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Mathematical programming methods for pressure management in water distribution systems

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Abstract

In this paper, we survey mathematical programming methods for the management of pressure in water distribution systems through optimal placement and operation of control valves. The optimization framework addresses the minimization of average zone pressure under multiple demand scenarios, enforcing hydraulic equations as nonlinear constraints. Binary variables are used to model the placement of valves. The derived nonlinear optimization problem is a non-convex MINLP. We implement and evaluate a direct MINLP solver and two different reformulation methods for MINLPs that solve sequences of regular NLPs. Moreover, we investigate the solutions under different design and operation loading conditions.

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1. Introduction

Pressure is a critical variable for water distribution systems since it is directly related to leakage and burst frequency [1]. Great benefits can be derived by keeping the operational pressure close to the minimum allowed by standard regulations. The subdivision of water distribution networks into small areas, called District Metered Areas (DMAs), has been successfully implemented in the last two decades for the management of pressure and leakage. The sectorization is realized through boundary valves, that are closed in order to form small metered areas, so as to easily monitor flow between zones and optimize pressure in a simplified way. This practice has improved leakage reduction, but at the same time has severely reduced the redundancy of the networks, affecting resilience and water quality [2]. Previous work on water distribution networks with adaptively reconfigurable topology [2] integrated the achievements in terms of leakage management provided by the DMAs structure with the benefits of high network connectivity, resulting in increased network redundancy and resilience. The network is segregated into small areas during the night, to take advantage of the sectorized topology for leakage localization. DMAs are then aggregated in order to achieve an effective pressure and resilience management in diurnal operation. To benefit from these emerging

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and advanced control schemes, the retrofit of existing networks requires the formulation and solution of both design and operational optimization problems.

In the present work we focus on mathematical optimization for network pressure management, minimizing average zone pressure through the optimal placement and operation of pressure reducing valves. Valve placement is a challenging problem because it combines binary variables (whether the valve is placed or not) with continuous variables representing nodal pressures and pipe flows. Valve control is embedded in the optimal placement problem, since the operational setting of a valve is identified with the pressure at the downstream node. The use of physically feasible models for real water distribution networks involves the formulation of hydraulic equations that lead to nonlinear constraints. Together with discrete decision variables, these result in a non-convex optimization problem with a large number of integer variables, belonging to the class of mixed integer nonlinear programs (MINLP).

Optimal valve placement has been studied using both mathematical programming and heuristic methods. In [3] a mixed integer linear programming method was applied to a linear approximation of the original optimization problem. A direct solution with a mixed integer nonlinear solver has been proposed in the report [4], considering just a steady-state snapshot of daily network operation. A faithful representation of real world water distribution systems requires the formulation of an extended time optimization problem, which includes multiple demand scenarios. These have been considered in [5], wherein is proposed a reformulation approach for mixed integer nonlinear programs. By relaxing the integer constraints, a penalty method is coupled with a heuristic rounding scheme for binary variables in order to improve the convergence of the method. Genetic algorithms and meta-heuristic approaches have also been applied to optimal valve placement in [6–8]. However, these methods present some fundamental drawbacks: they do not guarantee convergence to optimal solutions (not even to local optima) and they require a large number of function evaluations of objective and constraints in order to achieve good quality solutions. Therefore, with application to advanced and dynamic control schemes, it is necessary to study reliable and effective mathematical optimization methods for optimal valve placement and operation.

MINLPs are particularly challenging and several techniques have been applied to these problems, see for reference [9–11]. Since the integer variables involved are all binary, in the present work we investigate possible solutions by reformulating the problem as an equivalent mathematical program with complementarity constraints (MPCC). Th feasible set of an MPCC present a special structure that prevents solution via direct application of standard nonlinear programming solvers. Various methods have been proposed to handle this pathological nature; we focus on penalization and relaxation approaches. Convergence properties under suitable assumptions have been proved for both penalty methods [12–15] and relaxation methods [16–18]. We propose an algorithmic implementation of these reformulation approaches and we apply them to the case study.

