A COMPARISON OF POWER-STATION COOLING SYSTEMS INCORPORATING NATURAL-DRAUGHT COOLING TOWERS

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ABSTRACT

Three types of power-station cooling systems incorporating natural-draught cooling towers, viz. the Wet system, the Heller system and a novel system employing a refrigerant as heat-transfer medium are investigated. The object is to develop methods for determining their respective optimum design conditions and associated optimum merits so that comparisons between them can be made.

After choosing representative cooling-system circuits, an expression for the merit-factor, i.e. a novel criterion suitable for judging the merits of various cooling-system types, is developed. In addition, decisions are made as to which of the cooling-system characteristics have an appreciable effect on the merit of a cooling system and are to be regarded as design variables.

After expressing the merit-factor as a function of the adopted design variables, for each of the considered cooling systems, accurate and approximate optimization methods are devised to estimate design-variable values which can be used in determining optimum merit-factor values. Furthermore, simplified methods are established so that the least promising of a number of possible refrigerants and tower heat-exchanger types can be eliminated from further consideration prior to conducting time-consuming overall optimization procedures. These methods are successfully applied to a set of climatic conditions and cooling-system component prices.

considered to be representative of some regions in the United Kingdom in 1964. Examination of the ensuing results leads to the following chief conclusions:

The wet system is appreciably the best of the three considered cooling systems unless the capitalized price of providing its make-up water proves to be at least 1.5 to 1.8 times the total price of its cooling-system components. The Heller system can be reasonably expected to be slightly better than the Spalding system provided that the capitalized price of providing de-icing facilities does not exceed 17.6 per cent of the total price of cooling-system components for the Heller system.

CONTENTS LIST

TITLE 1

ABSTRACT 2

CONTENTS LIST 4

NOMENCLATURE 16

i) Symbols for subscripts 16

ii) General symbols 19

iii) Symbols for dimensionless groups 33

iv) Physical property symbols 34

v) Symbols for quantities containing constant-valued physical property groups 35

INTRODUCTION 37

1 Application of natural-draught cooling towers in power-station cooling systems 37

2 Historical development of power-station cooling systems incorporating natural-draught cooling towers 38

3 Object of present investigation 42

4 Outline of procedure 45

5 General information 48

6 Acknowledgements 48

1 DESCRIPTION OF COOLING SYSTEMS UNDER CONSIDERATION 49

1.1 Introduction 49

1.2 Wet system 49

1.3 Heller system 53

1.4 Spalding system 57

1.4.1 Function 57

1.4.2 Choices of some cooling-system components 59

1.4.3 Choices of promising refrigerant alternatives 60

1.4.4 Preliminary choices of surface steam condenser flow arrangements 61

1.4.5 Preliminary choices of surface steam condenser configurations 63

1.4.6 Discussion of tower heat exchanger alternatives 65

2 ESTABLISHMENT OF CRITERION OF MERIT FOR COOLING SYSTEMS 67

2.1 Deduction of merit-factor 67

2.2 Evaluation of temperature difference coefficient, a^* 68

2.2.1 Derivation of basic power-loss equation 68

2.2.2 Evaluation of P_u/q 72

2.2.3 Evaluation of $(\Delta P_u/P_u)$ 82

2.2.4 Evaluation of $£_{cap}/P_u$ 82

2.2.5 Evaluation of $(\Delta £_{fuel}/P_u)$ 83

2.2.6 Representative numerical values of a^* 87

2.3 Evaluation of capital price coefficient, c^* 89

2.4 Evaluation of pumping power coefficient, b^* 92

2.5 Summary 94

3 PRESENTATION OF MERIT-FACTOR AS A FUNCTION OF COOLING SYSTEM DESIGN VARIABLES 100

3.1 Introduction 101

3.2 Preliminary choice of fixed characteristics 101
3.3 Preliminary choice of design variables 102
3.4 Derivation of temperature difference term in merit-factor equation 109
3.4.1 Wet system 109
3.4.2 Heller system 111
3.4.3 Spalding system 114
3.5 Derivation of pumping-power term in merit-factor equation 118
3.5.1 Wet system 118
3.5.2 Heller system 120
3.5.3 Spalding system 124
3.6 Derivation of capital-price term in merit-factor equation 128
3.6.1 Wet system 128
3.6.2 Heller system 131
3.6.3 Spalding system 134
3.7 Final choice of fixed characteristics and design variables 137
3.7.1 Wet system 137
3.7.2 Heller system 142
3.7.3 Spalding system 145
4 CHOICE OF TOWER PACKING FOR WET SYSTEM AND TOWER HEAT EXCHANGERS FOR HELLER- AND SPALDING SYSTEMS 149
4.1 Introduction 149
4.2 Best tower packing type for Wet system 150

4.3 Best tower heat-exchanger types for Heller- and Spalding-systems . 154

4.3.1 Factors affecting choice of heat-exchanger types 154

4.3.2 Establishment of simplified merit-factor 157

4.3.3 Adopted method of determining the merits of heat-exchanger types 161

4.3.4 Description of considered heat-exchanger types 162

4.3.5 Presentation of merit-factors 171

4.3.6 Choice of tower heat-exchanger types 173

5 CHOICE OF SPALDING-SYSTEM REFRIGERANT AND MATERIALS FOR STEAM-CONDENSER TUBES AND TOWER HEAT EXCHANGER 178

5.1 Proposed method of choosing refrigerant 178

5.2 Evaluation of physical property groups 179

5.3 Choice of refrigerant and cooling-system materials 182

6 METHODS FOR THE PREDICTION OF OPTIMUM-DESIGN CONDITIONS FOR THE WET SYSTEM 185

6.1 Introduction 185

6.2 Accurate method of predicting optimum values of design variables 186

6.3 Approximate method of predicting optimum values of design variables 188

6.3.1 Establishment of method 188

6.3.2 Optimum Hot-side heat flux, Q_{H}, and loading factor, W_{H} 191

6.3.3 Optimum inside diameter of coupling-circuit duct, d_T 192

6.3.4 Optimum tower-base diameter, D_1 193

6.3.5 Optimum under-pack height to tower diameter ratio, r_p 195

6.3.6 Optimum throat to base diameter ratio, s 197

6.3.7 Optimum blanking factor, t 197

7 METHODS FOR THE PREDICTION OF OPTIMUM-DESIGN CONDITIONS FOR THE HELLER SYSTEM 200

7.1 Introduction 200

7.3 Approximate method of predicting optimum values of design variables 201

7.3.1 Establishment of method 201

7.3.2 Optimum inside diameter of coupling-circuit duct, d_T 204

7.3.3 Optimum heat-capacity factor ratio, C_F/C_G 205

7.3.4 Optimum Air-side heat-capacity factor, C_G 206

7.3.5 Optimum Cold-side loading factor, W_C 207

7.3.6 Optimum tower-base diameter, D_1 207

7.3.7 Optimum throat- to base-diameter ratio, s 208

7.3.8 Optimum partial number of Air-side transfer units, $n_{h,G}$ 209

8 METHODS FOR THE PREDICTION OF OPTIMUM-DESIGN CONDITIONS FOR THE SPALDING SYSTEM 211

8.1 Introduction 211

8.2 Approximate method of predicting optimum values of design variables 215

8.2.1 Establishment of method 215

8.2.2 Optimum Hot-side heat flux, Q_H, loading factor, W_H and exit vapour quality, X_H 217

8.2.3 Optimum inside diameters of coupling-circuit ducts, d_T and d_B 220

8.2.4 Optimum Air-side heat-capacity factor, C_G 221

8.2.5 Optimum tower-base diameter, D_1, and throat- to base-diameter ratio, s 222

8.2.6 Optimum partial number of Air-side transfer units, $n_{h,G}$ 223

9 PRESENTATION OF RESULTS 223

9.1 Abstract 225

9.2 Summary of decisions and values of quantities used in obtaining results 227

9.2.1 General decisions and values of quantities 227

9.2.2 Additional decisions and values of quantities 229

9.3 Tabular presentation of results 230

9.3.1 General tables 232

9.3.2 Additional tables 239

9.4 Graphical presentation of results 242

10 DISCUSSION OF RESULTS 279

10.1 Calculation of results 279

10.2 Appraisal and discussion of methods of determining optimum design-variable values 281

10.3 Variation of quantities with waste-heat rejection rate 286

10.4 Validity of assumptions made in carrying out methods of optimization 287

10.5 Appraisal of tower heat exchanger,type SG 1 290

10.6 Appraisal of tower heat exchanger, type LU 1 298

10.7 Surface steam condenser considerations 300

10.7.1 Wet system 300

10.7.2 Spalding system 301

10.8 Validity of adopted methods of predicting a^*, b^* and c^* values 303

10.9 Comparision of considered cooling systems 306

11 SUGGESTIONS FOR FUTURE RESEARCH AND GENERAL CONCLUSIONS 311

11.1 Suggestions 311

11.2 Conclusions 313

MAIN TEXT REFERENCES 316

APPENDICES

A PHYSICAL PROPERTIES OF COOLING SYSTEM MATERIALS 321

A.1 Introduction 321

A.2 Water and its vapour 322

A.3 Dry-air (one atmosphere) 323

A.4 Moist-air (one atmosphere) 323

A.4.1 Use of water-vapour mixture-fraction 323

A.4.2 Physical property equations in terms of water-vapour mixture-fraction 324

A.4.3 Determination of water-vapour mixture-fraction 324

A.5 Refrigerants and their saturated vapours 325

A.5.1 Introduction 325

A.5.2 Ammonia 326

A.5.3 Freon 12 327

A.5.4 Freon 22 328

A.5.5 Methyl Chloride 329

A.6 Thermal conductivities of metals and alloys 330

A.7 References 331

B FRICTION-FACTOR FORMULAE FOR FLUID FLOW INSIDE TUBES 334

B.1 Definition 334

B.2 Single-phase, single-component, turbulent fluid flow in tubes 334

B.2.1 Accurate formulae for transition and completely turbulent regimes 334

B.2.2 Approximate formulae for transition regime 335

B.3 Forced convection, two-phase, single-component fluid flow occurring during condensation in vertical tubes 336

B.4 References 337

C FRICTIONAL PRESSURE-DROP FORMULAE 339

C.1 Definition and basic equations 339

C.2 Forced convection, two-phase, single-component fluid flow during condensation in vertical tubes 339

C.3 Forced convection, two-phase, single-component fluid flow occurring during nucleate boiling in tubes 340

C.4 References 344

D HEAT TRANSFER-COEFFICIENT FORMULAE 346

D.1 Single-phase, single-component, turbulent liquid flow in tubes 346

D.2 Forced convection, two-phase, single-component fluid flow occurring during condensation in vertical tubes 346

D.3 Two-phase, single-component condensation of steam on axially horizontal tube bundles 347

D.4 Heat transfer during nucleate boilding under saturation conditions 350

D.4.1 Maximum heat transfer-coefficients inside horizontal tubes for Ammonia, Freon 12 and Methyl Chloride 350

D.4.2 General heat-transfer correlation for Freon 12, 22 and Methyl Chloride flowing through horizontal tubes 351

D.5 References 352

E TRANSFER UNIT-EFFECTIVENESS AND ASSOCIATED PERFORMANCE RELATIONSHIPS 353

E.1 Wet-system tower packing 353

E.1.1 Presentation of general transfer unit equation 353

E.1.2 Approximate h_M-h_C relationships 354

E.1.3 Transfer unit equation and other relationships using Merkel's approximation 354

E.1.4 Accurate transfer unit equation using quadratic approximation 355

E.2 Heller-system tower heat exchanger 356

E.3 Spalding-system tower heat exchanger 357

E.4 Wet-system steam surface condenser 358

E.5 Refrigerant-side temperature approach for Spalding-system steam surface condenser 359

E.5.1 Introduction 359

E.5.2 Temperature approach for single-phase, forced convection flow condition 360

E.5.3 Temperature approach under conditions of nucleate boiling heat transfer, independent of fluid velocity 360

E.5.4 Temperature approach under conditions of nucleate boilding heat transfer, dependent on fluid velocity. 361

E.6 References 362

F DERIVATION OF TERMS IN BUOYANCY EQUATION 363

F.1 Deduction of buoyancy equation 363

F.2 Determination of pressure-drop term 363

F.2.1 Wet system 363

F.2.2 Heller- and Spalding-systems 366

F.3 Theoretical determination of density-difference term 369

F.3.1 Wet system 369

F.3.2 Heller- and Spalding-systems 370

F.4 Approximate determination of density-difference term 370

F.4.1 Wet system 370

F.4.2 Heller- and Spalding-systems 372

F.5 Derived buoyancy equation expressed as explicit function of effective chimney length. 372

F.5.1 Wet system 372

F.5.2 Heller- and Spalding-systems 373

F.6 References 374

G DESCRIPTION OF NUMERICAL OPTIMIZATION TECHNIQUE 375

G.1 Introduction 376

G.2 Adopted numerical optimization technique 376

G.3 Modifications to basic master-program 376

G.4 References 378

H PRICE FORMUALE FOR STEAM SURFACE CONDENSERS 379

H.1 The base-price concept 379

H.2 Price of tubing 382

H.3 References 383

I PRICE FORMULAE FOR PIPING 384

I.1 Derivation of equations 384

I.2 References 385

J PRICE FORMULAE FOR COOLING TOWERS AND ASSOCIATED EQUIPMENT 386

J.1 Accurate evaluation of tower-shell outside surface area 386

J.2 Approximate evaluation of tower-shell outside surface area 388

J.3 Wet-system price formulae 389

J.4 Heller- and Spalding-system price formulae 392

K PRICE FORMULAE FOR TOWER HEAT EXCHANGERS 394

K.1 Introduction 394

K.2 Evaluation of total tower heat-exchanger price for helically wound fins (or wires) 395

K.3 Evaluation of total tower heat-exchanger price for circular tubes with continuous plate fins 397

K.4 References 398

L EQUATION FOR OPTIMUM NUMBER OF TUBE ROWS 400

NOMENCLATURE

The number of symbols employed in the present investigation is very large due to the wide field covered. To facilitate the identification of symbols, the nomenclature is split up into five sections as follows:

i) Symbols for subscripts.

ii) General symbols.

iii) Symbols for dimensionless groups.

iv) Physical property symbols.

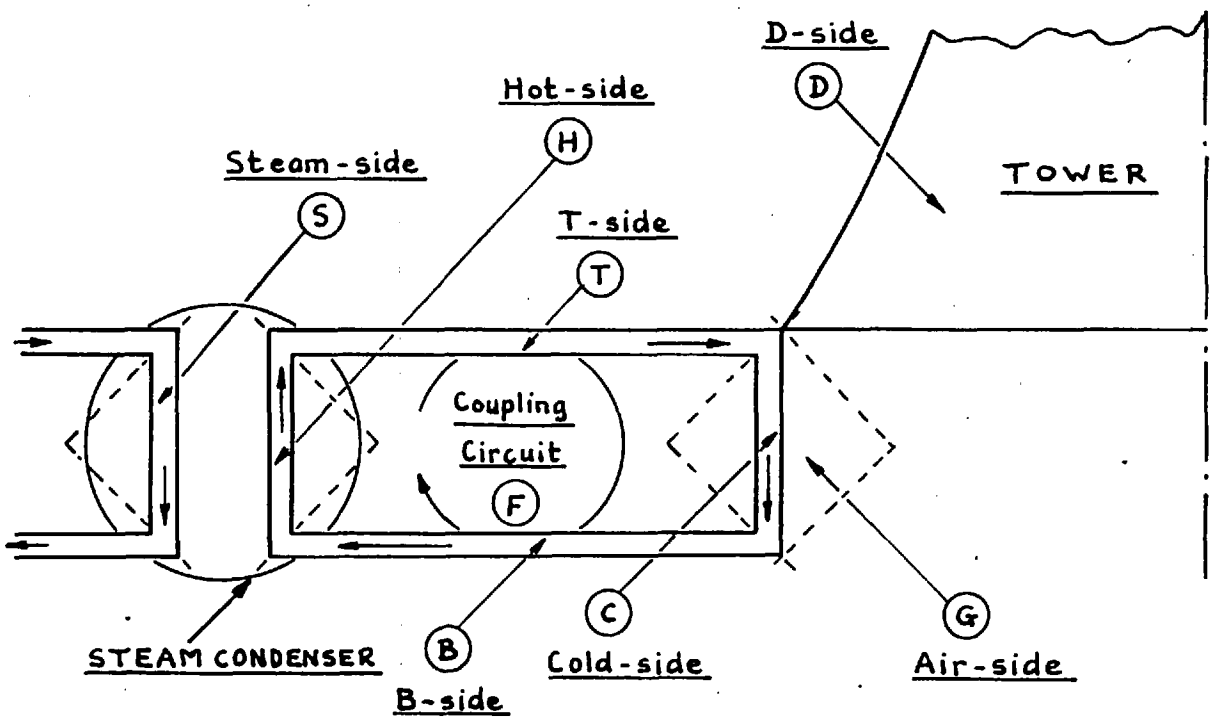
v) Symbols for quantities containing constant-valued physical property groups.

In each of these sections the symbols are arranged in alphabetical order; English letters are given first, followed by Greek letters.

i) Symbols for subscripts.

In analysing a cooling system, frequent references must be made to each of the sections comprising the cooling-system circuit. It has been found convenient to represent each of these sections by a single English letter. This enables symbols designating physical properties, dimensions and other characteristic quantities pertaining to a particular section to be identified by the assignation of a subscript representing the section concerned. For example, with reference to the following sketch and the subsequent nomenclature, the quantity $k_{f,H}$ represents the liquid-phase thermal conductivity of the coupling-circuit fluid as it

flows through the Hot-side, i.e. through the steam condenser. In particular, the subscript F is employed to denote "coupling circuit in general" so that those quantities which are practically constant for all of the coupling-circuit sections can be identified by a single subscript, viz.F. For example, for the practically isothermal coupling-circuit fluid in the Spalding system, the quantities $k_{f,H}$, $k_{f,T}$, $k_{f,C}$ and $k_{f,B}$, being equal, can all be replaced by the single quantity, $k_{f,F}$, thereby eliminating a possible cause of confusion.



A dry air.

B section represented by coupling-circuit duct returning from cooling tower, i.e. B-side.

C section in tower packing or heat exchanger through which coupling-circuit fluid flows while being cooled, i.e. Cold-side.

D section represented by the cooling tower, i.e. D-side.

F refers to coupling circuit in general and is assigned to equal quantities occurring in the individual coupling circuit sections.

f liquid phase.

fr friction or frontal, depending on the context.

G refers to section where atmospheric air flows through (or past) the tower packing or tower heat exchanger, i.e. Air-side.

g gas phase.

H section in steam condenser through which coupling-circuit fluid flows while being heated, i.e. Hot-side.

i inlet.

m metal or metal tube wall.

o outlet.

S section in steam condenser through which turbine exhaust steam flows, i.e. Steam-side.

s saturation conditions.

T section represented by coupling-circuit duct leading towards cooling tower, i.e. T-side.

W signifies water.

ii) General symbols

A outside surface area of cooling tower, sq ft.

A_A tower surface area coefficient defined in (J.2-1),(1).

A_{an} optimum turbine exhaust area, sq ft.

A_c minimum free-flow area (equals inside cross-sectional area for pipes and ducts), sq ft.

$A_{c,G}$ minimum free-flow area on Air-side of tower heat exchanger, sq ft.

$A_{fr,G}$ frontal area on Air-side for whole tower heat exchanger, sq ft.

A_h heat-transfer surface area, sq ft.

$A_{h,G}$ heat-transfer surface area on Air-side of whole tower heat exchanger, sq ft.

$A'_{h,G}$ heat-transfer surface area per unit tube length on Air-side of single tower heat-exchanger tube, ft.

a nucleate boiling heat-transfer performance constant given in (E.5-5), $(Btu/h)^{0.5}/(ft\ deg\ F)$.

$a_{a,H}$ see section v).

a_f linear friction-factor curve-fitting coefficient used in (B.2-3), (1).

$a_{f,C}$ see section v).

$a_{f,H}$ see section v).

a_g heat-transfer equation limit defined in (D.2-1), $lb_f^2/(ft\ lb)$.

a_{grav} $(=4.173 \times 10^8)$ gravitational acceleration, ft h^{-2}.

$a_{h,p}$ tower-packing heat-transfer performance coefficient given in (3.7-9) and 4.2, per ft.

$a_{h,s}$ spray heat-transfer performance coefficient given in (3.7-11) and 4.2, (1).

$a_{m,C}$ see section v).

$a_{m,H}$ see section v).

$a_{n,G}$ performance coefficient for Air-side of tower heat exchanger defined in (F.2-16), $\{Btu/(h\ sq\ ft\ deg\ F)\}^2$.

$a_{p,B}$ see section v).

$a_{p,H}$ see section v).

$a_{p,T}$ see section v).

a_p coefficient by which total coupling-circuit pressure drop must be multiplied to account for losses due to bends, contractions, expansions, entrances and exits, (1).

$a_{p,B}$ see section v).

$a_{p,C}$ see section v).

$a_{p,G}$ empirical pressure-drop performance coefficient for Air-side of tower heat exchanger, defined in (F.2-12).

$a_{p,H}$ see section v).

$a_{p.T}$ see section v).

$a_{q,G}$ performance coefficient for Air-side of tower heat exchanger defined in (3.7-13), Btu/(h sq ft deg F).

a_R price of refrigerant given in (3.6-21), pounds sterling per lb.

$a_{t,H}$ pressure to temperature conversion factor for refrigerant given in (3.4-26), deg F sq ft/lb_f.

$a_{u,C}$ see section v).

$a_{u,G}$ empirical heat-transfer performance coefficient for Air-side of tower heat exchanger, defined in (F.2-11).

$a_{u,H}$ see section v).

$a_{u,S}$ see section v).

$a_{v,e}$ eliminator velocity-head loss coefficient, given by (F.2-5) and F.2.1, (1).

$a_{v,i}$ tower inlet velocity-head loss coefficient given in (F.2-6) and (F.2-19), (1).

$a_{v,misc}$ miscellaneous velocity-head loss coefficient given in (F.2-8) and (F.2-21), (1).

$a_{v,p}$ tower-packing valocity-head loss coefficient given in (F.2-3), per ft.

$a_{v,s}$ spray velocity-head loss coefficient given in (F.2-4), per ft.

a_o, a_1 tower geometry factors defined by (J.1-7) and (J.1-8), (1).

a^* temperature-difference coefficient in merit-factor equation (2.1-8), defined by (2.1-2), per deg F.

B empirical heat-transfer performance exponent for Air-side of tower heat exchanger, given in (F.2-9),(1).

B_A tower surface area coefficient defined in (J.2-1),(1).

B^* pumping-power coefficient defined by (2.1-3), (1).

b empirical pressure-drop performance exponent for Air-side of tower heat exchanger, given in (F.2-10),(1).

b_f linear friction-factor curve-fitting exponent used in (B.2-3), (1).

$b_{h,p}$ tower-packing heat-transfer performance coefficient given in (3.7-10) and 4.2, (1).

$b_{h,s}$ spray heat-transfer performance exponent given in (3.7-11) and 4.2, (1).

$b_{p,C}$ see section v).

$b_{p,G}$ empirical pressure-drop performance coefficient for Air-side of tower heat exchanger, given in (F.2-12).

$b_{p,H}$ see section v).

$b_{u,G}$ empirical heat-transfer performance coefficient for Air-side of tower heat exchanger, given in (F.2-11).

$b_{v,i}$ tower inlet velocity-head loss coefficient given in (F.2-6) and (F.2-19), (1).

$b_{v,p}$ tower-packing velocity-head loss coefficient given in (F.2-3), (1).

$b_{v,s}$ spray velocity-head loss exponent given in (F.2-4), (1).

b_o to b_3 transfer unit coefficients given in (E.1-12).

b^* pumping power coefficient in merit-factor equation (2.1-8), defined by (2.1-7), (1).

C $\equiv q/(mc_f)$ heat-capacity factor, applicable to S-, H-, T-, C- and B-sides and introduced in 3.3., deg F.

C_G $\equiv q/(m_G c_G)$ Air-side heat-capacity factor, deg F.

C_A tower surface area coefficient defined in (J.2-1), (1).

$c_{h,p}$ tower-packing heat-transfer performance exponent given in (3.7-10) and 4.2, (1).

$c_{v,p}$ tower-packing velocity head loss exponent given in (F.2-3), (1).

c^* cooling-system price coefficient in merit-factor equation (2.1-8), defined by (2.1-4), (Btu/h) per pound sterling.

D outside diameter of ducts, pipes or tubes, ft.

$(D)_{chil}$ Chilton duty coefficient defined by (F.5-4).

D_G diameter over fins for spiral finned tubes of tower heat exchanger, ft.

D_t cooling-tower throat diameter, ft.

D_1 cooling-tower base diameter, ft.

$d_{h,p}$ tower-packing heat-transfer performance coefficient given in (3.7-10) and 4.2, (1).

$d_{v,p}$ tower-packing velocity-head loss coefficient given in (F.2-3), (1).

d_o to d_8 cooling-tower price coefficients given in (J.3-1), J.3, (J.4-1) and J.4.

E total effectiveness for a chosen reference-side of a heat exchanger, defined as the ratio of the temperature change on the reference-side to the maximum temperature difference across the heat exchanger, (1).

e partial effectiveness for a chosen reference-side of a heat exchanger, defined as the ratio of the temperature change on the reference-side to the maximum temperature difference between fluid and metal wall, on the reference-side, (1).

$e_{h,p}$ tower-packing heat-transfer performance coefficient given in (3.7-10) and 4.2, (1).

$e_{G,p}$ partial effectiveness of a single pass for the Air-side of a multi-pass tower heat exchanger, (1).

F_G water-vapour mixture fraction of moist air at dry-bulb temperature t_G, defined by (A.4-1), (1).

F_G^* water-vapour mixture fraction of saturated air at wet-bulb temperature t_G^*, (1).

f Fanning friction factor defined by (B.1-1), (1).

$(f)_{chil}$ empirical Chilton factor presented in graph in F.4.

$f_{h,p}$ tower-packing heat-transfer performance exponent given in (3.7-10) and 4.2, (1).

$(f)_{max}$ maximum friction factor introduced in B.3, (1).

f_n Fanning friction factor in completely turbulent regime, defined by (B.2-2), (1).

G_C/G_G ratio of water to air loading in Wet-system cooling tower, (1).

g_o (= 4.173×10^8) Newton's gravitational constant, lb ft/(lb_f hr^2).

I Interest rate presented in (2.2-13), fraction.

J (= 778) coefficient for converting Btu to lb ft.

J_H empirical, vapour-quality dependent factor defined by (C.3-8a), (1).

J_K (= 3413) coefficient for converting kWh to Btu.

j_o, j_1 empirical performance exponents for Air-side of tower heat exchanger, defined by (F.2-17) and (3.7-13), (1).

K,K' tower heat-exchanger price coefficients given by (K.2-6) and (K.3-3) respectively, (1).

L length of ducts, pipes or tubes, ft.

L_B length of coupling-circuit duct returning from tower, ft.

L_C total length of fluid flow path through the Cold-side of tower heat exchanger, ft.

L_H total length of fluid flow path through the Hot-side of surface steam condenser, ft.

L_T length of coupling-circuit duct leading towards tower, ft.

l_c transverse pitch of tower heat-exchanger tubes, ft.

l_r longitudinal pitch of tower heat-exchanger tubes, ft.

m mass flow rate of fluid, lb/h.

m_G mass flow rate of air through tower, lb/h.

N_h, N_{Pr}, N_{Re}, N_v, ΣN_v see section iii).

n'_f number of fins per unit length of tower heat-exchanger tube, per ft.

n_C number of tower heat-exchanger tubes, (1).

n_H number of surface steam-condenser tubes, (1).

n_h see section iii).

n_p number of passes in a heat exchanger, (1).

n_r number of rows of tower heat-exchanger tubes in direction of air flow, (1).

n_T (=1) number of coupling circuit ducts leading towards tower, (1).

n_v see section iii).

n_y expected life of power station, years.

$(NHR)_{t_{G,i}}$ station nett heat rate, calculated at atmospheric temperature, Btu/kWh.

$(NHR)_{t_{s,S}}$ station nett heat rate, calculated at exhaust steam temperature, Btu/kWh.

P required pumping power per cooling tower, in Btu/h.

P_L/q merit-factor, or overall power loss per unit waste heat rejection rate, given by (2.1-8), (1).

$(P_L/q)_1$ power-loss contribution due to temperature-difference requirement, defined by (2.1-2), (1).

$(P_L/q)_2$ power-loss contribution due to pumping-power requirement, defined by (2.1-3), (1).

$(P_L/q)_3$ power-loss contribution due to capital-price requirement, defined by (2.1-4), (1).

P_u generator output per cooling tower, kW.

$(P_{u,B} + P_{u,A})/P_u$ ratio of the sum of the boiler- and all other auxiliary power requirements - to generator output, (1).

$(P_u)_{pot}$ potential generator output per cooling tower, were the cooling system to cost nothing, used in (2.3-3), kW.

p pressure, $lb_f/(sq\ ft)$.

p_{atm} atmospheric pressure, $lb_f/(sq\ ft)$.

$p_{s,S}$ turbine exhaust steam pressure, $lb_f/(sq\ ft)$.

Q ($\equiv q/A_h$) heat flux applicable to S-, H-, T-, C- and B-sides and introduced in 3.3, Btu/(h sq ft).

Q_G ($\equiv q/A_{h,G}$) heat flux on Air-side of tower heat exchanger, Btu/(h sq ft).

q waste-heat rejection rate per cooling tower, Btu/h.

q_S heat supplied to steam by the boiler per cooling tower, Btu/h.

$r_{h,G}$ hydraulic radius of Air-side of tower heat exchanger (= 0.25 hydr.diam.), defined by (F.2-14), ft.

r_p ($\equiv z_p/D_1$) ratio of under-pack height to tower base diameter, (1).

r_1 ($\equiv z_1/D_1$) ratio of tower inlet height to tower base diameter, (1).

r_5 ($\equiv z_5/D_1$) ratio of height of top of tower to tower base diameter, (1).

S velocity based on minimum free-flow area, given by (3.3-3), ft/h.

S_G velocity of air flow through Air-side of tower heat exchanger, based on minimum free-flow area, ft/h.

s ($\equiv D_t/D_1$) ratio of tower throat diameter to tower base diameter, (1).

t (without subscript) Wet-system blanking factor, defined as the ratio of free-flow area at tower base to total tower base cross-sectional area, (1).

t (with subscript) temperature, deg F.

t_G dry-bulb temperature of moist air flowing through tower, deg F.

$t_{G,i}$ atmospheric dry-bulb temperature, deg F.

t^*_G wet-bulb temperature of moist air flowing through tower, deg F.

$t^*_{G,i}$ atmospheric wet-bulb temperature, deg F.

t_m temperature of metal, deg F.

t_s saturation temperature, deg F.

$t_{s,S}$ turbine exhaust steam temperature, deg F.

U overall heat-transfer coefficient, referred to the side designated by accompanying subscript, Btu/(h sq ft deg F).

u partial heat-transfer coefficient, referred to the side designated by accompanying subscript, Btu/(h sq ft deg F).

u_G partial heat-transfer coefficient for Air-side of tower heat exchanger, referred to the Air-side, Btu/(h sq ft deg F).

$u_{S,1}$ average condensing heat-transfer coefficient for uppermost steam-condenser tube, given in (D.3-2), Btu/(h sq ft deg F).

W ($\equiv mc_f/A_c$) loading-factor, applicable to S-, H-, T-, C- and B-sides and introduced in (3.3), Btu/(h sq ft deg F).

W_G ($\equiv m_G c_G/A_{c,G}$) loading-factor for Air-side of tower heat exchanger, Btu/(h sq ft deg F).

W_f, W_g partial loading-factors for liquid and gaseous phases of two-phase fluid flow, used in C.3, Btu/(h sq ft degF).

X vapour quality, fraction.

X_H total vapour quality change of refrigerant on Hot-side of Spalding system, fraction.

X_S vapour quality of turbine exhaust steam, fraction.

x correction factor to account for non-uniform air distribution across packing, used in (F.5-5), (1).

z_H effective height of water level in Heller-system jet condenser, ft.

z_p height of under-side of packing, ft.

z_R effective height of refrigerant liquid level above Hot-side entrance for Spalding-system steam condenser, ft.

z_1 inlet height of cooling tower, ft.

z_2 height of top of packing, ft.

z_3 height of top of spray, ft.

z_5 height of top of cooling tower, ft.

$£_{B/q}$ price of coupling-circuit duct returning from tower per unit waste-heat rejection rate, £h/Btu.

$£_{cap}/P_u$ capital cost of peak power, £/kW.

$£_{cs}$ total capital price of cooling system components per cooling tower, £.

$£_D$ capital price of a cooling tower and its associated equipment, £.

$(£_{fuel}/P_u)_{annual}$ annual price of fuel per unit generator output, introduced in (2.2-12), £/kW.

$(£_{fuel}/P_u)_{present\ worth}$ present worth of fuel price for useful life of power station, introduced in (2.2-13), £/kW.

$£_G/q$ total price of tower heat exchanger per unit waste-heat rejection rate, £h/Btu.

$£'_{G,tube}$ price per unit tube length of finned tubing for tower heat exchanger, £/ft.

$£''_G$ total price of tower heat exchanger per unit Air-side heat-transfer surface area, £/sq ft.

$£''_{G,base}$ base price of tower heat exchanger per unit Air-side heat-transfer surface area, introduced in K.2, £/sq ft.

$£''_{G,stand}$ price of standardized tower heat exchanger unit per unit Air-side heat-transfer surface area, introduced in K.3, £/sq ft.

$£''_{G,tube}$ price of finned tubing for tower heat exchanger per unit Air-side heat-transfer surface area, £/sq ft.

$£'_{pipe}$ price per unit length of pipes and ducts, £/ft.

$£_{pm}$ capital price of circulating pump - motor combination per cooling tower, £.

$£_R/q$ price of refrigerant per unit waste-heat rejection rate, £h/Btu.

$£_{rest}$ $(= £_{cap} - £_{cs})$ is the capital price difference introduced in (2.3-3), £.

$£_S/q$ total price of steam condenser per unit waste-heat rejection rate, £h/Btu.

$£_{S,base}/q$ base price of surface steam condenser per unit waste-heat rejection rate, £h/Btu.

$£''_S$ total price per unit Steam-side heat-transfer surface area of surface steam condenser, £/sq ft.

$£''_{S,base}$ base price of surface steam condenser per unit Steam-side heat-transfer surface area, introduced in H.1, £/sq ft.

$£^{n}_{S,tubes}$ price of tubing for surface steam condenser per unit Steam-side heat-transfer surface area, £/sq ft.

$£_T/q$ price of coupling-circuit duct leading towards tower per unit waste-heat rejection rate, £h/Btu.

$£_{turbine}/q$ price of Heller-system water turbine per unit waste-heat rejection rate, £h/Btu.

α ratio of condenser-shell radius to tube pitch, used in (D.3-3), (1).

α_f empirical Bergelin friction parameter given by (B.3-1).

α_M factor due to Merkel, defined by (E.1-7), (1).

α' cooling-tower geometry parameter introduced in J.1, (1)

β ratio of outside to inside diameter for pipes and tubes, (1).

β_A tower surface area coefficient defined in (J.2-1), (1).

ΔP_u decrease in generator output due to necessity of having $t_{s,S} > t_{G,i}$, kW.

Δp total pressure drop, lb_f/sq ft.

Δp_a acceleration pressure drop, lb_f/sq ft.

Δp_D total pressure drop of air flowing through cooling tower, lb_f/sq ft.

Δp_{fr} frictional pressure drop, lb_f/sq ft.

Δp_G pressure drop of air flowing through the Air-side of tower heat exchanger, lb_f/ sq ft.

Δp_x extra pressure drop introduced in Heller-system to maintain entire coupling circuit above atmospheric pressure, lb_f/sq ft.

Δt_F temperature variation of intermediary heat-transfer medium, used in (3.3-5), deg F.

Δt_G ($\equiv t_{G,o} - t_{G,i}$) dry-bulb temperature variation of air flowing through tower, used in (3.3-6), deg F.

$\Delta t_{s,H}$ $\equiv (t_{s,H})_i - (t_{s,H})_o$) saturation temperature variation in Hot-side of Spalding system, deg. F.

$\Delta £_{fuel}$ present worth of increase in fuel expenditure due to necessity of having $t_{s,S} > t_{G,i}$, introduced in (2.2-2), £.

δ thickness of duct, pipe or tube wall, ft.

δ_f thickness of fins, ft.

ε absolute roughness for ducts, pipes or tubes, ft.

η_B boiler efficiency, fraction.

η_{pm} overall efficiency of pump-motor combination for coupling circuit, fraction.

η_{pt} overall efficiency of pump-turbine combination for Heller system, fraction.

η_{st} steam turbine efficiency, fraction.

η_{TH} station nett thermal efficiency given by (2.2-3), fraction.

$\Sigma\Delta p$ total coupling-circuit pressure drop, lb_f/ sq ft.

σ_G ratio of free-flow area to frontal area on Air-side of tower heat exchanger, (1).

φ_f coefficient introduced in C.3, (1).

φ_1, φ_2 functions introduced in (4.3-1).

χ factor defined by (C.3-3), (1).

ϕ_1, ϕ_2 functions introduced in (4.3-2).

iii) Symbols for dimensionless groups

N_h ($\equiv UC/Q$) overall number of transfer units.

$N_{h,G}$ ($\equiv U_G C_G/Q_G$) overall number of Air-side transfer units.

n_h ($\equiv uC/Q$) partial number of transfer units.

$n_{h,G}$ ($\equiv u_G C_G/Q_G$) partial number of transfer units, introduced in 3.3.

$N_{Le,W}$ Lewis number for water-air.

N_{Pr} ($\equiv \mu c/k$) Prandtl number.

$N_{Pr,G}$ ($\equiv \mu_G c_G/k_G$) Prandtl number for air flowing through tower heat exchanger.

N_{Re} $\equiv dW/(\mu c_f)$ Reynolds number.

$N_{Re,f}$ local liquid-phase Reynolds number.

$N_{Re,G}$ $\equiv 4r_{h,G}W_G/(\mu_G c_G)$ Reynolds number for air flowing through tower heat exchanger.

$N_{Re,g}$ local gas-phase Reynolds number.

$(N_{Re})_f$, $(N_{Re})_g$ overall liquid- and gas-phase Reynolds numbers respectively.

N_v number of transformed velocity heads, obtained by converting number of local velocity heads to tower base area as reference plane.

$N_{v,G}$ number of transformed velocity heads for Air-side of tower heat exchanger.

n_v number of local velocity heads.

$n_{v,G}$ number of local velocity heads for Air-side of tower heat exchanger.

($N_{v,elim}$, $N_{v,i}$, $N_{v,misc}$, $N_{v,o}$, $N_{v,pack}$, $N_{v,spray}$) numbers of transformed velocity heads referring to eliminator,

tower inlet, miscellaneous losses, tower outlet, pack and spray respectively.

$(n_{v,elim}, n_{v,i}, n_{v,misc}, n_{v,o}, n_{v,pack}, n_{v,spray})$ numbers of local velocity heads referring to eliminator, tower inlet, miscellaneous losses, tower outlet, pack and spray respectively.

ΣN_v total number of transformed velocity heads for air flowing through cooling tower.

iv) Physical property symbols

c constant-pressure specific heat, Btu/(lb deg F).

c_A refers to dry air.

c_G refers to moist air flowing on Air-side.

$\bar{c}_G^*$ mean specific heat at wet-bulb temperature, defined by (E.1-9).

F_G, F_G^* see section ii).

h specific enthalpy, Btu/lb.

h_A refers to dry air.

h_C refers to water on Cold-side of Wet system.

h_G refers to moist air flowing on Air-side.

h_M specific enthalpy of saturated-air layer adjacent to water surface on Air-side of Wet system.

h_{fg} latent heat of vaporisation, Btu/lb.

k thermal conductivity, Btu/(h ft deg F).

k_A refers to dry air.

k_G refers to moist air flowing on Air-side.

v specific volume, cub ft/lb.

v_A refers to dry air.

v_G refers to moist air flowing on Air-side.

μ absolute viscosity, $lb_f/(h\ ft)$.

μ_A refers to dry air.

μ_G refers to moist air flowing on Air-side.

ρ density, lb/cub ft.

ρ_A refers to dry air.

ρ_G refers to moist air flowing on Air-side.

σ surface tension, lb_f/ft.

σ_W surface tension for water.

v) Symbols for quantities containing constant-valued physical property groups.

The procedure followed in this section is to present a symbol, followed by the number of the equation in which it is defined and then by the cooling system to which the symbol refers. To shorten the presentation, the Wet-, Heller- and Spalding-systems are represented by the letters W, H and S respectively.

$a_{a,H}$ (3.5-19), S.

$a_{f,C}$ (3.5-5), H.

$a_{f,H}$ (3.5-5), W.

$a_{m,C}$ (3.4-20), H,S.

$a_{m,H}$ (3.4-8), W,S.

$a_{p,B}$ (3.5-2),W.

$a_{p,H}$ (3.5-17), S.

$a_{p,T}$ (3.5-11), H.

$a_{p,B}$ (3.5-21), W,H,S.

$a_{p,C}$ (3.5-15), H.

$a_{p,C}$ (3.5-22), S.

$a_{p,H}$ (3.5-3), W.

(3.5-18), S.

$a_{p,T}$ (3.5-6), W,H.

(3.5-20), S.

$a_{u,C}$ (3.4-19), H.

(3.4-32), S.

$a_{u,H}$ (3.4-7), W,S.

$a_{u,S}$ (3.4-9), W,S.

$b_{p,C}$ (3.5-22), S.

$b_{p,H}$ (3.5-18), S.

INTRODUCTION

Three power-station cooling systems incorporating natural-draught cooling towers are investigated in this thesis with the object of helping power-station designers to choose between them. Before specifying the object of the investigation in greater detail, both the application of cooling towers in power-station cooling systems and the historical development of the three cooling systems under investigation are described in the following two sections.

1 Application of natural-draught cooling towers in power-station cooling systems.

Every thermal power station requires both a cooling system to remove waste heat in the power generation process, and a heat sink in which to deposit the waste heat. In areas where it is abundant, natural water is usually chosen as the heat sink. However, in areas where it must be conserved, or is too far from the proposed power-station site, atmospheric air is used instead. The present investigation is restricted to the examination of cooling systems for which atmospheric air must be used as a heat sink.

The transfer of waste heat to atmospheric air takes place in a heat exchanger, the efficiency of which increases with velocity of the air flowing through, or past the heat exchanger. The velocity is increased in practice, either by mechanically driven fans, or by utilizing the natural-draught forces caused by air-density differences occurring

in a type of chimney referred to as a natural-draught cooling tower.

The air-density differences are largely dependent on climate, and if this is such that the differences would be too small, fans must be used. The air-density differences are generally sufficiently large in areas of low atmospheric dry-bulb temperature, and sometimes even in areas of high dry-bulb temperature provided that the humidity is low and the waste heat is transferred to atmospheric air by an evaporative cooling process.

The task of choosing between the two foregoing alternatives is a large one involving the study of all possible cooling systems incorporating one or other of the alternatives. The present investigation concerns itself with one aspect of this task, viz. the investigation of power-station cooling systems incorporating natural-draught cooling towers.

The historical development and distinguishing features of these are briefly outlined in the next section, as a prelude to a more detailed description in the first chapter of the main text.

2 Historical development of power-station cooling systems incorporating natural-draught cooling towers.

Prior to 1956, there existed only one type of cooling system in this category, viz. the so-called Wet system in which natural water is employed as an intermediary heat-transfer medium to convey waste heat from a power station to

the atmosphere. After removing heat from turbine exhaust steam in a surface steam condenser, the water is made to trickle downwards through a layer of material referred to as the packing, this being situated in the lower part of a cooling tower. Draught-induced forces cause atmospheric air to flow upwards through the packing; the water and air are thus brought into direct contact, the water being both convectively and evaporatively cooled before returning to the steam condenser.

The Wet system offers the major advantage of a recooled water temperature close to the atmospheric wet-bulb temperature. On days of low humidity the latter is appreciably lower than the dry-bulb temperature, i.e. the lower limit for non-evaporative cooling.

The Wet system requires a continuous supply of natural water to make up for evaporation losses. Although the quantity required is a small fraction of that needed for cooling systems employing natural water as a heat sink, it can become disadvantageously large in water-scarce regions. Additional disadvantages are that the siting of power stations is restricted to areas close to natural water resources, and the cooling process can be adversely affected if the water freezes in cold weather.

At the 1956 World Power Conference in Vienna, Heller and Forgo announced the successful operation in Hungary of a closed-circuit power-station cooling system which, because it requires little or no make-up water, both conserves

natural-water resources and does not restrict the siting of power stations to areas close to them. This system, now referred to as the Heller system, uses boiler feed water as the intermediary heat-transfer medium. After removing waste heat in a direct-contact jet steam condenser, a fraction of the condenser water is circulated through a metal heat exchanger placed around the base perimeter of a cooling tower; the remaining fraction is sent to the boiler feed-heating system. Atmospheric air is caused to flow through the heat exchanger, and up the tower, by draught-induced forces, the water in the heat exchanger being convectively cooled before returning to the steam condenser. A water turbine is included in the cooling system circuit to impose an additional pressure drop; the water in the heat exchanger is consequently pressurized, thereby preventing contamination due to leakage into the cooling system.

The British Central Electricity Generating Board endorsed the merit of the Heller system by commissioning the English Electric Co. Ltd. to design and construct a power-station cooling system at Rugeley, based on the principles of Heller and Forgo.

The cooling system, although functioning satisfactorily since 1962, does have the following disadvantages.

i) The atmospheric dry-bulb temperature, and not the wet-bulb temperature, represents the lowest temperature limit to which the water can be cooled.

ii) A large mass flow rate of cooling water is required

and this results in high pumping costs.

iii) Expensive auxiliary equipment with complicated controls and ducts is necessary to drain water from the tower heat exchanger so as to prevent freezing in cold weather.

The last two disadvantages are eliminated in a closed-circuit cooling system proposed by D.B.Spalding, Professor of Heat Transfer, Imperial College of Science and Technology, London. In this system, hereafter referred to as the Spalding system, the proposed intermediary heat-transfer medium is a pressurized refrigerant operating at, or close to saturation temperature throughout the cooling system circuit. The refrigerant extracts heat from turbine exhaust steam in a surface steam condenser, being converted from liquid to saturated vapour in the process. The vapour is then circulated through a metal heat exchanger placed around the base perimeter of a cooling tower. It is condensed therein by transferring its latent heat of vaporization to atmospheric air, which is induced by draught forces to flow through, or past the heat exchanger and up the tower. The condensate is then returned to the steam condenser.

The advantages offered by the Spalding system relative to the Heller system, subject to the choice of a suitable refrigerant, are as follows:-

i) A pressure increasing device to keep the cooling system at a pressure higher than atmospheric is unnecessary, since the operating pressure of the refrigerant can be chosen to be higher than that of the atmosphere.

ii) The intermediary heat-transfer medium cannot freeze in the tower heat exchanger during cold weather.

iii) Operating under practically isothermal conditions ensures high thermodynamic efficiency.

iv) The removal of waste heat in the form of latent heat can lead to small refrigerant mass flow rates and small pumping costs.

The chief disadvantage of the Spalding system, relative to the Heller system, is that a surface steam condenser must be used in preference to a cheaper, direct-contact jet steam condenser so as to prevent the refrigerant from contaminating the boiler feed water.

No practical application of the Spalding system exists as it is a novel idea, mooted as recently as 1962. In fact, the present investigation contains the first detailed attempt to predict its performance and to compare it with other cooling systems.

3 Object of present investigation.

Power-station designers are usually supplied with information regarding the proposed site, climatic conditions and waste-heat rejection rate. For regions where natural-draught cooling towers are suitable, the following question arises:-

Which of the three cooling systems outlined in the previous section must be used to ensure the best overall power-station design, and what are the values of the design variables

characterizing it?

This question can be answered by evaluating and then comparing the relative cooling system merits according to the following procedure:-

i) Choose a representative circuit for each of the considered cooling systems.

ii) Establish a criterion for comparison in the form of an overall power-station merit-factor.

There are many design variables characterizing a cooling system and which affect its performance, and subsequently its merit-factor. Some, however, have relatively little effect and can be fixed for comparison purposes. Others are important and must be allowed to vary independently of each other so that their effect on the merit-factor can be examined.

iii) Therefore, choose the design variables which are to be allowed to vary independently and fix the unimportant ones.

iv) Express the merit-factor, adopted in ii), as a function of the independent design variables for each of the considered cooling systems.

For comparison purposes, only one of the many possible combinations of design variable values is of interest, viz. that combination which gives the optimum value of the merit-factor for a particular cooling system.

v) Therefore, develop methods for determining optimum design-variable values and the associated merit-factor value

for each of the considered cooling systems.

vi) Evaluate and compare the optimum merit-factor values. The cooling system having the best one must be used to ensure the best overall power-station design, provided that the optimum values of the design variables are employed when constructing the cooling system concerned.

There is only one published paper (ref.34) which attempts to compare different cooling systems incorporating natural-draught cooling towers and, even in this, the comparison is restricted to the Wet- and Heller-systems. The cost comparisons in this paper are based on unspecified design variable values. Since no attempt is made to establish an overall power-station merit-factor and to optimize it for each cooling system, the results must be considered unsatisfactory for comparison purposes.

The object of the present investigation is first of all to carry out the previously outlined steps i) to v) and, in so doing, to derive methods of evaluating the optimum values of the design variables and the associated optimum merit-factor values. In addition, it is proposed to carry out step vi) by using the derived methods to obtain quantitative results for a range of waste-heat rejection rates, a chosen climate and prices valid for the U.K. in 1964. The merit of each of the considered cooling systems will then be discussed in the light of these results.

4 Outline of procedure.

The first task, undertaken in chapter one, is the choice of simplified circuits for each of the considered cooling systems to make comparisons easier, without being unrealistic; the function and choice of components are described with the aid of diagrams. However, the task is only partially completed since the choices of the following components involve a number of alternatives, the merits of which can only be gauged in terms of their effect on overall power-station design:

i) tower packing type for Wet system;

ii) tower heat-exchanger types for both Heller- and Spalding-systems;

iii) type of surface steam condenser and

iv) refrigerant for Spalding system.

The effects of these alternatives can only be determined after establishing an overall power-station merit-factor, and expressing it as a function of cooling system design variables. The establishment of such a factor is carried out in chapter two, the merit-factor being presented as an overall power loss. The power-loss concept arises from the realization that a cooling system has certain requirements for its successful operation, all of which detract from the potential generator output. These requirements can be represented as individual power losses, the arithmetic sum of which constitutes an overall power loss. The overall power loss so obtained is suitable for use as a cooling system

merit-factor since its value will be smallest for the best of a number of cooling systems. In chapter two, the merit-factor is expressed as a function of these requirements after deriving methods for transforming them to their equivalent power losses.

The next step, in accordance with the object of the investigation, is to simplify comparisons by deciding which cooling-system characteristics are not important enough to be varied and which are to be treated as independent design variables. This is carried out in chapter three, an attempt being made to simplify comparisons still further by specifying the design variables in terms of quantities, or ratios thereof, which will be relatively invariant for cooling systems.

In choosing the design variables it transpires that their number is not as large as originally supposed. Due to the necessity of having to satisfy heat-balance relationships and the cooling tower buoyancy equation, some of the originally chosen design variables are dependent on others, thereby losing their status as independent design variables. The task of expressing the merit-factor as a function of the design variables is also undertaken in chapter three, after splitting the merit-factor equation into its constituent parts for ease of presentation.

It is then possible to gauge the merits of the alternatives for the previously ennumerated cooling system components, the choice between which was deferred until after the

development of a merit-factor equation. Accordingly, choices of tower-packing type and tower heat-exchanger types are made in chapter four, whereas choices regarding Spalding system surface steam condenser and refrigerant are made in chapter five.

Only at this stage can attention be paid to optimization, i.e. the task of finding that set of design variable values which, on substitution into the merit-factor equation, will give the minimum merit-factor value. In this respect, two objects are fulfilled. The first is the development of a numerical, "steep descent" method - suitable for use by an automatic digital computer - in order to determine accurately the values of the design variables at optimum conditions. Secondly, the results and trends accruing from the application of this method are used in developing an approximate method of determining the optimum design variables. The latter method is intended for use in circumstances where digital computers are not available. The development of the two methods is described in chapters six, seven and eight for the Wet-, Heller- and Spalding-systems respectively.

In chapter nine, both methods are used in the presentation of numerical results for a range of waste-heat rejection rates, a chosen climate and prices valid for the U.K. in 1964. The accuracy and applicability of the approximate solution, and the relative merits of each of the considered cooling systems, are discussed in the light of these results in chapter ten.

Finally, chapter eleven is devoted to suggestions for future research and general conclusions.

5 General information.

In the interests of readability, extensive use has been made of appendices, the purpose of which is to store detailed information and associated references in closed form without, it is hoped, incurring the disadvantage of excessive cross-referencing.

The wide field covered by the thesis has necessitated an elaborate nomenclature section. It is suggested that the reader should glance through this section, paying particular attention to the headings and the sub-section relating to sub-scripts, before proceeding to the main text.

6 Acknowledgements.

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1 DESCRIPTION OF COOLING SYSTEMS UNDER CONSIDERATION

1.1 Introduction

When cooling systems are compared in order to determine their relative merits, it is not necessary to enter into as detailed a design study as would be required for tendering purposes. For example, identical items of equipment common to each cooling system can be ignored. This chapter is accordingly devoted to describing simplified circuits suitable for comparison purposes, care being taken neither to exclude distinctive characteristics nor to become unrealistic. Diagrams are used in describing both the function of and the components comprising each cooling system circuit.

However, in choosing some of the cooling system components, there are a number of alternatives, the best of which can only be chosen after quantitatively calculating their relative effects on the as yet un-established cooling system merit-factor. In consequence, the present chapter restricts itself to discussing and recording the alternatives, choices between them being deferred to chapters subsequent to the establishment of a cooling system merit-factor.

1.2 Wet system.

A schematic diagram of the proposed Wet-system circuit is given in Fig.1, with a cross-sectional view through the tower to the left of the centre line and an itemized list of the cooling system components. With reference to Fig.1,

A Steam surface condenser
B Circulating water pump and motor
C Coupling-circuit ducts
D Tower shell
E Blanked off volume
F Eliminator
G Water distribution channel
H Spray pipes and sprayers
I Packing
J Tower-shell supports
K Packing supports
L Water collecting pond

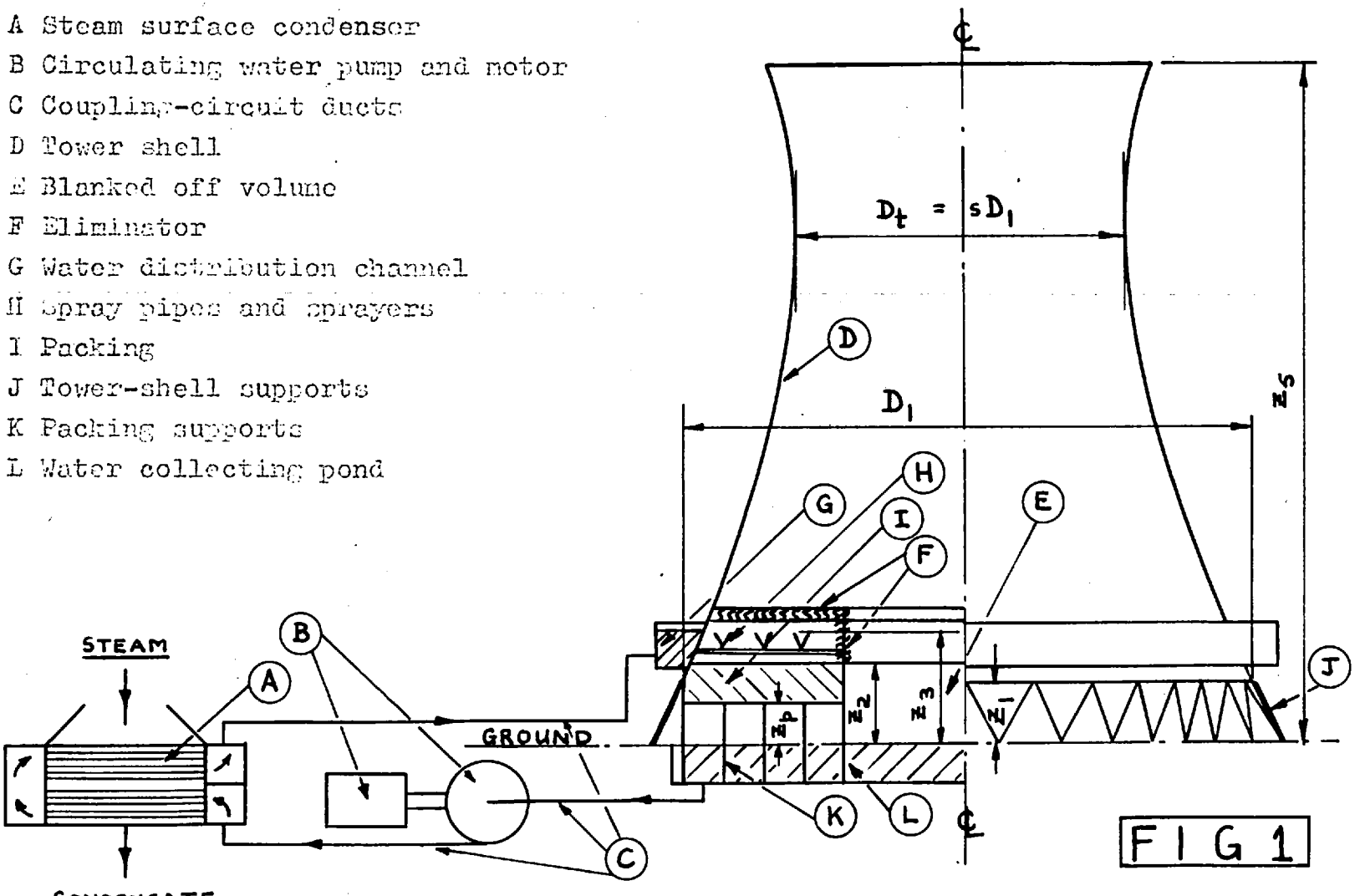


FIG 1

DIAGRAM OF ADOPTED WET-SYSTEM LAYOUT (NOT TO SCALE).

the Wet system operates as follows:-

After extracting heat from turbine exhaust steam in a surface steam condenser, A, natural water is circulated by a pump, B, along a duct, C, to a circumferential distribution channel, G. From here the heated water is distributed to a number of nozzles or sprayers, H, which spray it evenly over the top of the packing, I. As the water trickles downwards through the packed material comprising the packing, it comes into direct contact with atmospheric air. The air is induced by draught forces to enter the bottom of the packing via the cooling tower inlet, and then to flow upwards through the packing, past the sprayers and through the eliminator, F; this is designed to trap liquid water entrained by the air in passing through the pack and past the sprayers. The air then emerges from the top of the tower while the water, having been cooled by a combination of evaporative and convective heat transfer, falls into a storage pond, L, situated at the tower base; it is then returned to the steam condenser in a duct.

The choices of components for the Wet system involve a number of alternatives. However, experience gained from the design and operation of cooling systems has enabled other investigators to make the best choices for most of the alternatives. The following paragraphs contain descriptions of the components recommended for adoption by other investigators, followed by decisions regarding the choices of remaining alternatives.

With regard to the surface steam condenser, information in refs. 13 and 5 leads to the adoption of two water-side passes with horizontal condenser tubes constructed from admiralty brass. In addition, a mixed-flow circulating pump driven by an induction motor is chosen. Ducts are assumed to be constructed from mild steel in accordance with convention.

A departure from convention is made with regard to the packing arrangement in the cooling tower by introducing a blanked-off volume, formed by a hollow cylinder at the tower centre, as is shown by item E in Fig.1. The presence of this blanked-off volume, introduced by Spalding in ref. 38, results in the advantage of a reduction in packing, eliminator and sprayer requirements. This advantage can outweigh the disadvantage incurred by the resultant increase in pressure drop through the tower. The lower half of the packing is assumed to extend below the plane formed by the tower shell base, as can be seen from item I in Fig.1. According to ref. 38, this leads to a saving in cooling tower material costs. Water is distributed to the sprayer pipes by means of a circumferential channel. This method of distribution is chosen because, according to ref. 38, the channel can be made to act as a fairing, thereby increasing the aerodynamic efficiency of the tower.

A catenary profile is adopted for the concrete tower shell instead of the conventional hyperbola because of the resultant saving in shell material. A circular profile

would be just as satisfactory but is mathematically less manageable. Apart from its profile, the concrete cooling tower shell and associated equipment, viz. shell supports, packing supports, pond, eliminator, sprayers, walkways, etc., are assumed to be conventional.

There is one important alternative which remains, viz. the choice of packing type. There are many types and it is impossible to choose between them until some means of determining their relative merits has been established. Consequently, decisions regarding the choice of the best packing type are deferred until this has been done.

1.3 Heller system.

Fig.2 contains a schematic diagram of the proposed Heller-system circuit, with a cross-sectional view through the tower to the left of the centre line. The function of the Heller system is explained with the aid of Fig.2 as follows:-

Cooled water of boiler-feed purity is injected through nozzles into jet steam condenser, A, where it mixes with turbine exhaust steam. The steam transfers its latent heat to the water and is itself condensed. A circulating pump, B, both returns the condensate to the boiler feed heating system and circulates the heated water along a duct, C, and through a two-pass, cross-flow heat exchanger, G, placed around the base perimeter of a cooling tower, E. While flowing through the heat exchanger, the water is cooled

A Steam jet condenser
B Extraction pump and motor
C Coupling circuit ducts
D Water turbines
E Tower shell
F Tower heat exchanger
G Diagrammatic cross-section through tower heat exchanger
H Tower-shell supports

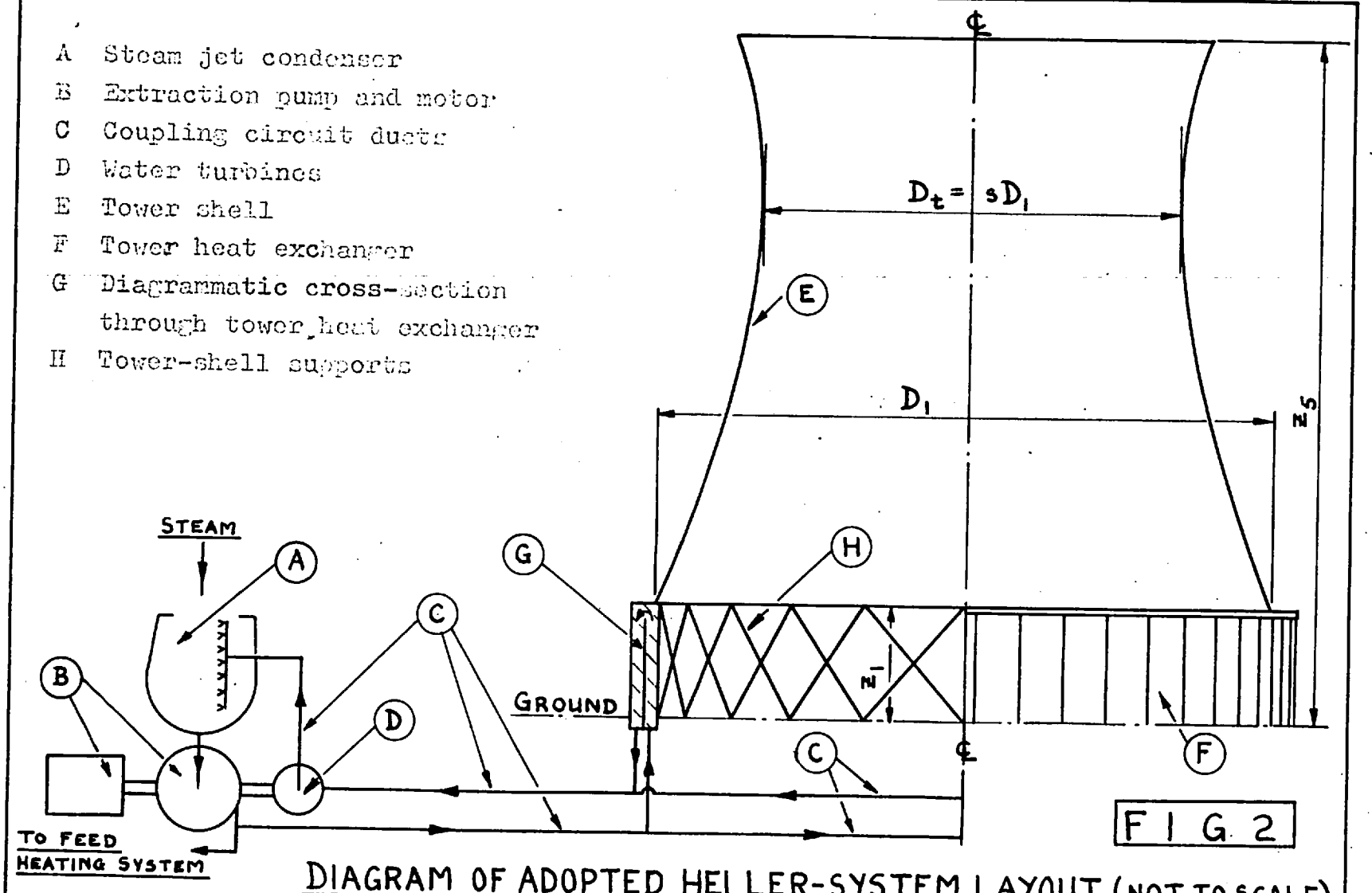


FIG 2

DIAGRAM OF ADOPTED HELLER-SYSTEM LAYOUT. (NOT TO SCALE)

convectively by atmospheric air which is induced by draught forces to flow through the heat exchanger and up the tower. The cooled water then travels along a duct, C, passing through a water turbine, D, before returning to the nozzles in the steam condenser. The function of the water turbine is to provide an additional pressure drop in the circuit to keep the pressure in the tower heat exchanger higher than atmospheric pressure; this prevents leakage into the system and consequent contamination of the boiler feed water. Some of the energy lost in providing the additional pressure drop is recovered by using the water turbine to help drive the circulating pump. Auxiliary equipment is provided to drain the tower heat exchanger so as to prevent freezing in cold weather.

There are not many alternatives when choosing components for the Heller system and the best choices for most of them are made as follows:-

As there is no risk of contaminating the boiler feed water, a jet steam condenser is suitable for use in the Heller system and is adopted because it is cheaper than an equivalent surface steam condenser. A centrifugal circulating pump, driven by an induction motor, and a propeller water turbine are chosen as being most suitable for Heller-system conditions according to information contained in refs. 13 and 31. Although, in practice, the circulating pump has the dual function of returning the condensed steam to the boiler feed-heating system and circulating the cooling

system water, it is assumed to have only the latter function for comparison purposes. This assumption follows from the fact that the condensed steam extraction pump is not included in the adopted Wet- and Spalding-system circuits.

Mild steel is the material chosen for ducts. The concrete tower shell profile is considered to be a catenary as in the Wet system. The greater Heller-system inlet height necessitates the use of X-formation tower supports, instead of the Wet-system V-formation supports, in order to reduce buckling tendencies to an acceptable level.

The remaining alternatives consist of choosing the best type of heat exchanger, the optimum number of water-side passes and the best way of fitting the heat exchanger around the base perimeter of the cooling tower. Although the tower heat exchanger in use at Rugeley has two water-side passes, with vertical tubes equal in length to the cooling tower inlet height, there is no indication in published literature whether this is the best arrangement or not. Consequently, an attempt will be made in the present investigation to examine a number of possible tower heat exchanger types in order to find the best type and the best arrangement. Since this can only be done after establishing a cooling system merit-factor and deriving heat-exchanger performance equations, this attempt is deferred to later chapers of the thesis.

An important point to note is that the auxiliary equipment which prevents freezing must be regarded as an additional

cooling system component when determining the Heller-system merit-factor. For the present investigation, this equipment is assumed to be the same as that employed at Rugeley and is described in ref. 9.

1.4 Spalding system

1.4.1 Function

A schematic diagram of the proposed Spalding- system circuit is given in Fig.3, with a cross-sectional view through the tower to the left of the centre line. With reference to Fig.3, the Spalding system operates as follows:-

A refrigerant in the liquid state and at saturation temperature extracts heat from turbine exhaust steam in a surface steam condenser, A, becoming vaporised in the process. The vapour is then transmitted along a duct, C, after passing through a vapour-liquid separator, B. The vapour is then circulated through a heat exchanger, H, placed around the base perimeter of a cooling tower, F. In passing through the tower heat exchanger tubes the vapour gives up its latent heat of vaporisation, gained in the steam condenser, to the atmospheric air which is induced by draught forces to flow past the heat exchanger and up the tower. The vapour is completely condensed during this process and is then returned to the steam condenser along a duct, C, by means of a circulating pump, E.

A Steam surface condenser
B Vapour-liquid separator
C Coupling-circuit ducts
D Separator-condenser re-circulation ducts
E Circulating pump and motor
F Tower-shell
G Tower heat exchanger
H Diagrammatic cross-section through tower heat exchanger
I Tower-shell supports

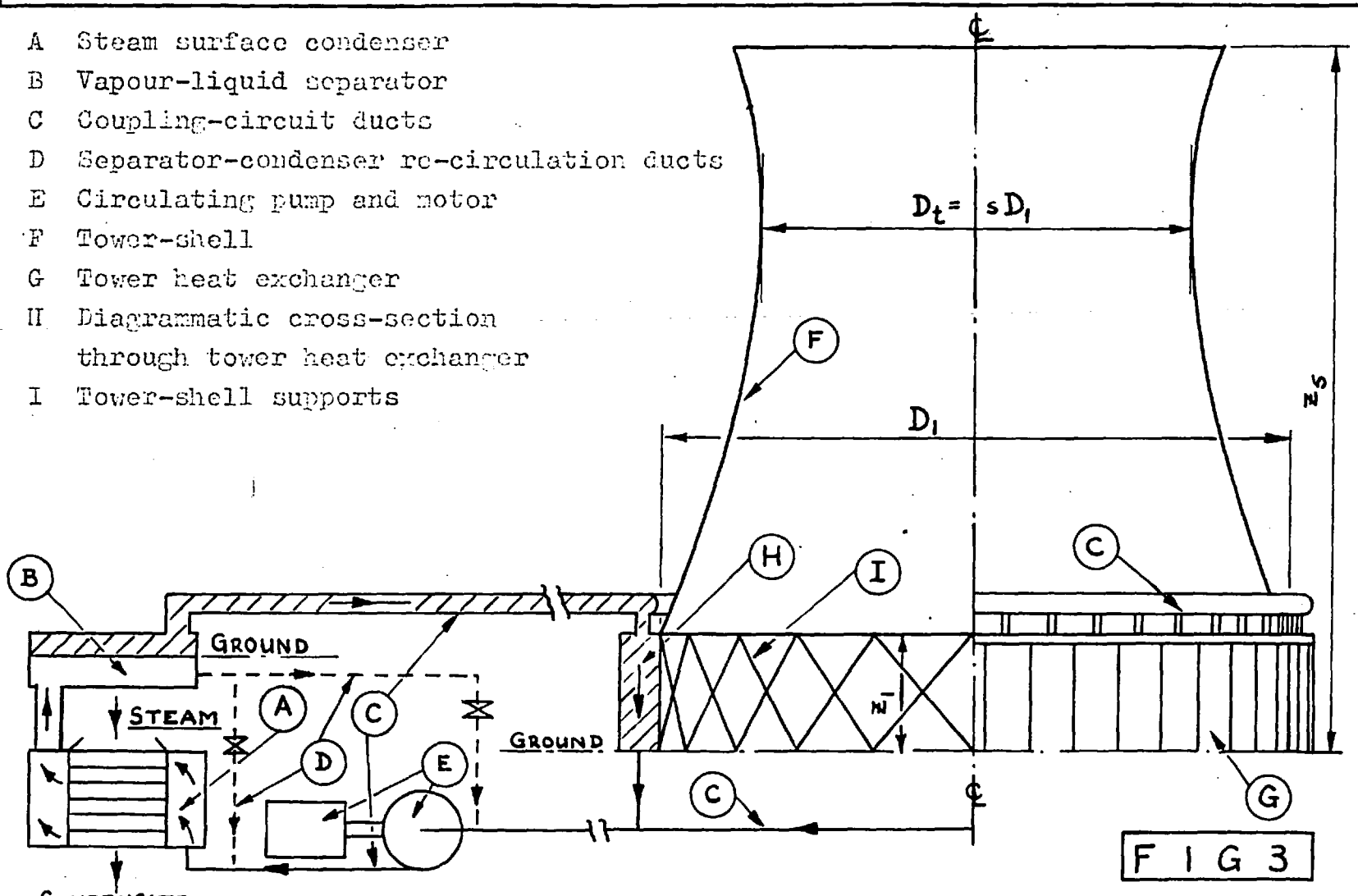


FIG 3

DIAGRAM OF ADOPTED SPALDING-SYSTEM LAYOUT. (NOT TO SCALE)

1.4.2 Choices of some cooling-system components

In choosing components for the Spalding system there are a number of alternatives, the best choices for some of which are carried out in this sub-section according to the following procedure.

A surface steam condenser must be employed in preference to a cheaper jet steam condenser, in order to prevent the refrigerant from contaminating the condensed steam. The circulating pump is assumed to be an axial-flow type, driven by an induction motor, according to the suggestion of Ellingen in ref. 13. Mild steel is considered a suitable material for ducts and the liquid-vapour separator. The concrete tower shell and supports are considered to be of the same shape and construction as for the Heller system.

The task of choosing between the remaining alternatives, relating to the refrigerant, steam condenser and tower heat exchanger, is complicated because there are so many of them. The object of the following sub-sections is to ease this task by first of all describing the alternatives, and then using both qualitative and quantitative reasoning to eliminate the least promising of them. Decisions as to which of the remaining alternatives are best are then deferred to later chapers of the thesis, when sufficient information will be available to enable the decisions to be made.

1.4.3 Choices of promising refrigerant alternatives

Refrigerants considered for use in the Spalding system must have the following feature:-

The saturation pressure at the lowest possible atmospheric wet-bulb temperature must be greater than atmospheric pressure.

This will prevent leakage into the system since the cooling system pressure will always be greater than that of the atmosphere. Also, the refrigerant will not freeze in practice. Investigation of a number of fluids and, in particular, the extensive list of refrigerants and their associated physical properties contained in ref. 1 reveals that the following refrigerants have the foregoing essential feature: freon 12, freon 13, freon 22, methyl chloride, ammonia, propane and propylene.

It is desirable for a Spalding-system refrigerant to have a high latent heat of vaporisation and high densities, both liquid and vapour, as this will ensure low refrigerant mass flow rates and low circulating power requirements. Freon 13 is unsuitable in these respects as its latent heat of vaporisation is relatively low. Propane and propylene, too, are unsuitable because of their low latent heats of vaporisation, combined with low densities.

Possible refrigerants are thus restricted to ammonia, methyl chloride, freon 12 and freon 22, the physical properties of which are presented in appendix A. A decision as to which of these refrigerants in best cannot be made until

the basis of cooling system comparisons has been established; this decision is accordingly deferred to a later chapter of the thesis.

1.4.4 Preliminary choices of surface steam condenser flow arrangements

There are four possible flow arrangements to be considered, viz.

i) natural-convection refrigerant flow with complete vaporisation,

ii) natural-convection refrigerant flow with partial vaporisation,

iii) forced-convection refrigerant flow with complete vaporisation, and

iv) forced-convection refrigerant flow with partial vaporisation.

The first possibility is represented in Fig.3 if the circulating pump, E, and both re-circulation ducts, D, are eliminated. It is assumed to be unsuitable as natural-convection forces are considered inadequate to overcome both the large flow resistance caused by complete vaporisation in the steam condenser, and the flow resistance in the remainder of the circuit.

However, the circulation task of the natural-convection forces can be made easier by re-circulating the refrigerant in the steam condenser. This is the second of the possible flow arrangements and is represented in Fig.3, with the eli-

mination of the right-hand re-circulation duct, D, and the circulation pump, E. If, after introduction of re-circulation, the natural-convection forces are only sufficient to overcome the flow resistance in the steam condenser, the circulating pump, E, must be inserted to overcome the flow resistance of the remainder of the circuit. Moreoever, if the natural-convection forces are insufficient even to overcome the flow resistance in the steam condenser, recourse must be had to the use of forced-convection refrigerant flow, i.e. one of the third and fourth possibilities.

The third possibility requires no re-circulation and is represented in Fig.3 by eliminating both re-circulation ducts, D. There is re-circulation in the fourth possibility, which can be represented in Fig.3 after eliminating the left-hand duct, D.

The choice between natural- and forced-convection is a difficult one because of the lack of satisfactory formulae for the evaluation of the natural-convection forces required to re-circulate the refrigerant in the steam condenser. This lack is due chiefly to the difficulty of accurately predicting pressure drops due to friction and acceleration for two-phase flow. In this investigation it is pessimistically assumed that natural-convection forces are inadequate, and that forced convection must be resorted to in order to circulate the refrigerant. This assumption will be acceptable, provided that the over-estimated circulating pump power, accruing from the application of the assumption, has a small

effect on the merit of a cooling system. As this can only be determined after carrying out numerical calculations in later chapters of the thesis, the justification of the foregoing assumption is accordingly deferred till later.

A forced-convection refrigerant flow arrangement having been decided upon, a choice between the third and fourth possibilities need not be made at this juncture. By adopting the fourth possibility, treating the amount of re-circulation as a design variable and then calculating the amount required for an optimized cooling system, the decision will be made automatically. For example, if the required amount of re-circulation is small, the third possibility is better; for a large amount, the fourth is to be preferred.

1.4.5 Preliminary choices of surface steam condenser configurations

The following variations in condenser configuration are considered possible:

i) extended surfaces on one of the sides,

ii) steam flowing outside tubes with refrigerant inside, or vice-versa,

iii) one or more passes on either the steam- or the refrigerant side,

iv) tubes either straight or serpentine in layout and

v) tubes at any angle of inclination to the horizontal.

If the heat-transfer coefficients on the steam- and refrigerant-side are similar, there is no need to depart from the conventional practice of having the steam in the

condenser shell, and the refrigerant flowing through horizontal bare tubes surrounded by the steam. However, if the two heat-transfer coefficients differ appreciably, the overall heat-transfer resistance can be decreased by employing fins to increase the tube surface area on the side having the smaller heat-transfer coefficient.

In the event of the refrigerant-side heat-transfer coefficient being the smaller of the two, the use of internal fins gives rise to manufacturing difficulties and large flow resistances in the tubes. These disadvantages can be overcome by departing from convention in having the refrigerant on the shell-side and the steam flowing through the tubes; the fins can then be more conveniently attached to the external surface of the tubes.

In the present investigation it is assumed that the difference between the two heat-transfer coefficients is not large enough either to warrant the use of extended surfaces or to re-route the steam through the condenser tubes. The validity of this assumption can only be tested in a later chapter of the thesis after calculating the steam condenser heat-transfer coefficients for optimized cooling systems.

As the refrigerant pressure drop must be kept low in order to preserve the near-isothermal nature of the Spalding-system, the length to diameter ratio of the condenser tubes will be relatively small. This, coupled with the fact that the condenser operates with the same efficiency as a counter-flow heat exchanger, leads to the conclusion that there is no

need to consider a serpentine tube layout or more than one pass on either side.

Examination of the calculated steam-condenser dimensions, e.g. length of tubes, after having optimized the Spalding-system, will reveal whether the adoption of straight tubes and only one pass is reasonable or not.

1.4.6 Discussion of tower heat exchanger alternatives

The alternatives consist of choosing the best type of heat exchanger, the optimum number of refrigerant-side passes and the best way of fitting the heat exchanger around the base perimeter of the cooling tower.

Decisions regarding the number of refrigerant-side passes can be made now. Since the Spalding-system operates under practically isothermal conditions, the tower heat exchanger has the same thermodynamic efficiency as a counter-flow heat exchanger. Consequently, there is no need for more than one refrigerant-side pass. This viewpoint is endorsed by the fact that the use of more than one pass poses problems of condensate drainage.

A possible tower heat exchanger arrangement, closely resembling that of the Heller system as used at Rugeley, is shown in Fig.3. The refrigerant vapour passes into the vertical tubes of a single-pass tower heat exchanger via a distribution duct encircling the tower shell. However, as in the case of the Heller system, no tower heat exchanger decisions can be made yet; the choices of the best type and

arrangement are accordingly deferred to later chapters of the thesis.

