A Comment on Monetary Policy and Rational Asset Price Bubbles*

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May 5, 2025

Abstract

Galí (2014) showed that a monetary policy rule that raises interest rates when bubbles exceed some steady-state benchmark can paradoxically lead to larger deviations from that steady state. This comment shows that a central bank that wants to dampen the current value of a bubble can always do so by setting a higher-than-expected interest rate, although it may have to raise the rate aggressively. This is a different point from the one in the Miao, Shen and Wang (2019) comment on Galí's paper. They showed that when the central bank targets a different steady state than the one Galí considered, raising interest rates when bubbles exceed this alternative steady-state leads to (weakly) smaller deviations from steady state. We study how the set of equilibria changes when a central bank sets an interest rate above what markets expect rather than when it sets out to target a different steady state.

*We are grateful to Fernando Arce, Yu Awaya, Bob Barsky, Macro Bassetto, John Conlon, Joel David, Jordi Galí, Tomohiro Hirano, Bart Hobijn, Ryo Jinnai, Jianjun Miao, Nicolas Werquin, and various seminar participants as well as Marzena Rostek (the editor), Ben Moll (the co-editor), and three anonymous referees for helpful comments.

Introduction

Policymakers have long argued about how to respond to a surge in asset prices that cannot be explained by changes in fundamental factors. This is sometimes framed as the *lean* versus *clean* debate over asset bubbles, i.e., whether policymakers should lean against asset prices by raising interest rates (or by restricting credit) or wait to see if asset prices fall and, if necessary, clean up afterwards.

In a provocative paper, Galí (2014) questioned the underlying premise of this debate. He observed that while raising the real interest rate lowers an asset's fundamental value due to the higher discounting of dividends, there is no analogous reason why a higher real interest rate should reduce the gap between an asset's price and its fundamental. Equilibrium only requires the expected gap to grow at the rate of interest. A higher interest rate should result in faster growing bubbles but not necessarily smaller ones. Galí then presented a model in which the central bank follows a rule that raises the interest rate when bubbles exceed their value at a particular deterministic steady state. He showed that raising the interest rate more aggressively when bubbles exceed this steady-state value can lead to larger deviations from steady state.

Miao, Shen, and Wang (2019) argued that Galí's results were specific to the particular steady states he focused on. They showed that if the central bank targets a different steady-state value from the one Galí considered, a more aggressive rule does not lead to larger deviations from steady state. They further showed that the equilibria they study Pareto dominate the equilibria Galí studied, and that the equilibria under their rule are expectations stable (E-stable) and learnable while the equilibria Galí studied are not. As such, the equilibria they study may be more natural for a central bank to pursue.

This comment examines the distinct but related question of what a central bank that faces an asset bubble can do to dampen it. Can it dampen the bubble by setting a high interest rate? Does this depend on which equilibrium we start at? We argue that *regardless* of the equilibrium it starts at, a central bank can always dampen the current bubble by setting an interest rate above what markets expect. However, given the forces Galí identifies, it may have to set a sufficiently high rate to ensure a smaller bubble.

The intuition for our result is as follows. Since monetary policy interventions must be neutral in the long run, actions today will have no effect on the long-run value of the bubble. If a higher interest rate implies a faster growing bubble that has the same value in the long run, the current value of the bubble must fall. When the economy admits multiple equilibria, as in Galí's model, a higher interest may act to select an equilibrium with a higher long-run value, undercutting this intuition. However, the central bank cannot keep selecting such equilibria indefinitely. Eventually, it will select the largest possible bubble. At that point, a more aggressive rule cannot keep selecting larger bubbles and must dampen them instead.

We begin with Galí's original model. Galí showed that a rule which responds more aggressively when bubbles exceed a steady-state benchmark can lead to bubbles that exceed the steady state even more. We show that once this rule turns infinitely aggressive, the bubble will never exceed the steady state, and

consequently the central bank will never end up setting a high interest rate. This gives our first glimpse that Galí's result may not extend to sufficiently aggressive rules. It also reveals that Galí's rule works not just by setting a high interest but also by *threatening* to set a high interest rate even without raising it. The effect of the rule combines the effects of raising rates with threats to raise them.

This motivates us to isolate the effects of a higher-than-expected interest rate by looking at a monetary policy shock rather than a rule that tells agents to expect higher interest rates whenever a bubble exceeds some benchmark. To obtain analytical results, we consider a related setting to Galí's that can be solved in closed form. We show that starting from any equilibrium, the central bank can reduce the bubble contemporaneously by setting an interest rate sufficiently above what agents expect.

Finally, we consider a variation of our model that generates a bubble as the unique equilibrium. In that case, we show that *any* increase in interest rates will dampen the bubble. This highlights the role of indeterminacy in driving Galí's result that a higher rate may lead to larger bubbles. It also shows that dampening bubbles does not inherently require aggressive policy. Aggressive policy to dampen a bubble is only necessary when dealing with multiple equilibria.

While we argue that a central bank can dampen bubbles by raising rates, we do not take a stand on whether dampening bubbles this way is desirable. That requires theoretical and quantitative analysis beyond what we do. The answer will also depend on which underlying model of bubbles we use.[1] Our concern here is only in whether raising interest rates above what markets expect can dampen bubbles.

1 Infinitely Aggressive Policy in the Galí (2014) Model

We begin with Galí's original 2014 model. His model featured an overlapping generations economy in which a mass 1 of agents is born each period and lives for two periods. The cohort born at age t derives utility from final goods consumption $C_{1,t}$ when young and $C_{2,t+1}$ when old according to the function

$$U\left(C_{1,t}, C_{2,t+1}\right) = \ln C_{1,t} + \beta \ln\left(C_{2,t+1}\right) \tag{1}$$

The consumption goods that agents value are produced from a continuum of intermediate goods according to a constant elasticity of substitution production function: Given Y_{it} units of intermediate good i, the amount Y_t of final goods that can be produced is

$$Y_t = \left[\int_0^1 Y_{it}^{1-\sigma} di\right]^{\frac{1}{1-\sigma}} \tag{2}$$

[1] In Galí's model that features dynamic inefficiency, bubbles play a useful role. Hence, as Miao et al emphasize, targeting the larger steady-state bubble Pareto dominates targeting the smaller steady-state value. In other models of bubbles, dampening a bubble can be welfare improving, e.g., by avoiding misallocation or mitigating a recession that would be more severe if a bubble were allowed to grow and collapse. Whether these benefits exceed the costs of tighter policy is a quantitative matter. For an overview of different models of bubbles and their policy implications, see Barlevy (2025).

Each agent is endowed with one unit of labor when young that they supply inelastically. When agents turn old, they lose the ability to work. However, they can hire labor and direct workers how to produce distinct intermediate goods. One unit of labor can produce one unit of any intermediate good in any period. Since young workers provide one unit of labor in total and employment at each intermediate goods producer is the same in equilibrium, equilibrium output of final goods is 1. All workers earn the same real wage, denoted W_t, while old agents who hire workers each earn a real profit of $1 - W_t$ after paying workers.

Each cohort of young agents is also endowed with one unit of a new intrinsically worthless asset that pays no dividends. We will refer to these assets as bubbles if they trade at a positive price. Let $Q^B_{t|t-k}$ denote the real price at date t of the vintage of assets that arrived k periods earlier for $k = 0, 1, ...$ Young agents at date t are thus endowed with $W_t + Q^B_{t|t}$ worth of resources. They can use these resources to buy goods or assets. In addition, they can issue one-period nominal debt or buy one-period nominal debt issued by other young agents. One-period nominal debt is available in zero net supply, and so any nominal debt purchased by young agents must be issued by other young agents. Let $Z^B_{t|t-k}$ denote the quantity of the asset which appeared k periods earlier that agents buy at date t. Let Z^M_t denote the real value of nominal debt that young agents at date t hold. The budget constraint of young households at date t is given by

$$C_{1,t} + Z^M_t + \sum_{k=0}^{\infty} Q^B_{t|t-k} Z^B_{t|t-k} = W_t + Q^B_{t|t} \tag{3}$$

When agents turn old, they will earn $1 - W_{t+1}$ in real profits. In addition, they will have the real value of any one-period nominal debt and any bubble assets they hold.[2] Let i_t denote the nominal interest rate on one-period debt purchased at date t and P_t denote the price level at date t. The realized real return on one-period nominal debt is then $\frac{1+i_t}{P_{t+1}/P_t} \equiv \frac{1+i_t}{\Pi_{t+1}}$, and so

$$C_{2,t+1} = 1 - W_{t+1} + \frac{1 + i_t}{\Pi_{t+1}} Z^M_t + \sum_{k=0}^{\infty} Q^B_{t+1|t-k} Z^B_{t|t-k} \tag{4}$$

For the goods market to clear, we need

$$C_{1,t} + C_{2,t} = 1 \tag{5}$$

For asset markets to clear, we need $Z^M_t = 0$ for all t and $Z^B_{t|t-k} = 1$ for all t and all $k = 0, 1, ...$ Following Galí, we define $U_t \equiv Q^B_{t|t}$ as the initial value of the assets that appear at date t and $B_t \equiv \sum_{k=1}^{\infty} Q^B_{t|t-k} Z^B_{t|t-k}$ as the value at date t of all available bubble assets inherited from the previous period. Substituting in $Z^M_t = 0$ and $Z^B_{t|t-k} = 1$ and canceling U_t on both sides of (3), we can rewrite the budget constraint of young households in equilibrium as

$$C_{1,t} + B_t = W_t \tag{6}$$

The first order condition with respect to Z^M_t yields

$$1 = \beta (1 + i_t) E_t \left[\frac{C_{1t}}{C_{2,t+1}} \frac{1}{\Pi_{t+1}} \right] \tag{7}$$

[2] Galí assumed a fraction δ of any existing asset vintage was destroyed each period, so agents who buy assets at date t only retain $1 - \delta$ of them at $t + 1$. Since this assumption has no impact on the results, we set $\delta = 0$ to conserve on notation.

