

IMPROVING THE ECONOMY-OF-SCALE OF SMALL ORGANIC RANKINE CYCLE SYSTEMS THROUGH APPROPRIATE WORKING FLUID SELECTION

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ABSTRACT

Organic Rankine cycles (ORC) are becoming a major research area within the field of sustainable energy systems. However, a major challenge facing the widespread implementation of small and mini-scale ORC systems is the economy-of-scale. To overcome this challenge requires single components that can be manufactured in large volumes and then implemented into a wide variety of different applications where the heat source conditions may vary. The aim of this paper is to investigate whether working fluid selection can improve the current economy-of-scale by enabling the same system components to be used in multiple ORC systems. This is done through coupling analysis and optimisation of the energy process, with a performance map for a small-scale ORC radial turbine. The performance map, obtained using CFD, is adapted to account for additional loss mechanisms not accounted for in the original CFD simulation before being non-dimensionalised using a modified similitude theory developed for subsonic ORC turbines. The updated performance map is then implemented into a thermodynamic model, enabling the construction of a single performance contour that displays the range of heat source conditions that can be accommodated by the existing turbine whilst using a particular working fluid. Constructing this performance map for a range of working fluids, this paper demonstrates that through selecting a suitable working fluid, the same turbine can efficiently utilise heat sources between 360 and 400 K, with mass flow rates ranging between 0.5 and 2.75 kg/s respectively. This corresponds to using the same turbine in ORC applications where the heat available ranges between 50 and 380 kW_{th}, with the resulting net power produced by the ORC system ranging between 2 and 30 kW. Further investigations also suggest that the same pump could also be used; however, the heat exchanger area scales directly with increasing heat input. Overall, this paper demonstrates that through the optimal selection of the working fluid, the same turbomachinery components (i.e. pump and turbine) can be used in multiple ORC systems. This offers an opportunity to improve the current economy-of-scale of small ORC systems, ultimately leading to more economical systems for the utilisation of low temperature sustainable heat sources.

NOMENCLATURE

a	Speed of sound, m/s
A	Area, m ²
A_r	Diffuser area ratio
c	Velocity, m/s
C_w	Windage torque loss coefficient
D	Turbine rotor diameter, m
g	Acceleration due to gravity, m/s ²
h	Enthalpy, J/kg
H	Pump head, m
$\dot{m}$	Mass flow rate, kg/s
N	Turbine rotational speed, rpm
P	Pressure, Pa
PP	Pinch point, K

PR	Pressure ratio
q	Heat transfer, W
Q	Volumetric flow rate, m ³ /s
r	Radius, m
Re	Reynolds number
s	Entropy, J/(kg K)
T	Temperature, K
U	Overall heat transfer coefficient, W/(m ² K)
W	Work, W
Y	Total pressure loss coefficient
η	Efficiency, %
θ	Diffuser divergence angle, °
μ	Dynamic viscosity, Pa s
ρ	Density, kg/m ³
ϕ	Pump flow coefficient
ψ	Pump head coefficient
ω	Rotational speed, rad/s
ω_s	Pump specific speed
Δh_s	Turbine isentropic enthalpy drop, J/kg
Δh_w	Turbine enthalpy loss due to windage, J/kg
ΔP_v	Volute pressure drop, %
$\Delta T_{\log}$	Log mean temperature difference, K
ΔT_{sh}	Amount of superheat, K

Subscripts

*	Choked (sonic) flow conditions
0	Total conditions
1-5	Turbine locations
6	Pump inlet/condenser outlet
7	Pump outlet/evaporator inlet
8	Evaporator pinch point
c	Heat sink
d	Design point
h	Heat source
hi	Heat source inlet
hp	Heat source pinch
p	Pump
n	Net
o	ORC
s	Conditions after isentropic expansion
ts	Total-to-static
tt	Total-to-total

Abbreviations

CFD	Computational fluid dynamics
EoS	Equation of state
ORC	Organic Rankine cycle

1 INTRODUCTION

The growing interest in organic Rankine cycles (ORC) can be attributed to its potential to effectively convert low temperature sustainable heat sources such as solar, geothermal, biomass and waste heat into mechanical power. However, low heat source temperatures imply low cycle thermal efficiencies, which places a greater pressure on the need to develop economically viable systems. Despite successful commercialisation for power outputs above a few hundred kilowatts, ORC technology has not been widely commercialised at the smaller-scale. However, a recent review [1] suggested that automotive waste heat recovery, combined heat and power, and concentrated solar power applications could be large potential markets for small-scale ORC systems. The authors of that paper also go on to say that the successful uptake of small-scale ORC systems within these sustainable energy systems can only be realised through the high volume production of modular systems, leading to lower system costs. To achieve this, it is necessary to widen the scope of existing systems by developing components that operate efficiently over a wide range of operating conditions, and with different working fluids. However, as stated in [2], many existing state-of-the-art ORC systems are designed for a nominal operating point and exhibit poor off-design. Clearly there is a need to develop new methods to understand and predict the design and off-design performance of ORC expanders, and also to investigate the impact of working fluid selection and replacement on the performance of both the expander and the whole ORC system.

The focus of many ORC studies within the literature has been thermodynamic modelling and optimisation. For clarification, the authors make a distinction here between design optimisation and cycle optimisation. In the former the aim is to optimise the design of the ORC system to deliver the best performance for the available heat source and heat sink. In this case the desired component efficiency can be specified during thermodynamic optimisation, and then during the component design phase the components are designed to achieve this performance. On the other hand, cycle optimisation concerns the case where pre-existing system components are available, and the cycle operating conditions are optimised to maximise performance. In this case, off-design component models are critical since it is no longer suitable to assume constant component efficiency. Many examples of design optimisation studies can be found within the literature, for example [3-5]. However, within the scope of this paper, cycle optimisation studies are more appropriate, where off-design models for the pump, evaporator, condenser and expander are implemented into thermodynamic models.

Even in the case of cycle optimisation, pump efficiency is often assumed constant. In [6] it was found that the pump could consume up to 15% of the power produced by the expander, demonstrating the large impact a change in pump efficiency can have on system performance. The few authors that have considered pump performance have considered it within dynamic models [7,8]. These studies construct non-dimensional performance maps based on pump similitude theory, but this requires performance data that is particular to a given pump and not always available. The same authors have also constructed dynamic heat exchanger models, which apply a one-dimensional differential energy and mass balance to establish temperature distributions as a function of space and time. For steady-state models, heat exchanger performance is often obtained by establishing the effectiveness as a function of the heat exchanger geometry and flow conditions (ϵ -NTU method), and this has been demonstrated for ORC systems in [9].

Arguably, the expander is the most critical component so this is the main focus within this paper. Particularly in small-scale systems it is not suitable to assume constant expander efficiency as the search for optimal cycle conditions may often move the expander performance away from the design point. Indeed, it has been highlighted that thermodynamic models are only accurate when expander performance is taken into account [10]. Performance maps can be used to model turbine performance, and these plot mass flow rate and turbine efficiency against pressure ratio and rotational speed. These maps are typically non-dimensionalised using similitude theory, which is well established for ideal gases [11]. Whilst similitude theory has been applied to ORC turbines as early as the 1980s [12], and has continued until more recently [13], these analyses focussed on turbine design rather than assessing off-design performance.

Furthermore, these studies concerned axial, rather than radial turbines. For off-design, similitude has been applied to ORC turbines [14-17]. However, these studies implemented a simplified similitude model that used ideal gas relationships that are not suitable for organic fluids. A recent study showed that these formulations cannot accurately predict turbine performance when using organic fluids [18]. This agrees with recent work conducted by the authors [19]. However, the authors' work also proposed a modification to the similitude model, which is found to accurately predict ORC turbine performance during subsonic operation, when compared to CFD simulations. It is worth noting that one-dimensional loss models could be used to assess turbine performance. These loss models have been applied to ORC turbines [20-22], however this is often for turbine design, rather than assessing off-design performance. Furthermore, these loss models are based on empirical data obtained for ideal gases, and have not been validated for organic fluids. However, if validated, these loss models could have a place in off-design modelling of ORC turbines.

Another important variable within an ORC system is the working fluid, and working fluid selection remains an important research area. The key selection criteria for an optimal working fluid have been discussed and reiterated within many research papers [23-25]. Furthermore, there have been many working fluid studies where a number of working fluid candidates have been evaluated for different applications, and this has also included considering different thermodynamic cycle configurations [26-28]. However, what is missing in all of these studies is a consideration of the impact that the working fluid has on the performance of the system components, both at design and off-design conditions. It should therefore be noted that the emphasis within this paper is to investigate this coupling between the working fluid and the turbine performance, rather than reiterating selection criteria and then repeating working fluid selection studies. A few authors have previously investigated this aspect. In [16] a performance model for an ORC turbine was coupled to an ORC model and the performance of the system was evaluated when using R134a and Isobutane. More recently, a similar model was also used to investigate the potential replacement of R134a with R1234yf or R1234ze(E) [29]. However, both papers use the similitude model formulated using the ideal gas law, which is not suitable for organic fluids. Furthermore, these models were applied to only a very limited group of working fluids. In [30] a model of a scroll expander is coupled to a thermodynamic model. These results show that the same scroll expander can be operated with a range of different working fluids, resulting in expander output powers between 21 W and 400 W and isentropic expander efficiencies around 50%. This is a promising result which demonstrates the potential to improve the economy-of-scale of small ORC systems. However, that paper did not mention if similar results are possible for ORC turbines.

