

Sequential Convex Optimization for Detecting and Locating Blockages in Water Distribution Networks

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ABSTRACT

Unreported partially/fully closed valves or other type of pipe blockages in water distribution networks result in unexpected energy losses within the systems, which we also refer to as faults. We investigate the problem of detection and localization of such faults. We propose a novel optimization-based method, which relies on the solution of a non-linear inverse problem with ℓ_1 regularization. We develop a sequential convex optimization algorithm to solve the resulting non-smooth non-convex optimization problem. The proposed algorithm enables the use of non-smooth terms within the problem formulation, and exploits the sparse structure inherent in water network models. The performance of the developed method is numerically evaluated to detect and localize blockages in a large water distribution network using both simulated and experimental data. In all experiments, the sequential convex optimization algorithm converged in less than three seconds, suggesting that the proposed fault detection and localization method is suitable for near real-time implementation. Furthermore, we experimentally validate the developed method for near real-time fault diagnosis in a large operational water network from the UK. The method is shown to successfully detect and localize blockages, with real system modelling uncertainties.

INTRODUCTION

Water distribution networks (WDNs) are faced with multiple challenges due to aging infrastructure, growing water demand, and more stringent environmental standards. Advanced pressure control schemes are being implemented to improve WDN management and operation (Wright et al. 2015; Pecci et al. 2019), and enable the dynamically adaptive control of both hydraulic conditions and network connectivity (Wright et al. 2014). These advances have resulted in an increased level of instrumentation (e.g. pressure control valves, isolation valves) throughout the network. In order to benefit from these technological innovations, accurate hydraulic models, together with monitoring and diagnostic tools, are critical to support intelligent and efficient network operation. In this work, we consider WDNs whose hydraulic models have been calibrated to achieve a level of accuracy that is compliant with model calibration guidelines (Water Research Centre 1989). After the initial model calibration, monitoring and diagnostic tools are necessary to continuously validate and maintain the hydraulic model performance.

This paper investigates the detection and localization of blockages, which are due to unreported changes to status or location of valves, following various network interventions, or other type of blockages (e.g. the accumulation of particles at strainers of flow meters). In the following, blockages are interchangeably referred to as faults. These faults cause unexpected energy losses (local losses), and result in inaccurate hydraulic models, affecting WDNs operation. We formulate the problem of detection and localization of blockages in WDNs as a non-linear inverse problem, where we estimate the blockage location using measured hydraulic data - for a review on inverse problems, see (Aster et al. 2012). Non-linear inverse problems have been previously studied to address a variety of estimation problems in WDNs. Some examples include hydraulic model calibration (Kapelan 2002; Piller et al. 2012; Kang and Lansey 2011; Do et al. 2016), state estimation (Preis et al. 2011; Piller et al. 2014), and leak detection (Pudar and Ligett James 1992; Sophocleous et al. 2019). Previous methods to identify partially/fully closed valves include Delgado and Lansey (2009), Wu et al. (2012), and Do et al. (2018). Delgado and Lansey (2009) implemented a transient model to detect a closed valve in a single pipeline. In comparison, studies by

Wu et al. (2012) and Do et al. (2018) formulated and solved an inverse problem to calibrate valve local loss coefficients in WDNs. In particular, Do et al. (2018) implemented a combination of a genetic algorithm with a Levenberg-Marquardt method to identify the location of partially/fully closed valves in a water network with 147 pipes, with a reported overall computational time of more than 4 hours. As observed by Do et al. (2018), methods implemented in previous literature require a significant computational effort and are not suitable for near real-time implementations of monitoring and diagnostic tools when large scale WDNs are considered.

The inverse problem of detection and localization of blockages in WDNs is ill-posed, due to the availability of fewer measurements than possible fault locations (Do et al. 2018). A standard technique to solve ill-posed inverse problems is based on ℓ_2 (or Tikhonov) regularization (Neumaier 1998), which was implemented by Piller et al. (2012) to regularize a joint hydraulic and water quality model calibration problem. The resulting optimization problem was then solved using a Gauss-Newton method. In this study, we assume that most of the potential faults (blockages) are not present at the same time. Therefore, the vector of fault states is expected to be sparse. In comparison to ℓ_2 regularization, ℓ_1 regularization techniques are more effective in promoting sparsity within the solution vector (Tibshirani 1996), and they are commonly employed for sparse signal recovery (Donoho 2006). The implementation of ℓ_1 regularization within inverse problem formulations for fault estimation has been considered in other engineering frameworks (Gorinevsky et al. 2009). However, to the best of the authors knowledge, ℓ_1 regularization techniques has not been applied to fault diagnosis problems in water networks.

In this paper, we propose a novel inverse problem formulation to detect and localize blockages in WDNs. The considered formulation aims to detect and localize a blockage by estimating the corresponding fault state, defined as deviation from the nominal state of the head difference across a given pipe or valve. Our problem formulation is simplified with respect to the non-linear model used by Wu et al. (2012, Do et al. (2018) to calibrate local loss coefficients. The objective function to be minimized relies on the sum of squares residuals (Pudar and Ligett James 1992; Kang and Lansey 2011; Piller et al. 2012; Piller et al. 2014; Do et al. 2018). Moreover, we add a ℓ_1

penalty term to promote sparsity within the fault state vector. Standard gradient-based optimization methods implemented in previous literature (Pudar and Ligett James 1992; Kang and Lansey 2011; Piller et al. 2012; Piller et al. 2014) are not suitable for solving the considered ℓ_1 regularized inverse problem, which result in a non-smooth non-convex optimization problem. Therefore, we propose a sequential convex optimization algorithm to solve the considered non-smooth non-convex optimization problem. The developed algorithm relies on the solution of a series of convex sub-problems within a trust region algorithm (Conn et al. 2000). The proposed solution algorithm exploits the sparse structure inherent in WDN models, which is retained by the constraints of the convex sub-problems. In contrast to previous literature (Kang and Lansey 2011; Piller et al. 2012; Piller et al. 2014), the developed algorithm does not require the computation of the full inverse of large matrices to evaluate the derivatives of the residual vector with respect to the unknowns.

Benefits and limitations of the proposed method are investigated using a large operational water network as a case study. Firstly, we test the ability of the method to accurately detect and localize single and multiple faults using simulated data. Furthermore, we demonstrate and validate a near real-time implementation of the developed fault detection and localization method using experimental data from the considered case study.

PROBLEM FORMULATION

In the following, we formulate the problem of detection and localization of blockages in WDNs by using a single snapshot of network operation. For multiple demand conditions, the proposed fault detection and localization method is implemented sequentially to solve an inverse problem at each time step. We consider a WDN with n_p links (e.g. pipes and valves), n_n demand nodes, and n_0 known head nodes (e.g. water sources, reservoirs). Known nodal demands are defined by the vector $\mathbf{d} \in \mathbb{R}^{n_n}$ (measured in m^3/s). The hydraulic heads at water sources (e.g. reservoirs, network inlets) are known. The vector of known hydraulic heads is given by $\mathbf{h}_0 \in \mathbb{R}^{n_0}$ (measured in meters). In addition, we assume that when the network model includes pumps and tanks, hydraulic heads at pump and tank outlets are measured. In this case, we can remove pumps and tanks from the network, modelling them as known hydraulic heads nodes. We denote with $\mathbf{h} \in \mathbb{R}^{n_n}$ the vector

of unknown hydraulic heads (measured in meters), while the vector $\mathbf{q} \in \mathbb{R}^{n_p}$ represents unknown flow rates across network links (measured in m^3/s). In addition, frictional head losses within link $j \in \{1, \dots, n_p\}$ are modelled by a function $\phi_j : \mathbb{R} \rightarrow \mathbb{R}$ defined as:

$$\phi_j(q_j) = r_j q_j |q_j|^{n_j-1}, \quad (1)$$

Resistance coefficient r_j and exponent n_j take different values depending on the energy loss model used. When link j corresponds to a valve, we set $n_j = 2$ and $r_j = 8K_j/(g\pi^2 D_j^4)$, where K_j and D_j are valve local loss coefficient and diameter, respectively (Larock et al. 1999, Section 2.2.5). In comparison, when link j represents a pipe, different frictional energy loss models can be used. Darcy-Weisbach (D-W) and Hazen-Williams (H-W) formulae are two commonly used frictional energy loss models (D'Ambrosio et al. 2015) - see also the Appendix. Both the D-W and H-W formulae have unbounded second order derivatives around the origin (D'Ambrosio et al. 2015). However, their first order derivatives are continuous (Abraham and Stoianov 2016). In the numerical experiments presented in this manuscript, we consider the hydraulic model of a large operational WDN from the UK, where D-W formula is used to represent frictional energy losses. In this case, we have $n_j = 2$, while r_j depends implicitly on the flow q_j . Here, we formulate the D-W friction head loss model using the explicit approximation implemented in the hydraulic modelling software EPANET (Rossman 2000) - see the Appendix and Simpson and Elhay (2010) for a detailed formulation. When other frictional energy loss formulae are used, the considered fault detection and localization problem results in analogous formulations to what described in the following.