Taking the first order condition with respect to $Z^B_{t|t-k}$ and combining the conditions for all k yields

$$B_t + U_t = \beta E_t \left[\left(\frac{C_{1t}}{C_{2,t+1}} \right) B_{t+1} \right] \tag{8}$$

The budget constraints (3) and (4) rely on the real wage W_t. This wage is determined in the labor market where intermediate goods producers hire the labor supplied by young workers. Each intermediate goods producer $i \in [0,1]$ sets the price P_{it} of their particular good one period in advance, before observing W_t. In particular, they choose P_{it} to solve

$$\max_{P_{it}} E_{t-1} \left[\frac{\beta C_{1,t-1}}{C_{2,t}} Y_{it} \left(\frac{P_{it}}{P_t} - W_t \right) \right]$$

where P_t is the price of the final good at date t. Demand for intermediate goods will come from competitive final goods producers who convert intermediate goods into final goods. Given the constant elasticity of substitution production function in (2), demand for each intermediate good is given by $Y_{it} = (P_{it}/P_t)^{-1/\sigma} Y_t$ where Y_t is the total amount of final goods (and is equal to 1 in equilibrium). Substituting in for this demand, differentiating with respect to P_{it} and equating with 0, and using the fact that all firms set the same price so $P_{it}/P_t = 1$ in equilibrium, we get

$$E_{t-1} \left[\frac{\beta C_{1,t-1}}{C_{2,t}} \left(1 - \frac{W_t}{1-\sigma} \right) \right] = 0 \tag{9}$$

Finally, we need to specify how the interest rate i_t on one period nominal debt is determined. Galí assumed the central bank directly sets this rate.[3] At each date t, the central bank observes the value of new assets U_t ($\equiv Q^B_{t|t}$) and then announces a nominal interest rate i_t. Galí posited a rule in which the central bank sets i_t as an increasing function of how the realized value of bubble assets $B_t + U_t$ compares to the value of bubble assets in some deterministic steady state, denoted $B + U$. Formally, let R denote the gross real interest rate associated with the deterministic steady state in which bubbles are equal to $B + U$ and let Π denote steady-state inflation (which is also the central bank's long-run inflation target). Galí assumed the central bank sets the nominal interest rate according to the rule

$$1 + i_t = R E_t \left[\Pi_{t+1} \right] \left(\frac{\Pi_t}{\Pi} \right)^{\phi_\pi} \left(\frac{B_t + U_t}{B + U} \right)^{\phi_b} \tag{10}$$

A higher value of ϕ_b corresponds to a more aggressive or larger interest rate increase whenever bubbles exceed the benchmark steady-state value that the central bank uses as its reference.

An equilibrium is a path for seven variables $\{C_{1t}, C_{2t}, W_t, \Pi_t, i_t, B_t, U_t\}_{t=0}^{\infty}$ that satisfies the six equilibrium conditions (5), (6), (7), (8), (9), and (10) at each date t. Galí (2014) and Miao et al (2019) solved for

[3] In practice, central banks don't set the rate on private debt but influence it indirectly through the nominal rate they charge to borrow from and lend money to the central bank. Modeling this transmission more realistically is delicate given that money can serve the same role as a bubble asset. We could allow the central bank to offer to borrow and lend money but restrict attention to equilibria in which no money circulates. See Galí (2014) for a discussion on allowing money in this model.

equilibrium paths given an exogenous process for $\{U_t\}_{t=0}^{\infty}$. Galí showed that a bubble equilibrium in which $B_t > 0$ for some t exists if and only if $(1-\sigma)^{-1} < 1+\beta$, which implies $R < 1$.

We begin with deterministic equilibria where $U_t = U$ for all t and is thus nonstochastic. In a steady-state equilibrium, $B_t = B$ for all t and the total value of all bubble assets will equal $B+U$ at each date. Assuming the rule in (10) is based on deviations from the same value for $B+U$ as the steady state we are solving for, the central bank would set $1+i_t = R\Pi$ at all t. Galí showed that there exists a value $\overline{U} > 0$ such that for any $U \in (0, \overline{U})$, if $U_t = U$ for all t, there exist two steady-state values B given U, each associated with a different steady-state interest rate R and supported by a monetary policy targeting a different steady-state value for bubbles. One of these steady states is locally indeterminate (which Galí describes as the stable steady state) and one is locally determinate (which Galí describes as the unstable steady state).

Galí used the locally indeterminate (stable) steady state to construct additional stochastic equilibria for a monetary policy rule that sets the nominal interest rate based on how B_t+U_t compares with $B+U$ at the locally indeterminate (stable) steady state. This construction involved introducing small iid fluctuations in U_t around the constant U. Log-linearizing the equilibrium system of equations, Galí showed that given a value of ϕ_b, there exists a stochastic equilibrium that can be approximated by

$$\ln(B_{t+1}/B) = \chi \ln(B_t/B) + (\phi_b + 1)(1-R)\ln(U_t/U) \tag{11}$$

where $\chi = R(1+(1-\beta)B/\sigma) < 1$.[4] To ensure that that B_t and U_t are not so large as to violate aggregate resource constraints, the process U_t cannot be too large in magnitude. The restrictions on U_t become stricter for higher values of ϕ_b. This means that given some value $\overline{\phi}$, we can use the same iid process for U_t that yields an admissible equilibrium for $\overline{\phi}_b$ to construct admissible equilibria for all $\phi_b \in [0, \overline{\phi}_b]$.

Suppose that for each value of $\phi_b \in [0, \overline{\phi}_b]$ we select the equilibrium in (11) holding fixed the iid process for U_t that we choose for $\overline{\phi}_b$. We can then ask what happens across these equilibria when we move towards more aggressive rules, or larger values of ϕ_b, within the interval $[0, \overline{\phi}_b]$. Galí's insight was that a rule that increases the interest rate more aggressively if the bubble deviates from steady state can paradoxically lead to a bubble that deviates from its steady state even more. For suppose $B_t = B$, meaning the economy starts at steady state, and consider a positive shock to U_t, i.e., the value of bubbles that arrive at date t exceeds U. Since $B_t + U_t$ exceeds $B+U$, the monetary policy rule in (10) will dictate an interest rate that is increasing in ϕ_b. At the same time, from (11), a higher value of ϕ_b would lead to a larger value for B_{t+1} when $U_t > U$. When the central bank sets a higher rate, bubbles will be larger one period later.

The results above rely on an arbitrary parameter $\overline{\phi}_b$ to construct a family of equilibria indexed by $\phi_b \in [0, \overline{\phi}_b]$ based on a single process U_t. As we increase ϕ_b between 0 and $\overline{\phi}_b$, the interest rate at date t and the value of B_{t+1} can both rise. But what if the central bank tried to be even more aggressive and set

[4] Galí considered a broader class of equilibria that include an additional martingale term ξ_t where $E_{t-1}[\xi_t] = 0$. The special case where $\xi_t = 0$ for all t suffices for our purposes.

$\phi_b > \overline{\phi}_b$? The set of stochastic processes that can be equilibria becomes smaller as ϕ_b increases, and the equilibrium associated with a fixed nondegenerate process U_t becomes unsustainable for sufficiently large ϕ_b. To see this, consider what happens in the limit as $\phi_b \to \infty$. In that case, $B_t + U_t$ can never exceed the steady state $B + U$ in equilibrium at any date t. For suppose $B_t + U_t > B + U$ at some date t. From (10), we know that $\lim_{\phi_b \to \infty} \frac{1+i_t}{E_t[\Pi_{t+1}]} = \infty$. The only way to satisfy (7) in this case is if $C_{1,t}/C_{2,t+1}$ tends to zero whenever $B_t + U_t > B + U$. Since $B_{t+1} \leq 1$ given that young agents cannot spend on existing assets more than the total resources of the economy, equation (8) implies that $B_t + U_t$ would have to equal 0. But a bubbly steady state requires that $B + U > 0$, which contradicts the assumption that $B_t + U_t > B + U$. More aggressive rules thus cannot keep leading to higher rates and larger bubbles as ϕ_b rises past $\overline{\phi}_b$. The equilibria that survive under more aggressive rules must feature smaller deviations above the deterministic steady state, and eventually no equilibrium that exceeds the deterministic steady state will remain.

If $B_t + U_t$ cannot exceed $B + U$ in the limit as $\phi_b \to \infty$, an infinitely aggressive rule will never actually raise the real interest rate above R. Infinitely aggressive rules affect the size of bubbles not by aggressive rate increases but by eliminating alternative equilibria using the threat to raise rates. To determine whether raising rates is effective in dampening a bubble, we need to study what happens when the interest rate exceeds what agents expect without the additional threat of high rates in the future if the bubble in the future is too large. In the next section, we shift to studying monetary policy shocks as opposed to monetary policy rules. We also switch to a model that can be solved in closed form.

Before we turn to this alternative approach, we note that Miao et al (2019) offered a separate comment on Galí's paper. They studied the same rule (10) as Galí but assumed that the central bank targets the locally determinate (unstable) steady state value of B for a given U rather than the locally indeterminate (stable) steady state value. In this case, Galí's result that a more aggressive rule can be associated with more volatile bubbles no longer holds. Miao et al show that if U_t is iid as Galí assumed and the central bank targets the locally determinate steady state, the volatility of bubbles is zero for any ϕ_b: A higher realization of U_t crowds out B_t to keep $B_t + U_t$ constant. If U_t is serially autocorrelated and the central bank targets the locally determinate steady state, a more aggressive rule will result in less volatile bubbles. Miao et al noted that targeting the larger unstable steady state leads to a Pareto superior equilibrium, consistent with how bubbles are generally welfare-improving in settings with dynamic efficiency. They also showed that this equilibrium is E-stable and learnable, in contrast to the equilibria Galí studied. It is thus arguably more natural for the central bank to target the locally determinate (unstable) steady state, in which case Galí's results no longer hold. Miao et al thus considered what happens when the central bank targets a different steady state, not what happens when the central bank sets a higher rate than agents expect for one period.

