

DETERMINATION OF EXHAUST SYSTEM EXTERNAL LOSSES FOR
A FRONT FAN JET ENGINE NACELLE HAVING TWO SEPARATE
CO-AXIAL EXHAUST STREAMS

- by -

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ABSTRACT

The pursuit of better economics for subsonic transport aircraft has led to the development of high by-pass ratio turbofan aero engines. The comparison between predicted and measured performance of these engine installations is difficult because the flow patterns are complex. The loss can only be predicted by summing the separately estimated component drags whereas the measured performance can only be determined in terms of the total loss. The high values of the ratio of gross to net thrust inherent in the high by-pass ratio cycle render losses in the exhaust system of special significance.

A feature of these installations is the primary afterbody which is immersed in the efflux from the fan nozzle. The direct measurement of the drag of this component is impossible in flight. The accurate prediction of afterbody drag is difficult because of the complex nature of the flow field generated by the under-expanded fan nozzle flow.

The research reported in this Thesis is in three parts. First, the drag components of the nacelle have been defined in terms of measurable quantities. This section represents an extension of work by the A.R.C. The second section of the research was the construction of a mathematical model of the flow field downstream of the fan nozzle which has enabled some afterbody predictions to be made. A novel feature of this work is the combination of boundary layer prediction with prediction of the potential flow field using the method of characteristics. The third part of the work is the presentation and discussion of some experimental measurements. Use is made of two experimental programmes. The first was carried out in support of a current engine project while the second was directed towards amplifying and confirming the results of the theoretical model.

PREFACE

The author declares that the contents of this Thesis are his own original work, except where stated to the contrary in the text. This Thesis has not previously been submitted to any other University.

The research was carried out during the period October 1968 to August 1973 at the National Gas Turbine Establishment and at Imperial College, London under the scheme for Registration as Internal Students of persons working for the M.Phil. or Ph.D. at research institutions.

Extracts from Chapter 2 of this Thesis have been published as A.R.C. Current Paper No. CP 1311 and part of Chapter 3 has been published as "Calculation of the flow field downstream of the fan nozzle of a turbofan aero engine" in the Aeronautical Journal, December 1970.

A handwritten signature in cursive script, reading 'P. G. Street', followed by a horizontal flourish line.

P. G. Street

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CHAPTER 1

An introduction

The research reported in this Thesis was concerned with the problem of predicting installation losses for the new generation of subsonic transport aero engines. These engines differ from the earlier types by employing considerably higher values for the by-pass ratio; typically, values range from 5 to 8 compared with a figure close to 1 for the previous generation. This large increase in by-pass ratio has led to new problems in predicting the installed performance of the powerplant. A novel feature of the new designs is the use of separate co-axial exhaust nozzles for the by-pass and gas generator streams. Thus, optimisation of the nacelle design has become more complicated. Douglass^{(1)*} has estimated that at by-pass ratios less than about 4 a long fan cowl has a superior performance but that at higher by-pass ratios the fan cowl should be shortened. Other features also have to be considered such as the need for adequate area inside the fan duct for noise attenuation purposes and the need to provide adequate space for thrust reversers⁽²⁾.

Complex interactions between the powerplant and the aircraft structure make any attempt to optimise the complete installation virtually impossible. Therefore progress can best be made by first optimising the powerplant nacelle in isolation and then finding the optimum location relative to the aircraft. The work described here contributes to the better understanding of the performance of the nacelle in isolation.

High by-pass ratio turbofan aero engines offer a significant improvement in the specific fuel consumption of the bare engine when compared with conventional turbojets. The improved efficiency of the high by-pass ratio engine results partly from an improvement in thermodynamic efficiency arising from the use of higher compressor pressure ratios and partly from an improvement in the propulsive, or Froude efficiency brought about by the use of an increased air mass flow rate,

*Figures in brackets refer to References, listed following Chapter 5.

with a smaller jet velocity, for a given thrust level. The specific net thrust (net thrust per unit airflow) for a high by-pass ratio engine is thus appreciably lower than for a conventional turbojet so that the ratio of engine gross thrust to installed net thrust is higher. Typically, for an engine of unit by-pass ratio, the ratio of gross to net thrust is about 1.6 under high subsonic speed cruise conditions, while for an engine of by-pass ratio 5 it is between 2.6 and 3.

One of the consequences of this lower specific thrust is that the installed performance becomes more sensitive to the pod drag level⁽³⁾. The magnitude of this effect has been shown by Neale⁽⁴⁾ who derived the curve shown in Figure 1. In view of the commercial pressures described by Swann⁽⁵⁾, whereby "a difference of 5 per cent in airplane drag can mean the loss of many millions of dollars in aircraft re-orders", there is obviously considerable incentive not only to reduce pod drag but to be able to predict, and measure the installation losses accurately.

One of the difficulties common to many examples of drag analysis is that while the installation loss can only be predicted by summing the separately estimated component drags, the measured performance can only be obtained in terms of the total loss. This difficulty is highlighted by Holdhusen⁽⁶⁾ who lists 6 aerodynamic factors which contribute to the installation loss. These are:-

1. Inlet internal recovery;
2. Inlet internal distortion;
3. Exhaust system thrust coefficient;
4. Inlet external drag;
5. Aft surface external drag;
6. Mutual interference between the engine and aircraft flow fields.

In Reference 6 these 6 components are separately estimated with the aid of both theoretical predictions and model experimental results so that the total installation loss can be estimated.

A particular feature of the short cowl installation which initially gave rise to some concern is the drag of the gas generator cowl which is immersed in the efflux from the fan nozzle - loss 5 in Holdhusen's list. The drag per unit wetted area of the afterbody is

higher than normal because of the increased velocity of the fan exhaust. The phrase "Afterbody scrubbing drag" is sometimes used to describe this added drag. Green⁽⁷⁾ notes that the term - scrubbing drag - has slightly ominous overtones, suggesting a problem area where there are large and perhaps unnecessary losses, but he goes on to point out that this is an essential feature of the minimum drag configuration. By virtue of common usage, scrubbing drag has come to mean the skin friction component of the afterbody drag though it has sometimes been used either for the extra drag arising from the velocity increment of the fan jet over the free stream velocity - sometimes called the blowing drag - or for the total afterbody drag.

A more complete analysis of the installation losses for an isolated nacelle is given by Frazier⁽⁸⁾ who divides the losses into 10 components as shown in Figure 2. He uses experimental data reported by Viall⁽⁹⁾ to estimate the inlet and fan cowl losses, data reported by Lawrence⁽¹⁰⁾ to estimate the scrubbing drag and data reported by Salemann⁽¹¹⁾ to estimate the boat-tail drag.

Lawrence measured the force on a parallel afterbody immersed in the efflux from an annular nozzle. The majority of his test points lie in a region where the flow is similar to a fully developed wall jet. That is to say, the afterbody was long compared with the height of the fan nozzle. Typically, most of Lawrence's results are for geometries where the ratio of the afterbody length to the fan nozzle height is 20 or greater. Current high by-pass ratio installations feature values of this ratio between 4 and 10. Green⁽¹²⁾ has made theoretical predictions of the friction drag and wall jet effects on cylindrical afterbodies. The trends of drag with jet Mach number exhibited by Lawrence's data differ significantly from Green's predictions. Green concludes that of a number of possible explanations for this discrepancy, a likely cause may have been in the experimental force measurements which varied with jet pressure level. Thus, use of Lawrence's data in performance calculations must be viewed with some reservation.

Frazier derived boat-tail drag from the results of a series of drag balance experiments, reported by Salemann⁽¹¹⁾, which investigated a series of circular arc afterbodies downstream from a range of annular

nozzles of various heights. Unfortunately, Salemann's paper is not generally available, but the results of one series of tests are reproduced by Swann, (Figure 7 of Reference 5). However, it is possible to deduce from the notes by Frazier and Swann that the boat-tail drag was probably computed by correcting the drag balance results using Lawrence's data. There is no mention of potential flow buoyancy - or interference effects, which will be discussed in Chapter 2, so once more, the data must be viewed with some reservation.

Thus, the Frazier analysis of afterbody drag is based on the summation of a skin friction drag measured on a parallel afterbody, the value of which is in some doubt, and a boat-tail drag term based on a drag balance result with a doubtful correction. It might be argued that some of the errors in this technique may be self cancelling but there must be some concern about the adequacy of the method. A further factor reducing the usefulness of Frazier's paper is the omission of vertical scales on the graphs, presumably for reasons of commercial security.

Poland and Schwanebeck⁽¹³⁾ have described the technique used to determine the thrust for the TF39 in the C-5A. This technique was based on a calculation of the scrubbing drag using fully developed pipe flow data with the fully expanded jet Mach number to predict the skin friction, plus an afterbody pressure drag term defined as - "the difference due to external flow in the pressure/area force term acting on the aft facing surfaces". The latter definition would seem to be incomplete since it ignores the form and wave drags experienced by the afterbody under quiescent conditions, (i.e., still air surrounding the nacelle).

Yarker and Stanhope⁽¹⁴⁾ refer briefly to the change of afterbody drag due to the effect of external flow. Otherwise, afterbody drag is hidden in an overall thrust coefficient for the complete exhaust system.

Thus the approach to afterbody drag recorded in the literature is somewhat varied. Methods which attempt to predict the overall installation loss use experimental data to estimate boat-tail drag and compute skin friction from the fully expanded jet conditions on a flat plate basis. Alternatively, the afterbody drag is treated as an internal loss

comprising a part of the thrust deficit of the fan stream as measured in model tests. None of the methods either measure or predict the true afterbody drag so that little or no data is available to confirm the accuracy of more comprehensive predictions.

A considerable quantity of experimental data has been produced which is relevant to the design of conventional single stream nacelles. Since there can be no difference in principle between the design of the inlet and fan cowl of a high by-pass ratio installation and that of a conventional nacelle, the author considers that adequate techniques already exist, or are under development for this part of the installation. Therefore the research has been concentrated on the problems associated with the gas generator afterbody and its surrounding flow field. The work falls into three parts.

In 1953, the Performance Sub-Committee of the Aeronautical Research Council set up a temporary Panel to consider the problem of defining powerplant thrust and drag and also the related problems of the definitions to be used in the description and analysis of the drag of aircraft. Two reports were published dealing with the problems of defining jet engine thrust⁽¹⁵⁾ and drag⁽¹⁶⁾. Chapter 2 of this Thesis describes an extension to the work of the Panel which considers the special problems of the two-stream nacelle. This section of the work forms the basis of analysis for the subsequent chapters.

Chapter 3 describes the computer implementation of a mathematical model of the flow field downstream from the fan nozzle. At subsonic cruising speeds current high by-pass ratio engines typically feature fan nozzle pressure ratios of between 2 and 3:1. Therefore, even with a convergent nozzle, the flow in the jet is supersonic, allowing prediction by the method of characteristics. The numerical application of a characteristics solution to the calculation of plane and axisymmetric supersonic flows was proposed by Guderly⁽¹⁷⁾ as a development of an approximate graphical method derived by Prandtl and Busemann⁽¹⁸⁾. The method used in the present program has been developed by Kantrowitz⁽¹⁹⁾ from Guderly's method. The Kantrowitz method has been applied in previous computer programs at N.G.T.E. to flow in axisymmetric ducts⁽²⁰⁾ and to supersonic plug nozzles⁽²¹⁾. A novel feature of this new program is the simultaneous calculation of the boundary layer.

The problem has here been simplified by restricting the solution to quiescent conditions so that the pressure at the free jet boundary is everywhere equal to ambient static pressure and the mixing or entrainment at the outer boundary of the jet has been ignored. Following the publication of this part of the work in Reference 22, Young⁽²³⁾ used a source/sink distribution method⁽³¹⁾ to predict the effects of a surrounding subsonic flow field. Young excluded the calculation of the boundary layer, arguing that its inclusion could not be justified unless the mixing region at the jet boundary could also be included. His results show lateral movements of the flow patterns, as the external flow Mach number is changed, which agree reasonably well with experimental observations made on the rig described in Chapter 4 of this Thesis.

The test rig described in the first part of Chapter 4 was commissioned during 1969. The design of this thrust rig was based on an earlier rig which has been described in Reference 24. A second thrust measuring rig was required so that programmes of work on supersonic propelling nozzles and high by-pass ratio nacelles could proceed in parallel. The opportunity was taken to modify the basic design in order to suit the rig better to the subsonic application; this is described in Section 4.2.

A two-stream research model was designed to enable the performance of a number of alternative gas generator afterbodies to be obtained quickly in support of a current engine programme. The ability to substitute a plug nozzle for the simple convergent primary nozzle was included, but no plug nozzle work is submitted here.

As the ideas outlined in Chapter 2 began to develop, it soon became clear that it would be desirable to be able to measure the force on the afterbody. This desire led to the design of a second model in which the afterbody was supported by linear bearings and restrained axially by a force transducer so that the net aerodynamic force on the outer surface could be deduced. Unfortunately, the scale of the model precluded inclusion of the primary jet pipe as well as the bearing and force measuring system, so the primary jet was represented in some tests by plume simulators⁽²⁵⁾.

In view of the extreme sensitivity of the powerplant to losses in the exhaust system, a high order of accuracy was required of the

experiments. Section 4.4 discusses the rig and model calibration techniques and is followed by a description of a typical test run. The chapter concludes with a brief review of the test results.

Chapter 5 presents a comparison between the test results and the theoretical methods described in Chapters 2 and 3. Areas in which the theoretical techniques could be improved are discussed. At the end of the chapter, the reader will find a discussion of the overall programme together with some concluding remarks.

It should be noted here that, due to circumstances which were not foreseen at the start of the research programme, the author was moved to another department within N.G.T.E. on the 1st of July, 1970. This change meant that the author was no longer directly responsible for the work described here. However, at this time, the test rig had been completed and commissioned as had the two models. The test programme was in the initial stages and had been planned in detail, the thrust/drag analysis described in Chapter 2 had been completed and the mathematical model described in Chapter 3 was working. The experimental work was taken over by Mr. A. R. Seed and Mr. J. T. Proffitt who kindly agreed to complete the experimental work according to the author's instructions. The author maintained close contact with Mr. Seed and Mr. Proffitt during the ensuing period and effectively decided with them the course to be followed as the experimental programme proceeded.

During the time of this research, the thrust measuring rig was used for other projects associated with the British Aero' Industry. The more academic nature of the work reported here meant that time on the rig was limited by commercial pressures with the result that the experiments were not completed until the Autumn of 1973. The authors engagement since that date on other work at N.G.T.E. has meant that the analysis and reporting of the experimental data have continued at a slower pace than had originally been intended.

CHAPTER 2

Thrust/drag analysis

2.1 Introduction

With the advent of the high by-pass ratio front fan engine as a commercial proposition, the problem of defining thrust and drag for a nacelle with two separate co-axial exhaust streams has become pressing. The problem is complicated by the fact that flow from the outer, or fan exhaust passes over the surface of the downstream part of the nacelle, the afterbody. Thus the methods adopted for single stream nacelles, outlined in References 15 and 16, need modifying. Furthermore, the ideas need extending so that values of component thrusts and drags can be determined from experimental measurements.

As the specific thrust decreases with rising by-pass ratio the ratio of gross to net thrust increases. One feature of powerplants with a high ratio of gross to net thrust is a marked sensitivity to exhaust system performance. Losses in the exhaust system effectively act on the gross thrust level. For an engine of by-pass ratio 5, the ratio of gross to net thrust at cruise at a high subsonic Mach number is between $2\frac{1}{2}$ and 3. Thus a 1 per cent decrease in exhaust system performance would cost between $2\frac{1}{2}$ and 3 per cent on net installed performance.

This Chapter will show that the drag of the afterbody, where it is scrubbed by the fan jet, is properly treated as a loss of gross thrust, so that the loss is effectively magnified by the gross to net thrust ratio. The drag of the part of the nacelle upstream of the fan nozzle, the intake and fan cowl, can be treated by established methods so attention will be focussed on afterbody drag in the latter part of the Chapter.

2.2 Drag in potential flow

The drag of a closed non-lifting body in isolation in infinite potential flow is zero; d'Alembert's paradox. Prandtl⁽²⁶⁾ extended the paradox to show that, in potential flow, bodies of semi-infinite or infinite extent in the streamwise direction also have zero drag. The only forces acting in potential flow would be normal pressure forces - there would be no skin friction. Thus the walls of an infinitely long stream tube would have no drag. Outside, the tube could represent an

infinite body, and since pressures on either side of a streamline are equal, there would be no drag on the inside. If one considers a part of a closed or an infinite body, then the normal pressure force will, in general, not be zero. Thus one can see that it is possible for a part of a body to experience a force which is not drag; the force would be balanced by equal and opposite components elsewhere on the body.

2.3 Drag in real flow

All bodies in real flow exhibit drag. If the flow is subsonic, this drag comprises two components; skin friction, (which is the integrated shear stress at the wall), and the pressure or form drag. Form drag arises from the modification of the pressure distribution due to boundary layer growth and sometimes breakaway of the flow from the surface.

Wave drag further modifies the pressure distribution when the flow is supersonic. Skin friction, form, profile and wave drags are discussed in detail in Appendix A.

2.3.1 Drag of parts of bodies

As the drag in potential flow is zero, the pressure drag in real flow can be expressed as the difference between the integrated pressure force in real flow and the integrated pressure force in potential flow, considering the same body shape in both cases. This concept is useful when considering parts of bodies because the integrated pressure force on a part of a body in potential flow would not be zero. Thus the drag of a part of a body is not the sum of the skin friction and the integrated pressure force. The pressure drag will be the difference between the non-zero integrated pressure force in potential flow and the integrated pressure force in real flow. The integrated pressure force in potential flow will be termed 'potential flow buoyancy' in this Thesis. Its magnitude will vary according to what portion of the whole body is being considered, tending to zero as the portion is extended to include the whole body.

2.4 Thrust and drag for a single stream nacelle

The A.R.C. Panel, set up to consider 'thrust and drag definitions for ducted bodies and jet engines'^(15,16), defined nacelle drag as the summation of forces on the outside of the stream tube bounding the flow which passes through the duct, and the thrust, or internal drag, as the summation of forces on the inside of the stream tube. These

definitions are consistent with Prandtl's approach because the stream tube is considered to extend from a station an infinite distance upstream from the duct inlet to a station an infinite distance downstream from the duct outlet. Thus the stream tube/body combination represents an infinite body which would have no drag in potential flow. The net force on the outer surfaces of the duct would not equal the drag because of buoyancy effects from the pre-entry and post-exit stream tubes. However, if all the forces are summed, then the buoyancy components cancel leaving the drag as the net result. A similar argument can be applied to the derivation of thrust.

Figure 3a shows a single stream nacelle in isolation. F_i and F_c^* represent the total forces, skin friction plus normal pressure forces, on the external part of the intake and boat-tail respectively, the directions being taken as shown in the figure. T_I is the intrinsic net thrust⁽¹⁵⁾ comprising the summation of friction and pressure forces on the engine and internal parts of the duct. As all the forces have been included, the net installed thrust-minus-drag must equal $T_I + F_i - F_c$. Note, however, that F_i and F_c are forces, not drags.

The pre-entry and post-exit stream tubes have been included in Figure 3b. Station 'o' is sufficiently far upstream for the static pressure to be undisturbed, i.e. upstream infinity, station 'i' is the plane of the intake, station 'e' is the nozzle exit plane and station 'j' is at downstream infinity where the static pressure once again reaches the undisturbed value.

The external drag, as defined in Reference 16, is the sum of the forces $F_{pre} + F_c - F_i - F_{post}$. Provided that the body is long compared with the diameter so that the forebody and afterbody do not interact, this can be broken down into two components. These would be the intake drag, $F_{pre} - F_i$, which is equal to the sum of the datum intake drag and the basic spill drag as defined in Reference 16, and the boat-tail drag, $F_c - F_{post}$, which is equal to the sum of the datum afterbody drag and the jet interference drag.

*All forces are taken to act in an axial direction. Lift is excluded. Symbols are defined after Chapter 5.

The internal thrust is the sum of the internal forces, $T_{pre} + T_I - T_{post}$. The net installed thrust ' X_n ' is the thrust minus drag so that:-

$$X_n = T_{pre} + T_I - T_{post} - F_{pre} - F_c + F_i + F_{post} \quad \dots(II,1)$$

However, forces on either side of a streamline must be equal so that:-

$$F_{pre} = T_{pre} \quad \dots(II,2)$$

and
$$F_{post} = T_{post} \quad \dots(II,3)$$

Combining Equations (II,2) and (II,3) with Equation (II,1),

$$X_n = T_I - F_c + F_i \quad \dots(II,4)$$

which is the same result as was obtained by considering the forces on the hardware. But note that expressions for drag components are now available.

2.5 Thrust and drag for a two-stream nacelle

Figure 4 shows a two-stream nacelle with co-axial exhausts. The case treated here is the common one of a single intake, the flows being split within the engine. However, it is convenient for the analysis which follows to consider the intake flow as two separate flows, i.e. the internal flows maintain their separate identities throughout the flow field. Stations 'o', 'i' and 'j' are taken as before but two exhaust planes are now required, 'ef' and 'ep', representing the nozzle outlet planes for the by-pass, or fan flow and the gas generator, or primary flow respectively. Two intrinsic net thrusts are specified, T_f and T_p , each representing a summation of forces within the duct as before. T_f represents the summation of forces between stations 'i' and 'ef' on the annular stream tube which passes through the by-pass duct and T_p represents the summation of forces between stations 'i' and 'ep' on the inner stream tube bounding the gas generator flow.

Maintaining the definitions of thrust and drag given in References 15 and 16, external drag, D_{ext} , is defined by:-

$$D_{ext} = F_{pre} - F_i + F_c - F_{post} \quad \dots(II,5)$$

The internal thrust of the fan stream, T_{int} , is given by:-

$$T_{int} = T_{pre} - f_{pre} + T_f - F_a - T_{post} + f_{post} \quad \dots(II,6)$$

The internal thrust of the gas generator is given by:-

$$t_{int} = t_{pre} + T_p - t_{post} \quad \dots(II,7)$$

The net installed thrust minus drag, X_n , is given by:-

$$X_n = T_{int} + t_{int} - D_{ext} \quad \dots(II,8)$$

Substituting Equations (II,5), (II,6) and (II,7) into (II,8), and noting that forces on either side of streamlines are equal, as before, one can show that:-

$$X_n = T_f + T_p + F_i - F_c - F_a \quad \dots(II,9)$$

The net installed thrust-minus-drag is equal to the algebraic sum of the internal and external forces on the fan cowl and afterbody. Again following the method for the single stream nacelle, intake drag is equal to $F_{pre} - F_i$, cowl drag is equal to $F_c - F_{post}$ and afterbody drag could be taken as equal to $F_a - f_{post}$. A point to note is that F_a and f_{post} are internal forces. They modify the thrust of the fan and primary streams.

While this is a viable definition of afterbody drag in that summation of thrusts and drags will give the correct net installed thrust-minus-drag, it omits to take account of the fact that the afterbody cannot reasonably be considered in isolation. Experimental results on certain geometries show the existence of potential flow buoyancy effects between the fan cowl and the gas generator afterbody. This takes the form of an apparent increase in cowl drag which is offset by an apparent reduction in afterbody drag. The mechanism of this buoyancy might be explained as follows. The external flow is directed towards the centreline as it leaves the cowl trailing edge and requires

a pressure force to turn it back to an axial direction. This pressure force is a part of F_{post} . It is transmitted through the fan efflux thus raising the pressure level on the afterbody. The pressure levels on the cowl are lower than they would be if the afterbody were omitted. This may be further illuminated by the following example. Consider the case where the afterbody diameter is increased so that it effectively becomes a continuation of the cowl. Then the pressure distribution over the cowl would be like the early part of the pressure distribution on a normal afterbody, that is a low pressure region. The pressure force on the latter part, the afterbody, would be positive, inducing a thrust nearly equal to the rearward force on the cowl.

Estimation of the potential flow buoyancy between the cowl and afterbody by calculation of the entire potential flow field would be extremely difficult. Assumptions of the relevant conditions at the fan nozzle throat would be open to question and, since the flow over the afterbody is supersonic, wave drag on the afterbody could not be excluded.

However, it should be noted that drag always appears as a deficit in momentum flux in the wake. Thus, if the drag of either the cowl or the afterbody could be estimated from conditions in the wake, then the difference between the drag and the net force on the body could be calculated. The drag of the afterbody is made complicated by virtue of the 3 components - skin friction drag, form drag (or pressure drag due to boundary layer growth) and wave drag. Calculation of the sum of the first 2 components can be attempted using boundary layer prediction methods but the extension of this calculation to downstream infinity, i.e. allowing for the continued growth of the form drag in the wake, is not at all well founded for supersonic flows and, since the difference in pressure levels between the afterbody trailing edge and free stream can be large, the correction term could also be large. No method is known for calculating the third component, wave drag, in this case.

However, if we turn our attention to the cowl, the situation is less difficult. The flow over the cowl afterbody is essentially subsonic so that the only contributions to drag are those due to boundary layer effects. Furthermore, the pressure level at the trailing edge of the cowl is not far different from free stream static pressure.

Another feature is that the pressure distribution is smooth in form; the pressure distribution on the afterbody is much more complicated due to the alternate expansion and compression regions. Thus the calculation of the momentum flux deficit in the cowl wake would be more reliable.

The thoughts expressed in the preceding paragraphs lead to the following technique for the derivation of afterbody drag from experimental measurements on a model representing the installation aft of the fan cowl maximum diameter.

The force on the primary post-exit streamline, f_{post} , can be found by considering the change in momentum flux in the primary stream between stations 'ep' and 'j'. Thus:-

$$\frac{Q_p}{g} (V_{jp} - V_{ep}) = A_{ep} (P_{ep} - P_{\infty}) - A_{jp} (P_{jp} - P_{\infty}) - f_{\text{post}} \quad \dots(\text{II},10)$$

where Q_p is the primary mass flow, V_{jp} is the primary stream velocity* at downstream infinity, V_{ep} is the velocity at outlet from the primary nozzle, A_{ep} is the primary nozzle outlet area, P_{ep} is the local static pressure at the primary nozzle outlet, P_{∞} is the free stream static pressure, A_{jp} is the area occupied by the expanded primary jet and P_{jp} is the pressure to which the primary jet expands. However, since station 'j' is at downstream infinity,

$$P_{jp} = P_{\infty} \quad \dots(\text{II},11)$$

and Equation (II,10) can be rewritten:-

$$f_{\text{post}} = \frac{Q_p}{g} (V_{ep} - V_{jp}) + A_{ep} (P_{ep} - P_{\infty}) \quad \dots(\text{II}.12)$$

Then noting that the drag of the afterbody^{*}, D_a , is equal to the momentum deficit in the wake at downstream infinity, one can write the

*The derivation of V_j is discussed in Section 2.5.3.

[†]It is shown in Section 2.5.3 that D_a may include a thrust recovery term equal to a proportion of the potential divergent thrust at supercritical pressure ratios.

momentum equation between stations 'ef' and 'j' in the fan stream
thus:-

$$\frac{Q_f}{g} (V_{jf} - V_{ef}) - D_a = A_{ef} (P_{ef} - P_{\infty}) - F_{post} + f_{post} - F_a$$

...(II.13)

which can be re-arranged to give

$$\begin{aligned} F_{post} &= \frac{Q_f}{g} (V_{ef} - V_{jf}) + A_{ef} (P_{ef} - P_{\infty}) \\ &\quad + \frac{Q_p}{g} (V_{ep} - V_{jp}) + A_{ep} (P_{ep} - P_{\infty}) - F_a + D_a \end{aligned}$$

...(II,14)

However, the cowl boat-tail drag, D_c , is given by

$$D_c = F_c - F_{post} \quad \text{...(II,15)}$$

Combining Equations (II,14) and (II,15)

$$\begin{aligned} F_c - D_c &= \frac{Q_f}{g} (V_{ef} - V_{jf}) + A_{ef} (P_{ef} - P_{\infty}) \\ &\quad + \frac{Q_p}{g} (V_{ep} - V_{jp}) + A_{ep} (P_{ep} - P_{\infty}) - F_a + D_a \end{aligned}$$

...(II,16)

so that

$$\begin{aligned} D_a &= \frac{Q_f}{g} (V_{jf} - V_{ef}) - A_{ef} (P_{ef} - P_{\infty}) \\ &\quad + \frac{Q_p}{g} (V_{jp} - V_{ep}) - A_{ep} (P_{ep} - P_{\infty}) \\ &\quad + F_a + F_c - D_c \end{aligned}$$

...(II,17)

This result can be demonstrated by an alternative method as follows. The gross thrust-minus-drag of the complete exhaust system can be derived in two ways, either by considering nozzle exit momenta and body forces or by considering momentum flux at downstream infinity.

$$\begin{aligned} \text{Thus} \quad T - D &= \frac{Q_f}{g} V_{ef} + A_{ef} (P_{ef} - P_{\infty}) \\ &\quad + \frac{Q_p}{g} V_{ep} + A_{ep} (P_{ep} - P_{\infty}) - F_c - F_a \quad \dots(\text{II},18) \end{aligned}$$

$$\text{or} \quad T - D = \frac{Q_f}{g} V_{jf} + \frac{Q_p}{g} V_{jp} - D_c - D_a \quad \dots(\text{II},19)$$

Combining the two equations, one can write:-

$$\begin{aligned} D_a &= \frac{Q_f}{g} (V_{jf} - V_{ef}) - A_{ef} (P_{ef} - P_{\infty}) \\ &\quad + \frac{Q_p}{g} (V_{jp} - V_{ep}) - A_{ep} (P_{ep} - P_{\infty}) \\ &\quad + F_a + F_c - D_c \quad \dots(\text{II},20) \end{aligned}$$

which is identical to Equation (II,17). All of the terms on the right hand side of this equation can either be measured or predicted. The following sections describe methods for estimating the individual terms.

2.5.1 The nozzle flows

In Equation (II,20) the mass flow rates, Q_f and Q_p , are presumed to be known either from direct measurements, as in model tests, or from engine performance curves in full-scale tests. Calculation of the mass flows from a knowledge of the nozzle discharge coefficients could lead to errors in some installations since, as demonstrated in the examples given in Section 2.6, the gas generator nozzle applied pressure ratio can be significantly lower than the exhaust pressure ratio so that the

nozzle may be operating in a regime where discharge coefficient varies rapidly with changes in the applied pressure ratio. In addition the effect of asymmetry in the installation may lead to a variation in base pressure around the nozzle exit plane. Thus precise values of discharge coefficient may be difficult to determine.

2.5.2 Exit velocities

These problems associated with base pressure also lead to difficulties in satisfactorily defining V_e . In the examples, V_e has been evaluated from the applied pressure ratio based on mean values of the jet pipe total and base pressures. Reference 15 gives details of integral methods for determining V_e , but these depend on a more detailed knowledge of the exit plane flow field than is usually available.

2.5.3 Calculation of the V_j 's

Both derivations assume that the V_j 's are computed using a conservation procedure from the exit velocities and the differences in base pressure from ambient static pressure.

The concept of "thrust-minus-drag" is well understood and there is no doubt that the "thrust-minus-drag" can be expressed in terms of exit momenta and body forces as shown in Equation (II,18). However, the expression given in Equation (II,19) depends on defining the thrust which would be obtained in the absence of drag - here written as $\frac{Q_f}{g} V_{jf} + \frac{Q_p}{g} V_{jp}$ (thus Equation (II,19) could be taken as a definition of the total drag $D_c + D_a$). The calculation of the V_j 's therefore depends on evaluating the thrust at downstream infinity which would be obtained in the absence of drag.

When the flow in the jet is wholly subsonic, it would seem reasonable to expect the flow to expand adiabatically and isentropically from the nozzle exit pressure P_e to the ambient static pressure P_∞ . Thus a value for V_j based on the Jones thrust⁽¹⁵⁾ might be expected.

When the flow in the jet is wholly supersonic it would be unreasonable to expect any more thrust at downstream infinity than that which exists at the nozzle exit because of shock losses in the jet. Thus a value for V_j based on the Standard thrust⁽¹⁵⁾ would seem reasonable. This implies that, if the body is shaped so that some further external expansion takes place, then the additional thrust is credited

to the body rather than to the internal thrust. This additional thrust, here termed "thrust recovery", would be a proportion of the potential divergent thrust. Thus we see that D_a , as now defined, is not the afterbody drag when the fan nozzle pressure ratio is supercritical, but is equal to the afterbody drag minus the thrust recovery. This, of course, leads to the possibility of obtaining a negative value for D_a when the thrust recovery exceeds the afterbody drag.

In the case when the A.P.R. is subcritical and the E.P.R. is supercritical it is suggested that V_j should be based on an isentropic expansion to sonic conditions with no thrust change in the supersonic part of the jet. This also has the advantage that the definition of thrust is continuous as the pressure ratios are changed.

2.5.4 Calculation of the cowl and afterbody forces, F_c and F_a

The force on the cowl, or afterbody, is given by:-

$$F = \int_0^x 2\pi r \tau dx - \int_A (P_L - P_\infty) dA \quad \dots(\text{II},21)$$

taking boat-tail projected areas as positive.

The second integral in Equation (II,21) is determined directly from the pressure distribution. The first integral depends on a prediction of shear stress τ .

In Reference 30, Art 27.5, Shapiro suggests that flat plate skin friction data is fitted within ± 2 per cent by*

$$C_{d\infty} = \frac{0.472}{\left(\text{Log}_{10} R_\infty\right)^{2.58} \left(1 + \frac{\gamma - 1}{2} \text{Mn}_\infty^2\right)^{0.467}} \quad \dots(\text{II},22)$$

$$\text{and} \quad C_{f\infty} = C_{d\infty} \left(1 - \frac{1.12}{\text{Log}_{10} R_\infty}\right) \quad \dots(\text{II},23)$$

* ∞ in this context means at the edge of the boundary layer, which for a flat plate at zero incidence is identical to free stream conditions.

In order to take into account pressure gradients, it is suggested here that the length used in computing R_∞ should be replaced by the equivalent length 'X' defined in Equation (II,31) below, i.e. replace R_∞ by Rx.

$$\text{Now,} \quad \tau = C_{f\infty} \times \frac{1}{2} \rho_e V_e^2 \quad \dots(\text{II},24)$$

Combining Equations (II,22), (II,23) and (II,24), and taking $\gamma = 1.4$, we can write

$$\tau = \frac{0.7 P_e M_{n_e}^2 \times 0.472 \left(1 - \frac{1.12}{\log_{10} Rx}\right)}{\left(\log_{10} Rx\right)^{2.58} \left(1 + \frac{M_{n_e}^2}{5}\right)^{0.467}} \quad \dots(\text{II},25)$$

Now we can re-write Equation (II,21) as

$$F = 0.6608\pi \int_0^x \frac{r P_e M_{n_e}^2 \left(1 - \frac{1.12}{\log_{10} Rx}\right)}{\left(\log_{10} Rx\right)^{2.58} \left(1 + \frac{M_{n_e}^2}{5}\right)^{0.467}} dx - \int_A (P_L - P_\infty) dA \quad \dots(\text{II},26)$$

Thus, using Equation (II,26), one can estimate F_c and F_a . Whilst the estimation of F_c should be fairly accurate, the estimation of F_a will be less so. A direct measurement of F_a , if this is possible, would be preferable.

2.5.5 Calculation of the cowl drag, D_c

It would be possible to calculate the boundary layer development along the cowl surface from the measured static pressure distribution, so that the boundary layer momentum thickness, θ_{TE} , at the trailing edge could be calculated. One could then calculate the momentum deficit at the trailing edge from:-

$$D = \rho_{TE} V_{TE}^2 2\pi r_{TE} \theta_{TE} \quad \dots(\text{II},27)$$

However, D would not equal the drag D_c since, while the first term of Equation (A11) of Appendix A is zero in the wake, the second

term may continue to grow. The continued activity of the latter term is accounted for by using the momentum thickness at downstream infinity rather than at the trailing edge. An exact analytical technique for calculating the momentum thickness at downstream infinity does not exist, but Squire and Young⁽²⁷⁾ have developed a semi-empirical method for incompressible flow which has been extended by Nash et al⁽²⁸⁾ for compressible flow. Nash's result is:-

$$\frac{\theta_{\infty}}{\theta_{TE}} = \left(\frac{V_{TE}}{V_{\infty}} \right)^{\frac{\bar{H}+5}{2}} \left(\frac{\Theta_{TE}}{\Theta_{\infty}} \right)^3 \quad \dots(\text{II},28)$$

where $\bar{H}$ is the transformed shape parameter; a value for $\bar{H}$ of 1.4 is suggested, θ_{∞} is the momentum thickness at downstream infinity, V_{∞} is the undisturbed free stream velocity and Θ is the static temperature.

Thus the total drag of the fan cowl due to boundary layer growth can be written:-

$$D_C = 2\pi r_{TE} \theta_{TE} \rho_{\infty} V_{\infty}^2 \left(\frac{V_{TE}}{V_{\infty}} \right)^{\frac{\bar{H}+5}{2}} \left(\frac{\Theta_{TE}}{\Theta_{\infty}} \right)^3 \quad \dots(\text{II},29)$$

Since the static pressure at the trailing edge of the cowl is not far different from the free stream static pressure, the 'Nash' correction term will have a value close to unity.

The momentum thickness at the trailing edge can be predicted by the method of Reference 29. This method has been chosen here simply to illustrate the technique. The quality of experimental data likely to be obtained may not justify more refined methods of boundary layer prediction. Stratford and Beavers' analysis is simple to carry out and gives results which are in fair agreement with later methods, particularly so far as θ prediction is concerned.

Summarising the method, the function $\bar{P}$, where

$$\bar{P} = \left[\frac{Mn}{1 + \frac{Mn^2}{5}} \right]^4 \quad \dots(\text{II},30)$$

and the radius, r , are tabulated as functions of x . The equivalent length, X , is then evaluated, where

$$X = \left(\bar{Pr}^\alpha \right)^{-1} \int_0^x \bar{Pr}^\alpha dx \quad \dots(\text{II},31)$$

given that $\alpha = 1.25$ if Rx is of order 10^6 , and

$\alpha = 1.2$ if Rx is of order 10^7

To assist in the evaluation of Equation (II,26), values of X should be tabulated along the surface. The Reynolds number at the trailing edge, Rx , is then evaluated from:-

$$Rx = \frac{9.888 \times 10^7 \times Mn \times X \times P_t \left(\Theta_t + 117 \left(1 + \frac{Mn^2}{5} \right) \right)}{\Theta_t^2 \times \left(1 + \frac{Mn^2}{5} \right)^{2.5}} \quad \dots(\text{II},32)$$

where X is in feet, P_t is in psi and Θ_t is in $^{\circ}\text{K}$. Then, according to Reference 29, if Rx is of order 10^6

$$\theta = 0.036 \left(1 + \frac{Mn^2}{10} \right)^{-0.70} X Rx^{-1/5} \quad \dots(\text{II},33)$$

and if Rx is of order 10^7

$$\theta = 0.022 \left(1 + \frac{Mn^2}{10} \right)^{-0.70} X Rx^{-1/6} \quad \dots(\text{II},34)$$

Having calculated θ at the trailing edge, D_c can be evaluated from Equation (II,29) which can be re-written in a more convenient form as

$$D_c = 2.8\pi r_{TE} \theta_{TE} P_\infty M_{n_\infty}^2 \left(\frac{M_{n_{TE}}}{M_{n_\infty}} \right)^{3.2} \left(\frac{1 + \frac{M_{n_\infty}^2}{5}}{1 + \frac{M_{n_{TE}}^2}{5}} \right)^{4.6} \dots (II,35)$$

2.5.6 Experimental measurements required

From Equation (II,20), the measurements required to determine D_a are listed below:

- (a) Total pressure in the two exhaust streams and in the free stream.
- (b) The base pressures of the two nozzles.
- (c) The static pressure in the free stream.
- (d) The static pressure distribution on the fan cowl (from which both F_c and D_c can be calculated, see Sections 2.5.4 and 2.5.5).
- (e) The force on the afterbody (or, as second best, the pressure distribution so that F_a may be computed, see Section 2.5.4).
- (f) All projected and nozzle exit areas.
- (g) The two nozzle air mass flow rates.

2.6 Example calculations

Data for these example calculations can be found in Appendix B. Three test points are used to illustrate the three different methods required for the calculation of the V_j 's.

2.6.1 A supercritical case

In Example 1 (Appendix B) both pressure ratios for the two nozzles are supercritical. Therefore, following Section 2.5.3, the gauge stream thrust at the nozzle exits are taken as equal to the gauge stream thrusts at downstream infinity and Equation (II,20) can be re-written as:-

$$D_a = F_a + F_c - D_c \dots (II,36)$$

so that following Appendix B,

$$\begin{aligned} D_a &= 0.617 + 1.420 - 1.121 \\ &= 0.916 \end{aligned}$$

Expressed as a proportion of the ideal gross gauge convergent thrust of the two nozzle streams separately expanded,

$$\frac{D_a}{F_{ID}} = 0.0064$$

2.6.2 An intermediate case

In Example 2, both pressure ratios for the fan nozzle are supercritical, but, while the E.P.R. for the primary nozzle is supercritical, the A.P.R. is subcritical. In this case, the change in stream thrust in the fan stream is taken as zero, but the change in the primary stream is taken as that from the A.P.R. to sonic conditions.

Thus Equation (II,20) can be re-written as:-

$$\begin{aligned} D_a &= \frac{Q_p}{g} (V_p^* - V_{ep}) - A_{ep} (P_{ep} - P_\infty) + A_p^* (P_p^* - P_\infty) + F_a + F_c - D_c \\ &\dots (II,37) \end{aligned}$$

A more convenient form of Equation (II,37) is:-

$$\begin{aligned} D_a &= \left(Q \sqrt{\Theta_t} \right)_p \left[\left(\frac{V}{g \sqrt{\Theta_t}} \right)_p^* - \left(\frac{V}{g \sqrt{\Theta_t}} \right)_{ep} \right] \\ &\quad - P_{tp} A_{ep} \left[\frac{P_{ep}}{P_{tp}} - \left(1 - \frac{A_p^*}{A_{ep}} \right) \frac{P_\infty}{P_{tp}} - \frac{A_p^*}{A_{ep}} \frac{P_p^*}{P_{tp}} \right] + F_a + F_c - D_c \end{aligned}$$

... (II,38)

or

$$D_a = \left(Q \sqrt{\Theta_t} \right)_p \left\{ 1.86501 - \left(\frac{V}{g \sqrt{\Theta_t}} \right)_{ep} \right\} \\ - P_{tp} A_{ep} \left\{ \frac{1}{APRF} - \left(1 - \frac{A_p^*}{A_{ep}} \right) \frac{1}{EPRF} - \frac{A_p^*}{A_{ep}} \times 0.52828 \right\} + F_a + F_c - D_c \\ \dots (II, 39)$$

so that

$$D_a = 0.728 - 0.775 - 0.513 + 1.697 - 1.119 \\ = 0.018$$

and

$$\frac{D_a}{F_{ID}} = 0.00016$$

2.6.3 A subcritical case

Example 3 is typical of a case where the pressure ratios are subcritical. In this example, Equation (II,20) is used in full. Again for convenience, Equation (II,20) may be re-written as:-

$$D_a = \left(Q \sqrt{\Theta_t} \right)_f \left\{ \left(\frac{V}{g \sqrt{\Theta_t}} \right)_{EPRF} - \left(\frac{V}{g \sqrt{\Theta_t}} \right)_{APRF} \right\} - P_{tf} A_{ef} \left(\frac{1}{APRF} - \frac{1}{EPRF} \right) \\ + \left(Q \sqrt{\Theta_t} \right)_p \left\{ \left(\frac{V}{g \sqrt{\Theta_t}} \right)_{EPRF} - \left(\frac{V}{g \sqrt{\Theta_t}} \right)_{APRF} \right\} \\ - P_{tp} A_{ep} \left(\frac{1}{APRF} - \frac{1}{EPRF} \right) + F_a + F_c - D_c \\ \dots (II, 40)$$

so that

$$D_a = 1.610 + 2.112 - 1.666 - 2.279 - 0.472 + 2.029 - 1.119 \\ = 0.215$$

and
$$\frac{D_a}{F_{ID}} = 0.003$$

2.7 Discussion

As shown in Section 2.5.3, the methods for estimating V_j lead, in some circumstances, to a negative value for D_a when the thrust recovery exceeds the drag. The opinion has been advanced that the afterbody should be credited with any thrust recovery. It would be equally reasonable, however, to suggest that the afterbody should always be expected to produce the full thrust recovery and that any shortfall in this respect should be added to the afterbody drag. This would mean that, at supercritical exhaust pressure ratios, the velocity at downstream infinity, V_j , should be the fully expanded (convergent-divergent) value.

The choice between the two methods clearly represents a choice between two different scales to represent D_a , the first benefiting the afterbody with any external expansion and treating the fan nozzle correctly, the latter penalising the afterbody for any failure to achieve thrust recovery.

The author can see no real reason for preferring one method to the other. Both approaches would give the same overall thrust-minus-drag - they merely vary the distribution of the losses between the internal thrust, the afterbody drag and the cowl drag.

It has not been found possible to separate the two components of D_a because the proportion of the potential thrust recovery actually achieved is not known. No absolute value of afterbody drag has been revealed in this chapter. Rather the work should be read as a definition of a DRAG-MINUS-THRUST term, D_a , closely related to afterbody drag.

It should be noted that all forces have been assumed to act in an axial direction. In a real installation the efflux from the fan nozzle may be inclined to the axis. This will introduce a cosine effect since the flow will exert an additional force on the afterbody by turning to an axial direction.

CHAPTER 3

Theoretical prediction of the afterbody flow field

3.1 Introduction

This chapter is concerned with a method of predicting the fan exhaust flow field over the afterbody so that the skin friction drag, (loss 5 in Figure 2), and the form drag, (loss 6 in Figure 2), can be estimated. At subsonic cruising speeds current high by-pass ratio engines typically feature fan nozzle pressure ratios of between 2 and 3:1. Therefore, even with a convergent nozzle, the flow in the jet is supersonic, allowing prediction by the method of characteristics. However, in order to fit the problem within the currently available computer, it was necessary to restrict the solution to quiescent conditions (i.e. still air surrounding the nacelle), so that the pressure at the free jet boundary is everywhere equal to ambient static pressure, and the mixing or entrainment region at the outer boundary of the jet has been ignored.

Following publication of this part of the work in Reference 22, Young⁽²³⁾ has used a source/sink distribution method⁽³¹⁾ to calculate the pressure distribution on the jet boundary which had been predicted using quiescent boundary conditions. This pressure distribution was then used as a boundary condition for the re-calculation of the jet flow using the method of characteristics. A new jet boundary shape was thus produced. In successive iterations the shape of the new jet boundary was used to re-compute the pressure distribution which then, using a further characteristics solution for the jet, led to an improved estimate for the jet boundary shape. In all cases attempted by Young the solution converged after 6 iterations. Viscous effects were not included either at the gas generator surface or in the mixing region between the supersonic jet flow and the surrounding subsonic flow. Young concluded that the agreement between the mathematical predictions and experimental results improved as the subsonic external Mach number was increased because of the reduced effect of the shear region at the jet boundary in the experiment.

It may be noted that current engine guarantee demonstrations are carried out in altitude test cells under quiescent conditions⁽¹⁴⁾. Thus the programme described in this chapter can be used to predict both model and full scale engine results in usefully realistic conditions.

A computer program has been written to predict the flow field using a method of characteristics technique modified to allow for the growth of the surface boundary layer. The program is written in Fortran IV to run on an SDS 9300 computer.

3.2 The characteristics solution

The flow from the fan nozzle is assumed to expand from sonic conditions at the throat through a Prandtl-Meyer expansion fan centred at the lip as shown in Figure 5. The strength of this Prandtl-Meyer fan is a function of the pressure ratio. The waves comprising the fan reflect from the afterbody surface and may be reinforced by body waves generated by surface curvature. On reflection from the free stream boundary, the expansion waves become compression waves. Depending on body shape, the compression waves may tend to focus producing a shock wave after reflection from the body surface. When the shock reaches the free stream boundary, it reflects as a Prandtl-Meyer expansion fan and thus a pattern of alternate expansion and compression regions is repeated along the body.

The hyperbolic partial differential equations governing supersonic potential flow can be solved numerically by the method of characteristics. Kantrowitz's derivation⁽¹⁹⁾ of the compatibility relations along characteristics is repeated in Appendix C and the characteristics are shown to be Mach waves. Thus characteristics produced by the calculations to be described give the Mach wave pattern in the fan exhaust. From Appendix C the compatibility relation in axisymmetric flow for a left running wave such as B-C in Figure 6 may be written:-

$$\delta(\omega - \lambda) = \frac{\sin(\lambda)}{r \sin(\lambda + \mu)} \frac{\delta r}{Mn} \quad \dots(\text{III},1)$$

Similarly for a right running wave such as A-C in the Figure, one can write:-

$$\delta(\omega + \lambda) = \frac{\sin(\lambda)}{r \sin(\lambda - \mu)} \frac{\delta r}{Mn} \quad \dots(\text{III},2)$$

The characteristic directions are at angles of μ and $-\mu$ respectively to the local streamline direction λ , where μ is the Mach angle. Equations (III,1) and (III,2) with suitable boundary conditions suffice to compute the potential flow field.

The principles of the calculation are as follows. Referring to Figure 6a, if the flow conditions are known at two points A and B, which are not on the same characteristic, then a first approximation to the position of a point C, located at the intersection of the right running characteristic through A and the left running characteristic through B, can be found by basing the characteristic directions on the local flow conditions at A and B. A first approximation to the flow conditions at C can be computed using Equation (III,2) along A-C and Equation (III,1) along B-C to calculate $(\omega + \lambda)$ and $(\omega - \lambda)$ at C, whence ω and λ can be found. In general, the flow conditions at C will differ from those at either A or B - the characteristics will be curved - so an iterative technique is used to improve both the location of C and the accuracy of the computed flow conditions. In successive iterations the mean of conditions at A and C and at B and C are used to re-draw the characteristics and then mean conditions are used in the compatibility relations to re-compute the local flow conditions. Thus this fundamental constituent of the flow network can be calculated to a high degree of accuracy.

Next, point C is used as point A for the calculation of another point in the flow field, so the computation proceeds along right running characteristics.

3.3 Boundary conditions

Three sets of boundary conditions are required. These are:-

- (a) an initial line,
- (b) an outer boundary
- and (c) an inner boundary.

Both flow direction and Mach number (or Prandtl-Meyer angle) must be specified along the initial line but the distribution of either flow direction or Mach number will suffice on the other two boundaries.

(a) The initial line

The total Prandtl-Meyer turn through the lip fan is computed from the pressure ratio. The fan is made up from N equal strength waves and the flow conditions at M initial points on the first wave are assumed constant in any particular case. N and M are pre-set values. Thus if the total Prandtl-Meyer turn through the fan is ω_c and the

inclination of the throat flow to the axis is λ_i , the flow direction at the 'M' initial points on the first wave will be $\left(\lambda_i + \omega_c/N\right)$, and the Prandtl-Meyer angle will be ω_c/N . The wave inclination will be $\left(\lambda_i + \omega_c/N - \mu_i\right)$ where μ_i is the Mach angle corresponding with the Mach number at the M initial points.

(b) The outer boundary

Under quiescent, or still air conditions the static pressure at the free stream boundary is constant so that the Mach number at this boundary is known as a function of the pressure ratio. In Figure 6b, A and C are points on the free stream boundary and B is on the right running characteristic through A. Conditions are known at A and B so that following an iterative technique similar to that described above, C can be located at the intersection of the streamline through A and the left running characteristic through B. Using Equation (III,1), $(\omega - \lambda)$ can be evaluated at C and, since ω is known, λ can be found.

