

Strengthening Reinforced Concrete Beams in Shear with Ultra High Performance Concrete Jackets

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DECLARATION

I hereby confirm that this thesis is the result of my own work carried out in the Department of Civil and Environmental Engineering at Imperial College London. Any work from other sources reported in this thesis is cited and referenced appropriately.

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ABSTRACT

The construction industry regularly faces the need to strengthen existing reinforced concrete (RC) members to maintain or extend service life. Shear failure in beams is brittle in nature and may occur without warning leading to catastrophic failure and loss of life. This thesis investigates the strengthening of RC T-beams in shear with ultra high performance concrete (UHPC) U-shaped jackets. Studies of RC beams strengthened with UHPC jackets have mainly focused on bending with only limited investigation of shear failure for which there is no agreed design method. This research studies the effectiveness of UHPC U-shaped jackets at increasing the shear strength of RC T-beams through an experimental program, nonlinear finite element analysis (NLFEA) and analytical work. Two groups of four beams were tested, with three of the beams in each group having UHPC U-shaped jackets that varied in geometry and embedded longitudinal reinforcement. Tests were monitored using digital image correlation (DIC) to determine the kinematics of the critical shear cracks. The contributions to shear resistance of aggregate interlock, fibre bridging, dowel action, stirrups and the uncracked compression zone, were determined from the crack kinematics. Fibre bridging was found to be the dominant shear resisting mechanism in the jacketed specimens, but its contribution became less dominant towards failure due to widening of the critical shear cracks. The measured strengths of the tested beams are compared with design code based predictions and numerical analysis. Direct superposition of individually calculated design strengths of the RC beam and UHPC jacket is shown to overestimate shear strength particularly for thicker jackets. Based on the analysis of the shear resisting mechanisms, a more conservative method for predicting the ultimate shear strength of RC beams with UHPC jackets is proposed. In addition to the beams tested in this research, a developed database of 36 RC beams with UHPC jackets from the literature is used to validate the proposed methodology. Overall, the capacities predicted with the proposed method compare reasonably with the measured strengths.

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LIST OF SYMBOLS

Latin upper-case letters:

A_{cz}	: Area of the uncracked compression zone
A_{fv}	: Area of the beam cross-section over which fibre reinforcement contributes to shear resistance
A_g	: Gross area of the concrete section
A_s	: Area of longitudinal reinforcement in tension
$A_{s,u}$	: Area of longitudinal reinforcement in tension in the jacket
A_{sc}	: Area of longitudinal reinforcement in compression
A_{sw}, A_v	: Area of transverse reinforcement
A_t	: Total surface area
$A_{v,min}$	: Minimum area of transverse reinforcement
A_x	: Projected contact area in the normal direction for a crack between the aggregate and cement matrix
A_y	: Projected contact area in the tangential direction for a crack between the aggregate and cement matrix
D	: Particle size of granular material
D_{min}	: Smallest particle size of granular material
D_{max}, d_{max}, d_g	: Largest particle size of granular material
E_c	: Elastic modulus of concrete
E_{cm}	: Mean of elastic modulus of concrete
E_f	: Elastic modulus of fibre
E_s	: Elastic modulus of reinforcement
E_{sh}	: Elastic strain hardening modulus of reinforcement
E_{sm}	: Mean elastic modulus of reinforcement
E_{sw}	: Elastic modulus of transverse reinforcement
$F_c, F_{c,i}, F_{c,1}, F_{c,2}$	: Internal compressive force of NSC
$F_{c,u}$	: Internal compressive force of UHPC
F_s	: Internal force of longitudinal tensile reinforcement in the beam
$F_{s,u}$	: Internal force of longitudinal tensile reinforcement in the jacket
F_{sc}	: Internal force of longitudinal compressive reinforcement in the beam
F_{nl}	: Force that corresponds to the loss of linearity in bending of UHPC

$F_{t,u}$	: Internal tensile force of UHPC
G	: Reduced shear modulus of concrete in ATENA
G_c	: Shear modulus of concrete in ATENA
G_f	: Fracture energy of concrete; Foundation modulus
H	: Height of the UHPC prism after deduction of the notch depth (AFNOR, 2016a, 2016b)
I	: Moment of inertia
I_c	: Moment of inertia of the concrete area below the longitudinal reinforcement bars contributing to dowel action
I_s	: Moment of inertia of the longitudinal reinforcement bars contributing to dowel action
I_v	: Moment of inertia of the longitudinal bars and concrete area below the longitudinal reinforcement bars contributing to dowel action
K, K_f, γ_c	: Reduction factor that considers the global mechanical effects of fibre orientation on the tensile strength of UHPC (AFGC, 2013; AFNOR, 2016a, 2016b; JSCE, 2006)
$K(\omega)$	: Effective ratio of the contact area in terms of displacement (ω)
$K_{eh}(w_n)$	: Average fibre pull-out stress coefficient due to mechanical anchorage in terms of crack opening displacement (w_n)
K_i	: Initial dowel shear stiffness
K_p	: Prestressed cracking moment factor
$K_{st}(w_n)$	: Average fibre pull-out stress coefficient due to frictional bond in terms of crack opening displacement (w_n)
L	: Virtual gauge length
L_c	: Compressive crush band size of concrete in ATENA; Characteristic length of UHPC
L_f, l_f	: Length of longest fibre
L_{ch}^c	: Characteristic length of concrete in compression
L_{ch}^t	: Characteristic length of concrete in tension
L_t	: Tensile crack band size in ATENA; Characteristic length of UHPC
M	: Bending moment
M_b	: Flexural resistance of the uncracked section
M_{calc}	: Calculated flexural strength
M_{Ed}	: Design bending moment that is taken at distance d away from the support or concentrated load but not closer than a distance d from the face of the support
M_f	: Flexural resistance of cracked section

M_p	: Ultimate plastic bending moment of longitudinal reinforcement bar
N	: Amount of constituent material used in the concrete mixture per one cubic metre
N_b	: Normal force of the uncracked section
N_{Ed}	: Axial force normal to the cross-section due to loading or prestressing
N_f	: Normal force of the cracked section
N_u	: Axial force acting normal to the cross-section of a beam
P	: Force; Prestressed force
P_{calc}	: Calculated strength
P_{ed}	: Effective tensile force in the prestressed longitudinal reinforcement
P_{exp}	: Experimental force
$P_i(D)$	: Particle size distribution of the constituent material
$P_{mix}(D)$	: Cumulative particle size distribution of the concrete mixture
$\bar{P}_{mix}(D)$	: Mean of the cumulative particle size distribution of the concrete mixture
P_{num}	: Numerical strength
$P_{tar}(D)$	: Target function for the cumulative particle size distribution
P_u	: Ultimate force
R^2	: Coefficient of determination
R_{ai}, C_f	: Reduction factor that accounts for aggregate fracture
R_s	: Elastic rigidity per length
RSS	: Residual sum of squares
TSS	: Total sum of squares
V	: Shear force; Absolute volume of the concrete mixture
V_{agg}	: Shear resistance contribution of aggregate interlock
V_b	: Total unit volume of the concrete mixture
V_{calc}	: Calculated shear strength
V_{cz}, V_c	: Shear resistance contribution of the uncracked compression zone
$V_{da}, V_d, V_{dow}, V_{Dowel}, V_l$	: Shear resistance contribution of dowel action
$V_{da,cr}$	: Dowel action cracking load
$V_{da,max}$	: Maximum shear resistance contribution of dowel action
$V_{da,s}$	: Stirrup shear resistance contribution to dowel action
V_{du}	: Ultimate dowel force limit

V_{Ed}	: Design shear force
V_{exp}	: Experimental shear strength
V_f	: Volume of fibres in terms of total concrete volume
V_{fib}	: Shear resistance contribution of fibre reinforcement (i.e., fibre bridging action)
$V_{fib+dow}$	: Experimentally derived shear strength of the UHPC jacket (i.e., sum of the fibre reinforcement and dowel action contributions to shear resistance)
V_n	: Nominal shear strength
V_p	: Volume of the compressed concrete mixture of granular materials
V_{ped}	: Shear strength contribution of prestressed longitudinal reinforcement in UHPC
V_{Rd}, V_d, V_{yd}	: Design shear strength
$V_{Rd,c}, V_c, V_{rpcd}$	: Shear strength contribution of concrete
$V_{Rd,f}, V_{Rd,U}, V_{fd}, V_{R,f}$	: Shear strength contribution of fibre reinforcement
$V_{Rd,max}, V_{u,max}, V_{wcd}$	: Shear strength of concrete web
$V_{Rd,s}, V_s, V_{R,s}$	: Shear strength contribution of the transverse reinforcement
V_{Res}	: Shear resistance contribution due to residual tensile stress of concrete
V_{stir}, V_s	: Shear resistance contribution of the stirrups
V_{Total}	: Summation of the absolute volumes for all granular materials of the concrete mixture
V_u	: Ultimate shear force or strength
V_w	: Shear strength contribution of the cracked concrete web
X	: Volume fraction of the constituent material

Latin lower-case letters:

a	: Dimension of UHPC prism (AFNOR, 2016a)
a_v, a	: Shear span measured as the distance between the loading and support plates
b, b_w	: Width of beam
b_f	: Flange width of beam
b_n	: Effective width of beam
b_v	: Effective flange width of beam
$b_{v,eff}$	: Effective width of beam
c_n	: Depth to the resultant force of the linear compressive stress profile located above the crack tip

c_m	: Depth to the resultant force of the rectangular compressive stress profile located below the loading plate
c_1	: Longitudinal reinforcement bar spacing coefficient
d, d_s	: Depth to the longitudinal reinforcement in tension; Distance to the neutral axis from the longitudinal tensile reinforcement
d'	: Depth to the longitudinal compressive reinforcement
d_b	: Diameter of longitudinal reinforcement bar
d_c	: Distance between the centroid of the total area of bars and concrete to the centroid of the rectangular block area of concrete below the longitudinal reinforcement bars contributing to dowel action; Distance to the neutral axis from the internal compressive force of NSC
$d_{c,u}$	: Distance to the neutral axis from the internal compressive force of UHPC
d_f	: Diameter of fibre
d_i	: Distance between the centroid of the total area of bars and concrete to the centroid of the longitudinal reinforcement bars contributing to dowel action
d_p	: Effective depth of the prestressed reinforcement
d_{sc}	: Distance to the neutral axis from the longitudinal compressive reinforcement
$d_{s,u}$	: Distance to the neutral axis from the longitudinal tensile reinforcement in the jacket
$d_{t,u}$	: Distance to the neutral axis from the internal tensile force of UHPC
d_u	: Depth to the longitudinal tensile reinforcement in the jacket
e	: Distance from the base of the notch to the extensometer; Eccentricity parameter in ATENA
f_c, f'_c	: Compressive strength of concrete
f_{cd}	: Design compressive strength of concrete
f'_{cd}	: Design compressive strength of concrete
f_{ck}	: Characteristic compressive strength of concrete
f_{cm}	: Mean compressive cylinder strength of concrete
$f_{cm,cu}$	: Mean compressive cube strength of concrete
f_{ct}, f_t	: Tensile strength of concrete
$f_{ct,el}$	: Direct tensile strength at the elastic limit of UHPC
$f_{ct,fl}$	: Limit of elasticity in bending of concrete
$f_{ct,sp}$	: Splitting tensile strength of concrete
f_{ctm}	: Mean direct tensile strength of concrete

$f_{ctm,el}$	: Mean direct tensile strength at the elastic limit of UHPC
$f_{ctm,sp}$	: Mean splitting tensile strength of concrete
$f_{ctm,1\%}$	: Tensile stress of UHPC that corresponds to a crack width of 0.01H
f_{cu}	: Compressive cube strength of concrete
f'_{c0}, f_{c0}	: Onset of concrete crushing in ATENA
$f_{L/R}(x, y)$	: Coordinates positioned on the division of a crack to monitor movements as the crack widens during loading
f_s	: Stress in the longitudinal tensile reinforcement in the beam
$f_{s,u}$	: Stress in the longitudinal tensile reinforcement in the jacket
f_{sc}	: Stress in the longitudinal compressive reinforcement
f_u	: Ultimate strength of reinforcement
$f_{u,sw}$	: Ultimate stress of transverse reinforcement
f_{Ucd}	: Design compressive strength of UHPC
f_{Uted}	: Direct design tensile strength at the elastic limit of UHPC
f_{Utud}	: Direct design tensile strength of UHPC
f_{vd}	: Design residual tensile strength of UHPC
f_y	: Yield strength of reinforcement
f_{ym}	: Mean yield strength of reinforcement
f_{yt}	: Yield strength of transverse reinforcement
f_{yw}	: Yield strength of transverse reinforcement
f_{ywd}, f_{sd}	: Design yield strength of transverse reinforcement
h	: Height of beam
h_c	: Height of rectangular block area of concrete below the longitudinal reinforcement bars contributing to dowel action
h_F	: Vertical distance between the crack tip and loading plate
h_f	: Flange height of beam
k	: Prestressed factor for UHPC contribution to shear strength
k_{avg}	: Average value of the local orientation factor for all active fibres
k_b	: Initial fibre bond stress coefficient
k_c	: Foundation modulus of the concrete surrounding the longitudinal reinforcement bars contributing to dowel action; Compression zone factor; Concrete strength reduction factor
k_v	: Factor for concrete contribution to shear strength
k_ε	: Strength reduction factor that accounts for strain in the web of a beam

k_1, k_1^*	: Factor related to the average stress of the parabolic-rectangular stress block in compression of concrete
k_2, k_2^*	: Factor related centroidal position of the parabolic-rectangular stress block in compression of concrete
l	: Span length of beam
l_{bf}	: Fibre embedment length
$l_{cr,i}$	: Length of crack i
l_{da}	: Effective length of the concrete in tension below the longitudinal reinforcement bars contributing to dowel action
l_i	: Distance between mechanically anchored ends for hooked end fibres
$l_{tot,i}$	: Length where tensile stresses are redistributed due to a crack
m	: Crack displacement coefficient
n, n_e, α_e	: Modulus ratio of steel to concrete
n_{bars}	: Number of longitudinal reinforcement bars in a single layer that contribute to dowel action
p_k	: Ratio of aggregate to total concrete volume
q	: Distribution modulus
r, ψ	: Crack sliding to opening ratio
r_c	: Reduction factor of concrete compressive strength in ATENA due to concrete cracking in the perpendicular direction
r_c^{lim}	: Lower bound of the reduction factor of concrete compressive strength in ATENA due to concrete cracking in the perpendicular direction
r_F	: Horizontal distance between the crack tip and centre of the loading plate
$r_{F,edge}$	: Horizontal distance between the crack tip and edge of the loading plate
r_g	: Shear retention factor of concrete in ATENA
S, S_{sw}	: Spacing of transverse reinforcement
S_{cr}	: Height of the flexural crack
S_{eh}	: Mechanical anchorage fibre bond slip
S_F	: Shear factor coefficient in the fixed crack model of concrete in ATENA
S_f	: Maximum frictional fibre bond slip
t_b	: Thickness of the bottom layer of the UHPC jacket
t_s	: Thickness of the side layer of the UHPC jacket
u, w, ω, w_n	: Crack width; Crack opening displacement

u_0	: Initial crack opening due to flexure
$\bar{u}$	: Relative crack displacement
v, s, w_t, δ	: Crack slip; Crack sliding displacement
v_c	: Dimensionless value of the shear strength contribution of the uncracked compression zone
v_l	: Dimensionless value of the shear strength contribution of dowel action
v_w	: Dimensionless value of the shear strength contribution of the cracked concrete web
w_{c0}	: Crack opening displacement where the fibre with the longest embedment depth begins to pull-out
w_d	: Displacement at which a complete loss of concrete compressive strength occurs in ATENA
$w_{L/R}(x, y)$	: Displacement fields of monitoring points positioned on the crack division ($f_{L/R}(x, y)$)
w_{peak}	: Crack width, or opening, that corresponds to the peak tensile stress of UHPC
w_{sw}	: Crack width located at the stirrups
w_u	: Crack opening obtained in bending at the ultimate limit state with axial force on the end fibre
$w_{1\%}$	: Crack width of UHPC that is equal to $0.01H$
w^*, w_{lim}	: Crack opening limit of UHPC
$\bar{w}$	: Shear crack displacement
x	: Depth to the neutral axis
z	: Internal lever arm
z_c	: Height of the uncracked compression zone

Greek upper-case letters:

Δ	: Shear displacement due to dowel action
Δe	: Eccentricity of the axial force
Δ_{stir}	: Shear displacement at the location of the dowel crack and nearest stirrup that is a distance (f) away from the inclined crack
$\Omega(\theta)$	: Contact density function in terms of inclination angle (θ)

Greek lower-case letters:

α	: Ratio of measured stress to yield stress of the longitudinal reinforcement; Inclination of the active reinforcement; Truss angle; Relative depth of the crack
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α_{cc}	: Coefficient that accounts for long term effects on compressive strength of UHPC
α_{cw}	: Coefficient that accounts for the state of stress in the compression chord
α_n	: Relative depth of the neutral axis
α_r	: Coefficient of thermal expansion
β	: Relative stiffness of the idealised foundation; Inclination of transverse reinforcement; Volume dilation plastic factor of concrete in ATENA
β_{eh}	: Coefficient that accounts for the fibres that do not reach the maximum mechanical anchorage force
β_f	: Coefficient that reflects the slip endured on the longer embedded end of the fibre
β_u	: Truss angle of fibre reinforcement
γ	: Inclination angle of combined crack opening and sliding displacement or fibre bending angle; Element orientation factor in ATENA
γ_b	: Member reduction factor
γ_c	: Partial safety factor for the compressive strength of concrete
γ_{cf}	: Partial safety factor for fibre reinforcement
γ_{crit}	: Critical fibre bending angle
γ_E	: Safety coefficient for the contribution to shear strength of UHPC
γ_{max}	: Maximum fibre bending angle
δ	: Combined crack opening and sliding displacement
δ_{exp}	: Experimental displacement
δ_{num}	: Numerical displacement
δ_u	: Ultimate displacement
ε	: Porosity of the concrete mixture (i.e., void fraction)
ε^*	: Strain limit of UHPC
ε_c	: Strain in compression of concrete
ε_c^p	: Plastic strain of concrete corresponding to the peak compressive stress (f'_c) in ATENA
ε_{cu}	: Strain in the extreme compression fibre of concrete
ε_{el}	: Strain at the elastic limit of UHPC
$\varepsilon_f, \varepsilon_1$	: Maximal fracturing strain of concrete in ATENA; Compressive strain at the web/flange junction
ε_{loc}^c	: Compression localisation onset which initiates compressive softening behaviour of concrete in ATENA

ε_{loc}^{sh}	: Shear localization onset of concrete in ATENA
ε_{loc}^t	: Tension localisation onset which initiates tensile softening behaviour of concrete in ATENA
ε_s	: Reinforcement strain
$\varepsilon_{t,el}$	: Tensile strain at the elastic limit of UHPC
$\varepsilon_{t,lim}$	: Ultimate tensile strain of UHPC
$\varepsilon_{t,peak}$	: Total tensile strain at peak stress of UHPC
$\varepsilon_{t,peak}^{el}$	: Tensile elastic strain at peak stress of UHPC
$\varepsilon_{t,peak}^f$	: Tensile fracturing strain at peak stress of UHPC
$\varepsilon_{t,1\%}$	: Total tensile strain of UHPC that corresponds to a crack width of $0.01H$
$\varepsilon_{t,1\%}^{el}$	: Tensile elastic strain of UHPC that corresponds to a crack width of $0.01H$
$\varepsilon_{t,1\%}^f$	: Tensile fracturing strain of UHPC that corresponds to a crack width of $0.01H$
ε_y	: Yield strain of reinforcement
ε_u	: Ultimate strain of reinforcement; Maximum elongation of concrete obtained in bending at the ultimate limit state with axial force
$\varepsilon_{u,lim}$	: Ultimate limit strain of UHPC
ε_x	: Longitudinal strain at mid-depth of the effective shear depth
ε_y	: Yield strain of reinforcement
ζ	: Size effect factor in the compression chord
$\eta_{\theta f}, \eta_{f,uvem}$	: Fibre orientation factor
θ	: Inclination angle; Crack inclination angle; Truss angle of transverse reinforcement; Truss angle of fibre reinforcement
$\theta_{cr,i}$	: Angle of crack i
θ_f	: Range of possible fibre orientation angles
θ_{min}	: Minimum truss angle
θ_w	: Fibre pull-out loading direction
$\theta_{1-3,2-4}$	: Orientation angle used to rotate the horizontal ($P_1 - P_3$) and vertical ($P_2 - P_4$) virtual gauges together
θ_{2-4}	: Orientation angle used to only rotate the vertical ($P_2 - P_4$) virtual gauge
κ	: Coefficient for determining the direct tensile strength at the elastic limit of UHPC ($f_{ct,el}$)
λ	: Coefficient that represents the relative stiffness of the idealised foundation; Modification factor that accounts for the weight of concrete

λ_s	: Size effect factor
μ	: Coefficient of friction
ν	: Poisson's ratio; Effectiveness factor for compressive strength of concrete
ν_1	: Strength reduction factor for concrete cracked in shear
ξ	: Reduction factor that considers the axial stress in the longitudinal reinforcement
ρ	: Density
ρ_s	: Longitudinal reinforcement ratio
ρ_w	: Shear reinforcement ratio
σ	: Normal stress
σ_c	: Reduced compressive strength of concrete in ATENA
σ_{cf}	: Tensile stress resisted by fibres
$\sigma_{cf,eh}$	: Fibre tensile stress component due to mechanical anchorage
$\sigma_{cf,st}$	: Fibre tensile stress component due to frictional bond
σ_{cf0}	: Fibre effectiveness
σ'_{con}	: Compressive contact stress
$\sigma_d(w)$	: Design tensile stress-crack opening response of UHPC
σ_f	: Post-cracking stress of UHPC
$\sigma_f(w), \sigma(w)$	: Tensile stress-crack opening response of UHPC
$\sigma_f(\varepsilon)$	: Tensile stress-strain response of UHPC
$\sigma_k(w)$	: Characteristic tensile stress-crack opening response of UHPC
σ_{pu}	: Yield strength of cement matrix
$\sigma_{Rd,f}, f_v, \sigma_{res}$	: Residual tensile strength of UHPC
σ_s	: Measured stress of longitudinal reinforcement
σ_{sw}	: Stress in the stirrups
$\sigma_{sw}(\Delta_{stir})$	: Stress based on the shear displacement (Δ_{stir}) at the location of the dowel crack and nearest stirrup that is a distance (f)
$\sigma_{sw}(\varepsilon_{sw})$	: Tensile stress-strain response of the transverse reinforcement
τ	: Shear stress
$\tau_{b,avg}$	: Average fibre bond stress
$\tau_{b,0}$	: Initial fibre bond stress when the fibre is parallel to the pull-out loading direction; Bond stress of transverse reinforcement before yielding
τ_{bf}	: Average fibre bond stress
τ_{b1}	: Bond stress of transverse reinforcement after yielding

$\tau_{eh,max}$	: Fibre pull-out stress due to mechanical anchorage
$\tau_{f,max}$	: Maximum frictional fibre bond stress
τ_{xz}	: Shear stress in the compression zone
τ_0	: Shear strength of the crack when the crack displacement is equal to zero
ϕ	: Fibre pull-out loading direction; Diameter of reinforcement
ϕ_s	: Diameter of longitudinal reinforcement
ϕ_{sw}	: Diameter of transverse reinforcement
ϕ_v	: Packing density of the concrete mixture
χ_e	: Equivalent elastic curvature
χ_m	: Curvature of the uncracked portion
ω'_{lim}	: Elastic limit
ω'_{max}	: Maximum compressive displacement in a direction θ that is taken as a positive value
ω'_{θ}	: Relative displacement
ω'_{θ_p}	: Local displacement

CHAPTER 1: INTRODUCTION

1.1 RESEARCH BACKGROUND

Strengthening reinforced concrete (RC) beams in shear is the focus of this research. Beams are normally designed to support other structural members such as slabs, columns, walls or other beams. When the load carrying capacity of a beam is insufficient it may fail in either flexure, shear or combined flexure-shear. Flexural failure is preferable to shear failure since for under reinforced sections it is ductile in nature. On the other hand, shear failure is brittle in nature and can occur without warning leading to catastrophic failure and loss of life. Shear failure is associated with diagonal cracking between the loading point and support, and it typically occurs prior to flexural yield. Flexural-shear failure arises when shear failure occurs subsequent to yield of flexural reinforcement.

The construction industry regularly faces the need to strengthen existing RC members to maintain or extend service life. The most common reasons for this include changes in use, inadequate design, damage, concrete deterioration and corrosion of steel reinforcement. Damage typically stems from excessive loading, design inadequacy or poor workmanship. Concrete deterioration and corrosion to steel reinforcement primarily result from exposure to severe environmental conditions exacerbated by factors such as poor choice of materials and lack of cover. Strengthening may also be required due to structural or functional alterations. In any of the aforementioned cases, the load carrying capacity of the RC member in question needs to be either restored or increased owing to change in use.

Various strengthening techniques for deficient RC beams are available. Selecting a suitable strengthening technique depends on the deficiency of the RC beam in question, whether it be in flexure, shear or a combined flexure-shear. Jacketing an existing beam by wrapping a repair material around its sides and soffit is a common technique for strengthening beams in both shear and flexure. The most common jacketing materials in practice are RC, steel plate or fibre reinforced polymer (FRP) composite U-shaped jackets, with an example of the latter presented in Figure 1.1. Despite their effectiveness and wide-spread use, they each possess some noteworthy disadvantages.



Installation of U-shaped jacket around beam



Installed U-shaped jackets

Figure 1.1 – Strengthening deficient RC beam with FRP composite U-shaped jacket (Alkhrdaji, 2015).
Images reproduced with permission of the rights holder, National Council of Structural Engineers Associations (NCSEA).

A reinforced concrete jacket involves enlarging the cross-sectional area of an existing beam by casting new to old standard concrete. Additional longitudinal and transverse reinforcement is placed within the jacket for flexural and shear strengthening, respectively. The cross-sectional area may increase excessively due to the need for additional reinforcement bars and a minimum required concrete cover. This increase in size and weight may lead to supporting RC members having to be strengthened, with knock on effects that can disrupt the global lateral load resisting system or necessitate significant architectural alterations (Karayannis et al., 2008; Zhang et al., 2018).

Steel plates can be mechanically fastened or adhesively bonded to an existing beam (Sharif et al., 1995). Longitudinal plates are fixed to the soffit of the beam for flexural strengthening and along the sides of the beam for shear strengthening. Much like RC jackets, beams with steel plate jackets may also experience significant increases in weight. Mechanically fixing the steel plates onto a beam may require the design and installation of a complex mechanical fastening system (Alaee and Karihaloo, 2003). On the other hand, adhesively bonded steel plates may fail due to debonding. As a material, steel lacks durability and fire resistance and is susceptible to corrosion and vandalism (Alaee and Karihaloo, 2003).

Fibre reinforced polymer (FRP) composite sheets and plates have been extensively researched and applied in practice for strengthening beams. There are national and international design guides for its use in concrete repair (ACI, 2017; fib, 2019; The Concrete Society, 2012). FRP is a lightweight material, which makes it easier to install when compared to steel plates, and it has a relatively high tensile strength as in carbon fibre reinforced polymers (CFRP). FRP sheets

and plates can be adhesively bonded onto the soffit of the beam for flexural strengthening, or sheets can be wrapped around the sides and soffit of the beam for shear strengthening. FRP lacks fire resistance in addition to the risk of vandalism and accidental damage if left unprotected, much like steel plates. Beams strengthened with FRP may also experience debonding failure.

Research has led to the development of ultra high performance concrete (UHPC). It is a cementitious material comprising fine granular constituent materials, high binder content and low water to cement ratio that combine to give compressive strengths in excess of 150 MPa. The differences in mixture proportions and compressive strength between UHPC and other more standard types of concrete such as normal strength concrete (NSC) and high performance concrete (HPC) are presented in Table 1.1. Steel fibres are normally used in UHPC to ensure ductile compressive and tensile behaviour, and they may also reduce the need for typical steel reinforcement bars (Toutlemonde and Resplendino, 2011). The direct tensile behaviour of UHPC with steel fibres is similar to other types of fibre reinforced concretes (FRCs). The post-cracking response in tension can be either strain-softening or strain-hardening as shown in Figure 1.2. The direct tensile strength of UHPC with steel fibres normally exceeds 6 MPa, with the post-cracking behaviour dependent on the concrete matrix and steel fibre dosage. Due to its fine granular constituent materials and low water to cement ratio, which form a compact microstructure, UHPC is deemed to have negligible fluid transport properties that provide it with high resistance to corrosion and chemical attacks (Ghafari et al., 2014; Graybeal, 2006).

Table 1.1 – General mixture proportions and compressive strength of NSC, HPC and UHPC

Type		NSC ^A	HPC ^B	UHPC ^C
Cement	[kg/m ³]	275	450	900
Fine aggregate	[kg/m ³]	550	675	1000
Coarse aggregate	[kg/m ³]	1100	1125	0
Water to cement ratio		0.6	0.3	0.2
Compressive strength	[MPa]	30	90	150

^A NSC (Cobb, 2015); ^B HPC (Megat Johari et al., 2011); ^C UHPC (Toutlemonde and Resplendino, 2011)

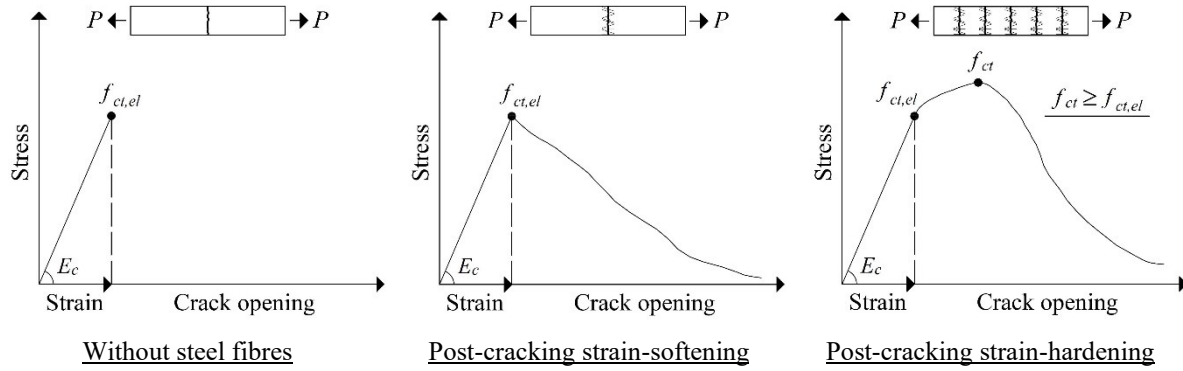


Figure 1.2 – Direct tensile response of fibre reinforced concrete. Adapted from Wille et al. (2014). Images reproduced with permission of the rights holder, Elsevier.

Strengthening RC beams with UHPC jackets has the potential to overcome several drawbacks of conventional techniques. However, due to the relatively recent emergence of UHPC and lack of recognized design guidance, its use for strengthening existing RC beams is not widely established in the construction industry. The compressive and tensile behaviour of UHPC can allow for a relatively thin jacket to be designed, which can possibly reduce both size and weight issues posed by jacketing with standard reinforced concrete (Toutlemonde and Resplendino, 2011). The relatively high direct tensile strength of UHPC can also reduce the demand for longitudinal or transverse steel reinforcement embedded in the jacket (Toutlemonde and Resplendino, 2011). Unlike beams strengthened with steel plates or FRP sheets or plates, failure due to debonding is not common with UHPC strengthened beams as UHPC has better strain compatibility with standard concrete (Alaee and Karihaloo, 2003; Liu and Charron, 2023). Given the negligible fluid transport properties of UHPC, it has the potential to also act as protective barrier to existing beams unlike standard types of concrete, steel or FRP.

Recent research shows that UHPC jackets can significantly enhance the ultimate flexural and shear strength of RC beams (Alkhoury, 2017; Al-Osta et al., 2017; Bahraq et al., 2019; Choi, 2016; Daugevičius et al., 2016; Hong and Lim, 2023; Liu and Charron, 2023; Martinola et al., 2010; Paschalis, 2017; Meda et al., 2012; Martins et al., 2021; Said et al., 2022; Sakr et al., 2019; Wang et al., 2013, 2015a, 2015b). However, further research is required to establish definitive design guidance. It is evident that multiple variables impact the shear design of beams with UHPC jackets such as the tensile strength of UHPC, thickness of the jacket and critical shear crack inclination angle of the strengthened beam. The relative lack of relevant experimental data from tests on strengthened beams hinders the development of rational models for assessing the strength of RC beams with UHPC jackets.

1.2 RESEARCH AIM AND OBJECTIVES

This research aims to advance the state of the art of strengthening RC beams with UHPC through the study of the shear resisting mechanisms in RC beams with UHPC jackets. The key objectives of this research are the following:

1. Investigate the shear strengthening of RC T-beams with UHPC U-shaped jackets through an experimental program, nonlinear finite element analysis (NLFEA) and analytical work.
2. Through study of crack kinematics, assess the contribution of key shear resisting mechanisms to the shear resistance of the tested jacketed beams. Hence, quantify the individual contributions of the RC T-beam and UHPC jacket to shear resistance from first cracking to the ultimate load.
3. Evaluate the applicability of current RC and UHPC design code equations to the assessment of ultimate shear and flexural strength of RC T-beams strengthened with UHPC jackets.
4. Develop recommendations for assessing the shear strength of RC T-beams with UHPC U-shaped jackets.

1.3 THESIS OUTLINE

This thesis is comprised of eight chapters, and they are as follows:

- Chapter 1 provides an overview of the research and its relevance. The aim of this research and its objectives are given. To conclude this chapter, an outline of the thesis is provided, including a brief description of each chapter.
- Chapter 2 reviews relevant literature on jacketing RC beams with UHPC and highlights the gaps in the literature that this research aims to address. The development of UHPC over the past couple of decades is described, including the general composition, production, material properties, current structural applications and environmental impact. The chapter describes the various theoretical models from the literature that are used in the thesis to determine the contribution of each shear resisting mechanism to overall shear resistance of the tested beams. The shear design methods of national and

international design codes and guides are presented, with focus on Eurocode 2, fib Model Code 2010, NF P18-710 and SIA 2052 amongst others (AFNOR, 2016b; BSI, 2014; EPFL, 2016; fib, 2013). To conclude this chapter, findings in the literature from tested RC beams with UHPC jackets are examined.

- Chapter 3 describes the experimental methodology adopted in this research. Two groups of four medium-scale RC beams with UHPC jackets were tested. Each group of beams included one control RC beam and three RC beams that were strengthened with UHPC U-shaped jackets with varying geometry and embedded longitudinal reinforcement. The chapter describes the reinforcement detailing of each specimen along with the manufacturing process, test arrangement, instrumentation and data acquisition. The chapter describes the main features of the digital image correlation (DIC) system used to monitor surface displacements in the sides of the beams during testing (LaVision, 2019). The development of a non-proprietary UHPC, which was used to manufacture the jackets, is also described in this chapter. The chapter also gives a description of splitting tensile tests on small-scale NSC-UHPC composite specimens, which were carried out to characterise the interfacial bond.
- Chapter 4 presents the test results of the eight beam specimens, which mainly include the material properties, crack development, mode of failure, load-displacement response and strain in the steel reinforcement. The splitting tensile test results of the NSC-UHPC composite specimens are also presented. To conclude this chapter, the depths of the critical cracks appearing on the specimens with jackets are assessed through the partial removal of the jackets and probe penetration.
- Chapter 5 reports the crack kinematics of the critical shear cracks of the tested beams at selected load steps until the ultimate load. The crack kinematics were determined using DIC, and the measured crack opening and sliding displacements were utilized to assess the contribution of each shear resisting mechanism to overall shear resistance of the tested beams. The contribution of each shear resisting mechanism is assessed along the critical shear crack using various theoretical models from the literature to ultimately quantify the influence of the UHPC jackets on the shear resistance mechanisms.
- Chapter 6 describes the numerical modelling methodology of this research. The NLFEA software package ATENA was utilised, and its main features are outlined in this chapter. Experimental results of RC, UHPC and UHPC strengthened beams from the literature were used to develop the adopted NLFEA approach before applying it to

the beams tested in this campaign. The modelling approach was developed in an iterative trial and error process that involved calibrating key input parameters. It was validated by assessing the accuracy of the numerical predictions according to the experimental load-displacement response, failure mode, reinforcement response and crack pattern.

- Chapter 7 expands on the results of the tested beams by comparing the results to design code predictions. A developed database comprised of test results of RC beams with UHPC jackets from the literature similar to those of this research is also used to assess code predictions. Prediction equations from national and international codes and guides are utilised to calculate the shear and flexural strength of RC beams strengthened with UHPC jackets. Based on these comparisons, recommendations are suggested for predicting the ultimate shear strength of RC beams with UHPC jackets.
- Chapter 8 summarises the conclusions of this research based on the experimental, numerical and analytical work. To conclude this chapter, recommendations for future research are presented.

CHAPTER 2: LITERATURE REVIEW

2.1 INTRODUCTION

This chapter reviews the literature pertinent to strengthening RC beams with UHPC jackets. The development of UHPC over the past couple of decades is described, including UHPC's general composition, production, material properties and current structural applications. The chapter explains the shear resistance mechanisms of reinforced concrete beams as well as UHPC beams. It also describes various theoretical models from the literature that can be used to estimate the contribution of each shear resisting mechanism to the total shear resistance of a given beam. The shear design methods of national and international standard reinforced concrete and UHPC design codes and recommendations are presented, with focus on Eurocode 2, fib Model Code 2010, ACI 318-19 and NF P18-710 amongst others (ACI, 2019; AFGC, 2013; AFNOR, 2016b; BSI, 2014; EPFL, 2016; fib, 2013; JSCE, 2006). To conclude this chapter, key observations and findings from tested RC beams with UHPC jackets from the literature are discussed, with the gaps in knowledge highlighted.

2.2 ULTRA HIGH PERFORMANCE CONCRETE

2.2.1 HISTORICAL BACKGROUND

The past few decades have seen significant advances in the development of UHPC which have resulted in it being produced more easily. When researchers first began producing what is now known as UHPC in the 1970s, preparation was only possible in laboratories, and it proved to be too difficult and energy consuming to be adopted by the construction industry (Roy et al., 1972). The 1980s saw the addition of supplementary cementitious material (SCM), steel fibres and theoretical particle packing models in UHPC preparation (Gu et al., 2015). Workability issues were still present at that time, which limited the applications of UHPC in practice, however the 1990s saw the addition of chemical admixtures in UHPC preparation. This significant advancement led to the first proprietary, or commercially available, UHPC product known as Ductal that is manufactured by Holcim Group based in Switzerland.

The use of UHPC by the construction industry has risen in recent years, including in rehabilitating existing structures (Brühwiler and Denarié, 2008). The Sherbrooke bridge in

Canada in 1997 was the first structure to be built entirely with UHPC, and UHPC has now been used in various types of structures across Asia, Australia, Europe and North America (Gu et al., 2015). Several case studies from the literature highlight the effectiveness of UHPC in enhancing the structural and durability performance of existing concrete structures (Brühwiler and Denarié, 2008). Cases include the repair of structural members that are exposed to harsh environmental conditions such as bridge decks, piers and crash barriers with UHPC overlays as shown in Figure 2.1 (Brühwiler and Denarié, 2008). Due to UHPC's negligible fluid transport properties, the relatively thin UHPC overlays were found to be effective in repair as service life was prolonged and the need for long-term continuous maintenance was reduced (Brühwiler and Denarié, 2008; Ghafari et al., 2014; Graybeal, 2006). Another notable case study includes the Hammersmith Flyover in London, UK, where UHPC structural components were used to successfully repair corrosion induced damage to its post-tensioning system (Cousin et al., 2017).



Bridge deck repaired with cast in-situ UHPC



Bridge pier repaired with precast UHPC

Figure 2.1 – Existing concrete structures repaired with UHPC (Brühwiler and Denarié, 2008)

The development of UHPC structural design codes and guidelines began in the early 2000s. The French Association of Civil Engineering (AFGC) was the first to publish interim recommendations in 2002 for the design of ultra high performance fibre reinforced concrete structural members (AFGC, 2002). Largely based off of these recommendations, the Japan Society of Civil Engineers (JSCE) published draft recommendations for the design and construction of ultra high strength fibre reinforced concrete structures initially in 2004, which were later updated in 2006 (JSCE, 2004, 2006). The latter is hereinafter referred to as JSCE 2006 (JSCE, 2006). The design recommendations of the AFGC and JSCE included properties, preparation and control of UHPC. The AFGC published updated recommendations in 2013,

hereinafter referred to as AFGC 2013 (AFGC, 2013). These developments eventually led to the French Standard Institute (AFNOR) issuing a UHPC design code in 2016 that is compatible with the nationally annexed concrete design code known as Eurocode 2, hereinafter referred to as EC2 (AFNOR, 2016b; BSI, 2014). The French codes include NF P18-470, NF P18-710 and NF P18-451 which cover the classification, structural design and construction of UHPC, respectively (AFNOR, 2016a, 2016b, 2018). The Swiss Federal Institute of Technology Lausanne (EPFL) also went on to publish recommendations in 2016 known as SIA 2052 for what they call ultra-high performance fibre reinforced cement-based composites (EPFL, 2016).

2.2.2 CONSTITUENT MATERIALS

Ultra high performance concrete generally consists of cement, fine aggregate, steel fibres mixed with a water to cement ratio of around 0.2 (Toutlemonde and Resplendino, 2011). The superior mechanical and durability properties of UHPC primarily stem from its constituent materials and the resulting porosity. The cross-sections shown in Figure 2.2 highlight the compact structure of a UHPC mixture as compared to a NSC mixture. The compact structure of UHPC leads to small and disconnected pores that enhance both strength and durability (DiPaolo, 2011).

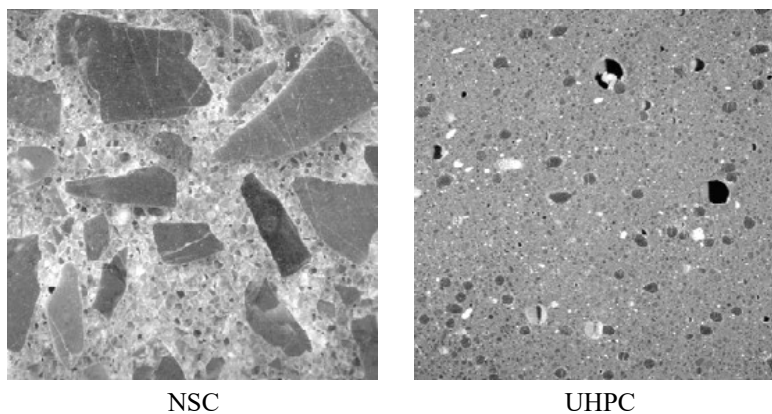


Figure 2.2 – Cross-section of NSC and UHPC (DiPaolo, 2011)

Cement is a key component in concrete as it acts as a binder for all other constituent materials. Cement of type CEM I is most commonly used in UHPC as it is low in tricalcium aluminate content, which reduces the need for water (BSI, 2019a; Sohail et al., 2018). Test results from the literature suggest that the type of cement in UHPC does not significantly impact the

resulting mechanical and durability properties, provided that the tricalcium aluminate content is lower than 8% (Alkaysi et al., 2016; Wille et al., 2011).

Supplementary cementitious materials (SCMs) are normally used in UHPC as a substitute for cement. The addition of SCMs, which include silica fume, fly ash and ground granulated blast-furnace slag amongst others, reduces the need for a very high cement content of around 900 kg/m³. Consequentially, the use of SCMs mitigates the negative environmental impacts of cement such as its notably high carbon dioxide emissions, and it creates a more sustainable building material (Randl et al., 2014). SCMs may improve the hydration process as they generate a pozzolanic reaction, and they may be used as fillers to fill the voids in a concrete mixture, which decreases porosity and enhances the packing density of a mixture (Sohail et al., 2018). SCMs are generally chosen depending on their local availability, cost and most importantly their respective physical and chemical composition (Sohail et al., 2018). Prior studies indicate that the SCM proportion in a UHPC mixture may range between 0.25 and 0.5 times the weight of cement (Sohail et al., 2018).

Sand between 150 µm and 4.75 mm in diameter is the only type of aggregate used to produce UHPC with coarse aggregate predominantly avoided (Sohail et al., 2018). According to Gu et al. (2015), coarse aggregate is avoided as it diminishes the homogeneity of a UHPC mixture and produces mixtures with a less-uniform stress distribution when loaded. Aggregate proportions may range between 0.9 and 1.3 times the weight of the cement in UHPC according to prior studies (Sohail et al., 2018). Quartz powder, with particle sizes of the order of micrometres, is normally used in addition to sand in UHPC (Sohail et al., 2018). It is suggested that sand and quartz powder with particles that cover a range of sizes rather than of a unified size be used to produce UHPC as it reduces porosity and increases strength (Park et al., 2008).

Discrete high strength steel fibres are regularly included in UHPC mixtures as they increase ductility and resistance to delamination, spalling and fatigue (Sohail et al., 2018). More importantly, steel fibres considerably enhance the post-cracking tensile behaviour of a UHPC mixture when compared to the same mixture without steel fibres (Wille et al., 2014). Steel fibres may come in various shapes and sizes such as straight or deformed, which include hooked end or spiral, and they are normally around 0.2 mm in diameter and between 13 mm and 30 mm in length. Relatively high tensile strength values in excess of 6 MPa can be achieved

with fibre dosages in terms of total concrete volume that range between 1% and 3% according to non-proprietary UHPC mixtures from the literature (Sohail et al., 2018; Wille et al., 2014). Increasing the fibre dosage may lead to an increase in tensile strength, however it will also lead to a decrease in workability (Sohail et al., 2018).

The water content in a UHPC mixture is quite low when compared to standard concrete mixtures. Non-proprietary UHPC mixtures from the literature suggest water to cement ratios between 0.12 and 0.25 may be used (Sohail et al., 2018). Much like standard concrete, the mechanical properties such as compressive strength decrease in a UHPC mixture as the water content is increased while workability is increased. To cater to the low water to cement ratio requirement of UHPC, chemical admixtures are used such as superplasticizers, or high-range water reducers (Sohail et al., 2018). Superplasticizers improve the rheological behaviour of UHPC, and as a result help reduce the need for excess water. A variety of other chemical admixtures may be used when preparing UHPC depending on the targeted properties as they help give special properties to both fresh and hardened concrete.

2.2.3 MIXING, CASTING AND CURING

Producing a UHPC mixture requires additional care when compared to standard concrete, especially when mixing and casting due to the fine granular particles and steel fibres. A UHPC mixture requires a comparatively higher input energy when mixing to avoid particle agglomeration and to ensure a homogenous dispersion of particles (Sohail et al., 2018). According to NF P18-470, the dry constituent materials should be mixed first up to homogenization followed by the addition of water and all or part of the chemical admixtures, and the steel fibres are then added gradually along with any remaining chemical admixtures (AFNOR, 2016a). The French and Swiss standards recommend that fresh UHPC should be cast from one end of the mould only without the use of internal or external vibration to avoid the disturbance of fibre orientation and dispersion (AFNOR, 2016a; EPFL, 2016). Much like standard concrete, exposed surfaces of a freshly cast UHPC mixture should be covered immediately after casting to allow for the continuation of the hydration process (Sohail et al., 2018). The effects of various curing methods such as steam, water, moist and ambient curing on the mechanical and durability properties of UHPC have been extensively studied (Graybeal, 2006; Randl et al., 2014; Wille et al., 2011). Test results suggest that steam and water curing

offer the most promising results in terms of compressive strength and fluid transport properties, yet moist and ambient curing were sufficient as well (Graybeal, 2006).

2.2.4 THEORETICAL PARTICLE PACKING MODELS

The design of a concrete mixture involves selecting the right combination of materials to achieve a targeted outcome. Kumar V and Santhanam (2003) suggest that mixture design should ultimately be treated as a volumetric-packing problem. This is due to the mechanical and durability properties of concrete, especially UHPC, being greatly affected by the degree of packing of the constituent granular materials. The degree of packing is a function of particle size distribution (PSD), minimum and maximum particle size, particle shape, particle texture, particle angularity and presence of liquids (Vogt, 2010). Various theoretical particle packing models have been applied when designing non-proprietary UHPC mixtures in order to minimize porosity and avoid excessive trial mixing, which may be tedious (Brouwers and Radix, 2005; Jones et al., 2002; Yu et al., 2015; Vogt, 2010). The basic principles of theoretical particle packing models are that particles are assumed to be spherical, and smaller particles fill the voids between larger particles to form a densely packed mixture as shown in Figure 2.3.

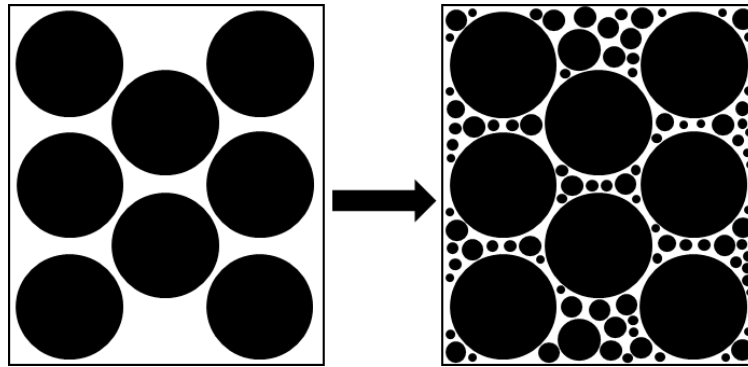


Figure 2.3 – Theory of particle packing

The packing density of a mixture (ϕ_v) is calculated using the following the equation:

$$\phi_v = V_p/V_b \quad (2.1)$$

where V_p is the volume of the compressed mixture of granular materials and V_b is the total unit volume. The porosity of a mixture (ε), or void fraction, is calculated as follows:

$$\varepsilon = 1 - \phi_v \quad (2.2)$$

where the porosity of a mixture (ε) can be minimized by maximizing the packing density (ϕ_v).

Several theoretical particle packing models have been developed over the years, which may be applied in concrete mixture design to minimize porosity. They can be categorized as either monodisperse systems, where all particles are the same size, or polydisperse systems, where particles cover a range of sizes (Funk and Dinger, 1994). Models of polydisperse systems of particles are better suited for concrete mixture design, and they can also be categorized as either discrete or continuous based on the particle size distribution (PSD) of the granular materials. The discrete approach refers to the packing of particles with size classes covering a narrow size range, and the continuous approach refers to the packing of particles with size classes of an infinite size range (Funk and Dinger, 1994). Figure 2.4 shows a chart of all the theoretical particle packing models of polydisperse systems that have been developed along with their respective theoreticians and commercially available software packages (Kırgız, 2020).

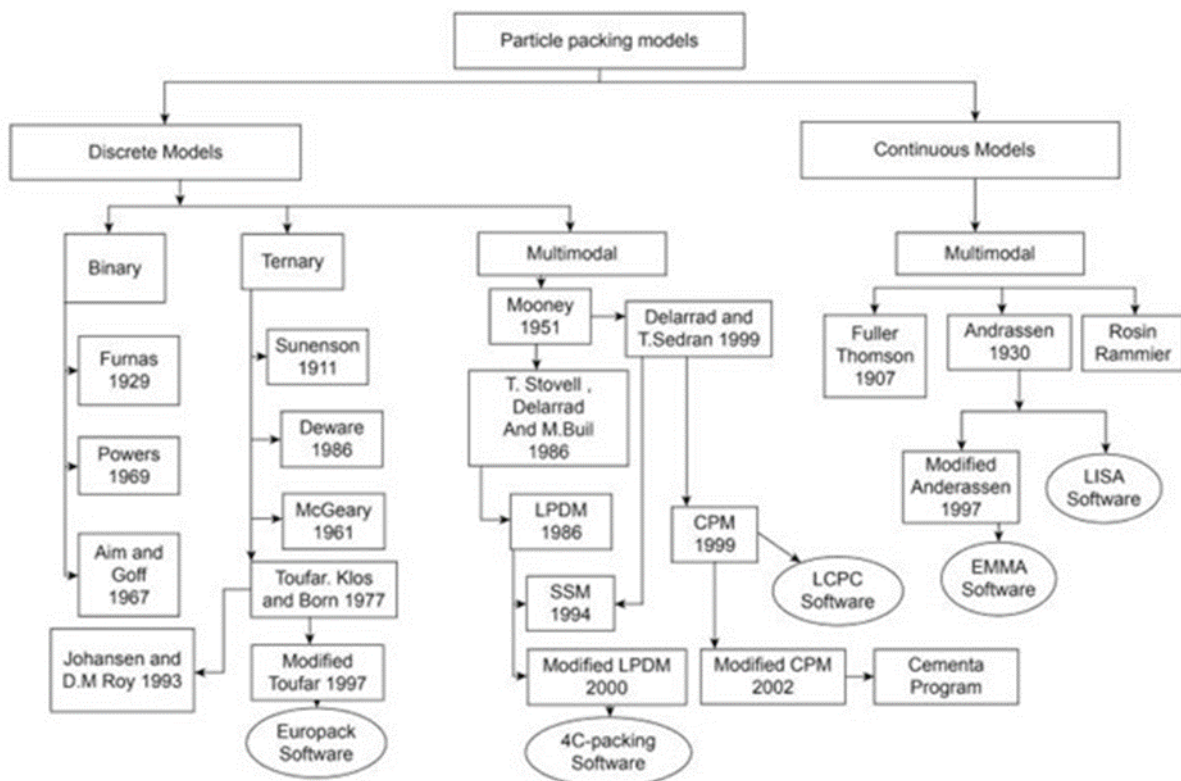


Figure 2.4 – Theoretical particle packing models for polydisperse systems (Kırgız, 2020). Image reproduced with permission of the rights holder, Elsevier.

The continuous approach is better suited for concrete mixture design seeing as it considers the infinite size range of the particle size classes for all granular materials. Nonetheless, prior studies have applied the ternary and multimodal discrete packing models shown in Figure 2.4 to help in successfully achieving a concrete mixture design with minimal porosity (Jones et al., 2002). Prior studies have also adopted the continuous approach, where mainly the modified Andreasen model was used (Brouwers and Radix, 2005; Yu et al., 2015; Vogt, 2010). According to Funk and Dinger (1994), Andreasen's work forms the basis for the packing of continuous PSDs where all particle sizes are represented in the distribution. Derived from dimensional analysis and geometry, the modified Andreasen model is based on a particle packing system where a similarity condition exists around particles of specific and very different sizes. Based on granular images, the modified Andreasen model suggests that the surrounding environment of packed particles is the same even when magnified to show the surrounding environment of even smaller packed particles.

Prior studies have adopted the modified Andreasen packing model, which may also be referred to as the model of Funk and Dinger (1994), when developing a UHPC mixture design (Brouwers and Radix, 2005; Yu et al., 2015; Vogt, 2010). Despite its successful use, a major drawback of the modified Andreasen packing model is that it does not directly consider the water content in the mixture, which greatly influences the porosity of a concrete mixture. The modified Andreasen packing model offers a theoretical method to determining a cumulative PSD representing all granular materials of a mixture that achieves the highest packing density. To do so, this model provides a target function for the cumulative PSD ($P_{tar}(D)$), and it is given by the following equation:

$$P_{tar}(D) = \frac{D^q - D_{min}^q}{D_{max}^q - D_{min}^q} \quad (2.3)$$

where D is the particle size of the granular material, $P_{tar}(D)$ is the cumulative percent of particles finer than size D , D_{min} is the smallest particle size, D_{max} is the largest particle size and q is the distribution modulus. The modified Andreasen packing model is derived from the original Andreasen packing model, and its equation for the targeted cumulative PSD is as follows:

$$P_{tar}(D) = \left(\frac{D}{D_{max}} \right)^q \quad (2.4)$$

where the only difference is that the smallest particle size (D_{min}) is not considered as in Eqn 2.3.

Funk and Dinger (1994) carried out a series of numerical analyses to determine the optimum value for the distribution modulus (q) that achieves the densest packing, and the results suggest that its value should not exceed 0.37. It was also found that at higher values for the distribution modulus (q), the resulting target mixture contains coarser particles and is less workable (Brouwers and Radix, 2005; Elkem, 2012; Yu et al., 2015; Vogt, 2010). For concrete or mortar mixtures with fine particles such as UHPC, the distribution modulus (q) should be less than or equal to 0.25 according to multiple prior studies (Brouwers and Radix, 2005; Elkem, 2012; Yu et al., 2015; Vogt, 2010). Yu et al. (2015) applied a distribution modulus (q) of 0.23 when designing a typical UHPC mixture with the modified Andreasen model, and the resulting mixture achieved self-compacting rheological behaviour along with a relatively high compressive strength.

The modified Andreasen model requires the calculation of a target PSD using Eqn. 2.3 along with the calculation of the cumulative PSD for a defined concrete mixture. To achieve the densest packing for the defined concrete mixture, the resulting PSD curves must be aligned. To calculate a mixture's cumulative PSD, the particle density (ρ) and amount used per one cubic metre (N) for each constituent material should be known to determine the absolute volume (V) of each component of the mixture using the following equation:

$$V = N/\rho \quad (2.5)$$

where the absolute volume for each component (V) should then be used to calculate the volume fraction of each component (X) using the following equation:

$$X = V/V_{Total} \quad (2.6)$$

where V_{Total} is the summation of the absolute volumes for all granular materials of the mixture. The cumulative PSD of a mixture ($P_{mix}(D)$) is ultimately calculated as follows:

$$P_{mix}(D) = \sum X_i \cdot P_i(D) \quad (2.7)$$

where at each size class, the cumulative PSD considers the contribution of all granular materials in the mixture. To quantify the difference between the cumulative PSD of the mixture with the target PSD, the following set of statistical equations may be used:

$$RSS = (P_{mix}(D) - P_{tar}(D))^2 \quad (2.8)$$

$$TSS = (P_{mix}(D) - \bar{P}_{mix}(D))^2 \quad (2.9)$$

$$R^2 = 1 - \frac{RSS}{TSS} \quad (2.10)$$

where RSS is the residual sum of squares, TSS is total sum of squares, $\bar{P}_{mix}(D)$ is the mean of the cumulative PSD of the mixture and R^2 is the coefficient of determination. The coefficient of determination (R^2) is given as a percentage, and a value of 100% indicates that the two curves are perfectly aligned. Figure 2.5 shows the application of the modified Andreasen model for a UHPC mixture prepared by Yu et al. (2015), which is referred to as mixture no. 3. Figure 2.5 shows the PSDs of all the dry granular material, which includes cement, silica fume, fly ash and two types of aggregate along with the target and cumulative PSD curves. The target and cumulative PSD curves are relatively close to one another, and the calculated coefficient of determination (R^2) was equal to about 98.0%, which suggests a high packing density as it is relatively close to 100%.

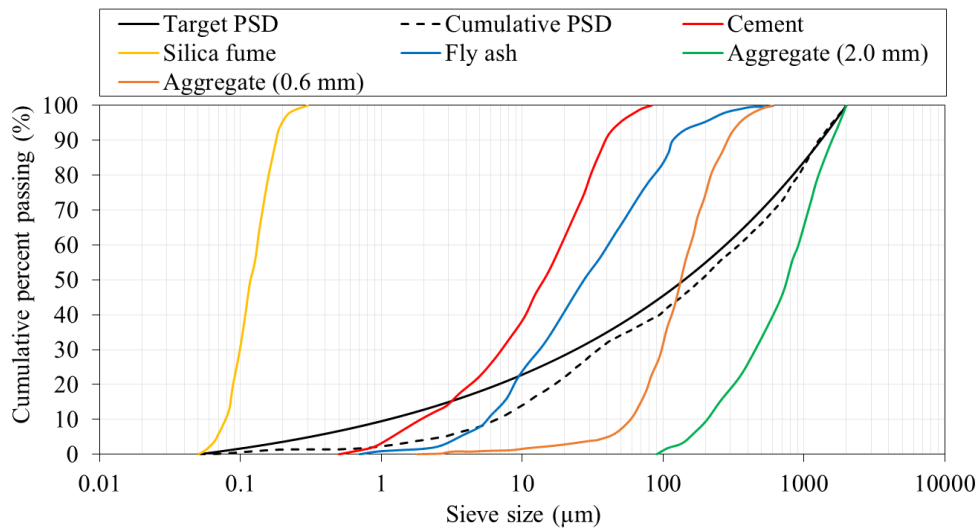


Figure 2.5 – Target and cumulative PSD of the modified Andreasen model for mixture no. 3 by Yu et al. (2015)

2.2.5 CHARACTERISATION OF MATERIAL PROPERTIES

Guidance is provided by both NF P18-470 and SIA 2052 for characterising the key rheological and mechanical properties of fresh and hardened UHPC such as workability, compressive strength, elastic modulus and direct tensile strength amongst others (AFNOR, 2016a; EPFL, 2016). NF P18-470 stipulates that the workability of fresh UHPC can be measured with either a flow table test in accordance with BS EN 12350-5:2009 or an adaptation of ASTM C230/C230M as well as a slump flow test based on BS EN 12350-8:2010 (AFNOR, 2016a; ASTM, 2014; BSI, 2009a, 2010). NF P18-470 provides consistence classes for UHPC based on the measured flow and fresh state condition for each of the previously mentioned tests as shown in Table 2.1 (AFNOR, 2016a). Classes Ca and Cv can generally be cast without the use of any vibration (i.e., self-compacting), while class Ct refers to UHPC that may be thixotropic in nature and more difficult to cast. Much like standard concrete, the compressive strength and elastic modulus of UHPC can be obtained using BS EN 12390-3:2009 and BS EN 12390-13:2013, respectively, according to NF P18-470 (AFNOR, 2016a; BSI, 2011, 2013b). NF P18-470 also stipulates that the smallest dimension of the compressive strength test specimens, which may either be cubes or cylinders, should be at least five times the length of the longest fibre or six times the maximum aggregate size (AFNOR, 2016a).

Table 2.1 – Consistence classes for fresh UHPC according to NF P18-470 (AFNOR, 2016a)

Class	Condition of fresh state	Flow table test (BS EN 12350-5) ^A [mm]	Flow table test (ASTM C230/C230M) ^B [mm]	Slump flow test (BS EN 12350-8) ^C [mm]
Ca	Self-compacting	≥ 560 (No shock)	≥ 270	≥ 760
Cv	Viscous	420 – 560 (No shock) > 560 (15 shocks)	230 – 270	660 – 760
Ct	Exhibiting a flow threshold	< 420 (No shock) > 560 (15 shocks)	< 230	< 660

^A BS EN 12350-5 (BSI, 2009a); ^B ASTM C230/C230M (ASTM, 2014); ^C BS EN 12350-8 (BSI, 2010)

Characterising the direct tensile strength of UHPC requires additional care due to its unique behaviour in tension caused by the steel fibre reinforcement. The tensile response of UHPC can be divided into elastic and plastic, or post-cracking, phases prior to failure. Micro-cracking follows the elastic phase where either a strain-softening or strain-hardening response may occur. The micro-cracks occur due to the fibre reinforcement bridging the cracks, with failure typically occurring after a single crack begins to localize and expand causing the fibre reinforcement to be pulled out. Direct tensile tests have been used by researchers to measure the tensile strength of UHPC, however indirect tensile tests are often more preferred due to certain complexities surrounding direct tensile test methods (Benson and Karihaloo, 2005). NF P18-470 and SIA 2052 give guidance for measuring the elastic and post-cracking tensile strength indirectly through bending tests, while SIA 2052 gives additional guidance for a direct test method (AFNOR, 2016a; EPFL, 2016).

Direct tensile test methods generally involve the tensile pulling of small-scale cylinder, prism or dumbbell-shaped specimens on their constrained ends (Benson and Karihaloo, 2005; Denarié et al., 2017; Graybeal and Baby, 2013; Maltais et al., 2018; Shen and Brühwiler, 2018; EPFL, 2016; Wille et al., 2014; Zhou and Qiao, 2018). Prior studies suggest that direct tensile tests are sensitive to specimen shape and loading arrangement, which often result in premature failure near the constrained ends due to unwanted stress concentration (Benson and Karihaloo, 2005). Common issues also include misalignment of the specimen before or during the test, specimen defects and ineffective monitoring locations (Benson and Karihaloo, 2005). Specimen ends are typically fixed or rotating, and specimens may include a notch (Wille et al., 2014). It has been reported that fixed ends with no rotation may initiate eccentric loading that may cause bending (Benson and Karihaloo, 2005). Ends with allowed rotation may reduce this

affect, although the resulting tensile strength may be lower (Benson and Karihaloo, 2005). Dumbbell-shaped specimens are more commonly used and preferred as they provide a geometry with a uniform tensile stress field that is unaffected by loading arrangement (Maltais et al., 2018; Shen and Brühwiler, 2018; Wille et al., 2014; Zhou and Qiao, 2018). Appendix D of SIA 2052 provides guidance for a direct tensile test method using dumbbell-shaped specimens, and it includes guidance on specimen dimensions and preparation along with loading test arrangement and procedures (EPFL, 2016).

Measuring the direct tensile strength of UHPC indirectly involves performing bending tests on small-scale prism specimens. Prior studies have performed three- or four-point bending tests on prisms, with some prisms including a notch (Baby et al., 2012; Denarié et al., 2017; Martínez, 2017). Obtaining the equivalent direct tensile strength through bending tests requires the application of an inverse analysis method on the bending test results to determine the direct tensile stress-crack opening behaviour. Annex D of NF P18-470 provides guidance on specimen dimensions and preparation along with loading test arrangement and procedures for thick UHPC members, which are those with thicknesses greater than three times the length of the longest fibre (AFNOR, 2016a). Annex E of NF P18-470 offers alternative guidance for thin UHPC members, which are those with thicknesses less than three times the length of the longest fibre (AFNOR, 2016a). Appendix E of SIA 2052 also provides a similar set of guidelines to Annex D of NF P18-470 for bending tests that are applicable to all types of UHPC members (EPFL, 2016). Both NF P18-470 and SIA 2052 provide guidance for the inverse analysis method that should be applied to the bending test results in order to obtain the equivalent direct tensile strength (AFNOR, 2016a; EPFL, 2016).

Annex D of NF P18-470 provides a comprehensive set of procedures for determining the direct tensile strength of UHPC through bending tests, and similar procedures are commonly used among researchers (AFNOR, 2016a; Baby et al., 2012; Denarié et al., 2017; Martínez, 2017). It stipulates that the tensile strength at the elastic limit should be measured through four-point bending tests on unnotched prisms, while the post-cracking response should be measured through three-point bending tests on notched prisms (AFNOR, 2016a). The prisms should be cast from one end of the mould only (AFNOR, 2016a). The loading arrangements of the four- and three-point bending tests are shown in Figure 2.6. As the specimens are loaded until failure, the force-deflection response should be measured in the four-point bending test, and the force-crack opening response at the notch, or load-crack mouth opening displacement (CMOD),

should be measured in the three-point bending tests. The prisms used for both tests consist of a square cross-section ($a \times a$), where the dimension (a) should be between 70 mm and 200 mm and at least five to seven times the length of the longest fibre (AFNOR, 2016a). For the notched prism used in the three-point bending test, the depth of the notch should be half of the length of the longest fibre with a width of 3 mm (AFNOR, 2016a).

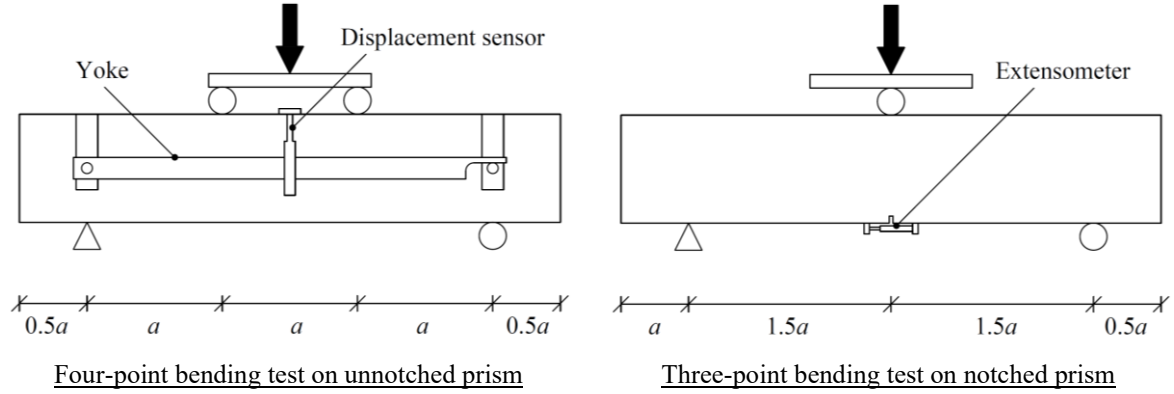


Figure 2.6 – Methods for characterising UHPC tensile strength according to NF P18-470 (AFNOR, 2016a)

The results of the four- and three-point bending tests on prisms enable the determination of the direct elastic and post-cracking responses. From the results of the four-point bending tests on unnotched prisms, which is the force-deflection response, the direct tensile strength at the elastic limit ($f_{ct,el}$) can be calculated. To do so, the limit of elasticity in bending ($f_{ct,fl}$) is required, and it is determined using the following equation:

$$f_{ct,fl} = 3F_{nl}/b \cdot a \quad (2.11)$$

where F_{nl} refers to the force that corresponds to the loss of linearity, a refers to height of the prism and b refers to the width of the prism. Annex D of NF P18-470 provides an empirical formula that estimates the direct tensile strength at the elastic limit ($f_{ct,el}$), and it is the following:

$$f_{ct,el} = f_{ct,fl} \cdot \frac{\kappa a^{0.7}}{1 + \kappa a^{0.7}} \quad (2.12)$$

where κ is a coefficient that is equal to 0.08.

An inverse analysis method is applied to results of the three-point bending test, more specifically the force-crack opening response, in order to determine the equivalent direct post-cracking stress-crack opening response. The iterative inverse analysis method given by Annex D of NF P18-470 is based on the cross-section analysis of a notched prism shown in Figure 2.7 (AFNOR, 2016a). The cross-section is split into uncracked and cracked sections.

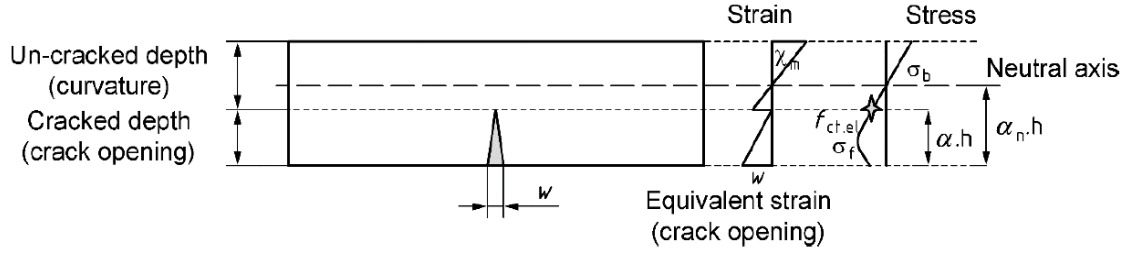


Figure 2.7 – Cross-section analysis of notched prism according to NF P18-470 (AFNOR, 2016a)

The main equations used to determine the post-cracking stress-crack opening response are the following:

$$N_b = \frac{E_{cm}\chi_m b h^2}{2} [(1 - \alpha_n)^2 - (\alpha - \alpha_n)^2] \quad (2.13)$$

$$N_f = \frac{\alpha h b}{w} \int_0^w \sigma_f dw \quad (2.14)$$

$$M_b = \frac{E_{cm}\chi_m b h^3}{3} [(1 - \alpha_n)^3 - (\alpha - \alpha_n)^3] + h \alpha_n N_b \quad (2.15)$$

$$M_f = \alpha h N_f - \frac{(\alpha h)^2 b}{w^2} \int_0^w \sigma_f w dw \quad (2.16)$$

where N_b is the normal force of the uncracked section, N_f is the normal force of the cracked section, M_b is the flexural resistance of the uncracked section, M_f is the flexural resistance of the cracked section, E_{cm} is the elastic modulus, χ_m is the curvature of the uncracked portion, b is the prism width, h is the prism height after the deduction of the notch depth, α is the relative depth of the crack, α_n is the relative depth of the neutral axis, w is the crack width and

σ_f is the post-cracking stress. According to the following relationship, the curvature (χ_m) and crack width (w) are linked:

$$\chi_m = \frac{3w}{2(\alpha h)^2} - 2\chi_e \quad (2.17)$$

where χ_e is the equivalent elastic curvature, and it is determined using the following equation:

$$\chi_e = \frac{M}{E_{cm}I} \quad (2.18)$$

where M is the measured moment and I is the moment of inertia of the prism. In addition, the relative depths of the neutral axis (α_n) and crack (α) are linked using the following relationship:

$$\alpha_n = \frac{f_{ctm,el}}{h\chi_m E_{cm}} + \alpha \quad (2.19)$$

If the extensometer that is measuring the crack width is offset by a distance (e) from the base of the notch, then the measured crack width should be corrected using the following equation:

$$w = w_{mes}[ah/(\alpha h + e)] \quad (2.20)$$

where w_{mes} is the measured crack width by the extensometer. Eqn. 2.20 is only applied when the crack opening is measured with an extensometer that is placed below the prism (AFNOR, 2016a).

The post-cracking tensile response is determined iteratively at each coordinate (w_i, σ_{f_i}). The curve should be discretized along the x-axis related to the crack opening at 20 μm steps. At the initial iteration ($i = 1$), the normal forces of the uncracked (N_b) and cracked (N_f) sections as well as the flexural resistance of the cracked section (M_f) are all equal to zero. Additionally, the flexural resistance of the uncracked section at the initial iteration ($i = 1$) is equal to the following:

$$M_b = \frac{-bh^2\sigma_f}{6} \quad (2.21)$$

The first coordinate of the tensile stress-crack opening curve can then be plotted. To determine the coordinates of the following iterations ($i + 1$), the stresses are calculated using the trapezoidal rule according to the following equation:

$$\int_0^{w_{i+1}} \sigma_f dw = \int_0^{w_i} \sigma_f dw + \left(\frac{\sigma_{f_i} + \sigma_{f_{i+1}}}{2}\right)(w_{i+1} - w_i) \quad (2.22)$$

The normal force of the cracked section is calculated using the following equation:

$$N_{f_{i+1}} = N_{f_i} \cdot \frac{\alpha_{i+1}}{\alpha_i} \cdot \frac{w_i}{w_{i+1}} + \alpha_{i+1} \cdot b \cdot h \cdot \left(\frac{\sigma_{f_i} + \sigma_{f_{i+1}}}{2}\right) \left(1 - \frac{w_i}{w_{i+1}}\right) \quad (2.23)$$

and the flexural resistance of the cracked section is calculated using the following equation:

$$M_{f_{i+1}} = M_{f_i} \cdot \left(\frac{\alpha_{i+1}}{\alpha_i} \cdot \frac{w_i}{w_{i+1}}\right)^2 + \alpha_{i+1} \cdot h \cdot N_{f_{i+1}} \cdot \left(1 - \frac{w_i}{w_{i+1}}\right) - \frac{(\alpha_{i+1} \cdot h)^2 \cdot b \cdot \sigma_{f_{i+1}}}{2} \cdot \left(1 - \frac{w_i}{w_{i+1}}\right)^2 \quad (2.24)$$

Using the set of previously mentioned equations, the two unknown variables, which are the relative depth of the crack (α_{i+1}) and the post-cracking stress ($\sigma_{f_{i+1}}$), can then be determined using the following equations:

$$N_{b_{i+1}} + N_{f_{i+1}} = 0 \quad (2.25)$$

$$M_{b_{i+1}} + M_{f_{i+1}} - M = 0 \quad (2.26)$$

where the normal force ($N_{b_{i+1}}$) and flexural resistance ($M_{b_{i+1}}$) of the uncracked section can be calculated using Eqns. 2.13 and 2.15, respectively. The resulting post-cracking stress-crack opening response may contain spikes, so NF P18-470 suggests applying a moving average at

the end of each iteration to smoothen the response curve (AFNOR, 2016a). The moving average equation is as follows:

$$\sigma_{f_i} = (2\sigma_{f_i} + \sigma_{f_{i+1}})/3 \quad (2.27)$$

The post-cracking tensile strength (f_{ct}) can be determined using the post-cracking stress-crack opening response, and it is dependent on whether a local peak is observed as shown in Figure 2.8. If the post-cracking stress-crack opening curve has a local peak, then the post-cracking tensile strength (f_{ct}) corresponds to the peak stress value (AFNOR, 2016b). If the post-cracking stress-crack opening response has no local peak, then the post-cracking tensile strength (f_{ct}) corresponds to the stress at a crack opening of 0.3 mm (AFNOR, 2016b).

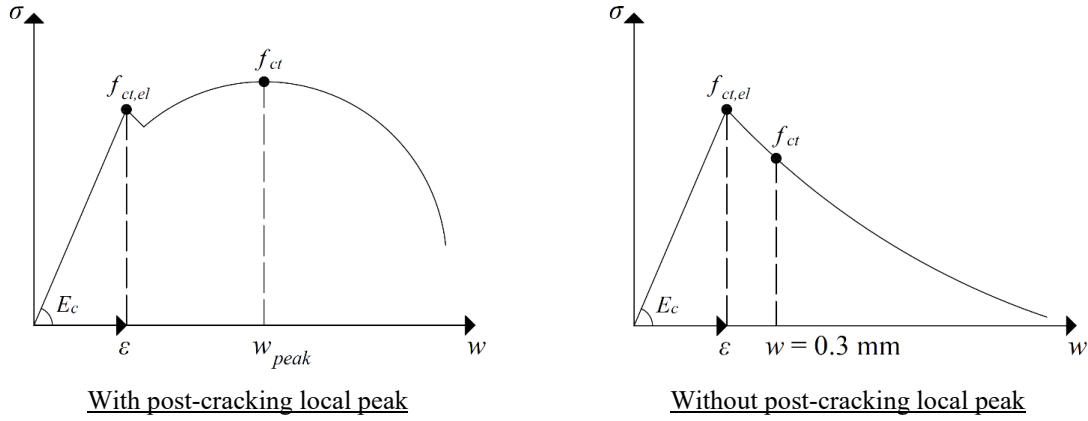


Figure 2.8 – Determination of post-cracking tensile strength according to NF P18-710 (AFNOR, 2016b)

2.3 SHEAR RESISTANCE MECHANISMS

2.3.1 INTRODUCTION

The complex behaviour of reinforced concrete beams failing in shear has been studied extensively over the past several decades. Although there is still no universally agreed theory describing shear behaviour, researchers agree that up to six mechanisms contribute to shear resistance in reinforced concrete beams (ASCE-ACI Committee 445, 1998). The main shear transfer mechanisms, or actions, in reinforced concrete beams with transverse reinforcement are shown in Figure 2.9, and comprise aggregate interlock, dowel action, stirrups and the uncracked compression zone. For beams with steel fibres, the shear force along the crack interface is resisted by the fibre reinforcement in addition to the previously listed mechanisms

(Matos et al., 2020). In UHPC beams, the shear resisted by aggregate interlock is neglected due to the absence of coarse aggregate. The contribution of each mechanism to total shear resistance depends on the crack kinematics and shape of the critical shear crack as well as its location, and the contributions of each mechanism may significantly vary between similar or identical beams (Campana et al., 2013; Huber et al., 2016; Pastore, 2020). This section describes each of the previously mentioned shear resistance mechanisms and introduces theoretical models from the literature that can be used to quantify the contribution of each mechanism. Other resistance mechanisms in reinforced concrete beams, which are not covered in this research, include the shear resisted by the residual tensile stresses in the crack tip as well as the shear resisted through arching action. The contribution to total shear resistance of the residual tensile stresses is only dominant in beams with fairly small crack openings that are between 0.05 mm and 0.15 mm, otherwise its contribution may be insignificant (ASCE-ACI Committee 445, 1998). The contribution to total shear resistance of arching action is dominant in non-slender beams ($a_v/d < 2.5$), and it is less dominant in slender beams ($a_v/d \geq 2.5$), which are the focus of this research as discussed later in Chapter 3 (Kani, 1964; Kani et al., 1979).

