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Analysis and Mitigation of Sub-Synchronous Oscillations Induced by Inverter-Based Resources

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Abstract

High shares of Inverter-Based Resources (IBRs) in power systems are triggering complex dynamic interactions and new stability challenges. One common problem is poorly damped Sub-Synchronous Oscillation (SSO) caused by adverse interaction among IBRs through the network. These oscillations are difficult to foresee, pose a serious threat to system security and often force grid operators to operate conservatively by limiting the instantaneous share of IBRs. This thesis focuses on the detection, analysis and mitigation of these SSO. First, it establishes the modelling adequacy to accurately capture SSO in time- and frequency-domain studies. The role of network dynamics, which is generally ignored in planning tools, is emphasised. Consequently, Electromagnetic Transient (EMT)- dq modelling is established as a middle-ground offering greater accuracy than positive-sequence Root Mean Square (RMS) models with significantly less computational burden than EMT- abc (point-on-wave) to allow more scenario checks. Next, a set of diagnostic indicators is developed to identify which IBR control loop is primarily responsible for a specific SSO mode using time-domain signal sensing. Building on these realisations, novel and practical control strategies for both grid-following (GFL) and grid-forming (GFM) inverters are developed where the known network model is made part of the plant for control design. Such a design ensures higher bandwidths and enhanced stability margins. Finally, to circumvent the opaqueness of proprietary ‘black-box’ IBR models, a data-driven technique is presented to estimate each IBR’s linearised model in the form of impedance or admittance spectra around a given operating point. This technique eliminates the need for dynamic frequency scans and enables system-wide stability assessment and root cause analysis (e.g., participation of each IBR in SSO) using only one-shot input-output measurements for each IBR. Overall, this thesis presents scalable modelling framework to capture and analyse IBR-driven SSO, proposes advanced control techniques to mitigate them, and introduces data-driven IBR estimation method to obtain operating point specific impedance/admittance spectra of ‘black-box’ IBR models for analysing SSO.

Statement of Originality

I, **Muhammad Sharjeel Javaid**, hereby declare that this thesis, entitled “*Analysis and Mitigation of Sub-Synchronous Oscillations Induced by Inverter-Based Resources*”, is the result of my own independent research and work, except where otherwise stated. All sources of information and literature used in the course of this work have been appropriately acknowledged.

This thesis has not been submitted, either in whole or in part, for any other academic degree or professional qualification at this or any other institution.

Muhammad Sharjeel Javaid

24 May 2025

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Parts of this thesis have appeared in the following publications:

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Contents

List of Publications	6
1 Introduction	20
1.1 Sub-Synchronous Oscillations	22
1.1.1 Understanding SSO: Characteristics and Observed Incidents	22
1.1.2 The Overlooked Role of Network Dynamics in SSO	24
1.2 Modelling Challenges for IBR-Dominated Systems	25
1.2.1 Limitations of Conventional Phasor-Domain Studies	25
1.2.2 The Need for Advanced modelling: Bridging the Gap with EMT-dq Sim- ulations	26
1.3 Addressing Industry Concerns: Black-Box IBR Models and SSO Unpredictability	28
1.3.1 Opaqueness of Proprietary IBR Models and its Impact on Grid Reliability	28
1.3.2 Challenges in SSO Diagnosis and Root Cause Identification	29
1.4 Thesis Contributions	30
1.5 Thesis Structure	32
2 Modelling Framework	34
2.1 Grid-Following Inverter	35
2.2 Grid-Forming Inverter	37
2.3 Loads	40
2.4 Floating Buses	41
2.5 Branches	41
2.6 Full System	41
2.7 Conclusion	43
3 Modelling Adequacy for Sub-Synchronous Oscillation Analysis	44
3.1 Modelling Tools Landscape	45
3.2 Benchmarking EMT- <i>dq</i> and EMT- <i>abc</i>	48

3.3	Interaction between IBRs and Network	51
3.3.1	Grid-Following Inverter	52
3.3.2	Grid-Forming Inverter	54
3.4	Test Cases	55
3.4.1	An IBR Connected to Thévenin Equivalent of a Power System	56
3.4.2	GFL and GFM Supplying a Load	58
3.4.3	9-bus Test System with Two GFMs and One GFL	61
3.4.4	39-bus test system with five GFLs and five GFMs	61
3.5	Conclusion	62
4	Characterisation and Mechanisms of Sub-Synchronous Oscillations	67
4.1	PLL-Induced SSO	68
4.1.1	PLL–Network Dynamic Interactions	69
4.1.2	Validation on Test Systems	74
4.1.3	Summary	77
4.2	GFM Voltage Control-Induced SSO	77
4.2.1	Small-Signal Analysis	78
4.2.2	Inner Control Sensitivity Analysis	82
4.2.3	Validation on a Multi-IBR Test System	85
4.2.4	Summary	87
4.3	Conclusions	87
5	IBR Control Design to Mitigate Sub-Synchronous Oscillation	88
5.1	Disk Margin as Index of Robustness	89
5.2	Enhanced GFL Control	90
5.2.1	Disk Margin vs Phase Margin	90
5.2.2	Identifying the Dominant Contributors to Peaks in Frequency Response .	91
5.2.3	Tuning Methods	94
5.2.4	Performance Comparison	96
5.2.5	Summary	105
5.3	Enhanced GFM Voltage Control	105
5.3.1	Tuning Methods	106
5.3.2	Performance Comparison	111
5.3.3	Summary	113

5.4	Conclusion	114
6	Root Cause Analysis of Sub-Synchronous Oscillation using In Situ Estimated IBR Models	115
6.1	Methodology for in situ Estimation	118
6.1.1	Pulse Response Measurements	119
6.1.2	In Situ Estimation by Inversion for IBR Model	119
6.1.3	IBR Model Order Reduction	119
6.1.4	Modal Analysis	120
6.2	Augmented ERA	121
6.2.1	Markov Parameters and Hankel Matrices	121
6.2.2	Automatic Selection of Model Order	123
6.2.3	The ERA-based Realization	124
6.3	Case Studies	125
6.3.1	Case 1: A Five-Bus Two-IBR System	125
6.3.2	Case 2: A 39-bus 10-IBR System	129
6.4	Conclusion	135
7	Conclusion and Future Work	136
	Appendix: Graphical Representation of Normalised Participation Factor	148

List of Acronyms

DAE Differential Algebraic Equation

DI Diagnostic Indicator

DS Direct Synthesis

EMT Electromagnetic Transient

ERA Eigensystem Realisation Algorithm

GFL Grid-Following Inverter

GFM Grid-Forming Inverter

IBR Inverter-Based Resource

MIMO Multiple-Input Multiple-Output

PCC Point of Common Coupling

PLL Phase-Locked Loop

QSP Quasi-Static Phasor

RMS Root Mean Square

RoS Rest of the System

SCR Short Circuit Ratio

SG Synchronous Generator

SISO Single-Input Single-Output

SO Symmetrical Optimum

SSO Sub-Synchronous Oscillation

List of Figures

2.1	Control blocks for a GFL with an LC filter	35
2.2	GFL model block diagram	37
2.3	GFL modular representation	37
2.4	Control blocks for a GFM with an LC filter	38
2.5	GFM model block diagram	39
2.6	GFM modular representation	39
2.7	Modular representations of loads, floating nodes, and branches. These modules do not have tunable parameters. Fixed parameters and states are shown in blue and green, respectively.	42
2.8	Circuit diagram with all modelled components where similar components are combined at virtual buses (solid line) through their possible interconnections (dashed line).	42
2.9	Modular block diagram of generalizable framework	43
3.1	EMT- dq model as a middle ground between EMT- abc and RMS+ to balance accuracy for IBR-driven SSO (y-axis) and the number of scenario checks possible (x-axis)	46
3.2	Test system to benchmark EMT- dq against EMT- abc simulations	48
3.3	Benchmarking dynamic responses from EMT- dq (blue) against EMT- abc (yellow) and RMS+ (red) for IBR in Figure 3.2 as (a) GFL and (b) GFM	49
3.4	Benchmarking frequency responses from EMT- dq (blue) against EMT- abc (yellow) and RMS+ (red) for for IBR in Figure 3.2 as (a) GFL and (b) GFM	50
3.5	Computational time ratio of EMT- abc to EMT- dq on single (left) and two (middle) IBRs system	51

3.6	An IBR connected to a Thévenin equivalent of a power system	52
3.7	(a) Mode shapes showing GFL states (gray markers) and network states (red markers) dominant in SSO modes 41.9 Hz (left) and 2.6 Hz (right); (b) Dynamic response of active power, terminal voltage, frequency and reactive power where the dominant oscillation mode is triggered by increasing reference power to 1.05 pu	53
3.8	Damping ratio (top) and network state participation (bottom) for the dominant SSO mode (41.9 Hz in base case) with varying SCR (left), and PLL bandwidth in Hz (right)	54
3.9	(a) Mode shapes showing GFM states (gray markers) and network states (red markers) dominant in SSO modes 12.8 Hz (left) and 1.5 Hz (right); (b) Dynamic response of active power, terminal voltage, frequency and reactive power where the dominant oscillation mode is triggered by increasing reference power to 1.05 pu	55
3.10	Damping ratio (top) and network state participation (bottom) for the dominant SSO mode (22.1 Hz in base case) with varying SCR (left), and GFM voltage control bandwidth in Hz (right)	56
3.11	(a) SSO mode trajectories for GFL connected to a Thévenin equivalent with EMT- dq (left), RMS+ (right), when SCR is varied from 1.06 (green) to 10 (red). SCR stability threshold (orange) is 1.6 for EMT- dq model and 1.2 for RMS+ model; (b) Dynamic response of active power, terminal voltage, frequency, and reactive power, when voltage reference is reduced from 1.0 pu to 0.95 pu at 0.1 s for SCR = 1.6	58
3.12	(a) SSO mode trajectories for GFM connected to a Thévenin equivalent with EMT- dq (left), RMS+ (right), when SCR is varied from 1.06 (green) to 10 (red). SCR stability threshold (orange) is 6.3 for EMT- dq model and 7.7 for RMS+ model; (b) Dynamic response of active power, terminal voltage, frequency, and reactive power, when voltage reference is reduced from 1.0 pu to 0.95 pu at 0.1 s for SCR = 5.3	59
3.13	A GFL and a GFM supplying a load	59

3.14	(a) SSO mode trajectories for the test case of Figure 3.13 with EMT- dq (left) and with RMS+ (right), when active power reference of GFL is varied from 0.8 (green) to 1.2 (red). The power threshold (orange) is 1.0 pu for the EMT- dq model, RMS+ model could not detect instability. Dynamic response of active power, terminal voltage, frequency and reactive power of (b) GFL, and (c) GFM when GFL voltage set point is changed to 1.0 pu at 0.1 s while GFL power reference is 0.965 pu.	60
3.15	9-bus test system with two GFMs and one GFL [1]	61
3.16	Dynamic response of active power, terminal voltage, frequency, and reactive power of GFM at (a) B1 and (b) B2 in the 9-bus system. The reference voltage of GFM at bus 2 is increased to 1.05 pu at 0.1 s.	62
3.17	39-bus test system with five GFLs and five GFMs [2]	63
3.18	Dynamic response of IBR active power and terminal voltage for selected buses in the 39-bus system. At 0.1 s, the active power reference of GFL at bus 30 (not shown) is decreased by 10% together with a 5% increase in voltage reference of GFM at bus 31.	66
4.1	(a) A controllable AC current source synchronized with an AC voltage source using a PLL. (b) PLL with linearized dynamic network equations. (c) Standard PLL structure used for control design.	69
4.2	Results from analytical models. (a) SSO mode dependency on active power injection–DI-(II) (b) Damping ratio comparison for variations in PLL bandwidth and line impedance for dynamic (left) and algebraic (right) RL representations–DI-(III).	73
4.3	(a) Low-magnitude 29 Hz oscillation in EMT model (left), modal composition of 29 Hz mode (right)–DI-(I). (b) Mode trajectory of SSO modes for variation in active power–DI-(II). (c) Mode trajectory comparison (algebraic vs dynamic RL) of SSO modes for variation in SCR–DI-(III).	75
4.4	11-bus test system with two GFMs and two GFLs [3].	76
4.5	Dynamic response of IBR terminal voltages for 11-bus system verifying all three diagnostic indicators.	76

4.6	For a GFM connected to a Thévenin equivalent: (a) SSO mode trajectories when the SCR is varied from 4 to 8 (square), τ_c is varied from 0.3 to 0.64 (diamond), and $K_{P_{dq}}$ is varied from 0.2 to 0.1 (circle); (b) active power and voltage reference tracking when the reference voltage is reduced to 0.95 pu at 0.1 s for three oscillatory scenarios, denoted by purple markers.	80
4.7	Mode shapes representing the signature of the oscillatory mode (~ 12.5 Hz) when (a) SCR = 5.4; (b) $\tau_c = 0.46$ ms; (c) $K_P = 0.14$	81
4.8	GFM simplification: (a) GFM as a voltage source with inner control regulating only the voltage at the PCC; (b) simplification of current control resulting in a current source representation of GFM; (c) dynamic block diagram linearized around an operating point with first-order network model.	83
4.9	Damping ratio for varying network parameters (left), and control parameters (right). At base case (100%), $\mathcal{S} = 3$, $\mathcal{X} = 10$, $\omega_0 C = 0.22$, $\tau = 0.3$ ms, $K_P = 0.2$, and $K_I = 4.67$	85
4.10	Dynamic response of terminal voltage of IBR buses for the 11-bus system. The reference voltage of B1 is increased to 1.05 pu at 0.1 s.	86
5.1	Evaluating performance metrics—(a)(b) bandwidths and (c)(d) phase margin while sweeping outer control gains with a scaling factor of 1 to 8. (a), (c) correspond to performance metrics of P-control whereas (b), (d) show the performance metrics of V-control.	91
5.2	Evaluating performance metrics in terms of (a) damping ratio of least-damped SSO mode, and (b) multi-loop disk margin when proportional gain of d- and q-axis are independently scaled. The multi-loop disk margin, unlike phase margin shown in Figure 5.1, corresponds with the actual stability.	92
5.3	Determining cause of instability in least-damped SSO. (a) SSO modes trajectory as line length is increased from 13 km to 130 km. (b) Contribution of states participating in the problematic mode. (c) Reducing gains at the 80 km line length to restore stability. (d) P and V tracking before (red) and after (blue) reducing gains when a step change of 5% is introduced in P reference.	93

5.4	Analysing high frequency peaks in the plant. (a) Envelope of the magnitude response when line length is swept from 13 km (green) to 130 km (red). (b) States participating in the the high frequency peaks—around 1852 Hz. (c) Same envelope as (a) but the inset highlights the set of other two mid-range frequency peaks. (d) States participating in the mid-range frequency peaks—around 600 Hz.	94
5.5	Formation of G_{sys} (3x3 MIMO system) for k^{th} GFL from the generalisable framework. Possible presence of infinite bus is handled by connecting a voltage source directly to the virtual bus (shown by dash-dot line at the bottom).	97
5.6	Formation of sub-systems for SISO designs. (a) Closing PLL with designed gains $(K_{P_\theta}^{gk}, K_{I_\theta}^{gk})$ to obtain G'_{sys} . (b) Closing P -control loop with designed gains $(K_{P_d}^{gk}, K_{I_d}^{gk})$ to obtain G''_{sys} . (c) Closing V -control loop with designed gains $(K_{P_q}^{gk}, K_{I_q}^{gk})$ to complete the design process for k^{th} GFL.	98
5.7	MIMO design process. (a) Appending notch filter with G'_{sys} of Fig. 5.6(a) to mitigate unwanted interfering network frequencies. (b) Obtaining control gains of P -control $(K_{P_d}^{gk}, K_{I_d}^{gk})$, V -control $(K_{P_q}^{gk}, K_{I_q}^{gk})$, and coupling gains $(C_{11}^{gk}, C_{12}^{gk}, C_{21}^{gk}, C_{22}^{gk})$ through co-optimisation of MIMO objectives for k^{th} GFL.	99
5.8	Magnitude response of the combination of four notch filters (left). Its impact on the envelope of plant's magnitude response (right).	99
5.9	A GFL connected to Thévenin's equivalent circuit. For CLD (left column), against a specific design bandwidth, operational SCR is swept and performance metrics; P -control bandwidth (top), V -control bandwidth (middle), and multi-loop disk margin (bottom) are observed. Similarly, for NLD (middle column) and proposed (right column), against a specific design carried out at a SCR of y -axis, operational SCR is swept and performance metrics are monitored.	100
5.10	A GFL connected to a large power system via transformer and a line. For CLD (left column), against a specific design bandwidth, operational SCR is swept and performance metrics; P -control bandwidth (top), V -control bandwidth (middle), and multi-loop disk margin (bottom) are observed. Similarly, for NLD (middle column) and proposed (right column), against a specific design carried out at a SCR of y -axis, operational SCR is swept and performance metrics are monitored.	102

5.11	9-bus system performance charts. Comparing P-control and V-control bandwidths of both GFLs for: Case 1 (top left), where B1 is GFM; Case 2 (top right), where B2 is GFM; and Case 3 (bottom left), where B3 is GFM. Comparing multi-loop disk margin (bottom right) for all three cases.	103
5.12	11-bus system reference tracking comparison for Case 1 (top), where both areas have mix of GFL and GFM; Case 2 (middle), where Area 1 is GFL and Area 2 is GFM; and Case 3 (bottom), where Area 1 is GFM and Area 2 is GFL. For Case 2 and Case 3, CLD and NLD cannot provide a stable design.	104
5.13	Comparing bandwidth (left) and phase margin (right) of two tuning methods—direct synthesis and symmetrical optimum—oblivious to network strength for three current control bandwidths	108
5.14	A GFM connected to a Thévenin equivalent of a power system	109
5.15	A MIMO plant $P(s)$ and its controller $K(s)$ with loop gain $L(s) = K(s)P(s)$. .	110
5.16	V_{dq} closed-loop bandwidth and disk margin comparison of three tuning methods under varying network strength for different target bandwidths	111
5.17	Eigenvalue trajectory of three tuning methods when SCR is varied from 4 to 8 (top); Dynamic response comparison of three techniques at SCR = 5.8 (bottom)	112
5.18	Bandwidth and disk margin comparison of proposed technique (prop.) with direct synthesis (DS), symmetrical optimum (SO) for 9-bus (top) and 11-bus (bottom) system.	114
6.1	The proposed methodology for <i>in situ</i> estimation of black-box IBRs connected to power system.	118
6.2	The circuit diagram of the pulse current injection at the POI of the m -th IBR connected to RoS. The subscript m refers to the index of the investigated IBR. .	118
6.3	The block diagram of the setup for measuring pulse response data. The investigated IBR, denoted by G_m , is regarded as black-box input-output dynamical system, where the input is the perturbed current and the output is the perturbed voltage.	118
6.4	Voltage (top) and current (bottom) responses when a pulse with an amplitude of 0.01 pu is applied at 0.1 s on GFL1 under the poorly-damped condition. . .	126

6.5	Comparison of the estimated singular value responses with that of actual model for GFL1 (left) and GFM2 (right) in the poorly-damped scenario. A frequency range of 1-1000 Hz is considered.	127
6.6	Comparison of the estimated closed-loop system eigenvalues with those of actual system (left); and the comparison of the estimated participation with the actual participation of each IBR in the poorly-damped mode of 21 Hz (right).	127
6.7	Voltage (top) and current (bottom) response when a pulse with an amplitude of 0.01 pu is applied at 0.1 s on GFL1 under the well-damped condition.	128
6.8	Comparison of the estimated singular value response with that of actual model for GFL1 (left) and GFM2 (right) in the well-damped scenario. A frequency range of 1-1000 Hz is considered.	128
6.9	Comparison of the estimated closed-loop system eigenvalues with those of actual system (left); and the comparison of the estimated participation with the actual participation of each IBR in the poorly-damped mode of 3.5 Hz (right).	129
6.10	For all 5 GFLs in Scenario I, the singular values of Hankel matrix are automatically selected for estimating $G_{e,V}$ (top left) and $G_{e,V}$ (top right); The normalized state energy $\mathcal{E}$ is considered as the metric for balanced truncation which determines the reduced order (bottom left); The subplot (bottom right) summarises three phases of the model order selection and reduction procedure.	131
6.11	For all 5 GFMs in Scenario I, the singular values of Hankel matrix are automatically selected for estimating $G_{e,V}$ (top left) and $G_{e,V}$ (top right); Similar to Fig. 6.10, the zoomed insets show the procedure of automatic selection of the model order for each GFM; the balanced truncation with normalized state energies to determine the reduced model order (bottom left); three-phase summary of the model order reduction process (bottom right).	133
6.12	Comparison of the singular values of the estimated frequency response with the singular values of actual frequency response of GFLs (top row) and GFMs (bottom row) for Scenario I of modified 39-bus system.	134
6.13	Comparison of the estimated participation with the actual participation of each IBR in poorly-damped mode(s) of each scenario. Scenario IV has two poorly-damped modes at 9.2 Hz and 10.5 Hz.	134

A1 Circuit diagram to explain participation factor polar plots. 149

A2 Eigenvalues and modal composition (participation factor with angle of right
 eigenvector) of poorly-damped mode. 151

List of Tables

3.1	Tunable Parameters	64
3.2	Fixed Control and Filter Parameters for GFL and GFM	65
3.3	Network Parameters for Each Case Study	65
4.1	GFM Parameters	79
5.1	Control Gains for Three Tuning Methods	112
6.1	System and Control Parameters for the Test Case 1	126
6.2	Configurations for the Four Scenarios in the Test Case #2	130

Chapter 1

Introduction

The global energy landscape is undergoing a profound transformation, driven by the imperative to decarbonize electricity generation and mitigate climate change. This transition is characterized by the rapid integration of renewable energy sources, such as solar photovoltaic and wind power, which are predominantly connected to the grid via power electronic inverters. These IBRs are fundamentally different from traditional synchronous generators, leading to a paradigm shift in power system dynamics and presenting novel challenges to grid stability and reliability.

The increasing penetration of IBRs is a defining feature of modern power systems. Traditional large-scale synchronous generators are being phased out and replaced by IBR technologies, fundamentally altering the operational characteristics of the grid [4]. This transition is not merely an increase in generation capacity but a qualitative change in how the grid operates, demanding IBRs to provide a wider range of essential services to maintain reliability, resilience, and security [4]. For instance, in the ERCOT system, IBRs constituted 43% of installed capacity and set a new renewable penetration record of 75.67% in March 2024 [5]. This rapid growth underscores the urgency of understanding and addressing the associated dynamic challenges. The replacement of traditional synchronous generators with IBRs signals a profound systemic shift in the fundamental paradigm of grid operation. Historically, power system stability analysis and operational strategies have been predicated on the inherent characteristics of synchronous machines, such as their rotational inertia, fault current contribution, and voltage regulation capabilities. As IBRs increasingly displace these conventional assets, the underlying assumptions of grid behaviour are no longer fully applicable. This necessitates a comprehensive re-evaluation of existing grid codes, planning criteria, and operational practices. The grid is transitioning from a system dominated by electromechanical dynamics to one increasingly

governed by fast power electronics controls. This change is not simply about adding more generation; it represents a fundamental alteration in the grid’s dynamic response, requiring new approaches to ensure its continued reliability and security.

The shift towards an IBR-dominant grid introduces complex dynamic interactions and new stability challenges that were less prevalent in traditional power systems [6]. These challenges stem from the inherent characteristics of IBRs, which interface with the grid through power electronic converters rather than direct electromechanical coupling. Consequently, conventional grid services such as inertia, grid strength, and voltage support, traditionally provided by synchronous generators, are declining, leading to undesired events at system and area levels [6]. One of the most critical and frequently observed issues is the increased susceptibility to poorly damped oscillations, particularly SSO [7]. These oscillations are difficult to foresee and pose a serious threat to system security, often forcing grid operators to limit the instantaneous share of IBRs to maintain stability [8]. NERC disturbance reports provide strong evidence of systemic deficiencies in IBR performance during grid events, highlighting issues in modelling and study accuracy for IBR integration [9]. These performance deficiencies are of greatest risk in bulk power system (BPS)-connected solar PV resources, leading NERC to issue a Level 2 alert in March 2023 with recommendations for generator owners [9]. Furthermore, Grid-Following Inverter (GFL), which currently constitute the majority of connected IBRs, are particularly vulnerable to instability under small disturbances in weak grids, where their performance significantly deteriorates [4]. This is because GFL inverters synchronize with the existing grid and inject constant current, making them akin to a dancer struggling to keep rhythm with faint, erratic beats in low-strength conditions [4]. The lack of sufficient short-circuit current (SCC) from IBRs also jeopardizes transient stability and protection coordination [6]. The decline in system inertia, grid strength, and voltage support, traditionally provided by synchronous generators, is not an isolated set of problems but rather a deeply interconnected set of stability issues. The reduction in these fundamental grid characteristics directly contributes to the emergence of new dynamic phenomena, including SSO. For instance, when grid strength diminishes, GFL inverters struggle to maintain synchronization, leading to potential instability. This cascading effect means that addressing SSO effectively requires a holistic approach that considers its interplay with other stability aspects, such as voltage stability, frequency stability, and transient stability, rather than treating it as an isolated phenomenon. Mitigation strategies, therefore, often need to provide multi-faceted support to the grid, improving overall system resilience.

The regulatory and operational urgency surrounding IBR performance is evident from NERC’s actions. The issuance of a Level 2 alert in March 2023 and the finding that “vol-

untary recommendations... are not being implemented” [9] reveal a significant gap between the recognized issues and their effective resolution within the industry. A particularly concerning aspect is the observation that IBR protection settings, such as those for Phase-Locked Loop (PLL) loss of synchronism, are often configured below maximum capability, leading to inadvertent tripping during grid faults. Crucially, these risks are frequently not modelled in positive sequence simulations or even historical EMT models, meaning they are not captured in interconnection studies or annual planning assessments [9]. This situation points to a critical failure in current planning and operational practices, where a lack of accurate modelling and adherence to best practices can lead to significant unaddressed vulnerabilities. This regulatory imperative suggests that academic research must not only identify these problems but also propose practical, implementable solutions that can be adopted by grid operators and generator owners, emphasizing the need for improved model validation and stricter enforcement of modelling standards.

1.1 Sub-Synchronous Oscillations

Sub-synchronous oscillations (SSO) represent a significant dynamic stability challenge in power systems with high IBR penetration. Unlike the traditional sub-synchronous resonance (SSR) associated with series capacitors and synchronous machines, modern SSO phenomena are predominantly driven by the complex control interactions within and between IBRs and the grid.

1.1.1 Understanding SSO: Characteristics and Observed Incidents

SSO refers to oscillations in power systems at frequencies less than the fundamental power system frequency [10]. These oscillations can occur under normal system conditions due to disturbances and, if poorly damped, can lead to undamped amplification, causing damage to transmission and generator assets, and resulting in generation loss and circuit trips [10]. The IEEE PES IBR SSO Taskforce has documented numerous real-world SSO events globally, with 19 examples reported between 2007 and 2021 [11]. More recently, the GB system observed 5-9 Hz SSO on multiple occasions in June and July 2023 [10].

A notable example comes from Australia, where SSOs have been frequently observed in areas with high IBR penetration, particularly in the West Murray Zone [8]. These oscillations typically range between 8 Hz and 20 Hz, exhibiting varying magnitudes and durations [8]. Specific incidents include an 8 Hz voltage oscillation in 2018 caused by five solar farms,

triggered by a 220 kV transmission circuit disconnection, and a 17 Hz voltage oscillation observed in 2020 [8]. These incidents fall under the category of IBR small-signal stability issues [8]. The ESIG Oscillations Guide (2024) elaborates on various types of SSO, emphasizing sub-synchronous and super-synchronous Control Interaction (SSCI) as a dominant contributor in IBR-dominated systems. SSCI arises when inverter controls are compromised by unexpected host system responses in fast frequency ranges, often manifesting as negative resistance at the unstable frequency [7].

The evolution of SSO mechanisms is a critical distinction in modern power systems. Historically, the term sub-synchronous resonance (SSR) was primarily used to describe interactions between series capacitors and the torsional mechanical systems of synchronous turbine-generators, leading to potentially catastrophic mechanical failures [7]. However, with the increasing penetration of IBRs, new types of oscillations have emerged. The ESIG guide explicitly identifies sub-synchronous and super-synchronous Control Interaction (SSCI) as the dominant contributor to IBR-related oscillations, highlighting that these are driven by inverter controls and their unexpected responses to fast frequency changes in the host system [7]. This indicates a fundamental shift in the underlying physical and control mechanisms driving SSO, moving beyond traditional electromechanical resonance to complex power electronics control interactions. Consequently, traditional SSR analysis tools and mitigation strategies, designed for synchronous machine characteristics, may not be effective for IBR-driven SSO, necessitating new approaches that specifically account for inverter control dynamics and their frequency-dependent impedance characteristics.

The geographic and systemic nature of SSO is underscored by its global observation. The repeated mention of SSO events occurring “globally” [8] and specific examples from Australia and the GB system [10] demonstrate that SSO is not an isolated or localized problem but a widespread, systemic issue affecting grids worldwide as IBR penetration increases. The Australian incidents, for instance, were specifically noted in regions with “large penetration of IBRs” and “low system strength levels” [8]. This correlation between SSO occurrences and areas of high IBR penetration and reduced system strength points to these factors as critical risk indicators. The global prevalence of this problem highlights the universal applicability and importance of research into SSO. Solutions developed and validated in one region could potentially be adapted and applied to others, fostering international collaboration in power system stability.

1.1.2 The Overlooked Role of Network Dynamics in SSO

A critical aspect of IBR-driven SSO that is often overlooked in conventional planning tools is the intricate role of network dynamics. While individual IBR characteristics are important, the adverse interactions among IBRs through the transmission network are a significant cause of poorly damped oscillations [8]. The ESIG guide highlights that SSCI often involves the effective impedances of active system components (inverters, IBR plants) interacting with transmission network impedances at sideband frequencies [7]. Instability results when the sum of driving-point reactances of an IBR plant and the transmission system sums to zero, and the sum of resistances is negative [7].

The widespread integration of IBRs, particularly in remote parts of the network and in close proximity to each other, exacerbates these control interactions [8]. For instance, in Australia, the impedance scan study revealed that certain IBRs could increase the effective grid impedance seen by another IBR in close proximity, making the resulting SSO mode more pronounced due to a combination of control interactions among IBRs through the network and internal IBR resonance modes [8]. This necessitates a comprehensive understanding of how the network's frequency-dependent characteristics influence IBR stability and vice versa. CIGRÉ Technical Brochure 909 (2023) provides guidelines for SSO studies in power electronics-dominated systems, emphasizing a systematic study procedure that implicitly accounts for network characteristics [12].

The understanding of SSO has moved beyond considering only local, single-device interactions to recognizing them as system-wide phenomena. The primary cause of SSO is not merely the behaviour of individual IBRs but rather the “adverse interaction among IBRs through the network”. This means that the collective behaviour of multiple IBRs, mediated by the frequency-dependent impedance of the transmission network, is a true root cause of these oscillations. The NREL document on impedance-based stability analysis further supports this by highlighting the challenges of evaluating “wide-area oscillations resulting from control interactions among numerous IBRs through the power system network” and the need for scalable methods to analyze the impact of each IBR on system stability [13]. This perspective implies that mitigation strategies cannot solely focus on individual IBR control tuning but must consider system-wide coordination and the impact of network topology and strength. Consequently, planning tools must evolve to capture these complex multi-IBR, multi-location interactions, moving beyond simplified representations of the grid.

The network is not simply a passive conduit for power flow; it is an active participant in

the dynamics that can lead to instability. The ESIG guide clarifies that instability arises when “the sum of driving-point reactances of an IBR plant and the transmission system sums to zero, and the sum of resistances is negativ” [7]. This framing highlights that the network’s impedance characteristics, particularly at various frequencies, can either dampen or exacerbate oscillations. The prevalence of SSO in regions with “low system strength levels” [8] further reinforces the network’s active role. In such weak grid conditions, the network’s impedance can interact unfavorably with IBR controls, leading to sustained oscillations. Therefore, power system planning and operation must incorporate detailed frequency-dependent network models and consider the dynamic interplay between IBR controls and grid impedance. This points to a crucial need for advanced analytical and simulation tools that can accurately represent the network’s dynamic behaviour across a wide frequency spectrum, ensuring that the network’s contribution to stability or instability is fully understood and accounted for.

1.2 Modelling Challenges for IBR-Dominated Systems

The complexity and speed of IBR dynamics, coupled with their interactions with the network, necessitate advanced modelling and simulation tools that go beyond traditional approaches.

1.2.1 Limitations of Conventional Phasor-Domain Studies

Conventional phasor-domain RMS simulation tools, while computationally efficient for large-scale system dynamic analysis, fall short in accurately capturing the fast and complex dynamic behaviours introduced by IBRs [14]. These tools are typically limited to frequencies up to 3-5 Hz and struggle with very fast or unbalanced phenomena [7]. A key limitation is their assumption that positive and negative sequence impedances are decoupled, which is not valid for active devices like IBRs that exhibit a frequency-coupled response due to their internal controller configurations [15]. Specifically, the phenomenon of Mirror Frequency Coupling (MFC), dominant in the sub-synchronous range, significantly impacts stability performance and is not captured by traditional positive sequence models [15].

NERC reports indicate that critical risks, such as inadvertent tripping due to PLL loss of synchronism protection, are often not represented in positive sequence simulations, leading to uncaptured risks in interconnection studies and annual planning assessments [9]. EirGrid, for instance, acknowledges that typical phasor-domain dynamic models may not be adequate to identify potential interactions and instabilities related to converter controls in weak grids,

such as those caused by interactions between the phase-locked loop (PLL) and inner current controller loops [16]. This highlights a critical gap: the positive sequence RMS (RMS+) studies, while useful for screening, lack the fidelity required for detailed SSO analysis and root cause identification.

This shortcoming in RMS+ leads to hidden risks within power system planning. The NERC report explicitly states that critical risks, such as inadvertent tripping of IBRs due to protection settings, “would not be captured in interconnection studies or during annual planning assessments” if reliance is placed solely on positive sequence simulations [9]. This implies that system planners and operators might be operating under a false sense of security, as potential instability issues are effectively masked by the limitations of conventional modelling tools. The consequences can be severe, including “unwanted equipment disconnections, supply interruptions, equipment damage, and other violations of reliability criteria” [7]. This situation emphasizes the critical need for more accurate and comprehensive modelling from the earliest stages of project interconnection and system planning to accurately identify and mitigate these uncaptured risks.

1.2.2 The Need for Advanced modelling: Bridging the Gap with EMT-dq Simulations

To accurately capture the fast dynamics and complex control interactions of IBRs, electromagnetic transient (EMT) simulation models are increasingly recognized as essential [8]. EMT simulations represent all three phases and solve differential equations for network elements, providing high accuracy for fast and unbalanced phenomena [7]. However, full EMT-*abc* (point-on-wave) simulations are computationally demanding and time-consuming, making them impractical for large-scale system-wide dynamic and operational analysis.

This computational burden highlights the critical need for a middle-ground simulation method that offers greater accuracy than positive-sequence RMS models with less computational burden than EMT-*abc*. EMT-*dq* modelling emerges as a promising solution. While the existing modelling landscape do not explicitly detail EMT-*dq* as a distinct simulation tool, they strongly advocate for EMT models that can capture *dq*-domain impedance [17] and acknowledge the need for detailed EMT models for IBR-driven dynamics while seeking computational efficiency. Hybrid EMT-TS (Transient Stability) simulations, for example, can greatly reduce computational burden while preserving detailed fast dynamics in the EMT zone, achieving speed-ups (e.g., 2.4x on a 240-bus system) while accurately capturing both slow electrome-

chanical and fast IBR-driven dynamics [18]. This aligns with the concept of a computationally efficient yet accurate method for wide-area analysis. The ability of tools like NREL’s Grid Impedance Scan Tool (GIST) to output scan data in rotating (dq) reference frames further supports the utility of dq -domain analysis within an EMT framework [17].

The challenge of balancing accuracy with computational efficiency, often referred to as the “accuracy-efficiency” dilemma, is a central issue in power system modelling for IBR-dominated grids. On one hand, RMS+ models are computationally fast but fundamentally inaccurate for capturing the fast, frequency-coupled dynamics of IBRs. On the other hand, full EMT-*abc* simulations provide high fidelity but are prohibitively slow for large-scale system studies [14]. This dilemma necessitates the exploration of intermediate modelling approaches that balance fidelity and computational efficiency. The emergence of hybrid EMT-TS simulations, which combine the detailed representation of EMT for critical areas with the computational speed of transient stability models for the rest of the system [18], directly addresses this need. Similarly, the focus on EMT- dq modelling, which captures the essential frequency-dependent characteristics of IBRs in a rotating reference frame while being more computationally tractable than full three-phase EMT, offers a viable pathway to achieve both fidelity and scalability. The thesis’s emphasis on EMT- dq as an intermediate modelling approach is therefore highly relevant and timely, as it directly responds to a critical industry need for scalable yet accurate simulation capabilities to perform more scenario checks and ensure grid reliability under high IBR penetration.

A concerted effort towards standardizing advanced modelling practices is evident across the industry. NERC and CIGRÉ documents frequently publish guidelines and recommended practices for EMT modelling and SSO studies [12]. For instance, the PNNL’s work on Generic EMT modelling for IBR in support of IEEE P2800.2 [19] highlights a collaborative effort to standardize and improve the quality and usability of EMT models for IBRs. This indicates that the industry is not just recognizing the problem of inadequate modelling but is actively working towards establishing best practices and common frameworks for advanced simulation tools. This standardization effort provides a strong foundation and validation for the modelling frameworks proposed in this thesis. It also implies that the research contributions, particularly in establishing modelling adequacy, will have a direct path to industry adoption and impact, facilitating a more reliable and secure integration of IBRs into the grid.

1.3 Addressing Industry Concerns: Black-Box IBR Models and SSO Unpredictability

The rapid integration of IBRs into power systems has introduced significant challenges related to the transparency and predictability of their dynamic behaviour, particularly concerning SSO. These challenges stem from the proprietary nature of IBR control systems and the inherent complexity of identifying the root causes of oscillations in a highly interconnected grid.

1.3.1 Opaqueness of Proprietary IBR Models and its Impact on Grid Reliability

A major concern for grid operators and planners is the opaqueness of proprietary IBR models, often referred to as “black-box” models [20]. Original Equipment Manufacturers (OEMs) and vendors typically provide these models without disclosing the intricate internal control logic, parameters, or protection functions due to intellectual property concerns [19]. This lack of transparency severely hinders accurate and validated modelling of IBRs, which is essential for maintaining grid reliability [20]. A 2024 NERC report highlights industry-wide challenges, noting that many IBR models used in planning studies fail to accurately reflect real-world behaviour, partly because they are not always validated against commissioning or disturbance data [20].

Furthermore, generator owners often rely on OEMs or consultants for modelling data, with limited follow-up or quality assurance before submission to transmission planners or planning coordinators [20]. This can lead to errors going undetected, with models submitted using default parameters, outdated templates, or lacking site-specific details and protection logic [20]. A NERC Level 2 alert response indicated that an overwhelming majority (78%) of generator owners submit only standard library models, with only 7% submitting both standard and manufacturer-specific models, despite NERC guidance [21]. This practice creates artificial limitations on ride-through capability, decreasing the IBRs’ ability to withstand transients and faults, and leaving a substantial portion of reactive power capability unused [20]. The dominance of a small number of manufacturers in the IBR fleet (e.g., top five account for over 83% in North America) also raises concerns about common vulnerabilities [21].

The issue of black-box IBR models is not merely a data access problem; it represents a fundamental impediment to effective system planning and operation, creating a knowledge vacuum for grid operators. The proprietary nature of IBR control logic means that system planners

often lack the necessary detailed information to accurately predict IBR behaviour under various grid conditions, especially during disturbances. This translates into a significant risk: without accurate and validated models that reflect real-world performance, planning studies can produce unreliable results, leading to misoperations and compromised system resiliency [20]. The prevalence of default parameters and outdated models further exacerbates this issue, as the simulated grid behaviour deviates significantly from actual performance. This situation underscores a critical need for methods that can infer or estimate IBR dynamic characteristics without requiring access to confidential internal control diagrams, enabling more robust stability assessments.

1.3.2 Challenges in SSO Diagnosis and Root Cause Identification

The unpredictability of SSO and the difficulty in identifying their root causes pose significant operational challenges. When oscillations are observed in the field, it is often challenging to determine whether a specific IBR or a group of IBRs is responsible, or which IBRs are positively or negatively contributing to the oscillations [8]. Time-domain simulations, while useful for replicating incidents, do not always reveal the root cause without extensive sensitivity analysis. Moreover, control mode changes or parameter adjustments in models to replicate an oscillation incident can sometimes lead to false identification of the source [8]. It is also difficult to identify the stability margin or distance to instability from time-domain simulation results alone [8].

The ESIG Oscillations Guide (2024) outlines various diagnostic approaches, including signal diagnostics (FFTs, spectrograms, Prony analysis) and diagnostic simulations [7]. However, white-box modelling methods, such as eigenvalue analysis, may not be practical due to the aforementioned confidentiality issues surrounding IBR control block diagrams and parameters [8]. This highlights a pressing need for diagnostic indicators that can identify the primary responsible IBR control loop using readily available time-domain signal sensing, circumventing the need for detailed internal models. Impedance-based frequency scans of IBRs and the network at their points of connection offer useful insights for root cause analysis, as they can identify potential resonance modes and reveal control interactions [8]. This approach relies on the IBR impedance response obtained from black-box EMT models, which helps to navigate confidentiality issues [8].

The challenge of SSO diagnosis and root cause identification is compounded by the inherent complexity of control interactions in IBR-dominated grids. Unlike traditional power systems where oscillations might be linked to specific synchronous machine parameters, IBR-driven

SSOs arise from intricate interplay among fast-acting inverter controls and the network. The difficulty in pinpointing the exact source or contributing factors from field measurements or even time-domain simulations means that mitigation efforts can be misdirected or ineffective [8]. This situation underscores the need for robust diagnostic tools that can accurately attribute oscillatory behaviour to specific IBR control loops or groups of IBRs, thereby enabling targeted and efficient mitigation strategies. Without such capabilities, grid operators are left to operate conservatively, potentially curtailing renewable generation unnecessarily, which impedes the energy transition.

1.4 Thesis Contributions

This thesis addresses the critical challenges posed by SSOs in IBR-dominated power systems. It proposes novel methodologies and tools to enhance the detection, analysis, and mitigation of SSO, thereby contributing to the secure and reliable operation of future grids. The specific contributions are outlined as follows:

Contribution 1: Establishing Modelling Adequacy for SSO Analysis

This thesis establishes a comprehensive modelling framework to accurately represent SSO in both time- and frequency-domain studies. It emphasizes the often-overlooked role of network dynamics, which are generally ignored or simplified in conventional planning tools. Consequently, the research establishes EMT- dq modelling as a crucial middle-ground simulation method. This approach offers significantly greater accuracy than positive-sequence RMS models, which fail to capture frequency-coupled responses and critical protection settings [9], while imposing substantially less computational burden than full EMT- abc (point-on-wave) simulations. This balance of fidelity and computational efficiency allows for a greater number of scenario checks, which is vital for comprehensive stability assessment in large-scale IBR-dominated systems. The framework ensures that the complex interactions between IBR controls and the frequency-dependent network impedance are adequately represented, moving beyond the limitations of traditional decoupled sequence models [15]. The novel contribution here is the establishment of EMT- dq modelling as a practical and scalable alternative that strikes a unique balance between fidelity and computational cost. While prior works have highlighted the importance of impedance-based methods for small-signal stability assessment, this thesis is the first to systematically demonstrate, through both time-domain and frequency-domain valida-

tion, that an EMT- dq representation is sufficient to capture critical SSO mechanisms without resorting to point-on-wave simulations. This opens up new possibilities for more scalable yet accurate stability screening in future zero-inertia grids.

Contribution 2: Developing Diagnostic Indicators for SSO Source Identification

Building upon the established modelling adequacy, this thesis develops a novel set of diagnostic indicators. These indicators are designed to identify which IBR control loop is primarily responsible for a specific SSO mode using time-domain signal sensing. Traditional root cause analysis often struggles with false identification or the inability to pinpoint the source of oscillations from time-domain simulations alone [8]. Furthermore, the confidentiality of internal IBR control block diagrams hinders the application of white-box analysis methods like eigenvalue analysis [8]. The proposed diagnostic indicators circumvent these limitations by providing a practical, non-invasive approach to attribute oscillatory behaviour, enabling system operators to eliminate the source of oscillations operationally and work with generator owners on long-term solutions [8].

Contribution 3: Advancing Control Design for GFL and GFM Inverters with Network Awareness

This thesis develops novel and practical control strategies for both GFL and Grid-Forming Inverter (GFM) to mitigate SSO. A key innovation lies in incorporating the known network model as an integral part of the plant for control design. Current GFL inverters often struggle in low-strength grids, where their performance deteriorates due to reliance on external grid synchronization [4]. While GFM inverters are gaining attention for their ability to provide robust dynamic support and operate in weak grids [4], their control design can be further enhanced in terms of bandwidth and efficient provision of grid services. By making the network model part of the plant for control design, this research ensures higher bandwidths and enhanced stability margins. This approach allows for more adaptive and robust control performance, particularly in scenarios where IBRs interact adversely through the transmission network, leading to poorly damped oscillations [8]. This network-aware control design represents a significant step towards enabling stable operation of IBRs in increasingly complex and weak power systems.

Contribution 4: Impedance Estimation for Black-Box IBRs

To circumvent the opaqueness of proprietary 'black-box' IBR models, this thesis presents a data-driven technique to estimate each IBR's linearised model in the form of impedance or admittance spectra around a given operating point. The confidentiality surrounding inverter controller information often hinders accurate stability analysis [22]. Existing methods often rely on invasive frequency sweeps [22]. This method is considered invasive because it requires deliberate injection of sinusoidal frequency sweeps into the system, thereby perturbing normal inverter operation across a range of frequencies to characterize impedance, rather than relying solely on passive measurements or natural fluctuations. The proposed technique eliminates the need for dynamic frequency scans and enables system-wide stability assessment and root cause analysis (e.g., participation of each IBR in SSO) using only one-shot input-output measurements for each IBR [22]. This data-driven approach provides interpretable dynamic structures, making it capable of identifying oscillation causes and enabling grid companies to determine which component is causing instability, even when internal control information is unknown [22]. This contribution is particularly valuable for integrating multi-provider components and performing impedance shaping to improve grid reliability, efficiency, and flexibility.

1.5 Thesis Structure

This thesis is organized into five main chapters, each building upon the preceding one to provide a comprehensive treatment of SSOs in IBR-dominated power systems.

Chapter 2 Modelling Framework: This chapter details the theoretical foundations and practical implementation of the proposed modelling framework. It delves into the specifics of EMT- dq modelling, explaining its advantages over traditional RMS+ and full EMT- abc methods for accurately capturing IBR dynamics and network interactions relevant to SSO studies. The chapter will cover the mathematical formulations and simulation considerations necessary for robust analysis.

Chapter 3 Modelling Adequacy for Sub-Synchronous Oscillation Analysis: This chapter analyses how network dynamics, particularly grid impedance characteristics, influence the occurrence and propagation of SSO in IBR-dominated power systems. It will present case studies and simulations to demonstrate the often-underestimated role of the transmission network in exacerbating or mitigating these oscillations, highlighting scenarios where adverse

control interactions through the network lead to instability.

Chapter 4 Characterisation and Mechanisms of Sub-Synchronous Oscillations: In this chapter, the focus is on the characterization of SSO phenomena, including their frequency ranges, damping levels, and the development of diagnostic indicators. It will present the methodology for identifying the specific IBR control loops responsible for observed SSO modes using time-domain signal processing techniques, providing a practical tool for root cause analysis in real-world scenarios.

Chapter 5 IBR Control Design to Mitigate Sub-Synchronous Oscillation: This chapter details the development and validation of advanced control strategies for both GFL and GFM inverters aimed at mitigating SSO. It elaborates on the integration of network model information into the inverter control design, demonstrating how such an approach can achieve higher bandwidths and improved stability margins, thereby enhancing the overall damping of sub-synchronous modes.