Since the problem is non-convex, like most nonlinear programming algorithms, under suitable assumptions all the methods guarantee convergence only to local minimum points and the quality of the solution would depend on the initial point. We have to take this into account when comparing different approaches. Therefore, we will consider a solution qualitatively good if it is obtained with various random initial guesses and provides an average zone pressure close to the best known solution.

2. Problem formulation

We address the minimization of average zone pressure through the placement of n_v control valves in a water distribution network with n_0 water sources (eg. reservoirs or tanks), n_n nodes and n_p pipes, represented as a directed graph (V, E), with $n_n + n_0$ vertices and $2n_p$ links. Note that the j^{th} physical pipe is modelled by two graph links j and $j + n_p$, for all $j = 1, ..., n_p$. The optimization problem is solved for extended time settings; we discretize the considered time interval into n_l subintervals. For each time step $k \in \{1, ..n_l\}$, we have vectors of known nodal demands $d^k \in \mathbb{R}^{n_n}$ and known fixed heads at water sources $h_0^k \in \mathbb{R}^{n_0}$. Moreover, each node $i \in \{1, ..., n_n\}$ has a known elevation e_i. Unknown pressure heads and flows, at each time step k, are represented by the vectors $p^k = [p_1^k, ..., p_{n_n}^k]^T$ and $q^k = [q_1^k, ..., q_{2n_p}^k]^T$, respectively. The friction headloss across every link $i_1 \xrightarrow{j} i_2$ is commonly represented by the Hazen-Williams (HW) or Darcy-Weisbach (DW) formula. In DW models the relation between friction headloss and flow is expressed by an implicit equation and has to be calculated through an iterative process. This complicates the implementation of these models in a mathematical optimization framework. On the other hand, the semi-empirical HW formula is given by $HW_f(q_j^k) = r_j(q_j^k)^{1.852}$, where r_j, the resistance coefficient of the pipe, is

defined by $r_j = \frac{10.670 L_j}{C_j^{1.852} D_j^{4.871}}$ with L_j, C_j, D_j length, roughness and diameter of the pipe, respectively. The HW formula involves a non-smooth exponential function, hence is difficult to handle for most nonlinear programming solvers. Therefore, a smooth and sufficiently accurate approximation of the headloss formulae is needed. Consider a generic pipe with length L, diameter D and H-W coefficient C. We model the friction head loss using a quadratic function $h_f(q) := a^\star q^2 + b^\star q$, with $(a^\star, b^\star)$ the minima of the integral of the square error $J_{q_1,q_2}(a,b) := \int_{q_1}^{q_2} \left(aq^2 + bq - rq^{1.852}\right)^2 dq$ over an approximation range $[q_1, q_2]$.

Once the quadratic model is calculated, it is possible to formulate the optimization problem. For each link j we need to introduce a new unknown, which assume value 0 or 1 depending whether a valve is present on j or not

$$v_j = \begin{cases} 1, & \text{if a valve is placed on link } j \\ 0, & \text{otherwise.} \end{cases}$$

The optimization problem for optimal valve placement and operation can be formulated as follows, for the sake of brevity we omit some details on the definition of constraints, which can be found in [3–5,19].

$$\min \sum_{k=1}^{n_l} \frac{1}{W} \sum_{i=1}^{n_n} w_i p_i^k \tag{1a}$$

$$\text{subject to:} \quad A_{12}^T q^k - d^k = 0, \quad \forall k = 1, ..., n_l, \tag{1b}$$

$$S(q^k)\left(-A_{12} p^k - A_{12} e - A_{10} h_0^k - h_f(q^k)\right) \geq 0, \quad \forall k = 1, ..., n_l, \tag{1c}$$

$$-A_{12} p^k - A_{12} e - A_{10} h_0^k - h_f(q^k) - M^k v \leq 0, \quad \forall k = 1, ..., n_l, \tag{1d}$$

$$v_j + v_{n_p + j} \leq 1, \quad \forall j = 1, ..., n_p, \tag{1e}$$

$$\sum_{j=1}^{2n_p} v_j = n_v, \tag{1f}$$

$$p_{min}^k \leq p^k \leq p_{max}^k, \quad \forall k = 1, ..., n_l, \tag{1g}$$

$$0 \leq q_j^k \leq \frac{\pi D_j^2}{4}, \quad \forall j = 1, ..., 2n_p, \ \forall k = 1, ..., n_l, \tag{1h}$$