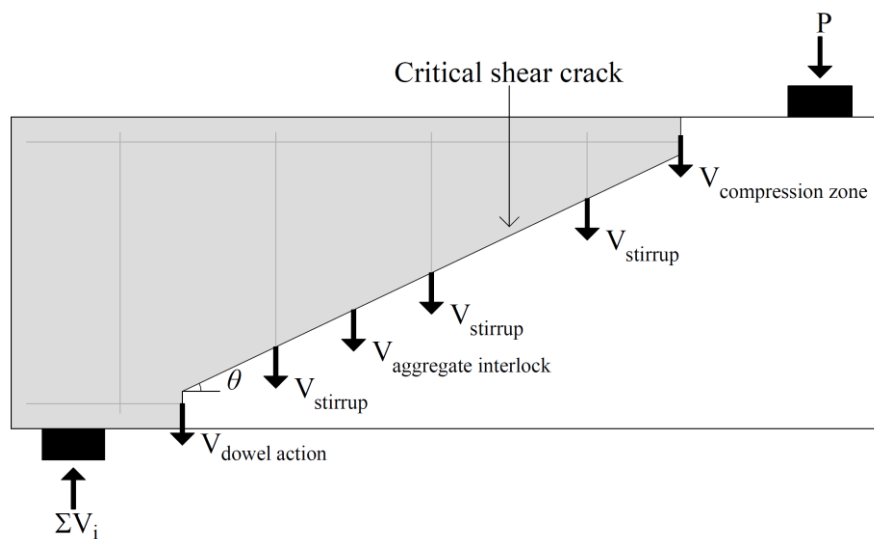


Figure 2.9 – Shear resistance mechanisms in reinforced concrete beams with transverse reinforcement

2.3.2 AGGREGATE INTERLOCK

Interface shear stresses are transferred when two cracked surfaces slide across one another, which results in the shear resistance mechanism known as aggregate interlock (ASCE-ACI Committee 445, 1998). Cracking in standard concrete typically occurs in the cement matrix around the circumference of the embedded coarse aggregate seeing as the interface bond zone

is the weakest link in the system. Normal and shear stresses are transferred when the aggregate of one cracked surface interlocks into the matrix of the other cracked surface. Extensive experimental work has led to the development of several analytical and empirical models that are aimed at estimating the shear force transferred through aggregate interlock. Proposed theoretical models considered various influential parameters that affect aggregate interlock such as concrete compressive strength, maximum aggregate size and crack kinematics. The general equation for estimating the shear force contribution (V_{agg}) of aggregate interlock in beams is the following:

$$V_{agg} = b_w \cdot \sum l_{cr,i} \cdot (\tau \cdot \sin \theta_{cr,i} - \sigma \cdot \cos \theta_{cr,i}) \quad (2.28)$$

where b_w is the width of the beam's web, $l_{cr,i}$ is the length of crack i , $\theta_{cr,i}$ is the angle of crack i , τ is the shear stress and σ is the normal stress.

Walraven (1980) idealised aggregate in concrete as rigid spherical objects embedded within a rigid plastic cement matrix based on the results of a series of push-off tests. Normal and shear stresses are transferred at the interface contact area where the aggregate crushes and slides into the cement matrix as shown in Figure 2.10. The stress conditions on the contact area are also shown in Figure 2.10. Based on the crack kinematics observed from push-off tests, Walraven (1980) assumed that a crack initially opens normal to the crack plane (w) and then slides tangential to the crack plane (Δ). Walraven (1980) proposed that the normal (σ) and shear (τ) stresses are based on the projected contact areas (A_x and A_y), the coefficient of friction between the aggregate and cement matrix (μ), yield strength of the cement matrix (σ_{pu}), maximum aggregate size (D_{max}) and the ratio of aggregate to total concrete volume (p_k).

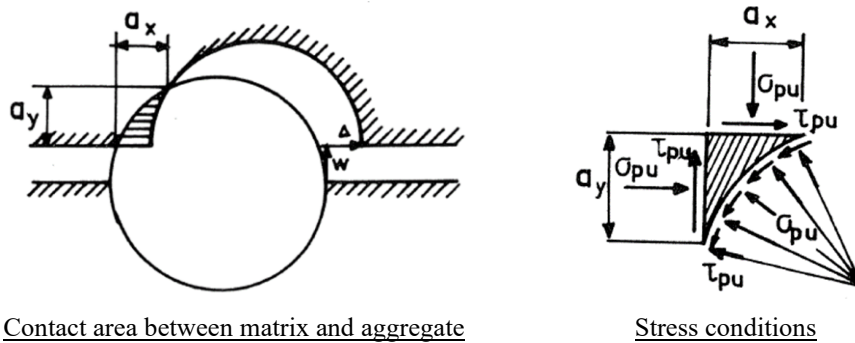


Figure 2.10 – Aggregate interlock model of Walraven (1980). Images reproduced with permission of the rights holder, Prof. Joost Walraven.

Expanding on the Two-Phase model of Walraven (1980), Ulaga (2003) and Guidotti (2010) developed their own respective theoretical models based on observed crack kinematics. As previously mentioned, the model of Walraven (1980) assumes that a crack initially opens and then slides as shown in Figures 2.11(a) and 2.11(b). The model of Ulaga (2003) assumes that crack opening and sliding (δ) occur simultaneously at the onset of cracking at an inclination angle (γ) as shown in Figures 2.11(a) and 2.11(c). Guidotti (2010) developed a model that assumes a crack undergoes an initial crack opening (u_0) due to flexure followed by simultaneous crack opening and sliding (δ) at an inclination angle (γ) as shown in Figures 2.11(a) and 2.11(d). Figure 2.11(a) shows the various aggregate interlock models in terms of the crack opening (u) and sliding (v). Based on Figure 2.11(a), the combined opening and sliding displacement (δ) as well as the inclination angle (γ) are calculated using the following equations:

$$\delta = \sqrt{(u - u_0)^2 + v^2} \quad (2.29)$$

$$\gamma = \tan\left(\frac{u - u_0}{v}\right) \quad (2.30)$$

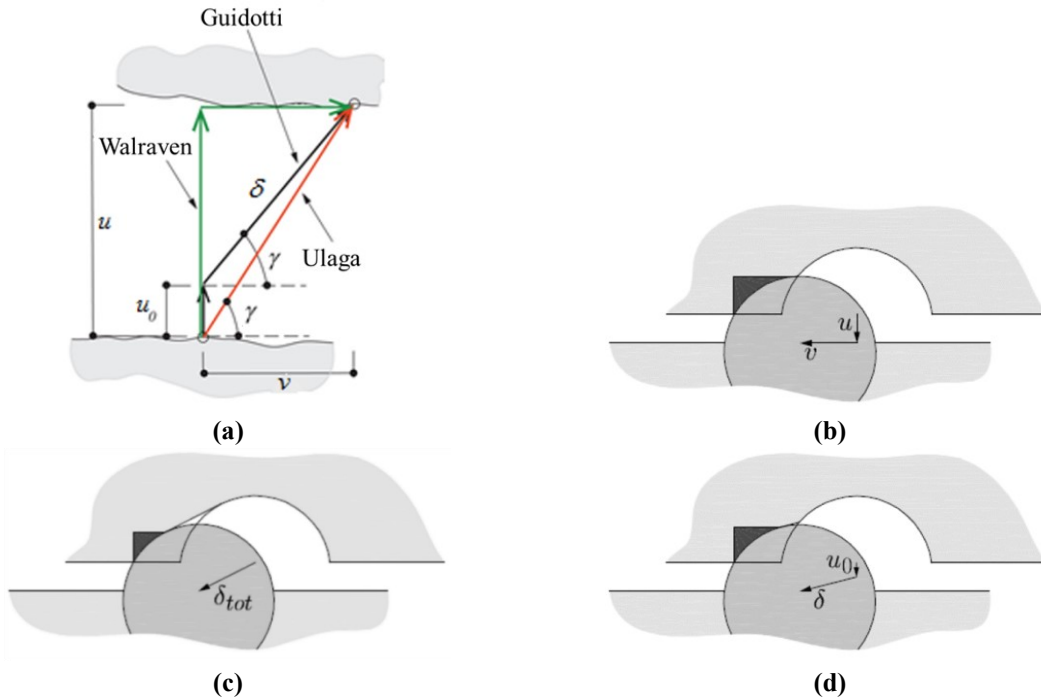


Figure 2.11 – Representation of aggregate interlock models: **(a)** Various models (Cavagnis et al., 2018), **(b)** Walraven (1980), **(c)** Ulaga (2003), **(d)** Guidotti (2010). **(b-d)** Guidotti (2010). Images reproduced with permission of the rights holders, John Wiley and Sons **(a)** and Dr. Robert Guidotti **(b-d)**.

The assumed kinematics of Walraven (1980) and Ulaga (2003) are specific cases of the comprehensive model of Guidotti (2010) (Pastore, 2020). When the crack displacement equals the initial crack opening displacement ($u = u_0$) and the inclination angle is equal to zero ($\gamma = 0$), then the set of equations proposed by Guidotti (2010) are equivalent to those of Walraven (1980). When the initial crack opening displacement is equal to zero ($u_0 = 0$), then the equations of Guidotti (2010) are equivalent to those of Ulaga (2003). The set of equations provided by Guidotti (2010) are therefore given in this section seeing as they are also applicable to the models of Walraven (1980) and Ulaga (2003). The normal (σ) and shear (τ) stresses resisted by aggregate interlock are calculated using the following equations:

$$\sigma = \sigma_{pu}(A_x - \mu \cdot A_y) \quad (2.31)$$

$$\tau = \sigma_{pu}(A_y + \mu \cdot A_x) \quad (2.32)$$

where the coefficient of friction (μ) equals 0.4. The yield strength of the cement matrix (σ_{pu}) is calculated as follows:

$$\sigma_{pu} = 6.39f_{cu}^{0.56} \quad (2.33)$$

where f_{cu} refers to the compressive cube strength of concrete. When the combined opening and sliding displacement is less than or equal to the initial opening displacement ($\delta \leq u_0$), the contact areas (A_x and A_y) are calculated using the following equations:

$$A_x = \int_{D_1}^{D_{max}} p_k \cdot \frac{4}{\pi} \cdot F\left(\frac{D}{D_{max}}\right) \cdot G_1(u, v, D) \cdot dD \quad (2.34)$$

$$A_y = \int_{D_1}^{D_{max}} p_k \cdot \frac{4}{\pi} \cdot F\left(\frac{D}{D_{max}}\right) \cdot G_2(u, v, D) \cdot dD \quad (2.35)$$

where the ratio of aggregate to concrete volume (p_k) is taken as 0.75 (Walraven, 1980). When the combined opening and sliding displacement is greater than the initial opening displacement ($\delta > u_0$), then the contact areas (A_x and A_y) are calculated using the following equations:

$$A_x = \int_{D_3}^{D_2} p_k \cdot \frac{4}{\pi} \cdot F\left(\frac{D}{D_{max}}\right) \cdot G_3(u, v, D) \cdot dD + \int_{D_2}^{D_{max}} p_k \cdot \frac{4}{\pi} \cdot F\left(\frac{D}{D_{max}}\right) \cdot G_1(u, v, D) \cdot dD \quad (2.36)$$

$$A_y = \int_{D_3}^{D_2} p_k \cdot \frac{4}{\pi} \cdot F\left(\frac{D}{D_{max}}\right) \cdot G_4(u, v, D) \cdot dD + \int_{D_2}^{D_{max}} p_k \cdot \frac{4}{\pi} \cdot F\left(\frac{D}{D_{max}}\right) \cdot G_2(u, v, D) \cdot dD \quad (2.37)$$

The following set of equations are used to calculate the variables required to determine the contact areas (A_x and A_y) according to Guidotti (2010):

$$D_1 = \frac{u^2 + v^2}{v} \leq D_{max} \quad (2.38)$$

$$D_2 = \frac{u^2 + v^2}{u \cdot \cos \gamma - v \cdot \sin \gamma} \leq D_{max} \quad (2.39)$$

$$D_3 = 2 \cdot \frac{v \cdot \tan \gamma + u_0}{\cos \gamma} \leq D_{max} \quad (2.40)$$

$$G_1(u, v, D) = \frac{e_{lim}}{D^3} \cdot \left(v - u \cdot \sqrt{\frac{D^2}{u^2 + v^2} - 1} \right) + \frac{e_{lim} + u}{2 \cdot D^3} \cdot \sqrt{D^2 - 4 \cdot (u + e_{lim})^2} - \frac{u}{2 \cdot D^3} \cdot \sqrt{D^2 - 4 \cdot u^2} + \frac{1}{4 \cdot D} \cdot \left[\text{asin} \frac{2 \cdot (u + e_{lim})}{D} - \text{asin} \frac{2u}{D} \right] \quad (2.41)$$

$$G_2(u, v, D) = \frac{v \cdot e_{lim}}{D^3 \cdot \sqrt{u^2 + v^2}} \cdot \sqrt{D^2 - u^2 - v^2} - \frac{e_{lim} \cdot (u + e_{lim})}{D^3} \quad (2.42)$$

$$G_3(u, v, D) = \frac{1}{4 \cdot D} \cdot \left[\frac{\pi}{2} - \left(\gamma + \text{asin} \frac{2u}{D} \right) + \cos \gamma \sin \gamma \right] - \frac{\sin \gamma}{D^2} \cdot \left(\frac{D}{2} \cdot \cos \gamma - u \right) - \frac{u}{2 \cdot D^3} \cdot \sqrt{D^2 - 4 \cdot u^2} \quad (2.43)$$

$$G_4(u, v, D) = \frac{1}{D^3} \cdot \left(\frac{D}{2} \cdot \cos \gamma - u \right)^2 \quad (2.44)$$

$$e_{\text{lim}}(D) = \frac{1}{2} \cdot \left(v \sqrt{\frac{D^2 - u^2 - v^2}{u^2 + v^2}} - u \right) \quad (2.45)$$

$$F\left(\frac{D}{D_{\text{max}}}\right) = \sum_{i=1}^{\infty} \alpha_i \left(\frac{D}{D_{\text{max}}}\right)^{\beta_i} \quad (2.46)$$

where the constants α_i and β_i are given in Table 2.2 according to Guidotti (2010).

Table 2.2 – Constants for aggregate interlock model of Guidotti (2010)

Constant	Iteration (<i>i</i>)								
	1	2	3	4	5	6	7	8	9
α_i	0.53916	-0.21429	-0.06818	-0.03750	-0.02467	-0.01783	-0.01367	-0.01092	-0.00337
β_i	0.5	4	6	8	10	12	14	16	18

To account for possible aggregate fracture, a reduction factor can be applied as proposed by Walraven and Stroband (1994), Yang (2014) or Yang et al. (2017). The reduction factors that were proposed depend on the concrete's compressive strength and aggregate strength, while Yang (2014) and Yang et al. (2017) also considered the crack roughness index. Aggregate fracture is considered by multiplying the determined normal and shear stresses by the proposed factors. The reduction factor (C_f) proposed by Walraven and Stroband (1994) equals 0.35 for high strength concrete or concretes with weak aggregates, otherwise it equals 1.0. The reduction factor (R_{ai}) proposed by Yang (2014) and Yang et al. (2017) is determined using the following equation:

$$R_{ai} = \begin{cases} 0.75 & \text{for lightweight concrete} \\ 0.85 \cdot \sqrt{\left(\frac{7.2}{f_c - 40}\right) - 1} + 0.34 & \text{for } f_c > 65 \text{ MPa} \\ 1 & \text{for } f_c < 65 \text{ MPa} \end{cases} \quad (2.47)$$

Bažant and Gambarova (1980) originally developed the Rough Crack model, which was based on a comparative analysis between semi-empirical results and the push-off test results of Paulay and Loeber (1974). The Rough Crack model idealises the crack surface as a regular array of trapezoids as shown in Figure 2.12. Gambarova and Karakoc (1983) expanded on the Rough Crack model by comparing theoretical results with the push-off test results of Daschner and Kupfer (1982) where confinement was kept constant. This development provided an improved formulation that considers the relationship between the confinement stress and crack displacement (Gambarova and Karakoc, 1983). The following set of equations are used to determine the normal (σ) and shear (τ) stresses attributed to aggregate interlock according to Gambarova and Karakoc (1983):

$$\sigma = -0.62 \cdot \sqrt{w} \cdot \frac{r}{(1 + r^2)^{0.25}} \cdot \tau_{ag} \quad (2.48)$$

$$\tau = \tau_0 \cdot \left(1 - \sqrt{\frac{2 \cdot w}{d_{max}}}\right) \cdot r \cdot \frac{a_3 + a_4 \cdot |r|^3}{1 + a_4 \cdot r^4} \quad (2.49)$$

$$r = s/w \quad (2.50)$$

$$\tau_0 = 0.25 \cdot f_c \quad (2.51)$$

$$a_3 = 2.45/\tau_0 \quad (2.52)$$

$$a_4 = 2.44 \cdot (1 - 4/\tau_0) \quad (2.53)$$

where r is the crack sliding (s) to opening (w) ratio, τ_0 is the shear strength of the crack when the crack opening displacement is equal to zero ($w = 0$ mm), a_3 is a coefficient, a_4 is a coefficient and d_{max} is the maximum aggregate size.

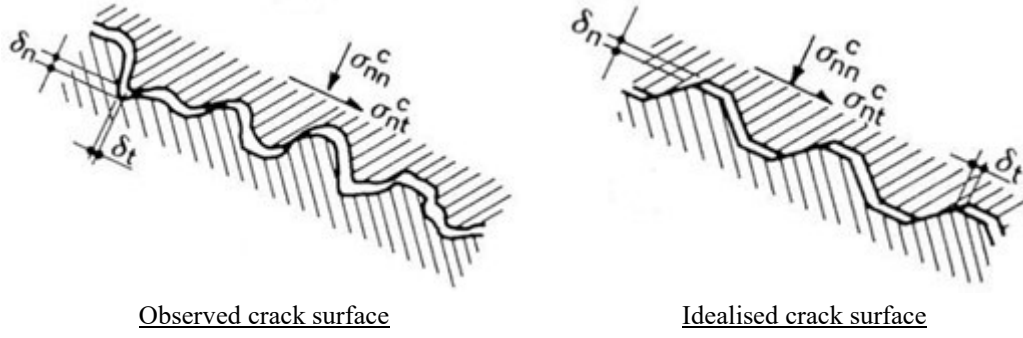


Figure 2.12 – Representation of Rough Crack model (Bažant and Gambarova, 1980). Images reproduced with permission of the rights holder, American Society of Civil Engineers (ASCE).

Li et al. (1989) developed the Contact Density model that idealizes the crack surface as infinitely small contact units with local angles as shown in Figure 2.13. The area of the contact unit (dA_θ) is represented by a contact density function ($\Omega(\theta)$), with an inclination angle (θ) that varies between -90° to 90° . Li et al. (1989) assumed that the stress state at the contact area is elasto-plastic as shown in Figure 2.13, and the aggregate interlock stresses (σ and τ) are therefore obtained through numerical integration using the following set of equations:

$$\sigma = \int_{-\pi/2}^{\pi/2} \sigma'_{con} K(\omega) A_t \Omega(\theta) \cos \theta d\theta \quad (2.54)$$

$$\tau = \int_{-\pi/2}^{\pi/2} \sigma'_{con} K(\omega) A_t \Omega(\theta) \sin \theta d\theta \quad (2.55)$$

$$\sigma'_{con} = \begin{cases} R_s \cdot (\omega'_\theta - \omega'_{\theta_p}) & \text{for } \omega'_\theta \geq \omega'_{\theta_p} \\ 0 & \text{for } \omega'_\theta < \omega'_{\theta_p} \end{cases} \quad (2.56)$$

$$\omega'_{\theta_p} = \begin{cases} \omega'_{max} - \omega'_{lim} & \text{for } \omega'_{max} > \omega'_{lim} \\ 0 & \text{for } \omega'_{max} \leq \omega'_{lim} \end{cases} \quad (2.57)$$

$$K(\omega) = 1 - \exp\left(1 - \frac{0.5 \cdot d_{max}}{\omega}\right) \geq 0 \quad (2.58)$$

$$A_t = \left(\int \Omega(\theta) \cos \theta d\theta \right)^{-1} = \frac{4}{\pi} = 1.27 \quad (2.59)$$

$$\Omega(\theta) = 0.5 \cdot \cos \theta \quad (2.60)$$

where σ'_{con} is the contact compressive stress, R_s is the elastic rigidity per length, ω'_θ is the relative displacement, ω'_{θ_p} is the local displacement, ω'_{max} is the maximum compressive displacement in a direction θ that is taken as a positive value, ω'_{lim} is the elastic limit, A_t represents the total surface area and $K(\omega)$ is the effective ratio of the contact area. Li et al. (1989) offers a simplified approach to the Contact Density model where a rigid-plastic behaviour is assumed for the compressive contact stress (σ'_{con}) as shown in Figure 2.13, and the effective ratio ($K(\omega)$) equals 1.0 assuming monotonic loading conditions. Due to this simplification, the following set of equations are used to determine the normal (σ) and shear (τ) stresses due to aggregate interlock:

$$\sigma = m \left[\frac{\pi}{2} - \text{acot} \psi - \frac{\psi}{1 + \psi^2} \right] \quad (2.61)$$

$$\tau = m \cdot \frac{\psi^2}{1 + \psi^2} \quad (2.62)$$

$$\psi = \delta / \omega \quad (2.63)$$

$$m = 3.83 \cdot f'_c{}^{1/3} \quad (2.64)$$

where ψ is the crack sliding (δ) to opening (ω) ratio and m is the crack displacement coefficient.

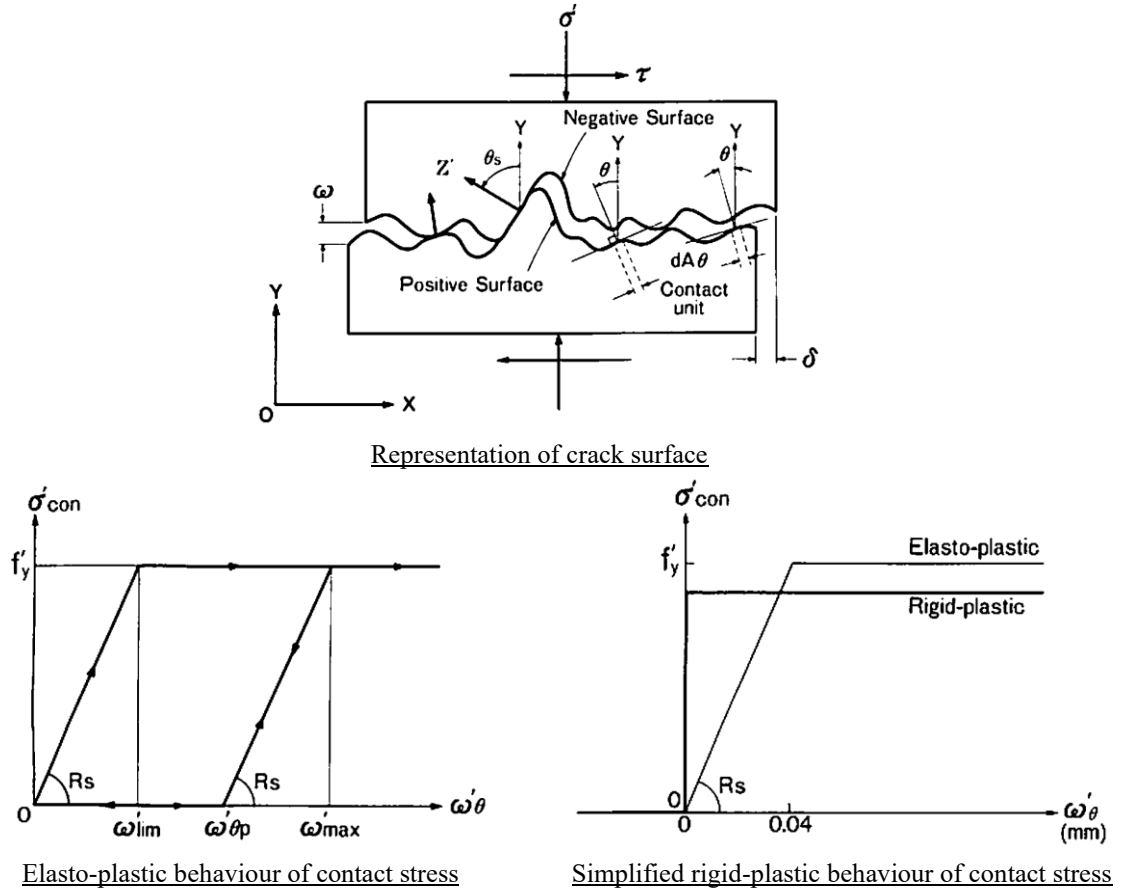


Figure 2.13 – Representation of Contact Density model of Li et al. (1989). Images reproduced with permission of the rights holder, Dr. Koichi Maekawa.

2.3.3 FIBRE BRIDGING

When the critical shear crack opens in a fibre reinforced concrete (FRC) member, the steel fibres bridge the crack between the two cracked surfaces and limit crack displacement until the crack eventually widens to a point where the fibres begin to pull-out from the concrete matrix. The opening of a crack in a FRC member is described in Figure 2.14, where the crack displacement (w) equals the following:

$$w = \sqrt{w_n^2 + w_t^2} \quad (2.65)$$

where w_n is the crack opening displacement, w_t is the crack sliding displacement and l_{bf} refers to the fibre embedment depth. The adhesion and friction of the fibres in relation to the surrounding concrete body act as shear resisting mechanisms during fibre bridging. Mechanical anchorage of the fibres may also be a contributing resisting mechanism, however this only

occurs with deformed types of fibres such as hooked end (Matos et al., 2020). Over the last couple of decades, several theoretical models have been developed that can quantify the shear resistance contribution of fibre reinforcement. The theoretical models are based on fibre pull-out behaviour, which is an influential characteristic of the total tensile behaviour of FRC. The general equation for estimating the shear force contribution (V_{fib}) of fibre reinforcement in beams is the following:

$$V_{fib} = b_w \cdot \sum l_{cr,i} \cdot \sigma \cdot \cos \theta_{cr,i} \quad (2.66)$$

where b_w is the width of the beam's web, $l_{cr,i}$ is the length of crack i , $\theta_{cr,i}$ is the angle of crack i and σ is the normal stress.

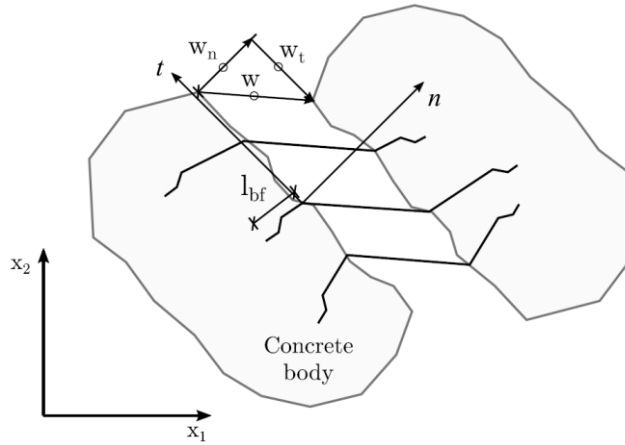


Figure 2.14 – Representation of crack opening in fibre reinforced concrete (Matos et al., 2020). Image reproduced with permission of the rights holder, Elsevier.

Pfyl (2003) proposed a theoretical model based on several key assumptions. The model of Pfyl (2003), or the Engagement model, assumes that fibres are uniformly distributed in the concrete matrix, and they are randomly oriented. The same loading direction is assumed for all of the fibres while the bond stress along the embedment length of the fibres is constant. Pfyl (2003) does not consider the flexural stiffness of the fibres nor any deformations in the surrounding concrete matrix. Pfyl (2003) proposed a set of two equations for determining the tensile stress resisted by the fibres, where the initial equation is attributed to fibre activation ($0 \leq w \leq w_{c0}$) while the other is attributed to the onset of fibre pull-out ($w_{c0} < w \leq l_f/2$). The onset of fibre pull-out is based on the crack displacement where the fibre with the longest embedment depth begins to pull-out (w_{c0}). When the crack displacement (w) is greater than half of the fibre length ($l_f/2$), then the fibres are no longer capable of resisting any tensile stress. The tensile stress

resisted by the fibres (σ_{cf}) according to Pfyl (2003) is calculated using the following set of equations:

$$\sigma_{cf} = \begin{cases} \left(2 \cdot \sqrt{\frac{w}{w_{c0}}} - \frac{w}{w_{c0}} \right) \cdot \sigma_{cf0} & \text{for } 0 \leq w \leq w_{c0} \\ \left(1 - \frac{2w}{l_f} \right)^2 \cdot \sigma_{cf0} & \text{for } w_{c0} < w \leq l_f/2 \end{cases} \quad (2.67)$$

$$w_{c0} = \frac{\tau_{bf} \cdot l_f^2}{d_f \cdot E_f} \quad (2.68)$$

$$\sigma_{cf0} = \eta_{\theta f} \cdot \frac{V_f \cdot \tau_{bf} \cdot l_f}{d_f} \quad (2.69)$$

$$\eta_{\theta f} = \frac{1}{\pi} \cdot \int_{\theta_w - \pi/2}^{\pi/2} \cos \theta_f \cdot d\theta_f = \frac{1 + \cos \theta_w}{\pi} \quad (2.70)$$

$$\theta_w = \left| \arctan \left(\frac{w_t}{w_n} \right) \right| \quad (2.71)$$

where l_f refers to the fibre length, d_f refers to the fibre diameter, E_f refers to the fibre elastic modulus, τ_{bf} refers to the fibre average bond stress, σ_{cf0} refers to the fibre effectiveness, V_f refers to the fibre volume in terms of the total concrete volume, $\eta_{\theta f}$ is the fibre orientation factor and θ_w refers to the pull-out loading direction. The fibre orientation factor ($\eta_{\theta f}$) is calculated based on a two-dimensional stress-strain representation according to the range of possible fibre orientation angles (θ_f). Figure 2.15 describes the pull-out loading direction along with its angle (θ_w) relative to the crack plane as well as the intervals with active and inactive fibres (Matos et al., 2020). Some fibres are deemed to be inactive due to their unfavourable orientation in relation to the pull-out loading direction (Matos et al., 2020). The average fibre bond stress (τ_{bf}) is meant to be taken from the fibre bond stress-slip response, which is determined by conducting fibre pull-out tests with an inclination angle equal to 0° on specimens consisting of a single fibre embedded in a concrete body. In the absence of experimental data,

Pfyl (2003) suggests that the following equation may be used for standard concrete with fibre dosages of around 30 kg/m^3 to 60 kg/m^3 to determine the average fibre bond stress (τ_{bf}) as it correlates well in comparison to experimental data:

$$\tau_{bf} = 2 \cdot f_{ct} = 0.6 \cdot f_c^{2/3} \quad (2.72)$$

where f_{ct} and f_c refer to the tensile and compressive strength of concrete, respectively.

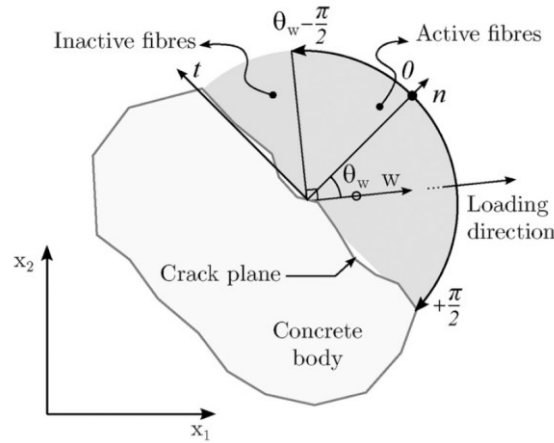


Figure 2.15 – Active and inactive fibres in relation to pull-out loading direction according to fibre bridging theoretical models (Matos et al., 2020). Image reproduced with permission of the rights holder, Elsevier.

Htut (2010) proposed a theoretical model, which is known as the Unified Variable Engagement model (UVEM). Voo and Foster (2003) initially introduced the Variable Engagement model for Mode I fractures (VEMI), and then Lee and Foster (2008) extended this approach for Mode II fractures (VEMII). The UVEM is a uniform bond stress model that combines the approaches of both previously proposed models to account for mixed mode fractures, which is the case for inclined cracks in normally loaded beams. The UVEM assumes that the total tensile resistance is the sum of the concrete matrix and fibre reinforcement contributions. Much like the Engagement model of Pfyl (2003), fibres are assumed to be uniformly distributed in the concrete matrix according to the UVEM, however they have equal probability of having any orientation. Figure 2.16 describes the fibre bending angle (γ) and pull-out loading direction angle (ϕ) as well as the fibre orientation angle (θ_f) that is positive in the clockwise direction. Pull-out is assumed to only occur on the shorter embedded end while a perfect bond is assumed between the longer embedded fibre end and surrounding concrete matrix. The UVEM does not consider any elastic strains sustained by the fibres nor energy lost as a result of fibre bending

(Matos et al., 2020). Similar to the Engagement model of Pfyl (2003), no tensile stress is resisted when the crack displacement (w) exceeds half of the fibre length ($l_f/2$).

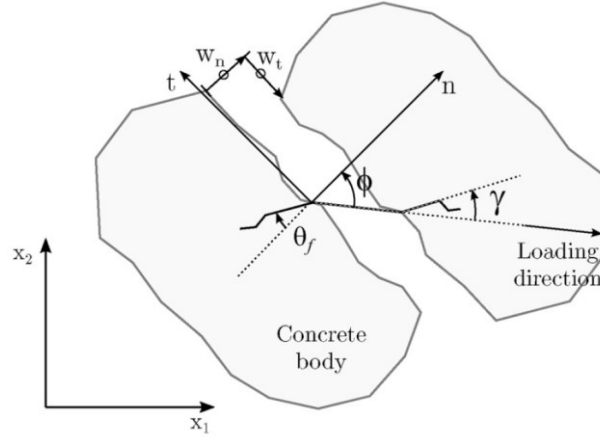


Figure 2.16 – Representation of fibre orientation and angle based on UVEM (Matos et al., 2020). Image reproduced with permission of the rights holder, Elsevier.

According to Htut (2010), the tensile stress resisted by the fibres (σ_{cf}) is calculated using the following set of equations:

$$\sigma_{cf} = \eta_{f,uvem} \cdot \frac{l_f}{d_f} \cdot V_f \cdot \tau_{b,avg} \quad (2.73)$$

$$\gamma_{crit} = \frac{2 \cdot \gamma_{max}}{\pi} \cdot \arctan \left(\sqrt[3]{\frac{3.5 \cdot w}{d_f}} \right) \quad (2.74)$$

$$\gamma_{max} = |\phi| + \frac{\pi}{2} \quad \text{for } \pi/2 \leq \gamma_{max} \leq \pi \quad (2.75)$$

$$\phi = \left| \arcsin \left(\frac{w_t}{w} \right) \right| = \left| \arctan \left(\frac{w_t}{w_n} \right) \right| \quad (2.76)$$

$$\eta_{f,uvem} = \left(\frac{a + b}{\pi} \right) \cdot \left(1 - \frac{2 \cdot w}{l_f} \right) \cdot k_{avg} \quad (2.77)$$

$$a = \min \left\{ \gamma_{crit}; \frac{\pi}{2} - |\phi| \right\} \quad (2.78)$$

$$b = \gamma_{crit} \quad (2.79)$$

$$k_{avg} = \frac{1}{2} - \frac{w}{l_f} \quad (2.80)$$

where $\tau_{b,avg}$ is the average fibre bond stress, γ_{max} is the maximum fibre bending angle, γ_{crit} is the critical fibre bending angle, $\eta_{f,uvem}$ is the fibre orientation factor, $\frac{a+b}{\pi}$ is the probability density function of a fibre crossing the crack and k_{avg} is the average value of the local orientation factor for all active fibres. In the absence of the relevant experimental fibre bond stress-slip response, the average fibre bond stress ($\tau_{b,avg}$) can be calculated using the following equation according to Htut (2010):

$$\tau_{b,avg} = \tau_{b,0} + 0.0625 \cdot \gamma_{crit}^3 \quad (2.81)$$

where $\tau_{b,0}$ is initial fibre bond stress when the fibre is parallel to the pull-out loading direction ($\gamma = 0$). Based on the experimental results of Htut and Foster (2012), it is suggested that the initial fibre bond stress ($\tau_{b,0}$) can be calculated using the following equation depending on the fibre type and mean concrete compressive cylinder strength (f_{cm}):

$$\tau_{b,0} = k_b \sqrt{f_{cm}} \quad (2.82)$$

where k_b is a coefficient that is given in Table 2.3, and it varies depending on the concrete and fibre type according to Htut and Foster (2012). If the concrete compressive strength results are unavailable, the initial fibre bond stress ($\tau_{b,0}$) can be calculated using the following equation:

$$\tau_{b,0} = 3 \cdot k_b \cdot f_{ct} \quad (2.83)$$

where f_{ct} is the mean tensile strength of the unreinforced concrete (Htut and Foster, 2012).

Table 2.3 – Initial fibre bond stress coefficient (Htut and Foster, 2012)

Concrete type	Fibre type	Coefficient (k_b)	Reference
Mortar	Straight	0.30	Voo and Foster (2003), Htut (2010)
	Hooked end	0.67	
Concrete	Straight	0.40	Foster (2010)
	Hooked end	0.80	
Reactive powder concrete (RPC)	Straight	0.60	Voo et al. (2010)
	Hooked end	1.00	

Lee et al. (2013) proposed a theoretical model that is known as the Simplified Diverse Embedment model (SDEM). It was derived from the original Diverse Embedment model (DEM) proposed by Lee et al. (2011). The DEM considers the behaviour of the fibres on both of their embedded ends across a single crack, while also considering random fibre orientations and embedment lengths. It assumes that the resisted tensile stress is the sum of the frictional bond behaviour between the fibres and surrounding concrete matrix as well as the mechanical anchorage, with each component evaluated separately (Lee et al., 2013). It is important to note that the mechanical anchorage component only applies to hooked end fibres, and it is neglected for straight fibres according to Lee et al. (2013). The implementation of the DEM requires a complex double numerical integration method to calculate the tensile stress resisted by the fibres. The SDEM provides a simpler and more practical method of calculation by replacing the double numerical integral with easily determined coefficients. In doing so, the SDEM assumes that the fibre pull-out behaviour is a rigid-body translation while the crack width is equivalent to the slip of the fibre with the shorter embedded end (Matos et al., 2020). Much like the previously mentioned theoretical models, no tensile stress is resisted by the fibres when the crack opening displacement (w_n) exceeds half of the fibre length ($l_f/2$).

The tensile stress resisted by the fibres (σ_{cf}) is calculated using the following set of equations according to Lee et al. (2013):

$$\sigma_{cf} = \sigma_{cf,st} + \sigma_{cf,eh} \quad (2.84)$$

$$\sigma_{cf,st} = \eta_{\theta f} \cdot V_f \cdot K_{st}(w_n) \cdot \tau_{f,max} \cdot \frac{l_f}{d_f} \cdot \left(1 - \frac{2 \cdot w_n}{l_f}\right)^2 \quad (2.85)$$

$$K_{st}(w_n) = \begin{cases} \frac{\beta_f}{3} \cdot \frac{w_n}{s_f} & \text{for } w_n < s_f \\ 1 - \sqrt{\frac{s_f}{w_n}} + \frac{\beta_f}{3} \cdot \sqrt{\frac{s_f}{w_n}} & \text{for } w_n \geq s_f \end{cases} \quad (2.86)$$

$$\sigma_{cf,eh} = \eta_{\theta f} \cdot V_f \cdot K_{eh}(w_n) \cdot \tau_{eh,max} \cdot \frac{2 \cdot (l_i - 2 \cdot w_n)}{d_f} \quad (2.87)$$

$$K_{eh}(w_n) = \begin{cases} \beta_{eh} \cdot \left(\frac{7 \cdot w_n}{15 \cdot s_{eh}}\right) & \text{for } w_n < s_{eh} \\ 1 + \left(\frac{7 \cdot \beta_{eh}}{15} - 1\right) \cdot \sqrt{\frac{s_{eh}}{w_n}} - \frac{2 \cdot (\sqrt{w_n} - \sqrt{s_{eh}})^2}{l_f - l_i} & \text{for } s_{eh} \leq w_n \leq \frac{l_f - l_i}{2} \\ \left(\frac{l_i - 2 \cdot w_n}{2 \cdot l_i - l_f}\right)^2 \cdot K_{eh,i} & \text{for } \frac{l_f - l_i}{2} < w_n < \frac{l_i}{2} \end{cases} \quad (2.88)$$

where $\sigma_{cf,st}$ is the tensile stress component due to frictional bond and $\sigma_{cf,eh}$ is the tensile stress component due to mechanical anchorage, $K_{st}(w_n)$ is the average fibre pull-out stress coefficient due to frictional bond, $K_{eh}(w_n)$ is the average fibre pull-out stress coefficient due to mechanical anchorage, $K_{eh,i}$ is the fibre pull-out stress when the crack opening displacement (w_n) is equal to $(l_f - l_i)/2$, $\eta_{\theta f}$ is the fibre orientation that is equal to 0.5 for general FRC members with dimensions much greater than the fibre length according to Lee et al. (2013) and l_i is the distance between mechanically anchored ends for hooked end fibres. The maximum frictional bond stress ($\tau_{f,max}$) and the corresponding slip (s_f) are meant to be taken from the experimental fibre bond stress-slip response, yet Lee et al. (2013) does not provide an empirical formula in the absence of experimental data such as Pfyl (2003) and Htut and Foster (2012). The fibre pull-out stress due to mechanical anchorage ($\tau_{eh,max}$) and the corresponding slip (s_{eh}) should be taken from the bond stress-slip response of hooked end fibres. The coefficient (β_f) reflects the slip endured on the longer embedded end of the fibre, and it has been analytically determined to be equal to 0.67 according to Lee et al. (2013). According to Lee et al. (2013), the coefficient (β_{eh}) accounts for the fibres that do not reach the maximum mechanical anchorage force, and it is equal to 0.76.

2.3.4 DOWEL ACTION

Shear force is transferred through dowel action when vertical displacement occurs in the longitudinal reinforcement that is caused by the extension and widening of the critical shear crack. Baumann and Rüsç (1970) proposed a comprehensive theoretical model for determining the dowel action contribution to shear resistance as it considers several influential factors. These factors include the diameter of the longitudinal reinforcement bars and the number of layers of reinforcement as well as the concrete cover. The theoretical model of Baumann and Rüsç (1970) was based on bending test results on beams with and without transverse reinforcement, with the loading arrangements, typical load-displacement response curves and proposed material models shown in Figure 2.17.

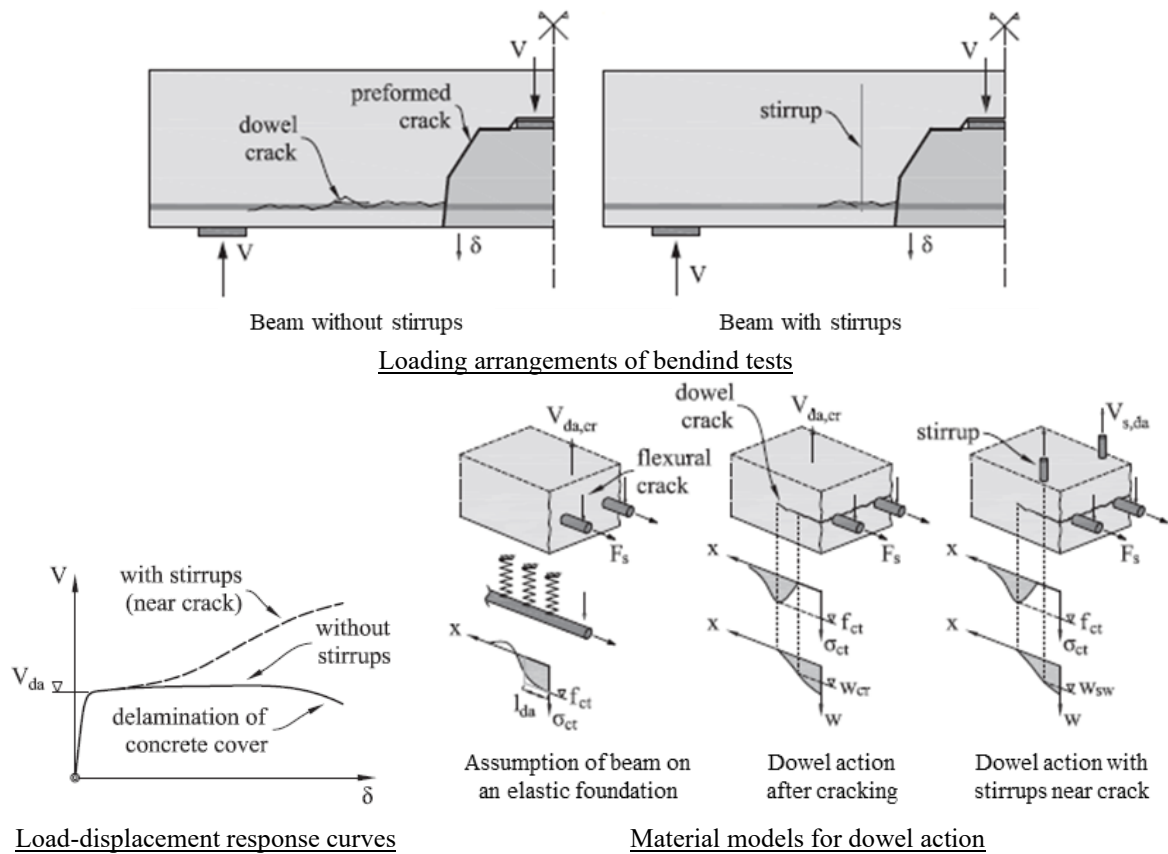


Figure 2.17 – Dowel action of longitudinal reinforcement according to Baumann and Rüsç (1970) (Huber et al., 2016). Images reproduced with permission of the rights holder, Elsevier.

According to Baumann and Rüsç (1970), the dowel action contribution (V_{da}) for beams without transverse reinforcement is determined using the following equation:

$$V_{da} = \begin{cases} \frac{\Delta}{0.08} \cdot V_{da,cr} & \text{for } \Delta < 0.08 \text{ mm} \\ V_{da,cr} & \text{for } \Delta \geq 0.08 \text{ mm} \end{cases} \quad (2.89)$$

where Δ is the shear displacement at the location of the inclined crack and uppermost layer of reinforcement and $V_{da,cr}$ is the dowel cracking load. The dowel cracking load is determined using the following set of equations:

$$V_{da,cr} = f_{ct} \cdot b_n \cdot l_{da} \quad (2.90)$$

$$b_n = b_w - \sum \phi_s \quad (2.91)$$

$$l_{da} = 40 + 0.032 \cdot I_v \cdot 10^{-4} \quad (2.92)$$

where f_{ct} is the tensile strength of concrete, f_c is the compressive cylinder strength of concrete, b_n is the effective beam width, b_w is the beam width, ϕ_s is the diameter of the longitudinal reinforcement, l_{da} is the effective length of the concrete in tension below the reinforcement bars that depends on the number of layers of reinforcement bars and I_v is the moment of inertia of the longitudinal bars and concrete area below the reinforcement bars. The moment of inertia (I_v) depends on the number and layers of longitudinal bars as well as the concrete area below the bars as shown in Figure 2.18. The moment of inertia (I_v) is calculated using the following equations:

$$I_v = I_c + I_s \quad (2.93)$$

$$I_c = \frac{n_{bars} \phi_s h_c^3}{12} + n_{bars} \phi_s h_c d_c^2 \quad (2.94)$$

$$I_s = \sum_i^{\text{layers}} \frac{n \cdot n_{bars} \pi \phi_s^4}{64} + \frac{n \cdot n_{bars} \pi \phi_s^2 d_i^2}{4} \quad (2.95)$$

where I_c is the moment of inertia of the concrete, I_s is the moment of inertia of the longitudinal bars, n_{bars} refers to the number of bars in i layers and n refers to the modular ratio of steel to

concrete (E_s/E_c) that is equal to 8 according to Baumann and Rüsch (1970). The dimension h_c refers to the height of the rectangular block area shown in Figure 2.18. The dimensions d_c and d_i refer to the distances between the centroid of the total area to the centroids of the rectangular concrete block areas and areas of the longitudinal bars, respectively.

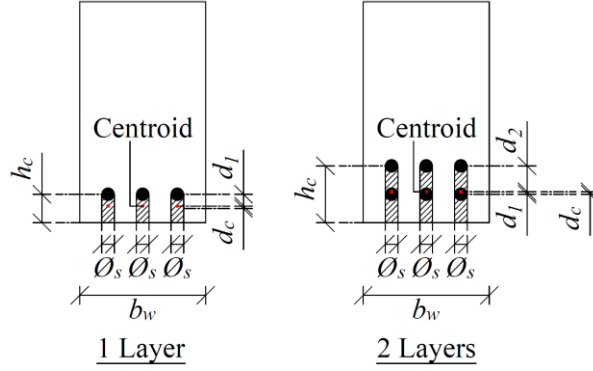


Figure 2.18 – Moment of inertia of reinforcement bar and concrete of RC beam. Adapted from Baumann and Rüsch (1970). Images reproduced with permission of the rights holder, German Committee for Structural Concrete (DAfStb).

For beams with transverse reinforcement, Baumann and Rüsch (1970) propose that the stirrup contribution ($V_{da,s}$) is only activated after the dowel crack opens horizontally leading to concrete delamination. The dowel contribution (V_{da}) of beams with stirrups is limited by the load carrying capacity of the stirrups ($V_{da,max}$). The following set of equations are proposed by Baumann and Rüsch (1970) for the dowel contribution of beams with transverse reinforcement:

$$V_{da} = \begin{cases} \frac{\Delta}{0.08} \cdot V_{da,cr} & \text{for } \Delta < 0.08 \text{ mm} \\ \max\{V_{da,cr}; V_{da,s}\} & \text{for } \Delta \geq 0.08 \text{ mm} \end{cases} \quad (2.96)$$

$$V_{da,s} = \sqrt{\frac{\Delta_{stir}}{\gamma}} \leq V_{da,max} \quad (2.97)$$

$$\gamma = 0.000045 \cdot f^2 / I_v \quad (2.98)$$

$$V_{da,max} = A_{sw} \cdot f_{u,sw} \quad (2.99)$$

where Δ_{stir} corresponds to the shear displacement at the location of the dowel crack and nearest stirrup that is a distance (f) away from the inclined crack as shown in Figure 2.19, A_{sw} is the area of transverse reinforcement and $f_{u,sw}$ is the ultimate stress of the transverse reinforcement.

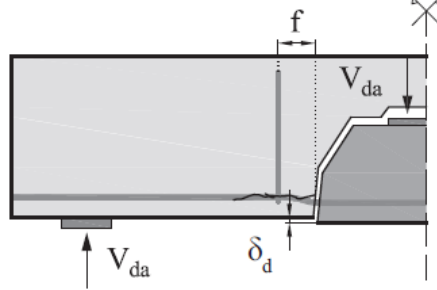


Figure 2.19 – Test setup of RC beams with stirrups highlighting the distance between the vertical crack and nearest stirrup according to Baumann and Rüschi (1970) (Huber et al., 2016). Image reproduced with permission of the rights holder, Elsevier.

Another method for estimating the stirrup contribution ($V_{da,s}$) was proposed by Huber et al. (2020). Huber et al. (2020) expanded on the Baumann and Rüschi (1970) model by suggesting that the stirrup contribution should be based on the force generated at the location of the dowel crack and nearest stirrup rather than using Eqn. 2.97 of Baumann and Rüschi (1970). Huber et al. (2020) proposed that the measured shear displacement at the intersection point between the dowel crack and nearest stirrup (Δ_{stir}) should be used to determine the stress in the stirrup (σ_{sw}), and the stress (σ_{sw}) should be determined using the theoretical model proposed by Sigrist (1995) that is described later in Section 2.3.5. Huber et al. (2020) also suggests that the dowel contribution (V_{da}) is the sum of the dowel cracking load ($V_{da,cr}$) and stirrup contribution ($V_{da,s}$) when the shear displacement (Δ) is greater than 0.08 mm. The following set of equations are proposed by Huber et al. (2020) for the dowel contribution of beams with transverse reinforcement:

$$V_{da} = \begin{cases} \frac{\Delta}{0.08} \cdot V_{da,cr} & \text{for } \Delta < 0.08 \text{ mm} \\ V_{da,cr} + V_{da,s} & \text{for } \Delta \geq 0.08 \text{ mm} \end{cases} \quad (2.100)$$

$$V_{da,s} = A_{sw} \cdot \sigma_{sw}(\Delta_{stir}) \leq V_{da,max} \quad (2.101)$$

where $\sigma_{sw}(\Delta_{stir})$ refers to the stress based on the shear displacement at the location of the dowel crack and nearest stirrup that is a distance (f) determined using the model of Sigrist (1995).

Walraven and Reinhardt (1981) developed a semi-empirical model used to estimate the shear force transferred by dowel action of the longitudinal reinforcement. It is based on the idealization of the longitudinal reinforcement as a beam on an elastic foundation, and the theoretical results were curve fit to the dowel test results of Paulay et al. (1974). Walraven and Reinhardt (1981) describe the derivation of the model, which stems from the following original equation that estimates the force due to dowel action (V_d):

$$V_d = \beta^3 \cdot E_s \cdot I_s \cdot \Delta \quad (2.102)$$

where β is the relative stiffness of the idealised foundation, E_s is the elastic modulus of the longitudinal reinforcement, I_s is the moment of inertia of the longitudinal reinforcement and Δ is the vertical shear displacement of the longitudinal reinforcement. The relative stiffness (β) is calculated using the following set of equations:

$$\beta = \sqrt[4]{\frac{\varphi_s \cdot G_f}{4 \cdot E_s \cdot I_s}} \quad (2.103)$$

$$G_f = 34 \cdot \sqrt{f_c} \cdot \Delta^{-0.85} \quad (2.104)$$

$$I_s = \frac{\pi \cdot \varphi_s^4}{64} \quad (2.105)$$

where G_f is the foundation modulus and φ_s is the diameter of the longitudinal reinforcement. Walraven and Reinhardt (1981) derived Eqn. 2.103 by observing that the foundation modulus (G_f) decreases inversely with the vertical shear displacement (Δ) from Paulay et al. (1974). Combining Eqns. 2.103 to 2.105 with Eqn. 2.102, the force due to dowel action (V_d) becomes the following:

$$V_d = 3.56 \cdot \varphi_s^{1.75} \cdot G_f^{0.75} \cdot \Delta \quad (2.106)$$

Walraven and Reinhardt (1981) considers that the force due to dowel action (V_d) should be multiplied by a reduction factor that considers the axial tensile stress in the longitudinal reinforcement. The reduction factor (ξ) is calculated as follows:

$$\xi = 0.2 \cdot (w + 0.2)^{-1} \quad (2.107)$$

where w is the crack displacement parallel to the longitudinal reinforcement. Considering the reduction factor (ξ), the force due to dowel action (V_d) according to Walraven and Reinhardt (1981) is ultimately calculated using the following equation:

$$V_d = 10 \cdot (w + 0.2)^{-1} \cdot \Delta^{0.36} \cdot \varphi_s^{1.75} \cdot f_c^{0.38} \quad (2.108)$$

Millard and Johnson (1984) proposed a model that estimates the force due to dowel action (V_d) that is also based on the theory of a beam on an elastic foundation, and the equation is as follows:

$$V_d = C \cdot G_f^{0.75} \cdot \varphi_s^{1.75} \cdot E_s^{0.25} \cdot \Delta \quad (2.109)$$

where C is an experimentally derived constant equal to 0.166 according to Millard (1983), G_f is the foundation modulus, E_s is the elastic modulus of the longitudinal reinforcement and Δ is the vertical shear displacement of the longitudinal reinforcement. While taking Eqn. 2.109 into consideration, Millard and Johnson (1984) also acknowledged that dowel failure can also occur as a result of concrete delamination or the longitudinal reinforcement reaching its bending capacity (Millard and Johnson, 1984). Due to the latter case, it is suggested that the plastic bending moment (M_p) be considered along with the axial stress in the reinforcement bar to limit the force due to dowel action using the following equations:

$$M_p = \frac{\varphi_s^{1.75} \cdot f_y}{6} \cdot (1 - \alpha^2) \quad (2.110)$$

$$\alpha = \sigma_s / f_y \quad (2.111)$$

where φ_s is the diameter of the longitudinal reinforcement α is the ratio of the measured stress (σ_s) to the yield stress (f_y) of the longitudinal reinforcement. Substituting Eqns. 2.110 in 2.109 ultimately gives the following set equations for estimating the force due to dowel action (V_d) according to Millard and Johnson (1984):

$$V_d = V_{du} \cdot [1 - \exp(-K_i \cdot \Delta/V_{du})] \quad (2.112)$$

$$V_{du} = 1.3 \cdot \varphi_s^2 \cdot f_{cu}^{0.5} \cdot [f_y \cdot (1 - \alpha^2)]^{0.5} \quad (2.113)$$

$$K_i = 0.166 \cdot G_f^{0.75} \cdot \varphi_s^{1.75} \cdot E_s^{0.25} \quad (2.114)$$

$$G_f = 126.26 \cdot f_{cu}^{0.5} = 126.26 \cdot \left(\frac{f_c}{0.8}\right)^{0.5} \quad (2.115)$$

where V_{du} is the ultimate dowel force limit, K_i is the initial dowel shear stiffness, f_{cu} is the concrete compressive cube strength and f_c is the concrete compressive cylinder strength. The theoretical results were curve fit to test results, where a shear load was applied via a hydraulic jack across a central crack on the surface of reinforced concrete prisms as in Figure 2.20 (Millard, 1983).

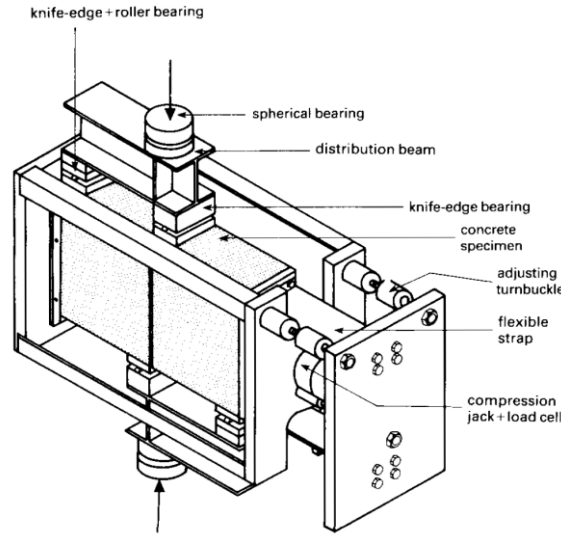


Figure 2.20 – Dowel test loading arrangement (Millard and Johnson, 1984). Image reproduced of the rights holder, Emerald Publishing Limited.

He and Kwan (2001) also developed a dowel action model while using the beam on an elastic foundation theory. He and Kwan (2001) suggest that the foundation may be treated as a bed of Winkler springs as shown in Figure 2.21, where the reaction force along the foundation is proportional to the vertical displacement of the beam at corresponding points. Similar to Millard and Johnson (1984), the plastic bending capacity is considered by He and Kwan (2001) to limit the force due to dowel action. According to He and Kwan (2001), the force due to dowel action (V_d) can be calculated using the following set of equations:

$$V_d = E_s \cdot I_s \cdot \lambda^3 \cdot \Delta \leq V_{du} \quad (2.116)$$

$$V_{du} = 1.27 \cdot d_b^2 \cdot \sqrt{f_c \cdot f_y} \quad (2.117)$$

$$\lambda = \sqrt[4]{\frac{k_c \cdot d_b}{4 \cdot E_s \cdot I_s}} \quad (2.118)$$

$$k_c = \frac{127c_1\sqrt{f_c}}{d_b^{2/3}} \quad (2.119)$$

$$I_s = \frac{\pi \cdot d_b^4}{64} \quad (2.120)$$

$$c_1 = \begin{cases} 0.6 & \text{for a clear bar spacing equal to 25 mm} \\ 1.0 & \text{for a clear bar spacing greater than 25 mm} \end{cases} \quad (2.121)$$

where V_{du} is the ultimate dowel force limit, E_s is the modulus of elasticity of the longitudinal reinforcement, I_s is the moment of inertia of the longitudinal reinforcement, λ is a coefficient that represents the relative stiffness of the foundation, Δ is the vertical shear displacement of the longitudinal reinforcement, d_b is the diameter of the longitudinal bar, f_c is the concrete compressive cylinder strength, f_y is the yield stress of the longitudinal reinforcement, k_c is the foundation modulus of the surrounding concrete and c_1 is the bar spacing coefficient.

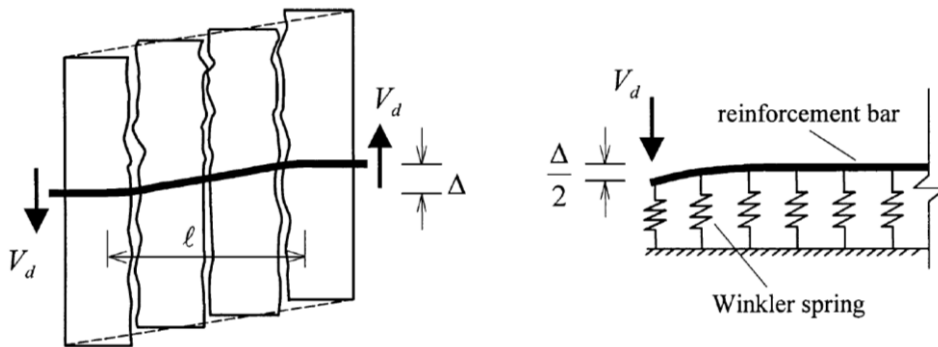


Figure 2.21 – Dowel action model (He and Kwan, 2001). Image reproduced with permission of the rights holder, Elsevier.

2.3.5 STIRRUPS

Transverse reinforcement in concrete beams become active upon the opening of diagonal shear cracks (Campana et al., 2013). Following diagonal cracking, tensile force develops in the transverse reinforcement that contributes to shear resistance. The bond condition between steel and concrete greatly impacts the development of tensile stress in the transverse reinforcement with increasing crack width. The simplified and widely used approach of Sigrist (1995) proposes a rigid-plastic interface bond behaviour with constant bond stresses for before (τ_{b0}) and after (τ_{b1}) the yielding of the stirrups as shown in Figure 2.22(a) (Campana et al., 2013; Huber et al., 2016; Pastore, 2020). This approach suggests that the stirrup carries all of the tensile stress at the location of the crack. Due to the assumption of constant bond stress, the stress carried by the stirrups decreases linearly with distance away from the crack towards the concrete regions. Figure 2.22(b) shows the corresponding variation in steel strain and stress along a RC tie with multiple cracks (Campana et al., 2013).

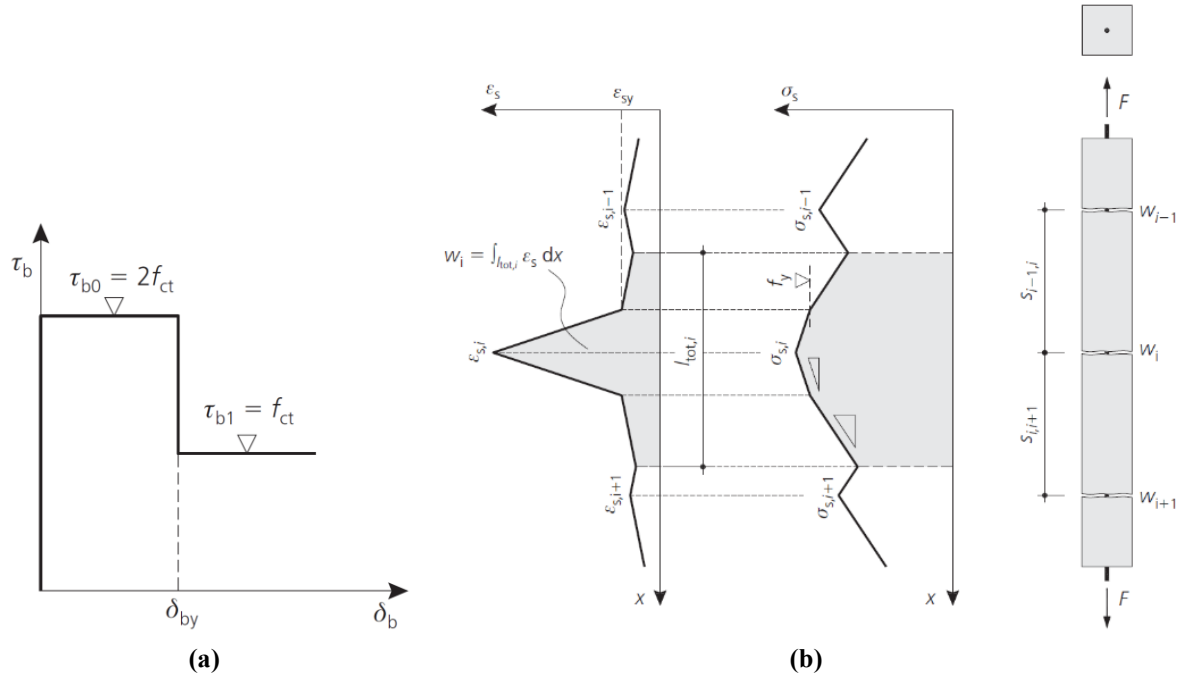


Figure 2.22 – Shear force contribution of stirrups according to Sigrist (1995): **(a)** Rigid-plastic bond behaviour, **(b)** Crack opening and steel strain and stress response in RC tie. **(a-b)** Campana et al. (2013). Images reproduced with permission of the rights holder, Emerald Limited Publishing.

The theoretical model of Sigrist (1995) relates the stress profile in the stirrups to the crack width. The crack width can be determined by integrating the strains in the steel over the considered length using the following equation:

$$w_{sw,i} = \int_{l_{tot,i}} \varepsilon_s dx \quad (2.122)$$

where $w_{sw,i}$ is the width of crack i , ε_s is the steel strain and $l_{tot,i}$ is the length where tensile stresses are redistributed from the concrete to the steel. Having adopted the proposed model of Sigrist (1995), Fang (2014) derived a set of equation for determining the stress response before and after yielding of an embedded stirrup with a single crack. Before yielding of the stirrup ($\sigma_{sw} \leq f_y$), the stress (σ_{sw}) is calculated using the following set of equations:

$$\sigma_{sw} = \sqrt{\frac{4 \cdot \tau_{b0} \cdot E_{sw}}{\phi_{sw}} \cdot w_{sw}} \quad (2.123)$$

$$\tau_{b0} = 2 \cdot f_{ct} \quad (2.124)$$

where τ_{b0} is the bond stress before yielding, E_{sw} is the elastic modulus of the transverse reinforcement, ϕ_{sw} is the diameter of the transverse reinforcement and f_{ct} is the tensile strength of concrete. After yielding of the stirrup ($f_y < \sigma_{sw} \leq f_u$), the stress (σ_{sw}) is calculated using the following set of equations:

$$\frac{\phi_{sw}}{2} \cdot \left(\frac{f_y^2}{2 \cdot E_{sw} \cdot \tau_{b0}} + \frac{(\sigma_{sw} - f_y)^2}{2 \cdot E_{sh} \cdot \tau_{b1}} + \frac{f_y \cdot (\sigma_{sw} - f_y)}{E_{sw} \cdot \tau_{b1}} \right) - w_{sw} = 0 \quad (2.125)$$

$$\tau_{b1} = f_{ct} \quad (2.126)$$

$$E_{sh} = \frac{f_u - f_y}{\varepsilon_u - \varepsilon_y} \quad (2.127)$$

where τ_{b1} is the bond stress after yielding, E_{sh} is the strain hardening modulus of the transverse reinforcement, f_u is the ultimate stress of the transverse reinforcement, ε_u is the ultimate strain of the transverse reinforcement and ε_y is the yield strain of the transverse reinforcement. The shear force (V_{stir}) resisted by the stirrups crossing the critical shear crack can then be calculated as follows:

$$V_{stir} = \sum \sigma_{sw,i} \cdot A_{sw} \quad (2.128)$$

where i refers to each stirrup crossing the critical shear crack.

2.3.6 UNCRACKED COMPRESSION ZONE

The uncracked compression zone also serves as a shear resisting mechanism (ASCE-ACI Committee 445, 1998). By integrating the shear stresses directly over the critical shear crack as shown in Figure 2.23, the shear resistance of the uncracked compression zone (V_{cz}) can be calculated using the following equation:

$$V_{cz} = \int_{A_{cz}} \tau_{xz} dA_{cz} \quad (2.129)$$

where τ_{xz} is the shear stress in the compression zone and A_{cz} is the area of the compression zone.

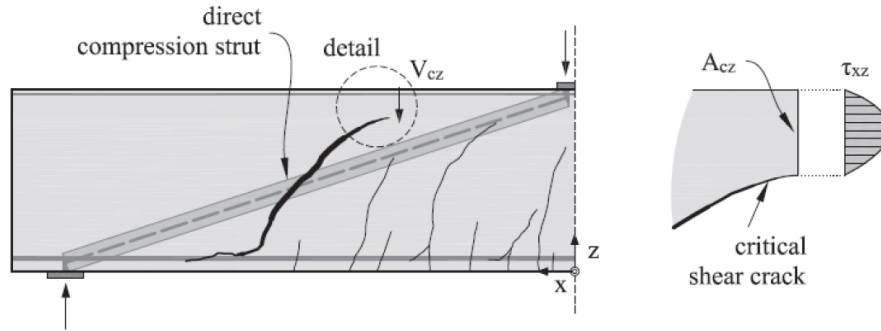


Figure 2.23 – Uncracked compression zone shear resistance mechanism (Huber et al., 2016). Image reproduced with permission of the rights holder, Elsevier.

Several approaches have been applied by researchers to estimate the shear stress and resulting force resisted by the uncracked compression zone (Pastore, 2020). Sherwood et al. (2007) determined the shear stress distribution in the compression zone based on compressive strain measurements recorded throughout the experiment by strain gauges installed in two consecutive cross-sections. Ultimately, Sherwood et al. (2007) found that the shear stress distribution in the compression zone estimated using the classical beam theory of Mörsh (1909) provided an accurate representation of the experimental shear stress distribution.

Expanding on this finding, Yang (2014) derived a method of calculation that estimates the shear force resisted by the uncracked compression zone (V_c) in slender RC beams using the following set of equations:

$$V_c = \frac{2}{3} \cdot \frac{z_c}{z} \cdot V \quad (2.130)$$

$$z_c = d - s_{cr} \quad (2.131)$$

$$z = \frac{2}{3} \cdot d + \frac{1}{3} \cdot s_{cr} = d - z_c/3 \quad (2.132)$$

$$s_{cr} = \left(1 + \rho_s \cdot n_e - \sqrt{2 \cdot \rho_s \cdot n_e + (\rho_s \cdot n_e)^2} \right) \cdot d \quad (2.133)$$

$$\rho_s = \frac{A_s}{b_w \cdot d} \quad (2.134)$$

where z_c is the height of the compression zone, z is the internal lever arm, V is the experimental shear force, d is the depth to the longitudinal reinforcement, s_{cr} is the vertical height of the crack, ρ_s is the longitudinal reinforcement ratio, n_e is the modular ratio of steel to concrete (E_s/E_c), A_s is the area of longitudinal reinforcement and b_w is the width of the beam. Alternatively, the varying height of the compression zone (z_c) can be measured throughout the experiment using non-optical monitoring techniques such as digital image correlation (DIC) rather than calculated using Eqn. 2.131 as demonstrated by Pastore (2020). The model of Yang (2014) assumes that the flexural tension is resisted entirely by the longitudinal tensile reinforcement. It also assumes that a crack spontaneously arises once the moment reaches the cracking moment, where the vertical height of the crack (s_{cr}) remains constant until failure.

Cavagnis et al. (2018a) offer an alternative approach for estimating the shear force resisted by the uncracked compression zone. Figure 2.24 describes the approach of Cavagnis et al. (2018a), and it shows the considered free-body diagram that includes the internal vertical and horizontal forces. The approach assumes an inclined parallel compression field between the critical shear crack tip and the edge of the loading plate with a linear compressive stress profile located above the crack tip and a rectangular compressive stress profile located below the loading plate. By applying moment equilibrium with respect to the end support to iteratively calculate the internal vertical and horizontal forces, the shear force of the compression zone (V_{Compr}) can be determined using the following equation:

$$V_{Compr} = f_c \cdot b \cdot 2 \cdot c_m \cdot \frac{c_n - c_m}{r_{F,edge}} \quad (2.135)$$

where f_c is the concrete compressive strength, b is the width of the beam, c_m is the depth to the resultant force of the rectangular compressive stress profile located below the loading plate, c_n is the depth to the resultant force of the linear compressive stress profile located above the crack tip and $r_{F,edge}$ is the horizontal distance between the crack tip and edge of the loading plate. After varying the initially assumed depth (c_m) in each iteration of the calculation process, Eqn. 2.135 represents the final iteration where the maximum compressive stress at the crack tip reaches the compressive strength of concrete. Cavagnis et al. (2018a) used a database of shear tested slender beams to validate Eqn. 2.135. They found the shear force resisted by the compression zone to be significantly dependent on the ratio of the vertical to horizontal distances of the compression zone (r_F/h_F). Therefore, the application of a constant (k_c) was suggested to improve the accuracy of the predicted shear force. The following equation was proposed to estimate the shear force resisted by the compression zone (V_{Compr}):

$$V_{Compr} = \frac{k_c \cdot h_F / r_F}{1 - k_c \cdot h_F / r_F} \cdot (V_{Res} + V_{agg} + V_{Dowel}) \quad (2.136)$$

where k_c is a constant equal to 0.5, h_F is the vertical distance between the crack tip and loading plate, r_F is the horizontal distance between the crack tip and centre of the loading plate, V_{Res} is the shear force due to residual tensile stress, V_{agg} is the shear force due to aggregate interlock and V_{Dowel} is the shear force due to dowel action. The vertical (h_F) and horizontal (r_F) distances from the crack tip to the loading plate can be measured during the experiment using a DIC system. Eqn. 2.136 can be extrapolated by adding other shear resisting forces that are not considered such as the shear resistance due to stirrups (Pastore, 2020).

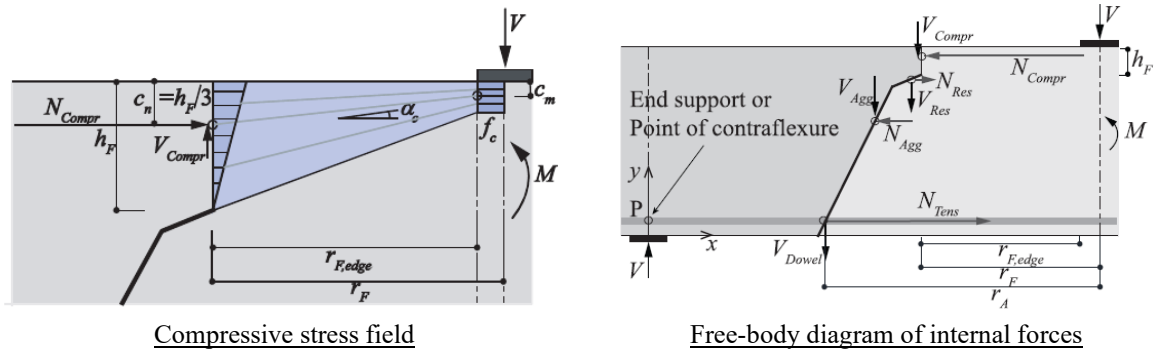


Figure 2.24 – Uncracked compression zone shear resistance mechanism (Cavagnis et al., 2018a). Images reproduced with permission of the rights holder, Elsevier.

Another notable approach was applied by Cavagnis et al. (2018b) where the shear stress in the compression zone was ultimately determined based on principal strains that were measured throughout the experiment using a DIC system. The approach involves converting the input principal strains to principal stresses and eventually to vertical stresses through the use of a relatively complex method of calculation. An adequate image size should be recorded by the DIC system during the experiment to apply the approach of Cavagnis et al. (2018b). Otherwise, DIC measurements may not be accurate enough to apply this approach as found by various researchers (Huber et al., 2016; Pastore, 2020).

2.4 SHEAR DESIGN CODE EQUATIONS AND ALTERNATIVE METHODS

2.4.1 INTRODUCTION

The shear strength of a beam equals the sum of its shear resisting mechanisms, which include aggregate interlock, fibre bridging, dowel action, stirrups and the uncracked compression zone as discussed in Section 2.3. Researchers have successfully measured the contribution of each shear resistance mechanism by using the latest technologies in non-contact optical monitoring techniques such as digital image correlation (DIC) (Campana et al., 2013; Huber et al., 2016; Pastore, 2020). The contributions of each shear resistance mechanism are based on the crack kinematics and shape of the critical shear crack as well as its location. National and international design codes for standard reinforced concrete and UHPC beams do not account for all of the shear resistance mechanisms directly as shear strength is entirely defined by only one or two mechanisms (ACI, 2019; AFGC, 2013; AFNOR, 2016b; BSI, 2014; EPFL, 2016; fib, 2013; JSCE, 2006). Sections 2.4.2 and 2.4.4 present the design code equations for predicting the shear strength at the ultimate limit state (ULS) of RC and UHPC beams, respectively. A mechanical based design model is presented in Section 2.4.3.1 that provides an alternative method to design codes as it can be used to determine the contribution of each shear resisting mechanism. Alternative methods that are also briefly presented include lower-bound plasticity based stress field models and the strut-and-tie method (STM) as well as compatibility based methods such as the Modified Compression-Field Theory (MCFT) model.

Reinforced concrete design codes assume that the shear force acting on a beam with transverse reinforcement is resisted by the transverse reinforcement, with some also considering the concrete contribution (ACI, 2019; BSI, 2014; fib, 2013). The contribution of the transverse

reinforcement is calculated based on an equilibrium truss model according to national and international design codes (ACI, 2019; BSI, 2014; fib, 2013). The truss model was first proposed by Ritter (1899) and it idealizes a beam with diagonal cracks as a parallel compressive and tensile chord truss system with compression diagonals inclined at 45° (ASCE-ACI Committee 445, 1998). The truss model is adopted in Eurocode 2 (EC2) as well as the next generation of Eurocode 2 from which Figure 2.25 is taken (CEN/TC250, 2022). According to equations given by the current version of Eurocode 2 (EC2) and fib Model Code 2010 (MC 2010), the truss angle is a variable that lies between a minimum value and 45° (BSI, 2014; fib, 2013). The truss angle is fixed at 45° according to the equations given by ACI 318-19 (ACI, 2019). The truss angle significantly influences the calculated contribution of the shear reinforcement, with a higher truss angle giving a lower strength value and vice versa. Test results indicate that the contribution of the transverse reinforcement given when a 45° truss angle is adopted is conservative seeing as the angle of the critical shear crack is less than that at failure (ASCE-ACI Committee 445, 1998). In addition to the contribution of the transverse reinforcement, fib MC 2010 and ACI 318-19 consider the concrete matrix contribution to shear resistance with the latter based on an empirical approach. The sum of the contributions to shear resistance are limited by the crushing strength of the diagonal struts in all of the previously mentioned design codes (ACI, 2019; BSI, 2014; fib, 2013).

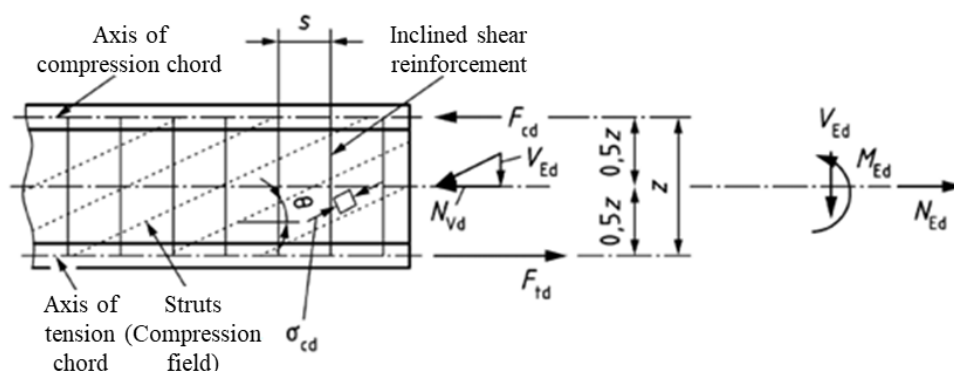


Figure 2.25 – Truss model for beams with transverse reinforcement (CEN/TC250, 2022)

Reinforced concrete design codes define shear strength in terms of one or two resistance mechanisms without directly accounting for each of the shear resistance mechanisms described in Section 2.3, which include aggregate interlock, dowel action, stirrups and the uncracked compression zone (ASCE-ACI Committee 445, 1998). More recently, mechanical based models have been developed by researchers to determine the contribution of each shear resistance mechanism (Cladera et al., 2016; Marí et al., 2015, 2016). The Multi-Action Shear

Strength Model (MASM) is a mechanical model that was initially developed for slender beams with and without transverse reinforcement (Cladera et al., 2016; Marí et al., 2015, 2016; Pastore, 2020). The MASM offers an alternative method to design codes that can be used to determine the shear strength of reinforced concrete beams by all of its shear resistance mechanisms.

For the design of UHPC beams, current codes and recommendations assume that the shear force is largely resisted by the fibre reinforcement (AFGC, 2013; AFNOR, 2016b; EPFL, 2016; JSCE, 2006). Shear force is also resisted by the transverse reinforcement if present with some codes also considering the contribution of the UHPC matrix as in AFGC 2013, NF P18-710 and JSCE 2006 (AFGC, 2013; AFNOR, 2016b; EPFL, 2016; JSCE, 2006). The contributions of the fibre and transverse reinforcement are calculated using an equilibrium truss model which is similar to that adopted in standard reinforced concrete design codes. All of the UHPC design codes agree that the truss angle is a variable that ranges from a minimum value of 30° to a maximum value of 45°, with the French codes suggesting that it can also be calculated (AFGC, 2013; AFNOR, 2016b; EPFL, 2016; JSCE, 2006). Much like standard concrete design codes, the sum of the contributions to shear resistance for UHPC beams are limited to the shear strength of the concrete web according to all UHPC design codes and recommendations.