Given a link $i \xrightarrow{j} k$, let u_j be the corresponding fault state (measured in meters). The energy conservation equation is written as

$$h_i - h_k = \phi_j(q_j) + u_j. \quad (2)$$

In contrast to previous literature (Wu et al. 2012; Do et al. 2018), we model faults caused by blockages as slack variables, which appear linearly within the energy conservation equations. As

a result, the proposed formulation has a reduced degree of non-linearity with respect to previously published methods (Wu et al. 2012; Do et al. 2018). When $u_j = 0$ in (2), there is no fault at link j and the head difference is equal to the frictional energy loss. On the contrary, when $u_j \neq 0$, the head difference does not correspond to the frictional energy losses. Hence, an anomaly is detected on link j . When modelling and measurement errors are present, to determine if an actual fault (blockage) occurred, we consider the magnitude of u_j , compared to the level of uncertainty - see Figure 9a and the related discussion in the Case Study section.

In order to write (2) for all links in matrix form, we introduce the edge-unknown head node incidence matrix $\mathbf{A}_{12} \in \mathbb{R}^{n_p \times n_n}$ defined as

$$A_{12}(j, i) = \begin{cases} 1 & \text{if link } j \text{ enters node } i \\ 0 & \text{if link } j \text{ is not connected to } i \\ -1 & \text{if link } j \text{ leaves node } i \end{cases} \quad (3)$$

Analogously, we define the edge-known head node incidence matrix $A_{10} \in \mathbb{R}^{n_p \times n_0}$. The energy conservation equations can be written in compact form as:

$$\phi(\mathbf{q}) + \mathbf{A}_{12}\mathbf{h} + \mathbf{A}_{10}\mathbf{h}_0 + \mathbf{u} = \mathbf{0} \quad (4)$$

where vector $\mathbf{u} \in \mathbb{R}^{n_p}$ represents the unknown fault states. Furthermore, the conservation of mass is formulated as:

$$\mathbf{A}_{12}^T \mathbf{q} - \mathbf{d} = \mathbf{0} \quad (5)$$

Let $\mathbf{x} = [\mathbf{q}^T \mathbf{h}^T]^T \in \mathbb{R}^{n_p + n_n}$ be the vector of hydraulic states. Then, the network conservation laws can be written as:

$$\mathbf{f}(\mathbf{x}, \mathbf{u}) := \begin{bmatrix} \phi(\mathbf{q}) + \mathbf{A}_{12}\mathbf{h} + \mathbf{A}_{10}\mathbf{h}_0 + \mathbf{u} \\ \mathbf{A}_{12}^T \mathbf{q} - \mathbf{d} \end{bmatrix} = \mathbf{0}. \quad (6)$$

We observe that function $\mathbf{f}(\cdot)$ is continuously differentiable (Abraham and Stoianov 2016) and we

have

$$\frac{\partial \mathbf{f}(\mathbf{x}, \mathbf{u})}{\partial \mathbf{x}} = \begin{bmatrix} \mathbf{G}(\mathbf{q}) & \mathbf{A}_{12} \\ \mathbf{A}_{12}^T & \mathbf{0} \end{bmatrix} \quad (7)$$

where $\mathbf{G}(\mathbf{q})$ denotes the Jacobian matrix of function $\phi(\cdot)$. The matrix in (7) has a saddle-point structure, so it is non-singular if and only if $\text{Ker}(\mathbf{A}_{12}^T) \cap \text{Ker}(\mathbf{G}(\mathbf{q})) = \{\mathbf{0}\}$ (Benzi et al. 2005, Theorem 3.2). Such condition holds if and only if the water network model does not include loops with all zero flows (Nielsen 1989; Elhay et al. 2014; Abraham and Stoianov 2016). When this is not verified, the links involved in the loop can all be removed from the network model.

In this manuscript, we assume that there exists a vector of flows satisfying the mass conservation laws (5). Moreover, we assume that matrix $\mathbf{A}_{12} \in \mathbb{R}^{n_p \times n_n}$ is full column rank. Under such assumptions, given a vector of fault states, there exists a unique vector of hydraulic states such that equation (6) is satisfied (Collins and Cooper 1978; D'Ambrosio et al. 2015) - for a detailed proof see Appendix. Therefore, the implicit function theorem guarantees that we can define a continuously differentiable function $\mathbf{c} : \mathbb{R}^{n_p} \rightarrow \mathbb{R}^{n_p + n_n}$ such that $\mathbf{f}(\mathbf{c}(\mathbf{u}), \mathbf{u}) = \mathbf{0}$. Function $\mathbf{c}(\mathbf{u})$ is evaluated by solving equation (6) for fixed $\mathbf{u}$. This can be achieved using standard algorithms for hydraulic analysis (Todini and Pilati 1988), which are implemented in various commercial or open source software packages (Rossman 2000). In this work, we use the null space algorithm proposed by (Abraham and Stoianov 2016), a computationally efficient implementation of the Newton method tailored for hydraulic analysis.

Assume that hydraulic head measurements $\hat{\mathbf{h}} \in \mathbb{R}^{m_1}$ and flow measurements $\hat{\mathbf{q}} \in \mathbb{R}^{m_2}$ are available at different locations throughout the network. Set $m := m_1 + m_2$ and let the vector of measured hydraulic states $\mathbf{y} \in \mathbb{R}^m$ be $\mathbf{y} := [\hat{\mathbf{q}}^T \hat{\mathbf{h}}^T]^T$. Define a selection matrix $\mathbf{A} \in \mathbb{R}^m \times \mathbb{R}^{n_p + n_n}$ to select the hydraulic state corresponding to each measurement. Typically, water networks are characterized by scarce measurement data, compared to the number of unknown parameters, i.e. $m \ll n_p$. In order to avoid over-fitting, we formulate the problem of fault detection and localization as the

following regularized non-linear inverse problem:

$$\underset{\mathbf{u}}{\text{minimize}} \quad \frac{1}{2} \|\mathbf{Ac}(\mathbf{u}) - \mathbf{y}\|_2^2 + \lambda \|\mathbf{u}\|_1 \quad (8)$$

where $\lambda \geq 0$ is a positive scalar. The ℓ_1 regularization term in (8) is expected to induce sparsity within the solution vector $\mathbf{u}^*$, i.e. only few components of $\mathbf{u}^*$ will be non-zero (Tibshirani 1996). For this reason, ℓ_1 regularization has been previously implemented for fault estimation in other engineering frameworks - as example, see (Gorinevsky et al. 2009). However, the formulation of fault detection and localization in water networks as a non-linear inverse problem with ℓ_1 regularization has not been previously investigated. The proposed novel formulation results in a non-smooth non-convex optimization problem.

SOLUTION METHOD

In this work, we investigate mathematical optimization methods for solving Problem (8). Since second order derivatives of $\mathbf{c}(\cdot)$ may not be well defined for some vectors $\mathbf{u} \in \mathbb{R}^{n_p}$, solution algorithms for Problem (8) rely only on first order derivatives of $\mathbf{c}(\cdot)$. Previously implemented methods for solving non-linear least-squares problems in WDNs include reduced gradient method (Pudar and Ligett James 1992; Kang and Lansey 2011), and Gauss-Newton based methods (Piller et al. 2012; Piller et al. 2014). However, these methods are not suitable when non-smooth terms are included within the objective function. In comparison, here we investigate solution methods for inverse problems whose formulations include convex non-smooth functions. The proposed method enables a variety of objective function modelling choices, which can be tailored for the applications in study - some examples can be found in (Boyd and Vandenberghe 2004, Chapters 6 and 7). Furthermore, numerical methods proposed in previous literature relied on the computation of the full Jacobian matrix of $\mathbf{c}(\cdot)$ at each iteration - as examples, see (Kang and Lansey 2011; Piller et al. 2012; Piller et al. 2014). The Jacobian matrix is computed as follows. By definition of $\mathbf{c}(\mathbf{u})$, we have

$$\mathbf{f}(\mathbf{c}(\mathbf{u}), \mathbf{u}) = \mathbf{0}. \quad (9)$$

Differentiating the above equation, we obtain:

$$\frac{\partial \mathbf{f}(\mathbf{x}, \mathbf{u})}{\partial \mathbf{x}} \mathbf{J}(\mathbf{u}) + \frac{\partial \mathbf{f}(\mathbf{x}, \mathbf{u})}{\partial \mathbf{u}} = \mathbf{0}, \quad (10)$$

where $\mathbf{J}(\mathbf{u})$ is the Jacobian matrix of function $c(\cdot)$. Water distribution networks have a sparse structure, which is retained by the network conservation laws (6) and matrices $\frac{\partial \mathbf{f}(\mathbf{x}, \mathbf{u})}{\partial \mathbf{x}}$ and $\frac{\partial \mathbf{f}(\mathbf{x}, \mathbf{u})}{\partial \mathbf{u}}$ - see (Abraham and Stoianov 2016). Nonetheless, matrix $\mathbf{J}(\mathbf{u})$ is not necessarily sparse. As a consequence, solving equation (10) requires significant computational effort when large water networks are considered. In this paper, we implement a solution algorithm that does not require the computation of $\mathbf{J}(\mathbf{u})$, offering a scalable method to solve the considered problem in large scale water networks.

