

H_2 enrichment of CH_4 blends in lean premixed gas turbine combustion: An experimental study on effects on flame shape and thermoacoustic oscillation dynamics

Efstathios Karlis^{1,*}, Yushuai Liu¹, Yannis Hardalupas¹, Alexander M. K. P. Taylor¹

Department of Mechanical Engineering, Imperial College London, Exhibition Road, London SW7 2AZ, UK

Abstract

Thermoacoustic instabilities under lean operation in gas turbine burners hinder the development of lean premixed combustion mode of operation, thus limiting the potential to decrease NO_x emissions. A method to improve stability of lean combustion while maintaining low thermal NO_x formation is to add hydrogen in typical gas turbine fuels such as natural gas. The present work examines the thermoacoustic dynamic characteristics of hydrogen-enriched methane blends in a swirl stabilized model gas turbine combustor. We blend CH_4 with increasing H_2 molar content (from 0% to 40%) at a global equivalence ratio of $\phi = 0.55$ on a constant Reynolds number $Re=19000$. The reference case of pure methane is susceptible to blow off at the same equivalence ratio. On increasing the hydrogen content at 10% H_2 , the flammability limits are extended. However, further increase of the H_2 content leads to manifestation of random short bursts of dynamic pressure and heat release. Thermoacoustic dynamics are intermittently injected between a quiescent state and a regime of high amplitude oscillations. On further increasing H_2 content, the dynamics are attracted towards a limit cycle; a fully established high amplitude regime, where no requiescence is observed. In this regime, heat release rate and dynamic pressure oscillate in phase and at the same frequency. Based on the observed dynamics, we seek a mixture property to characterize the dynamic state, the combustor operates in. We show that the extinction strain rate associated with each mixture can collapse the dynamic transitions from quiescent to intermittent instabilities and finally to fully established thermoacoustic oscillations. In each dynamic state, we examine the mechanism that affiliates coherent structures of the underlying thermodynamic flow field with the flame through the relation of the spatial distribution of the flow imposed strain rate over the extinction strain rate of each mixture. It is shown that the flame anchoring locations for a given mixture are dictated by the flow imposed strain rate and then that increased extinction resistance couples the flame with naturally excited swirling flows helical instabilities, that may instigate mechanisms responsible for intermittent heat release rate bursts. Finally, the combustor demonstrates a limit cycle behaviour where the flow-imposed strain rate oscillates about the extinction strain rate along the inner shear layers causing

local extinction of the flame at the root close to the centerbody.

Keywords: methane/hydrogen enrichment, extinction strain rate, Dynamic Mode Decomposition

1. Introduction

Gas turbine combustors operating in the lean premixed mode are inherently susceptible to the manifestation of thermoacoustic oscillations thus limiting the operational range and potential NO_x emission reduction [1]. Fuel flexibility provides a possibility to extend the operability range of gas turbine combustors by adjusting fuel properties such as turbulent flame speeds, extinction strain rate, heat release rate density *etc.* In this experimental campaign we consider hydrogen enriched methane mixtures. Hydrogen addition extends the lean blow off limit [2], hence providing the capacity to grant robust to extinction leaner flames and achieve lower adiabatic flame temperatures. However, the fuel properties that need to be taken into consideration to secure fuel interchangeability and thermoacoustically stable operation are a subject of research. In the context of this article we examine the applicability of the extinction strain rate as a scaling parameter of the observed combustion dynamics in an experimental swirl stabilized combustor. The extinction strain rate according to [3] is a mixture property that is a function of the equivalence ratio and the fuel composition but most importantly encapsulates hydrodynamic flowfield effects. Turbulent swirl stabilized combustors use complex three dimensional flows to provide suitable anchoring flame regions [4]. Therefore, it is of interest to examine how adjusting the extinction strain rate of a gaseous hydrogen/methane mixture might lead to changes in the topology of the flame and thereby to changes of the observed combustion dynamics.

In [5], the authors investigated combustion dynamics in a backward facing step combustor, where oscillations were driven by flame-vortex interactions. In the experiments, they enriched propane mixtures with hydrogen at elevated preheat temperatures and different equivalence ratios. They reported that it was possible to collapse dynamic mode transitions to high amplitude combustion dynamics at different regimes *via* a characteristic stretch rate value for each mixture. They calculated the characteristic stretch rate *via* numerical simulations and PIV experiments. They suggested that the characteristics of the interaction between the turbulent flow and the flame dynamics can be well represented by the flow imposed stretch rate on the flame front. The concept of extinction strain rate scaling with the flow imposed strain rate for studying the flame macrostructure topology has been

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Email address: ek3215@ic.ac.uk (Efsthios Karlis)

¹Corresponding author

translated in a model gas turbine combustor in [6]. The authors enriched methane mixtures with hydrogen and maintained a constant equivalence ratio. They identified a V-shaped flame on 0% hydrogen with the flame stabilizing between the inner and the outer shear layers. On increasing the hydrogen content, the flame was able to further penetrate into the outer recirculation zone (ORZ). The gradual transition from a V to an M shape was linked to high amplitude combustion dynamics. The same group in [7] examined the scaling of flow imposed timescales to the extinction strain rate and were able to identify an intermittency mechanism, where a flamelet originating from the outer shear layer of a V flame would penetrate into the outer recirculation zone and ignite reactants in this region. Hence, they were able to interpret the transition from a V to an M shape, through an intermittent regime where the flame would randomly appear in the ORZ. In [8] the authors quantified the effects of hydrogen addition to methane flames under elevated pressure conditions in a swirl stabilized burner. Increasing hydrogen content under elevated mean combustor pressure resulted in a reduction of the flame length and a shift of flame anchoring regions upstream towards the inlet. They linked this change to fundamental mixture properties such as increased laminar flame speed and increased burning rate due to low Lewis numbers. However, they did not conclude on the applicability of the extinction strain rate scaling in their geometry. In [9], the authors examine the flame topology in a swirl stabilized combustor, under thermoacoustically stable operating conditions, as a function of the extinction strain rate. They altered the extinction strain rate of the premixture by preheating the reactants and keeping the fuel consistency constant. They report that they were able to predict the flame shape transitions by extinction strain rate considerations.

In [10], the dynamic state of the combustor was found to shift from a quiescent state to a high amplitude limit cycle regime on increasing the equivalence ratio while maintaining a constant methane/hydrogen fuel consistency. They reported three operational regimes namely a stable, a local transient and a limit cycle regime. They suggested that the transition to instability was associated to the flame front location which was linked to an equivalence ratio increase in their combustor. Even though not explicitly reported in that work, the extinction strain rate of the mixtures increased for richer equivalence ratio cases. Based on the mean flame structure they presented for each dynamic state, the flame was lifted in the stable case but appeared to stabilize closer to the centerbody in the intermittent and unstable regimes. They reported that the intermittent regime is associated to hydrodynamic interaction of the flame with a vortical wave generated by the swirler, convected at the bulk flow velocity downstream in the combustor. They obtained further insight into the flow and flame interaction leading to transition to instability by applying an algebraic technique, namely the Dynamic Mode Decomposition (DMD) on natural luminosity images of the flame to extract dynamic modes, growth rates and frequencies of flow-flame interac-

tion phenomena in each regime. In the same work, the authors also compared the flame shapes between a pure methane mixture and a hydrogen/methane mixture, with 90% H_2 concentration. They studied the limit cycle behaviour, under a constant equivalence ratio. They showed that the hydrogen enriched flame was shorter and moved closer to the inlet of the combustor in comparison to the methane flame. The shape of the pure methane flame was a V lifted flame, whereas the hydrogen enriched flame attained an M shape. The hydrogen enriched flame also appeared to be susceptible to flame area modulations due to the convection of vortical structures identified *via* DMD.

The current research work reports on the way the relative value of the flow imposed strain rate to the extinction strain rate of the mixture, dictates coherent structures and flame interactions which can drive the combustor at different dynamic states. We identify three dynamic regimes: stable flames, transitional flames and fully thermoacoustically unstable flames. We show that these dynamic states cannot be collapsed by the widely used interchangeability parameter, the Lower Wobbe Index (LWI), but we are able to do so by considering the extinction strain rate of each mixture. The coherent structure dynamics that are dominant in the isothermal flowfield and are responsible for the intermittency observed in the geometry are defined by applying DMD on Particle Image Velocimetry (PIV) vector fields. We identify flame anchoring regions in respect to the flow field *via* separately conducted CH^* high speed imaging and PIV measurements. We are able to show that the flame anchoring regions are interpreted by flow imposed strain rate considerations. Flame topology changes lead to dynamic state bifurcations, hence we show that fuel properties play a significant role in dictating the eventual combustor dynamic operating regime.

2. Experimental configuration, operational parameters, mixture properties characterization

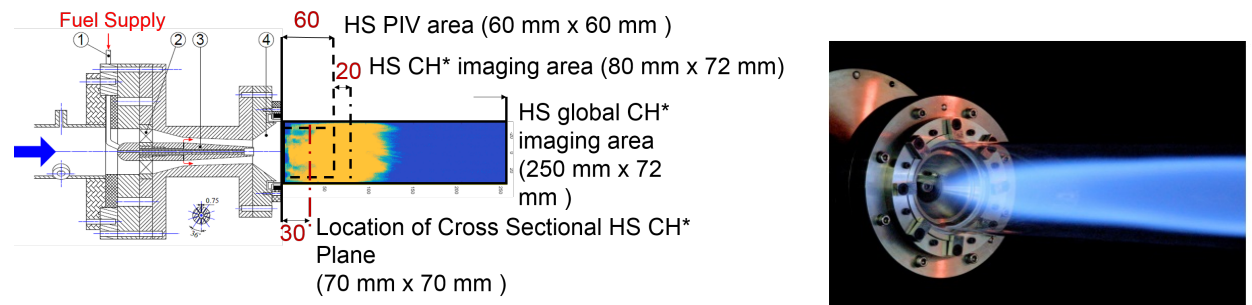


Figure 1: Left: The combustor geometry, with the dimensions of the high speed (HS) PIV and CH^* imaging planes in mm. 1: Fuel Supply Line, 2: Location of blades, 3: Centrebody, 4: optically non-accessible diffuser. Right: Long exposure image of a quiescent elongated flame, from [11].

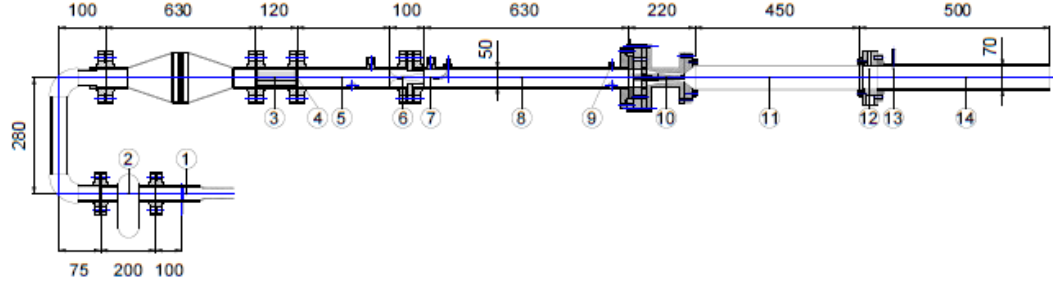


Figure 2: Lean premixed model gas turbine combustor rig, [12]. Dimensions in mm. 1: Upstream fuel supply (not used in this campaign), 2: Air flow supply backpressure regulator, 3: Flow straightener, 4: Wire mesh gauge, 5: Flow pre conditioner, 6: Critical Venturi nozzle, 7: Alumina particles seeding nozzle, 8: Sonic flow settling length, 9: Pressure tapping, 10: Swirler, 11: Fused silica window section, 12: Spark plug igniter, 13 Dynamic pressure microphone tube tapping, 14: Exhaust.

We provide an overview of the experimental configuration that is used in the current study in Fig. 1 and in Fig. 2. The configuration is similar to previous works [13], [14], [11]. The fuel mixtures consisted of bottled CH_4 and H_2 at variable molar percentages, as described in Table 1.

Compressed air was provided at 4 barg, measured through a thermal mass flow meter (M+W Instruments, Mass Stream D-6280) and controlled through a thermal mass flow controller (M+W Instruments, Mass Stream D-6383). The air flow is regulated through the following equipment. A pressure reducer is installed (Backpressure Regulator, Mackenberg UV 5.1) upstream of a critical Venturi (Cussons Technology Ltd, BS/ISO 9300:2005). The critical Venturi nozzle is used to choke the flow and provide a sonic boundary. The flow profile is conditioned before the nozzle through a tube bundle containing a wire mesh gauge and a flow straightener. Upstream and downstream of the Venturi nozzle two absolute pressure gauges are installed (Omega, PXM319-3.5A10V) to estimate the extent of chocking across the nozzle.

Two thermal mass flow meters (M+W Instruments, Mass Stream D-6250, measurement uncertainty ± 0.625 g/s, 3% factory accuracy at full scale) measure the mass flow rate of the two gaseous fuels. The mass flow streams are controlled through two thermal mass flow controllers (M+W Instruments, Mass Stream, D-6320, measurement uncertainty ± 0.01 g/s, 1% factory accuracy at full scale). The fuels streams are combined directly downstream the fuel controllers and a pipe length of 4 m secures homogeneous mixing between the two fuels. The fuel mixture enters the injector at the rear of the centerbody and is forced out of ten equally spaced holes in the swirling annular air flow. Another absolute pressure gauge (Omega, PXM319-3.5A10V) measures the pressure in the fuel delivery line to secure that in each experimental case fuel supply is uncoupled from the dynamics in the combustor. In Fig.