$$v \in \{0,1\}^{2n_p}. \tag{1i}$$

The objective is the minimization of *average zone pressure* over an extended time period, expressed by the weighted sum of nodal pressures (1a), where weights w_i are defined by $w_i := \sum_{j \in I(i)} \frac{L_j}{2}$, with $I(i)$ the set of indices of links that are connected to node i and $W := \sum_{i=1}^{n_n} w_i$. The optimization problem is primarily subject to hydraulic constraints, in order to ensure the physical feasibility of the solutions. The conservation of flow at each junction node is expressed by (1b), while the couple (1c), (1d) represents the conservation of energy across all links. The matrices $A_{12}^T \in \mathbb{R}^{n_n \times 2n_p}$ and $A_{10}^T \in \mathbb{R}^{n_0 \times 2n_p}$ are the node-edge incidence matrices for the n_n nodes and the n_0 water sources, respectively. Moreover, $S \in \mathbb{R}^{2n_p \times 2n_p}$ is the diagonal matrix of flows q^k, $S(q^k)_{j,j} = q_j^k$, and $h_f(q^k) = [h_f(q_1^k) \ ... \ h_f(q_{2n_p}^k)]^T$. Finally, $M^k \in \mathbb{R}^{2n_p \times 2n_p}$ is a diagonal matrix of sufficiently large constants. The optimization problem is also subjected to some specific physical, economic and operational constraints, expressed by (1e), (1f), (1g) and (1h), respectively.

Problem (1) is a non-convex MINLP, with polynomial nonlinear constraints and $N = n_l(n_n + 2n_p) + 2n_p$ variables. This class of mathematical programs is particularly challenging, especially when the scale of the problem is large. In fact, it combines the difficulties of dealing with nonlinear non-convex constraints, with the complication of optimizing in a discrete variable space. When the number of integer variables becomes large, it is in general impossible to find a solution. We aim to investigate specialized mathematical programming solution methods for (1).

3. Reformulation approaches

Let $x = [p^T, q^T, v^T]^T \in \mathbb{R}^N$ be the vector of unknowns. We set $B = \left\{n_l(n_n + 2n_p) + 1, ..., n_l(n_n + 2n_p) + 2n_p\right\}$, the set of indices corresponding to the binary variables. Similarly, let I and E be the index set of rows in (1) which correspond

to inequality and equality constraints, respectively. Moreover, let $g : \mathbb{R}^N \to \mathbb{R}^m$ with $m = |I|$ and $h : \mathbb{R}^N \to \mathbb{R}^n$ with $n = |E|$ be the functions that group inequality and equality constraints, respectively. Since its integer variables are binaries, Problem (1) can be equivalently reformulated as a mathematical program with complementarity constraints (MPCC):

$$\begin{aligned} \min_{x \in \mathbb{R}^N} \quad & f(x) && \text{(2a)} \\ \text{subject to} \quad & g_i(x) \geq 0, \quad \forall i \in I, && \text{(2b)} \\ & h_i(x) = 0, \quad \forall i \in E, && \text{(2c)} \\ & 0 \leq x_j \perp 1 - x_j \geq 0, \quad \forall j \in B. && \text{(2d)} \end{aligned}$$

In the framework of nonlinear programming (NLP), linear independence of the gradients of constraints, also referred to as linear independence constraint qualification (LICQ), is usually required in order to guarantee the convergence of NLP solvers to stationary points, see [20]. Complementarity constraints result in a special feasible set, which violates generic constraint qualifications, causing severe convergence issues to standard NLP solvers. Nonetheless, MPCCs have been solved using tailored algorithms in other engineering frameworks [15,21]. Various methods have been proposed in recent years, see [12–14,16–18,22] and the references therein.

We are interested in penalty and relaxation approaches, both employ the solution of sequences of regular nonlinear programs. The penalty method relies on the solution of a sequence of NLPs, each one obtained dropping the special constraint (2d) and penalizing in the objective deviations from the complementarity condition. It has already been applied to optimal valve placement in [5], coupled with an heuristic rounding scheme, in order to improve the convergence of their method. On the other hand, the relaxation method focuses on the geometry of the region of feasibility and solves a sequence of NLPs with a simplified feasible set. Although this relaxation approach has been adopted in other engineering problems with complementarity constraints, its application to water distribution systems is novel. In the following we present them with more details.