We formulate a sequential convex optimization algorithm for the solution of the non-convex optimization problem (8). Because of non-convexity, mathematical optimization methods to compute globally optimal solutions for Problem (8) requires the implementation of global optimization techniques (Tawarmalani and Sahinidis 2002). However, these algorithms require a large computational effort, and can be impractical when large water networks are considered (Pecci et al. 2018). For this reason, we investigate the implementation of gradient-based optimization algorithms, and develop a trust-region based sequential convex optimization algorithm for solving Problem (8). Trust-region techniques are used to evaluate the progress achieved by the sequential convex optimization algorithm, and modify the search space to achieve convergence to locally optimal solutions (Conn et al. 2000, Chapter 11).

The considered sequential convex optimization method is an iterative algorithm, which generates a sequence of approximate solutions to Problem (8). Given an iterate $\mathbf{u}_k$, a new trial iterate is obtained by minimizing a convex approximation of the original objective function in Problem (8):

$$\frac{1}{2} \|\mathbf{A}\mathbf{c}(\mathbf{u}) - \mathbf{y}\|_2^2 + \lambda \|\mathbf{u}\|_1 \approx \frac{1}{2} \|\mathbf{A}(\mathbf{c}(\mathbf{u}_k) + \mathbf{J}(\mathbf{u}_k)(\mathbf{u} - \mathbf{u}_k)) - \mathbf{y}\|_2^2 + \lambda \|\mathbf{u}\|_1 \quad (11)$$

where $\mathbf{J}(\mathbf{u}_k)$ is the Jacobian matrix of $\mathbf{c}(\cdot)$ evaluated at $\mathbf{u}_k$. The convex function in (11) is expected

to be a good approximation of the original objective function in a neighbour of $\mathbf{u}_k$. Therefore, the minimization of the approximated objective function is restricted to a trust region around $\mathbf{u}_k$:

$$\begin{aligned} \underset{\mathbf{u}}{\text{minimize}} \quad & \frac{1}{2} \|\mathbf{A}(\mathbf{c}(\mathbf{u}_k) + \mathbf{J}(\mathbf{u}_k)(\mathbf{u} - \mathbf{u}_k)) - \mathbf{y}\|_2^2 + \lambda \|\mathbf{u}\|_1 \\ \text{subject to} \quad & \|\mathbf{u} - \mathbf{u}_k\|_\infty \leq \Delta_k \end{aligned} \quad (12)$$

where $\Delta_k > 0$ is the trust region radius. As discussed in the previous Section, matrix $\mathbf{J}(\mathbf{u}_k)$ is the solution of the adjoint equation:

$$\frac{\partial \mathbf{f}_k}{\partial \mathbf{x}} \mathbf{J}(\mathbf{u}_k) + \frac{\partial \mathbf{f}_k}{\partial \mathbf{u}} = \mathbf{0} \quad (13)$$

where $\frac{\partial \mathbf{f}_k}{\partial \mathbf{x}} := \frac{\partial \mathbf{f}(\mathbf{c}(\mathbf{u}_k), \mathbf{u}_k)}{\partial \mathbf{x}}$ and $\frac{\partial \mathbf{f}_k}{\partial \mathbf{u}} := \frac{\partial \mathbf{f}(\mathbf{c}(\mathbf{u}_k), \mathbf{u}_k)}{\partial \mathbf{u}}$. Since matrix $\mathbf{J}(\mathbf{u}_k)$ is not necessarily sparse, solving (13) at each iteration can become impractical when large water networks are considered. In this manuscript, we implement an alternative approach. Firstly, we introduce auxiliary variables and rewrite Problem (12):

$$\begin{aligned} \underset{\mathbf{u}, \mathbf{x}}{\text{minimize}} \quad & \frac{1}{2} \|\mathbf{A}\mathbf{x} - \mathbf{y}\|_2^2 + \lambda \|\mathbf{u}\|_1 \\ \text{subject to} \quad & \mathbf{x} = \mathbf{c}(\mathbf{u}_k) + \mathbf{J}(\mathbf{u}_k)(\mathbf{u} - \mathbf{u}_k) \\ & \|\mathbf{u} - \mathbf{u}_k\|_\infty \leq \Delta_k \end{aligned} \quad (14)$$

As observed in the previous Section, we can assume without loss of generality that matrix $\frac{\partial \mathbf{f}_k}{\partial \mathbf{x}}$ is non-singular. Hence, Equation (13) implies that

$$\mathbf{J}(\mathbf{u}_k) = \frac{\partial \mathbf{f}_k}{\partial \mathbf{x}}^{-1} \frac{\partial \mathbf{f}_k}{\partial \mathbf{u}} \quad (15)$$

Consequently, Problem (14) is equivalent to the following convex problem:

$$\begin{aligned} \underset{\mathbf{u}, \mathbf{x}}{\text{minimize}} \quad & \frac{1}{2} \|\mathbf{A}\mathbf{x} - \mathbf{y}\|_2^2 + \lambda \|\mathbf{u}\|_1 \\ \text{subject to} \quad & \frac{\partial \mathbf{f}_k}{\partial \mathbf{x}}(\mathbf{x} - \mathbf{x}_k) + \frac{\partial \mathbf{f}_k}{\partial \mathbf{u}}(\mathbf{u} - \mathbf{u}_k) = \mathbf{0} \\ & \|\mathbf{u} - \mathbf{u}_k\|_\infty \leq \Delta_k \end{aligned} \quad (16)$$

where $\mathbf{x}_k = \mathbf{c}(\mathbf{u}_k)$. Observe that the equality constraints in Problem (16) preserve the sparsity structure inherent in water distribution network models. Hence, Problem (16) can be efficiently solved by state-of-the-art convex optimization methods (Boyd and Vandenberghe 2004), or specialized algorithms for large scale sparse convex problems (Zheng et al. 2017).

The implemented sequential convex optimization method is outlined in Algorithm 1 and Figure 1b. The algorithm starts from a given initial solution $\mathbf{u}_0$ and computes $\mathbf{x}_0 = \mathbf{c}(\mathbf{u}_0)$. At iteration k , we solve Problem (16) and obtain trial iterates $(\mathbf{x}^+, \mathbf{u}^+)$. Then, we compute the following ratio:

$$\rho_k = \frac{\frac{1}{2}\|\mathbf{A}\mathbf{x}_k - \mathbf{y}\|_2^2 + \lambda\|\mathbf{u}_k\|_1 - \frac{1}{2}\|\mathbf{A}\mathbf{c}(\mathbf{u}^+) - \mathbf{y}\|_2^2 - \lambda\|\mathbf{u}^+\|_1}{\frac{1}{2}\|\mathbf{A}\mathbf{x}_k - \mathbf{y}\|_2^2 + \lambda\|\mathbf{u}_k\|_1 - \frac{1}{2}\|\mathbf{A}\mathbf{x}^+ - \mathbf{y}\|_2^2 - \lambda\|\mathbf{u}^+\|_1}. \quad (17)$$

The value ρ_k compares the reduction in the original objective function achieved by $(\mathbf{x}^+, \mathbf{u}^+)$, with the predicted reduction. If the ρ_k is large enough, iteration k is defined as successful and $\mathbf{u}_{k+1} = \mathbf{u}^+$, $\mathbf{x}_{k+1} = \mathbf{c}(\mathbf{u}^+)$, while the trust region radius is increased. If ρ_k is too small, we conclude that the approximate model is not a good estimate of the original objective function within the current trust region, and the radius is reduced. Then, if a termination criterion is met, the algorithm stops. Otherwise, a new iteration is performed. We observe that each iteration of Algorithm 1 requires solving one sparse convex problem and one system of non-linear equations (6).

The proposed fault detection and localization method relies on the formulation of Problem (8), and the implementation of Algorithm 1 to solve the resulting non-convex optimization problem, with $\mathbf{u}_0 = \mathbf{0}$ and $\Delta_0 = 20$. The overall process is summarized in Figure 1. In the next Section, the developed method is tested using a large operational network as case study.

