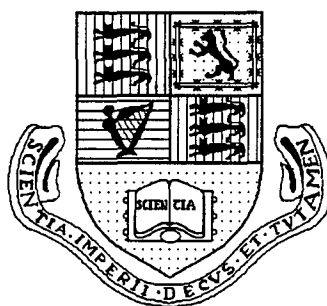

**Thesis submitted for the degree of
Doctor of Philosophy of the University of London
and the Diploma of Imperial College**

The High Speed Double Torsion Test

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Abstract

The high speed double torsion test is a high rate version of the standard double torsion fracture test. The test has been used by previous researchers to induce rapid crack propagation in tough engineering polymers in order to measure a material's dynamic fracture resistance as a function of crack velocity. Experimental results show that at medium to high impact speeds the crack propagates at a fairly constant velocity but at low impact speeds the crack propagates in a stick-slip manner.

In order to calculate the dynamic fracture resistance, a post-mortem analysis of the test is required. The analysis previous to this work consisted of modelling the test using a numerical finite difference scheme. The discretised equations of motion used in the scheme included the effects of axial stresses in the test specimen but ignored axial inertia. The equations of motion used have been modified to include axial inertia and the boundary conditions corrected.

This work examines and models the more transient characteristics of the test, exemplified by the stick-slip crack propagation mentioned above. In order to achieve this goal, the finite difference model has been re-formulated in propagation mode where the dynamic fracture resistance is prescribed and the crack history calculated, as opposed to the previously used generation mode where the crack history is prescribed and the dynamic fracture resistance calculated.

Previous work has examined the effects of the characteristic curved crack front in the double torsion test and the possible non-linear material behaviour of the test specimen. These aspects have been re-examined and previous assumptions found to be invalid. Correcting the errors had a significant influence on the calculated value of dynamic fracture resistance.

Contact stiffness and overhang effects have been analysed and their incorporation into the model has resulted in an accurate prediction of the experimental load history oscillations.

The resulting model was extensively validated and allowed the characteristics of the high speed double torsion test to be examined. A variety of polymers have been tested and analysed. The results show that the stick-slip behaviour is, at least in part, due to the dynamics of the HSDT test rather than the material's dynamic fracture resistance falling with crack velocity.

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Nomenclature

a	Axial crack length
$\dot{a}$	Axial crack velocity
$a\xi$	Distance of the crack front from the bottom surface of specimen
A	Region between load plane and crack front
A	Fracture surface area
B	Specimen thickness
c, c'	New and old foundation stiffness coefficients
C	Phase velocity
C_s	Shear wave speed
C_T	Saint-Venant's torsional wave speed
C_t^0	C_T calculated using μ_0
F	Reaction force at support
G	Crack driving force
G_D	Dynamic fracture resistance
G_P	Peak load toughness
h	Variable proportional to order of resonance
H	Specimen width
J	Section constant defined in equation (3.7c)
K	Stress intensity factor, Section constant defined in equation (3.9b)
K_D	Dynamic fracture toughness
l', m', n'	Direction cosines
L	Specimen length
L	Section constant defined in equation (3.7c)
M	Applied moment
ΔL	Effective shortening of torsion beam
m_e	Calibration factor for calculating striker velocity
m_n	Slope of linear regions modelling torsional impedance results
N	Normalised frequency
p	Circular frequency
p_{sv}	Resonant frequency predicted by the Saint-Venant
P	Overhang region
P	Section constant defined in equation (3.9b)
P_c	Critical pressure
r, θ	Polar co-ordinates

s	Remaining ligament thickness
s_T	Difference between B and 'V' groove depth
F	Reaction force at support point
R	Radius of circular bar
R	Region ahead of crack front
S	Striker force
$t_e _{34}$	Calibration factor for calculating striker velocity
T	Applied torque
u, v, w	Displacements in the x, y, z directions respectively
U_k	Internal kinetic energy
U_s	Internal strain energy
v_{impact}	Impact velocity of striker
W	External work
x, y, z	Cartesian co-ordinates
X, Y, Z	Body forces in the x, y, z directions respectively
$\bar{X}, \bar{Y}, \bar{Z}$	External force in the x, y, z directions respectively
β	Side ratio of rectangular section (≤ 1)
χ	Contact stiffness adjustment factor
δ	Beam separation at the crack front
$\epsilon_x, \epsilon_y, \epsilon_z$	Direct strains in the x, y, z directions
ϕ	Warping function
γ	Section constant
γ_e	Effective strain
γ_{max}	Maximum strain in section
Γ_c	First polar moment of area of section
Γ_m	Strain moment
γ^2	Non-dimensional section constant defined in equation (3.10)
$\gamma_{xy}, \gamma_{yz}, \gamma_{zx}$	Shear strains
$\dot{\eta}$	Crack velocity normal to the crack front
λ	Non-dimensional section constant defined in equation (3.10)
μ	Shear modulus
μ_0	High strain rate, low strain shear modulus
μ_s	Effective section secant shear modulus
μ_t	Effective section tangent shear modulus
ω	Frequency
ϖ	Angle subtended by the normal to the crack front to the z axis
$\dot{\theta}_{LP}$	Rotation rate imposed by the striker

ρ	Density
$\sigma_x, \sigma_y, \sigma_z$	Direct stresses in the x, y, z directions
τ_e	Effective shear stress
τ_o	Effective stress if $\mu = \mu_o$
$\tau_{xy}, \tau_{yz}, \tau_{zx}$	Shear stresses
ν	Poisson's ratio
ζ	Steady state variable, Section constant defined in equation (3.27)
ξ	y co-ordinate of bottom of the 'V' groove
ψ	Axial displacement function
Λ	Wavelength
Ω	Foundation stiffness/unit length

Abbreviations

ASTM	American society for testing materials
COD	Crack opening displacement
DCB	Double Cantilever beam
EPDM	Ethylene-propylene non-conjugated diene
SEN	Single edge notched
DT	Double torsion
FD	Finite difference
FE	Finite element
FV	Finite volume
HDPE	High density polyethylene
HSDT	High speed double torsion
LEFM	Linear elastic fracture mechanics
OCG	Optical crack gauge
MDPE	Medium density polyethylene
PE	Polyethylene
PE80	Grade of medium density polyethylene
PE100	Grade of high density polyethylene
PMMA	Polymethylemethacrylate
POM	Polyoxymethylene
RCP	Rapid crack propagation
S1, S2	Support points in the DT test
S4	Steady state, small scale
TTL	Transistor-transistor logic

Chapter 1

Introduction

1.1 What is rapid crack propagation ?

This work has been motivated by the need to understand and quantify rapid crack propagation in pressurised plastic pipes. Rapid crack propagation (RCP) is a dynamic fracture event where a crack propagates rapidly through a structure. The propagation is characterised by a steady crack velocity and a smooth, uniform fracture surface. Full scale field tests, performed by British Gas [1.1], have demonstrated that propagation of a fast crack, initiated in a gas pressurised plastic pipe, can be sustained indefinitely. Steady crack velocities of up to 350 m/s were observed. Figure 1.1 depicts one of these tests, the picture being dominated by the clouds of backfill blown upwards by the escaping gas.



Fig. 1.1 Rapid crack propagation in a full scale test on a gas pressurised pipe

Underlying this event is an apparently steady crack propagation displaying a constant frequency, sinusoidal crack path (figure 1.2). The fracture surface is typically of a smooth, brittle appearance with some ductility (shear lips) at the free surfaces, particularly at the bore of the pipe. Arrest marks indicate a curved crack front, generally having a well defined leading edge close to the bore of the pipe and a long trailing edge asymptotic to the outer surface.

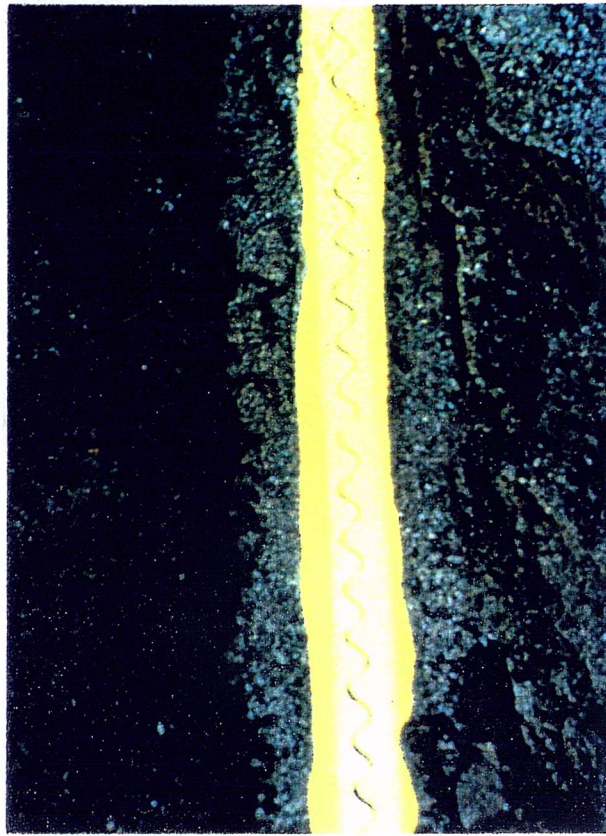


Fig. 1.2 Crack path in the full scale pipe test

The energy required for the fracture is predominantly supplied by the de-pressurising gas, forcing open the pipe wall behind the crack front, which in turn produces tensile, circumferential stresses at the crack front. A relatively small amount of additional energy is provided by the strain energy stored in the pipe wall prior to initiation. In order for the crack to continue propagating the crack front must therefore keep pace with the de-pressurisation wave front. For any particular pipe there is a critical pressure (P_c), above which RCP is sustainable and below which an initiated crack will quickly arrest.

The pipe industry needs to ensure that catastrophic RCP failure never occurs in service. The only certain guarantee is to design a pipe such that an RCP event, once initiated, will quickly arrest. An alternative approach is to design against initiation, but this is impossible for a pipe which is to remain in service for up to fifty years and is susceptible to accidental damage.

In order to investigate RCP failure in pressurised pipes a small scale, steady state (S4) pipe test which is fast and economical, relative to the full scale test, has been developed by Leever and Yayla [1.2]. From their work, and subsequent work by Venizelos [1.3], a much greater understanding has been gained of the RCP failure mode. This

understanding has led to the development, by Ivankovic [1.6], of a predictive finite volume model which is aimed at estimating a prospective material's performance, with regard to RCP in pressurised pipes, for any given temperature, geometry or pressure. The model involves simulation of the complicated dynamic interaction between the pressurised fluid and pipe wall. A main parameter required by the model is the pipe material's dynamic fracture resistance (G_D). The value of G_D is, in theory, obtainable from the model and the S4 test itself, but this requires production of pipe specimens and the use of elaborate testing techniques.

One of the major concerns of this thesis is to establish a reliable technique to determine the lower bound G_D of pipe grade materials, which is relevant to RCP in pressurised pipes. The technique should also provide a rapid, efficient means of testing so that it can be used in quality control by both material and pipe manufacturers.

By exact definition RCP is concerned only with continuous crack propagation, but in all practical situations there is also a start (initiation) and end point to the event. The end point can either be due to arrest or to the crack reaching a free surface. Under certain conditions a series of RCP events may occur in rapid succession denoted as 'stick-slip'.

Since, as was stated above, the only means to guarantee against RCP is to ensure arrest occurs quickly, this work also examines arrest criteria.

1.2 Experimental methods and analytical techniques for RCP

1.2.1 Dynamic Fracture Mechanics

In order to design against RCP one first needs to investigate and define the criteria under which it occurs. To be of a predictive value these criteria must be independent of the test geometry and loading conditions used to evaluate them. This is the role of dynamic fracture mechanics.

Dynamic fracture mechanics must be used for situations in which the inertia of the structure becomes a dominant feature of the fracture, as opposed to static fracture where the crack growth can be described by a series of static equilibrium states. As with static fracture mechanics, the foundation of dynamic fracture mechanics lies in Griffiths' theory. His theory is an energy balance approach which states that crack extension only occurs when the available energy for the crack propagation released by a body during crack extension (crack driving force), is equal to or greater than that energy required to separate

the material to form the new crack surfaces. For a propagating crack, the equality of equation (1.1) must hold if the global energy balance is to be satisfied.

$$G = G_D \quad (1.1)$$

where G is the crack driving force. For the dynamic case the crack driving force is normally expressed as:

$$G = \left(\frac{dW}{dt} - \frac{dU_s}{dt} - \frac{dU_k}{dt} \right) / \left(\frac{dA}{dt} \right) \quad (1.2)$$

where t is time, W is the external work done on the body, U_s and U_k are respectively the internal strain and kinetic energies of the body and A is the fracture surface area. This is generally described as the global energy approach. Continuum mechanics is used in the evaluation of G .

When using the global energy approach it is not necessary to evaluate the total energy of the body but rather just the energy balance of a volume enclosed by some contour around the crack tip.

The other major approach of fracture mechanics is to consider the stress field local to the crack tip. The stresses local to the crack tip are dominated by a singular term such that these stresses are of the same form, whatever the loading conditions. The magnitudes of the stresses are determined by the loading conditions and are characterised in terms of K , the stress intensity factor. For example, the stress intensity factor for mode I (opening mode) is defined as:

$$K_I = \lim_{r \rightarrow 0} \left[(2\pi r)^{\frac{1}{2}} \sigma_{xx} \right]_{\theta=0}$$

where r is the distance from the crack tip, θ is the angle from the crack path and σ_{xx} is the direct stress normal to the crack path. For crack propagation the equality:

$$K = K_D$$

holds, where K_D is the dynamic fracture toughness. The problem then becomes one of evaluating the appropriate value of K for the particular case in question. Although some analytical solutions do exist this is, in general, performed numerically, extrapolating the value of K at $r = 0$ (where, for a linear elastic solution, a stress singularity exists).

demanding, prone to programming errors which are difficult to detect and their results are difficult to analyse in terms of cause and effect. With today's availability of cheap and fast computing power, it is this last point which is the biggest draw-back of numerical methods and the greatest advantage of the first three methods.

A compromise is to use a combination of numerical and analytical solutions to understand any particular dynamic fracture problem. The analytical solution being used both as a precursor to the numerical solution in order to gain an insight into the characteristics of the problem and then, if feasible, as a final analysis once the assumptions made have been verified by a numerical solution. In general it is better to keep a numerical solution as transparent and as uncomplicated as possible in order to allow a better understanding of the problem.

The discussion of analysis techniques so far, has only been concerned with the dynamics of RCP. An equally important factor is the modulus used in the analysis for the test material. The modulus of most polymers is very strain and strain rate dependent. The fact that RCP occurs at high strain rates invalidates the use of a modulus value determined from static test methods. Ivankovic [1.10] matched his FE results to experimentally measured strains close to the crack path by adjusting the value of the modulus in a thin strip of elements containing the crack path. Using this procedure he calculated that the effective modulus in this strip was much lower than either that measured using static tests, or the dynamic modulus measured using low strain, high strain rate ultrasonic methods. With this adjustment he showed that the upswing in G_D at high crack velocities (>200 m/s), calculated using a single dynamic modulus, was eliminated giving a flat G_D - $\dot{a}$ characteristic.

1.2.2 Dynamic fracture test methods in tough polymers

In order to observe and investigate RCP, a reasonable amount of crack growth is required in an easy to perform and analyse test. When investigating tough polymers there is the additional problem of initiating a fast crack. Research [1.11] shows that RCP can only be initiated in tough polymers if the local loading rate and therefore local strain rate at the initiation point is comparable to that occurring during RCP itself. An ideal test would be one which is quick and cheap to perform, initiation and propagation effects are completely divorced, there are no transient effects such as reflection of stress waves from free boundaries, there is a reasonable length of crack propagation and the crack velocity can be accurately controlled and measured. In order to put the High Speed Double Torsion

(HSDT) test, which is used in this work, into the perspective of existing testing techniques the main characteristics of two alternative tests are described below. The tests are very different in both their form and associated analysis. They typify present day test methods.

1) Three point bend test

The test is one of the oldest and most simple tests, and includes the well known instrumented Charpy test. Recent advances have been made by Clutton [1.12] in the analysis and interpretation of this test. He used an instrumented tup to measure load and calculate and measure the energy absorbed up to the peak load in tough polymers. He showed that the point of peak load corresponded closely to the onset of fast fracture relative to the total time of the test. By integrating under the load-displacement curve up to peak load, he calculated the associated crack driving force (G_P). He showed that G_P was independent of thickness and decreased with increasing impact speed. He proved that increasing impact speed was accompanied by a reduction in size of the initial craze zone and postulated that the lower bound value of G_P (high strain rates) was equivalent to G_D . The test and its analysis are fast and easy to perform but a large number of tests are required to define the lower bound value of G_P . The combination of small crack propagation length and the presence of shear lips in tough polymers prevent any accurate measurement of crack velocity, and transient effects dominate the test at high loading rates.

2) Duplex single edge notch (SEN) (e.g. Genussov [1.11] and Ivankovic [1.10])

This test involves bonding a notched brittle material to the tough polymer to be tested. The sample is then put under slowly increasing tension until a fast crack initiates in the brittle material which, under the correct conditions, propagates across the interface and through the test material. The test shows a constant crack velocity, which can be controlled by the depth of pre-notch, and a fairly straight crack front. The disadvantages of the test are that the specimen preparation is time consuming requiring a good bond between the two materials to allow the transmission of stress waves emanating from the crack front. The analysis of the test is performed using a fully dynamic, two spatial dimension FE code, briefly described in section 1.2.1 in the context of appropriate modulus values in dynamic analysis.

The High Speed Double Torsion (HSDT) test, developed by Leever [1.13] and subsequently Wheel [1.14], is the test method used in this work. The HSDT test is a high

speed form of the standard double torsion (DT) test. The test is an ideal candidate for analysing RCP, particularly with reference to pressurised plastic pipes, because:

- 1) The test is quickly and easily performed apart from the specimen preparation required for the measurement of crack velocity.
- 2) The crack path is long (approximately 160 mm) over which there is a reasonably steady crack velocity.
- 3) The crack front shape is curved, intersecting the lower surface at an acute angle equal to and asymptotic to the upper surface. The fracture surface has a brittle, smooth appearance over much of its area but shows signs of ductility at the trailing edge of the crack front. The general features are therefore similar to that seen in the pipe test.
- 4) The test analysis is amenable to a one spatial dimension FD solution.
- 5) At low loading rates the crack history shows accelerations and decelerations which in the extreme become stick-slip propagation. Although this might be considered a disadvantage the arrest marks are clear and occur under well defined conditions.

The HSDT test can therefore be used to examine the process of arrest, provided that the analysis is capable of dealing with this transient behaviour.

1.3 The HSDT test

1.3.1 The test rig

The test specimen is a plate 100 mm by 200 mm and 9 mm thick. A 2 mm deep axial 'V' groove is machined along the axial centre line on the under side of the specimen to control the crack propagation direction. The specimen rests horizontally on four support points as shown in figure 1.3. A conductive grid is painted onto the underside of the specimen, consisting of twelve equally spaced conductive grid lines across the crack path connected by resistive grid lines to form a wheatstone bridge arrangement. As the propagating crack breaks the grid lines a staircase voltage output is produced from which the crack history, and thus $\dot{a}$, can be calculated. In order to obtain maximum, constant step changes in the output voltage the resistance of each grid line should be a factor of 1.2 larger than the permanent bridge arm resistors.

The specimen has an initial notch of 40 mm machined into it at one end. The crack driving force is provided by impacting a 1.336 kg striker vertically at this end. This sends equal and opposite torsional waves along the two opposing halves of the specimen. As they pass the crack tip the crack driving force begins to increase. At some point in time, a critical value is reached and the crack begins to propagate, 'surfing' on the torsional wave

towards the free end of the specimen. The two halves of the specimen behave as rectangular sectioned torsion beams. Beyond the crack front, each beam is subjected to a restoring force by the other. Since the displacement rate is constant, the crack is driven at a fairly uniform velocity along the specimen.

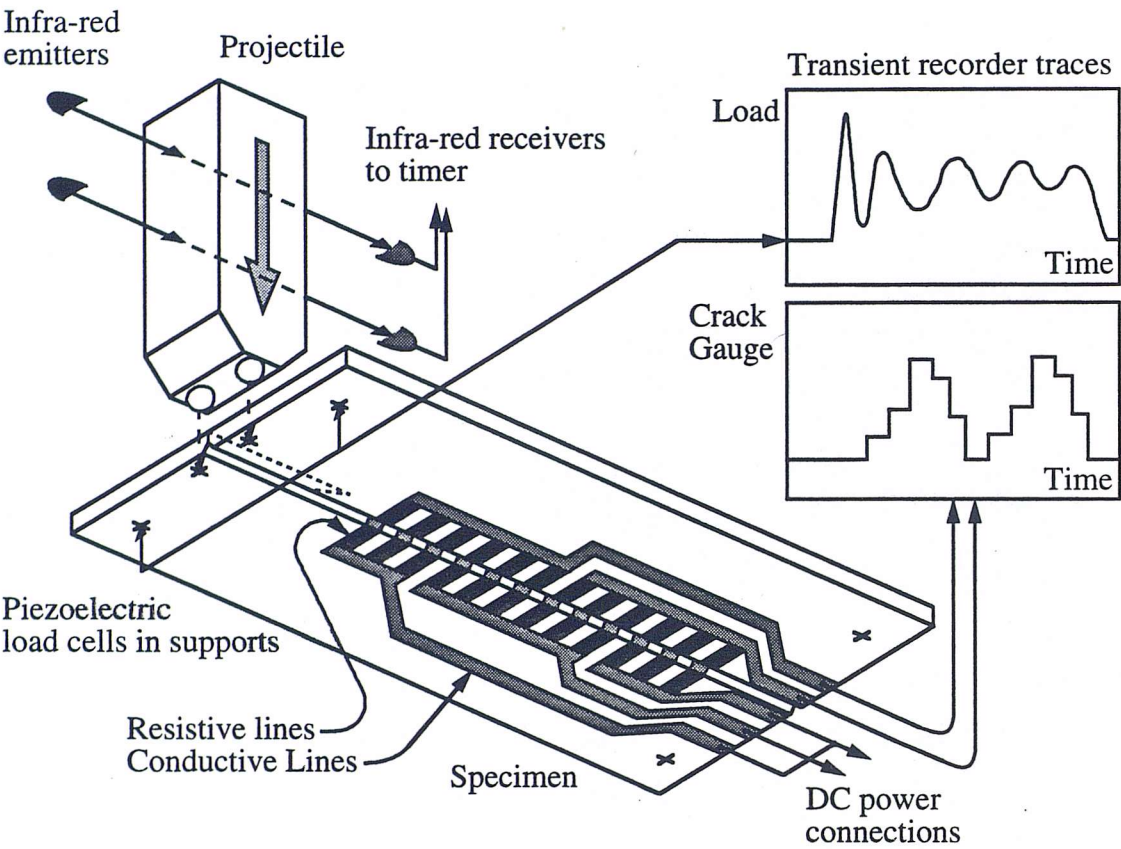


Fig. 1.3 Schematic of the High Speed Double Torsion test

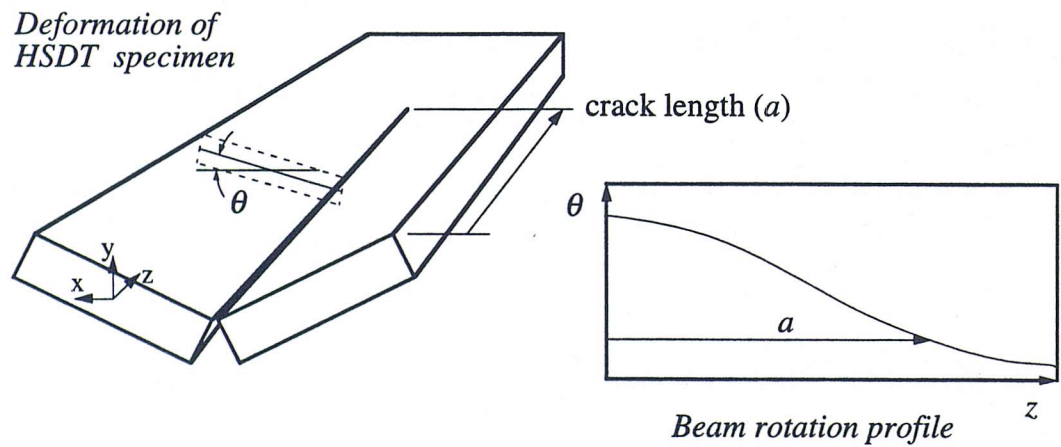


Fig. 1.4 Definition of the beam rotation profile

The deformation is symmetric about the future crack plane and can be described at any point in time by a beam rotation profile: this is a plot of section rotation (θ) against axial distance (z) as shown in figure 1.4. Section rotation is defined here as the rotation of a half-specimen section about the z axis, remote from the crack plane; i.e. the distortion of a section due to the restraint of the opposing half-specimen is ignored.

The striker is accelerated towards the specimen using a gas gun. The pressure can be adjusted (6 bar maximum) to control the impact velocity (5 m/s to 35 m/s). Just prior to impact the striker passes two infra-red sensors, 40 mm apart. The time that the striker takes to pass between the two sensors is recorded using an electronic timer, allowing the calculation of striker velocity. Crack velocity is dependent on the imposed displacement rate and therefore a range of crack velocities can be obtained by varying the gas gun pressure. A picture of the HSDT rig used in this work is shown in figure 1.5.

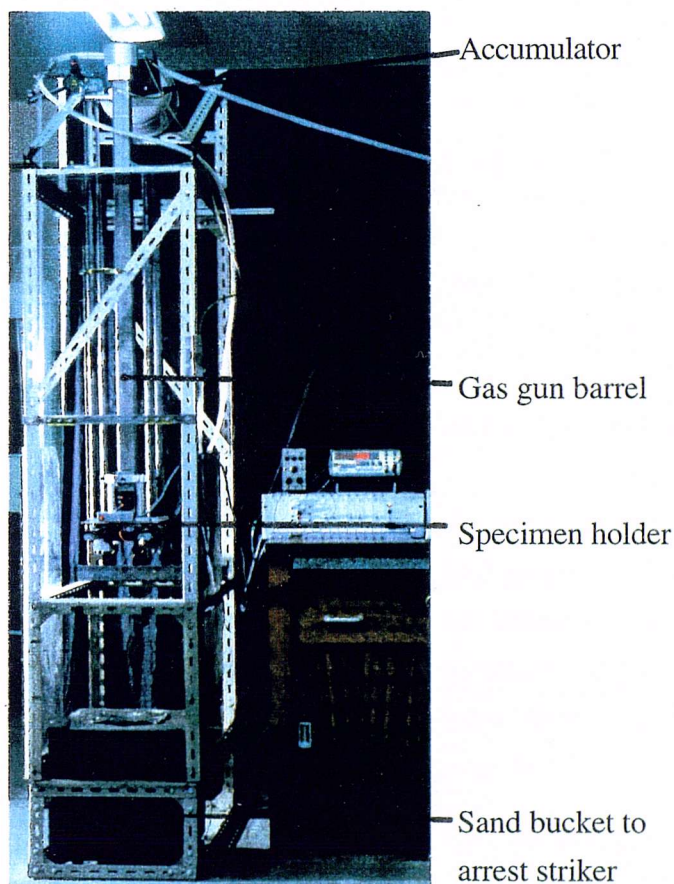


Fig. 1.5 The HSDT test rig

In addition to the crack history and striker velocity the reaction force at the support points, on which the impacted end of the specimen rests, is recorded by means of piezoelectric load cells.

The main advantage of the HSDT test are the long crack propagation distances under reasonably constant loading rate. The HSDT crack front shape is curved (see figure 7.18). Although this complicates the analysis, the curved shape has the advantage of promoting a plane strain mode of failure.

Most of the tests in this work were performed at 0°C, since this is the minimum expected operating temperature of buried pressurised plastic pipes and is deemed to be the worst case conditions. Each material was tested at a range of striker velocities to obtain G_D as a function of crack velocity.

During this work a number of improvements were made to the rig. The most notable of these was a new method of measuring crack velocity, the optical crack gauge (OCG), which reduces specimen preparation time to approximately ten minutes as opposed to over fifty minutes for the timing line technique described above. The improvements are discussed in detail in Chapter 2.

1.3.2 Analysis for the HSDT test

1.3.2.1 Review

To evaluate G_D a post-mortem analysis of the test is required, taking the experimental data as its input. Wheel [1.14] has written a review on the possible analysis techniques, which is briefly summarised here. In all the analyses only one half of the sample is modelled since the deformation is symmetric about the crack path.

One of the first analyses that attempted to take into account dynamic effects was a quasi static formulation using the torsional wave equation of Saint-Venant [1.15]. Saint-Venant was the first to account for the fact that the section of a rectangular beam subjected to torsion undergoes warping. That is, the section is subjected to displacements out of its plane. The quasi static solution for the DT test gave:

$$G_D = A \left(1 - \left(\frac{\dot{a}}{C_T} \right)^2 \right)$$

where A is a constant dependent on the geometry, modulus, loading rate and instantaneous crack length of the specimen and C_T is the torsional wave speed predicted by Saint-Venant. Although this quasi static solution gave very scattered results, it did show that the limiting crack speed corresponds to C_T and that the results for G_D are very

sensitive to crack velocity as the velocity approaches C_T . These predictions were borne out by experiment.

Popelar [1.16] produced one of the earliest dynamic analyses of the DT test, solving Saint-Venant's wave equation by the method of characteristics. The model indicated the important influence that stress waves in the specimen have on the crack driving force.

Leevers [1.13] introduced the concept of foundation stiffness into the analysis of the HSDT test. The previous models had always assumed the half-specimen torsional beams were built in at the crack tip, such that beyond it the section rotation was zero (see figure 1.6).

Leevers [1.17] proposed the use of a foundation stiffness per unit length, which was constant at sections beyond the crack tip. This allowed the development of a far more realistic model of the test, such that the section rotation decays towards zero in front of the crack. His solution therefore predicted that the beams in the region of the crack tip were subjected to non-uniform twist. Non-uniform twist of rectangular beams is accompanied by axial stresses which are not included in Saint-Venant's solution. Leevers incorporated a modified form of Timoshenko's solution for non-uniform twist into his analysis to account for the axial stresses.

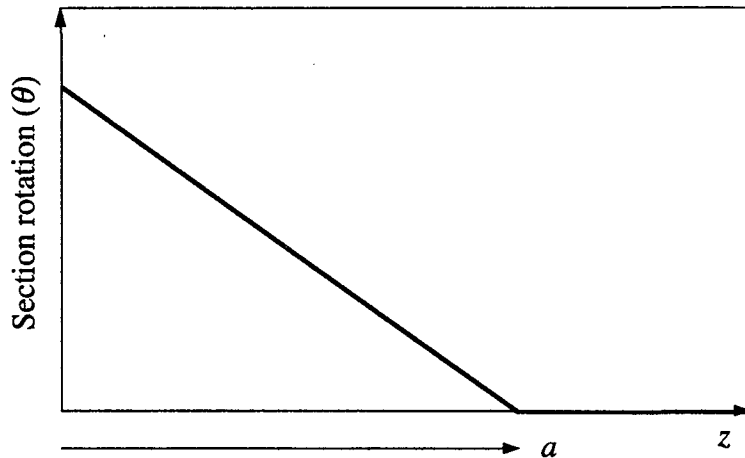


Fig. 1.6 Rotation profile for torsion beams modelled as being built in at the crack tip

The steady state analysis, with the inclusion of foundation stiffness and axial stresses, was used by both Leevers and Wheel [1.18], to analyse the HSDT test. The results were comparable to the quasi static model but showed less scatter.

Wheel modelled the HSDT test using a numerical FD scheme. He also incorporated the effects of material non-linearity, foundation stiffness, and axial stresses (via the solution of Gere[1.19]) into his model. He modelled the crack velocity as being constant and calculated the crack driving force using a local holding back force approach. His model

was effective in reducing the scatter of previous analyses. In addition, he modelled the crack front as curved, as opposed to straight, to examine the effect on predicted G_D . He came to the conclusion that any difference between the curved and straight crack fronts was negligible and therefore continued to use the simpler straight crack front model.

1.3.2.2 Adopted analysis

The present work was initially aimed at investigating the nature of the stick-slip crack propagation seen in the HSDT test at low loading rates. In order to achieve this the dynamic analysis of the test must be accurate.

The more basic approach of FE and FV methods, although of a more standard mathematical formulation, has not been pursued in the HSDT analysis since a large number of degrees of freedom would be required for an accurate model. With the inclusion of non-linear material properties a FE or FV model would take a considerable time to solve. By applying reasonably accurate assumptions the equation of motion can be reduced to one spatial dimension. The FD method can be used to integrate these equations and attain a solution within a few minutes on a modern personal computer. This allows a rapid assessment of any alterations made to the model. The fact that the equations of motion can be reduced to one spatial dimension allows their characteristics to be examined, which leads to a deeper understanding of the results from the FD model.

The FD model used in this work has been completely changed from that developed by Wheel. The major alterations are listed below:

- 1) Axial inertia was not included in Wheel's model. Localised axial disturbances were therefore modelled as having an infinite speed. Chapter 3 describes the equations of motion used to account for axial inertia.
- 2) The foundation stiffness was modified and the effects of the curved crack front re-examined (Chapter 4).
- 3) The non-linear material analysis of Wheel was examined and found to contain an invalid assumption which was corrected (Chapter 5).
- 4) The FD model was rewritten to run in propagation mode, such that the model predicted the crack history given the value of G_D . This removed the constraint of assuming the crack velocity to be constant.
- 5) Wheel's model predicted a fairly constant load imposed by the striker during the test and did not account for the oscillations seen in the load trace. The effect was examined using the new FD model with the incorporation of contact stiffness between the load point and the specimen.

The FD model and its characteristics are discussed in Chapter 6.

Three materials were examined during the course of this work: a high density pipe grade polyethylene (PE100), a pure and rubber toughened polyoxymethylene (Delrin) and a pure and rubber toughened polypropylene homopolymer. The results for these materials are discussed in Chapter 7.

1.4 References

- (1.1) Greig, J.M. and Ewing, L., 'Fracture propagation in polyethylene gas pipes', *Proc., Plastic Pipes V*, York, UK, (1982).
- (1.2) Greig, J.M., Leever, P.S. and Yayla, P., 'Rapid crack propagation in pressurised plastic pipe I: Full scale and small scale RCP testing', *Eng. Fracture Mech.*, **42**, p. 663, (1992).
- (1.3) Leever, P.S., Venizelos, G. and Ivankovic, A., 'The driving force for rapid crack propagation along pressurised pipelines', *Proc., 9th European Conference on Fracture*, Varna, Bulgaria, Sept. 21-25, (1992).
- (1.4) Ames, W.F., *Numerical methods for partial differential equations*, Nelson, (1969).
- (1.5) Zienkiewicz, O.C. and Taylor, R.L., *The finite element method*, McGraw-Hill, (1989).
- (1.6) Ivankovic, A., Demirdzic, I., Williams, J.G. and Leever, P.S., 'Application of the finite volume method to the analysis of dynamic fracture problems', *Int. J. of Fracture*, **66**, p. 357, (1994).
- (1.7) Mott, N.F., 'Fracture of Metals: Theoretical considerations', *Engineering*, **165**, p. 16, (1948).
- (1.8) Broberg, K.B., 'The Propagation of a Brittle Crack', *Arkiv for Fysik*, **18**, p. 159, (1960).
- (1.9) Williams, J.G., 'the analysis of dynamic fracture using lumped mass-spring models', *Int. J. of Fracture*, **33**, p.47, (1987).
- (1.10) Ivankovic, A., Rapid crack propagation in polymer multi-layer system, PhD Thesis, Univ. of London, (1991).
- (1.11) Genussov, R.M.S., Rapid crack propagation in pipe grades of polyethylene, PhD Thesis, Univ. of London, (1989).
- (1.12) Hemingway, A.J., Channell, A.D. and Clutton, E.Q., 'Instrumented Charpy impact testing of polyethylene', *Plastics Rubber and Composites Processing Applications*, **17**, p. 147, (1992).
- (1.13) Leever, P.S. 'Resistance of pipe grade polyethylenes to high speed crack propagation', *J. De Physique*, **49**, p. C3-231, (1988).
- (1.14) Wheel, M.A., High Speed Double Torsion Testing of Pipe Grade Polyethylenes, PhD Thesis, Univ. of London, (1991).
- (1.15) Saint-Venant, B. de, 'Mémoire sur les vibrations tournemantes des verges élastiques', *Comptes Rendus*, **28**, pp. 69, (1849).
- (1.16) Popelar, C.H., 'A model for dynamic fracture in a double torsion fracture specimen', *Crack Arrest Methodology and Applications*, ASTM, STP 711 (Eds. G.T. Hahn and M.F. Kanninen), p. 24, (1980).
- (1.17) Leever, P.S., 'Crack front shape effects in the double torsion test', **17**, p. 2469, (1982).
- (1.18) Wheel, M.A. and Leever, P.S., 'High speed double torsion tests on tough polymers. II: Non-linear elastic dynamic analysis', *Int. J. of Fracture*, **61**, p. 349, (1993).
- (1.19) Gere, J.M., 'Torsional Vibrations of Beams of Thin Walled Open Sections', *J. App. Mech.*, **21**, p. 381, (1954).

Chapter 2

An Optical Crack Gauge for the HSDT test

2.1 Introduction

One of the main disadvantages of the HSDT test was the long (fifty minutes) specimen preparation time needed. This was dominated by the application of a timing line crack gauge to allow measurement of crack velocity. A new optical technique of measuring crack velocity that reduces specimen preparation time to ten minutes, has been developed.

Further modifications have been made to the HSDT test rig during the course of this work. These include the redesign and testing of the infra red detector system to measure striker velocity, changes to the structures used to support and align the specimen in the rig and alterations to the specimen geometry.

All of the above modifications are discussed in the following sections.

2.2 The Optical Crack Gauge

In the HSDT test it is essential to record the crack length history throughout the test in order to calculate G_D . The existing timing line crack gauge technique required extensive specimen preparation. The Optical Crack Gauge (OCG) design avoids this completely by using an 'off-specimen' method, the optical sensors being resident on the rig, not the specimen.

2.2.1 Theory

The concept behind the OCG relies on the large out-of-plane displacements associated with torsion about an in-plane axis. Static finite volume [2.1] analysis indicates that local rotation adjacent to the crack path rapidly increases within a few millimetres of the crack front (see figure 2.1a). In the dynamic case it is reasonable to assume that any section would display a local angular acceleration in this region as the crack front passed. The specimen's surface displacement is therefore sensed optically at a series of points adjacent to the crack path.

The results in figure 2.1a relate to a straight crack front, whereas in reality the crack front is curved (see figure 2.1b). This fact does not invalidate the theory behind the OCG since:

- 1) The distance to which the local change in curvature extends into the section is proportional to the remaining ligament thickness to a power greater than unity.

- 2) At the leading edge of the crack front the remaining ligament thickness reduces much more rapidly than at the trailing edge.

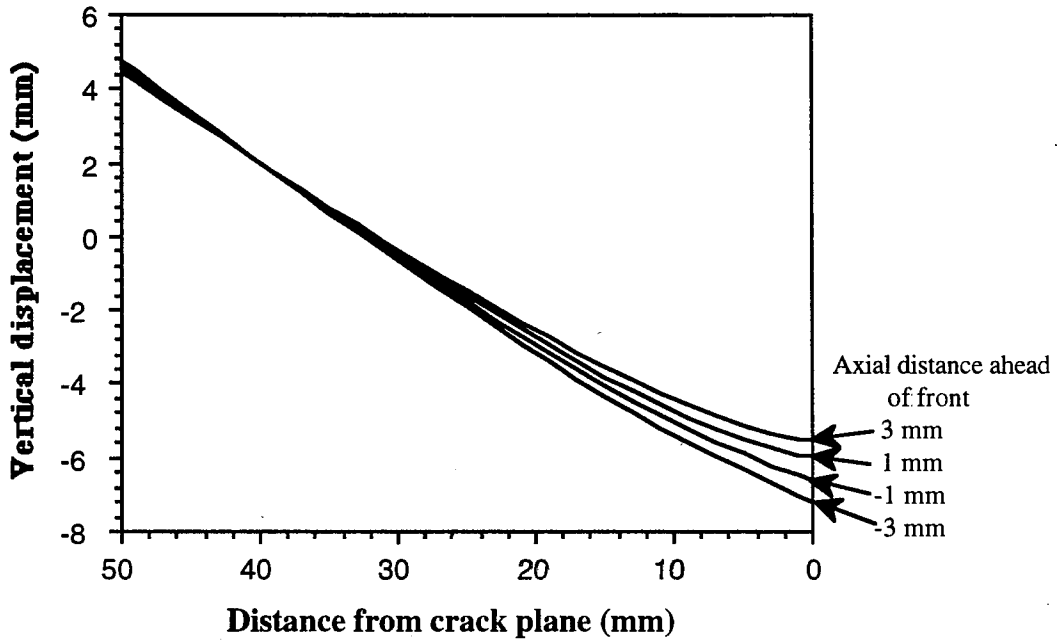


Fig. 2.1a Finite volume results showing top surface beam section rotations in one half of a static double torsion specimen at different axial distances (thickness=9 mm, 'V' Groove depth=2 mm, crack length = 100 mm, straight crack front)

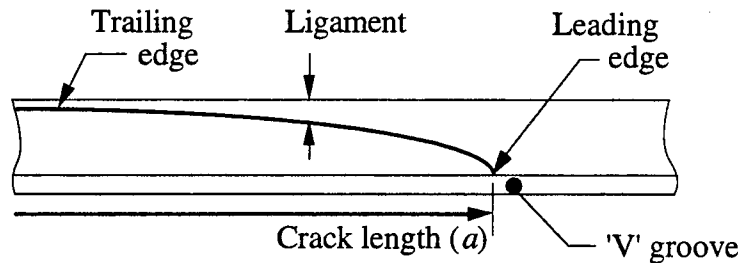


Fig. 2.1b Glossary of terms used to describe the curved crack front

Therefore, provided that the average section rotation over the region of the local change in curvature is considered an acceleration will be seen in the rotation rate as the leading edge of the crack front passes.

2.2.2 Design

The following discussion is divided into two areas. The first section relates to the design of the physical gauge and the correct positioning of the optical sensors that measure the local rotation. The second section describes the signal processing circuit.

2.2.2.1 Gauge

The deformation of the specimen is symmetrical about the major axis and therefore only one half of the specimen need be considered. To calculate the average section rotation in

the region close to the crack plane, vertical displacement must be measured at two points across the section and the results subtracted. Each sensor consists of two phototransistors (receivers) and a of photoemitter to illuminate the surface of the specimen. A schematic of one sensor is shown in figure 2.2.

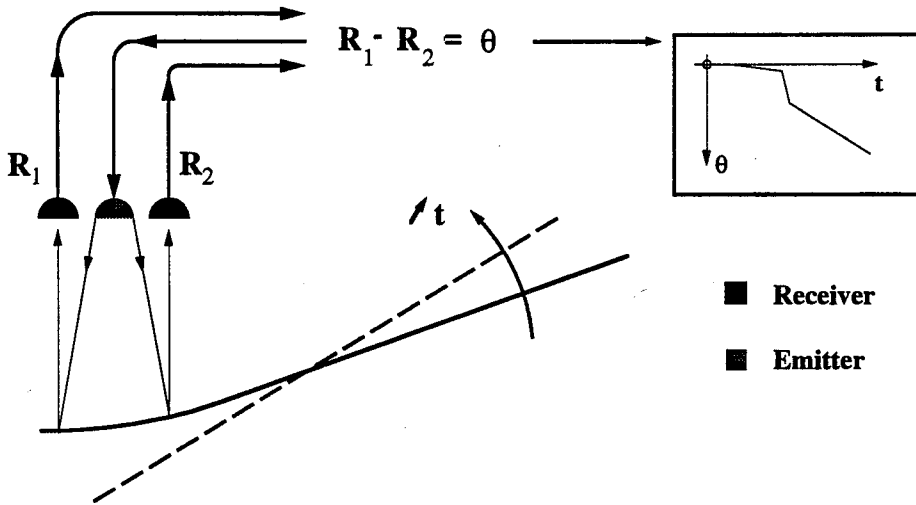


Fig. 2.2 Schematic of one sensor of the Optical Crack Gauge

White PVC insulating tape is applied to the top surface of the specimen on the sensor side of the crack path to scatter incident light. Since the light is scattered the intensity of radiation measured by each receiver is related to the vertical displacement of the region of specimen directly beneath it, not to the angle of incidence. The scattering also smoothes irregularities in the radiation distribution of the emitters over their illuminated area. Black PVC insulating tape is applied to the other side of the top surface to prevent interference and to normalise the optical characteristics.

Ten sensors are used in total, spaced at 10 mm axial intervals, starting at 60 mm from the loaded end of the specimen. Infra-red, spectrally matched silicon phototransistors and wide angle emitters were chosen since:

- 1) The use of an infra red, narrow spectral range (80 nm) minimised interference from ambient light.
- 2) Their low cost far outweighed the increased accuracy of, for example, fibre-optic/laser techniques used by Béguelin [2.2].
- 3) The intensity (20 mW at $I_F=100$ mA) met the requirements of linearity and noise levels.

Several geometric factors are taken into account in the gauge design (see figure 2.3):

- 1) The lateral distance between receivers (10 mm), and their field of view (2 mm diameter) are minimised, since only the rotation over only a small proportion of the section width is required.
- 2) The vertical separation between the specimen and the gauge must be minimised to maximise luminous intensity, but must allow for clearance of the specimen whilst it is rotating. This separation also determines the region of response linearity of the sensors.

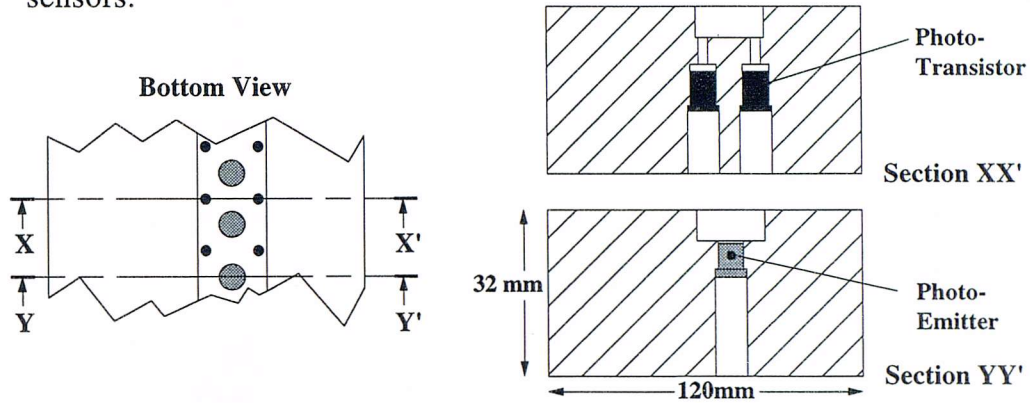


Fig. 2.3 Drawing of the OCG

The sensors are mounted in an aluminium block. This is secured to the test rig above the specimen by a tripod arrangement of spacers to allow adjustment of trim and height. The aluminium block is earthed to provide electromagnetic shielding of the sensors.

The sensors are individually connected to the signal processing circuit via shielded cables. A picture of the OCG in-situ is shown in figure 2.4. A detailed drawing of the gauge design is given in Appendix 1.

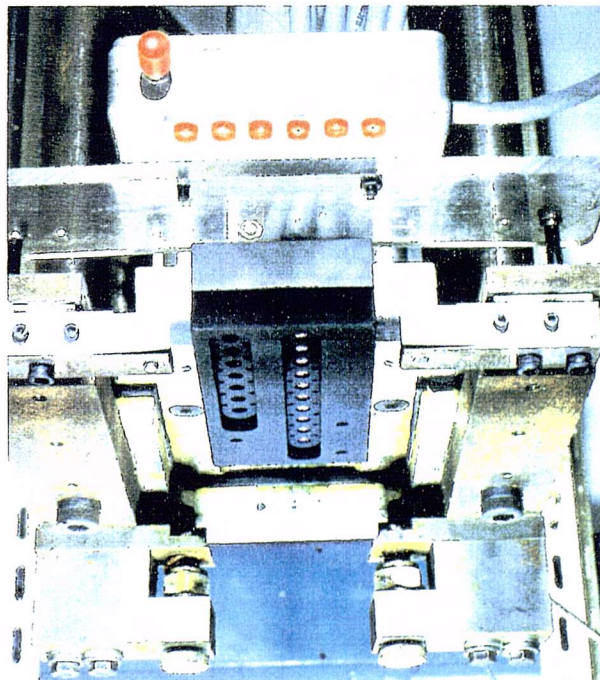


Fig 2.4 Picture of the OCG in situ on the HSDT rig, viewed from below

2.2.2.2 Signal Processing

The signal processing provides a composite output to one channel of a high speed transient recorder, showing the time at which the crack tip passes each sensor along its path. The front end stage for each sensor consists of a difference amplifier whose output represents local surface rotation. An example of the rotation outputs from sensors 3 and 7, at 70 mm and 110 mm respectively from the load point, is shown in figure 2.5, which also shows the composite output from the conventional timing line crack gauge.

The rotation signal is then passed through two differentiator stages to obtain the angular acceleration. Analogue differentiation, being frequency dependent, means that high frequency interference can completely obscure the signal. To avoid this, low value feedback capacitors were used to form low pass, first order filters to limit the frequency response of both differentiation stages. The output from this stage for sensors 3 and 7 is also shown in figure 2.5.

Both sensors indicate some rotation before the crack front reaches them, this is indicative of the rotation corresponding to the average over the region close to the crack plane, not the rotation actually at the crack plane itself. Comparing the two sensors, there is a more pronounced angular acceleration in the rotation history of sensor 7 as the crack front passes. This is because the dominant (Saint-Venant's) torsional wave and the crack front diverge as they propagate through the specimen.