Using the same notation of (2), we introduce the following penalty function $\Psi(x) = \sum_{j \in B} x_j(1 - x_j)$. Given $\rho > 0$, consider the nonlinear program PEN(ρ):

$$\begin{aligned} \min_{x \in \mathbb{R}^N} \ & f(x) + \rho\Psi(x) \ s.t. \\ & g_i(x) \geq 0, \quad \forall i \in I, \\ & h_i(x) = 0, \quad \forall i \in E, \\ & 0 \leq x_j \leq 1, \quad \forall j \in B. \end{aligned} \tag{3}$$

Penalty approaches have been studied in [12–15,21], the overall strategy consists in the generation of a sequence of stationary points $(x^k)_k$ of PEN(ρ^k), with $\rho^k \xrightarrow{k \to +\infty} +\infty$. Then it is possible to prove (see [12,15]) that the iterates (x^k) converge to a stationary point of Problem (2). We implement the penalty method is Algorithm 1. The initial guess g_0 is selected randomly and the complementarity violation is evaluated as $\texttt{Vio}(x) = \max_{j \in B}\big(\min(x_j, 1 - x_j)\big)$.

Algorithm 1 Penalty method

1: **Initialization:**
   Select random g_0, $\beta \leftarrow 1.1$, $\alpha \leftarrow 0.01$, $\epsilon \leftarrow 10^{-6}$;
2: Solve $PEN(0)$ with initial guess g_0 and get the solution x^0 and value of objective function z^0;
3: $k \leftarrow 0$; $\rho^0 \leftarrow \alpha \cdot z^0$;
4: **while** $\texttt{Vio}(x^k) > \epsilon$ **do**
5: Solve $PEN(\rho^k)$ with initial guess x^k and get the solution x^{k+1};
6: $\rho^{k+1} \leftarrow \beta \cdot \rho^k$;
7: $k \leftarrow k + 1$;
8: **end while**

Alternative approaches for the solution of Problem (2) include relaxation methods, see [14,17,18]. In the present work we focus on the technique proposed by [17], where theoretical proof for the convergence is provided. For $t > 0$

consider the nonlinear program REL(t)

$$\begin{aligned} \min_{x\in\mathbb{R}^N} \; & f(x) \; s.t. \\ & g_i(x) \geq 0, \quad \forall i \in I, \\ & h_i(x) = 0, \quad \forall i \in E, \\ & 0 \leq x_j \leq 1, \quad \forall j \in B, \\ & \sum_{j\in B} x_j(1-x_j) \leq t. \end{aligned} \tag{4}$$

The method generates a sequence of stationary points of (4), for decreasing values of the positive parameter t, solving the nonlinear problem REL(t). The sequence will convergence to a stationary point of Problem (2), for a rigorous and general proof see the appendix in [17]. We implement the relaxation method in Algorithm 2, where $\texttt{Vio}(x)$ is the same as above.

Algorithm 2 Relaxation method

1: **Initialization:**
   Select random g_0, $x^0 \leftarrow g_0$, $t^0 \leftarrow 1$, $t_{min} \leftarrow 10^{-15}$, $\epsilon \leftarrow 10^{-6}$, $\beta \leftarrow 10^{-4}$, $k \leftarrow 0$;
2: **while** $\left(\texttt{Vio}(x^k) > \epsilon \;\&\; t^k > t_{min}\right)$ OR $\left(k = 0\right)$ **do**
3: Solve $REL(t^k)$ with initial guess x^k and get the solution x^{k+1};
4: $t^{k+1} \leftarrow \beta \cdot t^k$;
5: $k \leftarrow k + 1$;
6: **end while**

All the NLP subproblems involved in the presented methods have sparse nonlinear structures and so can be solved using tailored techniques for large sparse nonlinear programs, offering a scalable approach for large scale water distribution systems. However, note that both penalty and relaxation approaches are local methods and so converge to stationary points of Problem (2). If the nonlinear constraints are non-convex, which is the case in our problem, then the convergence is highly influenced by initial guess and algorithmic parameters.

