

Study on energy flows in thermoacoustic engines utilizing two-temperature heat sources

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Abstract

The proposal of a novel thermoacoustic regenerator using multi-temperature heat sources (MTHS) makes it possible to utilize lower-grade energy and keep relatively high efficiency in a thermoacoustic engine (TE) simultaneously. Based on thermodynamic laws combined with linear thermoacoustic theory, the time-averaged total power, enthalpy, acoustic power, entropy flow, and exergy flow in each component are derived and calculated to further understand the mechanism of a TE with the regenerator using two-temperature heat sources (TTHS). The comparison of the energy flows between the traditional TEs and those utilizing TTHS shows that the improvement of the temperature gradient in the regenerator by adding a mid-heater with appropriate heating power increases the acoustic power and efficiency of a TE.

Key words: low-grade energy; thermoacoustic engine; enthalpy flow; entropy flow; exergy flow.

1. Introduction

Thermoacoustic engine is a new type of environmentally friendly engine which can utilize low-grade thermal energy and has no moving parts [1]. The regenerator is the key components of a TE which converts heat energy into acoustic power, thus it attracts more attention in the thermoacoustic study [2,3]. In general, the regenerator in a traditional TE makes use of heat source at only one temperature level. Obviously, one must increase the temperature of the heat source to obtain high efficiency. In fact, it is possible for a regenerator in a TE to utilize heat sources at several temperature

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levels instead of only one. We proposed a new thermoacoustic regenerator, so-called regenerator utilizing MTHS, which can utilize heat sources at different temperature levels [4]. The gas parcels in the new regenerator can utilize heat sources at different temperatures according to their working temperature levels. Taking the regenerator using heat sources at two temperatures as an example, we have carried out preliminary study on a traveling-wave TE using TTHS [5], which shows that the regenerator using TTHS can utilize lower-grade heat sources with high efficiency compared with the traditional one. The additional heat source at lower temperature can increase the pressure ratio, acoustic power, efficiency and exergy efficiency of the TE. However, the energy conversion mechanism in the TE using TTHS is not fully explained.

From the point of view of energy conversion, this paper gives the expressions of the total power, time-averaged enthalpy flow, acoustic power, entropy flow, and exergy flow in the compliance, heat exchangers at room and high temperatures, regenerator and thermal buffer tube of a traveling-wave TE. The energy flows are calculated and the expressions are used to analyze the difference between the traditional TE and TE with mid-heater utilizing TTHS. The conditions when the mid-heater is located at different positions and applied to different amount of power are compared to illustrate the energy conversion mechanism and the optimization method for the mid-heater.

2 Theoretical fundamentals

2.1 Expressions for energy flows in thermoacoustic systems

The thermodynamic fundamentals of an open system are shown in Appendix A, which shows that for ideal isothermal element: time-averaged enthalpy $\langle \dot{H} \rangle = 0$, time-averaged entropy $\langle \dot{S} \rangle = \text{constant}$; for ideal adiabatic element: $\langle \dot{H} \rangle = \text{constant}$, $\langle \dot{S} \rangle = 0$.

The time-averaged total power in thermoacoustic systems is [6]:

$$\langle \dot{H}_2 \rangle_x = \langle \dot{E} \rangle + T_m \langle \dot{S} \rangle - (Ak + A_{\text{solid}} k_{\text{solid}}) \frac{dT_m}{dt} \quad (1)$$

Change of the time-averaged acoustic power along the axial direction is [6]:

$$\frac{d\langle\dot{E}\rangle}{dx} = -\frac{r_v}{2}|U_1|^2 - \frac{1}{2r_k}|p_1|^2 + \frac{1}{2}\text{Re}[g\tilde{p}_1U_1] \quad (2)$$

Ignoring the heat conduction in the axial direction of the thermoacoustic element, Equation (1) shows that the time-averaged total power is equal to the enthalpy, i.e.,

$$\langle\dot{H}_2\rangle_x = \langle\dot{H}\rangle = \langle\dot{E}\rangle + T_m\langle\dot{S}\rangle \quad (3)$$

The definition of the time-averaged exergy flux associated with a thermoacoustic wave is [6]:

$$\langle\dot{X}\rangle = \frac{T_0}{T_m}\langle\dot{E}\rangle + \left(1 - \frac{T_0}{T_m}\right)\langle\dot{H}_2\rangle_x \quad (4)$$

2.2 Enthalpy, acoustic power, entropy, and exergy flux in a TE

There are some assumptions in the expressions discussed below:

1. There is no heat conduction in the axial direction of each element. In each control volume along the axial direction, the heater, mid-heater, and cooler are considered to be isothermal. Note that though there is temperature gradient along the axial direction of the regenerator, in each control volume the regenerator can be assumed to be isothermal because of its extremely short length.
2. The positive flow is to the right, as shown in Figure 1.