1 the air flows from left to right. The majority of the air (90%) is delivered to the swirler, while a smaller amount flows through the centerbody for cooling purposes. The swirler technically premixes the reactants with the oxidizer in a similar manner to industrial gas turbine combustors [4]. The resulting swirl number is approximately 0.7, based on the formulation suggested in [15].

Monitoring of the fuel and air flow rate stream provided an indication of the instantaneous global equivalence ratio. The assessment was carried out throughout the acquisition to secure that the global equivalence ratio remained within ± 0.01 range of the nominal equivalence ratio for a given operational condition. Nevertheless, it is important to note here that the actual instantaneous equivalence ratio is not oscillating under the influence of thermoacoustic instabilities. The fuel line is decoupled from acoustic instabilities as it is choked [12]. The pressure transducer installed upstream the injection point corroborates that the fuel mass flow rate is independent of the thermoacoustic instabilities in the combustor. In effect, as assessed in [11], equivalence ratio non-uniformities are not generated at the thermoacoustic frequency of oscillations. The Reynolds number was calculated using the diameter of the tube as a characteristic length scale, the mean velocity and the kinematic viscosity of the mixture were defined at standard conditions. The Reynolds number was kept constant at $Re=19000$ in all operational conditions. In each experiment we monitored the air flow temperature from the compressor. We started acquisition when the indication of the thermocouples installed on the section downstream the Venturi nozzle ensured the incoming air temperature was stabilized around $25^{\circ}C$. All experiments took place at ambient conditions. Each experiment was conducted four times.

2.1. Signal Acquisition

Downstream of the swirler, the air-fuel mixture is expanded through a diffuser. This is an important aspect of our geometry because, the gradual expansion does not allow for an outer recirculation zone, as encountered in some similar configurations in literature, hence allowing for flame-wall interactions. The premixture is introduced into a 1.2 m long tube combustor. A cylindrical fused quartz with 70 mm internal diameter and 2 mm wall thickness allows 360° optical access to the flame. The quartz tube length is 450 mm. At the end of the quartz tube section, a tapping allows a spark plug igniter to access the ignitable mixture. A tapping 50 mm downstream of the igniter accommodates the installation of a semi-infinite tube (SIT) configuration. It consists of a coiled seamless tube of 2.93 mm internal diameter, whose length is 2 m to avoid any interference with the acoustic field. It is connected to a plastic hose for nitrogen flow for cooling purposes of the installed dynamic microphone (Kulite MIC-190M, nominal sensitivity: 0.0035mV/Pa at 160 dBa excitation). The nitrogen cooling flow is not mixed with the reactants or the exhaust gases. The microphone is installed 130 mm over the bore of the exhaust and acoustic

waves from the combustor are measured through a 1 mm hole drilled on the SIT configuration.

Chemiluminescence signals are acquired using a photomultiplier developed in [11]. Global chemiluminescent emissions from the transparent part of the combustor are detected by a set of UV plano-concave (Thorlabs, focal length -30 mm, diameter 1 in) and UV plano-convex (Thorlabs, focal length 35mm, diameter 1 in) lenses, focusing the light on a 1.3 mm diameter lens of an optical fibre (Thorlabs-FT1500UMT-0.39 NA, 300-1200 nm). The spectroscopic unit acquires the light transmitted through the optical fibre. The light is split into three spectral fractions using two dichroic mirrors. The reflected light is directed onto filters, to acquire the light from the spectra of interest (308.5 nm for OH*-bandwidth 18.0 nm, 430.5 nm for CH*-bandwidth 1.9 nm, 516.0 nm for C₂* – bandwidth 5 nm). The pressure and chemiluminescence analogue signals are acquired and stored through a National Instruments data acquisition card. Data from the acquisition card in each experimental case are collected for 10 seconds at a sampling rate of 32768 Hz at segments of 1024 samples.

2.2. High Speed Chemiluminescent Images

For each experimental case, 5000 high speed CH* images of the flame were captured at each experimental case at 3000 frames per second. We selected to monitor the CH* spatial distribution as a surrogate of the spatial distribution of the heat release rate. In [16], it was shown experimentally in a premixed counterflow burner configuration, that increasingly strained methane flames preserve a linear relationship between the CH* intensity and heat release rate. The camera (Photron CMOS HighSpeed Star2) is equipped with a Nikon lense (f/4.5, focal length 105mm) which is fitted with a narrow bandpass interference filter (432 ± 5 nm). The quantum efficiency of the camera at the examined optical range is 30%. The triggering of the camera is synchronous to the dynamic pressure signal. The stochastic nature [17] of the intermittent phenomena that are encountered in the intermediate regimes between the quiescent and oscillatory regimes, which are referred to as intermittent in the rest of the article, require the development of a short term prediction technique to forewarn the acquisition system ahead of a pending high amplitude dynamic pressure and heat release burst. The method we adopted in the current context is the permutation entropy, which has been described in a previous work [18]. A dip in the permutation entropy signal signifies that the combustor dynamics is attracted from an unstructured low amplitude mode to a high amplitude low complexity limit cycle. The transition from a high entropy signal, characteristic during quiescence, to a low entropy one, as is the case during limit cycle combustion is essentially the triggering indication. A significant tuning parameter of the algorithm that affected the repeatable capturing of information ahead of the impending instability was the size of the interrogation window of the permutation entropy signal. The size of the window was selected so as to balance between sensitivity of the algorithm to a pending instability and the time the algorithm

needed to calculate the permutation entropy signal. A larger window was associated with higher computational requirements, but also yielded robust crossing of the predefined threshold and initiated the acquisition robustly. Shorter time windows did not provide a sharp decline of the permutation entropy signal; in that case precursors of the impending transition were not monitored. The characteristics of the algorithm were tuned to achieve capturing the transition at about 0.25 seconds ahead of the first burst of intermittent instability. The camera acquisition leads the permutation entropy triggering signal by 300 images in order to capture events before the onset of an instability. The resolution of each image is 832×1024 pixels, and the size was 72 mm by 80 mm hence each pixel resolves 0.0068 mm^2 of the imaging area. The images acquired were line of sight integrated, so in order to extract the flame topology in an axial cross section of the combustor we applied an Abel transformation, as described in [19]. The flame structure was time independent when the flame was quiescent, but was significantly oscillatory when the flame was unstable. Therefore, when presenting a quiescent flame topology, the mean line of sight integrated images describe adequately the flame anchoring locations. However, when the flame is oscillatory, the images are phase averaged conditionally in respect to the dynamic pressure signal phase and then each topology is Abel deconvoluted. The spatial location of the high speed CH^* imaging window is described in Fig. 1.

2.3. High Speed Particle Image Velocimetry

We resolved the time dependent structure of the flow field through high speed Particle Image Velocimetry (PIV) experiments. Micron alumina particles were used to seed the flow, using a cyclone seeder. The illumination source was an Nd: YAG laser double pulsed at a rate of 3 kHz, emitting light at 532 nm. Pulse energies were 4 mJ/pulse at a time distance between each pulse of $dt=4 \mu\text{s}$. The dt was selected so as to limit out of plane particle displacement between successive illuminations. The Mie scattering signal was captured with a Photron CMOS HighSpeed Star 2 equipped with a Nikon lense (f/4.5, focal length 105mm) at a resolution of 1024×992 . The lense was fitted with a narrowband filter (Edmund Optics, $532 \text{ nm} \pm 5 \text{ nm}$) to suppress flame chemiluminescence from contaminating the image. Unavoidably, reflection lines along the longitudinal axis were captured, due to the cylindrical shape of the optical confinement. They were suppressed using a polariser. It successfully suppressed reflections and secured the CMOS of the camera was not saturated, in the expense of the signal to noise ratio of the alumina particles. The time stamp of each illumination pulse was explicitly defined by extracting the high intensity peaks in the photomultiplier C_2^* signal.

Two cross sectional planes were considered. The radial-azimuthal circular cross-sectional plane whose stream-wise location is described in Fig. 1 and an axial-radial cross sectional plane downstream the outlet of the diffuser. For the first plane a 90° degree mirroring prism was mounted 30 cm downstream the far exit of the combustor

duct to avoid harming the camera by hot combustion products. The camera was fitted with a Nikon lense of focal length equal to 300 mm, and mounted perpendicularly to the combustor axis focusing at the far end radial-azimuthal field of view with the aid of the prism. The isothermal radial-azimuthal cross section was successfully measured, however it was not possible to measure in non isothermal conditions. We assume this is due to integral black body radiation emitted by the alumina particles along the whole length of the combustor, contaminating the captured images. The sides of the combustor upon which the laser sheet was impinging were masked out of the post processed vector fields, as the polariser did not suppress those reflections adequately. The axial-radial cross section had an effective spatial resolution of 60 mm by 60 mm. Synchronisation between the illumination source and the camera acquisition was achieved through LaVision's Davis 7.2 software and the post processing and vector extraction of the raw Mie scattering images was achieved through LaVision's 8.4 software. The size of the final interrogation PIV window was 32 pixel by 32 pixel with 75% overlap. Based on the 0.01 minimum detectable pixel shift, the incurring nominal uncertainty was 0.03 m/s.

2.4. Extinction Strain Rate Calculations

Central part in the current analysis is the characterization of the extinction strain rate of each mixture k_{ext} and the association of this quantity with the flow imposed strain rate k_{flow} . Extinction strain rate calculations were conducted in the laminar flame twin counter flow premixed burner configuration in Cantera [20]. The chemistry mechanism used was the GRI3.0. The relevance of the calculated extinction strain rates with the turbulent flame in our combustor can be justified *via* the following considerations: 1) The extinction strain rate calculations are conducted on laminar counter flow burner flames. 2) In [14] it was assessed that the flame in the current experimental configuration lies in the thin reaction zone regime. Hence, we conduct the rest of the analysis considering that the laminar extinction strain rates calculated in Cantera are representative of the actual extinction strain rate of the flames examined in the combustor.

2.5. Flow imposed strain rate

In [6] it was suggested that an upper estimation of the absolute values of the flow imposed strain rate that the flame experiences can be calculated considering a set of velocity gradients, in the axial-radial domain. The spatial distribution of the flow imposed strain rate is expressed in equation 1.

$$\kappa_{flow}(x, y) = \left| \frac{\partial V(z, r)}{\partial z} \right| + \left| \frac{\partial U(z, r)}{\partial z} + \frac{V(z, r)}{R} + \frac{\partial V(z, r)}{\partial r} \right| + \left| \frac{U(z, r)}{R} + \frac{\partial U(z, r)}{\partial r} \right| \quad (1)$$

U and V stand for the radial and axial components of velocity respectively. Symbols r and z stand for the radial and axial coordinates respectively and R is the combustor radius. The least square differentiating scheme has been employed away from the boundaries of the domain and the forward and backward difference schemes at the appropriate boundaries. The authors in [6] suggest that equation 1 is applicable to time averaged mean flowfields, that demonstrate cylindrical symmetry. Equation 1 was derived by considering the local flame normal of the time averaged flame structure. It was also assumed that the flame's angle was independent of the axial section. In the current article, as shown in subsection 4.3 we propose the expansion of the method presented in [6] by calculating the flow imposed strain rates on conditionally phase averaged flow fields and not only on the mean flow field. The difference between the two approaches lies on the fact that the conditionally time averaged flowfields contain the coherent component of each phase angle in addition to the mean component.

The current approach described by equation 1 considers solely strain rates on the axial-radial plane. It is worth discussing the effects of neglecting the out of plane shearing strain rate terms. In [21], the authors assessed the uncertainty introduced by neglecting the out of plane strain rate components. The study showed that these terms were significant when calculating the instantaneous spatial distribution of the strain rate. Individual flowfield PIV snapshots demonstrated circumferential asymmetries. Under such conditions, the azimuthal normal to the flame component increased considerably the contribution of shearing terms in the azimuthal direction. However, statistically converged time averaged flowfields were indeed circumferentially symmetric, hence diminishing the relative contribution of the out of plane shearing terms in respect to the in-plane components. In the current work, PIV measurements of axial sections of the combustor were possible but only for isothermal cases. Under reacting conditions, integral black body radiation contaminated the Mie signal. In effect, the out of plane strain rate can only be assessed via the isothermal axial-radial and radial-azimuthal measurements. The discussion of the relative significance between the resolved and unresolved strain rate terms is continued in section 4.1, as part of the discussion with regards the structure of the isothermal flowfield.

2.6. Operating Conditions

The experimental campaign refers to H_2 enrichment of the CH_4 blends. The molar content, equivalence ratio, dynamic state and the extinction strain rate of the mixtures are presented in Table 1. The effect of hydrogen enrichment at three different global equivalence ratios, namely $\phi = 0.55$, $\phi = 0.60$, $\phi = 0.70$ has been studied. The equivalence ratios were selected in order to conclude on the dynamic state and flame shape transitions for varying dynamic state reference cases (namely 100% CH_4). The $\phi = 0.55$ reference case was susceptible to blow off, the $\phi = 0.60$ reference case was robust to blowoff and quiescent and the $\phi = 0.70$ reference case was unstable

Table 1: Experimental campaign operational parameters. Three equivalence ratios have been tested ($\phi = [0.55, 0.60, 0.70]$) at a constant Reynolds number $Re = 19000$. The fuel molar content (χ : H_2 molar content), the dynamic state of the combustor, and the numerically calculated extinction strain rate k_{ext} are defined.