(c) The inner boundary

In the absence of a boundary layer, the distribution of inner surface flow direction is determined by the shape of the afterbody. Referring to Figure 6c, if conditions are known at a point A then conditions at a point B, where the right running characteristic through A intersects the boundary, can be obtained by computing $(\omega + \lambda)$ at B using Equation (III,2). Hence, knowing λ from the surface slope, ω can be found.

3.4 The effect of the boundary layer

The boundary layer can be regarded as replacing the body by an equivalent shape having its surface displaced a distance δ^* from the solid surface. The displacement thickness, δ^* , is calculated by the method of Reference 29. This method was chosen from the many available because it provides working formulae which are simple to use and reflect boundary layer behaviour to a fair degree. The technique for embodying the boundary layer into the characteristics solution is as follows. In Figure 7, the right running characteristic through A is first used to calculate conditions at point B on the solid surface, ignoring the boundary layer. Following Reference 29, conditions at B are used to compute the equivalent length X where:-

$$X = \left(\bar{Pr}^\alpha \right)^{-1} \int_0^x \bar{Pr}^\alpha dx \quad \dots(\text{II},31)\text{bis}$$

(For simplicity a constant value for α of 1.2 is used in the programme: Reference 29 suggests the possibility of using either $\alpha = 1.2$ or $\alpha = 1.25$ depending on Reynolds number.)

The distance x is measured from the nozzle throat, since, in view of the rapid contraction upstream, it is assumed that the throat boundary layer thickness is zero. $\bar{P}$ is computed using Equation (II,30). The local Reynolds number is then computed using Equation (II,32).

Now, according to Reference 29, if Rx is of order 10^6 ,

$$\delta^* = 0.046 (1 + 0.8 Mn^2)^{0.44} X Rx^{-1/5} \quad \dots(\text{III},3)$$

so that
$$\frac{d\delta^*}{dX} = 0.8 \frac{\delta^*}{X} \quad \dots(\text{III},4)$$

and if Rx is of order 10^7 ,

$$\delta^* = 0.028 (1 + 0.8 Mn^2)^{0.44} X Rx^{-1/6} \quad \dots(\text{III},5)$$

so that
$$\frac{d\delta^*}{dX} = 0.833 \frac{\delta^*}{X} \quad \dots(\text{III},6)$$

Thus by computing the co-ordinates of the point C in Figure 7, at a distance δ^* above B, and defining the displacement surface as a line through C inclined at an angle equal to that of the solid surface plus the angle represented by $d\delta^*/dx$, the co-ordinates of point D can be found at the intersection of the characteristic through A and the displacement surface. The slope at D is the slope of the displacement surface. Thus, the flow conditions at D can be computed from the known conditions at A in a similar manner to the calculations for B.

 $\frac{dX}{dx} = 1$ and so can be ignored.

It is usual to assume that static pressure is constant through the boundary layer. Thus, in the present program, one can transfer the static pressure at point D to a point E on the solid surface so that the boundary layer growth under D can be checked. This forms the basis of an iterative loop in that point E can be treated as a point B so that the entire calculation can be cycled. The iterations converge rapidly except where there are severe adverse pressure gradients in the potential flow field. The presence of severe adverse pressure gradients in the physical flow is likely to cause the reverse flow phenomenon known as boundary layer separation. A similar feature occurs in the mathematical model. As the iteration proceeds, the boundary layer displacement thickness increases in an unstable manner until a condition is reached where reverse flow is implied. The method then breaks down as, indeed, would the real flow. So far, no attempt has been made to correlate the mathematical separation with experimental data, as there is some difficulty in setting up limiting conditions, both experimentally and theoretically.

It is however known from wind tunnel experiments that afterbody shapes exhibiting this phenomenon are unlikely to have a high performance, and thus such shapes would not be of further practical interest.

3.5 Problem areas

The method of characteristics is only valid in supersonic potential flow regions. Two problems arise when this method is applied to afterbody flows.

3.5.1 Shock waves

The first problem is the occurrence of shock waves in the flow field. In principle, shock waves could be correctly treated using the Rankine-Hugoniot equations (see Reference 32 for example) but the increase in program organisational detail required precluded proper treatment from the present program.

Shock waves are manifest as intersections of characteristics of the same family. In the current program when intersections are detected, the upstream characteristic is deleted and the downstream characteristic allowed to continue with increased strength, as indicated in Figure 8. Such 'strong' characteristics were used with some success in Reference 21 in treating shock waves. A shortcoming of this

technique occurs when, if there are a number of shock waves in the flow field, the number of characteristics remaining is seriously depleted. Another shortcoming is that the reflection of the 'strong' characteristic, representing the shock wave, from the free stream boundary is incorrectly represented: instead of a Prandtl-Meyer fan, the reflection takes the form of a strong expansion wave - a negative shock! Attempts to overcome these shortcomings by identifying the number of waves lost in generating each shock so that this number could be fed into an expansion fan at the reflection point have not so far been successful. Two difficulties arise. The first is in the organisation of the counting and marking of the shock waves, and the second is in ascribing a shock strength to a characteristic - which, by definition, has zero strength.

3.5.2 Subsonic patches

The second problem arises if subsonic patches occur in the flow field. These patches are more likely to occur if the afterbody diameter decreases as the solution progresses downstream - as it does on a typical nacelle - notwithstanding the flow expansion introduced by the convex curvature of the nacelle in the axial direction. Moreover small rounding errors are sufficient to give a subsonic result when the true Mach number is close to unity.

Subsonic patches are associated with regions of high adverse pressure gradient so it is difficult to separate the effects of an approximate treatment of the subsonic region from the potential separation of the surface boundary layer. However, it was felt that forcing the local Mach number to remain slightly supersonic, and so enabling the characteristics solution to proceed, would be justified provided that the subsonic patches were small in extent. This does of course introduce an error into the program which propagates downstream. However, in the majority of cases tried so far, any subsonic patches are usually confined to regions near the trailing edge.

In concluding this brief discussion of problems it should be noted that high performance exhaust systems are unlikely to exhibit either strong shock waves or extensive subsonic patches.

3.6 The computer program

The calculations described above have been programmed for an SDS 9300 digital computer. The mathematics of the calculation are straightforward; the difficulty arises in the organisation of the logic. This is the reason for excluding use of the full Rankine-Hugoniot equations for shock waves at the present stage. In writing the program, attention was given to economy of storage to leave room for subsequent extensions either in the treatment of shock waves and shock - outer boundary reflections, or in the boundary layer prediction methods.

The main core of the logic centres round a scanning system whereby the solution progresses along successive right running characteristics. The flow conditions at each point are not printed out as they are calculated since they may subsequently be overwritten by a shock. In the latest version of the program 24 characteristics are carried in the store, each one being printed before it is overwritten by the next allocated to that store. When the computation comes to an end, the remaining 24 characteristics are printed.

The boundary layer functions $\bar{P}$, X and R_x form subroutines. A further subroutine has been written to calculate Mach angle from Prandtl-Meyer angle. Data for the program consist of an initial line and geometry specification, pressure ratio, total pressure and temperature. The geometry specification consists of a nozzle lip radius and up to 200 body co-ordinates, the surface being represented by a series of straight lines in the computer. In its current form, the program occupies 19571 words in the store, of which 14405 are used for storing the local flow conditions, and the running times average out at about 3 minutes when $N = 24$ and $M = 23$, their current maximum values.

Figure 9 shows a graphical representation of a typical set of results with $N = 6$ and $M = 5$. The pattern of characteristics, while exhibiting a lack of detail near the trailing edge due to the formation of shock waves, gives a useful picture of the flow. The boundary layer displacement thickness reacts in a realistic manner to the pressure distribution - here plotted as the surface Mach number distribution.

It was stated in Section 3.1 that the computer solution was limited to quiescent conditions and supercritical pressure ratios. Thus

it follows that Equation (II,36) can be used to predict the afterbody drag, but with the modification that F_c and D_c are zero; in other words, the afterbody drag is equal to the force on the afterbody. Therefore some versions of the program use Equation (II,26) to predict F_a which is then equated to D_a .

3.7 Discussion

The program described in this chapter is the only completely theoretical attack on the problem of determining afterbody drag known to the author. Boundary layer theory and the method of characteristics have been successfully combined in a step by step technique for predicting supersonic flow fields over bodies of revolution. The program is limited to cases where the fan nozzle pressure ratio is supercritical and the conditions surrounding the nacelle are quiescent. While these limitations prevent calculation of the flight conditions, they do permit prediction of the flow field and drag components in an altitude test cell experiment. This is important because the altitude test cell is used for guarantee demonstrations.

Furthermore, interference effects between the wing and nacelle, or the fuselage and nacelle in the aft mounting case reduce the usefulness of axisymmetric flow modelling of the complete flow field. Indeed, at the present time, little is known about interpretation of test results in the flight case because of the complexity of the flow field.

CHAPTER 4

Experimental programme

4.1 Introduction

Current high by-pass ratio engines for subsonic transport aircraft have a gross to net thrust ratio at cruise of about 2.5, (a figure incidentally similar to that for the Olympus engine in the Concorde at Mach 2), leading to about a 5 per cent change in long range payload for a 1 per cent change in nacelle gross thrust performance. Where the future trend is to engines of higher by-pass ratio and therefore lower specific thrust the installation losses for the same cruise thrust will begin to assume large proportions. Hence a small change in nacelle shape leading to a small improvement in nacelle gross thrust performance can have a significant effect on aircraft range and payload. Gross thrust performance of model exhaust systems must therefore be determined to a high degree of accuracy if the data are to be meaningful.

In order to meet this need an experimental programme was initiated. This chapter will describe first a test rig built at N.G.T.E. specifically for this programme, then 2 of the high by-pass ratio installation models used in the programme and finally a representative selection of the test results. The aim of the programme was to determine and validate the techniques needed to attain the required high degree of accuracy rather than to explore the performance of a particular design in complete detail.

4.2 The test rig

The thrust measuring rig shown diagrammatically in Figure 10 is based on an earlier rig developed to test supersonic propelling nozzles⁽²⁴⁾. The central feature of the new rig is a 54 inch length of 4 inch bore diameter pipe mounted horizontally in air bearings. Dry air at up to 3 atmospheres pressure - according to model pressure ratio required - is fed to the supply box, a large plenum chamber, from which it enters the pipe through radial inlet guide vanes and an intake flare. This arrangement ensures that the air enters the pipe with zero axial momentum. Pipe mass flow rate is measured at the airmeter which comprises a 3 head total pressure rake and 4 wall static tappings situated about 6 diameters downstream from the intake flare. Provision

has been made for ducting 2 separately metered flows to the model but the outer stream has not been used in the current test programme.

A detachable sting, onto which the model is mounted, is bolted to the rig pipe just downstream from the second air bearing whilst at the opposite end a dummy sting, blanked off internally, is permanently attached to the intake flare. This dummy sting bears onto the thrust meter which supports the rig 'out-of-balance' forces.

A load measuring system called the Precimeter, developed at N.G.T.E., was used at first. This consisted of an hydraulic load cell and a servo valve which regulated the hydraulic pressure inside the load cell so as to keep the floating part of the rig in a null position. This system was later superseded by an electronic load cell of the semi-conductor strain gauge variety. A pre-load arm keeps the dummy sting in contact with the thrust meter so that out-of-balance force measurement can be made even when, as occurs under certain operating conditions, the net force on the floating parts would pull the dummy sting out of contact with the load cell.

The test model is located in a slotted, octagonal wind tunnel, 17.5 inches across flats, illustrated in Figure 11. The tunnel flow area was matched to the available plant capacity so as to yield a maximum tunnel Mach number slightly in excess of unity. Tunnel mass flow at this condition is about 87 lb/s. Ambient air is induced into the tunnel via a bellmouth (Figure 10). The tunnel thus operates at ambient total pressure and temperature. Operation on a humid day can result in water condensation effects restricting the useful upper limit of tunnel Mach number to less than about 0.9.

In order to ensure good quality schlieren photography an octagonal tunnel design was adopted, based on that of Reference 33. To simplify construction the slots were made parallel for the whole length of the tunnel though slot liners could be fitted to taper the slots; the open area ratio is $12\frac{1}{2}$ per cent. The working section is surrounded by a 27 inch diameter circular plenum chamber connected to a short constant diameter exhaust pipe which leads to the exhaust diffuser. The detailed design was arranged to allow air leaving the working section through the slots to pass out to the diffuser.

For operation in the quiescent mode (Figure 12) the bellmouth intake is replaced by a seal plate which fits closely to the sting but

does not touch it. Model exhaust is ducted away via a collecting section leading to an exhaust diffuser sized to the model flow and thence through a silencer to atmosphere. This alternative exhaust system fits inside the tunnel as shown in the figure.

Schlieren quality glass windows are fitted towards the downstream end of the tunnel. The windows are 5 inches deep by 15 inches long thus allowing some variation in choosing the best model position in the tunnel. The schlieren system is of the standard lens type employing 4 inch diameter collimating and objective lenses mounted on an optical bench. The light source is a commercially available type containing a 10 kV Argon guided spark and a 12 V filament lamp. A virtual image of the lamp filament is focussed in the spark gap by a lens. The light unit produces a line source so that the complete unit must be rotated as the knife edge is rotated. The image produced by the schlieren system can be viewed on a ground glass screen which can be replaced by a photographic plate when a permanent record is required. For space saving reasons, two 45 degree mirrors are interposed in the optical path to bend the light rays through a right angle on either side of the tunnel. The whole system is rigidly mounted on a trolley so that it can be traversed over the length of the windows.

Instrumentation specific to the rig includes the airmeter, the load cell, a thermocouple to measure model air temperature and a number of wall static pressure tapings on the tunnel slats and in the surrounding plenum chamber. The control room is equipped with a large number of manometers, control gear for the gate valves controlling the model and tunnel flows and the necessary electronic units for the load cells.

4.3 The model exhaust systems

A number of high by-pass ratio exhaust system models have been constructed and run on the rig. Of these, 2 are of interest to this programme. The first is an axisymmetric, two-stream model which was based on a current engine design. This model was designed to explore the thrust-minus-drag performance of a range of different afterbody shapes.

The second model design had a more academic background. The thoughts expressed in Chapter 2 led to the desire to be able to measure the force on the primary afterbody directly. The model was designed

with the outer skin of the primary afterbody moving axially on linear bearings so that the force required to restrain it could be detected using an internal force transducer.

The models were approximately one-twentieth of full scale and were run at pressure levels of about 5 times those to be found under full scale cruise conditions. The temperature of the fan nozzle efflux was representative. Thus the test Reynolds numbers were about one-quarter of the full scale value. However, both the models and the full scale installation operate well into the turbulent regime where changes of viscous effects with Reynolds number are relatively slow*. Therefore the results of tests on these models should be applicable to the full scale installation.

4.3.1 The two-stream research model

The layout of the two-stream research model is shown in Figure 13. The single metered air supply from the sting is split into two streams within the model chassis. The inner, or primary duct is located by 3 spats across the outer annulus. A dummy turbine tail cone is similarly located within the primary duct. The fan cowl, afterbody and tail cone are exchangeable so that alternative geometries can be built up. For example, one range of builds, not reported here, investigated the performance when the tail cone/afterbody combination was replaced by a plug nozzle.

Six total head pressure rakes, with the tubes in each rake distributed on an equal area basis, are located at the simulated fan and turbine exhaust planes. Static tapings in the duct walls are fitted in the same planes as the rakes. The remaining model instrumentation consists of spirals of 12 static pressure tapings along the aft facing surfaces of the cowl and afterbody and tapings in the 2 nozzle bases. A feature of this model which caused some delay to the programme was the method of leading out the pressure lines from the afterbody. In order to maintain both the inner and outer profiles of the primary duct, the total thickness in the region of the fan nozzle exit had to be kept small - too small in fact to consider double skinning. Thus, if the afterbody were to be interchangeable, a 'plug-and-socket' scheme had to be devised to connect the afterbody

*Typical Reynolds numbers for the model tests based on mean jet Mach number and afterbody length were in the range 3 million to 6 million.

tappings back to the chassis. The scheme finally chosen used individual 'O' ring seals at each joint and was fairly satisfactory. However, great care was needed on assembly to avoid chipping the seals.

4.3.2 The floating afterbody model

The floating afterbody model layout is shown in Figure 14. Provision of the bearings and load cell for the floating skin left no room for a primary air duct. Thus the model has only the fan stream represented. The external shape of the fan cowl can be changed by fitting alternative parts and the primary base plate can be replaced by jet simulators consisting of initially conical or parallel extension pieces. The 3 fan cowls and 3 jet simulators are shown superimposed in Figure 15a.

Three different fan cowl shapes were chosen in an attempt to show the effect on the afterbody flow field of varying the buoyancy effect from the fan cowl during tests with external flow. The internal contours of the 3 cowls were designed to be identical though the results to be presented later show that the conical fan cowl probably had a slightly different internal form. Since all the fan cowls were to be mounted from the same sting and since the fan nozzle geometry was fixed the normal projected area of each cowl was the same.

The 3 plume simulators were designed on the following basis. At subcritical pressure ratios across the primary nozzle the efflux would initially be expected to flow towards the centreline reaching a vena-contracta some distance downstream from the nozzle outlet. At supercritical pressure ratios, a Prandtl-Meyer expansion at the nozzle lip would turn the flow initially away from the axis by an amount dependent on the pressure ratio. Thus an initially convergent simulator would simulate the jet plume at subcritical pressure ratios while an initially divergent simulator would simulate the plume at a supercritical pressure ratio. Only the initial part of the jet was thought to be important in determining conditions on the afterbody so that the simulators could be terminated a short distance downstream. Initial angles of 10 degrees were chosen for the convergent and divergent simulators largely on the basis of intuition. The third simulator was made parallel to represent an intermediate stage between the other two. Each simulator was equipped with a recessed, gauzed base to give steady

overall thrust-minus-drag readings. The simulators are illustrated alongside the relevant flow fields in Figure 15b.

A photograph of the model assembled with the conical fan cowl and the convergent jet simulator is shown in Figure 16 and part assemblies are shown in Figure 17.

Gaps were left at either end of the floating skin but these gaps were set as small as possible during assembly by using shims. Typical gap sizes found to be satisfactory were between 0.0015 and 0.002 inches. There will, of course, be a small leakage into one gap and out of the other but the small sizes of the gaps should reduce the effect on the local boundary layer to acceptable proportions. The flange connecting the floating skin to the linear bearing assembly has a number of holes through it in order to equalise the pressures in the 2 chambers. The pressure difference between the chambers has been measured on a water manometer and found to be negligible.

Model instrumentation was similar to that employed on the two-stream model. The fan nozzle entry conditions were measured using a 6 total head pressure rake with the tubes distributed on an equal area basis. Static tapings in the plane of the rake were located in the inner and outer walls. There were 3 base tapings on the fan cowl and 12 statics in a spiral on the cowl boat-tail. In order to compute the pressure force on the floating skin, the number of tapings on the primary boat-tail was increased to 24. Twelve tapings were located on the primary base plate in a spiral pattern. The connections for these tapings were used to "pressure plot" the primary jet simulators when fitted. Two tapings were located inside each slot to measure the pressures in the 2 internal chambers.

The floating skin was mounted on linear ball races. Care was needed on assembly because the very small movements involved could lead to the bearings feeling 'sticky' if the balls were on the point of entering the return run. Air bearings could not be used in this situation because the air discharged from the bearings would effect the overall thrust-minus-drag measurement.

Axial movement of the floating skin was restrained by a proprietary semi-conductor strain gauge load cell chosen for its high stiffness. The load cell was mounted on a movable plate which was located by a differential jacking screw to position the floating skin correctly.

4.4 Test technique

Four basic measurements are essential for the analysis of model performance on a thrust-minus-drag basis:-

- (a) model mass flow(s) which contribute to the calculation of ideal thrust,
 - (b) nozzle entry total pressure(s) which are also a part of the ideal thrust computation,
 - (c) the forces on the moving part of the rig from which the actual gross thrust minus drag of the model can be found
- and (d) the free stream static pressure which affects both actual and ideal thrust calculations.

The precision of the final result depends critically on accurate calibrations of the various measurement techniques. It is only by taking care with the calibrations that systematic errors, as opposed to random errors which can be reduced by statistical means, can be maintained at a reasonable level. Even then, it is not possible to ascribe with a high degree of confidence a magnitude to the likely systematic error.

Measurement of the force on the afterbody using the floating afterbody technique also depends on calibration of the internal forces so that the net force on the external skin can be determined.

4.4.1 Rig calibration

The thrust meter was used to determine calibration curves for all the other force terms occurring in the thrust equation, as well as for the air flow meter. Thus the thrust meter calibration is crucial to the overall accuracy of the rig. The calibration technique adopted for the Precimeter was to record the variation in thrust meter hydraulic pressure on a single limb mercury manometer as a series of accurately known weights were suspended from the pre-load arm. The velocity ratio of the pre-load arm was determined from inspected measurements so that the axial force applied to the sting could be deduced. Typical results for the Precimeter calibration in terms of manometer reading minus zero load reading are shown in Figure 18 which includes error boundaries for 0.1 and 0.25 per cent of full scale deflection (FSD). Experience with the rig over a period of about a year led to the conclusion that the thrust meter calibration over the

range of normal operation was maintained within the 0.1 per cent FSD boundary.

The Precimeter system was later superseded by a 200 lbf FSD electronic load cell. The output from the load cell was read on a digital voltmeter. Repeatability and hysteresis, both less than 0.05 per cent FSD, are a worthwhile improvement over the corresponding figures for the hydraulic system, as may be deduced from Figure 19.

Leakage paths where the sting passes through the walls of the pressurised supply box are reduced by labyrinth seals. While the sting diameters at the 2 seals are nominally the same, slight variations due to manufacturing tolerances have led to a small out-of-balance force, F_1 in Figure 10, acting so as to push the dummy sting onto the thrust meter. The variation of F_1 with supply box pressure was found by measuring the force on the sting at several supply box pressures, flow along the sting being prevented by a blanked end where the model is normally mounted. A calibration curve (Figure 20) was constructed in terms of seal pressure difference and measured force to yield an effective out-of-balance area. This effective area was found to be constant over the operating range of the rig and the scatter about the mean line is equivalent to ± 0.03 per cent FSD of the thrust meter.

The piston force, P , which is shown later to be large under both quiescent and external flow conditions, results from the difference in local pressures between the 2 ends of the moving pipe assembly. It can be calculated from a knowledge of the relevant pressures and the sting cross-sectional area. When the rig is set up for quiescent tests (Figure 12), an additional force, F_2 , arises from the scrubbing action of the air leaking into the tunnel past the seal plate. Direct measurement of the piston plus leakage force was made at a range of tunnel depressions, again with the sting blanked, and the results analysed in terms of an effective sting cross-sectional area, which is slightly larger than the physical area. The calibration curve given in Figure 21 shows the variation in effective sting area with the pressure difference ($P_a - P_\infty$). Scatter at tunnel depressions greater than 10 inches of mercury, corresponding to model pressure ratios greater than about 3.0 in the quiescent mode, is negligible but below this value it increases to the equivalent of ± 0.13 per cent of thrust meter

FSD at a pressure difference of 3 inches of mercury - about 1.4 model pressure ratio.

When the rig is set up for external flow tests, the leakage force, F_2 , is replaced by a sting drag term representing the skin friction drag of the tunnel flow as it passes over the sting. This force is the term DS in the force equation shown in Figure 10. Three possible methods of determining sting drag have been considered; a bluff base, a blown nozzle or a non-metric jet simulator technique.

Although the use of a bluff base at the downstream end of the sting gives the simplest method of determining sting drag, past experience has revealed a critical deficiency. Unsteady base pressures occur due to vortex shedding and the consequent uncertainty in base force leads to an unacceptable level of scatter in the calculated values of sting drag. Less scatter is obtained if a recessed base is used provided that the open end of the recess is fitted with a gauze to reduce internal recirculation. However, the net force on the sting still varies with time so that there is still significant scatter in the results. In addition, the method is open to criticism on the grounds of non-representation of actual operating conditions over the latter part of the sting when an exhaust system model is being tested.

In the non-metric jet simulator technique a solid body representing the model exhaust system and its associated jet boundary is mounted from the tunnel wall so that it adjoins the sting but does not actually touch it. This technique leads to a simple experiment but the engineering details are difficult. The simulator must be accurately located axially in the tunnel to give a sufficiently small gap at the sting/simulator interface and the floating assembly must be extremely stiff to maintain line-up of the flow surfaces. Any tendency of the sting to whip would lead to discontinuous surfaces and incorrect base force measurement.

The technique chosen for this exercise used a blown nozzle with an external shape typical of the fan cowl shapes to be tested later. The nozzle was mounted on the sting and run quiescently to establish its internal performance on the same pipe work on which it would operate during external flow testing. Thus velocity profile and skin friction effects would be accounted for. During sting drag calibration nozzle internal performance could be deduced from the quiescent

calibration leaving only the model external forces to be evaluated before the sting drag could be quantified. An advantage of this technique is that the jet plume is more realistically represented. However, the method is indirect, depending on 2 separate experiments and a number of calculation steps.

The model chosen for this exercise was a convergent-divergent nozzle with a design pressure ratio of 5, whose boat-tail shape was typical of that of the fan cowl on current high by-pass ratio engines. The internal geometry was selected from among those reported in Reference 3⁴ so that the discharge coefficient was known. Model instrumentation consisted of 4 jet pipe static tapings located 1 diameter upstream from the nozzle entry, 2 tapings in the base and 12 tapings distributed over the boat-tail in a single spiral. From a knowledge of the jet pipe and throat areas, the mean Mach number in the jet pipe could be calculated so that the total pressure could be derived from the static pressure reading. An incorrect assessment of nozzle total pressure here does not affect the calculation of sting drag; applied pressure ratio is used only as the abscissa of the internal thrust calibration curve so that, provided nozzle total pressure is assessed in exactly the same manner when the calibration curve is constructed as when it is used, any total pressure measurement errors will be self cancelling.

Figure 22 illustrates the forces acting on the sting in external flow; under quiescent conditions the boat-tail skin friction and sting drag forces can be regarded as zero. The internal performance obtained from the quiescent tests is shown in Figure 23. External flow tests were conducted at several values of tunnel Mach number and covered a range of nozzle pressure ratios above choking. Boat-tail skin friction force was calculated by an axisymmetric step integration process (see Chapter 2) using the measured pressure distribution, on which the calculation of boat-tail pressure force was also based. These forces, together with the piston force, nozzle base force and nozzle internal thrust as deduced from the calibration curve, were algebraically subtracted from the measured sting plus model force, leaving only the drag of the sting, D_s . The results were expressed in drag coefficient form as shown in Figure 24: although the scatter appears significant, this has not been reflected in tests of high by-pass ratio arrangements. The

figure shows sting drag coefficient to be independent of nozzle applied pressure ratio. It is also independent of tunnel Mach number.

Calibration of the rig air meter was by means of a range of standard convergent nozzles of known vacuum thrust efficiency⁽³⁴⁾, thereby making it dependant on the calibration of the thrust meter. With the rig in the quiescent configuration the standard convergent nozzles were run at pressure ratios above choking to ensure that the air meter pipe Mach number remained constant. Combination of the experimental data and the known values of vacuum thrust efficiency and discharge coefficient from Reference 34 enabled the flow $Q\sqrt{\Theta}$ passing through the air meter to be calculated. The flow function $\frac{Q\sqrt{\Theta}}{AP_t}$ was then calculated using one-dimensional flow theory, from the measured ratio of total to static pressure at the air meter. Thus an effective pipe area at the air meter could be found.

Four standard convergent nozzles of different throat diameter were used, each nozzle being run over a range of pressure ratios above choking. A value of air meter effective area was then deduced from the 4 closely grouped values obtained experimentally. Typical results are shown in Figure 25.

During the sting drag calibration the opportunity was taken to obtain a number of pressure distributions in the tunnel. Static pressure levels were measured on a slat, in a slot, in the plenum chamber surrounding the tunnel and along the sting (as indicated in Figures 26 and 27). Tunnel Mach number was based on slat tapping number 11 which is roughly opposite to the nozzle outlet. Figure 26 shows distributions along a slat over a range of model nozzle pressure ratios, at a nominal tunnel Mach number of 0.85. The results are expressed in terms of local Mach number based on the measured static pressure and the tunnel total pressure - taken as equal to ambient pressure. For the nominally constant tunnel Mach number of 0.85, slat Mach number increases steadily from entry to the model boat-tail region, then decreases rapidly in the vicinity of the nozzle exit before stabilising at the downstream end of the tunnel. Variation in model pressure ratio appears to have little effect at the higher pressure ratios. The differences shown in the figure can be largely attributed to experimental scatter. Later experiments have shown that at lower model

pressure ratios, there is a steady progression towards the nozzle-off curve. Care should be exercised when drawing conclusions from this latter curve; if tunnel Mach number had been controlled on a slat tapping further upstream or downstream the nozzle-off curve would lie close to the other curves with the exception of the model outlet region. Here the absence of the jet must alter the local distribution by effectively introducing a negative blockage effect.

Results from all 4 rows of static tappings are shown superimposed in Figure 27. These results were obtained with a model exhaust pressure ratio of 3.84 and a nominal tunnel Mach number of 0.85. Sting flow is still accelerating in the initial part of the tunnel but eventually levels off to an essentially constant level. Up to the model exit plane, sting surface and slat Mach numbers are reasonably similar and thus a slat tapping near the model exit plane could be used to indicate an average Mach number over the model. All the curves lie in a relatively narrow band and they all show relatively uniform conditions both upstream and downstream from the model exit plane. The change in pressure which is obtained in the region of the model exhaust can be attributed to the rapid change in blockage there.

4.4.2 Model calibration

Model exhaust system performance for the two-stream research model is assessed in terms of the ratio of actual gross thrust-minus-drag to ideal thrust, that is a gross thrust efficiency. Since current high by-pass ratio engines employ convergent nozzles in both the fan and primary streams, the ideal thrust has been defined on a choked convergent nozzle basis.

$$\frac{F_M}{F_{ID}} = \text{model gross thrust efficiency} = \frac{\text{net model force}}{\text{sum of the ideal convergent thrusts of the two streams assuming them to be choked and exhausting separately to free stream static pressure}}$$

Thus it can be seen that the assessment of model performance is divided into 2 distinct parts; the calculation of the ideal and actual thrusts. Consider first the calculation of the ideal thrust.

At subcritical pressure ratios the definition of F_{ID} is based on the actual exhaust pressure ratios.

In the general two-stream case, the ideal choked convergent thrust for the 2 nozzles separately expanding to free stream static pressure can be expressed as

$$F_{ID} = \frac{Q_p V_p}{g} + A_p^* (P_{ep} - P_\infty) + \frac{Q_f V_f}{g} + A_f^* (P_{ef} - P_\infty) \quad \dots (IV,1)$$

On a one-dimensional basis, taking $\gamma = 1.4$, Equation (IV,1) can be reduced to

$$F_{ID} = C_{D_p} A_{g_p} P_{t_p} \left(1.26789 - \frac{1}{EPRP} \right) + C_{D_f} A_{g_f} P_{t_f} \left(1.26789 - \frac{1}{EPRF} \right) \quad \dots (IV,2)$$

It can be seen from these equations that the separate measurement of fan and primary mass flows is required. However, only the total flow to the model is known from the rig air meter so that one of the nozzle flows must be independantly determined before the other can be deduced. It was considered simplest to measure the discharge coefficient of the primary nozzle, leaving the fan nozzle flow to be found by subtraction.

Referring to Figure 13, the flow velocity profile into the primary duct can be expected to be fairly uniform. It was therefore possible to consider using the rig of Reference 34, which has a relatively uniform velocity profile at nozzle entry, to calibrate a scale model of the primary exhaust system. This technique was preferred since flow rate determination from the pitot rake and static pressures at the simulated turbine exit plane would have excluded the flow deficit in the wakes of the spats holding the centrebody. Thus the variation of discharge coefficient with applied pressure ratio was determined using the rig of Reference 34 with a model about 60 per cent larger than the two-stream component. The increase in scale approximately corrected for Reynolds number effect since the early rig was sub-atmospheric.

The total pressure for each stream was defined as that measured at the respective rake planes. This had the effect of including the duct from the rake plane to the nozzle exit as a part of the nozzle. Particular importance was attached to this feature when comparing conventional nozzles with plug nozzles because one of the potential advantages of the plug nozzle is the removal of the diffusing section downstream of the turbine with its attendant losses.

The net model force is the thrust-minus-drag of the complete model. That is to say, it includes the nozzle base forces and the forces on the external cowling. It cannot be measured directly. The thrustmeter measures the net force on the moving rig pipe so that corrections must be made for the other forces before the net model force can be deduced. In external flow, the forces acting on the moving pipe assembly are shown in Figure 10 and the thrust equation can be written:-

$$FM = WT - WP - F1 + P + DS \quad \dots(IV,3)$$

where

FM = net model force

WT = thrustmeter load

WP = pre-load

F1 = supply box leakage force

P = piston force

and

DS = sting drag

Under quiescent conditions (Figure 12), Equation (IV,3) is modified by the deletion of the sting drag (DS) and the addition of the small force (F2) due to leakage flow around the seal plate at the entrance to the tunnel, thus:-

$$FM = WT - WP - F1 + P + F2 \quad \dots(IV,4)$$

Taking unit value for FM, typical values for the forces comprising the thrust balance are given in the following table. External flow values are for a representative cruise condition (2.6 fan exhaust pressure ratio at 0.85 external flow Mach number) whilst those for quiescent operation correspond to the same cruise fan exhaust pressure ratio.

Force	Quiescent	External flow
FM	1.00	1.00
WT - WP	0.60	0.30
F1	0.005	0.005
P	0.40	0.635
F2	0.005	-
DS	-	0.07

The calibration of the floating afterbody model was simplified by the exclusion of the primary stream. The discharge coefficient for the fan nozzle could be obtained directly from the rig air meter and model pressure measurements. The model internal force transducer was calibrated using weights. The calibration was performed with the load cell axis horizontal, as it is in use, and the load applied through a wire which passed over a pulley. The linear bearings were in place so that friction effects would show up as hysteresis. The curve shown in Figure 28 is typical of the results obtained when the load cell was working. However, a certain amount of trouble was experienced with electrical failure within the transducer - reliability was a problem.

The readings of the load cell corresponded to the total force on the floating skin so a correction was required to take account of the internal force. The pressures inside the 2 cavities under the floating skin were measured independently and found to agree but a calibration was needed to determine the effective area of the internal surface. The effective area may differ slightly from the geometric area because of flow effects through the slots. The base plate was replaced by a similar part having a number of $\frac{1}{4}$ inch diameter tubes passing through. Two tests were carried out. First, compressed air was blown into the cavity through the base plate tubes and the readings of the force transducer were recorded for a range of internal pressures. In this test air escaped from both slots. Second, the base plate tubes were connected to a suction pump so that the internal pressure could be lowered - in this test, air leaked in through both slots. Readings of the force transducer were again recorded over a range of internal

pressure. The results from these tests were expressed in terms of effective area and are shown in Figure 29. The small difference between the high and low pressure results reflects the flow effects through the slots. In use, air passes in through 1 slot and out through the other so that it would seem reasonable to use a mean value for the effective area as indicated on the figure. In fact, the difference between the 2 mean levels is far smaller than the basic scatter shown by the experimental points. This scatter is randomly distributed suggesting a basic limitation in the measurements.

4.4.3 A quiescent run

The sting used for this programme had 60 hypodermic 1 mm diameter tubes let into the outer surface. The selected test model was screwed onto the end of the sting and the instrumentation runs were completed by joining the model chassis lines to the sting lines using 1 mm I.D. Portex tubing. After leak tests, the joint region was bandaged with shim steel held in place by self-adhesive tape. The sting was then mounted on the rig and the instrumentation lines were connected to the rig bulkhead using long loops of flexible tubing arranged to give a minimum force in an axial direction on the moving assembly.

The rig was set up with the blanking plate and seal plates at the tunnel entry and the exhaust collector and diffuser, mounted in the tunnel downstream from the model, connected to the silencer - as indicated in Figure 12.

At the start of the run, the air bearings were turned on and the clearance between the sting and seal plates was checked with feeler gauges. The contact light, which is part of an electrical circuit which is completed when the moving part of the rig touches the fixed part, was then turned on and checked. This light usually stayed out throughout the run except for an occasional flicker caused either by traces of water in the bearing air or by sting bounce if the running condition was unsteady.

Air from the compressor hall at about 4 atmospheres pressure was fed to the rig through a silica-gel drier which reduced the frost point to between -30°C and -50°C . The frost point was monitored using a commercial hygrometer; moisture in the air is unlikely to affect the internal performance of these models but condensation effects could

alter the strength of the external shock waves on the afterbody. Pressure in the rig plenum was controlled by a gate valve in the supply line.

At a number of selected values of the fan exhaust pressure ratio, readings of all instruments were recorded, schlieren observations were made and photographs were taken. Readings of the thrust meter were recorded at the start and finish of the reading cycle as a check on rig stability. After the run, the recorded readings were transferred to punched paper tape for subsequent analysis by computer.

4.4.4 A run in external flow

The rig was converted to the external flow mode by replacing the blanking plate by the entry flare, removing the collector/diffuser assembly and connecting the tunnel exhaust to the suction plant; the set-up is indicated in Figure 10. The test procedure was similar to that described in the previous section except that the tunnel flow had to be set and maintained at the desired value for each test point. It proved difficult to set a particular value of Mach number quickly so the experimental programme was arranged to complete tests at a given Mach number value, testing over the desired range of model pressure ratio. Changes in model pressure ratio led to small changes in tunnel Mach number which was adjusted as necessary.

All tests, both under quiescent conditions and in external flow, were repeated on separate days; repeatability was good.

4.5 Test results

Graphical symbols used to differentiate between the various builds and test conditions are illustrated in Figure 30. Circular symbols denote quiescent tests and squares are used for tests with external flow for the floating afterbody model. The type of "fill-in" denotes the fan cowl and the "orientation" denotes the jet simulator configuration.

Figure 31 shows the order in which the tests were done.

4.5.1 Fan nozzle discharge coefficient

Figures 32, 33 and 34 show the measured fan nozzle discharge coefficients plotted against the fan nozzle applied pressure ratio (A.P.R.)* for the 10° , 12° and conical fan cowls respectively, on the

*APPLIED PRESSURE RATIO (A.P.R.) is the ratio of the jet total pressure to the nozzle base pressure which, on simple nozzles, controls the internal flow in the nozzle throat. EXHAUST PRESSURE RATIO (E.P.R.) is the ratio of the jet total pressure to the free stream ambient pressure which controls the final expansion of the jet.

floating afterbody model. The 3 graphs show similar levels, with the results for the conical cowl being slightly lower than the others. External flow leads to a slight decrease in discharge coefficient. This decrease suggests that the internal flow is not solely controlled by the ratio of total pressure to base pressure in this class of nozzle as it is in simple nozzles having no centrebody. The slightly lower value for the conical cowl probably arises from slightly different internal contours.

The fan nozzle discharge coefficient for the two-stream research model is shown plotted in Figure 35. Not only is the curve for external flow on the opposite side of the quiescent line but the level of the curves is about 1 per cent higher. If the floating afterbody model results are to be believed, then the primary nozzle flow in the two-stream model has been underestimated by about 3 percentage points. Such an error is possible because, even when the fan A.P.R. is as high as 2.6, the primary nozzle is not choked; primary flow was estimated from a knowledge of the primary A.P.R., total pressure level and a discharge coefficient curve determined on another model and the nozzle is operating at a condition where the discharge coefficient varies rapidly with pressure ratio.

These results serve to highlight the difficulties of determining the flows from nozzle performance in the two-stream situation.

4.5.2 Measurements on the floating skin

The net force on the floating skin, which was computed by correcting the load cell measurement for the internal force in the cavity, is plotted against fan nozzle E.P.R., (that is the ratio of total pressure to free stream static pressure), in Figures 36 to 39. The force is expressed in non-dimensional form by dividing by the product of the fully expanded jet dynamic head and the afterbody stream-wise projected area. Figure 36 shows the variation of afterbody force with no jet simulator. It has been possible to draw single curves through the points for all 3 fan cowls but there is a significant difference between quiescent and external flow results. In Figure 37, which is for the convergent simulator, the results for the quiescent tests on the 12° cowl stand out from the rest. The results for the parallel simulator, shown in Figure 38, are more complex, the points for the conical fan cowl being different from the rest both under

quiescent and external flow conditions. In this figure, the quiescent results give a higher force coefficient than do the external flow results. Finally, on Figure 39, the results for the divergent simulator can be fitted reasonably well using 2 curves, 1 for quiescent conditions and 1 for external flow.

Considering the end plate as the limit of a convergent simulator, comparison of Figures 36 to 39 shows that the net force on the afterbody decreases as simulator angle increases, even leading to negative values at the lower pressure ratios for the divergent simulator - that is a net force in the thrust direction rather than in the drag sense. The results with the end plate show higher forces in external flow than under quiescent conditions but the results with the simulators tend to show an increasing reduction in force with external flow as the simulator angle increases.

The net force on the afterbody can be divided into 2 components - the integrated pressure force and the skin friction. As discussed in Chapter 2, the integrated pressure force is the sum of the form drag, the wave drag and any buoyancy effects. Figures 40 to 43 show the variation of an integrated pressure force coefficient, defined as the ratio of the pressure force to the product of the fully expanded jet dynamic head and the streamwise projected area, with fan nozzle E.P.R. Figure 40 is for the end plate and Figures 41, 42 and 43 for the convergent, parallel and divergent simulators respectively.

The results with the end plate, shown in Figure 40, indicate that the effect of external flow is to increase the pressure force in the drag sense but that the conical fan cowl gives smaller values than do the profiled cowls. A difference in discharge coefficient was also noted for the conical cowl. Increasing simulator divergence leads to changes in the direction of the pressure force coefficient as shown by Figures 41 to 43. As the pressure force coefficient curves cross the axis, the effect of external flow also changes sign so that the modulus of the coefficient is always larger with external flow than under quiescent conditions.

If the integrated pressure force is subtracted from the total external force, then the remainder will equal the skin friction force. Figures 44 to 46 show plots of the skin friction coefficient based on

the residual skin friction force and the product of the fully expanded jet dynamic head and the skin wetted area versus the fan nozzle exhaust pressure ratio (E.P.R.). Figure 44 is for the 12° fan cowl, Figure 45 is for the conical fan cowl and Figure 46 is for the 10° fan cowl. Each figure exhibits considerable scatter which arises because the skin friction is the small difference of 2 large quantities. Typically, the skin friction is of the order of 1 lb whereas the correction due to the effect of internal pressure is of the order of 20 lb. Comparison of the 3 figures suggests that there is no effect of external flow, no effect of fan cowl external shape and very small, if any, effect of simulator geometry. A reasonable experimental value for the skin friction coefficient is judged to be about 0.0054 - this value is indicated on each figure.

4.5.3 Afterbody pressure distributions

The effect of the various jet simulators on the afterbody pressure distribution is shown in Figure 47. As the effective simulator angle is increased the pressure on the downstream end of the afterbody increases. The shock at the trailing edge moves upstream and strengthens as the boundary layer thickens because of the increasingly adverse conditions downstream.

Figure 48 shows the variation in afterbody pressure distribution as the external flow Mach number is increased. The effect of external flow is to move the first shock on the afterbody downstream, to reduce the strength of the subsequent expansion but to leave the trailing edge pressure virtually unchanged.

Figure 49 illustrates the effect of changing the fan cowl external geometry at an external flow Mach number of 0.8. The slight changes shown indicate that the potential flow buoyancy effects from the 3 cowls must be closely similar. Reference to Figure 41 shows that the pressure force coefficient values calculated for these 3 test points are virtually the same.

Repeatability of the pressure distribution data is shown in Figure 50 where the conditions are quiescent. In this case, one would expect identical curves. The scatter shown probably arises from the fact that the pressure ratio across the fan nozzle is not exactly repeated.

The effect on the distribution of varying the fan nozzle exhaust pressure ratio over a wide range is illustrated in Figure 51. At high pressure ratios the "saw tooth" distribution seen earlier prevails. As the pressure ratio is reduced the number of shocks on the afterbody increases but the strength of these shocks decreases. At subcritical pressure ratios a smooth pressure distribution is obtained.

In Figures 52 and 53 the pressure distributions obtained using the two-stream model are shown superimposed on results from the floating afterbody model at similar exhaust pressure ratios. One of the features which became clear when these curves were prepared was that the convergent jet simulator gave the closest approximation to the correct conditions on the afterbody surface. A more distasteful discovery was that the 2 afterbodies were not exactly similar. The floating afterbody skin had been made shorter than the two-stream afterbody skin by the thickness of the end plate. The two-stream afterbody skin was then cut back to the same length as the floating portion of the floating afterbody model and the tests repeated. The resulting pressure distributions are shown in Figure 54. Fortunately, this slight modification had no significant effect on the results.

The proposition that the convergent jet simulator gave the closest approximation to the two-stream conditions was further explored by plotting the non-dimensionalised trailing edge pressure for each build against fan nozzle exhaust pressure ratio. The resulting plot is shown as Figure 55. This figure clearly shows the effect of varying the initial angle of the jet simulator and indicates that the convergent simulator gives the closest correlation throughout the pressure ratio range. A better correlation might have been obtained had the initial angle of the simulator been increased from 10° to say 15° but this was not explored.

4.5.4 Schlieren observations

Figures 56 to 63 show a selection of schlieren photographs. All of the pictures were taken at a nominal exhaust pressure ratio of 2.5. Each figure shows a number of axial positions along the afterbody obtained by traversing the schlieren system.

Figure 56 shows the flow under quiescent conditions with no simulator fitted. In Picture a) the Prandtl-Meyer expansion fan from the lip of the fan nozzle is clearly visible. The shock wave formed

from the reflection of the expansion waves can be seen part way along the body. The mixing region at the jet boundary is less clearly shown. Picture b) shows the downstream part of the afterbody and can be located with reference to Picture a) by the location of the first shock wave. A second shock wave can be seen close to the trailing edge, but this shock probably does not reach the surface. The appearance near the surface is that of a family of focussing compression waves leading to a focussed shock about a $\frac{1}{4}$ inch out from the surface. Picture c) shows the flow downstream and a third shock can be discerned in the wake region. In interpreting these pictures, it should be recalled that the flow field is axisymmetric - the triangular appearance of the body shocks arises because the schlieren system reacts to the density variations all around the body so that the "vertical" shadow arises from the parts of the shock wave out-of-plane with the body silhouette. In all the pictures shown, the knife edge was vertical.

In Figure 57 the geometry is the same as in Figure 56 but an external flow at 0.8 Mach number has been superimposed. The first body shock appears to have weakened and moved downstream. The second shock is now located at the trailing edge.

Figures 58 and 59 show the patterns obtained with the convergent jet simulator. Comparison between these 2 figures shows the marked effect of external flow in weakening the first body shock and in moving the shocks downstream.

Figure 60 is interesting because Picture b) shows a third shock formed at the afterbody/simulator junction. In Figure 61 this third shock has merged with the second body shock which is now positioned further upstream than on the previous photographs. This upstream movement can only be attributed to a premature thickening of the boundary layer as discussed in the previous section.

Figures 62 and 63 show the divergent simulator under quiescent conditions with 2 different fan cowls. The close similarities between these 2 sets of pictures demonstrates that the flows from the 2 builds are virtually the same. However, it will be recalled that the conical fan cowl gave a slightly lower value for the discharge coefficient.

4.5.5 Overall thrust performance

The variation of the overall thrust coefficient, $\frac{F_M}{F_{ID}}$, with fan nozzle exhaust pressure ratio, E.P.R., is shown in Figures 64 to 67. Figure 64 shows the performance with the end plate fitted. Under quiescent conditions the performance is virtually independent of E.P.R. but with external flow there is a marked decrease in performance as E.P.R. falls. Broadly similar results were obtained using the simulators but the performance level decreases as the simulator angle, and hence base area, increases. With the end plate and the divergent simulator the results in external flow using the 12° fan cowl give a lower performance than the other cowls but with the convergent and parallel simulators it is the 10° fan cowl which exhibits the lower performance. No explanation has been found for this behaviour.

The overall thrust performance for the two-stream model is shown in Figure 68. The form of the results is similar to those reported for the floating afterbody model but the levels are considerably higher. These higher levels reflect the absence of the large primary base which imparts considerable drag to the floating afterbody model. The two-stream model exhibits a virtually constant performance under quiescent conditions and shows the performance dropping as E.P.R. is reduced under external flow conditions.

4.6 Discussion

It has not been possible to compute afterbody drag from the floating afterbody model test results because it has not been possible to quantify the drag of the large primary base. However, comparison of the pressure distributions along the afterbody surface with those obtained on the two-stream model indicates that the flows are closely similar. Thus it should be permissible to apply the force measurements made on the floating afterbody model to the two-stream model. The measured force can be separated into 2 components - the normal pressure force comprising the form drag, wave drag and buoyancy effects, and the skin friction force.

The measured skin friction results have been shown in coefficient form in Figures 44 to 46. These 3 graphs show a scatter band extending over a ± 50 per cent range (standard deviation, σ , 21 per cent). A part

of this scatter is attributed to the fact that the measurement comprises the small difference of 2 large quantities. However, close examination of the results suggests that there may be an effect of the jet simulator geometry. The average skin friction coefficient values obtained in both quiescent and external flow conditions are shown in the table below:-

TABLE 4.1

Skin friction results

Simulator	Average C_f	σ
None	0.005469	0.00124
Convergent	0.005611	0.00118
Parallel	0.005174	0.00110
Divergent	0.005224	0.00099
All	0.005369	0.00114

These results suggest that both the parallel and divergent simulators give rise to a slightly reduced skin friction. This observation is in line with the analysis of the pressure plots and the schlieren photographs which showed the trailing edge shock moving upstream as the simulator angle was increased - this movement being caused by premature thickening, or even separation of the boundary layer thus leading to a reduced skin friction force.

Another contributory factor to the scatter could arise from errors in the pressure force or overall force measurements. There were some variations in overall force coefficient which could not be explained and some of the pressure force coefficient curves behaved in an unexpected manner. Examination of the skin friction results reveals that the measurements obtained with the conical fan cowl, Figure 45, seem to behave in a more regular manner than do the results obtained with the other fan cowls, i.e., the end plate gives consistently high values, the parallel simulator gives consistently low values. No reasonable explanation has yet been found to explain why the conical fan cowl behaves differently from the other 2 cowls.

Comparison of the discharge coefficient results from the two-stream model with those from the floating afterbody model have served to highlight the difficulties in determining the mass flows from nozzle characteristics. Rather recourse should in future be made to separate measurements of the 2 flows, for which the test rig is already equipped. Figures 52, 53 and 55 illustrate that the use of jet simulators is reasonable although the range of simulators tested here did not include the "correct" geometry for this particular application. Extrapolation of the results expressed in Figure 55 suggests that a convergent simulator with an initial angle of about 15° might have given better results.

CHAPTER 5

General discussion and conclusions

5.1 Introduction

Chapter 2 has outlined a revised method of book-keeping needed to adequately describe those two-stream high by-pass ratio aero engine installations which use separate exhaust nozzles for the by-pass and gas generator flows. Methods were presented for estimating the drag of the gas generator afterbody taking into account potential flow buoyancy effects arising from the presence of the fan cowl and the post-exit stream tubes. Theoretical methods were described for estimating the force on the gas generator afterbody from a knowledge of the pressure distribution.