2 ESTABLISHMENT OF CRITERION OF MERIT FOR COOLING SYSTEMS

2.1 Deduction of merit-factor

Now that the chosen cooling system circuits have been described, the next step is to establish a suitable criterion for determining the merits of the considered cooling systems so that comparisons between them can be made. In establishing the merit of a cooling system in ref. 39, Spalding recommends that a cooling system be thought of as having the following requirements:

i) a temperature difference given by subtracting the atmospheric dry-bulb temperature from the turbine exhaust-steam temperature and which provides the necessary driving force for removal of power-station waste heat;

ii) power to circulate the intermediary heat-transfer medium round the cooling-system circuit;

iii) capital expenditure in providing the cooling-system components.

Spalding also submits that if these requirements did not exist the generator output could be increased. In consequence, these requirements can be regarded as constituting an overall power loss, the value of which would be equal to the gain in generator output were there no cooling-system requirements. This overall power loss can be used to indicate the merit of a cooling system since the best of a number of cooling systems will incur the smallest overall power loss.

The overall power loss is the arithmetic sum of the individual power-loss contributions arising from each of the

cooling-system requirements previously enumerated. In formulating an expression for the merit-factor, Spalding introduces coefficients to convert the cooling system requirements to their corresponding power-loss contributions. In addition, each power-loss term is rendered dimensionless by dividing it by the waste-heat rejection rate.

Consequently, the overall power loss per unit waste-heat rejection rate for one cooling tower, hereafter referred to as the merit-factor, is the sum of the three itemized power-loss contributions, i.e.

$$P_L/q = (P_L/q)_1 + (P_L/q)_2 + (P_L/q)_3, \qquad (2.1\text{-}1)$$

where

$$(P_L/q)_1 = a^*(t_{s,S} - t_{G,i}), \qquad (2.1\text{-}2)$$

$$(P_L/q)_2 = B^*(P/q), \qquad (2.1\text{-}3)$$

$$(P_L/q)_3 = c^*(£_{cs}/q) \qquad (2.1\text{-}4)$$

P_L/q is the merit-factor, or overall power loss per unit waste heat rejection rate, in dimensionless units.

$(t_{s,S} - t_{G,i})$ is the difference between turbine exhaust steam temperature and atmospheric dry-bulb temperature, in deg.F.

P is required pumping power per cooling tower, in Btu/h.

$£_{cs}$ is total capital price of cooling system components per cooling tower, in pounds sterling.

q is the waste-heat rejection rate per cooling tower, in Btu/h.

a^*, B^* and c^* are coefficients which convert cooling system requirements to power-loss contributions.

Spalding uses a modified version of (2.1-1) so that the power-loss contribution due to all pumping requirements can be more easily observed by inspection of a single term. Accordingly, the capital price of the circulating pump and motor is included in (2.1-3), so that

$$(P_L/q)_2 \equiv b^*(P/q), \tag{2.1-5}$$

$$(P_L/q)_3 \equiv c^*(£_{cs}/q - £_{pm}/q) \tag{2.1-6}$$

and

$$b^* \equiv B^* + c^*(£_{pm}/q)/(P/q). \tag{2.1-7}$$

$(£_{pm}/q)$ is the capital price of circulating pump and motor per unit waste-heat rejection rate, in pounds sterling per (Btu/h).

The merit-factor can thus be evaluated from the expression.

$$\boxed{P_L/q = a^*(t_{s,S}-t_{G,i}) + b^*(P/q) + c^*(£_{cs}/q - £_{pm}/q),} \tag{2.1-8}$$

provided that a^*, b^* and c^* are known. The following sections of this chapter contain the derivations of formulae suitable for the estimation of these coefficients; in addition, representative numerical values are calculated.

The method of using (2.1-8) in optimizing the design of a particular cooling system is first of all to choose the independent design variables characterizing the cooling system.

This done, each of the temperature difference, pumping power and capital price terms in (2.1-8) must be expressed as functions of the chosen design variables. Both of these tasks are undertaken in the following chapter. After substitution of the resultant functions into (2.1-8), the merit-factor will be a function of the chosen design variables. The single set of design variable values giving the minimum merit-factor value must then be determined, since it is this set which characterizes an optimum cooling system design. The optimum merit-factor thus obtained can then be compared with similarly determined optimum merit-factors for other cooling system types; the best one will have the smallest optimum merit-factor value.

2.2 Evaluation of temperature difference coefficient, a^*.

2.2.1 Derivation of basic power-loss equation.

As can be seen from (2.1-2),

$$a^* \equiv (P_L/q)_1/(t_{s,S} - t_{G,i}), \qquad (2.2\text{-}1)$$

where $(P_L/q)_1$ is the loss of useful generator output per unit waste-heat rejection rate, due to the necessity of having $t_{s,S} > t_{G,i}$. In evaluating $(P_L/q)_1$, the manner in which a power-station is affected by changes in exhaust-steam temperature must be determined.

According to Bartlett, on page 5 of ref. 7, an increase in turbine exhaust-steam temperature decreases the energy available, thereby decreasing the generator output and also detracting from the station thermal efficiency. This means

that, for a chosen generator output, an increase in turbine exhaust temperature represents, firstly, a power loss equal to the potential decrease in generator output. Secondly, there is additional capital expenditure equal to the present worth of the increase in fuel expenditure resulting from the decrease in plant thermal efficiency.

The additional capital expenditure can be converted to an equivalent power loss after assigning a capital value to power. Such an assignation can be carried out according to the procedure recommended on page 41 of ref. 3; a ratio, defined as the capital cost of peak power, is formed by adding the overall power-station capital cost per unit generator output to the present worth, per unit generator output, of the management, insurance and other works costs and the capital cost of the transmission connection. The additional capital expenditure can consequently be regarded as directly proportional to its equivalent power loss, the coefficient of proportionality being the previously mentioned ratio. Of the two items constituting the capital cost of peak power, the one involving station capital cost can be obtained from published tables, e.g. on page 5 of ref. 11, whereas the present worth item has a current value of 18 pounds sterling per kW generated, according to page 36 of ref. 3.

In accordance with the foregoing remarks, $(P_L/q)_1$ is given by the sum of two terms, viz.

$$(P_L/q)_1 = J_K(P_u/q)(\Delta P_u/P_u)+J_K(P_u/q)(£_{cap}/P_u)^{-1}(\Delta £_{fuel}/P_u). \qquad (2.2\text{-}2)$$

J_K(=3413) is a coefficient converting kWh to Btu.

P_u is generator output per cooling tower, in kW.

q is waste-heat rejection rate per cooling tower, in Btu/h.

ΔP_u is decrease in generator output due to necessity of having $t_{s,S} > t_{G,i}$, in kW.

$£_{cap}/P_u$ is capital cost of peak power, in pounds sterling per kW.

$\Delta £_{fuel}$ is present worth of increase in fuel expenditure due to necessity of having $t_{s,S} > t_{G,i}$, in pounds sterling.

The reason for the algebraic manipulation of terms in presenting (2.2-2) is to formulate groups which are comparatively easy to evaluate. The next step is to outline methods for the evaluation of each of these groups.

2.2.2 Evaluation of P_u/q

An expression for P_u/q may be derived from the definition of station net thermal efficiency, i.e. the ratio of electrical power sent out to total power supplied to a power station. This definition can be expressed as

$$\eta_{TH} = J_K \eta_B \{ 1 - (P_{u,B} + P_{u,A})/P_u) \} /(q_S/P_u). \qquad (2.2\text{-}3)$$

η_{TH} is station net thermal efficiency expressed as a fraction.

J_K (= 3413) is a coefficient converting kWh to Btu.

η_B is boiler efficiency expressed as a fraction.

$(P_{u,B} + P_{u,A})/P_u$ is the ratio of the sum of the boiler - and

all other auxiliary power requirements - to generator output, in dimensionless units.

q_S/P_u is the heat supplied to steam by the boiler per unit generator output, in Btu/kWh.

According to page 68 of ref. 7, q_S is obtained from a heat balance by subtracting the heat added in the boiler feed pump from the sum of the following items: mechanical and generator power losses, heat loss to steam-seal system, generator output and waste-heat rejection rate. Furthermore, heat balances for typical power-station turbines on pages 53 and 68 of ref. 7 indicate that the effect of the foregoing subtraction is to practically cancel out some of the power-loss items, thereby enabling q_S to be accurately evaluated from the following simplified equation:

$$q_S = q + J_K P_u.$$

q is the waste-heat rejection rate in Btu/h. The substitution of this equation into (2.2-3) results in

$$P_u/q = (\eta_{TH}/J_K)/\{\eta_B - \eta_{TH} - \eta_B(P_{u,B}+P_{u,A})/P_u\}. \quad (2.2\text{-}3a).$$

In order to obtain numerical values for P_u/q, the power ratio in the final term of this equation need not be computed accurately as P_u/q will only be increased by about 10 per cent if it is ignored; a fixed value of 0.04 is recommended on page 69 of ref. 7.

According to page 373 of ref. 28: "Boiler efficiency for a given steam condition, feed water-heating cycle, and steam output will not change with exhaust pressure or turbine

exhaust size. Generally 85 to 90 per cent efficiency may be used in making economic studies." In accordance with this quotation, a fixed value of boiler efficiency will be used in the present investigation.

Station nett thermal efficiency, on the other hand, does vary with exhaust pressure; the manner in which it varies must be determined in order to evaluate P_u/q from (2.2-3) for various exhaust-steam temperatures. The most accurate method of doing this would involve the preparation of optimized overall power-station heat balances for a range of turbine-exhaust pressures, so that a graph of station thermal efficiency vs exhaust-steam temperature could be drawn. This is a difficult and tedious task which can, however, be made easier by using existing simplified methods for the prediction of changes in power-station behaviour. These methods owe their existence to the fact that some power-station variables are relatively unimportant when predicting changes and can therefore be ignored.

In one such method, outlined in pages 160 to 195 of ref. 7, graphs and simple equations are provided so that the variation of station heat rate with exhaust-steam temperature can be estimated. The variation in station nett thermal efficiency can then be determined since

$$\eta_{TH} \equiv J_K \,/\, (\text{station nett heat rate, in Btu/kWh}). \quad (2.2\text{-}4)$$

The procedure is first of all to choose a representative power-station layout operating under typical conditions and to cal-

culate the turbine internal heat rate. Then, for various values of exhaust pressure, the graphs and simple equations will indicate corresponding changes in turbine internal heat rate, exhaust loss, throttle flow and condenser flow. The new values so obtained can then be used to calculate changes in station nett heat rate and station nett thermal efficiency.

A basic feature of this method is that the initial and reheat steam conditions and turbine exhaust area are fixed. This represents a major disadvantage in that the predicted variation of thermal efficiency with exhaust pressure will probably not be indicative of that for optimized values of initial and reheat steam conditions and turbine exhaust area, and it is the latter variation which is required for present purposes. According to the argument in the next paragraph, the turbine exhaust area must be optimized whereas initial and reheat steam conditions, being less important, can be fixed at representative values.

A method of carrying out economic comparisons between steam turbines is established in ref. 29, and the effect of turbine exhaust area is mentioned on page 364 as being more important than that of the initial and rehat steam conditions. This statement is endorsed by noting, on pages 123 to 130 of ref. 7, that the changes in turbine performance which accrue from changes in initial and reheat steam conditions are only slightly dependent on exhaust pressure. Therefore, it is assumed that optimum values of initial and reheat steam conditions are approximately equal to the fixed design values

of a representative, optimized power-station. The optimum turbine exhaust area, on the other hand, is strongly dependent on exhaust pressure, being governed by the following approximate relationship, presented on page 9 of ref. 7.

$$A_{an} \approx (\text{generator output, MW})/(\text{ave.exh.pressure, in Hg abs}), \quad (2.2\text{-}5)$$

where A_{an} is the optimum turbine exhaust area, in sq.ft.

From the knowledge in the previous paragraph, the variation of station thermal efficiency with exhaust-steam temperature can be estimated by means of the previously described simple method; the only modification required is to calculate the optimum exhaust area from (3.2-5) instead of treating it as a fixed quantity. The purpose of the following calculations is to carry out the simple method by evaluating quantities which will enable η_{TH} and P_u/q to be numerically evaluated from (2.2-4) and (2.2-3a) respectively.

The first step is to choose a turbine type, generator output and operating conditions which are representative of present practice. Important quantities chosen here are tabulated in the final section of this chapter, the purpose of which is to summarize the main results of the present chapter.

According to page 5 of ref. 11, turbines first commissioned in 1964 for the U.K. have an output of about 350 MW, initial steam conditions of 2400 psig and 1050 deg F with one reheat to 1050 deg F, a design station nett thermal

efficiency of 38.9 per cent and a station capital cost per unit generator output power of 35 pounds sterling per kW. In addition, the turbine type is assumed to be cross-compound, double-flow, 3600/1800 rpm with last-stage bucket lengths of 43 in and an exhaust area of 247.6 sq ft, since this type has the desirable feature of low turbine nett heat rates at 350 MW; this can be seen from pages 175 to 180 of ref. 7. A final feed-water temperature of 470 deg F, 7 feed-heaters and other representative minor assumptions, described on page 153 onwards of ref. 7, are adopted. An exhaust pressure of 1.5 in Hg is chosen since the resultant optimum exhaust area value of 233 sq ft, ensuing from (2.2-5), is in fair agreement with the originally specified value of 247.6 sq ft.

By making use of the foregoing values in following the procedure for evaluating turbine internal heat rate, as outlined on page 188 of ref. 7, the following table can be constructed.

net turbine heat rate, (Btu/kWh)	7480
h.p. and l.p. mechanical losses, kW	820
	850
h.p. and l.p. generator losses, kW	1730
	1810
throttle flow, lb/h	2.11×10^6
boiler feed-pump power, kW	8680
condenser flow, lb/h	1.44×10^6
exhaust loss, Btu/lb	12.0
corrected exhaust loss, Btu/lb	9.33
corrected exhaust loss, kW	3942
turbine internal heat rate, (Btu/kWh)	7120

Then, the turbine internal heat rate having been calculated under typical conditions, variations in turbine performance with changes in exhaust-steam temperature can be evaluated according to the procedure set out in the following table. The sources of information required to do this are given on pages 188 to 190 of ref. 7.

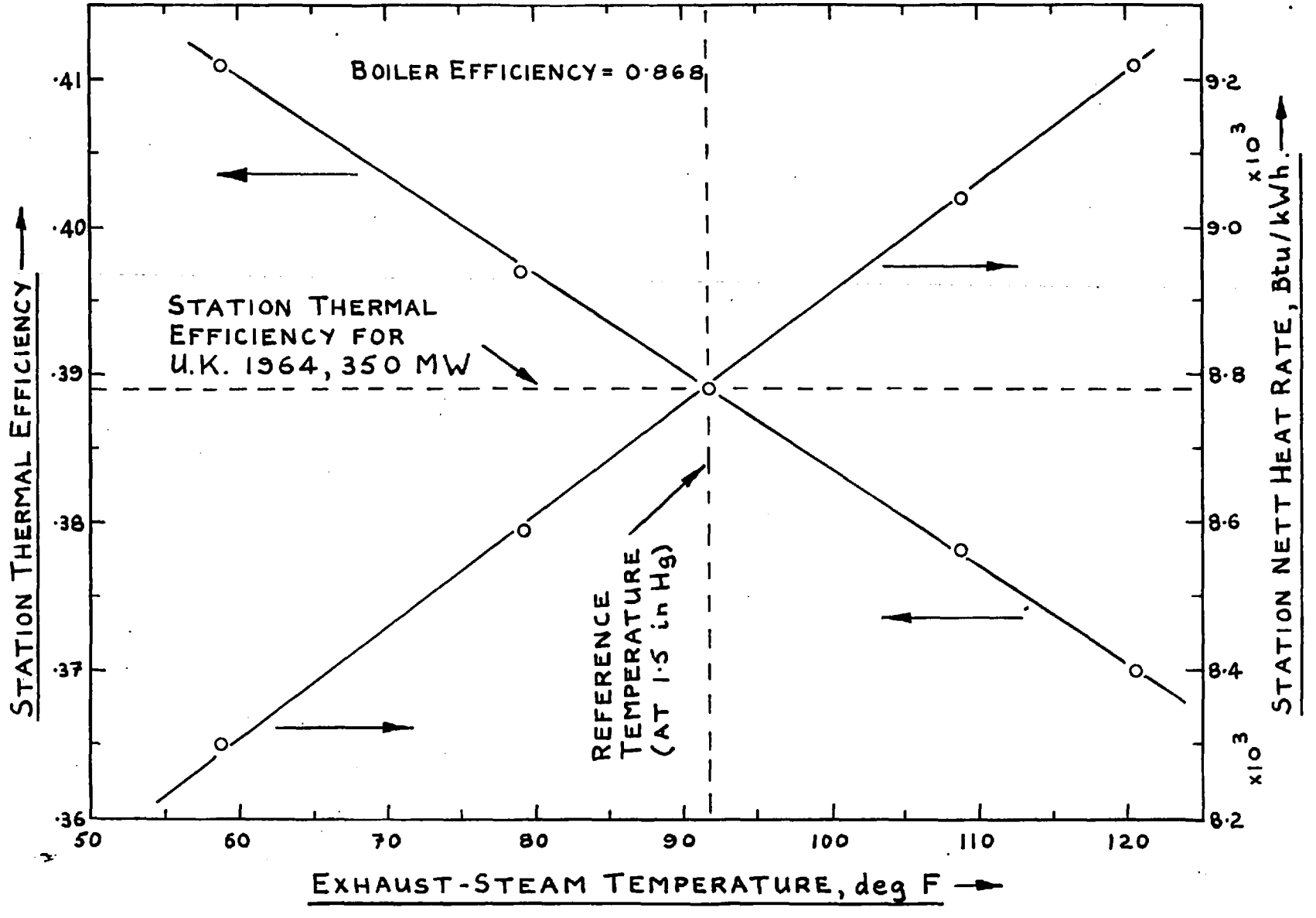
turbine exhaust pressure, in. Hg		0.5	1.0	1.5	2.5	3.5
exhaust-steam temperature, deg F		58.7	79.2	91.8	108.8	120.6
turbine internal heat rate, (Btu/kWh)		6760	6976	7120	7302	7424
optimum exhaust area, sq ft.		700	350	233	140	100
throttle flow, lb/h	$\times 10^6$	2.03	2.08	2.11	2.17	2.30
condenser flow, lb/h	$\times 10^6$	1.35	1.41	1.44	1.51	1.62
specific volume of steam (cub ft/lb)		1250	648	445	275	200
vapour end quality, fraction		0.869	0.893	0.904	0.921	0.931
exhaust velocity, ft/sec		579	645	692	760	840
exhaust loss, Btu/lb		8.5	10.6	12.0	14.7	18.1
corrected exhaust loss, Btu/lb		6.35	8.06	9.33	11.6	14.5
corrected exhaust loss, kW		2502	3323	3942	5157	6886
total mechanical losses, kW		1670	1670	1670	1670	1670
total generator losses, kW		3540	3540	3540	3540	3540
generator output, kW	$\times 10^6$	3.5	3.5	3.5	3.5	3.5
internal output power, kW	$\times 10^6$	3.577	3.585	3.592	3.604	3.621
turbine gross heat rate, (Btu/kWh)		6916	7153	7313	7526	7688
station nett heat rate, (Btu/kWh) (assuming $\eta_B=0.868, P_{u,B}=0.04P_u$)		8300	8590	8780	9040	9220
station thermal efficiency, fraction		0.411	0.397	0.389	0.378	0.370

In evaluating station nett heat rate, a numerical value of boiler efficiency must be adopted. The calculation of such a value is complicated and varies with the type of fuel used. A typical boiler efficiency value can, however, be deduced by stipulating that the calculated station thermal efficiency, at the representative exhaust pressure of 1.5 in Hg, should be equal to the value specified for U.K. power-stations commissioned in 1964, viz. 0.389. This simple approach is only permissible if the resultant boiler efficiency value, obtained from the tabled quantities for the adopted power-station characteristics, is representative of recently commissioned power-stations.

The foregoing stipulation is satisfied for a boiler efficiency of 0.868. This value lies between the previously mentioned limits of 0.85 and 0.90, and is also close to the modern boiler efficiency values given on page 542 of ref. 12, e.g. 0.88 for coal with a gross calorific value of 10,000 Btu/lb. Consequently, the simple approach is regarded as being satisfactory and a boiler efficiency of 0.868 is accordingly adopted for numerical calculations in the present investigation.

The tabled results are used to produce the curves shown overleaf. Both station nett thermal efficiency and station nett heat rate are seen to exhibit an almost linear relationship with exhaust-steam temperature.

Having reached this stage, numerical values of P_u/q can be calculated from (2.2-3a), with $\eta_B = 0.868$, a power ratio


BOILER EFFICIENCY = 0·868
STATION THERMAL EFFICIENCY FOR U.K. 1964, 350 MW
REFERENCE TEMPERATURE (AT 1·5 in Hg)
STATION THERMAL EFFICIENCY
STATION NETT HEAT RATE, Btu/kWh.
EXHAUST-STEAM TEMPERATURE, deg F
·41
·40
·39
·38
·37
·36
9·2
9·0
8·8
8·6
8·4
8·2
x10³
x10³
50
60
70
80
90
100
110
120

of 0.04 and a η_{TH} value read from the accompanying graph. For example, exhaust-steam temperatures of 90 deg F and 110 deg F result in the following respective values of P_u/q:

$$P_u/q = 2.62 \times 10^{-4}, \qquad (2.2\text{-}6)$$

$$P_u/q = 2.79 \times 10^{-4} \qquad (2.2\text{-}7)$$

2.2.3 Evaluation of $(\Delta P_u/P_u)$

According to ref. 23, the decrease in generator output due to the temperature difference $(t_{s,S} - t_{G,i})$ is related to station nett heat rate by the expression

$$\Delta P_u/P_u = \{(NHR)_{t_{s,S}} - (NHR)_{t_{G,i}}\}/(NHR)_{t_{G,i}} \qquad (2.2\text{-}8)$$

$(NHR)_{t_{s,S}}$ is the station nett heat rate, in Btu/kWh, and calculated at exhaust steam temperature, $t_{s,S}$, whereas $(NHR)_{t_{G,i}}$ is calculated at atmospheric dry-bulb temperature, $t_{G,i}$.

The graph presented in 2.2.2 can be used to provide station nett heat rate values for use in (2.2-8), e.g. if $t_{G,i} = 57$, exhaust temperatures of 90 and 110 deg F result in the following respective values:

$$\Delta P_u/P_u = (8760 - 8264)/8264 = 0.060, \qquad (2.2\text{-}9)$$

$$\Delta P_u/P_u = (9060 - 8264)/8264 = 0.0963 \qquad (2.2\text{-}10)$$

2.2.4 Evaluation of $£_{cap}/P_u$

As mentioned before, the capital cost of peak power is equal to the sum of the overall power-station capital cost

per unit generator output and a present worth value, currently equal to 18 pounds sterling per kW generated. Values of the former quantity are contained in published literature, e.g. for U.K. power stations commissioned in 1964 and with 350 MW generators, a value of 35 pounds sterling per kW generated is given on page 5 of ref. 11. Furthermore, it is recommended in the same reference that this value, valid when the Wet-system is employed for cooling purposes, be increased by 4 when dry cooling systems are employed. Therefore, $£_{cap}/P_u$ takes on the following corresponding values:

$$£_{cap}/P_u = 53, \qquad (2.2\text{-}11)$$

$$£_{cap}/P_u = 57. \qquad (2.2\text{-}11a)$$

2.2.5 Evaluation of $(\Delta £_{fuel}/P_u)$

The first step in evaluating this quantity is to determine the annual price of fuel from an expression of the form

$$(£_{fuel}/P_u)_{annual} = (\text{price of fuel per unit gross calorific value, pounds sterling per Btu}) \times (\text{load factor, fraction}) \times (\text{hours/year}) \times (\text{station nett heat rate, Btu/kWh}). \qquad (2.2\text{-}12)$$

The present worth of the fuel price for the useful life of the power-station can then be calculated from

$$(£_{fuel}/P_u)_{present\ worth} = (£_{fuel}/P_u)_{annual} \{(I+1)^{n_y} - 1\} / \{I\,(I+1)^{n_y}\}, \qquad (2.2\text{-}13)$$

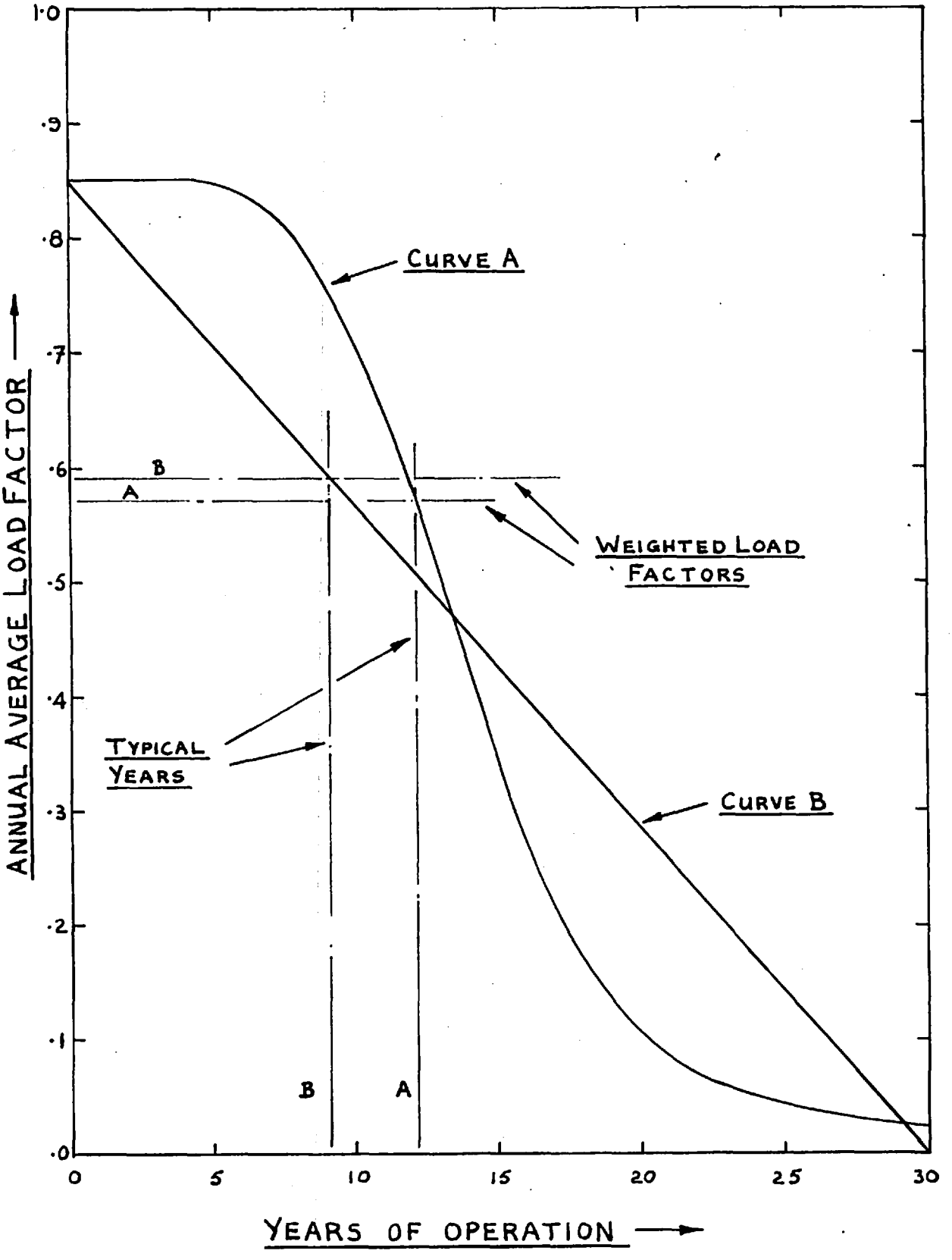
where I is the interest rate, expressed as a fraction, and

n_y is the assumed power-station life, in years. According to ref. 10, (2.2-13) is used by the Central Electricity Generating Board together with $n_y = 30$ and a present interest rate of 0.075 due to financial obligations arising out of the April 1961 Government White Paper.

In assigning a numerical value to load factor, account must be taken of the fact that it will decrease with age of a power station. Bartlett, on page 249 of ref. 7 recommends doing this by employing a curve, indicating the variation of load factor with time, in order to estimate a weighted load factor corresponding to a typical year in the life of a power station. The required load factor, weighted for present worth, is obtained by reading annual load factors from the curve and substituting them into the following equation:

$$\text{Weighted load factor} = \left\{ \sum_{n=1}^{n_y} (\text{annual load factor})(1+I)^{-n} \right\} / \sum_{n=1}^{n_y} (1+I)^{-n} . \qquad (2.2\text{-}13a)$$

The provision of a suitable load factor vs time curve is a difficult task involving predictions into the future. The best approach is to assume some representative curves based on past experience and to determine whether the shape of curve significantly affects the value of weighted load factor. For the representative curves A and B shown overleaf, given on page 249 of ref. 7 and page 41 of ref. 3 respectively, the weighted load factor takes on corresponding values of


1·0
·9
·8
·7
·6
·5
·4
·3
·2
·1
·0
ANNUAL AVERAGE LOAD FACTOR
CURVE A
B
A
WEIGHTED LOAD FACTORS
TYPICAL YEARS
CURVE B
B
A
0
5
10
15
20
25
30
YEARS OF OPERATION

0.59 and 0.57. The closeness of these two values indicates that the value of weighted load factor is not greatly dependent on shape of curve. However, the initial and final values of annual average load factor will greatly affect the weighted load-factor value. For the purposes of numerical calculations in the present investigation, curve A will be chosen together with its associated weighted load-factor value of 0.59.

If the price of fuel per unit gross calorific value is known, (2.2-12) and the station nett heat rate curve in 2.2.2 can be used in obtaining numerical values of $(£_{fuel}/P_u)_{present\,worth}$ from (2.2-13), for any exhaust steam temperature. In particular, $(\Delta £_{fuel}/P_u)$ is the difference between the two present worth values computed at the exhaust-steam temperature, $t_{s,S}$, and the atmospheric dry-bulb temperature, $t_{G,i}$. Unfortunately, the price of fuel per unit gross calorific value varies according to type of fuel, station locality, etc., thus making it difficult to assign a fixed value to it. However, use can be made of the predicted U.K. national average prices of coal delivered to power stations, as presented on page 3894 of ref. 18. These are 1.57×10^{-7} and 1.75×10^{-7} pounds sterling per Btu for the years 1965 and 1980 respectively. On page 250 of ref. 7, Bartlett recommends that the typical-year fuel price be substituted into (2.2-12). For curve A, the twelfth year of operation is the typical year and this corresponds to 1976 for the adopted commissioning year of 1964. If it is reasonably assumed that coal cost increases linearly

with time, the typical year fuel price is equal to 1.7×10^{-7} pounds sterling per Btu.

With the representative values in the previous paragraphs and an atmospheric dry-bulb temperature of 57 deg F, values of $(\Delta £_{fuel}/P_u)$ for exhaust-steam temperatures of 90 and 110 deg F are respectively calculated from (2.2-13) as

$$(\Delta £_{fuel}/P_u) = 5.15, \qquad (2.2\text{-}14)$$

$$(\Delta £_{fuel}/P_u) = 8.27. \qquad (2.2\text{-}15)$$

2.2.6 Representative numerical values of a^*

These values are calculated from (2.2-2) and (2.2-1) for $t_{G,i} = 57$ and two exhaust-steam temperatures, viz. 90 and 110 deg F, to give the following values:

$$a^* = (0.0536 + 0.0868)/33 = 0.00425, \qquad (2.2\text{-}16)$$

$$a^* = (0.0916 + 0.1485)/53 = 0.00453, \qquad (2.2\text{-}16a)$$

$$a^* = (0.0536 + 0.0808)/33 = 0.00407, \qquad (2.2\text{-}17)$$

$$a^* = (0.0916 + 0.1380)/53 = 0.00433. \qquad (2.2\text{-}17a)$$

(2.2-16) and (2.2-16a) refer to wet cooling towers at 90 and 110 deg F whereas (2.2-17) and (2.2-17a) refer to dry cooling towers at 90 and 110 deg F. a^* is seen to be relatively insensitive to variations in exhaust-steam temperature. This insensitivity is advantageous since it enables a^* to be given a fixed value, thereby resulting in simpler differential forms of (2.1-8) for use in optimization procedures. In fact, an error of 5 deg F in guessing the optimum exhaust-steam

temperature will not affect the value of a^* by more than 2 per cent.

A few comparisons with published data can be made in order to check the validity of the foregoing numerical values of a^*. On page 684 of ref. 35, Weinlich derives a formula involving the rate of change of output power of a given turbine with exhaust-steam temperature. This formula, claimed to be valid for normal vacuum ranges encountered in steam turbines, can be presented in the form

$$J_K^{-1} (P_u/q)(\Delta P_u/P_u)/(t_{s,S} - t_{G,i}) = 0.001783\ \eta_{st}, \qquad (2.2\text{-}18)$$

where η_{st} is the steam turbine efficiency, expressed as a fraction. The expression on the left hand side is seen to be the first term in (2.2-1), after the substitution of (2.2-2). In order to evaluate it from (2.2-18), a turbine efficiency value must be obtained. The adopted turbine in the present investigation, being approximately comparable to the turbine investigated on page 618 of ref. 8, will have a similar efficiency, viz. 0.85. With this value, the left hand side expression in (2.2-18) is equal to 0.00152. If, however, the numerical values obtained in the previous subsections of this chapter are substituted into it, the value of the expression is 0.00162, for an exhaust-steam temperature of 90 deg F; this temperature is regarded as lying within normal vacuum ranges. The reasonably close agreement between these two values serves as an indication that the method derived in the present investigation gives feasible

values of the first term in (2.2-1). It is emphasized here that the method of Weinlich is conceptually the less accurate of the two because it applies to a fixed turbine, and there is no attempt to predict the change in optimum exhaust area with vacuum.

A check on the variation of the station nett heat rate with exhaust-steam temperature, as predicted by the curve appearing in 2.2.2, can be made by comparing it with typical published values for fixed turbines operating over a range of exhaust pressures. The curve in 2.2.2 represents a variation of 1.73 per cent per deg F, using 1.5 in Hg as the base exhaust pressure. According to data on page 109 of ref. 33, the variation is approximately 1.5 per cent per deg F for large exhaust areas whereas page 168 of ref. 9 specifies a variation of 2.0 per cent per deg F. It can thus be concluded that the variation predicted in the present investigation seems to be reasonable, even though the values are not directly comparable because the published figures apply to fixed turbines.

Of course, another rough check on the validity of the adopted method of estimating a^* is to compare the optimum exhaust-steam temperatures accruing from its use either with those existing in practice or those predicted for the future.

2.3 Evaluation of capital price coefficient, c^*

As can be seen from (2.1-4),

$$c^* = (P_L/q)_3/(\pounds_{cs}/q), \qquad (2.3\text{-}1)$$

where $(P_L/q)_3$ is the loss of useful generator output power per unit waste-heat rejection rate, due to the capital expenditure involved in providing cooling system components. This power loss can be determined by relating the actual capital cost of peak power to that potentially obtainable were the cooling system to cost nothing. The former quantity, already defined in 2.2.1, can be represented as

$$£_{cap}/P_u = (£_{rest}/P_u) + (£_{cs}/P_u) \qquad (2.3\text{-}2)$$

whereas the latter quantity is given by

$$£_{cap}/(P_u)_{pot} = £_{rest}/P_u. \qquad (2.3\text{-}3)$$

$£_{cap}$ is the sum of overall power-station capital cost and the present worth of the management, insurance and other works costs and the capital cost of the transmission connection, in pounds sterling.

$£_{rest}$ is equal to $£_{cap}$ minus the capital cost of cooling system components, in pounds sterling.

$£_{cs}$ is the capital cost of cooling system components, in pounds sterling.

P_u and $(P_u)_{pot}$ are the respective actual and potential generator outputs, in kW.

All of the above quantities refer to one cooling tower.

(2.3-2) and (2.3-3) can be combined and manipulated algebraically to form the expression

$$J_K \{ (P_u)_{pot} - P_u \}/q = J_K(£_{cs}/q)(£_{cap}/£_{rest})(£_{cap}/P_u)^{-1}. \qquad (2.3\text{-}4)$$

Since the left hand side of this equation is an algebraic representation of the definition of $(P_L/q)_3$ in (2.3-1), (2.3-4) can be substituted into (2.3-1) to give

$$c^* = J_K \, (£_{cap}/£_{rest})(£_{cap}/P_u)^{-1} \qquad (2.3\text{-}5)$$

In evaluating c^* from (2.3-5), use can be made of the $(£_{cap}/P_u)$ values already presented in 2.2.4. The ratio $(£_{cap}/£_{rest})$ will be close to unity since the capital price of the cooling system components represents a small fraction of the total station capital price. In order to assign numerical values to this price ratio, itemized capital prices of the components comprising optimum or near-optimum power-station designs are required. A table containing such prices is given on page 65 of ref. 34 and the foregoing price ratio is seen to have the values

$$(£_{cap}/£_{rest}) = 1.06 \qquad (2.3\text{-}6)$$

and

$$(£_{cap}/£_{rest}) = 1.15 \qquad (2.3\text{-}7)$$

for wet- and dry-systems respectively. These values should be regarded as estimates only and should be checked after optimizing the merit-factor. This can be done by calculating $£_{cs}$, subtracting this from the $£_{cap}$ value - obtainable according to the procedure outlined in 2.2.4 - to get $£_{rest}$; then the $(£_{cap}/£_{rest})$ value determined from these values must be compared with those given by (2.3-6) and (2.3-7) to see whether the latter pair of values are sufficiently accurate.

With the values given both in this section and in 2.2.4, c^* is given as

$$c^* = 3413 \times 1.06/53 = 68.2, \qquad (2.3\text{-}8)$$

$$c^* = 3413 \times 1.15/57 = 68.9. \qquad (2.3\text{-}9)$$

for wet and dry cooling towers respectively.

2.4 Evaluation of pumping power coefficient, b^*

As can be seen from (2.1-3) and (2.1-7),

$$b^* = (P_L/q)_2/(P/q) + c^*(\pounds_{pm}/q)/(P/q), \qquad (2.4\text{-}1)$$

where $(P_L/q)_2$ is the loss of useful generator output per unit waste-heat rejection rate, due to the necessity of driving the intermediary heat-transfer fluid round the cooling system. This power loss is equal to that supplied by the generator to the pump motor and is accordingly related to the required pumping power by the expression

$$(P_L/q)_2 \; / \; (P/q) = 1/\eta_{pm}. \qquad (2.4\text{-}2)$$

η_{pm} is the efficiency of the pump-motor combination, expressed as a fraction. According to ref. 16, modern pumps for cooling system purposes have efficiencies of approximately 0.85. This value, in conjunction with a representative motor efficiency of 0.98, results in

$$(P_L/q)_2 \; / \; (P/q) = 1.2. \qquad (2.4\text{-}3)$$

The validity of the foregoing procedure, resulting in (2.4-3), can be checked by comparing it with the method of determining pumping costs outlined on page 168 of ref. 9.

In this reference, auxiliary power is said to be charged "at the rate earned per kW". This implies that power-station auxiliary power is equal to a loss of useful generator output as supplied to consumers and therefore constitutes an approach identical to that adopted in the present investigation.

An accurate relationship between required pumping power and combined price of the pump-motor combination is difficult to ascertain because the pump price, unlike the motor price, is dependent on volume flow rate as well as power. However, the combined price will be approximately proportional to required power if the motor price is considerably greater than that of the pump. This situation is, in fact, nearly achieved since the motor and pump prices are given respectively as 12,000 and 8,000 pounds sterling, in ref. 7, for the Heller-system pumping power requirement of 1480 horsepower specified in ref. 2. In consequence, the second term in (2.4-1) can be approximately evaluated according to the expression

$$c^* (£_{pm}/q)/(P/q) = 20000\ c^*/(1840 \times 2544) = 0.00531\ c^*. \qquad (2.4\text{-}4)$$

The substitution of (2.4-3) and (2.4-4) into (2.4-1) results in

$$b^* = 1.2 + 0.00531\ c^*. \qquad (2.4\text{-}5)$$

If the typical c^* values in (2.3-8) and (2.3-9) are substituted into (2.4-5), b^* takes on the values

$$b^* = 1.2 + 0.362 = 1.562 \qquad (2.4\text{-}6)$$

and

$$b^* = 1.2 + 0.366 = 1.566 \qquad (2.4\text{-}7)$$

for wet and dry cooling towers respectively. Since the second term in these equations is relatively small, any error caused by assuming the combined pump-motor price proportional to required pumping power will have a small effect on the value of b^*.

2.5 Summary

This section has been included in order to summarize the main points arising out of this chapter. In addition, important quantities chosen or calculated in the previous sections are presented in tabular form so that they can be easily referred to at later stages in the thesis. Where applicable, the validity and probable variation of some of these quantities are discussed.

In evaluating the temperature difference coefficient, a^*, in the merit-factor equation, it was necessary to predict the variation of station nett heat rate with exhaust-steam temperature. In order to do this, choices of numerical characteristics considered to be typical of present-day steam-turbines and power stations were made as follows:

Country:	United Kingdom
commissioning year:	1964
initial steam temperature, deg F:	1050

initial steam pressure, psig:	2400
single reheat temperature, deg F:	1050
generator output, kW:	350,000
turbine type:	cross-compound, double-flow, 3600/1800 rpm.
final feed-water temperature, deg F:	470
last stage bucket lengths, inches:	43
number of feed heaters:	7
boiler efficiency, fraction:	0.868
ratio of all auxiliary power requirements to generator output:	0.04
station nett thermal efficiency at 1.5 in Hg exhaust pressure, fraction:	0.389
other minor features:	pg.153 onwards, ref. 5
station capital cost per unit generator output, pounds sterling per kW:	35 (wet system) 39 (dry systems)
present worth value additional to previous item, pounds sterling per kW:	18
interest rate, fraction:	0.075
power station life, years:	30
price of delivered coal, pounds sterling per Btu:	1.57×10^{-7} (1965) 1.75×10^{-7} (1980)

In addition to the above information the following typical operating conditions were adopted:

atmospheric pressure, psf:	2117 (14.7 psia)
atmospheric dry-bulb temperature, deg F:	57
atmospheric wet-bulb temperature, deg F:	52
load-factor curve A shown in 2.2.5.	

From the above values and the tables in 2.2.2, the following quantities were calculated for wet- and dry-systems and exhaust-steam temperatures of 90 and 110 deg F:

weighted load factor fraction:	0.59
typical year is the twelfth year of operation.	
typical-year delivered-coal price, pounds sterling per Btu:	1.7×10^{-7}
P_u/q, kWh/Btu:	0.000262 (90 deg F)
	0.000279 (110 deg F)
$\Delta P_u/P_u$:	0.0600 (90 deg F)
	0.0963 (110 deg F)
$£_{cap}/P_u$, pounds sterling per kW:	53 (wet system)
	57 (dry system)
$\Delta £_{fuel}/P_u$, pounds sterling per kW:	5.15 (90 deg F)
	8.27 (110 deg F)
a^*, per deg F (wet system):	0.00425 (90 deg F)
	0.00453 (110 deg F)
a^*, per deg F (dry system):	0.00407 (90 deg F)
	0.00433 (110 deg F)

The tabulated values of a^* can be used to show that a^* increases with exhaust-steam temperature by 0.33 and 0.32 per cent per deg F for wet- and dry-systems respectively,

using a^* at 90 deg F as the base value. The a^* values for the wet system are equal to 1.045 times the dry-system values.

Some comments regarding probable variations of a^* from the foregoing values would be useful at this stage. Examination of the constituent terms of a^* in (2.2-2) and (2.2-1), and the equations from which these terms are evaluated, reveals that a^* is given by

$$a^* = J_K \{ \Delta(NHR)/(t_{s,S}-t_{G,i}) \} \; (P_u/q) \{ 1/(NHR)_{t_{G,i}} + (\text{coefficient})/(\pounds_{cap}/P_u) \} . \qquad (2.5\text{-}1)$$

$$\Delta(NHR) \equiv (NHR)_{t_{s,S}} - (NHR)_{t_{G,i}} .$$

coefficient $\equiv$ (price of fuel per unit gross calorific value) x (weighted load factor) x (hours/year) $\times \{ (I+1)^{n_y} - 1 \} / \{ I(I+1)^{n_y} \}$.

The ratio $\Delta(NHR)/(t_{s,S} - t_{G,i})$ is a constant due to the linear relationship between station nett heat rate and exhaust steam temperature, as shown by the curve in 2.2.2. For a constant value of the previously defined coefficient, a^* will increase in value as turbine units get larger. This is deduced from the table on page 5 of ref.11 in which station thermal efficiency and the reciprocal of station capital cost per kW generated - and hence P_u/q, $1/(NHR)_{t_{G,i}}$ and $1/(\pounds_{cap}/P_u)$ - are seen to increase with increase in turbine capacity. a^* will also increase with increase in value of the coefficient.

Due chiefly to the dependence of this coefficient on initial and final load factor and interest rate - these being quantities subject to considerable variation in value - it is impossible to specify tolerances (for a^* values) which are sufficiently small to be of practical use.

In evaluating c^* from (2.3-5) the following quantities have been adopted:

	Wet system	Dry systems
$£_{cap}/£_{rest}$:	1.06	1.15
$£_{cap}/P_u$, pounds sterling per kW:	53	57
c^*, (Btu/h) per pound sterling:	68.2	68.9

c^* will increase with increase in turbine capacity due to the dependence of $£_{cap}/P_u$ on turbine capacity as indicated in the table on page 5 of ref.11. However, the fact that this dependency is relatively slight, combined with the knowledge that $£_{cap}/£_{rest}$ is almost constant, means that c^* is subject to much smaller variations in value than a^*.

In evaluating b^* from (2.4-1) the following quantities have been adopted:

pump efficiency:	0.85
motor efficiency:	0.98
$(P_L/q)_2/(P/q)$:	1.2
$£_{pm}/P$, pounds sterling per (Btu/h):	0.00531
b^*:	1.562 (Wet system) 1.566 (Dry systems)

b^* will increase with increase in turbine capacity due to the dependence of c^* on turbine capacity. The increase will, however, be very slight because the term involving c^* in (2.4-1) is relatively unimportant. The fact that any errors in evaluating this term have little effect on the value of b^* means that the adopted values of b^* are probably accurate and, moreover, not subject to as much variation as c^* or a^*.

3 PRESENTATION OF MERIT-FACTOR AS A FUNCTION OF COOLING SYSTEM DESIGN VARIABLES.

3.1 Introduction

Now that a basis for comparing cooling systems has been established, the temperature difference, pumping power and capital price terms in (2.1-8) must be expressed as functions of the chosen design variables. In choosing design variables the following decisions have to be made.

i) Which cooling system characteristics are to be fixed, either because they are considered to have a small effect on the merit-factor, or because of other reasons, e.g. being forced to use standardized items of equipment supplied by manufacturers?

ii) Which are the characteristics whose values contribute significantly to the value of the merit-factor and, therefore, should be regarded as variables in order to study the effects of changes in their values?

A preliminary answer to the first question is given in the next section, viz. 3.2. Then, a first attempt at answering the second question is made in section 3.3 by arbitrarily choosing quantities which seem suitable for adoption as design variables. The individual cooling system requirements in the merit-factor equation (2.1-8) are then separately expressed as functions of the preliminary design variables, for each of the considered cooling systems, in sections 3.4, 3.5 and 3.6 respectively. Finally, the resultant expressions are examined in 3.7 with the following objects:

i) to see whether there are any further quantities which can be fixed for comparison purposes, and

ii) to make decisions regarding the final choice of design variables.

3.2 Preliminary choice of fixed characteristics

The former question has already been partially answered by virtue of the fact that many of the cooling system components have been chosen in chapter one. It will be even more fully answered at a later stage in the thesis when decisions regarding the deferred choices of cooling system components, including their dimensions and materials of construction, are made. However, there are still a few additional cooling system characteristics which are chosen to be fixed and they are outlined in the following paragraph, together with reasons for choosing them where necessary.

The lengths of the ducts coupling the steam condenser to the cooling tower are considered fixed and equal for each of the considered cooling systems. The Wet-system packing depth is assumed fixed since manufacturers construct packing sheets according to standardized dimensions. The diameters and thicknesses of steam-condenser tubes are also assumed to be fixed because of standardization by tube manufacturers. However, care should be taken, when assigning a numerical value to tube diameter, that the length of tube required does not fall outside the range covered by the adopted condenser price formulae. The temperature variation of the intermediary

heat-transfer medium, during circulation round the cooling system, is considered sufficiently small for physical properties to be calculated at a single temperature which is close to the mean operating temperature of the cooling system concerned. The vapour quality of turbine exhaust steam, necessary for evaluating heat-transfer coefficients as is seen in (D.3-4), is fixed since it is relatively invariant as shown in the table appearing in 2.2.2.

3.3 Preliminary choice of design variables

Having made a preliminary choice of fixed characteristics, the next step is to decide on the design variables governing the cooling systems under consideration. Design variables possess the property of being able to be varied independently of one another; a designer is thus free to choose any one of an infinite number of combinations of design variables in order to specify a cooling system, provided that the ranges of validity of associated performance formulae are not exceeded.

Difficulties in satisfying this provision can be experienced if the design variables are so poorly chosen that their acceptable values vary greatly with changes in operating conditions such as waste-heat rejection rate; a designer might then have to make many guesses before finding a satisfactory combination. To avoid such difficulties, use can be made of another property of design variables, i.e. a set of design variables specifying a cooling system can be trans-

formed to another set, serving the same purpose, merely by multiplying or dividing any of the original design variables by one or more of the others, in any desired combination. In consequence, this property can be used in the present investigation to specify design variables as ratios and/or products of certain quantities so that they will be insensitive to changes in operating conditions. For example, the ratio of tower throat diameter to tower base diameter would be a better design variable than tower throat diameter alone, as its acceptable values will be less sensitive to changes in operating conditions such as waste-heat rejection rate.

With the foregoing knowledge, a first attempt at choosing design variables is made by giving the following tentative list of the remaining unfixed quantities required to specify the considered cooling systems:

i) numbers of steam-condenser and heat-exchanger tubes;

ii) lengths of steam-condenser and heat-exchanger tubes;

iii) velocities of intermediary heat-transfer medium and air through the various cooling system components;

iv) diameters of ducts connecting steam condenser to cooling tower;

v) linear cooling tower dimensions;

vi) temperature of intermediary heat-transfer medium on entering tower heat exchanger or packing;

vii) temperature variation of intermediary heat-transfer medium;

viii) temperature variationof air flowing through the cooling tower;

ix) vapour quality of Spalding-system refrigerant at exit of steam-condenser;

x) number of rows of heat-exchanger tubes in direction of air flow;

xi) blanking factor for Wet system;

xii) mass flow rate of steam.

These items were deduced after formulating and examining equations from which the temperature difference, pumping power and capital cost terms in the merit-factor equation, (2.1-8), can be evaluated. The equations are not presented at this stage as they will be derived in the next sections as functions of more suitable design variables, obtained by modifying some of the foregoing tentative ones as follows:-

Of the tentative design variables, those appearing in i), ii), iv), v) and xii) will vary appreciably with changes in waste-heat rejection rate and should thus be replaced by less sensitive quantities where possible. There appear to be no convenient modifications which will decrease the sensitivity of the duct diameters in iv) and the tower base diameter in v). However, with regard to items i) and ii), there are three insensitive quantities which can be used to replace numbers and lengths of tubes and, for the sake of convenience, velocities as well. These quantities are C,

Q and W.

C ($\equiv q/mc_f$) is a heat-capacity factor with units of deg F.

Q ($\equiv q/A_h$) is heat-flux with units of Btu/(h sq ft).

W ($\equiv mc_f/A_c$) is a loading-factor with the same units as a heat-transfer coefficient, viz. Btu/(h sq ft deg F).

q is the waste-heat rejection rate, in Btu/h .

m is mass flow rate, in lb/h .

c_f is constant-pressure specific heat of liquid, in Btu/(lb deg F).

A_h is total heat-transfer surface area, in sq ft.

A_c is total minimum free-flow area, i.e. total cross-sectional area of tubes, in sq ft.

These three quantities are seen to be valid replacements for numbers and lengths of tubes, and also velocities of fluid flow, by examining the relationships formed from the foregoing definitions and the continuity equation for fluid flow, i.e.

$$n = 4q/(\pi d^2 CW), \qquad (3.3\text{-}1)$$

$$L = dCW/(4Q), \qquad (3.3\text{-}2)$$

$$S = vW/c_f. \qquad (3.3\text{-}3)$$

n is number of tubes.

L is length of tubes, in ft.

S is velocity of fluid flow, in ft/h .

d is fixed inner diameter of tubes, in ft.

v is specific volume of fluid, in cub ft/lb.

C, Q and W virtually suggested themselves as replacement design variables since similar quantities have been found convenient for use in comparision studies by other investigators, e.g. Kayes and London in ref. 22. In conducting the present investigation, some other alternatives were also considered but increased familiarity with the equations involved in the present investigation confirmed that C, Q and W are more suitable.

Since the remaining linear cooling tower dimensions in item v), viz. axial length of tower shell, entrance height, packing-base height and throat diameter - all of which are given in feet - are sensitive to changes in waste-heat rejection rate, they should be replaced by less sensitive design variables. This is easily accomplished by respectively dividing each of them by the tower base diameter, D_1, to give $r_5 - r_1$ ($\equiv z_5/D_1 - z_1/D_1$), r_1 ($\equiv z_1/D_1$), r_p ($\equiv z_p/D_1$) and s ($\equiv D_t/D_1$).

At this stage, the tentative design variables having been modified where possible, the important question of whether they really are independent of each other must be answered. In other words, are there any supplementary conditions to be satisfied which might relate one or more of the tentative list of design variables to any of the others? An appraisal of the considered cooling systems reveals that there are, in fact, four such supplementary conditions, viz. three heat-balance equations governing the transfer of waste-heat from exhaust steam, from intermediary heat-transfer medium and to atmos-

pheric air respectively. Finally, there is the buoyancy equation which stipulates that the draught-induced forces must of necessity be equal to the air-flow resistance through a natural-draught cooling tower.

It is here that further advantages offered by using the heat-capacity C as a replacement design variable are seen. For example, the heat-balance equations

$$q = m_S X_S h_{fg,S}, \tag{3.3-4}$$

$$q = m_F c_{f,F} \Delta t_F, \tag{3.3-5}$$

$$q = m_G c_G \Delta t_G, \tag{3.3-6}$$

can be used to replace the mass flow rate of steam in xii) and the temperature differences Δt_F and Δt_G in vii) and viii) by C_S, C_F and C_G respectively, where

$$C_s \equiv q/(m_S c_{f,S}), \tag{3.3-7}$$

$$C_F \equiv q/(m_F c_{f,F}), \tag{3.3-8}$$

$$C_G \equiv q/(m_G c_G), \tag{3.3-9}$$

Also the buoyancy equation can be used to replace $(r_5 - r_1)$ as a design variable as will be shown in the last section of this chapter.

By using the above replacements when expressing the merit-factor as a function of the design variables, unacceptable duplication of design variables is easily avoided.

The temperature of the water entering the packing, indicated in item vii), is a convenient design variable for the Wet system, due to the form of the adopted empirical packing

performance equations. However, for the Heller- and Spalding-system it is better to replace this temperature by the partial number of transfer units, referred to the air-side of the tower heat exchanger, viz.

$$n_{hG} \equiv u_G A_{h,G}/(m_G c_G) \qquad (3.3\text{-}10)$$

which, on making use of (3.3-9) and the previously given Q definition, becomes

$$n_{h,G} = u_G C_G/Q_G. \qquad (3.3\text{-}11)$$

$n_{h,G}$ has been used by many investigators to present thermodynamic performances of various heat-exchanger flow arrangements, thereby avoiding the calculation of log mean temperatures which were a feature of older methods of presentation. It possesses the advantages of being both dimensionless and relatively insensitive to changes in waste-heat rejection rate.

Now that modifications and replacements for the tentatively chosen design variables have been suggested, the next step is to derive equations from which the cooling system requirements in the merit-factor equation, (2.1-8), can be evaluated and, furthermore, to present these equations as functions of the modified design variables. This is done in the three following sections for the temperature difference, pumping power and capital-price terms respectively. Then, the resultant equations are inspected in the final section of this chapter with a view to making final recommendations regarding design variables and fixed characteristics.

3.4 Derivation of temperature difference term in merit-factor equation

This section is separated into three sub-sections which are reserved for the Wet-, Heller- and Spalding-systems respectively. For each system a basic equation is presented. The constituent terms of this equation are then expressed as functions of design variables. A final equation is then formed by grouping together all the newly obtained expressions.

3.4.1 Wet system

The temperature difference term in (2.1-8) can be subdivided into a number of constituents as follows:

$$t_{s,S} - t_{G,i} = (t_{s,S} - t_{H,o}) + (t_{H,o} - t_{C,i}) + (t_{C,i} - t_{G,i}) \quad (3.4\text{-}1)$$

An expression for $(t_{s,S} - t_{H,o})$ can be derived as follows:

$$\begin{aligned} t_{s,S} - t_{H,o} &= (t_{H,o} - t_{H,i})(1/E_H - 1) \\ &= (N_{h,H}\, Q_H/U_H)(1/E_H - 1) \\ &= (N_{h,H} Q_H/U_H)(1/N_{h,H} - 0.5 + 0.0833\, N_{h,H}) \\ &= Q_H/U_H - 0.5C_H + 0.0833\, U_H C_H^2/Q_H. \end{aligned} \quad (3.4\text{-}2)$$

(3.4-2) is arrived at by using the information in E.3 and incorporating both the equations in E.4 and the following definition for overall number of transfer units:

$$N_{h,H} \equiv U_H C_H/Q_H. \quad (3.4\text{-}3)$$

The subscript H refers to the region inside the steam-condenser tubes, viz. the Hot-side. U_H is the overall heat-transfer coefficient in the steam condenser, referred to the Hot-side.

Negligible heat loss to the atmosphere from the ducts connecting the steam condenser to the cooling tower can be reasonably assumed. Therefore, $t_{H,o} - t_{C,i} = 0.$ (3.4-4)

The substitution of (3.4-2) and (3.4-4) into (3.4-1) results in the final equation

$$\boxed{t_{s,S}-t_{G,i} = (Q_H/U_H-0.5C_H+0.0833U_HC_H^2/Q_H)+(t_{C,i}-t_{G,i}).} \quad (3.4-5)$$

U_H can be evaluated from the expression

$$1/U_H = (1/0.85)\{1/u_H+1/u_{m,H}+(Q_S/Q_H)/u_S\} . \quad (3.4-6)$$

The denominator 0.85 is an assumed cleanliness-factor, the value of which is recommended in ref.13 and on page 90 of ref. 24. The inverse heat-transfer coefficients for the Hot-side, condenser-tube wall and Steam-side can be respectively calculated from

$$1/u_H = a_{u,H}^{-1} W_H^{-0.8} \quad (3.4-7)$$

where

$$a_{u,H} = 0.0184k_{f,H}(\mu_{f,H}C_{f,H})^{-0.8}(N_{Pr,H})_f^{-0.4} d_H^{-0.2},$$

$$1/u_{m,H} = a_{m,H}^{-1} \quad (3.4-8)$$

where

$$a_{m,H} \equiv 2k_{m,H}/\{d_H(\beta_H - 1)\},$$

$$1/u_S = a_{u,S}^{-1}(C_H W_H)^{-0.0833}\ (C_S/Q_H)^{-0.333} \qquad (3.4\text{-}9)$$

and

$$a_{u,S} \equiv 0.647k_{f,S}\{g_o c_{f,S}(Q_H/Q_S)/(X_S \mu_{f,S} v_{f,S}^2 \beta_H d_H^{0.5}\ q^{0.25})\}^{\frac{1}{3}}.$$

(3.4-7) and (3.4-9) are standard equations presented in D.1 and D.3 respectively. Their ranges of validity are also presented in these sections and, in expressing (D.1-1) as (3.4-7), the following Reynolds number definition is incorporated:

$$(N_{Re,H})_f \equiv d_H W_H/(\mu_{f,H} c_{f,H}). \qquad (3.4\text{-}10)$$

3.4.2 Heller system

The temperature difference term in (2.1-8) can be subdivided into a number of constituents as follows:

$$t_{s,S} - t_{G,i} = (t_{s,S} - t_{H,o}) + (t_{H,o} - t_{C,i}) + \{t_{C,i} - (t_{m,G})_o\} + \{(t_{m,G})_o - t_{G,i}\}. \qquad (3.4\text{-}11)$$

$(t_{m,G})_o$ is the air-side tube-wall temperature at air-flow exit from the tower heat exchanger.

The jet steam-condenser temperature approach, $(t_{s,S} - t_{H,o})$, is considered to be a constant in the present investigation. However, it is difficult to assign a numerical value to it as there is no published information for jet condensers of the size associated with power-station cooling systems. According to Cheshire and Daltry on page

160 of ref.9, it should be close to zero since the use of a jet condenser eliminates "the temperature drop associated with the use of tubes in a conventional condenser". For counterflow barometric jet condensers, in which the temperature approach is probably comparable with the vacuum type considered here, Kern states on page 397 of ref.24 that the temperature approach is taken as 5 deg F. For the jet condenser employed in the geothermal power station at Wairakei in New Zealand, described in ref.17, the temperature approach is 11 deg F according to a private communication from the consulting engineers responsible for its design and construction (ref.43). In this reference it is also suggested that a temperature approach of 7 deg F represents a value which can be reasonably expected under practical conditions. Due to the uncertainty displayed by the foregoing figures, a range of constant temperature approach values will be adopted when carrying out numerical calculations at later stages in the thesis.

As in the Wet-system, the assumption of negligible heat loss from the connecting ducts results in

$$t_{H,o} - t_{C,i} = \dot{0}. \qquad (3.4\text{-}12)$$

The temperature difference between water at entry to the tower heat exchanger and tube outside-surface temperature at the same point can be evaluated from the following standard heat-transfer formula:

$$t_{C,i} - (t_{m,G})_o = (Q_C/Q_G)(1/u_C + 1/u_{m,C})Q_G \qquad (3.4\text{-}13)$$

where $Q_C \equiv q/A_{h,C}$, $Q_G \equiv q/A_{h,G}$.

The last temperature difference term in (3.4-11) can be expressed in terms of air-side heat-capacity factor and partial effectiveness as

$$(t_{m,G})_o - t_{G,i} = C_G/e_G \ , \tag{3.4-14}$$

since

$$C_G \equiv q/m_G c_G , \tag{3.4-15}$$

$$q = m_G c_G (t_{G,o} - t_{G,i}) , \tag{3.4-16}$$

$$e_G \equiv (t_{G,o} - t_{G,i})/\{(t_{m,G})_o - t_{G,i}\} . \tag{3.4-17}$$

The substitution of (3.4-12), (3.4-13) and (3.4-14) into (3.4-11) results in the final equation

$$\boxed{t_{s,S} - t_{G,i} = (t_{s,S} - t_{H,o}) + (Q_C/Q_G)(1/u_C + u_{m,C}^{-1})Q_G + C_G/e_G .} \tag{3.4-18}$$

The inverse heat-transfer coefficients for the Cold-side and heat-exchanger tube wall can be respectively calculated from

$$1/u_C = a_{u,C}^{-1} \ W_C^{-0.8} \tag{3.4-19}$$

where

$$a_{u,C} \equiv 0.0.184 k_{f,C} (\mu_{f,C} c_{f,C})^{-0.8} (N_{Pr,C})_f^{0.4} \ d_C^{-0.2} ,$$

$$1/u_{m,C} = a_{m,C}^{-1} \tag{3.4-20}$$

and

$$a_{m,C} \equiv 2k_{m,C} \ / \ \{d_C(\beta_C - 1)\} .$$

(3.4-19) is a standard equation taken from D.1 and is of the same form as that for the Hot-side.

Depending on whether one or two Cold-side passes are eventually adopted, C_G/e_G can be respectively calculated from the following equations:

$$C_G/e_G = 1.02\ C_G[2/n_{h,G}+0.5\ \{(C_C/C_G)\ +1\}]\ , \qquad (3.4\text{-}21)$$

$$C_G/e_G = 1.02\ C_G\{(1/n_{h,G}+0.5+0.0625n_{h,G})+(0.5-0.125n_{h,G})$$

$$(C_C/C_G)\ +0.0625n_{h,G}(C_C/C_G)^2\ \}\ . \qquad (3.4\text{-}22)$$

These equations are approximations to those originally derived in section E.2, the approximations being deemed necessary to simplify optimization studies. In arriving at (3.4-21) and (3.4-22) the same procedure as used in deriving (E.2-5) and (E.2-6) was adopted, except that the small, last term in (E.2-4) was omitted. The multiplying factor of 1.02 has been introduced to compensate for the omission, with the result that (3.4-21) and (3.4-22) are accurate to within 0.5 per cent for expected Heller-system values of $n_{h,G} \approx 1$ and $C_C/C_G \approx 0.5$.

3.4.3 Spalding system

The adopted temperature differences making up the total temperature term in (2.1-8) are:

$$t_{s,S}-t_{G,i} = \{t_{s,S}-(t_{m,H})_o\} + \{(t_{m,H})_o-(t_{s,H})_o\}$$
$$+ \{(t_{s,H})_o- t_{C,i}\} + \{t_{C,i} - (t_{m,G})_o\}$$
$$+ \{(t_{m,G})_o- t_{G,i}\}\ . \qquad (3.4\text{-}23)$$

The temperature drop from exhaust steam to tube outside wall temperature at refrigerant exit from steam condenser can be calculated from the following standard formula:

$$t_{s,S} - (t_{m,H})_o = Q_H \{1/u_{m,H} + (Q_S/Q_H)/u_S\} \qquad (3.4\text{-}24)$$

where $Q_H \equiv q/A_{h,H}$, $Q_S \equiv q/A_{h,S}$.

The derivation of the temperature approach between tube outside wall and refrigerant, $\{(t_{m,H})_o - (t_{s,H})_o\}$, is fully outlined in section E.5. There are three possible equations, viz. (E.5-4), (E5-6) and (E.5-8) from which this quantity may be evaluated. Decisions as to which one is most suitable can only be made in conjunction with the choice of refrigerant, and this has been deferred until after the completion of this chapter. It is to be noted, however, that the variation of saturation pressure on the Hot-side, viz. $\Delta t_{s,H}$, where

$$\Delta t_{s,H} \equiv (t_{s,H})_o - (t_{s,H})_i \ , \qquad (3.4\text{-}25)$$

appears in each of these equations. Before the foregoing temperature approach can be evaluated, a method of determining $\Delta t_{s,H}$ must be found. In order to do so, use is made of the relationship between refrigerant pressure and temperature under saturation conditions. Numerical calculations based on the physical property equations in appendix A indicate that the relationship is approximately linear between 90 and 110 deg F, i.e. over the temperature range in which practical refrigerant temperatures are considered to lie. Moreover, the linear equation governing the relationship is

$$t_{s,H} = a_{t,H} p_{s,H} , \qquad (3.4\text{-}26)$$

where $a_{t,H}$ is equal to 0.00207, 0.00379, 0.00238 and 0.00410 for ammonia, freon 12, freon 22 and methyl chloride respectively. (3.4-26) can be used in relating $\Delta t_{s,H}$ to the total Hot-side pressure drop according to the expression

$$\Delta t_{s,H} = a_{t,H}\{\Delta p_H + (a_{grav}/g_o) z_R / v_{f,H}\} . \qquad (3.4\text{-}27)$$

Δp_H is the sum of the Hot-side frictional and accelerative pressure drops whereas z_R is the effective difference in height between the Hot-side inlet and outlet.

Now, returning once again to the evaluation of temperature differences in (3.4-23),

$$(t_{s,H})_o - t_{C,i} = 0 \qquad (3.4\text{-}28)$$

due to the assumption of negligible heat loss from the connecting ducts to the atmosphere.

The fourth and fifth temperature difference terms in (3.4-23), like the Heller-system, can be calculated from the equations

$$t_{C,i} - (t_{m,G})_o = (Q_C/Q_G)(1/u_C + 1/u_{m,C})Q_G , \qquad (3.4\text{-}29)$$

$$(t_{m,G})_o - t_{G,i} = C_G/e_G . \qquad (3.4\text{-}30)$$

The substitution of (3.4-24), (3.4-25), (3.4-29) and (3.4-30) into (3.4-23) results in the final equation

$$t_{s,S} - t_{G,i} = Q_H\{1/u_{m,H} + (Q_S/Q_H)/u_S\} + \{(t_{m,H})_o - (t_{s,H})_o\} + (Q_C/Q_G)(1/u_C + 1/u_{m,C})Q_G + C_G/e_G . \qquad (3.4\text{-}31)$$

The inverse heat-transfer coefficients for the condenser-tube wall and steam-side are evaluated in the same way as for the Wet system, viz. from (3.4-8) and (3.4-9). As mentioned previously, $(t_{m,H})_o - (t_{s,H})_o$ is calculated from E.5 and (3.4-27). (3.4-20) is suitable for the calculation of the inverse heat-transfer coefficient, $1/u_{m,C}$, for the heat-exchanger tube wall.

In order to achieve a high condensing heat-transfer coefficient on the Cold-side, the refrigerant vapour is assumed to enter the tower heat exchanger at a high velocity. An applicable formula, with its associated ranges of validity, is derived in D.2 and can be re-expressed as

$$1/u_C = (1/a_{u,C})W_C^{-0.932} , \qquad (3.4\text{-}32)$$

where

$$a_{u,C} \equiv 0.00238 \{k_{f,C}/(\mu_{f,C}c_{f,C})\}\{d_C/(\mu_{g,C}c_{f,C})\}^{-0.0683} \{v_{g,C}(N_{Pr,C})_f/v_{f,C}\}^{0.5} .$$

A large design safety factor has been imposed on this formula since the investigators responsible for the correlation - apparently the only one in existence - upon which the formula is based, emphasise that their correlation is only preliminary in nature.

An equation derived in E.3 is suitable for use in evaluating C_G/e_G according to the expression

$$C_G/e_G = C_G(1/n_{h,G} + 0.5 + 0.0833\, n_{h,G}). \qquad (3.4\text{-}33)$$

3.5 Derivation of pumping-power term in merit-factor equation

As in 3.4, this section is separated into three sub-sections which are reserved for the Wet-, Heller- and Spalding-systems respectively. The pumping-power term, i.e. the required pumping power per unit waste-heat rejection rate, is expressed as a function of the various pressure drops occurring in each cooling-system circuit. Then, equations are derived in order to express the pressure drops as functions of design variables.

3.5.1 Wet system

In evaluating the total pressure drop it is assumed that:

i) the flow is incompressible;

ii) the diameters, lengths and fluid velocities relating to the two ducts connecting the surface steam condenser to the cooling tower are identical;

iii) with the exception of the head required to pump water up to the height of the spray, i.e. z_3, potential head effects may be ignored.

In consequence, the total pressure drop is given by

$$\Sigma\Delta p = \Delta p_{fr,H} + 2\Delta p_{fr,T} + (a_{grav}/g_o)z_3/v_{f,T} \quad . \qquad (3.5\text{-}1)$$

Since the circulating pump is situated on the B-side of the cooling system, the pumping-power term in the merit-factor equation is given as follows:

$$P/q = a_p \Sigma \Delta p \, m_B v_{f,B}/(Jq),$$

$$\therefore P/q = (a_{p,B}/C_B)\{\Delta p_{fr,H} + 2\Delta p_{fr,T} + (a_{grav}/g_o) z_3/v_{f,T}\}, \quad (3.5\text{-}2)$$

where $a_{p,B} \equiv a_p v_{f,B}/(J c_{f,B})$ and $C_B \equiv m_B c_{f,B}/q$.

In deducing (3.5-2) it is assumed that the extra pumping power required due to the losses incurred by bends, entrances and exits can be taken into account by introducing the constant multiplying factor, a_p.

The Hot-side frictional pressure drop is given by

$$\Delta p_{fr,H} = a_{p,H} f_{f,H} C_H Q_H^{-1} W_H^3 \quad (3.5\text{-}3)$$

where

$$a_{p,H} \equiv v_{f,H}/(2 g_o c_{f,H}^2),$$

after substituting (3.3-2) and (3.3-3) into the standard frictional pressure drop equation, (C.1-1). $f_{f,H}$ can be evaluated accurately or approximately from (B.2-1) and (B.2-4) respectively, provided that the indicated ranges of validity are not exceeded. (B2-1) and (B.2-4) can be respectively presented as

$$f_{f,H}^{-0.5} = -1.7372 \ln\{0.27027(\varepsilon_H/d_H) + 1.255/(N_{Re,H} f_{f,H}^{0.5})\}, \quad (3.5\text{-}4)$$

where

$$N_{Re,H} \equiv d_H W_H/(\mu_{f,H} c_{f,H})$$

and

$$f_{f,H} \approx a_{f,H} W_H^{-0.224}, \quad (3.5\text{-}5)$$

where

$$a_{f,H} \equiv 0.06 \{d_H/(\mu_{f,H} c_{f,H})\}^{-0.224}.$$

The T-side frictional pressure drop is given by

$$\Delta p_{fr,T} = a_{p,T} f_{f,T} C_T^{-2} d_T^{-5} \qquad (3.5\text{-}6)$$

where

$$a_{p,T} \equiv 3.24\ v_{f,T} L_T q^2/(g_o c_{f,T}^2),$$

after substituting for W_T and S_T in (C.1-1) from (3.3-1) and (3.3-3) respectively, noting that $n_T = 1$. $f_{f,T}$, like $f_{f,H}$, can be evaluated accurately from (B.2-1). However, the relatively high value of d_T is expected to result in values of $N_{Re,T}$ sufficiently high to warrant the use of the simplified equation, (B.2-2), viz.

$$f_{f,T} \approx [-1.7372 \ln \{0.27027 (\epsilon_T/d_T)\}]^{-2}, \qquad (3.5\text{-}7)$$

subject to the indicated range of validity.

3.5.2 Heller system

For this cooling system, the assumptions i) and ii), enumerated in 3.5.1 for the Wet system, are also applicable. In evaluating the total pressure drop, account must be taken of the extra water-turbine pressure drop, Δp_x, which is imposed in order to ensure that the cooling-system circuit will always be above atmospheric pressure. Most of the energy lost due to this pressure drop is recovered by utilising the water turbine to assist in driving the circulating pump. In consequence, the effective total pressure drop is given by

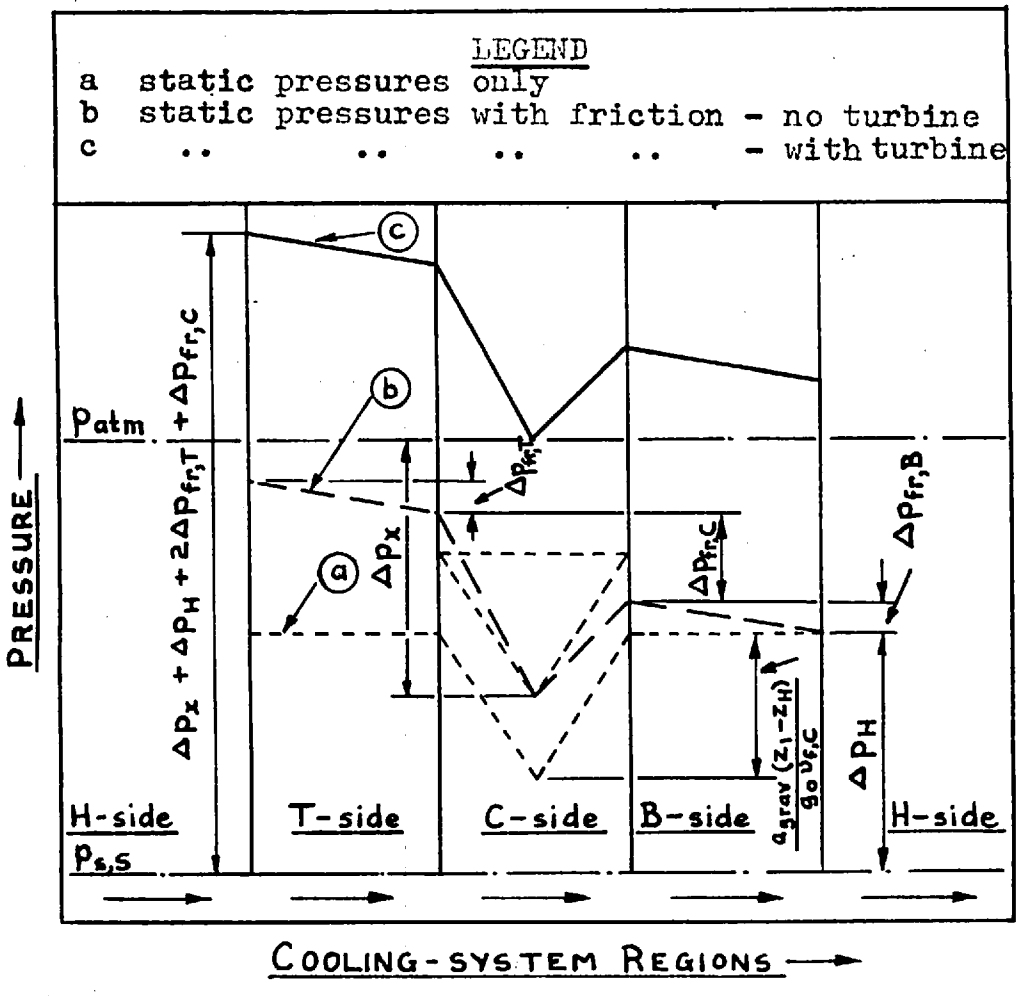
$$\Sigma \Delta p = (1-\eta_{pt})\ \Delta p_x + \Delta p_H + 2\Delta p_{fr,T} + \Delta p_{fr,C}. \qquad (3.5\text{-}8)$$

η_{pt} is the efficiency of the pump-turbine combination and Δp_H is the total Hot-side pressure drop, due both to frictional losses and the energy lost in creating a suitable spray for direct-contact heat transfer in the jet steam condenser.

With reference to the accompanying diagram, Δp_x may be represented as

$$\Delta p_x = p_{atm} - \{p_{s,S} + \Delta p_H + \Delta p_{fr,B} + 0.5\Delta p_{fr,C} - (a_{grav}/g_o)(z_1 - z_H)/v_{f,C}\}, \quad (3.5\text{-}9)$$

where z_H is the effective height of jet-condenser water level above ground, z_1 is the cooling tower entrance height and $p_{s,S}$ is the pressure in the jet condenser.



The substitution of (3.5-9) into (3.5-8) results in

$$\Sigma\Delta p = \eta_{pt}\Delta p_H + (1-\eta_{pt})\left[p_{atm} - p_{s,S} + \{a_{grav}/(g_o v_{f,C})\}(z_1 - z_H)\right] + (1+\eta_{pt})(\Delta p_{fr,T} + 0.5\,\Delta p_{fr,C}), \qquad (3.5\text{-}10)$$

noting that the T- and B-side frictional pressure drops are identical.

Since the circulating pump is situated on the T-side of the cooling system, the pumping-power term in the merit factor equation is given as follows:-

$$P/q = a_p \Sigma\Delta p m_T v_{f,T}/(Jq),$$

$$\therefore\ P/q = a_{P,T} C_G^{-1} (C_T/C_G)^{-1}\, \Sigma\Delta p \qquad (3.5\text{-}11)$$

where $a_{P,B} \equiv a_p v_{f,T}/(J c_{f,T})$, $C_G \equiv q/(m_G c_G)$, $C_T \equiv q/(m_T c_{f,T})$ and a_p is a multiplying factor accounting for additional pumping power due to bends, exits and entrances.

