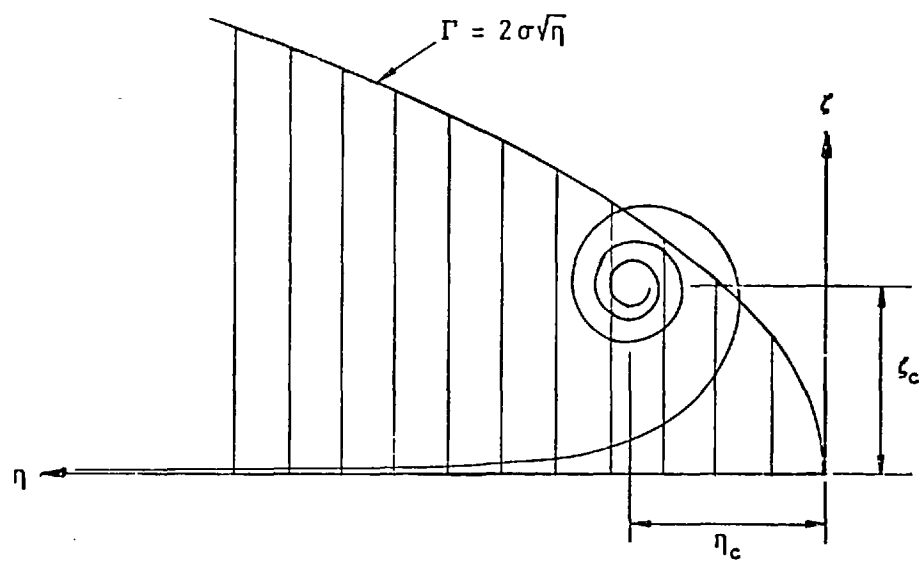


Fig.6 Vortex sheet and coordinate axes according to Kaden [3].

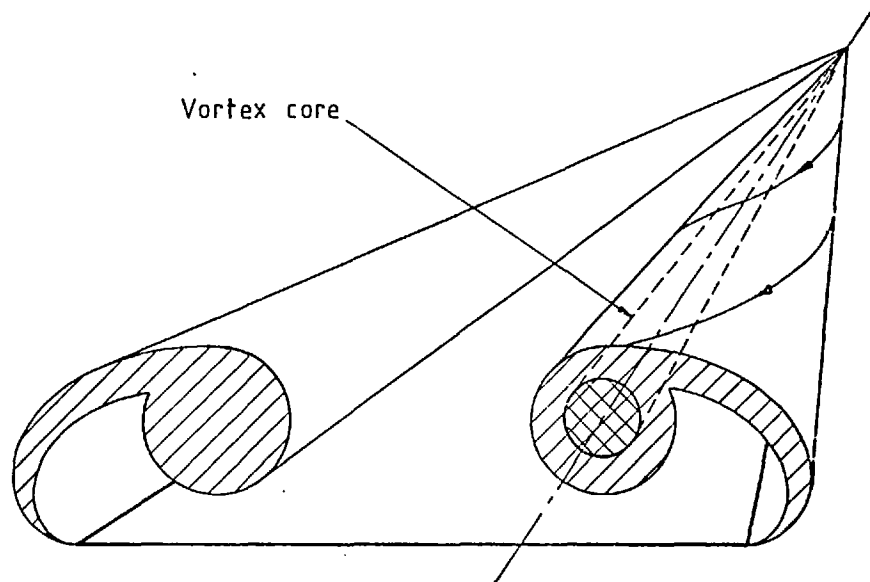


Fig.7 Vortex core over a slender delta wing

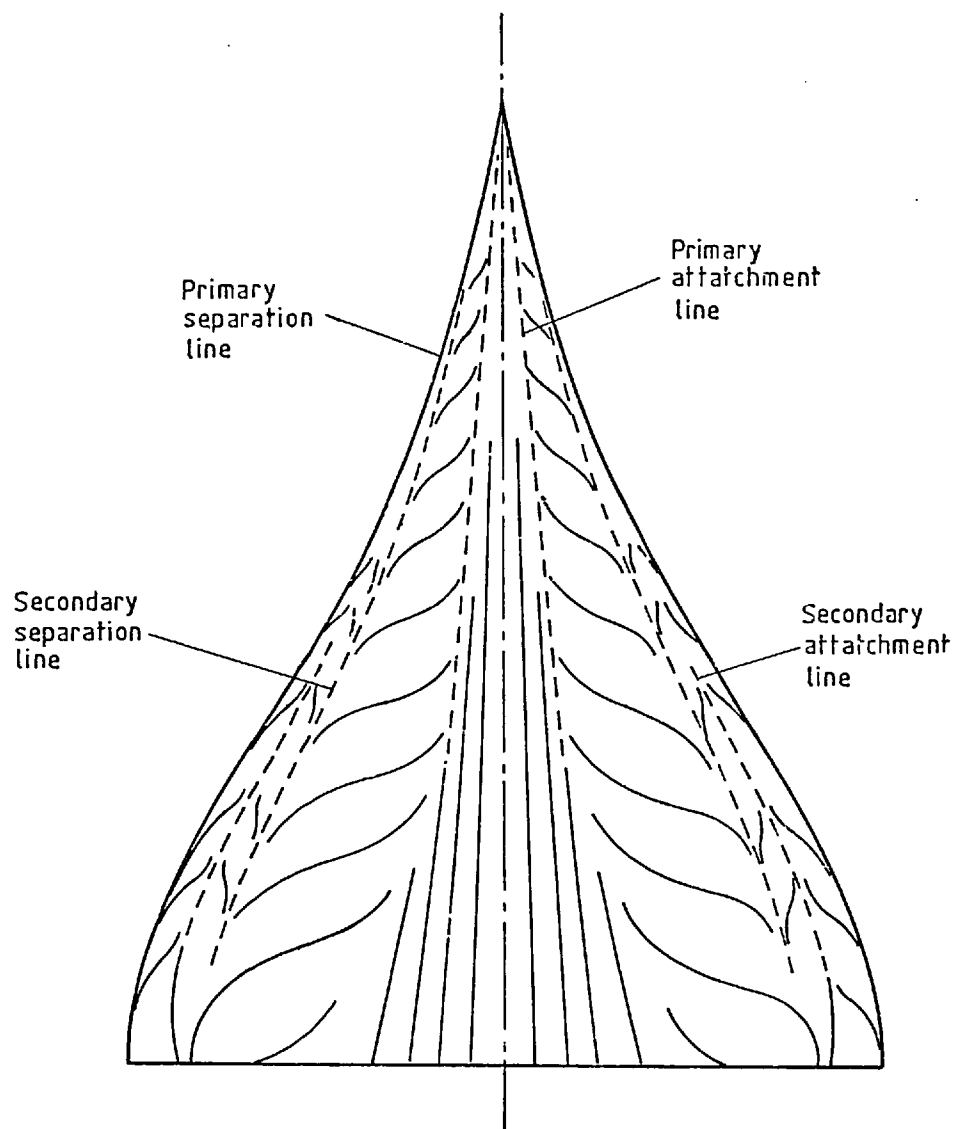


Fig. 8 Streamline pattern in the upper surface of a slender wing at low speeds; $\alpha = 15^\circ$.

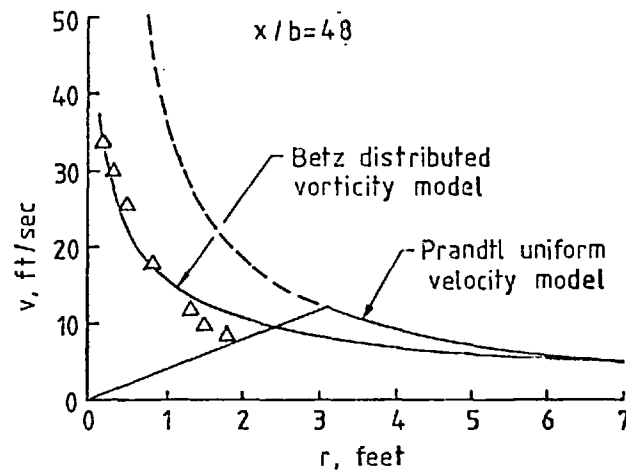


Fig.9 Tangential velocity in the vortex wake of an Army 0-1 aircraft [22].

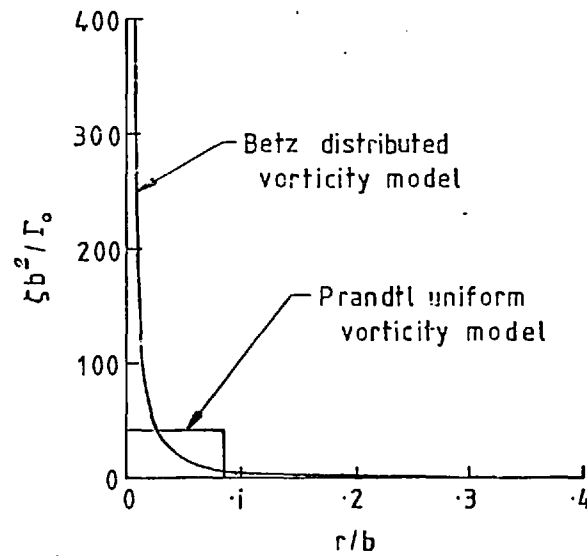


Fig.10 The vorticity distribution for the vortex velocities shown in Fig.9.

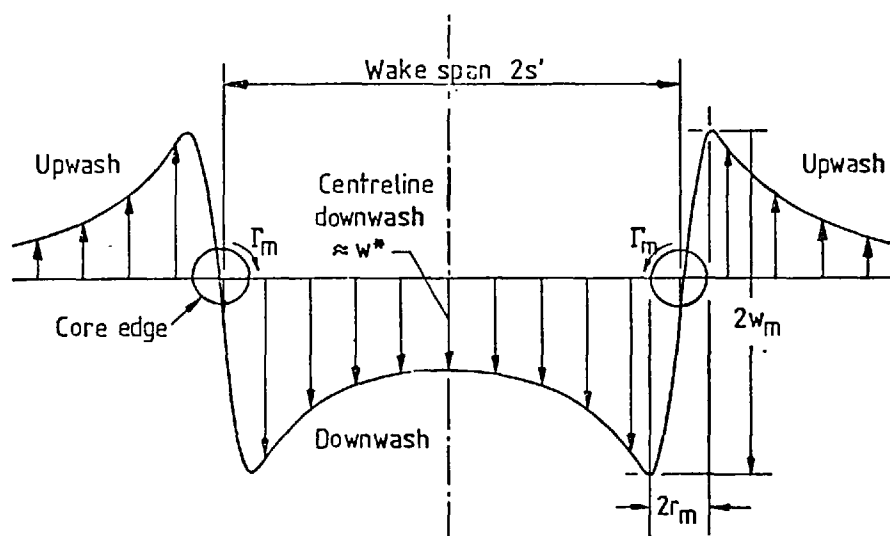


Fig.11 Cross-sectional profile of the downstream wake.

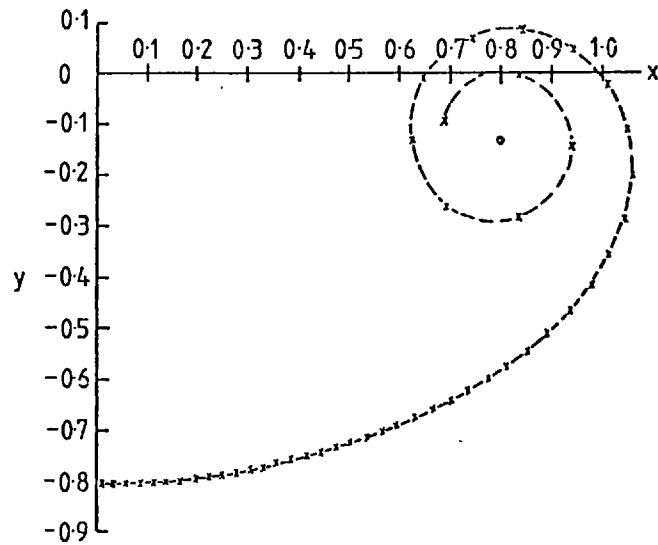


Fig.12 Shape of the spiral at $t^*=1.0$, $\theta_c=90^\circ$, $M=60$, $dt^*=2 \times 10^{-3}$, vortices initially equispaced for the right-hand half of the vortex sheet as calculated by Moore [36].

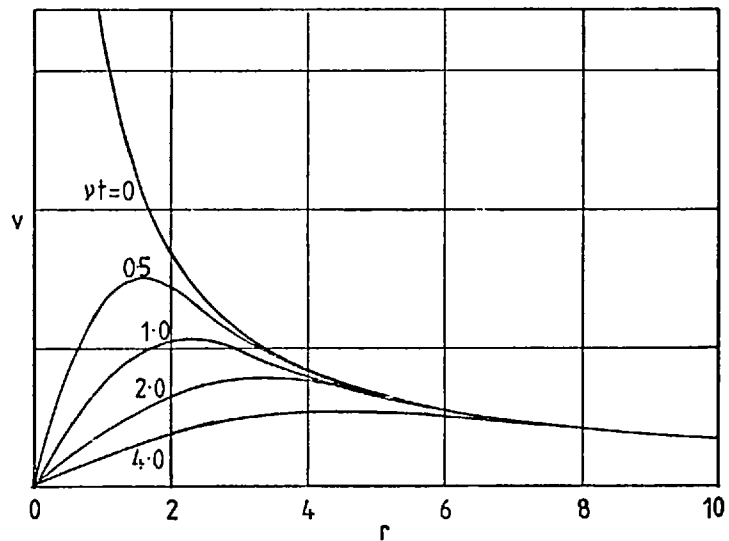


Fig.13 Velocity distribution associated with a spreading line vortex.

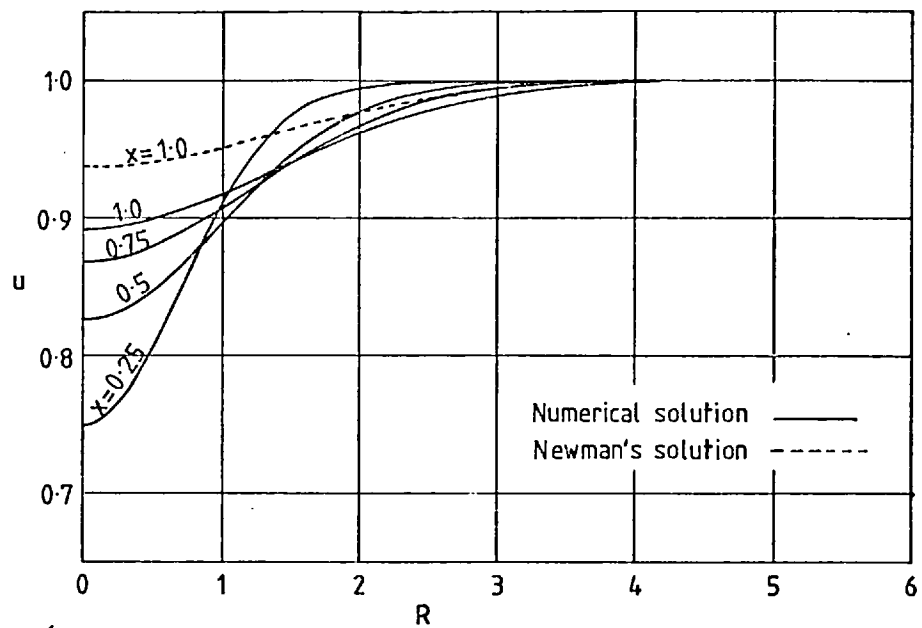


Fig.14a Profiles of axial velocity in a trailing vortex [62].

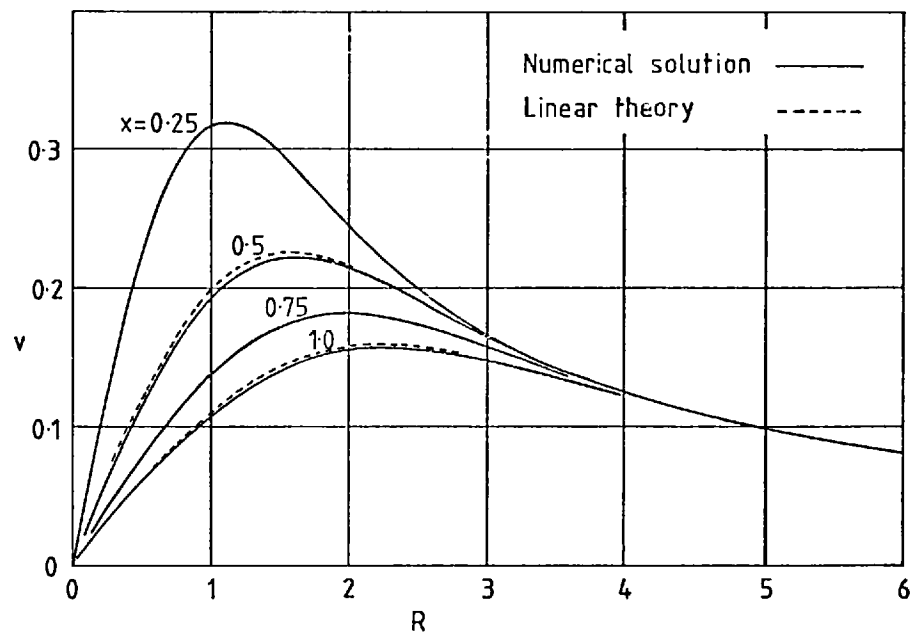


Fig.14b Profiles of circumferential velocity in a trailing vortex [62].

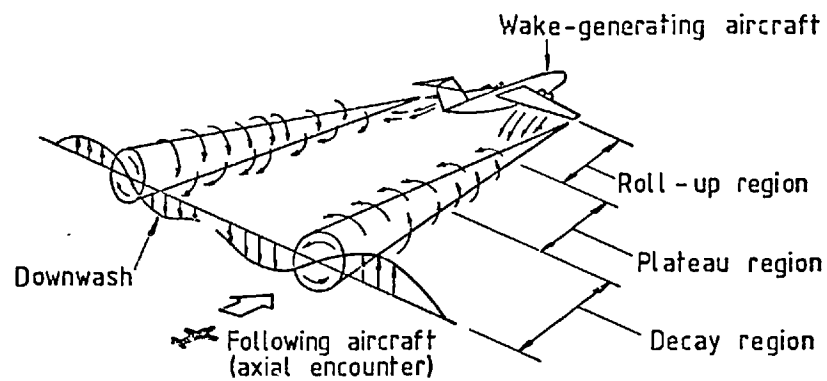


Fig.15 Flow field produced by lift-generated vortices.
(distances not drawn to scale.)

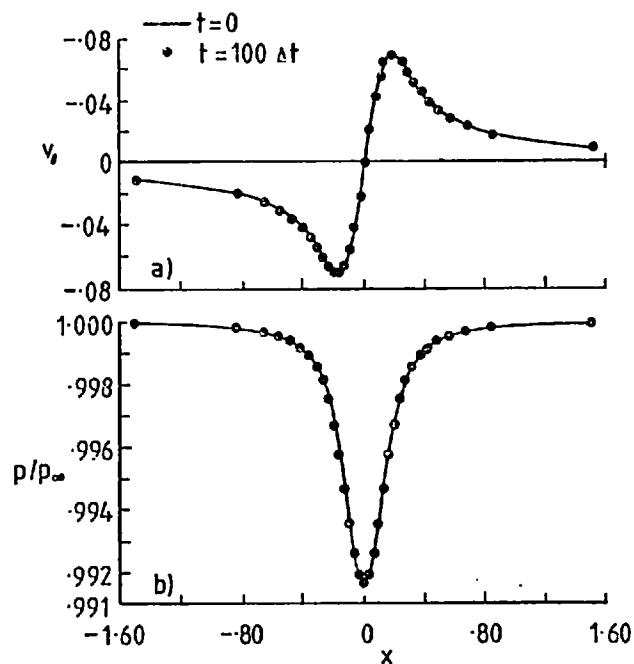


Fig.16 Accuracy check of compressible equations for single Lamb vortex: a) pressure; b) radial velocity;
 $\Gamma = 0.106$, $r_c = 0.15$, Courant Number = 5 [64].

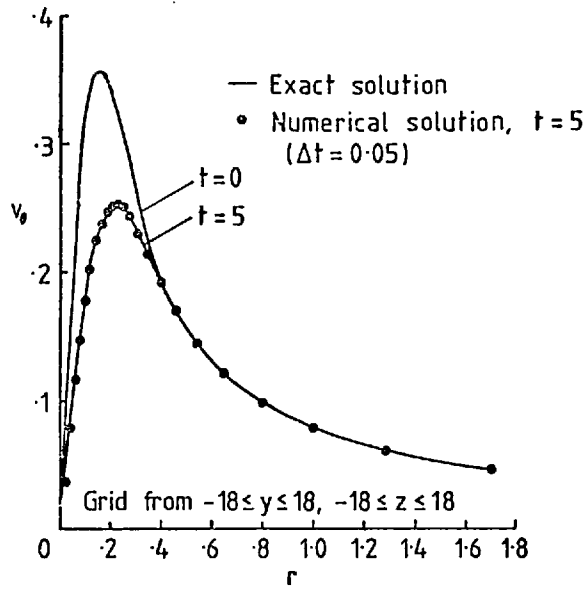


Fig. 17 Accuracy check of incompressible Navier-Stokes equations for single Lamb vortex; $\Gamma = 0.5$, $d_c = 2r_c = 0.28284$, $\nu = 0.001$ [64].

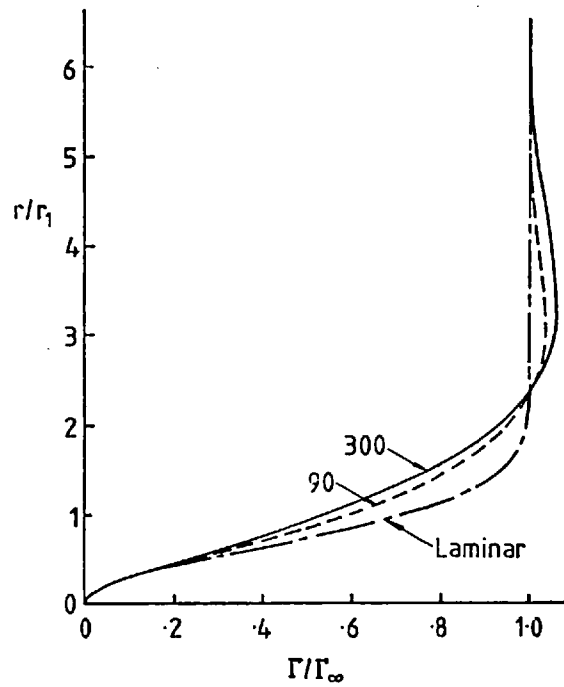


Fig. 18 Comparison of circulation profiles normalized to Γ_∞ for several values of $(\bar{v}_{10}/r_{10})t$ [79].

$\bar{v}_{10}$ initial core velocity
 r_{10} initial core radius

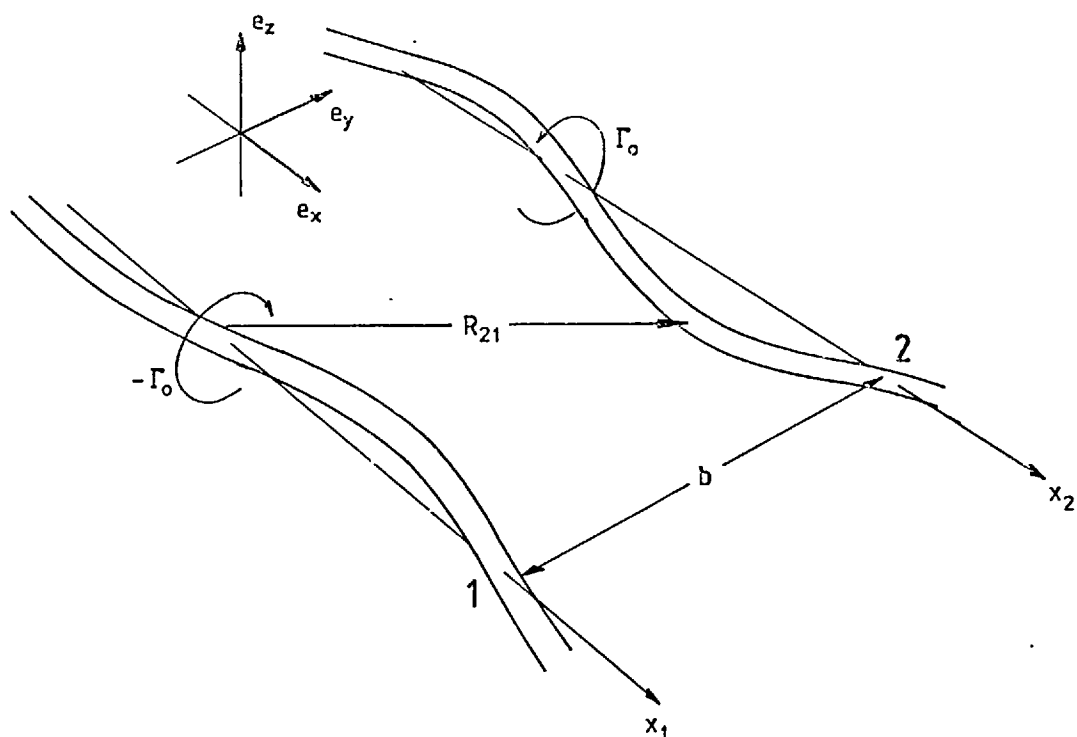


Fig. 19 Geometrical quantities entering Crow's analysis [98].

The vortices are viewed from above, so the aircraft generating them lies beyond the upper left-hand corner of the figure.

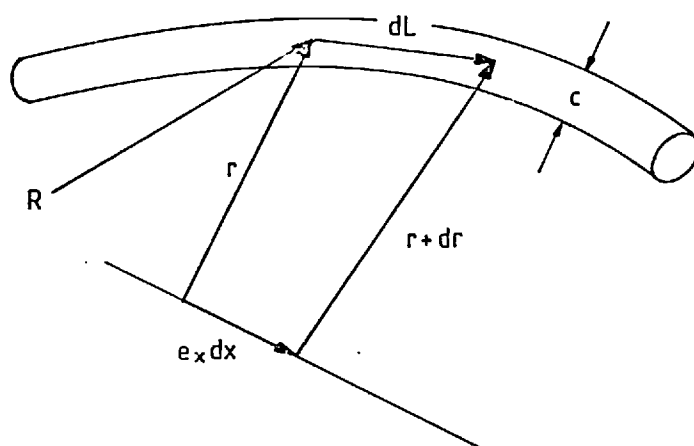


Fig. 20 Relation between an arc length dL and a displacement $e_x dx$ down the longitudinal axis in Crow's analysis [98].

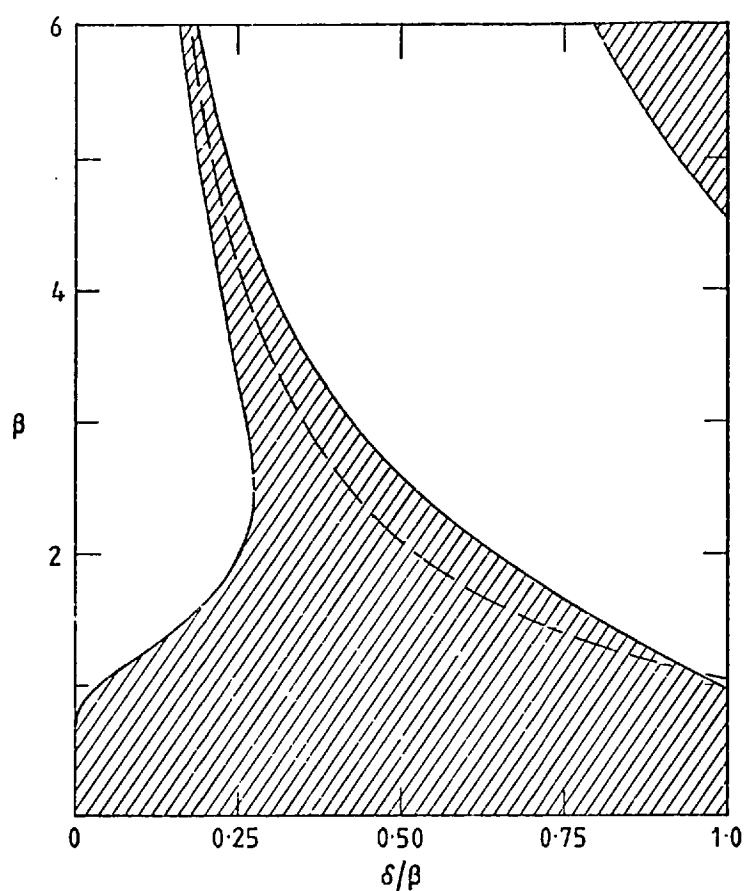


Fig. 21 Stability diagram for mode S. Regions of instability are shaded, and the locus of vanishing self-induction is shown as a dashed line. The shaded region in the upper right-hand corner is probably a spurious effect of the cut-off model.

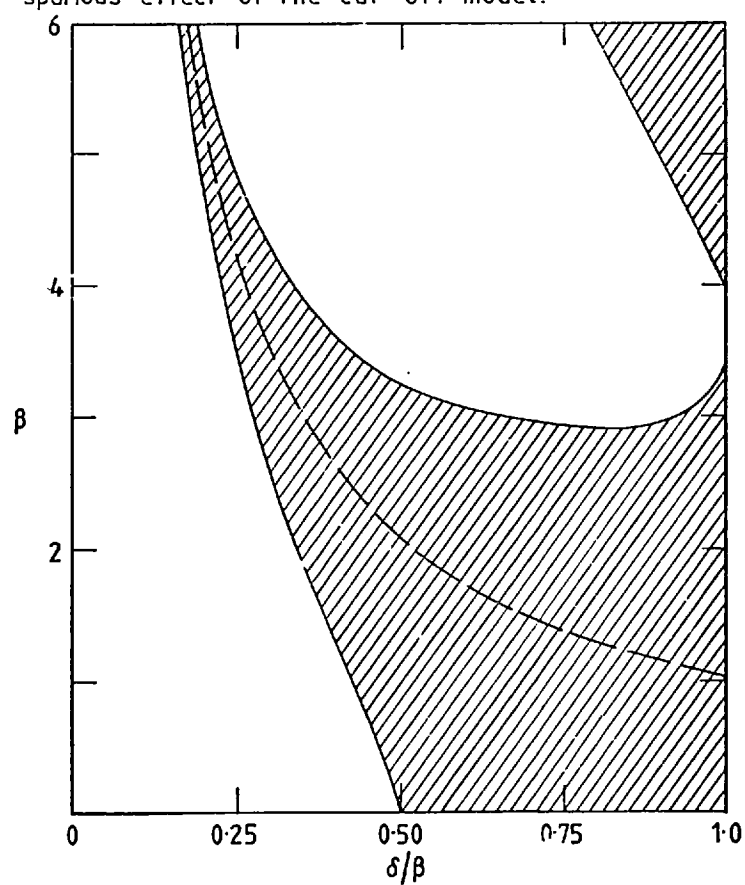


Fig. 22 Stability diagram for mode A [98].

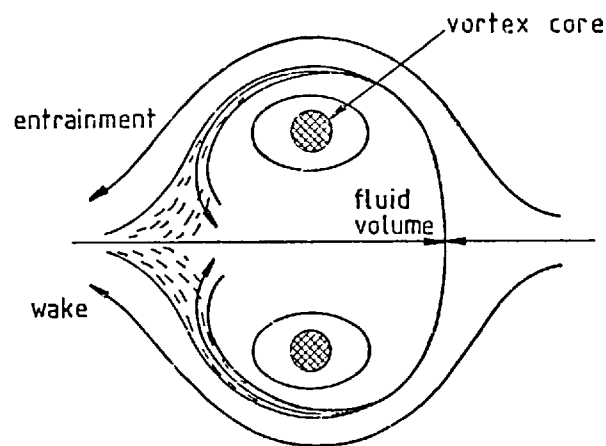


Fig. 23 General features of the flow field of a vortex ring in a real fluid.

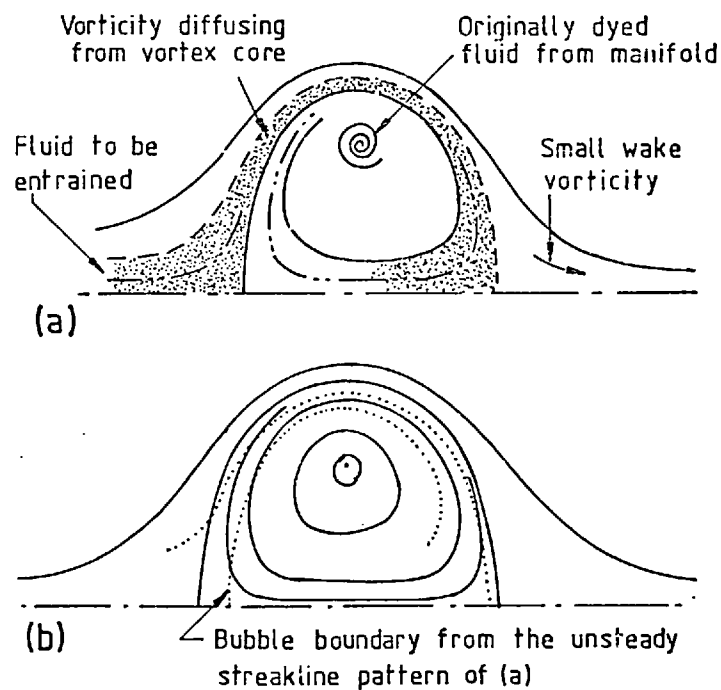


Fig. 24 (a) Diagrammatic view of entrainment by a vortex ring, showing fluid from upstream being entrained into the outer regions of the ring. (b) Diagrammatic view of instantaneous streamlines in a frame of reference moving with the mean velocity U of the ring.

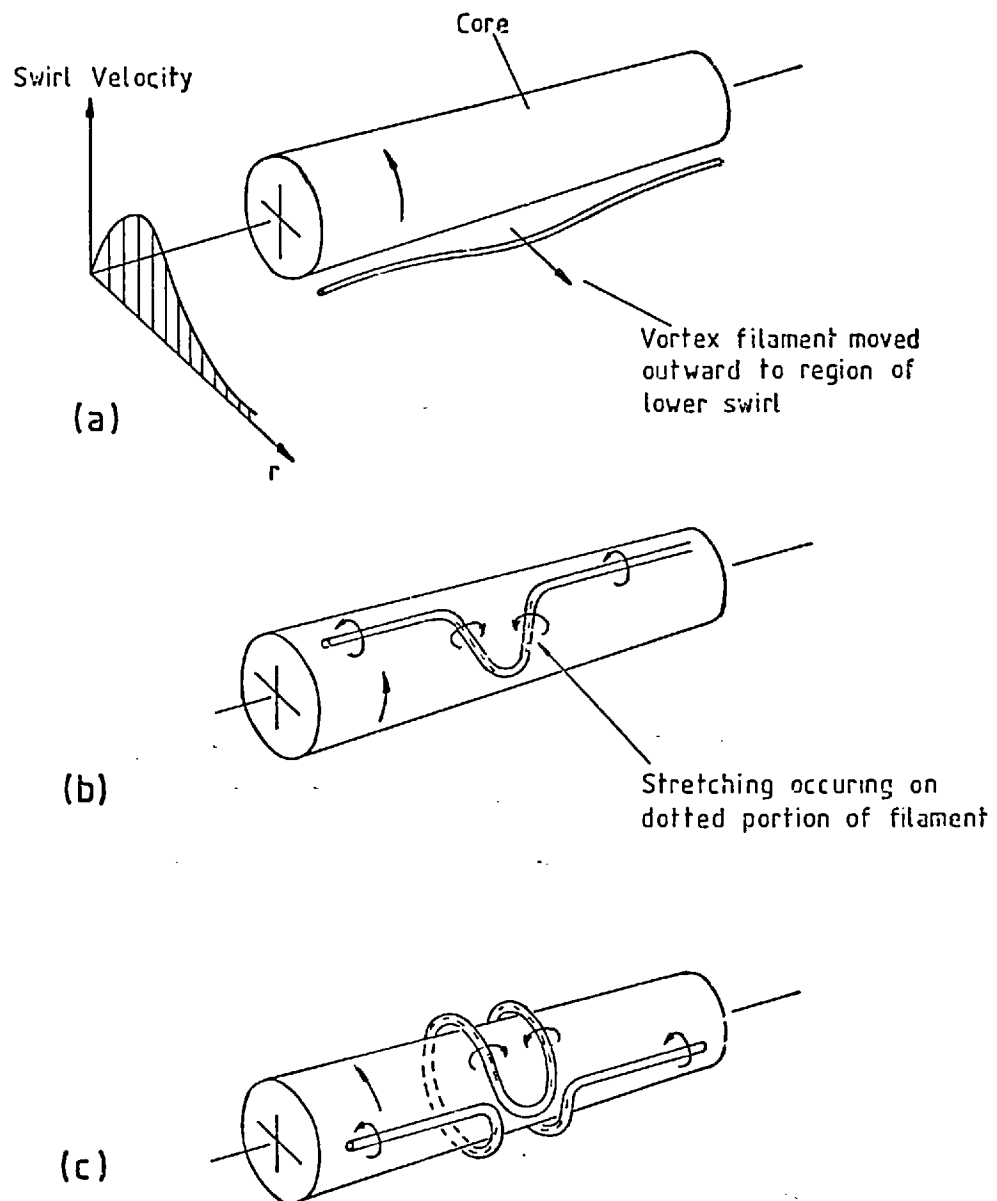


Fig. 25 A suggested mechanism for the development of the ring vortices [119].

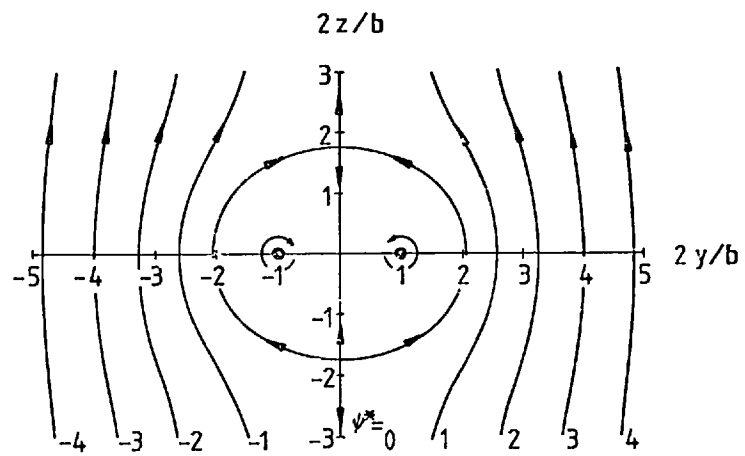


Fig. 26 Streamlines produced by a pair of vortex tubes of opposite sign as seen by an observer at rest relative to the two vortices.

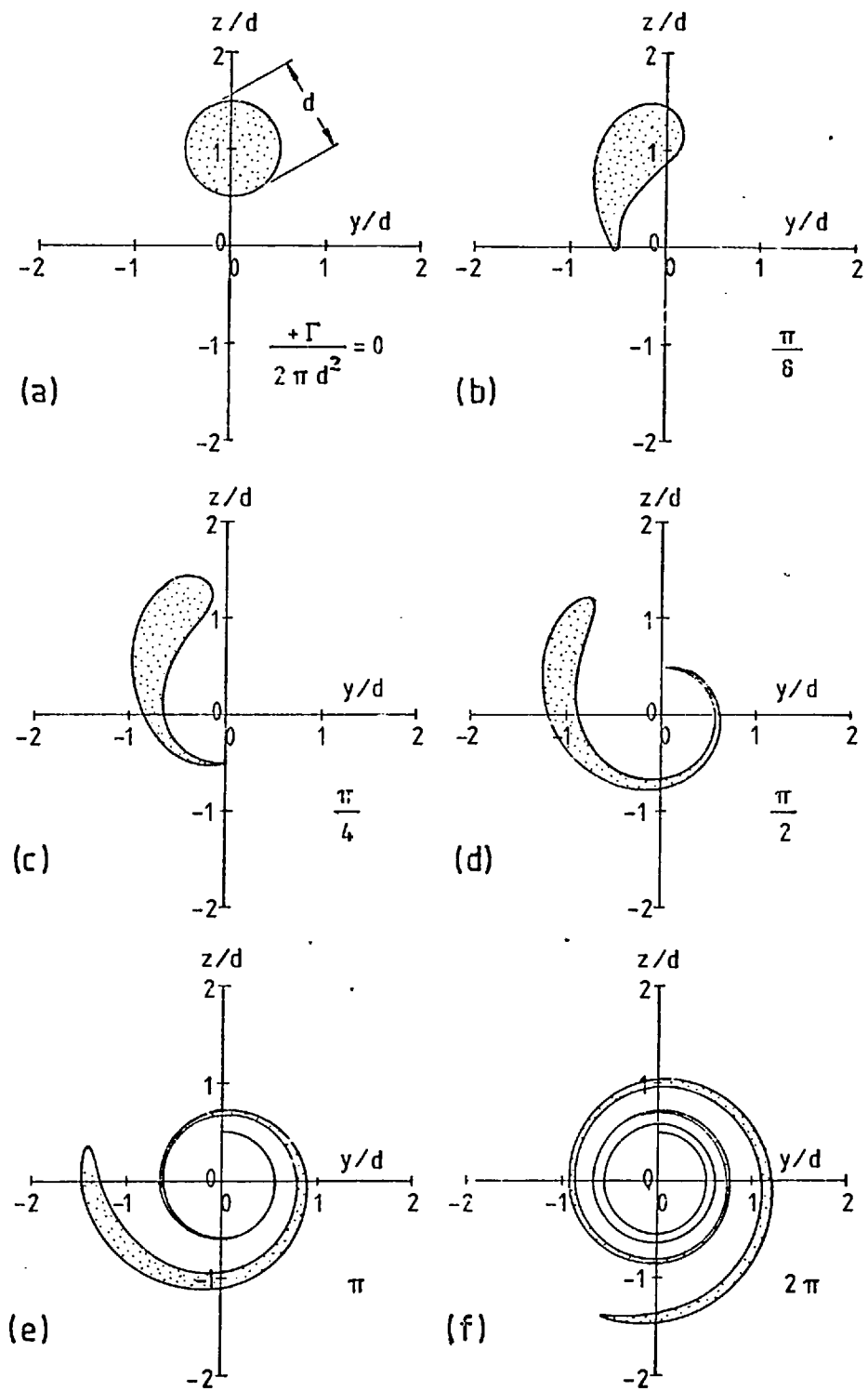
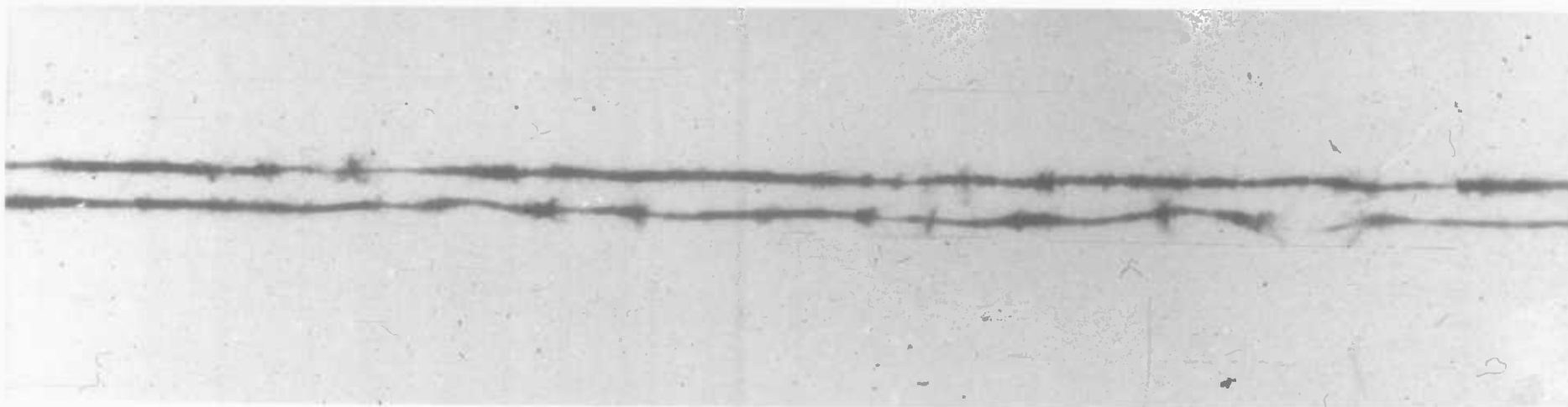
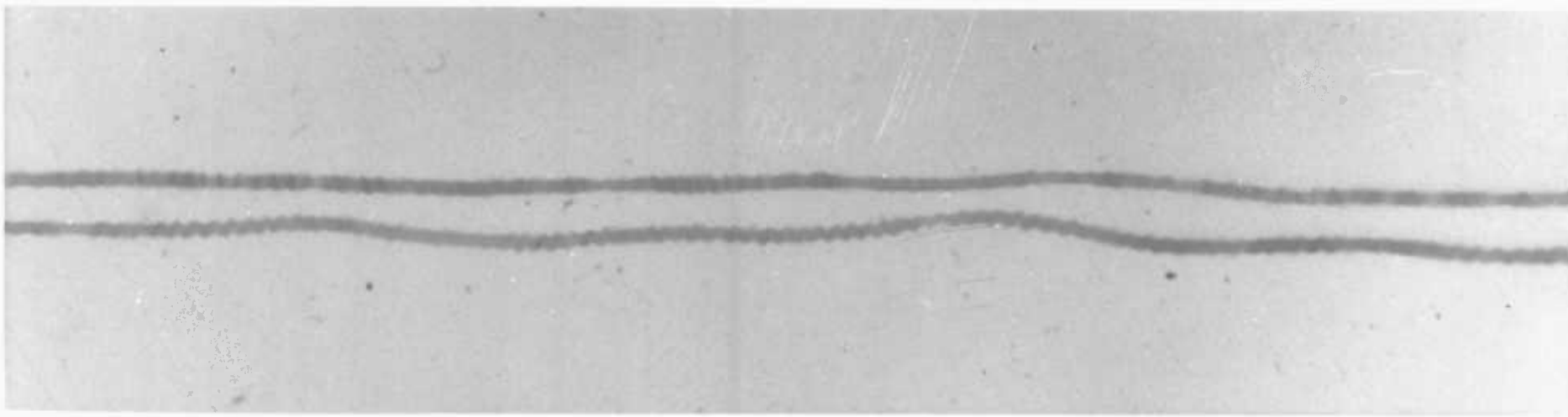


Fig. 27 The inviscid merging of a vortex of vanishingly small strength with a very strong vortex. [127].



a)



b)

Fig.28 Vapour trails behind a wide-bodied jet aircraft.