Chapter 6 Root Cause Analysis of Sub-Synchronous Oscillation using In Situ Estimated IBR Models: This chapter presents the data-driven methodology for estimating the linearised impedance spectra of 'black-box' IBR models using one-shot input-output measurements. It discusses the theoretical underpinnings of this relatively less invasive technique and demonstrates its application for system-wide stability assessment and participation factor analysis. This allows a deeper understanding of IBR contributions to SSO without proprietary IBR information.

Chapter 2

Modelling Framework

This chapter presents the modelling details of the electrical components employed in the design and testing processes of bulk power systems throughout this thesis. Here, a modular approach is detailed, where each module is represented by a set of differential and algebraic equations. All states, fixed parameters, tunable parameters, inputs, outputs, and references are explicitly mentioned. This approach facilitates scalability in small-signal stability analysis and control design process, discussed in Chapter 4 and 5, respectively. This chapter also explains how this modelling framework enable modelling of multi-IBR systems with complex topologies. The symbols presenting all quantities in this chapter and equations have two-tiered subscripts and superscripts. Each symbol carry different meaning depending on their position. For example, the subscript ‘m’ denotes ‘measured’, while the superscript ‘m’ indicates a GFM quantity. Since this thesis focuses on the zero-carbon power systems, the modelling details of Synchronous Generator (SG) are excluded. Existing SGs may have been the part of the large power system or of the Thévenin equivalent circuit; however, their impact on the modelling and design is minimal due to the timescale separation in control actions and high penetration of IBRs. Their exclusion simplifies the dynamic modelling process and allows for focused analysis of IBR-dominant behaviours.

The IBRs—both GFL and GFM—are modeled in their respective local dq rotating reference frames. The IBR current or voltage (X) in local dq frame is transformed onto a common network reference frame DQ using $X_{DQ} = X_{dq}e^{j\theta}$ where θ is the angle between the two frames. An aggregated model of the IBR plant is considered with only the grid-side converter equipped with an LC filter, as in [23]. The dynamics of the primary resource (wind turbine, solar panel etc.) is not considered to keep the study resource agnostic. As the focus of this thesis is small-signal stability, we assume balanced operation and narrow-band signals tightly centred

around the nominal frequency. While large-signal stability, concerned with system response under severe disturbances such as faults, islanding, or protection events, remains critical for overall system security, it typically requires nonlinear time-domain simulations and different analytical tools. This thesis, however, is motivated by the need for analytical insight into mechanisms of sub-synchronous oscillations in IBR-dominated systems. Such oscillations often emerge from small perturbations around an operating point and evolve within narrow frequency bands. Small-signal analysis thus offers a tractable and sufficiently accurate framework to study their initiation and propagation. A rigorous treatment of large-signal behaviour is outside the scope of this work but remains a complementary area for future research.

The superscripts correspond to the component type - g for GFL, m for GFM, δ for network, f , for floating node, and l for loads - and k denotes the bus number where the component is connected. Consider K as the complete set of bus indices in a system, with K_G and K_M being subsets of K , representing the bus indices where GFLs and GFMs are connected, respectively.

2.1 Grid-Following Inverter

A GFL regulates active power, P_m^{gk} , and V_m^{gk} , at Point of Common Coupling (PCC) of Bus k . The standard control loops—PLL, current control, outer control—of a GFL, driving a converter connected at Bus k are shown in Figure 2.1. The DC side of the converter is well-regulated, obviating the need to include DC side dynamics in this model.

The GFL model block diagram is shown in Figure 2.2. The C_f^{gk} dynamics of Figure 2.2 are described by (2.1a), (2.1b). The PLL action, captured in (2.1c)-(2.1e), aligns the d-axis of

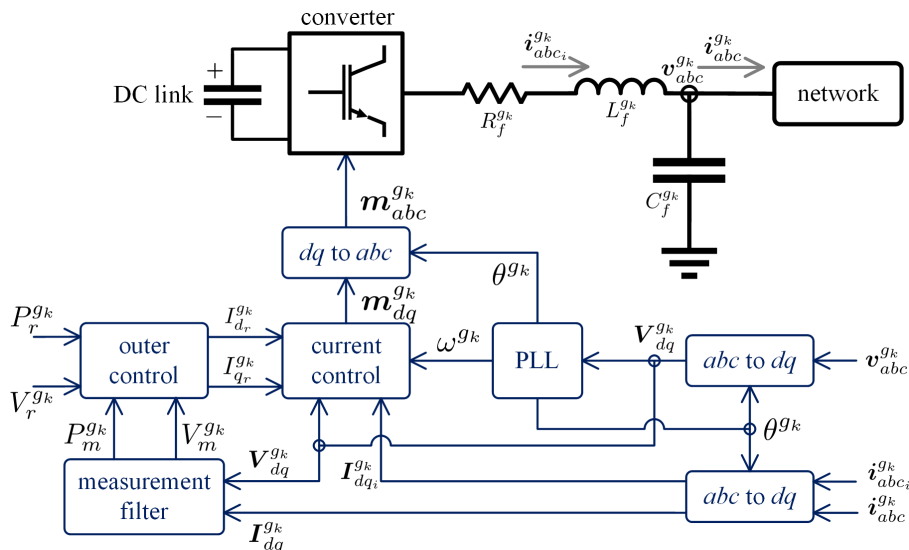


Figure 2.1: Control blocks for a GFL with an LC filter

the synchronously rotating dq frame with the GFL terminal voltage V^{gk} by forcing its q-axis component, V_q^{gk} , to zero. The corresponding state introduced by the integrator of PI state is denoted by x_θ^{gk} . We introduce a low-pass filter in PLL of a certain bandwidth ($\equiv 1/2\pi\tau_\theta^{gk}$), say 100 Hz, governed by (2.1d), to mitigate high frequency interference in the PLL response. The design of control gains for both PLL and outer control is discussed in Chapter 5. Moreover, considering perfect decoupling feed-forward, the closed-loop current control is simplified to first-order time lag with a specified closed loop bandwidth ($\equiv 1/2\pi\tau_c^{gk}$) as described by (2.1f), (2.1g). The outer control that tracks active power reference (d-axis) and voltage reference (q-axis) at PCC is defined by (2.1h), (2.1i), where x_d^{gk} and x_q^{gk} are corresponding states introduced by the integrator of PI controller of each path. Finally, (2.1j) and (2.1k) govern the measurement filter dynamics characterised by measurement bandwidth ($\equiv 1/2\pi\tau_m^{gk}$). Overall, (2.1) defines the dynamic model of all GFLs in G , where $G = \{g_k \mid k \in K_G\}$, for $K_G \subset K$.

$$\dot{\mathbf{V}}_d^G = (\mathbf{I}_{d_i}^G - \mathbf{I}_d^G)/\mathbf{C}_f^G + (\omega_0 + \Delta\omega^G)\mathbf{V}_q^G \quad (2.1a)$$

$$\dot{\mathbf{V}}_q^G = (\mathbf{I}_{q_i}^G - \mathbf{I}_q^G)/\mathbf{C}_f^G - (\omega_0 + \Delta\omega^G)\mathbf{V}_d^G \quad (2.1b)$$

$$\dot{\theta}^G = \Delta\omega^G \quad (2.1c)$$

$$\Delta\dot{\omega}^G = (\Delta\tilde{\omega}^G - \Delta\omega^G)/\tau_\theta^G \quad (2.1d)$$

$$\dot{\mathbf{x}}_\theta^G = (\Delta\tilde{\omega}^G - \mathbf{x}_\theta^G \mathbf{K}_{I_\theta}^G)/\mathbf{K}_{P_\theta}^G = 0 - \mathbf{V}_q^G \quad (2.1e)$$

$$\dot{\mathbf{I}}_{d_i}^G = (\mathbf{I}_{d_r}^G - \mathbf{I}_{d_i}^G)/\tau_c^G \quad (2.1f)$$

$$\dot{\mathbf{I}}_{q_i}^G = (\mathbf{I}_{q_r}^G - \mathbf{I}_{q_i}^G)/\tau_c^G \quad (2.1g)$$

$$\dot{\mathbf{x}}_d^G = (\Delta\mathbf{I}_{d_r}^G - \mathbf{x}_d^G \mathbf{K}_{I_d}^G)/\mathbf{K}_{P_d}^G = \mathbf{P}_r^G - \mathbf{P}_m^G \quad (2.1h)$$

$$\dot{\mathbf{x}}_q^G = (\Delta\mathbf{I}_{q_r}^G - \mathbf{x}_q^G \mathbf{K}_{I_q}^G)/\mathbf{K}_{P_q}^G = \mathbf{V}_r^G - \mathbf{V}_m^G \quad (2.1i)$$

$$\dot{\mathbf{P}}_m^G = (\mathbf{V}_d^G \mathbf{I}_d^G + \mathbf{V}_q^G \mathbf{I}_q^G - \mathbf{P}_m^G)/\tau_m^G \quad (2.1j)$$

$$\dot{\mathbf{V}}_m^G = (|\mathbf{V}_{dq}^G| - \mathbf{V}_m^G)/\tau_m^G \quad (2.1k)$$

In (2.1), $\mathbf{I}_{d_r}^G = \Delta\mathbf{I}_{d_r}^G + \mathbf{I}_{d_0}^G$, and $\mathbf{I}_{q_r}^G = \Delta\mathbf{I}_{q_r}^G + \mathbf{I}_{q_0}^G$, where $\mathbf{I}_{d_0}^G$ are initial condition of GFL current states. The model is initialised at a representative operating point, with all state initial conditions derived from load-flow results. The modular representation of a GFL block is given in Figure 2.3 where the fixed parameters, tunable parameters, and states are shown in blue, red and green respectively.

So, a nonlinear state equation for GFL model would be $\dot{\mathbf{x}}^G = f_G(\mathbf{x}^G, \mathbf{u}^G, \mathbf{r}^G)$, where $\mathbf{x}^G$, $\mathbf{u}^G$, and $\mathbf{r}^G$ can be read from Figure 2.3. The output equation for GFL, in the common network frame, is $\mathbf{y}^G = \mathbf{V}_{dq}^G e^{j\theta^G}$. One GFL comprises of 11 states, 6 tunable parameter, and 5 fixed

parameters. If $N_G = |G|$, the total number of states, tunable parameters, and fixed parameters would be $N_G \times 11$, $N_G \times 6$, and $N_G \times 5$, respectively. Each GFL having 11 states, 6 tunable parameter $(K_{P_d}^{gk}, K_{P_q}^{gk}, K_{I_d}^{gk}, K_{I_q}^{gk}, K_{P_\theta}^{gk}, K_{I_\theta}^{gk})$, and 5 fixed parameters $(\omega_0, C_f^{gk}, \tau_\theta^{gk}, \tau_c^{gk}, \tau_m^{gk})$.

2.2 Grid-Forming Inverter

For GFM dynamic model, M is the set of all buses identified as a GFM bus. Figure 2.4 shows that, instead of a PLL tracking the synchronously rotating frame, a power-frequency droop control sets the voltage angle. The role of voltage control is to set the current reference for the the control which then produces the modulation signal for the converter.

Figure 2.5 shows the dynamic model as defined by (2.2). Similar to GFL modelling, (2.2a) and (2.2b) represents the $C_f^{m_k}$ dynamics, and (2.2c) is the oscillator state equation. Here also, current control is simplified into a first order lag, characterised by its closed loop bandwidth ($\equiv 1/2\pi\tau_c^{m_k}$) and is defined by (2.2d), (2.2e). The state equations governing d - and q -axes

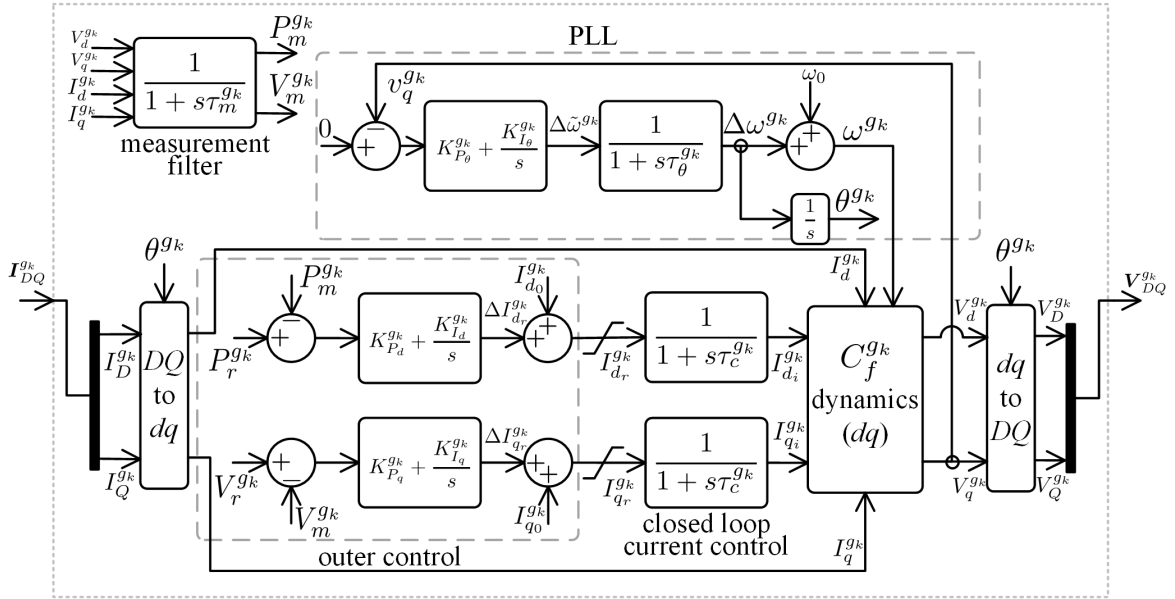


Figure 2.2: GFL model block diagram

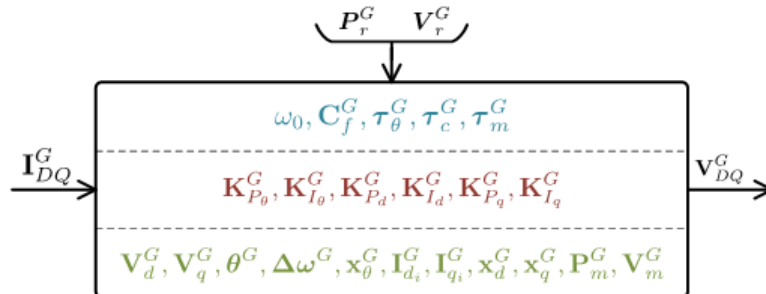


Figure 2.3: GFL modular representation

control (2.2f), (2.2g), set the corresponding reference current, $I_{dq_r}^{m_k}$ through (2.3a), (2.3b), where $x_d^{m_k}$, and $x_q^{m_k}$ are the integrator states of PI controllers of d - and q -axis, respectively. Lastly, (2.2h) is the measurement filter lag characterised by a fixed bandwidth ($\equiv 1/2\pi\tau_m^{m_k}$). For GFM too, the control design process is discussed in Chapter 5.

$$\dot{\mathbf{V}}_d^M = (\mathbf{I}_{d_i}^M - \mathbf{I}_d^M)/C_f^M + (\omega_0 + \Delta\omega^M)\mathbf{V}_q^M \quad (2.2a)$$

$$\dot{\mathbf{V}}_q^M = (\mathbf{I}_{q_i}^M - \mathbf{I}_q^M)/C_f^M - (\omega_0 + \Delta\omega^M)\mathbf{V}_d^M \quad (2.2b)$$

$$\dot{\theta}^M = \mathbf{D}_{P\omega}^M(\mathbf{P}_r^M - \mathbf{P}_m^M) = \Delta\omega^M \quad (2.2c)$$

$$\dot{\mathbf{I}}_{d_i}^M = (\mathbf{I}_{d_r}^M - \mathbf{I}_{d_i}^M)/\tau_c^M \quad (2.2d)$$

$$\dot{\mathbf{I}}_{q_i}^M = (\mathbf{I}_{q_r}^M - \mathbf{I}_{q_i}^M)/\tau_c^M \quad (2.2e)$$

$$\dot{\mathbf{x}}_d^M = (\Delta\mathbf{I}_{d_r}^M - \mathbf{x}_d^M \mathbf{K}_{I_d}^M)/\mathbf{K}_{P_d}^M = \mathbf{V}_r^M - \mathbf{V}_d^M \quad (2.2f)$$

$$\dot{\mathbf{x}}_q^M = (\Delta\mathbf{I}_{q_r}^M - \mathbf{x}_q^M \mathbf{K}_{I_q}^M)/\mathbf{K}_{P_q}^M = 0 - \mathbf{V}_q^M \quad (2.2g)$$

$$\dot{\mathbf{P}}_m^M = (\mathbf{V}_d^M \mathbf{I}_d^M + \mathbf{V}_q^M \mathbf{I}_q^M - \mathbf{P}_m^M)/\tau_m^M \quad (2.2h)$$

Following algebraic equations directly follow from the block diagram of Figure 2.5.

$$\mathbf{I}_{d_r}^M = (\omega_0 + \Delta\omega^M)\mathbf{C}_f^M \mathbf{V}_q^M + \Delta\mathbf{I}_{d_r}^M - \mathbf{I}_d^M \quad (2.3a)$$

$$\mathbf{I}_{q_r}^M = (\omega_0 + \Delta\omega^M)\mathbf{C}_f^M \mathbf{V}_d^M + \Delta\mathbf{I}_{q_r}^M + \mathbf{I}_q^M \quad (2.3b)$$

So, collectively for all GFMs a nonlinear state equation is given by $\dot{\mathbf{x}}^M = f_M(\mathbf{x}^M, \mathbf{u}^M, \mathbf{r}^M)$, where $\mathbf{x}^M$, $\mathbf{u}^M$, and $\mathbf{r}^M$ are mentioned in Figure 2.6, where the fixed parameters, tunable

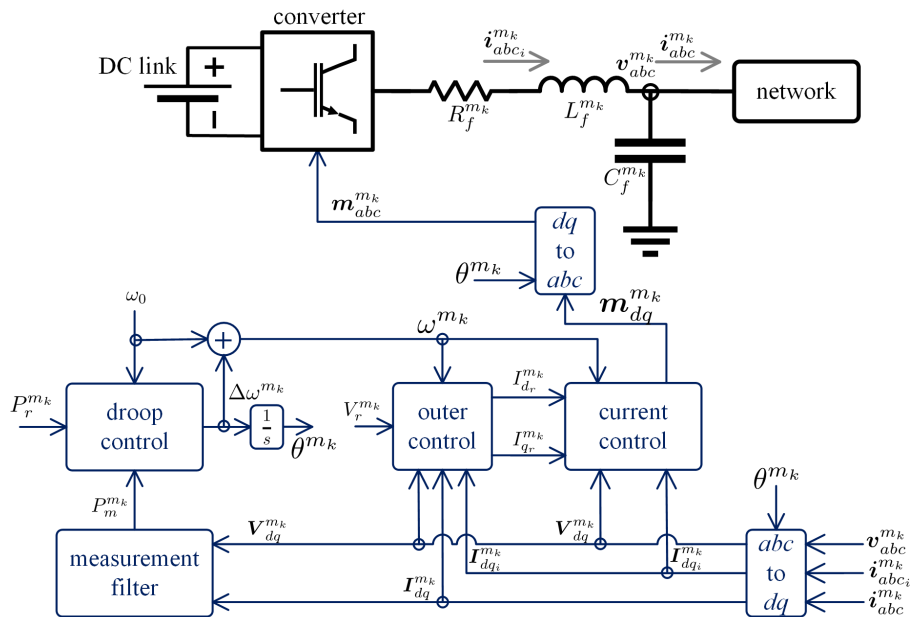


Figure 2.4: Control blocks for a GFM with an LC filter

parameters, and states are shown in blue, red and green respectively.. The output of the module is obtained in the common network frame i.e., $\mathbf{y}^M = \mathbf{V}_{dq}^M e^{j\theta^M}$.

If $N_M = |M|$, the total number of states (green) for GFM model are be $N_M \times 8$. Similarly, the number of fixed (blue) and tunable (red) parameters are $N_M \times 4$, and $N_M \times 5$, respectively. Each GFM having 8 states, 5 tunable parameter ($K_{P_d}^{m_k}, K_{P_q}^{m_k}, K_{I_d}^{m_k}, K_{I_q}^{m_k}, D_{P_\omega}^{m_k}$), and 4 fixed parameters ($\omega_0, C_f^{m_k}, \tau_c^{m_k}, \tau_m^{m_k}$).

The current limiters in both GFL and GFM can switch between active power (P) and reactive power (Q) priority modes. However, as most simulations focus on small-signal analysis, these limiters typically have no observable impact on the time-domain plots—unless the system becomes unstable.

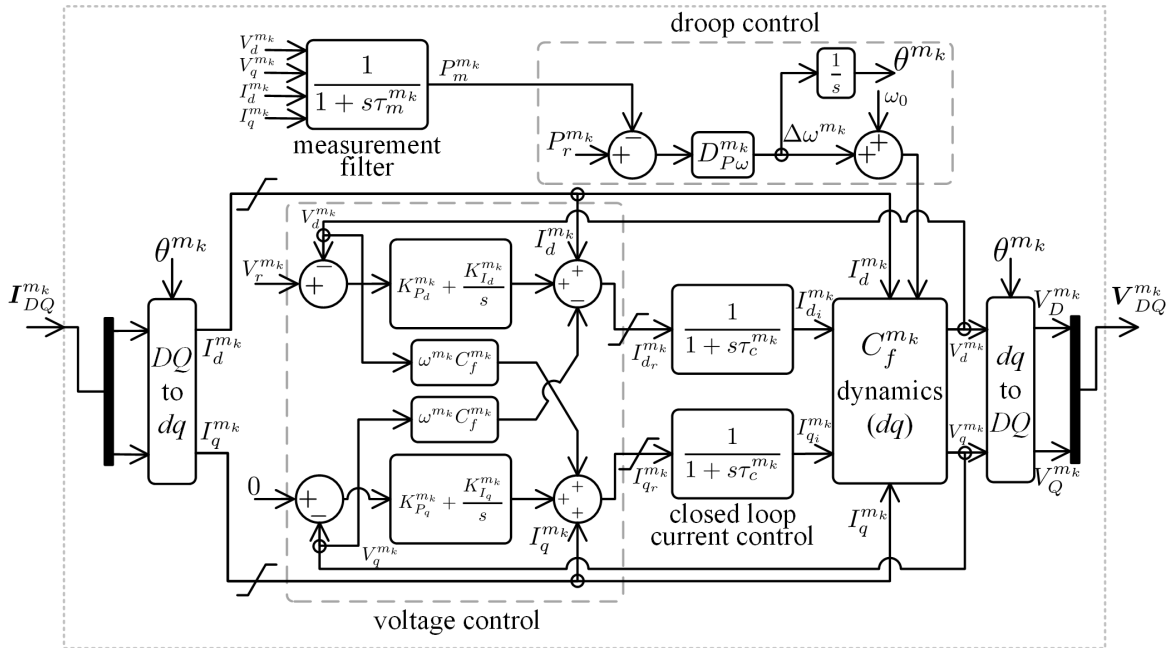


Figure 2.5: GFM model block diagram

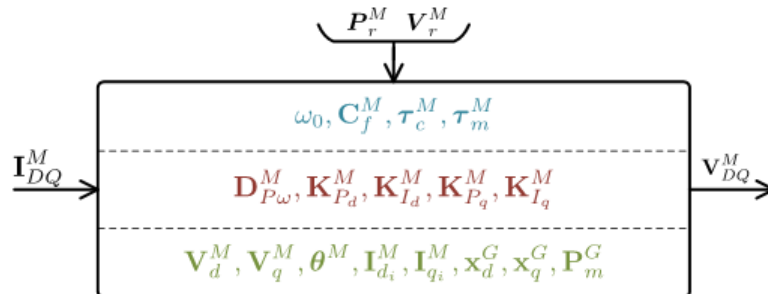


Figure 2.6: GFM modular representation

2.3 Loads

The ZIP model is used to represent all loads connected to load buses grouped in L . Owing to the much slower electromechanical and thermal time-constants of aggregated loads, their internal dynamics are neglected. Loads are therefore modelled as static ZIP elements that contribute algebraic power injections, introducing no additional differential states in the small-signal model. This assumption is valid for the SSO investigated, where load states vary quasi-statically compared with inverter and network dynamics. However, the cumulative line charging susceptance of the bus to which they are connected provides following two state equations:

$$\dot{\mathbf{V}}_D^L = (\mathbf{I}_{D_i}^L - \mathbf{I}_D^L)/\mathbf{C}_l^L + \omega_0 \mathbf{V}_Q^L \quad (2.4a)$$

$$\dot{\mathbf{V}}_Q^L = (\mathbf{I}_{Q_i}^L - \mathbf{I}_Q^L)/\mathbf{C}_l^L - \omega_0 \mathbf{V}_D^L \quad (2.4b)$$

For each element of the ZIP model (either constant Z , I , or P), the current consumed by the loads is governed by following set of algebraic equations.

$$\mathbf{I}_{D_i}^L = -(\tilde{\mathbf{P}}^L \mathbf{V}_D^L + \tilde{\mathbf{Q}}^L \mathbf{V}_Q^L)/(\mathbf{V}_D^{L^2} + \mathbf{V}_Q^{L^2}) \quad (2.5a)$$

$$\mathbf{I}_{Q_i}^L = (\tilde{\mathbf{Q}}^L \mathbf{V}_D^L - \tilde{\mathbf{P}}^L \mathbf{V}_Q^L)/(\mathbf{V}_D^{L^2} + \mathbf{V}_Q^{L^2}) \quad (2.5b)$$

where,

$$\tilde{\mathbf{P}}^L = \mathbf{P}^L (|\mathbf{V}_{DQ}^L|/\mathbf{V}_{DQ_0}^L)^{\Lambda_P^L} \quad (2.6a)$$

$$\tilde{\mathbf{Q}}^L = \mathbf{Q}^L (|\mathbf{V}_{DQ}^L|/\mathbf{V}_{DQ_0}^L)^{\Lambda_Q^L} \quad (2.6b)$$

In (2.6) $\Lambda_{PQ}^L \in \{0, 1, 2\}$ that dictates the load type i.e., $\Lambda_{PQ}^{l_k} = 0$ for constant power load; $\Lambda_{PQ}^{l_k} = 1$ for constant current load; and $\Lambda_{PQ}^{l_k} = 2$ for constant impedance load. The total capacitance at every bus where load is connected is $C_l^{b_k} = 0.5 \sum_{z \in Z} C_l^{\delta_z}$, where Z contains the indices of lines connected to bus k and $\mathbf{C}_l^L$ is the vector of net capacitance of all buses connected to loads.

Thus, a nonlinear state equation for loads is $\dot{\mathbf{x}}^L = f_L(\mathbf{x}^L, \mathbf{u}^L, \mathbf{r}^L)$, whereas the output equation, without any rotation, is $\mathbf{y}^L = \mathbf{V}_{DQ}^L$. If $N_M = |M|$. Its modular form is shown in Figure 2.7 indicating that for $N_L = |L|$, number of load states contributed to the full system is $N_L \times 2$, while number of fixed parameters is $N_L \times 1$. Because λ is a fixed coefficient that characterises the intrinsic type of the aggregated load rather than a controllable variable, the load model contains no tunable parameters at the system level.

2.4 Floating Buses

Floating buses do not have any active component connected. Therefore, they neither have current injection nor absorption. So the dynamics are solely depending on the current flowing into line charging susceptance and are linear, as shown in (2.7):

$$\dot{\mathbf{V}}_D^F = -\mathbf{I}_D^F/\mathbf{C}_l^F + \omega_0 \mathbf{V}_Q^F \quad (2.7a)$$

$$\dot{\mathbf{V}}_Q^F = -\mathbf{I}_Q^F/\mathbf{C}_l^F - \omega_0 \mathbf{V}_D^F \quad (2.7b)$$

$\mathbf{C}_l^F$ is the vector of net capacitance of all buses classified as floating buses. So, the linear state equation for floating buses is $\dot{\mathbf{x}}^F = f_F(\mathbf{x}^F, \mathbf{u}^L)$ with $N_L \times 2$ states and $N_L \times 1$ fixed parameters (for $N_F = |F|$), as shown in Figure 2.7. Just like loads, the output variables of floating bus module are already in network frame and are given by $\mathbf{y}^F = \mathbf{V}_{DQ}^F$.

There may be cases where two transmission lines terminate at the same bus where no other active device (generator or load) is connected. In such situations, the framework treats the combined shunt charging as a single state by aggregating the susceptance at that node. This handling has been explicitly hardcoded into the modelling framework.

2.5 Branches

Branch models operate on the voltage differentials of two buses they connect and give two current states, as shown in (2.8):

$$\dot{\mathbf{I}}_D^\Delta = (\mathbf{V}_D^\Delta - \mathbf{I}_D^\Delta \mathbf{R}_l^\Delta)/\mathbf{L}_l^\Delta + \omega_0 \mathbf{I}_Q^\Delta \quad (2.8a)$$

$$\dot{\mathbf{I}}_Q^\Delta = (\mathbf{V}_Q^\Delta - \mathbf{I}_Q^\Delta \mathbf{R}_l^\Delta)/\mathbf{L}_l^\Delta - \omega_0 \mathbf{I}_D^\Delta \quad (2.8b)$$

All branches are characterised by both resistive and inductive components and are shown in Figure 2.7. Figure 2.7 comprises of $N_\Delta \times 2$ fixed parameters, and $N_\Delta \times 2$ states. So $\dot{\mathbf{x}}^\Delta = f_\Delta(\mathbf{x}^\Delta, \mathbf{u}^\Delta)$ is the state equation for branch module. Its outputs are its states and hence is given by $\mathbf{y}^\Delta = \mathbf{x}^\Delta = \mathbf{I}_{DQ}^\Delta$.

2.6 Full System

Combining all these modules gives us a scalable dynamic model for zero-carbon power systems equipped with theoretically any number of GFL and GFM converters. Figure 2.8 shows the generalisable circuit diagram where all modules are shown as circuit elements. Their possible

electrical interconnections are shown with dotted lines. The solid bus bars in Figure 2.8 are virtual buses that depict the concept of clubbing all buses with same module type regardless of their bus index number.

In Figure 2.9, modular representation of full system is shown where the outputs of GFM, GFL, loads, and floating buses are stacked, reordered by operator $\mathcal{R}$ and then multiplied by the transpose of incidence matrix, T , to obtain the input of branch module, as shown in (2.9).

$$\mathbf{T}' \times \mathcal{R}([\mathbf{y}^G, \mathbf{y}^M, \mathbf{y}^L, \mathbf{y}^F]') = \mathbf{V}_{DQ}^\Delta \quad (2.9)$$

So, a nonlinear state equation for floating buses would be $\dot{\mathbf{x}} = f(\mathbf{x}, \mathbf{u})$. Linearising it around operating point would yield, $\Delta \dot{\mathbf{x}} = A\Delta \mathbf{x} + B\Delta \mathbf{u}$

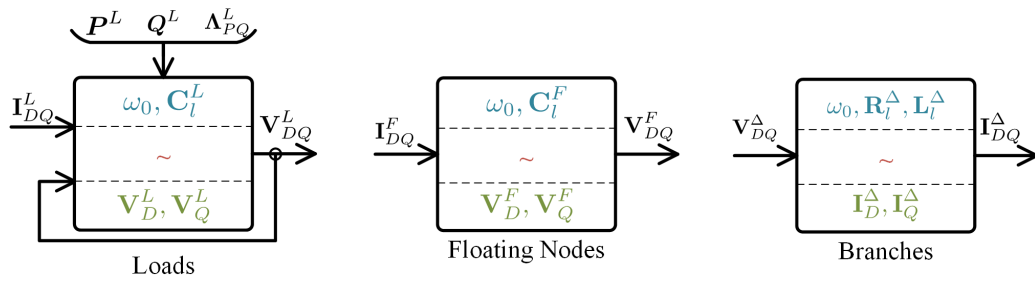


Figure 2.7: Modular representations of loads, floating nodes, and branches. These modules do not have tunable parameters. Fixed parameters and states are shown in blue and green, respectively.

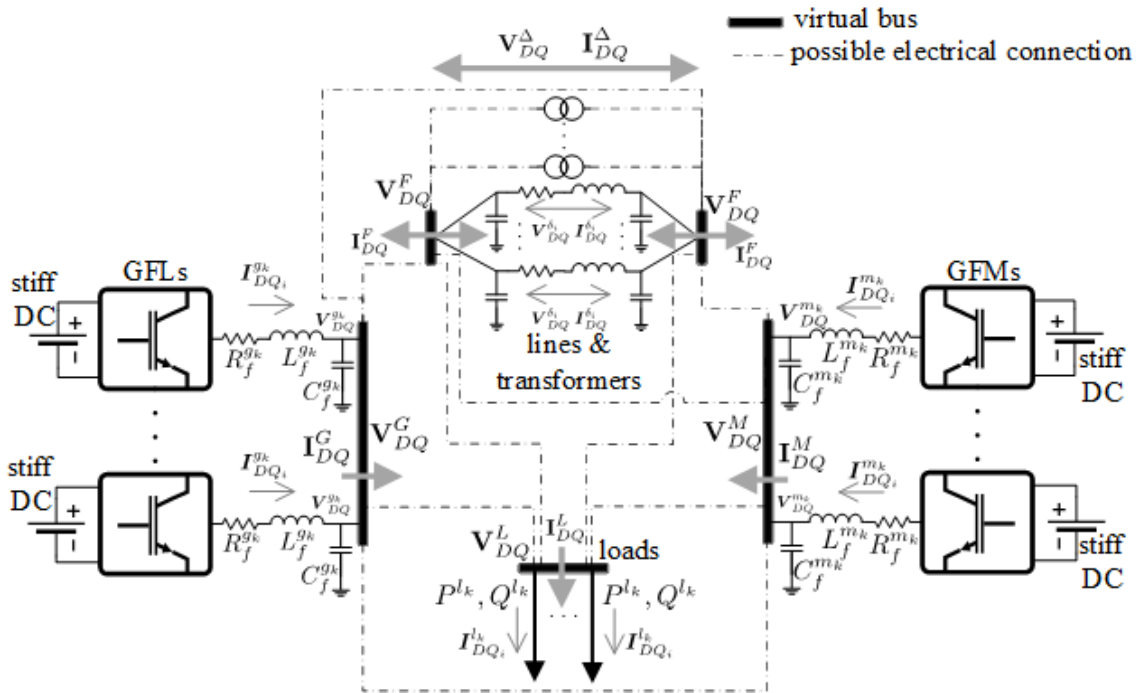


Figure 2.8: Circuit diagram with all modelled components where similar components are combined at virtual buses (solid line) through their possible interconnections (dashed line).

Chapter 3

Modelling Adequacy for Sub-Synchronous Oscillation Analysis

This chapter investigates the modelling requirements necessary to accurately capture SSO, with particular emphasis on evaluating the role of dynamic network representation in the detection and analysis of SSO in IBR-dominated systems. To ensure clarity and consistency throughout this chapter, let's first define the modelling frameworks used in this chapter. The dynamic model developed in Chapter 2, which captures both electromagnetic and control dynamics in the dq -domain, is referred to as the EMT- dq . This model preserves the dynamic behaviour of the network. However, when the network dynamics expressed in (2.7) and (2.8) are simplified to algebraic constraints the resulting model is termed the positive sequence RMS (henceforth denoted as RMS+). This model retains a quasi-steady-state representation. For benchmarking and validation, both EMT- dq and RMS+ models are compared with the EMT- abc (point-on-wave) model, which serves as the highest fidelity representation based on three-phase electromagnetic transient simulations.

While both EMT- abc and EMT- dq models preserve the dynamic behaviour of the network, they differ in how the system variables are represented and integrated. The EMT- abc model operates directly in the time-domain using three-phase (point-on-wave) signals, capturing all electromagnetic phenomena including harmonics, unbalances, and switching events. In contrast, the EMT- dq model applies a rotating reference frame transformation under the assumption of balanced conditions, leading to a time-invariant representation of dynamic phasors for each component. This transformation simplifies the system dynamics, reducing the computational burden while still retaining the essential frequency components needed for SSO studies. As discussed in [24], EMT- dq and EMT- abc are theoretically equivalent for balanced systems and

narrow-band phenomena, but differences may arise due to implementation choices, especially in the treatment of initialization routines, and numerical integration schemes. Therefore, in this work, EMT-*abc* is used as the benchmark for high-fidelity validation, while EMT-*dq* is portrayed as a computationally efficient yet suitable for analytical and numerical SSO studies, both in time and frequency domains. Moreover, the only difference between RMS+ and EMT-*dq* lies in the network representation: RMS+ uses algebraic equations for both shunt susceptance and *RL* branches, while EMT-*dq* retains their full dynamic differential equations.

3.1 Modelling Tools Landscape

The conventional RMS+ modelling approach relies on the assumption of a clear time-scale separation between slow electromechanical dynamics (e.g., generator rotor swings, frequency deviations) and fast electromagnetic transients (e.g., line charging, current waveforms). This separation allows electromagnetic effects to be neglected or simplified as algebraic constraints, enabling faster simulations and analytical tractability. However, in IBR-dominated systems, this distinction becomes blurred, particularly when studying SSO. Unlike synchronous machines, IBRs operate through tightly regulated fast control loops that interact directly with the grid at frequencies well above traditional electromechanical modes. As a result, some IBR-driven SSOs may manifest within frequency ranges that lie between the classical electromagnetic and electromechanical domains. This overlapping of time scales challenges the core assumption underpinning RMS+ models and necessitates dynamic network representations to accurately capture the coupled behaviour between IBR control and grid impedance across relevant frequency bands. While RMS+ might be inadequate for SSO, point-on-wave EMT simulation (henceforth denoted as EMT-*abc*) could be overly complex and computation-intensive, particularly when analysing system-wide SSO. This complexity limits the number of scenario checks possible and does not provide much insight into the oscillation pathway. Hence, a balanced strategy is required to maintain accuracy while enabling a large number of scenario assessments.

In the literature, power system models are broadly classified into two categories based on how the network is represented [24]: 1) the Quasi-Static Phasor (QSP) with an algebraic network model [25] and 2) electromagnetic transients with dynamic network model [26]. Both QSP and EMT models can be further divided into sub-categories, each tailored to cover different time scales and fidelity with different trade-offs between accuracy and computational burden [24].

The RMS simulation models are the most common type of QSP models that use Differential Algebraic Equation (DAE) with only positive-sequence considering balanced operation [27].

The RMS modelling is widely used in planning tools such as PTI PSS/E, GE PSLF, PowerFactory, PowerWorld, TSAT etc. High-fidelity point-on-wave simulation captures fast electromagnetic phenomena with sub-cycle granularity as in EMTP, PSCAD, PLECS, Simulink Simscape etc. EMT-*abc* requires special initialization and implicit integration techniques [24]. They typically use a detailed representation of transmission lines such as the Bergeron or other frequency-dependent models and converter switching dynamics [26]. An EMT-*abc* simulation, even with an averaged converter model and lumped line representation, involves a heavy computational burden and hence slow simulation speed [28]. As a result, only a limited number of scenarios can be studied in EMT-*abc*, especially if the problem is system-wide rather than localized. Additionally, EMT-*abc* does not readily lend to frequency-domain analysis for insight into the stability problem for effective mitigation.

Considering balanced operation and narrow-band signals for small-signal stability (scope of this thesis), the EMT-*abc* model can be transformed into an EMT-*dq* model in the synchronously rotating reference frame without the need for dynamic phasors [29, 24]. The EMT-*dq* model is initialized in the same way as RMS+ but EMT-*dq* solves a set of ordinary differential equations including network dynamics while RMS+ solves a set of differential-algebraic equations. The transformation from three-phase *abc* to the *dq* typically leads to a simpler and better-conditioned system by eliminating the need to track explicit phase variables at the fundamental frequency that reduces overall stiffness. This reduction in stiffness allows larger

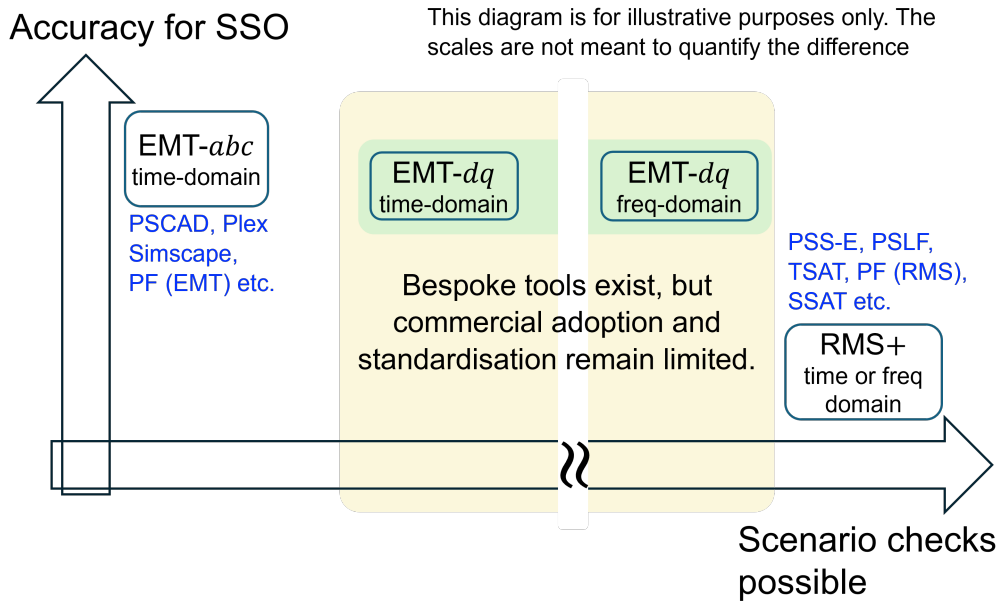


Figure 3.1: EMT-*dq* model as a middle ground between EMT-*abc* and RMS+ to balance accuracy for IBR-driven SSO (y-axis) and the number of scenario checks possible (x-axis)

time steps or fewer iterations per step [30]. The rotating frame can further exploit the system's structure via smaller and more easily factorized block matrices. In addition, EMT- dq often employs a simultaneous-implicit solution [31], which can be more efficient than the implicit numerical integration used in EMT- abc [32]. Thus, both the improved conditioning in the dq reference frame and the simultaneous-implicit integration technique provide EMT- dq the speed advantage over EMT- abc .

RMS+ tools are routinely employed for planning studies which includes preliminary stability assessments for various scenarios. The industry practice is to use appropriate screening methods to identify cases that require EMT- abc studies [33]. While screening methods are well-established for localized phenomena, their effectiveness in detecting wide-area SSO risks is limited. As a result, it is challenging to identify in advance which wide-area SSO-prone scenarios warrant full EMT- abc simulation. This practice can be improved if a computationally efficient alternative such as EMT- dq can deliver similar accuracy as EMT- abc in the sub-synchronous range to allow large number of scenario checks.

The role of network dynamics in IBR-dominated power systems is established. However, the underlying mechanism of the interaction of a GFL or GFM with the network is yet to be fully understood. Reference [34] highlights that in synchronous generator-dominated systems, network dynamics can lead to differences between EMT- dq and QSP/RMS+ for poorly damped high-frequency modes. However, it does not address interactions between IBRs across the network. Reference [35] considered GFM as a virtual synchronous machine [36] but the findings were not generalized for inter-IBR interaction and were not benchmarked against EMT- abc . The approach in [37] was to obtain the IBR control settings for a given operating point where QSP/RMS+ achieves similar accuracy as EMT- dq . However, this leads to a conservative control design even at relatively low IBR penetration. Since [37] considers a GFM at a low Short Circuit Ratio (SCR) where it is inherently stable, the role of network dynamics at more onerous high SCR conditions was not covered. Although [24] demonstrated that EMT- dq achieves similar levels of accuracy as EMT- abc for SSO, it is focused on GFM only and lacks frequency-domain insight into the interaction between the IBRs and the network.

Reference [38] provides a mathematically rigorous approach to finding the bounds on GFM (with virtual oscillator [39]) control gains and reference set-points for global asymptotic stability in the presence of a dynamic network model. However, it stops short of explaining the underlying reason that makes network dynamics crucial in analyzing SSO. Reference [40] claims that network dynamics do not impact stability assessments of GFMs with virtual synchronous generator (VSG) control but recognizes that this is not necessarily the case for GFLs and GFM

with other types of control (e.g., droop). As most IBRs currently operate as GFL and not many, let alone all, would use GFM with VSG control in the future, the network dynamics will influence IBR-driven SSO.

This chapter highlights that neglecting network dynamics could result in false stability assurance, especially at low SCR for GFL and high SCR for GFM while the actual SSO response is either poorly damped or even unstable. The analysis and insight into how network dynamics play a crucial role in SSO that renders RMS studies inadequate and necessitating the use of EMT is provided. As EMT- abc simulations are computationally intensive, EMT- dq provides an effective intermediate solution with comparable accuracy to EMT- abc (as in PSCAD) for SSO while enabling more scenario checks and offering valuable insight via frequency-domain analysis. This is illustrated in Figure 3.1 and substantiated throughout the rest of the chapter.

3.2 Benchmarking EMT- dq and EMT- abc

To validate the modelling approach presented in Chapter. 2, it is benchmarked against EMT- abc simulation. The test system of Figure 3.2 was used for benchmarking, where an IBR is connected to a large power system through a 33/400 kV transformer and a 400 kV transmission line. The IBR controls the active power at 600 MW while maintaining 33 kV at the PCC. The base case network parameters in pu are given in Table 3.3 for a 600 MVA system base. The tunable and fixed IBR parameters are provided in Table 3.1 and 3.2, respectively at the end of this chapter, together with the parameters of other case studies. GFL control gains are designed using the equations detailed in Chapter 5.2.3. Similarly, GFM control gains are designed using the process explained in Chapter 5.3.1.

Figure 3.3(a) corresponds to a GFL operating at an SCR of 1.3 and Figure 3.3(b) corresponds to a GFM operating at an SCR of 4.7. The disturbance is a step reduction in IBR voltage reference from 1.0 pu to 0.95 pu at 1.1 s. The responses from EMT- abc and EMT- dq virtually

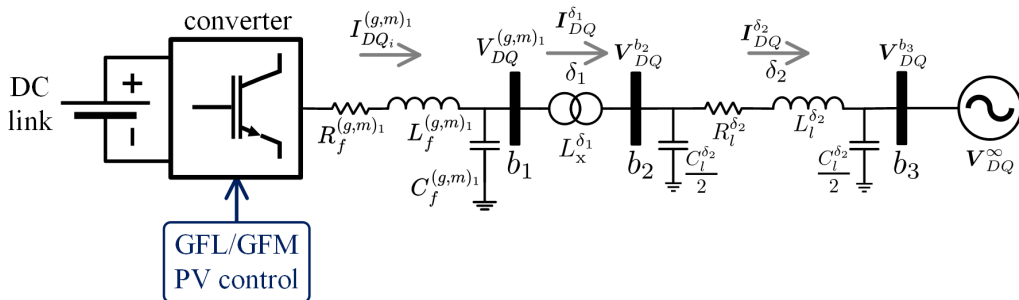


Figure 3.2: Test system to benchmark EMT- dq against EMT- abc simulations

overlap and confirm that EMT- dq captures the SSO adequately. Corresponding RMS+ traces are also shown where oscillations are not fully captured. The EMT- abc simulation is carried out in PSCAD with a step size of $50 \mu s$. The step size for EMT- dq is also $50 \mu s$. However, for RMS+, the step size is 0.1 ms which is one-third of the fastest time constant in the model (i.e., current control at $\tau_c = 0.3 \text{ ms}$), sufficient for numerical accuracy. However, with faster dynamics, the step size must be reduced accordingly to avoid integration-error buildup.