2 A Closed-Form Model of Bubbles

To determine whether a central bank that faces a bubble can dampen it by setting a high interest rate, we need to shift from studying the effect of a monetary policy rule as Galí did to studying the effect of

monetary policy surprises. Since the relevant question is how a higher-than-expected rate affects the entire set of possible equilibria, it will be convenient to work with a model where we can characterize this set in closed form. We now introduce a model of bubbles inspired by Galí's model that we can solve analytically.

We begin by illustrating the key features of our model assuming a pure endowment economy without a monetary authority. We then modify our model to incorporate monetary policy in the next section.

Consider an overlapping generations economy in which the cohort born at date t is endowed with e_t goods when young and none when old.[5] This is analogous to imposing $W_t = 1$ in Galí's model so that agents earn nothing when they turn old. We replace Galí's log utility in (1) with a linear specification in which agents only care about consumption when old:

$$u\left(c_{1,t}, c_{2,t+1}\right) = c_{2,t+1} \tag{12}$$

Under these preferences, each cohort will seek to convert their endowment when young into consumption when old. For now, we allow them two ways to do so. First, they can buy an intrinsically worthless asset when young to sell when old. We assume a single asset vintage that is endowed to the old at date 0 and yields no dividend. We denote its price at date t by p_t. This corresponds to the case where $U_t = 0$ for all $t > 0$ in Galí's model, while p_t corresponds to the price $Q^B_{t|0}$ at date t of the asset that arrived t periods earlier as well as the total value B_t of all bubble assets given all other vintages always trade at 0.

Second, we allow agents to store their endowment and consume it one-for-one when old. Let s_t denote the amount of their endowment that agents store. The cohort born at date t thus solves

$$\max_{s_t} E_t\left[c_{2,t+1}\right] \tag{13}$$

s.t.

1. $c_{2,t+1} = s_t + \frac{p_{t+1}}{p_t}\left(e_t - s_t\right)$
2. $0 \leq s_t \leq e_t$

The second constraint implies agents cannot borrow to buy assets (since storage s_t cannot be negative) nor sell assets short to buy goods (since storage s_t cannot exceed e_t).

Depending on how $\frac{p_{t+1}}{p_t}$ compares to 1, agents will either store all of their endowment ($s_t = e_t$), use it all to buy assets ($s_t = 0$), or will be indifferent between the two options. Storage s_t plays a similar role as consumption when young $C_{1,t}$ in Galí's model. Agents in our model allocate e_t between storage s_t and holdings of the bubble asset p_t in the same way agents in his model allocate W_t between consumption $C_{1,t}$ and bubble assets B_t per (6). Since we assume linear preferences, s_t is easy to characterize.

In Galí's model, a bubble arises when the steady-state real interest rate $R < 1$. Since agents in our model can store goods, the real return that young agents can earn must be at least 1. The analog to having a

[5]To minimize confusion between Galí's model and ours, we refer to the key variables in our model with lower case letters.

negative real rate in our setting is to allow the endowment to grow over time, i.e.,

$$e_t = (1+g)^t e_0 \tag{14}$$

for $g > 0$. Growth makes it possible for all agents to consume more than they could using storage alone. Suppose each cohort of young agents gave their endowment to the old agents they overlap with and received the endowment of the next cohort, which exceeds their own endowment when $g > 0$. A bubble in which an intrinsically worthless asset trades at a positive price implements these transfers.[6]

An equilibrium in this economy is a path of prices $\{p_t\}_{t=0}^{\infty}$ such that, when agents choose s_t optimally, the amount of assets that young agents want to buy at each date t is equal to the amount of assets that the previous cohort wants to sell. As in Galí's framework, the model admits many equilibria. The next result, whose proof is in Appendix A, fully characterizes the deterministic equilibria in this model:

Proposition 1 *The set of deterministic equilibrium asset prices $\{p_t\}_{t=0}^{\infty}$ can be parameterized by two variables, a date t^* and a price p_{t^*} at that date. Any deterministic equilibrium path is given by*

$$p_t = \begin{cases} e_t & \text{if } t < t^* \\ p_{t^*} & \text{if } t > t^* \end{cases} \tag{15}$$

where $0 \leq t^* \leq \infty$ and p_{t^*} can assume any value in $[e_{t^*-1}, e_{t^*}]$ with $e_{-1} \equiv 0$.

All deterministic equilibria have the property that once the price p_t falls below e_t, it remains constant. If p_t fell below e_t at some date t, young agents in that period both buy the asset and store goods. This requires $\frac{p_{t+1}}{p_t} = 1$, which means p_{t+1} will remain below e_{t+1} given the endowment continues to grow. Figure 1 illustrates several deterministic equilibrium price paths from the set of possible equilibria. At one extreme, we have an equilibrium where $p_t = e_t$ for all t. The return to buying the asset at these prices is $\frac{p_{t+1}}{p_t} = 1+g$, so buying the asset dominates storage. Since agents cannot spend more than their endowment, this is also the highest possible equilibrium price at each date. At the other extreme, we have an equilibrium where $p_t = 0$ for all t. If agents expect $p_{t+1} = 0$, they would not be willing to pay anything for the asset at date t.

While Proposition 1 characterizes deterministic equilibria, there also exist stochastic (sunspot) equilibria in which agents who buy the asset at date t are uncertain about the price p_{t+1} at which they can sell the asset at date $t+1$. Since agents are risk neutral, their decisions will depend on the expected return $\frac{E_t[p_{t+1}]}{p_t}$.

3 Adding Monetary Policy to the Closed-Form Model

To incorporate monetary policy into the model in Section 2, we need several additional elements. First, following Galí, we assume that in addition to the asset and storage, agents can trade a bond available in

[6] The fact that agents can be better off than they would be with storage implies the equilibrium with storage is dynamically inefficient. The connection between dynamic inefficiency and bubbles was emphasized in Tirole (1985).

zero net supply that pays those who buy it at date t a nominal interest rate $1 + i_t$ set by the central bank.

Formally, let P_t denote the price of goods at date t and $\Pi_{t+1} = P_{t+1}/P_t$ denote the inflation rate between dates t and $t+1$. The realized real return on nominal debt is $\frac{1+i_t}{\Pi_{t+1}}$. We continue to let p_t denote the real price of the asset in terms of goods. If we let z_t denote the amount of goods agents sell to buy the nominal bond, then agents will now solve

$$\max_{s_t, z_t} E_t \left[c_{2,t+1} \right] \tag{16}$$

s.t.

1. $c_{2,t+1} = \frac{1+i_t}{\Pi_{t+1}} z_t + s_t + \frac{p_{t+1}}{p_t} (e_t - s_t - z_t)$
2. $s_t + z_t \leq e_t$

The second constraint implies agents cannot short the intrinsically worthless asset to buy goods. However, they can now borrow by setting $z_t < 0$, effectively issuing nominal debt.

Since nominal debt is in zero net supply, the debt holdings of all young agents at date t must add up to 0. This implies agents cannot expect to profit either from borrowing to buy bubble assets or from holding debt instead of the bubble asset. If they did, agents would be net long or net short in nominal debt rather than sum to zero. Since agents at date t maximize their expected consumption at date $t+1$, this requires

$$E_t \left[\frac{1 + i_t}{\Pi_{t+1}} \right] = \frac{E_t \left[p_{t+1} \right]}{p_t} \tag{17}$$

Condition (17) is analogous to combining equilibrium conditions (7) and (8) in Galí's model. It implies that if the monetary authority changes i_t, either inflation Π_{t+1} or the expected return on the intrinsically worthless asset $\frac{E_t[p_{t+1}]}{p_t}$ must change to ensure (17) holds. To determine which of these variables changes requires moving beyond the endowment economy in the previous section and specifying how goods are produced and priced. Here we only sketch the key arguments. The details are left to Appendix B.

We introduce price rigidity by assuming final goods are produced from intermediate goods whose prices are set before the central bank sets i_t, similarly to the approach in Galí. This means P_t cannot depend on the realization of i_t, although it can depend on the distribution of i_t that agents expect at date t. We further assume that firms set prices in a forward-looking manner that does not depend on the past realization of i_t. Hence, P_{t+1} does not depend on i_t either. It follows that inflation $\Pi_{t+1} = \frac{P_{t+1}}{P_t}$ is independent of the realization of i_t and can be fully anticipated at date t. A high realization for the nominal rate i_t will thus have no effect on the inflation rate Π_{t+1}. Instead, it will be associated with a high expected real return on the asset $\frac{E[p_{t+1}]}{p_t}$. The same is true in Galí's model, although in his setting it is the central bank's monetary policy rule that ensures Π_{t+1} is independent of i_t. The prediction that an unexpectedly higher nominal rate leads to a higher real return rather than higher inflation is a staple of rigid-price models.

To be more concrete, suppose the central bank chooses i_t at random at date 0 but deterministically thereafter. More specifically, suppose i_0 can assume three values, $i_0^L < i_0^M < i_0^H$, with probabilities

$$\begin{aligned} \Pr(i_0 = i_0^L) = \Pr(i_0 = i_0^M) &= \tfrac{1}{2}(1 - q) \\ \Pr(i_0 = i_0^H) &= q \end{aligned}$$

We set $i_0^H = i_0^M + \Delta$ and choose i_0^L and q to ensure that $E_0[i_0] = i_0^M$. That is, we impose that

$$q = \frac{i_0^M - i_0^L}{i_0^M - i_0^L + 2\Delta}$$

One can verify that $E[i_0] = i_0^M$ for this value of q. Since the real rate cannot fall below the return on storage, we only consider equilibria in which $\frac{1+i_0^M}{\Pi_1} > 1$. This allows us to choose a fixed $i_0^L < i_0^M$ such that $\frac{1+i_0^L}{\Pi_1} \geq 1$. To capture large unexpected interest rate increases, we can let $\Delta \to \infty$ and $q \to 0$. Agents expect i_0 will equal i_0^M on average, but they understand that the realized value of i_0 can be higher or lower. A realization in which $i_0 \neq i_0^M = E[i_0]$ can be interpreted as a monetary policy shock akin to those identified in Christiano, Eichenbaum, and Evans (1999), Kuttner (2001), Romer and Romer (2004), Gürkaynak, Sack, and Swanson (2005), Gertler and Karadi (2015), and Nakamura and Steinsson (2018).