4. Case study

We have applied the different outlined mathematical programming approaches to Problem (1) and we compare their solutions with those provided by the MINLP solver BONMIN (v.1.8.1). The NLP subproblems within penalty and relaxation methods were solved using the interior point solver for large scale nonlinear optimization IPOPT (v3.11.8). Moreover, we directly provided to BONMIN and IPOPT sparse gradients and Jacobians, in order to exploit the very sparse structure of the problem. All computations were performed within MATLAB 2013b-64 bit for Windows 7, installed on a 2.50GHz Intel® Xeon(R) CPU E5-2640 0 with 18 Cores.

The selected benchmarking network has been already considered in [4–8,23,24], see Figure 1a. The network has 22 nodes, 37 pipes and 3 reservoirs, see [5,24] for details on pipes' characteristics, nodal demands and reservoir levels. In contrast to the report in [4], our formulation is extended time, we divide network's daily operation into 24 time steps, assigning a specific demand scenario to each hour. A similar model was considered in [5]. Daily demand profile is reported in Figure 1b.

In the code implementation, in order to accommodate the scale of the problem, we considered the minimization of the non-normalized version of (1a) : $\sum_{k=1}^{n_l}\sum_{i=1}^{n_n} w_i p_i^k$. We follow the recommendation in [9,25] and we apply BONMIN with branch and bound algorithm to our non-convex MINLP (1), all other input options for BONMIN were set to their default values. In [5] BONMIN was reported to fail when the placement of more than 2 valves was considered. Also in our initial implementation the solver encountered convergence issues which we have overcome with the substitution of the equality linear constraint (1f) with $\sum_{j=1}^{n_p} v_j \leq n_v$. Note that this inequality constraint results in an equivalent optimization problem, since we cannot reduce pressure further by having fewer valves. Moreover, the direct implementation of sparse gradients and Jacobians has improved the performance of BONMIN, allowing

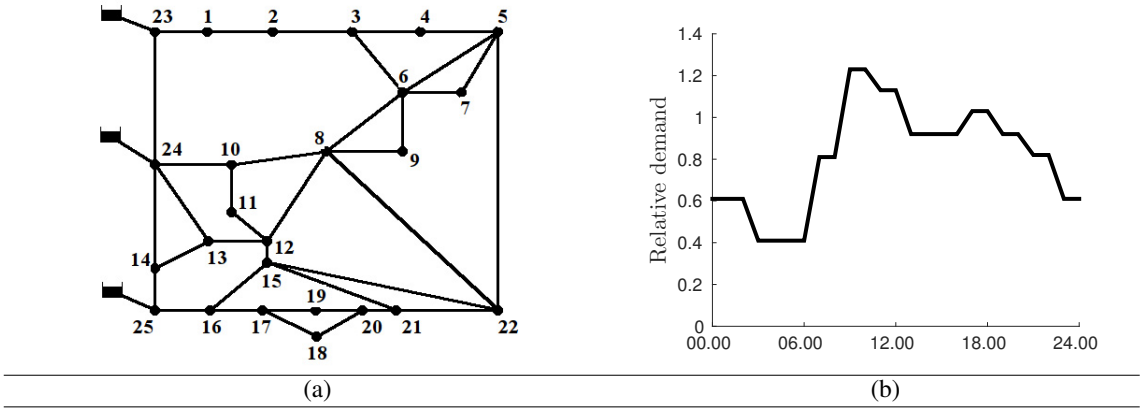


Figure 1. (a) network model from [23]; (b) demand profile

convergence to the best known optimal solutions for this network. In Table 1 we summarize the results, while in Figure 2 we present the average zone pressure profiles, corresponding to different number of optimized valves. Note that a relatively small decrease in pressure (p) can result in significant reduction in the amount of leakage (L), since $L \propto p^{\alpha_L}$ with $0.5 \leq \alpha_L \leq 2.5$ [1]. Moreover, the placement of an optimized control valve can provide other benefits, not directly related to the minimization of AZP. For example, when we move from the configuration with 4 valves to the optimized set of 5 valves we achieve a substantial decrease in night pressure, reducing pressure variability in the network (see Figure 2).