A method for predicting the pressure distribution on the afterbody under quiescent conditions when the fan nozzle exhaust pressure ratio is supercritical has been presented in Chapter 3. An extension to this method to include the effects of a subsonic external flow field has been reported by Young⁽²³⁾.

The design, calibration and use of a new exhaust system test rig has been described in Chapter 4. A selection of the test results obtained when using this rig with 2 exhaust system models has been presented.

It is the purpose of the first part of this chapter to bring together the ideas and observations recorded in the 3 previous chapters and to compare the theoretical predictions with the experimental observations. A general discussion of the place of this work in the overall scene then follows and the chapter finishes with some concluding remarks.

5.2 Pressure distributions

An attempt to illustrate the importance of the potential flow buoyancy effect has been made in Figures 40 to 43. However, the 3 different fan cowls employed in the test programme had to fit on a common sting so that the maximum diameter of each cowl was fixed. In order to maintain the internal geometry of the fan nozzle, since it was not intended that nozzle geometry should be among the variables, the nozzle lip diameter for each cowl had to be the same. Thus each fan

cowl had the same projected area and the test results suggest that each cowl produced essentially the same buoyancy effect.

The results do show most clearly the very significant effect of changes in the jet simulator shape in changing the static pressure level towards the aft end of the afterbody. This effect is brought about less by buoyancy changes than by viscous effects. The flow over the afterbody is essentially supersonic at cruise pressure ratios so, at first sight, it might seem surprising that downstream geometrical changes can lead to such large changes in the pressure force on an upstream component. However, the changes in local static pressure are confined to the last 30 per cent of the afterbody length and it is in this region, where the boat-tail angles are highest, that most of the projected area lies. These changes in local static pressure are caused by the variations in flow turning at the afterbody/simulator junction which lead to changes in shock strength and hence to changes in downstream pressure which feed upstream through the subsonic parts of the boundary layer. The displacement thickness of the boundary layer increases ahead of the downstream pressure rise thus causing the shock to move upstream softening the effect of the flow turning by smoothing out the pressure rise. It is this anticipation of the pressure rise by the boundary layer which leads to the changes in static pressure a considerable distance upstream.

The ability of the boundary layer to anticipate downstream effects leads to a problem both with the skin friction predictions described in Chapter 2 and with the displacement thickness calculation used in the characteristics solution described in Chapter 3. Examination of Figure 69 shows that the agreement between the characteristics solution and experiment is good up to the end of the first pressure rise at $x/L = 0.6$ and is still reasonable through the subsequent expansion up to $x/L = 0.75$. However, the theoretical method fails to predict the pressure rise in the region where the boundary layer anticipates the flow conditions at the trailing edge.

This behaviour can only be expected because the displacement thickness prediction method was based on a downstream stepping process which, while it could allow for upstream history, could have no knowledge of downstream effects. Indeed this criticism could be levelled at most boundary layer prediction techniques. The same problem has

been encountered by Maria-Sube et al⁽³⁵⁾ who used a modern "lag-entrainment" method presented by Green, Weeks and Brooman in Reference 36. Maria-Sube predicted the boundary shape of a jet exhausting into a compressible external flow. The method worked well except for a region near the trailing edge of the afterbody where the theory predicted local pressures which were significantly lower than those observed in practice. The only way of overcoming this problem would seem to be to make a completely new start basing the theoretical approach on the new time-dependent methods currently under development. Such an approach would have the advantage of dealing equally well with both the subsonic and supersonic parts of the flow but it may still need a 'fudge' to deal effectively with shock waves.

The extension of the Chapter 3 work reported by Young in Reference 23 shows that it is possible to predict the trends in pressure distribution brought about by the introduction of an external flow field. The weakening of the shock part way along the body and its movement downstream are adequately predicted but the method still fails to correctly predict the pressure at the trailing edge. Young makes the observation, which must be echoed by the present author, that "... although the pressure distribution is not accurate enough for estimating drag forces it could be useful in initial design studies".

5.3 Skin friction

Comparisons between the measured skin friction force and values predicted from the measured pressure distributions using Equation (II,25) are shown for quiescent conditions in Figure 70 and for external flow in Figure 71. These graphs show that the modified flat plate prediction method underestimates the skin friction by an amount which increases as the mean jet Mach number increases, though the agreement is fair at low Mach numbers.

It was considered possible that the predictions might be improved if Equation (II,25) were modified by including an additional factor containing Stratford's X. A linear correlation analysis relating the ratio of the measured and the predicted skin friction forces to the ratio of Stratford's X with the geometrical length of the afterbody gave a correlation coefficient of 0.039. With such a low correlation coefficient it is highly unlikely that any sensible relationship between the 2 ratios exists. Thus any attempt to improve the

predictions by using a simple combination of Equation (II,25) and Stratford's X must fail.

On the basis of the experimental results the best approach to the prediction problem would be to use the mean measured skin friction coefficient of 0.0054. This approach would predict the skin friction force to within about ± 20 per cent and, noting that the skin friction force represents about 1 per cent of the ideal gross gauge thrust, the resulting error in predicted performance would be of the order of ± 0.2 per cent on a gross thrust basis. Indeed, if the analysis of the experimental data were restricted to those results which were obtained with the convergent jet simulator under quiescent conditions, then the data exhibit a standard deviation of 11 per cent about a mean value for the coefficient of 0.0053. Thus the contribution to the error in overall performance due to skin friction predicted in this way is likely to be only about 0.1 per cent of the ideal gross thrust under engine test cell conditions.

5.4 Drag predictions

The example calculations given in Appendix B can now be repeated using the revised skin friction coefficient approach. For these revised calculations a skin friction coefficient of 0.0053 has been used and the wetted area of the afterbody has been computed at 20.5 square inches.

The equation for the afterbody skin friction force now becomes:-

$$F_{\tau} = C_f \cdot \frac{\gamma}{2} \cdot P_{\infty} \cdot M_j^2 \cdot A_{wet} \quad \dots(V,1)$$

In Example 1 we have

$$P_{tf} = 48.28 \text{ in.Hg abs}$$

$$P_{\infty} = 18.63 \text{ in.Hg abs}$$

so that $M_j = 1.2505$

and $F_{\tau} = 1.0846 \text{ lb}$

Since Equation (II,25) was shown to be in reasonable agreement with experiment at low Mach numbers, the calculations for the other factors need not be changed so that:-

$$F_a = 0.9982 \text{ lb}$$

$$D_a = 1.2972 \text{ lb}$$

and $\frac{D_a}{F_{ID}} = 0.0091$

Similarly, in Example 2,

$$M_j = 1.124$$

$$F_{\tau} = 0.879 \text{ lb}$$

$$F_a = 0.786 \text{ lb}$$

$$D_a = 0.291 \text{ lb}$$

and $\frac{D_a}{F_{ID}} = 0.00259$

Finally, in Example 3,

$$M_j = 0.949$$

$$F_{\tau} = 0.627 \text{ lb}$$

$$F_a = -0.301 \text{ lb}$$

$$D_a = 0.386 \text{ lb}$$

and $\frac{D_a}{F_{ID}} = 0.0054$

These results all show a significant increase in the computed afterbody drag when compared with the results given in Appendix B. At first sight these revised values would seem to be reasonable: the afterbody drag is calculated to lie in the range from $\frac{1}{4}$ to 1 per cent of the ideal gross gauge thrust. However, both Examples 2 and 3 give values for D_a which are less than F_{τ} . Clearly this result is untenable. Either the estimates of the cowl force or drag are in error or there could be a significant thrust recovery. While divergent thrust recovery is possible under the conditions of Example 2, no divergent thrust recovery should be observed in Example 3 since, under these conditions with the definition of V_j chosen, the afterbody should be debited with any failure to achieve full thrust recovery. This apparent anomaly might be taken to suggest that the methods for

predicting the force or drag of the cowl are not yet fully adequate. Indeed, a possible explanation for this anomaly might lie in the fact that the boundary layer on the cowl does not start at the front of the model. The boundary layer starts in the tunnel entrance and grows along the sting so that conditions at the front of the cowl are not those assumed in the prediction method.

However, the main contributory factor is thought to be that the efflux from the fan nozzle is not axial. At the fan cowl lip the flow is directed inwards towards the centreline at 10° . At the nozzle throat the afterbody surface is inclined inwards at 2° . Therefore the flow has to be turned through an average angle of about 6° , giving rise to a force, arising from the cosine effect, of about $\frac{1}{2}$ per cent of the fan stream gross thrust. Since, in these cold flow models, the fan stream comprises about three-quarters of the total flow, this effect would increase the computed afterbody drag by about 0.38 per cent of gross thrust which would be sufficient to remove the anomaly. However, it is not possible to quantify this factor exactly with the information available and further work is indicated.

5.5 General discussion

One unsought advantage of spending a long time completing this Thesis has been that it has proved possible to make some judgement of the value of some parts of the work. Following the author's departure from Engine Research Department, Mr. A. R. Seed modified the theoretical computer program by changing the boundary conditions at the afterbody surface. Instead of specifying the location and direction of the boundary, he specified the pressure distribution, thus inverting the problem. This new version of the program is thus able to design surfaces that will produce the specified pressure distribution. The program still contains approximate procedures since it deals with shocks and subsonic patches in exactly the same way as the original program. Use of this "design" program led to the development of a range of conical afterbodies. Following from this work, the adoption of a conical afterbody on a current commercial high by-pass ratio turbo-jet gave an improvement in installed performance worth about $1\frac{1}{4}$ per cent of net thrust. Attention to detail around the primary base and the deletion of the fairings covering the hot jet spoiler mechanism by the engine manufacturer yielded another $1\frac{1}{4}$ per cent. Testing of the

modified installation on an aircraft later revealed an apparent favourable wing interference effect worth another $2\frac{1}{2}$ per cent of net thrust. Thus the overall saving is about 5 per cent on installed specific fuel consumption. These significant advances followed directly from the work described in Chapter 3. It is to be hoped that Swann⁽⁵⁾ was right and that the commercial success of the aircraft will thus be made secure.

The work described in Chapter 2 resulted in this author being chosen to lead a sub-committee writing a chapter of the report of a joint U.K. Aero Industry/MoD(PE) working party which is currently addressing the problem of thrust measurement in flight. This chapter deals with book-keeping systems and the fundamentals of thrust and drag.

The test rig described in Chapter 4 continues to be used both for research projects and directly in support of Industry. The operating techniques have been further refined so that the scatter in gross thrust-minus-drag measurement is now of the order of 0.2 per cent. One of the features of the rig which has caused some concern is the problem of adequately defining the ambient static pressure in the wind tunnel. Attempts to improve the flow distribution within the tunnel by modifying the slots have not been successful.

5.6 Concluding remarks

1. Past methods for analysing and predicting the performance of those high by-pass ratio turbo-fan aero engines which use separate exhaust nozzles for the by-pass and gas generator flows have not been satisfactory.
2. No method has been available for predicting or measuring the drag, as distinct from the force, acting on the gas generator cowlings which is immersed in the efflux from the fan nozzle.
3. This work has highlighted the important differences between the drag and the force acting on a part of a body. A method of analysing the two-stream aero engine installation has been developed which properly accounts for potential flow buoyancy effects and which enables the component drags to be isolated.

4. A method of predicting the pressure distribution on the gas generator cowl when the fan nozzle pressure ratio is supercritical has been developed. This computer program combines the prediction of the potential flow field using the method of characteristics with a prediction of the surface boundary layer.
5. The computer program treats shock waves and subsonic patches in the flow field by approximate methods. These approximations lead to progressive errors which propagate downstream. Prediction of the boundary layer by a downstream stepping process has been shown to be inadequate since downstream disturbances have been shown to propagate a significant distance upstream.
6. This program adequately predicts the flow over the forward part of the cowl but fails to properly predict the pressure rise at the aft end. It has been suggested that the only way of overcoming this failure would be to employ new techniques based on the time-dependent solutions to the flow equations which are currently being developed.
7. This program has proved useful as a coarse sieve. Designs which prove difficult to analyse because of the presence of strong shock waves or of extensive subsonic patches are unlikely to have a high performance.
8. Modifications to this characteristics program have enabled other workers
 - a) to develop a method for predicting the effect of an external flow on the gas generator cowl pressure distribution,
 - and b) to write a program to design afterbody shapes of improved performance.
9. A rig to test scale models of high by-pass ratio exhaust systems has been designed, built and commissioned. Careful calibration of the rig has led to a high degree of confidence in the results from comparative tests. Some uncertainty about the correct value to ascribe to the ambient static pressure reduces confidence in the absolute magnitude of results obtained in external flow tests.

10. Two scale models of the exhaust system of a high by-pass ratio engine installation have been constructed and tested. One of these models incorporated the 2 exhaust streams and was designed to undertake comparative tests of alternative afterbody shapes. The other model was designed so that force measurements could be made on the gas generator cowl. The provision of bearings and a force measuring transducer precluded inclusion of the gas generator exhaust flow in this second model. The effects of the gas generator exhaust flow were simulated in some tests by plume simulators.

Tests on the 2 models have enabled the upstream part of the characteristics predictions to be confirmed. Schlieren observations have confirmed the structure of the flow field.

11. Tests on the floating afterbody model have yielded data on the skin friction forces to be found in this type of installation. The results show the skin friction coefficient to be independent of the pressure ratio across the fan nozzle and of the external flow conditions. A value for the skin friction coefficient of 0.0053 has been found from tests with a convergent plume simulator. The error from using this value in performance predictions for scale models is thought to be less than 0.2 per cent of the ideal gross gauge thrust. Under cruise conditions the Reynolds number based on afterbody length and mean jet Mach number was about 6×10^6 for these models, a value about a quarter of that relevant to the full-scale installation. Since, at these values of Reynolds number, the boundary layer is well into the turbulent region, it would not be unreasonable to use the above value of skin friction coefficient for predictions at full scale.

12. The foregoing discussion indicates 3 main areas where further work is required.

The first area is concerned with the book-keeping system where methods to correctly allow for non-axial force components and asymmetry in the flow need to be developed. In addition the most relevant way of calculating conditions at downstream infinity - not only to give a representative drag value but also to correctly depict the thrust - must be determined.

The second area is the need for a new approach to the theoretical flow prediction problem to allow for the upstream propagation of downstream disturbances through the subsonic part of the boundary layer. Techniques based on time dependent solutions to the flow equations have been suggested.

The third area is concerned with development of the experimental techniques so that floating afterbody force measurements can be effectively combined with overall thrust-minus-drag measurements with the primary flow present. Buoyancy effects from the primary jet would then be correctly simulated and the drag of the afterbody could be deduced using the methods of Chapter 2. This will probably require larger models than have been used in the present exercise. The use of larger models would be advantageous in that it should then be possible to increase the test Reynolds number to even more representative levels.

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NOTATION

A	projected area
A _{wet}	wetted area of a surface immersed in a flow
A.P.R.	applied pressure ratio = $\frac{\text{nozzle entry total pressure}}{\text{base pressure}} \cdot \frac{P_T}{P_b}$
A.P.R.F.	fan stream applied pressure ratio $\frac{P_{tf}}{P_{ef}}$
A.P.R.P.	primary stream applied pressure ratio $\frac{P_{tp}}{P_{ep}}$
C _d	drag coefficient
C _D	discharge coefficient
C _f	skin friction coefficient
D	drag force
D _a	afterbody drag-minus-thrust recovery
D _f	skin friction drag force
D _p	pressure (or form) drag force
D _s	sting drag
E.P.R.	exhaust pressure ratio = $\frac{\text{nozzle entry total pressure}}{\text{free stream static pressure}} \cdot \frac{P_T}{P_\infty}$
E.P.R.F.	fan stream exhaust pressure ratio $\frac{P_{tf}}{P_\infty}$
E.P.R.P.	primary stream exhaust pressure ratio $\frac{P_{tp}}{P_\infty}$
F, f	force
F1, F2	leakage forces
F _{ID}	gross ideal convergent thrust of two streams separately expanded
FM	net model force
g	acceleration due to gravity

$\bar{H}$	transformed shape parameter
L	afterbody length
M	number of initial points
Mn	Mach number
M_{SURF}	Mach number at the surface
N	number of characteristics in lip fan, distance along normal
P	pressure, piston force
$\bar{P}$	function of Mach number $\bar{P} = (Mn/(1 + 0.2 Mn^2))^4$
Q	mass flow rate
Rx	Reynolds number based on X
R_∞	Reynolds number referred to conditions at edge of boundary layer
r	radius
S	distance along surface
T,t	thrust
U,V	velocity
W_P	preload force
W_T	thrust meter force
X	equivalent length of boundary layer growth
	$X = \left(\bar{P}^a \right)^{-1} \int_0^x \bar{P}^a dx$
X_n	net thrust-minus-drag
y	distance from surface normal to axis
α	exponent in equivalent length formula ($\alpha = 1.2$)
β	angle of surface relative to axis
γ	ratio of specific heats ($\gamma = 1.4$)
δ^*	boundary layer displacement thickness, $\delta^* = \int_0^\infty \left(1 - \frac{\rho U}{\rho_e U_e} \right) dy$

θ	absolute temperature
θ	boundary layer momentum thickness, $\theta = \int_0^{\infty} \frac{\rho U}{\rho_e U_e} \left(1 - \frac{U}{U_e}\right) dy$
λ	flow direction relative to axis
μ	Mach angle
ν	kinematic viscosity
ξ	skin friction function = $\frac{r P_e M_{n_e}^2 \left(1 - \frac{1.12}{\log_{10} Rx}\right)}{\left(\log_{10} Rx\right)^{2.58} \left(1 + \frac{M_{n_e}^2}{5}\right)^{0.467}}$
ρ	density
σ	standard deviation
τ	shear stress
ψ	$P_L - P_{\infty}$
ω	Prandtl-Meyer angle
ω_c	total Prandtl-Meyer turn in lip fan

SUBSCRIPTS

a	afterbody
b	base
c	cowl boat-tail
e	edge of boundary layer
ext	external
ef	fan exit plane
ep	primary exit plane
f	fan stream
I	intrinsic
i	on first wave of lip fan (initial value), intake

SUBSCRIPTS (cont'd)

int	internal
jf	fan stream at downstream infinity
jp	primary stream at downstream infinity
L	local static
p	primary stream, profile
post	post exit streamline
pre	pre-entry streamline
t	stagnation
TE	trailing edge
w	wall, wave
τ	skin friction
∞	free stream static
ϕ	friction

SUPERSCRIPTS

*	at sonic conditions
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APPENDIX A

Drag components

In two-dimensional flow we can write:-

$$\text{Pressure drag on a complete body} = \int_0^{\infty} (P_L - P_{\infty}) \frac{dy}{dx} dx \quad \dots(A1)$$

and assuming δ^* is measured normal to the 'x' axis,

$$\text{Pressure drag on displacement surface} = \int_0^{\infty} (P_L - P_{\infty}) \frac{d(y + \delta^*)}{dx} dx \quad \dots(A2)$$

so that:-

(the pressure drag on the displacement surface) minus

$$\text{(the pressure drag on the body)} = \int_0^{\infty} (P_L - P_{\infty}) \frac{d\delta^*}{dx} dx \quad \dots(A3)$$

The drag of a complete body in isolation in potential flow is zero, so that the drag of the displacement surface is zero. Thus we can write Equation (A3) as:-

$$0 - \text{body pressure drag} = \int_0^{\infty} (P_L - P_{\infty}) \frac{d\delta^*}{dx} dx \quad \dots(A4)$$

so that

$$\text{Body pressure drag} = \text{form drag} = - \int_0^{\infty} (P_L - P_{\infty}) \frac{d\delta^*}{dx} dx \quad \dots(A5)$$

The above derivation can be transposed to axisymmetric flow to give:-

$$\text{Form drag, } D_p = - \int_0^{\infty} 2\pi r \left(P_L - P_{\infty} \right) \frac{d\delta^*}{dx} dx \quad \dots(A6)$$

Alternatively, working from the boundary layer momentum equation (26), the pressure drag due to boundary layer growth can be derived as:-

$$D_p = \int_0^{\infty} 2\pi r \delta^* \frac{dP_L}{dx} dx \quad \dots(A7)$$

which is the same as Equation (A6) since, in general

$$\int y dx = - \int x dy \quad \dots(A8)$$

Skin friction

Skin friction arises from tangential shear stress at the surface of the body. If shear stress is represented by τ_w , then the local shear force on the body is $\tau_w dS$ and the axial component is $\tau_w dS \cos \beta$, where β is the angle of the surface relative to the axis.

$$\text{Now,} \quad dS \cos \beta = dx \quad \dots(A9)$$

so that the axial component of local shear force is $\tau_w dx$ and the total drag in two-dimensional flow due to skin friction is $\int_0^x \tau_w dx$.

In axisymmetric flow, the skin friction drag D_f can be written:-

$$D_f = \int_0^x 2\pi r \tau_w dx \quad \dots(A10)$$

Profile drag

Profile drag, sometimes known as boundary layer drag, is the total drag due to boundary layer growth and thus equals the sum of the skin friction and form drags.

$$\text{Profile drag, } D = \int_0^x 2\pi r v_w dx - \int_0^\infty 2\pi r \left(P_L - P_\infty \right) \frac{d\delta^*}{dx} dx \quad \dots(A11)$$

Wave drag

When the flow over the afterbody is supersonic, the jet efflux will contain shock waves which, through the loss in total pressure, give rise to drag. The flows under consideration differ from those pertaining to supersonic aircraft because the flow over the afterbody is a supersonic jet surrounded by a subsonic flow field, rather than an infinite supersonic flow field. By way of explanation of the difference, first consider the boat-tail of a long fuselage in a supersonic flow field with no boundary layers. As the flow expands round the boat-tail, the velocity will increase - leading to sub-ambient pressures on the boat-tail surface. At the end of the boat-tail, the flow is suddenly straightened by a shock wave which extends out to infinity in the radial direction. The wave drag of the boat-tail is equal to the loss of momentum through this single infinite shock, which will equal the pressure integral over the boat-tail. Thus:-

$$D_w = - \int_A \left(P_L - P_\infty \right) dA \quad \dots(A12)$$

In the case of the front fan nacelle afterbody, instead of the single shock extending out to infinity, neglecting viscous effects at the jet boundary, there are an infinite number of bounded shocks in the jet. The momentum loss in this infinite number of shocks is the wave drag, which will in the absence of buoyancy effects again equal the pressure integral over the afterbody. In the presence of a boundary layer, the wave drag is the wave drag of the displacement surface.

APPENDIX B

Example calculations

1.0 Definition of geometry

The example calculations given in this Appendix are for a model exhaust system under test in a wind tunnel. The fan cowl geometry is given in Table B1 and the afterbody geometry is given in Table B2. The primary nozzle exit area, A_{ep} , is 1.755 square inches and the fan nozzle exit area, A_{ef} , is 5.348 square inches.

2.0 Example 1 - Supercritical pressure ratios

2.1 Readings

Tunnel	$P_t = 29.89$ in.Hg abs, $\Theta_t = 288^\circ\text{K}$
Fan stream	$P_{tf} = 48.28$ in.Hg abs, $\Theta_t = 291^\circ\text{K}$
Primary stream	$P_{tp} = 47.857$ in.Hg abs, $\Theta_t = 291^\circ\text{K}$
Fan nozzle base	$P_{bf} = 19.433$ in.Hg abs
Primary nozzle base	$P_{bp} = 22.187$ in.Hg abs
Free stream static	$P_\infty = 18.630$ in.Hg abs

so that

$$\text{A.P.R.F.} = 2.484$$

$$\text{E.P.R.F.} = 2.592$$

$$\text{A.P.R.P.} = 2.157$$

$$\text{E.P.R.P.} = 2.569$$

$$\text{Mn}_\infty = 0.851$$

Conversion factor at 21.5°C :- 2.0440 in.Hg = 1 psi.

Pressure distributions

<u>Fan cowl</u>		<u>Afterbody</u>	
Static No.	P _L in.Hg abs	Static No.	P _L in.Hg abs
1	18.72	1	28.33
2	18.02	2	21.78
3	17.60	3	17.30
4	17.40	4	14.20
5	17.20	5	14.47
6	17.22	6	15.81
7	17.60	7	20.70
8	17.60	8	21.58
9	18.50	9	20.00
10	18.69	10	16.10
11	20.12	11	20.50
12	20.12		

$$\left(Q \sqrt{\Theta_t}\right)_{\text{FAN}} = 49.094$$

$$\left(Q \sqrt{\Theta_t}\right)_{\text{PRIM}} = 15.836$$

$$F_{\text{ID}} = 144.282 \text{ lb}$$

A full tabulation of the steps in calculating F_c , D_c and F_a are given in this example only.

2.2 Calculation of F_c

$$F_c = 0.6608\pi \int_0^x \frac{r P_L M_n^2 \left(1 - \frac{1.12}{\log_{10} Rx}\right)}{\left(\log_{10} Rx\right)^{2.58} \left(1 + \frac{M_n^2}{5}\right)^{0.467}} dx - \int_A \left(P_L - P_\infty\right) dA$$

...(II,26)bis

Let

$$\frac{r P_L M_n^2 \left(1 - \frac{1.12}{\log_{10} Rx}\right)}{\left(\log_{10} Rx\right)^{2.58} \left(1 + \frac{M_n^2}{5}\right)^{0.467}} = \xi$$

and
$$(P_L - P_\infty) = \psi$$

then the tabulation is as shown in Table B3.

$$\int_A \psi \, dA = \int_A (P_L - P_\infty) \, dA = -0.446 \, \text{lb} \quad \text{i.e. a drag}$$

$$0.6608\pi \int_0^x \xi \, dx = 2\pi \int_0^x r \, \tau \, dx = 0.974 \, \text{lb}$$

so that
$$F_c = 1.420 \, \text{lb}$$

2.3 Calculation of D_c

$$D_c = 2.8\pi \, r_{TE} \, \theta_{TE} \, P_\infty \, Mn_\infty^2 \left(\frac{Mn_{TE}}{Mn_\infty} \right)^{3.2} \left(\frac{1 + \frac{Mn_\infty}{5}}{1 + \frac{Mn_{TE}}{5}} \right)^{4.6}$$

...(II,35)bis

and
$$\theta_{TE} = 0.036 \left(1 + \frac{Mn_{TE}^2}{10} \right)^{-0.70} \times Re^{-\frac{1}{5}}$$

...(II,34)bis

Thus, assuming trailing edge conditions are represented by static number 12, we get

$$D_c = 1.1213 \, \text{lb}$$

2.4 Calculation of F_a

Following Section 2.2 of this Appendix we get the tabulation for F_a shown in Table B4.

$$\int_A \psi \, dA = \int_A (P_L - P_\infty) \, dA = 0.0864 \, \text{lb} \quad \text{i.e. a thrust}$$

$$0.6608\pi \int_0^x \xi \, dx = 2\pi \int_0^x r \, \tau \, dx = 0.7037 \, \text{lb}$$

so that $F_a = 0.617 \, \text{lb}$

In this example, all pressure ratios are supercritical (1.89) so that

$$\begin{aligned} D_a &= F_a + F_c - D_c \\ &= 0.617 + 1.420 - 1.121 \\ &= 0.916 \, \text{lb} \end{aligned}$$

and $\frac{D_a}{F_{ID}} = 0.0064$

3.0 Example 2 - Intermediate pressure ratio

For the second example, a case where the primary nozzle pressure ratios are on either side of critical has been chosen.

Tunnel	$P_t = 29.880 \, \text{in.Hg abs}, \Theta_t = 288^\circ\text{K}$
Fan stream	$P_{tf} = 41.108 \, \text{in.Hg abs}, \Theta_t = 293^\circ\text{K}$
Primary stream	$P_{tp} = 40.797 \, \text{in.Hg abs}, \Theta_t = 293^\circ\text{K}$
Fan nozzle base	$P_{bf} = 19.34 \, \text{in.Hg abs}$
Primary nozzle base	$P_{bp} = 22.453 \, \text{in.Hg abs}$
Free stream static	$P_\infty = 18.68 \, \text{in.Hg abs}$

so that

$$A.P.R.F. = 2.126$$

$$E.P.R.F. = 2.201$$

$$A.P.R.P. = 1.817$$

$$E.P.R.P. = 2.184$$

$$M_{n_{\infty}} = 0.848$$

Conversion factor at 21°C:- 2.0438 in.Hg = 1 psi.

Pressure distributions

<u>Fan cowl</u>		<u>Afterbody</u>	
Static No.	P _L in.Hg abs	Static No.	P _L in.Hg abs
1	18.75	1	24.69
2	18.13	2	19.61
3	17.66	3	17.83
4	17.39	4	17.06
5	17.21	5	18.27
6	17.23	6	19.20
7	17.61	7	18.02
8	17.61	8	16.88
9	18.43	9	20.16
10	18.63	10	19.11
11	19.87	11	21.43
12	19.96		

$$\left(Q \sqrt{\theta_t} \right)_{FAN} = 41.767$$

$$\left(Q \sqrt{\theta_t} \right)_{PRIM} = 13.287$$

$$F_{ID} = 112.817 \text{ lb}$$

The integrations give:-

$$\begin{aligned}
 \text{Pressure force on fan cowl} &= -0.722 \\
 \text{Skin friction force on fan cowl} &= 0.975 \\
 F_c &= 1.697 \\
 D_c &= 1.119 \\
 \text{Pressure force on afterbody} &= 1.119 \\
 \text{Skin friction force on afterbody} &= 0.606 \\
 F_a &= 0.513
 \end{aligned}$$

The fan stream is supercritical so there is no change of momentum to be taken into account

$$\begin{aligned}
 \left(Q \sqrt{\Theta_t} \right)_P \left(1.85501 - \left(\frac{v}{g \sqrt{\Theta_t}} \right)_{APRP} \right) &= 0.728 \\
 P_{tp} A_{ep} \left\{ \frac{1}{APRP} - \left(1 - \frac{A_p^*}{A_{ep}} \right) \frac{1}{EPRP} - 0.5283 \frac{A_p^*}{A_{ep}} \right\} &= 0.775
 \end{aligned}$$

whence $D_a = 0.018$

and $\frac{D_a}{F_{ID}} = 0.00016$

4.0 Example 3 - Subcritical pressure ratios

Tunnel	$P_t = 29.87 \text{ in.Hg abs, } \Theta_t = 288^\circ\text{K}$
Fan stream	$P_{tf} = 33.395 \text{ in.Hg abs, } \Theta_t = 291^\circ\text{K}$
Primary stream	$P_{tp} = 33.169 \text{ in.Hg abs, } \Theta_t = 291^\circ\text{K}$
Fan nozzle base	$P_{bf} = 19.33 \text{ in.Hg abs}$
Primary nozzle base	$P_{bp} = 21.35 \text{ in.Hg abs}$
Free stream static	$P_\infty = 18.70 \text{ in.Hg abs}$

so that

$$A.P.R.F. = 1.727$$

$$E.P.R.F. = 1.786$$

$$A.P.R.P. = 1.553$$

$$E.P.R.P. = 1.774$$

$$Mn_{\infty} = 0.846$$

Conversion factor at 20°C:- 2.0434 in.Hg = 1 psi.

Pressure distributions

<u>Fan cowl</u>		<u>Afterbody</u>	
Static No.	P _L in.Hg abs	Static No.	P _L in.Hg abs
1	18.68	1	20.92
2	18.00	2	19.60
3	17.63	3	18.65
4	17.40	4	18.68
5	17.28	5	18.32
6	17.28	6	18.32
7	17.58	7	18.55
8	17.62	8	18.80
9	18.28	9	19.20
10	18.60	10	19.62
11	19.48	11	20.69
12	19.68		

$$(Q\sqrt{\Theta_t})_{FAN} = 33.265$$

$$(Q\sqrt{\Theta_t})_{PRIM} = 10.292$$

$$F_{ID} = 77.655$$

The integrations give:-

$$\text{Pressure force on fan cowl} = -1.050$$

$$\text{Skin friction force on fan cowl} = 0.979$$

$$F_c = 2.029$$

$$D_c = 1.119$$

$$\text{Pressure force on afterbody} = 0.928$$

$$\text{Skin friction force on afterbody} = 0.457$$

$$F_a = -0.472$$

Now both streams are subcritical, so we compute thrust changes on Jones' basis.

$$\frac{V_{jf}}{g\sqrt{\Theta_t}} = 1.786 \qquad \frac{V_{jp}}{g\sqrt{\Theta_t}} = 1.776$$

$$\frac{V_{ef}}{g\sqrt{\Theta_t}} = 1.738 \qquad \frac{V_{ep}}{g\sqrt{\Theta_t}} = 1.571$$

$$(Q\sqrt{\Theta_t})_f \left(\frac{V_{jf}}{g\sqrt{\Theta_t}} - \frac{V_{ef}}{g\sqrt{\Theta_t}} \right) = 1.610$$

$$(Q\sqrt{\Theta_t})_p \left(\frac{V_{jp}}{g\sqrt{\Theta_t}} - \frac{V_{ep}}{g\sqrt{\Theta_t}} \right) = 2.112$$

$$(A_{ep}) (P_{bf} - P_\infty) = 1.666$$

$$(A_{ef}) (P_{bp} - P_\infty) = 2.279$$

whence

$$D_a = 0.215$$

and

$$\frac{D_a}{F_{ID}} = 0.003$$

APPENDIX C

Derivation of compatibility relations

(Vide Reference 19)

Characteristics are mathematically defined as lines, or surfaces along which the partial differential equations of the flow field reduce to ordinary differential equations. Alternatively, they may be defined as lines or surfaces along any one of which known values of the flow are insufficient to determine the flow in any other region.

The compatibility relations are equations relating the flow properties which hold along characteristics of the flow field. They may be derived from the equations of irrotationality and mass continuity as follows:-

(1) Equation of irrotationality

$$\frac{\partial \lambda}{\partial S} - \frac{1}{\sqrt{Mn^2 - 1}} \cdot \frac{\partial \omega}{\partial N} = 0 \quad \dots (C1)$$

(2) Equation of continuity

From Figure 72(a) the area of the annulus is

$$A = 2\pi r \cdot \Delta N$$

Hence

$$\frac{dA}{dS} = 2\pi r \cdot \frac{d(\Delta N)}{dS} + 2\pi \cdot \Delta N \cdot \frac{dr}{dS}$$

and

$$\frac{1}{A} \frac{dA}{dS} = \frac{1}{\Delta N} \cdot \frac{d(\Delta N)}{dS} + \frac{1}{r} \cdot \frac{dr}{dS}$$

But from Figure 72(b)

$$\begin{aligned} \frac{d(\Delta N)}{dS} &= \lim_{\Delta S \rightarrow 0} \frac{(\Delta N_2 - \Delta N_1)}{\Delta S} \\ &= \lim_{\Delta S \rightarrow 0} \frac{\frac{\partial \lambda}{\partial N} \cdot \Delta N \cdot \Delta S}{\Delta S} \\ &= \frac{\partial \lambda}{\partial N} \cdot \Delta N \end{aligned}$$

so that
$$\frac{1}{A} \frac{dA}{dS} = \frac{\partial \lambda}{\partial N} \frac{1}{r} \cdot \frac{dr}{dS}$$

i.e.,
$$\frac{dA}{A} = \frac{\partial \lambda}{\partial N} \cdot dS + \frac{dr}{r}$$

But
$$dr = dS \cdot \sin \lambda$$

and therefore
$$\frac{dA}{A} = \frac{\partial \lambda}{\partial N} \cdot dS + \frac{\sin \lambda}{r} \cdot dS \quad \dots (C2)$$

The continuity equation is

$$\frac{dA}{A} + \frac{1}{\rho V} \frac{\partial}{\partial S} (\rho V) \cdot dS = 0 \quad \dots (C3)$$

Substituting Equation (C3) in Equation (C2)

$$-\frac{1}{\rho V} \cdot \frac{\partial}{\partial S} (\rho V) \cdot dS = \frac{\partial \lambda}{\partial N} \cdot dS + \frac{\sin \lambda}{r} \cdot dS \quad \dots (C4)$$

or
$$\frac{\partial \lambda}{\partial N} + \frac{\sin \lambda}{r} + \frac{1}{\rho} \cdot \frac{\partial \rho}{\partial S} + \frac{1}{V} \cdot \frac{\partial V}{\partial S} = 0$$

From the assumption of constant energy (see for example Reference 32)

$$\frac{dp}{\rho} = -Mn^2 \cdot \frac{dV}{V} \quad \dots (C5)$$

Equations (C4) and (C5) then give

$$\frac{\partial \lambda}{\partial N} - (Mn^2 - 1) \cdot \frac{1}{V} \cdot \frac{\partial V}{\partial S} = -\frac{\sin \lambda}{r}$$

But
$$\frac{dV}{V} = \frac{d\omega}{\sqrt{Mn^2 - 1}}$$

and so
$$\frac{\partial \lambda}{\partial N} - \sqrt{Mn^2 - 1} \cdot \frac{\delta \omega}{\delta S} = - \frac{\sin \lambda}{r} \quad \dots(C6)$$

Now in Reference 19, it is shown that on a second family characteristic

$$\delta(\omega + \lambda) = \left[- \frac{\partial \omega}{\partial N} \cdot \frac{1}{\sqrt{Mn^2 - 1}} + \frac{\partial \omega}{\partial S} - \frac{\partial \lambda}{\partial N} \cdot \frac{1}{\sqrt{Mn^2 - 1}} + \frac{\partial \lambda}{\partial S} \right] \cdot dS \quad \dots(C7)$$

Putting Equations (C1) and (C6) in Equation (C7)

$$\delta(\omega + \lambda) = \frac{\sin \lambda}{r \sqrt{Mn^2 - 1}} \cdot dS \text{ on a second family characteristic} \quad \dots(C8)$$

Similarly

$$\delta(\omega - \lambda) = \frac{\sin \lambda}{r \sqrt{Mn^2 - 1}} \cdot dS \text{ on a first family characteristic} \quad \dots(C9)$$

Furthermore, from Figure 72(c)

$$\frac{dS}{\cos \mu} = \frac{dx}{\cos (\lambda - \mu)} = \frac{dr}{\sin (\lambda - \mu)}$$

and

$$\cos \mu = \frac{\sqrt{Mn^2 - 1}}{Mn}$$

so that

$$\frac{dS}{\sqrt{Mn^2 - 1}} = \frac{dx}{Mn \cos (\lambda - \mu)} = \frac{dr}{Mn \sin (\lambda - \mu)} \quad \dots(C10)$$

Putting Equation (C10) into Equation (C8)

$$\delta(\omega + \lambda) = \frac{\sin \lambda}{r \cos (\lambda - \mu)} \cdot \frac{dx}{Mn} = \frac{\sin \lambda}{r \sin (\lambda - \mu)} \cdot \frac{dr}{Mn} \quad \dots(C11)$$

Similarly,

$$\delta(\omega - \lambda) = \frac{\sin \lambda}{r \cos (\lambda + \mu)} \cdot \frac{dx}{Mn} = \frac{\sin \lambda}{r \sin (\lambda + \mu)} \cdot \frac{dr}{Mn} \quad \dots(C12)$$

Equations (C11) and (C12) are known as the compatibility relations and they enable the variations of the flow properties to be calculated along second and first family characteristics respectively.

Simultaneous solution of Equations (C11) and (C12) along the 2 characteristics from 2 known points enables one to find the flow properties at a third point - which is at the intersection of the 2 characteristics.

TABLES

TABLE B1

Fan cowl geometry

	Static No.	x in.	r in.	dx in.	dA _{proj} in ²
Limit	1	0	2.2525	0.180	0
	2	0.36	2.2525	0.360	0
	3	0.72	2.2525	0.360	0.085
	4	1.08	2.2405	0.360	0.2005
	5	1.44	2.2240	0.360	0.3170
	6	1.80	2.1950	0.360	0.4610
	7	2.16	2.1570	0.360	0.5550
	8	2.52	2.1130	0.360	0.6780
	9	2.88	2.0545	0.360	0.7610
	10	3.24	1.9950	0.360	0.7735
	11	3.60	1.9310	0.345	0.7710
	12	3.93	1.8680	0.195	0.4690
Limit	T.E.	3.96	1.8600	-	-

TABLE B2

Afterbody geometry

	Static No.	x in.	r in.	dx in.	dA _{proj} in ²
Limit	1	-0.160	1.327	0.132	0.083
	2	0.103	1.307	0.263	0.192
	3	0.366	1.280	0.263	0.245
	4	0.629	1.246	0.263	0.285
	5	0.892	1.207	0.263	0.311
	6	1.155	1.164	0.263	0.339
	7	1.418	1.114	0.263	0.367
	8	1.681	1.059	0.263	0.388
	9	1.944	0.997	0.263	0.397
	10	2.207	0.932	0.263	0.403
	11	2.470	0.859	0.425	0.629
Limit	T.E.	2.763	0.776	-	-

TABLE B3

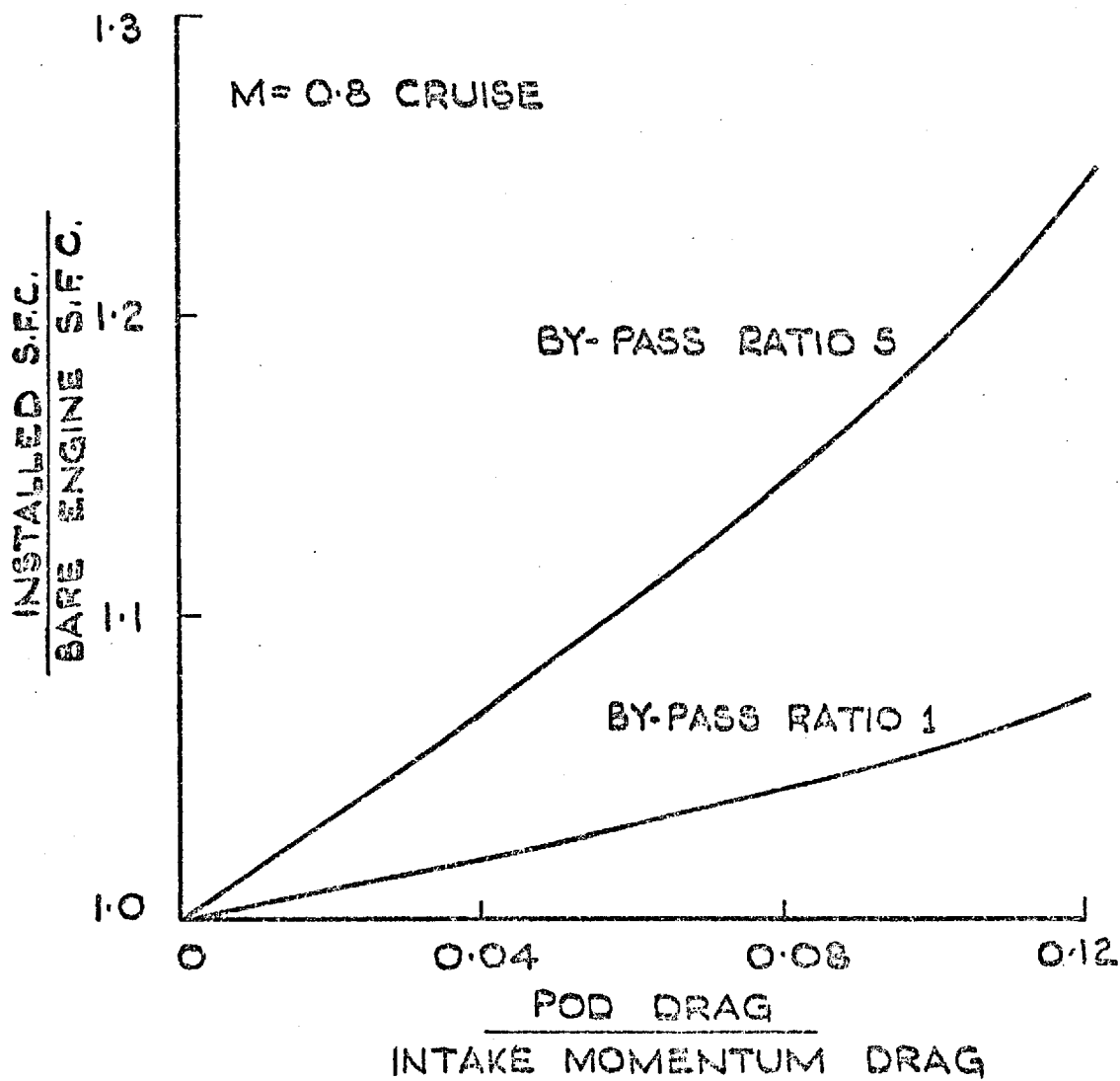
Parameters used in calculating cowl drag

P_L	r	dx	M_n	$\bar{P}$	$\bar{Pr}^a$	X	R_x	ξ	ξdx	ψ	dA
18.72	2.2525	0.180	0.8462	0.3001	0.8282	0.1800	6.68×10^4	0.1837	0.0331	0.09	0
18.02	2.2525	0.360	0.8824	0.3398	0.9377	0.5190	1.96×10^5	0.1547	0.0557	-0.61	0
17.60	2.2525	0.360	0.9043	0.3648	1.0067	0.8434	3.21×10^5	0.1443	0.0519	-1.03	0.085
17.40	2.2405	0.360	0.9148	0.3771	1.0336	1.1815	4.53×10^5	0.1363	0.0491	-1.23	0.2005
17.20	2.2240	0.360	0.9253	0.3895	1.0578	1.5144	5.83×10^5	0.1306	0.0470	-1.43	0.3170
17.22	2.1950	0.360	0.9242	0.3882	1.0373	1.9044	7.33×10^5	0.1237	0.0445	-1.41	0.4610
17.60	2.1570	0.360	0.9043	0.3648	0.9537	2.4314	9.28×10^5	0.1145	0.0412	-1.03	0.5550
17.60	2.1130	0.360	0.9043	0.3648	0.9294	2.8548	1.09×10^6	0.1092	0.0393	-1.03	0.6780
18.50	2.0545	0.360	0.8575	0.3123	0.7683	3.8139	1.42×10^6	0.0966	0.0348	-0.13	0.7610
18.69	1.9950	0.360	0.8477	0.3018	0.7155	4.4560	1.65×10^6	0.0904	0.0326	0.06	0.7735
20.12	1.9310	0.345	0.7741	0.2284	0.5198	6.4770	2.30×10^6	0.0752	0.0259	1.49	0.7710
20.12	1.8680	0.195	0.7741	0.2284	0.4987	6.9467	2.47×10^6	0.0719	0.0140	1.49	0.4690

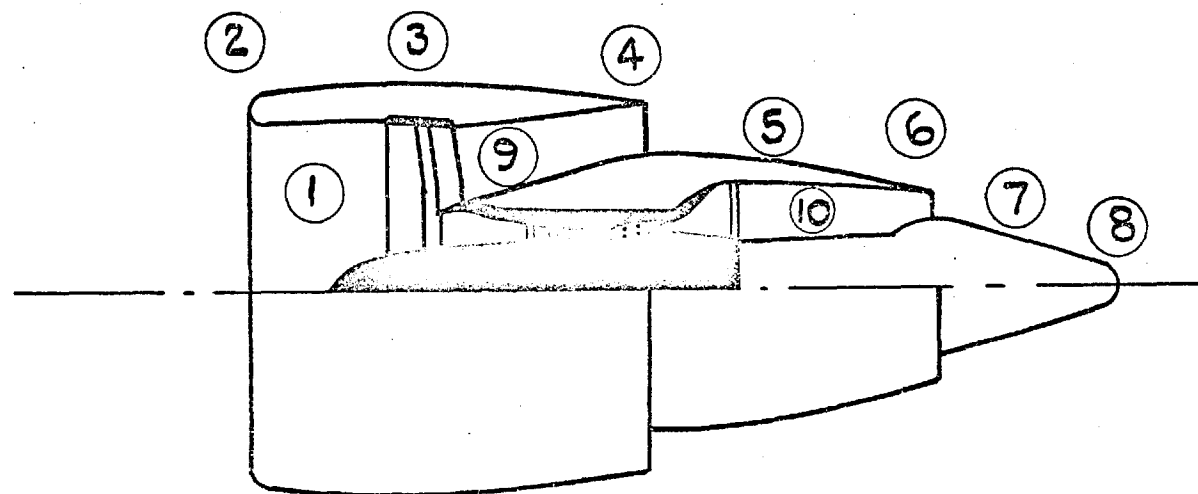
TABLE B4

Parameters used in calculating afterbody drag

P_L	r	dx	Mn	$\bar{P}$	$\bar{Pr}^a$	X	Rx	ξ	ξdx	ψ	dA
28.33	1.327	0.132	0.9075	0.3685	0.5249	0.1320	8.04×10^4	0.1799	0.0237	9.70	0.083
21.78	1.307	0.263	1.1306	0.6573	0.9186	0.3384	2.18×10^5	0.1680	0.0442	3.15	0.192
17.30	1.280	0.263	1.3060	0.8993	1.2244	0.5169	3.34×10^5	0.1563	0.0411	-1.33	0.245
14.20	1.246	0.263	1.4475	1.0827	1.4253	0.7071	4.48×10^5	0.1416	0.0372	-4.43	0.285
14.47	1.207	0.263	1.4343	1.0664	1.3491	1.0099	6.42×10^5	0.1291	0.0340	-4.16	0.311
15.81	1.164	0.263	1.3714	0.9863	1.1924	1.4057	9.02×10^5	0.1187	0.0312	-2.82	0.339
20.70	1.114	0.263	1.1706	0.7126	0.8156	2.3181	1.50×10^6	0.1031	0.0271	2.07	0.367
21.58	1.059	0.263	1.1379	0.6674	0.7170	2.8999	1.87×10^6	0.0936	0.0246	2.95	0.388
20.00	0.997	0.263	1.1972	0.7496	0.7468	3.0493	1.97×10^6	0.0887	0.0233	1.37	0.397
16.10	0.932	0.263	1.3583	0.9691	0.8875	2.8272	1.82×10^6	0.0846	0.0223	-2.53	0.403
20.50	0.859	0.425	1.1781	0.7231	0.5980	4.6208	2.99×10^6	0.0712	0.0303	1.87	0.629



EFFECT OF POD DRAG ON
INSTALLED S.F.C. (VIDE REE4)



- | | |
|--------------------------------|-----------------------|
| ① INTAKE INTERNAL FRICTION | ⑥ BOATTAIL DRAG |
| ② SPILL DRAG & SUPERVELOCITIES | ⑦ PLUG SKIN FRICTION |
| ③ SKIN FRICTION (FREE STREAM) | ⑧ BOATTAIL DRAG |
| ④ BOATTAIL DRAG | ⑨ FAN NOZZLE LOSS |
| ⑤ COWL SKIN FRICTION | ⑩ PRIMARY NOZZLE LOSS |

DRAG AND THRUST LOSSES

(WIDE REFERENCE 8)