Since inflation Π_1 between periods 0 and 1 does not depend on the realized interest rate i_0 in period 0, the real return on nominal debt $1 + r_1^H(\Delta)$ when $i_0 = i_0^H$ will be given by

$$1 + r_1^H(\Delta) = \frac{1 + i_0^M + \Delta}{\Pi_1} \tag{18}$$

and will exceed the real return on nominal debt $1 + r_0^M(\Delta)$ when $i_0 = i_0^M$ which is given by

$$1 + r_1^M(\Delta) = \frac{1 + i_0^M}{\Pi_1} \tag{19}$$

Condition (17) implies that a higher realization for i_0 will be associated with higher expected appreciation $\frac{E_0[p_1]}{p_0}$ on the bubble asset. This is consistent with Galí's observation that a higher real interest rate implies a faster growing bubble. However, this observation does not tell us how a higher realization for i_0 affects the size of the bubble p_0. Does a higher interest rate imply a smaller bubble or only a faster growing one?

To address this question, let us start with the case where $\Delta = 0$. In this case, the central bank sets $i_0 = i_0^M$ with probability 1 and i_0 is perfectly predictable. The model we construct in Appendix B is designed so that when i_t is perfectly predictable for all t, equilibrium output will grow at rate g, just as in the endowment economy. The set of equilibrium asset prices in this case will be the same as for the endowment economy. Let p_t^* denote the particular equilibrium price path when $\Delta = 0$. We want to know how the equilibrium asset price path $p_t^H(\Delta)$ when the central bank sets $i_0 = i_0^H$ for $\Delta > 0$ compares to the original path p_t^* when $\Delta = 0$.[7] Does the answer depend on the equilibrium path p_t^* we start with?

Our first observation is analogous to Galí's key finding: There exists an initial equilibrium p_t^* and a value of $\Delta > 0$ such that $p_t^H(\Delta) \geq p_t^*$ for all t, with strict inequality for $t \geq 1$. That is, a central bank

[7]Our approach thus compares the asset price under a higher-than-expected interest rate i_0 with the asset price when the central bank *always* sets the interest rate as expected. We could alternatively compare the price under different realizations of i_0 holding Δ fixed. However, the outcome when the central bank sets $i_0 = i_0^M$ can vary arbitrarily with Δ depending on how we select equilibria for different values of Δ. Our approach facilitates comparisons across Δ by using the same benchmark p_t^* for comparison regardless of how we select equilibria for different values of Δ.

that sets i_0 above what agents expect could face a larger bubble than when it commits to always setting the nominal i_0 to what agents expect. This possibility is best illustrated graphically, using the paths for deterministic equilibria in the endowment economy illustrated in Figure 1. By construction, these are also the deterministic equilibria in our production economy when $\Delta = 0$. The light blue line in Figure 2 represents a particular equilibrium path p_t^* in which $\frac{p_1^*}{p_0^*}$ is between 1 and $1 + g$. Since the return is below $1+g$, we can find other deterministic equilibria in the endowment economy in which the real return between periods 0 and 1 is higher than in the original equilibrium we choose. The dark blue line in Figure 2 is one example. Both equilibria start out with a price equal to e_0^*. But the dark blue equilibrium is associated with a larger bubble from period 1 on. The dark blue line can be supported as an equilibrium when $i_0 = i_0^H$ for small values of Δ. Essentially, for small Δ, setting $i_0 = i_0^H$ allows for the possibility that earnings remain unchanged while agents coordinate on a different deterministic equilibrium asset price path that is possible given this path of earnings. We confirm in Appendix B that if we start with an equilibrium where $1 < \frac{p_1^*}{p_0^*} < 1 + g$ and choose a small enough Δ, the dark blue line in Figure 2 can indeed correspond to an equilibrium path in which $p_t^H(\Delta) \geq p_t^*$ for all t. The full details are in Section B.5 in Appendix B.

The construction illustrated in Figure 2 will not work if we start with an initial equilibrium path p_t^* in which $E_0 \frac{[p_1^*]}{p_0^*} = 1 + g$. In that case, we would not be able to find an equilibrium in which earnings $e_t^H(\Delta)$ are the same as the original path of earnings e_t^*, the asset price $p_0^H(\Delta)$ in period 0 is equal to p_0^*, but the expected return $\frac{E_0[p_1^H(\Delta)]}{p_0^H(\Delta)}$ when $i_0 = i_0^H$ exceeds the expected return $\frac{E[p_1^*]}{p_0^*}$ when $\Delta = 0$. For suppose such an equilibrium existed. Since the expected return $\frac{E_0[p_1^H(\Delta)]}{p_0^H(\Delta)}$ would exceed $1 + g$, young agents at date 0 would prefer the asset to storage. With nominal debt in zero net supply, young agents spend all of their earnings on the asset. This would imply $p_0^H(\Delta) = e_0^*$. Since i_1 is perfectly predictable at date 1, earnings at date 1 must equal e_1^*. Since young agents cannot spend more than their earnings, $E_0[p_1^H(\Delta)] \leq e_1^*$. Combining these results, we have $E_0 \frac{[p_1^H(\Delta)]}{p_0^H(\Delta)} \leq \frac{e_1^*}{e_0^*} = 1 + g$. But that means the expected return when $i_0 = i_0^H$ is no higher than in the original equilibrium, which is a contradiction. Whether a higher than expected interest rate can lead to a larger bubble thus depends on what equilibrium price path p_t^* we start at when $\Delta = 0$.

The importance of which equilibrium we start with echoes Miao et al's result that more aggressive rules will not necessarily lead to larger deviations for all possible equilibria. However, the two results are distinct. Miao et al look at the effects of perturbations when the central bank commits to a rule that aims to stabilize the value of bubbles at the maximal steady state value, which is also the locally determinate (unstable) steady state. Our counterexample above involves the effect of a one-time interest rate shock starting at the equilibrium with the largest possible bubble.

Beyond requiring that we start from a particular equilibrium path p_t^*, the construction in Figure 2 also requires that we restrict attention to a small value of Δ that can ensure $\frac{1+i_0^H}{\Pi_1}$ is less than $1 + g$. We now argue that regardless of the equilibrium path p_t^* we start with when $\Delta = 0$ or which equilibria we select for different Δ, any equilibrium asset price $p_0^H(\Delta)$ will necessarily fall below p_0^* once Δ is sufficiently large.

We start with (17), which holds that when $i_0 = i_0^H$, the real interest rate on nominal debt must satisfy

$$1 + r_1^H(\Delta) = \frac{E_0[p_1^H(\Delta)]}{p_0^H(\Delta)} \tag{20}$$

Since $r_0^H(\Delta) > r_0^L(\Delta) \geq 0$, we know that storage will be dominated when $i_0 = i_0^H$. With nominal debt in zero net supply, this means $p_0^H(\Delta) = e_0^H(\Delta)$. That is, if the central bank sets a high real interest rate, young agents will spend whatever they earn in equilibrium on the asset. We can thus rewrite (20) as

$$1 + r_1^H(\Delta) = \frac{E_0\left[p_1^H(\Delta)\right]}{e_0^H(\Delta)} \tag{21}$$

In period 1, the most young agents can spend on the asset is their earnings e_1. Since we assume i_t is perfectly predictable for $t \geq 1$, equilibrium earnings e_1 at date 1 are uniquely pinned down and equal to e_1^*. It follows that $E_0[p_1^H(\Delta)] \leq e_1^*$, and so

$$1 + r_1^H(\Delta) = \frac{E_0\left[p_1^H(\Delta)\right]}{e_0^H(\Delta)} \leq \frac{e_1^*}{e_0^H(\Delta)} \tag{22}$$

Rearranging this equation implies $e_0^H(\Delta) \leq \frac{e_1^*}{1+r_1^H(\Delta)}$. Hence, if the realized real rate $1 + r_1^H(\Delta)$ is large, earnings $e_0^H(\Delta)$ in period 0 will be low. For any $p_0^* > 0$ we start with, we can find a sufficiently large $r_1^H(\Delta)$ such that earnings $e_0^H(\Delta)$ will be below p_0^*. Young agents cannot spend more than $e_0^H(\Delta)$ on the asset at date 0, and so $p_0^H(\Delta) \leq e_0^H(\Delta) < p_0^*$. It will thus always be possible to ensure a smaller bubble by setting a high enough interest rate. Appendix B fills in the details of how a high i_0^H leads to a high real rate $r_1^H(\Delta)$ and lower earnings $e_0^H(\Delta)$ in period 0: With rigid prices, it depresses the real wage, lowers labor supply, and causes output to fall. The key takeaway, which we summarize in a Proposition, is that regardless of which equilibrium path p_t^* we start with, the central bank can ensure the bubble at date 0 is smaller than p_0^* by setting the interest rate sufficiently above what markets expect.

Proposition 2: Given any initial equilibrium $p_0^* \in (0, e_0^*]$ when $\Delta = 0$, there exists a Δ large enough such that *any* equilibrium $p_0^H(\Delta)$ when $i_0 = i_0^M + \Delta$ must be below p_0^*. If the nominal interest rate sufficiently exceeds the expected nominal interest rate, the bubble will be smaller than when i_0 is perfectly predictable.