Case ID	$(1 - \chi)CH_4 + \chi H_2$	ϕ	Dynamic State	k_{ext} [1/s]	P[kW]
A	[100%, 0%]	0.55	Susceptible to blow off	273	32.58
B	[90%, 10%]	0.55	Quiescent	652	32.79
C	[80%, 20%]	0.55	Quiescent	759	33.10
D	[70%, 30%]	0.55	Intermittent	1127	33.47
E	[65%, 35%]	0.55	Intermittent	1250	33.68
F	[62.5%, 37.5%]	0.55	Intermittent	1350	33.79
G	[60%, 40%]	0.55	Limit Cycle	1540	33.91

$(1 - \chi)CH_4 + \chi H_2$	ϕ	Dynamic State	k_{ext} [1/s]	P[kW]
[100%, 0%]	0.60	Robust, Quiescent	1043	35.32
[90%, 10%]	0.60	Quiescent	1369	35.60
[80%, 20%]	0.60	Quiescent	1727	35.94
[77%, 23%]	0.60	Intermittent	1839	36.06
[75%, 25%]	0.60	Intermittent	1927	36.13
[65%, 35%]	0.60	Limit Cycle	2435	36.58
[62.5%, 37.5%]	0.60	Limit Cycle	2588	36.70

$(1 - \chi)CH_4 + \chi H_2$	ϕ	Dynamic State	k_{ext} [1/s]	P[kW]
[100%, 0%]	0.70	Limit Cycle	2191	40.83
[90%, 10%]	0.70	Limit Cycle	2781	41.17
[87%, 13%]	0.70	Limit Cycle	2967	41.28
[85%, 15%]	0.70	Limit Cycle	3099	41.35
[83%, 17%]	0.70	Limit Cycle	3120	41.43
[80%, 20%]	0.70	Limit Cycle	3263	42.10

demonstrating limit cycle Period-2 oscillations. In the current article we focus solely on the $\phi = 0.55$ mixtures and the respective flame shape and dynamic state transitions on increasing hydrogen content. For that matter, we identify only the flames for $\phi = 0.55$ in Table 1 to refer to in the following sections.

3. Dynamic Mode Decomposition

In the current article we have used the Dynamic Mode Decomposition (DMD) to retrieve dynamics in the isothermal and reacting flowfields. The algorithm has been introduced by Schmid in [22]. The version used in the current paper has been developed in [23]. Reviews of alternative DMD methods can be found in [24, 25, 26, 27, 28, 29]. A short description of the algorithm is provided in Table 2. DMD attempts to map a set of n observations $\mathbf{V}_1 = [\mathbf{z}_1, \mathbf{z}_2, \dots, \mathbf{z}_{n-1}]$ to $\mathbf{V}_2 = [\mathbf{z}_2, \mathbf{z}_3, \dots, \mathbf{z}_n]$, via $\tilde{\mathbf{A}}$. $\tilde{\mathbf{A}}$ approximates the actual Koopman operator $\mathbf{A}$ which satisfies $\mathbf{V}_2 = \mathbf{A}\mathbf{V}_1$, namely propagates an observation $\mathbf{z}_j$ onto a future one $\mathbf{z}_{j+1}$.

Table 2:

DMD algorithm [23]
1. Define $\mathbf{V}_1 = [\mathbf{z}_1, \mathbf{z}_2, \dots, \mathbf{z}_{n-1}]$ and $\mathbf{V}_2 = [\mathbf{z}_2, \mathbf{z}_3, \dots, \mathbf{z}_n]$, where $\mathbf{z}_i$ is a column vector.
2. Compute the Singular Value Decomposition (SVD) of $\mathbf{V}_1$, with rank r yielding: $\mathbf{V}_1 = \mathbf{U}\Sigma\mathbf{V}^*$
3. Calculate $\tilde{\mathbf{A}}$ as: $\tilde{\mathbf{A}} = \mathbf{U}^* \mathbf{V}_2 \mathbf{V} \Sigma^{-1}$
4. Calculate the eigenvectors $\mathbf{w}$ and eigenvalues λ of $\tilde{\mathbf{A}}$: $\tilde{\mathbf{A}}\mathbf{w} = \lambda\mathbf{w}$
5. For each eigenvector $\mathbf{w}$ and eigenvalue λ exists a DMD mode ϕ_i , stored as the i -th column vector in matrix Φ defined as $\Phi = \frac{1}{\lambda} \mathbf{Y} \mathbf{V} \Sigma^{-1} \mathbf{w}$

The discrete time eigenvalues λ of $\tilde{\mathbf{A}}$ are calculated following the algorithm presented in Table 2 and yield the frequency of each eigenmode, through $f = \text{Im}(\frac{\omega_i}{2\pi})$ and the respective growth rate through $\sigma = \text{Re}(\omega_i)$, where $\omega = \frac{\log \lambda_i}{\delta t}$ are the continuous time eigenvalues and δt the inverse of the sampling rate. In the current work, it is of interest to examine phenomena associated with a specific frequency range (e.g. the frequency associated with a coherent structure). At the same time, we seek to filter phenomena that contribute outside from the specified frequency range. For this purpose, the original data sets can be reconstructed through equation 2.

$$\mathbf{U}(\mathbf{x}, \mathbf{t}) = \Re \left[\sum_{i=1}^N \phi_i \mathbf{e}^{(\sigma_i + i2\pi f_i) \mathbf{t}} \right] \quad (2)$$

Further to the reduced order representation of the dynamics, an additional useful application of the algorithm is the capability to calculate the energy stored in each mode at different time instances of the acquisition. Adopting the approach developed in [30] by accounting for the energy growth/decay of each mode, the energy spectrum at the end of the acquisition is calculated according to equation 3.

$$\mathbf{E}_i = \|\phi_i\|_2 \frac{\mathbf{e}^{2\sigma_i T} - 1}{2\sigma_i T} \quad (3)$$

where T is equal to the total duration of the acquisition.

The algorithm presented in table 2, calculates the reduced singular value decomposition (SVD) of the original matrix of observations $\mathbf{V}_1$, by truncating the SVD matrixes (in step 2 of the algorithm) up to a rank r . Rank truncation of the data in such a manner ensures that the algorithm avoids corrupting the dynamics by fitting low energy and

Table 3: Indicative Wobbe index operational ranges for three different industrial gas turbine engines.

	Siemens [33]	General Electrics [32]	Alstom [34]
Type	SGT-300	9HA.02	GT13E2
Fuel	Natural Gas	Natural Gas	Natural Gas
Nominal Wobbe Index Range [MJ/m ³]	37-49	47-53	31-49

noisy modes [29]. In the current work the truncation rank r is selected based on a threshold $e_p = \frac{\sum_1^r \sigma_i}{\sum_1^h \sigma_i} < 0.0001$, where σ_i is the i -th singular value of matrix Φ that contains the DMD modes. By selecting this truncation threshold it was possible to calculate a full-rank Φ matrix for each examined case. The reader is directed to the supplementary material that complement the online version of this article for the singular values of the DMD modes of the cases we examine in sections 4.1 and 4.2.1.

4. Results and Discussion

The extinction strain rate of a mixture is proposed as a means of collapsing the dynamic behaviour of the combustor. Interchangeability between pairs of fuel in gas turbine applications is considered based on the Lower Wobbe Index (LWI) of each mixture. However, the LWI does not contain any information about the chemical properties of the mixture [31], hence it cannot be used as a reliable index of identifying the dynamic state of operation of the combustor. Fig. 3 (left) presents the LWI and the amplitude of the dynamic pressure of the fundamental mode of oscillation, for each experimental case examined. To contextualize this figure, Table 4 presents indicative nominal Wobbe indexes of operation of industrial gas turbines. The typical range of of LWI for Natural Gas type fuels is reported in the range 49-53 MJ/m³ [32]. Depending on adjustments on the LNG constituency and modifications of the engine configuration, the operational range may be reliably extended to a range of 30-50 MJ/m³ [33, 32, 34] . The ranges of LWI studied in the current work span from 42-49 MJ/m³. It is apparent that the dynamic state cannot be collapsed based on the LWI value. In [6] the authors suggested that scaling the dynamic pressure data based on the extinction strain (k_{ext}) can scale the flame shape anchoring regions. In Fig. 3 (right) the dynamic pressure amplitudes are reorganized based on their (k_{ext}) values. Fuels with k_{ext} greater than $1700 \frac{1}{s}$ give high amplitude dynamics. An intermediate regime can be recognised between $1200 \frac{1}{s}$ and $1700 \frac{1}{s}$ where fuels behaved in an intermittent manner namely they demonstrated high amplitude bursts and then they would return to a quiescent mode. Fuels with k_{ext} lower than $1200 \frac{1}{s}$ remained in the quiescent regime.

In the following subsections we will examine the interaction between the hydrodynamic coherent structures of the flowfield and the flame dynamics. We show that the flame anchoring locations are dictated by the flow

imposed strain rate and then that increased extinction resistance sets the flame susceptible to swirling flow instabilities, that may instigate mechanisms responsible for intermittent heat release rate bursts. For each operational regime we examine the observed dynamics by presenting phase space portraits of the dynamic pressure signal (see [17] for a description of the method) and by calculating the respective fast Fourier transformation.

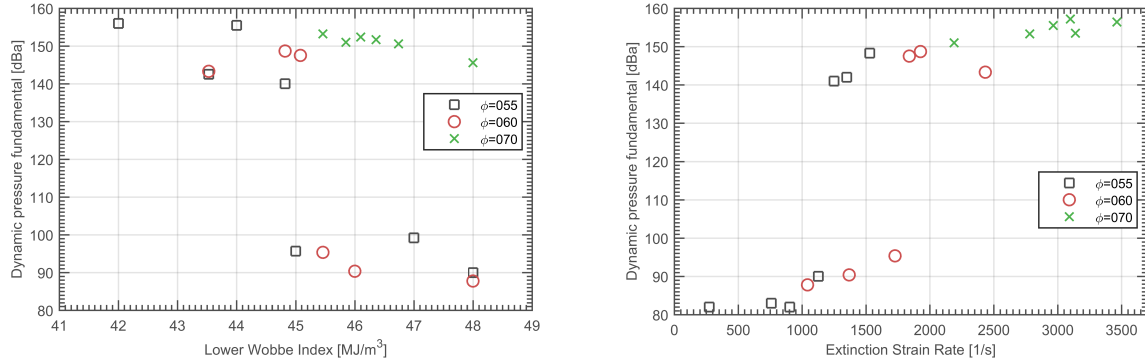


Figure 3: Amplitude of the dynamic pressure fundamental mode of oscillation plotted as a function of the Lower Wobbe Index (left) and to the numerically calculated extinction strain rate (right).

4.1. Isothermal Flow Field

The mean isothermal flow field is described in Fig. 4. The figure displays the relative position of the swirler in respect to the measurement PIV plane. Two dimensional streamlines of velocity vectors measured on the axial-radial plane at the inlet of the combustor are superimposed on the values of the axial velocity in Fig. 4 (left). The flowfield shows a central vortex core with reverse columnar flow along the combustor axis. This is a result of vortex breakdown due to the swirling motion imparted by the vanes of the centerbody. In between the jets and the free stream positive axial velocity areas, shear layers of zero axial velocity are produced, providing suitable flame anchoring time scales. In PIV measurements on field of views along the combustor centreline (not presented here) we can see that this recirculation zone extends until the end of the optically accessible part of the combustor, as expected in a supercritical flow. However, no downstream stagnation point can be observed. Moreover, two non-symmetric swirling jets are established. The jet on the right hand side extends until the end of the presented field of view (FOV), whereas the jet on the left hand side is longitudinally limited up to approximately 15mm. At the downstream end of the FOV, between $x=-10$ mm and $x=0$ mm of the radial dimension, the upper half of an ellipsoidal-like recirculation pattern can be observed. In contrast to some combustor geometries presented in literature, no outer recirculation zone can be observed in the isothermal field, due to the introduction of the pre-mixture in the combustor through a diffuser and not through a sudden step expansion.