THE SINGLE STREAM NACELLE

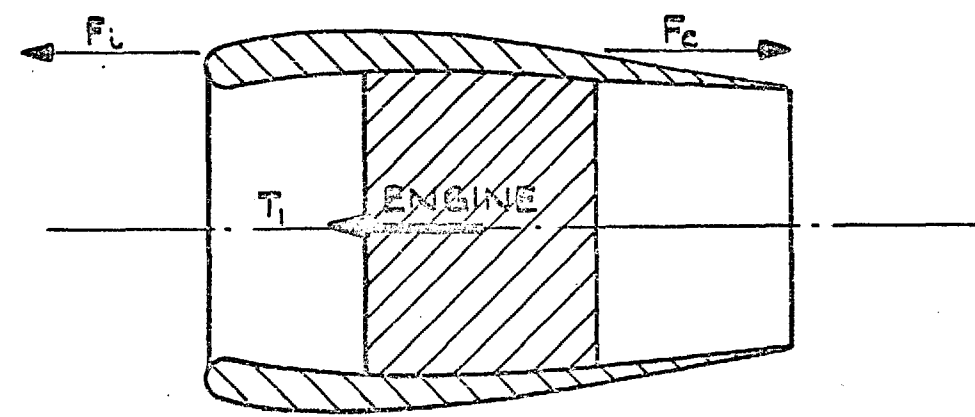


FIG. 3a FORCES ON A SINGLE STREAM NACELLE IN ISOLATION

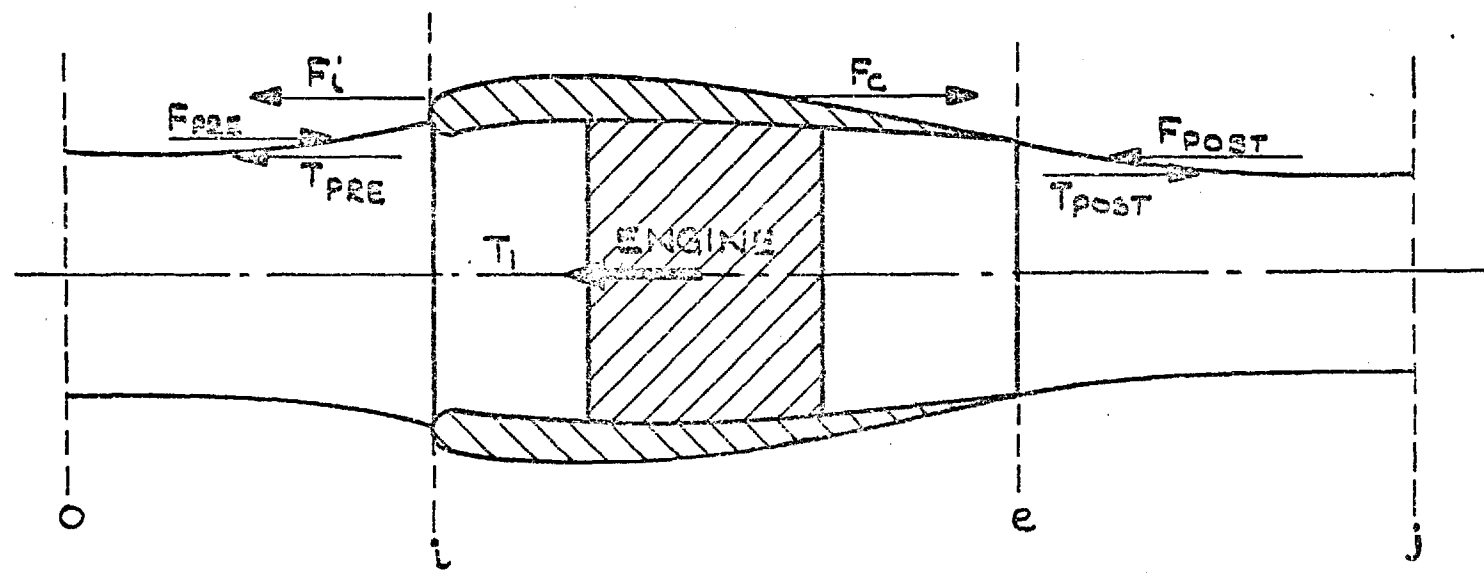


FIG. 3b FORCES USED IN DERIVATION OF THRUST AND DRAG

FIG. 3

THE TWO-STREAM NACELLE

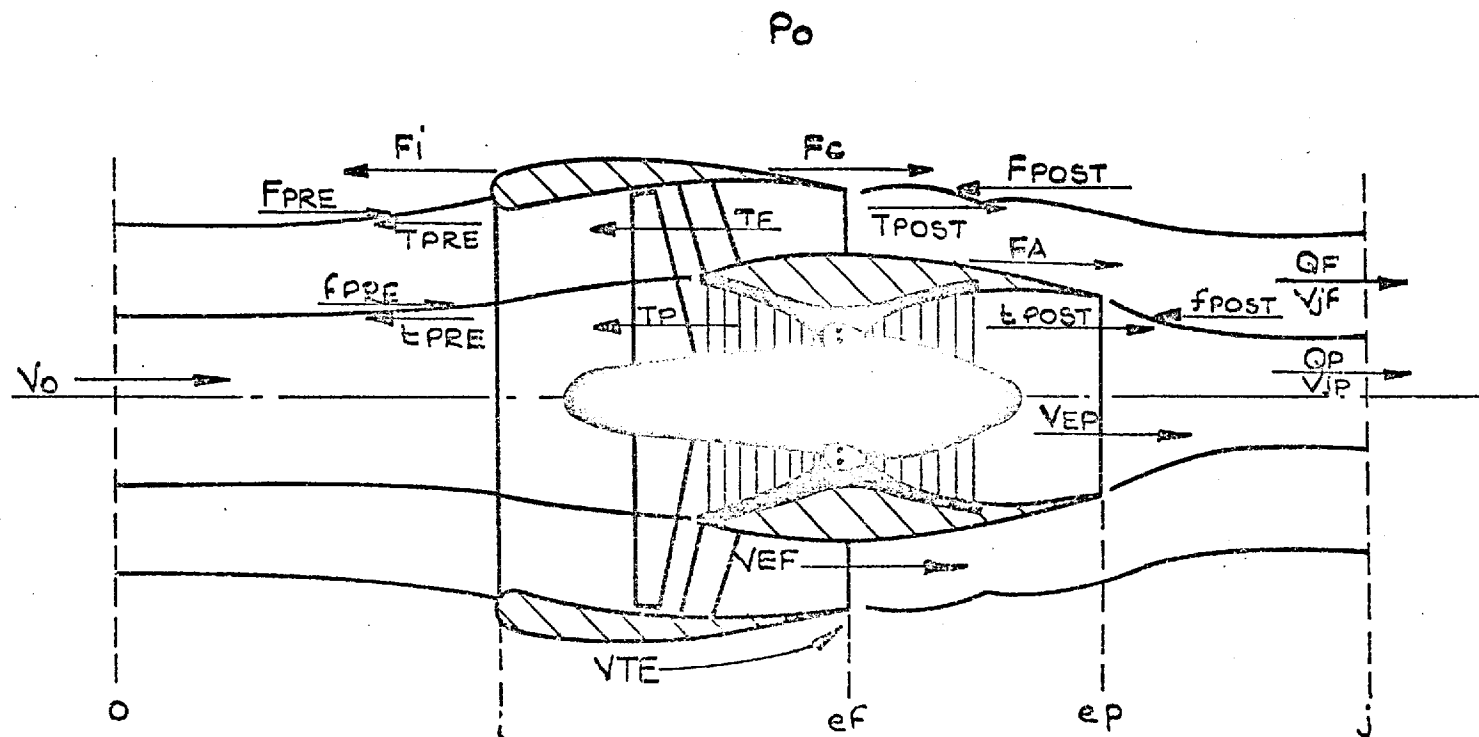
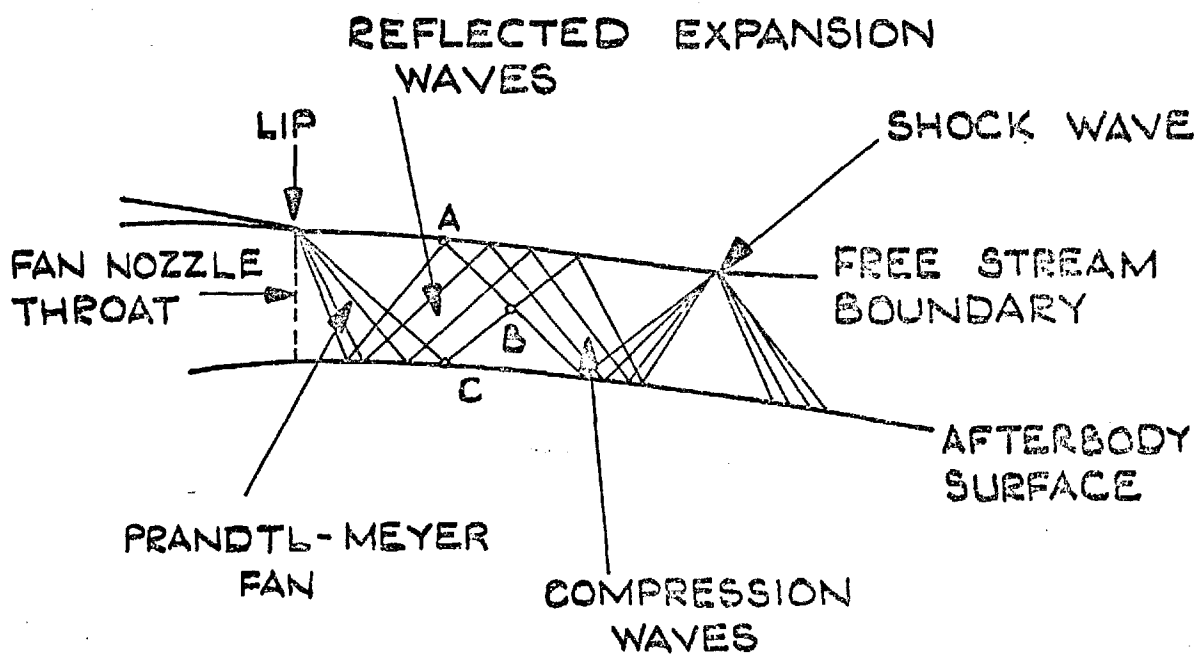


FIG. 4



THE INVISCID FLOW PATTERN

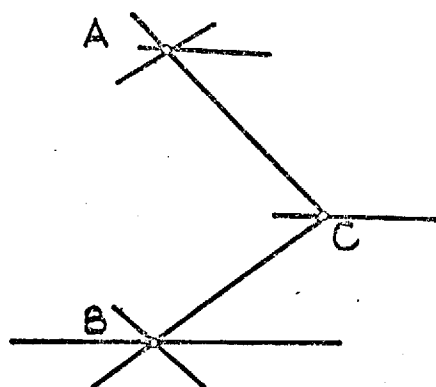


FIG. 6a GENERAL POINT IN THE FLOW

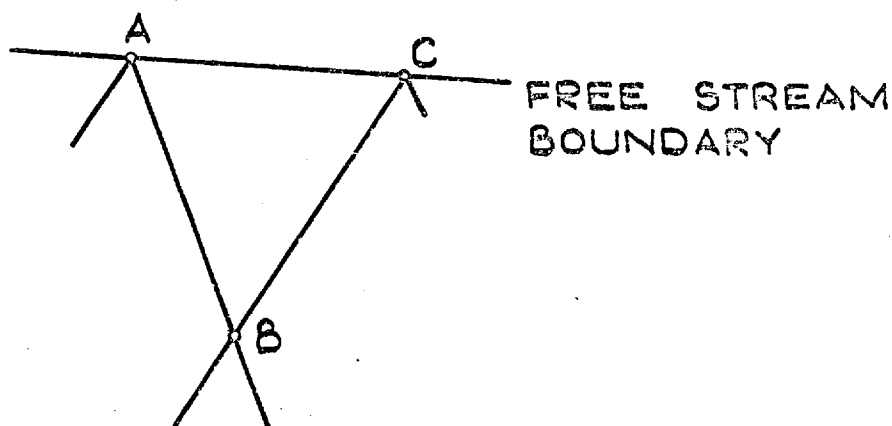


FIG. 6 b POINT ON OUTER BOUNDARY

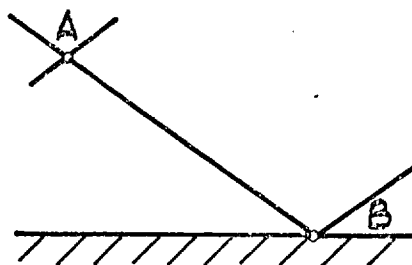
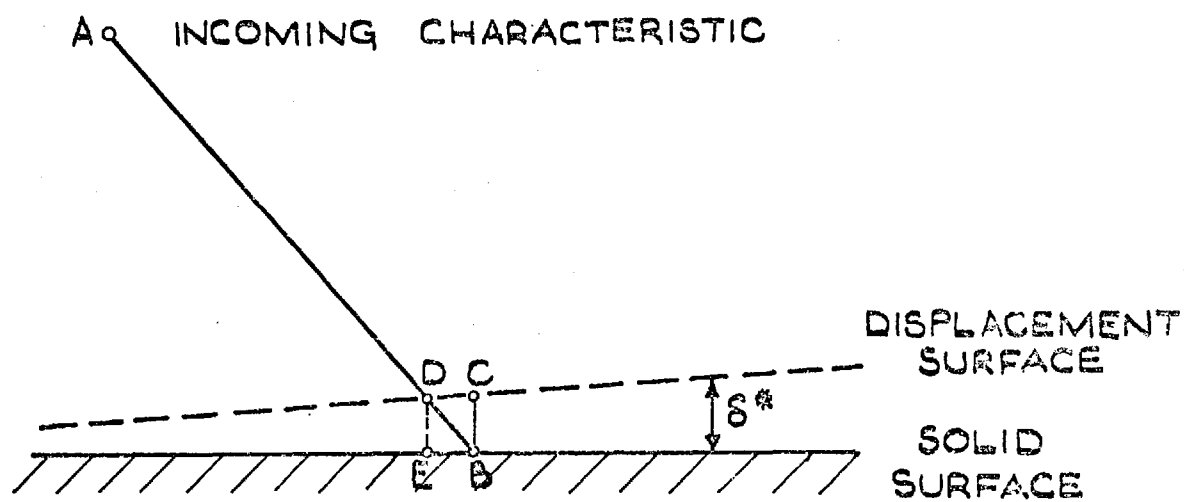


FIG. 6c POINT ON INNER BOUNDARY

MESH POINTS

FIG. 7



INNER SURFACE POINT WITH
BOUNDARY LAYER

FIG.8

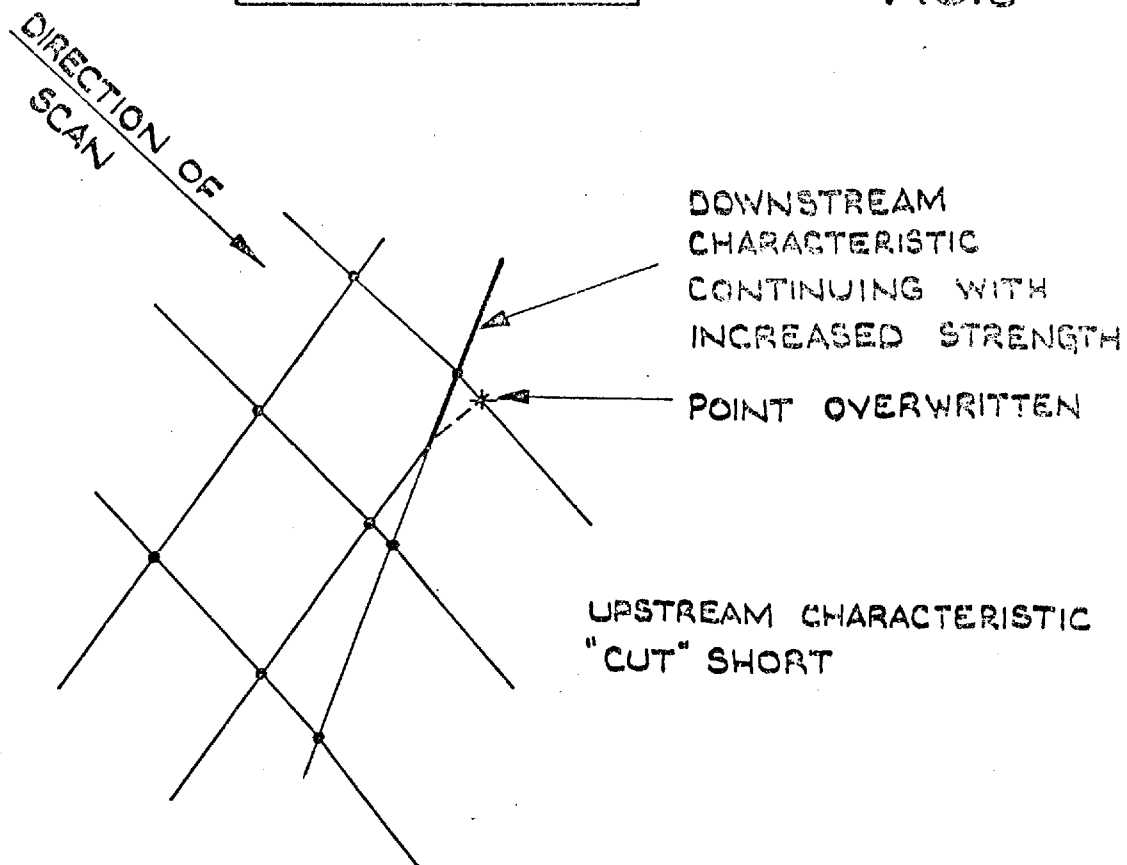


FIG.8a FORMATION OF LEFT RUNNING SHOCK WAVE

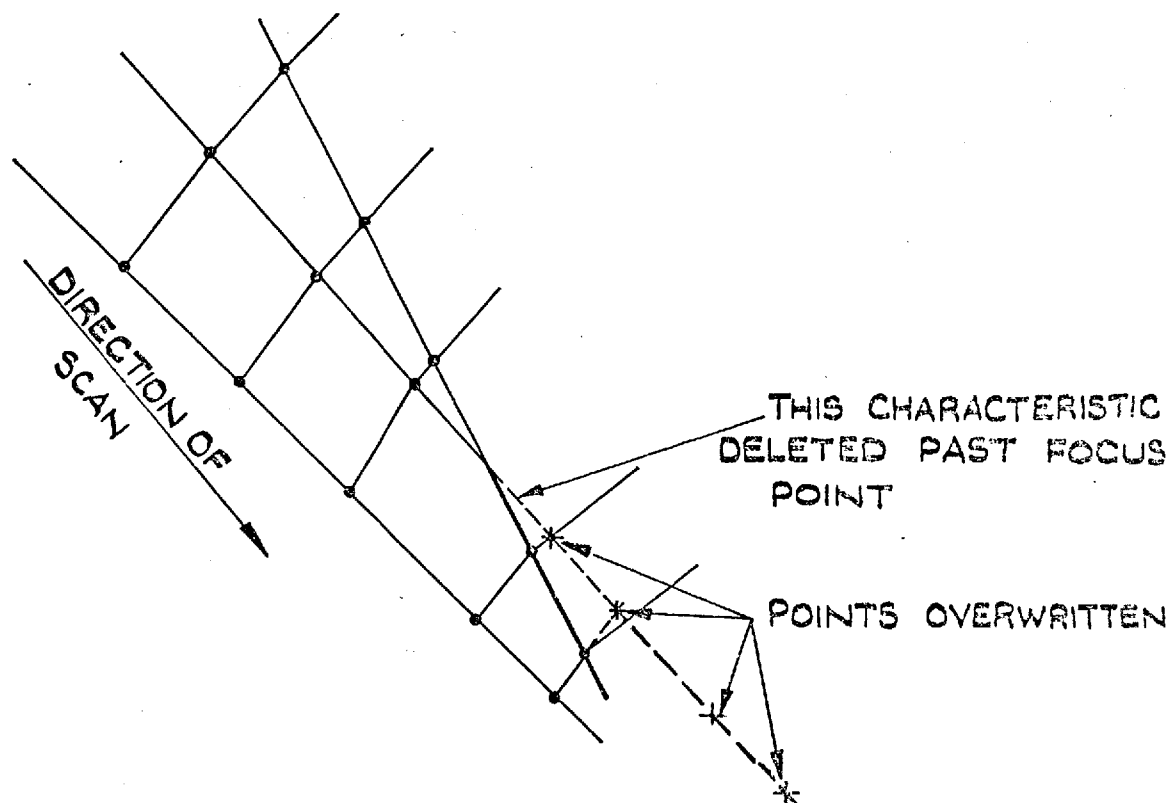


FIG.8b FORMATION OF RIGHT RUNNING SHOCK WAVE

CONSTRUCTION OF SHOCK WAVES

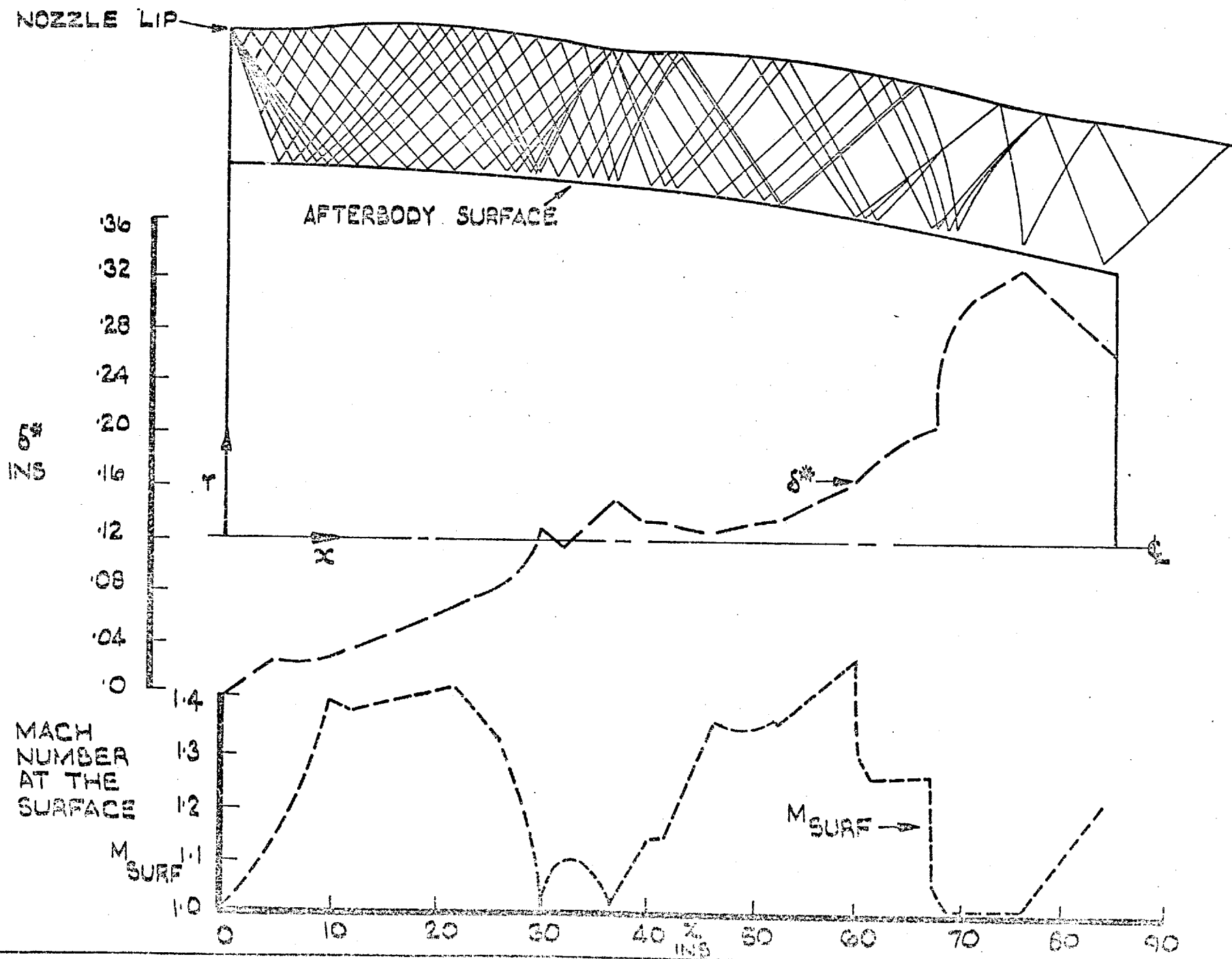
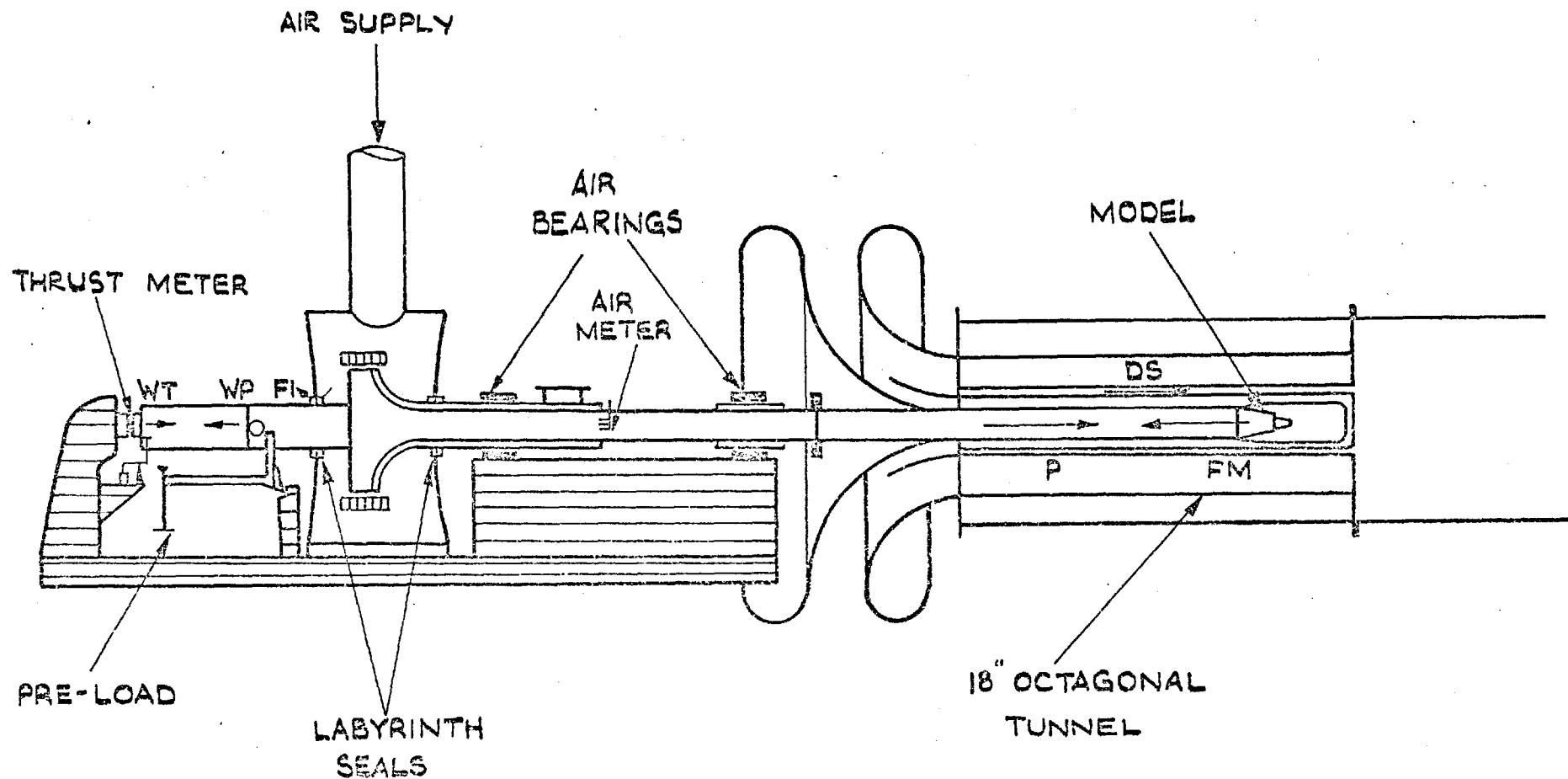


FIG. 9

THRUST RIG IN EXTERNAL FLOW MODE



$$FM = WT - WP - FI + P + DS$$

CONSTRUCTION OF TUNNEL

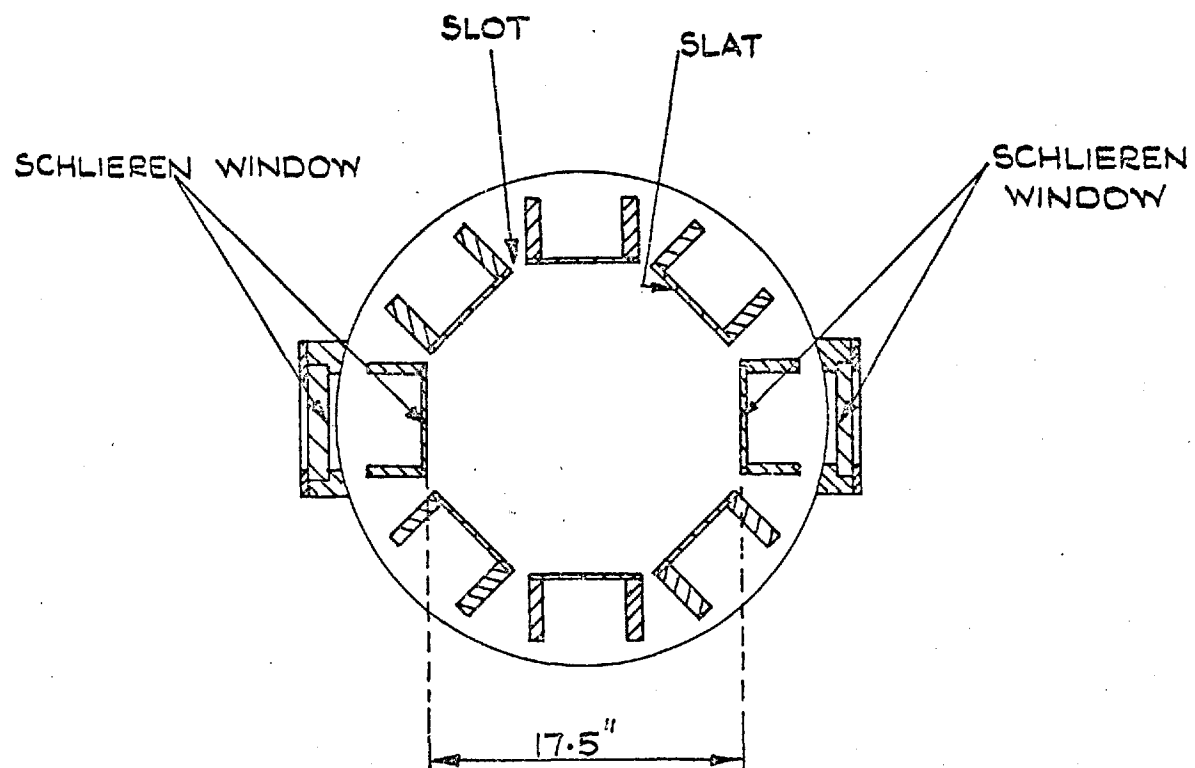
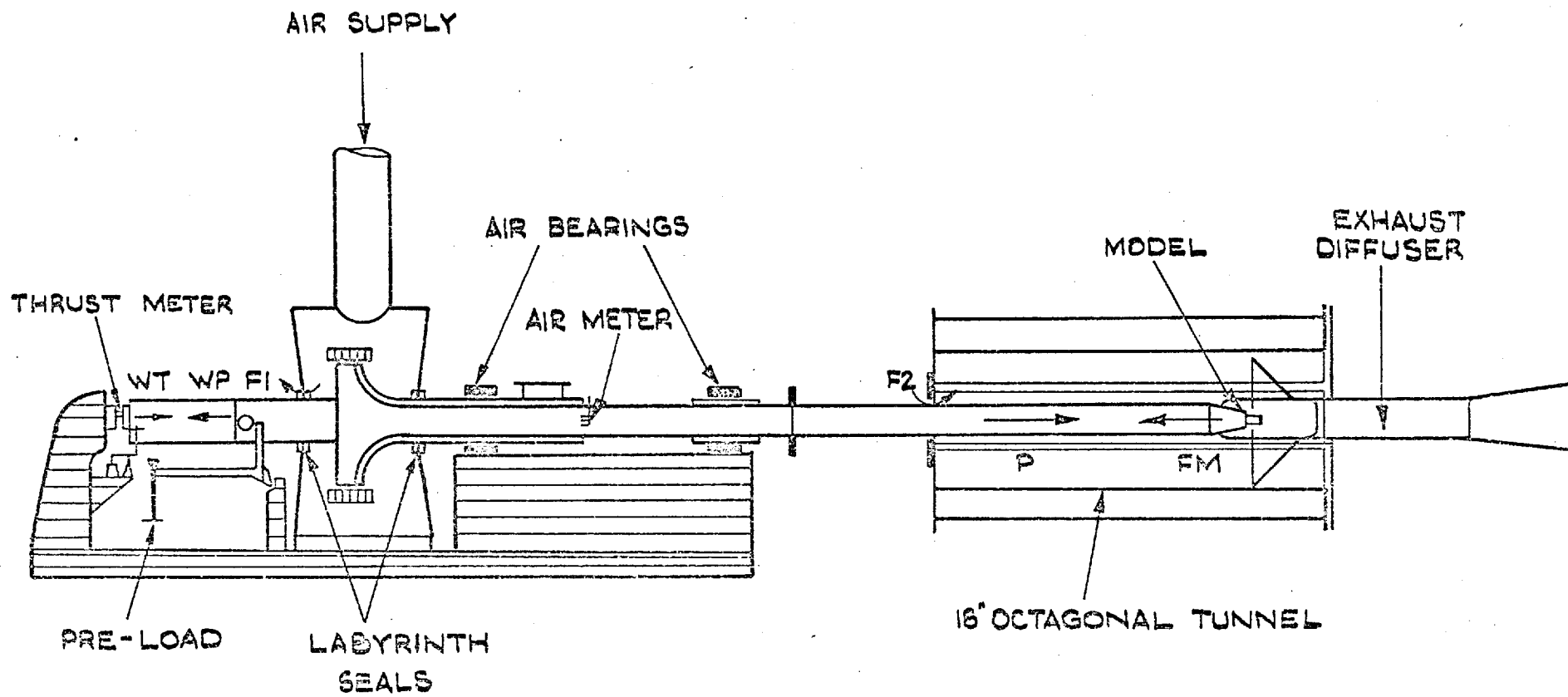
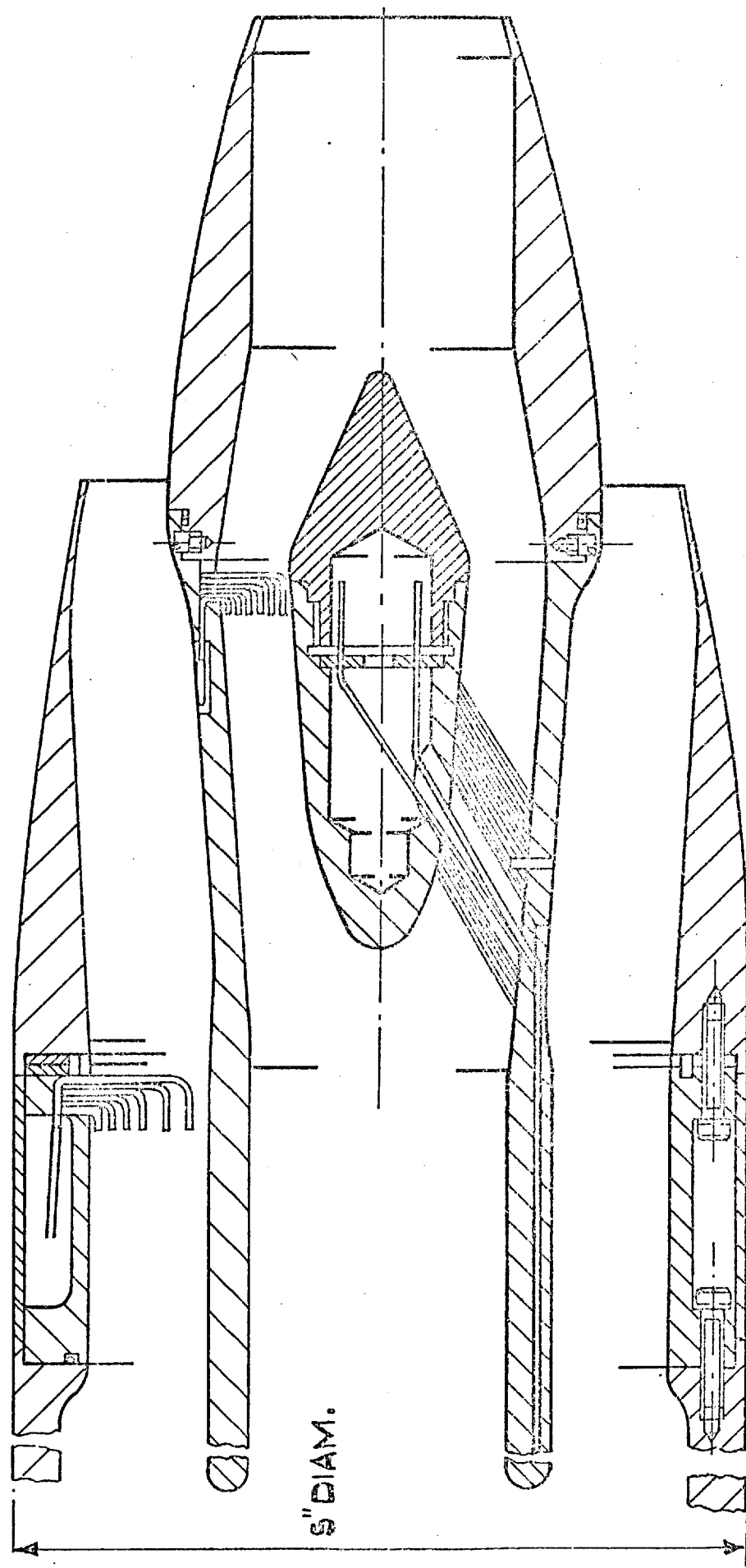


FIG. 11

THRUST RIG IN QUASISTATIC MODE



$$FM = WT - WP - FI + P + F2$$



8" DIAM.

THE TWO-STREAM RESEARCH MODEL

FLOATING AFTERBODY MODEL

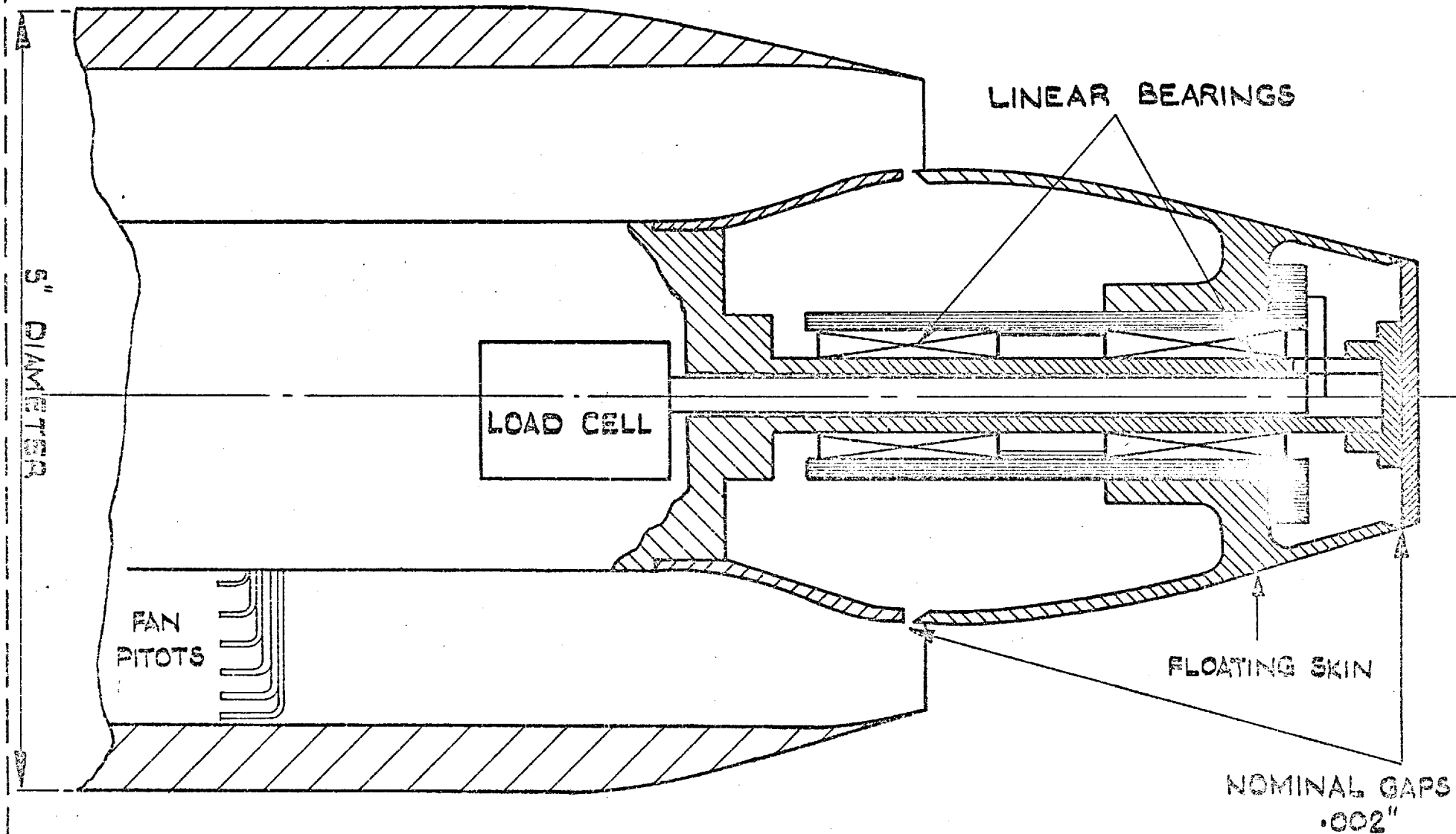


FIG. 14

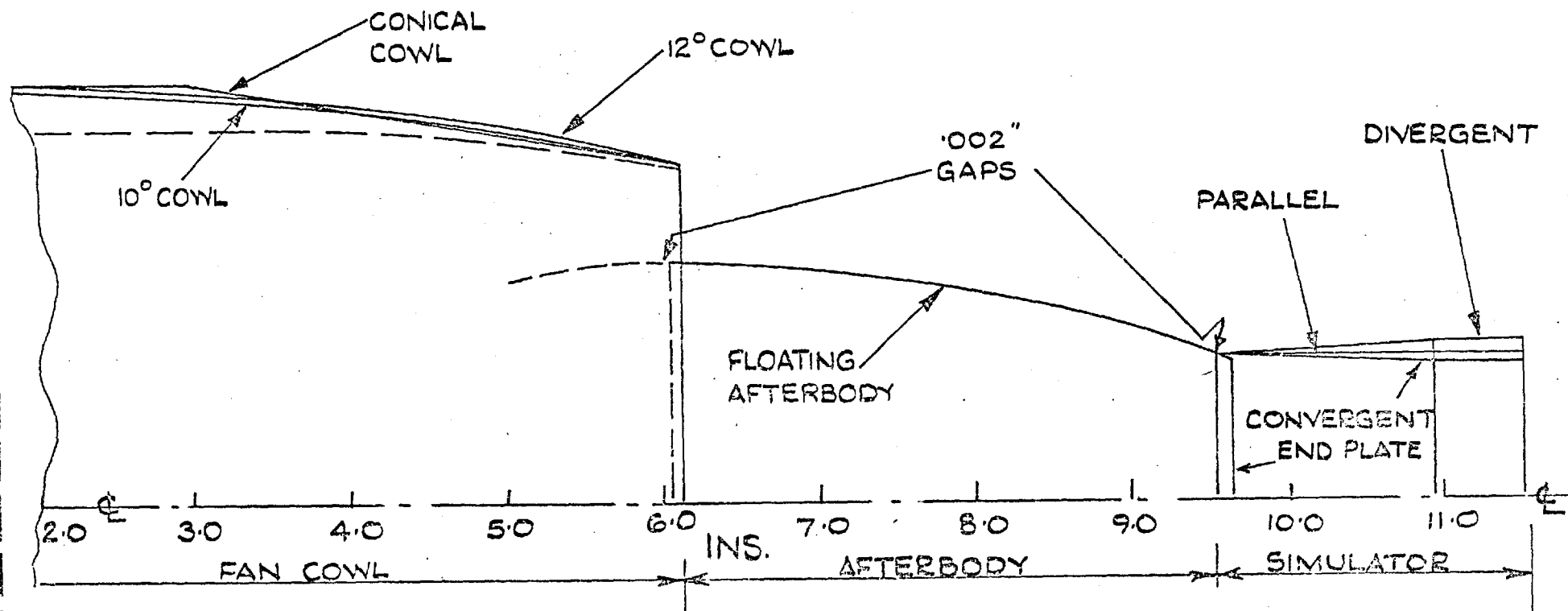
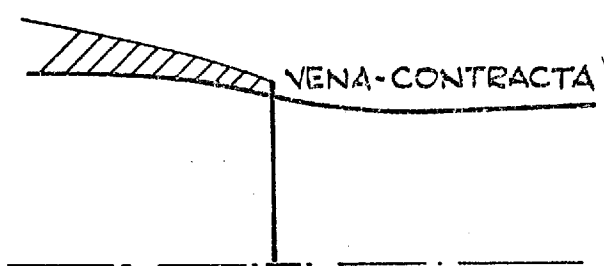
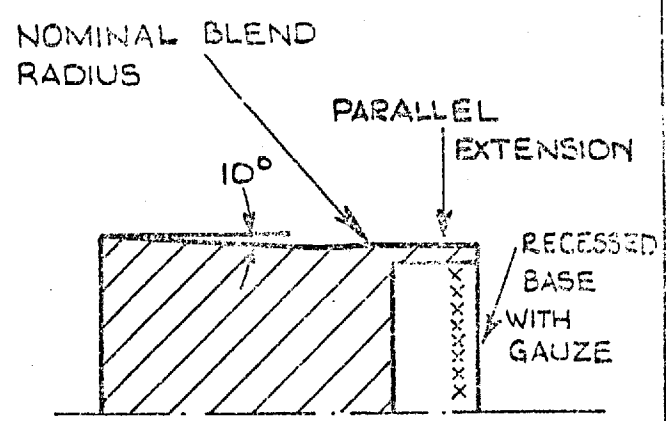


FIG. 15a

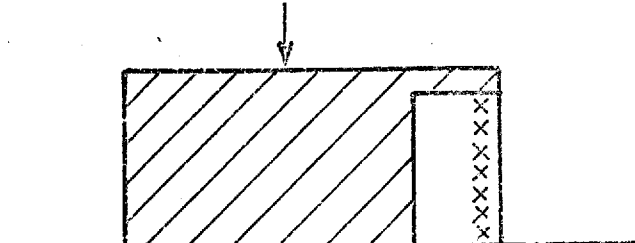


SUBCRITICAL PRESSURE RATIO

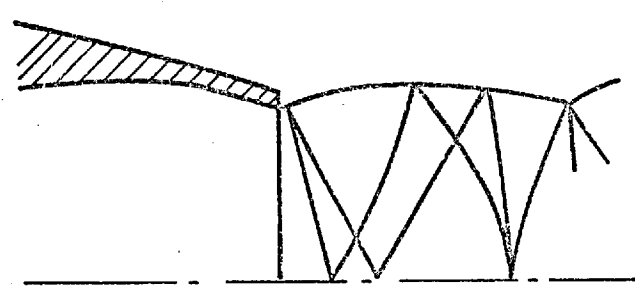


CONVERGENT SIMULATOR

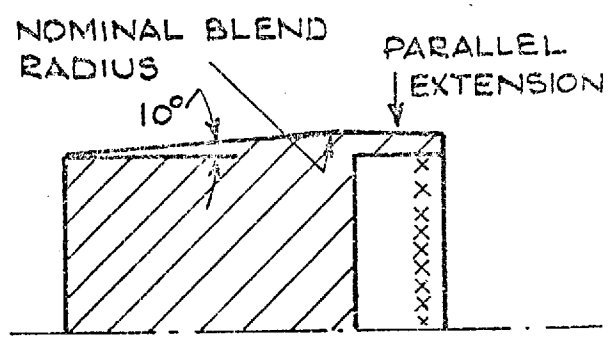
CYLINDRICAL SURFACE



PARALLEL SIMULATOR



SUPERCritical PRESSURE RATIO.