Figure 3 provides a graphical illustration. The light blue line once again corresponds to an initial equilibrium path p_t^* when $\Delta = 0$. Since i_t is perfectly predictable from date 1 on, earnings e_t for $t \geq 1$ must equal e_t^*, the same as when $\Delta = 0$. This means that for any realization of i_0, the set of equilibria from date 1 on will be the same as in the endowment economy. The solid lines in Figure 3 from date 1 on correspond to the same deterministic equilibrium price paths as in Figure 1. The dashed lines show how these equilibria would extend back to period 0 if $\Delta = 0$, including the price p_0^*. When $\Delta > 0$ and $i_0 = i_0^H$, the return on the asset between dates 0 and 1 will be $1 + r_1^H(\Delta) = \frac{1+i_0^H}{\Pi_1}$. In any deterministic equilibrium, the price $p_0^H(\Delta)$ at date 0 must equal $\frac{p_1^H(\Delta)}{1+r_1^H(\Delta)}$ by definition. This corresponds to the parallel solid lines in Figure 3 between dates 0 and 1. The maximum value that $p_0^H(\Delta)$ can assume in any equilibrium, including stochastic equilibria, is $\frac{e_1^*}{1+r_1^H(\Delta)}$. This is because e_1^* is the highest value that $p_1^H(\Delta)$ can assume in period 1. The red line in Figure 3 thus represents an upper bound on $p_t^H(\Delta)$. When the real rate $r_1^H(\Delta)$ is sufficiently high, the equilibrium

price $p_0^H(\Delta)$ at date 0 will necessarily be lower than the p_0^* we start with, regardless of which equilibrium we choose when $\Delta > 0$. That includes equilibria like the dark blue line in which the price $p_1^H(\Delta)$ at date 1 is higher than the price p_1^* when $\Delta = 0$. A central bank can always dampen the bubble on impact by setting a sufficiently high real interest rate.

Why does a sufficiently aggressive rate hike necessarily dampen the bubble on impact? As noted above, the set of equilibria from date 1 on is unaffected by what the monetary authority does at date 0. This is due to the fact that monetary policy shocks are neutral in the long run. For the particular model we describe in Appendix B, monetary policy shocks are neutral even after just one period. This means that the maximal equilibrium from date 1 on is unaffected by what the monetary authority does at date 0. Given the multiplicity of equilibria, setting an unexpectedly high interest rate at date 0 may select an equilibrium with a larger bubble in period 1, i.e., where $p_1^H(\Delta) > p_1^*$. However, setting a high enough interest rate will eventually select the largest possible bubble in that period. At that point, a higher interest rate can only imply a smaller bubble at date 0: A bubble that grows faster between dates 0 and 1 but is no higher at date 1 must be smaller in period 0.

We conclude our discussion by relating this result to our previous result about aggressive rules in Galí's setup. A rule specifies what interest rate the central bank will set in every state of the world. Since the value of new bubbles U_t in Galí's setup is stochastic, agents cannot perfectly anticipate the nominal interest rate that the central bank will set. The fact that a sufficiently large interest rate shock (large Δ) must dampen the bubble imposes a limit on what equilibria are possible under a sufficiently aggressive rule (large ϕ_b) which raises the interest rate in response to a shock. In particular, equilibria in which a shock U_t left B_t unchanged but resulted in both a higher real return on the bubble and a larger value of B_{t+1} become unsustainable: Equilibria that feature a high real interest rate would require either an impossibly large value of B_{t+1} in period $t+1$ or would require that B_t falls when U_t rises.

4 Monetary Policy and Bubbles with a Unique Equilibrium

The key insight from the previous section is that once the central bank attains the maximal possible bubble for a given earnings path, further increases in the real rate would depress earnings, which in turn depress asset prices. This suggests that if not for multiple equilibria, increasing interest rates would always dampen asset prices. We now confirm this by modifying our closed-form model so that it generates a bubble as a unique equilibrium and showing that a higher interest rate dampens the bubble for any $\Delta > 0$.

As in Section 3, we begin by solving a pure endowment version of the model before introducing monetary policy. Suppose that agents born at date t are endowed with e_t goods only when young but wish to consume only when old. We ignore nominal debt for now and assume agents can either store their endowment or use it to buy an asset endowed to the old at date 0. The only difference from Section 2 is that we now assume the asset pays a constant dividend of d goods per period. Our analysis in the previous two sections can be

viewed as a special case in which $d = 0$. As we show in Appendix A, when $d > 0$, the equilibrium price path in the endowment economy will be unique:

Proposition 3: If $d > 0$, the endowment economy admits a unique equilibrium, $p_t = e_t$ for all t.

Intuitively, a positive d ensures a strictly positive return on the asset as compared with a zero return on storage. Agents would then spend all their endowment on the asset, so the price is uniquely determined. Among the equilibria in Figure 1, only the red line corresponding to $p_t = e_t$ remains when $d > 0$. Since the price p_{t+1} is uniquely determined, the return $\frac{d+p_{t+1}}{p_t}$ is perfectly predictable.

Next, we show that the unique equilibrium in Proposition 3 corresponds to a bubble.[8] To confirm that this equilibrium represents a bubble, observe that since $p_t = e_t$, then $\lim_{t\to\infty} p_t = \infty$. At the same time, the fundamental value of the asset is given by the value of the dividends d per period, discounted by the deterministic return on savings $1 + r_{t+1} \equiv \frac{d+p_{t+1}}{p_t}$. That is,

$$f_t = \sum_{j=1}^{\infty} \left(\prod_{i=1}^{j} \frac{1}{1 + r_{t+i}} \right) d \tag{23}$$

Since $r_{t+1} = g + \frac{d}{e_t} > g$ for all t, we have

$$f_t < \sum_{j=1}^{\infty} \left(\frac{1}{1+g} \right)^j d = \frac{d}{g}$$

The fundamental value of the asset is bounded at all dates, while the price of the asset $p_t = (1+g)^t e_0$ grows without bound.[9] Thus, the price must eventually exceed the fundamental value of the asset. In fact, as we show in Appendix A, p_t exceeds f_t for all t:

Proposition 4: When $d > 0$, the bubble term $b_t = p_t - f_t > 0$ for all dates t.

To determine the effect of setting the interest rate higher than what agents expect, we again introduce a role for monetary policy by allowing agents to hold nominal debt that is available in zero net supply and whose interest rate i_t is set by the central bank. Once again, we assume i_0 in period 0 can take on one of three values but i_t is nonstochastic for $t \geq 1$. We also need to specify how goods are produced and priced. We sketch the argument here and leave the analysis to Appendix B.

[8] Tirole (1985) previously argued in a model of dynamic inefficiency that a unique equilibrium could be a bubble in part (c) of his Proposition 1. However, Hirano and Toda (2023) pointed out that his proof of this case is incomplete. They go on to derive conditions for when all equilibria feature a bubble, but without establishing conditions for uniqueness. While we highlight the role of $d > 0$ in getting a bubble as the unique equilibrium, Tirole emphasized the importance of assuming a zero (or negative) return on storage. Both conditions are necessary. If storage converted a unit of goods at date t into $1 + z$ units at date $t + 1$ for $z > 0$, the equilibrium in our model would no longer be unique even if $d > 0$.

[9] The finiteness of f_t requires $\lim_{t\to\infty} d/e_t = 0$. Rhee (1991) showed that $\lim_{t\to\infty} d/e_t > 0$ implies bubbles cannot arise.

Since our model features price rigidity that prevents inflation from adjusting to unexpected changes in i_0, the real return when $i_0 = i_0^H > E[i_0]$ and $\Delta > 0$ will be higher than when i_0 is perfectly predictable and $\Delta = 0$. To determine the real return on the asset in each of these cases, we first observe that since i_t is perfectly predictable beyond date 1, earnings e_t in the model in Appendix B are uniquely pinned down for $t \geq 1$ and equal to e_t^*. We further know from Proposition 3 that the unique equilibrium price from date $t = 1$ on when $d > 0$ is $p_t = e_t^*$, regardless of which value of i_0 was realized at date 0. Hence, the real return on the asset between periods 0 and 1 is $\frac{d+e_1^*}{p_0}$. Since this return exceeds 1 when i_0 is perfectly predictable and is even higher when $i_0 = i_0^H$ and $\Delta > 0$, agents will spend all of their earnings on the asset in both cases, implying $p_0 = e_0$ in both cases. For the real return $\frac{d+e_1^*}{e_0^H(\Delta)}$ when $i_0 = i_0^H$ and $\Delta > 0$ to exceed the real return $\frac{d+e_1^*}{e_0^*}$ when $\Delta = 0$, we need $e_0^H(\Delta) < e_0^*$. That is, when prices are rigid, earnings must fall. Since the asset price is equal to earnings in both cases, the asset price at date 0 also falls. Appendix B provides the full details of how an unexpectedly high nominal rate dampens earnings with rigid prices: A higher nominal rate increases the real wage, reduces labor supply, and drives down production.

Since the fundamental value of the asset is positive when $d > 0$, the price p_t no longer reflects the value of the bubble as when $d = 0$. To determine the effect of higher interest rates on the bubble term $b_0 = p_0 - f_0$, we can use the definition of the real rate $1 + r_1$ to solve for the asset price at date 0:

$$p_0 = \frac{d + p_1}{1 + r_1} \tag{24}$$

At the same time, the fundamental value of the asset defined in (23) satisfies the recursive equation

$$f_0 = \frac{d + f_1}{1 + r_1} \tag{25}$$

Rearranging and subtracting (25) from (24), together with the fact that $b_t = p_t - f_t$, yields

$$b_1 = (1 + r_1)\, b_0 \tag{26}$$

Since changing i_0 has no effect on real variables after date 0, neither the real asset price p_1 nor the real fundamental f_1 depend on the realization i_0. This implies b_1 does not depend on i_0 either. Since $r_1^H(\Delta) > r_1^*$, it follows $b_0^H(\Delta) = \frac{b_1}{1+r_1^H(\Delta)} < \frac{b_1}{1+r_1^*} = b_0^*$. Setting $i_0 = E[i_0] + \Delta$ will depress the bubble for any $\Delta > 0$.

The intuition once again involves the long-run neutrality of monetary policy. An intervention at date 0 in our model has no effect on the bubble beyond date 0. If a bubble grows more quickly but ends at the same value, its initial value must fall. By contrast, the multiplicity of equilibria when $d = 0$ allowed monetary policy to select different equilibria with different long-run values for the bubble allowing an intervention at date 0 to lead to a permanently larger bubble. Eliminating the possibility of multiplicity reveals that when the central bank cannot select equilibria, a higher nominal rate would necessarily dampen the bubble.