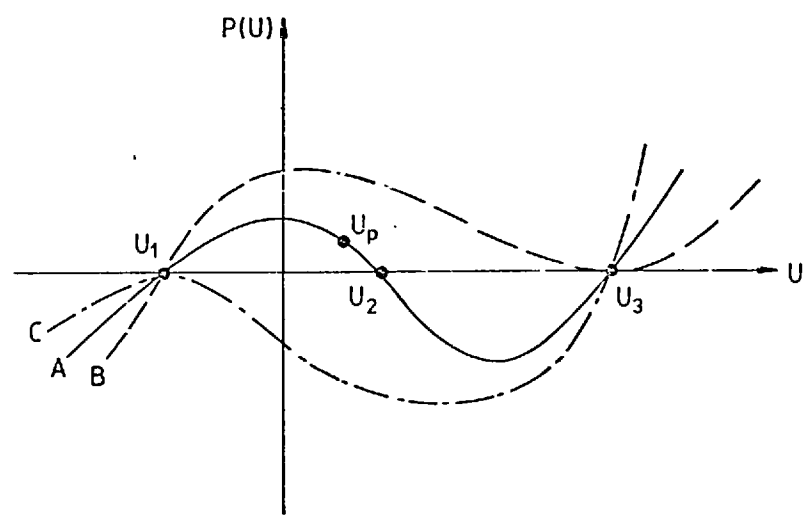


Fig. 29 The cubic $P(U)$ that determines the character of stationary waves.

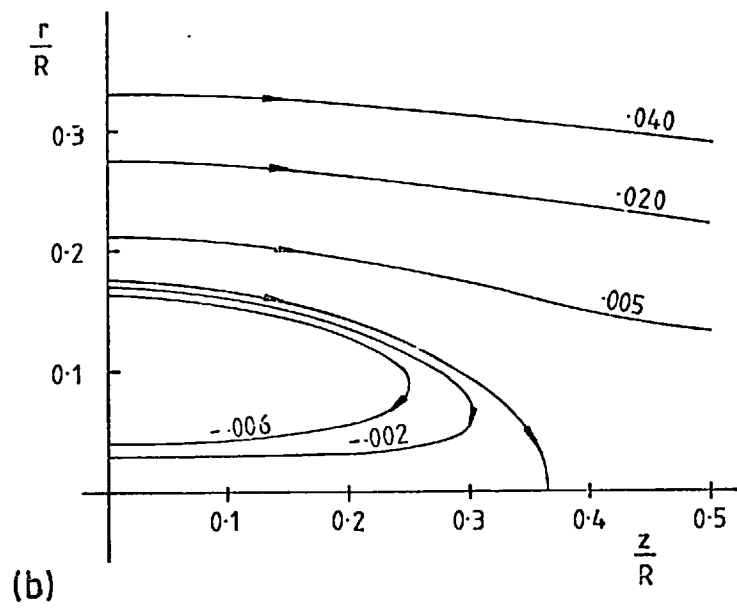
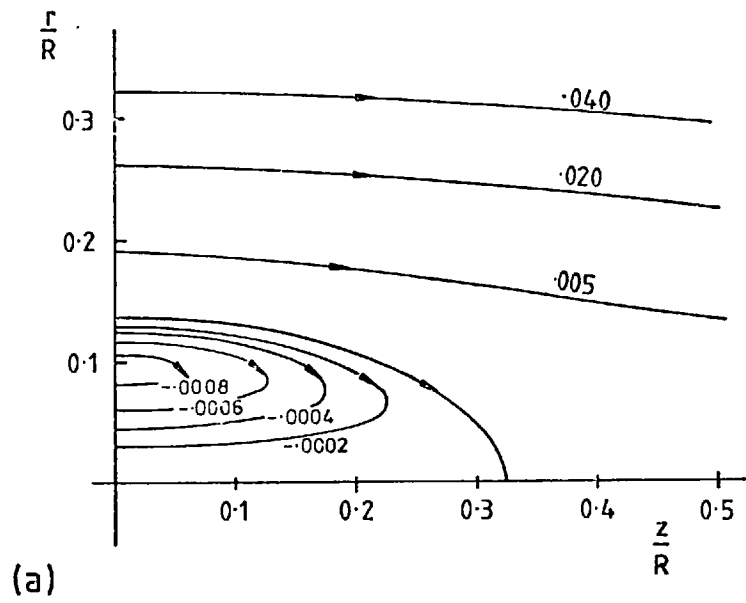


Fig. 30 Calculated vortex breakdown eddies [168].
 (a) $\epsilon = 0.8$
 (b) $\epsilon = 1.0$

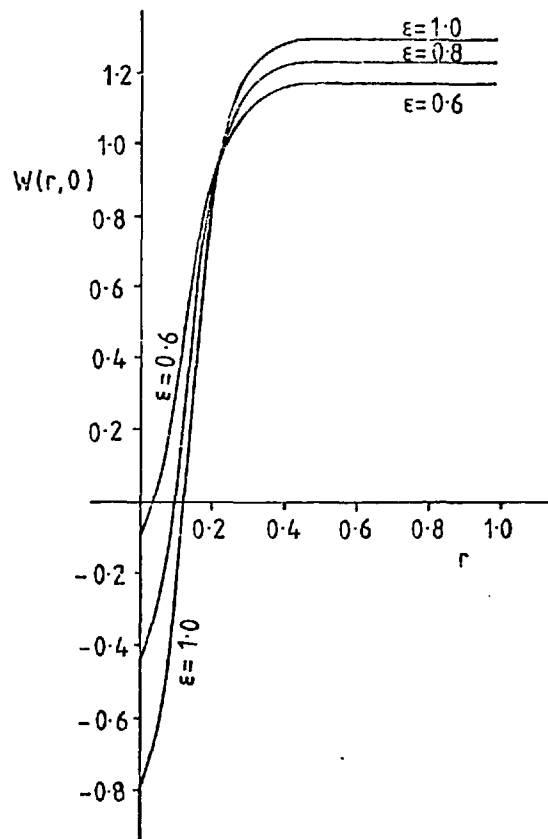


Fig. 31 Total axial velocity at the midplane [168].

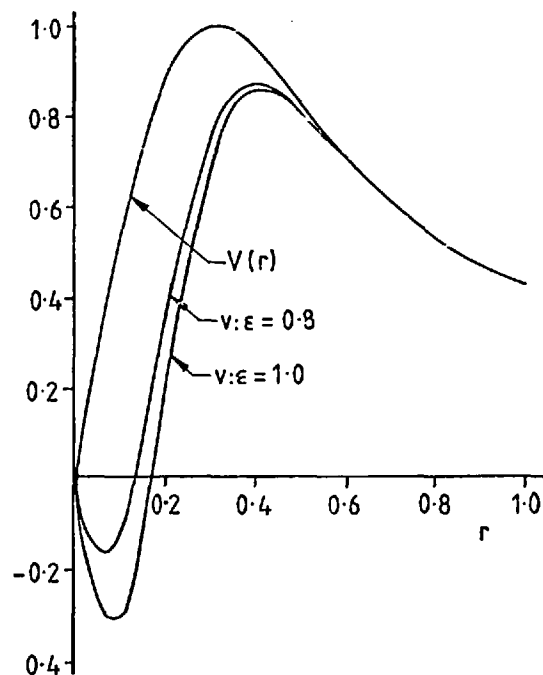


Fig. 32 Base flow and perturbed swirl velocities for an exponential variation of base flow circulation [168].

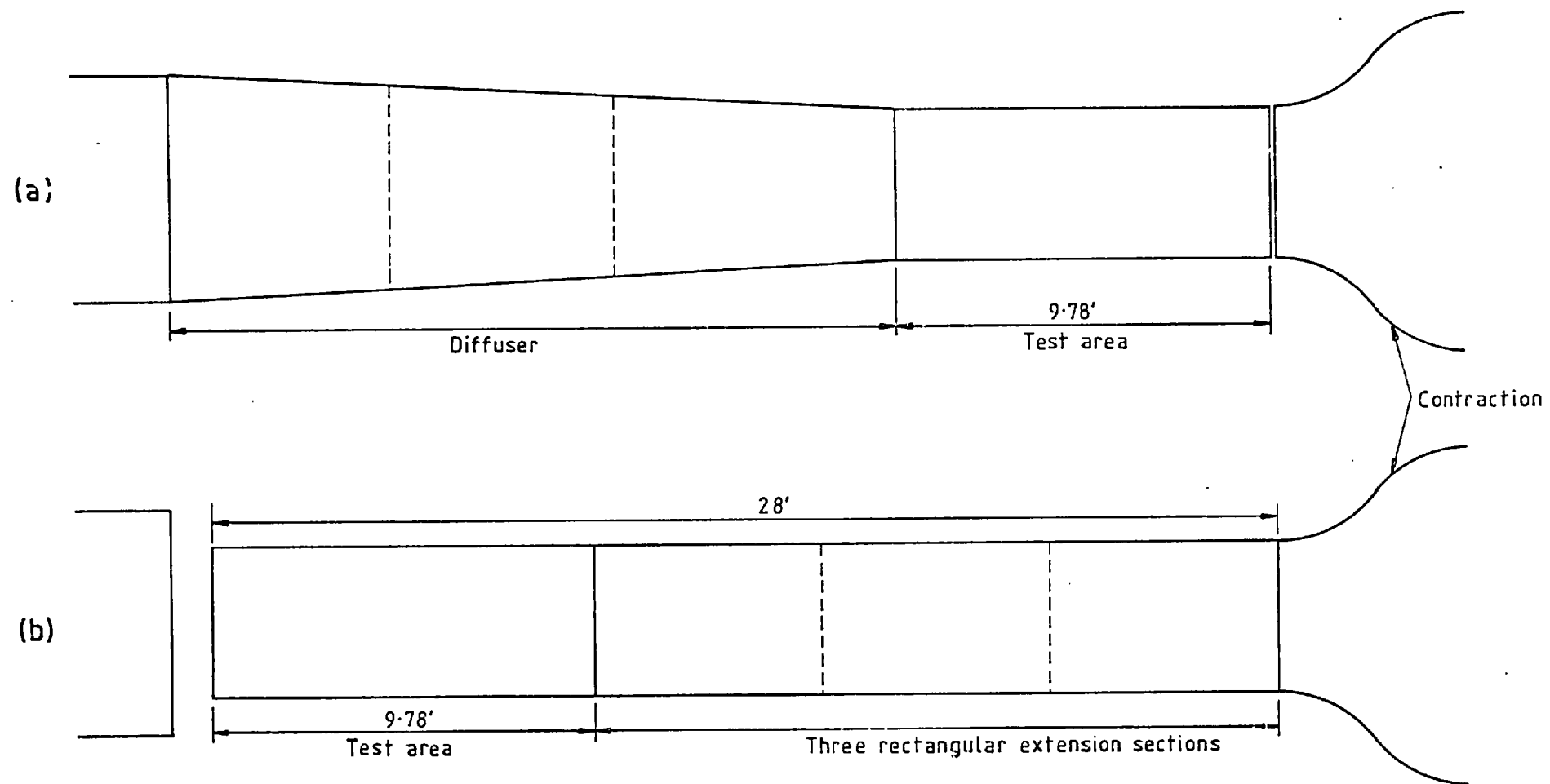
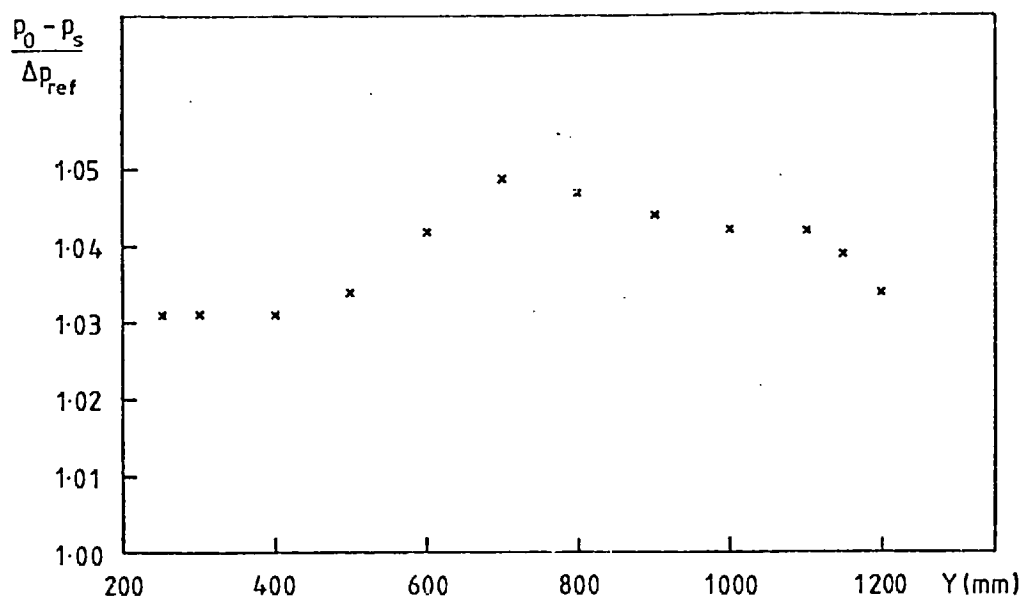
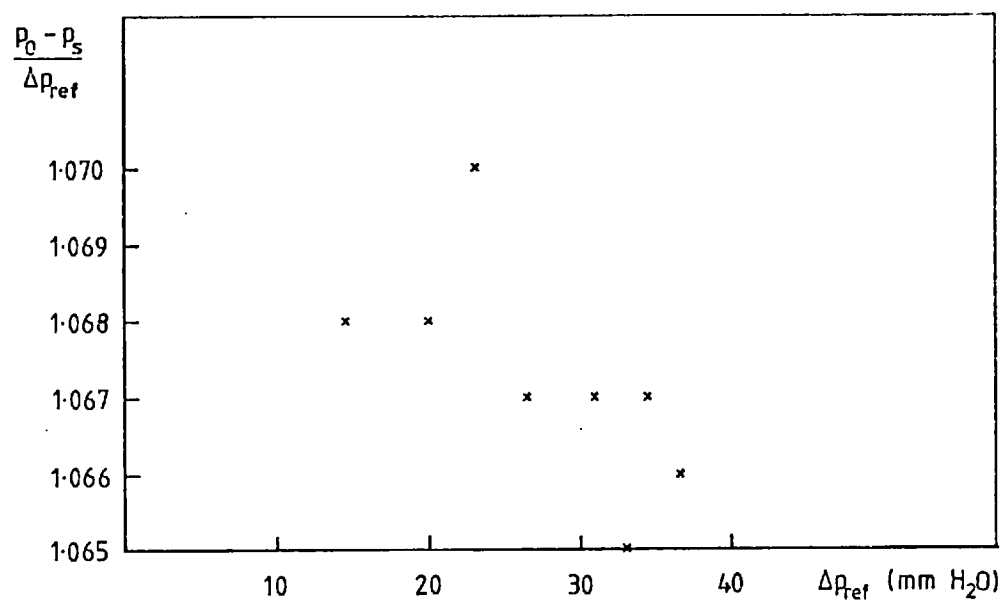


Fig. 33 Alternative configurations for 4.5' x 4' wind tunnel



(a) Calibration factor across the test area at height $Z = 609.6$ mm, at 23.8 m/sec.



(b) Calibration factor at $Y = 706$ mm, $Z = 1013$ mm.

Fig. 34 Calibration factor $\left(\frac{p_0 - p_s}{\Delta p_{ref}}\right)$, $20.4'$ from the start of the long working section.

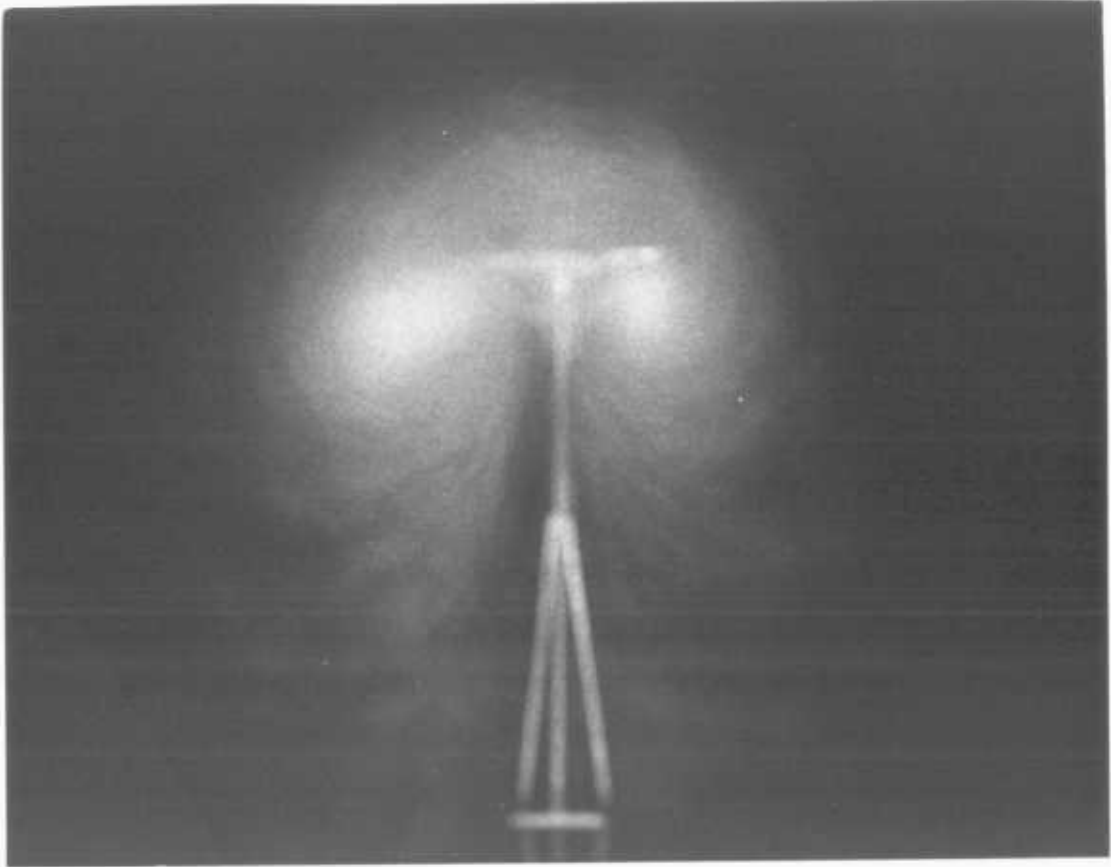


Fig. 35a Trailing vortices behind the 12.8" chord delta wing.

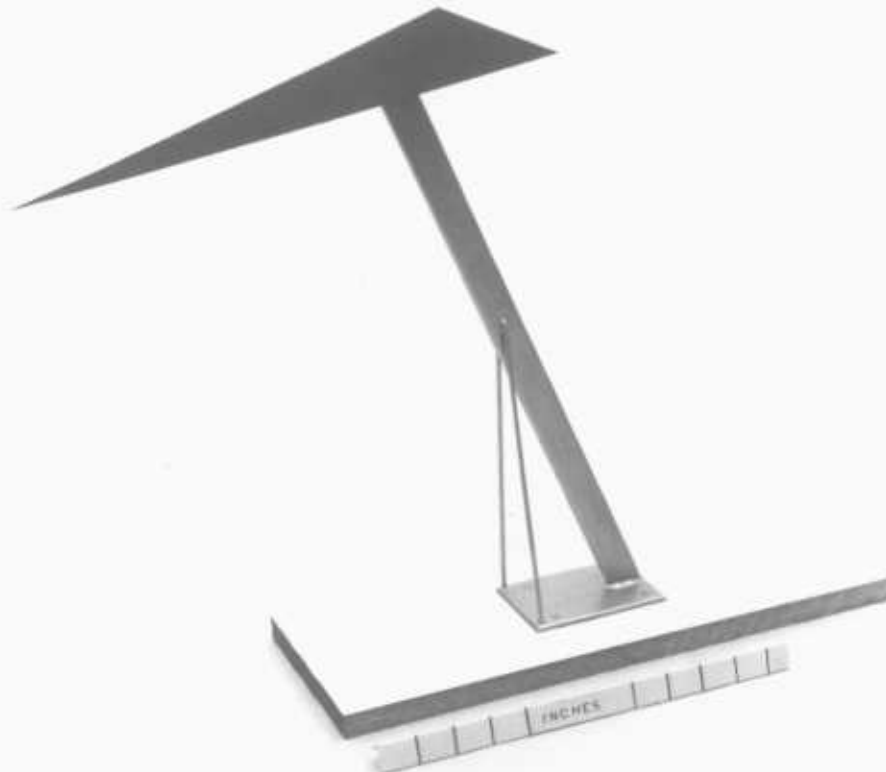


Fig. 35b 12.8" chord delta wing mounted on its support.



Fig.36 28" chord delta wing mounted on its support.

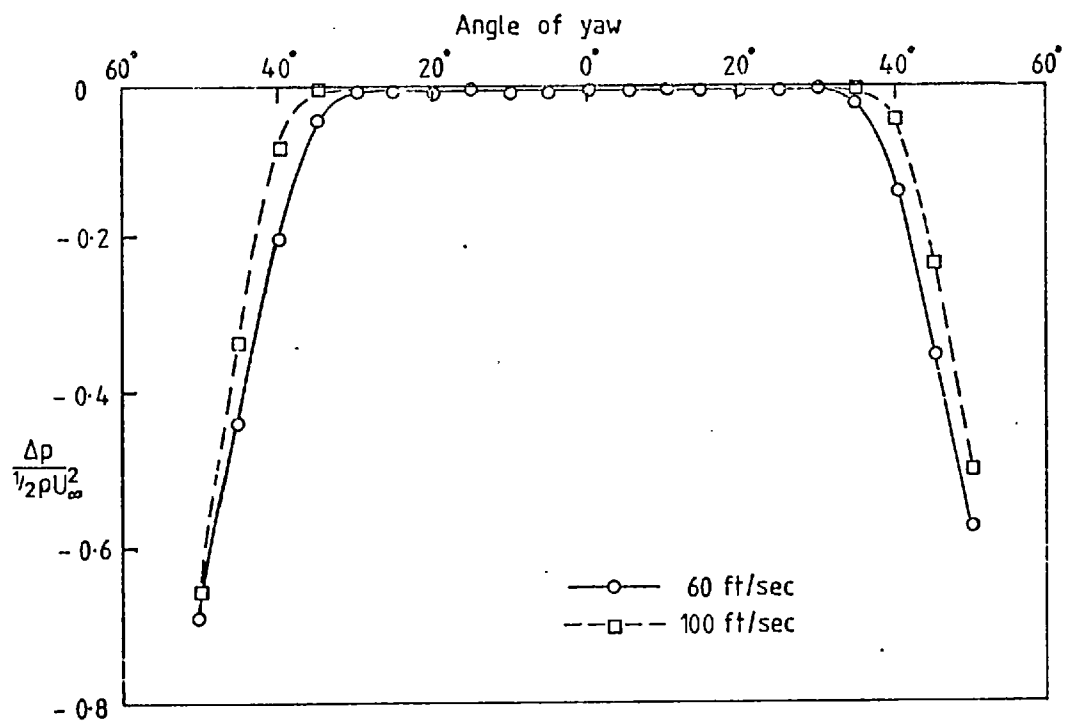
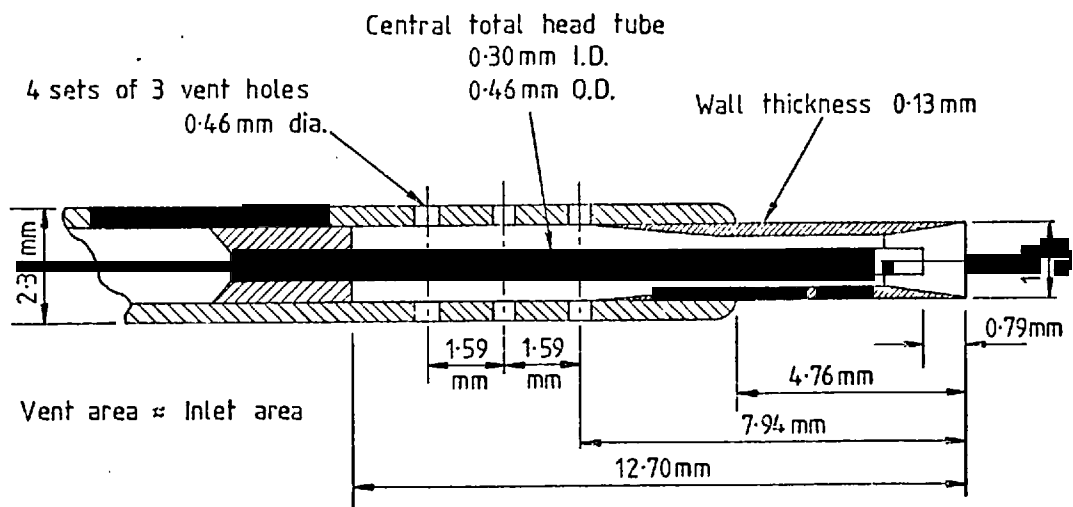


Fig. 37 Sketch of a Kiel type total head tube with typical calibration curve.

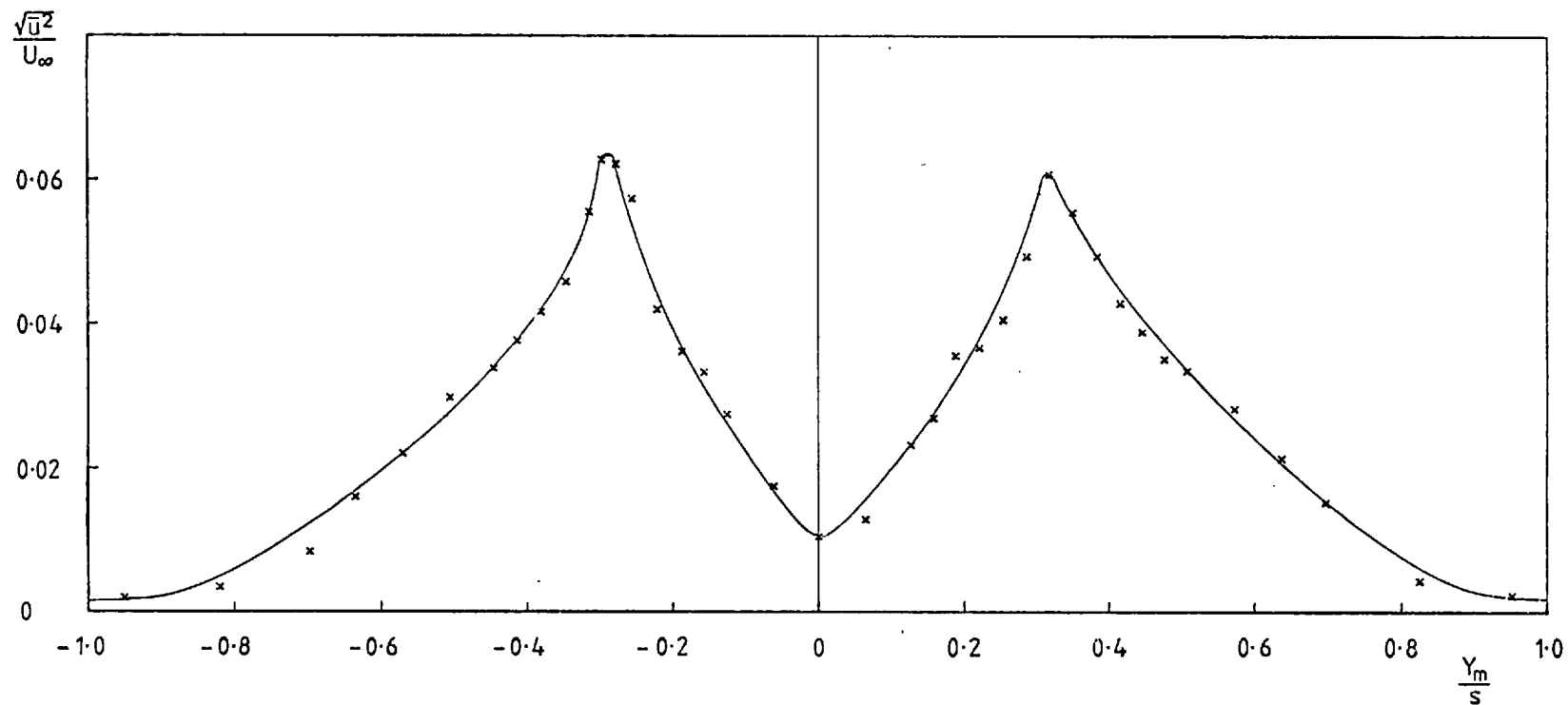


Fig. 38 Turbulence intensity, $\sqrt{u^2}/U_\infty$; a horizontal traverse through the vortices at $Y = 676.9$ mm (26.65"): 63" chord delta wing.

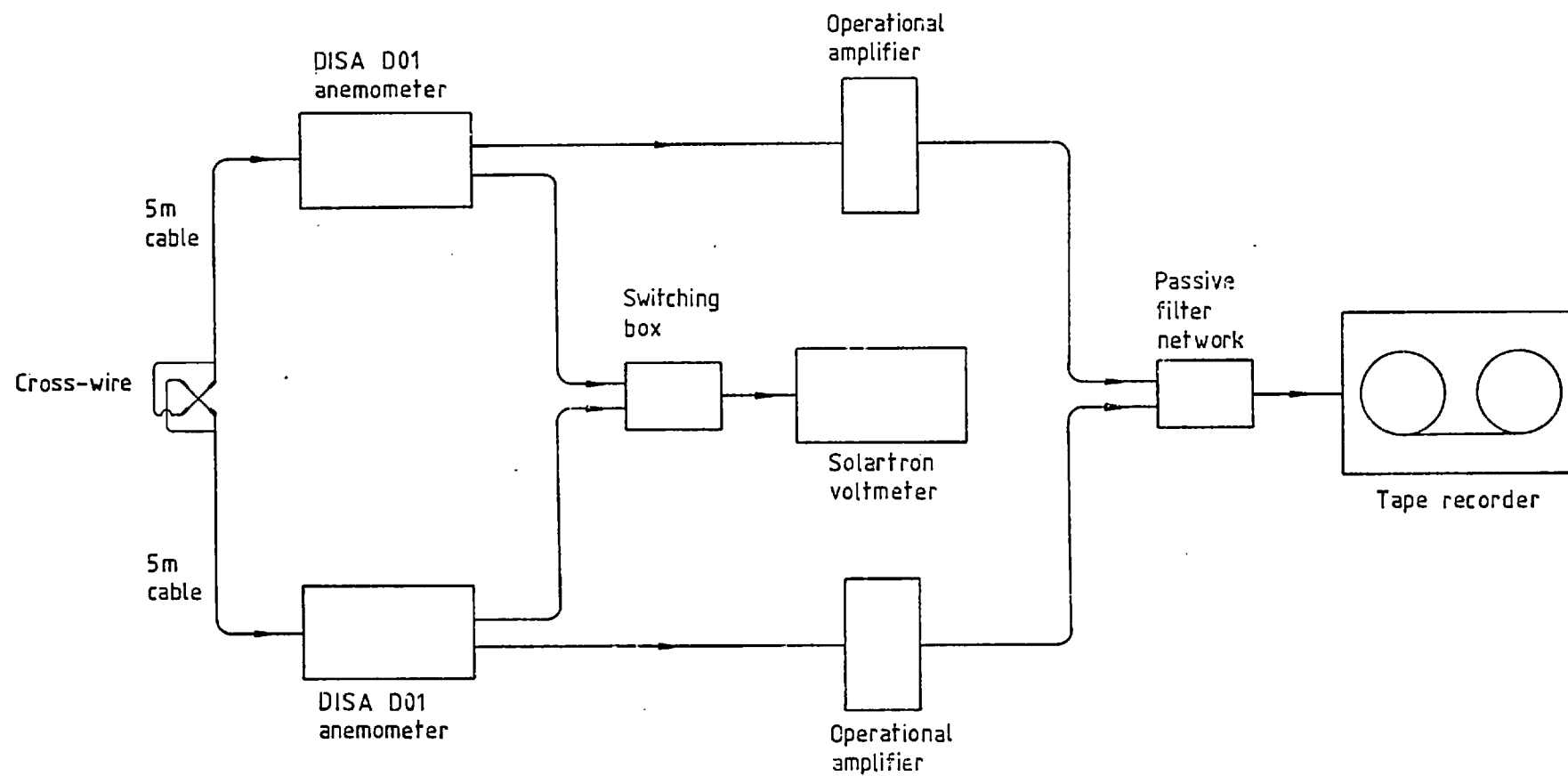


Fig. 39 Schematic diagram of the apparatus used for recording velocity measurements.

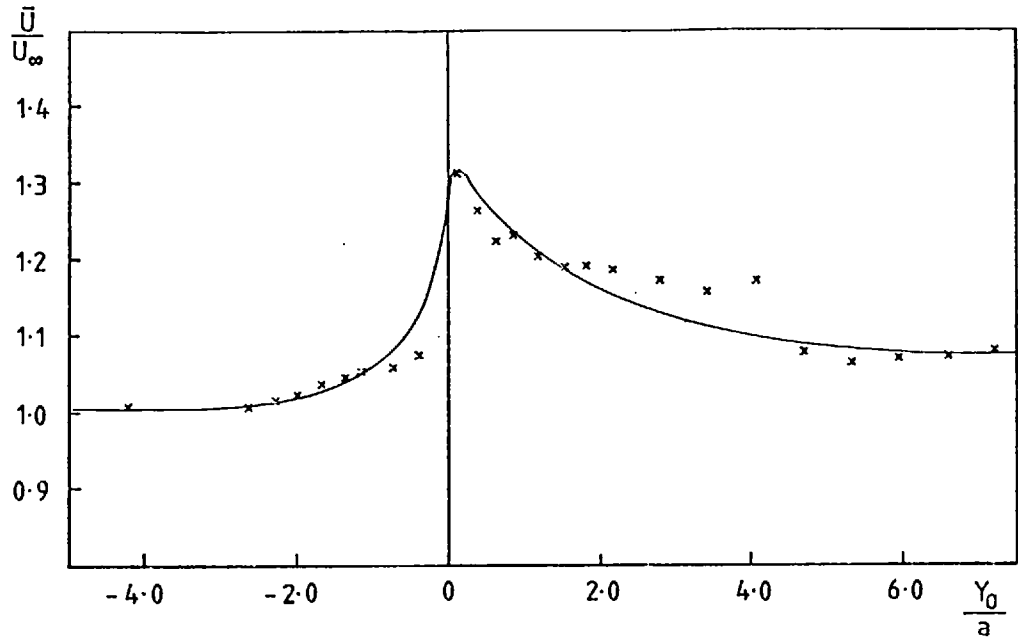


Fig. 40a Axial velocity profile of the vortex under study, (63" chord delta wing).

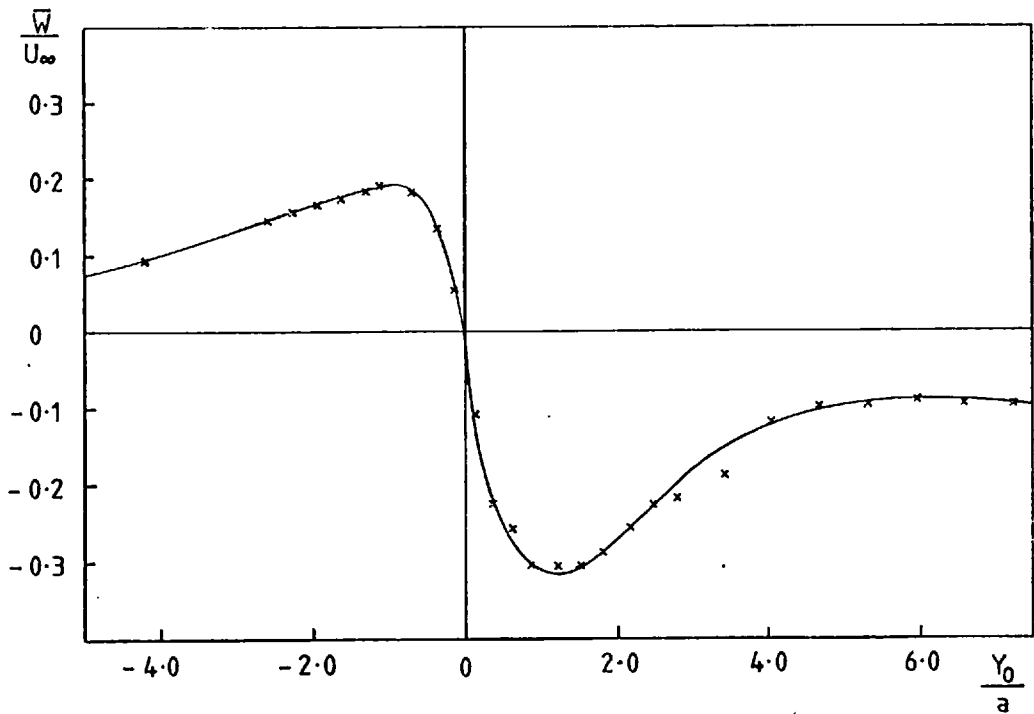


Fig. 40b Azimuthal velocity profile of the vortex under study, (63" chord delta wing).

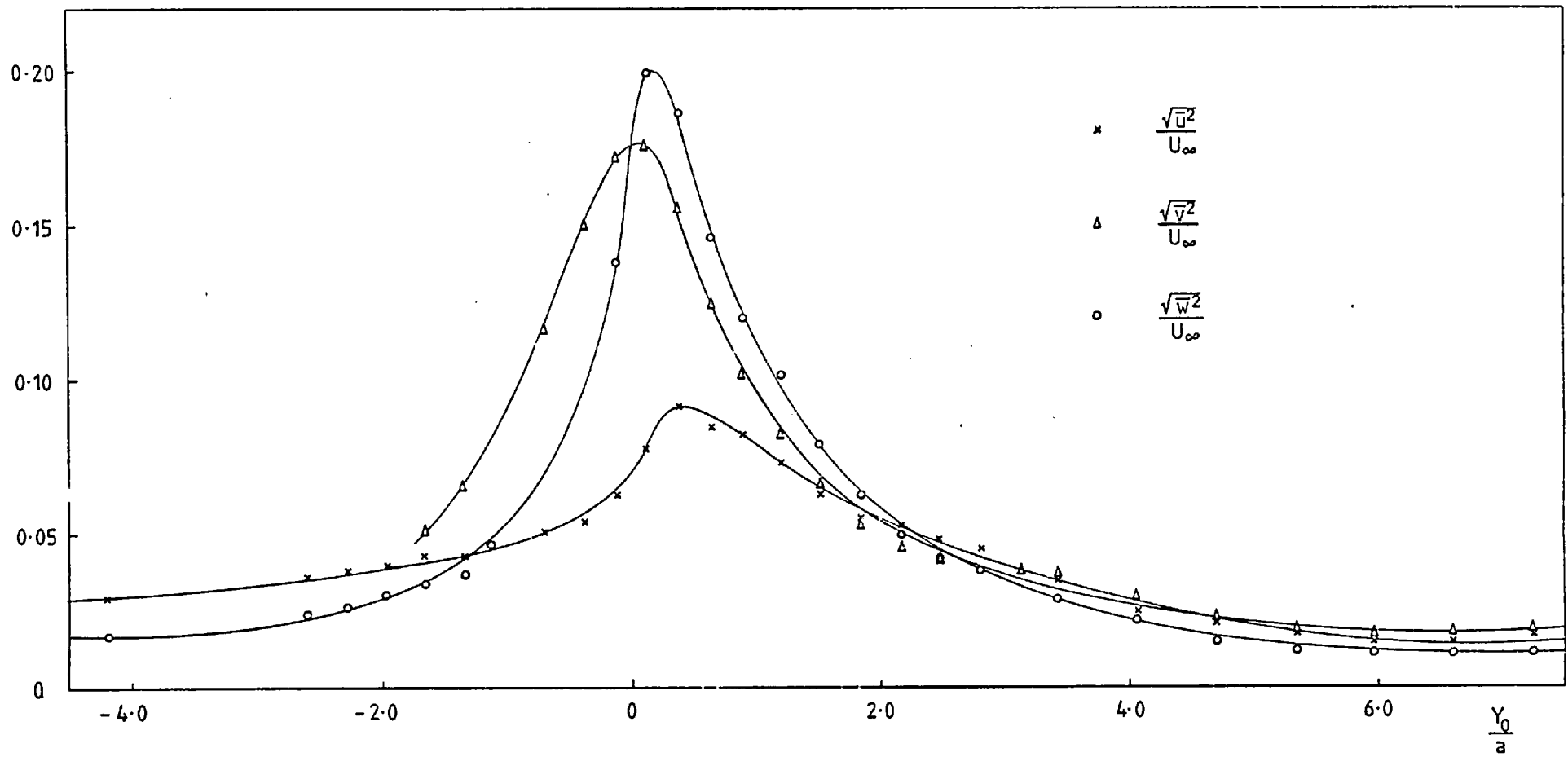


Fig. 41 Turbulence intensity levels of the vortex under study (63" chord delta wing).

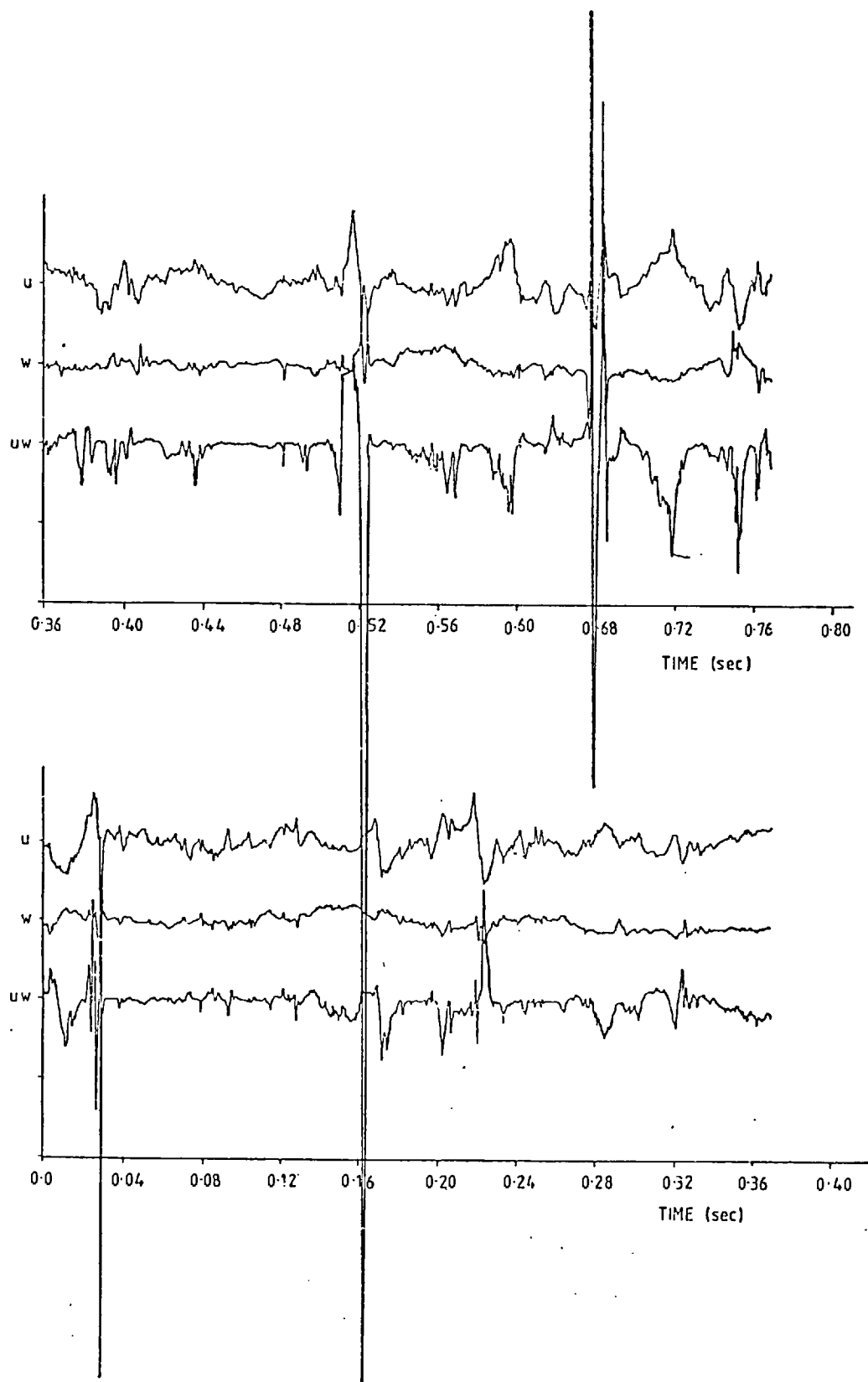


Fig. 42a Typical fluctuating velocity components and their instantaneous product as calculated by the DIG2W program.