CASE STUDY

The developed method is implemented to detect and localize blockages occurring in BWFLnet, the hydraulic model of a large operational water network from the UK operated by Bristol Water, Cla-Val, and Imperial College London (Wright et al. 2014) - see Figure 2. Frictional head losses across network pipes are modeled according to the D-W formula (Simpson and Elhay 2010). The network consists of 2546 nodes, 2606 pipes, 545 valves and 2 inlets (known head nodes). BWFLnet

Algorithm 1 Trust region based sequential convex optimization

- 1: **Initialization:** $\eta = 0.01$, $\gamma = 1.1$, $\alpha = 0.5$, $\varepsilon_1 = 10^{-3}$, $\varepsilon_2 = 10^{-4}$
- 2: Select $\mathbf{u}_0$ and Δ_0 ;
- 3: Set $k = 0$ and $\mathbf{x}_0 = \mathbf{c}(\mathbf{u}_0)$;
- 4: **repeat**
- 5: **Trial iterate computation.** Solve Problem (16), let $(\mathbf{x}^+, \mathbf{u}^+)$ be the computed solution;
- 6: **Acceptance of trial iterate.**
- 7: If $\rho_k \geq \eta$ then $\mathbf{u}_{k+1} = \mathbf{u}^+$, $\mathbf{x}_{k+1} = \mathbf{c}(\mathbf{u}^+)$, $\Delta_{k+1} = \gamma\Delta_k$, and

$$\text{rel}_{\text{change}} = \frac{\frac{1}{2}\|\mathbf{A}\mathbf{x}_k - \mathbf{y}\|_2^2 + \lambda\|\mathbf{u}_k\|_1 - \frac{1}{2}\|\mathbf{A}\mathbf{x}_{k-1} - \mathbf{y}\|_2^2 - \lambda\|\mathbf{u}_{k-1}\|_1}{\frac{1}{2}\|\mathbf{A}\mathbf{x}_{k-1} - \mathbf{y}\|_2^2 + \lambda\|\mathbf{u}_{k-1}\|_1},$$

- else $\mathbf{u}_{k+1} = \mathbf{u}_k$, $\mathbf{x}_{k+1} = \mathbf{x}_k$, and $\Delta_{k+1} = \alpha\Delta_k$;
- 8: **until** $\text{rel}_{\text{change}} \leq \varepsilon_1$ or $\min(\frac{1}{2}\|\mathbf{A}\mathbf{x}_k - \mathbf{y}\|_2^2 + \lambda\|\mathbf{u}_k\|_1, \Delta_k) \leq \varepsilon_2$
-

is equipped with a dynamically adaptive topology, where two dynamic boundary valves (DBVs) are open during the diurnal operation and closed at night (Wright et al. 2014), while 3 pressure control valves (PCVs) are optimally operated to minimize average zone pressure - see Figure 2. The dynamic boundary valves in BWFLnet are closed between 23:30 and 04:15, and open at 40% for the remaining part of network daily operation.

PCVs and DBVs are equipped with sensors, measuring inlet and outlet hydraulic heads, and flow rates. Moreover, flow from both inlets is measured. Additional 27 hydraulic head measurements are available from other locations throughout BWFLnet - see Figure 2. All measured hydraulic data used in this manuscript have time steps of 15 minutes. We observe that the network model in study is an order of magnitude larger than those considered in previous literature, and it has a lower density of measurement devices - as example see (Do et al. 2018). All experiments reported in this section are conducted in MATLAB 2018b-64 bit for Linux, installed on a 2.50 GHz Intel Xeon(R) CPU E5-2640 0 with 12 Cores and 12 GB of RAM. The convex sub-problems within Algorithm 1 are reformulated as quadratic optimization problems and solved using GUROBI (Gurobi Optimization 2017).

We expect the quality of the fault diagnosis to depend on sensors number and locations, and the value of the regularization parameter λ . The results reported in the next Sections were ob-

282 tained with $\lambda = 10^{-2}$. Future work should investigate strategies to select the best regularization
283 parameter. In many applications, a practical approach to select the parameter λ is cross-validation,
284 which requires solving the problem for different values of the regularization parameter, computing
285 a portion of the regularization path (Friedman et al. 2010).

286 In the experiments presented in sub-sections “Single fault scenarios” and “Multiple fault sce-
287 narios”, we assume perfect data information, and we do not explicitly consider the effect of uncer-
288 tainty. The ability of the proposed method to detect and localize blockages in presence of modelling
289 uncertainties is investigated using experimental data in the third sub-section, named “Experimental
290 validation”.

291 **Single fault scenarios**

292 In order to demonstrate our fault detection and localization method, we generate flow and hy-
293 draulic head measurements by simulating the water network model under different fault scenarios.
294 We set customer demands and hydraulic heads at the inlets to the values corresponding to 08 : 00
295 am, which corresponds to peak demand.

296 Firstly, we evaluate the performance of the method for detecting changes in network connec-
297 tivity, i.e. fault caused by closure of links that are assumed to be open (Wu et al. 2012; Do et al.
298 2018). In order to generate measurement data, we simulate the network under different fault sce-
299 narios. From the set of all network links, we select 1231 links whose closure does not result in
300 isolated nodes, i.e. nodes not connected to any water source - see Figure 2.

301 We simulate $N = 1231$ different faults, one for each considered link. To simulate each fault
302 (e.g. closed valve), we define a new network model, where the selected link is removed. Hydraulic
303 heads and flows are computed by solving the hydraulic equations for the modified network model,
304 using the null space newton method developed by (Abraham and Stoianov 2016). Let $\mathbf{y}^{(1)}, \dots, \mathbf{y}^{(N)}$
305 be the vectors of measurements generated for each fault scenario. Models of operational water
306 networks are subject to multiple sources of data and modelling errors. In these experiments, we do
307 not explicitly consider uncertainty. However, when faults result in residuals that are smaller than
308 the level of uncertainty experienced in WDNs models, methods based on hydraulic models can fail

to correctly detect and localize them. In addition, faults resulting in small residuals are expected to have limited impact on WDN operation. In this study, we focus our analysis on fault scenarios that result in large residuals. We define the analysed scenarios $\mathcal{S}$ as follows

$$\mathcal{S} := \left\{ i \in \{1, \dots, N\} \left| \frac{1}{2} \|\mathbf{A}\mathbf{c}(\mathbf{0}) - \mathbf{y}^{(i)}\|_2^2 \geq 1 \right. \right\} \quad (18)$$

For the considered case study, we have $|\mathcal{S}| = 334$ - see Figure 2. For each $i \in \mathcal{S}$, we formulate Problem (8) given measurements $\mathbf{y}^{(i)}$, where all network links are considered as possible fault locations (e.g. possible closed valves). Observe that, the formulation of Problem (8) includes $n_p = 2606$ unknowns. We implement Algorithm 1 to solve Problem (8) with $\lambda = 0.01$. The investigation of alternative strategies to select the regularization parameter for the considered problem is subject of future work.

Results and discussion

The developed sequential convex optimization method converge to a solution in all experiments, with an average computational time of 0.787 (s). This is a significant reduction compared to the method by [Do et al. \(2018\)](#), where a computational time of more than 4 hours was required to localize closed valves in a case study network smaller than BWFLnet. Let $\mathbf{u}^{*(i)}$ be the vector of parameters computed by solving Problem (8) for fault scenario $i \in \mathcal{S}$. We expect the ℓ_1 regularization to promote sparsity in $\mathbf{u}^{*(i)}$, and this is confirmed by the results reported in Figures 3b-3f. Since the number of possible fault locations is 2606, while the total number of available measurements is 44, we do not expect an absolute fault localization. Instead, we expect the solution of Problem (8) to be sparse and to provide a set of candidate fault locations. When the absolute value of the fault state assigned to a link is larger than 10^{-3} meters, we consider such link to be a candidate fault location (e.g. fault hotspot area). Since these experiments are conducted under the assumption of perfect data, such threshold is arbitrary, and it corresponds to the smallest magnitude of fault state that we aim to detect and localize. In presence of modelling uncertainty, the threshold can be identified using uncertainty quantification and state estimation techniques ([Puig](#)

2010; Vrachimis et al. 2018). Therefore, for $i \in \mathcal{S}$, we define the fault location candidates $\mathcal{C}^{(i)}$ as the index set:

$$\mathcal{C}^{(i)} := \left\{ j \in \{1, \dots, n_p\} \mid |u_j^{*(i)}| \geq 10^{-3} \right\}, \quad (19)$$

Figure 3a shows an example of successful fault detection and localization. In this case, the solution of Problem (8) corresponds to a link sequence, which includes the actual closed link. We now focus on inexact and wrong diagnoses. Figure 3c shows an inexact fault localization result, where the fault location was not included in the set of candidates but the correct network branch was identified. Hence, we do not consider this experiment as unsuccessful. In comparison, the results reported in Figure 3e corresponds to the worst fault localization performance. The fault location is not included within the set of candidates and the distance to fault is equal to the 16, the maximum value found in this experiment. Even though the correct part of the network was identified, we consider this case a wrong fault localization.

In order to evaluate the overall performance of the implemented fault detection and localization method, we introduce performance criteria based on graph theory. To do so, we first give some essential definitions.