Figure 3.4 presents the frequency-domain validation of the EMT- dq model against EMT- abc using a dynamic frequency scan approach for both GFL (Figure 3.4(a)) and GFM (Figure 3.4(b)). To validate the EMT- dq response, sinusoidal perturbations were injected into active power reference (P_r) and voltage reference (V_r) at discrete frequencies ranging from 1 Hz to

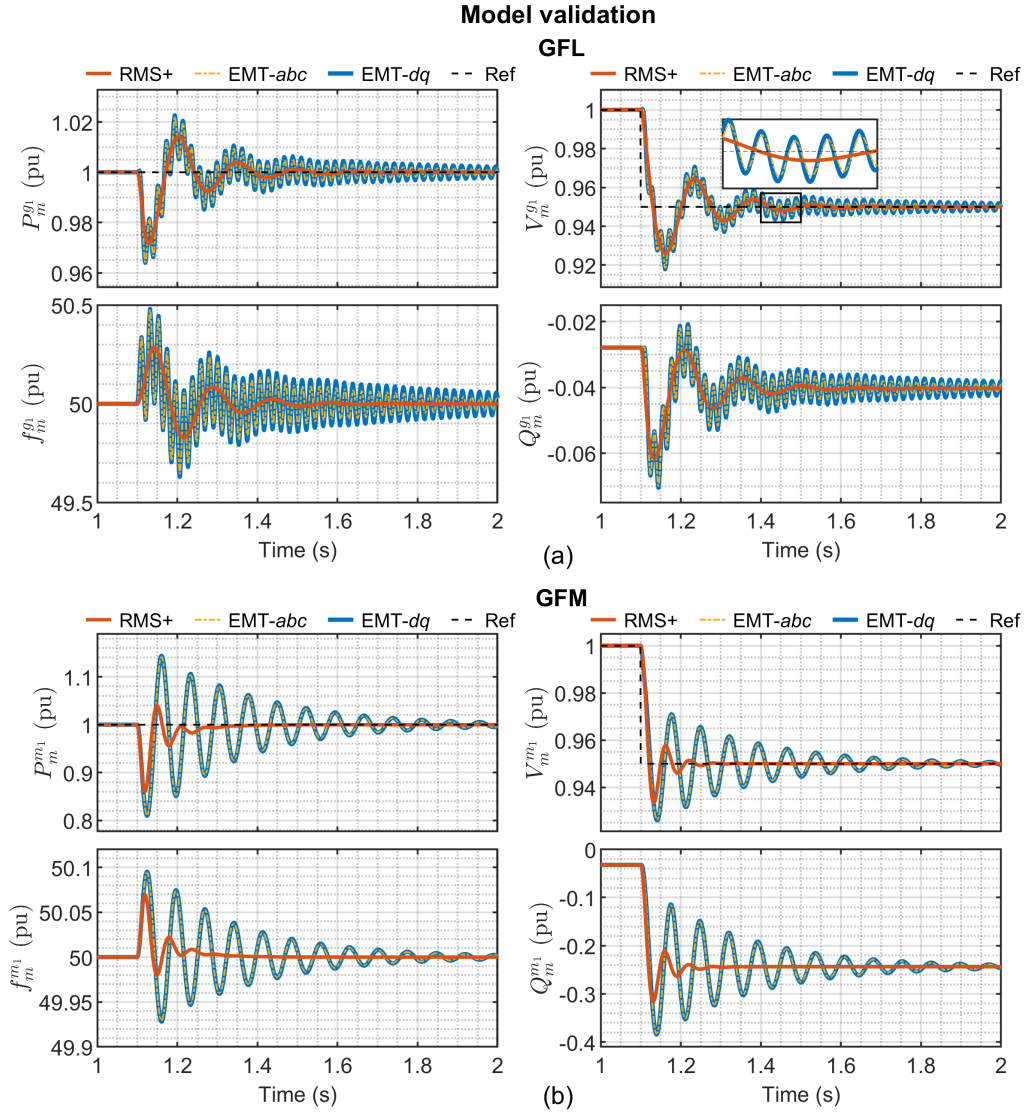


Figure 3.3: Benchmarking dynamic responses from EMT- dq (blue) against EMT- abc (yellow) and RMS+ (red) for IBR in Figure 3.2 as (a) GFL and (b) GFM

70 Hz in the EMT- abc model. The output signals, measured as active power (P_m) and voltage (V_m), were then extracted, and their magnitude and phase were overlaid on the Bode plot of the linearized EMT- dq model. The results demonstrate strong agreement between EMT- dq (blue) and EMT- abc (yellow markers) across the full frequency range. Notably, the EMT- dq model accurately captures the peak corresponding to the less-damped mode in the sub-synchronous frequency range. In contrast, RMS+ (red) deviates beyond 10 Hz. Since RMS+ does not accurately capture dynamic interactions above 10 Hz, it can not be a reliable tool to assess subsynchronous small-signal stability in IBR-dominated power systems.

To quantitatively support the claim that EMT- dq reduces computational burden compared to EMT- abc , the execution time required to run a time-domain simulation for both modelling approaches is evaluated. Figure 3.5 presents the ratio of execution time taken by EMT- abc

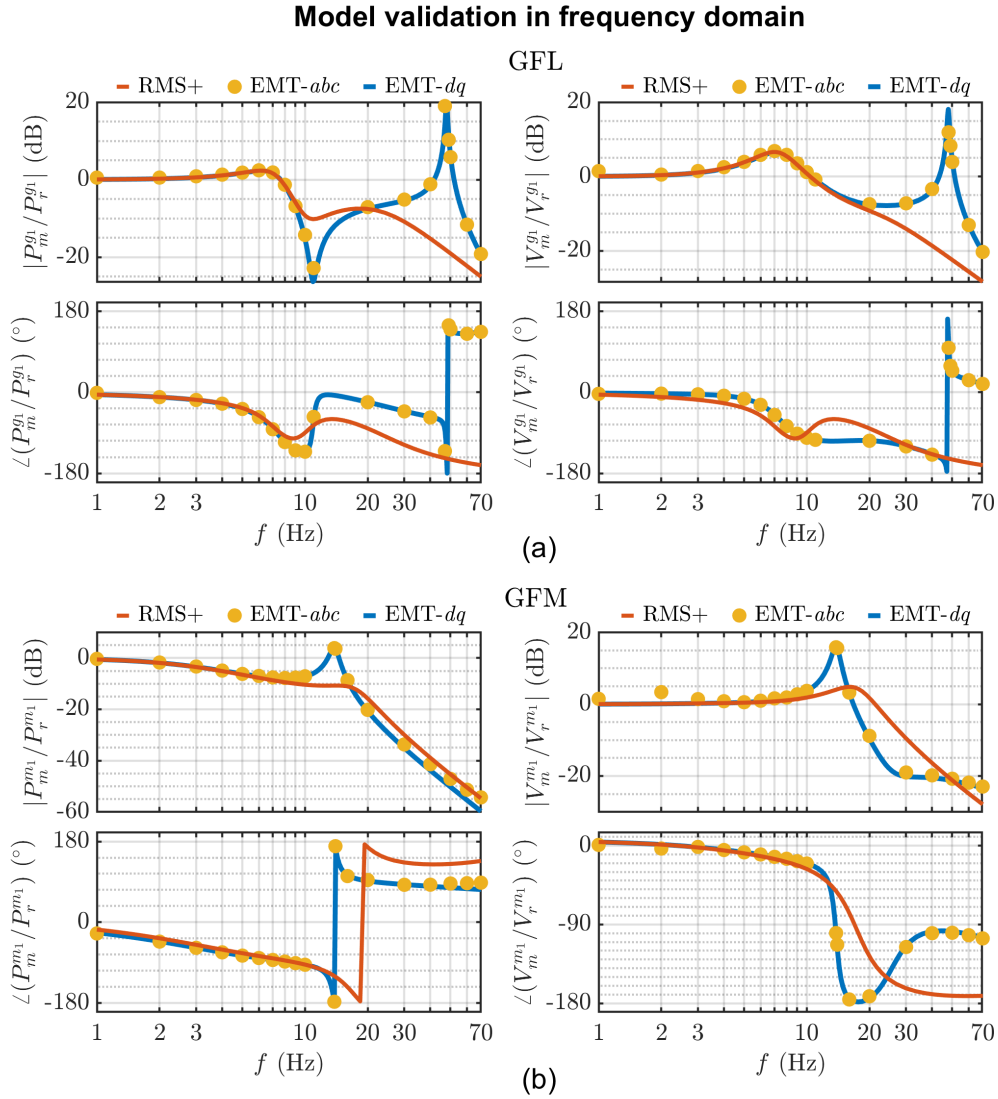


Figure 3.4: Benchmarking frequency responses from EMT- dq (blue) against EMT- abc (yellow) and RMS+ (red) for for IBR in Figure 3.2 as (a) GFL and (b) GFM

relative to EMT- dq for two test systems: a single IBR system (2-bus, Figure 3.6) and a two-IBR system (5-bus, Figure 3.13). The time ratio is computed for varying simulation times ranging from 0.1 s to 5 s. The results reveal that for the single IBR system, the time ratio starts at approximately 2 and increases to around 8 as the simulation time is increased. This trend highlights the growing computational advantage of EMT- dq over longer simulation periods. The effect becomes even more pronounced in the two-IBR system, where the time ratio starts at approximately 5 and increases to more than 20 at 5 s of simulation time. The increased complexity of the two-IBR system results in a higher computational overhead for EMT- abc . The rightmost panel of Figure 3.5 provides a statistical summary of the time ratio for both systems. The median time ratio for the single-IBR system is around 6, whereas for the two-IBR system, it exceeds 18. These results substantiate that EMT- dq reduces computational cost, making it a viable alternative to EMT- abc for screening oscillatory scenarios in IBR-dominated systems.

These validation studies confirm that EMT- dq provides a computationally efficient yet accurate alternative to EMT- abc for the small-signal study of SSO under the conditions in the first paragraph of Section 3.1. This makes EMT- dq preferable for studying SSO if both system size (geographical spread of SSO) and the number of scenarios to be studied are high. Henceforth, time- and frequency-domain results with only EMT- dq are shown in this chapter. As the scope of this thesis is small-signal analysis of SSO, large disturbances (e.g., faults) are excluded from benchmarking, where EMT- dq may deviate from EMT- abc .

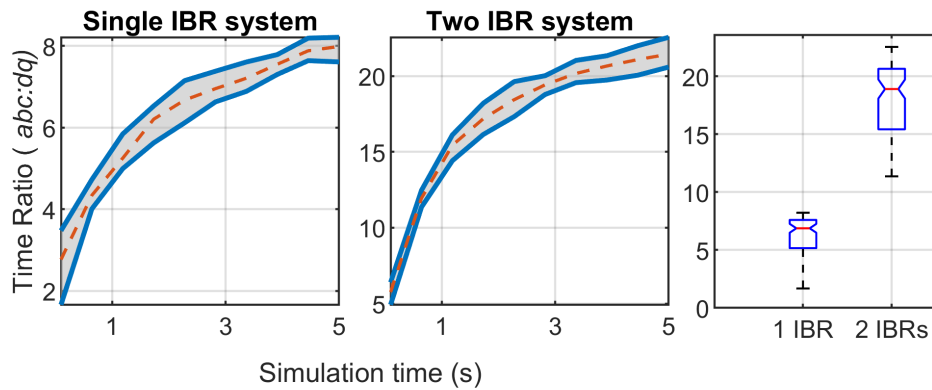


Figure 3.5: Computational time ratio of EMT- abc to EMT- dq on single (left) and two (middle) IBRs system

3.3 Interaction between IBRs and Network

The nature of the interaction between IBRs and the network is explained in this section for GFL and GFM separately, considering a single IBR - GFL or GFM - connected to a Thévenin

equivalent of a power system as shown in Figure 3.6. This simple case with one IBR is chosen to rule out inter-IBR oscillations and focus solely on the interplay between the IBR and the network states in an SSO mode.

3.3.1 Grid-Following Inverter

With a GFL in Figure 3.6, the mode shapes of the two SSO modes are shown in Figure 3.7. Such type of figures are introduced and explained in the Appendix, where a representative example is provided to illustrate how modal composition of an oscillatory mode is obtained, graphically represented and interpreted. Unlike the case with a GFL, the network states are not aligned with any GFM states. The GFL has been tuned in a way that one of the modes is poorly-damped so that modal composition of such mode can be investigated. The states are provided in Chapter 2.1. The area of each circle marker is proportional to the magnitude of the normalized participation factor of that state and the orientation is set by the angle of the corresponding right eigenvector (mode shape).

Figure 3.7(a) shows that for 41.9 Hz mode (left), the D -axis component of network current I_D^δ (marked red), oscillates in phase with the PLL oscillator state θ_g . Since the network current I_D^δ directly depends on PLL angle θ_g , their alignment reinforces the oscillation. Similarly, the filter capacitor state, V_d^g oscillates in-phase with the Q -axis component of network current, I_Q^δ . Without network dynamics, the significant in-phase contribution of the network states is not captured. The 2.6 Hz mode shown on the right in Figure 3.7(a) is not severely impacted since network states are not aligned with any of the GFL states and their participation is comparatively low. The corresponding dynamic response is shown in Figure 3.7(b).

For completeness of analysis and to see how the eigenvalue trajectories and time-domain response would appear in RMS+ tools, EMT- dq results are repeatedly compared with those

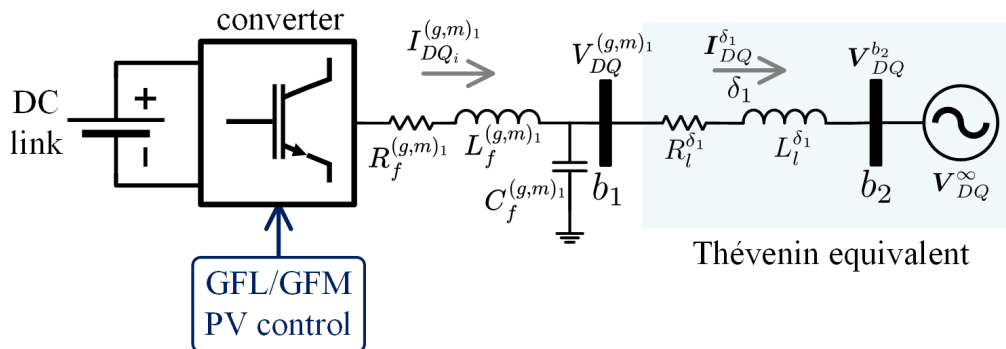


Figure 3.6: An IBR connected to a Thévenin equivalent of a power system

obtained with RMS+. The analysis of 41.9 Hz mode and its damping and frequency variation with grid strength (SCR) and PLL bandwidth is reported for EMT- dq and RMS+ in Figure 3.8 alongside the participation of network current in this mode. The vertical dotted line marks the actual stability threshold ($\text{SCR} = 1.57$) below which the damping ratio ζ_1 becomes negative based on EMT- dq . RMS+ shows higher damping across the range of SCR and fails to flag the instability below $\text{SCR} = 1.57$. The bottom-left subplot shows that the higher discrepancy in ζ_1 between EMT- dq and RMS+ at lower SCR is directly attributable to the increased participation of the network current I_D^δ .

The subplots on the right of Figure 3.8 show the effect of increasing the PLL bandwidth. In this particular case, the system becomes unstable beyond a PLL bandwidth of 18 Hz (vertical

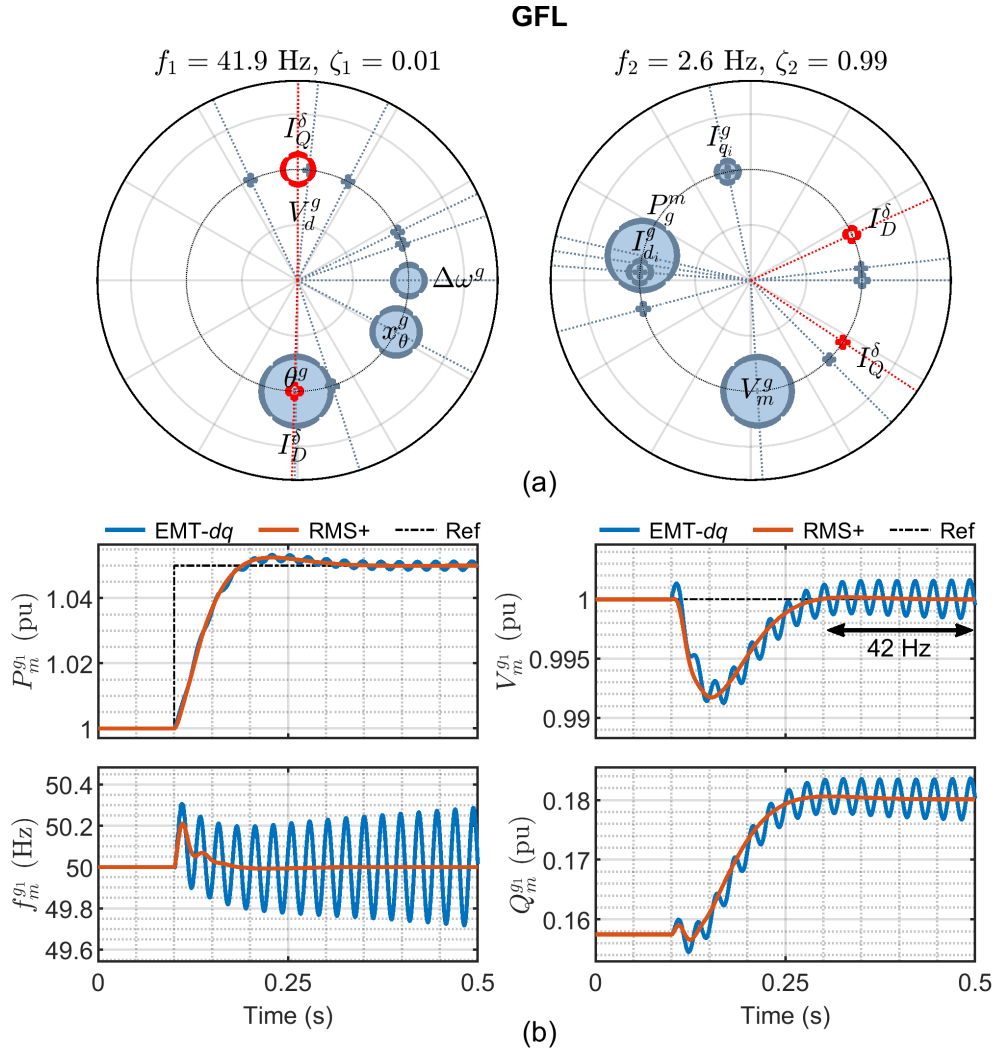


Figure 3.7: (a) Mode shapes showing GFL states (gray markers) and network states (red markers) dominant in SSO modes 41.9 Hz (left) and 2.6 Hz (right); (b) Dynamic response of active power, terminal voltage, frequency and reactive power where the dominant oscillation mode is triggered by increasing reference power to 1.05 pu

dotted line). This instability is not captured by the RMS+ model. As with SCR variation, this discrepancy at higher PLL bandwidth is associated with increased participation of the network current I_Q^δ .

3.3.2 Grid-Forming Inverter

With a GFM in Figure 3.6, the mode shapes of two SSO modes are shown in Figure 3.9(a). Here again, states can be understood by referring back to Chapter 2.2. However, they have higher participation as indicated by the size of the red markers in Figure 3.9. This suggests that network states are dominant contributors in these SSO modes. Figure 3.9(b) shows the corresponding dynamic response where the oscillation at a dominant mode of ~ 13 Hz is observed.

Figure 3.10 shows the damping ratio ζ_1 of 12.8 Hz mode for varying SCR (left) and GFM voltage control bandwidth (right). With GFM, the damping reduces at higher SCR before getting unstable above $\text{SCR} = 5.8$ (shown by the vertical dotted line). RMS+ model shows higher damping than EMT- dq across a range of SCR and fails to capture this instability. Similar to the case with GFL, the participation of network states increases as SCR is reduced (because of higher L_l^δ), which increases the difference in damping ratio with EMT- dq and RMS+. The actual damping captured by EMT- dq is much less than that suggested by the RMS+ model especially, at lower SCRs. For higher GFM voltage control bandwidth (right side of Figure 3.10), the participation of network states decreases and so does the difference between the damping with EMT- dq and RMS+.

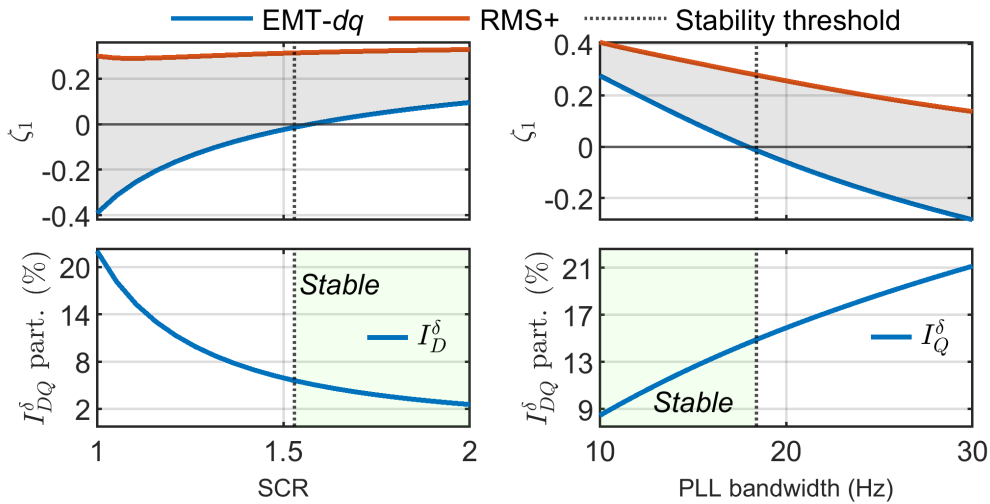


Figure 3.8: Damping ratio (top) and network state participation (bottom) for the dominant SSO mode (41.9 Hz in base case) with varying SCR (left), and PLL bandwidth in Hz (right)

3.4 Test Cases

In this section, the accuracy of EMT- dq in capturing SSO is demonstrated through mode trajectories and time-domain response on various test systems. For completeness, the corresponding RMS+ response is also shown which is unable to capture the SSO above 10 Hz, as expected. The response traces obtained using EMT- dq closely match those from EMT- abc , as demonstrated in Section 3.2. Therefore, EMT- dq is used throughout the remainder of the thesis for all SSO-related studies, with EMT- abc omitted for clarity. This choice is based on the validation in Section 3.2, which confirms that EMT- dq accurately captures sub-synchronous dynamics of interest. No simplifications were made in the control loops of GFL and GFM when

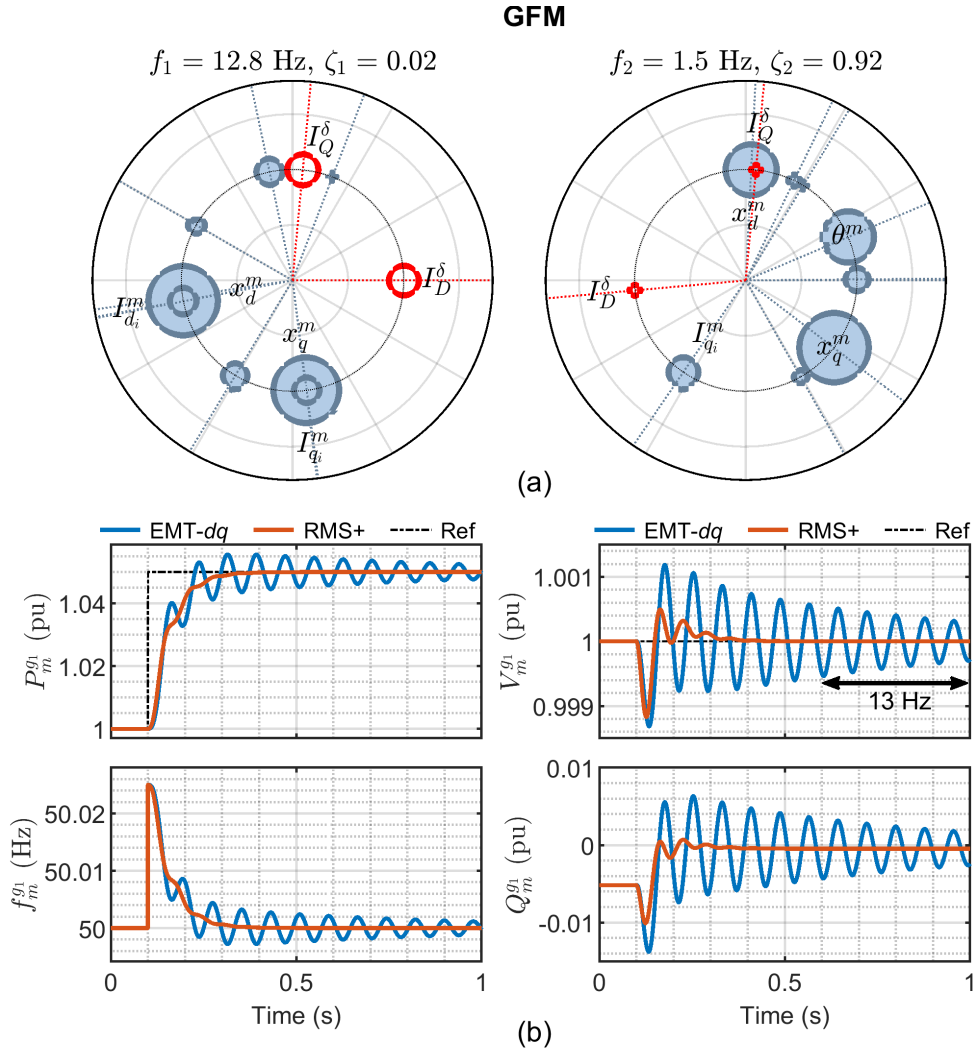


Figure 3.9: (a) Mode shapes showing GFM states (gray markers) and network states (red markers) dominant in SSO modes 12.8 Hz (left) and 1.5 Hz (right); (b) Dynamic response of active power, terminal voltage, frequency and reactive power where the dominant oscillation mode is triggered by increasing reference power to 1.05 pu

using RMS+. A simulation step size of $50 \mu s$ is used for EMT- dq and 0.1 ms for RMS+. Four separate test cases are considered:

1. Single IBR connected to Thévenin equivalent of a power system
2. A GFL and a GFM supplying a load
3. 9-bus test system with two GFMs and one GFL [1]
4. 39-bus test system with five GFLs and five GFMs [2]

For all cases, the IBR tunable and fixed parameters, and the network parameters are given in Table 3.1, 3.2, and 3.3, respectively at the end of this chapter. The operating condition for each case is shown in the respective network diagram.

3.4.1 An IBR Connected to Thévenin Equivalent of a Power System

The same test case discussed in Section 3.2 is considered, where a single IBR is connected to a Thévenin equivalent of a power system with varying grid impedance. This reflects a typical IBR connection compliance study where the IBR vendor or developer needs to comply with the connection requirements for a range of Thévenin equivalent impedances depicting strong to weak grid scenarios. For this case study, the IBR exports 1.0 pu active power and maintains 1.0 pu voltage at the PoC (Figure 3.6). Apart from comparing the time-domain response between

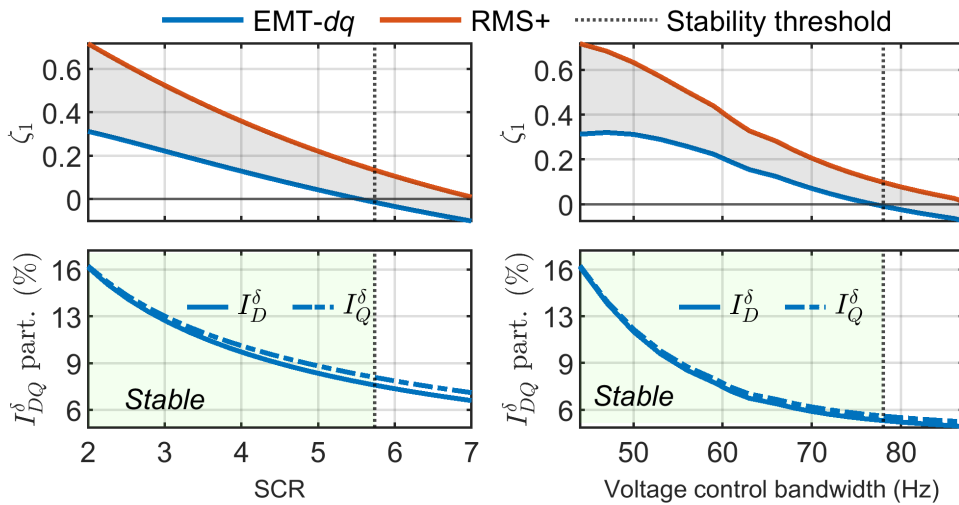


Figure 3.10: Damping ratio (top) and network state participation (bottom) for the dominant SSO mode (22.1 Hz in base case) with varying SCR (left), and GFM voltage control bandwidth in Hz (right)

EMT- dq and RMS+, the difference in the trajectories of the SSO modes with varying grid strength (i.e., SCR) is also observed.

IBR as Grid-Following Inverter

First, the SSO mode trajectories are compared for SCR ranging from 1.06 (green markers) to 10 (red markers) while network X/R is kept constant at 10. The mode trajectories with EMT- dq and RMS+ models are plotted in Figure 3.11(a) with numbers on the plots indicating the value of SCR at specific points. The orange markers denote the stability threshold. On the left of Figure 3.11(a), orange markers overlap with purple markers which indicate the test point for time-domain simulation. The SSO mode trajectories with RMS+ are very different from EMT- dq which reflects the actual situation. RMS+ fails to identify the pertinent instability of the 30-35 Hz mode below $SCR = 1.6$ and could only capture the instability of the 5-10 Hz mode below $SCR = 1.2$. This creates the false impression that the system is stable for $SCR > 1.2$, whereas it is actually stable for $SCR > 1.6$. Notably, when this low-frequency mode (5-10 Hz) becomes unstable in RMS+ at $SCR = 1.2$, the corresponding mode in EMT- dq remains on the left-hand side. Therefore, if high frequency PLL-driven mode is sorted by a careful control design, RMS+ may sometime trigger false stability concern.

The dynamic response of active power, terminal voltage, grid frequency and reactive power is shown in Figure 3.11(b), respectively for $SCR = 1.6$. An undamped SSO at 35 Hz is observed in the response from the EMT- dq model (blue trace) which is not present in the RMS+ response (red trace). This demonstrates that the stability threshold of $SCR = 1.2$ from RMS+ is not reflective of the actual stability limit. This is an example of a misleading stability outcome from RMS+.

IBR as Grid-Forming Inverter

For a GFM, including network dynamics is crucial to capture SSO mode due to the relatively higher participation of network states even under strong grid conditions. In Figure 3.12(a), SCR is again varied from 1.06 (green) to 10 (red). The SSO mode moves towards instability as SCR is increased. With EMT- dq , the SSO mode becomes unstable beyond $SCR = 6.3$. However, with RMS+, the SSO mode remains stable up to $SCR = 7.7$. Notably, the difference between EMT- dq and RMS+ SSO mode trajectories is more pronounced at lower SCR but due to high damping, this difference will be not observed in the time-domain.

The purple markers denote SSO modes at the SCR of 5.3, for which the time domain results

are shown in Figure 3.12(b). With the RMS+ model, a misleading well-damped stable response is seen while the actual response with EMT- dq exhibits poorly damped 14 Hz oscillations in line with the SSO mode trajectory shown on the left in Figure 3.12(a).

3.4.2 GFL and GFM Supplying a Load

To allow inter-IBR interactions across the network, a test system with two IBRs is considered as shown in Figure 3.13. Here a GFL and a GFM supply a load.

The mode trajectory is obtained by varying the GFL active power output from 0.8 pu to 1.2 pu with the GFM producing the rest to supply the fixed 2.0 pu load.

The numerical values in Figure 3.14(a) represent the GFL active power reference. In Figure

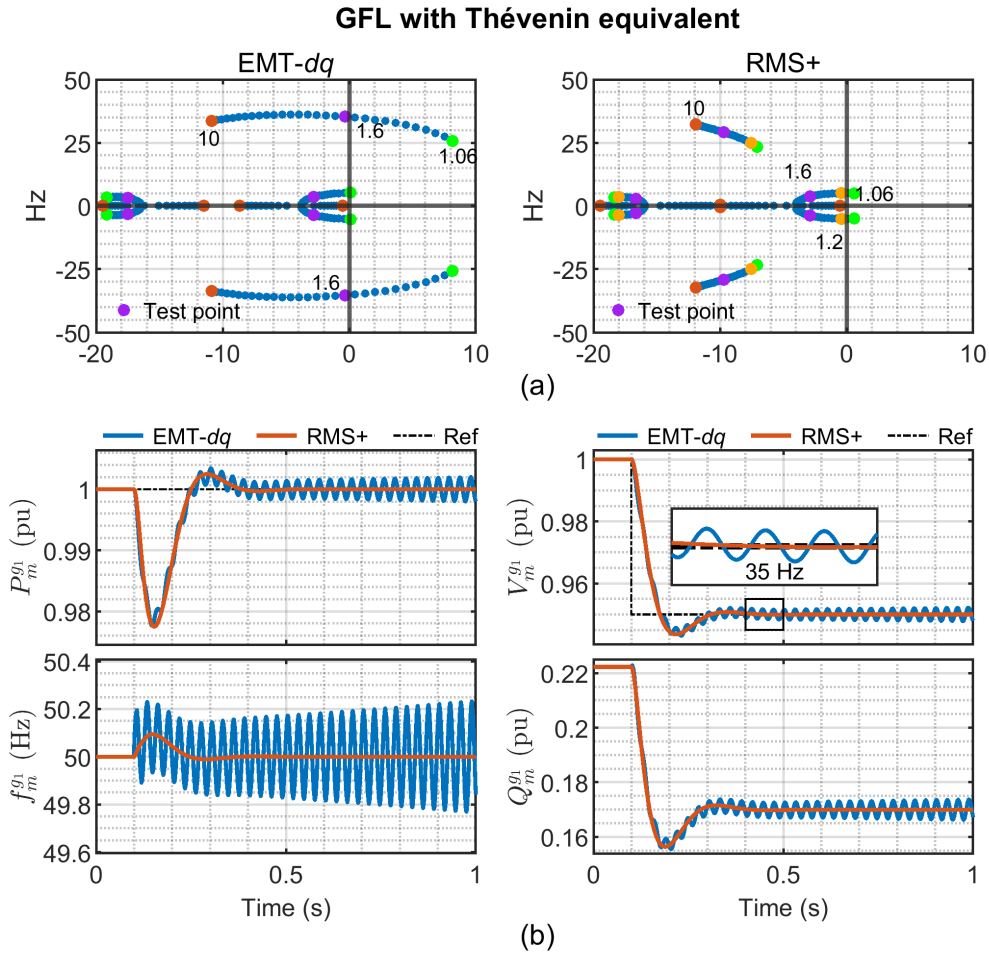


Figure 3.11: (a) SSO mode trajectories for GFL connected to a Thévenin equivalent with EMT- dq (left), RMS+ (right), when SCR is varied from 1.06 (green) to 10 (red). SCR stability threshold (orange) is 1.6 for EMT- dq model and 1.2 for RMS+ model; (b) Dynamic response of active power, terminal voltage, frequency, and reactive power, when voltage reference is reduced from 1.0 pu to 0.95 pu at 0.1 s for SCR = 1.6

3.14(a), a significant difference in the trajectory of the SSO mode having a frequency of around 33 Hz is observed. With EMT- dq , this SSO mode becomes unstable contrary to that with RMS+ where it hardly moves and remains stable. The effect of this discrepancy is manifested in time-domain plots, shown in Figure 3.14(b), when GFL power transfer is set at 0.965 pu (purple markers). At this operating point, the response with EMT- dq shows undamped oscillation at 33 Hz when the voltage reference of the GFL is decreased to 1.0 pu at 0.1 s. The oscillations

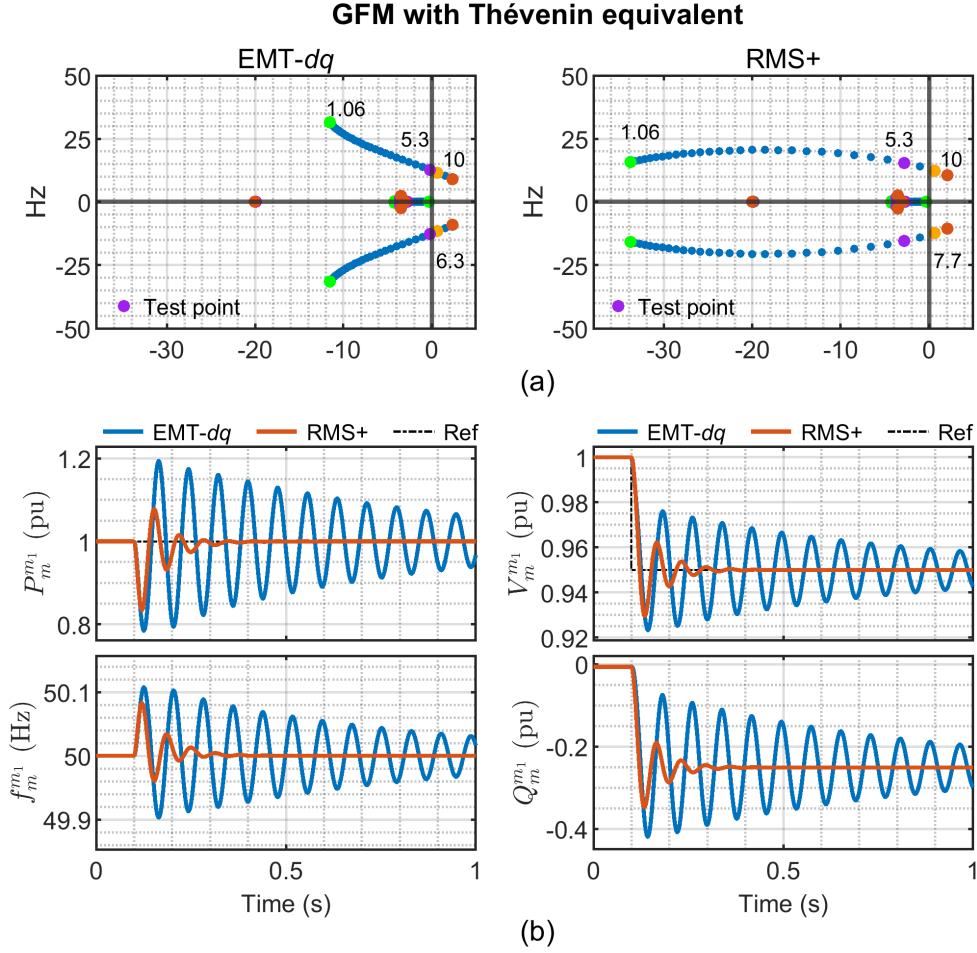


Figure 3.12: (a) SSO mode trajectories for GFM connected to a Thévenin equivalent with EMT- dq (left), RMS+ (right), when SCR is varied from 1.06 (green) to 10 (red). SCR stability threshold (orange) is 6.3 for EMT- dq model and 7.7 for RMS+ model; (b) Dynamic response of active power, terminal voltage, frequency, and reactive power, when voltage reference is reduced from 1.0 pu to 0.95 pu at 0.1 s for SCR = 5.3

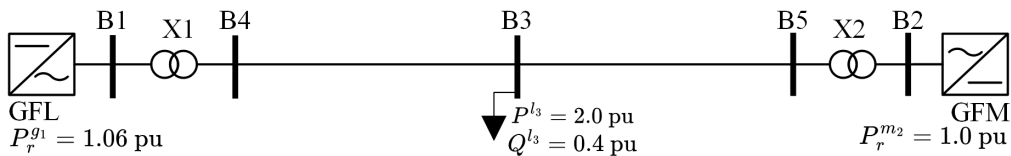


Figure 3.13: A GFL and a GFM supplying a load

are not visible in the RMS+ response.

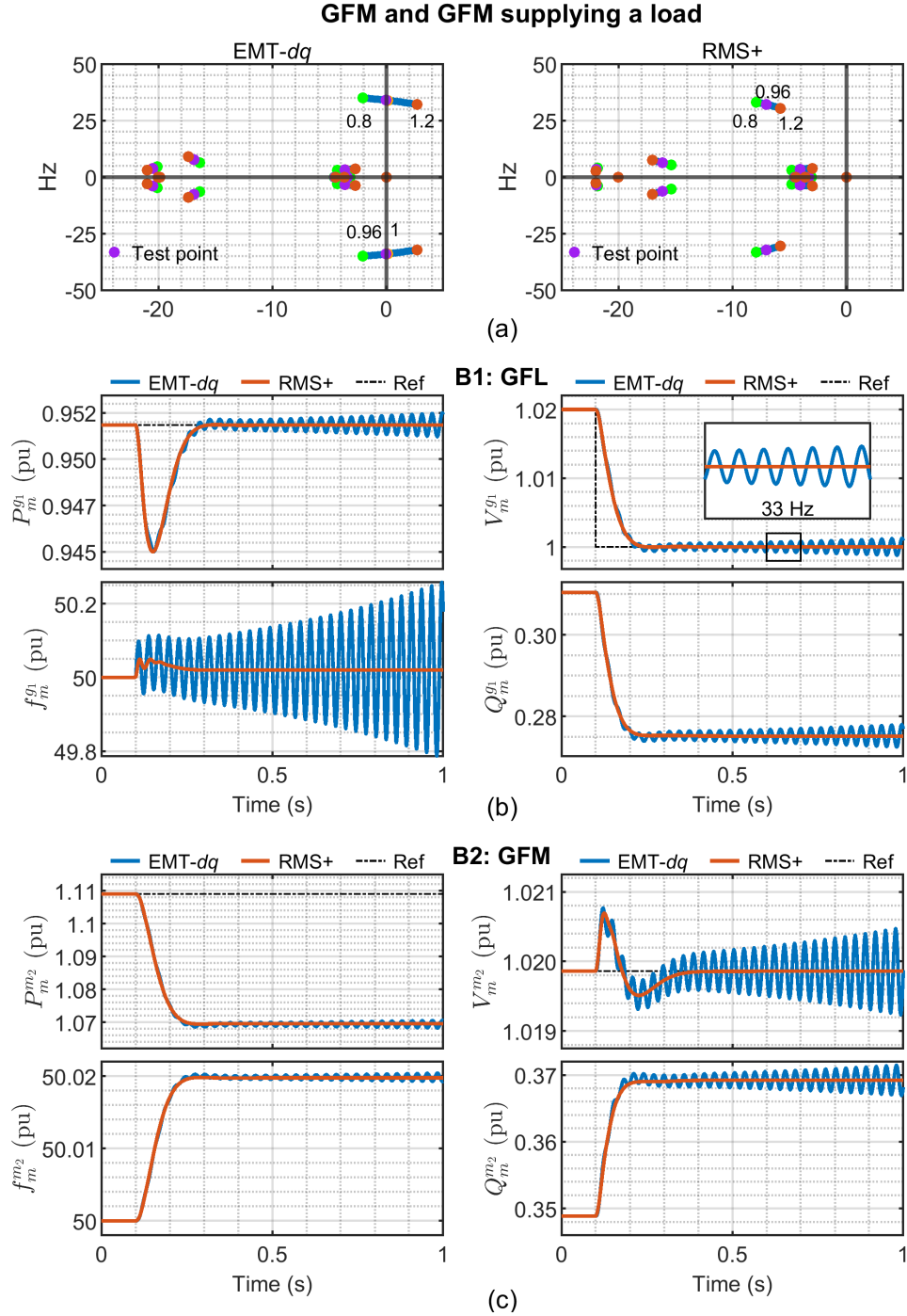


Figure 3.14: (a) SSO mode trajectories for the test case of Figure 3.13 with EMT- dq (left) and with RMS+ (right), when active power reference of GFL is varied from 0.8 (green) to 1.2 (red). The power threshold (orange) is 1.0 pu for the EMT- dq model, RMS+ model could not detect instability. Dynamic response of active power, terminal voltage, frequency and reactive power of (b) GFL, and (c) GFM when GFL voltage set point is changed to 1.0 pu at 0.1 s while GFL power reference is 0.965 pu.

3.4.3 9-bus Test System with Two GFMs and One GFL

The role of network dynamics is further substantiated on the 9-bus test system in Figure 3.15. All three synchronous generators in the original system [1] are replaced by IBRs - GFMs at bus 1 and bus 2, and a GFL at bus 3. The loads are impedance type and the operating condition is kept the same as in [1].

The voltage reference of the GFM at bus 2 is increased by 5% at 0.1 s. The dynamic responses in Figure 3.16 show poorly damped 8 Hz SSO for EMT- dq which is well damped in RMS+. The active powers of GFM deviate from their reference in post-disturbance steady-state due to droop control.

3.4.4 39-bus test system with five GFLs and five GFMs

The synchronous machines in the original 39-bus system from [2] are replaced by IBRs (five GFLs and five GFMs), as shown in Figure 3.17. The loads are impedance type and the operating (dispatch) condition is kept the same as in [2].

At 0.1 s, the active power reference of the GFL at bus 30 (not shown) is reduced by 10%, while the voltage reference of the GFM at bus 31 is increased by 5%. The dynamic responses of the GFMs at buses 31, 37, and 39, and the GFL at bus 32 which are near the disturbance location are shown in Figure 3.18. The responses with EMT- dq (blue trace) show poorly damped SSO around 14 Hz which is not reflected with RMS+. Post-disturbance, GFM active power deviates from its reference due to droop control, while GFL active power and both GFM and GFL voltage exhibit zero steady-state error due to PI control.

All four case studies substantiate that RMS+ studies cannot necessarily reveal poorly damped or unstable SSO. For the first two case studies, this is corroborated in the frequency domain with SSO mode trajectories for varying operating conditions.