In Figure 3 we show the optimal daily operation for 4 optimized valves. Valve on link $(23, 1)$ is always open and acts as a pressure reducing valve (PRV), maintaining outlet pressure close to the minimum. On the other hand, the control valve on $(22, 15)$ operates as a fixed closed boundary valve (BV). The remaining two valves have a similar dynamic behaviour; they are closed BVs during the night (minimum demand period) and active PRVs in diurnal operation. This benchmarking example demonstrates that our optimization constraints are very flexible and allow the modelling of control valves that operate as PRVs or BVs depending on hydraulic conditions. Therefore, the presented mathematical formulation is suitable for use in the design and implementation of the adaptive reconfigurable topology for water distribution networks proposed in [2].

Generally, the non-convexity of nonlinear constraint affects the convergence properties of tools like BONMIN for the solution of MINLPs, as noted in [25]. However, the optimal placements in Table 1 are obtained with different random initial guesses and are the best-known solutions for the case study. Therefore, the robustness of our implementation of BONMIN is satisfactory. Nonetheless, computational time suggests that more issues can arise when dealing with large scale networks, where the number of integer variables grows. This is the main motivation for the study of possible reformulation approaches.

Table 1. Optimal placement of control valves with Bonmin

No. of valves	Link	AZP/h	CPU time
1	$(13, 12)$	33.90 m	76 s
2	$(23, 1), (13, 12)$	32.94 m	962 s
3	$(23, 1), (13, 12), (16, 17)$	32.61 m	1345 s
4	$(23, 1), (22, 15), (12, 15), (13, 12)$	31.90 m	2124 s
5	$(23, 1), (22, 15), (12, 15), (25, 16), (13, 12)$	31.48 m	1892 s

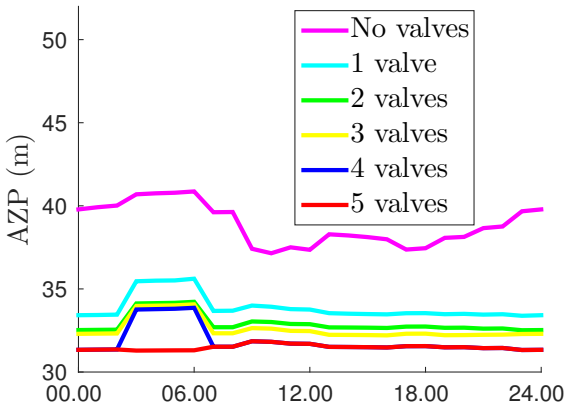


Figure 2. Average zone pressure after the optimal placement of a different number of control valves