Figure 1 shows the schematic of each individual element in the traditional TE and the one using TTHS. The energy balance in each element is shown in the control volume.

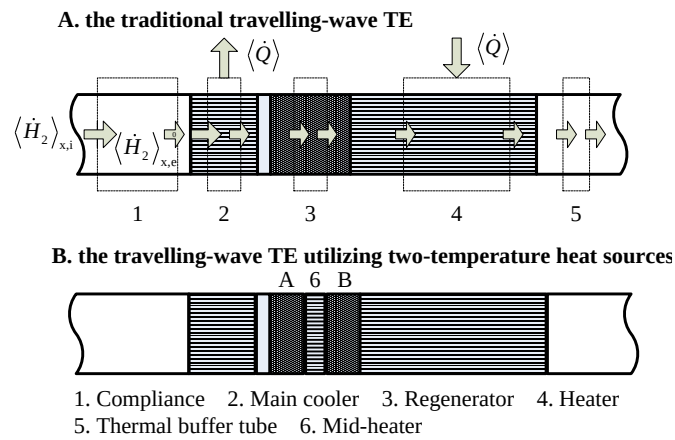


Figure 1 Schematic of a traveling-wave thermoacoustic engine.

The compliance, thermal buffer tube, and the space between the main cooler and regenerator in the traveling-wave TE shown in Figure 1 can be considered adiabatic. The space is unnecessary, but is unavoidable in our experimental making, so it is included in our calculation. The calculation model for the space is the same as that for the thermal buffer tube. As there is no exchange of heat and work for the control volume, thus Equation (A6) in Appendix A becomes:

$$\langle \dot{H}_2 \rangle_{x,e} = \langle \dot{H}_2 \rangle_{x,i}; \quad (5)$$

$$\text{Equation (5) combined with Equation (3) produces } \langle \dot{H} \rangle_e = \langle \dot{H} \rangle_i; \quad (6)$$

$$\text{According to Equation (3), there is } T_m \langle \dot{S} \rangle = \langle \dot{H} \rangle - \langle \dot{E} \rangle; \quad (7)$$

$$\text{Equation (3) and (4) produce } \langle \dot{X} \rangle = \frac{T_0}{T_m} \langle \dot{E} \rangle + (1 - \frac{T_0}{T_m}) \langle \dot{H} \rangle; \quad (8)$$

For ideal adiabatic element, $\langle \dot{S} \rangle = 0$ (Equation A9 in Appendix A);

$$\text{According to Equation (3) and (A9), } \langle \dot{H} \rangle = \langle \dot{H}_2 \rangle_x = \langle \dot{E} \rangle; \quad (9)$$

$$\text{Combination of Equation (9) and (4) is: } \langle \dot{X} \rangle = \langle \dot{E} \rangle = \langle \dot{H} \rangle. \quad (10)$$

There is no heat and work exchange for the control volume of the regenerator shown in Figure 1. Within the control volume, the regenerator in the axial direction of the TE is considered to be isothermal.

For the isothermal element, the relation between the inlet and outlet time-averaged total power is expressed as Equation (5), the enthalpy relation is Equation (6), expression of $T_m \langle \dot{S} \rangle$ is Equation (7), and relation of $\langle \dot{X} \rangle$ is Equation (8).

For the ideal isothermal element, $\langle \dot{H} \rangle = 0$; (Equation A8 in Appendix A);

$$\text{According to Equations (3) and (A8): } T_m \langle \dot{S} \rangle = -\langle \dot{E} \rangle; \quad (11)$$

According to Equations (8) and (11), there is:

$$\langle \dot{X} \rangle = \frac{T_0}{T_m} \langle \dot{E} \rangle = -T_0 \langle \dot{S} \rangle. \quad (12)$$

The heater, mid-heater, and main cooler in Figure 1 are heat exchangers. For

these elements, within the control volume, there is heat exchange for those open systems in Figure 1 in radial direction while there is no work exchange. Equation (A6) in Appendix A becomes:

$$\langle \dot{H}_2 \rangle_{x,e} = \langle \dot{H}_2 \rangle_{x,i} \pm \langle \dot{Q} \rangle; \quad (13)$$

$$\text{Combined Equation (3) with (13), there is: } \langle \dot{H} \rangle_e = \langle \dot{H} \rangle_i \pm \langle \dot{Q} \rangle; \quad (14)$$

On the right side of Equation (13) and (14), plus sign is used for heaters and minus sign for coolers. The expression of $T_m \langle \dot{S} \rangle$ is Equation (7) and $\langle \dot{X} \rangle$ is (8). The expressions discussed above are summed up in Table 1.