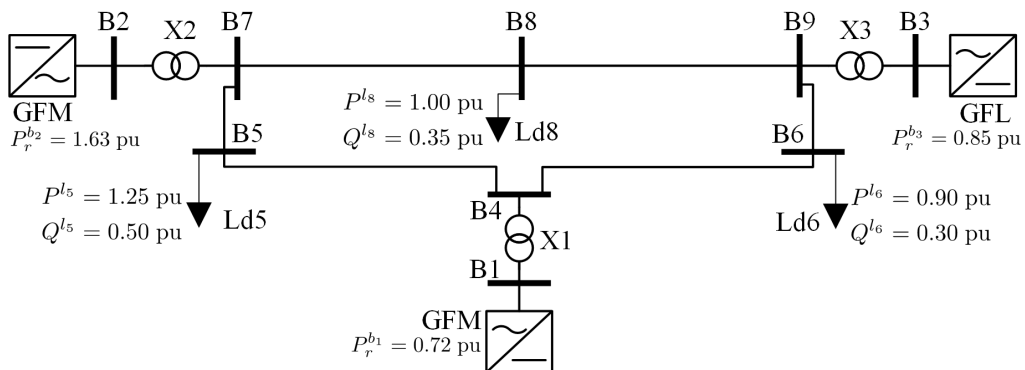


Figure 3.15: 9-bus test system with two GFMs and one GFL [1]

3.5 Conclusion

The chapter reveals that positive sequence RMS studies without network dynamics can offer false stability assurance, indicating well-damped IBR-driven SSO when the oscillations may actually be poorly damped or unstable. EMT modeling in a synchronously rotating reference frame (EMT- dq) is shown to effectively capture IBR-driven SSO with comparable accuracy as EMT- abc (in PSCAD) but significantly lower computational complexity. The reduction in complexity in EMT- dq enables a much larger number of scenario checks. Therefore, EMT- dq can be as fast as RMS+ and as accurate as EMT- abc for studying IBR-driven SSO. Furthermore, the ability of EMT- dq to facilitate frequency-domain analysis through a state-

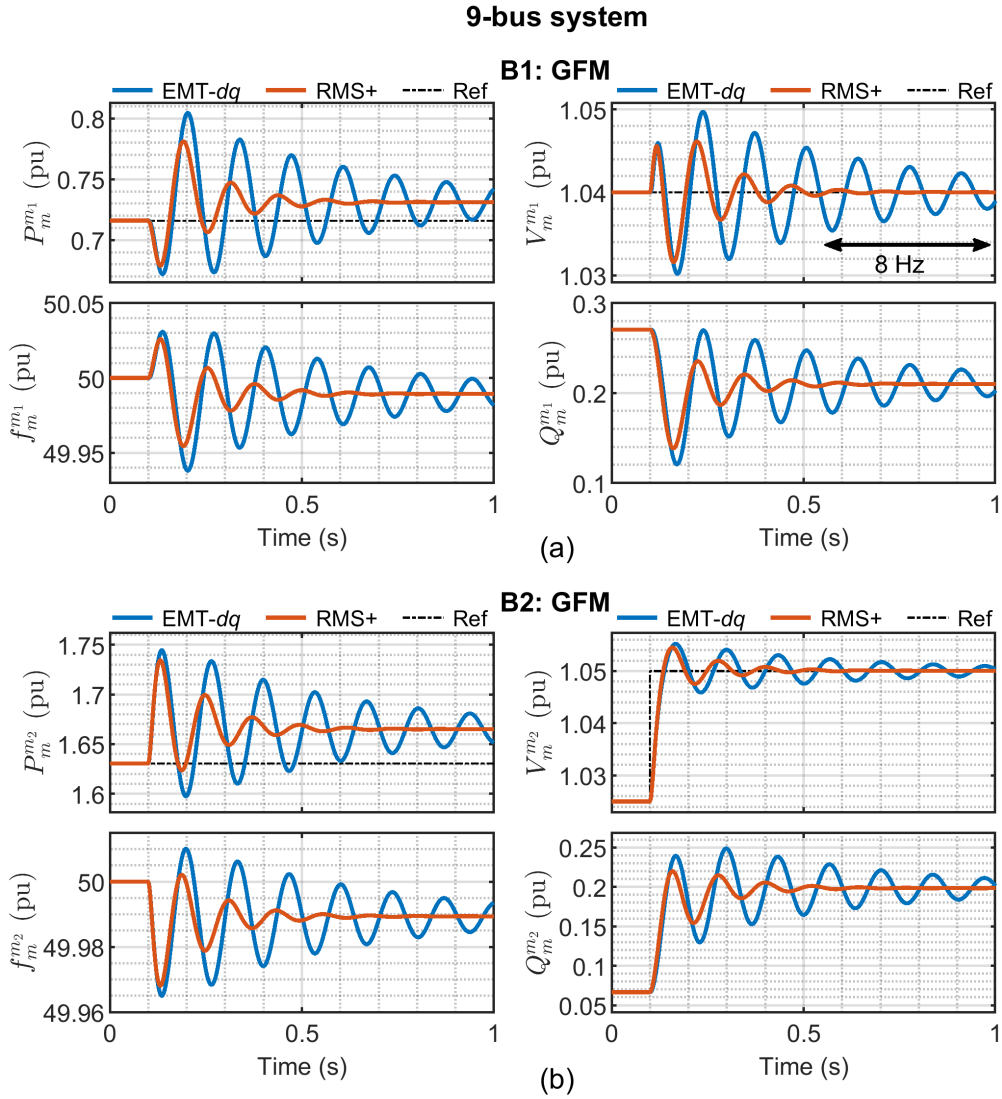


Figure 3.16: Dynamic response of active power, terminal voltage, frequency, and reactive power of GFM at (a) B1 and (b) B2 in the 9-bus system. The reference voltage of GFM at bus 2 is increased to 1.05 pu at 0.1 s.

space approach provides deeper insight into the pathways of SSO enabling effective mitigation. This chapter uncovers the adverse interactions between IBRs and the network for both GFL and GFM operation under various grid strengths and control bandwidths. While certain conditions, such as low PLL bandwidth in GFL or high voltage control bandwidth in GFM, reduce the discrepancy between RMS+ and EMT- dq , these should not dictate the control choices.

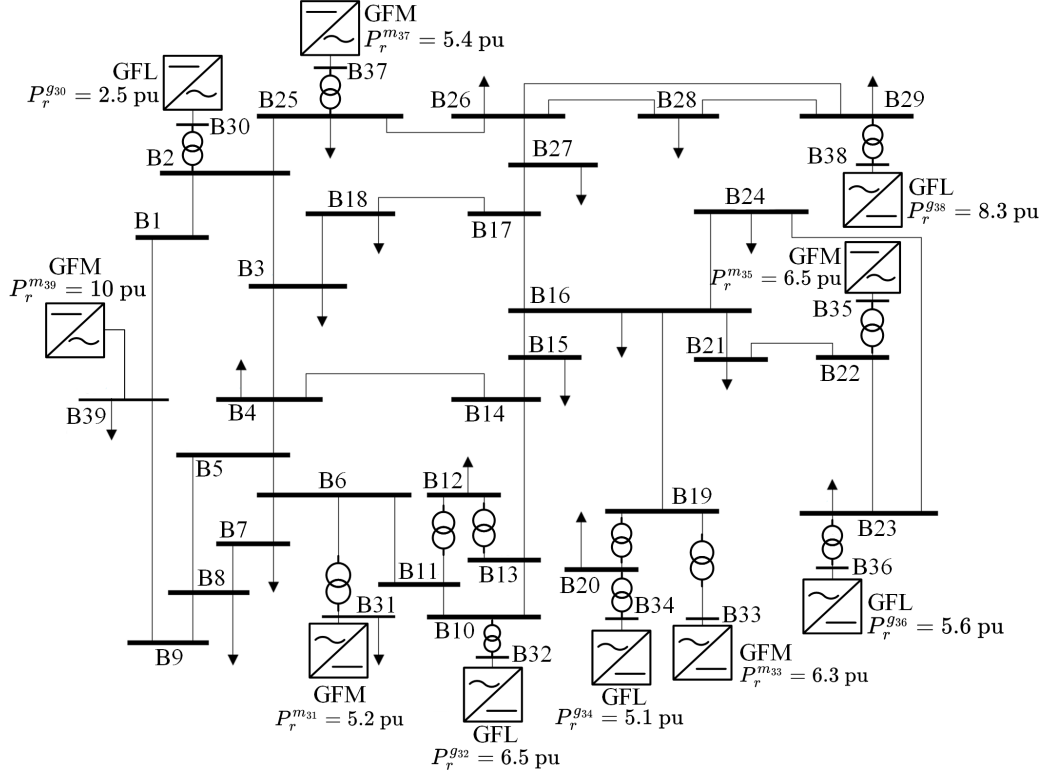


Figure 3.17: 39-bus test system with five GFLs and five GFMs [2]

Ultimately, this study underscores the inadequacy of RMS+ studies in assessing the risk of IBR-driven SSO and highlights the need for EMT- dq tools as they offer a better solution that achieves a crucial balance between accuracy, computational efficiency, and useful insight through frequency-domain analysis. Hence, it is recommended to move beyond bespoke EMT- dq tools used mainly by researchers and develop scalable and standardized EMT- dq tools to foresee the risk of SSO at planning and operational timescales.

Building on this modelling adequacy realisations, the next chapter delves deeper into the characterisation of specific SSO modes to investigate how different IBR control loops in GFL and GFM interact with network dynamics to drive these oscillations.

Table 3.1: Tunable Parameters

		GFL						GFM						
Section #	Description	$g\#$	$K_{P_\theta}^{gk}$	$K_{I_\theta}^{gk}$	$K_{P_d}^{gk}$	$K_{I_d}^{gk}$	$K_{P_q}^{gk}$	$K_{I_q}^{gk}$	$m\#$	$D_{P_\omega}^{mk}$	$K_{P_d}^{mk}$	$K_{I_d}^{mk}$	$K_{P_q}^{mk}$	$K_{I_q}^{mk}$
III.B.1/V.A.1	GFL w/ Thévenin equivalent	1	-188.5	-35531	0.01	31.4	-0.01	-31.4	-	-	-	-	-	-
	III.B.2/V.A.2 GFM w/ Thévenin equivalent	-	-	-	-	-	-	-	1	3.14	0.19	4.67	0.19	4.67
IV	GFL Simscape validation	1	-73.9	-1150	0.01	70.2	-0.01	-70.2	-	-	-	-	-	-
	GFM Simscape validation	-	-	-	-	-	-	-	1	3.14	0.02	0.37	0.02	0.37
V.B	GFL & GFM with a load	1	-226.2	-51164	0.01	31.4	-0.01	-31.4	1	3.14	0.19	4.67	0.19	4.67
V.C	9-bus system (GFL)	3	-188.5	-35531	0.02	62.8	-0.02	-62.8	1	4.39	0.09	0.27	0.09	0.27
	9-bus system (GFM)	-	-	-	-	-	-	-	2	1.93	0.22	0.61	0.22	0.61
	39-bus system	30							31	0.60	0.86	24.4	1.01	24.4
V.D		32							33	0.50	1.04	29.6	1.23	29.6
		34	-188.5	-35531	0.03	94.2	-0.03	-94.2	35	0.48	1.07	30.4	1.26	30.4
		36							37	0.58	0.89	25.2	1.05	25.2
		38							39	0.31	1.65	46.8	1.94	46.8

Table 3.2: Fixed Control and Filter Parameters for GFL and GFM

Controller	τ_c (ms)	τ_m (ms)	τ_θ (ms)	R_f (pu)	$\omega_0 L_f$ (pu)	$\omega_0 C_f$ (pu)
GFL	0.3	7.9	1.5	0.01	0.2	$0.02 S_r^{gk}$
GFM	0.3	7.9	–	0.01	0.2	$0.2 S_r^{mk}$

Table 3.3: Network Parameters for Each Case Study

Section	Description	$\delta_\#$	R_l (pu)	$\omega_0 L_l$ (pu)	$\omega_0 C_l$ (pu)
III.B/V.A	IBR with large power system	1	0.050	0.50	0.00
IV	GFL Simscape validation	1	0.063	0.63	0.63
		2	0	0.05	0
	GFM Simscape validation	1	0.016	0.16	0.16
		2	0	0.05	0
V.B	GFL & GFM with a load	1	0	0.05	0
		2	0.025	0.25	0.25
		3	0.025	0.25	0.25
		4	0	0.05	0
V.C	9-bus system	As per reference [1]			
V.D	39-bus system	As per reference [2]			

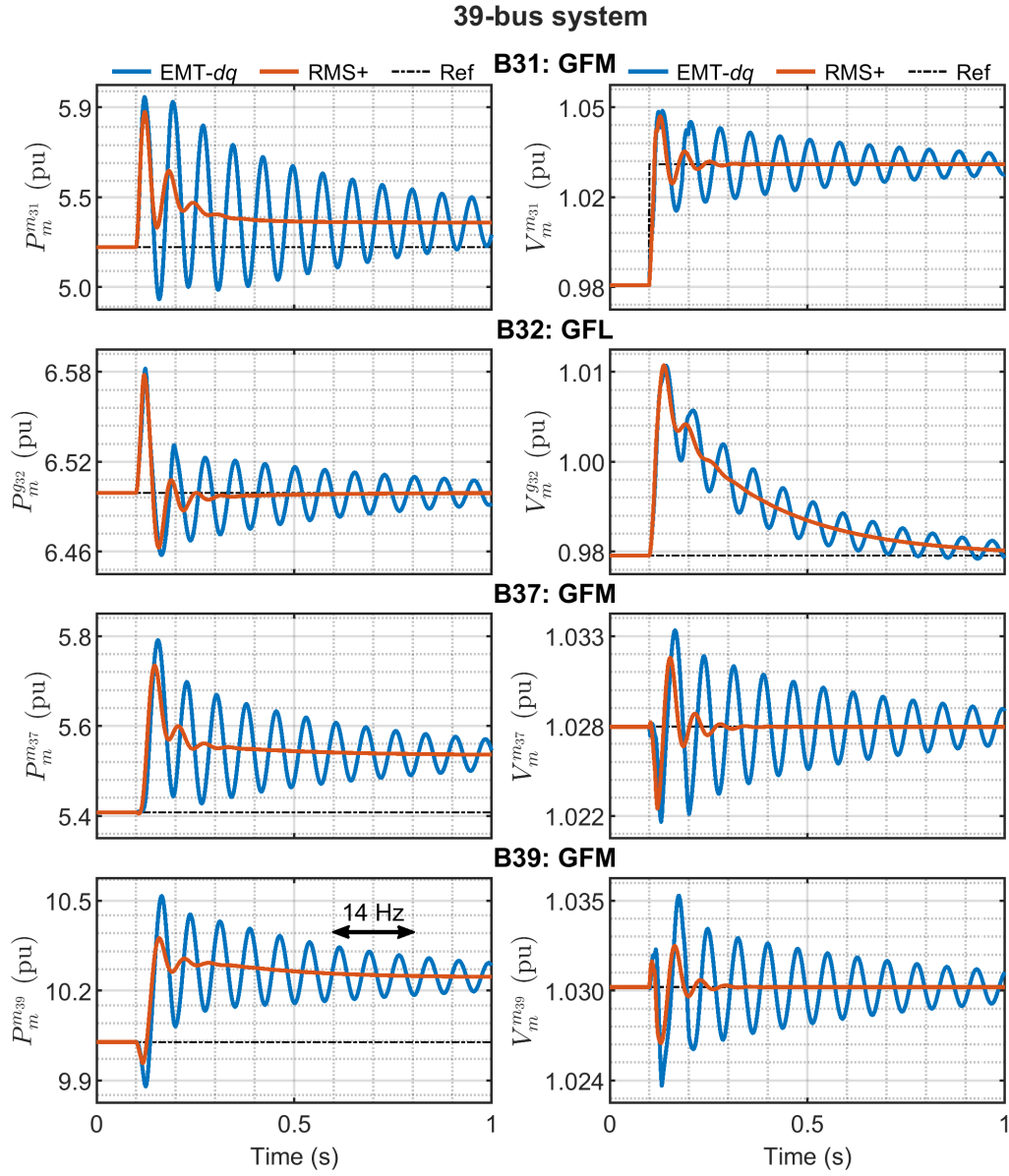


Figure 3.18: Dynamic response of IBR active power and terminal voltage for selected buses in the 39-bus system. At 0.1 s, the active power reference of GFL at bus 30 (not shown) is decreased by 10% together with a 5% increase in voltage reference of GFM at bus 31.

Chapter 4

Characterisation and Mechanisms of Sub-Synchronous Oscillations

The accurate characterisation and identification of SSO in IBR-dominated power systems are essential to develop better stability paradigms and implementing robust control strategies. This chapter investigates the distinctive mechanisms and characteristics behind two prevalent types of SSO: oscillations driven by PLL in GFLs, and those arising from voltage control interactions within GFM. This chapter presents the separate analytical treatment of both SSOs.

Firstly, PLL-driven SSOs are examined through an analytical and numerical lens. This results in the formation of unique diagnostic indicators that signify PLL involvement. It has been found that such oscillations are typically characterized by notably low amplitude variations in the voltage and power signals. Furthermore, it is shown that an increase in active power injection from the IBR decreases the damping associated with these modes. The analysis echoes with the message of Chapter 3 that incorporating dynamic network modelling is vital for accurately detecting and diagnosing PLL-induced oscillations. These findings are validated using EMT simulations conducted on representative power system test cases.

Subsequently, the analysis extends to oscillations triggered by GFM voltage control interactions with the network. Under strong grid conditions, GFMs exhibit susceptibility to SSO. Through modal and damping ratio sensitivity analyses, this chapter elucidates how the inner control loops, specifically current and voltage controllers, interact adversely with network dynamics, to cause poorly damped SSO. The analytical investigation highlights that retuning of inner control parameters, such as increasing current control bandwidth and voltage control proportional gains, effectively mitigates these sub-synchronous modes. The effectiveness of these tuning strategies is further corroborated by simulations in large-scale systems featuring high

penetrations of IBRs, including both GFLs and GFMs.

By characterizing these two distinct but interrelated types of SSO—both caused by IBRs, albeit via different mechanisms—the chapter provides foundational insights that aid in the targeted identification of the source of oscillation and then the mitigation of these oscillations.

4.1 PLL-Induced SSO

As IBRs continue to replace synchronous generation, new forms of SSOs have emerged in modern power systems. A broad survey of such events can be found in [11], which catalogues real-world SSO incidents attributed to factors like series compensation and low system strength. Of particular interest are 20 Hz modes reported at Hydro One, the Australian Energy Market Operator (AEMO), and Dominion Energy [41], where PLL interactions were implicated in their mechanism analysis. However, that study primarily relied on testing varied control settings and observing resultant Bode plots, leaving open questions on the deeper cause-and-effect pathways driving these oscillations.

A distinct type of SSO is reported in [42], becoming less damped with increased power injection and initially linked to the PLL. Closer examination shows that the mode also shifts markedly with changes in voltage control, indicating the PLL is not the sole driver of that mode from IBR side. While [43] presents a broad overview of diagnostic indicators for instabilities associated with sub- or super-synchronous control interactions, it neither isolates the internal control actions of the PLL nor provides a targeted analytical basis for attributing specific oscillatory behaviour to distinct control interactions.

This section addresses this gap by presenting an analytical investigation of PLL-driven SSOs in IBR-dominated systems. Specifically, it examines how the tight coupling between the PLL and network dynamics gives rise to low-amplitude sub-synchronous modes that may initially appear benign but can become increasingly unstable as the active power injection from the IBR is increased. The analysis leverages small-signal linearisation to uncover the root cause and frequency characteristics of these modes, providing insight into their sensitivity to operating conditions and control parameters. Based on this analysis, a set of diagnostic indicators is proposed designed to distinguish PLL-driven SSOs from other sub- or super-synchronous control interactions. Furthermore, it is demonstrated that how seemingly benign oscillations can escalate into severe grid-level disturbances if left unchecked, emphasizing the importance of correctly identifying the root cause to prevent system-wide instability.

Therefore, this section provides: (1) a clear theoretical explanation of the PLL’s role in

causing SSOs and (2) a practical checklist of diagnostic indicators enabling operators to detect PLL-driven SSO effectively. Specifically, it is shown that PLL-driven SSOs (i) exhibit low-amplitude oscillations, (ii) experience deteriorating damping with an increase in power injection, and (iii) require dynamic network modelling for emulation—they cannot be captured with purely algebraic network model. The application of these three DIs are validated on EMT simulations (with current limiters and average model of voltage source converter. on two representative test systems. These insights would potentially act as a screening tool to pinpoint PLL-driven SSOs before they threaten small-signal stability.

4.1.1 PLL–Network Dynamic Interactions

Figure 4.1 illustrates the configuration used to analyse the dynamic interaction between PLL and the network. This model is a simplified version of the detailed model presented in Chapter 2.1. Figure 4.1(a) represents a GFL G with controlled current source in parallel with a capacitor C (representing a filter capacitor) rotating in local dq frame. The parallel combination is connected to an AC voltage source (rotating in DQ frame) through an RL branch. Both sources are rotating in their respective local synchronous frames with constant frequency ω . The phase difference θ between them is determined by PLL based on current injection, such that $X_{dq} = X_{DQ}e^{-j\theta}$. Here X could be $(V, I)^{(G,N)}$.

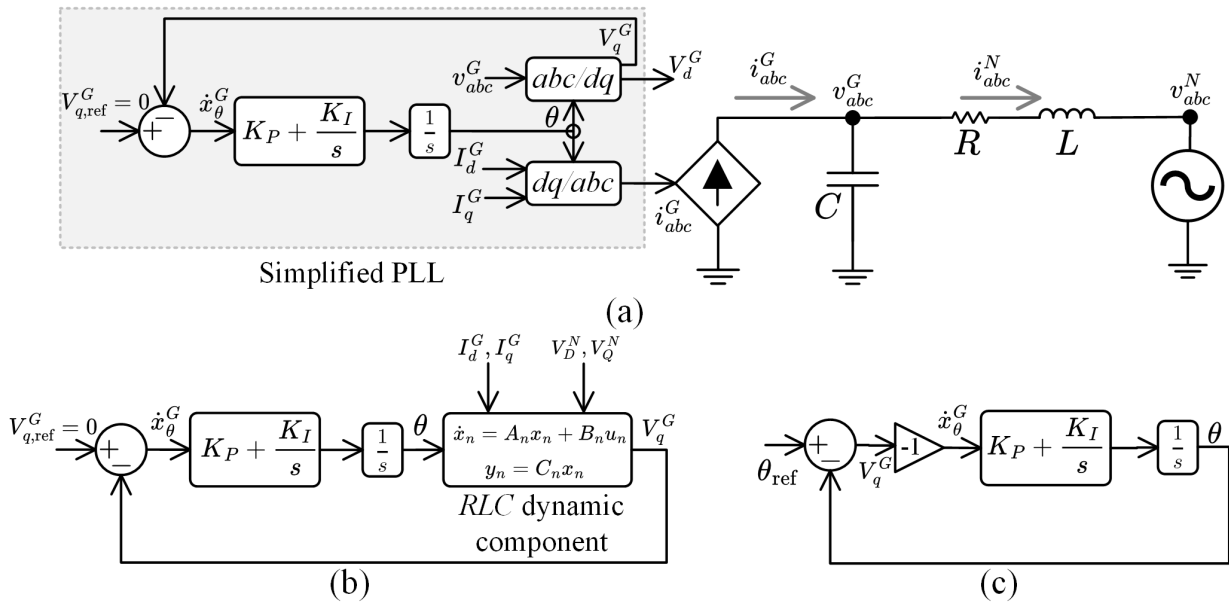


Figure 4.1: (a) A controllable AC current source synchronized with an AC voltage source using a PLL. (b) PLL with linearized dynamic network equations. (c) Standard PLL structure used for control design.

Linearizing frame transformation around an operating point and linking them to differential equations of *RLC* elements in their respective rotating frames gives following state space system with state vector $x_n = [V_d^G \ V_q^G \ I_D^N \ I_Q^N]^T$, input vector $u_n = [V_D^N \ V_Q^N \ I_d^G \ I_q^G \ \theta]^T$, and output $y_n = V_q^G$. For brevity, Δ is removed from linearized states, inputs and outputs.

$$A_n = \begin{bmatrix} 0 & w & -c_\theta/C & -s_\theta/C \\ -w & 0 & s_\theta/C & -c_\theta/C \\ c_\theta/L & -s_\theta/L & -R/L & w \\ s_\theta/L & c_\theta/L & -w & -R/L \end{bmatrix} \quad (4.1)$$

$$B_n = \begin{bmatrix} 0 & 0 & 1/C & 0 & (I_{D0}^N s_\theta - I_{Q0}^N c_\theta)/C \\ 0 & 0 & 0 & 1/C & (I_{D0}^N c_\theta + I_{Q0}^N s_\theta)/C \\ -1/L & 0 & 0 & 0 & (-V_{d0}^G s_\theta - V_{q0}^G c_\theta)/L \\ 0 & -1/L & 0 & 0 & (V_{d0}^G c_\theta - V_{q0}^G s_\theta)/L \end{bmatrix}$$

where $c_\theta = \cos \theta_0$, $s_\theta = \sin \theta_0$. Also, $C_n = \begin{bmatrix} 0 & 1 & 0 & 0 \end{bmatrix}$. Closing the loop in Figure 4.1(b) gives following A_{cl} for state vector $x_{cl} = [V_d^G \ V_q^G \ I_D^N \ I_Q^N \ x_\theta^G \ \theta]^T$:

$$A_{cl} = \begin{bmatrix} 0 & \omega & -\frac{c_\theta}{C} & -\frac{s_\theta}{C} & 0 & \frac{I_{D0}^N s_\theta - I_{Q0}^N c_\theta}{C} \\ -\omega & 0 & \frac{s_\theta}{C} & -\frac{c_\theta}{C} & 0 & \frac{I_{D0}^N c_\theta + I_{Q0}^N s_\theta}{C} \\ \frac{c_\theta}{L} & -\frac{s_\theta}{L} & -\frac{R}{L} & \omega & 0 & \frac{-V_{d0}^G s_\theta - V_{q0}^G c_\theta}{L} \\ \frac{s_\theta}{L} & \frac{c_\theta}{L} & -\omega & -\frac{R}{L} & 0 & \frac{V_{d0}^G c_\theta - V_{q0}^G s_\theta}{L} \\ 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & -K_P & 0 & 0 & K_I & 0 \end{bmatrix} \quad (4.2)$$

The closed-loop transfer function of a standard PLL (Figure 4.1(c)) is $-(K_P + sK_I)/(s^2 - sK_P - K_I^2)$. Comparing it with second-order canonical form, gives the expression of K_P , and K_I . Therefore, for desired closed-loop bandwidth, f_{PLL} (Hz), and damping ratio ζ , $K_P = -2\zeta\omega_n$, and $K_I = -\omega_n^2$, where $\omega_n = 2\pi f_{PLL}(2/((4\zeta^2 + 2)^2 + 4))^{0.5} + 4\zeta^2 + 2)^{0.5}$.

Using A_{cl} , the influence of PLL states on V_d^G can be quantified by examining the left eigenvector ψ_{ss0} corresponding to the only SSO mode λ_{ss0} . For valid operating conditions obtained on a practical range of grid impedance ($0.25 \text{ pu} < |R + j\omega L| < 1 \text{ pu}$, $3 < \omega L/R < 10$), filter capacitor ($0.01 \text{ pu} < \omega C < 0.05 \text{ pu}$) and PLL bandwidth ($10 \text{ Hz} < f_{PLL} < 40 \text{ Hz}$), A_{cl} gives the right (ϕ_{ss0}) and left (ψ_{ss0}) eigenvectors, associated with the only SSO mode, λ_{ss0} . The normalized magnitude of residual component $[C_{cl}\phi_{ss0}][\psi_{ss0}^T B_{cl}]$, varies between 0.01% and 6.3% for input I_d^G , and between 0.002% and 5.4% for input I_q^G . Such small ranges of residue indicate

very small oscillation magnitude in output voltage for variations in input current injections across a wide range of operating conditions. While it is true that other modes can also exhibit small amplitude variations, the distinguishing feature of PLL-driven SSOs is that this low-magnitude oscillation occurs specifically in the terminal voltage and active power signals of GFL units, often without corresponding network-wide observability at distant buses. This behaviour, linked to the tight coupling between the PLL and local network dynamics, serves as a useful indicator when interpreted in conjunction with other mode characteristics such as frequency range, damping sensitivity to power setpoints, and localisation in mode shapes. Hence, despite not being exclusive, the low-amplitude voltage oscillation becomes the first Diagnostic Indicator (DI), DI-(I), consistent with the observations in [41, Sec.IV.B, bullet 3].

The impact of power injection on PLL-driven SSO can be analysed through eigenvalue sensitivity analysis. For a small perturbation ΔA_{cl} in the closed-loop state matrix, first-order eigenvalue sensitivity theory yields: $\Delta \lambda_{ss0} \approx \psi_{ss0}^T \Delta A_{cl} \phi_{ss0} / \psi_{ss0}^T \phi_{ss0}$. The sign of $\Re(\Delta \lambda_{ss0})$ indicates the directional shift in modal damping, with a positive value signifying reduced damping. For a representative set of grid parameters (RLC), the sensitivity by varying the power injection is analysed. Figure 4.2(a) demonstrates that an increase in power injection corresponds to positive $\Re(\Delta \lambda_{ss0})$, indicating a reduction in damping. This relationship establishes our second diagnostic indicator, DI-(II), also reported as an observed feature in [41, Sec.IV.B, bullet 2].

The third diagnostic indicator for PLL-driven SSO stems from PLL's electromagnetic coupling with the network. To validate this characteristic, discriminating test is proposed: if the oscillations persist when the network is represented by algebraic equations rather than differential equations, the root cause likely lies elsewhere. Conversely, the well-damped response with an algebraic network model strongly indicates PLL as the source. To provide analytical validation of this indicator, the closed-loop system matrix is reformulated by reducing the dynamic network model to its algebraic equivalent. The DQ network current response to converter terminal voltages is then given by:

$$\begin{bmatrix} I_D^N \\ I_Q^N \end{bmatrix} = \frac{1}{D_R} \begin{bmatrix} R & \omega L \\ -\omega L & R \end{bmatrix} \begin{bmatrix} V_d^G \\ V_q^G \end{bmatrix}, \quad D_R := R^2 + (\omega L)^2. \quad (4.3)$$

The converter-frame inductor currents, I_{Bd} and I_{Bq} , are obtained by applying a rotational transformation:

$$F_d := I_{Bd} = I_D^N \cos \theta + I_Q^N \sin \theta, \quad (4.4)$$

$$F_q := I_{Bq} = -I_D^N \sin \theta + I_Q^N \cos \theta. \quad (4.5)$$

Substituting (4.3) into the above gives the following closed-form expressions:

$$F_d = \frac{R \cos \theta - \omega L \sin \theta}{D_R} V_d^G + \frac{R \sin \theta + \omega L \cos \theta}{D_R} V_q^G, \quad (4.6)$$

$$F_q = \frac{-R \sin \theta - \omega L \cos \theta}{D_R} V_d^G + \frac{R \cos \theta - \omega L \sin \theta}{D_R} V_q^G. \quad (4.7)$$

Next, we compute the Jacobian of F_d and F_q with respect to voltages and θ at the operating point $\theta = \theta_0$. This yields:

$$\left. \frac{\partial F_d}{\partial V_d^G} \right|_0 = \frac{R \cos \theta_0 - \omega L \sin \theta_0}{D_R}, \quad \left. \frac{\partial F_d}{\partial V_q^G} \right|_0 = \frac{R \sin \theta_0 + \omega L \cos \theta_0}{D_R}, \quad (4.8)$$

$$\left. \frac{\partial F_q}{\partial V_d^G} \right|_0 = \frac{-R \sin \theta_0 - \omega L \cos \theta_0}{D_R}, \quad \left. \frac{\partial F_q}{\partial V_q^G} \right|_0 = \frac{R \cos \theta_0 - \omega L \sin \theta_0}{D_R}. \quad (4.9)$$

$$\left. \frac{\partial F_d}{\partial \theta} \right|_0 = \frac{-R \sin \theta_0 - \omega L \cos \theta_0}{D_R} V_{d0}^G + \frac{R \cos \theta_0 - \omega L \sin \theta_0}{D_R} V_{q0}^G, \quad (4.10)$$

$$\left. \frac{\partial F_q}{\partial \theta} \right|_0 = \frac{-R \cos \theta_0 + \omega L \sin \theta_0}{D_R} V_{d0}^G - \frac{R \sin \theta_0 + \omega L \cos \theta_0}{D_R} V_{q0}^G. \quad (4.11)$$

Finally, incorporating these linearised relationships into the converter's capacitor dynamics and PLL controller equations yields the closed-loop state matrix for the algebraic network model:

$$A_{\text{alg}} = \begin{bmatrix} -\frac{R \cos \theta_0 - \omega L \sin \theta_0}{C D_R} & \omega - \frac{R \sin \theta_0 + \omega L \cos \theta_0}{C D_R} & 0 & -\left. \frac{\partial F_d}{C \partial \theta} \right|_0 \\ -\omega - \frac{-R \sin \theta_0 - \omega L \cos \theta_0}{C D_R} & -\frac{R \cos \theta_0 - \omega L \sin \theta_0}{C D_R} & 0 & -\left. \frac{\partial F_q}{C \partial \theta} \right|_0 \\ 0 & -1 & 0 & 0 \\ 0 & -K_P & K_I & 0 \end{bmatrix}, \quad (4.12)$$

This matrix A_{alg} captures the voltage and PLL dynamics while embedding the network behaviour via algebraic constraints. Its structural form contrasts with A_{cl} , where dynamic RL elements yield additional state couplings, especially significant when analysing sub-synchronous oscillatory behaviour.

In A_{cl} , network resistance indirectly dampens the voltages via distinctly available inductor states, which could potentially resonate with fast acting PLL. In contrast, A_{alg} collapses the inductor dynamics into direct constraints (F_d and F_q), causing the same resistance to more immediately dampen the voltage states. Consequently, A_{alg} lacks the ability to capture inductor states coupling with fast-acting PLLs, resulting in misleadingly high sub-synchronous damping. These distinctions in A_{cl} and A_{alg} explain the pronounced differences in subsynchronous damping characteristics, especially when the network impedance and the PLL bandwidth are increased, as shown in Figure 4.2(b). Therefore, dynamic RL model is necessary to capture the real risk of low damping or instability (top right corner Figure 4.2(b)). Such structural differences form the basis for the third diagnostic indicator, DI-(III): if an SSO is observed in EMT simulation but not observed with an algebraic network model during emulation, it is potentially a PLL-driven SSO.

Theoretical support for DI-(III) rests on the analytical structure of the PLL-GFL system as described in (4.12), where the PLL explicitly couples the GFL control dynamics with the network voltage angle and frequency through differential equations. In contrast, an algebraic (RMS-type) network representation removes this dynamic interaction, effectively decoupling the PLL from any resonant behaviour. Therefore, if an oscillatory mode is present in the EMT- dq model with a dynamic network but disappears when the same system is simulated with an algebraic network (RMS+), the origin of that mode must lie in the interaction between the

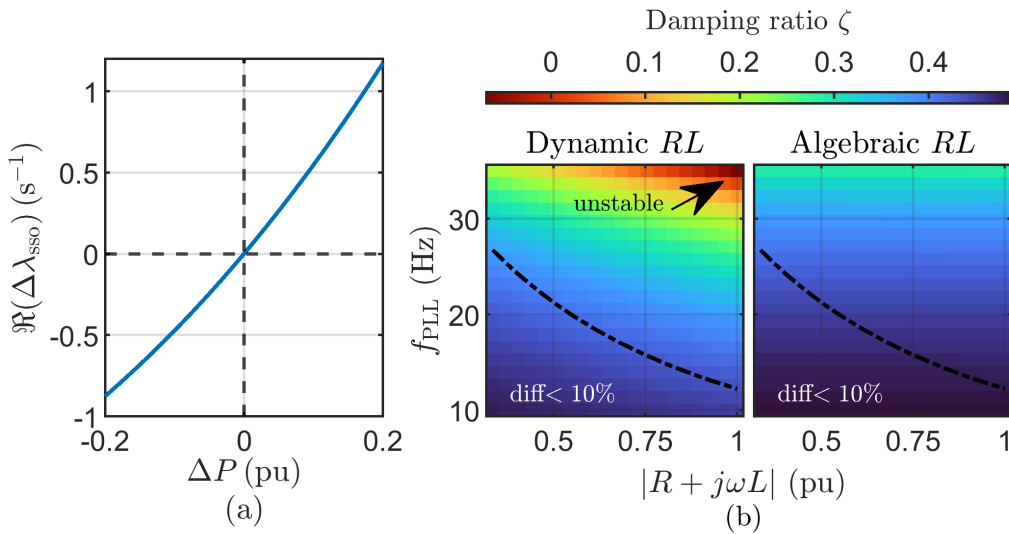


Figure 4.2: Results from analytical models. (a) SSO mode dependency on active power injection–DI-(II) (b) Damping ratio comparison for variations in PLL bandwidth and line impedance for dynamic (left) and algebraic (right) RL representations–DI-(III).

PLL and the network dynamics. Furthermore, if this behaviour coincides with the other two diagnostic indicator then DI-(III) provides a theoretical guarantee that the oscillation is indeed PLL-driven when voltage is observed at the GFL terminal,. This forms a sufficient condition for identifying PLL-induced SSO in GFL units.

All three DIs enlisted below:

DI-(I): The inverter's voltage and power signals oscillates with a very low magnitude at SSO range.

DI-(II): The damping of these oscillations worsens with increase in active power injection.

DI-(III): The oscillations are not observed when the network is represented using an algebraic model (offline indicator).

The simultaneous satisfaction of all three DIs is necessary to confidently identify PLL-driven SSO. As counterexamples, one can separately analyse the current and outer control loops to show that SSOs arising solely (or even partially) from these loops do not satisfy all three diagnostic indicators. While the complete derivation of these cases is reserved for future work, it is expected that such analysis would follow the same theoretical framework established in this chapter.

4.1.2 Validation on Test Systems

For both case studies, the dynamic model of GFL presented in Chapter 2.1 is used.

Single-GFL with Infinite Bus

The first validation test is conducted on the same system shown in Figure 3.6. For an SCR of 2, $X/R = 10$, and $f_{\text{PLL}} = 40$ Hz, a poorly damped mode exists at 29 Hz. In agreement to DI-(I), changing voltage reference to 0.95 pu at 0.05 s triggers a very low amplitude of oscillation in GFL's voltage and power measurement signal, as shown on the left side of Figure 4.3(a). On the right side of Figure 4.3(a), the mode shape of the 29 Hz mode is shown. All 13 states can be read from Figure 2.2. The area of each circle marker is proportional to the magnitude of the normalized participation factor of that state and the orientation is set by the angle of the corresponding right eigenvector. Small areas of V_d^G and V_q^G markers on the right side of Figure 4.3(a) show that these states contribute very little to the 29 Hz mode; accordingly, the oscillation manifests with only a faint amplitude in both voltage components.

Figure 4.3(b) illustrates DI-(II), where increasing power injection from 0.8 (green) to 1.2 (red) reduces the damping of the SSO. This indicates that a seemingly benign low-amplitude oscillation can threaten system dynamic stability. If such oscillations are dismissed merely due to their low amplitude, the system can drift toward instability under changing operating conditions. This underscores the importance of correctly identifying and diagnosing these modes before they evolve into more severe dynamic issues. Figure 4.3(c) is an example of DI-(III). For algebraic network, the PLL-driven mode (black markers) does not move rightwards, when SCR is decreased. On contrary, when network is modelled with dynamic equations, the mode swiftly moves rightwards capturing the interaction between PLL and network dynamics.

11-bus Test System with Two GFLs and Two GFMs

Our second test system is a modified two-area 11-bus system [3] incorporating two GFMs and two GFLs, as shown in Figure 4.4. The load type and operating conditions are maintained as

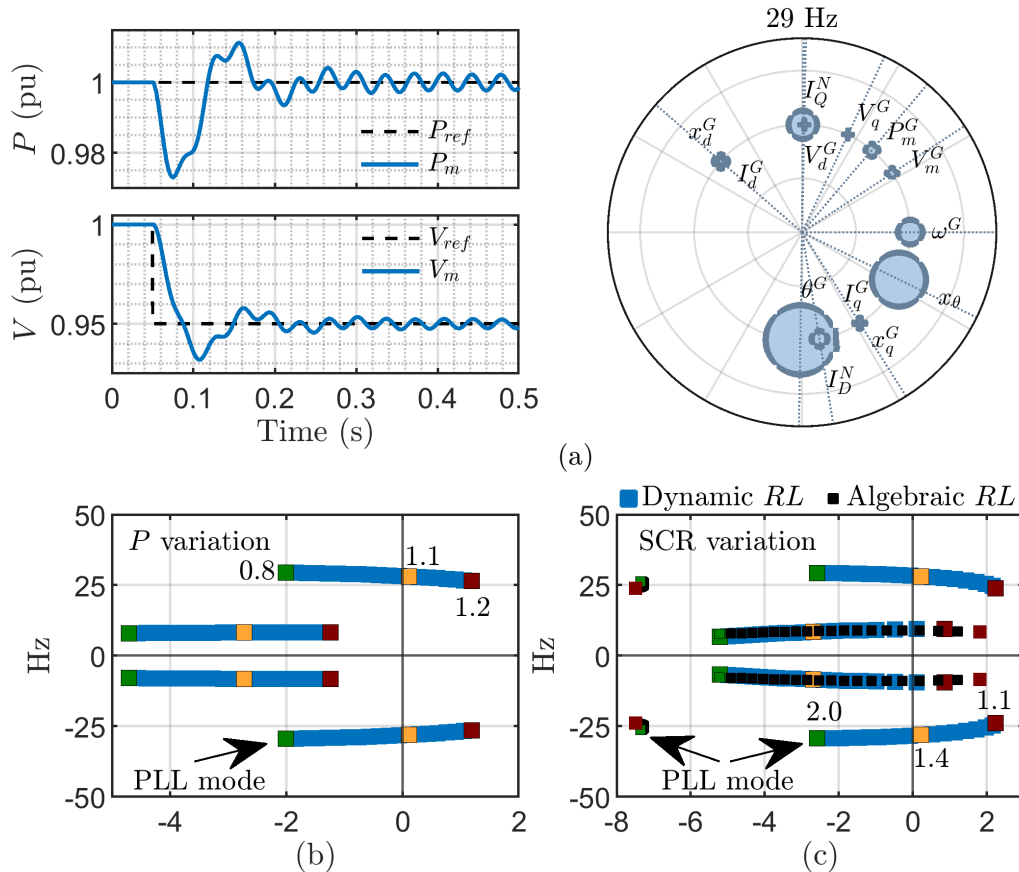


Figure 4.3: (a) Low-magnitude 29 Hz oscillation in EMT model (left), modal composition of 29 Hz mode (right)–DI-I. (b) Mode trajectory of SSO modes for variation in active power–DI-(II). (c) Mode trajectory comparison (algebraic vs dynamic RL) of SSO modes for variation in SCR–DI-(III).

in the original system, where the active power load is constant current and the reactive power load is constant impedance. Here, as well, the GFM model presented in Chapter 2.2 is used.

When a voltage step change is applied to the GFM at B1 (raising its reference to 1.05 pu at), a low-amplitude SSO at 25 Hz is observed in the terminal voltage (blue traces in Figure 4.5), aligning with DI-(I). It is also shown that as soon as power injection of GFL at B3 is reduced to 6.5 pu, the damping increases (yellow traces)–DI-(II). Lastly, in agreement with DI-(III), the response with algebraic network (red traces) is very well damped. With all three diagnostic indicators present, the SSO can be confidently attributed to the detuned PLL of GFL at B3.

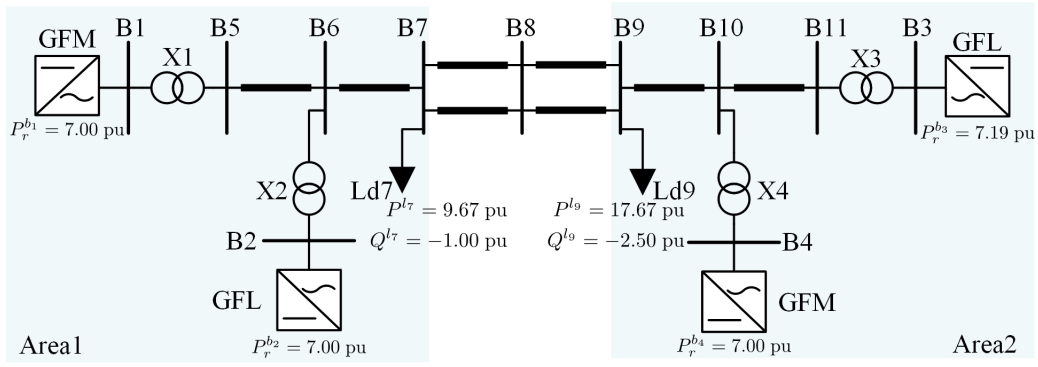


Figure 4.4: 11-bus test system with two GFMs and two GFLs [3].

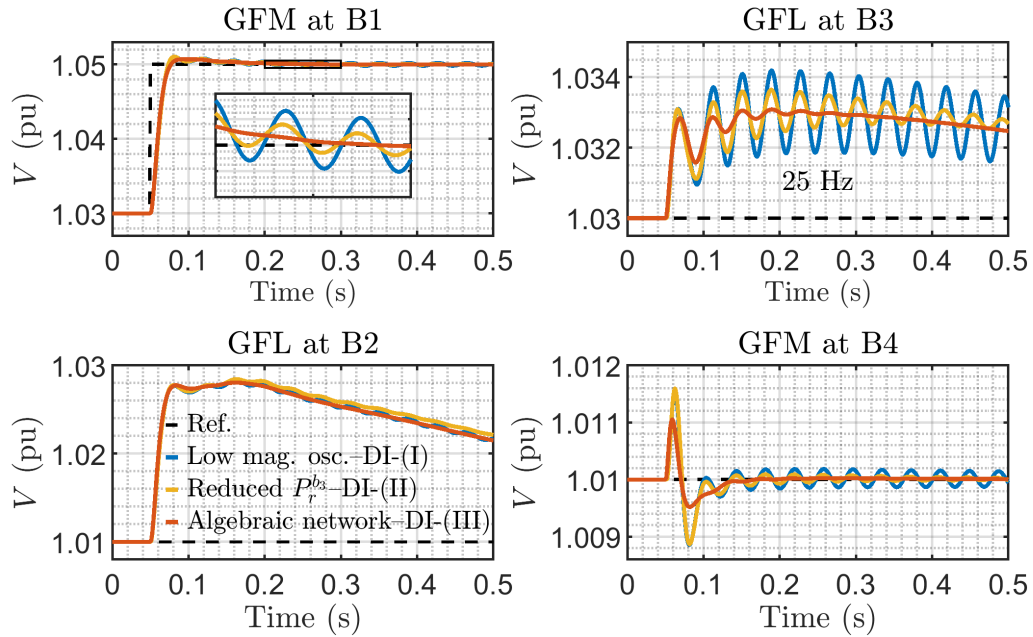


Figure 4.5: Dynamic response of IBR terminal voltages for 11-bus system verifying all three diagnostic indicators.

4.1.3 Summary

This section presented key diagnostic indicators for PLL-driven SSOs in IBR-dominated power systems. It has been established that these oscillations typically manifest with small amplitude in the inverter's voltage and current signals, become less damped with increased power injection, and require a fully dynamic network representation for accurate emulation. The absence of any single indicator precludes definitive conclusions about the role of PLL in the observed SSO. It is acknowledged that the results presented do not explicitly demonstrate cases where the absence of one or more DIs invalidates the attribution of an SSO to the PLL. The proposed DIs are intended to be used collectively as a sufficient condition for identifying PLL-driven SSOs. That is, when all three indicators are present, a strong diagnostic inference can be made. However, their individual absence does not conclusively rule out PLL involvement, as some borderline cases or overlapping mechanisms may partially satisfy the criteria. Establishing necessary and sufficient conditions would require a broader classification framework involving systematic counterexamples and other potential SSO drivers (e.g., outer-loop instabilities), which is identified as a direction for future work.

In the next section, SSO caused by GFM voltage control are investigated using analytical models. The insights obtained are then validated in EMT single- and multiple-IBR test cases.

4.2 GFM Voltage Control-Induced SSO

In GFM, as already discussed earlier in Chapter 2.2, the active and reactive power controllers determine the voltage magnitude and angle at PCC [44]. While much attention has been given to the primary control design (e.g., droop control [45] and virtual synchronous generator (VSG) control [36]), this work focuses on the impact of inner control—the voltage and current control. Their control gains are typically tuned locally. Current control is tuned using pole-zero cancellation [46] and voltage control is generally tuned using Symmetrical Optimum (SO) [47]. The voltage control design method, in particular, assumes idealized conditions, such as high current control bandwidth and resistive networks [48], which do not hold in practical high-power applications where switching frequency limitations impose constraints on bandwidth [49]. This discrepancy can lead to misleading stability assessments when simplified models are tested against real-world grid equivalents [50].

Several studies have investigated the sensitivity of eigenvalues to IBR control parameter variations, primarily relying on linearized numerical models. For GFL, such studies are con-

ducted in [51, 52, 53]. Similarly, for GFMs, [54, 55] have employed numerical models to study control interactions, but analytical insights into the underlying mechanisms remain largely unexplored. While these approaches can reveal modal behaviour and quantify sensitivity to parameter changes, they are typically scenario-specific and rely on simulation-backed interpretation [56, 57]. In contrast, general analytical insights, such as closed-form expressions or simplified theoretical models, that directly relate voltage control structure, network impedance, and oscillatory dynamics remain relatively scarce. This chapter contributes to that direction by formulating a compact analytical model that explains the core resonance mechanism and its dependence on controller and network parameters.

Therefore, an analytical formulation of a simplified GFM model, focusing on inner control dynamics is presented in this section. The analysis proposes two key mitigation strategies—increasing voltage control proportional gain or current control bandwidth—to enhance system stability. These analytical insights align with prior numerical observations and are also validated on a large-scale 100% IBR-driven system. This section reveals that modal participation patterns remain similar across variations in voltage control gains, current control bandwidth, and network strength, making it difficult to diagnose the exact source of SSOs using eigenvalue-based methods alone. While it is true that eigenvalue sensitivity analysis, well-established in the literature, can provide valuable insights into how specific control parameters influence modal properties, here the focus is on the practical challenge of diagnostic ambiguity: namely, that multiple destabilising mechanisms can produce similar eigenvalue loci and mode shapes. This overlap limits the ability to attribute the root cause solely based on eigenstructure. Our findings also highlight the need to incorporate network dynamics into control design.

