

Characterisation of Ductile Crack Initiation
and Growth in Mild Steel

by

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ABSTRACT

The effect of specimen size and geometry on ductile crack initiation and crack growth resistance of a mild steel has been investigated using compact, single-edge-notched three and four-point bend, centre-cracked and double-edge-notched tension specimens. In addition, the importance of various possible parameters for controlling fracture has been assessed.

The crack initiation toughness, as described by the J integral or the crack opening displacement at the original crack tip (δ), depended on the crack tip constraint. The latter was found to be influenced by specimen thickness, the position through the specimen thickness and, in sidegrooved specimens, the sidegroove depth. Criteria for the size of specimen to give plane strain constraint at the crack tip have been evaluated.

The slope of the J -resistance curve, dJ/da , in non-sidegrooved specimens was found to be dependent on the specimen size and geometry. This can be explained in terms of: the proportion of shear lip on the fracture surface, which is in turn governed by the geometry and the ratio of ligament to thickness; the crack growth resistance of the flat fracture region of the specimen, which is dependent on the crack tip constraint. In sidegrooved specimens, dJ/da was found to decrease with increasing sidegroove depth. For sidegrooving up to a depth of 25-50% this was because the influence of the specimen edge is removed and the crack tip constraint is increased up to that for plane strain. Deeper sidegrooving further lowered dJ/da by altering the deformation characteristics of the specimen but not the crack tip constraint.

The other possible crack growth characterising parameters investigated were: δ (at the original crack tip); crack opening displacement at the growing crack tip (CTOD); crack opening angle at the growing crack tip (CTOA). Of these, only CTOD was found to be geometry independent, although it was dependent, as was the initiation toughness, on the crack tip constraint. δ and CTOA were not suitable because they showed a distinct specimen geometry dependence, without a corresponding change in the crack growth resistance.

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Notation

a	crack length
$da, \Delta a$	increment of a
b	remaining ligament, $w-a$
b_0	initial b
B	specimen thickness
B_E	effective thickness, see equation 4.12
B_g	gross thickness
B_N	net thickness
C	elastic compliance
C_0	compliance of uncracked specimen
CE	complementary energy
e	$\ln(10)$
E	Young's modulus
E'	effective E ; E for plane stress, $E/(1-\nu^2)$ for plane strain
E_v	energy absorbed per unit volume of shear lip formed
$f(x), g(x)$	functions of x
$f'(x)$	first derivative of function $f(x)$
G	elastic energy release rate
G_0	G uncorrected for crack growth
h	crack face separation
$h_{r=\Delta a-SZW}$	h at start of fibrous crack growth region
$h_{r=\Delta a}$	h at the fatigue crack tip
J	J integral
J_f, J_s	J value at a given crack extension from the J - R curve for the flat fracture and shear lip regions of the specimen, respectively
J_m	Ernst modified J_r

$J_{\min}$	lowest measured J value associated with fibrous crack extension
J_o	J uncorrected for crack growth
J_r	J corrected for crack growth
$f_1(J), f_2(J)$	functions of J
K	stress intensity factor
L	plastic constraint factor
m	coefficient relating δ and J or δ and K^2
m'	coefficient relating dJ/da and $d\delta/da$
M	Q_L/Q_o
n, N	work hardening coefficients, $\sigma \propto \epsilon^n$ or $\epsilon \propto \sigma^N$
P	potential energy
q	displacement
Q	load
Q_L	limit load of cracked specimen
Q_o	limit load of uncracked specimen
r	distance from crack tip
r_i, r_g	rotation factors prior to and after crack initiation, respectively
s	contour path around which J is defined
S	shear lip width
T	traction vector
u	displacement vector
U	area under load versus displacement trace
V	mouth opening displacement
w	specimen width
W	internal strain energy
x, y	co-ordinates
X	distance across crack front from specimen edge
$\bar{Y}, Y$	LEFM compliance functions

Greek symbols

γ	coefficient relating CTOA and CTOD
γ_p	plastic work required to create a new surface
γ_s	surface energy
δ	crack opening displacement at the original crack tip
δ_a	crack opening displacement at a distance r behind growing crack tip
δ_g	increase in δ during fibrous crack extension
$\Delta\delta_{UL}$	change in δ on unloading
ϵ	strain
ζ	$2 - \frac{Qq_p}{W_p}$
η	eta factor
η_E, η_m	η values for compact specimens from the Ernst and Merkle and Corten solutions, respectively
η_o	η value appropriate to original crack length
$f(\eta)$	$1 + b \frac{d\eta}{\bar{\eta} da}$
$g(\eta)$	$f(\eta) - \eta$
θ	angle relative to crack direction
$f(\theta), g(\theta), g_1(\theta), g_2(\theta)$	functions of θ
ν	Poisson's ratio
σ	stress
σ_f	flow stress = $1/2(\sigma_y + \sigma_t)$
σ_m	mean or hydrostatic stress
σ_o	reference stress
σ_t	tensile stress
σ_y	yield stress
$\sigma_{xx}, \sigma_{yy}, \sigma_{zz}$	principal stresses

$\bar{\sigma}$ von Mises equivalent stress

ω $\frac{b}{J} \frac{dJ}{da}$

Commonly used suffixes

c critical value
e elastic component
i value at crack initiation
i j co-ordinates
p plastic component
I value under mode I loading
IC critical value as defined in a testing standard
()_i value at the "i" increment of crack extension
()_{i+1} value at the "i+1" increment of crack extension

Abbreviations

CCT centre-cracked tension
CTOA crack tip opening angle
CTOD crack opening displacement at growing crack tip
DEN double-edge-notched tension
EPFM elastic plastic fracture mechanics
LEFM linear elastic fracture mechanics
LVDT linear variable differential transformer
SEM scanning electron microscope
SZW stretch zone width
TOM travelling optical microscope

Chapter 1

Introduction

In recent years there has been considerable interest in the use of elastic-plastic fracture parameters to characterise the crack initiation and crack growth resistance of ductile materials, for which the micromechanism of fracture is microvoid coalescence. This has arisen because of the use of increasingly tougher materials for which the specimen size requirements, in order to use linear-elastic fracture parameters, have become prohibitively large. There have been many proposed elastic-plastic fracture parameters, as well as a considerable number of investigations to evaluate the conditions under which these parameters can characterise the crack initiation and/or crack growth resistance of the material. However, there still remains a considerable degree of uncertainty as to which parameters are most suitable, and under which conditions they may be applicable. In addition, a significant number of experimental investigations have been carried out to evaluate the effects of specimen size and geometry on the crack initiation and crack growth resistance of a material. These investigations often reveal that there are distinct effects, particularly on the crack growth resistance as characterised by a given parameter. However, rarely has it been shown whether these effects are due to changes in the intrinsic toughness of the material, or a violation of the conditions under which a fracture parameter can be used.

There are two aims of this study. The first is to determine the effects of specimen size and geometry on the crack initiation and crack growth resistance of a mild steel, BS4360 43A. The geometries which are considered are the compact, single-edge-notched three and four-point bend, and centre-cracked and double-edge-notched tension. The second aim is to assess the importance of various possible parameters for controlling fracture. The parameters which are considered up to crack initiation are the J integral and the crack opening displacement at the original crack tip (δ). After crack initiation the parameters which are considered, in addition to J and δ , are the crack opening displacement at the growing crack tip (CTOD) and the crack tip

opening angle (CTOA). δ , CTOA and CTOD are measured, in most cases, from silicone-rubber impressions of the crack tip profile.

The organisation of this dissertation is as follows. Chapter 2 contains a review of the mechanism of microvoid coalescence, a short review of linear-elastic fracture parameters, and a review of elastic-plastic parameters, both before and after crack initiation. The experimental techniques used in this study are described in Chapter 3, which includes those used to obtain silicone-rubber impressions of the crack profile. The procedures used to determine δ , CTOA and CTOD are also given. Chapter 4 describes the procedures used to determine J for the various specimen geometries investigated in this study. The experimental results pertaining to the effect of specimen size and geometry on the J at crack initiation (J_i) and the slope of the J versus crack growth curve (dJ/da), are given in Chapter 5. Chapter 6 reports the experimental results concerning the effect of specimen size and geometry on δ at crack initiation (δ_i), the slope of the δ versus crack growth curve ($d\delta/da$), CTOA and CTOD. A discussion of the effect of specimen size and geometry on J_i and δ_i is given in Chapter 7, together with the relationship between these two parameters. Chapter 8 presents a discussion on the effect of specimen size and geometry on dJ/da , $d\delta/da$, CTOA and CTOD, together with the relationship between these parameters. In addition, the capability of these various parameters to characterise the crack growth resistance of a material is discussed. A summary is given in Chapter 9, along with suggestions for further work. A list of references is given at the end of the dissertation.

Chapter 2

Literature Review

2.1 Microstructural Aspects of Microvoid Coalescence

2.1.1 Introduction

Microvoid coalescence is a fracture mechanism which involves void nucleation, growth and coalescence. It is often described as ductile fracture as it generally occurs in conjunction with large scale plasticity.

2.1.2 Void Nucleation

Voids generally form by the action of stress and/or strain on inclusions or second phase particles within the material. Hancock and Mackenzie (1976) suggested three mechanisms for void nucleation: decohesion of the particle-matrix interface; fracture of the particle; micro-cracking of the matrix surrounding the particle. The exact mechanism will depend on the type of particle and its orientation with respect to the direction of applied stress, as reported by Beremin (1981).

The conditions for void nucleation can be described by continuum mechanics when the nucleating particles are larger than about 20nm. Generally, it is reported that void nucleation occurs on obtaining a critical stress at the particle-matrix interface, as for example found by Beremin (1981) for manganese sulphide inclusions, although it has been suggested in some cases that a critical strain is required, see Hancock and Mackenzie (1976). If the criterion is based on a critical stress, rather than a critical strain, the onset of void nucleation will be governed by the imposed stress state, or triaxiality.

The ease of void nucleation will be determined by the strength of the particle-matrix interface. For inclusions, such as manganese sulphide, it is reported that void nucleation occurs easily, particularly ahead of a crack tip, see Lautridou and Pineau (1981), while carbides are generally considered to be well bonded to the matrix. For example, Devaux et al (1978) found that in a C-Mn steel, void nucleation around carbides only occurred at the high

strains present before the final linkage of large voids, formed around inclusions, to the crack tip.

2.1.3 Void Growth

Voids can grow by plastic deformation arising from the applied loading conditions following nucleation. A number of analytical equations for the rate of void growth have been derived considering an isolated void in a continuum matrix, see McClintock (1968) and Rice and Tracey (1969). They show that the growth rate is dependent on the imposed stress state or triaxiality, although the exact dependence is different in each case. However, it is unclear how applicable these equations are to the situation ahead of crack tips, where there is an intense strain gradient and a possible interaction between the void and the crack tip and between other voids.

2.1.4 Void Coalescence

The mechanism by which voids generally link to the crack tip, particularly in high strength steels, is called flow or shear localisation. This is where a thin band of intense shear deformation is formed between the void and the crack tip, which results in linkage after only a further small increase in applied strain. Flow localisation often occurs in conjunction with a "second" generation of voids formed around small particles such as carbides. However, it is still unclear whether this void formation results in or arises from flow localisation, Hancock and Cowling (1980). Hancock and Brown (1983) suggested that the conditions for flow localisation are dependent on the local strain hardening rate and strain state which are in turn governed by the global stress state and applied strain for a given material. Experimental studies find that the average void size when flow localisation occurs is either independent or decreases slightly with increasing triaxiality, see Thomson and Hancock (1981) and Pineau (1981).

2.1.5 Fracture Strain

The local fracture strain is found to depend on the stress state, for example see Mackenzie et al (1977). This is as expected as the stress state affects void growth rates and possibly the ease

of void nucleation and shear localisation. However, the dependence of fracture strain on stress state varies considerably from material to material, see Mackenzie et al (1977).

2.2 Linear-Elastic Fracture Mechanics

2.2.1 Introduction

Linear-elastic fracture mechanics (LEFM) has been shown to be a useful tool in predicting the conditions for failure when the component is behaving in an essentially elastic manner. The subject has been extensively reviewed by a number of other authors, for example Garwood (1976) and so will be covered only briefly in this review.

2.2.2 Stress Intensity Factor and Linear-elastic Energy Release Rate

Two LEFM parameters have gained general acceptance: the stress intensity factor (K) and the linear-elastic energy release rate (G). K controls the magnitude of the stresses and strains around the crack tip in a purely elastic material, for example under mode I loading:

$$\sigma_{ij} = \frac{K_I}{\sqrt{2\pi r}} f_{ij}(\theta) \quad 2.1$$

$$\epsilon_{ij} = \frac{K_I}{\sqrt{2\pi r}} g_{ij}(\theta) \quad 2.2$$

The stresses and strains are also governed by the crack tip constraint, which in fracture test specimens will be between that for plane strain (where the through thickness strain component is zero) and that for plane stress (where the through thickness stress component is zero). For a given crack tip constraint, K will give a one parameter description of the stress and strain field which will govern the onset of crack initiation, i.e. when $K=K_I$.

K can be calculated from:

$$K = \bar{Y} \sigma \sqrt{\pi a} \quad 2.3$$

$$K = \frac{QY}{B\sqrt{W}} \quad 2.4$$

$\bar{Y}$ and Y are dimensionless coefficients which are dependent on the component geometry, crack shape and size, and the type of loading.

G governs the rate of energy available for fracture per unit increment of crack area. G was originally defined in terms of the potential energy release rate, although it can also be considered in terms of the elastic absorbed energy (W_e) release rate:

$$G = -\frac{1}{B} \left. \frac{\partial W_e}{\partial a} \right|_q = \frac{1}{B} \left. \frac{\partial W_e}{\partial a} \right|_Q \quad 2.5$$

In a purely elastic material, the principle behind G as a crack initiation characterising parameter is that crack initiation will occur when there is enough elastic absorbed energy available to supply that required to create two new surfaces (i.e. 2x surface energy γ_s). Therefore, crack initiation occurs when:

$$G \geq 2 \gamma_s = G_i \quad 2.6$$

Irwin (1957) showed that K and G are related such that:

$$G = \frac{K^2}{E'} \quad 2.7$$

where $E' = E$ for plane stress

$$E' = \frac{E}{(1-\nu^2)} \quad \text{for plane strain}$$

Therefore, G and K can be considered as controlling the stress and strain field around the crack as well as the elastic energy release rate.

G can be calculated from equation 2.5; via K using equation 2.7; or by equation 2.8 which is obtained by rearranging equation 2.5:

$$G = \frac{\eta_e W_e}{Bb} \quad 2.8$$

where

$$\eta_e = \frac{b}{w_e} \left. \frac{\partial w_e}{\partial a} \right|_Q \quad 2.9$$

The elastic η factor (η_e) is dependent on the component geometry, crack shape and size, and type of loading. It can be calculated from equation 2.9 or from a number of other equations derived from equation 2.9, see Turner (1979):

$$\eta_e = \frac{b}{c} \frac{dc}{da} \quad 2.10$$

$$\eta_e = \frac{by^2}{(y^2da + \frac{E'C_0Bw}{2})} \quad 2.11$$

Equations 2.1 and 2.2 predict infinite stresses and strains at the crack tip. However, in metals the stress will be limited because of plastic deformation. As a consequence of this, there will be a region around the crack tip in which plastic deformation occurs which is called the plastic zone. Even though the stresses and strains within the plastic zone are no longer given by equations 2.1 and 2.2, it is generally considered that K and G can still characterise the conditions for the onset of crack initiation, provided the size of the plastic zone is small in comparison with the specimen dimensions. This is because K and G govern the size of the plastic zone and the characteristic conditions within it, such as the magnitude of the strain and the position of the maximum stress, see McMeeking (1977).

When there is local plastic deformation occurring during crack growth, the additional energy required for fracture above that required to create two new surfaces ($2\gamma_s$) is γ_p , and therefore crack initiation occurs when:

$$G \geq 2\gamma_s + \gamma_p = G_i$$

Figure 2.1 shows a schematic plot of the fracture toughness K_C or G_C , determined using the procedure given in BS5447, versus specimen thickness. This shows that with increasing specimen thickness, the fracture toughness decreases to a plateau value (K_{1C} or G_{1C}). The suggested minimum specimen thickness to obtain a K_{1C}

or G_{1C} value as given in BS5447 is:

$$a, B > 2.5 \left(\frac{K_C}{\sigma_y} \right)^2$$

A decrease in the fracture toughness with increasing specimen thickness would be expected as the crack tip constraint increases from plane stress to plane strain, resulting in a change in the local fracture strain, see section 2.1.5. However, for ductile materials as shown for example by Read and Reed (1977), the need to satisfy the above size criteria to obtain lower bound K_C or G_C values is not due to the need to obtain plane strain conditions at the crack tip, for which the size criteria are significantly less stringent, but arises because of the 5% offset procedure used in BS5447 to determine K_C and G_C values.

2.2.3 R Curves

It is found when testing thin sheets that the applied K or G has to be increased, above that required for crack initiation, to maintain crack growth. This increase in toughness with increasing crack extension is often attributed to an increasing proportion of shear lip on the fracture surface and may result in the onset of failure occurring after some amount of ductile crack extension. Therefore, to predict the failure conditions, it is important to characterise the crack growth resistance of the material. Both K and G have been tried as crack growth characterising parameters with a reasonable degree of success, see Wang and McCabe (1976). The K or G versus crack extension plots are commonly described as R curves.

2.3 Elastic-Plastic Fracture Parameters, up to Crack Initiation

2.3.1 Introduction

LEFM parameters can characterise the onset of crack initiation only when the plastic zone size is small in comparison with the specimen dimensions. When this is no longer the case, elastic-plastic fracture mechanics (EPFM) parameters have to be used instead. Two such parameters have gained general acceptance: the J integral and the crack opening displacement (δ).

2.3.2 J Integral

2.3.2.1 Introduction

Eshelby (1956) first suggested the use of a line integral to evaluate the energy release rate of a cracked body. However, the use of a line integral to characterise fracture under elastic-plastic conditions is credited to Rice (1968). He defined a two dimensional line integral called J.

$$J = \int_s (W \, dy - T \frac{du}{dx} \, ds)$$

The crack is aligned in the x direction as illustrated in Figure 2.2. W is the strain energy density, T is the traction vector which specifies the stresses on the contour s, and u is the displacement vector of points on s. Rice (1968) defined the potential energy of a body of area A as:

$$P = \frac{1}{B} \left(\int_A W \, dy \, dx - \int_{s_T} T \, u \, ds \right)$$

s_T refers to that portion of boundary over which stresses are prescribed as boundary conditions. Therefore, J is equivalent to the rate of change of potential energy with respect to crack extension:

$$J = \frac{-1}{B} \frac{dP}{da} \quad 2.12$$

Under displacement control, the potential energy of the system reduces to the area under the load-displacement curve, see Landes and Begley (1972). As shown in Figure 2.3, $JBda$ is thus given by the area between the load-displacement curves of two nominally identical specimens with crack lengths a and $a+\Delta a$, so J is also given by:

$$J = \frac{-1}{B} \frac{\partial W}{\partial a} \bigg|_q \quad 2.13$$

W is the internal absorbed energy which prior to crack initiation is equal to the area under the load-displacement curve.

Conversely, under load control, the potential energy is the negative of the area above the load deflection curve, see Landes and Begley (1972), which is the complementary energy CE, see Figure 2.3, and therefore J is also given by:

$$J = \frac{1}{B} \left. \frac{\partial CE}{\partial a} \right|_Q \quad 2.14$$

J has the property of being path independent for non-linear elastic materials, that is the same J value is obtained for any contour taken in an anti-clockwise direction from the lower to the upper crack face. Although metals do not exhibit non-linear elastic behaviour, provided they obey deformation or total theory of plasticity, J maintains its path independence behaviour, provided there is no unloading at the crack tip due to for example crack extension. Deformation or total theory of plasticity is where the stresses are in a fixed relationship with each other and are related to the total strain. However, J loses its interpretation as the energy release rate because not all the energy is elastic and therefore recoverable.

Hutchinson (1968) and Rice and Rosengren (1968), independently determined the stress and strain field around a crack tip for a material exhibiting power law hardening and obeying deformation theory of plasticity. This field is commonly described as the HRR field. The stress and strain equations for the HRR field were rewritten in terms of J by McClintock (1971):

$$\sigma_{ij} = f_1(J) r^{\left(\frac{-n}{n+1}\right)} g_1(\theta) \quad 2.15$$

$$\epsilon_{ij} = f_2(J) r^{\left(\frac{-1}{n+1}\right)} g_2(\theta) \quad 2.16$$

The functions f_1 and f_2 are dependent on whether the crack tip constraint is that for plane stress or plane strain.

Equations 2.15 and 2.16 show that the J local to the crack tip characterises the magnitude of the stresses and strains and therefore is likely to govern the conditions for crack initiation, for a given crack tip constraint. As J is path independent, the J calculated from any contour around the crack tip will also characterise the magnitude of the crack tip stresses and strains and therefore govern the conditions for crack initiation.

In practice, metals obey the incremental theory of plasticity, which is where the increment of strain is related to the increment of stress, rather than the deformation theory of plasticity. A number of finite element studies using incremental theory have shown that J maintains its path independence except very close to the crack tip, for example Hayes and Turner (1974) and Riccardella and Swedlow (1974). It is only in this region that the differences between using incremental and deformation theories become evident. McMeeking (1977) found that path independence broke down at distances less than about $6x\delta$ from the crack tip, for materials with a smooth yield surface. For materials with vertices on the yield surface, Needleman and Tvergaard (1983) showed that the distance was only $2x\delta$. In addition, Rice and Johnson (1970) and McMeeking (1977) showed that at distances less than about $1.9x\delta$ from the crack tip, the stress is no longer given by the HRR field, equation 2.15. However, convention has it that J will still govern the conditions for crack initiation, provided the region in which the HRR field does not apply is surrounded by a region in which the HRR field does apply and where J characterises the magnitude of the stresses and strains. This argument is similar to that used to justify the use of K in cases where there is a small plastic zone around the crack tip.

2.3.2.2 Experimental Determination

J is generally determined experimentally from equations 2.13 or 2.14 (or from equations derived from these) and Hollstein et al (1983) have confirmed that these J values are the same as those determined from the line integral definition of J , even in the presence of large scale plasticity. In early work by Begley and Landes (1972), J was calculated from equation 2.13 using a number of nominally identical specimens containing different crack lengths. However, this was found to be a costly process in both time and number of specimens. In recent times, a number of techniques have been suggested to calculate J directly from the load-displacement trace of a single specimen.

One technique is to calculate J from the area under the load-displacement trace, U , which is equal to the internal absorbed

energy W prior to crack initiation:

$$J = \frac{\eta W}{Bb} \quad 2.17$$

where

$$\eta = \frac{-b}{W} \frac{\partial W}{\partial a} \bigg|_q \quad 2.18$$

Note that equation 2.13 is reobtained by substituting equation 2.18 into 2.17. The η factor is dependent on the specimen geometry, crack shape and size, and type of loading. An adaptation of this technique involves splitting J into elastic and plastic components, see Sumpter and Turner (1976a).

$$J = J_e + J_p$$

J_e is taken as G to ensure compatibility between J and G under linear-elastic conditions, i.e.

$$J = G + J_p \quad 2.19$$

The plastic component of J is given by:

$$J_p = \frac{-1}{B} \frac{\partial W_p}{\partial a} \bigg|_{q_p} \quad 2.20$$

This can be rewritten as:

$$J_p = \frac{-1}{B} \int \frac{\partial Q}{\partial a} \bigg|_{q_p} dq_p \quad 2.21$$

or

$$J_p = \frac{1}{B} \int \frac{\partial q_p}{\partial b} \bigg|_Q dQ \quad 2.22$$

Alternatively, J_p can be calculated from the plastic work absorbed, W_p ,

$$J_p = \frac{\eta_p W_p}{Bb} \quad 2.23$$

where

$$\eta_p = \frac{-b}{W_p} \frac{\partial W_p}{\partial a} \bigg|_{q_p} \quad 2.24$$

Equation 2.24 is often simplified by assuming perfectly plastic conditions, where the maximum load is the limit load, Q_L :

$$\eta_p = \frac{-b}{Q_L} \frac{dQ_L}{da} \quad 2.25$$

Another technique to calculate J involves determining the relationship between J and the displacement. The J at a given

displacement can then be calculated from this relationship, see for example Bucci et al (1972).

Further discussion on the determination of J can be found in Chapter 4.

2.3.2.3 Three Dimensional Studies

The original definition of J was a two dimensional line integral. However, specimens and components are three dimensional in nature and a number of three dimensional finite element studies have been undertaken, see for example Miyamoto and Kikuchi (1981), Bakker (1982), de Lorenzi and Shih (1983) and Sakata et al (1983). Various adaptations of J have been suggested by the above mentioned authors to allow the variation of "J" across the crack front to be studied. However, little work has been carried out to investigate whether these adapted J values characterise the magnitude of the stresses and strains around the crack tip in three dimensional situations.

The studies show that in non-sidegrooved specimens, particularly under net section yielding conditions, the J increases from the surface to the centre of the specimen, which Miyamoto and Kikuchi (1981) showed was due to a corresponding change of crack tip constraint from plane stress to plane strain. This variation in applied J, together with a constant J for crack initiation (J_1) across the crack front, helps to explain the phenomenon of crack tunnelling, which is where the crack extension is larger in the centre than at the surface of the specimen, see Bakker (1982). The variation in J across the crack front was reported to increase with increasing applied displacement, although the degree of variation appeared to depend on the approach used to calculate J.

de Lorenzi and Shih (1983) showed that by sidegrooving a specimen by 25% of the gross thickness, the variation in J and crack tip constraint across the crack front was reduced significantly. This is consistent with a lower degree of crack tunnelling found in sidegrooved specimens, see Druce (1981).

2.3.3 Crack Opening Displacement

2.3.3.1 Introduction

Crack opening displacement (δ) is a measure of the crack face separation at the crack tip. Finite element computations by McMeeking (1977) suggest that δ gives a one parameter characterisation of the magnitude of the stress and strain field around the crack tip, at least for a given crack tip constraint. This is supported by the experimental results of Shoji et al (1978). Therefore, δ appears to be a useful parameter to characterise the conditions for crack initiation, for a given crack tip constraint.

2.3.3.2 Determination of δ

There are two main procedures used to determine δ . One procedure involves measuring δ from the crack tip profile which has been obtained by silicone-rubber infiltration of the crack, see Robinson and Tetelman (1974), metallographic sectioning, see You and Knott (1982) or by finite element analysis (provided the analysis allows large geometry changes to occur at the crack tip), see McMeeking (1977). In the case of metallographic sectioning, this is generally carried out after unloading the specimen, and therefore a correction has to be made to account for the change in δ during the unloading. Since the crack advances as it blunts, a suitable definition of δ could be the crack opening at the original crack tip. It is possible to determine δ in this way if the crack profiles are determined experimentally, although it would appear that Robinson (1976) has used a more arbitrary position on the crack profile from which to determine δ . However, this is not a suitable method for determining δ from the crack profiles obtained from finite element analysis because the position of the original crack tip is not evident. To overcome this, an alternative definition of δ can be used, illustrated in Figure 2.4, which involves measuring δ at the position of intersection between the crack profile and a line drawn from the blunted crack tip at a given angle to the crack plane. The angle generally used is 45°, see McMeeking (1977). This definition can also be used to determine δ from an experimentally determined crack profile, for example as used by Knauf and Riedel (1981). However, this definition is not suitable after crack initiation as the position of the blunted crack tip prior to crack

initiation is not known, although the definition based on the original crack tip position is still suitable.

The second type of procedure involves some form of extrapolation to determine δ . In the case of finite element studies, this often involves the determination of δ by extrapolation of the crack profile measured away from the crack tip, to the crack tip, as illustrated in Figure 2.5, see for example Hayes and Turner (1974). This overcomes the need for determining accurately the crack profile close to the crack tip. However, the δ value determined in this way is unlikely to be the same as that determined from the crack profile, see Figure 2.5.

The use of an extrapolation procedure to determine δ experimentally is very common, due to the difficult and time consuming process of measuring the crack profile. The procedure involves determining δ from the crack mouth opening displacement, V , which can be measured easily using a clip gauge. In the case of geometries where the loading is essentially bending, the specimen is assumed to rotate about a hinge point which is located ahead of the crack tip at a distance equal to the ligament length (b) multiplied by a rotation factor (r_i). δ , as illustrated in Figure 2.6, is then related to V by similar triangles:

$$\delta = \frac{r_i b V}{r_i b + a} \quad 2.26$$

Often, only the plastic component of the specimen rotation is assumed to occur about a hinge point and thus the plastic component of δ (δ_p) is calculated from the plastic component of the mouth opening displacement (V_p) using an equation similar to equation 2.26. The elastic component of δ (δ_e) is calculated from K . Thus δ is given by:

$$\delta = \frac{K^2}{m \sigma_y E'} + \frac{r_i b V_p}{r_i b + a} \quad 2.27$$

This type of equation is used in the δ testing standard BS5762:1979, where m is taken as 2.0 and r as 0.4. The disadvantage of using equations 2.26 and 2.27 is that there are uncertainties in the values of m and r_i . For example, Slatcher and Knott (1981) found that m was dependent on the yield stress of the material. Hayes and

Turner (1974) reported that r_i increased up to a plateau value with increasing displacement and Landes (1982) reported that r_i was dependent on the specimen dimensions. Part of these problems could be overcome for the bend geometries by measuring simultaneously the mouth opening and load-point displacements, from which an ' r_i ' value can be calculated, see for example Lin et al (1982).

Few additional studies have been undertaken to determine the relationship between δ and V for other geometries, although Robinson (1976), from his infiltration studies on centre-cracked tension specimens under net section yielding conditions, suggested that:

$$\delta = .46V$$

However, the disadvantage of using any of the extrapolation procedures to determine δ is that it is uncertain how close this δ is to that determined from the crack profile which is more likely to be a meaningful fracture parameter.

2.3.3.3 Three Dimensional Studies

The same definition of δ can be used in two and three dimensional situations, unlike J and therefore it is a suitable fracture parameter to investigate the three dimensional nature of specimens and components. The variation of δ across the crack front of non-sidegrooved specimens has been studied by Bakker (1982), and de Lorenzi and Shih (1983). They found that, as with J , δ increased towards the centre of the specimen. This is in agreement with the experimental findings of Robinson and Tetelman (1975). The variation of δ was also found to increase with increasing displacement. In addition, de Lorenzi and Shih (1983) showed that on sidegrooving the specimen by 25%, the variation of δ across the crack front was reduced significantly.

2.3.4 Relationship between J and δ

J and δ are generally related using equation 2.28, Turner (1979):

$$J = m \sigma_y \delta \quad 2.28$$

although in some cases a reference stress (σ_o) or a flow stress (σ_f) is used instead of the yield stress (σ_y) the latter as a first order

estimate for strain hardening. The reported values of m lie in the range 1 to 3, although part of this variation can be attributed to the different methods used to determine δ .

Based on the HRR field relationships, Shih (1981) suggested that m increased significantly with increasing work hardening coefficient, but decreased slightly with increasing σ_y/E . The crack tip constraint was also found to affect the m value. Shih (1981) also found that the m values determined from the HRR field equations were in agreement with those calculated from finite element calculations, except in the limit of non work-hardening, when the HRR field relationships break down. However, Knauf and Riedel (1981) showed that the crack tip openings predicted by the HRR field relationships are significantly smaller than those measured experimentally. This suggests that the m values predicted by Shih (1981) may be overly large, although the factors affecting m may be the same. However, results by Miyoshi and Shiratori (1979) suggest that J and δ should be related by Young's modulus (E) rather than σ_y , which suggests that m will be affected significantly by σ_y/E . Experimental studies show that m is lower for high strength steels compared to low strength steels, see Robinson (1976) and decreases with increasing amounts of prior cold work, see Chipperfield (1976). However, it is unclear from these studies whether the changes in m were due to changes in σ_y/E or work hardening coefficient, or both.

Xiao and Huang (1982) suggested that there is no unique relationship between J and δ , but there is between the plastic and elastic components of J and δ respectively. This suggests that m varies with increasing plastic deformation. However, Robinson (1976) found no change in m with increasing plastic deformation.

Robinson (1976) found a similar m value for the three-point bend and the centre-cracked tension geometries which suggests that m may be geometry independent.

2.3.5 Specimen Size Requirements for Plane Strain Conditions

2.3.5.1 Finite Element Studies

A number of finite element studies, mainly two dimensional, have been undertaken to investigate the size requirements to maintain plane strain conditions in the crack tip region. In these studies a criterion is chosen to decide when these conditions have broken down sufficiently so as to affect the fracture toughness of a material. McMeeking and Parks (1979) took as their criterion the maintaining over a distance of $8x\delta$ a stress distribution corresponding to that for small scale yielding under plane strain conditions. Their results suggest a minimum ligament size requirement for the deep-notched bending geometries of:

$$b > 25 \frac{J}{\sigma_f} \quad 2.29$$

and for the centre-cracked tension geometry:

$$b > 200 \frac{J}{\sigma_f} \quad 2.30$$

These size requirements were confirmed by Shih and German (1981) who took as their criterion the maintaining of the HRR field for plane strain over a distance of $6x\delta$ ahead of the crack tip. However, these requirements were determined for materials with a zero or low work hardening capacity. Different requirements may be applicable to materials with a medium or high work hardening capacity.

Sakata et al (1983) have carried out three dimensional finite element analyses of compact specimens of various thicknesses. Although they did not determine a thickness size requirement, it is possible to obtain one from their results. The criterion chosen for the departure from plane strain crack tip conditions was when J at the midsection of the specimen deviated from that for plane strain by more than 3% at a given displacement. The thickness size requirement obtained was:

$$B > 320 \frac{J}{\sigma_y} \quad 2.31$$

A lower coefficient value than 320 would be obtained if the size requirement was based on flow stress rather than yield stress. Although equation 2.31 must be considered somewhat speculative, it

does suggest that the thickness size requirement may be considerably larger than the ligament size requirement.

The specimen size requirements for plane strain crack tip conditions given by equations 2.29-2.31 can presumably be written in terms of δ rather than J , by noting the J - δ relationship given by equation 2.28.

2.3.5.2 Experimental Studies

There are two main problems in determining experimentally the specimen size requirements for plane strain crack tip conditions. The first is due to the variability in the initiation toughness values, arising from material inhomogeneity. This can often be a significant proportion of the mean toughness and can obscure the effects of specimen size which leads to optimistic size criteria being proposed. The second problem arises if "initiation" J values are determined using the ASTM J_{1C} testing procedure E813-81, (these values are subsequently referred to as J_{1C} values for convenience, although they may not have been determined from specimens which satisfy the size criteria given in the standard). For these overestimate the J values at the onset of crack initiation, J_i , particularly in high toughness materials, see for example Carlson and Williams (1981). Therefore, the size criteria based on J_{1C} values will appear apparently less restrictive than those based on J_i values. In addition, Carlson and Williams (1981) showed that the trends in J_i and J_{1C} with increasing sidegroove depth were different. This suggests that trends in J_i and J_{1C} with specimen size may also be different, and so the size requirements determined from J_{1C} values may not reflect those required to obtain plane strain J_i values. A similar problem can arise when determining the size criteria for δ , if δ is determined by one of the "extrapolation" methods, see section 2.3.3.2. For there may be differences between this δ and that measured from the crack profile, which can lead to different trends in the two δ values with specimen size as shown by Neale and Townley (1977). The above points have to be considered when examining the specimen size criteria which have been determined experimentally.

Paris suggested in a written comment on the early work by Begley and Landes (1972) that in order to ensure plane strain conditions within the fracture process zone around the crack tip, the specimen dimensions should be larger than a given multiple of δ , as δ represents the size of the process zone. By noting the relationship between J and δ , Paris suggested that the size criteria could be written in terms of J/σ_f . The initial size criteria suggested for the compact and bend geometries as given by Landes and Begley (1974) were:

$$B, b > 25 - 50 \frac{J}{\sigma_f} \quad 2.32$$

Most results seem to support these size criteria, for example Sunamoto et al (1977), Gudas et al (1979) and Huang and Gelles (1984). However, Marandet et al (1981) suggested a significantly more restrictive ligament criterion was required.

The corresponding size criteria based on δ , given $J \approx \sigma_f \delta$ are:

$$B, b > 25 - 50 \delta \quad 2.33$$

These size criteria are in agreement with those suggested by a number of authors for bend specimens, for example Robinson and Tetelman (1975) and Chipperfield (1976).

2.3.5.3 Effect of Specimen Geometry

In general, it is reported that the initiation values are the same for the essentially deep notched bending geometries, i.e. three and four-point bend and compact. For example Ingham and Morland (1979) and Carlsson et al (1981) in the case of J_i , and Chipperfield et al (1973) and Hancock and Cowling (1980) in the case of δ_i , although Gudas and Davis (1982) reported lower J_{1C} values for three-point bend specimens than compact specimens, whereas Keller and Munz (1977) reported the opposite. Therefore, it would appear that the size requirements for plane strain conditions, given by equations 2.32 and 2.33 are applicable to all the essentially deep notched bending geometries.

Keller and Munz (1977) reported similar J_{1C} values for compact and double-edge-notched tension specimens. On the other

hand, Hancock and Cowling (1980) reported that the δ_i values for double-edge-notched tension specimens could be higher or lower than that for compact specimens, depending on the specimen width to ligament ratio. However, as these δ_i values were probably determined indirectly, there must be some uncertainty about this result. Therefore, it is unclear from these studies what the specimen size criteria are for double-edge-notched tension specimens, although they are probably similar to those for the bend and compact specimens.

Begley and Landes (1976) reported similar J_{1C} values for centre-cracked tension and compact specimens of the same size, whereas Robinson (1976) reported that J_i and δ_i were slightly higher for centre-cracked tension than the three-point bend specimens. This suggests that the size criteria for the centre-cracked tension specimens may be more stringent than those for the bend and compact specimens. However, the results do not appear to suggest that the ligament size criterion for the centre-cracked tension specimens is significantly larger than that for the bend specimens, as suggested from the finite element calculations (compare equations 2.29 and 2.30).

2.4 Elastic-Plastic Fracture Parameters after Crack Initiation

2.4.1 Introduction

It is found that there is an increase in toughness with increasing ductile crack extension in thick specimens as well as in thin sheets. A major cause for this increase in toughness, as in the thin sheets, is the increasing proportion of shear lip on the fracture surface. However, there is still an increase in toughness when no shear lip is present, as for example in sidegrooved specimens, see Garwood and Turner (1977). One possible explanation for this is that the intrinsic toughness of the material increases with increasing crack extension, i.e. the process of ductile crack extension becomes progressively more difficult. However, a number of studies appear to show that the intrinsic toughness of the material remains constant with crack extension. For example, Garwood (1976) and Willoughby (1979) have shown that the crack opening displacement at the tip of the growing crack (CTOD) remains constant with crack

growth, and Shoji (1981) and Saka et al (1983) have shown that the size of the intense strain region around the crack tip remains constant with crack growth. Thus, it would appear that the measured increase in toughness is an apparent increase and not associated with any increase in the intrinsic toughness of the material. There does not appear to be any conclusive explanation for the apparent increase in toughness with increasing crack extension, although there have been a number of suggestions, see Rice and Sorensen (1978) and Chipperfield (1977). The principle behind both these suggestions is that during ductile crack growth the applied loading conditions have to be increased in order to maintain the appropriate critical fracture criterion required for further crack growth, which in turn results in an increase in apparent toughness; the critical fracture criterion being either a constant strain at a certain distance ahead of the crack tip or a constant crack tip profile. In addition, the work of Rice and Sorensen (1978) showed that there is a $\ln^{1/r}$ strain singularity ahead of a growing crack tip compared to a $1/r$ singularity ahead of the crack tip prior to crack growth. The authors suggest that the reduced singularity ahead of a growing crack tip may also partly explain the apparent increase in toughness with crack growth.

The fracture parameters which have been suggested to characterise the crack growth resistance of a material fall into two categories. One contains those parameters which measure the response of the specimen to the applied loading conditions, close to the growing crack tip (near field parameters), and these are likely to measure the intrinsic toughness of the material. In this category two parameters have been suggested, see Figure 2.7. These are the crack tip opening displacement (CTOD) and the crack tip opening angle (CTOA). The other category contains those parameters which measure the response of the specimen to the applied loading conditions, away from the growing crack tip (far field parameters), and these are likely to measure the increase in the apparent toughness of the material. The two main parameters which have been suggested in this category are J and δ (measured at the original crack tip).

2.4.2 Far Field Parameters

2.4.2.1 J Integral

The J integral was originally envisaged as only being applicable up to crack initiation because the unloading behind the crack tip during crack growth would result in J becoming path dependent in the crack tip region. However, Hutchinson and Paris (1979) suggested that J can characterise the crack growth resistance of a material provided that the size of the region in which J characterises the stress and strain field, dominates the near crack tip region where J is path dependent. They suggested that for this, two criteria have to be satisfied:

$$b \gg \Delta a \quad 2.34$$

$$\omega = \frac{b}{J} \frac{dJ}{da} \gg 1 \quad 2.35$$

A number of finite element studies have been undertaken to evaluate these criteria for compact specimens, see Shih et al (1979 and 1981). In each study a criterion was chosen for when J can no longer characterise the crack growth resistance of a material. Shih et al (1981) chose as their criterion for the breakdown of J controlled crack growth, when the J from a contour at a distance of $10x\delta_1$ ahead of the growing crack tip deviated by more than 10% from the J value from a contour well away from the crack tip. Using this criterion, Shih et al (1981) found that the maximum allowable crack extension to specimen ligament ratio for J controlled crack growth, increased with increasing crack growth resistance of the material (i.e. dJ/da). For high toughness materials the maximum allowable crack extension was:

$$\Delta a < 6\% b.$$

This criterion was reported previously by Shih et al (1979). In addition, the minimum ω value for J controlled crack growth was found to increase with increasing dJ/da and for high toughness materials the minimum ω was:

$$\omega \geq 10.$$

Not only does the stringency of these criteria depend on the crack growth resistance of the material, but it also depends on the criterion chosen for the breakdown of J controlled crack growth.

The Δa and ω criteria reported above for compact specimens are likely to be applicable to bend specimens, although for centre-cracked tension specimens Shih et al (1981) suggested a higher minimum ω value of 80.

Originally, the J value for a specimen which had undergone some ductile crack extension was calculated from the initial $\eta(\eta_0)$ and ligament length (b_0) values, and the area under the load versus displacement diagram, U , using equation 2.36; the J value so calculated being described as a J_0 value:

$$J_0 = \frac{\eta_0 U}{B b_0} \quad 2.36$$

This equation is based on equation 2.17. However, after crack initiation, as illustrated in Figure 2.8, U is no longer equal to the hypothetical internal absorbed energy W , which is given by the area under the load versus displacement curve of a specimen with a crack length equal to the final crack length, which has been loaded to the same load and displacement values. Garwood (1976) showed that U and W are related by:

$$\frac{dW}{da} = \frac{dU}{da} - BJ \quad 2.37$$

This equation can be used to determine W from the area under the load-displacement curve and therefore it can serve as the definition of W , although the value of W will depend on the method chosen to calculate J after crack initiation.

A number of authors, for example Garwood (1976), have suggested that a more appropriate J value than J_0 is one based on W , together with the final η and ligament length b , i.e.

$$J_r = \frac{\eta W}{Bb}$$

The principle behind this suggestion is as follows. J_r is the J value for a specimen with a crack length equal to the final crack length which has been loaded, without any crack growth, up to the final load and displacement values, see Figure 2.8. Therefore, J_r does not violate the original restriction on J which did not allow crack extension and so will be more likely to characterise the crack growth resistance of a material. However, it is unclear whether

this suggestion is valid. In general, J_r is determined from J_0 using one of a number of suggested crack growth correction equations, see for example Garwood et al (1975) and Ernst et al (1981). These equations are discussed in more detail in section 4.3.2.

Rice and Sorensen (1978) undertook an asymptotic analysis of steady state crack growth under small scale yielding and non work-hardening conditions. The analysis showed that the crack tip profile and the magnitude of the strain field around the crack tip were related to the far field loading conditions, which in turn could be described by a J like parameter. Rice and Sorensen (1978) showed that the crack opening was given by:

$$\frac{\delta_a}{r} = \frac{\alpha}{\sigma_y da} \frac{dJ}{E} + \frac{\beta}{E} \sigma_y \ln \left(\frac{eR}{r} \right) \quad 2.38$$

as $r \rightarrow 0$

where r is the distance behind the crack tip. The analysis determined the value of the coefficient β as 5.46 for $\nu = 0.3$. In addition, it appeared that the coefficient R scaled with the size of the plastic zone. A number of finite element analyses of crack growth under small scale yielding conditions have been undertaken to determine the values of coefficients α and R , see Dean and Hutchinson (1980), Parks et al (1981) and Sham (1983). In the latest study, Sham (1983) found that $\alpha = 0.58$ and:

$$R = A \times \frac{EJ}{\sigma_y^2}$$

where A depended on the crack growth history and ranged between 0.113 and 0.133. Drugan et al (1982) suggested a model for ductile crack growth based on a critical crack tip opening angle $(\delta_a/r)_c$ in which case equation 2.38 became:

$$\frac{\alpha}{\sigma_y da} \frac{dJ}{E} = \left(\frac{\delta_a}{r} \right)_c - \frac{\beta}{E} \sigma_y \ln \left(\frac{eR}{r} \right) \quad 2.39$$

This is often described as the Rice, Drugan and Sham (RDS) crack growth equation. Little work has been carried out to investigate whether equation 2.39 is valid under large scale yielding conditions and what the values of α, β and R might be, although it has been suggested that R may scale with the ligament size and the scaling factor may be geometry dependent, see Rice and Sorensen (1978). In

addition it has been suggested that α will be different under small and large scale yielding conditions, see Hermann and Rice (1980).

For relatively tough materials under large scale yielding conditions, the second term on the right hand side of equation 2.39 can be neglected, see Rice and Sorensen (1978) and therefore:

$$\frac{\alpha}{\sigma_y} \frac{dJ}{da} = \left(\frac{\delta_a}{r} \right)_c \quad 2.40$$

Smith (1981) obtained a similar result using Whuk's final stretch criterion, together with the Dugdale-Bilby-Cottrell-Swinden model, except that he found $\alpha=1$. In addition, from the relationship between the J-like parameter and the strain field ahead of a growing crack tip, Rice et al (1980) obtained an equation similar to equation 2.40 using a model based on a critical plastic strain at a characteristic distance, although in this case the critical plastic strain (ϵ_c) replaces the $(\delta_a/r)_c$ term.

Rice et al (1980) suggested that the J-like parameter which describes the far field loading conditions is independent of the crack growth rate, under rigid plastic conditions. Ernst (1983) suggested a correction to the J_r value to satisfy this criterion. The correction involves the subtraction of the plastic component of J_r which is dependent on the crack growth rate. The resulting J value is described as J_m and is given by:

$$\frac{dJ_m}{da} = \frac{dJ}{da} - \left. \frac{\partial(J-G)}{\partial a} \right|_{q_p} \quad 2.41$$

or

$$J_m = J - \int \left. \frac{\partial(J-G)}{\partial a} \right|_{q_p} da$$

For simplicity J_r is abbreviated to J in the above equations. Equation 2.41 can be simplified to:

$$\frac{dJ_m}{da} = \frac{dJ}{da} - \left. \frac{dJ_p}{da} \right|_{q_p} \quad 2.42$$

On differentiating equation 2.23 with respect to crack length at a

fixed plastic displacement:

$$\frac{\partial J_p}{\partial a} \bigg|_{q_p} = \frac{\eta_p}{Bb} \frac{\partial W_p}{\partial a} \bigg|_{q_p} + \frac{W_p}{Bb} \frac{d\eta_p}{da} + \frac{\eta_p W_p}{Bb^2}$$

Substituting in equations 2.20 and 2.23 gives:

$$\frac{\partial J_p}{\partial a} \bigg|_{q_p} = \frac{J_p}{b} g(\eta_p) \quad 2.43$$

$$\text{where } g(\eta_p) = 1 + \frac{b}{\eta_p} \frac{d\eta_p}{da} - \eta_p \quad 2.44$$

Substituting equation 2.43 into 2.42 gives:

$$\frac{dJ_m}{da} = \frac{dJ}{da} - \frac{J_p}{b} g(\eta_p) \quad 2.45$$

Integrating equation 2.45 with respect to crack length gives:

$$J_m = J - \int \frac{J_p}{b} g(\eta_p) da \quad 2.46$$

The experimental determination of J_m is discussed in more detail in sections 4.3.3 and 4.5.3. It has been suggested by Ernst (1983) that the use of J_m rather than J_r can extend significantly the amount of crack extension over which "J" can characterise the crack growth resistance of a material. Indeed, the results of Ernst (1983) would suggest that J_m is still applicable when $\omega > 0$ and when the amount of crack extension is up to 30% of the original ligament size.

2.4.2.2 Crack Opening Displacement, δ

The crack opening displacement, δ , measured at the original crack tip, has been used extensively to describe the crack growth resistance of a material. However, little work has been carried out to investigate the conditions under which this may be possible. Shih et al (1979) suggested that the following criterion needs to be satisfied:

$$\frac{E}{\sigma_f} \frac{d\delta}{da} \gg 1 \quad 2.47$$

although no minimum value for the left hand side of equation 2.47 was given.

Prior to crack initiation, δ could be determined from either the crack profile or by using an extrapolation procedure, see section 2.3.3.2. After crack initiation it is clear that δ can still be determined using procedures in which the crack profile is measured. However, it is not clear whether δ can be determined from the mouth opening displacement in the same way as before crack initiation. In the case of bend specimens it is uncertain whether the position of the hinge point is dependent on the original or current ligament size. In addition, it would appear that the rotation factor after crack initiation differs from that prior to crack initiation, as the results of Shang-Xian (1983) suggest that, based on the original ligament size, the rotation factor increases with increasing crack extension and Willoughby (1979) showed that, based on the current ligament size, the rotation factor decreases with increasing crack extension.

2.4.2.3 Relationship between dJ/da and $d\delta/da$

It has been suggested by a number of authors that the slopes of the J and δ resistance curves are related, for example, Willoughby et al (1981):

$$\frac{dJ}{da} = m' \sigma_y \frac{d\delta}{da} \quad 2.48$$

The results of Lautridou and Pineau (1981) and Willoughby et al (1981) suggest that the values of m' and m (which relates J/σ_y to δ , see equation 2.28) are similar for bend and compact specimens. However, using the HRR field equations, Shih (1981) suggested that m' and m are related by:

$$m' = \left(\frac{N+1}{N} \right) m$$

where $\epsilon \alpha \sigma^N$

2.4.2.4 Effect of Specimen Size on Crack Growth Resistance

A number of authors, for example Garwood et al (1978), have suggested that the overall crack growth resistance in non-sidegrooved specimens arises from two components. One is that due to the shear lip which is present at the edges of the specimens, and the other is that due to the flat fracture region in the centre of the specimen. Garwood (1976) extended the Krafft, Sullivan and Boyle (1961) model

to allow the overall J resistance curve to be determined. This in turn was later modified slightly by Green and Knott (1975) to become:

$$J = J_f \frac{(B - 2S)}{B} + E_v \frac{2S^2}{B} \quad 2.49$$

J_f is the J value, at a given crack extension, from the J-R curve for the flat fracture region; S, as illustrated in Figure 2.9, is the width of the shear lip and E_v is the energy absorbed per unit volume of shear lip. Garwood (1976) and Willoughby (1979) reported that E_v was independent of the amount of crack extension. The overall slope of the δ -R curve is also likely to comprise shear lip and flat fracture contributions, as it is related to the slope of the J-R curve, see equation 2.48.

Many studies have been carried out into the effect of specimen size on crack growth resistance. However, it is rarely possible to determine whether the reported effects of specimen size, if any, arise due to: a break down of J or δ controlled crack growth; the definition chosen for "J", the method chosen for the measurement of δ ; or a change in the intrinsic toughness of the material. Changes in the latter could arise due to variations in the crack tip constraint or the proportion of shear lip on the fracture surface. In most of the studies reviewed in this and the following section, the maximum amount of crack extension considered was relatively small, i.e. less than about 15% of the original ligament size.

Generally, it is reported that decreasing the thickness of specimens of a constant width results in a decrease in the slope of the J resistance curve dJ/da , see for example Read and Reed (1977) and Garwood and Turner (1977). Willoughby (1979) found a similar effect of specimen thickness on the slope of the δ resistance curve, $d\delta/da$. Garwood and Turner (1977) suggested that the decrease in dJ/da is due to a decrease in the proportion of shear lip on the fracture surface with increasing specimen thickness. However, some authors report finding little or no effect of thickness on dJ/da , see for example Marandet et al (1981).

Usually, it is reported that increasing the specimen thickness and width proportionally does not alter the dJ/da value, see for example Garwood (1976), Berger et al (1979) and Royer et al (1979). Garwood (1976) suggested this was because the proportion of shear lip on the fracture surface remained the same with increasing specimen dimensions. However, Shoji (1981) reported that both dJ/da and $d\delta/da$ decreased with increasing specimen dimensions.

Often it is reported that increasing the specimen width but keeping the thickness constant, results in an increase in dJ/da , see for example Garwood et al (1978) and Marandet et al (1981). Garwood (1976) suggested that this is due to an increase in the proportion of shear lip on the fracture surface with increasing specimen width. However, Vassilaros et al (1980) and Carlsson et al (1981) reported no effect of specimen width on dJ/da , while Etemad (1983) reported that dJ/da decreased with increasing specimen width.

It is generally reported that dJ/da is the same for a wide range of initial crack lengths to width (a/w) ratios, see for example Joyce and Gudas (1979) and Vassilaros et al (1980). However, at high initial a/w values, dJ/da would appear to increase with increasing a/w , see for example Joyce and Gudas (1979). In addition, there would appear to be an effect at low initial a/w values, as Chipperfield (1978a) showed that $d\delta/da$ increased with decreasing initial a/w .

Sidegrooving the specimen flanks is generally found to decrease dJ/da , see for example Garwood and Turner (1977) and Clarke (1980). Green and Willoughby (1980) reported a similar effect of sidegrooving on $d\delta/da$. However, Loss (1981) reported that dJ/da was not affected by shallow sidegrooves, while some authors report that there is no effect of sidegroove depth above a certain depth, see Garwood and Turner (1977) and Gudas et al (1979).

As illustrated above, the effects of specimen size on dJ/da and $d\delta/da$ are in the main unclear and so few conclusions can be made on the size requirements for plane strain crack tip conditions during crack growth, as well as those to maintain J and δ controlled

crack growth; although some suggested criteria, for compact and three-point bend specimens are given, in terms of J , in the draft ASTM J-R procedure by Albrecht et al (1982):

$$\Delta a < 0.1 b_0$$

$$B_N, b > \frac{20J}{\sigma_f}$$

2.4.2.5 Effect of Specimen Geometry on Crack Growth Resistance

In general, dJ/da is reported to be the same for the essentially deep-notched bending geometries, such as three and four-point bend and compact, see for example Royer et al (1979) and Carlsson et al (1981). However, Gudas and Davis (1982) did report higher dJ/da values for three-point bend compared to compact specimens. Chipperfield et al (1973) reported similar $d\delta/da$ values for four-point bend and compact specimens, but a higher value for three-point bend specimens. However, Green and Willoughby (1980) reported that when the small differences in the rotation factor between the three and four-point bend geometries are taken into account, $d\delta/da$ is the same for the two geometries.

It is generally reported that in non-sidegrooved specimens, the dJ/da value for the centre-cracked tension geometry is considerably larger than that for the essentially deep-notched bending geometries, see for example Begley and Landes (1976), Garwood (1979) and Sampath et al (1981). Garwood (1979) also reported a similar geometry dependence on $d\delta/da$ and suggested that the effect of geometry was due to its effect on the proportion of shear lip. Shih et al (1981) tested single-edge-notched specimens loaded in tension over the remaining ligament, so as to simulate the behaviour of centre-cracked tension specimens. They reported that the dJ/da value for these specimens was significantly higher than for compact specimens, which they explained in terms of the ligament sizes being insufficient in the single-edge-notched tension specimens to obtain J controlled crack growth. In sidegrooved specimens, Willoughby (1979) reported similar dJ/da values for both centre-cracked tension and three-point bend specimens, whereas Garwood (1982) reported that dJ/da and $d\delta/da$ values were slightly higher for the sidegrooved

centre-cracked tension specimens.

Shih et al (1981) reported that $d\delta/da$ for double-edge-notched tension specimens, although dependent on a/w , was always at least slightly larger than that for compact specimens.

In summary, it would appear that both dJ/da and $d\delta/da$ show a distinct geometry dependence. No conclusive explanation for this dependence has been proposed, although it has been suggested that it could be due to a corresponding dependence on the proportion of shear lip on the fracture surface or the criteria for J and δ controlled crack growth being significantly more stringent for certain geometries, for example the centre-cracked tension geometry.

2.4.3 Near Field Parameters

2.4.3.1 Introduction

One of the potential advantages of the near field parameters over the far field parameters is that they are likely to be capable of characterising the crack growth resistance of a material over a wider range of conditions. However, they have two serious disadvantages compared to far field parameters. One is that they are more difficult to measure and the other is that they are more difficult to use in a defect assessment procedure.