2.4.2 SHEAR DESIGN CODE EQUATIONS FOR RC BEAMS

2.4.2.1 EUROCODE 2

EC2 assumes that the shear force acting on a RC beam with transverse reinforcement is resisted entirely by the transverse reinforcement (BSI, 2014). The shear resistance contribution of the transverse reinforcement ($V_{Rd,s}$) is limited by the shear strength of the concrete web ($V_{Rd,max}$). Therefore, the following set of equations are used to determine the shear strength of a RC beam with transverse reinforcement according to EC2:

$$V_{Rd,s} = A_{sw} f_{ywd} z \cot \theta / s \leq V_{Rd,max} \quad (2.137)$$

$$V_{Rd,max} = \alpha_{cw} b_w z v_1 f_{cd} / (\cot \theta + \tan \theta) \quad (2.138)$$

where A_{sw} is the area of transverse reinforcement, f_{ywd} is the design yield strength of the transverse reinforcement, z is the shear lever arm that can be taken as $0.9d$, θ is the truss angle that is taken as 21.8° and must be less than or equal to 45° , s is the spacing of the transverse reinforcement, α_{cw} is a coefficient that accounts for the state of stress in the compression chord that can be taken as 1.0, b_w is the width of the beam's web and f_{cd} is the design compressive strength of concrete (BSI, 2014). The strength reduction factor for concrete cracked in shear (v_1) is calculated as follows:

$$v_1 = 0.6 \cdot (1 - f_{ck}/250) \quad (2.139)$$

where f_{ck} is the characteristic compressive strength of concrete (BSI, 2014).

2.4.2.2 FIB MODEL CODE 2010

The design method given by fib MC 2010 varies depending on the adopted level of approximation (LoA) (fib, 2013). The three LoAs for beams with transverse reinforcement offer different assumptions for the main design parameters, and each LoA represents the level of refinement of the design method (Pastore, 2020). The most accurate design method is given by the highest LoA, which is LoAIII. The shear force is resisted by the transverse reinforcement ($V_{Rd,s}$) and concrete ($V_{Rd,c}$) according to fib MC 2010, and their sum is limited by the shear strength of the concrete web ($V_{Rd,max}$) (fib, 2013). The following set of equations are used to calculate the shear strength (V_{Rd}) of RC beams with transverse reinforcement according to fib MC 2010 LoAIII:

$$V_{Rd} = V_{Rd,s} + V_{Rd,c} < V_{Rd,max}(\theta_{min}) \quad (2.140)$$

$$V_{Rd,s} = \frac{A_{sw}}{s_{sw}} z f_{ywd} \cot \theta_{min} \quad (2.141)$$

$$V_{Rd,c} = k_v \frac{\sqrt{f_{ck}}}{\gamma_c} b_w z \quad (2.142)$$

$$V_{Rd,max}(\theta_{min}) = k_c \frac{f_{ck}}{\gamma_c} b_w z \sin \theta_{min} \cos \theta_{min} \quad (2.143)$$

where z is the shear lever arm that can be taken as $0.9d$, s_{sw} is the spacing of the transverse reinforcement and γ_c is a partial safety factor for the compressive strength of concrete that is taken as 1.5 for transient and persistent situations and 1.2 for incidental situations (fib, 2013). The minimum truss angle (θ_{min}), which must be less than or equal to 45° , is calculated as follows:

$$\theta_{min} = 20^\circ + 10000\varepsilon_x \quad (2.144)$$

where the longitudinal strain at mid-depth of the effective shear depth (ε_x) is determined using the following equation:

$$\varepsilon_x = \frac{1}{2E_s A_s} \left(\frac{M_{Ed}}{z} + V_{Ed} + N_{Ed} \left(\frac{1}{2} \mp \frac{\Delta e}{z} \right) \right) \leq 0.003 \quad (2.145)$$

where E_s is the elastic modulus of the longitudinal reinforcement, A_s is the area of longitudinal reinforcement, V_{Ed} is the design shear force, M_{Ed} is the design bending moment that is taken at distance d away from the support or concentrated load but not closer than a distance d from the face of the support, N_{Ed} is the axial force normal to the cross-section due to loading or prestressing and Δe is the eccentricity of the axial force. The remaining parameters used to determine the shear strength (V_{Rd}) of RC beams with transverse reinforcement according to fib MC 2010 LoAIII are the following:

$$k_v = \frac{0.4}{1 + 1500\varepsilon_x} \left(1 - \frac{V_{Ed}}{V_{Rd,max}(\theta_{min})} \right) \geq 0 \quad (2.146)$$

$$k_c = k_\varepsilon \eta_{fc} \quad (2.147)$$

$$k_\varepsilon = \frac{1}{1.2 + 55\varepsilon_1} \leq 0.65 \quad (2.148)$$

$$\varepsilon_1 = \varepsilon_x + (\varepsilon_x + 0.002) \cot^2 \theta_{min} \quad (2.149)$$

$$\eta_{fc} = \left(\frac{30}{f_{ck}}\right)^{1/3} \leq 1.0 \quad (2.150)$$

where k_v is a factor, k_c is a strength reduction factor, k_ϵ is a strength reduction factor that accounts for strain in the web and η_{fc} is a strength reduction factor (fib, 2013). In the event that the determined shear strength (V_{Rd}) is greater than the shear strength of the concrete web based on the minimum truss angle ($V_{Rd,max}$) then the design method of fib MC 2010 LoAII should be used (fib, 2013).

2.4.2.3 ACI 318-19

Much like fib MC 2010, ACI 318-19 assumes the shear force is resisted by the transverse reinforcement (V_s) and concrete (V_c) (ACI, 2019; fib, 2013). The contribution of concrete to shear resistance depends on whether the amount of transverse reinforcement is greater than or equal to the minimum allowable amount (ACI, 2019). The truss angle is fixed at 45° in ACI 318-19, which calculates the shear strength (V_n) of a RC beam with transverse reinforcement as follows:

$$V_n = V_c + V_s \quad (2.151)$$

$$V_c = \begin{cases} \left[0.17\lambda\sqrt{f'_c} + \frac{N_u}{6A_g}\right] b_w d & \text{for } A_v \geq A_{v,min} \\ \left[0.66\lambda(\rho_w)^{1/3}\sqrt{f'_c} + \frac{N_u}{6A_g}\right] b_w d & \text{for } A_v \geq A_{v,min} \\ \left[0.66\lambda_s\lambda(\rho_w)^{1/3}\sqrt{f'_c} + \frac{N_u}{6A_g}\right] b_w d & \text{for } A_v < A_{v,min} \end{cases} \quad (2.152)$$

$$V_s = \frac{A_v f_{yd} d}{s} \quad (2.153)$$

where λ is a modification factor that accounts for the weight of concrete and it is taken as 1.0 for normal weight concrete, f'_c is the concrete compressive strength, λ_s is a size effect factor, A_v is the area of transverse reinforcement, ρ_w is the shear reinforcement ratio taken as $A_v/b_w s$, f_{yd} is the design yield strength of the transverse reinforcement and s is the spacing of the transverse reinforcement (ACI, 2019). The size effect factor (λ_s) is calculated as follows:

$$\lambda_s = \sqrt{\frac{2}{1 + 0.004d}} \leq 1 \quad (2.154)$$

The minimum area of transverse reinforcement ($A_{v,min}$) for beams with non-prestressed reinforcement is determined using the following equation:

$$A_{v,min}/s = \max \left\{ 0.062\sqrt{f'_c} \frac{b_w}{f_{yt}}; 0.35 \frac{b_w}{f_{yt}} \right\} \quad (2.155)$$

where f_{yt} is the specified yield strength of the transverse reinforcement. The axial force normal to the cross-section (N_u) and gross area of the concrete section (A_g) must satisfy the following condition according to ACI 318-19:

$$\frac{N_u}{6A_g} \leq 0.05f'_c \quad (2.156)$$

Additionally, the concrete contribution to shear resistance (V_c) is limited according to the following equation:

$$V_c \leq 0.42\lambda\sqrt{f'_c}b_wd \quad (2.157)$$

The determined shear strength is limited according to the following equation:

$$V_u \leq \phi(V_c + 0.66\sqrt{f'_c}b_wd) \quad (2.158)$$

where ϕ is a strength reduction factor for anchors in concrete and V_u is the factored shear strength.

2.4.3 ALTERNATIVE SHEAR DESIGN METHODS FOR RC BEAMS

2.4.3.1 MECHANICAL MODELS

The general approach of RC design codes relies on relatively simple analytical or empirical models in order to design new and safe structures rather than to predict the actual strength of

existing structures (Cladera et al., 2016). Research into shear behaviour has led to the development of multi-action mechanical models that define shear strength by all of its resistance mechanisms unlike design codes (ACI, 2019; BSI, 2014; fib, 2013). The Multi-Action Shear Strength model (MASM) is a mechanical model developed for slender beams with or without transverse reinforcement (Cladera et al., 2016; Mari et al., 2015, 2016). The MASM is described in Figure 2.26 for rectangular and T-shaped cross-sections. The MASM assumes that the critical shear crack is the closest crack to the point of zero bending moment, and it extends to the neutral axis of the cross-section with a horizontal projection equal to $0.85d$ (Cladera et al., 2016). The critical section where failure occurs is assumed to be located at the intersection of the critical shear crack and the neutral axis depth (Cladera et al., 2016). The shear strength according to the MASM is the sum of the following shear resistance mechanisms: uncracked compression zone, cracked concrete web, dowel action and stirrups if present. In addition to considering all resistance mechanisms, the MASM also considers the contribution to shear resistance of the flange in the uncracked compression zone for T-beams. Design codes such as EC2, fib MC 2010 and ACI 318-19 only consider the contribution of the web. Much like the design codes, the shear strength is limited by the shear strength of the concrete web according to the MASM.

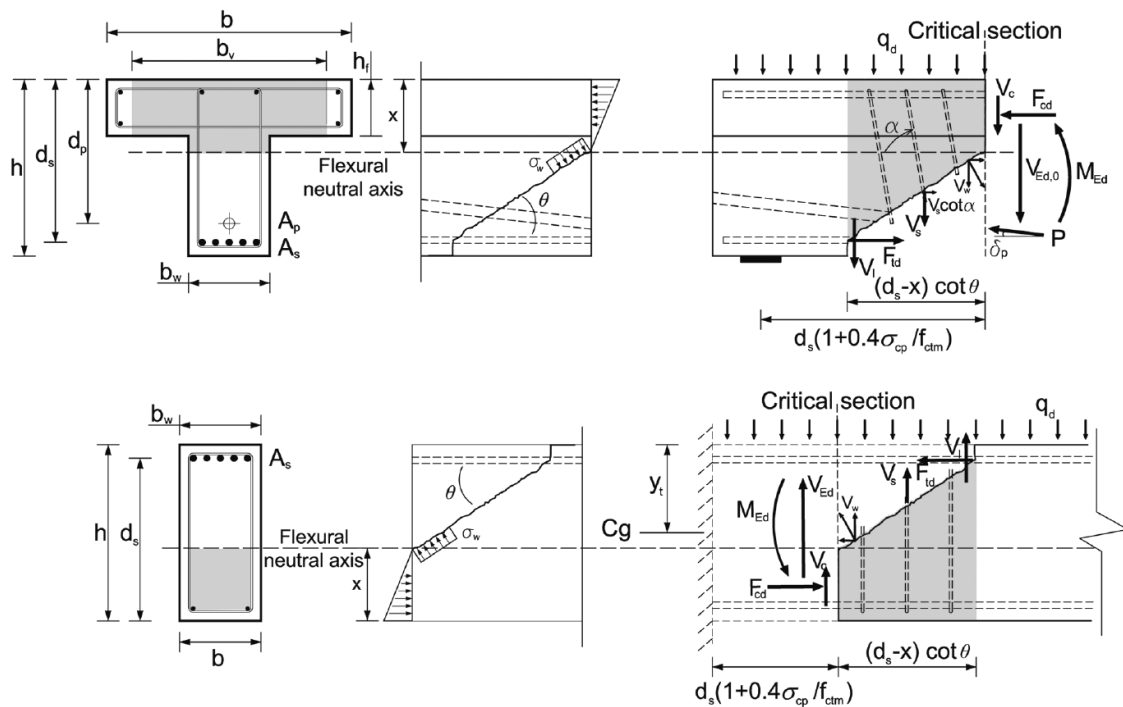


Figure 2.26 – Shear resistance contributions for simply supported and cantilever beams (Cladera et al., 2016).

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The main equations for calculating the shear strength (V_u) of a beam is as follows according to the MASM:

$$V_u = (V_c + V_w + V_l) + V_s = f_{ctm}bd(v_c + v_w + v_l) + v_sf_{ctm}bd \leq V_{u,max} \quad (2.159)$$

$$V_{u,max} = \alpha_{cw}b_wz v_1 f_{cm} \frac{\cot \theta}{1 + \cot^2 \theta} \quad (2.160)$$

where V_c is the contribution of the uncracked compression zone, V_w is the contribution of the cracked concrete web, V_l is the contribution of the dowel action, V_s is contribution of the stirrups, $V_{u,max}$ is the shear strength of the concrete web, f_{ctm} is the mean tensile strength of concrete and b is equal to the flange width for T-beams or web width (b_w) for beams with rectangular cross-sections. The dimensionless values of each of the shear resistance mechanisms are determined using the following set of equations:

$$v_c = \zeta \cdot \left[\left(0.88 + \left(0.20 + 0.50 \cdot \frac{b}{b_w} \right) \cdot v_s \right) \cdot \frac{x}{d} + 0.02 \right] \cdot \frac{b_{v,eff}}{b} \cdot K_p \quad (2.161)$$

$$v_w = 167 \cdot \frac{f_{ctm}}{E_{cm}} \cdot \frac{b_w}{b} \cdot \left(1 + \frac{2 \cdot G_f \cdot E_{cm}}{f_{ctm}^2 d} \right) \quad (2.162)$$

$$v_l = \begin{cases} 0 & \text{for } v_s = 0 \\ 0.23 \cdot \frac{\alpha_e \rho_l}{1 - x/d} & \text{for } v_s > 0 \end{cases} \quad (2.163)$$

$$v_s = (d_s - x) \cdot \cot \theta \cdot \frac{A_{sw} f_{yw}}{s f_{ctm} b d} \approx \frac{0.85 d_s A_{sw} f_{yw}}{s f_{ctm} b d} \quad (2.164)$$

where E_{cm} is the elastic modulus of concrete, α_e is the modular ratio of the longitudinal steel reinforcement to concrete (E_s/E_c), ρ_l is the longitudinal reinforcement ratio taken as A_s/bd , d_s refers to the effective depth, A_{sw} is the area of transverse reinforcement, f_{yw} is the yield strength of the transverse reinforcement and s is the spacing of the transverse reinforcement. The fracture energy of concrete (G_f) can be calculated using the following equation according to fib MC 2010:

$$G_f = 73 \cdot f_{cm}^{0.18} \quad (2.165)$$

where f_{cm} is the mean concrete compressive strength (fib, 2013). The crack inclination angle (θ), which must be less than or equal to 45° , is determined as follows:

$$\cot \theta = \frac{0.85d}{(d - x)} \leq 2.5 \quad (2.166)$$

where the neutral axis depth (x) is calculated using the following equation:

$$\frac{x}{d} = \alpha_e \cdot \rho_l \cdot \left(-1 + \sqrt{1 + \frac{2}{\alpha_e \cdot \rho_l}} \right) \quad (2.167)$$

The effective width of the beam ($b_{v,eff}$) shown in Figure 2.26, which applies only for T-beams, is calculated as follows:

$$b_{v,eff} = \begin{cases} b_w + 2h_f \leq b & \text{for } x \leq h_f \\ b_v \eta + b_w(1 - \eta) & \text{for } x > h_f \end{cases} \quad (2.168)$$

$$\eta = 3 \left(\frac{h_f}{x} \right)^2 - 2 \left(\frac{h_f}{x} \right)^3 \quad (2.169)$$

where h_f is the height of the flange, b_v is the effective flange width and η is a coefficient. The size effect factor in the compression chord (ζ) is calculated using the following equation:

$$\zeta = 1.2 - 0.2 \cdot a \geq 0.65 \quad (2.170)$$

where a is equal to the shear span in metres. If prestressed longitudinal reinforcement is present, then the prestressed cracking moment factor (K_p) should be calculated using the following equation:

$$K_p = 1 + 0.3 \frac{P \cos \alpha (x + d_s - d_p)}{f_{ctm} b d^2} \quad (2.171)$$

where P is the prestressed force, α is the inclination of the active reinforcement as shown in Figure 2.26 and d_p is the effective depth of the prestressed reinforcement.

2.4.3.2 PLASTICITY AND COMPATIBILITY BASED METHODS

The truss analogy initially proposed by Ritter (1899) is the basis of most of the codified shear design methods described in Section 2.4.2 for RC beams. Alternative methods that expand on the classical truss analogy include lower-bound plasticity based stress field models and the strut-and-tie method (STM). In addition, alternative shear design methods also include compatibility based methods such as the Modified Compression-Field Theory (MCFT) model developed by Vecchio and Collins (1986) and nonlinear finite element modelling.

Shear behaviour according to plasticity theory is based on the observation that concrete failure occurs along planes that slide against each other. The theory of plasticity was initially applied to reinforced concrete members loaded in shear by Nielsen and Braestrup (1975). In this approach, concrete is assumed to be a ductile material capable of redistributing stresses along the lines of discontinuity. Additionally, it is assumed that displacements at failure are small. Applying the theory of plasticity involves modelling concrete as a perfectly plastic material using the Mohr-Coulomb failure criterion. The tensile strength of concrete is considered to be equal to zero, and the compressive strength of concrete is reduced through an effectiveness factor (ν). The effectiveness factor (ν) accounts for the peak compressive strength not being achieved in all concrete areas crossed by cracks at failure. Steel reinforcement is typically considered to be rigid and perfectly plastic. The theory of plasticity includes both the upper-bound and lower-bound theorems that can be used to assess the ultimate shear strength of RC beams. The failure load in the upper-bound method is determined by considering a compatible failure mechanism where the rate of internal energy dissipation is equal to the rate of work done by the external loads. The failure load in the lower-bound method is determined through equilibrium between a system of internal resultants and the applied load that does not exceed the yield stress of the member. The estimated solution given by the lower-bound method is either less than or equal to the actual failure load while that given by the upper-bound method

can potentially be unsafe. Therefore, minimising the upper-bound solution may be necessary to identify the correct load capacity, or exact solution, where the solution of the upper-bound method is equal to the highest solution of the lower-bound method.

The strut-and-tie method (STM) is a notable application of the lower-bound theorem of plasticity and also an extension of the truss analogy. The STM is widely used to design assess the discontinuity regions, or D-regions, in RC beams where the stress distribution is not uniform and plane sections do not remain plane as is typically assumed in conventional shear design methods. The discontinuity regions, or D-regions, are the critical shear zones in a RC beam that are often located near the applied load or support (Albajar, 2008). The STM assumes that the applied load is transferred from the loading point to the support through a truss system. The compressive stresses in the concrete are resisted by discrete diagonal struts and the tensile stresses are resisted by the reinforcement bars that act as ties. Equilibrium between the external and internal forces of the truss system is assumed. Designing a RC beam using the STM is an iterative process with the aim of finding a solution where the adopted geometry of the truss system and the truss member forces are compatible. National and international codes such as EC2, fib MC 2010 and ACI 318-19 provide general guidance on the use of the STM, and it has been successfully used in ordinary RC beams as well as complex structures such as corbels and RC beams with openings (ACI, 2019; BSI, 2014; fib, 2013; Pastore, 2020).

The Modified Compression-Field Theory (MCFT) developed by Vecchio and Collins (1986) extends the equilibrium truss model to allow for strain compatibility. The MCFT expands on the Compression Field Theory (CFT) developed by Collins (1978). The MCFT accounts for the residual tensile stress in cracked concrete unlike the CFT which considers the principal tensile stress in concrete between the cracks equal to zero. Applying the MCFT model involves determining the average stresses and strains in the concrete and reinforcement along with the inclination of the diagonal stress fields through the use of a set of equilibrium and strain compatibility equations to ultimately predict the shear deformation response of a RC beam. The average stress-strain relationships of concrete used in the MCFT model were derived by Vecchio and Collins (1986) from test results of RC panels subjected to pure shear. The steel reinforcement in the MCFT model is assumed to be perfectly plastic. The MCFT assumes a rotating smeared crack model where the cracks are uniformly distributed within the material volume yet previous cracks are assumed to be inactive (Albajar, 2008). The MCFT does not consider the bond-slip relationship between concrete and the reinforcement bars which limits

accuracy of the local stresses at the reinforcement bars (Albajar, 2008). The MCFT model has been successfully used to accurately predict the ultimate shear strength of RC beams (Albajar, 2008).

2.4.4 SHEAR DESIGN CODE EQUATIONS FOR UHPC BEAMS

2.4.4.1 AFGC 2013 AND NF P18-710

The shear design equations for UHPC beams given by the current French code NF P18-710 are almost unchanged from the equations given by its predecessor AFGC 2013 (AFGC, 2013; AFNOR, 2016b). NF P18-710 assumes that the shear force is resisted by the UHPC ($V_{Rd,c}$), transverse reinforcement ($V_{Rd,s}$) and fibre reinforcement ($V_{Rd,f}$), and the total shear resistance (V_{Rd}) is limited by the shear strength of the UHPC web ($V_{Rd,max}$). The empirical approach for calculating the contribution of UHPC depends on whether the beam includes transverse reinforcement and prestressed longitudinal reinforcement. The shear strength of a UHPC beam (V_{Rd}) without prestressed reinforcement is calculated using the following set of equations according to NF P18-710:

$$V_{Rd} = V_{Rd,c} + V_{Rd,s} + V_{Rd,f} \leq V_{Rd,max} \quad (2.172)$$

$$V_{Rd,c} = \begin{cases} \frac{0.21}{\gamma_{cf}\gamma_E} k f_{ck}^{1/2} b_w d & \text{for with transverse reinforcement} \\ \frac{0.18}{\gamma_{cf}\gamma_E} k f_{ck}^{1/2} b_w h & \text{for without transverse reinforcement} \end{cases} \quad (2.173)$$

$$V_{Rd,f} = A_{fv} \sigma_{Rd,f} \cot \theta \quad (2.174)$$

where γ_{cf} is a partial safety factor for the fibre reinforcement that is equal to 1.3, γ_E is a safety coefficient that is equal to 1.15 such that $\gamma_{cf}\gamma_E$ is equal to 1.5, k is a prestressed factor that is taken as 1.0 if no prestressed reinforcement is present and θ is the inclination angle between the principal compression stress and the longitudinal axis where the stress tensor can be calculated at the neutral axis through an elastic calculation based on the force torsor (AFGC, 2013; AFNOR, 2016b). Unlike reinforced concrete design recommendations with variable

truss angles such as EC2, AFGC 2013 and NF P18-710 suggest that the truss angle in UHPC beams should be calculated as it differs to that in standard concrete beams due to low shear transfer through aggregate interlock (AFGC, 2013). The inclination of the initial shear crack in a standard concrete beam can differ from that of the final inclination of the shear crack at failure due to aggregate interlock, which is not the case in the UHPC beams according to AFGC 2013 (AFGC, 2013). Based on the elastic calculation for a UHPC beam without axial load or prestressed reinforcement, the inclination angle is equal to 45° , and the inclination is also equal to 45° for UHPC beams with transverse reinforcement (AFGC, 2013; AFNOR, 2016b). NF P18-710 along with its predecessor AFGC 2013 suggest that the minimum allowable inclination angle (θ), or truss angle, is equal to 30° (AFGC, 2013; AFNOR, 2016b). The area of the beam cross-section over which fibre reinforcement contributes to shear resistance (A_{fv}) is equal to $b_w z$, where z is the shear lever arm that is equal $0.9d$ for beams with longitudinal reinforcement or $7h/8$ for beams without longitudinal reinforcement (AFNOR, 2016b). According to NF P18-710, the contribution of the transverse reinforcement ($V_{Rd,s}$) if present is calculated using the same equation given by EC2, which is Eqn. 2.137. The method of calculation for the design residual tensile strength of UHPC ($\sigma_{Rd,f}$) depends on the tensile class of UHPC according to NF P18-710, which is based on the post-cracking behaviour under direct tension. For UHPC that exhibits post-cracking strain-softening (Class T1) or limited strain-hardening (Class T2) behaviour, the residual tensile strength ($\sigma_{Rd,f}$) is calculated using the following equation:

$$\sigma_{Rd,f} = \frac{1}{K\gamma_{cf}} \cdot \frac{1}{w^*} \int_0^{w^*} \sigma_f(w) dw \quad (2.175)$$

$$w^* = \max\{w_u; 0.3 \text{ mm}\} \quad (2.176)$$

where $\sigma_f(w)$ is the tensile stress-crack opening response obtained using the procedures given by Annex D of NF P18-470 that are discussed in Section 2.2.5, K refers to the global fibre orientation factor that may be taken as 1.25 and w^* is the crack opening limit, which is equal to the crack opening obtained in bending at the ultimate limit state with axial force on the end fibre (w_u) if it exceeds a crack opening of 0.3 mm (AFGC, 2013; AFNOR, 2016b). For UHPC that exhibits significant post-cracking strain-hardening behaviour (Class T3), the residual tensile strength ($\sigma_{Rd,f}$) is calculated as follows:

$$\sigma_{Rd,f} = \frac{1}{K\gamma_{cf}} \cdot \frac{1}{\varepsilon^* - \varepsilon_{el}} \int_{\varepsilon_{el}}^{\varepsilon^*} \sigma_f(\varepsilon) d\varepsilon \quad (2.177)$$

$$\varepsilon^* = \max\{\varepsilon_u; \varepsilon_{u,lim}\} \quad (2.178)$$

where $\sigma_f(\varepsilon)$ is the tensile stress-strain response obtained using the procedures given by Annex E of NF P18-470, ε_{el} is the strain at the elastic limit, ε^* is the strain limit, which equals to the maximum elongation obtained in bending at the ultimate limit state with axial force (ε_u) if it exceeds the ultimate limit strain ($\varepsilon_{u,lim}$), which is calculated as follows:

$$\varepsilon_{u,lim} = \frac{L_f}{4L_c} \quad (2.179)$$

where L_f is the length of the longest fibre and L_c is the characteristic length that equals $2h/3$ where h represents the height of the UHPC member (AFNOR, 2016b). The limiting shear strength of the UHPC web ($V_{Rd,max}$) is dependent on whether the beam includes transverse reinforcement according to NF P18-710. For a UHPC beam without transverse reinforcement, NF P18-710 gives the following equation for the shear strength of the UHPC web ($V_{Rd,max}$):

$$V_{Rd,max} = 2.3 \frac{\alpha_{cc}}{\gamma_c} b_w z f_{ck}^{2/3} \tan \theta \quad (2.180)$$

where α_{cc} is a coefficient that considers the long-term effects on compressive strength that may be taken equal to 1.0 and γ_c is a partial safety coefficient for UHPC that is equal to 1.5. For a UHPC beam with transverse reinforcement, the following equation is given by NF P18-710 for the shear strength of the UHPC web ($V_{Rd,max}$):

$$V_{Rd,max} = 2.3 \frac{\alpha_{cc}}{\gamma_c} b_w z f_{ck}^{2/3} \left[\frac{V_{Rd,s}(\cot \theta + \cot \alpha)}{(1 + \cot^2 \theta)} + V_{Rd,f} \tan \theta \right] \cdot \left[\frac{1}{V_{Rd,s} + V_{Rd,f}} \right] \quad (2.181)$$

where α is inclination angle of the transverse reinforcement. The equations given by AFGC 2013 are identical to those given by NF P18-710, excluding the equation for the shear strength of the UHPC web ($V_{Rd,max}$). The shear strength of the UHPC web ($V_{Rd,max}$) for beams with

and without transverse reinforcement is calculated using the following equation according to AFGC 2013:

$$V_{Rd,max} = 2 \cdot 1.14 \frac{\alpha_{cc}}{\gamma_c} b_w z f_{ck}^{2/3} / (\cot \theta + \tan \theta) \quad (2.182)$$

2.4.4.2 SIA 2052

The shear design equations for UHPC beams given by the Swiss recommendations SIA 2052 assume that the shear force is resisted by the fibre ($V_{Rd,U}$) and transverse reinforcement ($V_{Rd,s}$) only (EPFL, 2016). The shear strength (V_{Rd}) of a UHPC beam is calculated as follows according to SIA 2052:

$$V_{Rd} = V_{Rd,U} + V_{Rd,s} \quad (2.183)$$

$$V_{Rd,U} = 0.5(f_{Uted} + f_{Utud})b_w z / \tan \alpha \quad (2.184)$$

$$V_{Rd,s} = A_{sw} f_{sd} z (\cot \alpha + \cot \beta) \sin \beta / s \quad (2.185)$$

where f_{Uted} is the direct tensile strength of UHPC at the elastic limit, f_{Utud} is the direct tensile strength of UHPC, z is the shear lever arm that is taken as $0.9d$, A_{sw} is the area of transverse reinforcement, f_{sd} is the design yield strength of the transverse reinforcement and β is the inclination of the transverse reinforcement (EPFL, 2016). SIA 2052 provides guidance on how to derive the direct tensile strength of UHPC at the elastic limit (f_{Uted}) and at peak (f_{Utud}) from either direct or indirect tensile test results (EPFL, 2016). According to SIA 2052, the truss angle (α) should be greater than or equal to the minimum truss angle (α_{min}) and less than or equal to 45° (EPFL, 2016). The minimum truss angle (α_{min}) is equal to 30° for normal loading conditions, and it is equal to 25° when the web is subjected to considerable axial stress or 40° when the web is subjected to considerable tensile stress or if plastic deformations of the flanges are expected (EPFL, 2016). The determined shear strength (V_{Rd}) is limited by the shear strength of the UHPC web as it must satisfy the following condition according to SIA 2052:

$$\frac{V_d}{V_{Rd}} \cdot \left[\frac{V_{Rd,U}}{\tan \alpha} + \frac{V_{Rd,s}}{(\cot \alpha + \cot \beta \cdot \sin \alpha) \cdot \sin \alpha} \right] \leq 0.55 b_w z f_{Ucd} \quad (2.186)$$

where V_d is the design shear force and f_{Ucd} is the design UHPC compressive strength (EPFL, 2016).

2.4.4.3 JSCE 2006

The shear design equations for UHPC beams given by the Japanese recommendations JSCE 2006 assume that the shear force is resisted by the UHPC (V_{rpcd}), fibre reinforcement (V_{fd}) and prestressed longitudinal reinforcement (V_{ped}) if present (JSCE, 2006). The design recommendations given by JSCE 2006 are meant for UHPC beams without transverse reinforcement only, therefore the contribution to shear resistance of the transverse reinforcement is not considered. The shear strength (V_{yd}) is limited by the shear strength of the UHPC web (V_{wcd}). The shear strength (V_{yd}) of a UHPC beam without transverse reinforcement is calculated using the following set of equations according to JSCE 2006:

$$V_{yd} = V_{rpcd} + V_{fd} + V_{ped} \leq V_{wcd} \quad (2.187)$$

$$V_{rpcd} = 0.18 \sqrt{f'_{cd}} b_w d / \gamma_b \quad (2.188)$$

$$V_{fd} = (f_v / \tan \beta_u) b_w z / \gamma_b \quad (2.189)$$

$$V_{ped} = P_{ed} \sin \alpha_p / \gamma_b \quad (2.190)$$

$$V_{wcd} = 0.84 f'_{cd}{}^{2/3} \sin(2\beta_u) b_w d / \gamma_b \quad (2.191)$$

where f'_{cd} is the design compressive strength of UHPC, γ_b is the member reduction factor that is equal to 1.3 in Eqns. 2.188, 2.189 and 2.191 and 1.1 in Eqn. 2.190, z is the shear lever arm taken as $0.9d$, P_{ed} is the effective tensile force in the prestressed longitudinal reinforcement and α_p is the angle of the longitudinal reinforcement. The truss angle (β_u) should lie between a minimum value of 30° and a maximum value of 45° , and additional guidance is given by

JSCE 2006 for calculating the truss angle (β_u) for special cases such as for beams subjected to a combination of axial and shear stresses or beams with prestressed reinforcement (JSCE, 2006). The design tensile strength of UHPC (f_{vd}) is based on the AFGC guidelines published in 2002, which are similar to those of AFGC 2013 and NF P18-710 (AFGC, 2002, 2013; AFNOR, 2016b). According to JSCE 2006, the average tensile strength (f_v) was taken directly from AFGC 2002, and it is the following:

$$f_v = \frac{1}{K_f} \cdot \frac{1}{w_{lim}} \int_0^{w_{lim}} \sigma(w) dw \quad (2.192)$$

where $\sigma(w)$ is the tensile stress-crack opening response, K_f refers to global fibre orientation factor that is equal to 1.25 and w_{lim} is the limiting crack opening that is generally set to 0.3 mm (AFGC, 2002; JSCE, 2006). The average tensile strength (f_v) of JSCE 2006 in Eqn. 2.192 is almost identical to the residual tensile strength ($\sigma_{Rd,f}$) given by NF P18-710 in Eqn. 2.175 seeing as they are both based on the work of the AFGC (AFGC, 2002, 2013; AFNOR, 2016b; JSCE, 2006). The average tensile strength (f_v) is multiplied by a material factor that accounts for fibre orientation (γ_c) to form the equation for the design tensile strength (f_{vd}), and it is the following:

$$f_{vd} = \frac{1}{w_{lim}} \int_0^{w_{lim}} \frac{\sigma_k(w)}{\gamma_c} dw = \frac{1}{w_{lim}} \int_0^{w_{lim}} \sigma_d(w) dw \quad (2.193)$$

where γ_c is the fibre orientation factor that is equal to 1.3 and $\sigma_k(w)$ and $\sigma_d(w)$ are the characteristic and design tensile stress-crack opening responses (JSCE, 2006).

2.5 EXPERIMENTAL REVIEW OF RC BEAMS WITH UHPC JACKETS

2.5.1 INTRODUCTION

Several researchers have studied the behaviour of RC beams with UHPC jackets by performing bending tests on medium-scale specimens (Alkhoury, 2017; Al-Osta et al., 2017; Bahraq et al., 2019; Choi, 2016; Daugevičius et al., 2016; Hong and Lim, 2023; Liu and Charron, 2023; Martinola et al., 2010; Paschalis, 2017; Meda et al., 2012; Martins et al., 2021; Said et al.,

2022; Sakr et al., 2019; Wang et al., 2013, 2015a, 2015b). Experimental work primarily focused on the behaviour of jacketed beams that failed in flexure or combined flexure-shear, with fairly little work focused on the behaviour of jacketed beams failing in shear. Experimental campaigns from the literature normally consisted of a control RC beam along with a set of identical RC beams with various types of UHPC jackets in order to investigate the influence of the jackets on the shear and flexural behaviour of the RC beam. The beam specimens from the literature were all subjected to either three- or four-point bending tests with symmetrical or asymmetrical type monotonic loading until failure. Tested specimens generally consisted of a RC beam with either a rectangular or T-shaped cross-section that was strengthened with either a side or U-shaped UHPC jacket. The test specimens were typically manufactured by roughening the bonding surfaces of the beams through either particle impact removal, mechanical removal or hydrodemolition, and the jackets were produced with either cast in-situ UHPC or by bonding precast UHPC panels. The general jacket configurations are shown in Figure 2.27, with the jackets typically spanning across the entire beam length excluding the specimens with unconventional jacket configurations shown in Figure 2.28. The thicknesses of the side and bottom layers of the UHPC jackets ranged from a minimum of 7 mm to a maximum of 60 mm, with very few jackets including embedded longitudinal and transverse reinforcement.

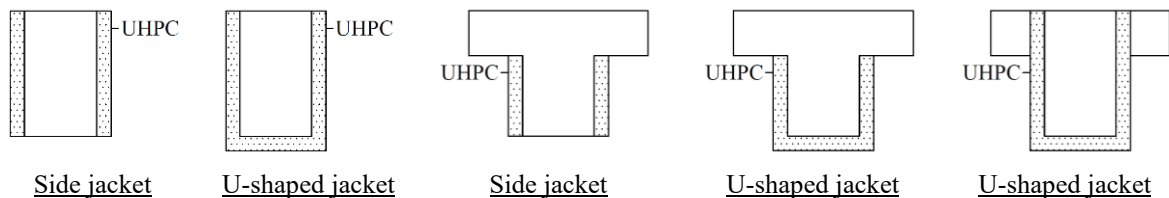
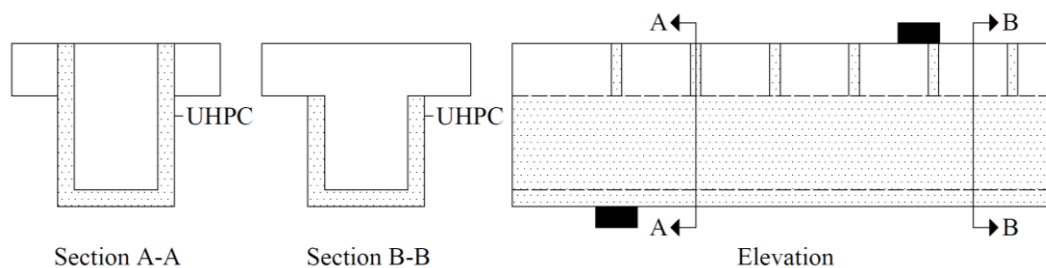


Figure 2.27 – Configurations of UHPC jackets in tested beams from the literature



Specimens tested by Alkhoury (2017)

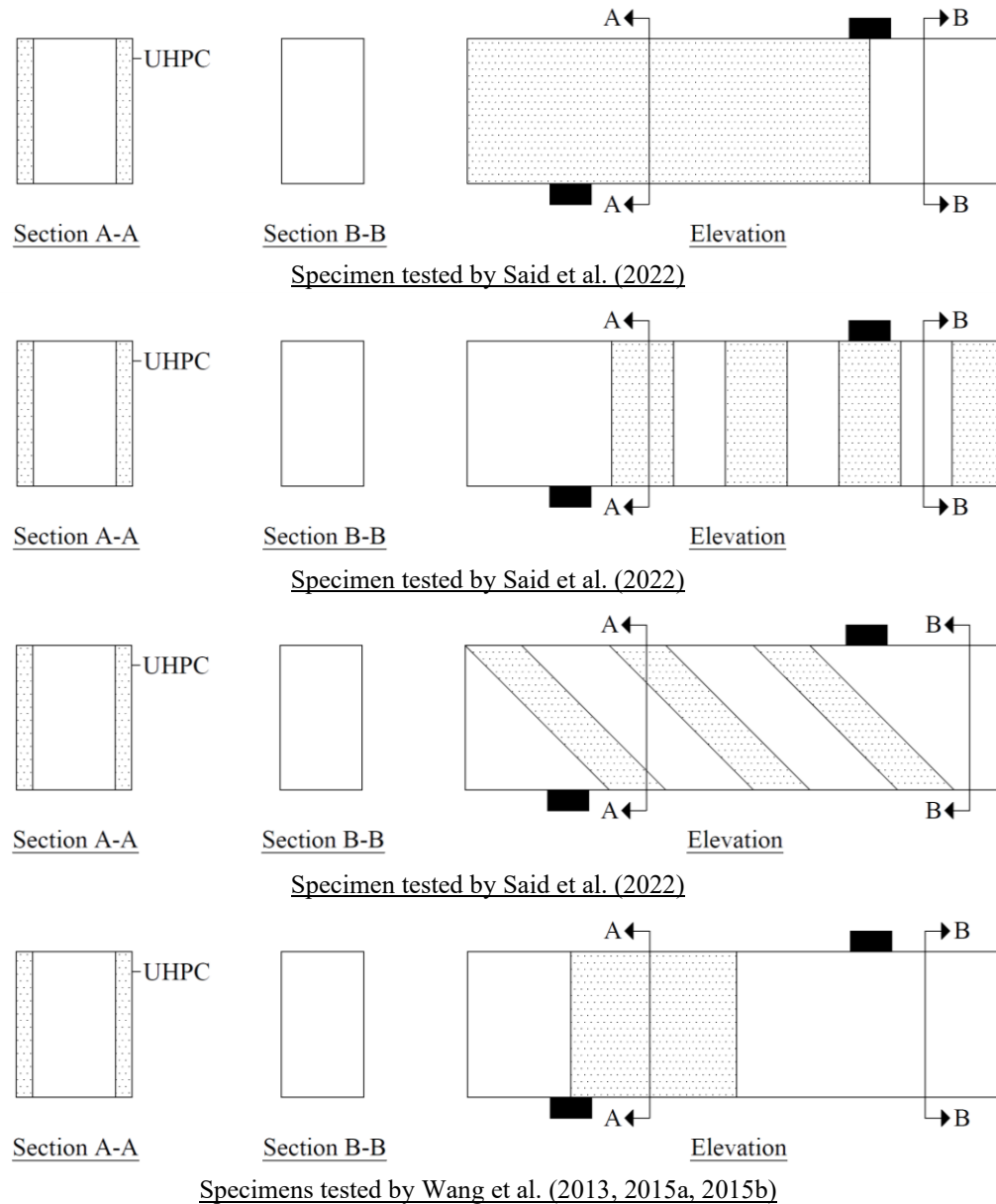


Figure 2.28 – Drawings of unconventional UHPC jacket configurations in tested beams from the literature

2.5.2 KEY OBSERVATIONS AND FINDINGS

Researchers have successfully demonstrated through bending tests the effectiveness of UHPC jackets in increasing strength and altering the failure modes of existing RC beams (Alkhoury, 2017; Al-Osta et al., 2017; Bahraq et al., 2019; Choi, 2016; Daugevičius et al., 2016; Hong and Lim, 2023; Liu and Charron, 2023; Martinola et al., 2010; Paschalis, 2017; Meda et al., 2012; Martins et al., 2021; Said et al., 2022; Sakr et al., 2019; Wang et al., 2013, 2015a, 2015b). The addition of a UHPC jacket without embedded longitudinal or transverse reinforcement bars was found to increase the strength of a RC beam with a rectangular cross-section up to 120%

as in specimen 3SJ1 of Paschalis (2017), which consisted of a U-shaped jacket with a 40 mm side layer thickness and 50 mm bottom layer thickness. Alkhoury (2017) reported a 270% increase in strength for a T-beam strengthened with a UHPC U-shaped jacket that reached the top surface of the beam as shown in Figure 2.27 (Specimen Nr. 7). The jacket, which included longitudinal and transverse reinforcement, had side and bottom widths of 40 mm and 60 mm respectively. The significant increase in strength due to the addition of the UHPC jackets can change the failure mode from brittle shear to preferable ductile flexure as reported by Bahraq et al. (2019), Choi (2016), Hong and Lim (2023) and Wang et al. (2013).

According to test results from the literature, the interfacial bond between the NSC beams and UHPC jackets is typically sufficient with significant jacket debonding not generally observed. Debonding throughout loading or at failure was generally not observed in the several beams with U-shaped jackets, apart from three specimens (TS-U25, TS-U50 and TS-U50A) tested by Liu and Charron (2023) and a single specimen (SC) tested by Meda et al. (2012). Liu and Charron (2023) and Meda et al. (2012) reported minor debonding of the jackets throughout loading and at failure, yet the test results were mostly uncompromised apart from specimen TS-U50 of Liu and Charron (2023) that failed prematurely. It is possible that debonding occurred as a result of the manufacturing of the jackets such as the use of two different types of UHPC (Thixotropic/Self-compacting) in specimen SC of Meda et al. (2012) to cast the side and bottom layers of the jacket, respectively. For beams with side jackets, Hong and Lim (2023) and Wang et al. (2013, 2015a, 2015b) observed minor debonding, or peeling, during loading prior to failure although the test results appear to be generally uncompromised. Debonding issues may have been related to the manufacturing of the jackets of Wang et al. (2013, 2015a, 2015b) as they were all manufactured by bonding precast UHPC panels onto the roughened surfaces of the RC beams, which heavily relies on the applied adhesive. Debonding may have also occurred in the beams with side jackets due to differences in deformability between the two materials as well as a lack of anchorage as the side jacket offers less anchorage than a U-shaped jacket due to the absence of a bottom layer.

Very little experimental work focused on jacketed RC beams failing in shear as focus was more on flexural failure. Beams with both side and U-shaped jackets that failed in shear were tested by the following researchers: Alkhoury (2017), Liu and Charron (2023), Martins et al. (2021), Said et al. (2022), Sakr et al. (2019) and Wang et al. (2013, 2015b). Prior studies generally reported on the load, displacement at midspan, strain development in the reinforcement bars

and general crack pattern at failure. Most prior studies focused on testing strengthened specimens with a set of jackets that varied in both side and bottom thickness. Therefore, the effect of the jacket side thickness alone on the ultimate shear strength was not identified. Non-optical measurement techniques such as DIC were not applied in most studies with exception of a single study by Liu and Charron (2023). As a result, the width and shape of the critical shear crack was not monitored during the tests. This is a notable omission, since the width and shape of critical shear crack are instrumental for determining the shear resistance mechanisms according to the theoretical models discussed in Sections 2.3. Therefore, the effects of the UHPC jacket on the shear resistance mechanisms have not been identified by prior studies. Some bending tests were unsuccessful in capturing the full potential shear force carrying capacity due to insufficient loading arrangements as the shear span to depth ratio (a_v/d) was less than 2.5 (Wang et al., 2013, 2015b). Ideally, beams should be tested with a shear span to depth ratio (a_v/d) that is greater than 3.5 to avoid any potential load transfer to the support through direct compression (Kani, 1964; Kani et al., 1979). Moreover, RC and UHPC design codes suggest the use of shear strength enhancement factors for beams that have been tested with a shear span to depth ratio (a_v/d) that is less than 2.0 (ACI, 2019; AFGC, 2013; AFNOR, 2016b; BSI, 2014; fib, 2013).

Researchers have yet to develop a mechanical model for the shear design or assessment of a RC beam with a UHPC jacket (Liu and Charron, 2023). Instead, researchers such as Choi (2016) and Wang et al. (2015b) attempted to verify their test results with little success by comparing the ultimate shear strength of the jacket with the predicted shear strength using equations given by design codes and recommendations. On the other hand, researchers appeared to be more successful when verifying the ultimate flexural strength achieved during the tests with the predicted flexural strength that was calculated using cross-section analysis based on simplified material models for NSC and UHPC (Al-Osta et al., 2017; Hong and Lim, 2023). Overall, the lack of relevant experimental data for jacketed beams failing in shear hinders the development of rational models for assessing the strength of RC beams with UHPC jackets. Available methods of calculation for the shear and flexural strength of RC beams with UHPC jackets are presented and discussed later in Chapter 7.

2.6 SUMMARY AND CONCLUSIONS

This chapter initially presents the background of ultra high performance concrete related to its development, composition and characterisation of its key material properties. Research in UHPC production since the 1970s has ultimately led to the development of several international UHPC design codes and recommendations such as NF P18-710 and SIA 2052 along with the recent global use of UHPC by the construction industry in bridge construction and the repair of existing concrete structures (AFNOR, 2016b; Brühwiler and Denarié, 2008; EPFL, 2016). A UHPC mixture typically consists of cement, fine aggregate, steel fibres and a water to cement ratio of around 0.2 (Toutlemonde and Resplendino, 2011). The use of a theoretical particle packing model is common when developing a UHPC mixture in order to minimize porosity and enhance mechanical and durability properties. The modified Andreasen particle packing model is commonly used in UHPC mixture design as it considers the infinite size range of the particle size classes for all granular materials, although it does not directly account for water content (Brouwers and Radix, 2005; Funk and Dinger, 1994; Vogt, 2010; Yu et al., 2015). Design codes and recommendations such as NF P18-470 and SIA 2052 offer guidance for characterising the key rheological and mechanical properties of fresh and hardened UHPC such as workability, compressive strength, elastic modulus and direct tensile strength amongst others (AFNOR, 2016a; EPFL, 2016). Properly characterising the direct tensile strength of UHPC, which is a key design parameter, is essential, and it requires additional care due to the unique behaviour of UHPC in tension caused by the steel fibre reinforcement. The direct elastic and post-cracking tensile strength can be determined indirectly through bending tests and the application of an inverse analysis method, with Annex D of NF P18-470 providing comprehensive procedures on specimen production, loading test arrangements and monitoring techniques (AFNOR, 2016a).

The behaviour of reinforced concrete beams failing in shear has also been introduced in this chapter. Despite the lack of consensus on a theory describing shear behaviour, the research community agrees that there are up to six mechanisms that contribute to shear resistance in reinforced concrete beams (ASCE-ACI Committee 445, 1998). For RC beams with transverse reinforcement, the shear resistance mechanisms mainly include aggregate interlock, dowel action, stirrups and the uncracked compression zone. Another main shear resistance mechanism may be fibre bridging if steel fibre reinforcement is present as is the case with UHPC, where aggregate interlock does not contribute to shear resistance due to the absence of coarse

aggregate. Other notable resistance mechanisms in RC beams that are not discussed in this chapter include the shear resisted by the residual tensile stresses in the crack tip as well as the shear resisted through arching action. Several theoretical models for each shear resistance mechanism are available in the literature, which can be used to estimate the contribution of each mechanism to the total shear resistance. This chapter presents a number of theoretical models for the main shear resistance mechanisms along with outlining each of their assumptions and methods of calculation.

Design codes define shear strength by either one or two resistance mechanisms as they do not directly account for all shear resistance mechanisms (ASCE-ACI Committee 445, 1998). Design codes for RC beams with transverse reinforcement assume that the shear force is mainly resisted by the transverse reinforcement, which is calculated based on a truss model. The truss model relies heavily on the adopted truss angle, which is a variable for EC2 and fib MC 2010 that ranges between a minimum value and 45° and it is fixed at 45° for ACI 318-19 (ACI, 2019; BSI, 2014; fib, 2013). Design codes such as fib MC 2010 and ACI 318-19 also consider the contribution of concrete to shear resistance (ACI, 2019; fib, 2013). UHPC design codes and recommendations assume that the shear strength is resisted by the fibre reinforcement, which is calculated based on a truss model with a suggested minimum truss angle of 30° (AFGC, 2013; AFNOR, 2016b; EPFL, 2016; JSCE, 2006). According to AFGC 2013 and NF P18-710, the truss angle should be determined through an elastic calculation of the uncracked section (AFGC, 2013; AFNOR, 2016b). Therefore, the inclination angle is equal to 45° based on the elastic calculation for a UHPC beam without axial load or prestressed reinforcement as well as beams with transverse reinforcement. UHPC design codes consider the contribution of the transverse reinforcement using the same truss model, although the equations given by JSCE 2006 do not consider the contribution of the transverse reinforcement (AFGC, 2013; AFNOR, 2016b; EPFL, 2016; JSCE, 2006). Design codes and recommendations such as AFGC 2013, NF P18-710 and JSCE 2006 also consider the contribution of the UHPC matrix, although SIA 2052 does not (AFGC, 2013; AFNOR, 2016b; EPFL, 2016; JSCE, 2006).

Researchers have successfully measured the contribution of each shear resistance mechanism in RC beams, but not UHPC strengthened RC beams, by using the latest technologies in non-contact optical monitoring techniques such as digital image correlation (DIC) (Campana et al., 2013; Huber et al., 2016; Pastore, 2020). The contributions of each shear resistance mechanism

are based on the crack kinematics and shape of the critical shear crack as well as its location. More recently, mechanical based models have been developed by researchers which explicitly consider the contribution of each shear resisting mechanism (Cladera et al., 2016; Marí et al., 2015, 2016). The Multi-Action Shear Strength Model (MASM) is a mechanical model that was initially developed for slender beams, which also considers RC beams with rectangular and T-shaped cross-sections (Cladera et al., 2016; Marí et al., 2015, 2016). The shear strength according to the MASM is the sum of the following shear resistance mechanisms: uncracked compression zone, cracked concrete web, dowel action and stirrups if present. Unlike design codes such as EC2, fib MC 2010 and ACI 318-19 that only consider the shear contribution of the web in the uncracked compression zone, the MASM considers the contribution of the flange as well. In addition to mechanical models, alternative methods can be used to assess the ultimate shear strength of RC beams such as lower-bound plasticity based stress field models and the strut-and-tie method (STM) as well as compatibility based methods such as the Modified Compression-Field Theory (MCFT) model.

A number of studies have investigated the behaviour of RC beams with UHPC jackets by subjecting medium-scale specimens to bending tests with monotonic loading until failure (Alkhoury, 2017; Al-Osta et al., 2017; Bahraq et al., 2019; Choi, 2016; Daugevičius et al., 2016; Hong and Lim, 2023; Liu and Charron, 2023; Martinola et al., 2010; Paschalis, 2017; Meda et al., 2012; Martins et al., 2021; Said et al., 2022; Sakr et al., 2019; Wang et al., 2013, 2015a, 2015b). Tests normally included a control RC beam along with a set of identical RC beams that were strengthened with relatively thin UHPC side or U-shaped jackets as shown in Figure 2.27 to study the influence of the jackets on the shear and flexural behaviour. Test results showed increases in strength of up to 120% and 270% due to the addition of UHPC jackets without and with embedded reinforcement, respectively (Alkhoury, 2017; Paschalis, 2017). Researchers also demonstrated that the addition of a UHPC jacket can alter the failure mode of a RC beam from a brittle shear failure to a more preferable ductile flexural failure (Bahraq et al., 2019). Overall, the interfacial bond between the NSC beams and UHPC jackets appeared to be sufficient as jacket debonding was generally not observed.

Very little experimental work focused on jacketed RC beams failing in shear as emphasis was more on flexural failure. The few prior studies that investigated the shear behaviour of jacketed beams did not apply any non-optical measurement techniques such as DIC in their experiments, which would allow for the estimation of each shear resistance mechanism (Alkhoury, 2017;

Sakr et al., 2019; Wang et al., 2013, 2015b). Therefore, the direct effect of the UHPC jacket on each shear resistance mechanism has not been identified. A few tests adopted a shear span to depth ratio (a_v/d) that was less than 2.5, which means that the full potential shear capacity may not have been captured as part of the applied load may have been transferred to the support through direct compression (Kani, 1964; Kani et al., 1979; Wang et al., 2013, 2015b). Researchers have yet to develop a mechanical model for the shear design or assessment of a RC beam with a UHPC jacket (Liu and Charron, 2023). Moreover, the lack of relevant experimental data for jacketed beams failing in shear hinders the development of rational models for assessing the strength of RC beams with UHPC jackets.

CHAPTER 3: EXPERIMENTAL METHODOLOGY

3.1 INTRODUCTION

This chapter describes the experimental methodology adopted in this research in which two groups of four beams were tested. Each group of beams consisted of four identical RC beams. Three beams of each set were strengthened with UHPC U-shaped jackets with variable geometry and amount of embedded longitudinal reinforcement. The fourth beam was not strengthened to enable the effect of the jackets to be determined. The aim of these tests was to investigate the impact of the UHPC jackets on shear behaviour, with focus primarily on the effect of the jacket on each shear resistance mechanism, angle of the critical shear crack and shear strength enhancement through the side layers. The RC beams of both groups were cast with NSC. Both groups of beams were designed to fail in shear but the RC beam in group 1 was more heavily reinforced than in group 2. The beams were loaded to failure under three-point loading. Full details of the loading arrangement, beam geometry, reinforcement details and instrumentation of each specimen are presented in this chapter. The manufacturing process of each specimen, which included the development of a non-proprietary UHPC through trial mixing, is also described. Additional small-scale tests on NSC-UHPC composite cylinder specimens were also carried out to characterise the interfacial bond. The experimental program of this research was carried out in the Structures Laboratory of the Department of Civil and Environmental Engineering at Imperial College London.

3.2 OVERVIEW OF EXPERIMENTAL PROGRAM

3.2.1 INTRODUCTION

A total of eight medium-scale T-shaped beams were tested. The beams were divided into two groups of four. Each group consisted of four identical RC beams of which one was tested as a control. The remaining three beams of each group were strengthened with UHPC U-shaped jackets with varying geometry and area of embedded longitudinal reinforcement. The RC T-beams were 2600 mm in length and 350 mm deep. The width of the web was 200 mm while the flange was 500 mm wide and 125 mm deep. The U-shaped jacket extended over the entire beam length and wrapped around the web of the beam up to the web/flange junction. The specimens were loaded asymmetrically in three-point bending until failure. The typical loading

arrangement is presented in Figure 3.1 along with the corresponding shear force and bending moment diagrams. The specimens were simply supported, and the test span length was equal to 2100 mm, with the left and right spans equal to 1200 mm and 900 mm, respectively. The theoretical loading ratios of the left and right supports are equal to $0.43P$ and $0.57P$, respectively, with the term P referring to the applied load. An asymmetrical loading arrangement was applied to focus investigation efforts and instrumentation on the left span, and to achieve a shear span to depth ratio (a_v/d) greater than 3.5 to avoid any potential load transfer to the support through direct compression (Kani, 1964; Kani et al., 1979).

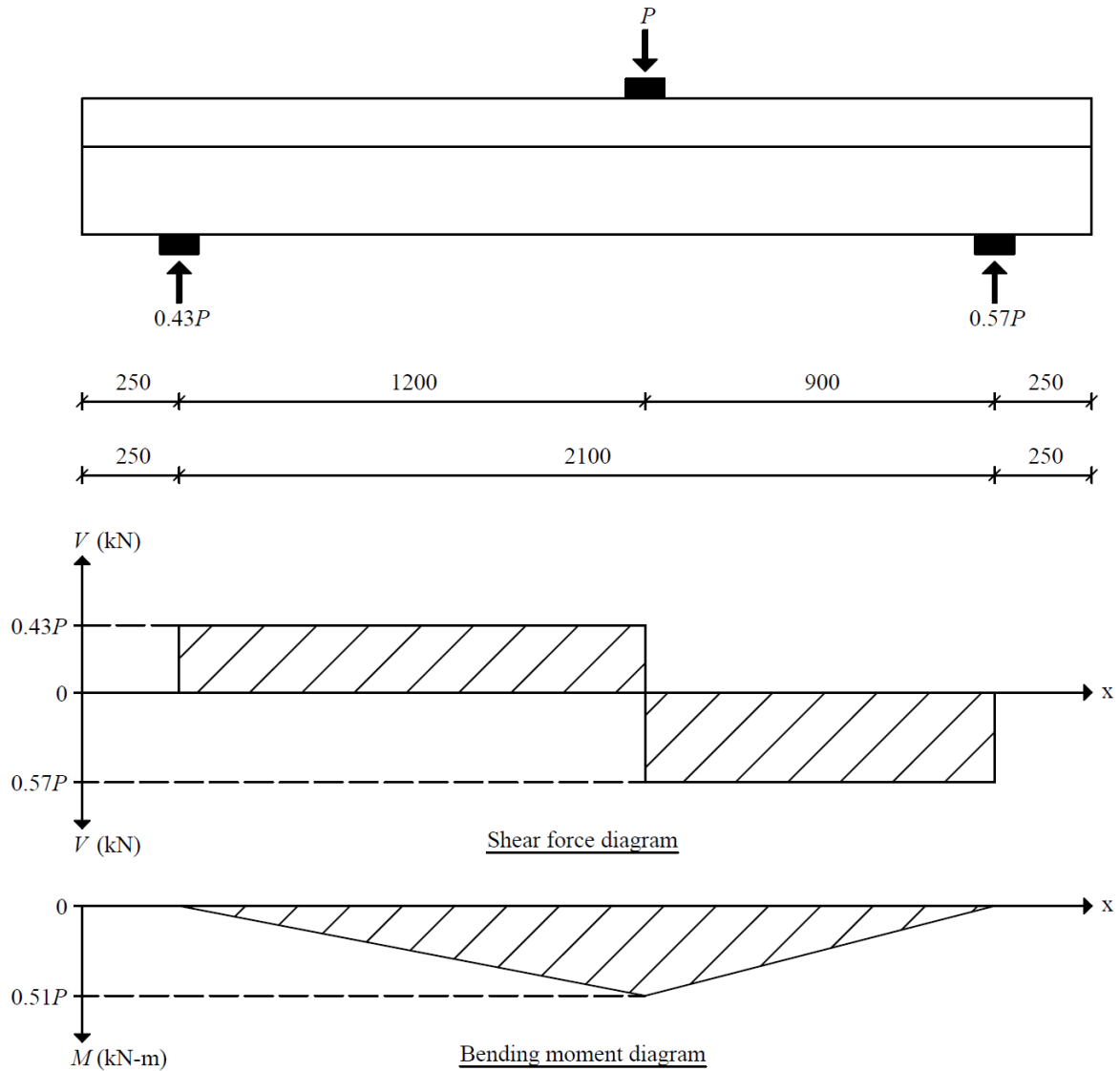


Figure 3.1 – Typical loading arrangement of beam specimens with shear force and bending moment diagrams (Dimensions in mm)

The number of tested RC beams with UHPC U-shaped jackets was limited to three per group to ensure time- and cost-effectiveness, which in turn limited the number of test parameters that

could be varied and studied. The aim in the tests of both groups 1 and 2 was to strengthen a RC beam such that shear failure occurs in order to investigate the shear resisting mechanisms at shear failure. The test parameters that were investigated in the jacketed beams include the side thickness of the jacket and amount of longitudinal reinforcement embedded in the jacket. The tests of group 1 focused on strengthening a RC beam that was over designed in flexure relative to shear. The side thickness of each jacket of group 1 was increased as it was thought at the time that the shear resistance would increase in proportion to the jacket side thickness. The jackets of group 1 were heavily reinforced in flexure and comprised of a constant amount longitudinal reinforcement bars to focus solely on the effect of the jacket side thickness in shear resistance. The tests of group 2 focused on strengthening a RC beam that was efficiently designed in both shear and flexure, thereby ensuring that shear failure occurred just before flexural failure. Consequently, the amount of flexural tension reinforcement embedded in each jacket was increased along with the jacket side thickness to ensure the strengthened beams reached their shear strength without premature flexural failure.

3.2.2 GROUP 1 SPECIMENS

The RC beam of group 1 was heavily reinforced in flexure, and the minimum required amount of shear reinforcement was provided in the critical shear span (BSI, 2014). It included two layers of $3\phi 20$ longitudinal reinforcement in tension and two layers of $2\phi 10$ longitudinal reinforcement in compression. The shear reinforcement in the critical span comprised $\phi 6$ stainless steel stirrups, and $\phi 10$ steel stirrups elsewhere. Stainless steel stirrups were used since 6 mm bars were unavailable in grade B500B steel (BSI, 2016). The cross-section and elevation of the control RC beam specimen (B1) is shown in Figures 3.2 and 3.3, respectively. The bottom layer of the UHPC jackets of group 1 (B2, B3 and B4) was 60 mm thick with a single layer of $4\phi 20$ longitudinal reinforcement. The side thickness of the jackets was 20 mm, 30 mm, and 40 mm for specimens B2, B3 and B4, respectively as shown in Figures 3.4 and 3.5, respectively. A summary of the main characteristics of each specimen is presented in Table 3.1.

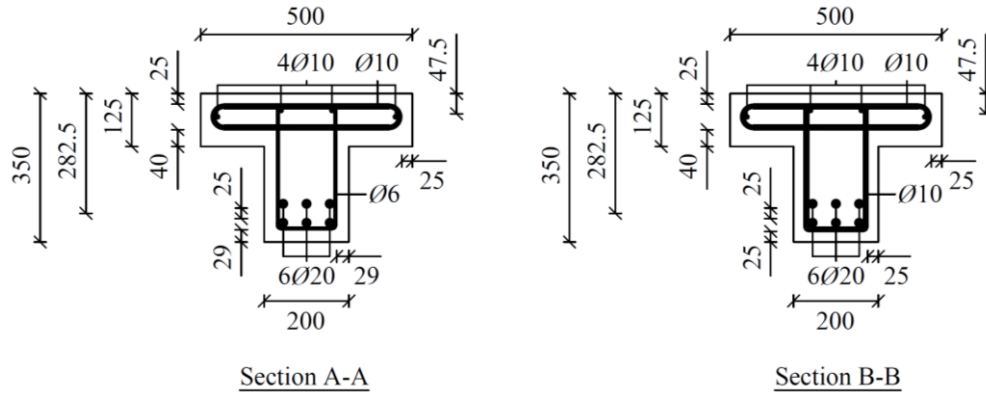


Figure 3.2 – Cross-section of specimen B1 (Dimensions in mm)

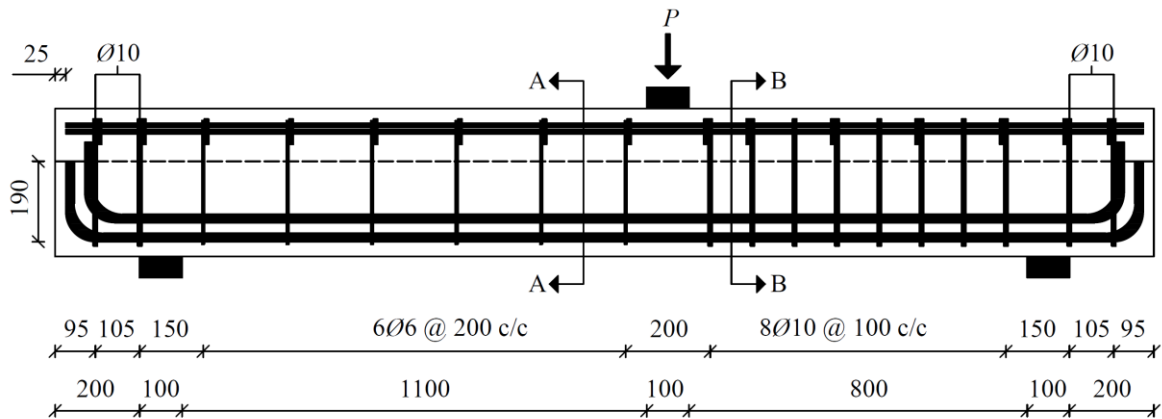


Figure 3.3 – Elevation of specimen B1 including test setup (Dimensions in mm)

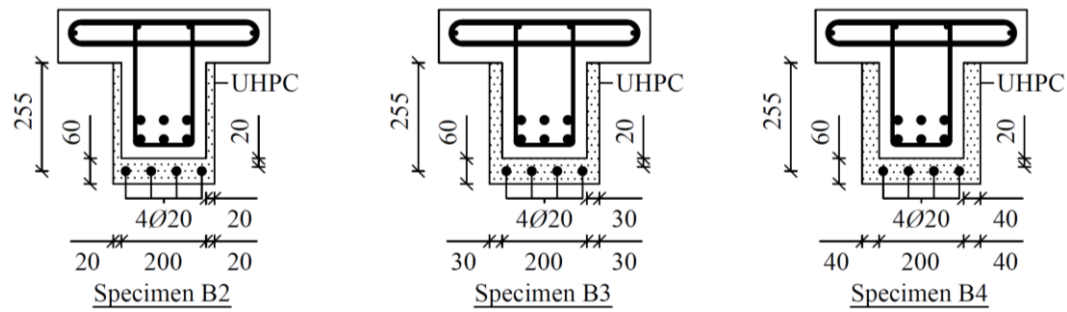


Figure 3.4 – Cross-sections of specimens B2, B3 and B4 of group 1 (Dimensions in mm)

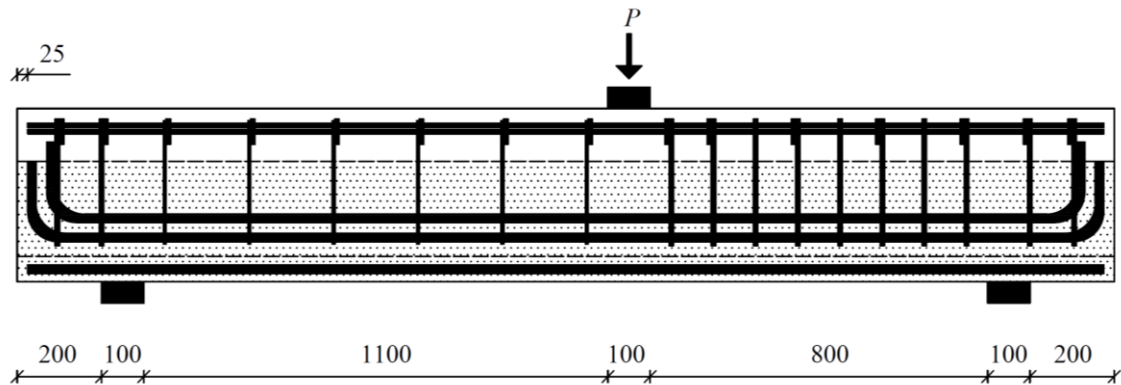


Figure 3.5 – Elevation of specimens B2, B3 and B4 of group 1 including test setup (Dimensions in mm)

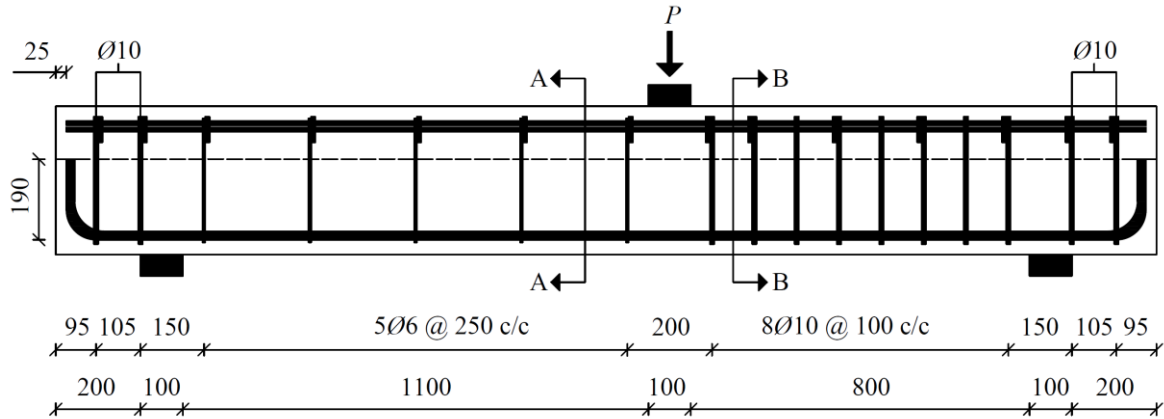


Figure 3.7 – Elevation of specimen B5 including test setup (Dimensions in mm)

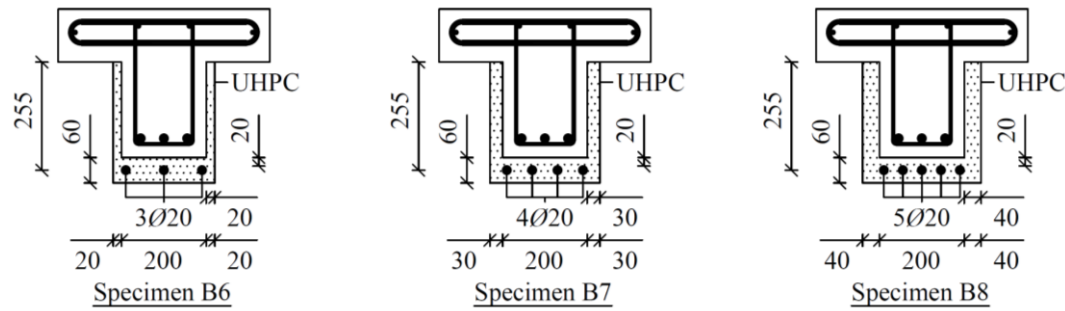


Figure 3.8 – Cross-sections of specimens B6, B7 and B8 of group 2 (Dimensions in mm)

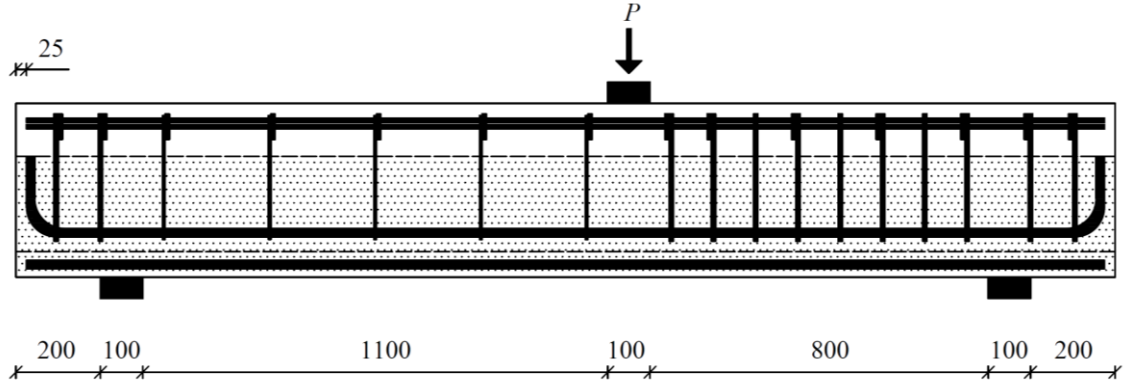


Figure 3.9 – Elevation of specimens B6, B7 and B8 of group 2 including test setup (Dimensions in mm)

Table 3.2 – Main characteristics of group 2 specimens

Specimen	Depth (d) [mm]	Load ratio (a_v/d)	Beam long. reinf. (A_s)	Beam shear reinf. (A_{sw})	Beam long. reinf. ratio ^A (ρ_l) [%]	Beam shear reinf. ratio ^B (ρ_w) [%]	Jacket side thickness (t_s) [mm]	Jacket reinf. (A_s)
B5	305.0	3.61	3Ø20	5Ø6 @ 250 mm	1.55	0.11	-	-
B6	342.5 ^C	3.21	3Ø20	5Ø6 @ 250 mm	1.55	0.11	20	3Ø20
B7	347.9 ^C	3.16	3Ø20	5Ø6 @ 250 mm	1.55	0.11	30	4Ø20
B8	351.9 ^C	3.13	3Ø20	5Ø6 @ 250 mm	1.55	0.11	40	5Ø20

^A Longitudinal reinforcement ratio (ρ_l) equals to $A_s/b_w d$; ^B Shear reinforcement ratio (ρ_w) equals to $A_{sw}/b_w s$; ^C Considers the depth to the longitudinal reinforcement in the beam and jacket

3.3 DEVELOPMENT OF NON-PROPRIETARY UHPC

3.3.1 OVERVIEW

A non-proprietary UHPC was developed for the jackets in this research. The developed UHPC was produced through trial mixing and testing. The trial mixture designs were roughly based on non-proprietary UHPC mixtures from the literature that aligned with the targeted material properties of this research. Local and readily available constituent materials were utilized to ensure time- and cost-effectiveness. The particle packing density of each trial mixture was characterised by applying a theoretical model to help minimise porosity and increase strength. Flow table, hardened cube density and compressive cube strength tests were performed for each trial mixture to define the general rheological and mechanical properties. Based on the achieved properties, an optimum UHPC mixture was selected and used to manufacture the jackets of the strengthened specimens of this research. The material properties of the optimum UHPC mixture were later characterised at 28 days and at the days of testing each primary specimen, with the results presented in Section 4.2.4.

The mixture designs targeted compressive (≈ 150.0 MPa) and tensile strength (≈ 6.0 MPa) grades that are typically associated with UHPC, while also considering the need for relatively high workability to manufacture the jackets. The jackets were cast with the beam upside down to ease preparation. Plywood formwork was clamped to the beam to form the sides of the jackets. As mentioned in Section 2.2.3, typical compaction methods for fresh concrete such as internal and external vibration should be avoided in fresh UHPC with steel fibres as they disturb the orientation and dispersion of the fibres (AFNOR, 2016a). Consequently, the UHPC was cast from one end of mould to the other with only the help of rodding or tapping of the sides of the mould if necessary. Therefore, it was essential to target a flow table result of around 230 mm in the trial mixtures. This value corresponds to fresh UHPC that is capable of placement without vibration (i.e., self-compacting) according to consistence class C_v of NF P18-470 as explained in Section 2.2.5 (AFNOR, 2016a).

3.3.2 MIXTURE DESIGN

A total of 50 trial mixtures were designed and tested to help select the optimum UHPC mixture. An iterative trial and error method was applied where the proportions of each constituent

material were trialled and possibly changed until the targeted material properties were achieved. Each mixture was designed using the absolute volume method where the summation of the constituent materials' absolute volumes was equal to one cubic metre. The absolute volume of each constituent material was determined by dividing the weight of a material in kilograms by its specific gravity multiplied by the density of water. The air content was considered to be equal to zero in the absolute volume method. The trial mixtures were prepared in four phases, where the initial phase was dedicated to preparing UHPC mixtures without steel fibres, and the remaining phases were dedicated to preparing UHPC mixtures with steel fibres. Only two phases were initially planned for the trial mixtures, however more phases were required due to significant drawbacks that were mainly caused by different batches of cement and silica fume as discussed later in Section 3.3.5.

The constituent materials and proportions of the developed UHPC mixture were roughly based on the non-proprietary UHPC mixtures from the literature presented in Table 3.3 (El-Tawil et al., 2016; Haber et al., 2018; Randl et al., 2014; Wille et al., 2011). It should be noted that proportions in terms of the total concrete volume were estimated using assumed specific gravities based on typical values for the cement (3.15), silica fume (2.20), aggregate (2.50), steel fibres (7.85) and superplasticizer (1.06) as they were not reported. Silica fume was used in this research as a partial replacement for cement, with both materials forming the binder of the mixture. The proportion of silica fume in relation to the weight of the binder was trialled while considering the percentages in Table 3.3, which ranged from 14.5% to 28.0%. The aggregate in UHPC normally contains quartz powder in addition to sand as in mixtures UHP-FRC, U-A and UM-5 shown in Table 3.3 (Haber et al., 2018; Randl et al., 2014; Wille et al., 2011). Quartz powder was not used in this research to ensure cost-effectiveness. In addition, mixture GG-25-00 by El-Tawil et al. (2016) shown in Table 3.3 demonstrates that quartz powder is not necessarily required in UHPC. The aggregate proportion in relation to the total concrete volume was trialled while considering the percentages in Table 3.3, which ranged between 39.6% and 48.2%. A steel fibre dosage of 2.5% in terms of total concrete volume was normally used in the trial mixtures. Mixture UHP-FRC by Wille et al. (2011) suggests that a 2.5% fibre dosage is capable of achieving a very high tensile strength along with post-cracking strain-hardening behaviour. The water to binder ratios varied between 0.170 and 0.234 for the mixtures shown in Table 3.3, with mixture UM-5 by Randl et al. (2014) demonstrating that a relatively high compressive strength can still be achieved when using a relatively high water to binder ratio in excess of 0.2. The water to binder ratio was trialled in addition to the

superplasticizer dosage with the aim of obtaining a mixture that met the targeted material properties.

Table 3.3 – Proportions and material properties of non-proprietary UHPC mixtures from the literature

Type		UHP-FRC ^A	U-A ^B	GG-25-00 ^C	UM-5 ^D
Cement	[kg/m ³]	767.2 ^E	788.0	775.0	729.0
Silica fume	[kg/m ³]	191.8 ^E	307.0	194.0	124.0
Quartz powder	[kg/m ³]	191.8 ^E	218.0	0	397.0
Sand (Type 1)	[kg/m ³]	199.5 ^E	764.0	245.0	833.0
Sand (Type 2)	[kg/m ³]	790.2 ^E	0	975.0	0
Steel fibres	[kg/m ³]	191.8 ^E	247.0	118.0	155.0
Water	[kg/m ³]	168.8 ^E	165.0	165.0	200.0
Water to cement ratio		0.220	0.209	0.220	0.274
Water to binder ratio		0.176	0.151	0.170	0.234
Superplasticizer	[% wt. of cement]	0.54	1.78	1.29	4.12
Binder (Cement and silica fume)	[% of vol.]	33.1	39.3	33.1	28.0
Quartz powder	[% of vol.]	7.7	8.8	0.0	15.4
Sand (Types 1 and 2)	[% of vol.]	39.6	30.8	48.2	32.1
Aggregate (Quartz powder and sand)	[% of vol.]	47.2	39.6	48.2	47.8
Steel fibres	[% of vol.]	2.5	3.0	1.5	2.0
Cement replacement (Silica fume)	[% wt. of binder]	20.0	28.0	20.0	14.5
Flow table	[mm]	-	146.0	-	280.0
Compressive cube strength ($f_{cm,cu}$)	[MPa]	201.0	-	173.8	166.1
Compressive cylinder strength (f_{cm})	[MPa]	-	147.3	-	-
Tensile strength (f_{ctm})	[MPa]	14.0	9.5	8.0	-

^A UHP-FRC (Wille et al., 2011); ^B U-A (Haber et al., 2018); ^C GG-25-00 (El-Tawil et al., 2016); ^D UM-5 (Randl et al., 2014);

^E Approximated based on reported proportions that were given in ratio of weight of material to weight of cement

The modified Andreasen particle packing model was adopted in this research and the coefficient of determination (R^2) was calculated for each trial mixture to characterise the packing density (Funk and Dinger, 1994). As discussed in Section 2.2.4, prior studies demonstrated that a high theoretical packing density can give a relatively high compressive strength. To recapitulate, the modified Andreasen model requires the calculation of a target particle size distribution (PSD) using Equation 2.3 along with the calculation of a cumulative PSD, which considers the proportions of each granular material. The highest packing density

theoretically occurs in a mixture when the targeted and cumulative PSD curves are perfectly aligned, which corresponds to a coefficient of determination (R^2) that is equal to 100%. A coefficient of determination (R^2) of at least 98.0% was targeted as prior studies with similar percentages achieved a relatively high compressive strength (Yu et al., 2015). A distribution modulus (q) equal to 0.23 was adopted when calculating the target PSD in Eqn. 2.3 as prior studies with similar constituent materials have applied a similar value (Yu et al., 2015).