Table 1 Expressions of energy flows under typical conditions

Expressions	$\langle \dot{H} \rangle = \langle \dot{E} \rangle + T_m \langle \dot{S} \rangle$ Eq. (3) $\langle \dot{X} \rangle = \langle \dot{E} \rangle T_0 / T_m + \langle \dot{H} \rangle (1 - T_0 / T_m)$ Eq. (8)				
	$\langle \dot{H} \rangle$	$T_m \langle \dot{S} \rangle$	$\langle \dot{E} \rangle$	$\langle \dot{X} \rangle$	Component
ideal adiabatic	constant	0	$\langle \dot{H} \rangle$	$\langle \dot{H} \rangle$	compliance; thermal buffer tube
ideal isothermal $\langle \dot{Q} \rangle = 0$	0	$\propto T_m$	$-T_m \langle \dot{S} \rangle$	$-T_0 \langle \dot{S} \rangle$	ideal regenerator
isothermal $\langle \dot{Q} \rangle = 0$	constant	$\langle \dot{H} \rangle - \langle \dot{E} \rangle$	$\langle \dot{H} \rangle - T_m \langle \dot{S} \rangle$	(8)	real regenerator
isothermal $\langle \dot{Q} \rangle \neq 0$	$\langle \dot{H} \rangle_e = \langle \dot{H} \rangle_i \pm \langle \dot{Q} \rangle$	$\langle \dot{H} \rangle - \langle \dot{E} \rangle$	constant	(8)	heater; mid-heater; main cooler

3 The energy flow calculation and analysis

3.1 Definitions

$$R_{\text{energy}} = \langle \dot{Q} \rangle_{\text{mid}} / \left| \langle \dot{H} \rangle_a \right| \quad (15)$$

R_{energy} will appear in the discussions of the regenerator using TTHS in Section 3.2. Where $\langle \dot{Q} \rangle_{\text{mid}}$ represents the mid-heater power, $\left| \langle \dot{H} \rangle_a \right|$ is the absolute value of

the enthalpy flow in Regenerator A. R_{energy} is the relative value of the mid-heater power to the enthalpy flow in Regenerator A. $R_{\text{energy}} = 1$ indicates the mid-heater power is equal to the enthalpy flow in Regenerator A and the inflow heat leads to a zero enthalpy flow in Regenerator B.

$$\eta = \langle \dot{E} \rangle_{\text{res}} / \sum \langle \dot{Q} \rangle_j \quad \text{and} \quad \eta_e = \langle \dot{E} \rangle_{\text{res}} / \sum \langle \dot{Q} \rangle_j (1 - T_0 / T_j) \quad (16)$$

η is the efficiency and η_e is the exergy efficiency, where $\langle \dot{E} \rangle_{\text{res}}$ is the acoustic power entering the resonator, $\langle \dot{Q} \rangle_j$ is each heater power, T_0 is the environment temperature, and T_j is each heat source temperature.

$$R = \langle \dot{E} \rangle_{\text{out}} / \langle \dot{E} \rangle_{\text{in}} \quad (17)$$

$\langle \dot{E} \rangle_{\text{out}}$ is the outlet acoustic power and $\langle \dot{E} \rangle_{\text{in}}$ is inlet acoustic power. We name R as the acoustic power amplifying ratio in a regenerator.

3.2 Calculation results and analysis

The time-averaged enthalpy flux $\langle \dot{H} \rangle$, acoustic power $\langle \dot{E} \rangle$, entropy flux $T_m \langle \dot{S} \rangle$, and exergy flux $\langle \dot{X} \rangle$ of the traveling-wave TEs are calculated based on the expressions in Section 2 and DeltaEC (Design Environment for low-amplitude Thermoacoustic Energy Conversion) [7]. DeltaEC can directly calculate the time-averaged total power, acoustic power, and exergy flow in an element, then the enthalpy and entropy flow could be obtained according to the abovementioned equations in Table 1. Our preliminary study [5] has introduced the details of the traveling-wave TEs and the calculation conditions (N_2 as working substance, filling pressure is 2.2 MPa, heater power is fixed at 1430 W).