Previous work conducted by the authors has led to the design of an ORC turbine [31], and the generation of the non-dimensional performance map using CFD. The focus of this paper is to combine this turbine performance map with thermodynamic cycle analysis in order to investigate the interaction between the selected working fluid and the turbine performance under different heat source conditions. Preliminary investigations have already been conducted by the authors [32], and this paper extends this analysis by implementing the modified and more accurate similitude model, updating the turbine performance map to account for additional loss mechanisms not accounted for during the CFD simulation, whilst also including a consideration of how the pump and heat exchanger performance varies with different working fluids under different heat source conditions. The main novelty in this work is the ability establish the full range of heat source mass flow rates that could be accommodated using a particular turbine design and working fluid. This information is presented on a single contour plot, which can be used to evaluate the suitability of using that turbine and working fluid for a particular application. The main aim of this research is to then establish the range of heat sources that could be effectively converted into mechanical power using the same turbine design, and to demonstrate how the turbine can be matched to the available heat source by selecting the most suitable working fluid. Ultimately, this is envisioned as a useful first step towards improving the economy-of-scale of small ORC systems, since the same turbine can be manufactured in large volumes and then implemented within a range of different ORC systems designed for a variety of different sustainable heat source conditions. This is considered to be an important

and necessary step to bridge the gap between component development and the economically feasible implementation of ORC systems within a number of different sustainable energy applications. To the authors' knowledge, this study is the first to couple the modified similitude theory to an ORC thermodynamic model, and to then explore methods to improve the economy-of-scale of small-scale ORC systems.

After this introduction, the modified similitude theory is introduced in Section 2 and the performance map obtained using CFD is updated to account for additional loss mechanisms that were not accounted for during the CFD simulation. In Section 3 the turbine performance map is implemented into the cycle model whilst models for the pump and heat exchangers are described in Section 4. In Section 5, a case study is considered which produces an example of the performance contour plot, and then the model is run for a range of heat source temperatures and working fluids. For each working fluid and heat source temperature the optimal operating point is established by evaluating the resulting contour plot, and a range of potential applications are obtained. Then, in Section 6 the conclusions of this research are outlined.

2 TURBINE MODELLING

Before discussing the turbine and system modelling in the next sections, it is necessary to define the notation used throughout this paper. This is shown in Figure 1.

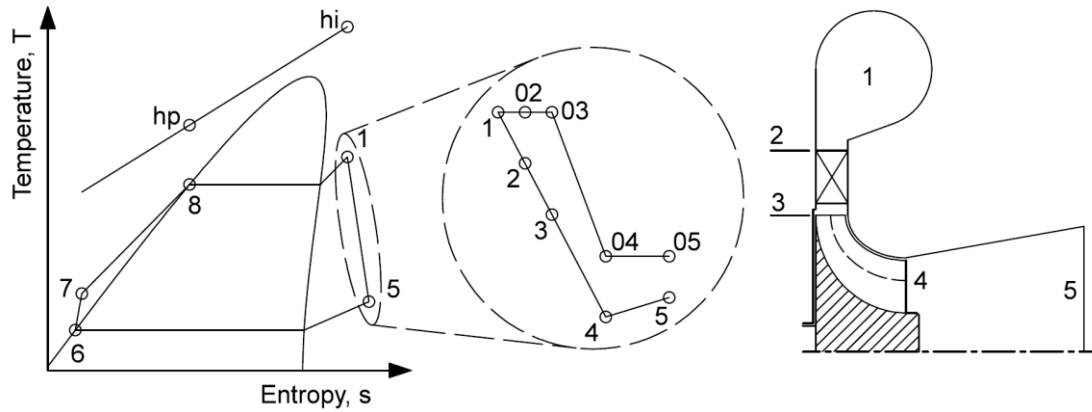


Figure 1. Notation used to model the turbine and ORC system.

2.1 Similitude theory

The authors have investigated the application of similitude theory to ORC turbines, and this led to a proposed modification to the existing model [19]. This modification is shown by Equation (1), and uses the density and speed of sound at the choked stator throat, denoted ρ^* and a^* with units of kg/m^3 and m/s respectively, instead of the turbine total inlet conditions; Δh_s is the isentropic total-to-total enthalpy drop across the turbine in J/kg , N is the rotational speed in rpm , D is the rotor diameter in m , η_{tt} is the turbine total-to-total isentropic efficiency, W is the power output in W , $\dot{m}_o$ is the working fluid mass flow rate in kg/s and μ is the dynamic viscosity in Pa s . Although the ratio of specific heats is used in the conventional similitude model, it has been neglected in Equation (1). For ideal gases ρ^* and a^* can be expressed using the ideal gas law, such that the ratio of specific heats is contained within the other non-dimensional groups. For a non-ideal gas, the ratio of specific heats has been removed as it is assumed that the variation in gas composition is accounted for by using a suitable equation of state to calculate ρ^* , a^* and Δh_s .

$$\left[\frac{\Delta h_s}{N^2 D^2}, \eta_{tt}, \frac{W}{\rho^* N^3 D^5} \right] = f \left(\frac{\dot{m}_o}{\rho^* N D^3}, \frac{ND}{a^*}, \frac{\rho^* N D^2}{\mu} \right) \quad (1)$$

Equation (1) can be simplified for a fixed turbine since the diameter cannot change. Furthermore, the term on the far right of Equation (1) is the rotational Reynolds number, and

for ideal gas turbines this term is often neglected. The previous study suggested this term can also be neglected for ORC turbines if the change in the Reynolds number is less than 200% [19]. At higher deviations, Reynolds number effects may become more prevalent, which might reduce turbine efficiency. Finally, the third term on the left hand side, the power coefficient, has been omitted for simplicity since $\dot{m}_o$, η_{tt} and Δh_s are all known. This simplification leads to Equation (2).

$$\left[\frac{\Delta h_s}{a^{*2}}, \eta_{tt} \right] = f \left(\frac{\dot{m}_o}{\rho^* a^*}, \frac{N}{a^*} \right) \quad (2)$$

Equation (2) shows that the reduced head coefficient ($\Delta h_s/a^{*2}$) and turbine efficiency η_{tt} are both functions of the reduced flow coefficient ($\dot{m}_o/\rho^* a^*$) and the reduced blade Mach number (N/a^*). Therefore, non-dimensional performance maps can be constructed based on these four parameters. It has been found that for a radial turbine operating with R245fa, R123 and R1234yf working fluids with a specified head coefficient and rotational speed, Equation (2) accurately predicts the turbine mass flow rate and isentropic efficiency to within 2% for all subsonic operating points, when compared to CFD simulations [19]. More recently, the similitude model has also been validated against unsteady CFD simulations for another radial turbine operating with these same working fluids in addition to R1234ze, pentane and isobutane [33]. In this study, Equation (2) was found to predict the mass flow rate and isentropic efficiency to within 1%. Furthermore, a comparison of the three-dimensional flow fields, in terms of blade Mach number triangles, mid-span velocity vectors and blade loading distributions also revealed very similar flow patterns within the turbine following working fluid replacement. Therefore, based on these detailed CFD simulations and given the lack of suitable experimental data concerning ORC turbines operating with different working fluids, the authors are reasonably confident that Equation (2) can be used to predict ORC turbine performance provided the conditions within the turbine remain subsonic. However, as soon as suitable experimental data becomes available it will be necessary to further verify the suitability of Equation (2). For further details on the CFD validation, readers can refer to [19,33].

Finally, it should be noted that currently the authors have focused on radial turbines for small ORC systems. However, there should be no reason why Equation (2) cannot be used to model the performance of different types of turbines, namely axial turbines, but future research efforts should investigate this further.

2.2 CFD turbine performance map

The design specification for an ORC turbine is given in Table 1. For the specified inlet conditions and working fluid the turbine performance was evaluated over a range of pressure ratios and rotational speeds using CFD. The turbine design and CFD analysis is documented in [31]. After completing each CFD simulation the mass flow rate and isentropic efficiency were obtained and then scaled using Equation (2). The turbine performance maps were then obtained by curve fitting the CFD results, and these are shown in Figures 2 and 3.

Table 1. Design point specification for the ORC turbine.

Working fluid	-	R245fa	
ORC condensation temperature	T_6	313.0	K
Total inlet temperature	T_{01}	350.0	K
Total inlet pressure	P_{01}	623.1	kPa
Pressure ratio	PR	2.5	
Mass flow rate	$\dot{m}_o$	0.7	kg/s
Rotational speed	N	37,525	rpm
Rotor diameter	D	66.7	mm

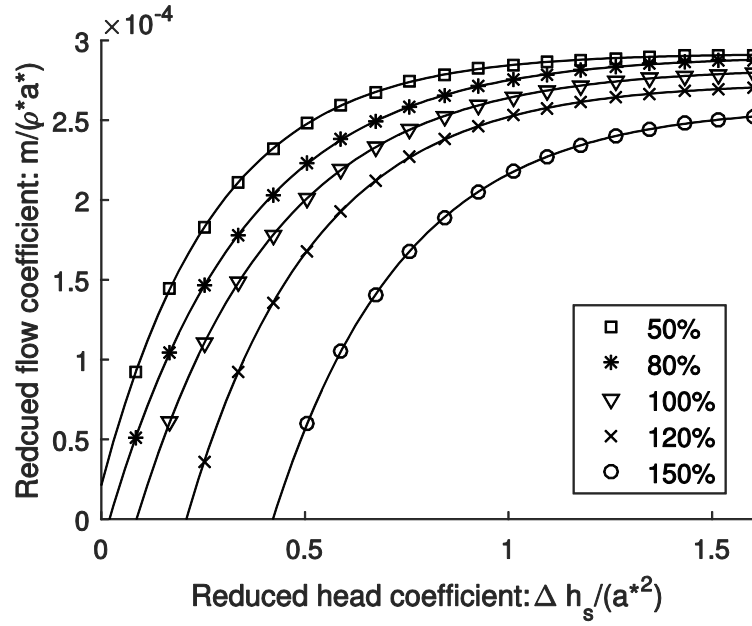


Figure 2. Variation in the reduced flow coefficient at different reduced head coefficients and reduced blade Mach numbers, as predicted using CFD simulations.