DIVERGENT SIMULATOR

PLUME SIMULATOR DESIGN

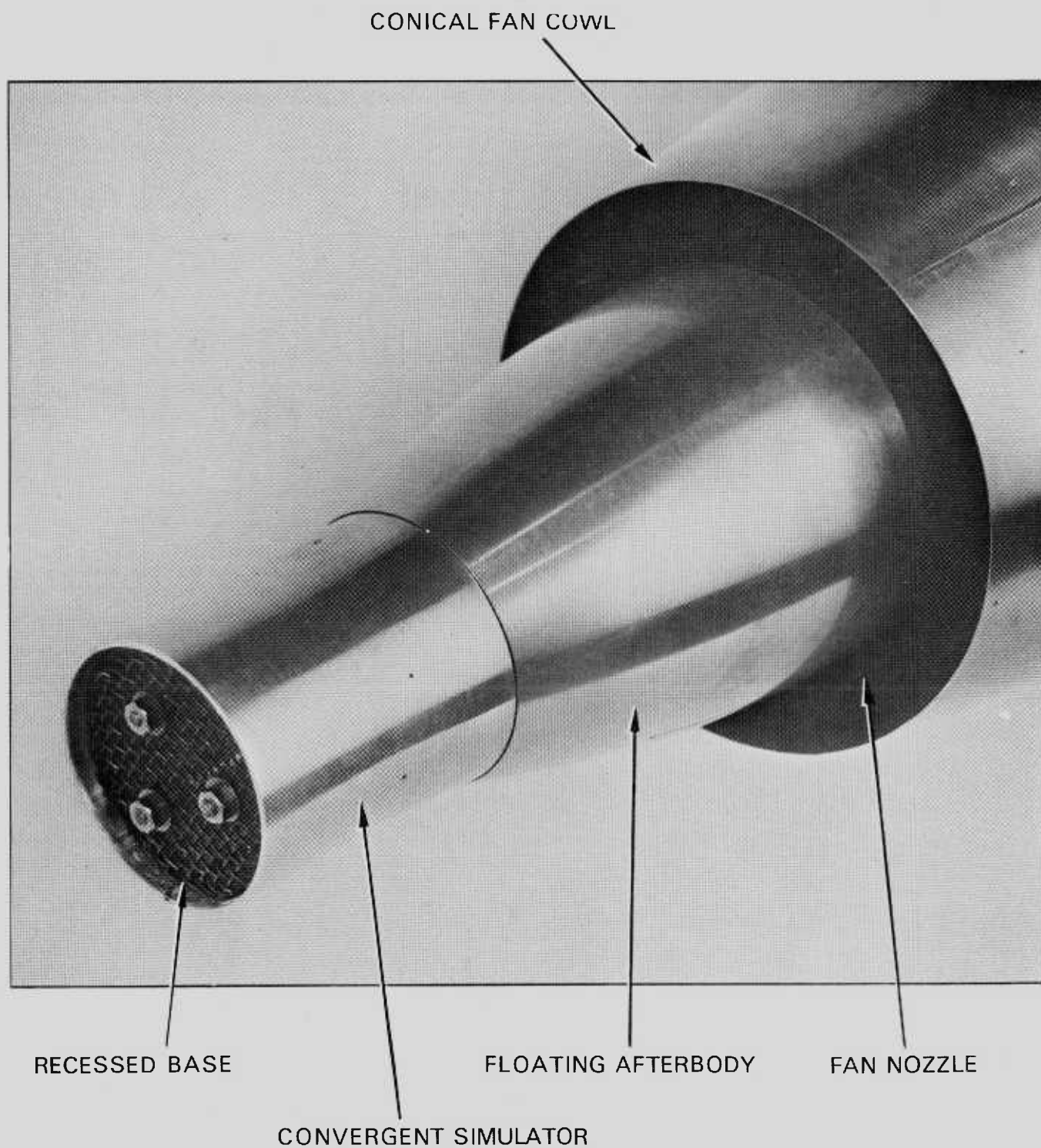
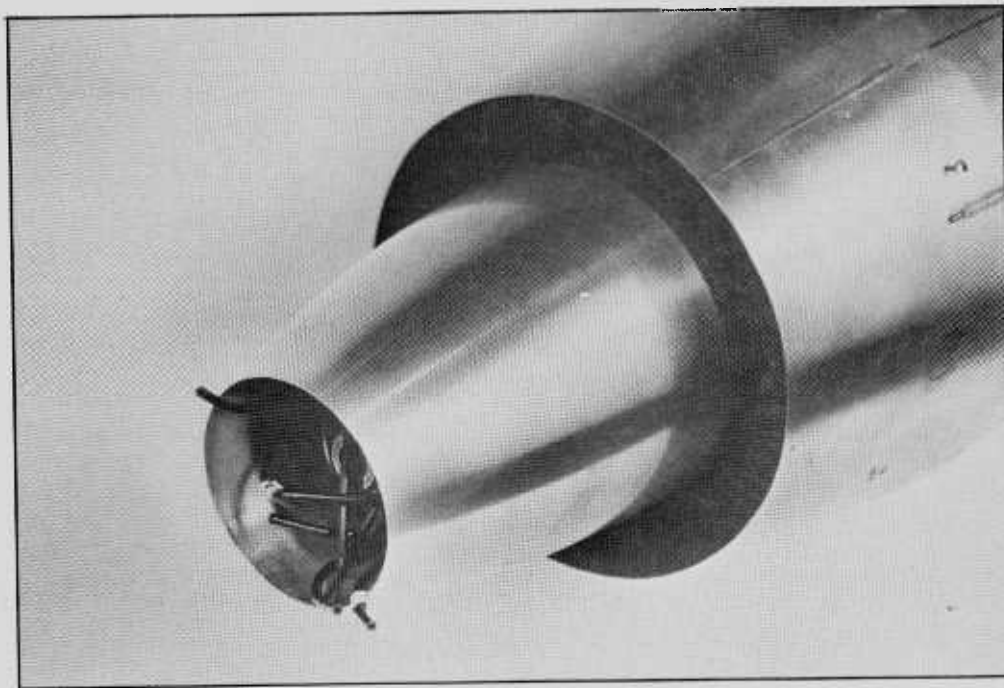
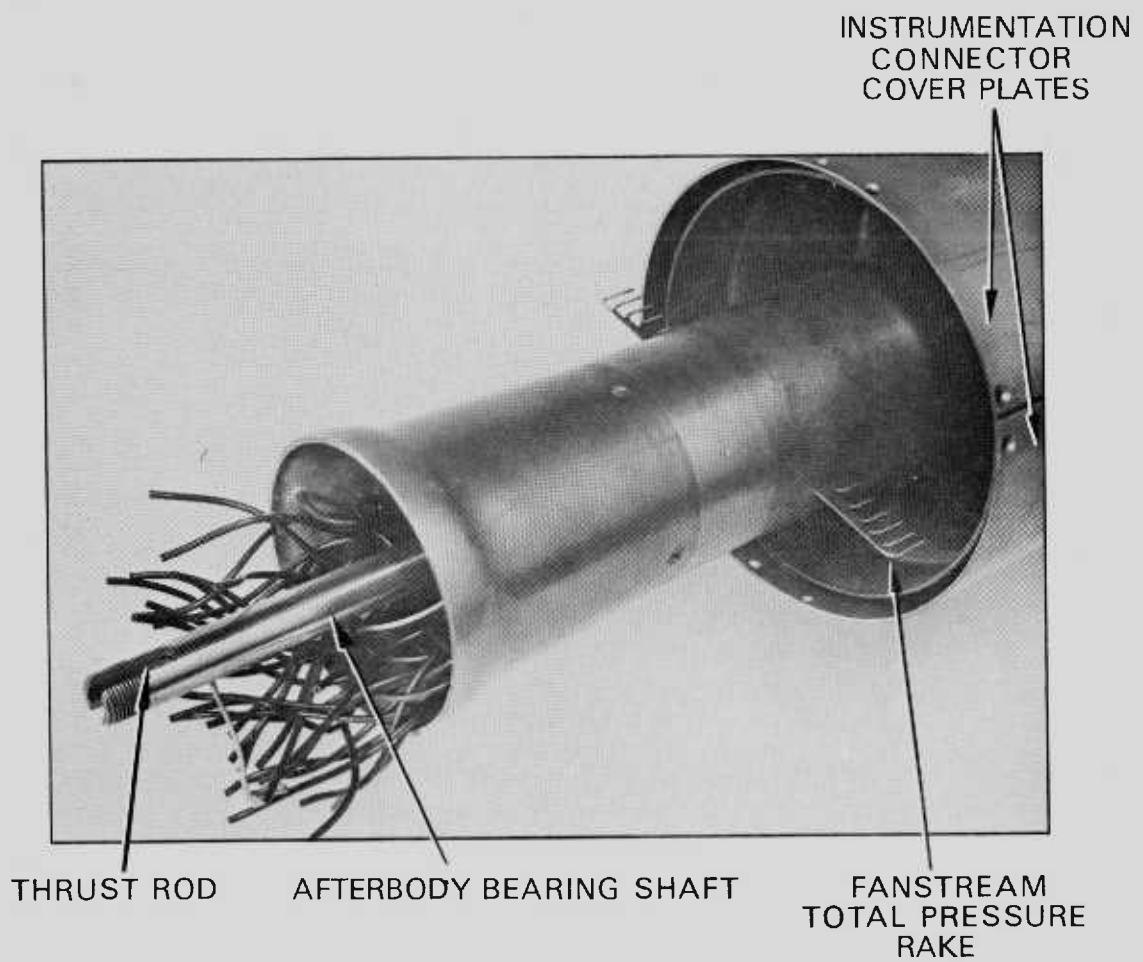


FIG.16 FLOATING AFTERBODY MODEL WITH
CONVERGENT JET SIMULATOR



(a) MODEL WITH SIMULATOR REMOVED



(b) BASIC CHASSIS

FIG.17 PART ASSEMBLIES OF FLOATING AFTERBODY MODEL

PRECIMETER CALIBRATION

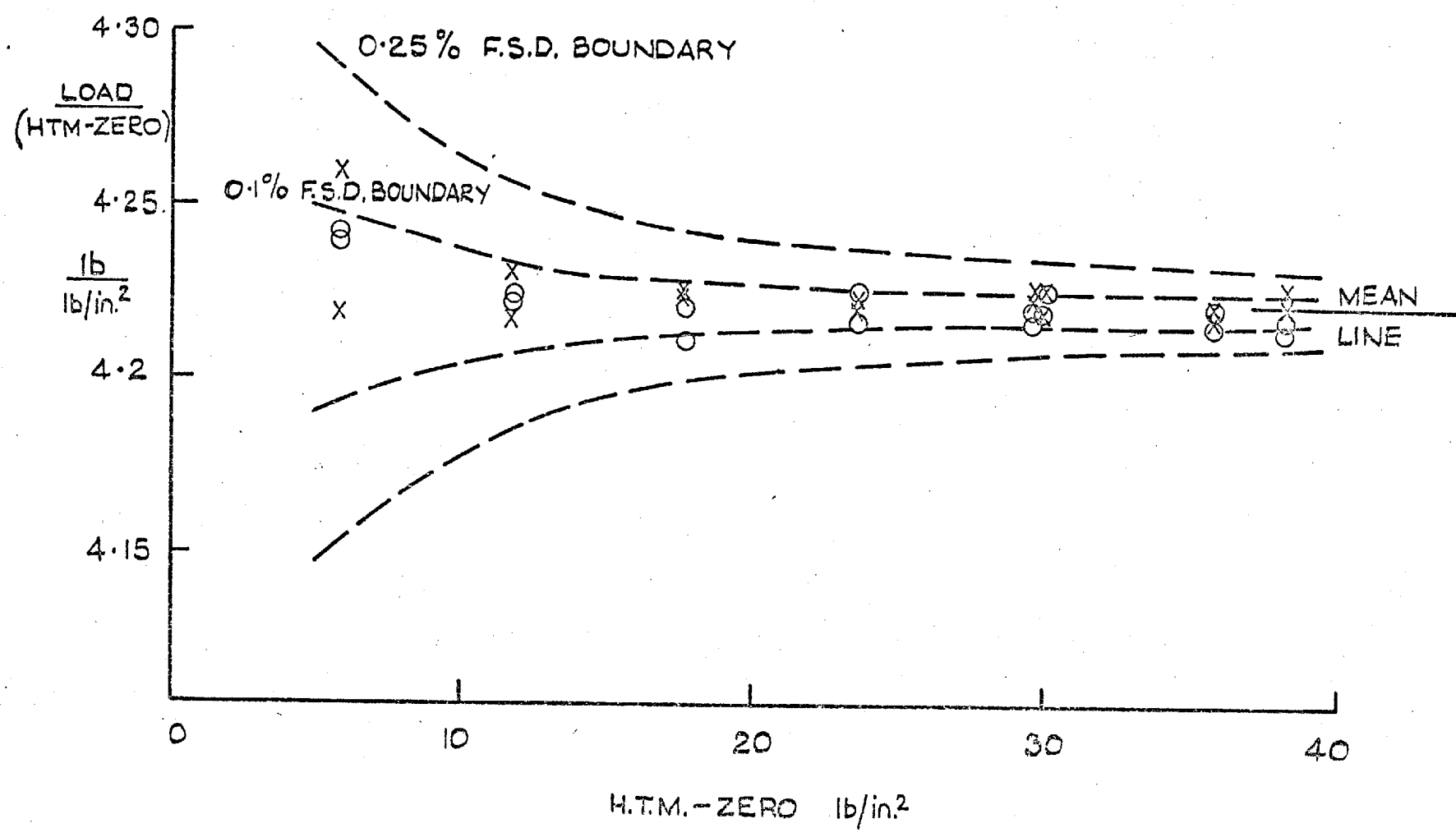


FIG. 10

RIG LOAD CELL CALIBRATION

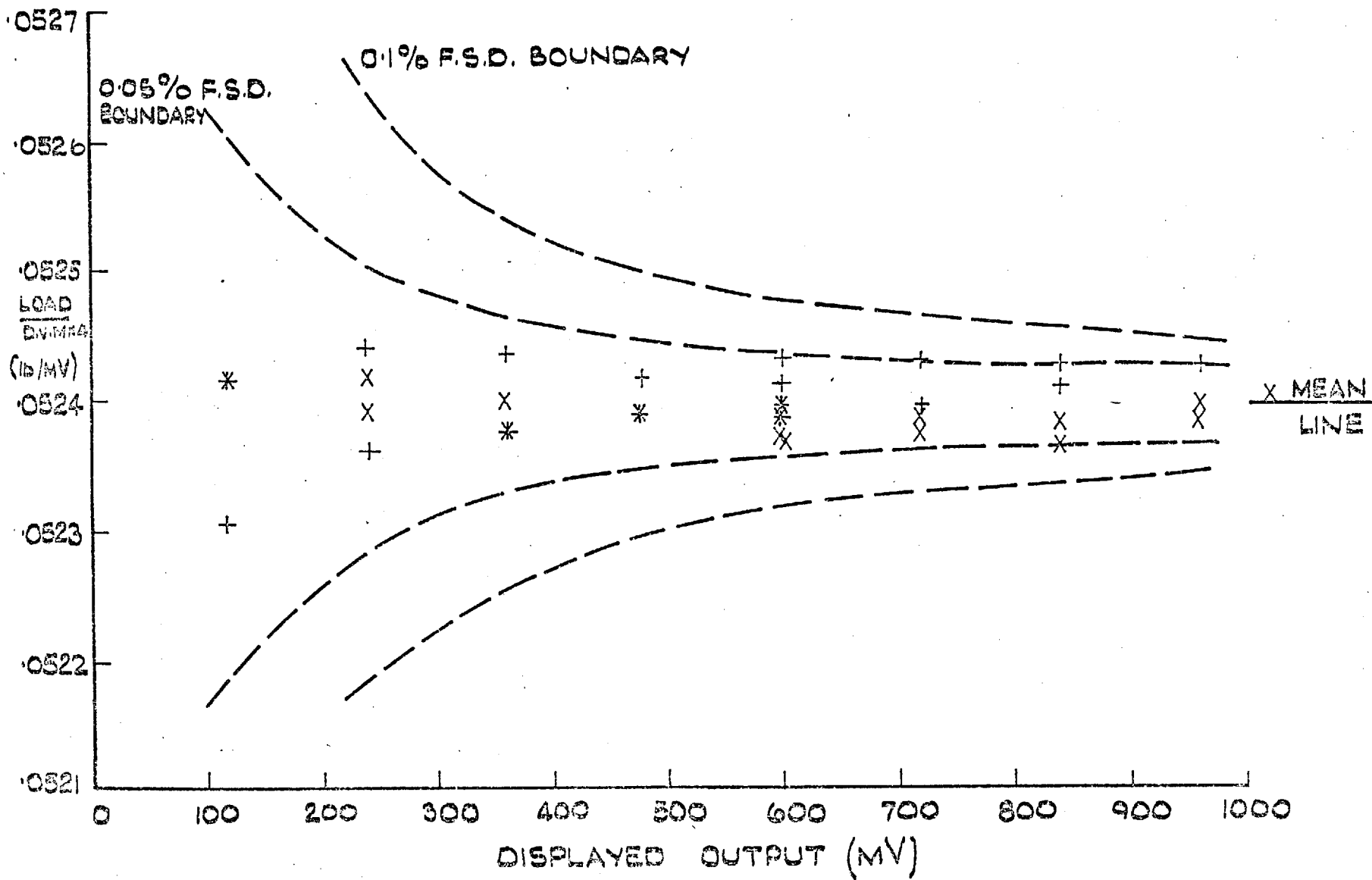
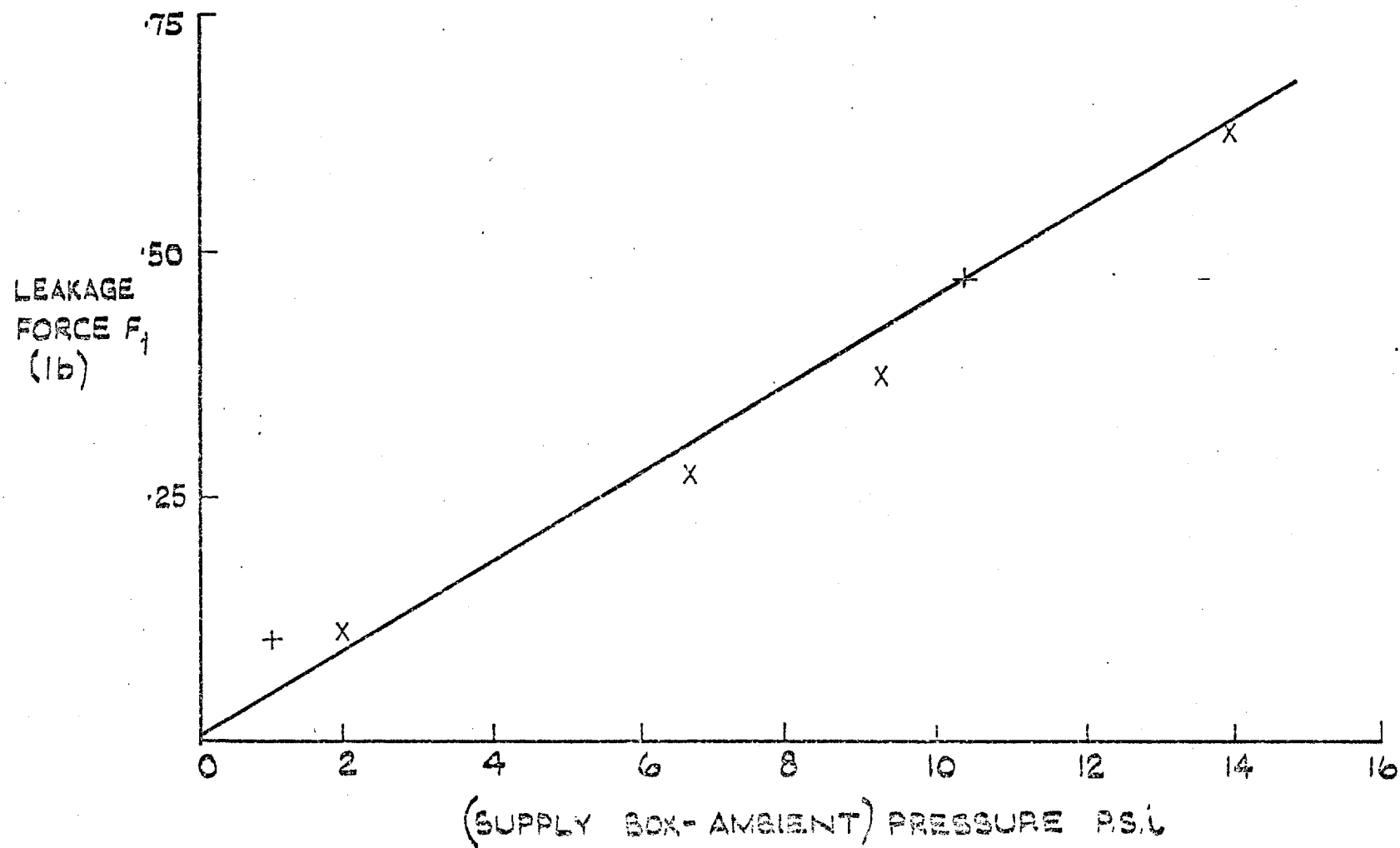
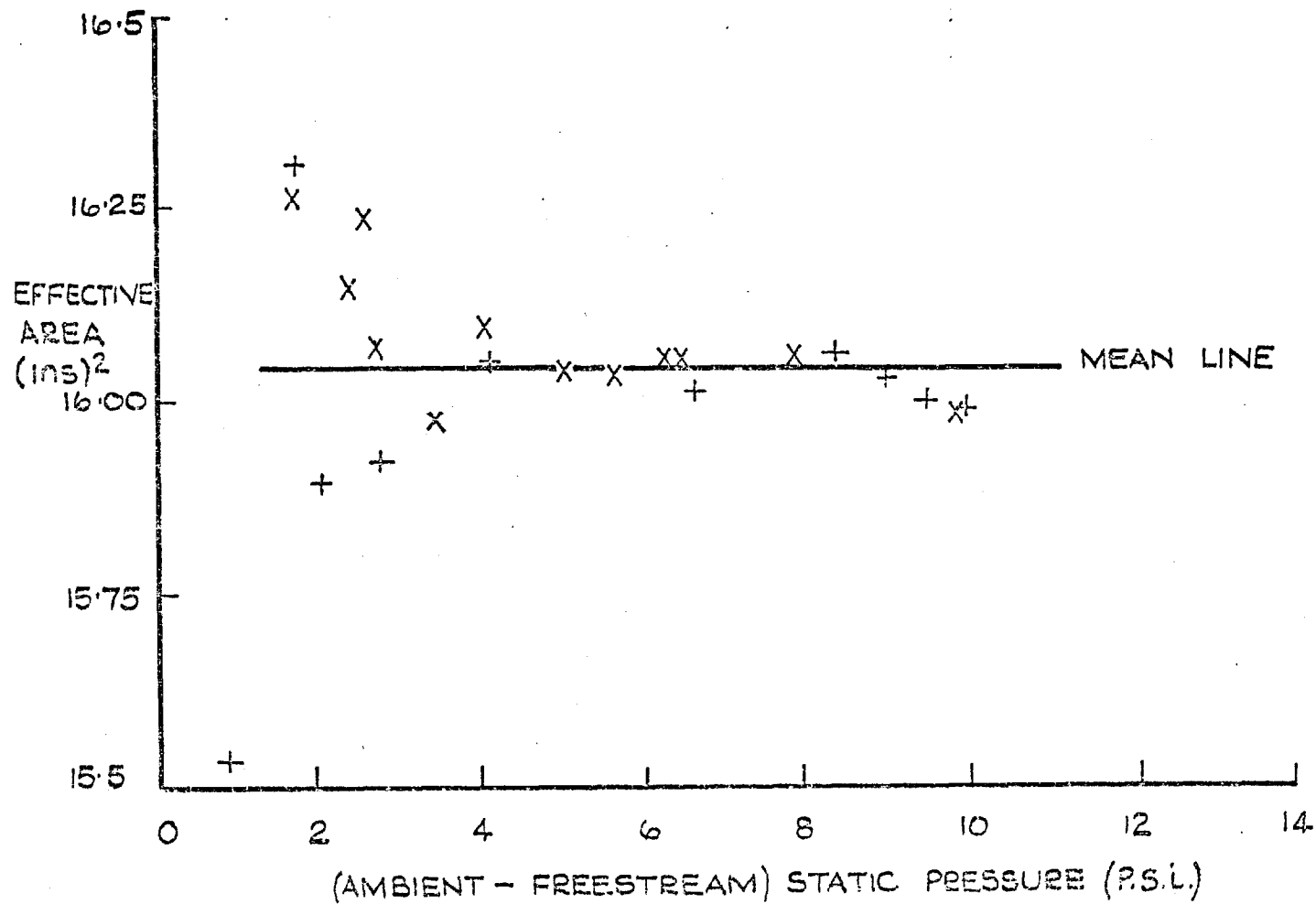


FIG. 19

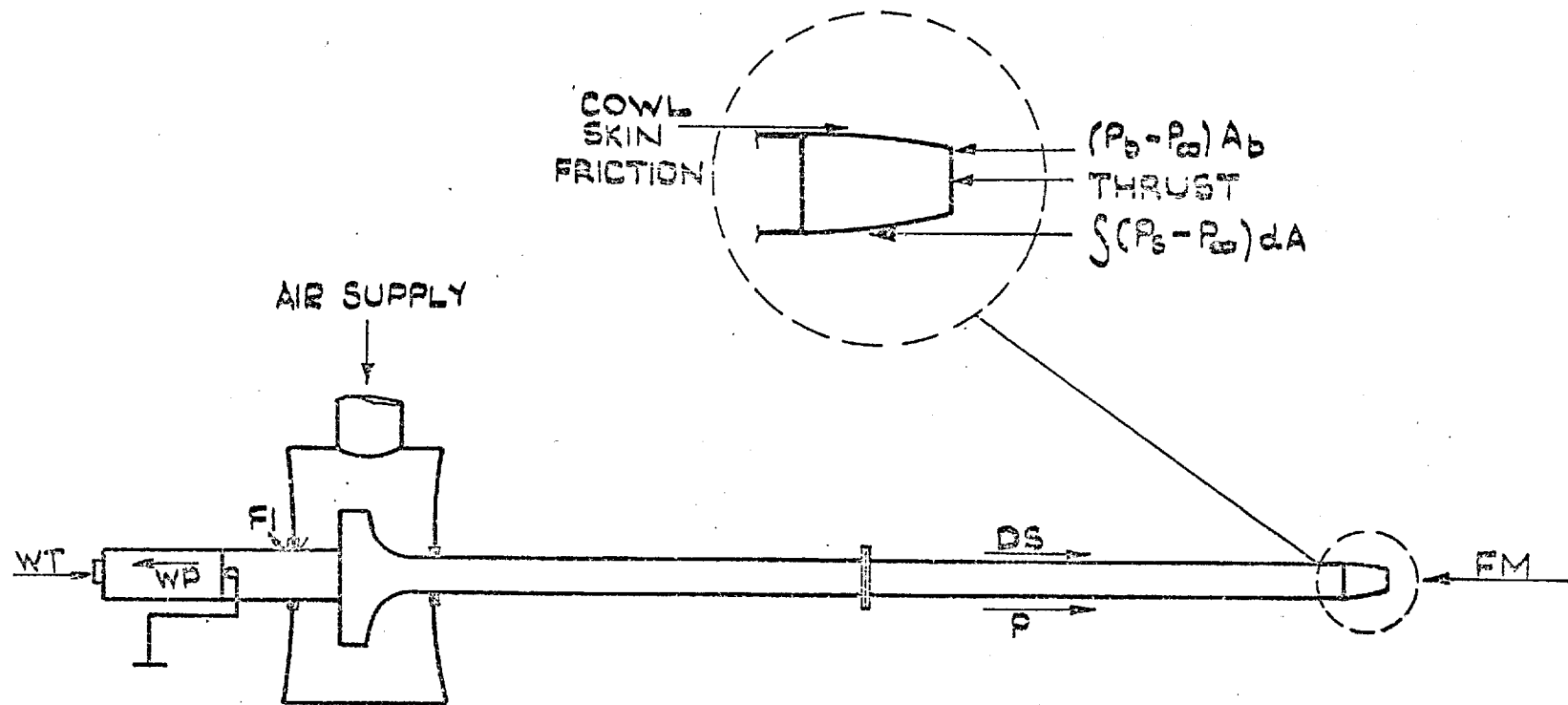
VARIATION OF LEAKAGE FORCE, F_L



STING EFFECTIVE AREA IN
QUICKSCENT CONCENTRATION

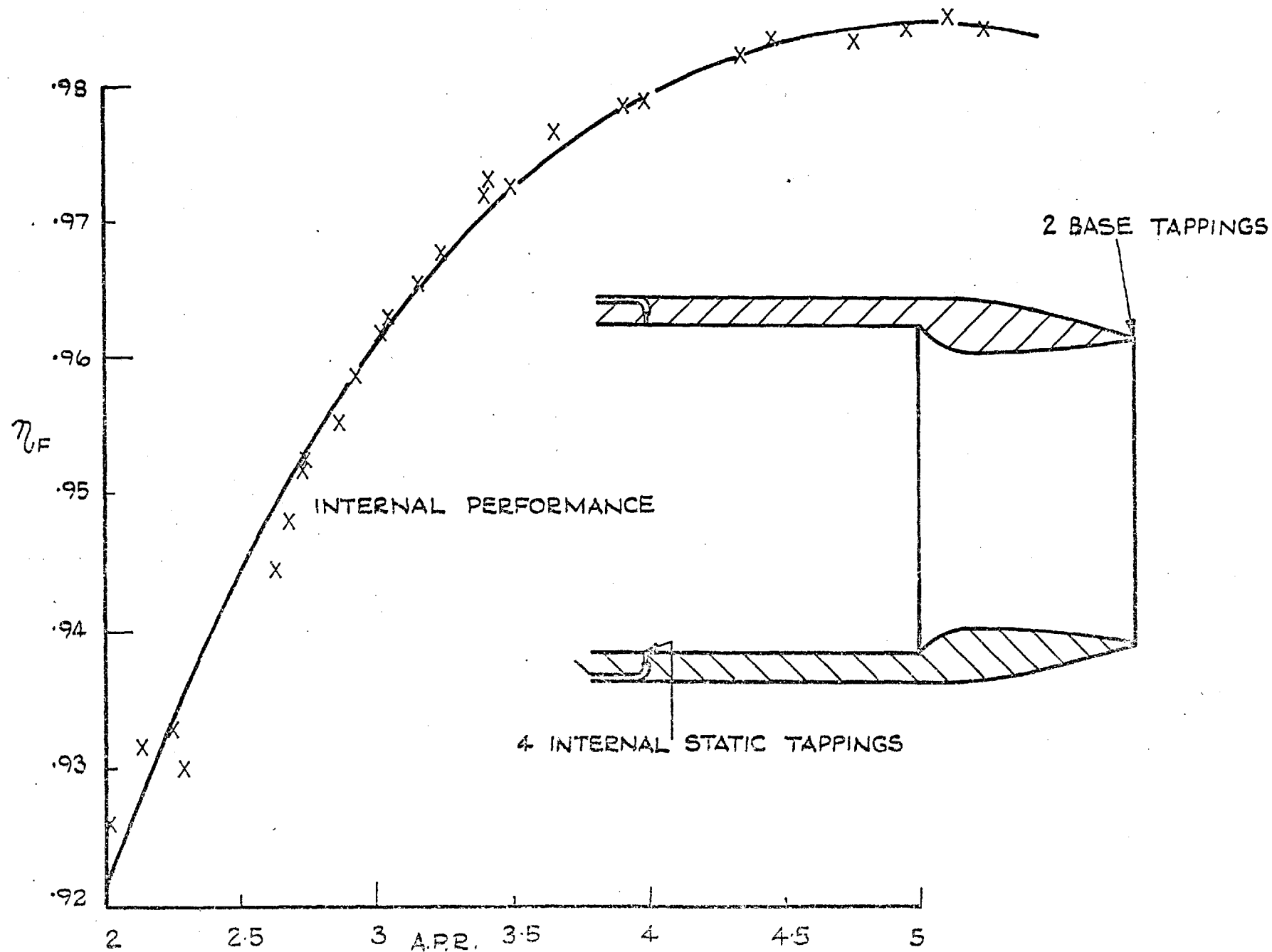


110.21



$$DS = WP + FI - WT - P + (\text{THRUST} + (P_s - P_\infty) A_b + \int (P_s - P_\infty) dA - \text{COWL SKIN FRICTION})$$

THRUST PERFORMANCE OF 5:1 CONVERGENT-DIVERGENT NOZZLE



STING DRAG CALIBRATION

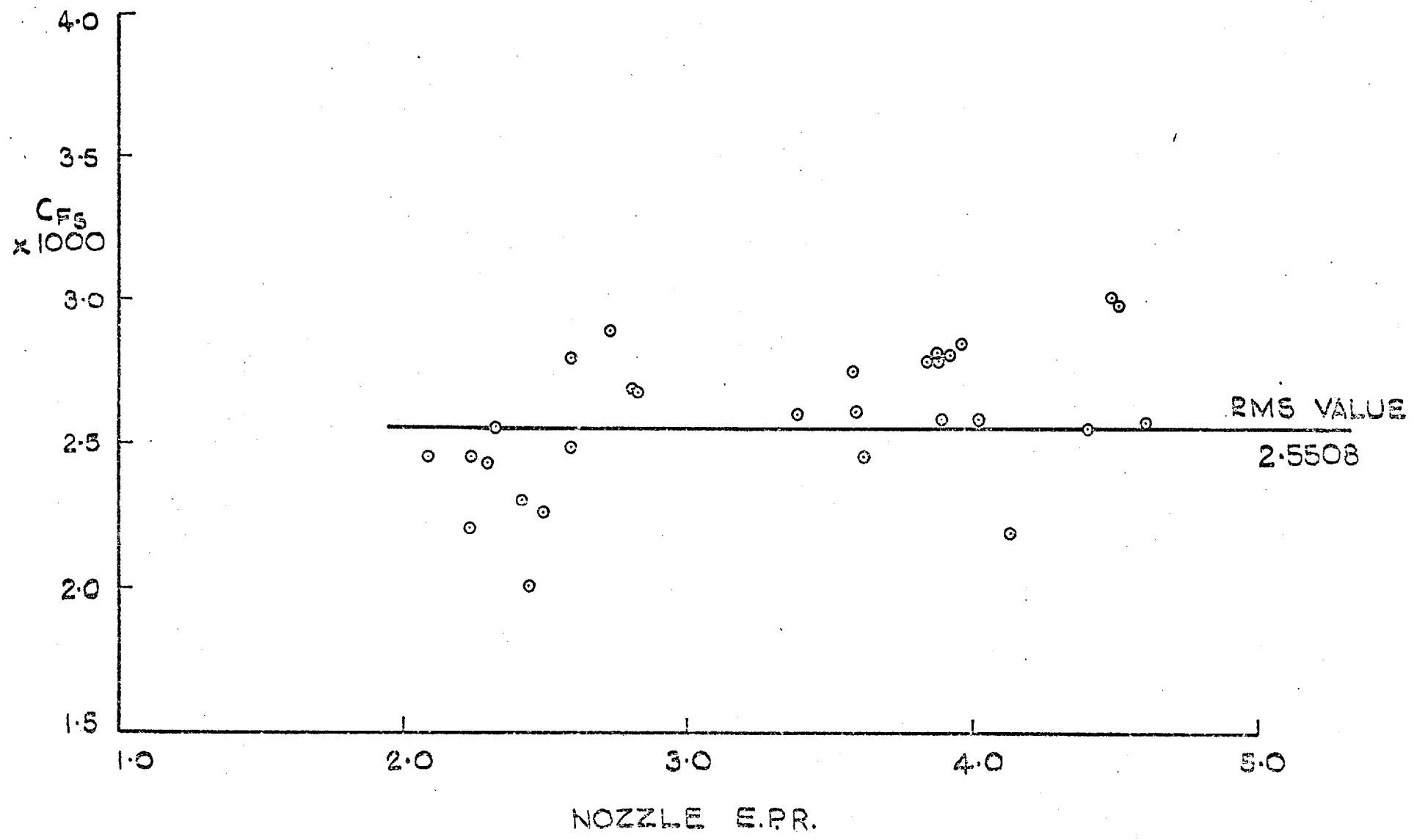


FIG. 24

ALBMEETER CALIBRATION

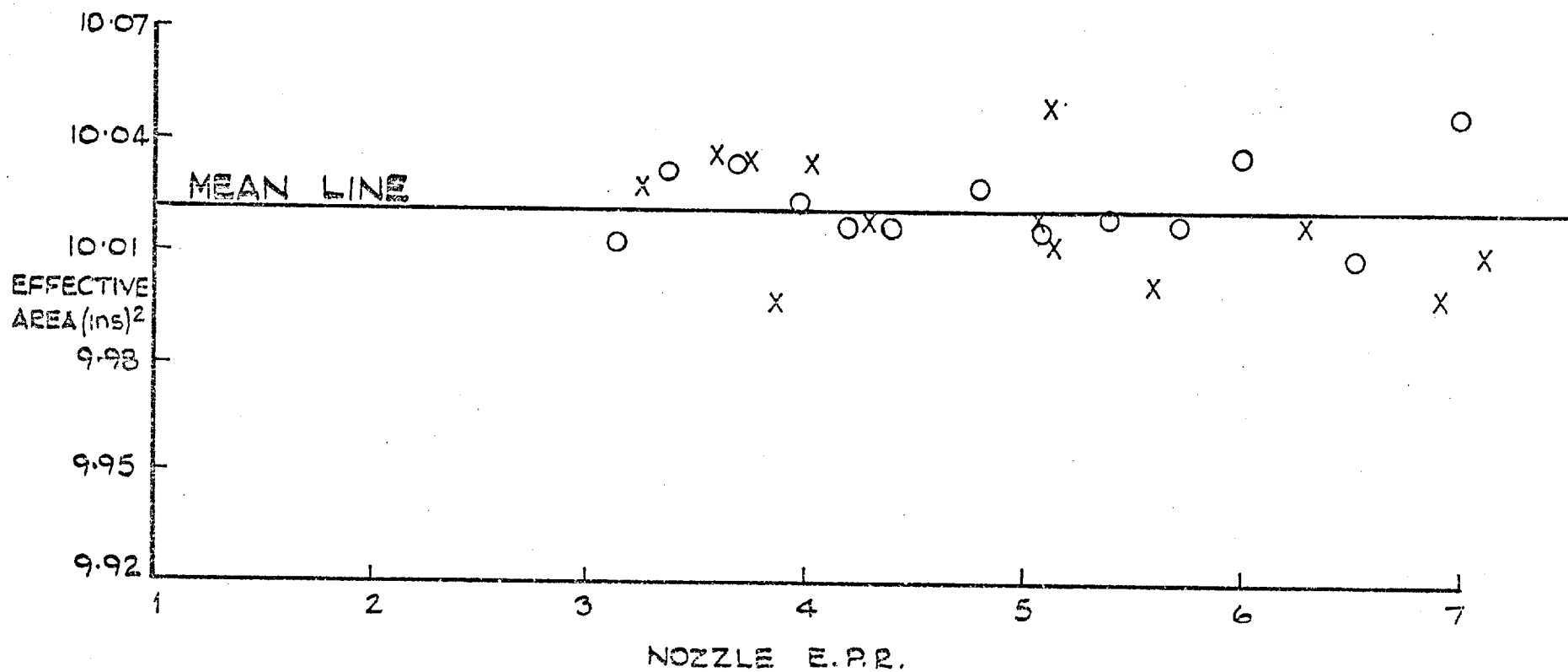
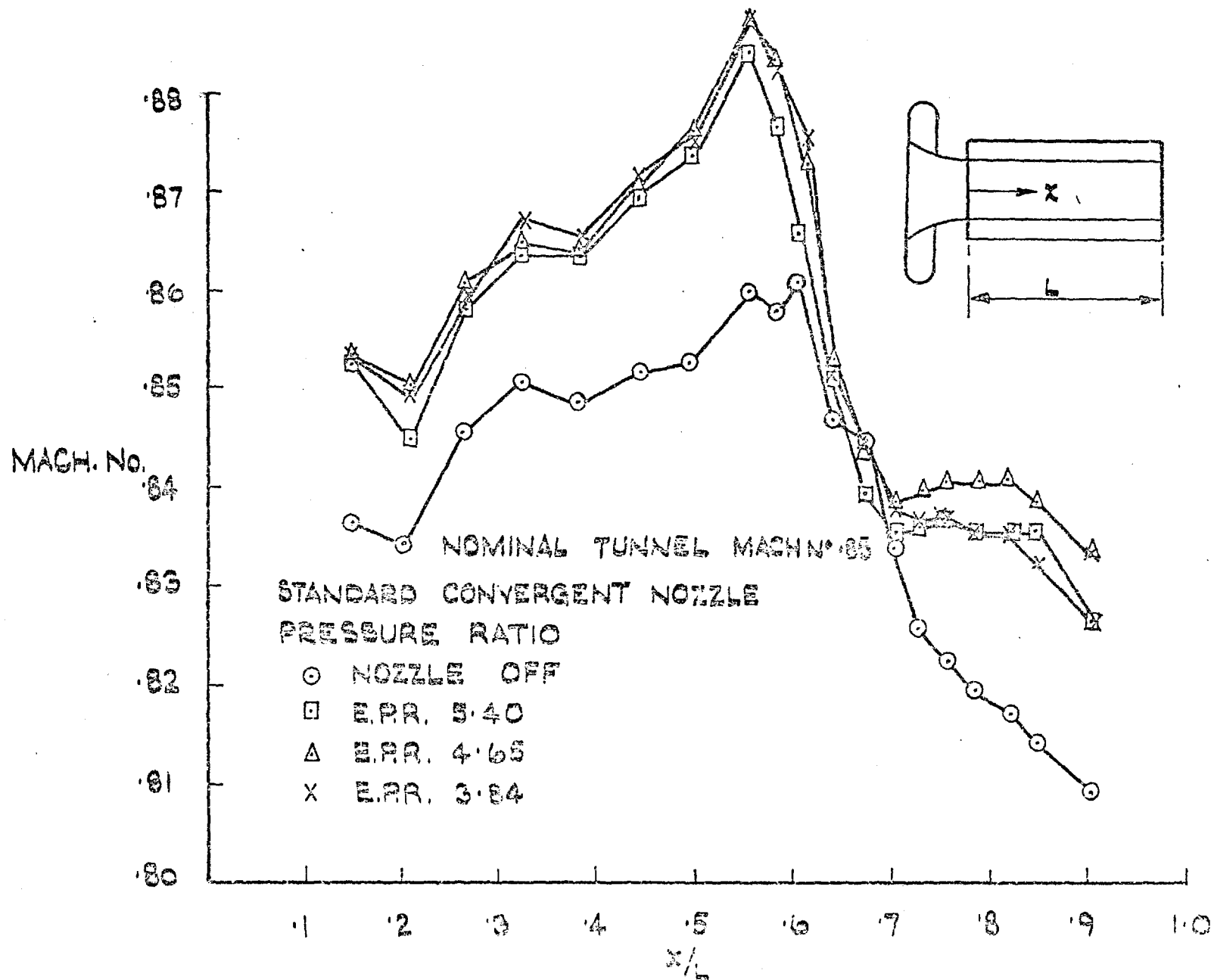
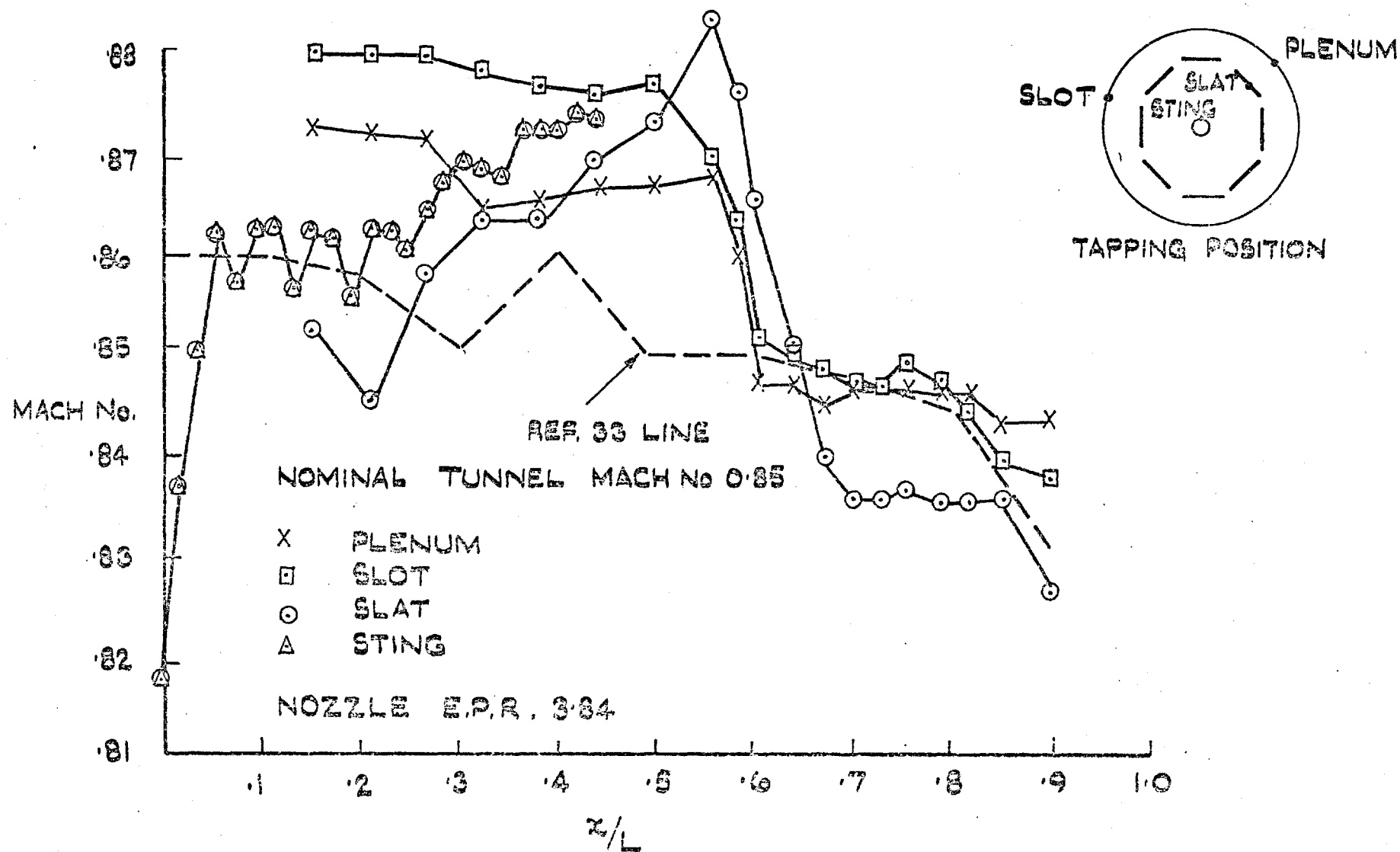


FIG. 2B

TUNNEL SLAT MACH. NO. DISTRIBUTION



TUNNEL PRESSURE DISTRIBUTIONS



MODEL LOAD CELL CALIBRATION

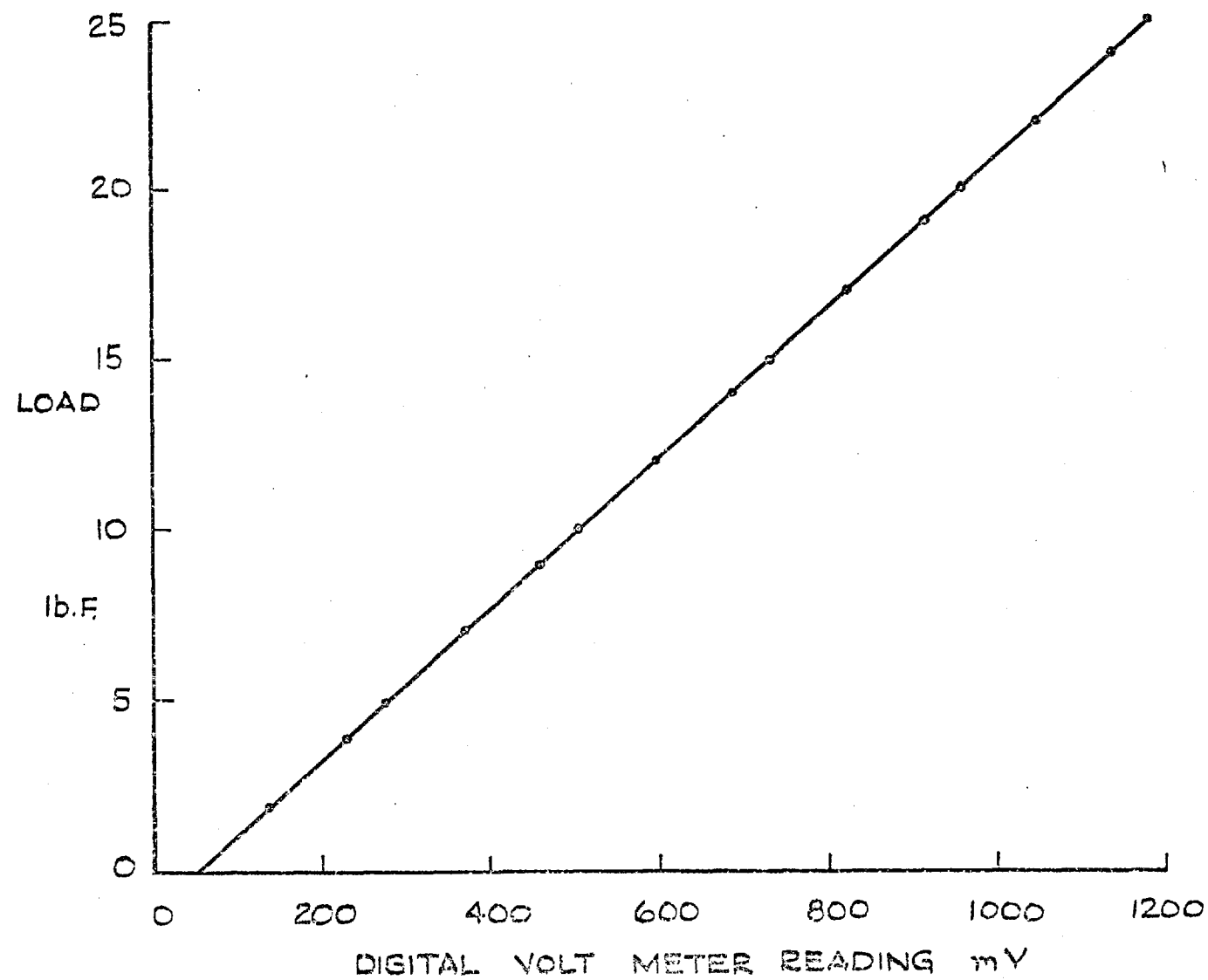
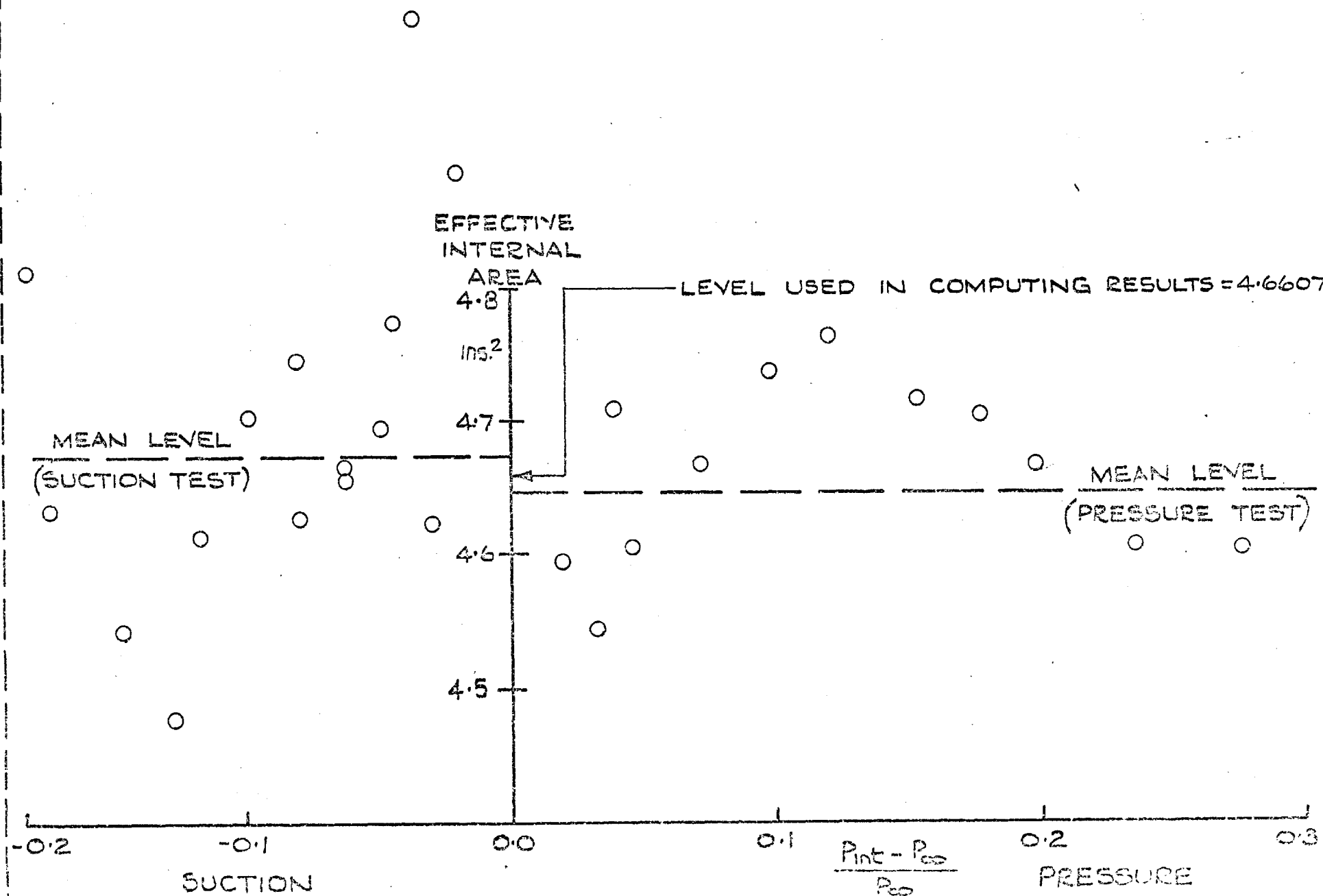