Definition 4.1 (Link adjacency). *Two links of a WDN graph are adjacent if they have an end in common.*

Definition 4.2 (Line graph). (*Diestel 2000, Section 1.1*) *The line graph of a WDN graph is a graph whose vertices correspond to the links of the WDN graph. Two vertices in the line graph are adjacent if and only if the corresponding links are adjacent.*

Definition 4.3 (Connected components). *A connected component of a graph (or sub-graph) is a maximal set of nodes such that each pair of nodes is connected by a path. Connected components of a graph (or sub-graph) form a partition of the set of graph (or sub-graph) vertices.*

Firstly, we introduce a metric to assess the ability of the method to correctly identify the actual fault location as a fault candidate. In particular, we consider the graph distance of the set of candidates to the actual fault location. Let $\delta(i)$ be the minimum length of the shortest paths between

the fault location and the links in $\mathcal{C}^{(i)}$, computed in the line graph of the graph of BWFLnet. We consider the following distribution function

$$F_1(t) := \frac{100}{|\mathcal{S}|} \left| \left\{ i \in \mathcal{S} \mid \delta(i) \leq t \right\} \right|. \quad (20)$$

Figure 4 reports the distribution of the distance to fault in all instances. The actual fault location was included in the set of candidate fault locations in the majority of experiments - resulting in $\delta(i) = 0$. Moreover, the distance from the set of candidates to the actual fault location is less than 5 in roughly 80% of the experiments.

In order to evaluate the precision of the fault localization method, we consider the cardinality of the set of candidate fault locations as second performance metric. Let $\chi(i) := |\mathcal{C}^{(i)}|$ be the number of candidate fault locations found in experiment i . We define the following distribution function:

$$F_2(t) := \frac{100}{|\mathcal{S}|} \left| \left\{ i \in \mathcal{S} \mid \chi(i) \leq t \right\} \right|. \quad (21)$$

As shown in Figure 5, the number of selected fault candidates in each experiment is never greater than 31. Considering that the number of possible fault locations is $n_p = 2606$, solving Problem (8) has considerably reduced the set of candidate fault locations, in all experiments.

Hydraulic models of operational water networks often include link sequences to represent long pipes, or to align hydraulic models and GIS information. As observed by [Do et al. \(2018\)](#), when a link sequence is considered, different combinations of head losses within individual links result in the same overall energy loss along the sequence. Therefore, when few measurements are available, all links involved in the sequence are reported as candidate fault locations - see Figure 3. As a consequence, we consider the fault localization method to be precise when the set of candidate fault locations includes the actual fault, and the sub-graph induced by the candidate links is connected. Hence, as third performance metric, let $\sigma(i)$ be number of connected components of the sub-graph

defined by links in $\mathcal{C}^{(i)}$. We set

$$F_3(t) := \frac{100}{|\mathcal{S}|} \left| \left\{ i \in \mathcal{S} \mid \sigma(i) \leq t \right\} \right|. \quad (22)$$

Figure 6 shows that $\mathcal{C}^{(i)}$ corresponds to a single link sequence in the majority of the experiments. In particular, in all successful experiments where the actual fault location is identified as a candidate, the sub-graph induced by the candidate links has a unique connected component, i.e. the set of candidate links is connected.

Multiple fault scenarios

We evaluate the proposed method to detect and localize multiple simultaneous faults in BWFLnet. In the first experiment, 3 faults were simulated at different locations - see Figure 7a. All links were assumed to be open in the nominal hydraulic model. Faults 1 and 3 correspond to incorrect modelling of two partially closed valves. To simulate Fault 1 we set the corresponding valve local loss coefficient to 1000, while Fault 3 corresponds to a valve whose local loss coefficient is set to 500. In addition, Fault 2 corresponds to a closed link, and it is simulated by removing such link from the model before performing the hydraulic simulation - analogously to the previous section. We formulate Problem (8) with $\lambda = 0.01$, where the measured hydraulic heads and flows are generated by simulating the 3 faults. Then, we implement Algorithm 1 to compute a solution and define the set of candidate fault locations as in (19) - see also Figure 7b.

Results and discussion

Similarly to the results reported in the previous section, Algorithm 1 required only 1.12 seconds to converge. The proposed method results in a successful fault detection and localization, with all 3 faults being identified. Moreover, the set of candidate fault locations has exactly 3 connected components - see Figures 7c - 7e. These results are not surprising, the considered links correspond to locations of individual faults that were correctly identified in the single-fault experiments.

In comparison, when the set of multiple faults includes a link corresponding to an individual fault that was not successfully identified, the method results in incorrect localization. This is

illustrated in the following. Analogously to the previous experiment, we simulate three faults in BWFLnet, namely Fault 4, 5 and 6 - see Figure 8a. Again, the nominal hydraulic model considers all links to be fully open. To simulate Fault 4, which corresponds to a partially closed valve, we set the local loss of the corresponding valve to 1000, while we simulate Fault 6 changing the local loss coefficient of the corresponding valve to 500. Finally, Fault 5 corresponds to a closed link. We formulate Problem (8) for fault detection and localization in BWFLnet, with $\lambda = 0.01$. Then, Algorithm 1 is implemented to compute a solution to Problem (8). The computational time required to converge is 0.87 seconds. The set of candidate fault locations is defined as in (19) and the results are reported in Figure 8. The implemented method has successfully identified Faults 4 and 6, but it has missed Fault 5. Note that the individual failure of the link corresponding to Fault 5 was also missed in the previous section experiments.

Experimental validation

We validate the proposed optimization based fault detection and localization method using experimental data from BWFLnet. In the following, we consider experimental data recorded in BWFLnet, including pressure data from 27 locations, together with inlet and outlet pressure and flow measurements from the PCVs and DBVs. All measured hydraulic data used in this manuscript have time steps of 15 minutes. A complete description of the dataset, together with the measured data, is available at <http://dx.doi.org/10.17632/srt4vr5k38.2>. The hydraulic model of BWFLnet was calibrated as shown in Waldron et al. (2019) using measured hydraulic data from a 24 hr *training day*. Then, recorded data from a different day (24 hours) was used to validate the developed method. In the formulation of Problem (8), we assume that pressure and flow measurements from DBVs are not available and that their opening level is 40% for the all day. As a result, the closure of the DBVs at night results in unexpected energy losses, captured by the recorded data. These energy losses can be considered as the effect of blockages occurring at the DBV locations. We investigate the application of the developed method to detect and localize such blockages using recorded experimental data.

At each time step, measured hydraulic heads at inlets and pressure control valves operation

were modelled within the network conservation laws (6). Moreover, the total measured demand was distributed according to the original demand allocation determined by the network operator, as described in Do et al. (2018). Observe that the main source of modelling errors is represented by demand uncertainty and variability within the system, which might differ from the assumptions made when modelling BWFLnet.

Results and discussion

Problem (8) was formulated and solved to detect and localize blockages in BWFLnet, considering all links as potential blockage locations. At each time step, Algorithm 1 required on average 2.7 seconds to converge, demonstrating that the proposed method is suitable for near real-time implementation in monitoring and diagnostic tools for WDNs. In Figure 9, we report the fault state for the two DBVs, computed by solving Problem (8), and the measured flow, which was recorded but not used within the implemented fault diagnosis process. As shown in Figure 9a, estimating the status of DBV 1 is challenging due to model uncertainty. In fact, DBV 1 experiences a small head difference during diurnal operation, with a limited measured flow. Therefore, the incorrect modelling of DBV 1 does not have a significant impact on the operation of BWFLnet. In comparison, Algorithm 1 resulted in an accurate estimation of the status of DBV 2, with the fault state equals to zero during daylight (i.e. the valve is open) and non-zero at night (i.e. the valve is closed). This is also confirmed by the flow data shown in Figure 9b. We conclude that the implemented method resulted in accurate diagnosis when this can be expected.

To further investigate the performance of the developed method to localize closed valves, we consider BWFLnet operation at 03:00. Both boundary valves are closed at this time of the day, but our model assumes that they are open at 40%. As discussed above, the effect of valve closure at DBV 1 is smaller than the level of modelling uncertainty experienced in BWFLnet. Therefore, the incorrect modelling of DBV 1 has little impact on the operation of BWFLnet, and it is not identified by the method. In contrast, as shown in Figure 10a, the fault vector computed by Algorithm 1 sharply identifies a set of candidate fault locations, which are shown in Figure 10b. The implemented fault detection and localization procedure has correctly identified a link sequence including

DBV 2.

CONCLUSION

We have proposed a novel optimization based method for detecting and localizing blockages in WDNs. These faults are caused by unreported partially/fully closed valves or other pipe blockages. We have formulated the considered problem as a non-linear inverse problem with ℓ_1 regularization. We have presented a sequential convex optimization algorithm for solving the resulting non-smooth non-convex optimization problem. The method is based on the solution of a sequence of sparse convex problems, which are solved efficiently using convex optimization techniques. Numerical experiments on a large operational water network show that the optimization based fault detection and localization performs well, achieving accurate fault diagnosis in most cases. In fact, in over 80% of the tested 334 fault scenarios, the actual fault location is separated from the set of estimated fault candidates by less than 5 links. In addition, in all experiments, the solution of the inverse problem via the proposed sequential convex optimization algorithm requires less than three seconds of computations, reducing the computational time by 4 to 5 orders of magnitude compared to Do et al. (2018). Furthermore, the developed method was validated for near real-time fault diagnosis using recorded data from a large operational water network with dynamically adaptive topology from the UK. The method is shown to successfully detect and localize network blockages using experimental data, despite unmodeled measurement noise and other modelling uncertainties present in the system. The results reported in this study suggest that the proposed optimization based method enables the implementation of real time fault diagnosis in water networks with a dynamically adaptive topology. Finally, we suggest that the developed framework is extended to other fault diagnosis problems in WDNs, including burst/leak detection and localization.