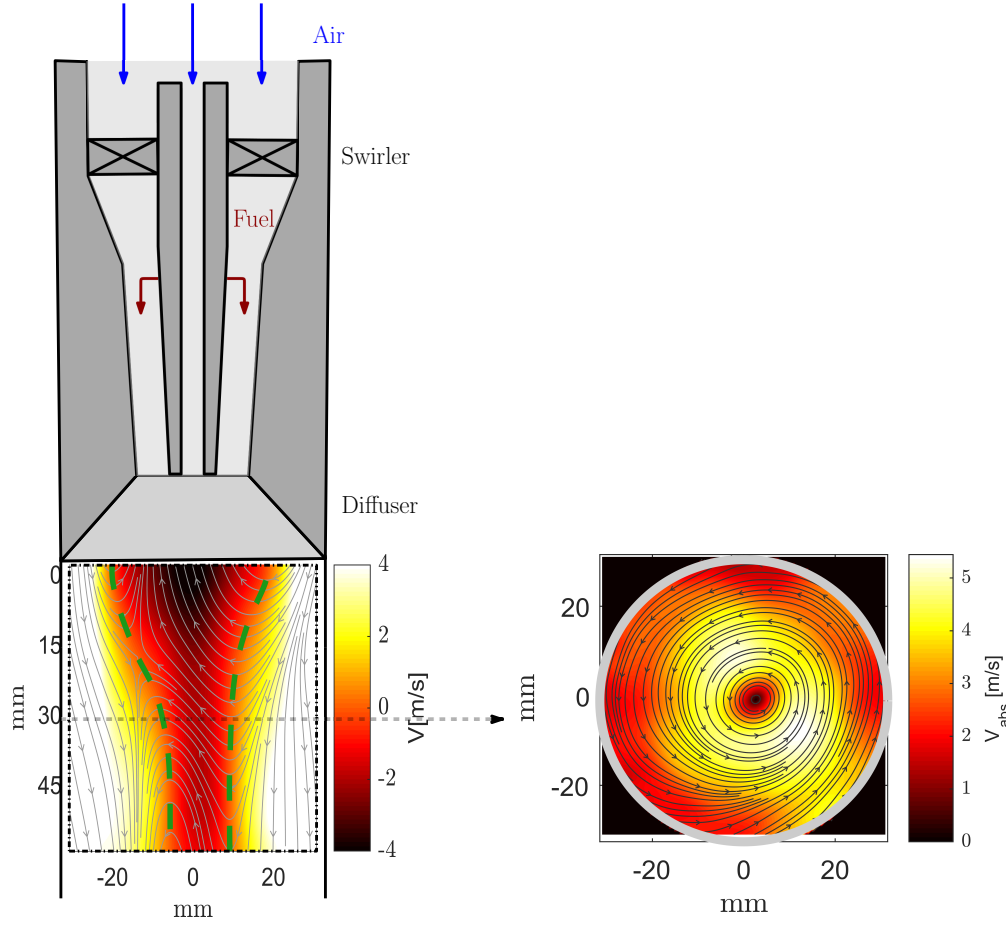


Figure 4: Description of the mean isothermal flow field. Left: Vector streamlines superimposed on contours of axial velocity. The schematic shows the relative position of the swirler and the diffuser in respect to the PIV axial-radial window (marked with dashed solid lines). The green dashed line is an isoline of zero axial velocity. Right: Transverse field of view at a streamwise location equal to 30 mm in reference to the axial field of view. Streamlines are superimposed on the two dimensional absolute values of velocity in the transverse field of view.

Fig. 4 (right) shows two dimensional streamlines of velocity vectors measured on an azimuthal plane located at an axial station at $y=30$ mm downstream the start of the optically accessible part of the combustor. High amplitudes of absolute velocity V_{abs} of the two dimensional transverse velocity components, are associated with the forced vortex region of the recirculating flow field. Within this regions a low amplitude V_{abs} region is located, which defines the core of the azimuthal recirculation and the extent of the free vortex region. The centre of the forced vortex region appears off the geometric centre of the transverse FOV.

Next, remember the discussion with regards the unresolved out of plane strain rate components in subsection 2.5. The measured isothermal field of views in Fig. 4 are currently used for estimating the significance of

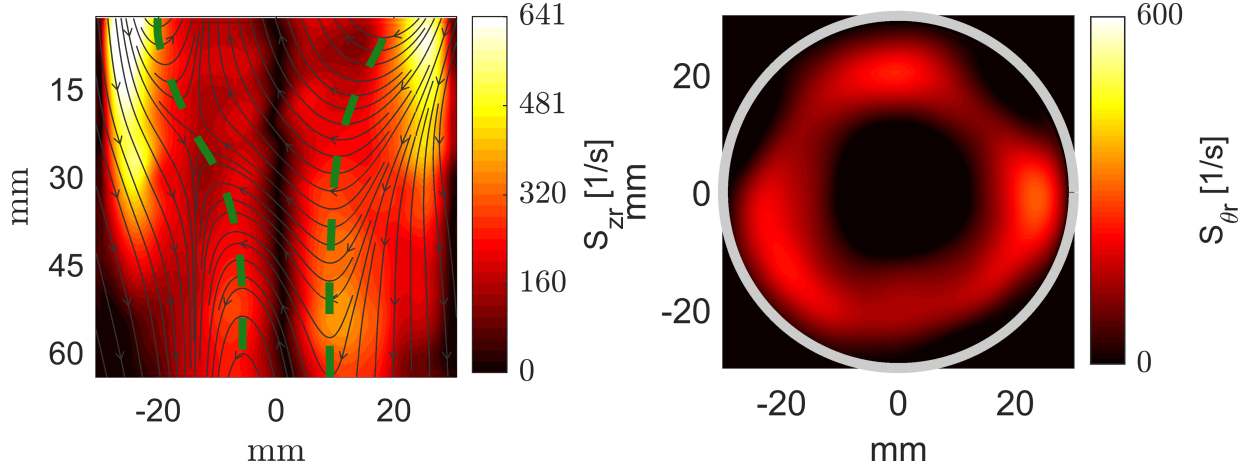


Figure 5: Strain rate contours of the mean isothermal flowfield. The same colourbar scale is used for both figures to aid comparison of the relative significance of the in plane and out of plane strain rate components. Left: Contours of axial-radial strain rate at isothermal conditions. Right: Contours of azimuthal-radial strain rates of the mean flowfield at an axial section of the combustor at $z=30\text{mm}$.

the out of plane terms relative to the resolved in plane ones. Figure 5 shows contours of the off diagonal shear strain rate terms. The axial-radial strain rate S_{zr} contours are shown on the left, defined by using equation 1, while the contours on the right describe the strain rate $S_{\theta r}$ on the azimuthal-radial plane. For the calculation of the strain rates on the azimuthal-radial plane the discussion in [35] and [36] has been followed. $S_{\theta r}$ is then equal to $S_{\theta r} = (r/2) \frac{\partial(U_\theta/r)}{\partial r}$, where U_θ is the azimuthal velocity, which is not retrievable by the two dimensional PIV axial-radial measurements. The purpose of this figure is to provide an estimation of the strain rates developed on the isothermal flowfield. To aid comparison we have used the same colourbar scales for both figures. As PIV measurements of the reacting flowfield at axial sections were not successful, we can only employ the isothermal measurements for an approximate assessment of the relative significance of the resolved and unresolved shear strain rate terms. The highest strain rates in the traverse section are located around the boundaries between the solid body rotation zone and the free stream zone. Correspondingly, high strain rate values can be found along the shear layers formed by the free stream and the recirculation zones of the combustor. The thermoacoustically stable flowfields corresponding to cases A, B and C demonstrate structure similar to the isothermal flowfield. The maximum strain rate in the traverse field is about 35% of the maximum axial-radial section. For these cases the out of plane strain rate that the flame is subjected to is expected to be underestimated by a factor of at least 35% of the maximum in plane and resolved strain rate. As will be shown later for cases B and C the radial extent of the recirculation zone increases. Under the influence of the cylindrical confinement, the gradients on the azimuthal-radial plane are expected to be higher, thereby increasing the contribution of the unresolved out of plane terms. Finally, the thermoacoustically unstable cases feature flowfields with distinctively different structure than the isothermal,

with a strong coherent component. Hence, conclusions for the unresolved strain rates cannot be deduced by considering the isothermal flowfield measurements.

The current configuration demonstrates inherent global instabilities that are helical in nature. This helical instability has been lengthy studied in literature and is referred to as a Precessing Vortex Core [37], [38]. First or second order azimuthal modes may become dominant and the associated coherent structure is wrapped around the shear layers [39], [40]. The helical mode advects the central vortex core off the geometric centreline of the combustor inducing a precessing motion of the recirculation zone in a direction co-rotating with the mean swirl direction [41], [38]. Proper Orthogonal Decomposition (POD) techniques have traditionally aided in identifying the nature of the coherent structure associated with the PVC. Results of the POD analysis on the azimuthal-radial and axial-radial isothermal PIV measurements on the current geometry are presented in section Appendix A. The reader is also directed to the on-line version of the current article, wherein an animation of the Mie-scattered signal shows the demonstration of the PVC in the isothermal flowfields.

To study the frequency of precession we apply the DMD algorithm on the isothermal flowfields. The energy content at each frequency is shown in Fig. 6, normalized over the energy content of the most energetic mode which is the one associated with the stationary 0 Hz component. The truncation rank r in this case is 1800. The rank has been calculated by satisfying the singular value thresholding criteria being described in section 3. The mode of interest is the most energetic non-stationary mode associated with the 170 Hz. In Fig. 7, we reconstruct the oscillations of the absolute velocity on the traverse field of view using solely the most non-stationary energetic dynamic mode. The precessing core displaces azimuthally the low amplitude V_{abs} azimuthal recirculation core, the eccentricity generating correspondingly higher absolute velocities in the respective near-wall confined regions. These higher absolute velocity regions also precess azimuthally, as indicated by the level of contours. The coherent structure has a high energy range of frequencies, as shown in Fig. 6 (left) at about half the dominant frequency of precession, at 75 Hz.

4.2. Quiescent Flames: Case B and C

In Fig. 8 we present representative phase space portraits, time series and frequency spectra for quiescent flames in cases B and C. The dynamics are attracted towards a low amplitude mode. Dynamic pressure fluctuations are random and stochastic, demonstrating the prevalence of turbulent broadband noise production, which is also indicated by the broadband Fourier spectra, that do not show any characteristic timescales. The axial veloc-

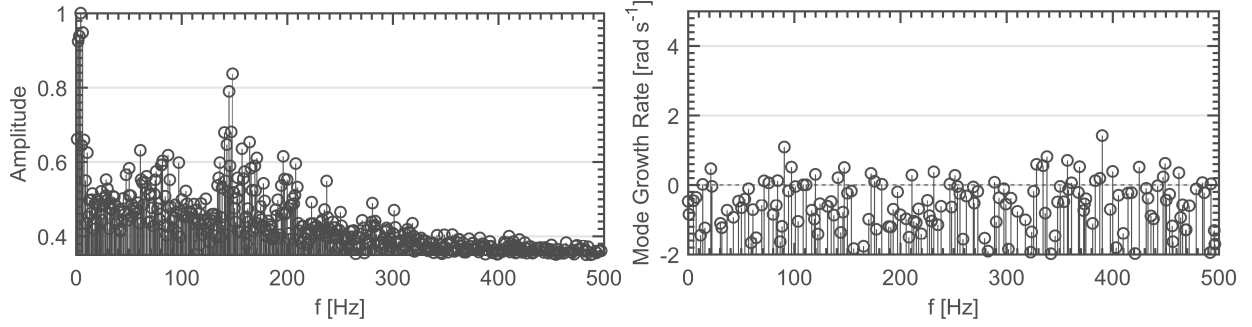


Figure 6: DMD spectrum of the isothermal transverse PIV measurements. Amplitude spectrum (left) and growth rate spectrum (right). Following the truncation method described in section 3 the rank of the Φ matrix containing the DMD modes is $r=1800$.

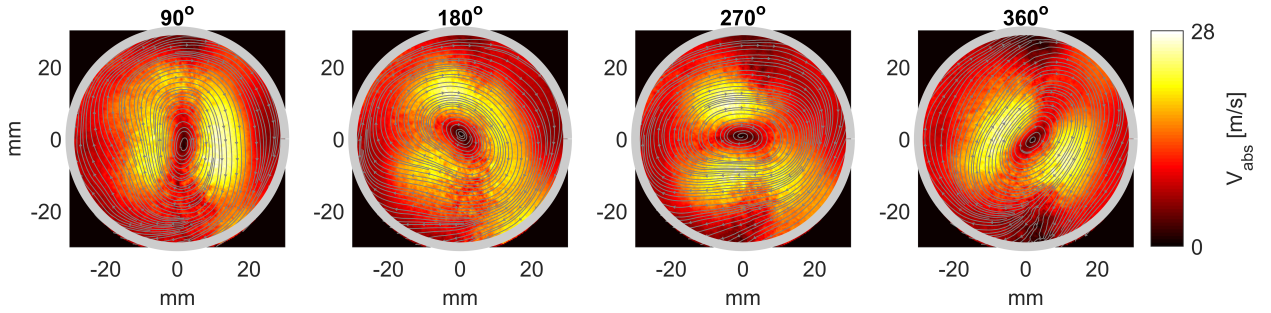


Figure 7: Phase averaged contours of absolute velocity spatial distribution. The contours and phases correspond to reconstructed dynamics using the DMD modes associated with the 170 Hz mode.

ity and flow imposed strain rate spatial distribution are presented in Fig. 9. Both figures are quite similar for both cases, therefore we present only the set of figures corresponding to case C. The mean flame anchoring regions in cases B and C are depicted in Fig. 10. The flow field resembles the isothermal flow field with increased back flow, due to flow dilatation caused by the flame. The central vortex core still extends until the end of the combustor. By examining the spatial distribution of k_{flow} in conjunction with the anchoring locations of the flame in cases B and C, we can conclude that the flame anchors upon the inner shear layers of the combustor at low k_{flow} locations. The maximum k_{flow} values are comparable to the k_{ext} of the mixture in Case C. When examining the anchoring regions of case B we observe a lifted, tubular flame with a maximum diameter within the FOV of about 5 mm. On increasing the H_2 content, the flame in case C was able to further penetrate through the high strain rate areas, anchor further upstream with a maximum diameter of about 20 mm but nevertheless it still appears to be lifted.