4.2.1 Small-Signal Analysis

The voltage control gains of a GFM are tuned using the SO method [46, 50]. Notably, the design in the SO method is carried out locally using only GFM parameters. This means that the SO method does not account for the static or dynamic strength of the network at the PCC. Assuming I_{dq} is independent of the network and τ_c is very small, the gain crossover frequency ω_m determines the proportional and integral gains, denoted by (K_{P_d}, K_{P_q}) and (K_{I_d}, K_{I_q}) , respectively, as below:

$$K_{P_d} = K_{P_q} = C_f \omega_m \quad (4.13)$$

$$K_{I_d} = K_{I_q} = \tau_c C_f \omega_m^3 \quad (4.14)$$

The frequency ω_m is proportional to the closed-loop bandwidth of the voltage control. Also, ω_m and τ_c are related to phase margin δ_m , as shown below:

$$\omega_m = \frac{1}{\tau_c} \sqrt{\frac{1 - \sin \delta_m}{1 + \sin \delta_m}} \quad (4.15)$$

From (2.2), the dynamic model of GFM has 8 states, and 8 performance parameters (K_{P_d} , K_{P_q} , K_{I_q} , K_{I_d} , τ_c , C_f , τ_m , $D_{P\omega}$). Among these parameters, τ_m and $D_{P\omega}$ are held constant throughout this chapter, set at $\tau_m = 7.9$ ms and $D_{P\omega} = 0.01 \cdot \omega_0 / P_{\text{rated}}$, as examining their influence is beyond the scope of this study. These are the typical values used in the literature for such parameters [50, 23]. Here, ω_0 represents the nominal system angular frequency ($\omega_0 = 2\pi \cdot 50$ rad/s). Since this study is exclusive to GFM, the superscript (m_k) is dropped from all quantities.

Here, the impact of interactions between GFM control and network states is explored by analysing a circuit in which the GFM is connected to a Thévenin equivalent of a power system, as shown in Figure 3.6. A simple case with one IBR is chosen to eliminate the possibility of inter-GFM oscillations and focus exclusively on the interaction between the GFM and network states in SSO modes. This setup reflects a typical GFM connection compliance study, where the GFM vendor or developer must comply with connection requirements across a range of Thévenin equivalent impedances, representing both strong and weak grid scenarios.

In this analysis, the GFM exports 1.0 pu active power and maintains a 1.0 pu voltage at the PCC. The GFM performance parameters are tabulated in Table 4.1. Due to simplifications in current control, the filter values of R_f and L_f are not explicitly required for small-signal analysis. This simplification, as discussed earlier, has been adopted from [46]. Since the current control loop forms a closed-loop oblivious to network parameters, the filter R_f and L_f values are not influenced by network strength, as discussed in Chapter 2. However, for EMT (point-on-wave) validation studies (not shown in this work), an R_f of 0.01 pu and $\omega_0 L_f$ of 0.2 pu has been used.

Table 4.1: GFM Parameters

$K_{P_{dq}}$	$K_{I_{dq}}$	τ_c (ms)	τ_m (ms)	$D_{P\omega}$	$\omega_0 C_f$ (pu)
0.2	4.67	0.3	7.9	3.14	0.22

Figure 4.6(a) examines the directional movement of the dominant SSO mode as three key parameters—SCR, K_P , and τ_c —are varied. In this analysis, when one parameter is varied to its extreme value, indicated by black markers, the other two parameters are held at their base values, denoted by green markers. The time constant τ_c and closed-loop bandwidth f_c of

the current controller are related by $\tau_c = 1/(2\pi f_c)$. The selected sweep ranges for SCR, K_P , and τ_c span realistic operating conditions for GFM inverters and are intended to capture the onset of instability without entering extreme regimes that are either non-practical from control viewpoint or where load flow does not converge. The trajectories of the SSO modes are plotted as each parameter is varied: SCR (blue square markers), τ_c (red diamond markers), and $K_{P_{dq}}$ (orange circle markers). Across all three variations, these modes remain in close proximity, especially before reaching instability, suggesting that the dominant SSO mode responds in a similar manner to changes in each parameter. This one-at-a-time parameter sweep offers a clear view of how each individual factor influences the modal trajectory which points towards the dominant sensitivities.

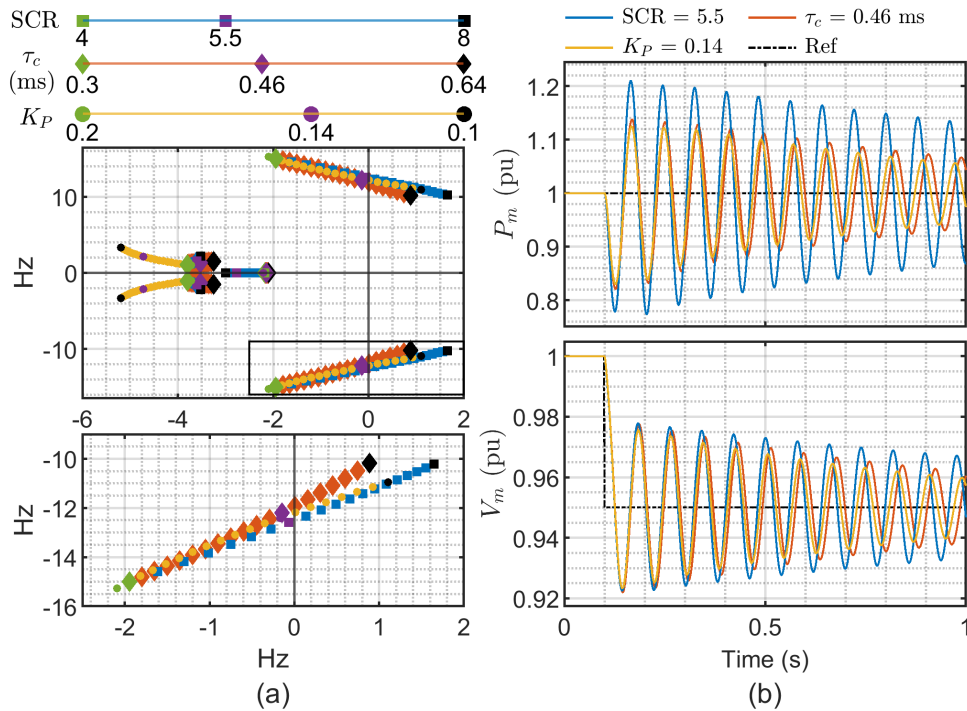


Figure 4.6: For a GFM connected to a Thévenin equivalent: (a) SSO mode trajectories when the SCR is varied from 4 to 8 (square), τ_c is varied from 0.3 to 0.64 (diamond), and $K_{P_{dq}}$ is varied from 0.2 to 0.1 (circle); (b) active power and voltage reference tracking when the reference voltage is reduced to 0.95 pu at 0.1 s for three oscillatory scenarios, denoted by purple markers.

Figure 4.6(b) shows the time-domain response (active power and voltage reference tracking) at the purple markers indicating conditions near the stability threshold for each parameter. These responses exhibit nearly identical frequencies and damping characteristics, making them practically indistinguishable. The critical stability points for each parameter are identified as follows: SCR at 5.5, τ_c at 0.46 ms, and $K_{P_{dq}}$ at 0.14. The droop gain remains constant across all three variations.

To further investigate the nature of this dominant SSO mode, centred around a frequency of approximately 12.5 Hz, its mode shapes are presented in Figure 4.7. This modal analysis is conducted at the stability threshold for each parameter, as indicated by the purple markers in Figure 4.7(a). The magnitude of the participation factor is represented by the size of the markers, while the angle of the markers reflects the mode shapes. This visual representation allows us to examine both the participation and the oscillation pathways simultaneously. The markers are colour-coded to distinguish the contributions from different states: blue for network states (I_D , I_Q), red for current control (I_{di} , I_{qi}), yellow for voltage control (x_d , x_q), and gray for other states (V_d , V_q , P_m , θ).

The results indicate that, although the SSO mode is caused by variation in different parameters, the mode shapes and participation factors remain consistent. The similarity in the mode shapes shown in Figure 4.7 indicates that a snapshot of a poorly-damped mode alone is

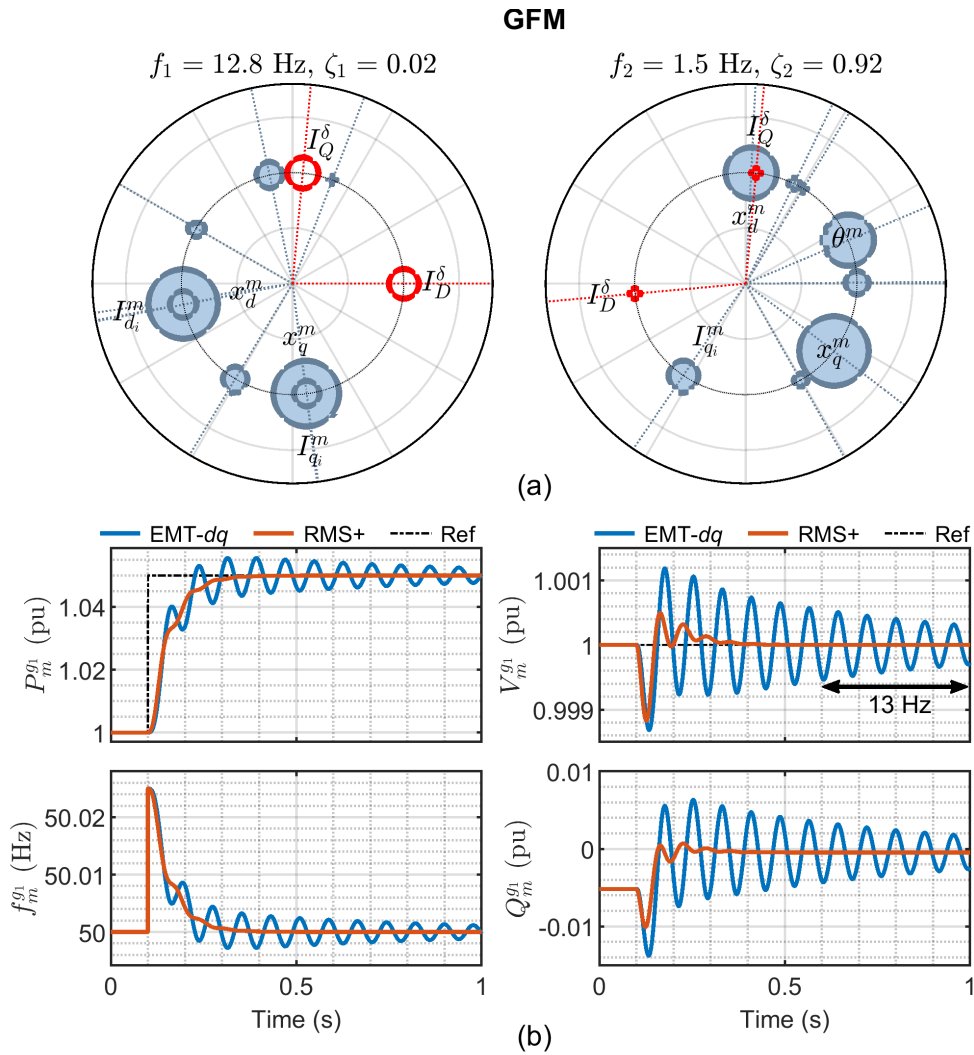


Figure 4.7: Mode shapes representing the signature of the oscillatory mode (~ 12.5 Hz) when (a) $\text{SCR} = 5.4$; (b) $\tau_c = 0.46$ ms; (c) $K_P = 0.14$.

insufficient to determine whether the oscillation is caused by a detuned current controller, an overly aggressive outer loop, or high SCR. While eigenvalue sensitivity analysis can reveal that which parameter is most capable of moving the mode back to stability, they do not completely resolve the ambiguity in cases where multiple parameters project similarly onto the dominant SSO mode. This observation is further explained later in the chapter through analytical expressions derived from a simplified system model. Thus, mode shape inspection without eigenvalue sensitivity analysis may lead to inconclusive or misleading attribution of the oscillation source. Moreover, since modal analysis reveals minimal participation from the state associated with the power droop (P_m), it is reasonable to conclude that for lower bandwidths of P -control (in this case, 0.96 Hz), the oscillation pathway observed in Figure 4.7 will remain largely unchanged, regardless of the specific control structure (droop, VSG, etc.).

The small-signal analysis of the GFM-Thévenin equivalent system has demonstrated how variations in network and control parameters can drive the system toward instability. The conclusions drawn here are not intended to capture transient or large-signal behaviours because such events involve non-linear dynamics and saturation effects, which extend beyond the applicability of the small perturbation assumption. A simplified GFM model, presented in the next section, allows us to analytically examine the sensitivity of system damping to inner control and network parameters.

4.2.2 Inner Control Sensitivity Analysis

To gain deeper insights into the specific influence of inner control parameters on system damping, this section simplifies the GFM model and focuses on the sensitivity analysis of its inner control dynamics.

The participation of dq states is similar in SSO mode, as shown by the consistent marker sizes for network, current control, and voltage control in Figure 4.7. This similarity indicates that the dq axes are reasonably decoupled. Therefore, either of the axes can represent the impact of inner control on SSO mode. This analysis also excludes droop control, as these oscillations arise predominantly from interactions between inner control and network states, with minimal droop control participation. For brevity, the subscripts and superscripts for all parameters and variables are also removed.

Figure 4.8 illustrates the step-by-step process leading to simplified GFM dynamic model. It begins with a representation of the GFM as a voltage source with an LC filter (Figure 4.8(a)). Next, after applying the current control simplification, as suggested by [46], the GFM

is represented by a current source (Figure 4.8(b)). Finally, Figure 4.8(c) presents the dynamic block diagram of the entire system, linearized around an operating point. Here, the network is modelled as a single-axis dynamic RL segment.

The GFM output voltage ΔV_{pcc} appears across the filter capacitor C , while the network determines the line current ΔI_n . The closed-loop transfer function of the system shown in Figure 4.8 is given by:

$$\frac{\Delta V_{pcc}}{\Delta V_r} = \frac{(R + sL)(K_I + sK_P)}{s^4 LC\tau + s^3(LC + \tau RC) + s^2(RC + \tau + LK_P)} \times \frac{1}{+s(RK_P + LK_I) + RK_I} \quad (4.16)$$

From (4.16), the impact of all six parameters (K_P , K_I , τ , R , L , and C) on the system roots can be measured. Under typical normalized GFM parameters (see Table 4.1), the coefficients of s^4 and s^3 are too small, indicating much faster response and hence can be ignored. Moreover, dividing numerator and denominator with the coefficient of s^2 repackages the denominator into

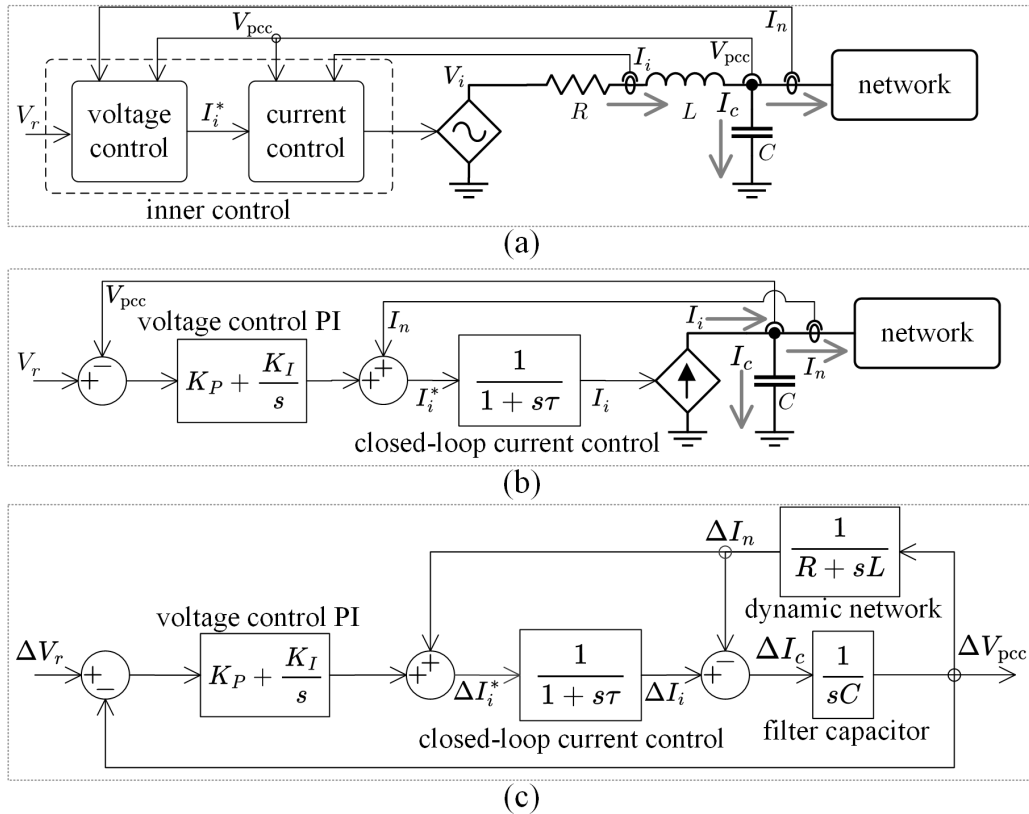


Figure 4.8: GFM simplification: (a) GFM as a voltage source with inner control regulating only the voltage at the PCC; (b) simplification of current control resulting in a current source representation of GFM; (c) dynamic block diagram linearized around an operating point with first-order network model.

the standard form of second order system with the following characteristic equation:

$$s^2 + \left(\frac{RK_P + LK_I}{RC + \tau + LK_P} \right) s + \left(\frac{RK_I}{RC + \tau + LK_P} \right) = 0 \quad (4.17)$$

Matching the coefficients of (4.17) with the standard second-order transfer function yields following expressions for the natural oscillation frequency ω_n and damping ratio ζ .

$$\omega_n = \sqrt{RK_I / (RC + \tau + LK_P)} \quad (4.18)$$

$$\zeta = \frac{LK_I + RK_P}{2\sqrt{R^2CK_I + \tau RK_I + RLK_PK_I}} \quad (4.19)$$

The network strength is generally characterized by SCR (denoted by $\mathcal{S}$), and X/R (denoted by $\mathcal{X}$), instead of R , and L , with $R = 1/(\mathcal{S}\sqrt{1 + \mathcal{X}^2})$ and $L = \mathcal{X}/(\omega_0\mathcal{S}\sqrt{1 + \mathcal{X}^2})$ at rated power of 1.0 pu. Substituting R and L in (4.18) gives:

$$\zeta = \frac{\mathcal{X}K_I + \omega_0K_P}{2\omega_0\sqrt{CK_I + \tau K_I\mathcal{S}\sqrt{1 + \mathcal{X}^2} + \mathcal{X}K_PK_I/\omega_0}} \quad (4.20)$$

Sensitivity analysis of ζ reveals distinct patterns across network and control parameters. The derivatives $\frac{d\zeta}{d\mathcal{S}}$, $\frac{d\zeta}{d\tau}$, and $\frac{d\zeta}{dC}$ are always negative, as shown in (4.21), (4.22), and (4.23), indicating that increases in network strength $\mathcal{S}$, current controller time constant τ , or filter capacitance C reduce system damping.

$$\frac{\partial \zeta}{\partial \mathcal{S}} = -\frac{(\mathcal{X}K_I + \omega_0K_P)\tau K_I\sqrt{1 + \mathcal{X}^2}}{4\omega_0\left(CK_I + \tau K_I\mathcal{S}\sqrt{1 + \mathcal{X}^2} + \frac{\mathcal{X}K_PK_I}{\omega_0}\right)^{3/2}} \quad (4.21)$$

$$\frac{\partial \zeta}{\partial \tau} = -\frac{(\mathcal{X}K_I + \omega_0K_P)K_I\mathcal{S}\sqrt{1 + \mathcal{X}^2}}{4\omega_0\left(CK_I + \tau K_I\mathcal{S}\sqrt{1 + \mathcal{X}^2} + \frac{\mathcal{X}K_PK_I}{\omega_0}\right)^{3/2}} \quad (4.22)$$

$$\frac{\partial \zeta}{\partial C} = -\frac{(\mathcal{X}K_I + \omega_0K_P)K_I}{4\omega_0\left(CK_I + \tau K_I\mathcal{S}\sqrt{1 + \mathcal{X}^2} + \frac{\mathcal{X}K_PK_I}{\omega_0}\right)^{3/2}} \quad (4.23)$$

Conversely, $\frac{d\zeta}{dK_P}\zeta$ in (4.24) is positive for small K_I , showing that higher proportional gain K_P enhances damping.

$$\frac{\partial \zeta}{\partial K_P} = \frac{2\omega_0^2CK_I + 2\omega_0^2\tau K_I\mathcal{S}\sqrt{1 + \mathcal{X}^2} + \omega_0\mathcal{X}K_PK_I - \mathcal{X}^2K_I^2}{4\omega_0^2\left(CK_I + \tau K_I\mathcal{S}\sqrt{1 + \mathcal{X}^2} + \frac{\mathcal{X}K_PK_I}{\omega_0}\right)^{3/2}} \quad (4.24)$$

The impact of integral gain K_I and network reactance $\mathcal{X}$, represented by $\frac{d\zeta}{dK_I}\zeta$ and $\frac{d\zeta}{d\mathcal{X}}$ in (4.25) and (4.26), respectively, is more complex and depends on the relative magnitudes of other system parameters.

$$\frac{\partial \zeta}{\partial K_I} = \frac{\mathcal{X}K_I - \omega_0 K_P}{4\omega_0 K_I \sqrt{CK_I + \tau K_I \mathcal{S} \sqrt{1 + \mathcal{X}^2} + \frac{\mathcal{X}K_P K_I}{\omega_0}}} \quad (4.25)$$

$$\frac{\partial \zeta}{\partial \mathcal{X}} = \frac{2K_I \left(CK_I + \tau K_I \mathcal{S} \sqrt{1 + \mathcal{X}^2} + \frac{\mathcal{X}K_P K_I}{\omega_0} \right) - (\mathcal{X}K_I + \omega_0 K_P) \left(\frac{\tau K_I \mathcal{S} \mathcal{X}}{\sqrt{1 + \mathcal{X}^2}} + \frac{K_P K_I}{\omega_0} \right)}{4\omega_0 \left(CK_I + \tau K_I \mathcal{S} \sqrt{1 + \mathcal{X}^2} + \frac{\mathcal{X}K_P K_I}{\omega_0} \right)^{3/2}} \quad (4.26)$$

These analytical findings are validated in Figure 4.9, where parameters are varied around their base values ($\mathcal{S} = 3$, $\mathcal{X} = 10$, $\omega_0 C = 0.22$, $\tau = 0.3$ ms, $K_P = 0.2$, and $K_I = 4.67$). Figure 4.9 demonstrates the reduction in damping as $\mathcal{S}$ (left) and τ (right) increase from their base values, while showing improved damping with increased K_P (right).

The analytical insights into parameter sensitivities provide clear understanding of actions conducive in stabilizing GFM-induced oscillations through appropriate selection of control and network parameters.

4.2.3 Validation on a Multi-IBR Test System

To validate our theoretical findings in a more realistic power system context, we examine the same two-area 11-bus system [3], as used in Section 4.1 incorporating two GFMs and two GFLs, one in each area. For easy referencing, the test system is shown in Fig. 4.4. The GFL dynamic model is adapted from Chapter 2.1, with control gains tuned for a closed-loop bandwidth of 10 Hz. For the GFMs, control gains are determined using the SO method. The K_{Pdq} of GFM at B1 is reduced to one-tenth of its original value to induce the oscillation in the base case.

Under base operating conditions, when a voltage step change is applied to the GFM at B1, raising its reference to 1.05 pu at $t = 0.1$ s, a poorly damped SSO at approximately

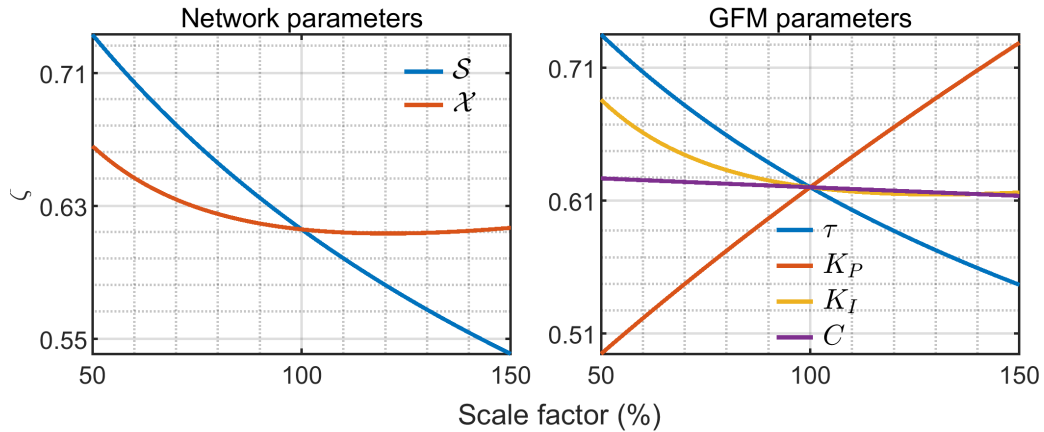


Figure 4.9: Damping ratio for varying network parameters (left), and control parameters (right). At base case (100%), $\mathcal{S} = 3$, $\mathcal{X} = 10$, $\omega_0 C = 0.22$, $\tau = 0.3$ ms, $K_P = 0.2$, and $K_I = 4.67$.

12 Hz is observed in the terminal voltage responses (red traces in Fig. 4.10). Consistent with our analysis, these SSO can be mitigated through three approaches (blue traces in Fig. 4.10). These approaches include increasing $K_{P_{dq}}$ to 0.2 from 0.1, reducing τ_c to 0.2 ms from 0.3 ms, or reducing the network strength by removing one tie line between buses 7 and 9. The effectiveness of these mitigation strategies demonstrates the strong relationship between controller parameters, network configuration, and system stability.

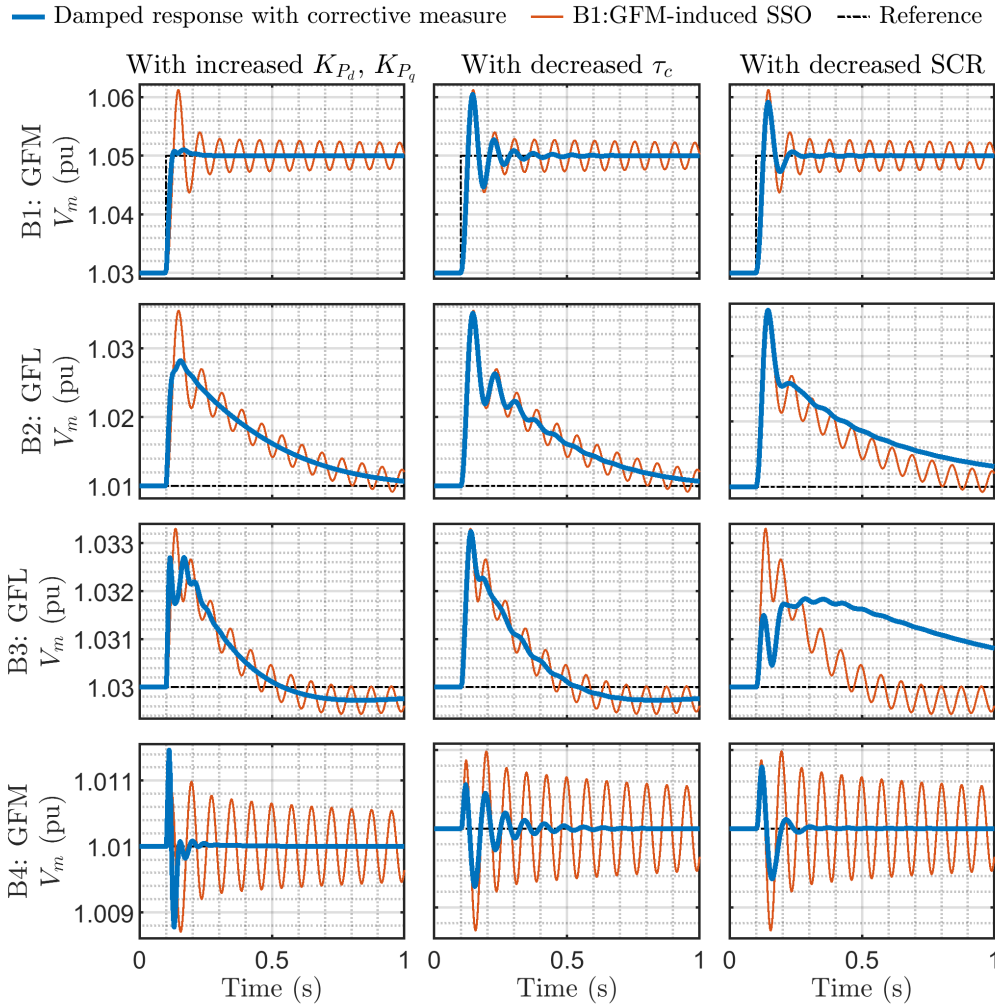


Figure 4.10: Dynamic response of terminal voltage of IBR buses for the 11-bus system. The reference voltage of B1 is increased to 1.05 pu at 0.1 s.

While this study primarily focuses on the interaction between GFM's inner control and network dynamics, inter-GFM oscillations may emerge in systems where GFMs are electrically close and operate with similar bandwidths. This interaction requires further investigation which is one of the areas of my ongoing research.

4.2.4 Summary

The work in this section presents the analysis of GFM-induced SSO, revealing that different parameter variations—SCR, voltage control gain, or current control bandwidth—lead to similar oscillatory patterns in terms of participation factors and mode shapes. This similarity makes it challenging to precisely diagnose the sources of oscillations using modal analysis alone. However, it reveals fundamental relationships between GFM control and network states. The analytical findings, validated on a multi-IBR test system, demonstrate that SSO can be effectively mitigated by increasing voltage control gain or current control bandwidth, or by reducing network strength, that is achieved in 11-bus system by opening a tie line, effectively increasing the line impedance between two areas.

4.3 Conclusions

This chapter presented a detailed characterisation of two dominant types of SSOs in IBR-dominated systems—those driven by the PLL in GFLs, and those arising from voltage control in GFMs. For PLL-driven SSOs, three analytical diagnostic indicators are developed to reliably identify these oscillations. The indicators are validated through EMT- dq simulations on both simplified and realistic test systems. This chapter not only pinpoints PLL as the root cause of low-magnitude, low-damping oscillations, but also highlights the importance of using dynamic network models for accurate detection.

On the other hand, GFM-induced SSOs are analysed by isolating the role of inner control parameters—voltage and current control—and their interaction with network dynamics. A small-signal model and a simplified analytical formulation are developed to reveal how variations in voltage control gain, current control bandwidth, and network strength lead to similar oscillatory behaviours. It is shown that such oscillations are difficult to distinguish using modal signatures alone. Instead, sensitivity analysis of the damping ratio yields key design insights, offering practical mitigation strategies. These are further validated on a multi-IBR test system, demonstrating the efficacy of tailored controller tuning or network reconfiguration.

Together, the analytical tools, indicators, and insights presented in this chapter provide a systematic way for both the diagnosis and mitigation of SSOs in IBR-dominated systems. These findings lay the groundwork for the next chapter, which focuses on integrating these insights into the design of control strategies that are robust to sub-synchronous interactions across a range of operating scenarios.

Chapter 5

IBR Control Design to Mitigate Sub-Synchronous Oscillation

This chapter presents advanced control tuning strategies for both GFL and GFM, with a particular focus on voltage control. The objective is to enhance controller robustness and dynamic performance under a wide range of SCRs, without departing from the classical PI control structure typically used in industry.

To achieve this, a MIMO control design framework is employed in both cases. The controller is synthesized to minimize the H_∞ norm, balancing two key performance metrics: closed-loop bandwidth (as a measure for speed of response) and disk margin (as a measure of robustness). Unlike traditional phase margin metrics, disk margin offers a more realistic robustness measure in MIMO systems, especially in the presence of model uncertainty or strong coupling [58]. The theoretical understanding of disk margin is given in the next section (Section 5.1).

In the GFL, at low SCR, a major challenge is the presence of high-frequency resonance peaks. These peaks can significantly degrade controller performance if not addressed properly. To mitigate their effect, a notch filter is introduced in the design loop to selectively suppress undesirable resonances. This preserves the desired integral action up to the target bandwidth. This tailored suppression strategy facilitates a robust controller that remains effective across a wide range of network strengths without the need for adaptive control.

For GFM, the design leverages the feedforward cancellation of bus current to effectively cancel the network's influence. The controller is then synthesized around this structure using the same MIMO H_∞ minimisation framework. The proposed GFM tuning is benchmarked against two commonly adopted strategies in literature and practice: SO and Direct Synthesis (DS).

Performance assessments across multiple test systems reveal that the proposed GFL and

GFM designs consistently outperform conventional approaches, both in terms of closed-loop performance and enhanced robustness.

5.1 Disk Margin as Index of Robustness

First, it is important to highlight that the voltage controls of both GFL and GFM are MIMO systems with two channels corresponding to d -axis and q -axis control. For a MIMO system, phase margin loses its significance as the index of robustness as it is unable to capture the cross-coupling effects and simultaneous variations in both channels. As an alternative, disk margin is generally used [58]. Consider a 2x2 MIMO open-loop transfer function $L(s) = K(s)P(s)$. To account for simultaneous gain and phase variations in both input and output channels, an uncertainty disk function $\Delta(\alpha)$ is defined in (5.1):

$$\Delta(\alpha) = \begin{bmatrix} \frac{1+\alpha\sigma_1/2}{1-\alpha\sigma_1/2} & 0 \\ 0 & \frac{1+\alpha\sigma_2/2}{1-\alpha\sigma_2/2} \end{bmatrix} \quad (5.1)$$

Here, both α and σ are complex numbers that characterize variations in amplitude and phase. The optimization problem is formulated to maximize α while ensuring that the closed-loop system of $L(s)\Delta(\alpha)$ remains stable for all variations in σ within the unit circle ($|\sigma_j| < 1$). Solving this optimization problem yields:

$$\alpha_{\max} = \frac{1}{\mu_{\Sigma}(\mathcal{M})} \quad (5.2)$$

where $\mathcal{M} = (\mathbb{I}_2 + L(s))^{-1} - \mathbb{I}_2/2$, and $\mu_{\Sigma}(\mathcal{M})$ represents the structured singular value of $\mathcal{M}$ with respect to diagonal uncertainties $\Sigma = \begin{bmatrix} \sigma_1 & 0 \\ 0 & \sigma_2 \end{bmatrix}$, defined as:

$$\mu_{\Sigma}(\mathcal{M}) = \frac{1}{\min_{\Sigma: \|\Sigma\| \leq 1} \{\|\Sigma\| : \det(\mathbb{I}_2 - \mathcal{M}\Sigma) = 0\}} \quad (5.3)$$

Therefore, $\mu_{\Sigma}(\mathcal{M})$ quantifies the worst-case diagonal perturbation that can render $(\mathbb{I}_2 - \mathcal{M}\Sigma)$ singular, thereby leading to closed-loop instability. Consequently, the disk margin in (5.2) is defined as the reciprocal of this worst-case perturbation severity, ensuring that for all $|\Sigma| < 1$, the system remains stable as long as $\alpha < \alpha_{\max}$. This concept can be extended to MIMO systems with dimensions larger than 2×2 .

5.2 Enhanced GFL Control

This section introduces a novel control strategy for GFL that ensures robust performance across varying grid strengths and network topologies. The design builds upon classical PI-based structures but leverages modern MIMO control objectives and robustness metrics to overcome limitations posed by weak grids and high-frequency resonances.

5.2.1 Disk Margin vs Phase Margin

To design controllers that remain stable and effective under varying grid conditions, a reliable metric of robustness is crucial. Traditionally, phase margin has been widely used as a measure of stability margin. However, in MIMO systems—such as those encountered in IBR control—phase margin can provide a misleading indication of system robustness. This section compares phase margin with disk margin, a more generalized robustness metric suitable for MIMO systems, using a test case involving a GFL connected to a Thévenin equivalent network of Figure 3.6, where it is regulating terminal voltage and active power injection. The system is linearized, and both performance and stability metrics are evaluated while scaling the outer loop PI gains (both P-control and V-control) with a factor ranging from 1 to 8.

In Figure 5.1(a) and (b), as per the expectation, bandwidth increases with control gains for both inner and outer loops, until the system becomes unstable. Phase margins of both P-control (Figure 5.1(c)) and V-control (Figure 5.1(d)), however, fail to track the actual damping behaviour of the least damped mode, shown in Figure 5.2(a)). Notably, there are ranges of controller gain where phase margin remains high while the system is approaching instability—i.e., when the sub-synchronous mode becomes poorly damped. This discrepancy arises because phase margin is derived from single-loop approximations and does not account for inter-loop interactions or the full dynamics of the coupled MIMO system. In contrast, the disk margin (Figure 5.2(b)) exhibits a strong correlation with the damping ratio of the least damped mode. As the outer control gain increases, the disk margin declines in sync with the sub-synchronous mode becoming less stable.

This analysis demonstrates that disk margin is a more reliable and physically meaningful measure of stability in systems where multiple feedback paths, strong coupling, or non-negligible network dynamics exist. Since SSO under study are inherently multi-mode and influenced by inverter-network interactions, disk margin is better suited to guide controller design. Therefore, in the remainder of this chapter, disk margin will be adopted as the primary index of robustness as it allows for robust tuning of voltage controllers in GFL systems across varying grid strengths.

The use of disk margin as performance criteria and its integration in network-inclusive control design process has not been explored in existing literature in the context of 100% IBR-driven power systems.

5.2.2 Identifying the Dominant Contributors to Peaks in Frequency Response

To further understand the origin of peaks in the plant model to be fed to control design process, this section investigates how physical network characteristics influence modal behaviour. A single GFL unit is connected to a large power system, and the transmission line length is swept from 13 km to 130 km. This setup emulates different SCRs and helps isolate the dominant dynamic contributors to the oscillatory modes.

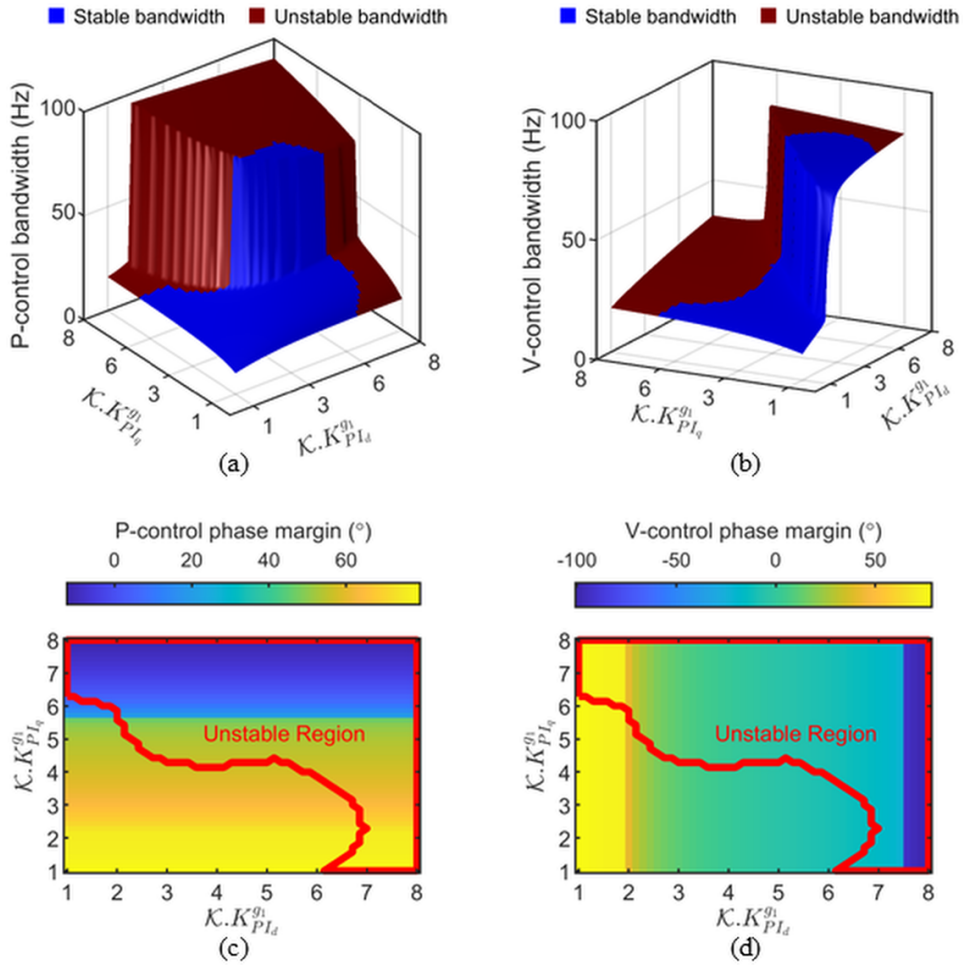


Figure 5.1: Evaluating performance metrics—(a)(b) bandwidths and (c)(d) phase margin while sweeping outer control gains with a scaling factor of 1 to 8. (a), (c) correspond to performance metrics of P-control whereas (b), (d) show the performance metrics of V-control.

Figure 5.3(a) presents the resulting eigenvalue sweep in the SSO frequency range. As the line length increases, the frequency of the critical oscillatory mode decreases from 21.3 Hz down to 2.9 Hz. Consistent with the mode-shape analysis discussed in Chapter 3, it is observed in Figure 5.3(b) that the PLL and network states are the primary contributors to this mode.

Next, one solution to the oscillation is presented whereby reducing the outer-loop control bandwidth increases the damping of the mode. As shown by the blue markers in Figure 5.3(c), lowering the control bandwidth improves the damping of the oscillatory mode, leading to a more stable system response. This impact is also shown in the time-domain response, shown in Figure 5.3(d), which highlights improved dynamic response under reduced control bandwidth.

To understand what limits bandwidth selection in GFL control design—especially under low SCR—Figure 5.4 explores the presence of high-frequency resonance peaks that can destabilize or constrain the control design. Two distinct sets of resonances are identified:

- The first set of higher-frequency peaks (red in Figure 5.4(a)) shifts from 1910 Hz to 1584 Hz and is attributed to the interaction between the filter capacitor and the transformer leakage reactance, as confirmed by Figure 5.4(b).
- The second set of lower-frequency peaks (green in Figure 5.4(c)) ranges from 635 Hz to 121 Hz and is traced to shunt susceptance and transmission line dynamics (Figure 5.4(d)), indicating that this resonance is a network-induced phenomenon.

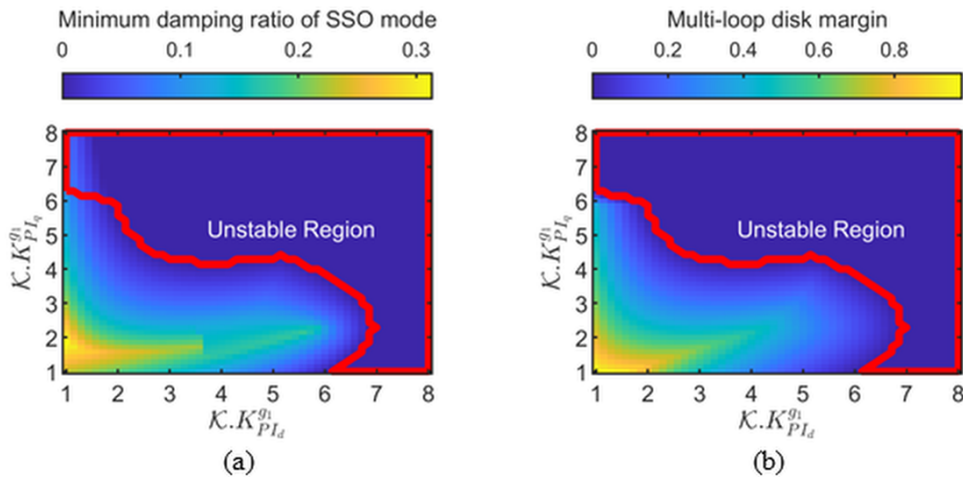


Figure 5.2: Evaluating performance metrics in terms of (a) damping ratio of least-damped SSO mode, and (b) multi-loop disk margin when proportional gain of d- and q- axis are independently scaled. The multi-loop disk margin, unlike phase margin shown in Figure 5.1, corresponds with the actual stability.

These resonances, particularly the second set, pose challenges in control tuning as they constrain the achievable bandwidth and force conservative tuning, especially in weak grids. Their presence justifies the need for targeted suppression strategies, such as the notch filter introduced in the next section. In summary, this analysis underscores that a high-performing GFL voltage control must account for the interfering high-frequency resonances inherent in the network and IBR configuration.

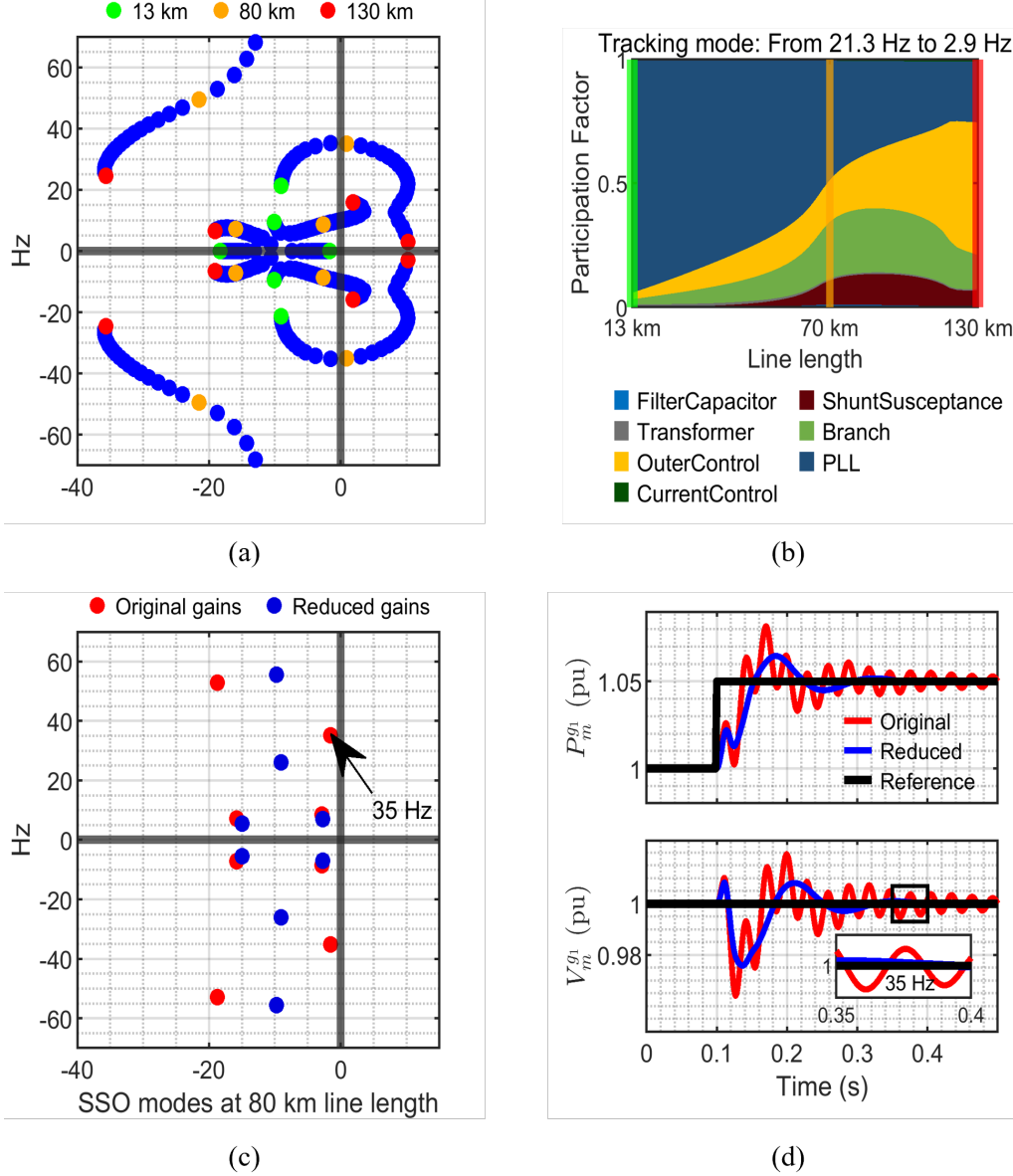


Figure 5.3: Determining cause of instability in least-damped SSO. (a) SSO modes trajectory as line length is increased from 13 km to 130 km. (b) Contribution of states participating in the problematic mode. (c) Reducing gains at the 80 km line length to restore stability. (d) P and V tracking before (red) and after (blue) reducing gains when a step change of 5% is introduced in P reference.