DATA AVAILABILITY STATEMENT

The hydraulic data and model of BWFLnet used during the study are available in a repository in accordance with funder data retention policies: <http://dx.doi.org/10.17632/srt4vr5k38.2>

ACKNOWLEDGEMENTS

This research was supported by EPSRC (EP/P004229/1, Dynamically Adaptive and Resilient Water

489 Supply Networks for a Sustainable Future). We thank Cla-Val and Bristol Water for their support in the
490 implementation and operation of BWFLnet.

APPENDIX I. EXISTENCE AND UNIQUENESS OF HYDRAULIC SOLUTIONS

The hydraulic conservation laws in (6) are:

$$\begin{aligned}\phi(\mathbf{q}) + \mathbf{A}_{12}\mathbf{h} + \mathbf{A}_{10}\mathbf{h}_0 + \mathbf{u} &= \mathbf{0} \\ \mathbf{A}_{12}^T \mathbf{q} - \mathbf{d} &= \mathbf{0},\end{aligned}\tag{23}$$

where function $\phi_j(q_j)$ represents the frictional head loss within link j , for all $j \in \{1, \dots, n_p\}$. As stated in the Problem Formulation section, we assume that the set $\{\mathbf{q} \in \mathbb{R}^{n_p} \mid \mathbf{A}_{12}^T \mathbf{q} - \mathbf{d} = \mathbf{0}\}$ is not empty. Moreover, we assume that the columns in matrix $\mathbf{A}_{12} \in \mathbb{R}^{n_p \times n_n}$ are linearly independent. We have that

$$\phi_j(q_j) = r_j q_j |q_j|^{n_j-1},\tag{24}$$

where r_j and n_j depends on the type of link (i.e. valve or pipe) and the frictional head loss formula that is used. In particular, when link j corresponds to a valve, we have:

$$\phi_j(q_j) = \frac{8K_j}{g\pi^2 D_j^4} q_j |q_j| \tag{25}$$

where K_j and D_j are valve local loss coefficient and diameter, respectively (Larock et al. 1999, Section 2.2.5). In comparison, when link j corresponds to a pipe, the frictional head losses can be modelled by either Hazen-Williams (H-W) or Darcy-Weisbach (D-W) formulae. When H-W is used, we set:

$$\phi_j(q_j) = \frac{10.67 L_j}{C_j^{1.852} D_j^{4.781}} q_j |q_j|^{0.852}, \tag{26}$$

where L_j , D_j and C_j are length, diameter and H-W coefficient of pipe j , respectively. In the case of D-W, the relation between $r_j = r_j(q_j)$ and q_j is defined implicitly. Here, we implement the explicit

approximation used in EPANET (Rossman 2000) and discussed in (Simpson and Elhay 2010):

$$\phi_j(q_j) = \begin{cases} \frac{128\nu}{\pi g} \frac{L_j}{D_j^4} q_j & |q_j| \leq 2000 \frac{\nu}{L_j} \\ \frac{8}{\pi^2 g} \frac{L_j}{D_j^5} \left(\sum_{l=0}^3 \frac{\alpha_j^l + \beta_j^l}{\theta_j} \eta^l \right) \cdot |q_j| q_j & 2000 \frac{\nu}{L_j} < |q_j| \leq 4000 \frac{\nu}{L_j} \\ \frac{2(\ln 10)^2}{\pi^2 g} \frac{L_j}{D_j^5} \frac{1}{(\ln \theta_j)^2} \cdot |q_j| q_j & 4000 \frac{\nu}{L_j} < |q_j|, \end{cases} \quad (27)$$

where ν is the kinematic viscosity, while L_j and D_j are length and diameter of pipe j , respectively.

Coefficients α_j , β_j , η_j and θ_j are defined as in (Simpson and Elhay 2010). In particular,

$$\theta_j = \frac{\varepsilon_j}{3.7D_j} + \frac{5.74(\pi\nu/4)^{0.9} D_j^{0.9}}{|q_j|^{0.9}} \quad (28)$$

where ε_j is the pipe roughness.

Consider the optimization problem:

$$\begin{aligned} & \underset{\mathbf{q}}{\text{minimize}} && E(\mathbf{q}) \\ & \text{subject to} && \mathbf{A}_{12}^T \mathbf{q} - \mathbf{d} = 0 \end{aligned} \quad (29)$$

where

$$E(\mathbf{q}) := \sum_{j=1}^{n_p} \int_0^{q_j} \phi(s) ds + \mathbf{q}^T (\mathbf{A}_{10} \mathbf{h}_0 + \mathbf{u}) \quad (30)$$

The next result follow was proved by Collins and Cooper (1978) in a different context. For the sake of completeness, we derive it in a our framework.

Theorem I.1. *A pair of vectors $(\mathbf{q}, \mathbf{h})$ solves (23) if and only if $\mathbf{q}$ is optimal solution of Problem (29) and $\mathbf{h}$ is the corresponding Lagrangian multiplier.*

Proof. Since function $\phi_j(\cdot)$ are monotonically increasing, the objective function $E(\cdot)$ is strictly convex. Hence, Problem (29) satisfies strong duality and the associated KKT condition are a necessary and sufficient condition for optimality (Boyd and Vandenberghe 2004, Chapter 5). Therefore, $\mathbf{q}$ is an optimal solution of Problem (29), with associated Lagrangian multiplier $\mathbf{h}$, if and only

if $(\mathbf{q}, \mathbf{h})$ satisfies the following set of non-linear equations:

$$\begin{aligned}\nabla E(\mathbf{q}) + \mathbf{A}_{12}\mathbf{h} &= 0 \\ \mathbf{A}_{12}^T \mathbf{q} - \mathbf{d} &= 0\end{aligned}\tag{31}$$

The above equations yield

$$\begin{aligned}\phi(\mathbf{q}) + \mathbf{A}_{10}\mathbf{h}_0 + \mathbf{u} + \mathbf{A}_{12}\mathbf{h} &= 0 \\ \mathbf{A}_{12}^T \mathbf{q} - \mathbf{d} &= 0\end{aligned}\tag{32}$$

Since we assume $\text{rank}(\mathbf{A}_{12}) = n_n$, given $\mathbf{q}^* \in \mathbb{R}^{n_p}$ optimal solution of Problem (29), there exists a unique associated vector of Lagrangian multipliers $\mathbf{h}^* \in \mathbb{R}^{n_n}$. $\square$

The following theorem holds.

Theorem I.2. *Problem (29) has a unique optimal solution.*

Proof. As observed in the previous Theorem, $E(\cdot)$ is strictly convex. Therefore, the solution of Problem (29) (if it exists) is unique.

In order to prove existence of a solution to Problem (29), it suffices to show that function $\mathbf{E}(\cdot)$ is coercive Bertsekas (1999, Proposition A.8), i.e. $\lim_{\|\mathbf{q}\| \rightarrow \infty} E(\mathbf{q}) = +\infty$. Assume that link j corresponds to a valve. We have

$$\int_0^{q_j} \phi(s) ds = r_j \frac{|q_j|^3}{3}.\tag{33}$$

When link j corresponds to a pipe, it is necessary to investigate separately H-W and D-W head loss formulae.

- H-W formula. In this case, we have

$$\int_0^{q_j} \phi(s) ds = r_j \frac{|q_j|^{2.852}}{2.852}\tag{34}$$

- D-W formula. Let $\omega_j = 4000 \frac{v}{L_j}$. Firstly, we observe that function $\phi_j(\cdot)$ is odd, i.e. $-\phi_j(s) = \phi_j(-s)$. Hence, Definition (27) implies that

$$\begin{aligned}
\int_0^{q_j} \phi(s) ds &= \int_0^{|q_j|} \phi(s) ds \\
&= \int_0^{\omega_j} \phi(s) ds + \int_{\omega_j}^{|q_j|} \phi(s) ds \\
&= \int_0^{\omega_j} \phi(s) ds + \int_{\omega_j}^{|q_j|} \frac{2(\ln 10)^2 L_j}{\pi^2 g D_j^5} \left(\ln \left(\frac{\varepsilon_j}{3.7 D_j} + \frac{5.74(\pi v/4)^{0.9} D_j^{0.9}}{|s_j|^{0.9}} \right) \right)^{-2} s^2 ds \\
&\geq \int_0^{\omega_j} \phi(s) ds + \int_{\omega_j}^{|q_j|} \frac{2(\ln 10)^2 L_j}{\pi^2 g D_j^5} \left(\ln \left(\frac{\varepsilon_j}{3.7 D_j} + \frac{5.74(\pi v/4)^{0.9} D_j^{0.9}}{\omega_j^{0.9}} \right) \right)^{-2} s^2 ds \\
&\geq I_j + \frac{2(\ln 10)^2 L_j}{\pi^2 g D_j^5} \left(\ln \left(\frac{\varepsilon_j}{3.7 D_j} + \frac{5.74(\pi v/4)^{0.9} D_j^{0.9}}{\omega_j^{0.9}} \right) \right)^{-2} \frac{|q_j|^3}{3}
\end{aligned} \tag{35}$$

where

$$I_j = \int_0^{\omega_j} \phi(s) ds - \frac{2(\ln 10)^2 L_j}{\pi^2 g D_j^5} \left(\ln \left(\frac{\varepsilon_j}{3.7 D_j} + \frac{5.74(\pi v/4)^{0.9} D_j^{0.9}}{\omega_j^{0.9}} \right) \right)^{-2} \frac{\omega_j^3}{3}. \tag{36}$$