2.4.3.2 Crack Tip Opening Displacement (CTOD)

Little experimental investigation of CTOD has been carried out. Studies by Garwood (1976), in which CTOD was measured from silicone-rubber impressions of the crack tip, and Willoughby et al (1981), in which CTOD was measured from metallographic sections of the crack tip, showed that CTOD remains constant with increasing crack extension. However, no study of the effect of specimen size and geometry on CTOD has been carried out and therefore it is unclear whether CTOD is a suitable fracture parameter.

2.4.3.3 Crack Tip Opening Angle (CTOA)

A number of finite element and experimental investigations of CTOA have been carried out. Experimental results obtained by Willoughby et al (1981) showed that CTOA increases to a plateau

value with increasing crack extension, whereas finite element calculations show that CTOA decreases to a plateau value with increasing crack extension, see for example Shih et al (1979). The difference in the two trends is probably because the finite element analyses do not consider the voids which link to the crack tip and which can, at small crack extensions, affect significantly the crack profile and thus the CTOA values.

Willoughby et al (1981) showed some results to suggest that the plateau value of CTOA is related directly to $d\delta/da$, which would suggest that the effect of specimen size and geometry on $d\delta/da$ and the plateau value of CTOA is the same. However, Abou-Sayed et al (1981) found that the plateau value of CTOA was not related to $d\delta/da$ at large crack extensions. In addition, they reported that CTOA was independent of specimen size, although so was $d\delta/da$ at small crack extensions. Therefore, as a number of other authors have reported that $d\delta/da$ is dependent on specimen size and geometry, see sections 2.4.2.4 and 2.4.2.5, so may CTOA.

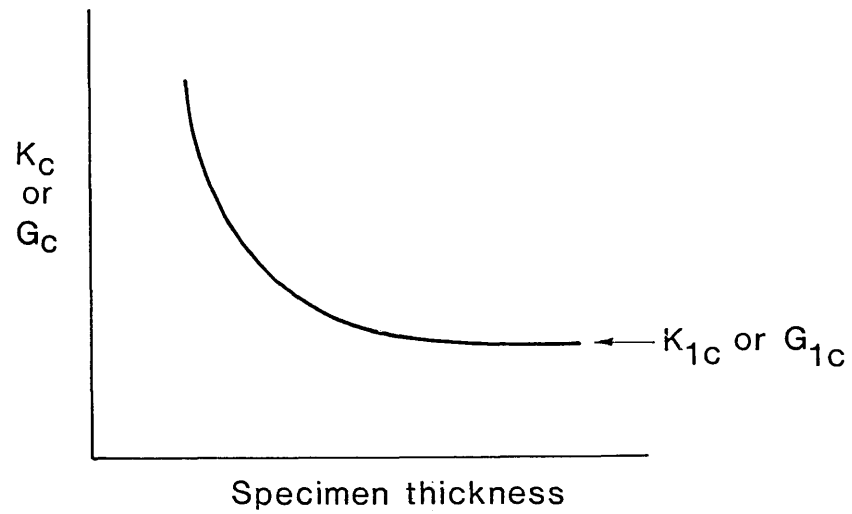


Figure 2.1 Schematic plot showing the effect of specimen thickness on the fracture toughness

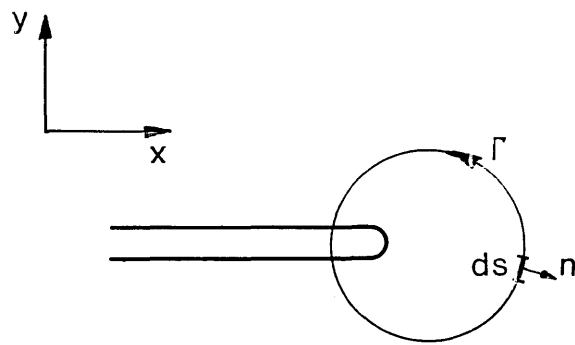


Figure 2.2 Definition of the J integral

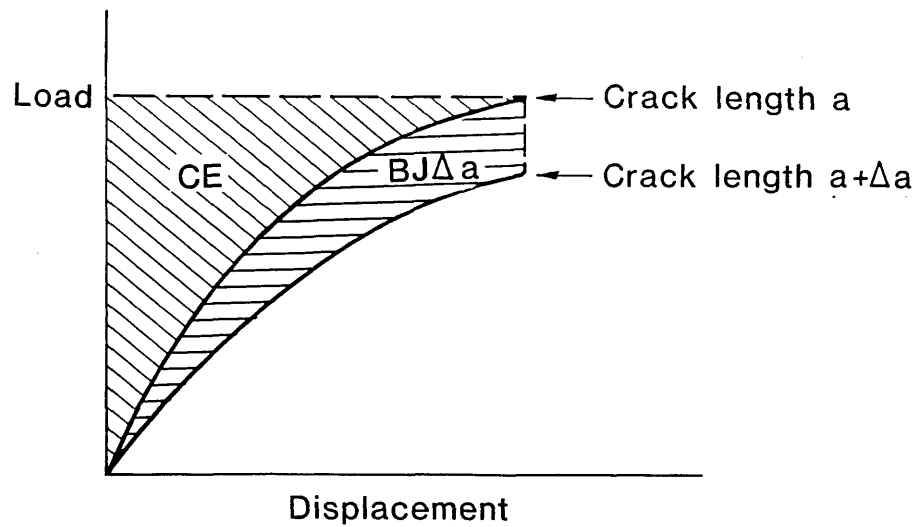


Figure 2.3 Load versus displacement plots for two nominally identical specimens with different crack lengths

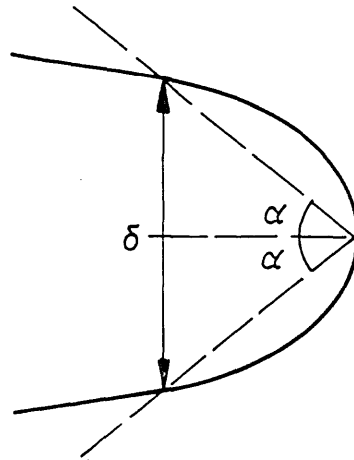


Figure 2.4 Schematic representation of a crack tip, illustrating a definition of δ

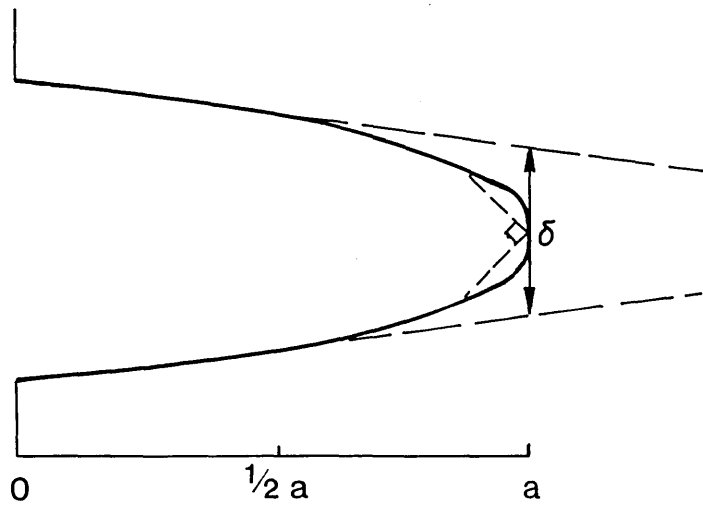


Figure 2.5 Schematic representation of a crack, illustrating an extrapolation procedure used to determine δ

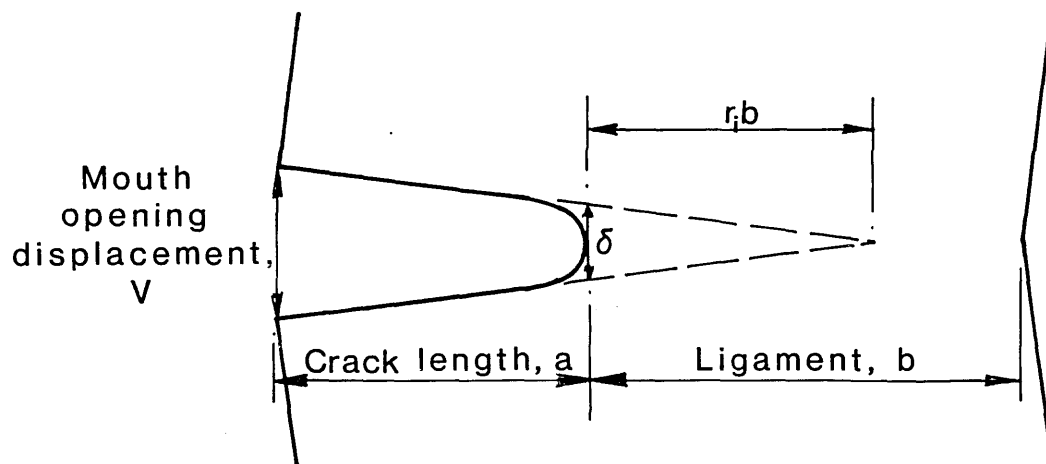


Figure 2.6 Hinge point construction used to determine δ

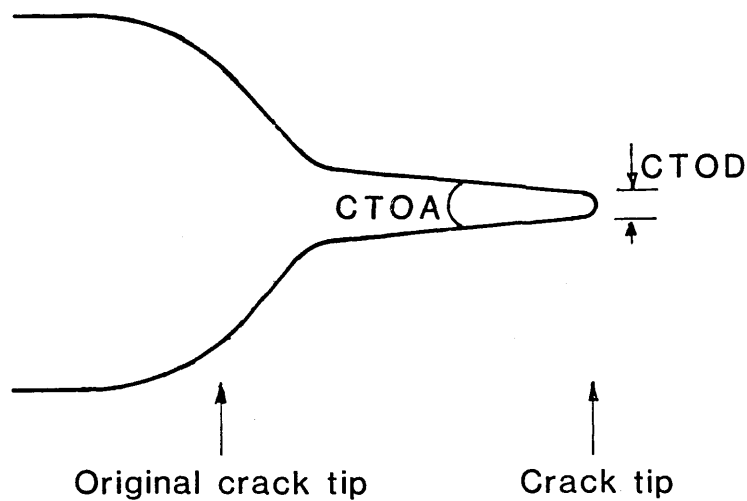


Figure 2.7 Schematic representation of a crack profile, illustrating various fracture parameters

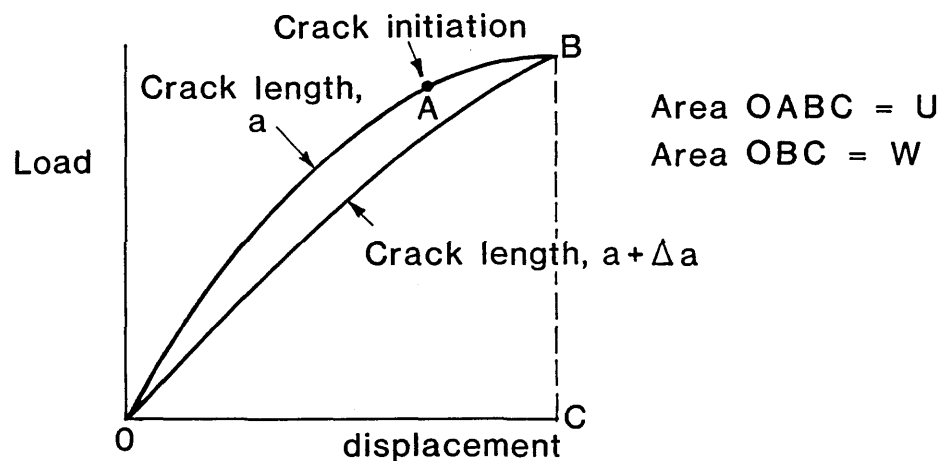


Figure 2.8 Loading curve for specimen with initial crack length, a and the hypothetical loading curve for a specimen with crack length, $a + \Delta a$

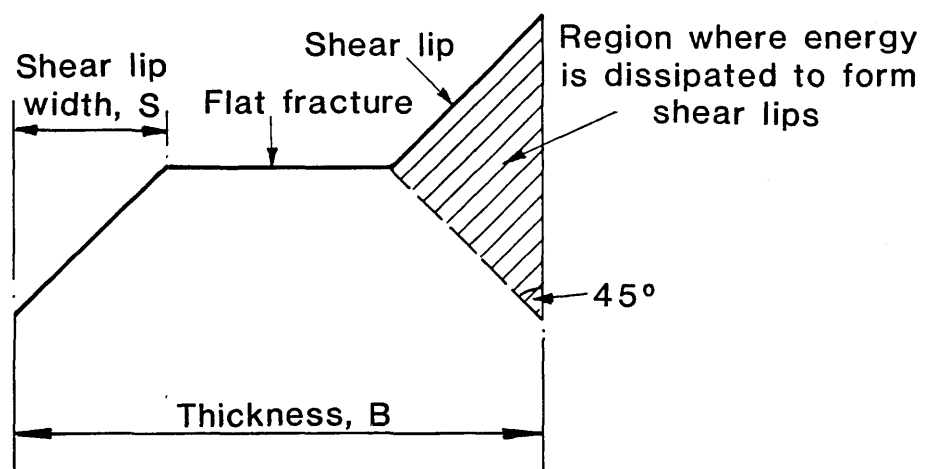


Figure 2.9 Section through the crack plane showing shear lip

Chapter 3

Experimental Details

3.1 Materials

The material used in this study was a normalised C-Mn steel, BS4360 43A. It was supplied in the form of a 2" thick rolled plate.

The chemical composition of the plate is given in Table 3.1. The sulphur content of this steel is relatively high and there are numerous inclusions in any given metallographic section, for example Plate 3.1. As can be seen from Plate 3.1, the inclusions are elongated in the rolling direction. An investigation was carried out into the variation of the carbon and sulphur content within the plate. This revealed that there was a significant variation; the extreme values found are given in Table 3.1. The implication of this variation on the variability of the upper shelf toughness is discussed in Chapter 5. Table 3.2 gives the average tensile properties of the steel in the rolling direction.

All the specimens were orientated so that the rolling direction was normal to the crack plane and the crack direction was in the width direction of the plate. Thus, the principal axis direction of the elongated inclusions was normal to the crack plane.

3.2 Specimen Design

The effect of specimen size and sidegroove depth on the ductile crack initiation and crack growth resistance was studied using compact geometry specimens. The specimen design, shown in Figure 3.1, was based on that given in the K_{IC} testing standard BS5447. The crack mouth region of the specimen was modified to allow direct measurement of the load line. In the specimens which were sidegrooved, a 60° V shaped notch was used; the sidegroove depth being expressed as a percentage:

$$\% \text{ sidegrooving} = \frac{B_N - B_g}{B_N} \times 100$$

Table 3.3 gives the thickness and width dimensions together with the sidegroove depth of the specimens tested in this study.

To investigate the effect of geometry on the ductile crack initiation and crack growth resistance, the following geometries were investigated: three and four-point bend, centre-cracked tension (CCT) and double edge-notched tension (DEN). Figures 3.2, 3.3 and 3.4 give the specimen design and dimensions for the various geometries.

A multispecimen approach was used to determine the crack initiation and crack growth resistance of the material, and so for each specimen size and geometry combination, 6-8 specimens were manufactured.

3.3 Specimen Pre-cracking

The specimen fatigue pre-cracking was carried out under load control, at room temperature, using a servohydraulic machine. The final crack length to width ratio aimed for was 0.55, although the actual ratio varied between 0.5-0.6.

In the case of compact and bend specimens the pre-cracking was carried out in accordance with the ASTM J_{1C} testing standard, E813-81, in which for the final 50% of the fatigue pre-crack extension, the maximum load was less than 0.4 of the limit load.

Prior to fatiguing the CCT specimens, spark machine slots (0.5mm wide) were cut from either side of the central hole to give a nominal $2a/w \approx 0.25$. This enabled the fatigue crack to initiate more easily.

When pre-cracking the CCT and DEN specimens under tensile loading, the fatigue crack length was found to be uneven across the crack front. To overcome this, pre-cracking was carried out from one side at a time, as illustrated in Figure 3.5, under four-point bending. The inner rollers were positioned so that the slight indentation of the specimen by the rollers was well away from the crack region. The maximum pre-cracking load was constant and such

that if the specimen was uncracked, the maximum stress on the upper specimen surface would be half the yield stress.

For the compact specimens the pre-cracking was automated using a crack propagation monitor unit (Servotest, type 945), and a non-linear programmer (Servotest, type 758). Briefly, the system operates as follows. A D.C. current is passed through the specimen to be pre-cracked and a "dummy" specimen. The voltage across the crack is measured from both specimens and the ratio of the two is determined. The "dummy" specimen is used to take into account changes in temperature and current during pre-cracking. The change in the ratio can be related to the change in crack length during pre-cracking from a previously determined calibration. When the non-linear programmer detects a certain change in the ratio, the pre-cracking loads are adjusted to those appropriate to the new crack length. For the other geometries, the crack length was measured optically from the specimen surface and the loads adjusted manually where appropriate.

3.4 Displacement Measurements

During the test, the load and one or two displacements, depending on the specimen geometry, were plotted on X-Y plotters. For the compact geometry only the load versus load-line displacement was recorded. The load-line displacement was measured by a linear variable differential transformer (LVDT) using a device shown in Plate 3.2. The arms of the device were located to the V machined notches on the load-line by a small ball bearing on each arm. A small level on the top of the device allowed the arms to be set up horizontally.

For the three and four-point bend specimens the load versus the ram displacement and the mouth opening displacement were measured. The mouth opening displacement was measured by a clip gauge mounted between knife edges. These knife edges were screwed to the top face of the specimens.

For the CCT and DEN specimens, the load versus the ram displacement and the mouth opening displacements were measured.

The mouth opening displacement on both sides of the specimen was measured by LVDT's attached by mountings to the specimen, although only the sum of the two mouth opening displacements was plotted. Plate 3.3 shows the LVDT mountings used for both geometries, although in Plate 3.3 the mountings are shown attached to a DEN specimen. The space between the LVDT's and the specimen allowed room for a heating tape. The mountings were screwed into the specimen to a depth of about 5mm, above and below the crack mouth, at a separation of 20mm for the CCT specimens and 40mm for the DEN specimens.

The LVDT's were regularly calibrated using one or two drum micrometers, while the clip gauge was calibrated using a robust clamp, the movement of which was monitored by a previously calibrated LVDT.

3.5 Testing Procedure

The specimens were loaded by one of a range of servohydraulic machines, depending on specimen size and geometry, whose maximum load capacity varied between 50-600kN. All the tests were conducted under displacement control at a constant ram speed of 1mm/min.

Generally, the test temperature was 30°C, although at this temperature it was found that the four-point bend, 50mm thick sidegrooved compact and the DEN specimens exhibited cleavage pop-in behaviour. A higher test temperature of between 35-45°C was used with these specimens to avoid pop-in. The implication of a slight change in testing temperature on the ductile crack initiation and crack growth resistance is discussed in Chapter 5. The specimens were heated by one or more heating tapes to within $\pm 2^\circ\text{C}$ of the required temperature which was measured by a thermocouple placed close to the crack tip.

After the specimens had been loaded to the required displacement, they were unloaded, removed from the machine, cooled in liquid nitrogen and then rapidly reloaded to promote failure by a cleavage micromechanism.

The amount of ductile crack extension, both from crack tip blunting and fibrous crack growth was measured from the fracture surface. For small crack extensions ($<0.5\text{mm}$) the measurements were made using a scanning electron microscope, SEM. With larger crack extensions the measurements were generally made using a travelling optical microscope, TOM, which was found sufficiently accurate at these larger crack extensions (i.e. within 2%). A nine point average crack extension was calculated from the measured crack extensions at nine evenly spaced positions across the crack front, in accordance with the ASTM J_{1C} testing procedure E813-81. The specimen dimensions and the nine point average initial crack length were measured using a TOM.

A significant proportion of the specimen testing, although no silicone-rubber crack infiltrations, was carried out by Dr S G Druce, see Table 3.3. In addition, he measured the nine point average crack extensions using a TOM and determined J values for these specimens which were reported by Druce (1981). The J integral values for these specimens were recalculated for this study to take into account recent suggested modifications to the J integral, see Chapter 4. In addition, in cases where the crack extension was small, ($<0.5\text{mm}$) it was remeasured using an SEM.

3.6 Stretch Zone Width Measurements

When the specimen is initially loaded, the crack tip blunts as illustrated in Figure 3.6, the amount of crack advance due to blunting being the stretch zone width, SZW.

It has been suggested, see for example Kobayashi et al (1979), that J at the onset of fibrous crack growth, J_i , can be calculated from SZW measurements using a procedure which involves two steps. The first involves the determination of the relationship between J and SZW prior to crack initiation, which is described as the material's blunting line. The second involves the measurement of the SZW corresponding to that at crack initiation, SZW_c . This can be measured from the fracture surface after crack initiation, see Chapter 5. The SZW_c is substituted into the blunting line relationship to give J_i .

The stretch zone is visible in the SEM and lies between the end of the fatigue region and the start of the cleavage or fibrous crack growth region. A typical example is given in Plate 3.4. Initially, a considerable time was spent tilting and rotating the specimen into various positions to show the stretch zone region more clearly. However, with sufficient experience the stretch zone was easily located without the need for additional tilting and rotating of the specimen.

The SZW was measured from photographs taken when the incident electron beam was normal to the fatigue crack plane, i.e. 0° tilt position. The magnification of the photographs was determined from a photograph of a mesh of known size, taken at the same magnification setting and focusing lens current. The average SZW for each specimen was determined from about 10 readings taken at various positions along the crack front in the central 2/3 of the specimen.

3.7 Silicone-Rubber Infiltration

3.7.1 Experimental Set-up

The silicone-rubber used to obtain crack tip replicas is a Kerr product called citricon. It has good dimensional stability, better than 0.001% shrinkage/hr, so that as the silicone-rubber replicas were photographed within 2 days, the dimensional change over this period was negligible. The setting time of this silicone-rubber was relatively fast and was adjusted to about 10 minutes by careful control of the proportion of hardener which was added to the base fluid. This gave sufficient time to get the fluid to the crack tip, but kept to a minimum any risk of ductile crack growth whilst the fluid set.

The procedure used to obtain silicone-rubber crack impressions was essentially the same for each geometry and was as follows. The specimen was loaded to the required displacement and then held at a constant ram displacement. The mouth opening or load-line displacement measuring device was removed. Then, the fluid was forced into the crack tip region using a syringe or plunger, the other openings to the crack tip region having been previously sealed

off to contain the fluid. The fluid had to be forced to the crack tip because of its relatively high viscosity. Once the fluid had set, after about another 3 minutes, the specimen was unloaded, cooled in liquid nitrogen and then broken apart by rapid loading. Generally, the silicone-rubber cast remained intact to one of the specimen halves. The cast was then removed and sectioned at 6 to 10 positions at various measured distances (X) from one specimen edge. The sections were then mounted on SEM stubs, coated with a thin layer of gold and photographed in the SEM.

The following sections describe in a little more detail the procedure used to force the silicone-rubber fluid to the crack tip for the various specimen geometries studied.

For the three and four-point bend specimens, the edges of the crack were sealed with adhesive tape prior to loading. The fluid was injected into the crack from a syringe and was forced towards the crack tip by using a wedge shaped piece of rubber. This procedure was found sufficient to get the fluid to the fatigue crack tip. However, full penetration to the fibrous crack tip was often not achieved, particularly when the crack extension was large ($>1\text{mm}$).

To increase the penetration of the fluid for the other geometries, a metal syringe and clamp, shown in Plate 3.5, was manufactured, which allowed a greater pressure to be applied to the fluid. The handle of the syringe could be unscrewed to allow fluid to be placed into the barrel of the syringe. There were rubber blocks on the syringe face and back plate which allowed a seal to be made when the syringe was clamped to the specimen. Various shapes and sizes of rubber blocks were used, depending on the specimen size and geometry in question.

For the CCT specimens the syringe was clamped over the crack mouth; the rubber blocks on the syringe face and the back plate were large enough to seal the crack region. For the DEN specimens the syringe was clamped over one of the crack mouths, for with this geometry silicone-rubber casts were only obtained for one crack. In addition, the sides of the crack were sealed off by

rubber blocks, backed by steel, which were held in place by clamps. The same procedure was used for the 13mm thick compact geometry specimens except that the crack sides were sealed off by metal shims which made a tight fit between the specimen and the clevis. For specimens which were sidegrooved, the sidegrooves were filled with rubber which was held in place by adhesive tape.

The approach used for the 13mm thick compact specimens was not found successful for the 50mm thick compact specimens, probably because of their larger size. So, to carry out infiltration on these larger specimens, they were first unloaded and the crack region up to the starter notch was filled with rubber. In addition, if the specimen was sidegrooved the sidegrooves were also filled. The syringe was clamped to the side of the crack so that the fluid could be fed into the fatigue crack region.

3.7.2 Measurement of Crack Opening Displacement

The crack opening displacement (δ) is defined in this study as the opening at the end of the original (or fatigue) crack, as illustrated in Figure 3.7. The fatigue region, as shown in Plate 3.6, was revealed as fine ripples on the top and bottom surfaces of the silicone-rubber casts. Thus, by observing the fine ripples, the end of the fatigue crack was detected. This allowed δ to be measured from the photographs of the sectioned casts, taken in the SEM when the incident electron beam was normal to the sectioned plane, (i.e. 0° tilt position).

The amount of crack extension, corresponding to the same distance from the specimen side (X) as that where the casts were sectioned, was measured from the fracture surface using a SEM.

3.7.3 Measurement of Crack Tip Opening Angle

The crack tip opening angle is the angle of the crack flanks close to the crack tip, as illustrated in Figure 3.7.

From the photograph of the crack profile, the crack face separation (h) was measured at 30-70 evenly spaced positions between the fibrous and fatigue crack tips at known distances (r) from the

fibrous crack tip. It was found, as schematically illustrated in Figure 3.8, that there was a 'r' range over which the gradient dh/dr was on average constant. Therefore, the gradient dh/dr close to the crack tip, from which CTOA should be calculated, could be determined over a much larger and thus more experimentally reasonable 'r' range. The CTOA was determined from this gradient (dh/dr) using equation 3.1:

$$CTOA = 2 \times \tan^{-1} \left(\frac{1}{2} \frac{dh}{dr} \right) \quad 3.1$$

3.7.4 Measurement of Crack Tip Opening Displacement

The crack opening displacement at the fibrous crack tip (CTOD) could in principle be measured from the silicone-rubber crack tip profiles, although there are a number of reasons against this: firstly, the silicone-rubber did not always penetrate to the crack tip; indeed, it was sometimes difficult to determine, particularly by eye, whether there was complete crack tip penetration, and secondly, when full penetration appeared to be achieved it was often not clear, as illustrated in Plate 3.7, at what position CTOD should be measured. Therefore, direct measurement of CTOD from the silicone-rubber crack profiles was not considered viable. However, it was possible to estimate the CTOD, as illustrated in Figure 3.8, by linear extrapolation of the measured crack profile to the fibrous crack tip, in the linear range where the gradient dh/dr was approximately constant. The CTOD was given by:

$$CTOD = h_{r=\Delta a} - \frac{dh}{dr} \times \Delta a \quad 3.2$$

$h_{r=\Delta a}$ is the intercept value at the fatigue crack tip, see Figure 3.8. CTOD could be estimated using equation 3.2 even when full crack tip penetration was not achieved, provided there was a reasonably sized linear region. There was some support for the accuracy of the estimated CTOD values, using equation 3.2, as they were found to be in good agreement with the CTOD values which could be directly measured from the crack profiles.

CTOD was also directly measured from the crack tip profiles, obtained after unloading of the specimen, by metallographic sectioning. The sections were taken parallel to the crack growth direction and perpendicular to the crack plane.

3.8 Measurement of Plastic Work in Sidegrooved Specimens

A sidegrooved specimen, as illustrated in Figure 3.9, can be considered as comprising two regions, a plane sided region of thickness B_N , and two specimen flanks. An investigation was carried out to determine the percentage of the total plastic work occurring in the plane sided region, as a function of sidegroove depth, using a procedure described below.

One half of a fracture surface was sectioned, as illustrated in Figure 3.10, at various distances ahead of the crack tip. A number of hardness traverses were taken across each section, with a greater concentration near to the crack plane. The hardness measurements were translated into equivalent plastic strains, using a calibration plot obtained by measuring the hardness of tensile specimens which had been prestrained to various levels of equivalent plastic strain. The plastic work per unit volume associated with each strain was obtained by measuring the area under the true stress versus true plastic strain tensile curve as a function of plastic strain. Each sectioned plane was divided into small regions, and from the hardness measurements a value of the plastic work per unit volume was allocated to each region. The average plastic work per unit depth, in each region, was given by multiplying the plastic work per unit volume by the area of the region. The total plastic work per unit depth, in the flanks and in the plane sided region of the specimen, were then given by summing up the individual values of the plastic work per unit depth associated with each region of the specimen. From this, the percentage of the total plastic work per unit depth in the plane sided region of the specimen was determined. This percentage was found to be the same for each section through the specimen, within experimental scatter, even though the total plastic work per unit depth was varying. Therefore, the percentage of the total plastic work in the plane sided region can be

considered as the average percentage of the total plastic work per unit depth, in the plane sided region.

Table 3.1
Chemical Composition in wt%

C	S	Mn	Si	P	Ni	Al	Cu
0.197-0.289	0.047-0.071	1.05	0.04	0.03	0.15	<0.01	0.34

Mo	Ti	V
0.03	<0.01	<0.01

Table 3.2
Tensile Properties

Lower yield stress σ_y = 283 MPa

Upper yield stress = 286 MPa

Tensile stress σ_t = 495 MPa

Flow stress = $1/2(\sigma_y + \sigma_t)$ = 389 MPa

Table 3.3
Compact Geometry specimen dimensions

Thickness mm	Width mm	% sidegrooving	Comments
13	26	0,10,25,50,75	
13	50,75,100	0	+
25	50	0,10,25,50	+
37.5	75,100	0	+
50	100	0	+
50	100	10,25,50	

⁺Tested by Dr S G Druce



Plate 3.1 Optical micrograph taken normal to the thickness direction of the plate

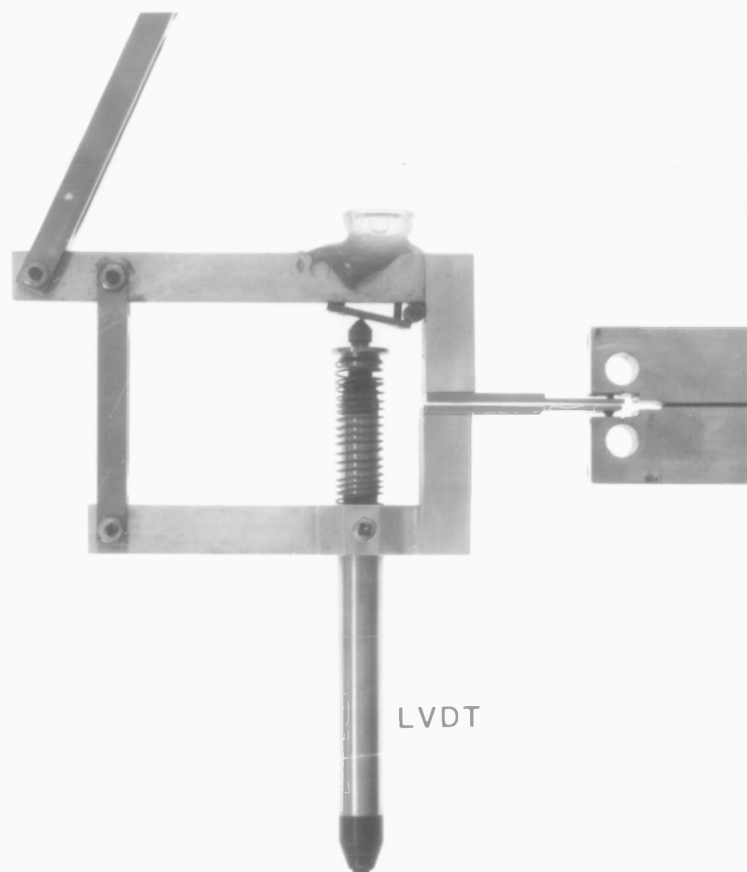


Plate 3.2 Apparatus used to allow the load-line displacement of compact specimens to be measured using an LVDT

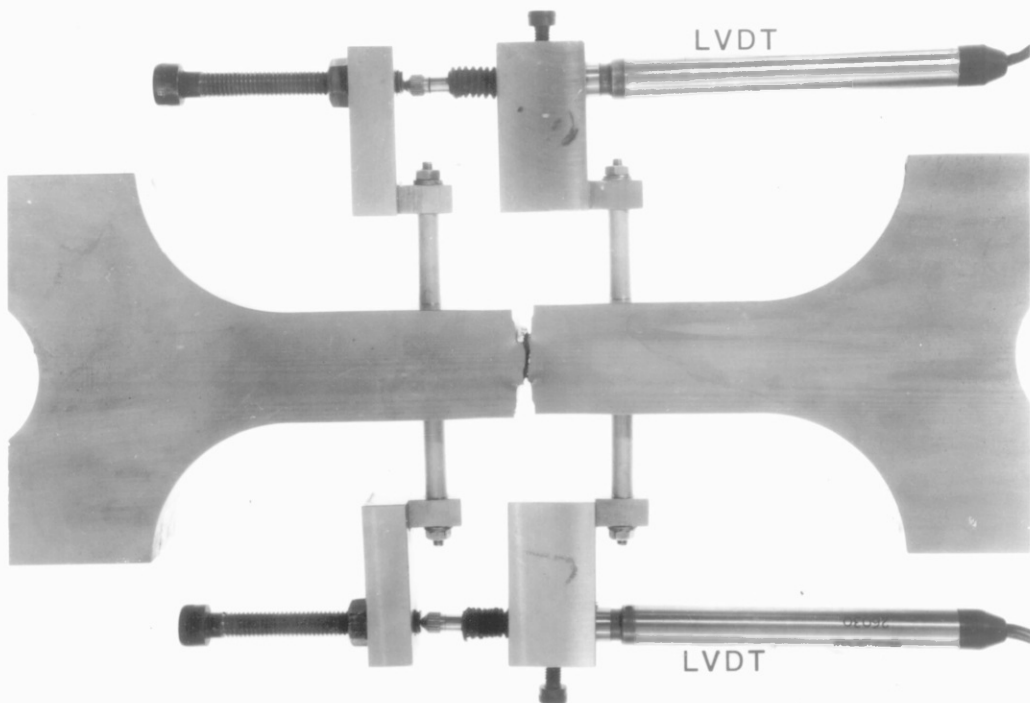


Plate 3.3 Apparatus used to allow the mouth opening displacement to be measured using two LVDTs

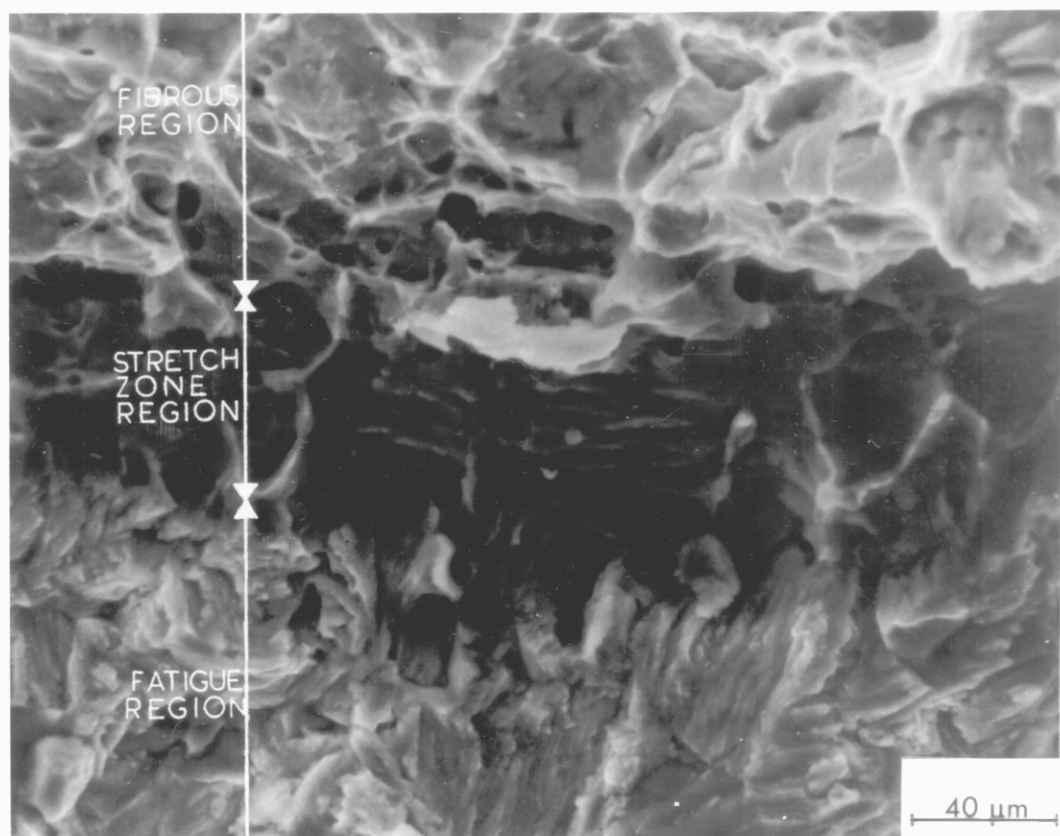


Plate 3.4 Scanning electron fractograph showing the stretch zone region

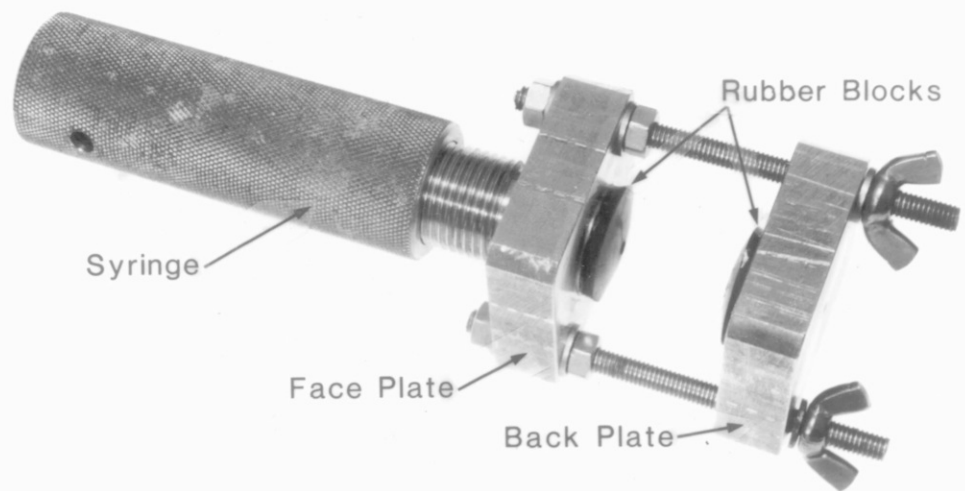


Plate 3.5 Apparatus used for silicone-rubber infiltration

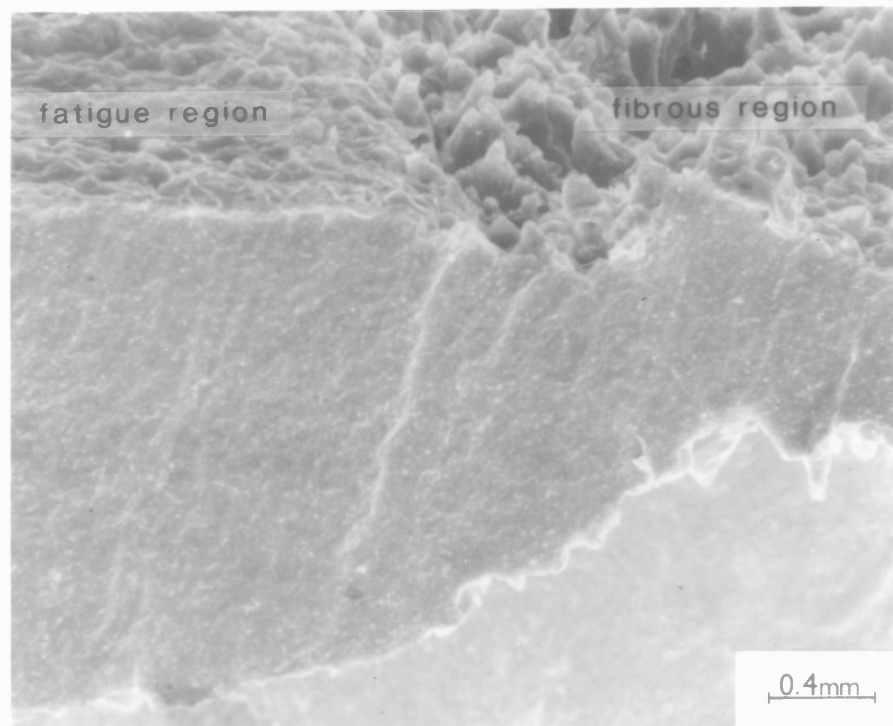


Plate 3.6 A sectioned silicone-rubber cast, showing fine ripples on the top surface associated with the fatigue region

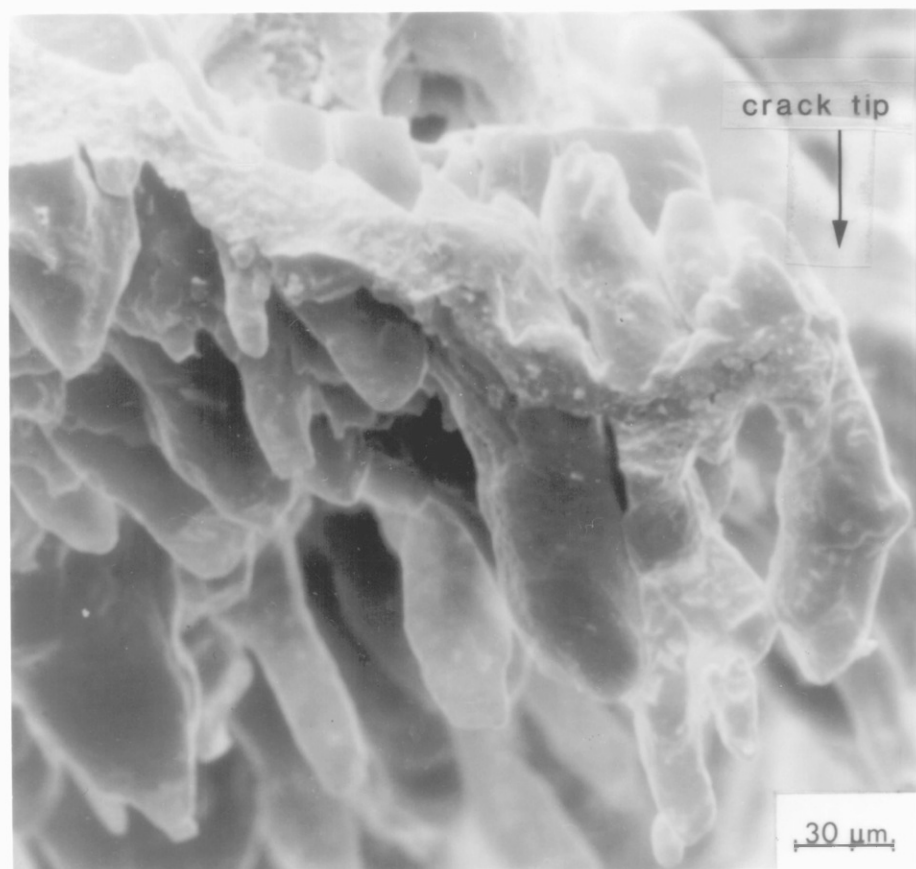


Plate 3.7 A sectioned silicone-rubber cast, illustrating the difficulty of measuring CTOD

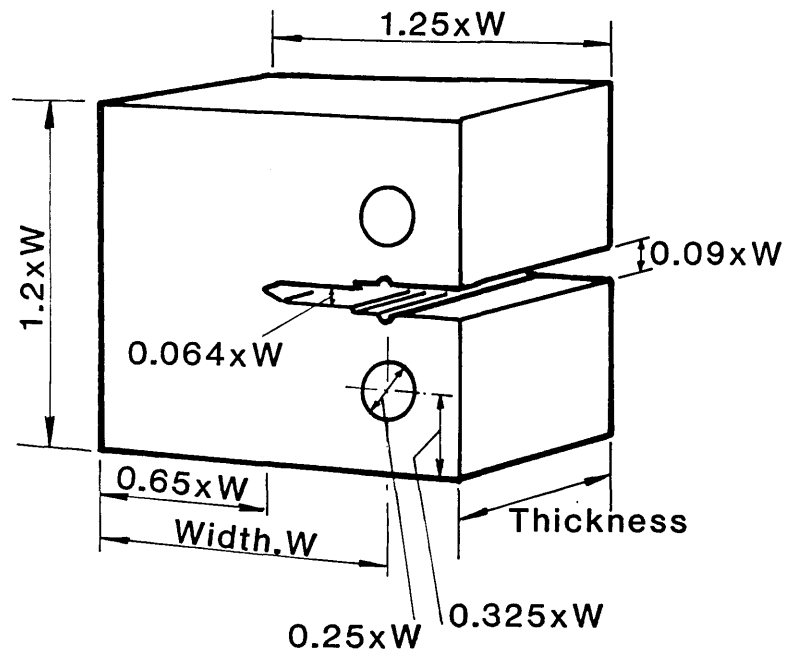
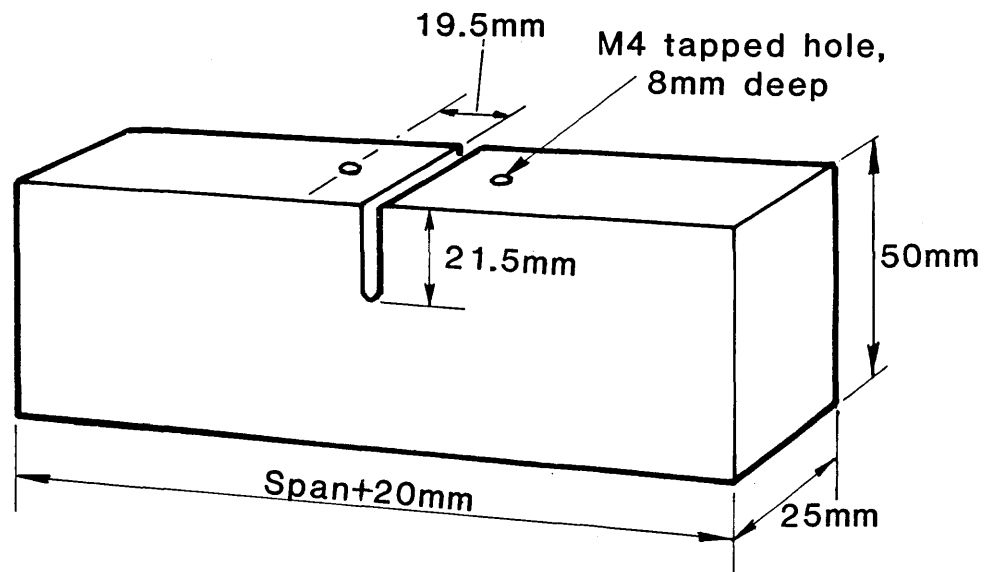


Figure 3.1 Design of compact specimens



Three-point bend Span=200mm
 Four-point bend Total span=280mm,inner span=150mm

Figure 3.2 Design of three and four-point bend specimens

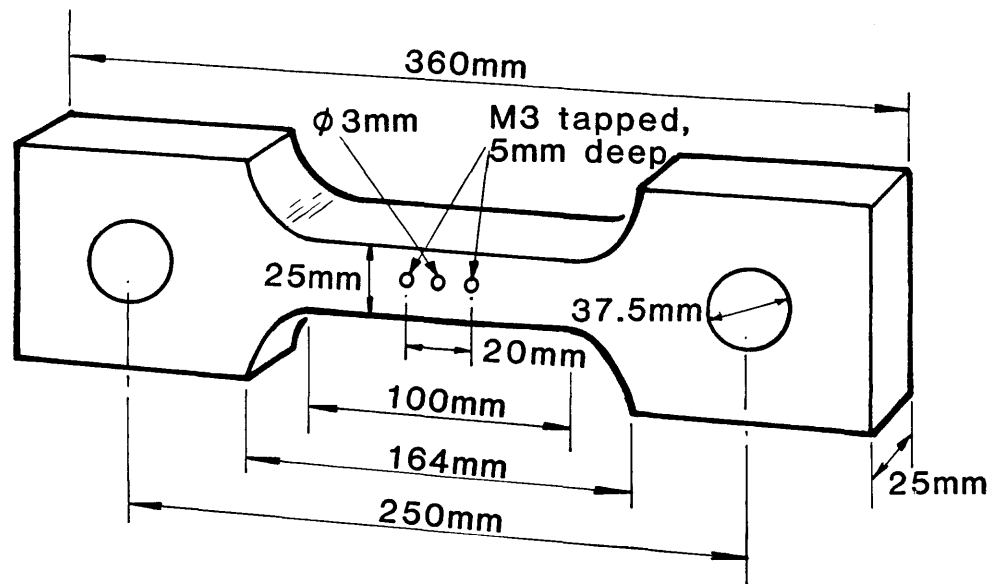


Figure 3.3 Design of centre-cracked tension specimens

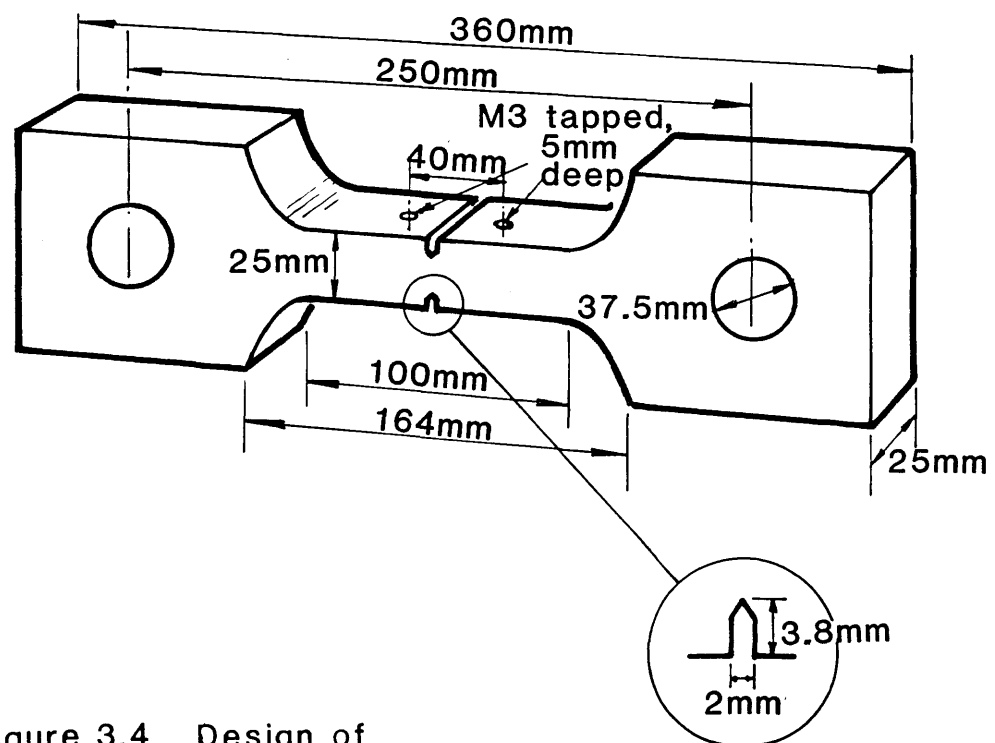


Figure 3.4 Design of double-edge-notched tension specimens

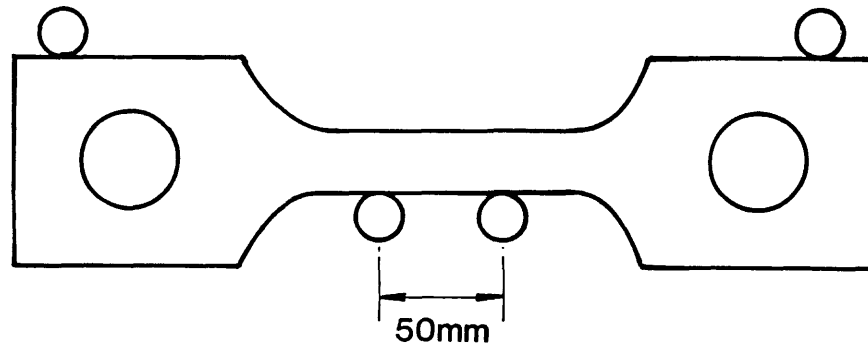


Figure 3.5 Illustration showing position of rollers during pre-cracking of centre-cracked and double-edge-notched tension specimens

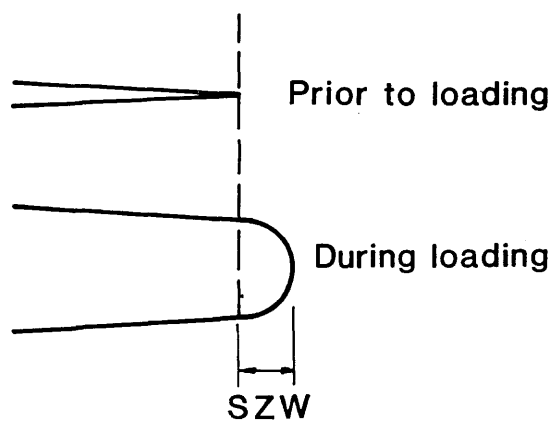


Figure 3.6 Illustration of crack tip blunting

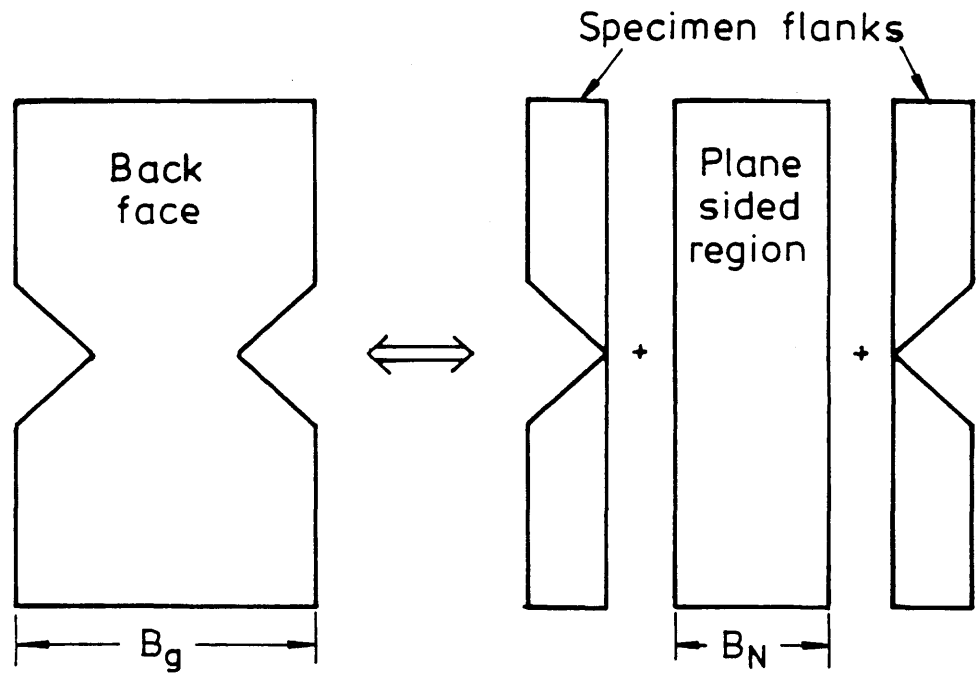


Figure 3.9 Illustration showing the component parts of a sidegrooved specimen

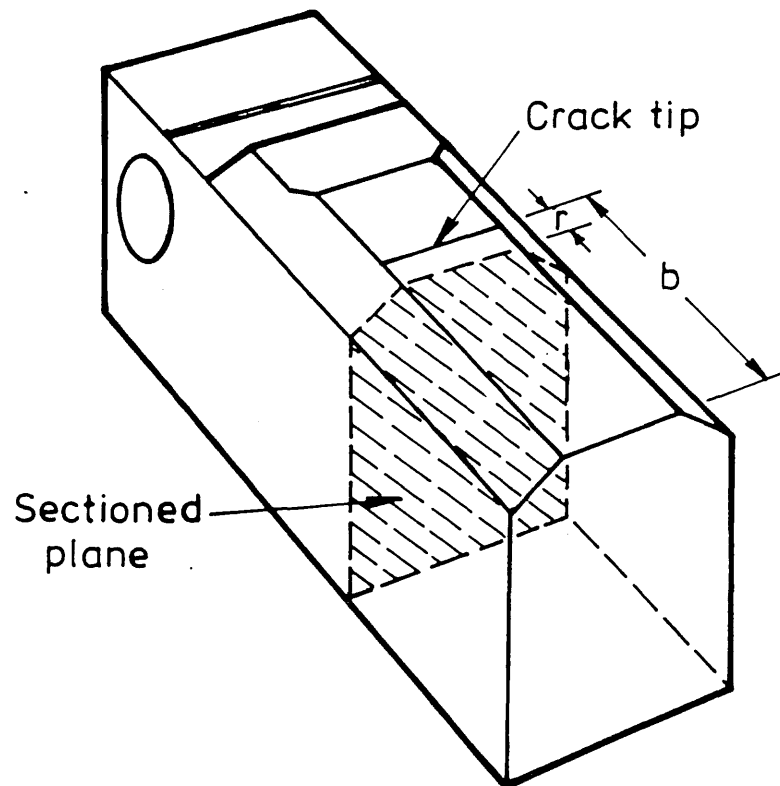


Figure 3.10 Illustration showing the orientation of the sections taken through the specimen

Chapter 4

Experimental Determination of the J Integral

4.1 Introduction

In this chapter the procedures used to determine J are described for the various specimen geometries investigated in this study. It is split into two main sections: the first considers the three and four-point bend, and compact geometries; the second considers the centre-cracked and double-edge-notched tension geometries. In each main section there are two subsections: the first considers the determination of J prior to any crack extension, and in the case of the compact geometry includes a study of the effect of sidegrooving. The second subsection considers the determination of "J" after crack initiation for which three different "J" values are considered: J without any correction for crack growth, J_0 , J with a correction for crack growth, J_r and the Ernst suggested modification to J, J_m , see section 2.4.2.1.