Equations for the evaluation of the individual terms in (3.5-10) are developed as follows:-

An accurate evaluation of the Hot-side pressure drop is difficult to achieve because of the absence of published information relating to the performance of jets or sprayers in jet condensers. However, for the jet condenser employed in the geothermal power station at Wairakei in New Zealand, described in ref.17, the pressure drop across the jets is about 500 p.s.f. (8ft.head) according to a private communication from the consulting engineers responsible for its design and construction (ref.43). In this reference it is also suggested that this value could go up to 624 p.s.f. (10 ft.

head). These values are in close agreement with those given in another private communication (ref.41), in which a pressure drop of 500 or 562 p.s.f. (8 or 9 ft. head) is suggested as being a reasonable practical value. Because these figures are reasonably close to each other it is considered that a value close to the midpoint between the two extremes will be suitable for adoption in the present investigation, viz.

$$\Delta p_H = 576. \qquad (3.5\text{-}12)$$

The efficiency of the pump-turbine combination is given by the product of the separate pump and turbine efficiencies. Each of these are quoted in ref.42 as being about 0.85 for the Heller- system cooling plant in operation at Rugeley. These figures are confirmed by ref.43 as being practically attainable, with the proviso that the optimum design conditions must be accurately predicted or the separate efficiences can fall to as low as 0.70, or less. For numerical calculations in the present work, it is assumed that efficiences of 0.85 are feasible, and therefore,

$$\eta_{pt} = 0.85 \times 0.85 = 0.72. \qquad (3.5\text{-}13)$$

$p_{s,S}$ may be evaluated at a constant representative exhaust-steam temperature of 100 deg F, say, without significant loss of accuracy since it represents a small fraction of the squre-bracketed terms in (3.5-10).

The evaluation of the T-side frictional pressure drop is identical to that for the Wet-system; equation (3.5-6)

is therefore applicable. It can be re-expressed as

$$\Delta p_{fr,T} = a_{p,T} f_{f,T} C_G^{-2} (C_T/C_G)^{-2} d_T^{-5}, \qquad (3.5\text{-}14)$$

where $a_{p,T}$ and $f_{f,T}$ are presented in the text following (3.5-6).

The Cold-side frictional pressure drop can be evaluated from an equation identical in form to (3.5-3). The equation can be conveniently expressed as

$$\Delta p_{fr,C} = a_{p,C} f_{f,C} C_G (C_C/C_G) Q_G^{-1} W_C^{3} \qquad (3.5\text{-}15)$$

where

$$a_{p,C} \equiv v_{f,C} / \{ 2 g_o c_{f,C}^{2} (Q_C/Q_G) \} .$$

In addition, $f_{f,C}$ can be evaluated accurately or approximately from equations (3.5-4) and (3.5-5) respectively, provided that the sub-script H is changed to C.

A single numerical value cannot be assigned to the effective height of jet-condenser water level above ground, since it depends, among other things, on the topography peculiar to a particular site. Its value will, however, have a significant effect on total pressure drop. Therefore, numerical calculations will be carried out for a range of z_H values to see how the merit-factor is affected.

3.5.3 Spalding system

In evaluating total pressure drop for this system the following assumptions are made:-

i) The refrigerant flow is incompressible except for the liquid-vapour mixture undergoing nucleate boiling in the steam-

condenser tubes, i.e. on the Hot-side.

ii) The lengths of the two ducts connecting the steam condenser to the cooling tower are equal.

iii) There is negligible frictional pressure drop in the condenser re-circulation circuit since a large duct can be employed without contributing significantly to capital expenditure.

iv) There is no hydrostatic-head gain due to the downward flow of condensate in the tower heat-exchanger tubes as the tubes will only run full of liquid near the bottom of the tubes, if at all.

The overall pressure drop is given in terms of its constituent pressure drops as

$$\Sigma\Delta p = \Delta p_H + \Delta p_{fr,T} + \Delta p_{fr,B} + \Delta p_{fr,C} , \qquad (3.5\text{-}16)$$

where Δp_H is the sum of the Hot-side friction and acceleration pressure drops.

The pumping-power term in the merit-factor equation is given by the following equations:

$$P/q = a_p \Sigma\Delta p \; m_H \; v_{f,H}/(Jq),$$

$$\therefore P/q = a_{p,H} C_H^{-1} (\Delta p_H + \Delta p_{fr,T} + \Delta p_{fr,B} + \Delta p_{fr,C}), \qquad (3.5\text{-}17)$$

where $a_{p,H} \equiv a_p v_{f,H}/(J c_{f,H})$, $C_H \equiv q/(m_H c_{f,H})$ and a_p is a multiplying factor accounting for additional pumping power due to bends, exits and entrances.

Equations for the evaluation of the individual pressure

drops in (3.5-17) are developed as follows:-

The flow of refrigerant through condenser tubes is considered to fall into the category of forced convection, two-phase, single-component fluid flow, accompanied by nucleate boiling. A suitable equation for calculating Hot-side frictional pressure drop under these conditions is derived in C.3. For values of refrigerant vapour quality at exit from condenser, X_H, less than 0.5, equation (C.3-8) must be used. However, for values of $X_H \leq 0.1$, (C.3-8) can be replaced by the simplified equation (C.3-8a) without significant loss of accuracy. On substitution of (3.3-2), (C.3-8a) becomes

$$\Delta p_{fr,H} = 0.25\, a_{p,H} d_H J_H C_H Q_H^{-1} W_H^{2.776} \qquad (3.5\text{-}18)$$

where

$$J_H \equiv 0.409 b_{p,H}^{2} X_H^{1.45} + 1.16 b_{p,H} X_H^{0.723} + 1,$$

$$a_{p,H} \equiv (0.12 v_{f,H} \mu_{f,H}^{0.224}) / (g_o c_{f,H}^{1.776} d_H^{1.224}),$$

$$b_{p,H} \equiv 3.85\, (v_{g,H}/v_{f,H})^{0.407} (\mu_{g,H}/\mu_{f,H})^{0.0911}.$$

The range of validity of (3.5-18) is given immediately after the presentation of (C.3-8a) in C.3.

The Hot-side acceleration pressure drop is evaluated from the equation recommended on page 699 of ref.27, viz.

$$\Delta p_{a,H} = a_{a,H} X_H W_H^{2} \qquad (3.5\text{-}19)$$

where

$$a_{a,H} \equiv (v_{g,H} - v_{f,H}) / (g_o c_{f,H}^{2}).$$

The equations from which the frictional pressure drops in the connecting ducts may be evaluated are similar to (3.5-6), viz.

$$\Delta p_{fr,T} = a_{p,T} f_{g,T} C_T^{-2} d_T^{-5} \qquad (3.5\text{-}20)$$

where

$$a_{p,T} \equiv 3.24\, v_{g,T} L_T q^2/(g_o c_{f,T}^2),$$

$$\Delta p_{fr,B} = a_{p,B} f_{f,B} C_B^{-2} d_B^{-5} \qquad (3.5\text{-}21)$$

where

$$a_{p,B} \equiv 3.24\, v_{f,B} L_B q^2/(g_o c_{f,B}^2).$$

The friction factors can be accurately evaluated from (B.2-1), but, as in the Wet system, the Reynolds numbers are expected to be sufficiently high to warrant the use of the simplified equation, (B.2-2).

The flow of refrigerant through tower heat-exchanger tubes is considered to fall into the category of forced convection, two-phase, single-component fluid flow, accompanied by condensation. A suitable equation for calculating Cold-side frictional pressure drop under these conditions is (C.2-2), the derivation of which appears in C.2. After incorporating (3.3-2), (C.2-2) becomes

$$\Delta p_{fr,C} = 0.25 d_C (Q_C/Q_G)^{-1} C_C Q_G^{-1} (0.333 a_{p,C} W_C^3 + 0.0833 b_{p,C} W_C^4) \qquad (3.5\text{-}22)$$

where

$$a_{p,C} \equiv (0.012 v_{g,C})/(g_o c_{f,C}^2 d_C),$$

$$b_{p,C} \equiv (0.00025 v_{g,C} v_{f,C} \sigma_W)/(g_o c_{f,C}^3 \sigma_C).$$

The range of validity of (3.5-22) may be obtained by referring to (C.2-2) and (B.3-1). The investigators responsible for the friction factor correlation, which is employed in sections B.3 and C.2 in order to derive (3.5-22), stress that their results are preliminary in nature. However, in the absence of more recent information, (3.5-22) is regarded as the best equation for use in the present investigation.

3.6 Derivation of capital-price term in merit-factor equation

As in the previous sections, this section is separated into three sub-sections which are reserved for the Wet-, Heller- and Spalding-systems respectively. In each case, the capital-price term is expressed in terms of constituent cooling-system component prices. Equations from which these constituent prices can be evaluated are then derived.

3.6.1 Wet system

The capital-price term can be presented as

$$(£_{cs} - £_{pm})/q = £_S/q + 2£_T/q + £_D/q. \qquad (3.6\text{-}1)$$

$£_S$ is the total surface steam-condenser price per cooling tower, in pounds sterling.

$£_T$ is the price of each of the connecting ducts per cooling tower, in pounds sterling.

$£_D$ is the price of one cooling tower and its associated equipment, in pounds sterling.

$(£_{cs}-£_{pm})$ is the total capital price of cooling system com-

ponents per cooling tower, excluding the capital price of the circulating pump-motor combination, in pounds sterling.

q is the waste-heat rejection rate for one cooling tower, in Btu/h.

A method of evaluating surface-steam condenser prices is presented in appendix H. The total price per unit bare tube surface area is assumed to be the sum of two constituent items, viz. a base price and the price of tubes, and it can therefore be represented by

$$\pounds_S'' = \pounds_{S,\text{base}}'' + \pounds_{S,\text{tubes}}''. \qquad (3.6\text{-}2)$$

The fact that two Hot-side passes have already been assumed, together with admiralty-brass tubes of fixed diameter and thickness (0.75 inches o.d. and 16 S.W.G., say), enables (H.1-2) and (H.2-1) to be substituted into (3.6-2) to give

$$\begin{aligned} \pounds_S/q &= \pounds_S'' Q_S^{-1}, \\ &= \pounds_S''(Q_S/Q_H)^{-1} Q_H^{-1}, \\ &= (2.33 + 66700\, q^{-1.3} Q_H^{1.3}) Q_H^{-1}, \end{aligned} \qquad (3.6\text{-}3)$$

where $Q_H \equiv q/A_{h,H}$ and (Q_S/Q_H) is the ratio of inside to outside heat transfer area. For approximate calculations, (3.6-3) may be simplified to

$$\pounds_S/q \approx 2.33\, Q_H^{-1}, \qquad (3.6\text{-}4)$$

which over-estimates $\pounds_S/q$ by not more than 3 per cent, provided that the bare-tube surface area, $A_{h,S}$, is not less than 50,000 sq.ft.

A check on the validity of these two equations can be made by comparing the value of the constant contained in (3.6-4), viz. 2.33, with that obtainable from the published values of bare-tube surface areas and prices of steam-condensers appearing in ref.34. A 1963 condenser price of 196,000 pounds sterling, for a bare-tube surface area of 100,000 sq ft, is arrived at by substracting a circulating pump and motor price of 20,000 pounds sterling, as recommended in ref.42, from the published condenser price which is inclusive of circulating pump and motor. Then, by using the price vs time graph in H.1, together with a Q_H/Q_S value of 1.11 computed from the published tube dimensions, the 1964 value of the constant is given by

$$\begin{aligned}(1964\text{price}/1963\text{price})(\pounds_S/q)Q_H &= 0.995\pounds_S/A_{h,H} \\ &= 0.995(Q_H/Q_S)\pounds_S/A_{h,S} \\ &= 0.995 \times 1.11 \times 196{,}000/100{,}000 \\ &= 2.17.\end{aligned}$$

The fairly close agreement between the values of the two constants indicates that (3.6-3) and (3.6-4) are suitable for the estimation of present-day, surface steam-condenser prices.

(I.1-2) is an equation suitable for the evaluation of connecting-duct prices per unit length and is derived in appendix I. It can be used for estimating the duct prices in (3.6-1) after re-arranging it to give

$$\pounds_T/q = (L_T/q)(0.8d_T^2 - 2.65d_T + 5.3). \qquad (3.6\text{-}5)$$

The range of validity is presented in (I.1-2).

The derivation of a formula for estimating the price of a cooling tower and its associated equipment (inclusive of packing) is outlined in J.3. The formula is given by (J.3-1), i.e.

$$\pounds_D/q = q^{-1}\left[d_o+d_1D_1^{\,2}+d_2tD_1^{\,2}+d_3D_1\{1-(1-t)^{0.5}\}+d_4D_1\right.$$
$$+d_5/D_1^{\,2}+d_6A+d_7A\{r_pD_1+0.5(z_2-z_p)\}$$
$$\left.+d_8tD_1^{\,2}\{r_pD_1+(z_z-z_p)+14.9\}\right] . \qquad (3.6\text{-}6)$$

The sub-scripted d symbols are price coefficients to which numerical values can only be assigned after the best packing has been chosen, and this is done in a later chapter of the thesis. Accurate and approximate formulae for evaluating the cooling tower surface area, A, are derived in J.1 and J.2 respectively.

3.6.2 Heller system

The capital-price term is presented as

$$\boxed{(\pounds_{cs}-\pounds_{pm})/q = \pounds_S/q+2\pounds_T/q+\pounds_G/q+\pounds_D/q+\pounds_{turbine}/q ,} \qquad (3.6\text{-}7)$$

where the individual price items refer respectively to jet steam condenser, connecting ducts, tower heat exchanger, cooling tower and, finally, water turbine.

A suitable price formula for jet condensers is difficult to derive because of their rarity in power-station applications and consequent lack of published information. However, ref. 43 gives a value of

$$£_S/q = 0.000148 \qquad (3.6\text{-}8)$$

for the jet condenser employed in the Wairakei geothermal power station described in ref.17.

A rough check on the validity of this value may be made by assuming that the price of a jet condenser is equal to that of the shell of an equivalent surface condenser, i.e. the so-called base-price. This assumption implies, fairly reasonably, that a jet condenser is a surface condenser without any condenser tubes. An estimate of base-prices for surface steam condensers can then be made from published information as follows:-

For large two-pass surface condensers, (H.1-2) can be employed to give

$$£_{S,base}/q = £''_{S,base} Q_S^{-1}$$
$$\approx 1.4 Q_S^{-1} \qquad (3.6\text{-}9)$$

after ignoring the second term in (H.1-2) as it will be small for large condensers. For two-pass condensers, 0.875 inch o.d. condenser tubes, a turbine exhaust steam vapour quality of 0.91 and an overall cleanliness factor of 0.85, page 20 of ref.13 states that Q_S will be close to the value

$$Q_S = 8480. \qquad (3.6\text{-}10)$$

Since an equivalent surface condenser for the Heller system would operate under clean conditions, a cleanliness factor of 1.0 may be assumed. The foregoing Q_S can therefore be modified by dividing it by 0.85. The substitution of the modified value into (3.6-9) gives a base-price of

$$£_{S,base}/q = 0.000141 \qquad (3.6\text{-}11)$$

which should be fairly close to the price of a jet condenser. Comparison of (3.6-8) and (3.6-11) shows that this is indeed the case. Consequently, (3.6-8) is considered to be suitable for estimating jet condenser prices and is accordingly adopted for use in the present investigation.

The price of connecting ducts is governed by the same formula as that for the Wet system, viz. (3.6-5).

Formulae suitable for the evaluation of tower heat-exchanger prices per unit air-side heat-transfer area, $£_G''$, are derived in appendix K. The total tower heat-exchanger price per unit waste-heat rejection rate can then be obtained from

$$£_G/q = £_G''/Q_G, \qquad (3.6\text{-}12)$$

where $Q_G \equiv q/A_{h,G}$. $£_G''$ is a function of the number of tube rows in the direction of air flow. It is given either by (K.2-7) or (K.3-5), depending on whether the heat exchanger consists of circular tubes with helically wound fins (or wires) or continuous fins. Before assigning numerical values to the terms in (K.2-7) and (K.3-5), a choice is made in a later chapter as to which of a number of tower heat-exchanger types is most suitable for adoption by the Heller system.

A price formula for the cooling tower and its associated equipment is derived in J.4 and given by (J.4-1) as

$$£_D/q = q^{-1}(d_o + d_1 A + d_2 Ar_1 D_1). \qquad (3.6\text{-}13)$$

The values of the coefficients are given in J.4 and the cooling tower surface area can be evaluated accurately or approximately from the equations in J.1 and J.2 respectively.

Since the water-turbine price constitutes a relatively small fraction of the whole capital-price term, a rough estimate of its value will be acceptable for the present investigation. Accordingly a value of

$$£_{turbine}/q = 4.55 \times 10^{-5} \qquad (3.6\text{-}14)$$

will be used on the basis of recommendations outlined in ref.42.

3.6.3 Spalding system

The capital-price term is presented as

$$\boxed{(£_{cs}-£_{pm})/q = £_S/q+£_T/q+£_B/q+£_G/q+£_D/q+£_R/q\ ,} \qquad (3.6\text{-}15)$$

where $£_R/q$ is the total price of refrigerant per unit waste-heat rejection rate. The prices of separator and recirculation ducts are ignored as they are considered to represent a negligible fraction of the total capital price of cooling system components.

The surface steam condenser price is given by (H.1-3), the derivation of which appears in appendix H. After incorporating (H.1-1) in (H.1-3) - since the condenser has been assumed to have one refrigerant or Hot-side pass - and making use of the fact that

$$£_S/q = £_S{}''Q_H^{-1}(Q_H/Q_S),$$

there results

$$£_S/q = (Q_H/Q_S)(1.55+1.68\times10^{10}q^{-2.5}Q_S^{2.5}+£_{S,tubes}'')Q_H^{-1}. \quad (3.6\text{-}16)$$

Numerical values can only be assigned to (Q_H/Q_S) and $£_{S,tubes}''$ after the tube material and dimensions have been decided upon. This is done in conjunction with the choice of refrigerant in a later chapter. For approximate calculations, (3.6-16) can be simplified to

$$£_S/q \approx (Q_H/Q_S)(1.55 + £_{S,tubes}'')\, Q_H^{-1} \quad (3.6\text{-}17)$$

which over-estimates $£_S/q$ by not more than 3 per cent, provided that the bare-tube surface area, $A_{h,S}$, is not less than 60,000 sq ft.

No recent published prices for single-pass steam condensers have come to light in order that the validity of these equations may be checked numerically. Nevertheless, they are considered to be reasonably accurate since the method of derivation and sources of information are identical to those employed in arriving at the two-pass steam condenser equations, the accuracy of which has already been shown to be satisfactory in 3.6.1.

The single equation (I.1-3) may be used in evaluating connecting-duct prices. Therefore

$$£_T/q = (L_T/q)(0.7d_T^2 - 1.3d_T + 3.2) \quad (3.6\text{-}18)$$

and

$$£_B/q = (L_T/q)(0.7d_B^2 - 1.3d_B + 3.2), \quad (3.6\text{-}19)$$

since $L_T \approx L_B$.

The equations for evaluating prices of tower heat

exchanger and cooling tower and its associated equipment are identical to those for the Heller system. Therefore equations (3.6-12) and (3.6-13) and the related comments are applicable to the Spalding system as well.

In deriving a price formula for the refrigerant used in the Spalding system, it is assumed that the steam-condenser and B-side duct are flooded, whereas the tower heat exchanger and T-side duct are assumed to contain vapour. A common temperature of 100 deg F is assumed for the whole coupling circuit in determining the mass of refrigerant required, the subscript F being introduced to denote: "coupling circuit in general". The simplifying assumptions are permissible since the price of refrigerant is a small fraction of the total cooling system capital price. In consequence, the price of refrigerant per unit waste-heat rejection rate is given by

$$f_R/q = 0.25 a_R \left[\left\{ (d_H/v_{f,F}) Q_H^{-1} + (Q_C/Q_G)^{-1} (d_C/v_{g,F}) Q_G^{-1} \right\} + (\pi L_T/q)(d_B^2/v_{f,F} + d_T^2/v_{g,F}) \right], \qquad (3.6\text{-}20)$$

where a_R is the price of refrigerant in pounds sterling per lb. For the four refrigerants under consideration, viz. ammonia, freons 12 and 22, and methyl chloride, a_R is given as

$$a_R = 0.0625,\ 0.125,\ 0.30 \text{ and } 0.075 \qquad (3.6\text{-}21)$$

respectively, according to ref.20.

3.7 Final choices of fixed characteristics and design variables

In this section, the physical quantities appearing in the previously derived equations are examined with a view to eliminating some of them from consideration as design variables. This done, a list of final design variables is presented at the end of the three following sub-sections, each of which is reserved for the Wet-, Heller- and Spalding-systems respectively.

3.7.1 Wet system

After perusal of the equations in 3.4.1, 3.5.1 and 3.6.1, the following quantities are unfixed and appear to be design variables:

Q_H, W_H, C_H, C_S, $t_{C,i}$, C_T, d_T, z_3, C_B, D_1, t, r_5-r_1, s and r_p.

However, a number of these are either dependent on, or approximately equal to, one or more of the others and can therefore be eliminated as design variables as follows:-

It has been previously assumed that the constant-pressure specific heat of intermediary heat-transfer medium is approximately constant throughout the cooling-system circuit. This assumption, combined with the knowledge that the mass flow rate of water is constant, can be used to show that

$$C_H = C_T = C_B = C_F \qquad (3.7\text{-}1)$$

after examining the following definitions:

$C_H \equiv q/(m_H c_{f,H})$, $C_T \equiv q/(m_T c_{f,T})$, $C_B \equiv q/(m_B c_{f,B})$ and $C_F \equiv q/(m_F c_{f,F})$.

In consequence, C_H, C_T and C_B can be replaced by the single design variable, C_F, viz. the heat-capacity factor referring to the coupling circuit as a whole. The heat-balance equation (3.3-5) can be used to show that

$$C_F = \Delta t_F = t_{C,i} - t_{C,o}. \qquad (3.7\text{-}2)$$

C_S is a design variable which can be seen from (3.3-4) and (3.3-7) to be given by

$$C_S = X_S h_{fg,S}/c_{f,S}. \qquad (3.7\text{-}3)$$

Due to the insensitivity of the physical properties to changes in exhaust-steam temperature, and the fact that X_S - the exhaust-steam vapour quality - has been assumed constant, C_S is practically constant. In consequence, C_S is assumed constant for the present investigation and is calculated at the representative values of $t_{s,S} = 100$ and $X_S = 0.91$, using (A2-4) and (A.2-9). Therefore,

$$C_S = 944.0. \qquad (3.7\text{-}4)$$

z_3 may be eliminated as a design variable after assuming that the maximum height of spray is four feet above the top of packing. This value is close to that normally used in practice as indicated on page 955 of ref.32. Therefore,

$$z_3 = r_p D_1 + (z_2 - z_p) + 4. \qquad (3.7\text{-}4a)$$

$(r_5 - r_1)$ can be eliminated as a design variable by introducing the buoyancy equation, one of the supplementary

conditions referred to in 3.3 subsequent to the presentation of (3.3-3). This is done by substituting (F.5-5) into

$$r_5 - r_1 = (r_5 - r_p) - 0.5(z_2 - z_p)/D_1 \qquad (3.7\text{-}5)$$

to get

$$r_5 - r_1 = 1.62\{(D)_{chil}/x\}^2 D_1^{-5}(N_{h,G}^{-1} + 0.5)^3 \Sigma N_v - 0.5(z_2 - z_p)/D_1. \qquad (3.7\text{-}6)$$

(F.5-5) is fully derived in F.5.1 and $(D)_{chil}$ can be evaluated from (F.5-4) after substituting the heat-balance equation

$$q = m_C c_{f,C}(t_{C,i} - t_{C,o})$$

into it to give

$$(D)_{chil}/x = (q/c_{f,C})x^{-1}\{0.5(h_{M,i} + h_{M,o}) - h_{G,i}\}^{1.5}(f)_{chil}^{0.5} . \qquad (3.7\text{-}7)$$

According to Spalding and Singham (ref.4 in appendix F) the value of x - introduced to account for loss in tower performance due to non-uniform air distribution across the packing - is approximately 0.8625. (3.7-7) is a function of the design variables C_F and $t_{C,i}$ as can be seen by examining (E.1-2), (E.1-4) and (3.7-2).

$N_{h,G}$ in (3.7-6) can be seen to be a function of C_F and $t_{C,i}$ after equating the transfer-unit equation (E.1-6) to the following empirical performance equation - for the tower packing - in order to eliminate G_C/G_G:

$$N_{h,G} = N_{h,pack} + N_{h,\ spray} \qquad (3.7\text{-}8)$$

where

$$N_{h,pack} = a_{h,p}\,(z_2 - z_p)/(\text{denominator}), \qquad (3.7\text{-}9)$$

$$\text{denominator} = \{ b_{h,p}(G_C/G_G)^{c_{h,p}} + d_{h,p} \}^{-1}$$

$$+ e_{h,p}(G_C/G_G)^{f_{h,p}} , \qquad (3.7\text{-}10)$$

$$N_{h,spray} = a_{h,s}(G_C/G_G)^{b_{h,s}}. \qquad (3.7\text{-}11)$$

The forms of the above equation have been adopted by Lieberam in ref.25 as they enable a large number of packing types to be classified, the constant values of the subscripted a to f symbols changing from one packing type to the next.

ΣN_v in (3.7-6) is dependent on the design variables and can be evaluated from (F.2-2) and the subsequent equations in F.2.1.

No more elimination of probable design variables is possible and the final choice of Wet-system design variables is presented as follows:

$$Q_H,\ W_H,\ C_F,\ t_{C,i},\ d_T,\ D_1,\ r_p,\ s,\ t.$$

In so far as fixed quantities are concerned, numerical values must be assigned to the as yet undetermined tower price constants and the tower packing heat-transfer and pressure-drop performance constants appearing in (3.6-6), (3.7-9) to (3.7-11) and (F.2-3) to (F.2-4). This can only be done after the best packing type has been chosen in a later chapter. The following table summarizes the numerical values

which have already been adopted for the remaining fixed quantities:

Quantity	value	equation	source
z_3-z_2, ft	4	F.2-3	ref.32
$a_{v,c}$	3.1	F.2-5	ref.32
$a_{v,i}$	.0853	F.2-6	sect.F.2.1
$b_{v,i}$	-2.2	F.2-6	sect.F.2.1
$a_{v,misc}$	2.3	F.2-8	ref.32
z_2-z_p, ft	7.125	3.7-9	arbitrary
d_H, ft ($x10^{-2}$)	5.183	H.2-1	ref.4,app.H
δ_H, ft ($x10^{-3}$)	5.33	H.2-1	ref.4,app.H
$k_{m,H}$, Btu/(hftdegF)	55.6	A.6-2	ref.2,app.A
L_T ($=L_B$), ft	300	3.5-6	arbitrary
X_S	0.91	3.7-4	table,sect.2.2.2
C_S, deg F	944	3.7-4	calculated at 100degF
x	.8625	3.7-7	ref.4,app.F
mean water temperature, degF	80		arbitrary
cleanliness factor	0.85	3.4-6	ref.13 and ref.24
a_p	1.2	3.5-2	arbitrary
ϵ_H/d_H, per ft	.0001	3.5-4	ref.1,app.B
ϵ_T ($=\epsilon_B$)	0.001	3.5-7	ref.1,app.B
$a_{v,s}$	0.16	F.2-2	F.2.1 and ref.25
$b_{v,s}$	1.32	F.2-2	F.2.1 and ref.25

3.7.2 Heller system

After perusal of the equations in 3.4.2, 3.5.2 and 3.6.2, the following quantities are unfixed and appear to be design variables:

Q_G, W_C, C_G, C_C/C_G, $n_{h,G}$, C_T/C_G, d_T, n_r, D_1, r_1, s, r_5-r_1.

However, some of these quantities are dependent on the others and can therefore be eliminated as design variables according to the following procedure:

Q_G can be seen to be a function of C_G, $n_{h,G}$ and r_r by substituting the tower heat-exchanger performance and geometrical relationships presented in appendix F, viz. (F.2-9) and (F.2-14), into the transfer unit definition.

$$n_{h,G} = u_G\, C_G\, Q_G^{-1} \qquad (3.7\text{-}12)$$

so as to eliminate W_G. The end result is

$$Q_G = a_{q,G}\, C_G\, n_{h,G}^{j_1} \qquad (3.7\text{-}13)$$

where

$$a_{q,G} \equiv b_{u,G}^{-j_1} \{v_G/(60c_G)\}^{-(1+j_1)} (n_r A'_{h,G}/l_c)^{-(1+j_1)}$$

and

$$j_1 \equiv 1/B.$$

As in the Wet system, the heat-capacity factors for the intermediary heat-transfer medium are practically equal and can thus be represented by the single factor C_F. Therefore

$$C_F/C_G = C_T/C_G = C_C/C_G \qquad (3.7\text{-}14)$$

where C_F is the heat-capacity factor referring to the coupling circuit as a whole.

After noting that $r_1 = r_p$ for the Heller system, (r_5-r_1) can be eliminated as a design variable by introducing the buoyancy equation (F.5-6) or its less accurate version, (E.5-7), to give

$$r_5-r_1 = \{0.81v_{G,i}/(g_o c_G^2 a_{grav}/g_o)\}\, q^2 \Sigma N_v C_G^{-2} D_1^{-5}/(\rho_{G,i}-\rho_{G,o}) \qquad (3.7\text{-}15)$$

where

$$\rho_{G,i} - \rho_{G,o} = \rho_{G,i}\{1-(t_{G,i}+460)/(C_G+t_{G,i}+460)\}\ ,$$

or

$$r_5-r_1 \approx \{438v_{G,i}^2/(g_o c_G^2 a_{grav}/g_o)\} q^2 \Sigma N_v C_G^{-3} D_1^{-5}\ . \qquad (3.7\text{-}16)$$

An equation for the evaluation of ΣN_v is derived in F.2.2; its final form is given by (F.2-18).

No more elimination of probable design variables is possible and the final choice of Heller-system design variables is presented as follows:

W_C,	C_G,	C_F/C_G,	$n_{h,G}$,	n_r,	d_T,	D_1,	r_1,	s.

In so far as fixed quantities are concerned, numerical values must be assigned to the as yet undetermined constants relating to the tower heat-exchanger price, performance and geometrical characteristics appearing in (3.6-12), (K.2-7), (K.3-5), (3.7-13) and (F.2-18). This can only be done after the best tower heat-exchanger type has been chosen in a later chapter. The following table summarizes the

numerical values which have already been adopted for the remaining fixed quantities:

Quantity	value	equation	source
$a_{v,i}$	0.167	F.2-19	app.F, F.2.2
$b_{v,i}$	0	F.2-19	app.F, F.2.2
$a_{v,misc}$	2.3	F.2-21	app.F, F.2.2
L_T $(=L_B)$, ft	300	3.5-14	arbitrary
mean water temperature, deg F	90		arbitrary
a_p	1.2	3.5-11	arbitrary
ε_T $(=\varepsilon_B)$	0.001	3.5-14	ref.1, app.B
η_{pt}	0.72	3.5-13	refs. 43 and 42
Δp_H, psf	576	3.5-12	refs.43 and 41
p_{atm}, psf	2116	3.5-10	sea level
$p_{s,S}$, psf	137	3.5-10	at 100 deg F
$£_S/q$, pounds sterling per Btu/h $(x10^{-4})$	1.48	3.6-8	ref.43
d_o	36600	3.6-13	app.J, J.4
d_1	0.582	3.6-13	app.J, J.4
d_2 $(x10^{-3})$	1.11	3.6-13	app.J, J.4

3.7.3 Spalding system

After examination of the equations in 3.4.3, 3.5.3 and 3.6.3, the following quantities are unfixed and appear to be design variables:

Q_H, C_H, W_H, C_S, z_H, X_H, Q_G, C_G, W_C, $n_{h,G}$, n_r, C_T, d_T, C_B, d_B, d_R, C_C, D_1, r_1, r_5-r_1, s.

However, some of these quantities are dependent on the others and can therefore be eliminated as design variables according to the following procedure:

Since the mass flow rates of refrigerant outside the steam condenser are equal and the refrigerant temperature is practically constant throughout the coupling circuit, the following relationships can be formed:

$$C_T = C_B = C_C = C_F \qquad (3.7\text{-}17)$$

where C_F is the general heat-capacity factor referring to the whole of the coupling circuit external to the steam condenser. By making use of the definition

$$C_F \equiv q/(m_F c_{f,F})$$

and the heat-balance relationship

$$q = m_F h_{fg,F} ,$$

C_F is seen to be given by

$$C_F = h_{fg,F}/c_{f,F}. \qquad (3.7\text{-}18)$$

Examination of the physical property equations in appendix A reveals that $h_{fg,F}/c_{f,F}$ is relatively insensitive

to variations in temperature and, in consequence, C_F may be regarded as fixed; it can therefore be eliminated as a design variable. A representative coupling circuit temperature of 100 deg F will be used in evaluating C_F. Therefore

$$C_F = 412,\ 232,\ 208 \text{ and } 397 \qquad (3.7\text{-}19)$$

for ammonia, freon 12, freon 22 and methyl chloride respectively.

C_H is not a design variable as it is dependent on C_F and X_H. This can be shown by relating the Hot-side mass flow rate relationship

$$m_H = m_F/X_H$$

to the definitions

$$C_H \equiv q/(m_H c_{f,F}) \text{ and } C_F \equiv q/(m_F c_{f,F})$$

to get

$$C_H = X_H C_F. \qquad (3.7\text{-}20)$$

C_S can be fixed at

$$C_S = 944 \qquad (3.7\text{-}21)$$

and is therefore eliminated as a design variable according to arguments similar to those used for the Wet system in 3.7.1.

z_R is not a design variable as it depends on the number of condenser tubes, and hence on C_F and W_H. This dependency may, however, be ignored with negligible loss of accuracy by

assigning a fixed value to z_R. This course of action is permissible since the value of z_R has only a slight effect on the value of $\{(t_{m,H})_o - (t_{s,H})_o\}$ in (3.4-23), as can be seen from (3.4-25), (E.5-4), (E5-6) and (E5-8). Therefore, the representative value

$$z_R = 15 \qquad (3.7\text{-}22)$$

is chosen for use in the present investigation.

Both Q_G and (r_5-r_1) can be eliminated as design variables, according to the arguments leading to the adoption of equations (3.7-13), (3.7-15) and (3.7-16) in 3.7.2 for the Heller system.

No more elimination of probable design variables is possible and it is concluded that the Spalding-system design variables are:

Q_H,	W_H,	X_H,	d_B,	d_T,	W_C,	C_G,	n_r,	$n_{h,G}$,	D_1,	r_1,	s.

In so far as fixed quantities are concerned, numerical values must be assigned to the as yet undetermined constants relating to the tower heat-exchanger price, performance and geometrical characteristics appearing in (3.6-12), (K.2-7), (K.3-5), (3.7-13) and (F.2-18). In addition, the best refrigerant must be chosen. These tasks are carried out in later chapters. The following table summarizes the numerical values which have already been adopted for the remaining fixed quantities:

Quantity	value	equation	source
$a_{v,i}$	0.167	F.2-19	app.F, F.2.2
$b_{v,i}$	0	F.2-19	app.F, F.2.2
$a_{v,misc}$	2.3	F.2-21	app.F, F.2.2
L_T (=L_B), ft	300	3.5-20	arbitrary
mean refrigerant temperature, deg F	100		arbitrary
a_p	1.2	3.5-17	arbitrary
ϵ_T (=ϵ_B)	0.001	3.5-20	ref.1, app.B
$a_{t,H}$, deg F per psf	see (2.4-26)	3.4-26	app.A
a_R, pounds sterling per lb	see (3.6-21)	3.6-20	ref.20
d_o	36600	3.6-13	app.J, J.4
d_1	0.582	3.6-13	app.J, J.4
d_2 ($\times 10^{-3}$)	1.11	3.6-13	app.J, J.4
d_H, ft ($\times 10^{-2}$)	5.183	H.2-1	ref.4, app.H
δ_H, ft ($\times 10^{-3}$)	5.33	H.2-1	ref.4, app.H
X_S	0.91	3.7-4	table, sect.2.2.2
C_S, deg F	944	3.7-21	calculated at 100 deg F
z_R, ft	15	3.7-22	arbitrary

4 CHOICE OF TOWER PACKING FOR WET SYSTEM AND TOWER HEAT EXCHANGERS FOR HELLER- AND SPALDING-SYSTEMS

4.1 Introduction

Now that a cooling-system merit-factor, and the equations from which it can be evaluated, have been presented, attention can be paid to choosing the best packing and heat exchangers for use in natural-draught cooling towers. The obvious way of ascertaining the best of a number of types would be to predict optimum design conditions for each of the possible types and then to calculate the optimum value of the adopted merit-factor from (2.1-8). The most suitable type of packing and/or heat exchanger will be that one which results in the smallest optimum merit-factor value. However, the prediction of optimum design conditions for all of the possible types involves so much labour that the obvious way is impractical.

A more fruitful approach would be to estimate the merits of the various types of packing and heat exchanger by means of simplified methods which are, nevertheless, sufficiently accurate to eliminate most of the possible types from further consideration.

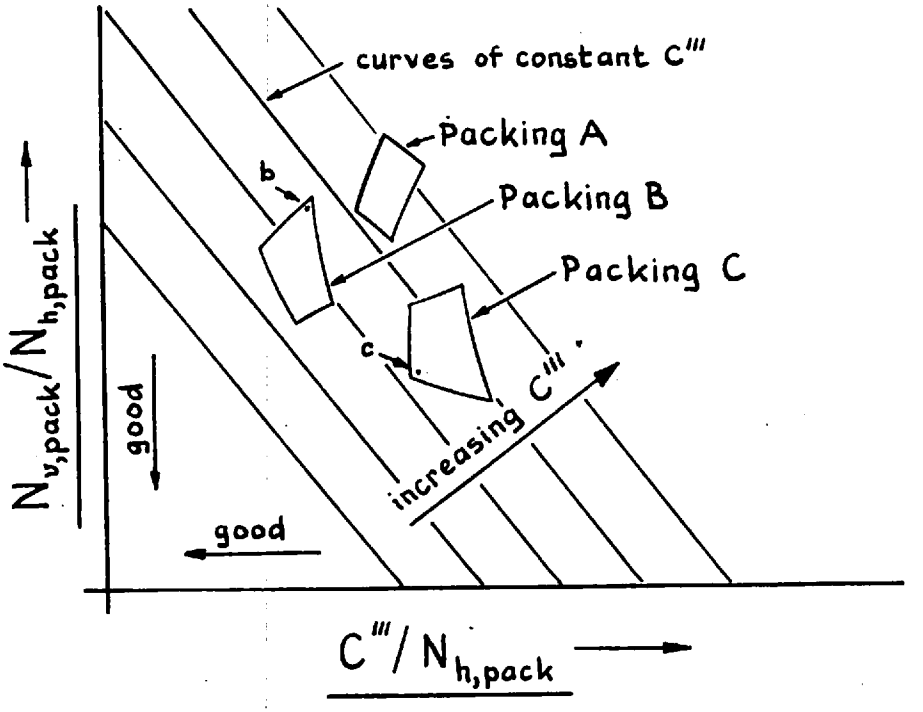
The following sub-sections are concerned with the choice of suitable packing and heat-exchanger types by means of simplified methods and, in the case of the Heller- and Spalding-systems, the development and application of one such method, applicable to the present investigation.

4.2 Best tower packing type for Wet system

A simplified method of determining the suitability of types of packing for use in natural-draught cooling towers has been devised by Spalding in ref.37. Using this method, Lieberam (ref.25) has examined many different packing configurations, constructed both from treated wood and asbestos cement. Lieberam concluded that packing No.21, referred to on page 937 of ref.26 and consisting of corrugated asbestos sheets, afforded the best compromise between heat transfer, pressure drop and price.

As a first step in deciding whether this conclusion is valid, a brief outline of the foregoing method is given as follows:

By making use of the empirical heat-transfer and velocity-head performance equations given in (3.7-9) and (F.2-3), and the costs both of packing and coupling-circuit pumping power, curves of $(N_{v,pack}/N_{h,pack})$ versus $(C'''/N_{h,pack})$ are drawn for a number of different packing types, over ranges of G_C/G_G and G_C values liable to occur in practice. C''' in this instance represents the total cost of packing and pumping power per unit volume of packing. In addition, curves of constant C''' are drawn, it being possible for the graph to have the following appearance:



One of the chief reasons for choosing the foregoing quantities as co-ordinates is that they will have low values for promising packing types. Of course, being interrelated, the merit of a packing type cannot be ascertained by examining just one of the co-ordinates in isolation, hence the need for a graph to display their combined effect.

The claim advanced is that the most promising packing types will have areas of operation - created by the ranges

of possible G_C/G_G and G_C values - near the curves of lowest constant C''' values. The following comments, concerning the three fictitious packing types in the graph, serve to illustrate the application of the method:

Packing B cannot be said to be more promising than packing C, even though this appears at first sight to be the case, since its merit at optimum cooling-system operating conditions, e.g. point b, could be worse than the corresponding merit for packing C, e.g. point c. However, packing B will always be better than packing A since its worst possible merit value can never be poorer than the best value for packing A.

As mentioned before, the described method is a simplified one. The original unsimplified version would requite the use of the co-ordinates $\Sigma N_v/N_{h,G}$ and $(P_L/q)/N_{h,G}$ - from (F.2-2), (3.7-8) and (2.1-8) - together with the superimposition of curves of constant P_L/q values. Examination of the adopted and the original co-ordinates reveals that the accuracy of the simplified method is dependent on the validity of the following inherent assumptions:

i) $t_{s,S} - t_{G,i}$ is constant in value.

ii) The sum of the ΣN_v constituents, apart from $N_{v,pack}$, is constant.

iii) The contribu tion of $N_{h,spray}$ to $N_{h,G}$ is negligible with respect to that of $N_{h,pack}$.

iv) The cost of pumping is chiefly dependent on the height to which the water must be raised before passing downwards through the packing.

The first assumption would be valid if, for a fixed atmospheric dry-bulb temperature, the exhaust steam temperature were to be fixed. The second assumption would hold good for cooling towers of unvarying proportions, viz. constant values of r, s and t, particularly since the value of $N_{v,spray}$ is negligibly small. The third and fourth assumptions can be seen from Wet-system design calculations, e.g. those appearing in chapter 9, to be generally valid.

In view of the foregoing comments, the simplified method established by Spalding is concluded as being suitable for the selection of promising packing types provided that optimum exhaust-steam temperatures and cooling-tower proportions do not vary significantly with different packing types. These provisions are considered to be satisfied in practice to within fairly narrow limits and it is therefore further concluded that the simplified method successfully fulfils its purpose, at least insofar as differentiating between very good and very poor packing types is concerned.

On returning once again to the choice of a particular packing type, Lieberam (ref.25) states that packing No.21 bears the same relationship to the next best type as packing B does to packing A in the foregoing graph, viz. it will always be better, whatever the optimum operating conditions are.

Accordingly, packing No.21 is adopted for the present investigation. The associated heat transfer and pressure drop characteristics can be represented by equations similar in form to (3.7-8) to (3.7-11) and (F.2-3). These forms have been adopted by Lieberam in ref.25 as they enable a large number of packing types to be easily classified. The constants in these equations relating to packing No.21 are:

$a_{h,p} = 16.13$, $b_{h,p} = 0.0096$, $c_{h,p} = 0.5$, $d_{h,p} = 0.052$

$e_{h,p} = 71.6$, $f_{h,p} = -0.25$, $a_{h,s} = 0.347$, $b_{h,s} = 0.5$

$a_{v,p} = 23.55$, $b_{v,p} = 0.0192$, $c_{v,p} = 0.5$, $d_{v,p} = 0.104$.

These values have been obtained by Singham in ref.4, app. F and are in reasonable agreement with those obtained by Lowe and Christie in ref.26.

4.3 Best tower heat-exchanger types for Heller- and Spalding-systems.

4.3.1 Factors affecting choice of heat-exchanger types.

The suitability of a particular type of heat exchanger for use in the Heller- or Spalding systems is chiefly dependent on its effect on the overall merit-factor in (2.1-8); the lower the optimum merit-factor, the more suitable the heat-exchanger type will be.

However, a low P_L/q value is not the sole criterion of merit since it is also important to choose a heat-exchanger type which, after optimizing a cooling system, will lead to the satisfaction of the following requirements:

i) The optimum length of heat-exchanger tubes must be such that it will be possible to construct individual heat-exchanger units having dimensions close to the previously adopted values of 16 feet long by 9 feet wide, in order to remain within the limits of applicability of the heat-exchanger price formulae presented in appendix K.

ii) The optimum physical dimensions of the whole tower heat-exchanger must be suitable for convenient insertion of the individual heat-exchanger units into and around the base of the cooling tower. In particular, it is desirable that the tower heat-exchanger frontal area should be greater than or equal to the tower inlet area in order to avoid the necessity for blanking off some of the tower inlet, thereby increasing inlet air-flow losses through the tower.

iii) The indicated value of the optimum number of heat-exchanger tube rows, arising from optimization techniques, should be close to an integer. This is desirable since the number of tube rows is chosen to be an integer in practice; if the indicated optimum value is not sufficiently close to the chosen integer value, the tower heat exchanger may operate under conditions which are so far removed from its indicated optimum conditions that its effective merit-factor will be worse than that of another type of heat exchanger which appears at first sight to be slightly the poorer of the two.

iv) In the case of the Heller system, the choice of the number of tube-side passes must be such as to result in con-

venient sizes of individual heat-exchanger units after optimization, bearing in mind that the greater the number of passes the more thermodynamically efficient the heat exchanger will be. The choice of the most suitable number of passes therefore depends on the application of engineering intuition in compromising between increase in efficiency, convenience of fitting into tower base and cost of manufacturing individual heat-exchanger units.

In this section, an attempt is made to choose the best of a number of considered heat-exchanger types, bearing in mind the comments in the previous paragraphs. The adopted procedure is to eliminate from the overall merit-factor equation, (2.1-8), the terms which are unnecessary for a comparison of heat-exchanger types. In addition, some of the remaining terms are found to be relatively unimportant and are therefore either fixed or ignored in order to simplify the merit-factor equation still further, care being taken not to destroy trends. In carrying out this latter task, two tube-side passes for the Heller-system tower heat-exchanger are assumed. This done, the resultant, simplified merit-factor equation is minimized with a view to determining its optimum value for each of the considered heat-exchanger types. The optimum values so obtained are then examined and the least promising heat-exchanger types are eliminated from further consideration.

Finally, an attempt is made to differentiate between the remaining promising types by determining which of them best

satisfy the previously enumerated requirements. In particular, a method of predicting the optimum number of tube rows is derived in order to ascertain accurately how well each of the heat-exchanger types satisfies the requirement in item ii).

4.3.2 Establishment of simplified merit-factor

Investigation of the Heller- and Spalding-system merit-factor equation, formed by substituting the power-loss terms derived in chapter 3 into (2.1-8), reveals that

$$P_L/q = \varphi_1 \text{ (terms containing heat-exchanger characteristics)} + \varphi_2 \text{ (terms not containing heat-exchanger characteristics).} \quad (4.3\text{-}1)$$

Furthermore,

$$\varphi_2 = \phi_1 \text{ (terms containing design variables appearing in } \varphi_1) + \phi_2 \text{ (terms not containing design variables appearing in } \varphi_1). \quad (4.3\text{-}2)$$

In so far as choice of tower heat exchanger is concerned, only the following proportionality need be considered:

$$P_L/q \propto \varphi_1 + \phi_1. \quad (4.3\text{-}3)$$

φ_1 and ϕ_1 are presented in the following paragraphs for both the Heller- and Spalding-systems by examining the equations in chapter 3. The relative magnitudes of the terms constituting φ_1 and ϕ_1 are then discussed with a view to fixing or eliminating unimportant quantities in order to derive a simple merit-factor suitable for comparing heat-exchanger types.

For the Heller system,

$$\varphi_1 = a^*(Q_C/Q_G)(u_C^{-1}+u_{m,C}^{-1})Q_G+0.5b^*a_{P,T}(1+\eta_{pt})\Delta p_{fr,C}C_G^{-1}(C_F/C_G)^{-1} + c^*(\pounds_G/q + \pounds_D/q) \qquad (4.3\text{-}4)$$

and the design variables involved are:

W_C, C_G, C_F/C_G, $n_{h,G}$, n_r, D_1, r, s.

Also,

$$\phi_1 = a^*C_G/e_G+ b^*a_{P,T}C_G^{-1}(C_F/C_G)^{-1}\left[\eta_{pt}\Delta p_H + (1-\eta_{pt})\left[p_{atm} - p_{s,S}+ \{a_{grav}/(g_o v_{f,C})\}(z_1-z_H)\right] +(1+\eta_{pt})\Delta p_{fr,T}\right]. \qquad (4.3\text{-}5)$$

For the Spalding system,

$$\varphi_1 = a^*(Q_C/Q_G)(u_C^{-1}+u_{m,C}^{-1})Q_G + b^*a_{P,H}C_H^{-1}\,\Delta p_{fr,C} + c^*(\pounds_G/q +\pounds_D/q + \pounds_R/q) \qquad (4.3\text{-}6)$$

and the design variables involved are:

W_C, C_G, n_r, X_H, D_1, r, s, $n_{h,G}$.

Also,

$$\phi_1 = a^*C_G/e_G+b^*a_{P,H}C_F^{-1}X_H^{-1}\,(\Delta p_H+ \Delta p_{fr,T}+ \Delta p_{fr,B}). \qquad (4.3\text{-}7)$$

In order to determine the relative orders of magnitude of the terms in the foregoing equations it is necessary to arbitrarily choose a heat-exchanger type, the number of water-side passes in the Heller-system heat exchanger and a refrigerant and to minimize P_L/q in (2.1-8) in order to obtain optimum values of the design variables. Only then can numerical values

be assigned to the terms in equations (4.3-4) to (4.3-7). This oourse of action has been followed after arbitrarily adopting ammonia as the refrigerant, two water-side passes and heat-exchanger type SG1, details of which are given later in this chapter. To avoid repetition, the resultant numerical values are not presented here but can be seen in the graphs appearing in the general result chapter, viz. chapter nine.

The results reveal that the sum of the terms containing b^* is less than 10 per cent of the sum of all the terms constituting φ_1 and ϕ_1. In consequence, the terms containing b^* are considered unimportant and will be ignored for heat-exchanger comparison purposes. The same applies to $£_R/q$, the refrigerant cost, as it is a very small fraction of the total cooling system cost. The ignoring of these terms results in the adoption of the following criterion of merit, valid for both the Heller- and Spalding-systems:

$$P_L/q \propto a^*\{(Q_C/Q_G)(u_C^{-1}+u_{m,C}^{-1})Q_G+C_G/e_G\}+c^*(£_G/q+£_D/q). \qquad (4.3\text{-}8)$$

From (3.7-13), Q_G can be represented as

$$Q_G = (a_{q,G}\, n_r^{1+j_1})\, C_G\, n_{h,G}^{j_1}\, n_r^{-(1+j_1)}. \qquad (4.3\text{-}9)$$

From (3.4-22) and (3.4-33), C_G/e_G is given by

$$C_G/e_G = 1.02\, C_G\{1/n_{h,G}+0.5+0.0625n_{h,G}+(0.5-0.125n_{h,G})(C_F/C_G) + 0.0625n_{h,G}(C_F/C_G)^2\} \qquad (4.3\text{-}10)$$

and

$$C_G/e_G = C_G(1/n_{h,G} + 0.5 + 0.0833n_{h,G}) \qquad (4.3\text{-}11)$$

for the Heller- and Spalding-systems respectively.

It is to be noted here that, in adopting (4.3-10), two water- or tube-side passes have been assumed for the Heller-system heat exchanger. Upon investigation, there appears to be no simple method of determining the optimum number of passes, thereby justifying the statement in the fourth requirement of the previous sub-section that: "The choice ... depends on the application of engineering intuition." However, the assumption of two passes was backed up numerically by noting from the results, previously referred to in the text between (4.3-7) and (4.3-8), that two water-side passes result in convenient optimum sizes for individual heat-exchanger units. The reasonability of the assumption is also endorsed by the fact that two water-side passes are employed for the Heller-system cooling plant in operation at Rugeley.

From (3.6-12), $\pounds_G/q$ is given as

$$\pounds_G/q = \pounds''_G/Q_G, \qquad (4.3\text{-}12)$$

where $\pounds''_G$ can be evaluated from (K.2-7) or (K.3-5). The substitution of (J.2-2), (3.7-16) and (F.2-18) into (3.6-13) results in

$$\pounds_D/q = q^{-1}\{d_o + (d_1 + d_2 r_1 D_1)\, A\} \qquad (4.3\text{-}13)$$

where

$$A = (2.19s+0.982)\{438v_{G,i}^2/(g_o c_G^2 a_{grav}/g_o)\} q^2 \Sigma N_v C_G^{-3} D_1^{-3} \quad (4.3\text{-}14)$$

and

$$\Sigma N_v = a_{n,G} n_{h,G}^{j_o} (0.25\pi q^{-1} C_G D_1^2)^2 + N_{v,o} + N_{v,i} + N_{v,misc}. \quad (4.3\text{-}15)$$

Examination of the magnitudes of the terms in the foregoing equations reveals that (4.3-8) may be further simplified by calculating u_C at representative, fixed values of d_C and W_C; this is permissible because the air-side of the heat exchanger is controlling. Also, the terms containing C_F/C_G are sufficiently unimportant for C_F/C_G to be fixed at a representative value. Furthermore, since $£_D/q$ is only about 50 per cent of $£_G/q$, the following quantities in equation (4.3-13) to (4.3-15) can be fixed at representative values:

$d_2 r_1 D_1$, s, $(N_{v,o} + N_{v,i})$.

The end result of the eliminations and simplifications carried out in this sub-section is that (4.3-8) is reduced to the following function:

$$P_L/q \propto \phi(C_G, D_1, n_{h,G}, n_r). \quad (4.3\text{-}16)$$

4.3.3 Adopted method of determining the merits of heat-exchanger types.

The merits of heat-exchanger types can be determined according to the following procedure:

Differentiate (4.3-8) with respect to each of the design variables in (4.3-16) and solve the following set of equations for each of the heat exchanger types:

$$\partial(P_L/q)/\partial x_i = 0 , \quad i = 0, 1, 2, 3 \qquad (4.3\text{-}17)$$

and $x_o = C_G$, $x_1 = D_1$, $x_2 = n_{h,G}$, $x_3 = n_r$,

in order to arrive at the optimum values of the design variables. Then substitute the values so obtained into (4.3-8); the heat-exchanger type giving the smallest value of P_L/q is the most promising type for use in cooling towers.

The foregoing procedure was attempted but, due to the nature of the set of equations in (4.3-17), a solution was not forthcoming. Since the equations are non-linear, an iterative method of solving for the design variables seemed suitable. However, no matter how the equations were arranged, the values of the design variables always diverged. In consequence it was decided to use the numerical "steepest-descent" method, briefly described in appendix G, in order to find minimum values of P_L/q in (4.3-8). This method proved successful and the results obtained thereby are presented in this section, after describing the tower heat-exchanger types which have been chosen for analysis.

4.3.4 Description of considered heat-exchanger types

Details of a number of possible tower heat-exchanger types have been obtained from manufacturers. Performance data, physical dimensions and prices of each of the types -

as supplied by the manufacturers - are presented on the following pages in the form of graphs and tables 1 and 2.

The various types are classified as follows:

i) Type CD consists of circular mild-steel tubes, electrical resistance welded, and spirally wound with wire coils; the tube rows are staggered. High tube-side operating pressures are permissible since the test pressure is at least 1000 p.s.i.

ii) Type LU consists of circular seamless, grooved mild-steel tubes with spirally wound aluminium fins; a staggered arrangement is employed with equidistant tubes. The operating pressure is in excess of 300 p.s.i.

iii) Type SG consists of circular copper tubes with spirally wound aluminium fins; the tube rows are staggered with equidistant tubes. The operating pressure is in excess of 200 p.s.i.

iv) Type ST consists of circular copper tubes with spirally wound copper fins soldered in position; staggered tube rows are employed. The spiral fins are corrugated to assist break-up of the air-side boundary layer and a working pressure of 200 p.s.i. is permissible.

v) Type HG consists of grooved, circular mild-steel tubes, electrical resistance welded, with spirally wound aluminium fins; the tube rows are staggered. Working pressures in excess of 300 p.s.i. are permissible.

vi) Type TR differs from the foregoing types in that continuous plate fins and circular tubes are employed.

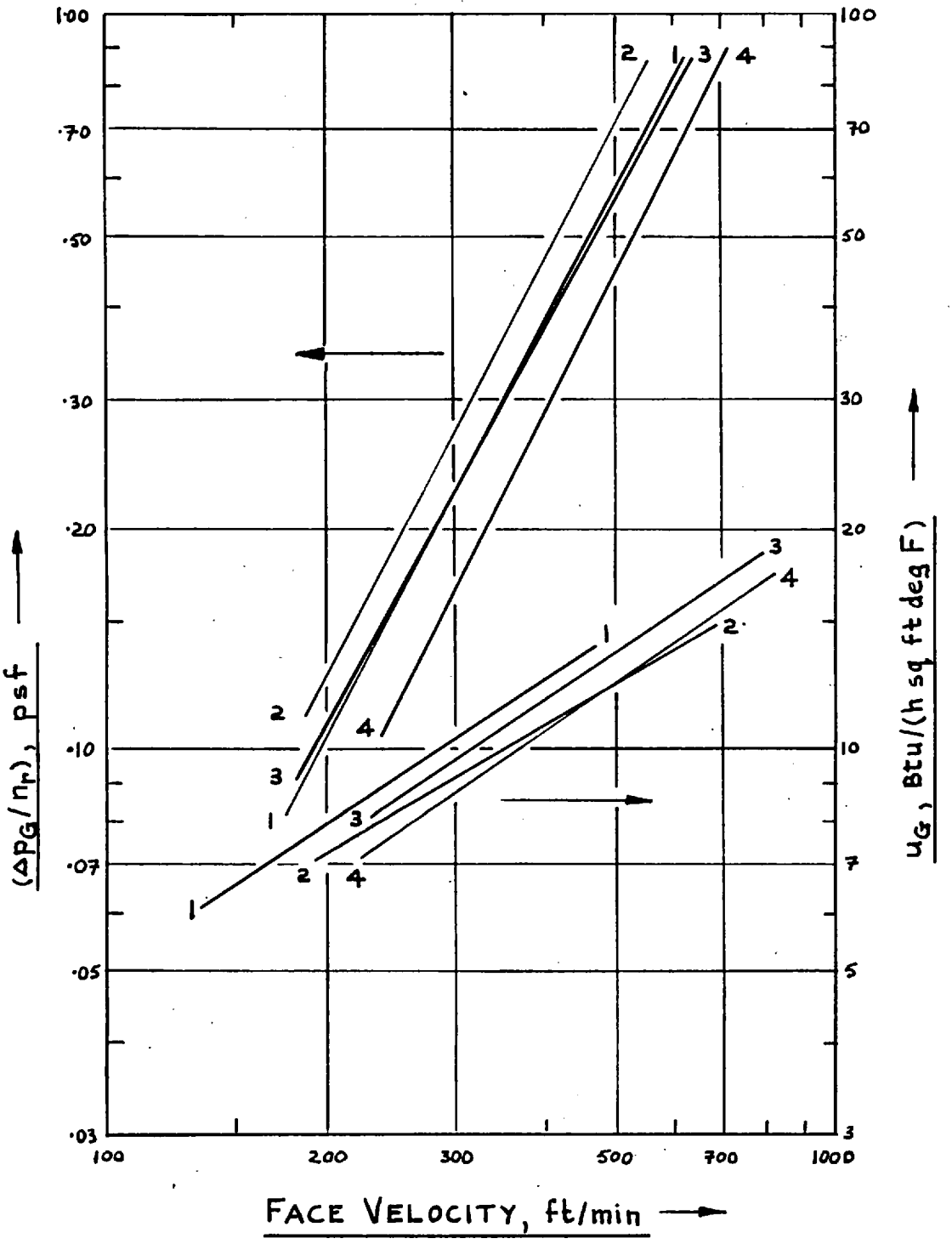
TYPE	TUBE	FIN	D_C ft $x10^{-2}$	δ_C ft $x10^{-3}$	D_G ft $x10^{-1}$	δ_f ft $x10^{-3}$	n'_f ft^{-1}	l_c ft $x10^{-1}$	l_r ft $x10^{-1}$
CD 1	MILD-	GALVA-	5.208	5.333	1.562	-	61.2	1.667	1.443
2	STEEL	NIZED	6.250	↑	1.667	-	61.2	1.771	1.533
3	↑	SOFT	8.333	↕	1.680	-	48.0	1.875	1.458
4	↕	IRON	8.333	↕	1.680	-	48.0	2.083	1.458
5	↕	Al	5.208	↕	1.374	-	48.0	1.458	1.198
6	↕	↑	5.208	↕	1.374	-	48.0	1.562	1.198
7	↕	↕	5.208	↕	1.374	-	48.0	1.667	1.198
8	↕	↕	6.250	↓	1.478	-	48.0	1.667	1.250
9	↓	↕	6.250	5.333	1.478	-	48.0	1.875	1.250
LU 1	MILD-	↕	8.333	8.667	1.875	1.333	132.0	2.187	1.897
	STEEL	↓							
SG 1	Cu	Al	5.208	5.333	1.562	1.333	156.0	1.583	1.371
ST 1	↑	Cu	5.208	5.333	1.458	-	48.0	1.875	1.624
2	↕	Cu	5.208	5.333	1.458	-	60.0	1.875	1.624
3	Cu	Cu	5.208	5.333	1.458	-	72.0	1.875	1.624
HG 1	MILD-	Al	8.333	6.000	1.665	1.167	132.0	1.771	1.771
2	↑	↑	8.333	↑	1.845	1.667	144.0	2.021	1.750
3	↕	↕	8.333	↕	1.875	1.667	120.0	2.021	1.750
4	↕	↕	8.333	↕	1.840	1.667	96.0	2.021	1.750
5	↓	↕	5.208	↓	1.562	1.333	96.0	1.667	1.443
6	STEEL	↓	5.208	6.000	1.562	1.333	120.0	1.667	1.443
TR 1	Cu	Al	5.208	5.333	-	1.667	144.0	1.250	1.250

TABLE 1

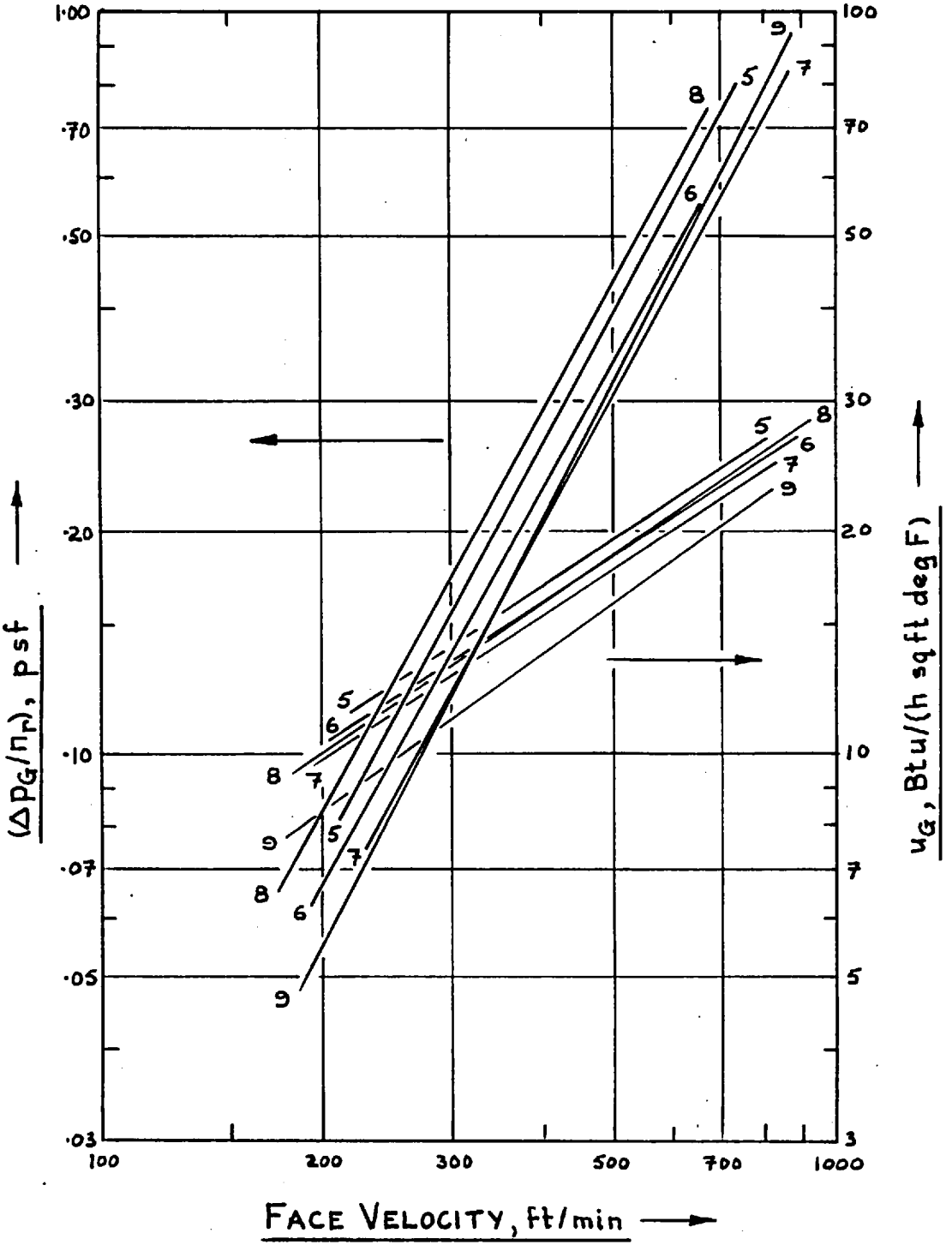
TYPE	$-j_o$	$-j_1$	$b_{u,G}$	$b_{p,G}$	$A'_{h,G}$	l_c	$k_{m,C}$	d_C	Q_C/Q_G	$£^n_{G,tu}$	$£^n_{G,ba}$
					ft	ft		ft		£/sq ft	£/sq ft
			10^{x-1}	10^{x-6}		10^{x-1}		10^{x-2}		10^{x-2}	10^{x-2}
CD 1	5.44	2.91	2.45	5.22	2.94	1.67	30.6	4.14	22.6	5.68	3.72
2	4.72	2.49	3.00	5.49	3.53	1.77	↕	5.18	21.7	4.85	3.10
3	5.50	3.05	2.11	7.47	2.74	1.87		8.33	12.0	6.40	3.99
4	5.99	3.04	1.89	2.16	2.74	2.08		8.33	12.0	6.40	3.99
5	5.27	2.88	3.37	4.55	1.72	1.46		4.14	13.2	11.62	6.36
6	4.94	2.75	3.54	4.82	1.72	1.56		4.14	13.2	11.62	6.36
7	5.12	2.80	3.28	3.54	1.72	1.67		4.14	13.2	11.62	6.36
8	5.68	3.14	2.69	5.67	2.04	1.67		5.18	12.5	11.05	5.36
9	6.38	3.42	1.97	2.00	2.04	1.87		5.18	12.5	11.05	5.36
LU 1	3.50	2.00	6.08	18.7	4.81	2.19	30.6	6.60	23.2	5.00	2.27
SG 1	2.77	1.79	6.10	25.2	5.07	1.58	218	4.14	39.0	4.51	2.16
ST 1	10.4	5.19	0.46	0.52	2.23	1.88	218	4.14	17.1	9.89	5.36
2	10.3	5.19	0.46	0.73	2.74	1.88	218	4.14	21.1	8.58	4.36
3	10.3	5.19	0.46	1.17	3.25	1.88	218	4.14	25.1	7.69	3.67
HG 1	3.99	2.43	2.06	15.4	4.57	1.77	30.6	7.14	20.4	3.27	2.39
2	4.37	2.66	1.34	17.1	6.38	2.02	↕	7.14	28.4	2.70	1.72
3	3.59	2.24	2.35	18.4	5.59	2.02		7.14	24.9	3.51	1.96
4	3.76	2.34	2.21	15.4	4.32	2.02		7.14	19.3	5.16	2.53
5	3.86	2.04	3.30	2.10	3.44	1.67		4.01	27.3	4.29	3.18
6	3.76	1.98	3.86	2.46	4.25	1.67	30.6	4.01	33.7	3.79	2.57
											$£^n_{G,st}$
											£/sqft 10^{x-2}
TR 1	3.49	2.07	5.00	20.6	3.73	1.25	218	4.14	28.7	-	10.5

TABLE 2

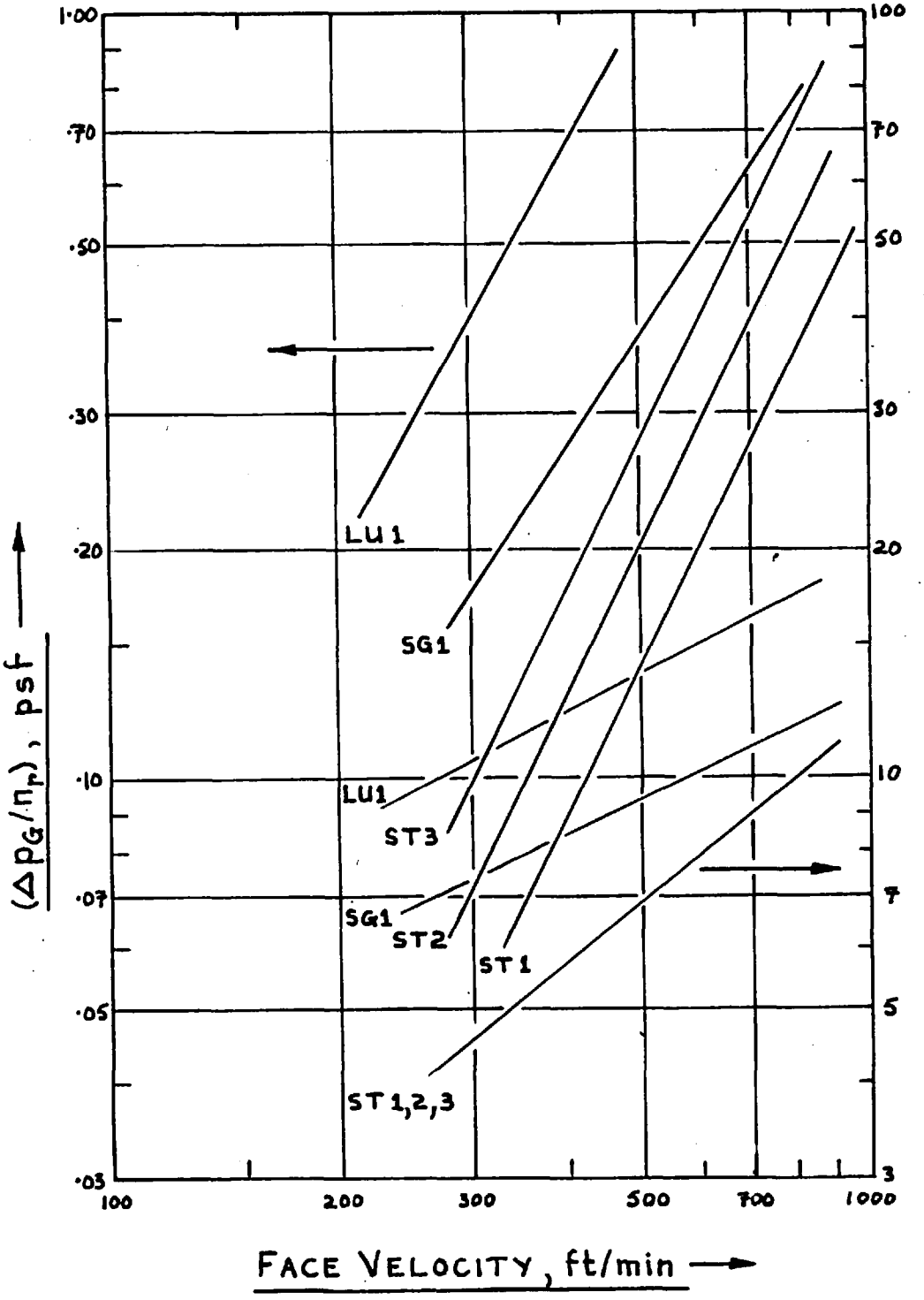
TYPES CD1 TO CD4.



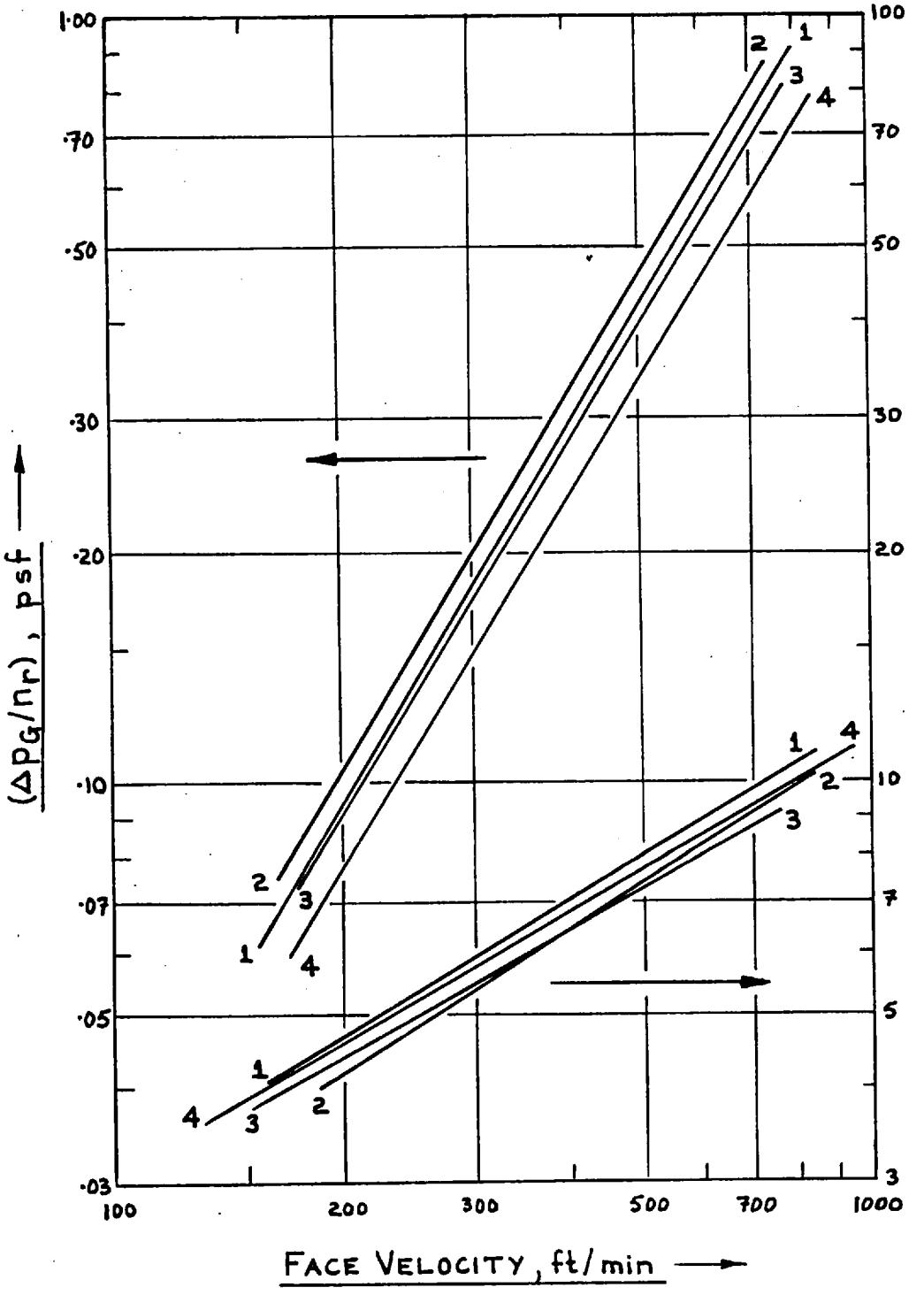
TYPES CD5 TO CD9



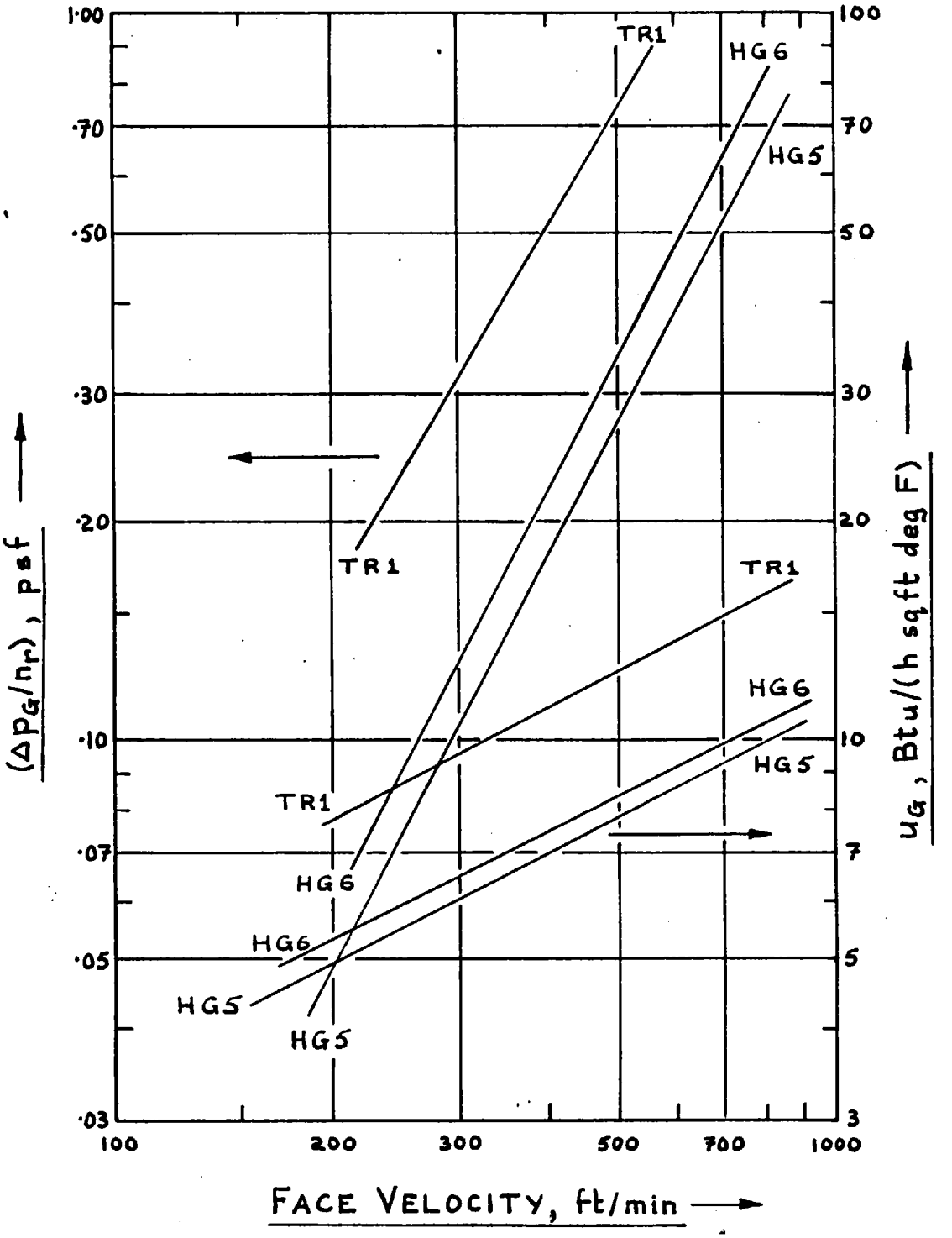
TYPES LU1, ST1 TO ST3 AND SG1



TYPES HG1 TO HG4



TYPES HG5, HG6 AND TR1



The construction is similar to that used at Rugeley Power Station except that provision is made for breaking up the boundary layer on the air-side by making the plate fins wavy instead of having punched slots. The tubes are of copper, the fins of aluminium and the tube rows are in line. The operating pressure is in excess of 200 p.s.i.