5.2.3 Tuning Methods

This section presents three different PLL and voltage control design methods for GFL. Unlike GFM design (next section), the superscripts are not dropped, as here the idea is generalisable to a GFL connected to a k^{th} bus.

- Converter-Level Design (CLD): A conventional tuning method relying on predefined bandwidth targets, without considering detailed network dynamics.

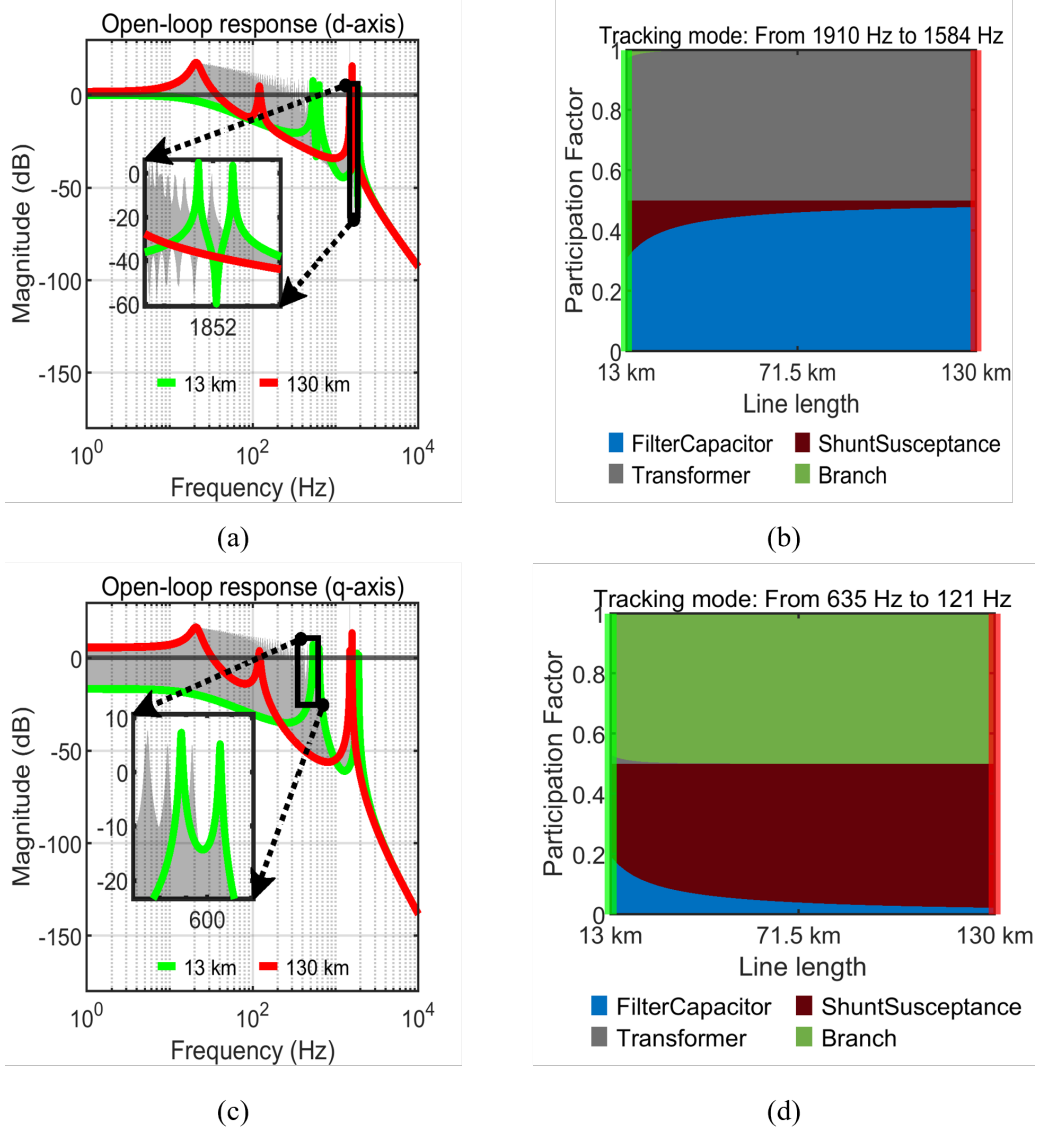


Figure 5.4: Analysing high frequency peaks in the plant. (a) Envelope of the magnitude response when line length is swept from 13 km (green) to 130 km (red). (b) States participating in the the high frequency peaks—around 1852 Hz. (c) Same envelope as (a) but the inset highlights the set of other two mid-range frequency peaks. (d) States participating in the mid-range frequency peaks—around 600 Hz.

- **Network-Level Design (NLD):** A control design framework that incorporates network strength and tunes for a Single-Input Single-Output (SISO) objective instead of MIMO objectives. It also does not address high-frequency resonances.
- **Proposed MIMO-Based Design:** A co-optimised control strategy using H_∞ objectives and notch filtering to suppress interfering network resonances.

Converter-Level Design

The CLD method follows classical tuning guidelines based on frequency-domain targets for different loops. This approach does not explicitly account for the interaction between the converter and the network but relies on a predefined set of design bandwidths to tune the control loops. These design bandwidths act merely as ballpark figures and serve as initial tuning references. The actual closed-loop bandwidths achieved in practice are highly dependent on the operating point and the network strength. If f_{PLL} , f_c , f_p , and f_v denote the target bandwidths PLL, inner current control, active power and voltage control, respectively, then corresponding control gains can be computed as under:

$$\begin{aligned}
 \text{Active Power Control: } K_{P_d} &= \frac{f_p}{f_c}, \quad K_{I_d} = 2\pi f_p \\
 \text{Voltage Control: } K_{P_q} &= -\frac{f_v}{f_c}, \quad K_{I_q} = -2\pi f_v \\
 \text{PLL Control: } K_{P_\theta} &= -4\pi f_{\text{PLL}}, \quad K_{I_\theta} = -\left(\frac{2\pi f_{\text{PLL}}}{\zeta}\right)^2
 \end{aligned} \tag{5.4}$$

where ζ is damping coefficient for second-order PLL approximation (Chapter 4.1.1).

Network-Level Design

The NLD approach incorporates the rest of the system into the design, as shown in Figure 5.6. It does not require the dynamic model of other IBRs. A 3×3 MIMO system model G_{sys} is formed for the k^{th} GFL converter. This model captures the IBR's interaction with the rest of the system.

The controller tuning is then carried out in a sequential loop-by-loop fashion, as illustrated in Figure 5.6. First, the PLL is closed using the designed gains $K_{P_\theta}^{g_k}, K_{I_\theta}^{g_k}$ to form G'_{sys} . Second, the P-control loop is closed to obtain G''_{sys} , followed by closing the V-control loop with gains $K_{P_q}^{g_k}, K_{I_q}^{g_k}$ to complete the design.

Although this method better reflects the influence of grid strength, it still does not consider cross-coupling effect and network resonances during the tuning process.

Proposed Design

The proposed control design framework enhances the NLD approach by addressing its primary limitation: vulnerability to high-frequency resonances introduced by network impedance. The first step is to append a notch filter to the model G'_{sys} (obtained after closing the PLL), as shown in Figure 5.7(a). This filter is specifically tuned to suppress resonances that could otherwise limit the achievable control bandwidth or even destabilize the loop, particularly under weak-grid conditions. A constrained bandwidth would lead to an overly conservative design, compromising fast tracking performance and slowing down reactive current injection during faults unless the control regime is switched altogether.

Next, a co-optimization process is used to simultaneously design the gains of the P -control and V -control loops along with four additional coupling gains (C_{11}^{gk} , C_{12}^{gk} , C_{21}^{gk} , C_{22}^{gk}) that reduces the cross-coupling interaction between d - and q - axes. The optimization objective, in principle, is similar to proposed GFM voltage control design and is detailed in Section 5.3.1.

A sample notch filter magnitude response and its impact on plant response envelope of Figure 5.4 is shown in Figure 5.8.

5.2.4 Performance Comparison

To evaluate the effectiveness and robustness of the proposed control design, four test systems are considered. Each test system involves performance comparison at different operating condition while monitoring key performance metrics—namely, the bandwidths of the P - and V -control loops and the multi-loop disk margin.

Single GFL Connected to Thevenin Equivalent

This test system considers a single GFL connected to a simple Thevenin equivalent system, as shown in Figure 3.6. Figure 5.9 illustrates a comparative assessment of three control design strategies applied to a GFL connected with a Thévenin equivalent system. Each row corresponds to a performance metric: the P -control bandwidth (top), V -control bandwidth (middle), and multi-loop disk margin (bottom). The colormaps indicate performance across a range of design bandwidths or design SCR (y-axis) and operational SCRs (x-axis), with dark blue regions denoting instability.

In the CLD case (left column), the controller is designed at a fixed bandwidth and tested across varying SCRs. Results show performance degradation and instability at lower SCRs. For instance, a design at 9.2 Hz becomes unstable below an SCR of approximately 1.25, while

a 15.3 Hz design remains stable only for SCRs above 1.7. These trends indicate CLD's poor robustness and sensitivity to grid strength.

In the NLD case (middle column), the controller is designed for each SCR value on the y-axis and evaluated across other SCR values. Although including network in the design improves performance in certain regions, the results remain fragmented, and bandwidth performance is inconsistent. Some designs become unstable even under modest SCR deviations, indicating limited robustness.

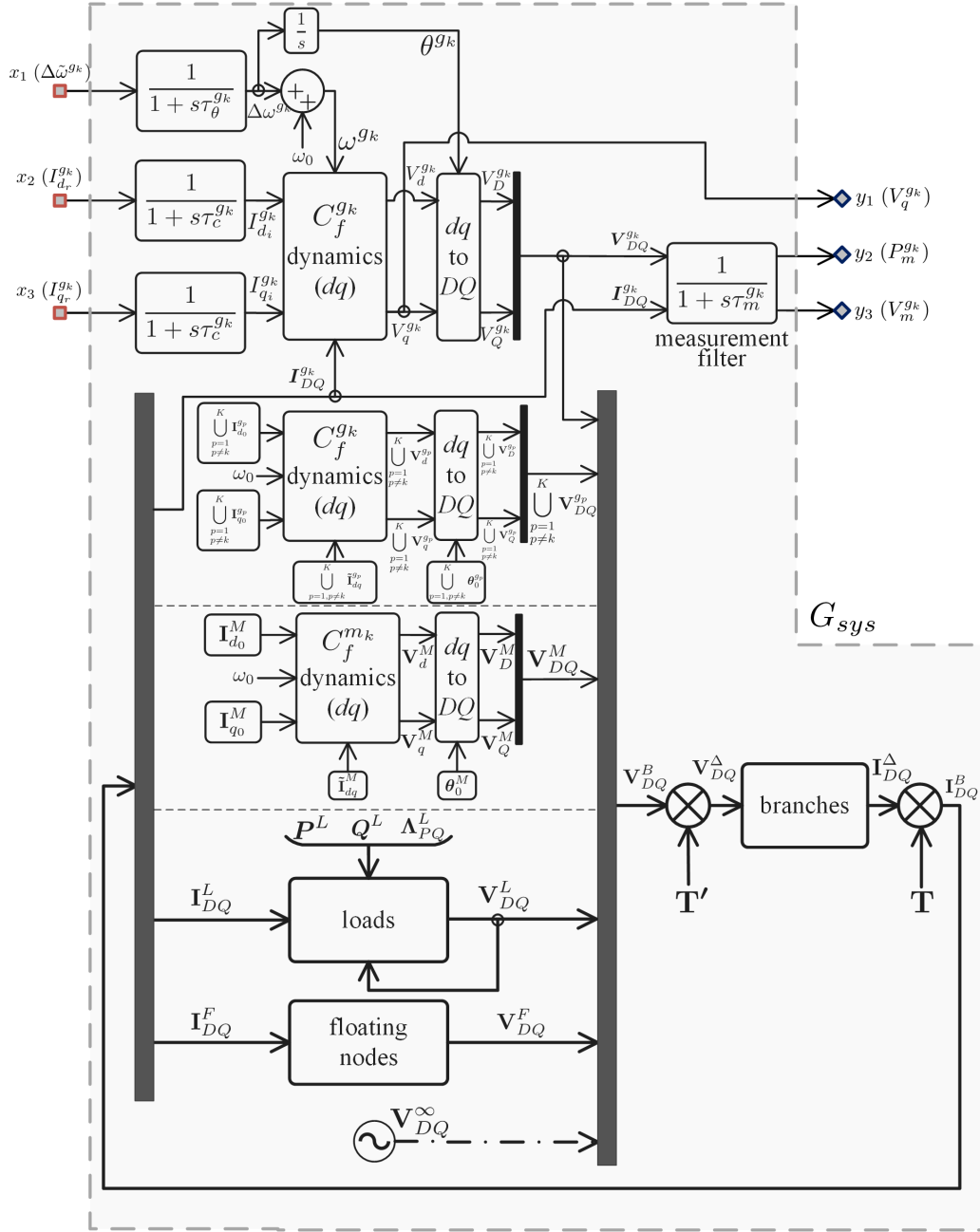


Figure 5.5: Formation of G_{sys} (3x3 MIMO system) for k^{th} GFL from the generalisable framework. Possible presence of infinite bus is handled by connecting a voltage source directly to the virtual bus (shown by dash-dot line at the bottom).

The proposed design (right column) demonstrates much improved and consistent performance. Both P - and V -control loops maintain high bandwidths across the entire SCR range. More importantly, the multi-loop disk margin remains well above the acceptable threshold of 0.5 throughout, with no observed instability.

Overall, the proposed approach clearly outperforms both CLD and NLD in terms of stability and robustness, particularly under weak-grid conditions.

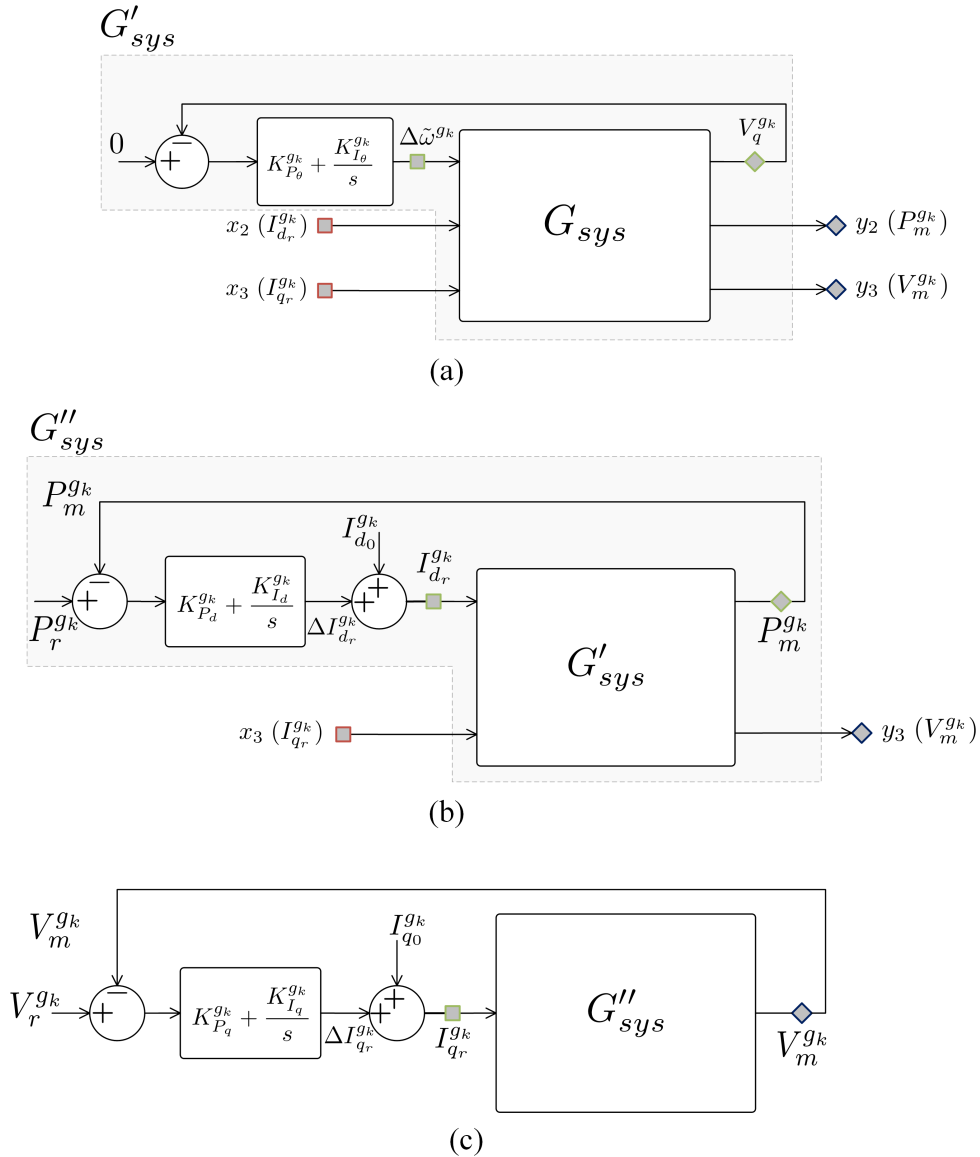


Figure 5.6: Formation of sub-systems for SISO designs. (a) Closing PLL with designed gains $(K_{P_\theta}^{gk}, K_{I_\theta}^{gk})$ to obtain G'_{sys} . (b) Closing P -control loop with designed gains $(K_{P_d}^{gk}, K_{I_d}^{gk})$ to obtain G''_{sys} . (c) Closing V -control loop with designed gains $(K_{P_q}^{gk}, K_{I_q}^{gk})$ to complete the design process for k^{th} GFL.

Single GFL Connected to a Large Power System

Here, a single GFL is connected to a more realistic large power system via a transformer and a line, as shown in Figure 3.2. Figure 5.10 presents a comparative analysis of the three control

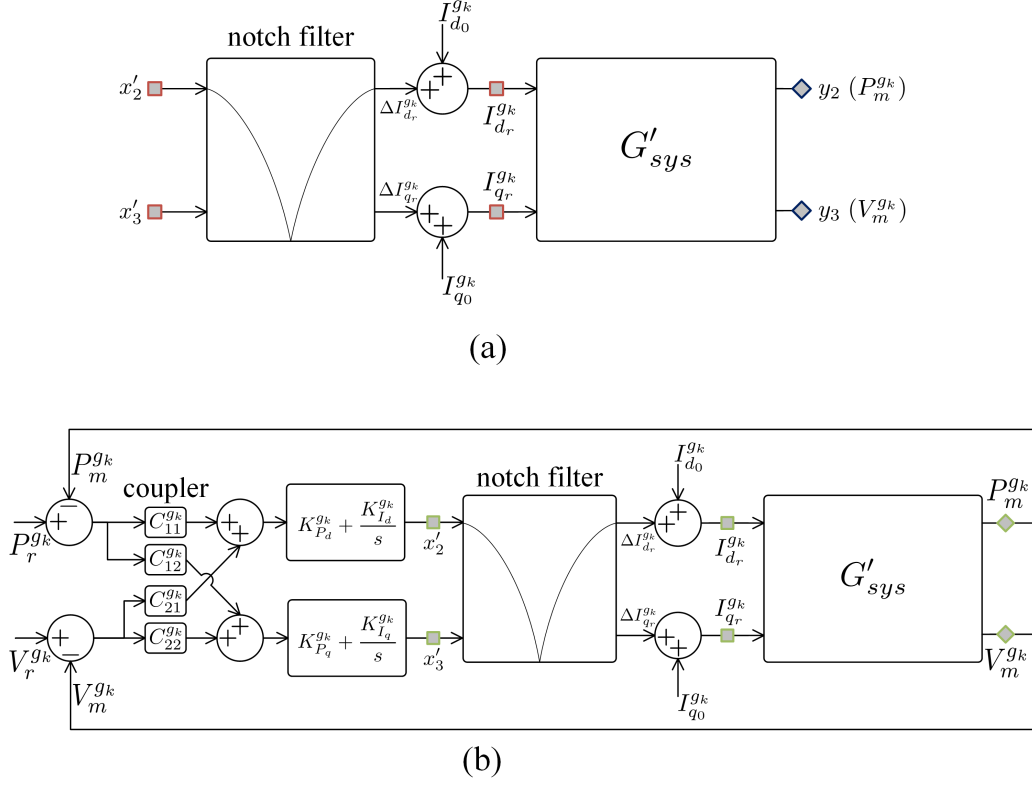


Figure 5.7: MIMO design process. (a) Appending notch filter with G'_{sys} of Fig. 5.6(a) to mitigate unwanted interfering network frequencies. (b) Obtaining control gains of P -control ($K_{P_d}^{gk}, K_{I_d}^{gk}$), V -control ($K_{P_q}^{gk}, K_{I_q}^{gk}$), and coupling gains ($C_{11}^{gk}, C_{12}^{gk}, C_{21}^{gk}, C_{22}^{gk}$) through co-optimisation of MIMO objectives for k^{th} GFL.

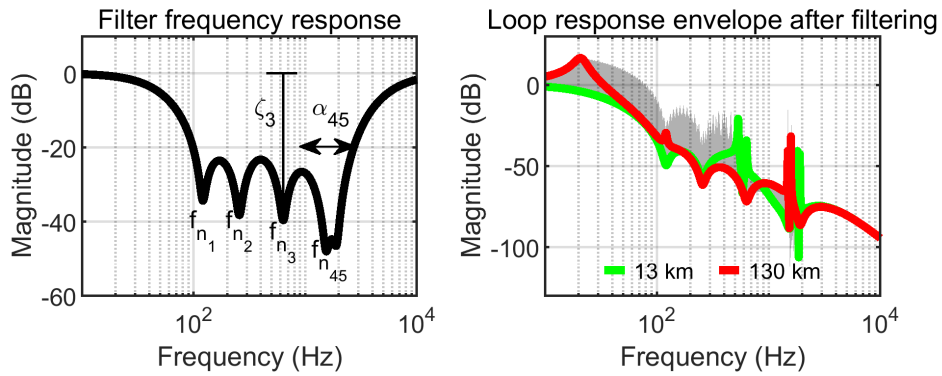


Figure 5.8: Magnitude response of the combination of four notch filters (left). Its impact on the envelope of plant's magnitude response (right).

strategies (CLD, NLD, and the proposed MIMO-based design) under varying transmission line lengths, capturing weak-grid effects through increased electrical distance. Each subplot maps the system's response across a grid of design bandwidths or design line lengths (y-axis) and operational line lengths (x-axis). Rows show P -control bandwidth (top), V -control bandwidth

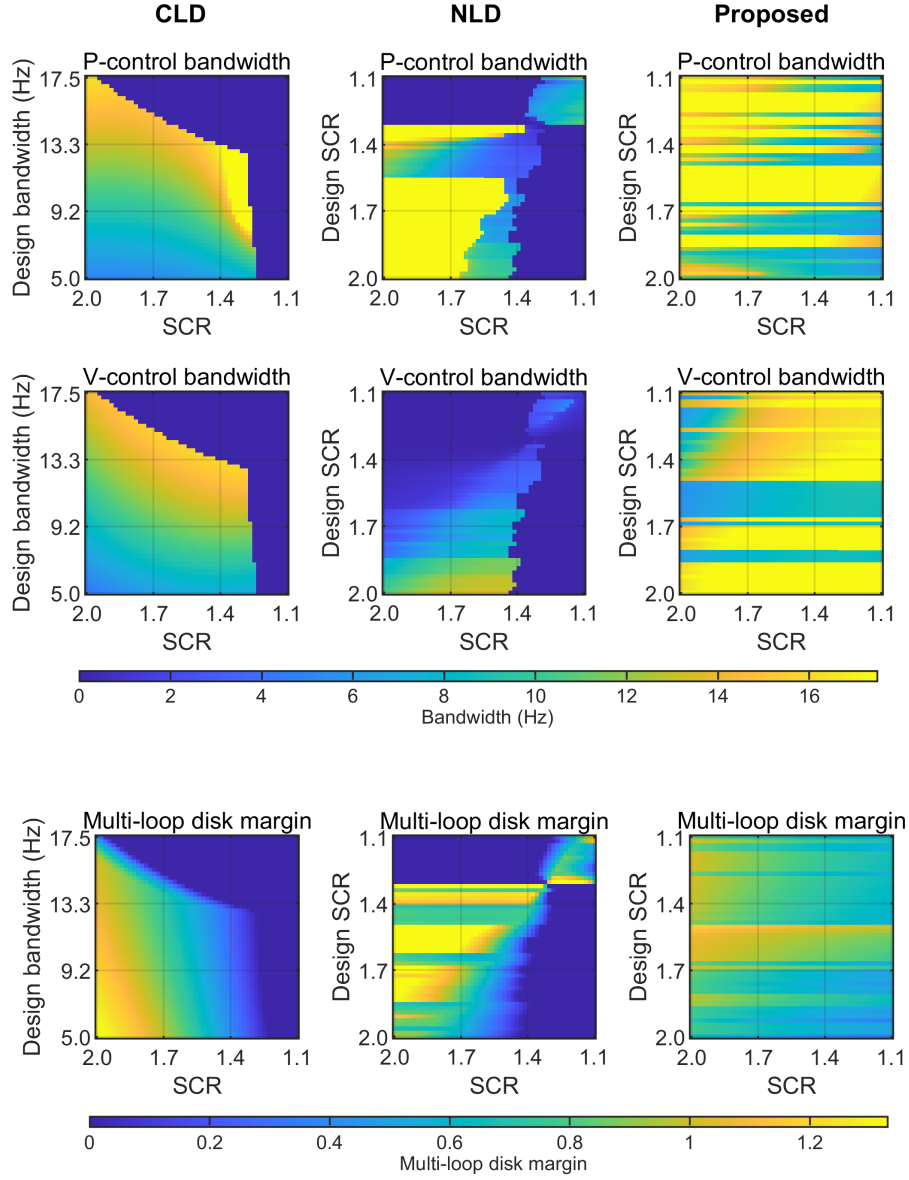


Figure 5.9: A GFL connected to Thévenin's equivalent circuit. For CLD (left column), against a specific design bandwidth, operational SCR is swept and performance metrics; P -control bandwidth (top), V -control bandwidth (middle), and multi-loop disk margin (bottom) are observed. Similarly, for NLD (middle column) and proposed (right column), against a specific design carried out at a SCR of y-axis, operational SCR is swept and performance metrics are monitored.

(middle), and multi-loop disk margin (bottom). As before, dark blue regions indicate instability.

For CLD (left column), the controller is tuned at a fixed bandwidth and evaluated over different line lengths. The results show that increasing line length causes a sharp deterioration in performance, with instability appearing beyond 70–80 km depending on the design bandwidth. Both P - and V -control loops suffer substantial bandwidth reduction in longer lines, and the disk margin collapses in a large region, exposing the CLD design’s vulnerability in weak-grid conditions. For NLD (middle column), the controller is designed at the specific line length shown on the y-axis and tested across other operating lengths. The inclusion of network dynamics in design process enhances performance for certain design points, but stability remains fragmented and bandwidths are inconsistent. The proposed method (right column) again delivers consistently high performance across all line lengths. Both control loops sustain higher bandwidths, and the multi-loop disk margin remains well above the critical stability threshold throughout the operating region. Even in long-line, weak-grid scenarios, the proposed design maintains stable and robust operation. Only in the lower right corner when design length is much smaller than the actual length, the system becomes unstable.

In summary, this figure highlights the clear advantage of the proposed method in accommodating variations in electrical distance, offering superior bandwidth and stability margins compared to both CLD and NLD across a wide range of line lengths.

9-Bus System with Different GFM Placements

This test evaluates the impact of GFM placement in a 9-bus system (Figure 3.15) on closed-loop control performance of three control design schemes of GFL. Three scenarios are considered: GFM located at Bus 1 (Case 1), Bus 2 (Case 2), and Bus 3 (Case 3), while the remaining buses host GFL. The comparison spans three controller design strategies: CLD, NLD, and the proposed MIMO-based design.

Figure 5.11 summarises the control performance using two key metrics: First, radar plots display the achieved closed-loop bandwidths (in Hz) of both active power (P) and voltage (V) control loops at the three GFL buses. Second, the bar chart quantifies the corresponding multi-loop disk margin for each case.

Across all three cases, the proposed design consistently delivers the highest overall bandwidths for both control loops. The radar plots reveal that the proposed method maintains a balanced and expanded control envelope across all buses. In Case 3, although the bandwidth of P_{b2} under NLD is slightly higher than that achieved by the proposed design, this advantage

is marginal and limited to a single control channel. In contrast, the proposed design maintains superior voltage control bandwidths across all GFL buses.

The bar chart further supports this assessment: the proposed design maintains a multi-loop disk margin above the critical stability threshold in all three scenarios, comparable to NLD

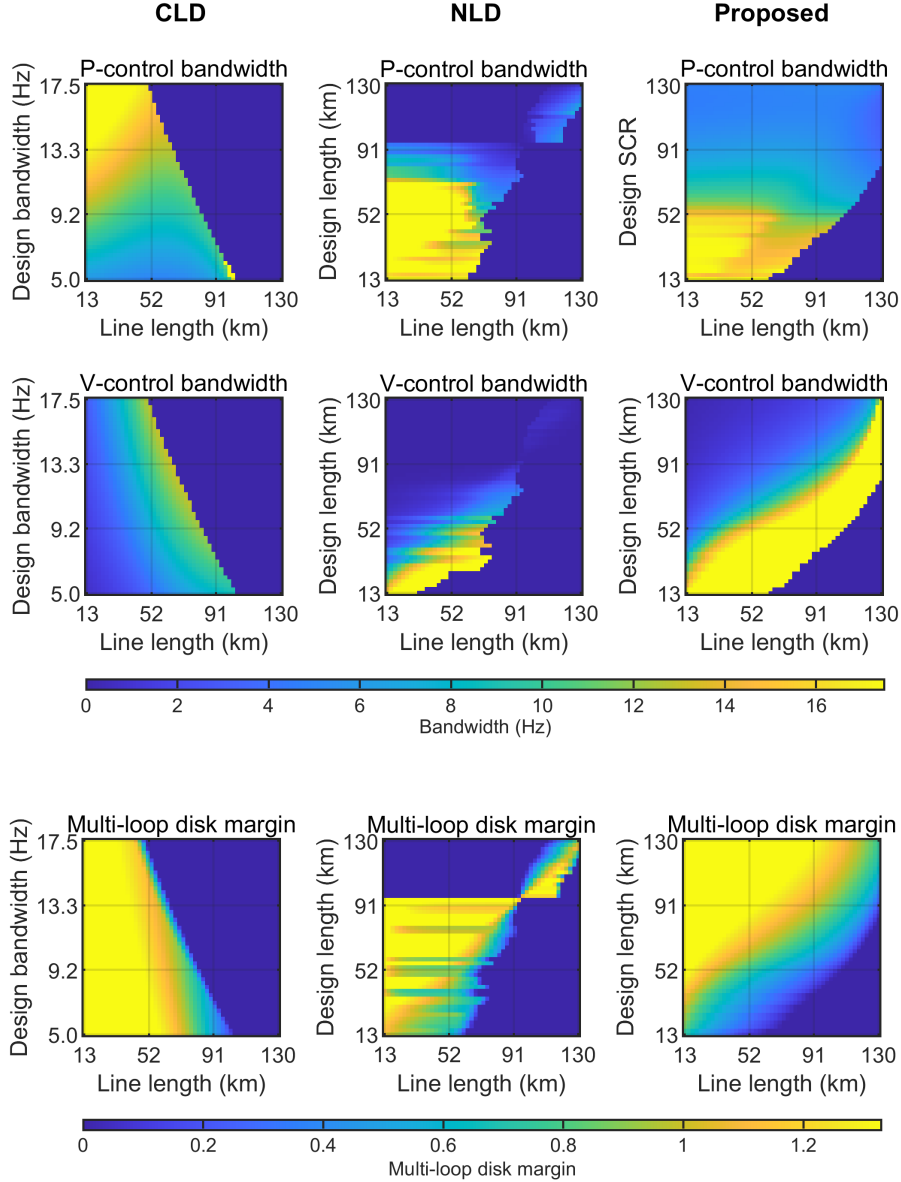


Figure 5.10: A GFL connected to a large power system via transformer and a line. For CLD (left column), against a specific design bandwidth, operational SCR is swept and performance metrics; P-control bandwidth (top), V-control bandwidth (middle), and multi-loop disk margin (bottom) are observed. Similarly, for NLD (middle column) and proposed (right column), against a specific design carried out at a SCR of y-axis, operational SCR is swept and performance metrics are monitored.

and much higher than CLD. This confirms that the proposed design does not compromise robustness while delivering higher bandwidth.

11-Bus System with Regional GFL/GFM Mix

The final test case involves an 11-bus 2-area system [3] with different regional mixes of GFL and GFM inverters. Figure 5.12, the radar plots illustrate the achieved bandwidths of the P - and V -control loops at selected buses in each area, while the bar chart summarizes the multi-loop disk margin across the three scenarios. In Case1, all three methods are compared, and while NLD offers higher bandwidths, the proposed design still provides a reasonably well-distributed response with acceptable disk margin. However, in Cases2 and 3, only the proposed design

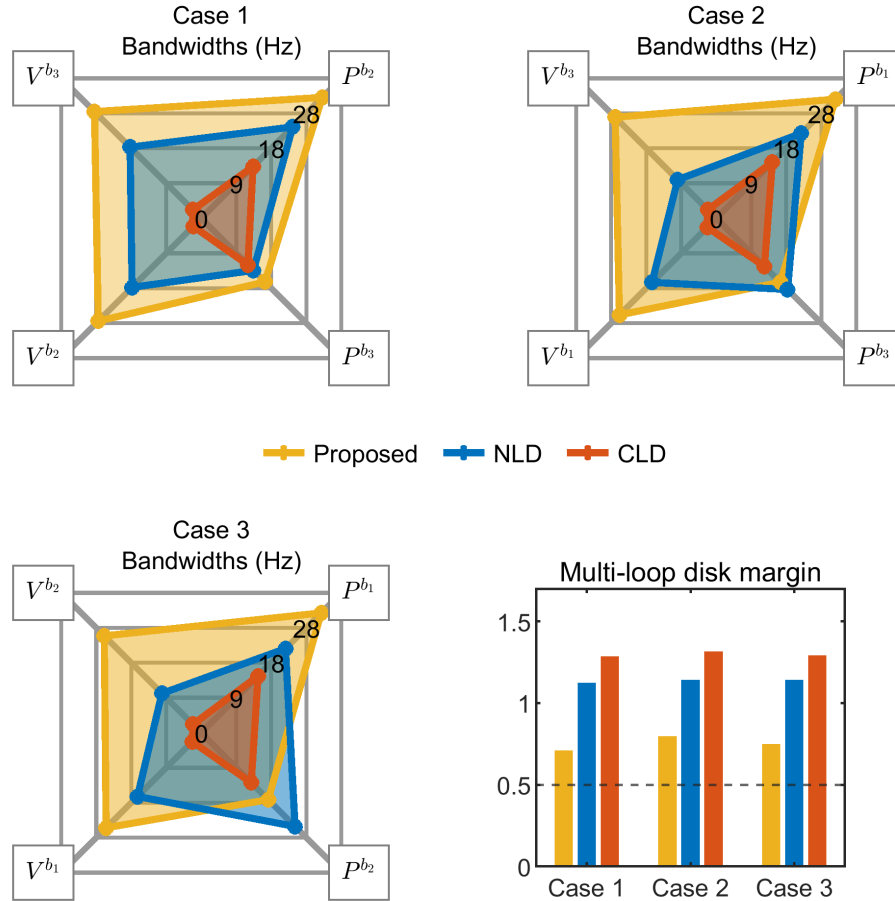


Figure 5.11: 9-bus system performance charts. Comparing P-control and V-control bandwidths of both GFLs for: Case 1 (top left), where B1 is GFM; Case 2 (top right), where B2 is GFM; and Case 3 (bottom left), where B3 is GFM. Comparing multi-loop disk margin (bottom right) for all three cases.

yields a stable controller i.e., both CLD and NLD fail to stabilise the system under these challenging configurations, likely due to weak inter-area coupling and unsuppressed resonance. The proposed method maintains robust performance by achieving consistent bandwidths across all loops and preserving disk margins. This demonstrates the superior robustness of the proposed control scheme in complex multi-area systems where conventional methods are no longer viable.

Overall, Figures 5.11 and 5.12 demonstrate that the proposed control strategy outperforms both NLD and CLD by offering a better balance between dynamic performance and stability margin, regardless of GFM location.

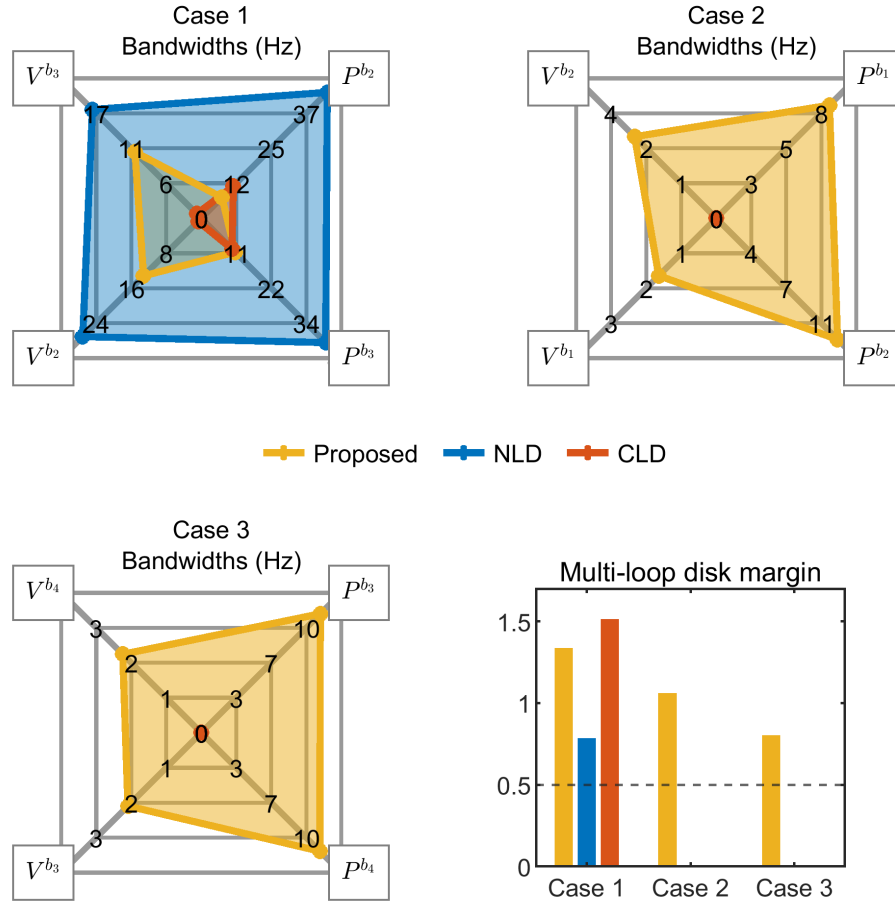


Figure 5.12: 11-bus system reference tracking comparison for Case 1 (top), where both areas have mix of GFL and GFM; Case 2 (middle), where Area 1 is GFL and Area 2 is GFM; and Case 3 (bottom), where Area 1 is GFM and Area 2 is GFL. For Case 2 and Case 3, CLD and NLD cannot provide a stable design.

5.2.5 Summary

Across all four test cases, a clear trend emerges: while CLD and NLD offer reasonable performance under specific conditions, their effectiveness is highly sensitive to variations in network strength and topology. The CLD approach, in particular, fails to maintain performance or robustness in weak-grid scenarios due to its lack of network awareness. NLD improves upon this by considering the SCR at the design stage, but it remains vulnerable to high-frequency network resonances that constrain bandwidth and degrade disk margin.

In contrast, the proposed MIMO-based design framework—with its incorporation of notch filtering and co-optimization of multi-loop gains—demonstrates consistent and superior performance. It delivers higher bandwidths for both P - and V -control loops and maintains robust disk margins across a wide range of SCRs and system configurations. Most critically, it is the only method that ensures stability and effective tracking in complex multi-area systems, such as those presented in the 11-bus case study.

5.3 Enhanced GFM Voltage Control

As already discussed in Chapter 2.2, in GFMs, the active power controller and reactive power controller determine the voltage angle and magnitude at the PCC, respectively [44]. This chapter adopts droop-based active power controller [59] while omitting reactive power controller, treating GFM buses as voltage-controlled nodes. Once the active power controller sets the voltage magnitude, inner control loops for voltage and current regulation generate the inverter's modulation index [60]. These controllers are typically tuned locally, with current control often relying on pole-zero cancellation [46], where PCC voltage feedforward compensates for filter dynamics. This approach simplifies current dynamics to a first-order lag with a predefined closed-loop bandwidth.

For voltage regulation, the SO method is commonly employed. SO tuning relies on network current feedforward terms to decouple grid dynamics from the inverter, theoretically improving disturbance rejection and transient response [46]. However, this decoupling is effective only in networks with predominantly resistive characteristics [48]. Additionally, SO tuning relies on a large current control bandwidth that is often considered impractical for high-power applications, where the need to minimise switching losses typically necessitates operating at lower switching frequencies [49]. Since current control bandwidth is typically maintained at least an order of magnitude lower than the switching frequency to prevent high-frequency interactions,

assumption of current control bandwidth to be very high might not reflect real-world conditions. Hence, voltage control designs based on these assumptions are prone to instability, particularly when the network's influence on feedforward currents becomes more significant.

Unlike GFLs, GFMs can encounter stability challenges in strong grid conditions due to their direct voltage regulation at the PCC. When multiple voltage sources are in close electrical proximity, conflicts in voltage regulation may arise [61]. To mitigate this issue, various strategies have been explored in the literature, including retuning power loop controller gains [62], implementing cross-coupling control techniques [63], and integrating virtual impedance methods [64]. The first method is a more of an ad-hoc fix that does not address the root cause of instability. The second and third one are applied on the active power controller and hence instability arising from poor voltage control design could not be directly addressed. In [50], feedforward attenuation method is proposed for improving GFM performance at strong grid by introducing an additional gain. However, its design relies on satisfying a necessary and sufficient condition based on the Routh-Hurwitz criterion, which depends on network equivalent resistance and inductance. These parameters are not fixed in practical systems. This section, in contrast, proposes a tuning method for standard PI control that remains robust across varying grid strengths, making it a more practical approach.

5.3.1 Tuning Methods

Here, two conventional control tuning approaches are presented: direct synthesis (DS) and symmetrical optimum (SO) methods. By examining their design characteristics, baseline for comparison with the proposed tuning methodology has been established. Here, too, since the study is solely focused on GFM voltage control, the superscripts shown in the full model (Figure. 2.6) are dropped. Therefore, the design takes place for K_{P_d} , K_{P_q} , K_{I_d} , and K_{I_q} . The classical tuning methods discussed in this section are based on two key assumptions: (1) the current feedforward terms are not influenced by the GFM voltage output, and (2) the current control bandwidth is sufficiently high to ensure effective feedforward cancellation.

Direct Synthesis Method

The direct synthesis (DS) is a classical approach where control gains are determined based on the desired phase margin and gain crossover frequency. With assumptions made above, GFM plant model can be extracted from Figure 2.5. The plant consists of a series of closed-loop current control (first-order lag) and a shunt filter capacitor. With a PI controller, the loop gain

$L(s)$ is given below:

$$L(s) = \left(K_{P_d} + \frac{K_{I_d}}{s} \right) \left(\frac{1}{1 + s\tau_c} \right) \left(\frac{1}{sC_f} \right) \quad (5.5)$$

Let f_{gc} denote the target gain crossover frequency (Hz) and δ_{PM} represent the desired phase margin (degrees). The corresponding design requirements can be mathematically expressed as follows:

$$|L(j2\pi f_{gc})| = 1 \quad (5.6a)$$

$$\angle L(j2\pi f_{gc}) = \delta_{PM} \quad (5.6b)$$

Solving (5.6a) and (5.6b) for positive K_{P_d} and K_{I_d} gives:

$$K_{P_d} = 2\pi f_{gc} C_f \sqrt{1 + (2\pi f_{gc} \tau_c)^2} \sin \phi \quad (5.7a)$$

$$K_{I_d} = (2\pi f_{gc})^2 C_f \sqrt{1 + (2\pi f_{gc} \tau_c)^2} \cos \phi \quad (5.7b)$$

where $\phi = (\pi\delta_{PM}/180) + \tan^{-1}(2\pi f_{gc} \tau_c)$. The q -axis gains can be determined in the same manner. Thus, $K_{P_q} = K_{P_d}$ and $K_{I_q} = K_{I_d}$.

This method prioritizes achieving the desired gain crossover frequency over the closed-loop -3 dB bandwidth, f_t . However, these parameters are interrelated, and for a broad range of bandwidth and phase margin combinations, they empirically satisfy the relation $f_{gc} = 0.84f_t$. This relation is also discussed in [65]. Therefore, DS uses (5.7a) and (5.7b) to calculate the proportional and integral gains, K_{P_d} , K_{P_q} , K_{I_d} , and K_{I_q} , for a given target closed-loop bandwidth, f_t , and phase margin δ_{PM} . It is important to note that this design is performed locally, without accounting for network strength or time constant.

Symmetrical Optimum Method

This design method has been discussed earlier in Chapter 4.2. But in this section, additional details are provided. As mentioned earlier, the SO method is also designed locally. It relies only on GFM parameters (τ_c and C_f). Gain crossover frequency f_{gc} is the only design specification required to determine the control gains $K_{P_{dq}}$ and $K_{I_{dq}}$, (4.13). These equations are once again given below for easy referencing:

$$K_{P_d} = K_{P_q} = 2\pi f_{gc} C_f \quad (5.8a)$$

$$K_{I_d} = K_{I_q} = (2\pi f_{gc})^3 \tau_c C_f \quad (5.8b)$$

In SO method, the phase margin δ_{PM} is not specified; instead, it is inherently related to f_{gc} , independent of the control gains, as given by:

$$f_{gc} = \frac{1}{2\pi\tau_c} \sqrt{\frac{1 - \sin \delta_{PM}}{1 + \sin \delta_{PM}}} \quad (5.9)$$

According to Figure 24 of [50], the SO method follows a rule of thumb where $f_{gc} = 0.63f_t$. Here too the design is not dependent on the scaling. However, it is important to emphasise that the effectiveness of the proposed control design is not critically dependent on this specific scaling. The role of this scaling factor is merely to align the achieved control bandwidth with the intended design bandwidth under ideal conditions, that is, in the absence of network-induced effects and cross-coupling between control loops. Consequently, in the SO method, the designer can specify either δ_{PM} or f_t to determine the control gains.

In Figure 5.13, the target closed-loop bandwidth f_t is varied from 20 Hz to 50 Hz for both tuning methods. Three different current control bandwidths (250 Hz, 500 Hz, and 750 Hz) are considered to assess their impact on the deviation between the measured and target bandwidths. With DS, the measured bandwidth f_m matches well with f_t , whereas with SO, f_m deviates from f_t . The phase margin plots (on the right) illustrate that DS successfully achieves the desired phase margin. In contrast, in the SO method, the phase margin follows (5.9), as depicted on the far right of Figure 5.13. This comparison demonstrates that between the two local tuning methods, DS outperforms SO in accurately achieving the target bandwidth and phase margin. Conversely, SO struggles to match the desired bandwidth, and its phase margin deteriorates as the target frequency increases.