3.3.3 CONSTITUENT MATERIALS

A summary of the constituent materials used in the developed UHPC is presented in Table 3.4, and photographs of each material are shown in Figure 3.10. The constituent materials included cement, silica fume, sand, steel fibres, water and superplasticizer. The binder was made up of cement and silica fume, with silica fume used as a partial cement replacement. The aggregate consisted of sand only, which had a maximum aggregate size of 1.25 mm. Dramix OL 13/.20 straight steel fibres by Bekaert were used, and they were 13 mm in length and 0.2 mm in diameter with a tensile strength of 2600 MPa. A modified polycarboxylate superplasticizer known as Viscocrete 30HE by Sika was used, which had a solid content of 32%. The specific gravity of each constituent material shown in Table 3.4 was provided by the suppliers, excluding the sand. Specific gravity tests were performed on two samples of the sand, and it was equal to 2.50 on average with a negligible standard deviation (BSI, 2013a).

Table 3.4 – Constituent materials of developed UHPC

Material	Specific gravity	Product
Cement	3.15	Breedon Portland cement plus (PC+)/Portland cement (PC)
Silica fume	2.20	Elkem microsilica 940U
Sand	2.50	Travis Perkins sharp sand (Max. aggregate size of 1.25 mm)
Steel fibre	7.85	Bekaert Dramix OL 13/.20
Water	1.00	Tap water
Superplasticizer	1.06	Sika Viscocrete 30HE

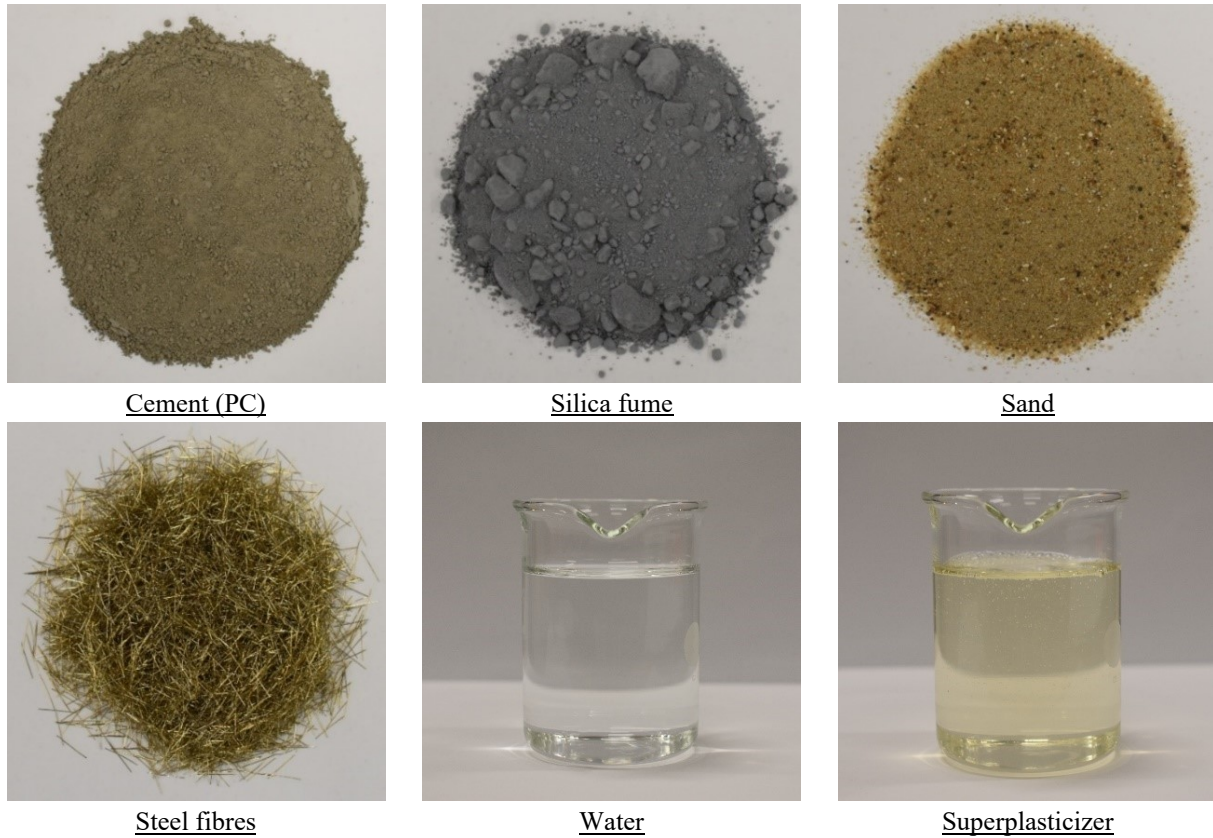


Figure 3.10 – Constituent materials of developed UHPC

Both Portland cement plus (PC+) and Portland cement (PC) by Breedon were investigated in the trial mix development. Both cement products are type CEM I 52.5N according to BS EN 197-1:2011 (BSI, 2019a). Portland cement plus (PC+) was used in the initial phases of trial mixing, but Portland cement (PC) was later used since it improved workability, as discussed later in Section 3.3.5. The adopted silica fume was dry undensified microsilica grade 940 by Elkem. Throughout trial mixing, different batches of cement and silica fume were used, which impacted the rheological properties of the produced UHPC, as discussed later in Section 3.3.5.

The sand in the developed UHPC was sieved from an original source known as sharp sand by Travis Perkins, which had a maximum aggregate size of 4.0 mm. Only the sand that passed through a 1.25 mm sieve was used. A sieve analysis test was performed on the original source of sand in accordance with BS EN 933-1:2012, and the results are shown Figure 3.11 (BSI, 2012). Water absorption tests were performed on three samples of the sieved sand in accordance with BS EN 1097-6:2013, and the 24-hour absorption was equal to 0.38% with a standard deviation of 0.13% (BSI, 2013a). The sieved sand used during trial mixing and the manufacturing of the jackets was kept covered and dry, and the moisture content was equal to approximately 0.37% (BSI, 2008a).

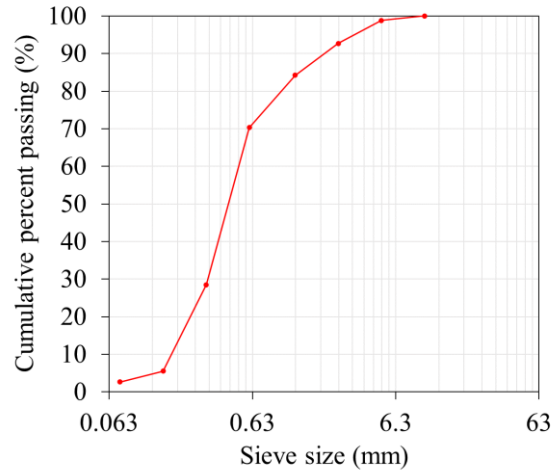


Figure 3.11 – PSD of original source of sand of developed UHPC obtained from sieve analysis

The particle size distributions (PSDs) of the granular materials were measured at the Materials Laboratory of the Department of Civil and Environmental Engineering at Imperial College London using a Mastersizer3000, which is a laser diffraction particle size analyser. Measurements were carried out by dispersing the silica fume and sieved sand in water, while the cement was dispersed in air. The average PSDs of three samples per material are shown in Figure 3.12. The average PSDs of each material were used to determine the coefficient of determination (R^2) of the modified Andreasen model for each trial mixture. The PSDs of the different types and batches of cement and silica fume showed marginal differences. The PSDs of each material shown in Figure 3.12 were applied in the calculations of the coefficient of determination (R^2) for all trial mixtures.

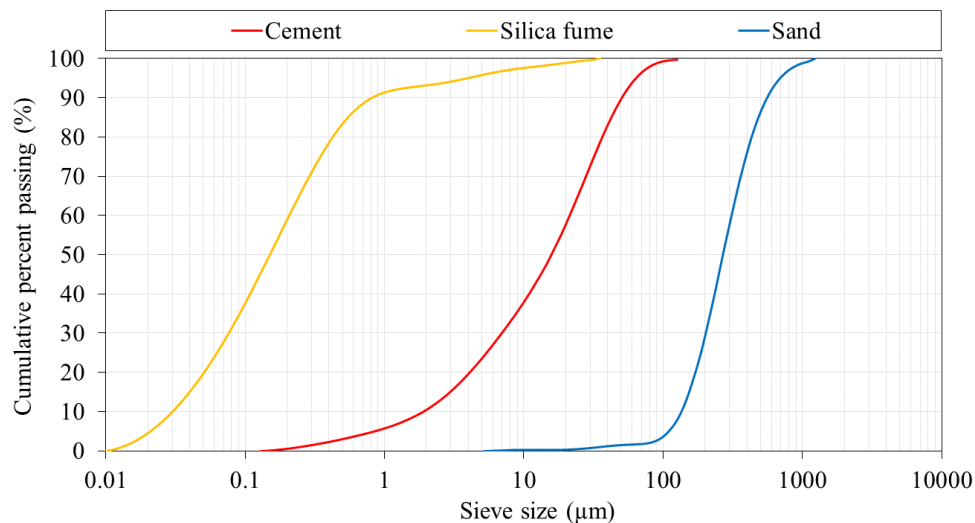


Figure 3.12 – PSDs of granular material of developed UHPC obtained from laser diffraction particle size analyser

3.3.4 TRIAL MIXING PROCEDURES

The UHPC mixtures were prepared using two different mixers. A fully automatic mortar mixer (AUTOMIX 65-L0006/AMZ) by Controls was used to prepare relatively small volumes of UHPC equal to 0.00055 m^3 . A rotating pan mixer (ZZ 30 HE Zyklus) by Pemat was used to prepare larger volumes of up to 0.021 m^3 . Only the Controls and Pemat mixers were used in trial mixing as the prepared volumes of UHPC did not exceed 0.021 m^3 . A forced action mixer (120L) by Soroto was used to prepare volumes of up to 0.065 m^3 . It was not used during trial mixing, but it was used to manufacture the jackets as discussed later in Section 3.4.4. Photographs of the various mixers are shown in Figure 3.13.



Figure 3.13 – Mixers used to prepare developed UHPC mixture

Each UHPC mixture was mixed using the same set of procedures. Each constituent material was initially batched by weight to the nearest 0.1 g based on the mixture design. The typical mixing phases are described in Figure 3.14. Cement, silica fume and sand were placed into the mixing pan and mixed up to homogenization for a period of 5 minutes. This phase of mixing is referred to as dry mixing. The superplasticizer was then placed into the water container, and they were both added to the dry mixture and mixed together for a period of 10 to 15 minutes. During this wet mixing phase, the mixture became increasingly more workable with time, therefore a relatively lengthy mixing period was often necessary. It was also necessary to hand mix all areas of the mixing pan using a hand trowel to break up agglomerated particles, and to ensure that the wet mixture appeared consistent overall. The steel fibres were then added gradually either by hand or with a hand trowel while the mixer continued to operate to avoid fibre bunching. The time required to gradually add the fibres depends on the volume of the mixture. After all of the fibres were added, the mixture was mixed for 5 minutes. Various

mixing speeds were applied with all available mixers where possible, however no significant differences were observed in the final mixture.



Phase 1: Dry mixing (Final)



Phase 2: Wet mixing (Initial)



Phase 3: Wet mixing (Intermediate)



Phase 4: Wet mixing (Final)



Phase 5: Gradual addition of fibres



Phase 6: Final mixture

Figure 3.14 – Typical mixing phases of developed UHPC mixture

Flow table tests were performed on the fresh UHPC immediately after mixing for each trial mixture to quantify the flow. Flow table tests were conducted in accordance with an adaptation of ASTM C230/230M as suggested by NF P18-470 (AFNOR, 2016a; ASTM, 2014). A conical mould was filled with fresh UHPC and then removed without shock to allow the UHPC to spread freely. When the UHPC stopped flowing, which typically occurred between 30 to 60 seconds, the average diameter of the spread was measured. The conical moulds shown in Figure

3.15 were used, and they included a miniature slump cone and an ASTM C230/230M cone (ASTM, 2014). The dimensions of the miniature slump cone are 40 x 90 x 75 mm, and the dimensions of the ASTM C230/230M cone are 75 x 100 x 50 mm (ASTM, 2014). The miniature slump cone was only used to measure the flow of UHPC without steel fibres, while the ASTM C230/230M cone was used for the UHPC with steel fibres as stipulated by NF P18-470 (AFNOR, 2016a; ASTM, 2014).



Miniature slump cone



ASTM C230/230M cone (ASTM, 2014)

Figure 3.15 – Measuring apparatus for flow table test of developed UHPC mixture

The fresh UHPC was cast in a single layer into cube moulds. At least three cube specimens were cast and tested for each trial mixture unless otherwise noted. Cubes of 50 mm dimensions were cast for the initial trial mixtures to avoid an excessive consumption of materials, and 100 mm cubes were cast for the remaining trial mixtures. Although typical compaction methods are meant to be avoided in fresh UHPC with steel fibres, compaction using a vibrating table was necessary for some trial mixtures that had low workability. The compaction time was recorded for each set of specimens, and it did not exceed 90 seconds. The specimens were covered with a polythene sheet immediately after casting, and then they were demoulded 24 hours after casting and stored in a curing room with a temperature of about 20°C and 95% relative humidity prior to the day of testing. Hardened cube density and compressive cube strength tests were performed using the test setups shown in Figure 3.16. The hardened cube density tests were performed in accordance with BS EN 12390-7:2009 as suggested by NF P18-470 (AFNOR, 2016a; BSI, 2008b). Compressive cube strength tests were normally performed at 28 days after casting in accordance with BS EN 12390-3:2009 and NF P18-470 (AFNOR, 2016a; BSI, 2011). Load rates of 0.3 MPa and 0.4 MPa were applied to the 50 mm and 100 mm cubes, respectively (AFNOR, 2016a).

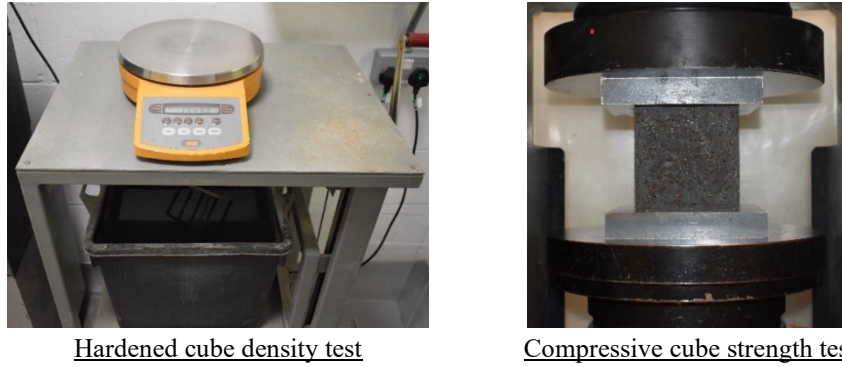


Figure 3.16 – Test setups for various mechanical properties of UHPC trial mixtures

3.3.5 TRIAL MIXING RESULTS

The initial phase of trial mixing (Phase 1) included no steel fibres, and the proportions and test results of each mixture are presented in Tables 3.5 and 3.6, respectively. The name of each mixture refers to the trial mixing phase and mixture number. The aim of trial mixing phase 1 was to determine suitable binder, aggregate, water and superplasticizer proportions. The aggregate proportion was trialled, and it ranged between 36.0% and 38.0% of the total concrete volume, which is close to the lower bound of 39.6% for the mixtures from the literature in Table 3.3. Consequentially, the binder proportion ranged between 39.0% and 41.3% of the total concrete volume, which is close to the upper bound of 39.3% for the mixtures from the literature in Table 3.3. The partial cement replacement was kept constant, with silica fume accounting for 20% of the binder, much like the mixtures of Wille et al. (2011) and El-Tawil et al. (2016) in Table 3.3. A similar water to binder ratio as Wille et al. (2011) of 0.184 was used for all mixtures. The superplasticizer dosage was trialled, and it ranged between 0.75% and 3.0% of the weight of cement. The coefficient of determination (R^2) of the modified Andreasen model was greater than 98.2% for all mixtures, which is greater than the targeted value of 98.0% and suggests a relatively high packing density (Yu et al., 2015).

Flow table test results for trial mixing phase 1 were near the targeted value of 230 mm. No segregation or bleeding was observed in the fresh UHPC, with a typical flow table test result shown in Figure 3.17. The highest flow of 220.0 mm was found in mixture 1-4, where the aggregate proportion was equal to 36.0% and the superplasticizer dosage was equal to 1.5%. The hardened density results were reasonable when compared to standard concrete. Compressive strength tests were conducted at 56 days after casting using 50 mm cubes, and all cubes failed in a brittle and explosive manner. The compressive cube strength varied significantly between specimens of the same mixture as highlighted by the relatively large standard deviations in Table 3.6. The inconsistencies are likely due to the absence of steel

fibres, which normally provide ductility. Inconsistencies were mainly observed in mixtures 1-2, 1-5, 1-6 and 1-8 where one out of the three tested specimens in each mixture exhibited significant cracks soon after loading that ultimately lead to premature failure unlike the remaining two specimens. For these cases, the outlier specimen that failed prematurely was not considered in the average result. The most extreme case was observed in mixture 1-7 where all three tested specimens failed prematurely. Mixture 1-7 achieved an average compressive cube strength of 58.0 MPa with an excessively high standard deviation of 32.3 MPa as all three tested specimens were considered. The average compressive cube strength for most of the mixtures without steel fibres ranged between 122.7 MPa and 147.1 MPa, which is acceptable.

Table 3.5 – Proportions of phase 1 of UHPC trial mixtures

No.	Cement (PC+) [kg/m ³]	Silica fume [kg/m ³]	Sand [kg/m ³]	Fibres [kg/m ³]	Water [kg/m ³]	Water to binder ratio	SP ^A [% wt.]	Binder vol. [%]	Agg. vol. [%]	Fibres vol. [%]	Silica fume ^B [% wt.]
1-1	905.5 (1)	226.4 (1)	951.0	0	208.3	0.184	2.50	39.0	38.0	0	20.0
1-2	934.7 (1)	233.7 (1)	900.9	0	215.0	0.184	2.50	40.3	36.0	0	20.0
1-3	928.3 (1)	232.1 (1)	900.9	0	213.5	0.184	3.00	40.0	36.0	0	20.0
1-4	947.8 (1)	237.0 (1)	900.9	0	218.0	0.184	1.50	40.9	36.0	0	20.0
1-5	957.8 (1)	239.5 (1)	900.9	0	220.3	0.184	0.75	41.3	36.0	0	20.0
1-6	951.1 (1)	237.8 (1)	900.9	0	218.8	0.184	1.25	41.0	36.0	0	20.0
1-7	944.5 (1)	236.1 (1)	900.9	0	217.2	0.184	1.75	40.7	36.0	0	20.0
1-8	949.8 (1)	237.4 (1)	900.9	0	218.5	0.184	1.35	41.0	36.0	0	20.0

[Note] – () refers to batch numbers; ^A Superplasticizer (SP) in terms of wt. of cement; ^B Cement replacement (Silica fume) in terms of wt. of binder

Table 3.6 – Test results of phase 1 of UHPC trial mixtures

No.	Volume [m ³]	Flow table [mm]	Compaction time [s]	Hardened density (ρ) [kg/m ³]	Compressive cube strength ($f_{cm,cu}$) [MPa]	Coefficient of determination (R^2) [%]
1-1	0.00055	170.00	60.0	2359.35 (11.74)	122.70 (7.14)	98.43
1-2	0.00055	202.50	90.0	2364.76 ^B (15.99)	126.49 ^B (5.95)	98.27
1-3	0.00055	177.50	90.0	n/a ^A	126.63 (7.17)	98.29
1-4	0.00055	220.00	10.0	n/a ^A	135.90 (5.42)	98.25
1-5	0.00055	192.50	10.0	n/a ^A	135.39 ^B (4.32)	98.22
1-6	0.00055	208.75	5.0	2336.64 ^B (2.75)	138.74 ^B (0.24)	98.24
1-7	0.00055	202.50	5.0	2328.83 (19.78)	58.02 (32.30)	98.25
1-8	0.00055	218.75	5.0	2373.97 ^B (0.02)	147.13 ^B (15.52)	98.24

[Note] – () refers to standard deviation; ^A Density not recorded due to fault in measuring apparatus; ^B Single outlier was removed due to premature failure of specimen

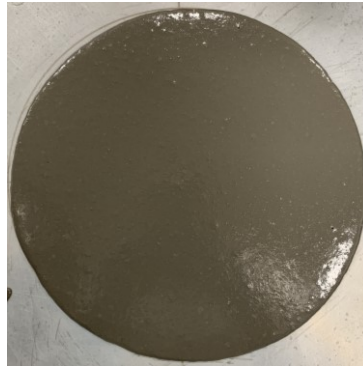


Figure 3.17 – Flow table test result for UHPC trial mixture without steel fibres (Mixture 1-8 – 218.75 mm)

The following phase of trial mixing (Phase 2) included steel fibres, and the proportions and test results of each mixture are presented in Tables 3.7 and 3.8, respectively. The aim of trial mixing phase 2 was to find a suitable UHPC mixture with steel fibres by trialling the silica fume and aggregate proportions as well as the water to binder ratio. A steel fibre dosage of 2.5% of the total concrete volume was adopted, which is similar to mixture UHP-FRC by Wille et al. (2011) in Table 3.3 that achieved a relatively high tensile strength.

The flow table test results of the initial five mixtures were satisfactory in relation to the targeted value of 230 mm, and no segregation, bleeding or fibre bunching was observed. For the following five mixtures of trial mixing phase 2, the use of a new batch of Portland cement plus (PC+) led to a significant decrease in workability. A loss of 21.25 mm occurred in the flow table test result of mixture 2-9 when compared to that of mixture 2-1. The two trial mixtures had an identical mixture design as shown in Table 3.7, with the key difference being the batches of cement. The hardened density results are marginally higher than those of standard concrete overall, and this is likely due to the addition of the steel fibres. The average compressive cube strength obtained at 28 days for trial mixing phase 2 ranged between 134.6 MPa and 163.3 MPa with a relatively low standard deviation overall. Cube dimensions of 50 mm were used for mixtures 2-1 to 2-4, and 100 mm cubes were used for mixtures 2-5 to 2-10. There appears to be no relationship between the coefficient of determination (R^2) of the modified Andreasen model, workability and compressive strength. In theory, a higher coefficient of determination (R^2) indicates a higher packing density, which in turn should lead to improved workability and compressive strength. However, the test results of trial mixing phase 2 suggest that a higher coefficient of determination does not necessarily lead to higher workability and compressive strength as shown in Figure 3.18 for mixtures with the same batch of materials. As discussed in Section 2.2.4, the modified Andreasen model only considers the granular materials in a

concrete mixture, and it does not directly consider the water content, which significantly influences the rheological and mechanical properties of concrete. Although the compressive cube strength results were acceptable in trial mixing phase 2, workability was insufficient as the targeted value of around 230 mm was not met.

Table 3.7 – Proportions of phase 2 of UHPC trial mixtures

No.	Cement (PC+) [kg/m ³]	Silica fume [kg/m ³]	Sand [kg/m ³]	Fibres [kg/m ³]	Water [kg/m ³]	Water to binder ratio	SP ^A [% wt.]	Binder vol. [%]	Sand vol. [%]	Fibres vol. [%]	Silica fume ^B [% wt.]
2-1	910.8 (1)	227.7 (1)	900.9	196	209	0.184	1.5	39.3	36.0	2.5	20.0
2-2	961.0 (1)	143.6 (1)	925.9	196	221	0.200	1.5	37.0	37.0	2.5	13.0
2-3	1101.6 (1)	58.0 (1)	925.9	196	213	0.184	1.5	37.6	37.0	2.5	5.0
2-4	1031.4 (1)	114.6 (1)	925.9	196	211	0.184	1.5	38.0	37.0	2.5	10.0
2-5	1031.4 (1)	114.6 (1)	925.9	196	211	0.184	1.5	38.0	37.0	2.5	10.0
2-6	851.5 (2)	212.9 (1)	1001.0	196	196	0.184	1.5	36.7	40.0	2.5	20.0
2-7	879.7 (2)	219.9 (1)	925.9	196	213	0.194	1.5	37.9	37.0	2.5	20.0
2-8	945.2 (2)	166.8 (1)	925.9	196	216	0.194	1.5	37.6	37.0	2.5	15.0
2-9	910.8 (2)	227.7 (1)	900.9	196	209	0.184	1.5	39.3	36.0	2.5	20.0
2-10	851.5 (2)	212.9 (1)	1001.0	196	196	0.184	1.5	36.7	40.0	2.5	20.0

[Note] – () refers to batch numbers; ^A Superplasticizer (SP) in terms of wt. of cement; ^B Cement replacement (Silica fume) in terms of wt. of binder

Table 3.8 – Test results of phase 2 of UHPC trial mixtures

No.	Volume [m ³]	Flow table [mm]	Compaction time [s]	Hardened density (ρ) [kg/m ³]	Compressive cube strength ($f_{cm,cu}$) [MPa]	Coefficient of determination (R^2) [%]
2-1	0.00055	207.50	10	2572.06 ^A (19.49)	163.31 ^A (10.84)	98.32
2-2	0.00055	245.00	10	2480.99 ^A (34.47)	146.84 ^A (7.03)	98.59
2-3	0.00055	215.00	10	2588.10 ^A (3.84)	136.01 ^A (5.73)	98.69
2-4	0.00055	223.75	10	2561.55 ^A (17.65)	146.30 ^A (0.72)	98.61
2-5	0.00440	237.50	0	2502.12 ^B (8.37)	144.00 ^B (4.22)	98.61
2-6	0.01570	195.00	10	n/a ^C	140.98 ^B (3.16)	98.57
2-7	0.00375	202.50	0	n/a ^C	142.48 ^B (4.33)	98.43
2-8	0.00375	216.25	0	n/a ^C	142.84 ^B (3.44)	98.53
2-9	0.00375	186.25	5	2452.27 ^B (11.94)	148.43 ^B (1.84)	98.32
2-10	0.00375	152.50	15	2460.11 ^B (6.50)	134.55 ^B (6.03)	98.57

[Note] – () refers to standard deviation; ^A 50 mm cube; ^B 100 mm cube; ^C Density not recorded due to fault in measuring apparatus

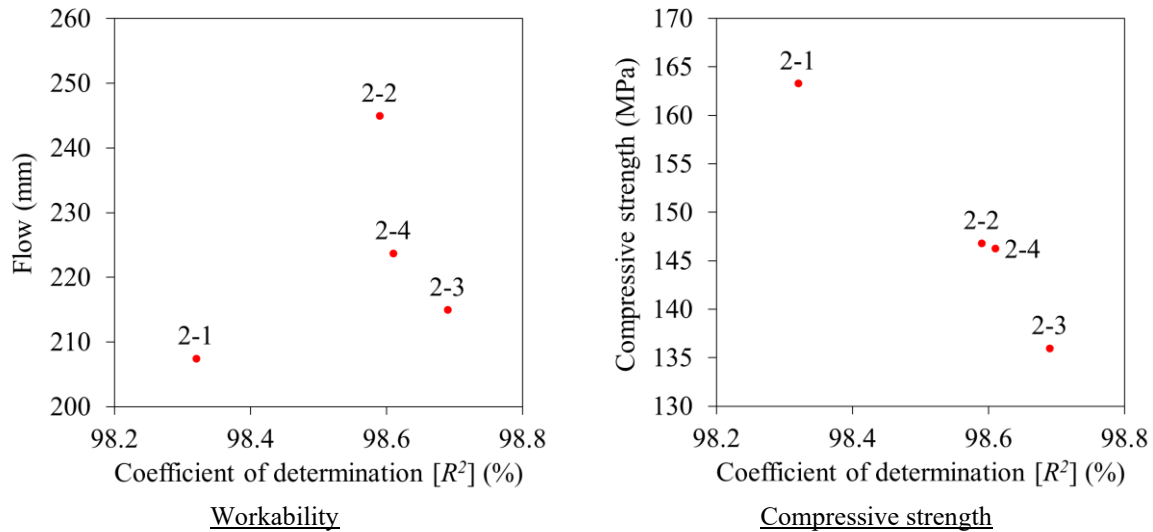


Figure 3.18 – Influence of theoretical particle packing on workability and compressive strength

In the next phase of trial mixing (Phase 3), Portland cement (PC) was used to help meet the targeted workability rather than Portland cement plus (PC+), which was used in the previous two trial mixing phases. The proportions and test results of each mixture are presented in Tables 3.9 and 3.10, respectively. Much like trial mixing phase 2, the aim of trial mixing phase 3 was to find a suitable UHPC mixture with steel fibres. The silica fume, aggregate, and steel fibre proportions were trialled as well as the water to binder ratio and superplasticizer dosage.

Figure 3.19 shows how workability was influenced when various proportions were independently changed in a set of mixtures with the same batches of materials. It is evident that workability decreases due to a decrease in the water to binder ratio, and it also decreases due to increases in the silica fume, aggregate and steel fibre proportions. It also appears that the optimum superplasticizer dosage lies between 1.4% and 1.8% of the weight of cement seeing as the difference in workability shown is minimal within this range. The flow table test results were well below the targeted value of 230 mm overall for trial mixing phase 3, and new batches of cement and silica fume led to significant decreases in workability, much like what occurred in trial mixing phase 2. A loss of up to 35 mm occurred in the flow table test results of mixtures 3-2, 3-9 and 3-23, which shared an identical mixture design, with the only difference being the batches of cement and silica fume. The compressive strength was not impacted by the different batches of cement and silica fume, with the average compressive strength at 28 days of 100 mm cubes ranging between 134.5 MPa and 152.5 MPa. Based on the results of trial mixing phase 3, a final trial mixing phase was required to achieve the targeted workability while using

the same batches of constituent materials that were to be used in the manufacturing of the jackets.

Table 3.9 – Proportions of phase 3 of UHPC trial mixtures

No.	Cement (PC) [kg/m ³]	Silica fume [kg/m ³]	Sand [kg/m ³]	Fibres [kg/m ³]	Water [kg/m ³]	Water to binder ratio	SP ^A [% wt.]	Binder vol. [%]	Sand vol. [%]	Fibres vol. [%]	Silica fume ^B [% wt.]
3-1	850.0 (1)	212.5 (1)	1001.0	196.3	196.6	0.1850	1.5	36.6	40.0	2.5	20.0
3-2	886.9 (1)	221.7 (1)	938.4	196.3	205.1	0.1850	1.5	38.2	37.5	2.5	20.0
3-3	923.9 (1)	231.0 (1)	875.9	196.3	213.6	0.1850	1.5	39.8	35.0	2.5	20.0
3-4	890.0 (1)	258.4 (1)	875.9	196.3	212.4	0.1850	1.5	40.0	35.0	2.5	22.5
3-5	919.9 (1)	195.1 (1)	938.4	196.3	206.3	0.1850	1.5	38.1	37.5	2.5	17.5
3-6	910.8 (1)	227.7 (1)	900.9	196.3	209.5	0.1840	1.5	39.3	36.0	2.5	20.0
3-7	932.9 (2)	197.9 (1)	938.4	196.3	200.7	0.1775	1.5	38.6	37.5	2.5	17.5
3-8	926.3 (2)	196.5 (1)	938.4	196.3	203.5	0.1813	1.5	38.3	37.5	2.5	15.0
3-9	885.7 (2)	221.4 (1)	938.4	196.3	204.8	0.1850	1.6	38.2	37.5	2.5	20.0
3-10	888.2 (2)	222.0 (1)	938.4	196.3	205.4	0.1850	1.4	38.3	37.5	2.5	20.0
3-11	886.9 (2)	221.7 (1)	938.4	196.3	205.1	0.1850	1.5	38.2	37.5	2.5	20.0
3-12	884.5 (2)	221.1 (1)	938.4	196.3	204.5	0.1850	1.7	38.1	37.5	2.5	20.0
3-13	883.2 (2)	220.8 (1)	938.4	196.3	204.2	0.1850	1.8	38.1	37.5	2.5	20.0
3-14	919.9 (2)	195.1 (1)	938.4	196.3	206.3	0.1850	1.5	38.1	37.5	2.5	17.5
3-15	854.4 (2)	248.0 (1)	938.4	196.3	203.9	0.1850	1.5	38.4	37.5	2.5	22.5
3-16	894.3 (2)	223.6 (1)	938.4	157.0	206.8	0.1850	1.5	38.6	37.5	2	20.0
3-17	879.5 (2)	219.9 (1)	938.4	235.5	203.4	0.1850	1.5	37.9	37.5	3	20.0
3-18	874.8 (2)	218.7 (1)	938.4	196.3	210.5	0.1925	1.5	37.7	37.5	2.5	20.0
3-19	863.0 (2)	215.8 (1)	938.4	196.3	215.8	0.2000	1.5	37.2	37.5	2.5	20.0
3-20	850.0 (2)	212.5 (1)	1001.0	196.3	196.6	0.1850	1.5	36.6	40.0	2.5	20.0
3-21	923.9 (2)	231.0 (1)	875.9	196.3	213.6	0.1850	1.5	39.8	35.0	2.5	20.0
3-22	885.7 (2)	221.4 (2)	938.4	196.3	204.8	0.1850	1.6	38.2	37.5	2.5	20.0
3-23	885.7 (2)	221.4 (2)	938.4	196.3	204.8	0.1850	1.6	38.2	37.5	2.5	20.0
3-24	885.7 (2)	221.4 (2)	938.4	196.3	204.8	0.1850	1.6	38.2	37.5	2.5	20.0

[Note] – () refers to batch numbers; ^A Superplasticizer (SP) in terms of wt. of cement; ^B Cement replacement (Silica fume) in terms of wt. of binder

Table 3.10 – Results of phase 3 of UHPC trial mixtures

No.	Volume [m ³]	Flow table [mm]	Compaction time [s]	Hardened density (ρ) [kg/m ³]	Compressive cube strength ($f_{cm,cu}$) [MPa]	Coefficient of determination (R^2) [%]
3-1	0.00375	178.75	5.0	2466.06 (11.83)	145.61 (4.78)	98.60
3-2	0.00375	192.50	5.0	2462.84 (16.93)	144.00 (5.20)	98.47
3-3	0.00375	206.25	0	2464.83 (17.73)	144.05 (3.92)	98.28
3-4	0.00375	188.75	5.0	2445.92 (12.84)	141.49 (6.81)	98.22
3-5	0.00375	206.25	0	2474.59 (10.91)	144.41 (5.39)	98.52
3-6	0.00375	200.00	0	2468.03 (16.81)	144.66 (4.42)	98.36
3-7	0.00375	161.25	25.0	2485.12 (11.39)	143.76 (4.07)	98.50
3-8	0.00375	176.25	25.0	2480.53 (7.08)	144.98 (3.53)	98.51
3-9	0.00375	176.25	5.0	2462.41 (3.58)	144.80 (2.46)	98.47
3-10	0.00375	166.25	25.0	2473.15 (3.76)	147.22 (2.66)	98.47
3-11	0.00375	170.25	10.0	2464.05 (3.48)	142.39 (3.22)	98.47
3-12	0.00375	178.75	20.0	2455.18 (3.43)	140.15 (2.76)	98.47
3-13	0.00375	175.00	30.0	2449.94 (6.97)	144.59 (1.11)	98.48
3-14	0.00375	175.00	20.0	2465.32 (5.42)	144.23 (4.99)	98.52
3-15	0.00375	158.75	30.0	2443.60 (10.75)	147.21 (9.02)	98.42
3-16	0.00375	175.00	30.0	2428.52 (2.93)	145.93 (5.32)	98.46
3-17	0.00375	158.75	30.0	2486.86 (5.60)	152.54 (6.09)	98.48
3-18	0.00375	178.75	25.0	2457.05 (8.81)	144.53 (3.98)	98.49
3-19	0.00375	187.50	20.0	2456.33 (2.71)	145.45 (2.30)	98.51
3-20	0.00375	146.25	75.0	2469.59 (7.05)	144.52 (3.59)	98.60
3-21	0.00375	178.75	25.0	2463.88 (4.40)	151.56 (4.11)	98.28
3-22	0.02120	165.00	30.0	2438.52 ^A (13.27)	134.46 ^A (9.72)	98.47
3-23	0.00375	157.50	30.0	2462.97 (11.80)	143.18 (4.25)	98.47
3-24	0.00375	153.75	30.0	2469.20 (14.38)	149.64 (4.89)	98.47

[Note] – () refers to standard deviation; ^A Tested 6 specimens

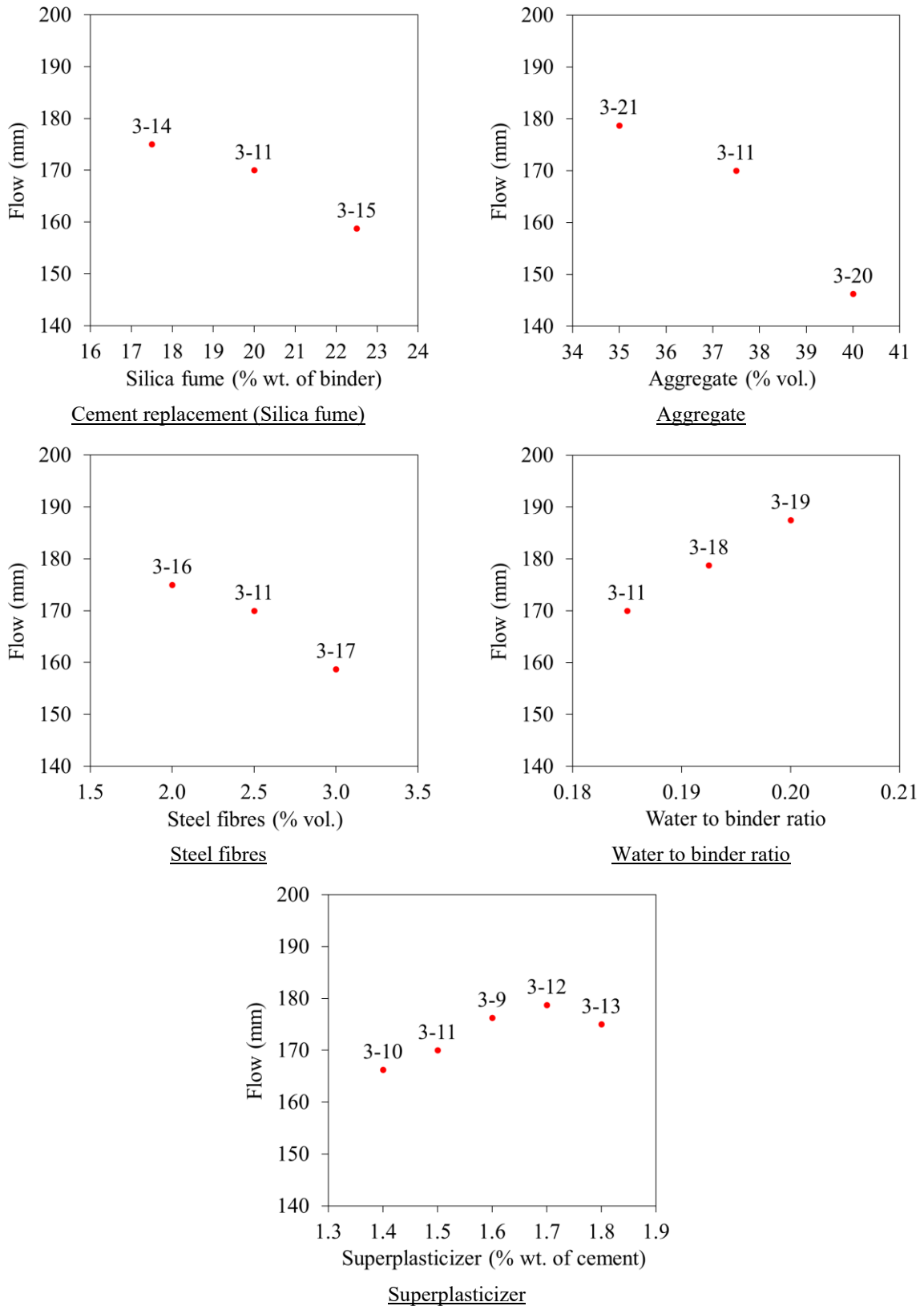


Figure 3.19 – Effect of varying constituent material proportions on workability of UHPC trial mixtures

The final phase of trial mixing (Phase 4) included the same batch of materials that were used to produce the UHPC of the jackets of the strengthened specimens of this research. Based on

the results of the previous trial mixing phases, workability still required improvement to meet the targeted flow table test result of 230 mm. To do so, it was necessary to increase the water to binder ratio above 0.2, and the silica fume, aggregate and superplasticizer proportions were trialled once again. The highest water to binder ratio was equal to 0.2225 in mixtures 4-2 and 4-3. Mixture 4-2 achieved a flow table test result equal to 257.5 mm, however significant fibre bunching was evident as shown in Figure 3.20, which is likely due to the silica fume and aggregate proportions that were lower than usual. A silica fume proportion of 20% of the weight of the binder gave acceptable flow table test results along with an aggregate proportion of 37.5% of the total concrete volume. A water to binder ratio of 0.22 and a superplasticizer dosage of 1.5% also gave acceptable flow table test results when compared to the targeted value of 230 mm. The flow table test result of mixture 4-5 was equal to 225 mm with no visible segregation, bleeding or fibre bunching, and it is shown in Figure 3.20. The average compressive cube strength at 28 days of 100 mm cubes ranged between 138.3 kN and 151.2 kN, which was satisfactory.

Table 3.11 – Proportions of phase 4 of UHPC trial mixtures

No.	Cement (PC) [kg/m ³]	Silica fume [kg/m ³]	Sand [kg/m ³]	Fibres [kg/m ³]	Water [kg/m ³]	Water to binder ratio	SP ^A [% wt.]	Binder vol. [%]	Sand vol. [%]	Fibres vol. [%]	Silica fume ^B [% wt.]
4-1	885.7 (3)	221.4 (2)	938.4	196.3	204.8	0.1850	1.6	38.2	37.5	2.5	20.0
4-2	913.0 (3)	161.1 (2)	900.9	196.3	239.0	0.2225	1.5	36.3	36.0	2.5	15.0
4-3	829.4 (3)	207.4 (2)	938.4	196.3	230.7	0.2225	1.5	35.8	37.5	2.5	20.0
4-4	836.7 (3)	209.2 (2)	938.4	196.3	227.5	0.2175	1.5	36.1	37.5	2.5	20.0
4-5	833.0 (3)	208.3 (2)	938.4	196.3	229.1	0.2200	1.5	35.9	37.5	2.5	20.0
4-6	834.1 (3)	208.5 (2)	938.4	196.3	229.4	0.2200	1.4	36.0	37.5	2.5	20.0
4-7	835.2 (3)	208.8 (2)	938.4	196.3	229.7	0.2200	1.3	36.0	37.5	2.5	20.0
4-8	836.3 (3)	209.1 (2)	938.4	196.3	230.0	0.2200	1.2	36.1	37.5	2.5	20.0

[Note] – () refers to batch numbers; ^A Superplasticizer (SP) in terms of wt. of cement; ^B Cement replacement (Silica fume) in terms of wt. of binder

Table 3.12 – Results of phase 4 of UHPC trial mixtures

No.	Volume [m ³]	Flow table [mm]	Compaction time [s]	Hardened density (ρ) [kg/m ³]	Compressive cube strength ($f_{cm,cu}$) [MPa]	Coefficient of determination (R^2) [%]
4-1	0.00375	158.75	30.0	2470.75 (4.67)	151.21 (3.23)	98.47
4-2	0.00375	257.50	0	2434.28 (7.71)	138.26 (2.66)	98.57
4-3	0.00375	215.00	0	2427.72 (4.54)	145.62 (8.26)	98.55
4-4	0.00375	205.00	0	2439.47 (2.20)	140.94 (0.44)	98.54
4-5	0.00375	225.00	0	2436.26 (6.85)	139.16 (1.36)	98.55
4-6	0.00375	215.00	0	2440.28 (0.62)	140.47 (0.65)	98.55
4-7	0.00375	218.75	0	2439.68 (4.60)	141.53 (1.95)	98.55
4-8	0.00375	207.50	0	2442.46 (4.46)	138.74 (2.65)	98.55

[Note] – () refers to standard deviation



Mixture 4-2 – 257.5 mm



Mixture 4-5 – 225.0 mm

Figure 3.20 – Flow table test results of UHPC trial mixtures with steel fibres

3.3.6 OPTIMUM UHPC MIXTURE DESIGN

Mixture 4-5 from the last phase of trial mixing (Phase 4) was selected as the optimum mixture as it met the targeted material properties. Proportions of this mixture are summarized in Table 3.13. The flow table test result was equal to 225 mm with no visible segregation, bleeding or fibre bunching. The flow table test result was very close to the targeted value of 230 mm, which suggests that the fresh UHPC is capable of placement without vibration (i.e., self-compacting) according to consistence class Cv of NF P18-470 (AFNOR, 2016). The average compressive cube strength at 28 days was equal to 139.2 MPa with a low standard deviation of 1.4 MPa. Although no relationship was established between the coefficient of determination of the modified Andreasen model (R^2), workability and compressive strength, the coefficient of

determination (R^2) was equal to 98.6%. The difference between the targeted and cumulative PSD curves of the modified Andreasen model appear to be minimal in Figure 3.21, which suggests a relatively high packing density (Yu et al., 2015). The mixture proportions presented in Table 3.13 were used to produce the UHPC of the jackets of the strengthened specimens of this research, and this is discussed later in Section 3.4.4.

Table 3.13 – Proportions of optimum UHPC mixture

Type		Proportion
Cement (Breedon Portland cement)	[kg/m ³]	833.0
Silica fume (Elkem microsilica 940U)	[kg/m ³]	208.3
Sand (Max. aggregate size of 1.25 mm)	[kg/m ³]	938.4
Steel fibres (Dramix OL 13/0.2)	[kg/m ³]	196.3
Water	[kg/m ³]	229.1
Water to cement ratio		0.275
Water to binder ratio		0.220
Superplasticizer (Sika Viscocrete 30HE)	[% wt. of cement]	1.5
Binder (Cement and silica fume)	[% vol.]	35.9
Aggregate (Sand)	[% vol.]	37.5
Steel fibres (Dramix OL 13/0.2)	[% vol.]	2.5
Cement replacement (Silica fume)	[% wt. of binder]	20.0

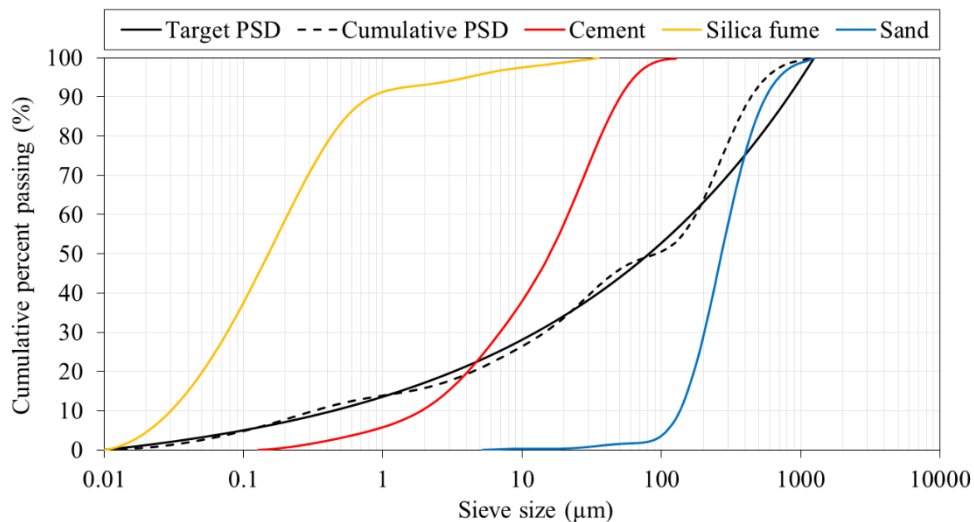


Figure 3.21 – Target and cumulative PSDs of the modified Andreasen model for developed UHPC

3.3.7 CHARACTERISATION OF MATERIAL PROPERTIES

The material properties of the developed UHPC presented in Table 3.13 were characterised at 28 days using control specimens that were cast in several batches. Control specimens such as the cubes, cylinders and prisms shown in Figure 3.22 were cast in batches using the Soroto and Pemat mixers, which were used to manufacture the jackets as discussed later in Section 3.4.4. The control specimens were cast in batches due to the limited volume of UHPC that can be cast at a time with the Soroto and Pemat mixers, and each batch was evaluated individually. The control specimens were covered with polythene sheets immediately after casting and demoulded 24 hours after casting. The control specimens were moist cured by spraying with water daily and covering afterwards with polythene sheets for a 28-day period, which is the same curing process used for the manufactured UHPC jackets. Control specimens consisted of 100 mm cubes and 100 x 200 mm cylinders that were used to measure the hardened cube density, compressive cube and cylinder strength, splitting tensile strength and elastic modulus. Control specimens also included 400 x 100 x 100 mm unnotched and notched prisms that were used to carry out four- and three-point bending tests to determine the tensile strength at the elastic limit and post-cracking tensile strength, respectively. A limited amount of 100 mm cubes and 100 x 200 mm cylinders were also cast alongside each UHPC jacket to characterise the material properties of UHPC at the day of testing each jacketed specimen, and this is discussed later in Section 3.4.4. The results of these tests are presented in Section 4.2.4.



Figure 3.22 – UHPC control specimens for testing of mechanical properties

3.4 MANUFACTURING OF RC BEAMS AND UHPC JACKETS

3.4.1 INTRODUCTION

The primary specimens of this research were manufactured in a set of processes that began with casting and curing the RC beams of groups 1 and 2. The bonding surfaces of the RC beams designed with jackets were then prepared by mechanical removal of the concrete substrate. The jackets were cast with the developed UHPC described in Section 3.3.6 as the RC beams laid inverted. The jacketed beam specimens were then cured prior to testing.

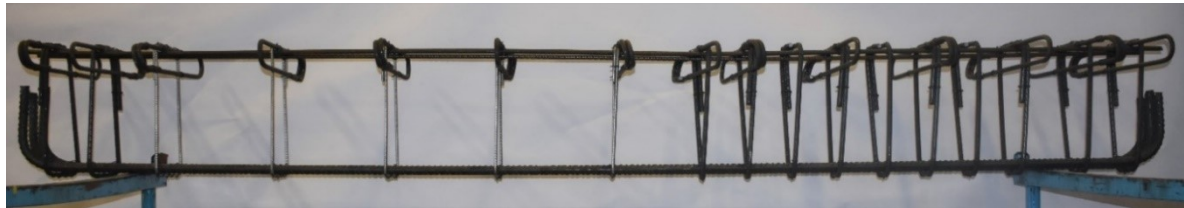
3.4.2 MANUFACTURING OF RC BEAMS

The manufacturing process of the RC beams is described in Figure 3.23. It consisted of initially assembling the steel reinforcement cage of each beam using steel tying wires. The steel reinforcement cages were placed into 19 mm plywood moulds and onto 25 mm plastic spacers, with the interior of the moulds covered in Adostrike by Adomast, which is a general-purpose mould release oil. Several lifting eyes located at the mid-height of the flange on either side of each beam were installed by threading through the two outer $\phi 10$ longitudinal reinforcement bars in compression so that the specimens can be carried. The beams were cast in two groups of four with two different batches of ready-mix concrete supplied by Cemex that targeted strength grade C30/37 and consistence class S3 (BSI, 2014). The concrete included limestone coarse aggregate with a maximum aggregate size of 10 mm, and the mixture design is presented in Table 3.14. The ready-mix concrete was loaded into a concrete skip and poured vertically into the moulds, and the concrete was compacted using a vibrating poker. A total 66 control specimens were cast alongside each group of beams, which included 100 mm cubes (27), 100 x 200 mm cylinders (21) and 150 x 300 mm cylinders (18) for hardened density, compressive strength, splitting tensile strength and elastic modulus tests. A 650 x 200 x 225 mm prism was also cast alongside each group of beams, with only one of them used as a beam to cast a trial UHPC jacket as discussed later in Section 3.4.4. An additional pair of 150 x 300 mm cylinders were also cast alongside each group of beams to characterise the NSC-UHPC interfacial bond through splitting tensile strength tests as discussed in Section 3.5. The beams and control specimens were covered with polythene sheets immediately after casting, and demoulded 24 hours after casting. The beams were moist cured as they were sprayed with water and covered afterwards with polythene sheets every day for the first two weeks and then every other day for

the remainder of the 28-day period. Several control specimens were cured alongside the beams while others were cured in water at 20°C.



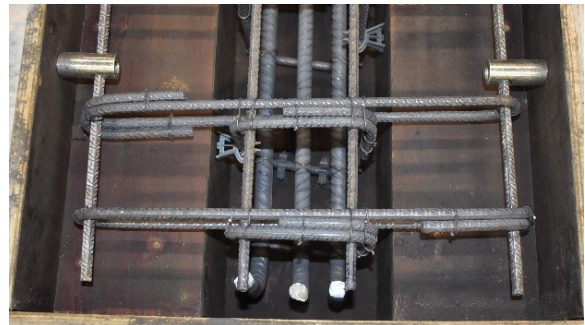
Assembled steel reinforcement cage of group 1 beam



Assembled steel reinforcement cage of group 2 beam



Placement of steel reinforcement cages in moulds



Steel reinforcement cage in mould



Ready-mix concrete loaded into concrete skip



Pouring and vibrating concrete in mould



Control specimens



Moist curing of all beams and control specimens

Figure 3.23 – Manufacturing and curing of RC beam specimens of groups 1 and 2

Table 3.14 – Proportions of ready-mix concrete supplied by CEMEX

Type		Proportion
Cement (CEM I)	[kg/m ³]	360.0
Coarse aggregate (4/10 Limestone)	[kg/m ³]	976.0
Fine aggregate (0/4)	[kg/m ³]	889.0
Superplasticizer (CEMEX WRA - CP105)	[mL/m ³]	2525.0
Water to cement ratio		0.460
Fine aggregate (0/4)	[% vol. of agg.]	48.0

3.4.3 PREPARATION OF BONDING SURFACES

Once the RC beams were adequately cured, the bonding surfaces of the beams designed with jackets were prepared. The most effective concrete substrate removal techniques that pose almost negligible risk for micro defects include particle impact removal by sandblasting and hydrodemolition through high pressure water jetting (Bissonnette et al., 2017; ICRI, 2013). The concrete substrate in this research was prepared by mechanical removal as it was simpler to execute in a laboratory setting when compared to particle impact removal techniques. The bonding surfaces were roughened using a heavy duty triple headed scabbling hammer (Cruciform version) by Trelawny and a composite air needle scaler by Sealey as shown in Figure 3.24 along with a typical roughened surface of a beam. The beams were placed invertedly as the bonding surfaces were roughened, and the roughened surfaces included the bottom and side surfaces of the beams along with the bottom surfaces of the flange. The scabbling hammer was mainly used while the needle scaler was only used along the edges of the beam as well as the bottom surfaces of the flange. The measured depth between the original surface and the highest point of the roughened surface was equal to approximately 3.0, with the lower points of the roughened surface appearing to be of the order of millimetres. The degree of roughness, which defines the macro depth of a concrete substrate's surface, can be evaluated using methods such as handheld concrete surface profile chips and sand spread tests amongst others (Bissonnette et al., 2017; ICRI, 2013). Prior studies have demonstrated that a degree of roughness of the order of millimetres (1.0 to 4.0 mm) is adequate for bonding UHPC jackets (Alaee and Karihaloo, 2003; Bahraq et al., 2019; Martinola et al., 2010; Meda et al., 2012; Paschalis, 2017). After the bonding surfaces were roughened, they were thoroughly cleared of dust and laitance using a 2.0 MPa to 18.0 MPa pressure washer (K7 Premium Full Control Plus Home) by Kärcher. Bissonnette et al. (2017) suggest that a clean surface enables

the cavities of the concrete substrate to become unobstructed, which in return allows the newly cast concrete to penetrate and harden through capillary absorption creating an anchorage effect.



Figure 3.24 – Method for roughening bonding surfaces of RC beams

3.4.4 MANUFACTURING OF UHPC JACKETS

The UHPC jacket manufacturing process began with spraying the bonding surfaces of each RC beam with water to obtain a saturated surface dry (SSD) condition. There is no reliable quantitative method for assessing the moisture condition of the substrate, so the beams were normally sprayed with water about an hour before casting. The optimal moisture condition of the concrete substrate is debated amongst researchers, with prior studies showing that an adequate bond can be achieved between old and newly cast concrete with relatively dry substrate conditions (Bissonnette et al., 2017). However, Bissonnette et al. (2017) suggest that an SSD moisture condition may improve the bond between old and newly cast concrete as it limits the rate of movement of water between the concrete substrate and new concrete. As each RC beam lay inverted, the 19 mm plywood mould of the jacket was assembled. The longitudinal steel reinforcement bars were placed into the mould and onto 20 mm plastic spacers, with the interior of the mould covered in Adostrike by Adomast. The jackets of the strengthened specimens were cast with the developed UHPC presented in Table 3.13. Prior to casting the jackets of the strengthened specimens, a trial jacket was cast onto a concrete beam to assess the workability of the developed UHPC in terms of filling the relatively thin jacket moulds without vibration. The cross-section and elevation of the jacketed trial specimen are shown in Figure 3.25, and it represented a quarter of the size of specimen B2 without the flange and steel reinforcement of the RC beam. The manufacturing process of the trial specimen is described in Figure 3.26, and the workability of the developed UHPC proved to be sufficient.

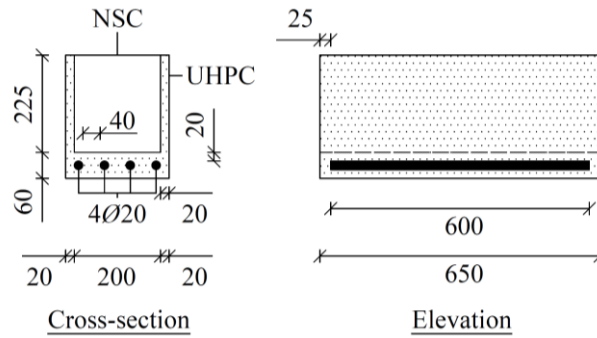


Figure 3.25 – Cross-section and elevation of trial beam with UHPC jacket (Dimensions in mm)

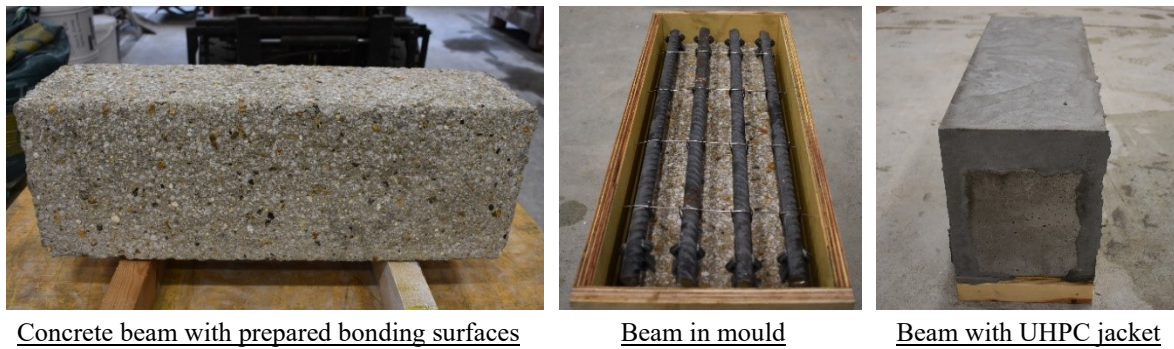


Figure 3.26 – Manufacturing of trial beam with UHPC jacket

Each UHPC jacket was manufactured separately using the process illustrated in Figure 3.27. The UHPC of each jacket was prepared simultaneously in separate batches using either two or three different mixers depending on the size of the jacket. Casting each jacket in batches was required due to the limited volume of UHPC that can be prepared at a time with the available mixers. The Pemat and Soroto mixers were used to produce the UHPC of the jackets as shown in Figure 3.13. The jackets of specimens B2 and B6 with 20 mm side thickness were each cast with two batches of UHPC prepared using a Pemat and a Soroto mixer. The remaining jackets, with side thicknesses of 30 mm and 40 mm, were each cast with three batches of UHPC prepared using two Pemat mixers and a Soroto mixer. The fresh UHPC was initially cast from the left end of each specimen using a handheld bucket or pan. It was necessary to gradually move the casting position along the length of each specimen towards the right end to avoid an overflow. The UHPC flowed from the left to the right side of each specimen with the help of a handheld trowel and the tapping of the sides of the mould with a hammer. Most of the jacket was cast with the batch of UHPC prepared using the Soroto mixer. A minimum of 5 control specimens were cast alongside each jacket due to the limited amount of volume that can be prepared at a time. They included 100 mm cubes and 100 x 200 mm cylinders for hardened density, compressive strength and splitting tensile strength tests. Control specimens were cast from each mixer where possible to ensure that the mechanical properties of each batch were

relatively uniform and consistent. The jackets and control specimens were covered with polythene sheets immediately after casting, and they were demoulded 24 hours after casting. The jackets and control specimens were moist cured as they were sprayed with water daily and covered afterwards with polythene sheets for a 28-day period.



Pre-wetting of RC beam



RC beam in mould



Casting UHPC from initial position



Casting UHPC from final position



Curing of jackets and control specimens of group 1



Curing of jackets and control specimens of group 2

Figure 3.27 – Manufacturing and curing of UHPC jackets of groups 1 and 2

3.5 INTERFACIAL BOND CHARACTERISATION

The interfacial bond between the NSC and UHPC was characterised to ensure that the bond capacity of the jacketed specimens was adequate. A variety of test methods can be applied to characterise the bond capacity of the NSC-UHPC interface, with the most common methods shown in Figure 3.28. The various test methods can be categorized based on the stress state of the bond interface, which includes tension, shear or combined shear and compression (Espeche

and León, 2011; Harris et al., 2015). The NSC-UHPC interfaces of the jacketed specimens are subjected to shear stresses when loaded, and the shear stresses may manifest into interfacial shear or tensile failure. Therefore, a combination of direct tensile and pure shear test methods should ideally be applied. The direct tensile test method was not considered due to its sensitivity to loading arrangement, specimen shape and misalignment of the specimen when loaded (Benson and Karihaloo, 2005). The various shear test methods were also not considered due to the complexities in preparing the test specimens, and the tests are also sensitive to misalignment of the specimen when loaded (Zhang et al., 2020a). The splitting tensile test method was applied in this research seeing as the test specimens were relatively simple to prepare and the test method was easy to carry out, with several prior studies also carrying out the same test (Al-Osta et al., 2017; Bahraq et al., 2019; Ganesh and Ramachandra Murthy, 2020; Harris et al., 2011, 2015; Tayeh et al., 2012; Zhang et al., 2020b). The adequacy of the interfacial bond between the NSC and UHPC was deemed sufficient if the composite splitting tensile strength was found to be greater than the splitting tensile strength of the NSC (Harris et al., 2015).

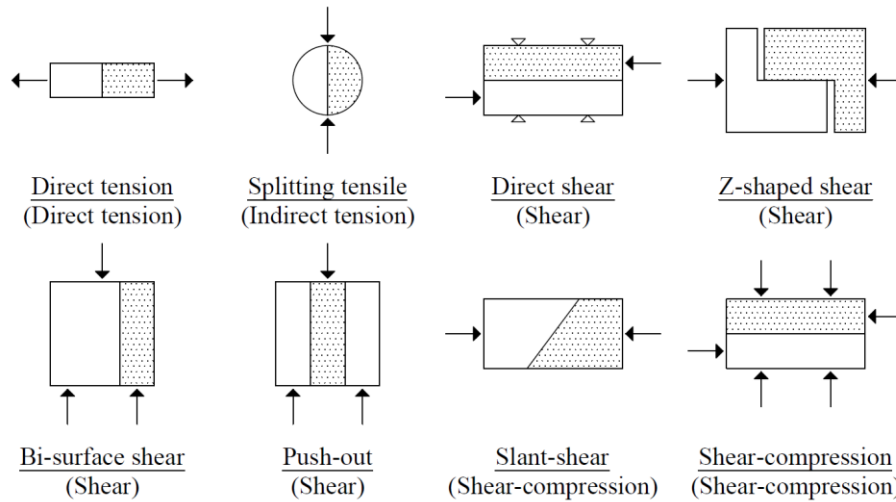


Figure 3.28 – Common interfacial bond strength test methods. Adapted from Espeche and León (2011). Image reproduced with permission of the rights holder, Elsevier.

The splitting tensile test method was carried out by applying an indirect tensile load at a constant rate of 0.04 MPa/s on 150 x 300 mm NSC-UHPC composite cylinders (BSI, 2009b). The manufacturing process of the NSC-UHPC composite cylinders is described in Figure 3.29. Two NSC cylinders were cast and moist cured alongside the RC beams of groups 1 and 2, with the four halves of the two cylinders used to produce four composite specimens per group. A thin metal sheet was placed in the middle of each mould prior to casting as shown in Figure 3.29, so that the two halves of each cylinder can be easily separated. The bonding interface of

each NSC half cylinder was prepared using the same set of processes that were applied to the bonding surfaces of the strengthened specimens, which are described in Section 3.4.3. The four NSC half cylinders of each group were placed into moulds as shown in Figure 3.29, and the remaining half of each mould was cast with a single batch of UHPC prepared using the Soroto mixer. The composite cylinders were covered with polythene sheets immediately after casting the UHPC, and they were demoulded 24 hours after casting. Much like the jackets of the strengthened specimens, the composite specimens were moist cured as they were sprayed with water daily and covered afterwards with polythene sheets for a 28-day period. The NSC-UHPC composite specimens are shown in Figure 3.30 along with the splitting tensile test setup.

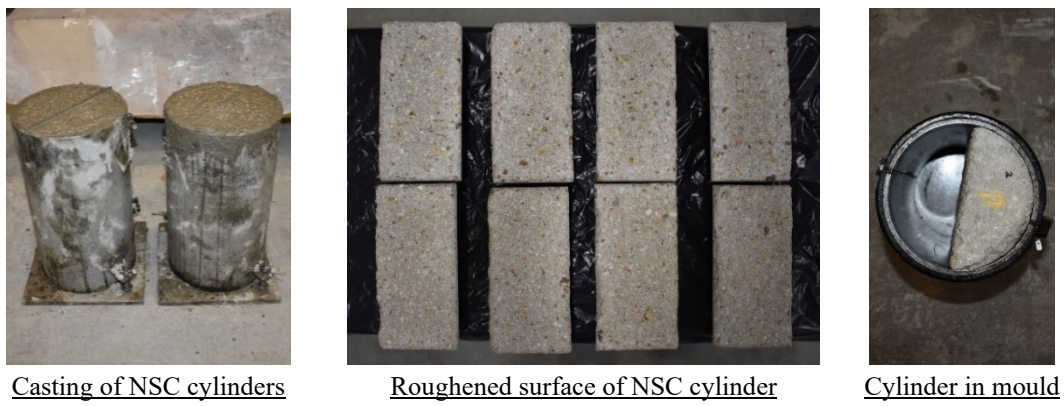


Figure 3.29 – Manufacturing of NSC-UHPC composite specimens

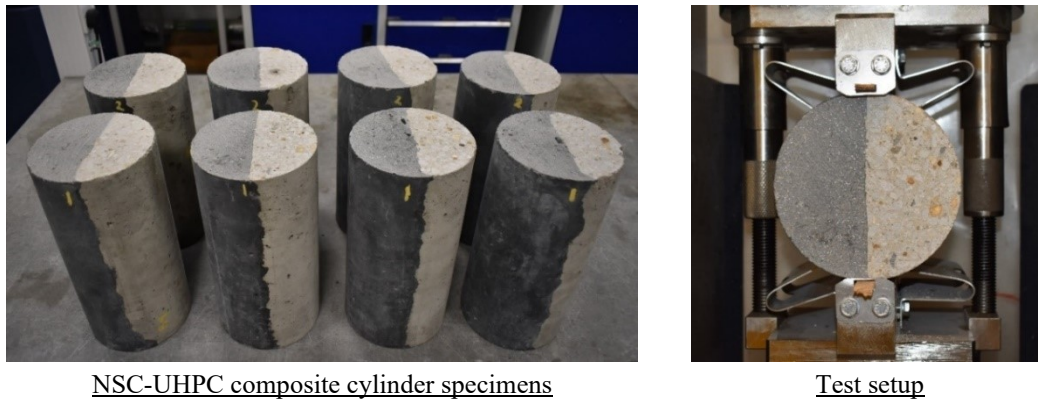


Figure 3.30 – NSC-UHPC composite specimens and splitting tensile test setup

3.6 TEST SETUP AND DATA ACQUISITION

The typical test setup for the beams of this research is described in Figure 3.31, with a schematic drawing presented in Figure 3.32. All specimens were loaded with a 1000 kN force capacity Instron actuator (Type A102404-1002). The load was applied in displacement control at a constant rate of 0.2 mm/min throughout the test for all specimens, excluding the control specimens of groups 1 (B1) and 2 (B5). To reduce the test duration, the load was applied at

initial rates of 0.1 mm/min and 0.2 mm/min for the control specimens of groups 1 (B1) and 2 (B5), respectively, and they were incrementally increased to 1.0 mm/min and 0.8 mm/min at various loading stages until failure. A load cell beneath the actuator measured the total applied load, and another load cell beneath the right support measured the reaction. The reaction of the left support was equal to their difference. A datalogger recorded all data from the instrumentation at a frequency of 1.0 Hz, and a computer recorded the data from each digital image correlation (DIC) system at a frequency of 1.0 Hz. Although the acquisition frequencies were similar, the data acquired by the datalogger and computer were not exactly synchronized.



Figure 3.31 – Typical experimental set up of beam specimens

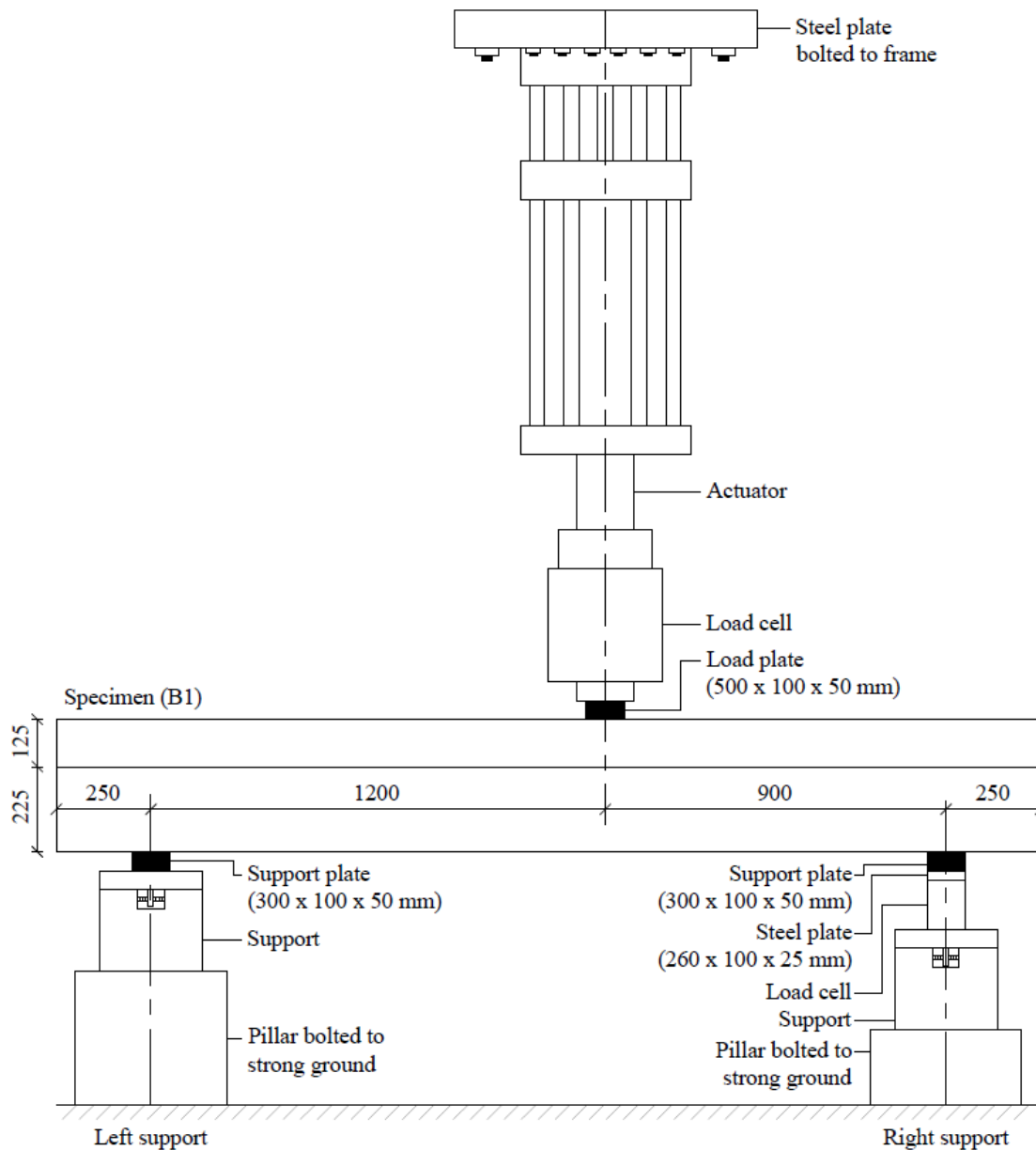


Figure 3.32 – Schematic drawing of typical experimental setup of beam specimens (Dimensions in mm)

Different loading plates were used for the specimens of groups 1 and 2 as shown in Figure 3.33. A rectangular load plate measuring 500 x 100 x 50 mm was used for the specimens in group 1. When using the rectangular load plate, the loading ratios measured from each support during the experiment were observed to be marginally different from the theoretical loading ratios that are based on the experimental setup as discussed in Section 4.4.5. Due to these differences in loading ratios, a knife edge load bearing plate was used for the specimens of group 2. The knife edge plate distributed the applied load more evenly resulting in actual loading ratios closer to the theoretical loading ratios. The knife edge load plate was 457 x 98 x 85 mm, and rectangular prism plates were placed beneath the knife edge plate, which were 500

x 100 x 50 mm and 500 x 100 x 10 mm for the control and strengthened specimens of group 2, respectively. The steel plates of the left and right supports were 300 x 100 x 50 mm, and each support is shown in Figure 3.34.

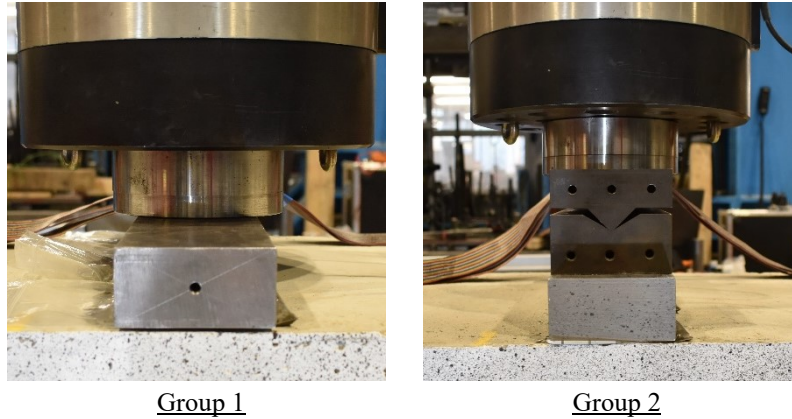


Figure 3.33 – Load plates of beam specimens of groups 1 and 2

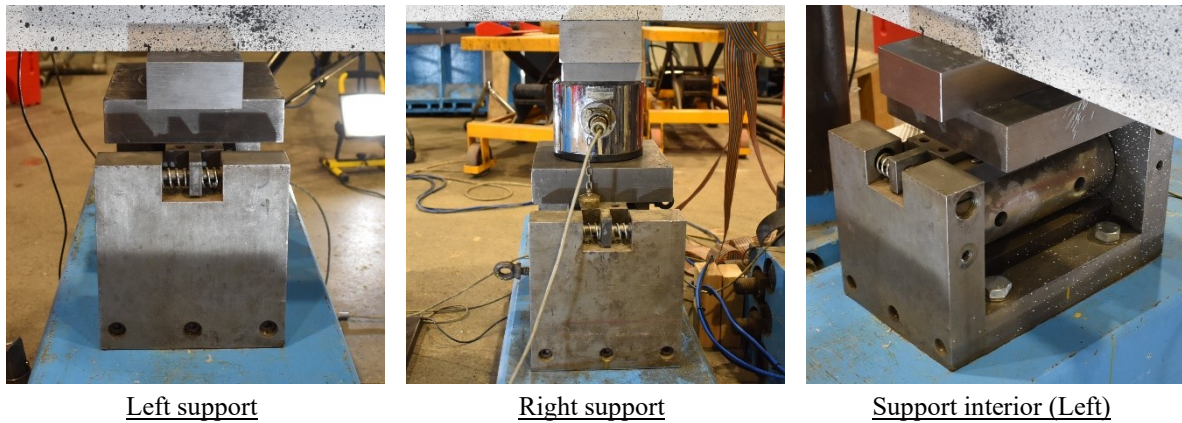


Figure 3.34 – Supports of beam specimens

3.7 INSTRUMENTATION

Each beam of this campaign was fitted with a similar set of instrumentation to monitor various aspects of structural performance throughout the experiment. Linear variable displacement transducers and inclinometers were used to record the global deformations and rotations of each specimen, respectively. Load cells at the loading point and right support recorded the force that is transmitted and resisted, respectively. The strains of the longitudinal and transverse reinforcement at key locations were measured by installed strain gauges. A digital image correlation (DIC) system was also used to monitor the cracked surfaces of each specimen throughout the experiment as described later in Section 3.8. The locations of the linear variable displacement transducers (LVDTs), load cells (LCs), inclinometers (INCs) and strain gauges

are shown in Figures 3.36 and 3.37 for the specimens of groups 1 and 2, respectively. Photographs of the typical locations of the LVDTs, LCs and INCs are shown in Figure 3.38.

The upward vertical displacements of the left and right ends of each specimen were measured by LVDT-1 and LVDT-3, respectively. They were both placed at the top surface of the beam over the centrelines of each respective support. Thin metal plates were adhesively bonded onto the top surface of each specimen to ensure a flat surface beneath LVDT-1 and LVDT-3. The downward vertical displacement at the loading point was measured with LVDT-2, which was placed on the bottom surface of each specimen and aligned at the centreline of the loading point. LVDT-1 and LVDT-3 were type DCTH1000A by RDP Group with a measurement range of 25 mm, and LVDT-2 was type DTCH2000A by RDP Group with a measurement range of 50 mm.

The total load acting on each specimen was recorded throughout the test by LC-1, which was located beneath the actuator. The load at the right support was recorded by LC-2, which was located beneath the support plate. The load at the left support was assumed to be equal to the difference between the total load (LC-1) and the load recorded at the right support (LC-2). The load cell within the actuator (LC-1) was Dynacell dynamic series 2527-120 by Instron, which was capable of measuring up to 1000 kN. The load cell at the right support (LC-2) was a 1000 kN NCB MRE type 440 by W. H. Mayes & Son (Windsor) Limited.

The rotations at the left and right ends of each specimen were measured by INC-1 and INC-2, which were magnetically mounted at the mid-height of the flange and aligned at the centrelines of each respective support. The inclinometers were DOG2 MEMS series type G-NSDOG2-001 by TE Connectivity with a measurement capacity of 25°. The inclinometers were placed on the back face of each specimen.

A set of strain gauges was installed onto the longitudinal and transverse steel reinforcement bars of each beam and jacket following the assembly of the steel reinforcement. The strain gauges were types YFLA-5-1LJC-F and FLAB-5-11-1LJC-F by Tokyo Instruments Laboratory that were capable of measuring strains of up to 20% and 5%, respectively. The notation N in the strain gauge identification number shown in Figures 3.36 and 3.37 refers to strain gauge type FLAB-5-11-1LJC-F. The strain gauge measuring lengths were equal to 5.0 mm and 1.5 mm for types YFLA-5-1LJC-F and FLAB-5-11-1LJC-F, respectively, and they

were both 10 mm to 12 mm in length. Prior to fixing the gauges, the surface of the bar was ground smooth with a disc grinder. The gauges were mounted on the sides of the bars between the ribs as shown in Figure 3.35 to minimise removal of material during grinding. The placement areas were thoroughly cleaned after grinding using acetone and a neutralizing acid to remove any lingering residue that may affect the bond. The strain gauges were adhesively bonded onto the bars as shown in Figure 3.35 using CN adhesive by Tokyo Instruments Laboratory, and then they were coated with epoxy for protection.