Figure 2 shows the time-averaged enthalpy flow $\langle \dot{H} \rangle$, entropy flow $T_m \langle \dot{S} \rangle$, acoustic power $\langle \dot{E} \rangle$, exergy flow $\langle \dot{X} \rangle$, the solid wall temperature T_{solid} , and mean gas temperature T_m along the axial direction of a real (not ideal) traveling-wave TE. The compliance is regarded as an ideal adiabatic element, thus $T_m \langle \dot{S} \rangle = 0$ (see Table 1), $\langle \dot{X} \rangle = \langle \dot{E} \rangle = \langle \dot{H} \rangle = 759 \text{ W}$. In the main cooler, $dT_m / dx = 0$ (see the lower graph in

Figure 2), Equation (2) becomes $\frac{d\langle\dot{E}\rangle}{dx} = -\frac{r_v}{2}|U_1|^2 - \frac{1}{2r_k}|p_1|^2$; Because of the small viscous resistance, $r_v \rightarrow 0$; In addition, $r_h \ll \delta_k$ in the main cooler, $1/r_k \rightarrow 0$ [6]; thus in the main cooler $d\langle\dot{E}\rangle/dx \approx 0$, which means that $\langle\dot{E}\rangle$ changes little. The exchanged heat to the environment from the main cooler leads to the decrease of $\langle\dot{H}\rangle$, $T_m\langle\dot{S}\rangle$, and $\langle\dot{X}\rangle$. The space between the main cooler and the regenerator could be considered as adiabatic element, where the length of the space is quite short, thus $dT_m/dx \approx 0$. The analysis of $\langle\dot{E}\rangle$ is similar to that in the main cooler, $\langle\dot{E}\rangle$ is constant. $\langle\dot{H}\rangle$ is constant, combined with unchanged $\langle\dot{E}\rangle$ cause little change in $T_m\langle\dot{S}\rangle$ and $\langle\dot{X}\rangle$. The time-averaged enthalpy $\langle\dot{H}\rangle$ in the regenerator does not change, as the regenerator is isothermal. When the traveling-wave regenerator is considered to be ideal isothermal, $\langle\dot{H}\rangle=0$, $\langle\dot{S}\rangle=\text{constant}$, $T_m\langle\dot{S}\rangle$ is proportional to the mean temperature T_m , $\langle\dot{E}\rangle=-T_m\langle\dot{S}\rangle$ indicates that $\langle\dot{E}\rangle$ is equal to the absolute value of $T_m\langle\dot{S}\rangle$. $\langle\dot{X}\rangle = -T_0\langle\dot{S}\rangle$ does not change. Actually, for the real regenerator, $\langle\dot{H}\rangle$ is not equal to 0 (but the value does not change), $\langle\dot{E}\rangle$ is smaller than the absolute value of $T_m\langle\dot{S}\rangle$, and $\langle\dot{X}\rangle$ decreases. The energy flows analysis in the heater is similar to that in the main cooler, $\langle\dot{E}\rangle$ is constant. The heater in the control volume absorbs heat from the heat source, which is different from that in the cooler. The heat absorption increases $\langle\dot{H}\rangle$, $T_m\langle\dot{S}\rangle$, and $\langle\dot{X}\rangle$. The characteristics of the thermal buffer tube is close to the ideal adiabatic element, $T_m\langle\dot{S}\rangle=0$, $\langle\dot{X}\rangle=\langle\dot{E}\rangle=\langle\dot{H}\rangle=1198 \text{ W}$.

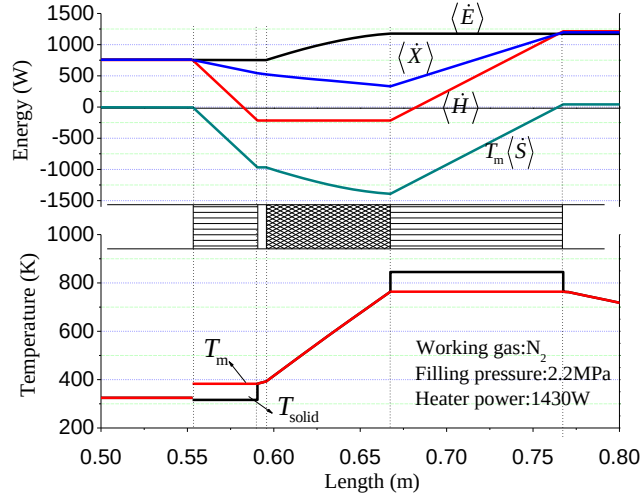


Figure 2 Energy flows and the temperature distribution of a traditional traveling-wave TE

Similar energy flow analysis is made in the traveling-wave TE utilizing TTHS. We assume that the traditional regenerator is divided into four equal parts in axial direction. Figure 3 shows that the mid-heater is located between the second and third part, which is labeled as Position 0.5. The analysis for the mid-heater is the same as the heater. When the mid-heater is applied in the TE, $\langle \dot{E} \rangle$ in the mid-heater does not change, the heat absorbed in the control volume of the mid-heater increases $\langle \dot{H} \rangle$, $T_m \langle \dot{S} \rangle$, and $\langle \dot{X} \rangle$. Figure 2 and 3 show the difference between the traditional TE and that using TTHS. In traditional TE, the time-averaged enthalpy $\langle \dot{H} \rangle$ is -215 W instead of zero. The mid-heater makes the enthalpy in the regenerator of the second half (Regenerator B, see Figure 1.B) close to zero. The traditional regenerator amplifies the acoustic power from 752 W at inlet to 1177 W at the outlet. The acoustic power amplifying ratio $R=1.57$ in the traditional regenerator. In the regenerator utilizing TTHS, Regenerator A amplifies the acoustic power from 765 W to 1188 W ($R_a=1.55$), and the Regenerator B amplifies the acoustic power to 1242 W ($R_b=1.05$), the total ratio $R=1.62$. Obviously, Regenerator A has better ability in acoustic power amplification than Regenerator B, which is attribute to the larger temperature gradient in Regenerator A (see the lower graph in Figure 3). As Equation