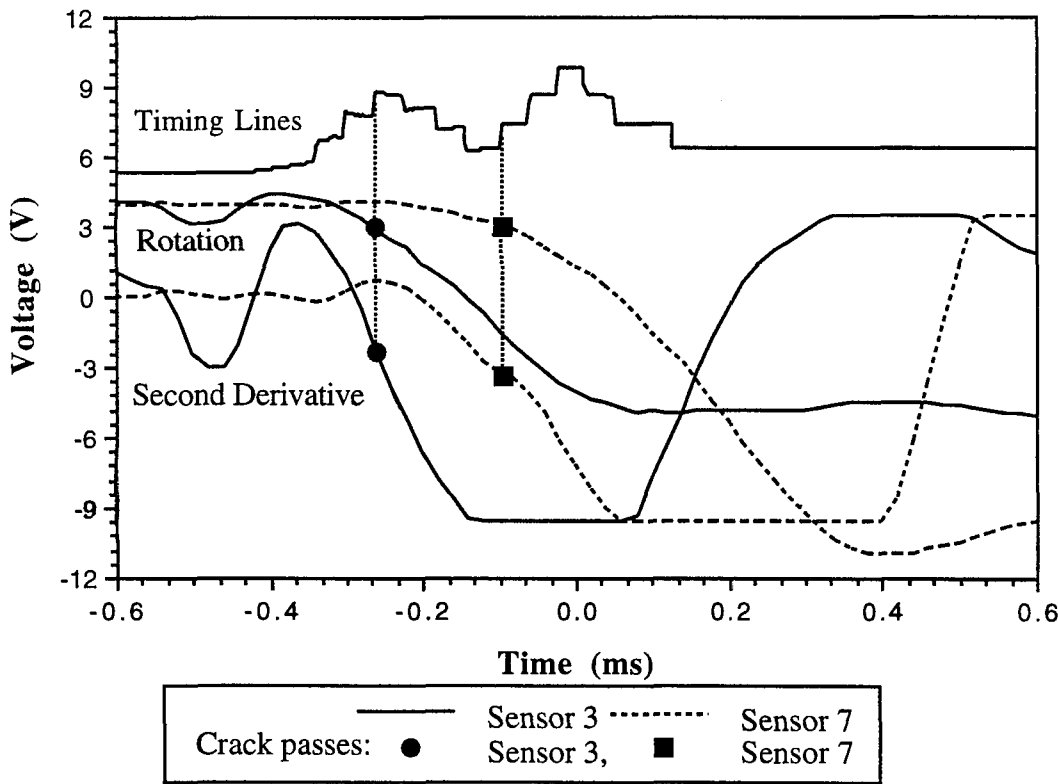


Fig. 2.5 Rotation and acceleration outputs from sensors 3 and 7.

In the frequency domain the majority of the information relevant to the angular acceleration is carried by frequencies up to 10 kHz. The value of 10 kHz might be expected to depend on crack speed, crack shape and loading rate but was found to be appropriate for all the cases observed in this work. In order to stop higher frequencies dominating the second differential output an ideal filter would have a brick wall response (rectangular filter) with a cut off frequency at 10 kHz. For a first order filter, as used in this work, the roll off is only 20 dB/decade and produces a 45° phase shift at the cut off frequency; it is therefore far from the brick wall response. A higher order filter would increase the roll off rate and could reduce the phase shift in the frequency region of interest, but was not used in this development work in order to keep the circuit as simple as possible. A compromise was therefore made between filtering unwanted high frequency interference and retaining the frequencies of interest. The best compromise was found by altering the feedback filter capacitors, in an iterative experimental approach, to obtain the best output. As can be seen from figure 2.5 the second differential output is not perfect, there being no well defined peak as the crack tip passes. However, very soon after the crack tip has passed each sensor (30 μ s), the second derivative outputs have the same gradient and are separated by the time it took the leading edge of the crack front to pass between them.

After the differentiation stage a Schmidt trigger is used to produce a step change in voltage output when the second differential signal crosses a pre-set level. Noting the above discussion it should not matter where the trigger level is taken provided that it is the same in each sensor circuit. Due to this method a small time shift (40 μ s) is apparent which is not important in terms of calculating $\dot{a}$. For the results shown here the level was set at -5V.

The output from the Schmidt trigger is then passed to a monostable multivibrator and amplitude control stage. The final output from each sensor circuit appears as a pulse of 10 μ s duration with an amplitude proportional to its position number, i.e. 0.1 V from sensor 1, 0.2V from sensor 2, etc. The outputs from all sensors are summed to provide a single composite signal (see figure 2.8). The circuit developed is shown as a block diagram in figure 2.6 and in detail in Appendix 2. The OCG with its associated circuitry is shown in figure 2.7.

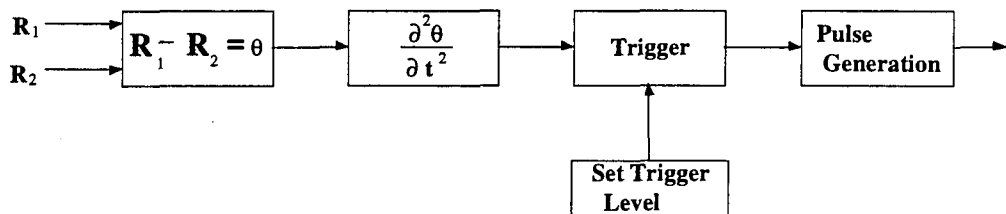


Fig. 2.6 Block diagram of the OCG circuit.

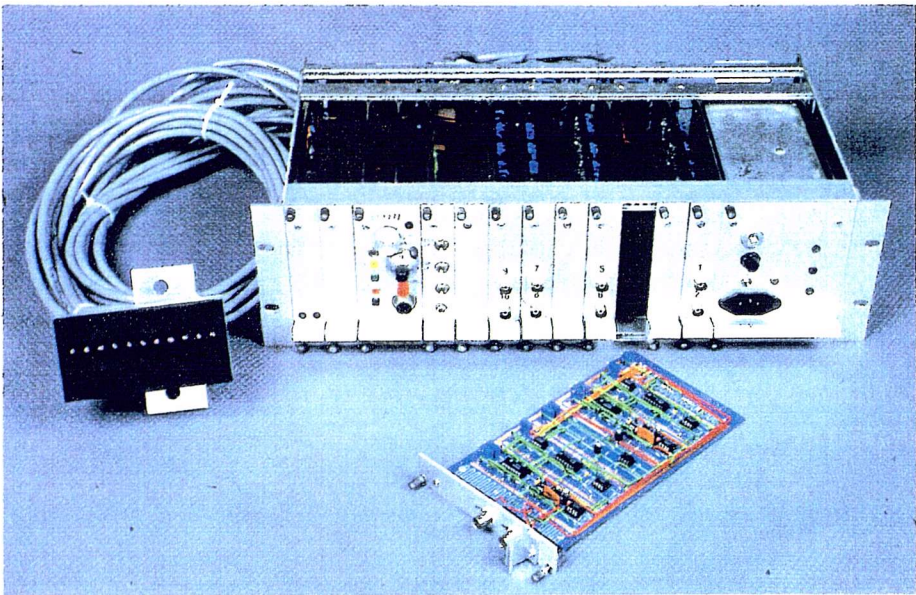


Fig. 2.7 Picture of the OCG and its associated circuitry

2.2.3 Comparison of timing line and OCG results

An example of the OCG output compared against the timing lines is shown in figure 2.8 for a toughened polyoxymethylene (Delrin ST). The crack histories from both outputs are plotted in figure 2.9. Only OCG gauges at crack lengths 100 and 110 mm show any noticeable deviation from the timing lines. On closer examination, however, it can be seen that both methods display a localised increase in $\dot{a}$ in this region.

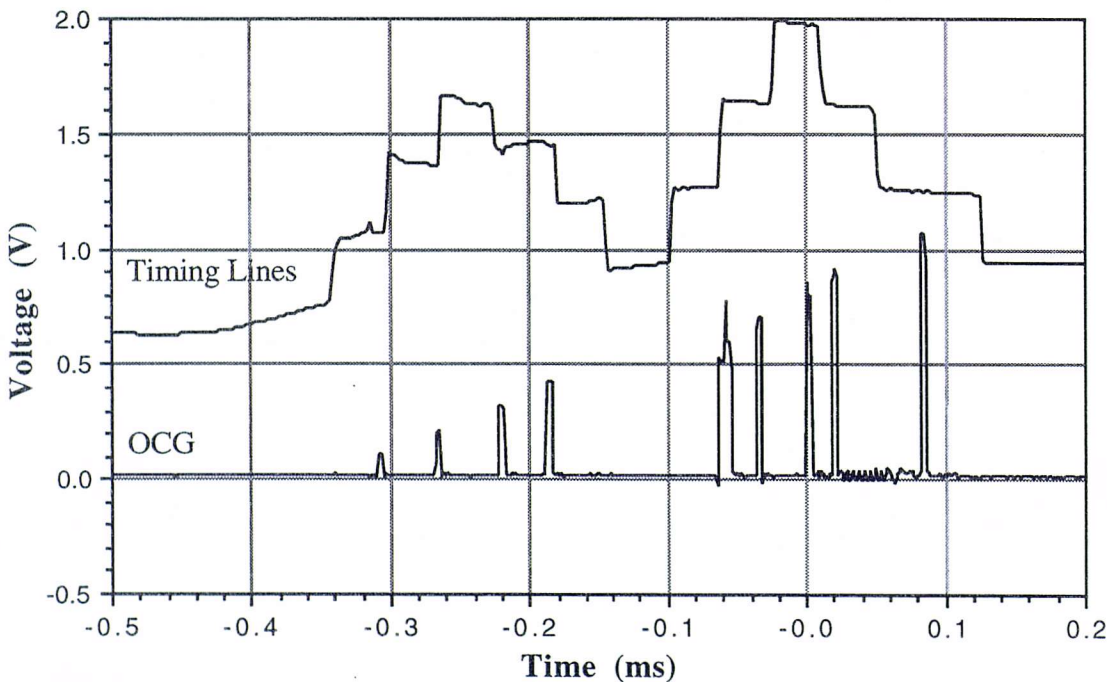


Fig. 2.8 Output from crack timing lines and OCG

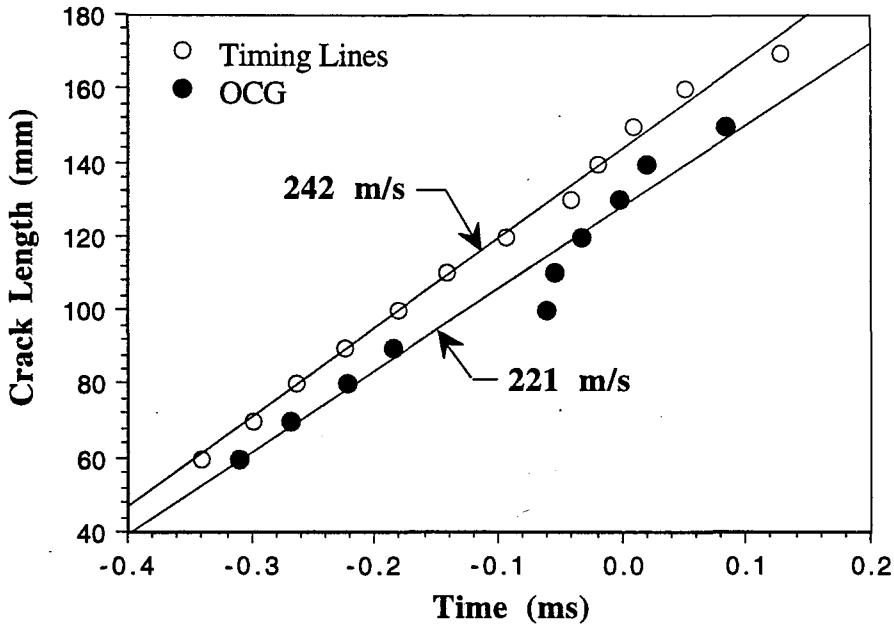


Fig. 2.9 Crack histories from crack timing lines and OCG outputs shown in figure 2.8

Tests were also performed on high (PE100) and medium (PE80) density polyethylenes which are summarised in Table 2.1.

As can be seen the results are reasonable with a 11% maximum error in crack velocity. There are still some improvements that could be made to the OCG circuit. The rotation traces all show a clearly defined acceleration as the crack tip passes but this could be further improved by reducing the spacing between the receivers of a sensor. A higher order, critically damped filter is also needed to produce a sharper roll off in its frequency response since the frequency of the interference is close to the frequencies of interest.

Material	Temperature (°C)	OCG crack velocity (m/s)	Timing lines crack velocity (m/s)
PE100	0	236	250
PE100	0	221	234
PE100	0	247	253
PE100	20	143	161
PE100	20	110	121
PE80	0	229	237
PE80	0	204	229
PE80	0	217	227
PE80	20	138	150

Table 2.1 Summary of OCG results on PE100 and PE80.

2.2.4 Section rotation measurements

A modified form of the OCG was designed to approximately measure the section rotation remote from the crack path (i.e. to measure the development of the beam rotation history). The physical arrangement of this gauge is exactly the same as described above except that the receivers and emitters are laterally shifted to lie over the centre line of the half-specimen. This gauge is incorporated into the OCG design which is shown in figures 2.4 and 2.7.

2.3 Striker velocity measurement system

The striker velocity measurement system, used by both Leever and Wheel [2.3], was described in section 1.3.1. The accuracy of the system relied on the assumption that the striker had achieved its maximum velocity by the time it passed the infra red sensors and is no longer accelerating. This assumption has never been checked. Simple ideal gas expansion calculations indicate that at high accumulator pressures, there is still a significant driving force energy remaining in the gas by the time the striker passes the sensors.

The system was prone to giving sporadic, erroneous results. The main cause was attributed to part of the associated circuitry of the sensors being mounted on the gas gun, which is subjected to vibrations each time a test is performed. At high firing pressures these vibrations disturbed both the circuit and the alignment of the sensors.

Taking these points into account the striker velocity measurement system was redesigned, both to improve the system performance, and to check the assumption of constant striker velocity. The basic concept of using infra red sensors to time the passing of the leading edge of the striker was retained.

2.3.1 Design

In order to measure acceleration, four rather than two sensor stages are used. Each sensor consists of an infra-red emitter and receiver, the emitter being on one side of the gas gun barrel and the receiver on the opposite side. Each emitter and receiver was attached independently to the gas gun via their own holders. The full assembly drawing for the holder design and location on the rig is shown in Appendix 3. In the following discussion the sensors are numbered 1 (the upper most sensor) to 4 (the lowest, nearest the impact point). The impact point is 77 mm below sensor 4.

The measurement of acceleration on a regular basis for each test was not considered to be necessary. The circuit was therefore designed such that all four sensors had associated

circuitry to allow the recording of their signals by a four channel transient recorder. Additional circuitry for the lower two sensors (i.e. those nearest the specimen) was developed to produce a pulse, the length of which corresponded to the time it took for the leading edge of the striker to pass between the two sensors. The response from all four sensors could then be used to calibrate the pulse response from the lower two in terms of striker velocity at the time of impact. For normal testing an independent electronic timer could then be used to time the pulse length and, using the pre-determined calibration, the striker velocity could be accurately calculated.

The circuitry for the lower two sensors is similar to the original design but with the following modifications:

- 1) All the circuit components, apart from the infra red emitters and receivers are located off the rig. The cables connecting the emitters and sensors to the circuit are fully shielded.
- 2) The emitters and sensors have been selected and implemented so as to be extremely sensitive, producing a sharp transition from 'on' to 'off' states. The most important features are a narrow field of view for the receivers and maximum intensity from the emitters.
- 3) After converting the analogue outputs to TTL logic compatible signals a latch is used to guard against any noise in the analogue signal. The latch is reset after the leading edge has passed both sensors by utilising the trailing edge signal.

The circuits for the first two sensors are identical to that described above but with the omission of the logic circuit required for pulse generation. The full circuit diagram for the lower two sensors is shown in Appendix 4.

2.3.2 Results

A detailed study to determine the calibration was performed by Traebert [2.4] under the author's supervision. The main results from her study are given below:

2.3.2.1 Response time

The response from the infra-red sensors consists of an initial high or 'on' steady voltage output. As the leading edge of the striker passes a sensor the infra-red beam from the opposing emitter is interrupted, producing a rapid decay to zero in sensor output voltage (the 'off' state). The timing pulse is generated by triggering of this decaying output when it reaches its mid-point between 'on' and 'off' states. The point at which the sensor output starts to decay, as supposed to the mid point of the decay, was found to give a more

accurate measurement of when the striker passes a sensor (see figure 2.10). However, the error introduced by using the mid point trigger method in calculating the time for the striker to pass any two sensors was found to be constant. The average striker velocity between sensors 3 and 4 is therefore:

$$v_{34} = \frac{\Delta s_{34}}{\Delta t_{34} + t_e|_{34}}$$

where $t_e|_{34}$ (48.4 μ s) is the error of the mid point trigger method for sensors 3 and 4, Δs_{34} is the distance between sensors 3 and 4, and Δt_{34} is the pulse length measured using the mid point trigger method.

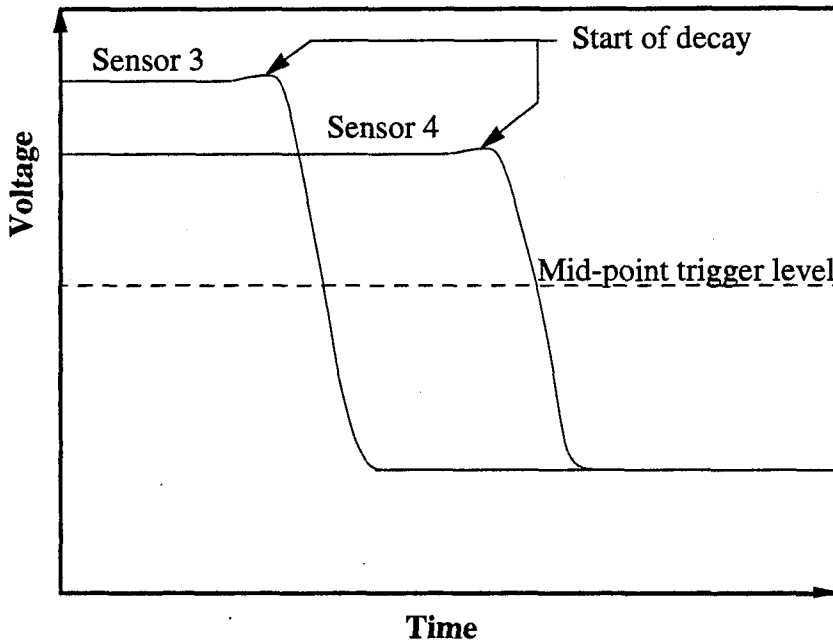
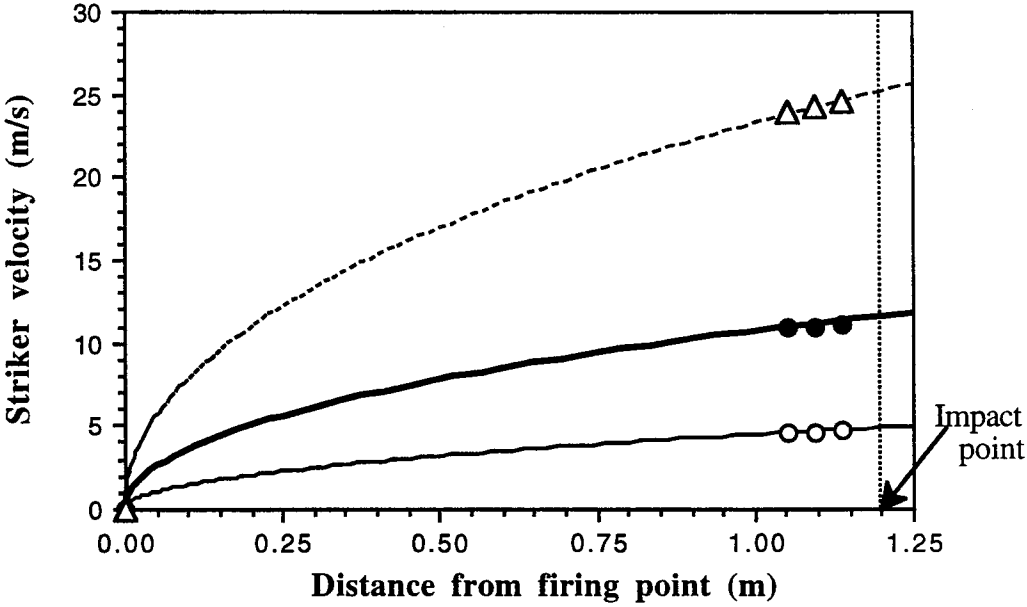


Fig. 2.10 Schematic of voltage output from sensors 3 and 4

2.3.2.2 Striker acceleration

At high gas gun pressures there is still a definite acceleration of the striker as it passes the sensors. In order to extrapolate the striker velocity to that at the impact point a one dimensional model of the gas gun was developed. The model assumed a perfect seal between the gas gun bore and the striker, isothermal expansion, no turbulence and constant cross sectional area of the gas gun tube. The results from this model were not expected to quantitatively agree with experiment but, by adjusting the assumed initial pressure before firing, a good fit could be obtained. Using this method the impact velocity could be extrapolated (see figure 2.11).



Initial accumulator pressure	
Predicted	Experimental
— 0 bar	○ 0.0 bar
— 0.4 bar	● 1.5 bar
····· 2.2 bar	△ 5.2 bar

Fig. 2.11 Comparison of experimental and predicted striker velocity results

In the range of interest the extrapolated impact velocity (v_{impact}) was found to be directly proportional to the measured velocity between sensors 3 and 4 such that:

$$\frac{dv_{impact}}{dv_{34}} = m_e$$

where m_e is the calibrated constant slope equal to 1.035. From the output pulse the impact striker velocity can therefore be calculated as:

$$v_{impact} = \frac{\Delta s_{34} m_e}{\Delta t_{34} + t_{e|34}}$$

The new striker velocity measurement system has proved to be very reliable in service.

2.4 Specimen geometry

The basic specimen geometry adopted by Wheel [2.5] has been changed in two ways:

2.4.1 Initial notch

A tapered initial notch was used to provide a smooth initiation. This has not been retained since it is difficult both to machine and to analyse. In its place, a straight initial notch is

used which is 40 mm long (see figure 2.12). This length was decided on as a compromise between allowing the torsional deformation to develop properly before initiation and retaining a reasonable length of crack propagation.

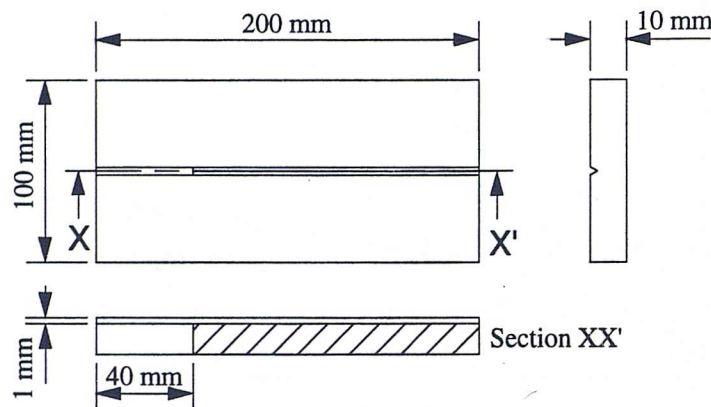


Fig 2.12 HSDT Specimen geometry

2.4.2 Razor blade slit

Wheel cut a 1 to 2 mm razor blade slit along the crack path on the top of the specimen in order to eliminate the effects of ductility seen in this region. This again is difficult both to machine and to analyse. The razor bade slit was not used in this work.

2.5 Specimen supports and alignment

The method of specimen alignment used by Wheel took the form of two 45° chamfers on the rig at the impact end of the specimen (see figure 2.13a). This system relied on the lateral sides of the specimen being oriented by eye parallel to the sides of the rig's specimen cage. A new, easier to use, system was developed where four hemi-spherical studs in the rig's specimen cage are used which together form a right angled contact surface (see figure 2.13b). The specimen is inserted into the rig so as to be in contact with all four studs.

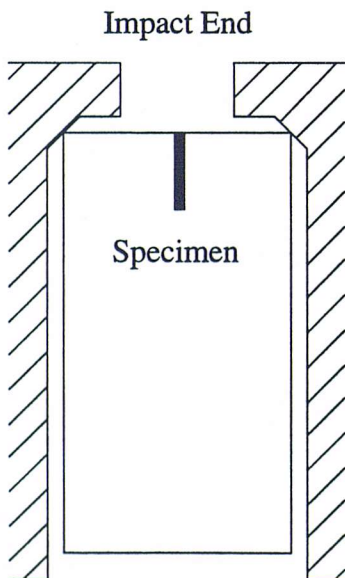


Fig. 2.13a Old specimen alignment

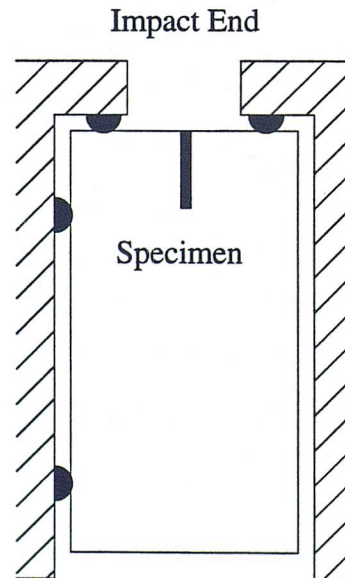


Fig. 2.13b New specimen alignment

Prior to impact the specimen rests on four contact points, one at each corner of the specimen positioned 10 mm in from the sides. In order to measure the imposed load on the specimen, the support points at the impacted end of the specimen consist of ball bearings which rest freely on piezoelectric load cells. The cage assembly used by Wheel to retain the ball bearings did not prevent them from moving vertically upwards (see figure 2.14a). At high impact speeds these ball bearings were completely dislodged, requiring time to be replaced. A conical cage was designed to retain the ball bearings shown in figure 2.14b.

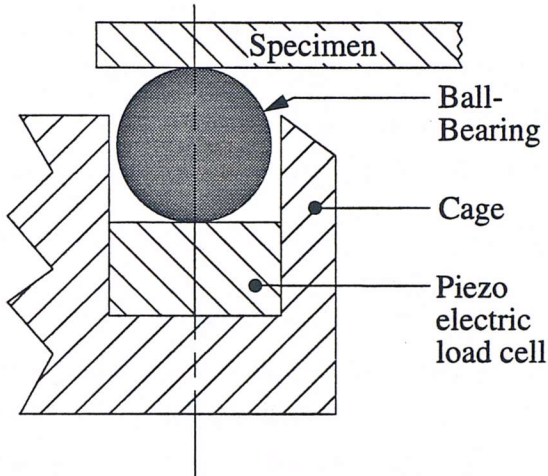


Fig. 2.14a Old support

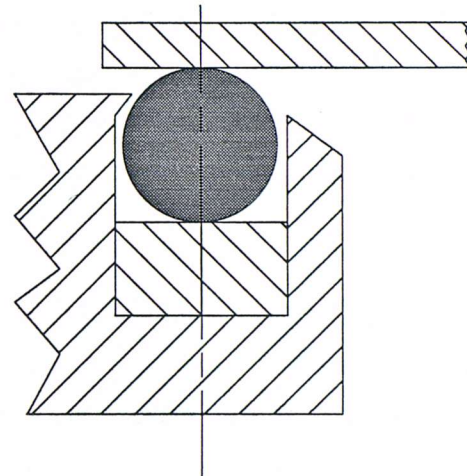


Fig 2.14b New support

2.6 References

- (2.1) Demirdzic, I., Ivankovic, A., Martinovic, D. and Muzaferija, S., 'Numerical methods for solving linear and non-linear solid body problems', *Proc., 1st Congress of Croatian Society of Mechanics*, Pula, (1994).
- (2.2) Béguelin, P., 'Mechanical characterisation of polymers and composites with a servohydraulic high-speed tensile tester', *J. De Physique*, **1**, p. 1867, (1991).
- (2.3) Wheel, M.A. and Leever, P.S., 'High speed double torsion tests on tough polymers I: Linear Elastic Steady State and Dynamic Analysis', *Int. J. of Fracture*, **61**, p. 331, (1993).
- (2.4) Traebert, A., 'Investigation of dynamic fracture toughness of rubber modified polymer blends', Imp. Coll. Mech. Eng. Dept., ERASMUS exchange student project report, (1995).
- (2.5) Wheel, M.A., High Speed Double Torsion Testing of Pipe Grade Polyethylenes, PhD Thesis, Univ. of London, (1991).

Chapter 3

Analysis of the high speed double torsion test

3.1 Introduction

The crack speeds in the high speed double torsion (HSDT) test are of the same order as the torsional wave speeds. The energy to propagate the crack is transmitted to the crack front by these waves. This means that the calculated value of G_D is strongly dependent on the accuracy with which the speed and amplitude of the torsional disturbances are modelled. This chapter describes the derivation and validation of equations used to model this transient behaviour.

Experiments have shown that torsional waves in prismatic bars undergo dispersion, which is a result of phase velocity increasing as the wavelength decreases. The classical solution for torsional waves was given by Saint-Venant, but his theory does not predict any dispersion. In regions of non-uniform twist, axial stresses must exist due to the restraint on the warping of the section. Barr [3.1] has proposed an approximate theory to model the dispersion, which accounts for axial stresses and also axial inertia associated with the warping. This theory differs from former theories which only take into account either axial stresses (Gere [3.2]) or axial inertia (Love [3.3]), or neither (Saint-Venant [3.4]). Barr's results lie in between those of the stress corrected and inertia corrected theories.

To check his theory Barr performed torsional vibration experiments. These appeared to show the requirement for an adjustment factor in his derived equations, to match theoretical prediction with experimental results. This correction had the overall effect of reducing the predicted asymptotic phase velocity at short wavelengths from the shear to the Rayleigh wave velocity.

The initial analysis presented in the following section closely follows that of Barr, but for the incorporation of a term modelling the foundation stiffness. Wheel [3.5] showed that the shear moduli of the materials he tested were not constant. Based on this observation, effective section secant and tangent shear moduli are defined. Their purpose is developed further in the discussion of non-linear material properties in chapter 5. In Section 3.3 a detailed discussion of the above mentioned correction factor is presented. The derived equations are then used to analyse static, non-uniform torsion of prismatic beams as in the classical case of Timoshenko [3.6]. These results are compared with those of a finite volume (FV) solution.

3.2 Differential equations of motion

3.2.1 General equations

Only one half of the double torsion (DT) specimen is modelled since the deformation is symmetric about the crack path. This can be clearly seen from high speed photographic results of Wheel [3.7].

The approximate theory is derived by using the variational equation of motion given by Love [3.3]:

$$\iiint_x \int_y \int_z [L(u)\delta u + L(v)\delta v + L(w)\delta w] dx dy dz = 0 \quad (3.1)$$

where u , v and w are respectively the displacements in the x , y and z directions and $L(u) = \frac{\partial \sigma_x}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} + \frac{\partial \tau_{xz}}{\partial z} - \rho \frac{\partial^2 u}{\partial t^2} + X$ ($L(v)$ and $L(w)$ being obtained by cyclic interchange of the co-ordinates).

The semi-inverse method of Saint-Venant is used to solve these equations. The displacements are assumed to be of Saint-Venant's form with the addition of an axial displacement function (ψ):

$$u = -y\theta(z, t) \quad v = x\theta(z, t) \quad w = \phi(x, y)\psi(z, t) \quad (3.2)$$

This form assumes that torsional and flexural motions are not coupled.

The z axis corresponds to the neutral axis of the beam. ψ models the axial stresses existing in areas of non-uniform twist by scaling the warping function (ϕ). The axial stresses produce a greater torsional stiffness than would be predicted by Saint-Venant's solution for the same twist. The function ϕ is assumed to be the same as Saint-Venant's warping function for static uniform torsion:

$$\frac{\phi}{H^2} = -\frac{xy}{H^2} + \beta^2 \left(\frac{2}{\pi}\right)^3 \sum_{n=0}^{\infty} \frac{(-)^n}{(2n+1)^3} \frac{\sinh \frac{(2n+1)\pi x}{H\beta}}{\cosh \frac{(2n+1)\pi}{2\beta}} \sin \frac{(2n+1)\pi y}{H\beta} \quad (3.3)$$

where $\beta < 1$ is the side ratio B/H . B and H are respectively the thickness and width of the specimen so that $-H/2 \leq x \leq H/2$, and $-B/2 \leq y \leq B/2$ (see figure 3.1)

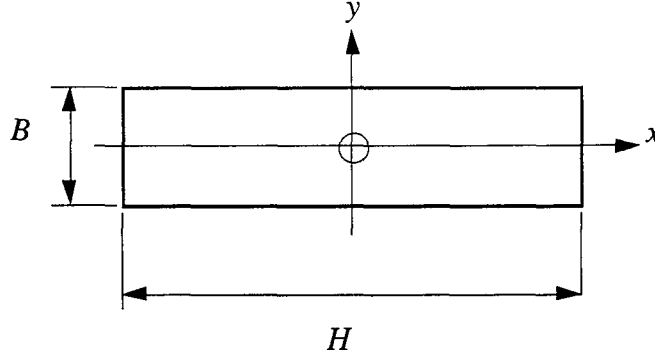


Fig. 3.1 Section definition of torsion beam

The variations of the displacements from equations (3.2) are:

$$\delta u = -y\delta\theta \quad \delta v = x\delta\theta \quad \delta w = \phi\delta\psi \quad (3.4)$$

Equation (3.1) will be satisfied for any variations if

$$\int_x \int_y [L(u)(-y) + L(v)(x)] dy dx = 0 \quad (3.5a)$$

$$\text{and } \int_x \int_y L(w)\phi dy dx = 0 \quad (3.5b)$$

The strains evaluated from equation (3.2) are:

$$\begin{aligned} \epsilon_z &= \phi \frac{\partial \psi}{\partial z}, \quad \epsilon_x = \epsilon_y = 0, \quad \gamma_{xy} = 0, \\ \gamma_{zx} &= \frac{\partial \phi}{\partial x} \psi - y \frac{\partial \theta}{\partial z}, \quad \gamma_{zy} = \frac{\partial \phi}{\partial y} \psi + x \frac{\partial \theta}{\partial z} \end{aligned} \quad (3.6)$$

Consider equation (3.5a). The terms containing σ_x and σ_y are found to be zero on integrating by parts and using the boundary conditions. Evaluating the shear stresses by Hooke's law gives the integral of the stress components as:

$$\int_x \int_y \left(-y \frac{\partial}{\partial z} (\mu \gamma_{zx}) + x \frac{\partial}{\partial z} (\mu \gamma_{zy}) \right) dy dx \quad (3.7a)$$

where μ is the shear modulus.

Substituting for the strain components from equation (3.6), this becomes:

$$\frac{\partial}{\partial z} \left(\int_x \int_y \left(\mu (y^2 + x^2) \frac{\partial \theta}{\partial z} + \mu \left(x \frac{\partial \phi}{\partial y} - y \frac{\partial \phi}{\partial x} \right) \psi \right) dy dx \right) \quad (3.7b)$$

In the case of a non-linear elastic material an effective section secant shear modulus (μ_s) is defined which is a function of the strain field of the section such that equation (3.7b) can be written as:

$$\frac{\partial}{\partial z} \left(\mu_s J \frac{\partial \theta}{\partial z} + \mu_s L \psi \right) \quad (3.7c)$$

where $L = \int_x \int_y \left[\frac{\partial \phi}{\partial y} x - \frac{\partial \phi}{\partial x} y \right] dy dx$ and $J = \int_x \int_y [x^2 + y^2] dy dx$

In the case of a linear elastic material μ_s is simply equal to the shear modulus.

In the beam root region, the foundation stiffness due to the opposing half is modelled as exerting a restoring moment (Ω) per unit length per radian rotation [3.8]. This is equated to a body force having components Y and X :

$$\Omega \theta = \int_x \int_y [Yx - Xy] dy dx \quad (3.8)$$

The derivation of Ω is discussed further in section 4.2.1.

Equation (3.5a) thus reduces to:

$$\mu_t J \frac{\partial^2 \theta}{\partial z^2} + \mu_t L \frac{\partial \psi}{\partial z} - \Omega \theta = \rho J \frac{\partial^2 \theta}{\partial t^2} \quad (3.9a)$$

where μ_t denotes an effective section tangent shear modulus with respect to the z axis.

Turning now to equation (3.5b), the integration of the shear stresses is carried out as follows:

$$\int_x \int_y \left(\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} \right) \phi dy dz = \int_x \int_y (\nabla \cdot \tau) \phi dx dy = \int_x \int_y (\nabla \cdot (\phi \tau) - \tau \cdot \nabla \phi) dx dy$$

where $\tau = \tau_{xz} \underline{i} + \tau_{yz} \underline{j}$. By Green's theorem, the right hand integral of the above equation is equivalent to:

$$\oint_C (\phi \tau) \cdot \underline{\hat{n}} ds - \int_x \int_y (\tau \cdot \nabla \phi) dx dy$$

Where s is the arc length of C , the boundary of the section. The first integral is zero since τ must be tangential to C at all points to satisfy free surface boundary conditions. Expanding the second integral gives:

$$\iint_{x,y} \left(\frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} \right) \phi dy dz = - \iint_{x,y} \left(\frac{\partial \phi}{\partial x} \tau_{xz} + \frac{\partial \phi}{\partial y} \tau_{yz} \right) dy dx$$

Using Hooke's law to evaluate the shear stresses in the integral on the right hand side, and substituting for the strain components from equations (3.6) gives :

$$- \iint_{x,y} \left(\mu \left(\left(\frac{\partial \phi}{\partial x} \right)^2 + \left(\frac{\partial \phi}{\partial y} \right)^2 \right) \psi + \mu \left(\frac{\partial \phi}{\partial y} x - \frac{\partial \phi}{\partial x} y \right) \frac{\partial \theta}{\partial z} \right) dy dx$$

As in the derivation of equation (3.9a) an effective section shear modulus (μ_s^*) is defined such that the above integral can be written as:

$$-\mu_s^* K \psi - \mu_s^* L \frac{\partial \theta}{\partial z}$$

where $K = \iint_{x,y} \left[\left(\frac{\partial \phi}{\partial x} \right)^2 + \left(\frac{\partial \phi}{\partial y} \right)^2 \right] dy dx$.

For the case of a linear elastic material μ_s^* is equal to μ_s . In the case of a non-linear material this will, in general, only be a first approximation. In the HSDT test the deformation of regions subjected to high shear strains, and therefore sensitive to material non-linearity, is governed largely by equation (3.9a). The approximation is therefore made that μ_s^* is equal to μ_s .

In calculating the axial stress in equation (3.5b), the lateral stresses are neglected, rather than their corresponding strains [3.9], so that the axial stress is simply equal to $E\sigma_z$ where E is the Young's modulus. Assuming that the Young's modulus is constant (Chapter 5), equation (3.5b) therefore reduces to:

$$EP \frac{\partial^2 \psi}{\partial z^2} - \mu_s K \psi - \mu_s L \frac{\partial \theta}{\partial z} = \rho P \frac{\partial^2 \psi}{\partial t^2} \quad (3.9b)$$

where $P = \iint_{x,y} \phi^2 dy dx$.

In order to reduce the number of section constants (J, K, L and P) the following non-dimensional section constants are defined: $\gamma^2 = P/(JH^2)$ and $\lambda = (J + L)/J$. Their evaluation is given by Davies [3.10] as:

$$\lambda = \frac{4\beta^2}{(1+\beta^2)} \left[1 - \frac{192\beta}{\pi^5} \sum_{n=0}^{\infty} \left(\frac{1}{(2n+1)^5} \tanh \frac{(2n+1)\pi}{2\beta} \right) \right] \text{ and}$$

$$\gamma^2 = \frac{\beta^2}{4(1+\beta^2)} \left\{ \frac{1}{3} + \frac{768\beta^2}{\pi^6} \sum_{n=0}^{\infty} \left[\frac{1}{(2n+1)^6} \left(\frac{6\beta \tanh \frac{(2n+1)\pi}{2\beta}}{(2n+1)\pi} + \tanh^2(2n+1)\pi \beta/2 - 3 \right) \right] \right\}$$

Since $K=-L$ (Love [3.3]) the following relationships can then be derived:

$$P/K = -P/L = \gamma^2 H^2 / (1 - \lambda)$$

With these substitutions equation (3.9a) becomes:

$$\mu_t \frac{\partial^2 \theta}{\partial z^2} + \mu_t (\lambda - 1) \frac{\partial \psi}{\partial z} - \frac{\Omega}{J} \theta = \rho \frac{\partial^2 \theta}{\partial t^2} \quad (3.10a)$$

and equation (3.9b) becomes:

$$E \frac{\partial^2 \psi}{\partial z^2} - \mu_s \frac{(1-\lambda)}{\gamma^2 H^2} \psi + \mu_s \frac{(1-\lambda)}{\gamma^2 H^2} \frac{\partial \theta}{\partial z} = \rho \frac{\partial^2 \psi}{\partial t^2} \quad (3.10b)$$

3.2.2 Boundary conditions

The surface condition arising from the variational equations of motion is given by Love [3.3] as:

$$\int_S \left(\left[\sigma_x l' + \tau_{xy} m' + \tau_{xz} n' - \bar{X} \right] \delta u + \left[\sigma_y m' + \tau_{yz} n' + \tau_{xy} l' - \bar{Y} \right] \delta v \right. \\ \left. + \left[\sigma_z n' + \tau_{xz} l' + \tau_{yz} m' - \bar{Z} \right] \delta w \right) dS = 0 \quad (3.11)$$

where l', m', n' are the direction cosines of the external normal to the surface S on the x, y , and z , directions and $\bar{X}, \bar{Y}, \bar{Z}$ are the components of external force per unit area.

For plane, normal ends to the rod where $l'=m'=0, n'=1$, equation (3.11) becomes:

$$\int_x \int_y \left\{ [\sigma_z \phi - \bar{Z} \phi] \delta \psi + [(\tau_{yz} x - \tau_{zx} y) + \bar{X} y - \bar{Y} x] \delta \theta \right\} dy dx = 0 \quad (3.12)$$

For equation (3.12) to be satisfied, either

$$\delta \psi = 0 \quad (3.13a)$$

$$\text{or } \int_x \int_y [(\sigma_z - \bar{Z}) \phi] dy dx = 0 \quad (3.13b)$$

and either

$$\delta \theta = 0 \quad (3.14a)$$

$$\text{or } \int_x \int_y (\tau_{yz} x - \tau_{zx} y) dy dx = \int_x \int_y (\bar{X} y - \bar{Y} x) dy dx = T \quad (3.14b)$$

Two sets of boundary equations which are important to this work can now be derived:

Clamped end

At a clamped end both rotation and axial displacement must be zero:

$$\theta = \psi = 0 \quad (3.15)$$

Free end

At a free end both the transmitted torque and the axial stresses must be zero. Equations (3.13b) and (3.14b) give respectively:

$$\frac{\partial \psi}{\partial z} = 0 \quad (3.16a)$$

$$\text{and } \mu_s J \left((\lambda - 1) \psi + \frac{\partial \theta}{\partial z} \right) = 0 \quad (3.16b)$$

Equation (3.14b) also defines the transmitted torque at any section:

$$T = \mu_s J \left((\lambda - 1) \psi + \frac{\partial \theta}{\partial z} \right) \quad (3.17)$$

3.2.3 Total energy

The analysis of total energy (strain energy plus kinetic energy) is important in that it can be used to provide a check on the numerical accuracy of the HSDT finite difference model in terms of conservation of energy.

From Love [3.3], the rate of change of kinetic energy is:

$$\frac{dU_k}{dt} = \int_x \int_y \int_z \rho \left(\frac{\partial^2 u}{\partial t^2} \frac{\partial u}{\partial t} + \frac{\partial^2 v}{\partial t^2} \frac{\partial v}{\partial t} + \frac{\partial^2 w}{\partial t^2} \frac{\partial w}{\partial t} \right) dx dy dz$$

and rate of change of strain energy is:

$$\begin{aligned} \frac{dU_s}{dt} = \int_x \int_y \int_z \left(\sigma_x \frac{\partial \epsilon_x}{\partial t} + \sigma_y \frac{\partial \epsilon_y}{\partial t} + \sigma_z \frac{\partial \epsilon_z}{\partial t} + \tau_{xy} \frac{\partial \gamma_{xy}}{\partial t} + \tau_{yz} \frac{\partial \gamma_{yz}}{\partial t} + \tau_{xz} \frac{\partial \gamma_{xz}}{\partial t} \right. \\ \left. + X \frac{\partial u}{\partial t} + Y \frac{\partial v}{\partial t} + Z \frac{\partial w}{\partial t} \right) dx dy dz \end{aligned}$$

Substituting for displacements from equations (3.2) and strains from equations (3.6), and evaluating the integrals with respect to the x and y axes, gives respectively:

$$\frac{dU_k}{dt} = \int_z \rho \left[P \frac{\partial^2 \psi}{\partial t^2} \frac{\partial \psi}{\partial t} + J \frac{\partial^2 \theta}{\partial t^2} \frac{\partial \theta}{\partial t} \right] dz$$

and

$$\frac{dU_s}{dt} = \int_z \left[EP \frac{\partial \psi}{\partial z} \frac{\partial^2 \psi}{\partial t \partial z} + \mu_s \left(K \psi \frac{\partial \psi}{\partial t} + L \psi \frac{\partial^2 \theta}{\partial t \partial z} + L \frac{\partial \theta}{\partial z} \frac{\partial \psi}{\partial t} + J \frac{\partial \theta}{\partial z} \frac{\partial^2 \theta}{\partial t \partial z} \right) + \Omega \theta \frac{\partial \theta}{\partial t} \right] dz$$

3.3 Resonance

Resonance analysis is useful in examining dispersion characteristics in the frequency domain. The analysis below relates to unrestrained prismatic beams and so the foundation stiffness term (Ω) is set to zero. The material is assumed to be linear elastic. The first part of this section considers the resonance of an infinite beam since it is simple to obtain the phase velocity as a function of frequency for this case. Consider an infinite train of torsional waves of unit amplitude. The displacement will be of the form:

$$\theta = \cos \left(pt - \frac{2\pi z}{\Lambda} \right)$$

Where Λ is the wavelength and p is the circular frequency. The phase velocity C is therefore:

$$C = \frac{p\Lambda}{2\pi}$$

Eliminating ψ between equations (3.10a) and (3.10b) and evaluating the derivatives of θ from its resonant form gives the following relationship between phase velocity and wavelength:

$$\left(\frac{C}{C_s}\right)^4 - \left(\frac{C}{C_s}\right)^2 \left(3 + 2\nu + \frac{1-\lambda}{(2\pi\gamma H/\Lambda)^2}\right) + \left(2(1+\nu) + \frac{\lambda(1-\lambda)}{(2\pi\gamma H/\Lambda)^2}\right) = 0 \quad (3.18)$$

where $C_s = \sqrt{\frac{\mu_s}{\rho}}$ is the shear wave speed.

The dispersion curve for a rectangular section ($\beta=0.25$) predicted from this equation is shown in figure 3.2. Also shown in the same figure are the dispersion curves predicted from the stress corrected theory used by Wheel, and the Saint-Venant theory. The stress corrected theory predicts an infinite phase velocity at very short wavelengths, whereas the present theory shows the phase velocity to become asymptotic to the shear wave speed.

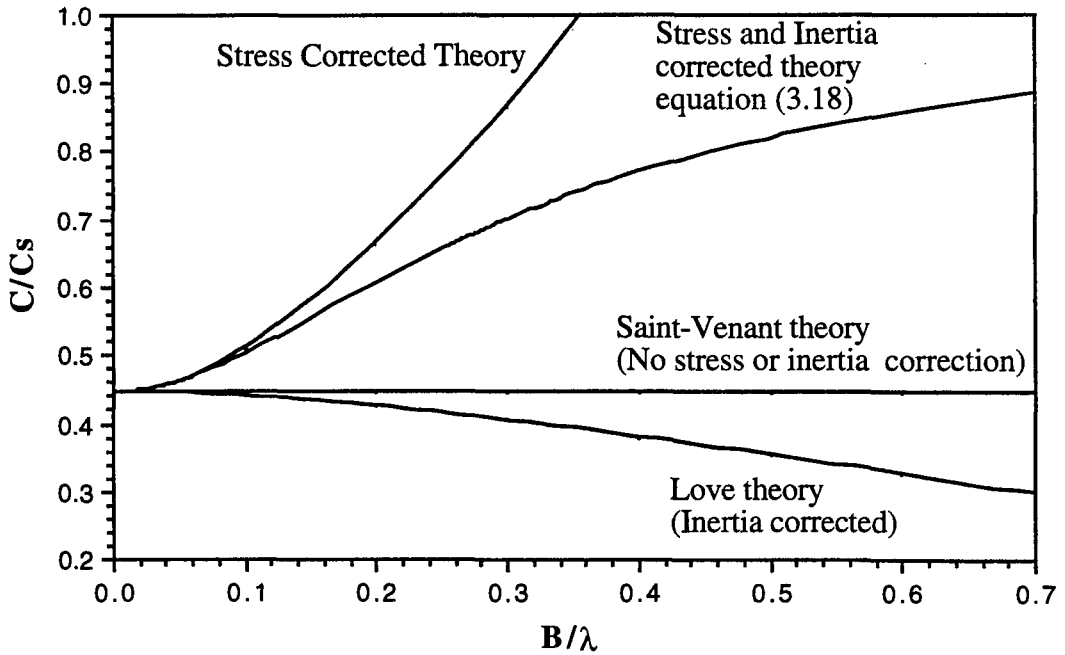


Fig 3.2 Dispersion curves for a rectangular section $\beta=0.25$ for various theories

In order to test his theory, Barr performed resonance experiments on prismatic, steel bars of finite length for a range of β values. He then compared these results with those predicted from the theory for infinite bars and found a notable discrepancy between the two. To correct for this he incorporated a correction factor into the differential equation for θ derived from equations (3.10a) and (3.10b). This effectively reduced the asymptotic phase velocity from the shear wave speed to the Raleigh wave speed.