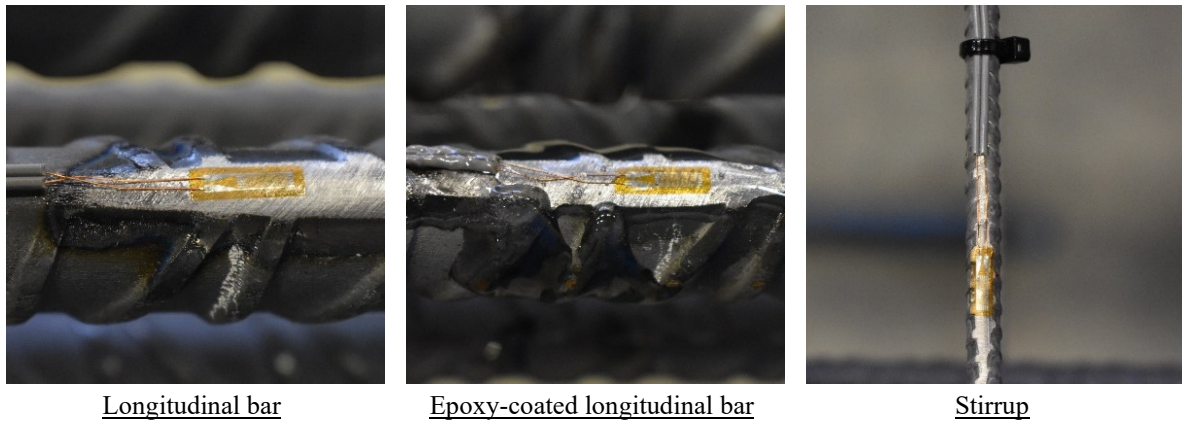


Figure 3.35 – Installed strain gauges on steel reinforcement

Strain gauges were installed onto the top and bottom longitudinal reinforcement bars at the loading point, which corresponds to the point of maximum bending. Strain gauges were also placed onto the longitudinal tension reinforcement mid-span of the critical shear span. Strain gauges were placed on the longitudinal reinforcement in tension near the left support to assess the dowel action. A pair of strain gauges were installed on diametrically opposite sides of the longitudinal reinforcement bars to offset the influence of bending. Strain gauges were also placed on one leg of each stirrup in the critical shear span. Their exact locations on the stirrups were selected based on the areas of maximum stress indicated by preliminary numerical modelling results of the beam specimens of this research.

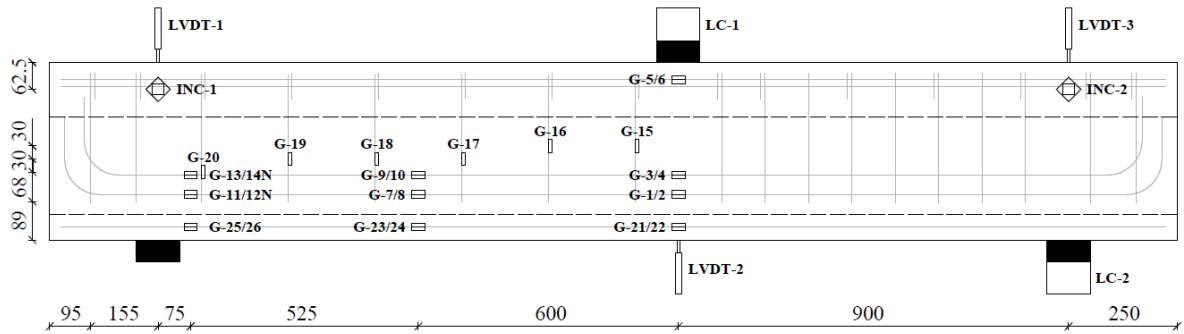
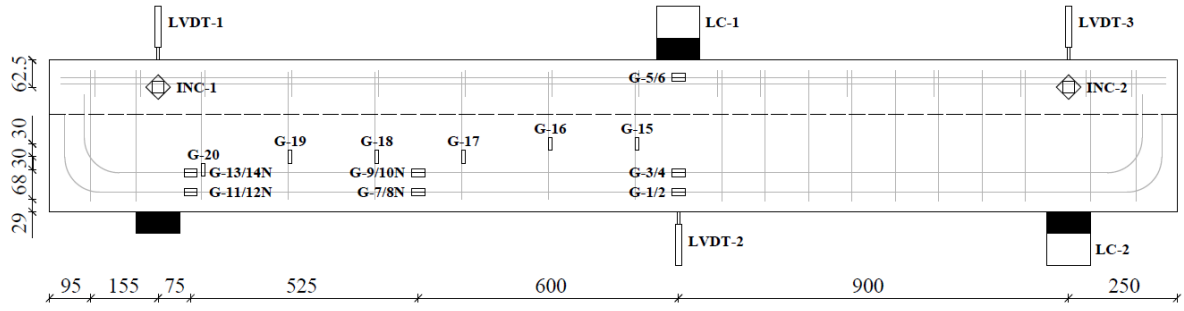


Figure 3.36 – Instrumentation of group 1 specimens (Dimensions in mm)

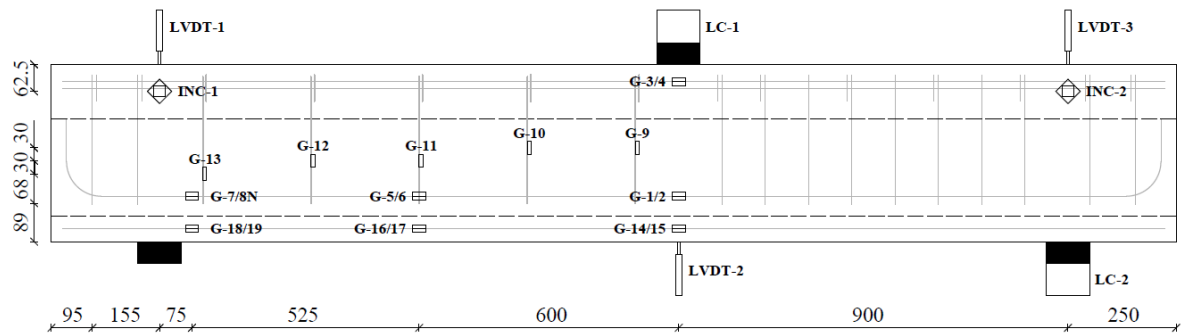
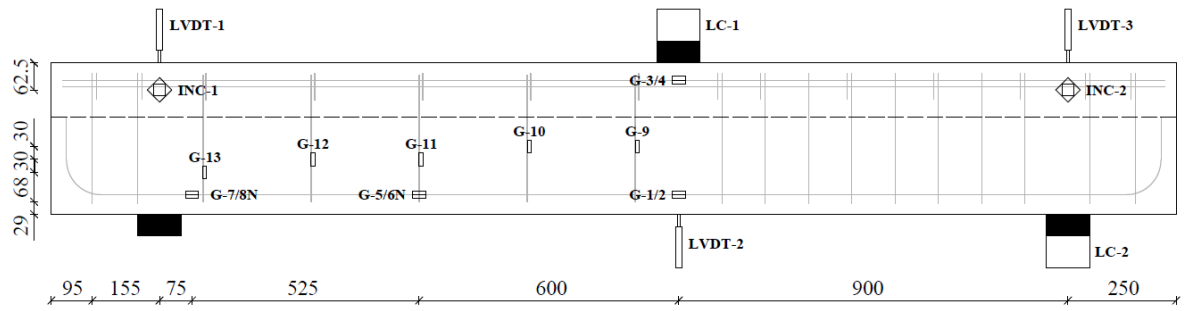
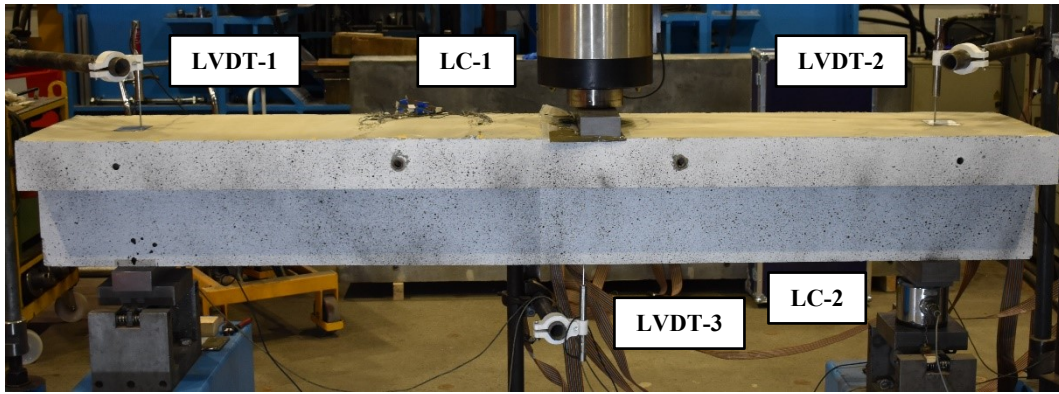
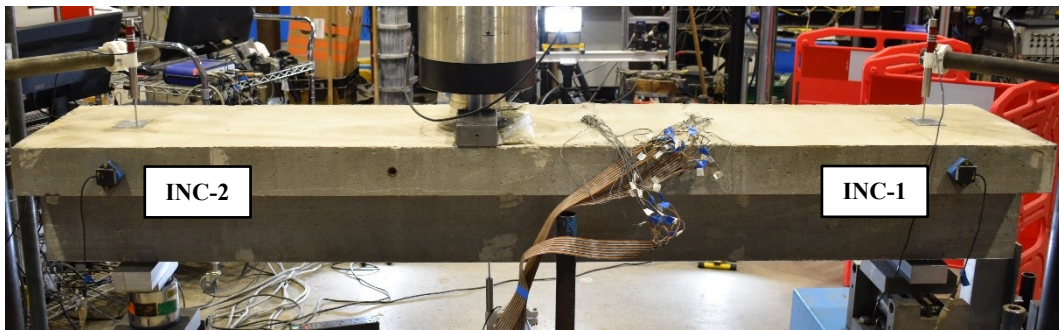


Figure 3.37 – Instrumentation of group 2 specimens (Dimensions in mm)



Locations of LVDTs and LCs (Front FOV)



Locations of INCs (Back FOV)

Figure 3.38 – Typical instrumentation of specimens of groups 1 and 2

3.8 DIGITAL IMAGE CORRELATION (DIC)

Digital image correlation (DIC) was used in this research to monitor the cracked surfaces of each primary specimen throughout the experiment until failure. DIC is a non-contact optical measurement method that tracks the movements and deformations of a targeted surface pattern of a specimen under loading by capturing high-resolution images at a specified frequency. Global deformations and crack displacements were ultimately calculated for each beam using the images that were recorded by the DIC system, and this is discussed later in Chapter 5. DIC was preferred over alternative methods for measuring crack displacements such as DEMEC mechanical strain gauges or extensometers as it is simpler, safer and less time-consuming (Pastore, 2020). The LaVision DIC system used in this research included a programmable timing unit that was connected to a set of high-resolution cameras and lighting units that were mounted on a metal rail, which were all supported by a tripod (LaVision, 2019). Several lighting units were also placed around each specimen to illuminate the targeted surface pattern. The programmable timing unit was connected to a computer, where the LaVision software package DaVis (Version 10.2.1) was used to control all data acquisition features as well as pre- and post-processing features (LaVision, 2019).

A DIC system was used to monitor the front face only for the initial two specimens that were tested, which were control specimen B1 and strengthened specimen B2. The development of the cracks appearing on the front and back faces of the UHPC jacket of specimen B2 appeared to vary during the test, and this is discussed later in Section 4.4.2. As a result of this observation, an additional DIC system was used to monitor the back face of the remaining specimens. The DIC systems used for the front and back face of views (FOVs) of each specimen are described in Figure 3.39. For the front FOV, four Imager M-lite 5M cameras by LaVision were used, and each camera had a 16 mm focal length lens with a resolution of 2464 x 2056 pixels. Cameras 1 and 2, which were mounted on separate ends of the metal rail in Figure 3.39, were focused on the critical span of each specimen. Cameras 3 and 4 in Figure 3.39 were focused on the entire specimen in case failure occurred outside of the critical span. For the back FOV, two Image E-lite 5M cameras by LaVision were used to focus on the critical span, and each camera had a 16 mm focal length lens with a resolution of 2448 x 2050 pixels. The distances between the specimen and the DIC systems of the front and back FOVs were approximately 2.4 m and 1.7 m, respectively.

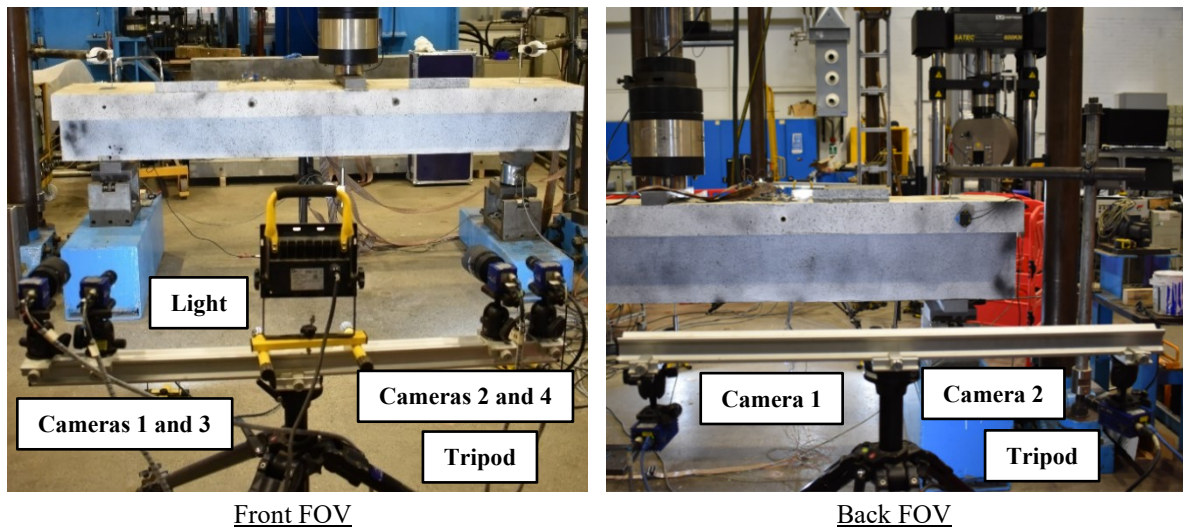
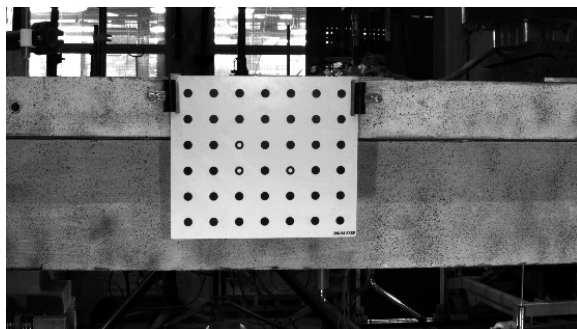


Figure 3.39 – Digital image correlation (DIC) systems for front and back FOVs of beam specimens

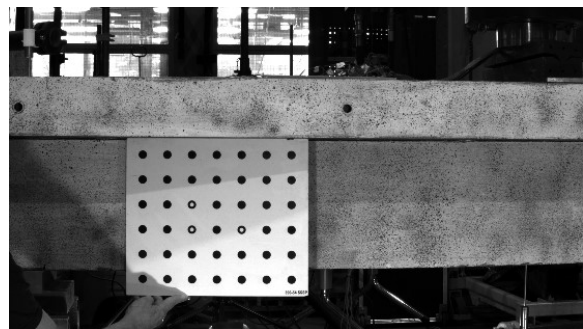
The strains and displacements of each targeted surface were processed in DaVis by comparing consecutive images that were captured at a specified rate throughout the test. Images were recorded at a frequency of 1.0 Hz for each DIC system, and they were synchronized with the applied load as the actuator was connected to each programmable timing unit. A speckle pattern was applied to the targeted surface of each specimen to establish identifiable characteristics in each image. Recorded images were discretized into small user-defined subsets, and the

displacements were obtained by tracking present movements and comparing them to past positions (LaVision, 2019; Pastore, 2020). A speckle pattern was applied to the targeted area of each beam as shown in Figure 3.39. The monitored surfaces were initially painted white using a paint roller, and random black dots of 1.0 to 3.0 mm were then gently spray-painted onto the white surfaces. The subset size that was used when processing the images in DaVis was 21 x 21 pixels with a step size of 7 pixels.

The DIC systems were calibrated before each test to correct perspective distortions and to define the image scale (LaVision, 2019). Calibration is essential as it determines the image scale that converts images from pixel to millimetres. The calibration process is described in Figure 3.40, and it involved placing a calibration plate (395-54 SSSP) on each section of the targeted surface while capturing an image for each plate position. The calibration plate was placed on the surfaces of the flange and web of each beam as they are in different planes. Over 10 images were normally captured during the calibration process for all specimens. Calibration was automatically processed in DaVis where a calibration mapping function was fitted to the marks on the calibration plate in each image (LaVision, 2019). According to LaVision (2019), the fit error should be checked as it indicates the average deviation between the mark positions on the plate in each image and the ideal grid. The fit error was found to be less than 0.1 pixel for all beams, which is considered excellent according to LaVision (2019), and the image scale was around 1.86 pixel/mm for all beams.



Calibration plate against flange



Calibration plate against web

Figure 3.40 – Typical calibration process of DIC system of beam specimens

DIC was also used to monitor the cracked surfaces of the notched UHPC prisms, which were subjected to three-point bending tests to characterise the post-cracking tensile strength of the developed UHPC as discussed in Section 3.3.7. Rather than using a mechanical extensometer, which may be tedious, DIC was used to measure the crack opening at the notch, or crack mouth

opening displacement (CMOD), of each prism throughout the test, and the method of calculation is discussed later in Section 4.2.4.1. The DIC system and calibration process are described in Figure 3.41. Images were recorded at a frequency of 1.0 Hz, and they were synchronized with the applied load and displacement of the actuator. The DIC system consisted of two Image E-lite 5M cameras by LaVision, and each camera had a 16 mm focal length lens with a resolution of 2448 x 2050 pixels. The distance between the specimen and the DIC system was approximately 0.8 m. Much like the beam specimens, a speckle pattern was applied onto the targeted surfaces of the UHPC prisms. The subset size that was used when processing the images in DaVis was 21 x 21 pixels with a step size of 7 pixels. Before each test, the DIC system was calibrated similarly to the beams of this campaign where a calibration plate (106-10) was placed on each section of the targeted surface while an image was captured for each plate position as in Figure 3.41. Three images were taken during the calibration process, and the fit errors were all less than 0.1 pixel while the image scale was around 6.01 pixel/mm.

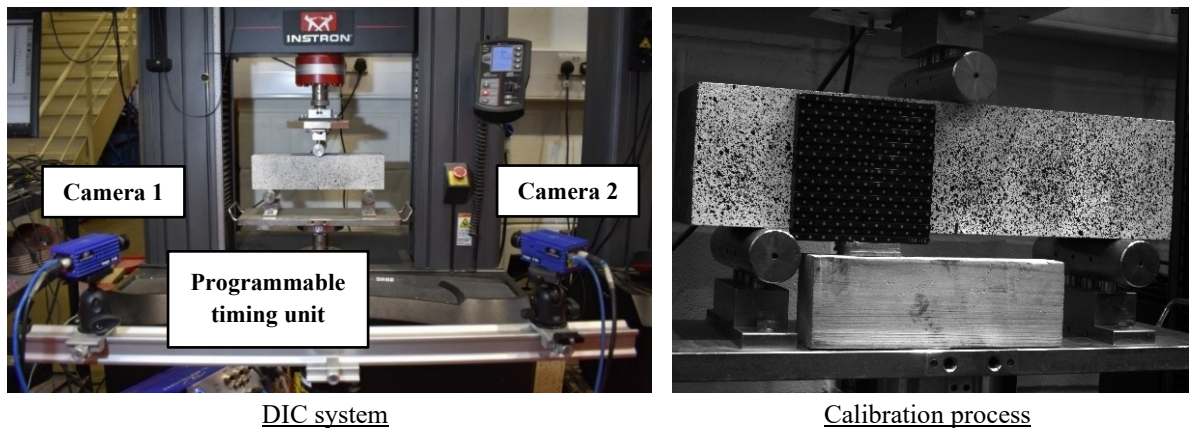


Figure 3.41 – Digital image correlation (DIC) system of UHPC prism specimens

3.9 SUMMARY

This chapter described the experimental program of this research that focused primarily on the testing of two groups of four beams. Each group of beams consisted of one control RC beam and three identical RC beams with UHPC U-shaped jackets that varied in geometry and amount of embedded longitudinal reinforcement. The specimens were subjected to three-point bending tests with asymmetrical type monotonic loading until failure, and the loading arrangement is presented in Figure 3.1. The aim of the tests was to investigate the influence of the UHPC U-shaped jackets on the ultimate shear and flexural strength enhancement of the RC beams. The RC beams of group 1 were shear critical as they were heavily reinforced in flexure. The RC

beams of group 2 were also shear critical but more lightly reinforced in flexure. The UHPC jackets of the three strengthened specimens of each group had a bottom thickness of 60 mm and side thicknesses of 20 mm, 30 mm, and 40 mm for each respective jacket. The longitudinal reinforcement embedded in the jackets of group 1 was the same for all the specimens in group 1 but varied for the jackets of group 2. The non-proprietary UHPC used to manufacture the jackets was developed through trial mixing and testing. Composite cylinder specimens were also manufactured to characterise the interfacial bond between the NSC and ensure that the bond capacity of the jacketed specimens was adequate.

This chapter also provides a description of the instrumentation used in each beam. Each beam was fitted with linear variable displacement transducers, inclinometers, load cells and strain gauges. A digital image correlation (DIC) system by LaVision was also used to monitor the cracked surfaces of each specimen throughout the experiment (LaVision, 2019). Each beam specimen had two DIC systems that monitored the front and back faces, excluding specimens B1 and B2, which were tested with only one DIC system monitoring the front face. The main features of the DIC system are described in this chapter including the hardware, software and calibration process. The DIC results were essential to calculating the global deformations and crack displacements of each specimen, and this is discussed later in Chapter 5. This chapter also described the DIC system that was used to monitor the cracked surfaces of the notched UHPC prisms, which were subjected to three-point bending tests to characterise the post-cracking tensile strength of the developed UHPC.

CHAPTER 4: EXPERIMENTAL RESULTS

4.1 INTRODUCTION

This chapter presents the experimental results for each of the two groups of four beams tested in this campaign. To recapitulate, each group of specimens consisted of one control RC beam specimen and three RC beams strengthened with UHPC jackets having varying dimensions and amounts of longitudinal reinforcement. This chapter presents the material properties of the adopted steel reinforcement, NSC and UHPC as well as the results of the tensile slitting tests carried out to characterise the interfacial bond between the NSC and UHPC. Also reported are the results of each beam test. The presented results include crack development, modes of failure, load-displacement responses and strains in the steel reinforcement. To conclude this chapter, the depths of the critical cracks appearing on the specimens with jackets are assessed through the partial removal of the jackets and probe penetration.

4.2 MATERIAL PROPERTIES

4.2.1 INTRODUCTION

Materials tests were conducted on at least three control specimens, unless noted otherwise, to establish the mechanical properties of the steel reinforcement, NSC and UHPC. The tensile strength of the steel reinforcement bars was determined through direct tensile tests on bars from the same batch as those used to manufacture the beam specimens. The hardened cube density, compressive cube and cylinder strength, splitting tensile strength and elastic modulus of both the NSC and UHPC were determined, in addition to the direct tensile strength of UHPC. The dimensions of each control specimen were measured prior to testing and considered when determining the mechanical properties. The mechanical properties of the supplied NSC were obtained at 28 days and at the day of testing each beam specimen using the control specimens cast alongside each group of beam specimens. Some NSC control specimens were moist cured alongside the beam specimens while others were cured in water at 20°C, so that an average can ultimately be taken between the results of both curing conditions. The mechanical properties of the developed non-proprietary UHPC were obtained at 28 days using moist cured control specimens that were cast separately from the jackets as only a limited volume of UHPC could

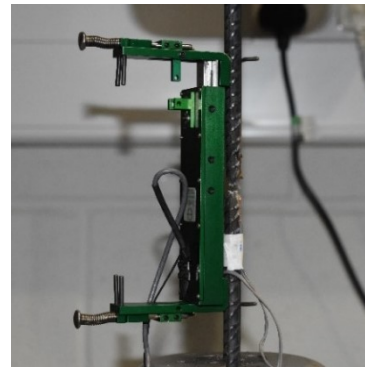
be cast at a time. The mechanical properties of UHPC on the day of testing were obtained using a limited amount of control specimens that were cast and moist cured alongside each jacket.

4.2.2 STEEL REINFORCEMENT

Direct tensile tests on the $\phi 20$, $\phi 10$ and $\phi 6$ reinforcement bars were conducted in accordance with BS EN ISO 6892-1:2019 as shown in Figure 4.1 (BSI, 2020). The lengths of the tested bars were equal to 400 mm, 300 mm and 250 mm for the $\phi 20$, $\phi 10$ and $\phi 6$ bars, respectively. The tests were conducted with a constant cross-head strain rate of 1.0 mm/min until failure. A clip-on strain gauge extensometer and a pair of strain gauges, which were adhesively bonded on diametrically opposite ends, were used to measure the strains for the $\phi 20$ and $\phi 10$ bars. The extensometer had a measuring length of 100 mm and the strain gauges were of type YFLA-5-1LJC-F supplied by Tokyo Instruments Laboratory. To adhesively bond the strain gauges, the procedures detailed in Section 3.7 were applied. Only a clip-on extensometer was used to measure the strains of the $\phi 6$ bar seeing as the cross-section diameter was too small to be ground down to install strain gauges.



Test setup and instrumentation – $\phi 10$ bar



Specimen at failure – $\phi 10$ bar

Figure 4.1 – Direct tensile test of steel reinforcement bar

Tensile tests were conducted for each size of reinforcement bar to determine the overall stress-strain responses. The stress-strain responses captured for the $\phi 20$, $\phi 10$ and $\phi 6$ bars are shown in Figure 4.2. The strain responses include those captured by the clip-on extensometer (E) and average of the pair of strain gauges (G) for each specimen as well as the determined average responses. The yield strength of all of the reinforcement bars was calculated using the 0.2% proof rule as shown in Figure 4.2 (BSI, 2020). Seeing as there is a well-defined yield plateau for the $\phi 20$ and $\phi 10$ steel reinforcement bars (see Figure 4.2), the yield strain was determined by dividing the yield stress by the elastic modulus. A well-defined yield plateau was not observed for the $\phi 6$ stainless steel reinforcement bar (see Figure 4.2), therefore the yield strain

was taken as the strain corresponding to the 0.2% proof stress. The average results and standard deviations of each bar size are shown in Table 4.1. For the $\phi 20$ and $\phi 10$ reinforcement bars, the yield strength and elastic modulus were determined using the strains measured by the strain gauges while the ultimate strength and corresponding strain were determined using the strains measured by the clip-on extensometer. For the $\phi 6$ reinforcement bar, all results were determined using the strains measured by the clip-on extensometer.

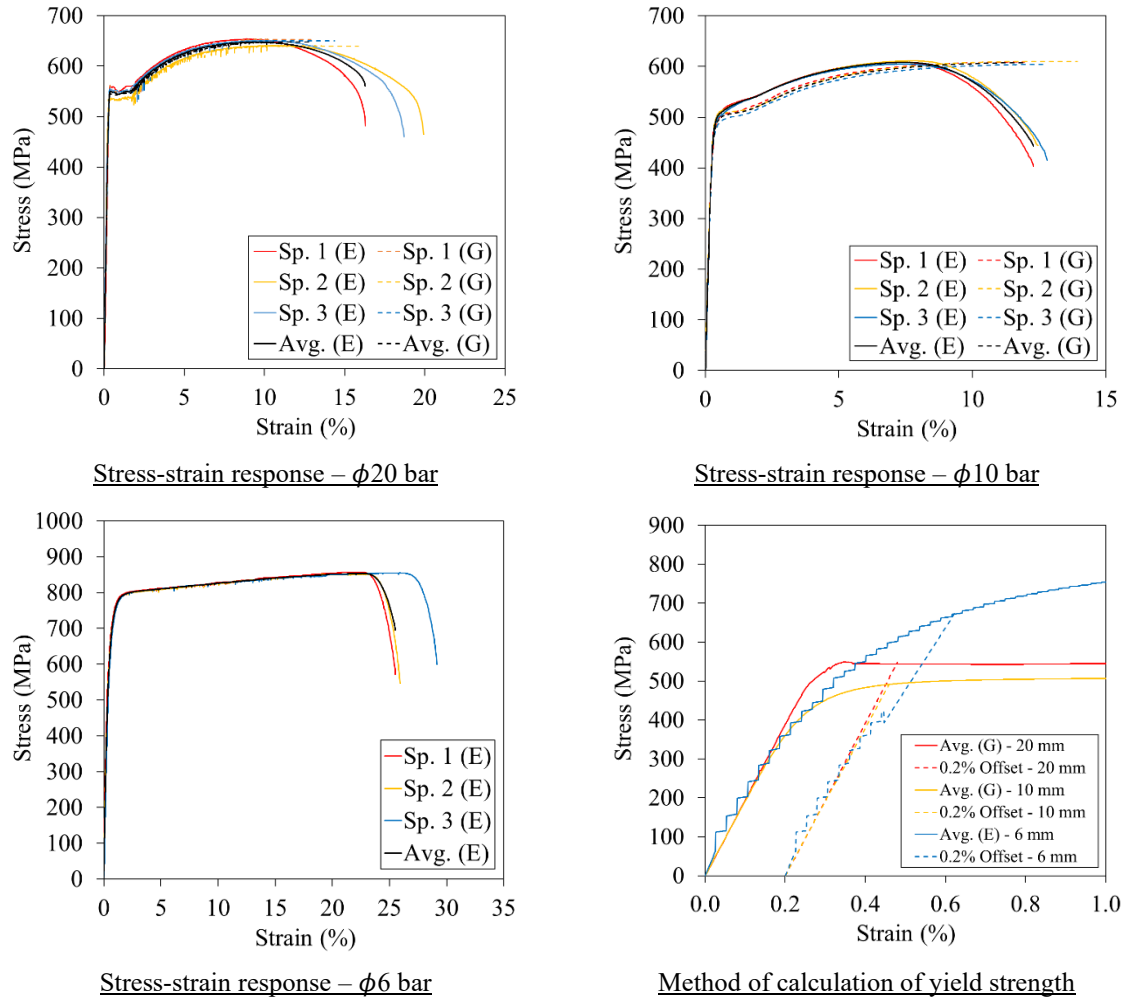


Figure 4.2 – Direct tensile response of steel reinforcement

Table 4.1 – Steel reinforcement mechanical properties

Diameter (ϕ) [mm]	Yield strength (f_{ym}) [MPa]	Yield strain (ϵ_y) [%]	Ultimate strength (f_u) [MPa]	Ultimate strain (ϵ_u) [%]	Elastic modulus (E_{sm}) [MPa]
20	547.94 (8.89)	0.28 (0)	647.70 (7.41)	9.68 (1.00)	195692.96 (259.13)
10	506.30 (6.88)	0.26 (0)	608.09 (3.30)	7.39 (0.30)	195484.55 (3980.00)
6	673.47 (8.93)	0.62 (0.01)	853.90 (2.89)	22.62 (1.61)	159212.43 (3181.60)

[Note] – () refers to standard deviation

4.2.3 NORMAL STRENGTH CONCRETE (NSC)

The ready-mix NSC supplied by CEMEX was specified as strength grade C30/37 with a consistence class S3 as detailed in Section 3.4.2. The slump of the fresh concrete was measured in accordance with BS EN 12350-2:2019 as shown in Figure 4.3 (BSI, 2019b). The slump was measured at 95 mm and 58 mm for the NSC of groups 1 and 2, respectively. The setups for the compressive cube and cylinder strength tests and the splitting tensile strength tests are shown in Figure 4.4. The compressive cube and cylinder strength was measured in accordance with BS EN 12390-3:2009, and the load was applied at a constant rate of 0.4 MPa/s on 100 mm cubes and 100 x 200 mm cylinders (BSI, 2011). The splitting tensile test, also known as the Brazilian test, was carried out in accordance with BS EN 12390-6:2009, with the indirect tensile load applied at a constant rate of 0.04 MPa/s on 150 x 300 mm cylinders (BSI, 2009b). The hardened cube density was determined in accordance with BS EN 12390-7:2009 (BSI, 2008b).

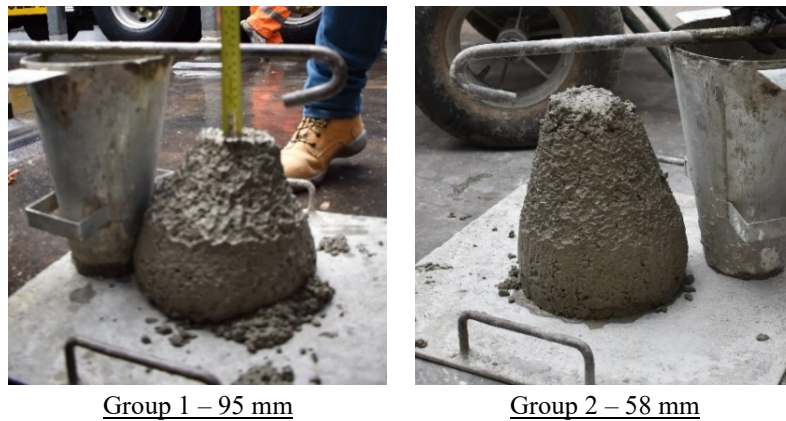


Figure 4.3 – Slump of NSC of groups 1 and 2

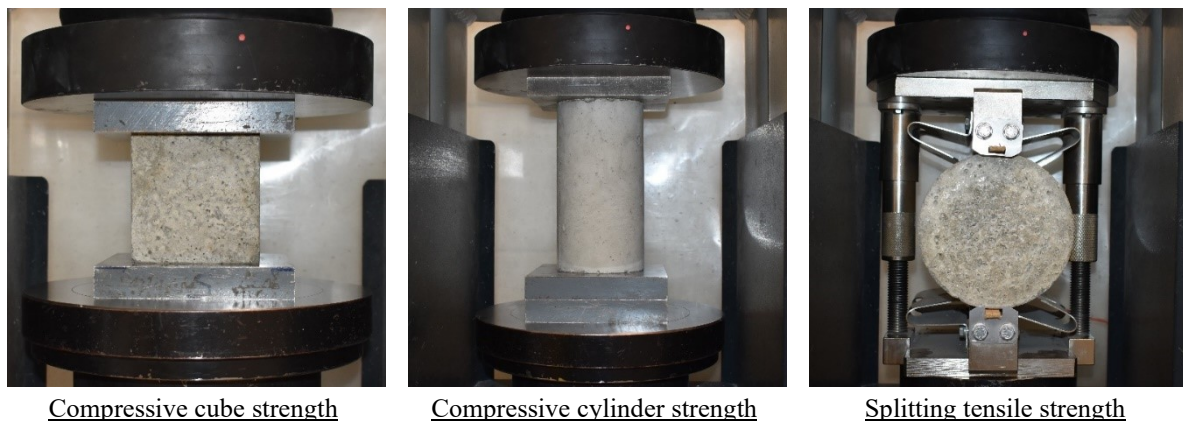


Figure 4.4 – Testing for various mechanical properties of NSC

Water cured cubes were used to evaluate the compressive strength development with age starting from day 3, 7, 14, 21 and 28 followed by the days of testing each beam specimen, and the results are shown in Tables 4.2 and 4.3 for groups 1 and 2, respectively. The compressive cube strength at 3 to 28 days after casting increased from 31.18 MPa to 42.11 MPa and 37.26 MPa to 49.27 MPa for groups 1 and 2, respectively. The beams were tested between 163 to 176 days after casting for group 1 and 163 to 191 days after casting for group 2. Between testing the first and last beams of each group, the compressive cube strength marginally increased from 49.09 MPa to 53.31 MPa and 62.48 MPa to 64.94 MPa for groups 1 and 2, respectively. The average hardened cube density for all specimens was equal to about 2350 kg/m³ and 2370 kg/m³ for groups 1 and 2, respectively, with both groups having a relatively low standard deviation of about 10 kg/m³.

Table 4.2 – NSC compressive cube strength development with age of group 1

Property	Age [Day]								
	3	7	14	21	28	163 ^A	169 ^A	172 ^A	176 ^A
Density (ρ) [kg/m ³]	2344.71 (3.74)	2327.66 (7.48)	2349.20 (5.16)	2354.6 ^B	2349.96 (3.20)	2359.37 (4.49)	2358.31 (2.43)	2356.19 (6.67)	2358.54 (6.04)
Compr. strength ($f_{cm,cu}$) [MPa]	31.18 (1.16)	32.67 (5.33)	41.96 (3.09)	43.29 ^B	42.11 (9.97)	49.09 (4.52)	51.49 (4.91)	52.78 (2.34)	53.31 (5.68)

[Note] – () refers to standard deviation; ^A Age at testing of specimens B1 to B4 in consecutive order following day 28; ^B Tested 1 specimen

Table 4.3 – NSC compressive cube strength development with age of group 2

Property	Age [Day]								
	3	7	14	21	28	163 ^A	171 ^A	176 ^A	191 ^A
Density (ρ) [kg/m ³]	2358.67 (9.44)	2363.61 (7.87)	2365.06 (1.67)	2360.97 (10.80)	2366.87 (3.40)	2373.87 (5.78)	2380.73 (6.14)	2383.70 (5.76)	2378.04 (5.46)
Compr. strength ($f_{cm,cu}$) [MPa]	37.26 (3.64)	38.76 (11.33)	48.89 (0.92)	53.57 (1.70)	49.27 (9.96)	62.48 (1.69)	59.26 (4.70)	61.98 (3.09)	64.94 (2.57)

[Note] – () refers to standard deviation; ^A Age at testing of specimens B5 to B8 in consecutive order following day 28

The compressive cylinder strength of moist and water cured specimens were taken at 28 days as well as the days of testing the first and last beam specimens of groups 1 and 2. The results are shown in Tables 4.4 and 4.5 for groups 1 and 2, respectively. The compressive cylinder strength at 28 days of the moist cured and water cured specimens of group 1 were equal to 41.69 MPa and 44.32 MPa, respectively, and the splitting tensile strength of the moist and water cured specimens were equal to 3.38 MPa and 3.54 MPa, respectively. The compressive cylinder strength at 28 days of the moist cured and water cured specimens of group 2 were

equal to 51.11 MPa and 49.80 MPa, respectively, and the splitting tensile strength of the moist and water cured specimens were equal to 3.35 MPa and 3.29 MPa, respectively. At the times of testing the first (B1) and last (B4) beams of group 1, the compressive cylinder strength of the moist cured specimens was 43.94 MPa and 42.87 MPa while the water cured specimens were 51.04 MPa and 50.00 MPa, respectively. The corresponding splitting tensile strength of the moist cured specimens was 3.57 MPa and 3.41 MPa while the water cured specimens were 3.65 MPa and 3.62 MPa, respectively. At the times of testing the first (B5) and last (B8) beams of group 2, the compressive cylinder strength of the moist cured specimens was 46.68 MPa and 46.53 MPa while the water cured specimens were 53.60 MPa and 55.30 MPa, respectively. The corresponding splitting tensile strength of the moist cured specimens was 3.38 MPa and 3.25 MPa while the water cured specimens were 3.81 MPa and 3.36 MPa, respectively.

Table 4.4 – NSC compressive and splitting tensile strength at 28 days of groups 1 and 2

Group	Compressive cylinder strength		Splitting tensile strength	
	Moist cured (f_{cm}) [MPa]	Water cured (f_{cm}) [MPa]	Moist cured ($f_{ctm,sp}$) [MPa]	Water cured ($f_{ctm,sp}$) [MPa]
1	41.69 (0.70)	44.32 (1.06)	3.38 (0.19)	3.54 (0.07)
2	51.11 (1.33)	49.80 (1.60)	3.35 (0.18)	3.29 (0.18)

[Note] – () refers to standard deviation

Table 4.5 – NSC compressive and splitting tensile strength at day of testing specimens of groups 1 and 2

Group	Specimen	Age [Day]	Compressive cylinder strength		Splitting tensile strength	
			Moist cured (f_{cm}) [MPa]	Water cured (f_{cm}) [MPa]	Moist cured ($f_{ctm,sp}$) [MPa]	Water cured ($f_{ctm,sp}$) [MPa]
1	B1	163	43.94 (1.59)	51.04 (1.03)	3.57 (0.05)	3.65 (0.09)
	B4	176	42.87 (0.77)	50.00 (2.60)	3.41 (0.15)	3.62 (0.02)
2	B5	163	46.68 (1.41)	53.60 (4.34)	3.38 (0.28)	3.81 (0.35)
	B8	191	46.53 (3.60)	55.30 (2.95)	3.25 (0.14)	3.36 (0.43)

[Note] – () refers to standard deviation

The elastic modulus at 28 days was measured according to BS EN 12390-13:2013 using moist cured 100 x 200 mm cylinders (BSI, 2013b). The test consisted of applying three consecutive compressive loading cycles at a constant rate of 0.5 mm/min from 5% to 35% of the compressive cylinder strength. The stress was determined using the load recorded by the actuator, and the corresponding strains were determined using the average measurements from

a pair strain gauges that were adhesively bonded on diametrically opposite ends of the cylinder as shown in Figure 4.5. The strain gauges supplied by Tokyo Instruments Laboratory were of type PL-90-11-1L, and they are capable of measuring strains of up to 2%. The elastic modulus of a single specimen was determined by averaging the inclined slopes of the three cycles' stress-strain responses. The average elastic modulus at 28 days was equal to 34997.87 MPa with a standard deviation of 649.77 MPa for the NSC of group 1, and it was equal to 37496.95 MPa with a standard deviation of 302.78 MPa for the NSC of group 2.

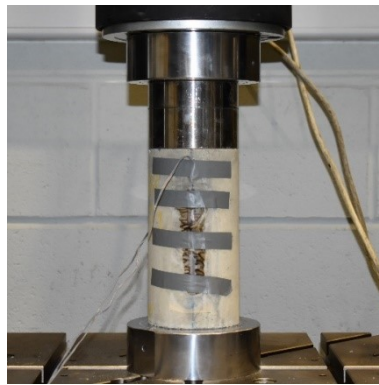


Figure 4.5 – Elastic modulus test setup and instrumentation for NSC

4.2.4 ULTRA HIGH PERFORMANCE CONCRETE (UHPC)

4.2.4.1 MATERIAL PROPERTIES AT 28 DAYS

The material properties of the developed non-proprietary UHPC were obtained at 28 days using moist cured control specimens that were cast in several batches prepared with the Soroto and Pemat mixers. Each batch of UHPC was evaluated individually. Flow table tests were performed on each batch of fresh UHPC in accordance with ASTM C230/230M as suggested by NF P18-470 (AFNOR, 2016a; ASTM, 2014). Control specimens consisted of 100 mm cubes and 100 x 200 mm cylinders that were used to measure the hardened cube density, compressive cube and cylinder strength, splitting tensile strength and elastic modulus. These tests were carried out using the same procedures as the NSC control specimens that are described in Section 4.2.3, and the test setups are shown in Figure 4.6 for the UHPC control specimens.

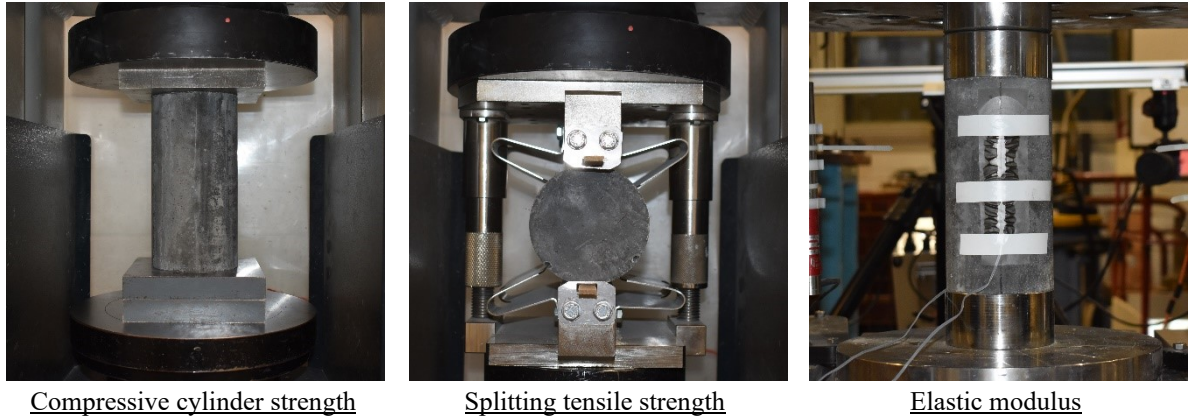


Figure 4.6 – Testing for various mechanical properties of UHPC

The tensile strength at the elastic limit ($f_{ct,el}$) and the post-cracking tensile strength (f_{ct}) were determined from four- and three-point bending tests, respectively, on UHPC prisms with dimensions of 400 x 100 x 100 mm. Unnotched prisms were used to obtain the direct tensile strength at the elastic limit ($f_{ct,el}$) and notched prisms to obtain the direct tensile strength at peak (f_{ct}). The test setups, which are shown in Figure 4.7, and the methods of calculation for the tensile strengths at the elastic limit ($f_{ct,el}$) and at peak (f_{ct}) comply with the requirements of Annex D of NF P18-470. These are described in Section 2.2.5 (AFNOR, 2016a). The load was applied in both tests with actuator displacement control at a constant rate of 0.25 mm/min (AFNOR, 2016a). The notches were formed using a handheld diamond disc blade. The resulting notch depth ranged between 6.5 mm to 10.1 mm with an approximate width of 3 mm. A notch width of 3 mm and a depth that is half of the length of the longest fibre is suggested by NF P18-470, which is equal to 6.5 mm given the fibres that are 13 mm in length used in this research (AFNOR, 2016a). DIC was used to measure surface displacements in one side of the prism throughout the experiment, with the setup of the DIC system described in Section 3.8. Depending on the duration of the experiment, the DIC system captured between 860 to 1600 images at a frequency of 1.0 Hz. The DIC images were mainly used to determine the crack mouth opening displacements (CMODs) at the notch of the notched prisms, so that the load-CMOD response can be determined. Global deformations corresponding to each image were analysed and produced in the software package DaVis, and a virtual horizontal gauge of 100 mm length was placed directly above the notch in DaVis to measure the CMODs throughout the experiment. An example of this procedure is shown in Figure 4.8.

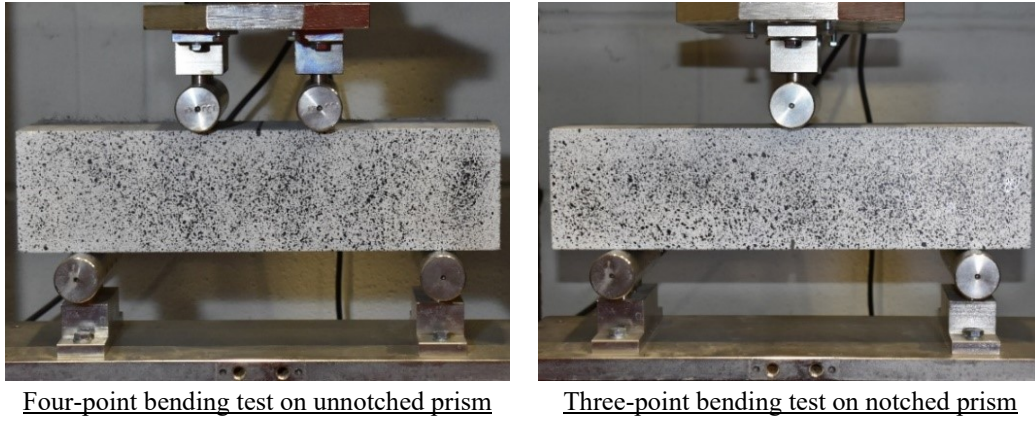


Figure 4.7 – Bending test setups for UHPC prisms

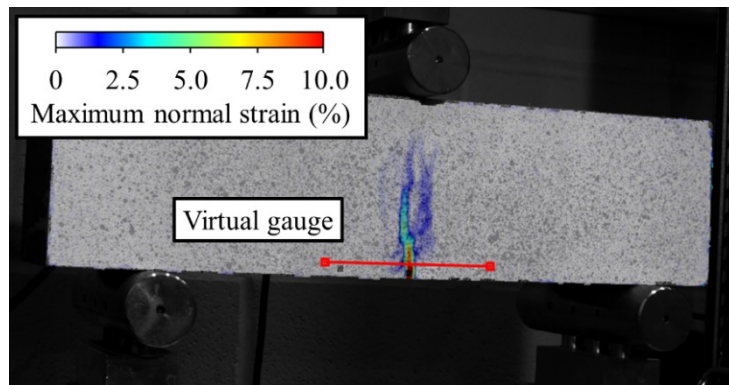


Figure 4.8 – Method for measuring crack openings in notched UHPC prisms based on processed DIC images of crack pattern in DaVis

The tensile strength at the elastic limit ($f_{ct,el}$) was calculated in terms of the flexural strength at the limit of elasticity ($f_{ct,fl}$). The latter is a function of the force at loss of linearity (F_{nl}) in the load-displacement response. The inverse analysis method given in Annex D of NF P18-470, (see Section 2.2.5), was applied to the determined load-CMOD response of each notched prism to obtain the post-cracking stress-CMOD response. In turn, the post-cracking tensile strength (f_{ct}) was determined from the post-cracking stress-CMOD response. To recapitulate, the post-cracking tensile strength (f_{ct}) corresponds to the peak stress of the post-cracking stress-CMOD response if there is a peak (AFNOR, 2016a). If the post-cracking stress-CMOD response has no peak, then the post-cracking tensile strength (f_{ct}) corresponds to the stress at a crack opening of 0.3 mm (AFNOR, 2016a).

The material properties of UHPC at 28 days are summarized in Tables 4.6 and 4.7 for the batches prepared using the Soroto and Pemat mixers, respectively. A total of 2 and 3 batches of UHPC were prepared using the Soroto and Pemat mixers, respectively, to obtain all of the required material properties. The volumes of each batch shown in Tables 4.6 and 4.7 were

intentionally similar to the volumes that were used when manufacturing the UHPC jackets in order to maintain similar conditions. The average flow table test results were about 230 mm for the UHPC prepared using both mixers. The hardened cube density was equal to 2433 kg/m³ and 2442 kg/m³ on average for the batches of the Soroto and Pemat mixers, respectively, which is about 100 kg/m³ greater than the hardened cube density of the NSC presented in Section 4.2.3. The compressive cube strength was equal to 131.13 MPa and 135.09 MPa on average for the batches of the Soroto and Pemat mixers, respectively. The compressive cylinder strength was equal to 119.89 MPa and 123.36 MPa on average for the batches of the Soroto and Pemat mixers, respectively. The average splitting tensile strengths for the batches from the Soroto and Pemat mixers were 19.85 MPa and 15.42 MPa, respectively. Seeing as the difference in compressive cylinder strength between both mixers was marginal, the elastic modulus was obtained from a single batch prepared using the Soroto mixer. The elastic modulus was equal to 39302.60 MPa with a minimal standard deviation of 212.31 MPa.

The average tensile strengths at the elastic limit in Tables 4.6 and 4.7 were derived from a total of 6 and 3 unnotched prism specimens for the Soroto and Pemat mixers, respectively. The load-displacement response of each specimen is shown in Figure 4.9, with the name of each specimen depicting the batch and specimen number. The displacement at midspan of the prism was recorded throughout the experiment by the actuator. The shapes of the load-displacement responses appear similar overall for all specimens, however the ultimate load varies by up to 20 kN between specimens of the same batch. Nonetheless, the average tensile strengths at the elastic limit were similar at 8.58 MPa and 8.50 MPa for the Soroto and Pemat mixers, respectively. Interestingly, the peak responses were considerably lower for specimens 2-1 and 2-3 from the Soroto mixer, which may possibly correspond to worse fibre dispersion.

The average post-cracking tensile strengths presented in Tables 4.6 and 4.7 were determined by testing 5 notched prism specimens from each mixer. The load-displacement response, load-CMOD response and post-cracking stress-CMOD response of each specimen is shown in Figure 4.10. The legend (e.g., 1-1) depicts the specimen batch and number within the batch. The displacement at midspan of the prism was recorded throughout the experiment by the actuator. The average post-cracking stress-CMOD response shown in Figure 4.10 was calculated by considering the responses of each notched prism specimen. The post-cracking tensile responses indicate that the softening phase initiated immediately after the crack opened. Therefore, the post-cracking tensile strength was taken as the stress corresponding to a crack

opening of 0.3 mm (AFNOR, 2016b). The average post-cracking tensile strengths were equal to 6.39 MPa and 7.88 MPa for the batches from the Soroto and Pemat mixers, respectively. This relatively large difference of about 1.5 MPa between the post-cracking tensile strength from each mixer is thought to arise from differences in the capabilities of each mixer to homogeneously disperse the steel fibres. Unlike the Soroto mixer, the Pemat mixer is capable of achieving a high input energy seeing as the pan rotates in the opposite direction to the rotating mixing blades. Although the frequencies of the mixing blades of both mixers were kept the same, the additional function of the smaller volume Pemat mixer makes it more suitable for UHPC as a high mixing energy is generally suggested to ensure homogenous dispersion of all the fine materials (Sohail et al., 2018). In addition, the mixing blades of the smaller Pemat mixer are manually retractable, which made it easier to hand mix all areas of the pan, and this may have possibly led to a more uniform dispersion of the steel fibres.

Table 4.6 – Material properties at 28 days of UHPC prepared using Soroto mixer

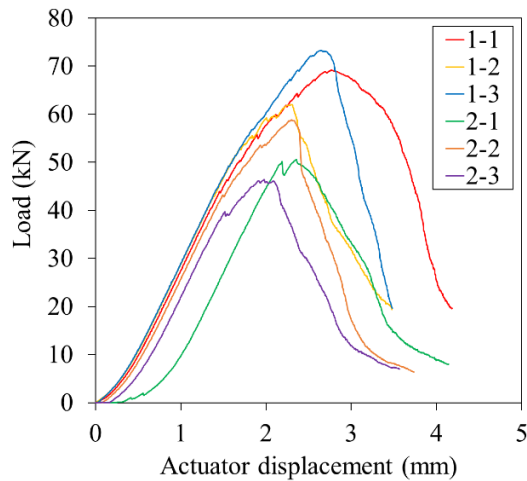
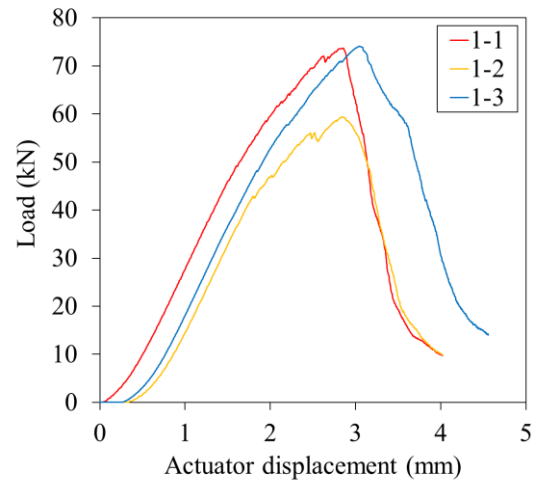
Property	Batch 1	Batch 2	Average
Prepared volume [m ³]	0.065	0.058	-
Flow table [mm]	220.00	237.50	228.75 (12.37)
Density (ρ) [kg/m ³]	2426.31 (8.24)	2441.58 (5.74)	2433.94 (10.51)
Compressive cu. strength ($f_{cm,cu}$) [MPa]	128.13 (1.60)	134.13 (3.16)	131.13 (3.98)
Compressive cyl. strength (f_{cm}) [MPa]	119.89 (2.72)	-	119.89 (2.72)
Splitting tensile strength ($f_{ctm,sp}$) [MPa]	19.85 (2.26)	-	19.85 (2.26)
Tensile strength elastic lim. ($f_{ctm,el}$) [MPa]	9.07 (0.78)	8.08 (0.60)	8.58 (0.83)
Tensile strength (f_{ctm}) [MPa]	5.96 ^A (0.54)	6.67 (0.57)	6.39 (0.62)
Elastic modulus (E_{cm}) [MPa]	39302.60 (212.31)	-	39302.60 (212.31)

[Note] – () refers to standard deviation; ^A Tested 2 specimens

Table 4.7 – Material properties at 28 days of UHPC prepared using Pemat mixer

Property		Batch 1	Batch 2	Batch 3	Average
Prepared volume	[m ³]	0.021	0.021	0.017	-
Flow table	[mm]	235.00	231.25	231.25	232.50 (2.17)
Density	(ρ) [kg/m ³]	2412.70 (0.72)	2454.60 (1.04)	2473.38 (7.85)	2442.01 (28.65)
Compressive cu. str.	($f_{cm,cu}$) [MPa]	126.24 (1.70)	139.15 (1.51)	144.30 (0.24)	135.09 (8.62)
Compressive cyl. str.	(f_{cm}) [MPa]	-	123.36 (1.42)	-	123.36 (1.42)
Splitting tensile str.	($f_{ctm,sp}$) [MPa]	15.42 (1.74)	-	-	15.42 (1.74)
Tensile st. elastic lim.	($f_{ctm,el}$) [MPa]	-	-	8.50 (0.68)	8.50 (0.68)
Tensile strength	(f_{ctm}) [MPa]	7.65 ^A (0.29)	8.03 (1.43)	-	7.88 (1.04)

[Note] – () refers to standard deviation; ^A Tested 2 specimens

Soroto mixerPemat mixer**Figure 4.9** – Load-displacement response at 28 days of unnotched UHPC prisms

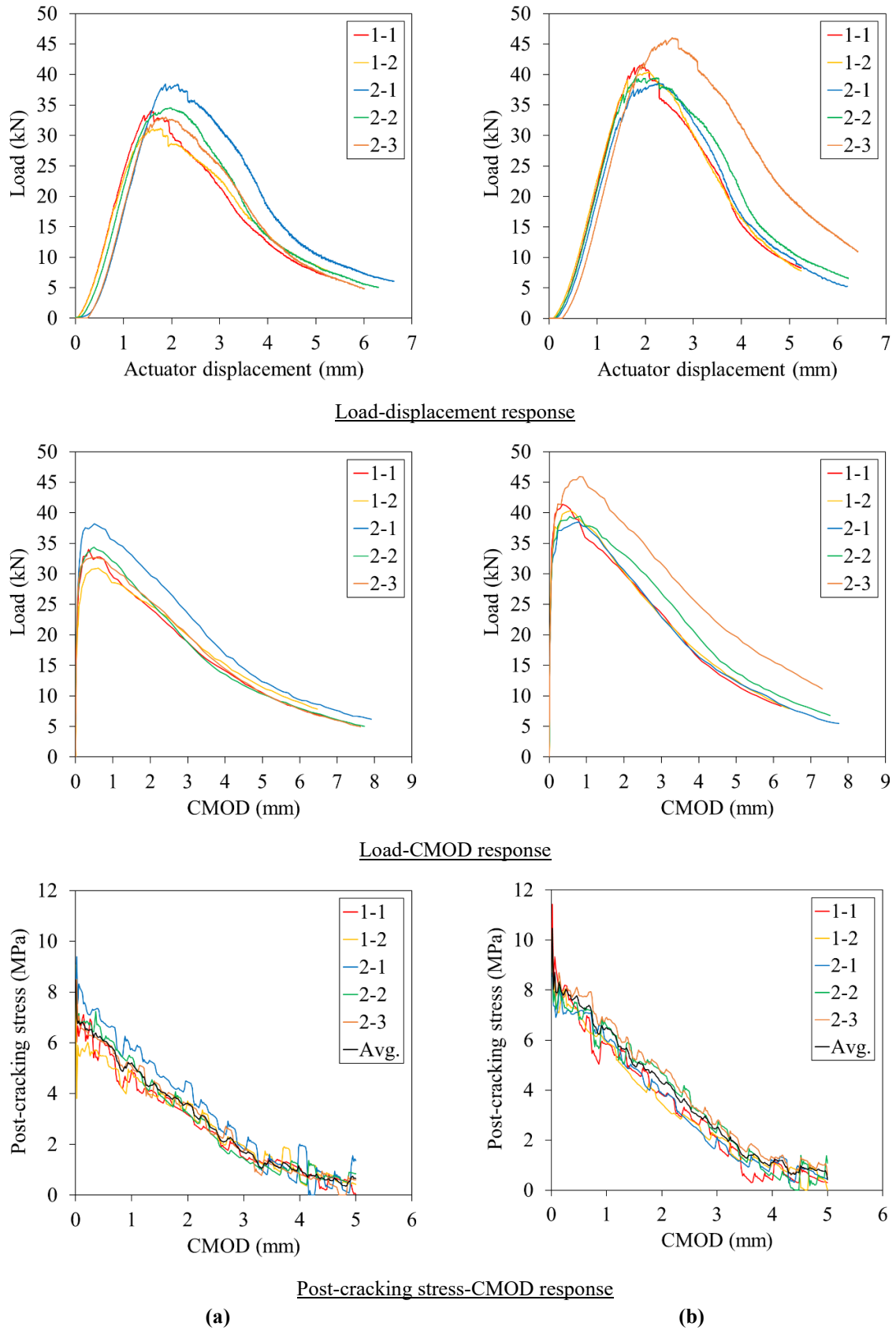


Figure 4.10 – Load-displacement response, load-CMOD response and post-cracking stress-CMOD response at 28 days of notched UHPC prisms: (a) Soroto mixer, (b) Pemat mixer

4.2.4.2 MATERIAL PROPERTIES AT DAY OF TESTING BEAM SPECIMENS

A limited number of control specimens were cast alongside each UHPC jacket to obtain the hardened cube density, compressive cube and cylinder strength and splitting tensile strength on the day of testing each strengthened beam specimen. Control specimens were cast from each mixer that was used to manufacture the jacket where possible to ensure that the mechanical properties of each mixer's batch are relatively uniform and consistent with the results obtained at 28 days. Control specimens included 100 mm cubes and 100 x 200 mm cylinders. Flow table tests were also performed on the fresh UHPC.

The material properties of UHPC at the days of testing each strengthened beam are summarized in Tables 4.8 and 4.9 for the specimens of groups 1 and 2, respectively. Materials tests were mostly conducted on less than 3 control specimens per test. The flow table test results ranged between 215.0 mm and 227.5 mm, which is consistent with the flow table test results achieved in previous batches that were around 230 mm. The strengthened beams were tested between 113 to 114 days after casting for group 1 and 45 to 59 days after casting for group 2. The hardened cube density and splitting tensile strength results between these ages were close to the results achieved in previous batches. The compressive cube and cylinder strength increased up to approximately 20 MPa with age when compared to the compressive strength obtained at 28 days. The compressive cube strength ranged between 146.55 MPa and 152.42 MPa for group 1, and 140.48 MPa and 152.90 MPa for group 2. The compressive cylinder strength ranged between 129.57 MPa and 137.33 MPa for group 1, and 124.44 MPa and 134.86 MPa for group 2. The results presented in Tables 4.8 and 4.9 for the specimens of groups 1 and 2, respectively, indicate that the material properties of each mixer's batch are reasonably uniform and consistent with the properties achieved at 28 days.

Table 4.8 – UHPC material properties at day of testing group 1 strengthened specimens

Specimen	Mixer	Volume [m ³]	Flow table [mm]	Age [Day]	Density (ρ) [kg/m ³]	Comp. cu. strength ($f_{cm,cu}$) [MPa]	Comp. cyl. strength (f_{cm}) [MPa]	Split. tens. strength ($f_{ctm,sp}$) [MPa]
B2	Soroto	0.065	222.50	113	-	-	135.97 ^B (3.60)	20.45 ^B (1.62)
	Pemat	0.017	217.50	113	2427.52 ^C (14.52)	152.42 ^C (4.48)	137.33 ^A	19.19 ^A
B3	Soroto	0.065	-	114	-	-	132.55 ^A	19.94 ^A
	Pemat 1	0.021	222.50	114	-	-	129.57 ^A	16.92 ^A
	Pemat 2	0.011	220.00	114	2439.32 ^A	152.22 ^A	136.14 ^A	16.91 ^A
B4	Soroto	0.065	215.00	-	-	-	-	-
	Pemat 1	0.021	217.50	-	-	-	-	-
	Pemat 2	0.021	226.25	113	2415.09 ^A	146.55 ^A	133.92 ^B (1.95)	22.76 ^B (0.31)

[Note] – () refers to standard deviation; ^A Tested 1 specimen; ^B Tested 2 specimens; ^C Tested 4 specimens

Table 4.9 – UHPC material properties at day of testing group 2 strengthened specimens

Specimen	Mixer	Volume [m ³]	Flow table [mm]	Age [Day]	Density (ρ) [kg/m ³]	Comp. cu. strength ($f_{cm,cu}$) [MPa]	Comp. cyl. strength (f_{cm}) [MPa]	Split. tens. strength ($f_{ctm,sp}$) [MPa]
B6	Soroto	0.065	227.50	45	-	-	124.44 ^A	17.82 ^B (4.28)
	Pemat	0.017	220.00	45	2448.52 ^B (13.47)	140.48 ^B (6.00)	129.24 ^B (1.26)	18.24 ^A
B7	Soroto	0.065	220.00	48	-	-	127.92 ^A	16.57 ^A
	Pemat 1	0.021	222.50	48	-	-	131.87 ^A	16.12 ^A
	Pemat 2	0.011	221.25	48	2460.77 ^A	152.90 ^A	134.86 ^A	19.89 ^A
B8	Soroto	0.065	218.75	59	-	-	132.04 ^A	21.66 ^A
	Pemat 1	0.021	222.50	-	-	-	-	-
	Pemat 2	0.021	223.75	59	2428.71 ^A	148.90 ^A	132.67 ^A	20.11 ^A

[Note] – () refers to standard deviation; ^A Tested 1 specimen; ^B Tested 2 specimens

4.3 INTERFACIAL BOND CHARACTERISATION

The interfacial bond between NSC and UHPC was characterised through a series of splitting tensile strength tests on 150 x 300 mm composite cylinders. A total of 8 composite specimens

were tested in two groups of four specimens. The NSC half of each specimen was cast first, and separately for each group, followed by the UHPC as described in Section 3.5. The same batch of UHPC was used for both groups of specimens. At testing, the age of the NSC was 322 days for group 1 and 307 days for group 2. The age of the UHPC at testing was 28 days for both groups of specimens. The UHPC was taken from batch 2 of the Soroto mixer (see Table 4.6 for material properties). The composite specimens were loaded in transverse compression as in the tensile splitting test shown in Figure 3.30 at a constant rate of 0.04 MPa/s (BSI, 2009b). The modes of failure and failure stresses were determined for each composite specimen.

The results of the splitting tensile strength tests are summarized in Table 4.10 for both groups of composite specimens. The name of each specimen in Table 4.10 depicts the group and specimen number. For the specimens of group 1, the composite splitting tensile strength ranged from 2.58 MPa to 5.88 MPa, and the average composite splitting tensile strength was equal to 3.93 MPa with a relatively high standard deviation of 1.40 MPa. For the specimens of group 2, the composite splitting tensile strength ranged from 3.22 MPa to 4.52 MPa, and the average composite splitting tensile strength was equal to 3.88 MPa with a relatively low standard deviation of 0.57 MPa. These results are consistent overall with results from the literature (Al-Osta et al., 2017; Bahraq et al., 2019; Ganesh and Ramachandra Murthy, 2020; Harris et al., 2011, 2015; Tayeh et al., 2012; Zhang et al., 2020b).

The composite splitting tensile strength was compared with the monolithic splitting tensile strength of the NSC specimens presented in Table 4.5. The monolithic splitting tensile strength of groups 1 and 2 correspond to the control specimens of beam specimens B4 (176 days) and B8 (191 days), respectively, as they were closest in age at testing when compared to the composite specimens. The monolithic splitting tensile strength was taken as the average between the results of the air and water cured control specimens presented in Table 4.5, and it was equal to 3.52 MPa and 3.30 MPa for groups 1 and 2, respectively. The composite splitting tensile strength of both groups of specimens surpass the monolithic splitting tensile strength on average, which suggests that the interfacial bond is sufficient.

The crack pattern at failure and the failure modes of the splitting tensile strength tests are shown in Figures 4.11 and 4.12, respectively. It appears that a combined failure mode occurred along the interfacial bond and the NSC substrate for all specimens. When both halves of the fractured composite cylinder specimens were separated as shown in Figure 4.12, it is clear that

significant portions of the NSC were still bonded to the UHPC. In addition to the relatively high splitting tensile strength results, this combined failure mode suggests that the interfacial bond between the NSC and UHPC is sufficient.

Table 4.10 – Splitting tensile strength of NSC-UHPC composite specimens

Specimen	Age of NSC [Day]	Age of UHPC [Day]	Comp. split. tens. str. ($f_{ct,sp}$) [MPa]	Mono. split. tens. str. ($f_{ct,sp}$) [MPa]	Comp. to mono. str. ratio ($f_{ct,sp}/f_{ctm,sp}$)	Average comp. split. tens. str. ($f_{ctm,sp}$) [MPa]	Average comp. to mono. str. ratio ($f_{ctm,sp}/f_{ctm,sp}$)
1-1	322	28	3.44	3.52	0.98	3.93 (1.40)	1.12 (0.40)
1-2	322	28	2.58	3.52	0.73		
1-3	322	28	3.82	3.52	1.09		
1-4	322	28	5.88	3.52	1.67		
2-1	307	28	3.63	3.30	1.10	3.88 (0.57)	1.18 (0.17)
2-2	307	28	4.15	3.30	1.26		
2-3	307	28	3.22	3.30	0.98		
2-4	307	28	4.52	3.30	1.37		

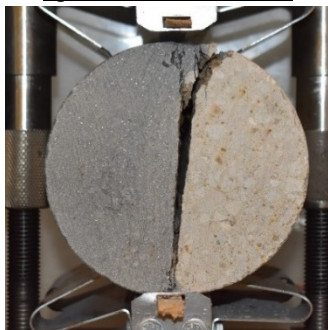
[Note] – () refers to standard deviation



Specimen 1-1 – Failure



Group 1 specimens



Specimen 2-1 – Failure



Group 2 specimens

Figure 4.11 – Crack pattern at failure of NSC-UHPC composite specimens

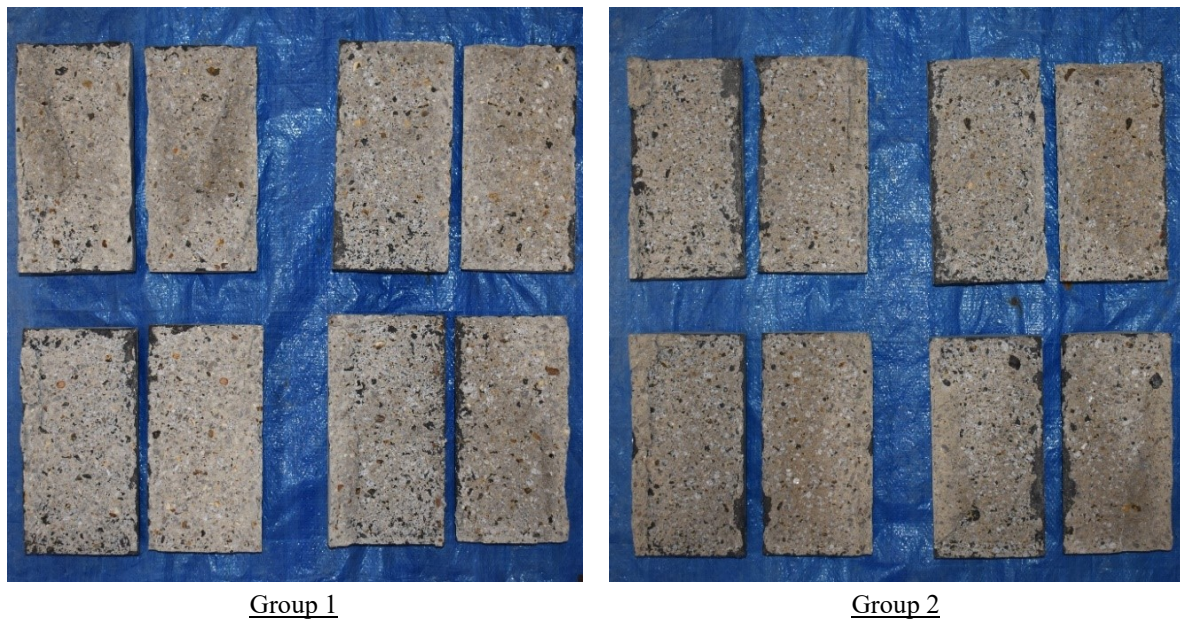


Figure 4.12 – Failure modes of NSC-UHPC composite specimens

4.4 RESULTS OF GROUP 1 BEAM SPECIMENS

4.4.1 INTRODUCTION

The specimens of group 1 included one control RC beam specimen (B1) along with three RC beam specimens with UHPC jackets (B2, B3 and B4). The identical RC beam specimens of group 1 were heavily reinforced in flexure while the minimum required amount of shear reinforcement was provided in the critical span. The UHPC jackets of specimens B2, B3 and B4 varied in side thickness only, as they included side thicknesses of 20 mm, 30 mm and 40 mm, respectively. The bottom thickness of the UHPC jacket was constant at 60 mm along with the amount of embedded reinforcement. Section 3.2.2 describes the specimens of group 1 while also presenting the cross-sections and elevations of each specimen. The material properties of the NSC and UHPC used in the assessments that are discussed later in Chapters 5, 6 and 7 are presented in Table 4.11 for each specimen of group 1. Minor differences are observed between the material properties of each specimen. The compressive cylinder and splitting tensile strength of the NSC of specimens B2 and B3 were taken as the average between the results of the air and water cured control specimens of the first (B1) and last (B4) tested beam specimens of the group presented in Table 4.5. The compressive cylinder strength of the UHPC was taken from the results presented in Table 4.8 of the control specimens prepared using the Soroto mixer that were cast and tested alongside each beam specimen. The compressive cylinder strength of the UHPC of specimen B4 was taken as the average between the results of the

control specimens of specimens B2 and B3 prepared using the Soroto mixer. The tensile strength at the elastic limit, post-cracking tensile strength and elastic modulus of the UHPC were taken from the average results obtained at 28 days from the batches of the Soroto mixer that are presented in Table 4.6. The elastic modulus of the NSC was also taken from the results at 28 days.

Table 4.11 – Summary of NSC and UHPC material properties of group 1 specimens

Specimen	Normal strength concrete (NSC)				Ultra high performance concrete (UHPC)				
	Age	Comp. cyl. strength (f_{cm})	Splitting tensile str. ($f_{ctm,sp}$)	Elastic modulus (E_{cm})	Age	Comp. cyl. strength (f_{cm})	Tensile strength ($f_{ctm,el}$)	Tensile strength (f_{ctm})	Elastic modulus (E_{cm})
	[Day]	[MPa]	[MPa]	[MPa]	[Day]	[MPa]	[MPa]	[MPa]	[MPa]
B1	163	47.49	3.61	34997.87	-	-	-	-	-
B2	169	46.96	3.56	34997.87	113	135.97	8.58	6.39	39302.60
B3	172	46.96	3.56	34997.87	114	132.55	8.58	6.39	39302.60
B4	176	46.44	3.52	34997.87	113	134.26	8.58	6.39	39302.60

The experimental results and main characteristics of each specimen of group 1 are summarized in Table 4.12. All of the specimens of group 1 failed in shear. Significant increases in the ultimate load were observed in the jacketed specimens in comparison to control specimen B1. The ultimate load increased by 57.16% (296.83 kN) due to the thinnest UHPC jacket (20 mm) of specimen B2, and it increased by 74.76% (379.86 kN) due to the thickest UHPC jacket (40 mm) of specimen B4. The intermediate UHPC jacket (30 mm) of specimen B3 led to an increase of 64.86% (337.02) in the ultimate load.

Table 4.12 – Summary of main experimental results of group 1 specimens

Specimen	Beam long. reinf. ^A (ρ_l) [%]	Beam shear reinf. ^B (ρ_w) [%]	Jacket thick. (t_s) [mm]	Area of long. reinf. ratio ^C [%]	Ult. load (P_u) [kN]	Ult. shear (V_u) [kN]	Ult. disp. (δ_u) [kN]	Ult. load incr. [kN]	Ult. load incr. [%]	Failure mode
B1	3.34	0.14	-	-	532.07	214.96	13.34	-	-	Shear
B2	3.34	0.14	20	66.67	828.90	337.84	10.40	296.83	57.16	Shear
B3	3.34	0.14	30	66.67	869.09	354.39	11.11	337.02	64.86	Shear
B4	3.34	0.14	40	66.67	911.93	375.66	10.49	379.86	74.76	Shear

^A Longitudinal reinforcement ratio (ρ_l) equals to $A_s/b_w d$; ^B Shear reinforcement ratio (ρ_w) equals to $A_{sw}/b_w s$; ^C Ratio of the area of longitudinal reinforcement in the UHPC jacket to that in the RC beam

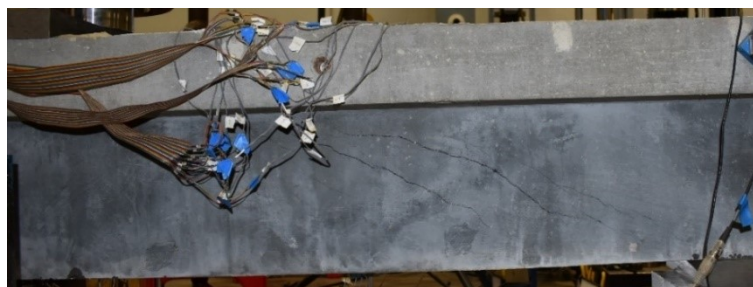
4.4.2 CRACK DEVELOPMENT AND MODE OF FAILURE

All of the specimens of group 1 cracked diagonally between the loading point and support in the critical span prior to eventual failure. The crack pattern at the ultimate load and post-peak failure of each specimen in group 1 is shown in Figure 4.14, where the critical shear crack is highlighted. The crack patterns in Figure 4.14 were traced onto the drawings of each specimen from superimposed DIC images for both the front and back face of views (FOVs) of each specimen where available. Only the front FOV of specimens B1 and B2 of group 1 were monitored by DIC from first loading to the end of the test. Photographs of each specimen were also taken after the experiment, and they are presented in Figure 4.15. Section A.1 shows the processed DIC images of each specimen of group 1, which include crack patterns and strain states starting from the load step $0.8P_u$ until post-peak failure.

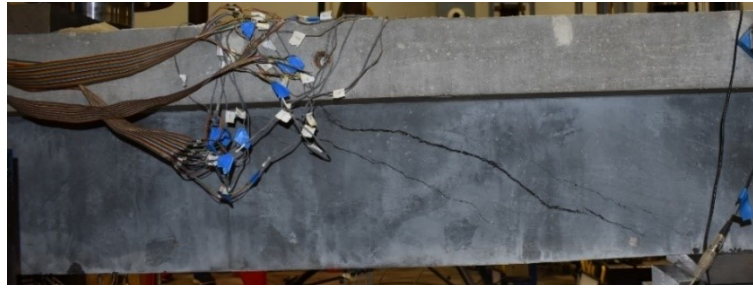
The critical shear crack and several almost parallel secondary shear cracks became visible in control specimen B1 around $0.4P_u$ (213 kN). The shear cracks initially appeared in the web and extended from around the mid-height of the web to the web/flange junction. The critical shear crack and secondary shear cracks widened and extended downward between the two layers of longitudinal reinforcement in tension as the ultimate load was reached (532 kN) as shown in Figure 4.14. After the ultimate load was reached, the critical shear crack merged with a nearby secondary shear crack near the mid-height of the web. The merged critical shear crack extended horizontally towards the support along the bottom layer of longitudinal reinforcement, which went on to form the delamination crack at post-peak failure (353 kN). The critical shear crack also extended beyond the web/flange junction and into the flange as it reached the loading plate at failure. There were minimal flexural cracks throughout the experiment in control specimen B1. Only the front FOV of control specimen B1 was monitored by DIC, with the results shown in Figure A.1. The crack patterns throughout the experiment and at failure were virtually the same for both FOVs as shown in the crack patterns of both FOVs after the test in Figure 4.15.

Initial cracking became visible in specimen B2 at around $0.5P_u$ (415 kN). Subsequently, it became apparent that the development of the critical and secondary shear cracks in the front and back faces of the jacket differed. Only the front FOV of control specimen B2 was monitored by DIC, with the DIC results shown in Figure A.2. The photographs in Figure 4.13 show the back FOV of specimen B2 near the ultimate load (829 kN). Only a single shear crack

formed in the front FOV. It initially appeared near the middle of the web in the critical span before growing to extend from the longitudinal reinforcement in the beam to just below the web/flange junction. A couple of short, less dominant, closely spaced and almost parallel secondary shear cracks formed about 80 mm to the left of the critical shear crack shortly before the ultimate load was reached. As the ultimate load was reached, the critical shear crack widened and extended upward to the web/flange junction and downward to the longitudinal reinforcement in the jacket as shown in Figure 4.14. After reaching the ultimate load, the critical shear crack continued to widen, and an inclined crack formed in the flange that extended to the loading plate at post-peak failure (464 kN). In the back FOV of specimen B2, a total of three closely spaced, less dominant and almost parallel shear cracks surrounded the critical shear crack with two of those cracks appearing about 100 mm to left of it and a single crack appearing about 100 mm to right of it. The critical shear crack in the back FOV appears to overlap with the critical shear crack in the front FOV based on visual comparison of their respective positions along the critical span as well as their respective crack shapes. The secondary shear cracks in the back FOV slightly widened as the ultimate load was reached (829 kN). At this point, they extended from almost the longitudinal reinforcement in the jacket to the web/flange junction as shown in Figure 4.13. The secondary cracks appearing in the back FOV are only partially visible in the front FOV where only a short segment of the secondary crack nearest to the support extends from the longitudinal reinforcement in the jacket to that in the beam. A similar inclined crack developed in the flange of the back FOV at post-peak failure (464 kN). The crack extended from the critical shear crack in the web to the loading plate as shown in Figure 4.15 for both FOVs.