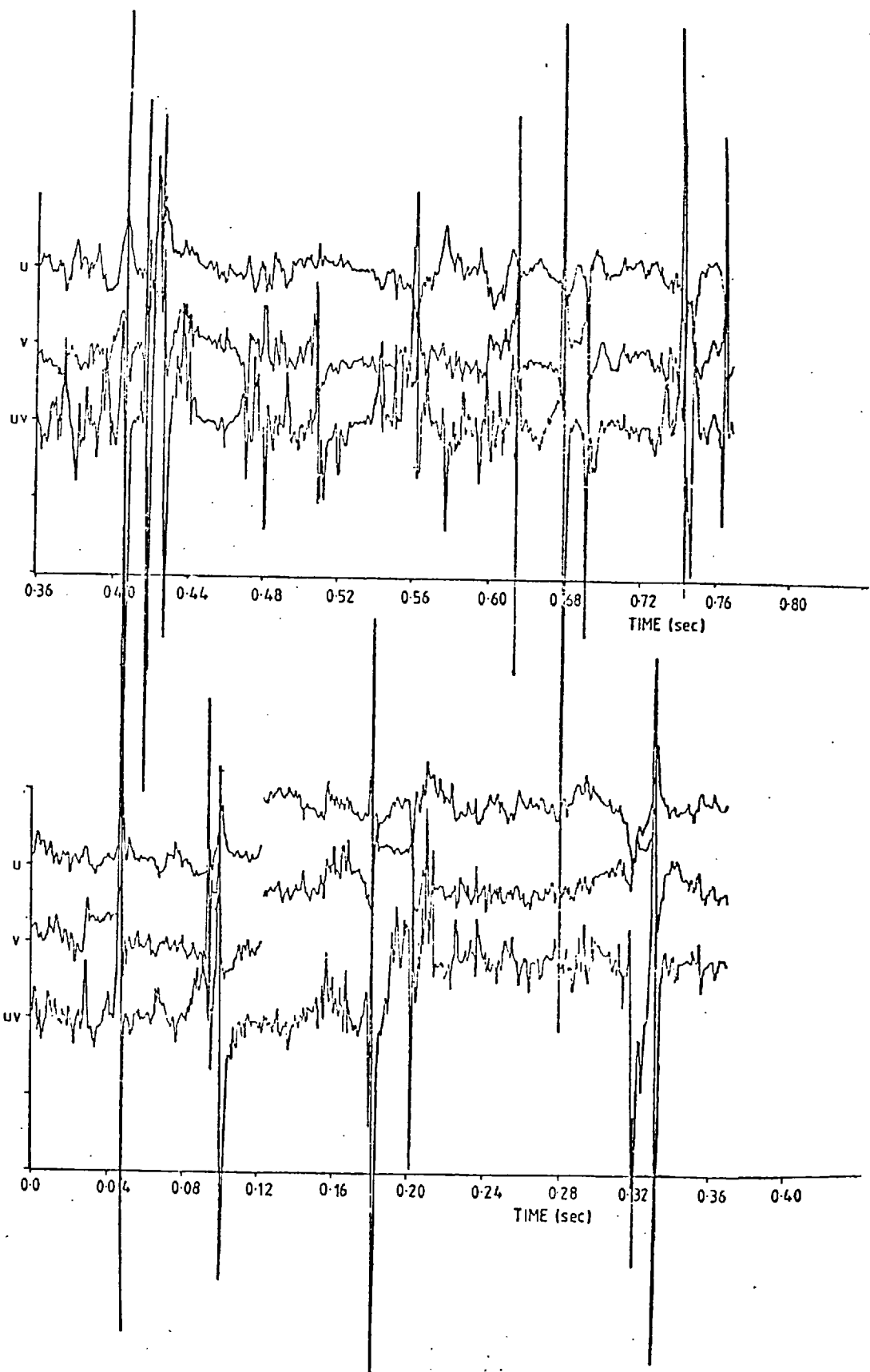


Fig. 42b Typical fluctuating velocity components and their instantaneous product as calculated by the DIG2W program.

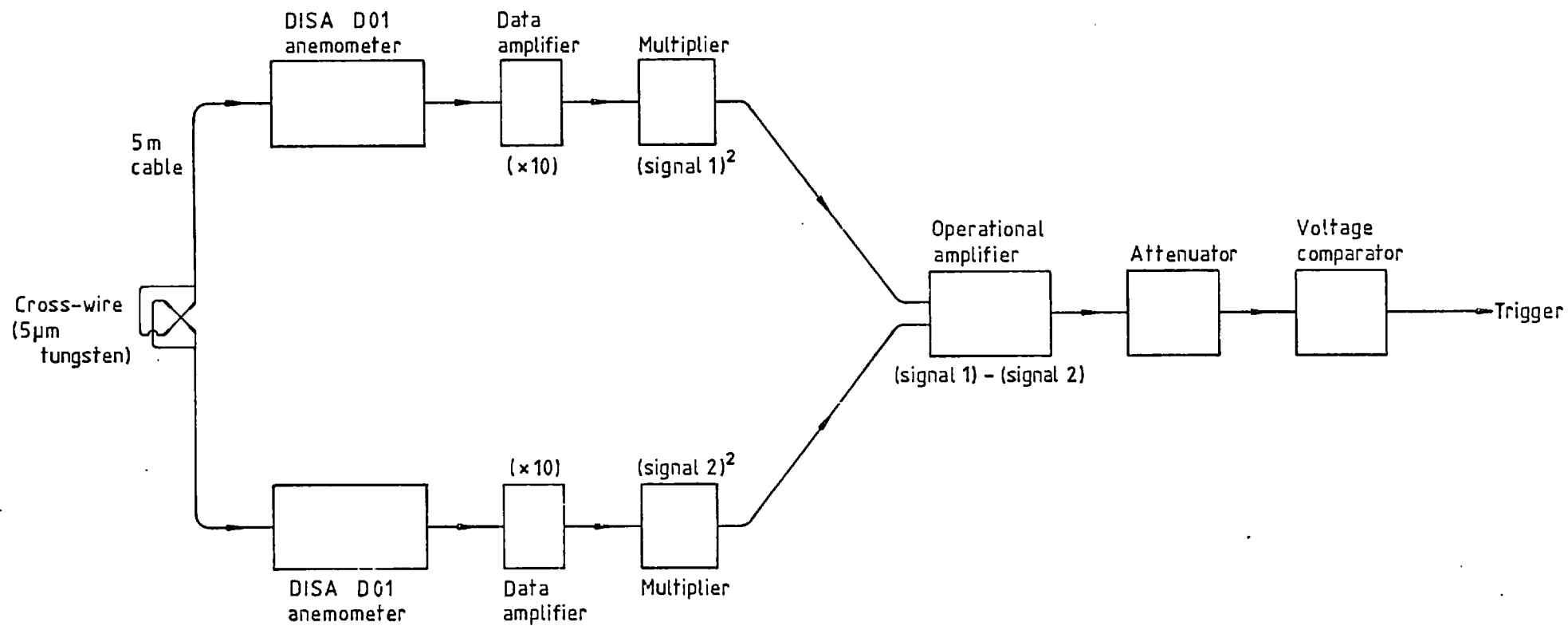


Fig. 43 Schematic diagram of the apparatus used to generate the trigger signal.

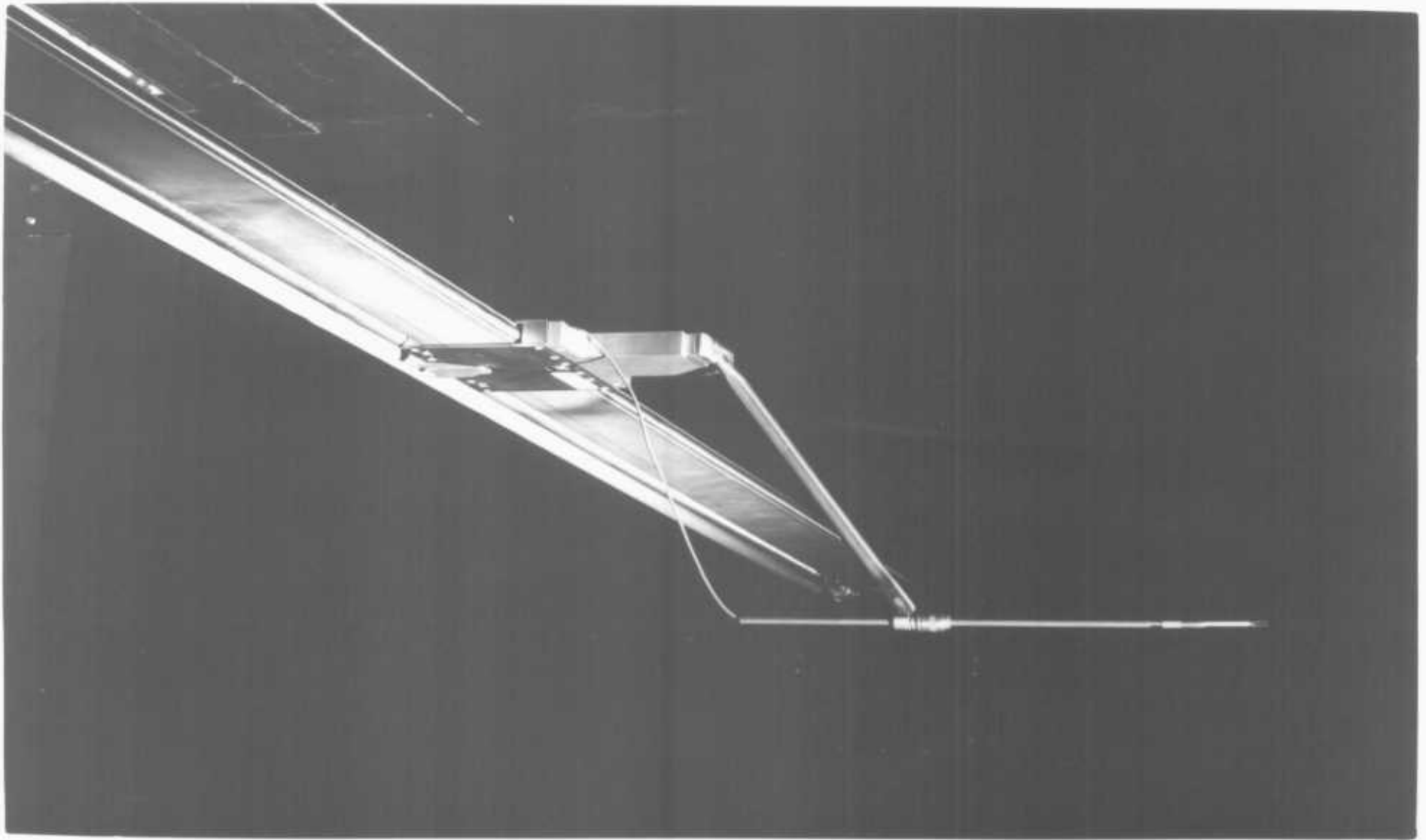


Fig.44. Single hot wire mounted on the traverse mechanism in the test area of the 4.5' x 4' wind tunnel.

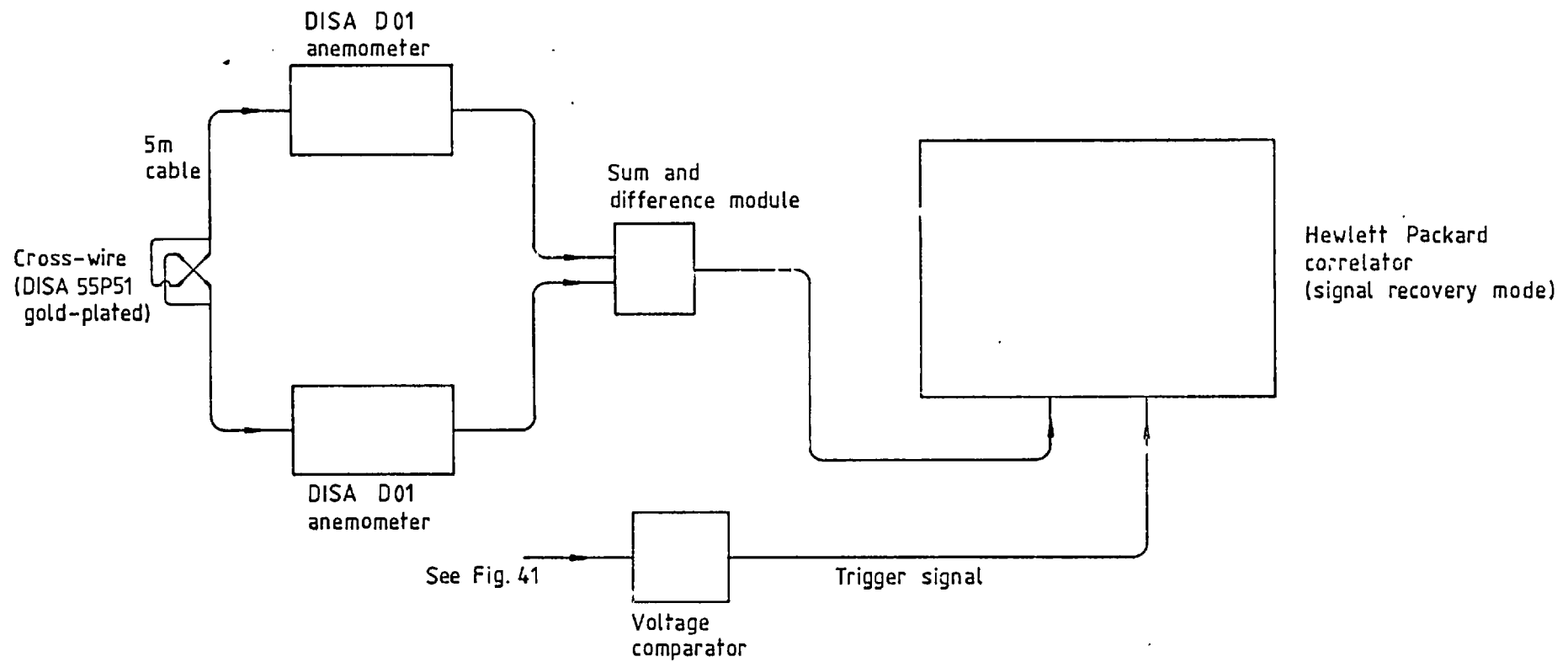


Fig. 45 Block diagram of instrumentation for recording fluctuating velocity data.

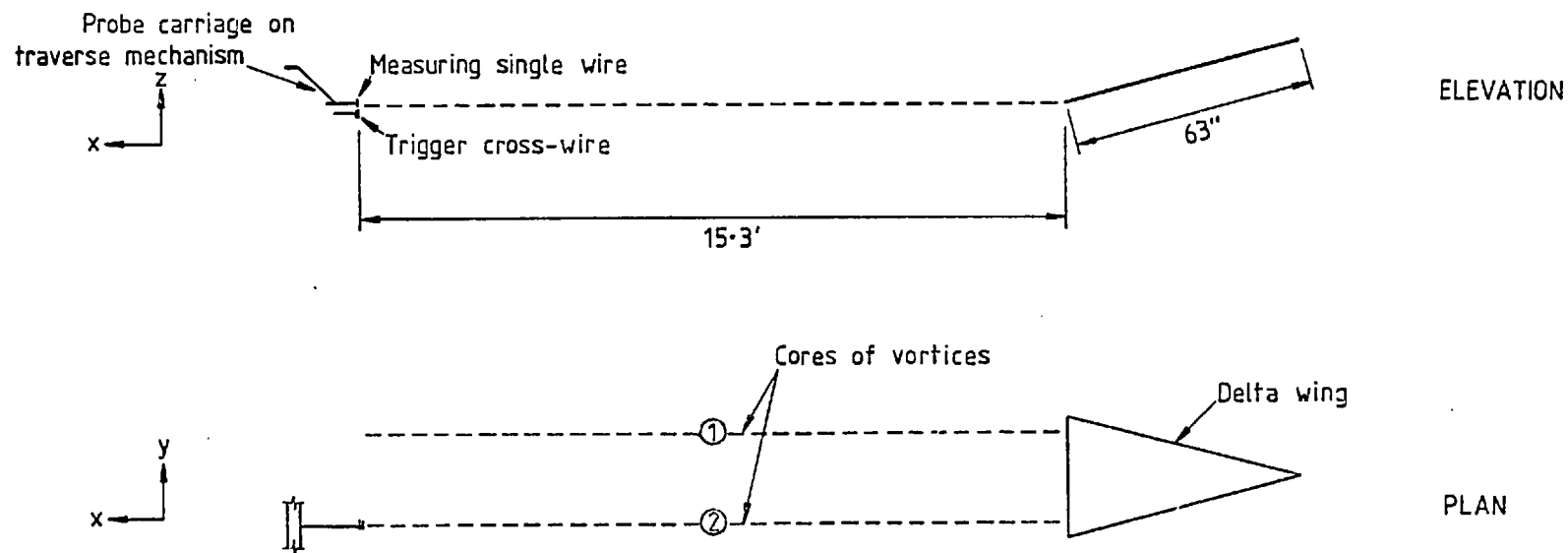


Fig. 46 Sketch of position of hot wire anemometers with respect to the delta wing model.
Experiment (i) of Table $\bar{V}$, see section 3.5.1.

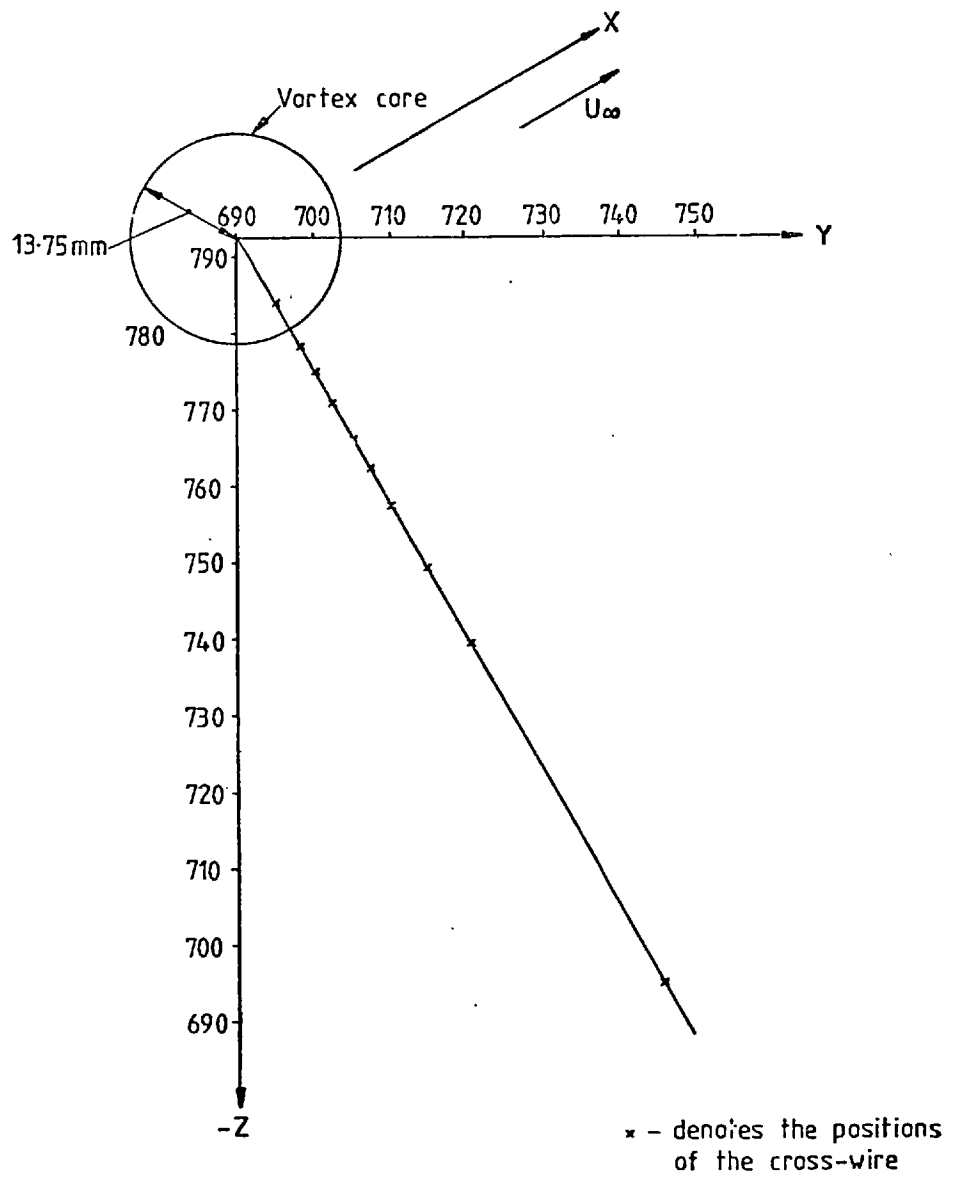


Fig. 47 Sketch showing the traverse of the measuring cross-wire: experiment (iii), Table $\bar{V}$.

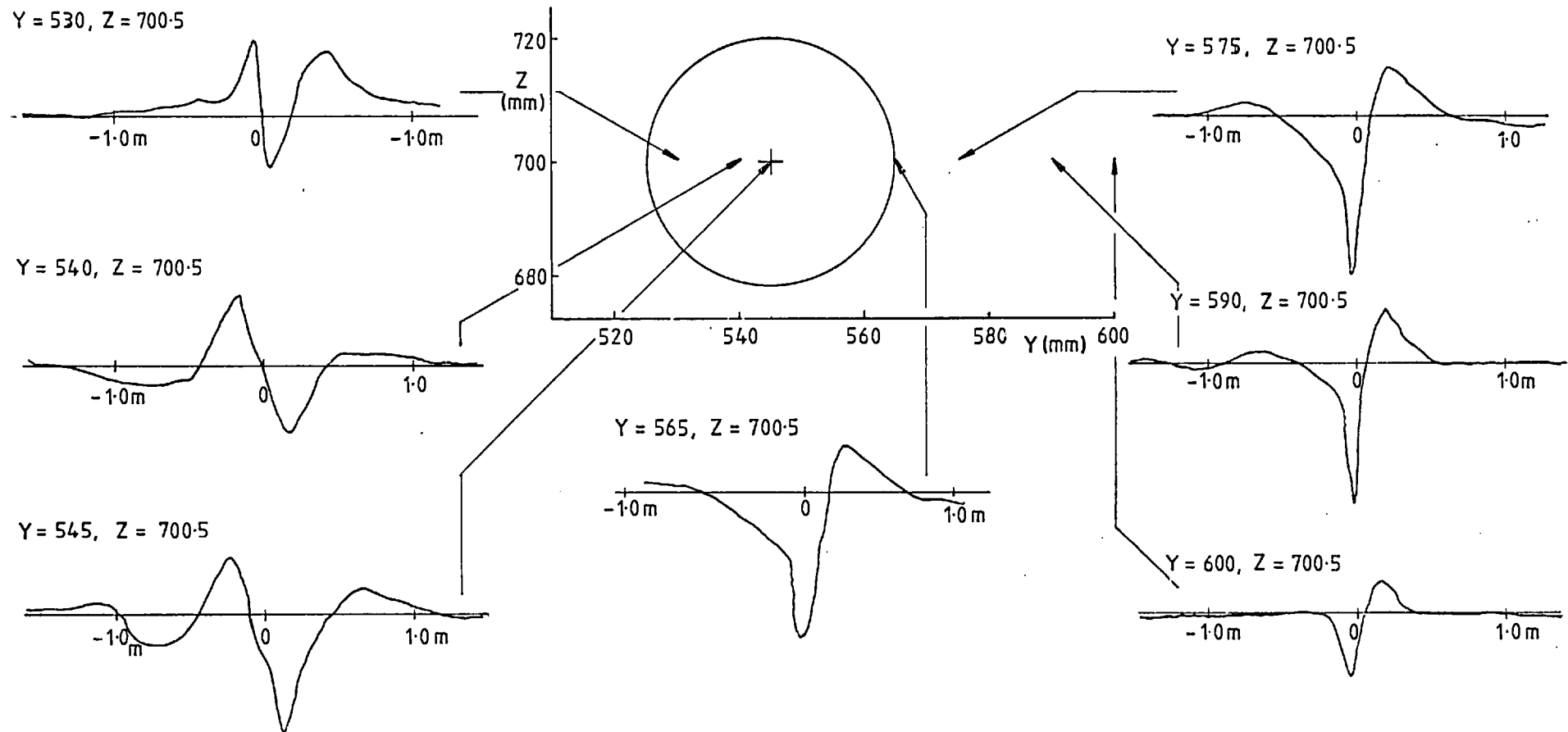


Fig. 48a The structured disturbance at positions through the centre of the vortex - a horizontal traverse with a single hot wire: experiment (i).
(hot wire vertical)

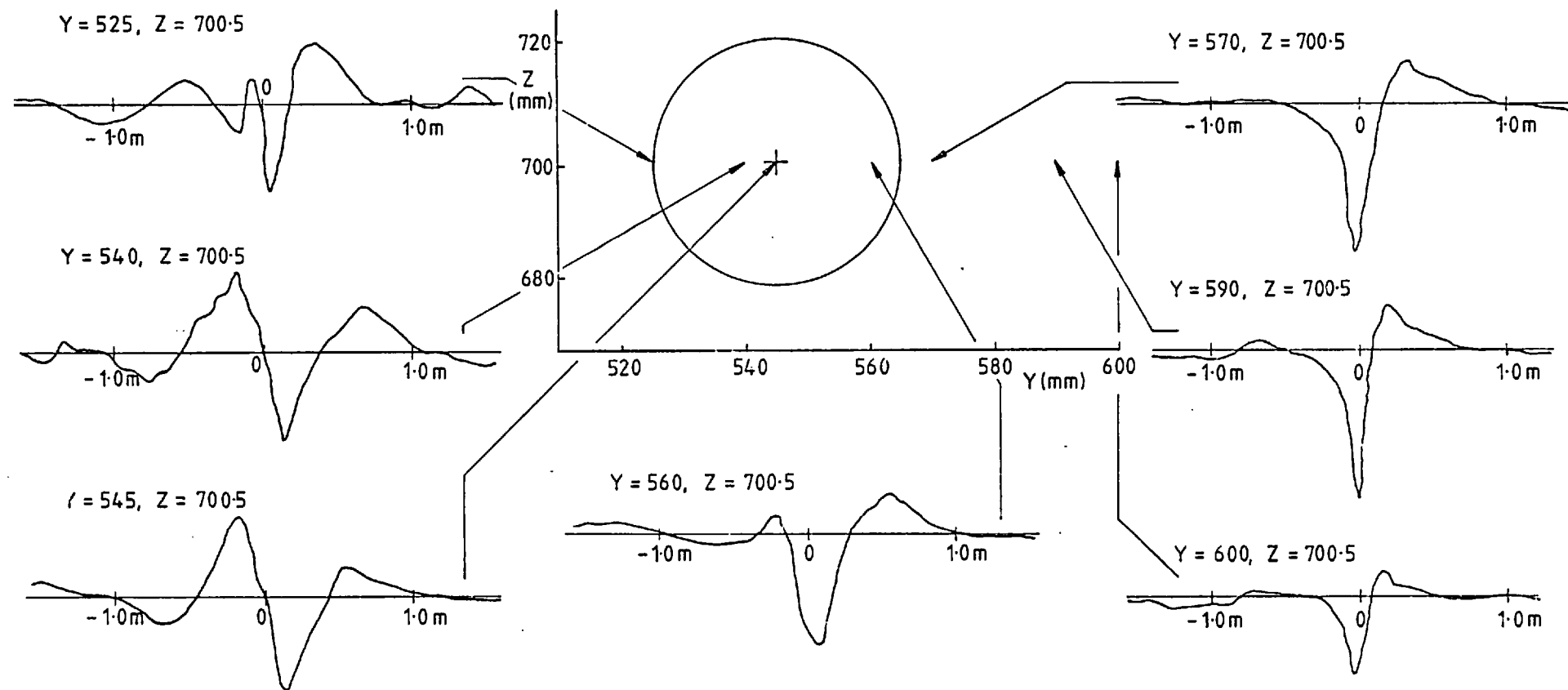


Fig. 48b The structured disturbance at positions through the centre of the vortex -- a horizontal traverse with a single hot wire: experiment (i).
(hot wire horizontal)

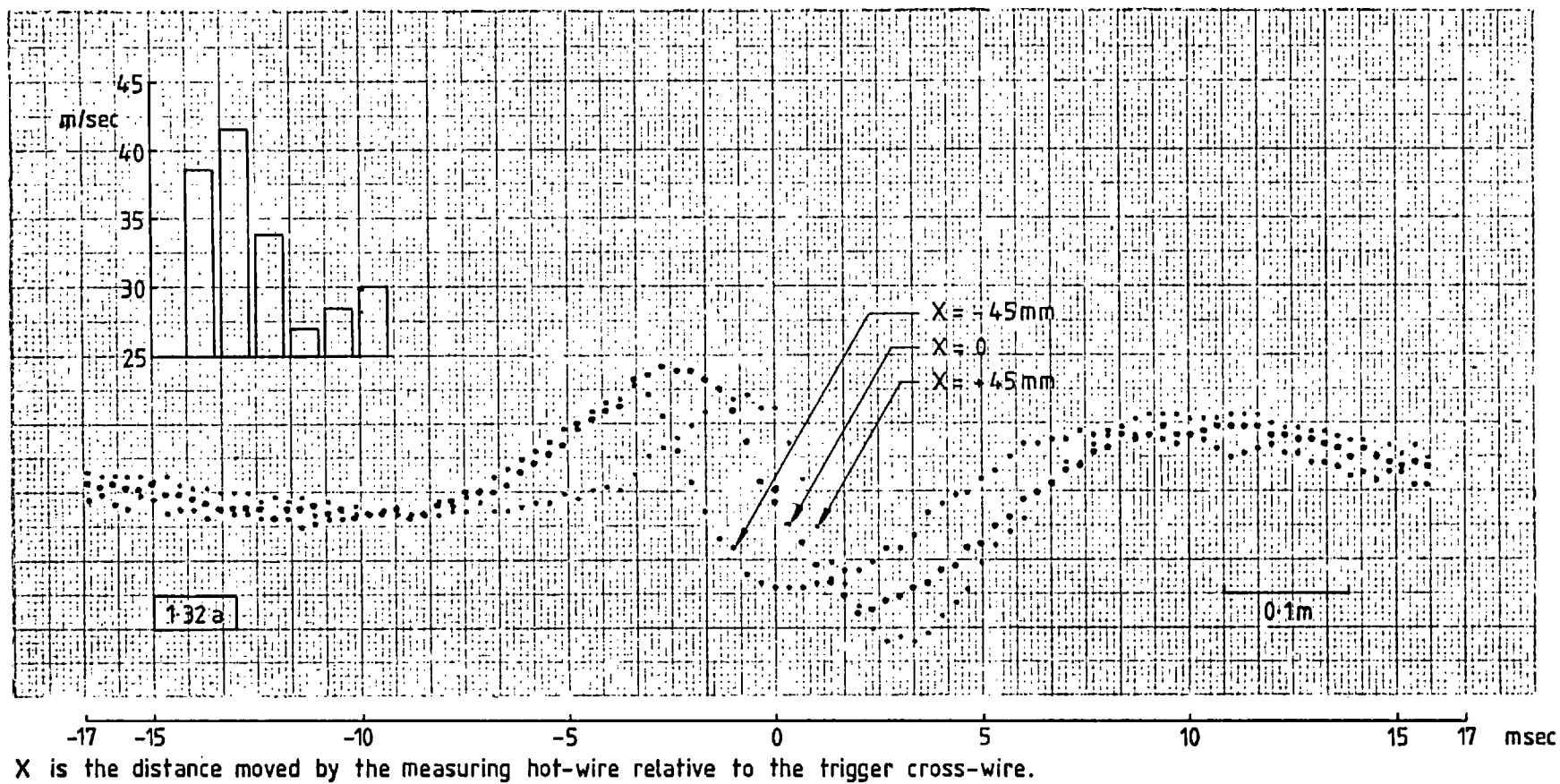


Fig. 49 Determination of the speed at which the structured disturbance travelled; the disturbance at three downstream positions.

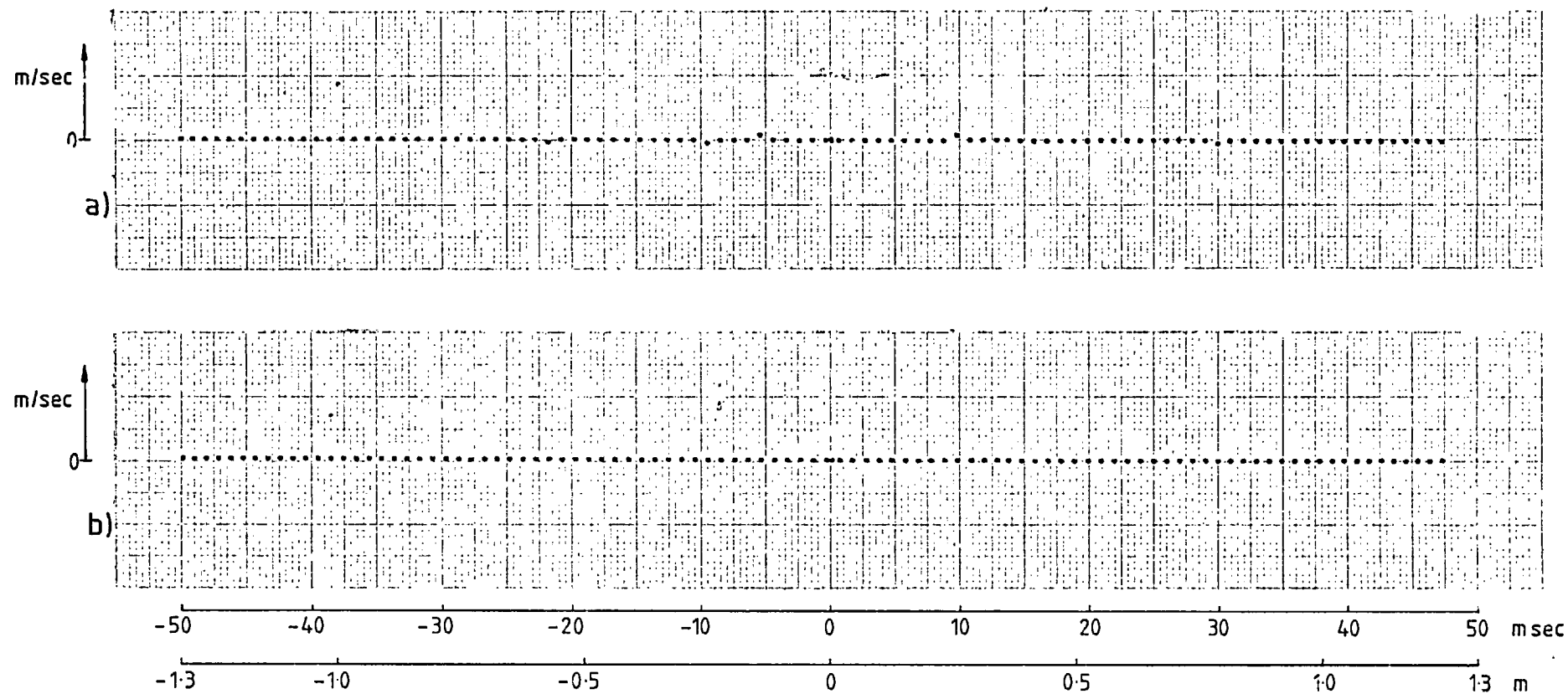


Fig. 50 Graph illustrating the absence of a structured disturbance in the flow outside the vortex.

At 24 core radii from the centre of the vortex: u_s - u_n plane.

a) u_s component, b) u_n component.

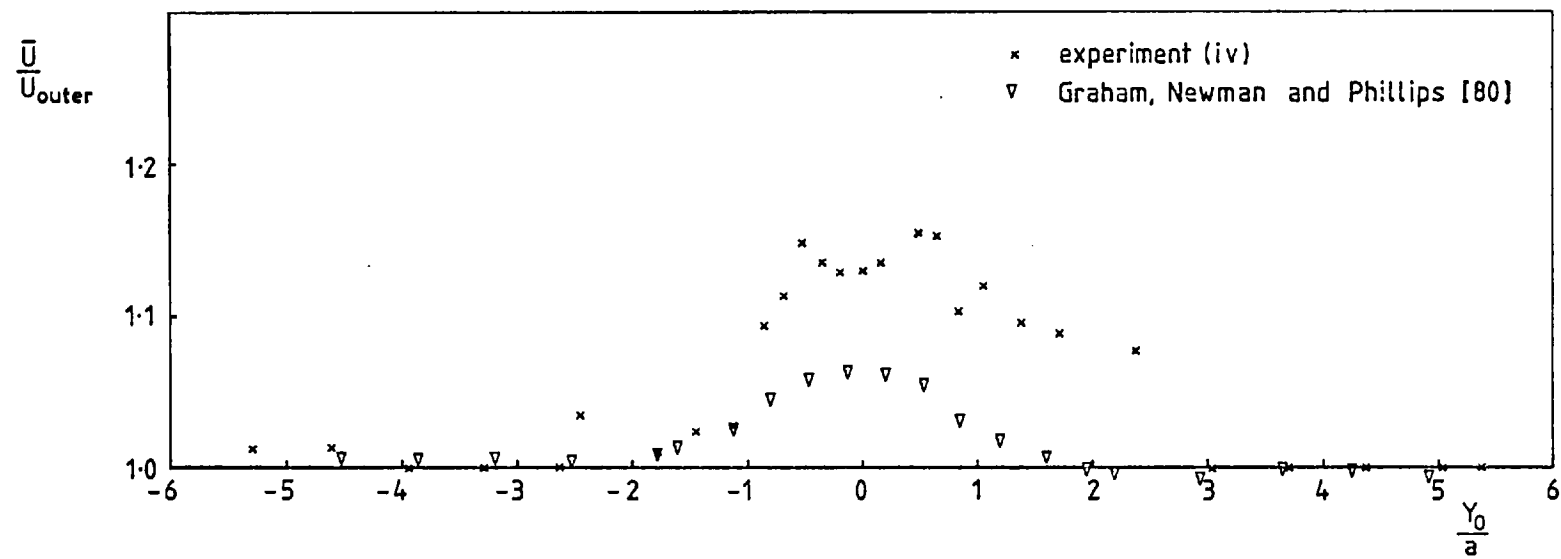


Fig. 51 Axial velocity profile of the vortex under study: experiment (iv).

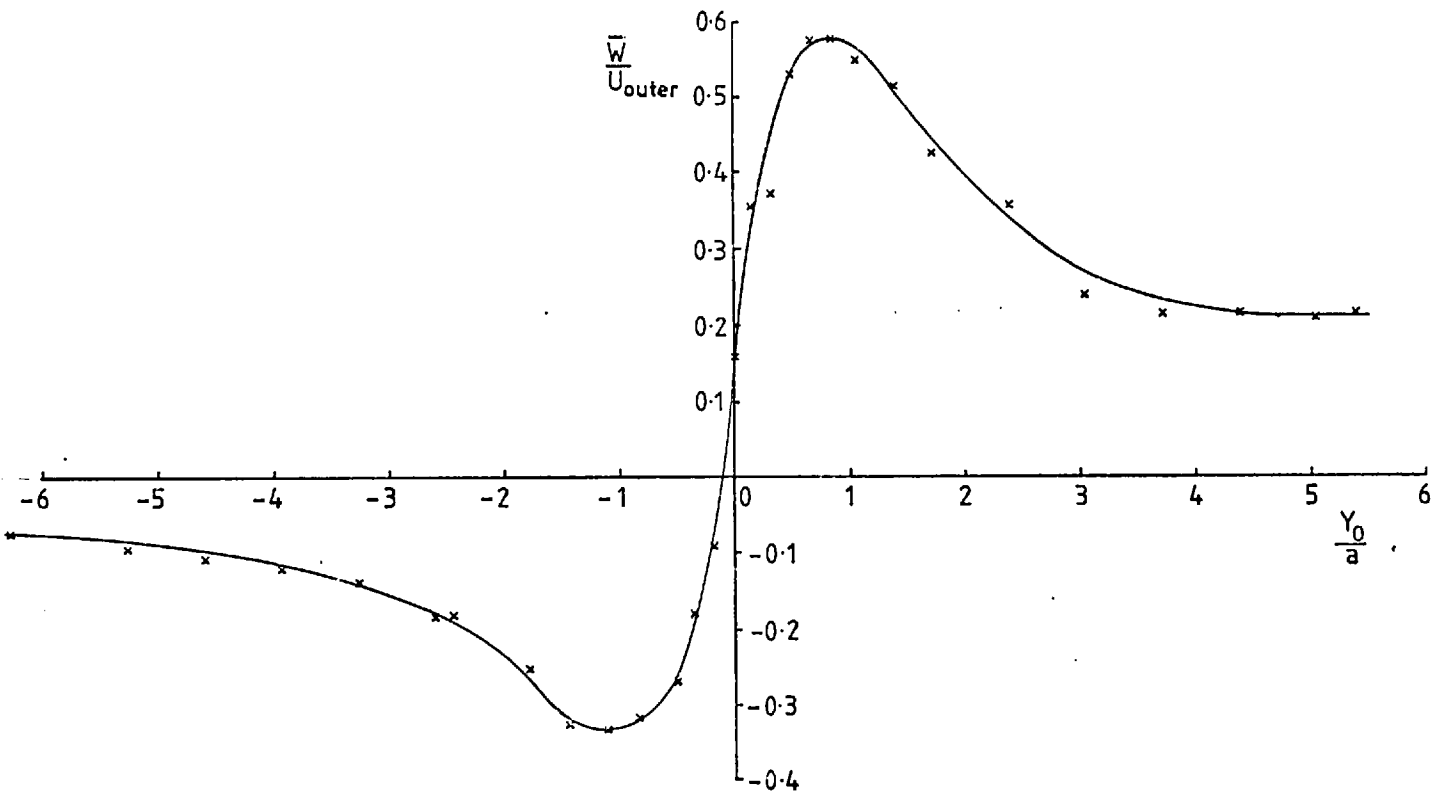


Fig. 52 a Azimuthal velocity profile of the vortex under study: experiment (iv).