Fig. 2

AFTERBODY INTERNAL AREA DETERMINATION



JET SIMULATOR					
MACH No	COWL	NONE	CONVERGENT	PARALLEL	DIVERGENT
0	10°				
	12°				
	CONICAL				
0.8	10°				
	12°				
	CONICAL				
0.85	10°				

FLOATING AFTERBODY MODEL

MACH.No.	COWL	SYMBOL
0	10°	+
.85	10°	X

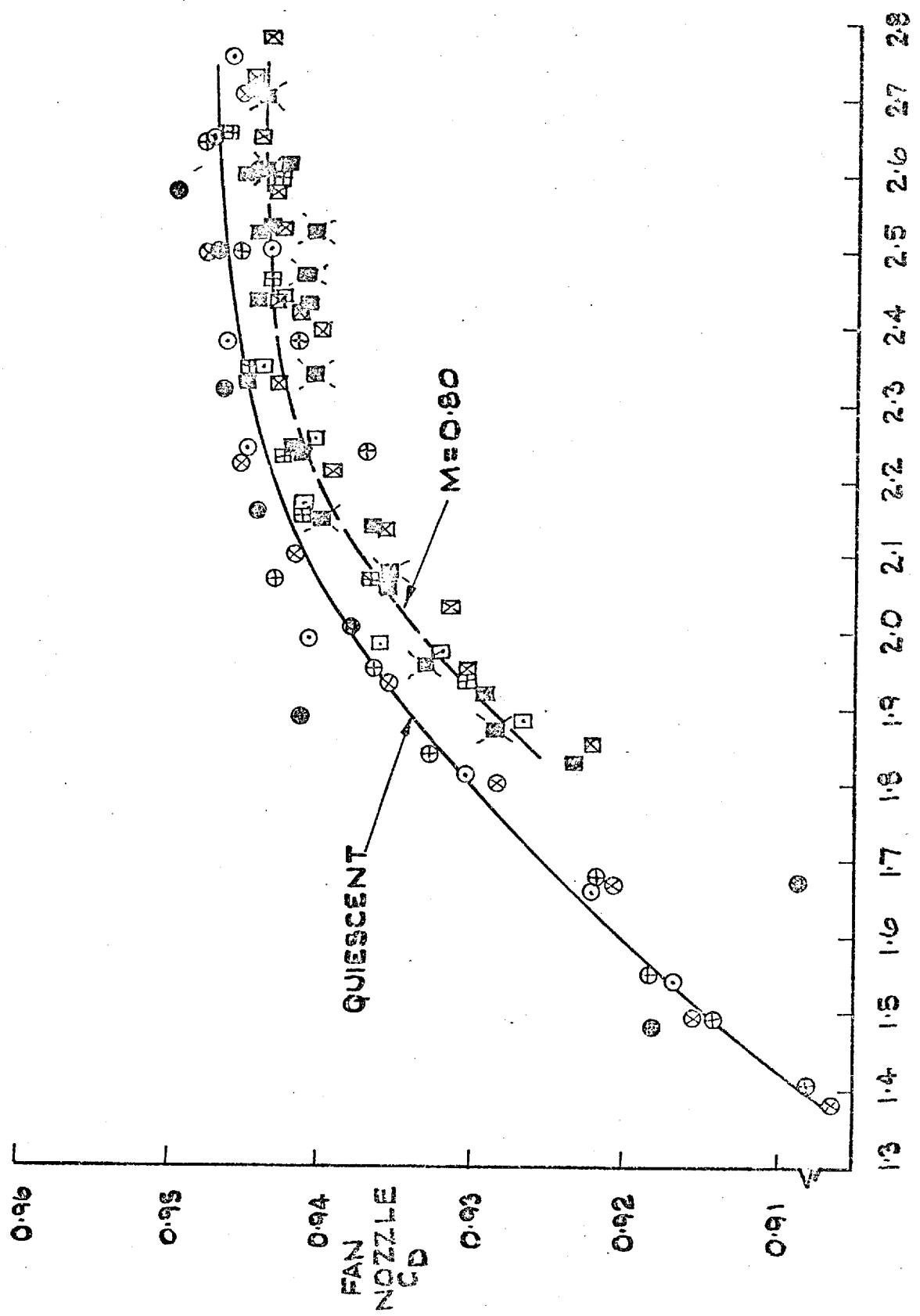
TWO-STREAM MODEL

GRAPHICAL SYMBOLS

RUN NO.	MACH	CONV.	SIMULATED	SYMBOL
168	0	10°	2-STREAM	+
169	0	10°	2-STREAM	+
212	.85	10°	CONV.	⊠
213	.85/.80	10°	CONV.	⊠ & ⊠
214	.80	10°	CONV.	⊠
215	.80	10°	DIV.	⊠
216	.80	10°	DIV.	⊠
217	.80	10°	PARA.	⊠
218	.80	10°	NONE	⊠
219	.80	12°	NONE	⊠
220	.80	12°	NONE	⊠
222	.80	12°	PARA.	⊠
223	.80	12°	DIV.	⊠
224	.80	12°	CONV.	⊠
226	.80	CONICAL	CONV.	⊠
227	.80	CONICAL	DIV.	⊠
228	.80	CONICAL	PARA.	⊠
229	.80	CONICAL	NONE	⊠
235	0	CONICAL	NONE	⊠
236	0	CONICAL	PARA.	⊠
237	0	CONICAL	DIV.	⊠
238	0	CONICAL	DIV.	⊠
239	0	CONICAL	CONV.	⊠
240	0	12°	CONV.	⊠
241	0	12°	DIV.	⊠
242	0	12°	PARA.	⊠
243	0	12°	NONE	⊠
244	0	10°	NONE	⊠
245	0	10°	PARA.	⊠
246	0	10°	DIV.	⊠
247	0	10°	CONV.	⊠
250	.85	10°	2-STREAM	X
251	.85	10°	2-STREAM	X

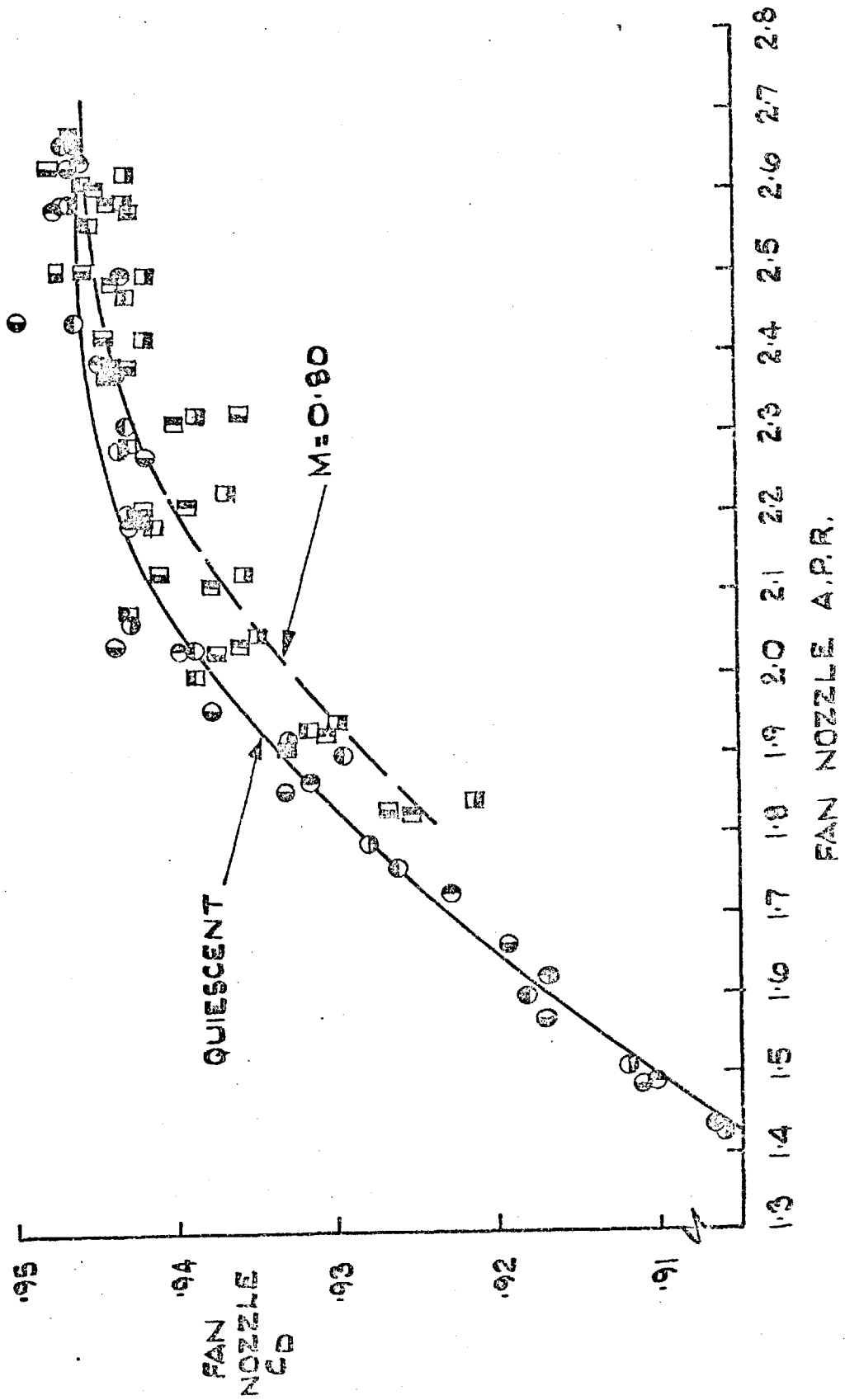
TEST RUN IDENTIFICATION

FIG. 32



FAN NOZZLE A.P.R.

FAN NOZZLE DISCHARGE COEFFICIENT
10° FAN COWL



FAN NOZZLE DISCHARGE COEFFICIENT

12° FAN COWL

FAN NOZZLE DISCHARGE COEFFICIENT, CONICAL FAN CONVL.

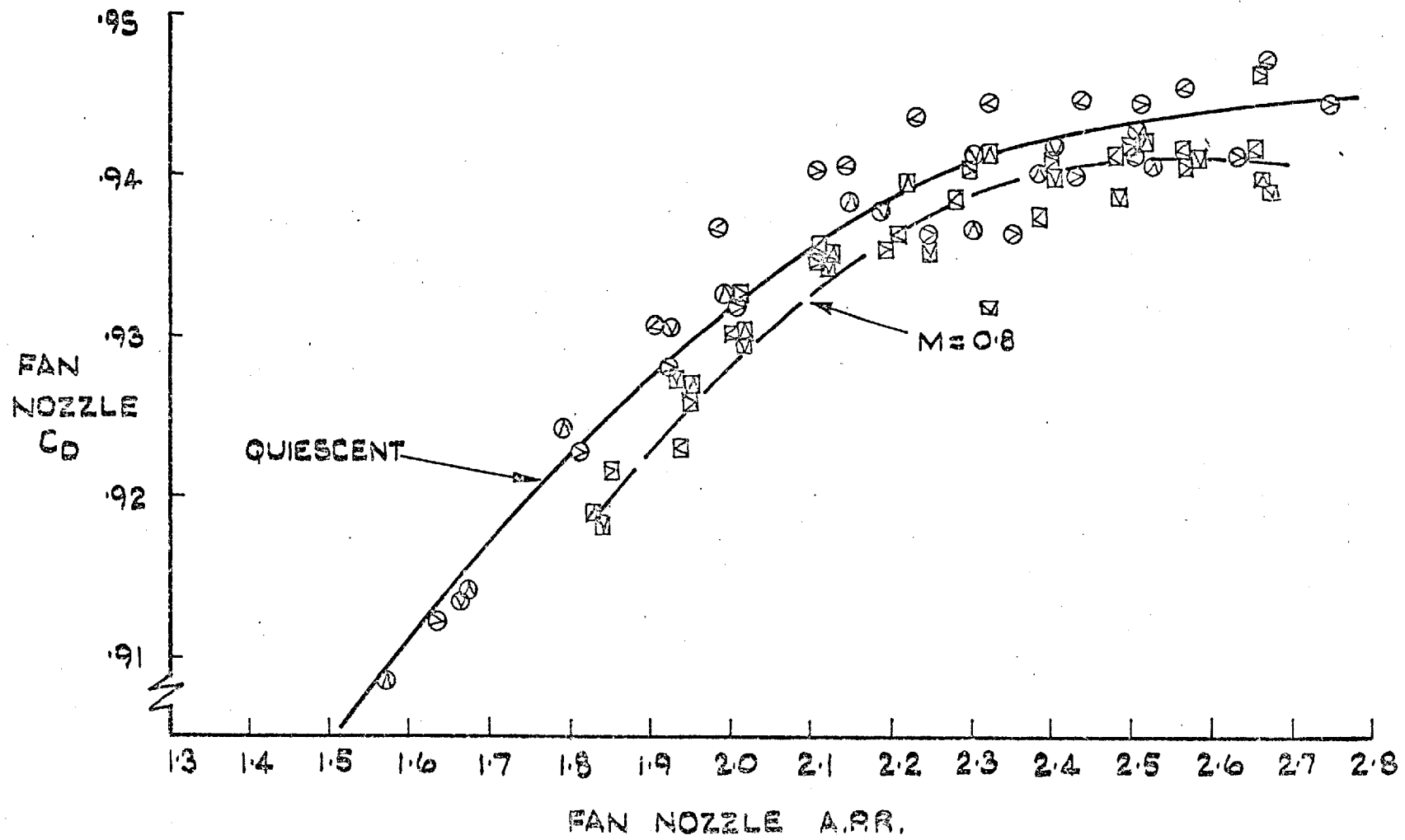
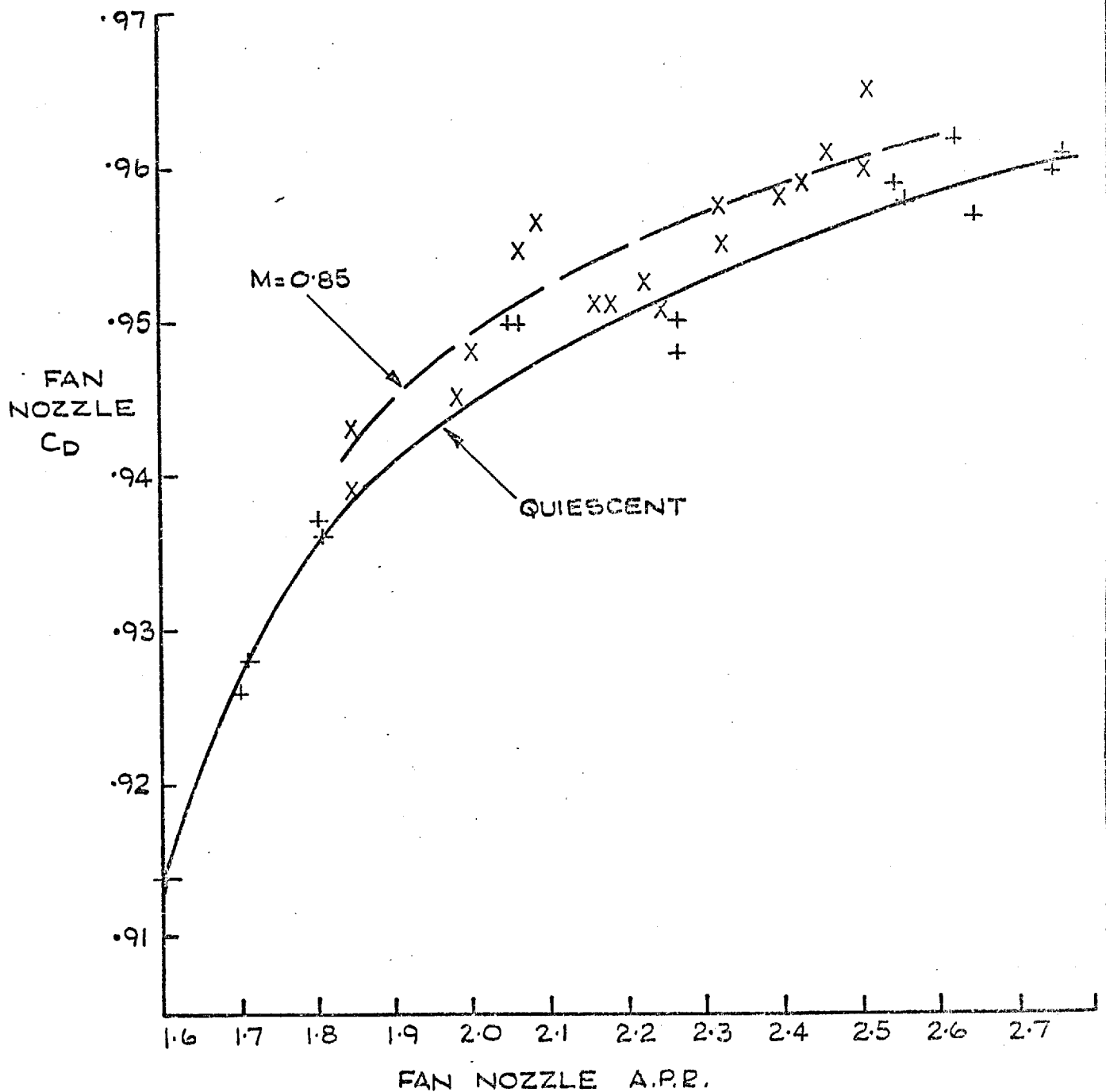


FIG. 34



TWO-STREAM MODEL FAN NOZZLE
DISCHARGE COEFFICIENT

$$C_{DFA} = \frac{FA}{\frac{\alpha}{2} P_{\infty} M_j^2 A_{proj}}$$

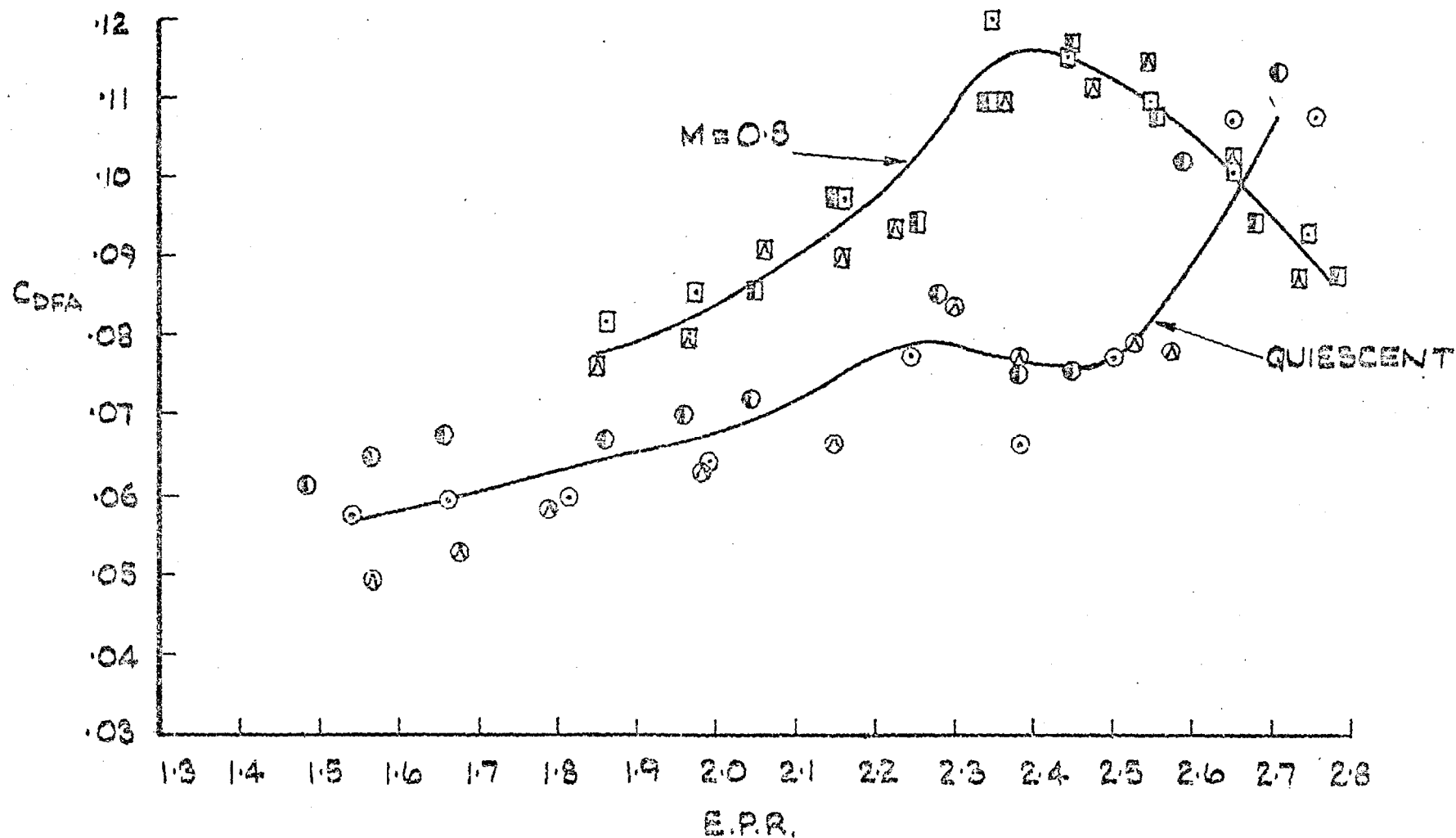
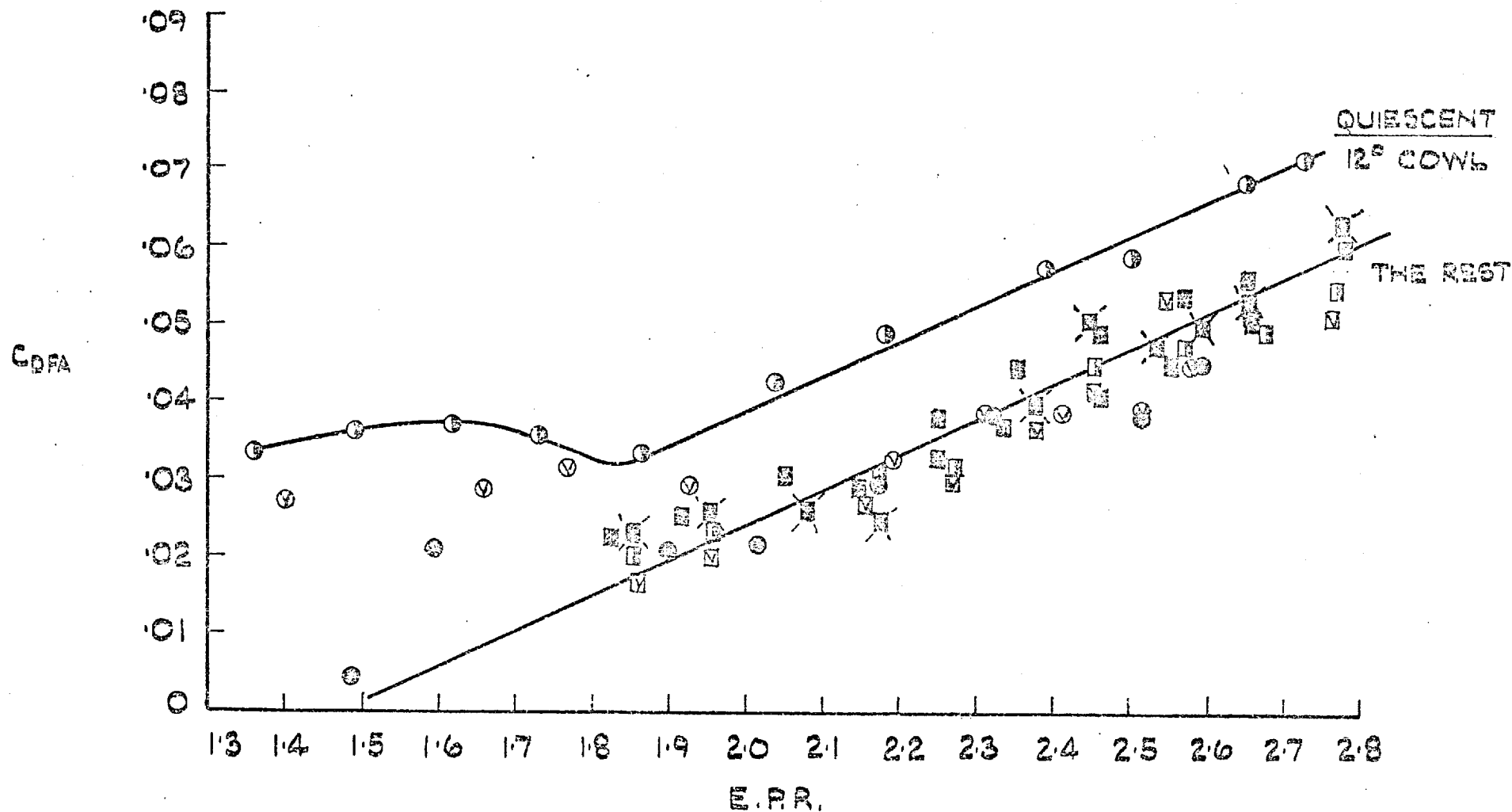


FIG. 66

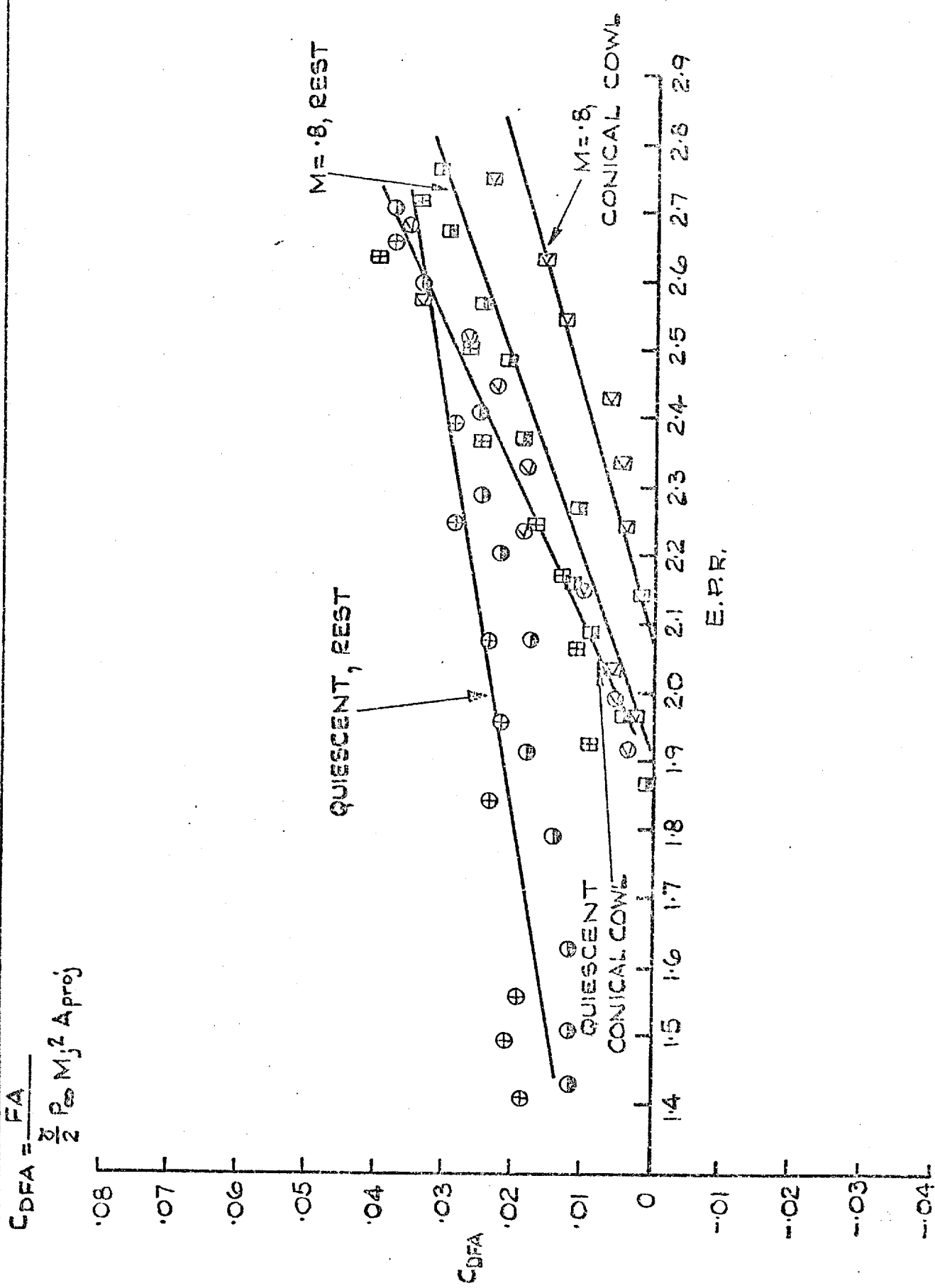
ATEROBY FORCE COEFFICIENT

ALTERBODY FORCE COEFFICIENT,
CONCURRENT SIMULATOR

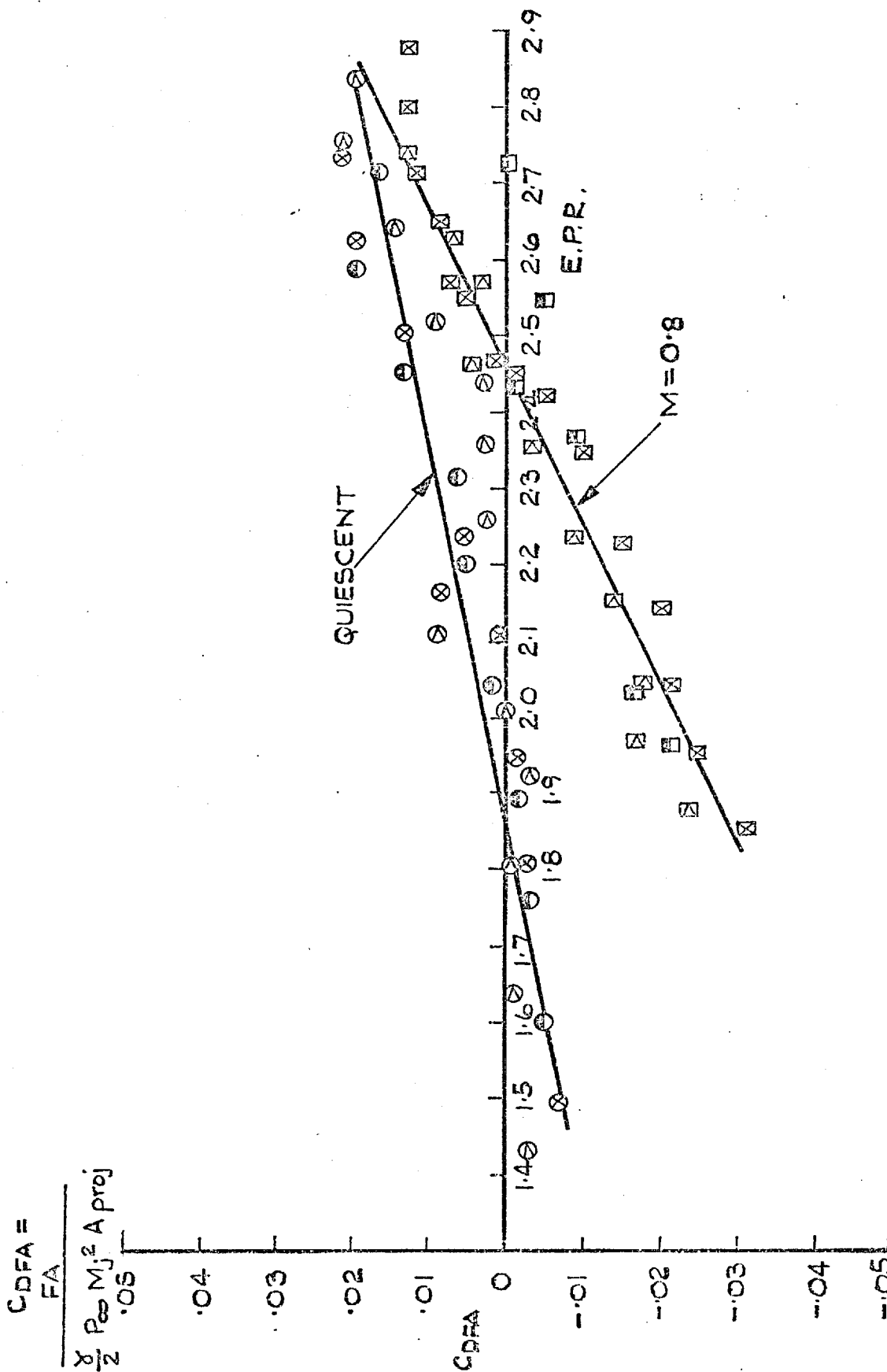
$$C_{DFA} = \frac{FA}{\frac{\pi}{2} P_{\infty} M_j^2 A_{proj}}$$



11.1.107



AFTERBODY FORCE COEFFICIENT,
PARALLEL SIMULATOR



AFTERBODY FORCE COEFFICIENT,
DIVERGENT SIMULATOR

AEROBODY PRESSURE FORCE COEFFICIENT END PLATE

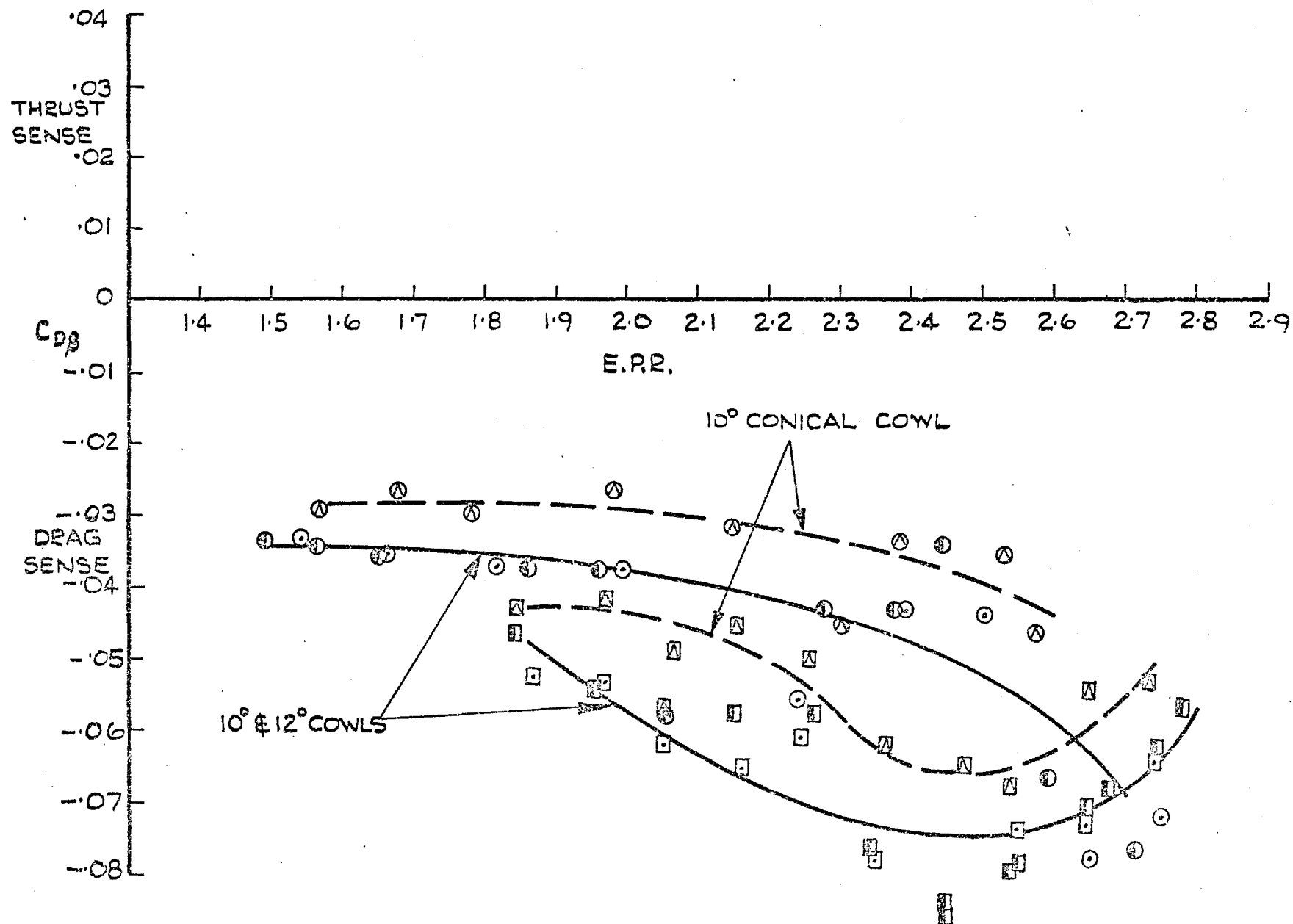
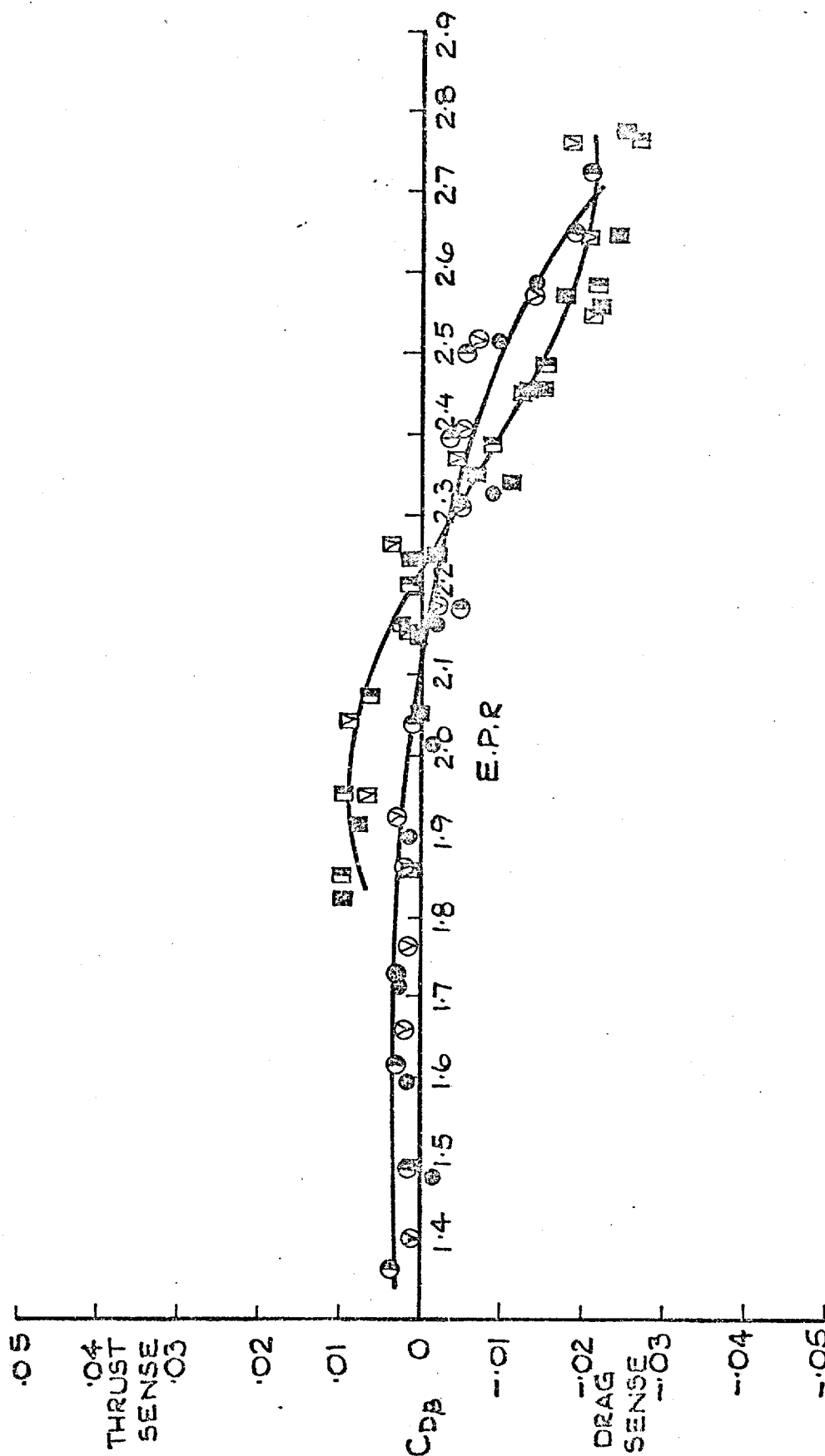


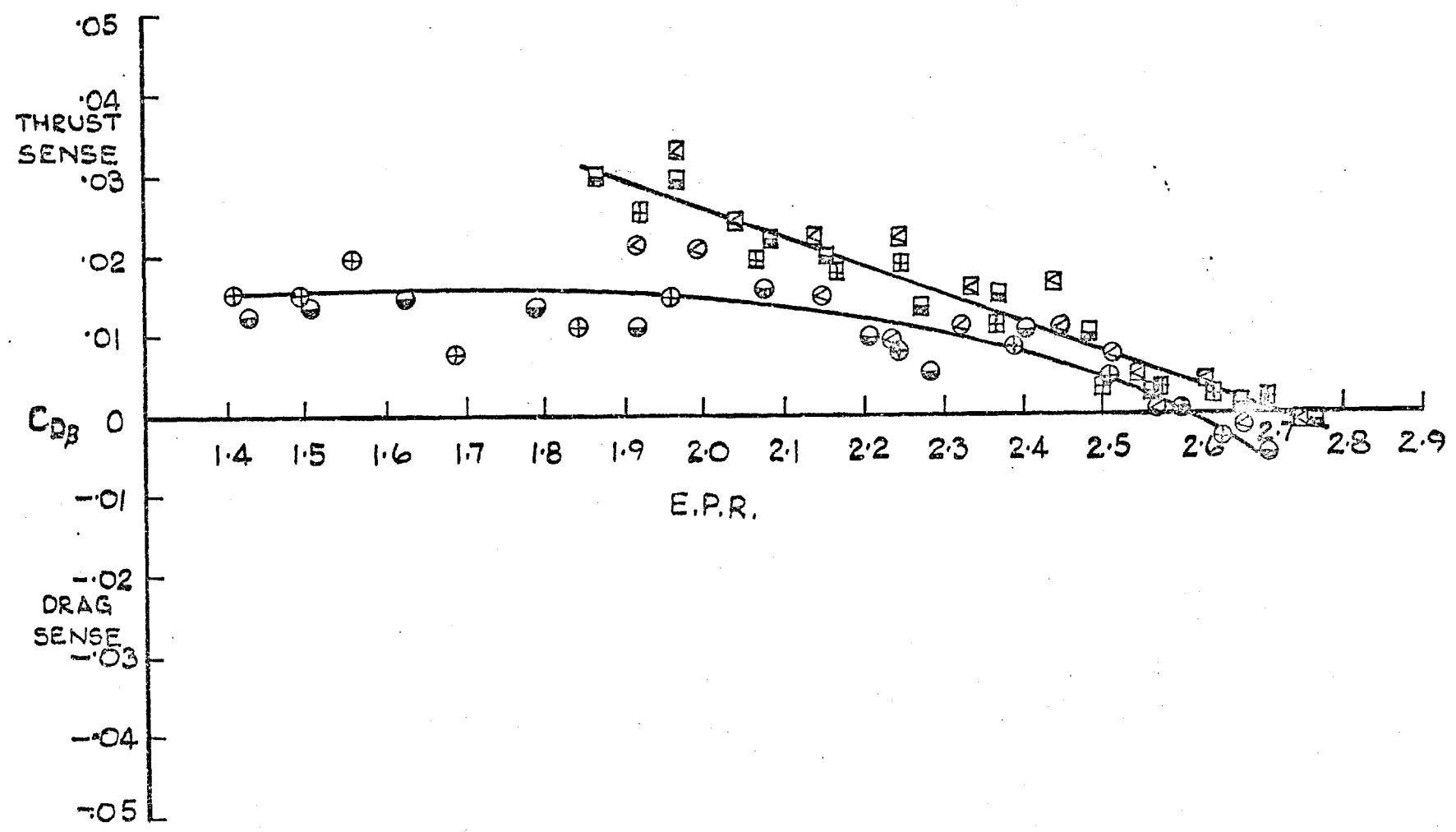
FIG. 40

FIG. 41



AFTERBODY PRESSURE FORCE
COEFFICIENT, CONVERGENT SIMULATOR

AFTERBODY PRESSURE FORCE COEFFICIENT
PARALLEL SIMULATOR



AFTERBODY PRESSURE FORCE COEFFICIENT,
DIVERGENT SIMULATOR

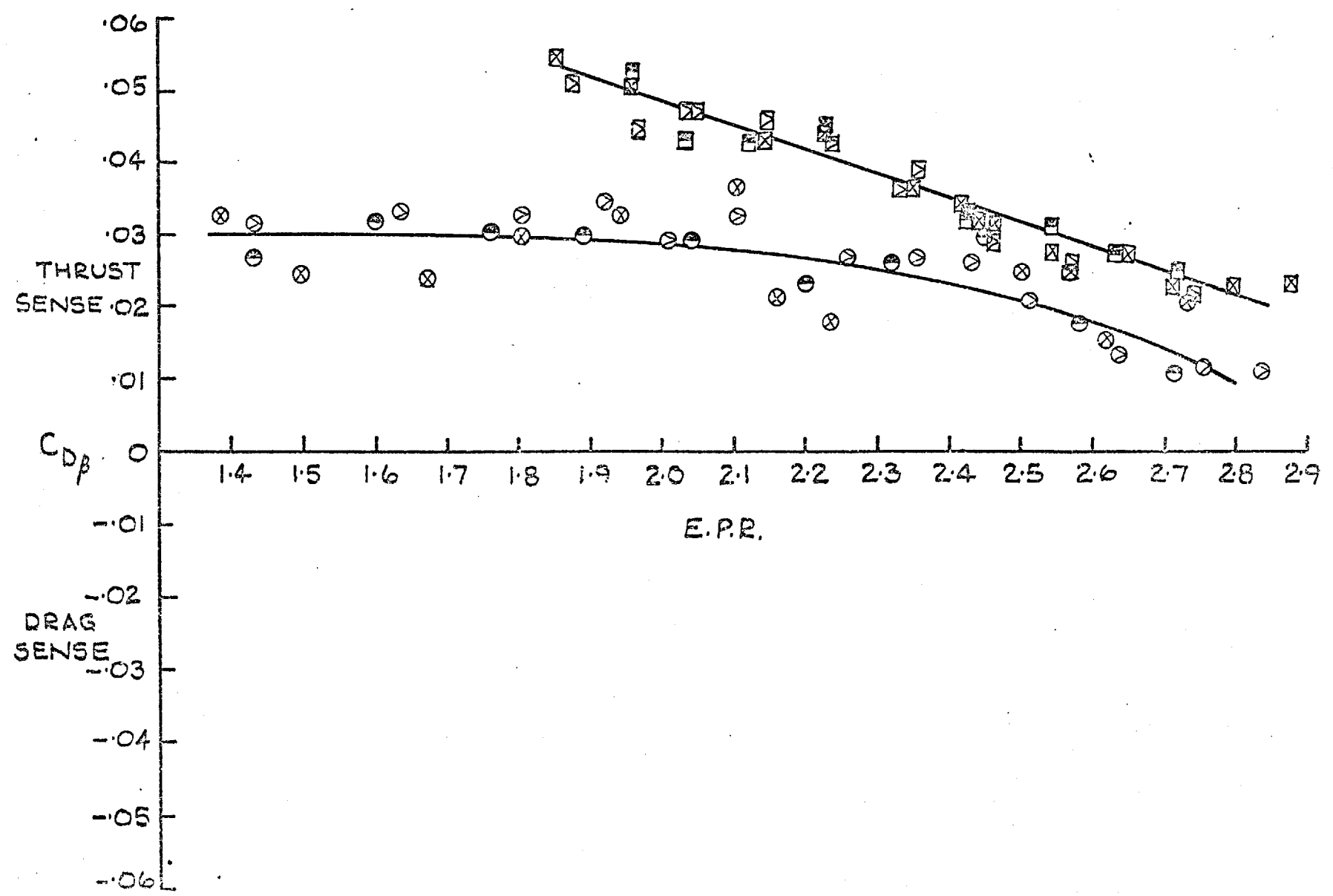
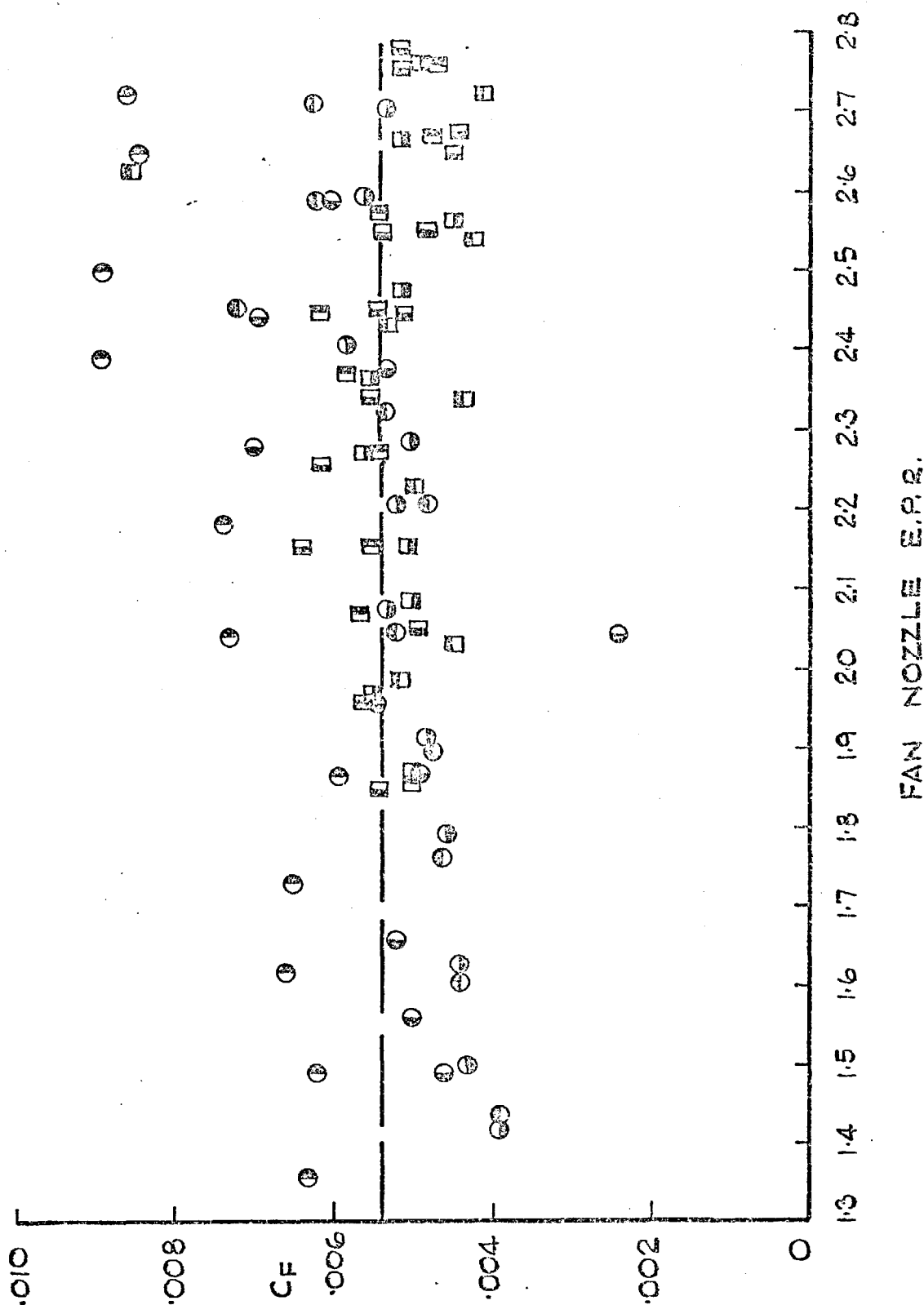
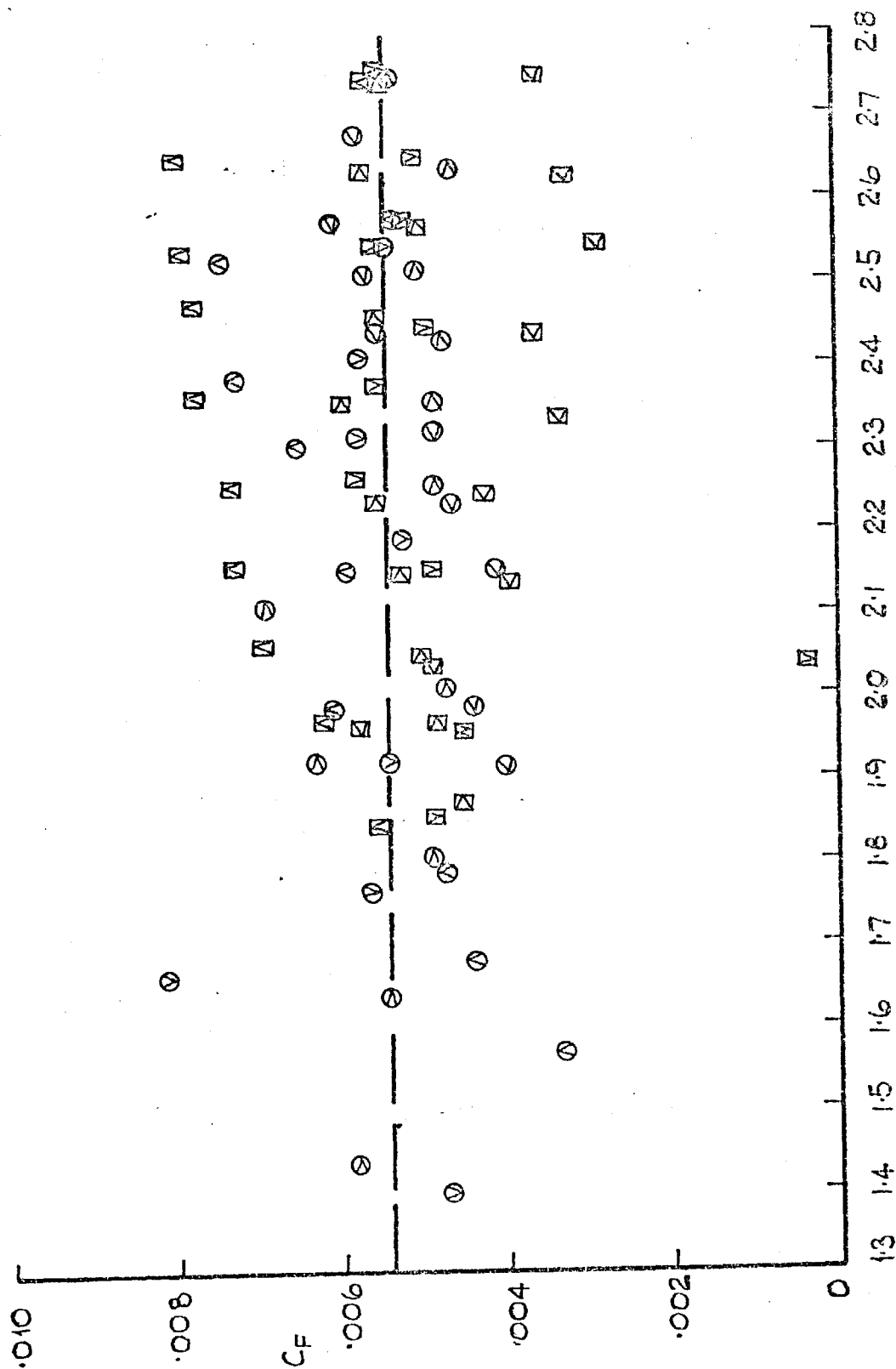


FIG. 45



AFTERBODY SKIN FRICTION
COEFFICIENT, 12° FAN COWL



FAN NOZZLE E.P.R.