4.3.5 Presentation of merit-factors

In evaluating the minimum merit-factors for the considered heat-exchanger types, the following values of fixed quantities were adopted:

Quantity	Value
a^*	0.0042
c^*	68.5
q	5×10^8
u_C	800
C_F/C_G	0.4
$d_2 r_1 D_1$	0.02
s	0.6
$N_{v,o}+N_{v,i}$	13
$t_{G,i}$	57
$t^*_{G,i}$	52

The minimum values of the merit-factor in (4.3-8) are presented in the following table:

Type	P_L/q		Type	P_L/q	
	Heller	Spalding		Heller	Spalding
CD 1	.224	.215	ST 1	.308	.290
2	.223	.213	2	.281	.272
3	.223	.212	3	.281	.272
4	.223	.217	HG 1	.213	.203
5	.235	.224	2	.207	.197
6	.233	.222	3	.213	.203
7	.232	.221	4	.226	.214
8	.230	.220	5	.222	.212
9	.231	.220	6	.217	.206
LU 1	.220	.211	TR 1	.256	.246
SG 1	.219	.210			

4.3.6 Choice of tower heat-exchanger type

Comparison of the results in the foregoing table indicates that types CD, ST have relatively high simplified merit-factor values and can therefore be eliminated from further consideration. Of the remaining types, the five most meritorious are HG2, HG1, HG3, HG6 and SG1, in descending order of merit for both the Heller- and Spalding-systems. The reasons for the HG types occupying the first four places are considered to be chiefly due to the fact that their tubes are constructed of cheap mild steel. The SG1 type, is considered to owe its relatively high status to the improvement in heat-transfer performance resulting from its unique feature of having an extremely high outside to inside heat-transfer surface ratio, in spite of having expensive copper tubes.

Choosing suitable heat-exchanger types for cooling-system applications does not only involve merit-factor considerations. The chemical compatabilities with possible heat-transfer fluids and corrosion characteristics in moist, polluted atmospheres of the constructional materials must also be examined. For example, the choice of aluminium fins renders all five of the high-merit types suitable insofar as atmospheric corrosion in industrial areas is concerned. That this is so can be seen on page 1020 of ref.36, provided that the aluminium is of greater than 99 per cent purity. However, in this context, the heat-exchanger types employing mild-steel tubes are regarded as

unsuitable, mild steel being stated on page 1020 of ref. 36 as having restricted applications in industrial areas. This disadvantage of mild steel can, at first sight, be offset to some extent by the use of various coatings on the outside of the tubes. For example, on page 655 of ref.14 it is stated that: "The long life of sprayed aluminium coatings on steel even in bad atmospheres is impressive" and "steel hot-dipped in aluminium is now being produced on a large scale". However, the extra cost involved in "hot-dipping" steel tubes in aluminium can be seen from ref.4 in appendix H to bring the tube cost nearly up to that for a copper tube. In other words, the extra cost of producing mild-steel tube coatings, such processes being further complicated in the present instance by the presence either of the aluminium fins or their seating grooves, almost completely offsets the apparent advantage offered by the use of mild-steel tubes.

In view of the foregoing comments it is concluded that the merit-factor values for types HG1, HG2, HG3 and HG6 should be made more representative by adjusting them to include the oost of cladding, assumed to be the difference in cost between a mild steel and copper tube. This can be done approximately by making use of (4.3-8) and (K1-1) in deducing that the extra quantities to be added onto the merit-factor values are given by

$$c^{*}\Delta\, \pounds_G/q \;=\; bc^{*}/\,\{(Q_C/Q_G)Q_G\}\;,$$

where b is temporarily defined as

b ≡ difference in cost between mild-steel and copper tubes per unit bare tube surface area, £/sqft.

By using b = 0.273, evaluated from ref.4 in appendix II, c^* = 68.9 from a table in 2.5, Q_C/Q_G values from table 2 in 4.3.4 and Q_G values associated with the merit-factors appearing in the foregoing table, the following table can be constructed:

TYPE	HELLER		SPALDING	
	$c^*\Delta\,£_G/q$	P_L/q	$c^*\Delta\,£_G/q$	P_L/q
HG1	.0121	.225	.0115	.215
2	.0109	.218	.0102	.207
3	.0106	.224	.0101	.213
6	.0112	.228	.0106	.217
SG1	-	.219	-	.210

These adjusted merit-factor values indicate that type HG2 is still the best but is now so closely followed by type SG1 that they can be regarded as equivalent for practical purposes. Type SG1 is chosen in preferance to HG1 for calculations in the present investigation because the larger Q_C/Q_G value associated with SG1 will result in a smaller frontal area for the whole tower heat-exchanger. This will be advantageous as it eases the awkward task of fitting the heat-exchanger into and around the cooling-

tower inlet area. In fact, if the frontal area is sufficiently small, the need to resort to a structurally inconvenient zig-zag or concertina formation, as employed in the Heller system tower at Rugeley (see ref.9), can be eliminated.

The use of copper tubes does not pose any chemical compatability or atmospheric corrosion problems for the Heller system as can be seen from pages 1020 and 1021 of ref.26. In the Spalding system, however, copper cannot be used should ammonia prove to be the most efficient heat-transfer medium. This is not considered to be serious since the tube can be made from mild-steel (page 1015, ref. 36), this material also being suitable should methyl chloride be contemplated (page 1019, ref.36). If freon is desirable in the Spalding system or if, for some obscure reason, copper is not considered suitable for the Heller system, aluminium tubes can be resorted to according to pages 1020 and 1021 of ref.36.

Since copper is expensive, the use of the foregoing alternative materials will either cause the price of the SG1 type to remain virtually unchanged or to be slightly reduced.

Table 1 in 4.3.4 shows that, along with a number of other heat-exchanger types, type SG1 has the smallest diameter tubes. The variation in optimum tower-heat exchanger dimensions is considered to be of practical interest. Therefore, type LU1, having tubes of larger diameter than

type SG1 is adopted for a restricted number of calculations in the present investigation so that the foregoing variations can be studied. Another reason for choosing type LU1 is that it is practically equivalent in merit to the SG1 type; after optimization of (2.1-8) it will be possible to see whether the merits of types SG1 and LU1 are as close together as the simplified method of distinguishing between them indicates.

Finally, no conclusions can be drawn here as to the ability of either type SG1 or type LU1 to satisfy the other heat-exchanger requirements enumerated in 4.3.1. This task must be deferred until the results of optimizing techniques have been presented in a later chapter of the thesis. However, it can be stated here that an expression for the optimum number of tube rows is derived in appendix L, in order to provide a means of testing whether the third of the enumerated requirements is satisfied.

5 CHOICE OF SPALDING-SYSTEM REFRIGERANT AND MATERIALS FOR STEAM-CONDENSER TUBES AND TOWER HEAT EXCHANGER

5.1 Proposed method of choosing refrigerant

Now that the constituent terms of the merit-factor equation have been completely derived, an attempt can be made to choose the most suitable refrigerant, the use of which will result in the lowest merit-factor, subject to considerations of chemical compatability, toxicity, etc. The simplest method of approach is to gather all the groups in the power-loss equation which contain refrigerant physical properties, ensuring that each group is proportional to the merit-factor. These groups are then evaluated for each of the four possible refrigerants after choosing an operating temperature, condenser-tube inside diameter and tower heat exchanger tube inside diameter which are representative of the Spalding system.

If one of the considered refrigerants has smaller values for all of these groups than any of the remaining refrigerants, it will be indisputably the best choice. It is even possible to come to a decision when a few of the physical property groups of an otherwise ideal refrigerant are slightly larger than those of a rival refrigerant, provided that the offending groups play a sufficiently small role in the evaluation of the merit-factor.

The next sub-section contains information leading to the presentation of a table of numerically evaluated

physical property groups. It is hoped that one of the possible refrigerants chosen in 1.4.3 will be of sufficient merit to respond to the simple method of approach outlined in this sub-section.

5.2 Evaluation of physical property groups

The required groups containing refrigerant physical properties are found by examining the equations presented in chapter 3. An English alphabet capital letter is used to designate each group as an aid in the compilation of tables. In addition, the reference numbers of the source equations are quoted for every case. Hence,

$$A = \{ 0.0184 \, (k_{f,H}/d_H)(N_{Pr,H})_f^{0.4} \}^{-1}, \qquad (D.1\text{-}1)$$

$$\left.\begin{aligned} B &= a_{t,H}/v_{f,H} \\ C &= a_{t,H} a_{p,H}^{C_F} b_{p,H}^2 \\ D &= a_{t,H} a_{p,H}^{C_F} b_{p,H} \\ E &= a_{t,H} a_{p,H}^{C_F} \\ F &= a_{t,H} a_{a,H} \end{aligned}\right\} \quad (3.4\text{-}27), (3.5\text{-}18), (3.5\text{-}19)$$

$$G = a^{-2}, \qquad (E.5\text{-}5)$$

$$H = a_{u,H}^{-1}, \qquad (D.4\text{-}4)$$

$$I = a_{u,C}^{-1}, \qquad (3.4\text{-}32)$$

$$\left.\begin{aligned} J &= v_{f,F} a_{p,H}/c_{f,F} \\ K &= v_{f,F} a_{p,H}^{b_{p,H}}/c_{f,F} \\ L &= v_{f,F} a_{p,H}^{b_{p,H}^2}/c_{f,F} \end{aligned}\right\} \quad (3.5\text{-}18)$$

$$M = v_{f,F} a_{a,H} C_F^{-1} / c_{f,F}, \quad (3.5\text{-}19)$$

$$N = v_{f,F} C_F^{-3} v_{g,F} / c_{f,F}^3, \quad (3.5\text{-}20)$$

$$O = v_{f,F} C_F^{-3} v_{f,F} / c_{f,F}^3, \quad (3.5\text{-}21)$$

$$\left.\begin{aligned} P &= v_{f,F} a_{p,C} / c_{f,F} \\ Q &= v_{f,F} b_{p,C} / c_{f,F} \end{aligned}\right\} \quad (3.5\text{-}22)$$

$$\left.\begin{aligned} R &= a_R / v_{f,F} \\ S &= a_R / v_{g,F} \end{aligned}\right\} \quad (3.6\text{-}20)$$

After assuming the values $d_H = 0.0518$, $d_C = 0.0414$ and a representative Spalding-system operating temperature of 100 deg F, appendix A may be used to calculate refrigerant properties necessary for the compilation of the following tables:

TABLE 1

Legend	Refrigerant			
	Ammonia	Freon 22	Methyl Chloride	Freon 12
$a_{t,H}$	2.07,-3	2.38,-3	4.10,-3	3.79,-3
$a_{p,H}$	1.93,-10	6.29,-10	9.07,-10	1.52,-9
$b_{p,H}$	13.84	9.20	13.49	10.32
C_F	412	208	397	232
$a_{a,H}$	2.48,-9	4.78,-9	1.36,-8	1.22,-8
$a_{p,C}$	8.42,-9	1.29,-8	3.59,-8	3.04,-8
$b_{p,C}$	1.11×10^{-24}	1.83,-22	2.06,-22	1.12,-21
a_R	0.0625	0.300	0.0750	0.125

TABLE 2

Legend	Refrigerant			
	Ammonia	Freon 22	Methyl Chloride	Freon 12
A	7.45	34.2	22.9	44.0
B	0.0753	0.170	0.227	0.298
C	3.15,-8	2.63,-8	2.69,-7	1.42,-7
D	2.28,-9	2.86,-9	2.00,-8	1.37,-8
E	1.65,-10	3.11,-10	1.48,-9	1.33,-9
F	5.13,-12	1.14,-11	5.71,-11	4.62,-11
G	0.0100	-	0.0204	0.0278
H	-	4.35	2.58	5.11
I	51.2	139	67.5	122
J	4.57,-12	2.55,-11	4.20,-11	8.00,-11
K	8.76,-10	2.16,-9	7.64,-9	8.52,-9
L	6.33,-11	2.34,-10	5.67,-10	8.26,-10
M	1.42,-13	9.31-13	1.59,-12	2.77,-11
N	3.58,-10	9.49,-9	4.30,-9	2.25,-8
O	6.93,-12	5.27,-10	8.80,-11	9.25,-10
P	2.00,-10	5.22,-10	1.66,-9	1.61,-9
Q	2.63,-26	7.41,-24	9.54,-24	5.91,-23
R	2.27	21.4	4.15	9.83
S	0.044	1.19	0.085	0.406

5.3 Choice of refrigerant and cooling-system materials

Table 2 shows that the physical property groups for Ammonia are smaller than those for the other refrigerants in all but one of the cases, viz. item C. After referring to the source equation of item C, i.e. (3.4-27), it is noted that $\Delta t_{s,H}$ is involved. Additional references to (E.5-4), (E.5-6) and (E.5-6) indicate that the value of $\Delta t_{s,H}$, which calculations show to be about 2 deg F, affects the value of the merit factor so slightly that the higher value of item C causes negligible detraction from the obvious merit of ammonia. In consequence, ammonia is regarded as the first choice with regard to thermodynamic performance.

This simple method of comparison does not provide an answer as to which of the remaining refrigerants is the next best. Inspection of table 2 shows that both freon 22 and methyl chloride are probably better than freon 12. Moreover, if the cost of refrigerant is a relatively unimportant item, as calculations indicate, then freon 22 probably has the edge on methyl chloride. In the event, however, of ammonia being unsuitable for some reason, it would be advisable to obtain minimum values of the merit-factor for both freon 22 and methyl chloride; only then would it be possible to decide which of the two is best for use in the Spalding system.

In the following paragraphs the chemical compatability of ammonia in relation to possible cooling-system materials is discussed.

As stated in 4.3.6, ammonia is chemically compatible with mild steel. Therefore, the use of mild-steel ductwork together with mild-steel steam-condenser and tower heat-exchanger tubes is not only permissible but also desirable because of the cheapness of mild steel. Two precautions to be observed are that copper alloys are to be avoided at all costs and that the outside surface of the heat-exchanger must be clad with aluminium to ensure long life (page 1015, ref.36 and page 655, ref.14). The use of mild-steel tubes is not expected to pose any problems whenin contact with steam and condensate on the Steam-side according to the information on page 1021 of ref.36.

The effects of another contingency must be considered before finally choosing a Spalding-system refrigerant. Namely, what happens if there is a leakage of refrigerant into the Steam-side due to condenser-tube or sealing failure? According to pages 182 and 183 of ref.14 this occurrence is not necessarily disastrous, if ammonia is the refrigerant, as the presence of some ammonia in boilers can actually be beneficial; it is apparently sometimes added to destroy unwanted acidity due to the presence of carbon dioxide, However, on page 465 it is stated that if copper or copper alloys are used in the boiler plant, the ammonia content in the boiler feed water must not exceed that amount necessary to combine completely with the carbon dioxide; this is found in practice to be less than 5 ppm. Therefore, if copper materials are used in the boiler circuit only a

small amount of ammonia leakage is permissible; if copper is not present a fairly large leakage is permissible.

Ammonia gas is fairly toxic, being of the same order of toxicity as carbon monoxide, chloroform, methyl chloride, but does not constitute a danger to life if its concentration in the atmosphere does not exceed 1 part in 10,000 (see page 277, ref.4). However, the fact that it is used in refrigeration plants indicates that it is safe to be used in the proximity of living creatures provided that reasonable precautions are taken to prevent excessive leakage. A fortunate feature in this context is that the pungent odour of ammonia makes it easy to detect when, and in many cases, where leakages occur.

In view of the foregoing comments, it is concluded that, in view of its obviously high thermodynamic efficiency, ammonia is the best of the considered refrigerants for use in the Spalding system, provided that:

i) the presence of copper is entirely eliminated in the coupling circuit, and restricted in the boiler circuit;

ii) excessive leakage of ammonia is eliminated.

Accordingly, ammonia is adopted for use in the present investigation.

6 METHODS FOR THE PREDICTION OF OPTIMUM-DESIGN CONDITIONS FOR THE WET SYSTEM

6.1 Introduction

As has been stated previously, a realistic comparison of different cooling-system types is possible only after the design of each one has been optimized. Since the criterion of comparison in the present investigation is the so-called merit-factor, established in chapter two, optimum design conditions will prevail when the merit-factor is at its minimum value. The classic method of determining the associated optimum values of the design variables is to solve the following set of equations:

$$\partial(P_L/q)/\partial x_i = 0. \qquad (6.1\text{-}1)$$

This set is derived by differentiating (2.1-8) with respect to each of the n design variables x_1, x_2, ... x_n.

Unfortunately, the equations (6.1-1) are so complicated that they cannot be easily solved unless they can be considerably simplified. In consequence, the aid of an alternative, numerical method of accurately ascertaining the optimum values of the design variables is enlisted, so that the orders of magnitude of the terms in equations (6.1-1) can be examined to see whether simplifications are possible.

Accordingly, this chapter is devoted, first of all, to describing the alternative numerical method which has the disadvantage of requiring the use of an automatic digital computer. Then, the numerical results resulting from the

application of this method are used in simplifying the equations (6.1-1) in such a manner that they can be easily solved, thereby providing an approximate method of determining optimum values of the design variables which can be easily carried out without the aid of a digital computer.

6.2 Accurate method of predicting optimum values of design variables.

The optimum values of the design variables are determined by the so-called "steep-descent" technique, briefly outlined in appendix G. The procedural requirements of this method, in the present context, are briefly outlined as follows:

i) Prepare a computer program which consists of the following two subsidiary programs:

a) a slave program which will calculate a value of the merit-factor from (2.1-8) and the most accurate versions of its constituent equations in chapter 3, given values of the waste-heat rejection rate, operating conditions, fixed characteristics and design variables;

b) a master program which will use the slave program to calculate merit-factor values for various, internally-generated combinations of design-variable values in a systematic search for that combination which will result in the minimum value of the merit-factor, this final combination representing the optimum values of the design variables.

ii) Choose a range of waste-heat rejection rates.

iii) Choose typical operating conditions such as appear in the second table in 2.5.

iv) Choose values for a^*, b^*, c^* and other fixed quantities such as fixed characteristics, constant physical property groups, performance constants and price constants.

v) Choose initial guesses (together with practical upper and lower limits) of the optimum values of the finally adopted design variables appearing in 3.7.

vi) Specify the upper and lower limits of the ranges of applicability of the various formulae used in analyzing the cooling system.

vii) Insert the numerical values of the quantities mentioned in steps ii) to vi) into the automatic digital computer.

viii) Allow the computer program mentioned in step i) to find the optimum values of the design variables and to check whether the ranges of applicability of any formulae have been exceeded.

ix) Finally, cause the optimum values of the design variables to be printed out, together with the associated minimum value of the merit-factor and any other auxiliary quantities which are considered important for future reference.

x) Go back to step ii), repeating steps ii) to x) until the range of waste-heat rejection rates has been exhausted.

The foregoing procedure was carried out over a range of waste-heat rejection rates for operating conditions, fixed characteristics and prices considered to be representative

of the United Kingdom in 1964. Values of these quantities and the results accruing therefrom are presented in chapter nine in the form of tables and graphs.

Notice is taken both of the numerical values and the trends, indicated by the use of this accurate numerical method of optimization, when establishing an approximate method of determining optimum values of design variables in the next section.

6.3 Approximate method of predicting optimum values of design variables.

6.3.1 Establishment of method

The first step in establishing an approximate method is to differentiate the merit-factor equation, (2.1-8), with respect to each of the Wet-system design variables finally adopted in 3.7.1, viz.

Q_H, W_H, C_F, $t_{C,i}$, d_T, D_1, r_p, s and t

in order to arrive at the set of equations represented by (6.1-1). Having done this, the solution of nine simultaneous, non-linear equations appears to be involved, chiefly because the design-variable, C_F, appears in nearly all of the individual equations constituting (6.1-1). However, further examination reveals that if a fairly rough estimate of the optimum value of C_F is known beforehand, then the three equations

$$\partial(P_L/q)/\partial Q_H = 0, \quad \partial(P_L/q)/\partial W_H = 0 \text{ and } \partial(P_L/q)/\partial d_T = 0 \qquad (6.3\text{-}1)$$

can be regarded as independent of the remaining equations in (6.1-1). In fact, optimum values of Q_H and W_H can be obtained by solving simplified versions of the first two equations by an iterative method, whereas d_T can be obtained directly from a simplified version of the third equation. This approach is considered suitable for doing this and the relevant equations are derived and presented in the following sub-sections.

Even though the three equations in the foregoing paragraph are regarded as independent of the rest, there is still no easy solution for the remaining six equations in (6.1-1). The reason for this is as follows:

The formulation of the equations

$$\partial(P_L/q)/\partial C_F = 0 \quad \text{and} \quad \partial(P_L/q)/\partial t_{C,i} = 0 \qquad (6.3\text{-}2)$$

involves obtaining analytical expressions for $\partial(r_5-r_1)/\partial C_F$ and $\partial(r_5-r_1)/\partial t_{C,i}$; this becomes evident upon noting that (r_5-r_1) is a term appearing in the merit-factor equation, as indicated in 3.7.1. Furthermore, (r_5-r_1) can be seen - from statements subsequent to the presentation of (3.7-6) - to bear such a complicated relationship to C_F and $t_{C,i}$ that it would be impracticable to try to formulate equations (6.3-2).

Consequently, it is concluded that unless optimum or near-optimum values of C_F and $t_{C,i}$ are known beforehand, no easy method of predicting optimum values of the remaining design variables, D_1, r_p, s and t exists. Therefore,

attempts at establishing an approximate method in the present investigation are restricted to the case where C_F and $t_{C,i}$ are both fixed at values considered to be close to their optimum values. Such values are easily obtained for present purposes since they are provided by the accurate numerical method described in the previous section. From graphs 3 and 4 of chapter 9, $t_{C,i}$ and C_F are seen to remain essentially constant with respect to q, the waste-heat rejection rate, although it is conceded that this applies to only one set of operating conditions (p_{atm}, $t_{G,i}$ and $t^*_{G,i}$).

The fixing of C_F and $t_{C,i}$ does not constitute a serious departure from reality as the Central Electricity Generating Board, when calling for cooling-tower design tenders, specifies fixed values of C_F and $t_{C,i}$, or fixed quantities from which these design variables can be calculated.

The simplification afforded by fixing C_F and $t_{C,i}$ is considerable. For example, in the (r_5-r_1) equation, given by (3.7-6), the factor $(D)_{chil}/x$ can be directly estimated from (3.7-7) and (E.1-4). Also, $N_{h,G}$ can be evaluated from equations (3.7-8) to (3.7-11), (E.1-6) and (E.1-4).

The establishment of an approximate method of obtaining optimum values of the remaining design variables, D_1, r_p, s and t now becomes relatively easy. First of all the equations

$$\partial(P_L/q)/\partial D_1 = 0, \quad \partial(P_L/q)/\partial r_p = 0,$$

$$\partial(P_L/q)/\partial s = 0, \quad \partial(P_L/q)/\partial t = 0, \qquad (6.3\text{-}3)$$

are formulated. Then they are simplified after examining the orders of magnitude of their constituent terms in the light of results obtained by the accurate numerical method described in the previous section. Each equation is then manipulated algebraically into a convenient final form so that an iterative method of solving for the design variables will result in rapid convergence.

The relevant equations are derived and presented in the following sub-sections. In all instances, wherever such choices exist, the least accurate forms of the formulae presented in 3.4.1, 3.5.1, 3.6.1 and 3.7.1 are employed; this is done to arrive at tractable equations. In the interests of brevity, explanations of the steps involved in the derivations are deliberately curtailed.

6.3.2 Optimum Hot-side heat flux, Q_H, and loading factor, W_H

In differentiating P_L/q with respect to Q_H and W_H, only those terms which are functions of Q_H and W_H need be considered. Thus, after perusing the equations in 3.4.1, 3.5.1 and 3.6.1, the expressions to be differentiated may be represented by the following proportionalities:

$$P_L/q \propto a^*\{Q_H/U_H - C_F(0.5 - 0.0833\ U_H C_F/Q_H)\} + b^* a_{P,B} C_F^{-1} \Delta p_{fr,H} + c^* \pounds_S/q, \qquad (6.3\text{-}4)$$

$$P_L/q \propto a^* Q_H/U_H + b^* a_{P,B} C_F^{-1} \Delta p_{fr,H}. \qquad (6.3\text{-}5)$$

The last term in the ordinary brackets in (6.3-4), being small, is given the constant value, 0.067. After using (6.3-4) and (6.3-5) in formulating

$$\partial(P_L/q)/\partial Q_H = 0 \quad \text{and} \quad \partial(P_L/q)/\partial W_H = 0 \qquad (6.3\text{-}6)$$

respectively, these two equations can be combined and rearranged to give

$$\boxed{Q_H = \left[(c^*/a^*)\mathcal{L}_S^n(Q_H/Q_S)/\{0.84/u_H + 1.18/u_{m,H} + 1.54(Q_S/Q_H)u_S\}\right]^{0.5}} \qquad (6.3\text{-}7)$$

and

$$\boxed{W_H = (a/b)^{\frac{1}{3}}} \qquad (6.3\text{-}8)$$

where the temporary symbols a and b are defined by

$$b \equiv c^*\mathcal{L}_S^n(Q_H/Q_S)\{0.0833(Q_S/Q_H)/u_S + 0.8/u_H\} ,$$

$$c \equiv b^* a_{P,B} a_{p,H} f_{f,H}\{1.98/u_H + 2.78/u_{m,H} + 3.62\ (Q_S/Q_H)/u_S\} .$$

Optimum values of Q_H and W_H are obtained by solving (6.3-7) and (6.3-8) iteratively, provided that representative numerical values of $q = 10^9$ and $C_F = 18$ are used in evaluating u_S; since these quantities occur to the one-twelfth power, u_S will be accurate even for large variations from the chosen values.

6.3.3 Optimum inside diameter of coupling-circuit duct, d_T

In formulating

$$\partial(P_L/q)/\partial d_T = 0 \qquad (6.3\text{-}9)$$

the required proportionality,

$$P_L/q \propto 2b^* a_{P,B} C_F^{-1} \Delta p_{fr,T} + 2\, c^* \pounds_T / q, \qquad (6.3\text{-}10)$$

can be used to give

$$d_T = \{(b^*/c^*) a_{P,B} a_{p,T} L_T^{-1} q C_F^{-3}\}^{1/6} (a)^{1/6} \qquad (6.3\text{-}11)$$

where the temporary symbol a is defined by

$$a \equiv (f_{f,T}) [5 - 2\{\ln(0.27\varepsilon_T/d_T\}^{-1}]/(1.6d_T - 2.65) \qquad (6.3\text{-}12)$$

After assuming a roughness factor of $\varepsilon_T = 0.001$ for bitumen-lined ducts, after examining the table on page 138 of ref.4, $(a)^{1/6}$ can be shown to be closely fitted by the expression

$$(a)^{1/6} = 0.63 d_T^{-0.304}, \quad \text{for } d_T \geq 3.0. \qquad (6.3\text{-}13)$$

The substitution of (6.3-13) into (6.3-11) results in the following explicit expression for d_T:

$$\boxed{d_T = 0.702\{(b^*/c^*) a_{P,B} a_{p,T} L_T^{-1} q C_F^{-3}\}^{0.128}.} \qquad (6.3\text{-}14)$$

The optimum value of d_T is independent of L_T as may be seen by referring to (3.5-6). A noteworthy point is that d_T must not be permitted to be greater than six feet in accordance with the restriction imposed by Stewarts and Lloyds in appendix I.

6.3.4 Optimum tower-base diameter, D_1

In formulating

$$\partial(P_L/q)/\partial D_1 = 0, \qquad (6.3\text{-}15)$$

the required proportionality,

$$P_L/q \propto b^* a_{P,B}\{(a_{grav}/g_o)/v_{f,T}\}\, C_F^{-1} r_p D_1 + c^*(\pounds_D/q), \quad (6.3\text{-}16)$$

can be used to obtain

$$0 = (c^*/q)\Big[d_3\{1-(1-t)^{0.5}\} + d_4 - 2d_5 D_1^{-3} + d_7 A r_p + 2d_1 D_1 + 3d_8 r_p t D_1^2 + 2\{d_2 + d_8(z_2 - z_p + 14.9)\}\, t D_1 + [d_6 + d_7\{r_p D_1 + 0.5(z_2 - z_p)\}]\,[\partial A/\partial D_1]\Big] + b^* a_{P,B} C_F^{-1}\{(a_{grav}/g_o)/v_{f,F}\}\, r_p \; ; \quad (6.3\text{-}17)$$

$\partial A/\partial D_1$ is given by

$$\partial A/\partial D_1 = (2.195 + 0.982)\Big[1.62\{(D)_{chil}/x\}^2 (N_{h,G}^{-1} + 0.5)^3 \{D_1^{-3} a_{v,s} (G_C/G_G)^{b_{v,s}} r_p - 3\Sigma N_v D_1^{-4}\} - 0.5(z_z - z_p)\Big]. \quad (6.3\text{-}18)$$

Numerical estimates of the orders of magnitude of the terms in the two foregoing equations indicate that:

i) Some ofthe terms in (6.3-17) are so small that they can be calculated at the representative values of $t = 0.75$, $D_1 = 300$, $A = 2.1 \times 10^5$, $r_p = 0.06$ and $z_2 - z_p = 7.125$ so that

$$d_3\{1-(1-t)^{0.5}\} + d_4 - 2d_5 D_1^{-3} + d_7 A r_p \approx 14, \quad (6.3\text{-}19)$$

$$d_2 + d_8(z_2 - z_p + 14.9) \approx 1.65, \quad (6.3\text{-}20)$$

$$d_7\{r_p D_1 + 0.5\,(z_2 - z_p)\} \approx 0.023. \quad (6.3\text{-}21)$$

ii) Some of the terms in (6.3-18) are negligibly small and may be ignored so that

$$D_1^{-3} a_{v,s} (G_C/G_G)^{b_{v,s}} r_p \approx 0, \qquad (6.3\text{-}22)$$

$$0.5\,(z_2 - z_p) \approx 0. \qquad (6.3\text{-}23)$$

After incorporating (6.3-19) to (6.3-23) in (6.3-17) and (6.3-18) and re-arranging, the optimum value of D_1 can be evaluated from

$$\boxed{D_1 = (a/b)^{0.25}\ ;} \qquad (6.3\text{-}24)$$

the temporary symbols a and b are defined as

$$a \equiv (10.64s + 4.77)\left[\{(D)_{chil}/x\}^2 (N_{h,G}^{-1} + 0.5)^3\, N_v\right],$$

$$b \equiv (d_6 + 0.023)^{-1}\left[14 + 2d_1 D_1 + 3d_8 r_p t D_1^2 + 3.3 t D_1 + q(b^*/c^*) a_{p,B} C_F^{-1}\{(a_{grav}/g_o)/v_{f,T}\}\, r_p\right].$$

6.3.5 Optimum under-pack height to tower diameter ratio, r_p

In formulating

$$\partial(P_L/q)/\partial r_p = 0, \qquad (6.3\text{-}25)$$

the required proportionality,

$$P_L/q \propto b^* a_{p,B} C_F^{-1}\{(a_{grav}/g_o)/v_{f,T}\}\, r_p D_1 + (c^*/q)\left[d_6 + d_7\{r_p D_1 + 0.5(z_2 - z_p)\} A + d_8 r_p t D_1^3\right], \qquad (6.3\text{-}26)$$

can be used to form

$$0 = b^{*}a_{p,B}C_F^{-1}\{(a_{grav}/g_o)/v_{f,T}\}D_1$$

$$+(c^{*}/q)\left[\left[d_6+d_7\{r_pD_1+0.5(z_2-z_p)\}\right](\partial A/\partial r_p)+d_7AD_1+d_8tD_1^{3}\right]; \quad (6.3\text{-}27)$$

$\partial A/\partial r_p$ is given by

$$\partial A/\partial r_p = (2.195+0.982)\left[1.62\{(D)_{chil}/x\}^2(N_{h,G}^{-1}+0.5)^3\right.$$

$$\left.\{D_1^{-2}a_{v,s}(G_C/G_G)^{b_{v,s}} - 2D_1^{-3}a_{v,i}r_p^{-3}-3\Sigma N_v D_1^{-4}\}\right]. \quad (6.3\text{-}28)$$

The relative insignificance of certain terms in these two equations makes it permissible to estimate them at the fixed representative values of $G_C/G_G = 1$, $D_1 = 300$, $r_p = 0.06$, $z_2-z_p = 7.125$, $t = 0.75$, $A = 2.1 \times 10^5$ and $a_{v,s} = 0.16$, so that

$$d_7\{r_pD_1 + 0.5(z_2-z_p)\} \approx 0.023, \quad (6.3\text{-}29)$$

$$d_7AD_1 + d_8tD_1^{3} \approx 1.3 \times 10^5, \quad (6.3\text{-}30)$$

$$D_1^{-2}a_{v,s}(G_C/G_G)^{b_{v,s}} \approx 1.8 \times 10^{-6}. \quad (6.3\text{-}31)$$

In addition the ΣN_v term in (6.3-28) is negligibly small and may be replaced by

$$3\Sigma N_v D_1^{-4} \approx 0. \quad (6.3\text{-}32)$$

After incorporating (6.3-29) to (6.3-32) in (6.3-27) and (6.3-28), and then re-arranging, the optimum value of r_p is given by

$$\boxed{r_p = (a/b)^{-\frac{1}{3}};} \quad (6.3\text{-}33)$$

the temporary symbols a, b, c and d are defined as follows:

$$a \equiv (c/d) + 1.8 \times 10^{-6},$$

$$b \equiv 2a_{v,i}D_1^{-3},$$

$$c \equiv (d_6 + 0.023)^{-1}[(b^*/c^*)qa_{p,B}C_F^{-1}\{(a_{grav}/g_o)/v_{f,T}\}D_1 + 1.3\times10^5],$$

$$d \equiv (3.55s + 1.59)\{(D)_{chil}/x\}^2 (N_{h,G}^{-1} + 0.5)^3.$$

6.3.6 Optimum throat to base diameter ratio, s.

In formulating

$$\partial(P_L/q)/\partial s = 0, \qquad (6.3\text{-}34)$$

the required proportionality,

$$P_L/q \propto (c^*/q)[d_6 + d_7\{r_p D_1 + 0.5(z_2 - z_p)\}]\, A, \qquad (6.3\text{-}35)$$

can be used to obtain

$$0 = \{(2.19A)/(2.195 + 0.982)\} - 6.48(2.19s + 0.982)\{(D)_{chil}/x\}^2 D_1^{-3}(N_{h,G}^{-1} + 0.5)^3 s^{-5}. \qquad (6.3\text{-}36)$$

The incorporation of (3.7-6) in (6.3-36), after ignoring the relatively small $(z_2 - z_p)$ term in (3.7-6), results in the following expression for the optimum value of s:

$$\boxed{s = \{0.547\Sigma N_v/(2.19s + 0.982)\}^{-1/5}.} \qquad (6.3\text{-}37)$$

6.3.7 Optimum blanking factor, t.

In formulating

$$\partial(P_L/q)/\partial t = 0, \qquad (6.3\text{-}38)$$

the required proportionality,

$$P_L/q \propto (c^*/q)\left[d_2 t D_1^2 + d_3\{1-(1-t)^{0.5}\}D_1 + [d_6 + d_7\{r_p D_1 + 0.5(z_2-z_p)\}]A + d_8 t D_1^2\{r_p D_1 + (z_2-z_p) + 14.9\}\right], \quad (6.3\text{-}39)$$

can be used to give

$$0 = (c^*/q)\left[d_2 D_1^2 + 0.5 d_3 D_1 (1-t)^{-0.5} + [d_6 + d_7\{r_p D_1 + 0.5(z_2-z_p)\}](\partial A/\partial t) + d_8 D_1^2\{r_p D_1 + (z_2-z_p) + 14.9\}\right] ; \quad (6.3\text{-}40)$$

$\partial A/\partial t$ is given by

$$\partial A/\partial t = -(7.1s+3.18)\left[\{(D)_{chil}/x\}^2 D_1^{-3}(N_{h,G}+0.5)^3 (n_{v,pack}+n_{v,elim})t^{-3}\right]. \quad (6.3\text{-}41)$$

Some terms in (6.3-40) are so small that they may be estimated at the fixed representative values of $D_1 = 300$, $t = 0.75$, $r_p = 0.06$ and $(z_2-z_p) = 7.125$ so that

$$0.5 d_3 D_1 (1-t)^{-0.5} \approx 2000, \quad (6.3\text{-}42)$$

$$d_8 D_1^2\{r_p D_1 + (z_2-z_p) + 14.9\} \approx 0.129 D_1^2, \quad (6.3\text{-}43)$$

$$d_7\{r_p D_1 + 0.5(z_2-z_p)\} \approx 0.023. \quad (6.3\text{-}44)$$

After the substitution of (6.3-42) to (6.3-44) into (6.3-40) and subsequent re-arrangement, the optimum value of t is given by

$$\boxed{t = (a/b)^{\frac{1}{3}},} \quad (6.3\text{-}45)$$

where the temporary symbols a and b are defined as

$$a \equiv (7.1s+3.18)\{(D)_{chil}/x\}^2 D_1^{-3}(N_{h,G}+0.5)^3(n_{v,pack}+n_{v,elim}),$$

$$b \equiv (d_6+0.023)^{-1}\{(d_2+0.129)D_1^{\,2}+2000\} .$$

7 METHODS FOR THE PREDICTION OF OPTIMUM-DESIGN CONDITIONS FOR THE HELLER SYSTEM

7.1 Introduction

The procedure followed in developing optimization methods for the Heller system is similar to that in the previous chapter, viz. for the Wet system. In the first instance, the accurate numerical method - the application of which has been described in 6.2 - is used to obtain optimum values of the following Heller-system design variables, finally adopted in 3.7.2:

W_C, C_G, C_F/C_G, $n_{h,G}$, n_r, d_T, D_1, r_1 and s.

Then, after noting the numerical values of and the trends displayed by various quantities, a method of determining approximate values of the design variables - not requiring the use of digital computer - is established in the same way as for the Wet system.

Attention is drawn here to the fact that, in following the above procedure, one of the design variables, viz. the number of tower heat-exchanger tube rows, n_r, is fixed at an even-integer value. This restriction on n_r is necessary otherwise optimizing procedures will generally result in non-integer values for n_r, and this does not happen in practice according to the requirements for tower heat exchangers ennumerated in 4.3.1. The assumption of an even-integer value for n_r arises from the fact that two Cold-side passes

have been assumed for the Heller system in chapter 4; the choice of an odd-integer value, not being a multiple of two, would give rise to construction problems.

In order to determine the required numerical value of n_r the following procedure is carried out:

i) Choose a range of even-integer values for n_r.

ii) Use the accurate numerical method of optimization in determining the minimum value of the merit-factor in (2.1-8) for each of the chosen n_r values. The required numerical value of n_r is that one which results in the smallest value of the merit-factor.

Numerical results accruing from the application of the accurate numerical method are given in chapter 9. The establishment of the approximate method, and the derivation of equations pertaining thereto, is described in the next section.

7.3 Approximate method of predicting optimum values of design variables

7.3.1 Establishment of method

The first step is to derive the set of equations, given by

$$\partial(P_L/q)/\partial x_i = 0, \qquad (7.3\text{-}1)$$

for each of the design variables listed in 7.1 - except for n_r - making use of the least accurate forms of the formulae presented in 3.4.2, 3.5.2, 3.6.2 and 3.7.2 in order to arrive

at the most tractable equations.

After formulating the equations (7.3-1), re-arranging them and then attempting to obtain optimum values of the design variables by an iterative method, no solution was forthcoming. Examination of the equations under consideration revealed that they were ill-conditioned, due chiefly to the fact that the forms of the two equations

$$\partial(P_L/q)/\partial n_{h,G} = 0 \qquad (7.3\text{-}2)$$

and

$$\partial(P_L/q)/\partial r_1 = 0 \qquad (7.3\text{-}3)$$

were such that simple iteration generally resulted in values of r_1 and $n_{h,G}$ which would not converge rapidly enough to prevent values of the remaining design variables from diverging.

One established method of increasing the probability of ultimate convergence is to obtain more accurate values of $n_{h,G}$ and r_1, for a particular iteration cycle, by carrying out a few iterations on each of the individual equations represented by (7.3-2) and (7.3-3). Unfortunately, this course of action requires more calculation as it involves iterations within an iteration.

Examination of the results in chapter 9, obtained by the accurate numerical method, reveal that insofar as $n_{h,G}$ is concerned the course of action suggested in the previous paragraph is unavoidable. However, the value of the merit-

factor can be seen from the results in chapter 9 to be relatively insensitive to the value of r_1, even although r_1 does have an optimum value. This occurrence is explained by the fact that the increase in cooling tower price, accruing from an increase in r_1, is almost completely compensated for by the accompanying decrease in inlet velocity-head loss of air flowing through the tower. The occurrence is a doubly fortunate one in that:

i) It enables r_1 to be fixed at a representative value, thereby eliminating it as a design variable; (7.3-3) can therefore be removed from (7.3-1) and will no longer be an obstacle to iteration processes.

ii) The ability to assign to r_1 numerical values which can vary over an appreciable range, without significantly affecting the value of the merit-factor, enables the inlet height of a cooling tower to be changed. This, in turn, gives a designer a certain amount of freedom if the optimum dimensions of the adopted heat exchanger are such that the tower inlet height has to be changed from its originally chosen value in order to ensure a successful fit into the tower base.

In view of the foregoing comments, it is assumed that r_1 can be fixed when establishing the approximate method of optimization in the present investigation.

Upon adopting the proposed modifications concerning $n_{h,G}$ and r_1, the equations in (7.3-1) did respond to an

iterative method of solution but, unfortunately, gave values of the optimum design variables which compared unfavourably with the values obtained by the accurate numerical method. Detailed examination of the equations revealed that the most likely source of error lay in the adoption of the approximate area equation (J.2-2) which, while proving satisfactory for the Wet system, was not suitable for the Heller system. In consequence, it was decided to adopt the more accurate equation (J.2-1). However, because it is considerably more complicated than (J.2-2), an attempt was made to restrict its use to the smallest possible number of equations in (7.3-1) consistent with required accuracy. In fact, it was found necessary to use (J.2-1) only in the formulation of the equation

$$\partial(P_L/q)/\partial s = 0. \qquad (7.3\text{-}4)$$

The derivation of equations from which approximate optimum values of the design variables can be evaluated now follows.

7.3.2 Optimum inside diameter of coupling-circuit duct, d_T

In formulating

$$\partial(P_L/q)/\partial d_T = 0 \qquad (7.3\text{-}5)$$

the required proportionality,

$$P_L/q \propto 2b^* a_{p,T} C_G^{-1} (C_F/C_G)^{-1} \Delta p_{fr,T} 2c^* \ell_T/q , \qquad (7.3\text{-}6)$$

can be used to obtain

$$d_T = 0.702\{(b^*/c^*)a_{P,T}a_{p,T}L_T^{-1}q\}^{0.128} C_G^{-0.384} (C_F/C_G)^{-0.384}, \quad (7.3\text{-}7)$$

after following a procedure analogous to that in 6.3.3 for the Wet system.

The optimum value of d_T is independent of L_T as may be seen by referring to (3.5-6). In applying (7.3-7), d_T must not be permitted to be greater than six feet in accordance with the restriction imposed by Stewarts and Lloyds in appendix I.

7.3.3 Optimum heat-capacity factor ratio, C_F/C_G.

In formulating

$$\partial(P_L/q)/\partial(C_F/C_G) = 0 \quad (7.3\text{-}8)$$

the required proportionality is

$$P_L/q \propto a^*C_G/e_G + b^*a_{P,T}C_G^{-1}(C_F/C_G)^{-1}\{d+(1+\eta_{pt})\Delta p_{fr,T}\}; \quad (7.3\text{-}9)$$

the temporary symbol d is defined by

$$d \equiv \eta_{pt}\Delta p_H + (1-\eta_{pt})[p_{atm} - p_{s,S} + \{a_{grav}/(g_o v_{f,C})\}(r_1 D_1 - z_H)],$$

after noting that $r_1 \equiv z_1/D_1$.

After differentiating (7.3-9) and then re-arranging, the following expression for the optimum value of C_F/C_G results:

$$C_F/C_G = C_G^{-1}[0.5(d/a) + \{0.25(d/a)^2 + (c/a)\}^{0.5}]^{0.5}; \quad (7.3\text{-}10)$$

the temporary symbols a and c are defined as

$$a \equiv \{1.02a^*/b^*a_{p,T})\}\{0.5-0.125n_{h,G}+0.125n_{h,G}(C_F/C_G)\} ,$$

$$c \equiv 3a_{p,T}(1+\eta_{pt})f_{f,T}d_T^{-5}.$$

7.3.4 Optimum Air-side heat-capacity factor, C_G

In formulating

$$\partial(P_L/q)/\partial C_G = 0 \qquad (7.3\text{-}11)$$

the required proportionality,

$$P_L/q \propto a^*[\{t_{C,i}-(t_{m,G})_o\}+ C_G/e_G]$$

$$+b^*a_{p,T}C_G^{-1}(C_F/C_G)^{-1}\Sigma\Delta p+(c^*/q)(d_1+d_2r_1D_1)A+c^*\mathcal{E}_G''Q_G^{-1}, \qquad (7.3\text{-}12)$$

can be used to obtain the following expression for the optimum value of C_G:

$$\boxed{C_G = [(0.5/a)\{f+(f^2+4ag)^{0.5}\}]^{0.5}.} \qquad (7.3\text{-}13)$$

The temporary symbols, a, d, e, f and g are defined as follows:

$$a \equiv a^*(Q_C/Q_G)(1/u_C+1/u_{m,C})a_{q,G}n_{h,G}^{j_1}+a^*/e_G ,$$

$$d \equiv \eta_{pt}\Delta p_H+(1-\eta_{pt})[p_{atm}-p_{s,S}+\{a_{grav}/(g_o v_{f,C})\}(r_1D_1-z_{II})] ,$$

$$e \equiv (d_1+d_2r_1D_1)(2.19s+0.982)\{438v_{G,i}^2/(g_o c_G^2 a_{grav}/g_o)\} ,$$

$$f \equiv b^*a_{p,T}d(C_F/C_G)^{-1}+0.5b^*a_{p,T}a_{p,C}a_{q,G}^{-1}(1+\eta_{pt})f_{f,C}W_C^3n_{h,G}^{-j_1}$$

$$+0.0625\pi^2c^*(e/q)a_{n,G}D_1n_{h,G}^{j_o}+c^*\mathcal{E}_G''a_{q,G}^{-1}n_{h,G}^{-j_1}$$

$$g \equiv 3b^* a_{p,T} a_{p,T} (1+\eta_{pt}) f_{f,T} d_T^{-5} (c_F/c_G)^{-3}$$

$$+ 3c^* eq D_1^{-3} (N_{v,o} + N_{v,i} + N_{v,misc}).$$

7.3.5 Optimum Cold-side loading factor, W_C

In formulating

$$\partial(P_L/q)/\partial W_C = 0 \qquad (7.3\text{-}14)$$

the required proportionality,

$$P_L/q \propto a^* \{t_{C,i} - (t_{m,G})_o\} + 0.5 b^* a_{p,T} c_G^{-1} (c_F/c_G)^{-1} (1+\eta_{pt}) \Delta p_{fr,C}, \qquad (7.3\text{-}15)$$

can be used to obtain the following expression for the optimum value of W_C:

$$\boxed{W_C = (a/b)^{0.28}.} \qquad (7.3\text{-}16)$$

The temporary symbols a and b are defined by

$$a \equiv 0.576 a^* a_{u,C}^{-1} a_{q,G}^2 (Q_C/Q_G) c_G^2 n_{h,G}^{2j_1},$$

$$b \equiv b^* a_{p,T} a_{p,C} a_{f,C} (1+\eta_{pt}).$$

7.3.6 Optimum tower-base diameter, D_1

In formulating

$$\partial(P_L/q)/\partial D_1 = 0 \qquad (7.3\text{-}17)$$

the relevant proportionality,

$$P_L/q \propto b^* a_{P,T} c_G^{-1} (c_F/c_G)^{-1} (1-\eta_{pt}) \{a_{grav}/(g_o v_{f,C})\} r_1 D_1$$

$$+ (c^*/q)(d_1 + d_2 r_1 D_1) A, \qquad (7.3\text{-}18)$$

can be used to arrive at the following expression for the optimum value of D_1:

$$\boxed{D_1 = q^{0.5}\,(a/b)^{0.25}\ ;} \qquad (7.3\text{-}19)$$

the temporary symbols a, b, d and e are defined as

$$a \equiv N_{v,o} + N_{v,i} + N_{v,\,misc}\ ,$$

$$b \equiv (qr_1 d/e)/(3d_1+2d_2r_1D_1) + 0.206\ a_{n,G}c_G^2 n_{h,G}^{j_o}\ ,$$

$$d \equiv b^* a_{p,T}c_G^{-1}(c_F/c_G)^{-1}(1-\eta_{pt})\{a_{grav}/(g_o v_{f,C})\}\ ,$$

$$e \equiv c^*\{438 v_{G,i}^2/(g_o c_G^2 a_{grav}/g_o)\}(2.195+0.982)c_G^{-3}\ .$$

In deriving (7.3-19) it is assumed that the ratio $(3d_1+2d_2r_1D_1)/(d_1+d_2r_1D_1)$ is equal to 3, an acceptable assumption in view of the fact that the terms containing d_2 are relatively small.

7.3.7 Optimum throat-to base-diameter ratio, s

As mentioned in 7.3.1, the area equation (J.2-1) is used in the formulation of

$$\partial(P_L/q)/\partial s = 0 \qquad (7.3\text{-}20)$$

instead of the less accurate (J.2-2) in order to obtain sufficiently accurate optimum values of the design variables. The use of the required proportionality,

$$P_L/q \propto (c^*/q)(d_1+d_2r_1D_1)A, \qquad (7.3\text{-}21)$$

then results in the following expression for the optimum value of s:

$$\boxed{s = (a/b)^{0.2}} \quad . \qquad (7.3\text{-}22)$$

The temporary symbols a and b are defined as

$$a \equiv 4\Sigma N_v^{-1}\left[\{B_A - A_A(s-1)^2\}s + C_A\right],$$

$$b \equiv A_A(s-1)(3s-1) + B_A \ .$$

7.3.8 Optimum partial number of Air-side transfer units, $n_{h,G}$

In formulating

$$\partial(P_L/q)/\partial n_{h,G} = 0 \qquad (7.3\text{-}23)$$

the required proportionality,

$$P_L/q \propto a^*\left[\{t_{C,i} - (t_{m,G})\} + C_G/e_G\right] + 0.5b^* a_{p,T}(1+\eta_{pt})c_G^{-1}(C_F/C_G)^{-1}\Delta p_{fr,C}$$

$$+ (c^*/q)(d_1 + d_2 r_1 D_1)A + c^* \pounds_G^n Q_G^{-1} \ , \qquad (7.3\text{-}24)$$

can be used to form the following expression for the optimum value of $n_{h,G}$:

$$\boxed{n_{h,G} = \{(1.62aq)/(c^* j_o d a_{n,G} D_1)\}^{1/(j_o-1)}} \quad . \qquad (7.3\text{-}25)$$

The temporary symbols a and d are defined by

$$a \equiv j_1 a_{q,G}^{-1}\{0.5b^* a_{P,T}(1+\eta_{pt})a_{p,C} f_{f,C} W_C^3 + c^* \pounds_G^n\} n_{h,G}^{-(1+j_1)}$$

$$- a^*(Q_C/Q_G)(1/u_C + 1/u_{m,C}) j_1 a_{q,G} c_G^2 n_{h,G}^{j_1}$$

$$-1.02a^* c_G^2\{0.0625 - n_{h,G}^{-2} - 0.125(C_F/C_G) + 0.0625(C_F/C_G)^2\} \ ,$$

$$d \equiv \{438 v_{G,i}^2/(g_o c_G^2 a_{grav}/g_o)\}(2.19s + 0.982)(d_1 + d_2 r_1 D_1).$$

As mentioned in 7.3.1, (7.3-25) requires to be iterated a few times in order to obtain a more accurate value of $n_{h,G}$ for use in the overall iteration cycle.

8 METHODS FOR THE PREDICTION OF OPTIMUM-DESIGN CONDITIONS FOR THE SPALDING SYSTEM

8.1 Introduction

The accurate numerical method of determining the values of optimum design variables for the Spalding system is carried out in a similar fashion to that for the Heller system.

The number of tube rows, n_r, is fixed at an integer value for the same reasons and according to the same procedure as outlined in 7.1. However, because one Cold-side pass has been previously adopted for the Spalding system, the integer value need not be an even number as is required for the Heller system.

Numerical results in chapter 9, accruing from the application of the accurate numerical method, indicate that there is no optimum value of r_1 for the Heller system. The form of the tower price equation (J.4-1) is such that the increase in price occasioned by an increase in r_1 is more than compensated for by the accompanying decrease in inlet velocity-head loss of the air flowing through the tower. The end result is that the merit-factor continues to decrease indefinitely with increase in r_1 until the tower-price equation and the inlet velocity-head formula are no longer valid. This situation - which does not arise in the Heller system due to the increase in pumping power with z_1 as shown in (3.5-10) and (3.5-11) - cannot happen in practice and something must be done to prevent r_1 from increasing to unaccept-

able values. Fortunately, the situation is not serious as numerical results obtained from the accurate numerical method and displayed in chapter 2 indicate that the merit-factor is relatively insensitive to changes in r_1. Therefore, r_1 can be eliminated as a design variable by fixing it at a value suitable for convenient fitting of the tower heat exchanger into the tower base. This procedure, applicable to both the accurate and approximate methods, is thus the same as that applied in the approximate method for the Heller system.

The application of the accurate method also shows that, for the chosen refrigerant, viz. ammonia, the optimum value of the design variable W_C (see 3.7.3) is so high that:

i) an impermissably high Cold-side pressure drop results, thereby causing a variation in saturation temperature along the tubes which will decrease the tower heat-exchanger efficiency to a significantly lower value than that for no variation;

ii) the range of applicability for (3.5-22), given in C.2, is exceeded.

Therefore W_C must also be removed as a design variable by fixing it at a value which will not give rise to the itemized situations. This is done according to the following procedure:

By examining (E.2-4) - with $n_p = 1$ - and (E.3-4), it can be seen that, for a representative value of $n_{h,G} = 1.3$ obtained from the results in chapter 9, the cross-flow effect-

iveness is practically equivalent to that of the counter-flow case for values of $C_C/C_G \leq 0.20$. Therefore, if the efficiency of the Spalding-system tower heat exchanger is not to fall significantly below the value resulting from the use of (E.3-4) - thereby invalidating the use of (E.3-4) in preference to (E.2-4) - C_C must obey the relationship $C_C \leq 0.20 C_G$. As C_C represents the variation of refrigeration saturation temperature on the Cold-side, and a representative value for C_G can be seen from the results in chapter 9 to be about 16, it can be said that the saturation temperature variation should be kept to a maximum of (0.20 x 16) = 3.2 deg F. This corresponds to a pressure drop of approximately 1500 psf for ammonia according to the physical property equations in appendix A.

The substitution of representative values accruing from the application of the accurate numerical method into (3.5-22) reveals that a value of

$$W_C = 1.53 \times 10^5 \qquad (8.1\text{-}1)$$

will give a Cold-side pressure drop of about 1500 psf without any of the ranges of applicability relating to (3.5-22) being exceeded. Accordingly, W_C is removed as a Spalding-system design variable in the present investigation as a result of adopting (8.1-1).

There is some doubt as to which formula to use when calculating heat-transfer coefficients from the tube inside walls to the refrigerant in the surface steam-condenser, i.e.

the Hot-side. Since ammonia has been chosen as the most suitable refrigerant, the choice of formulae is restricted to two, viz. (D.1-1), which is re-expressed in (3.4-7), and (D.4-1). The former formula is applicable to single-phase forced convection, whereas the latter refers to saturated-boiling conditions.

Due to the large variations in boiling heat-transfer coefficients for ammonia obtained by various investigators, as may be seen on page 439 of ref.19, it is considered unwise to consider (D.4-1) as providing more than just a qualitative estimate of the Hot-side heat-transfer coefficient. In consequence, (B.4-7) is adopted for the evaluation of the Hot-side heat-transfer coefficient in the approximate method of optimization. Results in chapter 9, accruing from the accurate method of optimization, indicate that this pessimistic choice results in heat-transfer coefficients comparable in value to those given by (D.4-1), at the expense of greater recirculation of refrigerant and a resultant increase in pumping power requirements. However, since the Spalding-system pumping power is small in any event, the merit-factor remains practically unaltered. The fact that there will be an increase in heat transfer rate, under practical conditions, due to agitation caused by bubble formation may be regarded as a design safety factor.

The derivation and presentation of equations for use in the approximate method of predicting optimum values of

design variables are carried out in the next section.

8.2 Approximate method of predicting optimum values of design variables.

8.2.1 Establishment of method

Having fixed the design variables n_r, r_1 and W_C and having decided to use a single-phase equation for predicting the Hot-side heat-transfer coefficient, the equations

$$\partial(P_L/q)/\partial x_i = 0 \qquad (8.2\text{-}1)$$

were derived for each of the following remaining Spalding-system design variables:

Q_H, W_H, X_H, d_B, d_T, C_G, $n_{h,G}$, D_1 and s,

making use of the least accurate formulae appearing in chapter 3. However, an attempt to re-arrange the equations in such a manner as to permit an interation procedure similar to that for the Heller system met with failure due to the ill-conditioned forms of the following equations:

$$\partial(P_L/q)/\partial Q_H = 0, \qquad (8.2\text{-}2)$$

$$\partial(P_L/q)/\partial W_H = 0, \qquad (8.2\text{-}3)$$

$$\partial(P_L/q)/\partial X_H = 0. \qquad (8.2\text{-}4)$$

It was found that the success of an iterative method could only be ensured by solving for Q_H, W_H and X_H according to the following procedure:

i) Substitute (8.2-4) into (8.2-3) and re-arrange in

order to arrive at an equation suitable for the direct evaluation of Q_{II}.

ii) Re-arrange (8.2-2) and (8.2-4) in such a manner that W_H and X_H can be evaluated by means of graphical interpolation. The details of the method of interpolation are given in the next sub-section after deriving and re-arranging the relevant equations.

Having carried out the above procedure, the same iterative procedure as employed in the Heller system - including internal iterations of the

$$\partial(P_L/q)/\partial n_{h,G} = 0 \qquad (8.2\text{-}5)$$

equation for each overall iteration cycle - was successfully used in arriving at a solution. However, the optimum values of the design variables were found to be in poor agreement with those resulting from the accurate method. As in the Heller system, the approximate tower area equation (J.2-2) was found to be the source of error and the more accurate equation, (J.2-1), had to be resorted to in formulating

$$\partial(P_L/q)/\partial D_1 = 0 \qquad (8.2\text{-}4)$$

and

$$\partial(P_L/q)/\partial s = 0 \qquad (8.2\text{-}5)$$

in order to obtain satisfactory results.

The derivations of the equations represented by (8.2-1) are carried out in the following sub-sections.

8.2.2 Optimum Hot-side heat flux, Q_H, loading factor, W_H, and exit vapour quality, X_H.

In formulating

$$\partial(P_L/q)/\partial Q_H = 0, \tag{8.2-6}$$

$$\partial(P_L/q)/\partial W_H = 0 \tag{8.2-7}$$

and

$$\partial(P_L/q)/\partial X_H = 0 \tag{8.2-8}$$

the relevant proportionalities are, respectively:

$$P_L/q \propto a^* Q_H\{u_{m,H}^{-1} + (Q_S/Q_H)/u_S\} + a^*\{(t_{m,H})_o - (t_{s,H})_o\}$$
$$+ b^* a_{P,H} C_F^{-1} X_H^{-1} \Delta p_H + c^* \pounds_S/q + c^* \pounds_R/q, \tag{8.2-9}$$

$$P_L/q \propto a^* Q_H (Q_S/Q_H) u_S^{-1} + a^*\{(t_{m,H})_o - (t_{s,H})_o\} + b^* a_{P,H} C_F^{-1} X_H^{-1} \Delta p_H, \tag{8.2-10}$$

$P_L/q \propto$ same as (8.2-10). (8.2-11)

After using (8.2-9) to (8.2-11) in formulating (8.2-6) to (8.2-8), and then substituting (8.2-8) into (8.2-6), the following expression results:

$$aQ_H^2 + bQ_H - c = 0, \tag{8.2-12}$$

where the temporary symbols a, b and c are defined by

$$a \equiv 2.776a^*\{u_{m,H}^{-1} + 1.303(Q_S/Q_H)u_S^{-1} + 0.712u_H^{-1}\},$$

$$b \equiv 2a_{a,H}(b^* a_{P,H} C_F^{-1} X_H^{-1} + 0.5a^* a_{t,H}) X_H W_H^2,$$

$$c \equiv 2.776c^*(Q_H/Q_S)(1.55 + \pounds''_{S,tubes}) + 0.694c^* a_R (d_H/v_{f,F}).$$

On substituting representative Spalding-system values into these definitions it is noted that b is so small that it can be ignored. In addition, the cost of ammonia is so small that the a_R term in c can also be ignored. In consequence, the optimum value of Q_H may be obtained from the following expression:

$$Q_H = \left[\{(c^*/a^*)(Q_H/Q_S)(1.55+\pounds''_{S,tubes})\}/\{u_{m,H}^{-1} + 1.303(Q_S/Q_H)u_S^{-1} + 0.712u_H^{-1}\}\right]^{0.5} \quad (8.2\text{-}13)$$

With regard to the determination of X_H and W_H, it has been found convenient to re-arrange (8.2-6) and (8.2-8) as follows:

$$W_H^{2.776} = d/e, \quad (8.2\text{-}14)$$

where the temporary varibles d and e are defined by

$$d \equiv a^*\{u_{m,H}^{-1} + 1.333(Q_S/Q_H)u_S^{-1} + u_H^{-1}\}Q_H^2 - 0.25c^* a_R(d_H/v_{f,F}) - c^*(Q_H/Q_S)(1.55+\pounds''_{S,tubes}),$$

$$e \equiv 0.25\, a_{p,H} d_H (b^* a_{P,H} C_F^{-1} X_H^{-1} + 0.5a^* a_{t,H}) C_F J_H X_H$$

and

$$W_H^{2.776} = f/g, \quad (8.2\text{-}15)$$

where the temporary symbols f and g are defined by

$$f \equiv 0.0833\, a^*(Q_S/Q_H)u_S^{-1}Q_H + b^* a_{p,H} C_F^{-1} X_H^{-1}(\Delta p_{fr,T} + \Delta p_{fr,B} + \Delta p_{fr,C}),$$

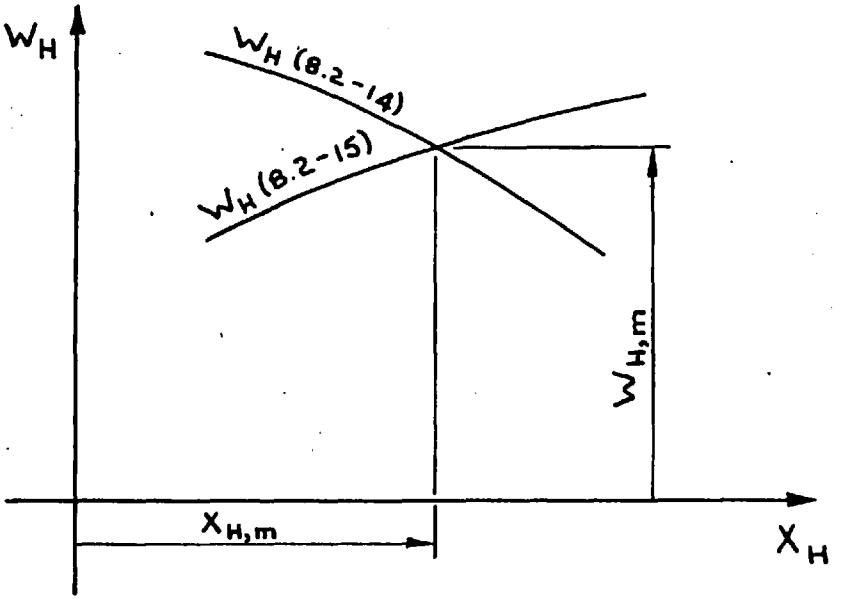
$$g \equiv 0{\cdot}125 a_{p,H} d_H [a^* a_{t,H} c_F (b_{p,H}^2 X_H^{1{\cdot}45} + 2 b_{p,H} X_H^{0{\cdot}723} + 1) X_H$$
$$+ 2 b^* a_{p,H} (0{\cdot}593 b_{p,H}^2 X_H^{1{\cdot}45} + 0{\cdot}839 b_{p,H} X_H^{0{\cdot}723})] Q_H^{-1}$$
$$+ 0{\cdot}5 a^* a_{t,H} a_{a,H} X_H W_H^{-0{\cdot}776} .$$

The following procedure is adopted when solving for W_H and X_H from (8.2-14) and (8.2-15):

i) Choose an arbitrary X_H value.

ii) Calculate W_H from both (8.2-14) and (8.2-15).

iii) Go back to i) and repeat i) to iii) until sufficient values have been recorded to plot a graph of W_H vs X_H as follows:



iv) The values $W_{H,m}$ and $X_{H,m}$ shown on the graph are the required values of W_H and X_H.

8.2.3 Optimum inside diameters of coupling-circuit ducts, d_T and d_B.

In formulating

$$\partial(P_L/q)/\partial d_T = 0 \qquad (8.2\text{-}16)$$

the relevant proportionality,

$$P_L/q \propto b^* a_{P,H} C_H^{-1} \Delta p_{fr,T} + c^* \mathcal{L}_T/q + c^* \mathcal{L}_R/q, \qquad (8.2\text{-}17)$$

can be used to form

$$d_T = \{(b^*/c^*) a_{P,H} a_{p,T} L_T^{-1} q C_F^{-3} X_H^{-1}\}^{1/6} (a)^{1/6}, \qquad (8.2\text{-}18)$$

where the temporary symbol a is defined by

$$a \equiv \left[\left[5-2\{\ln(0.27027\varepsilon_T/d_T\}^{-1}\right]/\left[f_{g,T}^{-1}\{(1.4d_T-1.3)+0.5\pi a_R v_{g,F} d_T\}\right]\right]^{1/6}.$$

With the assumption of a roughness factor $\varepsilon_T = 0.001$, given on page 138 of ref.4, a can be closely approximated by the formula

$$a = 0.561\, d_T^{-0.253}.$$

Substitution of a into (8.2-18) results in the following expression for the optimum value of d_T:

$$\boxed{d_T = \left[0.63\ (b^*/c^*) a_{P,H} a_{p,T} L_T^{-1} q C_F^{-3} X_H^{-1}\right]^{0.133}.} \qquad (8.2\text{-}19)$$

An expression for the optimum value of d_B can be derived in precisely the same way, so that

$$\boxed{d_B = 0.49\left[(b^*/c^*) a_{P,H} a_{p,B} L_T^{-1} q C_F^{-3} X_H^{-1}\right]^{0.137}.} \qquad (8.2\text{-}20)$$

(8.2-19) and (8.2-20) are both independent of L_T and are restricted to values of d less than or equal to 6 feet, as

for the other two cooling systems.

8.2.4 Optimum Air-side heat capacity factor, C_G

In formulating

$$\partial(P_L/q)/\partial C_G = 0 \qquad (8.2\text{-}21)$$

the required proportionality,

$$P_L/q \propto a^*[\{t_{C,i}-(t_{m,G})_o\}+C_G/e_G]+b^*a_{P,H}C_F^{-1}X_H^{-1}\Delta p_{fr,C} + c^*(\pounds_D/q+\pounds_G/q+\pounds_R/q), \qquad (8.2\text{-}22)$$

can be used to form the following expression from which the optimum value of C_G can be obtained:

$$\boxed{aC_G^4 - bC_G^2 - c = 0,} \qquad (8.2\text{-}23)$$

the temporary symbols a, b, c, d and e are defined as

$$a \equiv a^*(Q_C/Q_G)(u_C^{-1}+u_{m,C}^{-1})a_{q,G}n_{h,G}^{j_1}+a^*/e_G,$$

$$b \equiv \{b^*a_{P,H}C_F^{-1}X_H^{-1}d+c^*\pounds_G''+0.25c^*a_R(Q_C/Q_G)^{-1}(d_C/v_{g,F})\}a_{q,G}^{-1}n_{h,G}^{j_1} - 1.235(c^*/q)e\,a_{n,G}D_1n_{h,G}^{j_o},$$

$$c \equiv 3c^*eq\Sigma N_vD_1^{-3},$$

$$d \equiv \Delta p_{fr,C}Q_G,$$

$$e \equiv (d_1+d_2r_1D_1)(2.19s+0.982)\{438v_{G,i}^2/(g_oc_G^2a_{grav}/g_o)\}.$$

(8.2-23 can be treated as a quadratic in C_G with a, b and c as constants; therefore,

$$\boxed{C_G^2 = \{-b + (b^2-4ac)^{0.5}\}/(2a).} \qquad (8.2\text{-}24)$$

8.2.5 Optimum tower-base diameter, D_1, and throat - to base-diameter ratio, s.

As mentioned in 8.2.1, the area equation (J.2-1) is used in the formulation of

$$\partial(P_L/q)/\partial D_1 = 0 \qquad (8.2\text{-}25)$$

and

$$\partial(P_L/q)/\partial s = 0 \qquad (8.2\text{-}26)$$

instead of the less accurate (J.2-2) in order to obtain sufficiently accurate optimum values of the design variables.

The use of the proportionality,

$$P_L/q \propto (c^*/q)(d_1 + d_2\, r_1\, D_1)\, A, \qquad (8.2\text{-}27)$$

being of the same form as that for the Heller system in 7.3.7, results in an identical expression for the optimum value of s, viz.

$$\boxed{s = (a/b)^{0.2},} \qquad (8.2\text{-}28)$$

where the temporary symbols a and b are defined as

$$a \equiv 4\Sigma N_v^{-1}\left[\{B_A - A_A(s-1)^2\}s + C_A\right],$$

$$b \equiv A_A(s-1)(3s-1) + B_A.$$

The use of the proportionality,

$$P_L/q \propto c^*(\pounds_D/q), \qquad (8.2\text{-}29)$$

in conjunction with (8.2-25) results in the following expression for the optimum value of D_1:

$$D_1 = \left[-(a/c)(N_{v,o}+N_{v,i}+N_{v,misc})/(b+0.25a)\right]^{0.25}; \quad (8.2\text{-}30)$$

the temporary symbols a, b and c are defined as

$$a \equiv d_1\left[\{7A_A(s-1)^2 - 3B_A\}s - 3C_A\right] + d_2 r_1 D_1\left[\{8A_A(s-1)^2 - 2B_A\}s - 2C_A\right],$$

$$b \equiv (d_1+d_2 r_1 D_1)\left[\{B_A - A_A(s-1)^2\}s + C_A\right],$$

$$c \equiv 0.25(\pi C_G/q)^2 a_{n,G} n_{h,G}^{j_o} .$$

8.2.6 Optimum partial number of Air-side transfer units, $n_{h,G}$

In formulating

$$\partial(P_L/q)/\partial n_{h,G} = 0 \quad (8.2\text{-}31)$$

the required proportionality

$$P_L/q \propto a^*\left[\{t_{C,i}-(t_{m,G})_o\}+C_G/e_G\right] + b^* a_{P,H} C_F^{-1} X_H^{-1} \Delta p_{fr,C} + c^*(£_D/q + £_G/q + £_R/q), \quad (8.2\text{-}32)$$

can be used to form the following expression for the optimum value of $n_{h,G}$:

$$n_{h,G} = \{(1.62aq)/(c^* j_o d a_{n,G} D_1)\}^{1/(j_o-1)} . \quad (8.2\text{-}33)$$

The temporary symbols a and d are defined by

$$a \equiv j_1 a_{q,G}^{-1}\{b^* a_{P,H} C_F^{-1} X_H^{-1} \Delta p_{fr,C} Q_G + c^* £_G^n\} n_{h,G}^{-(1+j_1)} - a^*(Q_C/Q_G)(1/u_C+1/u_{m,C}) j_1 a_{q,G} C_G^2 n_{h,G}^{j_1-1} - a^* C_G^2(0.0833-n_{h,G}^{-2}),$$

$$d \equiv \{438 v_{G,i}^2/(\varepsilon_o c_G^2 a_{grav}/\varepsilon_o)\}\ (2.195+0.982)(d_1+d_2 r_1 D_1).$$

In presenting a, the term containing a_R has been ignored due to the fact that the relative cheapness of ammonia makes it negligible. As mentioned in 7.3.1 and 8.2.1, (8.2-33) requires to be iterated a few times in order to obtain a more accurate value of $n_{h,G}$ for use in the overall iteration cycle.

9 PRESENTATION OF RESULTS

9.1 Abstract

The achievements of this chapter, and the manner in which they are carried out, are enumerated as follows:

i) The previously described accurate and approximate methods of determining optimum design-variable values, and their associated optimum merit-factor values, are used to obtain general results for each of the considered cooling systems over a range of chosen waste-heat rejection rates. The adopted operating conditions and fixed characteristics, considered to be applicable to the United Kingdom in 1964, are summarized in 9.2.1 together with pertinent decisions made in previous chapters of the thesis. The results themselves are presented in tables 1 to 17 and graphs 1 to 32 in 9.3.1 and 9.4 respectively. The form of presentation is designed to permit easy comparison of values resulting from the accurate and approximate methods. In the tables, where two values of the same quantity appear, the former (or upper) value always derives from the accurate method; where only a single value occurs, it always derives from the accurate method unless stated otherwise.

ii) In obtaining the above results, r_1 is eliminated as a design variable in both the accurate and approximate methods for the Spalding system, and also in the approximate method for the Heller system, by being given a representative numerical value. The fact that this does not lead to an appreciable error in determining the optimum merit-

factor value is shown in graph 34.

iii) In order to obtain the results mentioned in item i), numerical values had to be arbitrarily chosen for two fixed characteristics of the Heller system which can conceivably vary over a range of values. The sensitivity of the optimum merit-factor to variations in these characteristics, viz. $t_{s,S} - (t_{s,H})_o$ and z_H, is shown in graphs 20 and 23 and graph 34 respectively.

iv) The results in item i) are based on the choice of the SG1 heat-exchanger type described in chapter 4. The effects of choosing a heat-exchanger type of similar potential merit but of larger tube diameter, viz. LU1, are calculated for the Heller system at $q = 5 \times 10^8$ and shown in table 21 and the following graphs: 20, 21, 22, 23, 28, 29, 30 and 31.

v) The Hot-side heat-transfer coefficient for the Spalding system is pessimistically assumed to be due to single-phase forced convection in obtaining the results mentioned in item i). Values resulting from the assumption of an over-optimistic nucleate boiling heat-transfer coefficient are shown in tables 22, 23 and 24 and graphs 20, 21, 22 and 23. Of importance is the emergent fact that no significant improvement in optimum merit-factor value results from the over-optimistic assumption.

vi) Graph 33 contains curves of P_u vs q so that generator output can be determined from a knowledge of waste-heat rejection rate for each of the considered cooling systems.

In addition, for the sake of general interest, tables 18, 19 and 20 are included to show the variation in Wet system results with atmospheric wet-bulb temperature.

vii) Finally, the results are examined and discussed in chapter ten.

9.2 Summary of decisions and values of quantities used in obtaining results

9.2.1 General decisions and values of quantities

Insofar as operating conditions are concerned, the values p_{atm} = 2117psf (= 14.7 psia), $t_{G,i}$ = 57 deg F and $t^*_{G,i}$ = 52 deg F are chosen as being representative of some regions in the United Kingdom. Results are then obtained for each of the cooling systems for a range of waste-heat rejection rates ranging from $q = 2.5 \times 10^8$ Btu/h to $q = 15 \times 10^8$ Btu/h.

Values of a^* are chosen by guessing the exhaust steam temperature to be 80 deg F and 100 deg F for the Wet- and Dry-systems respectively, and then applying linear interpolation to the a^* values given in 2.5. The result is that the following values of a^*, b^* and c^* accrue from 2.5:

$a^* = 0.0041$, $b^* = 1.562$, $c^* = 68.2$ for the Wet-system;

$a^* = 0.0042$, $b^* = 1.566$, $c^* = 68.9$ for the Dry-systems.

The equations appearing in chapter 3 are used in calculating results and most of the numerical values of fixed quantities contained therein are summarized in the tables

given in 3.7.1, 3.7.2 and 3.7.3 for the Wet-, Heller- and Spalding-systems respectively. The numerical values and decisions not covered by these tables are given as follows:

a) Wet system. Values of the performance constants of Packing No.21, the choice of which is made in chapter 4, are given in 4.2. The constants in the tower-price equation 3.6-6, inclusive of packing, can then be obtained from J.3. The value of $(f)_{chil}$ can be seen from the graph in F.4.1 to be $(f)_{chil} = 1.058 \times 10^4$.

b) Heller system. The characteristics and performance constants of the SG1 heat-exchanger type, the choice of which is made in chapter 4, appear in tables 1 and 2 in 4.3.4. The tube material is assumed in 4.3.6 to be constructed of aluminium, the thermal conductivity of which is given in (A.6-3), and two Cold-sides passes are adopted. The value of ε_C/d_C is assumed to be $\varepsilon_C/d_C = 0.0001$ after referring to page 138 of ref.4. The price of the water turbine is given in (3.6-14) as $£_{turbine}/q = 4.55 \times 10^{-5}$. After carrying out the procedure for determining the optimum number of tube rows outlined in 7.1, the value $n_r = 2$ was chosen. The representative value of $r_1 = 0.15$ was chosen for the approximate method as mentioned in 7.3.1. Finally, the values $t_{s,S} - t_{H,o} = 7$ and $z_H = 20$ were adopted. As mentioned in 3.4.2 and the latter part of 3.5.2 respectively, it is difficult to assign accurate values to these two quantities and, in consequence, calculations are carried

out for values differing from those arbitrarily adopted above in order to note the effect on the merit-factor. The chosen ranges of values are given in the next sub-section.

c) Spalding system. The SG1 heat-exchanger type is also used here except that the tube material is assumed in 4.3.6 to be constructed of mild steel, the thermal conductivity of which is given in (A.6-4), and a single Cold-side pass is adopted. The value of ε_C/d_C is also the same as that for the Heller system and the value of n_r resulting from the procedure outlined in 7.1 is $n_r = 2$. The representative value of $r_1 = 0.15$ was chosen in both the accurate and approximate methods and an upper limit of $W_C = 1530$ was imposed on the Cold-side loading factor according to the procedure outlined in 8.1. The intermediary heat-transfer medium was chosen to be ammonia in chapter 5 and, in consequence, C_F is given the value $C_F = 412$ according to (3.7-19). The steam condenser tubes are assumed to be constructed from mild-steel, the dimensions and price constant, $\pounds^n_{S,tubes}$, being given by (H.2-3). Finally, the mode of heat transfer on the Hot-side is pessimistically assumed to be due to single-phase forced convection.

The next sub-section contains details of other numerical values and decisions which are used to obtain results which justify some of the choices made in this sub-section.

9.2.2 Additional decisions and values of quantities

r_1 is given values ranging from 0.09 to 0.21 in order to determine the variation of the merit-factor for the

Heller- and Spalding-systems (at $q = 5 \times 10^8$), conditions otherwise remaining unchanged.

$t_{s,S} - (t_{s,H})_o$ and z_H are given values over the ranges 0 to 15 and 10 to 50 respectively in order to note the sensitivity of the Heller- system merit-factor (at $q = 5 \times 10^8$) to changes in these quantities; these have had arbitrary values assigned to them in the previous sub-section.

The accurate method is used in obtaining results for the Heller system (at $q = 5 \times 10^8$) after employing a tower heat-exchanger type similar in potential merit to type SG1 but with larger diameter tubes, viz. type LU1. By carrying out the procedure in 7.1, the value $n_r = 2$ was adopted as being the optimum number of tube rows.

For the Spalding system, the nucleate-boiling equation (E.5-6) is used in the accurate method in order to compare the ensuing results with those obtained from the use of the more pessimistic equation (E.5-4); the latter equation is used in the previous sub-section.

Finally, $t^*_{G,i}$ is varied from 48 to 56 deg F at $q = 10^9$ to note the variation in Wet-system results with wet-bulb temperature.

The results obtained by the use of the foregoing values and decisions are tabulated in the next section.

9.3 Tabular presentation of results

A contents list of the tables, followed by the tables themselves, is given as follows:

1,2,3: Design variables

4,5: Temperature-difference constituents

6,7: Pressure-drop constituents

8,9,10: Cooling-system price constituents

11,12: Merit-factor constituents

13,14,15: ΣN_v constituents and miscellaneous quantities

16,17: Miscellaneous quantities

18,19,20: Effect of variation in atmospheric wet-bulb temperature, $t^*_{G,i}$, on the Wet-system results at $q = 10^9$ (based on accurate method)

21: Results for Heller system at $q = 5 \times 10^8$ with LU1 heat-exchanger type (based on accurate method)

22,23,24: Results for Spalding system assuming nucleate boiling on the Hot-side and using equation (E.5-6) (based on accurate method).