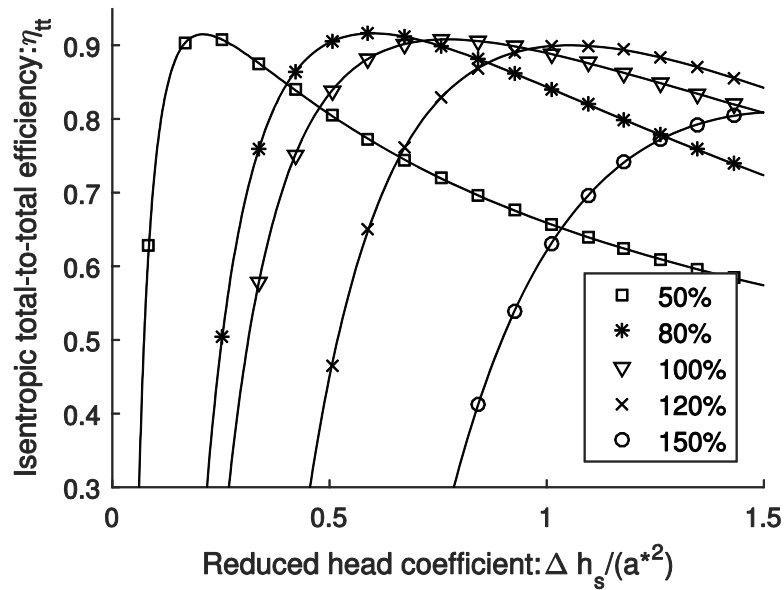


Figure 3. Variation in the turbine total-to-total efficiency at different reduced head coefficients and reduced blade Mach numbers, as predicted using CFD simulations.

2.3 Loss models

The CFD simulations used to construct Figures 2 and 3 were completed with periodic boundaries. Whilst this is necessary to reduce the computational expense of the simulations, this meant windage losses behind the rotor back face were not accounted for. Furthermore, these simulations did not consider the components upstream of the stator leading edge and downstream of the trailing edge, namely the volute and diffuser. Therefore, the performance maps should be updated to account for these additional losses before using them within further ORC studies. It should be noted that tip clearance was included within the CFD simulation and therefore tip clearance losses are already included.

2.3.1 Windage loss model

Within the clearance gap between the rotor back face and the rotor casing the circulation of fluid and the development of boundary layers on the rotor and casing walls results in a parasitic loss. As noted previously, the CFD simulation did not model this loss in an effort to reduce the simulation computational expense. Instead, a simple empirical model has been implemented for the sake of simplicity and cost. Of course, this empirical model was developed for ideal gases, so its validity for organic fluids should be confirmed through future computational and experimental studies.

This windage loss, expressed as an enthalpy loss Δh_w in J/kg, is defined by Equation (3) where C_w is a torque loss coefficient, ρ_3 is the density at the rotor inlet in kg/m³, ω is the rotational speed in rad/s, r_3 is the rotor inlet radius in m and $\dot{m}_o$ is the working fluid mass flow rate in kg/s.

$$\Delta h_w = \frac{\frac{1}{2} C_w \rho_3 \omega^3 r_3^5}{\dot{m}_o} \quad (3)$$

Four different flow regimes can occur, namely laminar and turbulent flow, both with merged and separated boundary layers respectively [34]. The flow within the clearance gap is laminar for $Re < 10^5$ and turbulent for $Re > 10^5$, where Re is the rotational Reynolds number (Equation 4). The design point Reynolds number for the developed turbine is $Re = 8.4 \times 10^6$, and therefore the flow is fully turbulent.

$$Re = \frac{\rho_3 \omega r_3^2}{\mu_3} \quad (4)$$

The ratio of the clearance gap ϵ , to the rotor inlet radius establishes whether the boundary layers are merged or separated. Following from Dixon [35], $\epsilon = 0.4$ mm was assumed which correlates to $\epsilon/r_3 = 0.012$. This is sufficiently small to assume merged boundary layers. In this instance the torque loss coefficient is given by Equation (5), which is an empirical correlation based on experimental results and is described in Glassman [34].

$$C_w = \frac{0.0622}{\left(\frac{\epsilon}{r_3}\right)^{\frac{1}{4}} Re^{\frac{1}{4}}} \quad (5)$$

2.3.2 Diffuser design and performance analysis

It is often beneficial to install a diffuser downstream of the rotor to reclaim some of the kinetic energy contained within the flow. However, the design and CFD analysis completed has not considered a diffuser, so it was necessary to design one. A simple straight-sided conical diffuser was assumed, where the geometry is controlled by the area ratio $A_r = A_5/A_4$, and the diffuser divergence angle θ . θ is a critical parameter governing diffuser performance and Aungier [36] suggested that optimal performance is obtained when $2\theta = 11^\circ$. Using this value for θ , a parametric study investigating a range of area ratios was conducted, and an empirical diffuser performance model [36] was used to assess the diffuser performance. From this study it was found that $A_r = 2.5$ provided sufficient energy recovery, increasing the isentropic total-to-static efficiency from 85.8% (no diffuser) to 88.1%. By comparison a further increase to $A_r = 4.0$ only resulted in a further increase of 0.3% to 88.4%.

It should be noted that the empirical diffuser performance model has not been validated for organic fluids. However real gas effects are generally more prevalent at the turbine inlet than at the outlet since the compressibility factor tends to reduce as the temperature and pressure increases, and the operating conditions approach the critical point.

2.4 Updated turbine performance map

Using the analysis discussed in Section 2.3, the CFD performance map was then updated. As a starting point the turbine inlet conditions were set to the original design point ($T_{01} = 350$ K, $P_{01} = 623.1$ kPa). To account for losses upstream of the stator leading edge a total pressure drop of $\Delta P_v = 1\%$ was assumed within the volute, immediately supplying the total temperature T_{02} in K and entropy s_{02} in J/(kg K) at the stator inlet using a suitable equation of state. Within this paper REFPROP has been used, which is a commercially available program containing state-of-the-art equations of state for a wide variety of different fluids [37]. However, for the sake of generality, the calculation is denoted with the notation ‘EoS’.

$$P_{02} = P_{01}(1 - \Delta P_v) \quad (6)$$

$$[T_{02}, s_{02}] = \text{EoS}(P_{02}, h_{01}, \text{fluid}) \quad (7)$$

Since the CFD performance map did not account for a volute, Figures 2 and 3 now apply to these updated stator inlet conditions (location 2) instead of the design inlet conditions (location 1). The choked conditions ρ^* and a^* are obtained by assuming an isentropic expansion from the stator inlet to the throat. An array of head coefficients consisting of 100 elements ranging from 0 to 1.6 was then constructed, and each value was converted into the isentropic total-to-total enthalpy drop from the stator inlet to the rotor outlet Δh_s . The size of this array is not critical, as it only affects the resolution of the resulting contour plot. At each head coefficient $\dot{m}_o$, η_{tt} and η_{ts} were established at 50%, 80%, 100%, 120% and 150% of the design reduced Mach number through interpolation of Figures 2 and 3. The total conditions at the rotor outlet (location 4) then follow for each combination of head coefficient and reduced blade Mach number. Here the subscript ‘s’ refers to the conditions following an isentropic expansion.

$$h_{04s} = h_{02} - \Delta h_s \quad (8)$$

$$P_{04} = \text{EoS}(h_{04s}, s_{02}, \text{fluid}) \quad (9)$$

$$h_{04} = h_{02} - \eta_{tt}(h_{02} - h_{04s}) \quad (10)$$

$$[T_{04}, s_{04}] = \text{EoS}(P_{04}, h_{04}, \text{fluid}) \quad (11)$$

Using the known value for η_{ts} the static conditions, and flow velocity c_4 in m/s, at the rotor outlet are obtained.

$$h_{4s} = h_{04} - \frac{h_{02} - h_{04}}{\eta_{ts}} \quad (12)$$

$$P_4 = \text{EoS}(h_{4s}, s_{02}, \text{fluid}) \quad (13)$$

$$[T_4, h_4] = \text{EoS}(P_4, s_{04}, \text{fluid}) \quad (14)$$

$$c_4 = \sqrt{2(h_{04} - h_4)} \quad (15)$$

With the rotor outlet conditions obtained, the diffuser performance model can then be run using the defined diffuser geometry. This supplies the total and static conditions at the diffuser outlet (location 5). The windage loss model is then run, and η_{tt} is reformulated as follows.

$$h_{05s} = \text{EoS}(P_{05}, s_{01}, \text{fluid}) \quad (16)$$

$$\eta_{tt} = \frac{(h_{01} - h_{05}) - \Delta h_w}{h_{01} - h_{05s}} \quad (17)$$

The choked flow parameters, ρ^* and a^* , associated with the original turbine inlet condition are then obtained, and the performance map is rescaled according to Equation (2). The resulting performance maps are shown in Figures 4 and 5, where they are also compared to the original CFD performance maps.

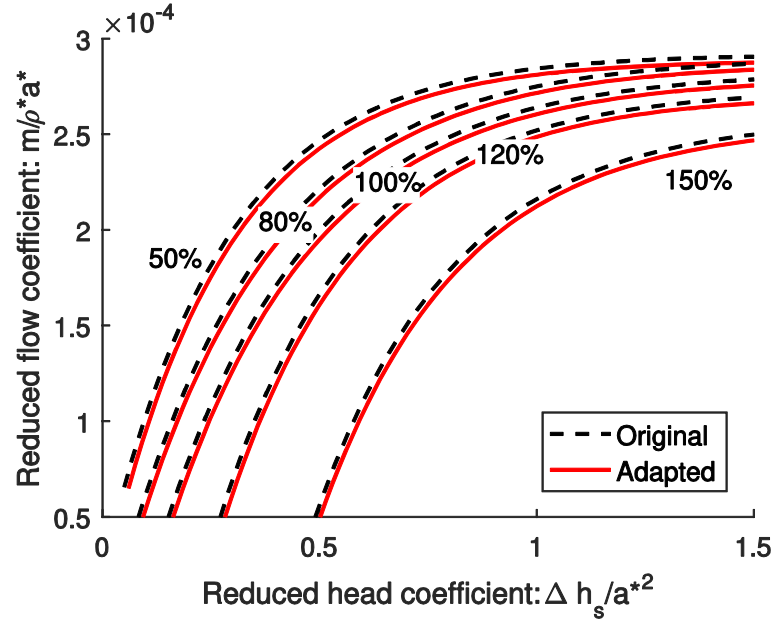


Figure 4. Updated turbine performance map showing the relationship between the reduced head coefficient and reduced flow coefficient for reduced Mach numbers ranging between 50% and 150% of the design value.