The analysis for the resonance of finite bars presented below reduces the above mentioned error without the need for a correction factor. To allow for end effects, no initial assumption is made of the mode shape. The harmonic motion of circular frequency p of a free rod is then taken to be:

$$\theta = f(z)\text{Cos}(pt) \quad (3.19)$$

where $f(z)$, the mode shape, is to be found. Let the length of the beam be L such that it lies in the region $-L/2 \leq z \leq L/2$.

Eliminating ψ between equations (3.10a) and (3.10b) gives the general torsional wave equation in rectangular bars:

$$\frac{\partial^4 \theta}{\partial z^4} - \left(\frac{\rho}{\mu} + \frac{\rho}{E} \right) \frac{\partial^4 \theta}{\partial z^2 \partial t^2} - \frac{\lambda(1-\lambda)\mu}{\gamma^2 H^2 E} \frac{\partial^2 \theta}{\partial z^2} + \frac{(1-\lambda)\rho}{\gamma^2 H^2 E} \frac{\partial^2 \theta}{\partial t^2} + \frac{\rho^2}{E\mu} \frac{\partial^4 \theta}{\partial t^4} = 0$$

Substituting for θ from equation (3.19) shows that the above equation is always satisfied provided that:

$$\frac{\partial^4 \theta}{\partial z^4} - A_1 \frac{\partial^2 \theta}{\partial z^2} + A_2 \theta = 0 \quad (3.20)$$

$$\text{where } A_1 = \frac{\lambda(1-\lambda)}{2(1+\nu)\gamma^2 H^2} - \frac{\lambda(3+2\nu)}{2(1+\nu)} \left(\frac{j\pi}{L} \right)^2 \left(\frac{p}{p_{sv}} \right)^2$$

$$\text{and } A_2 = \frac{\lambda^2}{2(1+\nu)} \left(\frac{j\pi}{L} \right)^4 \left(\frac{p}{p_{sv}} \right)^4 - \frac{\lambda(1-\lambda)}{2(1+\nu)\gamma^2 H^2} \left(\frac{j\pi}{L} \right)^2 \left(\frac{p}{p_{sv}} \right)^2$$

The term p_{sv} is the resonant frequency of mode j predicted by the Saint-Venant frequency equation:

$$p_{sv}^2 = \left(\frac{j\pi}{L} \right)^2 \frac{\mu\lambda}{\rho}$$

The general solution of equation (3.20) is:

$$\theta = C_1 e^{\omega_1 z} + C_2 e^{-\omega_1 z} + C_3 e^{\omega_2 z} + C_4 e^{-\omega_2 z} \quad (3.21)$$

where $\omega_1 = \left(\frac{A_1 \pm \sqrt{A_1^2 - 4A_2}}{2} \right)^{\frac{1}{2}}$ and $C_{1..4}$ are constants.

One root will be real and the other complex provided that:

$$\lambda < 1 \quad \text{and} \quad p^2 < \frac{G(1-\lambda)}{\rho \gamma^2 H^2}$$

The first inequality is always met and the second denotes a border between a higher and lower branch of torsional motion. The higher branch is predominantly axial motion and is not dealt with in this work.

Assuming that the above inequalities are satisfied, equation (3.21) can be transformed to:

$$\theta = C_1 \text{Sinh}(\omega_1 z) + C_2 \text{Cosh}(\omega_1 z) + C_3 \text{Sin}(\omega_2 z) + C_4 \text{Cos}(\omega_2 z) \quad (3.22)$$

where ω_2 is redefined as: $\omega_2 = \left(\frac{\sqrt{A_1^2 - 4A_2} - A_1}{2} \right)^{\frac{1}{2}}$

The coefficients C_3 and C_4 correspond to the resonance amplitude which is taken as unity here. Due to asymmetry about the mid-point of the beam for uneven modes and symmetry for even modes equation (3.22) can be expressed as two simpler equations:

$$\theta = C_1 \text{Sinh}(\omega_1 z) + \text{Sin}(\omega_2 z) \quad (3.23a)$$

(uneven Modes)

and $\theta = C_2 \text{Cosh}(\omega_1 z) + \text{Cos}(\omega_2 z) \quad (3.23b)$

(even modes)

At $z=L/2$, free end conditions from equations (3.16) give:

$$\frac{\partial^2 \theta}{\partial z^2} + B_1 \theta = 0 \quad (3.24a)$$

where $B_1 = \lambda \left(\frac{j\pi}{L} \right)^2 \left(\frac{p}{p_{sv}} \right)^2$

and $B_2 \frac{\partial \theta}{\partial z} - B_3 \frac{\partial^3 \theta}{\partial z^3} = 0$ (3.24b)

where $B_2 = \lambda - (3 + 2\nu) \frac{\lambda(\gamma H)^2}{(1 - \lambda)} \left(\frac{j\pi}{L} \right)^2 \left(\frac{p}{p_{sv}} \right)^2$ and $B_3 = \frac{2(1 + \nu)(\gamma H)^2}{(1 - \lambda)}$

Consider uneven modes: Substituting θ and its second derivative from equation (3.23a) into equation (3.24a) gives:

$$C_1 = \frac{(\omega_2^2 - B_1) \sin(\omega_2 L/2)}{(\omega_1^2 + B_1) \sinh(\omega_1 L/2)} \quad (3.25)$$

Substituting the first and third derivatives of θ from equation (3.23a) into equation (3.24b) gives:

$$C_1 = \frac{\omega_2 (B_3 \omega_2^2 + B_2) \cos(\omega_2 L/2)}{\omega_1 (B_3 \omega_1^2 - B_2) \cosh(\omega_1 L/2)} \quad (3.26)$$

Elimination of C_1 between equations (3.25) and (3.26) gives:

$$\frac{(\omega_1^2 + B_1)}{\omega_1 (B_3 \omega_1^2 - B_2)} \tanh(\omega_1 L/2) = \frac{(\omega_2^2 - B_1)}{\omega_2 (B_3 \omega_2^2 + B_2)} \tan(\omega_2 L/2)$$

A numerical technique was used to solve this equation, and the results for various values of β are shown in figure 3.3. This figure also depicts experimental results of Barr and the theoretical solution obtained when the bar is assumed to be infinite. The finite bar theory shows a considerably improved correlation with experiment, although the discrepancy with experiment does increase for squarer cross sections ($\beta \rightarrow 1$).

The finite beam effect can be seen when the mode shapes are plotted as in figure 3.4. End effects can clearly be seen to increase with decreasing wavelength.

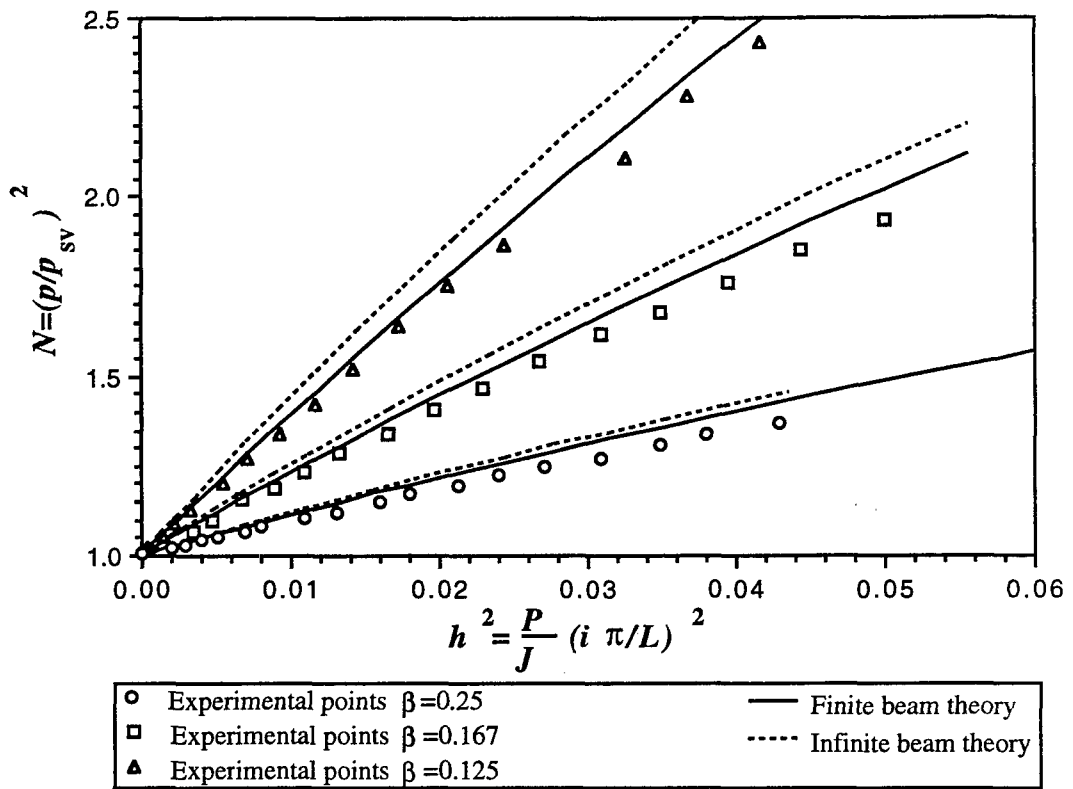


Fig. 3.3 Experimental values of $(p/p_{sv})^2$ compared with theory

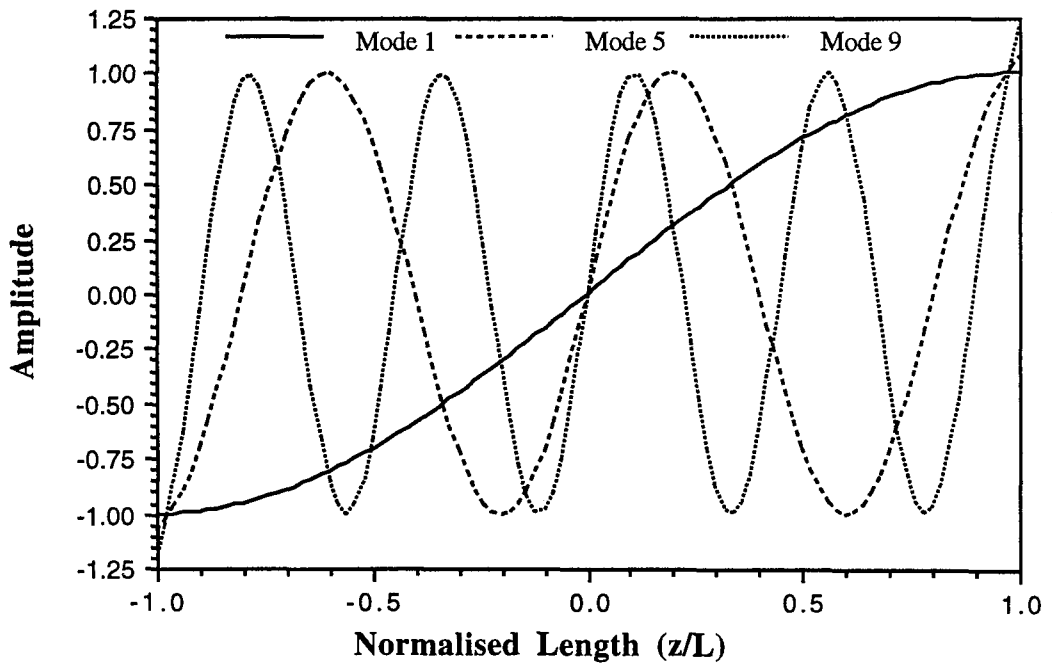


Fig. 3.4 Mode Shapes For Torsional Resonance with $\beta=0.12$.

3.4 Timoshenko Case

Timoshenko presented [3.6] a classical problem of static non-uniform torsion of rectangular beams, which he attempted to solve for thin sections. In this case the section at

one end is forced to remain plane (clamped end boundary condition). The other end is loaded by a pure torque. Timoshenko used a strain energy minimisation approach and the assumption that the axial stresses were proportional to the associated axial displacement (w) for the unrestrained case, decreasing exponentially from the clamped end.

This section derives the solution for this case using the static form of equations (3.10a) and (3.10b). The results are then compared with numerical results from a FV model written by Ivankovic [3.11]. The material is taken as being linear elastic.

Eliminating ψ from equations (3.10a) and (3.10b), and setting all derivatives with respect to t to zero gives the general differential equation describing the deformation of a static continuous prismatic beam:

$$\frac{\partial^4 \theta}{\partial z^4} - \left(\frac{\zeta}{H} \right)^2 \frac{\partial^2 \theta}{\partial z^2} = 0 \quad (3.27)$$

where $\zeta^2 = \frac{\lambda(1-\lambda)}{2(1+\nu)\gamma^2}$.

The general solution of equation (3.27) is:

$$C_1 + C_2 z + C_3 e^{\frac{\zeta}{H} z} + C_4 e^{-\frac{\zeta}{H} z} = \theta \quad (3.28)$$

where $C_{1..4}$ are constants.

The clamped end is taken to be at $z=0$ and the boundary conditions of equations (3.15) give:

$$\left. \frac{\partial^3 \theta}{\partial z^3} \right|_{z=0} - \frac{(\lambda-1)}{\lambda} \left(\frac{\zeta}{H} \right)^2 \left. \frac{\partial \theta}{\partial z} \right|_{z=0} = 0 \quad (\text{for } \psi = 0) \quad (3.29a)$$

$$\text{and } \theta = 0. \quad (3.29b)$$

Load plane boundary conditions ($z=L$):

$$\sigma_z = 0 \text{ gives } \frac{\partial^2 \theta}{\partial z^2} = 0 \quad (3.30a)$$

$$\text{and } \theta = \theta_{z=L}. \quad (3.30b)$$

Substituting for θ and its derivatives from equation (3.28) into equations (3.29) and (3.30) gives:

$$(\lambda - 1) \frac{H}{\zeta} C_2 - C_3 + C_4 = 0$$

$$C_1 + C_3 + C_4 = 0$$

$$C_4 + C_3 e^{2\frac{\zeta}{H}L} = 0$$

$$C_1 + C_2 L + C_3 e^{\frac{\zeta}{H}L} + C_4 e^{-\frac{\zeta}{H}L} = \theta_{z=L}$$

Eliminating C_1 , C_3 and C_4 from the above equations gives:

$$C_2 = \frac{\theta_{z=L}}{L - \Delta L} \quad (3.31)$$

$$\text{where } \Delta L = (1 - \lambda) \frac{\left(e^{2\frac{\zeta}{H}L} - 1 \right)}{\left(e^{2\frac{\zeta}{H}L} + 1 \right)} \frac{H}{\zeta}$$

The transmitted torque is the same at all values of z . The torque is most easily calculated at $z=0$ yielding:

$$T = \mu J \lambda C_2.$$

$$\text{Substituting for } C_2: \quad T = \mu J \lambda \frac{\theta_{x=L}}{L - \Delta L}.$$

This result was compared with FV results (a numerical method) for different section ratios. This problem is numerically difficult due to the high stress gradients across the section and the non uniform twist being concentrated within a small axial distance. In order to deal with these problems plates of 200 by 50mm and a range of thicknesses were modelled using very small elements (1mm sided cubes). The length ΔL corresponds to the effective shortening of the beam such that a beam of length $(L - \Delta L)$ subject to the same angle of rotation at the loaded end, but with a constant twist along its length, would display the same transmitted torque. The effective shortening was calculated from the FV results by using a straight line fit to the rotation profile in its linear region (see figure 3.5).

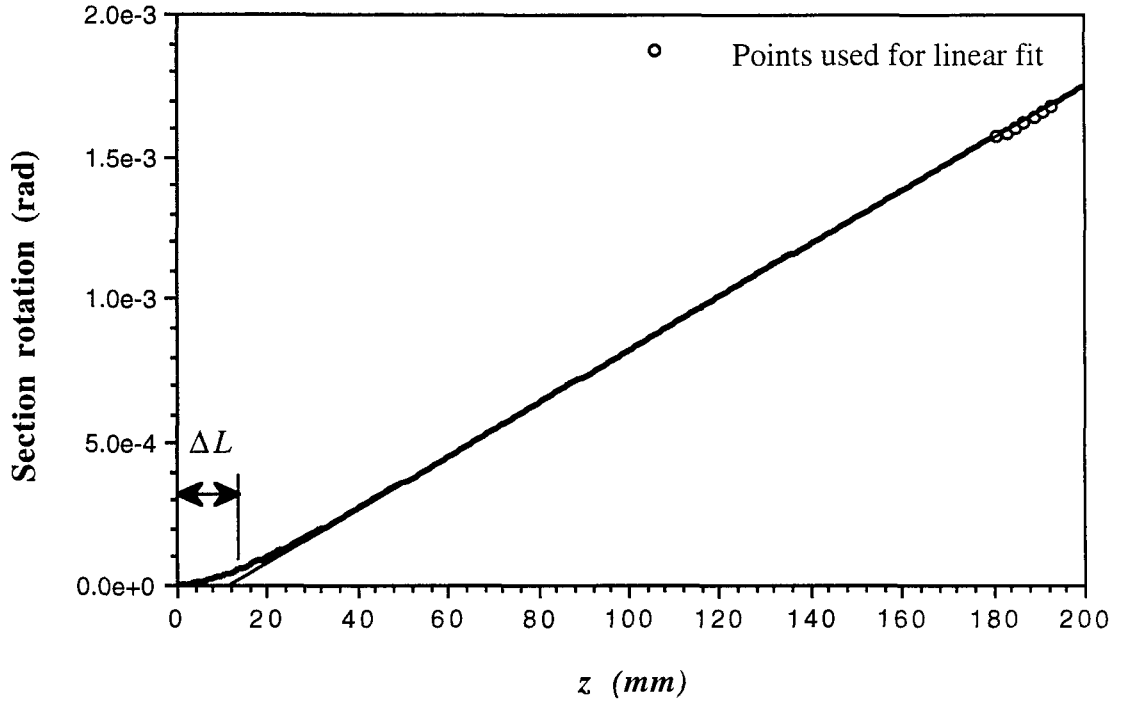


Fig. 3.5 Calculation of ΔL from the FV rotation profile ($\beta=0.04$)

The final results are shown in figure 3.6 and show excellent agreement between FV and analytical solutions. Timoshenko's solution is reasonable for very thin sections but rapidly diverges from the analytical solution with increasing values of β .

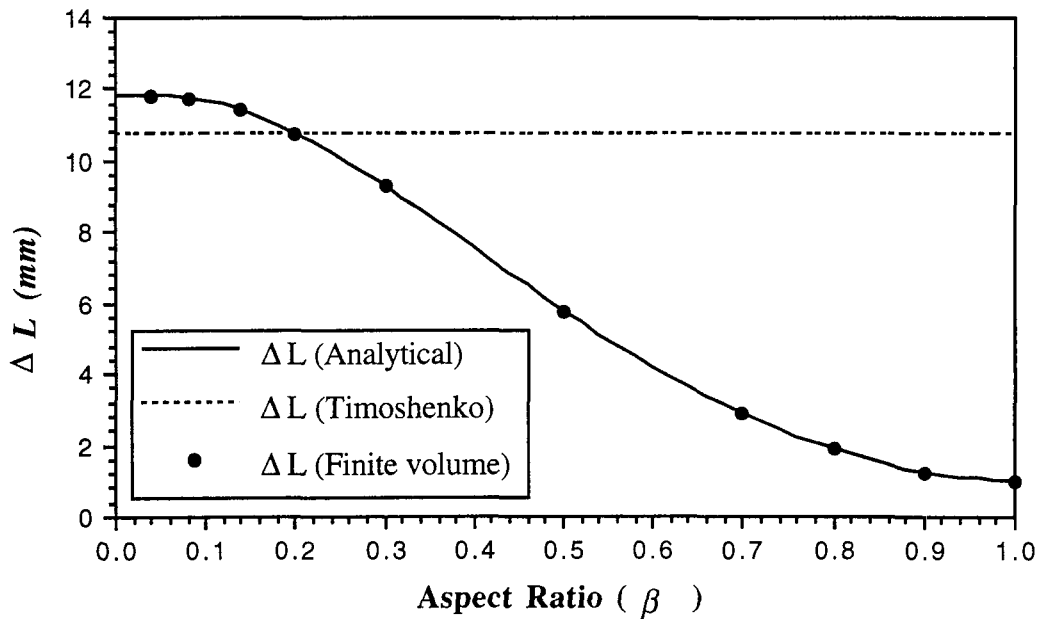


Fig 3.6 ΔL as a function of β calculated using both FV and analytical approaches

3.5 References

- (3.1) Barr, A.D.S., 'Torsional waves in uniform rods of non-circular cross section', *J. Mech. Eng. Sci.*, **4**, p.127, (1962).
- (3.2) Gere, J.M., 'Torsional vibrations of beams of thin walled open sections', *J. App. Mech.*, **21**, p. 381, (1954).
- (3.3) Love, A.E.H., *Mathematical theory of elasticity*, Fourth Edition, Cambridge University Press, (1927).
- (3.4) Saint-Venant, B. de, 'Mémoire sur les vibrations tournemantes des verges élastiques', *Comptes Rendus*, **28**, pp. 69, (1849).
- (3.5) Wheel, M.A. and Leevers, P.S., 'High speed double torsion tests on tough polymers. II: Non-linear elastic dynamic analysis', *Int. J. of Fracture*, **61**, p. 349, (1993).
- (3.6) Timoshenko, S.P. and Goodier, J.N., *Theory of elasticity*, Third Edition, McGraw-Hill, (1970).
- (3.7) Wheel, M.A. and Leevers, P.S., 'High speed double torsion tests on tough polymers I: Linear Elastic Steady State and Dynamic Analysis', *Int. J. of Fracture*, **61**, p. 331, (1993).
- (3.8) Leevers, P.S., 'Crack Front Shape Effects in the Double Torsion Test', *J. of Mat. Science*, **17**, p. 2469, (1982).
- (3.9) Reissner, E., 'On non-uniform torsion of cylindrical rods', *J. Math. Phys.*, **31**, p. 214, (1952).
- (3.10) Davies, R.M., 'Frequency of longitudinal and torsional vibration of unloaded and loaded bars', *Phil. Mag.*, **25**, p. 364, (1938).
- (3.11) Demirdzic, I., Ivankovic, A., Martinovic, D. and Muzaferija, S., 'Numerical methods for solving linear and non-linear solid body problems', *Proc., 1st Congress of Croatian Society of Mechanics*, Pula, (1994).

Chapter 4

The double torsion test and the curved crack front

4.1 Introduction

The Double Torsion (DT) test displays a pronounced curved crack front (see figure 7.18). This chapter demonstrates the different characteristics of the crack driving force (G) when analysing the DT test with a curved as compared to a straight crack front. Leever [4.1] has examined the shape of the curved crack front; he used a crack opening displacement (COD) approach to successfully predict this shape in low rate tests on thermosetting polyester resins. Wheel [4.2] used this approach in his dynamic analysis, examining the effect of the curved crack front on G . His results showed little difference in predicted dynamic fracture resistance (G_D) between straight and curved crack front models.

The initial aim of this part of the work was to quantify crack speed along the whole crack front, rather than just the axial crack velocity of the leading edge generated by the straight crack front model. To enable this calculation the shape of the evolving crack front had to be determined. At this stage a discrepancy was found with Wheel's statement that the calculated G_D was not affected by whether the crack front was modelled as straight or curved.

Dynamic effects have been avoided in most of the work of this chapter since they are difficult both to measure experimentally and to analyse. The displacement rates were therefore kept low (less than 1 mm/min) or at zero. The exclusion of dynamic behaviour does not preclude the use of the derived equations (for foundation stiffness) in the dynamic model as justified in section 4.5. Further simplifications are made by assuming the material to be linear elastic with a crack speed independent fracture resistance (G_D).

Section 4.2 derives the equations used to model the deformation of the DT specimen with a straight crack front. These include a description of Leever's analysis of the foundation of the DT specimen, which incorporated a calibration factor which was originally attributed a constant, geometry independent value. Here the calibration factor is determined by experiment and finite element (FE) analysis as a function of thickness and 'V' groove depth.

Section 4.3 examines the curved crack front case of the static DT test. This section is based on experimental measurements of the compliance of PMMA DT specimens with natural

(fracture induced) curved crack fronts. These results are compared with those measured for a machined straight crack front. The analytical model for the case of the curved crack front is then developed.

Section 4.4 derives equations to calculate the local crack driving force along the crack front and develops a new method of predicting the crack front shape. Finally, a method of implementing this work in the dynamic model is discussed.

4.2 The straight crack front DT test model

The full analytical solution of the straight crack front DT model is derived from the equations presented in chapter 3. This model forms the basis for all of the following work.

4.2.1 Foundation stiffness

The derivation of the equations of motion in chapter 3 assumes that foundation stiffness can be modelled as a restoring moment per unit length acting across the torsion beam section, such that the section does not change in shape. In reality, the moment is due to material continuity between the two half-specimen torsion beams, so that by symmetry $\partial v/\partial x$ must be zero along the future crack plane (see figure 4.1).

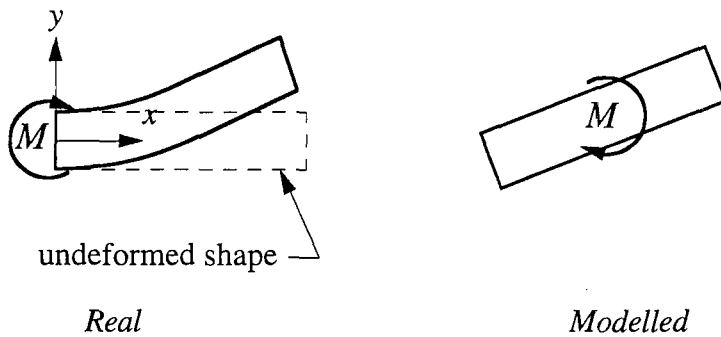


Fig. 4.1 Half-specimen section showing the real and modelled restoring moments due to the foundation.

To evaluate the restoring moment the real case of figure 4.1 is considered below.

Consider a section through the foundation in the xy plane as shown in figure (4.2). The foundation is assumed to deform as a simple elastic beam subjected to a pure bending moment about its built-in ends. The thickness of the beam is taken to be $(B/2+\xi)$, ξ being the y co-ordinate of the bottom of the 'V'. The effects of the material in the region $-B/2 < y < \xi$ and associated stress concentrations are therefore neglected in this initial analysis. The length of beam is defined here as $2c'H$ where c' is a non-dimensional factor

to be determined. The definition differs slightly from that of Leever and Wheel [4.2] in that the width of the beam (H) is used, rather than the remaining ligament thickness (s), as a normalising parameter.

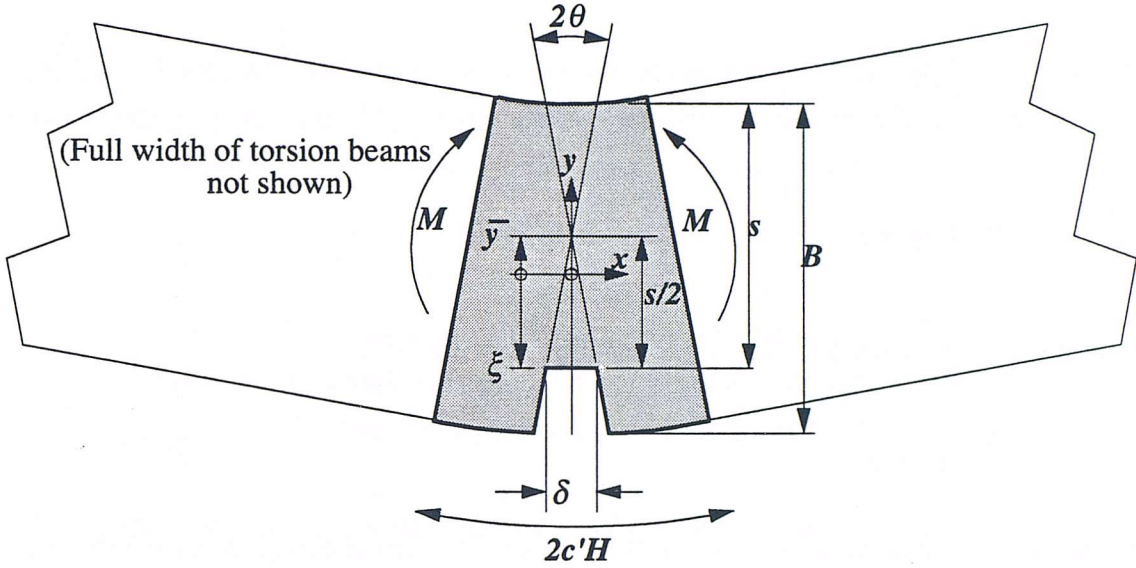


Fig. 4.2 Section of Specimen Showing the Elastic Foundation

The normal stresses and strains in the x direction can then be calculated by simple beam bending theory:

$$\sigma_x = (y - \bar{y}) \frac{E}{R} \quad (4.1)$$

$$\varepsilon_x = (y - \bar{y}) \frac{1}{R} \quad (4.2)$$

In this equation E is the flexural modulus and $\bar{y}$ is the position of the neutral axis. R is the local radius of curvature:

$$\frac{1}{R} = \frac{\theta}{c'H} \quad (4.3)$$

Where θ is the asymptotic foundation rotation at $x=H$.

The restoring moment (M) about $\bar{y}$, per unit length of foundation, is calculated by integrating across the thickness of the foundation in the normal manner:

$$M = \int_{\xi}^{\frac{B}{2}} \sigma_x (y - \bar{y}) dy$$

Evaluating this integral gives:

$$M = \frac{E\theta}{c'H} \left(\frac{B^3}{24} - \frac{\bar{y}B^2}{4} + \frac{\bar{y}^2B}{2} - \frac{\xi^3}{3} + \bar{y}\xi^2 - \bar{y}^2\xi \right) \quad (4.4)$$

For a constant ligament thickness in the root region equal to s , it is assumed that $\bar{y} = (B-s)/2$ (the centre of the remaining ligament). Since $\xi = (B-2s)/2$ equation (4.4) then reduces to:

$$M = \frac{Es^3}{12c'H} \theta \quad (4.5)$$

By definition from equation (3.8) the foundation stiffness is:

$$\Omega = \frac{Es^3}{12c'H} \quad (4.6)$$

The inclusion of this equation in the full analytical solution is described in section 4.2.2. The ligament thickness, s , in the case of the straight crack front, is equal to the difference between B and the depth of the 'V' groove. The validity of equation (4.6) is questionable when a 'V' groove is present because of the asymmetry between stresses at the groove root and the flat free surface. In this case equation (4.6) will be treated as a normalising function which allows the dependence of c' on s and B to be determined by iteratively matching the analytical compliance to FE and experimental results. The dual approach of FE and experiment was taken in order to provide a check on each method's reliability. The effect of two parameters was tested:

1) Specimen thickness dependency

In these tests there was no 'V' groove and B was varied between 7 mm and 9.525 mm.

2) 'V' groove dependency

In these tests B was kept constant at 9.525 mm and the depth of 'V' groove was varied between 0 mm and 7 mm.

In all tests the standard geometry of the DT test was used (see section 1.3.1) with a 100 mm long straight fronted crack. The tests are discussed in the sections 4.2.3 and 4.2.4.

4.2.2 Analytical model

The material is assumed to be linearly elastic. To model the double torsion test with a straight crack front the beam must be split into three discontinuous regions, as shown in figure 4.3.

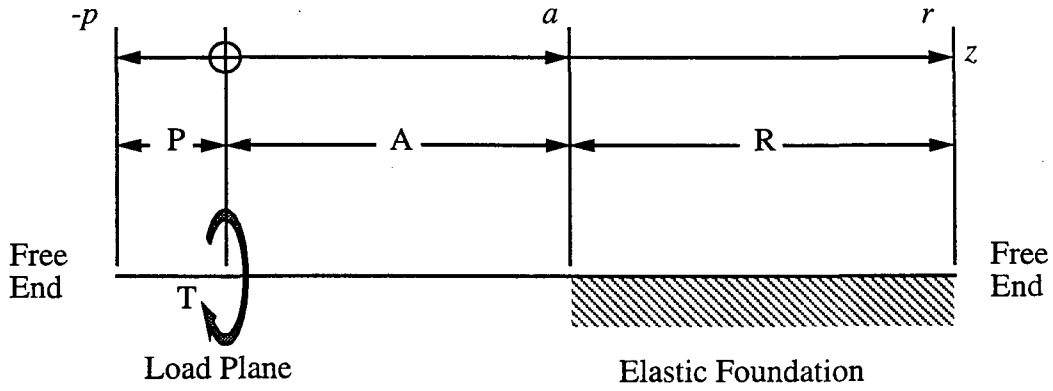


Fig. 4.3 Modelling of the static Double Torsion test

Region P : The region of overhang beyond the load point.

Region A : The region between the load point and the crack front.

Region R : The beam root region ahead of the crack front.

The general solution is derived first, followed by the boundary conditions.

4.2.2.1 General solution

The first step is to eliminate ψ between equations (3.10a) and (3.10b). Noting the partial derivatives with respect to time are zero for the static case, the equation used to model region P and A is identical to equation (3.27):

$$\frac{\partial^4 \theta}{\partial z^4} - \left(\frac{\zeta}{H} \right)^2 \frac{\partial^2 \theta}{\partial z^2} = 0$$

and to model region R is:

$$\frac{\partial^4 \theta}{\partial z^4} - \left(\frac{\omega}{H} \right)^2 \frac{\partial^2 \theta}{\partial z^2} + \left(\frac{\varphi}{H} \right)^4 \theta = 0$$

where $\omega^2 = \zeta^2 - \Omega^*$, $\varphi^4 = \Omega^* \frac{\zeta^2}{\lambda}$ and Ω^* is the non dimensional form of Ω such that:

$$\Omega^* = \frac{\Omega H^2}{\mu J}$$

The general solution for region P is therefore:

$$\theta = P_1 + P_2 z + P_3 e^{\frac{\zeta}{H} z} + P_4 e^{-\frac{\zeta}{H} z}, \quad (4.7a)$$

and region A is:

$$\theta = A_1 + A_2 z + A_3 e^{\frac{\zeta}{H} z} + A_4 e^{-\frac{\zeta}{H} z}, \quad (4.7b)$$

and finally region R is:

$$\theta = R_1 e^{q_1 z} + R_2 e^{-q_1 z} + R_3 e^{q_2 z} + R_4 e^{-q_2 z} \quad (4.7c)$$

where $q_{1,2} = \pm \sqrt{\frac{\omega^2 \pm \sqrt{\omega^4 - 4\phi^4}}{2H^2}}$ and P_n, A_n, R_n ($n=1..4$) are constants derived from the boundary conditions.

4.2.2.2 Boundary conditions

The boundary conditions are as follows:

z=-p (free end)

Eliminating ψ from equations (3.16) and substituting for θ and its derivatives from equation (4.7a) gives:

$$P_2 = 0 \quad (4.8a)$$

$$\& P_3 + P_4 e^{\frac{2\zeta}{H} p} = 0 \quad (4.8b)$$

z=0 (load plane)

Substituting the load plane rotation (θ_L) in equations (4.7a) and (4.7b) gives:

$$P_1 + P_3 + P_4 = \theta_L \quad (4.8c)$$

$$A_1 + A_3 + A_4 = \theta_L \quad (4.8d)$$

Axial displacements must be continuous across the regions P and A, so that:

$$\psi_b = \psi_a$$

Equations (4.7a) and (4.7b) therefore give:

$$P_3 - P_4 + \frac{(1-\lambda)H}{\zeta} P_2 = A_3 - A_4 + \frac{(1-\lambda)H}{\zeta} A_2 \quad (4.8e)$$

Similarly, axial stresses must be continuous across the regions P and A, so that:

$$\left(\frac{\partial \psi}{\partial z}\right)_b = \left(\frac{\partial \psi}{\partial z}\right)_a$$

Equations (4.7a) and (4.7b) therefore give:

$$P_3 + P_4 = A_3 + A_4 \quad (4.8f)$$

z=a (crack tip plane)

Continuity in all displacements and stresses across regions A and R is assumed. This is satisfied by matching θ and its first 3 derivatives with respect to z in equations (4.7b) and (4.7c):

$$R_1 e^{q_1 a} + R_2 e^{-q_1 a} + R_3 e^{q_2 a} + R_4 e^{-q_2 a} = A_1 + A_2 a + A_3 e^{\frac{\zeta}{H} a} + A_4 e^{-\frac{\zeta}{H} a} \quad (4.8g)$$

$$q_1 R_1 e^{q_1 a} - q_1 R_2 e^{-q_1 a} + q_2 R_3 e^{q_2 a} - q_2 R_4 e^{-q_2 a} = A_2 + \left(\frac{\zeta}{H}\right) A_3 e^{\frac{\zeta}{H} a} - \left(\frac{\zeta}{H}\right) A_4 e^{-\frac{\zeta}{H} a} \quad (4.8h)$$

$$q_1^2 R_1 e^{q_1 a} + q_1^2 R_2 e^{-q_1 a} + q_2^2 R_3 e^{q_2 a} + q_2^2 R_4 e^{-q_2 a} = \left(\frac{\zeta}{H}\right)^2 A_3 e^{\frac{\zeta}{H} a} + \left(\frac{\zeta}{H}\right)^2 A_4 e^{-\frac{\zeta}{H} a} \quad (4.8i)$$

$$q_1^3 R_1 e^{q_1 a} - q_1^3 R_2 e^{-q_1 a} + q_2^3 R_3 e^{q_2 a} - q_2^3 R_4 e^{-q_2 a} = \left(\frac{\zeta}{H}\right)^3 A_3 e^{\frac{\zeta}{H} a} - \left(\frac{\zeta}{H}\right)^3 A_4 e^{-\frac{\zeta}{H} a} \quad (4.8j)$$

z=r (free end)

As for $z=-p$, ψ is eliminated from equations (3.16) and substituting for θ and its derivatives from equation (4.7c) gives:

$$(mq_1 - nq_1^3)e^{q_1 r} R_1 - (mq_1 - nq_1^3)e^{-q_1 r} R_2 + (mq_2 - nq_2^3)e^{q_2 r} R_3 - (mq_2 - nq_2^3)e^{-q_2 r} R_4 = 0 \quad (4.8k)$$

where $m = \lambda \left(1 + \frac{\Omega^*}{\zeta^2}\right)$ and $n = \lambda \left(\frac{H}{\zeta}\right)^2$

and

$$\left(q_1^2 - \frac{\Omega}{\mu J}\right) R_1 e^{q_1 r} + \left(q_1^2 - \frac{\Omega}{\mu J}\right) R_2 e^{-q_1 r} + \left(q_2^2 - \frac{\Omega}{\mu J}\right) R_3 e^{q_2 r} + \left(q_2^2 - \frac{\Omega}{\mu J}\right) R_4 e^{-q_2 r} = 0 \quad (4.8l)$$

These twelve equations (4.8a-l) can then be solved using Gaussian elimination and the beam profile and compliance calculated for a given value of c' .

4.2.3 Finite element model

ABAQUS, a standard FE software package, was used to model the static case of the DT test. The package IDEAS was used as a post processor to obtain stress contour plots. The element type used was a twenty noded, quadratic, 3D brick (C3D20). As in the analytical case only one half of the DT specimen needed to be modelled due to symmetry. The mesh and boundary conditions were designed to match the experimental configuration as accurately as possible and are described below. Small displacement theory was used in all the solutions.

4.2.3.1 Mesh

The half-specimens modelled had the dimensions 200 by 50 mm and between 7 and 9.525 mm thickness. Each mesh consisted of 500 elements (five elements through the thickness and width and twenty elements along the length of the specimen). The minimum aspect ratio of any element was 100:100:14 (14 corresponding to the thickness direction). The notation used to define the support and load points is shown in figure 4.4.

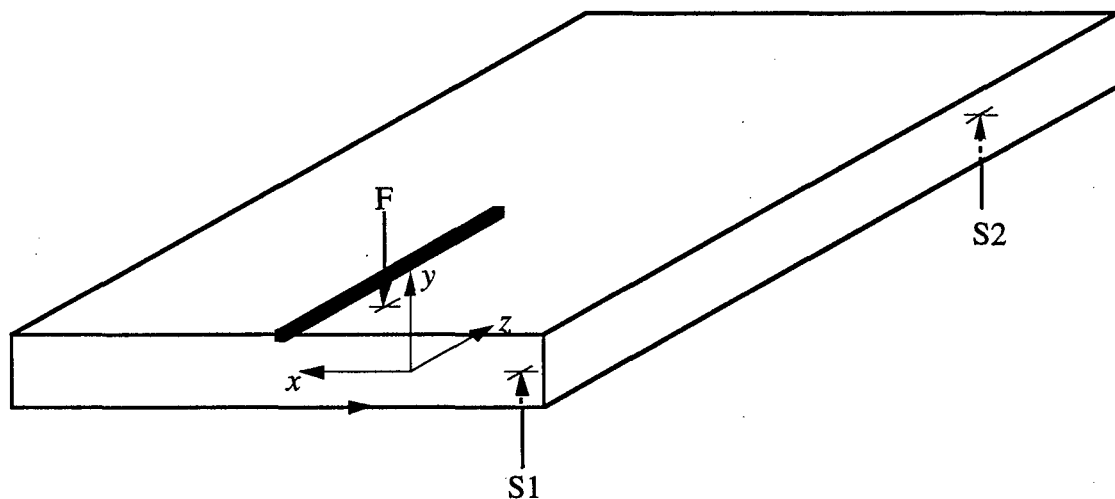


Fig. 4.4 Notation used in the FE model.

4.2.3.2 Boundary conditions

The boundary conditions can be divided into four areas:

- The loading (F) was simulated by imposing a 20 kN nodal force in the negative y direction at the appropriate node.
- The support at the load plane ($S1$) was modelled by restraining the appropriate node in the z and y directions. The restraint in the z direction was made to avoid a singularity due to rigid body motion. The solution predicted reaction forces in the order of 10^{-9} N

at this node in the z direction, which is negligible relative to the applied force. The magnitude of the y component of the reaction force was always within 0.1% of the applied force.

c) The rear support (S2) was modelled by restraining the appropriate node in the y direction.

d) The plane of symmetry was modelled by restraining all the element nodes in this plane in the x direction (rotational degrees of freedom are not applicable to this element type). Ligament thickness and crack length were simulated by releasing this restraint on the appropriate element nodes.

4.2.3.3 Finite element results

The results from the FE modelling that relate to the calibration of c' are presented in section 4.2.5 with the experimental results. Additional results specific to the FE modelling are presented in this section.

Initial FE calculations were performed to check the convergence of the solution in terms of the number of elements used in the z and y directions. The final spacing was set such that any further mesh refinement produced only a 0.05% change in the predicted compliance of the DT specimen.

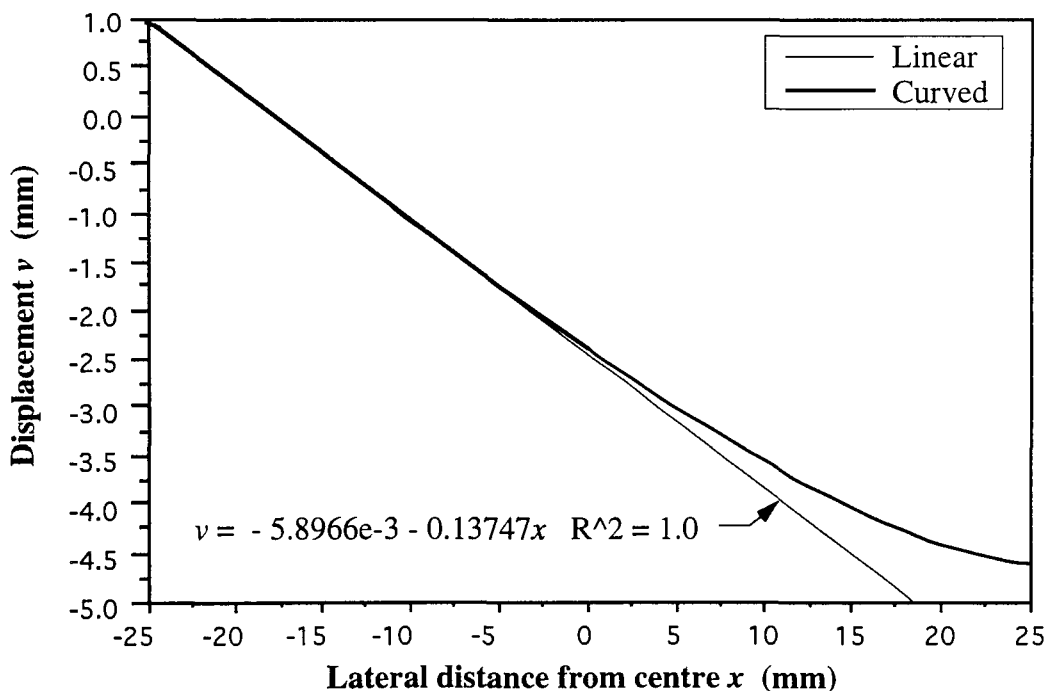


Fig. 4.5 Calculation of section rotation from nodal displacements

The outputs from selected FE solutions were converted into rotation profiles. This was calculated using the y component of the nodal displacements on the upper surface of the mesh to produce section rotation plots (see figure 4.5) at each cross-section. A linear least

squares fit was used to calculate the equation of the line through the nodes in the linear region. By small displacement theory the slope of this line gives the section rotation in radians.

a) Loading conditions

The axis of rotation in the analytical solution is assumed to be coincident with the centroidal axis of the half beam and the torque applied as shown in figure 4.6b. In reality the torque and displacement of the load plane are as shown in figure 4.6a, the axis of rotation passing approximately through the support points (S1 and S2 in figure 4.4).

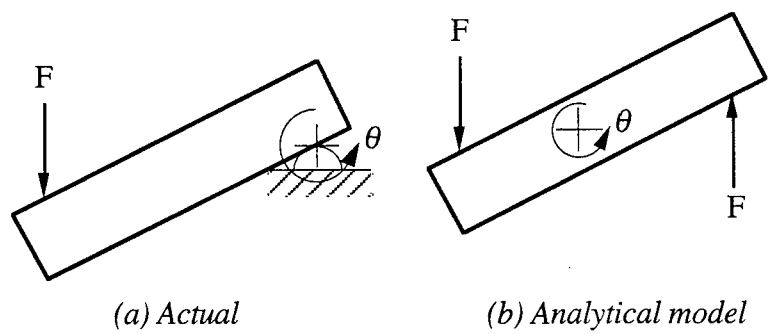


Fig. 4.6 Actual and assumed loading conditions on the DT half-specimen

Both cases (actual and modelled) were tested with the FE model. The results are shown in the form of rotation profiles in figure 4.7. There is only a slight disparity between the two cases (0.7% maximum difference in section rotation at 70 mm from the loaded end).

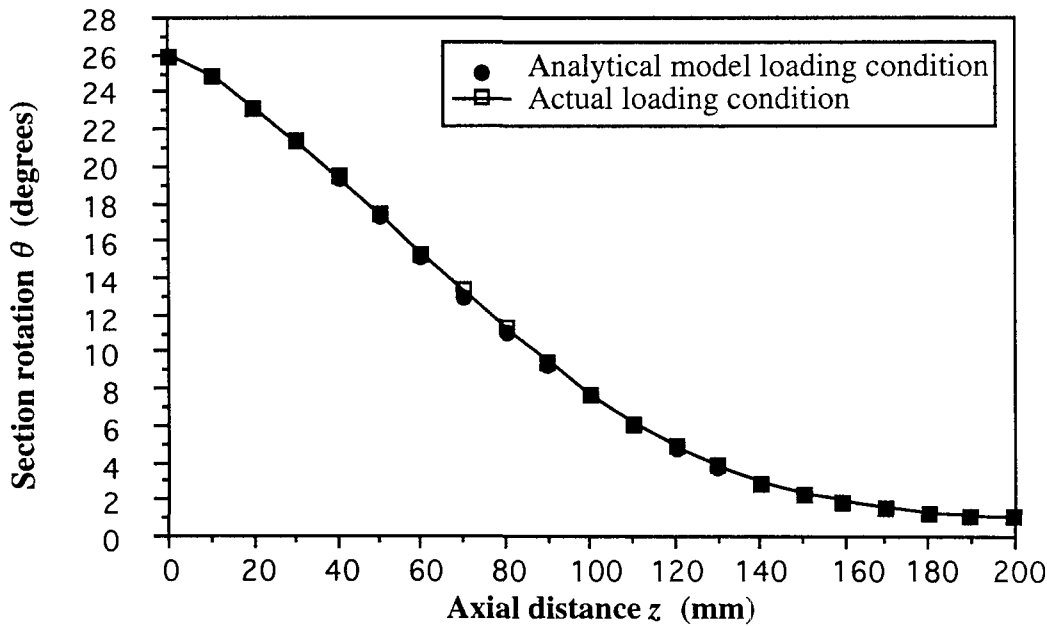


Fig. 4.7 Rotation profiles for the two loading cases shown in figure 4.6
(crack length = 100 mm)

b) The direct and shear stresses in the free beam

The shear stress distributions of τ_{xz} and τ_{yz} closely match those predicted analytically (see figures 4.8 and 4.9), τ_{xy} is negligible as assumed in the analytical solution. The distribution of the direct stresses along the axis of the plate (σ_z) closely follows that of the analytical solution as shown in figure 4.10. The direct stresses in the through thickness direction are negligible, as assumed, but in figure 4.11 the influence of the foundation can be seen on σ_x .