9.3.1 General tables

a) Design variables

Wet system (table 1)

q 10^8	Q_H 10^3	W_H 10^6	d_T	C_F	$t_{C,i}$	D_1	r_p 10^{-2}	s 10^{-1}	t 10^{-1}
15=	5.65	1.86	6.00	12.2	74.1	495	6.66	5.80	8.33
≈	5.31	2.24	6.00	13.0	75.0	503	6.51	5.73	8.42
10=	6.32	1.91	5.01	13.0	75.2	403	6.44	5.54	8.28
≈	5.31	2.24	5.65	13.0	75.0	419	7.41	5.91	8.78
7.5=	6.30	1.77	4.39	13.7	75.9	351	6.97	5.75	8.26
≈	5.31	2.24	5.06	13.0	75.0	363	8.00	6.00	8.96
5.0=	6.43	1.85	6.00	10.1	74.4	289	7.38	5.74	8.28
≈	5.31	2.24	4.33	13.0	75.0	300	8.68	6.10	9.12
2.5=	6.75	1.88	4.30	11.3	75.3	225	7.76	5.86	8.10
≈	5.31	2.24	3.32	13.0	75.0	225	9.19	6.15	9.09

Heller system (table 2)

q 10^8	d_T	C_F/C_G 10^{-1}	C_G	W_C 10^6	D_1	s 10^{-1}	$n_{h,G}$	r_1 10^{-1}
10=	5.78	4.86	16.3	1.79	505	6.64	1.19	1.44
≈	6.00	4.85	16.1	1.77	552	6.24	1.21	1.50
7.5=	6.00	4.08	16.6	1.87	435	6.67	1.21	1.47
≈	6.00	4.30	16.7	1.74	488	6.20	1.23	1.50
5.0=	6.00	4.09	16.8	1.70	358	6.68	1.25	1.67
≈	6.00	3.63	17.3	1.74	403	6.07	1.28	1.50
2.5=	5.14	3.17	18.3	1.83	263	6.62	1.32	1.61
≈	5.79	2.88	18.6	1.71	294	5.90	1.36	1.50

Spalding system (table 3)

q 10^8	Q_H 10^3	W_H 10^5	X_H 10^{-2}	d_B	d_T	C_G	D_1	s 10^{-1}	$n_{h,G}$
10=	-	-	-	-	-	-	-	-	-
≈	4.38	4.79	2.69	2.21	4.80	15.2	565	6.43	1.25
7.5=	4.89	7.90	3.38	1.97	4.26	15.5	478	6.66	1.21
≈	4.40	4.79	2.70	1.96	4.28	15.6	495	6.36	1.28
5.0=	4.93	7.91	3.40	1.66	3.63	16.2	396	6.54	1.25
≈	4.43	4.81	2.71	1.66	3.64	16.4	410	6.24	1.32
2.5=	4.96	7.89	3.48	1.24	2.75	17.6	286	6.37	1.33
≈	4.47	4.83	2.75	1.24	2.75	17.7	299	6.11	1.39

b) Temperature-difference constituents

Wet- and Heller- systems (table 4)

q 10^8	WET			HELLER			
	$t_{s,S}-t_{H,o}$	$t_{H,o}-t_{G,i}$	$t_{s,S}-t_{G,i}$	$t_{s,S}-t_{H,o}$	$t_{H,o}-(t_{m,G})_o$	$(t_{m,G})_o-t_{G,i}$	$t_{s,S}-t_{G,i}$
15	3.86	17.1	21.0	-	-	-	-
10	4.37	18.2	22.6	7.00	3.45	26.7	37.1
7.5	4.36	18.9	23.3	7.00	3.37	26.4	36.8
5.0	5.38	17.4	22.8	7.00	3.46	26.3	36.7
2.5	5.15	18.3	23.5	7.00	3.24	27.2	37.4

Spalding system (table 5)

q 10^8	$t_{s,S}-(t_{m,H})_o$	$(t_{m,H})_o-(t_{s,H})_o$	$(t_{s,H})_o-(t_{m,G})_o$	$(t_{m,G})_o-t_{G,i}$	$t_{s,S}-t_{G,i}$
10	-	-	-	-	-
7.5	3.39	6.06	5.26	22.1	36.8
5.0	3.33	6.09	5.19	22.8	37.4
2.5	3.21	6.15	5.05	24.0	38.4

c) Pressure-drop constituents

The following temporary definitions apply:

$A \equiv (a_{grav}/g_o)z_3/v_{f,T}$

$B \equiv \eta_{pt}\Delta p_H + (1-\eta_{pt})(p_{atm}-p_{s,S})$

$C \equiv (1-\eta_{pt})(a_{grav}/g_o)(z_1-z_H)/v_{f,C}$

$D \equiv (1+\eta_{pt})\Delta p_{fr,T}$

$E \equiv 0.5(1+\eta_{pt})\Delta p_{fr,C}$

Wet- and Heller-systems (table 6)

q 10^8	WET				HELLER				
	$\Delta p_{fr,H}$ 10^3	$2\Delta p_{fr,T}$	A 10^3	$\Sigma\Delta p$ 10^3	B	C	D	E	$\Sigma\Delta p$ 10^3
15	1.39	481	2.75	4.62	-	-	-	-	-
10	1.41	486	2.31	4.21	987	923	534	835	3.28
7.5	1.21	486	2.22	3.92	987	766	336	823	2.91
5.0	0.99	80	2.03	3.09	987	694	145	668	2.49
2.5	1.12	88	1.79	3.79	987	394	114	701	2.20

Spalding system (table 7)

q 10^8	Δp_{II}	$\Delta p_{fr,T}$	$\Delta p_{fr,B}$	$\Delta p_{fr,C}$ 10^3	$\Sigma \Delta p$ 10^3
10	-	-	-	-	-
7.5	681	26	20	1.53	2.26
5.0	687	27	21	1.55	2.29
2.5	703	28	26	1.59	2.35

d) Cooling-system price constituents

Wet system (table 8)

q 10^8	$£_S/q$ 10^{-4}	$2£_T/q$ 10^{-6}	$£_D/q$ 10^{-4}	$£_{cs}/q - £_{pm}/q$ 10^{-4}
15	4.14	7.29	5.59	9.80
10	3.71	7.26	5.83	9.61
7.5	3.72	7.27	6.00	9.80
5.0	3.67	21.9	6.38	10.3
2.5	3.57	20.9	7.94	11.7

Heller system (table 9)

For this table

$A \equiv £_{turbine}/q$

q 10^8	$£_S/q$ 10^{-4}	$2£_T/q$ 10^{-5}	$£_D/q$ 10^{-4}	$£_G/q$ 10^{-4}	A 10^{-5}	$£_{cs}/q - £_{pm}/q$ 10^{-3}
10	1.48	1.00	3.85	9.17	4.55	1.51
7.5	1.48	1.46	4.18	9.23	4.55	1.55
5.0	1.48	2.18	4.67	9.66	4.55	1.65
2.5	1.48	3.07	5.67	9.79	4.55	1.77

Spalding system (table 10)

For this table

$A \equiv \pounds_T/q + \pounds_B/q + \pounds_R/q$

q	$\pounds_S/q$	A	$\pounds_D/q$	$\pounds_G/q$	$\pounds_{cs}/q - \pounds_{pm}/q$
10^8	10^{-4}	10^{-5}	10^{-4}	10^{-4}	10^{-3}
10	-	-	-	-	-
7.5	4.58	1.48	4.50	9.96	1.92
5.0	4.55	1.59	4.90	10.1	1.97
2.5	4.56	1.89	5.92	10.4	2.10

e) Merit-factor constituents

For the following tables

$A \equiv a^*(t_{s,S} - t_{G,i})$

$B \equiv b^* P/q$

$C \equiv c^*(\pounds_{cs}/q - \pounds_{pm}/q)$

Wet system (table 11)

q	A	B	C	P_L/q
10^8	10^{-2}	10^{-2}	10^{-2}	10^{-1}
15 =	8.59	1.46	6.68	1.67
≈	8.29	1.72	6.89	1.69
10 =	9.25	1.25	6.55	1.71
≈	8.26	1.64	7.20	1.71
7.5 =	9.54	1.10	6.68	1.73
≈	8.36	1.60	7.35	1.73
5.0 =	9.34	1.18	7.00	1.75
≈	8.49	1.55	7.67	1.77
2.5 =	9.62	1.02	7.99	1.86
≈	8.57	1.45	8.78	1.88

Heller- and Spalding-systems (table 12)

q 10^8	A 10^{-1}	B 10^{-2}	C 10^{-1}	P_L/q 10^{-1}	A 10^{-1}	B 10^{-3}	C 10^{-1}	P_L/q 10^{-1}
10=	1.56	1.60	1.04	2.76	-	-	-	-
≈	1.53	1.67	1.06	2.76	1.53	8.86	1.37	2.99
7.5=	1.54	1.66	1.07	2.78	1.55	8.67	1.32	2.96
≈	1.55	1.59	1.08	2.78	1.55	8.91	1.40	3.03
5.0=	1.54	1.40	1.14	2.82	1.57	8.73	1.36	3.02
≈	1.54	1.55	1.13	2.83	1.57	9.00	1.43	3.09
2.5=	1.57	1.41	1.22	2.93	1.61	8.76	1.45	3.15
≈	1.57	1.46	1.24	2.95	1.62	9.02	1.51	3.22

f) ΣN_v constituents and miscellaneous quantities

For these tables

B ≡ $N_{v,pack}$

C ≡ $N_{v,elim}$

D ≡ $N_{v,spray}$

E ≡ $N_{v,misc}$

Wet system (table 13)

q 10^8	U_H	A/q 10^{-4}	B	C	D	E	$N_{v,i}$	$N_{v,o}$	ΣN_v
15	666	4.19	26.4	4.46	7.33	2.30	17.0	8.81	66.3
10	675	4.47	29.7	4.52	3.46	2.30	18.3	10.6	68.8
7.5	660	4.48	29.9	4.54	2.90	2.30	15.4	9.12	64.2
5.0	674	4.52	30.4	4.52	4.56	2.30	13.4	9.20	64.4
2.5	694	4.59	31.4	4.72	3.34	2.30	12.0	8.49	62.2

Heller system (table 14)

q 10^8	U_G	Q_G	A/q 10^{-4}	$N_{v,G}$	$N_{v,i}$	$N_{v,o}$	E	ΣN_v
10	7.00	120	5.26	23.9	8.02	5.15	2.30	39.3
7.5	6.98	119	5.65	23.4	7.68	5.05	2.30	38.4
5.0	6.74	114	6.06	22.7	5.96	5.02	2.30	36.0
2.5	6.56	112	6.69	26.9	6.43	5.20	2.30	40.8

Spalding system (table 15)

q 10^8	U_H	U_G	Q_G	A/q 10^{-4}	$N_{v,G}$	$N_{v,i}$	$N_{v,o}$	E	ΣN_v
10	-	-	-	-	-	-	-	-	-
7.5	599	6.11	110	6.08	29.3	7.42	5.08	2.30	44.1
5.0	607	6.00	109	6.44	30.9	7.42	5.46	2.30	46.1
2.5	618	5.81	106	7.09	33.7	7.42	6.08	2.30	49.5

g) Miscellaneous quantities

Wet system (table 16)

It is to be noted here that the values of $N_{h,G}$ and G_C/G_G accruing from the approximate method, being computed at fixed values of $t_{C,i}$ and C_F (= $t_{C,i}-t_{C,o}$), are themselves constant at $N_{h,G}$ = 1.54 and G_C/G_G = 0.769.

q 10^8	r_1 10^{-2}	$N_{h,G}$	G_C/G_G 10^{-1}	r_5-r_1	z_5	z_1	D_1^2/q 10^{-4}
15	7.38	1.55	7.90	1.14	601	36.6	1.63
10	7.31	1.55	7.82	1.26	538	29.5	1.62
7.5	7.99	1.54	7.60	1.22	456	28.1	1.64
5.0	8.60	1.70	10.9	1.22	377	25.0	1.67
2.5	9.35	1.65	9.79	0.99	243	21.0	2.03

Heller- and Spalding-systems (table 17)

	q 10^8	r_5-r_1 10^{-1}	z_5	z_1	L_C/n_p	$\frac{A_{fr,G}}{q}$ 10^{-4}	$\frac{\pi D_1 z_1}{q}$ 10^{-4}	D_1^2/q 10^{-4}
HELLER	10	8.36	495	72.8	15.7	1.30	1.16	2.55
	7.5	9.14	462	63.8	14.2	1.31	1.16	2.52
	5.0	9.66	406	59.7	13.7	1.37	1.34	2.56
	2.5	9.90	303	42.5	12.6	1.39	1.41	2.77
SPALDING	10	-	-	-	-	-	-	-
	7.5	8.03	456	71.7	161	1.41	1.44	3.05
	5.0	8.39	392	59.4	163	1.43	1.48	3.14
	2.5	9.00	300	42.9	167	1.47	1.54	3.27

9.3.2 Additional tables

a) Effect of variation in atmospheric wet-bulb temperature, $t^*_{G,i}$, on Wet-system results at $q = 10^9$ (based on accurate method).

Table 18

$t^*_{G,i}$	Q_H 10^3	W_H 10^6	d_T	C_F	$t_{C,i}$	D_1	r_p 10^{-2}	s 10^{-1}	t 10^{-1}
48	7.38	2.09	5.18	13.9	72.4	447	7.22	5.33	7.46
52	6.32	1.91	5.01	13.0	75.2	403	6.44	5.54	8.28
54	5.94	1.88	5.36	11.7	76.3	382	6.94	5.76	8.54
56	6.16	1.77	5.99	14.1	78.2	398	7.39	5.36	9.02

Table 19

$t^*_{G,i}$	$a^*(t_{s,S} - t_{G,i})$ 10^{-2}	b^*P/q 10^{-2}	$c^*(£_{cs}/q - £_{pm}/q)$ 10^{-2}	P_L/q 10^{-1}
48	8.49	1.32	6.54	1.63
52	9.25	1.25	6.55	1.71
54	9.65	1.34	6.51	1.75
56	10.4	1.09	6.60	1.81

Table 20

$t^*_{G,i}$	U_H	A/q 10^{-4}	ΣN_v	r_1 10^{-2}	$N_{h,G}$	$\frac{G_C}{G_G}$ 10^{-1}	z_5	z_1
48	687	4.69	74.1	8.01	1.49	6.81	519	35.8
52	675	4.47	68.8	7.31	1.55	7.82	538	29.5
54	673	4.28	63.4	7.86	1.60	8.87	533	30.1
56	656	4.09	59.7	8.29	1.49	6.89	509	33.0

b) Results for Heller system at $q = 5 \times 10^8$ with LU1 heat-exchanger type (based on accurate method).

Table 21

$d_T = 5.76$ $W_C = 1.62 \times 10^6$ $n_{h,G} = 1.46$	$C_F/C_G = 0.426$ $D_1 = 333$ $r_1 = 0.173$	$C_G = 18.1$ $s = 0.637$ $n_r = 2$
$a^*(t_{s,S}-t_{G,i}) = 0.153$ $c^*(£_{cs}/q-£_{pm}/q) = 0.114$	$b^*P/q = 0.0135$ $P_L/q = 0.281$	
$U_G = 7.97$ $z_5 = 469$ $A_{fr,G}/q = 1.83\times10^{-4}$	$Q_G = 124$ $z_1 = 57.8$ $\pi D_1 z_1/q = 1.2\times10^{-4}$	$\Sigma N_v = 39.7$ $L_C/n_p = 35.7$ $r_5-r_1 = 1.235$

c) Results for Spalding system assuming nucleate boiling on the Hot-side and using equation (E.5-6) (based on accurate method).

Table 22

q 10^8	Q_H 10^3	W_H 10^5	X_H 10^{-1}	d_B	d_T	C_G	D_1	s 10^{-1}	$n_{h,G}$
10	-	-	-	-	-	-	-	-	-
7.5	5.00	1.14	2.56	1.37	3.11	15.3	486	6.49	1.23
5.0	4.93	1.12	2.86	1.21	2.63	16.1	401	6.47	1.27
2.5	5.37	1.29	2.20	1.07	1.92	17.2	293	6.11	1.34

Table 23

q 10^8	$a^*(t_{s,S} - t_{G,i})$ 10^{-1}	b^*P/q 10^{-3}	$c^*(\pounds_{cs}/q - \pounds_{pm}/q)$ 10^{-1}	P_L/q 10^{-1}
10	-	-	-	-
7.5	1.60	1.01	1.34	2.95
5.0	1.62	0.92	1.38	3.00
2.5	1.67	1.19	1.46	3.14

Table 24

q 10^8	U_H	U_G	Q_G	N_v	z_5	z_1	L_C/n_p	$\frac{A_{fr,G}}{q}$ 10^{-4}	$\frac{\pi D_1 z_1}{q}$ 10^{-4}
10	-	-	-	-	-	-	-	-	-
7.5	475	6.10	107	44.8	453	72.9	166	1.47	1.48
5.0	478	5.93	105	46.0	384	60.1	169	1.49	1.51
2.5	495	5.81	103	51.6	306	43.9	173	1.52	1.62

9.4 Graphical presentation of results

A contents list of the graphs, followed by the graphs themselves, is given as follows:

1 Q_H and W_H for Wet- and Spalding-systems

2 Duct diameters, d_T and d_B

3 $t_{C,i}$ and t for Wet System

4 Heat-capacity factors, C_F and C_G

5 C_F/C_G and W_C for Heller System

6 D_1 and $n_{h,G}$

7 r_1 and s for all systems

7a X_H for Spalding System

8 Temperature-difference constituents for Wet System

9 Temperature-difference constituents for Heller System

10 Temperature-difference constituents for Spalding System

11 Pressure-drop constituents for Wet System

12 Pressure-drop constituents for Heller System

13 Pressure-drop constituents for Spalding System

14 Cooling-system price constituents for Wet System

15 Cooling-system price constituents for Heller System

16 Cooling-system price constituents for Spalding System

17 Merit-factor constituents for Wet System

18 Merit-factor constituents for Heller System

19 Merit-factor constituents for Spalding System

20 Temperature-difference term in merit-factor equation

21 Pumping-power term in merit-factor equation

22 Capital-price term in merit-factor equation

23 Merit-factor

24 $N_{h,G}$ and G_C/G_G for Wet System

25 Overall Hot- and Air-side heat-transfer coefficients

26 Air-side heat flux and ratio of tower surface area to waste-heat rejection rate

27 ΣN_v constituents for Wet System

28 ΣN_v constituents for Heller- and Spalding-systems

29 $r_5 - r_1$ and height of top of tower

30 Tower inlet height and length of tower heat-exchanger unit

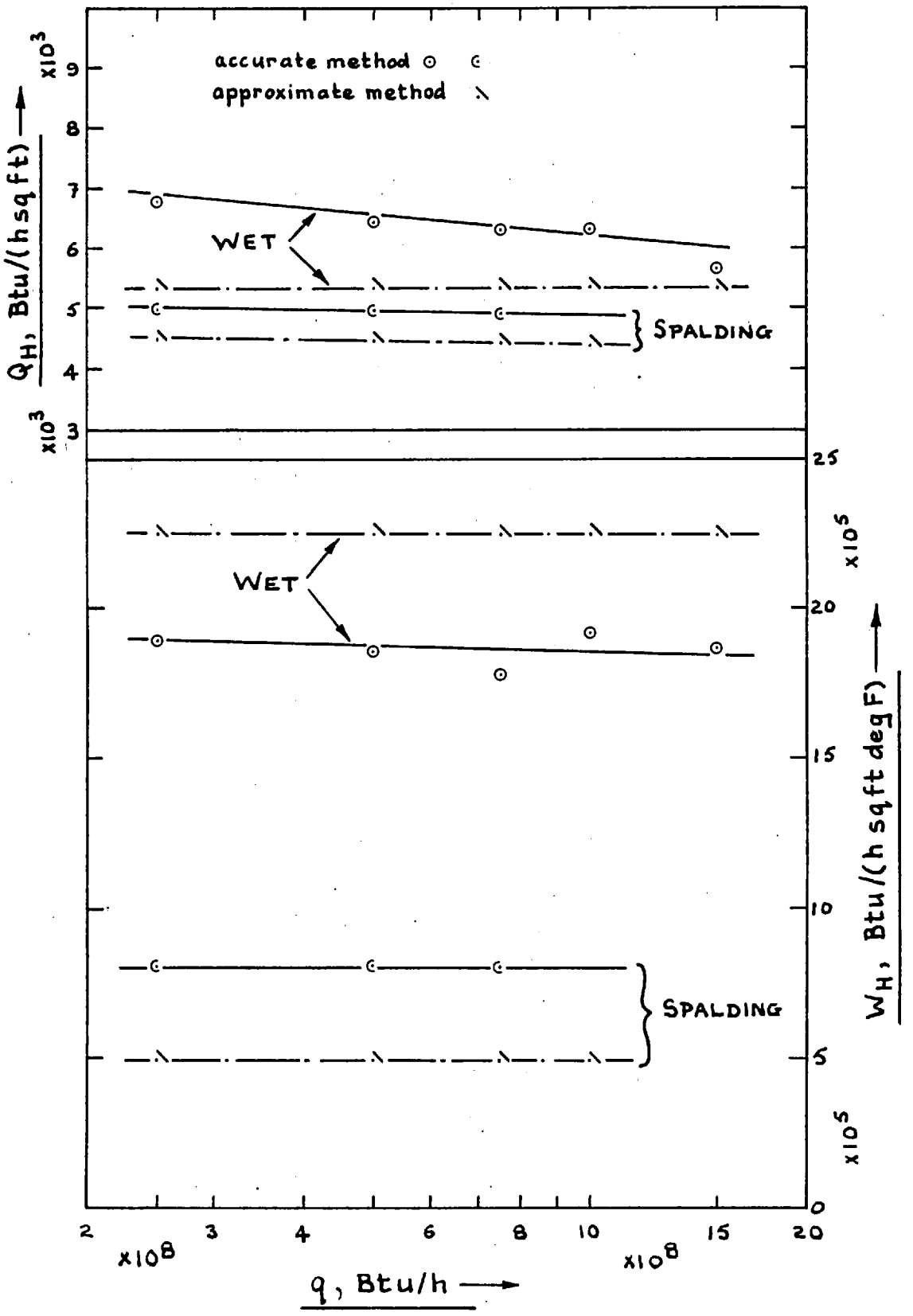
31 Ratios of heat-exchanger frontal area and tower inlet area to waste-heat rejection rate

32 Exhaust steam temperature and D_1^2/q

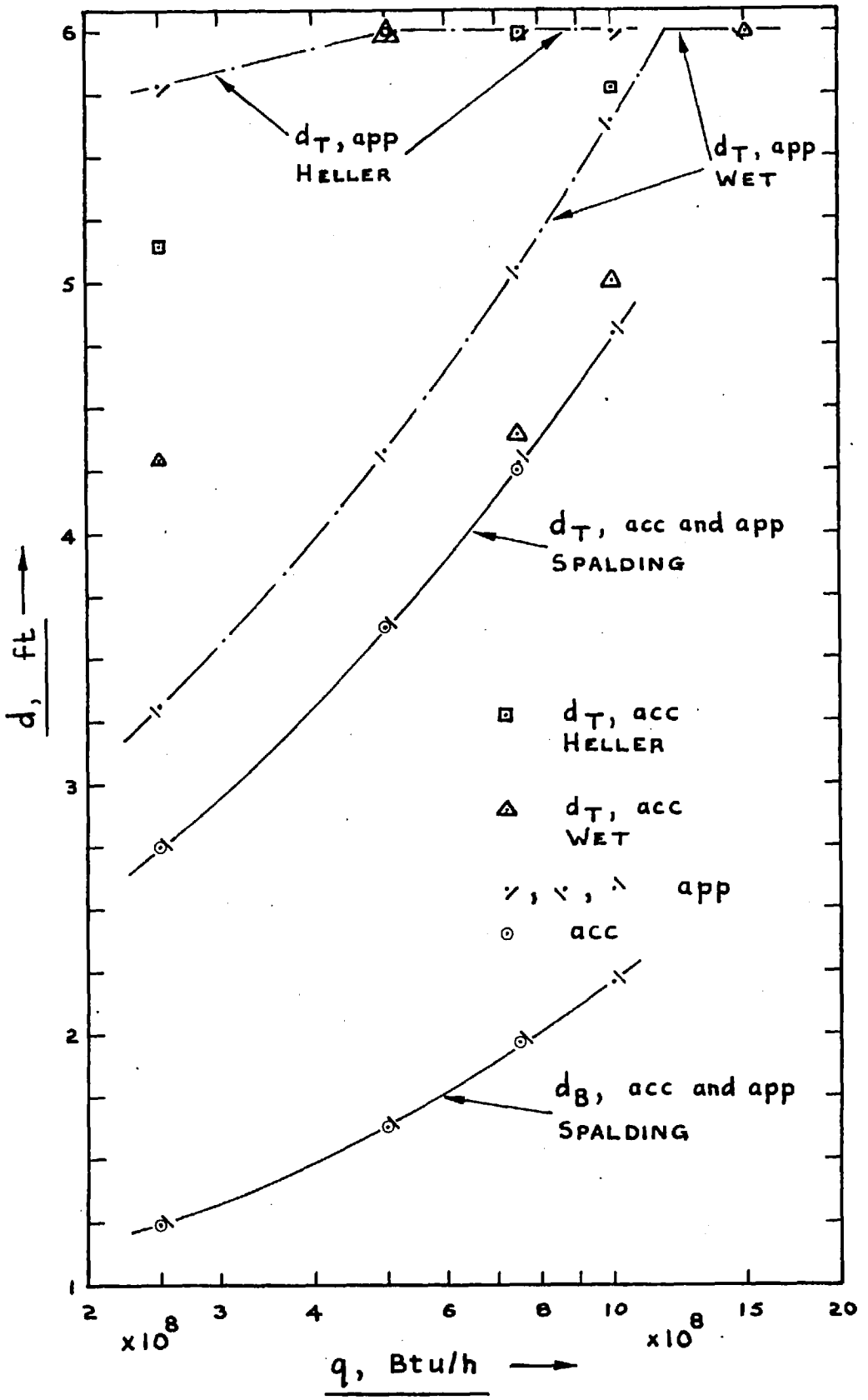
33 Generator output vs waste-heat rejection rate

34 Merit-factor vs z_H for Heller system and merit-factor vs r_1 for Heller- and Spalding-systems

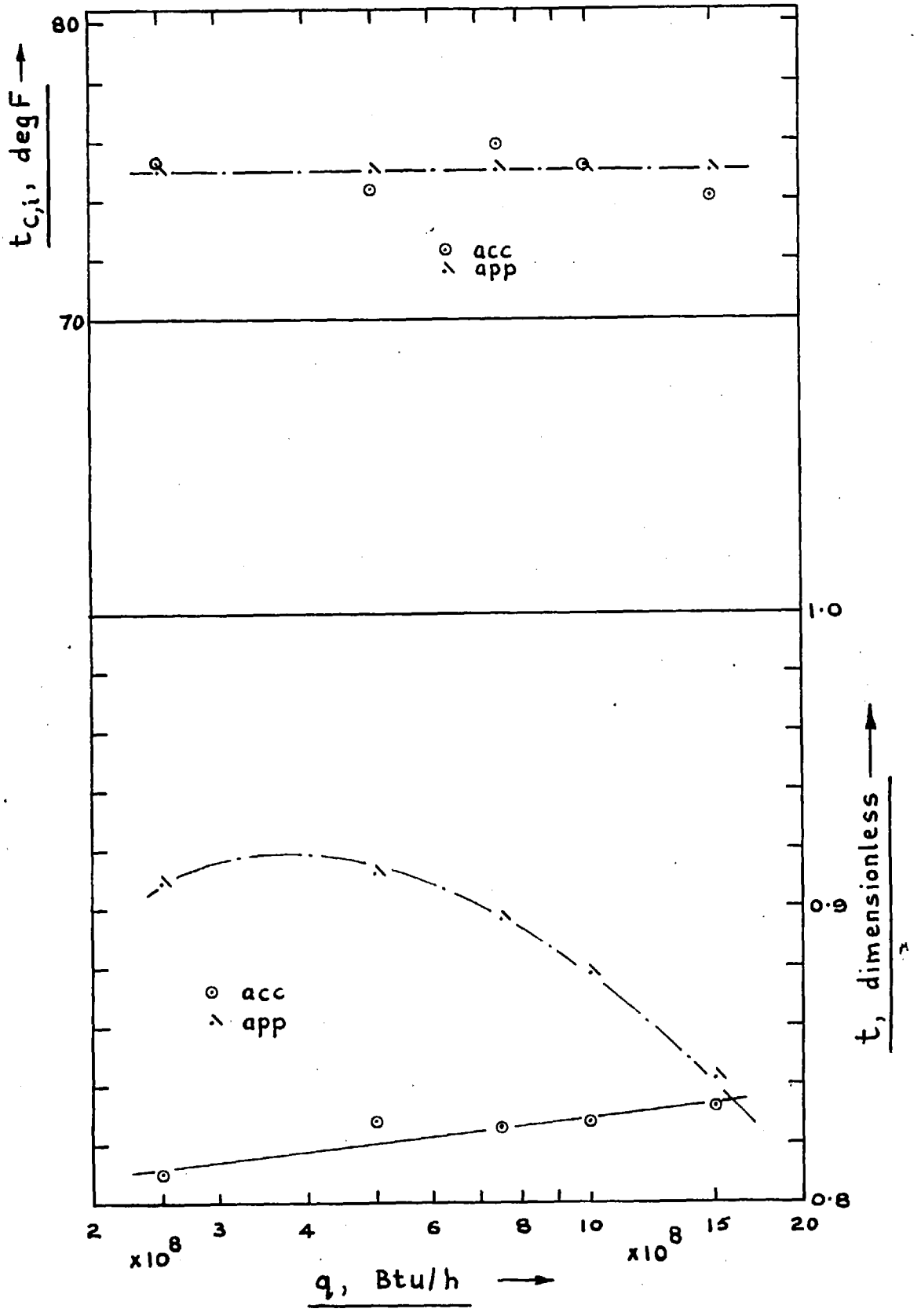
GRAPH I : Q_H and W_H for Wet- and Spalding-systems



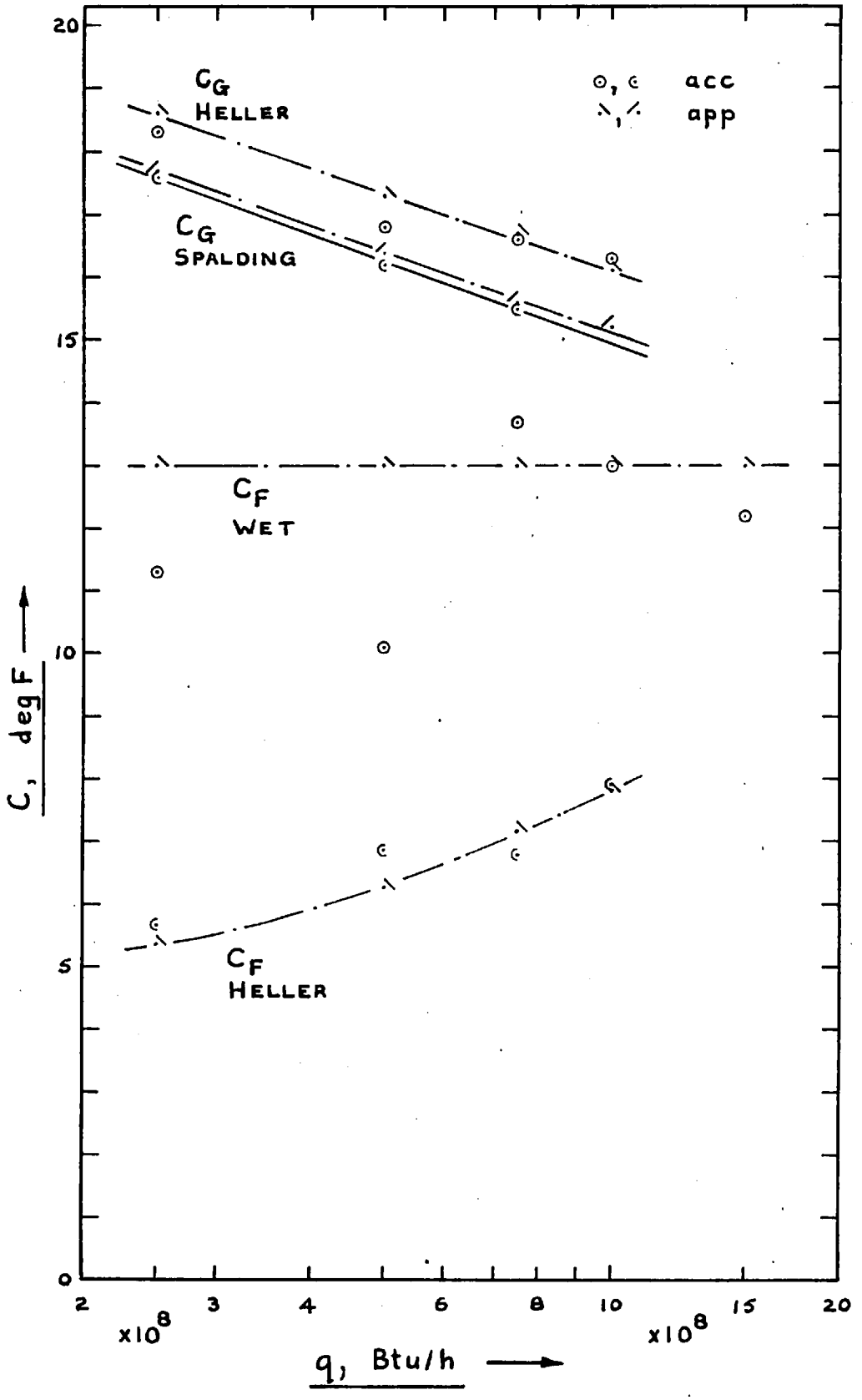
GRAPH 2: Duct diameters, d_T and d_B



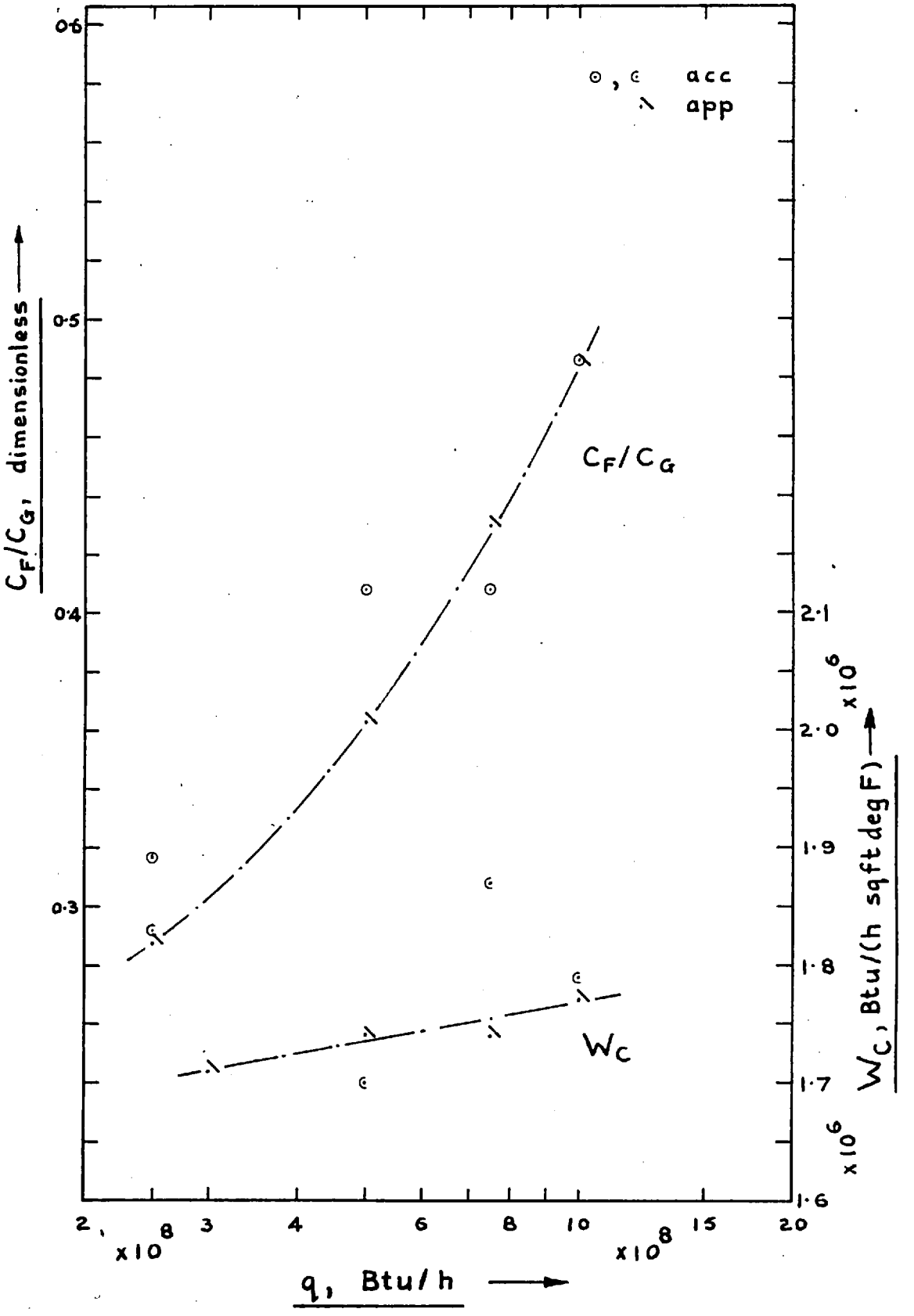
GRAPH 3: $t_{c,i}$ and t for Wet System



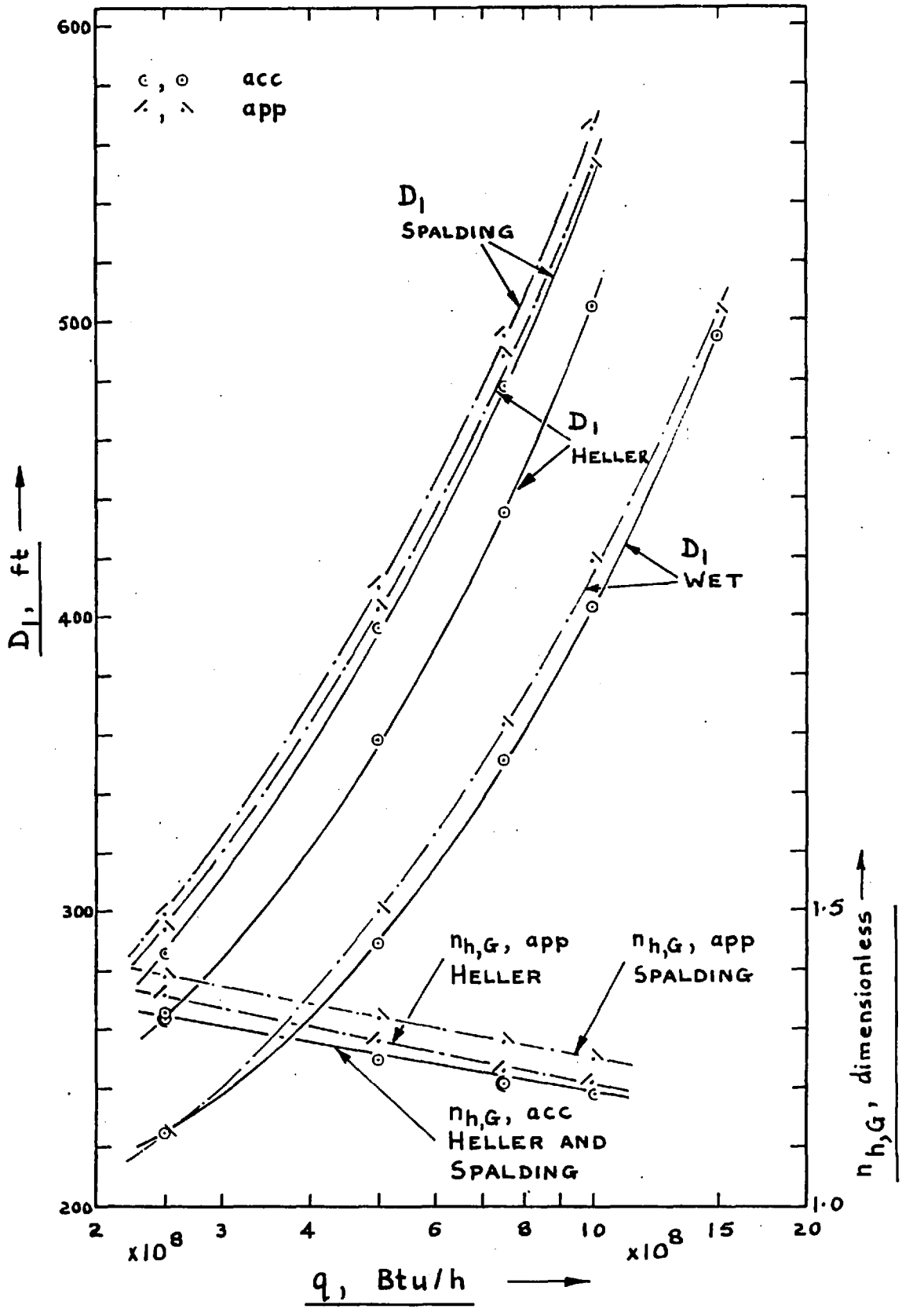
GRAPH 4: Heat-capacity factors, C_F and C_G



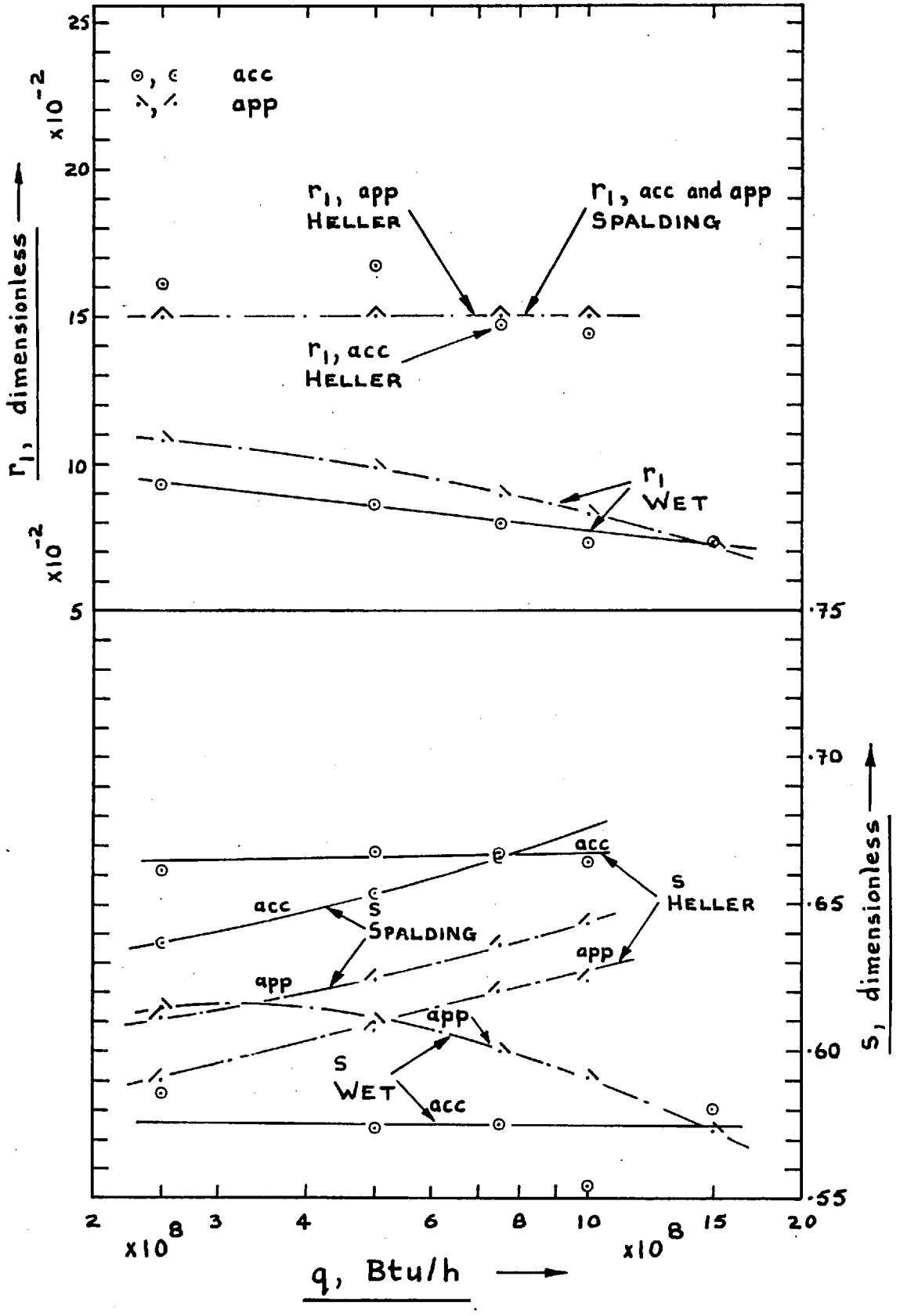
GRAPH 5: C_F/C_G and W_C for Heller System



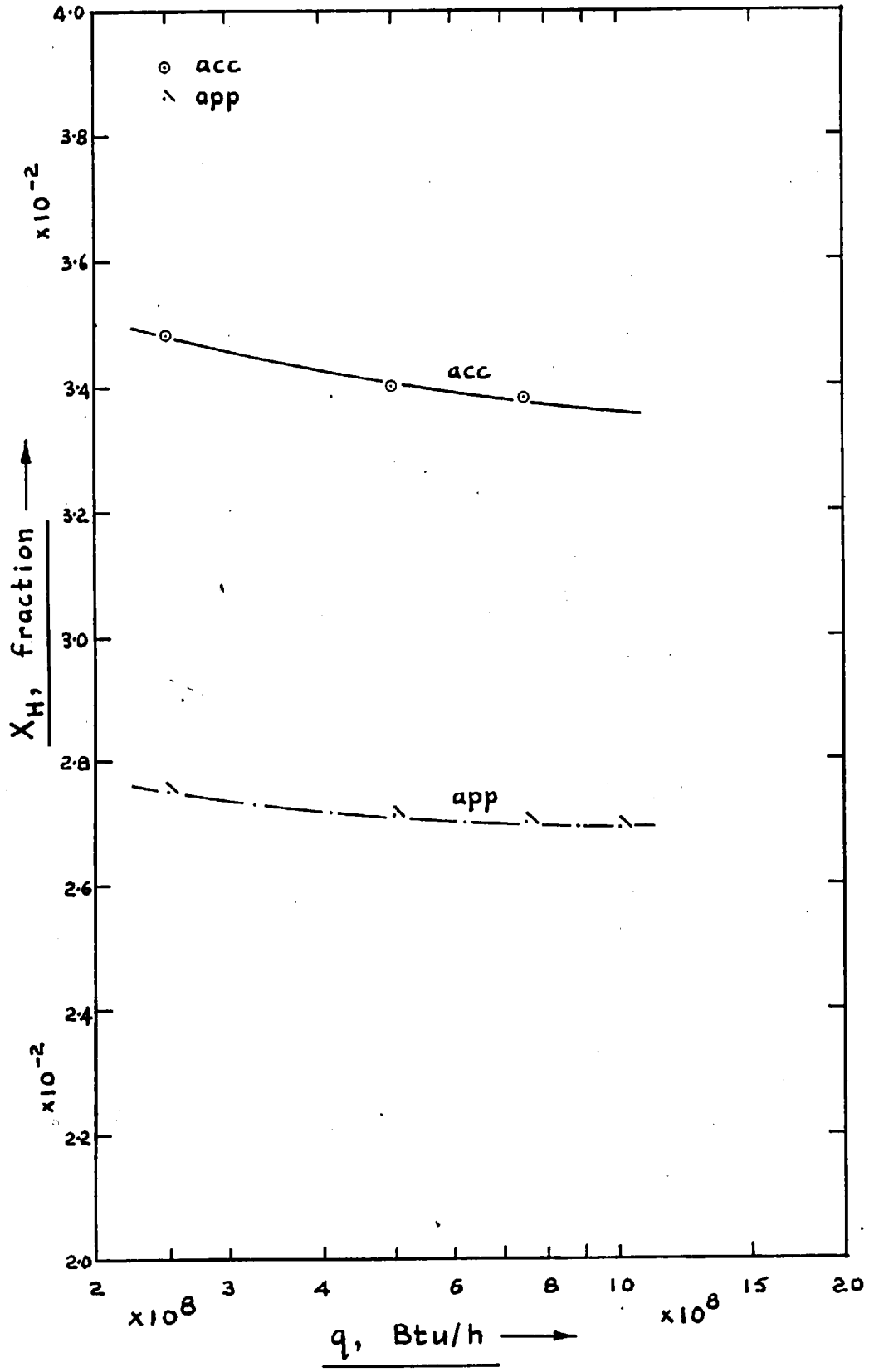
GRAPH 6 : D_l and $n_{h,G}$



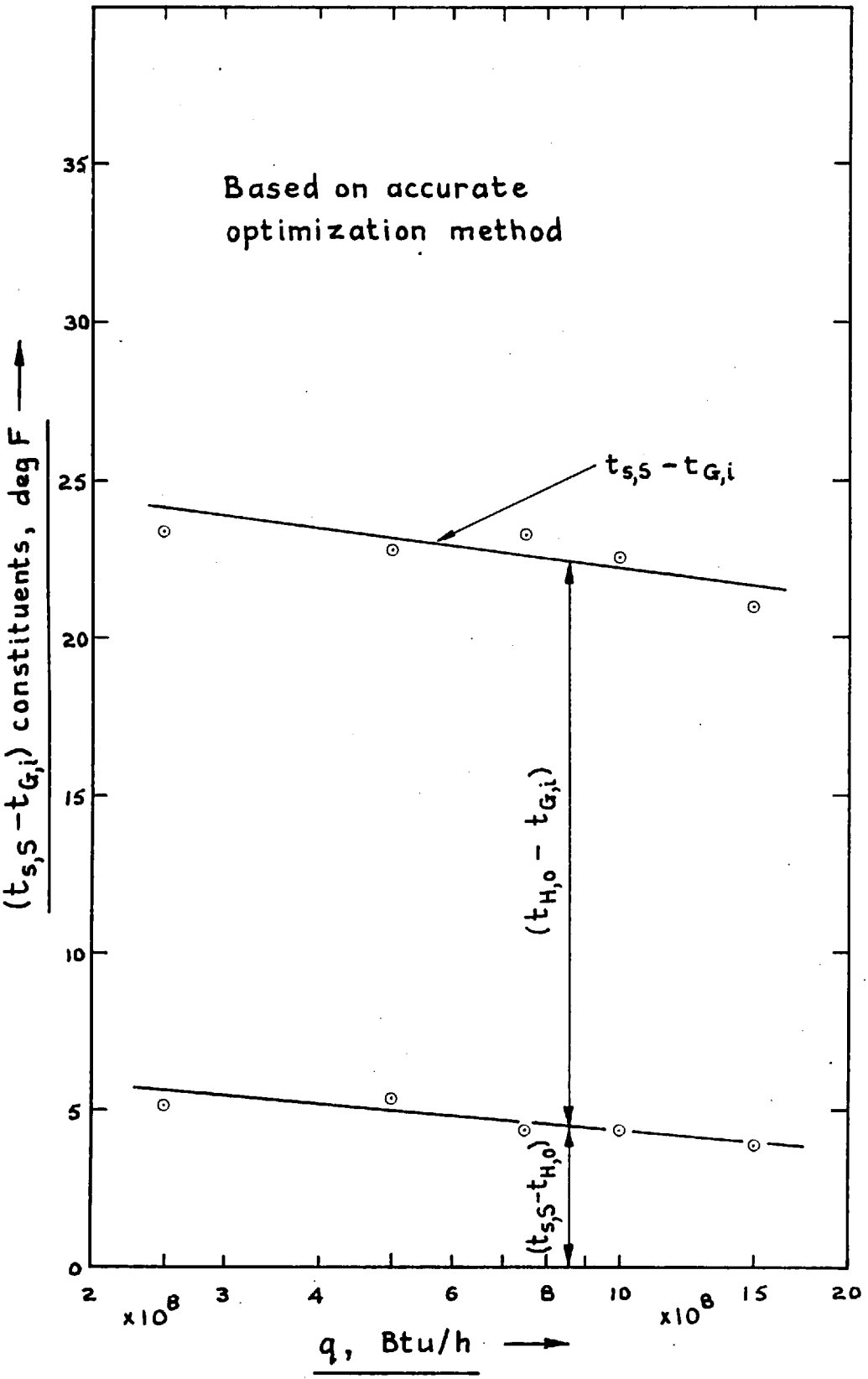
GRAPH 7 : r_1 and s for all systems



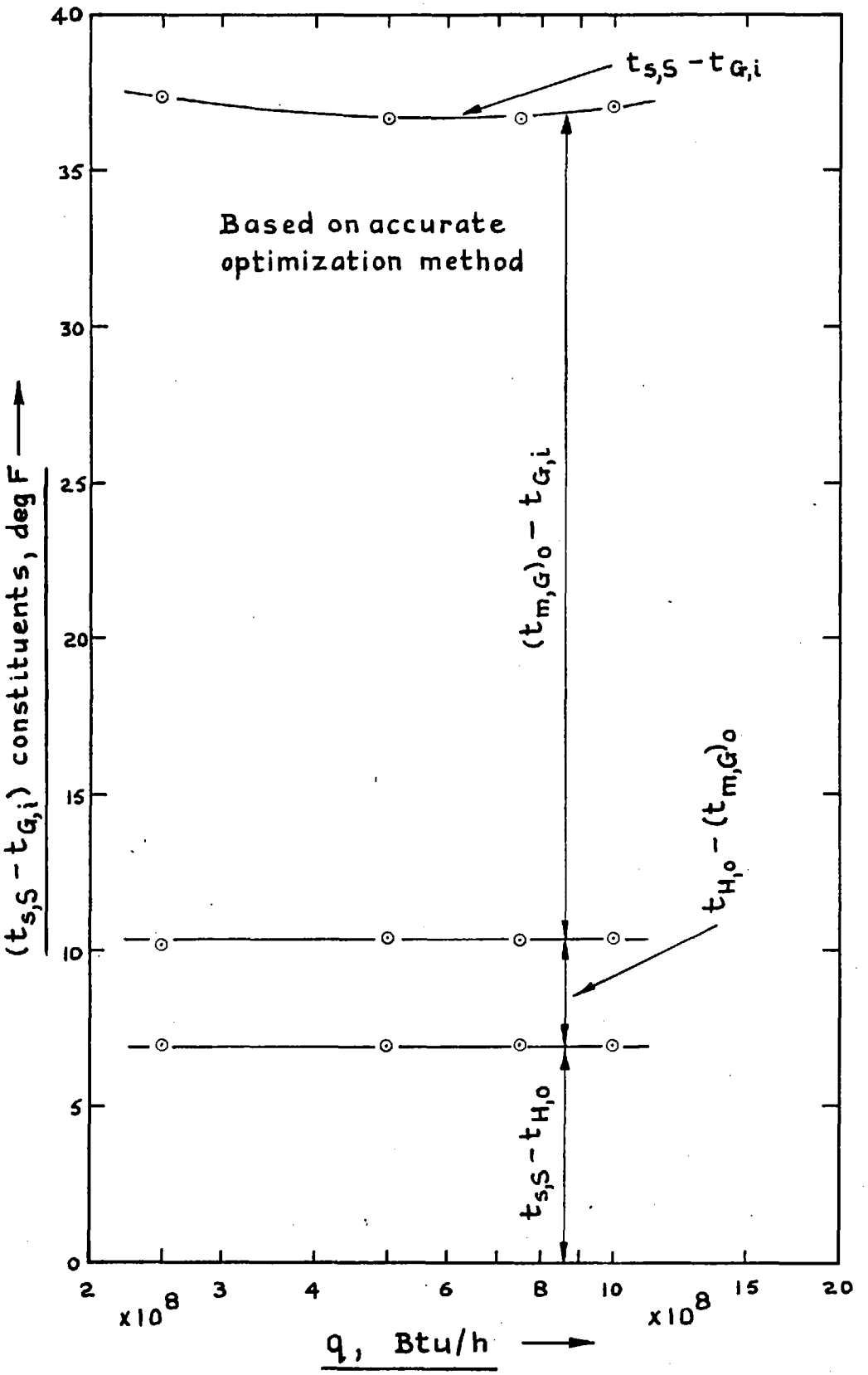
GRAPH 7a: X_H for Spalding System



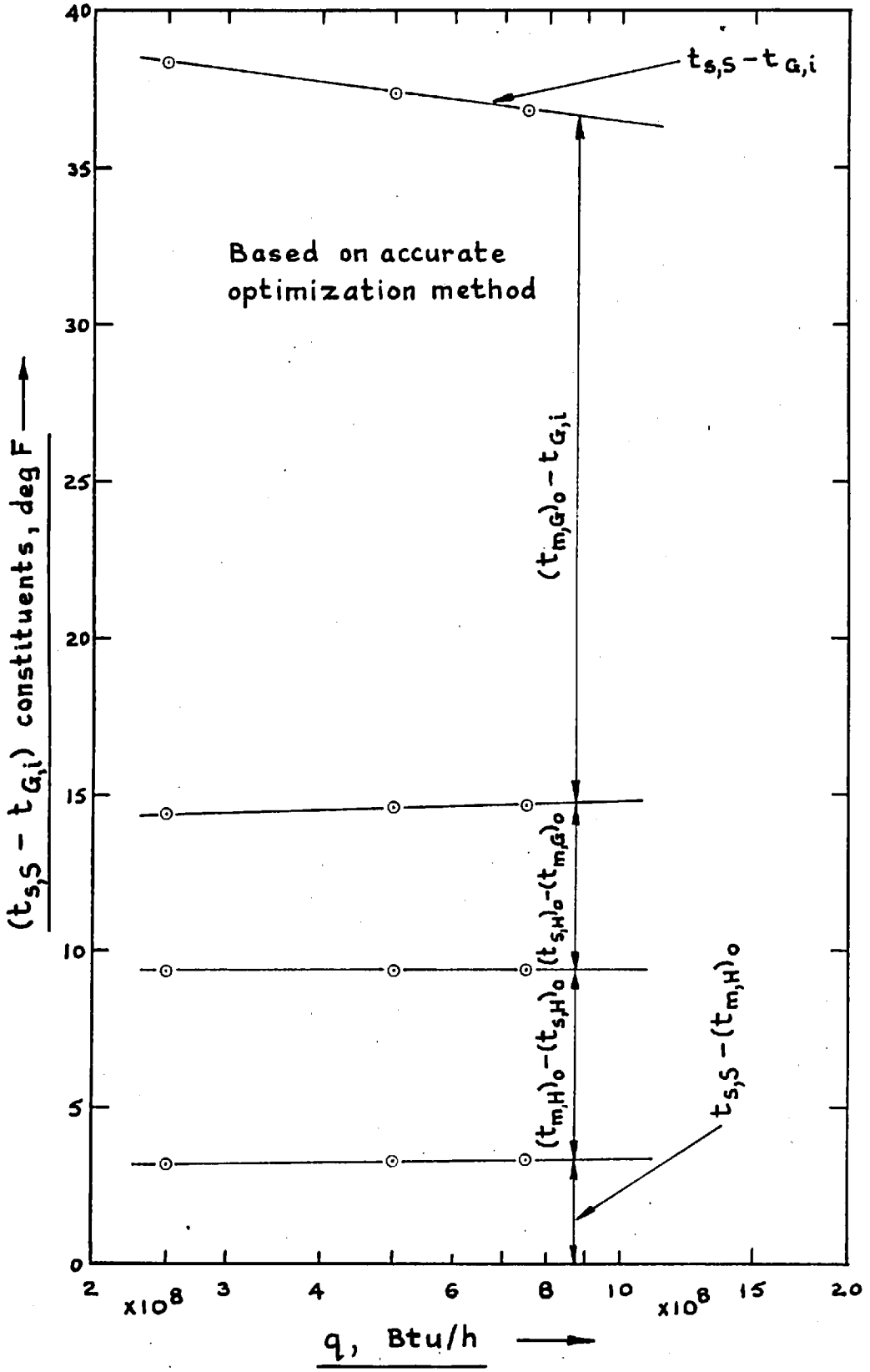
GRAPH 8: Temperature-difference constituents for Wet System



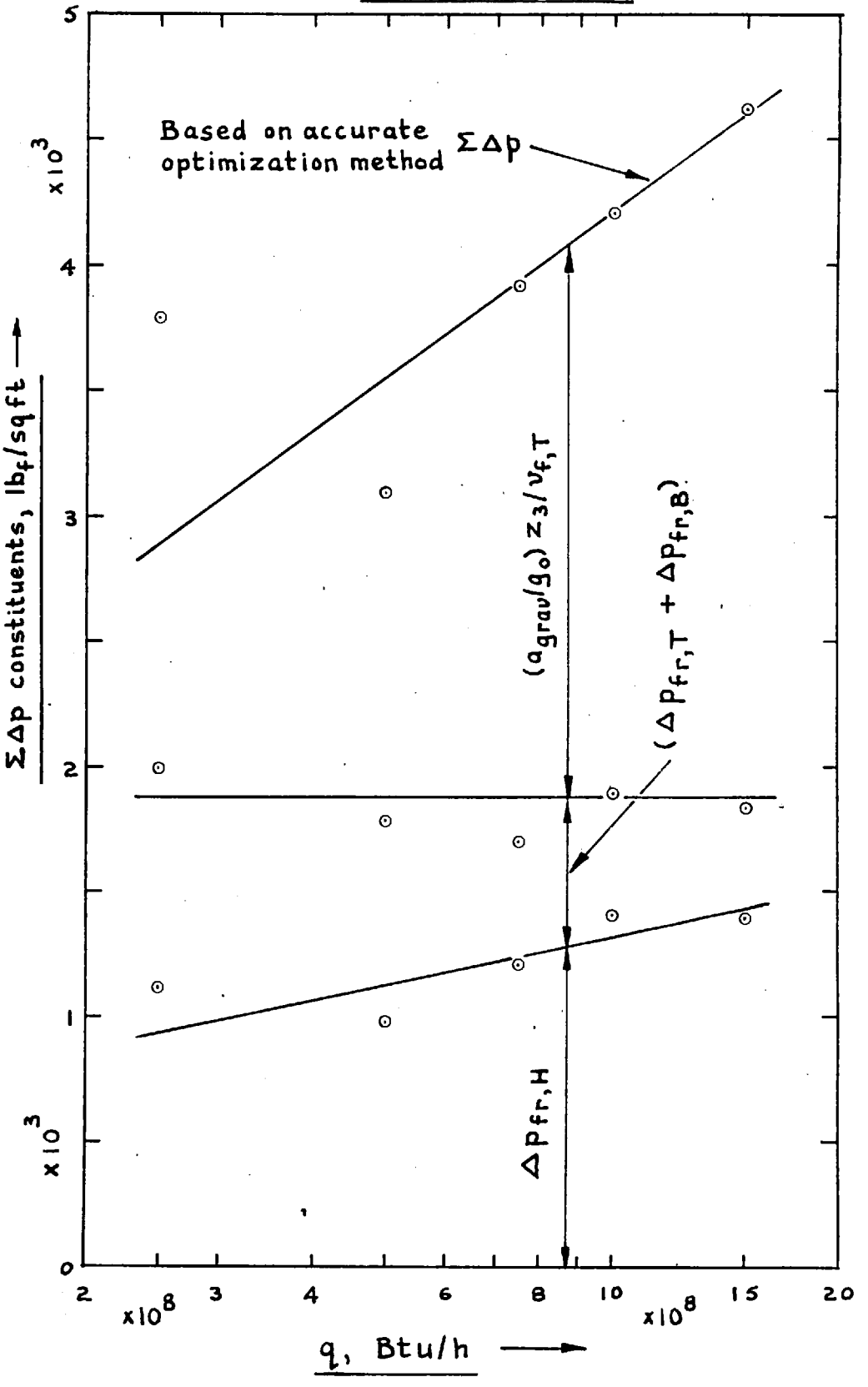
GRAPH 9 : Temperature-difference constituents for Heller System



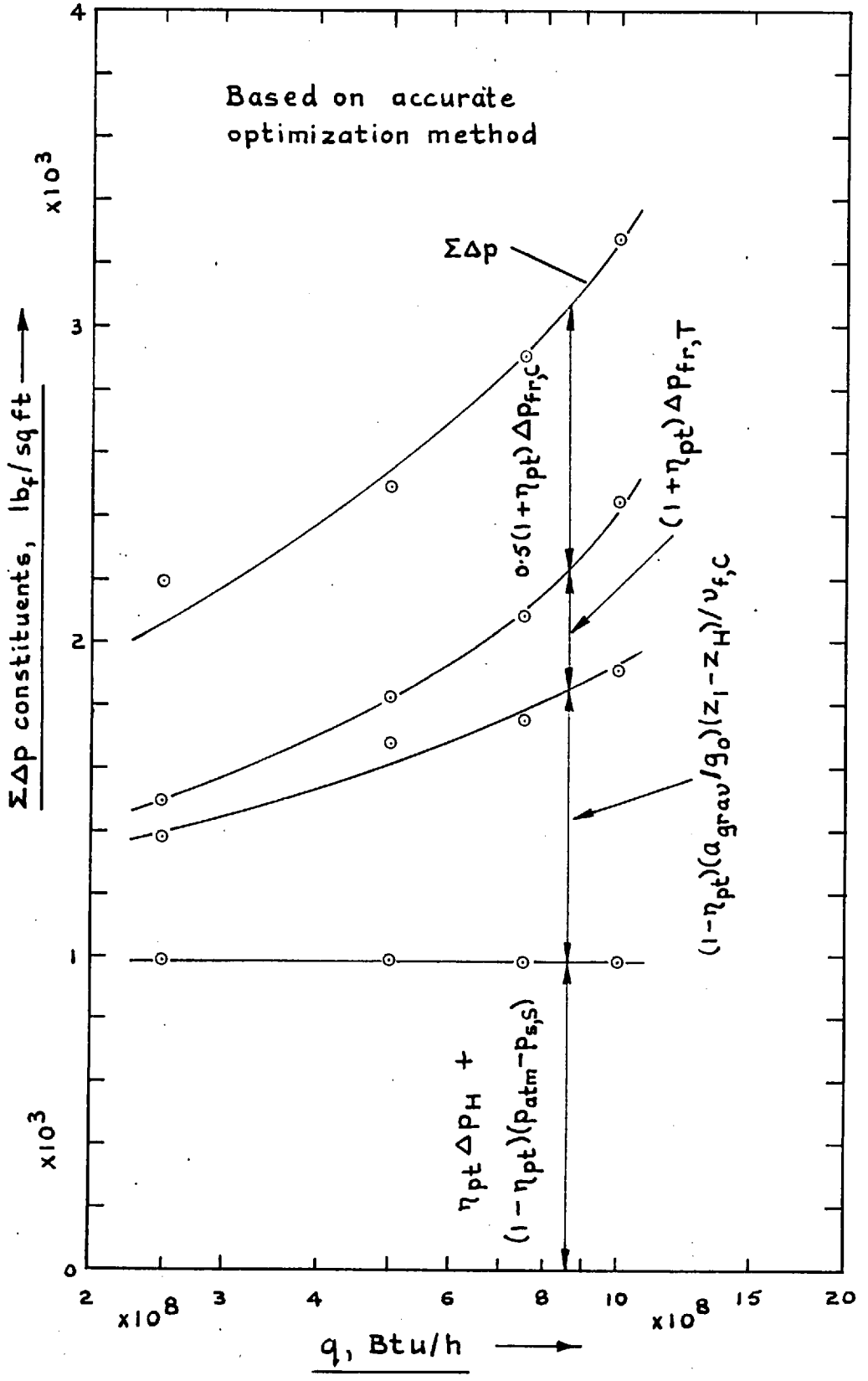
GRAPH 10: Temperature-difference constituents for Spalding System



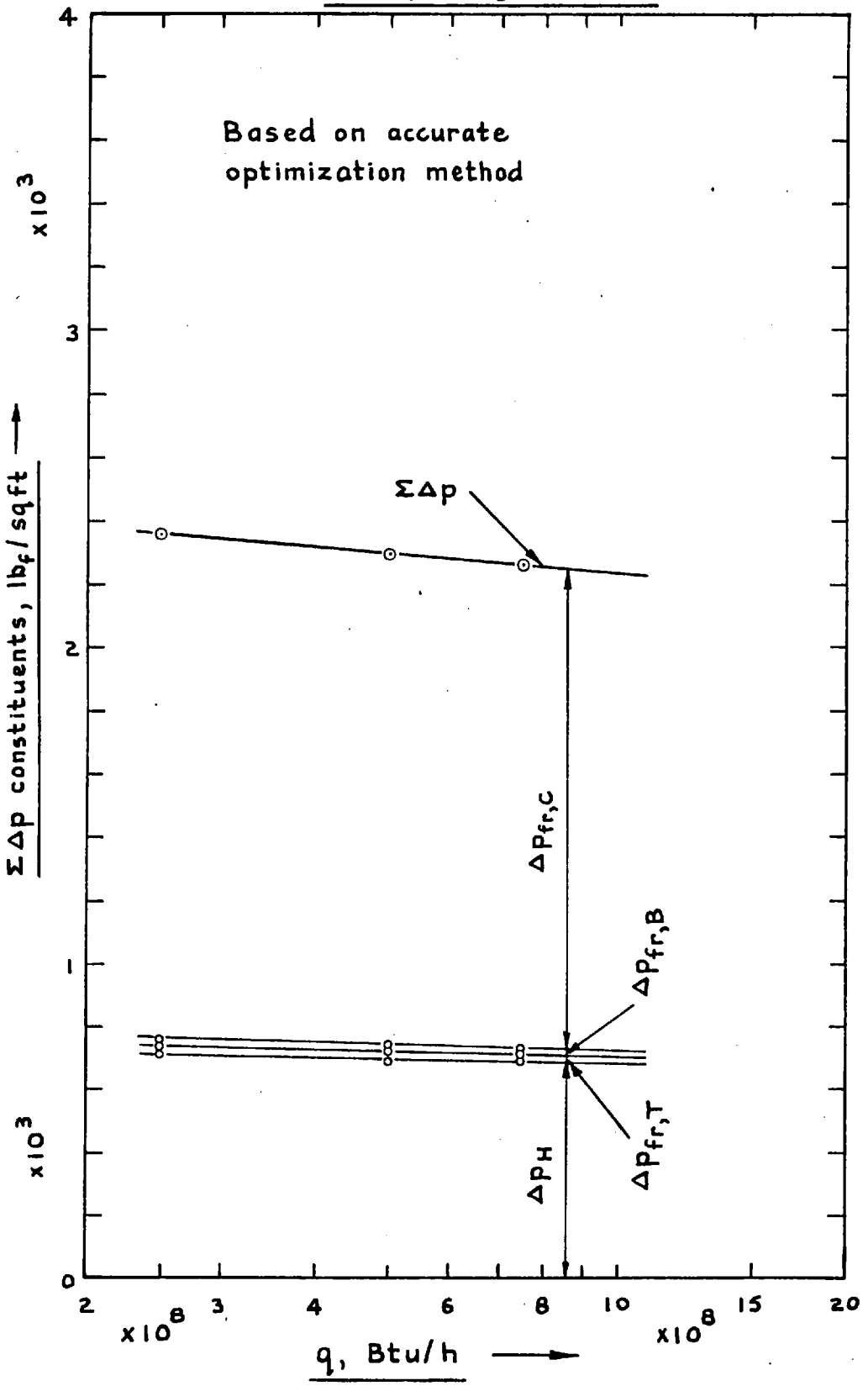
GRAPH II: Pressure-drop constituents for Wet System



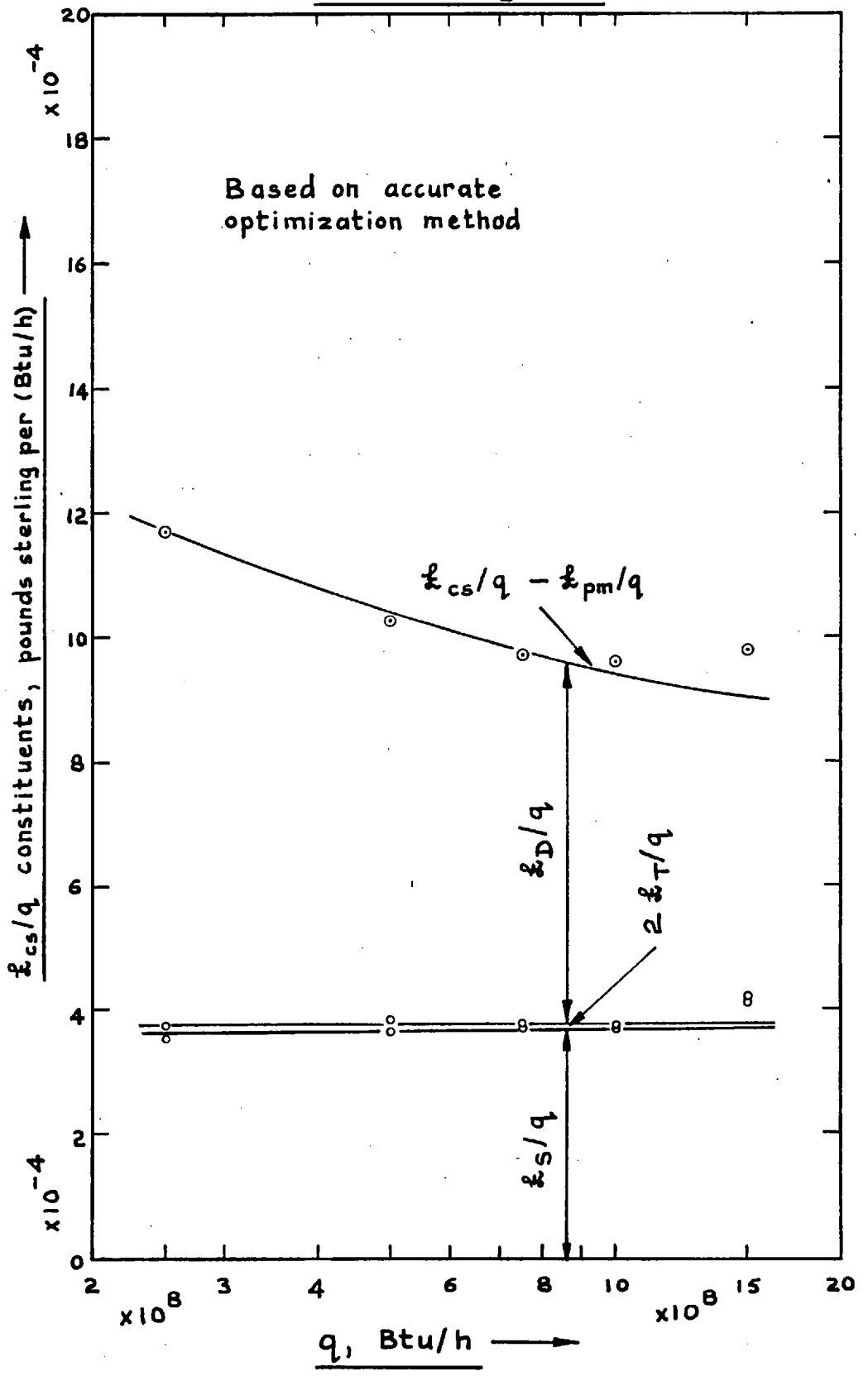
GRAPH 12 : Pressure-drop constituents for Heller System



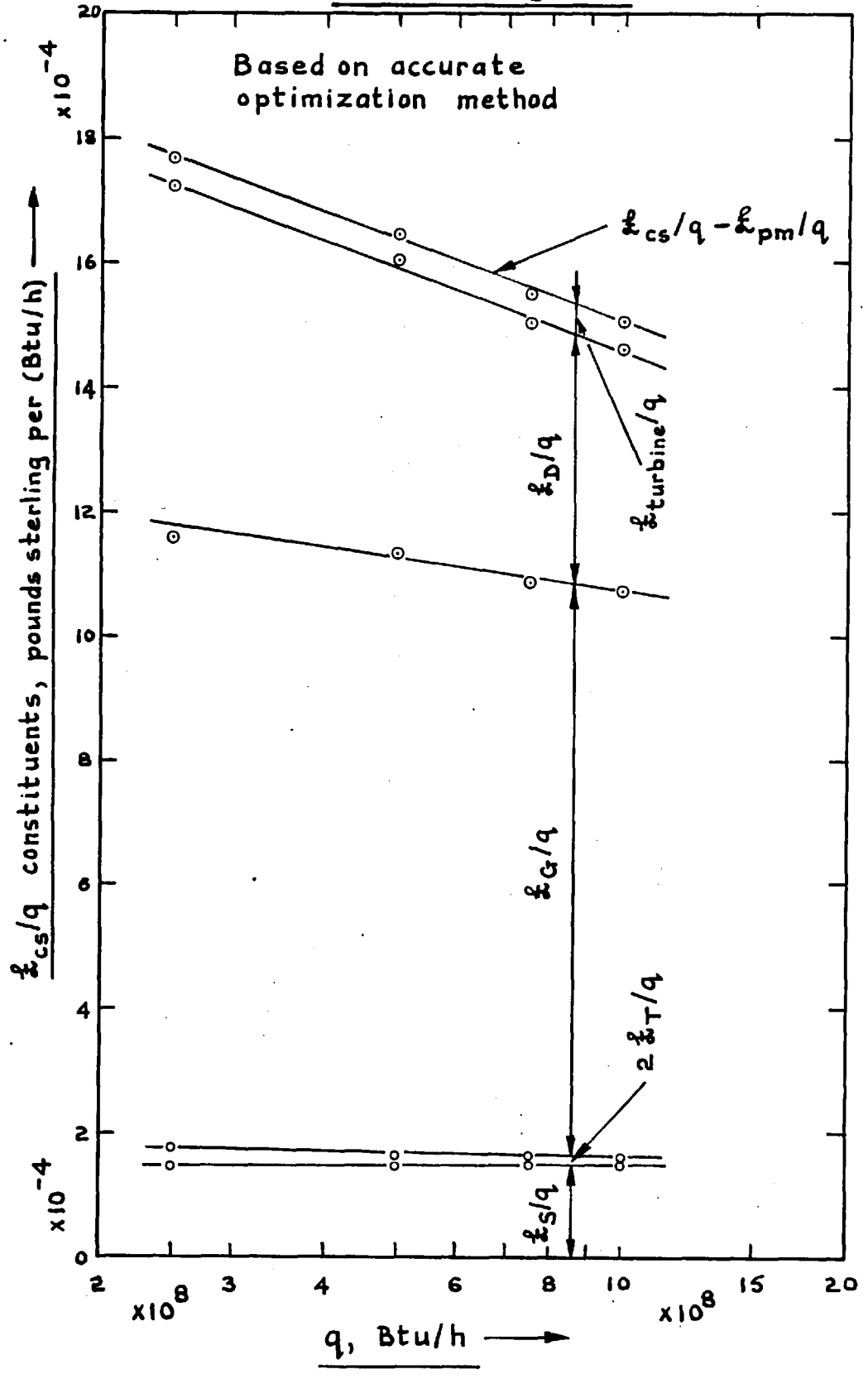
GRAPH 13 : Pressure-drop constituents for Spalding System