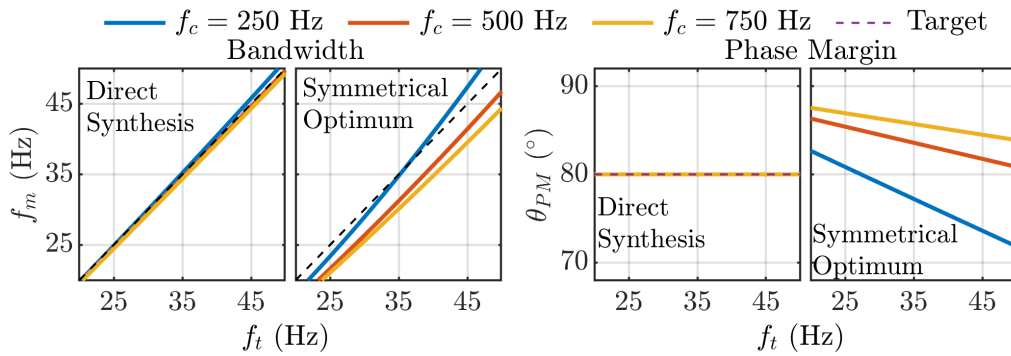


Figure 5.13: Comparing bandwidth (left) and phase margin (right) of two tuning methods—direct synthesis and symmetrical optimum—oblivious to network strength for three current control bandwidths

Proposed Tuning Method

Classical tuning methods rely on phase margin as the primary stability margin criterion, which may not be adequate for GFM voltage control which is essentially a multiple-input multiple-output (MIMO) system. To address this limitation, from this point onward in this section, disk margin is employed as the stability margin metric instead of phase margin.

In the proposed tuning method, the remainder of the system at the PCC of the GFM is represented using its Thévenin equivalent, as depicted in Figure. 5.14, where all quantities are expressed in GFM rotating reference frame unlike Figure. 3.6. During the design process, the filter inductor L_f and resistance R_f are excluded, as they are already accounted for in the closed-loop current control [46]. The value of power-frequency droop $D_{P\omega}$ governs θ in the frame transformation.

Figure 5.15 illustrates the plant model used in the proposed method. Both the d -axis and q -axis controllers are tuned concurrently to account for cross-coupling effects. The tuning objective is to ensure that both control loops closely achieve the target gain crossover frequency, f_{gc} , while preserving integral action at lower frequencies and ensuring a steep gain roll-off at higher frequencies. To meet these performance objectives, the design requirements are converted into weighting functions, which are then incorporated into an H_∞ optimization framework. The optimization aims to determine the control gains that minimize the H_∞ norm, aiming to balance performance and robustness [66].

The design requirements—achieving a target gain crossover frequency of f_{gc} , maintaining integral action for frequencies below f_{gc} , and ensuring a steep roll-off beyond f_{gc} —can be formulated using two weighting functions:

$$W_p(s) = \frac{s + 2\pi f_{gc}}{s}, \quad W_u(s) = \frac{s + 2\pi f_{gc}}{2\pi f_{gc}}. \quad (5.10)$$

Applying $W_p(s)$ to the sensitivity function $S(s) = (\mathbb{I}_2 + L(s))^{-1}$, enforces the target gain crossover frequency while preserving integral action below f_{gc} . Conversely, applying $W_u(s)$ to

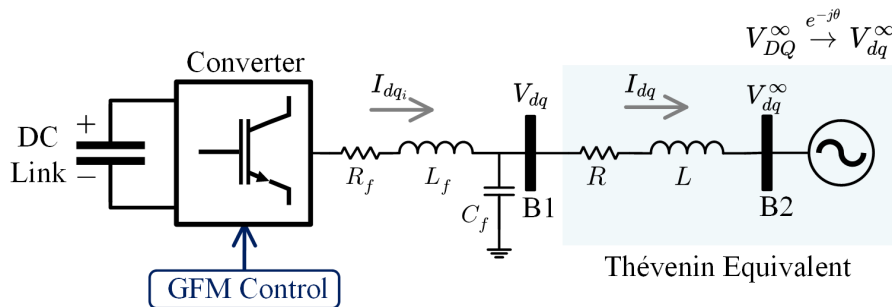


Figure 5.14: A GFM connected to a Thévenin equivalent of a power system

the complementary sensitivity function, $T(s) = L(s)(\mathbb{I}_2 + L(s))^{-1}$, enhances stability margins and ensures a rapid gain roll-off at higher frequencies.

Using these weighting functions, the H_∞ norm of the following stacked system is minimized:

$$\min_{K_{P_{dq}}, K_{I_{dq}}} \left\| \begin{bmatrix} W_p(s)S(s) \\ W_u(s)T(s) \end{bmatrix} \right\|_\infty$$

Figure 5.15 indicates the inclusion of dynamic network equivalent in the plant $P(s)$ (yellow box), governed by following set of equations in dq frame.

$$\dot{I}_d = (V_d - V_d^\infty - I_d R)/L + \omega_0 I_q \quad (5.11a)$$

$$\dot{I}_q = (V_q - V_q^\infty - I_q R)/L - \omega_0 I_d \quad (5.11b)$$

Instead of characterizing the network using R and L , it is defined in terms of SCR ($= 1/\sqrt{R^2 + (\omega_0 L)^2}$), and the reactance-to-resistance ratio (X/R). This representation makes the characterization of the dynamic network more directly comparable to bulk transmission network parameters. Consequently, in the proposed tuning method, control gains are determined not only by specifying the target gain crossover frequency f_{gc} but also by incorporating SCR and X/R as additional design parameters.

With the definition of three distinct tuning methods, three sets of voltage control gains are obtained to evaluate their performance across various test systems and grid conditions.

In the next section, sensitivity of each design to variations in grid strength is examined to assess how changes in network conditions influence bandwidth and robustness. Subsequently, the performance evaluation is extended to multi-IBR test cases, analysing the effectiveness of each tuning approach in a more realistic power system scenario.

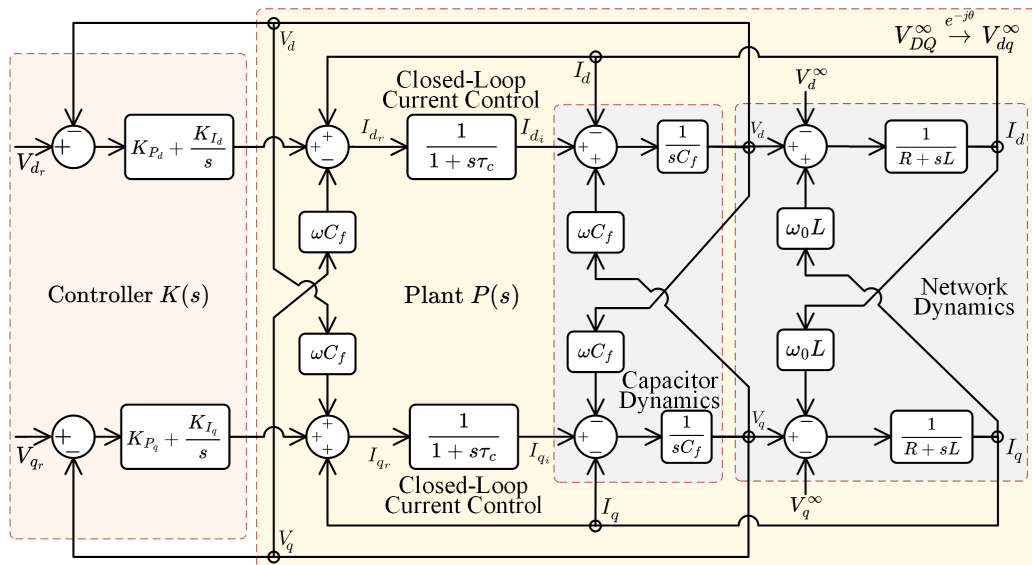


Figure 5.15: A MIMO plant $P(s)$ and its controller $K(s)$ with loop gain $L(s) = K(s)P(s)$.

5.3.2 Performance Comparison

First, the performance of the three tuning methods in a single-GFM-infinite bus system is compared. The evaluation is then extended to two test systems incorporating a mix of GFL and GFM inverters. The GFL control design and dynamic model follow the approach outlined in Chapter 2.1.

Impact of Network

To assess the impact of network variations, three target bandwidths (40 Hz, 50 Hz, and 60 Hz) are tested for each tuning method. The target bandwidths are intentionally set on the higher side to easily discern the differences in performance. For all designs, the filter capacitor is rated at 20% of rated power of IBR and current control closed-loop bandwidth is kept at 500 Hz. For DS, the target phase margin $\delta_{PM} = 80^\circ$. Similarly, for the proposed design, SCR and X/R are fixed at 6 and 10, respectively.

Table 5.1 presents the voltage control gains calculated using three tuning methods. These parameters are used to obtain Figure. 5.16 and Figure. 5.17 (50 Hz design).

Figure 5.16 illustrates the adverse effects of network variations on the two classical methods (yellow for DS, red for SO). Both methods exhibit lower bandwidth (indicating slower response) and become unstable at higher SCR values, as compared to the proposed one. This instability arises in DS and SO because network dynamics were not accounted for in the design. As

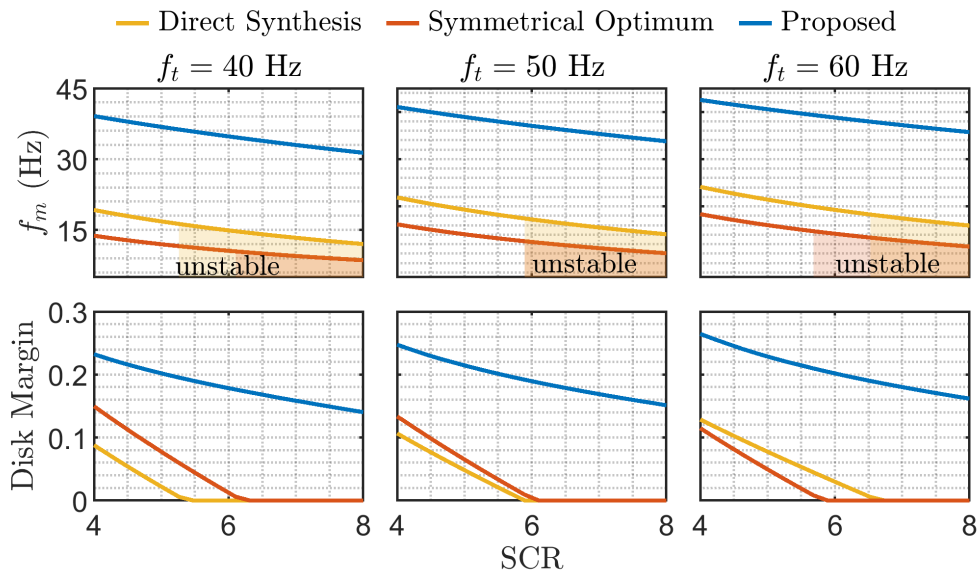


Figure 5.16: V_{dq} closed-loop bandwidth and disk margin comparison of three tuning methods under varying network strength for different target bandwidths

Table 5.1: Control Gains for Three Tuning Methods

f_t (Hz)	Direct Synthesis		Symmetric		Proposed	
	$\delta_{PM} = 80^\circ$		Optimum		SCR = 6, $X/R = 10$	
	$K_{P_{dq}}$	$K_{I_{dq}}$	$K_{P_{dq}}$	$K_{I_{dq}}$	$K_{P_{dq}}$	$K_{I_{dq}}$
40	0.22	5.24	0.14	1.70	1.24	0.00
50	0.27	5.57	0.17	2.94	1.48	0.00
60	0.31	4.89	0.19	4.78	1.73	0.00

a result feedforward cancellation becomes less effective when the impact of network grows. Consequently, the integral action of the PI controller interacts unfavourably with the network. Moreover, since the classical methods neglect dq -axis cross-coupling, the tracking performance may further be degraded. In contrast, the proposed design (blue traces) maintains higher bandwidth and disk margins across all variations.

The top row of Figure. 5.17 illustrates the eigenvalue trajectories for the three tuning methods at a target bandwidth of 50 Hz. As observed in Figure 5.16, both DS and SO become unstable at an SCR of 5.8 (orange marker), whereas the proposed method maintains stability across the entire variation range.

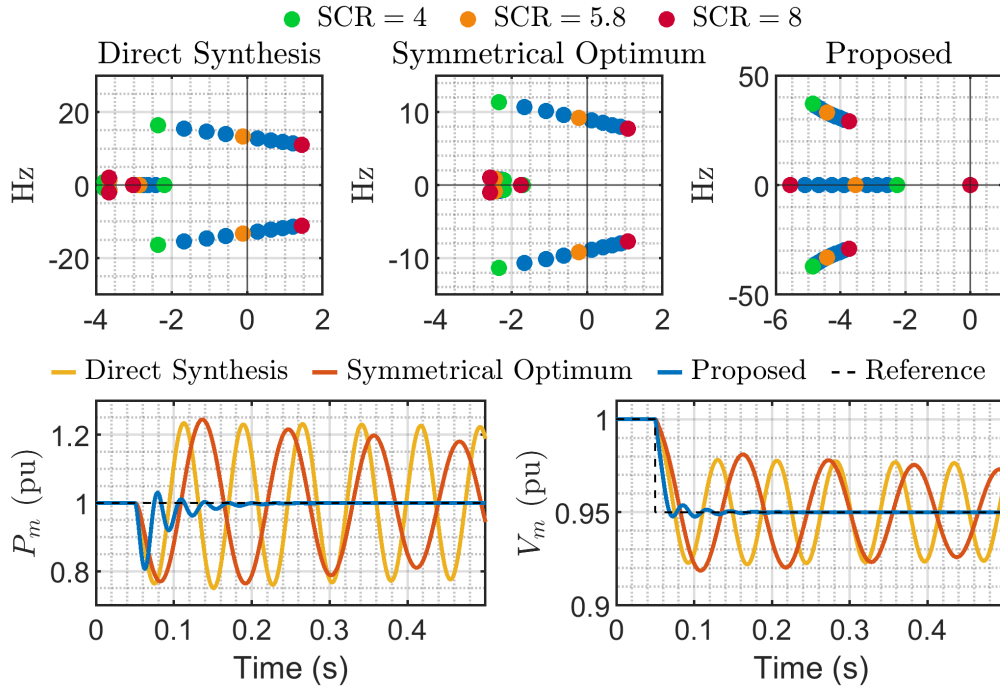


Figure 5.17: Eigenvalue trajectory of three tuning methods when SCR is varied from 4 to 8 (top); Dynamic response comparison of three techniques at SCR = 5.8 (bottom)

The bottom row of Figure 5.17 depicts the dynamic response when the voltage reference is stepped from 1 pu to 0.95 pu at $t = 0.05$ s, while active power injection remains at 1 pu. The DS (yellow) and SO (red) responses exhibit oscillations with a poorly damped 12 Hz and 8 Hz mode, respectively, whereas the proposed method delivers a fast, well-damped response.

Multi-IBR Test Cases

The performance of the proposed method is again evaluated on two well-established test systems widely employed for dynamic stability studies: the 9-bus WSCC system [1] and the 11-bus two-area system [3]. The operating conditions and load types remain consistent with the original references; however, synchronous machines are replaced with IBRs.

In the 9-bus system, GFMs are placed at Bus 1 and Bus 2, while a GFL is placed at Bus 3, as illustrated in Figure 3.16. Similarly, in the 11-bus system, each area contains one GFL and one GFM, as shown in Figure 4.4. For both cases, the target bandwidth and desired phase margin for DS are set to 20 Hz and 80° , respectively. The target bandwidth for SO is also kept at 20 Hz, as higher targets lead to system instability. In contrast, for the proposed method, the target crossover frequency is set to 35 Hz, with $X/R = 10$ and $SCR = 6$.

Figure 5.18 separately compares the d -axis (left) and q -axis (middle) bandwidths for all three tuning methods. The proposed method achieves, on average, a bandwidth that is 36.7 times higher than the other two methods in the 9-bus system (top) and 1.6 times higher in the 11-bus system (bottom). The disk margin comparison (right column) further indicates that the proposed design achieves higher stability margins despite offering superior bandwidths. It is important to note that these results are based on the highest achievable DS and SO target bandwidths while maintaining an individual loop phase margin above 60° . The significant degradation observed in DS and SO performance may be attributed to the large integrator gains inherent in these designs. A detailed investigation into the implications of large integrator gains and their necessity for other compliance studies remains an ongoing area of research.

5.3.3 Summary

This section presented a novel tuning method for grid-forming inverters (GFMs) that concurrently designs the dq -axis voltage control while explicitly accounting for network dynamics using the Thévenin equivalent at the point of connection. The proposed method translates design specifications into an H_∞ optimization framework, ensuring greater robustness and bandwidth. A comparative analysis against direct synthesis and symmetrical optimum methods demon-

strated that the proposed design consistently achieves higher bandwidth and superior stability margins across varying network conditions. In particular, the performance evaluation on 9-bus and 11-bus test systems highlighted the effectiveness of the proposed method, especially in the strong grid scenarios (9-bus system) where classical methods struggle with instability.

5.4 Conclusion

This chapter presented robust control design methodologies for both GFL and GFM, addressing the challenge of maintaining stability and performance across a wide range of network conditions. For GFLs, a novel MIMO-based tuning framework was developed, incorporating notch filtering and co-optimization to suppress interfering resonances and maximize disk margin. Comparative studies across four benchmark systems demonstrated the method's superiority over conventional tuning strategies, particularly in weak and dynamically varying grids.

For GFMs, a new design strategy was introduced that embeds the Thévenin equivalent of the network into the voltage control design. By translating performance objectives into an H_∞ optimization problem, the method ensures enhanced robustness and closed-loop performance. Simulation results on realistic power system test cases confirmed that the proposed design outperforms classical approaches in terms of both bandwidth and robustness, particularly under strong-grid conditions where GFM control typically faces limitations.

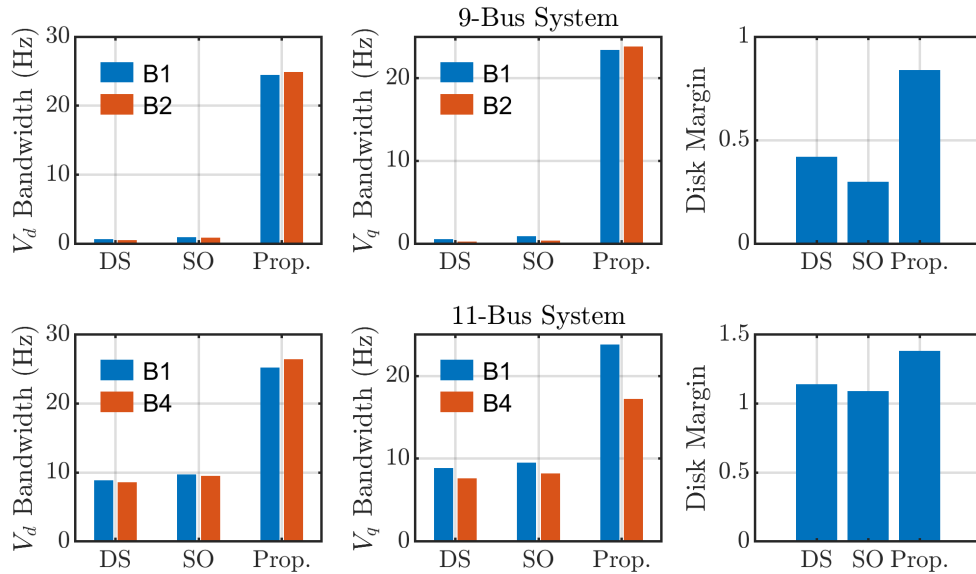


Figure 5.18: Bandwidth and disk margin comparison of proposed technique (prop.) with direct synthesis (DS), symmetrical optimum (SO) for 9-bus (top) and 11-bus (bottom) system.

Chapter 6

Root Cause Analysis of Sub-Synchronous Oscillation using In Situ Estimated IBR Models

Increasing penetration of IBR in power systems could trigger stability problems such as poorly damped SSO [67]. Several such SSO events have been reported in IBR-rich zones of power grids around the world [11, 68]. Among others, the 8 Hz SSO event reported by the National Energy System Operator (NESO) in Great Britain [69] drew significant attention, as it occurred in one of the most IBR-rich zones in the world, i.e., the north of Scotland. Such SSO events promoted guidelines for practitioners on diagnosis and mitigation of these oscillations [43].

The risk of poorly damped SSO events often forces system operators to operate conservatively by limiting the instantaneous share of IBRs. This could lead to curtailing wind or solar power generation with fossil-fuel generation on standby. Hence, it is crucial to foresee and mitigate the risk of IBR-induced SSO. The challenge, however, is the lack of transparency of vendor-specific “black-box” IBR models. Dynamic impedance scan is commonly used to estimate the IBR model under a given operating condition [70]. By injecting perturbations into the physical IBR or its vendor-provided black-box model at multiple frequencies within the range of interest, the frequency response is recorded and the corresponding linearized IBR model is estimated. This exercise is typically performed on a standalone IBR connected to a source that injects voltage/current perturbations. Once an IBR is connected to the system, this method cannot be used without disconnecting the IBR. This motivates the investigation on *in situ* estimation of an IBR model while the IBR remains connected to the Rest of the System (RoS).

One challenge with *in situ* estimation of IBR models is the scalability for realistic IBR-rich zones of a power grid. This is because an IBR model is initially estimated in closed-loop with RoS, making the model order of the initial estimation dependent on the size of overall system. This is not known a priori, as RoS includes black-box IBRs elsewhere. To overcome this issue, an augmented eigensystem realization algorithm is proposed which automatically selects an appropriate model order of the estimated closed-loop system. This, together with model order reduction, circumvents the scalability problem as demonstrated in the chapter.

Using the *in situ* estimated IBR models and the known network model, a state-space model of the overall power system can be constructed. Modal analysis of this model reveals the participation of each IBR in poorly damped SSO. The relative participation of IBRs points to the source of SSO, enabling targeted and effective mitigation.

Various methods have been reported for estimating black-box models of IBRs. One common approach is the dynamic impedance scan, which estimates an equivalent impedance model of an IBR over a specified frequency range [70, 71, 72]. These models are useful for system-level small-signal stability analysis [73]. While dynamic impedance scan is relatively straightforward for a standalone IBR connected to a perturbation source, it is not directly applicable for *in situ* estimation where the IBR remains embedded in a power system. Furthermore, the method relies on repeated sinusoidal injections which makes it computationally intensive and less practical for *in situ* estimation of IBR models.

A data-driven approach for analyzing SSO is Dynamic Mode Decomposition (DMD), which extracts dominant spatiotemporal features from time-series measurements [74, 75, 76, 77, 78]. DMD shows promise in identifying key modes that govern system behavior and oscillatory dynamics. However, its performance depends heavily on the choice of observables and degrades with limited measurements. Notably, DMD yields only the system's state matrix and does not provide a full state-space model that is essential for interconnecting subsystems.

The Eigensystem Realisation Algorithm (ERA) is a widely used method in system identification for deriving state-space models from time-domain pulse response data [79]. While ERA offers a systematic approach to estimate a system model, its accuracy critically depends on the selection of an appropriate “model order”. In theory, accurate estimation requires the model order to be used in the ERA-based estimation process to be close to that of the actual system. However, the actual order is typically unknown a priori, especially in the case of black-box systems such as IBRs. Several studies have applied ERA to power system oscillation analysis (e.g., [80, 81, 82]) and have dealt with the model order issue differently. For instance, [81] assumes prior knowledge of the actual model order and uses it directly in the estimation process. This

assumption limits the applicability for vendor-specific black-box IBRs. In contrast, [83] addressed model order selection through an iterative trial-and-error process starting from a guess of low-order. In each iteration, the step response of estimated model is compared to that of the actual system, and the process continues until the root-mean-square error falls below a specified threshold. While this approach avoids the need for prior knowledge of model order, it is not scalable due to the high computational cost of repeated time-domain simulations. Therefore, the limitation of model-order selection poses a significant barrier to applying ERA for *in situ* estimation of black-box IBRs in IBR-rich zones of the grid.

Some recent applications of ERA in power systems have used step-response data instead of pulse perturbations [81, 82, 83]. In these studies, to approximate the frequency response of the system, the estimated transfer function is multiplied by the Laplace variable ‘ s ’ and divided by the step amplitude. However, using step-response data for ERA may lead to fundamental issues. Note that ERA yields a discrete-time (DT) state-space model, which is first converted to a continuous-time (CT) state-space model and then to a CT transfer function, finally multiplied by the Laplace variable s . This series of conversions can lead to a cascading of numerical errors when using step-response data [84]. For instance, a nonzero feed-through matrix in the state-space model may result in an improper transfer function, leading to possible non-causal behavior [85]. Second, using step-response data measured from a nonlinear system may lead to a loss of minimality in the obtained system realization, which renders further stability analysis unreliable [86].

The above-mentioned limitations underscore a significant research gap in how an appropriate “model order” can be assigned to ERA for the estimation of black-box IBR models. These are particularly important in practical settings where system operators require an accurate *in situ* estimate of IBR models to analyse IBR-induced SSO.

In this chapter, an augmented ERA-based approach for *in situ* estimation of IBR models for analysing SSO is proposed. The original contributions are the following.

1. The proposed approach enables accurate *in situ* estimation of IBR models while the IBRs remain connected to the power system. This is accomplished by defining two intermediate transfer functions within the closed-loop formed by an IBR and the rest of the system, from which the IBR model is extracted via inverse operation. The inversion is well-defined due to a nonzero feed-through term in the transfer function.
2. An augmented ERA is proposed to ensure *scalability* for realistic IBR-rich zones of a power grid. The augmentation refers to automatic model order selection based solely on

the Hankel matrix constructed from measured data. Thus, an accurate estimation can be achieved without any prior knowledge of the actual model order and without requiring any iteration. The combination of augmented ERA and subsequent model reduction makes the overall approach well-suited for realistic IBR-rich zones.

6.1 Methodology for in situ Estimation

This section presents the proposed methodology for *in situ* estimation of black-box IBRs as impedance spectra. The main steps are illustrated in Fig. 6.1.

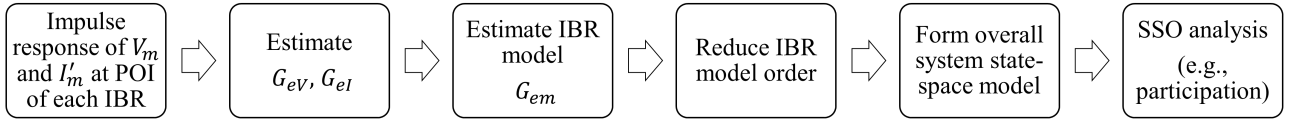


Figure 6.1: The proposed methodology for *in situ* estimation of black-box IBRs connected to power system.

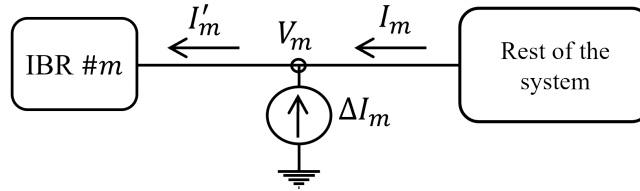


Figure 6.2: The circuit diagram of the pulse current injection at the POI of the m -th IBR connected to RoS. The subscript m refers to the index of the investigated IBR.

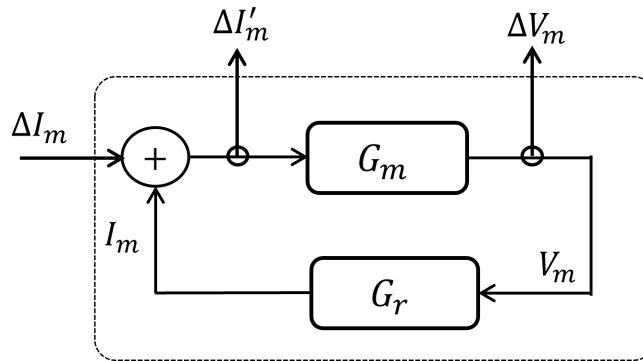


Figure 6.3: The block diagram of the setup for measuring pulse response data. The investigated IBR, denoted by G_m , is regarded as black-box input-output dynamical system, where the input is the perturbed current and the output is the perturbed voltage.

6.1.1 Pulse Response Measurements

Consider the m -th black-box IBR connected to the rest of the system (RoS) at its point of interconnection (POI), as shown in Fig. 6.2. The current and voltage in dq -frame, at the m -th POI, are denoted by:

$$I_m = \begin{bmatrix} I_{m,d} \\ I_{m,q} \end{bmatrix}, V_m = \begin{bmatrix} V_{m,d} \\ V_{m,q} \end{bmatrix}. \quad (6.1)$$

A pulse current, denoted by ΔI_m , is applied at the m -th POI. This pulse current is assumed to have sufficiently small amplitude and width, such that the investigated operating equilibrium is maintained. The variations in voltage and current due to the pulse, as shown in closed-loop feedback form in Fig. 6.3, are denoted by:

$$\Delta I_m' = \begin{bmatrix} \Delta I_{m,d}' \\ \Delta I_{m,q}' \end{bmatrix}, \Delta V_m = \begin{bmatrix} \Delta V_{m,d} \\ \Delta V_{m,q} \end{bmatrix}. \quad (6.2)$$

6.1.2 In Situ Estimation by Inversion for IBR Model

The discrete time-domain measurement data (6.2) are used for the augmented ERA-based estimation. Note that the theoretical foundation of augmented ERA will be discussed in Section 6.2. Two intermediate transfer functions are estimated, and they are denoted by:

$$G_{e,V} := \frac{\Delta V_m}{\Delta I_m}, \quad (6.3a)$$

$$G_{e,I} := \frac{\Delta I_m'}{\Delta I_m}. \quad (6.3b)$$

Then, the dynamic model of the m -th IBR can be calculated as:

$$G_{e,m} = G_{e,V} \times (G_{e,I})^{-1}. \quad (6.4)$$

The transfer function $G_{e,I}$ in (6.3b) is invertible thanks to the structure of direct feed-through from ΔI_m to $\Delta I_m'$. This ensures that the estimated IBR model $G_{e,m}$ in (6.4) is always well-defined, which contributes to the feasibility of the proposed methodology.

6.1.3 IBR Model Order Reduction

The inverse operation in (6.4) results in some uncontrollable and unobservable states. These states can be eliminated by a minimal realization of $G_{e,m}$. At this stage, the order of the estimated IBR model is typically large. To address this issue, the balance truncation technique is adopted to significantly reduce the order of the estimated IRB model. The normalized state energy is used as a metric in the balanced truncation process. This metric measures the energy

of each state relative to the state with maximum energy. This metric is denoted by $\mathcal{E} \in [0, 1]$. The states with smaller values of $\mathcal{E}$ contribute least to the input-output dynamics, and they are truncated. Those principal states with larger values of $\mathcal{E}$ are selected for obtaining the reduced-order model. Therefore, a reduced-order estimated model of the investigated IBR is obtained.

Despite its effectiveness in extracting system dynamics, the proposed method with its current implementation works with certain considerations. First, it relies on time-domain measurements with a sufficiently high sampling frequency to accurately capture fast dynamics after an impulse probing. Second, ERA can be sensitive to measurement noise, which may distort the estimated Markov parameters and affect the quality of the identified model. So a high-fidelity denoised signal is considered for the estimation process. Finally, it is important to highlight that the performance of ERA depends heavily on the nature of the excitation signal. The choice of impulse (whether ideal, shaped, or filtered) can significantly influence the observability of different modes.

To address these limitations, high-speed data acquisition sensors with large signal-to-noise ratio (SNR) are essential to ensure the fidelity of measurements. For cases where residual noise is still present, denoising techniques such as wavelet-based filtering applied after coherent signal averaging can be explored to suppress low-amplitude noise without compromising signal integrity. The sensitivity to the type of impulse input also opens avenues for future work. In addition to Gaussian impulses, other forms like chirp signals or multi-sine excitations could be evaluated to determine their effectiveness in exciting a broader range of system modes, particularly in multi-IBR scenarios.

6.1.4 Modal Analysis

The above-mentioned process is repeated for each IBR. Typically, the network topology and parameters are available to the system operators. In this regard, the state-space model of the overall power system can be obtained with a combination of the reduced-order estimated IBR models and the known network model. Therefore, stability analysis can be performed on the basis of the obtained overall power system model.

In this work, modal analysis to derive the participation factors of IBRs is presented, which identifies the root-cause of the IBR-induced SSO. The ideas are briefly revisited as follows. Let Z be the total number of IBR devices, n_z the number of state variables of the z -th device ($z = 1, \dots, Z$), and $p^i = [p_{1,1}^{(i)}, \dots, p_{z,n_z}^{(i)}]^\top$ the participation factor vector associated with i -th

mode, with entries identifiable by device and state number. For each device, summing the participation of all its states yields:

$$\mathcal{P}_z = \sum_{j=1}^{n_z} p_{z,j}^{(i)}, \quad (6.5)$$

for $z = 1, \dots, Z$, and

$$\mathcal{P} = (\mathcal{P}_1, \mathcal{P}_2, \dots, \mathcal{P}_Z)^\top \in \mathbb{R}^Z. \quad (6.6)$$

Normalizing the participation with respect to the largest contribution yields the relative contribution of each device, i.e.,

$$\mathcal{P}^N = \frac{\mathcal{P}}{\max_{1 \leq q \leq Z} \mathcal{P}_q}. \quad (6.7)$$

Note that $\mathcal{P}_z^N \in [0, 1]$, for all z .

6.2 Augmented ERA

This section presents the theoretical foundation of the proposed augmented ERA which is key to achieve an accurate estimation from pure measurement data. Notably, two schemes are suggested to automatically select an appropriate model order, without prior knowledge on the actual system. The ERA-based estimation is based on the selected model order. Note that the role of augmented ERA, within the proposed methodology, is to estimate $G_{e,V}$ in (6.3a), and $G_{e,I}$ in (6.3b), respectively, whereby the estimated *in situ* IBR model is calculated.

6.2.1 Markov Parameters and Hankel Matrices

The time-domain measurement data are collected at discrete instants, which naturally calls for the discrete-time system identification algorithm [79]. Consider a discrete-time linear time-invariant system:

$$x(k+1) = Ax(k) + Bu(k), \quad (6.8a)$$

$$y(k) = Cx(k) + Du(k), \quad (6.8b)$$

where $x \in \mathbb{R}^n$ is the state vector, $u \in \mathbb{R}^q$ is the input vector, $y \in \mathbb{R}^p$ is the output vector, $A \in \mathbb{R}^{n \times n}$ is the system matrix, $B \in \mathbb{R}^{n \times q}$ is the input matrix, $C \in \mathbb{R}^{p \times n}$ is the output matrix, $D \in \mathbb{R}^{p \times q}$ is the feed-through matrix, and $k \in \mathbb{N}$ is the discrete-time index of variables.

The initial state of (6.8) is assumed at the equilibrium, i.e., $x(0) = 0 \in \mathbb{R}^n$. Consider a pulse probed at the first input, i.e., $u_1(0) = 1$, and $u_1(k) = 0$, for all $k \geq 1$. Then, the pulse

response of (6.8) is derived as:

$$x(1) = Ax(0) + B_1 u_1(0) = B_1, \quad (6.9a)$$

$$x(2) = Ax(1) + B_1 u_1(1) = AB_1, \quad (6.9b)$$

$$\vdots$$

$$x(k) = Ax(k-1) + B_1 u_1(k-1) = A^{k-1} B_1, \quad (6.9c)$$

where $B_1 \in \mathbb{R}^{n \times 1}$ is the first column of the input matrix. Taking (6.9c) into (6.8b) yields the output vector $y(k) = CA^{k-1}B_1$, for all $k \geq 1$. Similarly, applying pulses to all inputs and arranging the corresponding output vectors yields:

$$y_k := \begin{bmatrix} CA^{k-1}B_1 & CA^{k-1}B_2 & \cdots & CA^{k-1}B_q \end{bmatrix} = CA^{k-1}B, \quad (6.10)$$

which is known as the Markov parameters. Similarly, arranging the initial output vectors, i.e., for $k = 0$, yields:

$$y_0 := \begin{bmatrix} D_1 & D_2 & \cdots & D_p \end{bmatrix} = D. \quad (6.11)$$

Stacking the Markov parameters yields the following Hankel matrix:

$$\mathcal{H} = \begin{bmatrix} y_k & y_{k+1} & \cdots & y_{k+n_c} \\ y_{k+1} & y_{k+2} & \cdots & y_{k+n_c+1} \\ \vdots & \vdots & \ddots & \vdots \\ y_{k+n_r} & y_{k+n_r+1} & \cdots & y_{k+n_r+n_c} \end{bmatrix}, \quad (6.12)$$

where $n_c \in \mathbb{N}$, $n_r \in \mathbb{N}$. Note that each row contains n_c+1 number of blocks, and each column contains n_r+1 number of blocks. Hence, the total dimension of the Hankel matrix (6.12) is $(n_r+1)p$ -by- $(n_c+1)q$.

Setting $k = 1$ and $k = 2$, respectively, in (6.12) yields two Hankel matrices:

$$\mathcal{H}_0 = \begin{bmatrix} y_1 & y_2 & \cdots & y_{n_c+1} \\ y_2 & y_3 & \cdots & y_{n_c+2} \\ \vdots & \vdots & \ddots & \vdots \\ y_{n_r+1} & y_{n_r+2} & \cdots & y_{n_r+n_c+1} \end{bmatrix}, \quad (6.13)$$

and

$$\mathcal{H}_1 = \begin{bmatrix} y_2 & y_3 & \cdots & y_{n_c+2} \\ y_3 & y_4 & \cdots & y_{n_c+3} \\ \vdots & \vdots & \ddots & \vdots \\ y_{n_r+2} & y_{n_r+3} & \cdots & y_{n_r+n_c+2} \end{bmatrix}. \quad (6.14)$$

Note that $\mathcal{H}_0$ and $\mathcal{H}_1$ are two key ingredients in the ERA-based realization. The singular value decomposition (SVD) of the Hankel matrix (6.13) is calculated as:

$$\mathcal{H}_0 := \mathbf{U}\mathbf{\Sigma}\mathbf{V}^\top, \quad (6.15)$$

where $\mathbf{U} \in \mathbb{R}^{(n_r+1)p \times (n_r+1)p}$ and $\mathbf{V} \in \mathbb{R}^{(n_c+1)q \times (n_c+1)q}$ are two square matrices composed by corresponding left and right singular vectors, respectively; $\mathbf{V}^\top$ is the transpose of $\mathbf{V}$. Note that $\mathbf{\Sigma}$ is a rectangular diagonal matrix with singular values listed in a descending order, that is,

$$\text{diag}(\mathbf{\Sigma}) = \{\sigma_1, \sigma_2, \dots, \sigma_{n_s}\}, \quad (6.16)$$

with $n_s \in \mathbb{N}$.

Now, at this stage a value (denoted by $r \in \mathbb{N}$) is determined and the singular values (from σ_1 to σ_r) are to be used for ERA-based realization.

The model order used for the system realization is typically an argument to ERA (see, e.g., the ‘era’ function in MATLAB [87]). This is however not feasible when such prior information is not available. In principle, to achieve an accurate estimation result, the value of r should be equal to the order of the actual linear system. Theoretically, the order of the actual linear system is equal to the rank of the Hankel matrix. This means that, in an ideal linear case, $r = \text{rank}(\mathcal{H}_0)$ can simply be selected. In such a case, the first r singular values (from σ_1 to σ_r) are positive, while the rest singular values (from σ_{r+1} to σ_{n_s}) are strictly zeros. However, nonlinearity of a practical system renders the Markov parameters not well-defined. In addition, numerical errors exist in the output measurements. These two practical issues make “ $r = \text{rank}(\mathcal{H}_0)$ ” an inappropriate selection for the ERA-based estimation. Therefore, an effective scheme to select an appropriate value of model order is needed.

6.2.2 Automatic Selection of Model Order

Two schemes to determine an appropriate model order for ERA-based realization are suggested based on the singular values in (6.16), .

Threshold-Based Scheme

Note that the singular values in (6.16) are ranked in descending order. One feasible approach is to select the r -number of largest singular values, such that:

$$\sigma_r = \epsilon \sigma_1, \quad (6.17)$$

with $\epsilon \in \mathbb{R}_{>0}$ a threshold to be defined by users.

Elbow-Based Scheme

Another feasible approach is by finding an “elbow-point” on the semi-logarithmic curve of singular values. One deterministic way to achieve this goal is as follows. Let $\mathbf{p}_1 = [1, \log_{10}(\sigma_1)]$ and $\mathbf{p}_n = [n_s, \log_{10}(\sigma_{n_s})]$ denote the first and last points, respectively. A straight line segment is determined by these two endpoints. For each singular value point $\mathbf{p}_i = [i, \log_{10}(\sigma_i)]$, its perpendicular distance to this line using the magnitude of the cross product is computed. The point with the “maximum” distance is then selected as the “elbow”. It is denoted by:

$$\mathbf{p}_r = [r, \log_{10}(\sigma_r)], \quad (6.18)$$

which represents the location where the curve transitions most sharply. Typically this is the point where a steep descent transitions to a more gradual decline. This method is computationally efficient and well-suited for large datasets, providing a robust geometric criterion for elbow detection.

Based on the information of singular values in (6.16), users have the freedom to choose either the threshold-based scheme (6.17) or the elbow-based scheme (6.18). Neither of the proposed schemes relies on any prior information on the actual model.

6.2.3 The ERA-based Realization

Once the value of r is determined by either of the above-mentioned schemes, the r -number of largest singular values is stored to form a diagonal matrix:

$$\mathbf{\Sigma}_r = \text{diag}\{\sigma_1, \sigma_2, \dots, \sigma_r\} \in \mathbb{R}^{r \times r}. \quad (6.19)$$

The r -number of corresponding left and right eigenvectors are used to form the rest of SVD matrices, denoted by $\mathbf{U}_r$ and $\mathbf{V}_r$, respectively.

Finally, one minimal realization of the system (6.8) can be calculated as (refer to [79, 88]):

$$\hat{G} := (\hat{A}, \hat{B}, \hat{C}, \hat{D}), \quad (6.20)$$

where

$$\hat{A} := \mathbf{\Sigma}_r^{-\frac{1}{2}} \mathbf{U}_r^\top \mathcal{H}_1 \mathbf{V}_r \mathbf{\Sigma}_r^{-\frac{1}{2}}, \quad (6.21a)$$

$$\hat{B} := \left(\mathbf{\Sigma}_r^{\frac{1}{2}} \mathbf{V}_r^\top \right) E_q, \quad (6.21b)$$

$$\hat{C} := E_p^\top \left(\mathbf{U}_r \mathbf{\Sigma}_r^{\frac{1}{2}} \right), \quad (6.21c)$$

$$\hat{D} := y_0, \quad (6.21d)$$

with $E_q = [\mathbf{I}_q, 0, \dots, 0]^\top$ and $E_p = [\mathbf{I}_p, 0, \dots, 0]^\top$ the selector matrices with corresponding dimensions; $\mathbf{I}_q \in \mathbb{R}^{q \times q}$ and $\mathbf{I}_p \in \mathbb{R}^{p \times p}$ are two identity matrices.

The realized input matrix $\hat{B}$ in (6.21b) is the segment of first q columns of the matrix $\Sigma_r^{\frac{1}{2}} \mathbf{V}_r^\top$. In (6.21c), the realized output matrix $\hat{C}$ is the segment of first p rows of the matrix $\mathbf{U}_r \Sigma_r^{\frac{1}{2}}$. In (6.21d), the feed-through matrix is given by (6.11).

6.3 Case Studies

This section shows the performance of the proposed method in estimating *in situ* black-box IBRs while in operation with power system. Two cases are presented. Note that the proposed augmented ERA does not rely on prior information of the actual models, such as IBR control structures or parameters. However, the information on actual models only for validating the accuracy of the estimated results has been used. The components of models (including GFL, GFM, loads and networks) are discussed in Chapter 2. In this chapter, the label and the index of an IBR indicates the inverter type and its corresponding bus number, respectively. For example, “GFL2” refers to an IBR which is GFL type and connected to bus 2.

6.3.1 Case 1: A Five-Bus Two-IBR System

In this case, the test system consists of two IBRs (i.e., GFL1 and GFM2) supplying a load at bus 3, as shown in Fig. 3.13. This case aims to show that the proposed method works well under both poorly-damped and well-damped conditions. The two black-box IBRs can be estimated sequentially for different operating points while they remain in operation. The system and control parameters for each operating point are given in Table 6.1. The current control bandwidth of both GFL1 and GFM2 is 500 Hz. The PLL bandwidth for GFL1 is 30 Hz. Note that these values are not required for the estimation algorithm; however, they are mentioned here for reference.

Poorly-Damped Scenario

A poorly-damped oscillatory mode at 21 Hz is observed in the two-IBR test system with the parameters in the left column of Table 6.1. In this scenario, the proposed method yields accurate estimation of the dynamic model for each IBR, and it accurately identifies the respective contribution of each IBR to the oscillatory mode. The accuracy of the estimation is validated by comparing the singular value response of the estimated models against those derived from

the actual models. In addition, the eigenvalues of the estimated and actual closed-loop systems are compared. The modal participation of each IBR is then computed using (6.7) and compared against the actual model to assess whether the estimated models accurately identify the source of the oscillatory mode.

Fig. 6.4 shows the responses to an injection of pulse signal with a small amplitude of 0.01 pu. This pulse signal is sequentially injected, at 0.1 s, first into the d-axis current and then into the q-axis current. For each injection, the window length of 1 s is recorded at a sampling interval of $50 \mu\text{s}$. Note that the dq components of the voltages and the currents are subtracted by their corresponding steady-state values. For the sake of brevity, Fig. 6.4 shows only the pulse response of GFL1, which is only up to 0.5 s. Note that the y-axes in all four subplots do not extend to the peak of the perturbation; however, the maximum response remains below the pulse magnitude of 0.01 pu. Similar responses are also obtained for GFM2. Note that the

Table 6.1: System and Control Parameters for the Test Case 1

	Poorly-damped system	Well-damped system
Line 3-4 and Line 3-5	$R=0.03$ pu	$R=0.025$ pu
	$X=B=0.3$ pu	$X=B=0.25$ pu
GFL1	Outer control bandwidth = 30 Hz	Outer control bandwidth = 5 Hz
GFM2	Voltage control bandwidth = 10 Hz	
Load	$P=2$ pu, $Q=0.4$ pu (constant Z type)	
Transformers	$X=0.05$ pu	

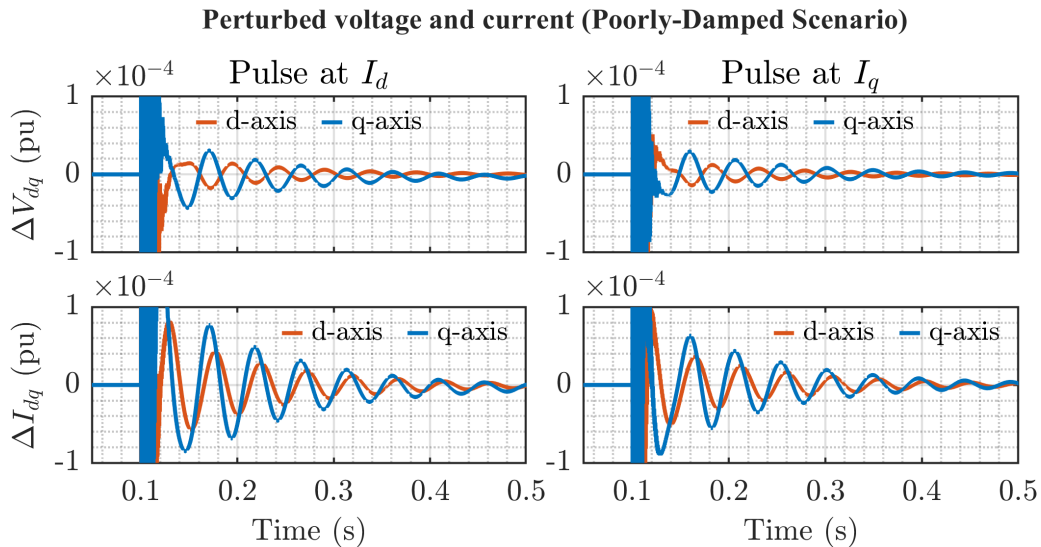


Figure 6.4: Voltage (top) and current (bottom) responses when a pulse with an amplitude of 0.01 pu is applied at 0.1 s on GFL1 under the poorly-damped condition.

measurement of pulse response data is obtained at the sampling rate of 20 kHz with a window length of 1 s. These data are fed into proposed method, which yields the linearized model of GFL1 and GFM2, respectively.