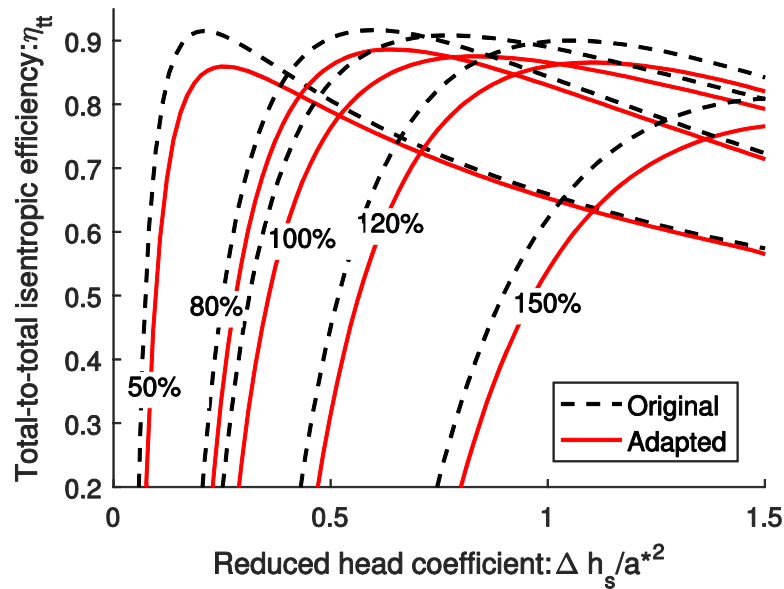


Figure 5. Updated turbine performance map showing the relationship between the reduced head coefficient and turbine efficiency for reduced Mach numbers ranging between 50% and 150% of the design value.

Figure 4 shows the variation in the reduced flow coefficient with the reduced head coefficient and reduced blade Mach number. The behaviour shown in Figure 4 can be explained by considering each additional loss that has now been modelled. Firstly, the windage loss is a parasitic loss that absorbs a fraction of the total power produced by the rotor. Therefore, it is not associated with a total pressure loss, so there is no effect on the reduced head coefficient.

To consider the diffuser performance, the total pressure loss coefficient Y is introduced (Equation 18). This is defined as the ratio of the total pressure drop through the diffuser, to the difference between the total and static pressures at the diffuser outlet.

$$Y = \frac{P_{05} - P_{04}}{P_{05} - P_5} \quad (18)$$

Across the operating conditions considered Y ranged between 0.05 and 0.3. Furthermore, the flow leaves the diffuser with a low velocity, which implies a small difference between P_{05} and P_5 . This implies a small total pressure drop within the diffuser, and a minimal change in the total-to-total isentropic enthalpy drop across the turbine. This will have a minimal effect on the reduced head coefficient. Therefore, the main shift seen in Figure 4 can be attributed to the 1% pressure drop applied upstream of the stator leading edge. This additional pressure drop increases the total-to-total pressure ratio across the whole turbine, and therefore increases the reduced head coefficient. Since the mass flow rate is unaffected, volute pressure drop simply shifts the constant blade Mach number lines to the right, as observed in Figure 4.

Figure 5 shows the variation in η_{tt} with the reduced head coefficient, and reduced blade Mach number. Considering first the diffuser, it has already been determined that there is a small total pressure drop within the diffuser, and a minimal change in total-to-total isentropic enthalpy drop. Furthermore, there is no energy transfer within the diffuser (i.e. $h_{04} = h_{05}$), so the change in η_{tt} is also minimal. Of course, if Figure 5 had plotted η_{ts} , a more significant shift would be observed since the purpose of the diffuser is to recover the kinetic energy and increase η_{ts} .

Using Equations (3) – (5) it can be shown that the windage loss is proportional to the rotational speed ω in rad/s, the meridional velocity at the rotor inlet c_{m3} and the fluid properties ρ_3 and μ_3 (Equation 19).

$$\Delta h_w \propto \frac{\omega^{\frac{11}{4}}}{c_{m3}(\rho_3\mu_3)^{\frac{1}{4}}} \quad (19)$$

Firstly, from Equation (19) it can be seen that windage loss increases with increasing rotational speed. This effect can be seen in Figure 5 where the constant reduced Mach number lines are increasingly shifted to the right with increasing speed. Secondly, Equation (19) implies that with increasing head coefficient, and therefore increasing mass flow rate, the windage loss will reduce. This is because a higher mass flow rate also implies a higher meridional velocity at the rotor inlet. This effect is also shown in Figure 5, where the original and adapted reduced Mach number lines appear to converge with increasing head coefficient.

Finally, we can consider the effect of applying a 1% pressure drop in the volute. This additional loss increases the total-to-total isentropic enthalpy drop across the turbine. Therefore, since there is no energy transfer in the volute the total enthalpy drop across the turbine remains constant, η_{tt} must reduce. Furthermore, throughout this analysis ΔP_v was kept constant, which means that at lower reduced head coefficients, which correspond to lower total-to-total pressure ratios, the volute total pressure loss is a higher fraction of the overall pressure drop across the turbine. This results in a more significant drop in efficiency at lower head coefficients, which further explains why the original and adapted reduced Mach number lines appear to converge at increasing head coefficients. It should be noted that in future studies it might be more beneficial to employ a more sophisticated volute performance model rather than applying a simple fixed value pressure drop.

3 SYSTEM MODELLING

A novel thermodynamic model has been developed which aims to establish the full range of heat source mass flow rates at a specified temperature that can be utilised using an existing turbine design, and present this information on a single contour plot. To obtain this contour plot, thermodynamic cycle analysis is coupled to the updated non-dimensional turbine performance curves (Figures 4 and 5). The result is a single contour plot that describes the performance of an ORC that utilises a particular heat source and operates with a specific turbine and working fluid. Ultimately, this plot can be used to determine the optimal heat source mass flow rates that can be effectively converted into useful power using this existing turbine. A simple subcritical ORC without a recuperator has been considered. Not only does this simplify the analysis, but it also reduces the overall cost of the system. Since the main focus is to investigate the interaction between turbine and cycle performance, additional aspects such as the required heat transfer areas, and pump performance are not considered, but instead are discussed later.

An ORC can be defined by the ORC condensation temperature T_6 , the pressure ratio and the amount of superheat ΔT_{sh} . If pressure drops within the pipes and heat exchangers are neglected, it is then simple to determine the working fluid properties at the pump inlet (location 6) and turbine inlet. For this analysis constant pump efficiency is assumed, from which the evaporator inlet conditions follow (location 7). The evaporator analysis is restricted to a simple energy balance when supplied with the evaporator pinch point PP_h (location 8). Since the aim of this analysis is to determine the optimal heat source mass flow rate, this parameter is unknown. However, the ratio of the working fluid mass flow rate $\dot{m}_o$, to the heat source mass flow rate $\dot{m}_h$, both in kg/s, is given by Equation (20), where the subscripts h_{hi} and h_{hp} refer to the heat source enthalpy in J/kg at the evaporator inlet and pinch point respectively.

$$\frac{\dot{m}_o}{\dot{m}_h} = \frac{h_{hi} - h_{hp}}{h_{01} - h_8} \quad (20)$$

With the turbine inlet conditions defined (i.e. T_{01} , P_{01}) the choked flow conditions (a^* and ρ^*) follow by assuming an isentropic expansion from the inlet to a Mach number of 1. Furthermore, the turbine outlet pressure is defined by T_6 , which in turn determines the reduced head coefficient $(h_{01} - h_{05s})/a^{*2}$. Referring back to Figure 4, for a known reduced head coefficient, there is a minimum and maximum flow coefficient that this turbine can accommodate, which correspond to the maximum and minimum reduced blade Mach numbers respectively. The minimum and maximum flow coefficients can be converted into the physical mass flow rate limits for the turbine and an array of mass flow rates can be constructed between these limits. For each value of $\dot{m}_o$ interpolation of Figure 4 supplies the reduced blade Mach number, whilst interpolation of Figure 5 supplies η_{tt} . This allows the turbine outlet conditions to be obtained, whilst $\dot{m}_h$ follows from Equation 20. A simple energy balance within the condenser, assuming a condenser pinch point PP_c , provides the cooling mass flow rate and completes the analysis. Ultimately, the result of this model is that for specified T_6 , PR, ΔT_{sh} and PP_h values there is a range of $\dot{m}_h$ values that can be converted into power using this existing turbine.

Although cycle performance could be evaluated by the net power W_n or the cycle thermal efficiency η_o , these evaluations do not give a clear indication of whether implementing the existing turbine design is a feasible solution. Instead, W_n is compared to the maximum net power that could be produced using the same heat source but with a turbine operating at an optimal efficiency. For fixed values of T_6 , ΔT_{sh} , PP_h , T_{hi} and $\dot{m}_h$ there exists an optimal pressure ratio at which optimal power can be produced. This optimum exists because, whilst a higher pressure ratio increases the cycle efficiency, a higher pressure ratio also leads to a higher evaporation temperature, and a smaller heat source temperature drop and ORC mass flow rate. Since W_n is the product of the specific power and the mass flow rate, there is a trade-off between maximising the cycle efficiency, and maximising the amount of heat absorbed by the working fluid. This trade-off has been investigated in Figure 6 for a range of heat source conditions,

where the following assumptions have been made: $T_6 = 313$ K, $\Delta T_{sh} = 10$ K, $PP_h = 15$ K, $\eta_p = 70\%$ and $\eta_{tt} = 85\%$. The top graph considers a range of heat source temperatures, all with a fixed $\dot{m}_h$, and clearly at higher heat source temperatures, the optimal pressure ratio increases. The bottom graph shows that for a fixed T_{hi} , the optimal pressure ratio is independent of $\dot{m}_h$, and W_n increases linearly with increasing $\dot{m}_h$. Therefore, when supplied with T_{hi} and $\dot{m}_h$ Figure 6 can be used to obtain the maximum potential power that could be obtained for a turbine operating at $\eta_{tt} = 85\%$. Here 85% was considered to be an achievable target at the design point. If W_n is greater than the maximum potential power this is the result of the turbine operating at a higher efficiency than 85%.