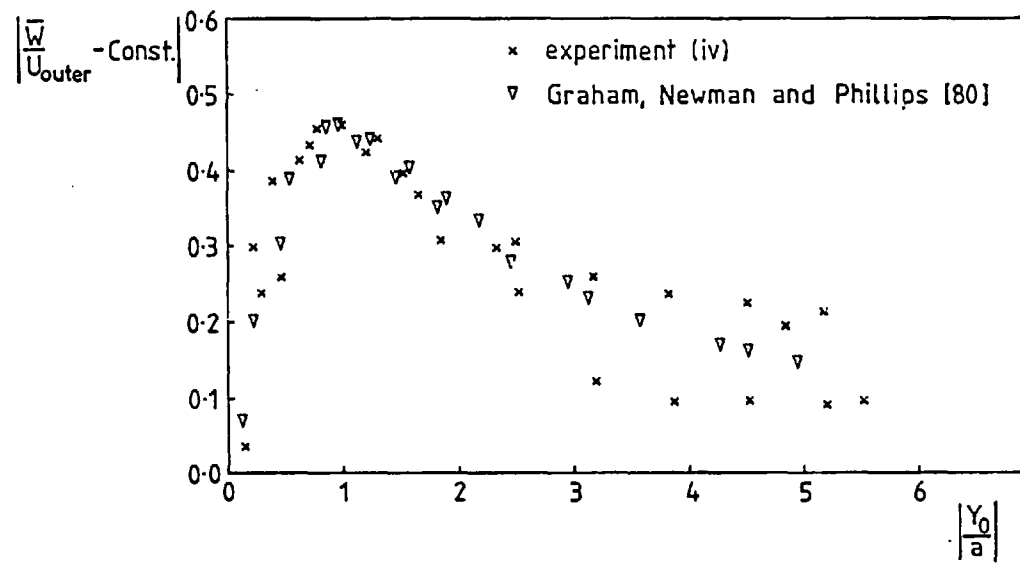


Fig. 52b Azimuthal velocity profile of the vortex under study: experiment (iv).

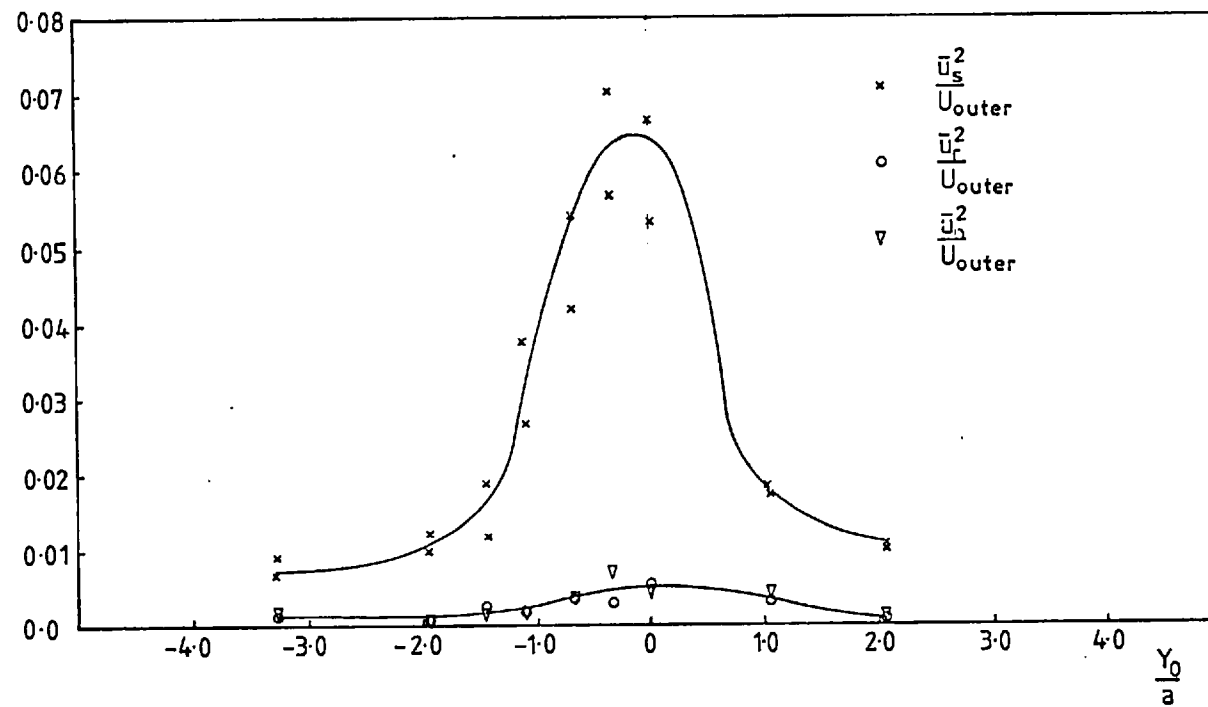


Fig. 53 Turbulence intensity levels: experiment (iv).

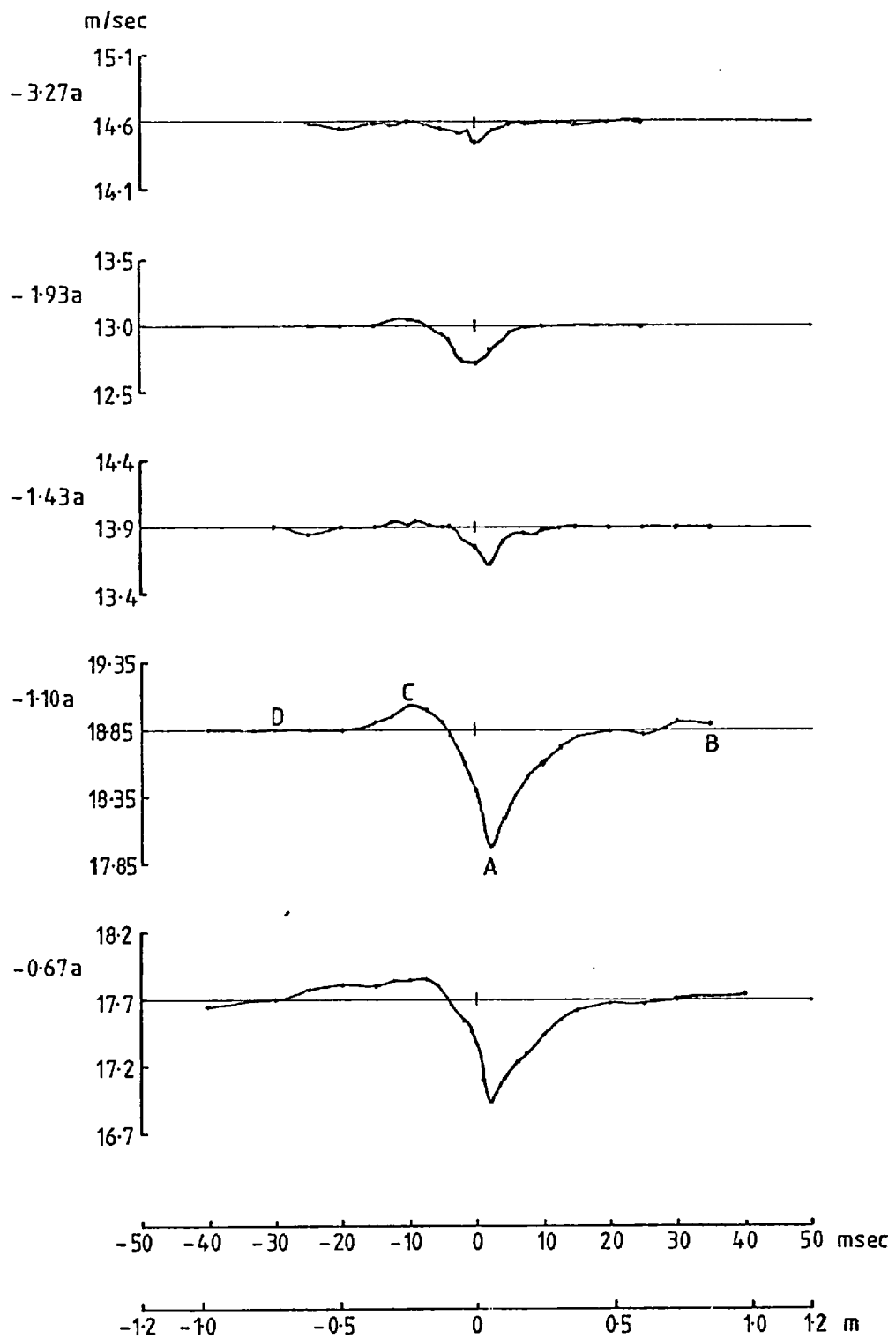


Fig. 54a Velocity profiles obtained using conditional sampling technique: experiment (iv).

Streamwise component, U_s , over a timescale of 100 msec, triggering on the negative peak of the uw signal. ($U_s - U_r$ plane)

Continued /.....

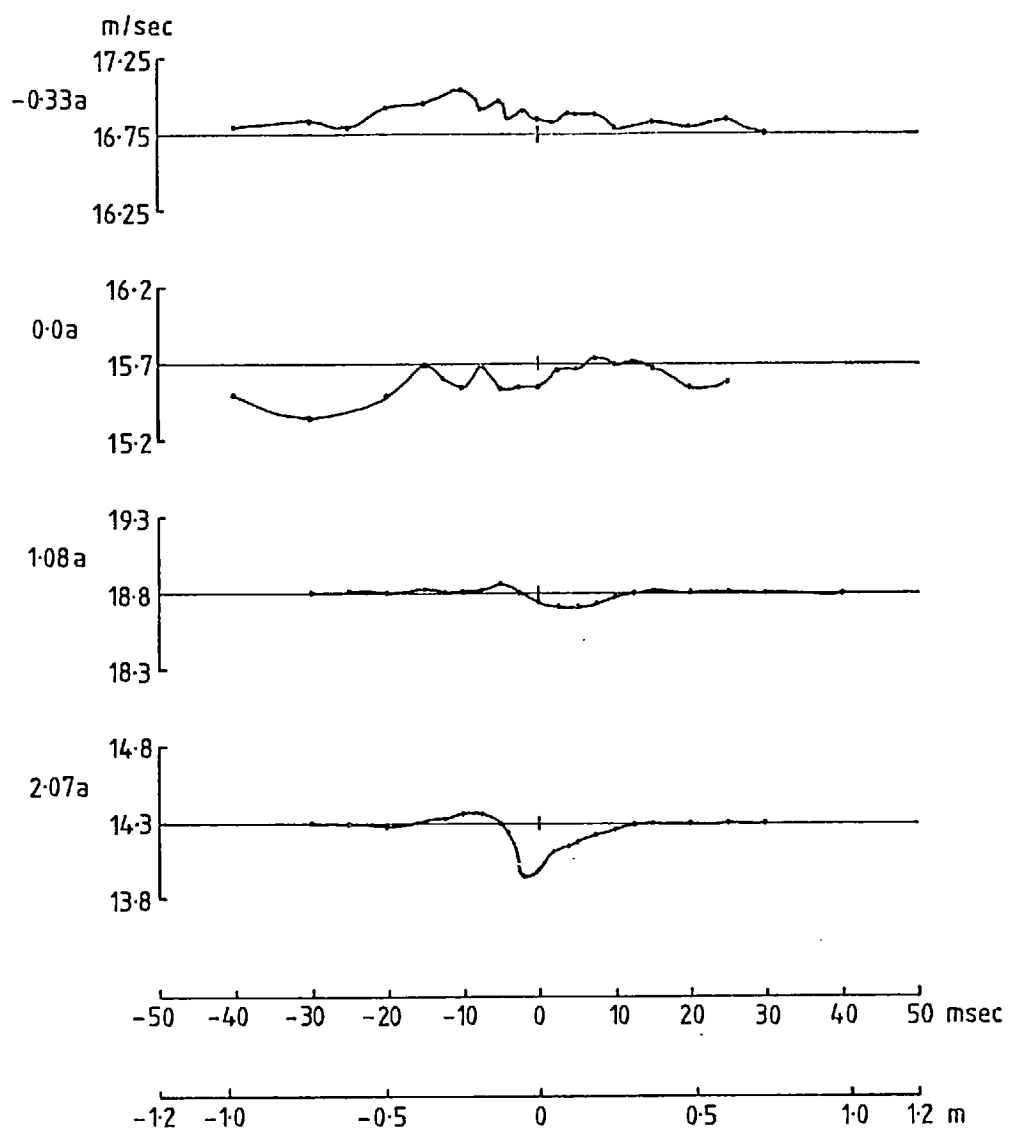


Fig. 54a continued.

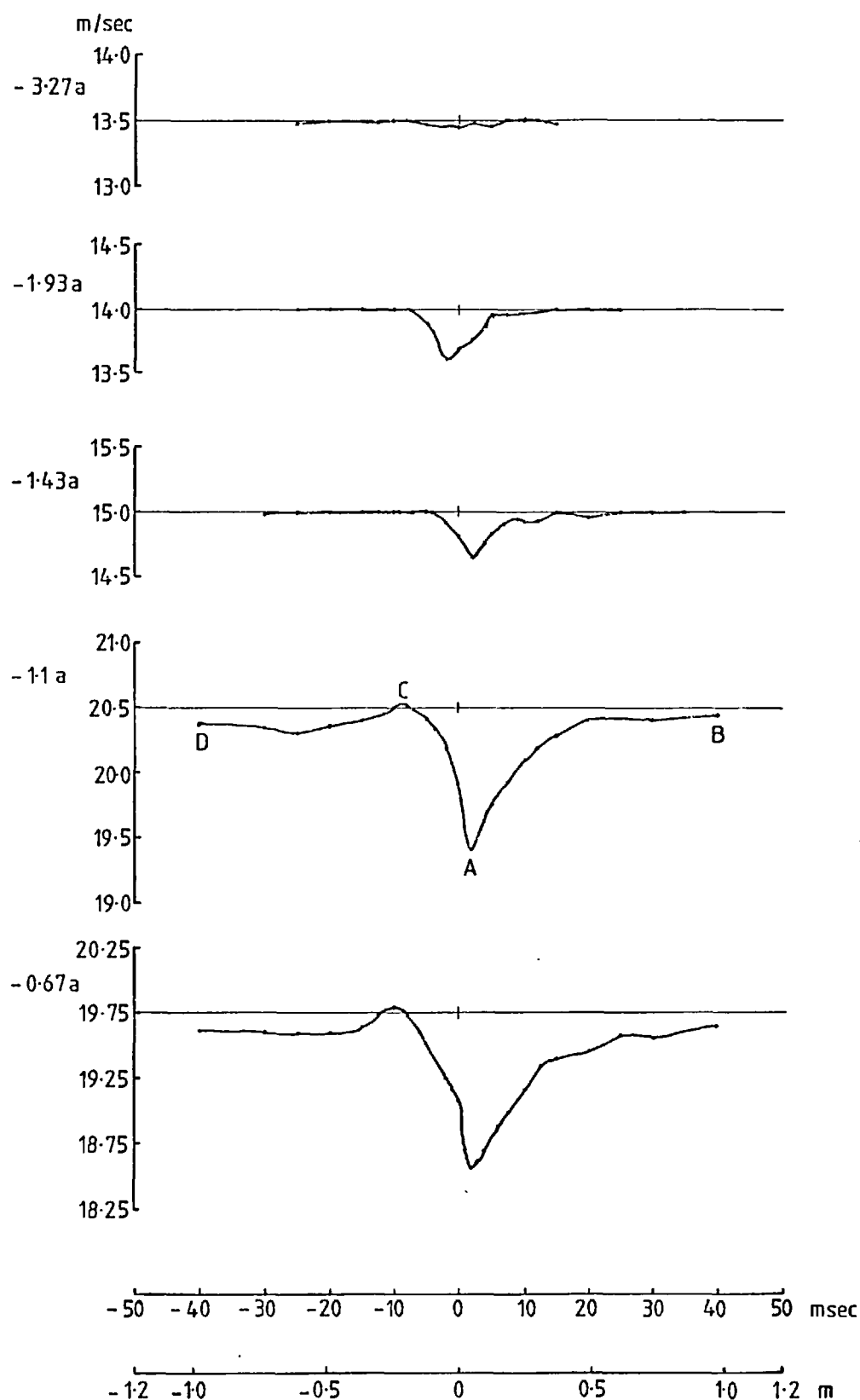


Fig. 54b Velocity profiles obtained using conditional sampling technique: experiment (iv).

Streamwise component, U_s , over a timescale of 100 msec, triggering on the negative peak of the uv signal. ($U_s - U_n$ plane)

Continued /

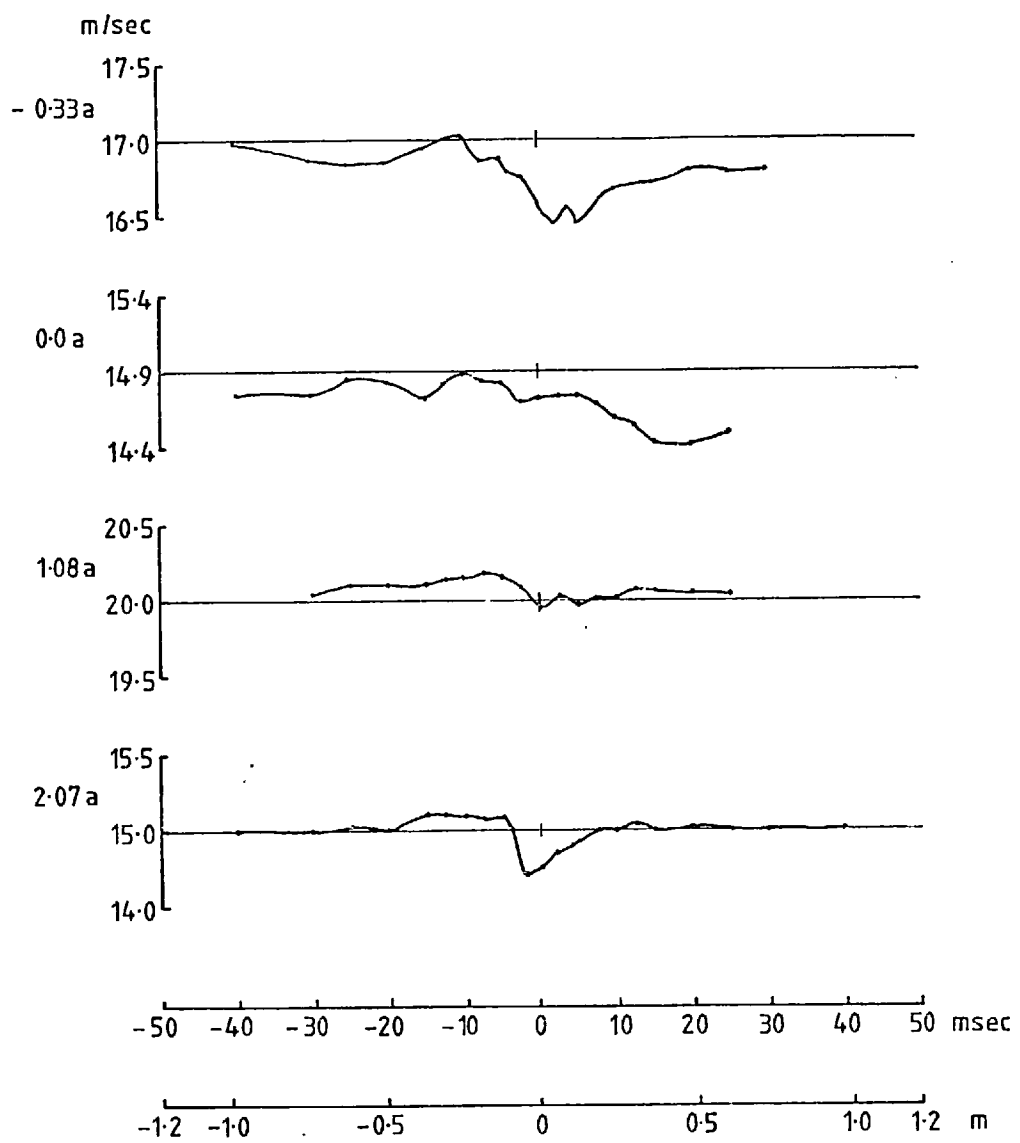


Fig. 54b continued.

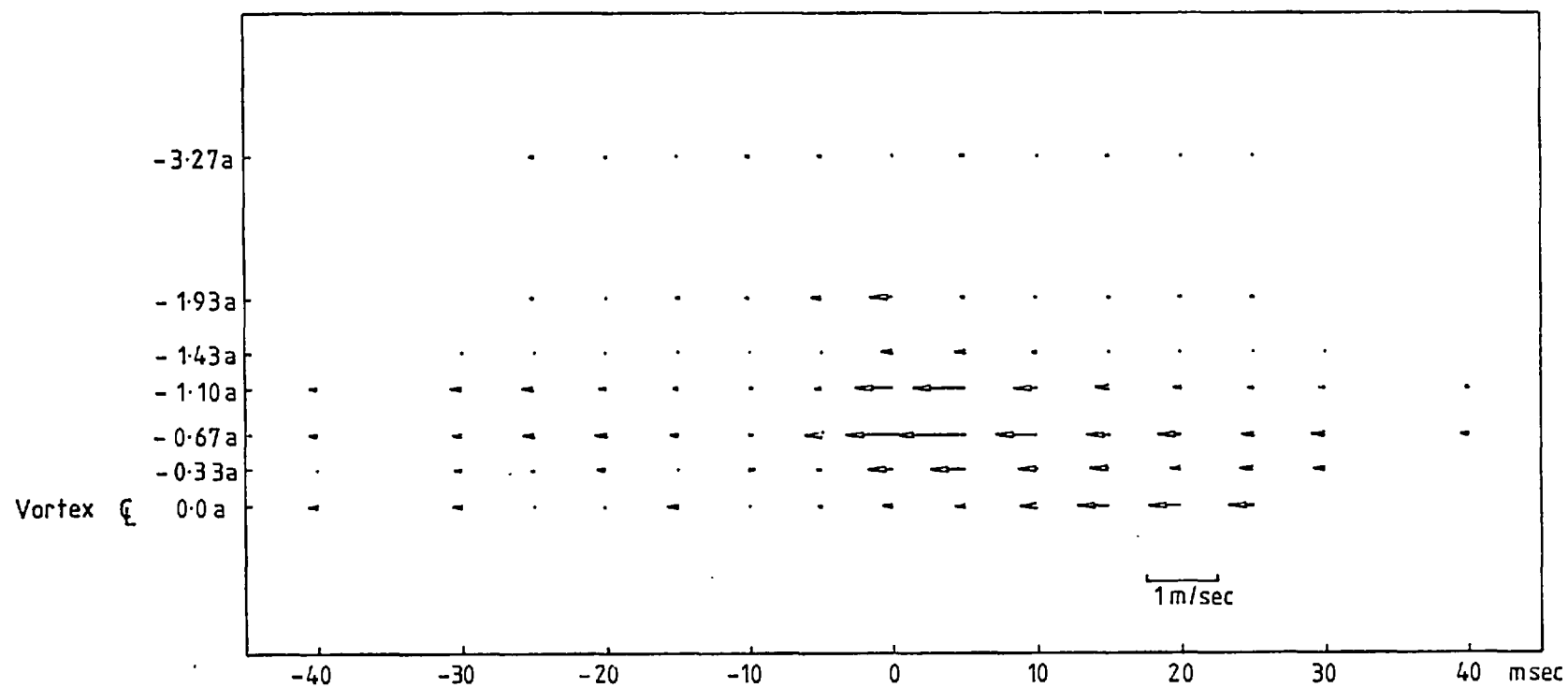


Fig. 55 Perturbations, u_s , (Figure 54b) for the $U_s - U_n$ plane at various positions in the vortex.

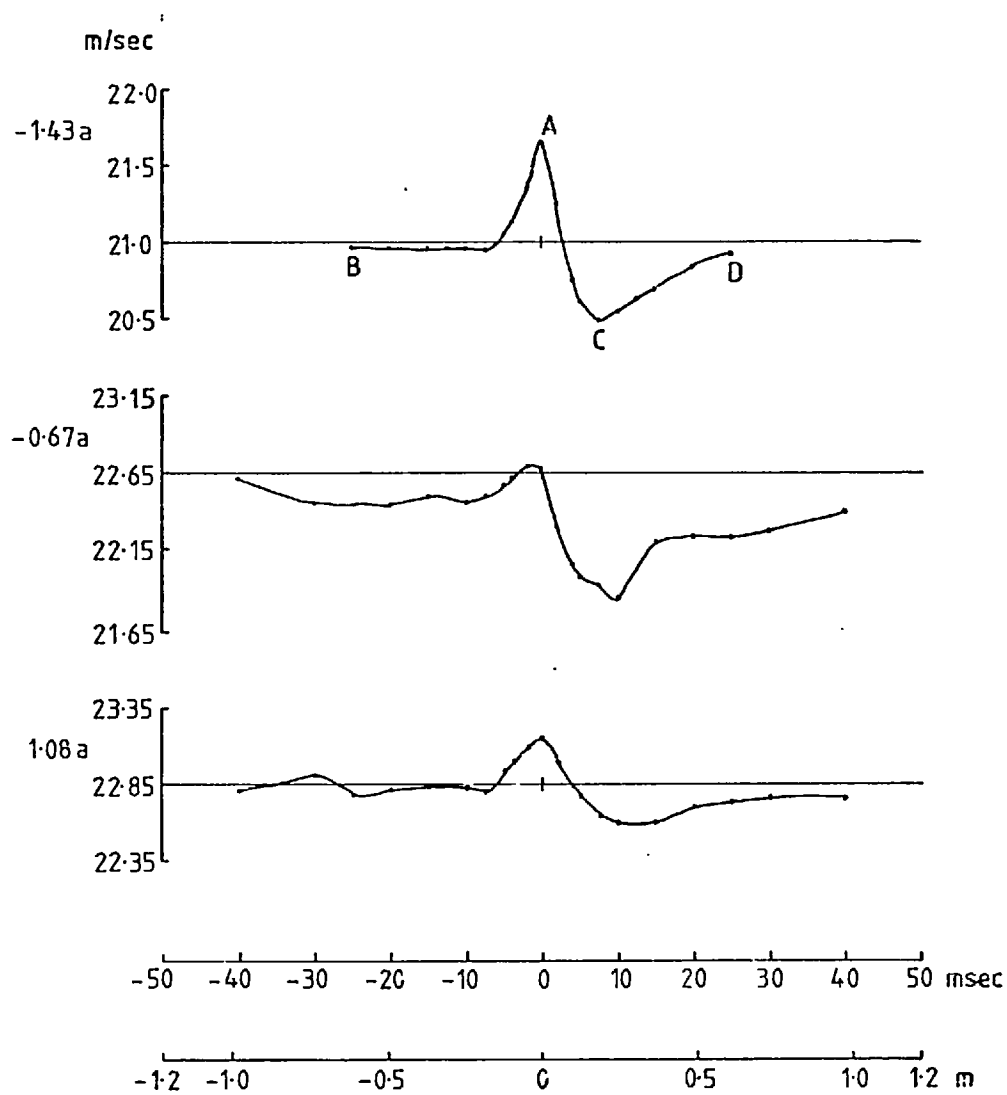


Fig. 56a Velocity profiles obtained using conditional sampling technique: experiment (iv).
Streamwise component, U_s , over a timescale of 100 msec, triggering on the positive peak of the uw signal. ($U_s - U_r$ plane)

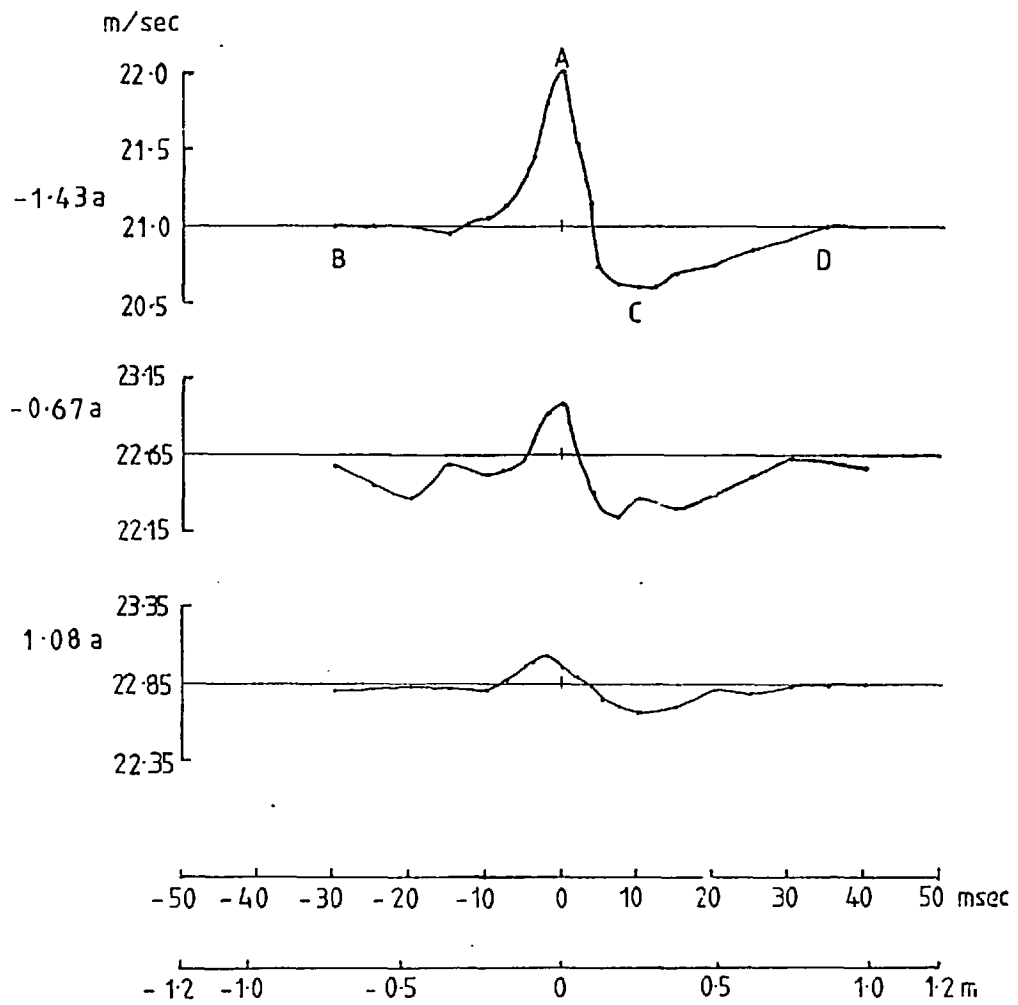


Fig. 56b Velocity profiles obtained using conditional sampling technique: experiment (iv).

Streamwise component, U_s , over a timescale of 100 msec, triggering on the positive peak of the uw signal. ($U_s - U_n$ plane)

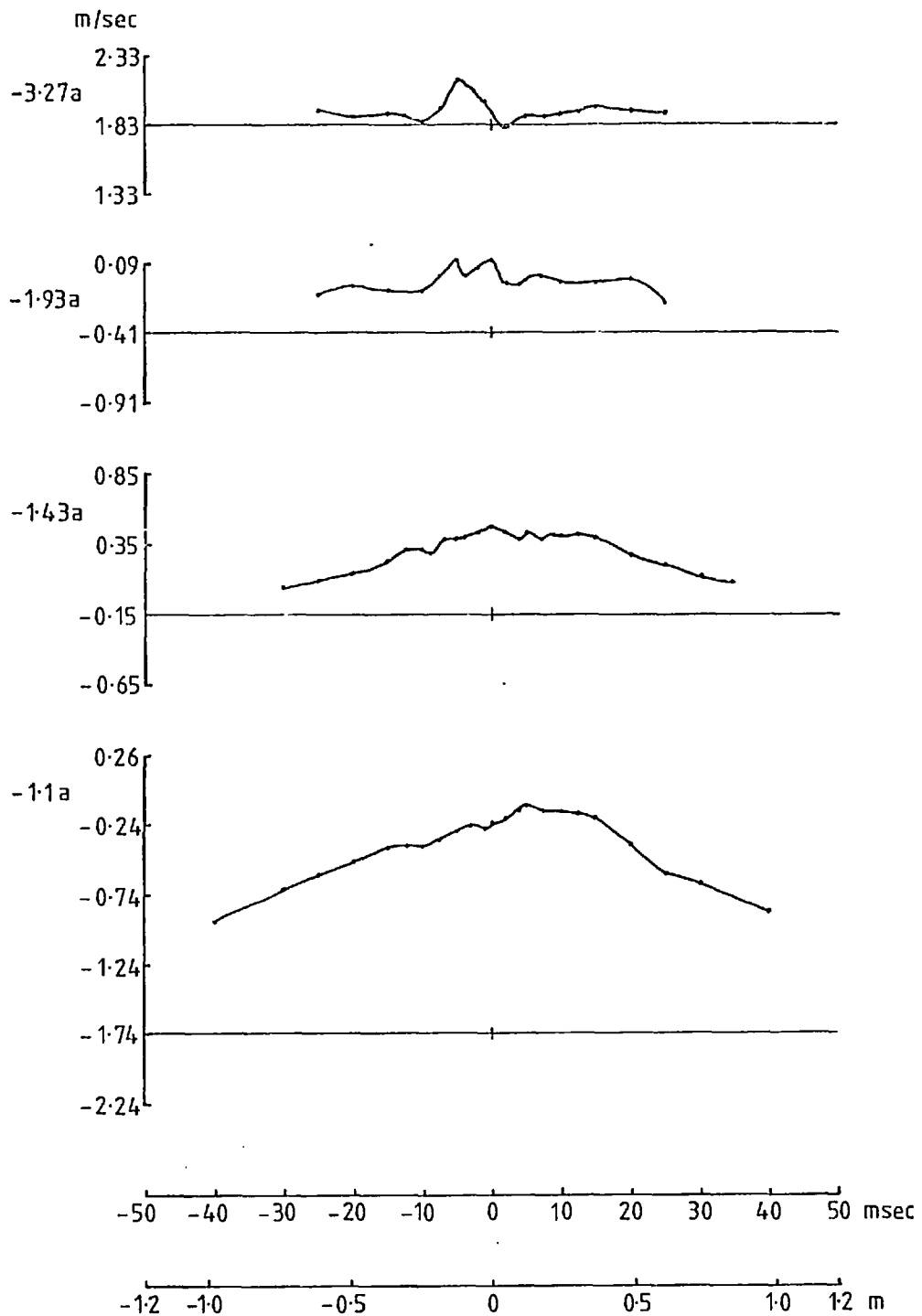


Fig. 57 Velocity profiles obtained using conditional sampling technique: experiment (iv).

Radial component, U_r , over a timescale of 100 msec, triggering on the negative peak of the uw signal.

Continued /

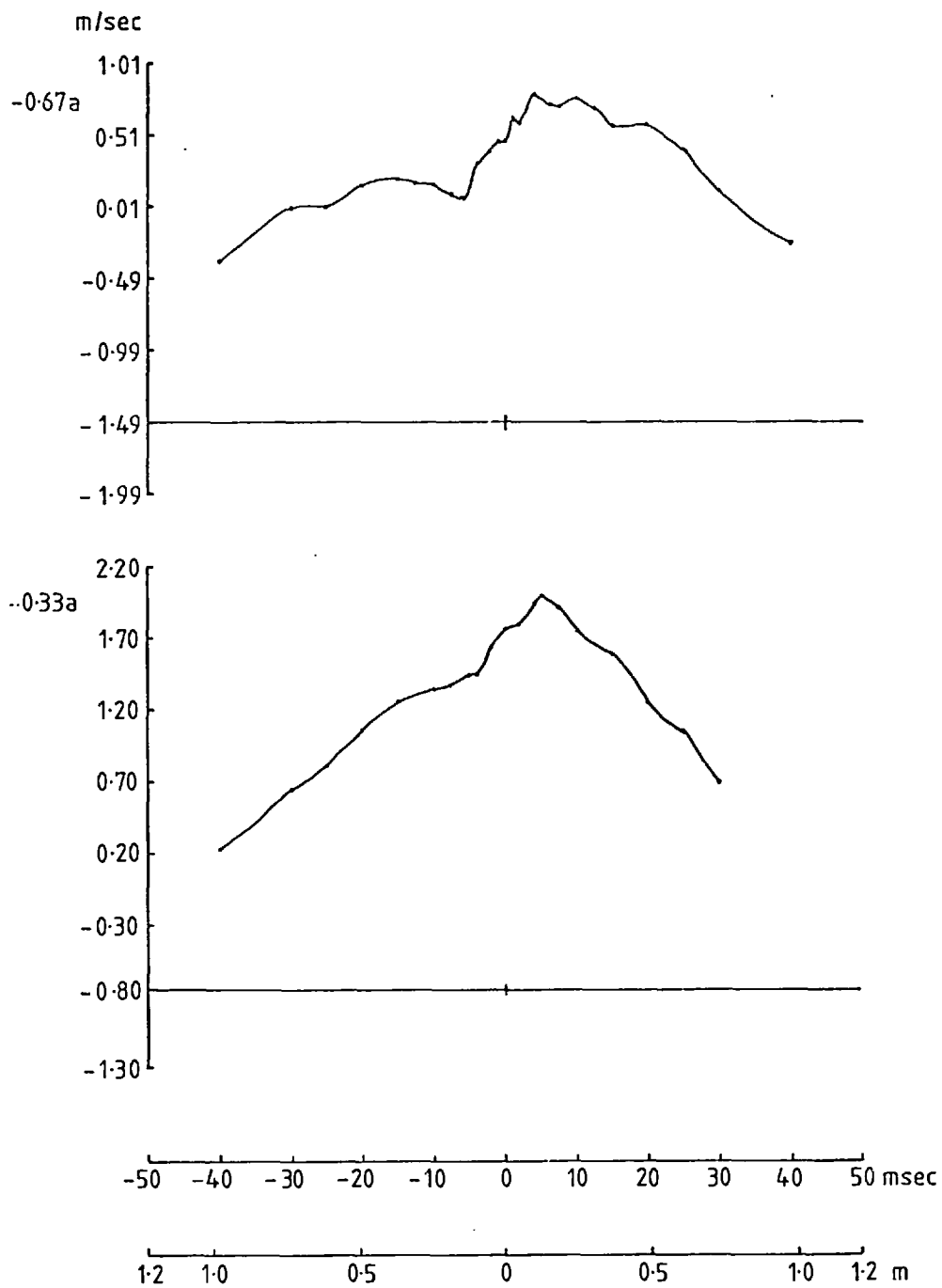


Fig. 57 Continued.

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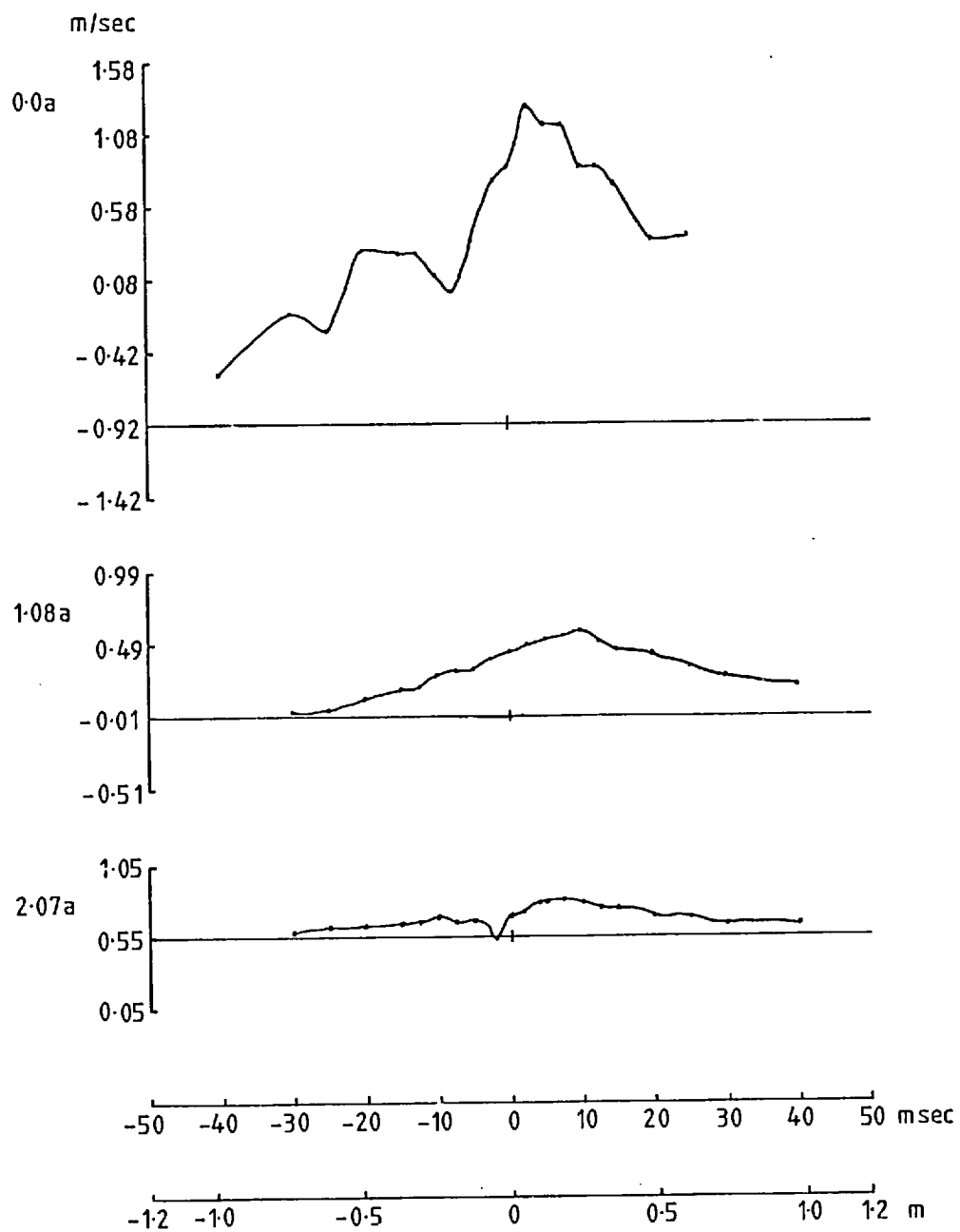


Fig. 57 Continued.

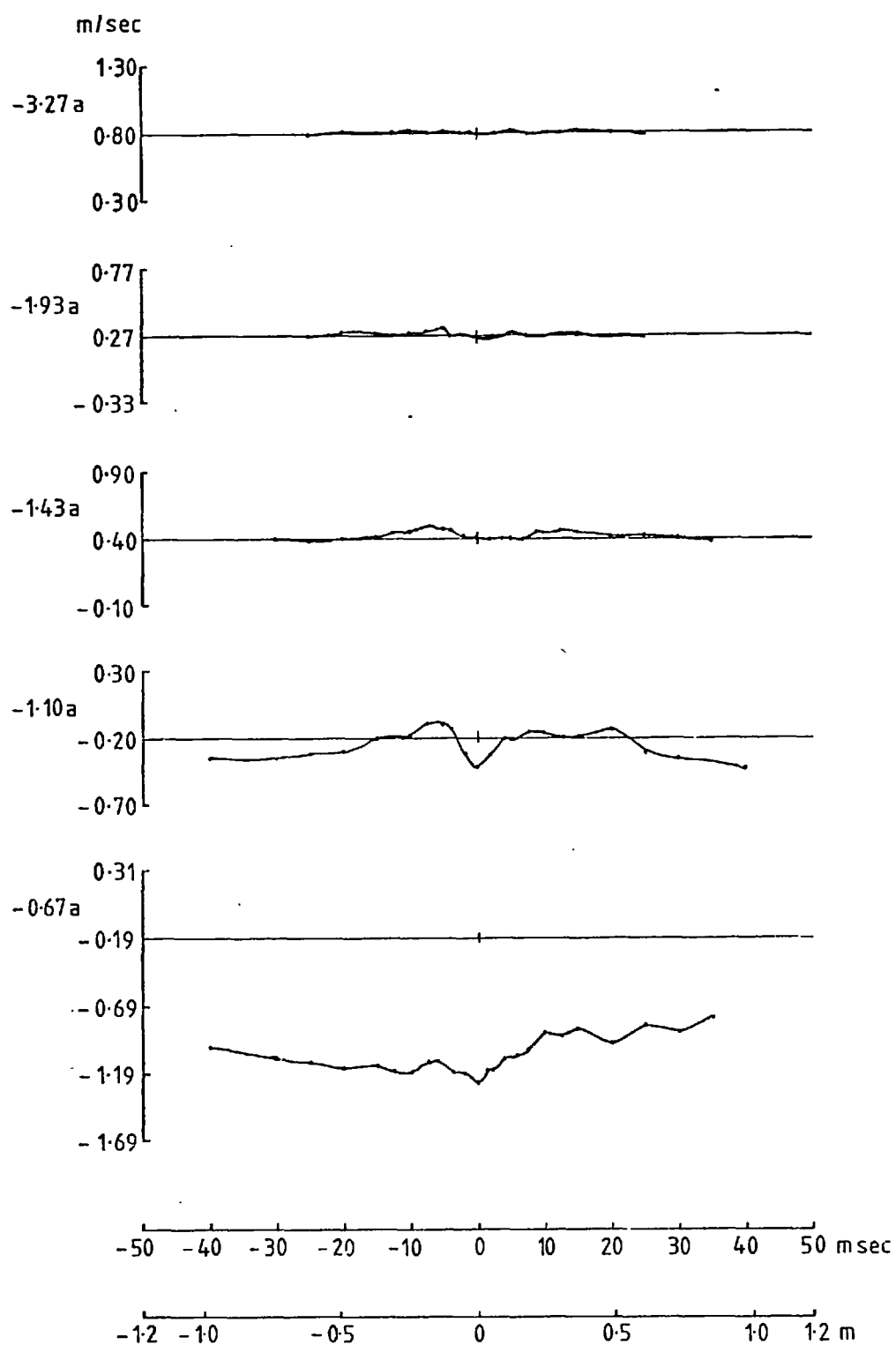


Fig. 58 Velocity profiles obtained using conditional sampling technique: experiment (iv).