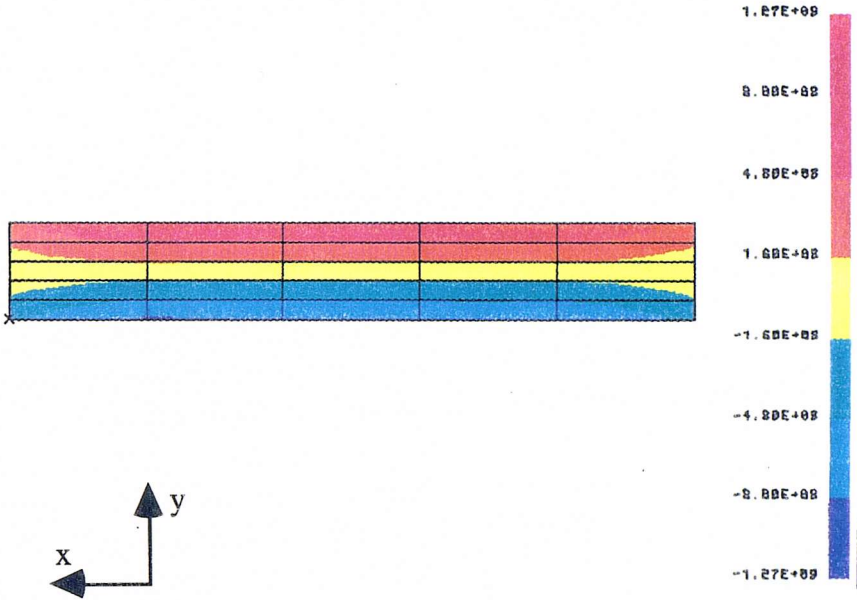


Fig. 4.8 τ_{xz} in the free beam ($z = 80 \text{ mm}$)

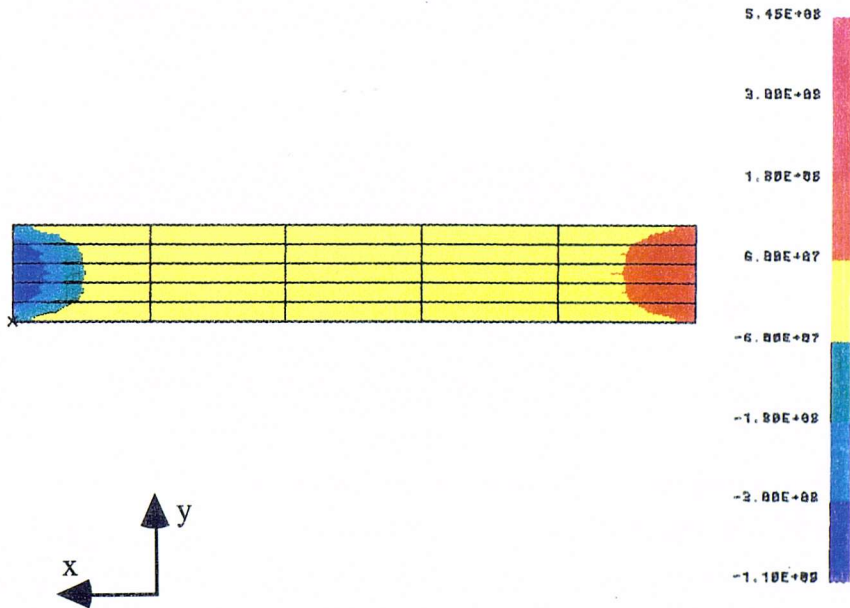


Fig. 4.9 τ_{yz} in the free beam ($z = 80 \text{ mm}$)

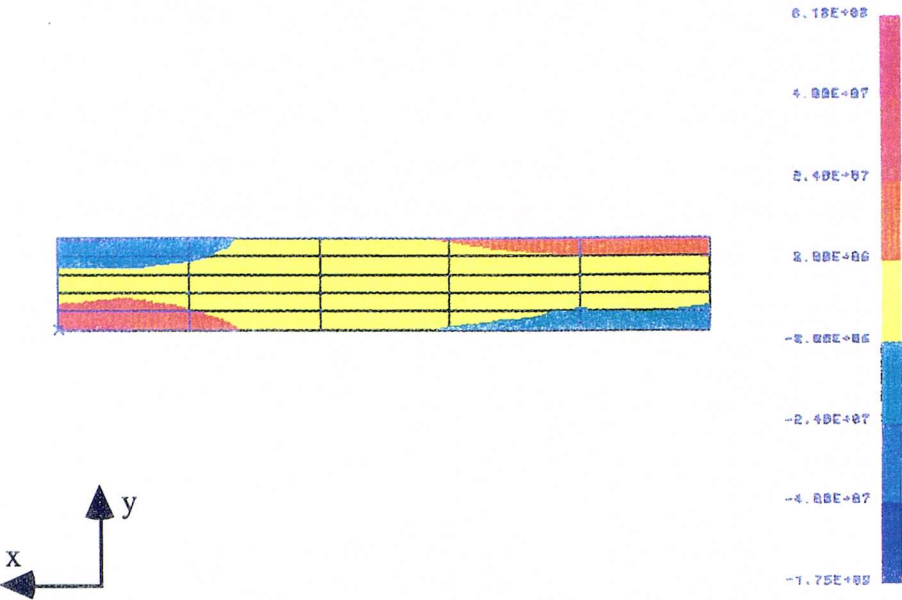


Fig. 4.10 σ_z in the free beam ($z = 80$ mm)

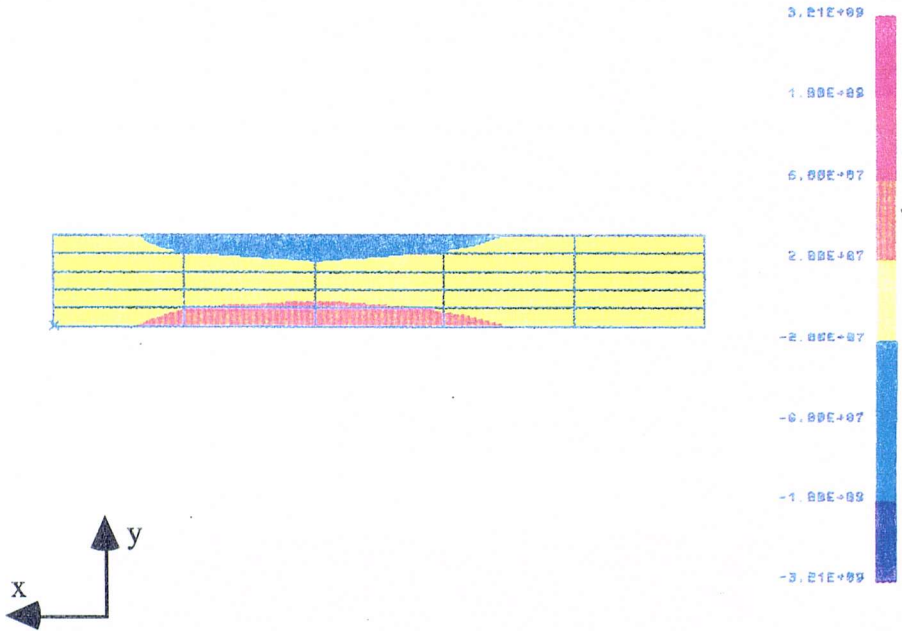


Fig. 4.11 σ_x in the free beam ($z = 80$ mm)

c) The direct and shear stresses in the beam root

In the beam root region σ_x acts as a pure moment on the plane of symmetry (see figure 4.12) as modelled in the analysis of the foundation. The foundation distorts the assumed distributions of τ_{xz} and τ_{yz} close to the plane of symmetry (see figures 4.13 and 4.14).

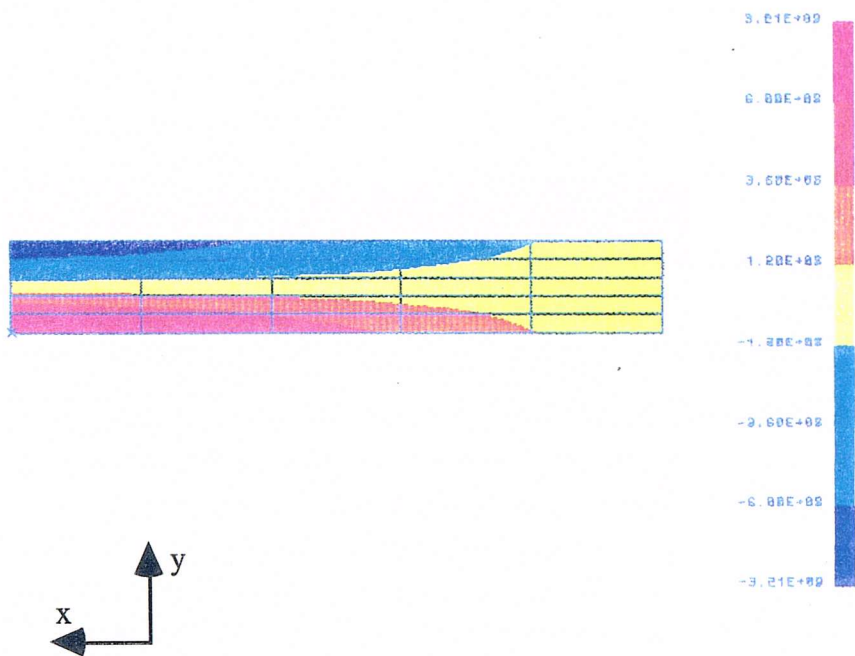


Fig. 4.12 σ_x in the beam root ($z = 150$ mm)

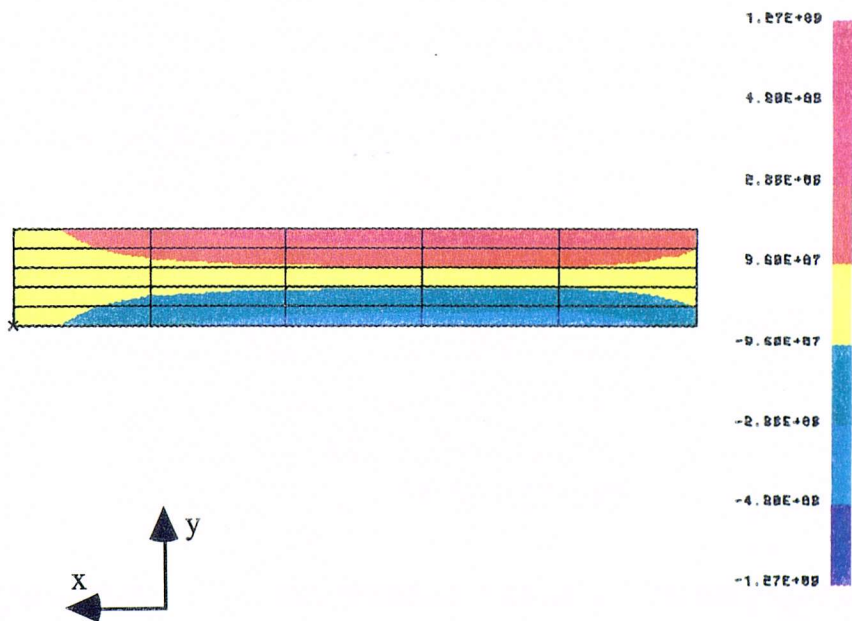


Fig. 4.13 τ_{xz} in the beam root ($z = 150$ mm)

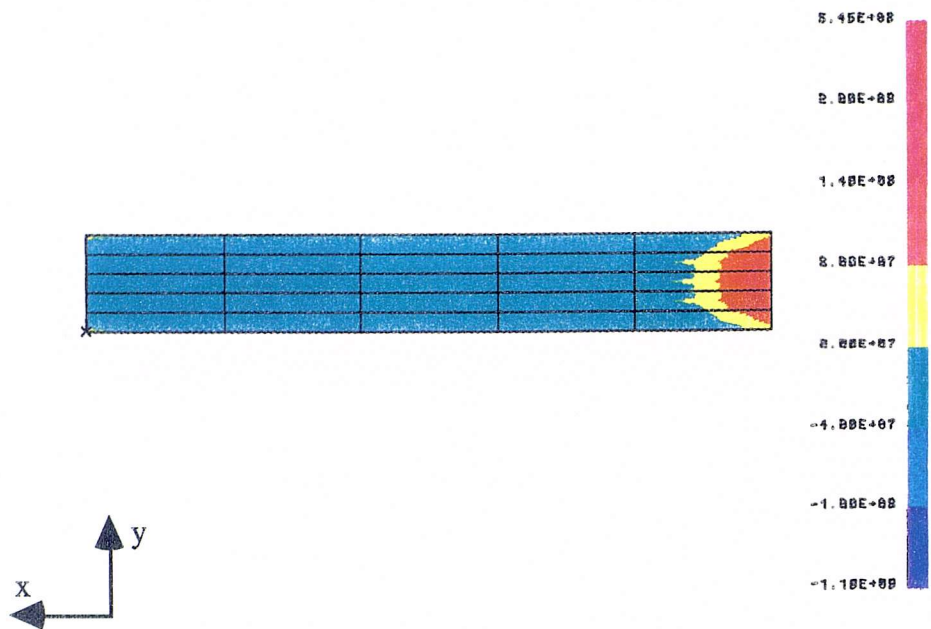


Fig. 4.14 τ_{yz} in the beam root ($z = 150$ mm)

4.2.4 Experimental Method

A static DT rig was designed by Sevsek [4.3] and later modified by Blumel [4.4] to the authors specification. The geometry of the rig conformed to that of the HSDT test described in section 1.3.1. An INSTRON 1168 low rate tensile testing machine was used to apply and measure the load. The test specimens were machined from aluminium alloy type 2014A-TF. This was chosen for its high yield strength to rigidity ratio and linear material properties. The experimental loads were chosen so that the elastic limit of the material was not exceeded (as calculated from the FE results). The material properties are quoted in table 4.1.

Aluminium type: 2014A-TF	
Density (kg/m^3)	2800
Poison's ratio	0.33
Young's Modulus (GPa)	74
Yield Stress (MPa)	420

Table 4.1 Material properties of aluminium alloy used in the experiment [4.5]

When examining the effects of 'V' groove depth and beam thickness the specimen compliance was measured. This was performed for a range of 'V' groove depths from 0 to

7 mm on a 9.525 mm thick specimen and for a range of thicknesses from 7 to 9.525 mm with no 'V' groove. The 'V' groove angle was 60° with a root radius of 0.15 mm.

For one case ($B = 7$ mm, no 'V' groove) the full rotation profile was determined. This required measurement of the section rotation, at a series of sections, as a function of load. The technique used consisted of measuring the upper surface vertical displacement at two points on each section with a linear displacement dial gauge. The two points were separated laterally by 25 mm and located within the outer half of the beam so as to be within the linear region (see figure 4.5). Since maximum rotation values never exceeded two degrees, the rotation could be calculated by dividing the difference of the two displacements by the point separation distance. This was repeated for a series of three loads. From these results a linear least squares fit was used to calculate the rate of section rotation with load. The specimen was always pre-loaded to the maximum load to be used, in order to minimise the effects of plastic deformation at the contact between the load points and the specimen.

4.2.5 Results

All the FE and experimental calibration results for c' are shown in figure 4.15. Good agreement is seen between the two methods. Although equation (4.6) is valid for the case of no 'V' groove, c' being approximately constant for the range of thicknesses used in the HSDT test, this is clearly not the case when a 'V' groove is present.

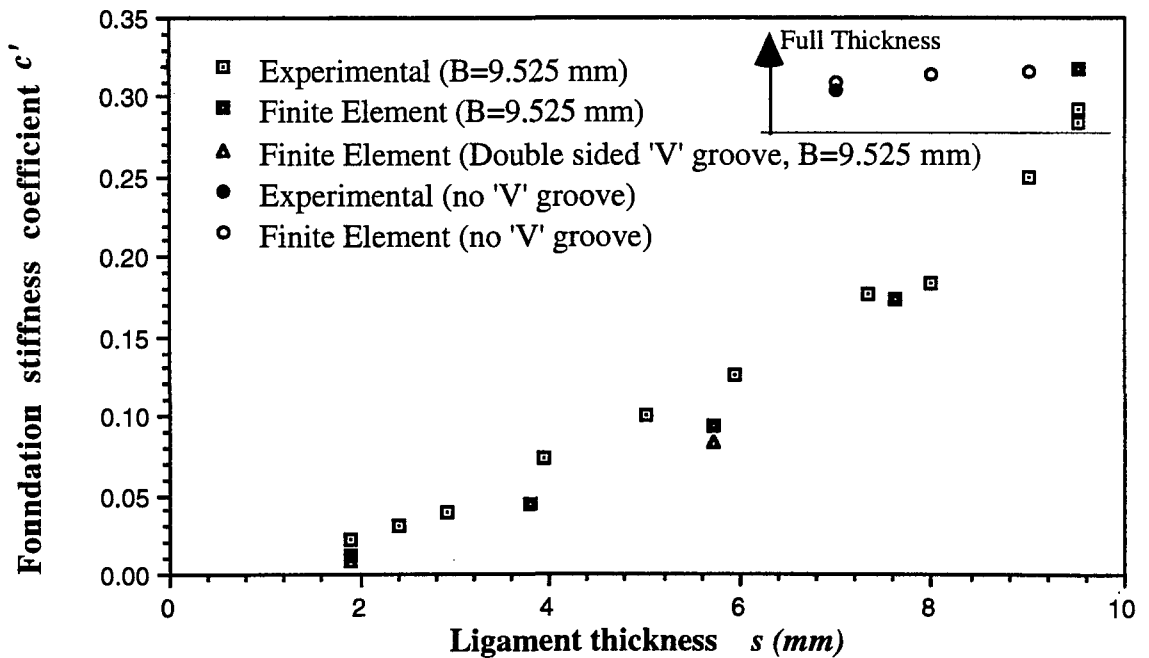


Fig. 4.15 Calibration tests for c'

Figure 4.16 shows a log-log plot of c' versus s/B , the function showing reasonable linearity. A good approximation of all the results is obtained if c' is replaced by the function $c(s/B)^{1.75}$ where this new c is equal to 0.29.

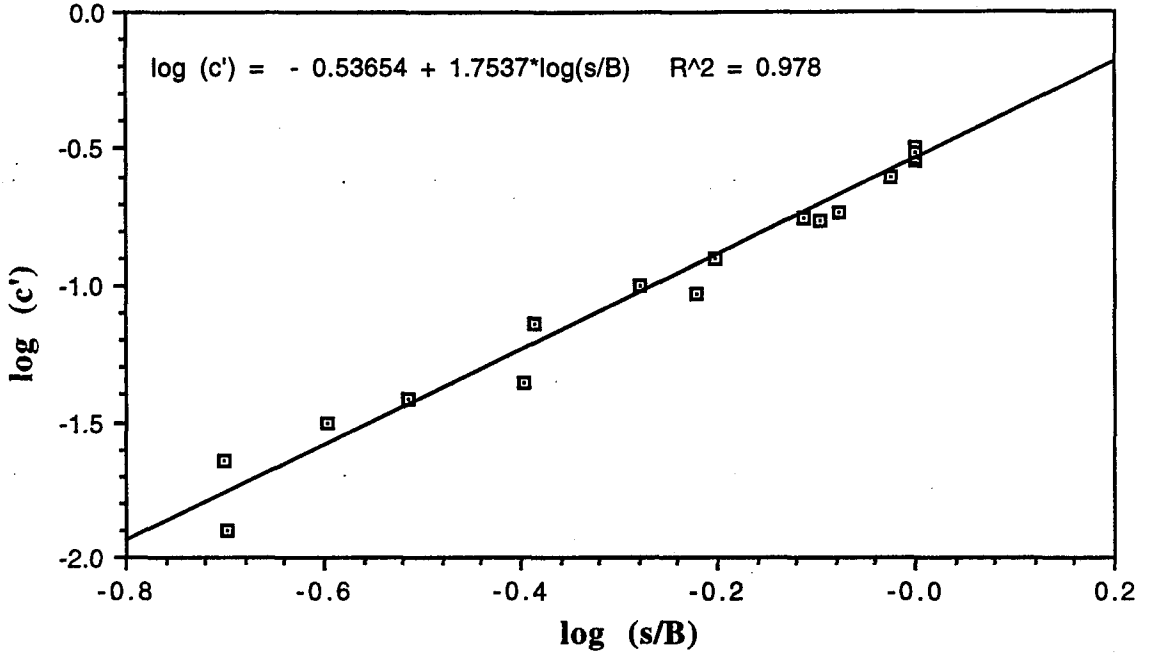


Fig. 4.16 All the calibration tests shown in figure 4.15 for c' on a log-log plot

Equation 4.6 now becomes:

$$\Omega = \frac{Es^3}{12c(s/B)^{1.75}H} \quad (4.9)$$

The effect of the stress concentration due to the 'V' groove can be seen when equations (4.9) and (4.6) are compared: the rotational stiffness is directly proportional to $s^{1.25}$, not to s^3 . In other words, the rate at which foundation stiffness decays with 'V' groove depth is much less than that predicted by the simple analysis which resulted in equation (4.6).

Full rotation profiles were obtained from the analytical, FE and experimental methods for the case of a 7 mm thick specimen with no 'V' groove. They are shown in figure 4.17.

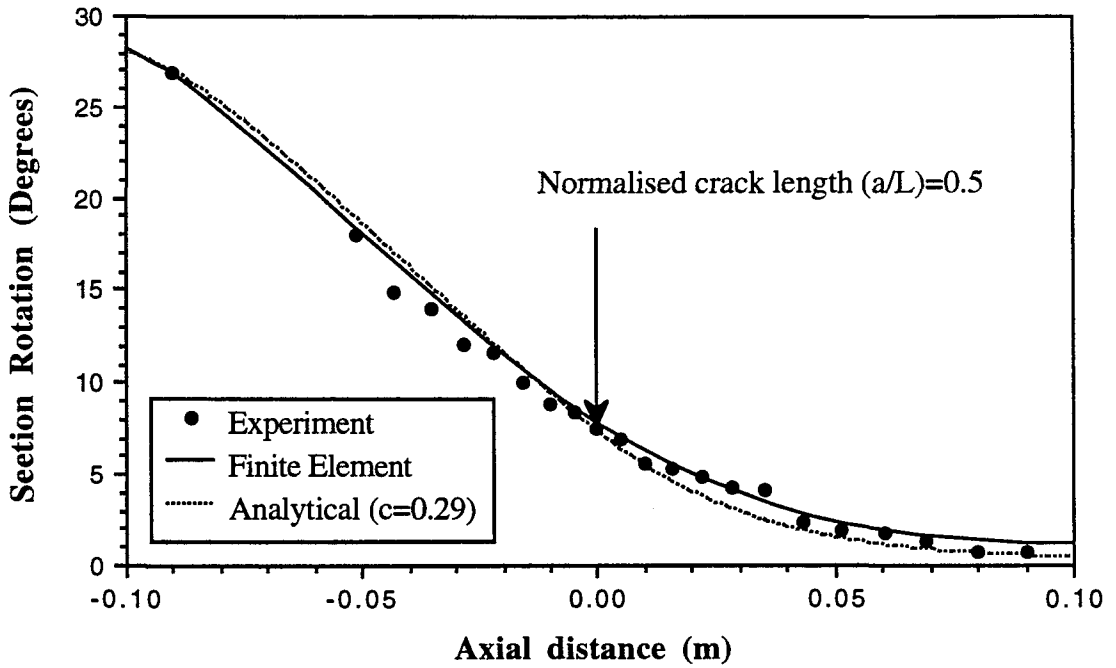


*Fig. 4.17 Rotation profiles from FE, experimental and analytic approaches
($B=7\text{ mm}$, Load= 20 kN)*

The agreement in compliance between the FE and experimental results was excellent (0.02% difference). Using the above calibrated value of c in the analytical solution generates a rotation profile which agrees well with experiment and FE results, especially at the section corresponding to the crack front. The agreement between analytical and experimental compliance was again very good (1.15% difference).

4.3 The curved crack front DT model

The preceding section described an accurate static model of the DT specimen with a straight crack front. The dependence of foundation stiffness on 'V' groove depth and thickness being accurately modelled. The work described below studied the dependence of the DT specimen compliance on the shape of the crack front. A method of calculating the crack front shape and the associated foundation stiffness is presented, along with experimental verification.

4.3.1 Experimental method

The experimental verification of the DT curved crack front analysis was performed on PMMA specimens. PMMA was considered ideal for this case since:

- 1) The material can be considered to deform linearly in low rate tests.

- 2) Slow rate DT tests on PMMA show reasonably steady crack propagation.
- 3) Due to its transparency and well defined crack front the propagation of the crack can be easily monitored.

The experiments compared the compliance, as a function of crack length, of a straight crack front specimen to that of a curved crack front. The experimental set-up was the same as that described in section 4.2.4. The procedure used for the straight crack front case was to machine a progressively longer crack in the specimen, measuring the compliance for each crack length by producing a load-displacement plot.

For the curved crack front case, the crack was grown at a constant displacement rate of 1 mm/min. At a series of crack lengths the specimen was unloaded and the crack length measured. During unloading it was noted that the crack shape altered, the leading edge of the crack front extending by up to 5 mm such as to reduce the angle of interception with the lower boundary (bottom of the 'V' groove). The specimen was then reloaded (during which there was no observable change in crack shape) to produce a load-displacement plot from which the compliance could be determined.

The compliance test results and crack front shape are shown in figures 4.18 and 4.19 respectively, together with the analytical results.

4.3.2 Analytical Model

The full solution of the static form of the DT test with the inclusion of the curved crack front is now considered. The shape was first modelled according to Leever's assumption [4.1] such that:

$$s = s_T \frac{\theta_T}{\theta} \quad (4.10)$$

Where θ_T is the section rotation at $z=a$ (the leading edge of the crack front) and s_T is the uncracked ligament thickness at $z=a$.

Leever's [4.1] showed equation (4.10) provided a good approximation to the shape of the crack when used in conjunction with his prediction of the rotation profile. Equation (4.10) can be considered to be based on a constant crack opening displacement (COD) criteria: Let the displacements of the free beam surfaces in the crack plane be described by a simple rotation about the centre of the remaining ligament, the beam separation at the crack front (δ in figure 4.2) is then equal to $s\theta$. Leever's showed that δ is several times larger than the

estimated critical COD, but that variations in critical COD between materials are reflected by variations in δ . He therefore demonstrated that δ could be taken as a criterion for fracture, which is applicable along the complete length of the crack front.

The general equations and boundary conditions used to model the deformation in this work are similar to those described in section 4.2.1. The only difference is in the model of the foundation stiffness in region P which was previously zero. The general equation modelling region R is now also used to model region P, the ligament thickness (s) in the foundation stiffness term (Ω) being a function of θ as described by equation (4.10), whereas in R it remains constant.

The simplest method of solving the system of equations to evaluate compliance was to use the dynamic finite difference model (described in chapter 6) with striker displacement held at a constant level and a small degree of damping added to stabilise the solution. The analysis was allowed to iterate until the change in predicted striker force was less than 0.1%.

The value of δ is required for the solution. This can be found by iterating on δ so that the analytically predicted crack length is equal to that in the experiment, for a given load displacement. This iterative procedure is not dependent on the value of the modulus used since the solution is fully displacement controlled. For a crack length of 100.14 mm δ was found to be 60 μm . From the results of Leever, this value can be taken to be constant for every crack length case analysed.

In order to compare these results with experiment the following procedure was used: All the experimental tests were modelled using the appropriate model (straight or curved crack front) with the approximate modulus for PMMA ($E=3.3 \text{ GPa}$). The ratio of the experimental and predicted compliances for the 99 mm crack length of the straight crack front was then calculated. This gave the correction factor for the modulus and load cell calibration, which was then applied to all the remaining straight and curved crack front results. The results are shown in figure 4.18.

The experimental accuracy of absolute load point displacement versus crack length on which the determination of δ is based is questionable due to non-linearity in the load displacement curve at small displacements (less than 0.2 mm). The sensitivity of compliance to δ was therefore estimated by repeating the analysis with a value of δ half that stated above. These results are also shown in figure 4.18 and it can be seen that the sensitivity is low.

The predicted compliances for the curved crack front case are much less than those measured experimentally. The predicted crack shape for a crack length of 100.14 mm is shown in figure 4.19 with reference to that actually measured. As can be seen the crack shapes are markedly different; the predicted shape producing a much larger remaining ligament thickness behind the leading edge. The reduction of the predicted compliance with reference to that measured is therefore unsurprising, the larger remaining ligament thickness providing a higher restoring moment.

A second curved crack front model was developed in which the experimentally measured curved crack front shape was used to define the shape, as supposed to employing equation (4.10). The compliance results are shown in figure 4.18 and good agreement is seen with experiment. Equation (4.10) is therefore inaccurate in modelling crack front shape and an alternative method is required, which is presented in the following section.

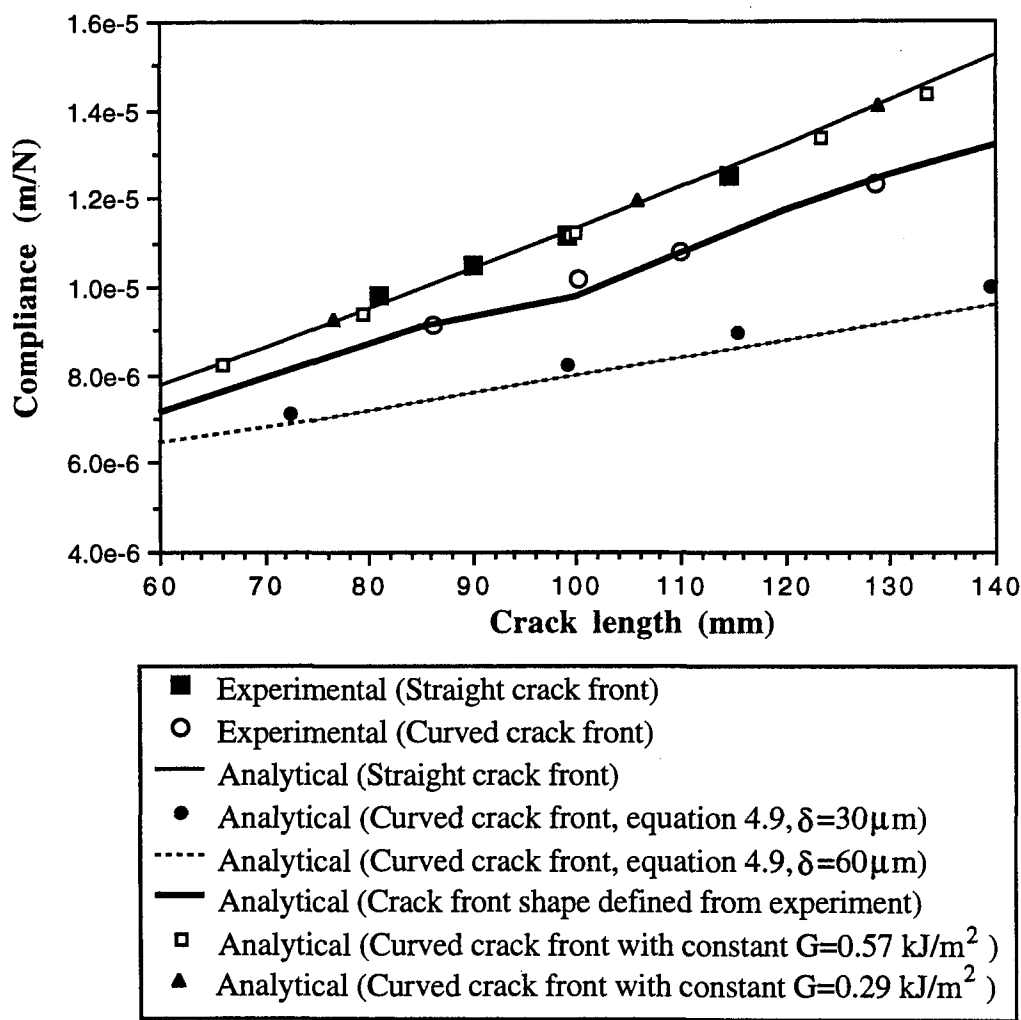


Fig. 4.18 Compliance versus crack length results for straight and curved crack fronts (The definition of crack length for the curved crack front cases is as shown in figure 2.1b)

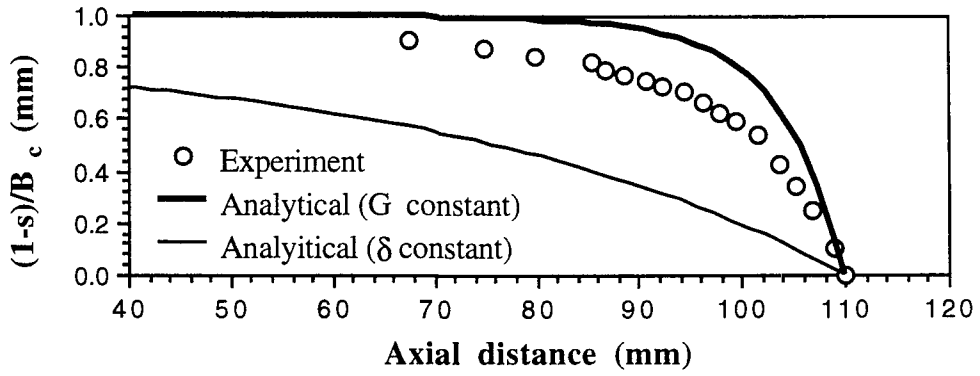


Fig. 4.19 Experimental and predicted crack shapes

4.4 Crack driving force model

This section works from the hypothesis that the crack shape during propagation can be predicted by assuming that the crack driving force (G) is constant along its length. In order to utilise this hypothesis a method of locally calculating the crack driving force along the front must be determined.

4.4.1 Static case

The dependence of G on the shape of the curved crack front has not yet been considered. This can be looked at in either a global or a local manner. In the global approach the total work done by the external load, the total strain energy of the specimen and the total crack surface area are calculated as functions of crack length to give the well known LEFM equation [4.6]:

$$G = \frac{P^2}{2} \frac{dC/da}{dA/da} \quad (4.11)$$

The local approach considers the local release of strain energy at a section. The two approaches are equivalent, but only the local approach is suitable for analysing the HSDT test in terms of predicting the crack shape.

Consider the generalised section shown in figure 4.2. At a section where the crack has started to propagate through the thickness, let ξ now denote the front of the advancing crack as opposed to the 'V' groove depth. Let the crack front advance through the section by $\delta\xi$. During this extension let the change in section rotation be $\delta\theta$. The work done by the two opposing moments on the foundation per unit length at this section is then:

$$\delta W = 2M\delta\theta$$

Substituting for $M=\Omega\theta$ from equation (4.9) and dividing through by $\delta\xi$ gives:

$$\frac{dW}{d\xi} = \frac{Es^{1.25}\theta}{6c\left(\frac{1}{B}\right)^{1.75}H} \frac{d\theta}{d\xi} \quad (4.12)$$

The strain energy per unit length of the foundation is:

$$U = 2 \int_0^{\theta} M d\theta$$

Substituting again for M from equation (4.9) and evaluating the integral gives:

$$U = \frac{Es^{1.25}\theta^2}{12c\left(\frac{1}{B}\right)^{1.75}H}$$

Differentiating with respect to ξ gives:

$$\frac{dU}{d\xi} = -\frac{1.25Es^{0.25}\theta^2}{12c\left(\frac{1}{B}\right)^{1.75}H} + \frac{Es^{1.25}\theta}{6c\left(\frac{1}{B}\right)^{1.75}H} \frac{d\theta}{d\xi} \quad (4.13)$$

The increase in fracture surface area (δA) per unit length is:

$$\delta A = \delta\xi \quad \therefore \frac{dA}{d\xi} = 1 \quad (4.14)$$

The crack driving force is defined by equation (1.2). In considering the crack driving force at the section the crack length variable (a_ξ) is defined here as the distance of the crack front from the bottom edge of the specimen:

$$a_\xi = B/2 + \xi \quad \therefore \frac{da_\xi}{d\xi} = 1 \quad (4.15)$$

The derivatives with respect to ξ in equations (4.12) to (4.14) are therefore equivalent to the derivatives with respect to a_ξ . Substituting for the derivatives of work done (equation (4.12)), strain energy (equation (4.13)) and fracture surface area per unit length (equation (4.14)) with respect to a_ξ in equation (1.2) gives, assuming changes in kinetic energy are negligible:

$$G = \frac{1.25E}{12c\left(\frac{1}{B}\right)^{1.75}H} s^{0.25}\theta^2 \quad (4.17)$$

Now, assuming G is constant along the crack front, equation (4.17) can be used to predict the compliance and crack shape of the static test for the experiments described in section 4.3.1. The appropriate value of G was found by iterating on its value until the predicted crack length matched that of the experiment for a given load displacement. The calculated value of G (0.57 kJ/m^2) was then used to predict compliance versus crack length. The compliance results are shown in figure 4.18 and show poor correspondence with the experimental curved crack front results but good correspondence to the straight crack front. The correspondence can be easily understood when the predicted crack shape from the constant G case is considered (see figure 4.19). The leading edge of the crack front is much blunter than seen in the experiment, so tending towards the straight crack front.

The accuracy of the assumption of constant G along the crack front together with the calculation of crack driving force (equation (4.17)) therefore appears inaccurate at first sight. If, however, the observation that the crack front shape sharpened as it was unloaded is taken into account, it is probable that the dynamic, propagating shape of the front is close to that predicted using the constant G assumption but the compliance results relate only to the unloaded, arrested shape.

4.4.2 Dynamic Case

The inertia term in equation (3.9a) includes the material constituting the foundation and the whole section is modelled as rotating without change in shape. In reality the foundation region adjacent to the future crack plane is prevented from rotating, this restraint decreasing with distance from the crack plane (see figure 4.1). In the dynamic case the contribution of the foundation ahead of the crack front to the section inertia is assumed to be negligible for two reasons: firstly, due to its relatively small size and, secondly, because it is subjected to low section rotation rates relative to the region behind the leading edge of the crack front. The neglect of dynamic effects in the previous sections does not therefore preclude the use of the derived equations for foundation stiffness being implemented in the dynamic model.

For completeness, the difference between the plane and curved crack front when considering the dynamic case is considered. For the case of a straight crack front, the foundation stiffness was modelled as:

$$\Omega = \frac{Es_T^{1.25}}{12c\left(\frac{1}{B}\right)^{1.75} H}$$

and the crack driving force by the local approach as:

$$G = \frac{E}{12c \left(\frac{s_T}{B} \right)^{1.75} H} (s_T \theta_T)^2 \quad (4.18)$$

where θ_T is the rotation at the crack front.

In Wheel's analysis [4.2] his c' can be considered as being $(s_T/B)^{1.75}$ grouped together with the constant c and H replaced by s_T . Calculating an equivalent c' for Wheel's analysis gives, for his specimen dimensions (7 mm thick specimen with a 1.5 mm deep 'V' groove), a value of 1.7. This is a 24% increase in the value he actually used of 1.3.

In order to compare the straight and curved crack front shapes the dynamic model described in chapter 6 with a straight crack front case was used to solve a case of specified geometry and boundary conditions for a non-linear material (HDPE at 0°C), with a G_D of 3.56 kJ/m². The dynamic model for the curved crack front case was then used to compute the value of G_D which gave the same crack velocity as that predicted by the straight crack front model, for the same geometry and boundary conditions. Two cases were analysed, one for a striker velocity of 7 m/s and another for 25 m/s.

For the 7 m/s case, the crack velocity predicted by the straight crack front model was 107 m/s and the associated value of G_D computed from the curved crack front model was 3.563 kJ/m². For the 25 m/s case, the crack velocity predicted by the straight crack front model was 187 m/s and the associated value of G_D computed from the curved crack front model was 4.188 kJ/m². The Saint-Venant's torsional wave speed for both cases was 371 m/s. It can be concluded that the difference between the straight and curved crack front analyses is negligible at low crack speeds but of the order of 20% at crack speeds approaching the torsional wave speed.

To explain this effect equations (4.17) and (4.18) must be compared. If rotation profiles for the straight and curved crack front cases were to be the same, then the value of the crack driving force would be a factor of 1.25 greater for the curved crack front case. This situation is indeed approached at high striker velocities, since the kinetic and strain energies of the torsion beams are far higher than the strain energy of the foundation and dominate the test. In this case the foundation behaves passively, conforming to, but not affecting, the imposed deformation of the torsion beams. As the crack velocity decreases, quasi static stresses from the relatively light foundation exert a greater relative effect on the overall deformation. The rotation at the crack front of the straight crack front model (and thus the predicted G_D) increases with respect to that at the leading edge of the curved crack front model for a given crack history.

4.5 Discussion

The analysis of the DT test presented in this chapter can be used to accurately model the deformation in the low rate test if the shape of the crack front is known. The use of Leevers' [4.1] constant COD approach to predict crack front shape does not agree with experiment. An alternative approach has been proposed to predict the crack shape by assuming G is constant along its length. This approach gives a much better agreement with experiment, but there is still some doubt about the shape of the propagating crack front.

Wheel's analysis of the curved crack front assumed Leevers constant COD approach throughout. This would have produced a marked variation of G along the curved crack front which was not observable since he only used a global energy balance approach in his analysis of the curved crack front. His results with regards to the curved crack front are therefore questionable, particularly so when it is noted that his assumed decay of restoring moment with ligament thickness was incorrect.

At crack velocities much lower than the characteristic wave speed of the material there is little difference in predicted G_D between the straight and curved crack front models, but at higher crack velocities the G_D predicted by the curved crack front model is significantly higher.

4.6 References

- (4.1) Leevers, P.S., 'Crack front shape effects in the double torsion test', *J. of Mat. Science*, **17**, p. 2469, (1982).
- (4.2) Wheel, M.A. and Leevers, P.S., 'High speed double torsion tests on tough polymers I: Linear elastic steady state and dynamic analysis', *Int. J. of Fracture*, **61**, p. 331, (1993).
- (4.3) Sevsek, B., Experimental and finite element verification of analytical equations used to model torsion of rectangular beam, Msc Thesis, Univ. of London, (1992).
- (4.4) Blumel, C., Development of the HSDT fracture test for brittle polymers, Imp. Coll. Mech. Eng. Dept., Third year project report, (1994).
- (4.5) Aluminium Federation, "The properties of aluminium and its alloys", ALFED, (1985).
- (4.6) Williams, G.J., Fracture mechanics of polymers, Ellis Horwood Ltd., (1987).

Chapter 5

Material properties

5.1 Introduction

As was stated in chapter 3, the accuracy of the calculated dynamic fracture resistance (G_D) from the HSDT analysis is strongly dependant on the accuracy to which the torsional waves are modelled. The velocity and amplitude of these waves are dependant on both the inertia and the stiffness of the specimen. The inertia and stiffness are functions of the specimen material's density and moduli respectively, as described by equations (3.10). The prime objective of this chapter is to establish a method to evaluate the moduli of a material appropriate to the HSDT test conditions.

Wheel [5.1] initially modelled the HSDT test specimen using a linear elastic material. He used a direct contact ultrasonic technique developed by Dioh [5.2] to determine both the tensile and shear moduli. These moduli correspond to a very low strain (0.15 %) and high strain rate ($20,000 \text{ s}^{-1}$). The values measured for the tensile and shear moduli using this technique are denoted in this work as E and μ_0 respectively. He measured the density, as in this work, using the ASTM standard D 792 water displacement test for plastics. His model over-predicted the experimentally measured load by approximately 50%, this disparity varying little with strain. The over prediction led him to the conclusion that the shear modulus was non-linear, decreasing with increasing strain.

Wheel [5.3] made the assumption, which is maintained throughout this work, that the tensile modulus was constant with strain and equal to the value determined by the ultrasonic method. This assumption was based on the fact that the appearance of the tensile modulus in the equations of motion is due only to the presence of axial stresses. Since the axial strains in the HSDT test are small it is reasonable to treat this material property as linear within these bounds.

The importance of shear modulus non-linearity can be clearly seen if an idealised case is considered, where the following assumptions are made:

- 1) Dispersion is negligible and the deformation is governed by Saint-Venant's wave equation.
- 2) The rotation profile, at any point in time, consists of a region of constant twist behind the wave front, such that the role the foundation stiffness plays in the deformation is negligible with respect to the torsional impedance (see figure 5.1).

- 3) The twist remains constant with time.
- 4) The crack velocity ($\dot{a}$) is constant.

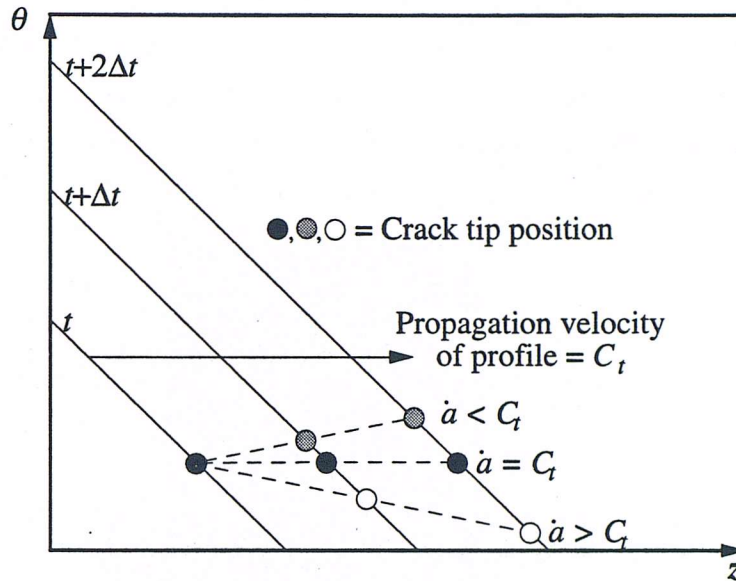


Fig. 5.1 Idealised Rotation Profiles

As will be seen in the following sections the propagation velocity of this rotation profile, along the specimen, is equal to the Saint-Venant torsional wave speed (C_t) which is directly proportional to the square root of the effective section tangent shear modulus (μ_t):

$$C_t = \sqrt{\frac{\lambda \mu_t}{\rho}} \quad (5.1)$$

Since G_D is directly proportional to the square of the section rotation at the leading edge of the crack front (see section 4.4) it is possible to identify three regions in the relationship between $\dot{a}$ and C_t :

- a) $\dot{a} < C_t$ would show G_D decreasing with crack length.
- b) $\dot{a} = C_t$ would show G_D constant with crack length.
- c) $\dot{a} > C_t$ would show G_D increasing with crack length.

Figure 5.1 demonstrates the case for constant C_t and a varying crack velocity. In the HSDT analysis the converse is true, the crack velocity being determined by experiment and C_t calculated, but the above equalities still hold. It is clear that in this highly simplified case an increase in C_t would produce an increase in predicted G_D for any given crack length or velocity.

In practice the foundation stiffness plays an important role and as $\dot{a}$ decreases the profile becomes governed by $\dot{a}$ rather than C_t , but the above case indicates how the calculated value of G_D depends strongly on the accuracy of the shear modulus used in the analysis.

5.2 The torsional impedance test

To evaluate the correct value of the shear modulus Wheel developed the torsional impedance test. The existing HSDT experimental set-up is used but rather than testing an unfractured specimen, a pre-fractured one is used. The test consists of measuring the impedance of the rectangular beams to the rotation rate imposed by the descending striker. Prior to loading, the two halves are held together by a small bulldog clip at the opposite end to that loaded (see figure 5.2). The clip is assumed to play no further role in the ensuing deformation.

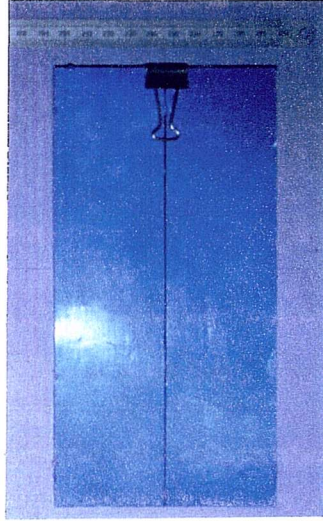


Fig. 5.2 Torsional impedance test specimen

The load trace recorded from a typical test is shown in figure 5.3. The trace shows that after an initial peak the load oscillates about a constant mean value. The nature of this oscillation is largely due to the overhang region behind the load plane: this can be proved by comparing the torque predicted using the finite difference model (see chapter 6) with the inclusion of the overhang, to that predicted without it. These results are also shown in figure 5.3.

Experiments show that as striker velocity is increased the mean value of the load increases. Since the deformation can be characterised by a mean load, it is reasonable to assume that in the region close to and in front of the load plane the transmitted torque does not change with axial distance. Applying this assumption to equation (3.17) implies that $\frac{\partial \psi}{\partial z} = 0$ and thus the axial stresses are zero. This is the special case of Saint-Venant [5.4] where $\psi = \frac{\partial \theta}{\partial z}$ and the wave speed is C_t (see equation (5.1)).