The case where $d > 0$ further reveals that dampening a bubble does not inherently require an aggressive monetary policy rule. An aggressive increase is only necessary to ensure the bubble is smaller when there are multiple equilibria. When the equilibrium is unique, any increase will dampen the bubble.[10]

[10]The possibility of a bubble as the unique equilibrium outcome is not limited to dynamic inefficiency models of bubbles.

5 Conclusion

This comment explores the implications of the results in Galí (2014) and Miao, Shen, and Wang (2019) for the question of whether policymakers can dampen bubbles by setting a high interest rate. We show that a central bank can always rely on a high interest rate to dampen a bubble on impact, regardless of what equilibrium we start at or end with. The logic behind this result is that monetary policy cannot affect the long-run value of the bubble, and a bubble that grows more quickly when the rate is higher but ends up at same value in the long-run must start out at a lower value. When there are multiple equilibria, monetary policy can in principle select equilibria where the long-run bubble is larger. However, even with multiple equilibria, the central bank cannot keep choosing larger bubbles indefinitely. A central bank may have to set rates sufficiently above what markets expect to ensure it dampens the bubble, but it can dampen it.

The fact that policymakers *can* dampen bubbles by raising the interest rate does not mean they *should*. Galí's model features a dynamically inefficient economy in which bubbles help mitigate the underlying inefficiency. As such, there is no benefit of deflating bubbles. However, there are other settings in which deflating bubbles may be useful. For example, Biswas, Hanson, and Phan (2020) consider a related model of bubbles based on credit market frictions in which bubbles help improve resource allocation. They show that when wages are rigid, a stochastic bubble that bursts may leave agents worse off because the collapse of the bubble can result in excessively high wages that create a prolonged recession. Allen, Barlevy, and Gale (2022) show that bubbles based on information frictions may reduce welfare by encouraging socially costly speculation. Once we acknowledge that leaning against bubbles can dampen them, we need to decide on which model of bubbles is best suited for studying whether we should in fact dampen them. We leave the broader question of whether leaning against bubbles is desirable to future work.

Other examples include models of bubbles based on overconfidence as in Scheinkman and Xiong (2003) and models of bubbles based on risk-shifting as in Allen, Barlevy, and Gale (2022).

Appendix A: The Endowment Economy

This appendix contains proofs concerning bubbles in an endowment economy that are discussed in the text.

Proof of Proposition 1: We wish to characterize all deterministic equilibria of the model in Section 2.

We first observe that the equilibrium price of the asset cannot be negative. With free disposal, demand for assets at a negative price would be infinite since young agents would like to be paid to buy assets they can just dispose, but old agents would refuse to sell assets at a negative price.

Next, suppose $p_t = 0$ at some date t. If $p_{t+1} > 0$, demand for the asset would be infinite at date t. Given the finite supply of the asset, this cannot be an equilibrium. Conversely, if $p_{t-1} > 0$, young agents would not want to buy the asset at date $t-1$ given storage provides a positive return, while the old would want to sell their assets, so this cannot be an equilibrium. The only deterministic equilibrium in which the asset price is 0 in some period is if the asset price is 0 in *all* periods. In any remaining deterministic equilibrium, $p_t > 0$ at all t, and the return to buying the asset at date t will equal $\frac{p_{t+1}}{p_t}$.

Suppose $p_t > 0$ and $\frac{p_{t+1}}{p_t} > 1$. The cohort born at date t would strictly prefer buying the asset to storage. Since the supply of the asset is 1, this implies $p_t = e_t$. The equilibrium price of the asset p_t is uniquely determined whenever $\frac{p_{t+1}}{p_t} > 1$.

Next, suppose $p_t > 0$ and $\frac{p_{t+1}}{p_t} = 1$. Young agents would be indifferent between storage and buying the asset, meaning any $s_t \in [0, e_t]$ is optimal. Since the amount agents spend on the asset is indeterminate, we cannot pin down the asset price p_t at date t. However, in this case we do know that

$$p_{t+1} = p_t \leq e_t < e_{t+1}$$

That is, if $\frac{p_{t+1}}{p_t} = 1$, then young agents must store some of their endowment at date $t+1$. This requires $\frac{p_{t+2}}{p_{t+1}} = 1$, since young agents at date $t+1$ must both store and buy the assets that old agents sell at $t+1$ given that the price at all dates, including $t+1$, is positive. This implies that a zero real return on the asset is an absorbing state: Once the real return falls to 0, it will remain there forever and the price of the asset will remain constant from that point on.

Finally, suppose $p_t > 0$ and $\frac{p_{t+1}}{p_t} < 1$. If $\frac{p_{t+1}}{p_t} < 1$, the young at date t would only choose storage. But the old would sell of their assets when the price is positive, so this cannot be an equilibrium.

The observations imply that a deterministic equilibrium must be associated with a cutoff date $0 \leq t^* \leq \infty$, before which $\frac{p_{t+1}}{p_t} > 1$ and after which $\frac{p_{t+1}}{p_t} = 1$. The asset price equals e_t before date t^* and is constant (and below e_t) after date t^*. At t^*, the price can assume any value $p_{t^*} \in [e_{t^*-1}, e_{t^*}]$. Although a deterministic equilibrium is associated with two parameters, t^* and p_{t^*}, each deterministic equilibria maps into a single value corresponding to the long-run limiting price $\lim_{t\to\infty} p_t$. This long-run value corresponds to p_{t^*} and allows us to deduce a unique interval $[e_{t^*-1}, e_{t^*}]$ from which we can deduce t^*. ■

Proof of Proposition 3: We prove a more general result than in the text: The equilibrium price p_t is unique for any sequence of dividends $\{d_t\}_{t=0}^{\infty}$ on the asset such that $\sum_{t=0}^{\infty} d_t = \infty$. A constant dividend $d_t = d > 0$ is just a special case of this more general condition.

In what follows, we allow for nondeterministic equilibrium price paths, meaning that p_{t+1} may not be perfectly predictable at date t. We show that the expected return on the asset $E_t\left[\frac{d+p_{t+1}}{p_t}\right]$ will strictly exceed 1 in any equilibrium. That would imply that storage is dominated by holding the asset, meaning $s_t = 0$ and $p_t = e_t$ for all t, establishing that the equilibrium is unique.

We first note that $p_t > 0$ for all t and any realization of the price. Since the asset yields a positive dividend, free disposal implies that demand for the asset at date t would be infinite if $p_t \leq 0$. Since the supply of the asset is finite, this cannot be an equilibrium.

Suppose that $p_t > 0$ and $E_t\left[\frac{d+p_{t+1}}{p_t}\right] < 1$. In that case, the old at date t would want to sell the asset but the young would refuse to buy. So this cannot be an equilibrium.

Next, suppose $p_t > 0$ and $E_t\left[\frac{d_t+p_{t+1}}{p_t}\right] = 1$. By the definition of an expectation, there must be a realization of p_{t+1} such that $p_{t+1} \leq E_t[p_{t+1}]$. For this realization, we have $p_{t+1} \leq E_t[p_{t+1}] = p_t - d_t$. In that realization, we would have $p_{t+1} < p_t \leq e_t < e_{t+1}$. This requires that $E_{t+1}\left[\frac{d_{t+2}+p_{t+2}}{p_t+1}\right] = 1$. By induction, there must be a path of price realizations such that the asset price declines by at least d_τ at each date τ relative to its value in the previous period. Since $\sum_{t=0}^{\infty} d_t = \infty$, there must be some date along this path in which the realized price of the asset is negative. But we argued above that this cannot occur in equilibrium. So there cannot be an equilibrium in which $p_t > 0$ and $E_t\left[\frac{d_t+p_{t+1}}{p_t}\right] = 1$ at some t.

The only case that remains is $p_t > 0$ and $E_t\left[\frac{d+p_{t+1}}{p_t}\right] = 1$ for all t. In this case, storage is dominated, and so $p_t = e_t$ for all t. This is the unique equilibrium. ■

Proof of Proposition 4: We prove a more general result than in the text: The equilibrium corresponds to a bubble at all dates for any path of nonnegative dividends $\{d_t\}_{t=0}^{\infty}$ where $\sum_{t=0}^{\infty} \frac{d_t}{(1+g)^t} < \infty$. To see this, we use the fact that from Proposition 3, the equilibrium return on savings at date t is uniquely determined and equal to $\frac{d+p_{t+1}}{p_t}$. If we define $1 + r_{t+1}$ as the return on savings, we have

$$p_t = \frac{d_{t+1} + p_{t+1}}{1 + r_{t+1}}$$

At the same time, the fundamental value f_t satisfies

$$f_t = \frac{d_{t+1} + f_{t+1}}{1 + r_{t+1}}$$

Subtracting the latter expression from the former reveals that the difference $b_t \equiv p_t - f_t$ must satisfy

$$b_t = \frac{b_{t+1}}{1 + r_{t+1}}$$

By repeated substitution, it follows that $b_0 = \left(\prod_{t=0}^{T-1} \frac{1}{1+r_{t+1}}\right) b_T$ for any $T > 0$. Hence, if $\lim_{T\to\infty} b_T > 0$, then $b_0 > 0$. Since f_t is finite when $\sum_{t=0}^{\infty} \frac{d_t}{(1+g)^t} < \infty$, we can find a T such that $p_T - b_T > 0$. The result then follows. ■

Appendix B: The Production Economy

This appendix describes a production economy with rigid goods prices which yields the endowment economy in the text as a reduced form when the nominal interest rate set by the central bank is perfectly predictable. We use this setup to confirm Proposition 2 for the production economy that is discussed in the text.

B.1 Production, and Earnings

The production economy setting we use is based on the Allen, Barlevy, and Gale (2022) model, which in turn builds on Galí (2014) but follows Adam (2003) in allowing for elastic labor supply.