AFTERBODY SKIN FRICTION
COEFFICIENT, CONICAL FAN COWL

AFTERBODY SKIN FRICTION COEFFICIENT,
10° FAN CONVL.

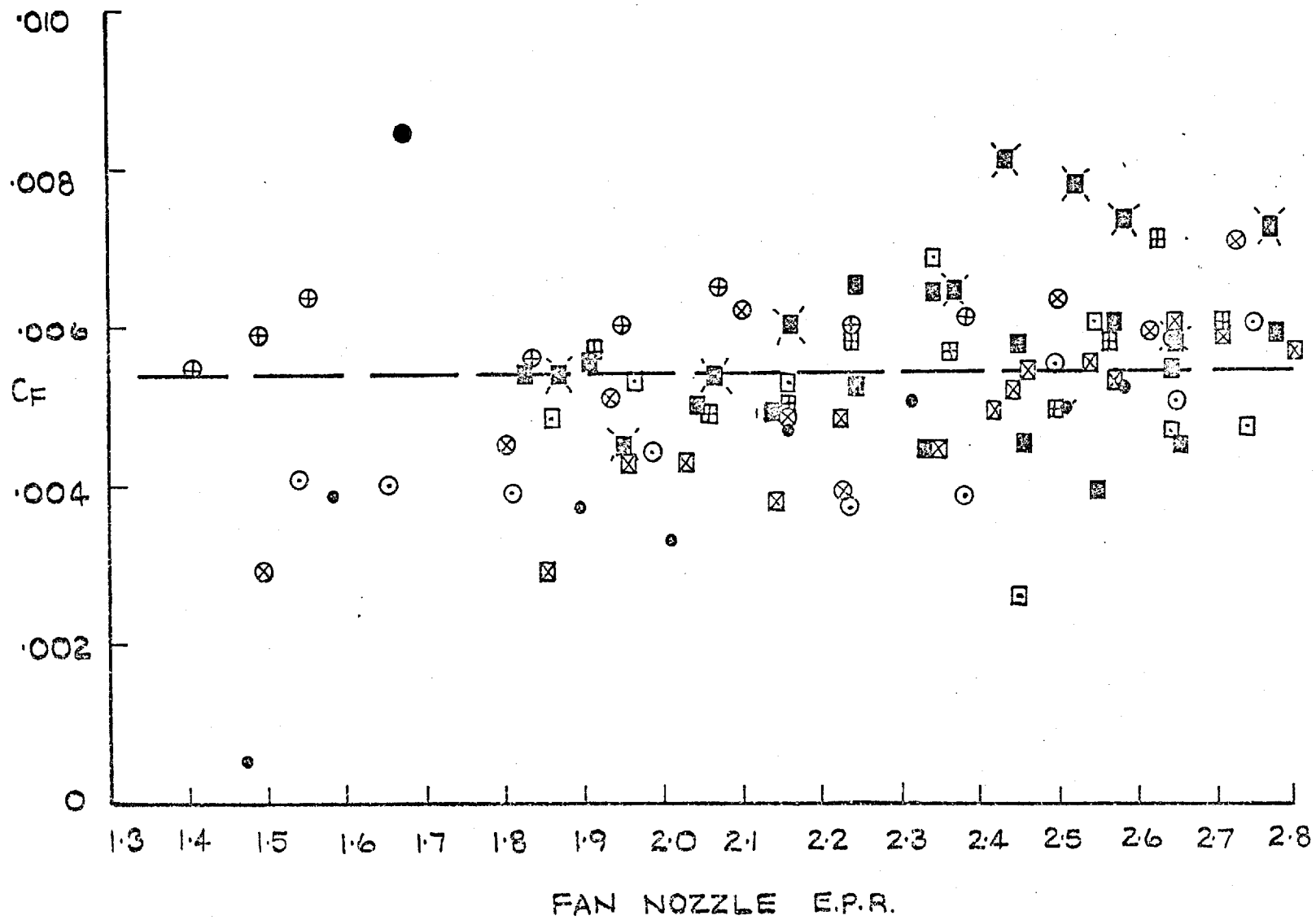
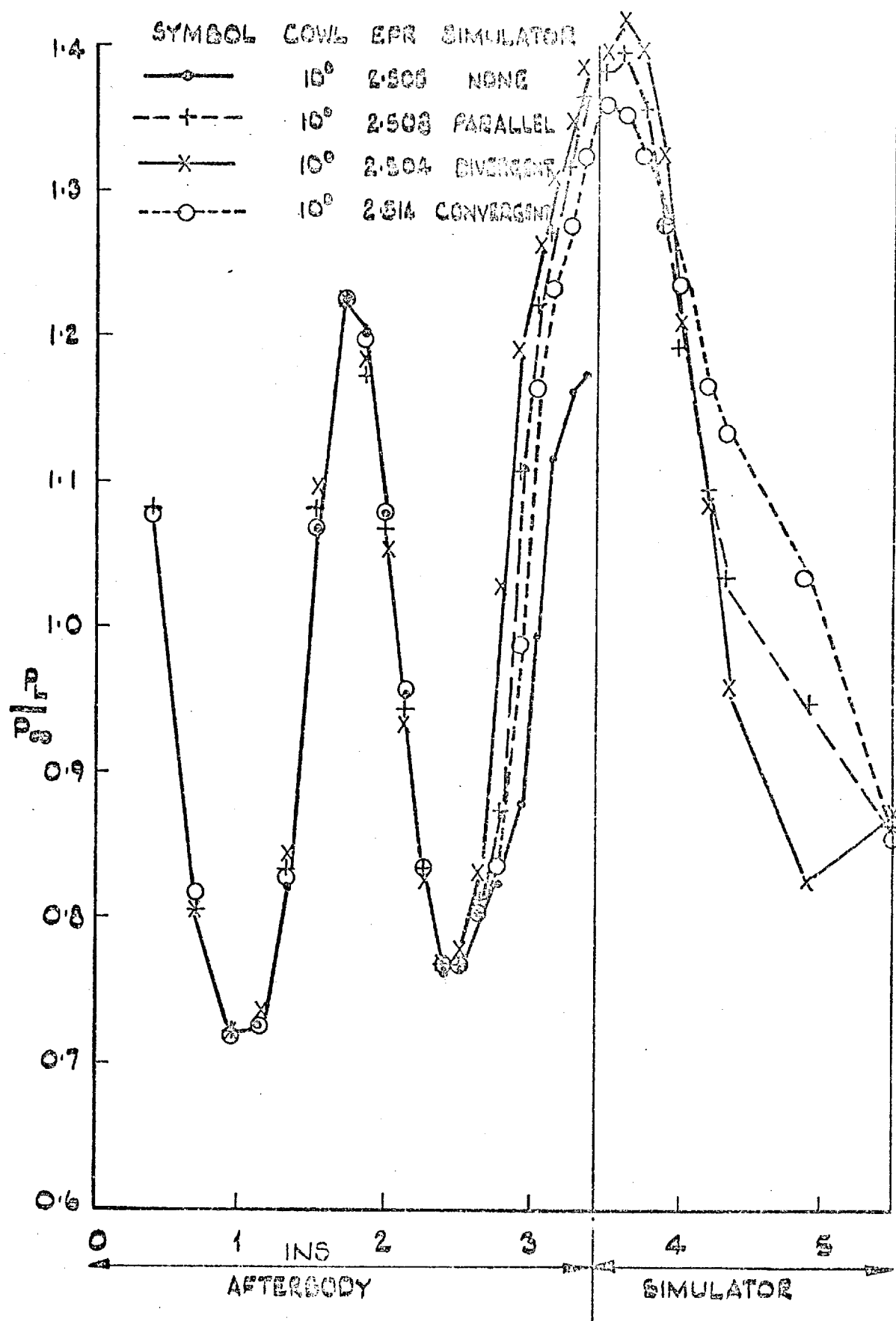
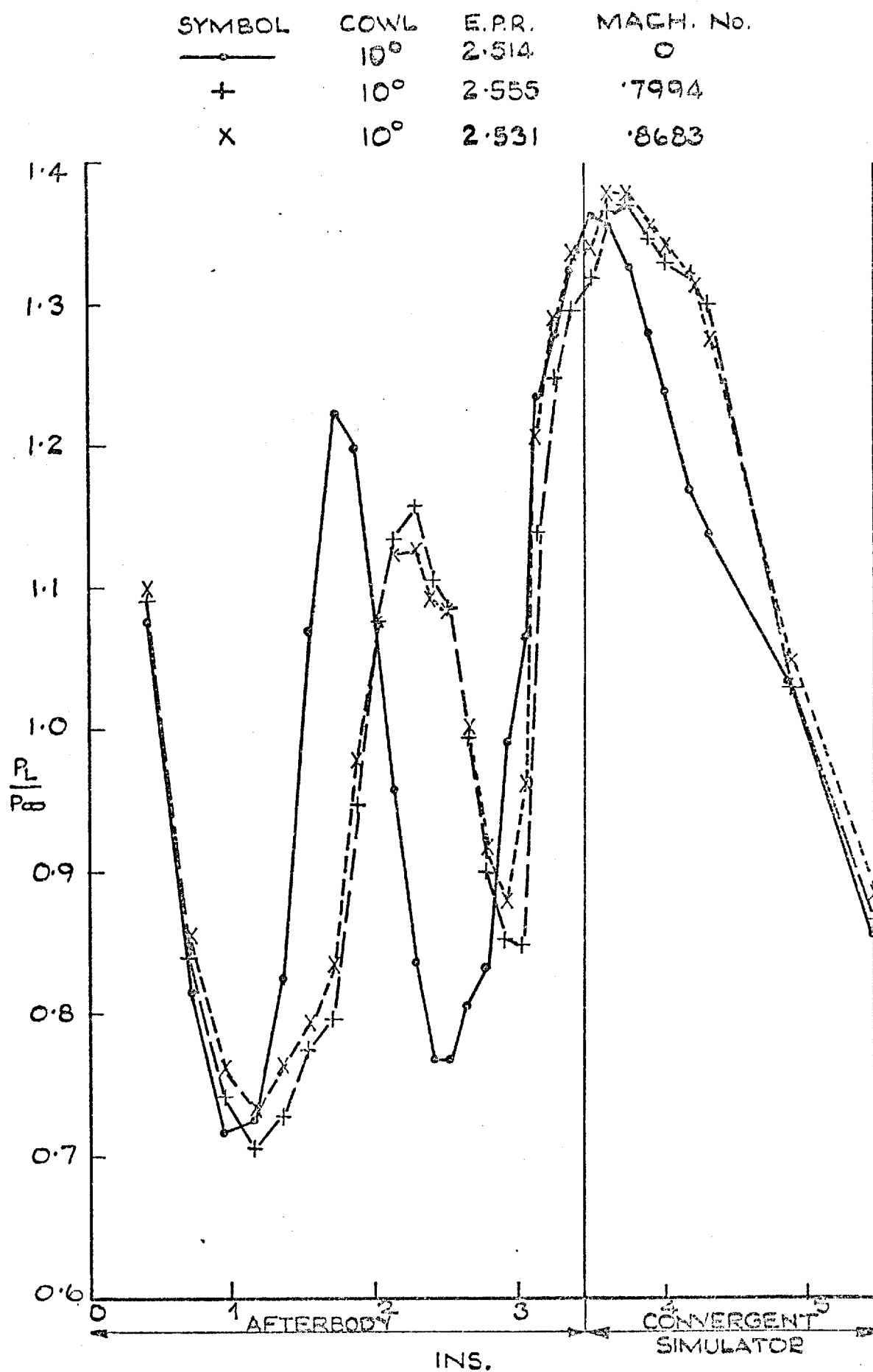


FIG. 4.6



EFFECT OF SIMULATORS ON PRESSURE DISTRIBUTIONS, QUIESCENT

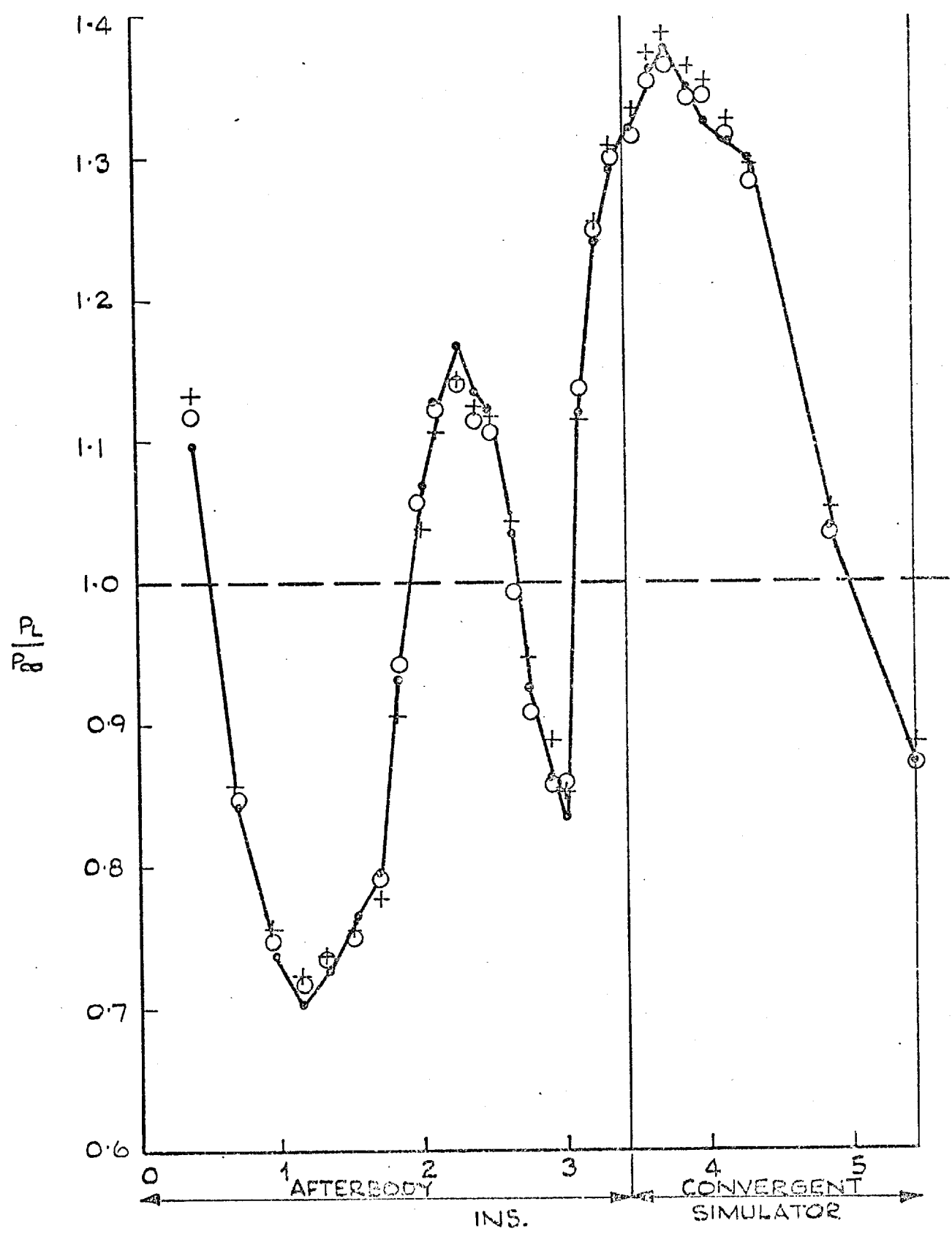
FIG. 48



EFFECT OF EXTERNAL FLOW ON
PRESSURE DISTRIBUTIONS,
CONVERGENT SIMULATOR

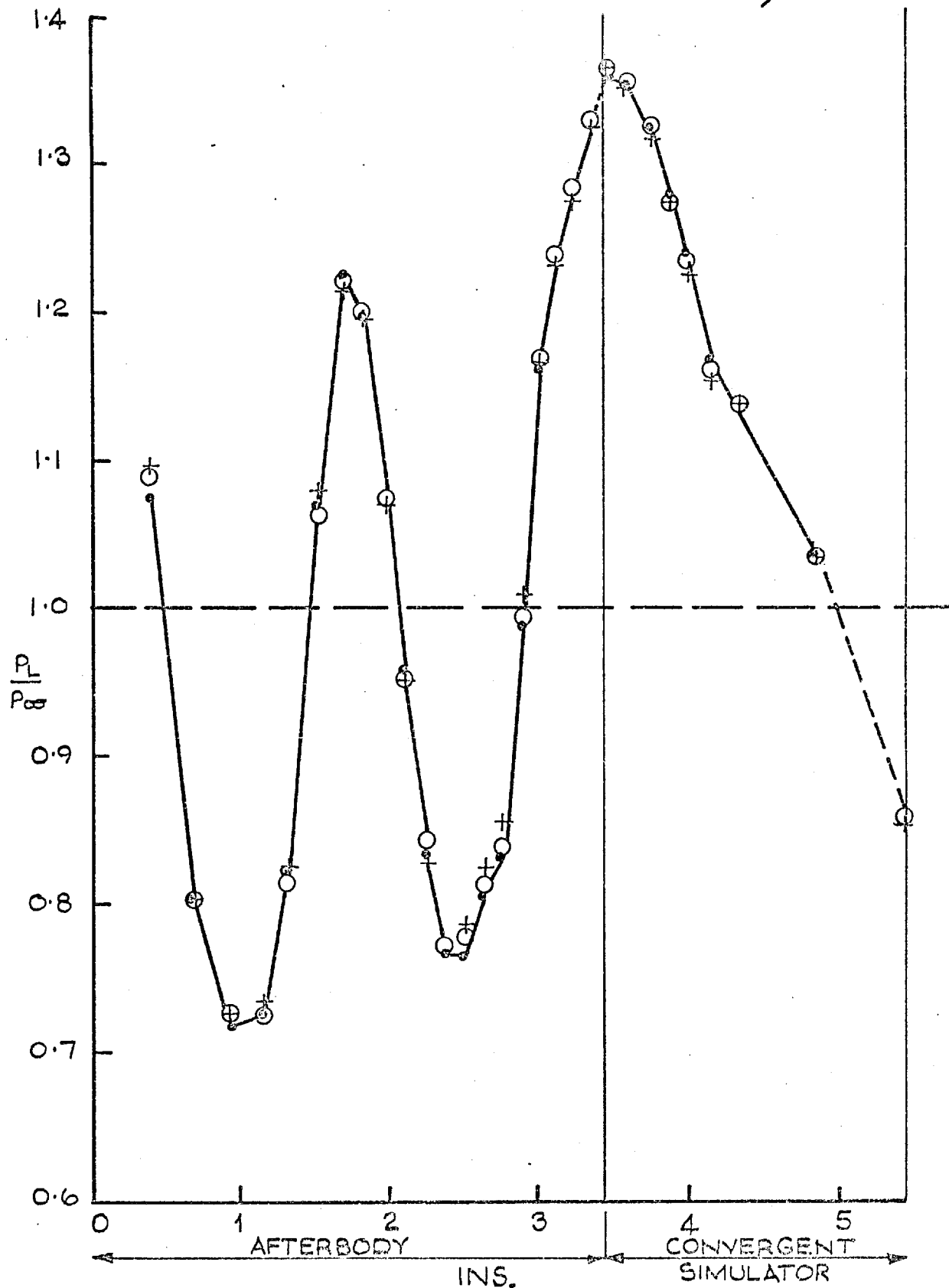
FIG. 49

SYMBOL	COWL	E.P.R.	SIMULATOR
—●—	10°	2.573	CONVERGENT
+	12°	2.567	CONVERGENT
○	CONICAL	2.544	CONVERGENT



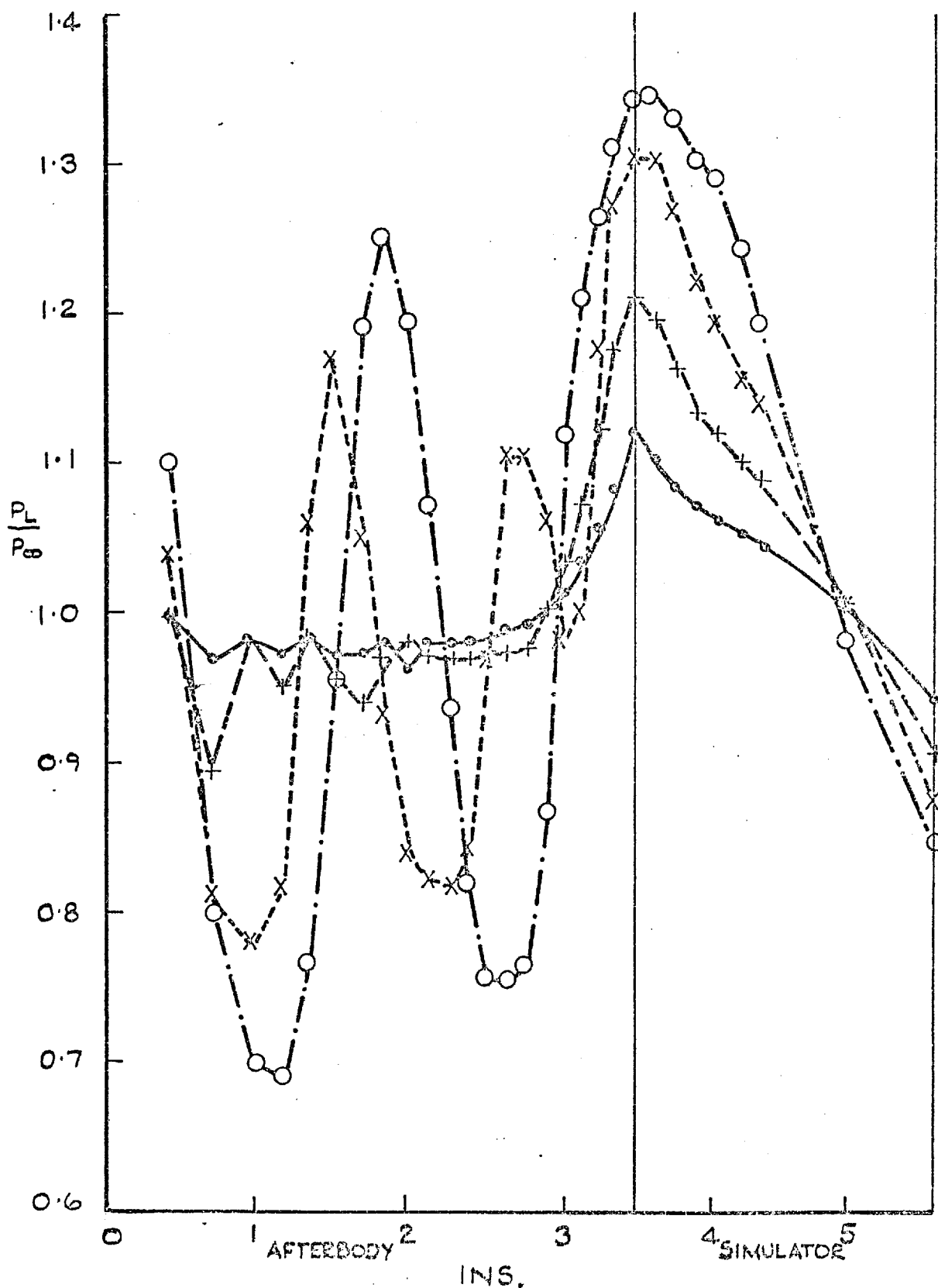
EFFECT OF FAN COWLS ON PRESSURE DISTRIBUTIONS, MACH No. 0.8

SYMBOL	COWL	E.P.R.	SIMULATOR	} QUIESCENT
—●—	10°	2.514	CONVERGENT	
+	12°	2.500	CONVERGENT	
○	CONICAL	2.513	CONVERGENT	



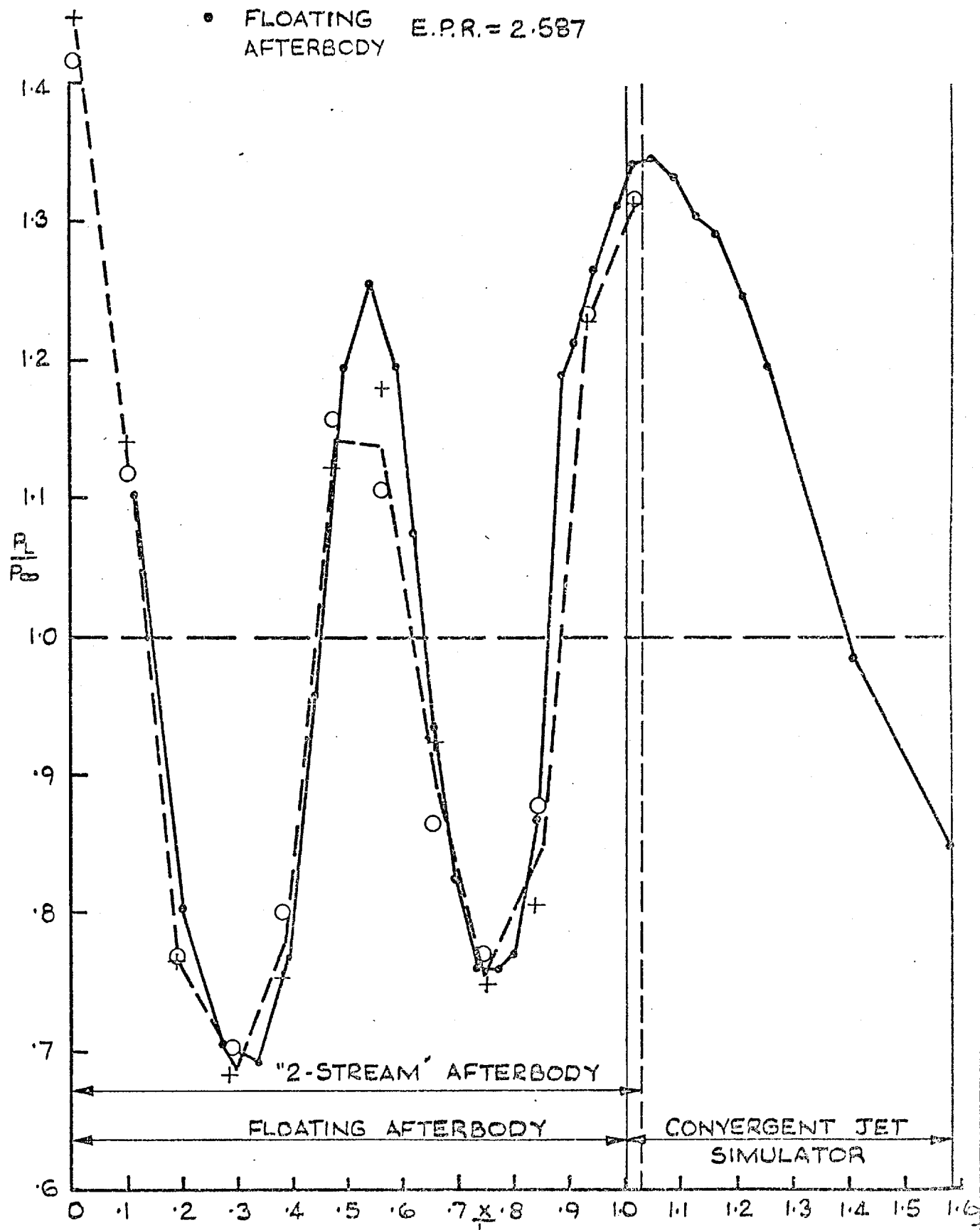
EFFECT OF FAN COWLS ON
PRESSURE DISTRIBUTIONS, QUIESCENT

SYMBOL	COWL	E.P.R.	SIMULATOR
—○—	10°	1.431	CONVERGENT
—+—	10°	1.899	CONVERGENT
—x—	10°	2.325	CONVERGENT
—○—	10°	2.587	CONVERGENT

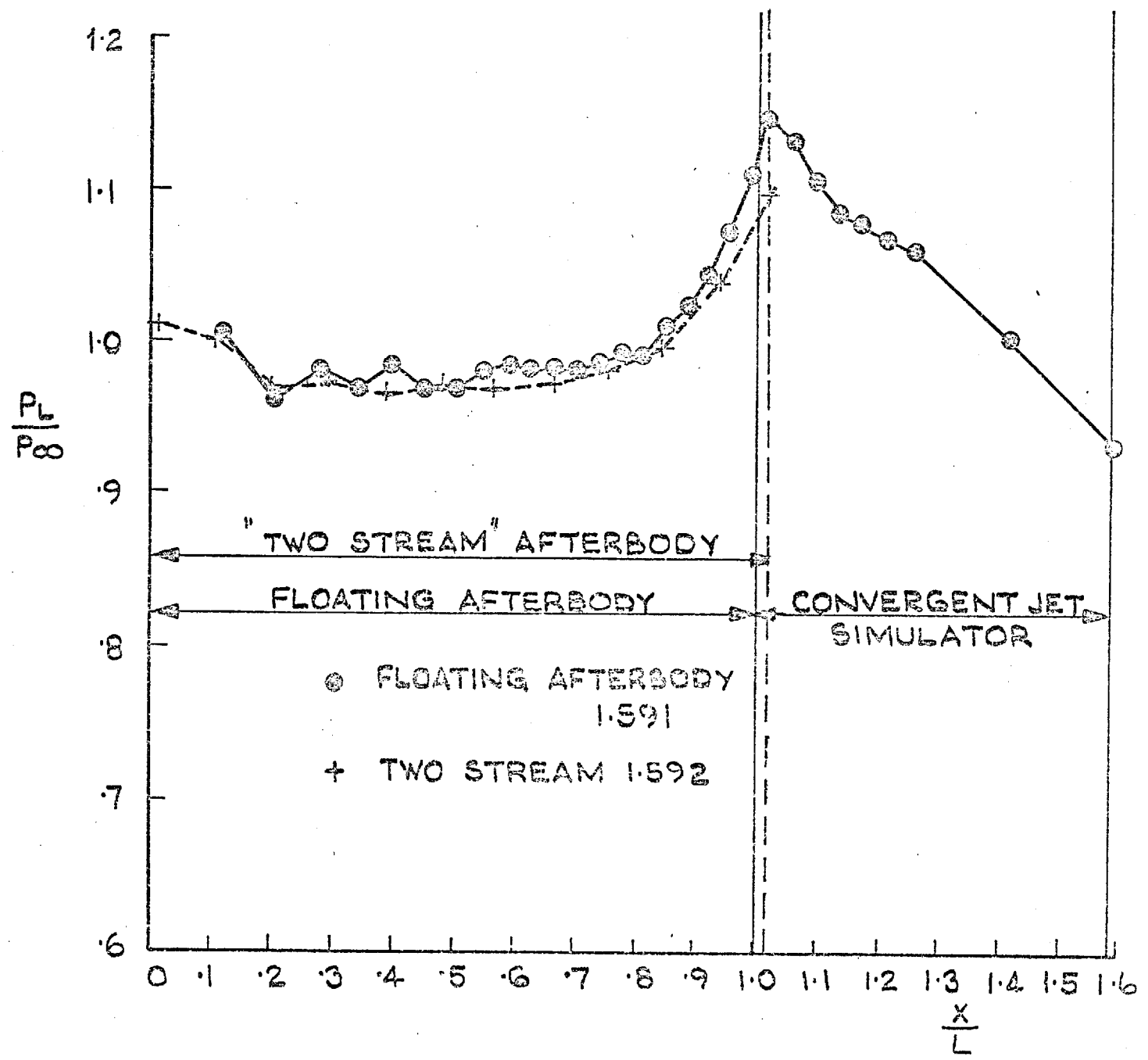


EFFECT OF PRESSURE RATIO ON
PRESSURE DISTRIBUTIONS, QUIESCENT

- + "2-STREAM" E.P.R. = 2.628
 o "2-STREAM" E.P.R. = 2.569
 • FLOATING AFTERBODY E.P.R. = 2.587

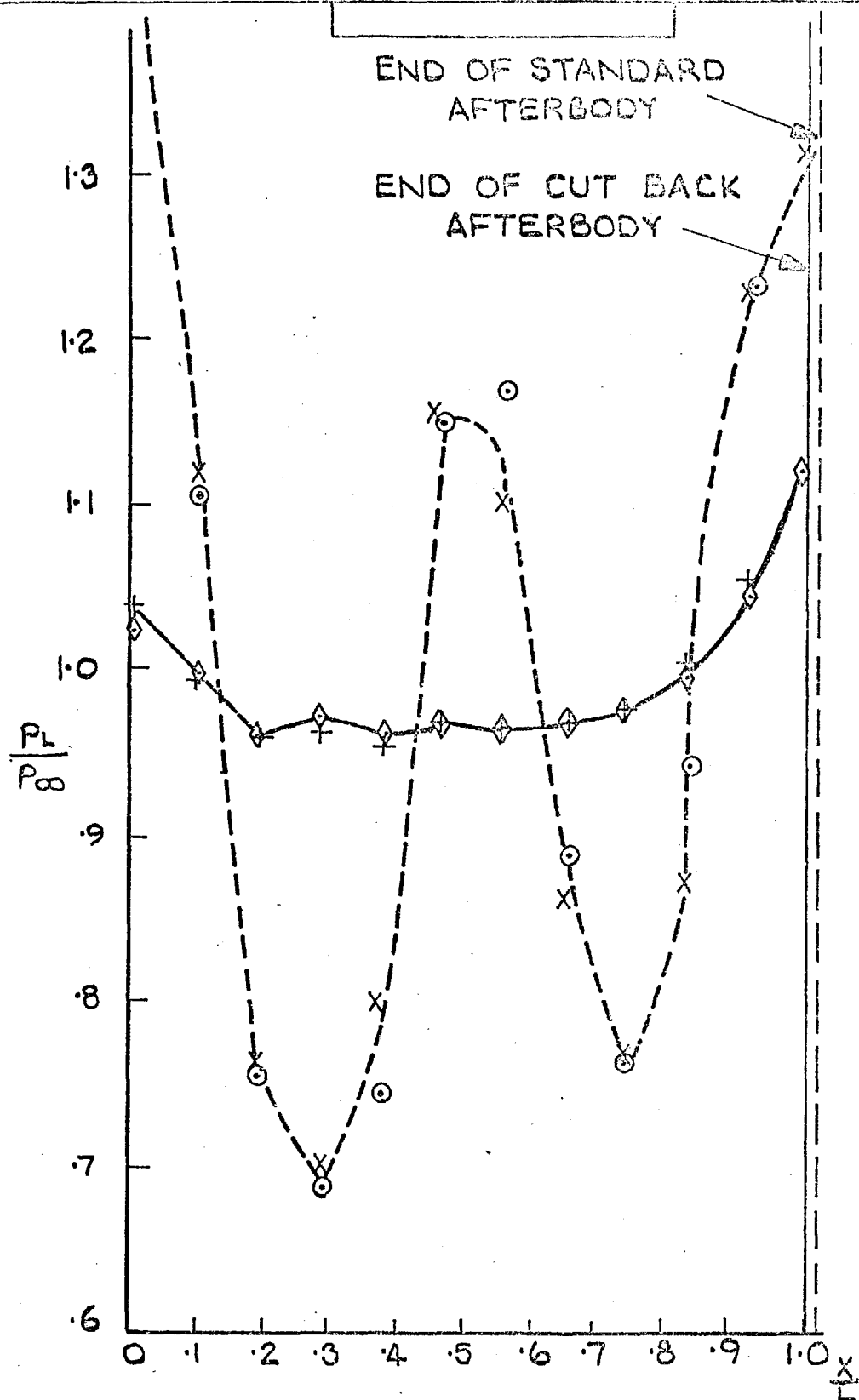


COMPARISON BETWEEN TWO-STREAM AND
 FLOATING AFTERBODY MODELS
 UNDER QUIESCENT CONDITIONS, E.P.R. 2.6



COMPARISON BETWEEN TWO-STREAM AND
 FLOATING AFTERBODY MODELS UNDER
 QUIESCENT CONDITIONS, APR 1959

FIG. 54



○ = CUT BACK E.P.R. = 2.582
 X = STANDARD E.P.R. = 2.569
 + = CUT BACK E.P.R. = 1.689
 ◇ = STANDARD E.P.R. = 1.700

COMPARISON BETWEEN STANDARD & CUT-BACK
 VERSIONS OF TWO-STREAM MODEL

COMPARISON OF AFTERBODY TRAILING

EDGE PRESSURES

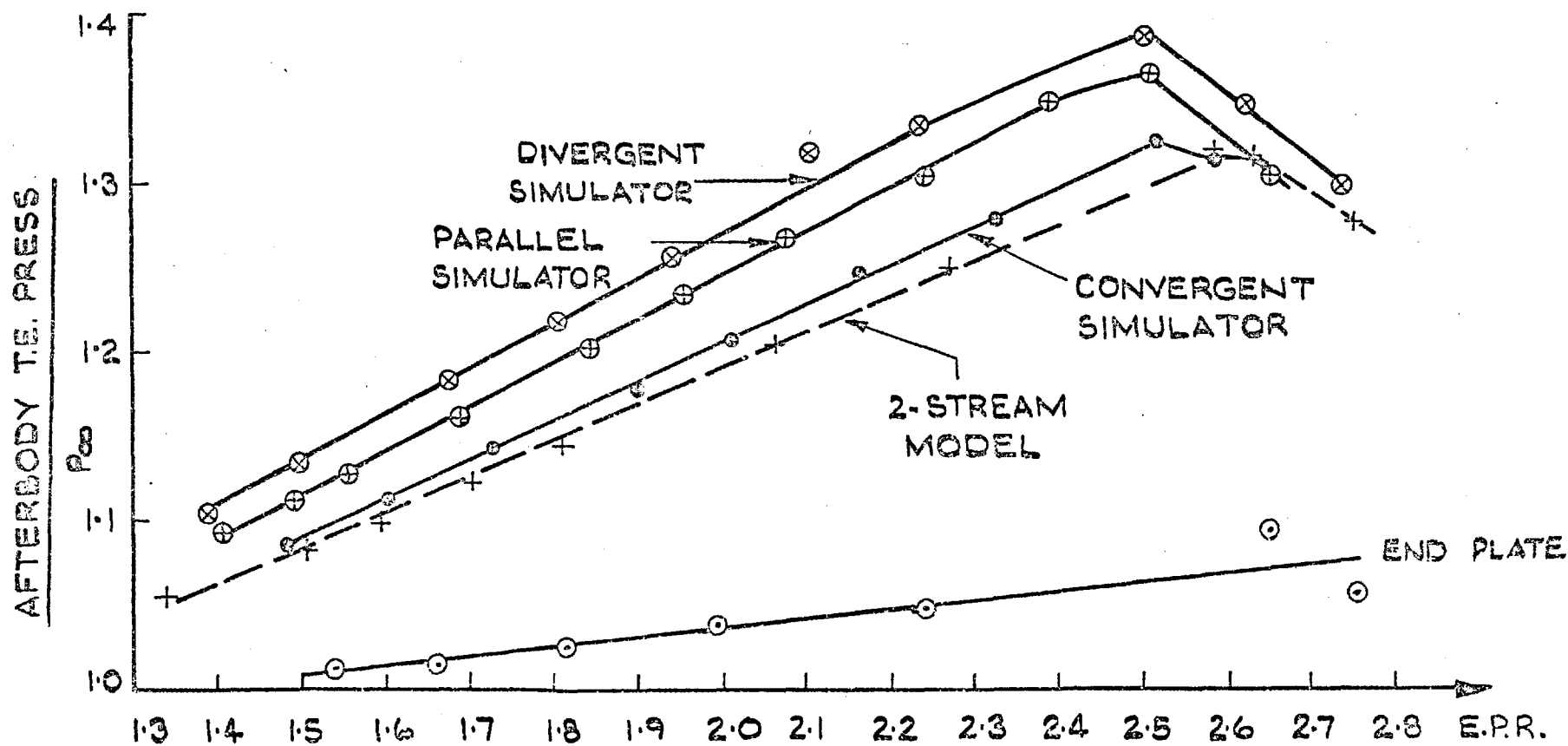
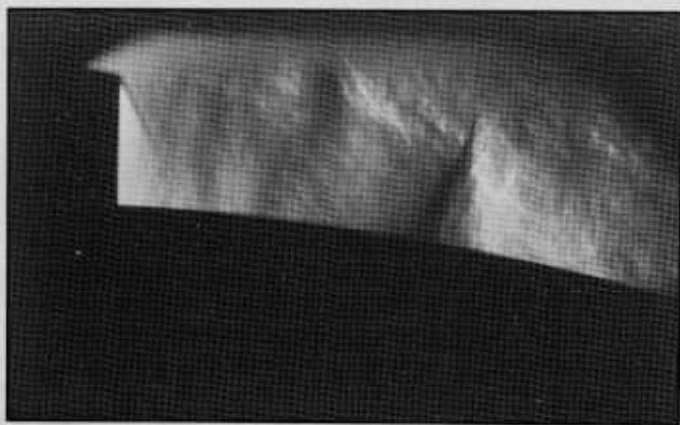
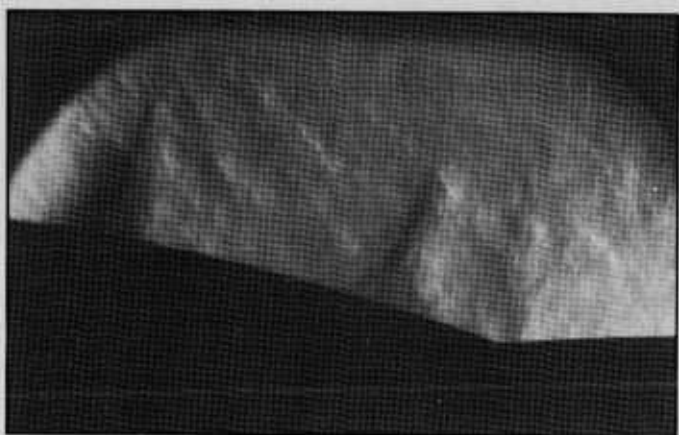


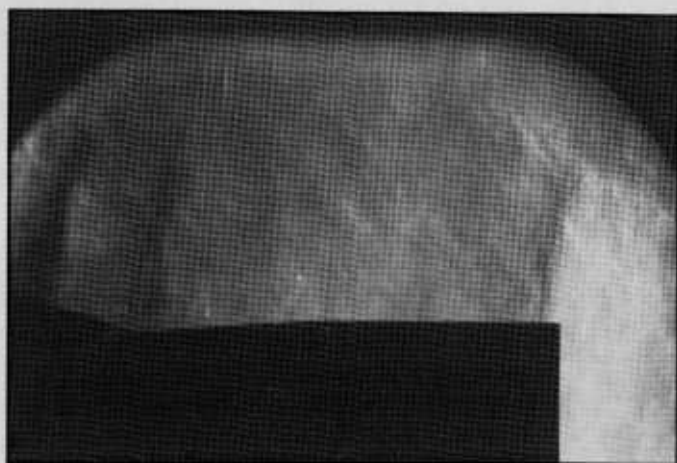
FIG. 53



a)



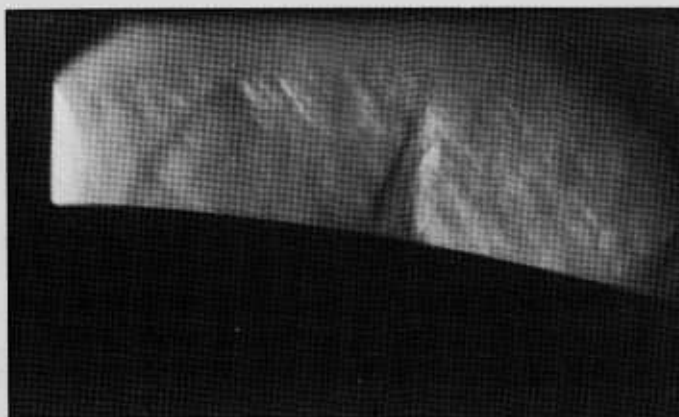
b)



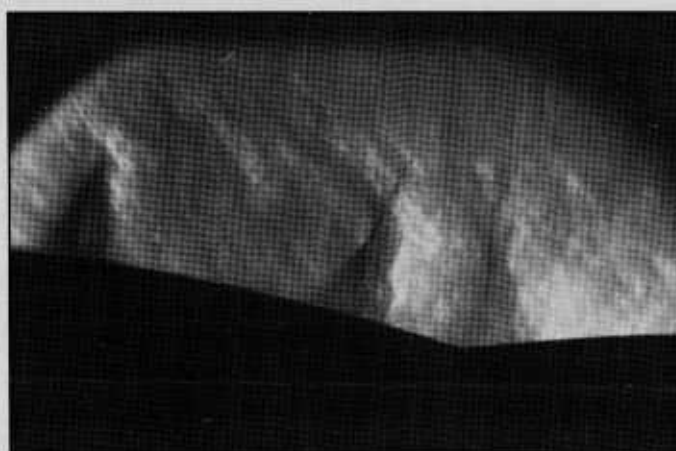
c)

10° FAN COWL
QUIESCENT
DIVERGENT
SIMULATOR
EPR 2.50

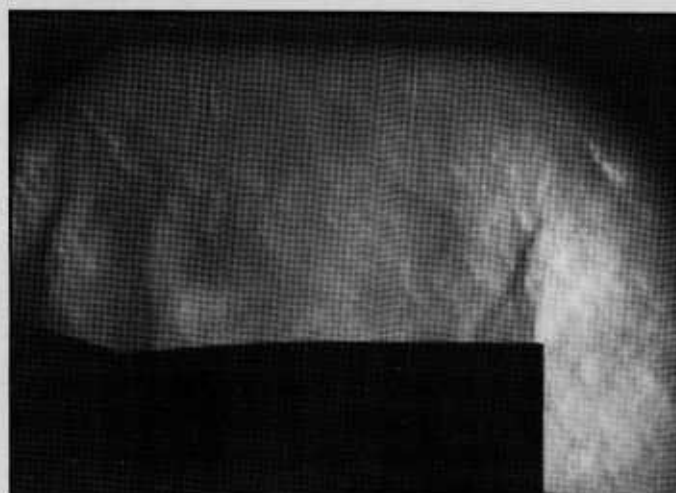
FIG.63 SCHLIEREN PHOTOGRAPHS



a)



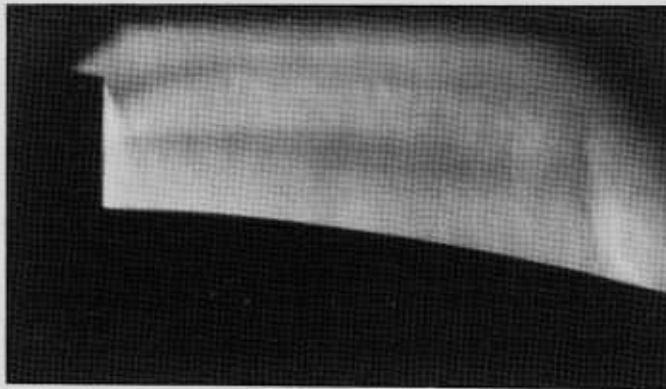
b)



c)