4.2.1. Intermittent Flame: Case E

The current section deals with intermittent dynamics observed for intermediate H_2 molar content. Dynamic transitions in swirl stabilized combustors have attracted significant attention in literature. A table with indicative studies of dynamic transitions in swirl stabilized combustors, on adjusting a given bifurcation parameter of the

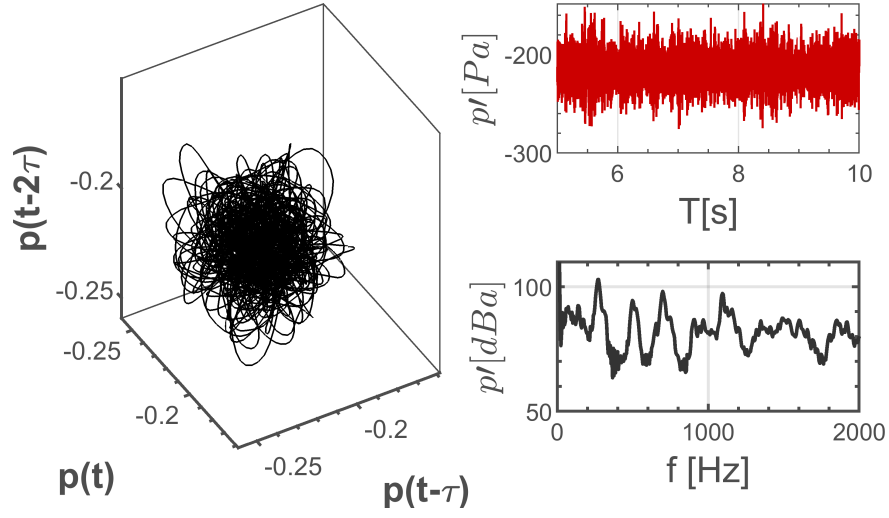


Figure 8: Case C: Left: phase space portrait of the dynamic pressure signal of a quiescent time series. Right: An indicative dynamic pressure signal and the respective FFT spectrum calculated on the whole time series.

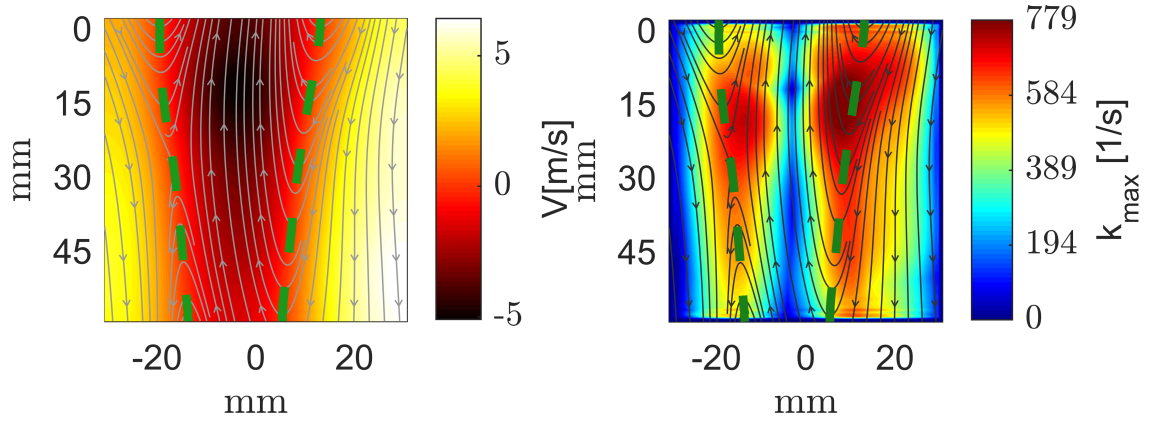


Figure 9: Case C: Streamlines superimposed on contours of mean axial velocity values(left) and the flow imposed strain rate (right). The numerically calculated extinction strain rate of the mixture is $k_{ext} = 759$ 1/s.

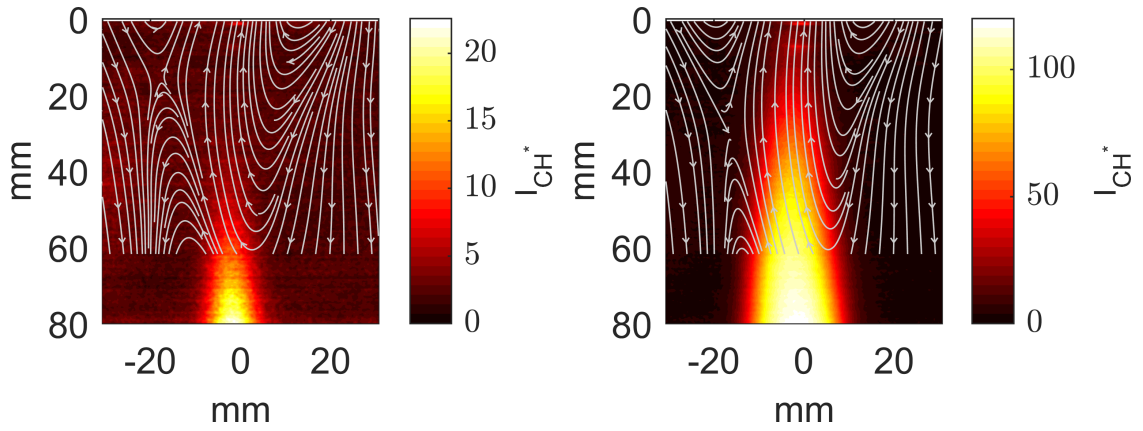


Figure 10: Case B (left) and C (right): Streamlines of the mean field superimposed on the mean CH^* intensity.

system is presented in Table B.4 in the Appendixes. The studies widely distinguish two transition types from quiescence to full blown limit cycles i.e. supercritical or subcritical Hopf bifurcations. The former type is associated with linearly unstable systems, whereas the latter characterizes non-linearly unstable systems [42]. Non-linearly unstable systems demonstrate bistability, i.e. for a finite interval of a bifurcation parameter both low and high amplitude modes are stable. Due to the superposition of the system over a turbulent background, the system may be attracted intermittently to either mode [43]. The intermittent dynamics observed in the current geometry demonstrate probability density functions (PDFs) of dynamic pressure amplitude that are bimodal (not presented in the context of the current work). However, the extensive assessment of the type of dynamic transitions observed in the current geometry is left for future work.

The intermittent dynamics that the flame demonstrated in Case E are depicted in Fig. 11. The representative phase space portrait of sudden bursts of dynamic pressure before a brief transition into a higher amplitude limit cycle shows that the dynamics follow a spiral motion while transitioning from the low amplitude mode towards the limit cycle. The Fourier spectra show a characteristic time scale which is equal to 160 Hz and its harmonic. Due to the fact that dynamic pressure signal is not ergodic, it is of further interest to compare the intermittent and limit cycle dynamics. Figure 12(left), focuses on the locus of attraction during three intermittent bursts, while Fig. 12(right) shows the respective dynamics during the following limit cycle state. The transitional dynamics also feature a higher frequency timescale at about 480 Hz whose magnitude is considerable in comparison to the 160 Hz most energetic mode. This is not the case for the limit cycle Fourier spectra wherein the amplitude of the 160 Hz mode is amplified and dominates the oscillations. The limit cycle dynamics are considered in section 4.3 further.

We now examine Fig. 13. The streamlines of the mean flow field are calculated solely from the part of the time series where the flame demonstrated intermittent behaviour not when it was attracted towards a limit cycle. The figures show that the mean flow field demonstrates a cylindrical asymmetry and a downstream stagnation point, hence the central vortex core has now formed a closed recirculation region. By examining the k_{flow} we can observe that there is a region of low strain rates leading towards the centerbody of the swirler. Hence, the flame can potentially anchor on the centerbody.

To examine the instantaneous behaviour of the flame we plot a sequence of CH^* images in Fig. 14, during an intermittent burst manifestation. These intermittent bursts were manifested in a stochastic manner. We can observe that when the flame is attached to the centerbody it precesses in a helical motion pattern similar to the

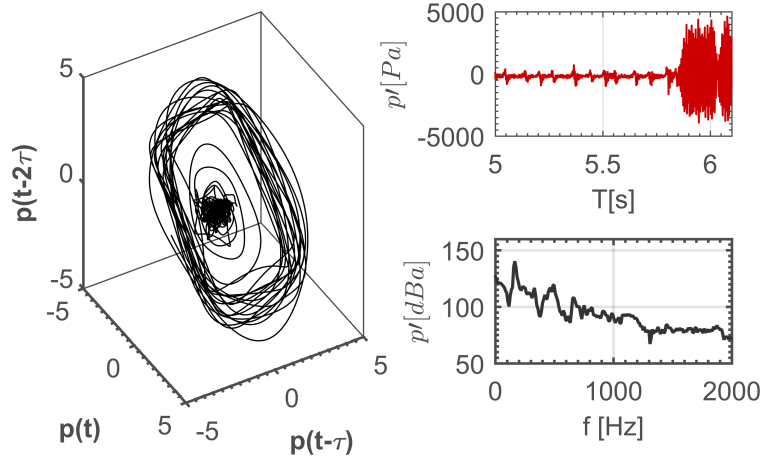


Figure 11: Case E: Phase space portrait of the dynamic pressure signal of a sample of intermittent dynamic pressure oscillations (left). An indicative dynamic pressure signal and the respective FFT spectrum calculated on the whole time series (right).

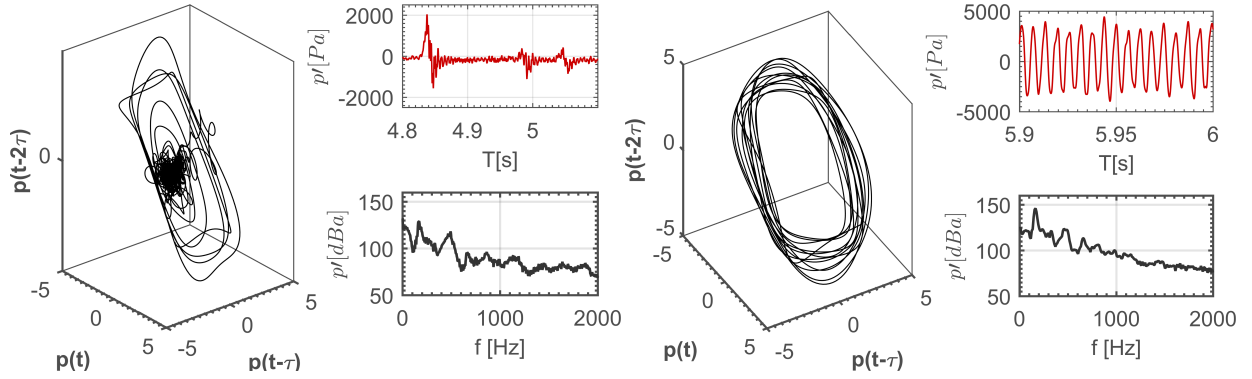


Figure 12: Case E: In reference to the time signal presented in Fig. 11, phase space portraits are presented of the time series during three events of intermittency (leftmost set of sub-figures) and during limit cycle operation (rightmost set of sub-figures).

naturally excited helical global hydrodynamic instability described in subsection 4.1. The flame is advected off the geometrical axis of symmetry and instantaneously is located in the regions of free stream velocity and it may even touch the wall of the combustor (see for example frames 10 and 12 in the presented time sequence). An explanation of the dependence of the demonstration of helical disturbances when the flame penetrates through high strain regions towards the centerbody, requires consideration of linear stability analysis in swirling flows. In [40], [44] and [45] linear stability analysis by Oberleithner and co-workers showed that the location of the helical mode wavemaker is located exactly downstream the combustor inlet. The wavemaker is a region of the flowfield that demonstrates absolute instability and drives the helical wave motion downstream the combustor. It would be reasonable to expect that this would hold true in our own geometry as well, however the presentation of a linear stability analysis on our geometry in the current article is beyond its scope.