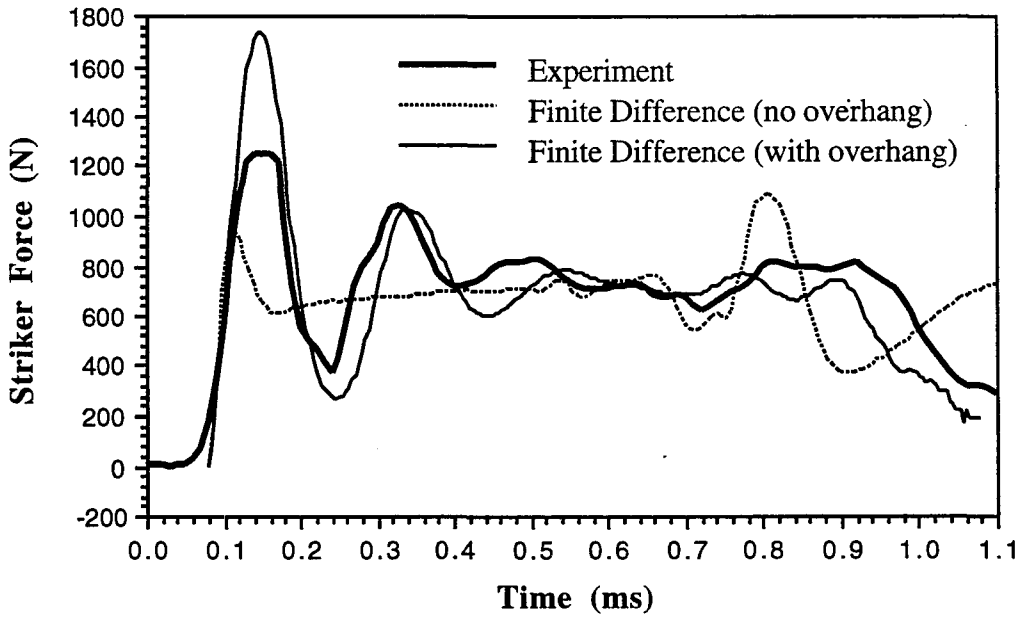


Fig 5.3 Experimental and Predicted Load Traces
(HDPE, 0°C, striker velocity = 22.2 m/s)

Wheel's [5.3] analysis consisted of calculating the low strain, high strain rate Saint-Venant torsional wave speed (C_t^o) using μ_o . Given the rotation rate imposed by the striker ($\dot{\theta}_{LP}$), the twist in the deformed region can then be calculated from:

$$\frac{\partial \theta}{\partial z} = \frac{\dot{\theta}_{LP}}{C_t} \quad (5.2)$$

In Wheel's analysis $C_t = C_t^o$, which is the torsional wave speed calculated from equation (5.1) using $\mu_t = \mu_o$. From the value of the twist he then defined the effective strain as the maximum shear strain in the section, which occurs at the centre of the longer edge of the section. The calculation of maximum shear strain assumed axial stresses to be negligible. The appropriate value of μ_s for the effective strain was found by substituting the experimentally measured torque into equation (3.17). The torque was calculated by integrating the load trace over time, excluding the initial peak, and dividing by the time period of the integration. At low striker velocities, the load trace becomes very oscillatory and the evaluation of mean torque is inaccurate. By repeating the test at different striker velocities, Wheel was able to define the secant shear modulus as a function of effective strain. From this function the tangent shear modulus could then be calculated as a function of effective strain.

As in this work, Wheel incorporated his results into the analysis of the HSDT test as a non-linear elastic model.

5.3 Problems with the analysis

The analysis described in the previous section is flawed by the fact that the wave speed in the specimen depends on the local tangent shear modulus which, assuming the material behaves non-linearly (which is the whole point of this analysis), will not be equal to μ_0 . Wheel's [5.3] analysis therefore predicted lower strains for a given striker speed than actually existed, and thus too fast a decay of μ_s with strain. Including this correction in the analysis means the twist can no longer be determined explicitly, since it depends on the value of μ_t which is to be calculated.

There is one other discrepancy in the analysis: the evaluation of the effective strain in regions of non-linear twist. Wheel's approach (see above) is adequate in regions of uniform twist (where $\psi = \frac{\partial \theta}{\partial z}$ is a constant) since, for this case, equation (3.6) shows that the strain at any point is directly proportional to twist. However, regions of non-uniform twist do exist in the fracture test, where the twist does not equal ψ . Wheel's definition of effective strain therefore does not correctly characterise the strain across the section. These problems are corrected by a new analysis detailed in the next section.

5.4 A revised analysis of the torsional impedance test

5.4.1 Definition of effective strain

In a full three dimensional solution the shear modulus at a point is defined as a function of the state of strain at that point. In a prismatic torsion beam all non-zero components of strain vary across the section. In order that the equations of motion involve only one spatial dimension, an effective modulus representative of the whole section was defined in deriving equations (3.9). This effective section modulus must be defined as a unique function of some effective strain, γ_e , which characterises the entire strain field at any section.

5.4.1.1 Circular Bar

To clarify the definition of γ_e the case of a circular bar subjected to a constant twist is considered before that of a prismatic bar. Let the material of the circular bar behave in a non-linear elastic manner. Assuming that sections rotate without change in shape the torque T is calculated as:

$$T = \int_0^R (2\pi r^2 \gamma \mu(\gamma)) dr. \quad (5.3)$$

where r is the distance from the centroidal axis of the bar, R is the radius of the bar and $\mu(\gamma)$ is the secant shear modulus, a function of the shear strain γ , which itself varies with r . An effective secant shear modulus is defined in a similar fashion to the derivation of equation (3.7) such that

$$T = \mu_e(\gamma_e)\Gamma_m \quad (5.4)$$

where the strain moment

$$\Gamma_m = \int r^2 \gamma(r) dr d\theta \quad (5.5)$$

is, in this case

$$\Gamma_m = \int 2\pi r^2 \gamma(r) dr.$$

The effective secant shear modulus has been assumed to be a function of γ_e which, as stated above, should completely define the strain field. This is achieved by using the following definition:

$$\gamma_e = \frac{\Gamma_m}{\Gamma_c} \quad (5.6)$$

where Γ_c is the first polar moment of area of the section and therefore a section constant

$$\Gamma_c = \int r dr d\theta$$

which is, in this case

$$\Gamma_c = \int_0^R (2\pi r^2) dr = \frac{2}{3} \pi R^3.$$

Equations (5.4) and (5.5) identify γ_e as that strain which, if considered constant across the section and substituted for γ in equation (5.3), would yield the same value of T as the actual strain distribution $\gamma(r)$. For a circular bar the strain distribution is

$$\gamma = r\theta' \quad (5.7)$$

where θ' is the twist, so that from equation (5.5)

$$\Gamma_m = \theta' \int_0^R (2\pi r^3) dr = \frac{1}{2} \pi \theta' R^4$$

and from equations (5.6)

$$\gamma_e = \frac{3}{4} R\theta' \quad (5.8)$$

Thus once $\mu_e(\gamma_e)$ has been measured, the torque can be calculated for any value of R and θ' , since, from equation (5.4)

$$T = \mu_e(\gamma_e) \Gamma_c \gamma_e \quad (5.9)$$

Where Γ_c is a function of geometry and γ_e is a function of geometry and twist. The preceding analysis therefore allows the true modulus and strain which vary across a section, once redefined as effective ones, to be brought outside of the integration term of equation (5.3). The most important characteristic of equation (5.7) is that it can be applied to any radius of bar, the function $\mu_e(\gamma_e)$ being independent of the radius of the bar. Once $\mu_e(\gamma_e)$ is found for one particular radius it can therefore be used to any to predict the torque/twist relationship for any radius of bar of the same material. The geometry independence is proved below.

Consider the case where μ_e as a function of γ_e is determined for a circular bar of radius R_1 and the torque/twist relationship is to be predicted for a circular bar of the same material but of radius R_2 .

The true torque-twist relationship, for the bar of radius R_2 can be determined from the substitution of shear strain from equation (5.7) into equation (5.3):

$$T_2 = 2\pi \int_0^{R_2} r^3 \mu(r\theta'_2) dr$$

where the subscripts 1 and 2 denote the bars with radii R_1 and R_2 respectively.

Changing the integration variable such that $r_e = \frac{R_1}{R_2} r$ gives:

$$T_2 = 2\pi \left(\frac{R_2}{R_1} \right)^3 \int_0^{R_1} r_e^3 \mu \left(\frac{R_2}{R_1} r_e \theta'_2 \right) dr_e. \quad (5.10)$$

To produce the same effective strain in the bar of radius R_1 the twist must be (from equation (5.8)):

$$\theta'_1 = \frac{R_2}{R_1} \theta'_2$$

From equations (5.4) and (5.3) the effective modulus, calculated from the bar of radius R_1 , for the same effective strain is therefore:

$$\mu_e = \frac{2\pi \int_0^{R_1} r_e^3 \mu \left(\frac{R_2}{R_1} r_e \theta'_2 \right) dr_e}{\Gamma_{m_1}}$$

Using equation (5.4) to now calculate the torque in the bar of radius R_2 for this effective strain and corresponding effective modulus gives:

$$T_2 = \frac{\Gamma_{m_2}}{\Gamma_{m_1}} 2\pi \int_0^{R_1} r^3 \mu \left(\frac{R_2}{R_1} r_e \theta'_2 \right) dr_e = 2\pi \left(\frac{R_2}{R_1} \right)^3 \int_0^{R_1} r_e^3 \mu \left(\frac{R_2}{R_1} r_e \theta'_2 \right) dr_e$$

The value for T_2 is identical to that given in equation (5.10), thus proving the geometry independence.

5.4.1.2 Prismatic Bar

In defining γ_e for a prismatic bar, it is first noted that from equations (3.6) the following is true:

$$\gamma_{xz}(x, y) = \gamma_{xz}(x, -y) \text{ and } \gamma_{yz}(x, y) = \gamma_{yz}(-x, y)$$

The effective strain is defined as that strain which, when assigned such that $\gamma_{xz} = \gamma_e$ for $y > 0$ and $\gamma_{yz} = \gamma_e$ for $x < 0$, and the previous definition is taken into account, the resulting value of the integral in equation (3.7a) is identical to that evaluated using the actual strain field. As for the circular bar

$$\gamma_e = \frac{\Gamma_m}{\Gamma_c}$$

where the strain moment is now

$$\Gamma_m = \int \int_{x,y} (\gamma_{yz}x - \gamma_{xz}y) dy dx = J \left((\lambda - 1)\psi + \frac{\partial \theta}{\partial z} \right)$$

and the first polar moment of area of the section is

$$\Gamma_c = 4 \int_0^{B/2} \int_0^{-H/2} (x - y) dx dy = \frac{1}{4} BH(B + H).$$

The definition of γ_e does not define a unique strain field since the field also depends on the ratio ψ to twist for a particular value of effective strain. It does, however, characterise the strain across the section for all possible ratios; the magnitudes of γ_{xz} increasing with the magnitude of y and γ_{yz} increasing with the magnitude of x in all cases. Given that there is some inaccuracy in the prediction of the strain field when ψ does not equal the twist (see derivation of equation (3.9b)), this approximation is reasonable.

The definition of γ_e also has another major advantage when the calculation of μ_t is considered, since the differentiation with respect to the longitudinal axis (z) in equation (3.7c) to derive equation (3.9a) yields the term:

$$\frac{\partial}{\partial z}(\mu_s \gamma_e \Gamma_c) = \Gamma_c \left(\gamma_e \frac{\partial \mu_s}{\partial z} + \mu_s \frac{\partial \gamma_e}{\partial z} \right)$$

By the chain rule this is equivalent to:

$$\begin{aligned} & \Gamma_c \left(\gamma_e \frac{d\mu_s}{d\gamma_e} \cdot \frac{\partial \gamma_e}{\partial z} + \mu_s \frac{\partial \gamma_e}{\partial z} \right) \\ &= \mu_t \Gamma_c \frac{\partial \gamma_e}{\partial z} \end{aligned}$$

$$\text{where } \mu_t = \frac{\partial}{\partial \gamma_e}(\mu_s \gamma_e). \quad (5.11)$$

The effective stress is defined by the normal rules of Hooke's law as:

$$\tau_e = \mu_s \gamma_e \quad (5.12)$$

Substituting into equation (5.11) gives:

$$\mu_t = \frac{\partial \tau_e}{\partial \gamma_e} \quad (5.13)$$

This definition of μ_t is the same as that in the standard calculation of the tangent modulus with the tangent shear modulus, shear stress and shear strain replaced by their corresponding effective section variables.

5.4.2 Calculation of effective strain in the torsional impedance test

Using equation (3.17) and putting $\psi = \frac{\partial \theta}{\partial z}$, the imposed torque can be written as:

$$T = \mu_s \Gamma_m \quad (5.14)$$

$$\text{and for this case } \Gamma_m = \lambda J \frac{\partial \theta}{\partial z}. \quad (5.15)$$

Applying the identity of equation (5.12) to calculate the geometry independent effective shear stress from the torque gives:

$$\tau_e = \frac{T}{\Gamma_c} \quad (5.16)$$

If the above assumptions are valid this relationship is independent of the section dimensions.

Substituting for μ_t from equation (5.13) in (5.1) and the resulting equation for Saint-Venant's torsional wave speed in equation (5.2) gives a function defining the twist. Substitution of this into equation (5.15) gives:

$$\frac{d\tau_e}{d\gamma_e} = \frac{1}{\mu_o} \left(\frac{\tau_o}{\gamma_e} \right)^2 \quad (5.17)$$

$$\text{where } \tau_o = \frac{\mu_o \lambda J}{\Gamma_c} \frac{\dot{\theta}_L}{C_t}. \quad (5.18)$$

The variable τ_o is the effective stress that would be produced for a particular geometry and rotation rate if the material was linear elastic and the shear modulus equal to the low strain high strain rate modulus (μ_o). The experimental results are used to define τ_e (equation (5.16)) as a function of τ_o (equation (5.18)) which should be independent of section dimensions:

$$\tau_e = f(\tau_o) \quad (5.19)$$

By the chain rule, $\frac{d\tau_e}{d\gamma_e} = \frac{d\tau_e}{d\tau_o} \cdot \frac{d\tau_o}{d\gamma_e}$.

Substituting from equation (5.17) for the derivative of effective stress with effective strain and integrating by separation of variables gives:

$$\frac{1}{\gamma_e} = -\mu_o \int \left(\frac{d\tau_e/d\tau_o}{\tau_o^2} \right) d\tau_o \quad (5.20)$$

Equation (5.20) thus allows the effective strain to be calculated as a function of effective stress.

5.5 Implementation of the analysis

In order to proceed with the evaluation of the shear moduli a mathematical form of the function in equation (5.19) must be stated. A characteristic set of normalised results from the torsional impedance test for an HDPE material is shown in figure 5.4.

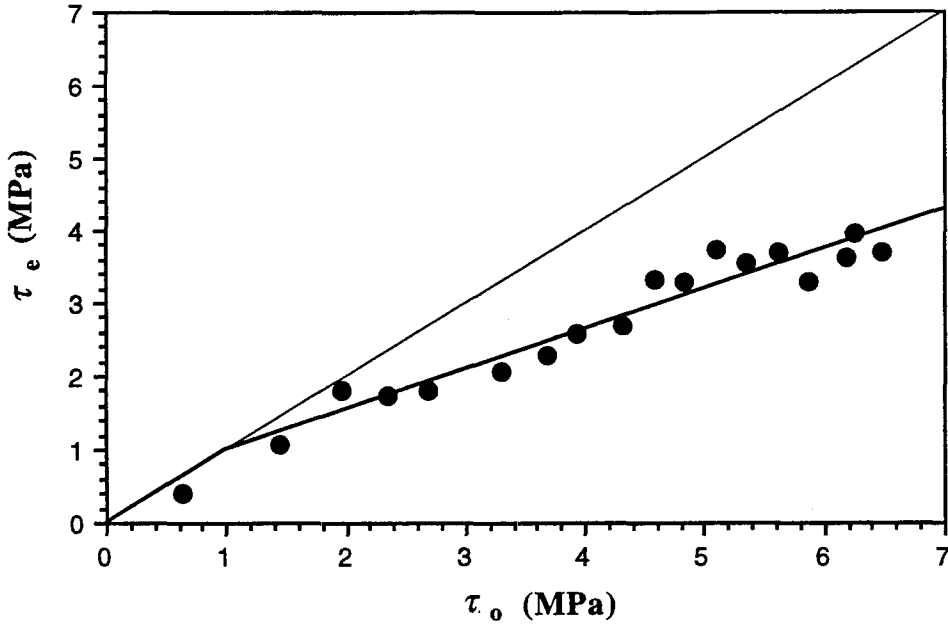


Fig 5.4 Torsional impedance results for an HDPE

At first sight the data appear linear and could thus be approximated by a simple straight line. This would imply that both μ_s and μ_t are constant and equal to the product of the square of this slope and μ_o (see equation (5.24)). Results from section rotation measurements (see section 5.6.2) will show this cannot be true at low values of strain and that in this region μ_s and μ_t are close to μ_o . The shear moduli must therefore decay from μ_o at low strains to a value which is determined from the slope of the results. A problem arises from this decay in that τ_e is not a single-valued function of τ_o . This is best demonstrated by performing the analysis of section 5.3 in reverse: in other words, assume a stress-strain relationship and calculate the dependency of τ_e on τ_o . Consider a stress-strain relationship which consists of two linear regions connected by a cubic spline as shown in figure 5.5.

Substituting for the slope of the effective stress-strain curve from equation (5.17) into equation (5.13) gives the equation:

$$\tau_o = \gamma_e \sqrt{\mu_o \mu_t} \quad (5.21)$$

From this equation and the stress-strain curve depicted in figure 5.5, τ_e as a function of τ_o can be calculated. The function is shown in figure 5.6. The figure clearly shows that τ_e is not single valued since, in the region corresponding to the spline, the rate of decay of μ_t with γ_e rapidly increases, producing a dip in τ_o as a function of γ_e (see equation (5.21)).

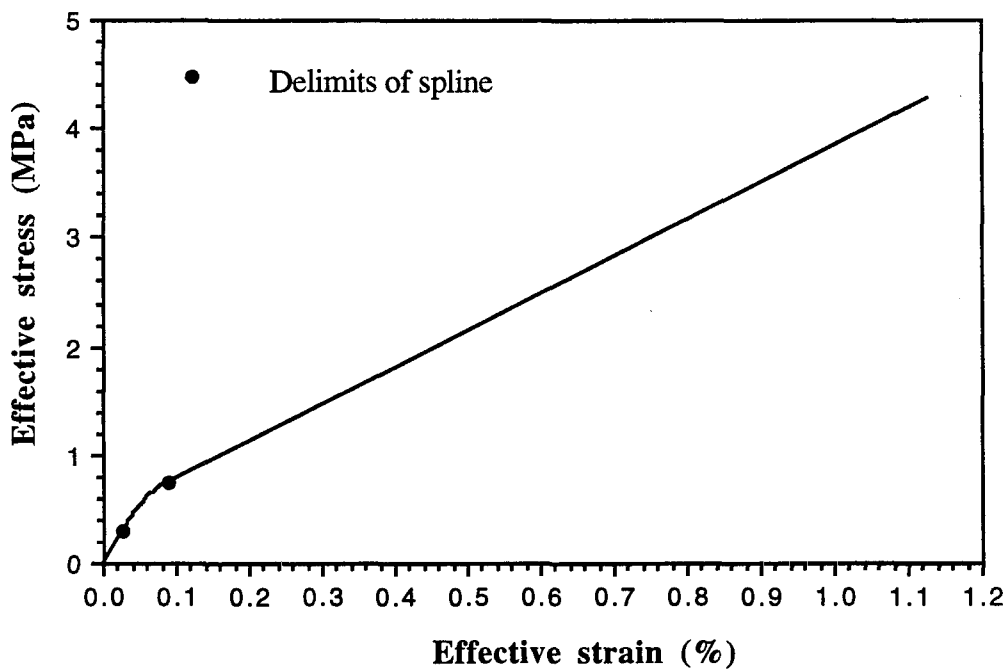


Fig 5.5 Stress-strain curve composed of linear and spline regions

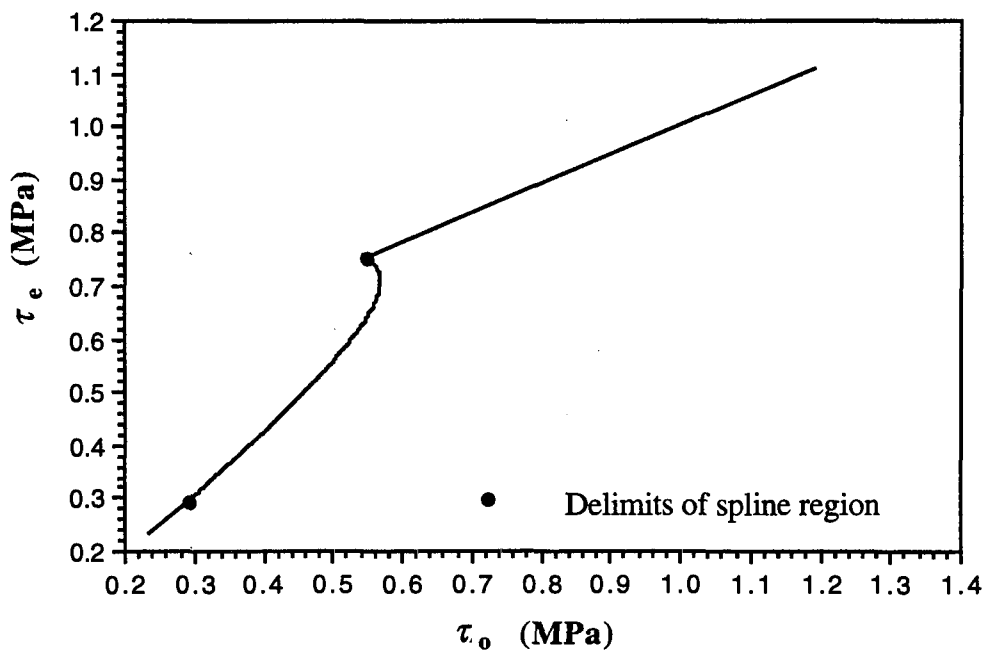


Fig 5.6 τ_e as a function of τ_o predicted from the effective stress-strain curve of figure 5.5

A function of the form depicted in figure 5.6 would therefore have to be fitted to the experimental results. This is not a viable approach, firstly because the experiment cannot be restrained to operate at a particular value of τ_e , and secondly the results are unreliable at low values of τ_o (low striker velocities).

The approach taken in this work is to consider the experimental data as a number of linear regions. The corresponding linear regions of the stress-strain curve can then be evaluated and connected by cubic splines, the start and end points of these splines and the slope at these points being defined by the adjacent linear regions. Spline fits between the linear regions are used to prevent instabilities that arise with a piecewise linear fit due to the singularity in μ_t that occurs at the strain demarking the cross-over from one region to another. The details are described below:

Let the experimental data be approximated to a series of linear regions, as shown in figure (5.7). The regions are numbered from 1 to N. Each region is defined by two points, such that region n is defined by the two points: $n-1$ and n , and is described by the following equation:

$$\tau_e = m_n(\tau_o - \tau_o|_{n-1}) + \tau_e|_{n-1} \quad (5.22)$$

where the slope, $m_n = \frac{\tau_e|_n - \tau_e|_{n-1}}{\tau_o|_n - \tau_o|_{n-1}}$ and $0 \leq n \leq N$.

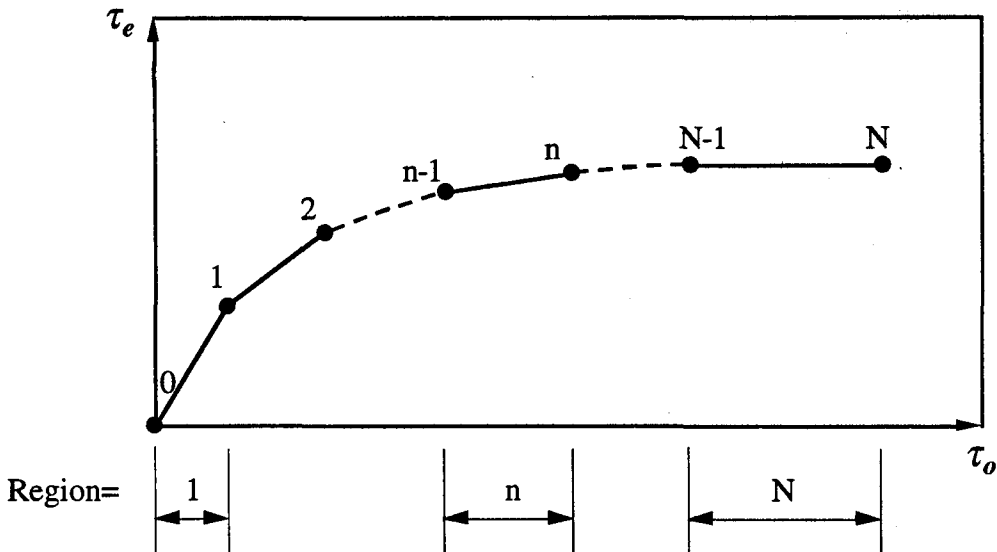


Fig. 5.7 Schematic of the piecewise linear approximation used to model the torsional impedance test results.

Substituting into equation (5.20) and evaluating the integral gives the reciprocal of the effective strain as:

$$\frac{1}{\gamma_e} = m_n \frac{\mu_o}{\tau_o} + A_n \quad (5.23)$$

Substituting for the effective strain from above in equation (5.17) gives the effective section tangent shear modulus from equation (5.13) as:

$$\mu_t = \frac{1}{\mu_o} (m_n \mu_o + A_n \tau_o)^2$$

Since this equation refers to individual, linear regions the effective section tangent shear modulus must be constant and therefore:

$$A_n = 0$$

The identity for μ_t thus reduces to:

$$\mu_t = \mu_o m_n^2 \quad (5.24)$$

Substituting for τ_e from equation (5.22) and τ_o from (5.23) into equation (5.16) gives, after rearranging, the effective section secant shear modulus as:

$$\mu_s = m_n^2 \mu_o + \frac{\tau_e|_{n-1} - m_n \tau_o|_{n-1}}{\gamma_e} \quad (5.25)$$

The strain which demarks the change between region $n-1$ and n can take one of two values depending on which region is considered. The difference in these two possible values of strain reduces as the difference in slope between the two regions reduces. Rather than using either of these two values the intercept is taken (point I in figure 5.8):

$$\gamma_e|_{n-1} = \frac{\tau_o|_{n-1}}{\mu_o (m_{n-1} + m_n)}$$

It now remains to define the spline connecting region $n-1$ and n . The mid strain of the spline is taken to correspond to the strain at the intercept (I). The interval of strain ($\Delta\gamma_e$) corresponding to the spline is equal to twice the difference in strain between the intercept and strain defined by point $n-1$ of region $n-1$:

$$\Delta\gamma_e = 2 \frac{\tau_o|_{n-1}}{\mu_o} \frac{m_n}{m_{n-1} (m_n + m_{n-1})}$$

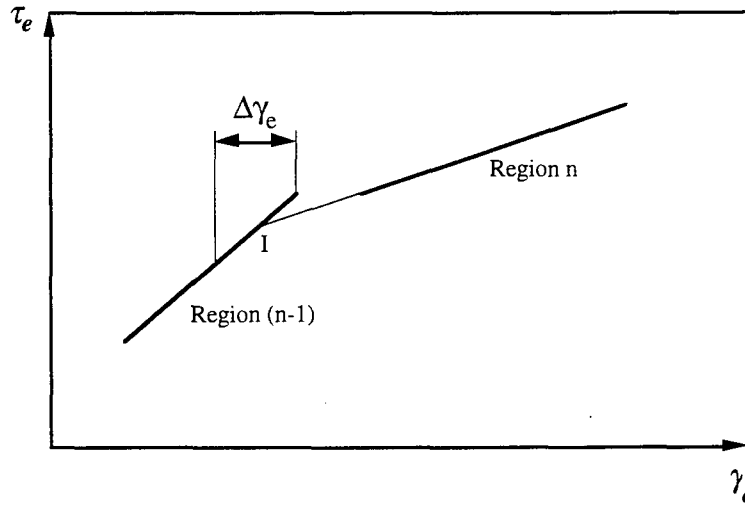


Fig. 5.8 Schematic of effective stress-strain regions: $n-1$ and n .

5.6 Validation

5.6.1 Finite difference model

One of the main validations of the analysis discussed in this chapter was to include the derived material stress-strain characteristic in the finite difference model (see chapter 6). The model was then used to predict the experimental results from which the characteristic was derived. The striker velocity and specimen dimensions were input in to the model and the mean striker force evaluated. These results were then transformed in exactly the same manner as the original experimental ones to produce a plot of τ_e versus τ_o . The two sets of results (finite difference and experimental) are shown in figure 5.9. As can be seen the two results show good agreement.

The validity of the low strain region of the stress-strain curve cannot be confirmed by this method since it is impossible to obtain reliable data at the low striker speeds, as explained earlier. To examine this further a second finite difference analysis of the experiments shown in figure 5.9 was performed, but the material model was changed to being linear, μ_s and μ_t being constant and equal to the product of the square of the experimentally determined slope ($d\tau_e/d\tau_o$) and μ_o (see equation (5.24)). This linear material model thus excludes the low strain, high gradient of the stress-strain curve corresponding to μ_o . The results from this model are also shown in figure 5.9.

There is a slight reduction in the calculated τ_e from the analysis as compared to experiment. This reduction is greater for the linear material model results than for the non-linear. For the non-linear case the error is probably due to the assumption made in deriving equation (5.2) that the rotation rate is constant. The rotation rate does in fact vary slightly through

the test and the value used in calculating τ_o was the rotation rate at the time when a wave travelling at C_t^o would pass the mid-length section of the torsion beam. This assumption could therefore be modified but on noting the scatter of the experimental results the apparent reduction in τ_e is relatively insignificant.

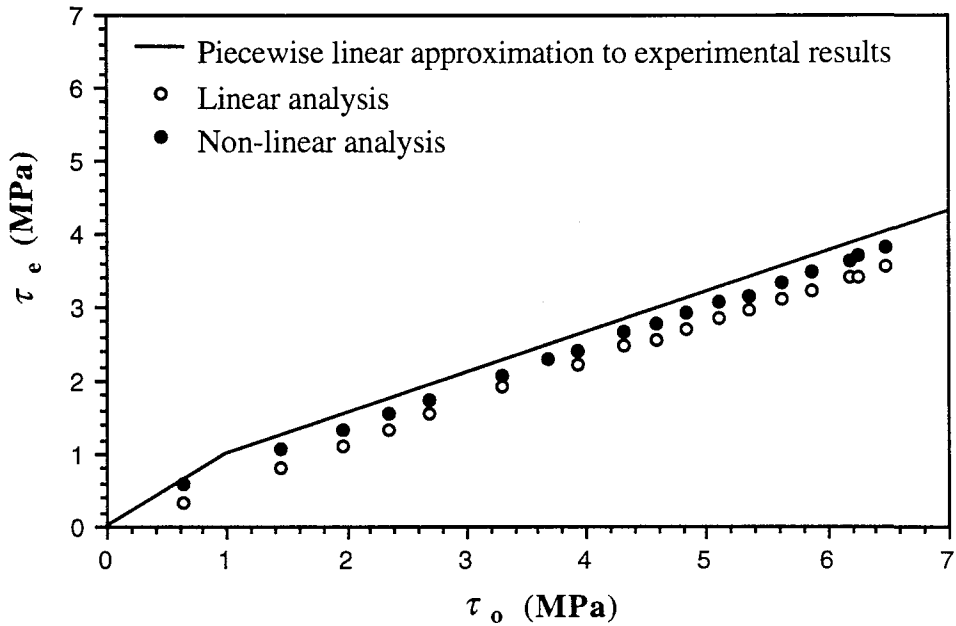


Fig 5.9 Normalised torsional impedance test results and the equivalent values predicted from the finite difference analysis

There is therefore little significant difference between the linear and non-linear cases. This can be explained once it is noted that the strains close to the load plane of the test specimen, throughout the time of the test, lie largely in the domain of the low gradient linear region of the non-linear stress-strain curve. The first point on figure 5.9 (lowest τ_o) is the most interesting in that it clearly shows the difference in the low strain modulus between the linear and non-linear models. Since experimental results are difficult to obtain in this region an experimental method, which is detailed in the next section, was developed to check the validity of using μ_o .

5.6.2 Section Rotation

The modified optical crack gauge (described in chapter 2) can be used to measure rotation at a series of sections along the specimen as a function of time. The technique provides a method of checking the above theory by comparing the modified OCG section rotation measurements during a torsional impedance test to those predicted using the finite difference method.

The maximum slew rate of the OCG sensor stage used in processing the signals was 13 V/ μ s (at least two orders of magnitude higher than the rates actually measured) and thus did

not affect the results. The output from seven sensors during a test were captured on a Nicolet 500 transient recorder with a sampling rate of 100 kHz. Sample results from two tests are shown in figure 5.12. The OCG was primarily designed to define times at which angular acceleration occurs at a section rather than to measure absolute rotation values. As such, the results for angular rotation show considerable scatter.

Software post-processing was used to determine the time at which each of the seven section rotations, recorded using the OCG, reached a set value. A ‘propagation rate’ (V) of that rotation along the specimen was then evaluated. An example of one test result is shown in figure 5.10 together with the prediction from the non-linear finite difference model without the modelling of the overhang.

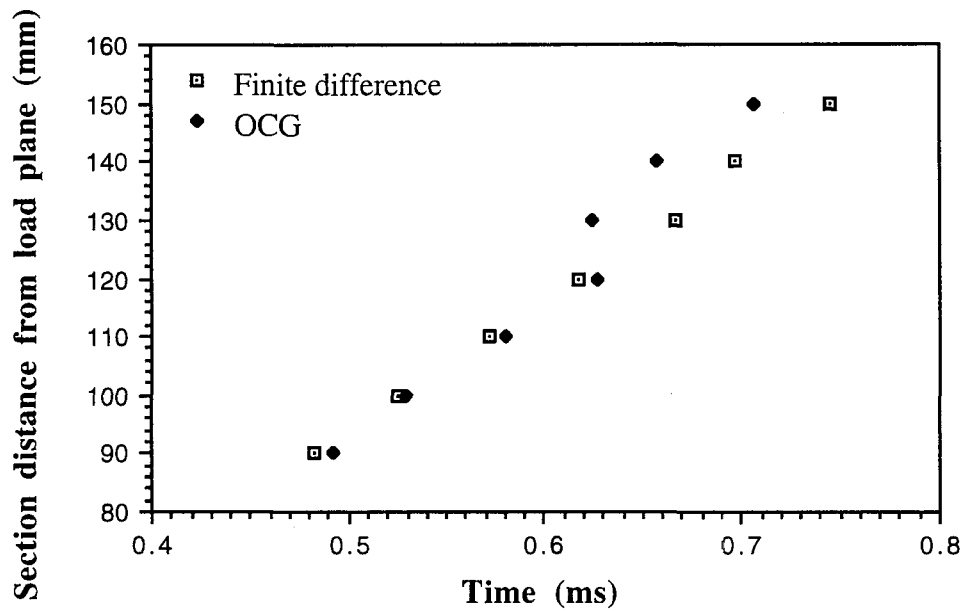


Fig. 5.10 Experimental and predicted propagation rates for a rotation amplitude of 8°

As can be seen the propagation rate is reasonably constant with an acceleration and deceleration at 120 mm due to a combination of overhang effects and reflected, unloading torsional waves reflected from the free end. Due to this reflection a linear least squares fit through only the first three points was used to determine the propagation rate from each set of experimental results. The corresponding propagation rate was then predicted using the finite difference model. The material was modelled as both linear and non-linear as described in section 5.6.1. The modelling of the overhang was omitted to reduce oscillations which would produce a scattering of the results (see section 5.2). The tests analysed correspond to those shown in figure 5.9, the propagation rate for an angular rotation of eight degrees and the minimum rotation (see figure 5.12) was calculated from both the finite difference model and experimental results.

The results from these experiments are shown in figures 5.11. There is good agreement between the non-linear finite difference model and experimental results for the propagation rate of the minimum rotation. This speed is, on average, just less than C_t^0 . The linear finite difference model results for this propagation rate do not show good agreement, the rate corresponding to the Saint Venant wave speed calculated using the high strain shear modulus input into this model. All results show reasonable agreement for the propagation rate of the 8° rotation amplitude.

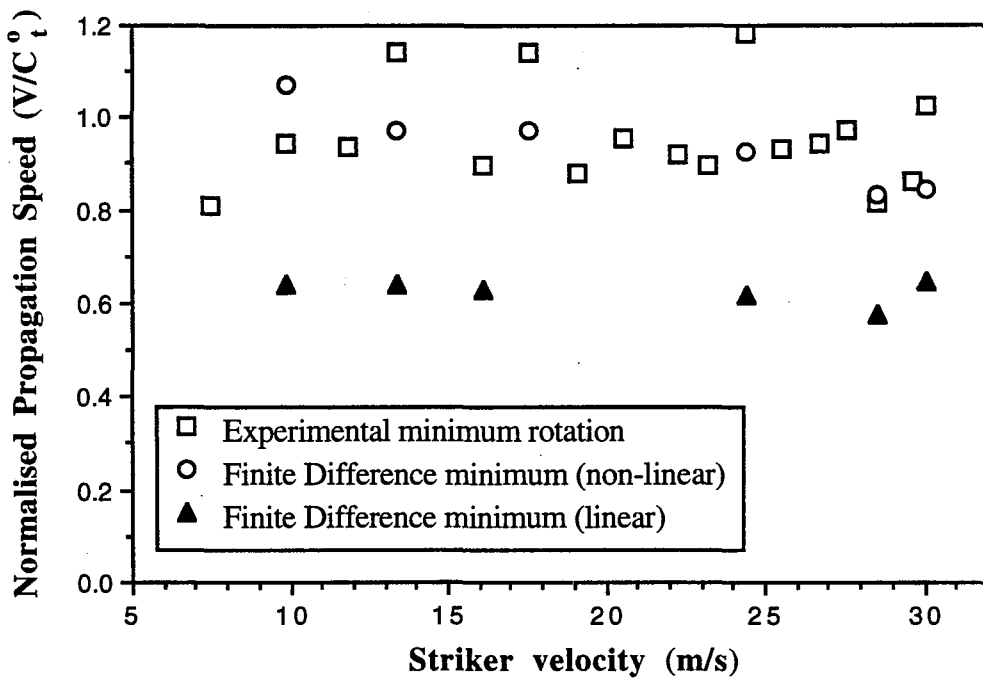


Fig. 5.11a Experimental and predicted propagation rates of the minimum rotation amplitude

It is conceivable that the propagation rate of the minimum amplitude is dependent on the dispersive nature of torsional waves in rectangular beams. If this amplitude is carried by high frequency components then the propagation rate could be higher than that predicted by Saint Venant's equation. The dispersion is due to the existence of axial stresses in the beam which are in turn dependant on the tensile modulus. The tensile modulus in the linear material finite difference model was set to the same value as that in the non-linear model and therefore the difference in propagation rate of the minimum amplitude cannot be due to dispersion. From these results it is clear that the non-linear material model is more appropriate for modelling the HSDT test.

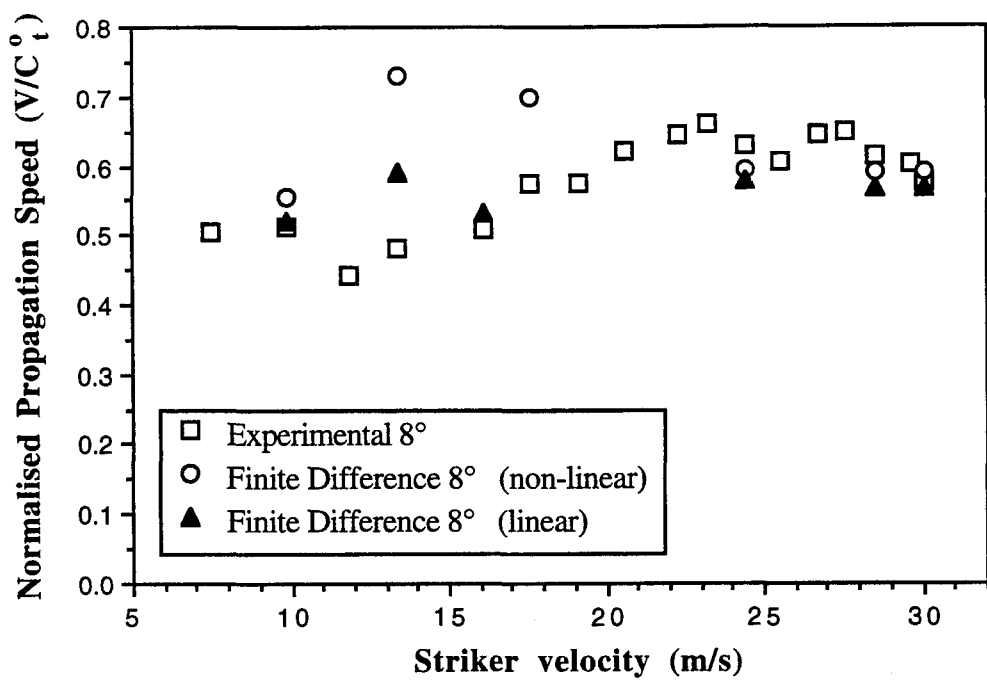


Fig. 5.11b Experimental and predicted propagation rates of the 8° rotation amplitude

A final observation can be made from the measured section rotation histories. This concerns the pronounced negative section rotation at the front of the torsional wave due to the dispersive nature of the deformation. Two sets of results are shown in figure 5.12. Both the predicted and the experimental results show an increase in magnitude of this negative rotation with striker velocity and with distance from the load plane.

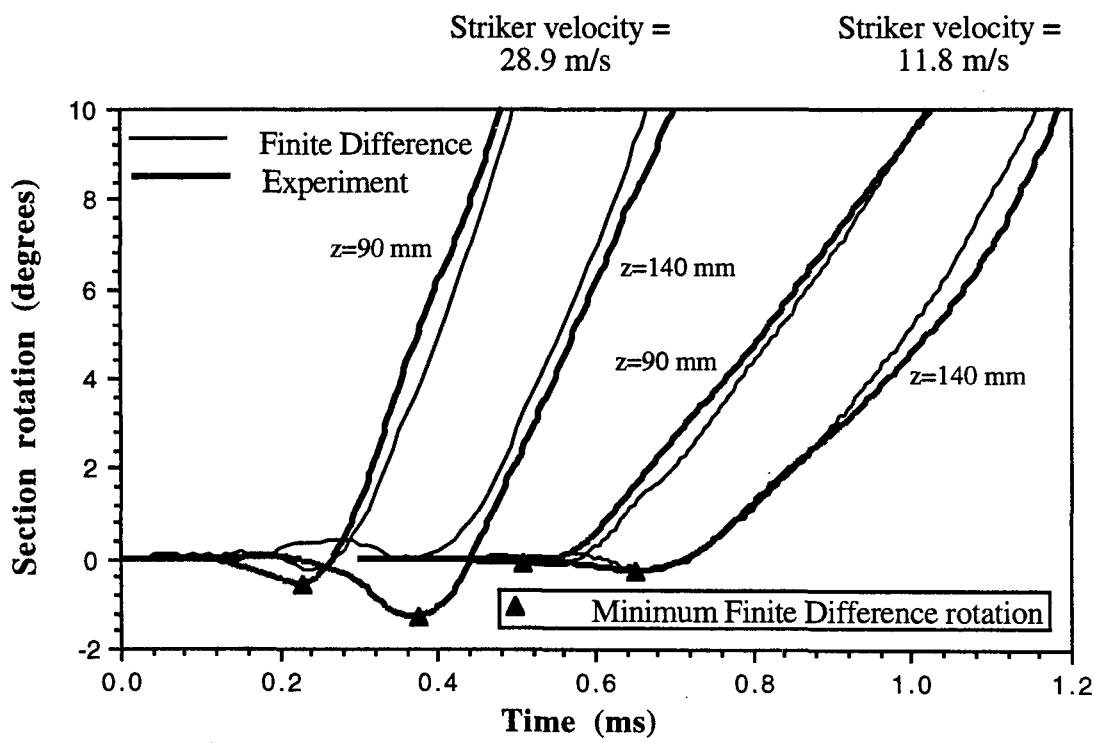


Fig. 5.12 Experimental and predicted rotation histories

5.6.3 Geometry Dependence

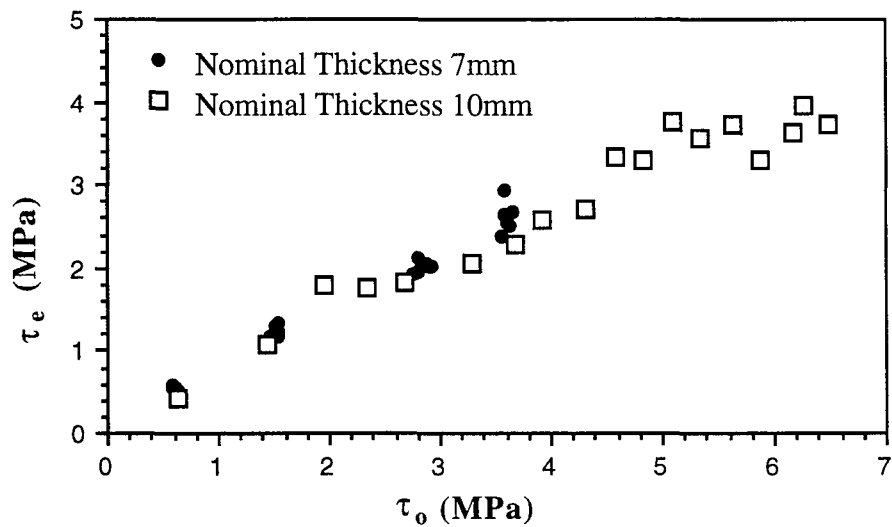


Fig. 5.13 Normalised torsional impedance results for specimens of both 7 and 10 mm nominal thickness.

Figure 5.13 shows torsional impedance test results for two thicknesses (7 and 10 mm) of the same grade of HDPE. The figure shows the results to be similar and confirms that the effective section modulus is geometry independent.

5.7 Results

Torsional impedance and ultrasonic modulus tests were performed on a range of materials. All the results could be modelled adequately by a bilinear fit. They are summarised in tables 5.1 and 5.2.

Polyethylene								
Material		HDPE				MDPE		
Temperature (°C)		-5	0	10	20	0	10	20
μ_o (GPa)		1.25	1.127	0.94	0.86	0.94	0.82	0.64
Poison's ratio (ν)		0.38	0.38	0.39	0.40	0.39	0.40	0.41
Density (kg/m ³)		960	960	960	960	940	940	940
Slope (m_1)		0.732	0.544	0.477	0.394	0.435	0.363	0.418
Point 1 (see figure 5.7)	τ_o (MPa)	1.734	1.047	0.672	0.794	0.648	0.519	0.164
	τ_e (MPa)	“	“	“	“	“	“	“
Point 2 (see figure 5.7)	τ_o (MPa)	7.00	7.00	6.00	6.00	5.00	6.00	5.00
	τ_e (MPa)	5.59	4.29	3.22	2.85	2.5	2.51	2.18
$(m_1)^2\mu_o$ (GPa)		0.67	0.33	0.21	0.13	0.18	0.11	0.11

Table 5.1 Summary of torsional impedance test results for polyethylene at a range of temperatures

		Polyoxymethylene		Polypropylene	
Material		Delrin NC (unmodified)	Delrin ST (modified)	Homo- polymer	30% weight EPDM
Temperature (°C)		0	0	0	0
μ_o (GPa)		1.783	1.209	1.79	0.856
Poisson's ratio (ν)		0.37	0.38	0.33	0.40
Density (kg/m ³)		1420	1330	912.3	894.9
Slope (m_I)		0.545	0.394	0.951	0.844
Point 1	τ_o (MPa)	0.706	0.172	0.0	0.593
	τ_e (MPa)	“	“	“	“
Point 2	τ_o (MPa)	10.00	8.00	6.0	5.00
	τ_e (MPa)	5.77	3.26	5.706	4.31
$(m_I)^2\mu_o$ (GPa)		0.53	0.19	1.61	0.61

Table 5.2 Summary of torsional impedance test results for polyoxymethylene and polypropylene

For all the results the high strain shear modulus is considerably less than μ_o . It is not easy to determine the physical relevance of the change from high to low modulus. The maximum strain in a section, where the axial stresses are zero ($\psi = \frac{\partial\theta}{\partial z}$), can be derived from equation (3.6):

$$\gamma_{\max} = B \frac{\partial\theta}{\partial z}.$$

For the HDPE results shown in figure 5.14 the maximum strain corresponding to the effective stress demarking the transition point between the two regions is approximately 0.8 %. Dioh [5.2] has investigated the stress-strain characteristics of HDPE at 0°C under compression at a range of strain rates; they are shown in figure 5.14.

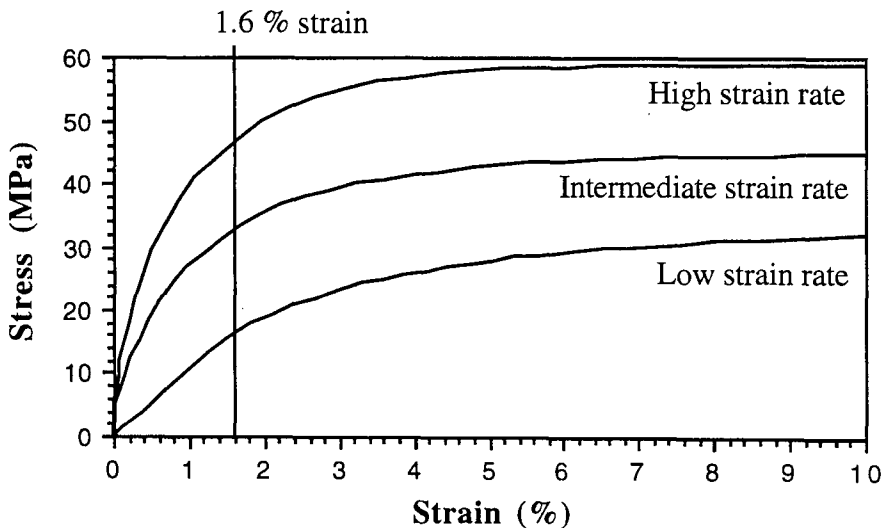


Fig. 5.14 Uni-axial stress-strain curves for HDPE at 0°C (Dioh [5.2])

A 0.8 % shear strain corresponds approximately to a 1.6 % normal strain (assuming pure shear stress conditions) which, from Dioh's data, is close to the yield point of the material. The transition may therefore be due to the onset of the section yielding. As the effective section shear strain increases beyond the transition a larger and larger proportion of the section undergoes yielding. This effect manifests itself as a reduction in the effective section shear modulus which remains constant after the initial transition period.