4.2 Determination of J, prior to crack initiation, for Bend and Compact Geometries

4.2.1 Introduction

J can be determined from its elastic and plastic components G and J_p , which are given by equations 2.8 and 2.23 respectively:

$$G = \frac{\eta_e W_e}{Bb} \quad 2.8$$

$$J_p = \frac{\eta_p W_p}{Bb} \quad 2.23$$

W_e and W_p are the elastic and plastic components of the internal absorbed energy, which prior to crack initiation can be determined, as illustrated in Figure 4.1, from the load versus displacement trace. In the case of the compact specimens the load versus load-line displacement trace was used, whereas for the three and four-point bend specimens the measured load versus ram displacement trace, corrected for extraneous displacement, was used. The extraneous displacement arises from deformation of the loading jig and the machine, and indentation of the specimen by the loading pins, and can be determined by loading an uncracked specimen with

the loading pins positioned to minimise any bending of the specimen.

If $\eta_e = \eta_p$, then J can be determined from the total internal absorbed energy using equation 2.17:

$$J = \frac{\eta W}{Bb} \quad 2.17$$

In the following sections the η_e , η_p and η values are determined for each geometry and these values were used with equations 2.8 and 2.23, or if appropriate, equation 2.17 to determine the J values for the bend and compact specimens.

4.2.2 Three and Four-point Bend Specimens

4.2.2.1 Elastic η Factors, η_e

Equation 2.11 was used to determine the η_e values for the three and four-point bend specimens used in this study. Table 4.1 gives the values which were determined using the LEFM Y values given by Haigh and Richards (1974), the unnotched compliance C_0 which included contributions from both bending and shear deformation, and the elastic properties data, given in Table 4.2. It can be seen from Table 4.1 that over the a/w range of interest in this study (0.5-0.65), $\eta_e \approx 2$ for the three-point bend specimens, while $\eta_e \approx 1.4$ for the four-point bend specimens.

4.2.2.2 Plastic η Factors, η_p

η_p can be determined from equation 2.25 if perfectly-plastic conditions are assumed. Haigh and Richards (1974) reported the limit load (Q_L) for various specimen geometries in terms of the yield load of the uncracked specimen under plane stress conditions (Q_0) and a factor M :

$$Q_L = MQ_0 \quad 4.1$$

As Q_0 is not a function of the crack length, equation 2.25 can be written as:

$$\eta_p = \frac{-b}{M} \frac{dM}{da} \quad 4.2$$

Haigh and Richards (1974) reported the M values for three and four-point bend specimens as:

$$M = A(1 - a/w)^2 \quad 4.3$$

where

$A = 1.26$ for four-point bend

$A = 1.22$ for three-point bend.

The η_p value for both geometries, obtained by substituting equation 4.3 into 4.2 is:

$$\eta_p = 2 \quad 4.4$$

Comparison of the η_p value and the η_e values given in Table 4.1 shows that $\eta_e \approx \eta_p$ for the three-point bend geometry but not for the four-point bend geometry. So, for the three-point bend geometry, J was calculated from equation 2.17, using $\eta=2$, see section 4.2.2.3, whereas for the four-point bend geometry, J was calculated from equations 2.8 and 2.23 using the η_e values given in Table 4.1 and the η_p value given by equation 4.4.

When correcting J for crack growth using the procedure described in section 4.3.2, the η value was required. In the case of the four-point bend geometry this was determined by substituting the measured J values without a correction for crack growth (see section 4.3.1) into equation 2.36.

4.2.2.3 η Factor for the Three-point Bend Geometry

Comparison of Table 4.1 and equation 4.4 shows that for the three-point bend geometry (with span/width ratio=4 and $a/w > 0.5$) the appropriate η value is:

$$\eta = 2.0 \quad 4.5$$

This is in agreement with that reported by Rice et al (1973) who undertook an analytical analysis of deep-notched bending. In addition, it is also in agreement with that determined from finite element studies, see for example Turner (1979) and experimental studies, see Chipperfield (1978b).

4.2.3 Compact Geometry

4.2.3.1 Introduction

A number of analytical analyses have been undertaken to determine J_p and thus η_p values for the compact geometry. The first was carried out by Merkle and Corten (1974) who adapted the

deep-notched bending relationships of Rice et al (1973) to take into account the different limit load relationships for the compact geometry compared to the pure bending case. The analysis showed that J_p was given by the sum of two terms: the second term, which was based on the complementary energy, can be ignored when $a/w > 0.5$, see Merkle and Corten (1974) and the first term was:

$$J_p = \frac{\eta_m W_p}{Bb} \quad 4.6$$

where η_m is the Merkle and Corten solution for η_p and is given by:

$$\eta_m = \frac{2(1 + \alpha)}{(1 + \alpha^2)} \quad 4.7$$

where

$$\alpha = \left(\frac{4a^2}{b^2} + \frac{4a}{b} + 2 \right)^{1/2} - \left(\frac{2a}{b} + 1 \right) \quad 4.8$$

Ernst (1981) showed that the Merkle and Corten (1974) solution for η_p , although giving the correct limiting value for pure bending (i.e. as $a/w \rightarrow 1$), gave the wrong limiting value for pure tension as given by Rice et al (1973). This suggests that in the a/w range of interest for this study (0.5-0.7) the η_p values given by equation 4.7 may be slightly inaccurate. Ernst (1981) then carried out an analytical analysis in which the overall deformation behaviour of the compact specimen was split into bending and tensile components, which were considered separately. The analysis showed that J_p was again given by the sum of two terms: the second term, which was based on the complementary energy can be ignored for $a/w > 0.5$, leaving J_p as:

$$J_p = \frac{\eta_E W_p}{Bb} \quad 4.9$$

where η_E is the Ernst solution for η_p and is given by:

$$\eta_E = \left(\frac{2(1+\alpha)(1+a/w) + \alpha(1-a/w)}{(1+\alpha^2)} \right) \times \left(1 + \frac{a}{w} + \alpha(1 - \frac{a}{w}) \right)^{-1} \quad 4.10$$

α is given by equation 4.8.

Table 4.3 gives the η_m values (from equation 4.7) and the η_E values (from equation 4.10) for a range of a/w values. Comparison of η_m and η_E shows that they differ by only 1-3% over

the a/w range of interest in this study (i.e. 0.5-0.7) and so the possible error in using either solution for η_p is small.

The elastic η values for the compact geometry were determined from the relationship between the elastic compliance and the crack length using equation 2.10. Table 4.4 gives the η_e values for a range of a/w values, which were determined from the elastic compliance relationships of Saxena and Hudak (1978). A comparison of Tables 4.3 and 4.4 shows that η_p (from either analytical solution) is about 7-10% lower than η_e . So, when the plastic component of J is larger than the elastic component, (i.e, when $J_p > G$) it is reasonably accurate to calculate J from the total absorbed work using equation 2.17 and with η given by one of the two analytical solutions for η_p . This approach was adopted for the ASTM J_{1C} testing standard E813-81, which uses the Merkle and Corten (η_m) for η .

When determining G , J_p or J for sidegrooved specimens, using equations 2.8, 2.23 and 2.17 respectively, the denominator term in all these equations, which is the uncracked ligament area, will be given by $B_N b$, where B_N is the distance between the sidegrooves (i.e. the net thickness). However, little investigation of the effect of sidegroove depth on the η_e and η_p values has been carried out, although in the ASTM J_{1C} testing standard E813-81 it is suggested that for up to a sidegroove depth of 25% the Merkle and Corten solution for η_p can be used for η .

In the following section an experimental investigation is described which was carried out to determine which of the two analytical solutions for η_p is the most accurate for non-sidegrooved specimens, and the effect of sidegroove depths up to 75% on η_e and η_p values.

4.2.3.2 Experimental details

Compact specimens were manufactured with various sidegroove depths: 0, 25, 50 and 75% (three in each category), from a second plate of BS 4360 43A steel. This plate had a slightly lower carbon content (≈ 0.15 wt%) and a significantly lower S content (≈ 0.013 wt%)

than the first plate from which the specimens used to evaluate the R curves were manufactured. The specimens with sidegroove depths of 0, 25 and 50% had a gross thickness = 25mm and a width = 50mm, while the specimens with a sidegroove depth of 75% had a gross thickness = 13mm and a width = 26mm. To reduce the effect of any material variability within the plate, all the specimens were manufactured from the centre of the plate and specimens with the same sidegroove depth came from the same region within the plate. For each batch of specimens with the same sidegroove depth, spark machine slots, approximately 1mm wide, were cut to various depths corresponding to nominal a/w values of 0.5, 0.55 and 0.6. Slots instead of fatigue cracks were used to retard the onset of crack extension during loading of the specimens. The specimens were loaded as described in section 3.5 to a load-line displacement which would have given rise to about 2.5mm of ductile crack extension if the specimens had been pre-fatigued; this displacement was dependent on the sidegroove depth. The 9 point average crack length (including the contribution due to blunting) was measured later from the fracture surface.

4.2.3.3 Elastic η Factor, η_e

As η_e can be determined from the elastic compliance using equation 2.10, the effect of sidegroove depth on the compliance was investigated to evaluate the effect on η_e .

The elastic compliance for the various a/w values and sidegroove depths was determined from the initial linear portion of the load versus load-line displacement traces. These cannot be directly compared to those given by Saxena and Hudak (1978) (used to determine η_e values in Table 4.4) to evaluate the effect of sidegroove depth, as they reported normalised values $ExCxB$. To allow a comparison, the Young's Modulus E was determined from the measured compliance values of the non-sidegrooved specimens and the normalised compliance values of Saxena and Hudak for the three different a/w values. The average E value determined was 201 GPa. Table 4.5 gives the ratio of the measured compliance to that given by Saxena and Hudak, using this value for E and taking B as the gross thickness, for the various a/w values and sidegroove depths. It can be seen

from Table 4.5 that, within experimental scatter, the compliance ratio, although dependent on sidegroove depth, is independent of a/w , at least over the range investigated of 0.5-0.6. Figure 4.2 shows a plot of the average compliance ratio versus sidegroove depth.

Shih et al (1977) have studied the effect of sidegrooving up to a depth of 50% on the elastic compliance, although in their case the angle of the sidegroove notches was 45° compared to the 60° used in this study. They showed that the compliance was increased by a factor which was dependent on the sidegroove depth but not the a/w ; the compliance ratio was given by:

$$\text{Compliance ratio} = \frac{B_G}{B_E} \quad 4.11$$

where

$$B_E = B_G - \frac{(B_G - B_N)^2}{B_G} \quad 4.12$$

The suggested relationship between compliance ratio and sidegroove depth given by equation 4.11 is shown in Figure 4.2, together with compliance ratios determined by Shih et al (1977), from a finite element analysis. From Figure 4.2, it can be seen that there is good agreement between the relationship given by equation 4.11 and the results obtained in this study, and those of Shih et al. It would be expected that the compliance ratios determined in this study for a 60° notch angle would be higher than those given by Shih et al for a 45° notch angle, as more material is removed to make a 60° notch; however, this is not clear from the figure.

As the increase in compliance with increasing sidegroove depth appears to be independent of the crack length, the η_e values (calculated from equation 2.10) will be the same as those given in Table 4.4 over the range of sidegroove depths investigated (i.e. up to 75%).

4.2.3.4 Plastic η Factor, η_p

To determine η_p values, the measured load versus load-line displacement traces for the various specimens were converted to load

versus plastic load-line displacement traces by subtracting the elastic contribution. Then the plastic absorbed energy W_p was obtained by measuring the area under the trace up to various plastic displacements. A few of the specimens showed small amounts of fibrous crack extension (up to $1/4$ mm), so in these cases W_p measurements were only taken up to a plastic displacement of about 0.4mm less than the final value, at which point no fibrous crack extension was likely to have occurred. From the W_p values for the three different a/w values (a_1/w , a_2/w , a_3/w), the η_p values were calculated using equation 2.24 for the two midrange a/w values between a_1/w and a_2/w , and a_2/w and a_3/w .

The η_p values, for each a/w value and sidegroove depth, were found to be independent, within experimental scatter, of the level of plastic displacement. As an example, Figure 4.3 shows a plot of η_p versus plastic load-line displacement for two a/w values determined from specimens with a sidegroove depth of 50%.

Table 4.6 gives the average η_p values determined from non-sidegrooved specimens, together with the η_m values (from equation 4.7) and the η_E values (from equation 4.10). It would appear from the table that the analytical solution for η_p by Ernst (1981) is more accurate than that by Merkle and Corten (1974). As η_p values determined experimentally are often subject to significant scatter (e.g. 5-10%, see for example Chipperfield (1978b)), this result would not be conclusive if it had not been additionally supported by the finite element results of de Lorenzi and Shih (1983). They found that at large plastic displacements when $\eta \approx \eta_p$, the J value calculated from equation 2.17, taking η_m as η overestimated the actual J value, calculated from equation 2.13, by about 2% for a specimen with an $a/w = 0.6$, which is consistent with the fact that η_m is about 2% higher than η_E when $a/w = 0.6$, see Table 4.3. Therefore, the Ernst solution was used to determine η_p values for the non-sidegrooved specimens.

Table 4.7 gives the average η_p values determined over the measured plastic displacement ranges and the η_p/η_E ratio, for the various a/w values and sidegroove depths. From the table it can be

seen that, within experimental scatter, the η_p/η_E ratio is independent of a/w over the range investigated, i.e. 0.53-0.59. Figure 4.4 shows a plot of the average η_p/η_E ratio versus sidegroove depth. This shows that η_p is only affected significantly by sidegrooving above a sidegroove depth of 25%. The finite element results of de Lorenzi and Shih (1983) give some support for the results given in Figure 4.4. They showed that for a specimen sidegrooved to a depth of 25% and with an $a/w = 0.6$, the J value calculated from equation 2.17, taking η_m as η , equals the actual J value calculated from equation 2.13, which, as η_m is 2% higher than η_E when $a/w = 0.6$, suggests that sidegrooving to a depth of 25% elevates η_p by 2%. This is in good agreement with that given in Figure 4.4. Therefore, η_p was determined for sidegrooved specimens from the η_E value and the η_p/η_E ratio for the relevant sidegroove depth given in Figure 4.4.

4.2.3.5 Determination of J

In this study, the plastic component of J was generally larger than the elastic component and so η_p values given in section 4.2.3.4 were used for η in equation 2.17 to calculate J (see section 4.2.3.1). However, when the material's blunting line was determined (see section 3.6), the elastic component of these low J values was significant; in these cases J was determined from the sum of its elastic and plastic components, which were in turn determined from equations 2.8 and 2.23 respectively.

4.3 Determination of J, after crack initiation, for Bend and Compact Geometries

4.3.1 J without any Crack Growth Correction, J_0

J_0 is the J value determined without any correction for crack growth, see section 2.4.2.1. It was determined for the three-point bend and compact specimens from the area under the load versus displacement trace U , using equation 2.36:

$$J_0 = \frac{\eta_0 U}{Bb_0} \quad 2.36$$

η_0 and b_0 are the initial values of η and b respectively. For the four-point bend specimens J_0 was calculated from elastic and plastic components of U (U_e and U_p) using equation 4.13:

$$J_o = \frac{\eta_{eo} U_e}{Bb_o} + \frac{\eta_{po} U_p}{Bb_o} \quad 4.13$$

η_{eo} and η_{po} are the initial values of η_e and η_p respectively.

4.3.2 J corrected for Crack Growth, J_r

It has been suggested that after crack initiation, J_r rather than J_o is the appropriate "J" to use to characterise the crack growth resistance of a material, see section 2.4.2.1. J_r can be determined from the area under the load versus displacement trace, using a "crack growth correction" equation, of which there are two types. The first is the "incremental" type in which J_r is determined knowing the previous determined J_r value associated with a smaller amount of crack extension. An equation of this type is used in the ASTM J_{1c} testing standard E813-81 to determine J_r . Incremental type equations are very suitable when the crack growth resistance of the material is determined using a single specimen approach. However, they are not suitable with a multispecimen approach, used in this study, because of problems of material inhomogeneity and variations in the initial crack length of the specimens. The second type of equation is the "single point correction" type, in which J_r is determined without reference to the previous determined J_r value. An equation of this type, derived below, was used to determine the J_r values in this study. Differentiation of equation 2.17 with respect to crack length, gives:

$$\frac{dJ}{da} = \frac{\eta}{Bb} \frac{dW}{da} + \frac{J}{b} f(\eta) \quad 4.14$$

where

$$f(\eta) = 1 + \frac{b}{\eta} \frac{d\eta}{da} \quad 4.15$$

The subscript r on J has been dropped for convenience. A substitution of equation 2.37 into 4.14 gives:

$$\frac{dJ}{da} = \frac{\eta}{Bb} \frac{dU}{da} + \frac{J}{b} g(\eta) \quad 4.16$$

where

$$g(\eta) = f(\eta) - \eta \quad 4.17$$

noting that

$$J = J_i + \frac{dJ}{da} \bigg|_i \Delta a \quad 4.18$$

The subscript i refers to the value at the onset of crack initiation. Substitution of equations 2.17 and 4.16 into 4.18 gives:

$$J = \frac{\eta_o}{b_o} \left(\frac{W_i + dU}{B} \right) + \frac{\Delta a}{b_o} J_i g(\eta) \quad 4.19$$

noting that up to crack initiation $W=U$, the first term on the right hand side of equation 4.19 is equal to J_o , see equation 2.36. Therefore:

$$J = J_o + J_i \frac{\Delta a}{b_o} g(\eta) \quad 4.20$$

Taking as an approximation to J_i , the lowest measured J value associated with fibrous crack extension, J_{min} , equation 4.20 becomes:

$$J = J_o + J_{min} \frac{\Delta a}{b_o} g(\eta) \quad 4.21$$

As the amount of crack extension in any of the bend or compact specimens in this study is small, i.e. less than about 25% of the original ligament length, the second term on the right hand side of equation 4.21 is less than about 5% of J_o . Therefore, the choice of J_{min} does not affect significantly the value of J_r .

The single point correction type equations are less accurate than the incremental type, particularly at large crack extensions. To check the accuracy of equation 4.21, a number of comparisons of the J_r values from equation 4.21 and those determined using the incremental type equation given in the ASTM J_{1c} testing standard E813-81, were carried out. These showed that the differences in the two J_r values were less than 1% up to the maximum crack extensions investigated in this study.

4.3.3. Ernst modified J , J_m

Ernst (1983) suggested a parameter J_m , a modification of J_r , which satisfies the conditions given by Rice et al (1980) for a J like parameter to characterise the crack growth resistance of a material, see section 2.4.2.1. J_m can be determined from equation 2.45. Substituting equation 4.16 into equation 2.45 gives:

$$\frac{dJ_m}{da} = \frac{\eta}{Bb} \frac{dU}{da} + \frac{1}{b} \left(J g(\eta) - J_p g(\eta_p) \right) \quad 4.22$$

The subscript r has been dropped from J and J_p in equation 4.22 for convenience.

The second term on the right hand side of equation 4.22 was neglected for the bend and compact specimen used in this study, because after crack initiation it was found that $J \approx J_p$ and so:

$$\frac{dJ_m}{da} = \frac{\eta}{Bb} \frac{dU}{da} \quad 4.23$$

Equation 4.23 was integrated to give an "incremental" type equation, from which J_m was determined, i.e.

$$(J_m)_{i+1} = (J_m)_i + \left(\frac{\eta}{b} \right)_i \frac{((U)_{i+1} - (U)_i)}{B} \quad 4.24$$

where $()_i$ refers to the value after the "i" increment of crack extension and $()_{i+1}$ refers to the value after the "i+1" increment. If the initial crack length of the specimen associated with the crack extension increment i+1 (specimen A), differed from that of the specimen associated with the increment i (specimen B), to take this into account $(U)_{i+1}$ was calculated from:

$$(U)_{i+1} = J_o \text{ (for specimen A)} \times \frac{Bb_o}{\eta_o} \text{ (for specimen B).}$$

4.4 Determination of J, prior to crack initiation, for Centre-Cracked and Double-Edged-Notched Tension Geometries

4.4.1 Introduction

Rice et al (1973) suggest that for deeply cracked, centre-cracked tension (CCT) and double-edge-notched tension (DEN) geometries, the load and the plastic displacement are related by:

$$q_p = b f \left(\frac{Q}{Bb} \right) \quad 4.25$$

Using this relationship, Rice et al (1973) derived analytically a relationship for J_p :

$$J_p = \frac{(2W_p - Qq_p)}{Bb} \quad 4.26$$

J is then given by the sum of J_p and G (calculated from K).

The assumption involved with the relationship given by equation 4.25 is that the plastic constraint factor, L , is independent of crack length. Haigh and Richards (1974) suggested that this was the case for the CCT geometry and also for the DEN geometry, if either $2a/w > 0.884$ or plane stress conditions prevailed. However, under plane strain conditions, and if $2a/w < 0.884$, the suggested constraint factor for the DEN geometry was:

$$L = 1 + \ln \left(\frac{w - a}{w - 2a} \right) \quad 4.27$$

As the $2a/w$ values of interest in this study were less than 0.884, a more appropriate relationship between the load and the plastic displacement for the DEN geometry would be:

$$q_p = b f \left(\frac{Q}{LBb} \right) \quad 4.28$$

where L is given by equation 4.27.

The relationship for J_p , derived analytically in Appendix A, using equation 4.28, is:

$$J_p = \frac{W_p}{Bb} \left(2 - \frac{1}{L(2 - 2a/w)} \right) - \frac{Qq_p}{Bb} \quad 4.29$$

4.4.2 Load versus Plastic Displacement Traces

An experimental investigation was carried out to determine the applicability of equations 4.25 and 4.28 for the CCT and DEN specimens used in this study. Three specimens of each geometry were manufactured from the second plate of the BS 4360 43A steel. Spark machined slots, 1mm wide for the DEN specimens and 1.5mm wide for the CCT specimens, were cut to give crack lengths corresponding to nominal $2a/w$ values of 0.45, 0.5 and 0.55 for the CCT geometry and 0.5, 0.55 and 0.6 for the DEN geometry. The specimens were loaded up to a ram displacement of about 2.6mm, at which point no fibrous crack growth had occurred. The load versus plastic displacement traces were determined from the measured load versus ram displacement traces by subtracting the elastic contribution and the contribution arising from extraneous plastic deformation, due to for example, loading pin indentation. The extraneous plastic contribution was determined by loading an uncracked specimen. The 9 point average crack length of each specimen was later measured from the fracture

surfaces. Figures 4.5 and 4.6 give the normalised load, (Q/b for the CCT specimens and Q/bL for the DEN specimens) versus normalised plastic displacement (q_p/b) for the two geometries. The load was not normalised by the thickness, as this was the same for all the CCT and DEN specimens in this study. The final normalised load and plastic displacement values for the pre-fatigue cracked specimens are also plotted in Figures 4.5 and 4.6; these were determined as above from the final point on the measured load versus displacement traces, together with the final measured crack lengths.

It can be seen from Figures 4.5 and 4.6 that there is a single normalised curve, within experimental scatter, over the $2a/w$ range of interest (i.e. $0.45 < 2a/w < 0.75$), for each geometry. This suggests that the relationships given by equation 4.25 for the CCT geometry, and equation 4.28 for the DEN geometry are appropriate. A best fit line was drawn through the data points given in Figures 4.5 and 4.6.

4.4.3 Experimental Determination of J_p

As the relationship given by equation 4.25 appears to be appropriate for the CCT geometry, J_p can be calculated from equation 4.26, which can be rearranged to give:

$$\frac{J_p B}{b} = \frac{2W_p}{b^2} - \frac{Qq_p}{b^2} \quad 4.30$$

As there is a unique relationship between Q/b and q_p/b , see Figure 4.5, there is a corresponding unique relationship between W_p/b^2 and q_p/b , and therefore from equation 4.30, between $J_p B/b$ and q_p/b . Figure 4.7 gives the relationship between $J_p B/b$ and q_p/b determined from Figure 4.5. This figure was used to determine J_p at the end of the test, from the final plastic displacement and ligament size, and the specimen thickness.

In the case of the DEN geometry, the relationship given by equation 4.28 appears to be appropriate, and so J_p can be determined from equation 4.29, which can be rearranged to give:

$$\frac{J_p B}{Lb} = \left(\frac{2W_p}{Lb^2} - \frac{Qq_p}{Lb^2} \right) - \frac{W_p}{Lb^2} \times \frac{1}{(2-2a/w)} \quad 4.31$$

Using a similar argument to that above, it can be shown that for the DEN geometry there is a unique relationship between the first term on the right hand side of equation 4.31 and q_p/b , and also between W_p/Lb^2 and q_p/b . Figure 4.8 gives these two relationships, which were determined from Figure 4.6. This figure was used, together with equation 4.31, to determine J_p at the end of the test, from the final plastic displacement and ligament size, and the specimen thickness.

4.4.4 Experimental Determination of J

J was determined from the sum of J_p (calculated using the procedure described in section 4.4.3) and G. G was calculated from K using equation 2.7 (assuming plane strain conditions), which was in turn calculated from equation 2.4, using the final load and crack length, the specimen dimensions and the LEFM Y values given by Haigh and Richards (1974).

4.4.5 Mouth Opening Displacement Measurements

The mouth opening and ram displacements were measured for the CCT and DEN specimens, although only the latter was used in the determination of J. In the case of the DEN geometry, this was because the measurements of the mouth opening displacement obtained using LVDT's were inaccurate because of slight necking of the specimen where the LVDT's were attached. This problem did not occur with the CCT specimens, although the ram displacement was still used so as to maintain compatibility with the procedure used to determine J for the DEN specimens.

4.5 Determination of J, after Crack Initiation, for Centre-Cracked and Double-edge-notched Tension Geometries

4.5.1 J corrected for Crack Growth, J_r

The J values determined using the procedure described in section 4.4.4 are already corrected for crack growth, i.e. are J_r values. G was calculated from the final load and crack length, and the value of J_p calculated was that for a specimen with a crack length equal to the final crack length which has been loaded up to the final load, and plastic displacement values without any crack extension.

4.5.2 J without any Crack Growth Correction, J_o

J_o can be determined from its elastic and plastic components, which are in turn determined, assuming no crack extension had occurred, i.e.

$$J_o = G_o + J_{po} \quad 4.32$$

where

$$J_{po} = \frac{\eta_{po} U_p}{b_o B} \quad 4.33$$

G_o was calculated using the procedure described in section 4.4.4 from the final load and initial crack length. U_p is the area under the load versus plastic displacement trace. This trace was determined from the measured load versus ram displacement trace by subtracting the elastic and extraneous plastic contributions. η_{po} is the η_p value, see section 4.5.4, corresponding to the initial crack length and the normalised plastic displacement (q_p/b) value at crack initiation. The latter was determined from a plot of normalised plastic displacement versus crack extension by linear extrapolation to zero fibrous crack extension.

4.5.3 Ernst Modified J, J_m

The Ernst (1983) modified J , J_m is given by equation 2.45. This equation can be rearranged to give:

$$\frac{dJ_m}{da} = \frac{dG}{da} + \frac{dJ_p}{da} - \frac{J_p}{b} g(\eta_p) \quad 4.34$$

dJ_p/da is given by equation 4.35. This equation was obtained in a similar way to equation 4.16 for dJ/da :

$$\frac{dJ_p}{da} = \frac{\eta_p}{Bb} \frac{dU_p}{da} + \frac{J_p}{b} g(\eta_p) \quad 4.35$$

Substituting equation 4.35 into 4.34 gives:

$$\frac{dJ_m}{da} = \frac{dG}{da} + \frac{\eta_p}{Bb} \frac{dU_p}{da} \quad 4.36$$

Equation 4.36 was integrated to give an "incremental" type equation from which J_m was determined, i.e.

$$(J_m)_{i+1} = (J_m)_i + (G)_{i+1} - (G)_i + \left(\frac{\eta_p}{b} \right)_i \frac{((U_p)_{i+1} - (U_p)_i)}{B} \quad 4.37$$

G was calculated using the procedure described in section 4.4.4 from the final load and crack length values, and η_p was determined from the final crack length and plastic displacement values using the

procedure given in section 4.5.4. If the initial crack length of the specimen associated with the crack extension increment $i+1$ (specimen A), differed from that of the specimen associated with the increment i (specimen B), this was taken into account by calculating $(U_p)_{i+1}$ from:

$$(U_p)_{i+1} = J_{p0} (\text{specimen A}) \times \frac{B b_0}{\eta_{p0}} (\text{specimen B})$$

4.5.4 Plastic η Factor, η_p

J_p is given by equation 4.26 for the CCT geometry. This equation can be rearranged to give:

$$J_p = \frac{\zeta W_p}{Bb} \quad 4.38$$

where

$$\zeta = 2 - \frac{Qq_p}{W_p} \quad 4.39$$

By noting equation 2.23, η_p , for the CCT geometry, is:

$$\eta_p = \zeta \quad 4.40$$

Figure 4.9 shows a plot of ζ versus normalised plastic displacement, which was determined from Figure 4.5.

J_p is given by equation 4.29 for the DEN geometry. This equation can be rearranged to give:

$$J_p = \left(\zeta - \frac{1}{L(2^{-2a/w})} \right) \frac{W_p}{Bb} \quad 4.41$$

where ζ is given by equation 4.39.

By noting equation 2.23, η_p for the DEN geometry is:

$$\eta_p = \zeta - \frac{1}{L(2^{-2a/w})} \quad 4.42$$

The plot ζ versus normalised plastic displacement determined from

Figure 4.6 for the DEN geometry, is also given in Figure 4.9. From Figure 4.9 it can be seen that ζ is different for each geometry and that it is only slightly dependent on the level of plastic displacement for either geometry. Using this figure, η_p was determined for the relevant plastic displacement and crack length from equation 4.40 or 4.42 (depending on the geometry).

Table 4.1

Elastic η Factors for the Three and Four-point Bend Specimens used in this Study

a/w	0.50	0.55	0.60	0.65
η Three-point bend	1.96	2.00	2.02	1.99
η Four-point bend	1.29	1.39	1.48	1.54

Table 4.2

Elastic Properties

Young's Modulus E = 201 GPa, from section 4.2.3.3

Shear Modulus = 80 GPa

Poisson's Ratio ν = .3 from ASM (1978)

Table 4.3

η_m Values from equation 4.7 and η_E Values from equation 4.10 for a range of a/w values

a/w	0.5	0.55	0.60	0.65
η_m	2.265	2.239	2.213	2.186
η_E	2.200	2.190	2.176	2.160

Table 4.4

Elastic η factors, η_e for the compact geometry

a/w	0.50	0.55	0.60	0.65
η_e	2.50	2.40	2.34	2.30

Table 4.5

Ratio of the measured compliance to that given by Saxena and Hudak (1978) for various a/w values and sidegroove depths

a/w	Sidegroove depth %		
	25	50	75
0.50	1.04	1.31	1.93
0.55		1.30	1.94
0.56	1.04		
0.60		1.29	1.95
0.61	1.03		
Average value	1.04	1.30	1.94

Table 4.6

η_p values determined experimentally from non-sidegrooved compact specimens

η_m values from Equation 4.7 and η_E values from Equation 4.10 for two a/w values.

a/w	.52	.58
η_p	2.22	2.16
η_m	2.25	2.23
η_E	2.20	2.18

Table 4.7

η_p values determined experimentally and the η_p/η_E ratio, for various a/w values and sidegroove depths

a/w	sidegroove depth %	η_p	η_p/η_E
.53	25	2.20	1.00
.59	25	2.23	1.02
.53	50	2.37	1.08
.58	50	2.34	1.07
.53	75	2.50	1.14
.58	75	2.55	1.17

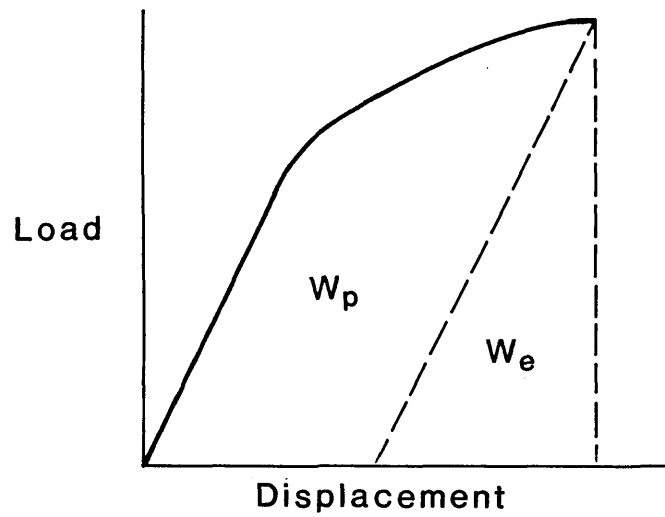


Figure 4.1 Elastic and plastic components of the internal absorbed energy , W

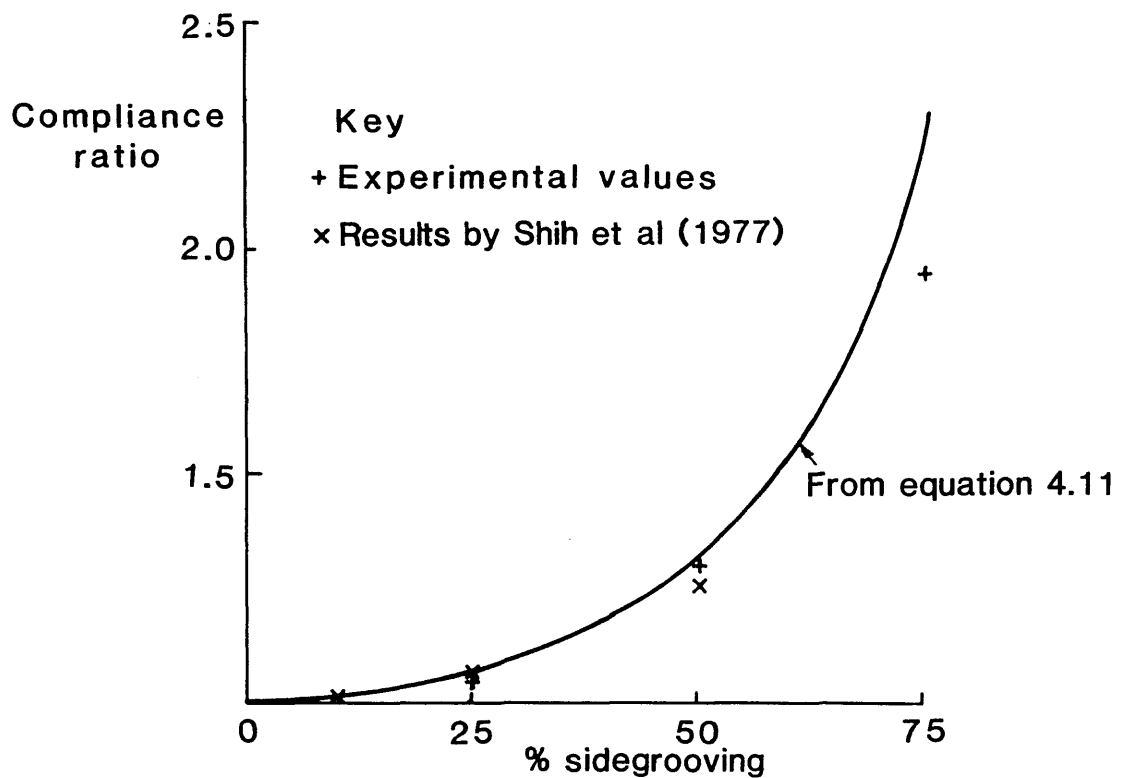


Figure 4.2 Effect of sidegroove depth on the elastic compliance of compact specimens relative to that for non-sidegrooved specimens

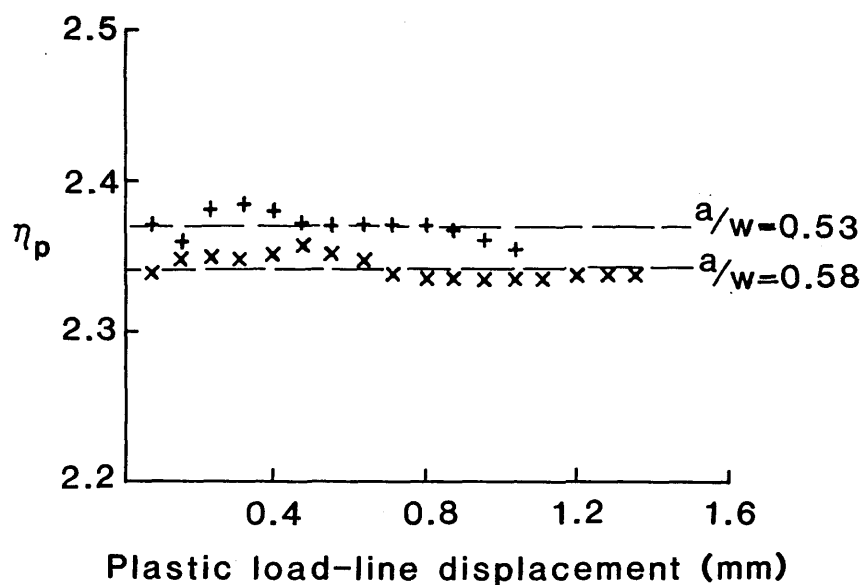


Figure 4.3 Effect of plastic load-line displacement on the η_p for compact specimens sidegrooved to a depth of 50%

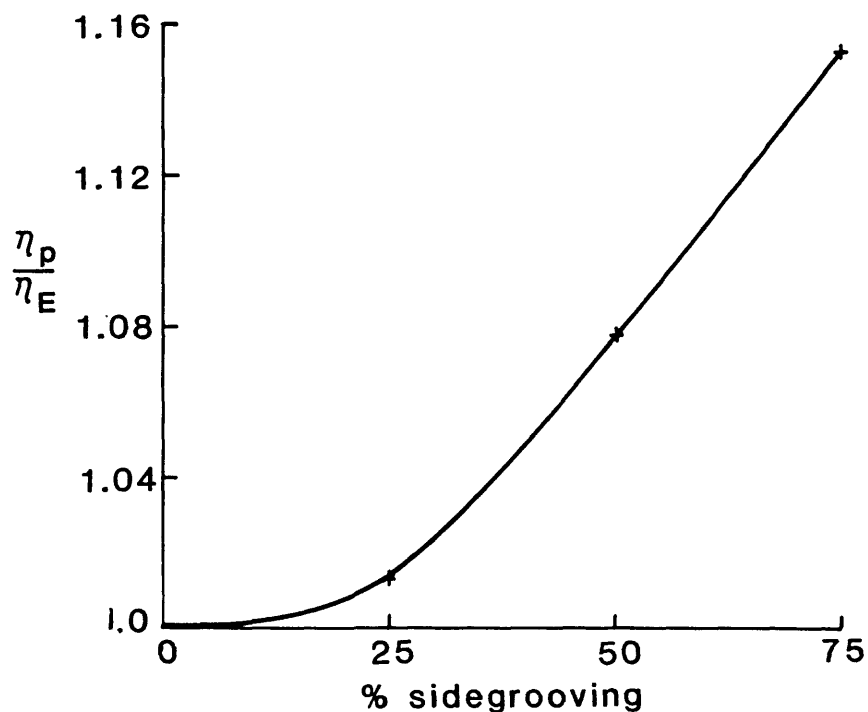


Figure 4.4 Effect of sidegroove depth on the η_p for compact specimens relative to η_E

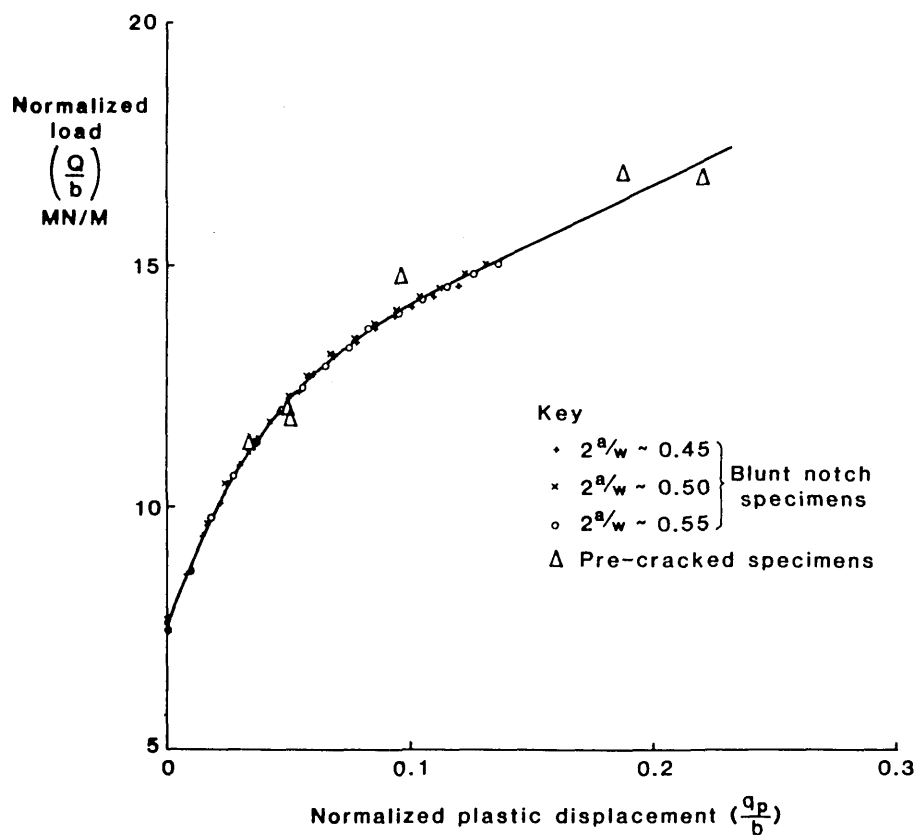


Figure 4.5 Normalized load versus plastic displacement for centre-cracked tension specimens

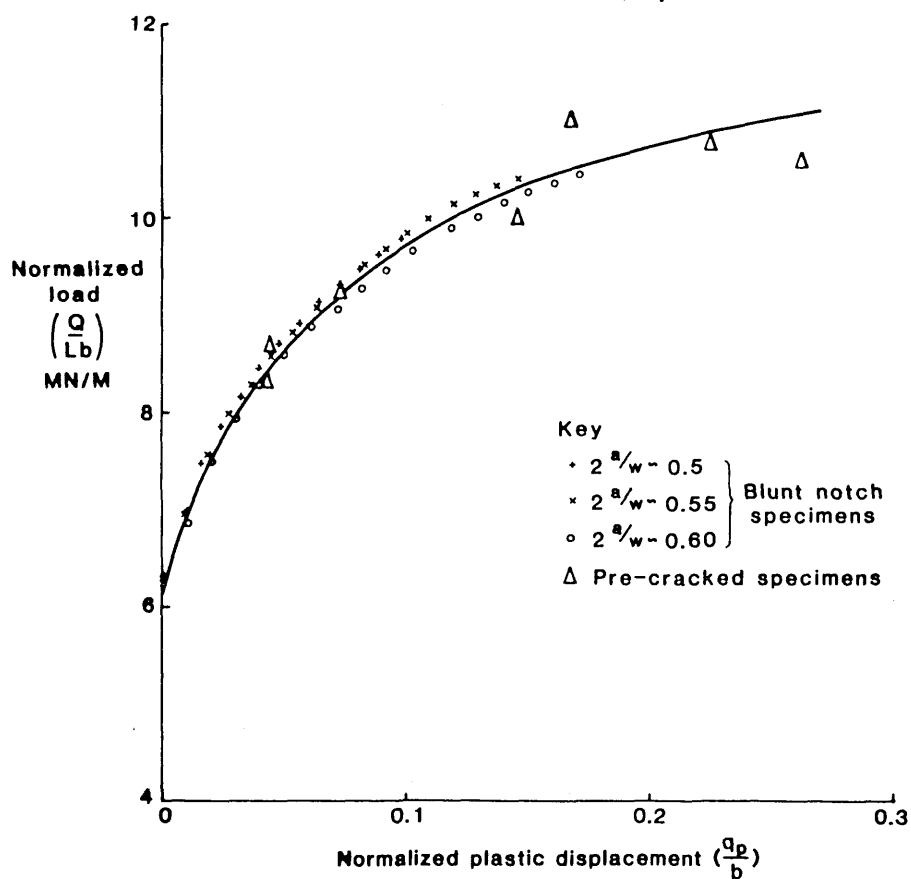


Figure 4.6 Normalized load versus plastic displacement for double-edge-notched tension specimens

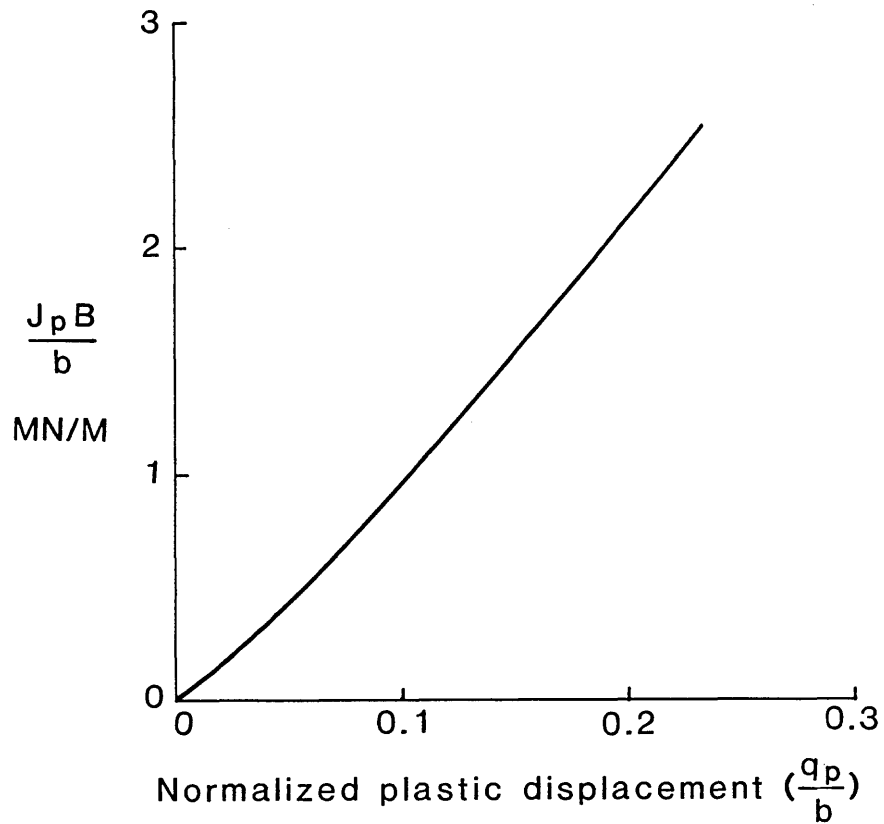


Figure 4.7 Normalized J_p versus plastic displacement for centre-cracked tension specimens

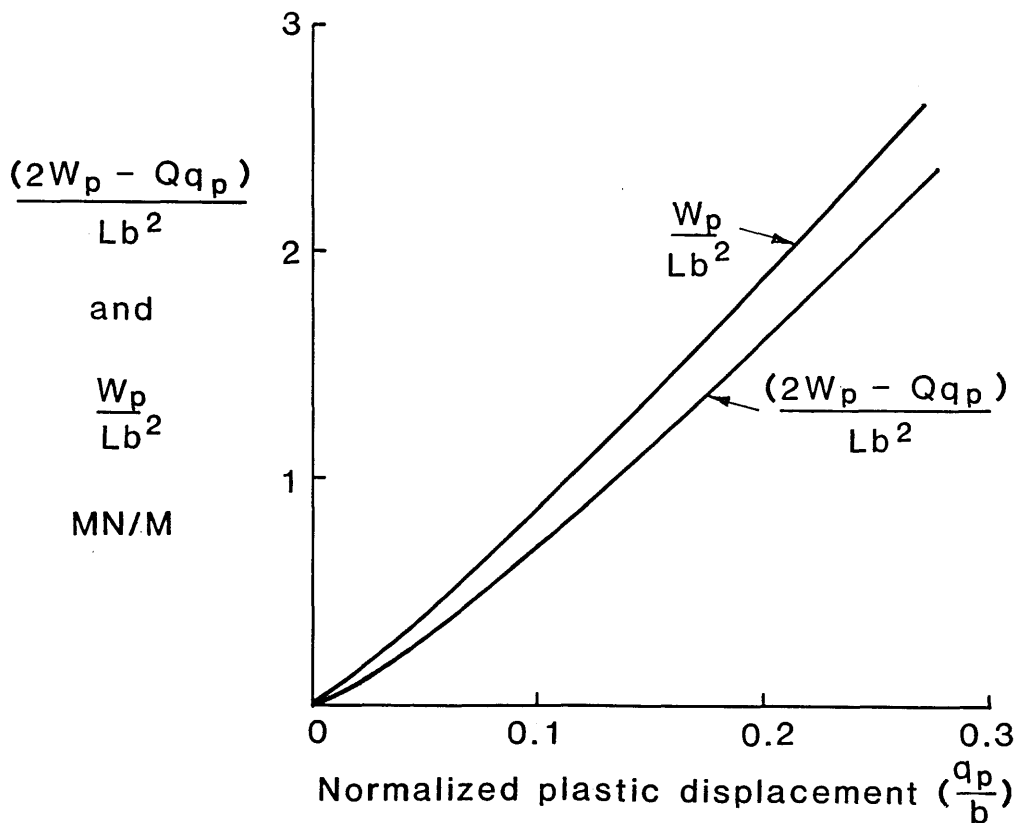


Figure 4.8 Normalized plastic absorbed energy values versus plastic displacement for double-edge-notched tension specimens

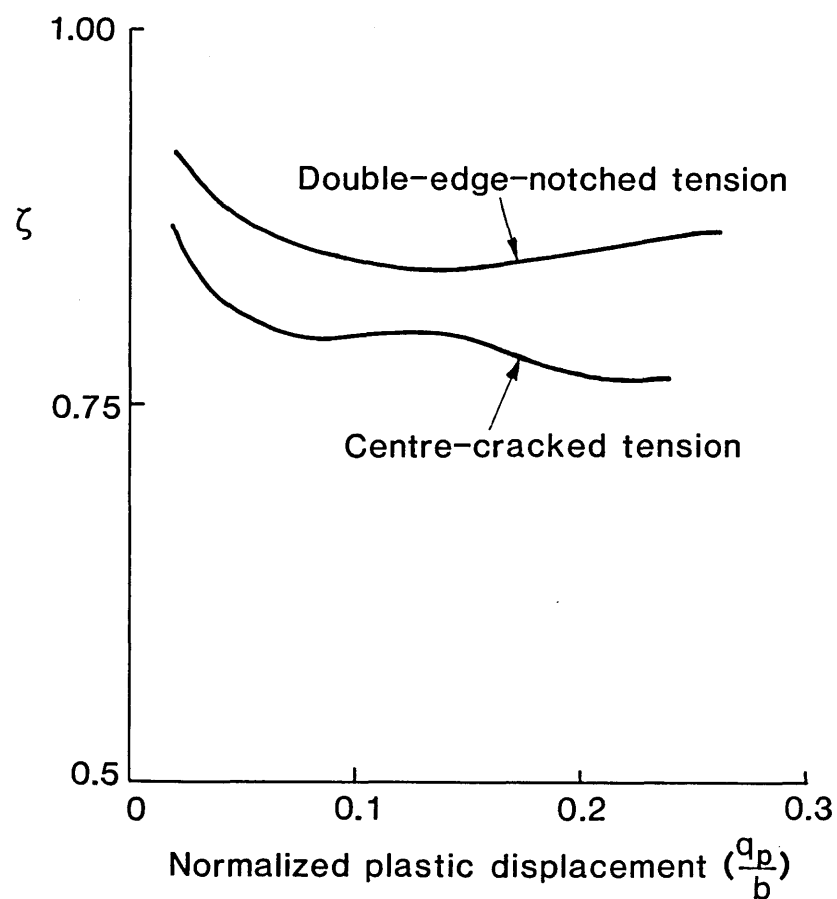


Figure 4.9 Effect of normalized plastic displacement on the ζ for centre-cracked and double-edge-notched tension specimens

Chapter 5

Results on J-R Curves

5.1 Effect of Variations in Chemical Composition and Test Temperature

The fracture toughness test specimens were manufactured from a plate within which variations in the carbon and sulphur levels were found, particularly through the plate thickness, see section 3.1. A number of investigations were carried out to determine the effect of these chemical variations on the fracture toughness. These all showed that there was no detectable link between the carbon or sulphur level, within the range found in the plate, and the fracture toughness. For example, Figure 5.1 gives the J_0 -R curve determined from 13mm thick (B), 26mm wide(w) non-sidegrooved compact specimens, together with the carbon and sulphur levels of each specimen. It can be seen that although both the carbon and sulphur levels vary, almost to the extreme values found within the plate as given in Table 3.1, there is no corresponding variation in the fracture toughness. Therefore no allowance is required for variations in chemical composition between specimens when comparing their J-R curves.

The test temperature was usually 30°C, although in some cases it was in the range 35-45°C to avoid a cleavage "pop-in" during the test. Figure 5.2 shows the J_0 -R curve determined from non-sidegrooved four-point bend specimens with B=25mm, w=50mm, which have been tested at various temperatures. It would appear that there is no significant effect of temperature, over the range 30-45°C, on the fracture toughness, and so no allowance has to be made for variations in test temperature when comparing J-R curves.

5.2 Comparison of the Various Approaches for Calculating "J" after Crack Initiation

Three approaches for calculating "J" after crack initiation have been used in this study, see Chapter 4. These are: the J without any correction for crack growth, J_0 ; the J with a correction for crack growth, J_r , and the modification to J_r suggested by Ernst

(1983), J_m . As an illustration to show the numerical differences between these "J" values in this study, Figures 5.3, 5.4 and 5.5 show the J_R -R, J_O -R and J_m -R curves respectively, for the various specimen geometries tested. All the specimens were 25mm thick and non-sidegrooved and for the bend and compact specimens $w=50\text{mm}$, while for the centre-cracked tension (CCT) and double-edge-notched tension (DEN) specimens $2w=25\text{mm}$. For one of the CCT specimens, the length of the fatigue region was particularly short, only 0.8mm compared to the 1.5–2.4mm for the other specimens. This may have caused the higher observed toughness of this specimen, and so the results from this specimen are not used in any subsequent analysis. Comparison of the three figures shows that except for the CCT and DEN specimens, the numerical differences between J_O , J_m and J_R are small, i.e. less than 10%, up to the maximum crack extensions of interest in this study (about 2mm). Indeed, this is the case for the rest of the specimens tested in this study. The numerical differences between the three types of "J" are larger for the CCT and DEN specimens because they have the smallest ligament sizes.

To investigate the effect of specimen size and geometry on the "J"-resistance curve, "J" was taken as J_O rather than J_R or J_m . J_O was used instead of J_R because, as will be shown and discussed in Chapter 8, J_O and J_m can characterise the crack growth resistance of a material over a wider range of conditions than J_R . In addition, J_O was used instead of J_m because unlike J_O , J_m was calculated from an "incremental" type equation, see Chapter 4. This means that J_m was determined from the previous J_m value associated with a lower increment of crack extension. As a multispecimen approach was used to determine the "J"-R curve, a particularly high or low value of J_m , arising from material inhomogeneity, would inappropriately affect the J_m values associated with larger crack extensions. However, as the numerical differences between the three types of "J" are usually small, the results from the J_O -R curves given in the subsequent sections will generally be similar to those which would be obtained from the J_m -R and J_R -R curves .

5.3 Analysis of the J_0 -R Curves

The ASTM J_{1C} testing standard E813-81 describes a procedure by which J "initiation" values can be determined from J -R curves. In the procedure the J at "crack initiation" is defined as the J value at the intersection of the theoretical blunting line ($J=2\sigma_f\Delta a$) and a linear fit to the (J - Δa) data points lying between two prescribed crack extension offsets. These offsets are at 0.15 and 1.5mm of crack extension from the theoretical blunting line. The J "initiation" values are called J_{1C} values, if the specimens from which the J - Δa data points are determined, satisfy certain size criteria. The procedure given in ASTM E813-81 has been modified slightly in two aspects for this study. The first is that as the number of J_0 - Δa points is limited (i.e. up to 6-7) because a multispecimen approach is used, those data points lying just outside the two offsets were included in the linear analysis. The second is that as the blunting line determined experimentally, see section 5.4.3, differs slightly from the theoretical blunting line, the experimental blunting line was used instead, as it was thought that this would result in more accurate J "initiation" values. As this second modification alters the J "initiation" values by less than 5%, it was justifiable to continue to describe these values as J_{1C} values. In addition, the description J_{1C} is still used for this study, even if the specimens do not satisfy the size criteria given in ASTM E813-81.