CONICAL FAN COWL
QUIESCENT
DIVERGENT
SIMULATOR
EPR 2.52

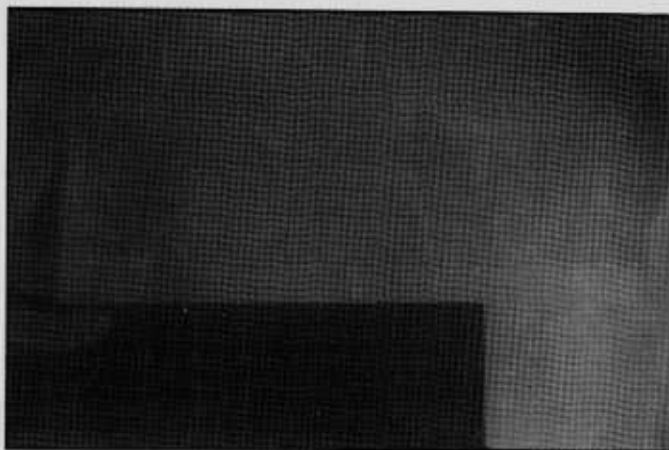
FIG.62 SCHLIEREN PHOTOGRAPHS



a)



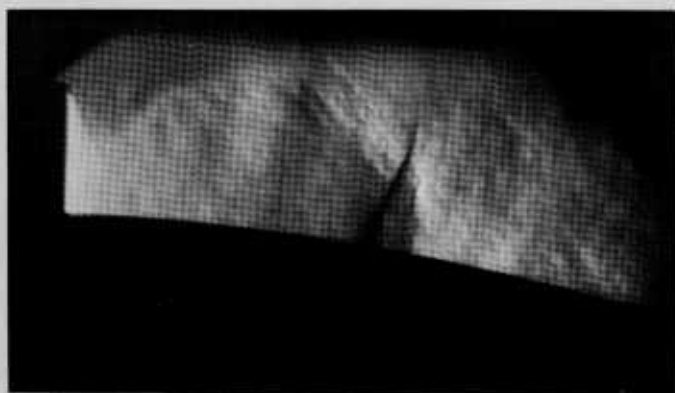
b)



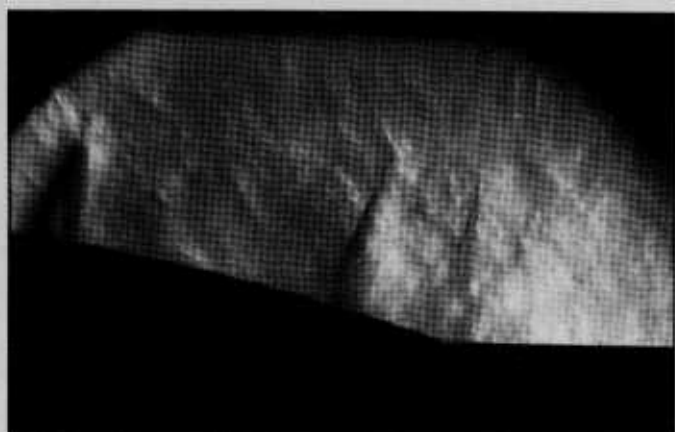
c)

12° FAN COWL
Moo = 0.8
PARALLEL
SIMULATOR
EPR 2.54

FIG.61 SCHLIEREN PHOTOGRAPHS



a)



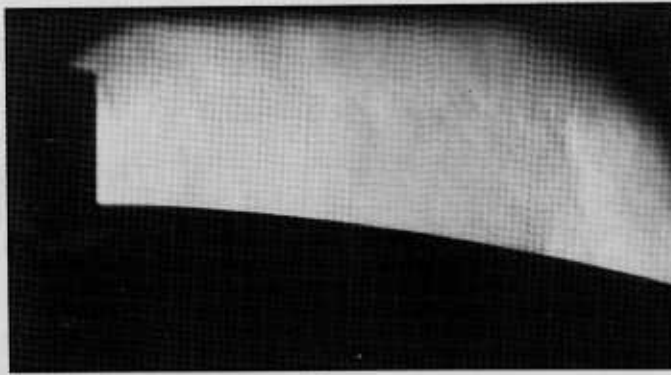
b)



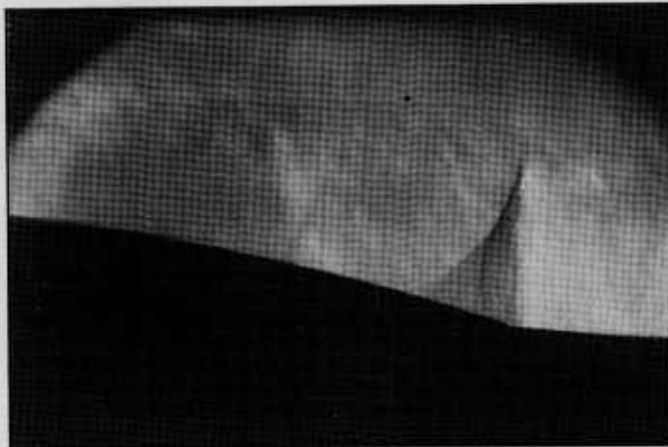
c)

10° FAN COWL
QUIESCENT
PARALLEL
SIMULATOR
EPR 2.49

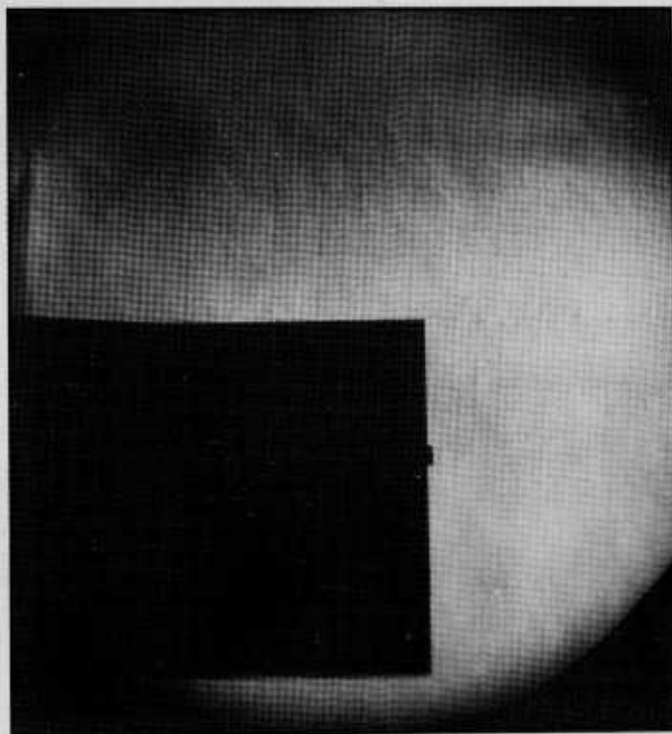
FIG.60 SCHLIEREN PHOTOGRAPHS



a)



b)



c)

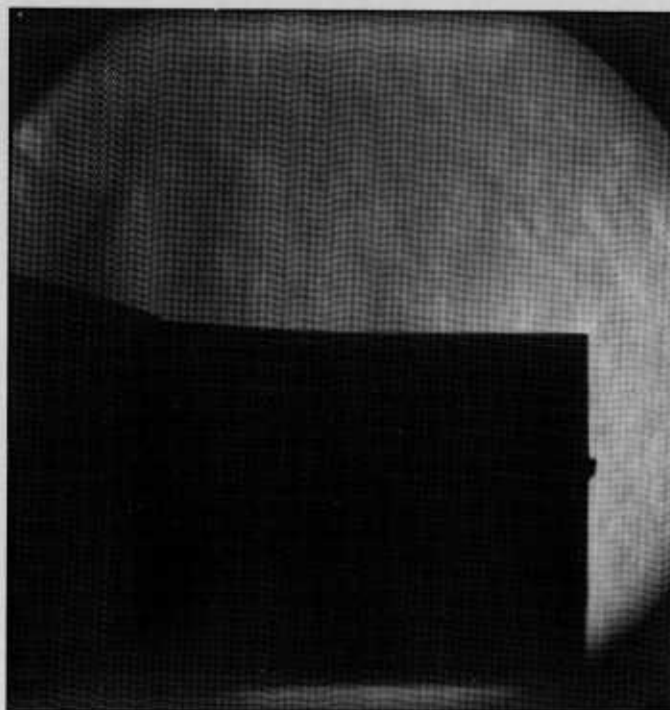
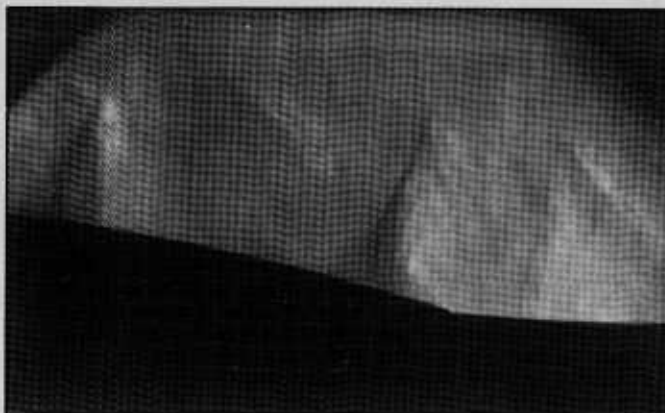
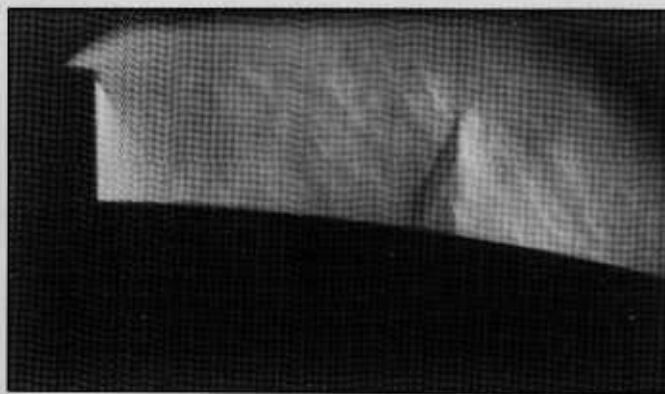
12° FAN COWL

$M_{\infty} = 0.8$

CONVERGENT
SIMULATOR

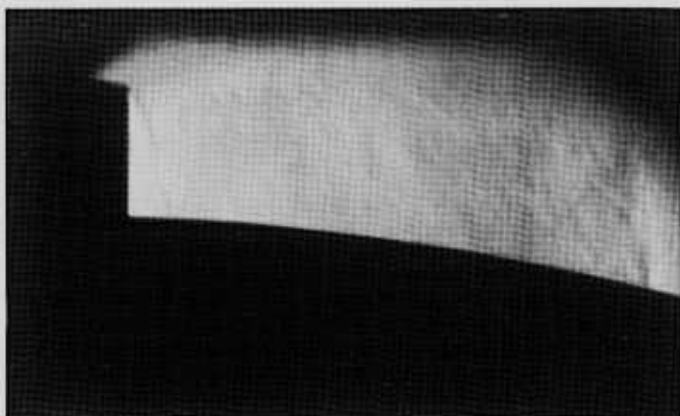
EPR 2.54

FIG.59 SCHLIEREN PHOTOGRAPHS



12° FAN COWL
QUIESCENT
CONVERGENT
SIMULATOR
EPR 2.50

FIG.58 SCHLIEREN PHOTOGRAPHS



10° FAN COWL

$M_{\infty} = 0.8$

NO SIMULATOR

EPR 2.54

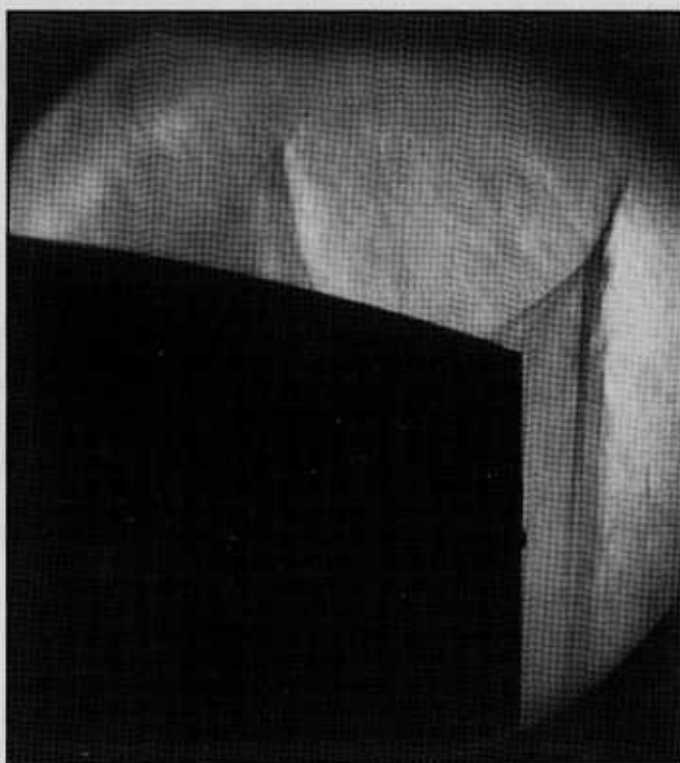
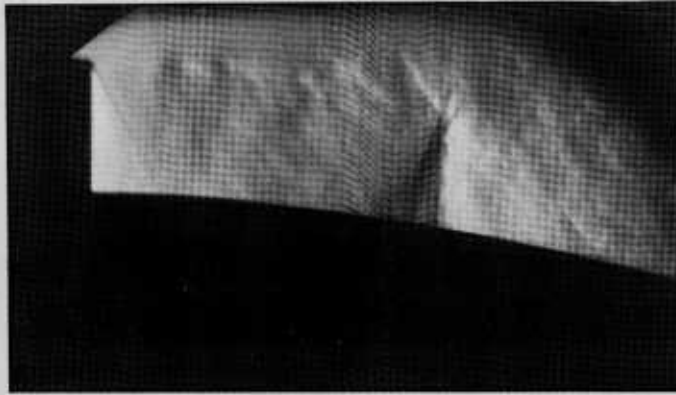


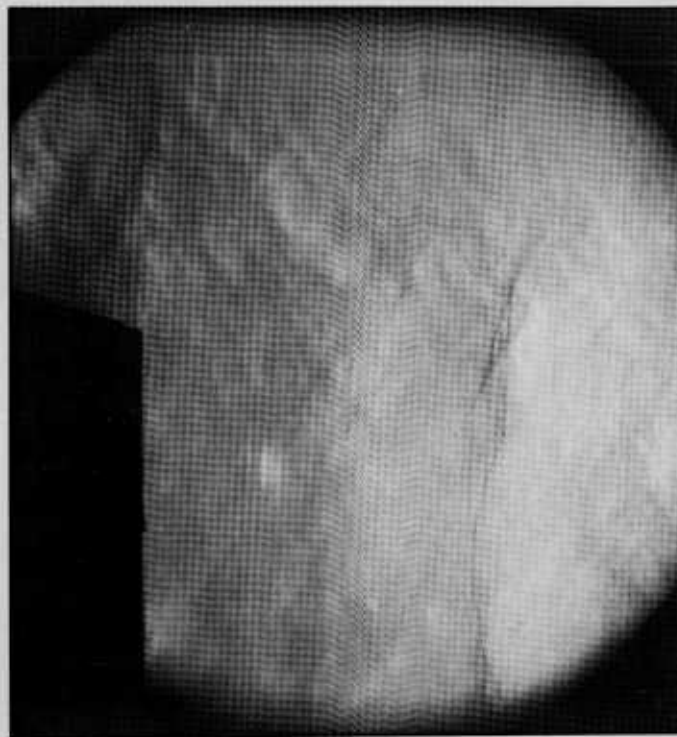
FIG.57 SCHLIEREN PHOTOGRAPHS



a)



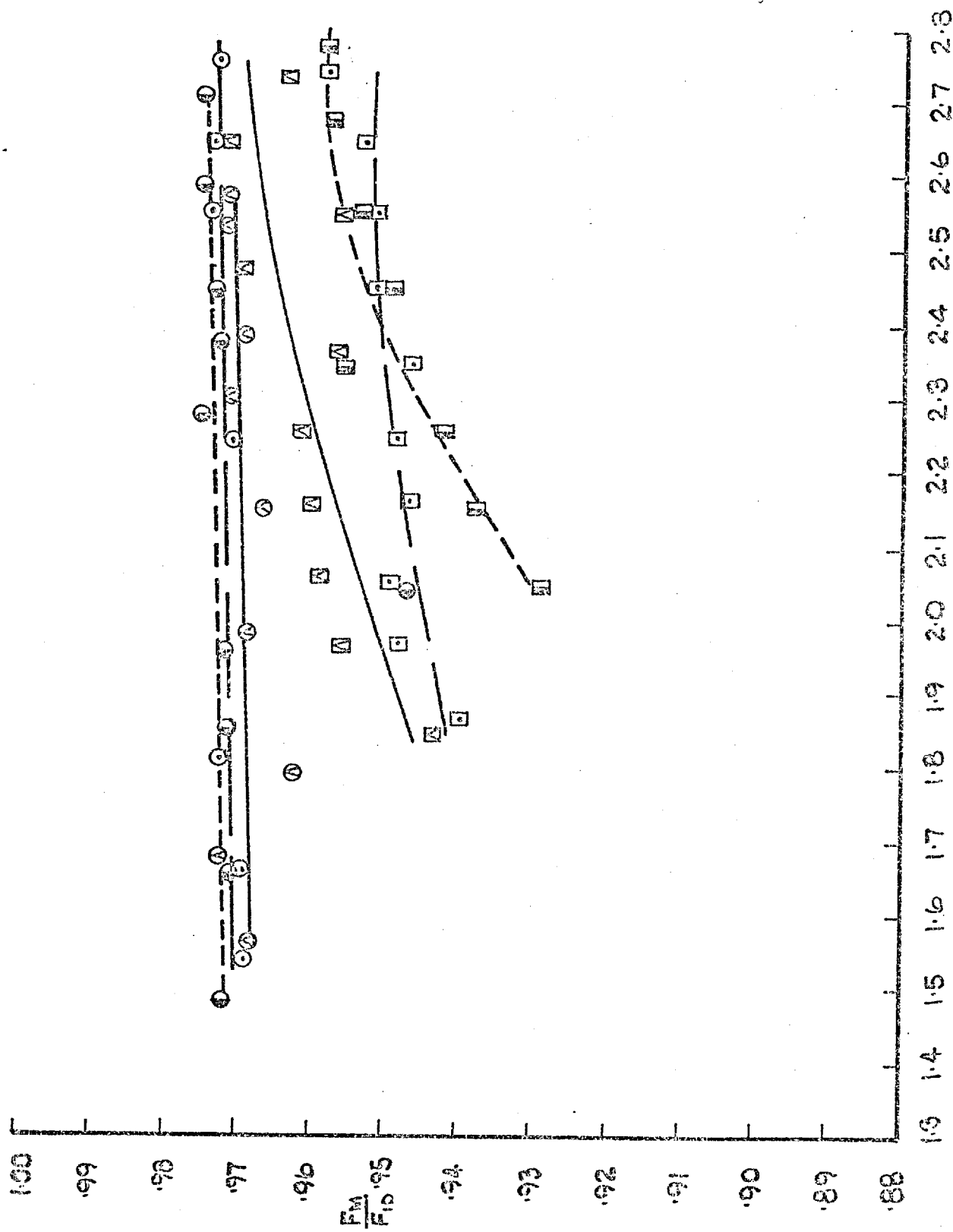
b)



c)

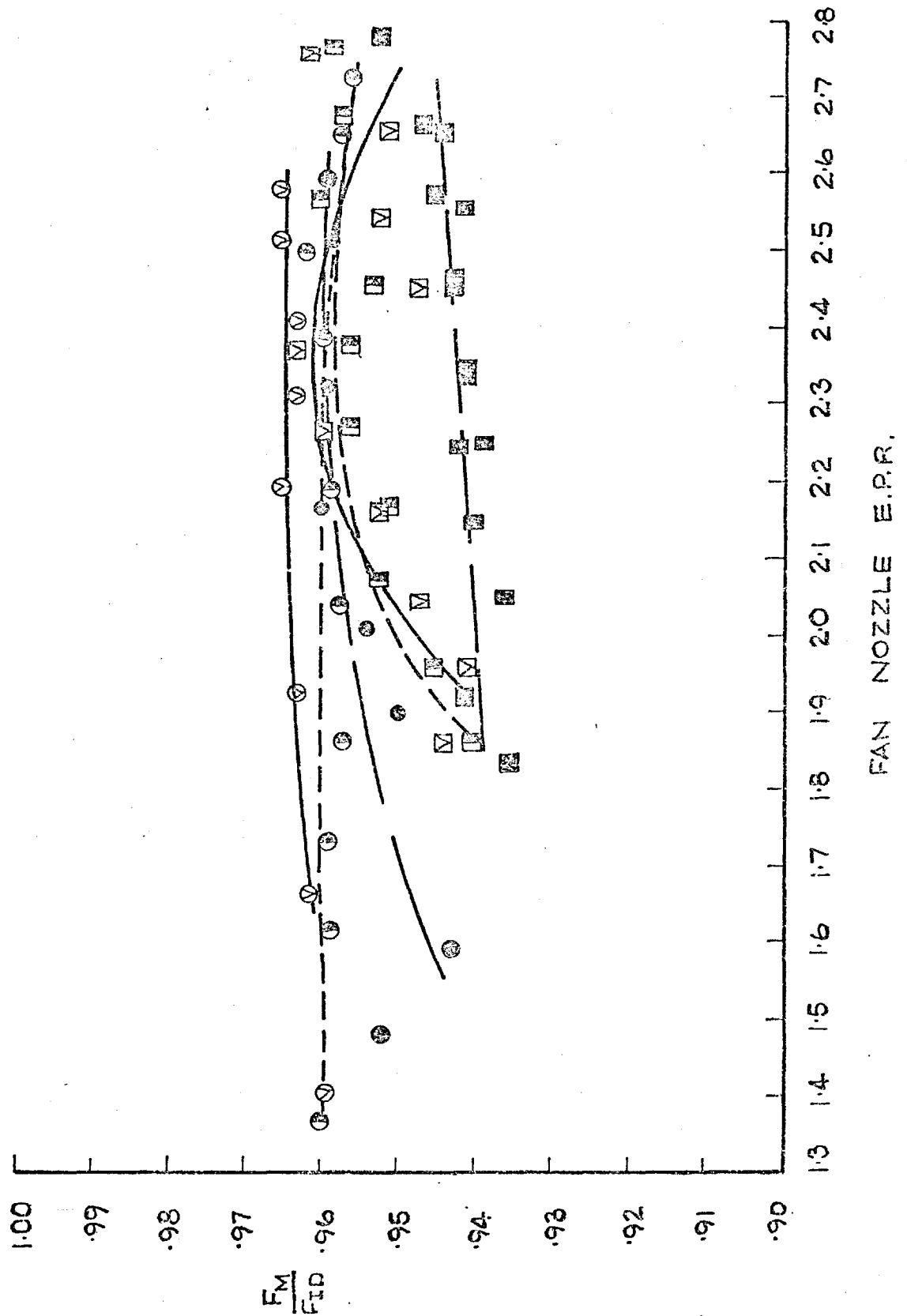
10° FAN COWL
QUIESCENT
NO SIMULATOR
EPR 2.50

FIG.56 SCHLIEREN PHOTOGRAPHS



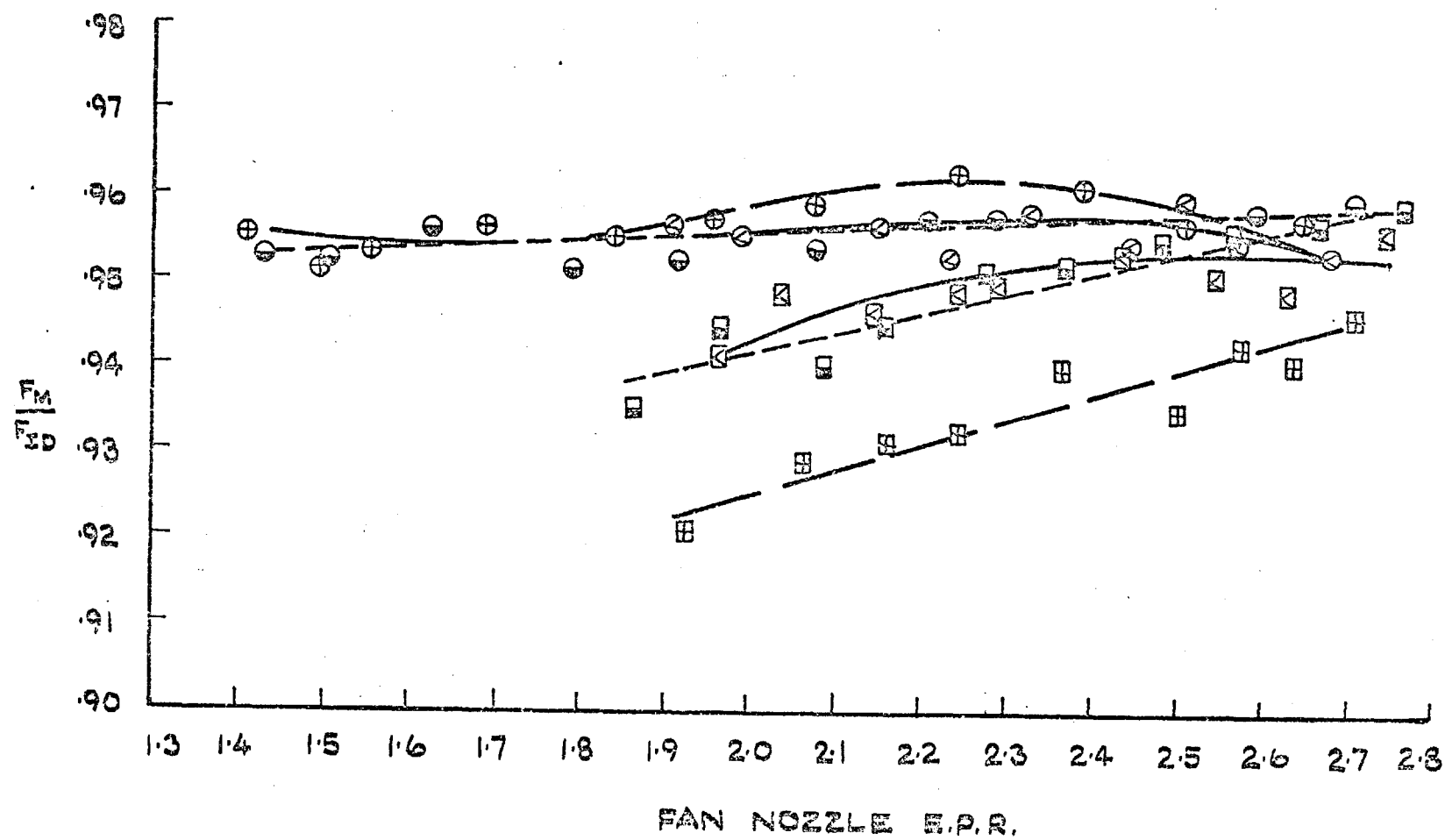
FAN NOZZLE E.P.R.

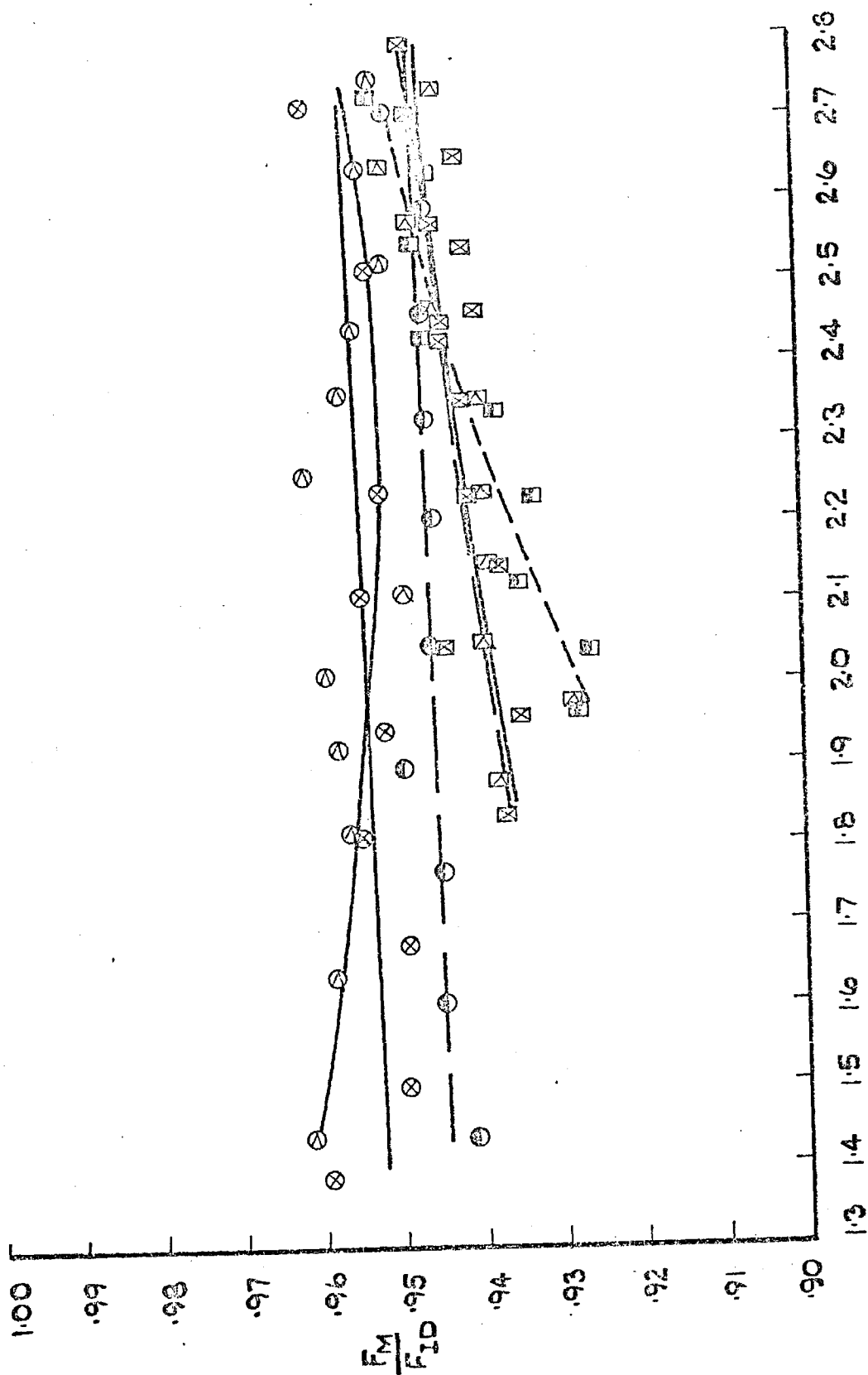
OVERALL THRUST PERFORMANCE, END PLATE



OVERALL THRUST PERFORMANCE,
CONVERGENT SIMULATOR

OVERALL THRUST PERFORMANCE
PARALLEL SIMULATOR





FAN NOZZLE E.P.R.

OVERALL THRUST PERFORMANCE
DIVERGENT SIMULATOR

OVERALL THRUST PERFORMANCE
TWO-STREAM MODEL

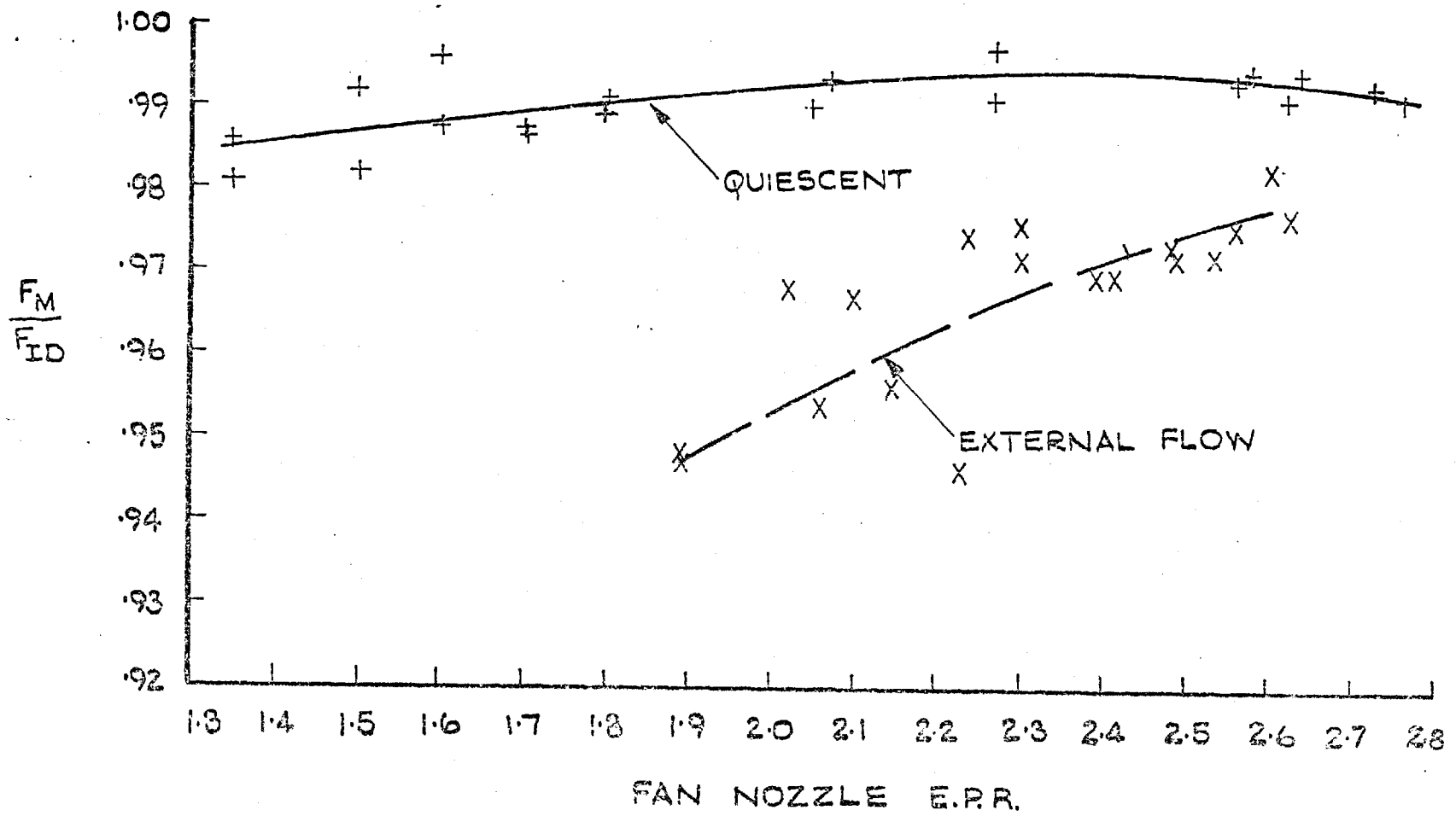
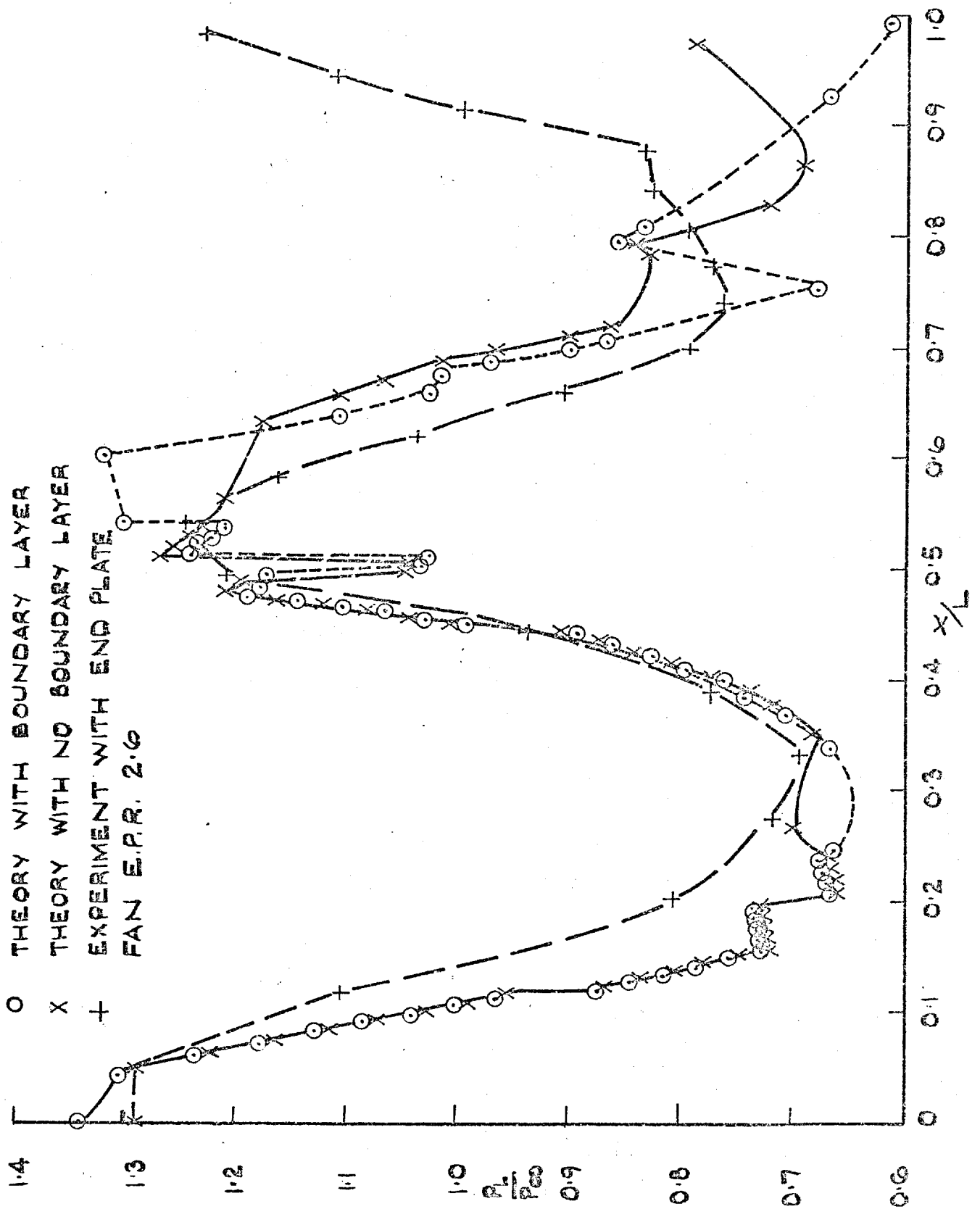
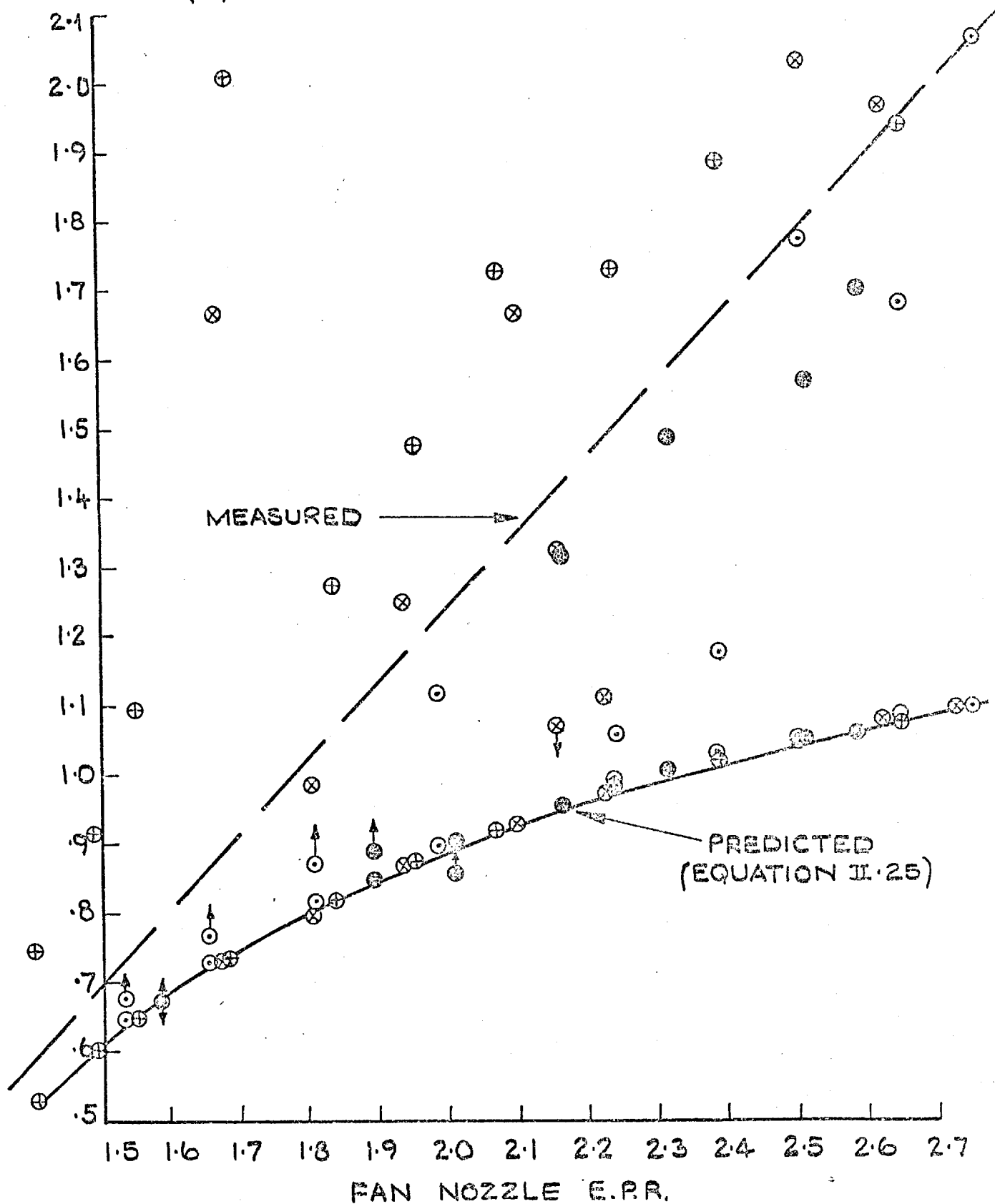


FIG. 6B

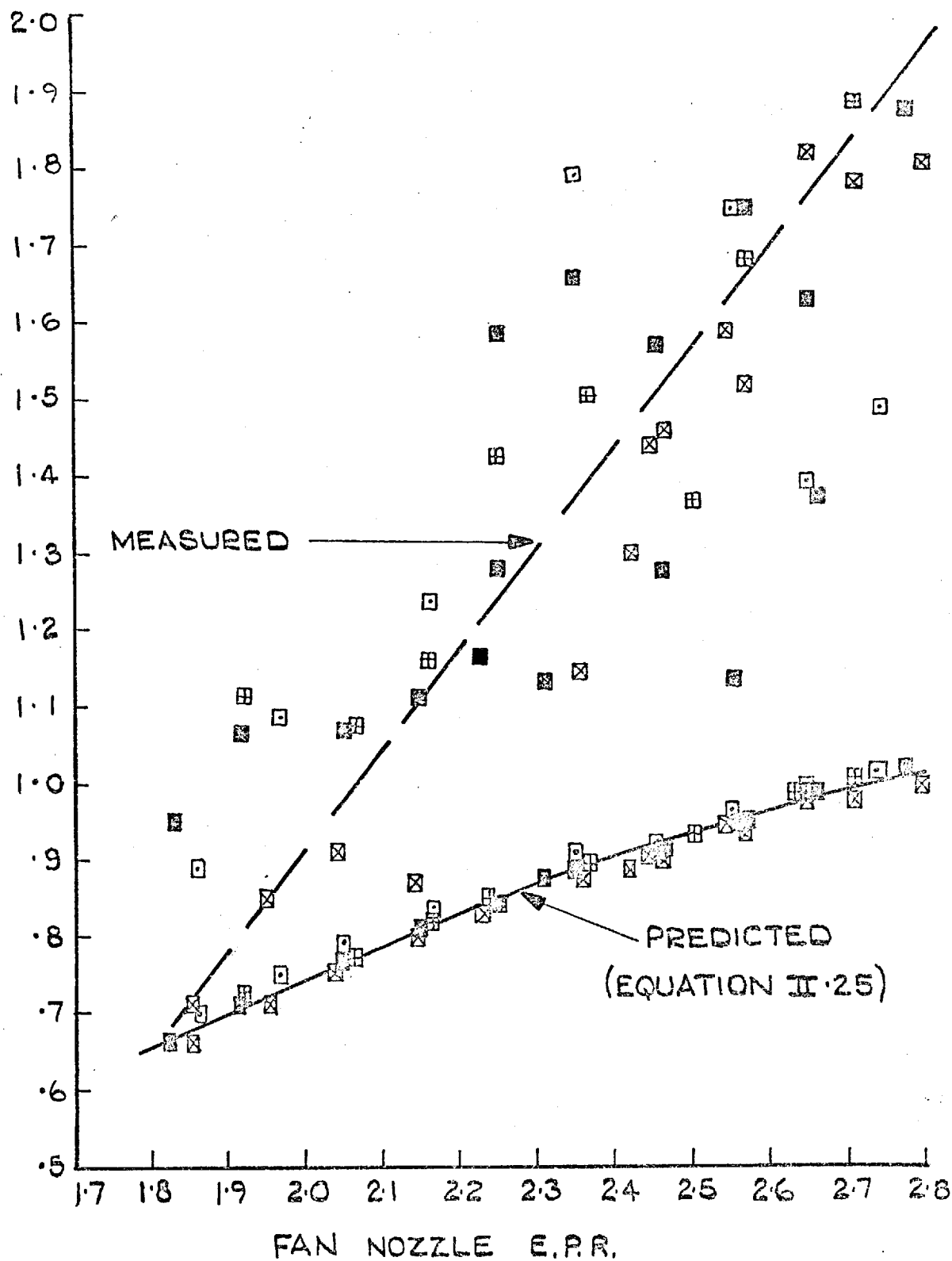
FIG. 69



COMPARISON BETWEEN THEORETICAL
& EXPERIMENTAL PRESSURE DISTRIBUTIONS

SKIN FRICTION
FORCE (lb)

COMPARISON BETWEEN MEASURED & PREDICTED
SKIN FRICTION FORCE (10° FAN COWL, QUIESCENT)

SKIN FRICTION
FORCE (lb)

COMPARISON BETWEEN MEASURED &
PREDICTED SKIN FRICTION FORCE
(10° FAN COWL, $M=0.8$)

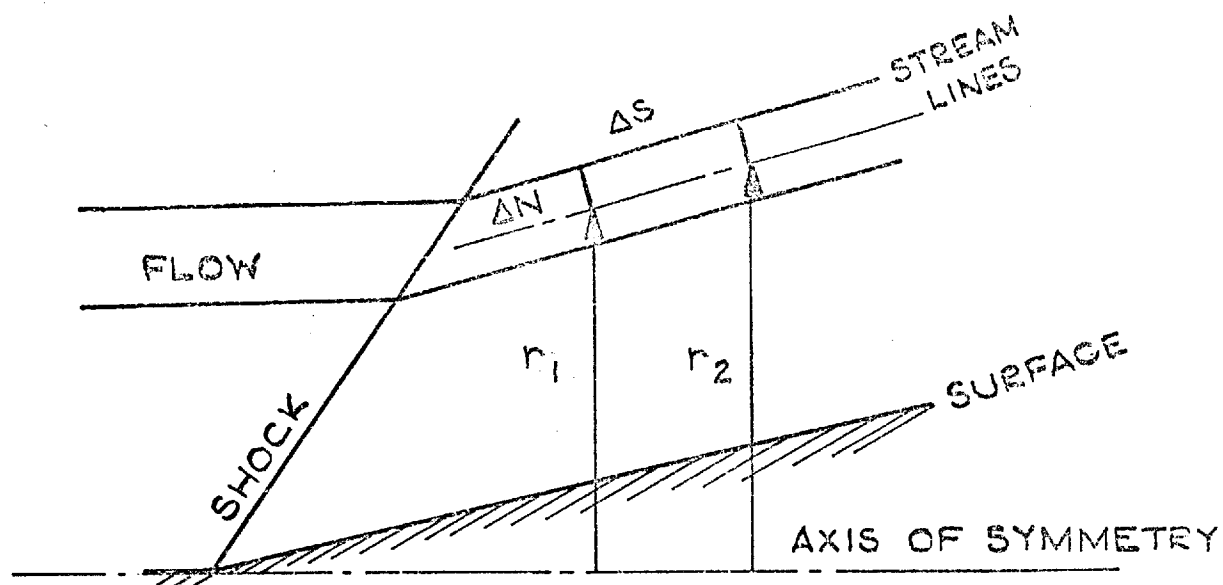


FIG. 72 a

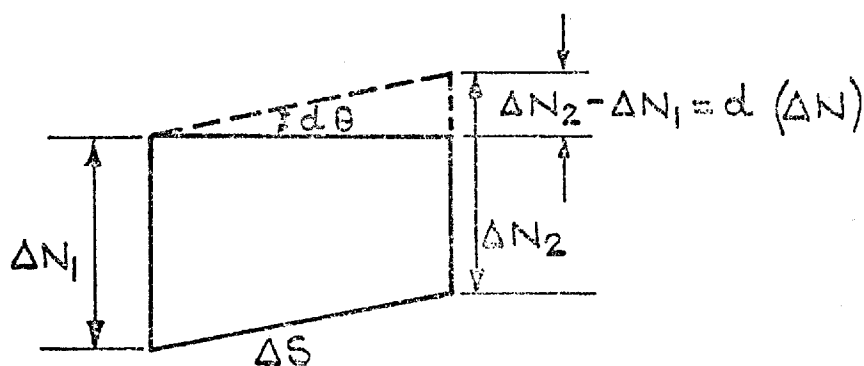


FIG. 72 b

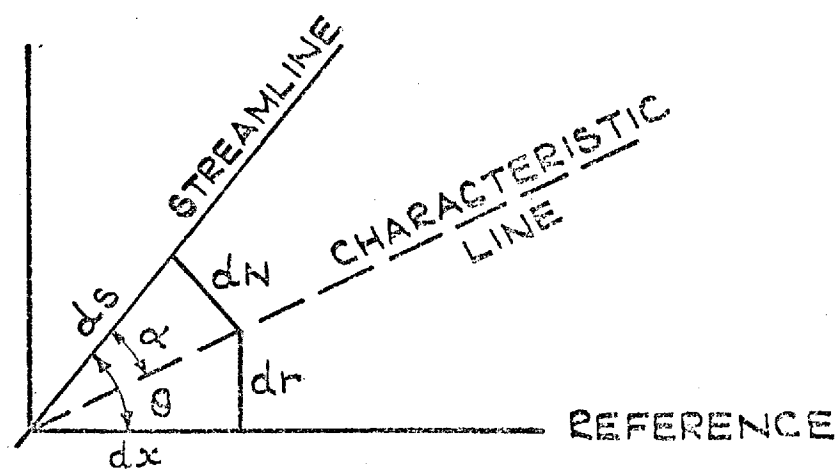


FIG. 72 c

GEOMETRIC RELATIONS IN CHARACTERISTICS THEORY