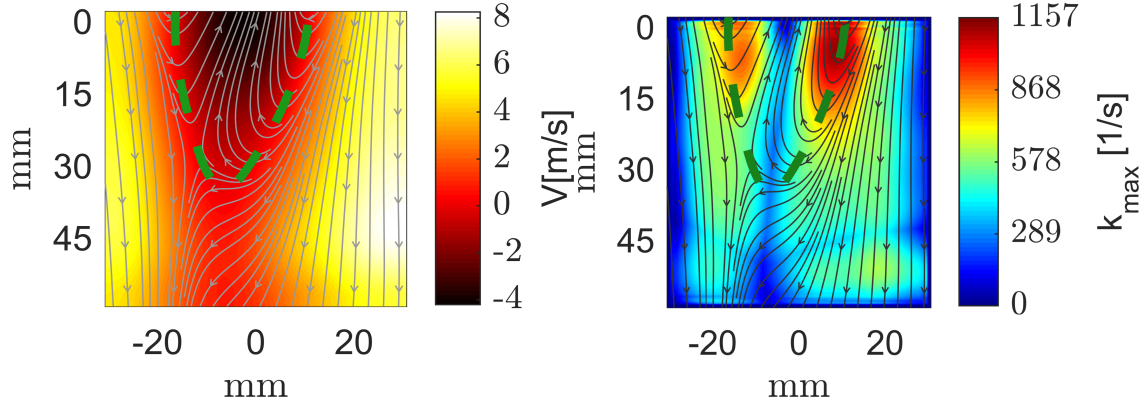


Figure 13: Case E: Streamlines superimposed on axial velocity values of the mean flow field (left) and the flow imposed strain rate (right). The numerically calculated extinction strain rate of the mixture is $k_{ext} = 1250$ 1/s

The dynamics of the underlying flowfield while the flame demonstrates intermittent behaviour are analysed by, again applying DMD. The application of the algorithm gives amplitude spectra and growth rates that are presented in Fig. 15. The truncation rank r in this case is 900. Again, r was selected with the scope of satisfying the singular value thresholding criteria being described in section 3. The maximum amplitude and growth rate is located at 137 Hz, which in comparison to the isothermal frequency of precession confirms that the precession rate has decreased in the reacting case. Having defined the frequency of the most energetic coherent structure, we reconstruct the dynamics using the zero growth rate mode which coincides with the mean flowfield and the most energetic mode using equation 2. The results of the reconstruction are presented in 16. Each flowfield shows a phase shifted reconstruction of the dynamics in respect to the period of the 137 Hz mode. The flowfields are described by superimposing reconstructed streamlines on contours of axial velocity. The recirculation zone is indicated with green dashed lines to aid the visualization of the precessing motion. Further to the observable precession, the axial extent of the recirculation zone oscillates as well.

To demonstrate that the intermittent bursts are associated with the flame advection off the axial centerline of the combustor, we plot in Fig. 17 the normalized intensity fluctuations in the outer windows of the combustor. The regions of consideration are denoted on the mean CH^* spatial distribution on the right. They are drawn by defining the boundaries of mean flame intensity where a background noise intensity threshold was exceeded. The normalization parameter is the mean flame intensity when the flame is quiescent. We can observe that the *integral* high amplitude bursts may reach up to 10 times the quiescent integral heat release. The normalized intensity of the *outer* windows can reach up to 2 times the quiescent integral heat release contributing significantly to the total high amplitude heat release burst even though the volume of the windows under consideration is disproportionate in relation to the total volume of the combustor. The authors in [7] have observed that the flame may

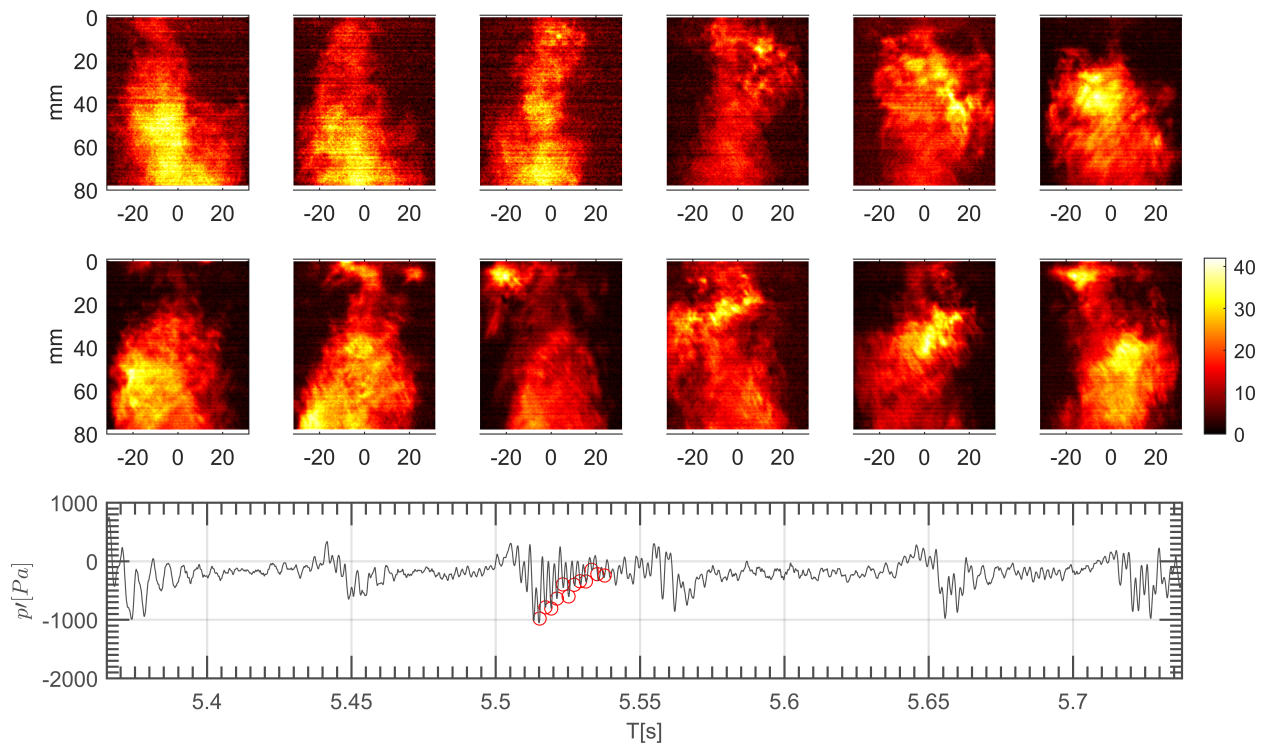


Figure 14: Case E: Sequence of CH^* spatial distribution while the flame is advected off the longitudinal axis of symmetry in a helical pattern of motion. The first row of images is temporally succeeded by the second row. The flame demonstrates intermittent behaviour. The respective time stamps of each instance on the dynamic pressure signal are indicated with a circle on the lower figure.

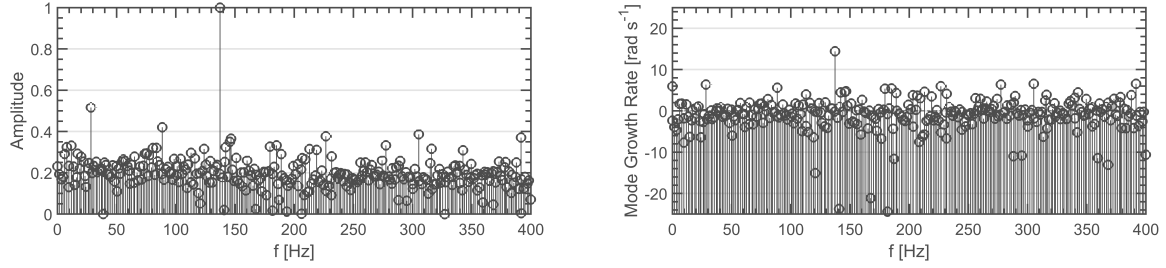


Figure 15: Case E: DMD spectra of the intermittent state. Amplitude spectrum (left) and growth rate spectrum (right). Following the truncation method described in section 3 the rank of the Φ matrix containing the DMD modes is $r=900$.

expand in the outer recirculation zone *via* flamelet penetration, provided that the ratio of the extinction strain rate to a local flow spinning time scale exceeds a threshold. However, in the current configuration the diffuser hinders the formation of outer recirculation zones, therefore the underlying mechanism must be different.

The current observations may lead to additional considerations on top of the existing intermittent regime interpretations. If the extinction strain rate of the mixture is high enough, so that the flame anchors upon the centerbody and be able to sustain high strain rates at the root, then helical disturbances might lead to two mechanisms of intermittency: 1) It might lead the flame in locations of freshly introduced combustible reactants in the free stream region. Reactants in this region would not otherwise start to react if the flame was not advected off the combustor centerline, as we have shown in section 4.2. Eventually, under this proposed mechanism the effect of the helical motion of the flame is an increase in the overall flame surface area. 2) It might also lead the flame towards impinging on the combustor walls. This would decrease the flame surface, in effect create fuel richer regions of unburned combustor products, which would be burned in a latter time. The validity of either of these hypothesis is subject of future work, involving high speed PLIF measurements.

4.3. Limit Cycle Period 1: Case G

On further increasing the extinction strain rate the high amplitude bursts become more frequent until the dynamics are permanently attracted to the limit cycle, which for case G are represented in Fig. 18. The frequency spectra show that the dynamics oscillate at 170 Hz and its harmonics. The fundamental coincides with the first quarter wave mode of the combustor [13]. The attractor topology is a period 1 limit cycle which means that there exists a single dominant time scale in the oscillations, which in this case is associated with the excited acoustic eigenmode of the duct. This is an indication, that at this operational condition hydrodynamic time scales are not excited, hence hydrodynamic effects do not affect the global sound or heat release rate production. What we can infer by this statement is that the helical motion may be associated with events *preceding* the consistent attraction

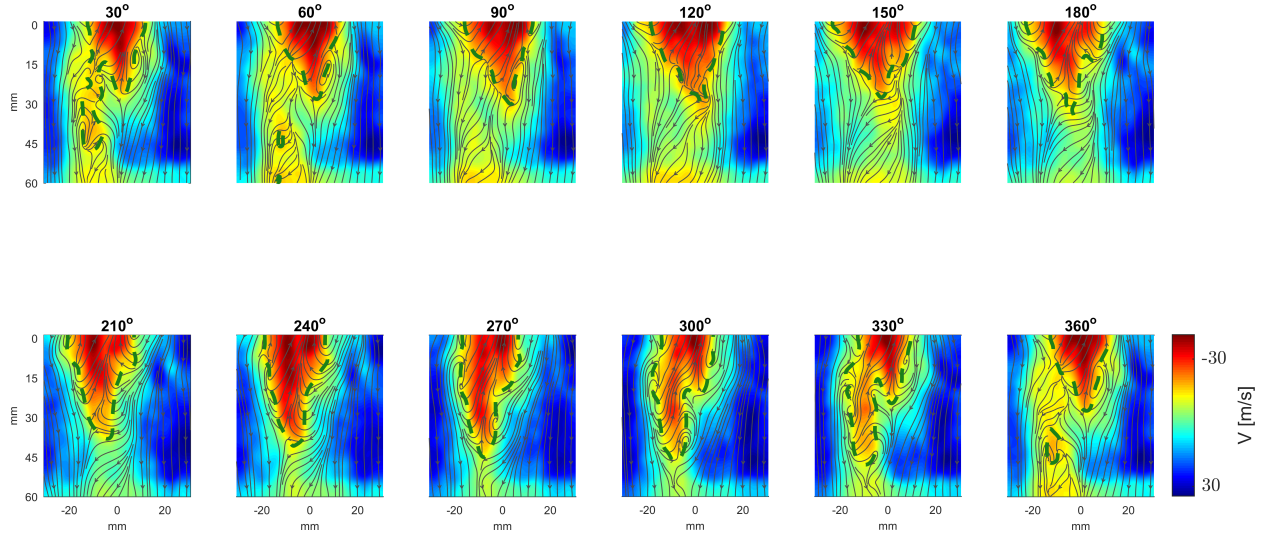


Figure 16: Case E: Reconstruction of the precessing flowfield using the dynamic mode corresponding to the 137 Hz frequency. Streamlines are superimposed on contours of axial velocity. Green dashed lines mark the location of zero axial velocity.

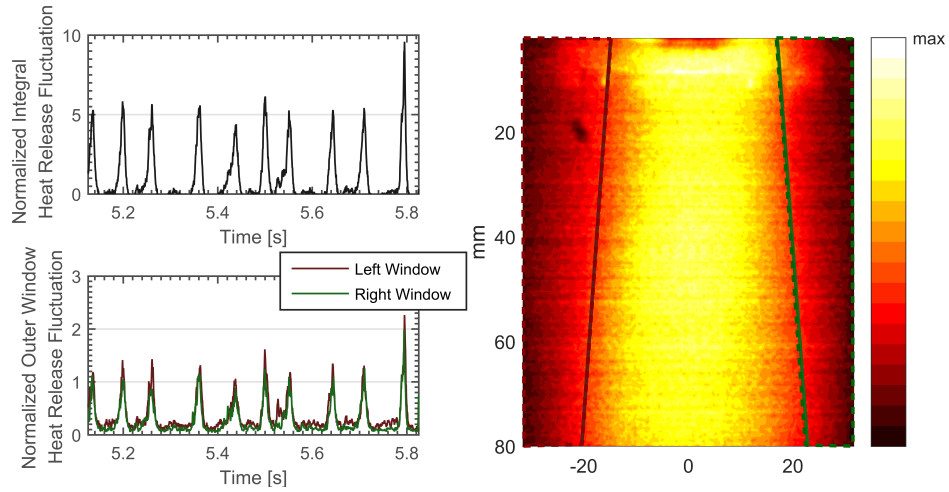


Figure 17: Case E: Integral CH^* intensity normalized over the mean intensity conditionally calculated when the flame is quiescent (top). Outer window CH^* intensity normalized by the mean intensity conditionally calculated when the flame is quiescent (bottom). Right: The regions of the outer windows are marked on the averaged intensity distribution of the flame.

towards a limit cycle. However, they do not necessarily survive or contribute to the perpetuation mechanism of the limit cycle oscillation in the operational condition we examine.