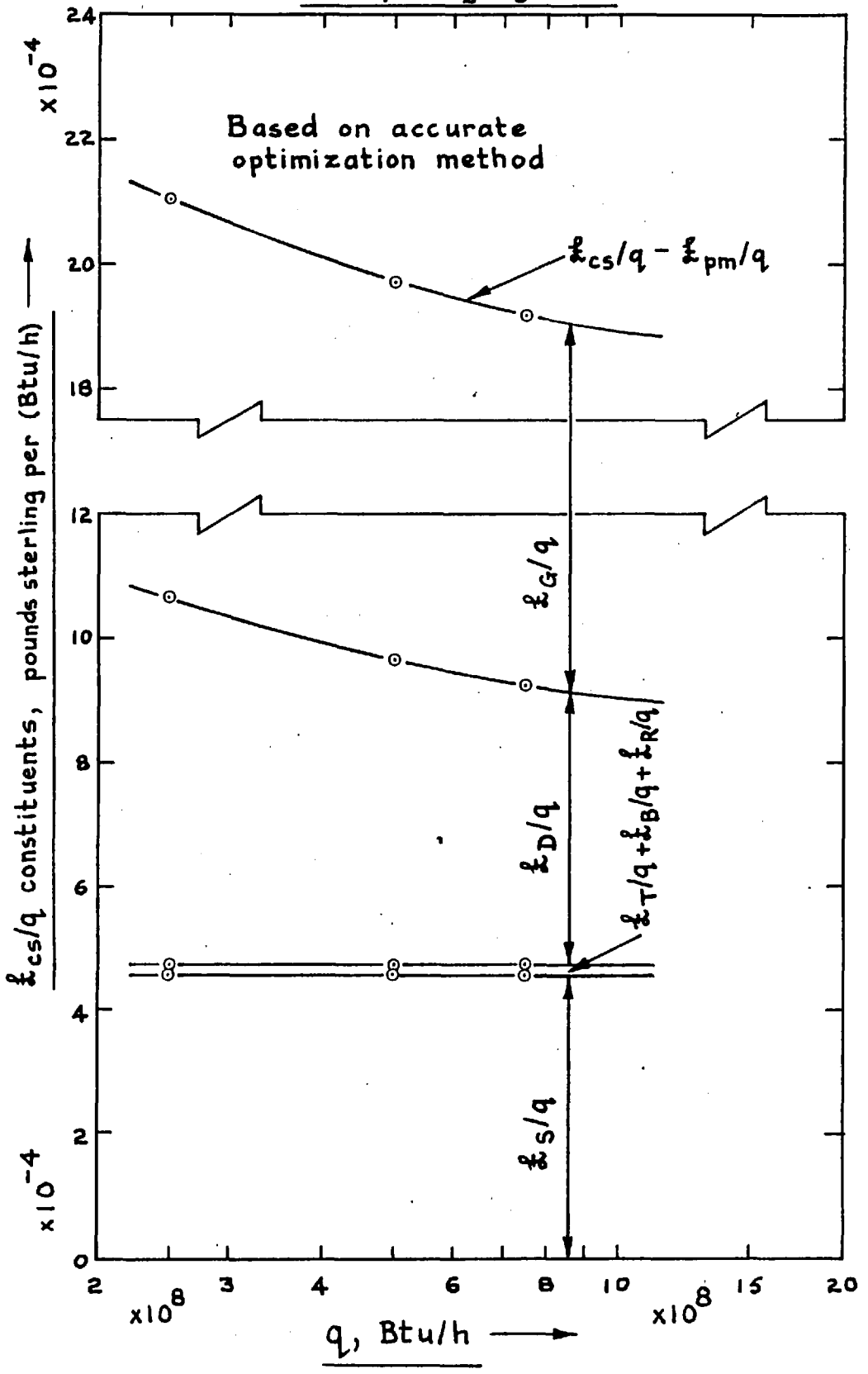
GRAPH 14: Cooling-system price constituents for Wet System



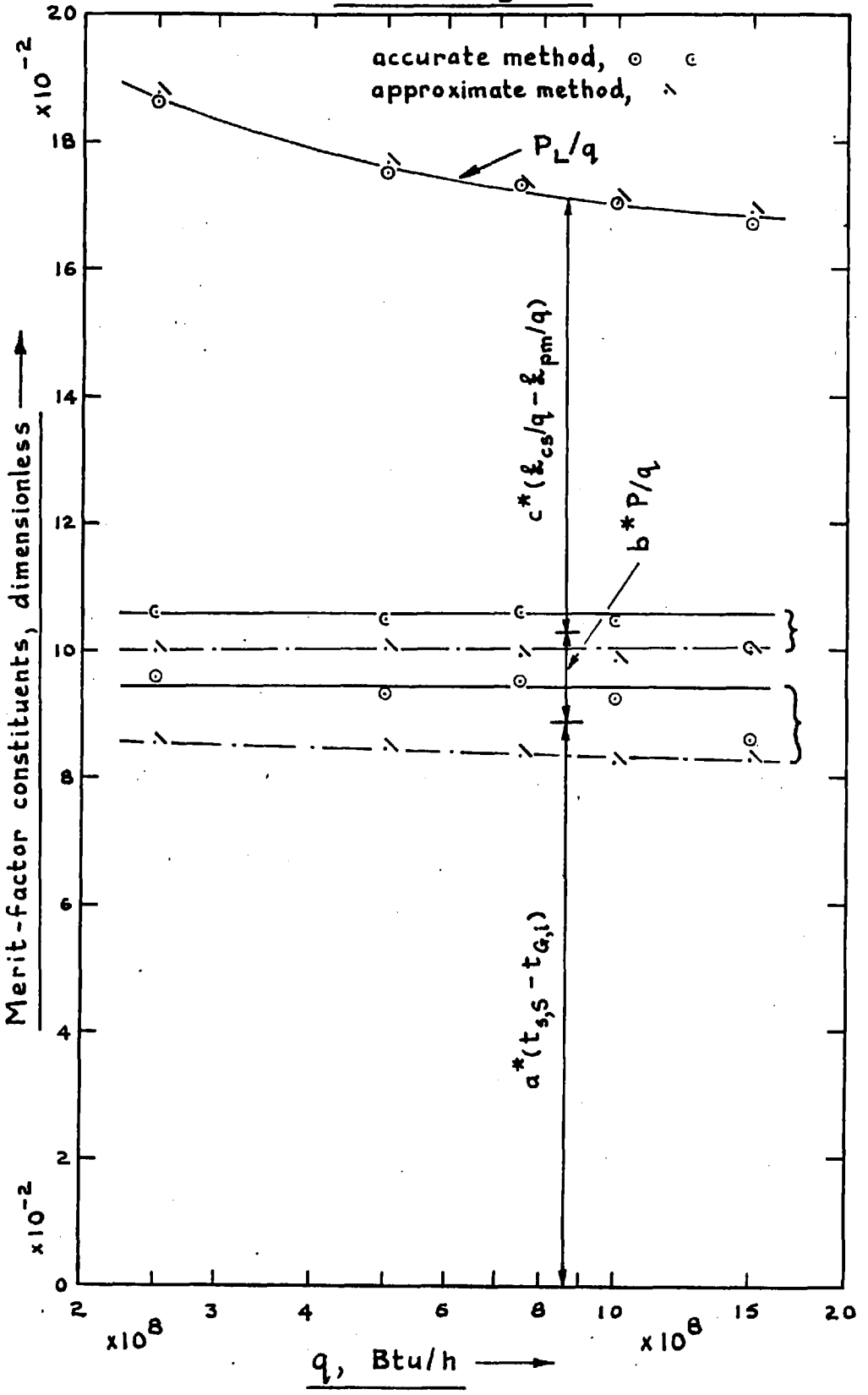
GRAPH 15: Cooling-system price constituents for Heller System



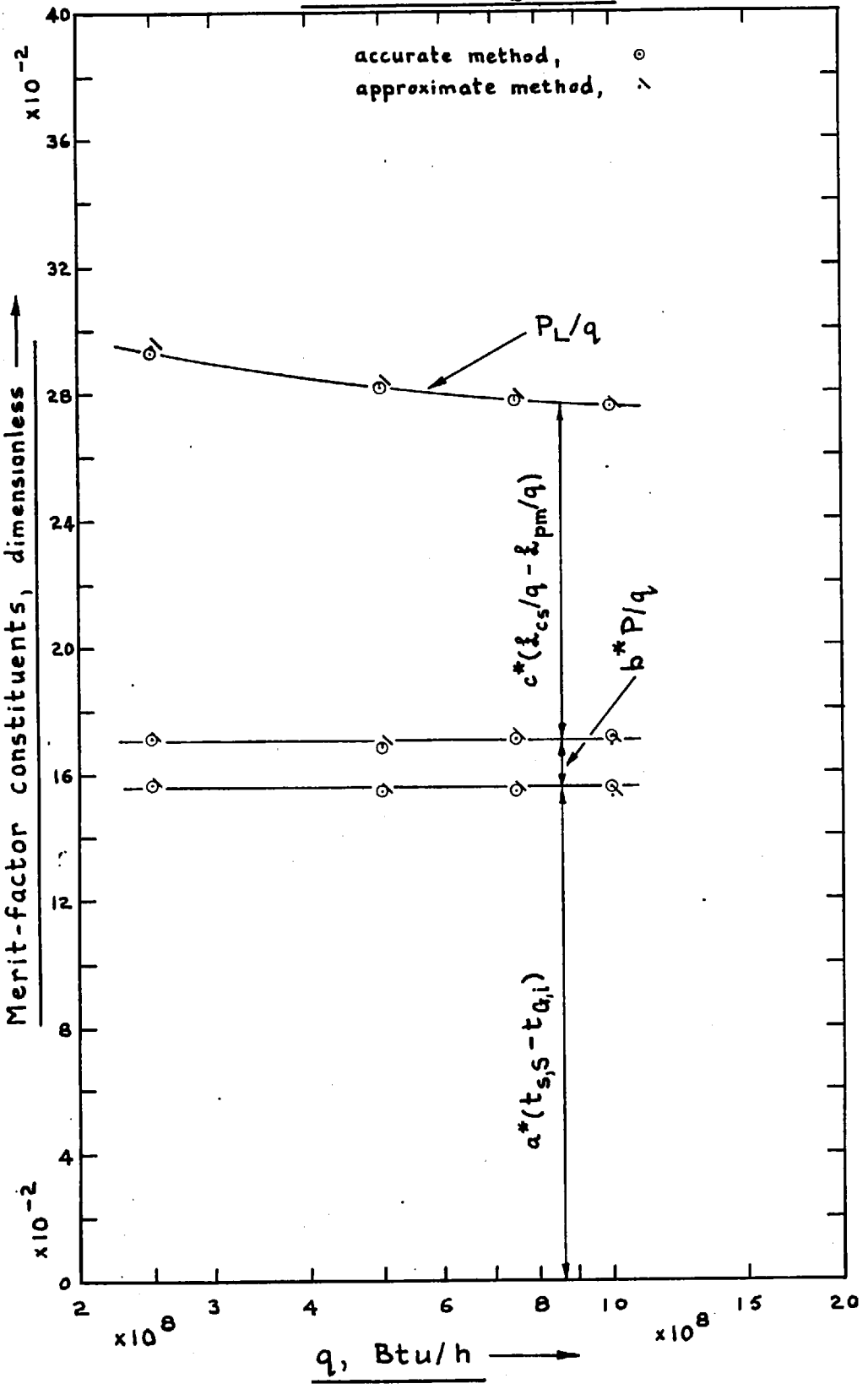
GRAPH 16 : Cooling-system price constituents for Spalding System



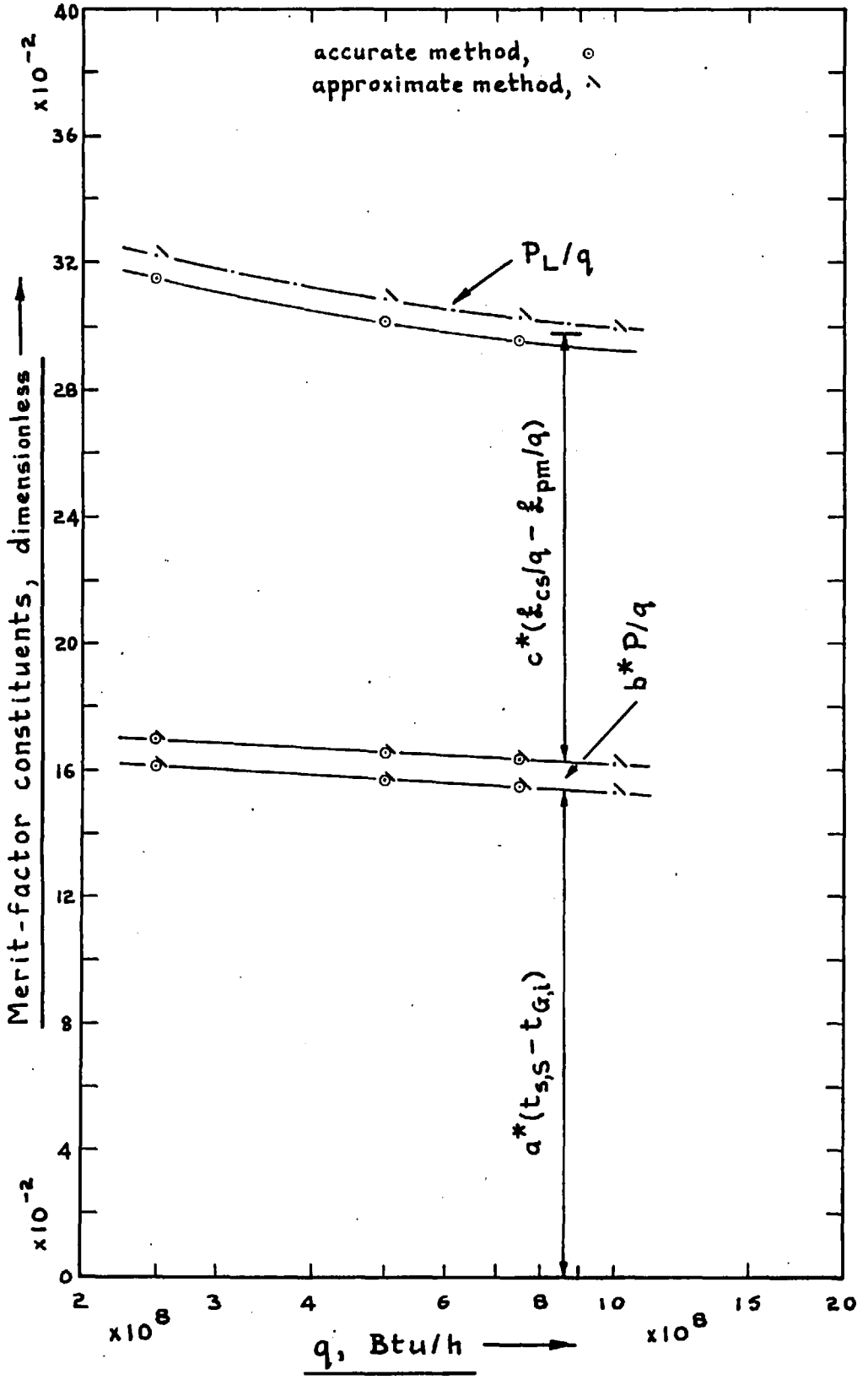
GRAPH 17: Merit-factor constituents for Wet System



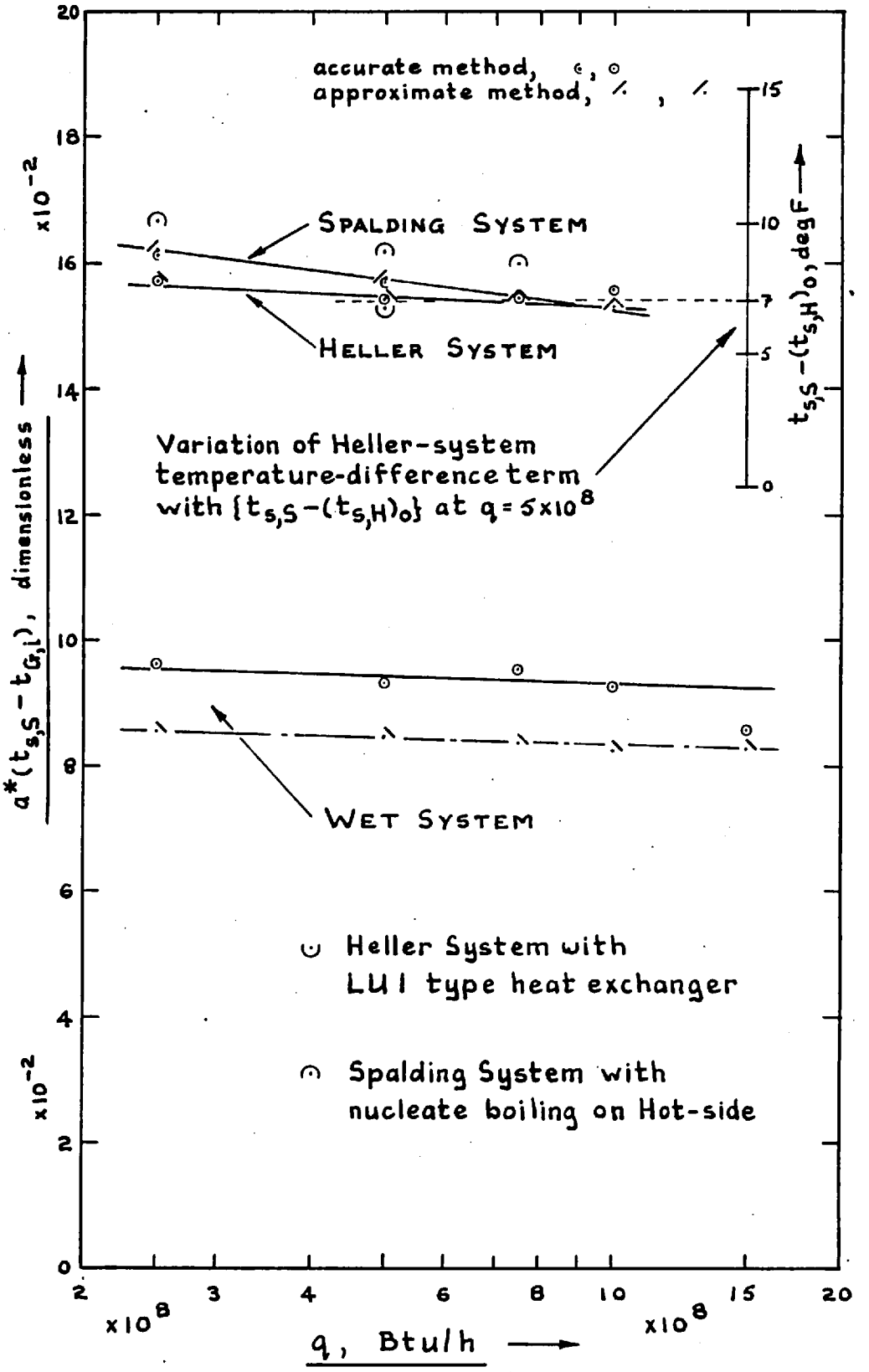
GRAPH 18: Merit-factor constituents for Heller System



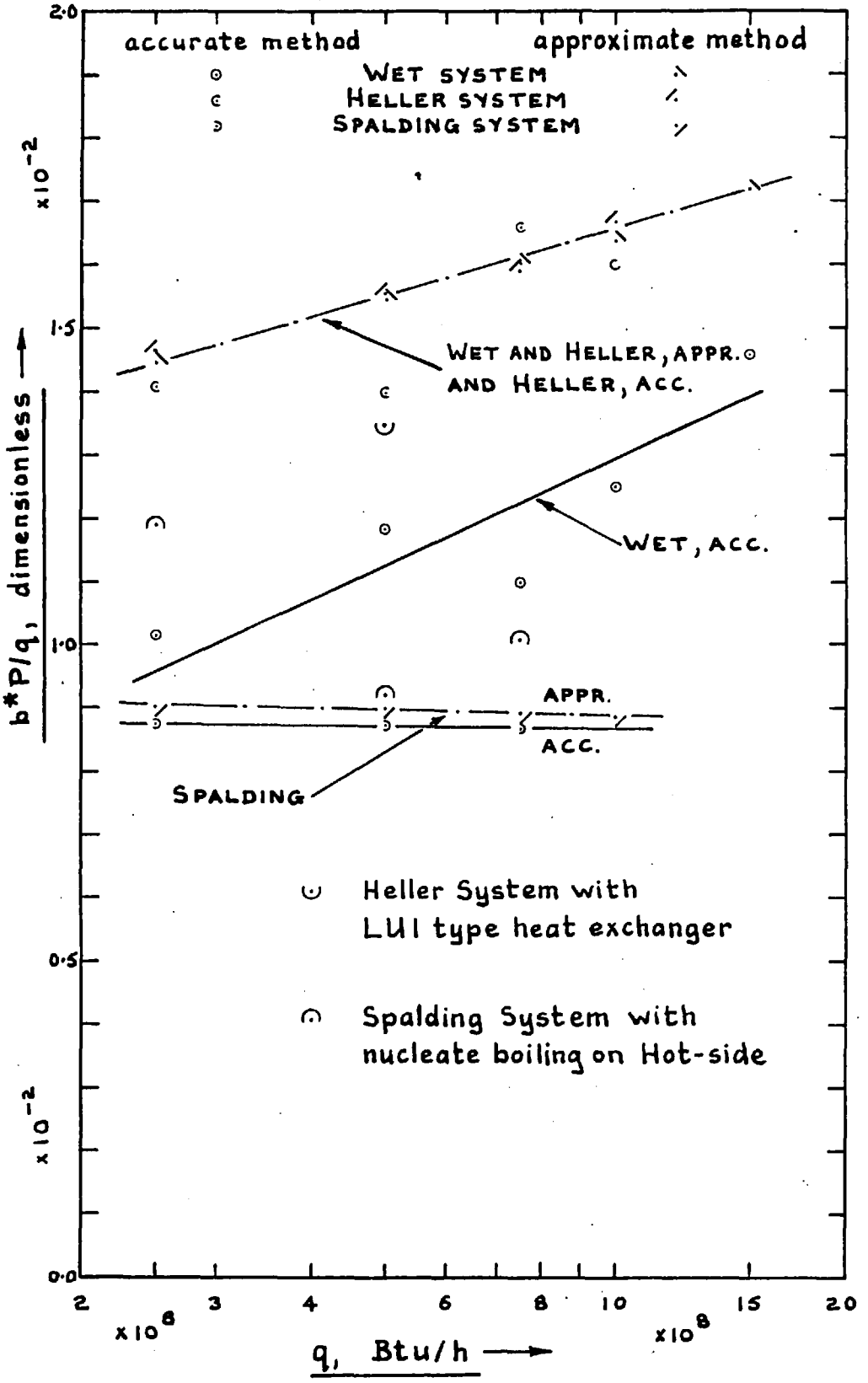
GRAPH 19 : Merit-factor constituents for Spalding System



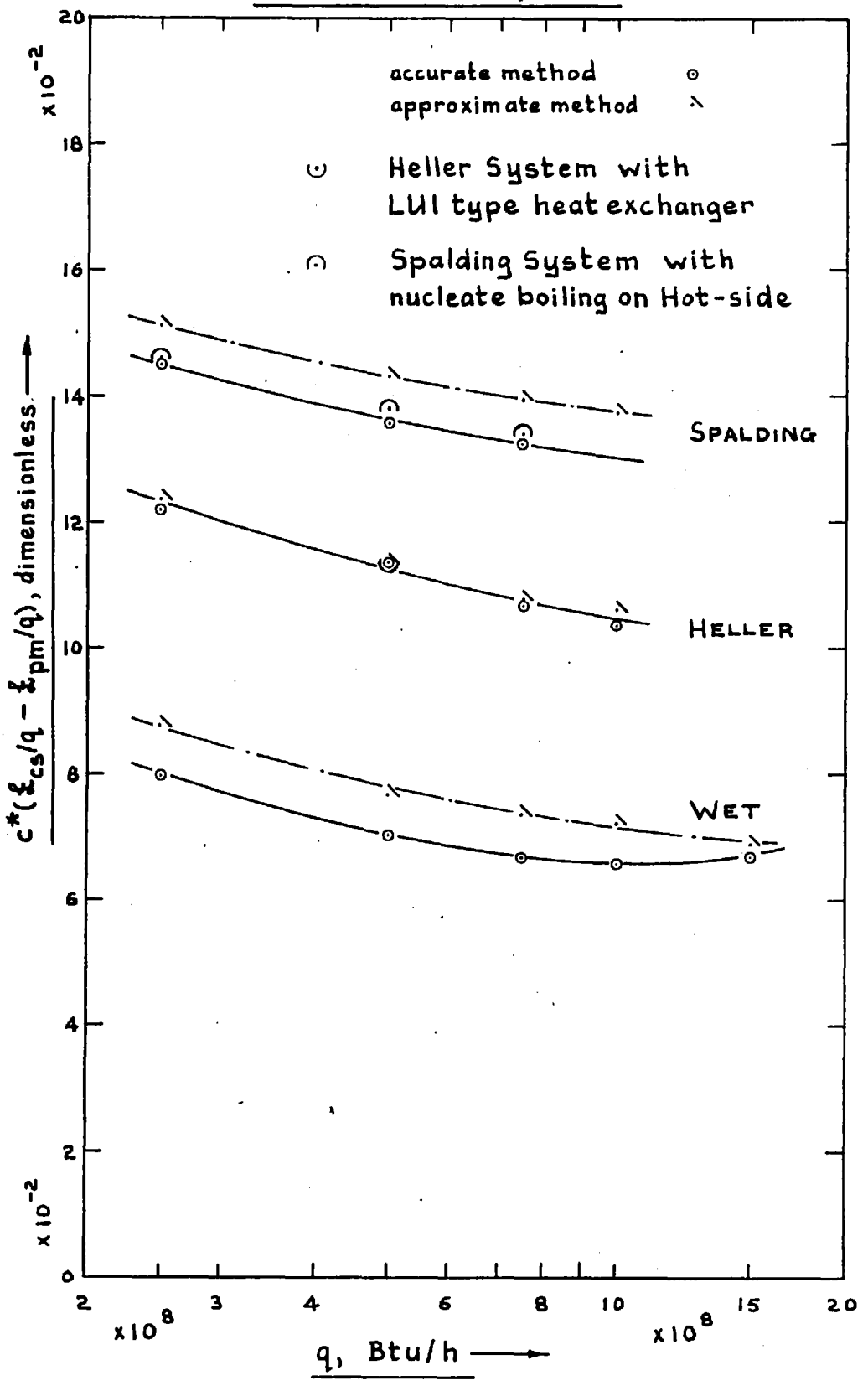
GRAPH 20 : Temperature-difference term in merit-factor equation



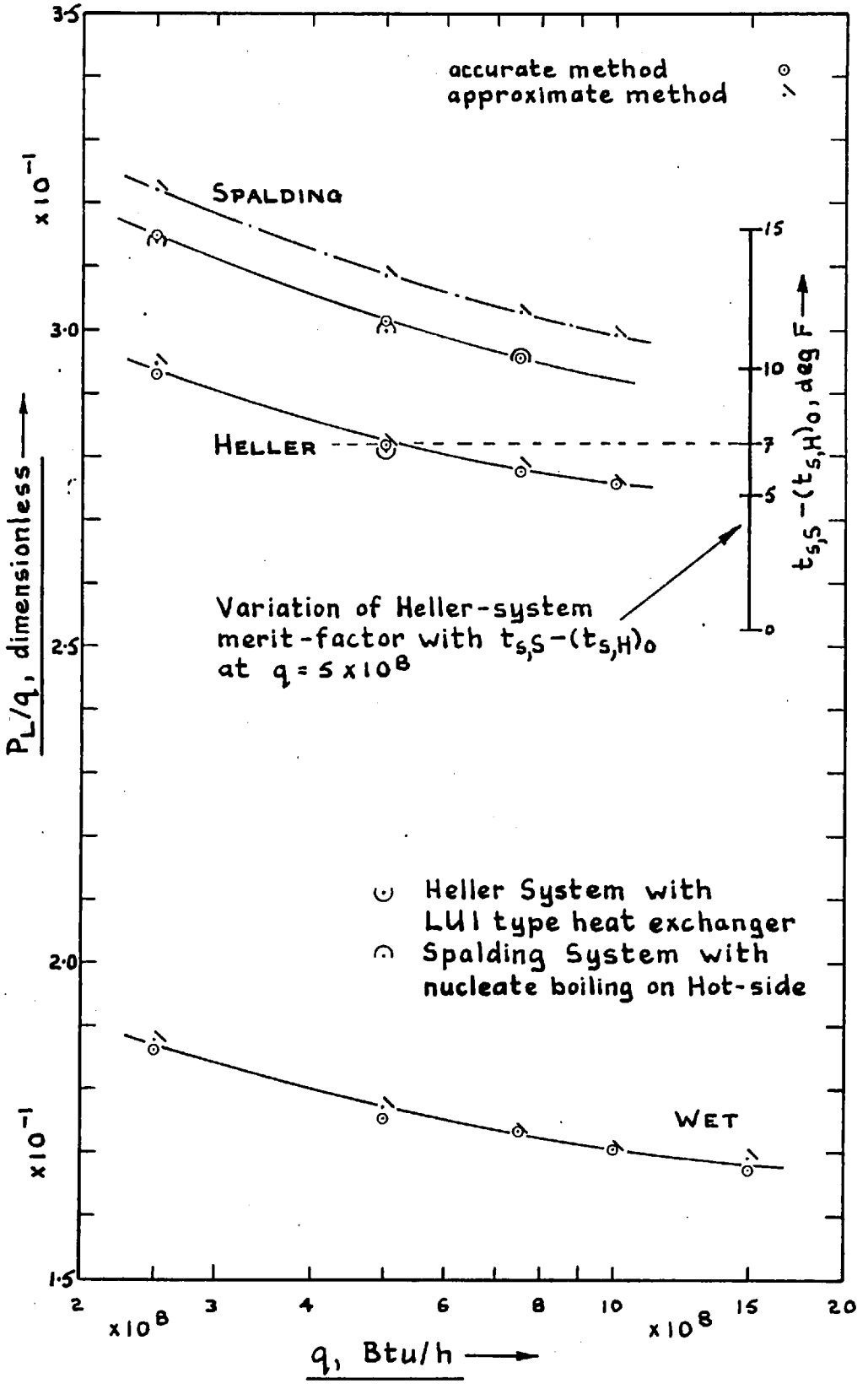
GRAPH 21 : Pumping-power term in merit-factor equation



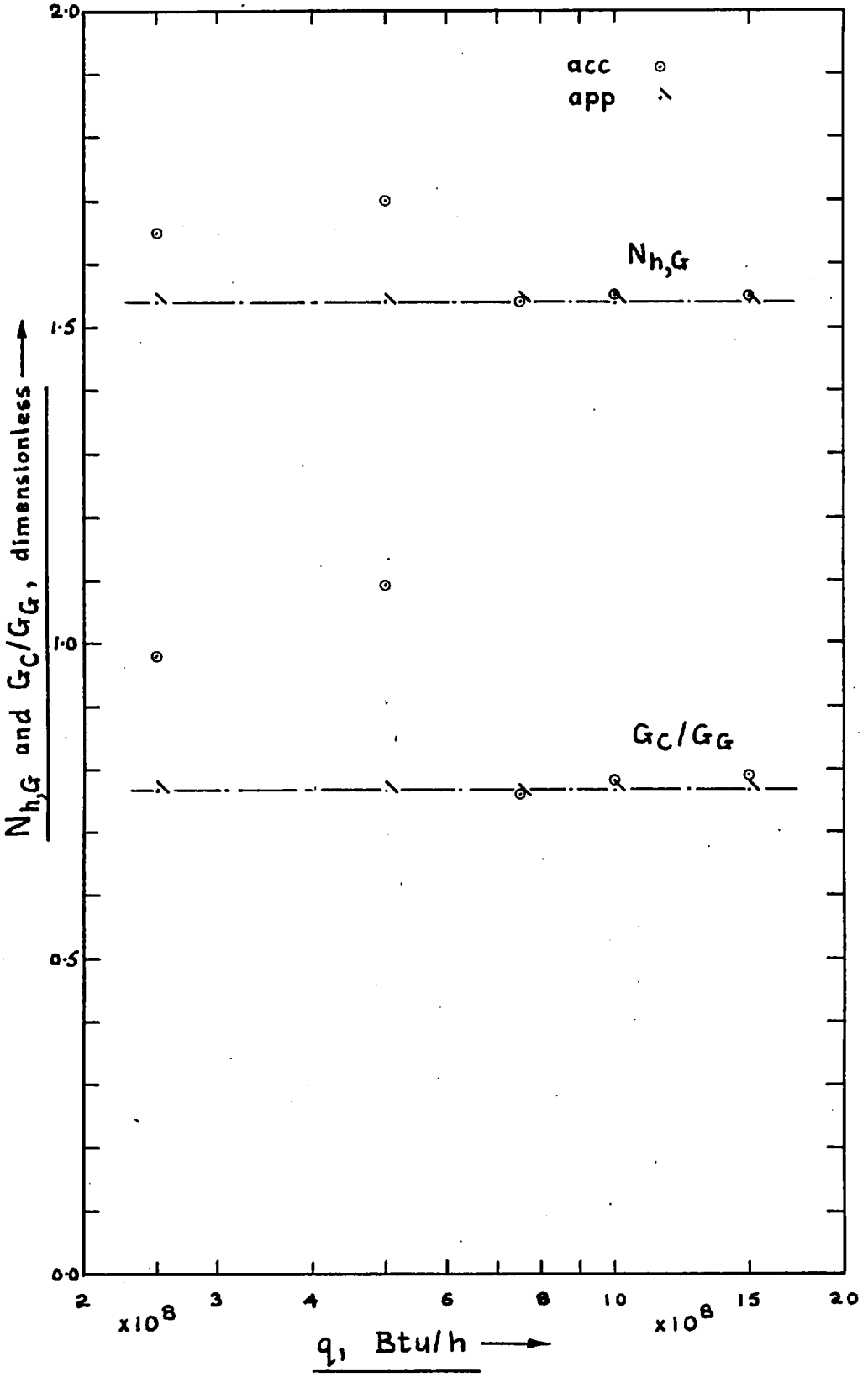
GRAPH 22: Capital-price term in merit-factor equation



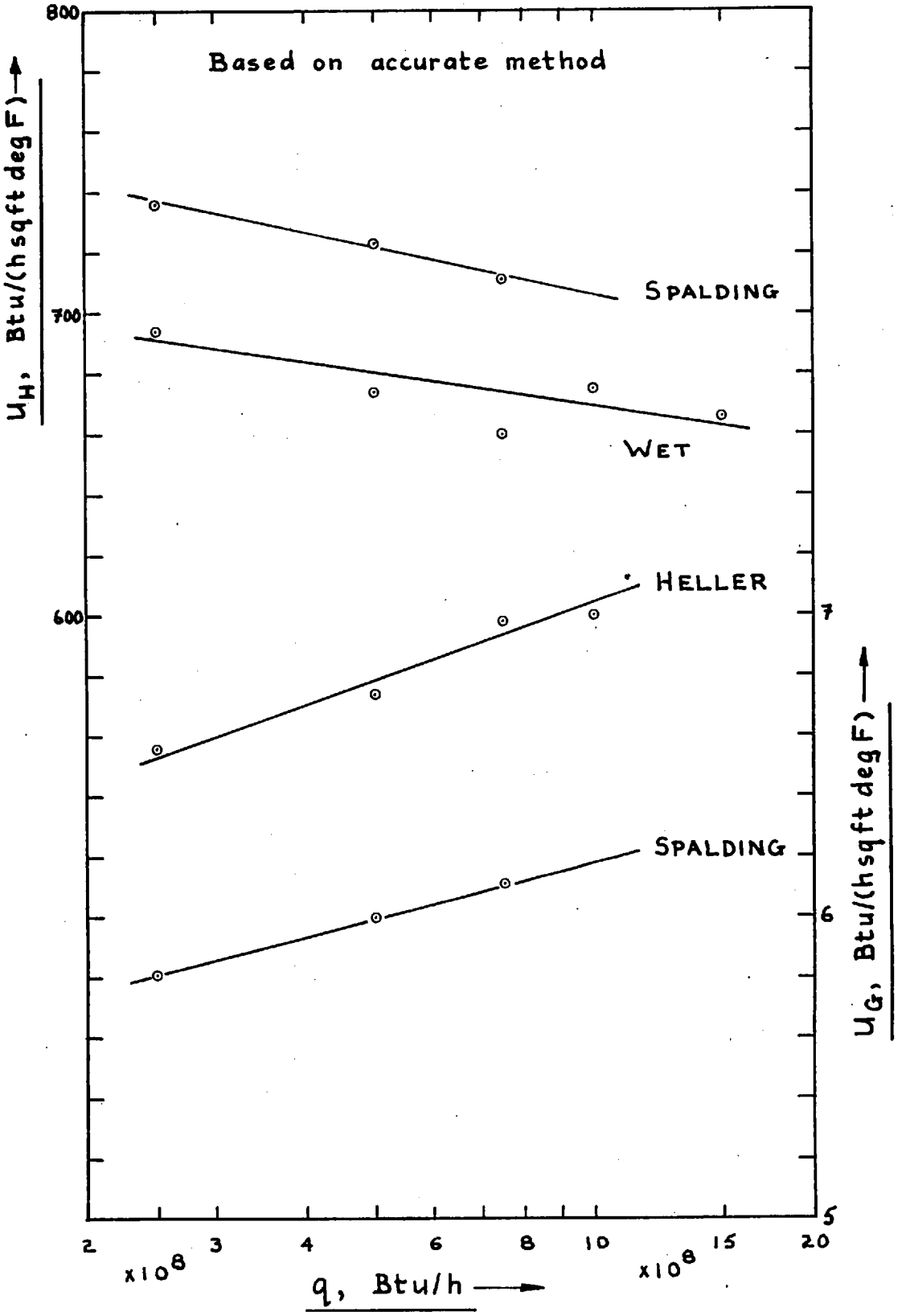
GRAPH 23 : Merit-factor



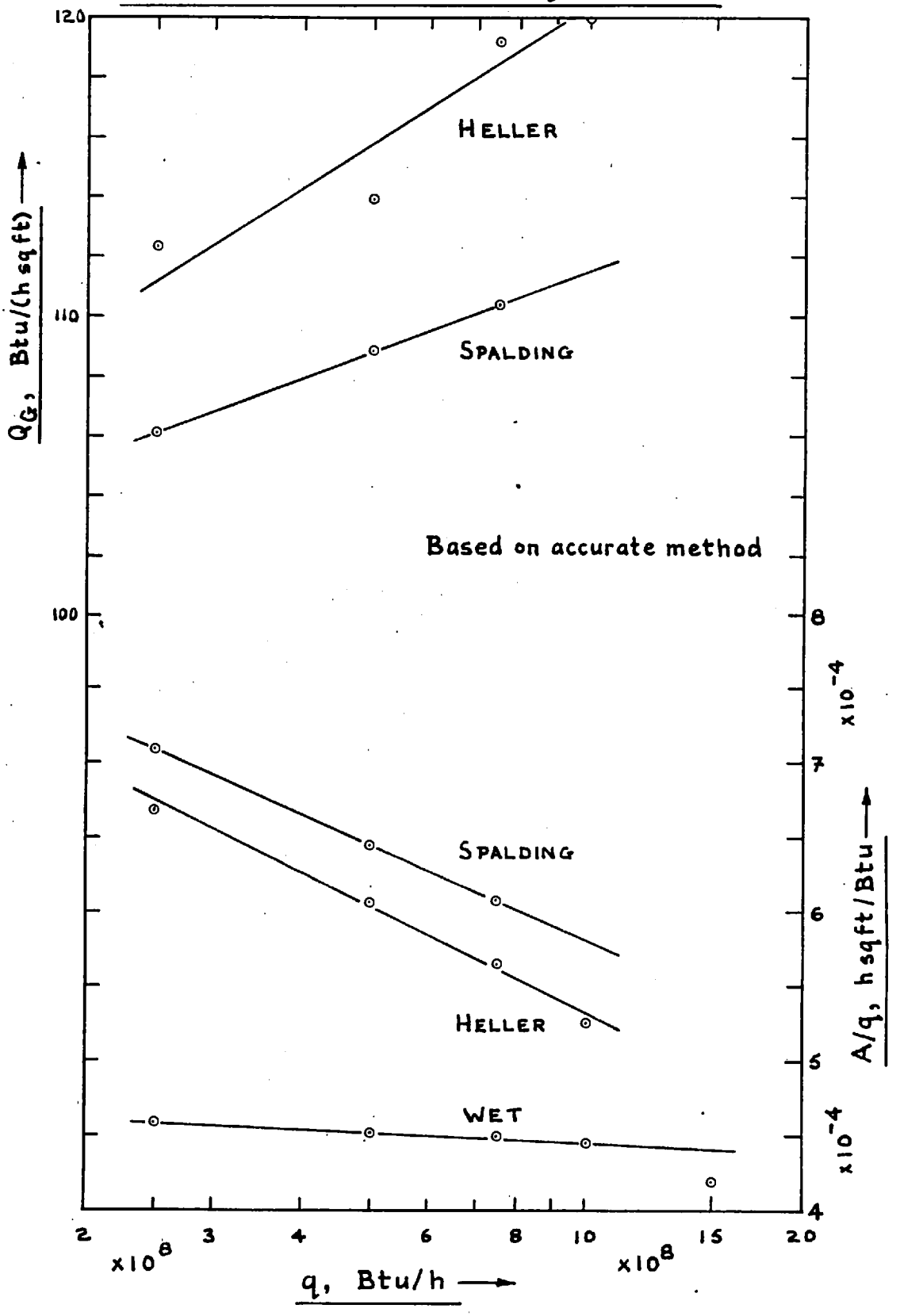
GRAPH 24: $N_{h,G}$ and G_C/G_G for Wet System



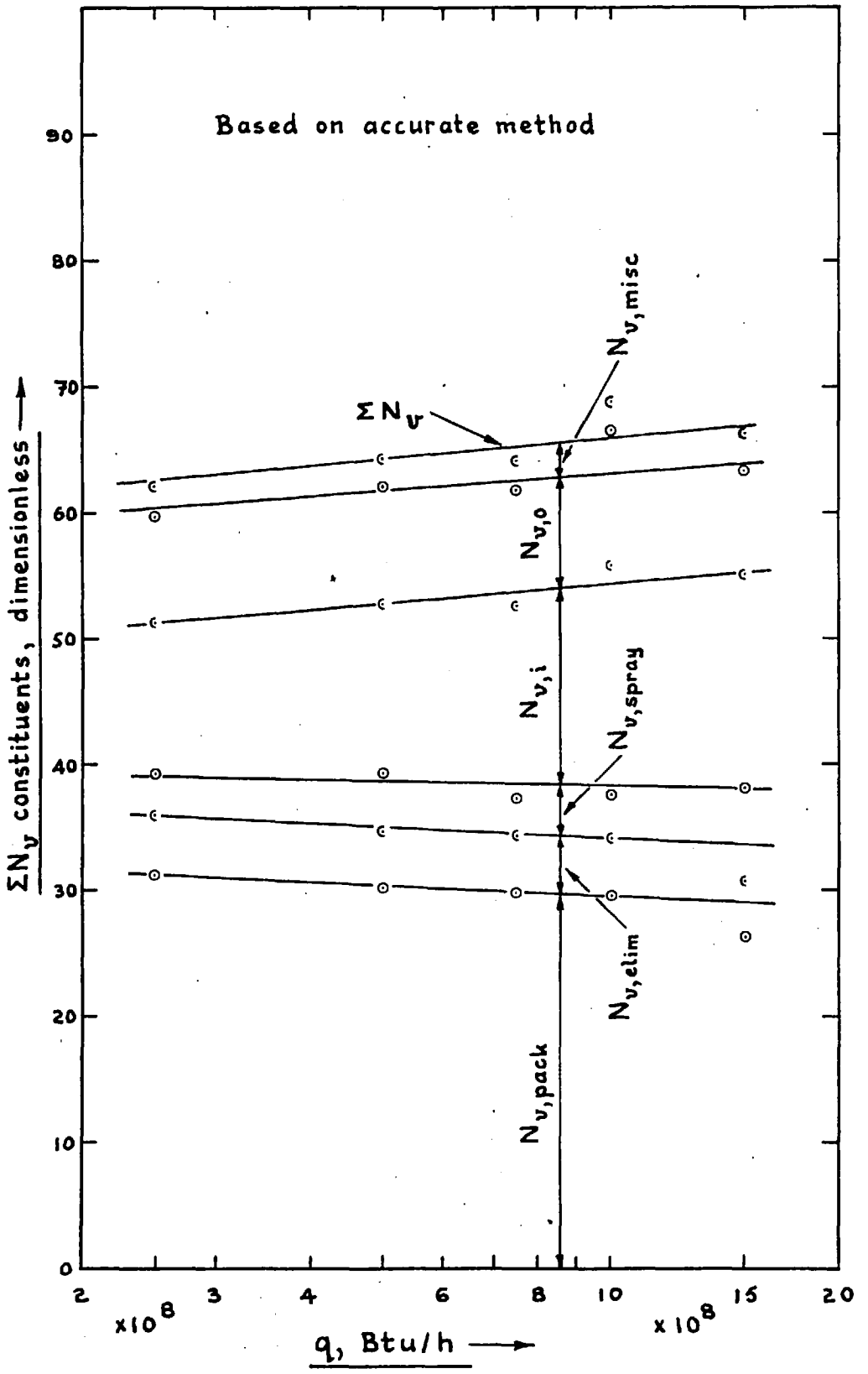
GRAPH 25 : Overall Hot- and Air-side heat-transfer coefficients



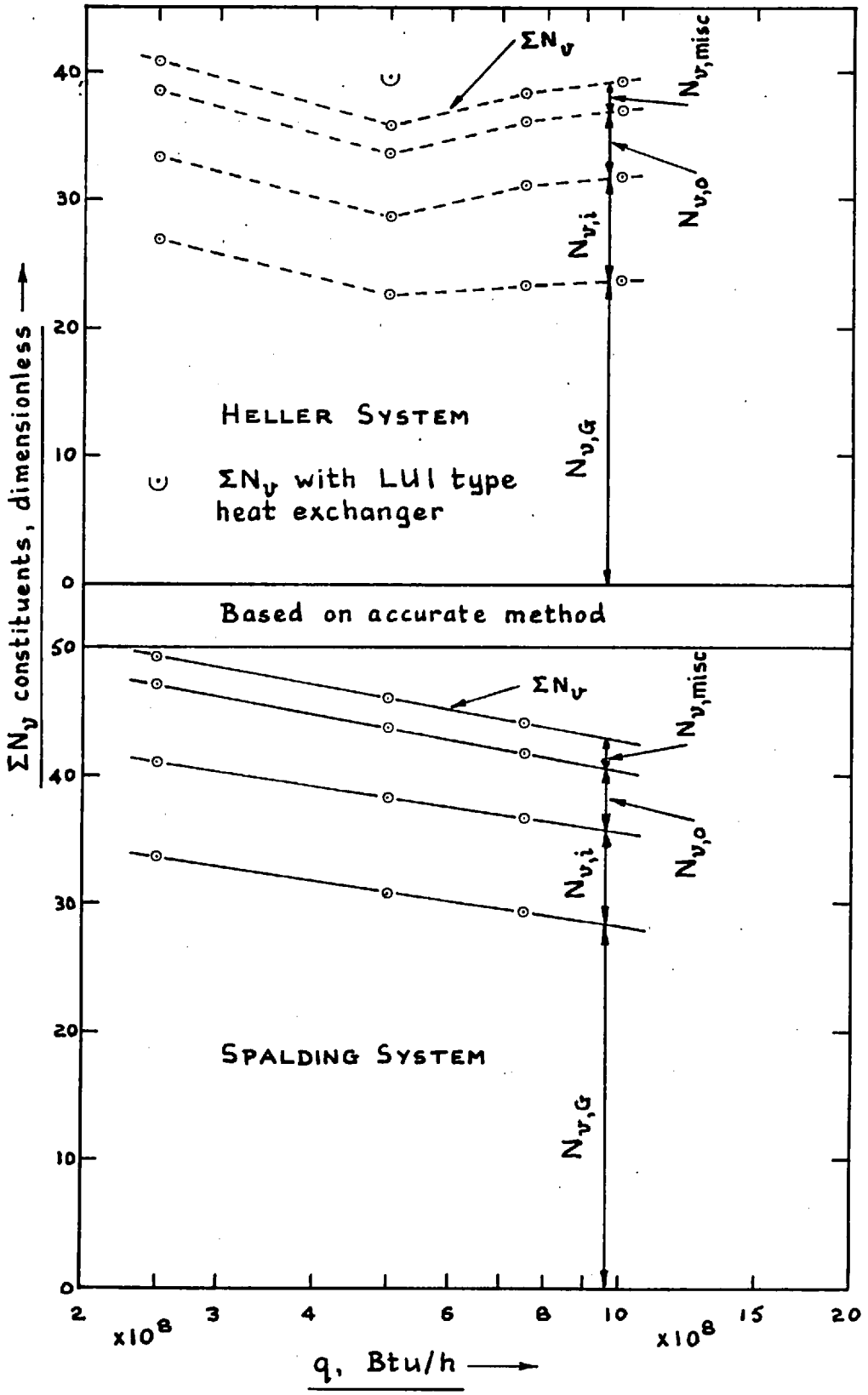
GRAPH 26: Air-side heat flux and ratio of tower surface area to waste-heat rejection rate



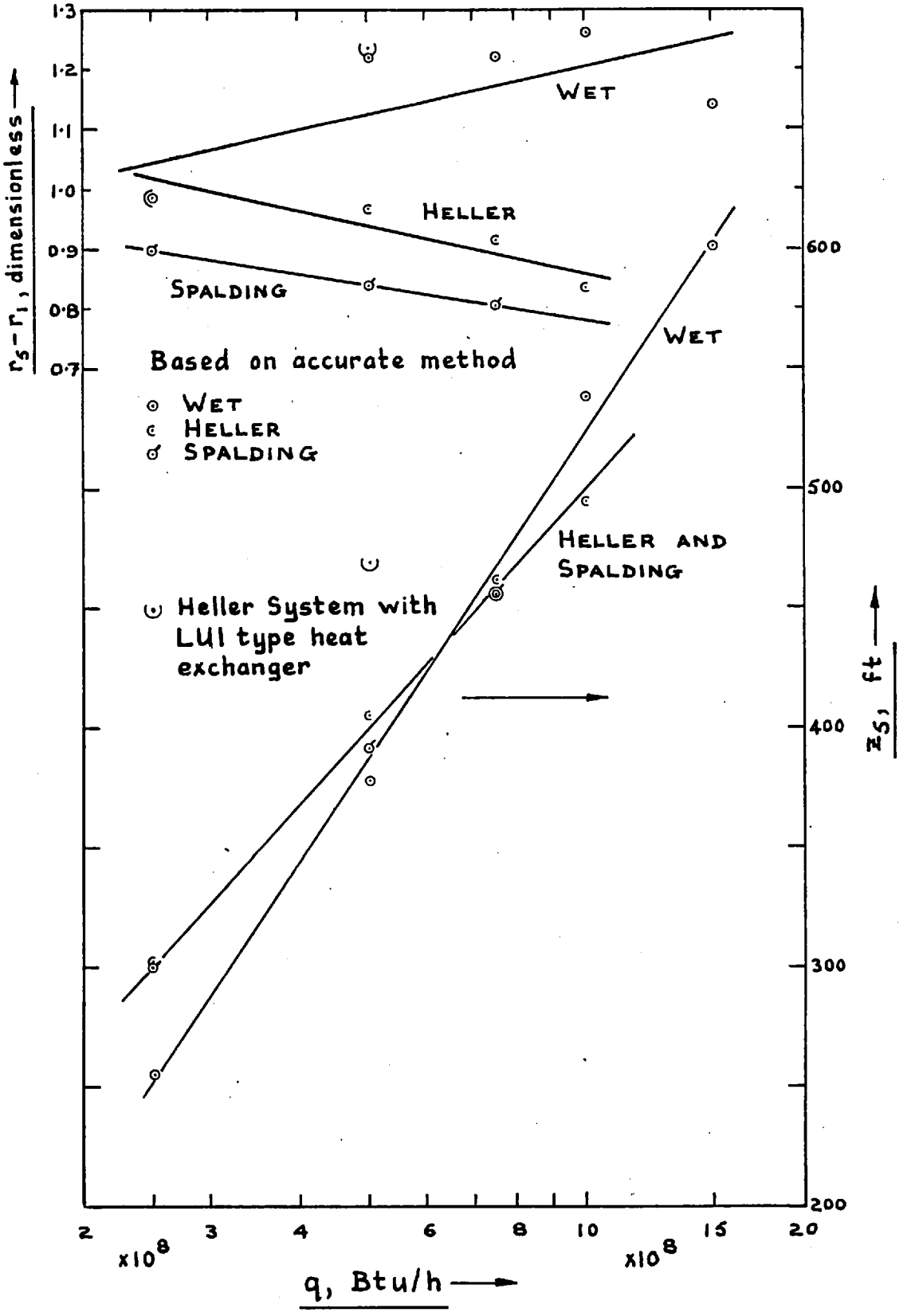
GRAPH 27 : ΣN_v constituents for Wet System



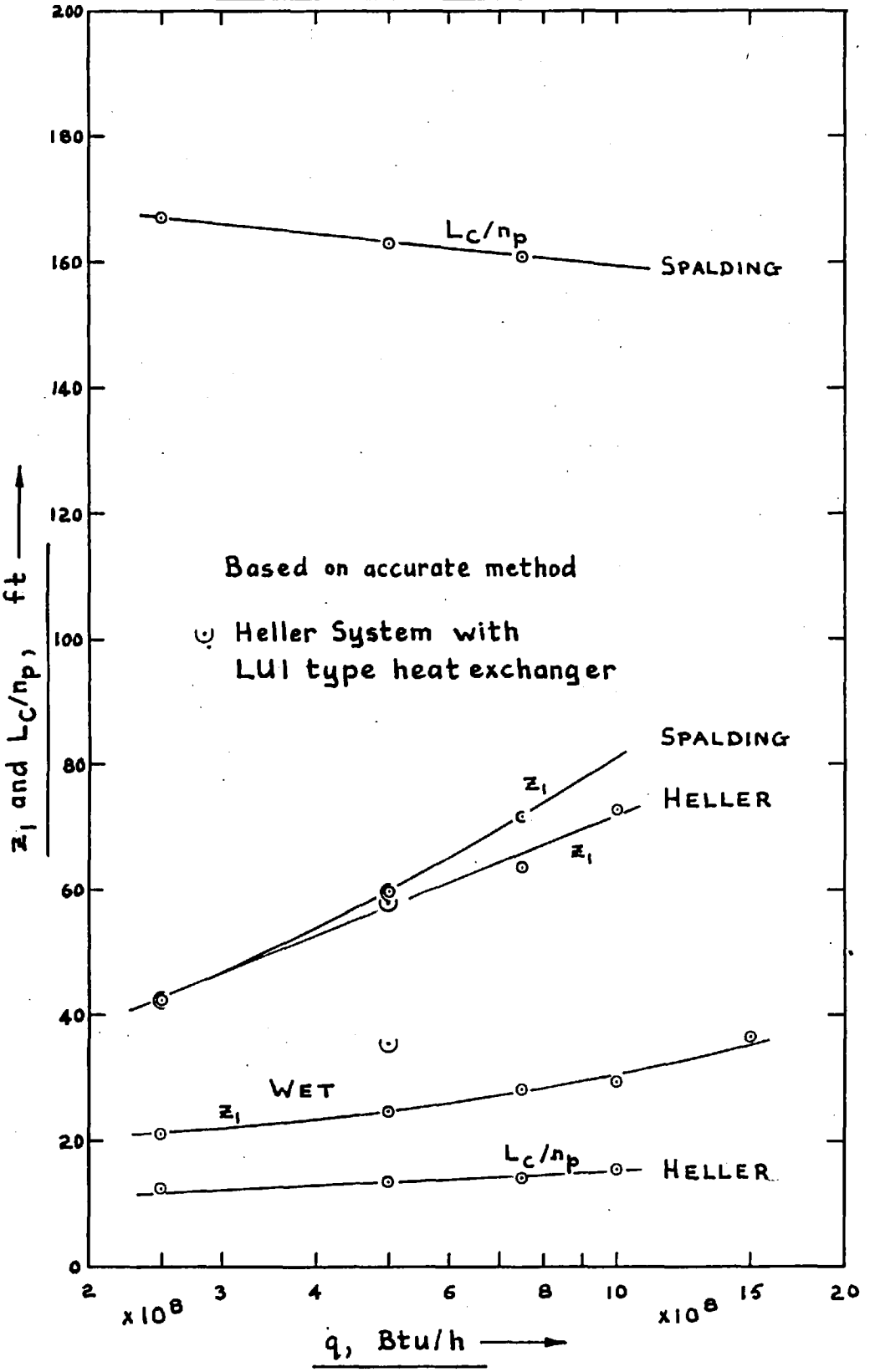
GRAPH 28: ΣN_v constituents for Heller- and Spalding-systems



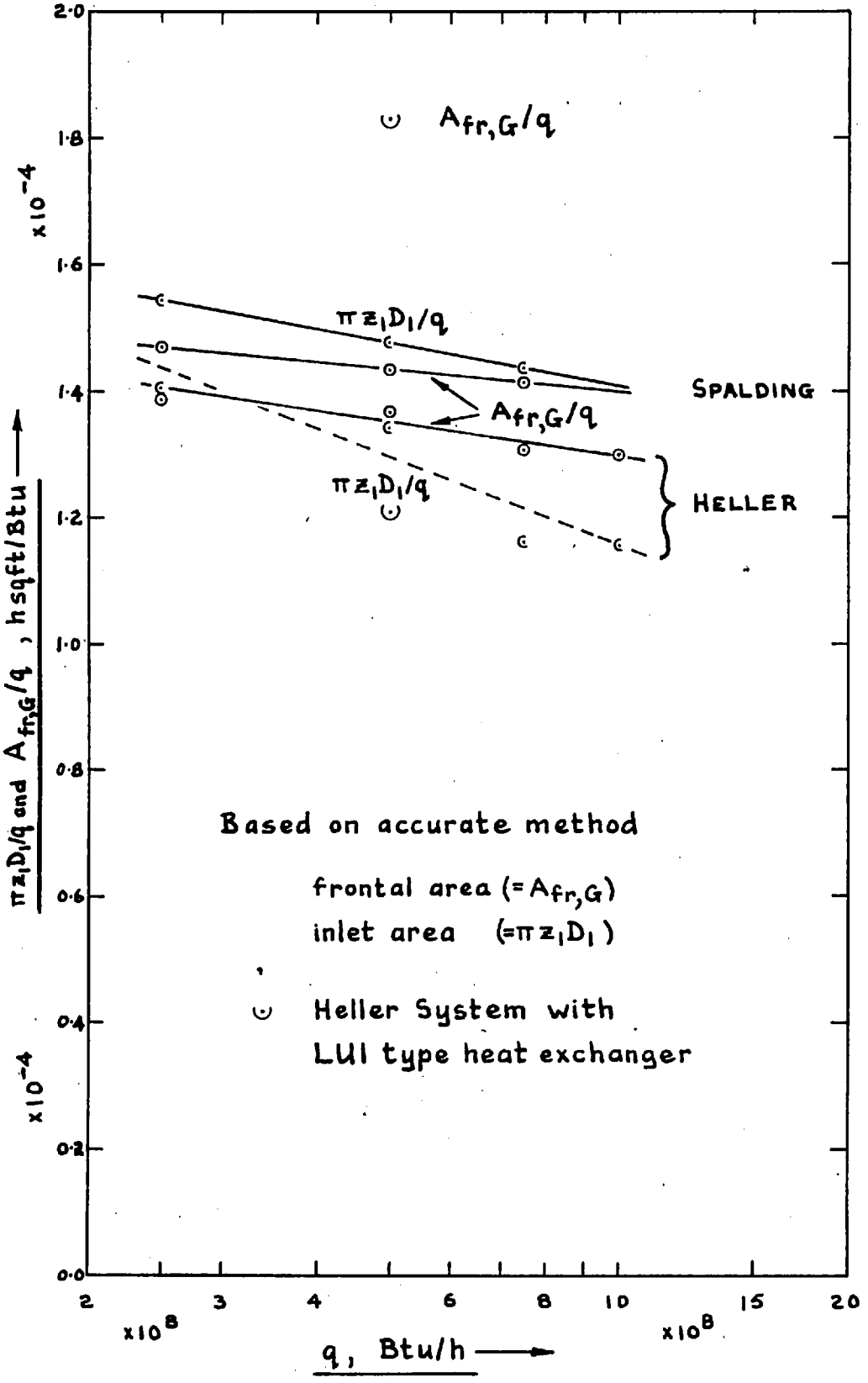
GRAPH 29: $r_s - r_1$ and height of top of tower



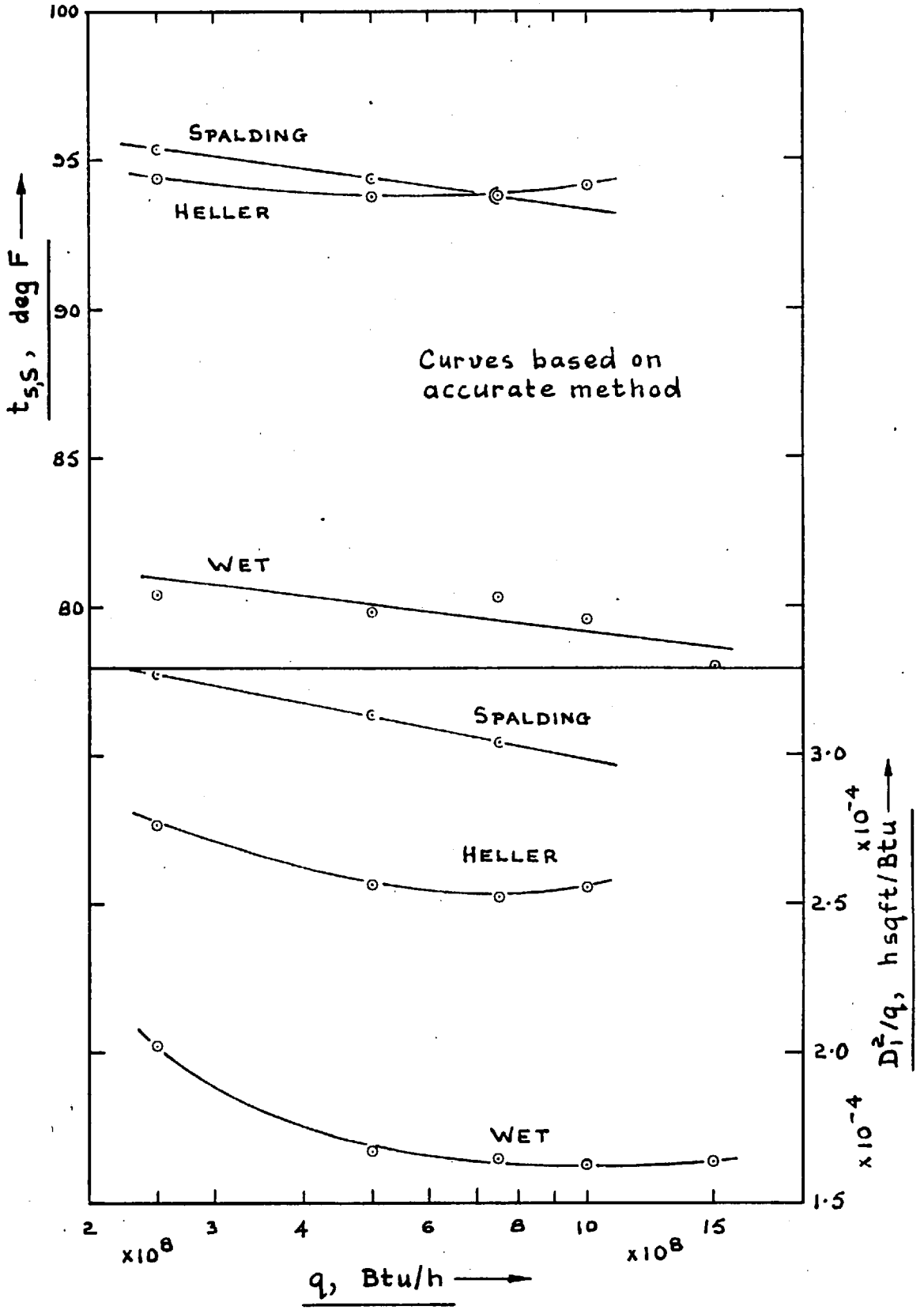
GRAPH 30: Tower inlet height and length of tower heat-exchanger unit



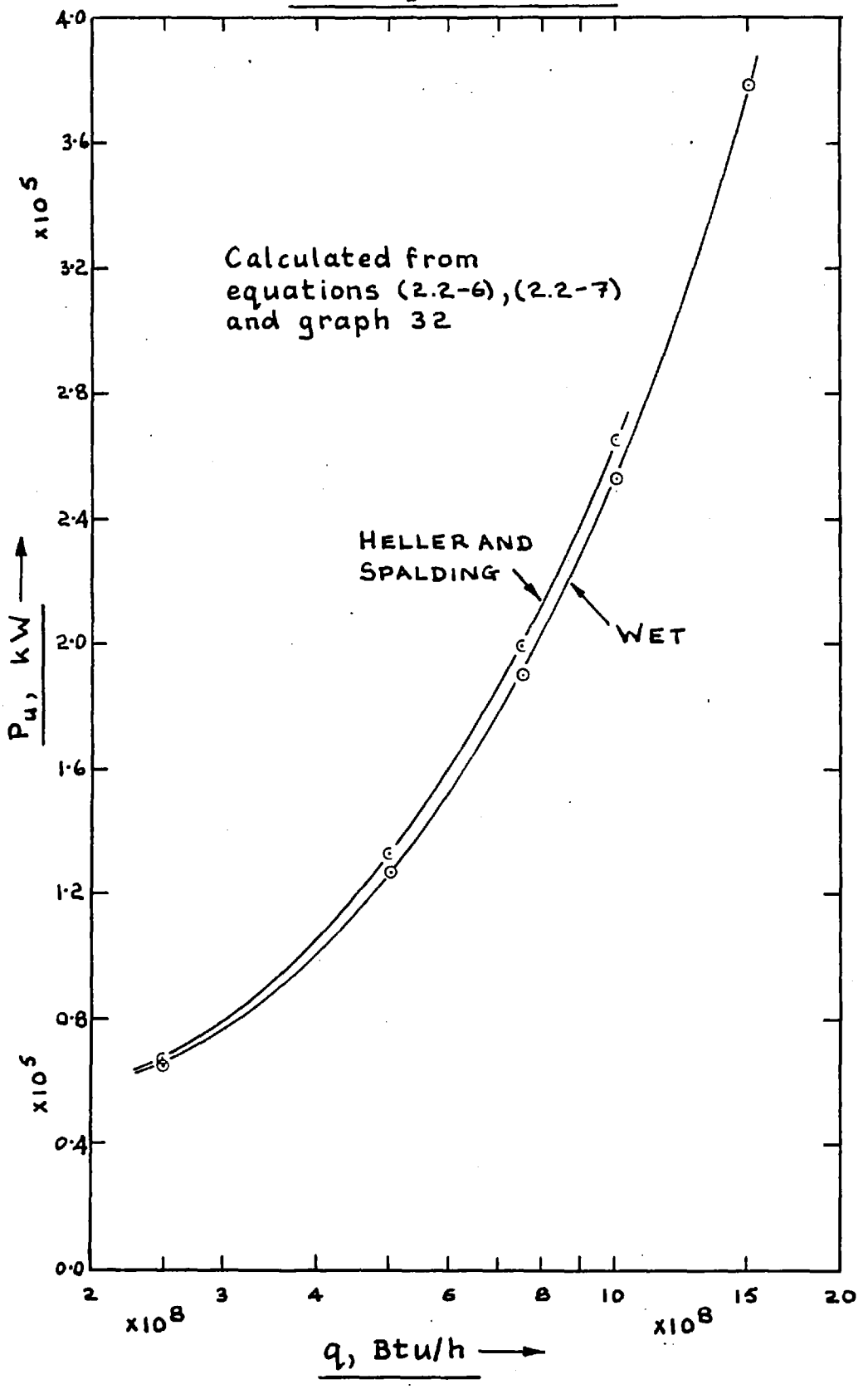
GRAPH 31: Ratios of heat-exchanger frontal area and tower inlet area to waste-heat rejection rate



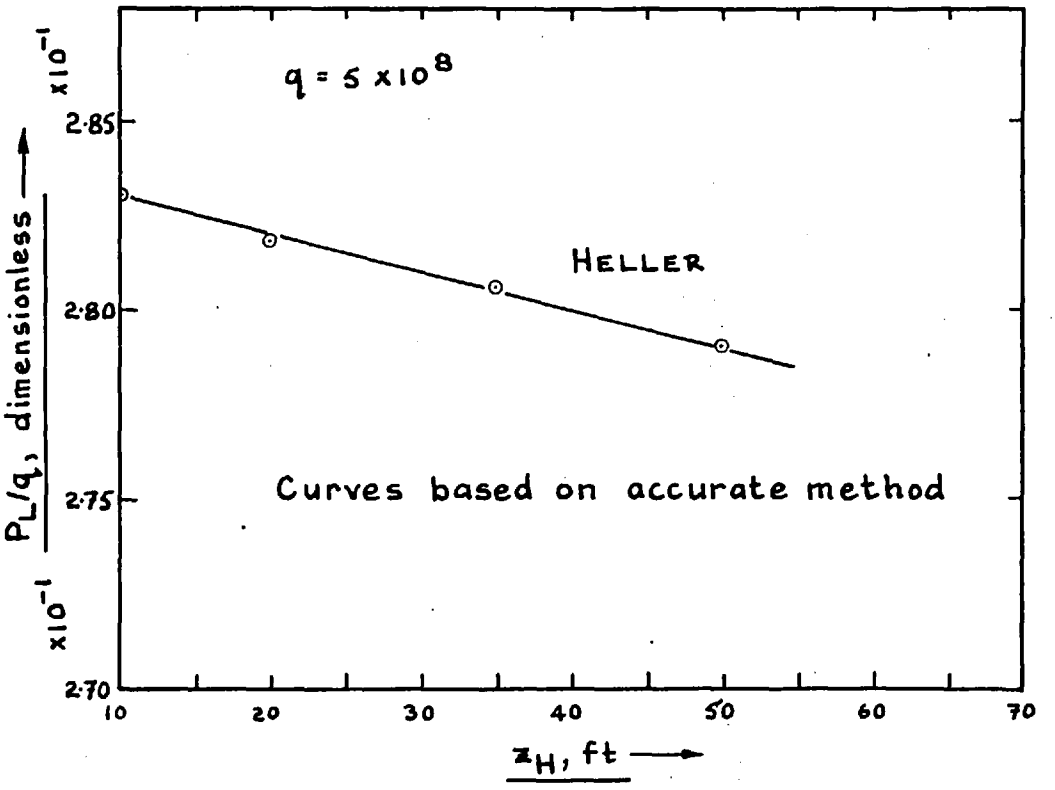
GRAPH 32 : Exhaust steam temperature and D_1^2/q.

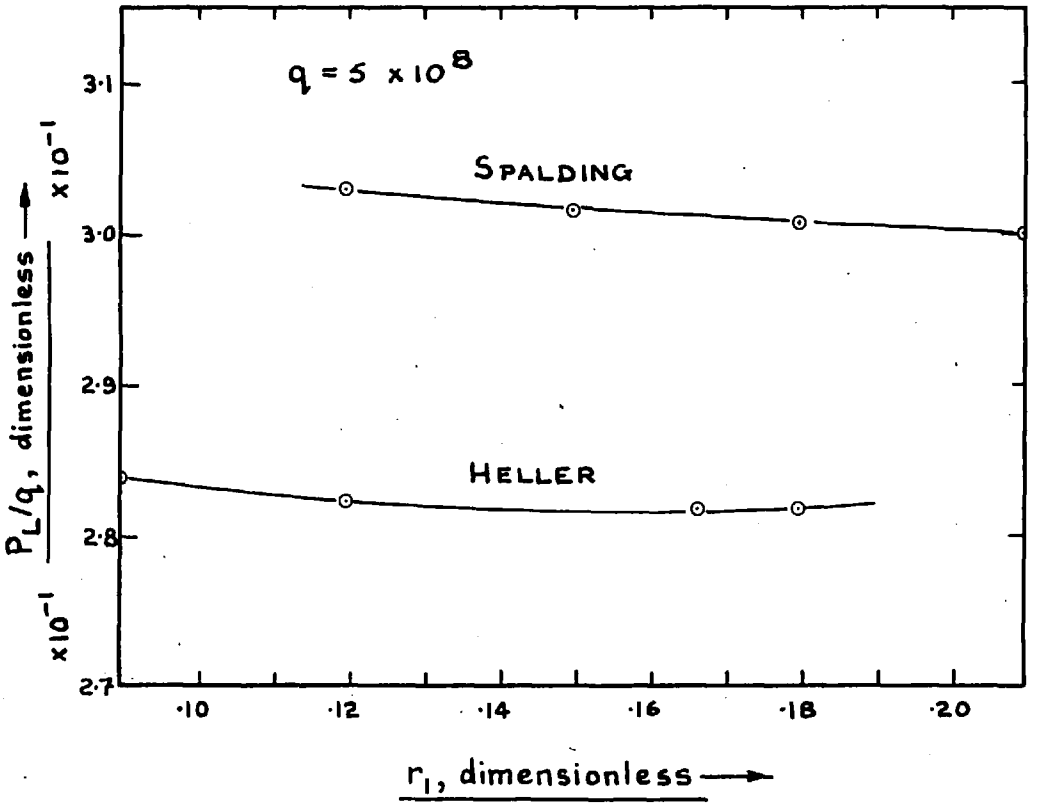


GRAPH 33: Generator output vs waste-heat rejection rate



GRAPH 34 : Merit-factor vs z_H for HELLER SYSTEM
and merit-factor vs r_1 for HELLER- AND SPALDING-SYSTEMS





10 DISCUSSION OF RESULTS

In this chapter, a comparison of the three considered cooling systems is left until the final section, viz. 10.9. The preceding sections are devoted, among other minor items, to justifications of the various assumptions, decisions and chosen numerical values necessary to the establishment and application of the accurate and approximate methods of optimization. It is left to the reader to decide which sections take priority.

10.1 Calculation of results

The results appearing in the previous chapter were calculated by the Mercury automatic digital computer of the University of London; the programs were written in CHLF3 program language. Although, as has been stated before, a computer is not essential in applying the approximate methods, the Mercury computer was used, nevertheless, in order to eliminate the tedium of carrying out iterative calculations for a large number of cases by hand.

When applying the approximate methods, the design-variable values were considered to have converged sufficiently close to their optimum values when the following formula was satisfied for all of the design variables:

$$\left| \begin{array}{l} (\text{design-variable value at } n^{\text{th}} \text{ iteration}) - \\ (\text{design-variable value at } (n+1)^{\text{th}} \text{ iteration}) \end{array} \right| \leq a$$

where

a ≡ one per cent of design-variable value at n^{th} iteration.

The times taken by the computer to satisfy this formula for each case and to print out the results were about 15, 30 and 40 seconds for the Wet-, Heller- and Spalding-systems respectively.

Due to various administrative reasons, only a small amount of computer time was available when applying the accurate methods. In consequence, the searching technique inherent in the accurate "steep-descent" method had to be stopped at a certain stage in each case so that it was not possible to obtain optimum design-variable values to the same degree of accuracy as in the application of the approximate methods. However, on printing results at various stages up to and including the last stage, the calculating time was seen to have been sufficient for the merit-factor to have converged rapidly to a value very close to its optimum value.

Since the prime object of both the accurate and approximate methods is to provide design-variable values which result in optimum or near-optimum merit-factor values, the shortage of computer time did not constitute a serious disadvantage. In fact, the scattered points in some of the graphs in 9.4 - arising from the non-convergence of some of the design variables - are useful in that they serve to show that optimum merit-factor values are relatively insensitive to the values of certain design variables. This point is enlarged upon in the next sub-section. A final noteworthy comment is that the calculation times for each case, up to the

stopping points, were about 7, 3 and 4.5 minutes - these values constituting computer calculating times of approximately 30, 6 and 7 times greater than those for the approximate methods - for the Wet-, Heller- and Spalding-systems respectively.

10.2 Appraisal and discussion of methods of determining optimum design-variable values

As can be seen from tables 11 and 12 and graph 23 (chapter 9), the substitutions of the design-variable values resulting from the approximate methods into the accurate versions of (2.1-8) give merit-factor values which exceed the accurately obtained optimum values by not more than 1.2, 0.7 and 2.4 per cent of the Wet-, Heller- and Spalding-systems respectively. Therefore, these design-variable values are considered to be sufficiently accurate for the prediction of optimum merit-factor values.

The foregoing small errors in merit-factor values are a little surprising, especially after noticing in graphs 1 to 7a that the accurate and approximate methods give widely differing design-variable values in several instances. An an illustration, the following table shows the maximum percentage differences occurring between design-variable values:

WET	Q_H	W_H	d_T	C_F	$t_{C,i}$	D_1	r_p	s	t
	32	22	40	30	1	4	16	8	12
HELLER	d_T	C_F/C_G	C_G	W_C	D_1	s	$n_{h,G}$	r_1	
	12	14	3	7	11	14	3	13	
SPALDING	Q_H	W_H	X_H	d_{B_1}	d_T	C_G	D_1	s	$n_{h,G}$
	9	60	27	1	1	1	5	5	6

The explanation for the small errors in merit-factor values is considered to be that P_L/q must be relatively insensitive to changes in value of several of the design variables. It would, of course, be advantageous to know what these relatively unimportant design variables are. However, the classic method of finding this out is a difficult and, in the present circumstances, an impractical task which involves differentiating the merit-factor equation twice with respect to each of the design variables, thereby arriving at expressions indicative of the sensitivity of P_L/q to changes in design-variable values.

Fortunately, some of the relatively unimportant design variables can be detected by examining the amount of scatter displayed in graphs 1 to 7a by the results accruing from the accurate methods. It is contended that the importance of a design variable is probably indicated by the amount of scatter displayed by its values; the less important the design variable, the greater will be the scatter. This

contention arises from the knowledge that: in generating a design-variable value, the searching technique inherent in the accurate "steep-descent" method produces fluctuations, the magnitudes of which are inversely proportional to the effect of the design variable on the merit-factor. The conclusion is that a design variable is unimportant if it displays widely scattered results. However, the absence of scatter does not necessarily indicate an important design variable; the reason for little or no scatter could merely be that the initial choices of design-variable values, necessary in applying the accurate methods, are so close to their optimum values that little change in their values is deemed necessary by the searching technique.

The design-variables which can be seen from graphs 1 to 7a to have widely scattered values are: d_T and C_F for the Wet system and d_T, C_F/C_G, W_C and r_1 for the Heller system. In view of the comments in the previous paragraph these design-variables are considered to be unimportant. That this is true for d_T is easily confirmed by the knowledge that values of d_T affect the prices of ducts and the pumping power, the values of both of which can be seen in graphs 14, 15, 17 and 18 to constitute a very small contribution to the merit-factor. For the same reason, d_T and d_B in the Spalding system are also considered unimportant although this cannot be divined by observation of scatter. The initially chosen d_T and d_B values were those resulting from the approximate method and, as can be seen from graph 2, are

so close to the values resulting from the accurate method that the special case referred to in the previous paragraph, where no scatter occurs, is considered to apply here. The conclusion that W_C is unimportant also seems reasonable since its value affects the Cold-side heat-transfer coefficient and the pumping power, both of which have a small effect on the merit-factor. r_1 has also been shown to be unimportant in both the Heller- and Spalding-systems by a different method as can be seen in graph 34. The fact that C_F for the Wet system and C_F/C_G for the Heller system are unimportant is not so evident but can probably be explained as follows:

If C_F and C_F/C_G are fixed at values which are different from their optimum values, then, during the course of optimization, the values of the remaining design variables will be re-adjusted so as to negate most, or all, of the effects on the merit factor occasioned by the differences between fixed and optimum values of C_F and C_F/C_G. The fact that C_F is relatively unimportant serves as an additional assurance that the fixing of C_F in the approximate method of optimization for the Wet system does not constitue a serious source of error.

The reason for paying so much attention to the weeding out of unimportant designvariables in the previous paragraphs is that a designer can assign fixed values to them, thereby considerably easing the task of finding the optimum values of the remaining design variables. For example, the

fixing of a design variable results in one less equation to be iterated in the approximate method; if the accurate method is used, a considerable saving in computer time is ensured since calculating time is roughly proportional to the cube of the number of design variables.

In comparing the accurate and approximate methods insofar as convenience of application is concerned, the conclusion is reached that if a computer is available, the accurate methods are to be preferred as they involve the least amount of effort on the part of a designer. In particular, it is unnecessary to obtain and re-arrange the differential forms of the merit-factor equation, these being processes which have to be repeated should it be necessary to replace existing formulae by new ones of different form.

Of course, if a computer is unavailable, the approximate methods must be used. In such events, unless fairly good initial guesses of the optimum design-variable values are made, it may be necessary to carry out more than four complete iterations before the design variables converge sufficiently close to their optimum values.