Agents preferences are the same as in the endowment economy, i.e.,

$$u\left(c_t, c_{t+1}\right) = c_{t+1}$$

Young agents are endowed with 1 unit of labor rather than goods. They can use their labor to produce goods at home or to work for another agent. They cannot work as their own employee. If a household allocates ℓ units of labor to home production, they will produce $(1+g)^t h(\ell)$ consumption goods where $h(\cdot)$ is increasing and concave. For convenience, we assume $h(\ell) = (\ell - \frac{1}{2}\ell^2)$, which implies $h'(\ell) = 1 - \ell$ and $h'(0) = 1$ and $h'(1) = 0$.

Agents are indexed by $i \in [0,1]$. As in Galí's model, each can produce a distinct intermediate good using the labor of others. All intermediate goods use the same linear production technology: If producer i hires n_{it} units of labor at time t, they will produce $y_{it} = (1+g)^t n_{it}$ units of intermediate good i.

As in (2), consumption goods are produced from intermediate goods via a constant elasticity of substitution technology: Given y_{it} of each $i \in [0,1]$, the amount of final goods y_t that can be produced is

$$y_t = \left(\int_0^1 y_{it}^{1-\sigma} di\right)^{\frac{1}{1-\sigma}} \tag{27}$$

with $\sigma < 1$. The market for producing consumption goods from intermediate goods is competitive. Let P_t denote the price of the final good and P_{it} denote the price of intermediate good i. An agent who purchases intermediate goods to produce and sell final goods at price P_t would solve

$$\max_{y_{it}} P_t \left(\int_0^1 y_{it}^{1-\sigma} di\right)^{\frac{1}{1-\sigma}} - \int_0^1 P_{it} y_{it} di$$

The first-order condition with respect to x_{it} yields the producer's demand for each intermediate good

$$y_{it} = y_t \left(\frac{P_{it}}{P_t} \right)^{-1/\sigma} \tag{28}$$

Since any agent can produce final goods, the price P_t must equal the per unit cost of producing a good in equilibrium. Setting $y_t = 1$, this means P_t must equal $\int_0^1 P_{it} \left(\frac{P_{it}}{P_t} \right)^{-1/\sigma} di$, and so

$$P_t = \left(\int_0^1 P_{it}^{\frac{\sigma-1}{\sigma}} di \right)^{\frac{\sigma}{\sigma-1}} \tag{29}$$

Let W_t denote the wage per unit labor and $w_t = (1+g)^{-t} W_t$ denote the cost of producing one unit of an intermediate good, both at time t. Each intermediate goods producer chooses their price P_{it} to maximize expected profits given demand (28) and production costs w_t. We assume producers must set their price before knowing what the monetary authority does. Producer i's choice thus depends on her information set Ω_{it}, and they will set P_{it} to solve

$$\max_{P_{it}} E \left[(P_{it} - w_t) y_t \left(\frac{P_{it}}{P_t} \right)^{-1/\sigma} \middle| \Omega_{it} \right]$$

The optimal price is given by

$$P_{it} = \frac{E\left[w_t y_t | \Omega_{it} \right]}{(1-\sigma) E\left[y_t | \Omega_{it} \right]} \tag{30}$$

By symmetry, all producers will charge the same price, produce the same amount, and hire the same amount of labor, i.e., $n_{it} = n_t$ for all $i \in [0,1]$. The output of consumption goods is thus

$$y_t = (1+g)^t \left(\int_0^1 n_t^{1-\sigma} di \right)^{\frac{1}{1-\sigma}} = (1+g)^t n_t$$

Each agent will optimally choose labor supply n_t to satisfy

$$(1+g)^t h'(1-n_t) = \frac{W_t}{P_t}$$

which given our functional form for $h(\cdot)$ implies

$$1 - n_t = \frac{w_t}{P_t} \tag{31}$$

Each agent earns $(1+g)^t \frac{w_t}{P_t} n_t$ worth of goods in wages, $(1+g)^t \left(1 - \frac{w_t}{P_t} \right) n_t$ worth of goods in profits, and $(1+g)^t h(1-n_t)$ goods produced at home. Given our functional form for $h(\cdot)$, total earnings are given by

$$e_t = (1+g)^t \left[1 - \tfrac{1}{2}(1-n_t)^2 \right]$$

Observe that real earnings e_t are increasing in n_t for $n_t \in [0,1]$, and are maximized when $n_t = 1$.

B.2 Savings

Agents who earn e_t while young will need to convert their earnings into consumption when old. As in the endowment economy, they can store goods or buy an asset. As in the endowment economy, there is a single asset available in unit supply and endowed to the old at date 0 that pays a fixed dividend $d \geq 0$. Let p_t denote the real price of the asset relative to goods. The expected return to buying the asset is

$$\frac{d + E_t[p_{t+1}]}{p_t}$$

Agents can also hold nominal debt, which is in zero net supply. To ensure agents are willing to hold zero nominal debt, the expected real rate paid by the central bank must equal the expected real rate from buying the asset, as in the endowment economy, i.e.,

$$E_t\left[\frac{1+i_t}{\Pi_{t+1}}\right] = E_t\left[\frac{d+p_{t+1}}{p_t}\right] \tag{32}$$

where $\Pi_{t+1} = \frac{P_{t+1}}{P_t}$. We will refer to $1 + r_{t+1}$ as the realized real rate on nominal debt at date t, i.e.,

$$1 + r_{t+1} = \frac{1+i_t}{\Pi_{t+1}} \tag{33}$$

In a symmetric equilibrium, $z_t = 0$, implying that s_t must maximize

$$\max_{s_t \in [0,e_t]} E_t\left[c_{t+1}\right]$$

s.t.

$$c_{t+1} = s_t + \frac{d+p_{t+1}}{p_t}\left(e_t - s_t\right)$$

B.3 Defining Equilibrium

Given a path of nominal interest rates $\{1+i_t\}_{t=0}^{\infty}$, an equilibrium consists of a path of prices $\{P_t, w_t, p_t\}_{t=0}^{\infty}$ for goods, wages, and the asset, together with a path for employment $\{n_t\}_{t=0}^{\infty}$ and a path for storage $\{s_t\}_{t=0}^{\infty}$ such that agents behave optimally and markets clear. Collecting the relevant conditions above yields the following five conditions:

(i) Optimal pricing: $$P_t = \frac{E\left[w_t y_t | \Omega_{it}\right]}{(1-\sigma)\, E\left[y_t | \Omega_{it}\right]}$$

(ii) Optimal labor supply: $$1 - n_t = \frac{w_t}{P_t}$$

(iii) Optimal savings: $$s_t = \begin{cases} e_t & \text{if } E_t\left[\frac{d+p_{t+1}}{p_t}\right] > 1 \\ \in [0, e_t] & \text{if } E_t\left[\frac{d+p_{t+1}}{p_t}\right] = 1 \\ 0 & \text{if } E_t\left[\frac{d+p_{t+1}}{p_t}\right] < 1 \end{cases}$$

(iv) Asset market clearing: $$p_t = e_t - s_t$$

(v) Debt market clearing: $$E_t\left[\frac{1+i_t}{\Pi_{t+1}}\right] = E_t\left[\frac{d+p_{t+1}}{p_t}\right]$$

B.4 Equilibrium with Perfectly Predictable Nominal Interest Rates

We begin with the case where i_t is nonstochastic, meaning intermediate goods producers set their prices P_{it} knowing i_t. In line with the notation we use in in the text, we refer to this equilibrium with a star. If i_t is known, producers will be able to deduce employment n_t, wages w_t, and output y_t. This means $\{w_t, y_t\} \subset \Omega_{it}$, and so $E[w_t y_t|\Omega_{it}] = w_t y_t$ and $E[y_t|\Omega_{it}] = y_t$. The optimal pricing rule (i) implies the equilibrium condition

$$P_t^* = \frac{w_t^*}{1-\sigma}$$

The equilibrium real wage per unit of effective labor $\frac{w_t^*}{P_t^*}$ is constant and equal to $1-\sigma$. Using the optimal labor supply condition (ii), we have

$$n_t^* = \sigma \tag{34}$$

Equilibrium real earnings e_t^* are thus equal to $(1+g)^t[1-\frac{1}{2}(1-\sigma^2)]$. These earnings grow at a constant rate g, just as in the endowment economy.

Next, we turn to asset prices. Since earnings e_t^* grow at a constant rate g, the equilibrium is the same as for the endowment economy. Hence, if $d > 0$, Proposition 3 implies the unique equilibrium is one in which $p_t^* = e_t^*$. for all t. The real return the asset is uniquely determined and given by $\frac{d+p_{t+1}^*}{p_t^*} = 1 + g + \frac{d}{e_t^*} > 1$. Since the real return is positive, $s_t = 0$ for all t. We can derive the equilibrium inflation rate $\Pi_{t+1}^* = \frac{1+i_t}{1+g+d/e_t^*}$ given the nominal rate i_t. If we assign a value to P_0, we can use Π_{t+1}^* to solve for P_t^* for all $t \geq 1$ and for $w_t^* = (1-\sigma)P_t^*$. The equilibrium $\{P_t^*, w_t^*, p_t^*, n_t^*, s_t^*\}_{t=0}^{\infty}$ is thus uniquely determined given an initial value of P_0. The indeterminacy of the price level in this model is in line with the well-known result by Sargent and Wallace (1975) that a pure nominal interest rate rule does not pin down the price level.

If $d = 0$, there exist multiple equilibrium asset price paths. Any given equilibrium pins down paths for p_t^* and s_t^*. Given a path for p_t^*, we can determine the expected return $E_t\left[\frac{p_{t+1}}{p_t}\right]$ at each date t. We can then derive the inflation rate $\Pi_{t+1} = \frac{1+i_t}{E_t[p_{t+1}/p_t]}$ using the path for i_t. Finally, for a given initial price level P_0, we can use Π_{t+1}^* to derive P_t^* and $w_t^* = (1-\sigma)P_t^*$ for all $t \geq 1$. Thus, for each equilibrium path of asset prices $\{p_t^*\}_{t=0}^{\infty}$, the equilibrium $\{P_t^*, w_t^*, p_t^*, n_t^*, s_t^*\}_{t=0}^{\infty}$ is uniquely determined up to the initial value of P_0.