Normal component, U_n , over a timescale of 100 msec, triggering on the negative peak of the uw signal.

Continued /

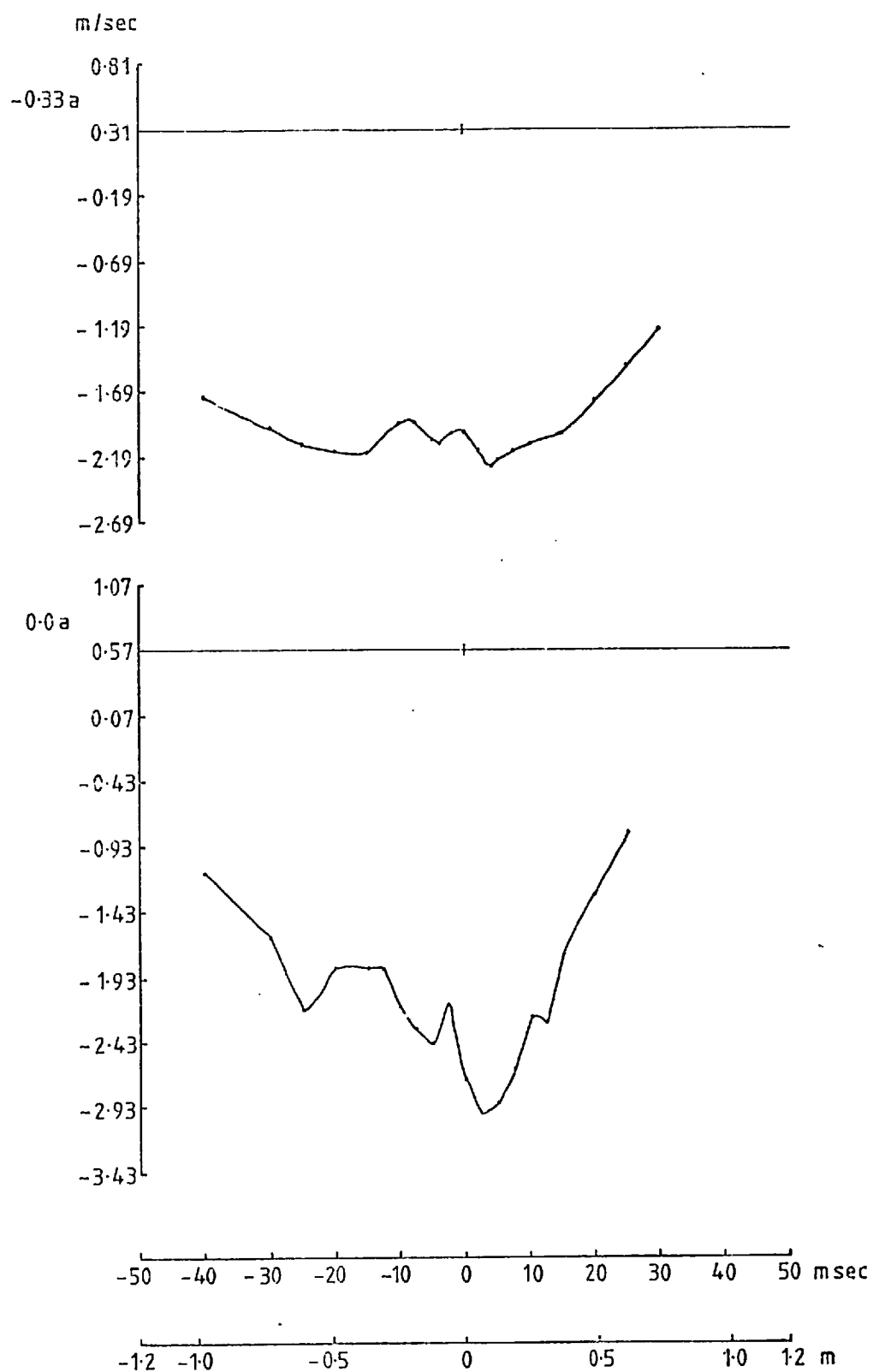


Fig. 58 Continued

Continued /

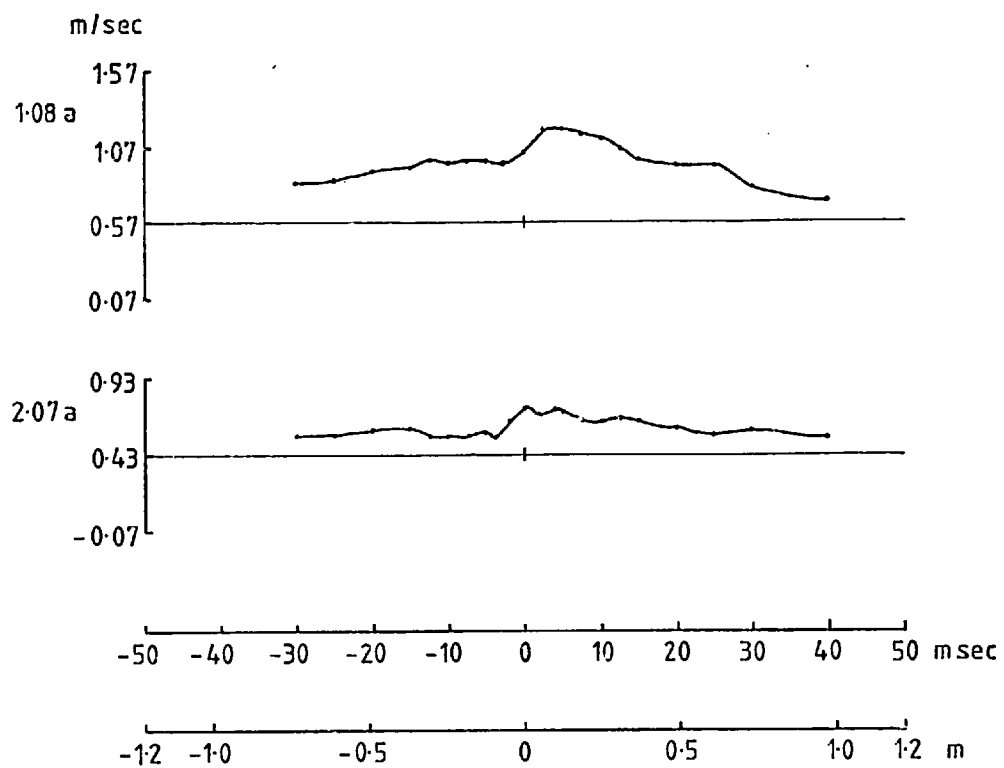


Fig. 58 Continued.

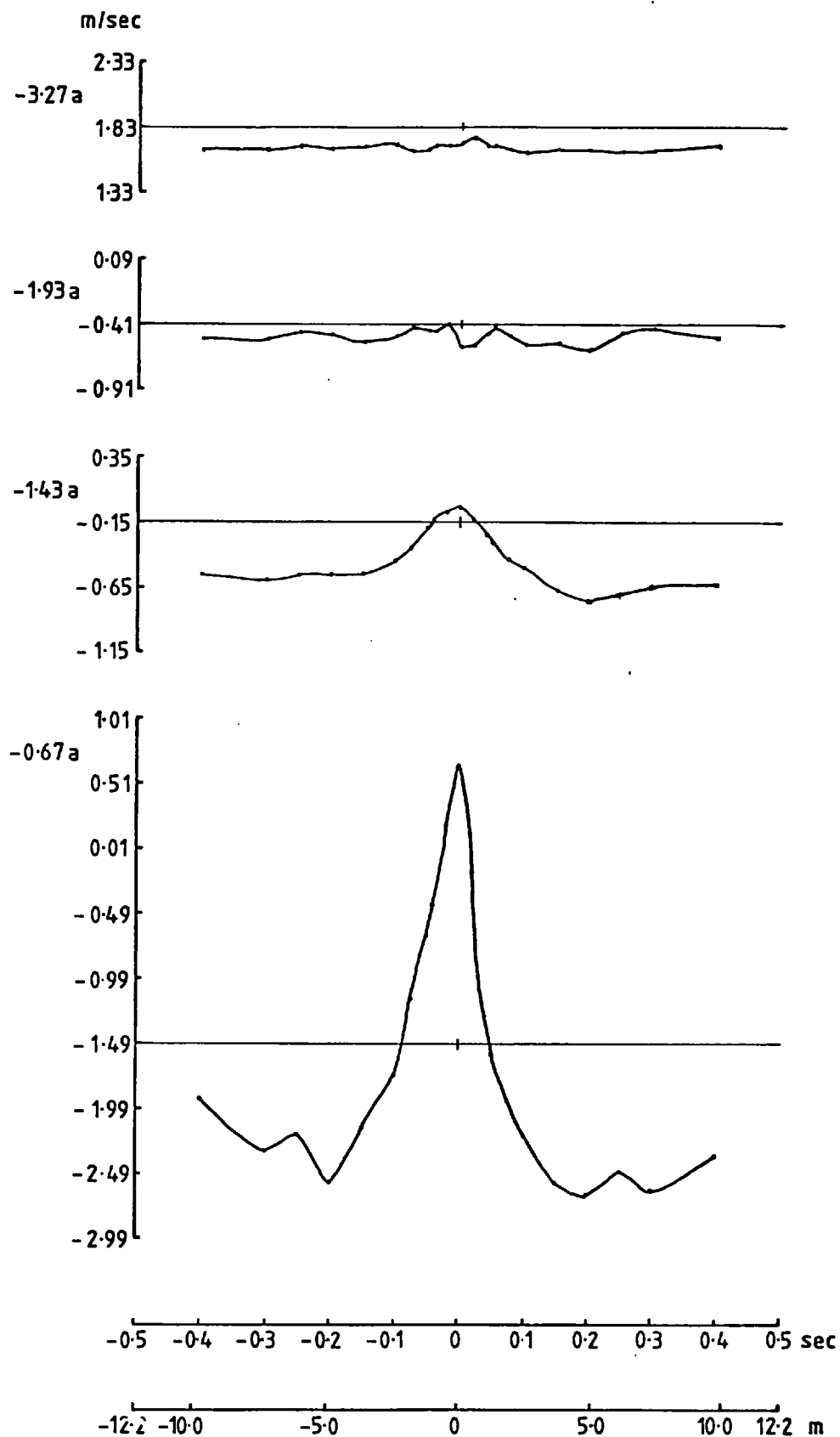


Fig. 59 Velocity profiles obtained using conditional sampling technique: experiment (iv).
 U_r , as in Figure 57, over a timescale of 1 sec.

Continued / , ...

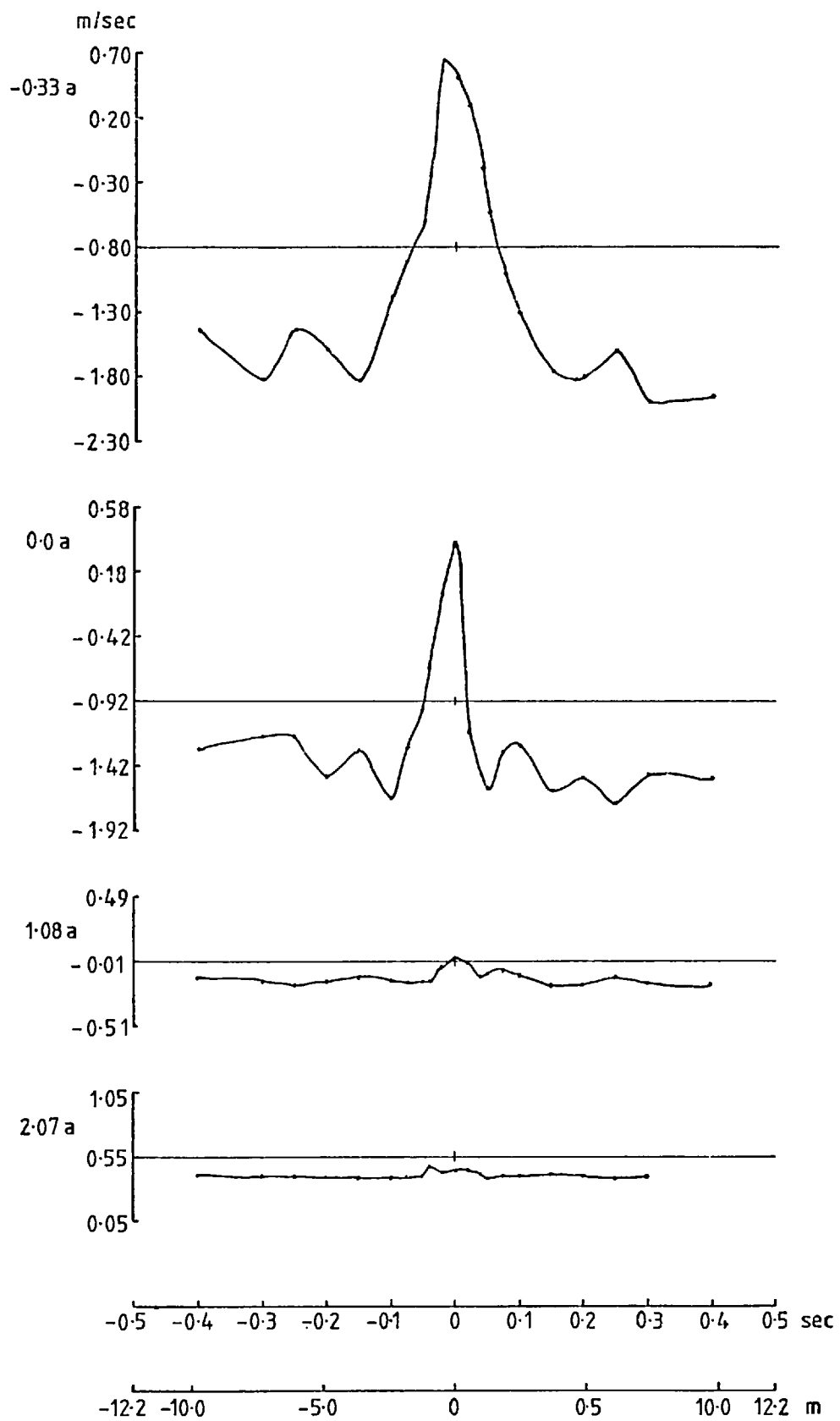


Fig. 59 Continued

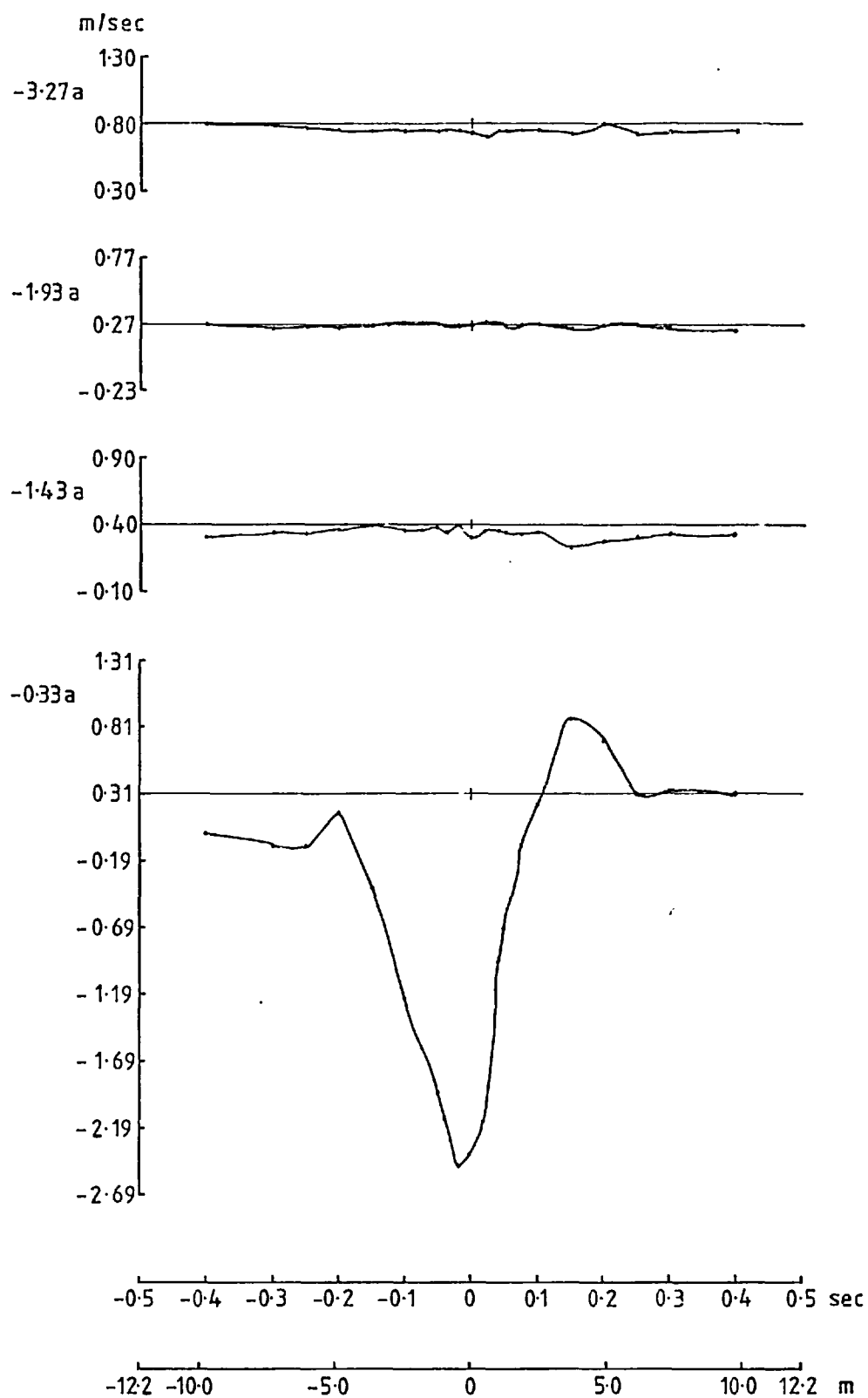


Fig. 60 Velocity profiles obtained using conditional sampling technique: experiment (iv).
 U_n , as in Figure 58, over a timescale of 1 sec.

Continued /

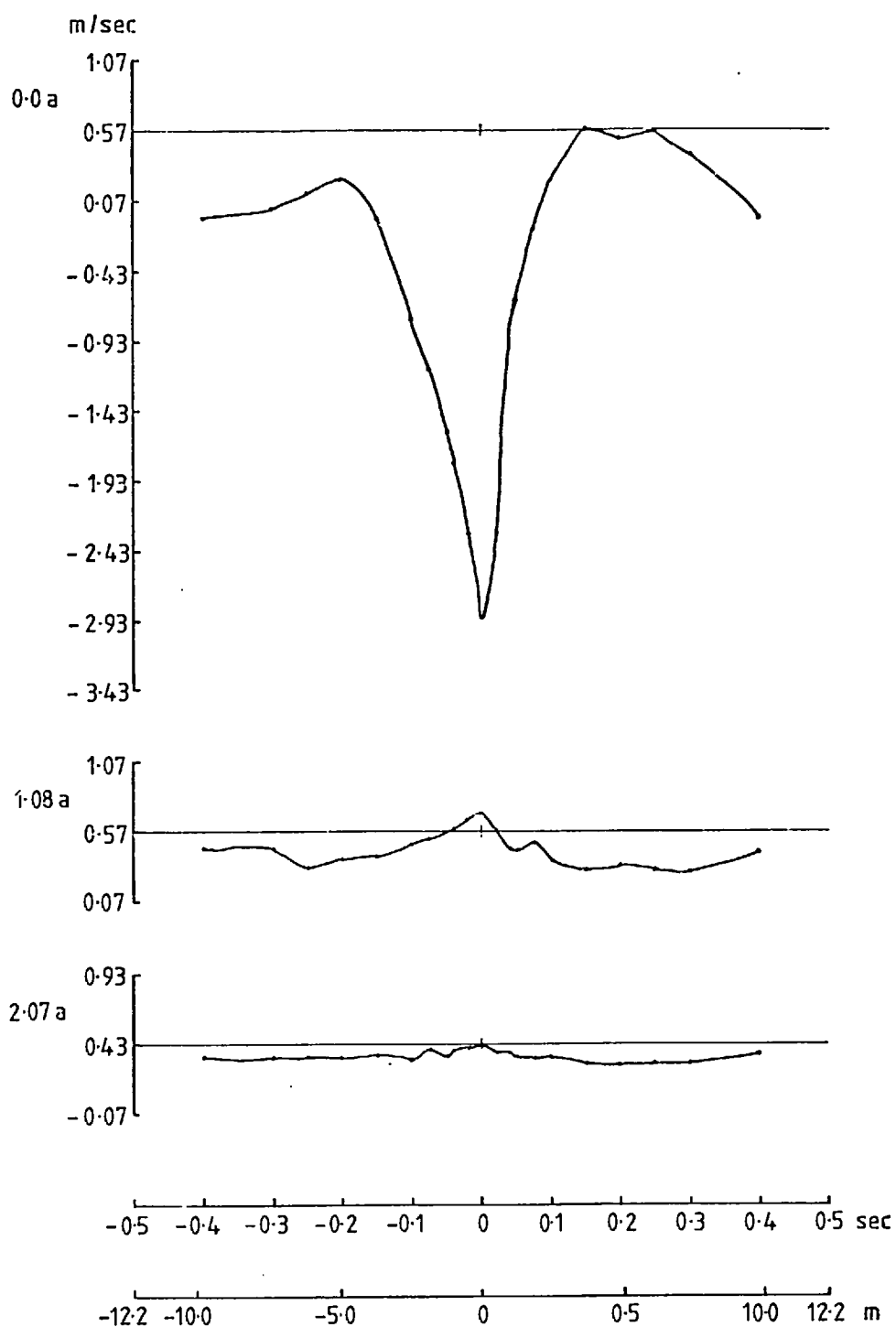


Fig. 60 Continued.

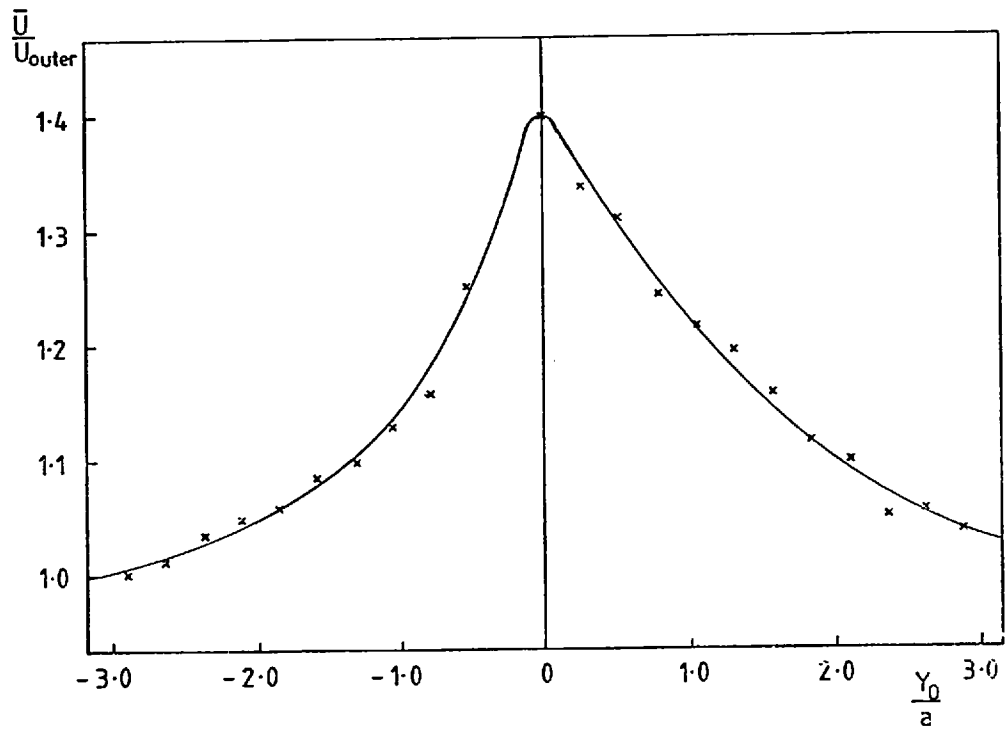


Fig. 61 Axial velocity profile of the vortex under study: experiment (ii).

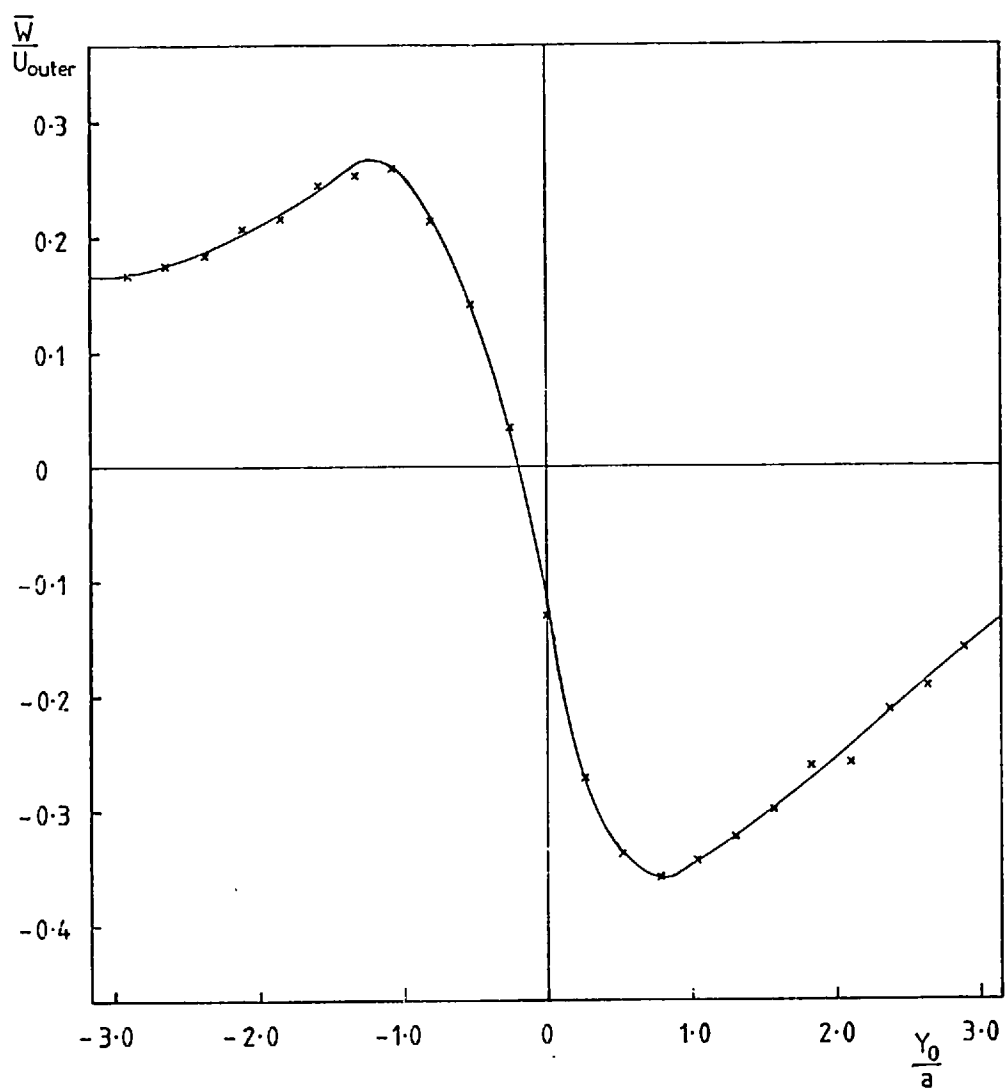


Fig. 62 Azimuthal velocity profile of the vortex under study: experiment (ii).

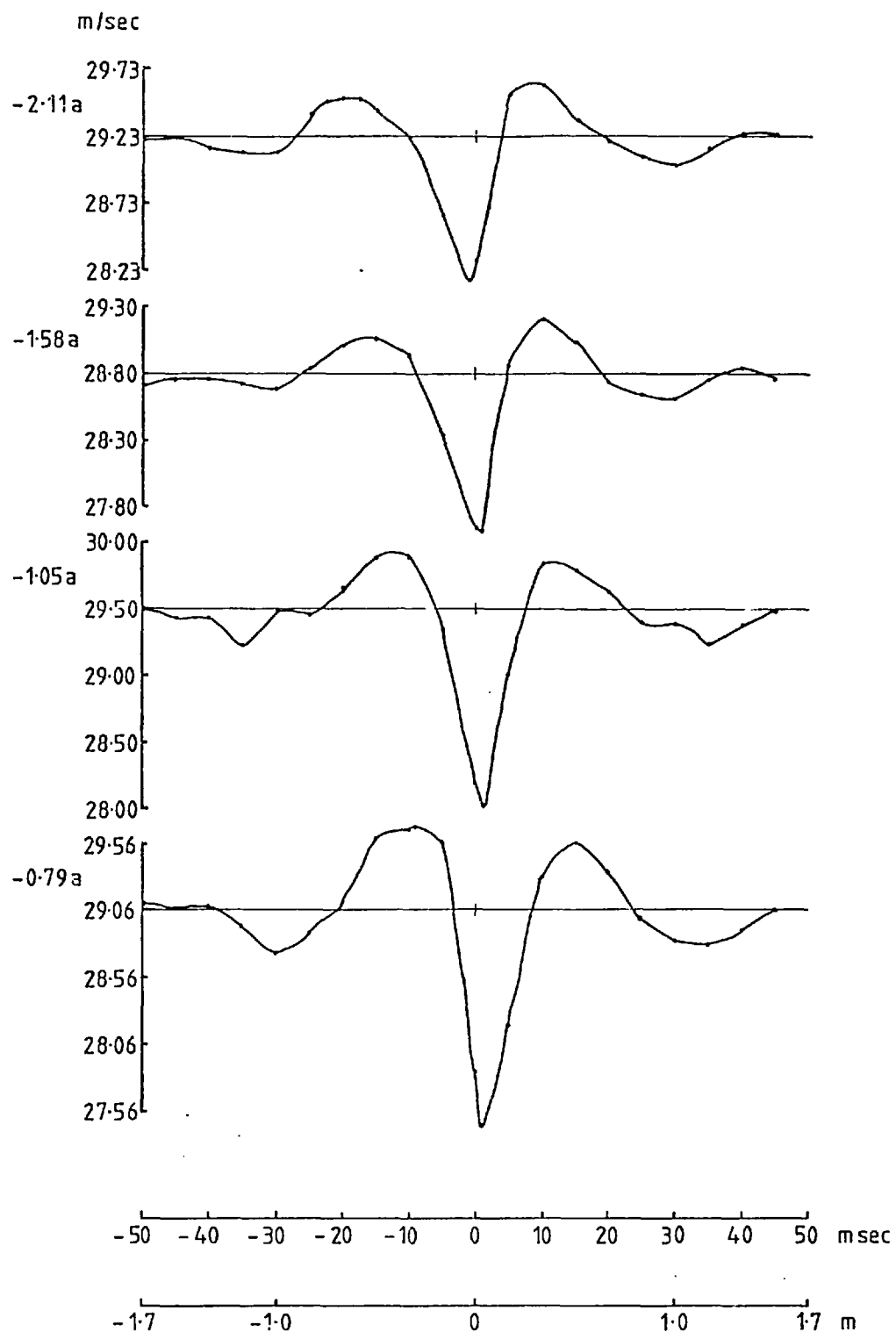


Fig. 63a Velocity profiles obtained using conditional sampling technique: experiment (ii).
Axial component, U . ($U - V$ plane)

Continued /

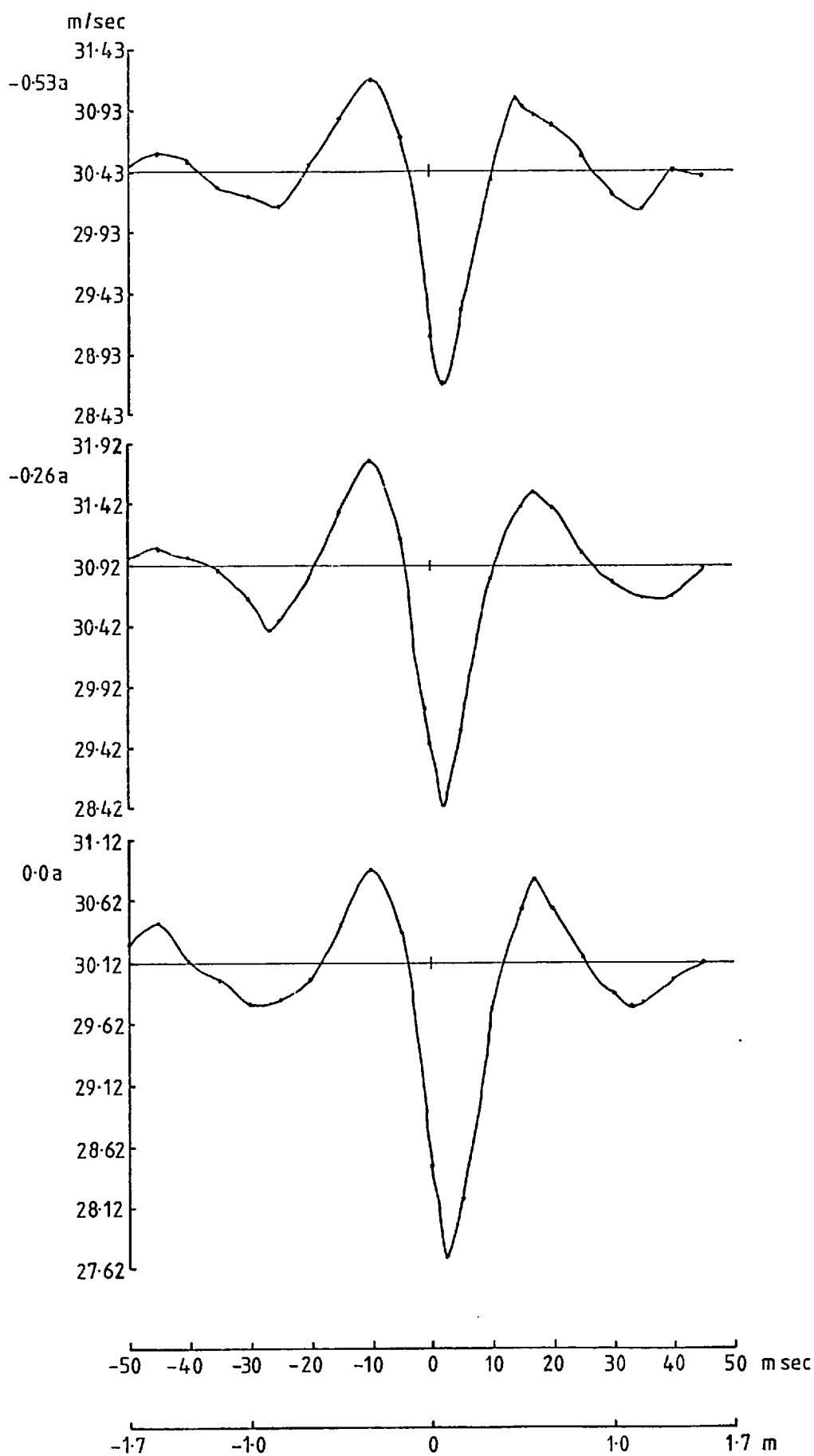


Fig. 63a Continued.

Continued /

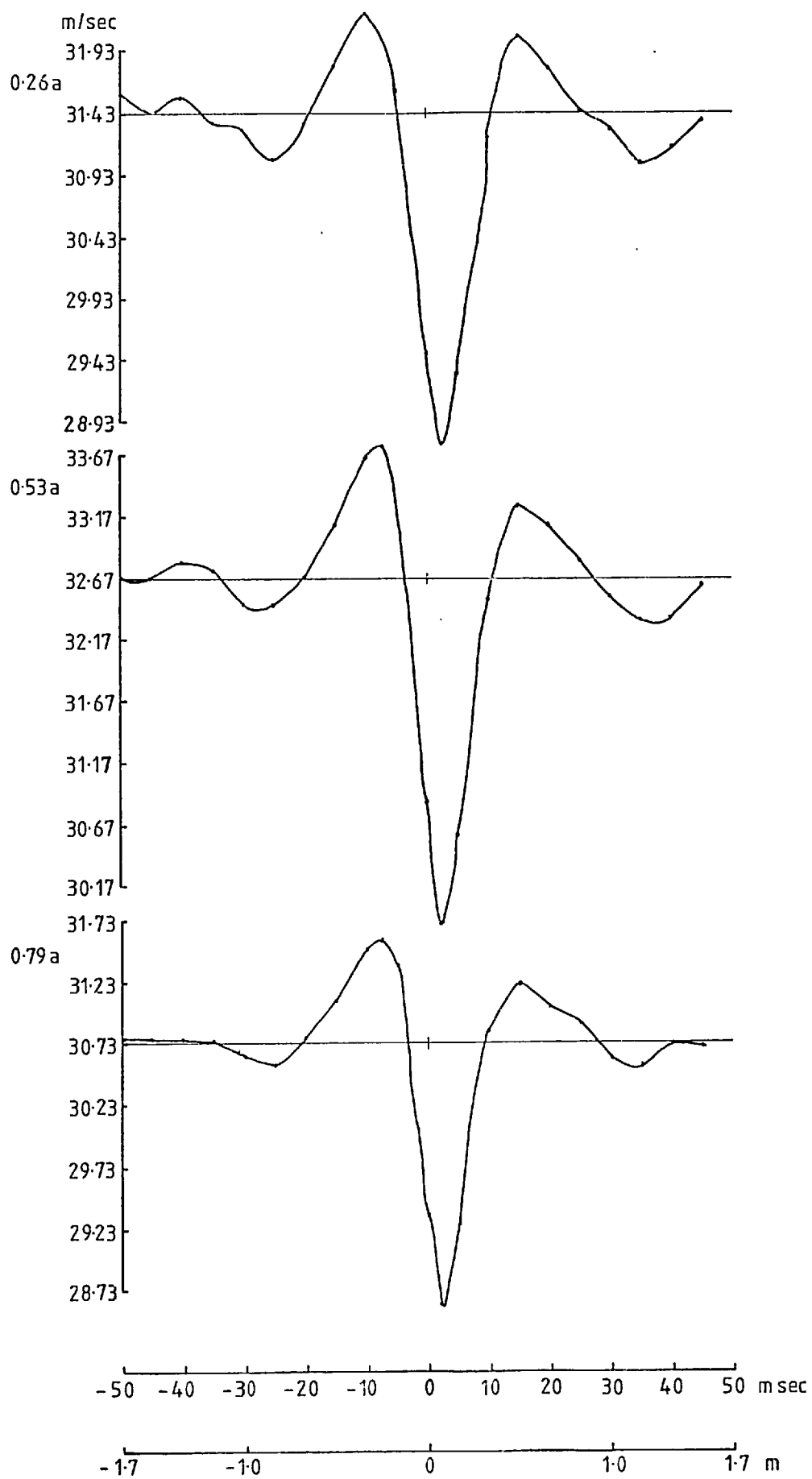


Fig. 63a Continued.

Continued /

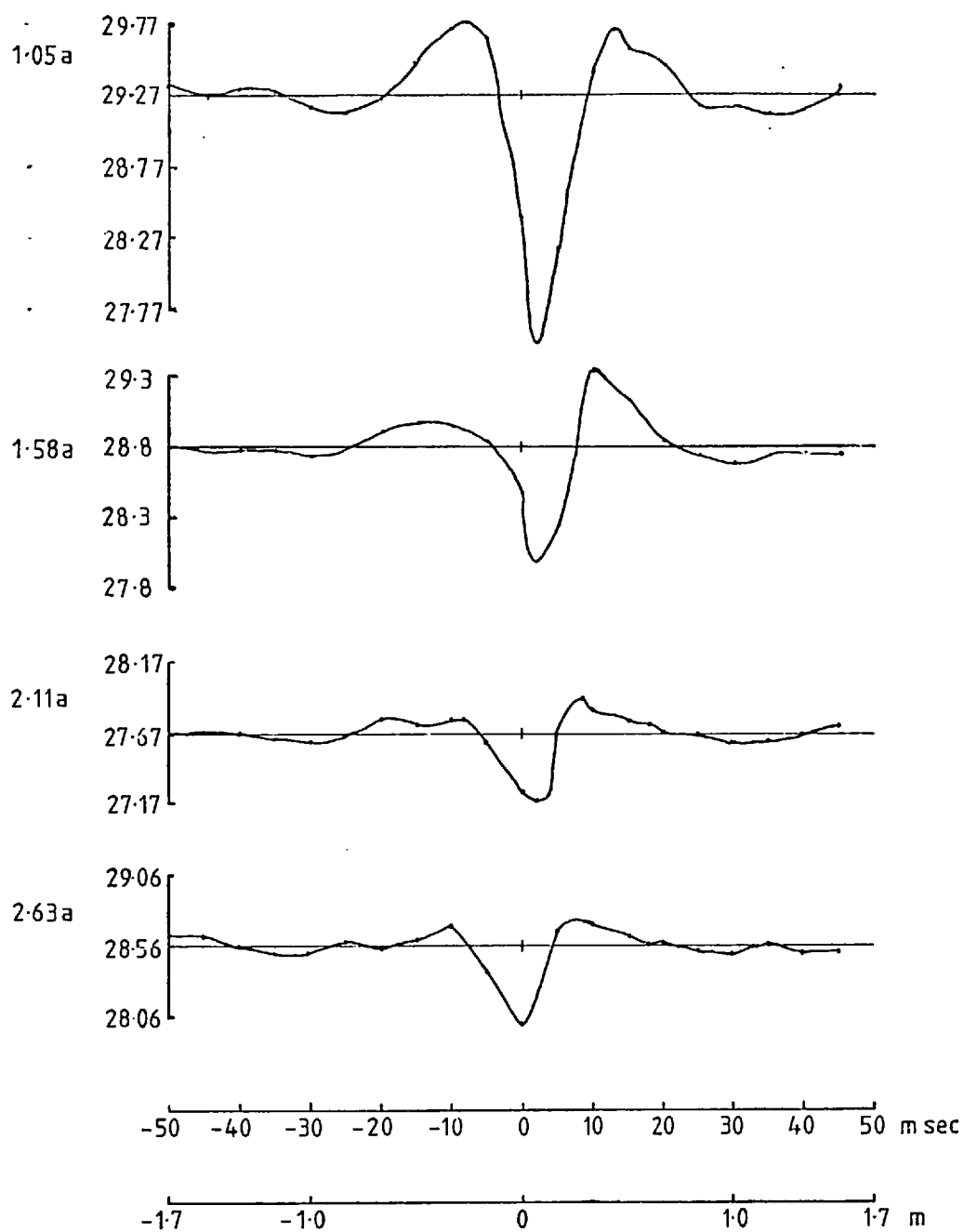


Fig. 63a Continued.