To give a single value representation of the initial slope of the J -resistance curve, dJ/da , the slope of the linear line through the J_0 - Δa data points lying within or just outside the two crack extension offsets was used.

5.4 J at Crack Initiation Determined from Stretch Zone Width Measurements

5.4.1. Introduction

J at crack initiation can be determined from stretch zone width (SZW) measurements by substituting the critical SZW (i.e. the SZW at crack initiation) into the J blunting line relationship, see section 3.6. The initiation J values determined in this way are described as J_i values in this study. All the measurements of SZW

were made along the central $2/3$ of the fatigue crack front. In the case of sidegrooved specimens this was because the SZW was difficult to measure near the specimen edges, where the fatigue crack front was curved due to the influence of the sidegrooves during pre-cracking. In the case of non-sidegrooved specimens, this was to avoid measurements being taken in the region of lower crack tip constraint which is present near the sides of the specimens.

The SZW at crack initiation, SZW_c , was measured from the fracture surface of specimens which had undergone some fibrous crack extension. This was possible because it was found by a number of investigations that the size of the stretch zone, after crack initiation, was not altered by subsequent fibrous crack extension.

In the following sections the two aspects which are important in the determination of J initiation values from SZW measurements are considered. These are the accuracy of the SZW measurements and the J blunting line relationship.

5.4.2 Estimation of Experimental Errors

The standard error (standard deviation/ $\sqrt{\text{number of data points}}$) is used in this study to indicate the likely accuracy of the mean of a set of data points. Indeed, there is about a 68% chance that the estimated mean is within ± 1 standard error of the actual or true mean, if the distribution of the data about the true mean is a normal distribution. However, in measuring the SZW, there was a degree of subjectivity involved because of the uncertainty in the exact starting and finishing positions of the stretch zone. Therefore, it was unclear in this case whether the standard error was still a suitable estimate of the likely accuracy of the estimated mean. To investigate this, 7 sets of measurements (10 measurements within each set) of the critical SZW (SZW_c) were obtained from a single specimen by various investigators and at different positions across the central $2/3$ of the crack front. For each set of measurements the mean and standard error were obtained while the true mean was taken as the mean of all the measurements.

It was found from the results that 5 out of 7 (about 71%) of the mean values were within ± 2 standard error of the true mean. This suggests that a more reasonable estimate of the likely accuracy of the mean of a set of SZW measurements is 2 standard error rather than just the standard error. So, for this study the mean SZW values and the subsequently determined J initiation values are reported with error bands of ± 2 standard error.

To check the general accuracy of the SZW measurements, SZW_C values were compared to δ values at crack initiation (δ_i) which can be determined reasonably accurately, see Chapter 6, and which are related because they are both measurements of the crack tip profile at crack initiation. Figure 5.6 shows the δ_i values ($\pm$ standard error) and the SZW_C values (± 2 standard error) determined from compact specimens with gross $B=13\text{mm}$ and $w=26\text{mm}$ which have been sidegrooved to various depths. This shows that both δ_i and SZW_C show a similar trend with increasing sidegroove depth, which suggests that the measurements of SZW are reliable.

5.4.3 J Blunting line Relationship

Figure 5.7 shows a plot of J integral versus SZW (± 2 standard error) determined from compact specimens which have been sidegrooved to various depths, but in which crack initiation has not occurred. The specimen dimensions were mainly gross $B=13\text{mm}$ and $w=26\text{mm}$. There appears to be a single linear relationship between J and SZW for all sidegroove depths, given by:

$$J = (2.21 \pm 0.05) \sigma_F SZW \quad 5.1$$

This is despite the fact that the SZW_C was found to be dependent on the sidegroove depth, see Figure 5.6.

Although Equation 5.1 has been determined using compact specimens, there is some evidence to suggest it is applicable to the other specimen geometries investigated in this study, because it appeared that for specimens of the same thickness, the SZW_C and the J at crack initiation were similar for the various geometries investigated. The latter can be seen by comparing their J_0 -R curves, which are similar at small crack extensions, see Figure 5.4.

5.5 Comparison of J_i Values from SZW and J_{1c} Values from ASTM E813

It was found that for the material used in this study, J_i determined from SZW measurements gave a good estimate of the J at the onset of crack initiation, as given by the deviation of the J - R curve from the blunting line, compared to J_{1c} from ASTM E813-81 which was generally significantly higher. As an example, Figure 5.8 shows the initial part of the J_o - R curve determined from non-sidegrooved compact specimens with $B=13\text{mm}$, $w=26\text{mm}$, together with the J_i ($\pm 2\times\text{standard error}$) and J_{1c} ($\pm \text{standard error}$) values. The reason why J_{1c} overestimates the J at crack initiation is discussed in section 7.3. Therefore, in this study only the J_i values determined from the SZW measurements are used to evaluate the effect of specimen size and geometry on J at crack initiation, although J_{1c} values are given for completeness.

5.6 Effect of Specimen Size and Geometry on the J_o - R Curve

Figures 5.9 to 5.15 give J_i values ($\pm 2\times\text{standard error}$) determined from SZW measurements, J_{1c} values ($\pm \text{standard error}$) determined from ASTM E813-81, and dJ/da values ($\pm \text{standard error}$), for the various specimen sizes and geometries investigated in this study.

Figures 5.9 and 5.10 show the effect of specimen thickness on J_i , J_{1c} and dJ/da for non-sidegrooved compact specimens with the width constant at 100mm or proportionally increasing, respectively. From the figures it can be seen that J_i decreases by about 30% with increasing specimen thickness to a plateau value which is assumed to be that corresponding to plane strain crack tip conditions. The plateau value occurs at specimen thicknesses greater than between 37.5-50mm, which suggests a plane strain thickness size criterion of:

$$\text{Thickness } B \geq \alpha \frac{J_i}{\sigma_f} \quad \text{where } 270 \leq \alpha \leq 360 \quad 5.2$$

From Figure 5.9 it can be seen that dJ/da decreases significantly with increasing specimen thickness, for specimens of the same width. In the case where the thickness and width are

increased proportionally, see Figure 5.10, there appears to be little effect on dJ/da over the range of thicknesses between 13 and 37.5mm, but the dJ/da does decrease significantly between a thickness of 37.5 and 50mm.

Figure 5.11 shows the effect of sidegroove depth on J_i , J_{lc} and dJ/da for compact specimens with Gross $B=13\text{mm}$ and $w=26\text{mm}$. It can be seen that J_i decreases with increasing sidegroove depth to a plateau value for sidegroove depths greater than between 25 to 50%. This plateau J_i value is the same as that obtained in larger non-sidegrooved specimens (see Figures 5.9 and 5.10), and shows that plane strain crack tip conditions can be obtained in smaller specimens if they are suitably sidegrooved. From this result the plane strain thickness size criterion for compact specimens sidegrooved to a depth of 25-50% would appear to be:

$$\text{Gross thickness, } B_g \geq 90 \frac{J_i}{\sigma_f} \quad 5.3$$

This may be a conservative estimate of the actual criterion, for the same plateau J_i value might be obtained from even smaller specimens if they were sidegrooved to a depth of 25 to 50%. On the other hand, dJ/da decreases continually with increasing sidegroove depth and there is no indication that dJ/da is decreasing to a lower bound plateau value, see Figure 5.11.

Figure 5.12 shows the effect of sidegroove depth on J_i , J_{lc} and dJ/da for compact specimens with gross $B=25\text{mm}$ and $w=50\text{mm}$. By comparing Figures 5.11 and 5.12 it can be seen that the trends in J_i and dJ/da with sidegroove depth are the same for both specimen sizes, except that the plateau J_i value appears to be obtained after sidegroove depths of between 10 and 50% for the larger specimen size.

Figure 5.13 shows the effect of sidegroove depth on J_i , J_{lc} and dJ/da for compact specimens with gross $B=50\text{mm}$ and $w=100\text{mm}$. It can be seen that there is no effect of sidegroove depth on J_i for this specimen size and that the J_i value is the same as the lower

bound plateau or plane strain J_i value given in Figures 5.9 to 5.12. Therefore, from the results given in Figures 5.11 and 5.13, it would appear that once the plane strain J_i value is obtained; further increases in sidegroove depth do not alter the J_i value and thus presumably the crack tip constraint.

From Figure 5.13 it can be seen that with increasing sidegroove depth there is little or no effect on dJ/da until the sidegroove depth is increased above 10 to 25% when dJ/da appears to decrease slightly. As with the smaller specimen sizes, see Figures 5.11 and 5.12, it would appear that there is no lower bound plateau dJ/da value for sidegrooved specimens.

Figure 5.14 shows the effect on J_i , J_{1c} and dJ/da of increasing the width of non-sidegrooved compact specimens of a constant thickness of 13mm. It would appear that J_i is unaffected by the specimen width over the range investigated. This suggests that the smallest ligament size tested was sufficient to satisfy the ligament size criterion for plane strain crack tip conditions. Based on the result of the smallest ligament size tested, this criterion would be:

$$\text{Ligament, } b \geq 65 \frac{J_i}{\sigma_f} \quad 5.4$$

This may be a conservative estimate of the actual criterion, for the same J_i value may be obtained from specimens with even smaller ligament sizes.

From Figure 5.14 it can be seen that dJ/da increases marginally with increasing specimen width over the range investigated. This trend is opposite to that when the specimen thickness is increased, see Figure 5.9.

Figure 5.15 shows the effect of specimen geometry on J_i , J_{1c} and dJ/da . All the specimens were non-sidegrooved and 25mm thick, the width was 50mm for the bend and compact specimens and 25mm for the CCT and DEN specimens. It can be seen that J_i is

independent of the specimen geometry, although it is higher than the lower bound plateau value or plane strain value given in Figures 5.9 to 5.13. This would suggest tentatively that specimen geometry does not affect the crack tip constraint, and thus the same specimen size criteria as those determined for the compact specimens are applicable to the other geometries investigated in this study. Further tests on specimens of different sizes would be required to verify this suggestion.

From Figure 5.15 it can be seen that the dJ/da value is the same, within experimental scatter, for the three and four-point bend and compact geometries. However, it would appear lower for the DEN geometry and higher for the CCT geometry.

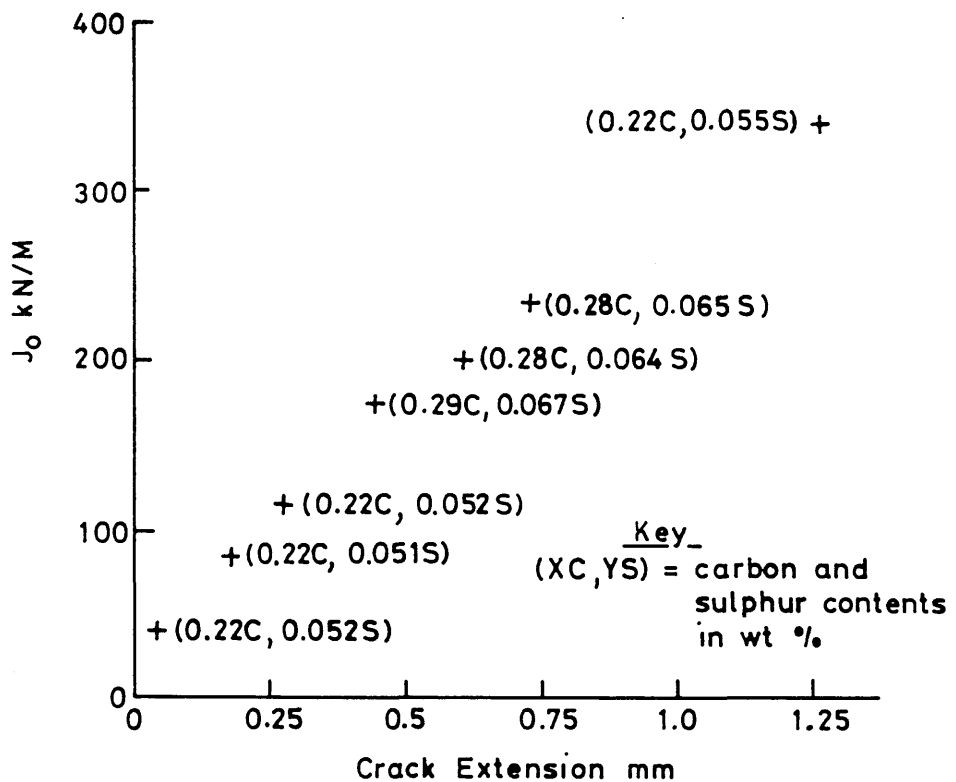


Figure 5.1 J_0 -R curve for non-sidegrooved compact specimens, $B=13\text{mm}$, $w=26\text{mm}$, together with carbon and sulphur levels of specimens

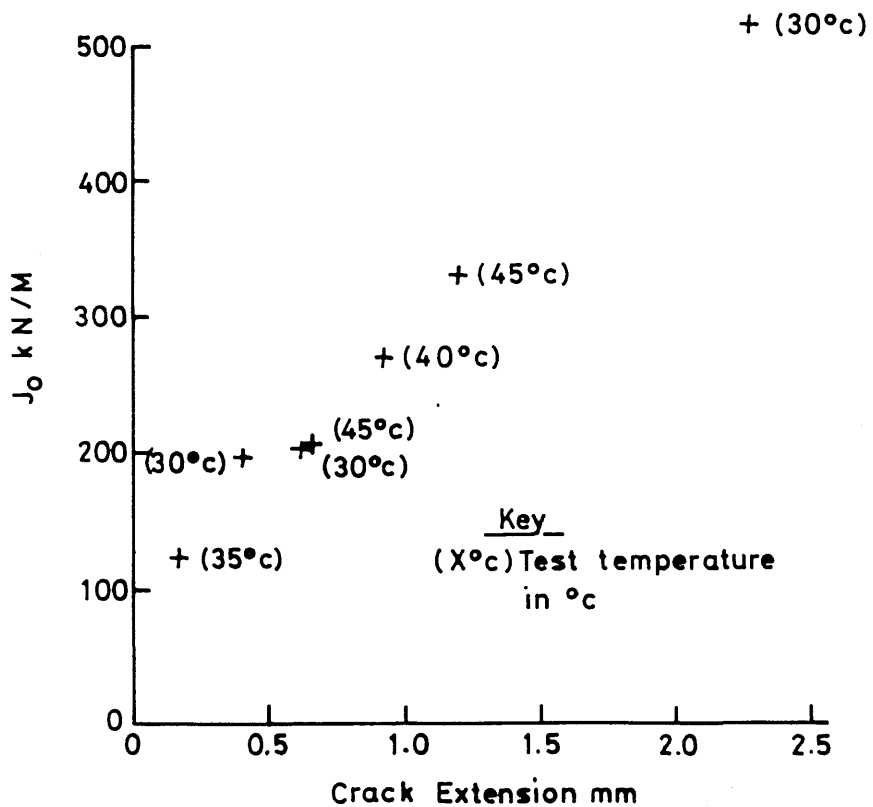


Figure 5.2 J_0 -R curve for non-sidegrooved four-point bend specimens, $B=25\text{mm}$, $w=50\text{mm}$ tested at various temperatures

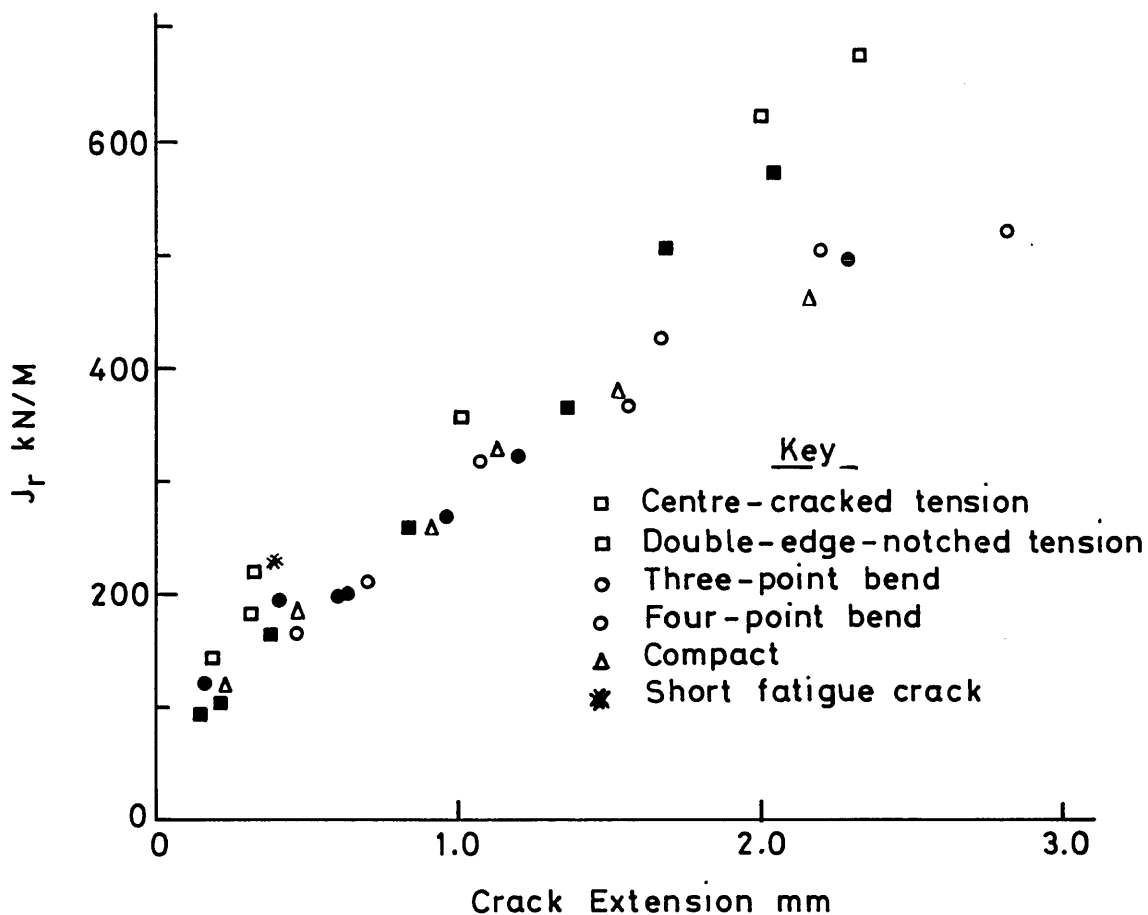


Figure 5.3 J_R -R curves for non-sidegrooved specimens, $B=25\text{mm}$, of various geometries

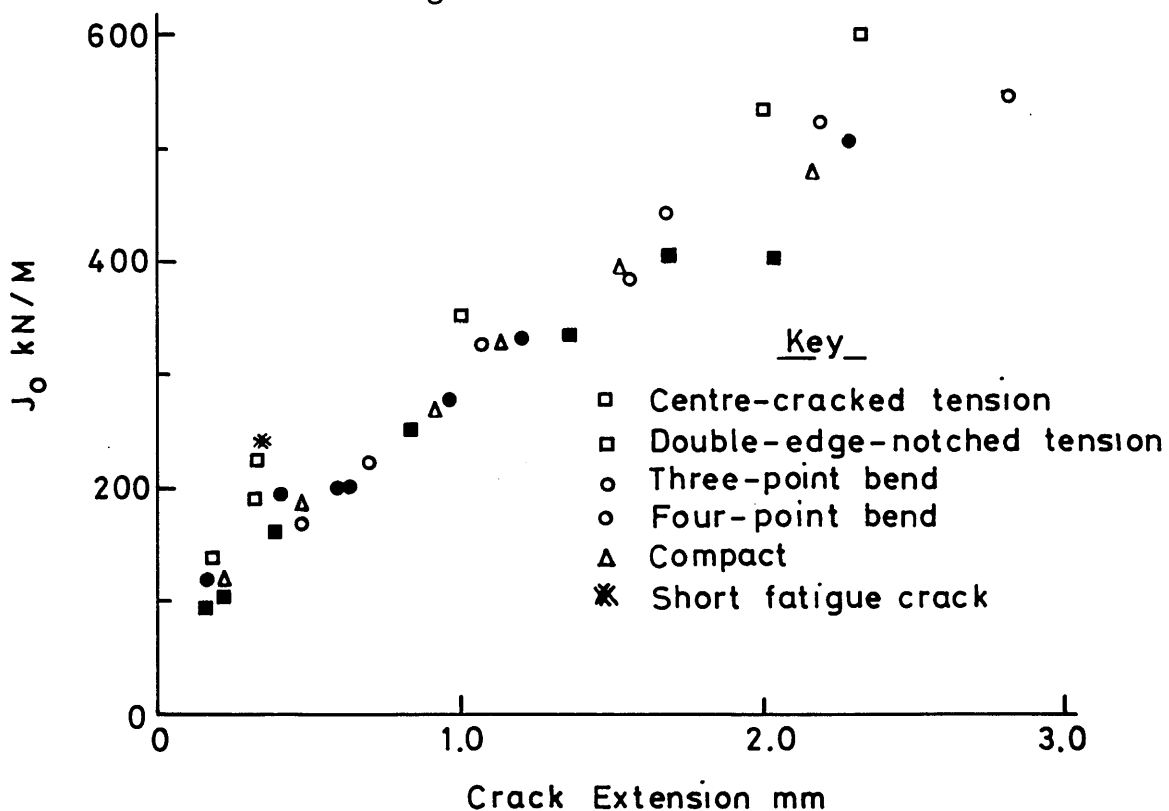


Figure 5.4 J_0 -R curves for non-sidegrooved specimens, $B=25\text{mm}$, of various geometries

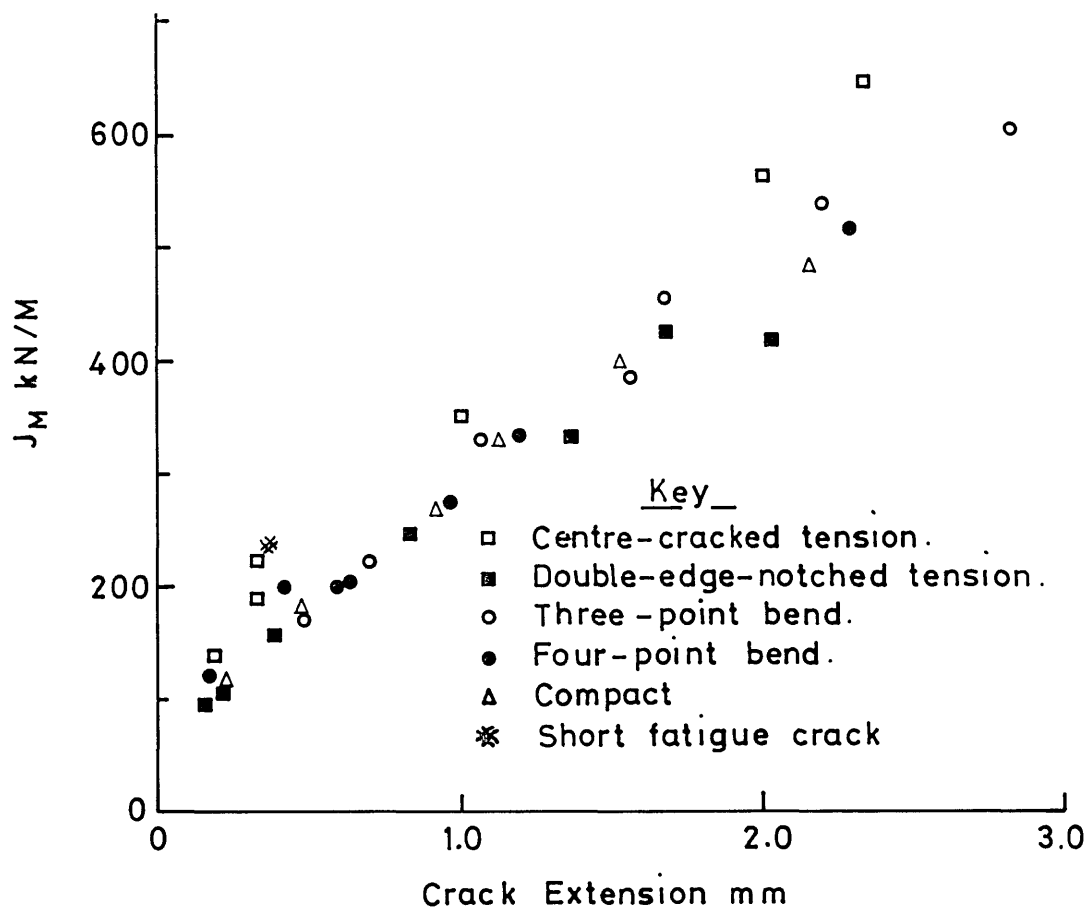


Figure 5.5 J_m -R curves for non-sidegrooved specimens, $B=25\text{mm}$, of various geometries

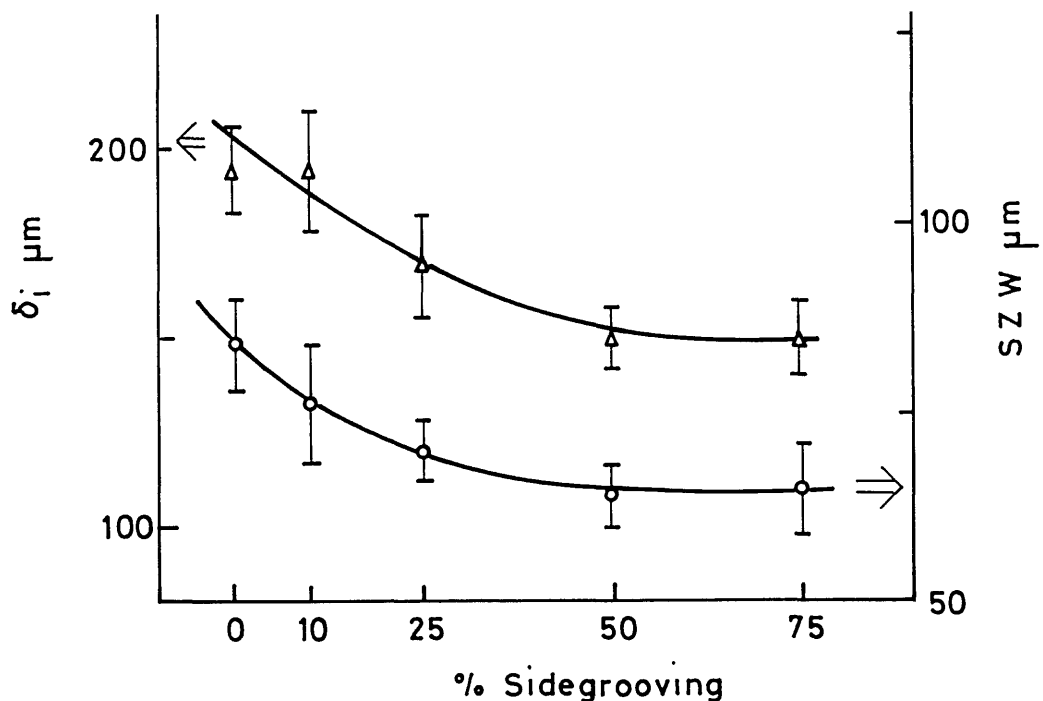


Figure 5.6 δ_i ($\pm$ standard error) and SZW_c (± 2 standard errors) versus sidegroove depth for compact specimens, gross $B=13\text{mm}$, $w=26\text{mm}$

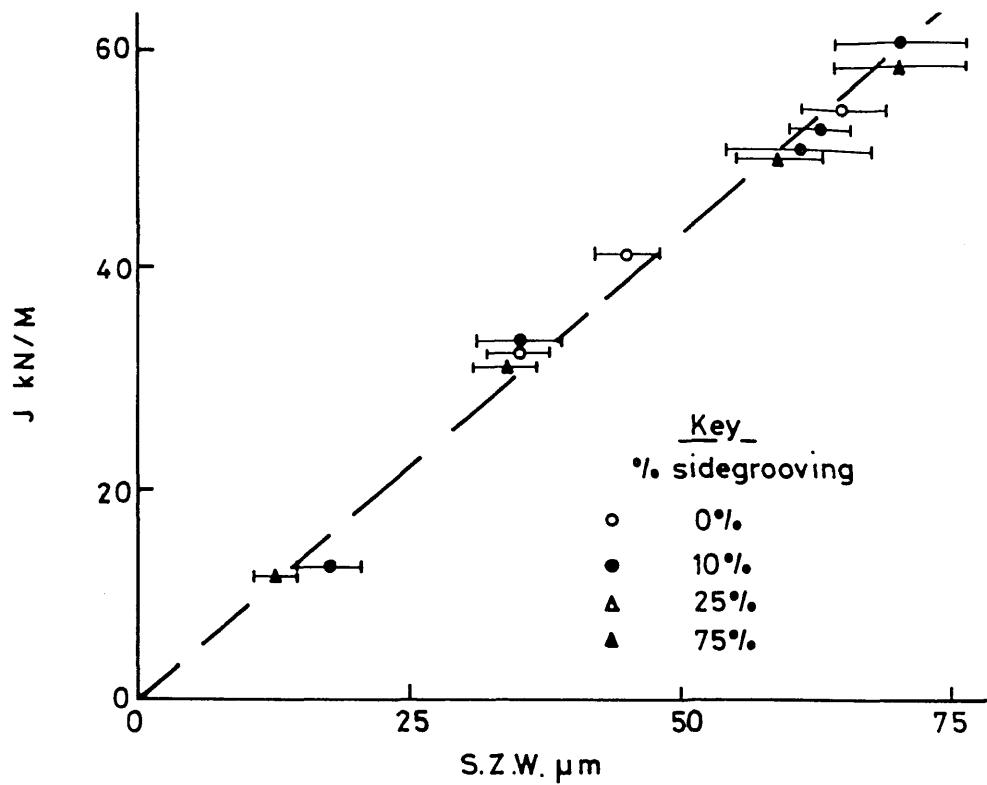


Figure 5.7 J versus SZW (± 2 standard errors), prior to crack initiation

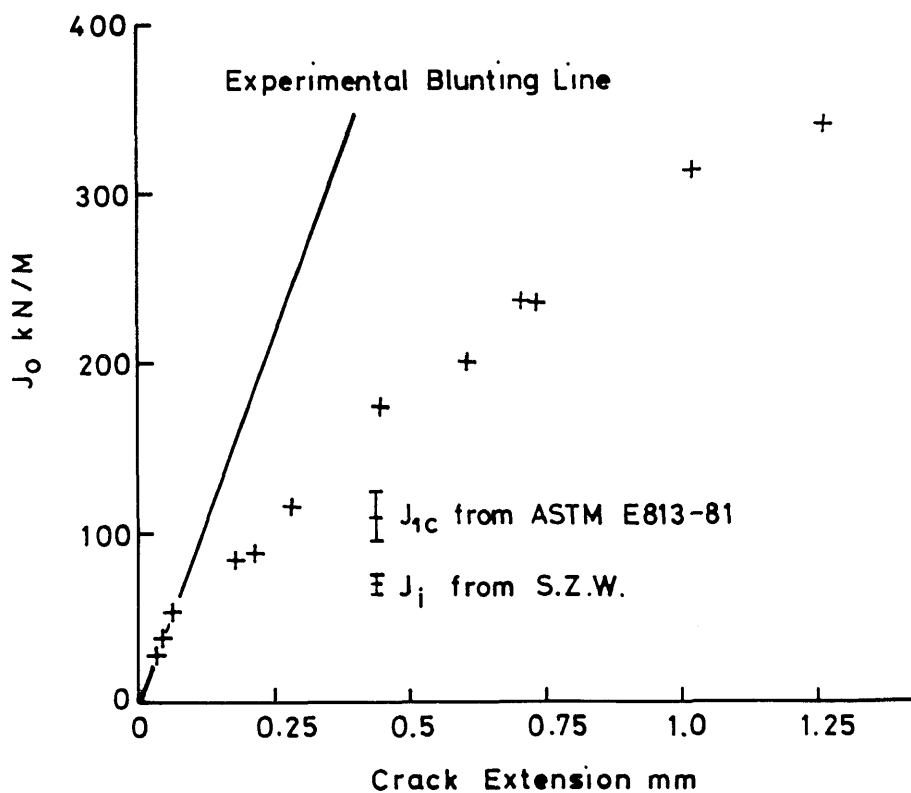


Figure 5.8 J_0 -R curve, J_i (± 2 standard errors) and J_{1c} ($\pm$ standard error) for non-sidegrooved compact specimens, $B=13\text{mm}$, $w=26\text{mm}$

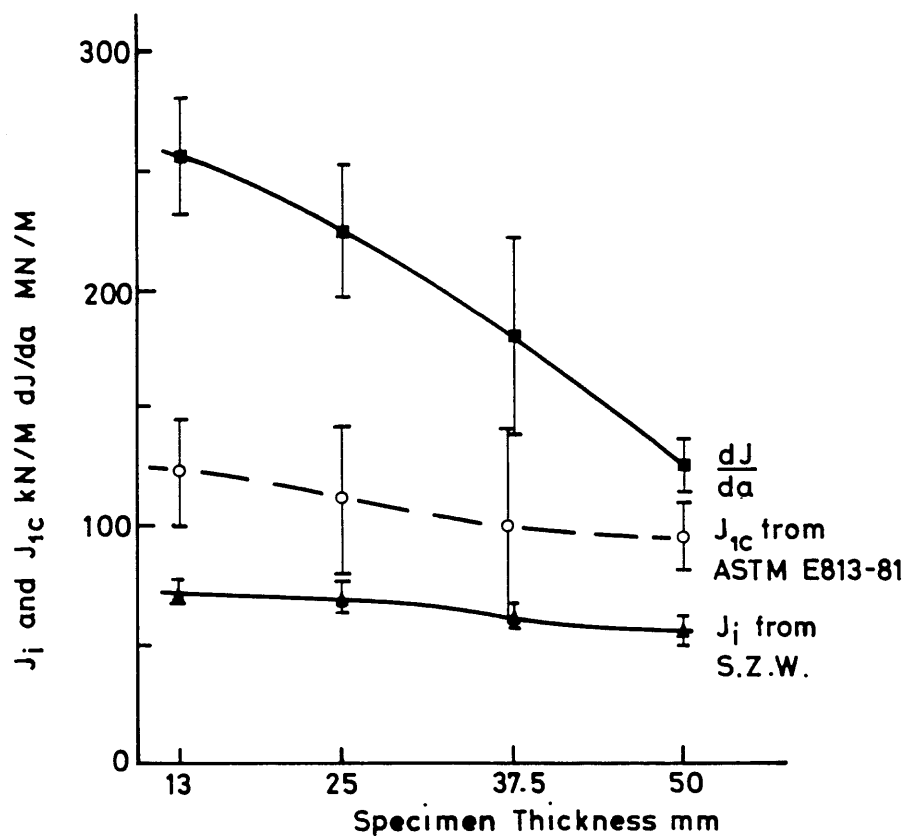


Figure 5.9 Effect of specimen thickness on J_i (± 2 standard errors), J_{1c} (± 1 standard error) and dJ/da (± 1 standard error) for non-sidegrooved compact specimens, $w=100\text{mm}$

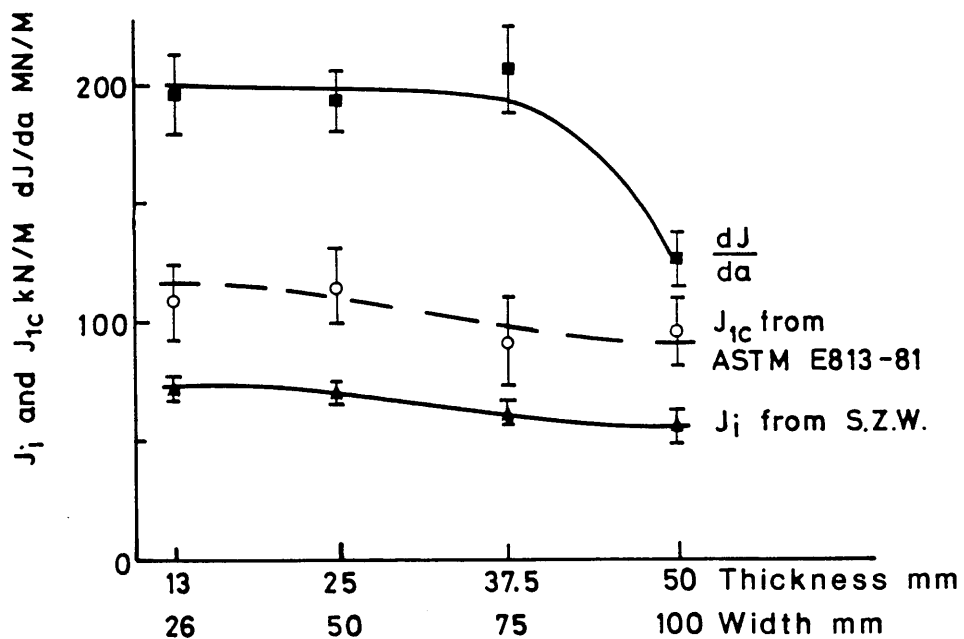


Figure 5.10 Effect of specimen size ($2B=w$) on J_i (± 2 standard errors), J_{1c} (± 1 standard error) and dJ/da (± 1 standard error) for non-sidegrooved compact specimens

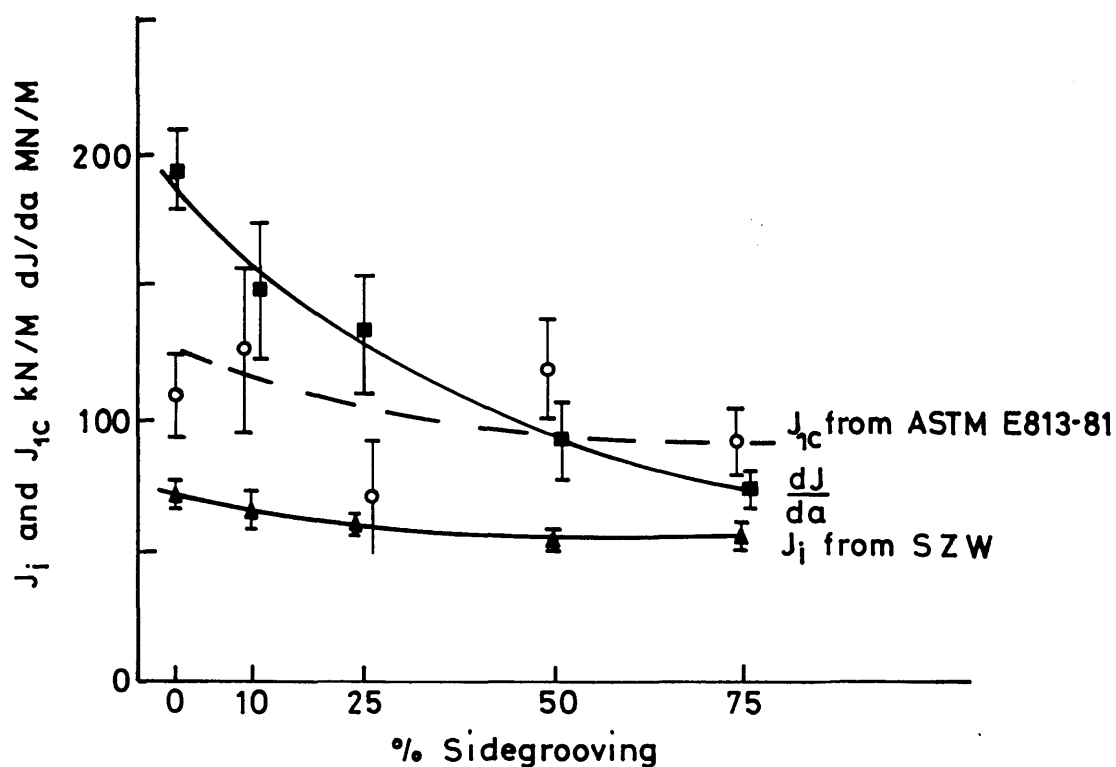


Figure 5.11 Effect of sidegroove depth on J_i (± 2 standard errors), J_{1c} ($\pm$ standard error) and dJ/da ($\pm$ standard error) for compact specimens, gross $B=13\text{mm}$, $w=26\text{mm}$

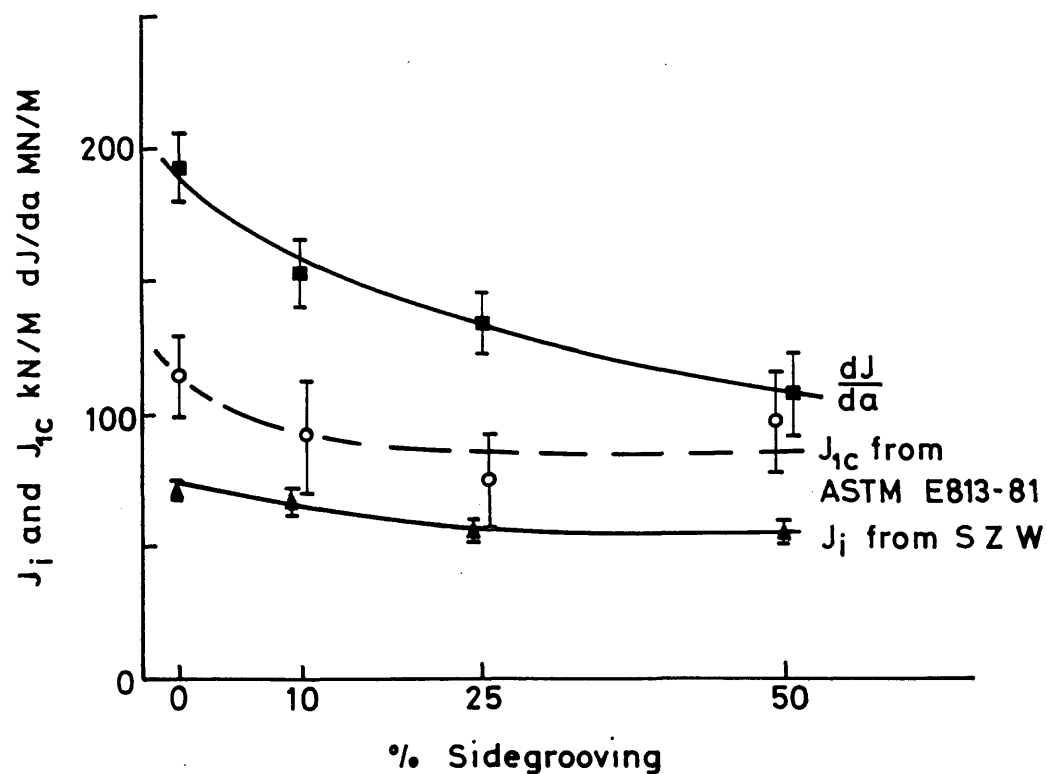


Figure 5.12 Effect of sidegroove depth on J_i (± 2 standard errors), J_{1c} ($\pm$ standard error) and dJ/da ($\pm$ standard error) for compact specimens, gross $B=25\text{mm}$, $w=50\text{mm}$

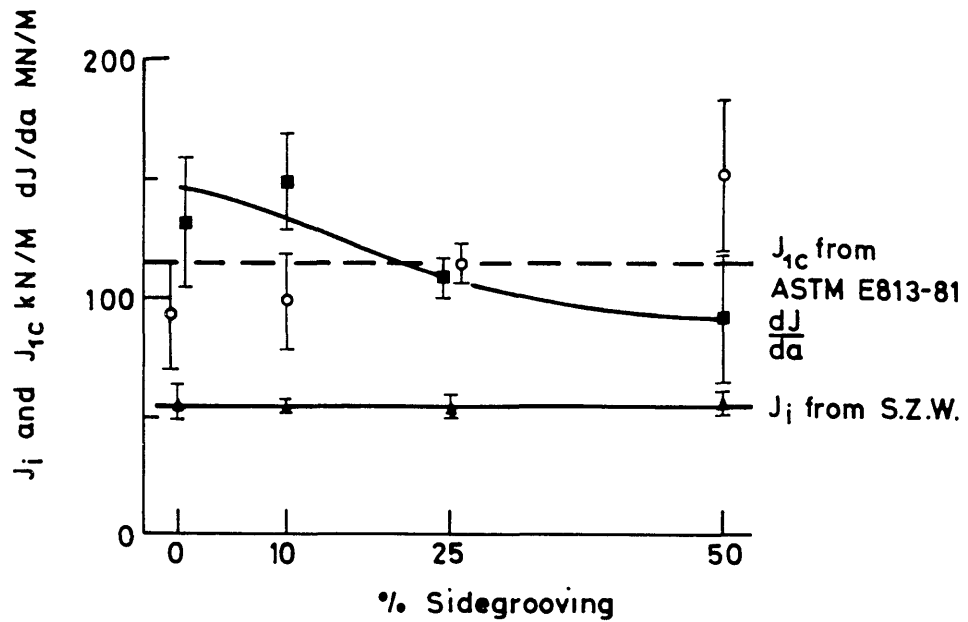


Figure 5.13 Effect of sidegroove depth on J_i (± 2 standard errors), J_{1c} (± 1 standard error) and dJ/da (± 1 standard error) for compact specimens, gross $B=50\text{mm}$, $w=100\text{mm}$

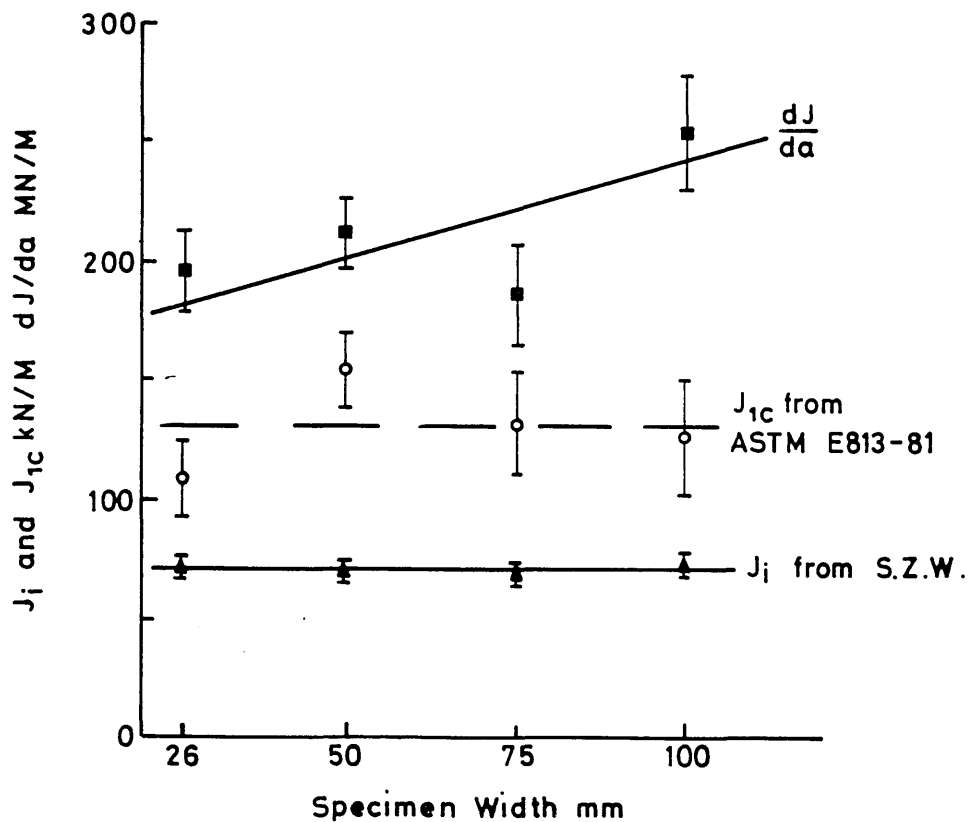


Figure 5.14 Effect of specimen width on J_i (± 2 standard errors), J_{1c} (± 1 standard error) and dJ/da (± 1 standard error) for non-sidegrooved compact specimens, $B=13\text{mm}$

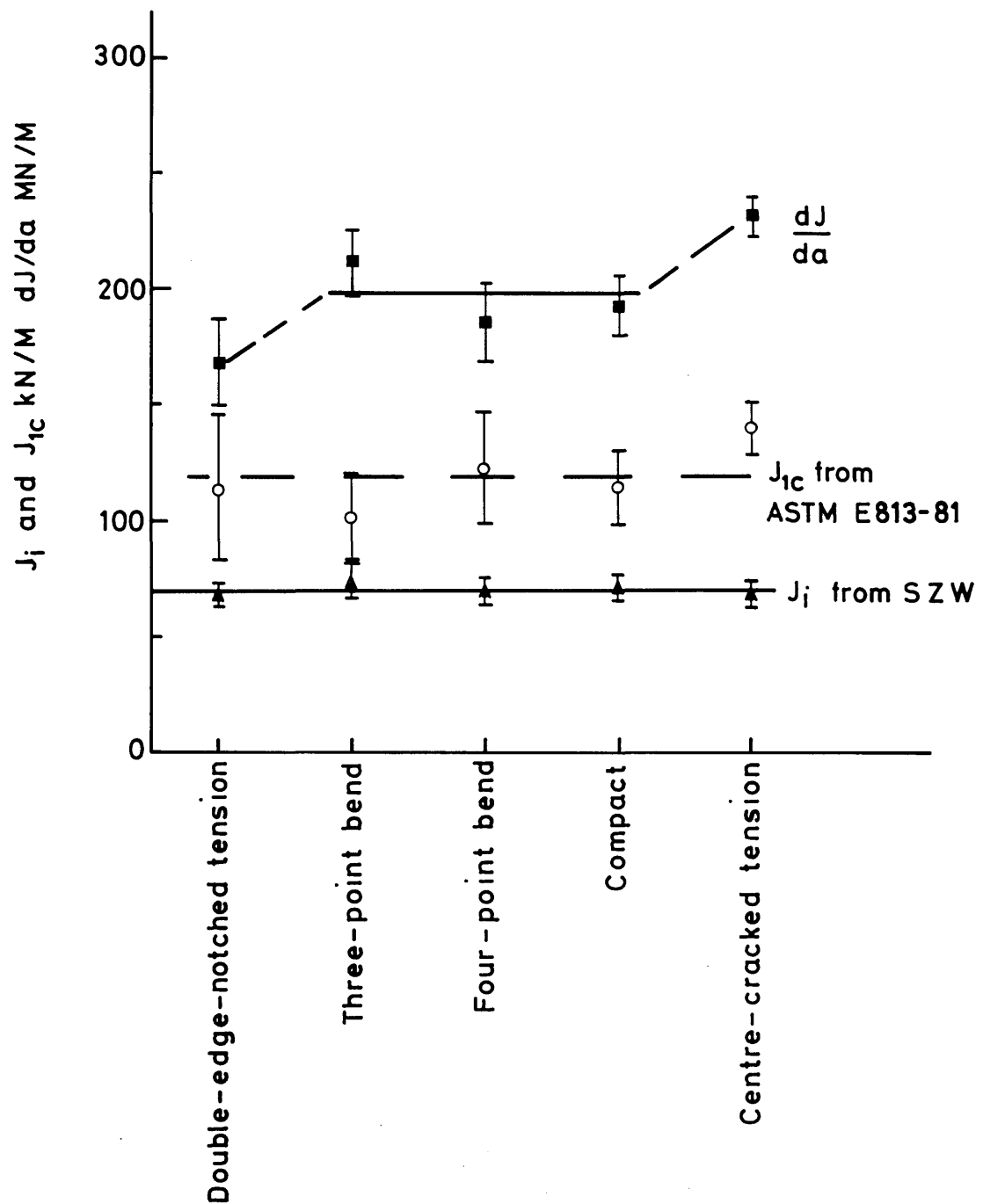


Figure 5.15 Effect of specimen geometry on J_i (± 2 standard errors), J_{Ic} ($\pm$ standard error) and dJ/da ($\pm$ standard error) for non-sidegrooved specimens, $B=25\text{mm}$

Chapter 6

Results from Silicone-Rubber Crack Impressions

6.1 Crack Opening Displacement, δ

6.1.1 Introduction

In this section the results on the effect of specimen size and geometry on the δ -R curve are given. In all cases, δ was taken as the crack opening displacement of a specimen whilst it was loaded. This was measured directly from silicone-rubber crack impressions in those cases where the impressions were obtained when the specimen was loaded, see section 3.7.2. In the other cases, where the impressions were obtained after unloading the specimen, the "onload" δ value was determined from the sum of the δ measured from the impression and a contribution, determined using a procedure given in section 6.1.3, for the change in δ during unloading.

The δ -R curve was found to be dependent on the distance across the crack front from the specimen edge, subsequently referred to as X, for specimens with a sidegroove depth of 10% or less. In these cases the δ - Δa points were grouped into various nominal X ranges, the size of which depended on the degree of variation of the δ -R curve with X. This allowed the δ -R curves at the average X values of the various ranges to be estimated.

The δ crack growth resistance, $d\delta/da$ and δ at the onset of fibrous crack initiation, δ_i , were determined from the δ -R curve. $d\delta/da$ was taken as the slope of the linear line through the δ - Δa data points as the δ -R curves were found to be linear in all cases, at least up to the maximum crack extensions investigated of about 4mm. δ_i was determined at the intersection of the linear δ -R curve and the relationship between δ and SZW (determined prior to crack initiation). The determination of latter relationship is described in section 6.1.2.

6.1.2 Relationship between δ and SZW

Figure 6.1 shows the relationship between δ and SZW, prior to crack initiation, determined from specimens of differing geometry

and sidegroove depth. The data points were obtained from various positions across the crack front, although none were from positions very close to the specimen edge (i.e. $X \leq 0.2\text{mm}$) due to the experimental difficulty in obtaining good silicone-rubber impressions in this region. The relationship between δ and SZW appears linear and the same for the CCT, DEN and compact geometries, and for specimens both non-sidegrooved and sidegrooved to a depth of 25%, and is given by:

$$\delta = (2.38 \pm 0.03) \times \text{SZW} \quad 6.1$$

This relationship was assumed applicable for the bend geometries, as their δ_i and SZW_C values are the same as those for the CCT and DEN geometries of the same thickness. In addition, this relationship was assumed valid for all sidegroove depths between 10 and 75%, as the trends in δ_i and SZW_C with increasing sidegroove depth were found to be similar, see Figure 5.6. The relationship was found to be independent of X over the range investigated ($X \geq 0.2\text{mm}$), although as discussed in section 7.2, a different relationship will be applicable close to the edge of non-sidegrooved specimens as the size of the stretch zone reduces to zero in this region. However, the relationship given by equation 6.1 was used for all X values because very few δ - Δa points were obtained at X values less than 0.2mm and also because the δ_i values are not altered significantly with changes in the relationship which is used.

6.1.3 Change in δ on Unloading Specimen

Based on a rigid-plastic contained yielding model, Rice (1965) suggested that prior to crack initiation the change in δ on unloading, $\Delta\delta_{UL}$, was 50% of δ . Extending this result to the non-contained yielding case, this would suggest that $\Delta\delta_{UL}$ is 50% of the elastic component of δ , δ_e . However, experimental results of Lewis et al (1975) suggest that after crack initiation and under contained yielding conditions, $\Delta\delta_{UL}$ is 30% of δ_e . As will be shown subsequently this later result appears to be the more accurate and so was used in this study.

δ_e can be determined from G knowing the relationship between J and δ . The latter is determined from the relationship between δ and SZW , and from the J blunting line relationship, see section 7.2, and is given by:

$$J = 1.27 \sigma_y \delta.$$

Therefore δ_e is given by:

$$\delta_e = \frac{G}{1.27 \sigma_y}$$

Using the result of Lewis et al (1975), $\Delta\delta_{UL}$ is given by:

$$\Delta\delta_{UL} = 0.236 \frac{G}{\sigma_y} \quad 6.2$$

Equation 6.2 was used in conjunction with the δ measured from silicone-rubber crack impressions, which were obtained after unloading the specimen, to determine the "onload" δ values which in turn were used to construct δ - R curves. Table 6.1 gives the δ_i values determined from these δ - R curves and those determined from substituting the SZW_c into the δ - SZW relationship, equation 6.1. The close agreement between the two δ_i values for each specimen confirms the accuracy of equation 6.2 and thus the result of Lewis et al (1975) because the δ_i values determined from SZW_c , unlike those from the δ - R curves, do not depend on the value of $\Delta\delta_{UL}$ and thus the accuracy of equation 6.2.

6.1.4 Effect of Specimen Size and Geometry, and Distance X on δ_i and $d\delta/da$

Figure 6.2 gives δ_i and $d\delta/da$ as a function of the average X value of each range, for non-sidegrooved compact specimens with $B=13\text{mm}$, $w=26\text{mm}$. From the figure it can be seen that δ_i is similar over most of the central region of the specimen, but is about twice as high near the specimen edge (i.e. $X < 1-1.5\text{mm}$), whereas $d\delta/da$ increases progressively across the crack front, although the value at the specimen edge is again about twice that at the mid-section.