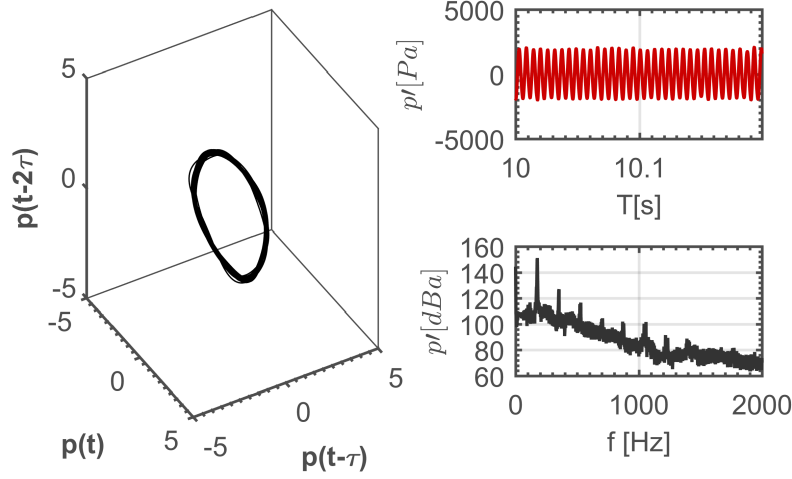


Figure 18: Case G: Phase space portrait of the dynamic pressure signal under limit cycle oscillations (left). An indicative dynamic pressure signal and the respective FFT spectrum calculated on the whole time series (right).

In Fig. 19 we present phase averaged streamlines superimposed on contours of axial velocity. The flowfields are conditionally averaged in respect to the phase angle of the dynamic pressure signal with maximum value at 360° and lowest value at 180° . The dashed green line indicates the zero axial velocity region, hence provide an indication of the extent of the recirculation zone. We can observe that the recirculation zone demonstrates a breathing motion with oscillating streamwise downstream stagnation point extent. From $210^\circ - 360^\circ$ a swirling jet is introduced in the combustor chamber and propagates along the walls of the combustor. As the streamwise position of maximum positive axial velocity is located further downstream along the combustor axis, the recircula-

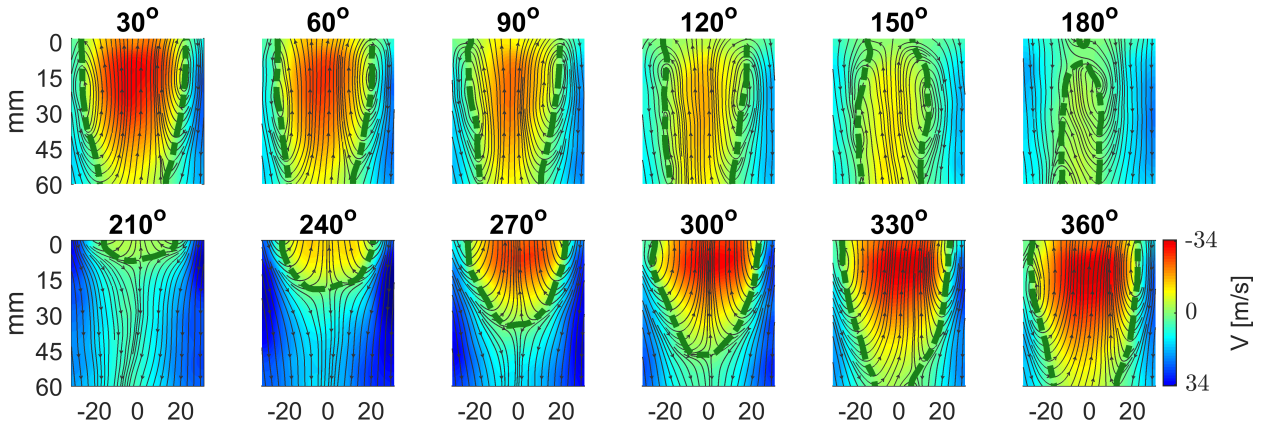


Figure 19: Case G: Streamlines superimposed on phase averaged values of axial velocity. The coherent components have been calculated in reference to the phase of the dynamic pressure signal. The green dashed lines mark regions of zero axial velocity.

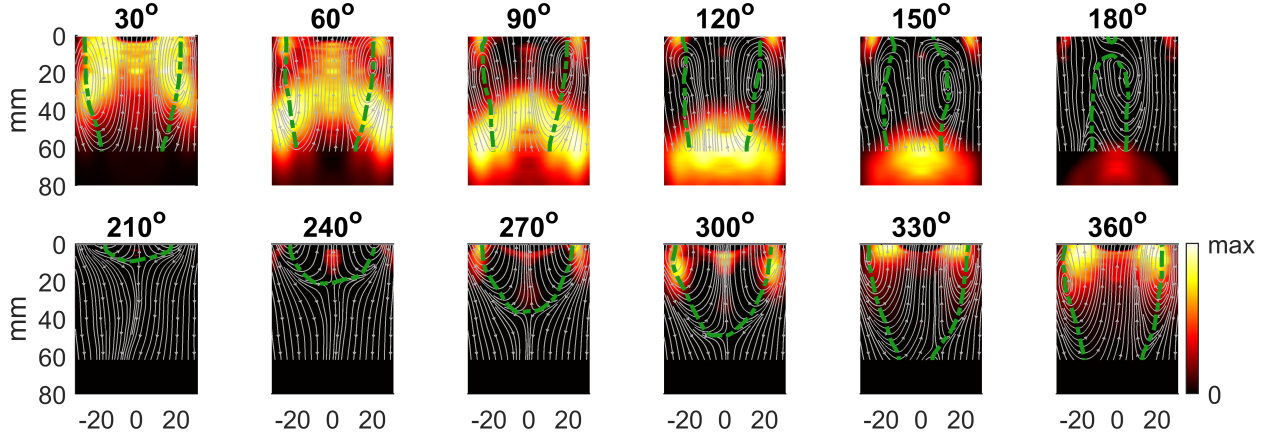


Figure 20: Case G: Streamlines superimposed on the phase averaged Abel deconvoluted CH^* spatial distribution. The coherent component has been acquired by phase averaging the flame images in respect to the phase of the dynamic pressure signal. The green dashed lines mark regions of zero axial velocity.

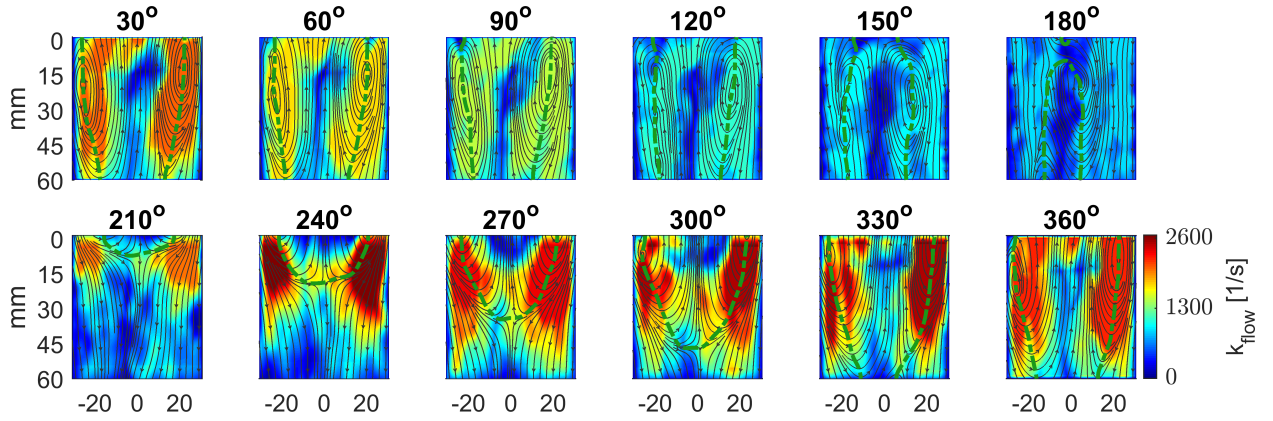


Figure 21: Case G: Streamlines superimposed on phase averaged flow imposed strain rate. The coherent component has been acquired by phase averaging the flow vectors in respect to the phase of the dynamic pressure signal. The green dashed lines mark regions of zero axial velocity. The numerically calculated extinction strain rate of the mixture is $k_{ext} = 1540 \text{ 1/s}$

tion volume increases and the location of the downstream stagnation point also moves further downstream. This occurs because the increased axial velocity in the free stream region induces increasing adverse pressure gradients along the combustor centreline, thereby enhancing the recirculation velocities. The effect is a recirculation region of increasing extent up to 360° . At 360° , when pressure reaches its maximum value, a pair of vortices is shed from the combustor inlet, propagating downstream with their centres demonstrating cylindrical symmetry. While the vortex pair is convected downstream from 30° to 180° , the recirculation velocity axial values and the backflow volume decrease.

In Fig. 20 and Fig. 21 the phase averaged CH^* intensity and the phase averaged flow imposed strain rate are presented respectively. During the second half of the oscillation period, namely from 210° until 360° the introduction of the swirling flame jet induces high k_{flow} values. These contours can be observed between the shear layers of the recirculation zone and the combustor walls. The flame jets propagate along the inner shear layers, where lower strain rate values appear. The high k_{flow} values are associated with increased velocity gradients due to the flow acceleration by the introduction of the reacting swirling jets. During the first half of the period of oscillations, namely from 360° to 180° , the vortex propagation downstream the combustor, induces significant strain and also increases the flame angle during the phase period between 360° to 60° . Increased strain due to the vortex-flame interaction causes extinction at the root, as observed from 30° until 60° . It then causes local extinction of the flame close to the axial section $x=0$ mm from 90° until 180° . From 30° to 90° k_{flow} contours attain maximum values 1.5 times higher than the k_{ext} of the specific mixture enough to extinguish the flame locally close to the centerbody.

The phase averaged flame shape can be interpreted based on the k_{flow} contours as well. The flame from 30° to 60° anchors along the inner shear layers, because it cannot penetrate through the high strain rate regions, assuming a V-shape. The outer-edges of the V shape are very close to the combustor walls, which would lead to local heat losses from the flame through the combustor walls to the environment and would also limit the growth of the flame surface area. We also expect that direct contact with the combustor walls would decrease the flame surface area during that period. The periodic shape changes of the flame during an oscillatory cycle, are hence considered to be a result of the local k_{ext} to k_{flow} ratio in our geometry.

5. Conclusions

In the current work we examined the dynamic state transitions while enriching methane blends with hydrogen in a technically premixed swirl stabilized model gas turbine combustor. We show that for a given combustor

geometry the relative k_{ext} and k_{flow} values will define the dynamic state the combustor will operate in.

It was showed that the extinction strain rate of methane hydrogen mixtures on a constant equivalence ratio and Reynolds number can dictate the interaction of the flame with coherent structures of the underlying flowfield and eventually, the dynamic state of the combustor. The extinction strain rate of each mixture was numerically calculated and each presented case was compared to the experimentally defined spatial distribution of the flow imposed strain rate. It was found that keeping all other operational parameters constant and increasing the H_2 content, thereby increasing the extinction strain rate leads to dynamic state transitions, from a stable to a limit cycle regime through an intermittent regime where the dynamics were intermittently injected between the two states. The quiescent flame was confined in a tubular shape by high strain rate regions, but was able to penetrate through them, towards the centerbody as the H_2 content was increased.

Once the H_2 content increased extinction robustness of the mixture, enough for the flame to attach to the centerbody we observe that the flame is driven by helical instabilities. They are typical naturally excited coherent structures in swirling flowfields. We have applied a DMD analysis on PIV vector fields while the flame demonstrated intermittent behaviour. We were able to showcase that the recirculation zone precesses under the influence of helical instabilities. The flame being anchored to the centerbody precesses as well under the helical motion influence. These helical disturbances, when interacting with the flame, can trigger mechanisms of high amplitude heat release rate and dynamic pressure bursts. We suggest two mechanisms that might lead to random dynamic pressure and heat release rate bursts, which are subject of future work: The first consideration suggests direct increase of flame surface area because the flame is advected in regions with reactants that would not otherwise ignite. The second consideration suggests that a reduction in flame surface area is induced by the flame-wall interaction. This would produce richer regions of unburned reactants but they would effectively react at a later time.

On further increasing the H_2 content, the dynamics are attracted to a Period 1 limit cycle. The flow imposed strain rate oscillated periodically throughout a period of thermoacoustic oscillation. The flame anchoring regions and eventually its shape during the limit cycle were shown to be dictated by the spatial relationship of the flow imposed strain rate with the mixture extinction strain rate. In the regions where the flow imposed strain rate exceeded the numerically calculated extinction strain rate the flame would not penetrate, hence flame anchoring regions are observed in low strain rate topologies. The flame was robust to high strain rates at the second half of dynamic pressure oscillation period, while the swirling jets were introduced in the combustor. Nevertheless,

the flame did not manage to sustain high strain rates caused by vortex propagation along the combustor inlet at maximum dynamic pressure during the first half of the oscillation. Hence, this vortex-flame interaction, sustains the limit cycle oscillation *via* the flow imposed and mixture extinction strain rate spatial relationship.