5.8 Summary

Once effective section shear stress, strain and moduli are defined the torsional impedance test provides an ideal method for measuring the effective stress-strain characteristic at the strain rates appropriate to the analysis of the HSDT test. The ultrasonically measured shear modulus is relevant at low shear strains, but at high strains it can over-estimate the shear modulus by a factor of two to three. The majority of the deformation is controlled by the Saint-Venant wave equation. The dispersive nature of torsional waves in rectangular beams is a secondary effect, which is most apparent in its production of a negative section rotation close to the wave front.

The modified OCG provides a convenient alternative to measuring μ_0 which does not require the apparatus of the ultrasonic method and produces a result more appropriate to the HSDT test.

5.9 References

- (5.1) Wheel, M.A. and Leevers, P.S., 'High speed double torsion tests on tough polymers I: Linear elastic steady state and dynamic analysis', *Int. J. of Fracture*, **61**, p. 331, (1993).
- (5.2) Dioh, N.N., High strain rate behaviour of polymers at various temperatures, PhD Thesis, Univ. of London, (1993).
- (5.3) Wheel, M.A. and Leevers, P.S., 'High speed double torsion tests on tough polymers. II: Non-linear elastic dynamic analysis', *Int. J. of Fracture*, **61**, p. 349, (1993).
- (5.4) Saint-Venant, B. de, 'Mémoire sur les vibrations tournemantes des verges élastiques', *Comptes Rendus*, **28**, pp. 69, (1849).

Chapter 6

The finite difference model

6.1 Introduction

In order to calculate the dynamic crack driving force from the HSDT test results the equations of motion and appropriate boundary conditions derived in chapter 3 must be solved. These are non-linear differential equations due to non-linear material properties. To obtain a solution to these equations a two dimensional (one spatial, one time), finite difference model is used throughout this work.

The finite difference (FD) method has been used in many areas of engineering in the solution of differential equations of motion. The most notable researchers in this area with regard to RCP are Kanninen [6.1] who simulated the Double Cantilever Beam (DCB) fracture test and Wheel [6.2] who simulated the HSDT test. Both researchers employed a two dimensional (one spatial, one time) FD solution and due to symmetry about the crack plane, only modelled a half-specimen. They modelled the restraint imposed by the opposing half using an elastic foundation.

Kanninen used his FD scheme in propagation mode. That is, he assumed that for crack advance the crack driving force at the node corresponding to the crack front, must satisfy the equality of equation (1.1) and at this point the crack front was advanced by one node. The fracture resistance was adjusted until the predicted crack history matched that measured experimentally.

Wheel's approach was to use a FD model in generation mode, where the experimental crack history is input in to the model and the crack driving force predicted. Wheel stated that it was not possible to accurately measure initiation time from the experimental results and so iterated on this parameter to find the initiation time which produced minimum variation in crack driving force during the test.

The FD model that is used in this work is discussed in the next sections. Its important features are described and its numerical accuracy and sensitivity to important parameters are discussed. The results from the model are used to examine the characteristics of the HSDT test and so determine the most suitable operating points to achieve accurate results for fracture resistance.

6.2 The finite difference model of the HSDT test

6.2.1 General features

An explicit, second order accurate, central difference finite difference scheme is used. This was shown to be both efficient and stable by Wheel [6.2], provided both the time step and node spacing are below the size required for convergence. Before discretising the equations of motion derived in Chapter 3, they were non dimensionalised using the following definitions:

$$z^* = \frac{z}{H} \quad \psi^* = \psi H \quad t^* = t \frac{C_t^0}{H}$$

Where t^* and z^* correspond to non-dimensional time and distance respectively and ψ^* is the non-dimensional axial displacement function.

Equations (3.10a) and (3.10b) were discretised in their second order forms and both ψ and θ solved at each time step. An alternative approach would have been to combine the two equations to eliminate ψ and discretise the resulting fourth-order partial differential equation. This approach was not taken since there is limited literature on using the FD method to solve fourth-order equations and because it would have increased the complexity of the coding. The resulting discretised equations are as follows:

$$\frac{\mu_t}{\mu_0} \frac{\partial^2 \theta}{\partial z^{*2}} + \frac{\mu_t}{\mu_0} (\lambda - 1) \frac{\partial \psi^*}{\partial z^*} - \Omega^* \theta = \lambda \frac{\partial^2 \theta}{\partial t^{*2}}$$

and $2(1 + \nu) \frac{\partial^2 \psi^*}{\partial z^{*2}} - \frac{\mu_s}{\mu_0} \frac{(1 - \lambda)}{\gamma^2} \psi^* + \frac{\mu_s}{\mu_0} \frac{(1 - \lambda)}{\gamma^2} = \lambda \frac{\partial^2 \psi^*}{\partial t^{*2}}.$

In order to ensure stability the time step (Δt) must be less than the time taken for a disturbance, travelling at the maximum possible velocity, to traverse the distance between two adjacent nodes. The time step chosen was one fifth of the time it takes a shear wave (asymptotic phase velocity of dispersive torsional waves at high frequencies) to travel between two adjacent nodes:

$$\Delta t = \frac{1}{5} \frac{\Delta z}{C_s^0}$$

where $C_s^0 = \sqrt{\mu_0/\rho}$ (the shear wave speed) and Δz is the spatial distance between nodes. 120, equally spaced nodes were used to define the test specimen. The dependence of the numerical accuracy of the model on Δt and Δz is discussed further in section 6.4.5.

6.2.3 Initial test case

A test case used for the above FD scheme was that of resonance, for which the analytical solution is known (see section 3.3). This was implemented by setting the initial nodal values of ψ and θ to that of a particular mode, predicted from equation (3.23), and then using the FD model to predict the resulting deformation. The model geometry corresponded to the standard HSDT half-specimen and the material properties to HDPE, with a constant shear modulus equal to that measured using the direct contact ultrasonic method (see section 5.1). The predicted frequency of oscillation was within 0.01 % of that calculated analytically and the mode shape showed no change over twenty oscillations. This procedure was repeated for all mode numbers up to and including the sixteenth, with no increase in the error between analytical and FD results.

6.2.2 Specific features

This section describes the specific features of the FD model of the HSDT test not already covered in section 6.2.1. Each feature that will be described influences the dependence of crack velocity on G_D . Where appropriate, default values for parameters are given. In section 6.4 the sensitivity of the model to each of these parameters is discussed in turn.

6.2.2.1 Load plane boundary conditions

The load trace from the HSDT test shows a characteristic oscillation about a mean load. Work by Williams [6.3] and Crouch [6.4] show that two possible causes for the oscillation are contact stiffness and overhang effects. These effects were therefore included in the FD model.

The load plane boundary conditions are exactly the same as those in the static case (see section 4.2.2.2) with the additional modelling of contact stiffness between the planar specimen surface and both the spherical contact points of the striker and support. The contact stiffness was modelled, according to Hertz [6.5], assuming no friction or relative slip between the contacting surfaces and linear material properties:

$$\alpha = \left(\frac{9}{16} \frac{\pi^2 (c_1 + c_2)^2}{R} \right)^{\frac{1}{3}} P^{\frac{2}{3}}$$

where R is the radius of the contact point, α is the distance that the two contacting bodies approach one another, outside of the region locally deformed by the contact. $c_n = \frac{1 - \nu_n^2}{\pi E_n}$ is

a compliance term, the subscript n having a value of 1 or 2, denoting the different material properties of the two bodies; E being the tensile modulus and ν the Poisson's ratio. In the

FD model E for the specimen material is equal to the product of the direct contact ultrasonically measured modulus (see section 5.1) and an adjustment factor (χ). The adjustment factor was introduced since the value of E appropriate to the strain and strain rate at the contact point is unknown prior to performing the analysis. The value of χ is adjusted to match experimental with predicted load histories.

The deceleration of the striker due to the contact force was included in the model.

6.2.2.2 Free end boundary conditions

The free end boundary conditions are exactly the same as those in the static case (see section 4.2.2.2)

6.2.2.3 Non-linear elastic material

Non-linear material behaviour is discussed in detail in Chapter 5. At each time step the effective strain at each node is calculated from the discretised form derived in section 5.4.1.2 and the associated stress-strain region identified and recorded. The corresponding nodal shear moduli are then calculated and used in calculating the future values of θ and ψ . The effective strain region is recorded at each time step so that at the proceeding time step a reasonable first guess can be made at the region in which the effective strain lies. This procedure considerably shortens the execution time if there are a large number of stress-strain regions.

An instability problem due to non-linear material effects occurs with some materials when there is a large difference between the low and high strain tangent shear moduli. To explain this effect it is convenient to consider the development in time of an oscillating rotation profile, as shown in figure 6.1a. The effective strain is largely governed by the twist and is therefore of the form shown in figure 6.1b. If the variation in effective strain is of the right value and amplitude to span the region between the low and high strain tangent shear moduli, there will be a dramatic difference in local torsional wave speeds between the regions L and H. This effect shows a form of positive feedback, in that the twist then tends to increase further in high strain regions, promoting a kind of 'shock wave', as shown in figure 6.1c. The rotation profile can become unrealistically 'noisy', since in reality the natural damping of the material will tend to smooth these fluctuations.

To overcome this problem a smoothing function is applied to the effective strain before the nodal shear moduli are calculated. 'The concept of smoothing lies in a murky area' [6.6]. The approach used here was to calculate the smoothed strain at a node as the 'windowed median' of 5 nodal strains: the node itself and the two nodes in front and the two nodes

behind it. This method prevents any instability and produces little change in results from the unsmoothed method when no instability occurs (see table 6.1). A more elegant method is to use Fast Fourier Transforms to perform filtering in the frequency domain, but this is very costly in terms of run time, compared to the method used.

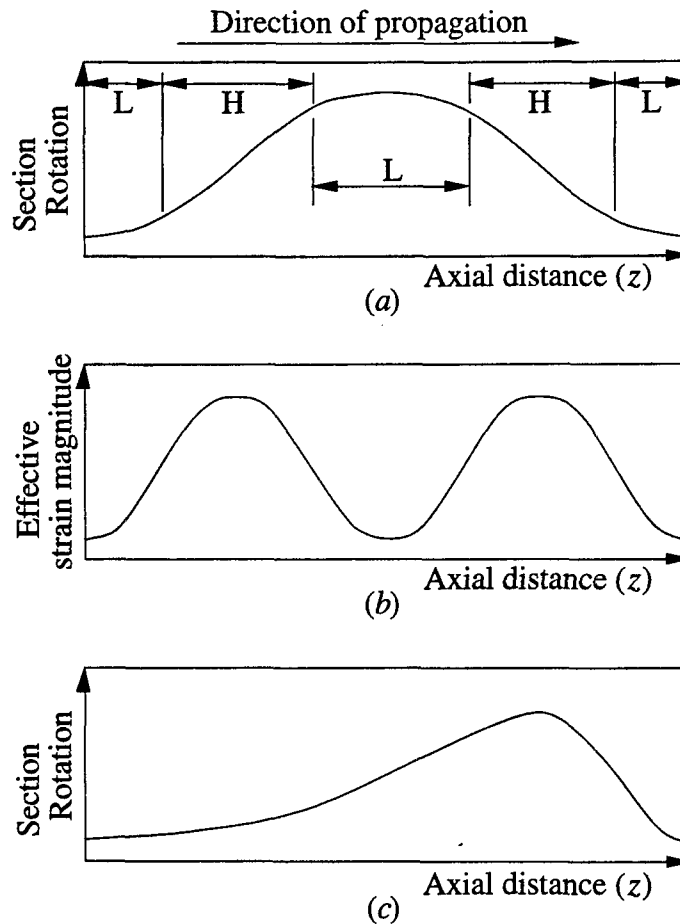


Fig. 6.1 Development of instabilities due to the non-linear material model

(a): propagating rotation profile

(b): effective strain associated with the rotation profile of (a)

(c): development of rotation profile shown in (a)

6.2.2.4 Crack propagation and the curved crack front

Propagation mode is used in the model for the following reasons:

- 1) Propagation mode requires iteration on G_D to find that value which predicts the same crack history as recorded from the experiment. The iteration time involved is comparable to that in generation mode when finding the correct initiation time.
- 2) Propagation mode produces fewer constraints in the model, since the crack velocity is allowed to fluctuate. In the test itself the crack histories all show some oscillation and in the extreme case of stick-slip crack propagation, the crack velocity actually drops to zero for a short period of time.

3) In order to determine G_D from an experimental test using propagation mode the value of G_D must be assumed to be constant during the analysis. This limitation is discussed further in section 6.2.2.5.

The crack front shape in the Double Torsion test is noticeably curved as discussed in chapter 4. The curved crack front was included in the model by employing equation (4.18) to calculate the shape of the crack front at each time step, assuming G_D to be constant along the crack front throughout analysis. Knowing the remaining ligament thickness at each node, the restoring moment due to the foundation can then be calculated, and thus the nodal values of θ and ψ at the proceeding time step.

The iterative method used by the model, to find the value of G_D that produces a predicted crack history the same as that measured experimentally, is as follows:

- 1) A straight line is fitted through the experimental crack history points using a least squares approach. The slope of this line gives the steady state crack velocity.
- 2) The model then steps through values of G_D , calculating the corresponding predicted steady state crack velocity at each step. The same number and spatial positions of crack history points as obtained experimentally are used in the calculation of predicted steady state crack velocity.
- 3) After identifying the step interval that encompasses the correct value of G_D , the method of bisection is used to converge to a value of G_D within the required accuracy.

This rather simplistic iterative scheme was used since $\dot{a}$ is not a smooth function of G_D (see figure 6.17). A more refined scheme such as the Newton-Raphson method would not be reliable since it is dependant on the smooth increase (or decrease) of the first derivative of the function to be solved.

6.2.2.5 Dynamic fracture resistance as a function of crack velocity

The existence of a curved crack front complicates the definition of crack velocity. Consider the propagation of a curved crack front and assume its shape does not change as it translates along the specimen length, as shown in figure 6.2.

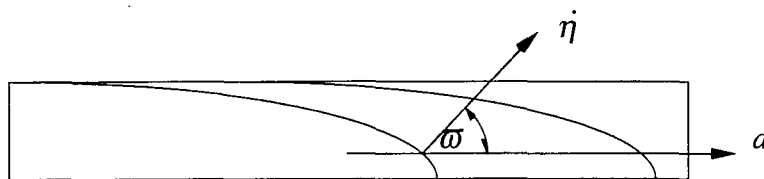


Fig. 6.2 Schematic of the HSDT fracture surface showing the idealised propagation of the curved crack front

Let the translation velocity be constant and equal to $\dot{a}$. The normal velocity to the crack front ($\dot{\eta}$) at any point is then described by:

$$\dot{\eta} = \dot{a} \cos(\varpi)$$

where ϖ is the angle subtended by the normal to the crack front to the z axis of the specimen. For this simplified case the true local crack velocity along the crack front reduces from a maximum at the leading edge to a minimum at the trailing edge. It is commonly accepted that as crack velocity reduces to low values there must be an increase in G_D .

Leevers [6.7] has developed a thermal decohesion model of the fracture process in thermoplastic polymers where the crack propagates by a micron scale melting process. The model assumes that G_D is the energy required to melt a material thickness equal to a weight averaged, half chain length of material either side of the future crack plane. On achieving this criterion the material then poses no resistance to separation. Leevers has developed this theory in order to calculate G_D as a function of crack velocity. The results for PE100 at 0°C are shown in figure 6.3. At 215 m/s there is a minimum which corresponds to a value for G_D of 3.56 kJ/m². If the crack velocity reduces from this minimum point there is more time for heat to be conducted away from the crack front and if the crack velocity is increased there is less time for the heat to be conducted to the required depth.

In the HSDT test it would therefore appear that a higher crack driving force is required to propagate a crack at the trailing edge of the crack front (where the $\dot{\eta}$ is low) as compared to the leading edge (where $\dot{\eta}$ is high). In order to evaluate the influence of this effect in the HSDT analysis, a second version of the code was written that predicted crack propagation, given G_D as a function of crack velocity. The routine used to predict crack propagation is outlined below.

Consider a rotation and crack front profile at time t and the calculation of the profiles at the next (future) time step. The future rotation profile is easily calculated since both the present and past rotations and the foundation stiffness are known at every node. The local propagation direction of the crack front (trajectory) at any node is assumed to be normal to the slope of the front. The angle of this trajectory to the z axis is calculated in the FD model by central difference in space as:

$$\varpi_{|i}^j = \tan^{-1} \left(\frac{2\Delta z}{s_{|i+1}^j - s_{|i-1}^j} \right)$$

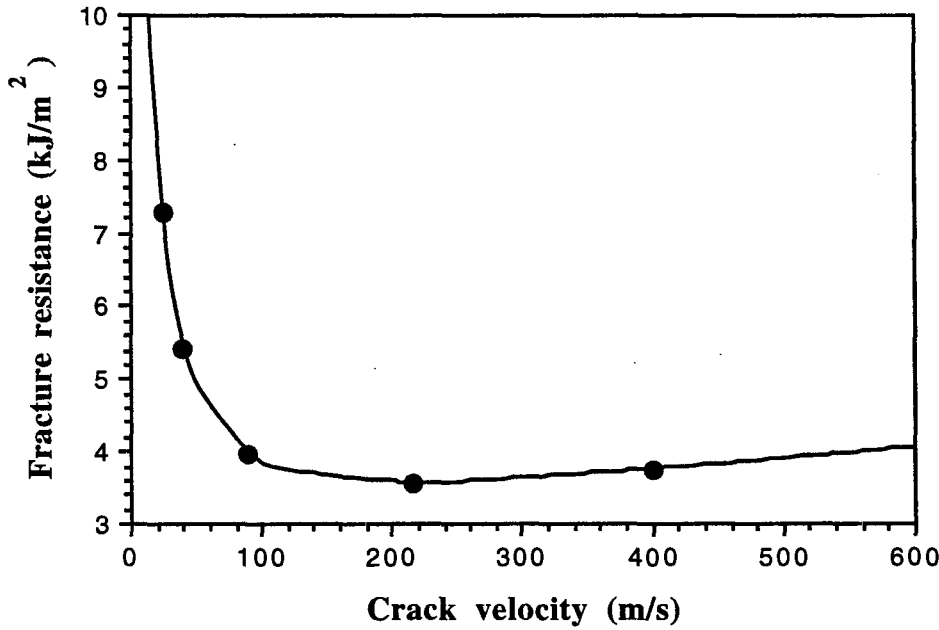


Fig 6.3 Prediction of fracture toughness as a function of crack velocity from the thermal decohesion model for PE100 at 0°C

The crack driving force (G) versus true crack velocity function is then calculated along this trajectory. The method used in this function is as follows: consider that the crack propagates from its present point (P) to a future point (F) along the trajectory. The crack driving force (during the propagation from P to F) is calculated by taking the average of G at the present time step at P and future time step at F. The value of G at F is found by interpolating the rotation at F from the two adjacent nodes and then applying equation (4.18). Equation (4.18) can be applied directly to calculate G at P since P lies at a node. The crack velocity during the propagation from point P to F is assumed to be constant. This procedure is then repeated for a series of points along the trajectory to define the G - $\dot{\eta}$ function. The intercept between the G - $\dot{\eta}$ and the G_D - $\dot{\eta}$ functions can then be found. In this work, if two intercepts occur the one corresponding to the higher crack velocity is taken unless otherwise stated. From the calculated trajectory and crack velocity at each node the new crack front position can then be mapped back onto the nodes and the next iteration started.

This method requires G_D as a function of $\dot{\eta}$ to be known in advance, and so rules out its use as a method of calculating G_D from the experimental results. This routine does, however, provide a means of examining the inaccuracies in the model as described in section 6.2.3.3 due to G_D actually being a function of $\dot{\eta}$.

6.2.2.6 Energy balance

In addition to solving the equations of motion at each time step, an additional routine calculates the incremental internal energy and work done on the specimen during each time step, from the discretised form of the equations given in section 3.2.3. The integration is performed using Simpson's method. These results provide a check that both the model is coded correctly and that the solution has converged such that the results from the model are independent of the size of Δz and Δt .

6.3 Software

Before coding the model certain objectives were set for the final program. These were as follows:

- 1) The model should be user friendly.
- 2) The model could be modified by anyone with a basic knowledge of the Pascal programming language.
- 3) The large amount of numerical data produced by the analysis could be displayed in a graphical form in order to ease their interpretation.

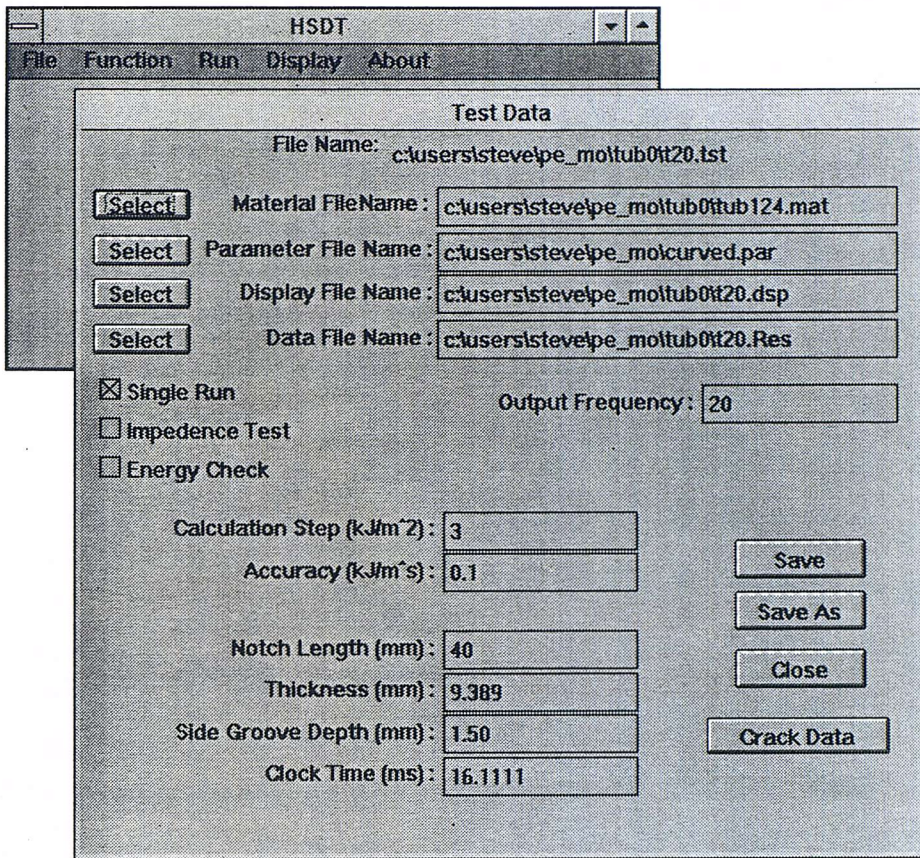
In order to meet these objectives the program was written in a modular form using Borland's object oriented Turbo Pascal for Windows. This compiler facilitates writing mouse driven software that is compatible with Microsoft Windows, and is of a standard 'look and feel' form which anyone familiar with Windows could easily use.

The input files for the model are dialogue boxes which are activated from a simple menu (see figure 6.4). Once the data has been entered it can be easily saved, loaded and edited. The data files created can then be selected and run in batches. Output files are of two types:

- 1) A display file which the application uses to produce an animated image of both the rotation profile and crack front shape as they develop in time (see figure 6.5).
- 2) An ASCII results file detailing the components of the internal energy, work done, striker force and crack length as a function of time. This file is compatible with standard graphing packages such as Cricket Graph and Excel.

Although the input and output routines are written using object oriented techniques, they are designed to be easily updated with little programming expertise. The core analysis routine is written in standard Pascal apart from dynamic memory allocation of the nodal arrays. Two versions of the executable program, plus source code, are contained on the attached disk. To run, 'double click' on `hsdt.exe` seen from File Manager of Microsoft Windows whilst viewing the directories of the floppy disk drive. The programs are:

- 1) `const_gd\hsdt.exe` (G_D is assumed to be constant with $\dot{a}$)
- 2) `vary_gd\hsdt.exe` (G_D as a function of $\dot{a}$ is prescribed in the input)



HSDT

File Function Run Display About

Test Data

File Name: c:\users\steve\pe_moltub01t20.tst

Select Material File Name: c:\users\steve\pe_moltub01t24.mat

Select Parameter File Name: c:\users\steve\pe_molcurved.par

Select Display File Name: c:\users\steve\pe_moltub01t20.dsp

Select Data File Name: c:\users\steve\pe_moltub01t20.Res

☒ Single Run
☐ Impedance Test
☐ Energy Check

Output Frequency: 20

Calculation Step (kJ/m^2): 3

Accuracy (kJ/m^2): 0.1

Notch Length (mm): 40

Thickness (mm): 9.389

Side Groove Depth (mm): 1.50

Clock Time (ms): 16.1111

Save
 Save As
 Close
 Crack Data

Fig. 6.4 Input dialogue box for test data

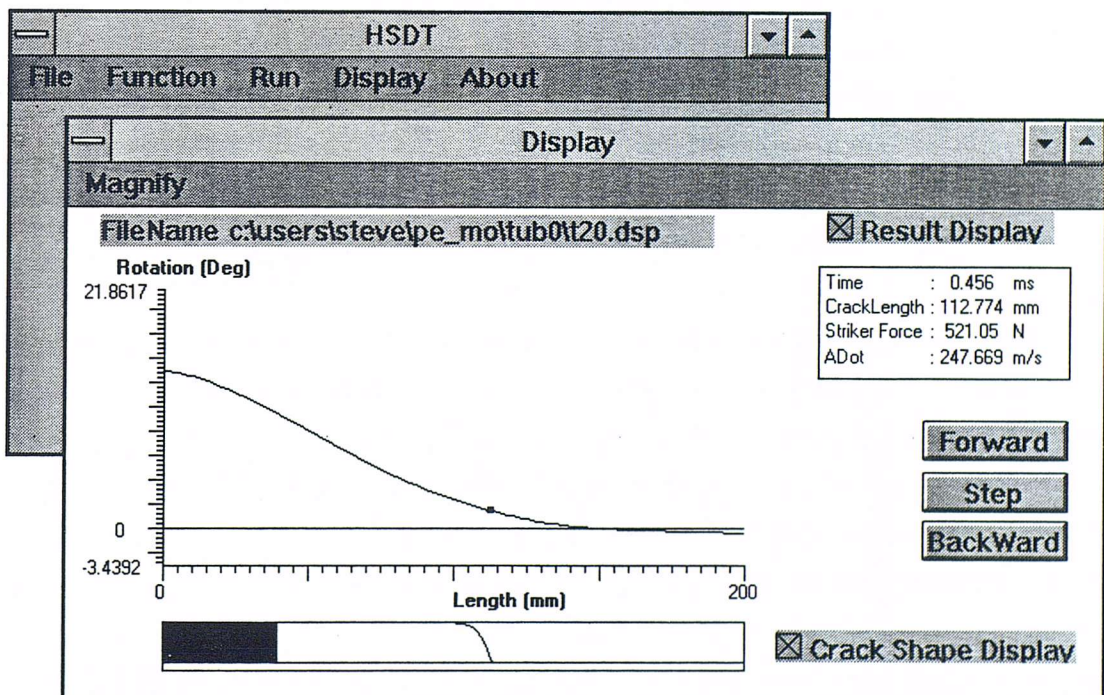


Fig. 6.5 Display screen show crack and rotation profiles
 (The forward and backward buttons cause the display to step through time)

6.4 Validation and Testing

Section 6.2 described the various components of the finite difference model used to analyse the HSDT test. The following section examines each of these components in turn. Unless otherwise stated the results refer to the following conditions:

- Contact stiffness adjustment factor (χ) = 0.2
- Non-linear material properties of PE100 0°C (see section 5.7)
- Smoothed effective strain formulation
- Constant G_D - $\dot{a}$ formulation, $G_D = 3.56 \text{ kJ/m}^2$
- Foundation stiffness coefficient (c) = 0.29
- Number of crack points = 10
- Striker velocity = 17 m/s
- Thickness (B) = 9.27 mm
- Side groove depth = 1.80 mm
- Initial notch = 46 mm

The definition of crack points is as follows: The first crack point is always at 60 mm from the loaded end of the specimen (50 mm from load plane). The subsequent points are then spaced at 10 mm intervals i.e. if 4 points are used then they lie at 60, 70, 90 and 100 mm from the loaded end of the specimen. These points are plotted against the time that the crack passes them. The crack velocity for this case refers to the slope of the line fitted to these points using the least squares method.

6.4.1 Contact stiffness

The dependence of predicted load on contact stiffness is shown in figure 6.5.

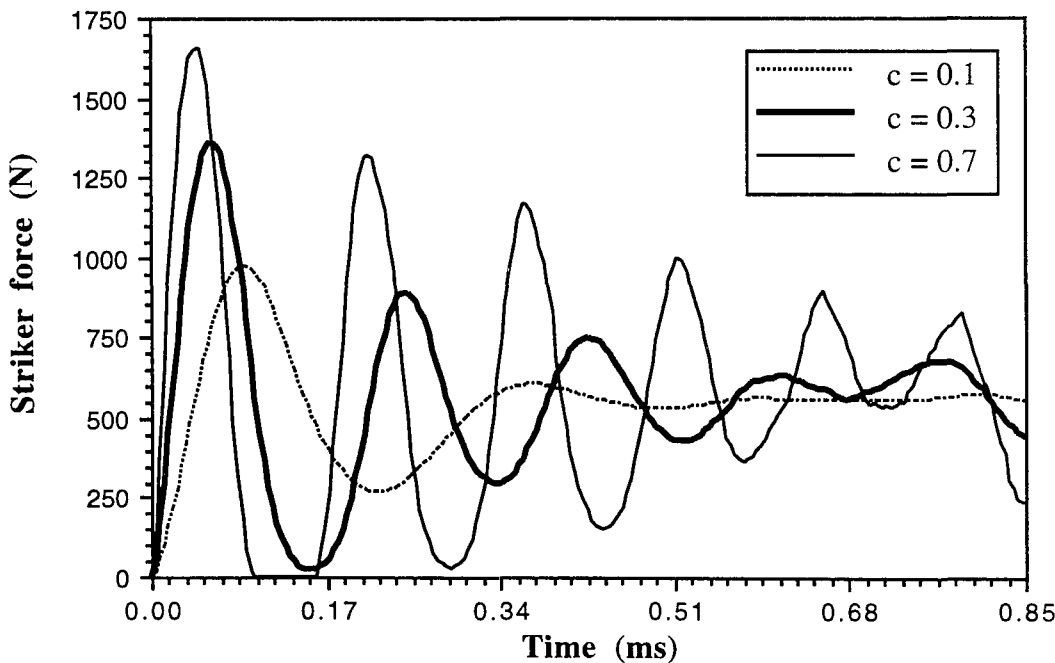


Fig. 6.6 Dependence of predicted load on contact stiffness

The main cause of the oscillations is not the contact stiffness but the overhang region behind the contact point as is shown in figure 5.3. Decreasing the contact stiffness reduces both the amplitude and frequency of the striker force oscillations whilst the mean force remains constant. This result would be expected from a mass-spring model [6.3]. The effect of contact stiffness on crack velocity for low, medium and high velocity cases for a variety of crack point numbers is shown in figure 6.7. The predicted crack histories for the minimum and maximum values of contact stiffness are shown in figure 6.8.

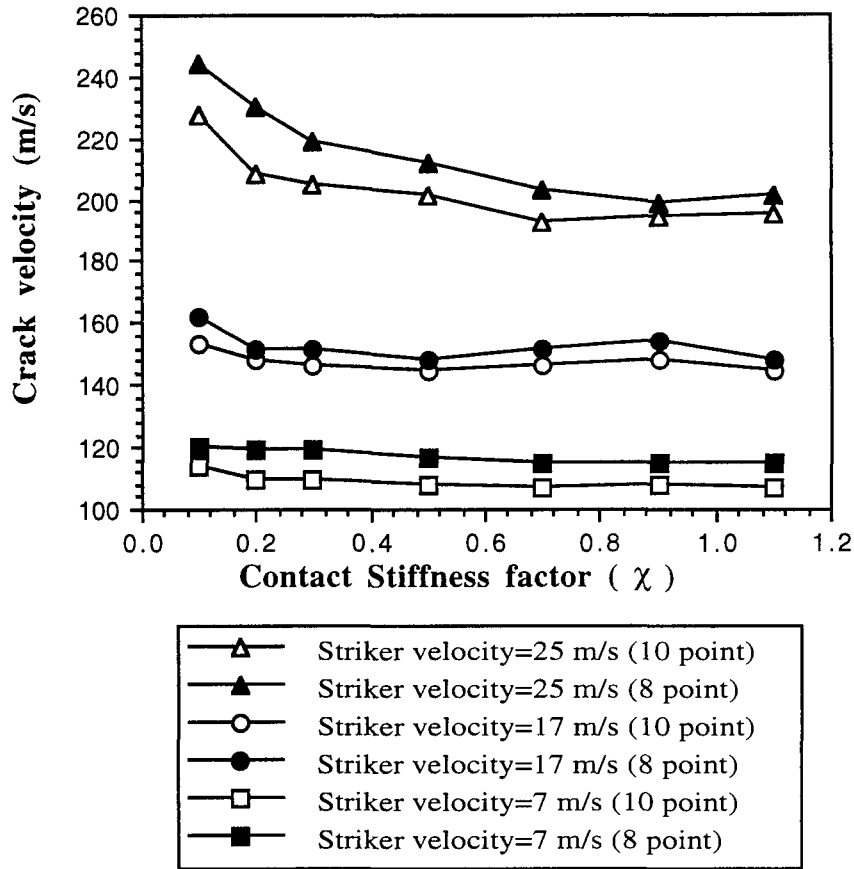


Fig. 6.7 Dependence of crack velocity on contact stiffness

In general, decreasing the contact stiffness increases the crack velocity, an effect which is mediated by increasing the delay in initiation and subsequent crack acceleration (see figure 6.8). Figure 6.7 also shows the dependence of crack velocity on contact stiffness calculated using 10 and 8 crack points, demonstrating that the calculated steady state velocity is dependant on the number of points used. The dependence on the number of points is due to the combination of stress waves produced by the load plane oscillations and reflections from the free end. In other words, the contact stiffness can have a strong influence on the points at which the crack accelerates and decelerates in the latter region of propagation. As a general rule it is better to take sample points over as wide a range as possible since low sampling numbers give a poor distinction between tests at high striker velocities (see results in figure 6.10).

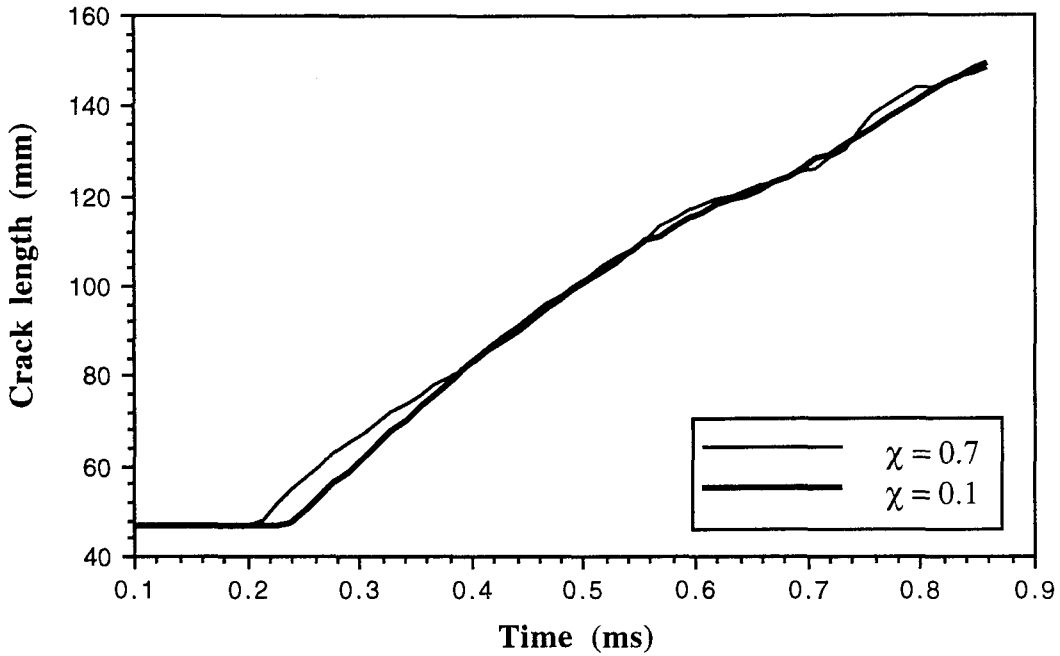


Fig. 6.8 Crack histories for two different values of contact stiffness

6.4.2 Non-linear material properties

In chapter 4 it was stated that all torsional impedance test results could be represented by a bi-linear function. Due to the limitation of the test at low striker velocities the transition point is not well defined from the experimental results. In order to examine the sensitivity of the predicted crack velocity to the transition point, a series of different non-linear material properties were examined. Each case is shown in table 6.1, together with the predicted crack velocity from the model at low and high striker velocities. In order to assess the smoothing function, each of these cases was tested using smoothed and unsmoothed versions of the model.

The definition in case 1 is that measured and described in chapter 5. Cases 2 and 3 are similar but with, respectively, a lower and a higher transition point (point 1). Cases 4 and 5 are effectively linear material properties; case 4 being the high strain modulus and case 5 the low strain modulus. Case 6 is similar to case 1 but with a more refined transition from the low strain to the high strain modulus.

None of the cases using the unsmoothed calculation of the effective shear strain showed signs of instability and as such comparison between unsmoothed and smoothed results therefore gives an indication of the error introduced by smoothing. As can be seen from the results the error is low apart from the medium striker velocity results of case 2 (9% increase from the smoothed to the unsmoothed result).

The crack velocity shows a low sensitivity to the modulus transition strain particularly at the low and high striker speeds. At the medium striker velocity, for the smoothed results, there is a 15% reduction from case 1 to case 3. Providing the transition effective stress is accurate to within 50% the error produced in predicted crack velocity should be less than 10%.

Case Number		1	2	3	4	5	6
μ_0 (GPa)		1.13	1.13	1.13	0.334	1.13	1.13
Point 0	τ_0 (MPa)	0.0	0.0	0.0	0.0	0.0	0.0
	τ_e (MPa)	0.0	0.0	0.0	0.0	0.0	0.0
Point 1	τ_0 (MPa)	1.05	0.5	2.0	1.0	1.0	0.52
	τ_e (MPa)	1.05	0.5	2.0	1.0	1.0	0.52
Point 2	τ_0 (MPa)	7.0	7.0	7.0	7.0	7.0	3.0
	τ_e (MPa)	4.29	4.29	4.29	7.0	7.0	2.11
Point 3	τ_0 (MPa)	-	-	-	-	-	7.0
	τ_e (MPa)	-	-	-	-	-	4.29
Smoothed (s) / Unsmoothed (u)	Striker velocity (m/s)	Predicted crack velocity (m/s)					
s	7	110	119	105	183	99	120
u	7	110	120	109	183	99	120
s	17	148	158	126	272	148	157
u	17	147	172	127	272	148	158
s	25	208	216	194	368	199	214
u	25	208	217	194	368	199	220

Table 6.1 Sensitivity of crack velocity to different non-linear shear modulus definitions

6.4.3 Curved crack front

6.4.3.1 Sensitivity to foundation stiffness coefficient

The sensitivity of predicted crack velocity to the foundation stiffness coefficient for low and high striker velocity cases is shown in figure 6.9.

As can be seen the crack velocity shows a complex sensitivity to c with a maximum error of 16% in predicted crack velocity over the range considered. The data in figure 6.9 also shows the sensitivity of the model to the value of E used in the foundation, since a change in the value of c can alternatively be considered as an inversely proportional change in E , in both the foundation stiffness and crack driving force terms (equations (4.9) and (4.18))

respectively). The actual value of c should be reasonably accurate but the value used for E in the foundation is questionable. An estimate of the possible errors can be made from the work of Morgan [6.8] who examined the plane strain value of the flexural modulus in the Charpy test. His results show that the flexural modulus could fall by up to 50% for the PE 80 and PE 100 materials at the maximum strains in the HSDT foundation. If it is noted that the strain rates in the HSDT test are 50-100 times greater than the rates at which Morgan performed his tests, and noting Diah's prediction of strain rate sensitivity of these materials (see figure 5.14), it is unlikely that E would be less than 75% of the direct contact ultrasonic measured value (an increase in c from 0.29 to 0.39). This error in E would only produce a maximum error in crack velocity of 4%.

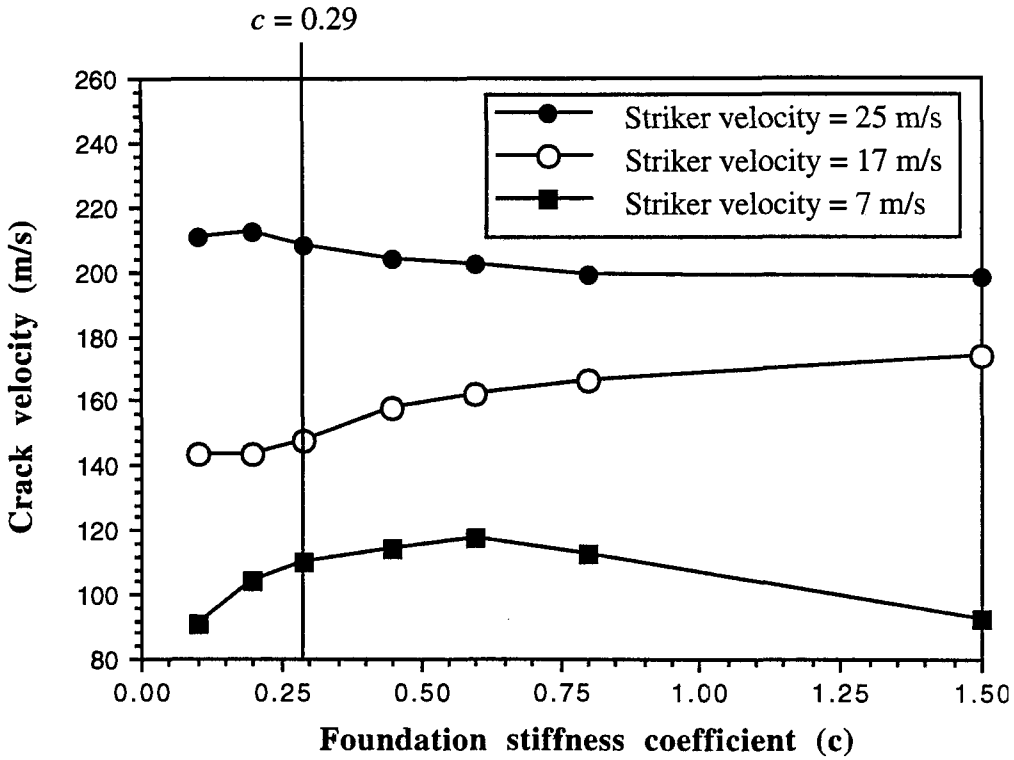


Fig. 6.9 Sensitivity of crack velocity to foundation stiffness coefficient

6.4.3.2 Dynamic fracture resistance as a function of crack velocity

The alternative crack propagation model described in section 6.2.2.5 was used to examine the effect of G_D as a function of true crack velocity. The crack histories for both low, medium and high striker velocities, calculated using Levers [6.7] predicted relationship of G_D to $\dot{\eta}$, are shown in figure 6.10. For comparison, the same cases, but with a constant G_D - $\dot{\eta}$ relationship ($G_D = 3.56 \text{ kJ/m}^2$) for the low and high striker velocities are also shown in figure 6.10. The results remain unaltered if Δt and Δz are reduced.

As can be seen the results are very similar due to the crack front propagating, at all points along it, near the minimum value of the G_D (3.56 kJ/m²).

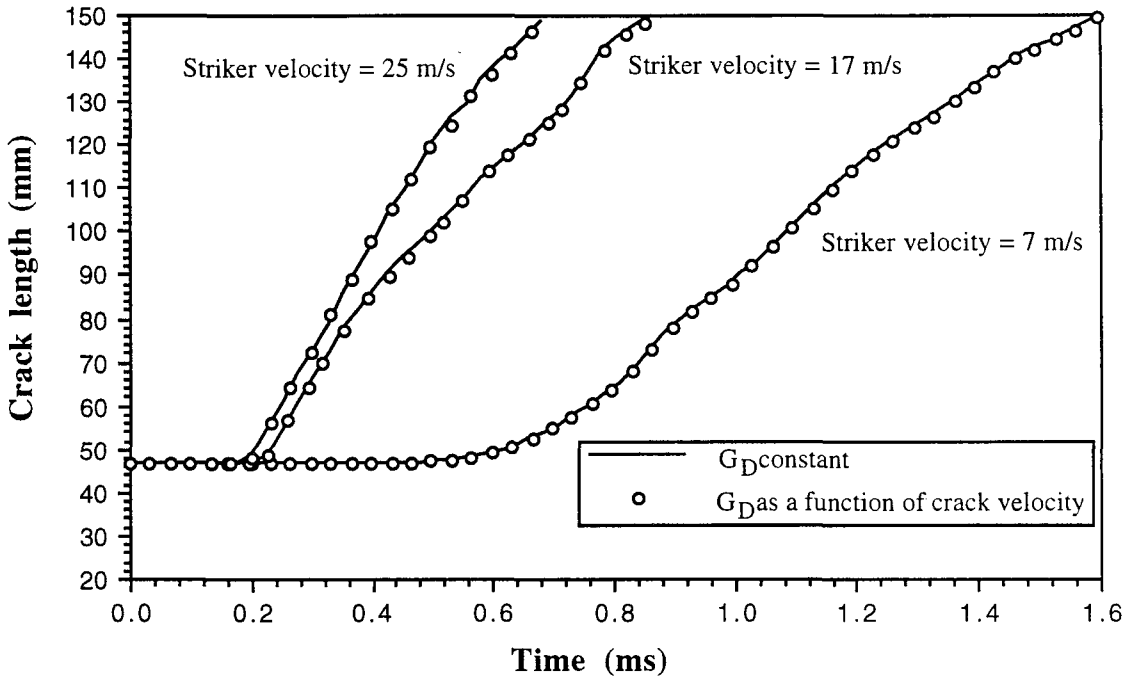


Fig. 6.10 Comparison of crack histories from different propagation models

Consider any point on the crack front where the crack driving force (G) is below the minimum value of G_D . Clearly the crack cannot be propagating at this point. By the nature of the deformation of the HSDT test, G must eventually increase, provided that the striker is not brought to a standstill. After G reaches the minimum value of G_D the crack will propagate when one of two conditions is met:

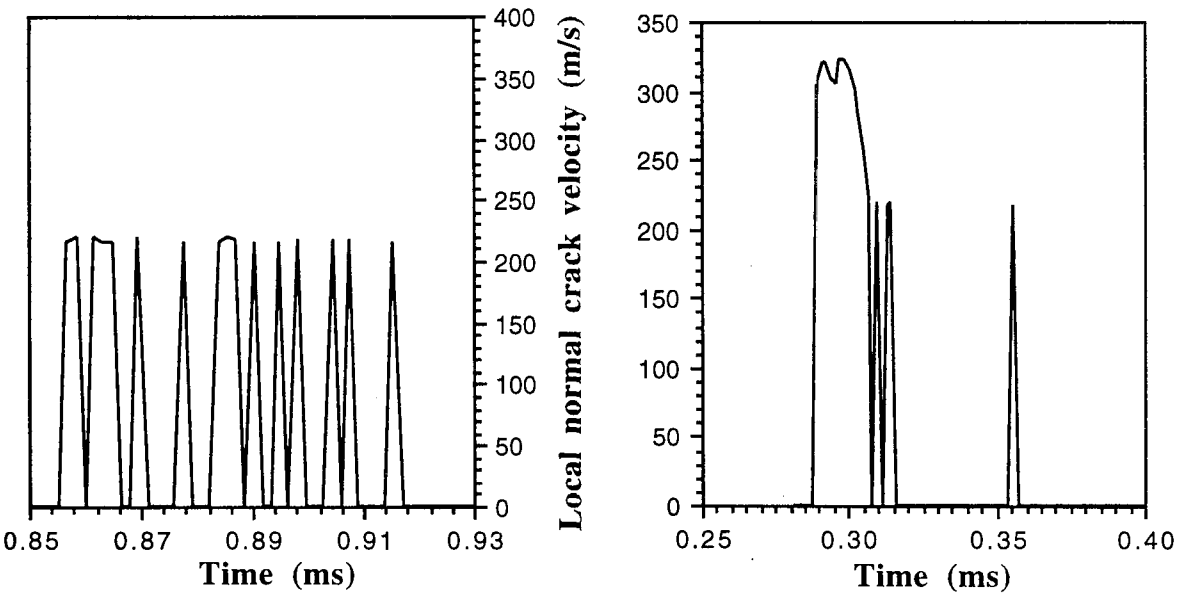
- 1) The crack propagates immediately at a velocity corresponding to the minimum position of G_D .
- 2) The crack remains stationary whilst the average G increases over the propagation distance to a value large enough to sustain propagation at a lower crack velocity.

Case 1 is always satisfied first since the rate of increase in G never keeps pace with the increase in crack velocity necessary to satisfy criteria 2. The fact that the crack does not propagate in regions where $dG/d\dot{a} < 0$ is a recognised instability problem [6.9].

HSDT results for rubber toughened polypropylene (section 7.4) indicate that in reality a material can operate in regions where $dG/d\dot{a} < 0$. Therefore there must be some additional relationship for the G_D - $\dot{\eta}$ function that is not included in the model. The missing link

could be the fact that the rate of change of G is not included in the model. In other words, the model does not distinguish between a slow foundation rotation rate and a fast one. At slow rotation rates there is more time for heat to be conducted away from the crack front into the underlying material. It should be this time dependency of G_D that is included in the model rather than the more simplistic $G_D-\dot{\eta}$ function that was actually used.