Figure 6.3 gives δ_i and $d\delta/da$ as a function of the average X value of each range, for non-sidegrooved DEN specimens with $B=25\text{mm}$, $2w=25\text{mm}$. The δ_i and $d\delta/da$ values close to the specimen edge were determined from only two δ - Δa points and so the accuracy of these values is questionable. Although, it does appear that δ_i is higher close to specimen edge, since the silicone-rubber crack impressions showed that no fibrous crack extension had occurred at a δ of $240\mu\text{m}$. The trend in δ_i across the crack front is similar to that for the thinner compact specimens, compare Figures 6.2 and 6.3. The δ_i values in the central and near edge region are also similar in the two cases. The trend in $d\delta/da$ across the crack front for the DEN specimens is also similar to that for the thinner compact specimens, although the overall variation in $d\delta/da$ appears larger for the thicker DEN specimens, as the overall increase in this case is by a factor of about 2.5. In addition, the $d\delta/da$ value at the midsection appears slightly higher than that for the thinner compact specimens.

Figure 6.4 gives δ_i and $d\delta/da$ as a function of the average X value in each range, for non-sidegrooved CCT specimens, with $B=25\text{mm}$ and $2w=25\text{mm}$. As in the DEN case, only two δ - Δa points were available to determine the δ_i and $d\delta/da$ values nearest to the specimen edge, although again the δ_i value in this region would appear higher, as crack initiation had not occurred at a δ of $250\mu\text{m}$. The trend in δ_i across the crack front is similar to that for the DEN and thinner compact specimens, compare Figures 6.2, 6.3 and 6.4. In addition, the δ_i values in the central and near edge regions are similar for these three cases. The trend in $d\delta/da$ across the crack front for the CCT specimens is also similar to that for DEN and thinner compact specimens, and the overall increase in $d\delta/da$ across the crack front (about 2.5x) is similar to that for the DEN specimens. However, the $d\delta/da$ value at the midsection of the CCT specimens is significantly higher than that for the DEN or thinner compact specimens.

Few δ - Δa data points were obtained for the four-point bend geometry, as a cleavage "pop-in" occurred during loading of many of the specimens, and so it was not realistic to construct any δ -R

curves for this geometry. However, a comparison of the δ - Δa points from both three and four-point bend specimens showed that within experimental scatter, they fell on the same δ - R curve for each X range. Therefore, the combined set of data points was used to determine the δ_i and $d\delta/da$ values for the two geometries. Figure 6.5 shows δ_i and $d\delta/da$ as a function of the average X value in each range, for the non-sidegrooved bend specimens, with $B=25\text{mm}$, $w=50\text{mm}$. δ_i appears to be similar at all values of X , although no value was obtained close to the specimen edge (i.e. $X < 1\text{mm}$), whereas for the other geometries, compact, DEN and CCT, see Figures 6.2, 6.3 and 6.4, δ_i was significantly higher near the specimen edge. This would suggest that the δ_i was also higher in this near edge region for the bend specimens. In addition, the δ_i in the midsection of the bend specimens is similar to that in the DEN, CCT and thinner compact specimens. The trend in $d\delta/da$ across the crack front in the bend specimens would appear similar to that for the other geometries, although the full extent of the variation in $d\delta/da$ is unknown. The $d\delta/da$ value in the midsection of bend specimens is lower than that in the DEN and in particular the CCT specimens. It would also appear to be marginally smaller than that in the thinner compact specimens, although this could be due to material inhomogeneity.

Figure 6.6 gives δ_i and $d\delta/da$ as a function of the average X value in each range, for compact specimens sidegrooved by 10%, with gross $B=13\text{mm}$ and $w=26\text{mm}$. It would appear that δ_i is similar across the whole crack front, except possibly very close to the sidegroove notch root (i.e. $X < 0.2\text{mm}$) where no δ - Δa data points were obtained. This contrasts with the behaviour in non-sidegrooved specimens, where δ_i increases significantly near the specimen edge (i.e. $X < 1-1.5\text{mm}$), see Figure 6.2. In addition, the δ_i at the midsection of the specimen is also lowered slightly by sidegrooving to a depth of 10%. Therefore, it would appear that shallow sidegrooves can increase the crack tip constraint across the whole crack front and so lower δ_i . From Figure 6.6 it can be seen that there is only a marginal variation in $d\delta/da$ across the crack front, which contrasts markedly with that found in non-sidegrooved specimens, see Figure 6.2. In addition, $d\delta/da$ at the midsection of the specimen is decreased slightly by sidegrooving.

Figure 6.7 shows δ_i and $d\delta/da$ as a function of the average X value in each range, for compact specimens sidegrooved to a depth of 10%, with gross B=50mm and w=100mm. δ_i is similar across the whole crack front, as in the smaller sidegrooved specimens, compare Figures 6.7 and 6.6, although in both cases no δ - Δa data points were obtained very close to the sidegroove notch root ($X < 0.2\text{mm}$). However, δ_i is higher in the smaller sidegrooved specimens, which suggests that the crack tip constraint is not that for plane strain in these specimens. Perhaps surprisingly, the variation in $d\delta/da$ across the crack front is larger in the larger sidegrooved compact specimens, although the $d\delta/da$ at the midsection of the larger specimens is marginally lower.

Figure 6.8 shows the effect of sidegroove depth on δ_i and $d\delta/da$ at the midsection of compact specimens with gross B=13mm, w=26mm. For specimens with a sidegroove depth greater than 10%, no variation in δ_i and $d\delta/da$ was found across the crack front, and so in these cases the δ_i and $d\delta/da$ values correspond to the average values across the whole crack front. From the figure it can be seen that δ_i decreases with increasing sidegroove depth to a plateau value, assumed to be that for plane strain, after a sidegroove depth of between 25-50%. This suggests that sidegrooving can increase the crack tip constraint in specimens which are of an insufficient size to meet the plane strain size requirements for non-sidegrooved specimens. In addition, from the figure it can be seen that $d\delta/da$ also decreases with increasing sidegroove depth, although there is no indication of a lower bound plateau value.

Figure 6.9 shows the effect of sidegroove depth on δ_i and $d\delta/da$ at the midsection of compact specimens with gross B=50mm, w=100mm. As with the smaller compact specimens, no variation in δ_i and $d\delta/da$ was found across the crack front of specimens with a sidegroove depth greater than 10%. It can be seen from the figure that δ_i is unaffected by sidegroove depth and is the same as the plateau value obtained in the smaller specimens with sidegroove depths greater than 25-50%, compare Figures 6.8 and 6.9. This suggests that as the specimen size increases and the crack tip constraint increases towards that for plane strain, a lower

sidegroove depth is required to promote plane strain crack tip conditions. From the Figure, it can be seen that the $d\delta/da$ in the midsection of specimen is similar for sidegroove depths between 10 and 25%, but decreases for sidegroove depths above 25%, which again suggests that there is no lower bound $d\delta/da$ value in sidegrooved specimens. Although there is no decrease in the $d\delta/da$ in the midsection of the specimen with increasing sidegroove depths between 10 and 25%, there is a decrease in the average $d\delta/da$ (calculated from the values at 9 evenly spaced points across the crack front), as the specimens sidegrooved to a depth of 10% exhibit a variation in $d\delta/da$ across the crack front, see Figure 6.7.

In summary, δ_i is found to be similar across the whole crack front, except near the edge of non-sidegrooved specimens (i.e. $X < 1-1.5\text{mm}$), where it is significantly higher. As illustrated in Figure 6.10, δ_i is independent of specimen geometry, although it decreases with increasing specimen size and sidegroove depth to a lower bound plateau value.

$d\delta/da$ varies across the crack front in specimens sidegrooved to a depth of 10% or less. The overall variation in $d\delta/da$ appears to be independent of specimen geometry, but increases with increasing specimen size and decreasing sidegroove depth. $d\delta/da$ at the midsection of the specimen shows a marked dependence on specimen geometry and a decrease with increasing specimen size and sidegroove depth but for large specimens only at depths above 25%, see Figure 6.10. There appears to be no lower bound plateau $d\delta/da$ value, unlike that for δ_i , in sidegrooved specimens.

6.2 Crack Tip Opening Angle

6.2.1 Analytical Determination of Crack Tip Opening Angle

Based on the assumption that the crack grows with straight crack flanks, Willoughby et al (1981) suggested that the crack tip opening angle (CTOA) can be determined from

$$\text{CTOA} = 2 \tan^{-1} \left(\frac{\delta - \delta_i - \text{CTOD}}{2(\Delta a - \text{SZW}_c)} \right) \quad 6.3$$

As δ was found to be linearly dependent on crack extension, see section 6.1.1.

$$\delta = \delta_i + \frac{d\delta}{da} (\Delta a - SZW_c) \quad 6.4$$

On substituting equation 6.4 into 6.3

$$CTOA = 2 \tan^{-1} \left(\frac{1}{2} \left(\frac{d\delta}{da} - \frac{CTOD}{\Delta a - SZW_c} \right) \right) \quad 6.5$$

This equation allows CTOA to be determined as a function of crack extension from $d\delta/da$, SZW_c (determined by substituting δ_i into the relationship between δ and SZW , equation 6.1), and the crack tip opening displacement (CTOD), see section 6.3.

At large crack extensions, equation 6.5 simplifies to

$$CTOA = 2 \tan^{-1} \left(\frac{1}{2} \frac{d\delta}{da} \right) \quad 6.6$$

6.2.2 Effect of Specimen Geometry and Sidegroove Depth on CTOA

CTOA was measured from those silicone-rubber crack impressions which were obtained prior to unloading of the specimen. Thus, CTOA was measured for the non-sidegrooved CCT, DEN and bend specimens, all with $B=25\text{mm}$, and the compact specimens, with gross $B=13\text{mm}$, $w=26\text{mm}$ which were sidegrooved to a depth of between 0 and 75%. A typical example of the results obtained is given in Figure 6.11. This shows the measured and predicted (from equation 6.5) CTOA values as a function of crack extension, for the various X ranges, determined from non-sidegrooved compact specimens with $B=13\text{mm}$, $w=26\text{mm}$. This shows, as in all cases, that there is good agreement between the measured and predicted CTOA values over a wide range of CTOA values and up to about 4mm of crack extension, the maximum investigated. CTOA is found to increase with increasing crack extension up to a plateau value, which is obtained after about 1.5mm of crack extension, the plateau value being given by equation 6.6. As the plateau CTOA value is related directly to $d\delta/da$, the

effect of specimen geometry and sidegroove depth on the plateau CTOA is the same as that found on $d\delta/da$, see section 6.1.4.

6.2.3 Single Specimen Determination of δ_i and $d\delta/da$

A procedure is outlined below which forms the basis of a single specimen method to determine δ_i and $d\delta/da$ from the crack profile of a specimen exhibiting more than about 1.5mm of crack extension.

If the crack extension is larger than about 1.5mm, $d\delta/da$ can be determined from the measured CTOA, for as shown in section 6.2.2, the measured plateau CTOA value is related to $d\delta/da$ by equation 6.6. An implication of the good agreement between the measured and predicted CTOA values, found in section 6.2.2, is that the increase in δ during crack growth, δ_g ($\delta - \delta_i$) is equal to $h_{r=\Delta a - SZW}$, which is the crack face separation determined by extrapolating the essentially linear region of the crack profile back to the start of the fibrous crack growth region (i.e. at a distance of SZW_c ahead of the fatigue crack tip) in a similar way to the determination of $h_{r=\Delta a}$, see Figure 3.8. Therefore, δ_i can be determined from the measured δ and $h_{r=\Delta a - SZW}$, i.e.

$$\delta_i = \delta - h_{r=\Delta a - SZW} \quad 6.7$$

As an example to show the accuracy of this procedure for determining δ_i , table 6.2 compares the δ_i values determined from this single specimen technique with those determined from the δ -R curves, for various X ranges for non-sidegrooved compact specimens with B=13mm, w=26mm. There is good agreement between the two sets of δ_i values.

6.3 Crack Tip Opening Displacement

6.3.1 Effect of Crack Extension

In all cases investigated, there was no consistent variation in CTOD with crack extension at least up to the maximum crack extensions investigated of about 4mm. However, as there was marked variability in the CTOD, which was probably mainly due to the

wide range of void nucleating particle sizes, as discussed in section 8.2, only a marked dependence of CTOD on crack extension would be detected. As an example, Figure 6.12 shows a plot of CTOD versus crack extension for the X range 10-12.5mm determined for non-sidegrooved DEN specimens. Therefore, as no variation in CTOD with crack extension is apparent, it is appropriate to use the mean value ($\pm$ standard error) to investigate the effect of specimen size and geometry on CTOD.

6.3.2 Variation of CTOD across the crack front

To investigate the variation of CTOD across the crack front of non-sidegrooved specimens, CTOD was measured from metallographic sections obtained at various positions through the thickness of one non-sidegrooved DEN specimen. The specimen had been previously loaded to give a 9 point average crack extension of about 3.5mm and the sections were taken perpendicular to the fatigue crack plane and parallel to the fatigue crack growth direction. Figure 6.13 shows CTOD as a function of the through thickness position X. A comparison of Figures 6.12 and 6.13 shows that the mean CTOD in the midsection of the specimen, determined from the silicone-rubber crack impressions, is slightly higher than that obtained from metallographic sections. This is as expected as the metallographic sections, unlike the silicone-rubber crack impressions, were obtained after unloading of the specimen. Therefore, as the CTOD from the metallographic sections can be measured reliably, this suggests that the extrapolation procedure used to determine CTOD from silicone-rubber crack impressions, see section 3.7.4, yields accurate values.

Close to the specimen edge of non-sidegrooved specimens there is a shear region where, as illustrated in plate 6.1, the crack growth occurs at an angle to the fatigue crack direction, resulting in the formation of a shear lip on the fracture surface. It would appear from Figure 6.13 that in this region the CTOD is lower than that in the central region of the specimen. Next to the shear region there is an intermediate region in which, as illustrated by plates 6.2a and b, there appears fewer voids around the crack tip than in the central region of the specimen. This is

presumably due to a lower crack tip constraint in this region, see section 7.2. It would appear from Figure 6.13 that the CTOD in this "low" crack tip constraint region is higher than that in the central or "high" crack tip constraint region. However, further CTOD values from the same specimen would be required before the size of this region and the average CTOD value within it can be determined accurately.

It was not possible from the CTOD values obtained from silicone-rubber crack impressions, to determine the variation in CTOD across the crack front of any one non-sidegrooved specimen, as there are too few values from each specimen in relation to the large variation found in the CTOD values. In addition, it was not possible to determine the variation in CTOD by using the results from a number of specimens associated with different amounts of crack extension, as the size of the shear lip was found to increase with increasing crack extension and therefore also the position across the crack front of the "low" crack tip constraint region. Therefore, it was felt appropriate to determine only the average CTOD value associated with a "high" crack tip constraint region for each specimen type, using all the CTOD values measured at positions such that $X > 5\text{mm}$ from various specimens of the same type. The reason behind the criterion $X > 5\text{mm}$ is as follows. The shear lip region is where the crack tip constraint is low enough so that the shear crack growth mechanism can operate. Therefore, the size of the shear region is likely to be proportional to the size of the region of "lower" crack tip constraint, i.e. the combined size of the shear and "low" crack tip constraint regions. As the size of the shear lip for the DEN specimen, used to determine Figure 6.13, is comparable to the largest measured in this study, it is expected from the results in Figure 6.13 that the CTOD values from positions where $X > 5\text{mm}$ will be those associated with the "high" crack tip constraint region.

In the case of sidegrooved specimens, there is no evidence of shear lip on the fracture surface. Therefore, it is expected that there is no region close to the edge of sidegrooved specimens exhibiting low CTOD values as in non-sidegrooved specimens, see

Figure 6.13. However, there could be a region of high CTOD values close to the specimen edge associated with a "low" crack tip constraint region. To investigate this the CTOD values were grouped into various X ranges, which allowed the average CTOD in each range to be determined as a function of the average X in each range. As an example, Figure 6.14 shows the plot obtained from compact specimens with gross $B=13\text{mm}$ and $w=26\text{mm}$ which have been sidegrooved to a depth of 10%. This shows, as in all cases investigated for sidegrooved specimens, that there is no evidence of a region close to the specimen edge associated with higher CTOD values. Therefore, it would appear that there are no shear or "low" crack tip constraint regions in sidegrooved specimens, and so in these cases the CTOD associated with the "high" crack tip constraint region can be determined from all the CTOD values from each specimen type.

6.3.3. Effect of Specimen Geometry and Sidegroove Depth on the CTOD in the Region of High Crack Tip Constraint

Figure 6.15 shows the average CTOD in the region of "high" crack tip constraint (i.e. $X>5\text{mm}$) for various specimen geometries. All the specimens were non-sidegrooved and 25mm thick, the CCT and DEN specimens were 25mm wide, while the bend specimens were 50mm wide. It can be seen from the figure that the CTOD in the "high" crack tip constraint region is unaffected by specimen geometry within experimental scatter. It is worth noting that δ_i is similar for each of these geometries, see Figure 6.10.

Figure 6.16 shows the average CTOD in the region of "high" crack tip constraint (i.e. $X>5\text{mm}$ for non-sidegrooved specimens and across the whole crack front for sidegrooved specimens) as a function of sidegroove depth in compact specimens with gross $B=13\text{mm}$ and $w=26\text{mm}$. This shows that CTOD decreases with increasing sidegroove depth to reach a plateau value after a depth of 25-50%; this trend is the same as that for δ_i for this specimen size, see Figure 6.8.

Table 6.1

δ_i values determined from δ -R curves and from SZW_C values for compact specimens with $B_{gross} = 50\text{mm}$, $w=100\text{mm}$.

sidegroove depth %	δ_i from δ -R curve $\pm$ standard error	δ_i from SZW_C $\pm 2 \times$ standard error
10	152 ± 15	155 ± 7
25	159 ± 29	159 ± 7
50	160 ± 17	155 ± 7

Table 6.2

δ_i values ($\pm$ standard error) from the single specimen technique and from the δ -R curve, for non-sidegrooved compact specimens with $B=13\text{mm}$, $w=26\text{mm}$.

	$\delta_i \mu\text{m}$	
X range mm	single specimen technique	from δ -R curve
.2-.7	408 ± 12	420 ± 23
1.2-2.4	246 ± 16	244 ± 49
2.7-4.8	217 ± 11	230 ± 45
5.3-6.3	196 ± 13	194 ± 24

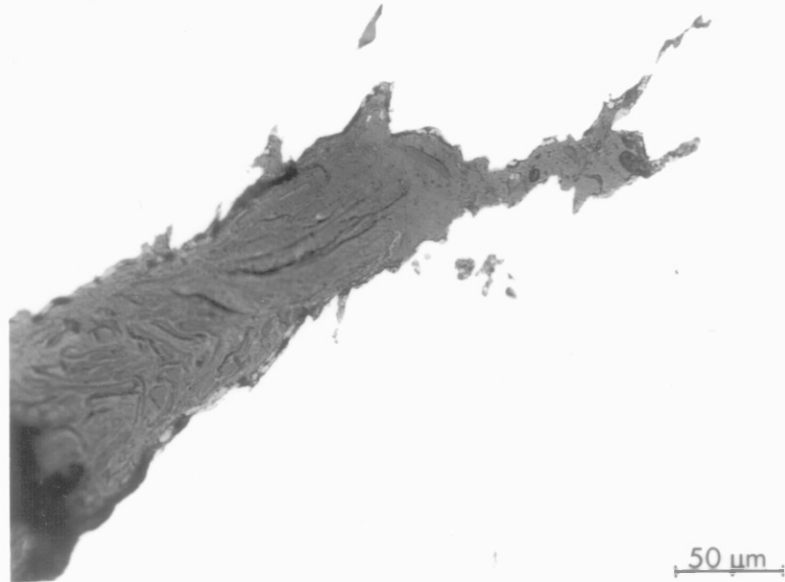
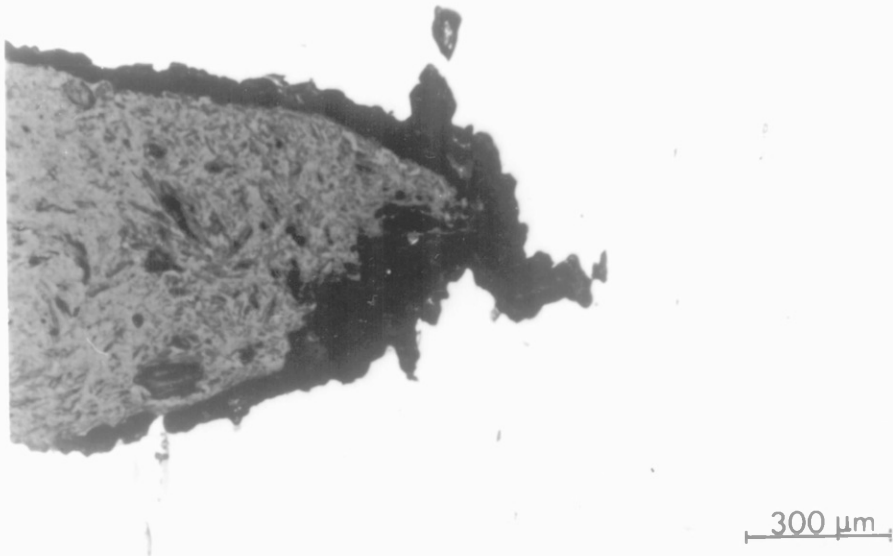


Plate 6.1 Optical micrograph taken perpendicular to the fatigue crack plane and parallel to the fatigue crack direction, near the edge of a non-sidegrooved double-edge-notched tension specimen



a)



b)

Plate 6.2 Optical micrograph taken perpendicular to the fatigue crack plane and parallel to the fatigue crack direction and a) in the intermediate or "low" crack tip constraint region and b) in the central region, of a non-sidegrooved double-edge-notched tension specimen

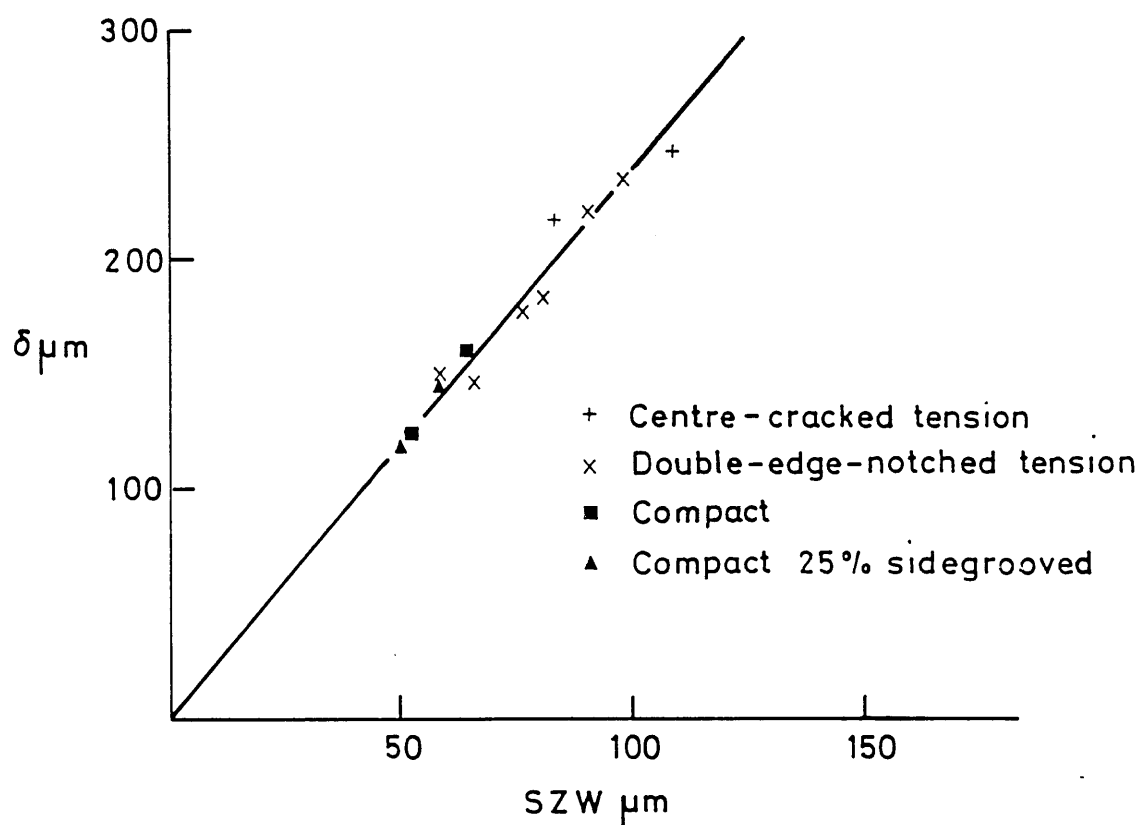


Figure 6.1 δ versus SZW, prior to crack initiation

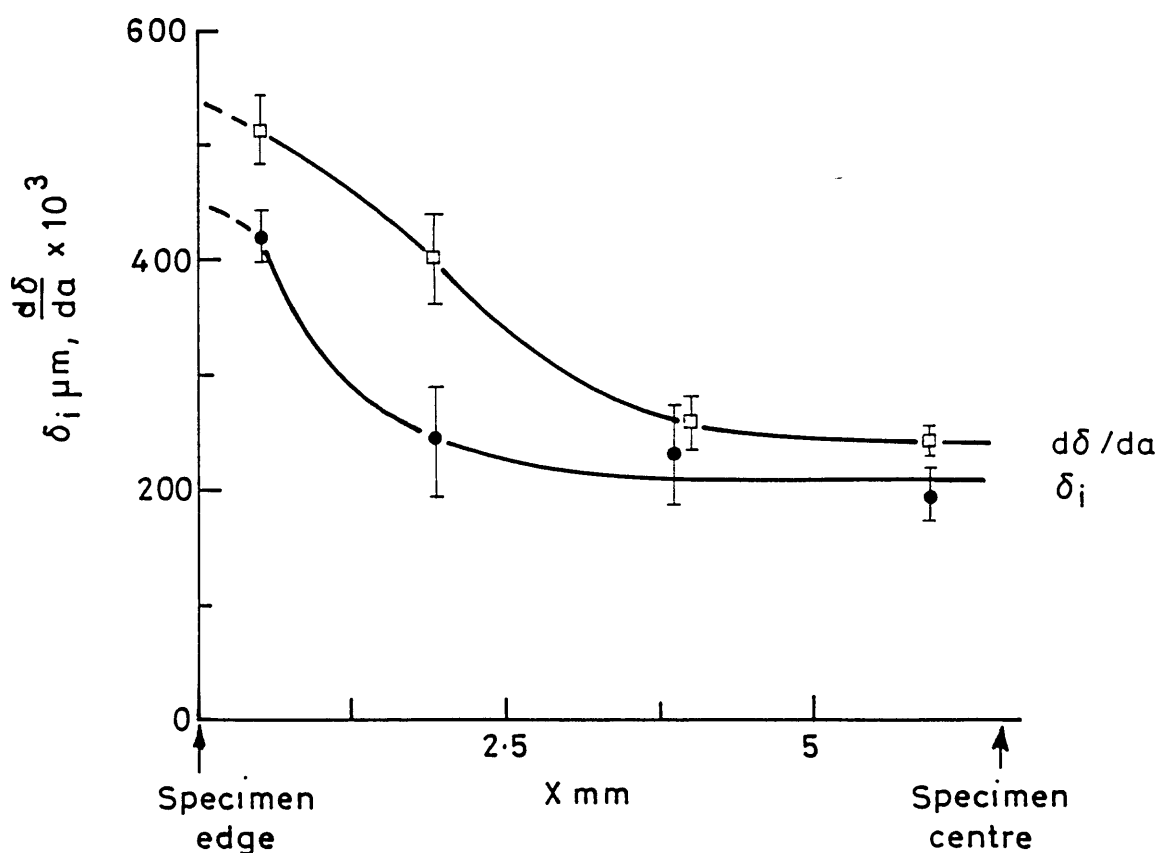


Figure 6.2 δ_i ($\pm$ standard error) and $d\delta/da$ ($\pm$ standard error) versus average X value in each range for non-sidegrooved compact specimens, $B=13\text{mm}$, $w=26\text{mm}$

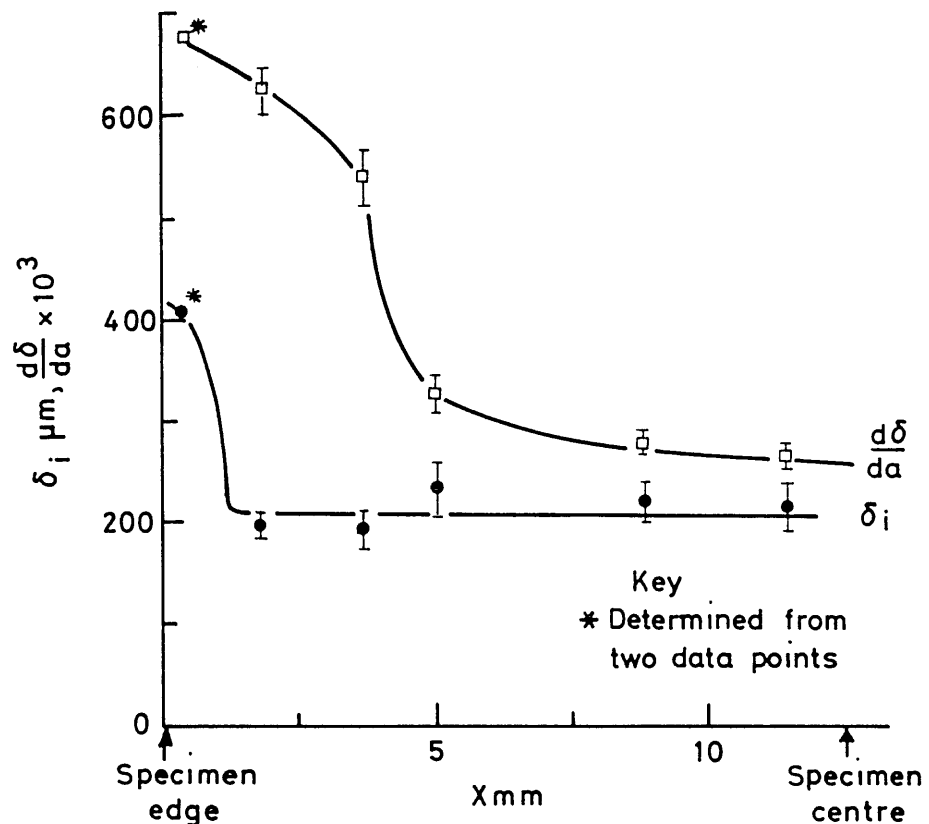


Figure 6.3 δ_i ($\pm$ standard error) and $d\delta/da$ ($\pm$ standard error) versus average X value in each range for non-sidegrooved double-edge-notched tension specimens, $B=25\text{mm}$, $2w=25\text{mm}$

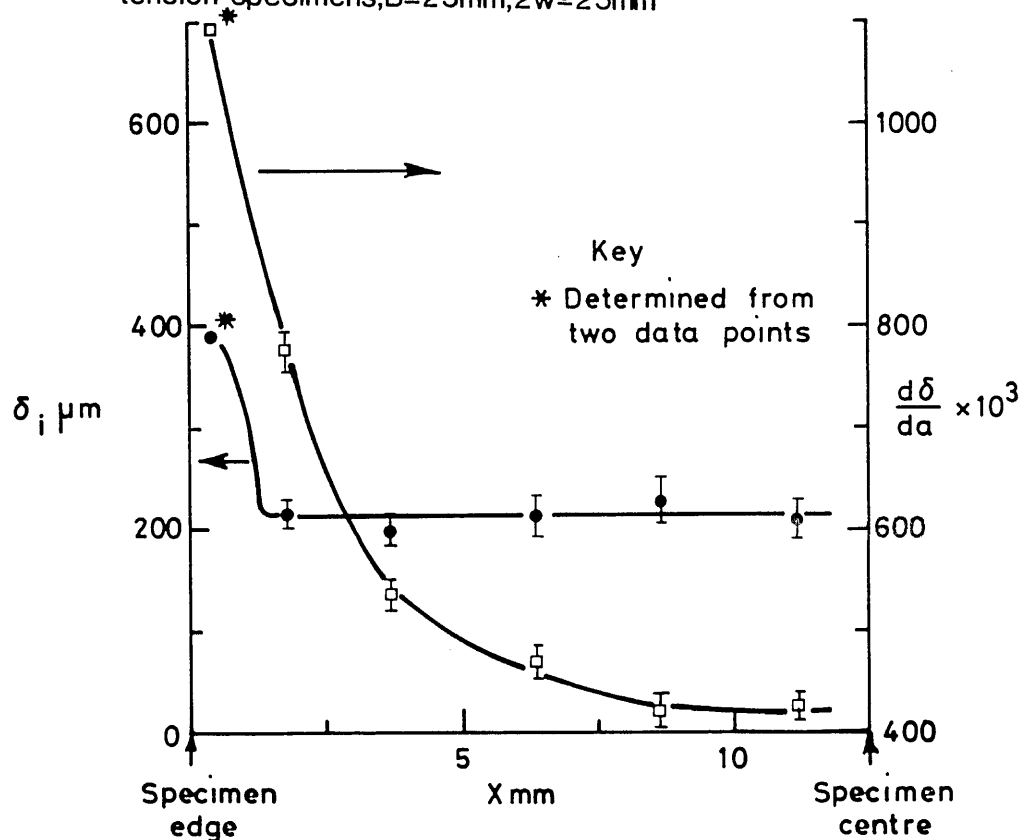


Figure 6.4 δ_i ($\pm$ standard error) and $d\delta/da$ ($\pm$ standard error) versus average X value in each range for non-sidegrooved centre-cracked tension specimens, $B=25\text{mm}$, $2w=25\text{mm}$

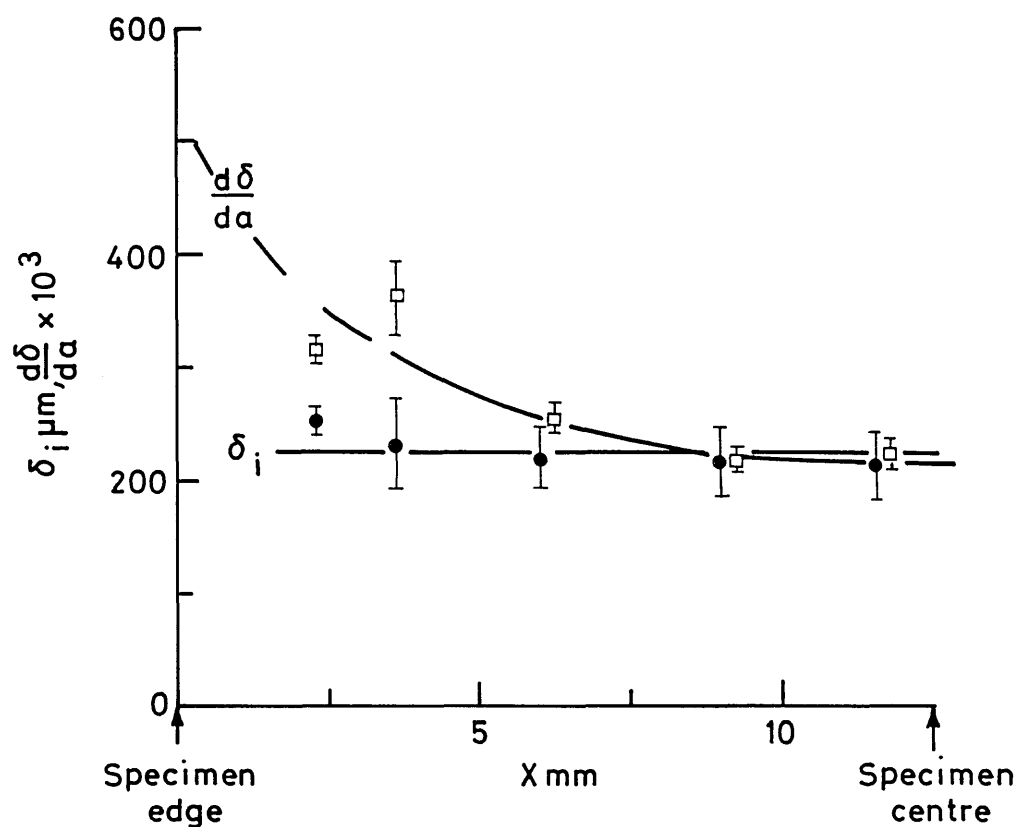


Figure 6.5 δ_i ($\pm$ standard error) and $d\delta/da$ ($\pm$ standard error) versus average X value in each range for non-sidegrooved three and four-point bend specimens, $B=25\text{mm}$, $w=50\text{mm}$

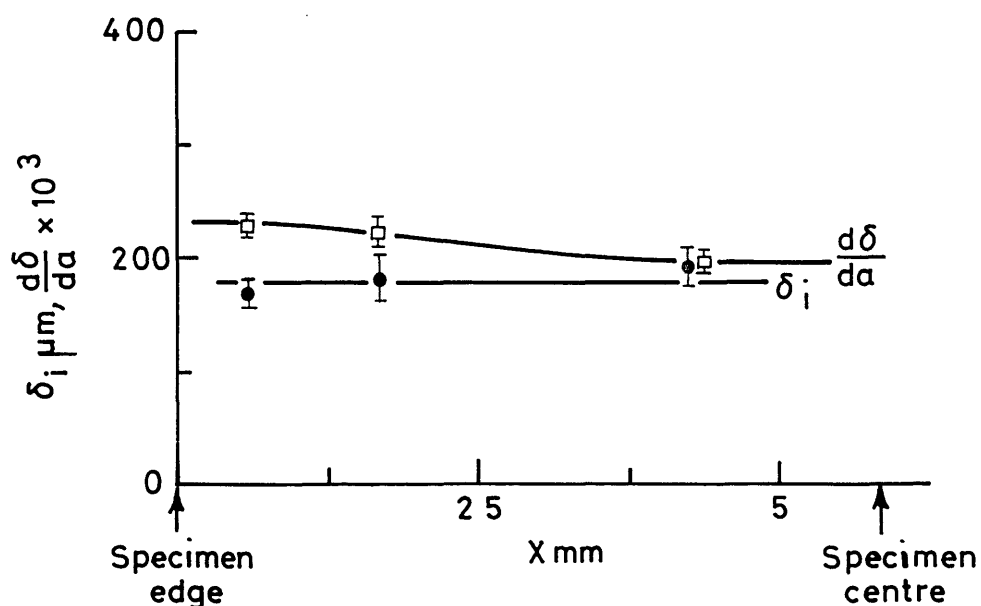


Figure 6.6 δ_i ($\pm$ standard error) and $d\delta/da$ ($\pm$ standard error) versus average X value in each range for compact specimens sidegrooved to a depth of 10% with gross $B=13\text{mm}$, $w=26\text{mm}$

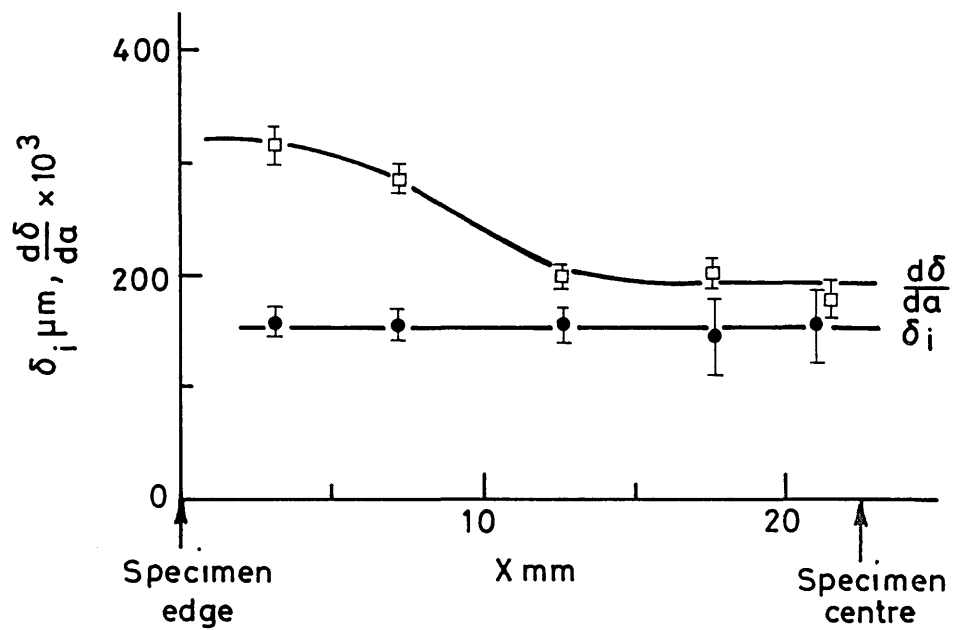


Figure 6.7 δ_i ($\pm$ standard error) and $d\delta/da$ ($\pm$ standard error) versus average X value in each range for compact specimens sidegrooved to a depth of 10% with gross $B=50\text{mm}$, $w=100\text{mm}$

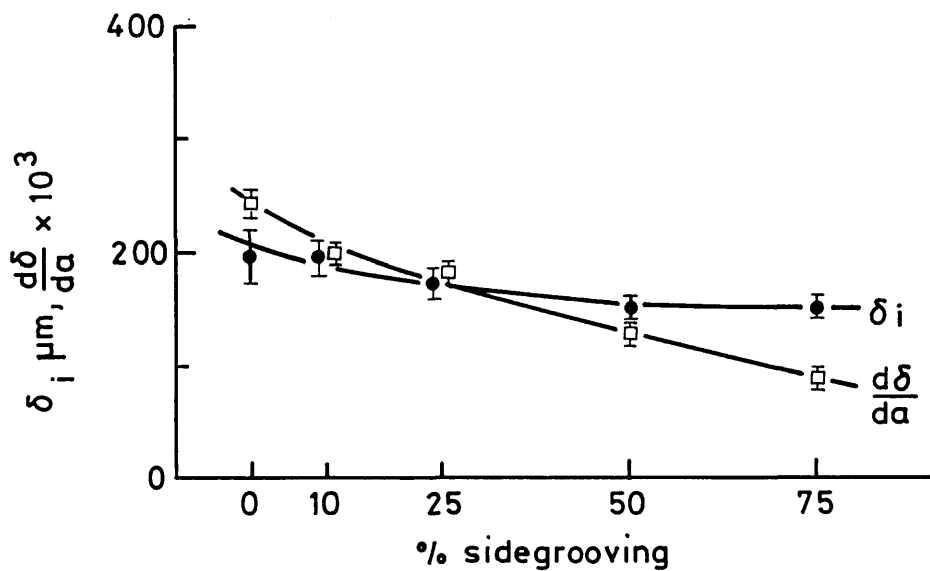


Figure 6.8 Effect of sidegroove depth on δ_i ($\pm$ standard error) and $d\delta/da$ ($\pm$ standard error) in the midsection of compact specimens, gross $B=13\text{mm}$, $w=26\text{mm}$

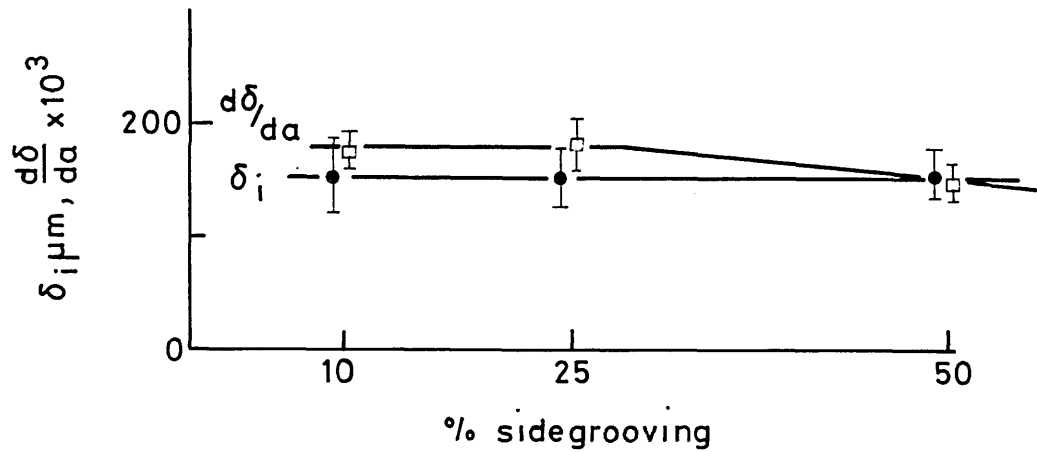


Figure 6.9 Effect of sidegroove depth on δ_i ($\pm$ standard error) and $d\delta/da$ ($\pm$ standard error) in the midsection of compact specimens, gross $B=50\text{mm}$, $w=100\text{mm}$

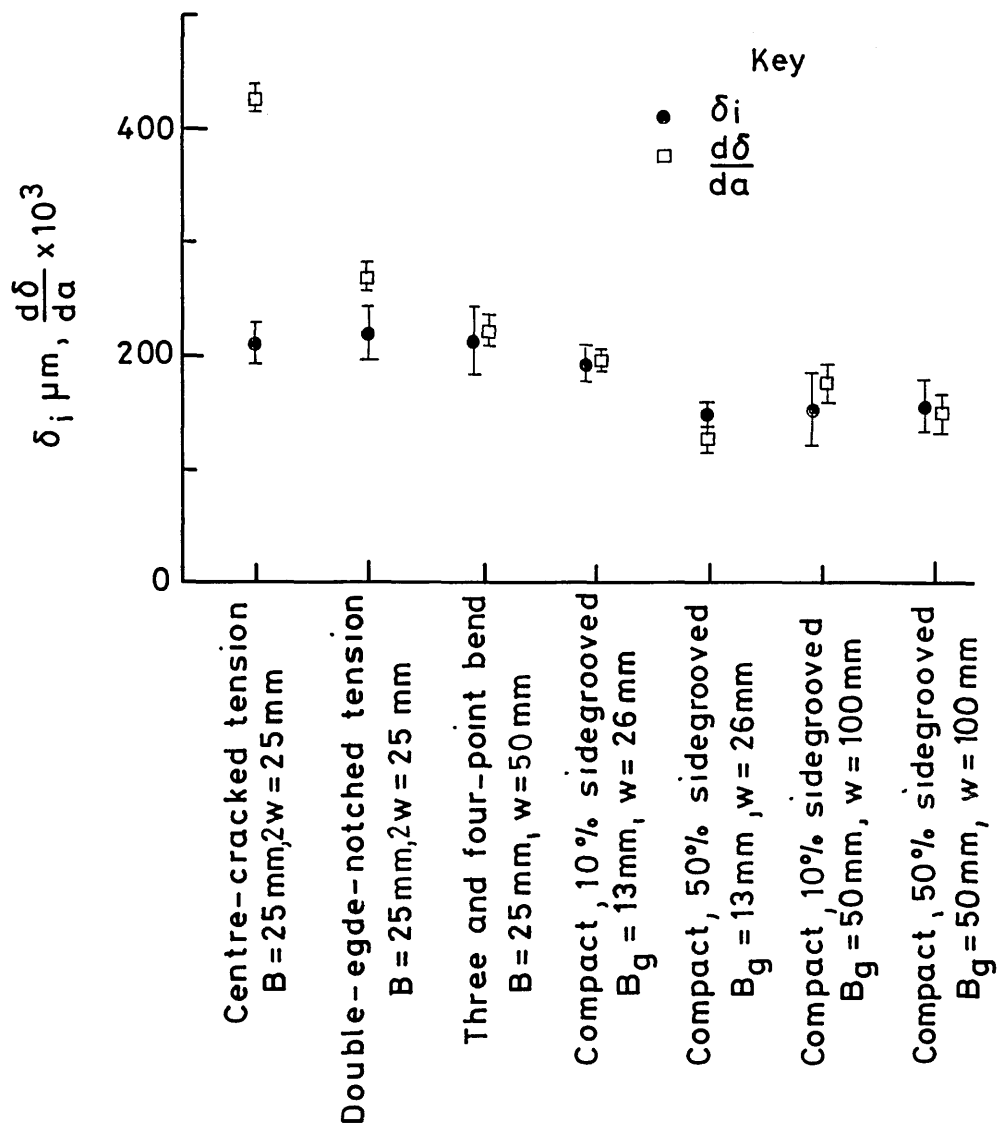


Figure 6.10 Effect of specimen size and geometry on δ_i ($\pm$ standard error) and $d\delta/da$ ($\pm$ standard error) in the midsection of the specimen

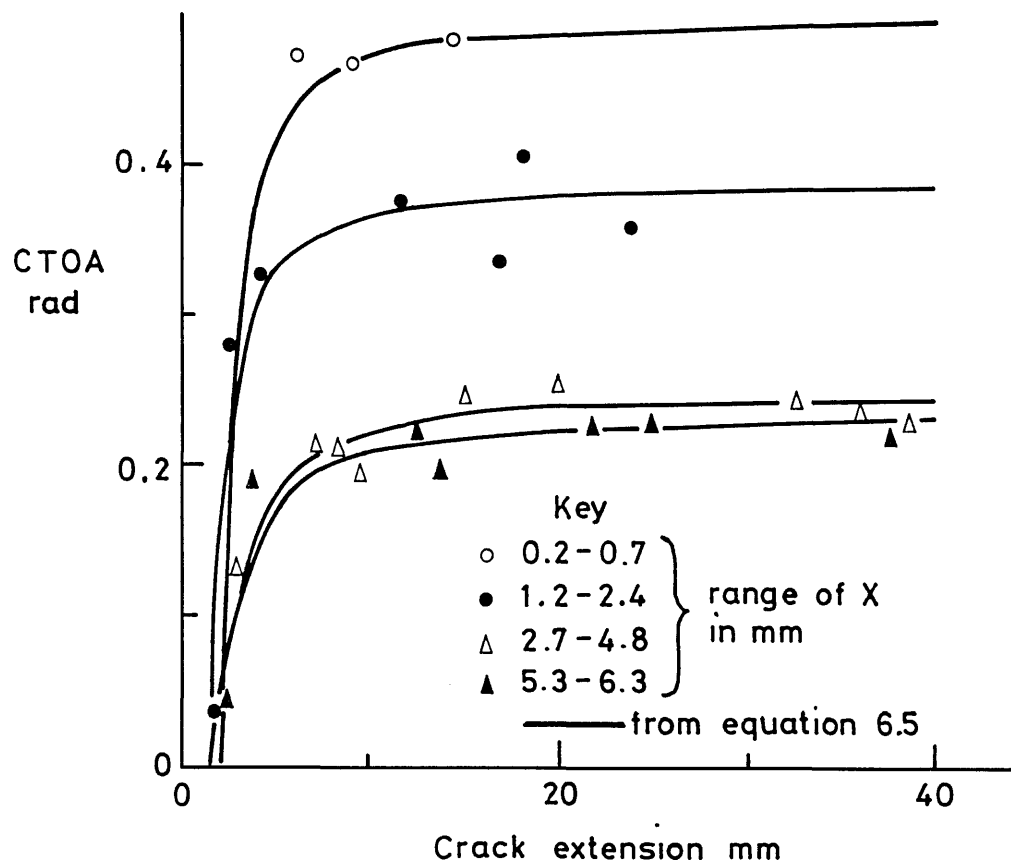


Figure 6.11 Experimentally measured and estimated CTOA values versus crack extension for various X ranges, for a non-sidegrooved compact specimen, $B=13\text{mm}$, $w=26\text{mm}$

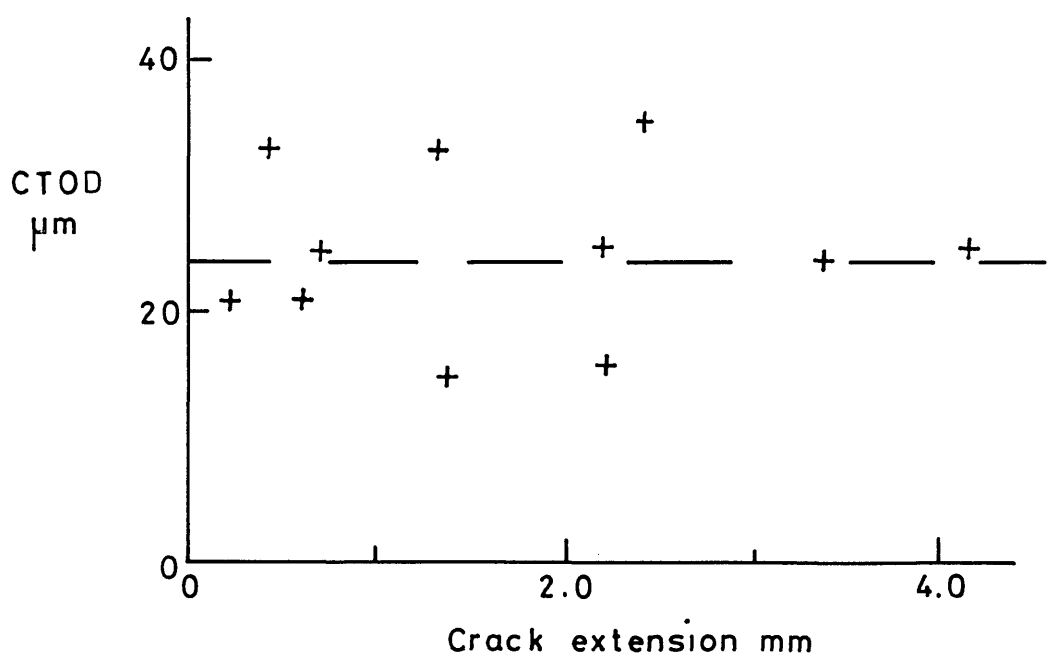


Figure 6.12 CTOD versus crack extension for non-sidegrooved double-edge-notched tension specimens in the X range 10-12.5mm

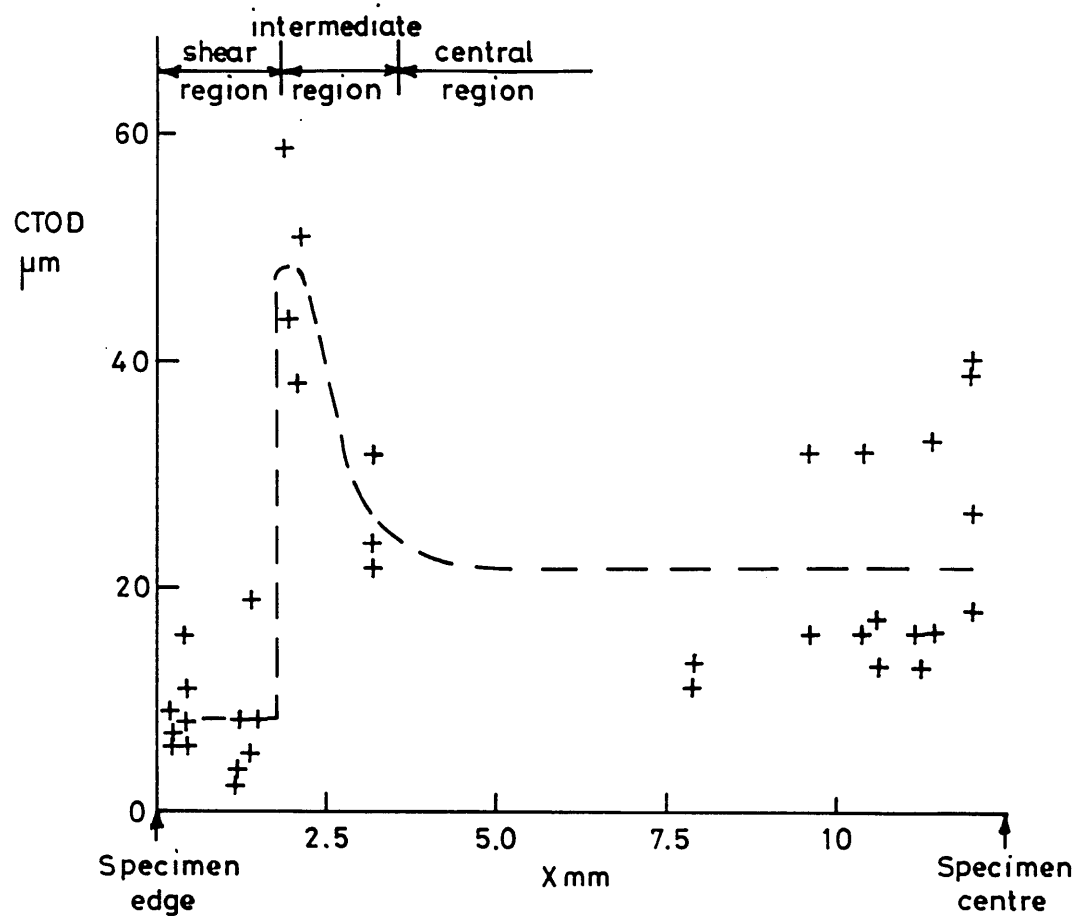


Figure 6.13 CTOD versus X for a non-sidegrooved double-edge-notched tension specimen, $B=25\text{mm}$, $2w=25\text{mm}$

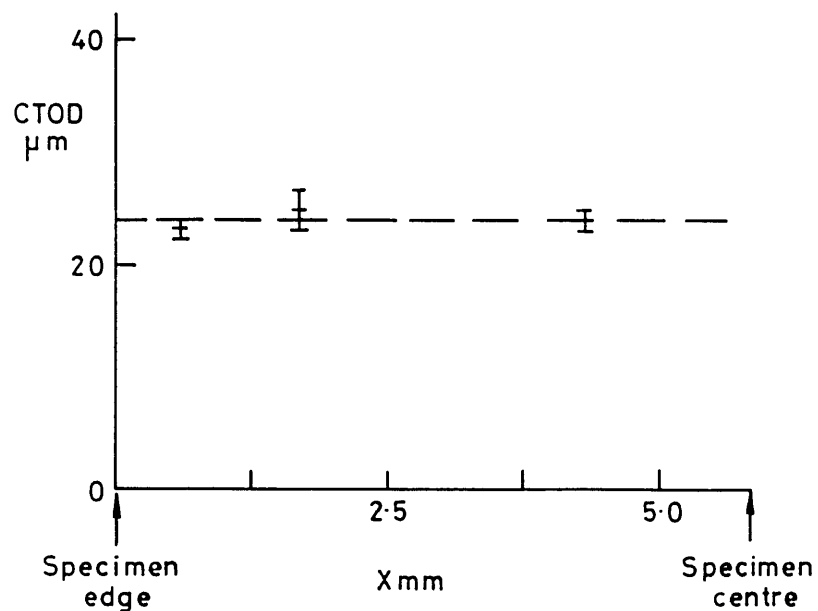


Figure 6.14 CTOD ($\pm$ standard error) versus X for compact specimens sidegrooved to a depth of 10% with gross $B=13\text{mm}$, $w=26\text{mm}$

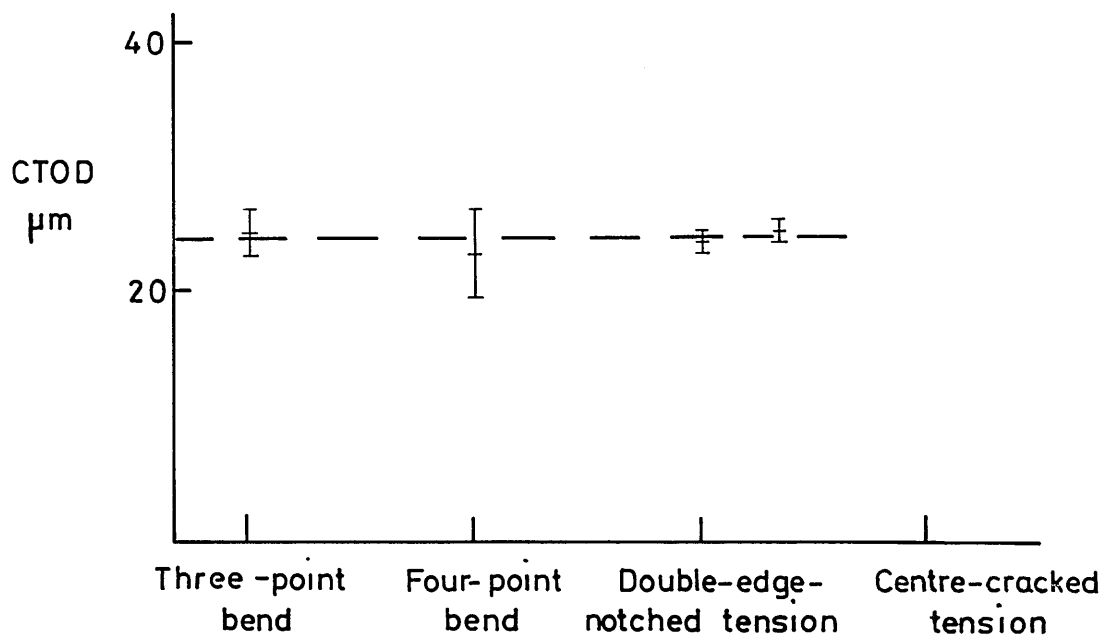


Figure 6.15 Effect of specimen geometry on CTOD($\pm$ standard error) in the central region of the specimen

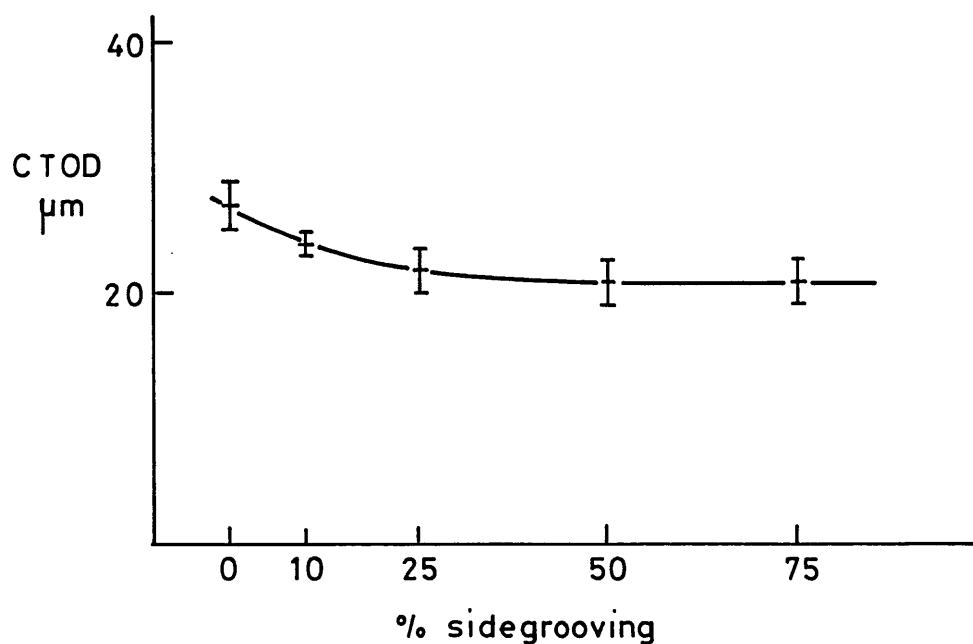


Figure 6.16 Effect of sidegroove depth on CTOD($\pm$ standard error) for compact specimens, gross $B=13\text{mm}$, $w=26\text{mm}$

Chapter 7
Discussion on Crack Initiation

7.1 Introduction

This chapter is divided into three main sections: the first section considers the effects of specimen size and geometry on the crack tip constraint and the crack initiation toughness of the material as characterised by J and δ , and the corresponding effect on the relationship between J and δ up to crack initiation; the second section considers the specimen size requirements to ensure plane strain crack tip conditions; the third section considers the determination of J prior to crack initiation.