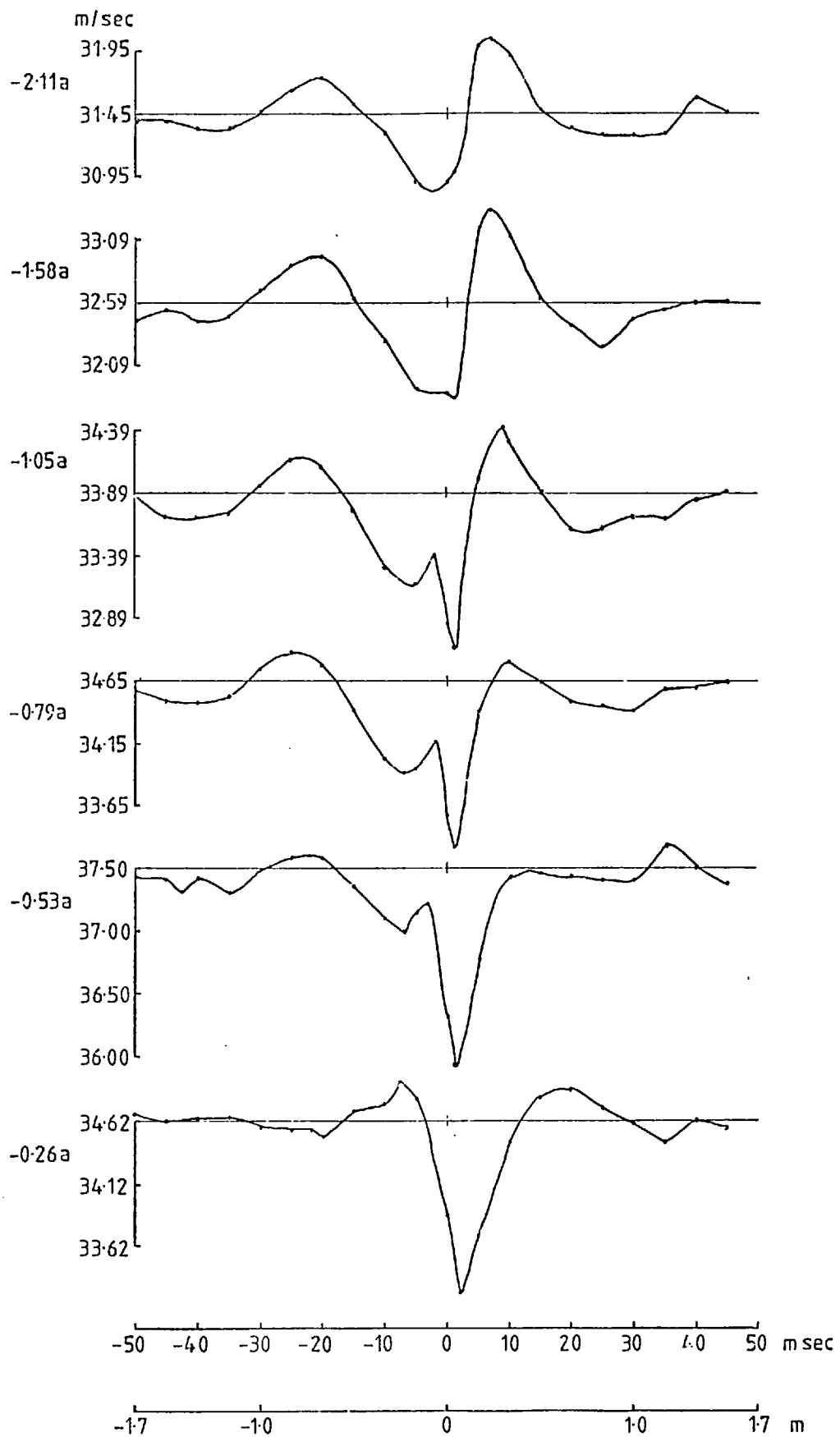


Fig. 63b Velocity profiles obtained using conditional sampling technique: experiment (ii).
Axial component, U . ($U-W$ plane)

Continued /

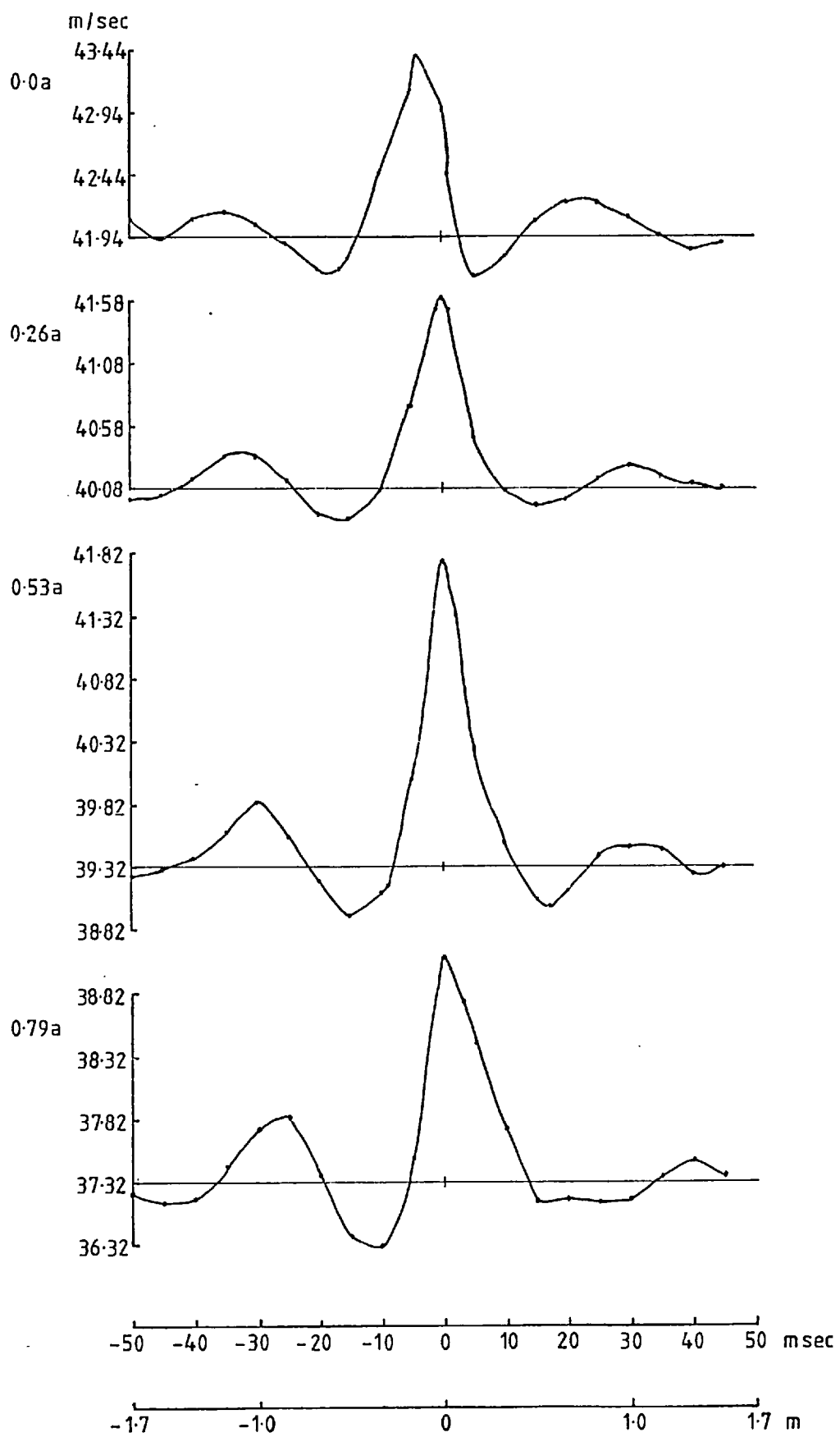


Fig. 63b Continued.

Continued /

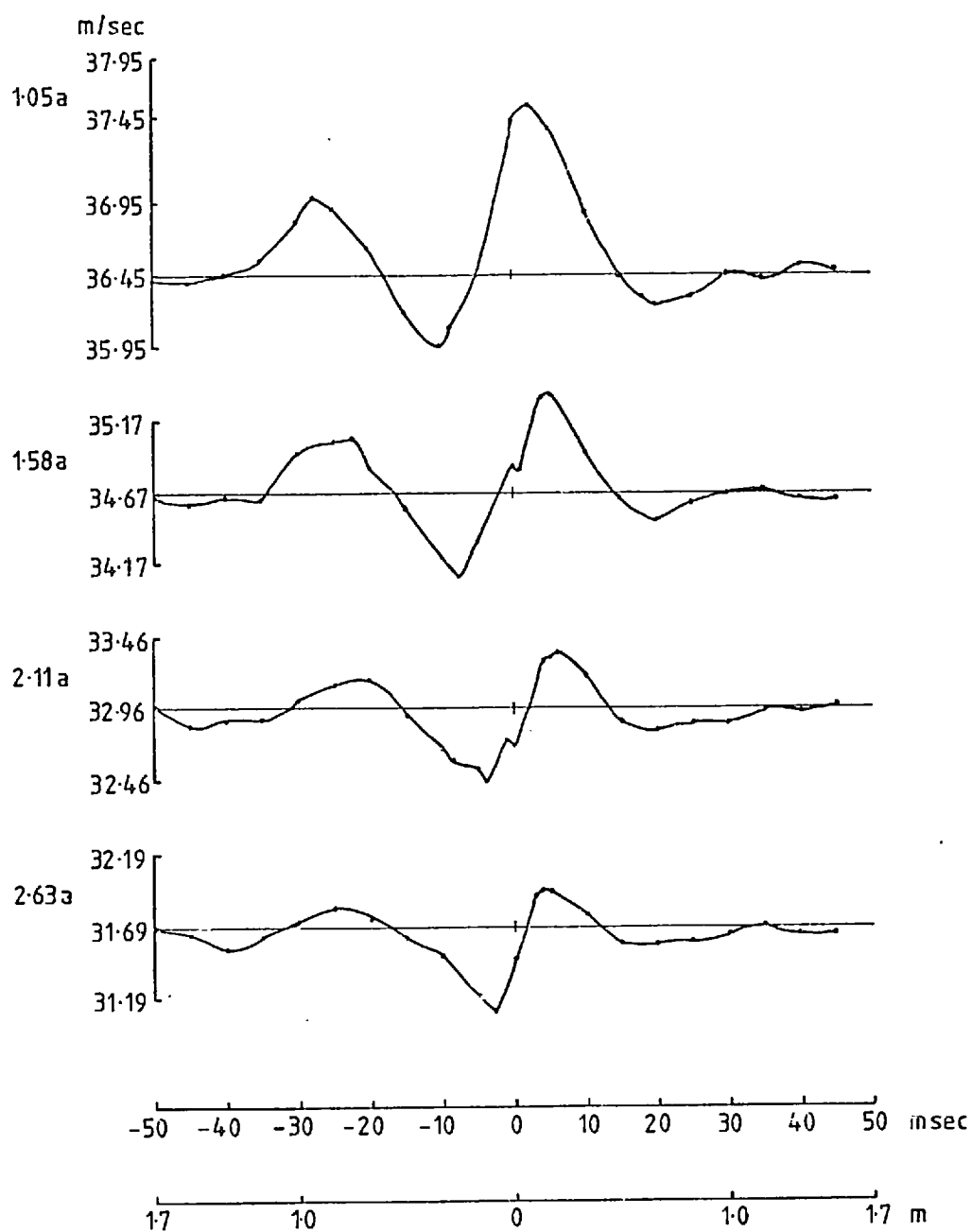


Fig. 63b Continued.

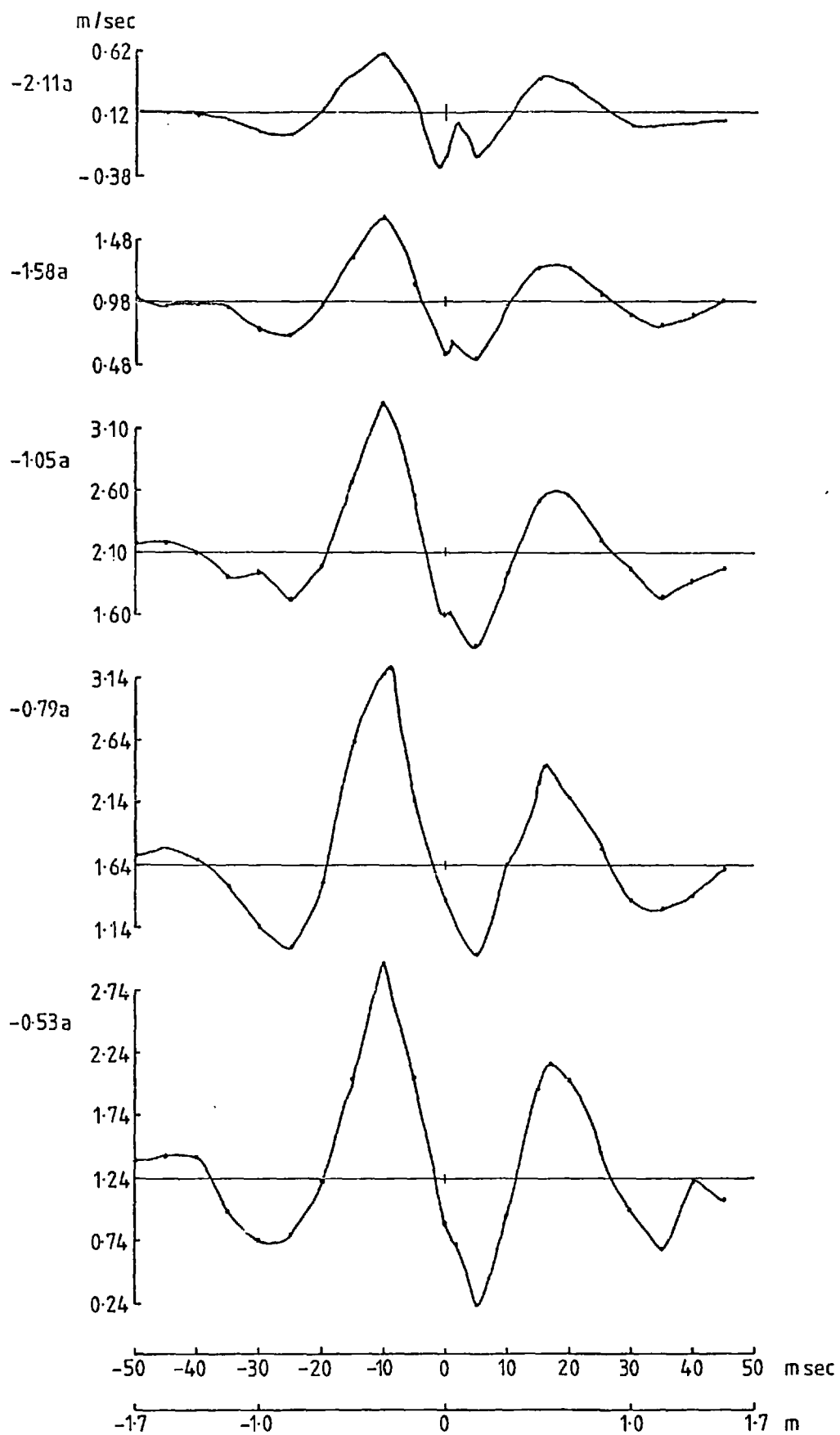


Fig. 64 Velocity profiles obtained using conditional sampling technique: experiment (ii).
Radial component, V_r .

Continued /

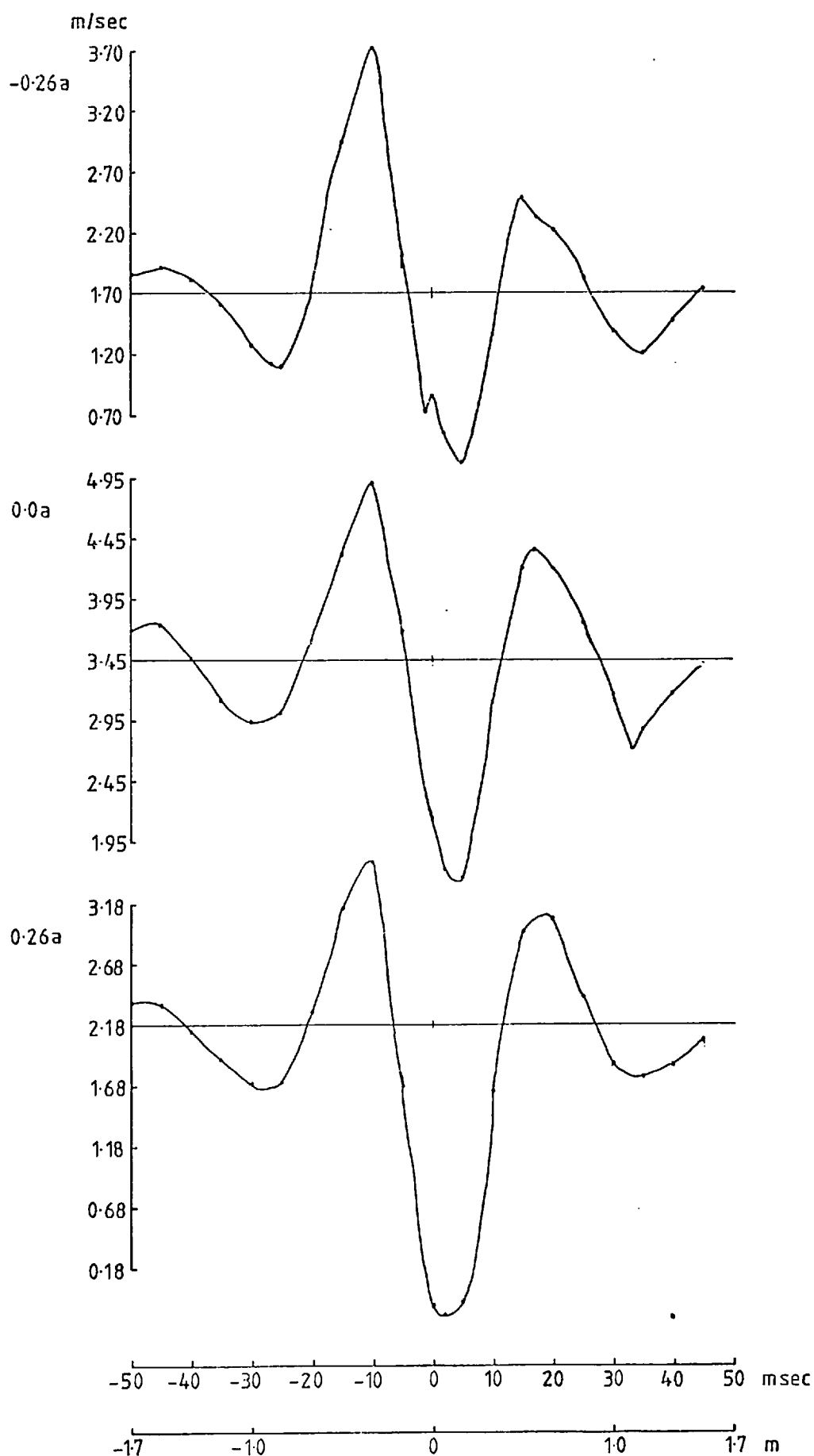


Fig. 64 Continued.

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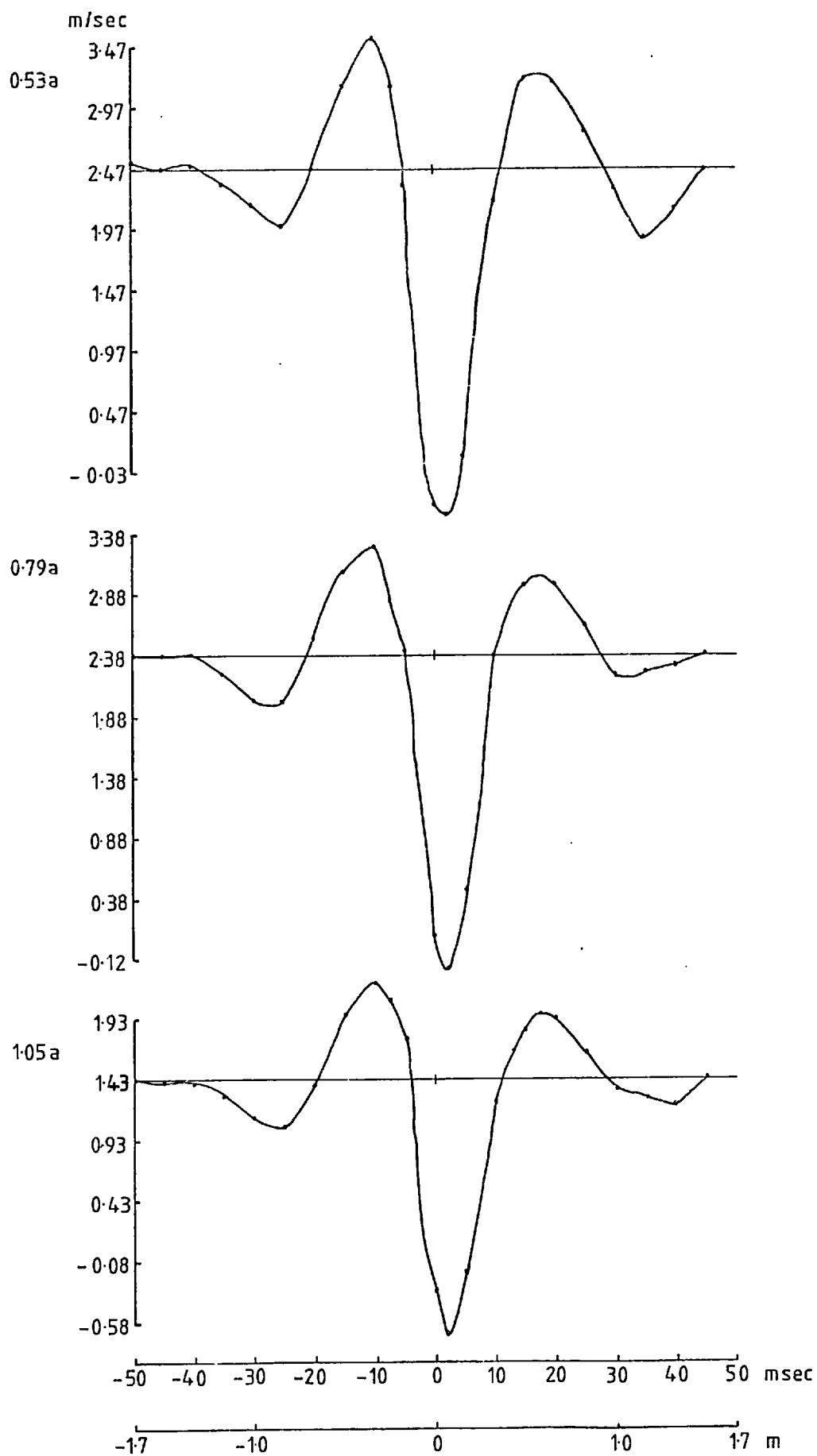


Fig. 64 Continued.

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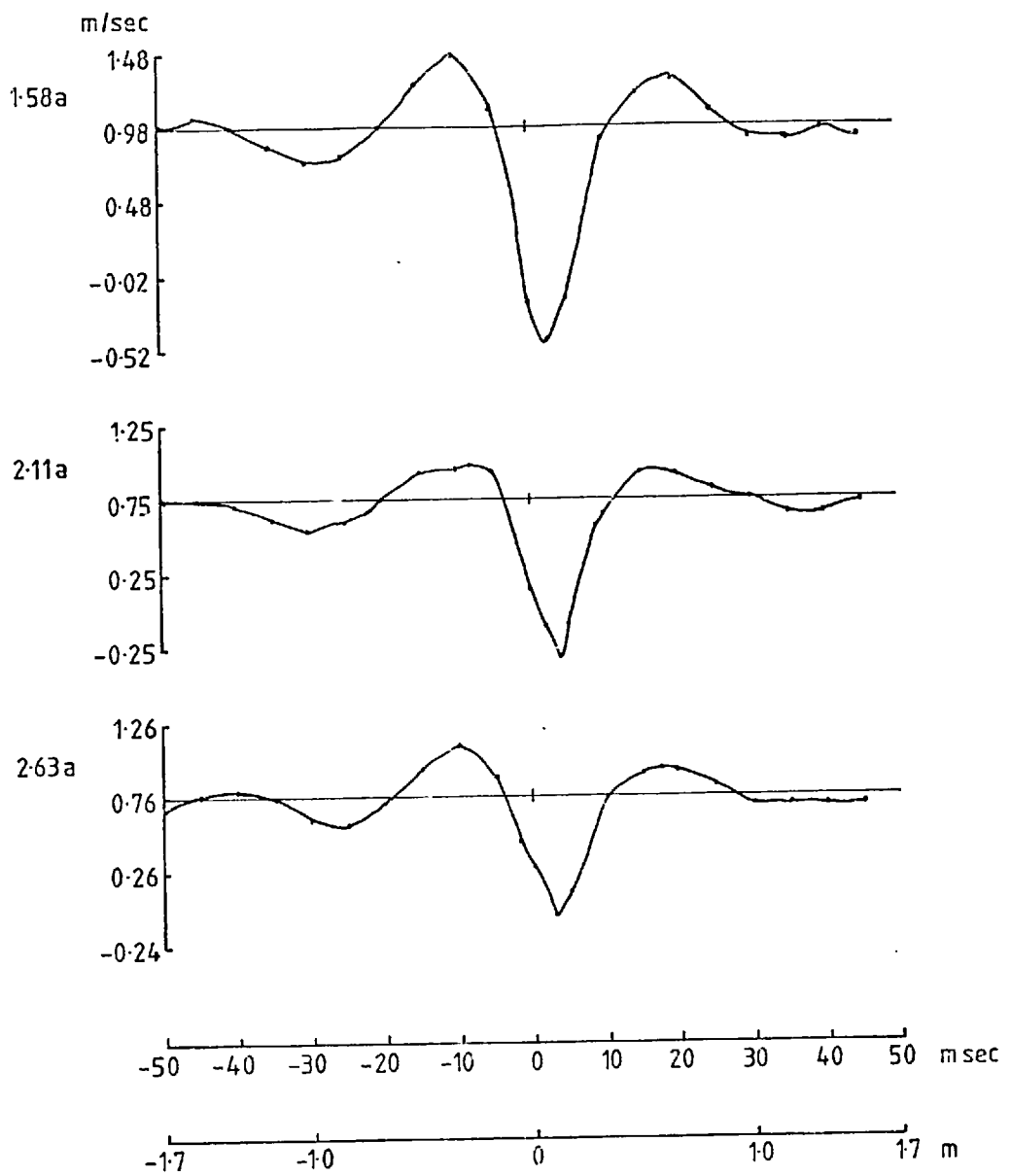


Fig. 64 Continued.

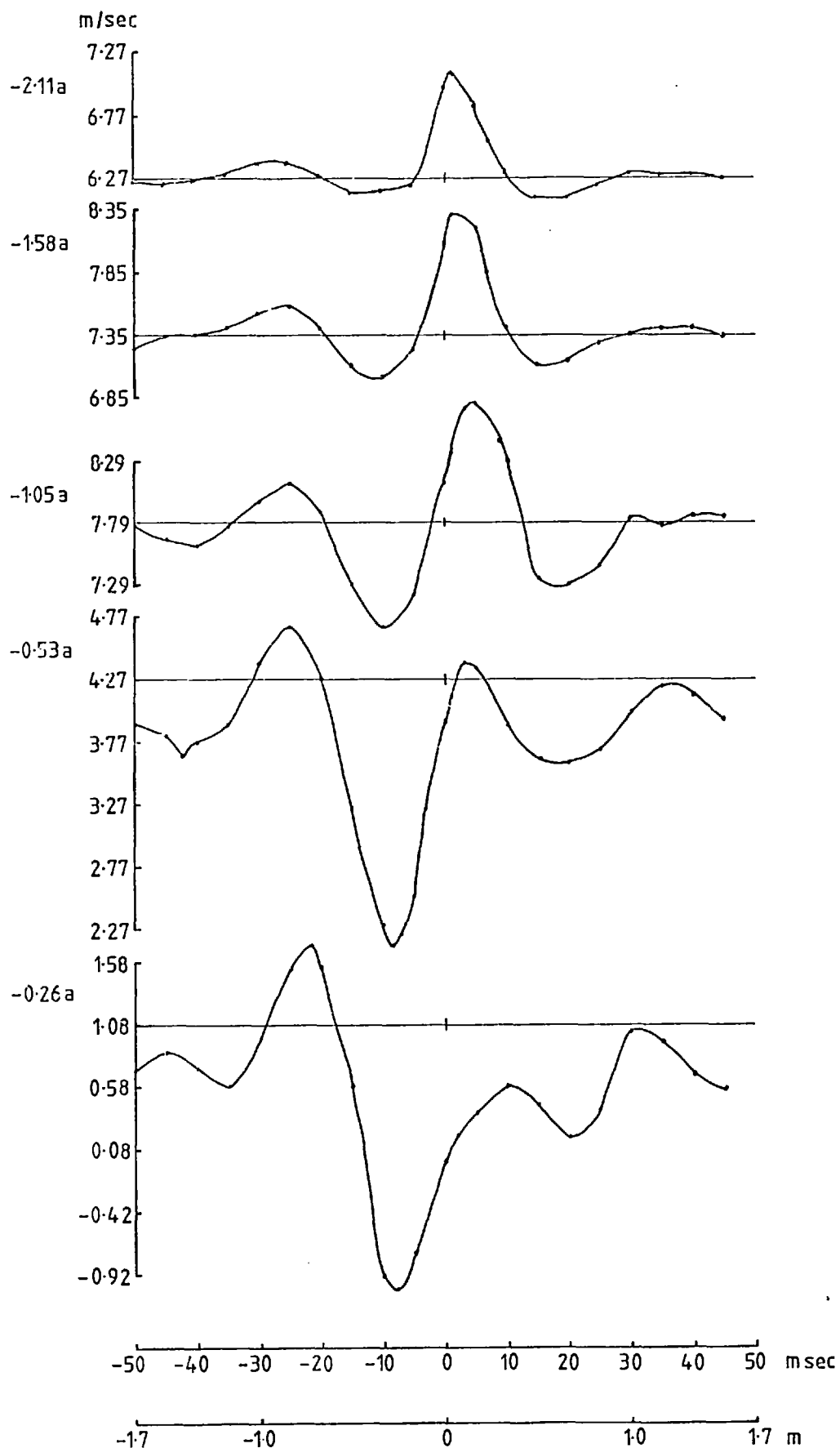


Fig. 65 Velocity profiles obtained using conditional sampling technique: experiment (ii)
Azimuthal component, W .

Continued /

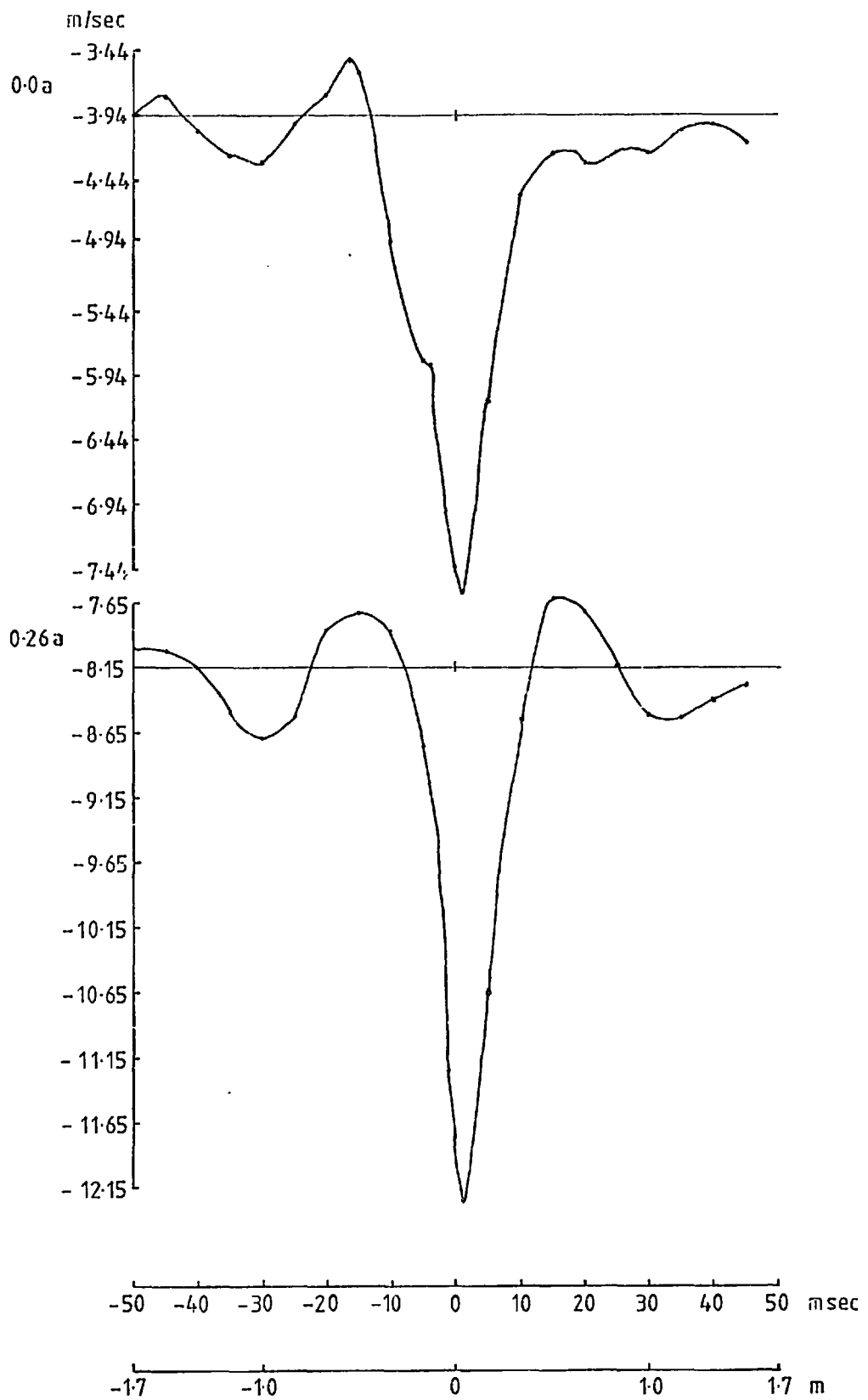


Fig. 65 Continued

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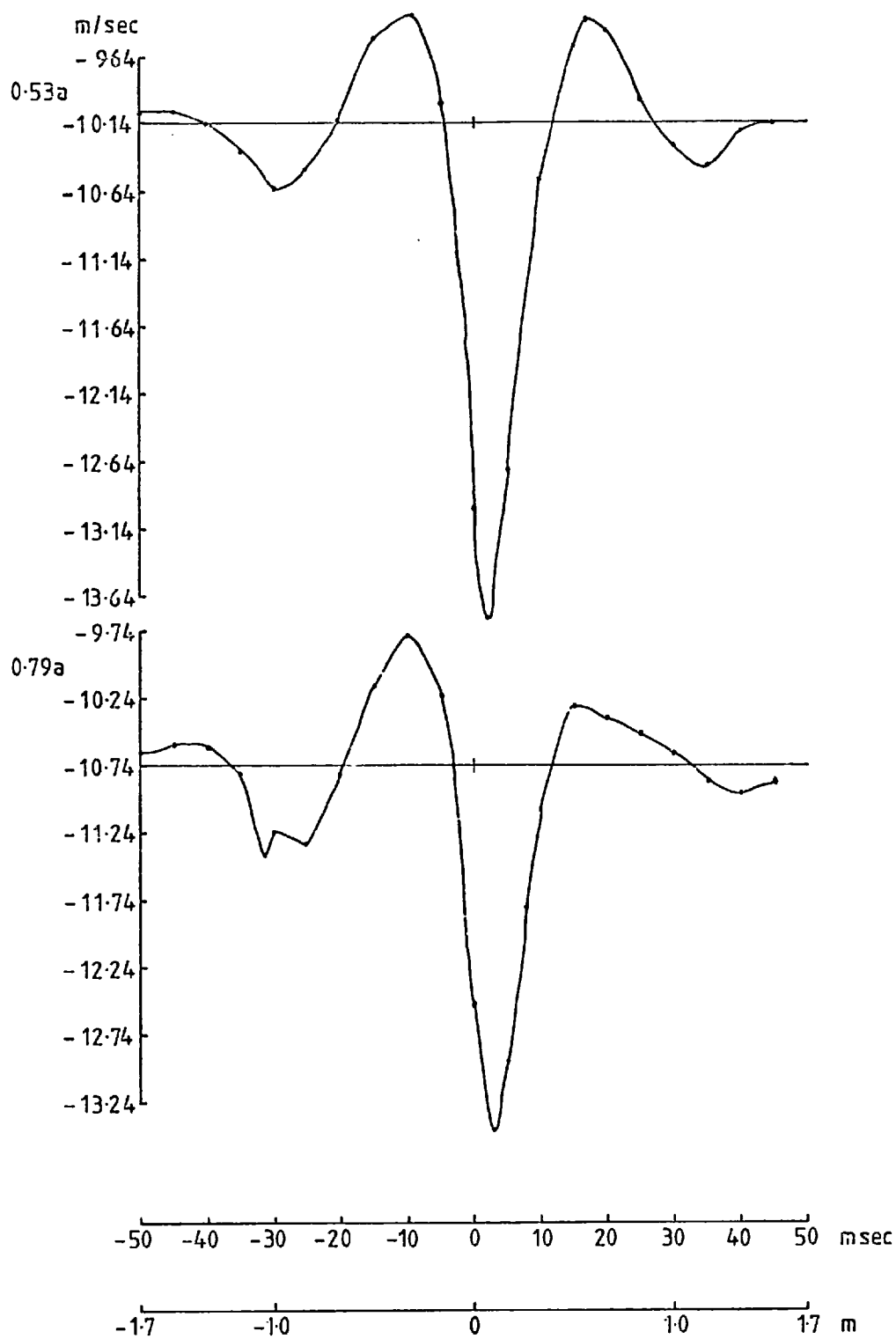


Fig. 65 Continued

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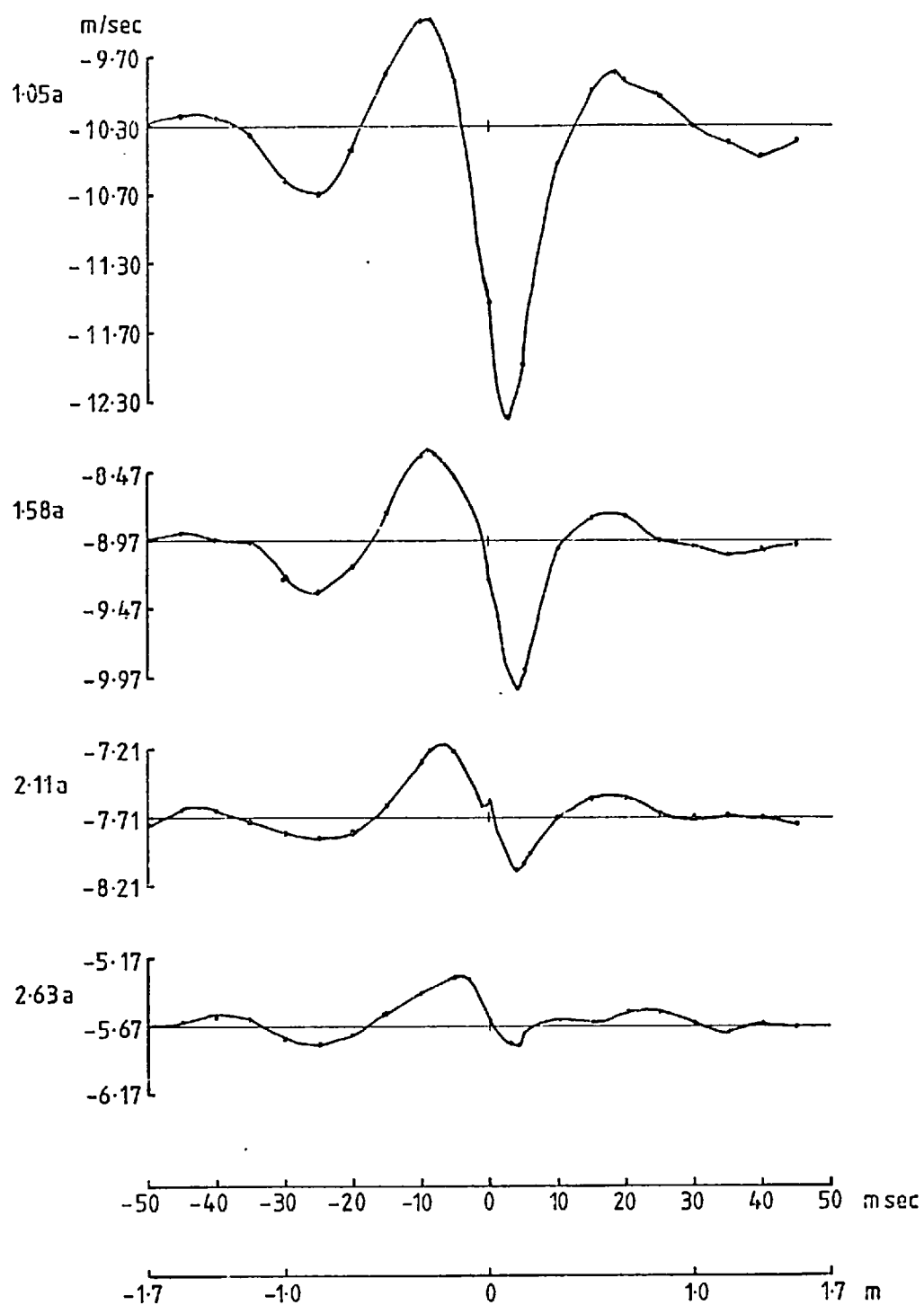


Fig. 65 Continued

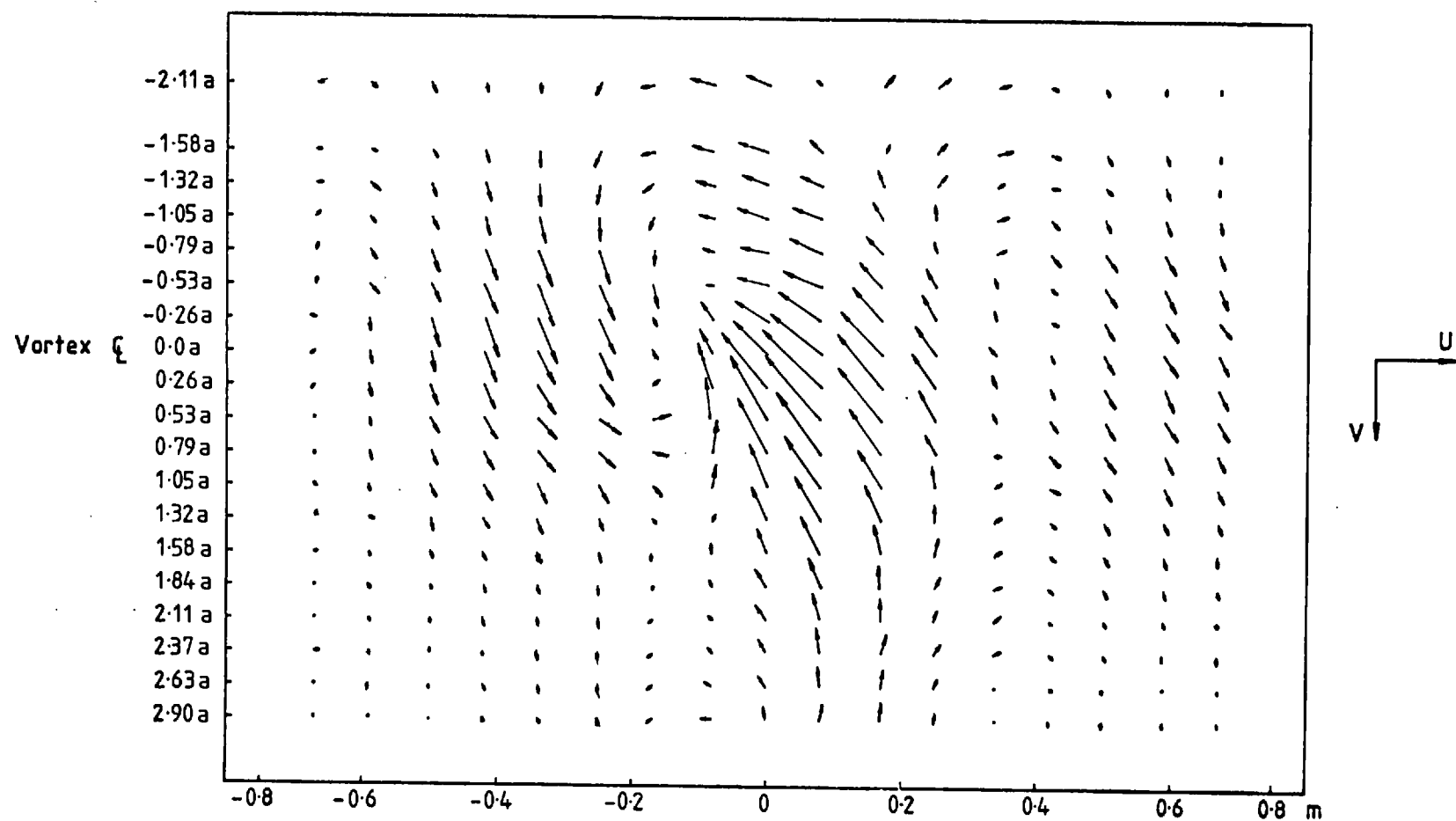


Fig. 66 Perturbations in the $U - V$ plane at various positions in the vortex.