However, in regions where the model predicts a fairly low rate of change of crack velocity, the approximation of the thermal decohesion analysis by using a $G_D-\dot{\eta}$ function is reasonable. This situation is true in the most important area close to the bottom of the 'V' groove where the foundation plays an important role in the ensuing deformation. As the remaining ligament thickness decreases, the remaining foundation plays a decreasing role in the overall deformation. Furthermore, the $G_D-\dot{\eta}$ formulation of the model mimics the experimental fracture surface quite well, in that arrest lines are predicted towards the upper region of the fracture surface at high striker velocities while the lower surface has no such lines. As the striker velocity is reduced, the arrest lines extend down towards the bottom of the 'V' groove. Figures 6.11a and 6.11b show the local normal velocity of the crack front ($\dot{\eta}$) at 71.7 mm from the loaded end of the specimen. The results refer to two striker velocities (7 and 25 m/s), the $G_D-\dot{\eta}$ material definition being that predicted by Leever from his thermal decohesion model where the minimum value of G_D occurs at 215 m/s.



(a) Striker velocity = 7 m/s

(b) Striker velocity = 25 m/s

Fig. 6.11 Local normal velocity of the crack front at 71.7 mm from the loaded end of the specimen

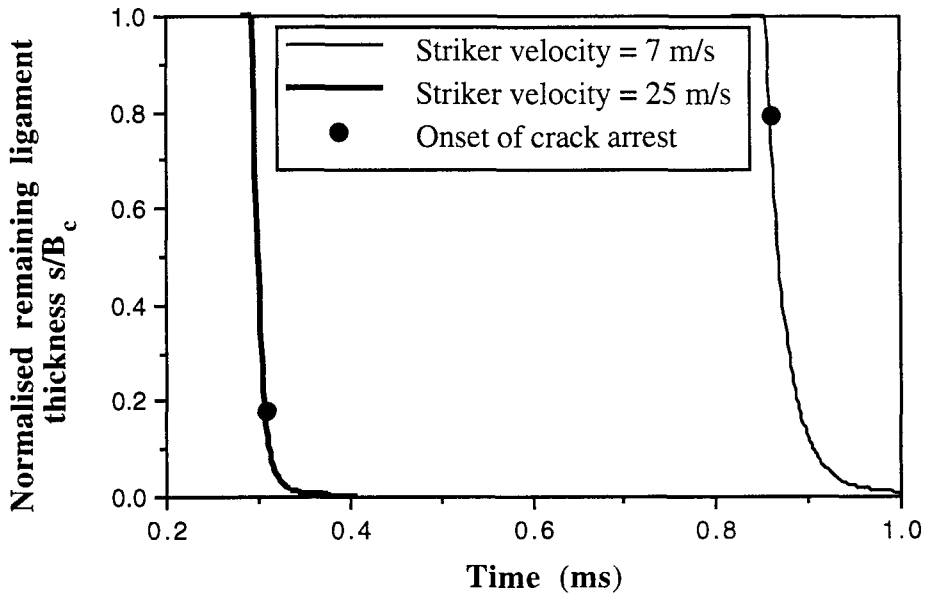


Fig. 6.12 Reduction in ligament thickness at 71.7 mm from the loaded end of the specimen

Figure 6.12 shows the change in ligament thickness with time. As can be seen, arrests begin at a lower ligament thickness for the higher striker velocity as is seen in experiments. For the high striker velocity case the crack initially propagates at a value of G_D above the minimum but as the ligament thickness reduces G_D decays to the minimum value, at which point crack arrest is predicted. In the low striker velocity case, where the rate of supply of energy to the crack front is much lower, the crack front is predicted as propagating at the minimum value of G_D in a stick-slip manner throughout the time of the test.

6.4.4 Oscillations in the crack history

The oscillations in the crack history (accelerations and decelerations of the crack) are due to a complex combination of a number of properties of the HSDT test. In the extreme these oscillations can cause the crack front to arrest either partially or along its whole length. The most important of these properties are discussed, in turn, below:

6.4.4.1 Oscillations due to the overhang

The overhang, in effect, superimposes an oscillation on the linear increase in rotation at the load plane. The oscillations are transmitted to the crack front through the torsion beams behind it producing localised (in time) accelerations. The comparison of the predicted crack histories with and without the overhang are shown in figure 6.13.

Since the contact stiffness modulates the load plane oscillations, it also plays a role in the oscillations seen in the crack history.

6.4.4.2 Stress wave reflections from the free end.

The torsional wave emanating from the load plane, which provides the crack driving force, does not stop at the crack front, but continues past it and is reflected back as an unloading wave. The reflected unloading wave then interferes with the loading wave at the crack front producing localised decelerations. This can be demonstrated by modelling a specimen of twice the standard length ($L=400$ mm) and comparing the predicted crack history with that for the standard length under otherwise identical conditions (see figure 6.13).

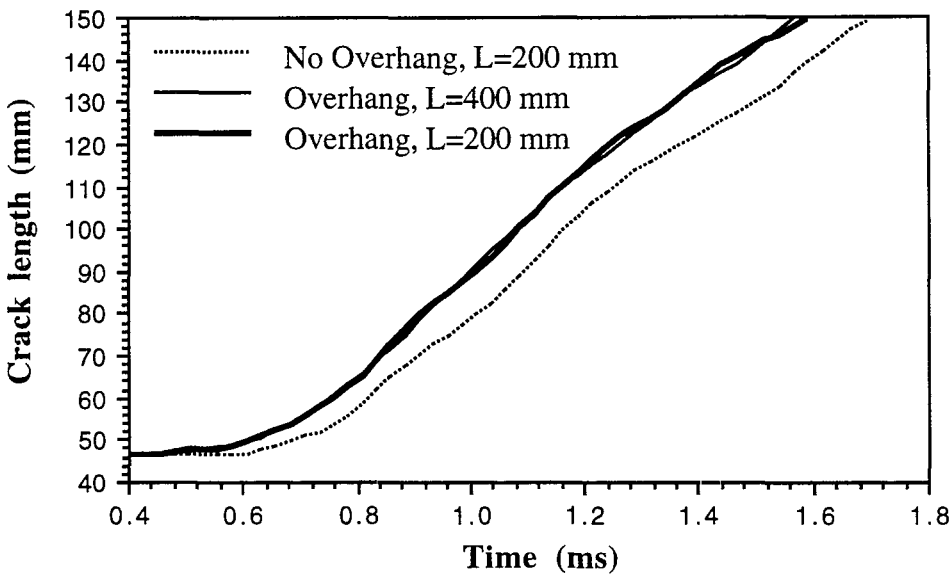


Fig 6.13 Dependence of oscillations in the crack history on overhang and specimen length

6.4.4.3 Unloading waves from the crack front

The crack front propagating along the specimen produces unloading waves which travel away from it in both directions. These waves are reflected from the ends of the specimen and return to the crack front, affecting its propagation.

6.4.5 Energy balance

The crack driving force calculated using the energy balance approach for the default case is shown in figure 6.14. Also shown are results using 1000 nodes with the default calculation of Δt plus the case for 120 nodes with a value of Δt reduced by a factor of 50 from the default case.

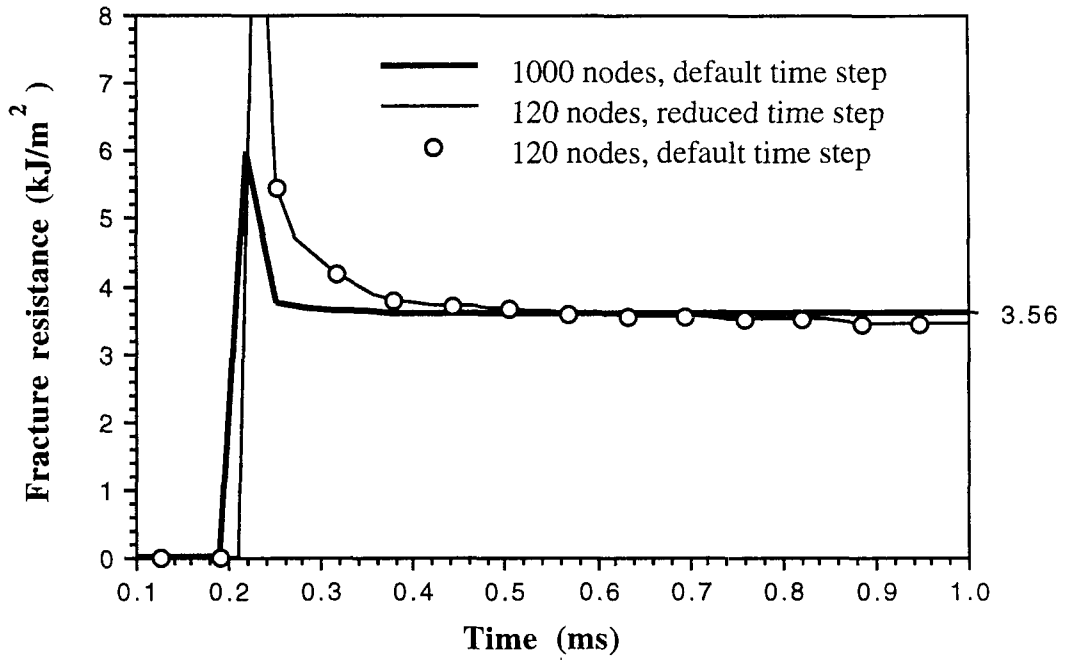


Fig. 6.14 Dynamic fracture resistance calculated from the global energy balance

The results all show an initial error but soon settle close to the expected value of input G_D value of 3.56 kJ/m^2 . The finer mesh results are significantly more accurate. The initial error is due to only a small number of nodes being used to model the curved crack front just after initiation. As the crack propagates along the specimen more and more nodes are used and resulting in a more accurate model of then crack shape. The crack histories from these two results are shown in figure 6.15, showing only a 0.01 % difference in the steady state crack velocity. The coarser mesh is therefore suitable for analysing the HSDT test when a local crack driving force approach is taken.

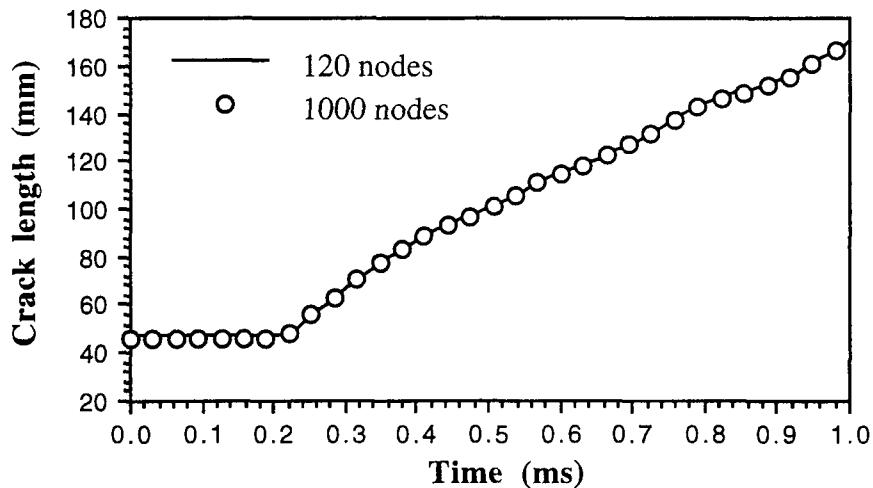


Fig. 6.15 Crack histories from a coarse and fine mesh

A greater understanding of the energy balance results can be gained if the work done and internal energy (kinetic, strain, foundation and contact energies) are examined (see figure 6.16). The crack driving force is calculated as the difference between the work done and internal energy divided by the fracture surface area (see equation (1.2)).

Since the crack driving force is calculated from the difference of two large numbers any small numerical errors are compounded by the energy balance approach. This observation demonstrates the usefulness of the local crack driving force approach in minimising errors.

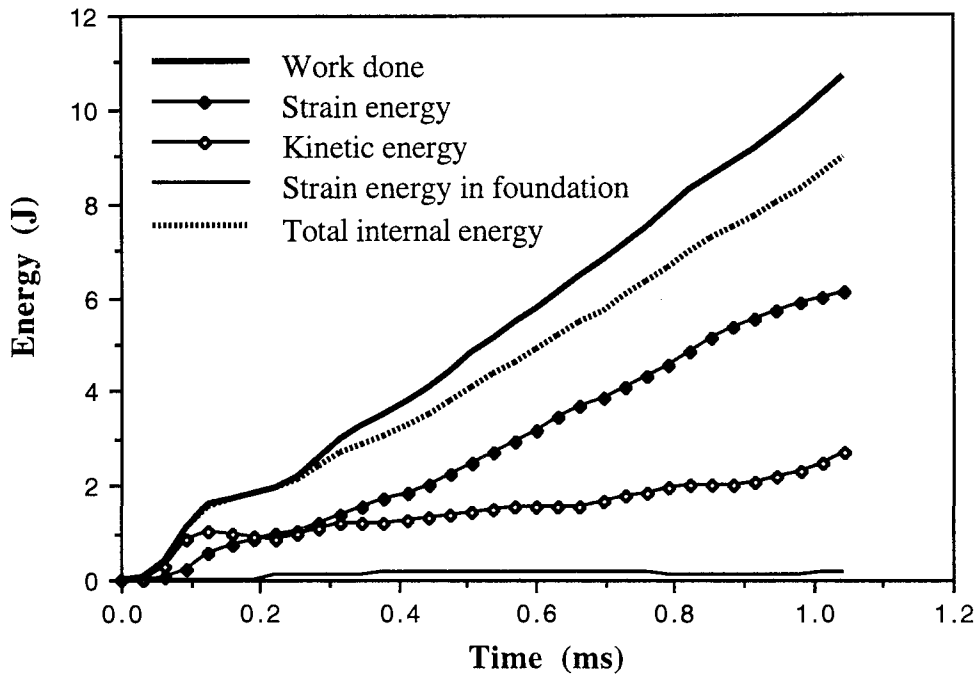


Fig. 6.16 Energy components in the HSDT test

6.4.6 Sensitivity of crack velocity to striker velocity and G_D

The previous sections (6.4.1 to 6.4.5) have looked at the sensitivity of the model's predicted crack history to various parameters. The final aim of the model is to enable the calculation of G_D from the experimental results of striker velocity and crack history. To examine the characteristics of the HSDT test, the default test case was used to predict crack velocity over a wide range of values for G_D and striker velocity. The results are shown in figure 6.17.

All of the results initially show a maximum crack velocity corresponding to the Saint-Venant torsional wave speed in the half beams, calculated using μ_0 . There is a much slower decay of crack velocity from with increasing G_D for the higher striker velocities. At low striker velocities and high values of G_D the crack velocity bottoms out and even

becomes multi-valued due to the effects of stick-slip propagation. The sensitivity of the model's calculated value of G_D to inaccuracies in the experimental and predicted results for crack velocity is therefore at a minimum in the mid-range striker velocities.

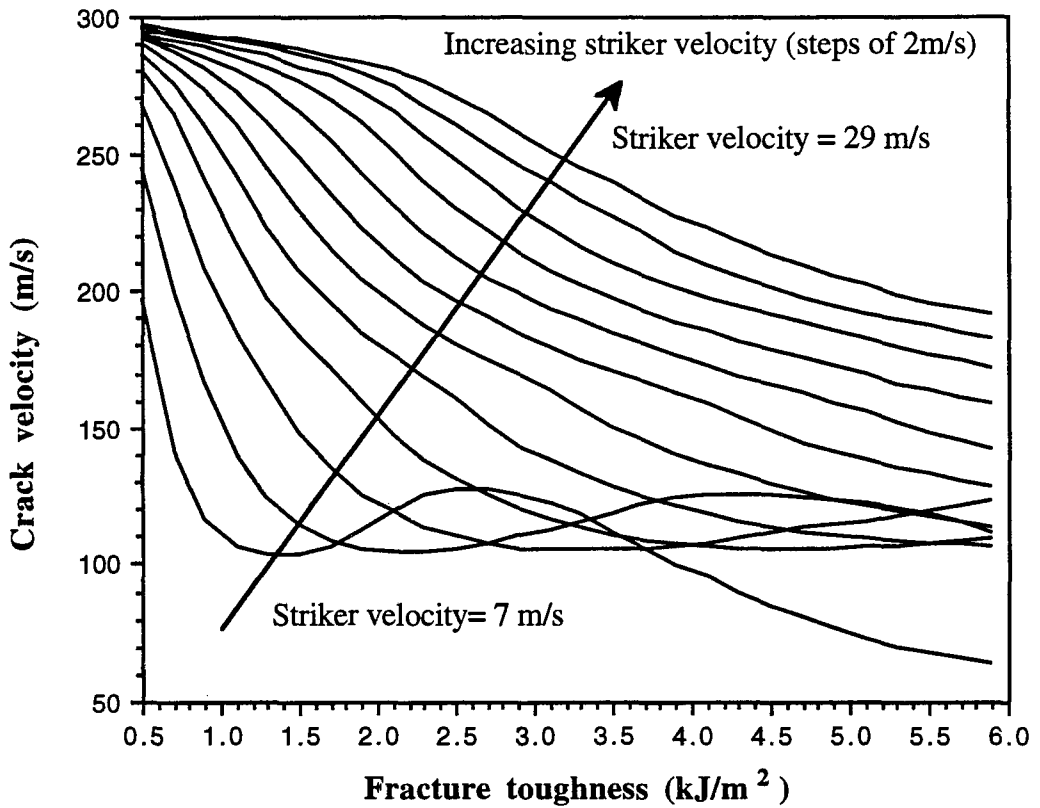


Fig. 6.17 Typical characteristics of the HSDT test

With regards to the low striker velocities, where stick-slip occurs, the points of acceleration and deceleration of the curved crack front provide a much improved convergence criterion to determine the correct value of G_D . This approach is discussed further in section 7.3.

6.5 Summary

From the results presented here, the maximum possible error in predicted crack velocity due to inaccuracies in the various model parameters is unlikely to exceed 10%. If the low crack velocities are ignored, a 10% error in crack velocity can produce an error in G_D of approximately 1 kJ/m².

The predicted crack velocity is dependant on the correct modelling of the oscillations in the crack history. The main cause of these oscillations is dynamic nature of the HSDT test itself rather than the G_D - $\dot{a}$ characteristic of the material. The only parameter on which these oscillations depend but cannot be directly calculated is the contact stiffness. However, the contact stiffness can be easily adjusted to match predicted and measured load

histories. Since the model can therefore be adjusted to impose the same load on the specimen as occurs experimentally the fluctuations in crack velocity due to the load plane oscillations should be realistic.

The steady state crack velocity is a slightly misleading conception since the crack velocity tends to reduce as the crack approaches the end of the specimen. In addition there are superimposed oscillations at the lower striker velocities. Although this result was only shown here from the FD model predictions it is also seen in the experimental results [6.10]. The calculated steady state crack velocity is therefore dependant on the number of crack points used. This dependency should not be important in terms of predicted G_D provided the same crack points are used in the FD model as in experiment and the FD model accurately simulates the fluctuations in crack velocity seen in the experiment.

The possibility that G_D is a function of crack velocity should not invalidate the use of a constant value of G_D if the crack is operating at the lower shelf region of the G_D - $\dot{a}$ characteristic of the material under test whilst it is propagating through the lower part of the foundation. This is because of the rapid decay in foundation stiffness as the crack front moves upwards through a section.

6.6 References

- (6.1) Kanninen, M.F., 'A dynamic analysis of unstable crack propagation and arrest in the DCB test specimen', *Int. J. of Fracture Mechanics*, **10**, p. 415, (1974).
- (6.2) Wheel, M.A. and Leever, P.S., 'High speed double torsion tests on tough polymers I: Linear elastic steady state and dynamic analysis', *Int. J. of Fracture*, **61**, p. 331, (1993).
- (6.3) Williams, J.G., The analysis of dynamic fracture using lumped mass-spring models, *Int. J. of Fracture*, **33**, p. 47, (1987).
- (6.4) Crouch, B., 'Finite element modelling of the three-point bend impact test', *Computers and Structures*, **48**, p. 167, (1993).
- (6.5) Timoshenko, S.P. and Goodier, J.N., *Theory of elasticity*, Third Edition, McGraw-Hill, (1970).
- (6.6) Press, W.H., Flannery, B.P., Teukolsky, S.A., Vetterling, W.P., *Numerical recipes in Pascal*, Cambridge Univ. Press, (1990).
- (6.7) Leever, P.S. and Greenshields, C.J., A model for predicting the dynamic fracture and impact fracture resistance of tough thermoplastics, *Proc., 53rd Annual Technical Conference of the Soc. of Plastic Engineers*, Boston, MA, (1995).
- (6.8) Morgan, R.E., Ductile-brittle transitions in pipe grade polyethylenes, PhD Thesis, Univ. of London, (1994).
- (6.9) Leever, P.S., 'Crack front shape effects in the double torsion test', *J. of Mat. Science*, **17**, p. 2469, (1982).
- (6.10) Wheel, M.A., High speed double torsion testing of pipe grade polyethylenes, PhD Thesis, Univ. of London, (1991).

Chapter 7

Results

7.1 Introduction

During this work the HSDT test has been used to determine the dynamic fracture resistance, as a function of axial crack velocity, of a number of polymers: high density polyethylene and pure and rubber toughened blends of both polypropylene homopolymer and polyoxymethylene. All the HSDT results relating to these materials are presented in this chapter.

During the tests on the high density polyethylene (PE100) at -5°C additional measurements of the section rotation at a series of points were made using the modified form of the OCG (section 2.2.4). The OCG was therefore unavailable to measure the crack velocity and so the timing line technique was used. The tests on PE100 are presented in more detail than the other results, to validate the theory presented in the preceding chapters.

Before discussing the HSDT results, experimental results and conclusions relating to the characteristic oscillations seen in the HSDT test load history are presented.

7.2 Oscillation in the load trace

7.2.1 Effects of overhang

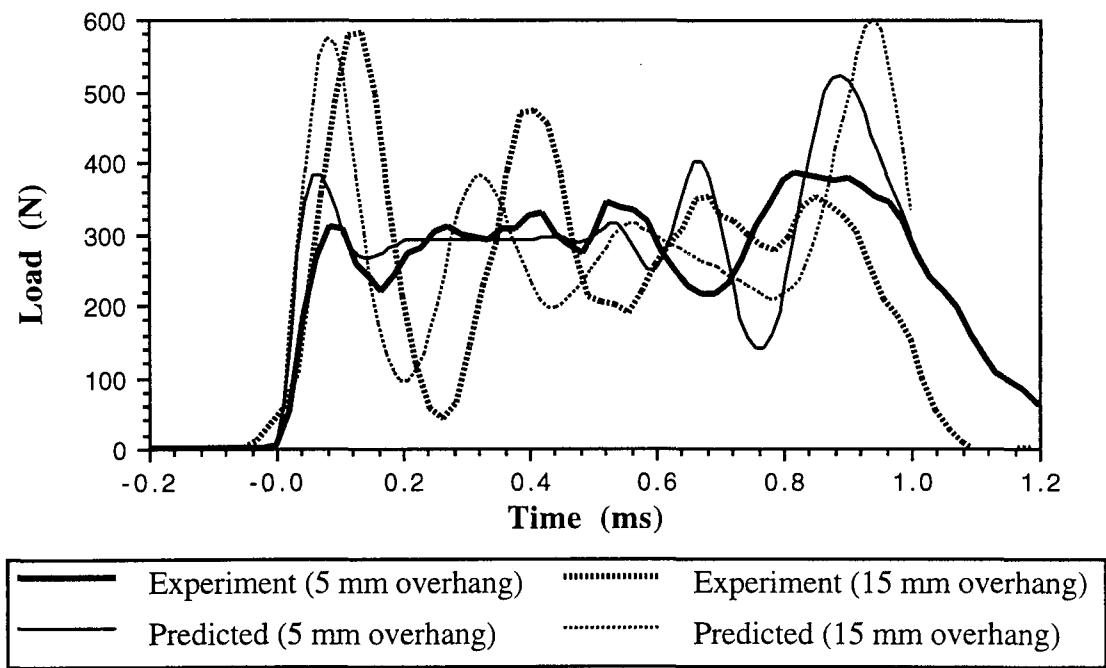


Fig. 7.1 Dependence of load oscillations on overhang length

Torsional impedance tests were performed on PE100 at -5°C with two different overhang lengths of 5 mm and 15 mm. The load histories for each test were recorded and compared with those predicted by the FD model. Exactly the same material definition was used to model each sample. The results, in figure 7.1, show an increase in both the frequency and amplitude of the load trace as the overhang length is increased.

7.2.2 Dependence of load history on measurement point

The load history in the HSDT test has always been measured via a piezo-electric load cell situated beneath the support point at the loaded end of the specimen (see section 1.3.1). Torsional impedance tests have also been performed by Venizelos [7.1] using an accelerometer positioned in the striker itself. The output from the accelerometer was then converted to applied load at the striker point using Newton's second law of motion. Both the applied load at the striker contact point (S) and the reaction force at the support point (F) for one test are shown in figure 7.2. The load trace from the accelerometer has been considerably smoothed to suppress the 'ringing' of the striker at 13 kHz.

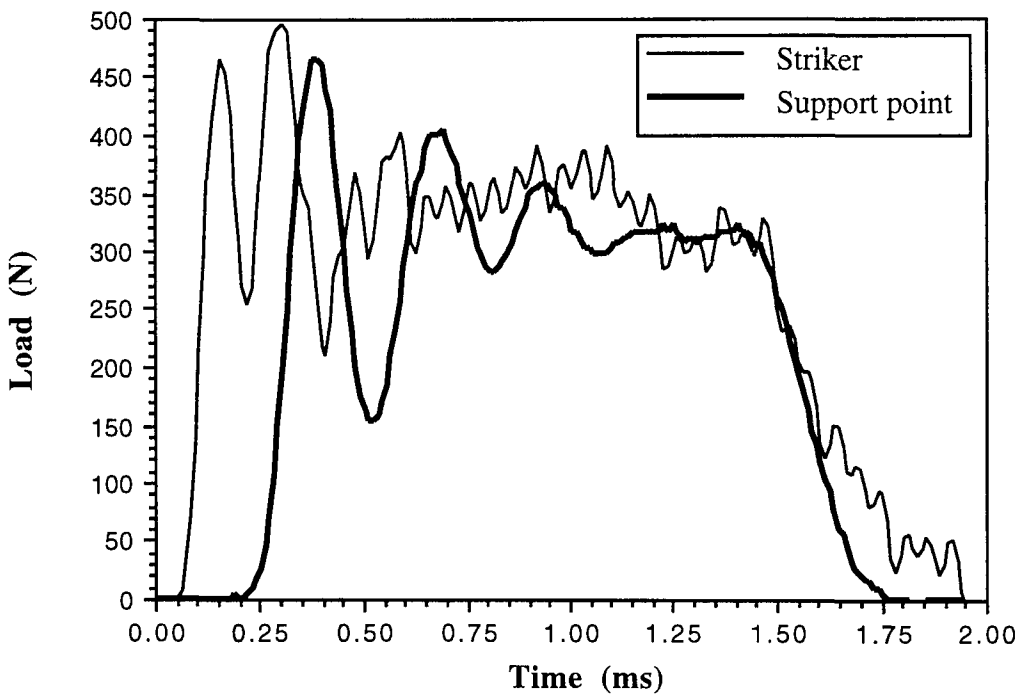


Fig. 7.2 Striker and Support loads during the same test

The analysis of the HSDT test assumes that $S=F$; a necessary assumption to reduce the equations of motion to one spatial dimension. Three important facts can be seen from these results:

- 1) The time at which the striker initially impacts the specimen is significantly (150 μ s) earlier than the time at which any response can be seen on the support point load.

- 2) The duration of the event is apparently longer if the striker, as opposed to the support point, load is considered. Both results indicate that the response ends at a well defined time.
- 3) The average load, after the initial peak, calculated from the two measurements is the same (within the calibration error of the accelerometer).

Before discussing the reasons for the differences between the two load traces the results from high speed photographic results are presented.

7.2.3 High speed photographic results

Wheel [7.2] examined the HSDT test using high speed photography. He used the results to measure beam section rotation directly. The photographs were used in this work to measure the vertical translation of a section in addition to its rotation. The results are not very accurate, due to the low resolution of the photographic film, but do indicate the general nature of the deformation. The results are shown in figure 7.3 for the beam section rotation at 25 mm from the loaded end of the specimen (15 mm from the load plane).

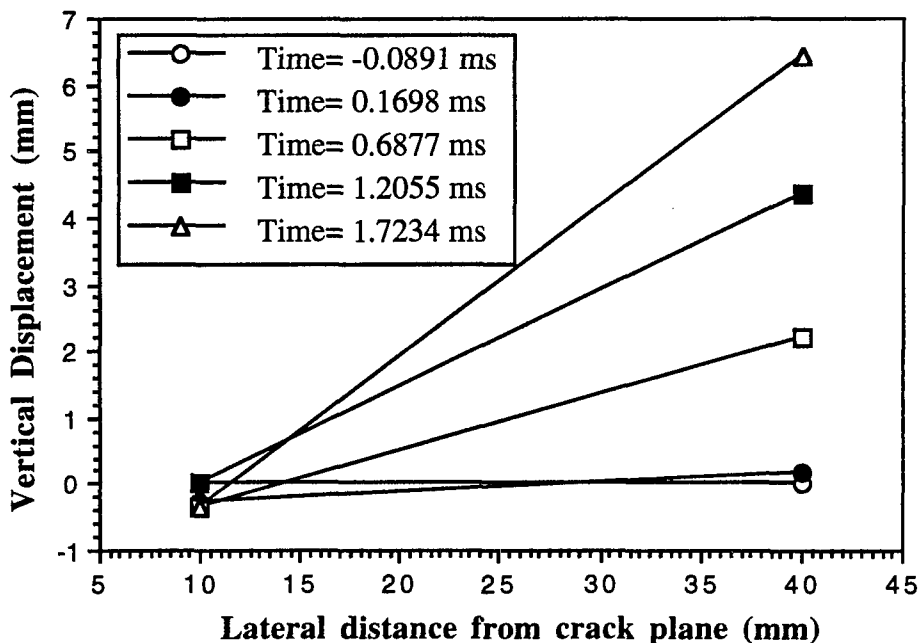


Fig. 7.3 Vertical displacement of a section 25 mm from the loaded end of the specimen

As can be seen there is a definite initial rotation of the section about an axis close to the centroid which quickly changes to a rotation about the support point.

7.2.4 Conclusions

The above results allow the following conclusions to be drawn about the nature of the deformation close to the load plane. As the striker impacts the specimen, the load plane section begins to rotate about a point close to the centroidal axis and the specimen actually lifts off the support point (Figure 7.4b). Torsional loading waves emanate from the load plane, in both directions, along the axis of the specimen. The wave travelling towards the impacted free end of the specimen is reflected back towards the load plane as an unloading wave (figure 7.4c). During this time the rotating sections also gain a net downward velocity component due to the continuing downwards motion of the striker.

As the reflected unloading wave reaches the load plane, contact between the specimen and the support point occurs and the reaction force at this point begins to increase (figure 7.4d), whilst the contact force between the striker and the specimen reduces. This process is then repeated, producing the characteristic oscillations in the load trace. The support and striker force oscillations are therefore in anti-phase and the initial increase in support load does not correspond to the time of impact but is delayed by approximately one time period of the load oscillations.

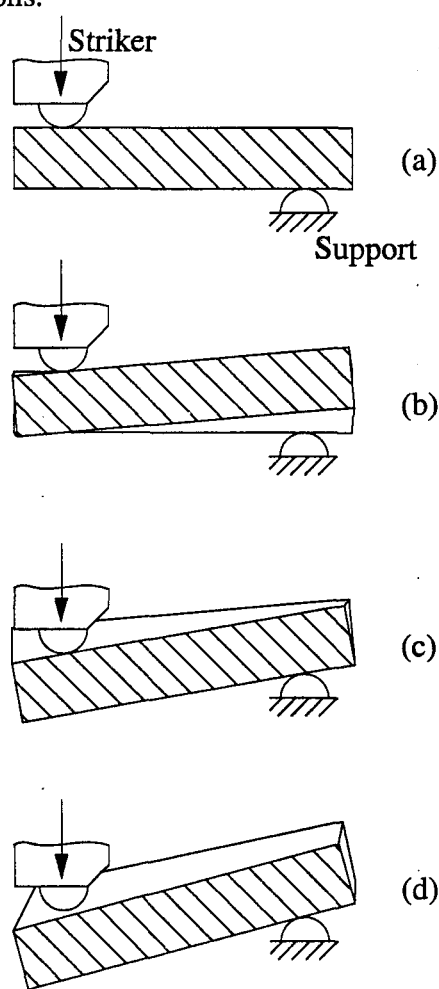


Fig. 7.4 Schematic of the deformation of the overhang region proceeding striker contact

The single spatial dimension FD analysis is unable to fully model this deformation, but does qualitatively account for the contact stiffness and overhang effects in the nature of the oscillations in the load trace. The predicted load trace resembles the striker contact force as opposed to the support point load. As was shown in section 6.4.4, the oscillations are only one factor in the accelerations and decelerations of the crack during an HSDT test. The question remains of what errors are introduced by modelling the striker and support forces as being equal. This question can only be answered by examining the correlation between the measured and predicted crack length histories (see section 7.3.3.3).

7.3 HSDT results for PE100

7.3.1 Effective modulus

The effective modulus for PE100 at -5°C was given in section 5.7, using three points to fully characterise the stress-strain curve. Since the HSDT results for this material are examined in depth a more complete, ten point definition of the characteristic was used, as shown in figure 7.5.

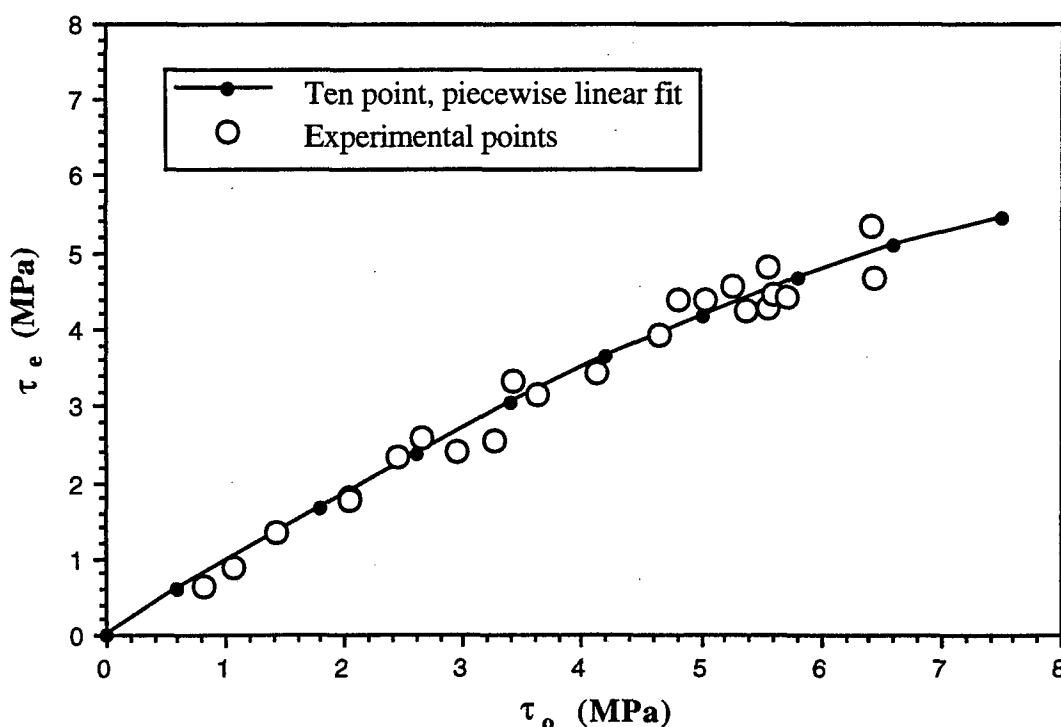


Fig. 7.5 Ten point definition of τ_e as a function of τ_o for PE100 at -5°C

7.3.2 Dynamic Fracture resistance of PE100

Nineteen HSDT tests were performed on PE100 at $-5^{\circ}\text{C} \pm 1^{\circ}\text{C}$. The material was supplied in granule form by BP Chemicals plc and was then compression moulded into 9.5 mm

thick sheets. The overhang for these tests was 5 mm, with a 30 mm long, straight fronted initial notch. The overhang was reduced to 5 mm from the standard 10 mm in order to minimise any errors due to the associated oscillations in the load trace. The average thickness of the specimens was 9.2 mm with a side groove depth of 1.6 mm.

The section rotation at distances 85, 95, 105, 115, 125 and 135 mm along the length of the specimen were measured using the modified OCG. Using the section rotation results in conjunction with the timing lines allowed the approximate average section rotation at the leading edge of the crack front to be calculated for each test. Equation (4.17) was then used to calculate G_D directly without the need for the FD analysis. The results for G_D from this section rotation method and from the FD analysis are compared in figure 7.6.

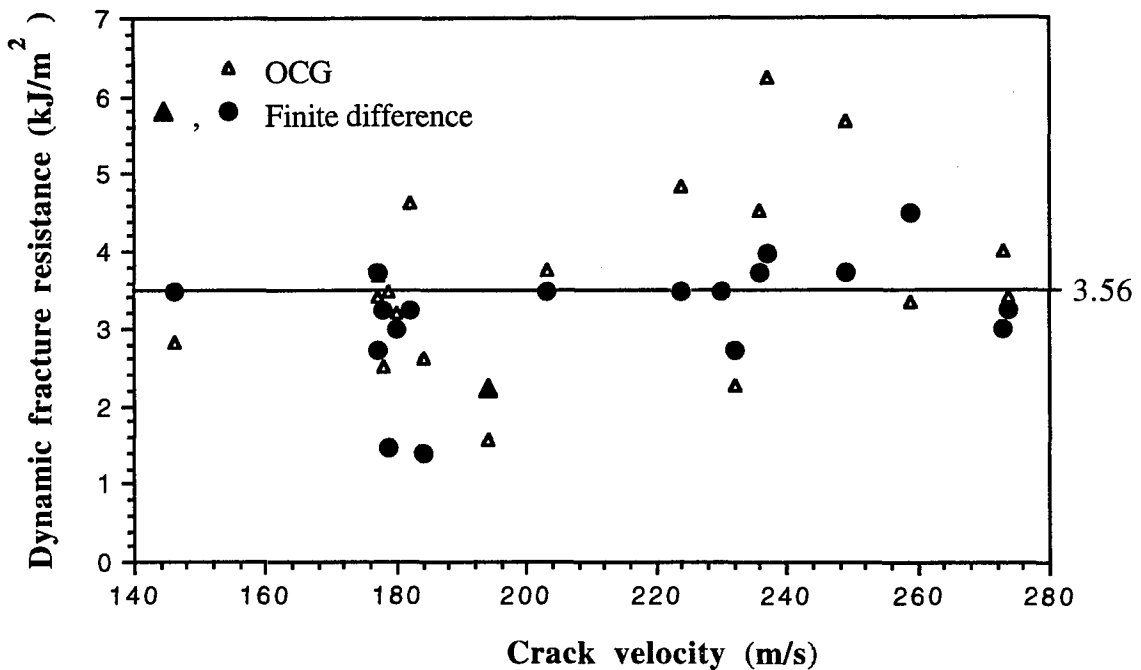


Fig. 7.6 G_D for PE100 calculated from both the FD method and directly from the crack front section rotation

The modified OCG results show a greater amount of scatter (as expected due to the inaccuracies of using the OCG to measure absolute section rotation), but the mean value (3.86 kJ/m²) correlates well with that from the FD results (3.20 kJ/m²). The correlation is very significant since it verifies all the main assumptions used in the HSDT analysis and FD model presented in the preceding chapters. The results correlate well with the minimum G_D prediction from Levers' model [7.3] (3.56 kJ/m²).

The FD results show pronounced scatter at the low crack velocities and this is discussed in detail in the following sections.

7.3.3 Accuracy of the HSDT test analysis

The accuracy of HSDT test analysis is discussed below by comparing predicted results to experimental results for three typical cases covering the range of striker velocities used: High (25.7 m/s), medium (19.1 m/s) and low (11.9 m/s) rates. The results are presented below by comparing the measured and predicted crack histories, load traces and section rotations for each case.

7.3.3.1 Rotation histories

The rotation histories for the medium and high striker velocity cases are shown in figures 7.7 and 7.8 respectively. For the medium and high striker velocity cases, the FD results correspond to the value of G_D which produced the same steady state crack velocity as that measured experimentally (an automated method described in section 6.2.2.4). As explained in section 7.2.4, the time that the striker made contact with the specimen could not be determined from the experimental load history. The correct time offset for the results from an experiment was calculated as the time difference between the occurrence of the predicted minimum in section rotation of the 85 mm section and that measured experimentally. This time offset was then applied to all the experimental results for that case. The calculated time offset is independent of the value of G_D since the minimum rotation is due to the low strain, dispersive nature of the high frequency components in the torsional wave, which travel much faster than the crack.

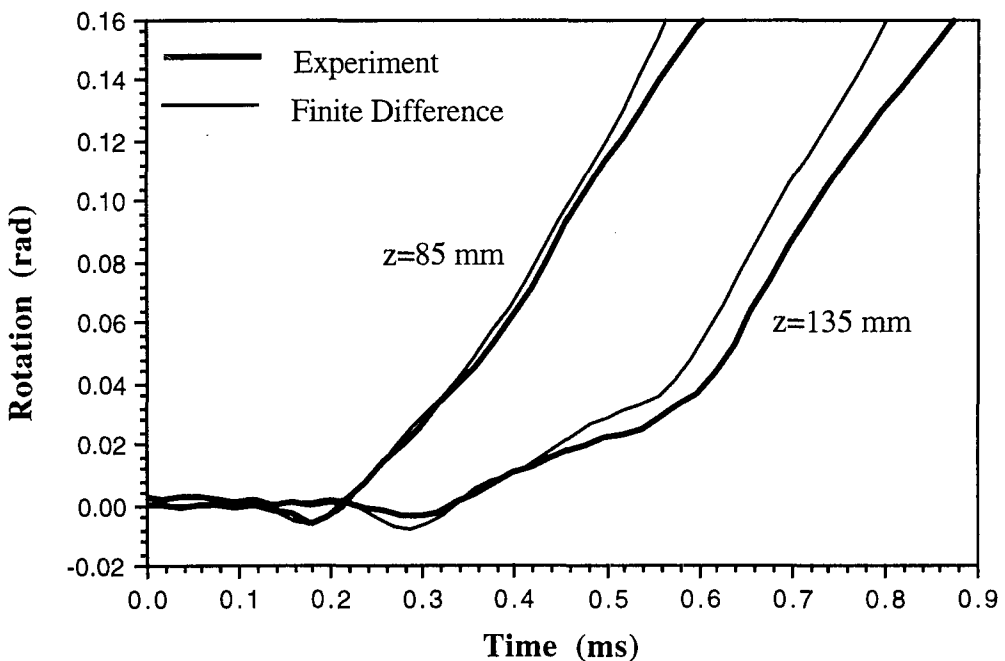


Fig. 7.7 Experimental and predicted section rotation histories at two positions
(Striker velocity = 19.1 m/s, Crack tip rotation = 0.042 rad)

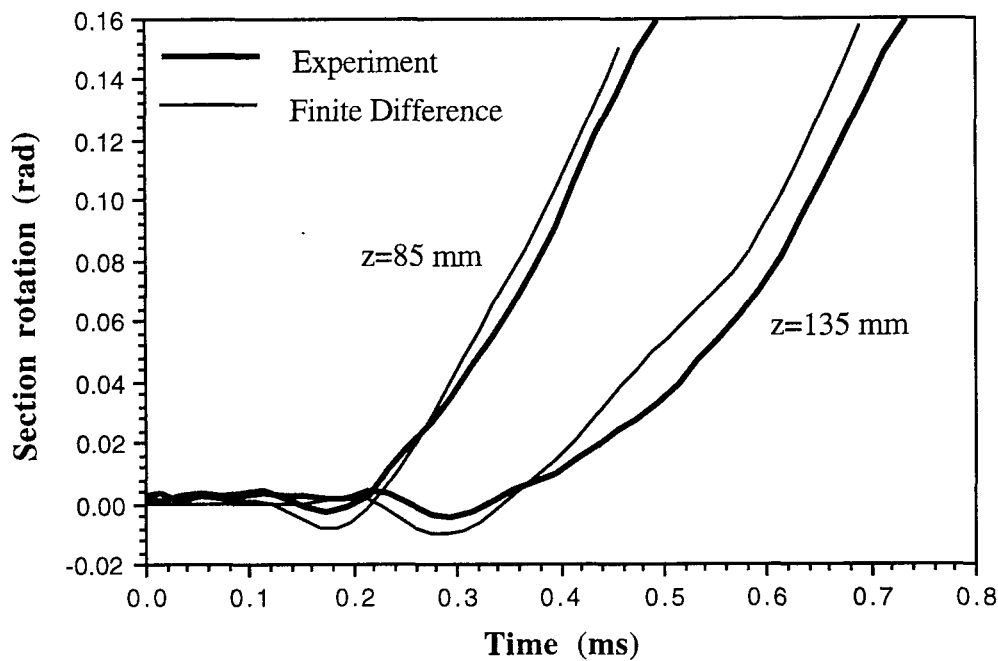


Fig. 7.8 Experimental and predicted section rotation histories at two positions
(Striker velocity = 25.7 m/s, Crack tip rotation = 0.046 rad)

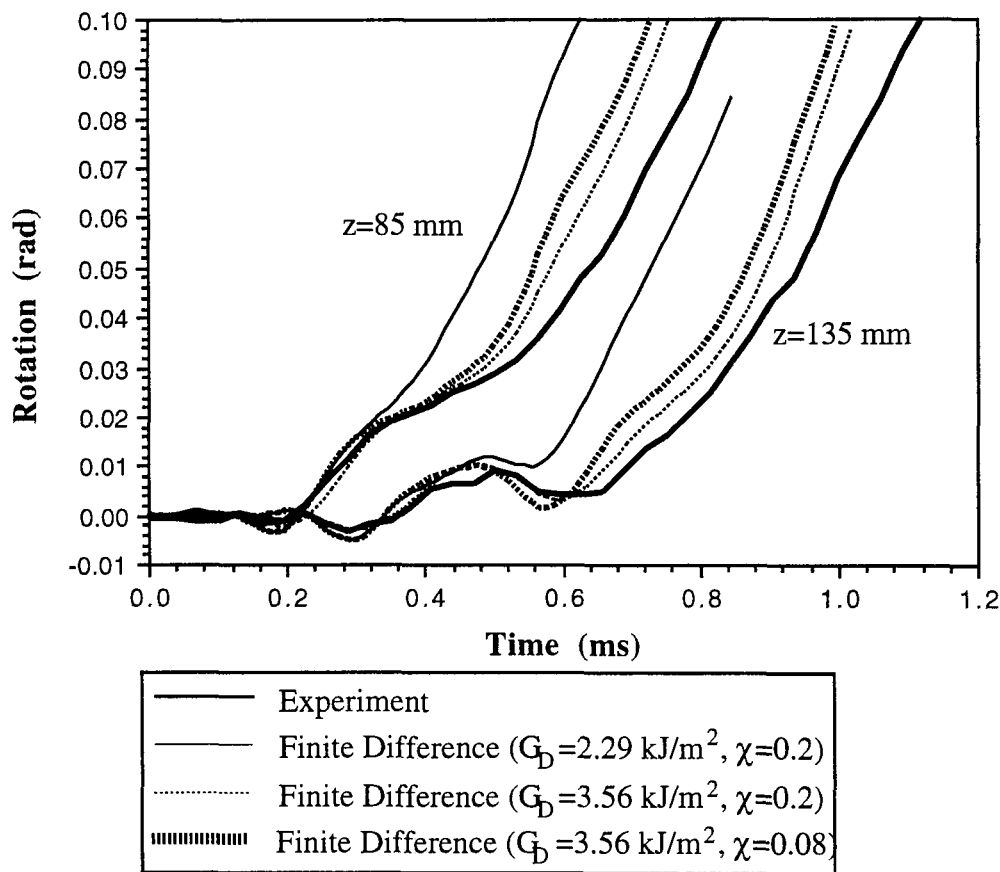


Fig. 7.9 Experimental and predicted section rotation histories at two positions
(striker velocity = 11.9 m/s)
(Crack tip rotation = 0.035 rad for $G_D=2.29 \text{ kJ/m}^2$)
(Crack tip rotation = 0.043 rad for $G_D=3.56 \text{ kJ/m}^2$)

For the medium and high striker velocity cases there is a good correlation between experimental and predicted results. It can be seen that section rotation begins to increase significantly before the leading edge of the crack tip passes the section. There is little, if any, discontinuity in the rotation history as the leading edge actually passes.

The low striker velocity case corresponds to the solid triangular point (▲) shown in figure 7.6. For the crack velocities typified by this test, there is a considerable increase in the scatter of the calculated G_D values. The measured rotation histories, together with the predicted results from the automated FD method are shown in figure 7.9. The calculated value of G_D for the predicted results is 2.29 kJ/m². As can be seen there is a considerable disparity between the FD and experimental rotation histories.

The FD model was then used to predict the rotation histories for the case of $G_D = 3.56$ kJ/m². The results, also shown in figure 7.9, have a much improved correlation with the experimental results. There seems, therefore, to be considerable doubt as to the validity of the predicted result for G_D of 2.29 kJ/m² for this test. This is examined further in section 7.3.3.3. For completeness, the effect of contact stiffness was also examined and, as can be seen from the results shown in figure 7.9, a reduction in contact stiffness by a factor of more than two has little effect on the rotation histories.

7.3.3.2 Load histories

The measured and predicted load histories for low, medium and high striker velocity cases are shown in figures 7.10 to 7.12. There is a good correlation between the FD predicted and measured loads, for all tests. The experimental load trace measured at the support points is delayed by one period of oscillation as explained in section 7.2.4.

For the high striker velocity case (figure 7.12) the predicted load is less than that measured. The most probable reason is that the effective stress-strain data for the material is limited for the high striker velocity and corresponding high strain tests (due to the limitation in maximum striker velocity). The initially increasing load in the low striker velocity case is due to the relatively long time required to achieve initiation.