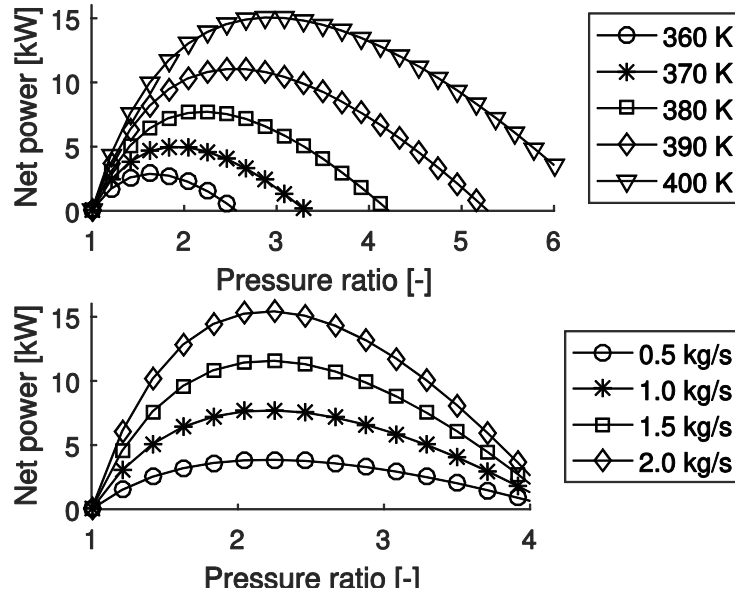


Figure 6. Variation in net power produced as a function of pressure ratio for different heat source conditions. Top: fixed heat source mass flow rate of 1.0kg/s; Bottom: fixed heat source temperature of 380K.

4 OTHER SYSTEM COMPONENTS

The motive behind the system model is to establish the range of heat source conditions that can be converted into power using the existing turbine. By simplifying the pump and heat exchanger analysis this stops the analysis being restricted by, for example, the pump performance. Therefore, it is assumed that whilst the same turbine could be used within a number of different systems, thus improving the economy-of-scale, alternative pumps and heat exchangers may be required. However, after completing the analysis, it is interesting to investigate the feasibility of also using the same pump and heat exchangers.

4.1 Pump modelling

The pump can also be modelled using similitude laws. This is expressed by Equation (21), where the pump head coefficient $\psi = gH/(r\omega)^2$, and pump efficiency η_p , are functions of the flow coefficient $\phi = Q/\omega r^3$; g is the acceleration due to gravity in m/s^2 , H is the pump head in m, r is the pump radius in mm, ω is the rotational speed in rpm, and Q is the volumetric flow rate in m^3/s .

$$\left[\frac{gH}{(r\omega)^2}, \eta_p \right] = f \left(\frac{Q}{\omega r^3} \right) \quad (21)$$

Following from [38], the relationships between ψ and ϕ , and η_p and ϕ , can be expressed using a simple quadratic expression of the form $y = ax^2 + bx + c$. Along with the

design point data (i.e. ϕ_d , ψ_d , $\eta_{p,d}$) the maximum head coefficient and maximum flow coefficient are needed to determine the quadratic coefficients for each expression. These are denoted as ψ_0 and ϕ_0 respectively, and correspond to pump operation when $Q = 0$ and $H = 0$ respectively. At these operating points $\eta_p = 0$.

Before modelling pump performance, a pump design is required. Conveniently ψ and ϕ can be combined to obtain pump specific speed ω_s (Equation (22)). Karassik [39] suggested that for a centrifugal pump ω_s can be as low as 0.2 and for this value, $\psi = 0.6$. For the ORC defined in Table 1, this corresponds to a design rotational speed of $\omega_d = 5,300$ rpm and a pump radius of $r = 37.5$ mm. The design point efficiency is assumed to be $\eta_{p,d} = 70\%$.

$$\omega_s = \frac{\phi^{\frac{1}{2}}}{\psi^{\frac{3}{4}}} = \frac{\omega_d Q^{\frac{1}{2}}}{(gH)^{\frac{3}{4}}} \quad (22)$$

To construct the pump performance map, values for ψ_0 and ϕ_0 are needed. A typical value for ψ_0 is 0.585 [39], whilst ϕ_0 is assumed to be $2\phi_d$. Whilst these are primitive assumptions, this facilitates the construction of the pump performance map (see Figure 11), which can be used during a preliminary assessment of pump performance following a change in working fluid. Future efforts should establish the performance map for a specific ORC pump.

4.2 Heat exchanger modelling

The required heat exchanger area A is given by Equation (23), where q is the heat transferred in W, $\Delta T_{\log}$ is the log mean temperature difference in K, and U is the overall heat transfer coefficient in W/(m² K). Whilst q and $\Delta T_{\log}$ follow from the cycle analysis completed in Section 3, U is dependent on the heat exchanger geometry. Since the heat exchanger design is not a focus of this study characteristic values for U have been estimated, as is typical during preliminary heat exchanger sizing. For this analysis $U = 50$ W/(m² K) is used during superheating and precooling, whilst $U = 1000$ W/(m² K) is used during preheating, evaporation and condensation. These values are set according to [40].

$$A = \frac{q}{\Delta T_{\log} U} \quad (23)$$

With fixed U values, it is easy to deduce from Equation (23) that it is unlikely that the same heat exchangers can be used within a range of different systems. Assuming that a similar temperature profile is maintained (i.e. $\Delta T_{\log}$), the required heat exchanger area should scale directly with the heat input.

5 RESULTS AND DISCUSSION

5.1 R245fa case study

An initial case study demonstrates the thermodynamic model developed in Section 3. A heat source of pressurised water ($T_{hi} = 380$ K, $P_h = 400$ kPa) has been defined and the ORC working fluid has been kept as R245fa. The ORC parameters were fixed according to Table 2. Both T_c and PP_c dictate the condenser area and the heat sink mass flow rate. The heat sink temperature is $T_c = 288$ K, whilst $T_g = 313$ K and $PP_c = 10$ K corresponds to an approximate 15 K temperature rise in the heat sink through the condenser. The value for PP_h has been estimated to be 15 K. Pinch points represent a trade-off between performance and cost and the values selected have been found to provide a reasonable balance. It has been widely shown that superheating is not necessary for organic fluids, but a small superheat of $\Delta T_{sh} = 2$ K ensures full vaporisation at the turbine inlet. Since the pump performance is not considered at this stage $\eta_p = 70\%$ is assumed.

Table 2. Fixed inputs for the R245fa case study.

Heat source fluid		water	
Heat source temperature	T_{hi}	380	K
Heat source pressure	P_h	400	kPa
Heat sink fluid		water	
Heat sink temperature	T_c	288	K
Heat sink pressure	P_c	101	kPa
Pump isentropic efficiency	η_p	70	%
ORC condensation pressure	T_6	313	K
Amount of superheat	ΔT_{sh}	2	K
Evaporator pinch point	PP_h	15	K
Condenser pinch point	PP_c	10	K

The ORC model was then run over a range of pressure ratios, and a range of $\dot{m}_o$ values were established at each pressure ratio. At each combination of $\dot{m}_o$ and PR, $\dot{m}_h$ was determined allowing the maximum potential power to be obtained. The result of this analysis is a performance map that shows the variation of W_n , as a percentage of the maximum potential power, with PR and $\dot{m}_o$ (Figure 7). The black lines, overlaid on the contour plot, indicate the resulting $\dot{m}_h$ values in kg/s.

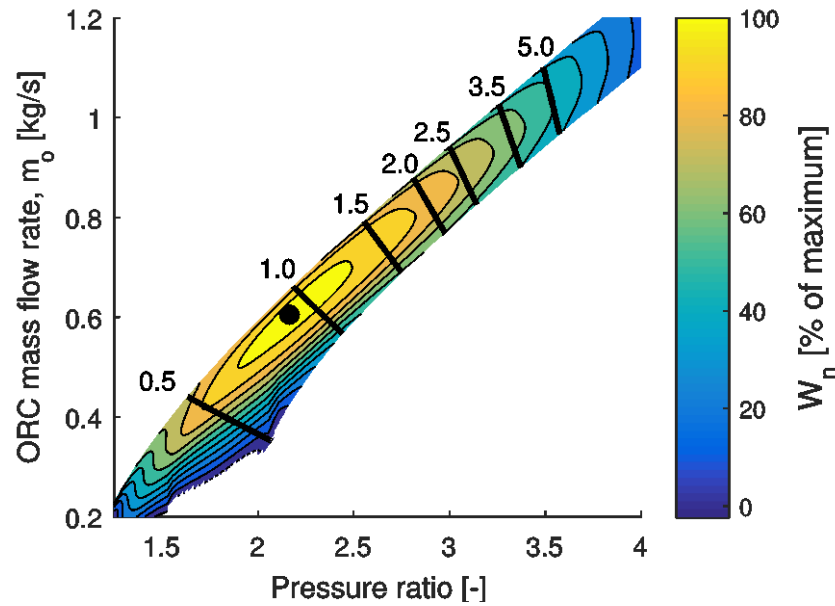


Figure 7. Contour of the net power produced by an ORC operating with the candidate turbine as a percentage of the maximum potential power. Heat source of water at 380K, and R245fa as working fluid. The black lines indicate the heat source mass flow rate in kg/s, whilst the black dot represents the point of optimal operation.