7.2 Effect of Crack Tip Constraint

This section is further divided into three parts: the first gives an introduction into what is meant by crack tip constraint and its importance in affecting the conditions for crack initiation as characterised by δ ; the second considers the relationship between J and δ up to the onset of crack initiation; the third considers the effects of specimen size and geometry on the crack tip constraint and correspondingly the initiation toughness.

Microvoid coalescence, as discussed previously in section 2.1, involves void nucleation, growth and coalescence, of which void growth and also possibly void nucleation and coalescence are affected by the stress state or triaxiality. Triaxiality is a one value description of the stress state at a given point and is given by

$$\text{Triaxiality} = \frac{\sigma_m}{\bar{\sigma}}$$

where

$$\sigma_m = \frac{1}{3} (\sigma_{xx} + \sigma_{yy} + \sigma_{zz})$$

and

$$\bar{\sigma} = \left(\frac{1}{2} \left((\sigma_{xx} - \sigma_{yy})^2 + (\sigma_{xx} - \sigma_{zz})^2 + (\sigma_{yy} - \sigma_{zz})^2 \right) \right)^{1/2}$$

As the degree of triaxiality affects the ease of microvoid coalescence, and thus the local fracture strain, see section 2.1.5,

in principle the triaxiality ahead of the crack tip will affect the crack initiation toughness of the material. In practice, the situation is more complicated because there is rarely a constant level of triaxiality ahead of the crack tip. As an example, Figure 7.1 shows the level of triaxiality directly ahead of a blunted crack tip, under plane strain conditions, as a function of the distance from the crack tip (normalised by δ). This was determined from the opening stress (i.e. the stress normal to the crack plane, σ_{yy}) distribution ahead of a blunted crack tip under non work-hardening conditions, which was derived by Rice and Johnson (1970) using a Prantl field modified to take account of crack tip blunting. σ_{yy} can be related to triaxiality under plane strain conditions, as shown by Hancock and Cowling (1980), by

$$\frac{\sigma_{yy}}{\sigma} = \text{Triaxiality} + \frac{1}{\sqrt{3}}$$

It can be seen from Figure 7.1 that the level of triaxiality decreases in the immediate crack tip vicinity, and so will vary across the process zone where void nucleation, growth and coalescence occur. However, the crack tip constraint can be considered as the effective triaxiality in the process zone, that is the level of triaxiality which, if constant across the process zone, would result in the same local fracture strain. Therefore, any factors which affect the level of triaxiality in the process zone and thus the crack tip constraint, will affect the crack initiation toughness.

One factor which will affect the crack tip constraint is the through thickness stress component, σ_{zz} . Under plane stress conditions, i.e. in thin specimens or close to the specimen edge, $\sigma_{zz}=0$, while under plane strain conditions, i.e. in the centre of thick specimens

$$\sigma_{zz} = \nu(\sigma_{yy} + \sigma_{xx}) \quad \text{under elastic conditions}$$

$$\sigma_{zz} = \frac{1}{2}(\sigma_{yy} + \sigma_{xx}) \quad \text{under fully plastic conditions.}$$

where ν is Poisson's ratio which is about 0.3 for steel. The effect of σ_{zz} on the crack tip constraint can be seen by considering the level of triaxiality in the plastic zone under small scale yielding

and plane stress conditions. For a non work-hardening material, the triaxiality will be $1/3$ at the blunted crack tip, but may increase up to $2/3$ within the plastic zone, given $0 \leq \sigma_{xx} \leq \sigma_{yy}$. However, on comparison with Figure 7.1, it can be seen that the level of triaxiality ahead of a blunted crack tip, and thus the crack tip constraint, is significantly lower under plane stress than plane strain conditions.

Another factor which could affect the crack tip constraint under net section yielding conditions is the specimen geometry. A number of authors, for example, Hancock and Cowling (1980), have suggested that under these conditions, the modified Prantl field of Rice and Johnson (1970) resulting in a high crack tip constraint, will only occur in geometries with high plastic constraint factors, such as the deeply-notched DEN geometry and possibly also the deeply-notched bend and compact geometries. In the case of the CCT geometry, which has a low plastic constraint factor, the slip line field which is suggested, see Figure 7.2, would only give a triaxiality of 0.577 in the plastic region under plane strain conditions. Therefore, on comparison with Figure 7.1, which gives the level of triaxiality ahead of the crack tip suggested for geometries with high plastic constraint factors, it can be seen that specimen geometry may affect the crack tip constraint.

The through thickness stress, σ_{zz} , will vary across the crack front, being zero at the specimen edge where plane stress conditions prevail, but increasing towards the centre of the specimen, up to that for plane strain conditions if the specimen is sufficiently thick. Therefore, the crack tip constraint will vary across the crack front, particularly if there is a modified Prantl field present in the central region of the specimen. De Lorenzi and Shih (1983) have determined from finite element analysis the "degree of plane strain" ($\sigma_{zz}/(\sigma_{xx} + \sigma_{yy})$) across the crack front of a non-sidegrooved compact specimen, $B=127\text{mm}$, $w=254\text{mm}$, see Figure 7.3. It is important to note that with increasing displacement and thus plasticity, this stress ratio for plane strain conditions will increase from a value of ν (≈ 0.3) under elastic conditions to 0.5 under fully plastic conditions. It can be seen from Figure 7.3 that

the change from plane stress to plane strain conditions occurs over a relatively narrow region close to the specimen edge. This can be compared to the profiles of δ_i across the crack front which were determined from the compact, DEN and CCT specimens, see Figures 6.2, 6.3 and 6.4 respectively. These figures show that there is a relatively narrow region close to the specimen edge where δ_i increases significantly. Therefore, it would appear that towards the specimen edge, there is a region in which the crack tip constraint decreases, as a result of the decrease in the through thickness stress component, σ_{zz} , which causes an increase in the initiation toughness and thus δ_i . It is worth noting that the size of the transition region in Figure 7.3 is much larger than that in Figures 6.2, 6.3 and 6.4; this is a result of differences in specimen size as discussed later. The dependence of δ_i on crack tip constraint has been suggested by a number of other authors, after finding that δ_i decreased with increasing specimen thickness, see for example Smith and Knott (1971).

Near the edge of non-sidegrooved specimens, the crack initiation and subsequent growth occurs by a shear mechanism, as illustrated in plate 7.1, so that crack growth occurs at an angle to the fatigue crack direction, as illustrated in plate 6.1. This mechanism contrasts with that operating in the rest of the specimen where the crack growth occurs in the fatigue crack direction. An interesting phenomenon of this shear mechanism is that it is not associated with a stretch zone, for as shown in plate 7.1, there is no stretch zone very close to the specimen edge. Therefore, this would suggest that when the crack tip constraint is sufficiently low, around that for plane stress, the deformation processes occurring at the crack tip are such that no crack advance occurs as the crack opens on initial loading of the specimen. Further into the specimen, there is a small stretch zone, even though crack initiation occurred by the shear mechanism, see plate 7.1. This would suggest that in this region the crack tip constraint is initially sufficiently high, that the deformation processes occurring at the crack tip are those found in the central region of the specimen, which result in the formation of a stretch zone as the crack opens. However, on further loading, the crack tip constraint

decreases, so that the crack tip deformation processes change to those associated with the shear mechanism, and thus no further increase in the stretch zone occurs. This decrease in crack tip constraint with further loading, in the region close to the specimen edge, is consistent with the observation that with increasing loading after crack initiation, the shear lip size and thus the size of the region of low crack tip constraint increases. This aspect is considered further in section 7.3.

The size of the shear region at crack initiation, i.e. where the shear mechanism operates, is typically 0.1-0.2mm, for the range of specimen sizes investigated in this study. However, the size of the region in non-sidegrooved specimens, in which δ_i is higher than in the central region of the specimen, would appear to be in the range 1-1.5mm, see Figures 6.2, 6.3 and 6.4; although, the actual size of this region is difficult to determine accurately in each case, as the number of δ - Δa points are limited in the near surface region. Therefore, there is a region between the shear and the central regions of the specimen in which the mechanism of crack initiation is the same as in the central region, but the crack tip constraint is lower and so consequently δ_i is higher. This region is called the "low" crack tip constraint region for convenience. There is additional evidence to confirm that the crack tip constraint is lower in this region, as less voids were found around the crack tip in this region compared to the central region of the specimen, although the number of inclusions seemed to be similar in both cases, see plate 6.2. A smaller number of voids is indicative of a lower crack tip constraint because void growth, and probably also void nucleation, are affected by triaxiality.

The situation in non-sidegrooved specimens contrasts markedly with that in sidegrooved specimens. This is well illustrated by the finite element results of de Lorenzi and Shih (1983), which show the "degree of plane strain" ($\sigma_{zz}/(\sigma_{xx} + \sigma_{yy})$) across the crack front of a compact specimen sidegrooved to a depth of 25%, with gross $B=127\text{mm}$, $w=254\text{mm}$, see Figure 7.4. Comparison of Figures 7.3 and 7.4 shows that by sidegrooving a specimen to a depth of 25%, the stress ratio is now apparently similar across the whole

crack front. Although not evident in Figure 7.4, the stress ratio will decrease to zero at the specimen edge as it is a free surface. However, it would appear that the size of the transition region over which the stress ratio decreases is very small. Therefore, the results in Figure 7.4 would suggest that in specimens sidegrooved to a depth of 25%, the crack tip constraint is similar across almost all the crack front. This is in agreement with the experimental finding that there is no apparent variation in δ_i across the crack front of specimens sidegrooved to a depth of 25%, although no δ - Δa data points were obtained very close to the specimen edge, i.e. $X < 0.2\text{mm}$. The agreement between these two results again confirms the dependence of δ_i on crack tip constraint. In fact, no variation of δ_i across the crack front was found in any of the sidegrooved specimens investigated, see for example Figures 6.6 and 6.7. Therefore, it would appear that for specimens sidegrooved to a depth of between 10 and 75%, there is no significant variation in crack tip constraint across the crack front, except possibly very close to the specimen edge. As a consequence of this, there is no significant sized region of low crack tip constraint in which the shear mechanism can operate, which explains why no shear lip is evident on the fracture surfaces, even for specimens sidegrooved to a depth of only 10%.

Summarising so far, the crack tip constraint has been shown to be important in governing the initiation toughness of a material, as characterised by δ_i . The crack tip constraint has been shown to be affected by the through thickness stress component and it may also be affected by the specimen geometry under net section yielding conditions. There is a variation in the crack tip constraint across the crack front of specimens, from that for plane stress at the specimen edge up to that for plane strain in the central region of the specimen. The transition in crack tip constraint occurs in a relatively narrow region close to the specimen edge, although the size of this region is very much smaller in sidegrooved specimens. As a result, δ_i was found to increase significantly near the specimen edge of non-sidegrooved specimens, but not in sidegrooved specimens, at least when the sidegrooved depth was 10% or more. In addition, there is a sufficiently sized region close to the edge of

non-sidegrooved specimens, where the crack tip constraint is sufficiently low, about that for plane stress, so as to allow a shear mechanism to operate.

In this study, two parameters J and δ were used to characterise the initiation toughness of the material. However, only δ_i could be determined at various positions across the crack front. J_i derived from SZW_c could only be determined as an average value in the central $2/3$ of the specimen over which SZW was measured. It would appear from the δ_i profiles across the crack front that this J_i value is that associated with the "high" crack tip constraint region of the specimen. It was only possible to determine J_i from SZW measurements taken in the central region of the specimen for the following reason. There is a variation in applied J across the crack front of a loaded specimen particularly for non-sidegrooved specimens, see section 2.3.2.3, and as shown by Miyamoto and Kikuchi (1981), it is only in the central region of the specimen that the actual applied J is approximately equal to that calculated experimentally from the area under the load-displacement trace. Therefore, it is only possible to construct a reasonably accurate J blunting line, from which J_i can be determined from SZW_c , for the central region of the specimen.

The relationship between J and δ up to crack initiation can be determined from the J blunting line and the relationship between δ and SZW , i.e.

$$J = m \sigma_y \delta \quad 7.1$$

where $m=1.28$

The results of this study suggest that the relationship between δ and SZW is independent of specimen geometry, sidegroove depth and also the position across the crack front, except very close to the specimen edge in the shear region (i.e. $X < 0.2\text{mm}$), as the shear mechanism, is not associated with the formation of a stretch zone, see section 6.1.2. Therefore, it would appear that the relationship is independent of crack tip constraint, provided the constraint is not low enough so that the shear mechanism operates.

This is because the relationship is the same in the "low" crack tip constraint region (between the shear and central regions) and the "high" crack tip constraint region (central region) of non-sidegrooved specimens, and because the relationship is independent of sidegroove depth, even though δ_i and thus the crack tip constraint are affected, see Figure 5.6. This is perhaps not a surprising result as both δ and SZW are measured from the crack tip profile. The fact that the δ -SZW relationship was found independent of specimen geometry conflicts with the finite element results of Sumpter et al (1973) which suggested that the crack tip profile was geometry dependent. However, as these authors admitted that they could not model accurately the crack tip profile, their findings of a geometry dependence may be inaccurate.

The results of this study suggest that the J blunting line relationship is independent of specimen geometry and sidegroove depth, see section 5.4.3. As the δ -SZW relationship up to crack initiation is also independent of these factors, the same will apply to the J- δ relationship up to crack initiation, given by equation 7.1. This suggests that this J- δ relationship is independent of crack tip constraint, as δ_i , and thus crack tip constraint, are affected by sidegroove depth, see Figure 5.6. However, a number of other authors have suggested that the J- δ relationship is different under plane stress and plane strain conditions, see for example Sumpter and Turner (1976b) and Shih (1981). Sumpter and Turner (1976b) showed that for the same J, the displacements at the crack tip and the strains ahead of the crack tip are much larger under plane stress than plane strain conditions. These apparently inconsistent results can be explained if the J- δ relationship is only affected by crack tip constraint when it results in a significant change in the deformation characteristics around the crack tip. This is plausible, because when this occurs, it is likely that a change in δ and thus level of plastic strain around the crack tip, can occur at a given J value and thus a given amount of absorbed energy in the specimen. As a stretch zone is formed at all positions across the crack front in all the specimens, except where the shear mechanism is operating, it would appear that the deformation characteristics around the crack tip are only

significantly changed when the shear mechanism is operating. Therefore, this would suggest that the J - δ relationship is only affected by crack tip constraint when the constraint is sufficiently low that the shear mechanism can operate.

The finding that the J - δ relationship up to crack initiation is independent of specimen geometry is in agreement with the results of Robinson (1976), who also measured δ from silicone-rubber crack impressions and found that the J - δ relationship for the three-point bend geometry was similar to that for the centre-cracked tension geometry. In this study, the geometry independence of the J - δ relationship can not be explained by suggesting that the loading mode in the CCT specimens is not tensile, because as shown in plate 7.2, the crack faces of a CCT specimen with a 1.5mm wide spark machined slot instead of a crack, were found to remain parallel, even after the specimen had been loaded to a relatively large displacement. However, a number of other authors have reported a geometry dependence of the J - δ relationship, for example Turner (1979) showed analytically, by relating J and δ via their respective relationships with the displacement, that

$$J = (1.67 - 1.87) \sigma_y \delta \text{ for three-point bend}$$

$$J = \sigma_y \delta \text{ for CCT.}$$

The main error in this type of analysis is the assumed relationship between δ and displacement for the CCT geometry. Turner (1979) assumed that δ was equal to the plastic displacement. The relationship found between the load-point displacement (q) and δ for the CCT specimens used in this study is

$$\delta = 0.53 q \quad 7.2$$

A similar relationship is found between the mouth opening displacement and δ , which is in turn similar to that found by Robinson (1976). If the relationship between J and δ for the CCT geometry was to be determined using equation 7.2, it would be similar to that for the three-point bend geometry, in agreement with

the experimental findings of this study. Sumpter and Turner (1976b) have shown by finite element analysis that the J - δ relationship is geometry dependent. However, as with their findings of a geometry dependent crack tip profile, their results may be incorrect due to inaccurate modelling of the crack tip region.

The value of m of 1.28 in equation 7.1 is within the range 1 to 3, which is usually reported, see section 2.3.4. It is not readily possible to check the accuracy of this m value by comparing it with other reported values because in most cases either the method of determining δ or the material is different, both of which are likely to affect the value of m . The exception to this are the values of m reported by Robinson (1976) who used a similar material to that in this study, and who also measured δ from silicone-rubber crack tip impressions. However, the m values quoted by Robinson of 2.2 and 2.6 are markedly different to that found in this study. The only likely reason for this is that Robinson measured δ from a different position on the crack profile which can, prior to crack initiation, result in a significantly different δ being recorded.

Therefore, summarising the results on the J - δ relationship up to crack initiation, it would appear that the relationship is independent of specimen geometry and crack tip constraint, at the higher crack tip constraints up to that for plane strain. The evidence suggests that when the crack tip constraint is sufficiently low, so that the shear mechanism can operate (i.e. about that for plane stress), there is a different J - δ relationship, possibly due to a change in the deformation characteristics around the crack tip. The value of m relating J and δ , determined in this study, is within the range commonly found by other authors, although it would appear that the value of m is highly dependent on the exact procedure by which δ is determined.

In the following section, the effects of specimen size and geometry on the initiation toughness, J_i and δ_i , and thus the crack tip constraint, are discussed. In doing so, the relationship between J and δ , and thus J_i and δ_i , given by equation 7.1, will be

assumed applicable for the central region of the specimens investigated, as would appear appropriate from the previous discussion.

J_i and δ_i in the central region of non-sidegrooved specimens of a given thickness, would appear to be independent of specimen geometry, see Figures 5.15 and 6.10: although these values are not those for plane strain crack tip conditions, as can be seen by comparison of these figures with Figures 5.6 and 5.11. Therefore, it would appear that specimen geometry, and thus the plastic constraint factor, do not affect the crack tip constraint in the central region of specimens of a given thickness. However, there is evidence to show that the size of the shear region at crack initiation is affected by specimen geometry. The size of the region where crack initiation occurred by the shear mechanism was about 0.2mm for the CCT geometry, 0.16mm for the compact geometry, and 0.13 for the DEN geometry for specimens of a given thickness. This effect of specimen geometry on the size of the shear region is confirmed by the fact that the size of the shear lip, at a given 9 point average crack extension, is larger for the CCT specimens than the same sized DEN specimens, see section 8.4.2.2. Therefore, it would appear that as the plastic constraint factor of the geometry decreases, it is progressively more difficult to maintain high crack tip constraint, and so as a consequence the size of the region of low crack tip constraint is increased. From Figures 6.2, 6.3 and 6.4, there is no apparent evidence of an effect of specimen geometry on the size of the region of higher δ_i values, and thus lower crack tip constraint. However, this could be easily attributed to the fact that the size of this region is difficult to determine accurately from the figures, because of the limited number of δ - Δa points and thus δ_i values in this region.

J_i and thus presumably δ_i , in the central region of non-sidegrooved specimens, was found to be affected by specimen thickness but not ligament size, see Figures 5.9 and 5.14. J_i was found to decrease with increasing specimen thickness to a plateau value, assumed to be that for plane strain; indeed, the same plateau J_i value was obtained from suitably sidegrooved specimens,

compare Figures 5.9 and 5.11. The ligament size, although not important over the range investigated, would presumably affect the J_i value if sufficiently small, as suggested by the finite element results of McMeeking and Parks (1979). However, it would appear that the thickness criterion for plane strain crack tip conditions is significantly more stringent than the ligament criterion. This would suggest that a larger thickness dimension is required to obtain the through thickness stress component for plane strain in the central region to the specimen, than the ligament dimension to shield the crack tip region from the influence of the back face of the specimen. An interesting point to note concerns the profile of δ_i across the crack front. As shown in Figure 6.2, δ_i and thus the crack tip constraint, is similar across the central region of the specimen, even when plane strain crack tip conditions have not been obtained, whereas it might have been expected that the crack tip constraint would increase continually towards the midsection of the specimen.

The size of non-sidegrooved specimens would appear to affect the size of the region, where the crack tip constraint is lower than that in the central region of the specimen, for from the δ_i measurements on compact specimens, $B=13\text{mm}$, $w=26\text{mm}$, the size of this region is about 1.5mm, see Figure 6.2, whereas from the finite element results on a compact specimen, $B=127\text{mm}$, $w=254\text{mm}$, the size of this region is about 15-25mm, see Figure 7.3 This result is consistent with the effect of specimen dimensions on the size of the shear lip region, see section 8.4.2.2, because when the ligament to thickness ratio, b/B , is about unity, as in these two cases for which $b/B \approx 0.9$ and 0.8 respectively, the shear lip size increases with ligament size, over the range investigated.

The presence of sidegrooves affects significantly the variation of crack tip constraint across the crack front, for even shallow sidegrooves (i.e. to a depth of 10%) induce a relatively uniform δ_i and thus crack tip constraint across the crack front, see for example Figure 6.6. However, very close to the specimen edge plane stress conditions will prevail, although the size of this region is so small that shear lips do not form. In addition, the

presence of sidegrooves can increase the crack tip constraint up to that for plane strain, as shown by the decrease in δ_i to a plateau value with increasing sidegroove depth, see Figure 6.8.

In summary, the crack tip constraint has been shown to be important in governing the crack initiation toughness of a mild steel as characterised by J and δ . The relationship between these two parameters has been shown to be independent of specimen geometry and it is probably independent of crack tip constraint provided the crack tip constraint is not low enough so that the shear mechanism can operate. (i.e. about that for plane stress). It has been suggested that a change in this relationship, is a result of a change in the deformation characteristic around the crack tip.

δ_i and thus crack tip constraint, are found to vary across a relatively narrow transition region in non-sidegrooved specimens, and close to the specimen edge a shear mechanism operates. There is evidence to suggest that the size of the transition region is dependent on the geometry and dimensions of the specimen.

The crack tip constraint in the centre of the specimen is more dependent on specimen thickness than ligament size, but it is not dependent on specimen geometry for specimens of a given thickness.

Sidegrooving the specimen induces a relatively uniform δ_i and thus crack tip constraint across the whole crack front, except very close to the specimen edge, where the crack tip constraint will be lower. This has the effect of stopping the shear mechanism from operating at the specimen edge. In addition, the crack tip constraint is increased up to that for plane strain, except very close to the specimen edge.

7.3 Specimen Size Requirements for Plane Strain Crack Tip Conditions

In this section, the specimen size requirements for plane strain crack tip conditions are discussed. These are conventionally written in terms of J/σ_f or δ , following the suggestion of Paris in

a written comment on the early work of Begley and Landes (1972). Paris suggested that in order to ensure plane strain crack tip conditions within the fracture process zone around the crack tip, the specimen dimensions should be larger than a given multiple of δ or J/σ_f . This is because δ or J/σ_f represents the size of the process zone in which voids nucleate, grow and coalesce to the crack tip. The size requirements given in the ASTM J_{1C} testing standard E813-81 are

$$b, B \geq \alpha \frac{J_{1C}}{\sigma_f} \quad 7.3$$

where $\alpha=25$.

This value of α is in the range 25 to 50 which is generally reported, see section 2.3.5.2. In this study, lower bound J_i values in non-sidegrooved specimens were only obtained when the thickness was in excess of 37.5 to 50mm, see Figure 5.9. This suggests a specimen thickness size requirement for plane strain crack tip conditions in non-sidegrooved specimens given by equation 5.2

$$B \geq \alpha \frac{J_i}{\sigma_f} \quad 5.2$$

where $270 \leq \alpha \leq 360$.

This thickness size requirement in terms of δ , using equation 7.1, is

$$B \geq \beta \delta_i \quad 7.4$$

where $250 \leq \beta \leq 330$.

The thickness size requirement can be reduced by sidegrooving the specimen to a depth of 25-50%, indeed specimens of thickness 13mm or possibly less are then of sufficient size to promote plane strain crack tip conditions, see Figure 5.11. This suggests a conservative estimate of the thickness size requirement for specimens sidegrooved to a depth of 25-50% given by equation 5.3

$$\text{Gross thickness, } B_g > 90 \frac{J_i}{\sigma_f} \quad 5.3$$

This thickness size requirement in terms of δ_i is

$$\text{Gross thickness, } B_g > 85 \delta_i \quad 7.5$$

The thickness requirements found in this study are considerably more stringent than those given in ASTM E813-81, compare equations 5.2 and 5.3 with 7.3. In the case of the requirement for sidegrooved specimens, this may be because equation 5.3 is only a conservative estimate of the actual requirement, although this is not the case for the thickness requirement for non-sidegrooved specimens, given by equation 5.2. However, this requirement is in agreement with that suggested from the results of Sakata et al (1983), who have carried out the only known finite element study of the effect of specimen thickness on the J value in the centre of the specimen. By assuming that plane strain crack tip conditions break down when the J value at the midsection of the specimen deviates by more than 3% from the plane strain value, at a given displacement, a thickness size requirement for plane strain crack tip conditions can be determined from their results, see section 2.3.5.1, i.e.

$$B > 320 \frac{J}{\sigma_y} \quad 2.31$$

The coefficient in equation 2.31 would be lower if the requirement was written in terms of the flow stress rather than the yield stress, and also if a deviation larger than 3% between the J values is permitted. For example, if plane strain crack tip conditions only break down when the J values deviated by more than 6%, the coefficient would be about 200. Although this result gives some support for a thickness size requirement in excess of that given in the J_{1C} testing standard E813-81, it does not explain why the thickness size requirement found in this study is considerably larger than that reported by other authors, whose results were used to obtain the requirement given in ASTM E813-81.

Part of this discrepancy is because the requirement given in ASTM E813-81 is based on J_{1C} rather than J_i values, which in this study overestimate the J_i values by typically a factor of two, see for example Figure 5.14. This has the effect of apparently making the thickness criterion, when based on J_{1C} , less stringent than that based on J_i . The reason why J_{1C} overestimates J_i is because in determining J_{1C} , a linear line is fitted to the J- Δa points lying

between the two exclusion lines; although as the J-R is non linear, particularly at small crack extensions, see Figure 5.8, the linear line when extrapolated to the blunting line, deviates significantly from the actual J-R curve at small crack extensions. A possible additional reason arises from the use of the $2\sigma_f \Delta a$ blunting line in determining J_{1C} , which may differ from the actual J blunting line. However, this is not the case in this study, as the experimentally determined J blunting line was used to determine J_{1C} .

Basing the thickness size requirements determined in this study on J_{1C} rather than J_i values does not totally explain the differences between the requirements found in this study and that given in ASTM E813-81. A possible explanation for this is that, as the thickness size requirement given in ASTM E813-81 has been mainly verified on either higher strength or higher toughness materials than that used in this study, the thickness size requirement based on J/σ_f is material dependent. An alternative possibility discussed below is that the J/σ_f term is inappropriate to characterise the thickness size requirement for plane strain crack tip conditions: for a criterion based on J/σ_f suggests that as J/σ_f increases, the crack tip constraint will decrease. However, this does not appear to be the case in the central region of the specimen, because CTOD, which is dependent on crack tip constraint, shows no change with increasing crack extension and thus J/σ_f , in this region, see section 8.2. Although as J/σ_f increases, the size of the shear region increases, as does the region over which the crack tip constraint is lower than in the central region of the specimen, see section 7.2. Therefore, a criterion based on J/σ_f is required to ensure that the region of low crack tip constraint does not extend across the whole crack front, but the thickness size requirement for plane strain crack tip conditions will not be based on J/σ_f . There is no evidence to suggest what form of thickness criterion is appropriate. A possibility is that the thickness should be greater than a given multiple of $1/\sigma_f$. From the results in this study, the requirement for non-sidegrooved specimens would be

$$B > \frac{\alpha}{\sigma_f}$$

7.6

where $14.6 < \alpha \text{ (MN.m)} < 19.5$.

An important point to remember when comparing the thickness size requirements determined experimentally for different materials is that they will depend on the degree to which changes in crack tip constraint, and thus crack initiation toughness, with variations in specimen size, can be detected. This will depend both on the accuracy with which the crack initiation toughness can be measured and the effect of variations in crack tip constraint on the crack initiation toughness. The former will depend on the technique used to determine the crack initiation toughness and the degree of material inhomogeneity. The latter will be dependent on the material, because the dependence of the local fracture strain and thus crack initiation toughness, on the stress state is material dependent, see section 2.1.5. Therefore, it is likely that the criteria to obtain apparently plane strain initiation values will be material dependent. To illustrate this, the specimen thickness for plane strain crack tip conditions suggested by equation 7.6 for a ASTM A533B material is 30-40mm. However, Ingham et al (1983) reported that the crack initiation toughness was apparently independent of specimen thickness over the range 10 to 100mm for this material. Robinson and Tetelman (1975) showed that for this material, δ_1 was not affected significantly by specimen size, even when there was marked transverse strain and thus a deviation from plane strain crack tip conditions. This suggests that the dependence of crack initiation toughness on crack tip constraint is much less for this material than the mild steel used in this study, which explains why the thickness suggested by equation 7.6 for plane strain crack tip conditions is more stringent than that suggested by Ingham et al.

As discussed in section 7.2, the crack tip constraint in the central region of specimens of a given thickness would appear to be independent of specimen geometry. This would tentatively suggest that the thickness size requirements to obtain plane strain crack tip conditions given above for the compact geometry will also be applicable to the CCT, DEN and bend geometries.

The ligament size requirement for plane strain crack tip conditions would appear to be based on J/σ_f . This follows from the results of a number of finite element studies, see for example McMeeking and Parks (1979), which show that as J/σ_f increases, the crack tip constraint eventually deviates from that for plane strain, in a specimen of given ligament size. No effect of ligament size within the range investigated was found on the J_i value in the central region of compact specimens, see Figure 5.14. Thus, from the smallest ligament size tested, a conservative estimate for the ligament size requirement for plane strain crack tip conditions can be determined, i.e.

$$b \geq 65 \frac{J_i}{\sigma_f} \quad 5.4$$

The ligament size requirement in terms of δ is

$$b \geq 60 \delta_i \quad 7.7$$

As the J_i and δ_i values were found to be independent of specimen geometry, see section 7.2, this suggests that the ligament size criterion for the CCT and DEN geometries were satisfied. From the results, a conservative estimate of the ligament size requirement for the CCT and DEN geometries can be determined, i.e.

$$b \geq 35 \frac{J_i}{\sigma_f} \quad 7.8$$

$$b \geq 30 \delta_i \quad 7.9$$

As the plastic constraint factor for the compact and bend geometries is higher than that for the CCT geometry, it is likely that equations 7.8 and 7.9 will also be conservative estimates of the ligament size requirement for the compact and bend geometries.

The ligament size requirement for the bend and compact specimens given by equation 7.8 is similar to that given in the J_{1C} testing standard E813-81, i.e. equation 7.3, particularly if the ligament size requirement found in this study is also written in terms of J_{1C} , which is about twice J_i for this material, see Figure

5.14. However, the ligament size requirement suggested for the CCT geometry, equation 7.8, is significantly less stringent than that suggested from the finite element results of McMeeking and Parks (1979), and Shih and German (1981) of

$$b > 200 \frac{J}{\sigma_F}$$

One possible explanation for this is that significant violations of the ligament size requirement for plane strain crack tip conditions only result in small changes in J_i , which remain undetected within experimental scatter. However, this explanation would appear unlikely to be able to account for the large differences in the two suggested criteria. A more likely explanation is as follows. The ligament size requirement predicted from finite element results depends critically on the size of the region around the crack tip in which it is assumed that plane strain conditions need to apply, in order to obtain plane strain crack initiation toughness values, particularly for the CCT geometry. The smaller the size of the region in which plane strain crack tip conditions need to be maintained, the smaller the ligament size requirement. The size of this region was $8 \times \delta$ in the study by McMeeking and Parks (1979) and $6 \times \delta$ in the study by Shih and German (1981). However, from a finite element analysis, Aoki et al (1984) reported that most of the void nucleation and growth occurred in a region $1 \times \delta$ in size. Therefore, if the size of the plane strain region need only be for example $2 \times \delta$, this would considerably reduce the ligament size requirement, in particular for the CCT geometry. Although it is not possible to determine from the results of for example McMeeking and Parks (1979), what the ligament size requirement would be, partly because their results are not sufficiently accurate very close to the crack tip; it is likely that the ligament size requirement for the CCT geometry would be close to that given by equation 7.8.

In summary, the thickness and ligament size requirements for plane strain crack tip conditions would appear to be similar for the CCT, DEN, bend and compact geometries. In addition, the

thickness size requirement is considerably reduced by sidegrooving the specimen to a depth of 25-50%.

The ligament size criterion suggested for the bend and compact geometries is in agreement with that given in the ASTM J_{1C} testing standard E813-81. The criterion suggested for the CCT geometry is significantly less stringent than that suggested from finite element results. This may be because of the excessive size of the region in which it is assumed in the finite element studies, that plane strain conditions need to apply in order to obtain plane strain crack initiation toughness values.

The thickness size requirement, based on J/σ_f , found in this study for non-sidegrooved specimens, is significantly more stringent than that given in the ASTM J_{1C} testing standard E813-81. Although, it is possible that the thickness size requirement for specimens sidegrooved to a depth of 25-50% is in agreement with that given in ASTM E813-81. It has been suggested that J/σ_f is not a suitable factor to specify the thickness size requirement; a factor such as l/σ_f may be more suitable.

It has been recognised that in order to obtain apparently plane strain initiation toughness values, the size requirements will be material dependent. This is because the degree to which variations in the crack tip constraint affect the initiation toughness is material dependent.

7.4 Determination of J Integral Prior to Crack Initiation

In this section, two aspects of the determination of the J integral prior to crack initiation are discussed: the first is the method used to determine J in this study; the second considers the effect of extraneous elastic and plastic work in the structure on the determination of J.

One of the original definitions of J is the potential energy release rate, equation 2.12, which can be written in terms of the area under the load displacement diagram, U, up to crack initiation, as suggested by Landes and Begley (1972).

$$J = \frac{-1}{B} \frac{\partial U}{\partial a} \bigg|_Q \quad 7.10$$

$$J = \frac{+1}{B} \frac{\partial U}{\partial a} \bigg|_Q \quad 7.11$$

All the J values in this study, up to crack initiation, have been determined from formulae which are directly equivalent to equation 7.10 or 7.11. This is illustrated below for the various geometries.

To determine J for the compact and bend specimens, equation 7.10 was rewritten as

$$J = \frac{\eta_e U_e}{Bb} + \frac{\eta_p U_p}{Bb} \quad 7.12$$

where η_e is given by equation 2.9 and η_p is given by equation 2.24. Note that equation 7.10 can be recovered by substituting equations 2.9 and 2.24 into 7.12, given that $W=U$ prior to crack initiation. The η_e values for the bend and compact specimens were determined from equations which are directly equivalent to equation 2.9, and the η_p values were determined directly from equation 2.24, analytically in the case of the three and four-point bend specimens, and experimentally in the case of the compact specimens. The experimental approach allowed η_p to be determined even for deeply sidegrooved compact specimens.

To determine J for the CCT and DEN specimens, equation 7.11 was rewritten as

$$J = G + J_p \quad 7.13$$

$$\text{where } J_p = \frac{-1}{B} \int \frac{\partial q_p}{\partial b} \bigg|_Q dQ \quad 7.14$$

G was determined from the load via K and J_p was determined from equation 7.14 as a function of the plastic load-line displacement, from the relationship between the load and the plastic load-line displacement for each geometry.

It would appear when J is determined from its definition as the potential energy release rate, via equation 7.10 or 7.11, that it is a suitable crack initiation characterising parameter because J_i was unaffected by changes in specimen size and geometry unless there was a change in the crack tip constraint, see section 7.2. This suggests that the reasons previously put forward to justify why J can characterise the initiation toughness of a material are true, at least in this study, see section 2.3.2.1. These are that J from its potential energy release rate definition, is equivalent to J from its contour definition, for a contour close to the boundary of the specimen, and that this J is in turn equivalent to a J calculated around a contour close to the crack tip, which is in turn capable of characterising the magnitude of the crack tip stresses and strains.

In the determination of J from the area under the load versus displacement trace, U , it is usually important to know the amount of extraneous plastic work contributing to U . Extraneous plastic work is the plastic work which is not required for the fracture process, that is the plastic work which occurs outside the crack region. Thus, in the most simple case, it is the plastic work which occurs in the structure if there was no crack. As an example, consider a specimen loaded in tension with a thin bar. Provided the load is sufficient, plastic deformation will occur in the thin bar, whether or not the specimen contains a crack, and thus must be considered as extraneous. The situation is more complicated when plastic deformation occurs in the crack region whether or not the specimen contains a crack. In these cases the determination of the amount of extraneous plastic work is difficult. Although, if the size of the crack region is small in comparison with the structure, it would be reasonable to take the extraneous plastic work again as the total plastic work which would occur if there was no crack. The crack region does not just include the region around the crack tip, but includes that region in which plastic deformation needs to occur for crack extension. Thus, in the case of a bend specimen in which the ligament must deform in order to extend the crack, the crack region will include the whole ligament region.

As the amount of extraneous plastic work is a function of the load on the structure, a correction for extraneous plastic work is required if J is calculated from equation 7.10 or equivalent equations, such as equation 7.12. Thus in the present study, the amount of extraneous plastic work occurring in the bend specimens due to roller indentation, was subtracted prior to calculating J . The extraneous plastic work was determined by loading an uncracked specimen. If the amount of extraneous plastic^{work} can not be determined experimentally or analytically (by, for example, finite element analysis), then J has to be calculated from alternative equations such as equation 7.11, or from its contour definition. It is not usually accurate to determine J from equation 7.11 under net section yielding conditions because the gradient of the load-displacement curves is low and so small errors in determining these curves can result in large errors in J . In addition, to calculate J from its contour definition will invariably require a finite element analysis of the cracked structure.

As with the extraneous plastic work, it is important to know the amount of extraneous elastic stored energy when calculating J from equation 7.10 or equivalent equations, such as equation 7.12. The extraneous elastic stored energy is that stored in the uncracked structure and can be determined from the load and the elastic compliance of the uncracked structure. Therefore, if J is determined from equation 7.12, either the extraneous elastic stored energy should be subtracted from the elastic component of U , or η_e values should be correspondingly lowered, knowing the elastic compliance of the uncracked structure, so that the extraneous elastic stored energy need not be subtracted. The latter approach was used in this study for the bend specimens, see section 4.2.2.1. If the amount of extraneous elastic stored energy is not known, the elastic component of J (taken as G) can be calculated either from equation 7.11 or from load on the structure via K , as was done for the CCT and DEN specimens, see section 4.4.4.

A sidegrooved specimen can be considered as consisting of two regions, as illustrated in Figure 3.9, a plane sided region and two specimen flanks. An investigation was carried out to determine

whether the plastic work occurring in the specimen flanks should be considered as extraneous or not. In this investigation, the percentage of the total plastic work per unit depth, occurring in the plane sided region, was determined at various positions ahead of the crack tip from hardness measurements, as described in section 3.8. This was carried out for specimens with an $a/w \approx 0.55$ and a sidegroove depth of 25, 50 or 75%. The percentage of the total plastic work per unit depth in the plane sided region was found to be independent, within experimental scatter, of the level of the total plastic work per unit depth, for each sidegroove depth, see Section 3.8. Therefore, the overall percentage of the plastic work occurring within the plane sided region can be taken as the average percentage of the plastic work absorbed per unit depth in the plane sided region, and can be considered as constant during loading in the post yield region. Figure 7.5 gives the percentage of the total plastic work in the plane sided region ($\pm$ standard error) as a function of the sidegroove depth. It can be seen from the figure that for specimens sidegrooved to a depth of 75%, only about 50% of the plastic work is absorbed in the plane sided region of the specimen. Given that this percentage is likely to be independent of a/w , the η_p values calculated from just the plastic work occurring in the plane sided region using equation 2.24 will be the same as those determined previously from the total plastic work. Therefore, the J values for specimens sidegrooved to a depth of 75%, determined from equation 7.12, will be significantly lower (up to about 50%), if U_p is just taken as the plastic work occurring in the plane sided region rather than the total plastic work. Thus, with reference to Figure 5.7, the J -SZW and thus the J - δ relationship will only be independent of sidegroove depth if, as in Figure 5.7, J is determined from the total plastic work rather than just that occurring in the plane sided region. For J to be a suitable fracture parameter, there should be a direct relationship between J and the near field parameters such as δ , which is independent of sidegroove depth. Therefore, the total plastic work rather than just that occurring in the plane sided region, should be used to calculate J and so the plastic work occurring in the specimen flanks of sidegrooved specimens should not be considered as extraneous plastic work. An additional reason justifying this

conclusion is that as the proportion of the total plastic work occurring in the specimen flanks is apparently fixed, for a given sidegroove depth, this would suggest that the plastic work occurring in the specimen flanks is required for crack extension and thus not extraneous.

In summary, a number of different equations have been used to determine J up to crack initiation for the various geometries investigated in this study. However, it has been shown that all these equations are equivalent to one of the original definitions of J as the potential energy release rate. It would appear that J defined in this way, is capable of characterising the initiation toughness of the material, for a given crack tip constraint, in a range of specimen geometries, i.e. bend, compact, DEN and CCT geometries.

In the determination of J from the rate of change with respect to crack extension of the area under the load displacement trace, U , at a constant displacement, or from equivalent equations, the importance of taking into account extraneous elastic and plastic work has been considered. Procedures for determining these contributions have been summarised, along with the alternative approaches for determining J when these contributions cannot be determined or estimated.

It has been shown that the plastic work absorbed in the flanks of sidegrooved specimens is not extraneous and should be included when determining J .

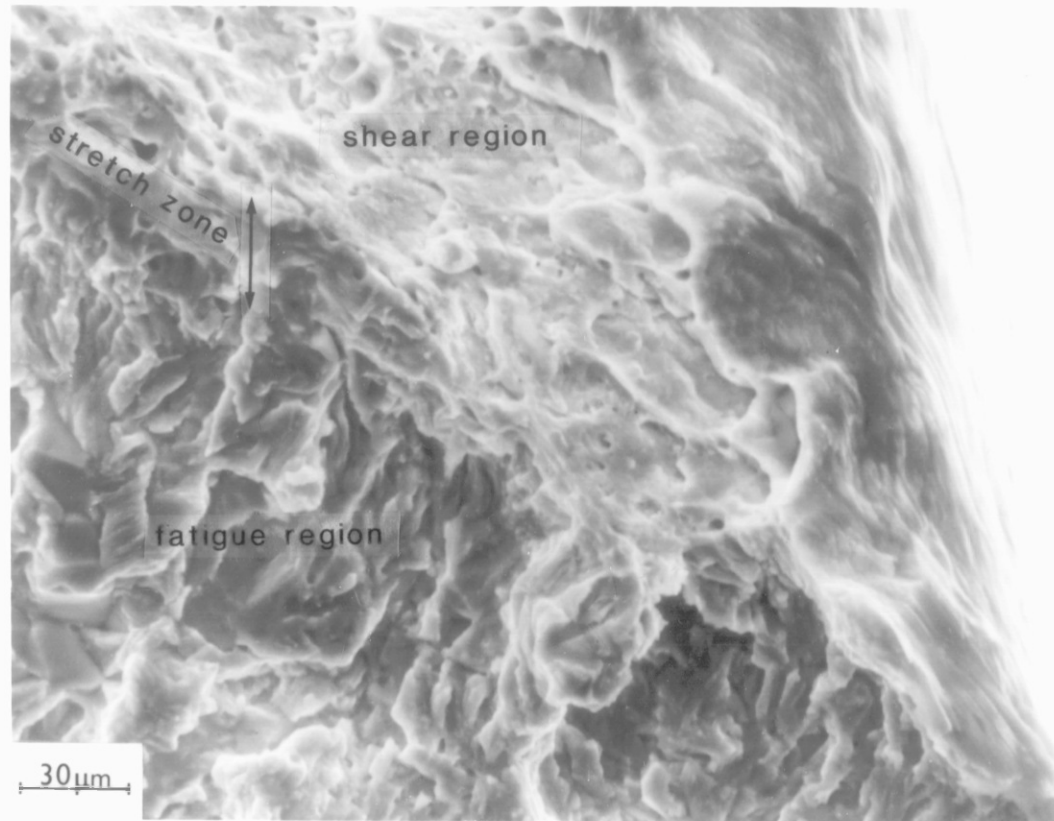


Plate 7.1 Scanning electron fractograph taken close to the edge of a non-sidegrooved compact specimen, $B=13\text{mm}$, $w=26\text{mm}$

↑
specimen
edge

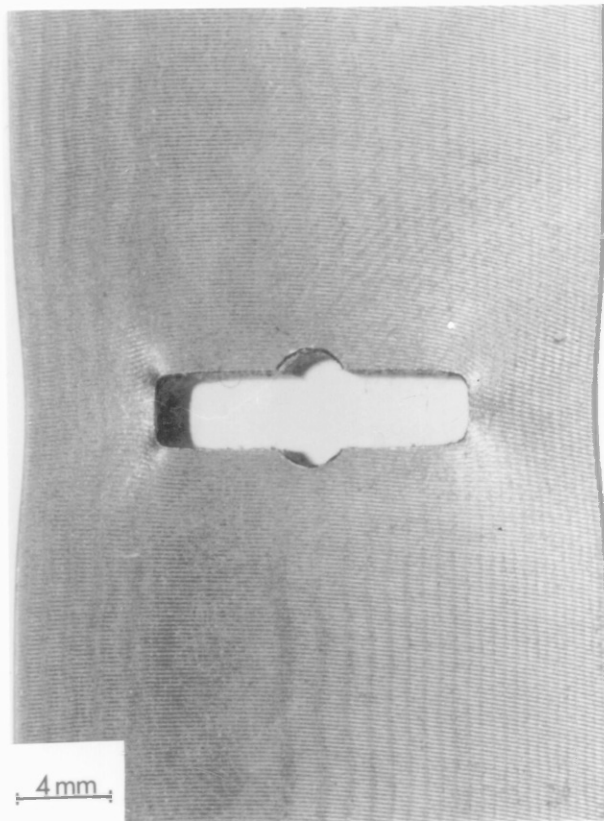


Plate 7.2 Profile of a spark machined slot, initially 1.5mm wide, after loading of a non-sidegrooved centre-cracked tension specimen

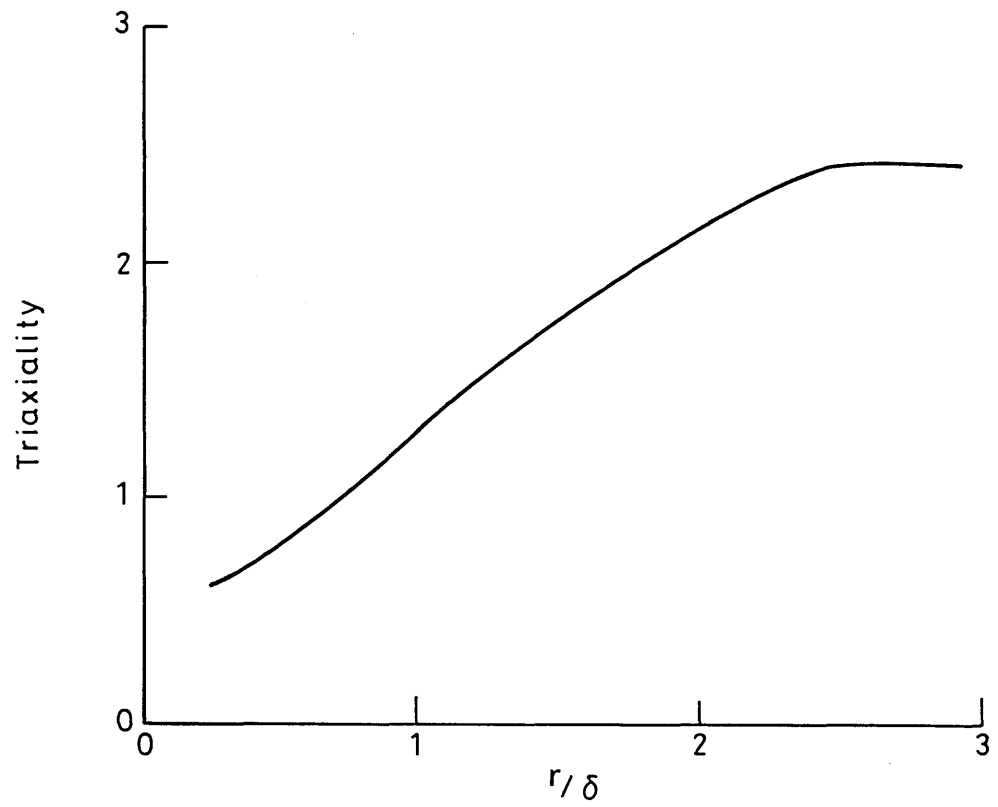


Figure 7.1 Triaxiality as a function of the distance ahead of a blunted crack tip, r , normalised by δ

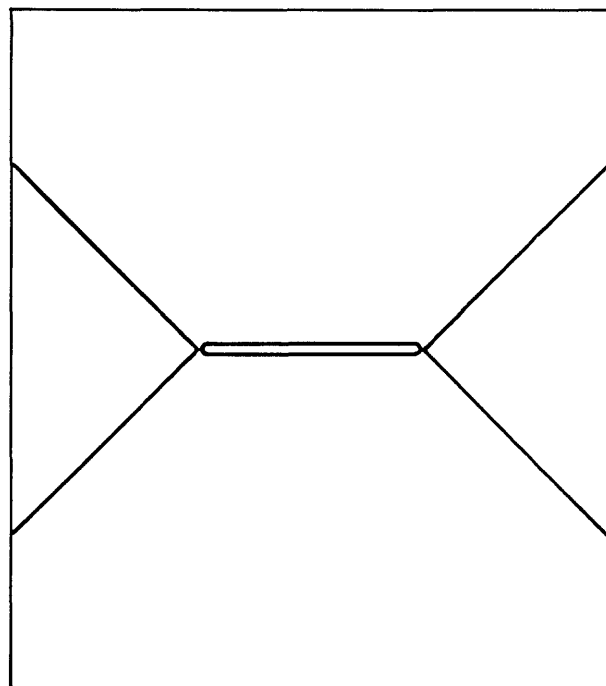


Figure 7.2 Proposed slip line field for the centre-cracked tension geometry

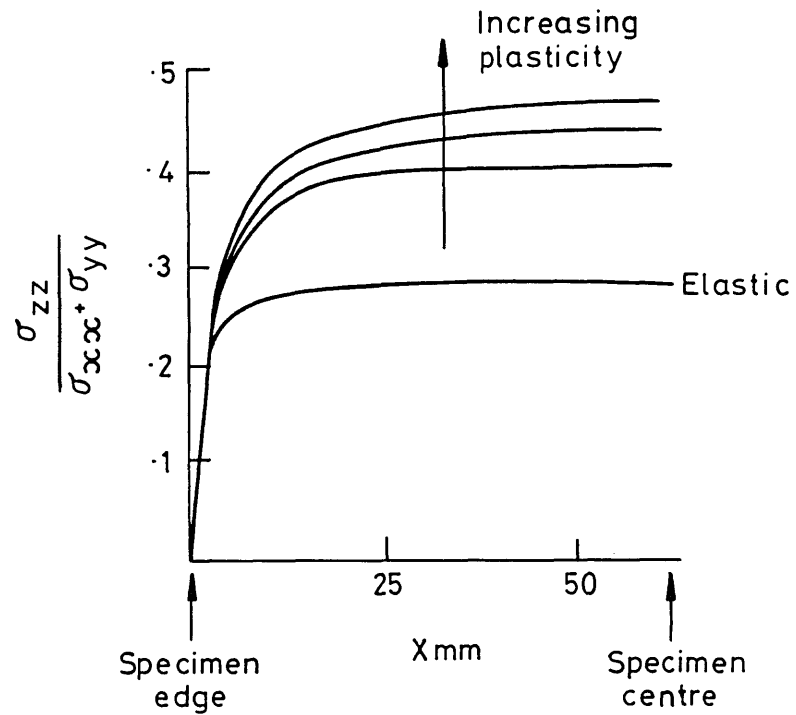


Figure 7.3 "Degree of plane strain" versus X for non-sidegrooved compact specimen, $B=127\text{mm}$, $w=254\text{mm}$

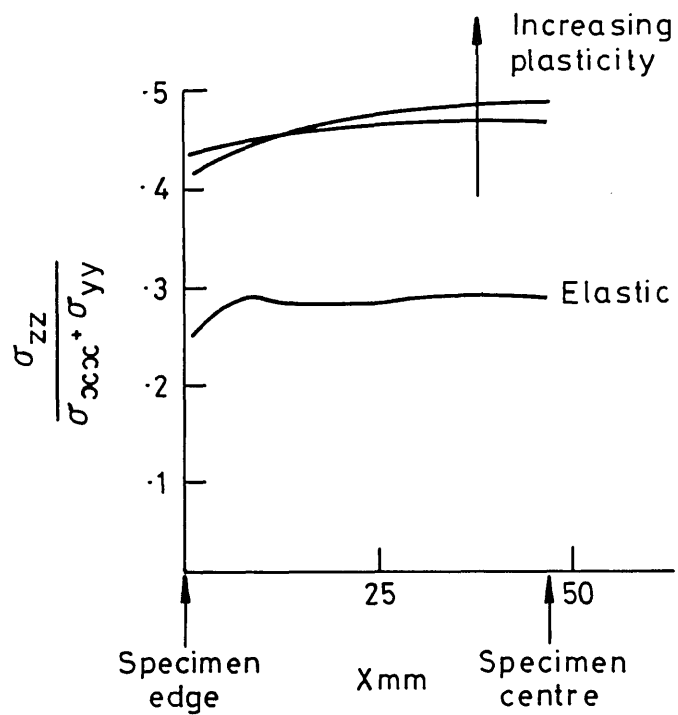


Figure 7.4 "Degree of plane strain" versus X for compact specimen sidegrooved to a depth of 25% with gross $B=127\text{mm}$, $w=254\text{mm}$

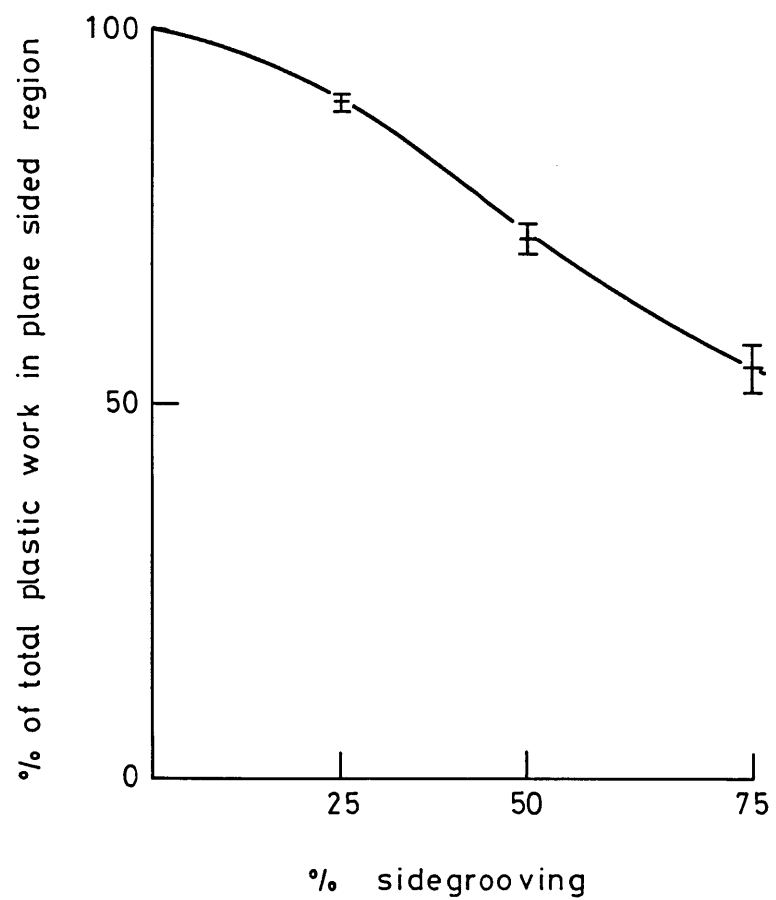


Figure 7.5 Effect of sidegroove depth on the proportion of the total plastic work in the plane sided region of the specimen

Chapter 8

Discussion on Crack Growth Resistance

8.1 Introduction

In this chapter, the capability of the various parameters investigated, to characterise the crack growth resistance of a material, is discussed. The capability of a parameter is assessed by determining whether changes in the parameter with, for example, changes in specimen size and geometry, can be explained in terms of a corresponding change in the crack growth resistance of the material, as a result of a change in the crack tip constraint. The crack growth resistance is expected to be dependent on the crack tip constraint, as the processes occurring during crack growth - void growth, and possibly also void nucleation and coalescence - are dependent on the triaxiality. This chapter is divided into four main sections: the first deals with CTOD; the second with CTOA and δ ; the third with J , and the fourth with the relationship between the various parameters.