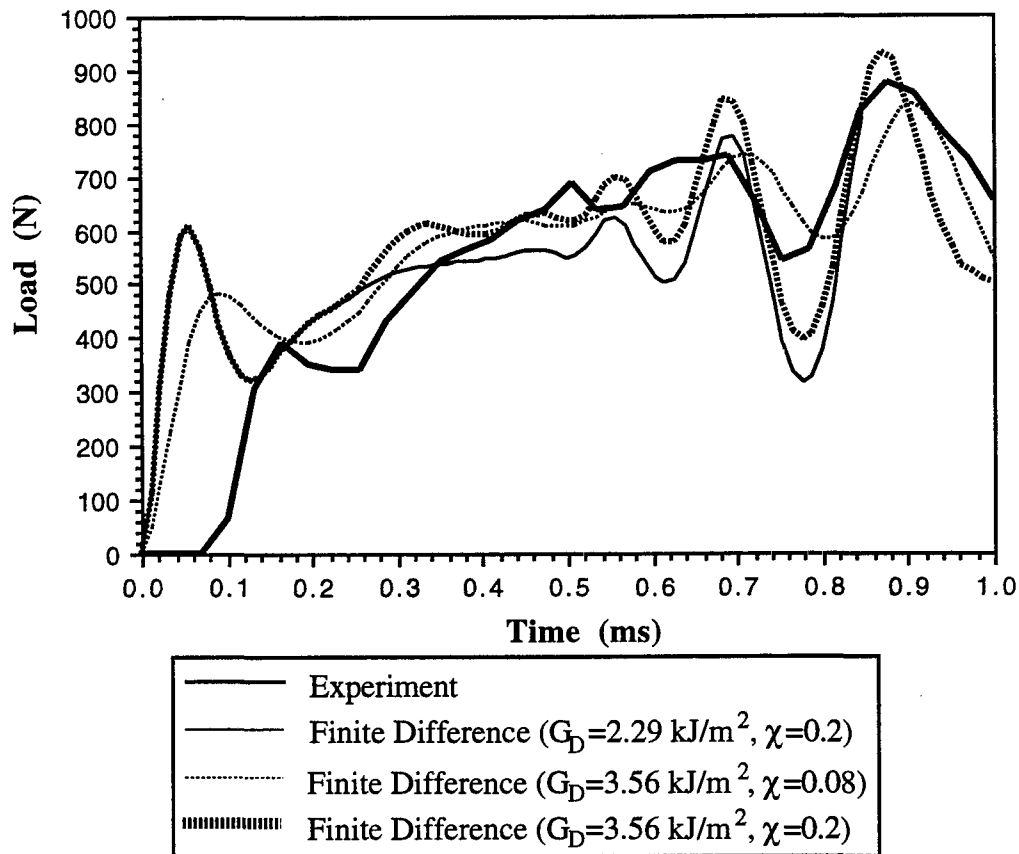


Fig. 7.10 Experimental and predicted load histories (striker velocity = 11.9 m/s)

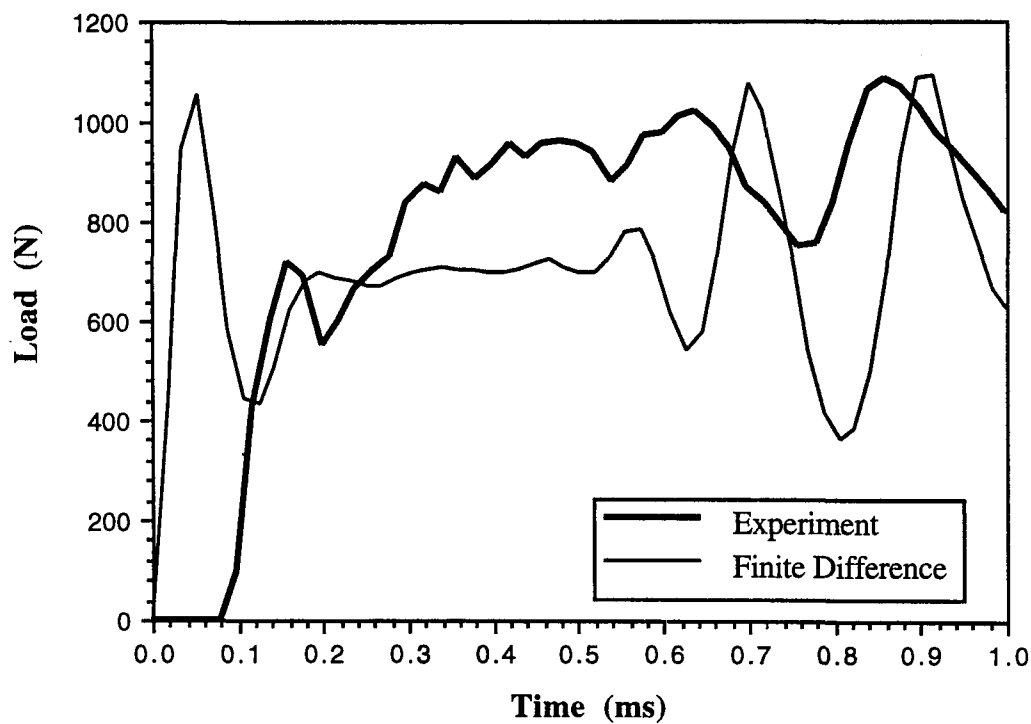


Fig. 7.11 Experimental and predicted load histories (striker velocity = 19.1 m/s)

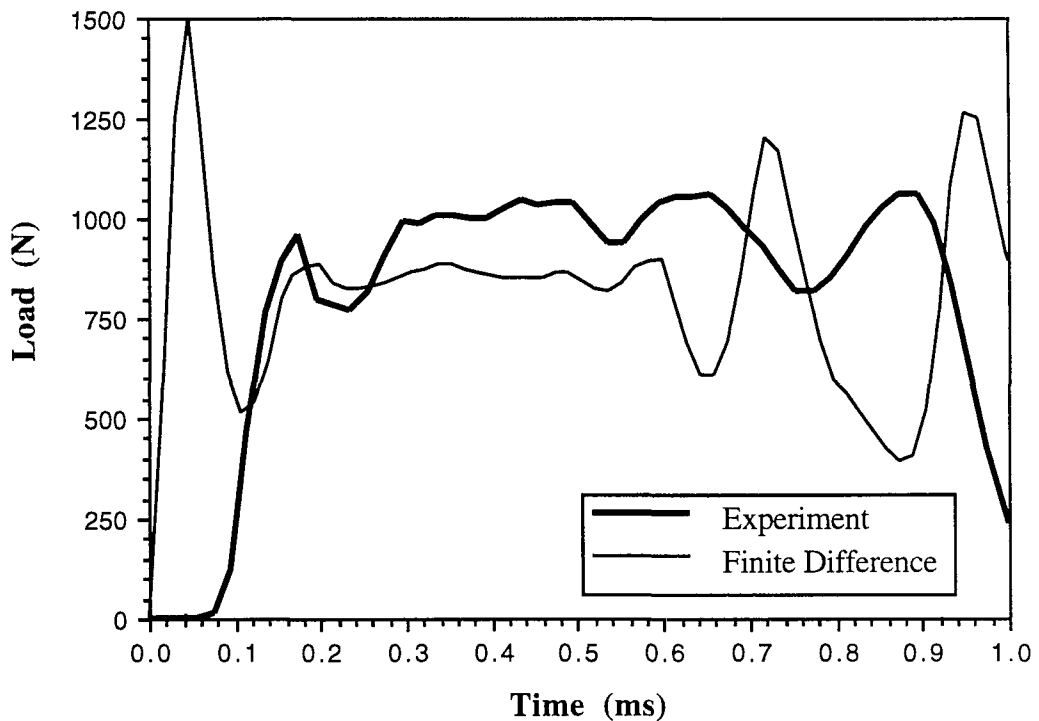


Fig. 7.12 Experimental and predicted load histories (striker velocity = 25.7 m/s)

7.3.3.3 Crack histories

The measured and predicted crack histories for the medium and high striker velocity cases are shown in figures 7.13 and 7.14. The time origin of the experimental histories has been offset as discussed in section 7.2.4, in order to facilitate comparison. As can be seen there is a good correlation between the FD predictions and the measured data.

Examining the low striker velocity case, the automated FD solution calculated G_D to be 2.29 kJ/m². The corresponding predicted crack history is shown in figure 7.15 together with the time offset measured crack history. The predicted crack history when G_D is changed to 3.56 kJ/m² is also shown in figure 7.15. The predicted steady state crack velocity for $G_D = 2.29$ kJ/m² is within 0.5% of that measured, but for $G_D = 3.56$ m/s the difference is increased to 12 % (194 to 171 m/s). The difference in crack velocity would at first seem to be very significant until the form of the crack histories is examined. As can be seen, both the predicted and experimental crack histories show pronounced oscillations about the steady state velocity. Since the steady state velocity is calculated from twelve discrete points, its value is strongly dependant on the position of the points along the crack path and the correct prediction of the oscillations. The results show that the oscillations are reasonably accurate apart from the initial few points which produce an apparently slower calculated steady state velocity.

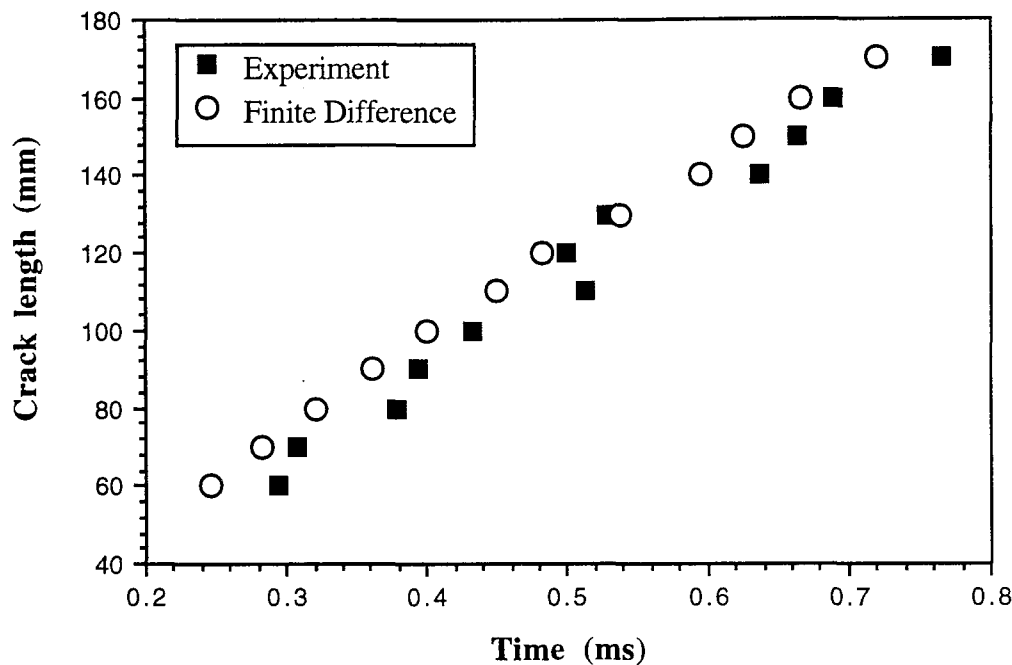


Fig. 7.13 Experimental and predicted crack histories (striker velocity = 19.1 m/s)
(crack velocity = 230 m/s)

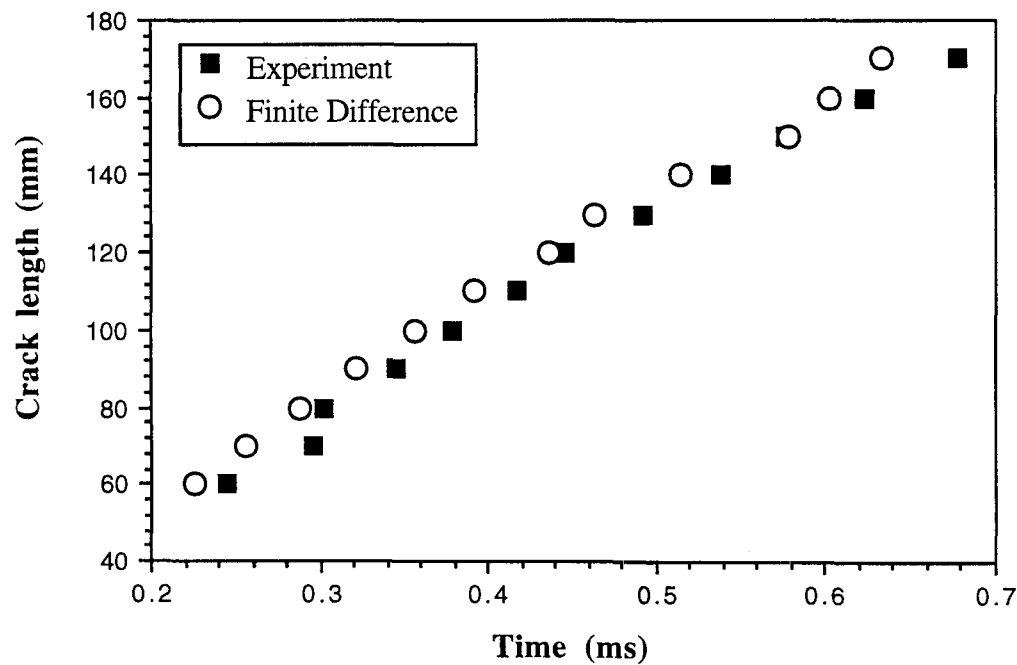


Fig. 7.14 Experimental and predicted crack histories (striker velocity = 25.7 m/s)
(crack velocity = 259 m/s)

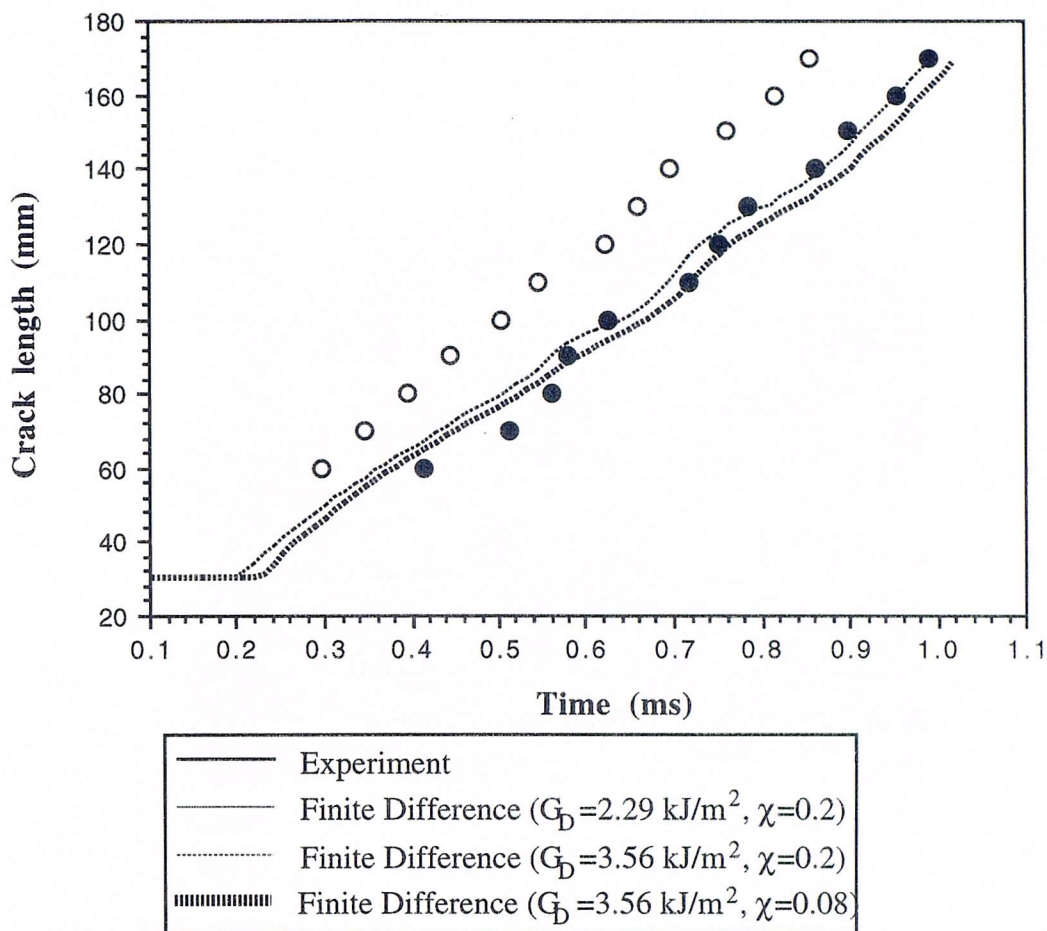


Fig. 7.15 Experimental and predicted crack histories (striker velocity = 11.9 m/s)

7.3.3.4 Fracture surfaces

A typical fracture surface is shown in figure 7.16. A fairly featureless surface is seen in the lower 80% of the surface with some frosting (micro ductility).

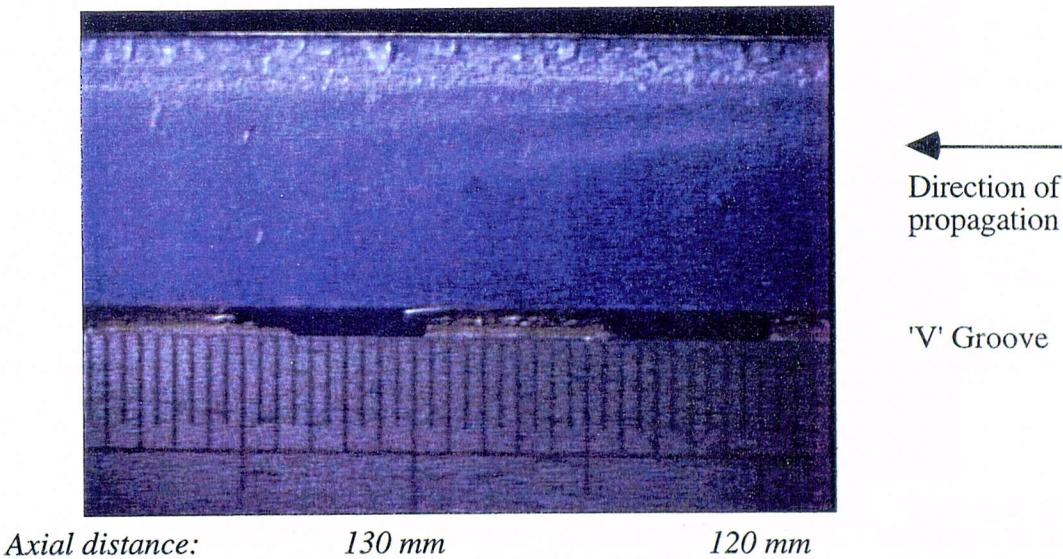


Fig 7.16 Picture of characteristic fracture surface (striker velocity = 9.0 m/s)

Partial arrest lines approach, but do not reach, the lower surface boundary. At the upper boundary an increase in surface roughening can be seen, shown magnified in figure 7.17. The roughening corresponds to very low, or highly oscillatory, crack speeds along the trailing edge of the crack front. (see figure 6.11).

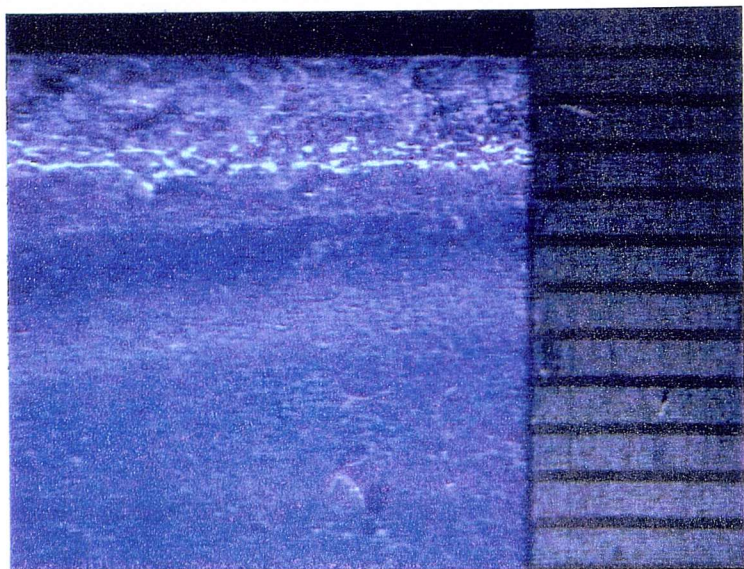


*Fig 7.17 Magnification of picture shown in figure 7.16 detailing the increased ductility close to the upper surface at 130 mm from the loaded end of the specimen
(Scale increments = 0.5 mm)*

A full arrest line is shown in figure 7.18 and the corresponding predicted crack shape is shown in figure 7.19.



Axial distance: 110mm 100 mm

Fig 7.18 Picture of arrest line seen on specimen tested at a striker velocity of 7.5 m/s

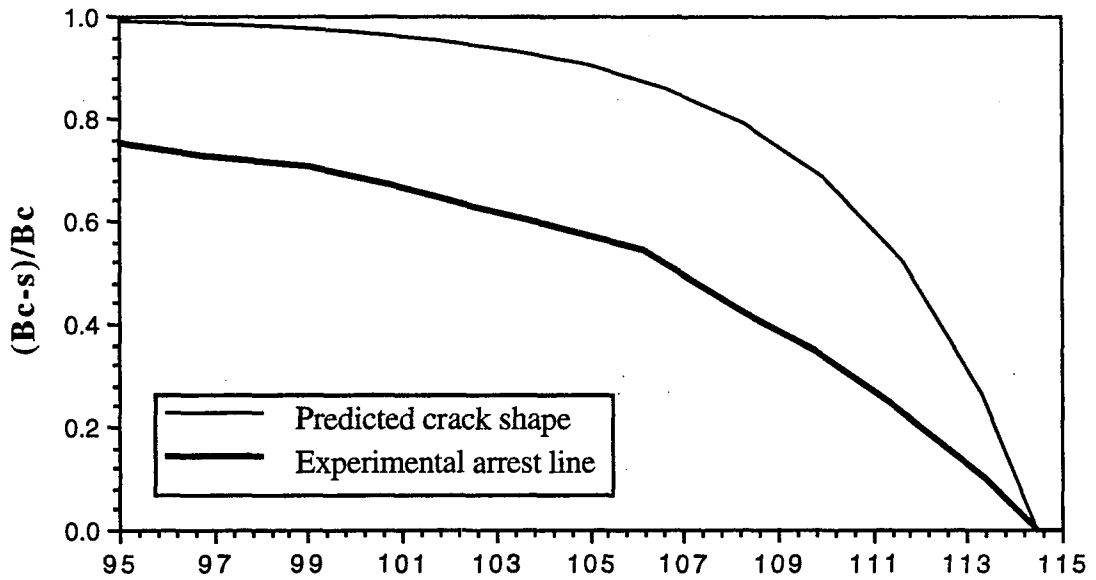


Fig 7.19 Predicted crack shape from the FD model compared to arrest line depicted in figure 7.18

The predicted shape does not match the arrest line observed, the leading edge of the predicted shape being much blunter. To investigate this further the arrest line depicted in figure 7.18 was used to define the dynamic crack front shape in the FD model. The steady state velocity used was the same as in the experiment. The section rotation histories predicted from the model for this crack front shape showed a very poor correlation with those measured, the foundation being far too stiff. The good agreement between the predicted and experimental low striker velocity crack history results suggests that the predicted shape is correct. There is no reason why the shapes of the arrest lines and the propagating crack front should match however. An arrest line does not represent an instantaneous position, but rather the locus of arrest points along a front which continues to propagate. Arrest can be viewed as the final stage of crack deceleration (with little change in G_D) and is caused by an interaction of stress waves which reduces the rate of energy supply to the crack tip. A loading stress wave travelling towards the load plane, or an unloading wave travelling away from the load plane, can produce a localised deceleration (and arrest) of the crack front which, as it propagates along the crack front, leaves an arrest line which has a sharper (more acute) leading edge than the dynamic crack front shape.

7.3.4 Summary

From the above results for PE100 it can be concluded that the HSDT analysis can be used to determine a material's G_D as a function of crack velocity using the FD model described

in chapter 6, provided that the striker velocity is high enough to overcome the dominance of the oscillations in the crack length history. At low crack velocities the test must be instrumented so as to provide a method for determining the time of contact between the striker and the specimen.

At low striker velocities the crack velocity oscillations are predominantly due to fluctuations in the rate of supply of energy to the crack front caused by local interactions of loading and unloading torsional waves.

The high striker velocity results presented above demonstrate a non-linear material characteristic seen in the HSDT test. The crack velocity for one test was 259 m/s with a striker velocity of 25.9 m/s. A lower striker velocity test (21.4 m/s) showed a crack velocity of 274 m/s. That is, a higher striker velocity produced a lower crack velocity. This can be understood by considering the rate of energy supply to the crack front. At higher striker velocities, as the crack velocity approaches its limit, the effective strain in the torsional beams behind the crack tip is approximately proportional to striker velocity. Therefore, as the striker velocity is increased the effective strain increases and the torsional wave speed decreases, decreasing the rate of supply of energy to the crack front. Thus the crack speed is reduced.

7.4 Polypropylene

The HSDT test was used to determine G_D as a function of crack velocity of a pure polypropylene homopolymer (PP) material and a rubber toughened form (PP/EPDM, 30% by weight EPDM). The material was supplied in granule form by the University of Twente, Netherlands and was then compression moulded into 9.5 mm thick sheets.

Both the pure and blended forms have been tested by Gaymans [7.4] using a notched tensile impact test to calculate G_D . His analysis of the test did not account for dynamic effects and as such his results, which showed the blended material to be superior, are dubious. The blended materials showed a rapid fall in G_D from over 100 kJ/m² to 12 kJ/m² as the loading rate was increased from 10⁻⁵ to 10⁻¹ m/s. At 10⁻¹ m/s a transition occurred and G_D increased with further increases in piston speed. The G_D of the pure material appeared to be virtually zero throughout the range of loading rates.

Further Scanning Electron Microscopy work by Gaymans revealed a melt layer 3-5 μ m thick close to the fracture surface in the blended material subjected to high rate tests, but this was not evident in the low rate tests. Gaymans then proposed that the transition in the

frequency oscillations in the crack velocity and is unable to account for the experimental results presented above. The stability of crack propagation at low rates therefore requires further investigation.

7.5 Polyoxymethylene

In this section, HSDT results for Polyoxymethylene (POM, DuPont 'Delrin') in both a toughened (ST) and a pure (NC) form are presented.

Injection moulded, 12 mm thick Delrin plates were provided for testing. To decrease the thickness and remove surface residual stresses 1.5 mm was machined off each face. All tests were performed at 0°C and at impact velocities between 7 m/s and 22 m/s. At impact velocities of less than 10 m/s, Delrin ST showed stick-slip propagation with characteristic arrest lines. All other tests showed a smooth crack surface.

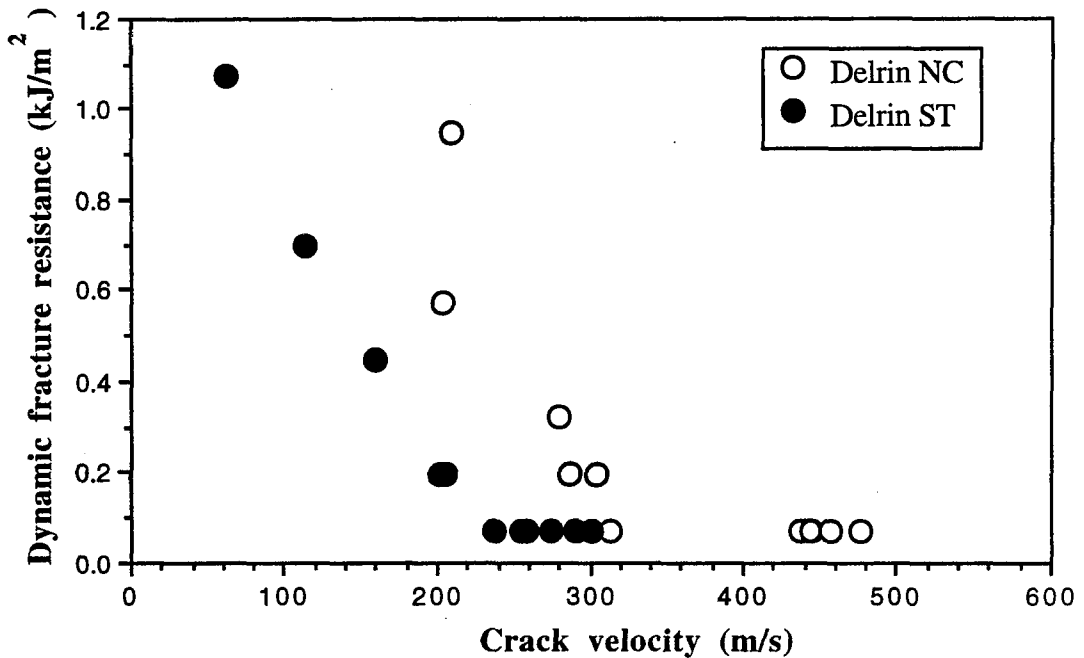


Fig. 7.22 G_D as a function of crack velocity for Delrin (ST and NC)

The results of tests are shown in figure 7.22. The toughened Delrin specimens show no improved resistance to RCP but in fact a lower resistance than the pure form at crack velocities less than 300 m/s. Both materials show a fall in their G_D versus a characteristic until they reach a lower limit. The figure indicates very little scatter in this lower plateau region but this is only an artefact of the FD model as G_D was only calculated to an accuracy of ± 0.05 kJ/m².

The fact that the toughened Delrin exhibited a lower G_D than the pure material could be qualitatively explained in terms of Leever's thermal decohesion model [7.3]: The rubber is

present in small particles which are regarded as having no intrinsic structural effect. The presence of the rubber therefore merely reduces the energy to melt a one-chain-length thick layer, since the overall enthalpy to melt is reduced.

Both Delrin materials have significantly lower G_D values than PE100. These results suggest that POM would exhibit a low critical pressure in pipe tests. Tests on 50 mm diameter, 6 mm thick pipe have indeed shown this to be true [7.5]. In the presence of an axial internal notch, introduced to induce plane strain conditions, RCP occurred spontaneously driven by residual stress only, i.e. at zero internal pressure. Without the axial notch initiation of the crack was unstable, being very sensitive to impact speed. Similar tests on PE100 pipe gave a finite critical pressure with no such sensitivity to impact speed.

7.6 References

- (7.1) Venizelos, G.P., Investigation of the S4 test for RCP in thermoplastics pipe, PhD Thesis, Univ. of London, (1995).
- (7.2) Wheel, M.A. and Leever, P.S., 'High speed double torsion tests on tough polymers I: Linear Elastic Steady State and Dynamic Analysis', *Int. J. of Fracture*, **61**, p. 331, (1993).
- (7.3) Leever, P.S. and Greenshields, C.J., A model for predicting the dynamic fracture and impact fracture resistance of tough thermoplastics, *Proc., 53rd Annual Technical Conference of the Soc. of Plastic Engineers*, Boston, MA, (1995).
- (7.4) Gaymans, R.J. Dijkstra, K., Janik, H., 'Polyamide-rubber blends: Influence of deformation speed on crack propagation process', *Proc., 9th International conference on deformation yield and fracture of polymers*, Cambridge (1994).
- (7.5) Leever, P.S., Freeman, P.N., Arthur, M.M., 'Rapid crack propagation in small diameter thermoplastic pipe', *Plastics, Rubber and Composites Processing and Applications*, **24**, p. 113, (1995).

Chapter 8

Conclusions and recommendations

The HSDT test, although initially conceived as a simple, straightforward rapid fracture test, possesses many complicating dynamic characteristics. It is essential to accurately model the local deformations in order to correctly partition the work done into strain, kinetic and fracture energies. A previous FD model by Wheel [8.1] was found to be flawed and did not account for many of the important dynamic characteristics of the test. The majority of this work has been devoted to understanding the dynamic characteristics of the HSDT test and developing a new model to account for them. The new model has been validated extensively. No other existing analysis of an experimental test used to investigate rapid crack propagation can match this validation. The HSDT test now provides an efficient and accurate tool to investigate a polymer's G_D - $\dot{a}$ characteristic.

The HSDT test is unrivalled in terms of continuous crack propagation length and its plane strain nature of fracture. The fact that the crack velocity oscillates during a test at low striker velocities can be viewed as an advantage, since it provides a method of investigating propagation stability in a test of known dynamic characteristics.

The most important achievements and conclusions of this work are summarised in the next section followed by recommendations for future work.

8.1 Summary of conclusions

8.1.1 Experimental test improvements

The HSDT experimental rig was modified in a number of ways. The most notable were the redesign of the striker velocity measurement system and the design and construction of an optical crack gauge (OCG) to measure crack velocity. The basic OCG design is now proven. It reduces specimen preparation to a few minutes, but would benefit from some minor alterations to improve its accuracy. A modified form of the OCG allows approximate section rotations to be measured as a function of time.

8.1.2 Derivation of analytical equations to model the HSDT test

To calculate the dynamic fracture resistance (G_D) of a material from the test results, a post mortem analysis that models the deformation during the test given the striker and crack velocities is required. A previous two dimensional (one spatial, one time) analysis, made

by Wheel [8.1], included the effects of axial stress but not axial inertia. Since the duration of the test is of the same order as the time taken for the axial stress waves to travel the length of the specimen, axial inertia is important and has now been included. The derivation of the fourth-order differential equations followed that of previous researchers but, by examining end effects, it was shown that an adjustment factor, resorted to previously, was not necessary. The restoring moment, due to the elastic foundation between the two torsion beams beyond the crack front, was included in a more rigorous and accurate manner than in previous analyses. The end boundary conditions used by Wheel were also corrected, allowing the model's load plane region to deform more realistically. The predictions from the resulting analytical solution correlated well with experimental and numerical results for both non-uniform twist of simple torsion beams and static deformation of the DT specimen.

The resulting equations were used to accurately determine the restoring moment of the elastic foundation as a function of 'V' groove depth. With the accuracy for the relatively simple static, straight crack front case proven, it was then possible to move on to the analysis of the curved crack front. It was shown that a simple model could be used to model the foundation where, along a curved crack front it is only partly separated. This model was then used to test the hypothesis that the crack shape could be predicted by assuming the crack driving force along a propagating curved crack front to be constant. The hypothesis was in part proved, but is still in doubt due to lack of data on the shape of the propagating crack.

Comparing results from the dynamic analysis of the straight and curved crack fronts, showed that the straight crack front model under predicted G_D . This effect became more pronounced as the crack velocity approached the torsional wave speed in the half-specimens.

8.1.3 Non-linear material properties

To examine the non-linear behaviour of polymers in the HSDT test, Wheel [8.1] developed the torsional impedance test to measure the shear modulus data required by the FD model. His analysis of the test was deeply flawed because he did not account for the reduction in torsional wave speed at the high strains due to the corresponding reduction in tangent modulus. A new, corrected analysis of the torsional impedance test has been derived, the assumptions of which have been fully tested and verified. The results from the analysis are thickness independent.

The results show that a material, whose modulus shows a low sensitivity to temperature at high strain rates and low strains, may possess a modulus at intermediate strain rates and high strains which is very sensitive to temperature. This may prove to be very important when considering RCP in pipes since the critical pressure appears to be strongly dependant on the relationship between decompression rate and crack speed. The crack speed is closely linked to modulus via the dominant wave speed in the pipe.

8.1.4 Propagation mode

Crack propagation can be modelled in one of two modes: generation or propagation. The previous analysis of Wheel [8.1] used generation mode, in which the crack history is prescribed and G_D calculated throughout the test. The disadvantage of generation mode is that the discrete experimental crack length data indicate that the crack velocity is not constant, but the resolution is not high enough to define the oscillations accurately. A prescribed history will therefore put unrealistic constraints on the model. The main advantage of generation mode is that G_D need not be assumed to be constant during a test. Wheel was unable to capitalise on this advantage since the initiation time of the crack cannot be measured accurately. He calculated the initiation time for a test by finding the value which produced minimum variation in predicted G_D from the model during the test; a method which assumes, *a priori*, that G_D is constant during the test.

This work used propagation mode, which, from the outset, assumes a constant G_D during a test, and proceeds to calculate G_D by finding that value which produces the best correlation between predicted and experimental crack histories.

8.1.5 Finite difference model

The computational finite difference model used to analyse the HSDT test is fast (less than 5 minutes), efficient and easy to use, generating an animated display of the deformation to ease interpretation of the results. Due to the speed of the model (as compared to a full 3D solution) the characteristics of a test can be easily examined, and the sensitivity to any parameter quickly investigated. The model has been used to demonstrate that the characteristics of the test are dominated by its dynamic nature and has led to an understanding of features such as the oscillating load and crack velocities and the curved crack front.

The term 'steady state crack velocity' (calculated as the slope of the best fit straight line through the crack history points) originated from the assumption that the crack velocity could be considered constant during an HSDT test. The constant crack velocity assumption

is not valid but the steady state crack velocity is still used to characterise the crack velocity during the test. The steady state crack velocity is therefore dependant on the number and position of discrete points used to determine the crack velocity, if the spacing between the points is of the same order as the wavelength of oscillations in the crack history.

8.1.6 Experimental results

Detailed tests were performed on an HDPE. A large amount of additional data was gathered on the deformation of the test, which was then used to validate the dynamic FD model. The detailed checking of the model has no precedent in other fracture tests where only the minimum amount of data is gathered to model the deformation. The characteristic stick-slip nature of HSDT test seen at low striker velocities has been shown to be due largely to the dynamic characteristics of the geometry and loading conditions, as opposed to the $G_D-\dot{a}$ characteristic of the material.

Comparisons between predicted crack front shapes and observed arrest lines indicate that arrest lines seen on the HSDT fracture surface do not reflect the shape of the propagating crack front. In other words, arrest is a dynamic process which does not necessarily occur simultaneously along the whole length of the front.

At low crack speeds the steady state crack velocity is insensitive to G_D . To calculate G_D for these cases the relationship between the crack history and the time of striker impact must be measured. A single section rotation history, measured using the optical crack gauge, allows this to be performed, with the added benefit of providing an extra check on the analysis.

Results for polyoxymethylene and polypropylene materials showed G_D to decrease with crack velocity. Although this result is intuitively sensible it was not expected that the crack could propagate with any stability whilst operating on a falling $G_D-\dot{a}$ characteristic. There are two known inaccuracies in the results:

- 1) The variation in crack velocity along the crack front is not accounted for.
- 2) The region where G_D is falling most rapidly with crack velocity corresponds to low rate tests for which the analysis has been shown to give unacceptable scatter.

However, the above inaccuracies are insufficient to explain the dramatic fall in the measured $G_D-\dot{a}$ characteristic of these materials. The results therefore show the importance of considering the loading rate in a fracture problem dealing with these materials.

At the loading rates considered in this work, the 'toughening' of a material by blending it with rubber appears to have the opposite effect to that intended. The rubber toughened POM actually has a lower G_D for a given crack velocity than the pure form. This result agrees with the prediction from Leever's thermal decohesion model [8.2].

8.2 Future directions

8.2.1 Dynamic fracture resistance as a falling function of crack velocity

The next stage in the development of the HSDT test is to investigate why a material, with a falling G_D - $\dot{a}$ characteristic, can fracture at a controlled rate without large oscillations in crack velocity. The approach should be to simulate a material with a known, or assumed, falling G_D - $\dot{a}$ characteristic and find which parameters can generate, or affect, crack propagation stability. The model described in section 6.1.1 should be used with the inclusion of rate effects in the G_D - $\dot{a}$ characteristic. The sensitivity to rate effects can, as a first approximation, be deduced from Leever's thermal decohesion model.

8.2.2 Reduced striker velocities

In order to cover a more complete range of crack velocities for any material, the present minimum striker velocity is inadequate. Currently the minimum striker velocity is limited by the rig since the free fall distance (zero accumulator pressure) is set at 1.5m. This can simply be overcome, but problems of striker retardation will become a problem due to its low momentum and the long crack propagation time. The best solution for these cases is to use the static rig described in section 4.2.4, in conjunction with a high rate tensile testing machine.

8.2.3 Rate sensitivity of modulus

The effective modulus for the HSDT test has been defined without accounting for rate effects. This was justified by the torsional impedance test operating at approximately the same rate as the HSDT test. To check this approximation, a form of torsional pendulum test could be used, with the standard cylindrical bar specimen replaced by a rectangular bar, similar in geometry to the HSDT test half-specimen.

8.2.4 Steady state analysis

In the past, the steady state analysis has not proved to be an effective tool in analysing the HSDT test. Now that the important parameters of the test have been evaluated it may well be possible to use a steady state analysis to calculate at least an approximate G_D from the HSDT test results. The two most important improvements that could be made are the use

of the correct time offset for the crack history results, and of the correct non-linear material properties.

The steady state analysis is unable to account for wave reflections from boundaries, or the oscillating load and crack histories, but it does offer a far simpler and faster analysis of the HSDT test than that presented in this work. The steady state solution will be most accurate at intermediate crack speeds since:

- 1) At high crack speeds, and therefore high strains, the section rotation rates will be characterised by the high strain torsional wave speed as well as the crack velocity.
- 2) The steady state analysis will be unable to account for stick-slip behaviour which is common at low crack speeds.

8.2.5 Improvements to the HSDT experimental procedure

The HSDT test procedure requires some adaptations to be able to measure a material's G_D at low crack velocities. The main improvement required is to measure initiation time. There are a number of methods available, such as incorporating a load cell in the striker itself, but the most practical would be to use the modified OCG, as described in section 7.3.3.1, to measure a single rotation history at a known distance from the load plane. If a single crack timing line was also used, positioned at the same distance from the load plane, the rotation history would provide an additional check on the analysis, by measuring the section rotation at the leading edge of the crack front.

8.3 References

- (8.1) Wheel, M.A., High Speed Double Torsion Testing of Pipe Grade Polyethylenes, PhD Thesis, Univ. of London, (1991).
- (8.2) Leever, P.S. and Greenshields, C.J., A model for predicting the dynamic fracture and impact fracture resistance of tough thermoplastics, *Proc., 53rd Annual Technical Conference of the Soc. of Plastic Engineers*, Boston, MA, (1995).

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MATERIAL: ALUMINIUM
REMOVE ALL SHARP
EDGES.

13

10

SECTION XX'

13

6

22

Ø6.4

Ø5.3

Ø6.4

Ø5.3

13

21

Ø2

1 HOLE DRILL
Ø4.2*15 DEEP
& TAP M5*12 MIN

HOLE SPACING
REGULAR AS
INDICATED

X

120

3 HOLES
DRILL
Ø4.2 &
TAP M5
THRO'

12

35

10

X'

100

10

5

40

10

10

40

60

105

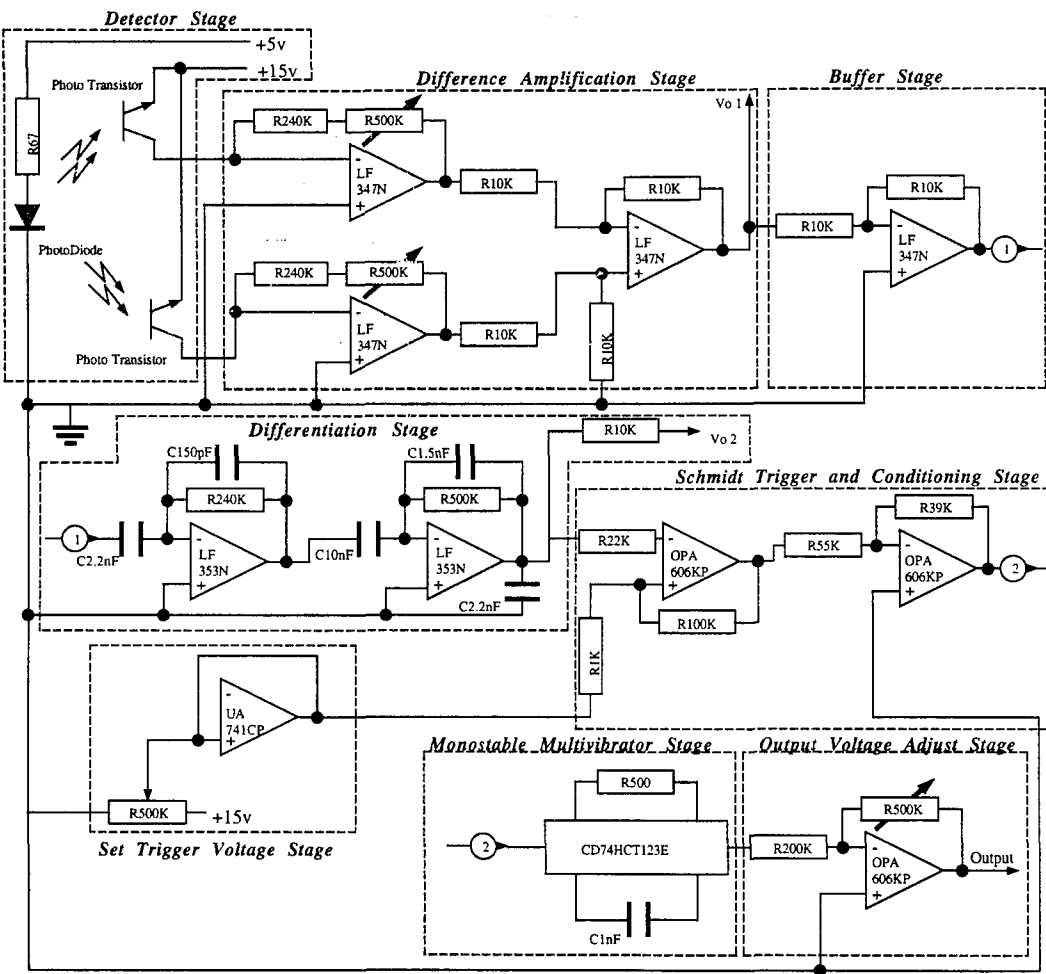
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MACHINE ALL OVER DO NOT SCALE FROM DRAWING	TOLERANCES ON MACHINED SURFACES		DATE: 7/3/93 SCALE: 1/1	CRACK GAUGE	DRAWN BY: S.RITCHIE DRAWING NO: CG07
	UPTQ 500	+0.2			
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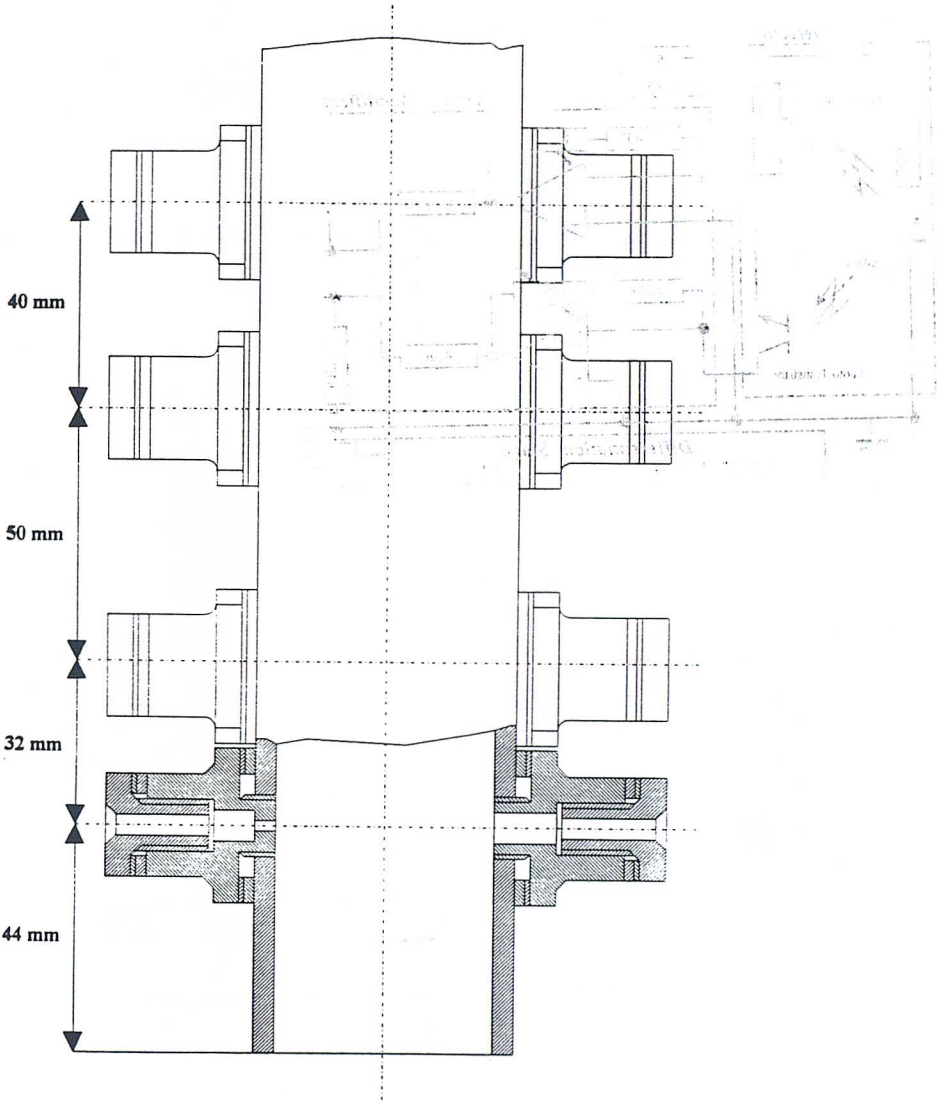
Appendix 2

Circuit for optical crack gauge



Appendix 3

Location of sensors to measure striker velocity



Appendix 4

Circuit for measuring striker velocity