Load ≈ 780 kN ($0.94P_u$)



Load ≈ 829 kN (P_u)

Figure 4.13 – Crack pattern of specimen B2 (Back FOV) throughout the latter stages of loading until the ultimate load

The critical shear crack became visible in specimen B3 soon after the load step $0.6P_u$ (522 kN). Both FOVs of specimen B3 were monitored by DIC with results shown in Figures A.3 and A.4 for the front and back FOVs, respectively. The critical shear crack in the front and back FOVs overlapped and developed similarly throughout the test to failure as shown in Figure 4.14. The critical shear crack initiated near the support around the mid-height of the web and extended to the web/flange junction. Two less dominant, almost parallel and closely spaced secondary shear cracks formed near the middle of the critical span of the beam at a flatter angle than the critical shear crack. Near the ultimate load (869 kN), the critical shear crack widened and extended downward to the longitudinal reinforcement in the jacket. The critical shear crack extended through the flange to the loading plate after reaching the ultimate load at post-peak failure (559 kN). Only one of the two secondary shear cracks in the front FOV grew significantly after the ultimate load, while both secondary shear cracks in the back FOV significantly grew as shown in Figure 4.15.

Much like specimen B3, the critical shear crack became visible near the middle of the critical span in specimen B4 shortly after the load step $0.6P_u$ (548 kN). Both FOVs of specimen B4 were monitored by DIC, and the DIC results are shown in Figures A.5 and A.6 for the front and back FOVs, respectively. The critical shear crack in the front and back FOVs overlapped and developed similarly overall throughout the experiment as shown in Figure 4.14. For the front FOV, the critical shear crack began to form between the longitudinal reinforcement in the beam to the mid-height of the web. Two secondary shear cracks developed as well. The first was located about 230 mm to the left of the critical shear crack and was near the support. It extended from between the longitudinal reinforcement in the jacket to the web/flange junction. The second secondary shear crack was located just above the critical shear crack and extended downward from the web/flange junction towards the secondary shear crack nearest to the support. As the ultimate load was reached (912 kN), the two secondary shear cracks merged

with the critical shear crack, while the critical shear crack extended downward reaching the bottom surface of the beam. The critical shear crack extended upward into the flange reaching the loading plate after the ultimate load was reached at post-peak failure (547 kN). The main difference in the back FOV was that the tip of critical shear crack did not reach the web/flange junction at the ultimate load (912 kN). The differences in the crack pattern in the front and back FOVs at failure can be seen by comparing Figures 4.14 and 4.15.

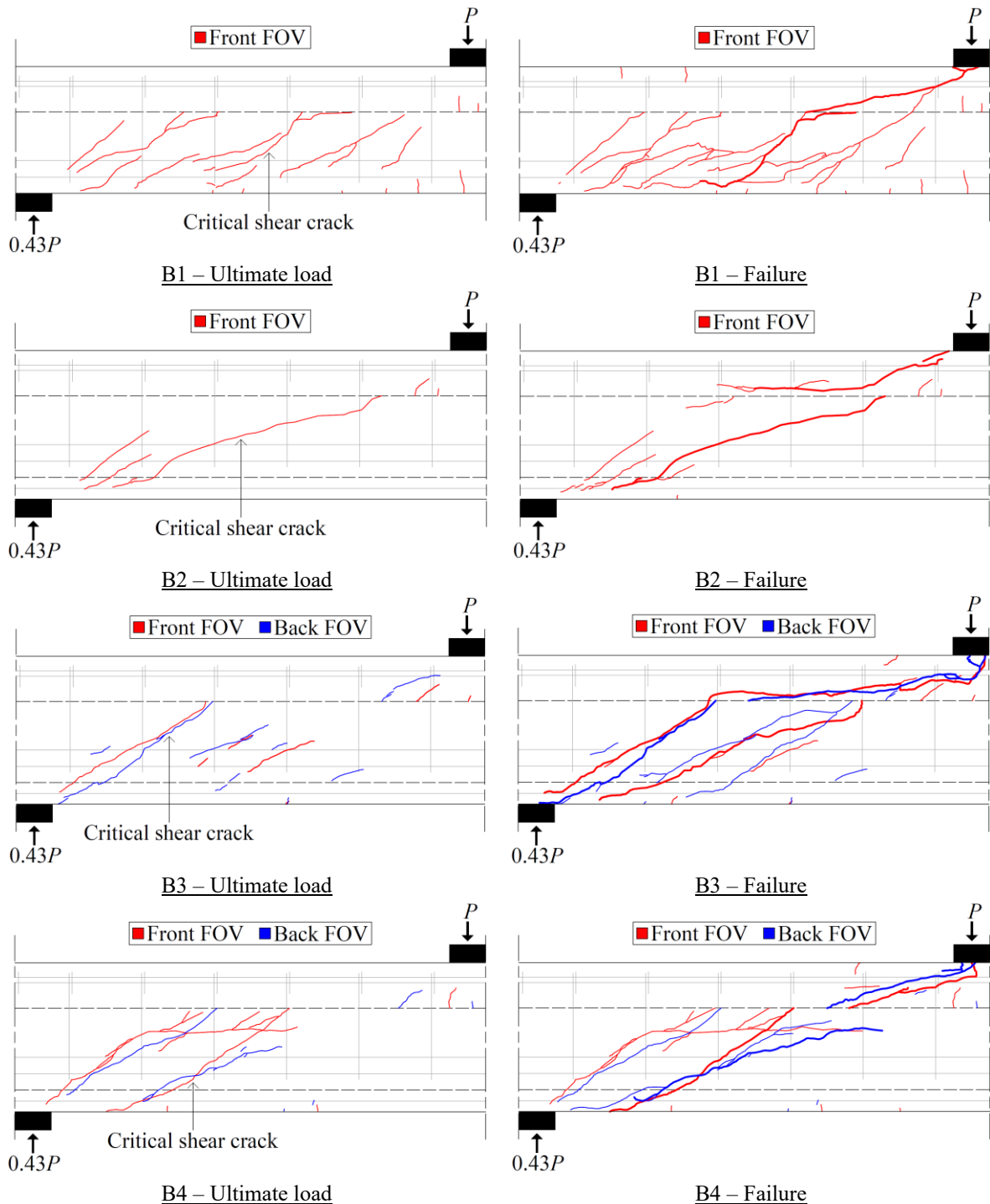


Figure 4.14 – Traced crack pattern of group 1 specimens at the ultimate load and failure

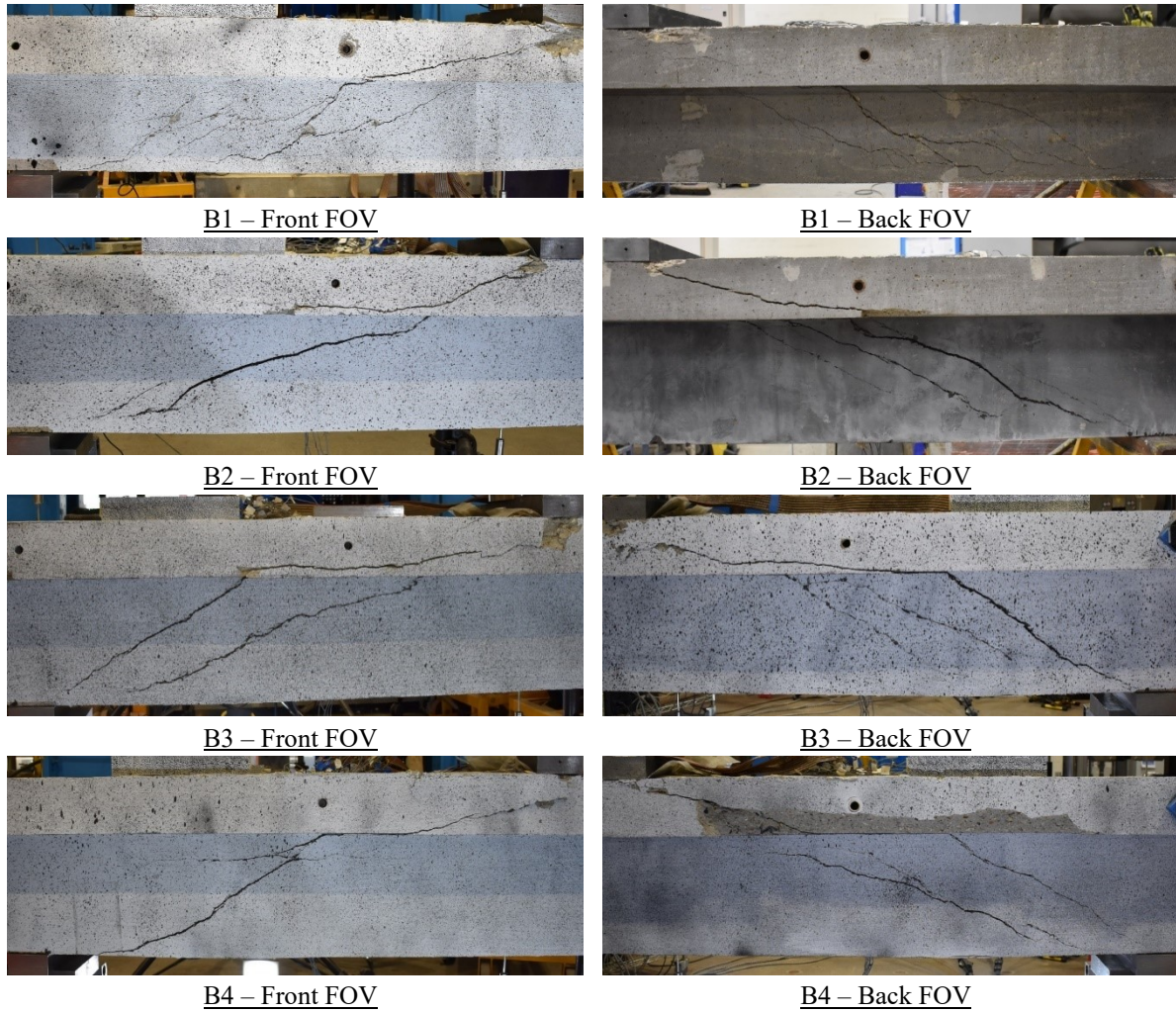


Figure 4.15 – Crack pattern of group 1 specimens after experiment

4.4.3 CRITICAL SHEAR CRACK INCLINATION ANGLES

The inclinations of the critical shear cracks in the beams of group 1 at the ultimate load are shown in Figure 4.16. The cracks at the ultimate load were traced from superimposed DIC images for the front and back FOVs. The inclination angle of the critical shear crack was taken as the angle between the diagonal failure plane measured between the crack ends and the longitudinal axis. The locations of the critical shear cracks are seen to vary between specimens. The critical shear cracks formed around the middle of the critical span, although the critical shear crack of specimen B3 formed much closer to the support. The crack inclination angles also varied, with control specimen B1 having the steepest angle of 34° . The flattest crack inclination angle was 19° in specimen B2. The inclinations of the critical shear cracks were similar in the front and back FOVs of specimen B3 at 32° and 33° , respectively. The inclination of the critical shear cracks in the front and back FOVs of specimen B4 differed by 10° probably

due to the differences in the development of the cracks in each face of the UHPC jacket described in Section 4.4.2. The inclination angles were equal to 31° and 21° for the critical shear cracks in the front and back FOVs of specimen B4, respectively.

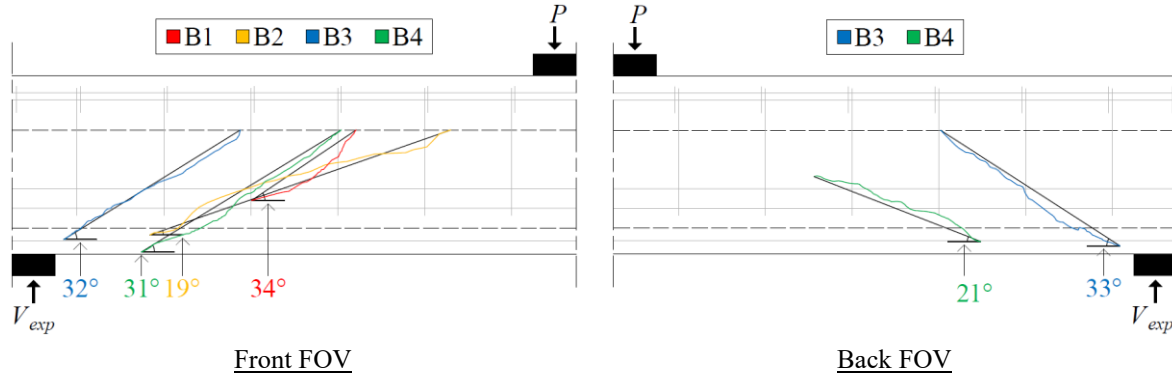


Figure 4.16 – Inclination angles of critical shear cracks at the ultimate load of group 1 specimens

4.4.4 LOAD-DISPLACEMENT RESPONSE

The load-displacement response of each specimen of group 1 is shown in Figure 4.17. The total load acting on each specimen in Figure 4.17 was recorded throughout the experiment by the actuator (LC-1), and the load at the right support was recorded by the load cell located at the underside of the support (LC-2). The locations of the load cells are given in Figure 3.36. The load at the left support in Figure 4.17 equals the difference between the total load (LC-1) and the load recorded at the right support (LC-2). The displacement in Figure 4.17 is that recorded by LVDT-2 which was positioned underneath the beam at the centreline of the loading point. The load-displacement responses of the group 1 specimens were brittle, with failure occurring almost immediately after reaching the ultimate load as is typical for shear failure. It is evident that the load-carrying capacity of specimens B2, B3 and B4 was increased relative to the control specimen B1 due to the addition of the UHPC jackets. The shapes of the load-displacement responses were similar for all specimens, while the displacements corresponding to the peak load were all within approximately 3.0 mm of one another.

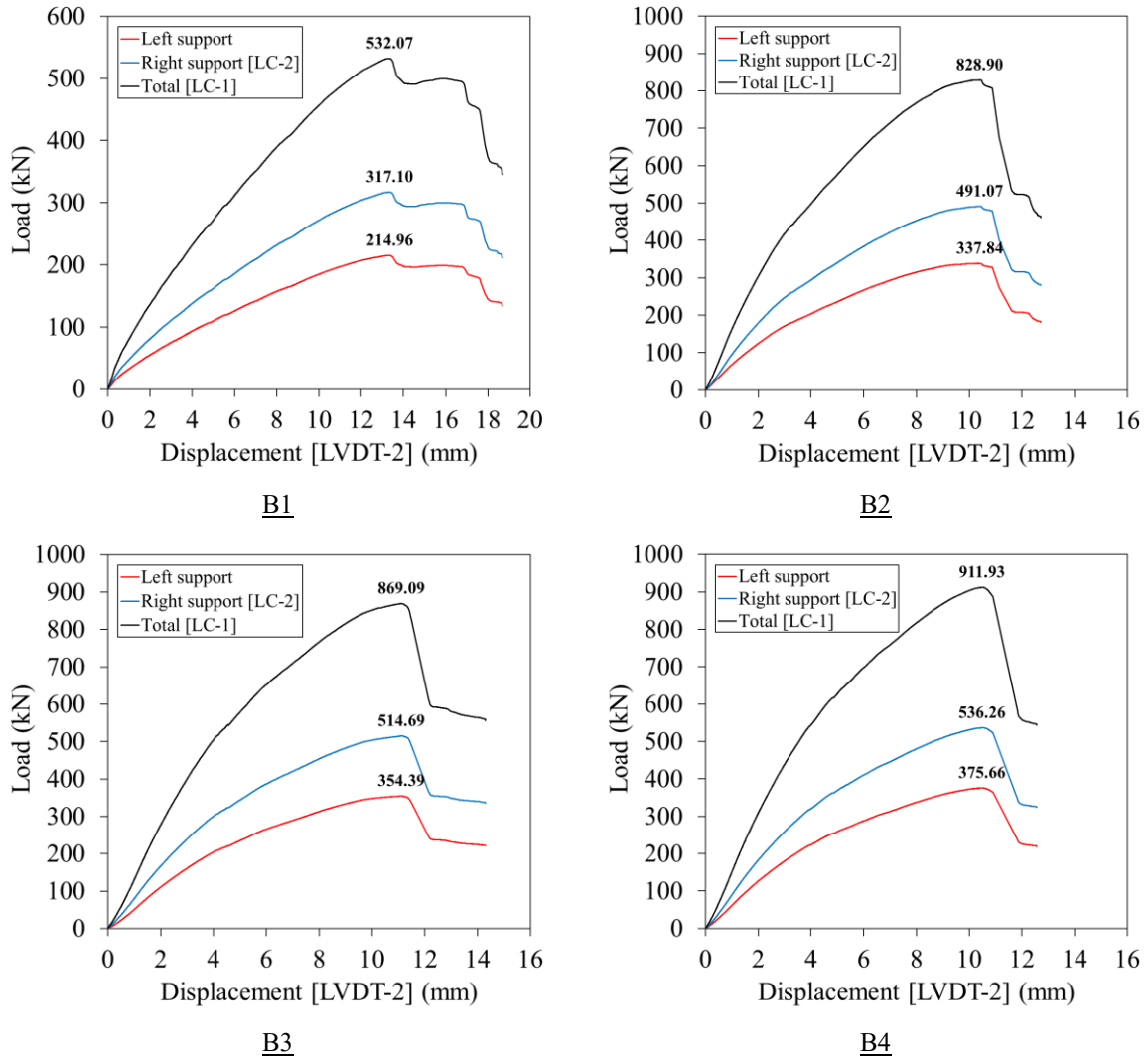


Figure 4.17 – Load-displacement response of group 1 specimens

4.4.5 LOADING RATIOS

Figure 4.18 shows the experimentally derived loading ratios at the left and right supports with respect to Figure 3.36 alongside the respective theoretical loading ratios of 0.429 and 0.571. The initial spikes prior to the load step $0.1P_u$ are due to bedding in of the specimen onto the supports. The actual loading ratios for the specimens of group 1 were approximately 0.407 and 0.593 on average throughout the experiments for the left and right supports, respectively. This is equivalent to percent differences of 4.98% and 3.74% for the left and right supports, respectively. Consequently, the shear force in the critical left span is proportionally slightly less than planned based on the experimental setup. These differences between the measured and theoretical loading ratios may have arisen due to the use of the rectangular loading plate shown in Figure 3.33 for the group 1 tests. The rectangular loading plate may have caused a

marginal difference between the intended and actual point of application of the load on the beam (Pastore, 2020).

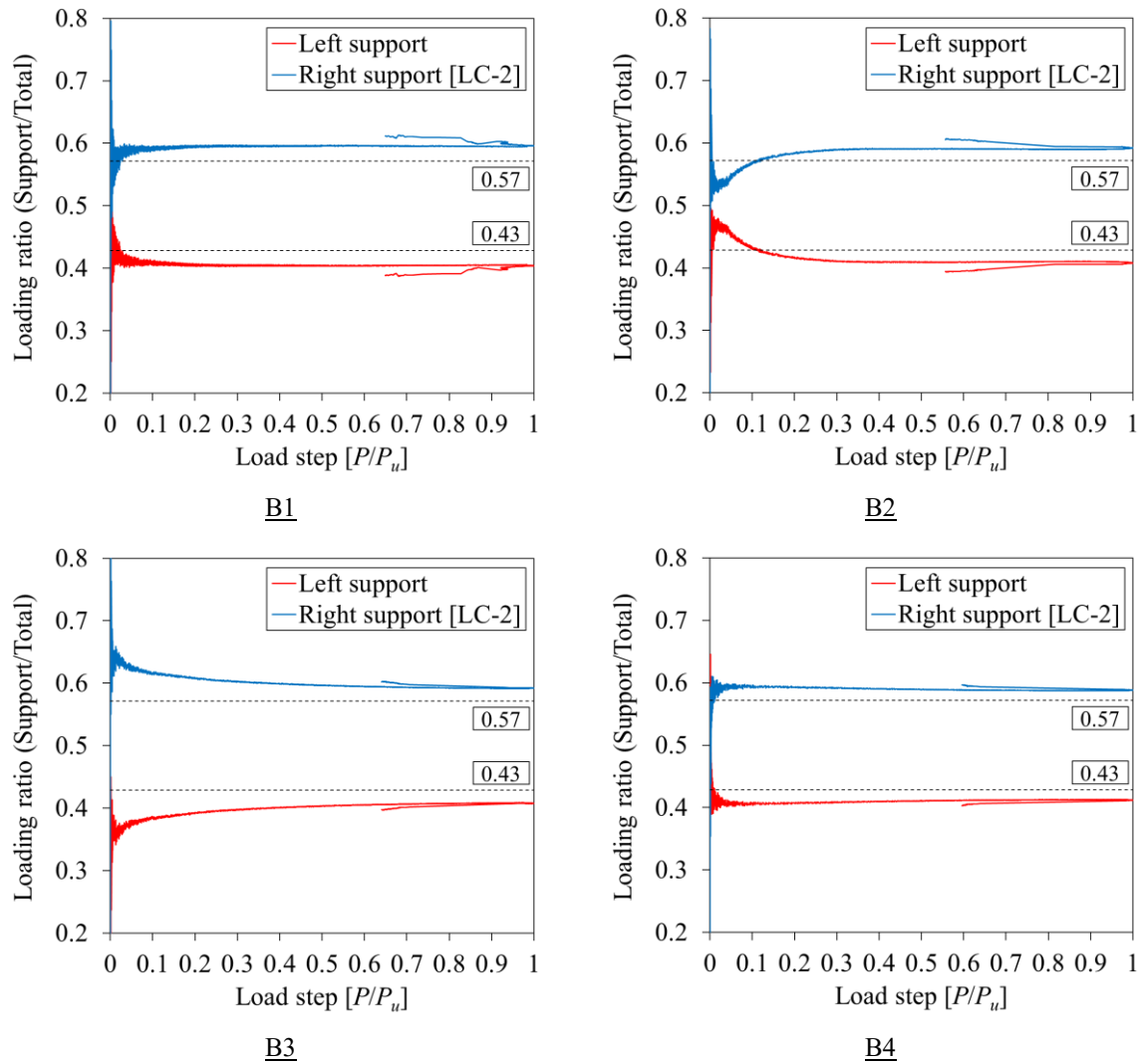


Figure 4.18 – Loading ratio response of group 1 specimens

4.4.6 STRAIN IN STEEL REINFORCEMENT

The measured reinforcement strains of control specimen B1 are shown in Figure 4.19. It appears that at the load step $0.9P_u$ the bottom layer of longitudinal reinforcement in tension yielded according to the two gauges located at the loading point (G-1/2). A single gauge at the midspan (G-7N) also indicates that the bottom layer of longitudinal reinforcement yielded as the ultimate load was reached, however the gauge located on the diametrically opposite side (G-8N) shows that the reinforcement was not near yielding, much like the other strain gauge measurements that are vertically aligned as well. Therefore, it is probable that the bottom layer

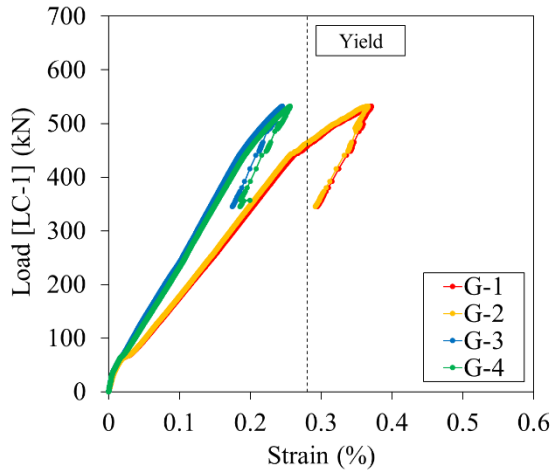
of longitudinal reinforcement did not yield at the midspan. The top layer of the longitudinal reinforcement in tension as well as the longitudinal reinforcement in compression remained elastic up to the peak load and failure. Yielding occurred at the second (G-19) and fifth (G-16) stirrups to the right of the support between load steps $0.7P_u$ and $0.9P_u$. The third (G-18) and fourth (G-17) stirrups to the right of the support were close to yielding at around load step $0.8P_u$, but strain measurements were not recorded beyond this point as the gauges failed. The four previously mentioned stirrups were intercepted by the critical shear crack as well as other secondary shear cracks. Strain measurements were not recorded for a single gauge (G-14) in specimen B1 that was located at the longitudinal reinforcement near the support as the gauge was likely damaged during concreting.

Figure 4.20 shows reinforcement strains for specimen B2. The longitudinal reinforcement in the jacket at the loading point yielded at the ultimate load. A single strain gauge in the jacket near the support (G-26) indicates yielding after the ultimate load was reached although the strain gauge located on the diametrically opposite side (G-25) does not. Two strain gauges located in the beam at the midspan (G-9) and near the support (G-12) indicate that the longitudinal reinforcement in tension yielded at around the load steps $0.6P_u$ and $0.8P_u$, respectively. However, the strain gauges that are located on diametrically opposite sides of these particular strain gauges at midspan (G-10) and near the support (G-11) show that the reinforcement was not near yielding, and this agrees with the other strain gauge measurements in the beam and jacket that are vertically aligned as well. Therefore, it is likely that the longitudinal reinforcement in tension in the beam did not yield at any point during the experiment. Additionally, the longitudinal reinforcement in compression did not yield throughout the experiment. A total of two stirrups yielded between the load steps $0.6P_u$ and $0.8P_u$, and they included the third (G-18) and fifth (G-16) stirrups to the right of the support. The fourth stirrup (G-17) to right of the support failed at around load step $0.6P_u$, and it was not close to yielding at that point. The three previously mentioned stirrups were intercepted by the critical shear crack. The second (G-19) stirrup to right of the support yielded shortly after the ultimate load was reached prior to failure.

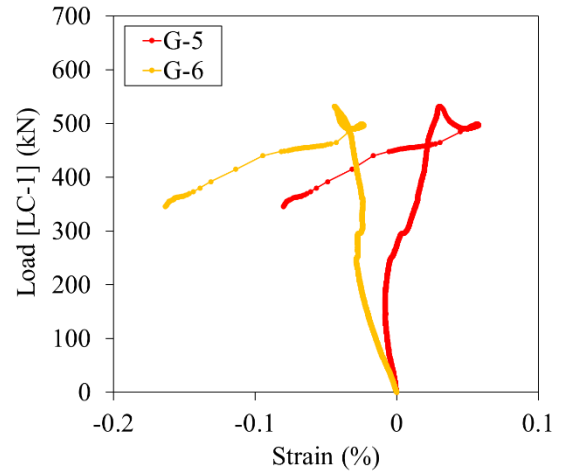
The reinforcement strains are shown in Figure 4.21 for specimen B3. The longitudinal reinforcement in the jacket at the loading point yielded at the ultimate load. A single strain gauge in the jacket at midspan (G-23) indicates yielding at the ultimate load yet the strain gauge

located on the diametrically opposite side (G-24) does not. Surprisingly, two strain gauges in the beam located on the bottom (G-12N) and top (G-13N) layers of longitudinal reinforcement in tension near the support indicate yielding between the load step $0.6P_u$ and the ultimate load. However, the strain gauges that are located on diametrically opposite sides of these particular strain gauges on the bottom (G-11N) and top (G-14N) layers of the longitudinal reinforcement in tension show that they were not near yielding, and this agrees with the other strain gauge measurements in the beam and jacket that are vertically aligned as well. Therefore, it is likely that the longitudinal reinforcement in tension in the beam did not yield at any point during the experiment. The longitudinal reinforcement in compression in the beam did not yield throughout the experiment as well. A total of three stirrups yielded between the load steps $0.7P_u$ and $0.9P_u$, and they included the third (G-18), fourth (G-17) and fifth (G-16) stirrups to the right of the support. These stirrups were intercepted by the same secondary shear crack as shown in Figure 4.21. The sixth (G-15) stirrup to right of the support yielded shortly after the ultimate load was reached, and the second (G-19) stirrup to the right of the support, which was intercepted by the critical shear crack, was very close to the yield point as the ultimate load was reached.

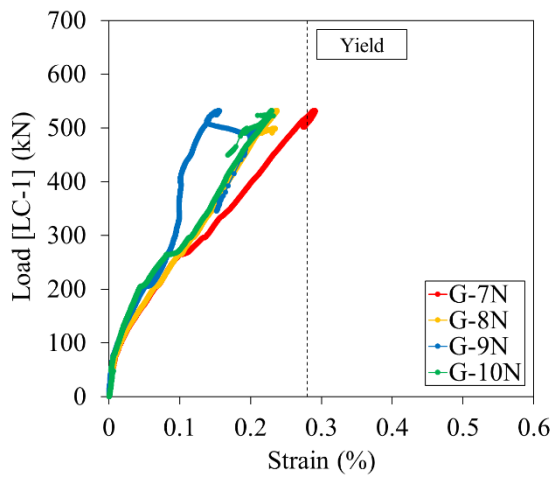
The strain responses of the steel reinforcement in specimen B4 are shown in Figure 4.22. The longitudinal reinforcement in tension and compression did not yield throughout the experiment. All of the stirrups yielded between the load steps $0.8P_u$ and $0.9P_u$ apart from the first (G-20) stirrup to the right of the support. The second (G-19), third (G-18), fourth (G-17) and fifth (G-16) stirrups to the right of the support were intercepted by the critical shear crack and secondary shear crack.



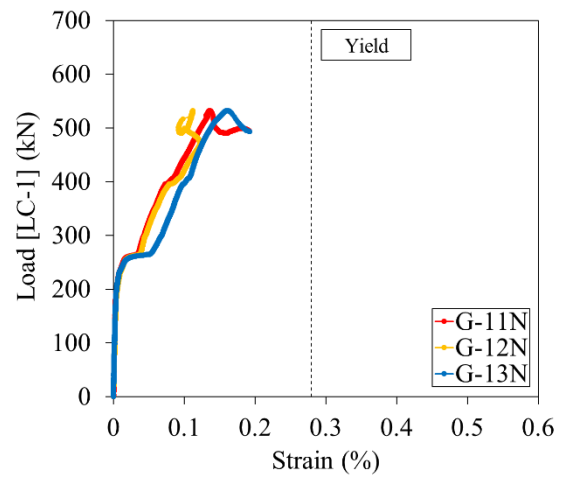
Long. reinforcement in tension at loading point



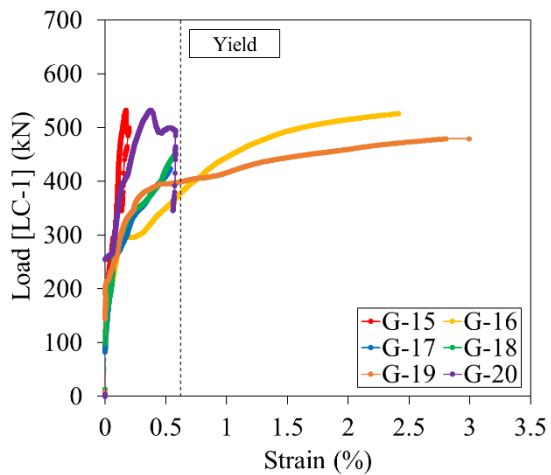
Long. reinforcement in compression at loading point



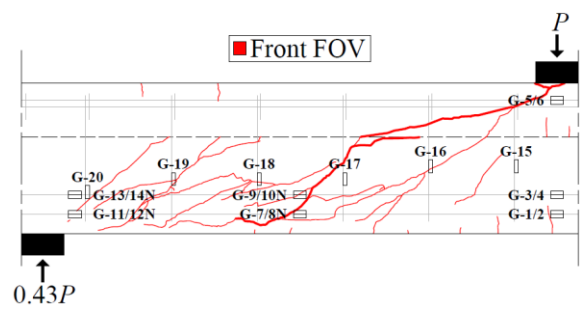
Longitudinal reinforcement at midspan



Longitudinal reinforcement near support

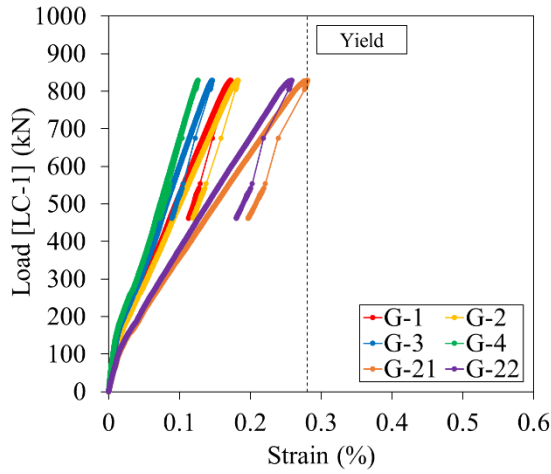


Transverse reinforcement

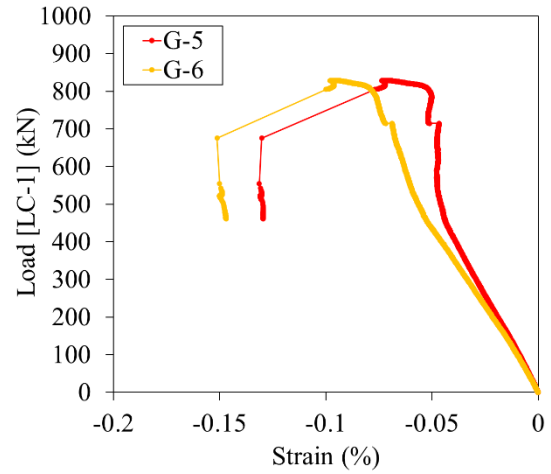


Gauge locations in relation to crack pattern at failure

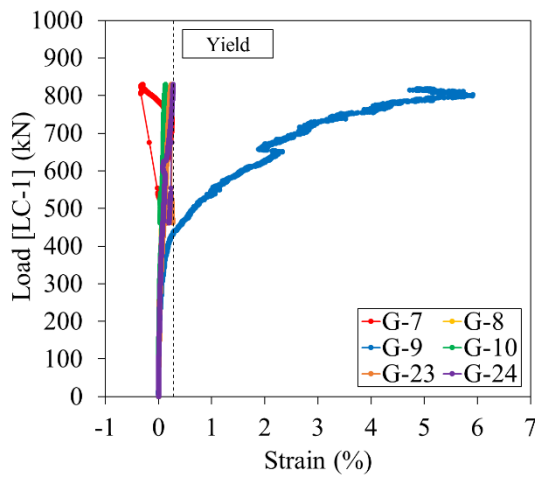
Figure 4.19 – Strain in steel reinforcement of specimen B1



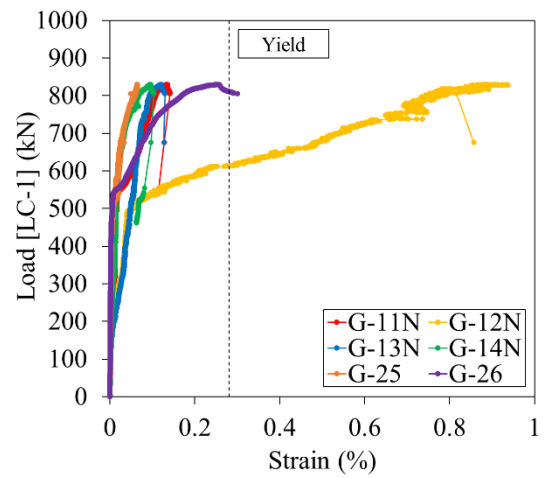
Long. reinforcement in tension at loading point



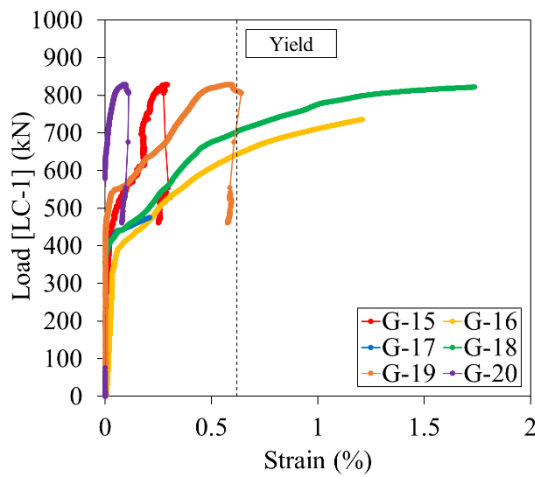
Long. reinforcement in compression at loading point



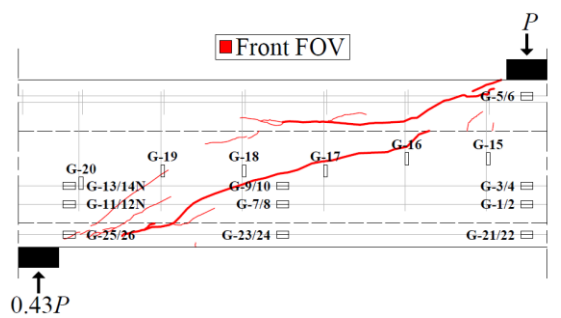
Longitudinal reinforcement at midspan



Longitudinal reinforcement near support

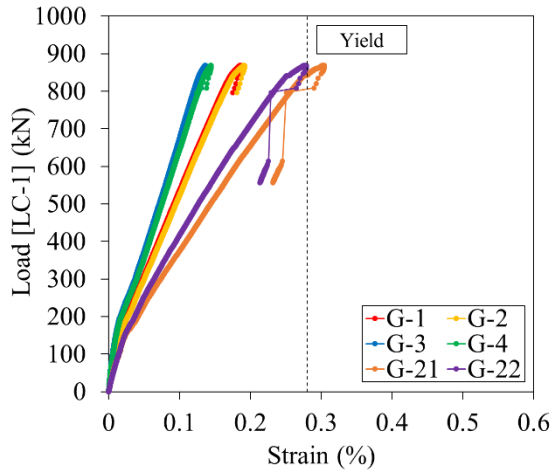


Transverse reinforcement

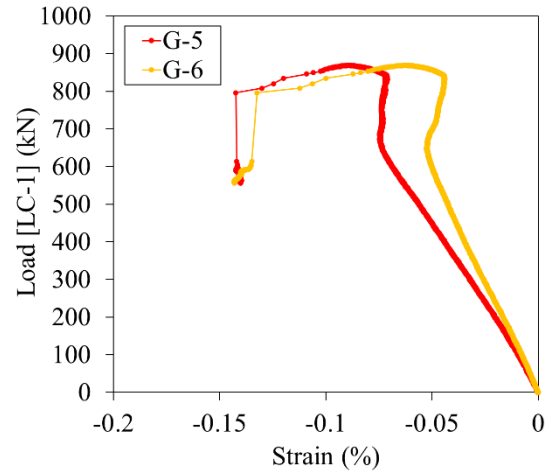


Gauge locations in relation to crack pattern at failure

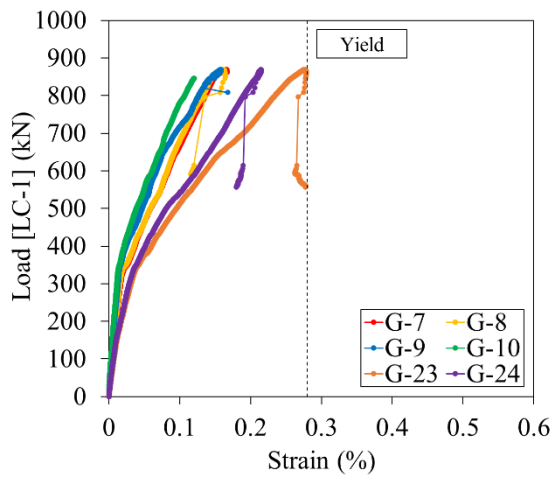
Figure 4.20 – Strain in steel reinforcement of specimen B2



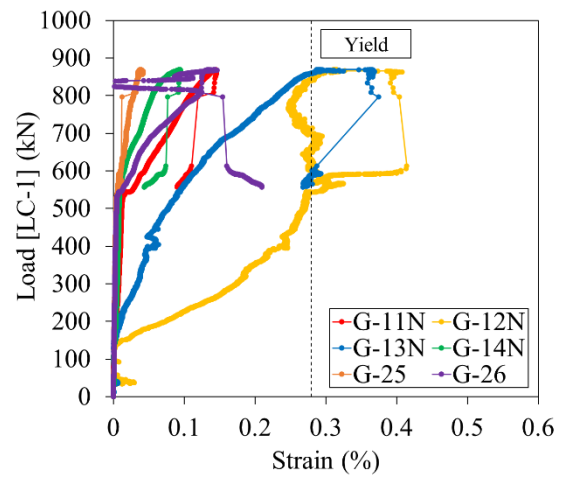
Long. reinforcement in tension at loading point



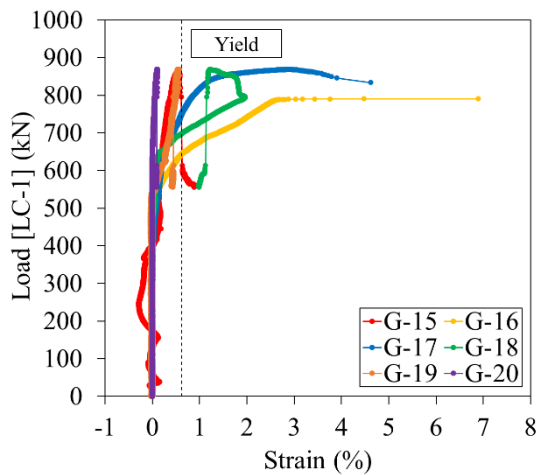
Long. reinforcement in compression at loading point



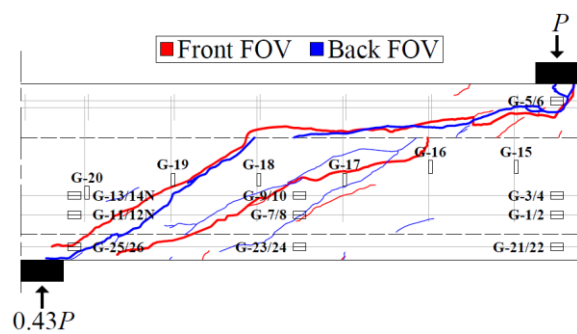
Longitudinal reinforcement at midspan



Longitudinal reinforcement near support

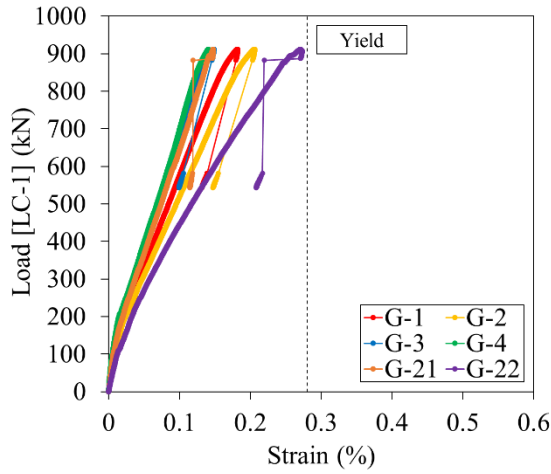


Transverse reinforcement

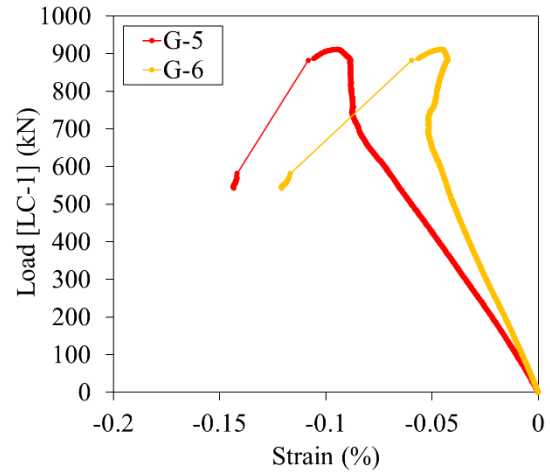


Gauge locations in relation to crack pattern at failure

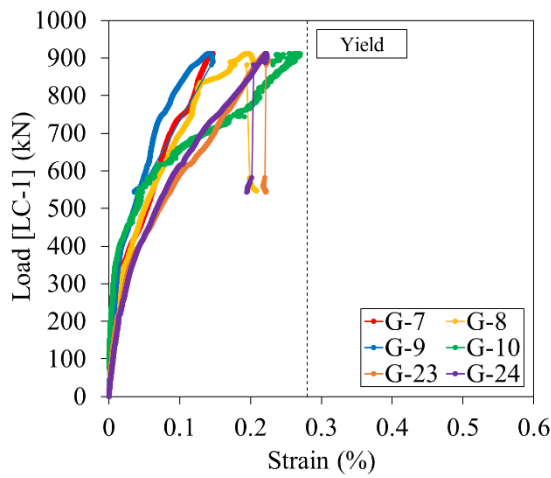
Figure 4.21 – Strain in steel reinforcement of specimen B3



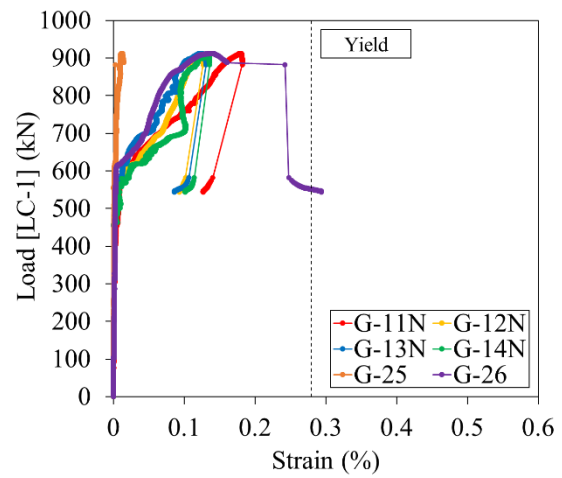
Long. reinforcement in tension at loading point



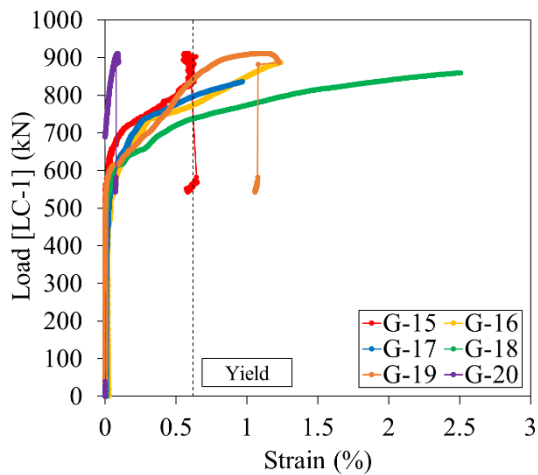
Long. reinforcement in compression at loading point



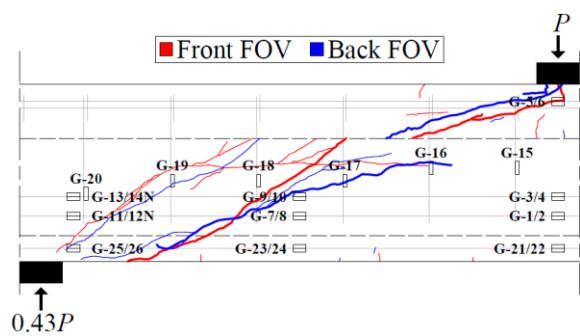
Longitudinal reinforcement at midspan



Longitudinal reinforcement near support



Transverse reinforcement

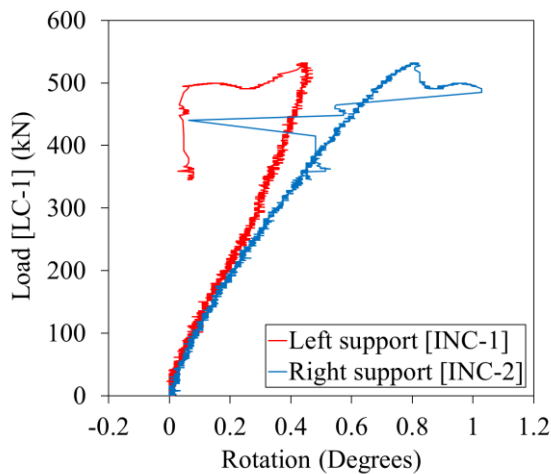


Gauge locations in relation to crack pattern at failure

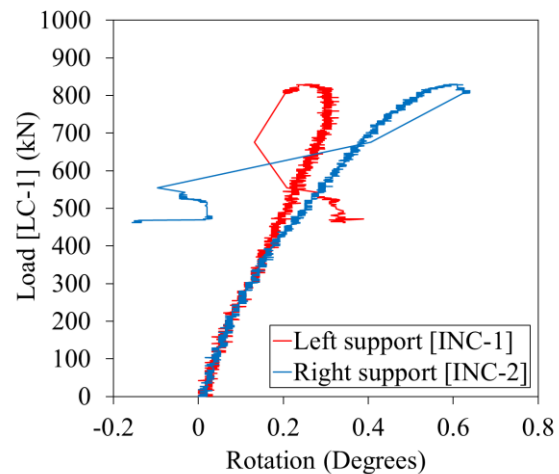
Figure 4.22 – Strain in steel reinforcement of specimen B4

4.4.7 ROTATIONS AND VERTICAL DISPLACEMENTS

The rotations and vertical displacements recorded by the inclinometers and LVDTs at the left and right ends of the beams in group 1 with respect to Figure 3.36 are shown in Figures 4.23 and 4.24, respectively. The rotations were measured by inclinometers positioned at mid-height of the flange directly over the centrelines of the left (INC-1) and right supports (INC-2). The exact locations of the inclinometers and LVDTs are presented in Figure 3.36. The upward vertical displacements of the left and right ends of each specimen were measured by LVDTs positioned on the top surface of each beam directly over the centrelines of the left (LVDT-1) and right supports (LVDT-3). The rotations at the left and right ends of each specimen were similar, but of opposite sign, until around $0.5P_u$ when shear cracks formed in the left span as described in Section 4.4.2. At this point, the direction of rotation switched to anticlockwise at the left end due to the opening of the shear cracks. The vertical displacements of the right end appear to be larger than those of left end for almost all of the specimens of group 1 as shown in Figure 4.24. This is possibly due to the higher degrees of rotation exhibited in the shorter right span of each specimen or a larger reaction at the right support which may have caused a larger vertically upward movement compared to the longer left hand span.



B1



B2

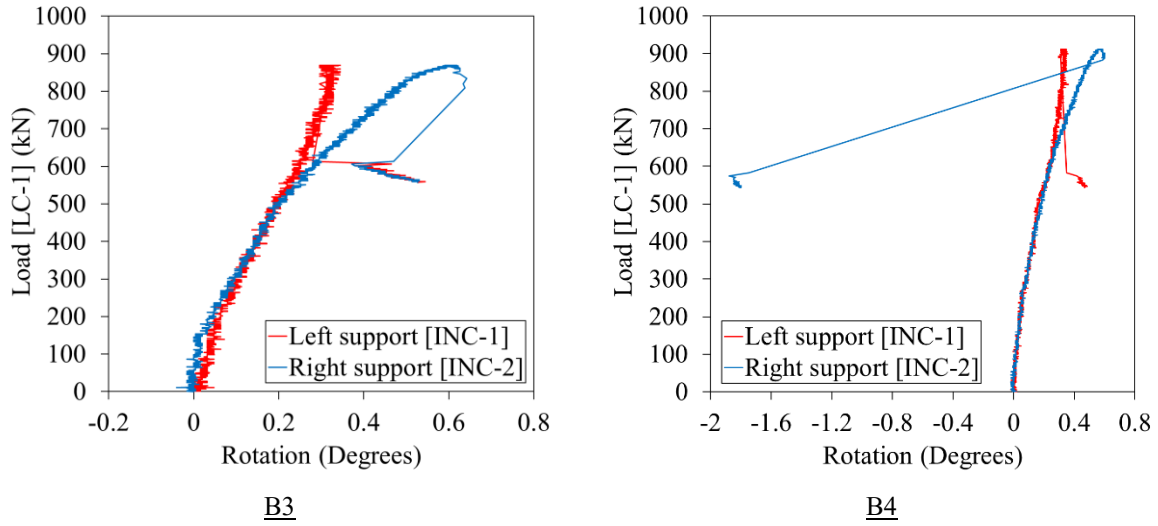


Figure 4.23 – Rotation response of group 1 specimens

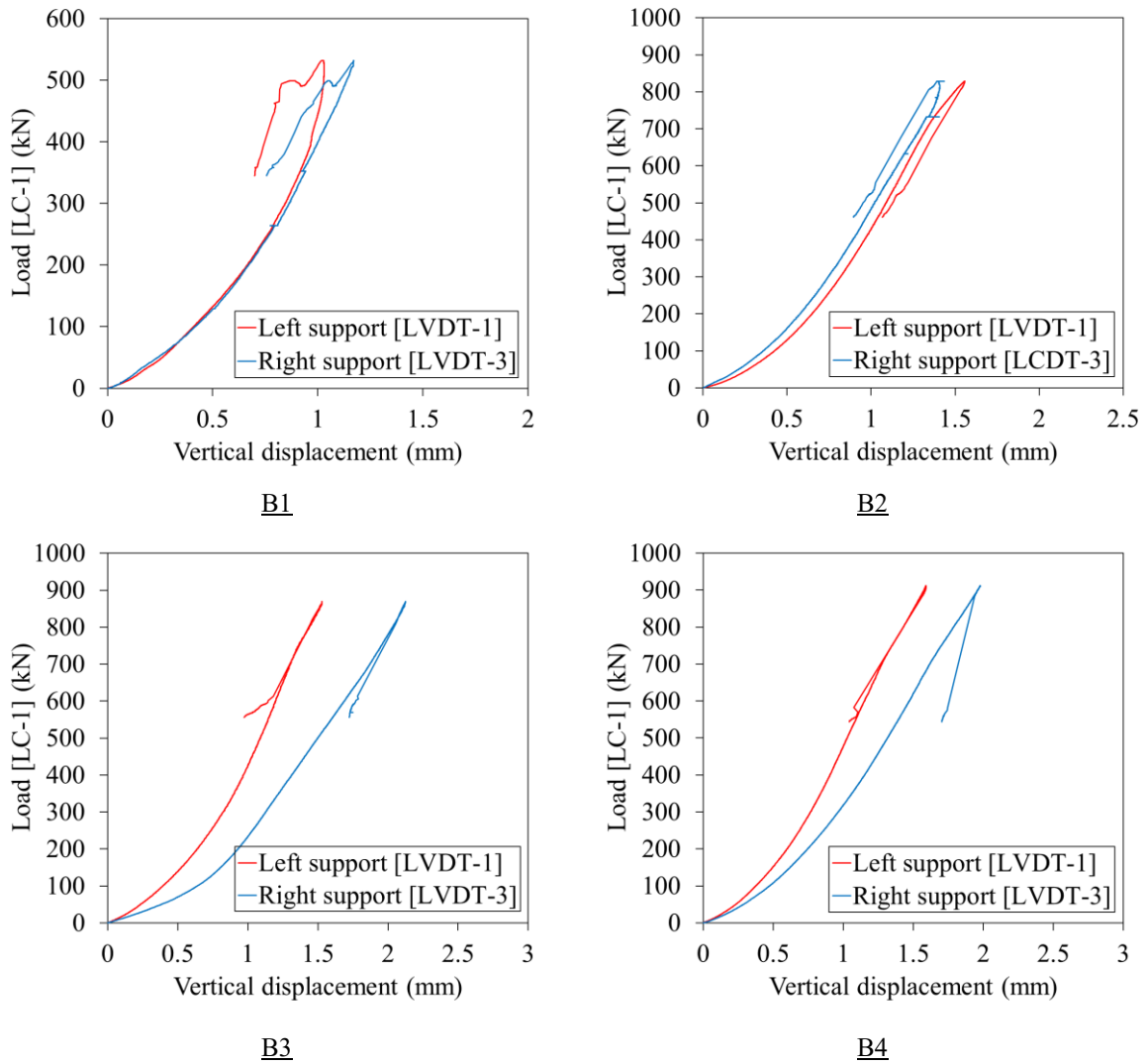


Figure 4.24 – Vertical displacement response of group 1 specimens

4.5 RESULTS OF GROUP 2 BEAM SPECIMENS

4.5.1 INTRODUCTION

Group 2 consisted of one control RC beam (B5) and three identical RC beams with UHPC jackets (B6, B7 and B8). The RC beams of group 2 were lightly reinforced in flexure. As described in Section 3.2, the critical shear span was reinforced with marginally less than the minimum amount of shear reinforcement required by EC2 (BSI, 2014). The side layer thickness of the UHPC jackets were 20 mm, 30 mm and 40 mm for specimens B6, B7 and B8, respectively. The bottom thickness of the jacket was constant at 60 mm, and it was reinforced with $3\phi 20$, $4\phi 20$ and $5\phi 20$ longitudinal reinforcement bars in specimens B6, B7 and B8, respectively. The material properties of the NSC and UHPC used in the assessments presented in Chapters 5, 6 and 7 are listed in Table 4.13 for each specimen of group 2. The differences are marginal between the material properties of each specimen. The compressive cylinder and splitting tensile strength of the NSC of specimens B6 and B7 were taken as the average between the results of the air and water cured control specimens of the first (B5) and last (B8) tested beam specimens of the group presented in Table 4.5. The compressive cylinder strength of the UHPC was taken from the results presented in Table 4.9 of the control specimens prepared using the Soroto mixer that were cast and tested alongside each specimen. The tensile strength at the elastic limit, post-cracking tensile strength and elastic modulus of the UHPC were taken from average results obtained at 28 days from the batches of the Soroto mixer in Table 4.6. The elastic modulus of the NSC was also taken from the results at 28 days.

Table 4.13 – Summary of NSC and UHPC material properties of group 2 specimens

Specimen	Normal strength concrete (NSC)				Ultra high performance concrete (UHPC)				
	Age [Day]	Comp. cyl. strength (f_{cm}) [MPa]	Splitting tensile str. ($f_{ctm,sp}$) [MPa]	Elastic modulus (E_{cm}) [MPa]	Age [Day]	Comp. cyl. strength (f_{cm}) [MPa]	Tensile strength ($f_{ctm,el}$) [MPa]	Tensile strength (f_{ctm}) [MPa]	Elastic modulus (E_{cm}) [MPa]
B5	163	50.14	3.59	37496.95	-	-	-	-	-
B6	171	50.53	3.45	37496.95	45	124.44	8.58	6.39	39302.60
B7	176	50.53	3.45	37496.95	48	127.92	8.58	6.39	39302.60
B8	191	50.91	3.30	37496.95	59	132.04	8.58	6.39	39302.60

The experimental results and main characteristics of each specimen in group 2 are summarized in Table 4.14. Control specimen B5 failed in combined flexure-shear, while all of the specimens with jackets failed in shear. Much like the specimens of group 1, significant increases in the ultimate load were observed in the jacketed specimens when compared to control specimen B5. The thinnest UHPC jacket (20 mm) of specimen B6 caused an increase of 96.06% (308.32 kN) in the ultimate load. The ultimate load increased by 103.80% (330.95 kN) due to the intermediate UHPC jacket (30 mm) of specimen B7. The largest UHPC jacket (40 mm) of specimen B8 caused an increase of 136.78% (448.66 kN). The increases in the ultimate loads of the strengthened specimens of group 2 were significantly higher than those of group 1 due to the proportionally greater area of reinforcement in the jacket relative to the beam.

Table 4.14 – Summary of main experimental results of group 2 specimens

Specimen	Beam long. reinf. ^A (ρ_l) [%]	Beam shear reinf. ^B (ρ_w) [%]	Jacket thick. (t_s) [mm]	Area of long. reinf. ratio ^C [%]	Ult. load (P_u) [kN]	Ult. shear (V_u) [kN]	Ult. disp. (δ_u) [kN]	Ult. load incr. [kN]	Ult. load incr. [%]	Failure mode
B5	1.55	0.11	-	-	325.46	134.17	24.92	-	-	Flexure-Shear
B6	1.55	0.11	20	100.00	633.78	263.07	8.98	308.32	96.06	Shear
B7	1.55	0.11	30	133.33	656.41	273.45	7.60	330.95	103.80	Shear
B8	1.55	0.11	40	166.67	774.12	317.69	8.06	448.66	136.78	Shear

^A Longitudinal reinforcement ratio (ρ_l) equals to $A_s/b_w d$; ^B Shear reinforcement ratio (ρ_w) equals to $A_{sw}/b_w s$; ^C Ratio of the area of longitudinal reinforcement in the UHPC jacket to that in the RC beam

4.5.2 CRACK DEVELOPMENT AND MODE OF FAILURE

Control specimen B5 developed flexural cracks in the web around the loading point in addition to shear cracks, unlike the jacketed specimens. Significant flexural and shear cracks only formed in the critical span of control specimen B5. The crack pattern at the ultimate load and post-peak failure of each specimen of group 2 is shown in Figure 4.25 in which the critical shear crack is highlighted for each beam. As mentioned in Section 4.4.3, the crack patterns in Figure 4.25 were traced from superimposed DIC images. Photographs of each specimen taken after the experiment are presented in Figure 4.26. Section A.2 shows the processed DIC images of each specimen of group 2, which include crack patterns and strain states starting from the load step $0.8P_u$ until post-peak failure.

Vertical flexural cracks first appeared in the web of the control specimen (B5), below the loading point, at around load step $0.4P_u$ (130 kN). The critical shear crack and a couple of secondary shear cracks developed soon after at around the load step $0.6P_u$ (196 kN). The critical shear crack formed around the middle of the critical span, while a secondary crack formed about 275 mm to left of it and another secondary shear crack formed about 250 mm to right of it. The shear cracks initially grew in the web between the longitudinal reinforcement and the web/flange junction before widening and extending downward and horizontally along the longitudinal reinforcement bar towards the support as the ultimate load (325 kN) was approached. The vertical flexural cracks widened in the web around the loading point and extended upward into the flange as the ultimate load was approached. The critical shear crack continued to grow until the concrete cover completely delaminated and broke off from the single layer of longitudinal reinforcement after reaching the ultimate load at post-peak failure (182 kN). The critical shear crack also extended into the flange and reached the loading plate at failure. Both FOVs were monitored by DIC for control specimen B5, and the results are shown in Figures A.7 and A.8 for the front and back FOVs, respectively. The crack patterns in B5 were virtually the same for both FOVs as shown in the traced crack patterns in Figure 4.25 as well as the photographs taken after the test in Figure 4.26.

Initial cracking in specimen B6 became visible around the load step $0.6P_u$ (381 kN). At this loading stage, the critical shear crack developed near the middle of the critical span in the web between the longitudinal reinforcement in the beam to just below the web/flange junction. A more dominant and almost parallel secondary shear crack developed at load step $0.6P_u$ (381 kN). The crack initiated approximately 300 mm to the left of the critical shear crack and from there extended to the longitudinal reinforcement in the beam and the web/flange junction. The critical and secondary shear crack continued to grow until a second almost parallel secondary shear crack formed approximately 150 mm to the left of the initial secondary shear crack at around load step $0.8P_u$ (508 kN). The second secondary shear crack extended from the longitudinal reinforcement in the jacket to around the mid-height of the web. The secondary shear cracks continued to widen as the ultimate load was reached (634 kN), much like the critical shear crack that also extended upward to reach the web/flange junction. After the ultimate load was reached, the critical shear crack became much more dominant as it extended downward to longitudinal reinforcement in the jacket as well as upward into the flange reaching the loading plate after the ultimate load was reached and as post-peak failure occurred (326

kN). The crack development of both FOVs of specimen B6 were monitored by DIC, with the results shown in Figures A.9 and A.10 for the front and back FOVs, respectively. The critical and secondary shear cracks were overlapping for both FOVs as shown in the traced crack patterns in Figure 4.25 as well as the photographs taken after the test in Figure 4.26.

The critical shear crack in specimen B7 first appeared about 400 mm to the right of the support at around the load step $0.7P_u$ (460 kN). Although the critical shear cracks overlap for both FOVs as shown in Figures 4.25 and 4.26, the crack development in the front and back FOVs of the jacket slightly differed. The differences in crack development for both FOVs are shown in the DIC results of the front and back FOVs in Figures A.11 and A.12, respectively. At around the load step $0.7P_u$ (460 kN), a secondary shear crack in the front FOV formed from just above the inner corner of the support and extended to the web/flange junction. The critical shear crack developed from the secondary shear crack at around mid-height of the web and extended downward towards the longitudinal reinforcement in the jacket as the ultimate load was reached (656 kN). For the back FOV, the critical shear crack appeared to form between the longitudinal reinforcement in the jacket to the web/flange junction at around the load step $0.7P_u$ (460 kN). As the ultimate load was reached (656 kN), the critical shear crack in the back FOV widened while a short secondary shear crack formed about 90 mm to the left of it, which appears to overlap with the secondary shear crack in the front FOV as shown in Figure 4.25. After the ultimate load was reached, the critical shear cracks in both FOVs extended into the flange and reached the loading plate as post-peak failure occurred (451.17 kN).

The critical shear crack of specimen B8 first appeared near the middle of the critical span at around the load step $0.6P_u$ (465 kN). Much like specimen B7, the crack development differed in each face of the jacket although the critical shear cracks overlap for both FOVs as shown in Figures 4.25 and 4.26. The DIC images of the front and back FOVs of specimen B8 in Figures A.13 and A.14, respectively, highlight the differences in crack development. At load step $0.6P_u$ (465 kN), the critical shear crack extended from the longitudinal reinforcement in the jacket to below the web/flange junction in each face of the jacket. The critical shear crack in the front FOV widened and extended upward past the web/flange junction and into flange as the ultimate load was reached (774 kN), with a relatively short and narrow secondary shear crack also forming about 220 mm to left of it. For the back FOV, the critical shear crack also widened and extended upward into the flange as the ultimate load was reached (774 kN). However, a

more dominant secondary shear crack developed, which stemmed from the critical shear crack around the longitudinal reinforcement in the beam and extended to the web/flange junction. In addition, a second almost parallel secondary shear crack formed in the back FOV near the support that extends to the web/flange junction, with a small segment of it also appearing in the front FOV. After the ultimate load was reached, the critical shear cracks in both FOVs reached the loading plate as post-peak failure occurred (456 kN).

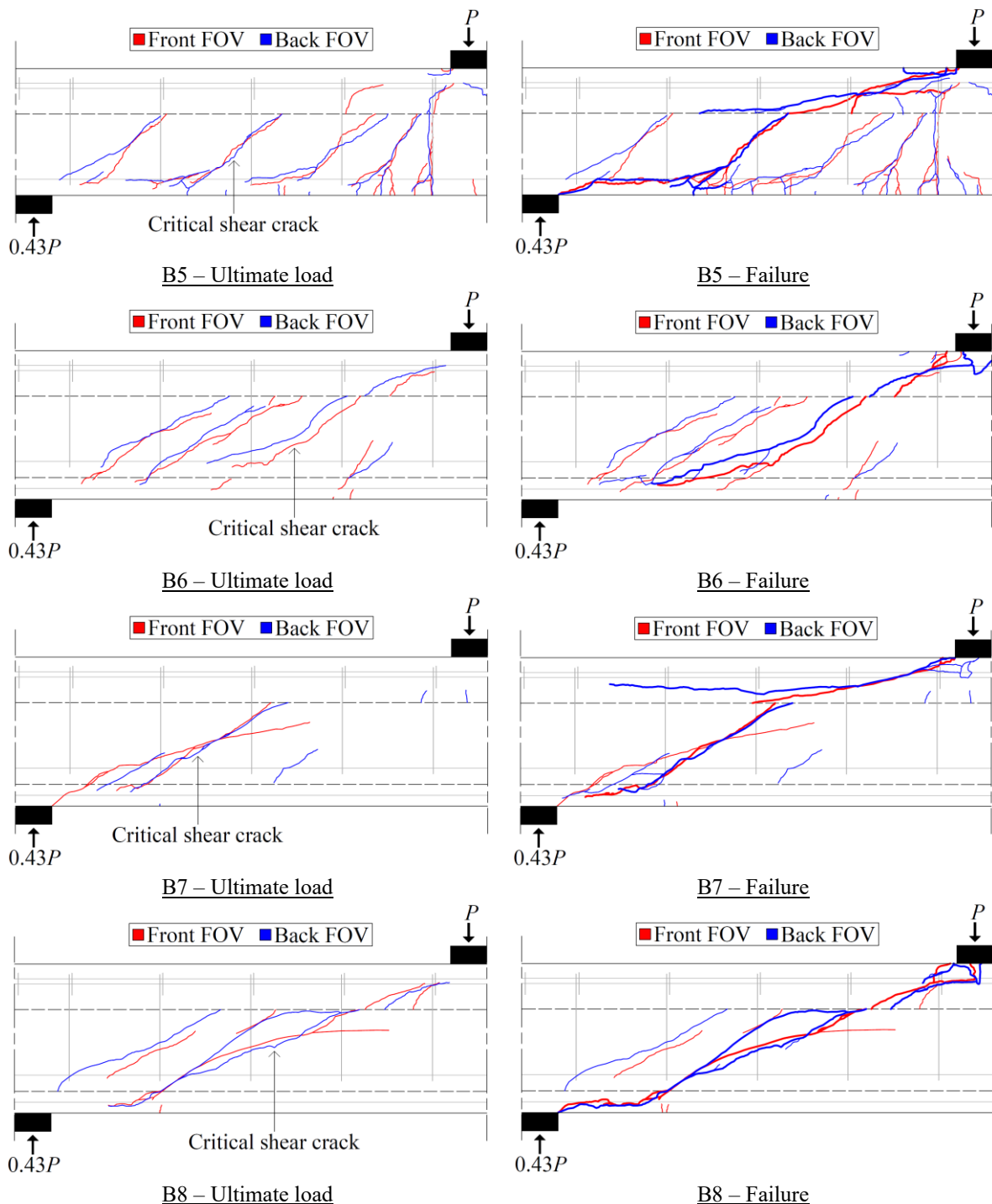


Figure 4.25 – Traced crack pattern of group 2 specimens at the ultimate load and failure

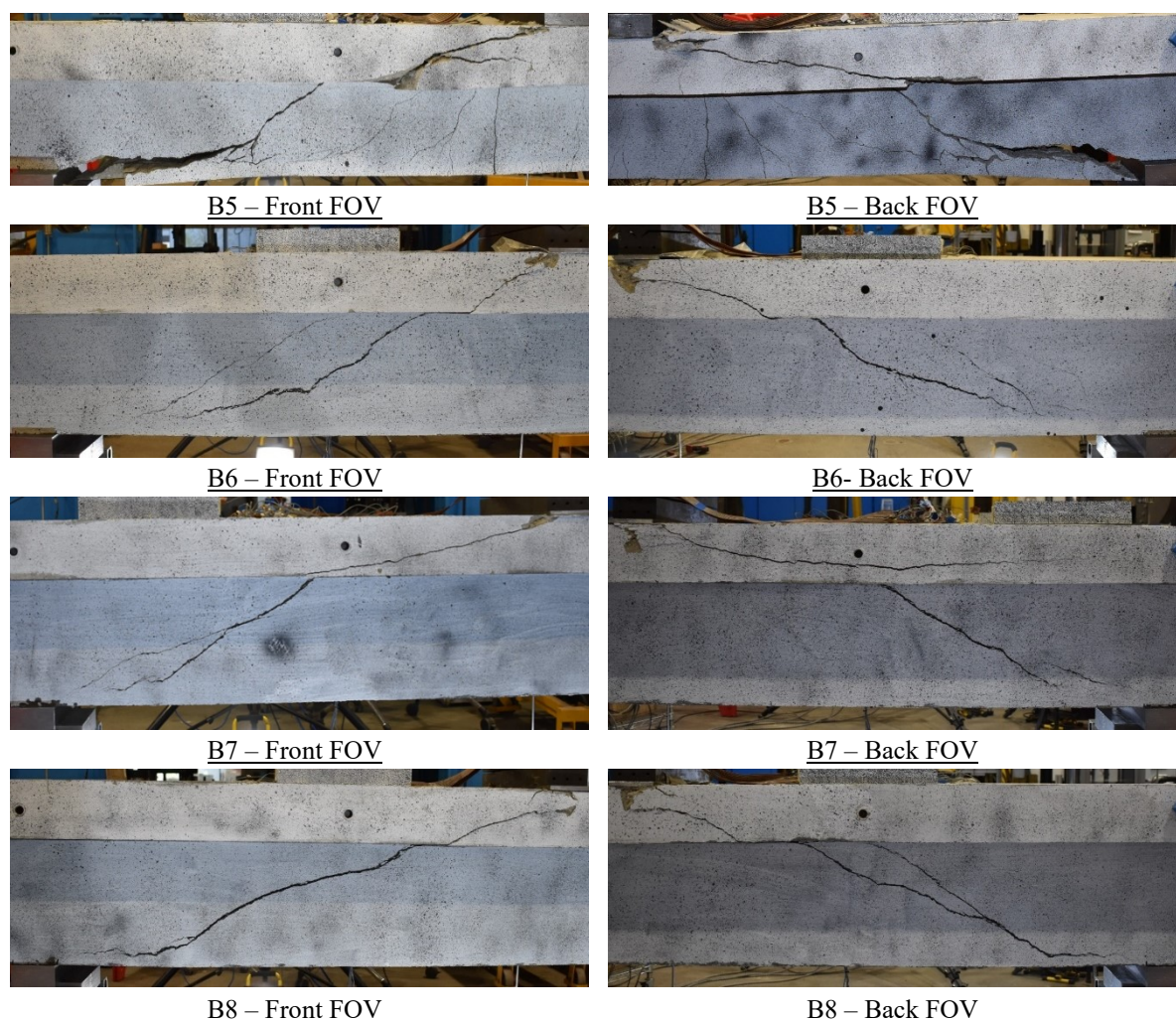


Figure 4.26 – Crack pattern of group 2 specimens after experiment

4.5.3 CRITICAL SHEAR CRACK INCLINATION ANGLES

The critical shear crack inclination angles at the ultimate load of the specimens of group 2 are shown in Figure 4.27. As described in Section 4.4.3, the cracks at the ultimate load were traced from superimposed DIC images for the front and back FOVs, and the inclination angle was taken as the angle between the diagonal failure plane measured between the crack ends and the longitudinal axis. In group 2, the critical shear cracks are located close to one another near the middle of the critical span. Much like group 1, the critical shear crack was steepest in the control beam (B5) where the inclination was 38° in both FOVs. The critical shear crack inclinations of the jacketed specimens are relatively similar in both FOVs with a maximum difference of 3° in specimen B6. The crack inclination angles in the front and back FOVs were 29° and 26° for specimen B6 and 30° and 29° for specimen B7. The critical shear crack inclination was flattest in specimen B8 where the angles were 21° and 20° in the front and back

FOVs, respectively. In specimen B8, the critical shear cracks extended into the flange at the ultimate load unlike the other specimens.

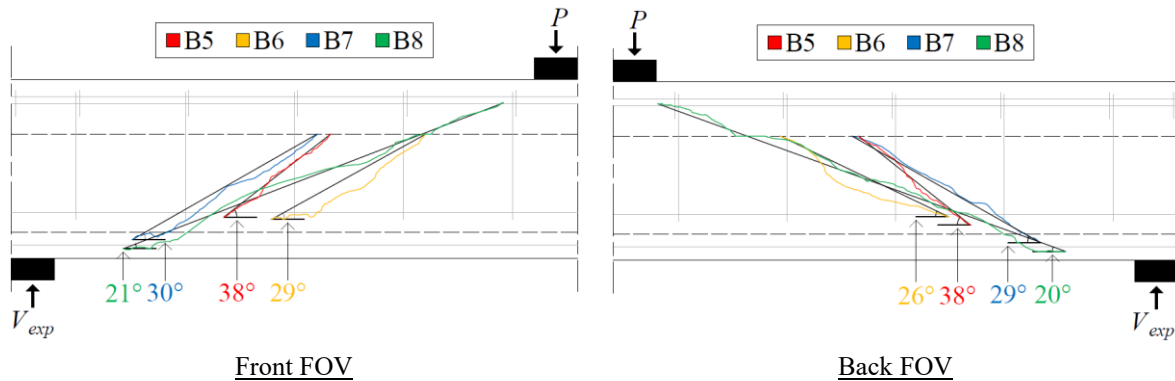


Figure 4.27 – Inclination angles of critical shear cracks at the ultimate load of group 2 specimens

4.5.4 LOAD-DISPLACEMENT RESPONSE

The load-displacement response of each specimen of group 2 is shown in Figure 4.28. As described in Section 4.4.4, the total load and the load at the right support were recorded throughout the experiment by the actuator (LC-1) and the load cell at the support (LC-2), respectively, with the load at the left support equal to their difference. The displacement in Figure 4.28 corresponds to the displacement measured by LVDT-2 that was aligned at the centreline of the loading point. The load-displacement response of control specimen B5 had a long almost horizontal plateau from a displacement of 10 mm to failure at a displacement of 30 mm indicating flexural yield. Unlike the control specimen, the strengthened specimens of group 2 were brittle, with failure occurring almost immediately after reaching the ultimate load. Due to difference in failure modes between the control specimen and strengthened specimens, the shapes of the load-displacement responses are significantly different. Although the load-carrying capacities of specimens B6, B7 and B8 were significantly increased when compared to control specimen B5, the displacements at failure of the strengthened specimens are almost 20 mm less than that of control specimen B5.

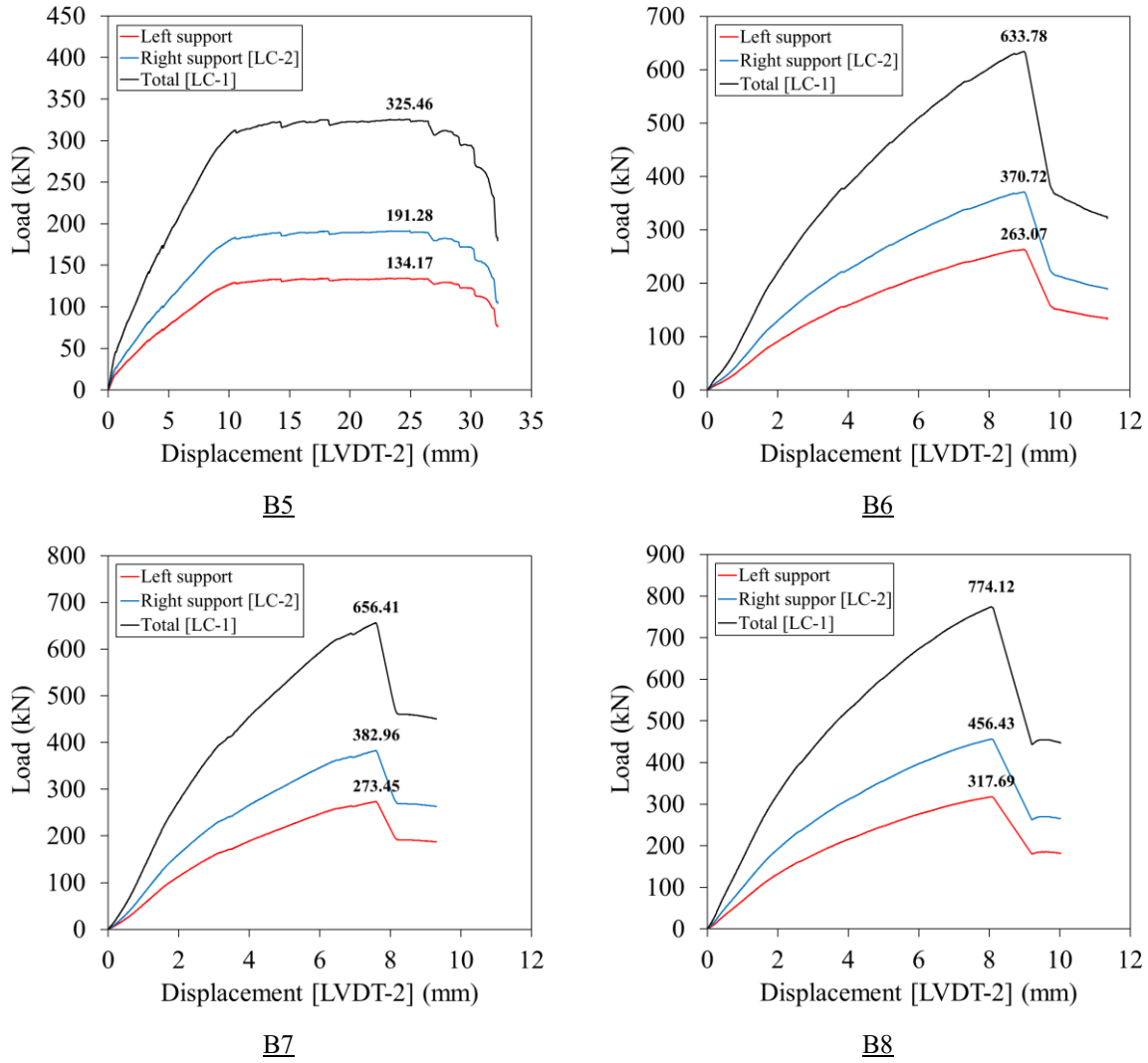


Figure 4.28 – Load-displacement response of group 2 specimens

4.5.5 LOADING RATIOS

Figure 4.29 presents the experimentally derived loading ratios at each support of the specimens of group 2 with respect to Figure 3.37 alongside the theoretical loading ratios corresponding to the experimental setup presented in Figure 3.1. As mentioned in Section 4.4.5, the initial spikes prior to the load step $0.1P_u$ in Figure 4.29 are due to the specimen settling in the testing rig. The knife edge load bearing plate shown in Figure 3.33 was used for all of the specimens of group 2 instead of the rectangular plate that is also shown in Figure 3.33 that was used for the specimens of group 1. This was done to locate the point of application of the applied load more accurately. The actual loading ratios for the specimens of group 2 were approximately 0.414 and 0.586 on average throughout the experiments for the left and right supports, respectively. This is equivalent to percent differences of 3.49% and 2.61% when compared to the theoretical

loading ratios of 0.429 and 0.571 for the left and right supports, respectively. This compares with differences of 4.98% and 3.74% for the left and right supports, respectively, for group 1. The marginal differences that remain between the actual and theoretical loading ratios are likely due to marginal variations in the loading geometry during the experiments (Pastore, 2020).