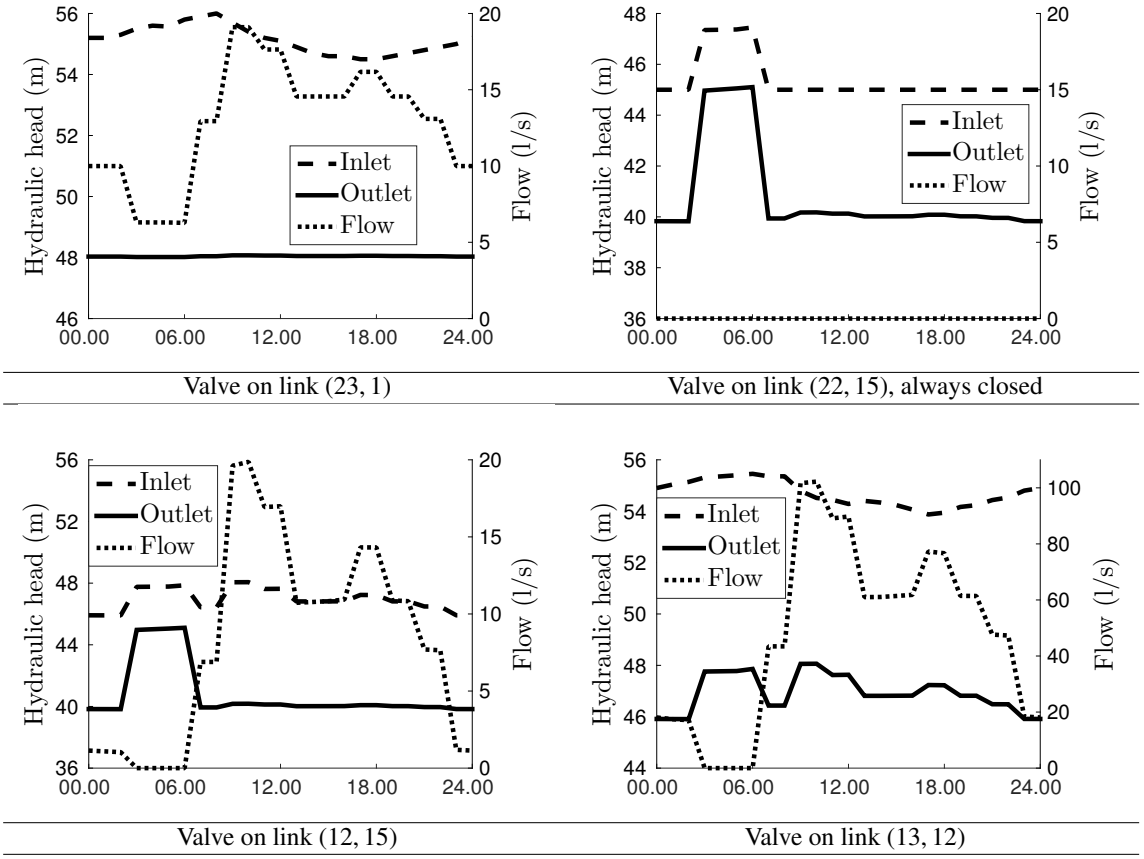


Figure 3. Operation of 4 optimized valves

4.1. Reformulation approaches

We applied to the case study the penalty and relaxation methods outlined in Section 3. We minimize average zone pressure, with 24 time steps, the results are reported in Table 2. A ⋆ highlights when the algorithm converged to the BONMIN solution. The penalty approach results in good quality solutions, although these do not always coincide with the BONMIN optima; however, our implementation of the penalty method results in improved performance with respect to a similar approach used in [5], both in terms of quality of solutions and computational time. On the other hand, the relaxation method was observed to achieve the same solution as BONMIN in most cases, with only slightly sub-optimal solutions where they are different.

Table 2. Optimal placement of control valves with reformulation approaches

No. of valves	Link IDs	AZP/h	CPU time	
Solution with Penalty method				
1	(13, 12)	33.90 m	126 s	⋆
2	(24, 10), (13, 12)	33.54 m	178 s	
3	(23, 1), (24, 10), (13, 12)	32.65 m	225 s	
4	(23, 1), (12, 15), (13, 12), (13, 12)	31.90 m	235 s	⋆
5	(23, 1), (22, 15), (12, 15), (24, 10), (13, 12)	31.62 m	101 s	
Solution with Relaxation method				
1	(13, 12)	33.90 m	66 s	⋆
2	(23, 1), (13, 12)	32.94 m	29 s	⋆
3	(23, 1), (12, 15), (13, 12)	32.93 m	77 s	
4	(23, 1), (22, 15), (12, 15), (13, 12)	31.90 m	15 s	⋆
5	(23, 1), (22, 15), (12, 15), (24, 10), (13, 12)	31.62 m	23 s	

Enabled by its superior computational performances, we applied the relaxation method to the problem in (1), with the number of valves ranging from 0 to 37. We report the average AZP for each number of optimized valves in Figure 4. We can consider Figure 4 as an approximation of the *p*areto front for a multiobjective optimization problem with objectives the minimization of AZP and the installation cost, expressed here by the number of valves. Note that, as reported also in [6], the region corresponding to large numbers of valves exhibits high instability and some of those points can not be optima; in fact, it is impossible to increase optimal AZP by adding more control valves. However, using Figure 4, we can realize a cost-benefit analysis: it appears that any solution with more than 7 valves should not be considered for this network, since the corresponding reductions in AZP are negligible.