Figure 7 is useful since, for a specified heat source at $T_{hi} = 380$ K, it is easy to assess the feasibility of using this turbine. For example, for $\dot{m}_h = 1.0$ kg/s and pressure ratio of 2.2, the turbine efficiency is high and 100% of the maximum potential net power can be achieved. The optimal operating point corresponds to $PR = 2.17$, $\dot{m}_o = 0.60$ kg/s and $\dot{m}_h = 0.91$ kg/s. At this operating condition the turbine operates at 88.7% of the design reduced rotational speed (N/a^*), which is within feasible limits.

As $\dot{m}_h$ moves away from this optimal point, the ORC performance deteriorates leading to a lower percentage of the maximum power being produced. However, it is found that for this heat source at 380 K, this existing turbine, operating with R245fa, can effectively operate with pressure ratios between 1.75 and 2.75. This corresponds to heat source mass flow rates between

0.5 kg/s and 1.75 kg/s, whilst N/a^* remains between 80% and 110% of the design value. Within these limits W_n should remain above 90% of the maximum potential power. At alternative heat source conditions an alternative turbine design may offer improved performance, and further analysis would be required to establish whether the improved performance would outweigh the increased costs of developing an alternative design.

5.2 Alternative working fluids

The analysis discussed in Section 5.1 can now be repeated for different heat source temperatures and working fluids. Reiterating that working fluid selection criteria is not a focus of this paper, 15 typical ORC working fluids have been arbitrarily selected. The heat source temperatures were then selected as 360 K, 380 K and 400 K. It is expected that below 360 K the cycle thermal efficiency would reduce which would lead to uneconomical systems. On the other hand, higher temperature heat sources above 400 K could result in higher pressure ratios across the turbine, and likely lead to supersonic flow within the turbine. Under these conditions it is likely that an alternative turbine design with a supersonic stator would be required. Hence at this stage it can already be hypothesised that the advantage of running the same turbine with different working fluids will be that the same turbine can be used for different heat source mass flow rates, but at similar operating temperatures.

For these studies the heat sink conditions, T_6 , η_p , ΔT_{sh} , PP_h and PP_c were all fixed according to Table 2. For each combination of working fluid and heat source temperature the performance contour plot was obtained (i.e. Figure 7), allowing the optimal operating point to be obtained. Figure 8 displays the results in terms of the optimal $\dot{m}_h$ and W_n values for each working fluid. The top-right plot in Figure 8 shows a summary all of the results, with each marker representing the result obtained for a particular working fluid at the respective heat source temperature. The remaining plots expand on these results by showing which working fluid each marker represents.

It is clear that a large spread of heat sources can be effectively utilised by this turbine. For example, for $T_{hi} = 400$ K this turbine can convert heat sources between 0.5 kg/s and 1.65 kg/s, with W_n ranging between 7.9 kW and 30.2 kW, by simply changing the working fluid. Furthermore, across all of the operating points it was found that the optimal point is consistently close to 100% of the maximum potential power, thus corresponding to turbine isentropic efficiencies close to 85%. This confirms that at the corresponding heat source conditions, the ORC is operating at an optimal pressure ratio that corresponds to the optimal head coefficient. In other words, it would be unlikely that an alternative turbine would offer much improvement on the turbine, and cycle, performance.

The optimal operating point for each working fluid and heat source have been plotted onto the turbine performance maps in Figure 9. This is useful to see how close to the design point the turbine is operating for each combination of working fluid and heat source temperature. Ultimately it is observed that as the heat source increases and the pressure ratio, and therefore reduced head coefficient increases, the reduced rotational speed is increased to ensure that the turbine efficiency remains close to the maximum. This ensures the turbine operates close to its design point and therefore operates efficiently over the range of conditions considered. Furthermore, for the range of heat source temperatures considered, the reduced rotational speed remains between 82% and 116% of the original design, confirming feasible turbine operation. Figure 9 also validates the selection of $T_{hi} = 360$ K and $T_{hi} = 400$ K as the limits of operation for this turbine. For lower heat source temperatures optimal operating points would shift to the left leading to lower reduced rotational speeds, and low turbine efficiencies. A similar scenario can be seen for increasing head coefficients, which correspond to higher heat source temperatures. Hence this confirms that the same turbine cannot be used with significantly different heat source temperatures, but can be used across a wide range of heat source mass flow rates.

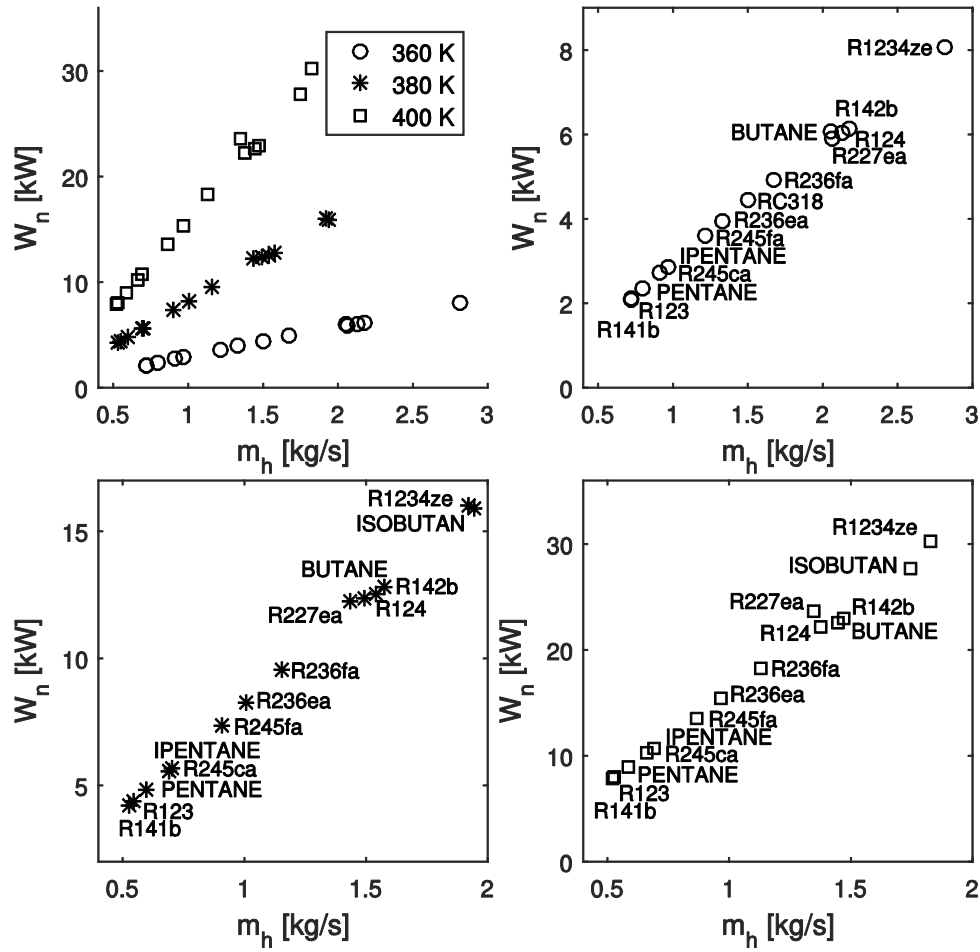


Figure 8. Cycle analysis results showing the heat source mass flow rates that can be accommodated by an ORC utilising the candidate turbine at each combination of heat source temperature and working fluid. Top left: summary of all results; top right: 360K; bottom left: 380K; bottom right: 400K.

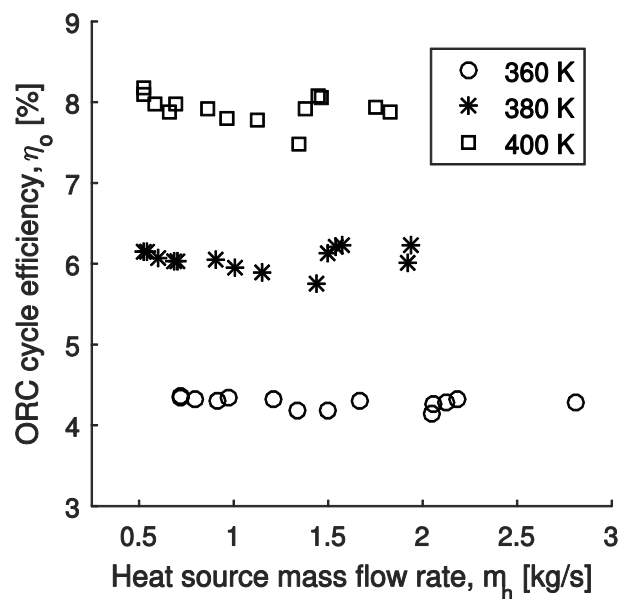


Figure 9. Results from each combination of heat source temperature and working fluid overlaid onto the turbine performance map.

The resulting cycle efficiencies η_o are shown in Figure 10. η_o increases with increasing heat source temperature, however there is only a small variation in η_o amongst the different working fluids. This is largely due to the optimal pressure ratio for a given heat source temperature being independent of the working fluid mass flow rate. It is arguable that at $T_{hi} = 360$ K, η_o is too low to develop an economically feasible system.