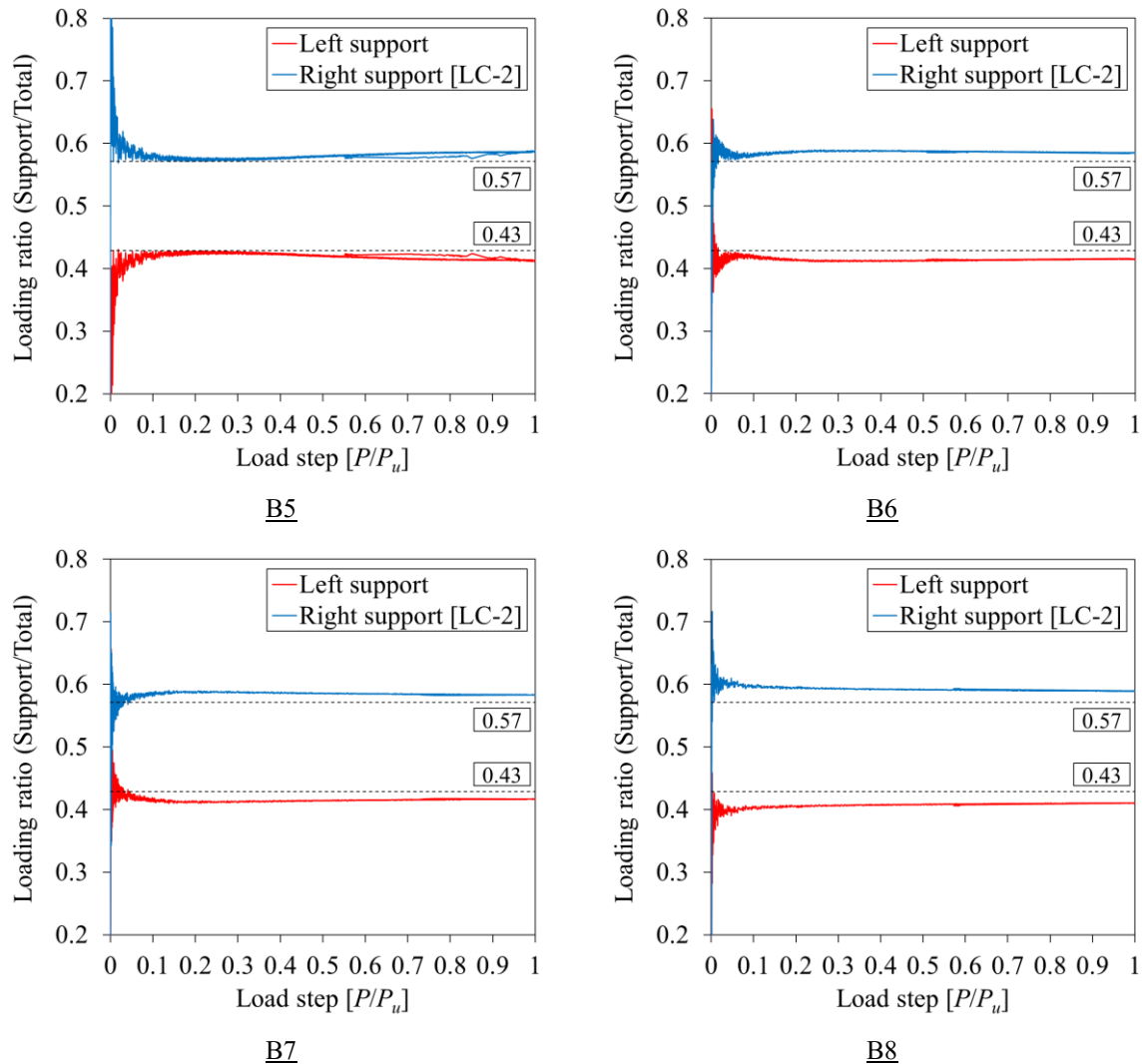


Figure 4.29 – Loading ratio response of group 2 specimens

4.5.6 STRAIN IN STEEL REINFORCEMENT

The reinforcement strains measured in control specimen B5 are shown in Figure 4.30. The longitudinal reinforcement in tension at the loading point yielded at load step $0.9P_u$ according to a single strain gauge (G-2), with the gauge (G-1) on the diametrically opposite side failing at load step $0.7P_u$ prior to reaching the yield point. The longitudinal reinforcement at midspan also yielded although it did not yield near the support throughout the experiment along. The

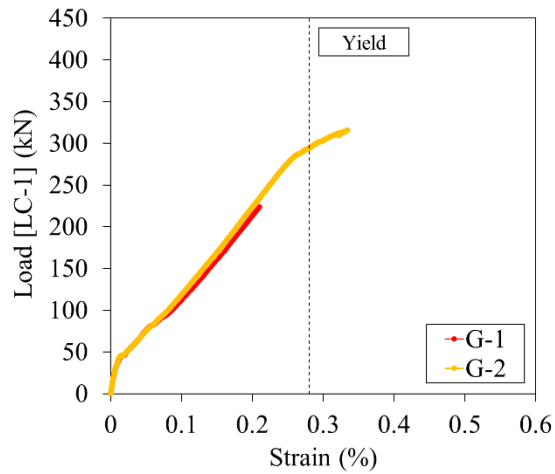
longitudinal reinforcement in compression was very close to the yield point as the ultimate load was reached. The fourth (G-10) stirrup to the right of the support, which was intercepted by a secondary shear crack, yielded at around load step $0.7P_u$. The third (G-11) stirrup to the right of the support, which was intercepted by the critical shear crack, was close to yielding before the gauge failed at around load step $0.8P_u$. Strain measurements were not recorded for two gauges in specimen B5 that were located at the longitudinal reinforcement at the midspan (G-5N) and the support (G-7N) as the gauges probably malfunctioned due to damage during concreting.

Reinforcement strains are shown for specimen B6 in Figure 4.31. The longitudinal reinforcement in the jacket at the loading point and at midspan yielded at the ultimate load. The longitudinal reinforcement in tension and compression in the beam did not yield throughout the experiment. Between load step $0.8P_u$ and the ultimate load, yielding occurred at the second (G-12) and third (G-11) stirrups to the right of the support. At around the load step $0.9P_u$, the fourth (G-10) stirrup to the right of the support was relatively close to yielding before the gauge failed. The third (G-11) and fourth (G-10) stirrups to the right of the support were intercepted by the critical shear crack, and the second (G-12) stirrup to right of the support was intercepted by the secondary shear crack nearest to the support.

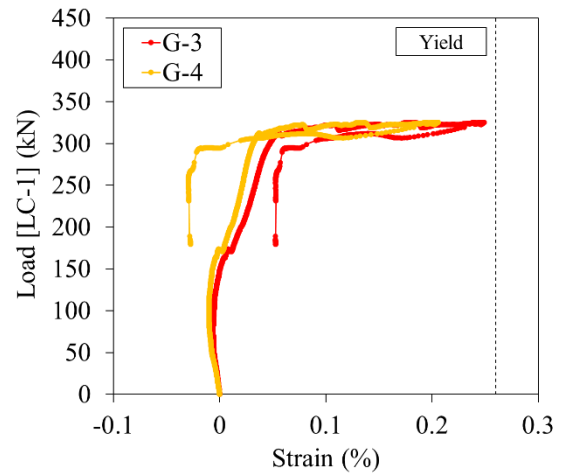
The reinforcement strains are shown for specimen B7 in Figure 4.32. The longitudinal reinforcement in tension and compression did not yield throughout the experiment. Between load step $0.7P_u$ and the ultimate load, yielding occurred in the second (G-12) and fifth (G-9) stirrups to the right of the support. The third (G-11) and fourth (G-10) stirrups to the right of the support were very close to yielding before the gauges failed as the ultimate load was reached. Only the second (G-12) and third (G-11) stirrups to the right of the support were intercepted by the critical shear crack. Strain measurements were not recorded for two gauges in specimen B7 that were located at the longitudinal reinforcement at the loading point in tension (G-2) and compression (G-3) as the gauges were likely damaged during concreting.

The reinforcement strains in specimen B8 are shown in Figure 4.33. The longitudinal reinforcement in tension and compression did not yield throughout the experiment. Surprisingly, a single strain gauge on the longitudinal reinforcement in the beam at the midspan (G-5) indicates yielding as the ultimate load is reached yet the gauge located on the diametrically opposite side (G-6) does not, much like the other strain gauge measurements that

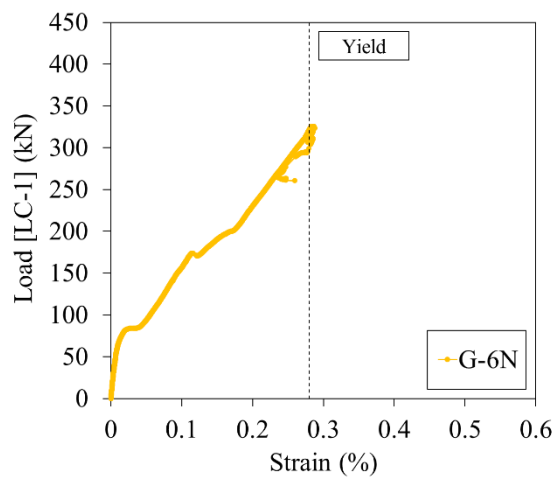
are vertically aligned. Therefore, it is probable that the longitudinal reinforcement in the beam did not yield. Between the load step $0.8P_u$ and the ultimate load, yielding occurred in the second (G-12), third (G-11) and fourth (G-10) stirrups to the right of the support. The third (G-11) and fourth (G-10) stirrups to the right of the support were intercepted by the critical shear crack, while the second (G-12) stirrup to the right of the support was intercepted by the secondary shear crack nearest to the support. Strain measurements were not recorded for two gauges in specimen B8 that were located at the longitudinal reinforcement in tension at the loading point (G-2) and near the support (G-8N) as the gauges probably malfunctioned due to damage during concreting.



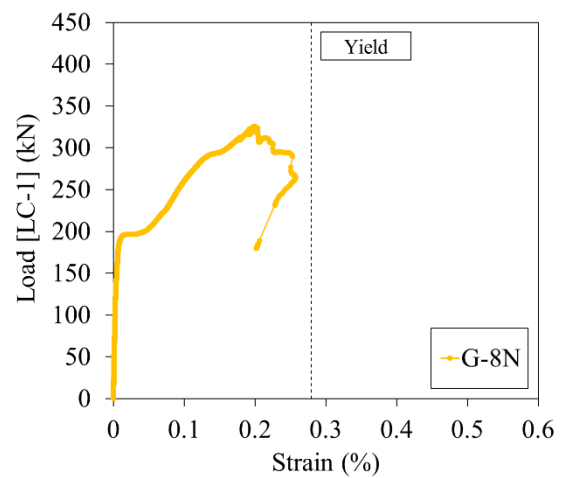
Long. reinforcement in tension at loading point



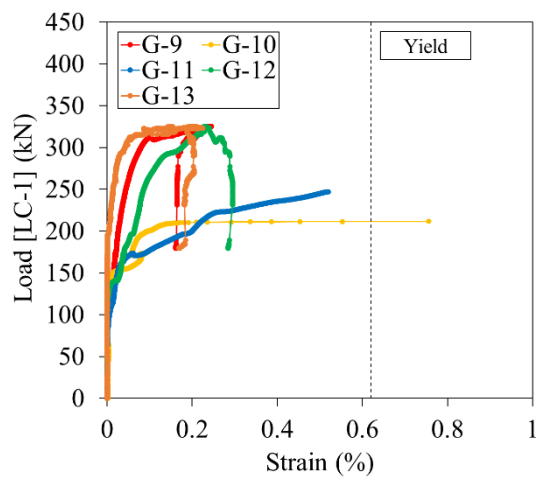
Long. reinforcement in compression at loading point



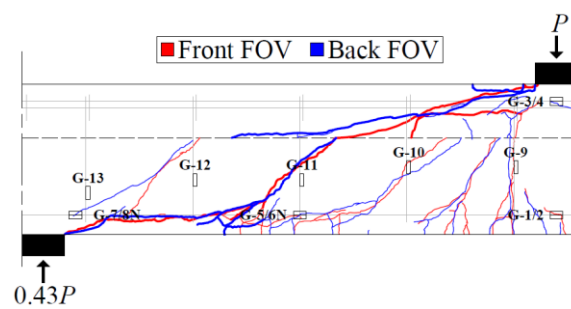
Longitudinal reinforcement at midspan



Longitudinal reinforcement near support

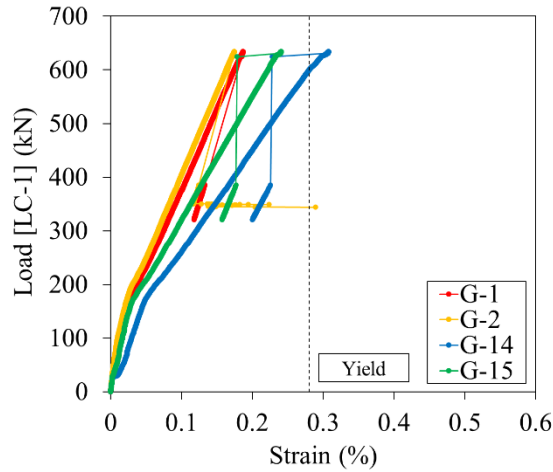


Transverse reinforcement

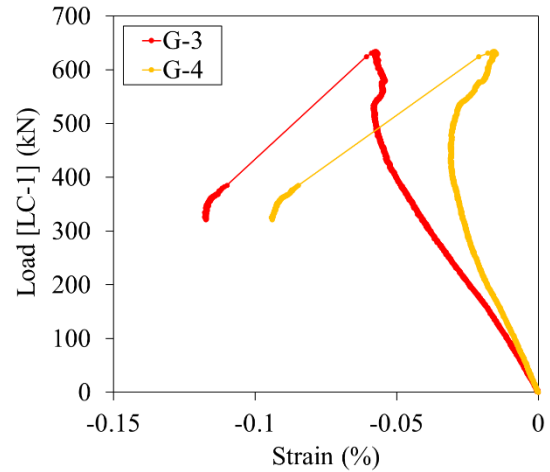


Gauge locations in relation to crack pattern at failure

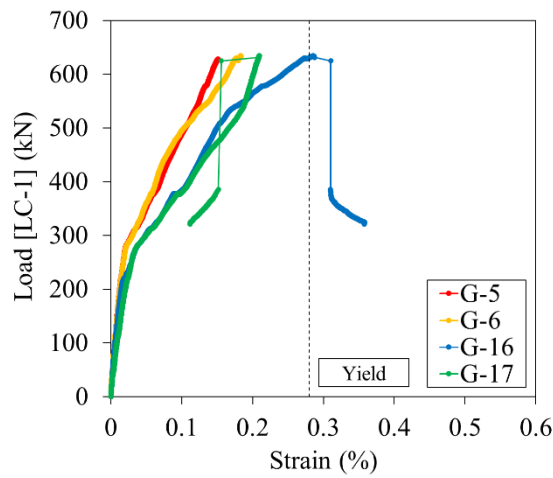
Figure 4.30 – Strain in steel reinforcement of specimen B5



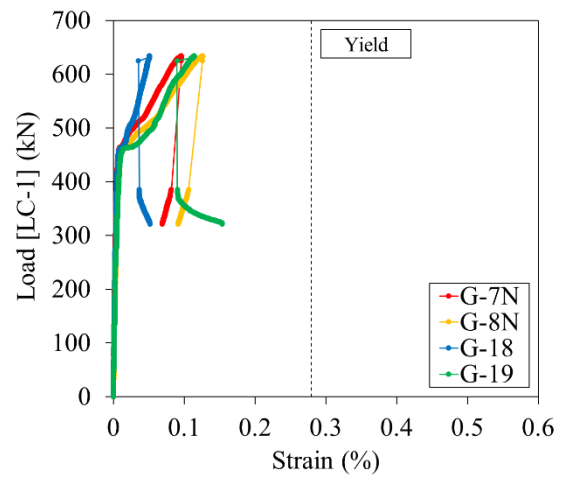
Long. reinforcement in tension at loading point



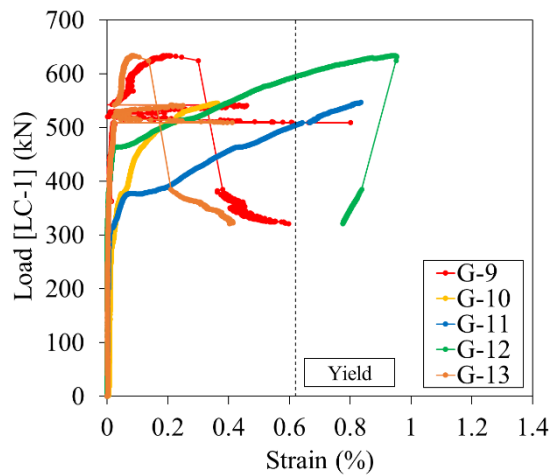
Long. reinforcement in compression at loading point



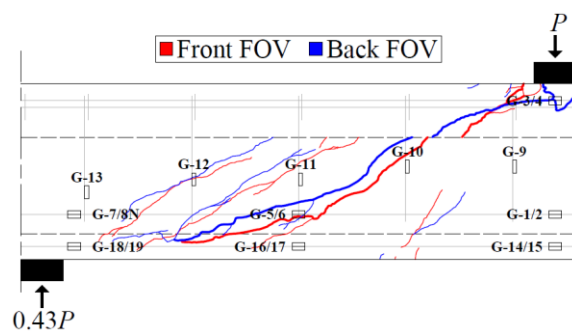
Longitudinal reinforcement at midspan



Longitudinal reinforcement near support

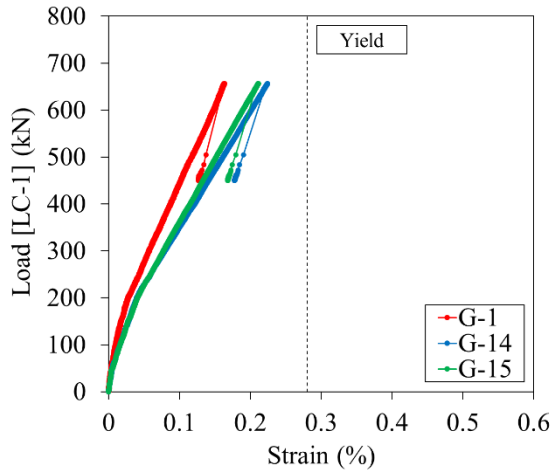


Transverse reinforcement

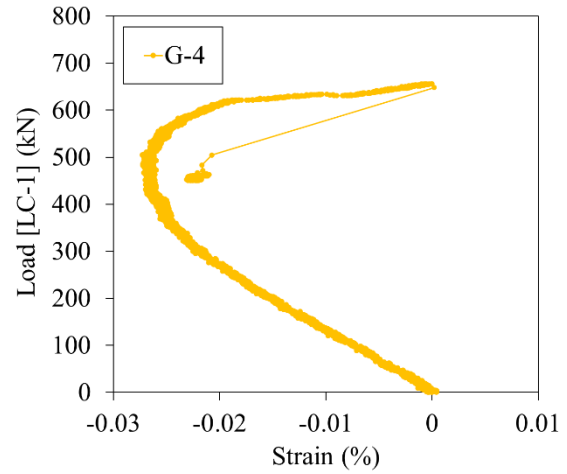


Gauge locations in relation to crack pattern at failure

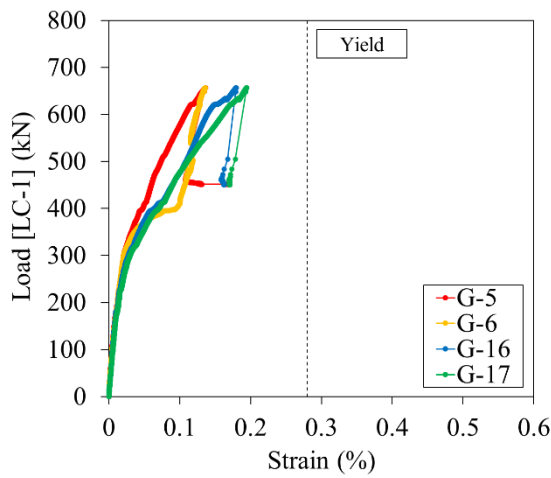
Figure 4.31 – Strain in steel reinforcement of specimen B6



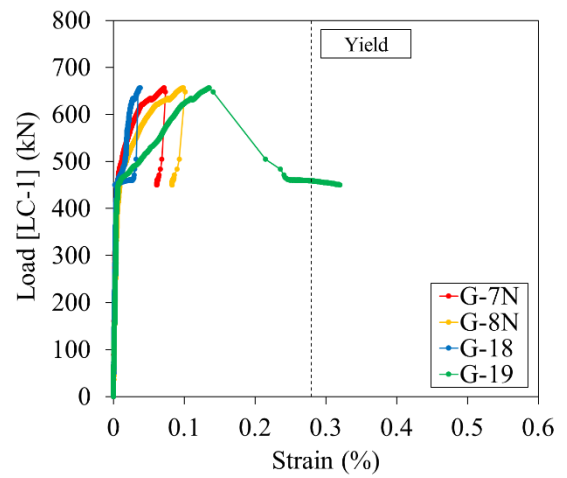
Long. reinforcement in tension at loading point



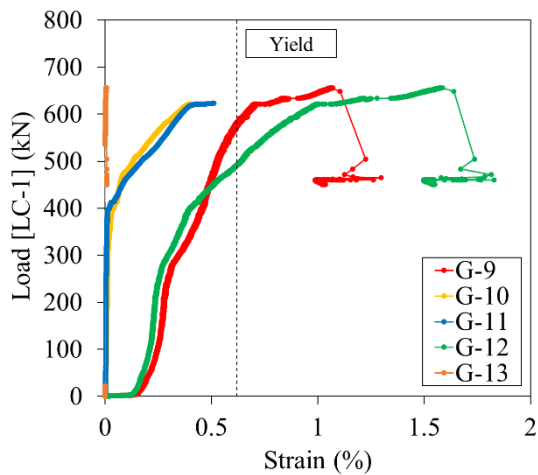
Long. reinforcement in compression at loading point



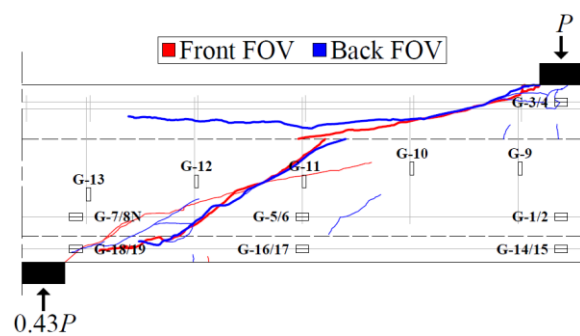
Longitudinal reinforcement at midspan



Longitudinal reinforcement near support

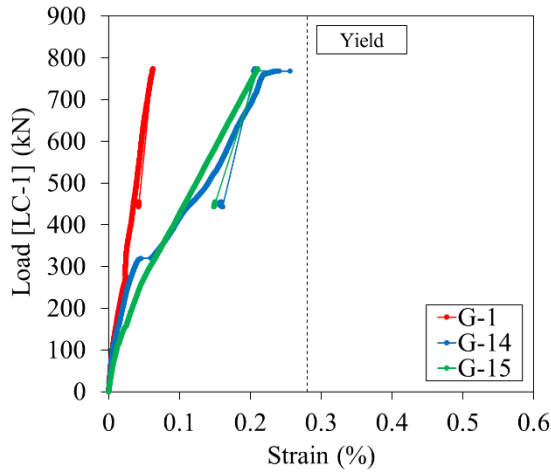


Transverse reinforcement

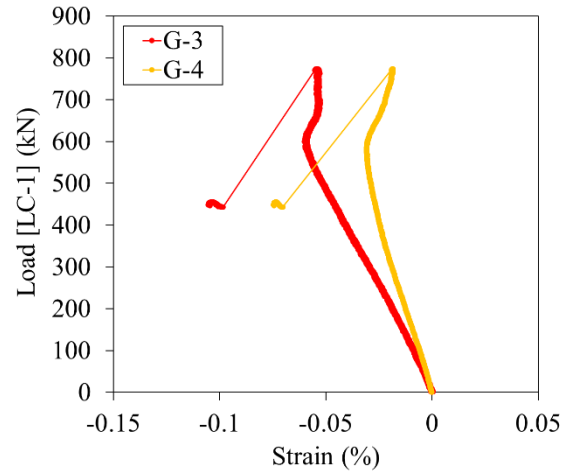


Gauge locations in relation to crack pattern at failure

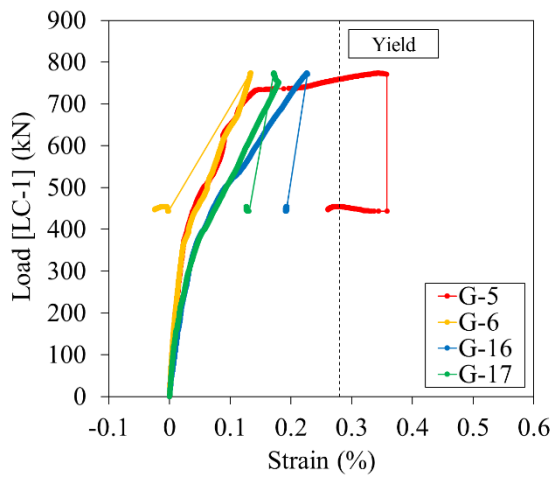
Figure 4.32 – Strain in steel reinforcement of specimen B7



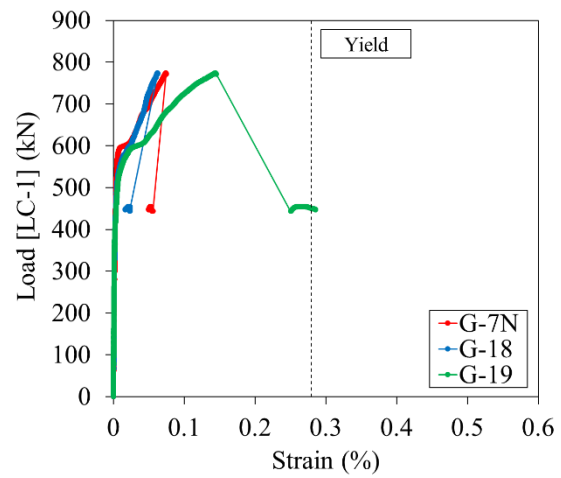
Long. reinforcement in tension at loading point



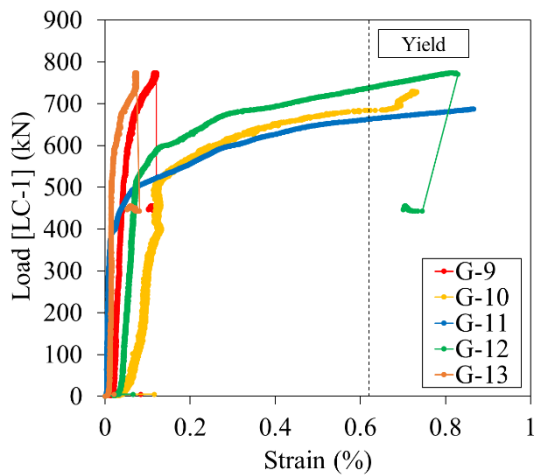
Long. reinforcement in compression at loading point



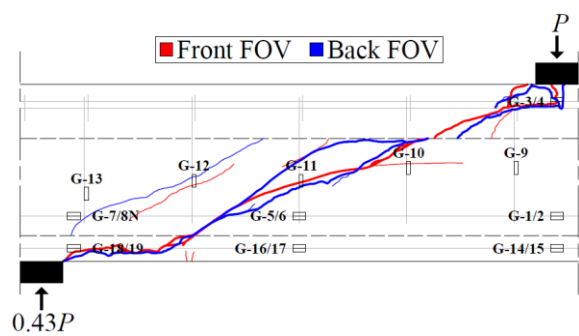
Longitudinal reinforcement at midspan



Longitudinal reinforcement near support



Transverse reinforcement

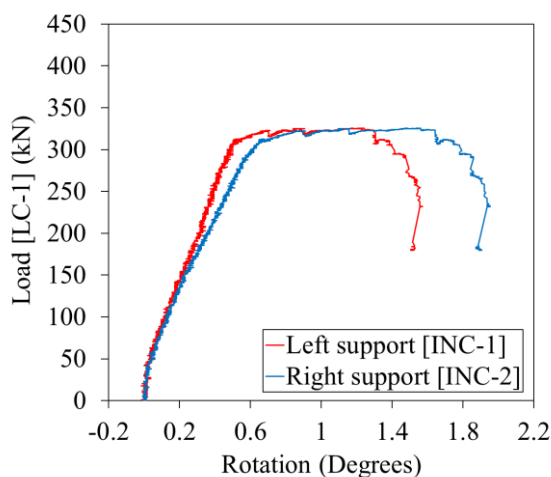


Gauge locations in relation to crack pattern at failure

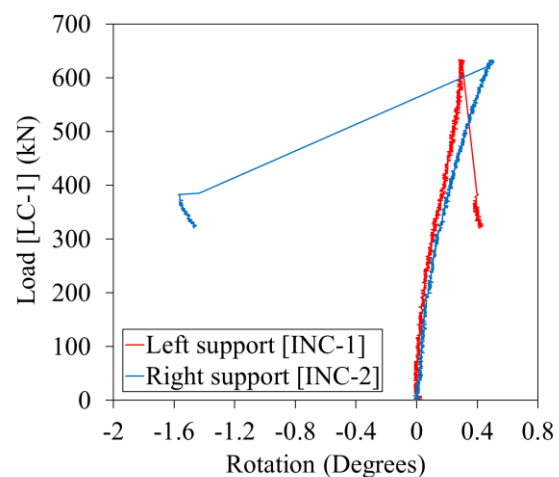
Figure 4.33 – Strain in steel reinforcement of specimen B8

4.5.7 ROTATIONS AND VERTICAL DISPLACEMENTS

The rotations and vertical displacements at the left and right end of the beams of group 2 with respect to Figure 3.37 are shown in Figures 4.34 and 4.35, respectively. As described in Section 4.4.7, the rotations were measured by inclinometers positioned at the centrelines of the left (INC-1) and right (INC-2) supports. The vertical displacements were measured by the LVDTs positioned on the top of the beam at the centrelines of the left (LVDT-1) and right (LVDT-3) supports. The rotation response of control specimen B5 in Figure 4.34 is notably different than that of the strengthened specimens due the differences between the failure modes and load-displacement responses. In control specimen B5, the clockwise and anticlockwise rotations of the respective left and right ends of each specimen are similar throughout the experiment until failure. Much like the specimens of group 1, the clockwise and anticlockwise rotations of the respective left and right ends of each strengthened beam of group 2 are similar until around load step $0.5P_u$. Subsequently, the formation of the shear cracks caused the rotational direction of the left end to switch from clockwise to anticlockwise. The right end of the specimens continued to rotate anticlockwise throughout the experiment. Much like the specimens of group 1, the vertical displacements of the right end appear to be larger than those of the left end overall for almost all specimens of group 2 as shown in Figure 4.35. This is thought to be due to the greater rotation of the shorter right hand span of each specimen or a greater reaction at the right support.



B5



B6

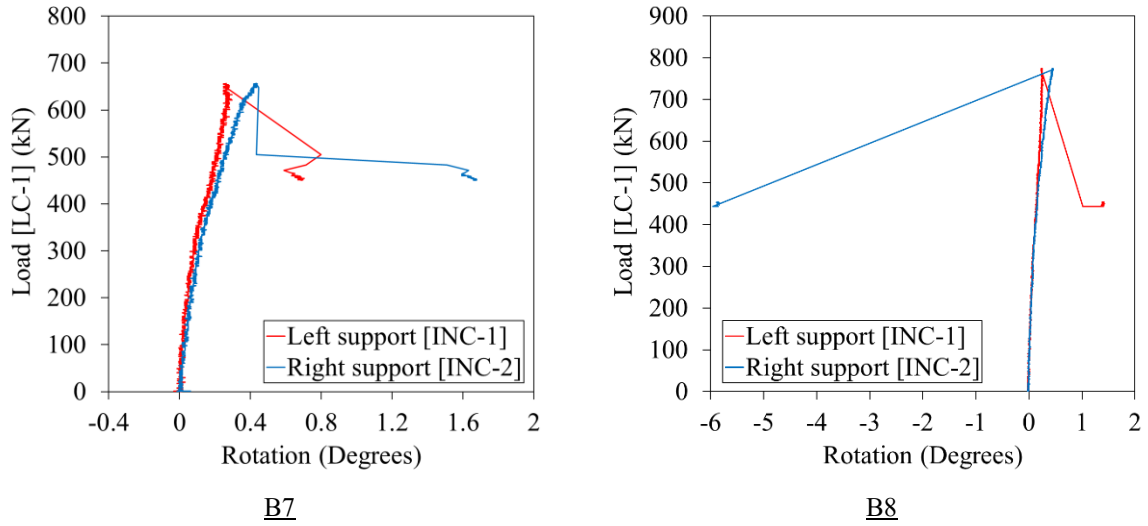


Figure 4.34 – Rotation response of group 2 specimens

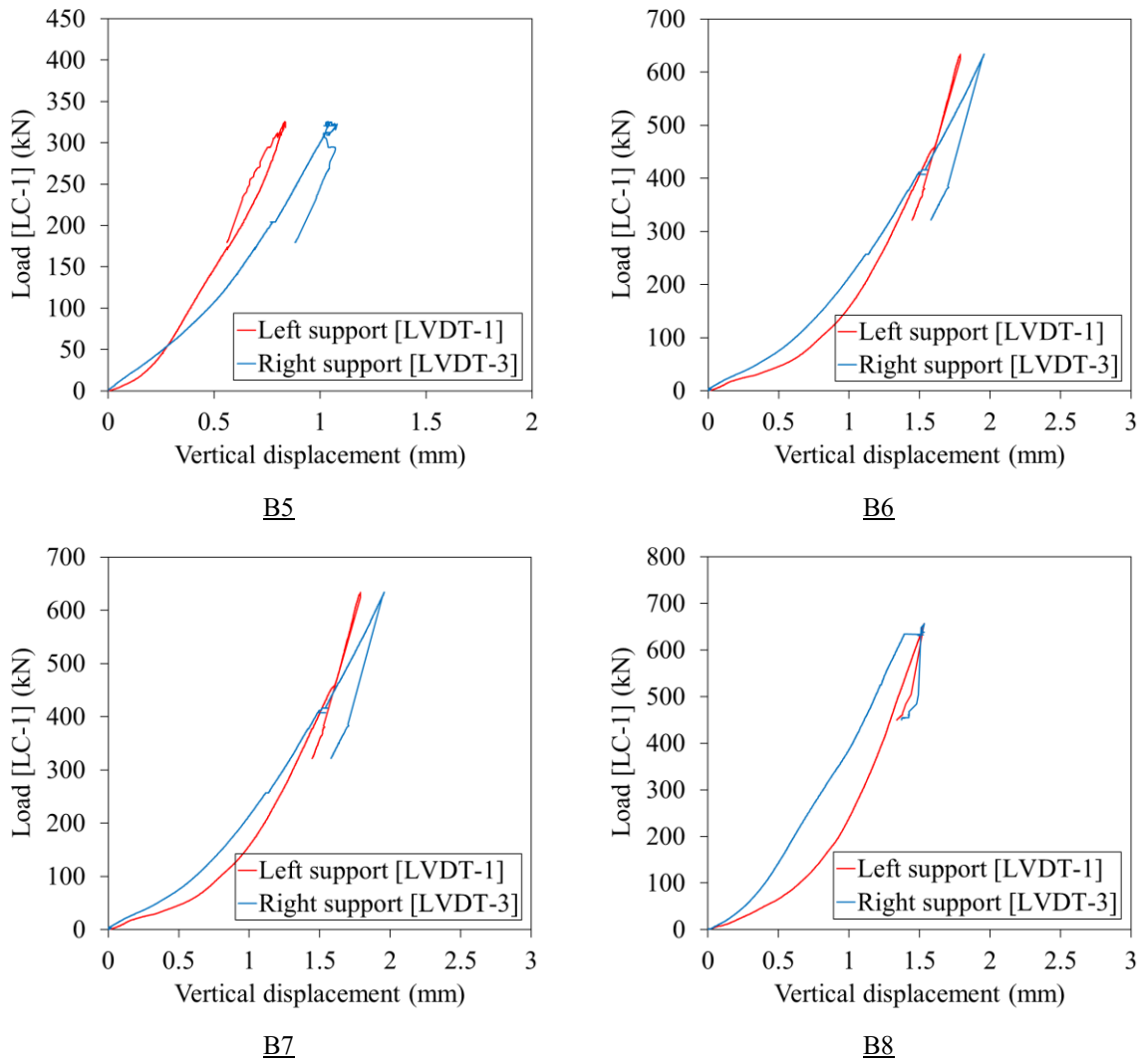


Figure 4.35 – Vertical displacement response of group 2 specimens

4.6 CRITICAL SHEAR CRACK DEPTH AND PARTIAL JACKET REMOVAL

4.6.1 INTRODUCTION

The depths of the critical shear cracks in the UHPC jackets were investigated to determine whether the cracks penetrated the RC beams. If so, it would suggest that the UHPC jackets and RC beams were well bonded and acting monolithically throughout the experiment. The depths of the critical shear cracks on both FOVs of each specimen were initially measured using a thin steel probe with dimensions of 150 x 4 x 1 mm. The measurement probe was inserted by hand with the aid of pliers into points along the length of each critical shear crack, and the depth was recorded for each point along with the height of the point in relation to the bottom surface of the specimen. Figure 4.36 shows the measurement probe that was used along with the measurement points along the critical shear crack of a strengthened specimen. The limitation of this approach is that the depth measurements are subject to errors as the probe can flex both vertically and horizontally within each crack. Therefore, the jackets of selected strengthened specimens were partially removed using an electric breaker by DeWalt (D25899K) to expose the beam surface. The exposed beam surface was visually inspected, and photographs were taken from the same position before and after the jacket removal to help determine whether the critical shear crack on the jacket passed through to the beam. Figure 4.37 shows the jacket removal process.



Measurement probe



Measurement points on critical shear crack (Specimen B6 – Front FOV)

Figure 4.36 – Critical shear crack depth measurement procedure



Initial phase of jacket removal

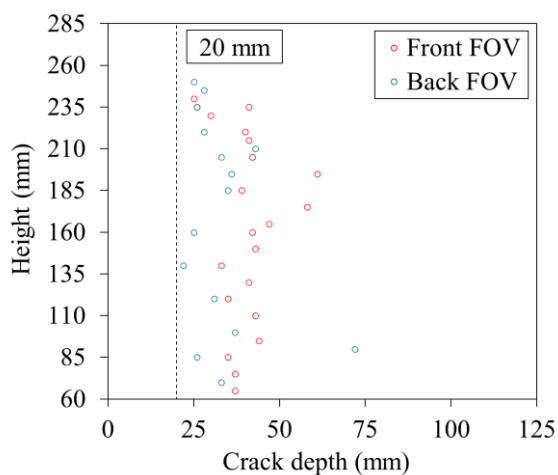
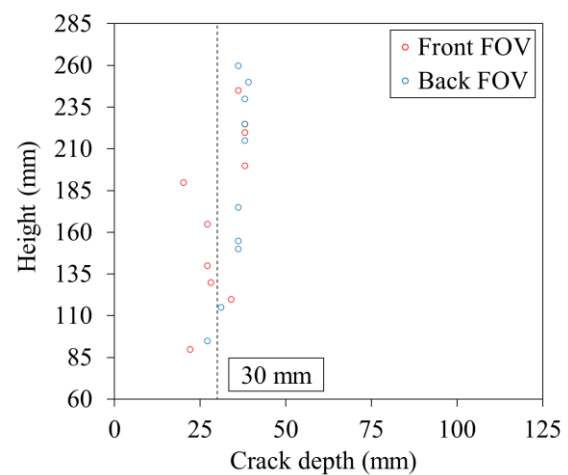


Intermediate phase of jacket removal

Figure 4.37 – Partial UHPC jacket removal using electric breaker (Specimen B6 – Front FOV)

4.6.2 CRITICAL SHEAR CRACK DEPTH MEASUREMENTS

The results of the critical shear crack depth measurements are shown in Figure 4.38 for all strengthened specimens. The depth measurements appear to exceed the depth of the jacket side thickness for all specimens. The critical shear crack depth measurements reach up to approximately 125 mm as in specimens B6 and B8. For specimen B3, there were several measurement points where the depth of the critical shear crack was less than the depth of jacket thickness. This is likely due to the inability of the probe to pass the rough grooves within the crack. In conclusion, the crack depth measurements shown in Figure 4.38 indicate that the cracks on the surfaces of the jackets penetrated the beams.

B2B3

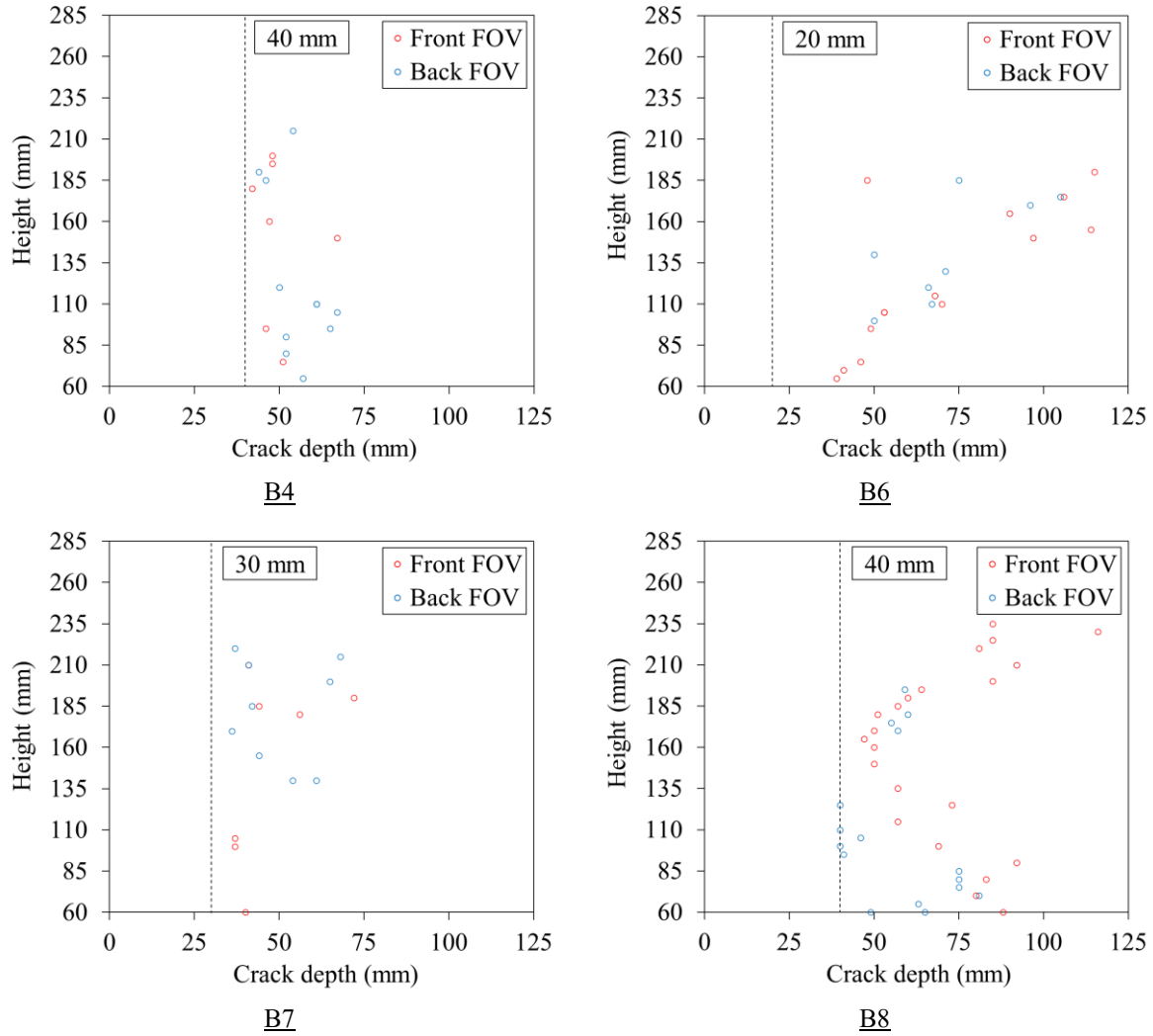
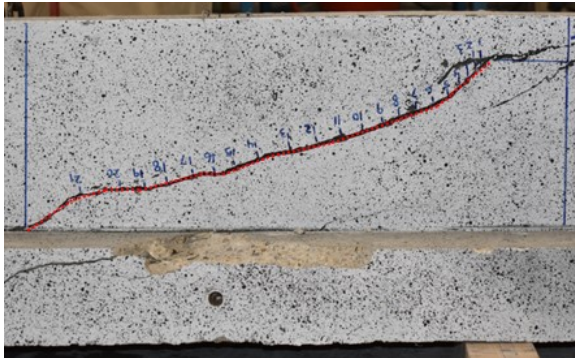


Figure 4.38 – Crack depth measurements of strengthened specimens of groups 1 and 2

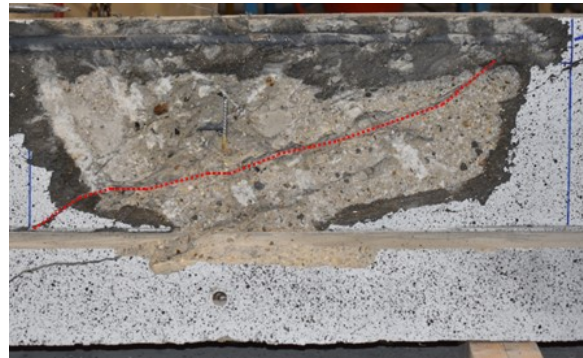
4.6.3 PARTIAL JACKET REMOVAL

Photographs taken before and after the partial UHPC jacket removal are shown in Figure 4.39 for the specimens with the thinnest and thickest jackets of groups 1 and 2. These specimens include specimens B2 (Front FOV), B4 (Front FOV), B6 (Front FOV) and B8 (Front and back FOV), with each specimen laid inverted in the photograph. The shapes of the critical shear cracks were highlighted in the photographs taken before and after jacket removal to indicate the locations of the critical shear cracks on the exposed beams. Figure 4.39 shows that the critical shear cracks in the jackets penetrated the beams in all specimens. It is also evident that there are shear cracks in the beam that do not fully appear in the jacket as in the front FOV of specimen B8. Interestingly, a segment of the jacket for the front FOV of specimen B6 that is located between the critical shear crack and the secondary shear crack was removed relatively

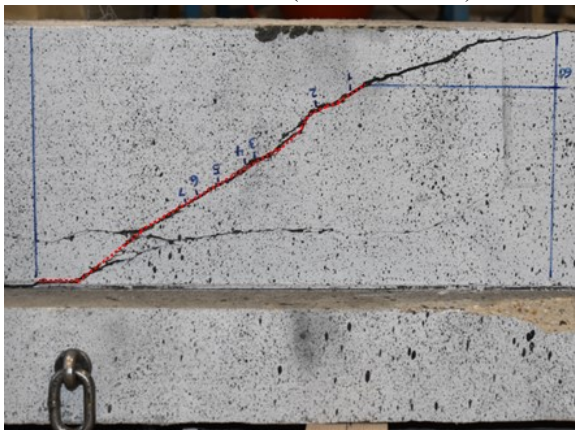
easily as shown in Figure 4.40. It appears that this segment may have been protruding outwards as the surrounding shear cracks were developing, which seems to have led to the minor separation of the jacket although the NSC of the beam has also broken away with the jacket indicating a good bond. Overall, it can be concluded that the critical shear cracks on the jackets pass through to the beams, indicating that the UHPC jacket and RC beam were acting monolithically throughout the experiment until failure.



B2 – Front FOV (Before removal)



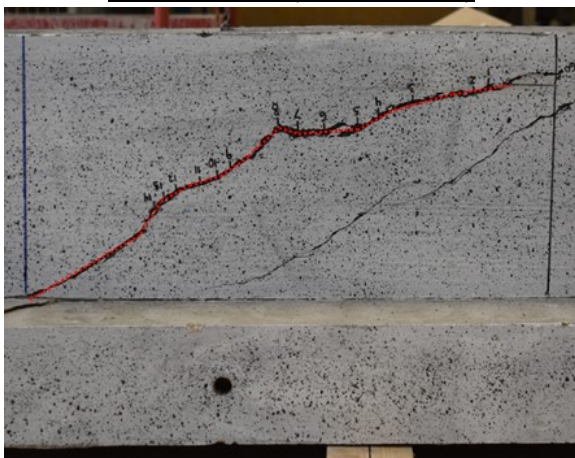
B2 – Front FOV (After removal)



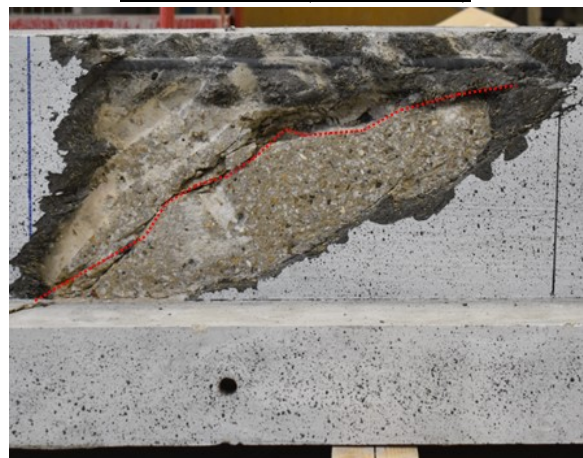
B4 – Front FOV (Before removal)



B4 – Front FOV (After removal)



B6 – Front FOV (Before removal)



B6 – Front FOV (After removal)

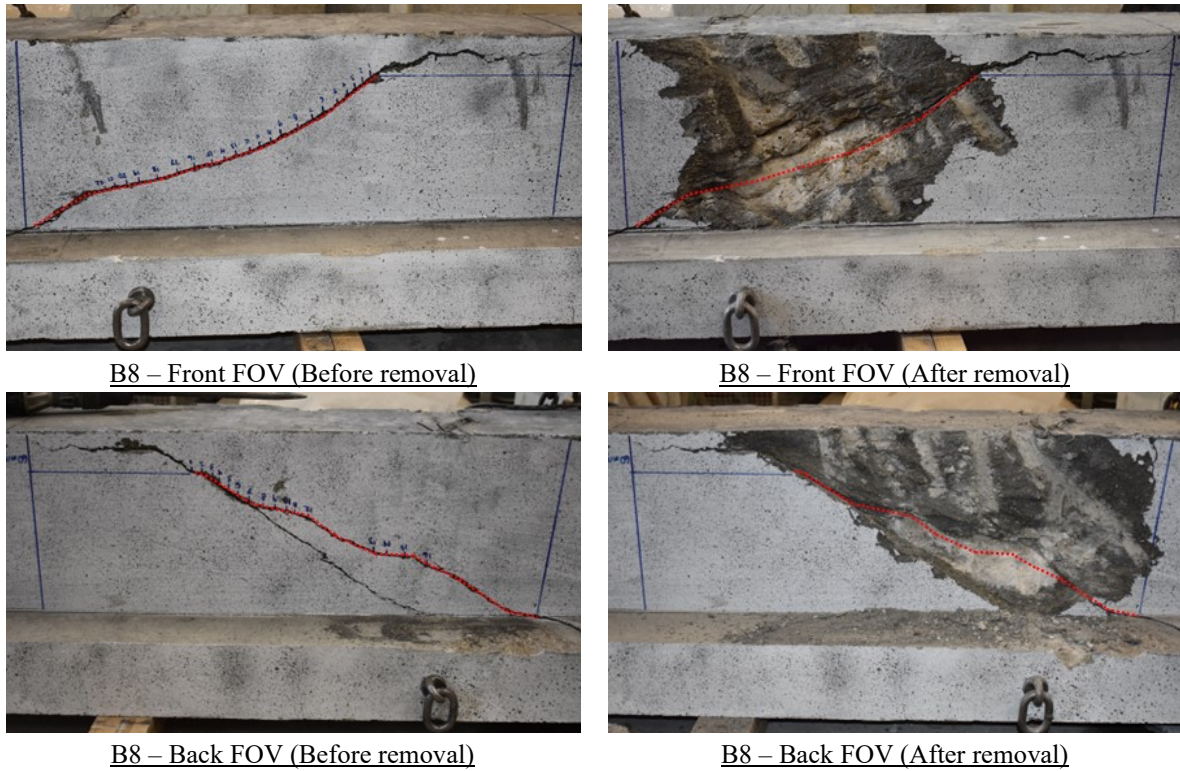


Figure 4.39 – Photographs of before and after partial UHPC jacket removal for strengthened specimens of groups 1 and 2



Figure 4.40 – Partial removal of UHPC jacket of specimen B6 (Front FOV)

4.7 CONCLUSIONS

This chapter presents and discusses the experimental results from this campaign. The beams were tested in two groups of four with each group consisting of one control RC beam and three RC beams strengthened with UHPC jackets. The materials properties of the reinforcement, NSC and UHPC are presented at the start of the chapter. The interfacial bond between the NSC and UHPC was characterised through a series of splitting tensile tests on composite cylinder specimens. The relatively high composite splitting tensile strength results and the combined

failure modes, which occurred along the interfacial bond and the NSC substrate, indicate that the interfacial bond between the NSC and UHPC was at least as strong as the NSC. Test results including a description of crack development, modes of failure, load-displacement responses and reinforcement strains are presented for each beam. The experimental results and main characteristics of each specimen are summarized in Tables 4.12 and 4.14 for the specimens of groups 1 and 2, respectively. After the tests, the depths of the critical shear cracks were measured and a few jackets were partially removed, and it was concluded that the critical shear cracks on the jackets passed through to the beams, which indicates monolithic performance.

The RC beams in group 1 were all reinforced with 3.34% flexural reinforcement. The critical shear span was reinforced with 6 mm diameter stainless steel stirrups at 250 mm spacing. Specimens B2, B3 and B4 were strengthened with UHPC jackets with side thickness of 20 mm, 30 mm and 40 mm and a bottom thickness of 60 mm. All the jackets were reinforced with 4 ϕ 20 longitudinal reinforcement bars. All of the specimens in group 1 failed in shear. The main conclusions from the group 1 tests are as follows:

- Jacketing the beams increased the ultimate load carrying capacity of specimen B2 with a 20 mm thick jacket by 57.16% (296.83 kN) and specimen B4 with a 40 mm thick jacket by 74.76% (379.86 kN).
- The development of the critical and secondary shear cracks varied in the front and back faces of the UHPC jacket throughout the test. However, the critical and secondary shear cracks mostly overlapped at the ultimate load.
- The critical shear cracks formed near the centre of the critical span in all beams except specimen B3 where the crack formed much closer to the support.
- The critical shear crack was steepest at 34° in control specimen B1. The crack inclination varied in the strengthened beams. The flattest crack inclination angle of 19° was in specimen B2. The critical shear crack inclination angles were equal to 32° and 33° for the front and back FOVs of specimen B3, respectively. Due to the differences in crack development in both faces of the jacket of specimen B4, the critical shear crack inclination angles were equal to 31° and 21° for the front and back FOVs, respectively.

Group 2 consisted of a control RC beam (B5) with 1.55% flexural reinforcement. The critical shear span was reinforced with 6 mm diameter stainless steel stirrups at 200 mm spacing. The

three identical RC beams strengthened with UHPC jackets (B6, B7, and B8) had side thicknesses of 20 mm, 30 mm and 40 mm and a bottom thickness of 60 mm. The longitudinal reinforcement in the bottom face of the jacket was $3\phi 20$ in specimen B6, $4\phi 20$ in specimen B7 and $5\phi 20$ in specimen B8. Control specimen B5 failed in combined flexure-shear while the jacketed specimens of group 2 all failed in shear. The main conclusions of the experimental results of the specimens of group 2 are as follows:

- The ultimate load carrying capacity increased by a minimum and maximum of 96.06% (308.32 kN) and 136.78% (448.66 kN) as a result of the thinnest (20 mm) and thickest (40 mm) UHPC jackets, respectively.
- Much like the specimens of group 1, the development of the critical and secondary shear cracks varied between the front and back faces of the UHPC jacket. However, the critical shear cracks and secondary shear cracks mostly overlapped at the ultimate load and failure.
- The critical shear cracks of all the specimens of group 2 formed around the middle of the critical span.
- The steepest critical shear crack inclination angle was equal to 38° for control specimen B5. Much like the strengthened specimens of group 1, the crack inclination angles varied for the strengthened specimens of group 2. The flattest inclination angles were equal to 21° and 20° for the critical shear cracks in the front and back FOVs of specimen B8, respectively. The crack inclination angles were equal to 29° and 26° for the critical shear cracks in the front and back faces of specimen B6, respectively. The inclination angles were equal to 30° and 29° for the critical shear cracks in the front and back FOVs of specimen B7, respectively.

CHAPTER 5: CRACK KINEMATICS AND SHEAR TRANSFER ACTIONS

5.1 INTRODUCTION

This chapter utilises the experimentally determined crack kinematics to assess the contribution of each shear resisting mechanism to the overall shear resistance of the tested beams. The contribution of each shear resisting mechanism is evaluated along the critical shear crack, in terms of the crack kinematics, using various analytical models from the literature. The shear resistance mechanisms that were considered include aggregate interlock (V_{agg}), fibre bridging (V_{fib}), dowel action (V_{dow}), stirrups (V_{stir}) and the uncracked compression zone (V_{cz}). The assessments helped quantify the influence of the UHPC jackets on the shear resistance mechanisms, and they were essential to evaluating the design code methods for predicting the ultimate shear strength of RC beams with UHPC jackets in Chapter 7.

5.2 CRACK KINEMATICS OF CRITICAL SHEAR CRACKS

5.2.1 INTRODUCTION

The crack opening and sliding displacements were determined along the critical shear crack of each beam using DIC. Typically, both the front and back face of the beams were monitored, from first loading to the end of the test, using the DIC system setup described in Section 3.8. The exceptions were specimens B1 and B2 where only one face was monitored. Depending on the duration of the experiment, the DIC system captured between 3400 to 6000 images at a frequency of 1.0 Hz per FOV for each specimen. Each image identified the targeted concrete surface with a constant x-y coordinate system, and global deformations corresponding to each image were analysed and produced in the software package DaVis. Crossed virtual gauges were placed at intervals along the critical shear crack of each beam to determine relative displacements between opposite sides of the investigated crack. The crack kinematics, or crack opening and sliding displacements, were determined for each DIC image based on the relative displacements measured by the virtual gauges.

Calculations were carried out using an in-house developed computational code that depended on the global deformations obtained from DaVis (Elwakeel, 2019). The adopted procedures

including virtual gauge arrangements and equations used are described in Section 5.2.2. The opening and sliding displacements along the critical shear cracks were determined for all the beams for both the front and back face where images were available. The load steps corresponding to each DIC image were rounded to within two decimal places. The calculated crack kinematics were refined to reduce scatter by averaging the results of the captured DIC images that share the same load step. Figure 5.1 depicts the data refinement procedure for the crack opening and sliding displacements by comparing the full data with the refined data.

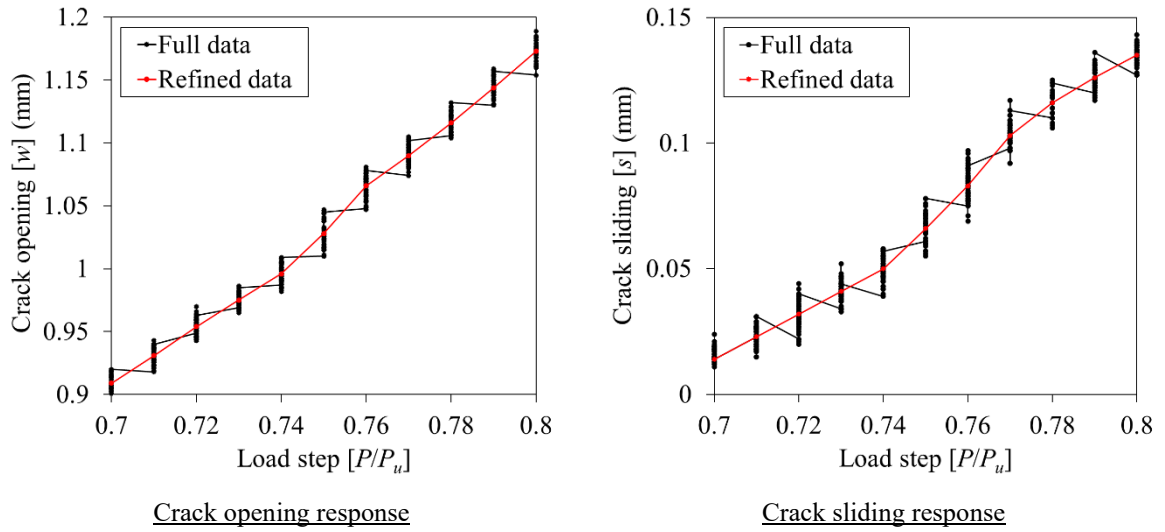


Figure 5.1 – Refinement of calculated crack kinematics for crack division 5 of specimen B1

To ensure the validity and reliability of the calculated crack kinematics, a couple of key aspects were considered. When placing each individual crossed virtual gauge, it was vital that only the critical shear crack was targeted throughout the experiment (Cladera et al., 2016). Therefore, the placed gauges varied in length and orientation to avoid intersecting secondary cracks and image distortions caused by concrete spalling, as well as ensuring gauges lay within the web of the beam. It was also recognized that small displacements between gauge ends may correspond to elastic deformation of uncracked concrete rather than actual fracture (Pastore, 2020). To distinguish between the two possibilities, it was assumed that concrete fracture occurs once a strain of 0.1 mm/m is reached. The resulting crack displacement equals 0.01 mm when measured with the adopted virtual gauge length of 100 mm (Pastore, 2020). Therefore, crack opening and sliding displacements less than 0.01 mm were taken as zero.

5.2.2 METHOD OF CALCULATION OF CRACK KINEMATICS

The crack kinematics were calculated using the approach of Campana et al. (2013) that is widely adopted (Cladera et al., 2016; Elwakeel, 2019; Huber et al., 2016; Pastore, 2020). The concrete on each side of the crack is treated locally as separate rigid bodies located on the left (L) and right (R) sides of the crack with each body behaving independently. The relative displacement between the two bodies ($\bar{u}$) is determined in terms of the displacements of each rigid body parallel to the x-axis (u) and y-axis (v). The relative displacement ($\bar{u}$) was calculated in terms of the movements of two initially coincident points with coordinates ($f_{L/R}(x, y)$) positioned on the crack. The position of the points ($f_{L/R}(x, y)$) was tracked throughout the experiment until the ultimate load was reached. The displacement fields of these two points ($w_{L/R}(x, y)$) were calculated at each load step, and their difference yielded the relative displacement ($\bar{u}$). The displacement fields ($w_{L/R}(x, y)$) were calculated from the coordinates of crossed virtual gauges ($P_1 - P_3$ and $P_2 - P_4$) initially centred at a given point on the investigated crack. From the relative displacement ($\bar{u}$), the shear crack displacement ($\bar{w}$), which refers to the crack opening (w) and sliding (s) displacements, was determined in terms of the inclination angle of the crack (θ). The crack opening (w) and sliding (s) displacements refer to the movements normal and tangential to the crack, respectively.

The method for calculating the crack kinematics is illustrated in Figure 5.2 where an undeformed and a deformed configuration of a cracked concrete surface are shown. The equations for calculating the crack displacements changes for cracks running from the bottom left to the top right and vice versa, corresponding to the front and back FOVs of the tested specimens. The critical shear crack was divided into linear segments at 5 mm intervals along the height of the crack. Crossed virtual gauges were centred on each segment of the crack as shown in Figure 5.2. The crack inclination angle (θ) of each linear segment was chosen when there was a clear deviation in orientation to avoid widely varying angles between adjacent crack segments (Pastore, 2020). Various methods for discretising the crack inclination angle (θ) were investigated by Pastore (2020), and the method adopted in this research was found to give the most realistic crack kinematics. Due to congestion caused by secondary cracks, DIC image distortions caused by concrete spalling or the critical crack nearing the web/flange junction, virtual gauges could not be placed at every crack division. Therefore, the gauges used varied in length (L) and orientation angles ($\theta_{1-3,2-4}$ and θ_{2-4}). A gauge length (L) of 100 mm was

adopted and used where possible based on the findings of Pastore (2020), which suggest the scatter in measured crack displacement is minimal when using such a gauge length or greater. Gauge lengths (L) of 50 mm and 75 mm were also used sparingly as they were necessary at certain crack divisions, despite the findings of Pastore (2020) suggesting that they may produce significant differences in the crack displacements when compared to results obtained with gauge lengths (L) of 100 mm or greater. The orientation angle ($\theta_{1-3,2-4}$) used to rotate the horizontal ($P_1 - P_3$) and vertical ($P_2 - P_4$) gauges together varied from 285° to 345° in 15° intervals for the front FOV calculations, and it varied from 15° to 75° in 15° intervals for the back FOV calculations. The orientation angle (θ_{2-4}) that only rotated the vertical gauge ($P_2 - P_4$) varied from 15° to 75° in 15° intervals. Virtual gauges that were not rotated were used where possible, however rotated gauges were often necessary to avoid intersecting secondary cracks or distorted DIC images caused by concrete spalling. The findings of Pastore (2020) suggest that varying the gauge orientation angles has less of an effect in terms of scatter in results in comparison to varying the gauge lengths.

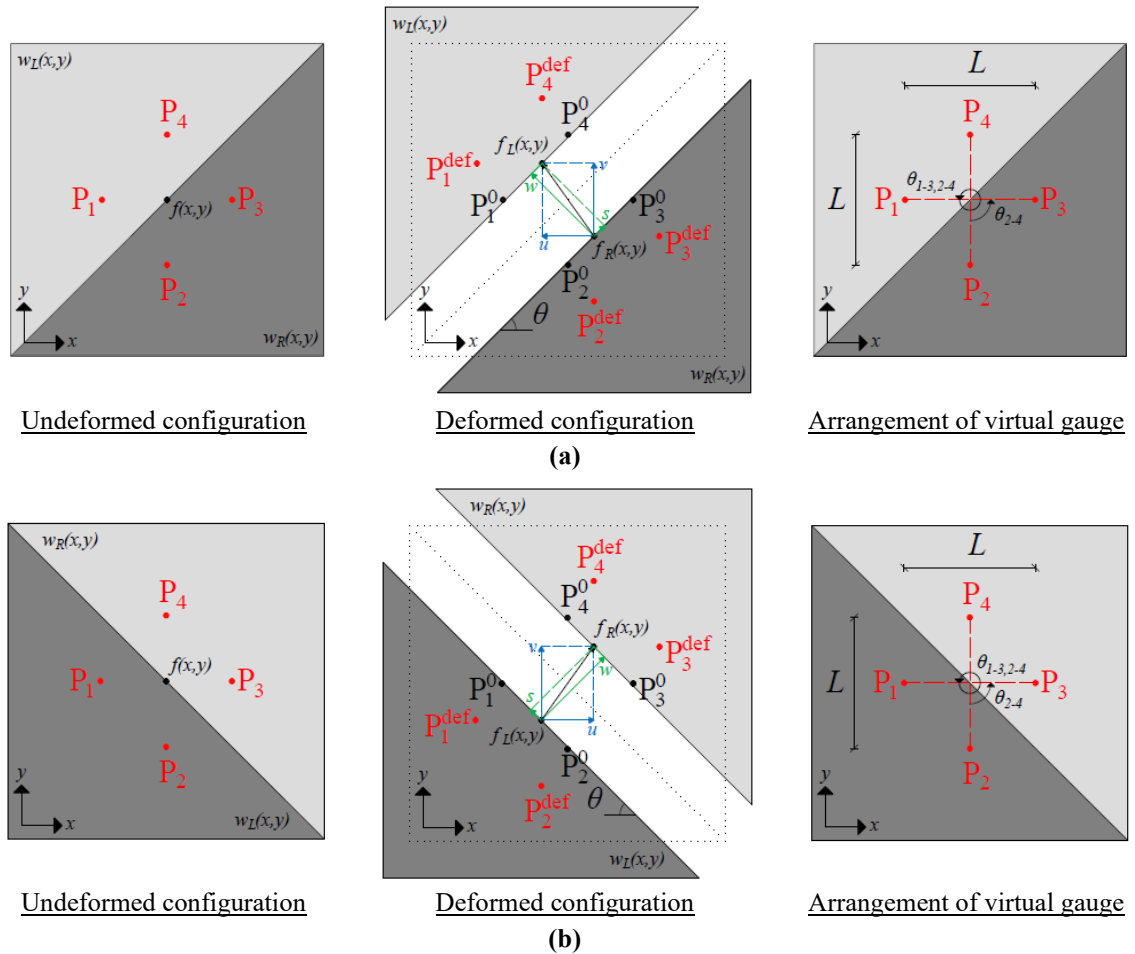


Figure 5.2 – Calculation of crack kinematics and virtual gauge arrangement: (a) Front FOV, (b) Back FOV

The following equations were used to calculate the displacement fields ($w_{L/R}(x, y)$) and coefficients responsible for translation and rotation ($r_{L/R}$):

$$w_{L/R}(x, y) = \begin{cases} w_{x_{L/R}} = a_{L/R} - e_{L/R} \cdot y \\ w_{y_{L/R}} = b_{L/R} + e_{L/R} \cdot x \end{cases} \quad (5.1)$$

$$r_{L/R} = \begin{bmatrix} a_{L/R} \\ b_{L/R} \\ e_{L/R} \end{bmatrix} = (A_{L/R}^T \cdot A_{L/R})^{-1} \cdot (A_{L/R}^T \cdot c_{L/R}) \quad (5.2)$$

where $a_{L/R}$ and $b_{L/R}$ are coefficients responsible for translation and $e_{L/R}$ is a coefficient responsible for rotation. For the front FOV calculations where the shear crack stems from bottom left to top right, the coefficients $A_{L/R}$ and $c_{L/R}$ were calculated as follows:

$$A_{L/R} = \begin{bmatrix} 1 & 0 & -Y_{P_1/P_2} \\ 0 & 1 & X_{P_1/P_2} \\ 1 & 0 & -Y_{P_4/P_3} \\ 0 & 1 & X_{P_4/P_3} \end{bmatrix} \quad (5.3)$$

$$c_{L/R} = \begin{bmatrix} X_{P_1/P_2}^{def} - X_{P_1/P_2}^0 \\ Y_{P_1/P_2}^{def} - Y_{P_1/P_2}^0 \\ X_{P_4/P_3}^{def} - X_{P_4/P_3}^0 \\ Y_{P_4/P_3}^{def} - Y_{P_4/P_3}^0 \end{bmatrix} \quad (5.4)$$

For the back FOV calculations where the shear crack stems from bottom right to top left, the coefficients $A_{L/R}$ and $c_{L/R}$ were calculated as follows:

$$A_{L/R} = \begin{bmatrix} 1 & 0 & -Y_{P_2/P_3} \\ 0 & 1 & X_{P_2/P_3} \\ 1 & 0 & -Y_{P_1/P_4} \\ 0 & 1 & X_{P_1/P_4} \end{bmatrix} \quad (5.5)$$

$$c_{L/R} = \begin{bmatrix} X_{P_2/P_3}^{def} - X_{P_2/P_3}^0 \\ Y_{P_2/P_3}^{def} - Y_{P_2/P_3}^0 \\ X_{P_1/P_4}^{def} - X_{P_1/P_4}^0 \\ Y_{P_1/P_4}^{def} - Y_{P_1/P_4}^0 \end{bmatrix} \quad (5.6)$$

The relative displacement ($\bar{u}$) and shear crack displacement ($\bar{w}$) were then ultimately calculated using the following equations for the front FOV:

$$\bar{u} = \begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} w_{x_L} - w_{x_R} \\ w_{y_L} - w_{y_R} \end{bmatrix} \quad (5.7)$$

$$\bar{w} = \begin{bmatrix} s \\ w \end{bmatrix} = \begin{bmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix} \cdot \bar{u} \quad (5.8)$$

For the back FOV calculations, the following equations were used to determine the relative displacement ($\bar{u}$) and shear crack displacement ($\bar{w}$):

$$\bar{u} = \begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} w_{x_R} - w_{x_L} \\ w_{y_R} - w_{y_L} \end{bmatrix} \quad (5.9)$$

$$\bar{w} = \begin{bmatrix} s \\ w \end{bmatrix} = \begin{bmatrix} -\sin \theta & \cos \theta \\ \cos \theta & \sin \theta \end{bmatrix} \cdot \bar{u} \quad (5.10)$$

5.2.3 RESULTS OF CRACK KINETMATICS OF CRITICAL SHEAR CRACKS

5.2.3.1 INTRODUCTION

The calculated crack kinematics of the critical shear cracks of each specimen include both front and back FOVs where available. The results are presented in Sections 5.2.3.2 and 5.2.3.3 for groups 1 and 2, respectively. The results include a vector plot of the kinematics of the critical shear crack showing the resultant crack displacement ($\bar{w}$) at each crack division beginning from the load steps $0.6P_u$ or $0.7P_u$ until the ultimate load (P_u). The crack kinematics plots also show the crack pattern at the ultimate load as well as the free-body diagram used to determine the internal shear forces discussed in Section 5.3. There are relatively large gaps that appear between crack divisions for some specimens, for example between divisions 9 and 10 of

specimen B1 in Figure 5.3. This is a result of not being able to place virtual gauges at every 5 mm interval along the height of the crack for the reasons described in Section 5.2.2. The results of each specimen also include the crack opening (w) and sliding (s) displacement responses throughout loading for all crack divisions in addition to the crack opening to sliding ratios (w/s) at specified crack divisions.

5.2.3.2 GROUP 1 SPECIMENS

The crack kinematics plot for the front FOV of control specimen B1 in Figure 5.3 indicates a fairly smooth crack profile of parabolic shape. The critical shear crack at the ultimate load extends from a delamination crack located between the two layers of longitudinal reinforcement. The crack progresses upward towards the top layer of longitudinal reinforcement at approximately 16° to the horizontal. The crack orientation increases to 27° from the top layer of longitudinal reinforcement to the mid-height of the web. From there, the crack goes on to cross a single stirrup at an angle of 45° prior to ending at the web/flange junction. The greatest crack openings occur in the region between the top layer of longitudinal reinforcement (Division 5) and mid-height of the web (Division 9). The maximum crack opening was 2.2 mm at the ultimate load. The non-continuous linear crack sliding response shows minimal crack sliding in the region between the two layers of longitudinal reinforcement. Crack sliding becomes significantly greater above the top layer of longitudinal reinforcement (Division 4) onward, with the maximum crack sliding at the ultimate load reaching 1.4 mm near the web/flange junction. The relatively low crack opening to sliding ratios (w/s) caused by the excessive sliding shown for division 10 indicate significant aggregate interlock action in the steeper region of the crack. The ratios (w/s) are much higher in the lower crack divisions due to less crack sliding.

The crack kinematics plot for the front FOV of specimen B2 shown in Figure 5.4 reveals a relatively smooth crack profile of parabolic shape. The critical shear crack at the ultimate load begins just above the longitudinal reinforcement in the jacket and progresses towards the top layer of longitudinal reinforcement at an inclination angle of 42° . From there, the crack crosses three stirrups in the web of the beam at an approximate inclination angle of 15° prior to reaching the web/flange junction at an increased angle of 33° . Interestingly, the crack opening and sliding displacements significantly increase between the load steps $0.8P_u$ to $0.9P_u$ and $0.9P_u$ to

the ultimate load, which suggests that the fibre bridging action is reducing as the fibres are beginning to be pulled out. The greatest crack openings occurred in the region between the longitudinal reinforcement in the beam (Division 10) and the flange (Division 35), with the maximum crack opening reaching 4.9 mm at the ultimate load. The greatest crack sliding occurred between the lower crack end (Division 1) and the top layer of longitudinal reinforcement in the beam (Division 16), and there is almost no sliding beyond this region. The crack sliding reached a maximum of 2.0 mm at the ultimate load. For the region of the crack where the greatest sliding occurs, the crack opening to sliding ratios (w/s) significantly decrease during the latter stages of loading as shown for divisions 2 and 13 in Figure 5.4.

Similarities, but also differences, are apparent in the crack displacements for the front and back FOVs of specimen B3 shown in Figure 5.5. The critical shear cracks at the ultimate load featured on both FOVs overlap. They both begin from the longitudinal reinforcement in the jacket much like specimen B2. The cracks progress in both FOVs towards the web/flange junction at an approximate inclination angle of 30° while also crossing a single stirrup. The crack kinematics plots of both FOVs show similar crack widths and crack profiles overall, although the profile of the crack in the back FOV is marginally rougher. The differences in crack roughness between both FOVs can be seen in the crack opening responses, where the responses are comparatively much smoother in the front FOV. More spikes are visible in the back FOV responses between adjacent crack segments, which highlight the deviations in crack orientations caused by the rougher crack profile. The greatest crack openings occurred in the uppermost region of the crack near the flange with the maximum openings reaching 3.6 mm and 3.4 mm at the ultimate load for the front and back FOVs, respectively. The crack sliding responses of both FOVs are non-continuous linear in shape, however the crack sliding for the back FOV is about two times greater than that for the front FOV overall throughout the experiment. The ratios (w/s) of the back FOV are almost constantly equal to 1.0 throughout the latter stages of loading while the ratios of the front FOV are generally greater than 1.0 as shown in Figure 5.5 for overlapping front and back FOV crack divisions.

Much like specimen B3, similarities and differences were observed between the crack kinematics of the front and back FOVs of specimen B4 shown in Figure 5.6. The critical shear cracks at the ultimate load on both FOVs overlap at the level of the longitudinal reinforcement in the jacket and progress at an approximate inclination angle of 26° to the bottom layer of

longitudinal reinforcement in the beam. From this point, the crack in the front FOV progresses to the web/flange junction at a steep angle of 35° while the crack in the back FOV only progresses to the mid-height of the web at a flatter angle of 18° . Due to these differences in crack development, the cracks in the front and back FOVs both cross the third stirrup to the right of the support, while the crack in the front FOV also just crosses the fourth stirrup to the right of the support. The crack widths are significantly smaller in the back FOV, which also has a comparatively more jagged crack profile than the front FOV. The greatest crack opening and sliding displacements occurred near the top layer of longitudinal reinforcement with maximum crack openings reaching 2.9 mm and 1.1 mm at the ultimate load for the front and back FOVs, respectively. The crack sliding responses of both FOVs are piecewise linear, however the crack sliding is more prevalent in the back FOV than the front FOV. This difference is highlighted in the crack sliding to opening responses where the ratios (w/s) of the back FOV are almost constantly equal to 1.0, while those of the front FOV are generally greater than 1.0.