In conclusion, for all cases, we have

$$E(\mathbf{q}) \geq \sum_{j=1}^{n_p} \left(\gamma_j + G_j |q_j|^{\mu_j} \right) + \mathbf{q}^T (\mathbf{A}_{10} \mathbf{h}_0 + \mathbf{u}) \tag{37}$$

where $\gamma_j \in \mathbb{R}$ and $G_j \geq 0$ and $\mu_j > 2$ are opportunely defined. Since

$$\lim_{\|\mathbf{q}\| \rightarrow +\infty} \sum_{j=1}^{n_p} \left(\gamma_j + G_j |q_j|^{\mu_j} \right) + \mathbf{q}^T (\mathbf{A}_{10} \mathbf{h}_0 + \mathbf{u}) = +\infty, \tag{38}$$

inequality (37) implies that

$$\lim_{\|\mathbf{q}\| \rightarrow +\infty} E(\mathbf{q}) = +\infty, \tag{39}$$

□

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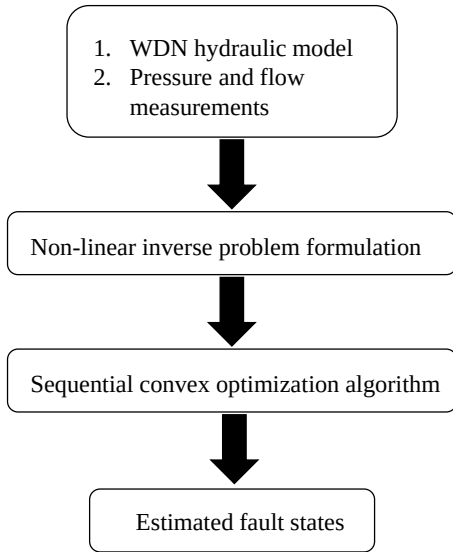
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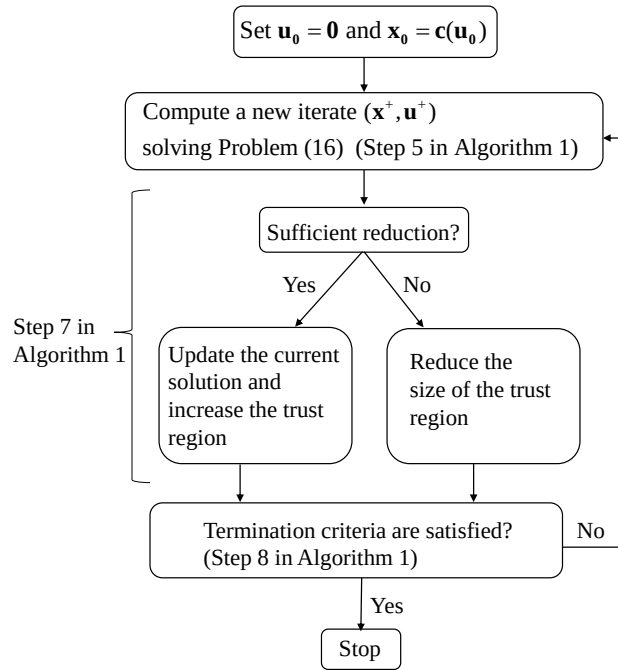
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(a) Proposed fault detection and localization method.



(b) Sequential convex optimization algorithm.

Fig. 1. Flow charts describing the implemented fault detection and localization method.

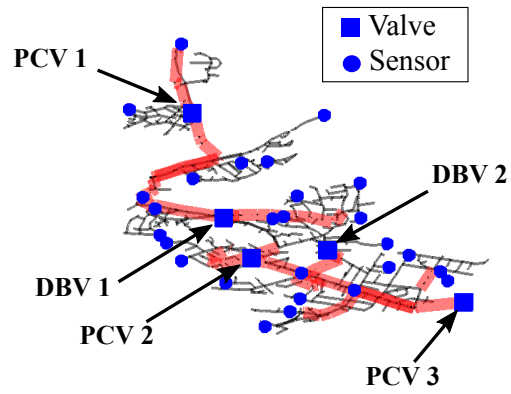
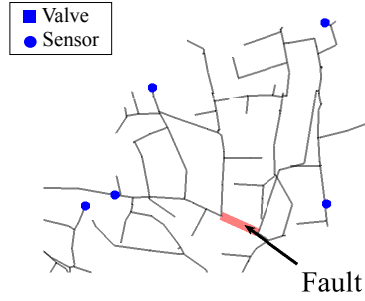
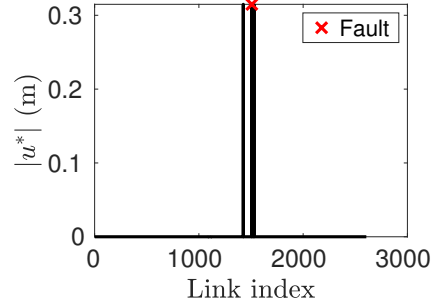


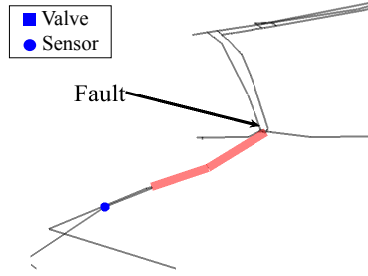
Fig. 2. Control valves and pressure sensors locations. Locations of the faults corresponding to links in $\mathcal{S}$ are highlighted.



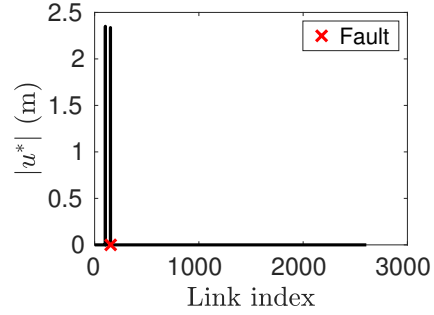
(a) Successful fault localization: fault location and candidate links are highlighted.



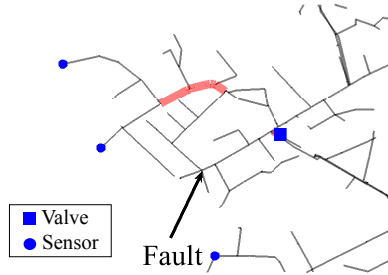
(b) Successful fault localization: $\mathbf{u}^*$ is the estimated fault state computed by solving Problem (8).



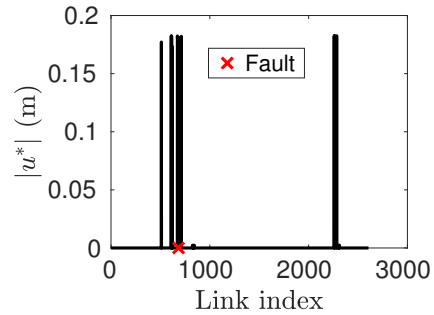
(c) Inexact fault localization: fault location and candidate links are highlighted.



(d) Inexact fault localization: $\mathbf{u}^*$ is the estimated fault state computed by solving Problem (8).



(e) Wrong fault localization: fault location and candidate links are highlighted.



(f) Wrong fault localization: $\mathbf{u}^*$ is the estimated fault state computed by solving Problem (8).

Fig. 3. Fault estimation results for three example cases: successful fault detection and localization (Top); inexact fault localization (middle); wrong fault localization (bottom).

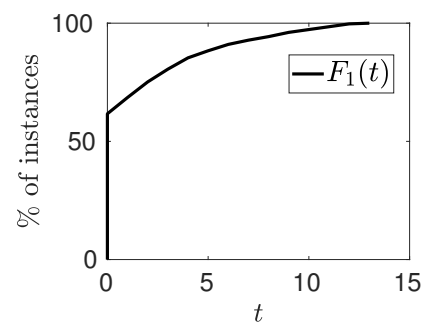


Fig. 4. Distribution of the distance of the set of candidates to the actual fault location.