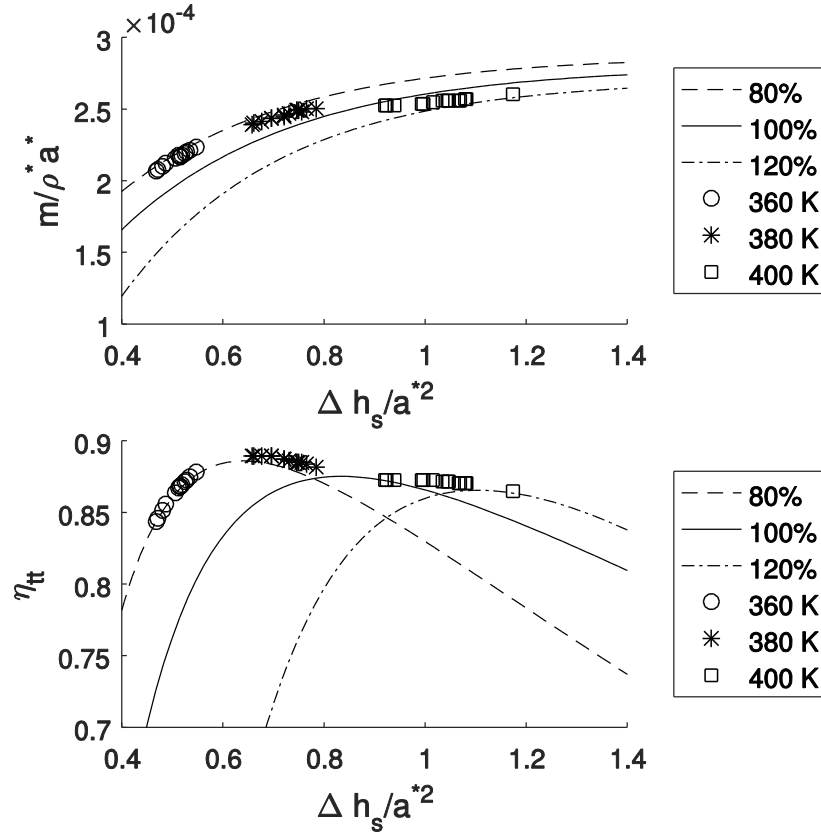


Figure 10. Cycle analysis results showing variation in cycle at the three different heat source temperatures.

Overall, Figure 8 suggests that the same turbine can be utilised within a number of different ORC applications with different heat source mass flow rates by selecting a suitable working fluid to match the available heat source. For example, for a heat source of 1.0 kg/s at 380 K, R245fa could be selected as the working fluid and power generated would be around 8 kW. However, for a heat source of around 1.75 kg/s at 400 K, R1234ze or isobutane could be selected and the power generated would increase to 30 kW. In Figure 11, the thermal input that each operating point corresponds to is also shown. This clearly shows that for a 360 K heat source that has between 50 and 200 kW_{th} of heat available, the same turbine can be used if the working fluid is matched to the heat available. Similarly, a heat source temperature of 380 K corresponds to heat inputs ranging between around 70 and 270 kW_{th}, whilst a heat source of 400 K corresponds to values between 100 and 380 kW_{th}. Hence, Figure 11 gives a clear indication of the range of potential applications that this turbine could be utilised within. Ultimately, this allows the same turbine to be manufactured in large volumes, thus facilitating an improvement in the economy-of-scale, and an improvement in the economic feasibility of implementing such a system.

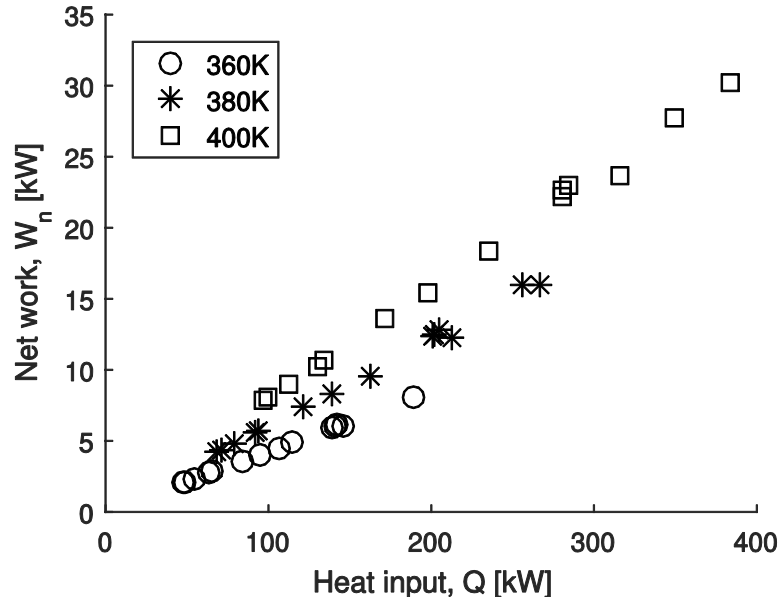


Figure 11. Net work plotted against the thermal heat input into the ORC system for each heat source temperature and working fluid. Each marker represents a particular working fluid.

Before progressing, it is important to discuss possible limitations to implementing the same turbine within a number of different systems. Firstly, the results in Figure 8 were obtained by varying only the pressure ratio. Therefore, the effects of T_6 , ΔT_{sh} , PP_h and PP_c were not considered. Therefore, it could be argued that the same turbine and working fluid could be used in different ORC systems by optimising these cycle parameters rather than changing the working fluid. However, whilst this might be true for fluids with similar performance, (i.e. they lie close to each other in Figure 8), it is unlikely that this would be possible when $\dot{m}_h$ changes significantly (i.e. from 0.5 kg/s to 1.5 kg/s). Secondly, additional factors, such as the bearing system and generator, are not taken into consideration during this study, and this may limit the feasibility of using the same turbine assembly across a wide range of power outputs. However, in these instances, even if modifications to the mechanical design are required, the costs associated with the aerodynamic design and manufacture of the stator and rotor assembly can still be avoided. Finally, within this study a wide range of working fluids were considered, which in reality may not be suitable due to availability, cost and legislative restrictions. Nonetheless, this work may be a novel contribution to the ORC community, demonstrating how non-dimensional turbine maps can be implemented within cycle analysis studies, and ultimately how the economy-of-scale of small-scale ORC systems could be improved.

5.3 Pump and heat exchanger performance

Having established the possibility of implementing the turbine within a number of different ORC configurations, the performance of the pump and heat exchanger performance can now be investigated. For each working fluid, at each heat source temperature, the optimal $\dot{m}_o$ and PR values are already known, which supplies the pump volumetric flow rate and the pump head. Using the pump performance map this provides the required rotational speed ω and pump efficiency η_p . Figure 12 displays the results of this analysis plotted onto the pump performance map for the pump design discussed in Section 4.1. Here ϕ and ψ have been normalised by the design values (i.e. ϕ_d , ψ_d). It is clear that for all the operating points considered ϕ remains between $0.6\phi_d$ and $1.5\phi_d$, which corresponds to values of $0.6\psi_d$ and $1.1\psi_d$ respectively. Under these conditions, the pump operates far enough away from the shut-off head, and run-out flow rate that η_p remains above 50%.

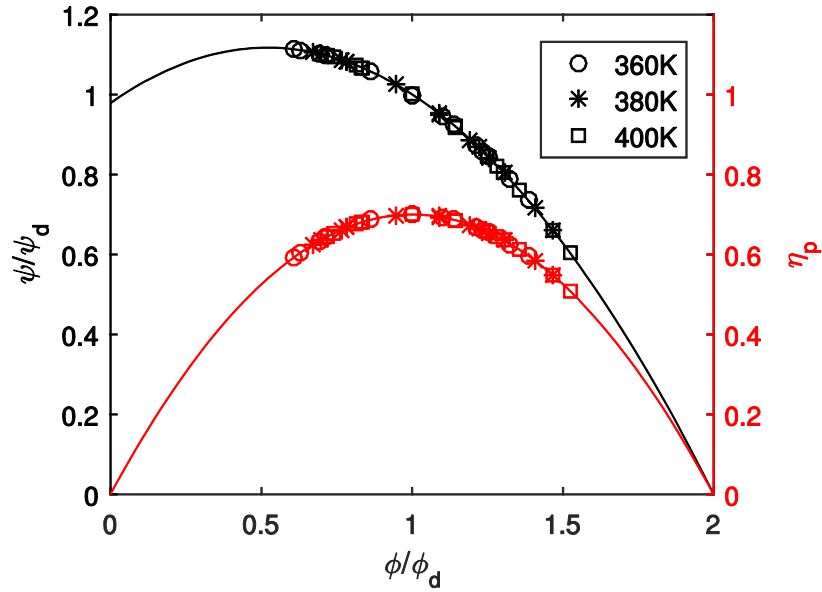


Figure 12. Non-dimensional pump performance map, overlaid with operating points for each heat source temperature.

Figure 13 displays the ω for each case and clearly, as T_{hi} and $\dot{m}_h$ increase, ω increases. The maximum rotational speed is around 14,000 rpm, which with $r_d = 37.5$ mm, corresponds to a maximum pump impeller tip speed of 55 m/s. The maximum allowable tip speed is governed by the mechanical design, and the prevention of cavitation within the pump. However, a typical maximum is around 50 m/s. Therefore, at this maximum rotational speed, the pump may be operating at the limit of feasible operation.

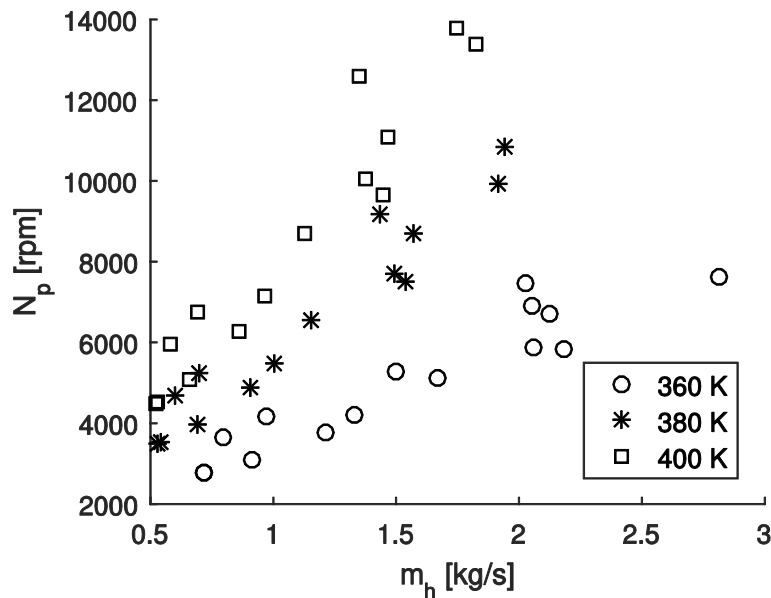


Figure 13. Pump rotational speed for each heat source temperature and mass flow rate. Each marker represents a particular working fluid.