(2) implies, $d\langle\dot{E}\rangle/dx$ is mainly determined by dT_m/dx . The regenerator using TTHS has larger R and acoustic power (see Table 2).

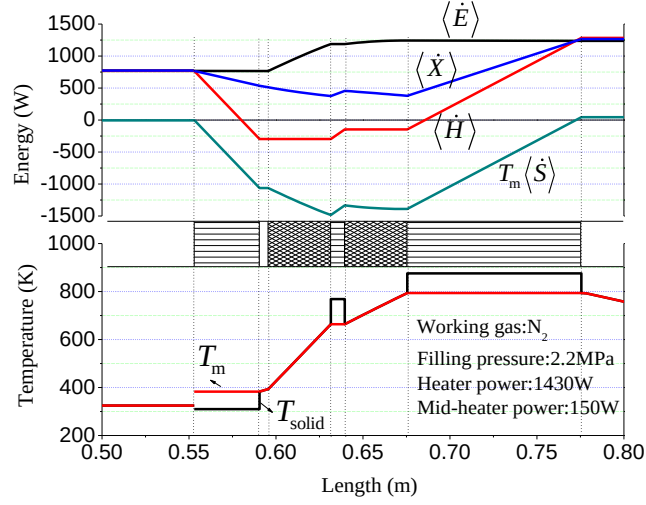


Figure 3 Energy flows and the temperature distribution of the TE using TTHS for Position 0.5, $R_{\text{energy}} = 0.5025$.

Figure 4 shows the mid-heater located between the first and second part, labeled as Position 0.25. While Figure 5 shows that the mid-heater is located between the third and fourth part, labeled as Position 0.75. At Position 0.25, both Regenerator A and B increase the acoustic power, R_a and R_b are both larger than 1 (see Table 2). At Position 0.75, Regenerator A increases the acoustic power but B decreases the acoustic power, $R_a > 1$ and $R_b < 1$. In Regenerator B, though $dT_m/dx > 1$, the dissipation loss is larger than the acoustic power generated in the form of $[g\tilde{p}_1 U_1]/2$ (see Equation 2), thus $R_b < 1$. When the acoustic power entering the resonator of the TE is taken into consideration in Figures 4 and 5, both η and η_e at Position 0.75 are larger than those at Position 0.25, which means that the mid-heater located at Position 0.75 improve the temperature gradient of the regenerator better than that at Position 0.25. For the same reason, the performance at Position 0.5 is better than that at Position 0.25. At Position 0.75 the TE has the best performance, because of the higher heating temperature. A heat source with lower grade (whose temperature is lower than the heater temperature) can be utilized to improve the temperature gradient in the

regenerator, so as to increase the acoustic power, efficiency and exergy efficiency of a TE.

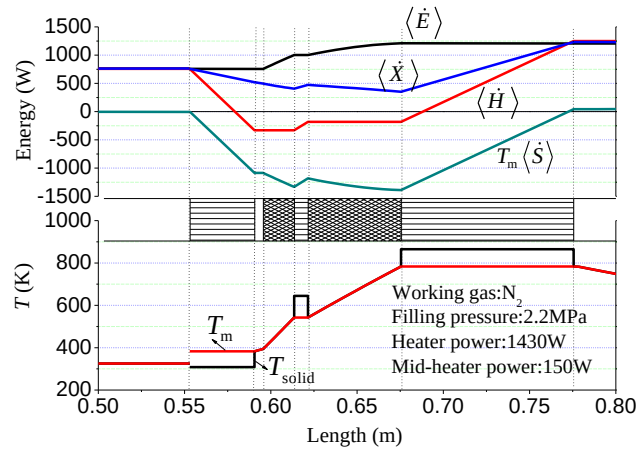


Figure 4 Energy flows and the temperature distribution in the TE using TTHS for Position 0.25. $R_{\text{energy}} = 0.4533$.