8.2 Crack Tip Opening Displacement, CTOD

The discussion on CTOD is divided into three main parts: the first covers the measurement and variability of CTOD; the second covers the effect of crack tip constraint on CTOD and considers the variation of CTOD across the crack front of non-sidegrooved and sidegrooved specimens; the third covers the effect of specimen size and geometry on CTOD. From the conclusions of the second and third parts of this section, the capability of CTOD to characterise the crack growth resistance is assessed.

In this study, CTOD was either measured directly from metallographic sections through the crack tip, or estimated by an extrapolation procedure from silicone-rubber casts, see section 3.7.4. It would appear that the CTOD values estimated by the extrapolation procedure are reasonably accurate because they are in good agreement with those which could be measured directly from the silicone-rubber casts and also those measured from metallographic sections, compare Figures 6.12 and 6.13.

The CTOD values from any one specimen were found to exhibit significant variation, as was also found by Willoughby et al (1981). This is to be expected, as ductile crack growth involves void nucleation, growth and coalescence. Therefore, CTOD will depend on the size of the void which last coalesced to the crack tip and thus in turn the height of the void nucleating particle normal to the crack plane. As the height of void nucleating particles vary significantly, i.e. from that for large inclusions to that for small carbides, in any one steel, there will be a significant variation in CTOD.

In the following part of the section, the variation of CTOD across the crack front of non-sidegrooved and sidegrooved specimens and the effect of crack extension on CTOD are discussed. The aim is to determine whether CTOD is dependent on crack tip constraint as is the crack growth resistance of the material, and thus whether CTOD is likely to be capable of characterising the crack growth resistance of the material. In the case of non-sidegrooved specimens, it is expected that there will be a variation in the crack tip constraint, see section 7.2, and thus the crack growth resistance across the crack front. Therefore, if CTOD were a suitable crack growth characterising parameter, there should be a corresponding variation in CTOD across the crack front. The variation in CTOD across the crack front of one non-sidegrooved DEN specimen has been determined, see Figure 6.13. It can be seen from the figure that on average the CTOD increases, from that in the central region of the specimen, as the specimen edge is approached, until the shear region is reached, where crack growth is occurring by a shear mechanism. Therefore, it would appear that a variation in crack tip constraint and thus crack growth resistance results in a variation in CTOD. There is some evidence to support the suggestion that the crack tip constraint is lower in the region where the CTOD is measurably higher, as it would appear that there are fewer voids around the crack tip in this region, even though there would appear to be the same number of inclusions around the crack tip, see Plates 6.2a and b. A smaller number of observed voids is indicative of a lower crack tip constraint because void growth and probably also void nucleation are affected by triaxiality. However, the CTOD on

average is lower in the shear region than in the central region of the specimen. This result would appear apparently to conflict with the above conclusion that a decrease in crack tip constraint results in an increase in CTOD. However, it is suggested below that the lower CTOD in the shear region is a result of the difference in the crack growth mechanism, rather than a conflicting trend of CTOD with crack tip constraint. Plates 8.1a and b show the crack tip profiles in the shear and central regions of the specimen, respectively. It can be seen that when the shear mechanism is operating, voids which have linked to the crack tip are less evident. This would suggest that the height of the voids, in the direction normal to the fatigue crack plane, is less when the shear mechanism is operating, presumably because of less void growth in this direction. As CTOD is dependent on the height of the voids, it will be dependent on the crack growth mechanism. The smaller void height, and thus CTOD associated with the shear mechanism, should not be considered as indicative of a lower crack growth resistance, but rather due to the different deformation characteristics associated with this mechanism, resulting in a different amount of accompanying void growth in the direction normal to the fatigue crack plane, which occurs prior to the void linking to the crack tip.

In the case of sidegrooved specimens, it would appear that up to crack initiation, there is little or no variation in crack tip constraint across almost all the crack front, even in specimens sidegrooved to a depth of only 10%, see section 7.2. This would also appear to be the case after crack initiation, as no shear lip was found on any of the fracture surfaces of sidegrooved specimens. As there is also no apparent variation of CTOD across the crack front of sidegrooved specimens, see for example, Figure 6.14, this result gives further support to the suggestion that variations in CTOD arise solely from changes in crack tip constraint, provided the same crack growth mechanism operates.

The investigation into the effect of crack extension on CTOD showed that there was no consistent variation in CTOD with crack extension, see for example, Figure 6.12, which is in agreement with some earlier results of Garwood (1976) and Willoughby et al

(1981). Although, close to the specimen edge no reliable trends in CTOD could be determined because of the limited number of data points in this region. This result suggests that there is no change in the crack tip constraint or the intrinsic crack growth resistance of the material with increasing crack extension, at least up to about 4mm, the maximum investigated. However, near to the specimen edge of non-sidegrooved specimens, the crack tip constraint would be expected to decrease with increasing crack extension, as the size of the shear lip and thus the region of low crack tip constraint is found to increase.

It is worth noting that CTOD is significantly smaller than δ_i , for example, compare Figures 6.8 and 6.16. However, this change in the critical crack opening between that for crack initiation and that for crack growth is not because the crack tip constraint increases considerably after crack initiation, but is due to the fact that less plastic strain and thus crack opening, is required to continue crack growth than to initiate it. This is because further crack growth occurs in previously strained material in which voids have already nucleated and started to grow.

In the next part of the section, the effect of specimen geometry and sidegroove depth on the CTOD in the central region of the specimen and its relationship with δ_i is discussed. The aim of this section is to assess whether variations in CTOD with, for example, changes in sidegroove depth, are consistent with expected changes in crack tip constraint, and thus whether CTOD is a suitable crack growth characterising parameter.

There would appear to be no effect of specimen geometry on the CTOD in the central region of non-sidegrooved specimens of a given thickness, see Figure 6.15. This result is consistent with the finding of no effect of specimen geometry on the δ_i value in the central region of the specimen, see section 7.2. Therefore, it would appear that specimen geometry does not affect the crack tip constraint in the central region, for a given specimen thickness and that CTOD is not affected by specimen geometry unless there is a change in the crack tip constraint.

CTOD was found to decrease with increasing sidegroove depth to reach a plateau value after a sidegroove depth of 25-50%, in compact specimens, $B_g=13\text{mm}$, $w=26\text{mm}$, see Figure 6.16. A plateau J_i and δ_i value was also obtained after a sidegroove depth of 25-50% for this sized specimen, see Figures 5.11 and 6.8. As the same plateau J_i value was obtained in larger non-sidegrooved specimens which would appear to be of sufficient size to promote plane strain crack tip conditions, see section 7.2, the CTOD results suggest that when the specimen size is insufficient to obtain crack growth under plane strain crack tip conditions in the central region of the specimen, sidegrooving can be used to increase the crack tip constraint up to that for plane strain.

It would appear from the previous discussion that CTOD can characterise the crack growth resistance of a material when the shear mechanism is not operating. This is because variations in CTOD with, for example, changes in specimen geometry or sidegroove depth, can be explained in terms of changes in crack tip constraint. To show that CTOD can characterise the crack growth resistance when the shear mechanism is operating, it would have to be shown that CTOD in the shear region is independent of specimen size and geometry. This is because the energy required for shear lip formation is not dependent on specimen size and geometry, as is shown in Section 8.4.2.1, and so it would appear that the crack growth resistance associated with the shear mechanism is also independent of specimen size and geometry. There are insufficient CTOD results obtained in the shear region to determine whether this is the case. However, as CTOD can characterise the crack growth resistance when the shear mechanism is not operating, and thus give a measure of the increase in crack opening and thus strain required for further crack growth under these conditions. It is likely that CTOD will have this capability when the shear mechanism is operating, which would suggest that CTOD can characterise the crack growth resistance when the shear mechanism is operating, although the value of CTOD will be different.

As the trends in CTOD and δ_i with specimen geometry and sidegroove depth are similar, as shown previously in this section,

it would appear that there is a direct relationship between CTOD and δ_i , provided the crack tip constraint and thus CTOD do not change during crack growth. In sidegrooved specimens and in the central region of non-sidegrooved specimens, the relationship suggested from the results is

$$\delta_i = (8.1 \pm 0.1) \text{ CTOD} \quad 8.1$$

A relationship between these two parameters would be expected, because both have been shown to be suitable fracture controlling parameters. The relationship is independent of sidegroove depth and specimen geometry, but it will be different for the shear region of non-sidegrooved specimens, as δ is higher and CTOD is lower in this region than in the central region, compare Figures 6.3 and 6.13.

In summary it would appear that CTOD can characterise the crack growth resistance of the material at least provided the shear mechanism does not operate, as variations in CTOD with, for example, changes in specimen geometry and sidegroove depth, can be explained in terms of a variation in the crack tip constraint in the region where the shear mechanism does not operate. In addition, it has been argued that CTOD should also be capable of characterising the crack growth resistance when the shear mechanism is operating. Although, the lower CTOD associated with the shear mechanism does not reflect a lower crack growth resistance. There is a direct relationship between δ_i and CTOD when the crack tip constraint and thus CTOD remain the same during crack growth. For sidegrooved specimens and in the central region of non-sidegrooved specimens, this relationship is independent of specimen geometry and sidegroove depth, but there is a different relationship for the shear region.

8.3 Crack Tip Opening Angle (CTOA) and δ Crack Growth Resistance

This section contains a discussion on the capability of CTOA and δ to characterise the crack growth resistance of a material and is divided into two main parts. The first considers the relationship between CTOA and the δ crack growth resistance ($d\delta/da$): the second considers the relationship between CTOD and these two parameters. The aim of the second part is to determine whether

there is a unique relationship between CTOD and CTOA or $d\delta/da$, at least for a given crack growth mechanism, and thus whether CTOA and/or $d\delta/da$ can characterise, as can CTOD, the crack growth resistance of a material.

CTOA can be estimated from $d\delta/da$ using equation 6.5 by assuming that over the fibrous crack region the crack flanks are essentially linear. Good agreement was found between the estimated CTOA values and those measured from silicone-rubber crack tip impressions, see for example Figure 6.11. At crack extensions greater than about 1.5mm, CTOA reaches a plateau value for this material, which is related to $d\delta/da$ by equation 6.6.

$$CTOA = 2 \tan^{-1} \left(\frac{1}{2} \frac{d\delta}{da} \right) \quad 6.6$$

The good agreement between the estimated and measured CTOA values means that equation 6.6 holds at least up to 4mm of crack extension, the maximum investigated in this study. However, Abou-Sayed et al (1981) showed that at larger crack extensions (typically 10mm or more, depending on specimen size) $d\delta/da$ increases, whereas CTOA remains the same, with increasing crack extension. This suggests that equation 6.6 breaks down at larger crack extensions, presumably because the crack flanks over the fibrous crack growth region are no longer essentially linear.

As illustrated in Figure 8.1, CTOA is related to CTOD by a distance γ i.e.

$$CTOA = 2 \tan^{-1} \left(\frac{CTOD}{2\gamma} \right) \quad 8.2$$

As CTOD is found independent of crack extension, except possibly near to the edge of non-sidegrooved specimens, see section 8.2, γ will decrease as CTOA increases with increasing crack extension, to reach a plateau value after about 1.5mm of crack extension for this material. The plateau value of γ will relate CTOD and $d\delta/da$ provided equation 6.6 holds,

$$\frac{d\delta}{da} = \left(\frac{CTOD}{\gamma} \right) \quad 8.3$$

when $\Delta a > 1.5\text{mm}$.

Figure 8.2 gives the plateau γ values ($\pm$ standard error) determined from the CTOD and $d\delta/da$ values associated with the midsection of the specimen, as a function of sidegroove depth, for two sizes of compact specimens, i.e. $B_g=13\text{mm}$, $w=26\text{mm}$ and $B_g=50\text{mm}$, $w=100\text{mm}$. In the case of the larger specimens, the CTOD values were inferred from δ_i values, using equation 8.1. It can be seen that the plateau γ value in the midsection of specimen is independent of specimen size and sidegroove depth up to about 25-50%. As δ_i and thus crack tip constraint are affected by specimen size and for the smaller specimens also sidegroove depth, see Figures 6.8 and 6.10, it would appear that the midsection γ value is independent of crack tip constraint, at least over the range investigated. Therefore, at least under these conditions, there is a direct relationship between CTOD and $d\delta/da$ or the plateau CTOA value. As an example to illustrate this, sidegrooving of specimens, 13mm thick and 26mm wide, up to a sidegroove depth of 25-50%, results in a decrease in both the CTOD and $d\delta/da$ values associated with the midsection of the specimen, see Figures 6.8 and 6.16.

It can be seen from Figure 8.2 that the plateau γ value increases with increasing sidegroove depths above about 25-50%, even though CTOD and thus crack tip constraint have reached a plateau value assumed to be that for plane strain crack tip conditions, see Figure 6.16. As a result of this, $d\delta/da$ decreases with increasing sidegroove depths above about 25-50%, see Figure 6.8. The effect of deep sidegrooving on γ is shown below to be a result of its effect on the plastic hinge point and thus the deformation characteristics of the specimen. Up to crack initiation the mouth opening displacement (V) is related to δ via the hinge point construction, as illustrated in Figure 8.3a.

$$\delta = \frac{r_i b V}{(r_i b + a)} \quad 8.4$$

After crack initiation the increase in mouth opening displacement (dV) and the increase in δ ($d\delta$) above their respective initiation values, V_i and δ_i may be related similarly by a hinge construction, as illustrated in Figure 8.3b.

$$d\delta = \frac{r_g b dV}{(r_g b + a)} \quad 8.5$$

r_{ib} and r_{gb} are the hinge point positions in the ligament, prior to and during crack growth respectively. Rearranging equation 8.5. gives an expression for r_g

$$r_g = \frac{a d\delta}{b(dV-d\delta)} \quad 8.6$$

Figure 8.4 gives the r_g values ($\pm$ standard error) calculated from equation 8.6 using the original crack length and ligament values, dV/da values and the 9 point average $d\delta/da$ values as a function of sidegroove depth for the two compact specimen sizes. Although there are large errors associated with the r_g values, it would appear that the trends in r_g and γ with sidegroove depth are similar, compare Figures 8.2 and 8.4. This result would be expected as there is essentially only elastic deformation behind the crack tip. Therefore, it would appear that sidegroove depths, above about 25-50%, alter the hinge position and thus the deformation characteristics of the specimen during crack growth, and this results in a change in γ . The change in the deformation characteristics is probably due to the restricting of plastic deformation to between the sidegroove notches. It is worth noting that the change in the deformation characteristics also results in a change in η_p see Figure 4.4, as η_p is dependent on specimen geometry and thus the deformation characteristics of the specimen.

Figure 8.5 gives the midsection plateau γ values ($\pm$ standard error) determined from the CTOD and $d\delta/da$ values associated with the midsection of the specimen using equation 8.3, as a function of specimen geometry. There are differences in the dimensions of the specimens, in that the bend specimens are $B=25\text{mm}$, $w=50\text{mm}$; the compact specimens are $B=13\text{mm}$, $w=26\text{mm}$ and the DEN and CCT specimens are $B=25\text{mm}$, $2w=25\text{mm}$. However, it is thought that the differences in dimensions are not important, as no effect of specimen size on the midsection γ value was found. It can be seen from Figure 8.5 that the midsection γ values are the same, within experimental scatter, for the bend and compact specimens, which suggests that γ is the same for geometries with similar deformation characteristics. However, the midsection γ values for the CCT and DEN geometries are lower than those for the bend and compact geometries, although only marginally in the case of the DEN

geometry. Therefore, this result again shows that γ is dependent on the deformation characteristics of the specimen and so that at least under net section yielding conditions, there is no unique relationship between CTOD and $d\delta/da$ or the plateau CTOA value. This in turn means that $d\delta/da$ and CTOA are geometry dependent even when CTOD and thus crack tip constraint are unaffected, compare Figures 6.10 and 6.15. This is in agreement with the results of Shih et al (1981) which were that $d\delta/da$ values for the DEN geometry, determined from specimens with a wide range of sizes and a/w values, were always higher than that determined from a reasonably sized compact specimen (presumably sufficient to promote plane strain crack tip conditions).

As γ is dependent on the deformation characteristics of the specimen, it is likely that γ under plane strain and plane stress conditions will be different. This apparently conflicts with the previous result which showed that γ was independent of changes in crack tip constraint in the central region of the specimen. However, it is likely that γ , as with m which relates J and δ prior to crack initiation, see section 7.2, is only affected by crack tip constraint when the deformation characteristics of the specimen are affected significantly.

The previous results show that at least under net section yielding conditions $d\delta/da$ and CTOA, unlike CTOD, are not capable of characterising the crack growth resistance of a material. This is because variations in midsection CTOA and $d\delta/da$ values, with for example changes in specimen geometry, do not arise solely as a result of changes in crack tip constraint, since the relationship between CTOD and $d\delta/da$ or CTOA is dependent on the deformation characteristics of the specimen.

The previous part of the section has considered the $d\delta/da$ and CTOA in the midsection of the specimen. In non-sidegrooved specimens or in specimens sidegrooved to a depth of less than 10-25%, there is a variation in $d\delta/da$ across the crack front. The degree of variation appears to increase with decreasing sidegroove depth and increasing specimen size, compare Figures 6.2, 6.6 and

6.7, but the ratio of $d\delta/da$ at the edge to that at the centre of the specimen does not appear to be affected by specimen geometry, compare Figures 6.3 and 6.4. The variation in $d\delta/da$ does not arise solely from a variation in CTOD, because in specimens sidegrooved to a depth of 10% there was no variation in CTOD, but there was in $d\delta/da$, see Figures 6.6, 6.7 and 6.14. This would suggest that there is a variation in γ across the crack front, possibly caused by a variation in the deformation characteristics.

So in summary it would appear that after about 1.5mm of crack extension, CTOA reaches a plateau value which is related directly to $d\delta/da$ at least up to 4mm of crack extension, the maximum investigated in this study. Although, there is evidence to suggest that at larger crack extensions (10mm or more) this relationship breaks down. $d\delta/da$ and the plateau CTOA value are related to CTOD by a distance γ . It has been shown that γ for the midsection of the specimen is dependent on the deformation characteristics of the specimen, at least under net section yielding conditions, which are in turn dependent on the specimen geometry and the sidegroove depth when above 25-50%. However, the midsection γ value is independent of specimen size and sidegroove depth up to about 25-50%. It would appear that γ is independent of crack tip constraint. Unless the crack tip constraint is low, about that for plane stress, when there is a change in γ as a result of a significant change in the deformation characteristics of the specimen.

CTOA and $d\delta/da$ are not suitable crack growth controlling parameters because variations in $d\delta/da$ and CTOA with for example, changes in specimen geometry, do not arise solely as a result of variations in crack tip constraint.

There is a variation in $d\delta/da$ and thus CTOA across the crack front of non-sidegrooved specimens, and in specimens sidegrooved to a depth of less than 10 to 25%. The degree of variation would appear to increase with decreasing sidegroove depth and increasing specimen size, but it would appear to be unaffected by specimen geometry. The variation in $d\delta/da$ and CTOA is likely to

be due to a variation in γ and also in CTOD in non-sidegrooved specimens.

8.4 J Crack Growth Resistance

8.4.1 Introduction

Three approaches for calculating "J" after crack initiation have been used in this study, i.e. J uncorrected for crack growth (J_O), J corrected for crack growth (J_R), and the Ernst modified $J_R(J_m)$. The capability of these three "J" values to characterise the crack growth resistance of a material is discussed in this section, which is divided into three main parts. The first considers the capability of J_O as a crack growth controlling parameter for non-sidegrooved specimens; the second considers the case of J_O for sidegrooved specimens; the third compares the characterisation capabilities of J_O , J_R and J_m .

8.4.2. J_O -R Curves for Non-Sidegrooved Specimens

This part of the section is subdivided into two. The first considers the capability of J_O as a crack growth controlling parameter for non-sidegrooved specimens; the second considers the effect of specimen size and geometry on J_O -R curves as found in this study and as reported by other authors.

8.4.2.1 J_O as a Characterising Parameter in Non-Sidegrooved Specimens

To evaluate whether J_O can characterise the crack growth resistance of a material in non-sidegrooved specimens, it has to be determined whether changes in the slope of the J_O crack growth resistance curve, dJ/da (the subscript o is not used in this section for convenience) can be explained in terms of changes in the crack growth resistance of the material arising from changes in the crack tip constraint. As J_O in this study is determined from the total absorbed energy in the specimen, it can be considered as the average value across the whole crack front. Therefore, any changes in dJ/da have to be compared to changes in the average crack growth resistance across the whole crack front. If there is only one crack growth mechanism operating, then CTOD will govern the crack growth resistance of the material. However, in non-sidegrooved specimens

there are two crack growth mechanisms operating; a shear mechanism giving rise to shear lips at the edges of the specimen, and that giving rise to the flat fracture region in the rest of the specimen, see Figure 8.6. Therefore, in this case the average CTOD across the crack front will not reflect the average crack growth resistance. To overcome this problem, the overall J resistance curve is split into two components; that for the flat fracture region and that for the shear lip region, using an equation suggested by Green and Knott (1975). This equation is very similar to that suggested by Garwood (1976), see section 2.4.2.4.

$$J = J_f \left(1 - \frac{2S}{B} \right) + E_v \frac{2S^2}{B} \quad 8.7$$

The second term is the shear lip contribution, and it is based on the assumption, at least in contained yielding, that the contribution arises from uniform plastic deformation occurring in a region bounded by 45° shear lip lines, see Figure 2.9. E_v is the energy required per unit volume for shear lip formation and it should be independent of specimen size and geometry, and S is the shear lip width.

The first term of equation 8.7 is the flat fracture contribution which is assumed to be governed by the J-R curve for the flat fracture region, J_f -R. Previous authors, for example Garwood (1976), have taken the J_f -R curve to be that for plane strain crack tip conditions. This assumption is not reasonable in this study because it is known that plane strain crack tip conditions are not always present in the central region of the specimen. Therefore, the J_f -R curve should be that appropriate to the crack tip constraint in the central region of the specimen. As will be discussed in section 8.4.3, the J_f -R curve can be determined from specimens sidegrooved to a depth of 25-50% because the shear lip contribution is removed at these sidegroove depths. However, for the specimen sizes investigated, sidegrooving to these depths also results in plane strain crack tip conditions, and so only the J_f -R curve appropriate to plane strain crack tip conditions could be measured. To allow the J_f -R curves appropriate to other crack tip constraints to be determined, equation 8.8 was used.

$$J_f(\text{crack tip constraint A}) = J_f(\text{plane strain}) \times \frac{J_i(\text{crack tip constraint A})}{J_i(\text{plane strain})} \quad 8.8$$

where J_i is the J value at crack initiation in the central region of the specimen.

Clearly, equation 8.8 is valid at crack initiation, when $J_f = J_i$. The justification for using this equation after crack initiation is as follows. J_i is related to the crack growth resistance, in the central region of the specimen, as characterised by CTOD, because J_i is related to δ_i , equation 7.1, and δ_i is related to CTOD, equation 8.1. If J_f can characterise the crack growth resistance in the central region of the specimen, i.e. only dependent on crack tip constraint, then dJ_f/da will be related to CTOD and thus J_i . Although this is an assumption, its use is justified because equation 8.8 is being used to determine the J_f values for use in equation 8.7, which, in order to be capable of determining the overall J_0 -R, relies on the same assumption.

Ideally, to investigate whether J_0 can characterise the flat fracture crack growth resistance, J_f -R curves would be determined from their overall J_0 -R curves using equation 8.7. This is not possible, as E_v is unknown. However, an indirect assessment can be made by determining E_v using equation 8.7 from the overall J_0 -R curves, measured shear lip widths and the estimated J_f -R curves from equation 8.8. This is because if it can be shown that E_v is independent of specimen size and geometry when using the estimated J_f -R curves, it shows that J_f -R curves calculated from equation 8.8 are accurate and thus, as stated above, that J_0 can characterise the flat fracture crack growth resistance in non-sidegrooved specimens. Figure 8.7 gives E_v determined from specimens of various sizes and geometries as a function of shear lip width. It shows that E_v decreases to a plateau value, with increasing shear lip width. However, as shown in Figure 8.8, there is a direct relationship in this study between the shear lip width and the amount of crack extension at the specimen edge, i.e. the length of the shear lip in the fatigue crack direction. Therefore, it is not possible to determine whether E_v is a function of shear lip width or length. However, this could be determined from tests on thin specimens, as

the relationship given in Figure 8.8 will break down when the shear lip width approaches its maximum value of half the specimen thickness. Therefore, it would appear that E_v decreases with increasing shear lip width or length to reach a plateau value after a shear lip width of 0.8mm or a shear lip length of 0.6mm for the material used in this study. This result is consistent with those of Garwood (1976), which showed no effect of shear lip width on E_v , as in this case all the shear lip widths were greater than 1mm.

It can be seen from Figure 8.7 that the E_v -S relationship is independent of specimen size and geometry. Therefore, it would appear that, as discussed previously, J_O can characterise the flat fracture crack growth resistance of a material in non-sidegrooved specimens. This capability can be illustrated in another way by considering the effect of specimen geometry on the J_f -R curve. The J_f -R curves were determined for each geometry, using equation 8.7, from their overall J_O -R curves, shear lip widths and the best fit to the E_v -S relationship given in Figure 8.7. Figure 8.9 gives the J_f -R curves for the various geometries, the specimen thickness being 25mm in each case. It can be seen that within experimental scatter there is no effect of specimen geometry on the J_f -R curve, at least up to about 2.5mm of crack extension, the maximum investigated. This is consistent with there being no apparent effect of specimen geometry on CTOD and thus crack tip constraint, in the central region of the specimen, see Figure 6.15, and therefore this result again demonstrates the capability of J_O to characterise the flat fracture crack growth resistance of a material in non-sidegrooved specimens.

It is worth noting the relative importance of the shear lip contribution to the overall J_O -R curve in this study. This can be seen by comparing Figures 8.9 and 5.4. The shear lip contribution is only relatively large at the larger crack extensions, typically being about 15% at about 2.2mm of crack extension, although it is 30% for the CCT geometry. At small crack extensions the shear lip contribution is very small, being typically about 2% at about 1mm of crack extension.

As it would appear that J_0 can characterise the flat fracture crack growth resistance, this suggests that the criteria for J controlled crack growth proposed by Hutchinson and Paris (1979) have not been violated by any of the specimens, at least up to about 2.5mm of crack extension. Therefore, from the results, conservative estimates for the criteria for the CCT and DEN geometries can be determined, i.e.

$$\Delta a \leq 60\% b \quad 8.9$$

$$\omega \geq 1.0 \quad 8.10$$

These criteria have been determined by taking b as half the final total ligament length for the tension specimens. Conservative estimates for the criteria for bend and compact specimens determined from the results are more stringent than those above, because the ligament sizes for these specimens were four times those for the DEN and CCT specimens. However, as the bend and compact geometries have higher plastic constraint factors than the CCT geometry, it is expected that equations 8.9 and 8.10 will be conservative estimates of the criteria for J_0 controlled crack growth in bend and compact specimens.

Summarising this part of the section, it has been shown that J_0 can characterise the flat fracture crack growth resistance of a material in non-sidegrooved specimens, at least up to the conditions given by equations 8.9 and 8.10. In doing so, it has been shown that the overall J_0 -R curve can be determined from equation 8.7 and the J_f -R curve can be determined from equation 8.8. In addition, it was found that the energy for shear lip formation, E_v , is independent of specimen size and geometry, but it does decrease with either increasing shear lip width or length to reach a plateau value after a shear lip width of 0.8mm, or a shear lip length of 0.6mm for the material used in this study. The shear lip width and length were found to be related directly, although only because the shear lip widths were all smaller than half the specimen thickness.

The fact that J_0 can characterise the crack growth resistance in the central region of the specimen, means that the J_f -R curve can quantify the increase in applied loading conditions (i.e. the combination of load and displacement) required for further crack growth and thus quantify the global crack growth resistance in the central region. As equation 8.8 has been shown to be valid, it implies that the slope of the J_f -R curve is related directly to CTOD which quantifies the local crack growth resistance, i.e. the magnitude of the crack tip stresses and strains required for further crack growth. Therefore, the J_0 -R curve for the central region of the specimen can characterise the global and local crack growth resistances in this region. An important question to ask is whether the overall J_0 -R curve for non-sidegrooved specimens can characterise either the average local or global crack growth resistance across the whole crack front? For the latter to be true, J_0 would need to be capable of characterising the global crack growth resistance in the shear region. The shear lip contribution to the overall J_0 -R curve, given in equation 8.7, can be rewritten as

$$\text{shear lip contribution} = \frac{2S}{B} \times (S \times E_v)$$

The $S \times E_v$ term can be considered as the J value from the J-R curve for the shear lip region (i.e. J_s -R). As E_v is independent of specimen size and geometry, this suggests that when the shear lip width is proportional to the shear lip length, there is a unique J-R curve for the shear lip region for a given material. Therefore, when this is the case, J_0 can characterise the global crack growth resistance in both the flat fracture and shear lip regions, and thus the average global crack growth resistance of non-sidegrooved specimens. For J_0 to characterise also the average local crack growth resistance of non-sidegrooved specimens, there needs to be a unique relationship between the local and global crack growth resistances, for both the flat fracture and shear lip regions of the specimen. Although there is no direct evidence, it would appear, as discussed below, that there is no unique relationship between the local and global crack growth resistances. At crack initiation it appears that the relationship between the global crack initiation toughness as characterised by J, and the local crack initiation

toughness as characterised by δ , is different for high crack tip constraints, as found in the central region and low crack tip constraints, as found in the shear region, see section 7.2. It was suggested that this was because the deformation characteristics around the crack tip were significantly different in the two cases. This is also likely to be the case during crack growth, which would suggest that the relationship between the local and global crack growth resistances is different in the flat fracture and shear lip regions of the specimen. Therefore, it would appear unlikely that J_0 can characterise the average local crack growth resistance of the material in non-sidegrooved specimens.

The basis of equation 8.7 is that the overall J_0 -R curve arises from two contributions, the shear lip and flat fracture contributions. In determining the flat fracture contribution, it was assumed that the crack tip constraint is uniform across the whole region, and is the same as that in the central region of the specimen. This is because the J_f -R curve derived from equation 8.8 is determined from the J-R curve for specimens sidegrooved by 25-50%, for which the crack tip constraint is known to be uniform across the crack front, see section 8.2, and from the J_1 values for the central region of the specimen. However, in non-sidegrooved specimens the CTOD is higher and thus the crack tip constraint lower, near the edges of the flat fracture region, i.e. in the "low" crack tip constraint region, compared to the central region, see section 8.2. Therefore, there is a contribution to the overall J_0 -R curve arising from the fact that the global crack growth resistance is higher in the "low" crack tip constraint region than in the central region, which has not been included in the flat fracture contribution. This contribution has been incorporated into the E_v values, and so the values calculated in this study will be higher than those solely associated with the energy for shear lip formation. The degree to which the E_v values have been increased will depend on the difference in the global crack growth resistance for the "low" crack tip constraint and central regions of the specimen, and the ratio of the shear lip width to the size of the "low" crack tip constraint region. Therefore, the reported trend of E_v with shear lip width may be a result of a variation in the ratio

of shear lip width to the size of the "low" crack tip constraint region, with increasing shear lip width. As the E_V -S relationship was found to be independent of specimen size and geometry, it would appear that the additional contribution from the "low" crack tip constraint region is independent of specimen size and geometry, for a given shear lip width. Therefore, it would appear still feasible to determine the overall J_O -R curve, from equation 8.7, using the J_F -R curve appropriate to the crack tip constraint in the central region of the specimen, and the E_V values, which include the additional contribution from the "low" crack tip constraint region. It is worth noting that it is not practical to modify equation 8.7 so that there is a separate term for the "low" crack tip constraint region, because of the difficulty in determining the size of this region. In addition, it is not practical to determine the J_F -R curve for the flat fracture region, which includes the contribution from the "low" crack tip constraint region. This is because the size of the "low" crack tip constraint region is not related to the size of the central region, as the former is likely to be related to the shear lip width, and the latter is likely to be related to (B-2S).

In summary, it would appear that J_O can characterise the local and global crack growth resistance in the central region of non-sidegrooved specimens, at least up to the conditions given by equations 8.9 and 8.10. Therefore, variations in the J_O -R curve in the central region, with for example, changes in specimen size and geometry, will only arise if there is a change in crack tip constraint or CTOD in this region. In the case of the effect of geometry for specimens of a constant thickness, it was found that both the J_O -R curve and CTOD, in the central region, were independent of specimen geometry. When the shear lip width and length are related, there is a unique J_O -R curve for the shear lip region. Under these conditions, J_O can also characterise the global crack growth resistance in the shear lip region. As the overall J_O -R curves for non-sidegrooved specimens comprises two components, the shear lip and flat fracture components, J_O can characterise the average global crack growth resistance of non-sidegrooved specimens, when it characterises the global crack growth resistance in the shear lip and flat fracture regions.

However, it is unlikely that J_0 can characterise the average local crack growth resistance of non-sidegrooved specimens, as it would appear that the relationship between the local and global crack growth resistances is different in the flat fracture and shear lip regions of the specimen.

The energy for shear lip formation, E_v , determined in this study, included a contribution arising from the fact that the higher crack growth resistance of the "low" crack tip constraint region, compared to that of the central region, had not been included in the flat fracture contribution.

8.4.2.2 Effect of Specimen Size and Geometry on the J_0 -R Curve

To understand the effect of specimen size and geometry on the J_0 -R curve, there is a need to evaluate their effect on the shear lip and flat fracture contributions. The shear lip contribution is dependent on the shear lip width, and so an investigation was carried out to determine the factors which affected the shear lip width. It was found that the shear lip width in the compact specimens, at a given 9 point average crack extension, was proportional to the square root of the ligament size. This is illustrated in Figure 8.10, which shows a plot of $S/\sqrt{b}$ versus the 9 point average crack extension obtained from specimens of various sizes and geometries. It can be seen that the shear lip width increases approximately linearly with crack extension, at least over the range of shear lip widths investigated. In addition, it would appear that the shear lip width is not dependent on specimen thickness. However, these results were obtained from specimens where the initial thickness to ligament ratio (B/b) was about unity or less. Etemad (1983) has carried out tests on a different steel using three-point bend specimens of various sizes in which the B/b ratio was unity or more. Although shear lip widths were not measured from the fracture surfaces, it was reported that when the thickness was increased for a constant ligament size or when the ligament size was decreased for a constant thickness, the degree of crack tunnelling also decreased, Etemad (1984). Crack tunnelling is where the amount of crack extension is larger in the centre than at the edge of the specimen. For a given 9 point average crack

extension, a reduced degree of crack tunnelling indicates that the shear lip length is larger, and thus from Figure 8.8 also the shear lip width. Therefore, the results of Etemad suggest that as the thickness is increased or the ligament size is decreased, when $B/b > 1$, there is an increase in the shear lip width.

It can be seen from Figure 8.10 that the shear lip width for a given 9 point average crack extension is similar for the three and four-point bend and compact geometries. This suggests that the size of the region of low crack tip constraint for a given specimen size, is similar for geometries with similar plastic constraint factors. However, the shear lip width is larger in CCT specimens than the same sized DEN specimens. It is not possible to compare directly the shear lip widths for the CCT and DEN specimens with those for the bend and compact specimens, as the CCT and DEN specimens were of a different size and have a B/b ratio > 1 so that their shear lip widths will not be normalised by $1/\sqrt{b}$, as discussed previously. However, it would appear that geometries with lower plastic constraint factors have comparatively larger shear lip widths and thus larger regions of low crack tip constraint, which might be expected, as these geometries have more difficulty in maintaining high triaxiality, see section 7.2. This is in agreement with the results of Garwood (1979), which showed that CCT specimens have a proportionally larger shear lip than bend specimens.

The shear lip contribution to the overall J_O-R is dependent on the energy for shear lip formation E_v and the S^2/B ratio, see equation 8.7. E_v is initially dependent on the shear lip width or length. However, when the shear lip width or length is such that the shear lip contribution is significant (above about 5%), E_v has reached a plateau value, at least for the material used in this study. The S^2/B term for a given average crack extension will be dependent on specimen geometry. In addition, when B/b is about unity or less, the S^2/B term will be proportional to b/B and also w/B if a/w is constant, as it would appear that $S \propto \sqrt{b}$. In the case when B/b is greater than unity, when it would appear that the shear lip width is dependent on the specimen thickness and inversely dependent on the ligament size, the S^2/B term is likely to be

inversely dependent on b/B or w/B if a/w is constant.

The flat fracture contribution will be primarily dependent on the J_f -R curve, at least in this study, as $B > 2S$, and thus the J_i value in the central region of the specimen, see equations 8.7 and 8.8. Therefore, factors which affect J_i will affect the flat fracture contribution. In this study, J_i is dependent on specimen thickness reaching a plane strain value after a thickness of between 37.5-50mm, but it is not apparently dependent on the ligament size or the specimen geometry, see section 7.2.

The effects of specimen size and geometry on the shear lip and flat fracture contributions described above, are used in the following part of the section to explain the effects of specimen size and geometry on the overall J-R curve found in this study and that reported by other authors. In most of the studies by other authors which are mentioned, the largest crack extensions are relatively small (i.e. less than 15% of the ligament size) and so the J values calculated from the various approaches (i.e. J_o , J_r and J_m) will be similar.

In this study, the effect of increasing the width of specimens of a constant thickness was to increase dJ/da marginally, see Figure 5.14. This is a result of an increase in the shear lip contribution as the w/B ratio is increased, because for these specimens B/b was about unity or less. There is little change in the flat fracture contribution as J_i is unaffected by specimen width over the range investigated. This trend in dJ/da with specimen width is as reported by a number of other authors, for example Garwood et al (1978) and Marandet et al (1981). However, a number of authors reported finding no effect of specimen width on dJ/da , for example Vassilaros et al (1980) and Carlsson et al (1981). There are a number of possible explanations for these results. In the case of the study by Vassilaros et al (1980), the crack extension was predicted using an unloading compliance method, and it would appear from their results that the agreement between the final predicted and measured crack extensions in each test was dependent on the w/B ratio. This suggests that if the crack extensions had

been predicted accurately in each case, an effect of width on dJ/da may have been found. In the case of the study by Carlsson et al (1981) there are two possible explanations. One is that, as the material used by Carlsson et al was different to that used in this study, it may be that E_v and thus the shear lip contribution is smaller for some materials than others, and so the effect of width on dJ/da may remain undetected within experimental scatter, particularly when dJ/da is calculated from data points associated with small crack extensions (i.e. less than 2mm) as in this case. Alternatively, it may be that not all the ligament sizes satisfied the ligament criterion for plane strain crack tip conditions in this case, and so increasing the width resulted in a reduction of J_i and thus the flat fracture contribution which offset the increase in the shear lip contribution, so that there was no detectable effect of specimen width over the range investigated. Etemad (1983) reported that there was a decrease in dJ/da with increasing width. This is expected in this case, as B/b was about unity or greater, and so the shear lip contribution will decrease as the width and thus w/B decrease, as discussed previously.

In this study, the effect of increasing specimen width and thickness proportionally was to leave dJ/da apparently unaltered over the thickness range 13-37.5mm, but to decrease dJ/da on going from 37.5 to 50mm, see Figure 5.10. A change in dJ/da over the thickness range 13-50mm is expected, as J_i decreases to the plane strain value resulting in a decrease in the flat fracture contribution. No change in the shear lip contribution is expected, as the w/B ratio remains the same. It would appear that the trends in J_i and dJ/da over the thickness range 13-50mm are different, but it is possible that they are the same within experimental scatter. If the specimen size were further increased, little change in dJ/da would be expected, as J_i in the central region of the specimen is now the plane strain value. This latter trend of dJ/da with specimen size is in agreement with that reported by many authors, for example Garwood (1976) and Royer et al (1979).

In this study, the effect of increasing the thickness of specimens of a constant width is to decrease dJ/da significantly,

see Figure 5.9. This is a result of a decrease in the shear lip contribution as w/B decreases, because for these specimens the B/b ratio was about unity or less, and a decrease in the flat fracture contribution, because J_i in the central region decreases to the plane strain value over this thickness range 13-50mm. A further increase in specimen thickness would be expected to increase dJ/da as the shear lip contribution would increase as w/B decreases, because the B/b ratio is now greater than one. Little change to the flat fracture contribution would be expected as J_i in the central region is the plane strain value. The decrease in dJ/da with increasing thickness when the B/b ratio is about unity or less, is in agreement with that reported by a number of other authors, for example Read and Reed (1977) and Garwood and Turner (1977). Marandet et al (1981) reported no effect of thickness, although in this study the effect of an increase in specimen thickness was to change the B/b ratio from being less than one to greater than one. Therefore, it would be expected that with increasing specimen thickness, dJ/da would decrease then increase. However, as the B/b ratio was not varied significantly either side of unity, it is likely that the expected changes in dJ/da were not detected due to experimental scatter.

Specimen geometry is expected to affect the J_0 -R curve because of the effect on the shear lip width and thus the shear lip contribution. The flat contribution is not expected to be affected significantly by specimen geometry because there appears to be no effect on J_i in the central region of specimens of a given thickness. Therefore, geometries with lower plastic constraint factors are expected to have higher J_0 -R curves, at least for specimens of a given size. This trend is in agreement with the results of this study, which show that the J_0 -R curve for the CCT geometry is higher than that for the bend and compact geometries, which in turn is higher than that for the DEN geometry, see Figure 5.4. The effect of geometry is not large in this case, as the shear lip contribution is relatively small, (on average 15%) at the maximum crack extensions investigated in this study. A higher J-R curve for the CCT geometry, compared to the bend and compact geometries, is as reported by a number of other authors, for example

Garwood (1979) and Sampath et al (1981).

The previous discussion has considered the effect of specimen size on the shear lip width for relatively small specimens (i.e. $B, b < 100$ to 200mm). In the following section the situation for large specimens is discussed. The effect of increasing the shear lip width for a given 9 point average crack extension, by, for example, increasing the specimen size when the B/b ratio is kept constant, is to reduce the degree of crack tunnelling because of the relationship between shear lip width and length, as discussed previously. However, there will come a point when there is no crack tunnelling and the crack extension is uniform across the whole crack front. When this point is reached, it is not expected that the shear lip width and length, at a given 9 point average crack extension, will increase any more, such that the amount of crack extension is larger at the specimen edge than at the specimen centre. This will mean that the shear lip width can no longer increase with, for example, increasing specimen size. Therefore, if the specimen dimensions were to further increase, then the S^2/B ratio and thus the shear lip contribution, see equation 8.7, would decrease, until in the limit of very large specimens, the overall J_0-R will just comprise of the flat fracture contribution, which might be as expected.

In summary, it would appear that the effect of specimen size and geometry on the J-R curve, found both in this study and that reported by other authors, can be explained in terms of the effect on the shear lip and flat fracture contributions. The flat fracture contribution is primarily dependent on the J_f-R curve, as least in this study as $B > 2S$. This is in turn dependent on the crack tip constraint in the central region of the specimen and J_1 , which in this study is dependent on specimen thickness but not specimen width or geometry. The shear lip contribution is primarily dependent on the S^2/B ratio, at least when the shear lip contribution is significant (above about 5%) when E_v has reached a plateau value. S^2/B is in turn dependent on specimen geometry, as geometries with low plastic constraint factors such as the CCT geometry have larger shear lip widths than geometries with high plastic constraint

factors, such as the DEN, bend and compact geometries. In addition, it was found that in relatively small specimens ($B, b < 100$ to 200mm), $S\alpha\sqrt{b}$ when the B/b ratio of the specimen was about unity or less. Under these conditions the shear lip contribution increased with increasing b/B ratio or w/B ratio when the a/w is constant. However, in cases when the B/b ratio of the specimen is larger than unity, it would appear that the shear lip contribution decreases with increasing b/B or w/B when the a/w is constant. In relatively large specimens, it has been suggested that the shear lip width will reach a maximum size for a given 9 point average crack extension, after which point the shear lip contribution will decrease with increasing specimen dimensions.

8.4.3 J_0 -R Curves for Sidegrooved Specimens

This section considers the effect of sidegroove depth on the slope of the J_0 -R curve and is split into two main parts: the first considers the effect of shallow sidegrooves, i.e. up to a depth of 25-50%; the second considers the effect of deep sidegrooves.

There are two obvious reasons for an effect of shallow sidegrooves on the J_0 -R curve. The first is that in specimens which are insufficiently large to promote plane strain crack tip conditions in the central region of the specimen, sidegrooving can increase the crack tip constraint and thus decrease CTOD and the J_0 -R curve in this region. The second is that sidegrooving to at least a depth of 10% would appear to be sufficient to remove the shear lip and "low" crack tip constraint regions which are present near the edge of non-sidegrooved specimens, see section 7.2. It would appear from the results presented in section 8.4.2 that the removal of these regions will result in a decrease in the overall J_0 -R curve. Therefore, it would be expected that sidegrooving to a depth of 10% will result in a decrease in dJ/da , even when plane strain crack tip conditions have been obtained in the central region of the specimen. However, this is not evident from the results, for as shown in Figure 5.13, it would appear that dJ/da is not altered significantly by sidegrooving to a depth of 10% for compact specimens, $B_g=50\text{mm}$, $w=100\text{mm}$, which from the J_i values would appear

to be sufficiently large to obtain plane strain crack tip conditions. A possible explanation for this result is as follows. In non-sidegrooved specimens there is a region of low triaxiality near the specimen edge: sidegrooving will raise the level of triaxiality close to the crack tip, near the specimen edge, but it is unlikely to raise the level of triaxiality outside the crack vicinity near the specimen edge. Therefore, close to the specimen edge of sidegrooved specimens, the triaxiality in the crack tip vicinity will be similar to that in the central region of the specimen. However, remote from the crack tip the triaxiality will be lower near the specimen edge than in the central region of the specimen. As a result, it is likely that there will be more accompanying remote plastic deformation during crack growth near the edge of the specimen than in the centre of the specimen, even though the local crack tip conditions will be the same. Therefore, the relationship between the local crack growth resistance, as characterised by CTOD, and the global crack growth resistance, as characterised by J_0 , will be different near the edge and centre of the specimen. This will mean there will be a variation in dJ/da even when there is no variation in CTOD across the crack front of specimens with shallow sidegrooves. Given that there is a relationship between dJ/da and $d\delta/da$, see section 8.5, this suggestion is in agreement with the variation in $d\delta/da$ found across the crack front of specimens sidegrooved to a depth of 10%, see Figures 6.6 and 6.7. Therefore, on sidegrooving to this depth, the regions of high local crack growth resistance, the shear lip and the "low" crack constraint regions are removed, but the effect of this on the overall dJ/da is reduced because there is now a higher associated global crack growth resistance for a given local crack growth resistance near the specimen edge. As the contribution of the shear lip region to the overall J_0 -R curve for the compact specimens, $B=50\text{mm}$, $w=100\text{mm}$, is relatively small, only up to 13% for the largest crack extension used to determine dJ/da , it is perhaps not surprising that no effect of sidegrooving to a depth of 10% on dJ/da was detected within experimental scatter for this specimen size, see Figure 5.13.

As the sidegroove depth is increased, plastic deformation is progressively prevented from occurring in the region of low triaxiality, i.e. in a region near the specimen edge but outside the crack tip vicinity. The result is that the higher global crack growth resistance near the specimen edge is reduced by further sidegrooving. Therefore, the overall dJ/da will decrease with increasing sidegroove depth, even when plane strain crack tip conditions have been obtained. This trend is in agreement with that shown in Figure 5.13. However, there will come a point when all the remote plastic deformation is occurring in a region of high triaxiality when the relationship between the global and local crack growth resistance will be the same across the whole crack front. It is under these conditions that the overall J-R can be considered as the flat fracture J-R curve, (i.e. the J_f -R curve), which can be used to determine the overall J-R for non-sidegrooved specimens via equation 8.7. Increasing the sidegroove depth beyond this point will result in the plastic deformation being progressively prevented from occurring outside a narrow region above and below the crack plane. This will in turn result in a change in the plastic hinge point and the distance γ as discussed in section 8.3. It would appear from Figure 8.2 that γ starts to increase after a sidegroove depth of between 25-50%. This suggests that specimens sidegrooved to this depth can be used to obtain a flat fracture J-R curve. It is worth noting that for the specimen sizes investigated in this study, sidegrooving to this depth also promoted plane strain crack tip conditions.

As the sidegroove depth is increased above about 25-50%, there is a continual decrease in dJ/da with no indication of a lower bound value, even when plane strain crack tip conditions have been achieved, see Figures 5.11 and 6.16. This is a result of an increase in γ and thus a decrease in $d\delta/da$, see section 8.3, which, because the relationship between dJ/da and $d\delta/da$ is not affected significantly by sidegroove depth, see section 8.5, results in a decrease in dJ/da . Therefore, it would appear that the relationship between the global crack growth resistance, as characterised by J_O , and the local crack growth resistance, as characterised by CTOD, is

dependent on the sidegroove depth when above about 25-50%, even in the central region of the specimen.

In summary, it would appear that the effect of sidegrooving on the global crack growth resistance as characterised by J_0 , can be considered in terms of the effect of sidegrooving on the local crack growth resistance and the relationship between the local and global crack growth resistances. Sidegrooving can lower the average local crack growth resistance by raising the crack tip constraint up to that for plane strain, across almost all the crack front. However, it would appear that for sidegroove depths less than 25-50% there is a higher global crack growth resistance near the specimen edge than in the specimen centre, even though the local crack growth resistances are the same. It has been suggested this is because there is more remote plastic deformation occurring near the specimen edge as a result of some of it occurring in a region of low triaxiality outside the crack tip vicinity. The effect of this is to reduce the expected decrease in the average dJ/da as a result of lowering the average local crack growth resistance by sidegrooving. As the sidegroove depth is increased up to 25-50%, less plastic deformation is occurring in the region of low triaxiality and so the global crack growth resistance near the specimen edge is reduced until the global crack growth resistance is also the same across the whole crack front. Increasing the sidegroove depth above about 25-50% results in a change in the relationship between the average global and local crack growth resistances as a result of a change in the deformation characteristics of the specimen. This causes a decrease in dJ/da with increasing sidegroove depth, even when plane strain crack tip conditions have been obtained.

8.4.4. Comparison of the Approaches for Calculating "J" after Crack Initiation

In the previous sections it has been shown that J_0 can characterise the global crack growth resistance of a material, at least up to the conditions given by equations 8.9 and 8.10. In this section the capability of J_m and J_r for characterising the global crack growth resistance is determined and compared to that for J_0 .

In this study the numerical difference between J_O and J_m values was less than 10%, even though the amount of crack extension was up to 40% of the original ligament size for the DEN and CCT specimens, see section 5.2. Therefore, it would be expected that the capability of J_O and J_m to characterise the global crack growth resistance of the material will be the same, at least over the range of conditions investigated in this study. To confirm this, the flat fracture J_m -R curves were determined for the various specimen geometries investigated, using the same approach by which the flat fracture J_O -R curves were determined, see section 8.4.2.1. The relationship between energy for shear lip formation and shear lip width or length determined from J_m -R curves, was found to be independent of specimen size and geometry as was that determined from J_O -R curves, see section 8.4.2.1. Figure 8.11 gives the J_f -R curves for the various specimen geometries determined from their J_m -R curves. It can be seen that there is no effect of specimen geometry on the J_f -R curves, within experimental scatter, at least up to about 2.5mm, the maximum crack extension investigated. This is in agreement with the apparent specimen geometry independence of the J_f -R curve determined from J_O -R curves, see Figure 8.9, and this illustrates that J_O and J_m can characterise the global crack growth resistance of a material, at least up to 2.5mm of crack extension. As the J_f -R curves from the J_m -R and J_O -R curves are similar, compare Figures 8.9 and 8.11, the conservative estimates for the criteria for J_m controlled crack growth will be similar to those for J_O controlled crack growth, given by equations 8.9 and 8.10. These criteria are similar to those suggested by Ernst (1983) for J_m controlled crack growth of

$$\Delta a \leq 30\%b_O$$

$$\omega \geq 0$$

The above criteria were deduced from the J_m -R curves for compact specimens sidegrooved to a depth of 20%. It is expected that this sidegroove depth is such that the J_m -R curves can be considered as the flat fracture J_m -R curves and so the criteria suggested by Ernst can be compared to those suggested in this study.