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Appendix A. Existence of PVC

The effect of the helical coherent structure on the swirling flow has been studied in [40]. It was showed that this hydrodynamic coherent structure can be detected in eigenmodes calculated by a Proper Orthogonal Decomposition applied on instantaneous snapshots of the flowfield. A similar analysis has been conducted in the current geometry on the isothermal flowfield. The eight orthogonal eigenmodes with the highest kinetic energy on the azimuthal radial PIV field of view are presented in Figure A.22. The analysis shows that modes one and two are azimuthally shifted by 45° and feature an $m=2$ azimuthal wavenumber. Modes three and five are shifted by 90° and form an $m=1$ azimuthal wave. The dynamic properties of the modes can be assessed by examining the frequency spectra of the temporal coefficients of the POD modes. Modes one and two demonstrate a strong temporal peak at 170 Hz. Modes three and five attain high peaks at about 70 Hz. The peak of the third mode however is significantly stronger than the respective peak of the fifth mode. This might indicate a decreasing rate of rotation of the $m=1$ wave.

To further assess the helical coherent structure we then focus on the POD modes extracted by PIV measurements of the axial radial field of view. The results of this analysis are presented in Fig. A.24. Furthermore, results of a Fast Fourier Transformation of the respective temporal coefficients of the same modes are presented in Fig. A.25. We start the analysis, by observing that the first mode features a dominant oscillation peak at 170Hz. Vortical structures with harmonically oscillating vorticity in the streamwise direction can be identified in mode pairs 2-3 and 4-5. Vortices appear to emanate from the inlet with decreasing vorticity towards the axial direction. In contrast to the first mode, the modes that are detected in pairs demonstrate subharmonic contribution at 70 Hz in their spectra as well. Vorticity appears to be skewed for mode pair 2-3 towards the radial half with negative dimensions. The reason for this asymmetry is not clear, but we can argue that vorticity is rather distributed on

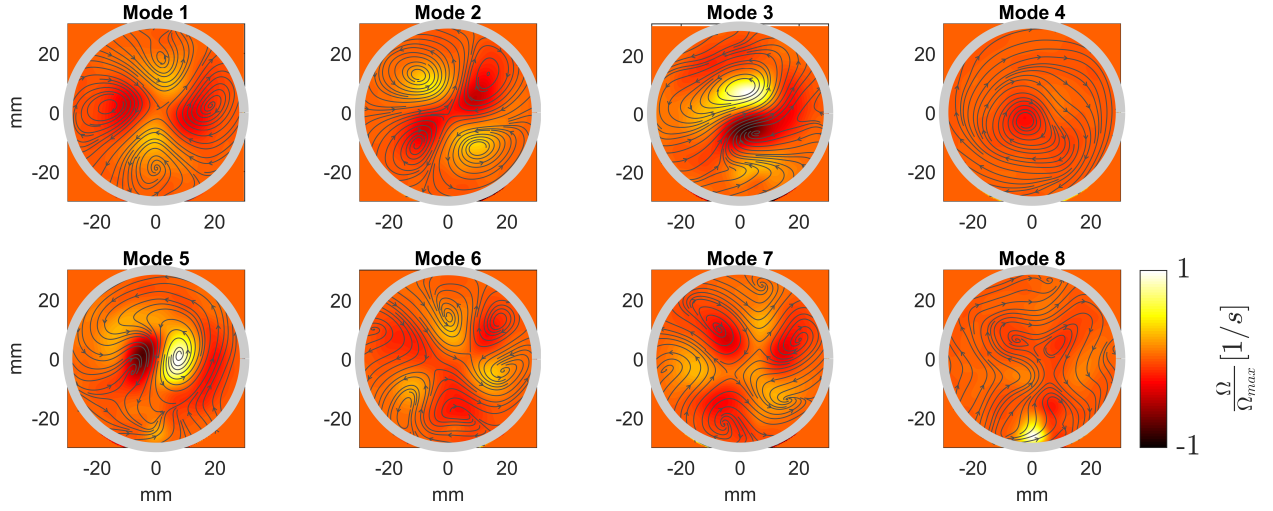


Figure A.22: The eight most energetic POD eigenmodes of the cross sectional plane of measurement. The contours represent normalized vorticity superimposed on two dimensional streamlines.

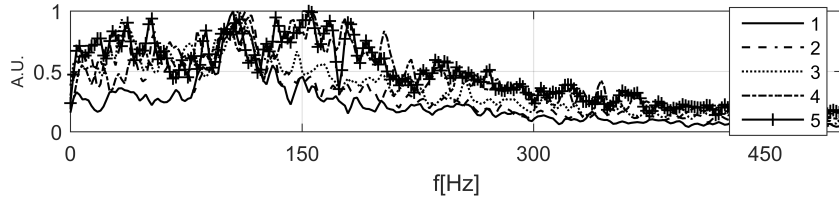


Figure A.23: Results of a Fast Fourier Transformation on the temporal coefficients of the first five POD modes on the azimuthal radial plane, in reference to Fig. A.22

the corresponding positive radial half for both modes 2 and 3. Mode pair 4-5 demonstrates higher wavenumber vorticity oscillations, in an alternating pattern that clearly indicates downstream convection of vortical structures. In the described context, the structure depicted in Mode 1, with topologies of high vorticity being concentrated mostly close to the inlet, can be interpreted as the wavemaker of the helical coherent structure. Referring again to the work by [40] the topology of the wavemaker is expected to be located close to the inlet and feature a single dominant eigenfrequency. In the current geometry, since the mode with the highest kinetic energy features contribution solely at 170 Hz, this timescale can be described as the forcing frequency of the wavemaker. This observation is consistent with the results of the DMD analysis on the isothermal flowfield presented in section 4.1. We remind the reader that the amplitude of the DMD mode with continuous frequency 170 Hz is the most prominent one as described in Fig. 6. Nevertheless, the complete structure of the wavemaker is not fully optically accessible, since the inlet of the combustor is partly obscured by the material of the diffuser.

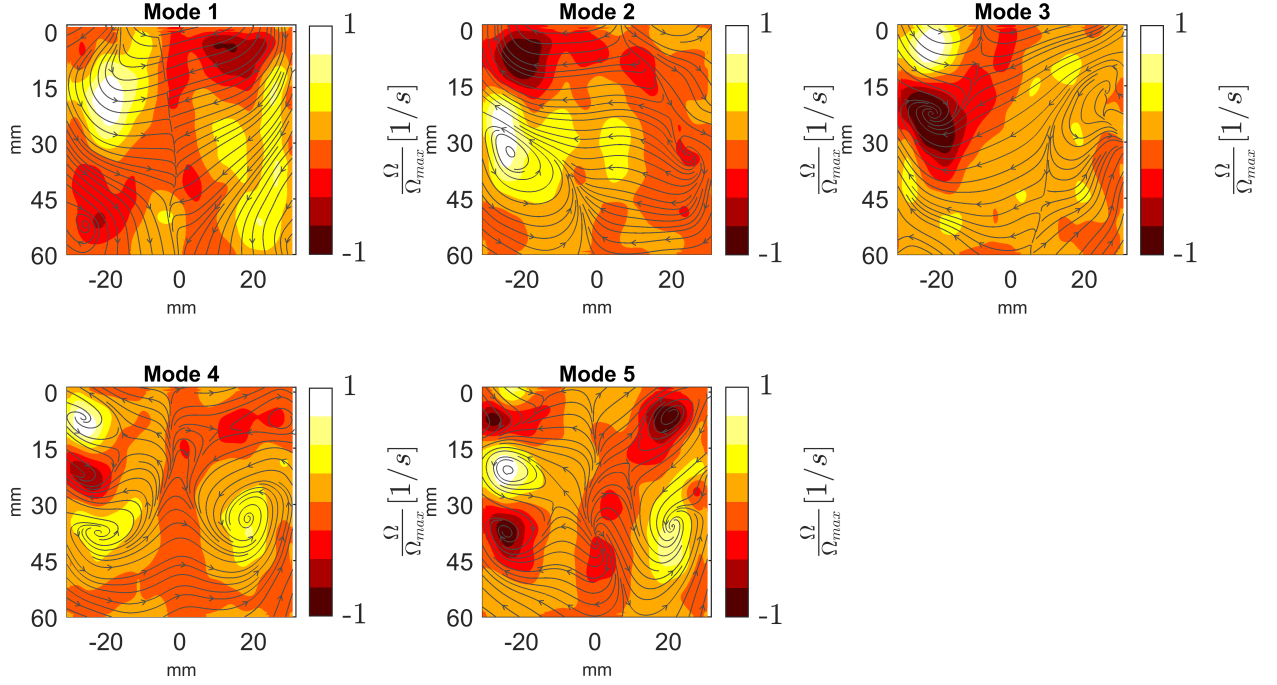


Figure A.24: The first five most energetic POD eigenmodes of the axial radial plane of measurement. The contours represent normalized vorticity superimposed on two dimensional streamlines.

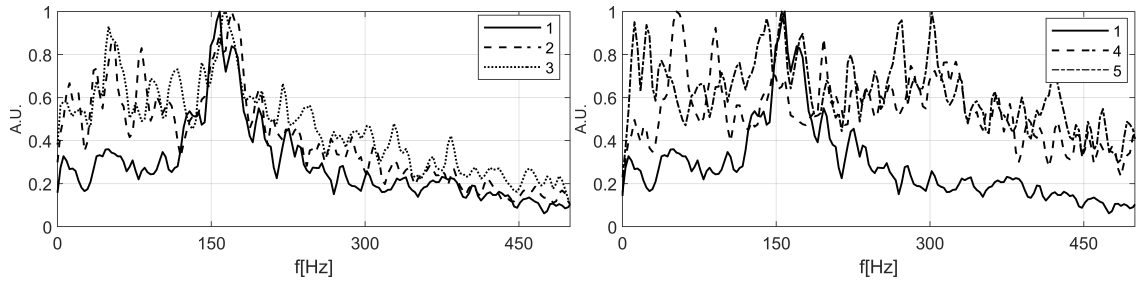


Figure A.25: Results of the Fast Fourier Transformation on the temporal coefficients of the first five POD modes, in reference to Fig. A.24.

Appendix B. Studies on transitional dynamics in model gas turbine combustors

The following studies describe transitions from stable to limit cycle operation. In some of these works, transitions occur via an intermittent regime, wherein the dynamics demonstrate the phenomenon of intermittency. Intermittency is demonstrated in non-linearly unstable systems and is associated with subcritical Hopf bifurcations. In contrast, supercritical Hopf bifurcations come about for linearly unstable systems. Nonlinearities are responsible for the saturation of the limit cycle and dictate the effective amplitude of instabilities. The list in Table B.4 is a non exhaustive account of observed transitions from quiescence to limit cycle instabilities. The bifurcation parameter is reported along with the reported succession of transitions between dynamic states. In all cases the terminology adopted in each work for the encountered dynamic states is followed. Most works adopt the equivalence ratio ϕ as their bifurcation parameter. It is interesting to notice that both supercritical and subcritical bifurcations may be observed on the same geometries. The conditions under which either type may be observed, depend on the system parameters that are kept nominally constant.

Table B.4: Indicatively mentioned studies on transitions observed in swirl-stabilized model gas turbine combustors. The bifurcation parameters and the reported sequence of dynamic transitions are provided. Experimental parameters such as the swirl number (S) of the geometry, the equivalence ratio (ϕ), the Reynolds number (Re) and the fuel are provided if explicitly reported by the authors.

Authors	Bifurcation Parameter	Transition Sequence	S Re ϕ Fuel
Lieuwen [42]	Bulk velocity 18-30m/s	Stable- Supercritical Bifurcation- Limit Cycle	- - 0.89 CH ₄
Lieuwen [42]	Bulk velocity 13.5-15.5m/s	Stable- Intermittent (Subcritical Bifurcation)- Limit Cycle	- - 0.85 CH ₄
Gotoda et al. [46]	ϕ enrichment	Stochastic- Low Dimensional Chaotic- (Intermittent)-Limit Cycle	- - 0.44-0.52 CH ₄
Kim and Hochgreb [47]	Impulse perturbation amplitude	Stable- Limit Cycle (Subcritical Bifurcation)	0.55 - 0.60 CH ₄
Taamallah et al. [48]	ϕ enrichment H ₂ enrichment	Stable- Intermittent- Limit Cycle	0.7 20000 0.50-0.75 CH ₄ 90%-10% CH ₄ -H ₂ 80%-20% CH ₄ -H ₂
Taamallah et al. [7]	ϕ enrichment	Stable- Intermittent- Limit Cycle	0.7 20000 0.56-0.66 CH ₄
Noiray and Denisov [49]	ϕ enrichment	Low amplitude (Linearly Stable) - Intermediate (Marginally Linearly Unstable) Stochastic Limit Cycle	0.5 - 0.47-0.50 Natural Gas
Palies et al. [10]	ϕ enrichment	Low amplitude - Transient- Limit Cycle	Low swirl injector - 0.30-0.40 90%-10% CH ₄ -H ₂
Bonciolini et al. [50]	ϕ enrichment	Low amplitude - Limit Cycle (Supercritical Hopf)- Low amplitude 34	- - 0.71-0.90 Natural Gas
Ebi et al. [51]	Load ramp	Low amplitude - Intermittent (Subcritical Hopf)- Limit cycle	- - 0.58-0.635 Natural Gas

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