4.2. Different loading conditions

Finally, we aim to investigate different demand conditions for the placement of valves and then for the determination of optimal control settings. Consider the *static* problem with just one fixed demand condition for the optimal placement of 3 valves. We solve with BONMIN three *static* optimization problems with demand fixed to the minimum, medium and maximum demand conditions from the diurnal demand profile. Then, enforcing the resulting optimal valve placement, we use IPOPT to solve the resulting nonlinear programs for the optimal operation of each valve. In Figure 5 compares the average zone pressure corresponding to the different configurations. Since the optimal placement found in Table 1 corresponds to the solution with medium demand, we conclude that an optimal solution for difficult extended time problem (1) can be obtained through the solution of a smaller MINLP using only the medium demand snapshot. Subsequently, we solve for an extended time NLP with the valve locations fixed.

5. Conclusions

We have presented a rigorous mathematical framework for the minimization of average zone pressure in water distribution networks, through optimal placement and operation of control valves. The mathematical formulation is

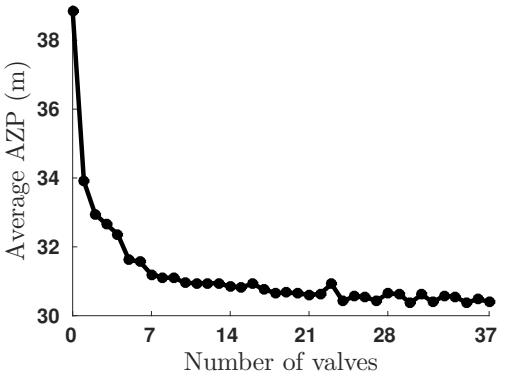


Figure 4. Average daily AZP corresponding to a different number of optimized valves

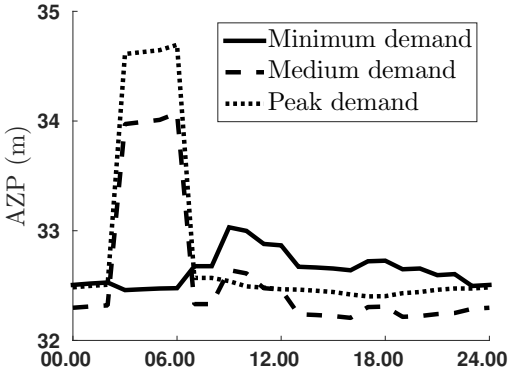


Figure 5. AZP corresponding to optimal placements obtained with different loading conditions: low demand: $(23,1),(24,10),(13,12)$; medium demand: $(23,1)$, $(13,12)$, $(16,17)$; peak demand: $(23,1),(22,15),(13,12)$

sufficiently flexible to incorporate different physical actuators, such as PRVs and BVs, within the same model. The optimization problem belongs to the class of non-convex mixed integer nonlinear programs. We have implemented and evaluated a direct MINLP solver BONMIN, using a benchmarking network model. Although our BONMIN implementation performed better than previous applications to the same problem, the CPU time required by the algorithm in this relative small network model suggests that infeasible computational time would arise when dealing with large scale networks, which would involve a large number of integer variables. We have therefore presented and implemented two reformulation approaches that reduce the MINLP to an NLP, the penalty and relaxation methods. The two methods rely on the solution of a sequence of sparse nonlinear problems which can be solved using tailored techniques for large sparse nonlinear programs. They have been both applied to the case study. The penalty method results in improved performances with respect previous work [5], where a similar scheme was applied to the problem of optimal valve placement for the same benchmarking network. However, the relaxation approach is shown to have superior performance both in quality of the solutions and CPU time. This relaxation method has been adopted in other engineering frameworks when dealing with complementarity constraints, however, its application to water distribution networks is novel. The numerical results are promising and suggests that our relaxation approach can be successfully applied to large-scale non-convex optimization problems with binary variables related to optimal design and operation of water distribution systems. Finally, we have investigated the influence of different loading conditions for the placement of valves. Results reveal that optimal pressure management is achieved when the placement of the valves is optimized under medium demand condition. In conclusion, the presented benchmarking study demonstrates that our mathematical optimization framework can provide effective tools to support design and operation of smart water distribution systems.

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