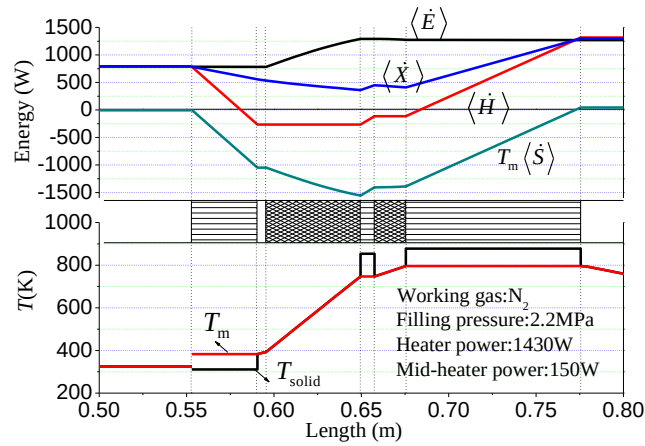


Figure 5 Energy flows and the temperature distribution in the TE using TTHS for Position 0.75. $R_{\text{energy}} = 0.5681$.

The energy flows in the new TE when $R_{\text{energy}} = 1$ is shown in Figure 6.A. Because the inflow heat into mid-heater is equal to the enthalpy flow in Regenerator A, the enthalpy flow in Regenerator B is 0. Regenerator A amplifies the acoustic power from 779 W to 1574 W ($R_a = 2.02$). The axial temperature gradient in Regenerator B is zero, which does not increase the acoustic power. Instead, Regenerator B dissipates the acoustic power to a value of 1381 W ($R_b = 0.88$). The total amplifying ratio $R = 1.77$. Figure 6.B shows the energy conversion when $R_{\text{energy}} = 1.1$. The heat flows into the

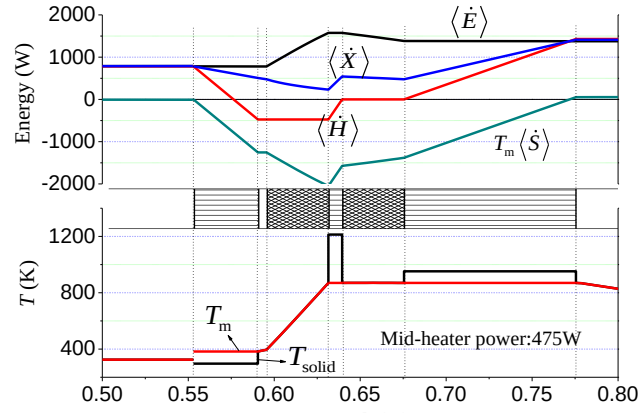
mid-heater is larger than the enthalpy flow in Regenerator A, thus the enthalpy flow in Regenerator B is larger than 0. Regenerator A amplifies the acoustic power from 779 W to 1735 W ($R_a=2.23$). The temperature gradient in Regenerator B is negative, which makes the acoustic power decrease to 1434 W ($R_b=0.83$). The total amplifying ratio $R=1.84$. Under those two conditions, the temperature in the mid-heater is larger than that in the main heater, the use of the mid-heater makes η_e even lower than that in traditional TE (see Table 2), which worsens the temperature gradient of the regenerator. It is necessary to find a way to optimize the mid-heater.

Table 2 Performance of different regenerators

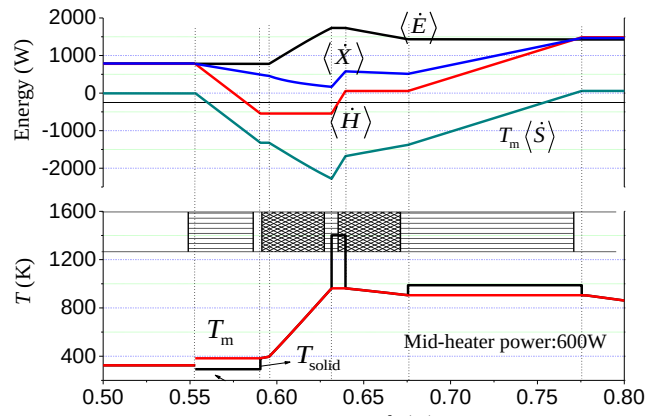
	Traditional	Position 0.25	Position 0.5	Position 0.75	Position 0.5	Position 0.5
$Q_m(W)$		150	150	150	475	600
R_{energy}		0.4533	0.5025	0.5681	1	1.1
R_a		1.326	1.55	1.649	2.02	2.23
R_b		1.207	1.05	0.987	0.88	0.83
R	1.57	1.60	1.62	1.63	1.60	1.84
$\eta(\%)$	19.31	18.77	19.76	20.34	20.94	21.47
$\eta_e(\%)$	29.55	28.87	29.89	30.58	29.54	29.44