B.5 Equilibrium with Stochastic Interest Rates

Next, suppose producers set prices P_{it} before the central bank chooses i_t at random. As in the text, we assume i_t is perfectly predictable from date $t = 1$ on. The equilibrium from date 1 on is the same as in Section B.4. The only issue is the equilibrium at date 0.

We begin with the case where $d > 0$ and then discuss the case where $d = 0$.

The optimal price setting condition (i) is given by

$$P_0 = \frac{1}{1-\sigma} \frac{\frac{1}{2}(1-q)(n_0^L w_0^L + n_0^M w_0^M) + q n_0^H w_0^H}{\frac{1}{2}(1-q)(n_0^L + n_0^M) + q n_0^H}$$

Dividing by P_0, a particular weighted average of the real wages must equal $1-\sigma$, i.e.

$$\sum_{\omega \in \{L,M,H\}} \alpha^\omega \left(\frac{w_0^\omega}{P_0} \right) = 1 - \sigma \tag{35}$$

where $\alpha^\omega = \frac{\Pr(i=i^\omega) n_0^\omega}{\sum_{\omega \in \{L,M,H\}} \Pr(i=i^\omega) n_0^\omega}$. Since the real wage must average $1-\sigma$, it must weakly exceed $1-\sigma$ in some state and weakly fall below $1-\sigma$ in another. The labor supply condition (ii) implies

$$n_0^\omega = 1 - \frac{w_0^\omega}{P_0} \tag{36}$$

Equation (36) relates employment n_0^ω to the real wage $\frac{w_0^\omega}{P_0}$. Real earnings are given by

$$e_0^\omega = 1 - \tfrac{1}{2}(1 - (n_0^\omega)^2)$$

which is increasing in n_0^ω given $n_0^\omega \leq 1$.

When the nominal interest rate is perfectly predictable, we can solve for employment n_t independently of asset prices. The two variables now interact. From date $t=1$ on, the nominal interest rate is perfectly predictable, and so we know from Section B.4 that $p_1 = e_1 = (1+g)[1 - \frac{1}{2}(1-\sigma^2)]$.

We assume agents are forward looking and do not set prices as a function of irrelevant variables from the past. Thus, when firms set their prices at date 1, they will not set their prices as a function of the realization of i_0 back at date 0. This is akin to a Markov perfection requirement. If neither P_0 nor P_1 depend on i_0, the inflation rate $\Pi_1 \equiv \frac{P_1}{P_0}$ will also be independent of i_0.

If we do not impose forward-looking price-setting, we can have equilibria in which Π_1 varies with i_0 in such a way that implies $\frac{1+i_0}{\Pi_1}$ is constant. Monetary policy will then be neutral even in the short run. Non-neutrality requires that the inflation rate not vary with $1+i_0$, at least not one-for-one. Forward-looking behavior seems reasonable given firms have no reason to pay attention to past variables in setting prices. In Galí's original model, what pins down inflation not forward-looking behavior on the part of agents but a forward-looking monetary policy rule which which sets the nominal interest rate as a function of current inflation and future expected inflation but not as a function of past variables.

Since inflation $\Pi_1 = \frac{P_1}{P_0}$ cannot depend on i_0, we have

$$\Pi_1 = \frac{(1+i_0^H)p_0^H}{d+e_1} = \frac{(1+i_0^M)p_0^M}{d+e_1} = \frac{(1+i_0^L)p_0^L}{d+e_1} \tag{37}$$

Given that $i_0^H > i_0^H > i_0^L$, we know that $p_1^L > p_1^M > p_1^H$. A higher realized nominal rate will be associated with a lower real asset price.

Since $r_1^H > r_1^M > r_1^L \geq 0$, we know that $p_0^H = e_0^H$, $p_0^M = e_0^M$, and $p_0^L \leq e_0^L$. This implies $e_0^H < e_0^M < e_0^L$. Since $e_0 = 1 - \frac{1}{2}(1-(n_0)^2)$, it further follows that $n_0^H < n_0^M < n_0^L$. Since n_0 is increasing in the real wage,

this implies $w_0^H < w_0^M < w_0^L$. Since the real wage must average $1-\sigma$, it follows that $\frac{w_0^H}{P_0} < 1-\sigma$. That in turn implies that $n_0^H < \sigma = n_0^*$, i.e., employment when $i_0 = i_0^H$ is below the level of employment when the central bank sets the interest rate in a perfectly predictable way. This in turn implies that $e_0^H < e_0^*$. In other words, if a central bank sets a higher interest rate than expected, earnings e_0^H will be lower than if the central bank sets the interest rate as expected with certainty.

We first look for an equilibrium in which $p_0^L = e_0^L$. In that case, we have

$$\frac{e_0^H}{e_0^M} = \frac{1+i_0^M}{1+i_0^H} \text{ and } \frac{e_0^L}{e_0^M} = \frac{1+i_0^M}{1+i_0^L} \tag{38}$$

We can use equations (35), (36) for each $\omega \in \{L, M, H\}$, and the two equations in (38) to solve for employment n_0^ω and real wages $\frac{w_0^\omega}{P_0}$ for each realization of i_0. We can use n_0^ω to compute e_0^ω and hence asset prices p_0^ω. Finally, we can use (37) to solve for Π_1. The equilibrium is uniquely determined up to P_0.

If $r_1^L = 0$, we can find additional equilibria in which $p_0^L < e_0^L \leq 1$. In that case, condition (35), combined with (38), imply that a higher value for n_0^L will be associated with lower values for n_0^M and n_0^H. The equilibrium in which $e_0^L = p_0^L$ is thus the one for which e_0^H is maximized, providing an upper bound on any equilibrium price p_0^H.

Finally, we turn to the case where $d = 0$. For each realization of i_0, the set of continuation equilibria from date $t = 1$ on is exactly the same as for the endowment economy. Let $E_0[p_1^\omega]$ denote the equilibrium expected asset price at date 1 if $i_0 = i_0^\omega$. In that case, we would replace (37) with

$$\Pi_1 = \frac{(1+i_0^H)p_0^H}{E_0[p_1^H]} = \frac{(1+i_0^M)p_0^M}{E_0[p_1^M]} = \frac{(1+i_0^L)p_0^L}{E_0[p_1^L]} \tag{39}$$

and for the equilibrium in which $p_0^L = e_0^L$, we would replace (38) with

$$\frac{e_0^H}{e_0^M} = \frac{(1+i_0^M)E_0[p_1^H]}{(1+i_0^H)E_0[p_1^M]} \text{ and } \frac{e_0^L}{e_0^M} = \frac{(1+i_0^M)E_0[p_1^L]}{(1+i_0^L)E_0[p_1^M]} \tag{40}$$

We can now verify the claim in the text regarding the example in Figure 2. Recall that we defined $e_t^* = (1+g)^t[1-\frac{1}{2}(1-\sigma^2)]$ as the path of equilibrium earnings when i_t is perfectly predictable at all dates. Given a path for the nominal interest rate $\{i_t\}_{t=0}^\infty$ and an equilibrium path $\{p_t^*\}_{t=0}^\infty$, equilibrium inflation is given by $\Pi_{t+1}^* = \frac{p_t^*}{p_{t+1}^*}(1+i_t)$.

Suppose we start with a deterministic initial equilibrium $\{p_t^*\}_{t=0}^\infty$ where $1 < \frac{p_1^*}{p_0^*} < 1+g$. Let i_0^M denote the nominal interest rate set by the central bank in this deterministic equilibrium. The real return on the nominal bond $\frac{1+i_0^M}{\Pi_1^*}$ must also be between 1 and $1+g$. We can find a value of $i_0^L < i_0^M$ such that $\frac{1+i_0^M}{\Pi_1^*} > 1$ and $\Delta > 0$ such that $\frac{1+i_0^M+\Delta}{\Pi_1^*} < 1+g$. Suppose we set $p_0^L = p_0^M = p_0^H = e_0^*$ and then set $p_1^L = \frac{1+i_0^L}{\Pi_1^*}e_0^*$, $p_1^M = p_1^*$, and $p_1^H = \frac{1+i_0^M+\Delta}{\Pi_1^*}e_0^*$. Given our assumptions on Δ, all three prices assume values between e_0^* and e_1^*. We can then verify that the case in which the real wage is equal to $(1-\sigma)^{-1}$ for all three realizations

of i_0, employment equals σ for all three realizations, earnings equal e_0^* for all three realizations, together with these asset prices, satisfy all of the equilibrium conditions. Hence, there exists an equilibrium in which $p_t^H \geq p_t^*$ for all t with strict equality for $t > 1$.

B.6 The Effect of Tighter Monetary Policy

Finally, we can show that in the production economy where the central bank can set interest rates to be arbitrarily high, it can always ensure a smaller bubble than in some original equilibrium.

Proof of Proposition 2: Let p_0^* denote the equilibrium when $\Delta = 0$ and $i_0 = i_0^M$ with probability 1. Likewise, let $1 + r_1^L(\Delta)$ denote the real rate when $i_0 = i_0^L$ given Δ, and let $1 + r_1^H(\Delta)$ denote the real rate when $i_0 = i_0^H$. If we let $\Delta \to \infty$, the ratio $\frac{1+r_1^H(\Delta)}{1+r_1^L(\Delta)} = \frac{1+i_0^H}{1+i_0^L} = \frac{1+i_0^M+\Delta}{1+i_0^L}$ will tend to ∞. Given storage, we also know that $1+r_1^L \geq 1$. Hence, $\lim_{\Delta\to\infty} 1+r_1^H = \infty$. At date $t = 1$, we know $e_1 = (1+g)[1-\frac{1}{2}(1-\sigma^2)]]$. Since $p_1^H(\Delta) \leq e_1$, the expectation $E_0[p_1^H(\Delta)] \leq e_1$. For agents at date 1 to afford the asset requires that $(1 + r_1^H)p_0^H \leq e_1$. Rearranging, we have $p_0^H < \frac{e_1}{1+r_0^H(\Delta)}$ which for sufficiently large Δ must be below p_0^*. ■

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