Attempts at simplifying the equations appearing in chapters 6, 7 and 8 still further in order to reduce the number of calculations per iteration cycle did not meet with success. The design-variable values which did emerge after simplification were so far removed from their actual optimum values that the ranges of validity of some of the various heat-transfer, pressure-drop and price formulae

appearing in the merit-factor equation were exceeded. This situation stems from the fact that P_L/q is relatively insensitive to changes in value of several design-variables. From the mathematical point of view, this insensitivity makes it easy to predict optimum merit-factor values to a high degree of accuracy, but difficult even to make approximate predictions of optimum design-variable values.

10.3 Variation of quantities with waste-heat rejection rate

It would be advantageous to be able to make fairly accurate predictions of design-variable values, and those of various other important quantities, at different waste-heat rejection rates from a knowledge of the values at a particular waste-heat rejection rate. This can be done after noting the trends in the graphs in 9.4 and making the following observations:

The designvariables which can be regarded as having values which do not vary with q are:

Q_H, W_H, C_F, $t_{C,i}$, t and s for the Wet system;

W_C, r_1 and s for the Heller system and

Q_H, W_H, r_1, s and X_H for the Spalding system.

For the remaining design variables, the following relationships appear to have values which are sufficiently invariant with respect to q to be adopted as constants:

d_T^2/q, d_B^2/q, dC_G/dq, $d(C_F/C_G)/dq$, D_1^2/q (see graph 32), $dn_{h,G}/dq$ and dr_1/dq (for the Wet system).

In addition, the following important auxiliary quantities can be regarded as constant with respect to q for all practical purposes:

$t_{s,S}-t_{G,i}$, ΣN_v, l_C/n_p, $A_{fr,G}/q$, $t_{s,S}$ and, to a lesser extent, Q_G.

A noteworthy comment is that both $(\pounds_{cs}/q-\pounds_{pm}/q)$ and P_L/q decrease as q increases, thereby emphasising that there are gains to be made, insofar as both cooling-system capital price and power-station merit are concerned, by making cooling systems as large as is practically possible.

The variation with respect to q of several other quantities which are considered tb be of interest to a designer can be seen in graphs 17 to 34.

10.4 Validity of assumptions made in carrying out methods of optimization

The values of the design variables obtained by both the accurate and approximate methods do not cause any of the ranges of validity of the various adopted heat-transfer, friction factor and pressure-drop formulae to be exceeded. Sufficiently good guesses of the exhaust-steam temperatures are considered to have been made for the evaluation of a^* since their values, viz. 80 and 100 deg F for the Wet- and two Dry-systems respectively, are close to the values calculated after optimization, viz. 80 and 95 deg F. Also, the physical properties of the intermediary heat-transfer fluids are considered to have been evaluated sufficiently

accurately at the guessed mean temperatures of 80, 90 and 100 deg F for the Wet-, Heller- and Spalding-systems respectively, the corresponding calculated temperatures being 70, 80 and 85 deg F.

The assumptions made in deriving the equations for the approximate optimization methods in chapters 6, 7 and 8, and the adoption of the various approximate equations in chapter 3, are justified since the optimum design-variable values accruing from the use thereof give accurate estimates of optimum merit-factor values. In particular, the fixing of r_1 for the approximate Heller-system method, and both Spalding-system methods, is justified by the fact that r_1 values have little effect on optimum merit-factor values as is shown in graph 34.

Tables 12 and 23 and graph 23 show that the choice of a single-phase, forced-convection Hot-side heat-transfer coefficient formula for the Spalding system does not give rise to optimum merit-factor values significantly different from those arising from the use of a more optimistic nucleate-boiling heat-transfer coefficient formula. With regard to optimum design variable values, the only noticeable differences are in W_H and X_H, the use of the latter formula resulting in smaller values of Hot-side refrigerant velocity and recirculation. Accordingly, the choice of the former single-phase formula in the present investigation is considered to be justified.

Graph 34 illustrates the variation of optimum merit-

factor with effective height of the water-level in the Heller-system steam condenser. The plotted curve makes it possible to determine whether the arbitrary adopted assumption, viz., $z_H = 20$, is a possible source of serious error in evaluating optimum merit-factor values. As expected, P_L/q decreases with increase in z_H; that this should be so can be deduced from (3.5-9), (3.5-8) and (3.5-11) wherein it is seen that increased z_H values are accompanied by decreases in extra pressure drop and, in consequence, required pumping power. The smallest possible value of P_L/q occurs when z_H has increased to the point where no extra pressure drop need be imposed by a water turbine, i.e. for the condition where $\Delta p_x = 0$ in (3.5-9). By substituting values obtained from tables 6 and 17 (at $q = 5 \times 10^8$) into (3.5-9), the value of z_H for the foregoing condition is found to be 76 feet. After linearly extrapolating the curve in graph 34, P_L/q can be shown to decrease by 2 per cent when z_H is increased from 20 to 76 feet. This, of course, represents a maximum decrease in P_L/q as no allowance has been made for the extra cost which would be involved in elevating the steam condenser up to the greater height. The foregoing possible maximum error in P_L/q is so small that the assumption of $z_H = 20$ in the present investigation is considered to be justified.

The adopted assumption for the Heller-system Hot-side "temperature approach", viz. $t_{s,S} - t_{H,o} = 7$ deg F, is regarded as a possible source of significant errors in evaluating

optimum merit-factor values. This is deduced as follows:

An error, Δt, in the adopted value of $t_{s,S}-t_{H,o}$ can be seen from the merit-factor equation to result in a corresponding error, $\Delta P_L/q$, in the merit-factor value, according to the expression

$$\Delta P_L/q = a^*\Delta t.$$

In consequence, the percentage error in P_L/q corresponding to an error of 1 deg F is given by

$$\{(\Delta P_L/q)(100)\} / \{(P_L/q)(\Delta t)\} = 100a^*/(P_L/q)$$

$$\approx 1.68$$

for $a^* = 0.0042$ and the reference value, $P_L/q = 0.25$. This means that an error of only 3 deg F will give an error in P_L/q of about 5 per cent.

The foregoing comments, coupled with the fact that various sources of information indicate that $(t_{s,S}-t_{H,o})$ could be anywhere between 0 and 11 deg F (see 3.4.2), lead to the conclusion that the Heller-system optimum merit-factor values in chapter 9 can only be regarded as accurate to within +6.7 and -11.8 per cent.

10.5 Appraisal of tower heat exchanger, type SG1

In deriving simple merit-factor equations in 4.3.2 to indicate the merit of tower heat-exhhanger types it is assumed that: of all the terms comprising φ_1 and ϕ_1 in (4.3-3), those containing b^* make a contribution of less than 10 per cent. Since the maximum value of b^*P/q, viz. 0.0167, can be seen from table 12 to be less than 9 per cent of the

minimum simplified merit-factor value, viz. 0.197, in the second table in 4.3.5, this assumption is certainly valid. As a justification for making several other assumptions it is also assumed that $£_D/q$ is only about 50 per cent of $£_G/q$; this is seen to be true in tables 9 and 10. Therefore, the conclusion is reached that the simplified merit-factor equations derived in 4.3.2 are sufficiently accurate to be used in determining the merits of tower heat-exchanger types.

As mentioned in 4.3.1, a tower heat-exchanger type must satisfy the enumerated requirements in 4.3.1, in addition to having a low merit-factor value, before it can be regarded as suitable for use in the Heller- and/or Spalding-systems. The ability of the SG1 heat-exchanger type, chosen for the Heller- and Spalding-systems in the present investigation, to satisfy these requirements is examined as follows:

i) The optimum overall lengths of tower heat-exchanger tubes, per unit number of Cold-side passes, are seen from graph 30 to be almost invariant with respect to waste-heat rejection rate, and have values of approximately 14 and 164 feet for the Heller- and Spalding-systems respectively. These values are such that the first requirement in 4.3.1, viz. that individual heat-exchanger units should have tube lengths close to 16 feet, is satisfied. For example, tube lengths of Heller-system units can be 14 feet long whereas those of the Spalding system can be 18.2, 16.4 or 14.2 feet in length depending on whether 9, 10 or 11 units are to be

preferred in making up the total tube length of 164 feet.

ii) A requirement stipulated in the second item in 4.3.1, viz. that the tower heat-exchanger frontal area should be greater than or equal to the tower inlet area, can be seen from graph 31 to be satisfied for the Heller system. For the Spalding system, on the other hand, $A_{fr,G}$ is a little less than $\pi z_1 D_1$. However, as mentioned before, the value of r_1 (fixed at 0.15 in the present investigation) can be varied without significantly affecting optimum merit-factor values. In consequence, r_1 can be adjusted by the small amount necessary to make $A_{fr,G} \geq \pi z_1 D_1$ and the SG1 heat-exchanger type is, therefore, considered to satisfy the foregoing requirement for both the Heller- and Spalding-systems. In fact, the values of the frontal and inlet areas are so close to each other that the whole tower heat exchanger can be fitted exactly into the hollow cylinder representing the tower inlet area. There is therefore no need for the awkward zig-zag or concertina arrangement as employed in the cooling system at Rugeley (see ref.9), the latter arrangement being made necessary by the fact that the heat-exchanger frontal area is nearly twice that of the tower inlet.

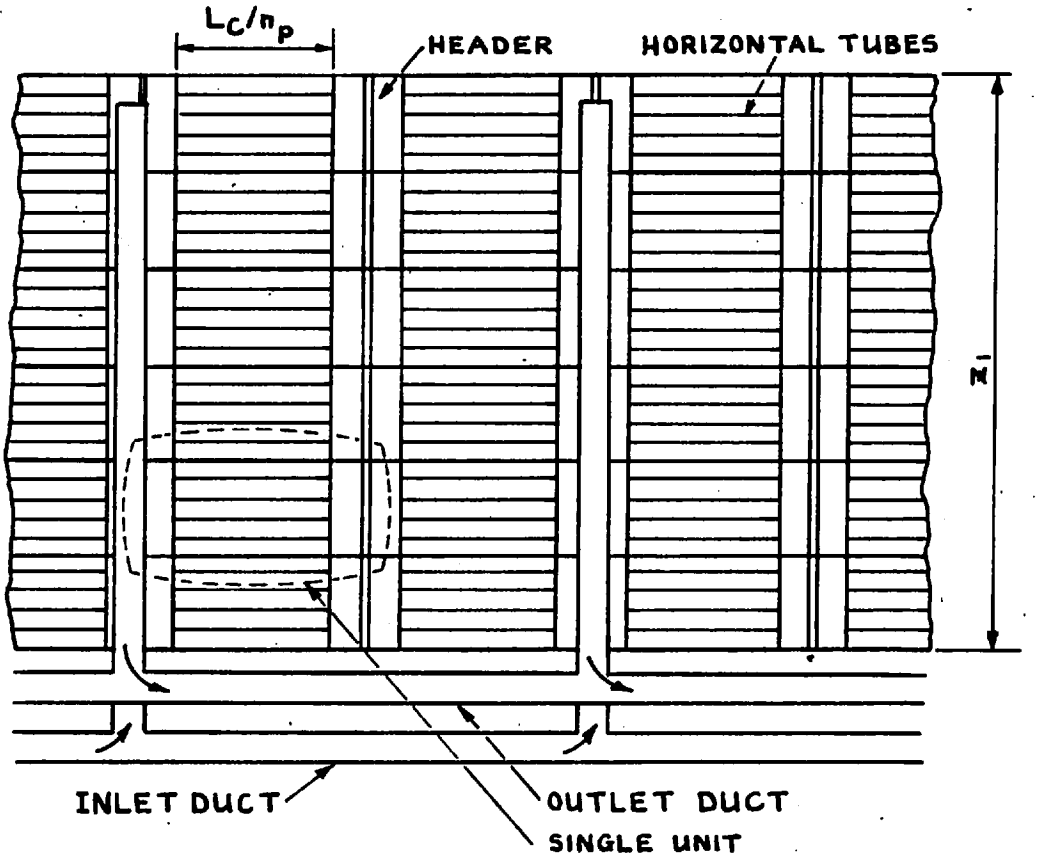
Anotherrequirement stipulated in the second item in 4.3.1 is that: "the optimum physical dimensions of the whole tower heat exchanger must be suitable for convenient insertion of the individual heat-exchanger units into and around the base of the cooling tower". It is claimed that the SG1 heat-exchanger type does fulfil this requirement and the two

possible arrangements sketched in the following pages are given in support of this claim.

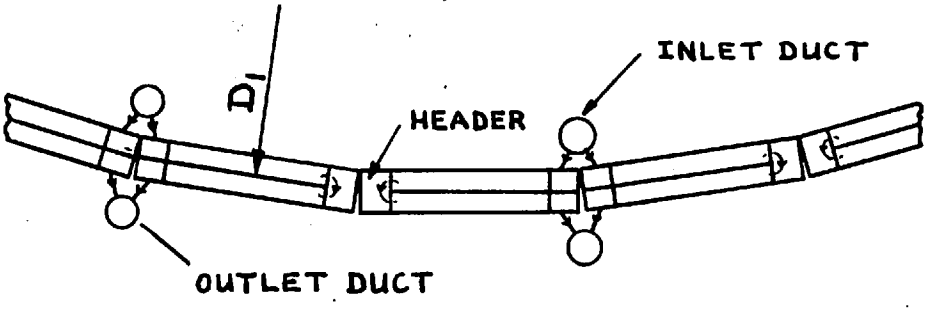
As can be seen from the Heller-system sketch, a number of individual heat-exchanger units are stacked in a vertical column so that the tubes are horizontal. Assuming that the units are about 9 feet wide, the choice of six units per column in the sketch would be suitable for an inlet height of about 54 feet; for greater inlet heights, a larger number of units per column would be required. Heated water from the steam condenser passes into the header of each unit via inlet ducts situated on the inside of the tower-base perimeter. After flowing through the two Cold-side passes the water, now cooled, returns to the steam condenser via outlet ducts situated on the outside of the tower-base perimeter.

In the Spalding-system sketch a number of individual heat-exchanger units are stacked in vertical columns so that the tubes are vertical. Assuming that the units are about 16 feet long, the choice of three units per column in the sketch would be suitable for an inlet height of about 48 feet; as in the Heller system a larger number of units per column would be required for greater inlet heights. Since the length of the Cold-side flow path , being about 164 feet, is much greater than the inlet height, more than one vertical column of stacked heat-exchanger units is required in order to make up the total length of flow path. The number of columns required is given by the integer nearest to the

HELLER SYSTEM

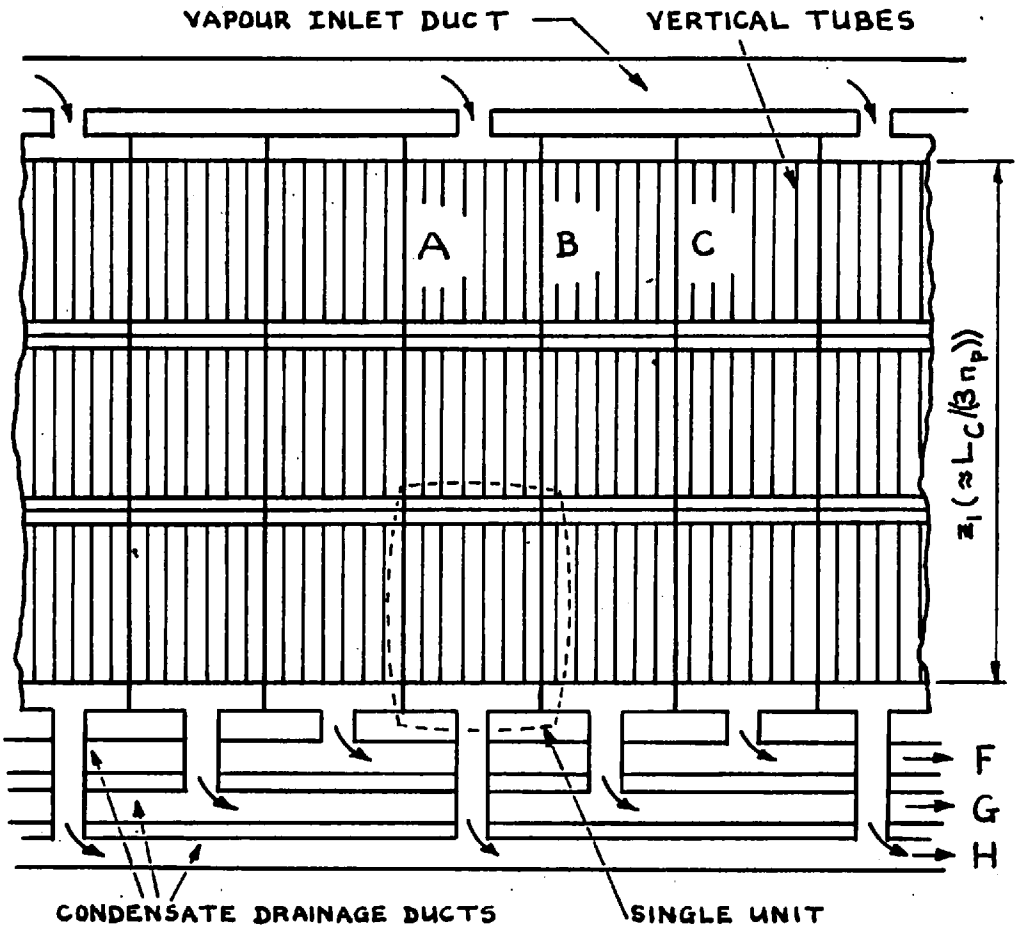


FRONT ELEVATION

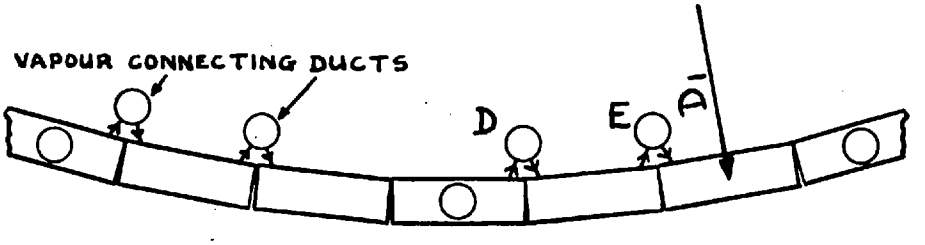


PLAN

SPALDING SYSTEM



FRONT ELEVATION



PLAN

number represented by the ratio $(L_c/n_p)/z_1$; for an inlet height of 48 feet the integer is 3. Therefore, a single Cold-side flow path is obtained by connecting together three of the vertical columns of stacked units, e.g. A, B and C in the sketch, by means of vapour connecting ducts, e.g. D and E. Of course, this arrangement necessitates the provision of three separate condensate drainage ducts, e.g. F, G and H, at the base of each column.

Refrigerant vapour from the steam condenser passes via a vapour inlet duct into the uppermost headers of the first of the sets of three columns comprising Cold-side flow paths, e.g. column A. After flowing downwards through the tubes the remaining vapour passes into the uppermost headers of the second columns in each set, e.g. column B, via the vapour connecting ducts, e.g. D, and the condensate is drained by duct H. This process is repeated for the second and third columns in each set until all of the vapour is condensed, the condensate thereafter being returned to the steam condenser via the condensate drainage ducts F, G and H.

iii) The third requirement in 4.3.1 to be satisfied is that the fixed integer values chosen to represent the optimum number of tube rows, viz. $n_r = 2$, for both Heller- and Spalding-systems, should be close to those values which would result from optimization were n_r to be regarded as a variable and not restricted to integer values. This would be so if the values of n_r, obtained by substituting values

from the tabulated results into the equation developed in appendix L, were to agree closely with the previously adopted integer values, viz. $n_r = 2$. After carrying out this substitution for $q = 5 \times 10^8$, n_r values of 3.2 and 2.5 were obtained for the Heller- and Spalding-systems respectively. Unfortunately, the fact that these values are not very close to 2, especially for the Heller system, leads to the conclusion that the conditions under which the SG1 heat-exchanger type operates are such that its potential merit is not fully exploited. The possibility therefore exists that there is another type of heat exchanger, of equal potential merit, which will result in a lowering of the Heller system and, to a lesser extent, the Spalding system optimum merit-factors, subject to the satisfaction of the other heat-exchanger requirements. However, it is contended that, in this context, any decreases in merit-factor values are likely to be small and that the foregoing discrepancies in n_r values do not render the SG1 heat-exchanger type unsuitable.

iv) Finally, the fourth requirement in 4.3.1 is satisfied since the choice of two Cold-side passes for the Heller system does result in convenient sizes of individual heat-exchanger units.

In view of the foregoing comments, the SG1 heat-exhhanger type is concluded as being suitable for use in both the Heller- and Spalding-systems since, in addition to being of high potential merit, it satisfies all but one of the

requirements enlisted in 4.3.1, the exception being regarded as relatively unimportant.

10.6 Appraisal of tower heat exchanger, type LU1

As mentioned in 9.3.1 the accurate method is used in obtaining results for the Heller system (at $q = 5 \times 10^8$) for the LU1 heat-exchanger type in order to note the effect of using another heat-exchanger type which has a potential merit similar to that of the SG1 type, but with larger diameter tubes. By comparing values in table 21 with those in tables 2 and 12 (chapter 9), no significant differences in the values of design variables or merit-factor values agree to within 0.36 per cent. However, further examination of table 12, tables 14 and 17, and the graphs subsequent to graph 27, reveals that the values of r_5-r_1, z_5, L_c/n_p and $A_{fr,G}/q$ for the LU1 heat-exchanger type are all appreciably greater than their counterparts for the SG1 type, whereas the value of $\pi z_1 D_1/q$ is only slightly smaller.

The results are used in discussing the suitability of the LU1 heat-exchanger type, in relation to type SG1, as follows:

i) The LU1 type is at a disadvantage because, as is shown in graph 31, the heat-exchanger frontal area is nearly twice that of the tower inlet area and, therefore, necessitates the use of a less convenient zig-zag or concertina arrangement in order to accommodate the whole heat exchanger in the tower inlet area.

ii) The LU1 type is also considered to suffer slightly in that L_c/n_p is approximately 36 feet. For example, the use of the Heller-system arrangement as proposed in the sketch in the previous sections (except for the inclusion of a zig-zag heat-exchanger layout in the plan view) will result in the tower-inlet air striking the finned tubes at an angle of appreciably less than 90 degrees with respect to the longitudinal axes of the tubes, thereby altering the heat-transfer and pressure-drop performance characteristics of the heat exchanger. Furthermore, the inlet height can be seen from graph 30 to be greater than L_c/n_p for all values of q greater than $q = 2 \times 10^8$, so that the arrangement as used for the Heller system at Rugeley (see ref.9) cannot be employed as an alternative in this instance.

iii) The LU1 type does, however, possess an advantage in that the number of tube rows evaluated from the equation in appendix L, viz. $n_r = 2.4$, is very much closer to the fixed integral number adopted during optimization, viz. $n_r = 2$, than is the case for the SG1 type. This indicates that the LU1 type operates under conditions such that its potential merit is more fully exploited than that of the SG1 type. The better exploitation of the LU1 type explains the fact that, although the potential merit of the LU1 type is 0.46 per cent worse than that of the SG1 type as can be seen from the second table in 4.3.5, the overall merit-factor resulting from its use is 0.36 per cent better than that for the SG1 type. Incidentally, the foregoing small decrease

in merit-factor value lends more weight to the contention in item iii) of the previous section that any improvements in merit-factor values, arising from the better exploitation of one of a number of heat-exchanger types of equal potential merit, will be small, at least insofar as the fixing of n_r at integer values is concerned.

iv) The LU1 type is considered unsuitable for use in the Spalding system because the results of calcuations, not shown in the present investigation, indicate that L_c/n_p would be approximately twice that for the SG1 type. The total Cold-side flow path would then have to be made up of six columns of stacked heat-exchanger units as opposed to the three columns proposed in the sketch in the previous section. This arrangement is considered to be more difficult to construct without offering any significant advantages.

In view of the foregoing comments the LU1 heat-exchanger type, employing tubes of larger diameter than the SG1 type, is considered to be less suitable for both the Heller- and Spalding-systems.

10.7 Surface steam condenser considerations

10.7.1 Wet system

In order to see whether the steam-condenser dimensions at optimum operating conditions are suitable for construction purposes, optimum Wet- system design variable values are substituted into (3.3-1), (3.3-2) and the equations for determining shell radius in D.3, to get L_H = 50.8 feet, n_H = 1906 and shell diameter = 12 feet. The length of the condenser

shell - resulting from the adoption of two Hot-side passes in 1.2, as recommended by Ellingen in ref.13, and 3/4 inch o.d. tubes - is therefore given as (50.8)/2 = 25.4 feet. That the proportions represented by these dimensions do occur in practice has been observed by the author and can be further authenticated by reference to various sources of information such as ref.24. The choice of two Hot-side passes and 3/4 inch o.d. tubes is therefore regarded as justified.

Furthermore, the adopted heat-transfer coefficient formulae are considered to be accurate because the predicted Hot-side overall heat-transfer coefficient, viz. $U_H = 675$ in table 13, compares favourably with the value $U_H = 640$, which is obtained from the graph of the Heat Transfer Institute on page 308 of ref.24.

10.7.2 Spalding system

In determining whether the steam-condenser dimensions are suitable for construction purposes, the same procedure as employed in the previous sub-section results in the following values (at $q = 5 \times 10^8$) for one Hot-side pass, 3/4 inch o.d. horizontal tubes and assuming single-phase, forced convection heat transfer on the Hot-side:

Length of condenser	$(=L_H/1.0)$	= 29.1 ft
number of tubes	$(=n_H)$	= 21,300
diameter of shell		= 12.8 ft
Hot-side overall heat-transfer coefficient	$(=U_H)$	= 607 Btu/(hr sq ft degF)

The corresponding values resulting from the use of the optimistic nucleate-boiling heat-transfer coefficient formula are similar to the foregoing ones and are calculated as

34.6 ft, 18,000, 11.8 ft and 478 Btu/(hr sq ft deg F)

respectively.

The foregoing dimensions, like those for the Wet system, are considered to constitute no difficulties insofar as construction is concerned. Therefore, the assumptions made in 1.4.5 that there is no need to consider the use of a serpentine tube layout or more than one pass are justified. Furthermore, the fixing of z_R, viz. the effective height of refrigerant liquid level above the Hot-side entrance, at 15 feet in 3.7.3 in order to calculate the total Hot-side pressure drop seems reasonable as the shell diameters, being about 12 or 13 feet, allow for a duct 2 or 3 feet in length to carry liquid refrigerant containing vapour up to the separator.

No reliable published values of heat-transfer coefficients have been found for ammonia under conditions of forced-convection nucleate boiling in order to check which of the foregoing two U_H values is the more accurate. However, as stated before, the point is not important as merit-factor values are essentially the same whichever of the two Hot-side heat-transfer coefficient formulae are used. After separating the respective U_H values into their constituent

Steam- and Hot-side partial heat-transfer coefficients the following two sets of values follow:

$$u_S = 1670, \quad u_H = 1410,$$

$$u_S = 1670, \quad u_H = 702.$$

The similarity between u_S and u_H in the former set means that the use of extended surfaces on one of the steam-condenser sides need not be considered should the single-phase, forced convection heat-transfer coefficient formula be the more reliable formula for the Hot-side. However, should the nucleate-boiling formula be better, then it is just feasible that the use of extended surfaces might be of some advantage. However, any discussion on these lines would be pure conjecture at this stage due to the absence of a formula which is known to be accurate.

One final comment is that the adoption of a refrigerant flow arrangement allowing for recirculation of the refrigerant, viz. the fourth possibility in 1.4.4, appears to be justified in view of the fact that the results indicate that large amounts of recirculation are required.

10.8 Validity of adopted methods of predicting a^*, b^* and c^* values

An approximate check on the validity of the adopted methods of estimating a^*, b^* and c^* is to compare the optimum values of exhaust-steam temperatures and cooling tower dimenions, obtained in the present investigation, with those for comparable cooling systems already in existence. This

statement is an expanded version of that appearing at the end of 2.2.6. Of course, comparisons can only be made if the following requirements are satisfied:

i) The existing cooling systems must be of similar design to the ones adopted in the present investigation.

ii) They must have been recently commissioned so that both the a^*, b^* and c^* values and the various price formulae employed in the present investigation can be regarded as equally applicable to them.

The first of the foregoing requirements is not satisfied in the case of Wet systems because the cooling system adopted in the present investigation incorporates a number of novel design features, e.g. blanking and a channel increasing the efficiency of air-flow, which are likely to introduce unforeseeable changes in the values of optimum cooling tower dimensions. It is therefore contended that the Wet system results cannot be used in order to ascertain the validity of the adopted a^*, b^* and c^* values.

The results for the Spalding system are no good for this purpose either as there is no cooling system in existence which is based on the Spalding-system design.

Fortunately, there is one cooling system in existence which is considered to satisfy the foregoing requirements sufficiently well to allow for comparisons, viz. the cooling system at Rugeley which is based on Heller-system design principles (see ref.9). This cooling system, commissioned as recently as 1962, is similar in design to the adopted one

in the present investigation, except for a different tower-heat exchanger layout. The cooling system serves a turbo-generator unit with a generator output of 120 MW, i.e. $P_u = 1.2 \times 10^5$ kW, and the atmospheric dry-bulb temperature is given as $t_{G,i} = 52$ deg F. These values, together with the following optimum values, are given on page 41 of ref.2:

$$D_1 = 325 \text{ ft}, \quad r_1 = 0.148, \quad s = 0.63, \quad z_5 = 350 \text{ ft},$$

$$t_{s,S} - t_{G,i} = 33 \text{ deg F}, \quad t_{s,S} = 85 \text{ deg F}.$$

In obtaining corresponding values from the results in the present investigation, a value of $P_u = 1.2 \times 10^5$ kW is seen from graph 33 to be equivalent to $q = 4.5 \times 10^8$ Btu/hr. Graphs 6, 7, 29 and 32 thereupon show that for $t_{G,i} = 57$ deg F,

$$D_1 = 336 \text{ ft}, \quad r_1 = 0.15, \quad s = 0.665, \quad z_5 = 380 \text{ ft},$$

$$t_{s,S} - t_{G,i} = 37 \text{ deg F}, \quad t_{s,S} = 94 \text{ deg F}.$$

Comparison of the physical dimension values indicates that the Rugeley cooling tower is only slightly smaller than that obtained in the present investigation. As the adopted value of $t_{G,i}$ is 5 deg F greater than the Rugeley value, the value of $t_{s,S}$ should also be greater by an approximately equivalent amount. In fact, the indicated difference is 9 deg F but it is to be remembered that a Hot-side temperature approach of 0 deg F has been assumed for Rugeley (see ref.9) as opposed to 7 deg F in the present investigation. The effective difference in exhaust-steam temperatures is

therefore 2 deg F. The fact that this value differs by only 3 deg F from the expected value, coupled with the fact that the cooling tower dimensions are very similar in both cases, provides an excellent indication that the adopted methods of estimating a^*, b^* and c^* values are valid.

10.9 Comparision of considered cooling systems

Overall merit-factor and cooling-system price constituents are considered to be of chief interest to a designer and, therefore, comparisons of the considered cooling system types are restricted to considerations of these two quantities. Since the rates of change of P_L/q and ($£_{cs}/q$ - $£_{pm}/q$) with respect to q can be seen in graphs 23, 14, 15 and 16 to be similar for all three systems, only one q value need be considered for comparison purposes. Therefore, a representative value of $q = 5 \times 10^8$ is chosen in presenting the following table to illustrate the differences in merit-factor and cooling-system price values.

		Wet	Heller	Spalding
P_L/q, (1)	$x10^{-1}$	1.75	2.82	3.02
$£_{cs}/q - £_{pm}/q$, £/(Btu/h)	$x10^{-3}$	1.03	1.65	1.97

These values show that the Wet system is far better than the other two systems, both with regard to overall merit and cooling-system price. There would therefore have

to be very good engineering before choosing one of the other two types in preference to the Wet system. For example, the capitalized price per unit waste-heat rejection rate of providing make-up water for the Wet system, temporarily identified by the letter a, would have to satisfy the relationship

$$a \geq \{(P_L/q)_{other} - (P_L/q)_{Wet}\} / c^*$$

for one of the other systems to become competitive. The tabled values indicate that, for $c^* = 68.2$, a would have to exceed the respective values of 0.00157 and 0.00186, i.e. 1.5 and 1.8 times the price of cooling system components, for the Heller- and Spalding-systems to become competitive.

If the above values are found to be exceeded for a chosen power-station site, a choice has then to be made between the Heller- and Spalding-systems. In making this choice, an examination of the local climate must be conducted in order to find out whether there is a possibility of water freezing in the Heller-system heat-exchanger tubes.

If such a possibility does not exist, the inclusion of de-icing facilities in the Heller system is not necessary and the tabulated cooling-price value remains valid. The tabulated Heller- and Spalding-system price values can then be directly compared, the conclusion being that the price of cooling-system components for the Spalding system is 1.19 times greater than that for the Heller system.

When comparing merit-factor values it is to be remembered

that the tabulated Heller-system P_L/q value has tolerances of +6.7 and -11.8 per cent according to the conclusion at the end of 10.4. Therefore, the worst P_L/q value for the Heller system is 0.301. This value, when compared with the tabulated Spalding-system value of 0.302, leads to the conclusion that, where de-icing facilities are not necessary, the Heller system will always be better than the Spalding system, even if only by a small amount.

However, if de-icing facilities are necessary for the Heller system, the capitalized price of de-icing facilities per unit waste-heat rejection rate, temporarily identified by the letter b, must satisfy the relationship

$$b \geq \{(P_L/q)_{Spalding} - (P_L/q)_{Heller}\} / c^* .$$

If the most optimistic Heller-system merit-factor value, viz. 0.249, is attainable in practice, without introducing extensive modifications to the jet steam condenser and thereby invalidating the adopted price formula, then b is given by

$$b = (0.302 - 0.249)/68.9 = 0.00077,$$

or 46.5 per cent of the value of tabulated cooling-system price for the Heller system.

Alternatively, the most pessimistic value of the Heller-system merit-factor, viz. 0.301, results in

$$b = 0.0000145$$

or 0.88 per cent of the tabulated cooling-system price.

It can therefore be concluded that, where de-icing facilities are necessary, the Heller-system will always have better merit-factor values than the Spalding system if the capitalized price of de-icing equipment is less than 0.88 per cent of the price of cooling-system components for the Heller system, whereas the converse is true for values greater than 46.5 per cent. These two limiting percentages, arising from the tolerances on the Heller-system P_L/q value, are considered to be too widely variant to be of much practical use.

In an effort to obtain a percentage value which can be reasonably expected in practice it is assumed that the Heller-system Hot-side temperature approach of 7 deg F adopted in the present investigation, and which is recommended by consultants responsible for the construction of large power-station jet condensers in New Zealand (refs. 43 and 17), does in fact constitute a fairly accurate value. In this instance the associated Heller-system merit-factor value, viz. 0.282, results in

$b = 0.00029$

or 17.6 per cent of the tabulated cooling system price.

It is thereupon concluded that the Spalding system can be reasonably expected to be better than the Heller system if the capitalized price of de-icing facilities is greater than approximately 17.6 per cent of the price of cooling system components. Unfortunately, there is no published

information available to indicate what this percentage is for the Heller system at Rugeley. It is, however, considered that the percentage is not likely to be as large as 17.6 per cent and that the Heller system will therefore always be slightly better than the Spalding system.

11 SUGGESTIONS FOR FUTURE RESEARCH AND GENERAL CONCLUSIONS

11.1 Suggestions

The various suggestions are given in point form as follows:

a) Heller system

i) The merit of the Heller system is appreciably affected by errors in evaluating the Hot-side temperature approach, i.e. $t_{s,S} - t_{H,o}$. This, coupled with the fact that little is known about the relationship between performance and cost of jet steam condensers large enough for power-station applications, means that a worthwhile project would be to predict this relationship accurately.

ii) Due to insufficient published information the capitalized price of de-icing facilities has not been evaluated in the present investigation. It is, however, considered important to conduct an independent survey in the future with the object of assigning a capitalized value to the additional equipment, controls and inconvenience caused by the necessity of providing de-icing facilities, in order to estimate how much the Spalding system profits by not requiring such facilities.

b) Spalding system

i) The empirical formulae for predicting Hot-side heat transfer and pressure drop in the present investigation, while appearing to be the best currently available, are regarded by their formulators as of limited application and to be used with care. Therefore, a fruitful

field of investigation is considered to be the checking of the adopted correlations, or the derivation of more accurate ones, using the results of experiments conducted with refrigerants at saturation temperature under conditions applicable to the Spalding system, viz. forced convection, two-phase fluid flow through tubes with high liquid refrigerant recirculation rates. Furthermore, this investigation could well be extended to the obtaining or derivation of accurate heat-transfer and pressure-drop correlations for steam flowing through, and refrigerant flowing past, condenser tubes. Possession of such correlations would enable a designer to predict with precision the merits of various other steam-condenser configurations in addition to that adopted in the present investigation.

ii) The comments in i) regarding limited application of correlations also apply to the formulae for predicting Cold-side heat transfer and pressure drop adopted in the present investigation. It is therefore suggested that the obtaining or derivation of Cold-side correlations known to be accurate for possible Spalding-system refrigerants is also a worthwhile project. Furthermore, the extension of such correlations to include the effects of making the tubes non-vertical, or even horizontal, would help a designer in choosing between various tower heat-exhhanger configurations.

The suggestions in the foregoing two items are worthy of consideration, particularly with regard to heat-transfer correlation, since even small errors in heat-transfer co-

efficient predictions can have a significant effect on the Spalding-system merit-factor value.

c) General

i) There is thought to be considerable scope for improving on the steam-condenser and tower heat-exchanger price formulae adopted in the present investigation by taking into account more of the many factors which significantly affect prices.

ii) The derived, simplified method of determining the potential merits of tower heat-exchanger types, so that the least promising of them can be eliminated from further consideration, requires the use of a computer in applying it. This is no good when computers are unavailable and, therefore, there is considered to be scope for the establishment of another method not requiring the use of a computer.

11.2 Conclusions

These are given in point form as follows:

i) The approximate optimization methods compare favourably with the accurate ones in estimating design-variable values for the purpose of evaluating accurate optimum merit-factor values.

ii) The various assumptions adopted both in deriving and applying the accurate and approximate optimization methods, and the simplified method of choosing promising tower heat-exchanger types, are justified.

iii) The choice of the SG1 heat-exhhanger type for adoption

in the Heller- and Spalding-systems is a good one in view both of its high merit and the relative ease with which it can be fitted into and around the inlet area of a cooling tower.

iv) Ammonia is the best of the refrigerants considered for use in the Spalding system.

v) The fact that, for the Heller system, the accurately calculated optimum values of cooling-tower dimensions, and difference between exhaust steam and atmospheric dry-bulb temperatures, are close to the corresponding values for a comparable cooling system already in operation indicates that:

a) the use of the novel criterion of merit, i.e. the so-called merit-factor, in predicting optimum design conditions is justified;

b) the estimated value of the fixed coefficients appearing in the merit-factor equation are valid for the place and period assumed, viz. the United Kingdom in 1964.

vi) For the adopted price and atmospheric conditions, considered to be representative of some regions in the United Kingdom in 1964, the Wet system has by far the highest merit-factor of the considered cooling systems. The Heller system will only be competitive if the capitalized value of supplying Wet-system make-up water exceeds 1.5 to 1.8 times the total price of cooling-system components. On the other hand, the merits of the Heller- and Spalding-systems are much closer together. However, the Heller

system can be reasonably expected to be always slightly the better of the two unless the capitalized value of the Heller-system de-icing facilities exceeds 17.6 per cent of the total price of cooling-system components.

vii) Finally, it is concluded that further research is required with regard to refrigerant heat-transfer and pressure-drop formulae and, in particular, to the relationship between the cost and performance of large jet steam condensers, in order to predict the Spalding- and Heller-system merits more precisely.

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A PHYSICAL PROPERTIES OF COOLING SYSTEM MATERIALS.

A.1 Introduction.

Data necessary for the evaluation of physical properties encountered in the design of cooling systems are fully summarized in this appendix. The data are presented in equation form to suit the requirements of automatic digital computers, of which extensive use was made in carrying out numerical calculations. Since works of reference usually specify physical property information in tabular form, the requisite equations were obtained by drawing graphs, postulating a suitable form of equation and then applying standard curve-fitting techniques to determine the equation constants.

In order to arrive at simple equations it is assumed that pressure has no effect on viscosity, thermal conductivity, specific-enthalpy or constant pressure specific-heat under non-saturation conditions. This is a justifiable assumption for the ranges of pressure and temperature encountered in the present work.

Where applicable, the physical property equations are accompanied by the range of validity, the maximum deviation between calculated and tabulated values, and the reference number and page of the source of information. A complete list of the source references appears at the end of this appendix.

A.2 Water and its vapour.

$$\ln p_{s,W} = -6.24 + \{402.8 - 4\times10^{-4}\,(t_{s,W}-934.4)^2\}^{0.5}.$$

32- 150 deg.F ; 1.0%; ref.6, p.9. (A.2-1)

$$v_{f,W} = 2.101\times10^{-8}t_{f,W}^2 - 1.303\times10^{-6}t_{f,W} + 0.01602.$$

32- 212 deg.F; 0.1%; ref.6, p.9. (A.2-2)

$$\ln v_{g,W} = 16.44 - \{334.7 - 4\times10^{-4}(t_{g,W}-846.1)^2\}^{0.5}.$$

32 -150 deg.F; 1.0%; ref.6,p.9; small superheating. (A.2-3)

$$h_{fg,W} = -1.949\times10^{-4}t_{s,W}^2 - 0.5386\,t_{s,W} + 1093.0.$$

32 -212 deg.F; 0.5%; ref.6, p.9. (A.2-4)

$$\mu_{f,W} = (1.207\times10^{-5}t_{f,W}^2 + 3.863\times10^{-3}t_{f,W} + 0.09461)^{-1}$$

32 -212 deg.F; 1.0%; ref.13, p.62. (A.2-5)

$$\mu_{g,W} = 4.946\times10^{-5}t_{g,W} + 0.02112.$$

32 -400 deg.F; 0.5%; ref.10, p.471. (A.2-6)

$$k_{f,W} = 4.722\times10^{-4}t_{f,W} + 0.3149.$$

32 -176 deg.F; 0.2%; ref.9, p.800. (A.2-7)

$$k_{g,W} = 2.233\times10^{-5}t_{g,W} + 7.945\times10^{-3}.$$

32 -392 deg.F; 1.0%; ref.13, p.199. (A.2-8)

$$c_{f,W} = 1.337\times10^{-6}t_{f,W}^2 - 3.374\times10^{-4}t_{f,W} + 1.018.$$

32 -212 deg.F; 0.3%; ref.13, p.151. (A.2-9)

$\sigma_W \approx 5.14 \times 10^{-3}$ (water against air).

60 deg.F; approximate spot value; ref.9,p.376. (A.2-10)

$c_{g,W} = 3.5 \times 10^{-5} t_{g,W} + 0.4431.$

10 -190 deg.F; 0.3%; ref.8, p.112. (A.2-11)

$(h_{f,W} + h_{fg,W}) = 0.4467\ t_{g,W} + 1062.0.$

32 - 212 deg.F; 0.3%; ref.6,p.9; small superheating. (A.2-12)

A.3 Dry-air (one atmosphere).

$\mu_A = -2.673 \times 10^{-8} t_A^2 + 6.819 \times 10^{-5} t_A + 0.03936.$

(-10) - 242 deg.F; 0.1%; ref.7, p.62. (A.3-1)

$k_A = -6.154 \times 10^{-9} t_A^2 + 2.591 \times 10^{-5} t_A + 0.01313.$

(-10) - 242 deg.F; 0.1%; ref.7, p.70. (A.3-2)

$c_A = 7.540 \times 10^{-6} t_A + 0.2401.$

(-10) - 242 deg.F; 0.2%; ref.7, p.39. (A.3-3)

$h_A = 0.2406\ t_A - 7.698.$

32 -212 deg.F; 0.2%; ref.8, p.4. (A.3-4).

A.4 Moist air (one atmosphere).

A.4.1 Use of water-vapour mixture-fraction.

According to page 271 of ref.12, moist-air can be regarded as an ideal gaseous mixture for present purposes. Its physical properties may be ascertained according to proportional representation by mass of the water-vapour and air constituting a moist-air mixture. The following equation can thus be formed:

(moist air physical property) = F_G (water-vapour physical property) $+(1-F_G)$(dry-air physical property),

where

$F_G \equiv$ (mass of water-vapour in moist-air mixture)/(total mass of moist-air mixture). (A.4-1)

In ref.12, F_G is referred to as the "water-vapour mixture-fraction".

A.4.2 Physical property equations in terms of water-vapour mixture-fraction.

The physical property equations presented here follow directly from the previous sub-section and are functions of F_G and the dry-bulb temperature t_G.

$$\mu_G = F_G \mu_{g,W} + (1-F_G)\mu_A. \quad (A.4-2)$$

$$k_G = F_G k_{g,W} + (1-F_G) k_A. \quad (A.4-3)$$

$$c_G = F_G c_{g,W} + (1-F_G) c_A. \quad (A.4-4)$$

$$h_G = F_G (h_{f,W} + h_{fg,W}) + (1-F_G)h_A, \quad \text{(small superheating)}. \quad (A.4-5)$$

The specific-volume of moist-air can be calculated from the following thermodynamically derived equation:

$$v_G = (R_A/p_{atm})(t_G + 460)(1 + 0.611\ F_G), \quad (A.4-6)$$

where R_A (≈ 53.35) is the individual gas-constant for dry-air and p_{atm} is the atmospheric pressure.

A.4.3 Determination of water-vapour mixture-fraction.

The deduction of F_G is outlined on page 283 of ref.12, the result being

$$F_G = F_G^* - (1-F_G^*)N_{Le,W}^{-2/3} \, c_G(t_G-t_G^*)/(h_{fg,W})_{t_G^*} \qquad (A.4\text{-}7)$$

where F_G^* is the water-vapour mixture fraction at the corresponding wet-bulb temperature t_G^*; $N_{Le,W}$ is the Lewis Number for water and has a value of 1.167 for moderate concentrations of water-vapour.

F_G is implicit in (A.4-7) since $c_G = \phi(F_G, t_G)$. However, examination of (A.4-4) shows that little error will ensue by placing $F_G = 0$ in (A.4-4) to give

$$c_G = c_A. \qquad (A.4\text{-}8)$$

In fact, the error in c_G will be less than 2% for dry-bulb temperatures up to 85 deg. F, even if the moist-air is completely saturated.

The substitution of (A.4-8) into (A.4-7) and evaluation of the Lewis Number term results in the following explicit relationship, from which F_G may be directly determined:

$$F_G = F_G^* - 0.902 \, c_A(1-F_G^*)(t_G-t_G^*)/(h_{fg,W})_{t_G^*} \qquad (A.4\text{-}9)$$

F_G^* can be deduced by assuming saturated-air conditions and manipulating (A.4-1) thermodynamically to give the following function of t_G^* :

$$F_G^* = \{1.611 \, (p_{atm}/p_{s,W}) - 0.6111\}^{-1}. \qquad (A.4\text{-}10)$$

A.5 Refrigerants and their saturated-vapours.

A.5.1 Introduction.

Investigations indicate that the four refrigerants

referred to in this sub-section show the greatest promise for adoption by the Spalding-system. The physical property equations are applicable to saturation, or near-saturation conditions, in accordance with Spalding-system requirements.

In applying the standard curve-fitting techniques mentioned in A.1, it was noted that a single equation was suitable for the prediction of all the physical properties needed for the present investigation, viz.

$$\ln(\text{physical property}) = a + bt_s + ct_s^2 + dt_s^3,$$

where a, b, c and d are constants and t_s is the saturation temperature.

The advantages offered by this simple equation made possible the development of a single computer-program to calculate the constants in the above equation for every physical property of the refrigerants under consideration. The Mercury computer of the University of London was used to calculate the constants appearing in the following sub-sections.

A.5.2 Ammonia.

$$\ln p_s = 8.385+0.02407t_s-5.570\times10^{-5}t_s^2+9.025\times10^{-8}t_s^3$$

0 - 120 deg.F; 0.1%; ref.6, p.20. (A.5-1)

$$\ln v_f = -3.721+9.796\times10^{-4}t_s+3.454\times10^{-6}t_s^2-4.197\times10^{-9}t_s^3$$

0 - 120 deg.F; 0.2%; ref.6, p.20. (A.5-2)

$$\ln v_g = 2.210-0.02266t_s+5.086\times10^{-5}t_s^2-1.026\times10^{-7}t_s^3$$

0. - 120 deg.F; 0.2%; ref.6, p.20. (A.5-3)

$$\ln h_{fg} = 6.344 - 1.363\times10^{-3}t_s - 2.314\times10^{-6}t_s^2 - 1.536\times10^{-8}t_s^3$$

0 - 120 deg.F; 0.1%; ref.6, p.20. (A.5-4)

$$\ln \mu_f = -0.5029 - 2.002\times10^{-3}t_s - 1.896\times10^{-6}t_s^2 - 3.962\times10^{-9}t_s^3$$

40 - 100 deg.F; 0.1%; ref.1, p.275, table 3. (A.5-5)

$$\ln \mu_g = -3.876 + 2.021\times10^{-3}t_s + 2.667\times10^{-7}t_s^2 - 2.218\times10^{-9}t_s^3$$

40 - 150 deg.F; 0.1%; ref.1, p.275, table 4. (A.5-6)

$$\ln k_f = -1.238$$

5-86 deg.F; constant value given; ref.1,p.275, table 5. (A.5-7)

$$\ln k_g = -4.565 + 4149\times10^{-3}t_s - 7.985\times10^{-6}t_s^2 + 1.281\times10^{-8}t_s^3$$

122 - 302 deg.F: 0.1%; ref.1, p.275, table 6. (A.5-8)

$$\ln c_f = 0.07691 + 4.404\times10^{-4}t_s + 2.662\times10^{-6}t_s^2$$

0 - 120 deg.F; 0.3%; ref.3, p.30. (A.5-9)

$$\ln\sigma = -6.436$$

77 deg.F; constant value given; ref.1, p.274, table 2. (A.5-10)

A.5.3 Freon 12.

$$\ln p_s = 8.142 + 0.02114t_s - 4.856\times10^{-5}t_s^2 + 8.137\times10^{-8}t_s^3$$

0 - 120 deg.F; 0.2%; ref.6, p.22. (A.5-11)

$$\ln v_f = -4.510 + 1.314\times10^{-3}t_s - 3.296\times10^{-7}t_s^2 + 1.703\times10^{-8}t_s^3$$

0 - 120 deg.F; 0.2%; ref.6, p.22. (A.5-12)

$$\ln v_g = 0.4756-0.01981t_s+4.132x10^{-5}t_s^2-8.696x10^{-8}t_s^3$$

0 - 120 deg.F; 0.2%; ref.6, p.22. (A.5-13)

$$\ln h_{fg} = 4.230-1.609x10^{-3}t_s-1.687x10^{-6}t_s^2-2.913x10^{-8}t_s^3$$

0 - 120 deg.F; 0.1%; ref.6, p.22. (A.5-14)

$$\ln \mu_f = -0.2102-4.695x10^{-3}t_s+2.097x10^{-5}t_s^2-6.133x10^{-8}t_s^3$$

0 - 120 deg.F; 0.4%; ref.1, p.275 (A.5-15)

$$\ln \mu_g = -3.600+1.109x10^{-3}t_s+6.427x10^{-6}t_s^2-4.054x10^{-8}t_s^3$$

0 - 120 deg.F; 0.2%; ref.1, p.275 (A.5-16)

$$\ln k_f = -2.977-2.604x10^{-3}t_s-2.801x10^{-6}t_s^2-1.102x10^{-8}t_s^3$$

0 - 120 deg.F; 0.1%; ref.1, p.276. (A.5-17)

$$\ln k_g = -5.423+3.036x10^{-3}t_s-4.388x10^{-6}t_s^2+5.760x10^{-9}t_s^3$$

0 - 120 deg.F; 0.1%; ref.1, p.276. (A.5-18)

$$\ln c_f = -1.538+6.165x10^{-4}t_s+5.667x10^{-6}t_s^2-5.037x10^{-9}t_s^3$$

0 - 120 deg.F; 0.1%; ref.1, p.277. (A.5-19)

$$\ln\sigma = -7.391.$$

50 deg.F; constant value given; ref.1, p.274. (A.5-20)

A.5.4 Freon 22.

$$\ln p_s = 8.628+0.02102t_s-4.781x10^{-5}t_s^2+7.677x10^{-8}t_s^3$$

0 - 120 deg.F; 0.3%; ref.1, p.305. (A.5-21)

$$\ln v_f = -4.430+1.376x10^{-3}t_s+4.155x10^{-7}t_s^2+2.154x10^{-8}t_s^3$$

0 - 120 deg.F; 0.1%; ref.1, p.305. (A.5-22)

$$\ln v_g = 0.3170-0.01986t_s+3.861\times10^{-5}t_s^2-9.605\times10^{-8}t_s^3.$$

0 - 120 deg.F; 0.1%; ref.1, p.305. (A.5-23)

$$\ln h_{fg}= 4.549-1.670\times10^{-3}t_s-5.693\times10^{-6}t_s^2-4.764\times10^{-8}t_s^3.$$

0 - 120 deg.F; 0.1%; ref.1, p.305. (A.5-24)

$$\ln \mu_f= -0.3511-3.668\times10^{-3}t_s+1.289\times10^{-5}t_s^2-3.048\times10^{-8}t_s^3.$$

0 - 120 deg.F; 0.4%; ref.1, p.275. (A.5-25)

$$\ln\mu_g = -3.712+1.679\times10^{-3}t_s+8.315\times10^{-7}t_s^2-1.891\times10^{-8}t_s^3.$$

0 - 120 deg.F; 0.8%; ref.1, p.275. (A.5-26)

$$\ln k_f= -2.766-2.442\times10^{-3}t_s-2.603\times10^{-6}t_s^2-8.227\times10^{-9}t_s^3.$$

0 - 120 deg.F; 0.1%; ref.1, p.276. (A.5-27)

$$\ln k_g= -5.236+3.051\times10^{-3}t_s-4.338\times10^{-6}t_s^2+5.389\times10^{-9}t_s^3.$$

0 - 120 deg.F; 0.1%; ref.1, p.276. (A.5-28)

$$\ln c_f= -1.309+1.985\times10^{-3}t_s-5.969\times10^{-6}t_s^2-1.119\times10^{-8}t_s^3.$$

0 - 120 deg.F; 0.1%; ref.1, p.277. (A.5-29)

$$\ln\sigma = - 7.391$$

77 deg.F; constant value given; ref.1, p.274. (A.5-30)

A.5.5 Methyl Chloride.

$$\ln p_s= 7.909+0.02265t_s-5.170\times10^{-5}t_s^2+7.287\times10^{-8}t_s^3.$$

0 - 120 deg.F; 0.1%; ref.1, p.299. (A.5-31)

$$\ln v_f= -4.127+1.058\times10^{-3}t_s+1.966\times10^{-7}t_s^2+7.049\times10^{-9}t_s^3.$$

0 - 120 deg.F; 0.1%; ref.1, p.299. (A.5-32)

$$\ln v_g = 1.603 - 0.02110 t_s + 4.488 \times 10^{-5} t_s^2 - 6.759 \times 10^{-8} t_s^3.$$

0 - 120 deg.F; 0.2%; ref.1, p.299. (A.5-33)

$$\ln h_{fg} = 5.203 - 1.277 \times 10^{-3} t_s - 2.548 \times 10^{-6} t_s^2 - 7.458 \times 10^{-9} t_s^3.$$

0 - 120 deg.F; 0.1%; ref.1, p.299. (A.5-34)

$$\ln \mu_f = -0.3273 - 3.427 \times 10^{-3} t_s + 8.083 \times 10^{-6} t_s^2 - 1.297 \times 10^{-8} t_s^3.$$

0 - 120 deg.F; 0.2%; ref.1, p.275. (A.5-35)

$$\ln \mu_g = -3.784 + 1.846 \times 10^{-3} t_s - 1.321 \times 10^{-6} t_s^2 - 4.868 \times 10^{-10} t_s^3.$$

0 - 120 deg.F; 0.5%; ref.1, p.275. (A.5-36)

$$\ln k_f = -2.186 - 2.420 \times 10^{-3} t_s - 3.085 \times 10^{-6} t_s^2 - 4.496 \times 10^{-9} t_s^3.$$

0 - 120 deg.F; 0.1%; ref.14, p.459. (A.5-37)

$$\ln k_g = -5.387 + 4.866 \times 10^{-3} t_s - 9.291 \times 10^{-6} t_s^2 + 1.152 \times 10^{-8} t_s^3.$$

0 - 120 deg.F; 0.1%; ref.1, p.276. (A.5-38)

$$\ln c_f = -1.002 + 6.356 \times 10^{-4} t_s - 2.792 \times 10^{-7} t_s^2 + 7.352 \times 10^{-10} t_s^3.$$

0 - 120 deg.F; 0.2%; ref.1, p.277. (A.5-39)

$$\ln \sigma = -6.803$$

68 deg.F; constant value given; ref.1, p.274. (A.5-40)

A.6 Thermal conductivities of metals and alloys.

Cooling water for the Wet-system steam surface-condenser is usually in the form of river, lake, or similar natural-water. For clean natural-water, 70/30 brass condenser tubes are suitable, according to page 8, ref.2. Where the natural water is brackish or polluted, the more expensive admiralty

brass tubing offers a slightly better corrosion resistance. For a temperature of 68 deg.F, page 18 of ref.2 gives thermal conductivity values of

$$k_m = 67.7 \qquad \text{(A.6-1)}$$

and

$$k_m = 55.6 \qquad \text{(A.6-2)}$$

for 70/30 brass and admiralty brass respectively.

From the data on pages 1015 and 1021 of ref.11 and discussion with Hart of ref.5, it is concluded that consideration of materials for use in the Heller- and Spalding-systems can be restricted to aluminium (99.5%), mild steel (En3A) and stainless steel (304). The respective thermal conductivity-temperature equations are:

$$k_m = 128.7 - 0.013\ t. \qquad \text{ref.4, p.22.} \qquad \text{(A.6-3)}$$

$$k_m = 31.22 - 0.0081\ t. \qquad \text{ref.4, p.22.} \qquad \text{(A.6-4)}$$

$$k_m = 8.17 + 0.0057\ t. \qquad \text{ref.5.} \qquad \text{(A.6-5)}$$

A.7 References.

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B FRICTION-FACTOR FORMULAE FOR FLUID FLOW INSIDE TUBES.

B.1 Definition.

The friction-factor used in the present investigation is the so-called Fanning friction-factor, defined by the following relationship:

$$f \equiv (2 g_o \Delta p_{fr} d)/(4 \rho S^2 L). \qquad \text{(B.1-1)}$$

B.2 Single-phase, single-component, turbulent fluid flow in tubes.

B.2.1 Accurate formulae for transition and completely turbulent regimes.

According to page 134 of ref. 1, the following Colebrook-White formula represents the best theoretical approach in determining f for the whole turbulent flow regime:

$$f^{-0.5} = -1.7372 \ln\{0.27027(\epsilon/d) + 1.255 N_{Re}^{-1} f^{-0.5}\}, \qquad \text{(B.2-1)}$$

valid for: $N_{Re} < 3000$.

For the completely turbulent regime where f approaches a constant value, f_n, N_{Re} values are so large that the last term in the foregoing equation can be ignored to give

$$f_n^{-0.5} = -1.7372 \ln\{0.27027(\epsilon/d)\}, \qquad \text{(B.2-2)}$$

valid for: $(f/f_n) < 1.01$.

For the transition regime, the implicit nature of (B.2-1) renders it unsuitable both for calculating f directly and in applying theoretical optimization techniques involving f calculations. In the former case, direct numerical calculation of f can be, and usually is, achieved by repre-

senting (B.2-1) graphically. For the latter case, the best solution is to replace (B.2-1) by approximate, explicit formulae which are valid for the ranges covered during optimization investigations.

B.2.2 Approximate formulae for transition regime.

For the present investigation, preliminary calculations show that approximate, explicit friction-factor formulae are required for the transition regime bounded by $10^4 \leq N_{Re} \leq 10^6$. Furthermore, these formulae must apply to extruded tubes of about 0.5 inches inner-diameter, for which the relative roughness is $\varepsilon/d = 0.0001$ according to page 38 of ref. 1.

Examination of the graphical representation, on page 137 of ref. 1, of f vs N_{Re} with ε/d as parameter, suggests the use of two approximate formulae of the following linear form:

$$f = a_f N_{Re}^{b_f}, \qquad (B.2\text{-}3)$$

where a_f and b_f are constants calculated during curve-fitting procedures.

Thus, for $\varepsilon/d = 0.0001$, there results

$$f = 0.06 N_{Re}^{-0.224},$$

valid for: $10^4 \leq N_{Re} \leq 10^5$ (B.2-4)

and

$$f = 0.022 N_{Re}^{-0.137},$$

valid for: $10^5 \leq N_{Re} \leq 10^6$. (B.2-5)

Both of these formulae over-estimate f with a maximum error not exceeding 2.5 per cent.

B.3 Forced convection, two-phase, single-component fluid flow occurring during condensation in vertical tubes.

On page 28 of ref. 2, Bergelin et al. present a graph of vapour-phase f vs N_{Re}, from which f may be evaluated for condensation in vertical tubes with annular flow of condensate. The authors stress that their results are preliminary in nature but, due to the absence of more recent information, their graph will be used as a basis for calculations in the present investigation.

Bergelin's graph is in the form of a family of f vs $N_{Re,g}$ curves with α_f as the parameter, where

$$\alpha_f = (0{\cdot}25\, v_f \mu_f \sigma_w / \sigma) N_{Re,f} \qquad \text{(B.3-1)}$$

and $N_{Re,g}$ is the vapour-phase Reynold's number at any tube cross-section, based on the local mass flow rate of vapour, and $N_{Re,f}$ is similarly defined for the liquid-phase.

It is desirable to specify the family of curves as a single equation, so as to permit the derivation of formulae during optimization procedures. Since each curve exhibits a maximum value of f, accompanied by non-symmetrical variations in slope, this is difficult to do. One way round the difficulty is to remove the dependence of f on N_{Re}, for the purposes of the present investigation, by replacing each f vs $N_{Re,g}$ curve, associated with a particular value of α_f, by a straight line giving a constant f value equal in magnitude to the maximum f value for that curve.

This assumption results in over-estimated f values,

thereby introducting a design safety factor. After plotting a curve of $(f)_{max}$ vs α_f, as shown overleaf, it is seen that an almost linear relationship exists, given by

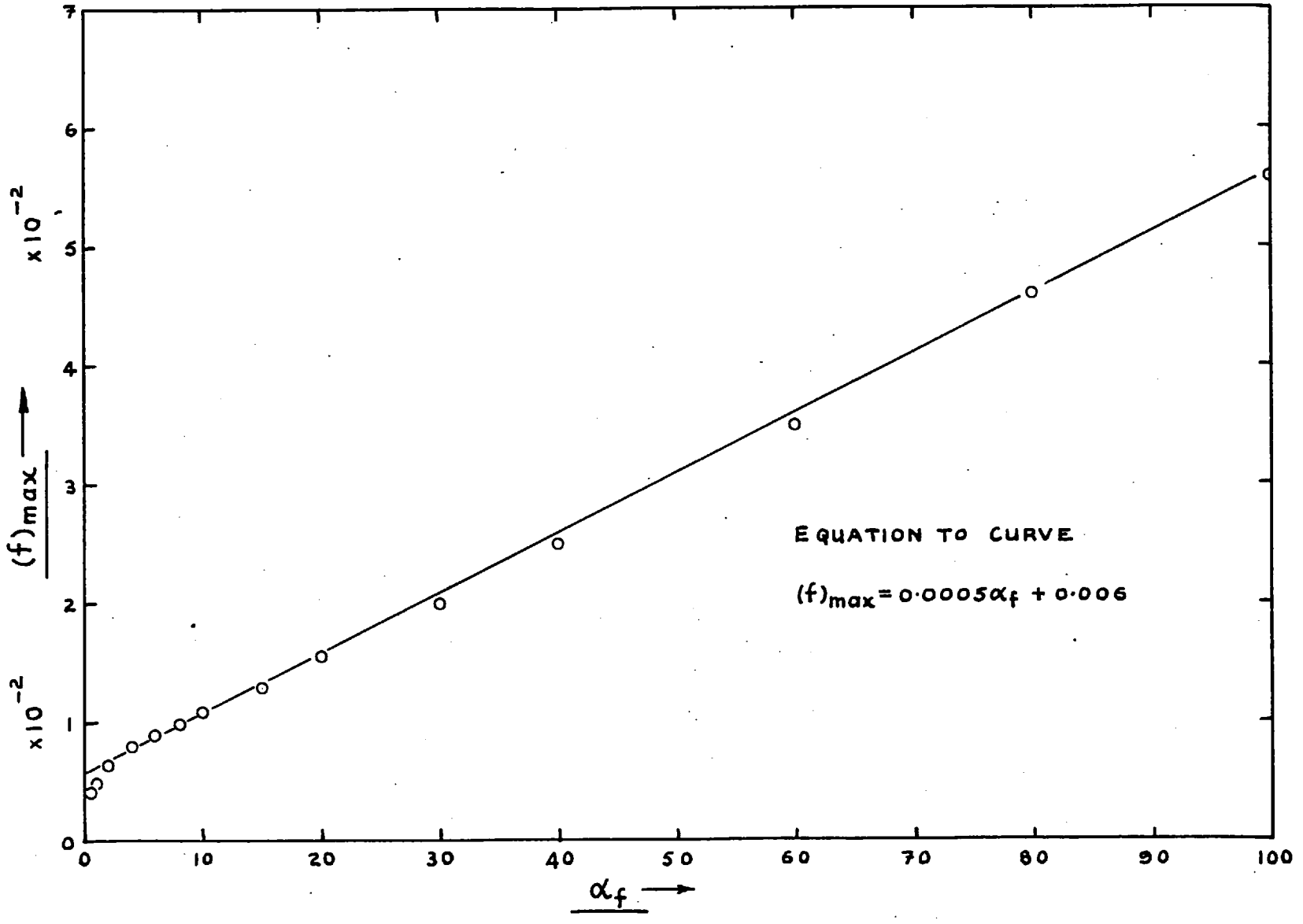
$$(f)_{max} = 0{\cdot}006 + 0{\cdot}0005\,\alpha_f, \qquad \text{(B.3-2)}$$

valid for: $\alpha_f \leq 100$. (B.3-2a)

- B.4 References.

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7
6
5
4
3
2
1
0
$\times 10^{-2}$
$(f)_{max}$
$\times 10^{-2}$
0
10
20
30
40
50
60
70
80
90
100
α_f
EQUATION TO CURVE
$(f)_{max} = 0.0005\alpha_f + 0.006$

C FRICTIONAL PRESSURE-DROP FORMULAE.

C.1 Definition and basic equations.

If the Fanning friction-factor is known, the frictional pressure-drop can be deduced by re-arranging (B.1-1) to give

$$\Delta p_{fr} = (4 f \rho S^2 L)/(2 g_o d). \qquad (C.1\text{-}1)$$

The differential form of (C.1-1), giving local pressure gradient, is

$$(dp_{fr}/dx) = (4 f \rho S^2)/(2 g_o d). \qquad (C.1\text{-}2)$$

The substitution of the continuity equation,

$$S = W v / c_f, \qquad (C.1\text{-}3)$$

results in

$$(dp_{fr}/dx) = (4 v f W^2)/(2 g_o c_f^2 d). \qquad (C.1\text{-}4)$$

C.2 Forced convection, two-phase, single-component fluid flow occurring during condensation in vertical tubes.

The overall frictional pressure-drop cannot be calculated directly by substituting (B.3-2) into (C.1-1), due to the changes in flow conditions produced by progressive condensation along the tubes. Instead, the local gas-phase pressure gradient, (dp_{fr}/dx), must first be determined by the substitution of (B.3-2) into (C.1-4) to give

$$(dp_{fr}/dx) = a_{p,C} W_g^2 + b_{p,C} W_g^2 W_f, \qquad (C.2\text{-}1)$$

where

$$a_{p,C} \equiv (0{\cdot}012\, v_g)/(g_o c_f^2 d), \qquad (C.2\text{-}1a)$$

$$b_{p,C} \equiv (0{\cdot}00025\, v_g v_f \sigma_W)/(g_o c_f^3 \sigma), \qquad \text{(C.2-1b)}$$

after inserting the following definitions:

$$N_{Re,g} \equiv (d W_g)/(\mu_g c_f), \quad N_{Re,f} \equiv (d W_f)/(\mu_f c_f).$$

The dependence of W_f and W_g on x/L, the fractional distance along the tubes, must be ascertained before (C.2-1) can be integrated. The theoretical pressure gradient calculated from (C.2-1) agrees closely with the experimental profile, for a typical condenser-tube, presented in fig.9 on page 28 of ref. 2, if the condensate mass flow rate is assumed to increase linearly along the tubes.

The adoption of this assumption, together with conditions of no liquid at entry and no vapour at exit, results in

$$W_g = W(1-x/L), \; W_f = Wx/L, \text{ where } W = W_f + W_g$$

and

$$\Delta p_{fr} = a_{p,C} L W^2 \int_o^1 (1-x/L)^2 d(x/L) + b_{p,C} L W^3 \int_o^1 (x/L)(1-x/L)^2 d(x/L).$$

The above equation can be integrated to give

$$\Delta p_{fr} = 0{\cdot}333 a_{p,C} L W^2 + 0{\cdot}0833 b_{p,C} L W^3, \qquad \text{(C.2-2)}$$

valid for: $\alpha_f \leq 100$,

where $a_{p,C}$ and $b_{p,C}$ are defined in (C.2-1) and α_f is given by (B.3-1).

C.3 Forced convection, two-phase, single-component fluid flow occurring during nucleate boiling in tubes.

According to page 696 of ref. 4, the overall frictional pressure gradient at any section x, during forced convection

boiling in a tube, is given by

$$dp_{fr}/dx = \phi_f^2 (dp_{fr}/dx)_f, \qquad (C.3\text{-}1)$$

where $(dp_{fr}/dx)_f$ is the local one-phase frictional pressure gradient calculated as if the local liquid constituent alone flows through the tube and ϕ_f is a multiplication factor. Before (C.3-1) can be integrated to give the overall frictional pressure drop, the terms on the right hand side must be expressed as functions of x according to the following procedure.

Preliminary calculations applicable to the present investigation indicate that the liquid-phase Reynolds Number, based on total mass flow rate, lies in the range $10^4 \leq (N_{Re})_f \leq 10^5$ and that friction factors may be evaluated with sufficient accuracy from (B.2-4). The substitution of (B.2-4) into (C.1-4) results in

$$(dp_{fr}/dx)_f = \{(0.12\, v_f \mu_f^{0.224})/(g_o c_f^{1.776} d^{1.224})\} W_f^{1.776}, \qquad (C.3\text{-}2)$$

where W_f refers to the local liquid constituent alone.

According to page 698 of ref. 4 and page 45 of ref. 3, ϕ_f can be evaluated from an empirically determined expression,

$\phi_f = \phi(\chi, \text{operating pressure}, \text{types of flow regimes})$,

where

$$\chi \equiv (v_f/v_g)^{1/(2-n)} (\mu_f/\mu_g)^{n/(2-n)} (W_f/W_g), \qquad (C.3\text{-}3)$$

with W_g based on the local vapour constituent alone and n being the exponent in (B.2-4). Therefore

$$\chi = (v_f/v_g)^{0.564}(\mu_f/\mu_g)^{0.126}(W_f/W_g)\,. \qquad (C.3\text{-}4)$$

ϕ_f can be reduced solely to a function of χ if the operating pressure is assumed to be one atmosphere and the flow regimes for liquid and vapour are both taken as turbulent. The resultant single curve, shown on page 698 of ref. 4, can be fitted by an equation of the form

$$\phi_f = 3{\cdot}85\,\chi^{-0.723} + 1\,, \qquad (C.3\text{-}5)$$

the use of which will result in an over-estimate of overall frictional pressure drop, thus providing a safety margin for design calculations. The substitution of (C.3-4) into (C.3-5) gives

$$\phi_f = 3{\cdot}85(v_g/v_f)^{0.407}(\mu_g/\mu_f)^{0.0911}(W_g/W_f)^{0.723} + 1\,. \qquad (C.3\text{-}6)$$

In establishing W_f and W_g as functions of x, it is assumed that the mass flow rate of vapour increases linearly with distance along the tube, in accordance with the procedure in ref. 4. This is a reasonable assumption, in the absence of specific information, as the tube wall and bulk liquid temperatures remain nearly constant in the present investigation. Thus, for conditions of no vapour at entry and a dryness fraction of X at the tube exit,

$$W_f = (1 - Xx/L)W, \quad W_g = X(x/L)W \quad \text{for} \quad W \equiv W_f + W_g.$$

The substitution of these equations into (C.3-2) and (C.3-6) and subsequent inclusion in (C.3-1), together with a transformation of variable, results in

$$dp_{fr}/dy = a_{p,H} W^{1.776} (L/X)[b_{p,H}^2 y^{1.446}(1-y)^{0.33} + 2 b_{p,H} y^{0.723}(1-y)^{1.053} + (1-y)^{1.776}],$$

where

$$a_{p,H} \equiv (0.12 \nu_f \mu_f^{0.224})/(g_o c_f^{1.776} d_i^{1.224}),$$

$$b_{p,H} \equiv 3.85 (\nu_g/\nu_f)^{0.407} (\mu_g/\mu_f)^{0.0911}$$

and

$$y \equiv X x/L . \qquad \text{(C.3-7)}$$

After binomial expansion of the (1 - y) terms, the foregoing equation can be presented as the analytically integrable expression

$$\Delta p_{fr} = a_{p,H} W^{1.776} (L/X) \int_o^X \{b_{p,H}^2 (y^{1.45} - 0.33 y^{2.45} - 0.11 y^{3.45}) + 2 b_{p,H} (y^{0.723} - 1.05 y^{1.723}) - 1.78 y + 1\} dy,$$

which, after integrating, becomes

$$\Delta p_{fr} = a_{p,H} W^{1.776} L \{b_{p,H}^2 X^{1.45} (0.409 - 0.099 X - 0.025 X^2) + 2 b_{p,H} X^{0.723} (0.58 - 0.39 X) + (1 - 0.89 X)\}. \qquad \text{(C.3-8)}$$

For values of X not exceeding 0.1, the terms containing X in the small brackets can be ignored to give

$$\Delta p_{fr} = a_{p,H} W^{1.776} L J_H,$$

where

$$J_H \equiv 0.409 b_{p,H}^2 X^{1.45} + 1.16 b_{p,H} X^{0.723} + 1 . \qquad \text{(C.3-8a)}$$

(C.3-8) and (C.3-8a) are valid for $10^4 \leq N_{Re} \leq 10^5$ where $N_{Re} \equiv (dW/\mu_f c_f)$; $a_{p,H}$ and $b_{p,H}$ are defined in (C.3-7).

Errors in using (C.3-8) to evaluate Δp_{fr} are small for $X < 0.5$ because of the properties of binomial expansions.

For higher values of X_H the errors become significant and reach a maximum at X = 1. Calculations based on (C.3-8a) compare favourably with theoretical calculations both for water on page 698 of ref. 4 and for freon 22 on page 195 of ref. 1; for operating pressures not exceeding 200 psia, (C.3-8a) over-estimates the frictional pressure drop by less than 25 per cent. However, an over-estimate of 100 per cent is obtained when using (C.3-8) for freon 22 at X = 1 and an operating pressure of 200 psia; for X = 0.5 the over-estimate is 45 per cent.

The foregoing figures indicate that (C.3-8) should be used with caution for $X > 0.5$. However, bearing in mind the difficulties involved in postulating a satisfactory frictional pressure drop formula for two-phase flow, (C.3-8) is a useful equation for design purposes, even for $X > 0.5$. It also possesses the desirable feature of being pessimistic, thus rendering it safe for design calculations.

C.4 References

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D HEAT-TRANSFER COEFFICIENT FORMULAE

D.1 Single-phase, single-component, turbulent liquid flow in tubes.

Since the wall to bulk liquid temperature differences are small in the present investigation, the Dittus-Boelter correlation as presented on page 219 of ref. 4 is applicable. On re-arranging the equation and multiplying the constant by 0.8, the heat-transfer-coefficient is given by

$$u = 0{\cdot}0184(k_f/d)(N_{Re})_f^{0{\cdot}8}(N_{Pr})_f^{0{\cdot}4},$$

valid for: $(N_{Re})_f \geqslant 10^4$,

$L/d \geqslant 60$,

$0{\cdot}7 \leqslant (N_{Pr})_f \leqslant 120$, (D.1-1)

where all the liquid physical properties are evaluated at the bulk liquid temperature.

D.2 Forced convection, two-phase, single-component fluid flow occurring during condensation in vertical tubes.

In the present investigation it is advantageous to increase condensation heat-transfer coefficients by operating under conditions of high vapour velocity. An applicable heat-transfer correlation, accurate to within 40 per cent, is presented on page 6 of ref. 2. After re-arranging the given equation and multiplying by 0.6 to introduce a design safety factor, heat-transfer coefficients may be evaluated from

$$u = 0{\cdot}016(k_f/d)\{v_g(N_{Pr})_f f_g/v_f\}^{0{\cdot}5}(N_{Re})_f,$$

valid for : $2\times10^6 \leqslant a_g \leqslant 2\times10^8$,

where $(N_{Re})_f \equiv \{dW/(\mu_f c_f)\}$, $(N_{Re})_g \equiv \{dW/(\mu_g c_f)\}$,

$$a_g \equiv (0{\cdot}17 v_g \mu_g^2/d^2) f_g (N_{Re})_g^2, \qquad \text{(D.2-1)}$$

and f_g is a superficial vapour-phase friction-factor computed as if there were no condensation. Preliminary calculations indicate that the vapour-phase Reynolds Number, based on total mass flow rate, lies in the range $10^5 \leq (N_{Re})_g \leq 10^6$ and that f_g may be evaluated with sufficient accuracy from (B.2-5).

The substitution of (B.2-5) into (D.2-1) results in

$$u = 2{\cdot}38 \times 10^{-3} (k_f/d)\{v_g (N_{Pr})_f / v_f\}^{0.5} (N_{Re})_f (N_{Re})_g^{-0.0683},$$

valid for: $2 \times 10^6 \leq a_g \leq 2 \times 10^8$,

$10^5 \leq (N_{Re})_g \leq 10^6$;

where

$$a_g \equiv (0{\cdot}00374 v_g \mu_g^2/d^2)(N_{Re})_g^{1{\cdot}864},$$

$$(N_{Re})_f \equiv \{dW/(\mu_f c_f)\}, \quad (N_{Re})_g \equiv \{dW/(\mu_g c_f)\}. \qquad \text{(D.2-2)}$$

D.3 Two-phase, single-component condensation of steam on axially horizontal tube bundles.

A formula recommended for the evaluation of average heat-transfer coefficients on the steam-side of plain or low-fin surface condensers, presented on page 464 of ref.3, is of the form

$$u_S = 0{\cdot}9222\, \alpha^{-1/6} u_{s,1} (1 - 0{\cdot}395 \alpha^{-1{\cdot}42} + 0{\cdot}375 \alpha^{-1{\cdot}5}), \qquad \text{(D.3-1)}$$

where α is the ratio of condenser-shell radius to tube-pitch and $u_{s,1}$ is the average condensing heat-transfer coefficient for the uppermost condenser tube, determined from

$$u_{s,l} = 0.651\, k_{f,S}\{2\pi g_o (Q_H/Q_S)/(\mu_{f,S}^2 v_{f,S}^2 \beta_H)\}^{1/3} (N_{Re})_{f,S}^{-1/3},$$

valid for: $(N_{Re})_{f,S} \leq 2.1 \times 10^3$,

where

$$(N_{Re})_{f,S} \equiv \{2\pi d_H X_S/(\mu_{f,S} c_{f,S})\}\, C_S^{-1}\, Q_H,$$

$$C_S \equiv q/(m_S c_{f,S}), \quad Q_H \equiv q/A_{h,H}. \qquad \text{(D.3-2)}$$

Q_H/Q_S is the ratio of outside to inside heat transfer area.

X_S is the steam dryness fraction at condenser inlet.

β_H is the ratio of outside to inside bare tube diameter.

S refers to steam-side.