Take the TE using TTHS at Position 0.5 as example to illustrate the optimization method. The mid-heater power is increased gradually and the heating power is fixed at 1430 W. Figure 7 is obtained with R_{energy} as x-coordinate, the acoustic power entering the resonator $\langle \dot{E} \rangle_{res}$ in the traveling-wave TE, efficiency η , exergy efficiency η_e and solid wall temperature T_{solid} as y-coordinate. From Figure 7 we can see that $\langle \dot{E} \rangle_{res}$ and η increase with R_{energy} , and η_e has a peak value when $R_{energy} = 0.5$. $R_{energy} < 0.71$ must be satisfied in order to ensure the mid-heater temperature lower than heater temperature. Obviously, $R_{energy} = 0.5$ is the optimal

condition under which a lower heat source can be utilized to get the larger acoustic power, efficiency, and exergy efficiency.



A. $R_{\text{energy}} = 1$, Position 0.5.



B. $R_{\text{energy}} = 1.1$, Position 0.5.

Figure 6 Energy flows and the temperature distribution in the TE using TTHS.

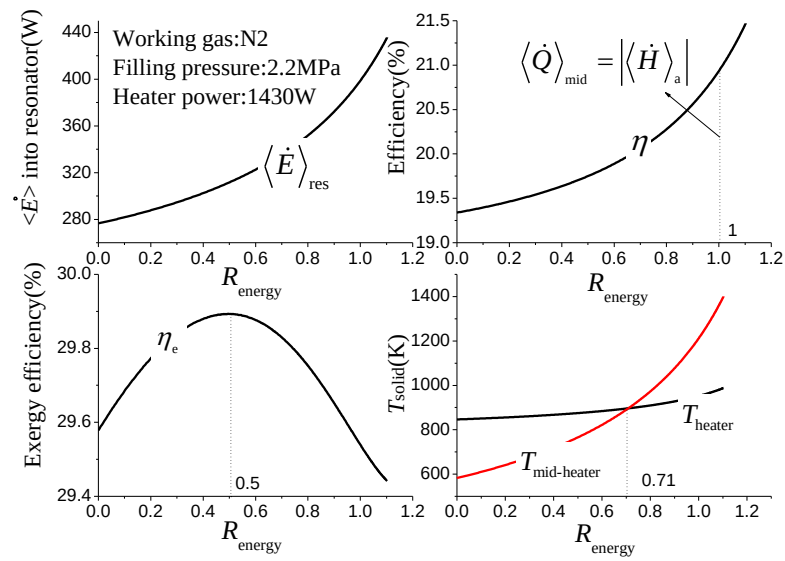


Figure 7 Dependence of $\langle \dot{E} \rangle_{\text{res}}$, η , η_e , and T_{solid} on R_{energy} in the TE using TTHS.

4 Conclusion

The use of the mid-heater changes the temperature gradient and the energy distribution of the regenerator. It mainly increases the temperature gradient in Regenerator A, which significantly increases the acoustic power. However, Regenerator B can produce a small increase when R_{energy} is small, which reduces the acoustic power when $R_{\text{energy}} \geq 1$. When an additional heat source with lower grade (compared with the temperature of the main heater) is available, it is possible to improve the performance of a TE using a mid-heater with appropriate heating power.

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Appendix A: Thermodynamic fundamentals of open systems [8]

For the open system shown in Figure A.A, the equation below can be obtained according to the First Law:

$$\dot{Q} = \dot{W} + (\dot{m}_e h_e - \dot{m}_i h_i) + \frac{d(mu)}{dt} \quad (\text{A1})$$

The Second Law for an open system shown in Figure A.B is:

$$\frac{\dot{Q}}{T_j} = (\dot{m}_e s_e - \dot{m}_i s_i) - \dot{S}_{\text{irr}} + \frac{d(ms)}{dt} \quad (\text{A2})$$

Time-averaged parameters are commonly used to describe the oscillating flow. The time-averaged mass flow is given by

$$\langle \dot{m} \rangle = \frac{1}{\tau} \int_0^{\tau} \dot{m} dt \quad (\text{A3})$$

Under the steady-state conditions in oscillating flow without the DC flow [9] (In TE study, DC flow is commonly called Gedeon streaming [6]) in the system, $\langle \dot{m} \rangle = 0$. The systems we discuss in this paper are all considered as steady state without Gedeon streaming. The time-averaged enthalpy and entropy flow can be expressed

using specific enthalpy h and specific entropy s , respectively:

$$\langle \dot{H} \rangle = \frac{1}{\tau} \int_0^\tau \dot{m} h dt \quad (\text{A4})$$

$$\langle \dot{S} \rangle = \frac{1}{\tau} \int_0^\tau \dot{m} s dt \quad (\text{A5})$$