For validation, Fig. 6.5 shows the accuracy of the estimated IBR models by comparing the singular values against those of actual models. It can be seen that, for a poorly-damped system, the singular values of both GFL1 and GFM2 are virtually overlapped within the frequency range of 1-1000 Hz. Fig. 6.6 shows that the eigenvalues of estimated and actual closed-loop systems are also closely matched. The zoomed inset shows that the magnitude difference between the poorly-damped mode of estimated and actual systems is less than 0.18 %. The right-hand side

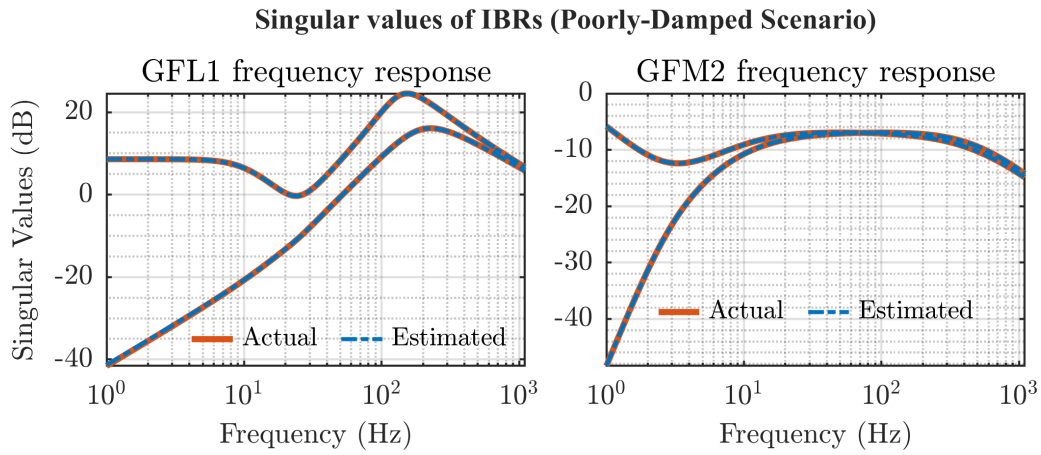


Figure 6.5: Comparison of the estimated singular value responses with that of actual model for GFL1 (left) and GFM2 (right) in the poorly-damped scenario. A frequency range of 1-1000 Hz is considered.

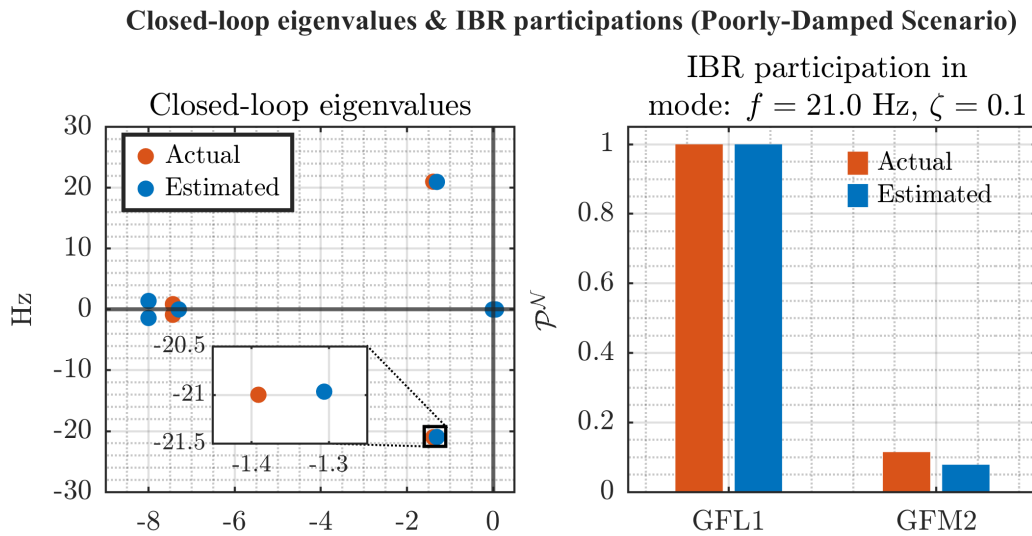


Figure 6.6: Comparison of the estimated closed-loop system eigenvalues with those of actual system (left); and the comparison of the estimated participation with the actual participation of each IBR in the poorly-damped mode of 21 Hz (right).

of Fig. 6.6 shows the participation of both IBRs in the oscillatory mode of 21 Hz. This confirms that the estimated model accurately identifies GFL1 to be the major contributor to the 21 Hz mode, which is aligned with the result obtained by actual model-based modal analysis.

Well-Damped Scenario

The two-IBR test system, with the parameters given in the right column of Table 6.1, operates in a well-damped condition. Fig. 6.7 shows pulse response of voltage and current (both d- and q-axes) at the POI of GFL1 when a pulse with a magnitude of 0.01 pu is injected at 0.1 s. The zoomed insets in all four subplots show high-frequency oscillations, which indicates the need of sensors with high sampling rates.

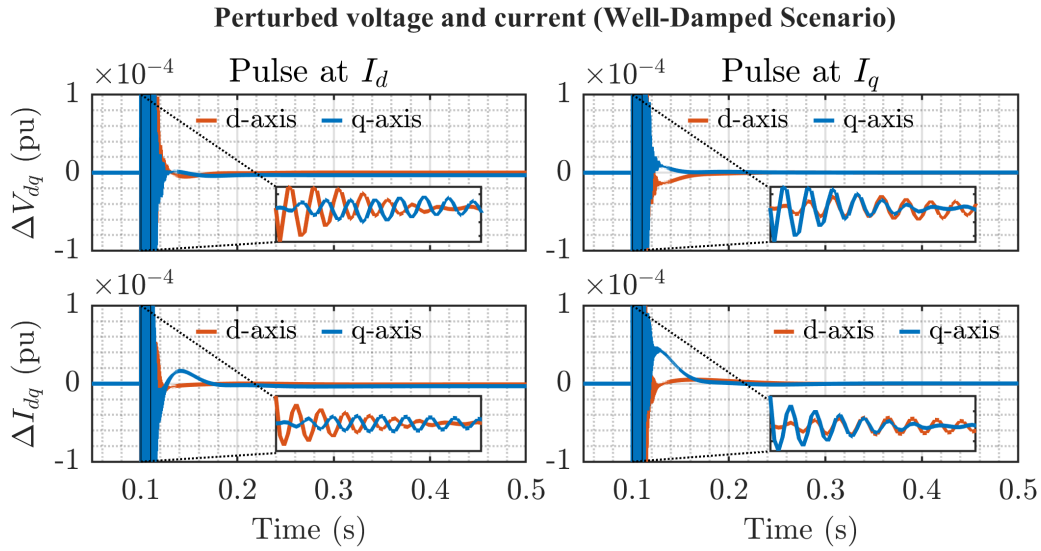


Figure 6.7: Voltage (top) and current (bottom) response when a pulse with an amplitude of 0.01 pu is applied at 0.1 s on GFL1 under the well-damped condition.

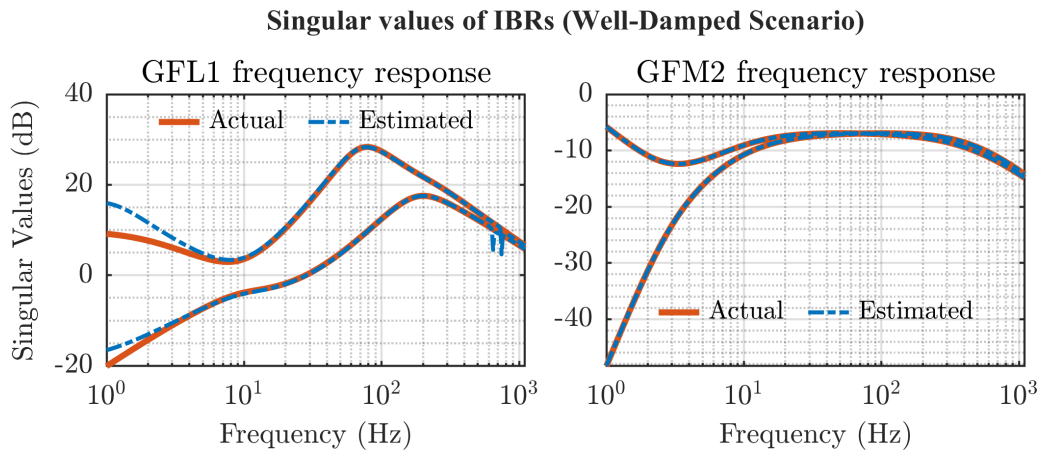


Figure 6.8: Comparison of the estimated singular value response with that of actual model for GFL1 (left) and GFM2 (right) in the well-damped scenario. A frequency range of 1-1000 Hz is considered.

Fig. 6.8 validates the accuracy of estimated models by comparing the singular values of the estimated frequency response with the actual models of GFL1 (left) and GFM2 (right). Despite a mismatch at lower frequencies (less than 3 Hz) in GFL1, the closed-loop eigenvalues shown on the left of Fig. 6.9 are aligned with the actual model. The percentage magnitude difference between the estimated and actual mode is 2.35 %, as shown in the zoomed inset. The right-hand side of Fig. 6.9 shows the participation factor bars in the well-damped mode of 3.5 Hz, where the participation bars obtained using the estimated model remain closely aligned with those obtained using actual model.

To sum up, the first test case has demonstrated the performance of the proposed method in estimating an in situ IBR embedded within the power system. The proposed method is shown to be feasible, in both poorly-damped and well-damped scenarios, for the stability analysis of black-box IBR-dominated power systems.

6.3.2 Case 2: A 39-bus 10-IBR System

Preliminaries

This test case extends the proposed method to a 39-bus 10-IBR test system (as shown in Fig. 3.17), which aims to demonstrate the scalability and effectiveness of the proposed method in analyzing a realistic power system. To be specific, the test case assesses the following indicators of the proposed method: 1) the capability to provide an accurate identification of reduced-order models; 2) the ability to select an appropriate model order with only measurement data, i.e.,

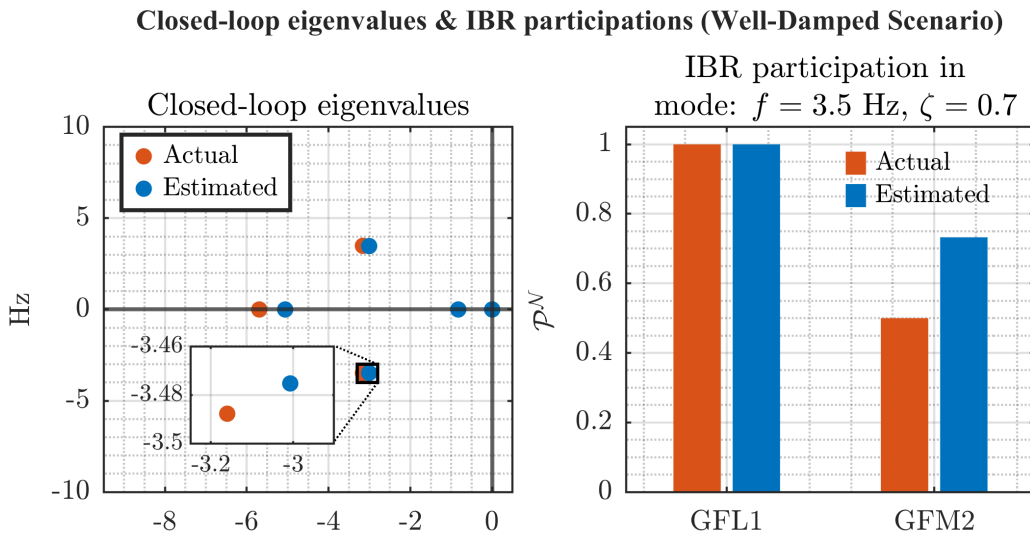


Figure 6.9: Comparison of the estimated closed-loop system eigenvalues with those of actual system (left); and the comparison of the estimated participation with the actual participation of each IBR in the poorly-damped mode of 3.5 Hz (right).

without prior knowledge of the actual models; 3) the effectiveness in identifying the root-cause of SSO through modal participation analysis.

Four scenarios are considered, each of which has a different mix and locations of GFLs and GFMs, as listed in Table 6.2. Note that these four scenarios exhibit at least one poorly-damped SSO. For brevity, the control parameters and bandwidth settings of each IBR as well as other system configurations are provided in [89]. Note also that information of these parameters is not required for the process of system identification. However, these parameters are provided merely for the validation of estimated results, i.e., the estimated models are to be compared against the actual models.

Table 6.2: Configurations for the Four Scenarios in the Test Case #2

Bus	Scenario I		Scenario II		Scenario III		Scenario IV	
	Type	P (pu)	Type	P (pu)	Type	P (pu)	Type	P (pu)
30	GFL	3.55	GFM	2.75	GFL	2.5	GFM	2.5
31	GFM	-5.86	GFL	4.99	GFL	8.76	GFM	12.35
32	GFL	9.23	GFM	6.5	GFL	6.5	GFM	6.5
33	GFM	6.32	GFL	6.32	GFL	6.32	GFM	6.32
34	GFL	7.21	GFM	5.08	GFL	5.08	GFM	5.08
35	GFM	6.5	GFL	6.5	GFM	5.85	GFL	5.2
36	GFL	7.95	GFM	5.6	GFM	5.04	GFL	4.48
37	GFM	5.4	GFL	5.4	GFL	4.86	GFL	4.32
38	GFL	11.79	GFM	8.3	GFM	7.47	GFL	6.64
39	GFM	10	GFL	10	GFM	9	GFL	8
GFL	54.1%		64%		47.5%		46.7%	
GFM	45.9%		36%		52.5%		53.3%	

Results of Scenario I

The results of Scenario I are presented here to avoid repetition, because the application of the proposed method in other scenarios are the same, and the results are similar. Fig. 6.10 illustrates, for all five GFLs, the procedure of model order sections and of model order reductions. The top two subplots illustrate the automatic selection process for a number of largest singular values; these selected singular values are then used for realizing two transfer functions,

i.e., $G_{e,V}$ and $G_{e,I}$, respectively. Notably, the zoomed-in regions illustrate the proposed elbow-based scheme for automatically determining an “elbow-point” among the singular value points (6.16). In this scenario, 9000 singular values are listed in a descending order, most of which are insignificant when compared to the largest singular value. For each GFL, a sharp change in the descending trend of singular values is observed, which corresponds to the value of selected model order. Such a model order is used for system realization.

Due to inevitable nonlinearities and numerical errors in practice, it is difficult to identify an ideal model order that is equal to the order of actual model. However, this test case shows that the proposed elbow-based scheme can automatically select an “appropriate” model order

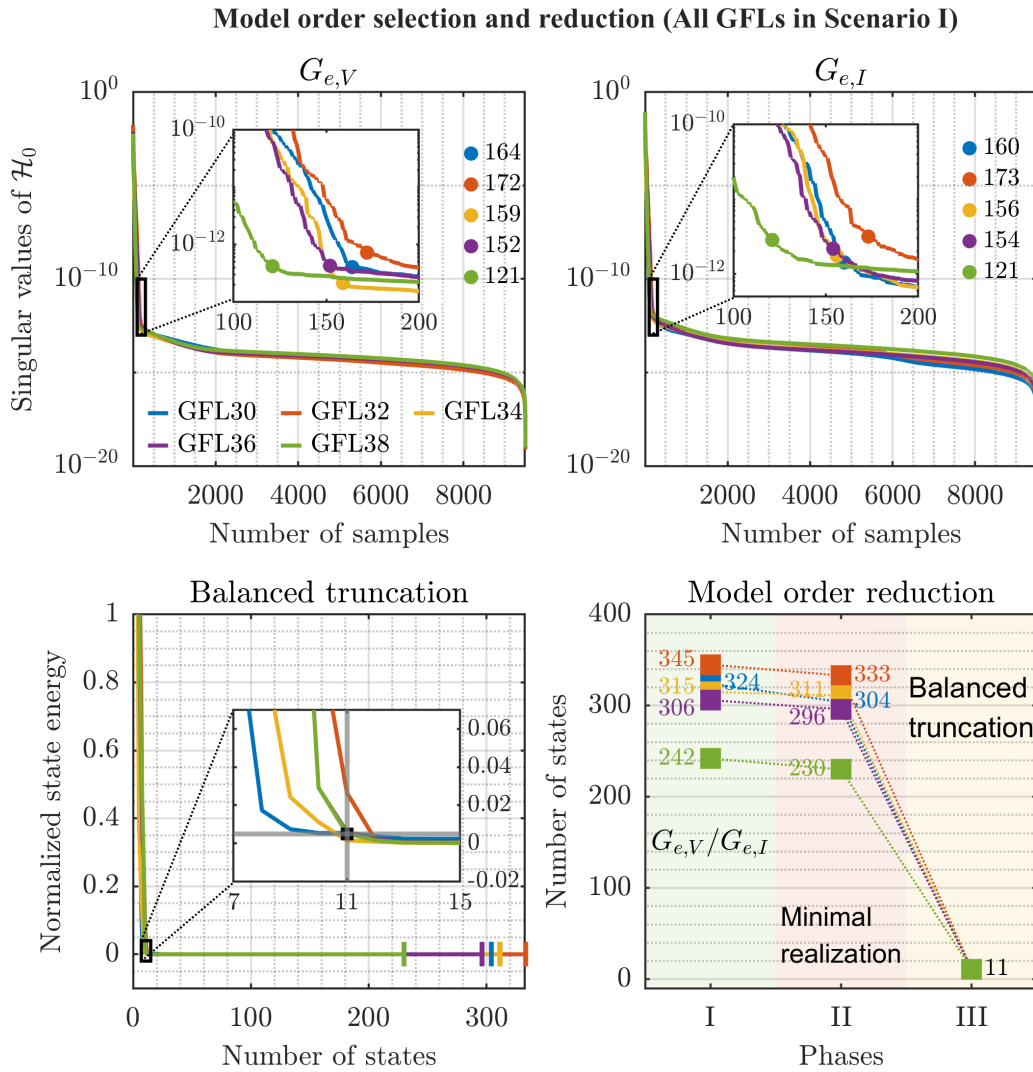


Figure 6.10: For all 5 GFLs in Scenario I, the singular values of Hankel matrix are automatically selected for estimating $G_{e,V}$ (top left) and $G_{e,V}$ (top right); The normalized state energy $\mathcal{E}$ is considered as the metric for balanced truncation which determines the reduced order (bottom left); The subplot (bottom right) summarises three phases of the model order selection and reduction procedure.

that contributes an accurate estimation.

The bottom-left plot of Fig. 6.10 shows the normalized state energy $\mathcal{E}$ for the balanced truncation process. A steep decline is observed in the first few states, with over 95% of the system energy captured within the first 11 states for all GFLs. The zoomed inset confirms that the energy contribution beyond the 11-th state is negligible (below the threshold 1%) which informs the choice of reduced model order as 11. The bottom-right plot of Fig. 6.10 summarizes the complete procedure of model order reduction in the following three phases. Phase I represents the initial order of the estimated IBR model, i.e., G_e in (6.4), which ranges from 242 (GFL38) to 345 (GFL30). Phase II illustrates a preliminary reduction by minimal realization. This phase performs a minimal realization by removing overlapping poles and zeros that arise from numerical artifacts introduced during inverse operations. This step ensures a more compact yet accurate representation. After this process, the model order of each GFL ranges between 296 (GFL38) and 333 (GFL30). Notably, Phase III shows a significant reduction of the model order after balanced truncation, where only 11 states are retained for each GFL. So far, the estimated reduced-order models (order value of 11) for all five GFLs are obtained.

Similar to the above-mentioned procedure for all GFLs, Fig. 6.11 presents the model order selection and reduction procedure for all five GFMs in Scenario I. Fig. 6.11 also includes the automatic selection of the model order by determining the elbow-point of singular values (top), balanced truncation from the distribution of normalized state energies (bottom-left), and the three-phase of model order reduction procedure (bottom-right). These confirm that the proposed method can effectively realize a reduced model order. Notably, the final model order is reduced significantly (to 8 states) for each GFM. Fig. 6.11 also demonstrates a key advantage of the proposed method, i.e., it does not require prior knowledge of the order of the actual models.

Fig. 6.12 compares the singular values of the estimated models with those of the actual models, for all GFLs (top row) and GFMs (bottom row) in Scenario I. In general, all singular values of the estimated models are closely matched to those of the actual models, within a wide frequency range of 0.1-1000 Hz. For all GFLs, the singular value curves of the estimated models are virtually indistinguishable from the actual responses. Similarly, the GFM estimations demonstrate excellent alignment, despite the models being of significantly different dynamic character and having much lower magnitudes in the low-frequency range (as low as -150 dB), where there is a slight mismatch. This strong correlation confirms that the proposed identification method yields accurate impedance models, regardless of the inverter type (GFL or GFM), validating its general applicability without any prior knowledge.

Validation for All Scenarios

Fig. 6.13 compares the estimated participation of each IBR in the poorly-damped modes, of the modified 39-bus system, across all four scenarios.

In Scenario I, a 5.3 Hz poorly-damped mode is observed. In this scenario, the proposed method accurately identifies GFL38 as the dominant contributor (tallest blue bar), matching the result of the actual model-based modal analysis (tallest red bar). In contrast, a purely data-driven (model-free) approach, DMD, incorrectly identifies GFM37 as the primary contributor (tallest yellow bar); GFL38 is mis-identified as the second contributor (second tallest yellow

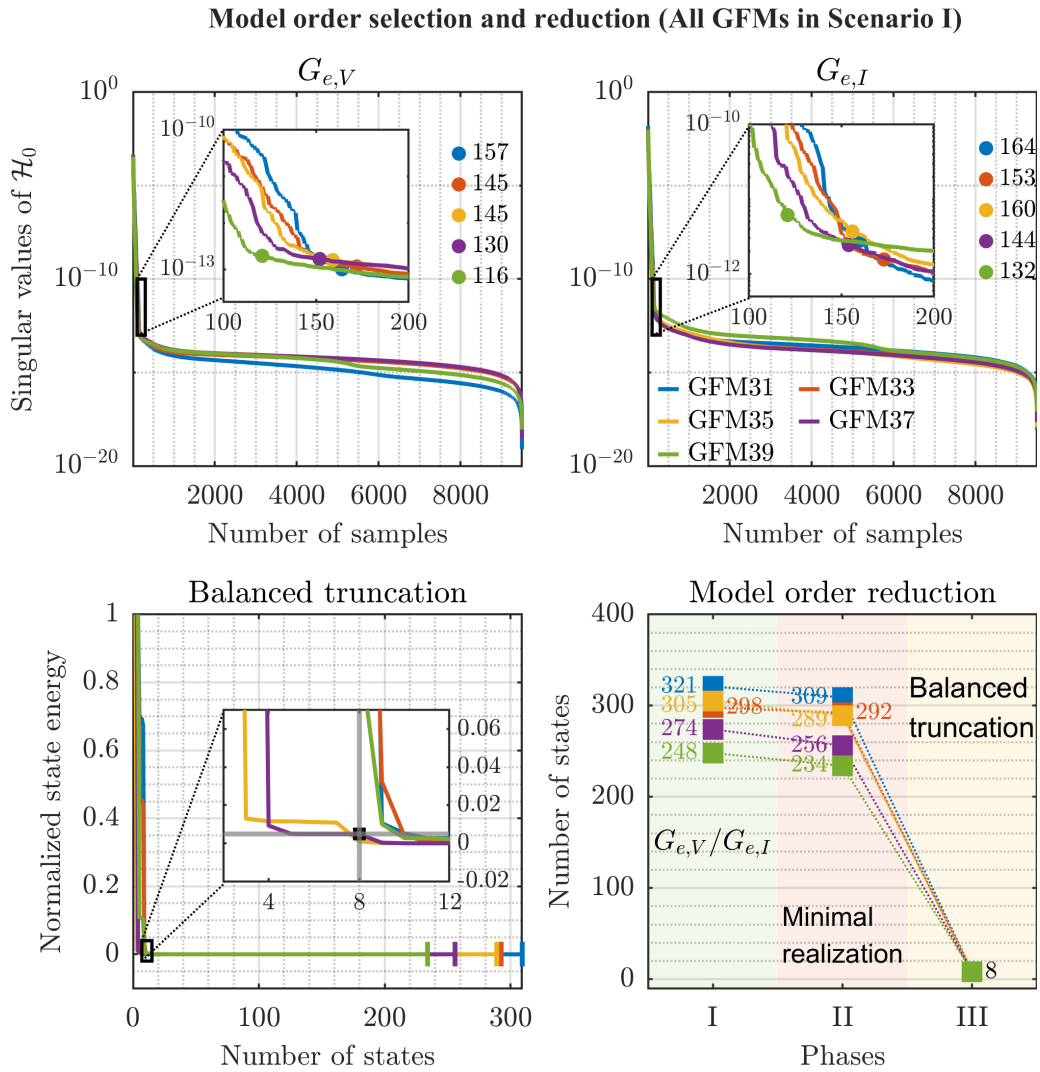


Figure 6.11: For all 5 GFMs in Scenario I, the singular values of Hankel matrix are automatically selected for estimating $G_{e,V}$ (top left) and $G_{e,V}$ (top right); Similar to Fig. 6.10, the zoomed insets show the procedure of automatic selection of the model order for each GFM; the balanced truncation with normalized state energies to determine the reduced model order (bottom left); three-phase summary of the model order reduction process (bottom right).

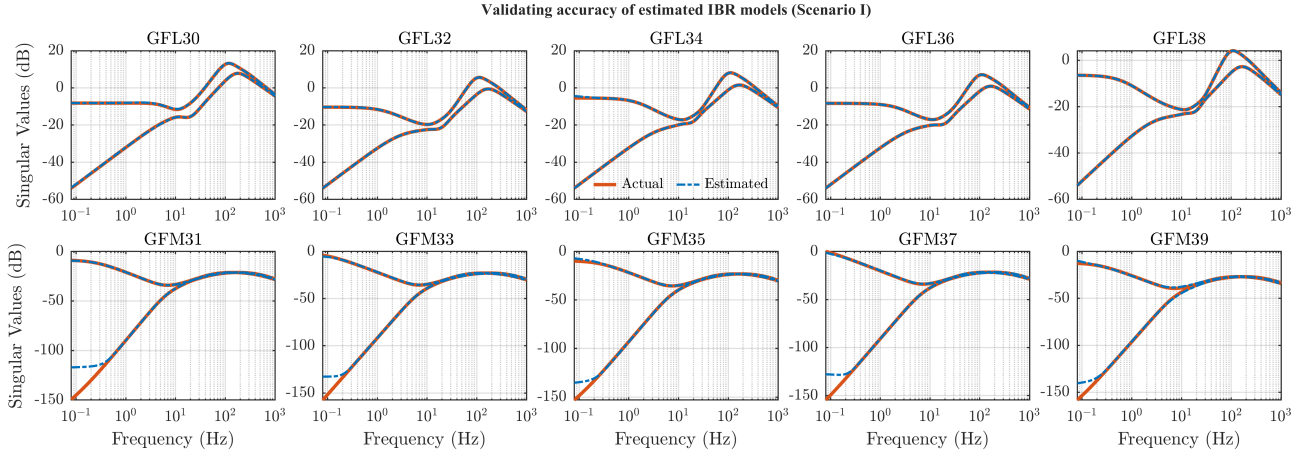


Figure 6.12: Comparison of the singular values of the estimated frequency response with the singular values of actual frequency response of GFLs (top row) and GFMs (bottom row) for Scenario I of modified 39-bus system.

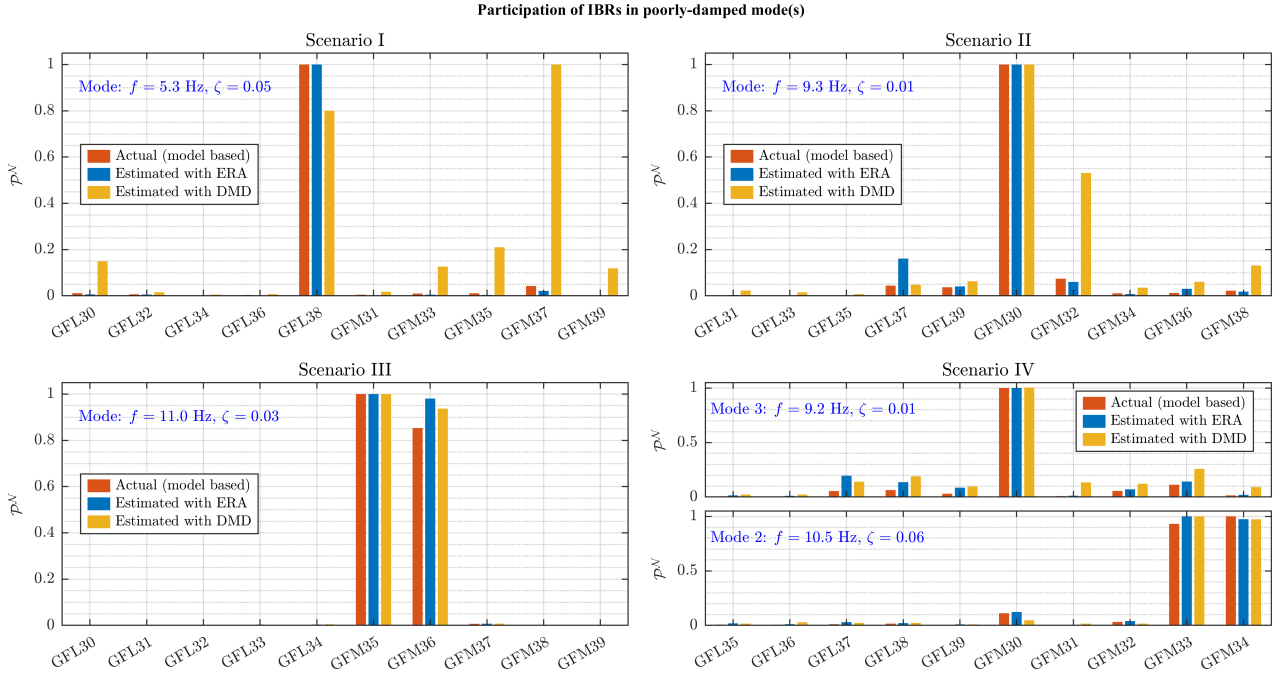


Figure 6.13: Comparison of the estimated participation with the actual participation of each IBR in poorly-damped mode(s) of each scenario. Scenario IV has two poorly-damped modes at 9.2 Hz and 10.5 Hz.

bar), which indicates the limitation of DMD-based approach in root-cause analysis. In Scenario II, a 9.3 Hz poorly-damped mode is observed, where the proposed method correctly identifies GFL30 as the main contributors. Besides, the DMD approach exaggerates the contribution of GFM32, leading to a less accurate analysis result. In Scenario III, the proposed method identifies GFM35 and GFM36 as the dominant participants for the 11.0 Hz poorly-damped

mode. In this scenario, the accuracies of augmented ERA-based results are very close to the DMD-based and actual model-based results. In Scenario IV, two poorly-damped modes are observed at 9.2 Hz and 10.5 Hz, receptively, with distinct contributing IBRs. For the 9.2 Hz mode, both DMD and augmented ERA-based analyses correctly identifies GFM30 as the leading contributors, which are in agreement with the actual model-based results; however, the DMD-based participation for other IBRs is slightly less accurate. For the 10.5 Hz mode, both augmented ERA and DMD-based analyses correctly identify that GFM33 and GFM34 are the dominant sources. These results demonstrate the satisfying performance and robustness of the proposed method in identifying the IBR(s) with dominant effect in poorly-damped SSO mode(s). This allows targeted and effective mitigation of the SSO problem.

6.4 Conclusion

This chapter has demonstrated the feasibility of *in situ* estimation of IBR models while the IBRs remain connected to the rest of the power system. By leveraging the proposed augmented ERA method, accurate IBR models are extracted from the closed-loop voltage and current responses to a small pulse perturbation. Thanks to automated model order selection and subsequent model reduction, prior information on actual model order or any iteration process is not necessary. This ensures scalability for realistic IBR-rich zones of a power grid. The frequency response of the resulting state-space model of the overall power system, obtained from estimated IBR models and a known network model, is accurate in the frequency range of interest. Modal analysis on this estimated model successfully identifies the IBR with dominant participation in a poorly damped oscillatory mode, which enables targeted and effective mitigation.

Future work will be devoted to dealing with noise in measurement data and using advanced system identification techniques that can better handle nonlinearities.

Chapter 7

Conclusion and Future Work

This thesis presents a comprehensive framework for analysing and mitigating sub-synchronous oscillations (SSOs) in power systems dominated by inverter-based resources (IBRs). Through modular modelling, detailed characterisation, robust control design, and data-driven estimation, the work addresses both the root causes and practical challenges associated with SSO risk in zero-carbon grids. The key contributions and insights are summarised below.

Chapter 2 develops a modular, state-space-ready EMT- dq modelling framework that enables scalable construction and analysis of IBR-rich systems. The model captures internal dynamics of GFLs, GFMs, and network components in a unified reference frame, supporting both time-domain and frequency-domain analysis. This sets the foundation for stability studies and control design undertaken in subsequent chapters.

Chapter 3 benchmarks the EMT- dq model against EMT- abc simulations and RMS+ tools. It demonstrates that RMS+ studies can provide misleading stability assessments due to the absence of network dynamics. EMT- dq modelling offers a superior trade-off between computational efficiency and accuracy, enabling a greater number of scenario checks and revealing frequency-coupled IBR-network interactions.

Chapter 4 characterises two distinct SSO mechanisms: (i) those induced by PLL dynamics in GFLs, and (ii) those arising from voltage control interactions in GFMs. For the PLL-driven SSO, diagnostic indicators are proposed based on analytical framework and validated with EMT- dq simulations to reliably characterise low-magnitude oscillations. For SSOs rooted in suboptimal design of GFM voltage control, the sensitivity of damping to voltage and current control gains is analysed. The modal analysis reveals the limitations of modal signatures alone in ascribing GFM-induced instabilities to a specific cause. Collectively, these tools offer both physical insight and actionable diagnostics for identifying and mitigating SSO risk.

Chapter 5 uses the analytical insights gained in Chapter 3 and Chapter 4 to develop advanced control design techniques. A robust MIMO-based framework is developed for GFLs, combining notch filtering and gain co-optimisation to ensure stability across changing grid strengths. For GFMs, a novel controller design embeds the network’s Thévenin equivalent into the voltage control loop, formulated via H_∞ optimisation. Both strategies are validated across benchmark systems, and outperform conventional designs in terms of tracking performance (bandwidth), and robustness (disk margin), particularly under stressed grid conditions.

Chapter 6 addresses the challenge of model estimation. A novel system identification methodology is proposed, leveraging pulse perturbations and an augmented eigensystem realisation algorithm (ERA) to extract accurate state-space models of IBRs *in situ*. This approach requires no prior model order and remains scalable to large systems. It enables frequency-domain modal analysis without requiring full EMT simulations or detailed knowledge of IBR model, providing a practical pathway to constructing digital twins of IBR-dominated grids.

Taken together, the thesis delivers an integrated framework for SSO diagnosis, analysis, and mitigation. From fundamental modelling to practical control and estimation, each contribution advances the state-of-the-art in understanding the dynamic behaviour of modern power systems with high IBR penetration. The work not only identifies specific vulnerabilities but also offers engineering solutions that are computationally viable and analytically grounded.

Recommendations

Based on the findings across all chapters, the following key recommendations are offered:

- **Modelling tools:** Utilities and system operators should adopt EMT- dq modelling tools as a viable intermediate approach between computationally intensive EMT- abc simulations and oversimplified RMS+ approaches. Standardisation of EMT- dq platforms is essential to enable wider adoption.
- **Planning studies:** Dynamic network representations must be retained in SSO studies. Positive-sequence impedance models are inadequate for capturing high-frequency instabilities induced by IBR-network interactions.
- **Diagnostic tools:** Deployment of PLL-based indicators and sensitivity-based metrics for controller tuning should be encouraged to facilitate SSO diagnosis.

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- **Control design:** GFL and GFM controllers should be designed with explicit consideration of the grid impedance profile. MIMO-based designs and Thévenin-aware control structures offer superior robustness under varying grid conditions.
 - **Model estimation:** Where models are unavailable, utilities should employ *in situ* estimation methods like augmented ERA to perform frequency-domain stability assessments.

Future Work

Although this thesis addresses key challenges, it also uncovers important gaps that require further investigation. They are:

- **Nonlinear behaviour and large-signal stability:** The current framework focuses on small-signal analysis. Extending the modelling and control strategies to account for nonlinear behaviour under large disturbances would enhance the applicability of the results to fault scenarios and protection design.
- **Hardware-in-the-loop (HIL) validation:** Experimental validation of EMT- dq using HIL platforms would help establish the practical viability of the proposed methods and bridge the gap between simulation and deployment.
- **Real-time implementation of control designs:** Further work is needed to integrate the proposed control designs into DSPs or FPGA-based platforms for deployment in commercial converter controllers.
- **Uncertainty-aware control design:** While the H_∞ framework addressed worst-case stability in control design process of GFL and GFM, future work can incorporate structured uncertainties in the network model (e.g., Thévenin impedance bounds, renewable intermittency) to develop probabilistic or robust adaptive control strategies.
- **Noise-robust estimation:** Chapter 6 assumes clean measurement conditions. Future research will explore noise-resilient identification techniques, for example coherent averaging and wavelet transform filtering, on real-world field data.

In closing, this thesis offers a multi-faceted approach to tackling the growing challenge of SSO in IBR-dominated systems. It is hoped that the tools, insights, and recommendations presented herein will aid both researchers and practitioners in ensuring the secure and stable operation of future zero-carbon power grids.

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Appendix: Graphical Representation of Modal Composition of Oscillatory Mode

The mode shape diagrams presented throughout this thesis (Figures 3.7, 3.9, 4.3, and 4.7) use a graphical representation that combines magnitude information of participation factors and phase information of eigenvector onto a polar chart. Each state is represented by a circular marker whose area is proportional to the magnitude of the normalised participation factor, and whose angular orientation reflects the phase angle of the associated entry in the mode's right eigenvector.

To formalise this, consider the small-signal state-space model of a linearised system around an operating point:

$$\dot{\mathbf{x}} = A\mathbf{x}, \quad \mathbf{x} \in \mathbb{R}^n,$$

where $A \in \mathbb{R}^{n \times n}$ is the system matrix. The eigen-decomposition of A yields

$$A\mathbf{v}_k = \lambda_k \mathbf{v}_k, \quad \mathbf{w}_k^\top A = \lambda_k \mathbf{w}_k^\top,$$

with $\lambda_k = \sigma_k \pm j\omega_k$ denoting the k -th eigenvalue, and $\mathbf{v}_k \in \mathbb{C}^n$, $\mathbf{w}_k \in \mathbb{C}^n$ representing the corresponding right and left eigenvectors. The oscillation frequency and damping ratio associated with the k -th mode are given by

$$f_k = \frac{\omega_k}{2\pi}, \quad \zeta_k = -\frac{\sigma_k}{\sqrt{\sigma_k^2 + \omega_k^2}}.$$

The participation factor of state x_i in mode k is defined as

$$P_{ik} = w_{ki} v_{ik},$$

which gives a scalar quantity reflecting how strongly the state both excites and responds to the mode. While P_{ik} is complex, only its magnitude is typically used:

$$|P_{ik}| \in [0, 1], \quad \tilde{P}_{ik} = \frac{|P_{ik}|}{\max_j |P_{jk}|}.$$

Each mode shape diagram is then constructed by plotting every state x_i as a disk on a unit-radius polar chart. The angular position is given by the phase of v_{ik} ,

$$\phi_{ik} = \arg(v_{ik}),$$

and the area of the disk is set proportional to $\tilde{P}_{ik}$. If radius-based scaling is used, the radius is proportional to $\sqrt{\tilde{P}_{ik}}$, ensuring that the area remains linearly proportional to participation magnitude.

These visualisations offer rapid insights. Large disks indicate states with strong modal involvement. When disks lie close in angular position, the associated states oscillate in phase. States on opposite sides of the circle are in anti-phase. Thus, angular spread reflects the coherency of the mode. Narrow clustering implies coherent group dynamics; wide spread suggests a more diffuse interaction pattern.

A Representative Example illustrating this procedure for a four-state arbitrary system is discussed below.

Consider the circuit shown in Figure A1. The inductor currents, node voltages and component variables are labelled as in Figure A1. The input is the source voltage $u(t) = v_{\text{in}}(t)$ and the output is the second-node voltage $y(t) = v_2(t)$.

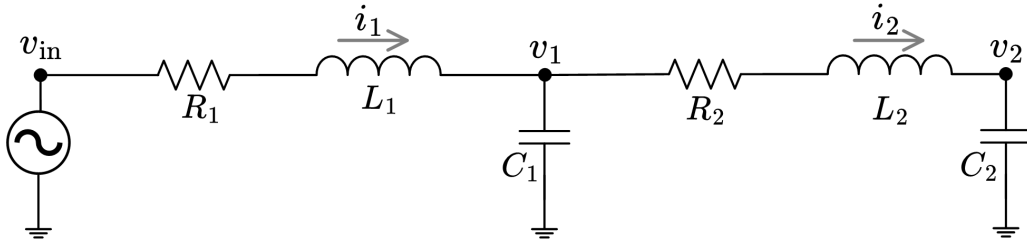


Figure A1: Circuit diagram to explain participation factor polar plots.

Constitutive relations of each component give following set of equations:

$$v_{L_1} = L_1 \dot{i}_1, \quad v_{R_1} = R_1 i_1,$$

$$v_{L_2} = L_2 \dot{i}_2, \quad v_{R_2} = R_2 i_2,$$

$$i_{C_1} = C_1 \dot{v}_1, \quad i_{C_2} = C_2 \dot{v}_2.$$

Subsequent application of Kirchhoff's laws yield the following:

$$\text{KVL (left section): } v_{\text{in}} = v_{R_1} + v_{L_1} + v_1,$$

$$\text{KVL (right section): } v_1 = v_{R_2} + v_{L_2} + v_2,$$

$$\text{KCL (node } v_1) : i_1 = i_{C_1} + i_2,$$

$$\text{KCL (node } v_2) : i_2 = i_{C_2}.$$

Using the constitutive relations inside the KVL/KCL statements gives

$$L_1 \dot{i}_1 = v_{\text{in}} - R_1 i_1 - v_1,$$

$$L_2 \dot{i}_2 = v_1 - R_2 i_2 - v_2,$$

$$C_1 \dot{v}_1 = i_1 - i_2,$$

$$C_2 \dot{v}_2 = i_2.$$

With $\mathbf{x} = [i_1 \ i_2 \ v_1 \ v_2]^\top$ and $u = v_{\text{in}}$, state space form can be obtained

$$\dot{\mathbf{x}} = A\mathbf{x} + B u, \quad y = C\mathbf{x} + D u,$$

$$A = \begin{bmatrix} -\frac{R_1}{L_1} & 0 & -\frac{1}{L_1} & 0 \\ 0 & -\frac{R_2}{L_2} & \frac{1}{L_2} & -\frac{1}{L_2} \\ \frac{1}{C_1} & -\frac{1}{C_1} & 0 & 0 \\ 0 & \frac{1}{C_2} & 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} \frac{1}{L_1} \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad C = \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix}, \quad D = 0.$$

Hence the circuit of Figure A1 is a 4th-order LTI system driven by the source voltage v_{in} and delivering the load-side capacitor voltage v_2 as its output.

Using the following values for the circuit components:

$$R_1 = 1 \ \Omega, \quad R_2 = 1 \ \Omega, \quad L_1 = 0.05 \ \text{H}, \quad C_1 = 0.5 \ \text{F}, \quad L_2 = 0.08012 \ \text{H}, \quad C_2 = 0.003182 \ \text{F},$$

the four eigenvalues of the system matrix A are computed as

$$\lambda_1 = -17.76, \quad \lambda_2 = -2.24, \quad \lambda_{3,4} = -6.24 \pm j \ 62.51.$$

Dividing by 2π gives the normalised eigenvalues (in Hz), as shown on the left in Figure A2. The modal frequency and damping ratio of λ_3 indicate a poorly-damped mode:

$$f_3 = \frac{\omega_3}{2\pi} = 9.95 \ \text{Hz}, \quad \zeta_3 = -\frac{\sigma_3}{\sqrt{\sigma_3^2 + \omega_3^2}} = 0.09,$$

Using the left and right eigenvectors associated with λ_3 ($\mathbf{v}_3$ and $\mathbf{w}_3$) the participation vector is computed as

$$\mathbf{P}_3 = \begin{bmatrix} -0.0000 + 0.0000j \\ -0.0187 + 0.1870j \\ 0.0001 - 0.0012j \\ 0.0186 - 0.1865j \end{bmatrix}.$$

Normalising the magnitudes by the largest magnitude yields the relative participation magnitude:

$$\tilde{\mathbf{P}}_3 = \frac{|\mathbf{P}_3|}{\max |\mathbf{P}_3|} = \begin{bmatrix} 0.0001 \\ 1.0000 \\ 0.0065 \\ 0.9936 \end{bmatrix}.$$

Finally, combining $\tilde{\mathbf{P}}_3$ with the angles of the corresponding right eigenvector entries gives the polar representation used for mode shape visualisation in Figure A2 (right), as shown below:

$$\tilde{P}_{i_1 3} \angle \phi_{i_1 3} = 0.0001 \angle 102.5^\circ, \quad \tilde{P}_{i_2 3} \angle \phi_{i_2 3} = 1.0000 \angle -84.3^\circ,$$

$$\tilde{P}_{v_1 3} \angle \phi_{v_1 3} = 0.0065 \angle -0.067^\circ, \quad \tilde{P}_{v_2 3} \angle \phi_{v_2 3} = 0.9936 \angle 180^\circ.$$

These results highlight that the mode is dominated by the interaction between i_2 and v_2 , with nearly quadrature phase oscillation, indicating that the resonance is shaped by the energy exchange between the inductor L_2 and capacitor C_2 .

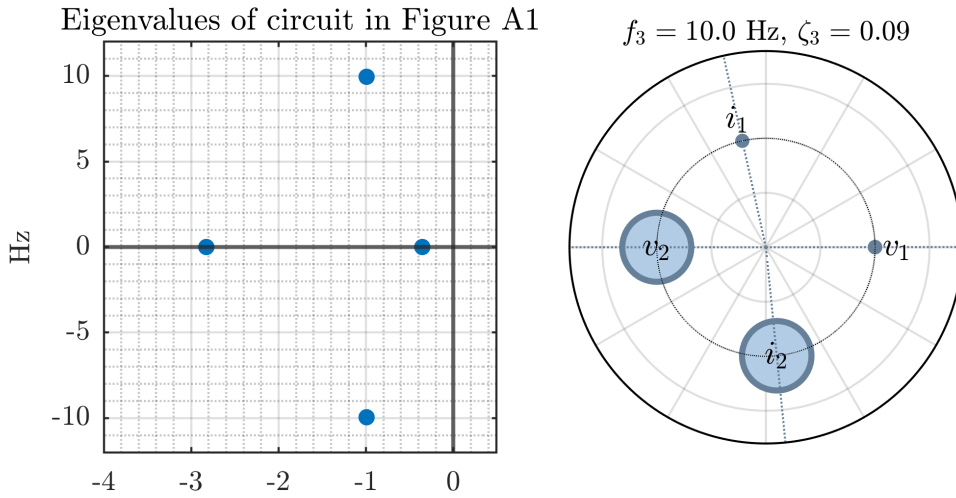


Figure A2: Eigenvalues and modal composition (participation factor with angle of right eigenvector) of poorly-damped mode.

While the polar radius is a purely visual guide (unit norm), colour or marker type can optionally group states (e.g., converter currents vs. network states). These plots are used across the thesis to distinguish and interpret oscillatory modes. This graphical framework is not only intuitive but also mathematically grounded.