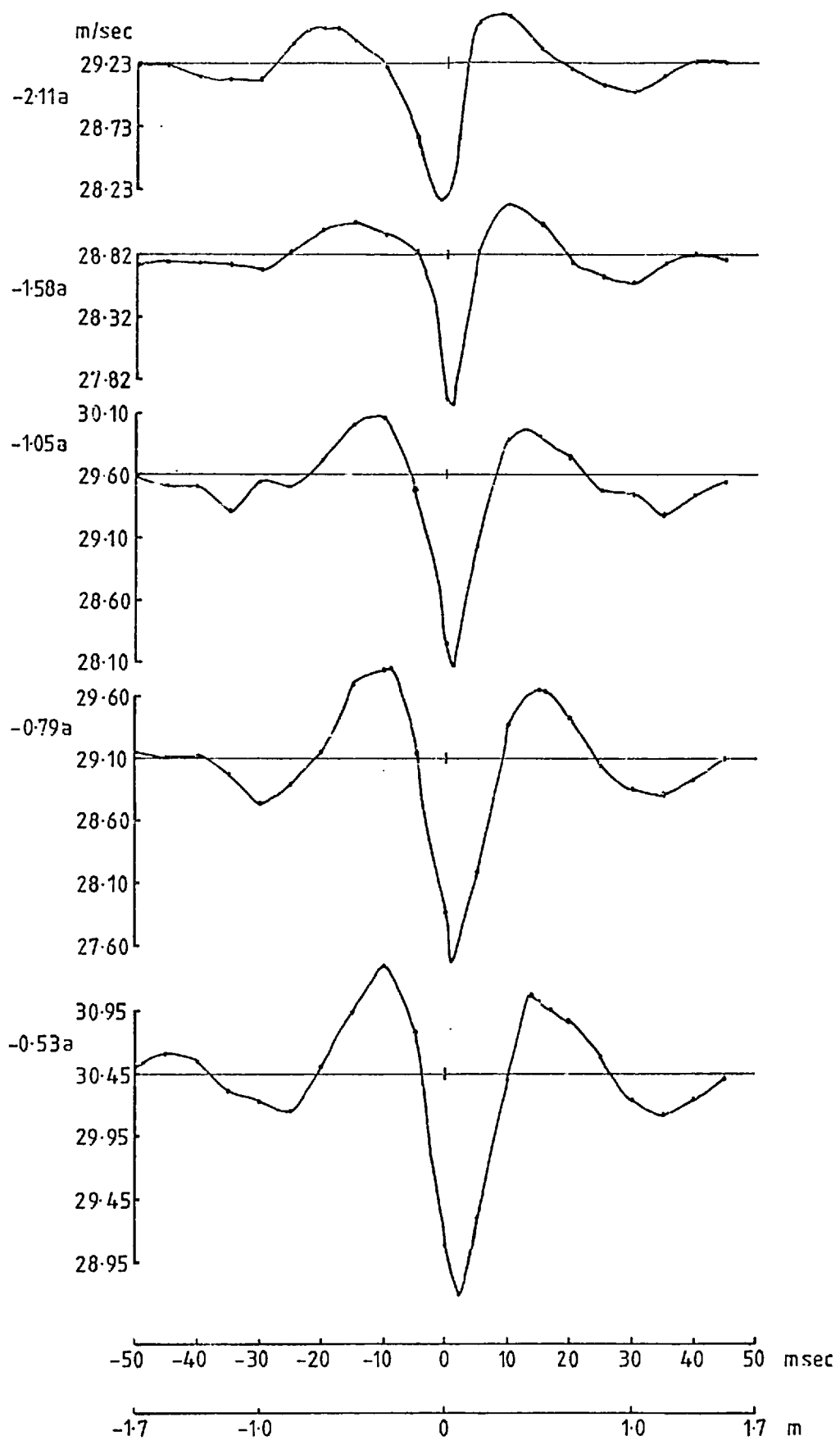


Fig. 67 Velocity profiles obtained using conditional sampling technique: experiment (ii).
Resultant component of fluctuating velocity, U_{res} , in the U-V plane.

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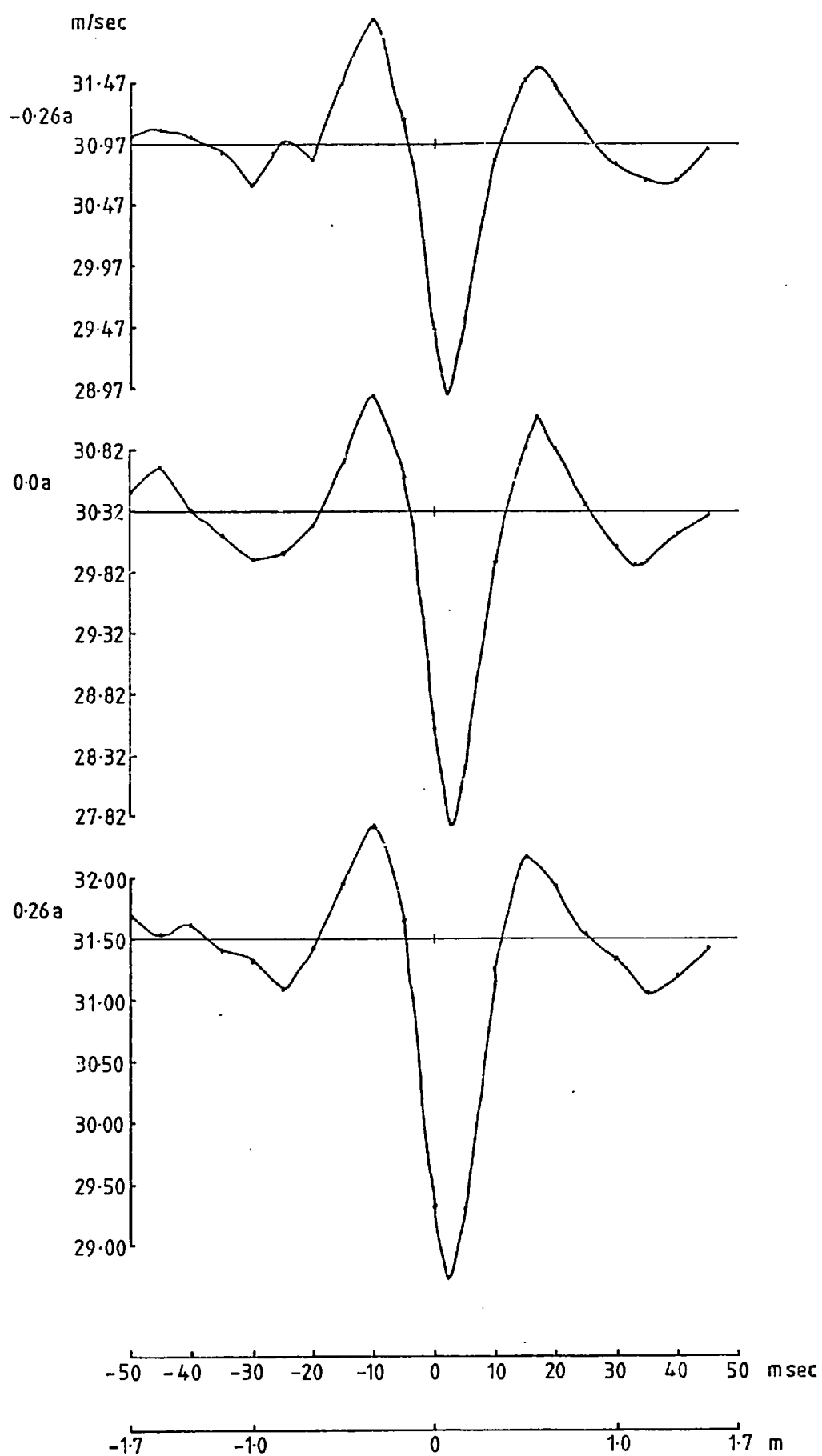


Fig. 67 Continued

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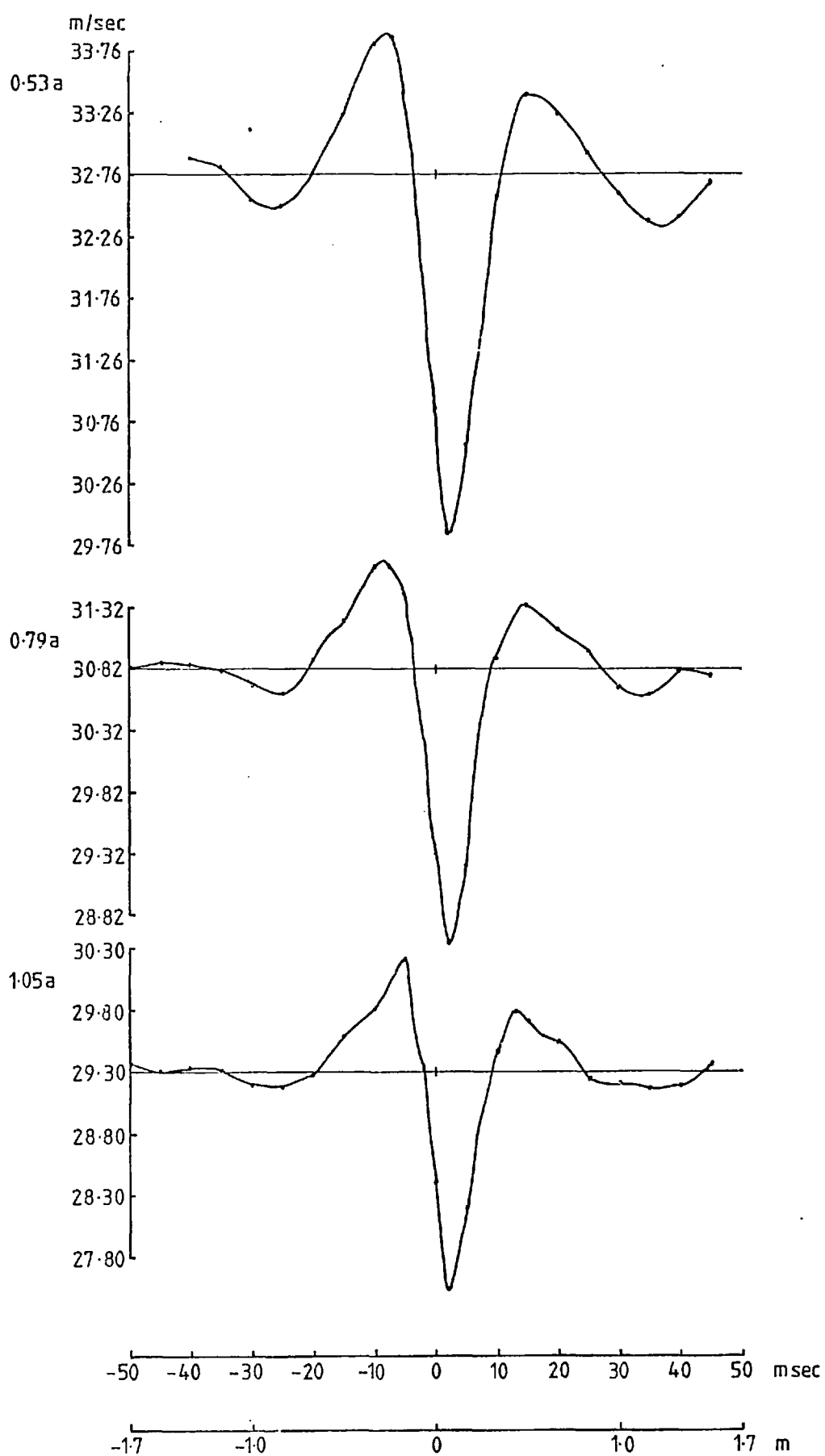


Fig. 67 Continued

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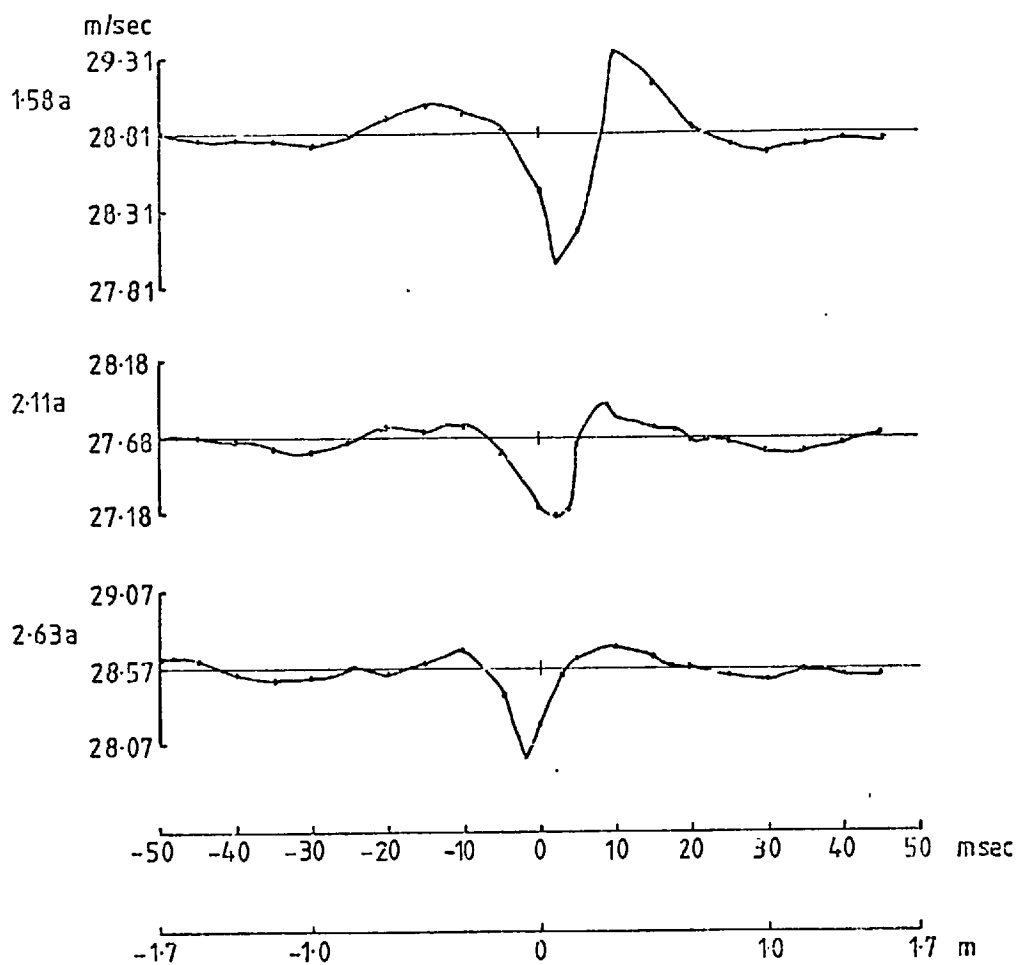


Fig. 67 Continued.

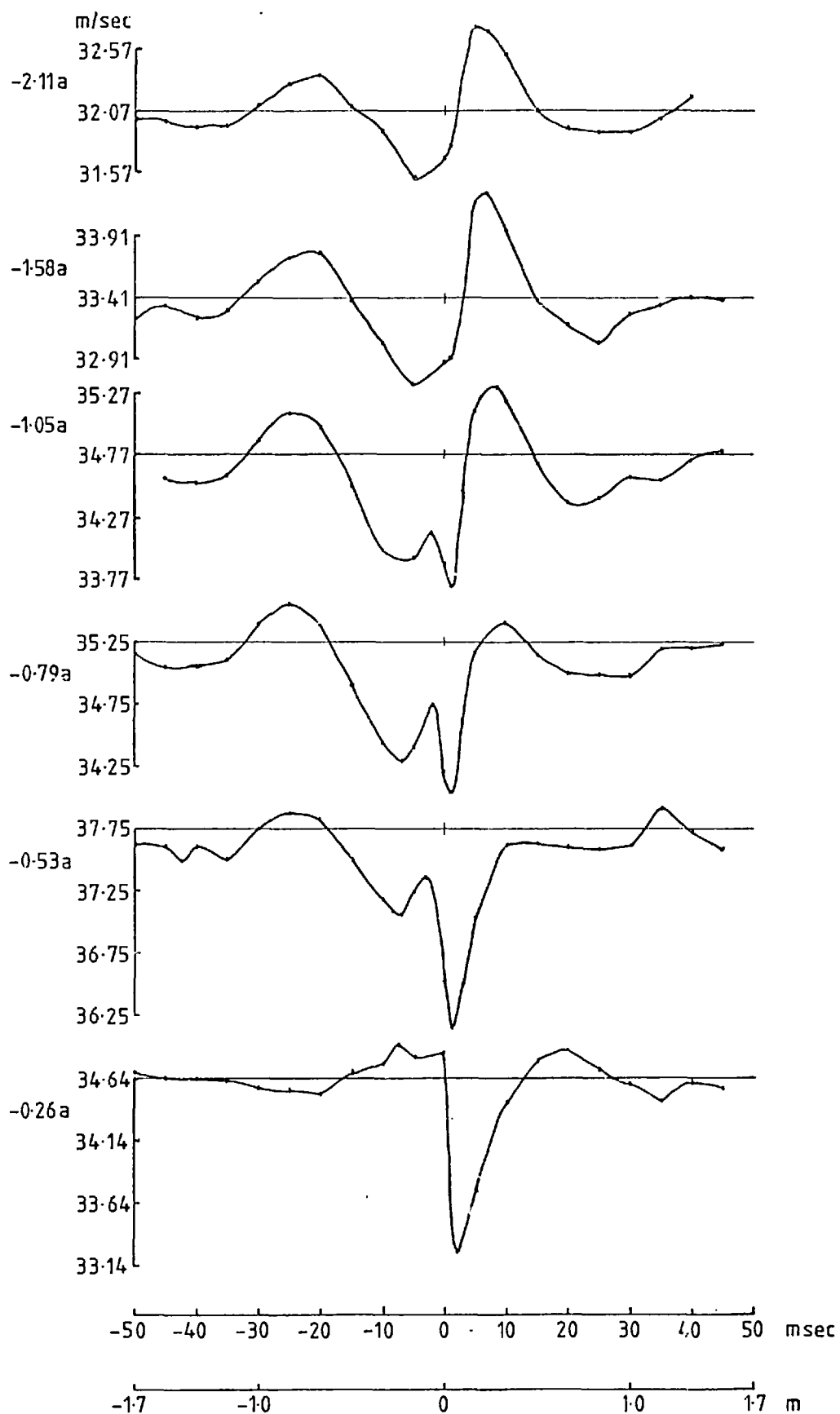


Fig. 68 Velocity profiles obtained using conditional sampling technique: experiment (ii).
Resultant component of fluctuating velocity, U_{res} , in the U-W plane.

Continued /

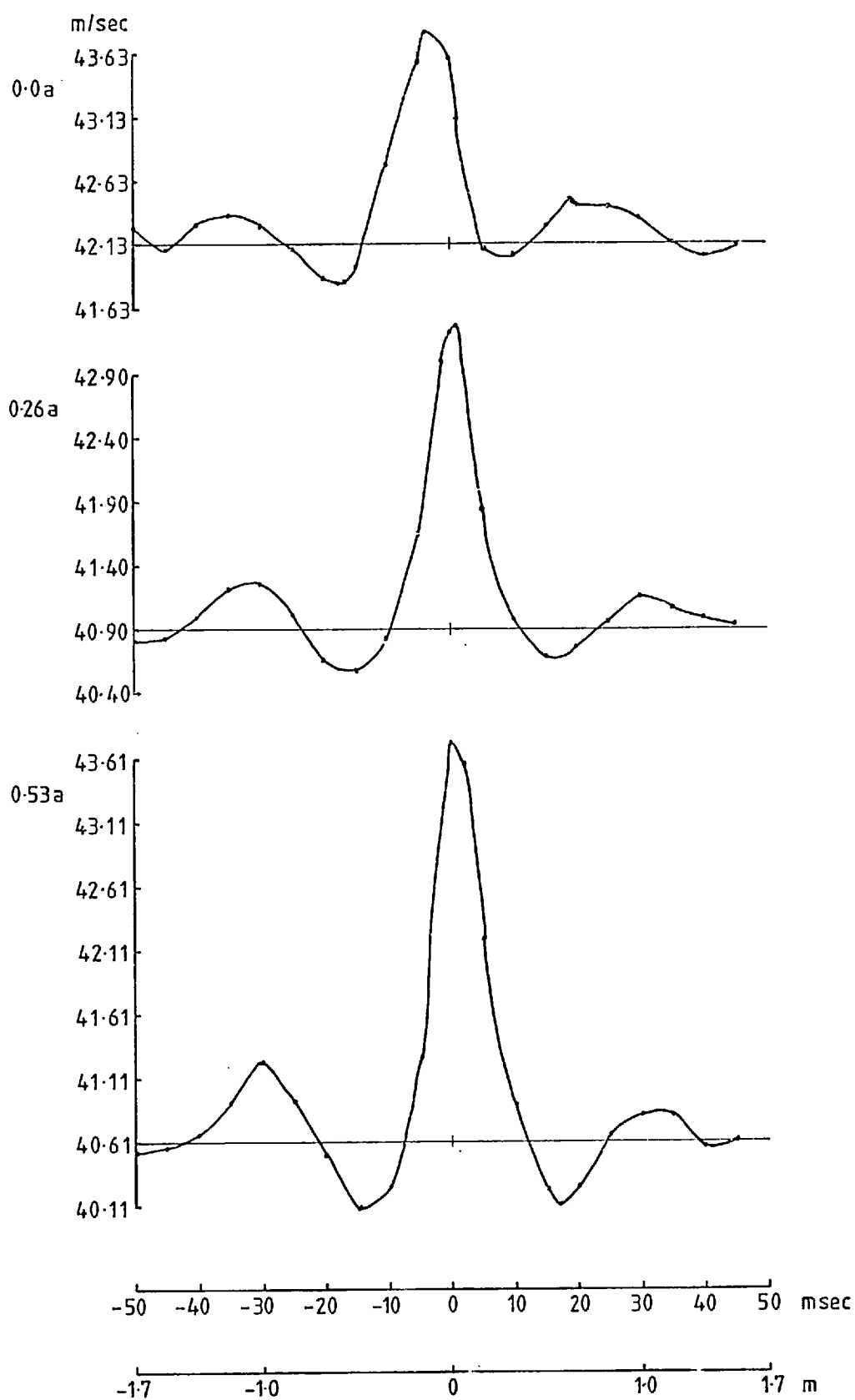


Fig. 68 Continued

Continued /

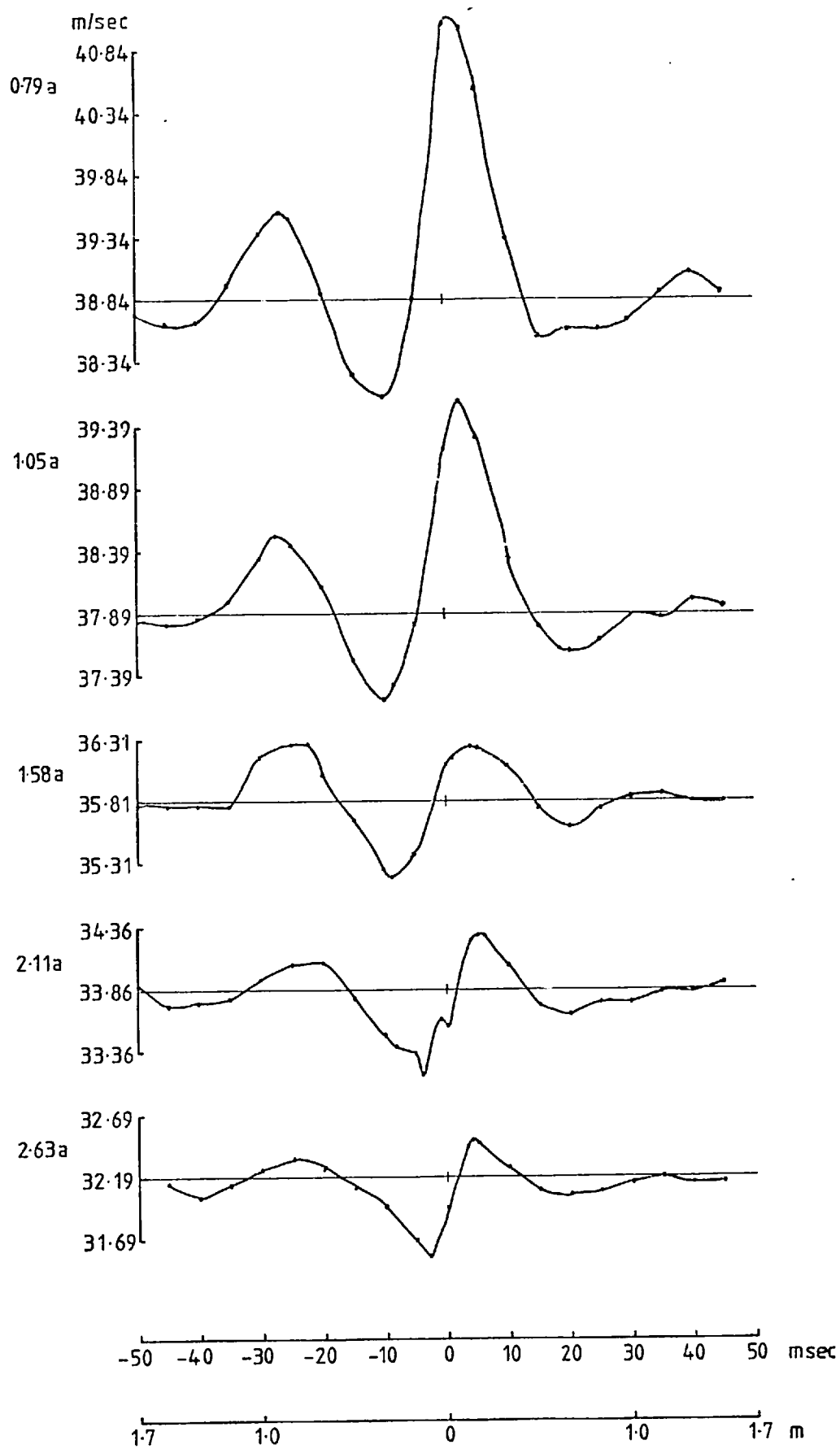


Fig. 68 Continued.

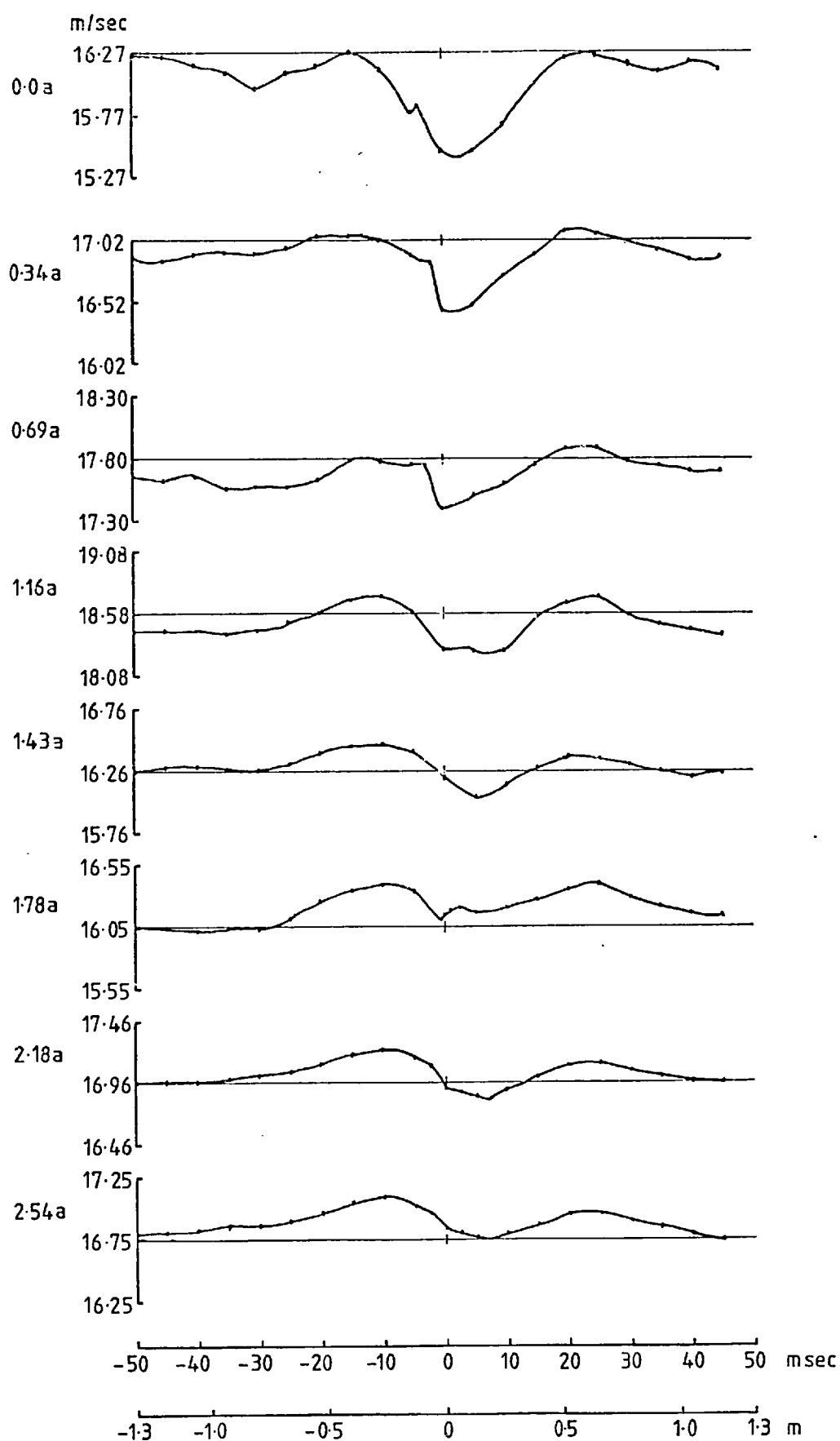


Fig. 69a Velocity profiles obtained using conditional sampling technique: experiment (iii).
Axial component, U_1 . ($U_1 - U_2$ plane)

Continued /

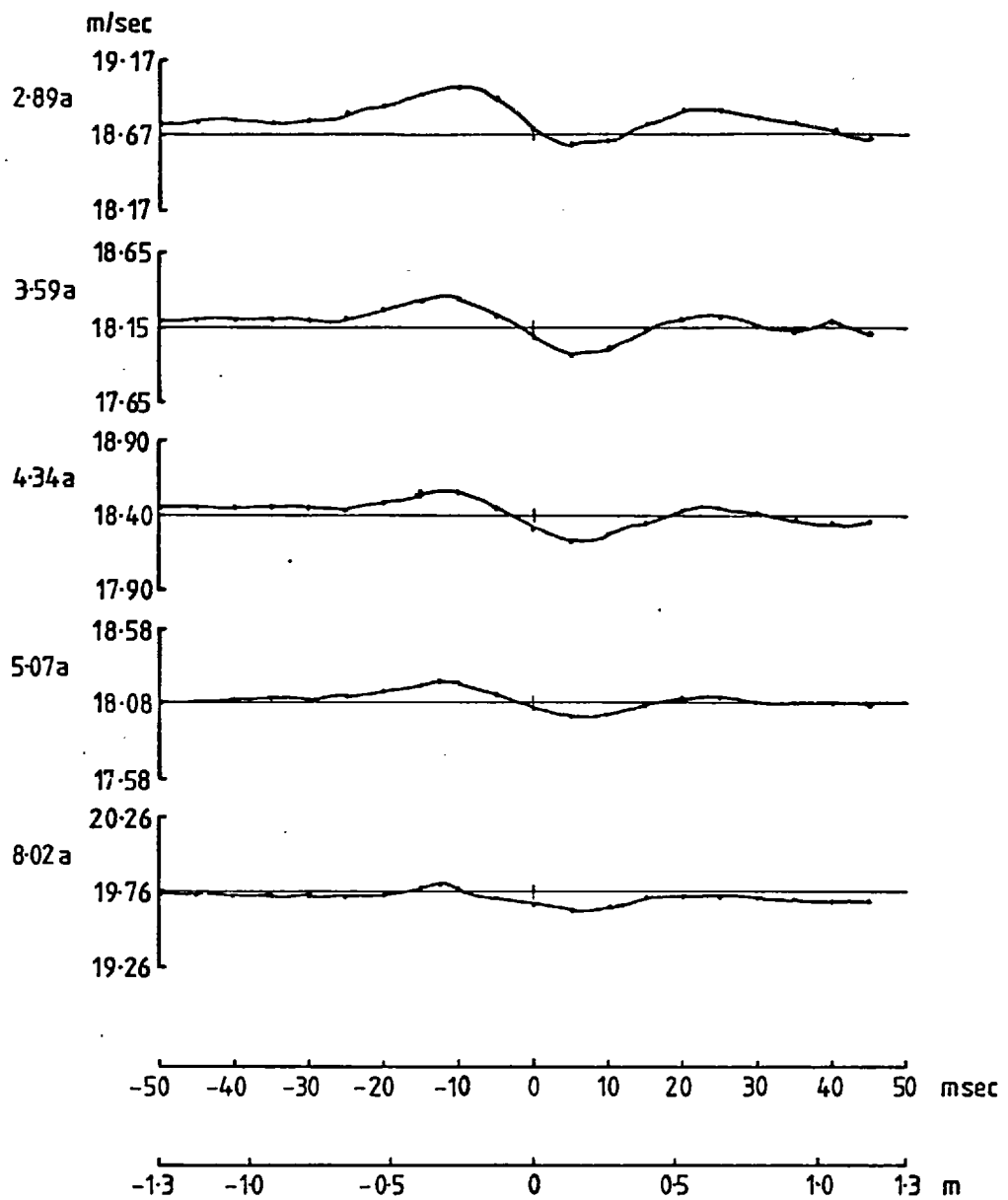


Fig. 69a Continued.

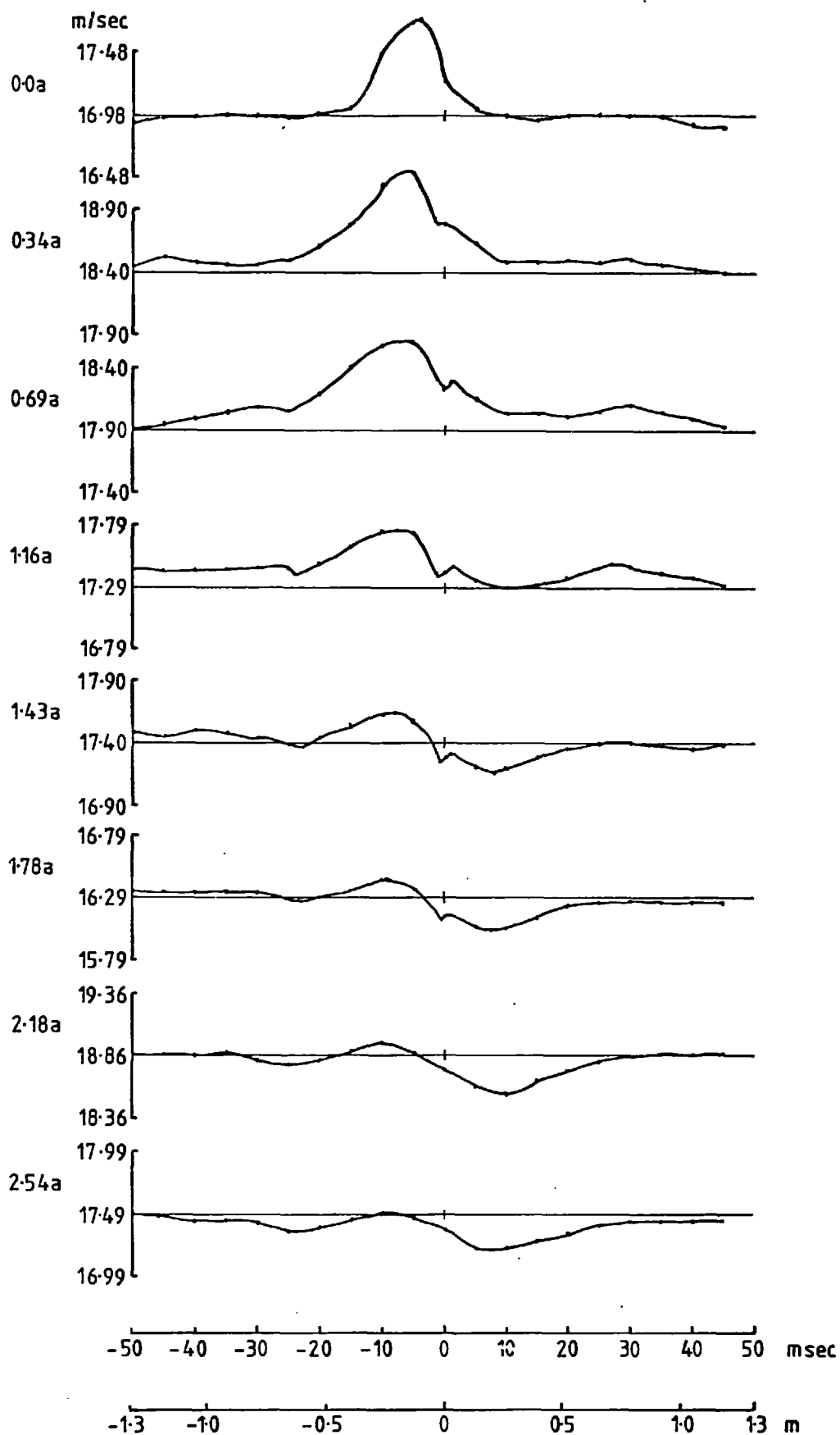


Fig. 69b Velocity profiles obtained using conditional sampling technique: experiment (iii).
Axial component, U_1 . ($U_1 - U_3$ plane)

Continued /

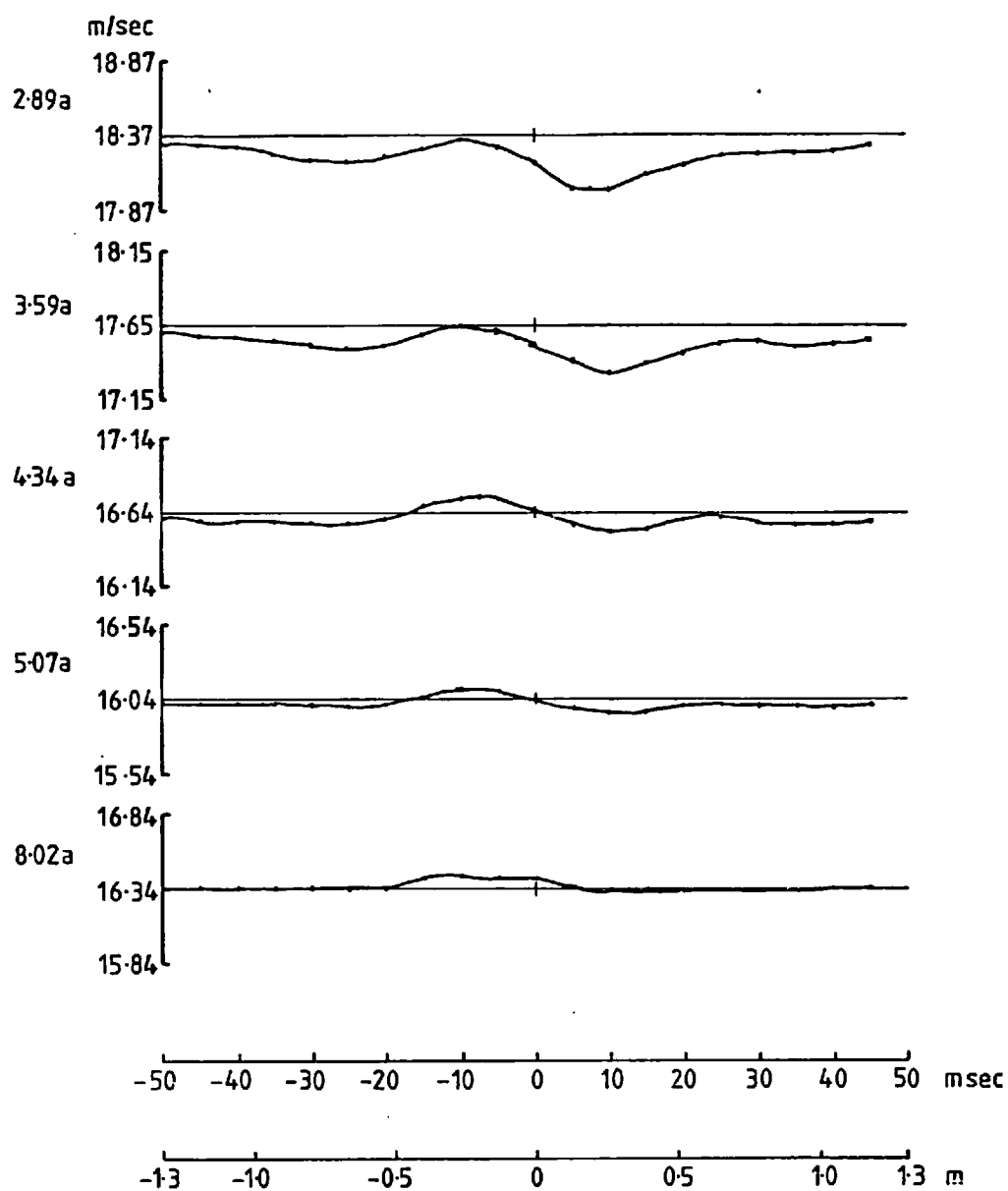


Fig. 69b Continued.

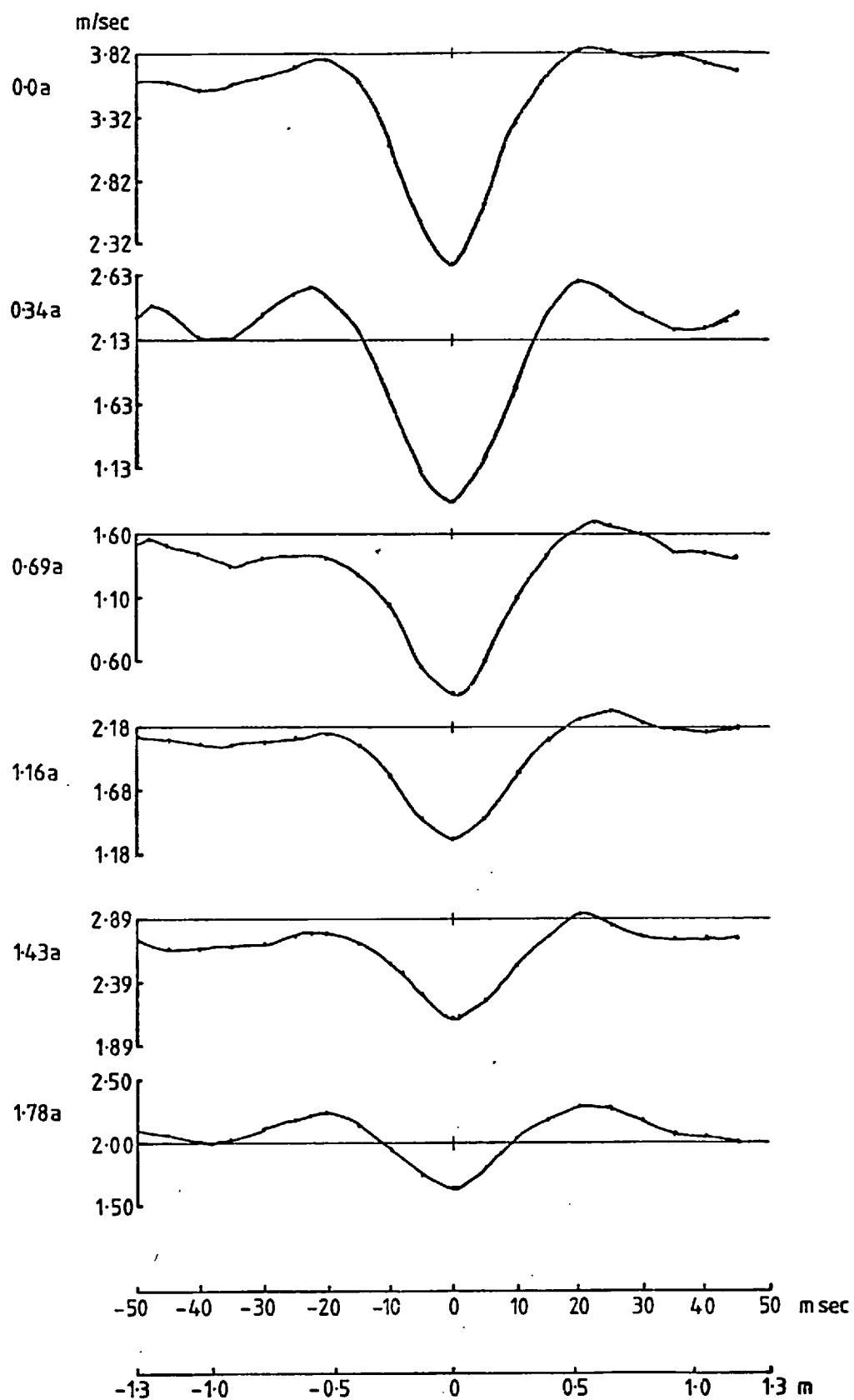


Fig. 70 Velocity profiles obtained using conditional sampling technique: experiment (iii).
Azimuthal component, U_3 .