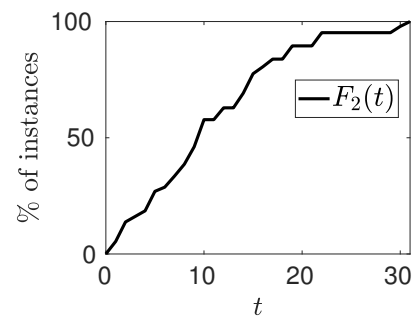


Fig. 5. Distribution of the number of candidates

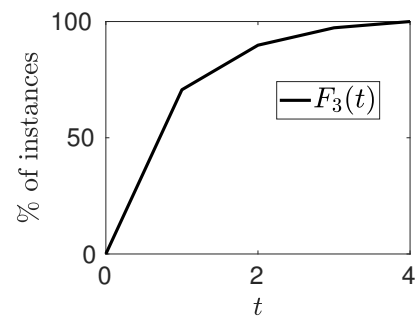
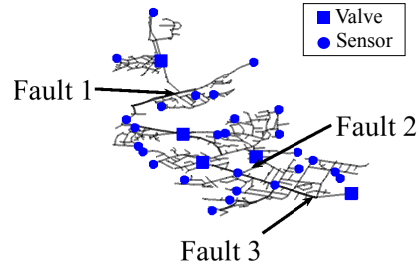
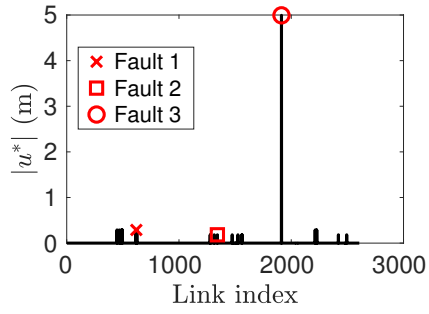


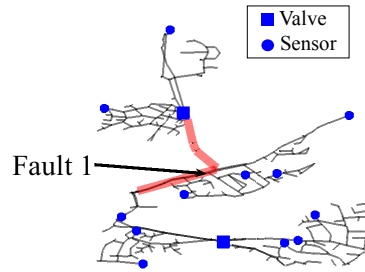
Fig. 6. Distribution of the number of connected components of the set of candidates.



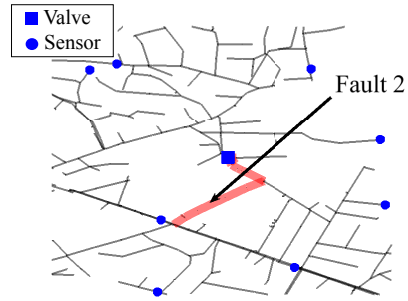
(a) Location of the 3 simulated faults.



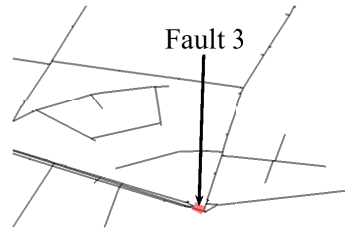
(b) Estimated fault state $\mathbf{u}^*$ computed by solving Problem (8).



(c) Localization performance.

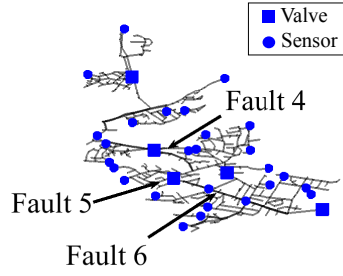


(d) Localization performance.

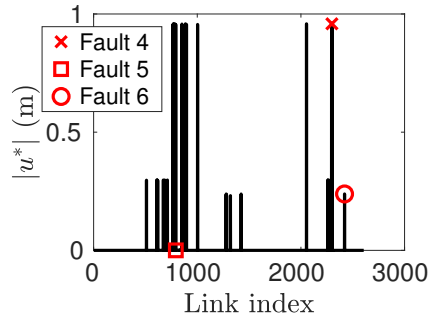


(e) Localization performance.

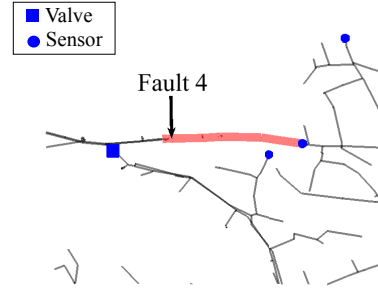
Fig. 7. Results of the first experiment with multiple faults, where candidate links are highlighted.



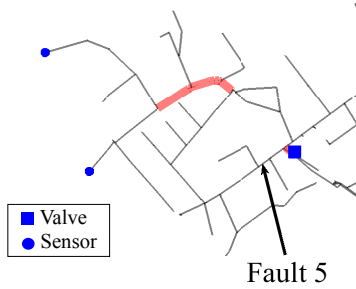
(a) Locations of the 3 simulated faults.



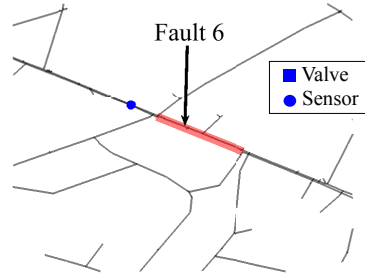
(b) Estimated fault state $\mathbf{u}^*$ computed by solving Problem (8).



(c) Localization performance.



(d) Localization performance.



(e) Localization performance.

Fig. 8. Second experiment with multiple faults, where candidate links are highlighted.

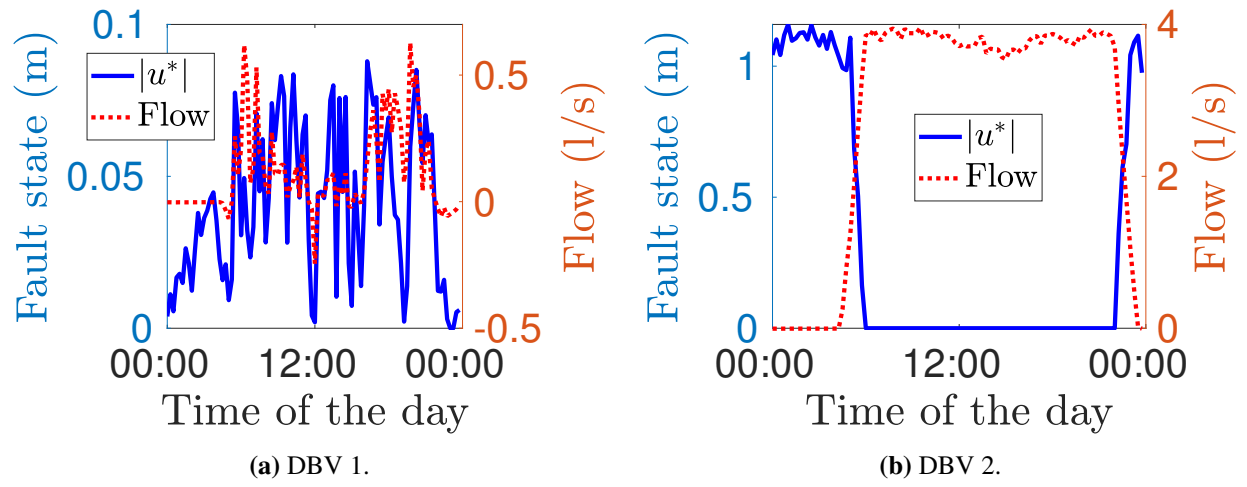
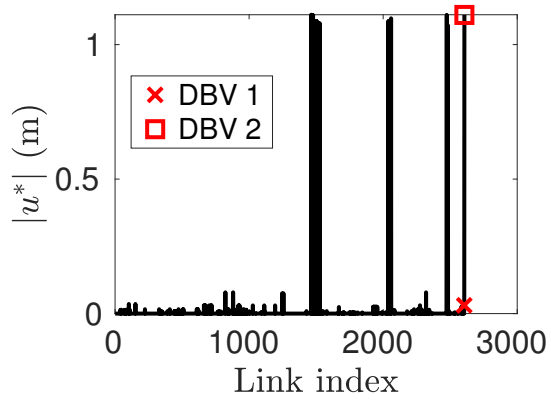
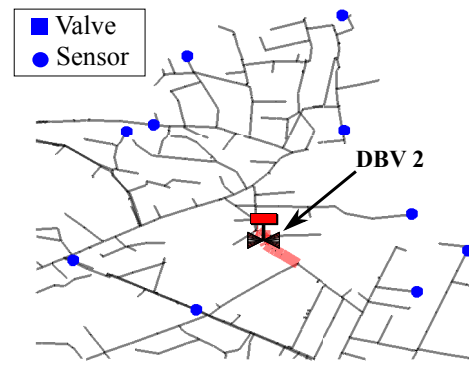


Fig. 9. Estimated fault state $\mathbf{u}^*$ at all time steps, and measured flow across the two dynamic boundary valves.



(a) Estimated fault state $\mathbf{u}^*$ computed by solving Problem (8).



(b) Localization performance.

Fig. 10. Results of the experiment at 03:00, where candidate links are highlighted.