For open oscillating systems, applying time averages to Equations (A1) and (A2) yields (where T_j is the temperature of the heat source):

$$\langle \dot{Q} \rangle = \langle \dot{W} \rangle + \langle \dot{H} \rangle_e - \langle \dot{H} \rangle_i + \frac{d\langle U \rangle}{dt} \quad (\text{A6})$$

$$\frac{\langle \dot{Q} \rangle}{T_j} = \langle \dot{S} \rangle_e - \langle \dot{S} \rangle_i - \langle \dot{S}_{\text{irr}} \rangle + \frac{d\langle S \rangle}{dt} \quad (\text{A7})$$

where $\langle U \rangle$ and $\langle S \rangle$ are the internal energy and entropy of the system averaged over one cycle. $d\langle U \rangle/dt=0$ and $d\langle S \rangle/dt=0$ for steady-state systems. When the gas is

considered as ideal gas, according to Equation (A4), $\langle \dot{H} \rangle = \frac{1}{\tau} \int_0^\tau \dot{m} c_p T dt$, thus for the

isothermal element, $\langle \dot{H} \rangle = 0$; In an ideal isothermal element without time-averaged

heat flow to the system, Equation (A7) becomes $\langle \dot{S} \rangle_e = \langle \dot{S} \rangle_i$, i.e., $\langle \dot{S} \rangle = \text{constant}$. In

an ideal adiabatic element, specific entropy s does not change, Equation (A5)

becomes $\langle \dot{S} \rangle = \frac{1}{\tau} \int_0^\tau \dot{m} ds$, equal to 0; For an adiabatic element, because $\langle \dot{Q} \rangle = 0$, $\langle \dot{H} \rangle$

is constant throughout the element according to Equation (A6) as long as there is no

work being done along the element, i.e., $\langle \dot{H} \rangle_e = \langle \dot{H} \rangle_i$. In summary:

$$\text{ideal isothermal element: } \langle \dot{H} \rangle = 0, \langle \dot{S} \rangle = \text{constant}; \quad (\text{A8})$$

$$\text{ideal adiabatic element: } \langle \dot{H} \rangle = \text{constant}, \langle \dot{S} \rangle = 0. \quad (\text{A9})$$

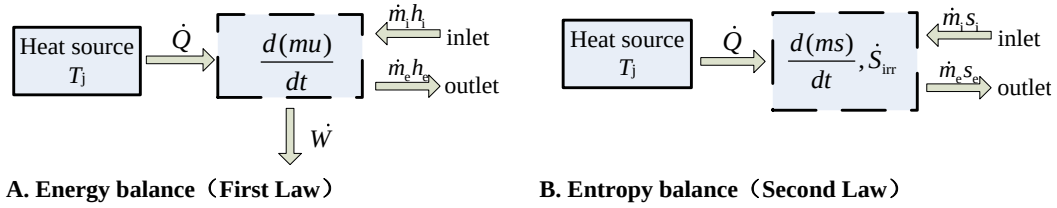


Figure A Schematic of a thermoacoustic component as an open thermodynamic system

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List of symbols:

English letters

A	area, m^2
c_p	isobaric heat capacity per unit mass, $\text{J/kg} \cdot \text{K}$
$\dot{E}$	acoustic power, W
g	complex gain factor arising in continuity equation
$\langle \dot{H}_2 \rangle_x$	time-averaged total power, W
$\dot{H}$	enthalpy, J
h	specific enthalpy, J/kg
k	thermal conductivity, $\text{W/m} \cdot \text{K}$
$\dot{m}$	mass flow, kg/s
p	pressure amplitude, MPa
$\dot{Q}$	heat transfer, W
Q_m	heating power of the mid-heater
R	ratio of output to input acoustic power
r_h	hydraulic radius, m
r	acoustic resistance per unit length, $\text{Pa} \cdot \text{s/m}^4$
$\dot{S}$	entropy, J/K
$\langle S \rangle$	entropy averaged over one cycle
s	specific entropy, $\text{J/kg} \cdot \text{K}$
T	temperature, K
t	time, s
U	volumetric flow rate, m^3/s
$\langle U \rangle$	internal energy averaged over one cycle
u	internal energy per unit mass, J/kg
u_x	x component of velocity, m/s
$\dot{W}$	work, W

$\dot{X}$ exergy flow, W

Greek letters

δ penetration depth, m

η efficiency

η_e exergy efficiency

ρ density, kg/m³

τ time over one cycle, s

Subscripts

m mean

e outlet

i inlet

j integer

k thermal

ν viscous

0 “environmental” or “ambient”

1 first order

mid middle

res into resonator of a TE

Special symbols

Re[] real part of

$\langle \rangle$ time-averaged

$||$ magnitude of complex number

overbar time average