In most cases the difference between the J_r and J_O values was less than 10%, although for the CCT and in particular the DEN

specimens, the difference was up to about 40% after about 2mm of crack extension. To evaluate the capability of J_r to characterise the global crack growth resistance of the material, the J_f -R curves for the various specimen geometries were also determined from their J_r -R curves. In this case the relationship between the energy for shear lip formation and the shear lip width or length was apparently geometry dependent at the larger crack extensions. As this relationship was independent of specimen geometry when determined from J_o -R or J_m -R curves, it was assumed that the apparent geometry dependence in this case was due to a breakdown in J_r controlled crack growth at larger crack extensions. Therefore in this case, the relationship determined from the bend and compact specimens was used for all the geometries. Figure 8.12 gives the J_f -R curves for the various specimen geometries, determined from their J_r -R curves. It can be seen from the figure that the J_f -R curves for the CCT and DEN specimens deviate from those for the bend and compact specimens at crack extensions greater than 1.5mm. Therefore, it would appear that above about 1.5mm of crack extension, one or both of the criteria for J_r controlled crack growth have been violated. This would suggest that these criteria are

$$\Delta a \leq 30\%b \quad 8.11$$

$$\omega \geq 2.2 \quad 8.12$$

although one of these criteria may be a conservative estimate of the actual criterion. As the CCT geometry has a lower plastic constraint factor than that for compact and bend geometries, it is expected that equations 8.11 and 8.12 are at least conservative estimates of the criteria for bend and compact geometries. A comparison of equations 8.11 and 8.12 with equations 8.9 and 8.10 shows that J_o and J_m can characterise the global crack growth resistance of a material over a wider range of conditions than J_r . The reasons for this are discussed in the following section.

The principle behind using J_r as a crack growth characterising parameter is different from that for J_o and J_m . In the case of J_r it is assumed that the J controlling crack growth and thus the magnitude of the stress and strain field ahead of a growing crack, is the same as that controlling the stresses and strains ahead of a stationary crack of the same length. This may be the

case for a purely elastic material (linear or non-linear), but from the work of Rice et al (1980), it is not the case for an elastic-plastic material, which is more relevant in the case of ductile crack growth, as plastic deformation always occurs. This is because although the stress field is similar ahead of a stationary and growing crack tip for an elastic-plastic material, the strain singularity is reduced from a $1/r$ to a $\ln 1/r$ type ahead of a growing crack tip, due to irreversible straining ahead of the crack tip. Rice et al (1980) have suggested that a J-like parameter can characterise the near field crack tip conditions, such as the magnitude of the strain field ahead of the growing crack tip, provided that this J is not a function of the rate of change of crack length under rigid-plastic conditions. Both J_0 and J_m satisfy this condition, unlike J_r , and so this explains why J_0 and J_m can characterise the global crack growth resistance of a material over a wider range of conditions than J_r .

Hutchinson and Paris (1979) suggested that J (presumably J_r) can characterise the crack growth resistance of a material, provided that the size of the region in which J does characterise the stress and strain fields dominates the near crack tip region where J does not. They suggested that this will be the case if the following criteria are satisfied.

$$b \gg \Delta a$$

$$\omega \gg 1$$

Shih et al (1980) from finite element analyses of growing cracks, quantified these criteria as

$$\Delta a \leq 6\%b$$

and

$$\omega > 10 \text{ for compact and bend geometries}$$

$$\omega > 60 \text{ for CCT geometry.}$$

Comparison of the above criteria with those found in this study for J_r controlled crack growth, equations 8.11 and 8.12, shows that J_r can characterise the global crack growth resistance of a material over a wider range of conditions than suggested by Shih et al (1980). The reason for this is presumably because J_r values are numerically similar to J_0 and J_m values up to the conditions given by equations 8.11 and 8.12.

The following part of the section considers whether J_O or J_m is likely to be better in characterising the crack growth resistance of a material. When $J \gg G$, the only numerical difference between J_O and J_m arises because the ligament and η values are updated to those corresponding to the current crack length when calculating J_m , but the original values are used when calculating J_O , see Chapter 4. Although it is not possible to determine from the results in this study whether η and b values should be updated, and thus whether J_O or J_m is a better crack growth characterising parameter, it is thought likely that these values should be updated. Under fully elastic conditions, J_m is equal to J_r , see equation 2.41, unlike J_O , and therefore the principle behind using J_m and J_O as a crack growth controlling parameter is now different. As discussed previously, it would appear that under fully elastic conditions, J_r and thus J_m , is a more suitable crack growth controlling parameter than J_O . However, the relevance of this is unclear as the limit of fully elastic conditions is never reached during ductile crack growth, as plastic deformation is required for void growth and coalescence.

Summarising this section, it would appear that J_O and J_m can characterise the crack growth resistance of a material over a wider range of conditions than J_r . This would appear to be because J_O and J_m , unlike J_r , satisfy the condition suggested by Rice et al (1980) for a J -like parameter to characterise the near field crack tip conditions under rigid, perfectly plastic conditions. It was not possible from the results in this study to determine whether J_O or J_m is more capable of characterising the crack growth resistance of a material, although it is more likely to be J_m .

8.5 Relationship between the Various Investigated Parameters

In this study, the capability of four parameters to characterise the crack growth resistance of a material has been investigated. These parameters can be split into two groups, depending on whether they measure the response of the specimen during crack growth, near to or far from the crack tip, i.e. near or far field parameters.

The two near field parameters investigated were CTOD and CTOA. CTOD has been shown to be capable of characterising the local crack growth resistance of a material, provided only one crack growth mechanism operates, at least over the range of conditions investigated, see section 8.2. However, CTOA does not appear to be a suitable crack growth controlling parameter, at least under net section yielding conditions. This is because γ , which relates CTOA and CTOD, see equation 8.2, is dependent on specimen geometry and sidegroove depth when above 25-50%, even when the crack tip constraint is not affected, see section 8.3.

The two far field parameters investigated are J and δ . J (i.e. J_m or J_o) has been shown to be capable of characterising the global crack growth resistance of the material, at least up to the conditions given by equations 8.9 and 8.10, see section 8.4. However, δ has been shown to be an unsuitable crack growth controlling parameter, because $d\delta/da$, which is related directly to the plateau value of CTOA (via equation 6.6) is dependent on specimen geometry even when the crack tip constraint is not affected, see section 8.3. The J and δ crack growth resistances can be related by equation 2.48.

$$\frac{dJ}{da} = m' \sigma_y \frac{d\delta}{da} \quad 2.48$$

Table 8.1 gives the m' values calculated from dJ/da (based on J_o) and the 9 point average $d\delta/da$ across the crack front, for various specimen geometries. No effect of specimen size or sidegroove depth up to 50% on m' was found within experimental scatter for the compact geometry and so an average value ($\pm$ standard error) is given in the table. There does appear to be an effect of specimen geometry and sidegroove depth when above 50%, on m' , which contrasts with the geometry invariance of m , which relates J and δ prior to crack initiation, see section 7.2. This suggests that m' is affected by the deformation characteristics of the specimen. m' for the bend and compact geometries is similar because these geometries are all essentially bending geometries under deeply notched conditions.

Rice et al (1980) carried out some simple calculations, assuming rigid, perfectly plastic conditions, which showed that m'

was geometry dependent. These calculations suggested a m' of 2.0 for bend specimens, 1.5 for DEN specimens and 1.1 for CCT specimens. These values are in reasonable agreement with those found in this study, see Table 8.1, particularly when one considers the approximate nature of the calculations carried out by Rice et al. Shih (1981), from the HRR field equations, suggested that m' and m are related by

$$m' = m \times \frac{N+1}{N}$$

where $\sigma^N \propto \epsilon$.

Using an approximate value for N of 5 for the material used in this study, this relationship would suggest that m' should be about 20% higher than m in this case, i.e. about 1.54. Therefore, this relationship is in reasonable agreement with the results for the DEN and CCT specimens, but not those for the compact and bend specimens.

It is worth noting that the J-R curves, unlike the δ -R curves, were non-linear over the range of crack extensions investigated. Therefore, dJ/da and thus m' will be dependent on the crack extension range over which dJ/da is calculated. For example, if the crack extension range was 1.5 to 3.0mm the m' values would be up to 50% lower than those given in Table 8.1.

The following part of the section considers the relationship between J and the near field parameters, CTOD and CTOA. Rice and Sorensen (1978) suggested a relationship between a J-like parameter, either J_0 or J_m and the crack opening profile, for a perfectly plastic material and under contained yielding conditions, see section 2.4.2.1.

$$\frac{\delta_a}{r} = \frac{\alpha}{\sigma_y} \frac{dJ}{da} + \frac{\beta \sigma_y}{E} \ln \left(\frac{eR}{r} \right) \quad 2.38$$

$r \rightarrow 0$

This equation is commonly described as the Rice, Drugan and Sham (RDS) equation for crack growth. The second term on the right hand side can be neglected for a relatively tough material as used in this study, except at very small r values, such that

$$\frac{\delta_a}{r} = \frac{\alpha}{\sigma_y} \frac{dJ}{da} \quad 8.13$$

An equation of this form is expected, given the relationship between dJ/da and $d\delta/da$, equation 2.48, the relationship between $d\delta/da$ and the plateau CTOA value, equation 6.6 and that $\delta_a/r = 2 \tan (1/2 \text{ CTOA})$. Indeed, α in equation 8.13 is equal to $1/m'$ which relates dJ/da and $d\delta/da$. Therefore, the relationship between CTOA and dJ/da will be geometry dependent as m' and thus α are geometry dependent, at least under net section yielding conditions. This is the result of CTOA, unlike J , being an unsuitable crack growth controlling parameter.

Equation 8.13 can be rearranged to give a relationship between dJ/da and CTOD if CTOD is taken as the δ_a at a constant r value behind the crack tip.

$$\text{CTOD} = A \frac{dJ}{\sigma_y da} \quad 8.14$$

A relationship of this form is expected, as CTOD was assumed to be proportional to dJ/da in deriving equation 8.8, which was later shown to be accurate, see section 8.4.2.1. The coefficient A would appear to be constant for the central region of both non-sidegrooved specimens and specimens sidegrooved up to a depth of 25-50%, from the results given in section 8.4. As a result, " A " is independent of specimen geometry, compare Figures 6.15 and 8.9. As discussed previously, the relationship between the local crack growth resistance as characterised by CTOD, and the global crack growth resistance as characterised by J_0 or J_m , is not unique, and thus neither is A , see section 8.4. It would appear that A will be dependent on three factors: the first is whether the crack tip constraint is such that the shear mechanism operates; the second is the sidegroove depth when above about 25-50%; and the third is whether the level of triaxiality, in the region where the remote plastic deformation is occurring, is significantly lower than that in the crack tip region.

In summary, this section has considered the relationship between the various possible crack growth controlling parameters

which have been investigated in this study. The main conclusions are:

- i) There is a direct relationship between the plateau value of CTOA and $d\delta/da$, at least up to 4mm of crack extension, the maximum investigated.
- ii) There is a relationship between CTOD or dJ/da and either the plateau CTOA value or $d\delta/da$, although these relationships are dependent on the deformation characteristics of the specimen as affected by specimen geometry and sidegroove depth when above about 25-50%, at least under section yielding conditions.
- iii) There is a relationship between CTOD and dJ/da which would appear to be unique for the central region of both non-sidegrooved specimens and specimens sidegrooved up to 25-50%, and thus independent of specimen geometry. However, it would appear to be affected by three factors: the first is whether the crack tip constraint is such that the shear mechanism operates; the second is the sidegroove depth when above about 25-50%; and the third is whether the level of triaxiality, in the region where the remote plastic deformation is occurring, is significantly lower than that in the crack tip region.

Table 8.1

m' ($\pm$ standard error where appropriate) for various specimen geometries.

Geometry	m'
Compact (sidegroove depth less than 50%)	2.4 $\pm$ 0.1
Compact (sidegroove depth of 75%)	3.0
Three and four-point bend	2.3 $\pm$ 0.1
Double-edge-notched tension	1.4
Centre-cracked tension	1.5

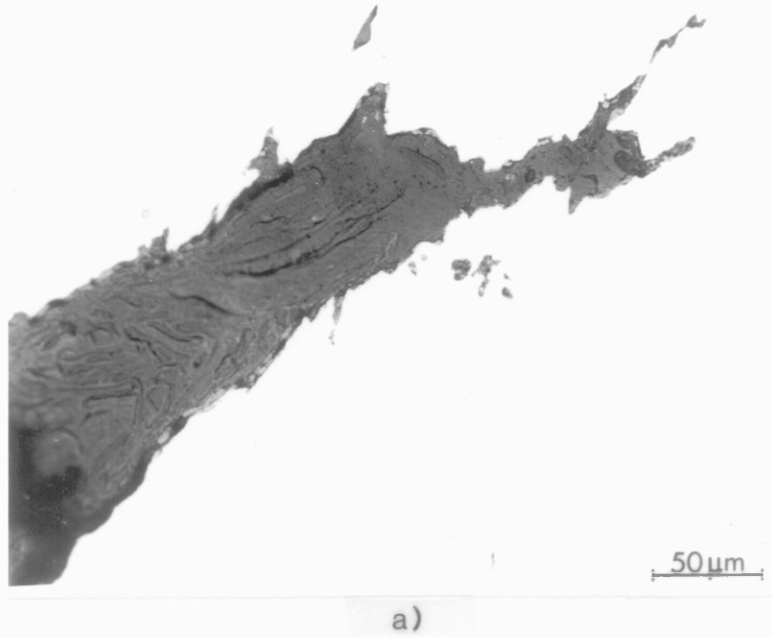


Plate 8.1 Optical micrograph taken perpendicular to the fatigue crack plane and parallel to the fatigue crack direction, a) near the edge and b) in the centre, of a non-sidegrooved double-edge-notched tension specimen

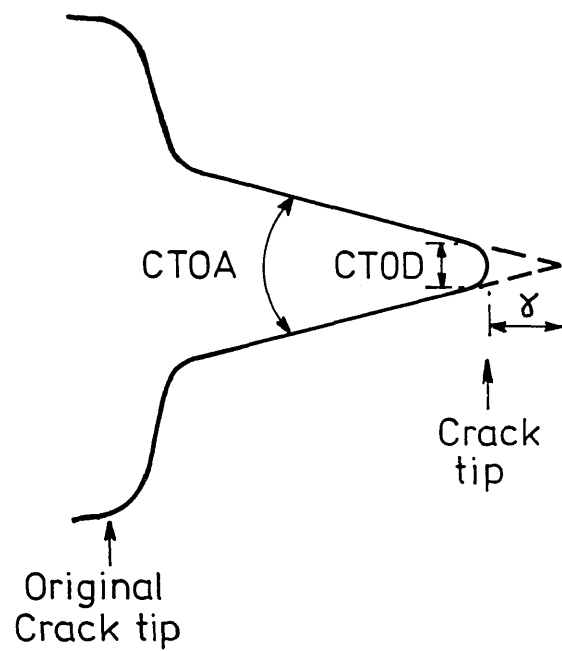


Figure 8.1 Illustration showing the relationship between CTOA, CTOD and Y

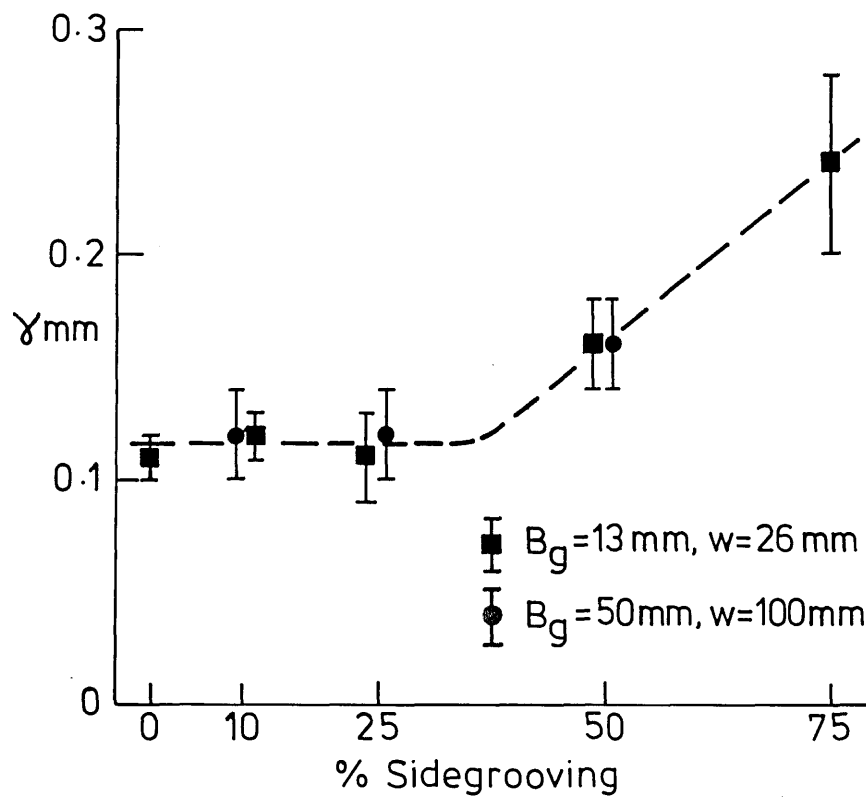


Figure 8.2 Effect of sidegroove depth on Y ($\pm$ standard error) for two compact specimen sizes

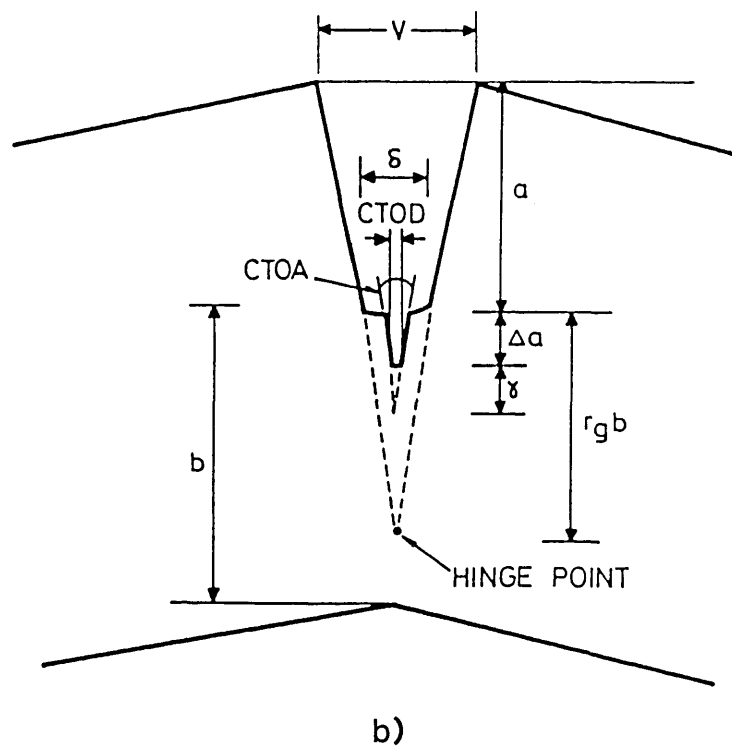
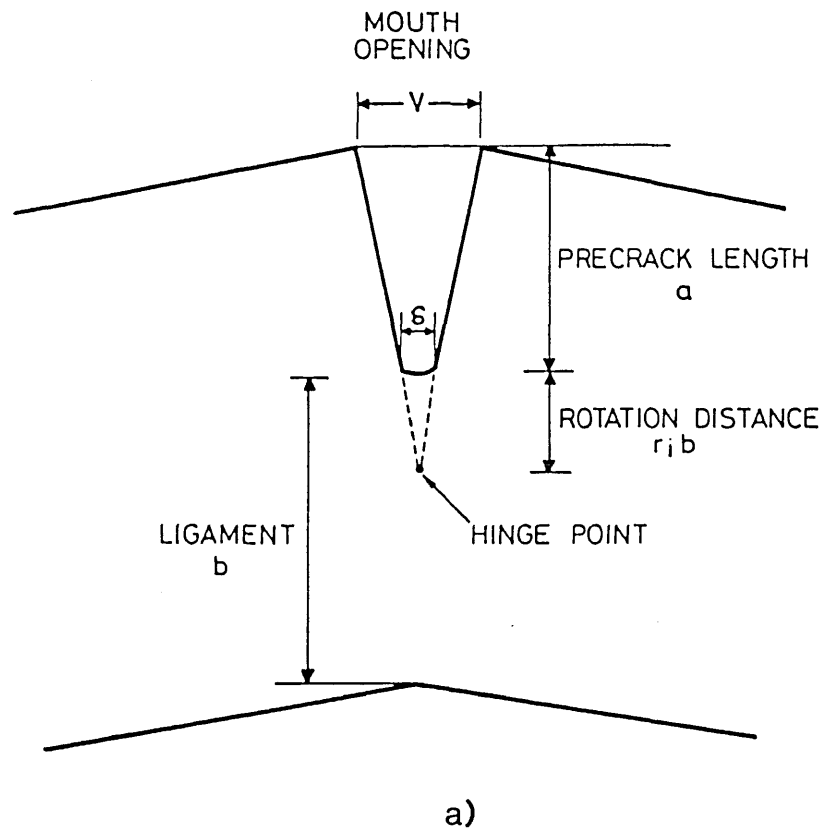


Figure 8.3 Illustration showing the hinge construction, a) prior to crack growth and b) during crack growth

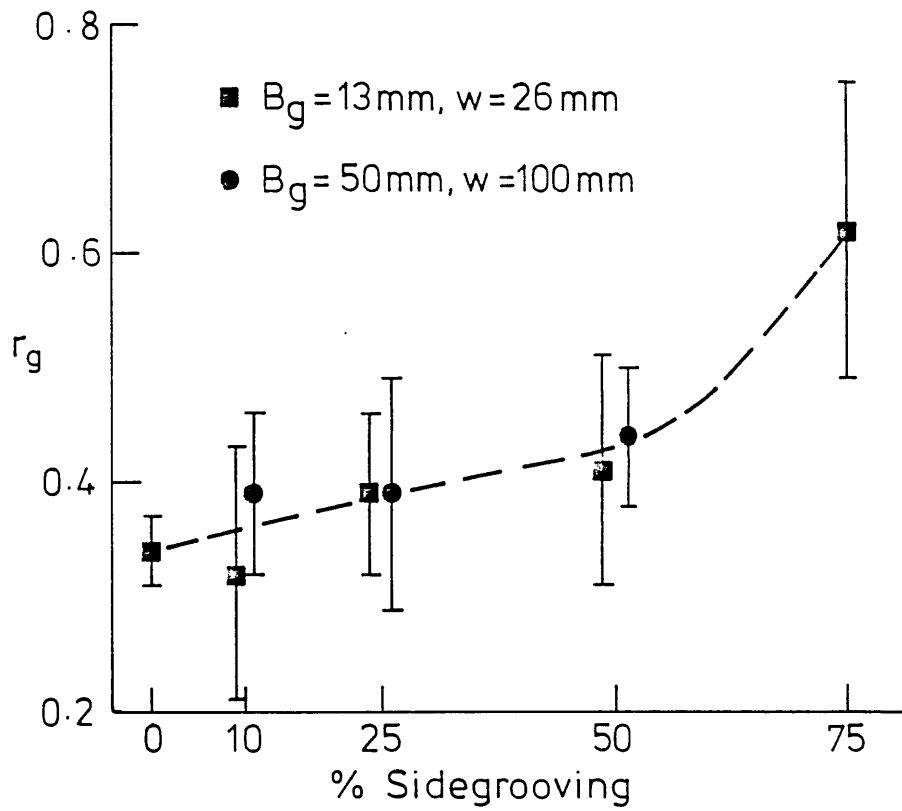


Figure 8.4 Effect of sidegroove depth on the rotation factor during growth, r_g ($\pm$ standard error) for two compact specimen sizes

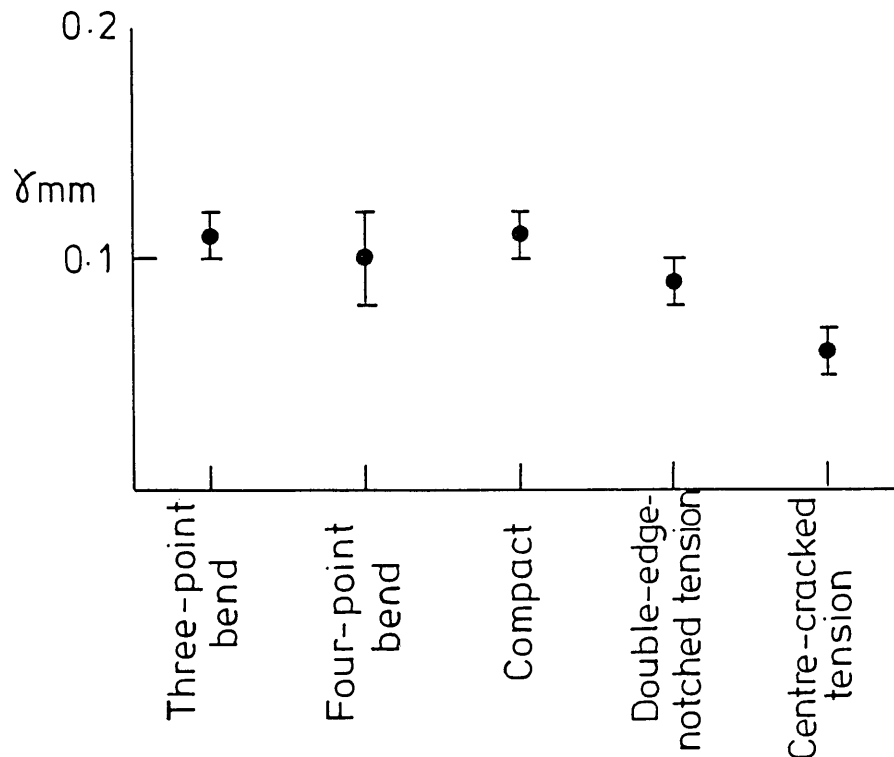


Figure 8.5 Effect of specimen geometry on Y ($\pm$ standard error)

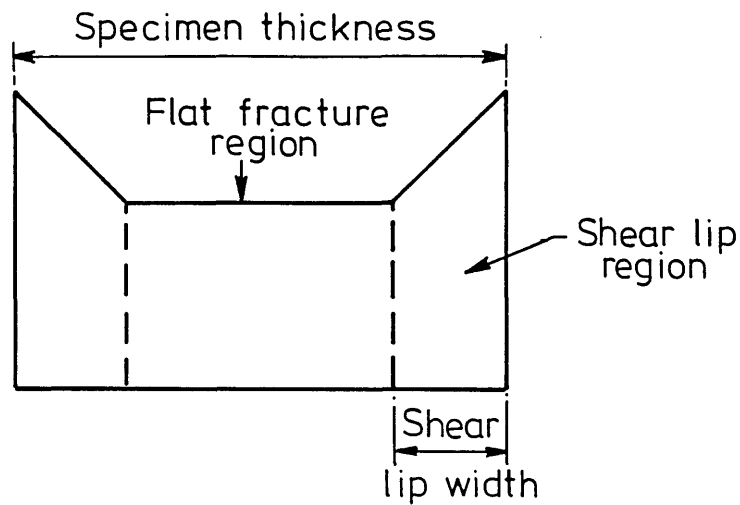


Figure 8.6 A section through a fracture surface illustrating the shear lip and flat fracture regions

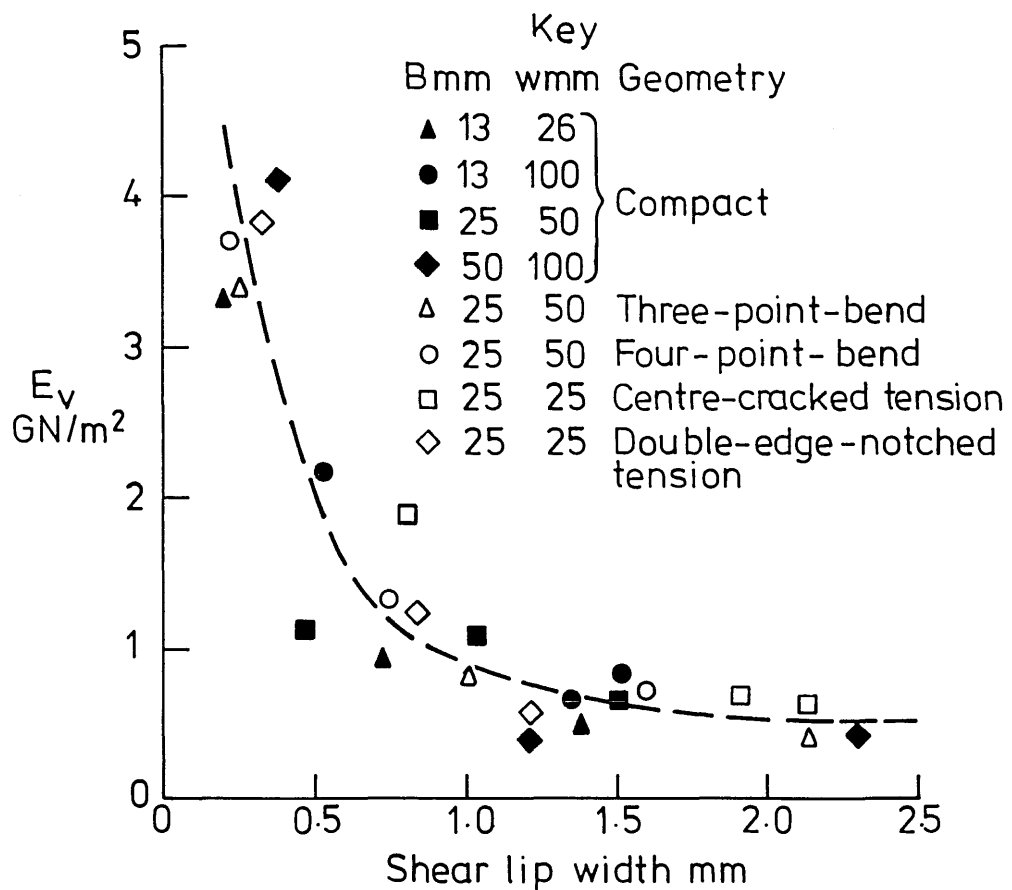


Figure 8.7 Energy per unit volume, for shear lip formation, E_v , versus shear lip width, for various specimen sizes and geometries

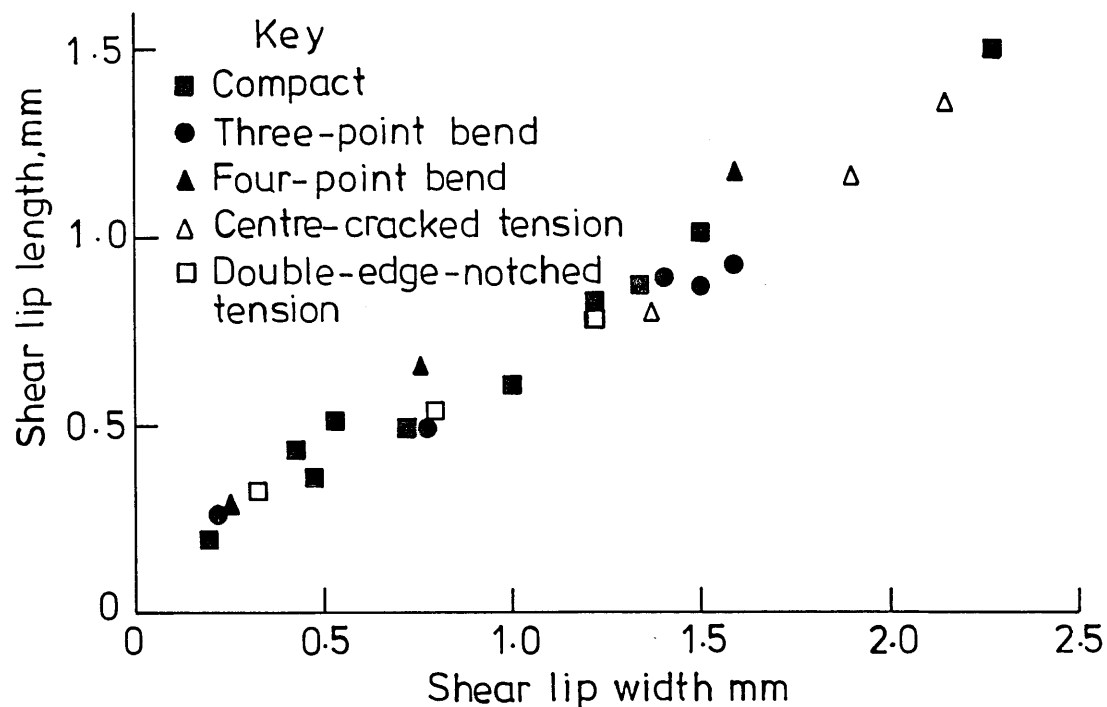


Figure 8.8 Shear lip length versus shear lip width, for various specimen geometries

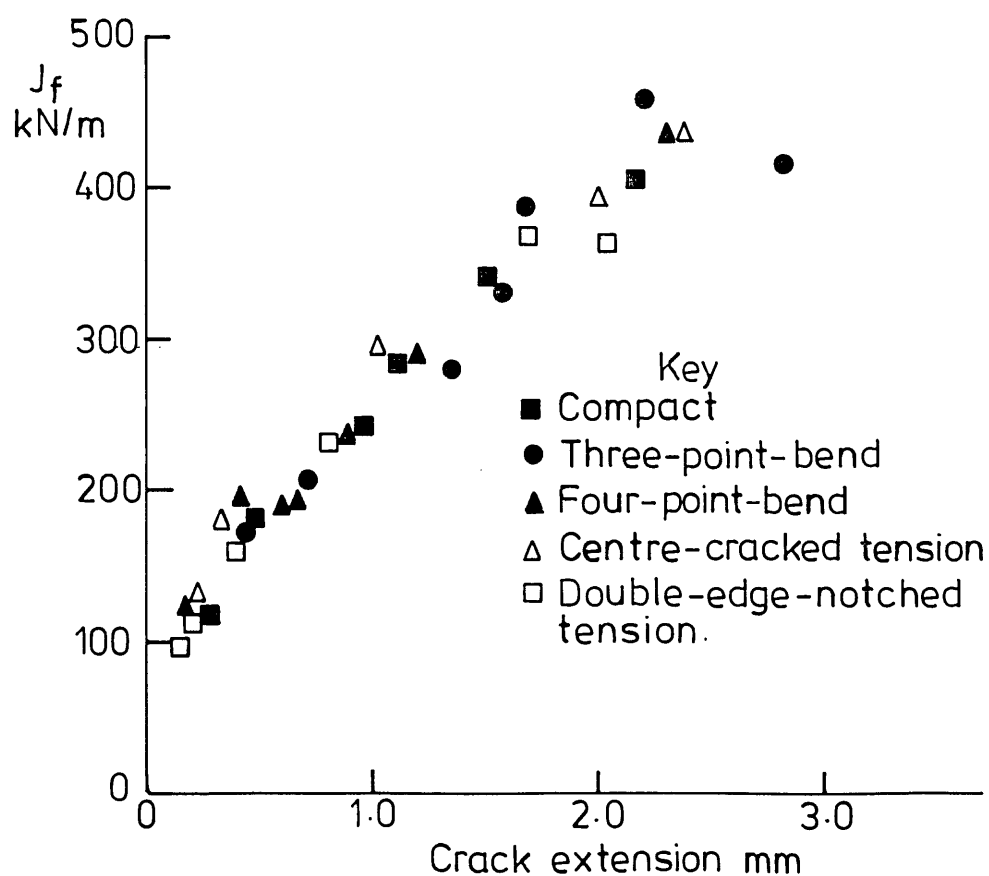


Figure 8.9 J_f -R curves determined from J_0 -R curves for various specimen geometries

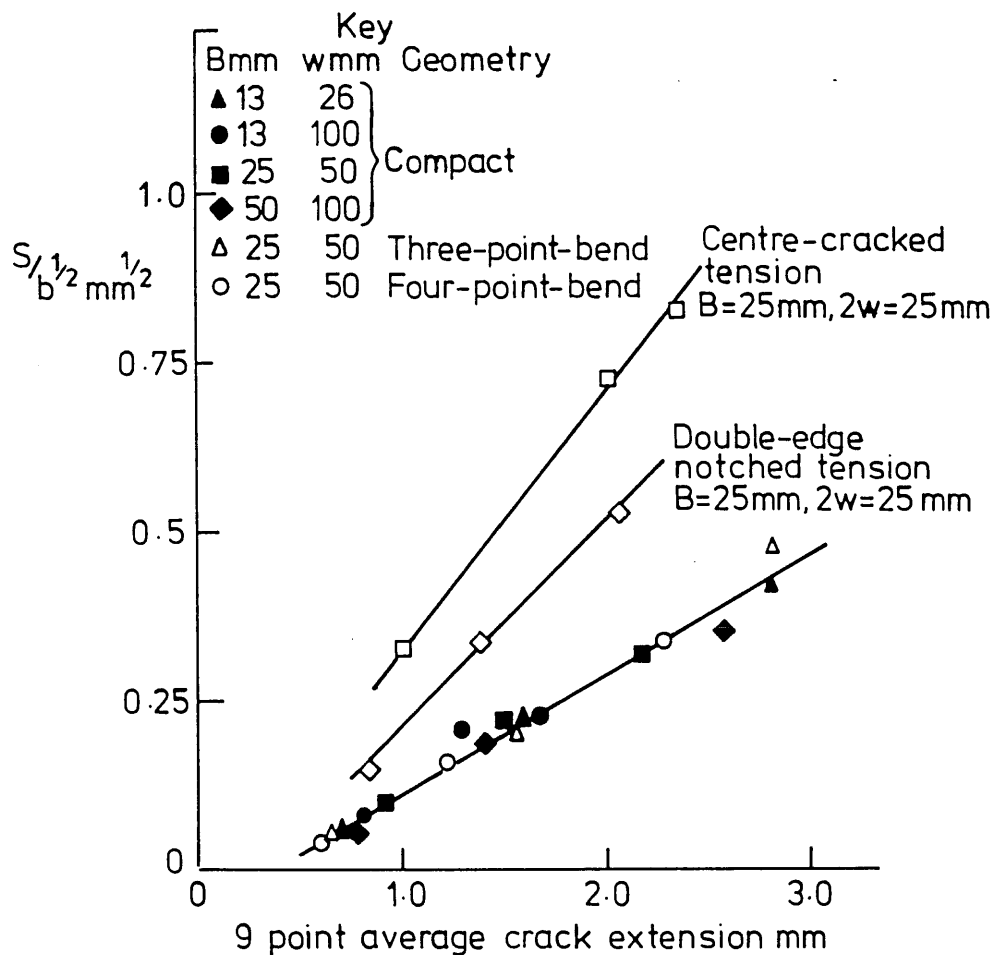


Figure 8.10 $S/b^{1/2}$ versus 9 point average crack extension for various specimen sizes and geometries

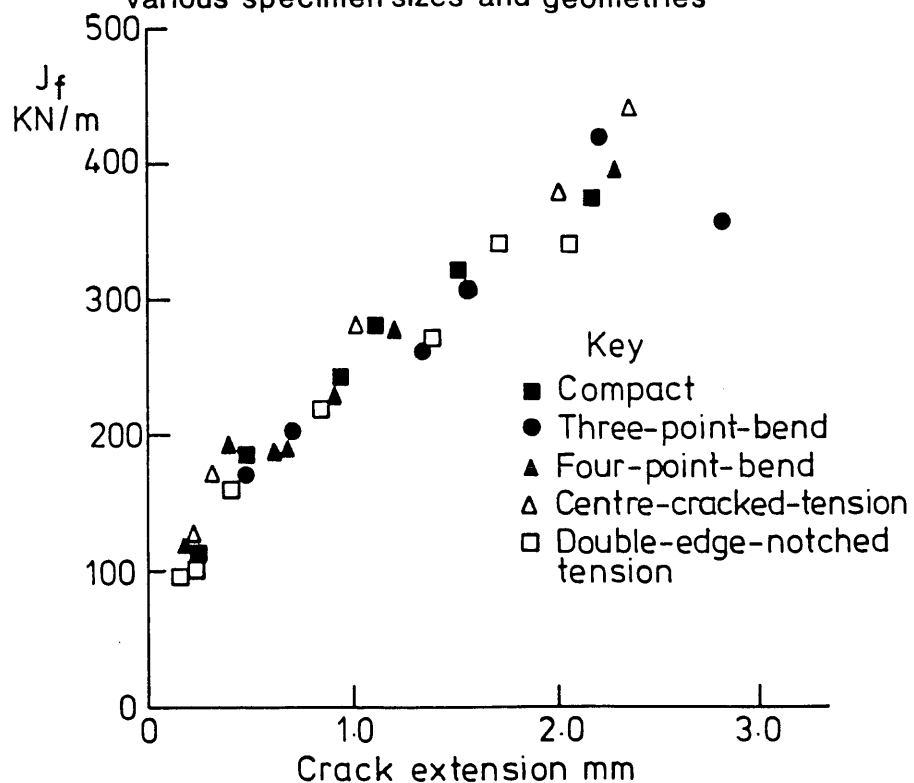


Figure 8.11 J_f -R curves determined from J_m -R curves for various specimen geometries

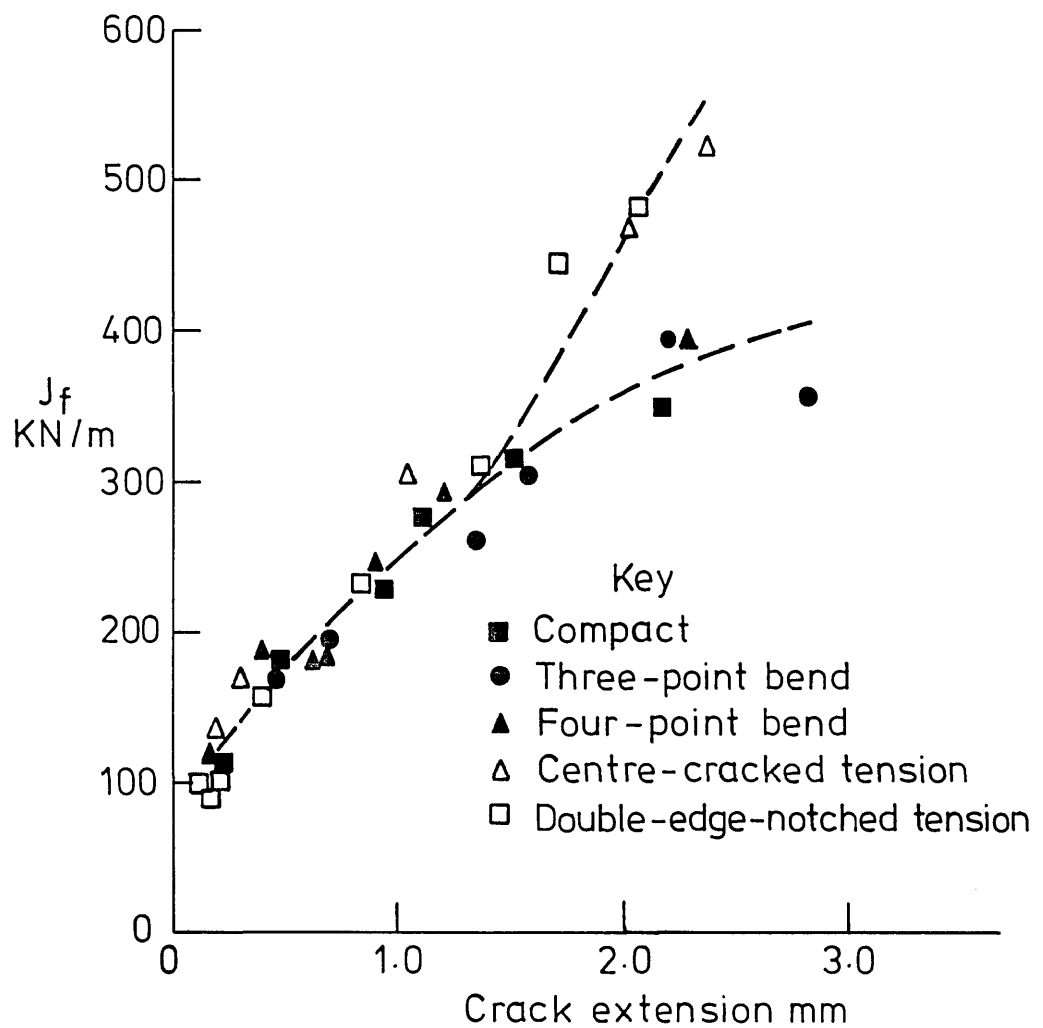


Figure 8.12 J_f -R curves determined from J_r -R curves for various specimen geometries

Chapter 9

Conclusions and Further Work

9.1 Conclusions

An investigation into the effect of specimen size and geometry on the ductile crack initiation and crack growth resistance of a mild steel has been carried out, using compact, single-edge-notched three and four-point bend, centre-cracked tension (CCT) and double-edge-notched tension (DEN) specimens. In addition, the capability of various parameters to characterise the crack initiation and/or the crack growth resistance of a material has been assessed. The parameters investigated were the J integral, the crack opening displacement at the original crack tip (δ), the crack tip opening angle (CTOA) and the crack opening displacement at the growing crack tip (CTOD). The main conclusions are:

The relationship between J and δ , up to crack initiation, was found to be independent of specimen geometry and also crack tip constraint, when it is near the value for plane strain. However, it would appear that there is a different relationship when the crack tip constraint is near the value for plane stress.

J and δ at crack initiation (J_i and δ_i) were found to be dependent on the crack tip constraint. δ_i and thus crack tip constraint, were found to vary across only a relatively narrow transition region at the edge of non-sidegrooved specimens. This transition region consists of two sub regions: a shear region, next to the specimen edge, where the crack tip constraint is sufficiently low so that a shear mechanism can operate; and a "low" crack tip constraint region, where the crack tip constraint is lower than in the central or "high" crack tip constraint region, but where the crack growth mechanism is the same. Sidegrooving the specimen induces an apparently uniform δ_i and thus crack tip constraint across the whole crack front, although very close to the specimen edge, the crack tip constraint will be lower.

J_i and δ_i values in the central region of the specimen would appear to be dependent primarily on specimen thickness and sidegroove depth, rather than ligament size, and they appear to be independent of specimen geometry for specimens of a given thickness. Increasing the sidegroove depth or specimen thickness decreases J_i and δ_i to a lower bound plateau value, assumed to be that for plane strain crack tip conditions.

The thickness and ligament size requirements for plane strain crack tip conditions appear to be similar for the CCT, DEN, bend and compact geometries. In addition, the ligament size requirement and possibly also the thickness size requirement for specimens sidegrooved to a depth of 25-50% were in agreement with those given in the ASTM J_{1C} testing standard E813-81. However, the thickness size requirement for non-sidegrooved specimens was significantly more stringent than that given in the standard, which could be because the factor J/σ_f used to specify the thickness size requirement is not suitable.

The capability of "J", δ , CTOA and CTOD to characterise the crack growth resistance of the material, has been assessed by determining whether variations in these parameters, with for example, changes in specimen size and geometry, can be explained in terms of a corresponding variation in the crack tip constraint. The results show that CTOA and δ are not suitable parameters, but CTOD can characterise the local crack growth resistance (i.e. the magnitude of the crack tip stresses and strains required for further crack growth) and "J" the global crack resistance (i.e. the increase in applied loading conditions required for further crack growth). Three approaches for determining "J" after crack initiation have been investigated: J uncorrected for crack growth, J_0 ; J corrected for crack growth, J_r ; the Ernst modified J_r , J_m . It would appear that J_0 and J_m can characterise the crack growth resistance of a material over a wider range of conditions than J_r . Whether J_0 or J_m is a better crack growth controlling parameter could not be determined from the results, although it is more likely to be J_m . The conservative estimates for the criteria for J_0 or J_m controlled

crack growth are:

$$\Delta a \leq 60\%b$$

$$\omega = \frac{b}{J} \frac{dJ}{da} \geq 1.0$$

CTOA increased with increasing crack extension to reach a plateau value after about 1.5mm of crack extension for the material used in this study. The plateau CTOA value was found to be related directly to the slope of the δ -R curve, $d\delta/da$, at least up to 4mm of crack extension, the maximum investigated. The plateau CTOA value and $d\delta/da$ are related to both CTOD and the slope of the "J"-R curve. However, these relationships are dependent on the deformation characteristics of the specimen, which are in turn affected by specimen geometry and sidegroove depth when above about 25-50%, at least under net section yielding conditions.

CTOD varied near the edge of a non-sidegrooved specimen. The CTOD in the shear region was lower than in the central region of the specimen, although this was due to a change in the crack growth mechanism rather than an indication of a lower crack growth resistance in the shear region. In the "low" crack tip constraint region, the CTOD was higher than in the central region of the specimen, indicating that CTOD is dependent on the crack tip constraint. Sidegrooving the specimen induces an apparently uniform CTOD and thus crack tip constraint across the whole crack front, although the crack tip constraint will be lower very close to the specimen edge. CTOD showed no consistent variation with crack extension in all cases investigated, although close to the specimen edge no reliable trends in CTOD could be determined. Therefore, it appears that the crack tip constraint remains the same with increasing crack extension, at least up to 4mm, the maximum investigated, except near the edge of non-sidegrooved specimens as the size of the shear region increases with increasing crack extension. There was a single relationship between CTOD and δ_i for sidegrooved specimens and in the central region of non-sidegrooved specimens; as a result the factors such as sidegroove depth which were found to affect δ_i , also affected CTOD.

The overall measured J-R curve (based on either J_O , J_R or J_m) for non-sidegrooved specimens was found to comprise two contributions, the flat fracture and shear lip contributions, i.e. Overall J = flat fracture contribution + shear lip contribution

$$= J_f(1 - \frac{2S}{B}) + E_v \frac{2S^2}{B}$$

$$= J_f(1 - \frac{2S}{B}) + J_s \frac{2S}{B}$$

J_f and J_s are the J values at a given crack extension from the J-R curves appropriate to the flat fracture and shear lip regions, respectively. E_v is the energy per unit volume required for shear lip formation, although in this study the E_v values included a contribution arising from the fact that the higher crack growth resistance in the "low" crack tip constraint region, compared to the central region, was not included in the flat fracture contribution. Under conditions of J controlled crack growth, E_v was independent of specimen size and geometry but decreased with either increasing shear lip width or length to reach a plateau value after a shear lip width of 0.8mm or a shear lip length of 0.6mm for the material used in this study. The shear lip width and length were found to be related directly, although only because the shear lip widths were all smaller than half the specimen thickness. When this is the case, there is a unique J-R curve for the shear lip region under conditions of J controlled crack growth. The slope of the J_f -R curve was related directly to the CTOD in the central region of the specimen, and under conditions of J controlled crack growth both the J_f -R curve and CTOD were the same, within experimental scatter, for CCT, DEN, bend and compact specimens of a given thickness. The relationship between the local and global crack growth resistances is likely to be different in the shear and flat fracture regions of the specimen. Therefore J can only characterise the average global crack growth resistance of non-sidegrooved specimens and not the average local crack growth resistance.

The effects of specimen size and geometry on the J-R curve (based on either J_O , J_R or J_m) for non-sidegrooved specimens, both found in this study and that reported by other authors, could be

explained in terms of the effect on the shear lip and flat fracture contributions under conditions of J controlled crack growth. The flat fracture contribution is primarily dependent on the J_f - R curve and thus the CTOD in the central region of the specimen, at least in this study, as the shear lip width is small in comparison with the specimen thickness. As CTOD was related directly to δ_i , which was in turn related directly to J_i , the factors such as specimen thickness which affect J_i also affect the whole J_f - R curve. The shear lip contribution is dependent on the ratio S^2/B and E_v ; although, at least in this study, E_v had reached a plateau value when the shear lip contribution to the overall J value was significant (above about 5%). The S^2/B ratio was dependent on the specimen geometry, and in relatively small specimens ($B, w < 100-200\text{mm}$) would appear to increase with increasing ligament to thickness ratio (b/B) when $b \geq B$ and decrease with increasing b/B ratio when $B > b$. In relatively large specimens, it has been suggested that the shear lip contribution will decrease gradually to zero with increasing specimen size.

The effect of sidegrooving the specimen on the J - R curve (based on either J_o , J_r or J_m) and thus the global crack growth resistance under conditions of J controlled crack growth, can be considered in terms of the effect on the local crack growth resistance and the relationship between the local and global crack growth resistances. Sidegrooving can lower the average local crack growth resistance by raising the crack tip constraint in the central region of the specimen, up to that for plane strain and by removing the regions of high local crack growth resistance, the shear and the "low" crack tip constraint regions. However, the decrease in the slope of the J - R curve as a result of sidegrooving to a depth of less than 25-50% is smaller than the decrease in the average local crack growth resistance. This is because the relationship between the local and global crack growth resistances in a region near the specimen edge is affected, resulting in a higher global crack resistance near the edge than in the centre of the specimen, even though the local crack growth resistances are the same. This arises because although there is a high stress state near to the crack tip in a region close to the specimen edge, induced by the

sidegrooves, the plastic deformation remote from the crack tip is still occurring in a region with a low stress state, compared to that in the central region of the specimen. As the sidegroove depth is increased up to 25-50%, this effect is reduced until the global crack growth resistance is the same across the whole crack front. Increasing the sidegroove depth above about 25-50% results in a change in the relationship between the local and global crack growth resistances across the whole crack front as a result of a change in the deformation characteristics of the specimen and this causes a decrease in dJ/da even when plane strain crack tip conditions have been obtained.

9.2 Suggestions for Further Work

The results of this study indicate a number of useful lines of research in order to gain a better understanding of the factors which affect the toughness of the material as characterised by parameters such as J , these are:

i) Thickness criterion for plane strain crack tip conditions: The results of this study suggest that a criterion based on J/σ_f is inadequate to describe the thickness requirement for plane strain crack tip conditions, although there is no obvious alternative. Further work involving evaluating the thickness requirement for plane strain crack tip conditions for a range of materials would help to evaluate a suitable criterion.

ii) Limits for J_0 and J_m controlled crack growth: The results of this study show that J_0 and J_m can characterise the global crack growth resistance of the material, at least provided certain criteria are satisfied, for which conservative estimates have been obtained. As large amounts of crack growth may be invoked when carrying out failure analyses of certain structures, it is important to know accurately the limiting conditions for J_0 and J_m controlled crack growth. Further work determining J_0-R and J_m-R curves up to large crack extension in both small and large specimens would allow these conditions to be evaluated. However, it would be important to ensure that the crack tip constraint in the central region of both specimens is the same in both cases, and that either the shear lip

size is the same at each level of crack extension, or there is no shear lip contribution, in both cases.

iii) Shear lip contribution to crack growth resistance: This study has shown the usefulness of splitting the J-R curve into shear lip and flat fracture components in order to explain the effects of specimen size and geometry. To enable this approach to be useful in a structural integrity analysis, it is important to be able to determine the J-R curve appropriate to complex crack shapes, such as semi-elliptical cracks, from that determined from conventional test specimens. Therefore, there is a need to confirm that the approach of splitting the J-R curve into two components can be applied to semi-elliptical cracks, as well as to determine the appropriate shear lip sizes for these crack shapes. This would require testing specimens containing both semi-elliptical and through thickness cracks.

Appendix A

Derivation of the Plastic Component of J for the Double-edge-notched Tension Geometry

J_p is given by equation A.1 from Rice et al (1973):

$$J_p = \frac{-1}{B} \int \frac{\partial q_p}{\partial b} \bigg|_Q dQ \quad A.1$$

The relationship between the load and plastic displacement is given by equation 4.28. Differentiation of the plastic displacement with respect to (wrt) ligament gives:

$$\frac{\partial q_p}{\partial b} \bigg|_Q = \frac{-Q}{BLb} f' \left(\frac{Q}{LBb} \right) + \frac{-Q}{BL^2} \frac{dL}{db} f' \left(\frac{Q}{LBb} \right) + f \left(\frac{Q}{LBb} \right) \quad A.2$$

Differentiation of the plastic displacement with wrt load gives:

$$\frac{\partial q_p}{\partial Q} \bigg|_b = \frac{1}{BL} f' \left(\frac{Q}{LBb} \right) \quad A.3$$

Substituting equations A.3 and 4.28 into A.2 gives:

$$\frac{\partial q_p}{\partial b} \bigg|_Q = \frac{-Q}{b} \frac{\partial q_p}{\partial Q} \bigg|_b - \frac{Q}{L} \frac{dL}{db} \frac{\partial q_p}{\partial Q} \bigg|_b + \frac{q_p}{b} \quad A.4$$

Substituting equations A.4 and A.5 into equation A.1 gives equation A.6:

$$W_p = \int Q dq_p = Qq_p - \int q_p dQ \quad A.5$$

$$J_p = \frac{1}{Bb} \left(2 + \frac{b}{L} \frac{dL}{db} \right) W_p - \frac{Qq_p}{Bb} \quad A.6$$

Differentiation of L, which is given by equation 4.27, wrt ligament, noting that $b = w - 2a$, gives:

$$\frac{dL}{db} = \frac{-1}{b(2 - 2a/w)} \quad A.7$$

Substitution of equation A.7 into A.6 gives:

$$J_p = \frac{1}{Bb} \left(2 - \frac{1}{L(2 - 2a/w)} \right) W_p - \frac{Qq_p}{Bb} \quad A.8$$

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