Continued /

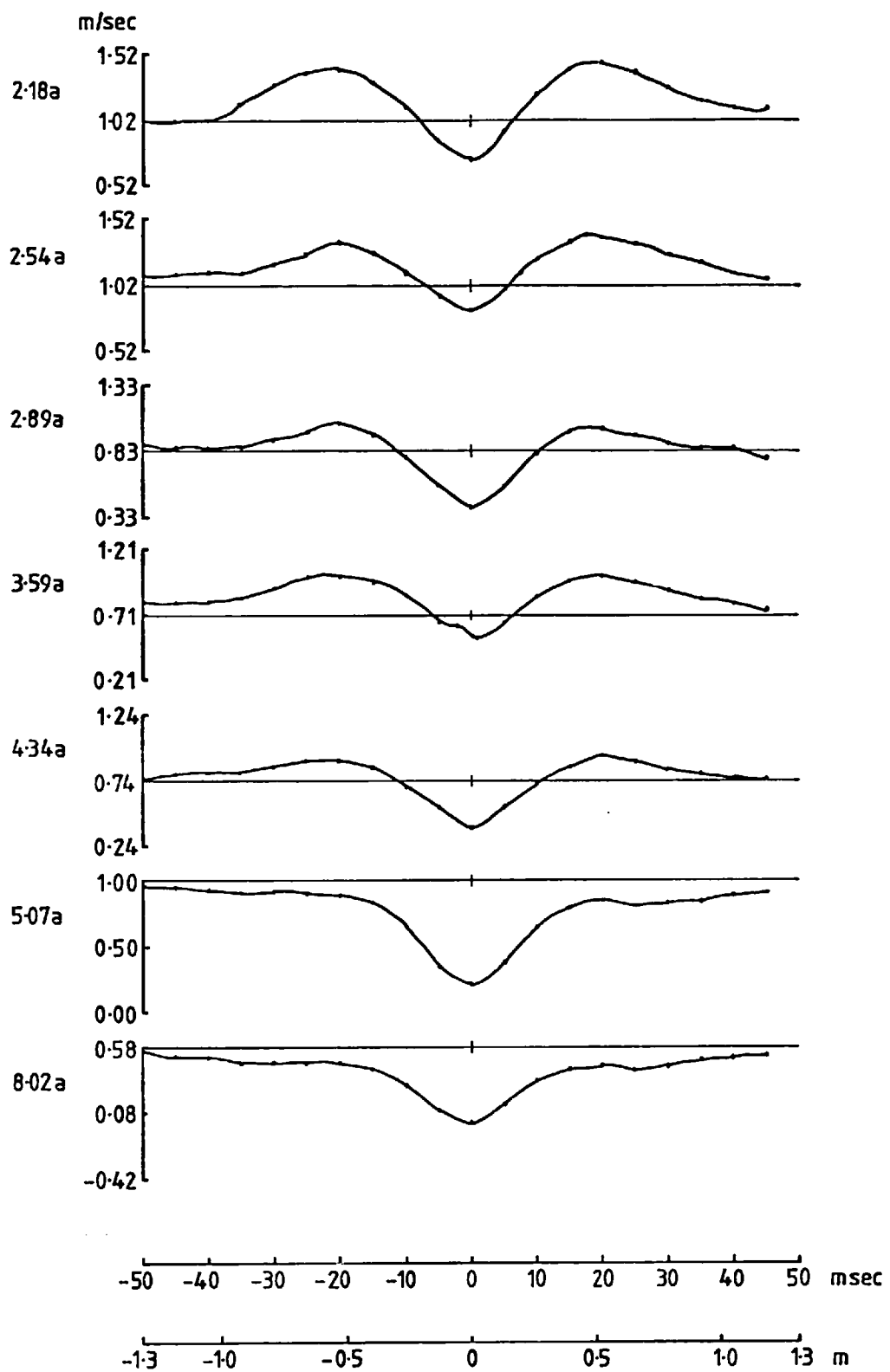


Fig. 70 Continued.

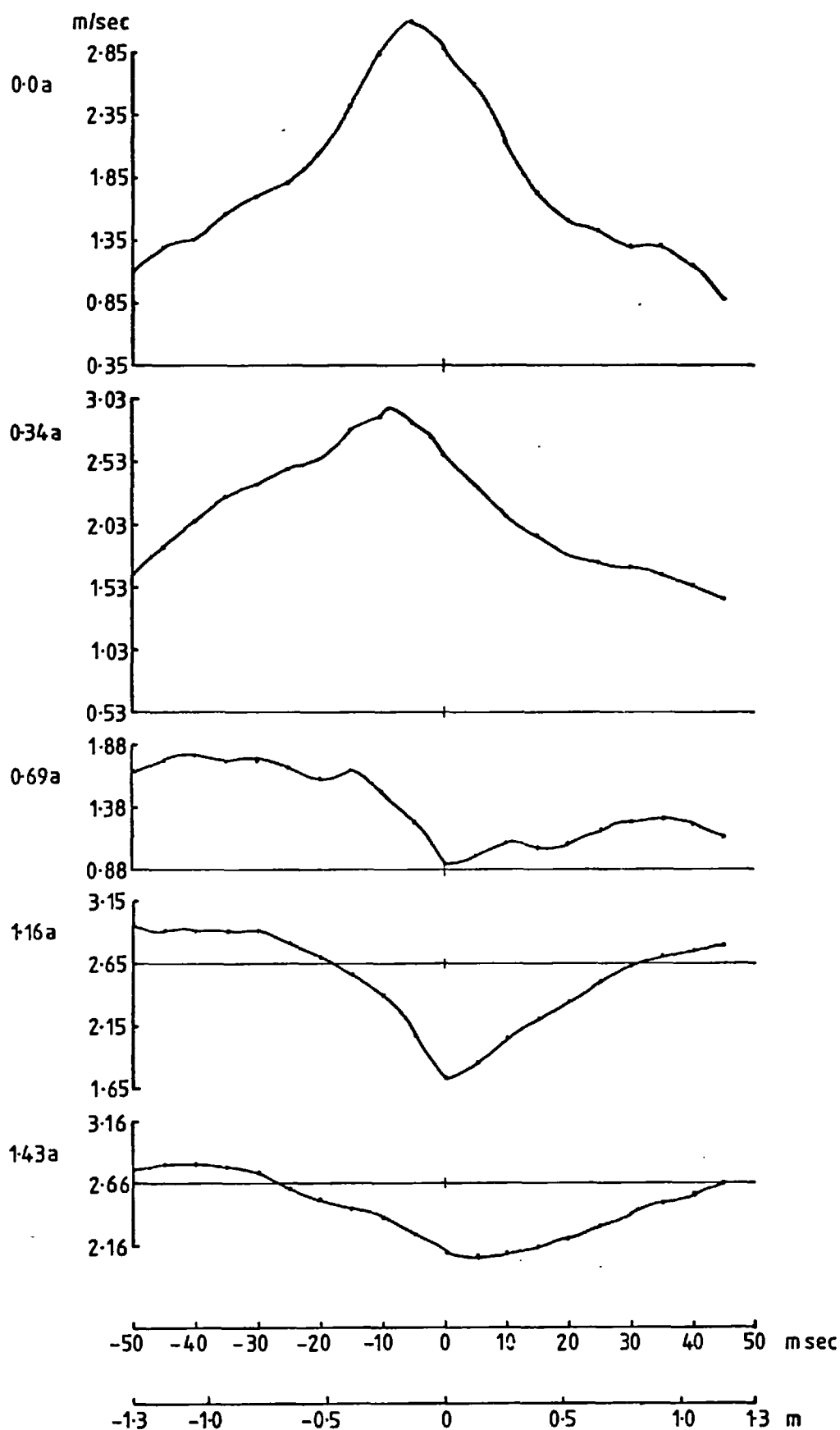


Fig. 71 Velocity profiles obtained using conditional sampling technique: experiment (iii).
Radial component, U_2 :

Continued /

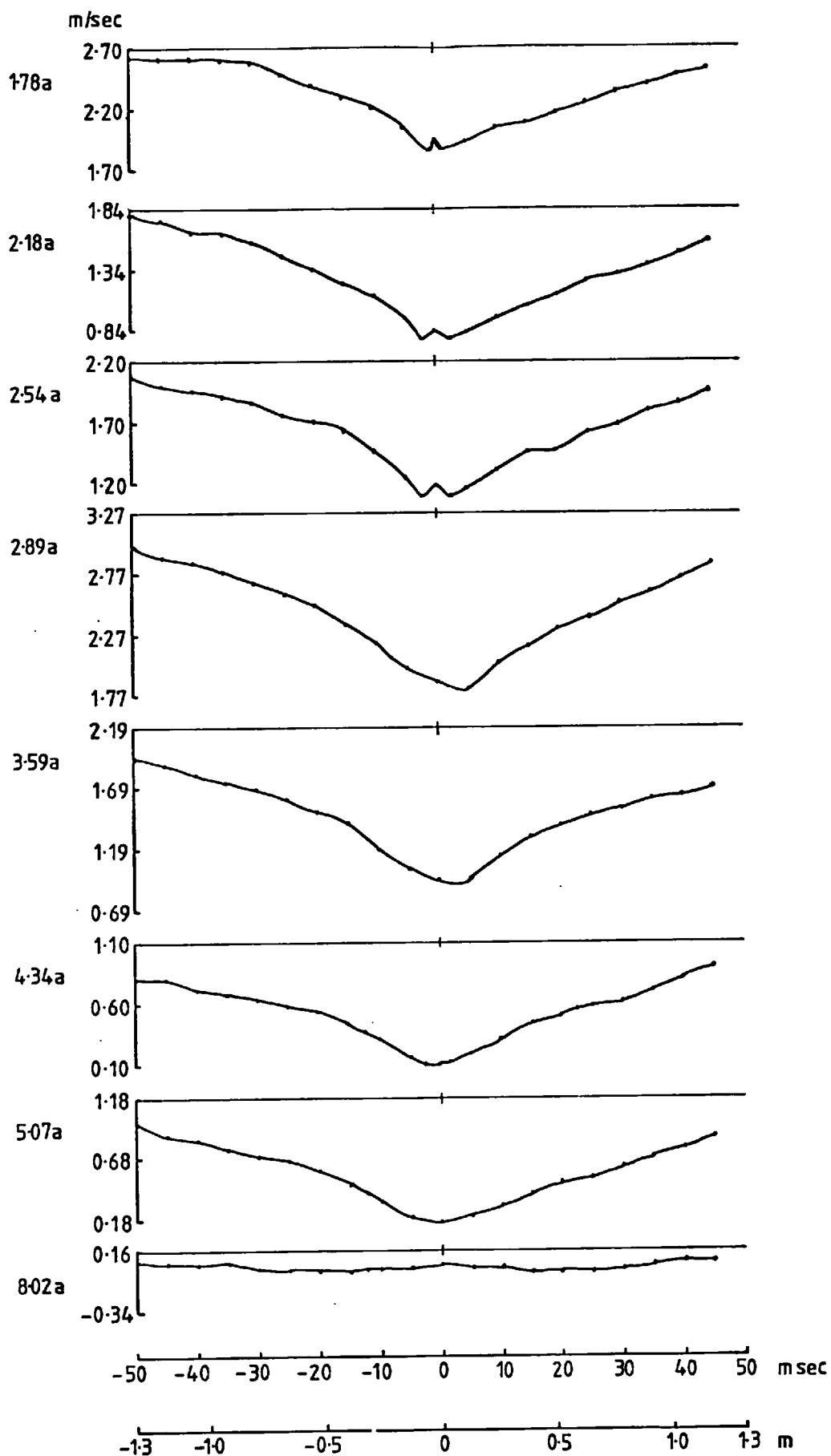


Fig. 71 Continued.

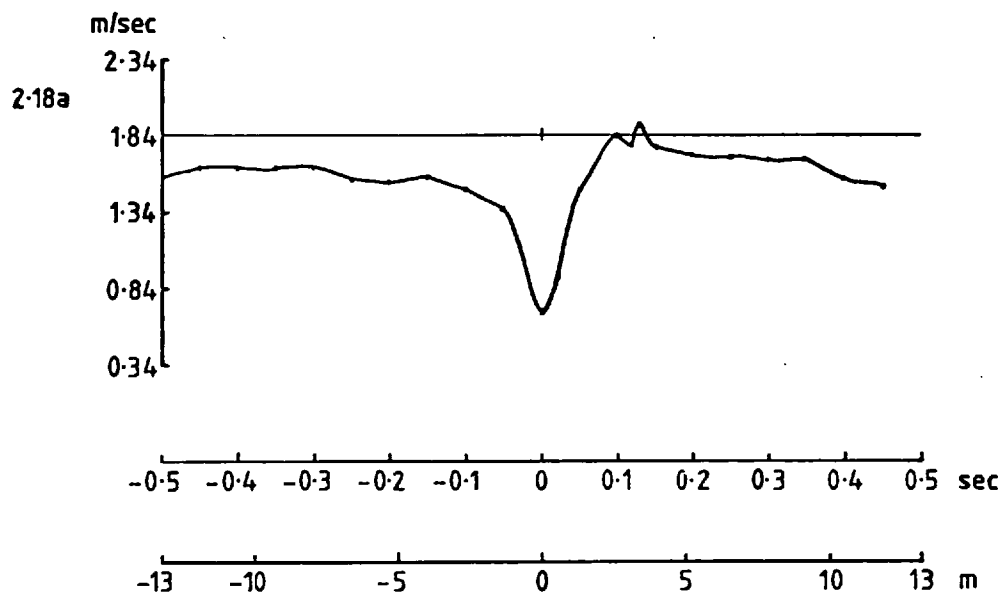


Fig. 72 Velocity profiles obtained using conditional sampling technique: experiment (iii).
Radial component, U_2 , at 2.18 core radii (a) for a timescale of 1 sec.

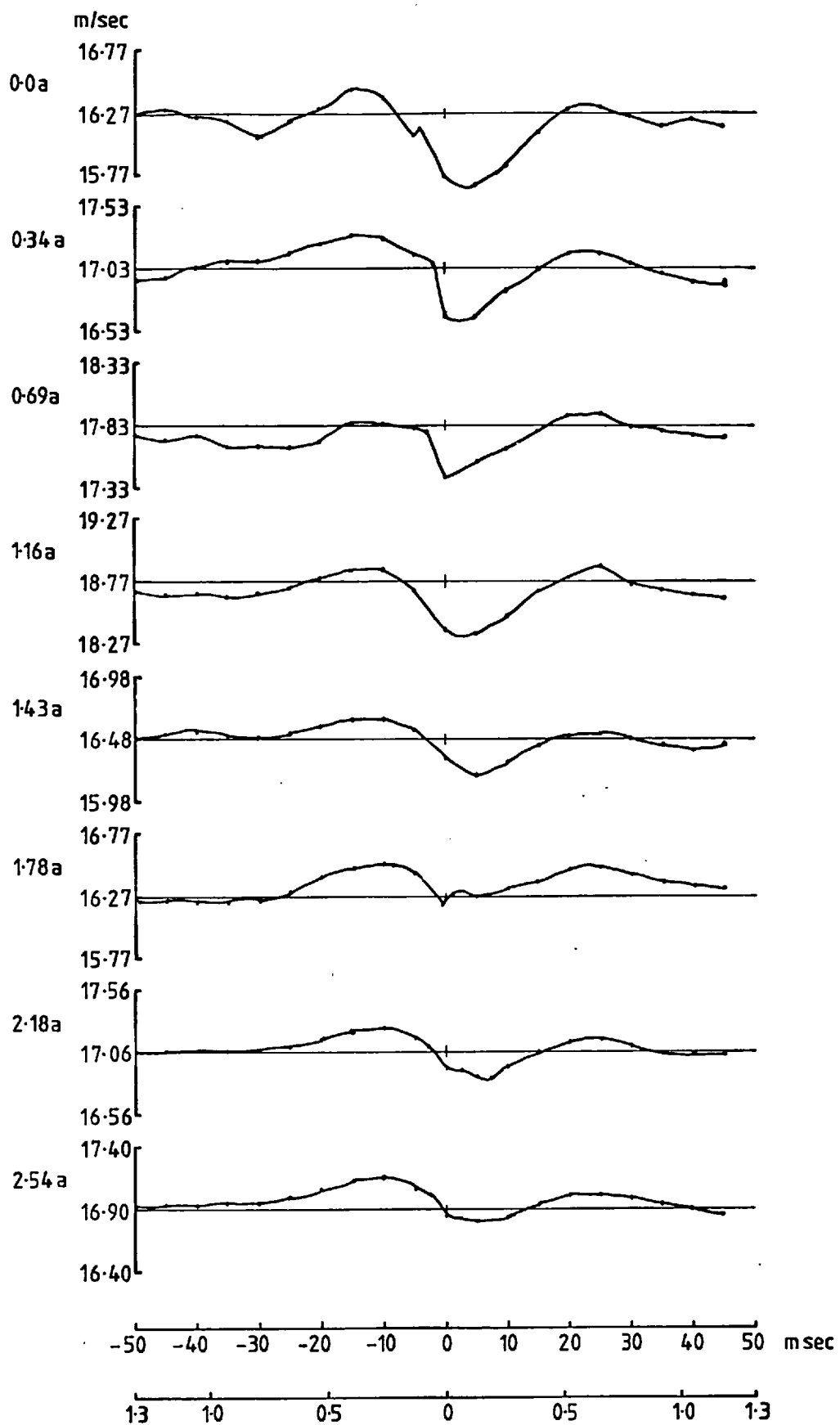


Fig. 73 Velocity profiles obtained using conditional sampling technique: experiment (iii).

Resultant component of velocity, U_{res} , in the $U_1 - U_2$ plane.

Continued /

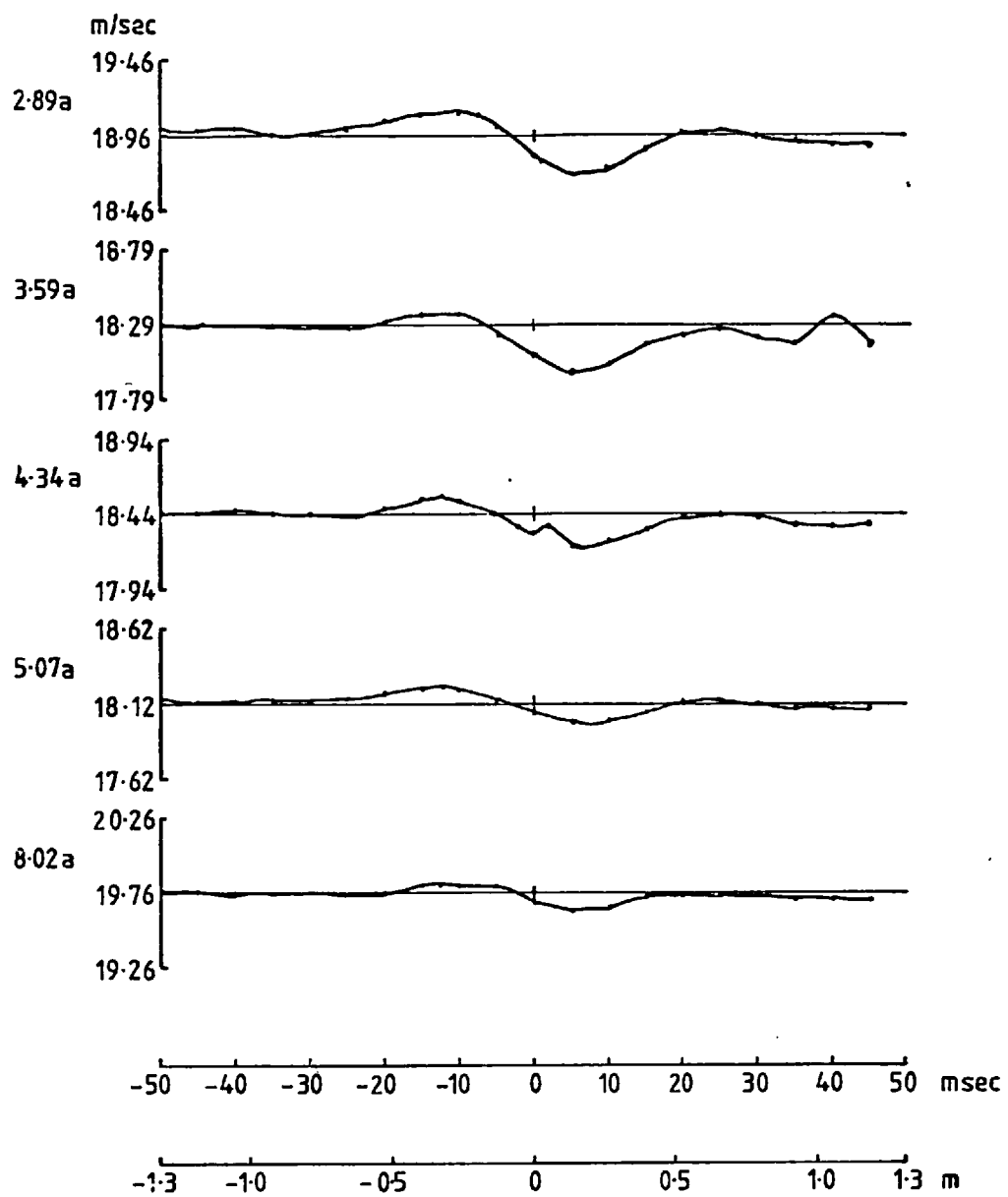


Fig. 73 Continued.

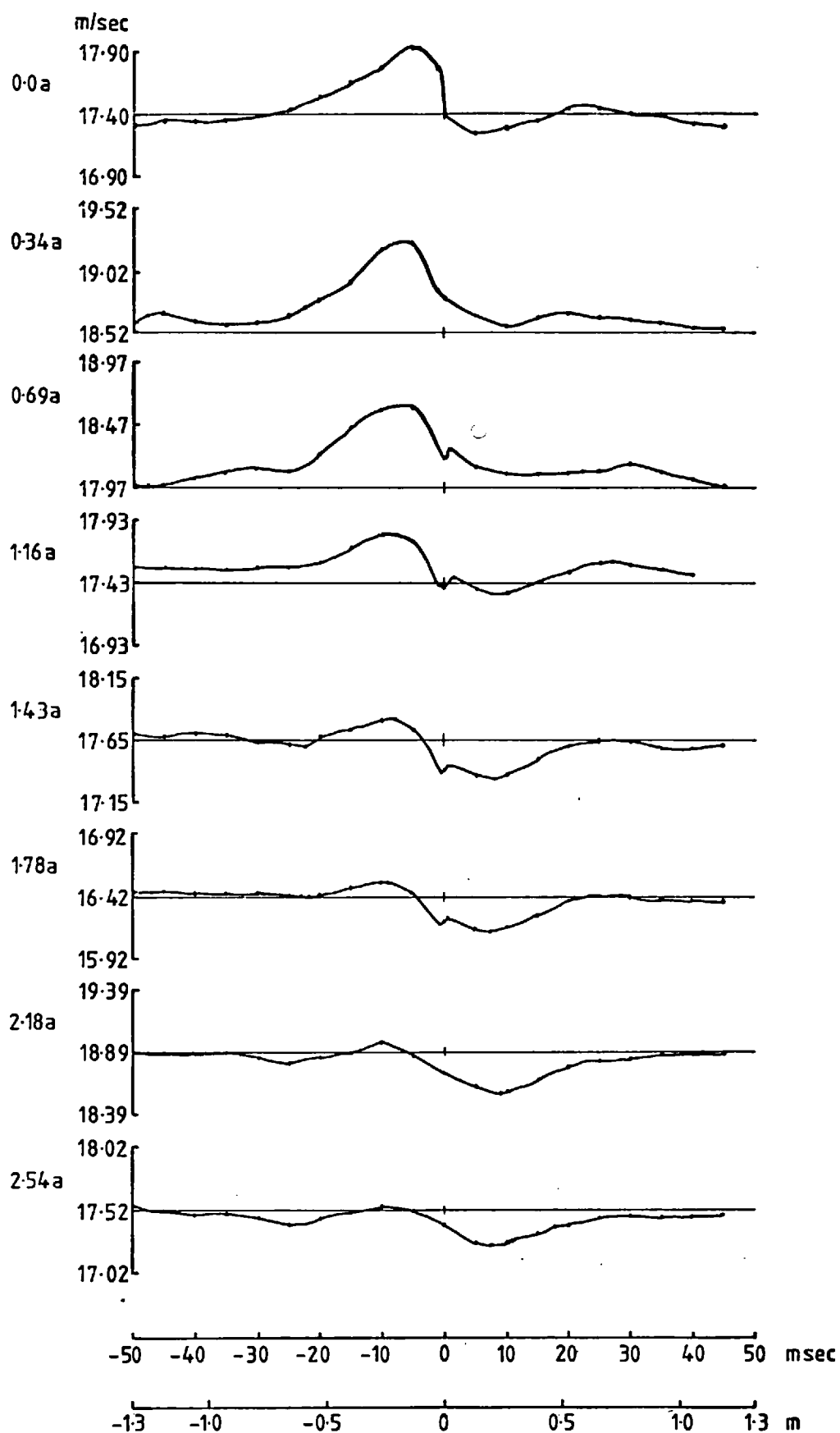


Fig. 74 Velocity profiles obtained using conditional sampling technique: experiment (iii).

Resultant component of velocity, U_{res} , in the $U_1 - U_3$ plane.

Continued /

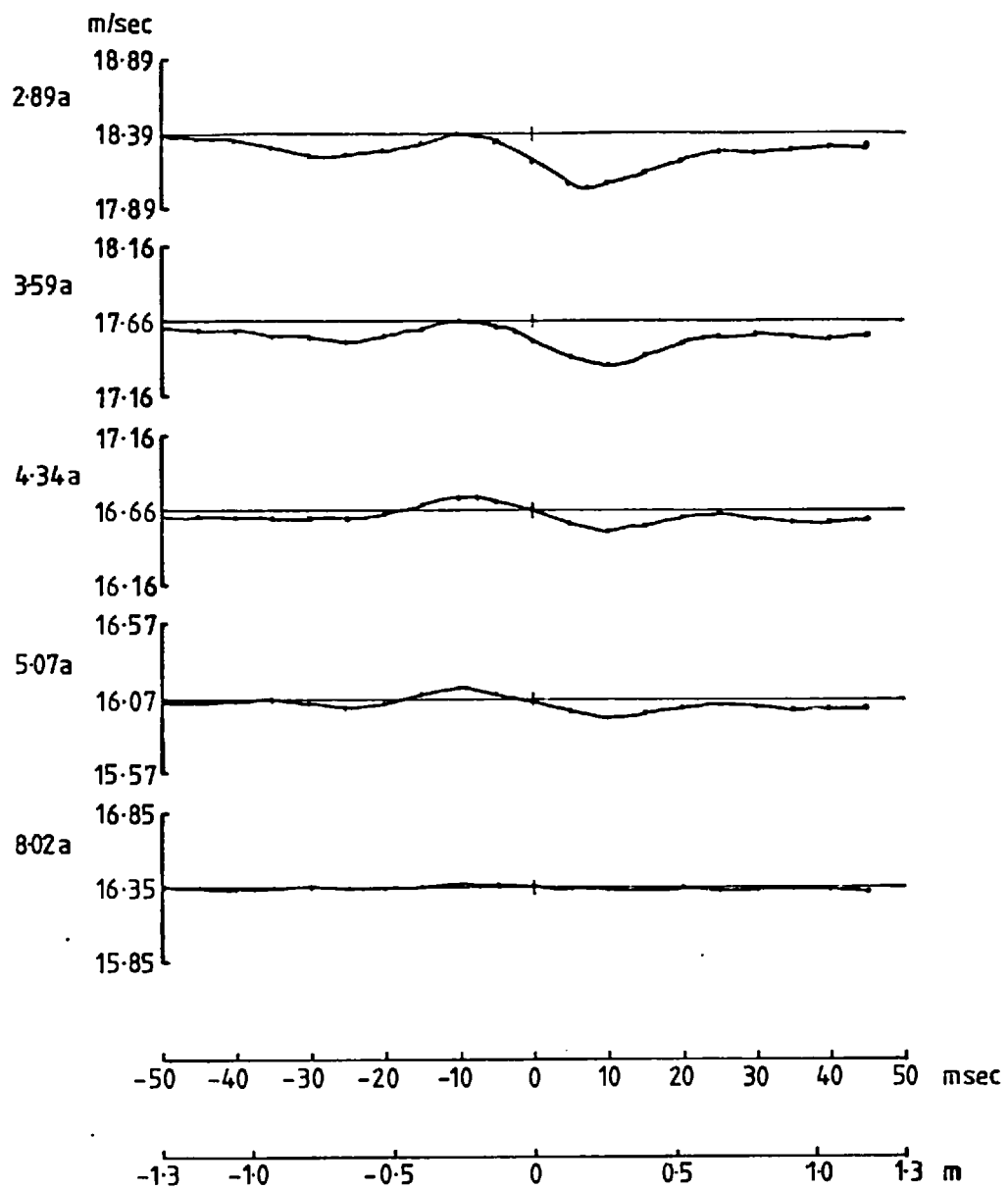
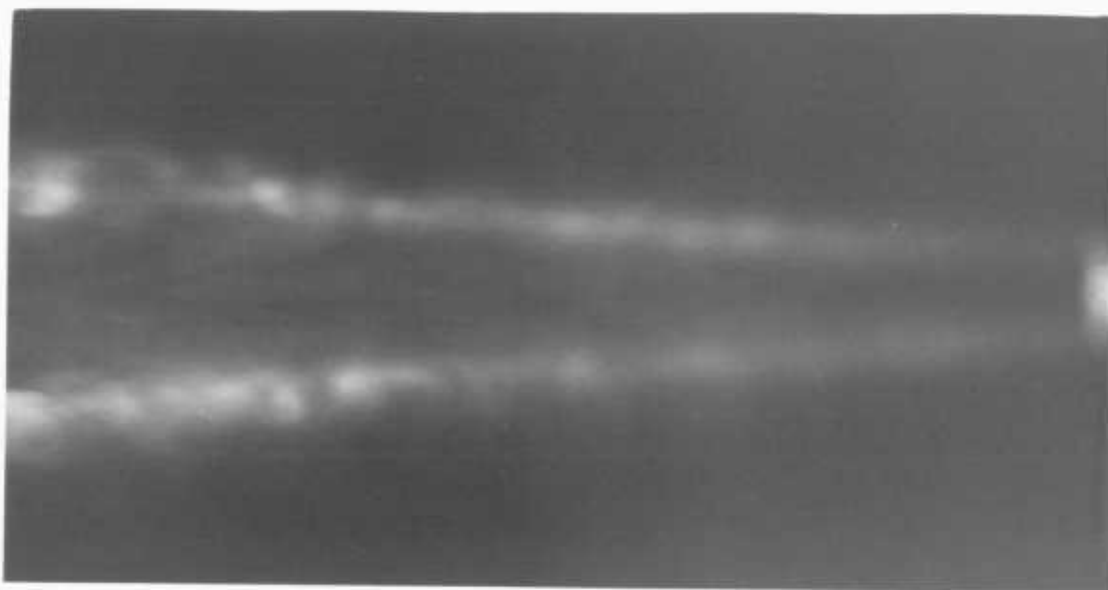
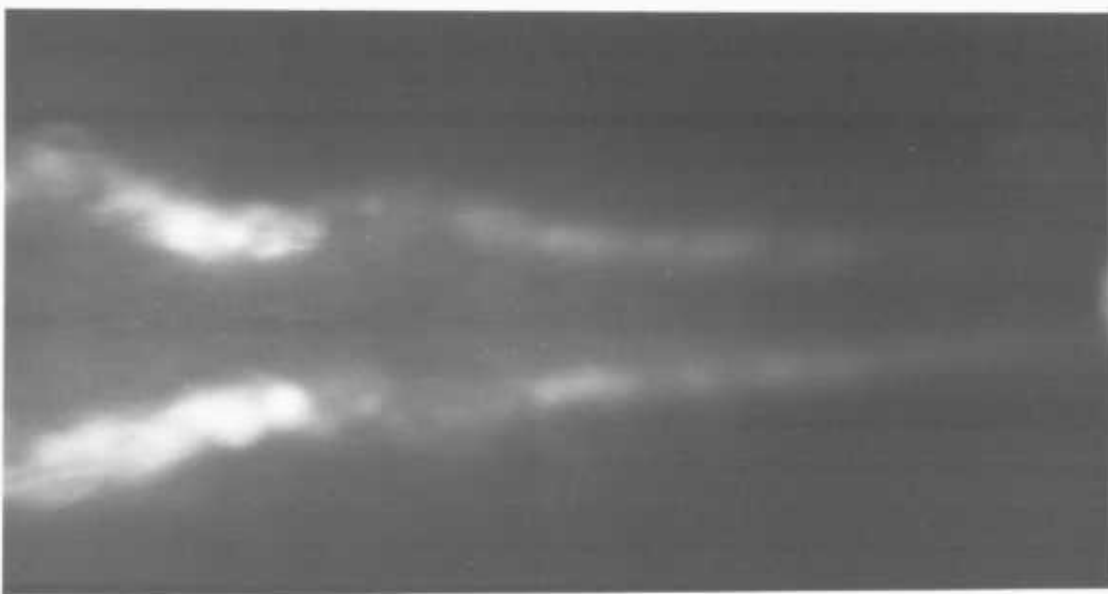


Fig. 74 Continued.



a)



b)

Fig. 75 Trailing vortices behind the 12.8" chord delta wing.

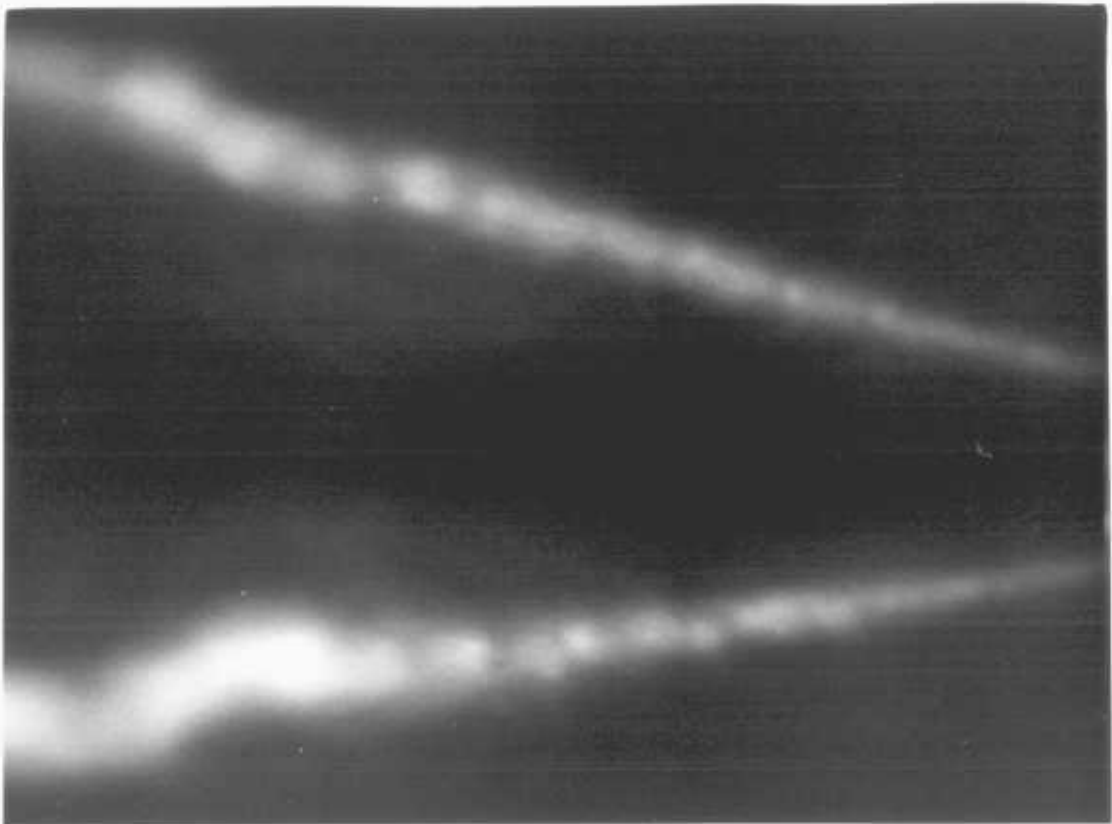


Fig. 76a Trailing vortices behind the 28" chord delta wing.

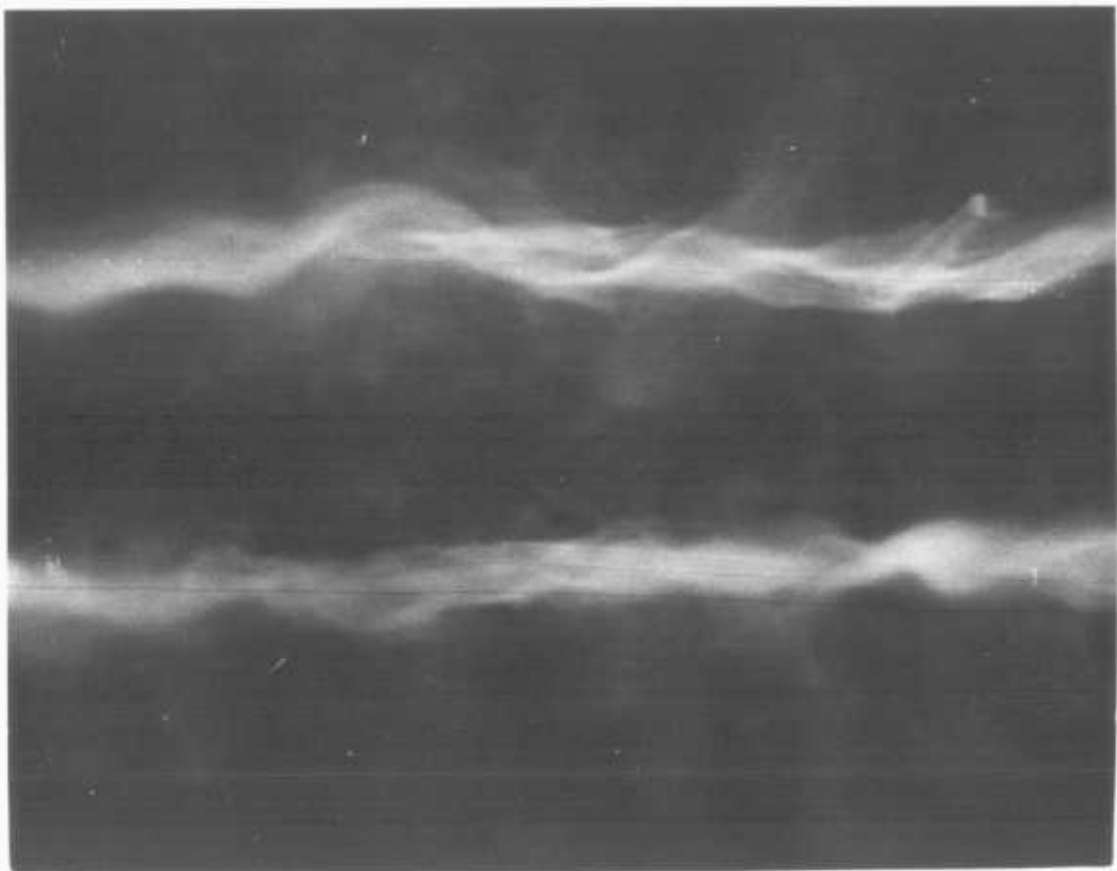


Fig. 76b The fully rolled up trailing vortices behind the 12.8" chord delta wing.

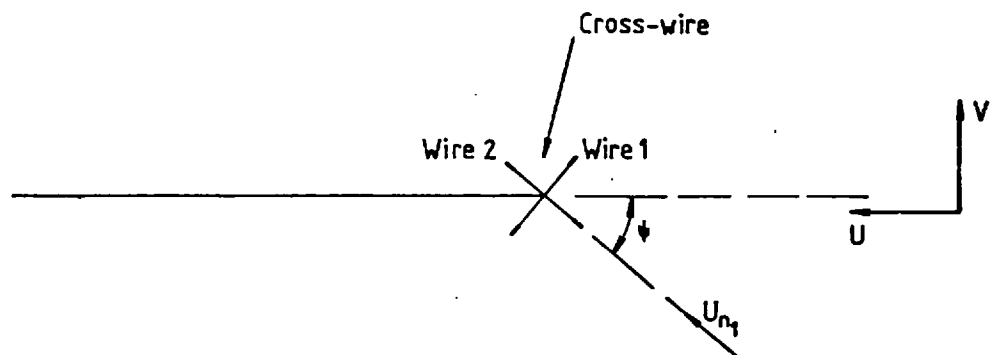


Fig. 77 Sketch defining the effective angle, ψ , of a hot wire.

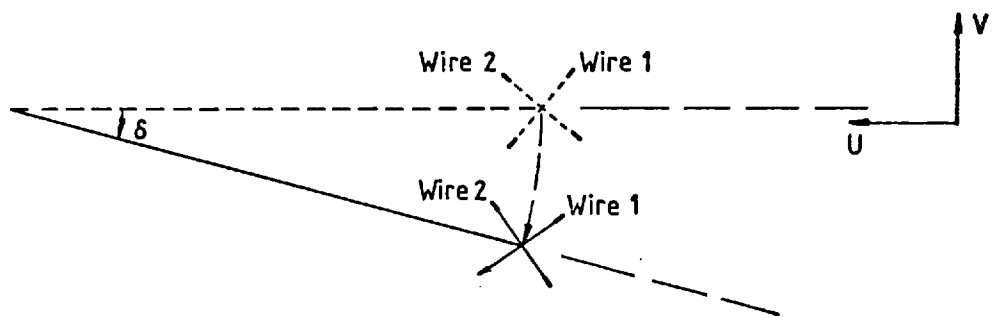


Fig. 78 Sketch defining the yaw angle, δ .

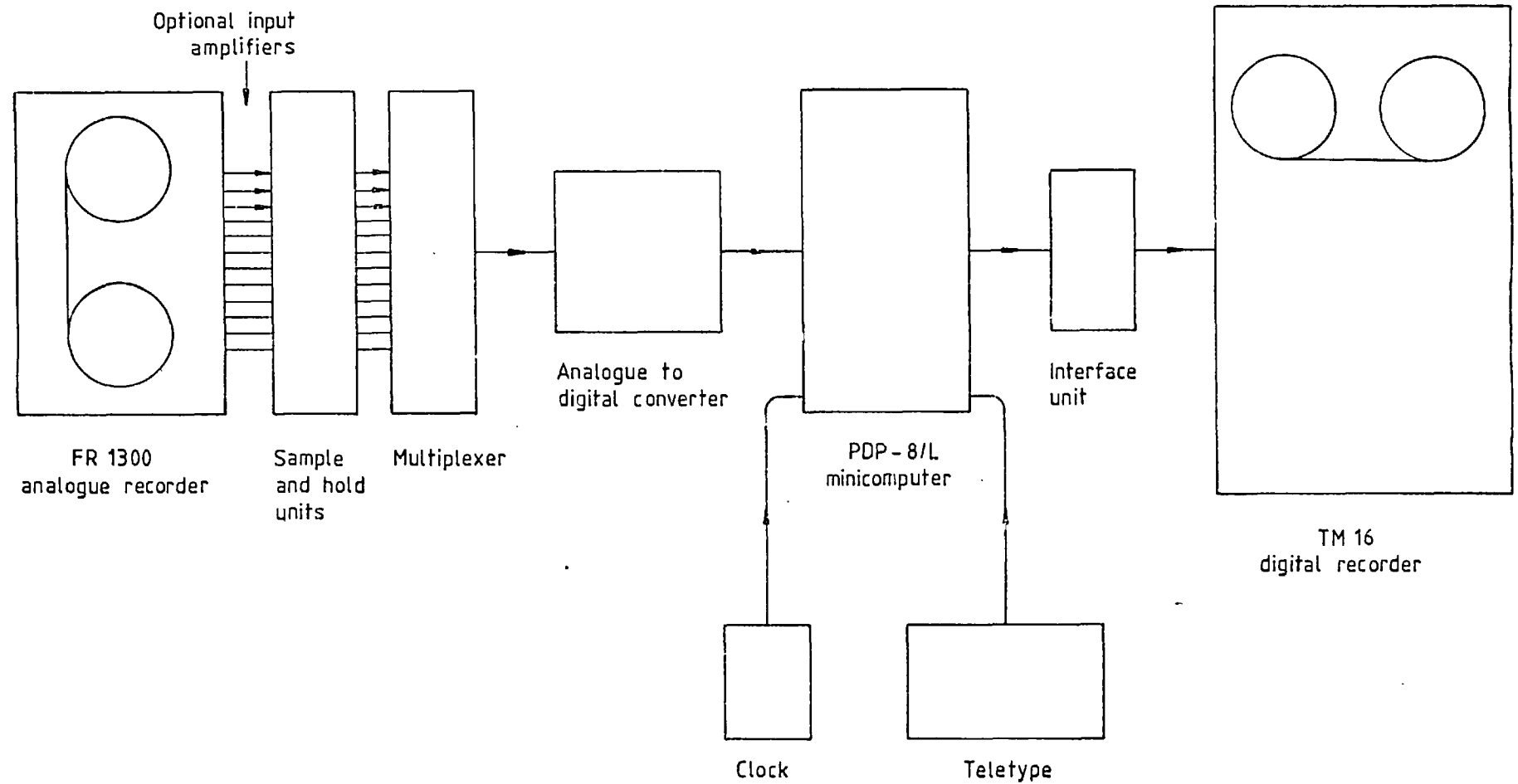


Fig. 79 Block diagram of analogue to digital system (hardware).