H refers to tube-side.

(D.3-2) is a re-arrangement of Nusselt's single tube result, presented on page 461 of ref. 3, after substituion of the heat balance vs heat rate equation for steam.

Although (D.3-1) applies to square pitch arrangements it can be used for the more commonly adopted triangular pitch arrangements when slight under-estimates of heat-transfer coefficient will accrue, a desirable feature for design calculations.

In deducing a formula for α it is assumed that the tube pitch is 1.5 d_H and that the summated areas of all the squares, of side 1.5 d_H, represent the condenser shell cross-sectional area. This is a reasonable assumption especially since any errors in α incurred thereby are rendered insignificant for the following reasons:

i) For the present investigation, α values are so large

that the last two bracketed terms in (D.3-1) may be ignored, thus rendering the bracketed expression insensitive to errors in α.

ii) As the remaining α term has an exponent of -1/6, errors are greatly diminished.

After deducing the radius of the condenser shell and dividing by the tube pitch, there results

$$\alpha = (n_H/\pi)^{0.5},$$

where n_H is the number of tubes. This equation can also be expressed as

$$\alpha = \{2 q^{0.5}/(\pi d_H)\} (C_H W_H)^{-0.5} \qquad \text{(D.3-3)}$$

after incorporating

$$n_H = q/(0.25 \pi d_H^2 C_H W_H),$$

which follows from the tube-side definitions

$$C_H \equiv q/(m_H c_{f,H}), \quad W_H \equiv m_H c_{f,H}/A_{c,H}.$$

Therefore, after the substitution of (D.3-2) and (D.3-3), (D.3-1) becomes

$$u_S = a_{u,S} (C_H W_H)^{1/12} (C_S/Q_H)^{1/3},$$

$$\text{valid for: } (N_{Re})_{f,S} \leq 2.1 \times 10^3,$$

where

$$a_{u,S} = 0.647 k_{f,S} \{g_o c_{f,S} (Q_H/Q_S)/(X_S \mu_{f,S} v_{f,S}^2 \beta_H d_H^{0.5} q^{0.25})\}^{1/3} \qquad \text{(D.3-4)}$$

and $(N_{Re})_{f,s}$ is defined in (D.3-2).

D.4 Heat transfer during forced convection nucleate boiling under saturation conditions.

D.4.1 Maximum heat-transfer coefficients inside horizontal tubes for Ammonia, Freon 12, and Methyl Chloride.

The results of various investigators are presented in graphical form on page 439 of ref. 5.

For Ammonia, the results of Cleis, Danilowa and Masukewitsch, applicable to boiling inside and outside vertical tubes, give the highest heat-transfer coefficients and can be correlated by the single expression

$$u_H = 10\, Q_H^{0.5},$$

valid for: $d_H \approx 0.06$,

$$300 \leq Q_H \leq 10^4. \qquad \text{(D.4-1)}$$

(D.4-1) may be used as a guide for boiling inside horizontal tubes, in the absence of specific information, provided that the fluid velocity is high enough to prevent a build-up of vapour inside the tubes.

For Freon 12, the highest heat-transfer coefficients for boiling inside horizontal tubes are given by the equation

$$u_H = 6.0\, Q_H^{0.5},$$

valid for: $d_H \approx 0.06$,

$$200 \leq Q_H \leq 5000. \qquad \text{(D.4-2)}$$

From indications in the overall review on page 438 of ref. 5 and the experimental results presented in the graph on page 439, the following equation is adopted for methyl

chloride:

$$u_H = 7{\cdot}0\, Q_H^{0{\cdot}5},$$

$$\text{valid for: } d_H \approx 0{\cdot}06,$$

$$3000 \leq Q_H \leq 10{,}000. \qquad \text{(D.4-3)}$$

These equations should be used only as rough guides due to the uncertainty of the experimental results and the fact that they were obtained for vapour quality changes much larger than those encountered in the present investigation.

D.4.2 <u>General heat transfer correlation for Freon 12, 22 and Methyl Chloride flowing through horizontal tubes</u>.

An equation for the determination of heat-transfer coefficients, for incomplete vaporization, is presented on page 194 of ref. 1. Observation of the scatter of experimental points around the curve plotted from the equation suggests the adoption of a multiplication safety factor of 0.7. After substituting the heat balance-heat rate equation for the refrigerant and incorporating the definitions

$$Q_H \equiv q/A_{h,H}, \quad W_H \equiv m_H c_{f,H}/A_{c,H},$$

the equation finally becomes

$$u_H = a_{u,H}(Q_H W_H)^{0{\cdot}375},$$

$$\text{valid for: } 2{\cdot}5\times10^{10} \leq \{3{\cdot}1\times10^{3}(c_{f,H}\mu_{f,H}^{2})^{-1} d_H Q_H W_H\} \leq 1{\cdot}5\times10^{12},$$

where

$$a_{u,H} \equiv 0{\cdot}32\, k_{f,H}(c_{f,H}\mu_{f,H}^{2})^{-0{\cdot}375}\, d_H^{-0{\cdot}625} \qquad \text{(D.4-4)}$$

and the subscript H refers to the tube-side.

This equation, like those in the previous sub-section, should be used with caution since the experimental results leading to the correlation were for higher vapour quality changes than those encountered in the present investigation. (D.4-4) is not applicable to boiling Ammonia, giving gross over-estimates for the boiling heat-transfer coefficient.

D.5 References.

1. Altman, M. et al. "Local and Average Heat Transfer and Pressure Drop for Refrigerants Evaporating in Horizontal Tubes", Trans. A.S.M.E., Jrnl. of Heat Transfer, August, 1960.

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E TRANSFER UNIT-EFFECTIVENESS AND ASSOCIATED PERFORMANCE RELATIONSHIPS

E.1 Wet-system tower packing.

E.1.1 Presentation of general transfer unit equation.

The heat-transfer potential between water and air in the wet-system tower packing is universally accepted as being proportional to enthalpy difference rather than temperature difference, a fact first recorded in 1925 by Merkel (ref.2). The postulates and assumptions of Merkel are outlined on pages 401 onwards of ref. 3 in deducing an expression for the number of overall air-side transfer units, given by

$$N_{h,G} = (G_C/G_G)\int_{h_{C,o}}^{h_{C,i}} \{h_M - h_{G,i} + (G_C/G_G)(h_C - h_{C,i})\}^{-1} dh_C, \qquad \text{(E.1-1)}$$

where (G_C/G_G) is the ratio of water- to air-side loading,

h_G and h_C are moist-air and water enthalpies respectively,

i and o refer to inlet and outlet respectively

and h_M is the enthalpy of the layer of saturated air adjacent to and at the same temperature as the water surface.

h_M is related to the water surface temperature according to (A.4-5), evaluated for saturated air. Using 32 deg. F as the enthalpy base-temperature, the relationship between h_M and h_C can be established by incorporating the following enthalpy-temperature expression for water,

$$h_C \approx t_C - 32. \qquad \text{(E.1-2)}$$

As the resultant h_M vs h_C relationship is complicated, (E.1-1) cannot be integrated analytically unless some simplifying assumptions are made.

E.1.2 Approximate $h_M - h_C$ relationships.

The most widely accepted approximation is that of Merkel, whereby the h_M vs h_C relationship is linearized by postulating an average enthalpy difference, given by

$$h_M - h_G = 0.5\{(h_{M,i}+h_{M,o})-(h_{G,i}+h_{G,o})\}. \qquad (E.1-3)$$

A more accurate approximation is obtained by replacing the actual h_M vs h_C relationship by a quadratic expression, valid over the range of enthalpies encountered in the present investigation. For one atmosphere, the equation

$$h_M = 8.124 + 0.09784\,h_C + 0.009743\,h_C^2,$$

$$\text{valid for: } 30 \leq h_C \leq 60, \qquad (E.1-4)$$

gives h_M values accurate to within 1.0%.

E.1.3 Transfer unit equation and other relationships using Merkel's approximation.

The substitution of (E.1-2) and (E.1-3) into (E.1-1) and the incorporation, after integration, of the heat balance relationship,

$$G_C/G_G = (h_{G,o}-h_{G,i})/(h_{C,i}-h_{C,o}), \qquad (E.1-5)$$

results in

$$N_{h,G} = \{\alpha_M/(G_C/G_G) - 0.5\}^{-1}, \qquad (E.1-6)$$

$$\text{where } \alpha_M \equiv \{0.5(h_{M,i}+h_{M,o}) - h_{G,i}\}/(t_{C,i}-t_{C,o}). \qquad (E.1-7)$$

(E.1-6) can also be expressed as

$$t_{C,i} - t^{*}_{G,i} = (\bar{c}^{*}_{G})^{-1}\{(G_C/G_G)(t_{C,i} - t_{C,o})(N^{-1}_{h,G} + 0{\cdot}5) + 0{\cdot}5(h_{M,i} - h_{M,o})\}, \quad \text{(E.1-8)}$$

$$\text{where} \quad \bar{c}^{*}_{G} \equiv (h_{M,i} - h_{G,i})/(t_{C,i} - t^{*}_{G,i}). \quad \text{(E.1-9)}$$

The asterisk super-script denotes conditions at wet-bulb temperatures.

$\bar{c}^{*}_{G}$ can be accurately determined by substituting (E.1-4) and (E.1-2) into (E.1-9), after noting that the enthalpy of unsaturated air is equal to the enthalpy at the associated wet-bulb temperature. Thus,

$$\bar{c}^{*}_{G} = 0{\cdot}009743(t_{C,i} + t^{*}_{G,i}) - 0{\cdot}5257. \quad \text{(E.1-10)}$$

E.1.4 Accurate transfer unit equation using quadratic approximation.

After substituting (E.1-4) and integrating, (E.1-1) becomes

$$N_{h,G} = [2(G_C/G_G)/b_3^{0{\cdot}5}]\left[\arctan\{(2b_2 h_C + b_1)/b_3^{0{\cdot}5}\}\right]_{h_{C,o}}^{h_{C,i}},$$

valid for: $30 \leq h_C \leq 60$,

where

$$b_3 \equiv 4\,b_0 b_2 - b_1^2,$$

$$b_0 \equiv 8{\cdot}124 - h_{G,i} + (G_C/G_G)\,h_{C,o},$$

$$b_1 \equiv 0{\cdot}09784 - (G_C/G_G),$$

$$b_2 \equiv 0{\cdot}009743. \quad \text{(E.1-12)}$$

E.2 Heller-system tower heat exchanger.

The tower heat exchanger is of the cross-flow variety where the water flowing through the tubes falls into the category of "unmixed-flow". On the air-side, the flow probably falls somewhere between the categories of "completely-mixed" and "completely-unmixed" flow. For the range of operating conditions encountered in cooling towers, the choice of either of these extremes leads to almost identical transfer unit-effectiveness relationships. Since algebraic analysis is easier for the former extreme, the tower heat exchanger is considered to fall into the cateory "cross-flow, one fluid mixed".

According to page 13 of ref. 1, the air-side partial effectiveness, defined as

$$e_G \equiv (t_{G,o} - t_{G,i}) / \{(t_{m,G})_o - t_{G,i}\}, \qquad \text{(E.2-1)}$$

where $(t_{m,G})_o$ is the air-side metal temperature at the heat-exchanger air-side outlet, is given by

$$e_G = [\{(C_C/C_G)a - b\}/(a-b)]^{-1},$$

where

$$a \equiv (1 - e_{G,p})^{n_p}, \quad b \equiv (1 - e_{G,p} C_C/C_G)^{n_p},$$

$$C_C \equiv q/(m_C c_C), \; C_G \equiv q/(m_G c_G) \; ; \qquad \text{(E.2-2)}$$

subscripts G and C refer to air- and water-side respectively, n_p is the number of water-side passes and $e_{G,p}$ is the partial air-side effectiveness per pass.

$e_{G,p}$ can be derived by a procedure similar to that outlined on page 151 onwards of ref. 1, the final result being

$$e_{G,P}^{-1} = (1 - e^{-a})^{-1},$$

where

$$a \equiv (C_C/C_G)^{-1}\{1 - e^{-(C_C/C_G)\,n_{h,G}/n_p}\} \qquad \text{(E.2-3)}$$

and $n_{h,G}$ is the air-side partial number of transfer units, defined by

$$n_{h,G} \equiv u_G A_{h,G}/(m_G c_G).$$

(E.2-3) can be accurately represented by the truncated series expression

$$e_{G,P}^{-1} = (n_p/n_{h,G}) + 0.5\{(C_C/C_G)+1\} + \{n_{h,G}/12\,n_p\}\{(C_C/C_G)^2+1\}, \qquad \text{(E.2-4)}$$

with negligible loss of accuracy for cooling tower applications.

For two water-side passes, (E.2-2) and (E.2-4) become

$$e_G^{-1} = \{e_{G,P}^{-2} - (C_C/C_G)\}/\{2\,e_{G,P}^{-1} - (C_C/C_G) - 1\} \qquad \text{(E.2-5)}$$

and

$$e_{G,P}^{-1} = 2\,n_{h,G}^{-1} + 0.5\{(C_C/C_G)+1\} + 0.04167\,n_{h,G}\{(C_C/C_G)^2+1\}. \qquad \text{(E.2-6)}$$

E.3 Spalding-system tower heat exchanger.

A feature of the Spalding-system is that the temperature of the refrigerant flowing through the tower heat-exchanger tubes remains practically constant. Since the temperature drop through the metal wall is relatively small in cooling tower applications, little error will be introduced by assuming that the air-side metal temperature also remains constant. Under these conditions, the heat exchanger can be theoretically treated as lying in the category of "counterflow with constant temperature on one side".

The associated transfer unit-effectiveness relationship can be derived according to the procedure outlined on page 11 of ref. 1, the final result being

$$e_G^{-1}=(1-e^{-n_{h,G}})^{-1}, \qquad \text{(E.3-1)}$$

where e_G, the air-side partial effectiveness, is defined as

$$e_G \equiv (t_{G,o}-t_{G,i})/\{(t_{m,G})_o - t_{G,i}\}. \qquad \text{(E.3-2)}$$

$(t_{m,G})_o$ is the air-side metal temperature at the heat-exchanger air-side outlet and $n_{h,G}$ is the air-side partial number of transfer units defined as

$$n_{h,G} \equiv u_G A_{h,G}/(m_G c_G). \qquad \text{(E.3-3)}$$

(E.3-1) can be represented by the following truncated series expansion with negligible loss of accuracy for $n_{h,G}$ values encountered in the present investigation:

$$e_G^{-1} = n_{h,G}^{-1} + 0{\cdot}5 + 0{\cdot}0833\, n_{h,G}. \qquad \text{(E.3-4)}$$

E.4 Wet-system surface steam condenser.

Since the steam temperature remains practically constant, the condenser can be theoretically treated as lying in the category of "counterflow with constant temperature on one side", as is the case in the previous sub-section. Similar procedures lead to the expression

$$E_H^{-1}=(1-e^{-N_{h,H}})^{-1}, \qquad \text{(E.4-1)}$$

where E_H, the hot-side overall effectiveness, is defined as

$$E_H \equiv (t_{H,o}-t_{H,i})/(t_{s,S}-t_{H,i}). \qquad \text{(E.4-2)}$$

$N_{h,H}$, the overall number of hot-side transfer units, is defined as

$$N_{h,H} \equiv U_H A_{h,H}/(m_H c_{f,H}), \qquad (E.4\text{-}3)$$

the subscripts H and S referring to the hot- or water-side and steam-side respectively.

E.5 Refrigerant-side temperature approach for Spalding-system steam surface condenser.

E.5.1 Introduction.

Little error is incurred by assuming that the refrigerant-side metal temperature, $t_{m,H}$, like the steam temperature, $t_{s,S}$, remains practically constant along the tubes as the resistance to heat transfer through the steam and tube wall is small. This assumption does not apply to the refrigerant saturation-temperature, $t_{s,H}$, which will vary due to changes in total pressure, with respect to tube length.

The first step in determining the refrigerant-side temperature approach is to equate the heat-rate equation to the heat balance equation for a typical elemental section of the condenser. Thus

$$u_H(t_{m,H} - t_{s,H})\, dA_{h,H} = dq \ , \qquad (E.5\text{-}1)$$

where $dA_{h,H}/dx = A_{h,H}/L_H$, $dq/dx = q/L_H$.

dx and L_H are the respective elemental-section and total tube lengths.

Linear decrease of refrigerant saturation-temperature with respect to tube length is assumed, this being reasonable

for the small temperature variations encountered in the present investigation. This assumption, together with the condition of no vapour at inlet, results in

$$(t_{m,H})_o - t_{s,H} = \{(t_{m,H})_o - (t_{s,H})_i\} + \Delta t_{s,H}\, x/L_H, \qquad \text{(E.5-2)}$$

where x/L_H is the fractional distance from tube inlet and

$$\Delta t_{s,H} \equiv (t_{s,H})_i - (t_{s,H})_o . \qquad \text{(E.5-2a)}$$

After substituting the foregoing equations, (E.5-1) becomes

$$q = (A_{h,H}/L_H)\int_o^{L_H} u_H[\{(t_{m,H})_o - (t_{s,H})_i\} + \Delta t_{s,H}\, x/L_H]\, dx . \qquad \text{(E.5-3)}$$

E.5.2 Temperature approach for single-phase, forced convection flow condition.

This condition applies if the tube wall to refrigerant temperature difference is insufficient to initiate nucleate boiling, the heat transfer then being that due to forced convection, given by (E.1-1). As the heat-transfer coefficient is independent of distance along the tubes, (E.5-3) can be integrated and re-arranged to give the temperature approach

$$(t_{m,H})_o - (t_{s,H})_o = u_H^{-1} Q_H + 0{\cdot}5\, \Delta t_{s,H} , \qquad \text{(E.5-4)}$$

where $Q_H \equiv q/A_{h,H}$

and u_H^{-1} is evaluated from (D.1-1).

E.5.3 Temperature approach under conditions of nucleate boiling heat transfer, independent of fluid velocity.

This condition pertains when (D.4-1), (D.4-2) and

(D.4-3) are valid for the refrigerant side. These equations can be transformed into the single form

$$u_H = a^2\{(t_{m,H})_o - t_{s,H}\}, \qquad \text{(E.5-5)}$$

where $a = 10, 6$ and 7

for Ammonia, Freon 12 and Methyl Chloride respectively, after substituting the heat balance-heat rate equation

$$Q_H = u_H\{(t_{m,H})_o - t_{s,H}\}.$$

After substituting (E.5-5) into (E.5-3), integrating and rearranging, the temperature approach is given by

$$(t_{m,H})_o - (t_{s,H})_o = 0{\cdot}5\Delta t_{s,H} + 0{\cdot}5\left[4a^{-2}Q_H - 0{\cdot}333\,\Delta t_{s,H}^2\right]^{0{\cdot}5}. \qquad \text{(E.5-6)}$$

This equation can be accurately represented as

$$(t_{m,H})_o - (t_{s,H})_o \approx 0{\cdot}5\,\Delta t_{s,H} + a^{-1}Q_H^{0{\cdot}5}, \qquad \text{(E.5-6a)}$$

for the small values of $\Delta t_{s,H}$ encountered in the present investigation.

E.5.4 <u>Temperature approach under conditions of nucleate boiling heat transfer dependent on fluid velocity</u>.

For these conditions, (D.4-4) is applicable and can be transformed into

$$u_H = a_{u,H}^{1{\cdot}6}\,W_H^{0{\cdot}5}\{(t_{m,H})_o - t_{s,H}\}^{0{\cdot}6}. \qquad \text{(E.5-7)}$$

The substitution of (E.5-7) into (E.5-3) results in the temperature approach being given by

$$(t_{m,H})_o - (t_{s,H})_o = \left[(Q_H u_H^{-1})^{1{\cdot}6} + 0{\cdot}8\{\Delta t_{s,H}\}\{(t_{m,H})_o - (t_{s,H})_o\}^{0{\cdot}6}\right]^{0{\cdot}625}, \qquad \text{(E.5-8)}$$

where u_H is calculated from (D.4-4).

E.6 References

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F DERIVATION OF TERMS IN BUOYANCY EQUATION.

F.1 Deduction of buoyancy equation.

The total pressure drop through the tower must be exactly counter-balanced by the draught induced pressure difference, i.e.

$$\Delta p_D - (z_5 - z_p)(\rho_{G,i} - \rho_{G,o})(a_{grav}/g_o) = 0 \quad , \qquad \text{(F.1-1)}$$

where the first and last terms are called the pressure-drop term and the density-difference term respectively. $(z_5 - z_p)$ is the effective draught length of the tower with z_5 as the height of the top of the tower. For Wet- system cooling towers, z_p is the height of the packing base whereas, for dry cooling towers, z_p is assumed to coincide with z_1, the height of the tower base.

(F.1-1) can be rewritten in terms of the dimensionless parameters r_5, r_p and the tower base diameter, D_1, so that

$$r_5 - r_p = \Delta p_D / \{D_1(\rho_{G,i} - \rho_{G,o})(a_{grav}/g_o)\}, \qquad \text{(F.1-2)}$$

where $r_5 \equiv z_5/D_1$, $r_p \equiv z_p/D_1$.

F.2 Determination of pressure-drop term.

F.2.1 Wet-system.

The pressure-drop term can be conveniently defined according to the velocity head convention as

$$\Delta p_D \equiv \Sigma N_v (0{\cdot}5\, v_{G,i}/g_o)\{m_G/(0{\cdot}25\,\pi D_1^2)\}^2, \qquad \text{(F.2-1)}$$

where ΣN_v is the arithmetical sum of all local velocity head losses through the tower, after they have been trans-

formed to the same reference plane, here chosen as the tower base cross-sectional area. ΣN_v can be expressed in terms of local and transformed velocity head losses after modifying the equations of Rish in ref.3 according to the suggestions of Spalding and Singham in ref.4. Therefore

$$\Sigma N_v = (n_{v,pack} + n_{v,elim})t^{-2} + n_{v,spray} + N_{v,i} + N_{v,o} + N_{v,misc}. \quad \text{(F.2-2)}$$

$n_{v,pack}$, $n_{v,elim}$ and $n_{v,spray}$ are the local velocity head losses due to the packing, eliminator and spray respectively. t, the blanking ratio, is the ratio of tower-base free-flow area to total tower-base cross-sectional area; its incorporation in (F.2-2) transforms local velocity head losses by referring them to the common reference plane represented by the tower-base cross-sectional area. $N_{v,i}$, $N_{v,o}$ and $N_{v,misc}$ are tower inlet, outlet and miscellaneous velocity head losses respectively.

According to Spalding and Singham, the respective velocity head losses can be evaluated from formulae of the following forms:

$$n_{v,pack} = a_{v,p}(z_2 - z_p)\{b_{v,p}(G_C/G_G)^{c_{v,p}} + d_{v,p}\}, \quad \text{(F.2-3)}$$

$$n_{v,spray} = a_{v,s}\{r_p D_1 + (z_3 - z_2)\}(G_C/G_G)^{b_{v,s}}, \quad \text{(F.2-4)}$$

$$n_{v,elim} = a_{v,e}, \quad \text{(F.2-5)}$$

$$N_{v,i} = a_{v,i}(r_p^{-2}) + b_{v,i}, \quad \text{(F.2-6)}$$

$$N_{v,o} = s^{-4}, \quad \text{(F.2-7)}$$

$$N_{v,misc} = a_{v,misc}. \quad \text{(F.2-8)}$$

(G_C/G_G) is the ratio of water to air loading.

(z_2-z_p) is the thickness of the packing, in feet.

r_pD_1 is the height of the underside of the packing, in feet.

(z_3-z_2) is the effective height of spray above the top of the packing, in feet.

r_p is the ratio of the height of the underside of the packing, z_p, to the tower base diameter, D_1.

s is the ratio of tower throat to tower base diameter.

The sub-scripted letters a, b, c and d are constants obtained by fitting curves to experimental results for particular tower configurations and packing types under consideration.

Experimental investigations by Lieberam at Imperial College, London, indicate that the asbestos sheet packing, referred to as type 21 by Lowe and Christie in ref.2, is one of the best packing types for cooling tower applications. The values of the performance constants pertaining to type 21 and those for the eliminator and miscellaneous items are:

$a_{v,p} = 23{\cdot}55$, $b_{v,p} = 0{\cdot}0192$, $c_{v,p} = 0{\cdot}5$, $d_{v,p} = 0{\cdot}104$,

$a_{v,s} = 0{\cdot}16$, $(z_3-z_2) = 4{\cdot}0$, $b_{v,s} = 1{\cdot}32$,

$a_{v,e} = 3{\cdot}1$, $a_{v,misc} = 2{\cdot}3$.

In determining the inlet velocity head loss, Ioannides, also of Imperial College, has conducted experiments on cooling tower models which suggest the use of

$a_{v,i} = 0{\cdot}0853$, $b_{v,i} = -2{\cdot}2$,

valid for : $0{\cdot}055 \leq r_p \leq 0{\cdot}070$.

F.2.2 Heller- and Spalding-systems.

The Wet-system pressure drop equation, (F.2-1), is also applicable to dry cooling towers, but the total velocity head loss is given by

$$\Sigma N_v = n_{v,G}(0{\cdot}25\pi D_1^2/A_{c,G})^2 + N_{v,o} + N_{v,i} + N_{v,misc},$$

where $n_{v,G}$ is the local velocity head loss and $A_{c,G}$ is the free flow area for the tower heat-exchanger.

$A_{c,G}$ may be eliminated by substituting the following definitions into the foregoing equation:

$$C_G \equiv q/(m_G c_G), \quad W_G \equiv m_G c_G/A_{c,G},$$

so that

$$\Sigma N_v = n_{v,G} W_G^2 (0{\cdot}25\pi q^{-1} C_G D_1^2)^2 + N_{v,o} + N_{v,i} + N_{v,misc}. \quad \text{(F.2-8a)}$$

In order to evaluate $n_{v,G}\, W_G^2$, use must be made of the empirical heat-transfer and pressure-drop performance data on the air-side of heat-exchangers suitable for cooling tower applications. These data are usually provided by manufacturers in the form of curves which can be easily and accurately converted to equations of the following form:

$$u_G = a_{u,G} W_G^{B+1}, \quad \text{(F.2-9)}$$

$$\Delta p_G = a_{p,G} n_r W_G^{b+2}, \quad \text{(F.2-10)}$$

where

$$a_{u,G} \equiv b_{u,G}\{v_G \sigma_G/(60 c_G)\}^{B+1}, \quad \text{(F.2-11)}$$

$$a_{p,G} \equiv b_{p,G}\{v_G \sigma_G/(60 c_G)\}^{b+2}. \quad \text{(F.2-12)}$$

$b_{u,G}$, $b_{p.G}$, B and b are the necessary curve-fitting parameters.

u_G is the air-side heat-transfer coefficient, referred to the air-side, in (Btu)/(hr sq ft deg F).

Δp_G is the air-side pressure-drop through the heat-exchanger in $(lb_f)/(sq\ ft)$.

W_G is the air-side loading factor, in (Btu)/(hr sq ft deg F).

n_r is the number of tube rows transverse to the direction of air-flow.

σ_G is the ratio of air-side free-flow area to heat-exchanger frontal area.

c_G and v_G are physical property symbols defined in the thesis nomenclature.

By incorporating the air-side partial number of transfer units and velocity head definitions

$$n_{h,G} \equiv u_G C_G Q_G^{-1}, \quad \Delta p_G \equiv n_{v,G} W_G^2 / (2 g_o c_G^2 / v_G)$$

and introducing the following heat-exchanger geometrical relationship:

$$4 r_{h,G} / \sigma_G = 4 l_r l_c / A'_{h,G}, \tag{F.2-13}$$

where

$$4 r_{h,G} \equiv 4 n_r l_r C_G^{-1} Q_G W_G^{-1}, \tag{F.2-14}$$

into (F.2-9) and (F.2-10), the following equation results.

$$n_{v,G} W_G^2 = a_{n,G} n_{h,G}^{j_o}, \tag{F.2-15}$$

where

$$a_{n,G} \equiv (2 g_o c_G^2 / v_G) b_{p,G} \{ b_{u,G} v_G / (60 c_G) \}^{-j_o} n_r^{1-j_o} (A'_{h,G} / l_c)^{-j_o}, \tag{F.2-16}$$

$$j_o \equiv (b+2)/B. \tag{F.2-17}$$

$n_{h,G}$ is the air-side partial number of transfer units, referred to the air-side.

Q_G ($\equiv q/A_{h,G}$) is the air-side heat flux, in (Btu)/(hr sq ft).

$A_{h,G}$ is the total air-side heat-transfer surface area, in sq ft.

$4r_{h,G}$ is the air-side hydraulic diameter, in ft.

l_c is the transverse tube pitch, in ft.

l_r is the tube pitch in the direction of air-flow, in ft.

$A'_{h,G}$ is the total air-side heat-transfer area of one tube per unit length of tube, in ft.

(F.2-15) can now be substituted into (F.2-8) to give

$$\Sigma N_v = a_{n,G} n_{h,G}^{j_o} (0.25\pi q^{-1} C_G D_1^2)^2 + N_{v,o} + N_{v,i} + N_{v,misc}. \quad \text{(F.2-18)}$$

In evaluating $N_{v,o}$, $N_{v,i}$ and $N_{v,misc}$, the following equations may be used, as in the case of the Wet-system:

$$N_{v,i} = a_{v,i}(r_1^{-2}) + b_{v,i}, \quad \text{(F.2-19)}$$

$$N_{v,o} = s^{-4}, \quad \text{(F.2-20)}$$

$$N_{v,misc} = a_{v,misc}. \quad \text{(F.2-21)}$$

Since the value of r_1 is much greater than that for the Wet-system, Ioannides recommends the use of

$a_{v,i} = 0.167$, $b_{v,i} = 0$,

valid for: $r_1 > 0.13$.

and the Wet-system value of

$a_{v,misc} = 2.3$.

F.3 Theoretical determination of density-difference term.

F.3.1 Wet-system.

The air density-difference between tower inlet and outlet follows directly from (A.4-6), i.e.

$$\rho_{G,i} - \rho_{G,o} = \rho_{G,i}\left[1 - \{(t_{G,i}+460)(1+0.611F_{G,i})\}/\{(t_{G,o}+460)(1+0.611F_{G,o})\}\right]. \quad (F.3\text{-}1)$$

$F_{G,i}$ can be directly evaluated from (A.4-9) and (A.4-10) since $t_{G,i}$, $t^*_{G,i}$ and p_{atm}, the atmospheric conditions, are usually specified. For the evaluation of $F_{G,o}$, $t_{G,o}$ and $t^*_{G,o}$ must be known.

To solve exactly for $t_{G,o}$ is difficult because the complicated heat and mass transfer relationships lead to a differential equation, which can only be solved by tedious numerical methods. However, considerable simplification results by assuming that the outlet air is saturated, this assumption causing little error for the relatively high atmospheric humidities in the United Kingdom.

Consequently, with the assumption of saturated outlet air and incorporating (E.1-2), (E.1-4) can be used to give

$$h_{G,o} \approx 8{\cdot}124 + 0{\cdot}09784(t_{G,o}-32) + 0{\cdot}009743(t_{G,o}-32)^2 . \quad (F.3\text{-}2)$$

After re-arranging, (F.3-2) becomes

$$t_{G,o} \approx 32 - 51{\cdot}32\{0{\cdot}09784 - (0{\cdot}03897\,h_{G,o} - 0{\cdot}307)^{0.5}\}. \quad (F.3\text{-}3)$$

$h_{G,o}$ can be evaluated by substituting (E.1-2) into the heat balance relationship

$$G_C/G_G = (h_{G,o} - h_{G,i})/(h_{C,i} - h_{C,o})$$

and re-arranging to get

$$h_{G,o} = (G_C/G_G)(t_{C,i} - t_{C,o}) + h_{G,i} . \qquad (F.3\text{-}4)$$

F.3.2 Heller- and Spalding-systems.

(F.3-1) is equally applicable to dry cooling towers but with $F_{G,o}$ equal to $F_{G,i}$ as there is no increase in air moisture content. Therefore,

$$\rho_{G,i} - \rho_{G,o} = \rho_{G,i}\{1 - (t_{G,i}+460)/(t_{G,o}+460)\}. \qquad (F.3\text{-}5)$$

By incorporating the heat-balance relationship,

$$C_G = t_{G,o} - t_{G,i} ,$$

where $C_G \equiv q/(m_G c_G)$,

(F.3-5) becomes

$$\rho_{G,i} - \rho_{G,o} = \rho_{G,i}\{1 - (t_{G,i}+460)/(C_G + t_{G,i}+460)\}. \qquad (F.3\text{-}6)$$

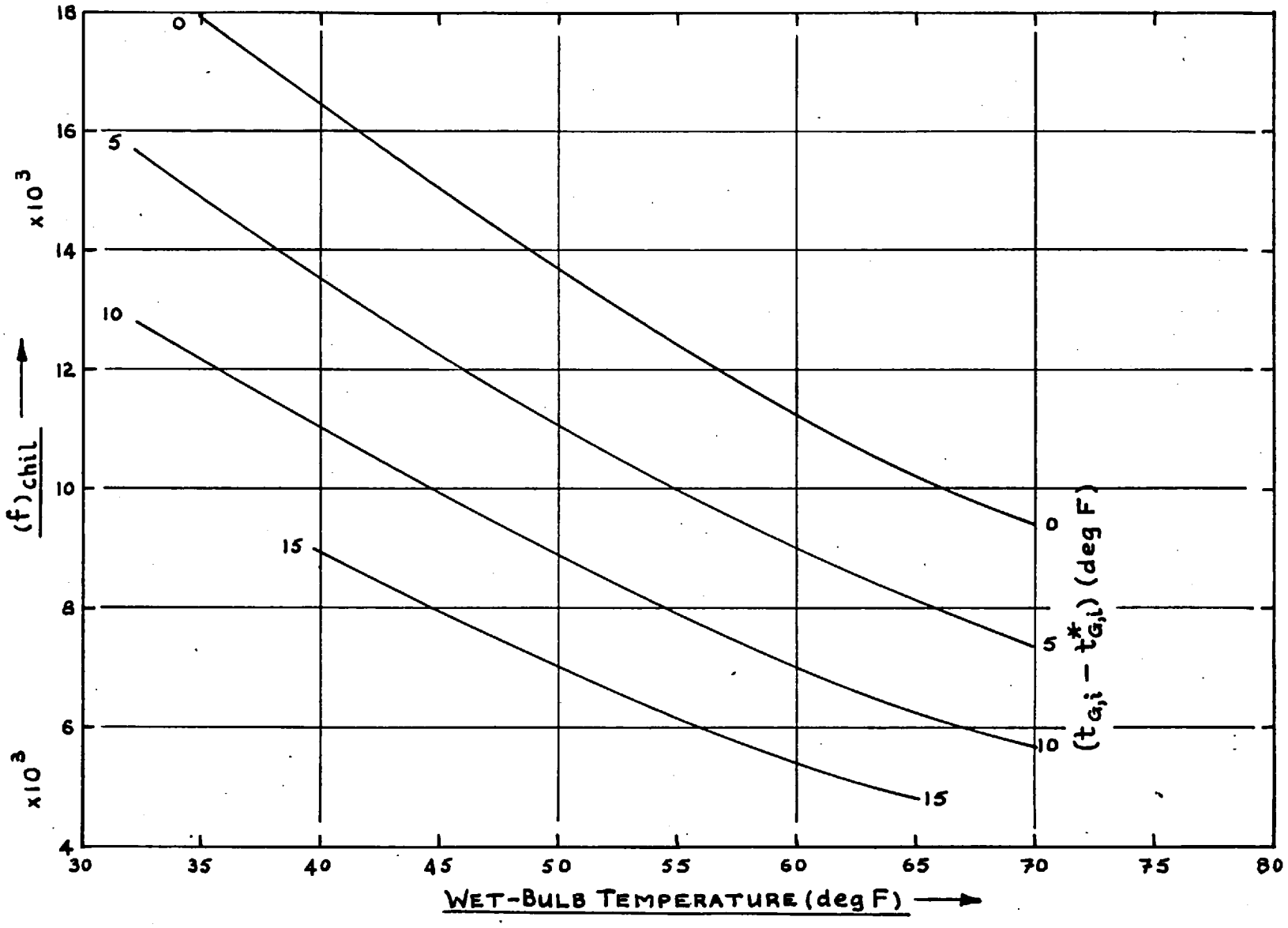
F.4 Approximate determination of density-difference term.

F.4.1 Wet-system.

On page 451 of ref.1, Chilton derives an expression for moist-air density difference which can be presented as

$$\rho_{G,i} - \rho_{G,o} = (f)_{chil}(G_C/G_G)(t_{C,i} - t_{C,o})/\{\Sigma g_o \rho_{G,i}(a_{grav}/g_o)\}. \qquad (F.4\text{-}1)$$

$(f)_{chil}$ is an empirically determined factor, valid if the outlet air is saturated and is dependent solely on the atmospheric wet- and dry-bulb temperatures. $(f)_{chil}$ may be estimated from a graph on page 444 of ref.1, a reproduction of which is presented overleaf. Caution must be exercised in using this graph as $(f)_{chil}$ can be shown theoretically to


18
16
14
12
10
8
6
4
x10^3
x10^3
(f)chil
0
5
10
15
0
5
10
15
(tG,i − t*G,i) (deg F)
30
35
40
45
50
55
60
65
70
75
80
WET-BULB TEMPERATURE (deg F)

depend on both the outlet wet- and dry-bulb temperatures.

F.4.2 Heller- and Spalding systems.

For the temperature ranges of interest in the present investigation, (F.3-6) is closely approximated by the expression

$$\rho_{G,i} - \rho_{G,o} = 0.00185\, \rho_{G,i}\, C_G,$$

valid for: $10 \leq C_G \leq 30$. (F.4-2)

F.5 Derived buoyancy equation expressed as explicit function of effective chimney length.

F.5.1 Wet-system.

After substituting (F.2-1), (F.1-2) becomes

$$r_s - r_p = \{(0.5\, v_{G,i}/g_o)\, \Sigma N_v\, G_G^2\} / \{D_1 (\rho_{G,i} - \rho_{G,o})(a_{grav}/g_o)\},$$

where $G_G \equiv m_G/(0.25\pi D_1^2)$. (F.5-1)

and the evaluation of ΣN_v is outlined in F.2.1.

(F.5-1) may be accurately determined by using (F.3-1) and the procedure in F.3.1 to estimate the air density-difference.

A simpler relationship accrues by following Chilton's procedure in substituting (F.4-1) into (F.5-1), the resultant equation being

$$r_s - r_p = (\Sigma N_v\, G_G^3)/\{(f)_{chil}\, D_1\, G_C (t_{C,i} - t_{C,o})\}. \quad \text{(F.5-2)}$$

This equation can be re-arranged still further by introducing Chilton's duty coefficient, $(D)_{chil}$, and incorporating (E.1-6), (E.1-7) and the definition

$$G_C \equiv m_C/(0{\cdot}25\pi D_1^2). \qquad \text{(F.5-2a)}$$

Thus

$$r_5 - r_p = 1{\cdot}62(D)_{chil}^2 D_1^{-5}(N_{h,G}^{-1} + 0{\cdot}5)^3 \Sigma N_v, \qquad \text{(F.5-3)}$$

$$\text{where } (D)_{chil} \equiv m_C(t_{C,i} - t_{C,o})/[\{0{\cdot}5(h_{M,i} + h_{M,o}) - h_{G,i}\}^{1{\cdot}5}(f)_{chil}^{0{\cdot}5}]. \qquad \text{(F.5-4)}$$

(F.5-3) has been used by the Central Electricity Generating Board as a basis for cooling tower design. However, in ref.4, Spalding and Singham suggest that the buoyancy equation, as represented by (F.5-3), becomes more accurate after introducing a correction factor, x, to account for loss in tower performance due to non-uniform air distribution across the packing.

With this modification, (F.5-3) becomes

$$r_5 - r_p = 1{\cdot}62\{(D)_{chil}/x\}^2 D_1^{-5}(N_{h,G}^{-1} + 0{\cdot}5)^3 \Sigma N_v. \qquad \text{(F.5-5)}$$

F.5.2 <u>Heller- and Spalding-systems</u>.

The substitution of (F.2-1) and the definition

$$C_G \equiv q/(m_G c_G)$$

results in

$$r_5 - r_p = \{0{\cdot}81 v_{G,i}/(g_o c_G^2 a_{grav}/g_o)\} q^2 \Sigma N_v C_G^{-2} D_1^{-5}/(\rho_{G,i} - \rho_{G,o}), \qquad \text{(F.5-6)}$$

where ΣN_v is evaluated in F.2.2.

(F.5-6) may be accurately represented by the substitution of (F.3-6) into it.

A simpler, slightly less, accurate relationship accrues by substituting (F.4-2) to give

$$r_s - r_p = \{438{\cdot}0\, v_{G,i}^2 / (g_o c_G^2 a_{grav}/g_o)\} q^2 \Sigma N_v C_G^{-3} D_1^{-5}. \qquad (F.5\text{-}7)$$

F.6 References.

1. Chilton, G. "Performance of Natural-Draught Water-Cooling Towers", Jrnl. Instn. Elec. Engrs., Part 2, Vol.99, Oct., 1952.

2. Lowe, H.J. and Christie, D.G. "Heat transfer and pressure drop data on cooling tower packings, and model studies of the resistance of natural-draught towers to airflow". International Developments in Heat Transfer, A.S.M.E., part 5, 1961, pp.933-950.

3. Rish, R.F. "The design of a natural-draught cooling tower", International Developments in Heat Transfer, A.S.M.E., part 5, 1961, pp. 951-958.

4. Spalding, D.B. and Singham, J.R. Private Communication. Heat Transfer Dept., Imperial College of Science and Technology, London, Jan., 1963.

G DESCRIPTION OF NUMERICAL OPTIMIZATION TECHNIQUE.

G.1 Introduction.

The analytical method of minimizing a function of several variables, e.g.

$$f = f(x_1, x_2 \ldots\ldots x_n), \qquad (G.1\text{-}1)$$

is to solve for each of the variables from the set of simultaneous equations given by

$$(\partial f/\partial x_i) = 0, \quad i = 1, 2, \ldots\ldots n. \qquad (G.1\text{-}2)$$

If the (G.1-2) equations are all linear, direct solution by matrix methods is possible. In the present investigation, where f is represented by the power-loss equation, the (G.1-2) equations are not linear and it is impossible to solve directly for the design variables.

However, approximate solutions are obtainable by simplifying (G.1-1), the power-loss equation, so that the resulting (G.1-2) equations can be solved by iterative methods. The accuracy of these approximate solutions must be tested before they can be accepted for general design work.

Theoretical error analysis is impractical because of the complicated nature of the power-loss equation. The remaining course of action is to compare the approximate solutions with accurate solutions obtained after minimizing the unsimplified power-loss equation by a numerical optimization technique carried out on an automatic digital computer.

G.2 Adopted numerical optimization technique.

The technique adopted for the present investigation is basically that of D.J. McConalogue, Lecturer in the Nuclear Power Dept. of the Imperial College of Science and Technology. This technique, in the form of a computer master-program specially adqpted for use on the Mercury computer of the University of London, was developed according to the principles outlined in ref. 1.

The master-program requires a slave-program in which the power-loss, f, can be calculated provided that values of the design-variables, x_1, x_2 ....., x_n, are made available to it.

The first function of the master-program is to make the slave-program calculate an f value with initial x_i values, specified by the designer. Then the master-program will create combinations of x_i values according to the procedural requirements of the optimization technique, each combination being used to calculate an f value, until that combination has been obtained which results in a minimum value of f.

G.3 Modifications to basic master-program.

There are no facilities for placing upper and lower limits on the design-variables in the basic master-program. To have such facilities is desirable because, in certain design optimization problems, poor choice of initial design variables may cause the master program to obtain a minimum

value for f for a mathematically acceptable combination of design-variables which is meaningless in practice. Provision is thus made in the master-program for limiting the field of search by restricting the design-variables to a practically acceptable range. Internal modifications are also incorporated to determine accurately the minimum value of f if it lies at one or more of the imposed limits, whether the derivative of f is zero or not. The latter case might arise, say, if one were required to find the minimum of a single-variable, linear function between two limits.

In the basic master-program the step-lengths, by which each of the design-variables are changed when determining new design-variable combinations, are functions of the initial design-variable values chosen by the designer. It was decided, instead, to specify the step-lengths as functions of the differences between the chosen upper and lower limits of the design-variables. This provides the useful facility of being able to fix one or more of the design-variables merely by making the upper and lower limits the same; the step-length is then zero.

The basic master-program compares a new f value with that for the previously determined design-variable combination; the minimum f value is considered to be found when the difference between the two f values is smaller than a predetermined amount. This sensing method is unsuitable for accurately evaluating the values of design-variables giving minimum f if f is relatively insensitive to changes

in the design-variables. As this situation pertains in the present investigation, the sensing method was made dependent on the values of the design variables instead.

Finally, in order to determine the sensitivity of f with respect to each design-variable, a finite difference approximation to the first derivatives at the minimum f value is included.

G.4 References.

1. McConalogue, D.J. and Strickland, R.E. "Some Experimental Mathematical Methods for the Solution of Some Analytically Intractable Optimization Problems in Engineering", Instn. of Mech. Engrs., Symp. on the Use of Computers in Mech. Eng., paper 4, Dec., 1962.

H PRICE FORMULAE FOR STEAM SURFACE CONDENSERS.

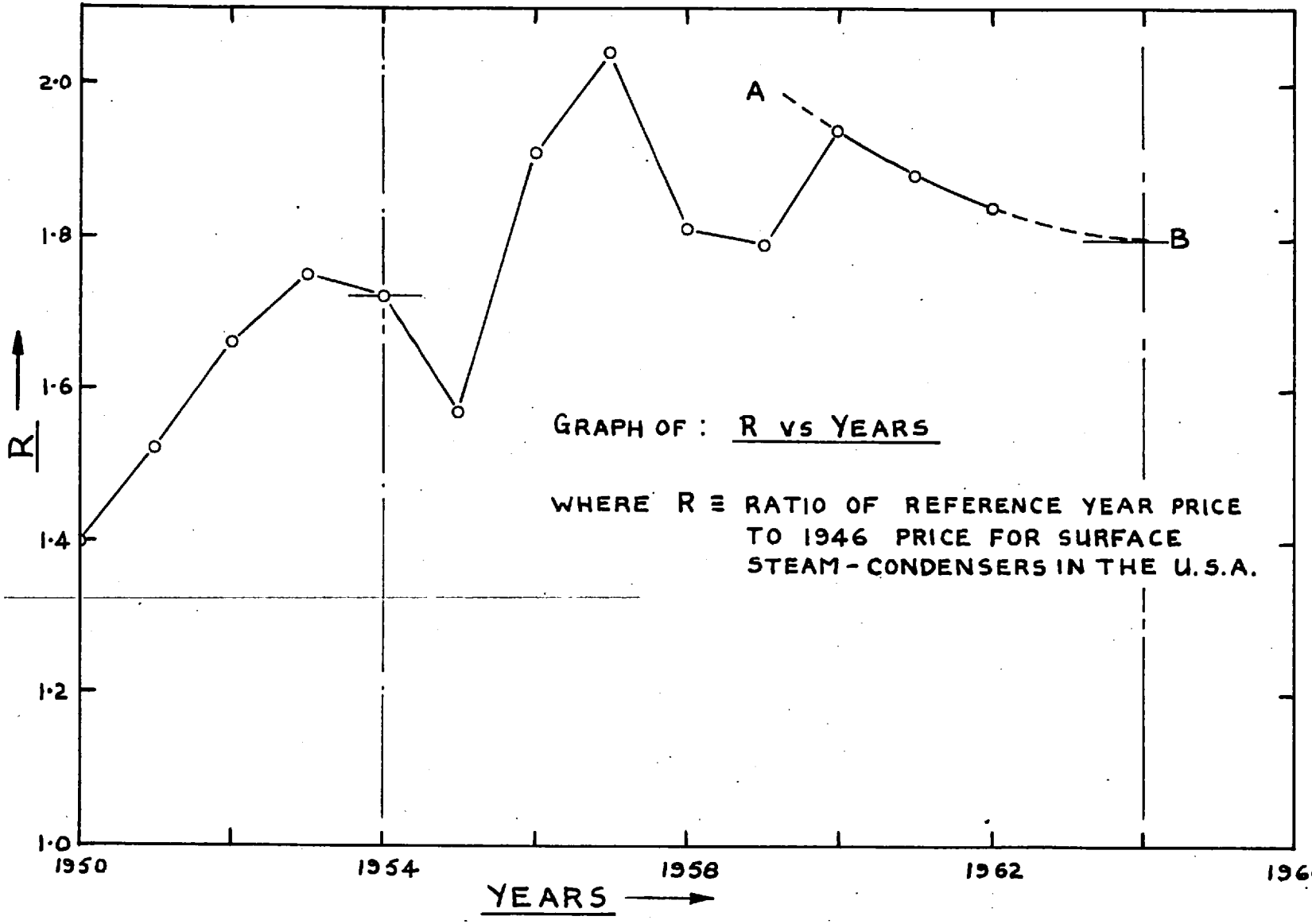
H.1 The base-price concept.

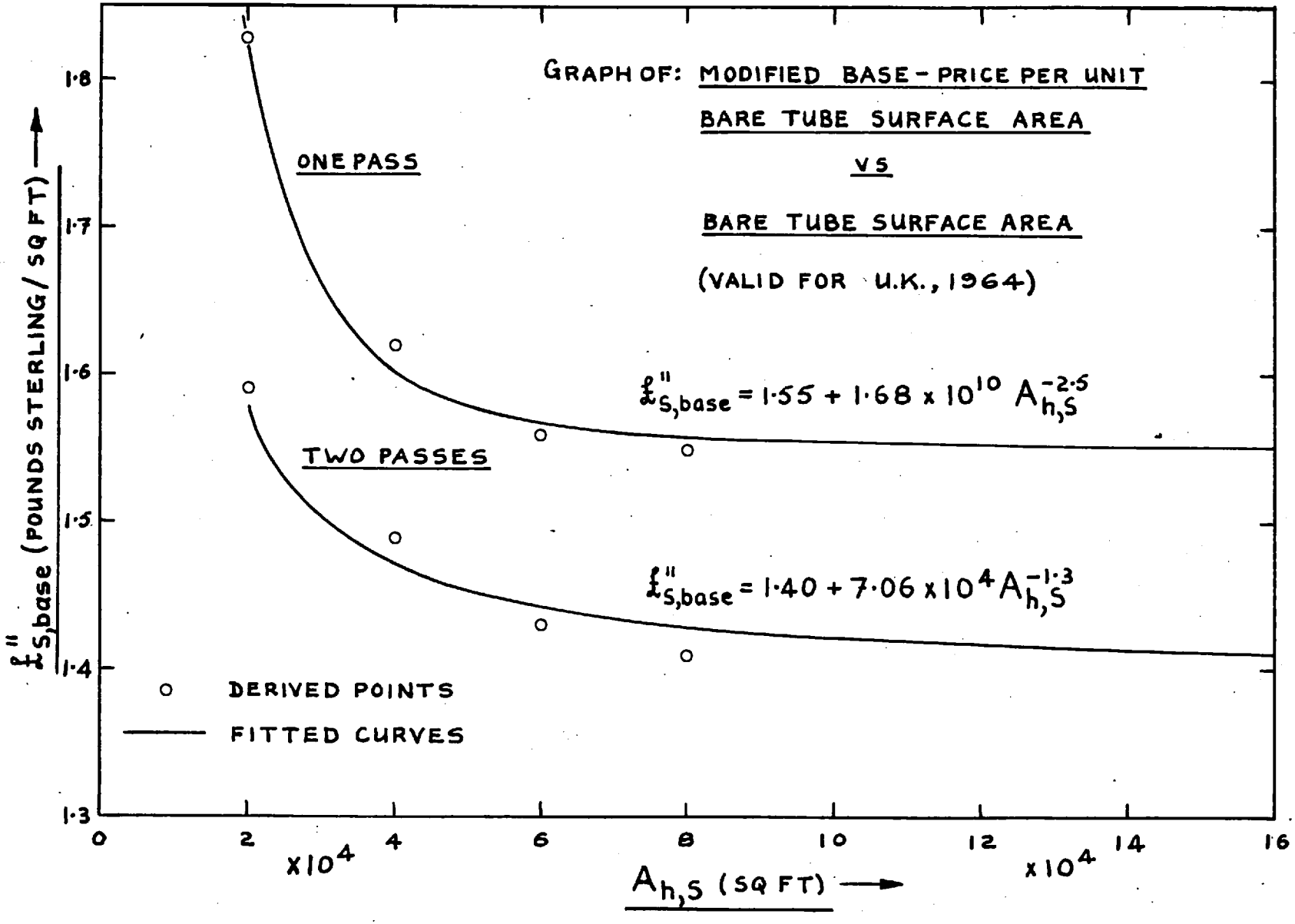
The adopted method of determining condenser prices is basically that described on page 116 of ref. 1. A feature of the method is the specification of a base-price, exclusive of tube price, expressed as a function of bare tube surface area. Included in this base-price, in addition to the condenser itself, are air-removal equipment, circulating pumps and motors, condensate pumps and motors and spring supports with jacks or an expansion joint.

Since the present investigation is in the nature of a comparison, it would be convenient if the costs of all items common to the condensers under consideration could be excluded. Consequently, with the information presented in refs. 1 and 2, it is possible to derive a modified base-price, exclusive of tube price, which includes only the price of the condenser itself, air-removal equipment and spring supports with jacks or an expansion joint.

As the modified base-price so derived applies to the U.S.A. in 1954, it must be converted so as to apply to the United Kingdom in 1964. Ref. 5 contains a graph, reproduced overleaf, showing the U.S.A. variation in steam surface condenser prices over the years. The reference also indicates that the 1964 U.S.A. price, obtained from extrapolated curve A-B, may be taken as equal to the 1964 U.K. price with little error.

With the use of the graph overleaf and the information


2·0
1·8
1·6
1·4
1·2
1·0
R
A
B
GRAPH OF : R vs YEARS
WHERE R ≡ RATIO OF REFERENCE YEAR PRICE TO 1946 PRICE FOR SURFACE STEAM-CONDENSERS IN THE U.S.A.
1950
1954
1958
1962
1966
YEARS


GRAPH OF: MODIFIED BASE - PRICE PER UNIT
BARE TUBE SURFACE AREA
VS
BARE TUBE SURFACE AREA
(VALID FOR U.K., 1964)
ONE PASS
TWO PASSES
$f''_{s,base} = 1.55 + 1.68 \times 10^{10} A_{h,s}^{-2.5}$
$f''_{s,base} = 1.40 + 7.06 \times 10^{4} A_{h,s}^{-1.3}$
DERIVED POINTS
FITTED CURVES
$f''_{s,base}$ (POUNDS STERLING / SQ FT)
1·8
1·7
1·6
1·5
1·4
1·3
0
2
4
6
8
10
12
14
16
x10^4
x10^4
$A_{h,s}$ (SQ FT)

in refs. 1 and 2, a second graph showing the variation in condenser modified base-price per unit bare tube surface area, with bare tube surface area, is presented. The derived points on the second graph may be fitted by the equations

$$£''_{S,base} = 1{\cdot}55 + 1{\cdot}68 \times 10^{10} q^{-2{\cdot}5} Q_S^{2{\cdot}5}, \qquad \text{(H.1-1)}$$

$$£''_{S,base} = 1{\cdot}4 + 706 \times 10^{4} q^{-1{\cdot}3} Q_S^{1{\cdot}3}, \qquad \text{(H.1-2)}$$

where $Q_S \equiv q/A_{h,S}$.

These equations refer to one and two passes respectively and are applicable to the U.K. in 1964.

The total condenser price per unit bare tube surface area is obtained from the expression

$$£''_S = £''_{S,base} + £''_{S,tubes}, \qquad \text{(H.1-3)}$$

where $£''_{S,tubes}$ is the price per unit bare tube surface area of condenser tubing.

H.2 Price of tubing.

The following prices are valid for quantities of tubing found in large surface condensers, and are given in pounds sterling per unit bare tube surface area in square feet, the values being valid for the U.K. in 1964.

For Admiralty-Brass

0·75" o.d., 16 S.W.G., $£''_{S,tubes} = 0{\cdot}535$, ref. 4. (H.2-1)

For Mild-Steel (En 3A)

0·5" o.d., 16 S.W.G., $£''_{S,tubes} = 0{\cdot}334$, ref. 3, (H.2-2)

0·75" o.d., 16 S.W.G., $£''_{S,tubes} = 0{\cdot}308$, ref. 3, (H.2-3)

For Stainless-Steel (304)

0·5" o.d., 16 S.W.G., $£''_{S,tubes} = 0·578$, ref. 3, (H.2-4)

0·75" o.d., 16 S.W.G., $£''_{S,tubes} = 0·870$, ref. 3. (H.2-5)

For Aluminium (99.5%)

0·50" o.d., 16 S.W.G., $£''_{S,tubes} = 0·218$, ref. 3, (H.2-6)

0·75" o.d., 16 S.W.G., $£''_{S,tubes} = 0·198$, ref. 3. (H.2-7)

H.3 References.

1. Anon. "To Figure Condenser Costs", Chem. Engng., March, 1955.

2. Ellingen, W.E. "Selecting Steam Surface-Condensers", Allis-Chalmers Elec. Rev., First Quarter 1955.

3. Hart. "Private Communication", Babcock and Wilcox Ltd., London, 1963.

4. Whessoe Ltd. "Heat Exchanger Design", Tube Cost Specification Sheet, 1963.

5. Anon. "Material and Equipment Cost Data", Book 11, Cost Control, Caltex Service Co., London, 1963.

I PRICE FORMULAE FOR PIPING

I.1 Derivation of equations.

According to ref. 1, Mild Steel Welded Pipes are suitable for the duct-work of the cooling-systems under consideration. It is recommended that the pipes be lined internally and externally with Stewarts and Lloyds standard bitumen lining, except for the conveyance of Freons 12 and 22. Instead, an internal lining of stoved epoxy/phenolic, to a minimum thickness of 0.005 inches, should be used.

A single equation can be fitted to the tabled price information given in ref. 1, so that the price per foot length of pipe is given as

$$\pounds'_{pipe} = (1{\cdot}1 - 1{\cdot}39\times10^{-5}p)d^2 - (6{\cdot}7 - 1{\cdot}875\times10^{-4}p)d + (11{\cdot}6 - 2{\cdot}92\times10^{-4}p), \quad \text{(I.1-1)}$$

valid for: $1{\cdot}5 \leq d \leq 6{\cdot}0$,

$$2{\cdot}16\times10^{4} \leq p \leq 4{\cdot}32\times10^{4},$$

where p is the maximum operating pressure, in p.s.f., and d is the internal pipe diameter in feet.

For operating pressures less than p = 21,600, (I.1-1) remains valid provided that p = 21,600 is used. This convention must be observed as Stewarts and Lloyds regard p = 21,600 as their lowest maximum operating pressure for design purposes.

For the Wet- and Heller-systems, maximum operating pressures are about 7,200 p.s.f. and 21,600 p.s.f. respectively. For these pressures, (I.1-1) reduces to the single equation

$$\pounds'_{pipe} = 0{\cdot}8d^2 - 2{\cdot}65d + 5{\cdot}3, \qquad \text{(I.1-2)}$$

valid for: $1{\cdot}5 \leq d \leq 6{\cdot}0$.

For the Spalding-system, where maximum operating pressure is about 28,800 p.s.f., (I.1-1) reduces to

$$\pounds'_{pipe} = 0{\cdot}7d^2 - 1{\cdot}3d + 3{\cdot}2, \qquad \text{(I.1-3)}$$

valid for: $1{\cdot}5 \leq d \leq 6{\cdot}0$.

I.2 References.

1. Stewarts and Lloyds Quotations, ref. numbers M.02838/62/JMe/AWD and M.02427/63/JMe/BTD.

J PRICE FORMULAE FOR COOLING TOWERS AND ASSOCIATED EQUIPMENT.

J.1 Accurate evaluation of tower-shell outside surface area.

A hyperbolic profile is usually chosen for large natural-draught cooling towers. However, small departures from this shape may be considered in order to save on tower-shell material. Investigations show that two profiles, viz. circular-arc and catenary, are practically identical when applied to cooling towers and their use results in cooling-towers slightly more slender near the base than the equivalent hyperbolic profile, thus constituting a saving in shell material cost.

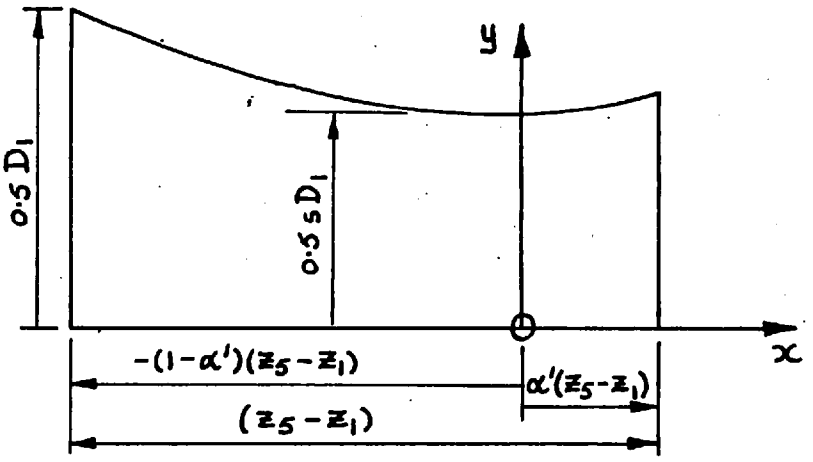
Since the use of the catenary profile results in equations which are mathematically more tractable than the circular arc profile, it is adopted for the present investigation. Thus, with reference to the accompanying sketch, the relevant equation for the profile is assumed to be

$$y = a + b(z_5 - z_1)\cosh\{x/b(z_5 - z_1)\}, \qquad \text{(J.1-1)}$$

where a and b are coefficients to be derived from the following boundary conditions:

$x = -(1-\alpha')(z_5 - z_1)$, $y = 0{\cdot}5\,D_1$,

$x = 0$, $y = 0{\cdot}5\,s\,D_1$.



After substituting these equations into (J.1-1) and solving for b, the following implicit relationship ensues:

$$b = [0{\cdot}5\, D_1(1-s)/(z_5-z_1)]/[\cosh\{-(1-\alpha')/b\}-1]. \qquad \text{(J.1-2)}$$

By expanding the cosh term and truncating the resultant series, (J.1-2) can be represented as

$$b = (1-\alpha')^2(z_5-z_1)/\{(1-s)\,D_1\}, \qquad \text{(J.1-3)}$$

with negligible loss of accuracy for cooling tower applications. The substitution of this equation into (J.1-1), together with the boundary conditions, results in

$$a = 0{\cdot}5\, s D_1 - (1-\alpha')^2(z_5-z_1)^2/\{(1-s)D_1\}. \qquad \text{(J.1-4)}$$

The outside surface area of the tower-shell is given by

$$A = \int_{-(1-\alpha')(z_5-z_1)}^{\alpha'(z_5-z_1)} 2\pi y\, dS, \qquad \text{(J.1-5)}$$

where $dS = dx\{(dy/dx)^2+1\}^{0.5}$.

On integrating, (J.1-5) becomes

$$A = \pi a_0 D_1^2 \left[(5-2a_0)\{\sinh(\alpha'/a_1) - \sinh a_1^{-1}(\alpha'-1)\}\right.$$
$$\left. + 0{\cdot}5\, a_0\{\sinh(2\alpha'/a_1) - \sinh a_1^{-1}(2\alpha'-2)\} + (r_5 - r_1)\right], \qquad (J.1\text{-}6)$$

where $a_0 \equiv (1-\alpha')^2 (r_5 - r_1)^2/(1-s)$, (J.1-7)

$a_1 \equiv (1-\alpha')^2 (r_5 - r_1)/(1-s)$, (J.1-8)

$(z_5 - z_1) \equiv D_1 (r_5 - r_1)$. (J.1-9)

For the present investigation, α' is assigned the value 0.25 since this will ensure a satisfactory tower shape.

The complicated form of (J.1-6) makes a simpler, approximate equation desirable for analytical optimization studies.

J.2 Approximate formulae for tower-shell outside surface area.

J.2.1 Truncated series formula.

For values of $(r_5 - r_1)$ and s normally encountered in cooling system design, the sinh terms in (J.1-6) can be replaced by the following truncated series expansion, with an error not exceeding one per cent:

$$\sinh x \approx x + x^3/3! .$$

In consequence, (J.1-6) can be re-written as

$$A \approx [\{A_A(s-1)^2 + B_A\}s + C_A](r_5 - r_1) D_1^2 , \qquad (J.2\text{-}1)$$

where

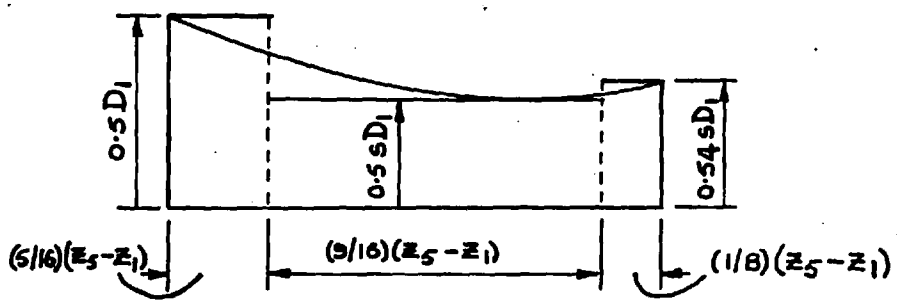
$$A_A \equiv 0{\cdot}167\,\pi(1-\alpha')^{-4}(r_5 - r_1)^{-2},$$

$$B_A \equiv \pi(r_5 - r_1)^{-2} - 0{\cdot}333\,\pi\beta_A(1-\alpha')^{-2},$$

$$C_A \equiv 0{\cdot}333\,\pi\beta_A(1-\alpha')^{-2},$$
$$\beta_A \equiv \alpha'^3 - (\alpha'-1)^3.$$

J.2.2 Hollow cylinder formula.

A formula which is even simpler, though less accurate than (J.2-1), can be derived if the tower-shell outside area is assumed to be equal to the surface area of three hollow cylinders, as shown in the accompanying sketch.



Having made this assumption, originally suggested by Prof. B.G. Neal of Imperial College, London, the axial lengths of each hollow cylinder must be chosen so that the shell outside surface area is approximately equal to that of the three hollow cylinders for practical cooling tower configurations.

For the axial lengths indicated in the sketch, the tower-shell outside surface area is given by

$$A = (2{\cdot}19\,s + 0{\cdot}982)(r_5 - r_1)\,D_1^2 \qquad \text{(J.2-2)}$$

and is accurate when $(r_5 - r_1) = 1.0$ and $s = 0.6$. (J.2-2) becomes increasingly unreliable as values depart from those given above. In particular, (J.2-2) should be used with

caution if the derivative of A with respect to cooling system design variables is required. (J.2-1), being a good approximation to the exact area equation, is more suitable when differentiation is required.

J.3 Wet-system price formulae.

In the Wet-system, the packing cannot be considered independently of the tower since the functions of these items are inter-related. Therefore the packing price is included in the price of cooling tower and associated equipment.

The price of each item of equipment may be regarded as proportional to one of the following quantities:

i) a constant,

ii) tower-shell base-area, D_1^2,

iii) tower-base free-flow area, tD_1^2,

iv) radial width of packing area, $D_1\{1 - (1 - t)^{0.5}\}$,

v) tower-shell base diameter, D_1,

vi) reciprocal of tower-shell base area, D_1^{-2},

vii) tower-shell outside surface area, A,

viii) tower-shell weight multiplied by tower base height, $A\{z_p + 0.5\,(z_2 - z_p)\}$,

ix) tower base free-flow area multiplied by height to top of eliminator, $tD_1^2\{z_p + (z_2 - z_p) + 14.9\}$, where 14.9 is a value accounting for pond depth and a 6.5 feet walking space under the eliminator.

The following items of equipment are considered to fall

into the foregoing categories as follows:

i) earth-moving equipment, site-workshops and miscellaneous items,

ii) pond floor,

iii) distribution piping, stack beams, eliminator beams, sheeting and spacers, eliminator sheeting and spacers,

iv) walk-ways,

v) channel, channel structure, de-icing equipment,

vi) pond wall,

vii) tower shell, piling, footings,

viii) diagonal shell-supporting columns,

ix) stack columns.

After introducing constants of proportionality, the total price of cooling tower and associated equipment, per unit rate of heat rejection, can be expressed as

$$\begin{aligned} \mathfrak{L}_D/q = q^{-1}\big[& d_0 + d_1 D_1^2 + d_2 t D_1^2 + d_3 D_1\{1-(1-t)^{0.5}\} \\ & + d_4 D_1 + d_5 D_1^{-2} + d_6 A + d_7 A\{r_p D_1 + 0.5(z_2 - z_p)\} \\ & + d_8 t D_1^2\{r_p D_1 + (z_2 - z_p) + 14.9\}\big], \end{aligned} \qquad (J.3\text{-}1)$$

after incorporating

$$r_p \equiv z_p / D_1. \qquad (J.3\text{-}2)$$

Numerical values may be assigned to the constants of proportionality from a knowledge of the itemized prices of a typical Wet-system cooling tower. Precise values of these constants are not used in the present investigation in

order to protect commercial interests. Instead, the following approximate quantities are used:

$d_o = 36{,}600$ $\quad$ $d_5 = 3.93 \times 10^8$

$d_1 = 0.135$ $\quad$ $d_6 = 0.582$

$d_2 = 1.59$ $\quad$ $d_7 = 0.00111$

$d_3 = 6.64$ $\quad$ $d_8 = 0.00322.$

$d_4 = 27.1$

These numerical values, while only approximate, are nevertheless accurate enough to retain realism and will not detract appreciably from the validity of cooling system comparison studies.

J.4 Heller- and Spalding-system price formulae.

In the present investigation, the prices of cooling towers and associated equipment for closed cooling systems are attributed solely to the following items:

i) earth-moving equipment, site workshops and other incidental miscellaneous items,

ii) tower-shell, pilings and footings and

iii) diagonal shell-supporting columns.

The prices of these items are considered to be proportional to:

i) a constant,

ii) tower-shell outside surface area, A, and

iii) tower-shell weight multiplied by tower-base height, Az_1 ($= Ar_1D_1$), respectively.

On the introduction of constants of proportionality, the total price of cooling tower and associated equipment, per unit rate of heat rejection, is given by

$$\pounds_D/q = q^{-1}(d_o + d_1 A + d_2 A r_1 D_1). \qquad \text{(J.4-1)}$$

As for the Wet-system, approximate values for the constants of proportionality are given in order to protect commercial interests. Therefore,

d_o = 36,600,

d_1 = 0.582, and

d_2 = 0.00111,

while approximate, are sufficiently accurate for cooling system comparison studies.

K PRICE FORMULAE FOR TOWER HEAT EXCHANGERS.

K.1 Introduction.

In order to achieve realistic price formulae for air-cooled heat exchangers it is not permissible to consider the price as being dependent solely on the amount and type of heat-transfer surface required. Account must be taken also of the configuration of the units comprising a heat exchanger, in so far as width, length, number of tube rows, headers and supporting framework are concerned.

For the present investigation, the length and width of units will be fixed at 16 feet and 9 feet respectively as, according to page 164 of ref.2, these dimensions represent a good compromise between convenience of manufacture, ease of transport and adaptability for cooling-tower purposes.

In consequence, the total tower heat-exchanger price, $£''_G$, expressed in pounds sterling per square foot of air-side heat-transfer surface, is assumed to be a function of:

i) price of finned tubing, $£''_{G,tube}$, expressed in pounds sterling per square foot of air-side heat-transfer surface,

ii) price of headers and framework, $£''_{G,base}$, expressed in similar units, which must be added to $£''_{G,tube}$ after being multiplied by

iii) the row factor, K, dimensionless, to account for variations in number of rows, so that

$$£''_G = £''_{G,tube} + K\,£''_{G,base}. \qquad (K.1\text{-}1)$$

Having formulated (K.1-1), the next step is to specify formulae for the three terms contained therein, valid for heat-exchanger matrix types applicable to cooling towers.

Investigations indicate that the matrix types to be considered fall into two categories, viz.

i) circular tubes with helically wound fins (or wire) on the outside and

ii) circular tubes with continuous plate fins.

K.2 Evaluation of total tower heat-exchanger price for helically wound fins (or wires).

Manufacturers of finned tubing usually supply prices as a function of foot run of tubing so that

$$£''_{G,tube} = £'_{G,tube} / A'_{h,G} . \qquad (K.2\text{-}1)$$

$£'_{G,tube}$ is the price per foot run, in pounds sterling per foot and

$A'_{h,G}$ is the air-side heat-transfer surface area per foot run, in (sq. ft/ft).

$£''_{G,base}$ may be obtained from the manufacturers of air-cooled heat exchangers. In order to protect commercial interests an approximate, through realistic, formula for this quantity, viz.

$$£''_{G,base} = 0.1094 / A'_{h,G} , \qquad (K.2\text{-}3)$$

valid for six rows of tubes, is chosen on the advice of T.F.Thomas, ref. 5.

In determining a suitable expression for the row-factor, K, the selling price curves appearing in Fig.4 of

ref.1 are utilized. The variations in total heat-exchanger prices with number of rows, for very large installations, are multiplied by 0.488 in order to convert them to present-day prices in accordance with the procedure specified on page 65 of ref.4.

From a knowledge of the geometrical characteristics of the tubing used in the compilation of the curves in Fig.4 of ref.1 and the present-day tube price, given as

$$£'_{G,tube} = 0.195 \qquad \text{(K.2-4)}$$

in ref.3, the following table can be compiled after employing (K.1-1).

$£''_G - £''_{G,tube} (= K£''_{G,base})$	0.0791	0.0593	0.0487	0.0424	0.0353	0.0294
n_r (no. of tube rows)	3	4	5	6	8	12

Now K = 1 at $n_r = 6$, since $£''_{G,base}$ is specified at $n_r = 6$ in (K.3-3). Therefore, from table at $n_r = 6$, $£''_{G,base} = 0.0424$. K can then be found at each n_r value from the expression

$$K = (K£''_{G,base})/(0.0424), \qquad \text{(K.2-5)}$$

which can be used to form the table

n_r	3	4	5	6	8	12
K	1.866	1.399	1.149	1.000	0.833	0.693

By employing curve-fitting procedures, the tabled relationship between K and n_r can be accurately represented by the equation

$$K = 6.983\, n_r^{-1.5} + 0.525. \qquad \text{(K.2-6)}$$

After collecting terms, (K.1-1) becomes

$$£''_G = (£'_{G,tube}/A'_{h,G}) + K\,(0.1094/A'_{h,G}). \qquad (K.2\text{-}7)$$

K.3 Evaluation of total tower heat-exchanger price for circular tubes with continuous plate fins.

For this type of heat exchanger the price of finned tubing, $£''_{G,tube}$, has no meaning since the heat-exchanger is constructed "en bloc" from bare tubes and many large parallel plates, punched to receive the tubes. In consequence a more realistic price formula results by placing $£''_{G,tube} = 0$ and re-expressing (K.1-1) as

$$£''_G = K' £''_{G,stand}\,, \qquad (K.3\text{-}1)$$

where K' is a modified row-factor and $£''_{G,stand}$ is the total price of a standardized heat-exchanger unit per unit air-side heat-transfer surface area.

As in the previous sub-section, Fig.4 of ref.1 is used in the determination of K'. By noting that $£''_{G,tube} = 0.0745$ for the associated tubing, the table preceding (K.2-5) becomes

$£''_G\ (= K' £''_{G,stand})$	0.1536	0.1338	0.1232	0.1169	0.1098	0.1039
n_r	3	4	5	6	8	12

If the standardized heat-exchanger unit is defined as having 6 rows of tubes, then K' = 1 at $n_r = 6$ and, from table, $£''_{G,stand} = 0.1169$. K' can then be found at each n_r value from the expression

$$K' = (K' £''_{G,stand})/(0.1169), \qquad (K.3\text{-}2)$$

which can be used to form the table

n_r	3	4	5	6	8	12
K'	1·314	1·145	1·054	1·000	0·939	0·889

The table can be represented accurately by

$$K' = 2{\cdot}53\, n_r^{-1{\cdot}5} + 0{\cdot}827\,. \qquad \text{(K.3-3)}$$

If the standardized heat-exchanger unit is defined as having n_r rows where $n_r \neq 6$, (K.3-3) is no longer valid and must be changed to

$$K' = \{(K')_6/(K')_{n_r}\}(2{\cdot}53\, n_r^{-1{\cdot}5} + 0{\cdot}827), \qquad \text{(K.3-4)}$$

where $(K')_6$ and $(K')_{n_r}$ are evaluated from (K.3-3) for 6 and n_r rows respectively.

After collecting terms, (K3-1) becomes

$$\pounds''_G = \pounds''_{G,stand}\{(K')_6/(K')_{n_r}\}(2{\cdot}53\, n_r^{-1{\cdot}5} + 0{\cdot}827), \qquad \text{(K.3-5)}$$

where $\pounds''_{G,stand}$ must be obtained from the manufacturers for the heat exchanger concerned and n_r is the associated number of tube rows.

K.4 References.

1. Anderson, R.J. and Smith, E.C. "Cooling with Ambient Air", The Petroleum Times, London, 29th Aug.1958.

2. Cheshire L.J. and Daltry, J.H. "A Closed Circuit Cooling System for Steam Generating Plant", The South African Mech.Engr., Feb.1960.

3. Clayton Dewandre Co., Ltd., London, W.1. Price Quotation, 1964.

4. Krzymuski, J.I. "Economics of Air Cooling in a Refinery", Jrnl. Inst. Fuel, Vol. 36, No.265, Feb. 1963.

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L EQUATION FOR OPTIMUM NUMBER OF TUBE ROWS

L.1 Theory

In the Heller- and Spalding-systems, the number of tube rows, n_r, is fixed prior to carrying out optimization procedures. Consider the case where the fixed value of n_r is equal to its optimum value which would have resulted after optimization had n_r been allowed to vary. For such a contingency it is true to say that there would be no difference between the value of n_r obtained from the equation

$$\partial(P_L/q)/\partial n_r = 0, \qquad (L.1\text{-}1)$$

after substituting into it various associated quantities which prevail after optimization, and the original fixed value. Furthermore, the greater the difference between the fixed value of n_r and its true optimum value, the greater will be the difference between the fixed value and that which accrues from the use of (L.1-1).

The above theoretical discussion is introduced because the size of the foregoing difference is used in the present investigation as an indication of whether or not the fixed n_r values used in various calculations, and of necessity fixed at integer values as explained in 4.3.1, are close to their actual optimum values. The contention is that the differences between the fixed n_r values and those evaluated from (L.1-1) should be small if the potential performance of a considered heat-exchanger type is to be

regarded as having been fully exploited.

An approximate expression for (L.1-1) is derived and re-arranged in the next section so that n_r can be directly evaluated.

L.2 Derivation of equation

In differentiating the merit-factor equation (2.1-8) with respect to n_r, only the following proportionality need be considered:

$$P_L/q \propto a^*\{t_{C,i}-(t_{m,G})_o\} + b + c^*\{\pounds_G/q + \pounds_D/q + d\} . \quad (L.2\text{-}1)$$

The temporary symbols b and d are defined as

$$b \equiv 0.5 b^* a_{P,T} C_F^{-1} (1+ \eta_{pt}) \Delta p_{fr,C},$$

$$d \equiv 0$$

for the Heller system and

$$b \equiv b^* a_{P,H} C_F^{-1} \chi_H^{-1} \Delta p_{fr,C},$$

$$d \equiv \pounds_R/q.$$

for the Spalding system.

After substituting the least accurate of the various formulae in chapter 3 into (L.2-1), differentiating with respect to n_r to obtain an approximate version of (L.1-1) and then re-arranging, the following expression for the optimum number of tube rows results:

$$n_r = \left[\{Q_G/(10.5\varepsilon''_{G,base})\}\left[(1+j_i)\{\varepsilon_G/q-(a^*/c^*)\{t_{C,i}-(t_{m,G})_o\}\right.\right.$$

$$\left.\left.+\ (1-j_o)(d_1+d_2r_1D_1)(A/q)(N_{v,G}/\Sigma N_v)\right]\right]^{-1/1.5} \qquad (L.2\text{-}2)$$

In formulating (L.2-2) use is made of the fact that the terms containing b^* and ε_R/q are negligible as can be shown by using the results in chapter 9 to examine relative orders of magnitude.