Overall, this analysis suggests that it would be possible to use the same pump within the majority of operating points shown in Figure 8, and under these conditions η_p would remain between 50% and 70%. Further analysis is required to establish the impact of this reduction in η_p on the whole system. More detailed research is also required for the design and analysis of

ORC pumps to obtain more accurate performance maps, and to validate the use of similitude theory to ORC pumps. Nonetheless, the analysis presented here is believed to be an important first step.

The required head transfer areas for the evaporator and condenser for each working fluid and heat source combination have been calculated and are presented in Figures 14 and 15. Ultimately these results confirm that it is not feasible to use the same heat exchanger across a range of different operating conditions. As discussed previously, it was expected that the required heat transfer area would directly scale with increasing heat input q . Furthermore, since $q = W_n/\eta_o$, and Figure 10 has already shown that η_o is independent of T_{hi} , this means that the required evaporator heat transfer area directly scales with W_n , and therefore $\dot{m}_h$. This relationship is clearly observed in Figure 14.

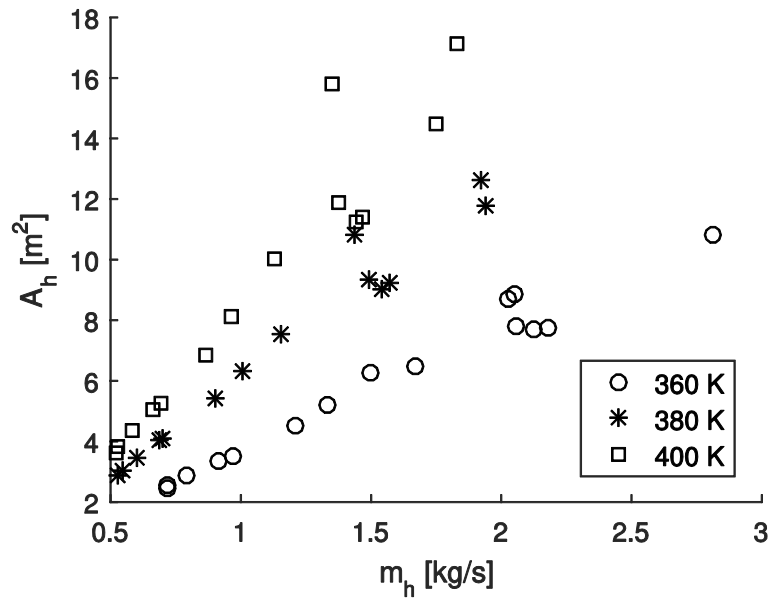


Figure 14. Required evaporator heat transfer area for each heat source temperature and mass flow rate. Each marker represents a particular working fluid.

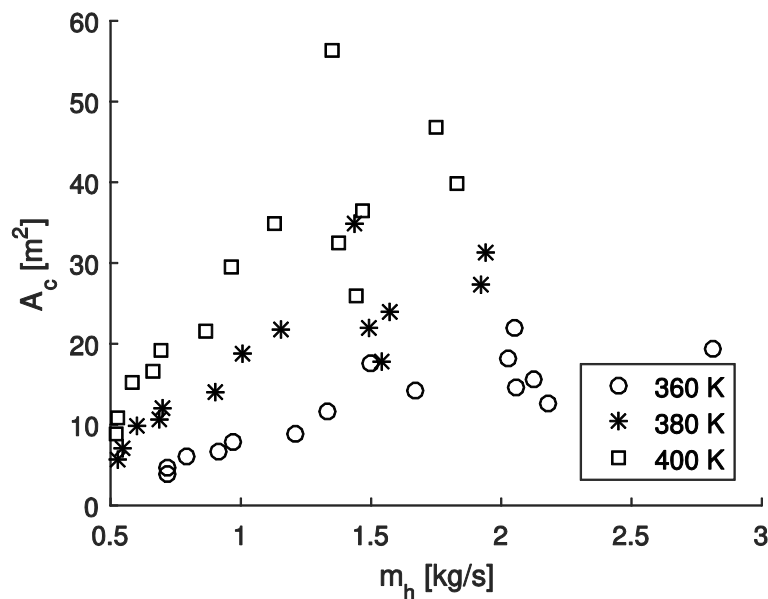


Figure 15. Required condenser heat transfer area for each heat source temperature and mass flow rate. Each marker represents a particular working fluid.

5.4 Current limitations to applying the similitude model to ORC systems

Within the previous sections the performance map for a particular turbine has been considered, and the heat source temperature was kept below 400 K to ensure subsonic turbine operation. More generally, the similitude model should be valid for any heat source temperature so long as the working fluid conditions within the turbine remain subsonic. However, for a single stage turbine, higher heat source temperatures facilitate higher pressure ratios and a larger enthalpy drop across the turbine. Therefore, at a certain heat source temperature, supersonic conditions will be encountered. To obtain this limiting heat source temperature consider the results generated in Figure 16, where the required rotor inlet Mach number is determined for three different working fluids across a range of heat source temperatures. For this analysis the same ORC parameters used in Section 5.1 are defined (hot water heat source at $P_h = 400$ kPa; $T_6 = 313$ K; $\Delta T_{sh} = 2$ K; $PP_h = 15$ K; $\eta_p = 0.7$ and $\eta_{tt} = 0.85$). For each heat source temperature, the optimal pressure ratio is determined using Figure 6, whilst a simple turbine design is obtained using the same design parameters described in [33]. From Figure 16 it is clear that whilst there is a slight variation between working fluids, supersonic conditions are typically expected for heat source temperatures greater than 400 K. Hence, the method described is at least suitable for low temperature heat sources below this limit. Above this limit a supersonic turbine is needed, which requires complex stator geometries that are dependent on the working fluid polytropic index [41]. The effects of working fluid replacement on these supersonic geometries is not well understood and is currently under investigation by the authors. Once these investigations are complete, an extension of the existing similitude model to supersonic turbines may be possible.

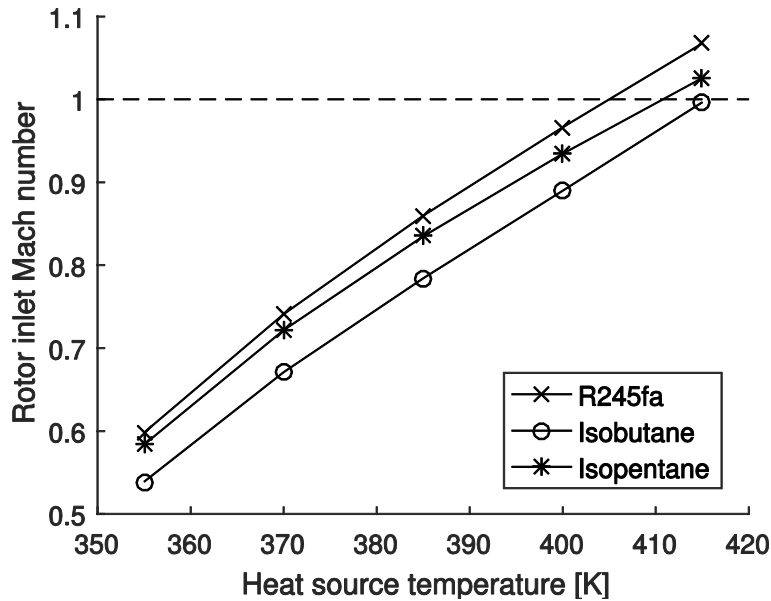


Figure 16. Required rotor inlet Mach number for varying heat source temperatures and different working fluids. Results obtained for; hot water heat source at $P_h = 400$ kPa; $T_6 = 313$ K; $\Delta T_{sh} = 2$ K; $PP_h = 15$ K; $\eta_p = 0.7$; $\eta_{tt} = 0.85$; pressure ratio optimised to maximise work output from the cycle (i.e. Figure 6).

6 CONCLUSIONS

To improve the economy-of-scale of small ORC systems, it may be necessary to implement the same system components into a range of different applications. This paper has investigated improvements in this area by combining component performance models with thermodynamic cycle analysis. First a turbine performance map, obtained using CFD, was adjusted to account for additional loss mechanisms, before being non-dimensionalised using a modified similitude theory. A novel thermodynamic model was then constructed, and a case study was considered.

This study showed that for a given heat source temperature and working fluid there exists an optimal heat source mass flow rate that can be efficiently converted into power using the existing turbine design. Repeating this analysis for different heat source temperatures and working fluids has demonstrated the possibility of utilising the same turbine for a range of different heat source flow rates. In particular, this study demonstrated that through selecting a suitable working fluid the existing turbine could convert heat sources ranging from 360 K and 400 K, with mass flow rates between 0.5 kg/s and 2.75 kg/s, into power outputs between 2 kW and 30 kW without compromising on turbine performance. Whilst the required heat exchanger areas were found to scale directly with increasing heat input, the possibility of also using the same pump within a number of different applications was also demonstrated. Therefore, this study has demonstrated the possibility of using the same pump and turbine within a number of different ORC systems. This is expected to improve the economy-of-scale of small ORC systems, allowing the same components to be manufactured in large volumes and then implemented within different applications, thus reducing costs and facilitating a move towards more economically viable ORC systems. Further efforts should investigate whether these findings are equally applicable to higher temperature ORCs, which are expected to introduce more uncertainties into the modelling process. Firstly, these systems will require alternative working fluids that are operated closer to their critical point and exhibit more extreme real gas behaviour. Furthermore, due to the low speed of sound supersonic turbines may be required, which will also require the modified similitude model to be investigated for supersonic flows. Finally, more effort is needed to validate both numerically and experimentally the use of similitude theory, and give due consideration to its validity to other types of turbines and ORC pumps.

7 ACKNOWLEDGMENTS

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