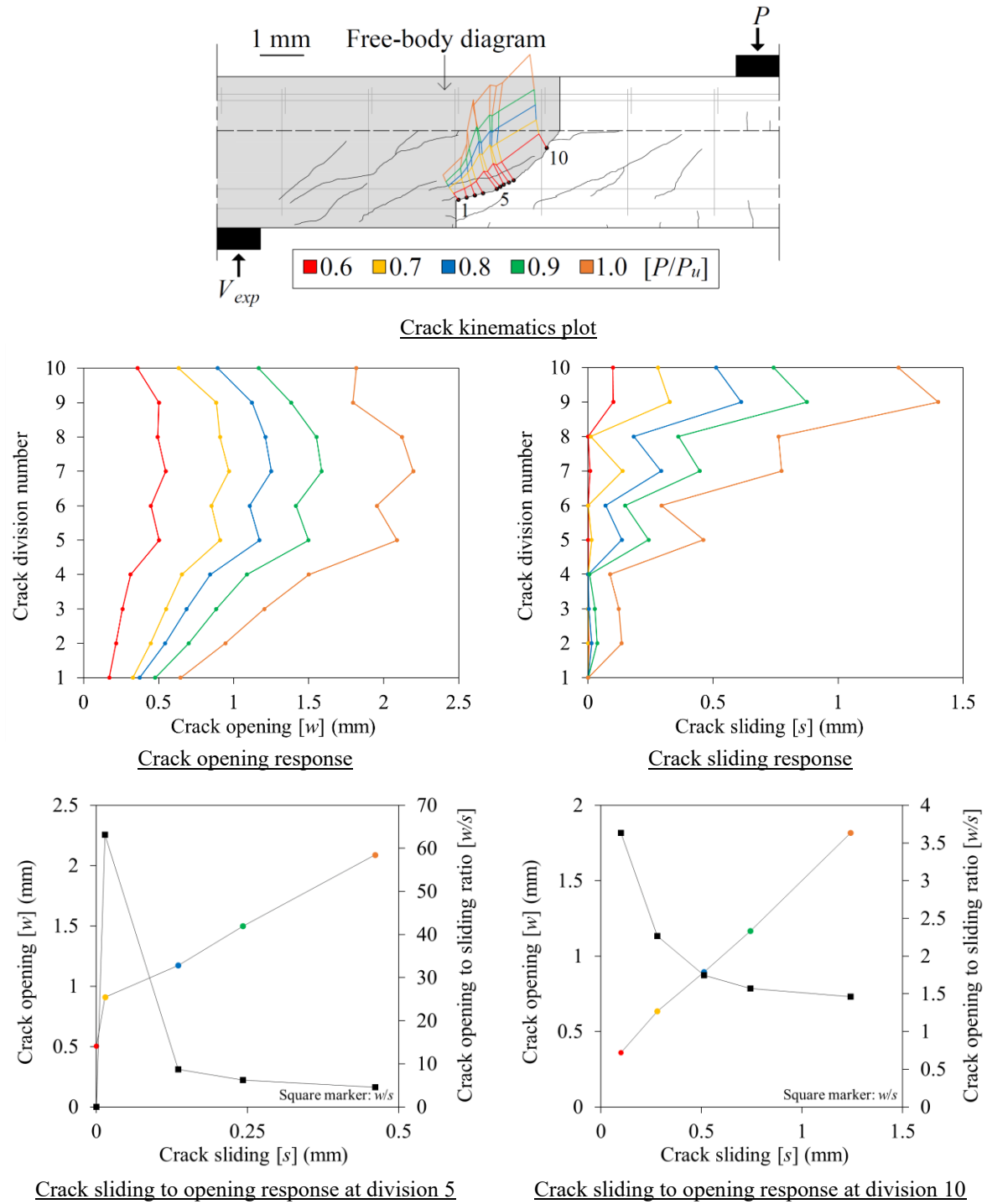


Figure 5.3 – Summary of crack kinematics of critical shear crack of specimen B1 (Front FOV)

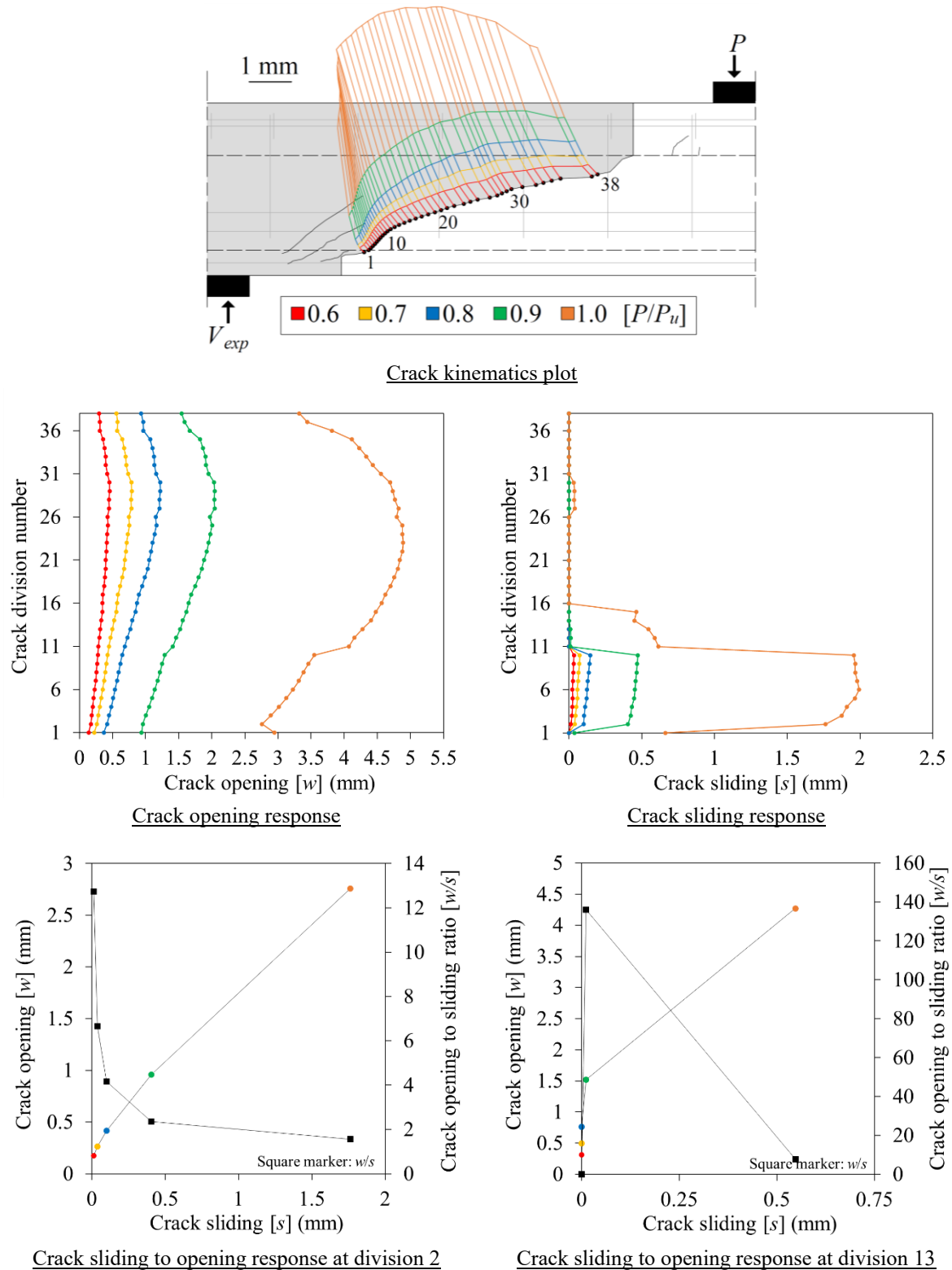


Figure 5.4 – Summary of crack kinematics of critical shear crack of specimen B2 (Front FOV)

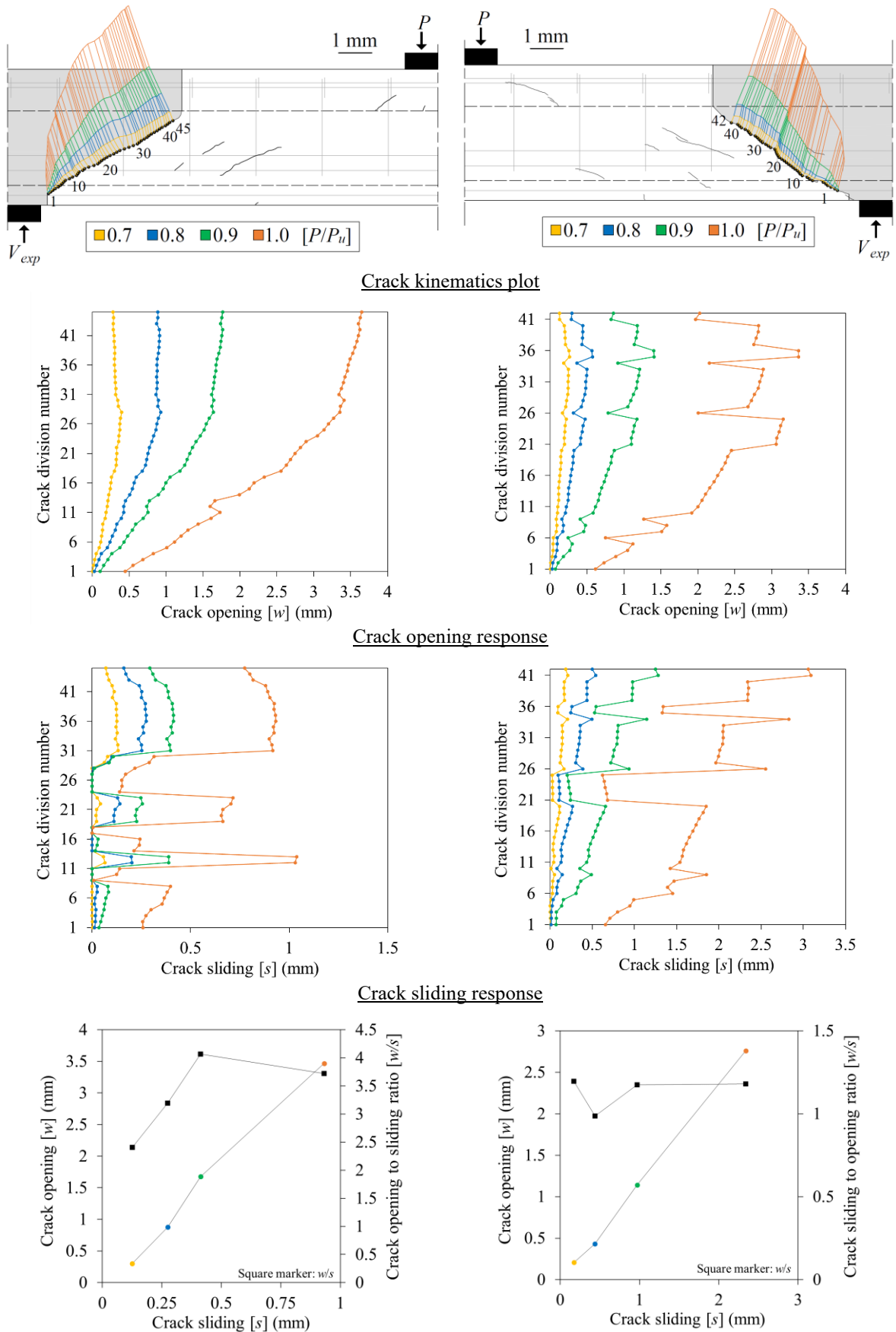


Figure 5.5 – Summary of crack kinematics of critical shear crack of specimen B3 (Front and back FOV)

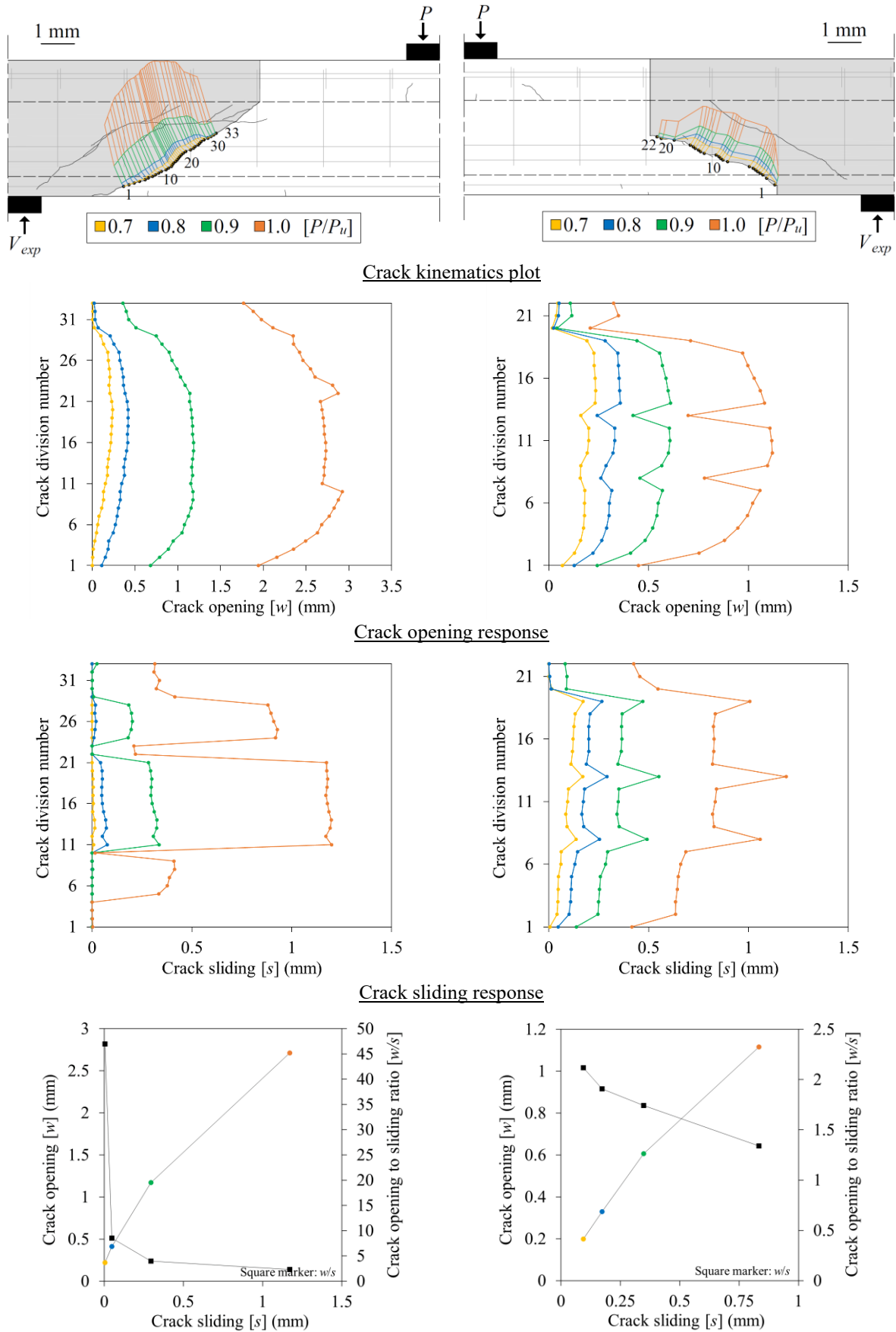


Figure 5.6 – Summary of crack kinematics of critical shear crack of specimen B4 (Front and back FOV)

5.2.3.3 GROUP 2 SPECIMENS

The crack kinematics plots for control specimen B5 in Figure 5.7 show similar crack profiles and crack widths for each face of the beam. In both FOVs, the critical shear crack extends upward from a delamination crack at the level of the longitudinal reinforcement. The crack intersects a single stirrup as it progresses upward towards the web/flange junction at an approximate inclination angle of 38° . The crack profile is fairly smooth for both FOVs, but the crack inclination is marginally steeper in the back FOV in the region between the mid-height of the web and web/flange junction. The greatest crack openings occurred in the region between the longitudinal reinforcement and mid-height of the web for both FOVs of specimen B5 as shown in Figure 5.7. The maximum crack openings reached 1.7 mm and 1.4 mm at the ultimate load for the front and back FOVs, respectively. The crack sliding responses for both FOVs are piecewise linear, with comparatively less crack sliding occurring in the back FOV. The maximum crack sliding at the ultimate load reached 1.2 and 1.0 mm for the front and back FOVs, respectively, and they both occurred near the mid-height of the web. The ratios (w/s) of the front and back FOV were similar and relatively low as they were typically equal to about 1.0 in the latter stages of loading as shown in Figure 5.7 for overlapping front and back FOV crack divisions.

The overlapping critical shear cracks in both FOVs of specimen B6 share a similar rough crack profile and crack kinematics as shown in the crack kinematics plots of Figure 5.8. The critical shear cracks at the ultimate load both begin at the longitudinal reinforcement in the beam and progress upward toward the web/flange junction while crossing two stirrups. The critical crack inclination angle in the front FOV is approximately 36° for the region between the longitudinal reinforcement in the beam (Division 4) to the web/flange junction. For the back FOV, the crack inclination angle is initially 17° for the region between the longitudinal reinforcement in the beam (Division 1) to the mid-height of the web (Division 20) prior to increasing to 46° for the following region just below the flange (Division 31) and eventually shifting to approximately 27° for the small region reaching the web/flange junction. The greatest crack openings occur in the mid-height of the web for the front FOV between divisions 10 and 20, and they occur in the steepest region of the crack that is nearest to the flange for the back FOV that is between divisions 20 to 31. The greatest crack openings are around 1.5 mm for both FOVs. The greatest crack sliding reaches about 1.5 mm for both FOVs, however crack sliding is more prominent

in the crack in the back FOV, especially in the less steep region of the crack between divisions 1 and 20. Therefore, the ratios (w/s) are slightly lower overall for the back FOV when compared to those of the front FOV during the latter stages of loading as highlighted by the overlapping front and back FOV crack divisions shown in Figure 5.8.

In specimen B7, much like specimen B6, the critical shear cracks in the front and back FOVs overlap and have similar profiles and crack widths as highlighted in the crack kinematics plots of Figure 5.9. At the ultimate load, both critical shear cracks extend along the longitudinal reinforcement in the jacket and progress upward to the web/flange junction with similar approximate inclination angles of 33° and 29° , respectively. The critical shear cracks on the front and back FOVs both cross the third stirrup to the right of the support, while the crack in the back FOV also just barely crosses the second stirrup to the right of the support. The greatest crack openings occur between the mid-height of the web and the flange for both FOVs, and the maximum crack openings at the ultimate load reached 2.6 mm and 2.2 mm for the front and back FOVs, respectively. For the region between the initial crack division and mid-height of the web, the crack openings at the ultimate load are larger in the back FOV by approximately 1.0 mm. The crack sliding is overall significantly greater in the back FOV as it reaches around 1.5 mm while the crack sliding in the front FOV only reaches about 0.75 mm. This significant difference in crack sliding is highlighted by the lower ratios (w/s) for the back FOV in comparison to those of the front FOV as shown in Figure 5.9 for overlapping front and back FOV crack divisions.

The crack kinematics of the overlapping critical shear cracks are significantly different for the front and back FOVs of specimen B8 as indicated in Figure 5.10. The difference in crack kinematics is likely due to the state in which each critical crack developed throughout loading as described in Section 4.5.2. For the front FOV, the critical shear crack extends from the longitudinal reinforcement in the jacket to the longitudinal reinforcement in compression. The inclination angle begins at about 31° between the lower end of the crack (Division 3) to the mid-height of the web (Division 20), and from there it shifts to about 15° as the crack approaches the web/flange junction (Division 34) until increasing to about 26° as the crack reaches the web/flange junction. There were no significant secondary cracks surrounding the critical shear crack in the front FOV as shown in the DIC results of Figure A.13. The critical shear crack in the back FOV can be divided into two segments. The first segment is between

the longitudinal reinforcement in the jacket to that in the beam (Divisions 1 to 13) with an inclination angle of 31° , and the second segment begins from there (Division 13) until the web/flange junction with an approximate inclination angle of 21° . There is a secondary shear crack stemming from the first segment of the critical shear crack near the longitudinal reinforcement in the beam (Division 13) that reaches the web/flange junction at a steeper angle of 35° . This secondary crack seems visually wider than the critical shear crack throughout loading until the ultimate load was reached as shown in the DIC results of Figure A.14. However, the critical shear crack becomes more dominant than the secondary crack between the ultimate load and failure as its width increases significantly, and this is also shown in the DIC results of Figure A.14. It should be noted that the critical shear cracks in both FOVs of specimen B8 reached the longitudinal reinforcement in compression at the ultimate load, which is located in the flange, however the crack kinematics of this region were not calculated nor considered when estimating the shear resistance contributions discussed later in Section 5.3.

The crack kinematics plots shown in Figure 5.10 highlight the differences between the crack kinematics of the critical shear cracks featured in the front and back FOVs of specimen B8. The crack profile is smooth for the front FOV and rough for the back FOV. The shape of the crack widths is parabolic for the front FOV, while the two distinct segments of the critical shear crack in the back FOV are emphasized by the sharp decline in crack width occurring around the mid-height of the web. The greatest crack openings occur at the mid-height of the web for the front FOV with the maximum crack opening reaching 3.6 mm. The greatest crack opening in the first segment of the critical shear crack in the back FOV reached 1.9 mm, and that of the second segment reached 1.3 mm. Crack sliding is significantly greater in the back FOV as crack sliding reaches about 2.0 mm and 1.0 mm in the first and second segments of the crack, respectively. For the front FOV, crack sliding only reached about 1.0 mm for the region of the crack between the longitudinal reinforcement in the jacket and mid-height of the web, and almost no sliding occurred beyond this region. Interestingly, the crack opening and sliding responses are similar in shape and value throughout each load step for the back FOV. This produced ratios (w/s) close to 1.0 for the back FOV as opposed to those of the front FOV, which were generally much higher than 1.0, as shown in Figure 5.10 for overlapping front and back FOV crack divisions.

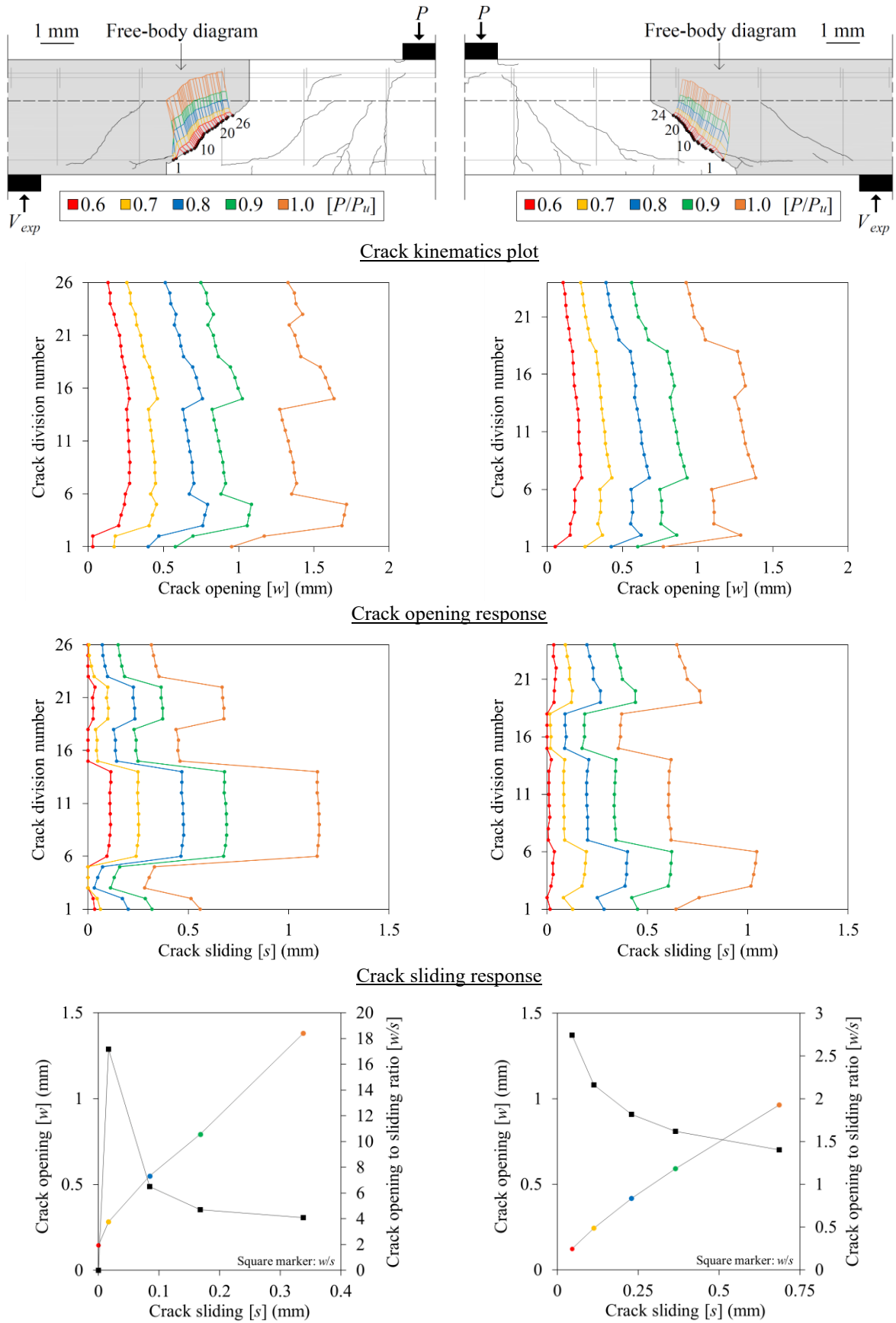


Figure 5.7 – Summary of crack kinematics of critical shear crack of specimen B5 (Front and back FOV)

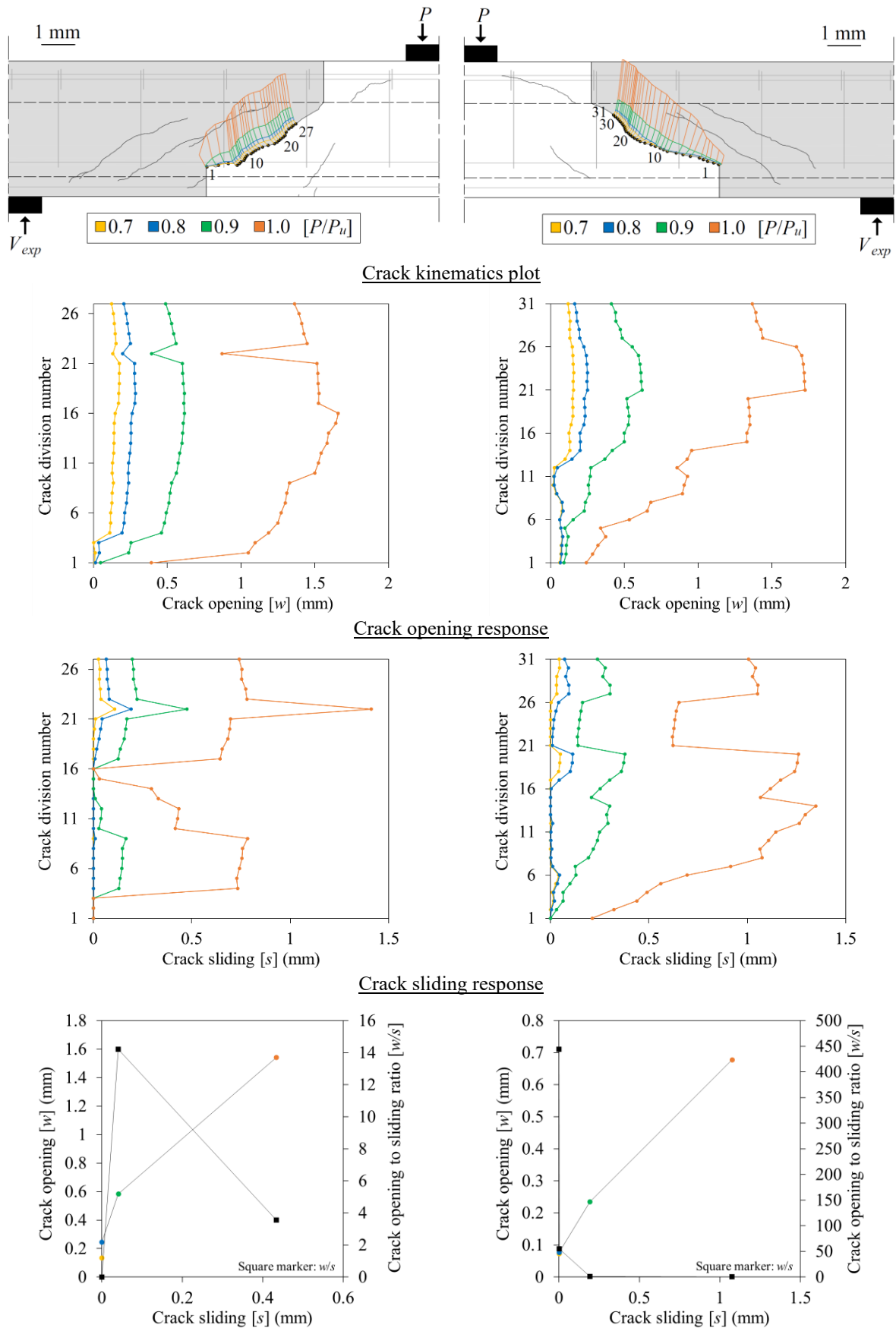


Figure 5.8 – Summary of crack kinematics of critical shear crack of specimen B6 (Front and back FOV)

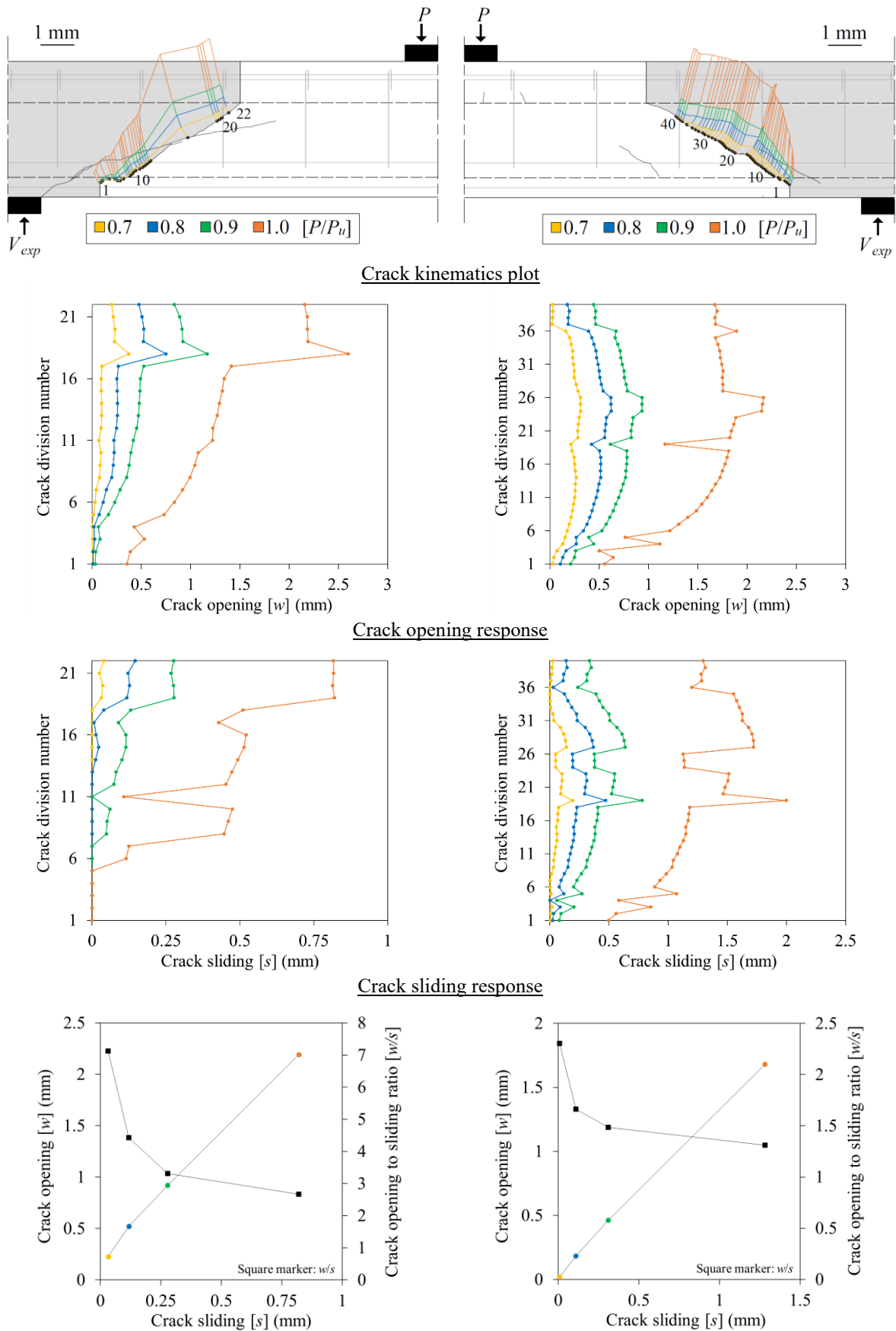


Figure 5.9 – Summary of crack kinematics of critical shear crack of specimen B7 (Front and back FOV)

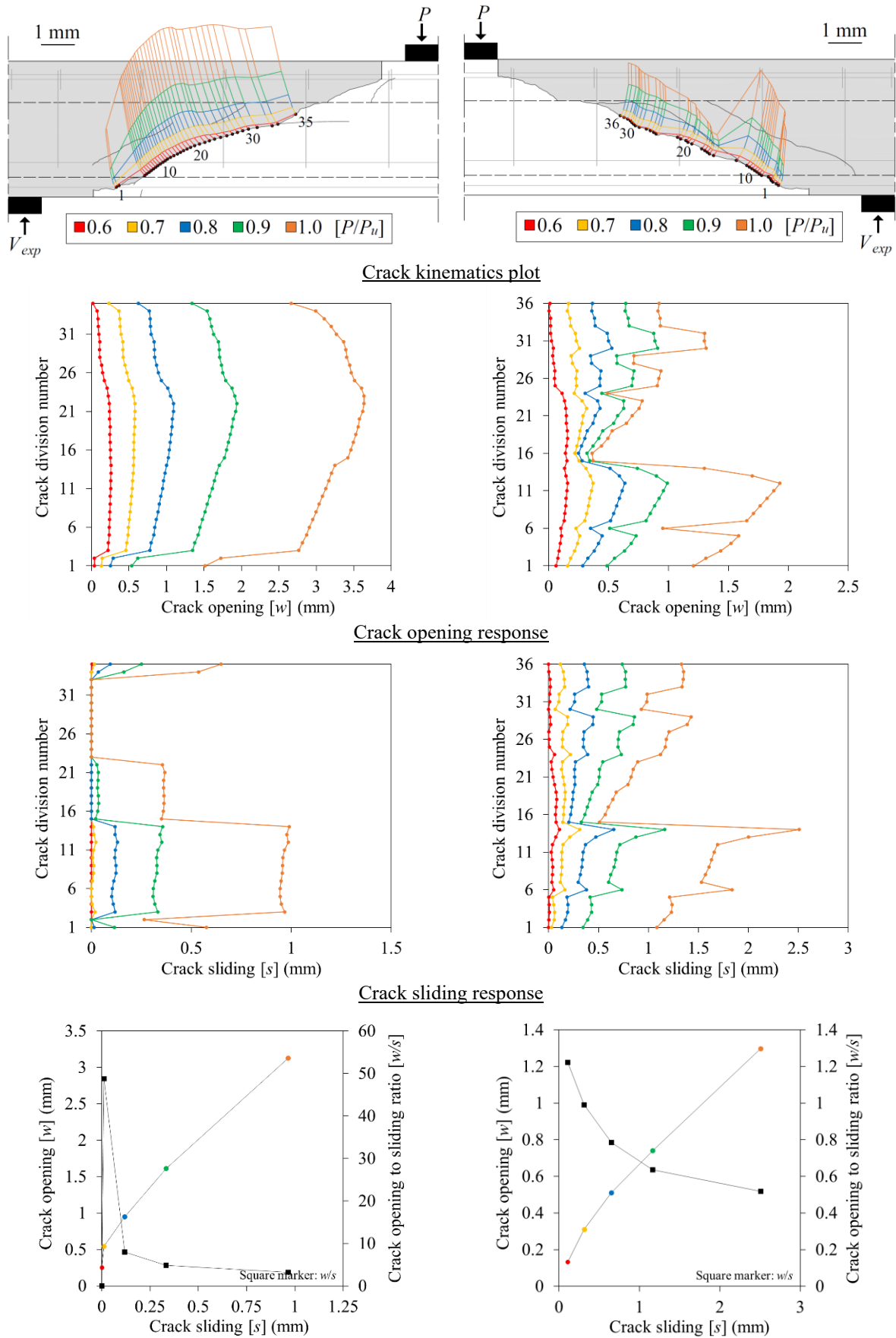


Figure 5.10 – Summary of crack kinematics of critical shear crack of specimen B8 (Front and back FOV)

5.3 CONTRIBUTIONS OF SHEAR RESISTANCE MECHANISMS

5.3.1 INTRODUCTION

The contributions of the various shear resistance mechanisms were estimated individually for each specimen using several models from the literature. The contributions to shear resistance were estimated from first cracking to failure. Estimations were carried out by substituting the calculated crack kinematics of the critical shear cracks into the various theoretical models. For each shear resisting mechanism, the results of a single model were eventually selected for the overall assessment of each beam as discussed in Section 5.4. The contributions of the various shear resistance mechanisms were estimated for each FOV where available and averaged as discussed in Section 5.4.

The considered shear resisting mechanisms in the jackets were fibre bridging (V_{fib}) and dowel action (V_{dow}) of the longitudinal reinforcement. In the RC beam, the considered shear resisting mechanisms were aggregate interlock (V_{agg}), dowel action (V_{dow}), stirrups (V_{stir}) and the uncracked compression zone (V_{cz}). As discussed in Section 5.2, the calculated crack kinematics were based on the DIC measurements taken from the surfaces of each specimen throughout the experiment. For the specimens with jackets, it was necessary to assume that the calculated crack kinematics were applicable to the RC beams as well as the jackets. Tentative support for this is provided in Section 4.6 where it is shown that the critical shear cracks passed through both the jackets and beams. However, despite this, the crack kinematics within the RC beams may have differed from those at the surface due to unobserved differences in cracking between the two. Consequently, there is some uncertainty in the estimated contributions of the shear resisting mechanisms within the RC beams of the specimens with jackets.

In addition to using the calculated crack kinematics to estimate each shear resistance mechanism, free-body diagrams based on the shapes of the critical shear cracks were developed for each specimen to visualize the internal shear forces. Figures 5.11 and 5.12 show the free-body diagrams for the specimens of groups 1 and 2, respectively, which include both FOVs where available. The free-body diagrams assisted in highlighting the exact locations of the shear forces, namely the forces due to dowel action and stirrups, which were required to estimate their contributions as discussed later in Sections 5.3.4 and 5.3.5. The free-body

diagrams also reference the locations of the fixed strain gauges used during the experiment on the stirrups that were intercepted by the critical shear cracks for comparison purposes, and this is also discussed later in Section 5.3.5.

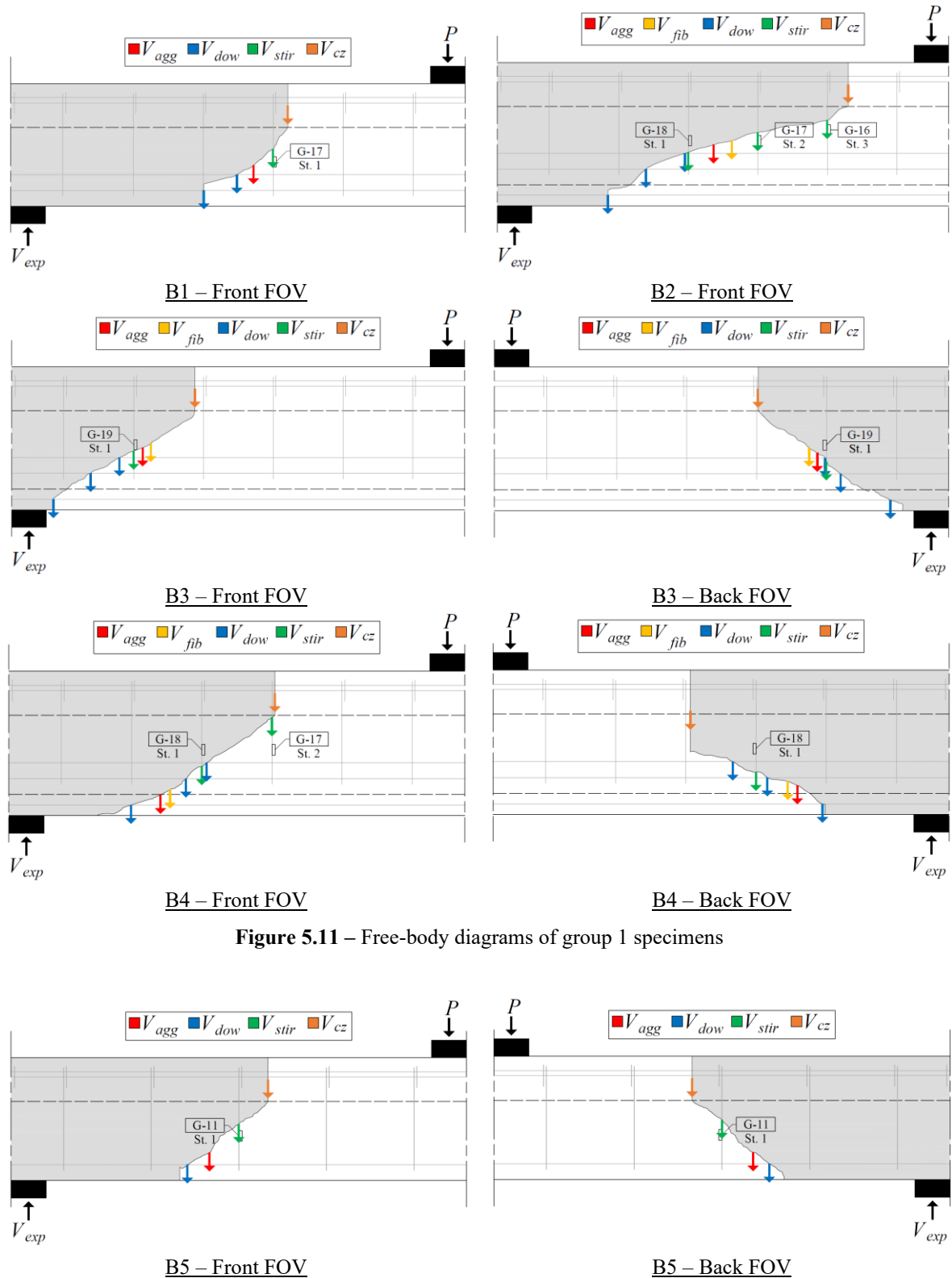


Figure 5.11 – Free-body diagrams of group 1 specimens

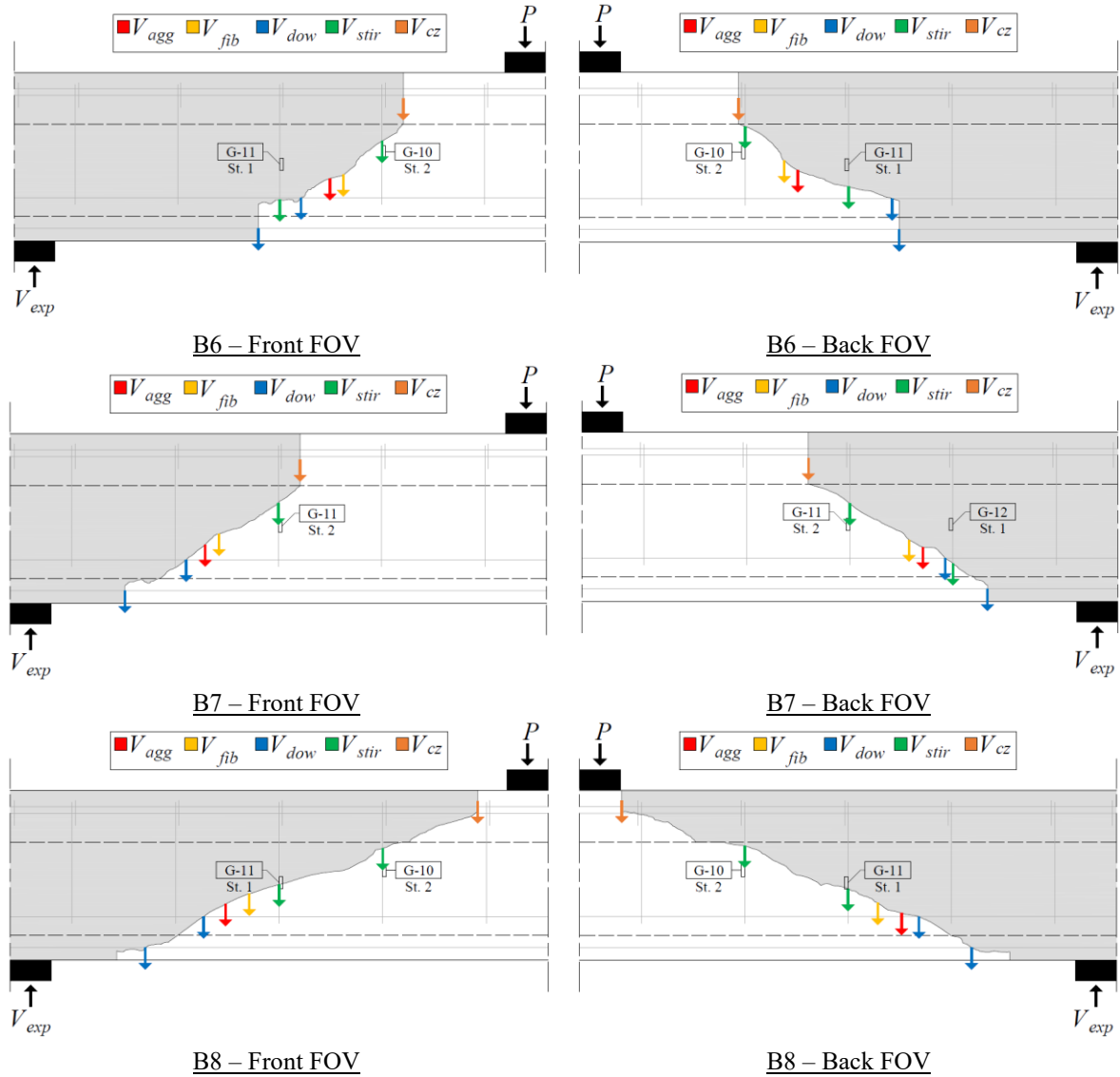


Figure 5.12 – Free-body diagrams of group 2 specimens

5.3.2 AGGREGATE INTERLOCK

The aggregate interlock action attributed to the RC beams was estimated throughout the experiment until the ultimate was reached for each specimen using various theoretical models from the literature. The theoretical models include the Two-Phase models based on the crack kinematics of Walraven (1980), Guidotti (2010) and Ulaga (2003) along with the Rough Crack model of Gambarova and Karakoc (1983) and the Simplified Contact Density model of Li et al. (1989). Section 2.3.2 provides an overview of each theoretical model including methods of calculation. As mentioned in Section 5.3.1, the calculated crack kinematics for the specimens with jackets were assumed to be representative of the crack kinematics of the beams, and not only the jackets. The obtained material properties of each specimen presented in Chapter 4

were considered in the estimations along with the calculated crack kinematics presented in Section 5.2.3. As discussed in Section 3.4.2, limestone aggregate with a maximum size of 10 mm was used for the specimens of groups 1 and 2.

The Two-Phase model based on the crack kinematics of Guidotti (2010) requires the initial crack opening displacement (u_0) as detailed in Section 2.3.2. To recapitulate, Guidotti (2010) and Ulaga (2003) adopted the Two-Phase model of Walraven (1980), which assumes that a crack initially opens and then slides. Guidotti (2010) assumes that a crack undergoes an initial crack opening (u_0) followed by simultaneous crack opening and sliding, while Ulaga (2003) suggests that crack opening and sliding occur simultaneously at the onset of cracking. The initial crack opening displacement (u_0) was estimated for each crack division through least square linear regression to determine the best fit line for the crack sliding versus crack opening displacements plot (Pastore, 2020). The interception of the best fit line with the y-axis gives the initial crack opening displacement (u_0) as illustrated in Figure 5.13, which shows that the initial crack opening displacements (u_0) varied between crack divisions for each specimen. This suggests that the approach of Guidotti (2010) is most representative of the crack kinematics observed in the tests as the crack opening at first sliding varies.

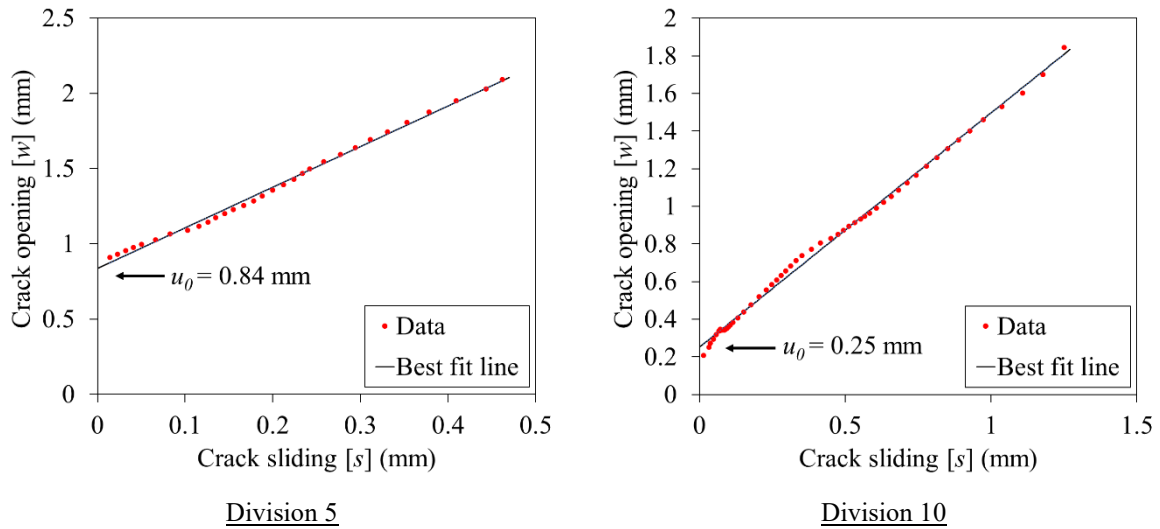


Figure 5.13 – Determination of initial crack opening displacement for specimen B1 based on Guidotti (2010)

The estimated aggregate interlock contributions to shear resistance for the specimens of groups 1 and 2 are presented in Figures 5.14 and 5.15, respectively. The results show the estimated aggregate interlock shear force (V_{agg}) based on each theoretical model and its contribution to

the total shear resistance (V_{agg}/V_u) throughout each load step (P/P_u) until the ultimate load (P_u). The estimated results include both front and back FOVs of each specimen where available.

The aggregate interlock contributions of control specimen B1 in Figure 5.14 show a trend that is also shared with several other specimens in both groups 1 and 2. The estimated aggregate interlock contributions at the ultimate load ranged from 0.8% to 20.7%. The Simplified Contact Density model of Li et al. (1989) gives the highest contribution when compared to the other models. This is likely due to the lack of reduction in the aggregate contact area when large crack widths occur relative to the maximum aggregate size, unlike in the other models (Pastore, 2020). The Rough Crack model of Gambarova and Karakoc (1983) gives the second highest contribution at the ultimate load with 7.0%. The Two-Phase models based on the crack kinematics of Walraven (1980), Guidotti (2010) and Ulaga (2003) give the lowest contributions at the ultimate load, with all less than 6.5%. The original Two-Phase model of Walraven (1980) gives the highest contribution between them followed by the models based on the crack kinematics of Guidotti (2010) and Ulaga (2003) in consecutive order. The scatter between the various adaptations of the Two-Phase model of Walraven (1980) is dependent on the crack kinematics. The observed trends in the results for control specimen B1 agree with the observations of Campana et al. (2013), Cavagnis et al. (2018a, 2018b), Huber et al. (2016) and Pastore (2020).

The aggregate interlock contributions of both FOVs of control specimen B5 in Figure 5.15 are comparable due to the similar crack kinematics. The estimated aggregate interlock contributions at the ultimate load range from 3.1% to 39.5% and 9.8% to 34.7% for the front and back FOVs, respectively. A similar trend to control specimen B1 was observed, with the Simplified Contact Density model of Li et al. (1989) giving the highest contribution for both FOVs when compared to the other models. A marginal difference of about 8.0 kN was observed between both FOVs for the contributions at the ultimate load given by the model of Li et al. (1989). This is also the case for the contributions given by the model of Gambarova and Karakoc (1983), where the difference at the ultimate load was about 1.0 kN between both FOVs. Much like control specimen B1, the Rough Crack model of Gambarova and Karakoc (1983) gives the second highest contribution for the front FOV followed by the various adaptations of the Two-Phase model of Walraven (1980). The Two-Phase model of Walraven (1980) gives the second highest contribution for the back FOV followed by the models of

Gambarova and Karakoc (1983), Guidotti (2010) and Ulaga (2003) in consecutive order. The contributions given by the various adaptations of the Two-Phase model are significantly higher for the back FOV than the front FOV, with the difference between the two reaching 80.4% (23.81 kN) for the model of Walraven (1980). These large discrepancies are likely due to the marginally steeper crack inclination angles of the crack segments featured in the region between the mid-height of the web to the web/flange junction for the back FOV. In addition, the crack opening to sliding ratios (w/s) are marginally greater for the back FOV. Prior studies found that the Two-Phase model of Walraven (1980) is sensitive to steep crack inclination angles and crack surfaces with crack sliding (s) that is greater than or equal to crack opening (w), and this may explain the overestimated contributions (Campana et al., 2013; Huber et al., 2016).

The aggregate interlock contributions shown in Figures 5.14 and 5.15 show a trend for the front FOV of all of the strengthened specimens that is similar to the trend described for control specimen B1. It can be seen that the model of Li et al. (1989) gives the highest contribution followed by the model of Gambarova and Karakoc (1983) and the various adaptations of the Two-Phase model of Walraven (1980). The contributions for the front FOV of the specimens with jackets are less than the contributions of their respective control specimens, which seems reasonable given the fairly large crack widths that generally appeared on their surfaces in relation to the maximum aggregate size used in the beams. The contributions are generally lower overall for the strengthened specimens of group 1 when compared to those of group 2, and this may be due to the additional shear reinforcement in the RC beams of group 1. Campana et al. (2013) suggests that the aggregate interlock contributions may vary depending on the amount of shear reinforcement amongst other contributing factors such as the crack profile and crack kinematics.

There are notable similarities and discrepancies between the contributions of both FOVs for the strengthened specimens of groups 1 and 2 shown in Figures 5.14 and 5.15, respectively. The models of Li et al. (1989) and Gambarova and Karakoc (1983) give reasonably comparable contributions between both FOVs of each strengthened specimen, where differences of up to 24 kN and 11 kN were observed at the ultimate load, respectively. On the other hand, the Two-Phase models based on the crack kinematics of Walraven (1980), Guidotti (2010) and Ulaga (2003) give very different contributions for each FOV. Large differences between estimated

aggregate interlock contributions for each FOV were found for specimens B3, B4, B6, B7 and B8. In the most extreme case, which is the back FOV of specimen B8, the model of Walraven (1980) gives an aggregate interlock contribution at the ultimate load of 135%, which is unrealistic as it exceeds the experimental shear force (V_u). These discrepancies may be due to the Two-Phase model of Walraven (1980) being overly sensitive to steep crack angles and crack surfaces where the crack sliding (s) is greater than or equal to the crack opening (w) (Campana et al., 2013; Huber et al., 2016). The profiles of the cracks featured on the back FOV of the strengthened specimens were marginally rougher causing steeper inclinational angles when compared to the cracks on the front FOV. In addition, crack sliding was much more prevalent and significant in the cracks on the back FOV of the strengthened specimens than in the front FOV. These factors may have led to the unrealistic overestimations given by the Two-Phase model of Walraven (1980) and its adaptations. Interestingly, Huber et al. (2016) also found the Two-Phase model of Walraven (1980) to be overly sensitive to steeply inclined cracks with excessive sliding as it gave aggregate interlock contributions that were close to 100% of the experimental shear force for RC beams that are similar to those of this research. In conclusion, the predictions of the Two-Phase model of Walraven (1980) and its adaptations are considered overly sensitive to the crack kinematics as shown in Figures 5.14 and 5.15.

The Rough Crack model of Gambarova and Karakoc (1983) was adopted to evaluate the aggregate interlock contribution (V_{agg}) for each specimen in Section 5.4. The Rough Crack model was observed to be the least sensitive to the small variations in crack opening and sliding displacements. The contributions given by the Rough Crack model were either the lower bound or moderately close to the lower bound of the considered aggregate interlock models, which agrees with the findings of Sagaseta and Vollum (2011). Overall, the Rough Crack model gives reasonable and consistent aggregate interlock contributions for all specimens when compared to similar RC beams with shear reinforcement from the literature (Campana et al., 2013, Huber et al., 2016; Pastore, 2020). The resulting aggregate interlock contributions for control specimens B1 and B5 were 7.0% and 20.2%, respectively. The contributions of the strengthened specimens were less overall as they ranged between 0.7% to 2.0% and 2.3% to 5.6% for groups 1 and 2, respectively. The contributions of each FOV estimated using the Rough Crack model were averaged throughout loading as discussed later in Section 5.4.

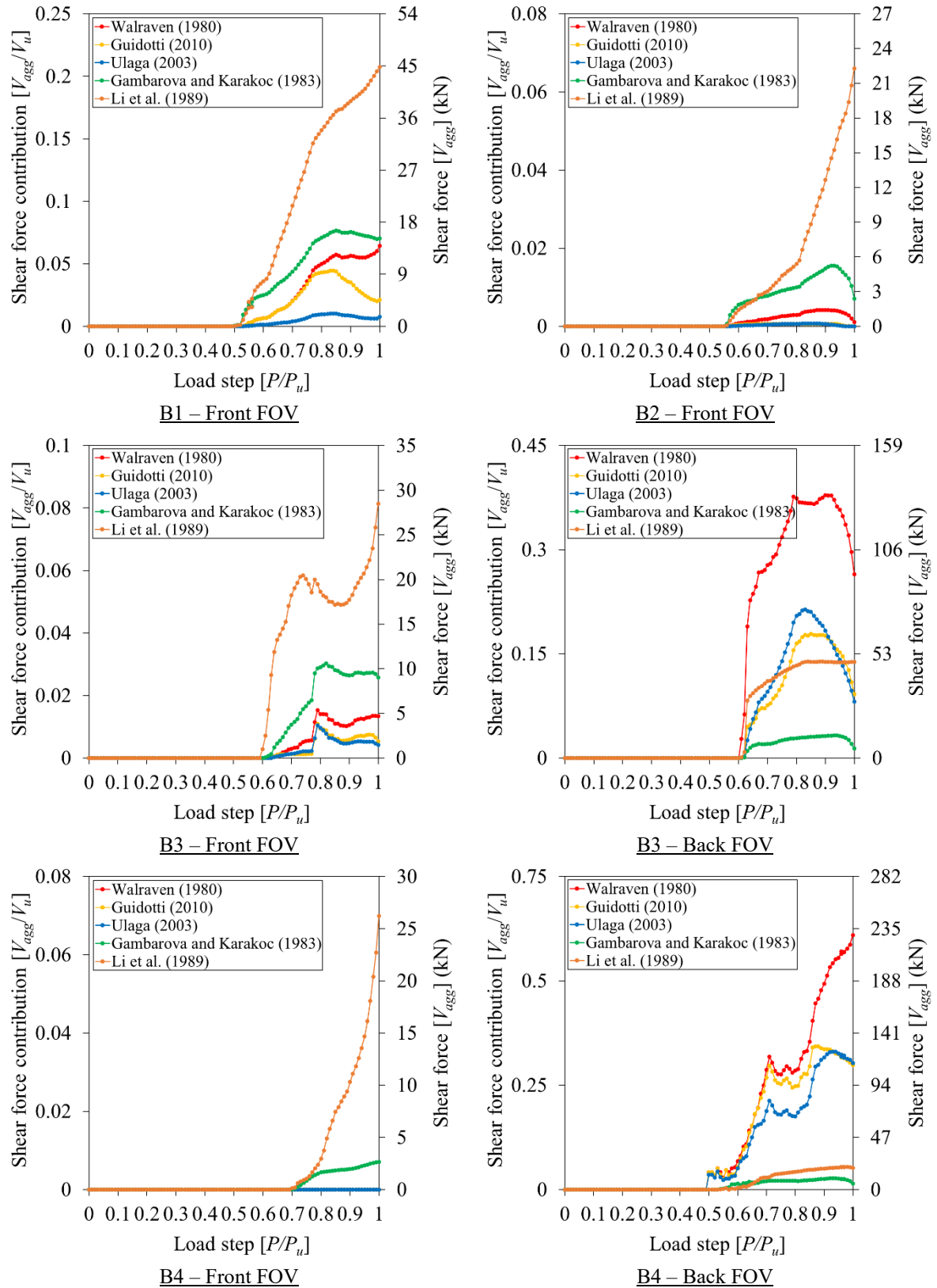
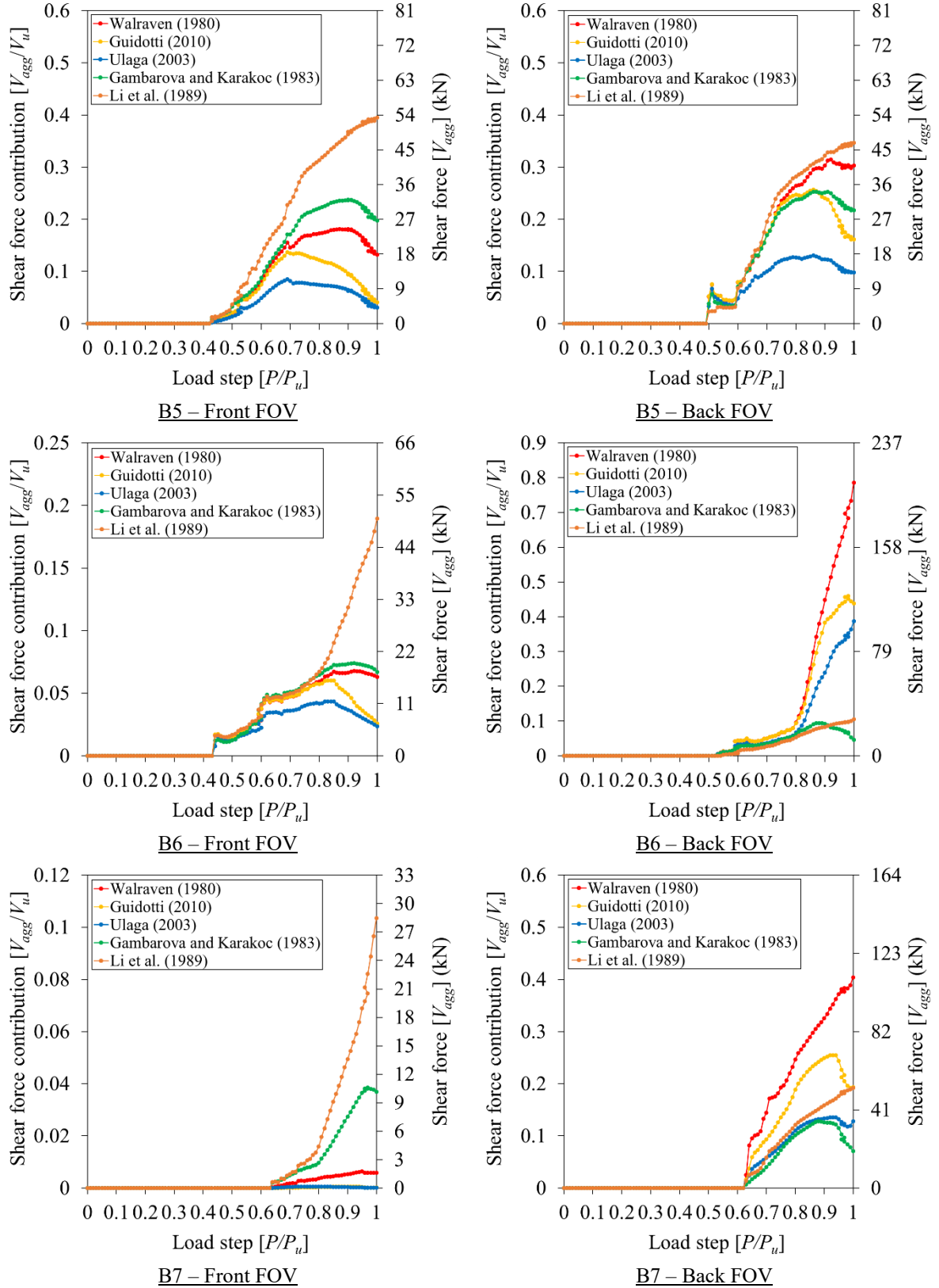


Figure 5.14 – Contribution of aggregate interlock of group 1 specimens based on various theoretical models



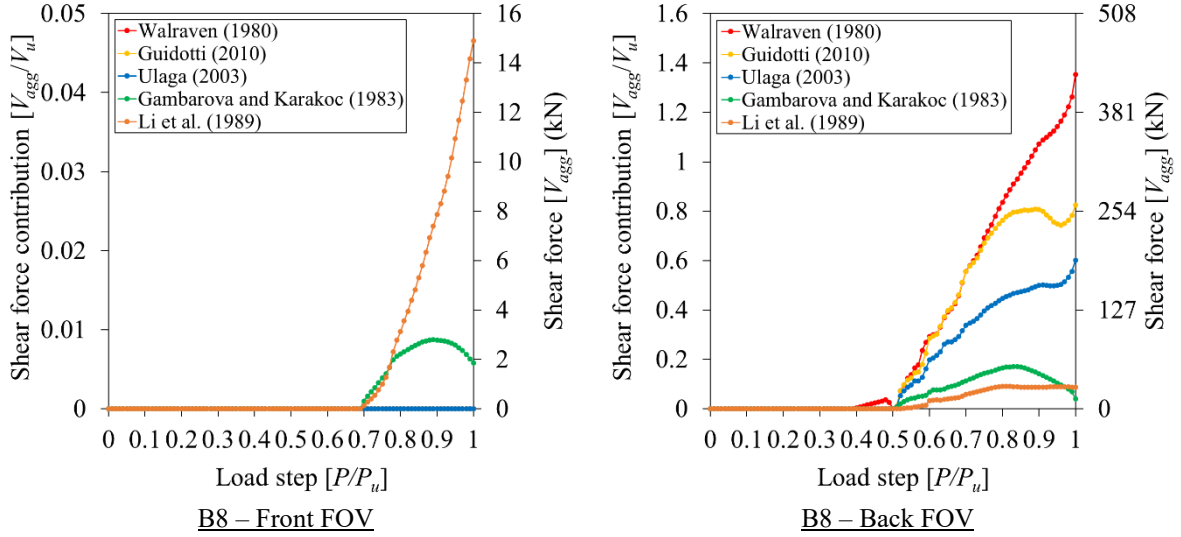


Figure 5.15 – Contribution of aggregate interlock of group 2 specimens based on various theoretical models

5.3.3 FIBRE BRIDGING

The contribution to shear resistance due to fibre bridging (V_{fib}) was estimated using the experimental post-cracking stress response of UHPC that is discussed in Section 4.2.4.1. In addition, the Engagement model of Pfy (2003), UVEM of Htut (2010) and SDEM of Lee et al. (2013) (see Section 2.3.3) were considered. The average post-cracking tensile stress-CMOD response of UHPC obtained from the batch of the Soroto mixer presented in Figure 4.10 was adopted. Estimating the fibre bridging contribution to shear resistance (V_{fib}) was done by using the post-cracking stress-CMOD response to determine the stress (σ) corresponding to the calculated crack opening displacement (w) for each division of the critical shear crack. With the stress (σ) known, Eqn. 2.66 was then used to calculate the shear resistance contribution of the fibre reinforcement (V_{fib}). As discussed in Section 3.3, straight steel fibres were used in the UHPC of the jackets, and the fibre volume in terms of the total concrete volume (V_f) was equal to 0.025 (2.5%). The maximum steel fibre length (l_f) was 13 mm with a diameter (d_f) of 0.2 mm, and the elastic modulus (E_f) was assumed to be equal to 200 GPa.

The use of the experimental post-cracking tensile response of UHPC to determine the fibre bridging contribution to the total shear resistance was compared with models from the literature. All of the theoretical models from the literature depend on the ultimate fibre bond stress, which should ideally be obtained experimentally from fibre pull-out tests as discussed in Section 2.3.3. The ultimate fibre bond stress refers to the average fibre bond stress (τ_{bf}) in

the model of Pfyl (2003), the initial fibre bond stress ($\tau_{b,0}$) in the model of Htut (2010) and the maximum frictional bond stress ($\tau_{f,max}$) in the model of Lee et al. (2013). In the absence of experimental data, the ultimate fibre bond stress was calculated using Eqn. 2.72 based on the compressive cylinder strength of UHPC, which is equal to 119.89 MPa on average for the batches of the Soroto mixer presented in Table 4.6. The ultimate fibre bond stress was found to be equal to 6.57 MPa. The SDEM of Lee et al. (2013) requires an additional input parameter, which is the slip (s_f) that corresponds to the ultimate fibre bond stress. There is no calculation method for the ultimate fibre bond slip (s_f) in the absence of experimental data, therefore the value of 0.46 mm was taken from the fibre pull-out test results of Wille and Naaman (2012) that used a similar type of UHPC and steel fibre as in this campaign.

Figure 5.16 compares the experimental post-cracking response of UHPC with the estimated responses given by the various fibre bridging models from the literature. It can be seen that the Engagement model of Pfyl (2003) is very accurate when compared to the experimental response of UHPC. The models of Htut (2010) and Lee et al. (2013) underestimate the post-cracking tensile response of UHPC. This is likely due to the input fibre bond stress and slip values that were used in the calculations. The input fibre bond stress and slip values were calibrated for the models of Htut (2010) and Lee et al. (2013) to achieve the best fit with the experimental response as shown in Figure 5.16. It appears that higher bond stress values of 10.40 MPa and 8.60 MPa, respectively, were required for both models, while an unrealistically low fibre bond slip of 0.00001 mm was required for the model of Lee et al. (2013) to achieve the best fit with the experimental response. Although the Engagement model of Pfyl (2003) seems capable of accurately capturing the post-cracking tensile response of UHPC, the experimental post-cracking tensile response of UHPC was adopted to estimate the shear resistance contribution due to fibre bridging (V_{fib}).

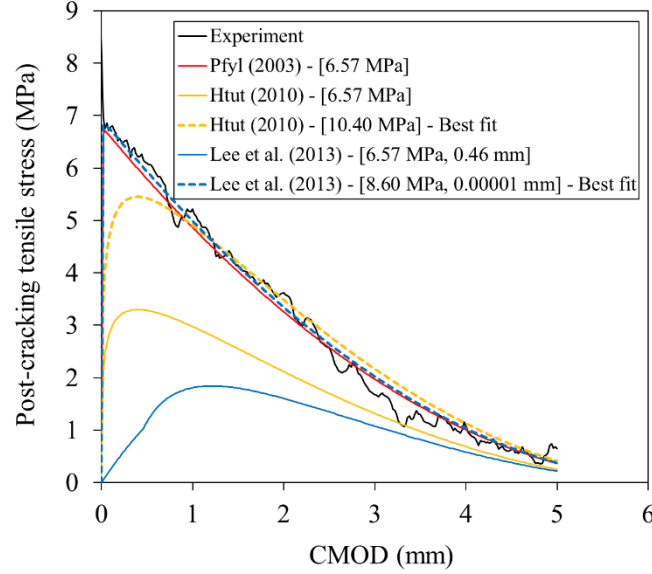


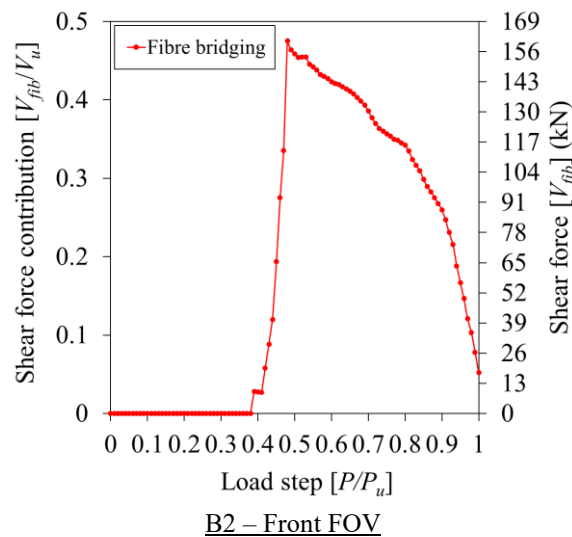
Figure 5.16 – Comparison between experimental and estimated post-cracking tensile stress response of UHPC based on various theoretical models

The estimated fibre bridging contributions to shear resistance for the strengthened specimens of groups 1 and 2 are presented in Figures 5.17 and 5.18, respectively. The results show the estimated fibre bridging shear force (V_{fib}) and its contribution to the total shear resistance (V_{fib}/V_u) throughout each load step (P/P_u) until the ultimate load (P_u). The estimated results include both front and back FOVs of each specimen where available.

The fibre bridging contributions for all of the strengthened specimens appear to reach their peak around the load steps $0.5P_u$ to $0.8P_u$. This is reasonable seeing as the crack opening displacements are less than 0.5 mm around these load steps for all strengthened specimens, which can be seen in the results of the crack kinematics in Section 5.2.3. According to the experimental post-cracking tensile response of UHPC presented in Figure 4.10, the highest tensile resistance occurs at low crack opening displacements. The upper and lower bounds of the peak contributions were between 22.7% to 54.3%, which represent the peak contributions of the front FOV of specimen B6 and back FOV of specimen B4, respectively. The upper and lower bounds exclude the extreme case of specimen B8, where the peak contributions for the front and back FOVs reached 96.7% and 82.6%, respectively. The peak contributions for specimen B8 are unrealistically high given that there are other contributing shear resistance mechanisms. This is likely due the large crack opening displacements (w) occurring along the entire length of the crack as well as the relatively low crack inclination angles.

After reaching their peak, the fibre bridging contributions generally descend in a concave downward shape until the ultimate load is reached as shown in Figures 5.17 and 5.18. This is due to the development of relatively large crack opening displacements (w) during the latter stages of loading, with openings (w) reaching approximately 1.5 mm to 5.0 mm. The fibre bridging action is significantly reduced when large crack openings occur seeing as the fibres bridging the critical shear crack begin to pull out, and this loss of tensile resistance is also highlighted by the experimental post-cracking tensile response of UHPC. The upper and lower bounds of the contributions at the ultimate load were between 5.2% and 61.7%, which represent the contributions of the front FOV of specimen B2 and back FOV of specimen B8, respectively.

The fibre bridging contributions of both FOVs are dependent on their respective crack opening displacements (w) throughout loading and crack profile. The similarities and differences can be observed by comparing the shapes of the fibre bridging contribution responses for each FOV of an individual specimen shown in Figures 5.17 and 5.18. The responses of both FOVs are fairly similar in shape for most specimens, although the contributions may differ. The highest percent differences in contributions between both FOVs at the ultimate load are seen in specimens B4 and B8, which are equal to 96.4% (103.62 kN) and 86.3% (118.26 kN), respectively. The percent differences for the remaining strengthened specimens range between 6.4% and 55.1%, which is equivalent to a maximum difference of 30 kN. The fibre bridging contributions of each FOV were averaged in order to evaluate each specimen individually, and this is discussed later in Section 5.4.



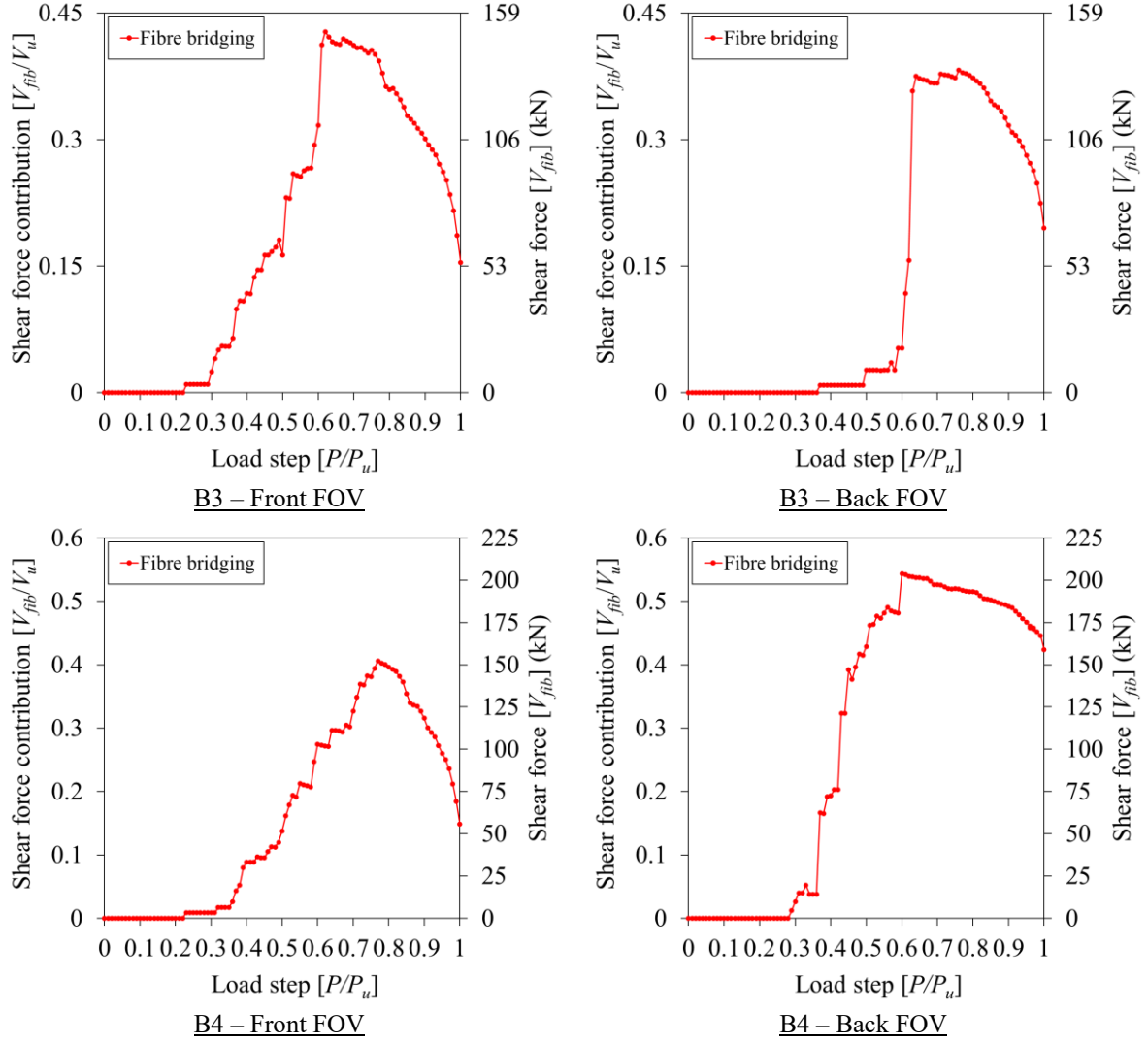
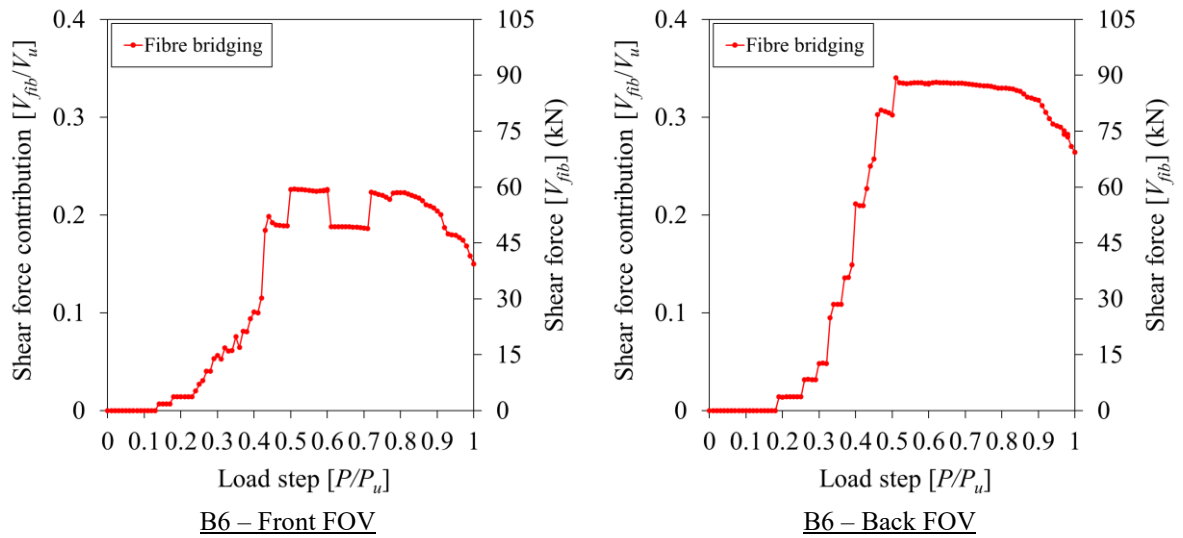


Figure 5.17 – Contribution of fibre bridging of group 1 specimens based on experimental post-cracking tensile stress of UHPC



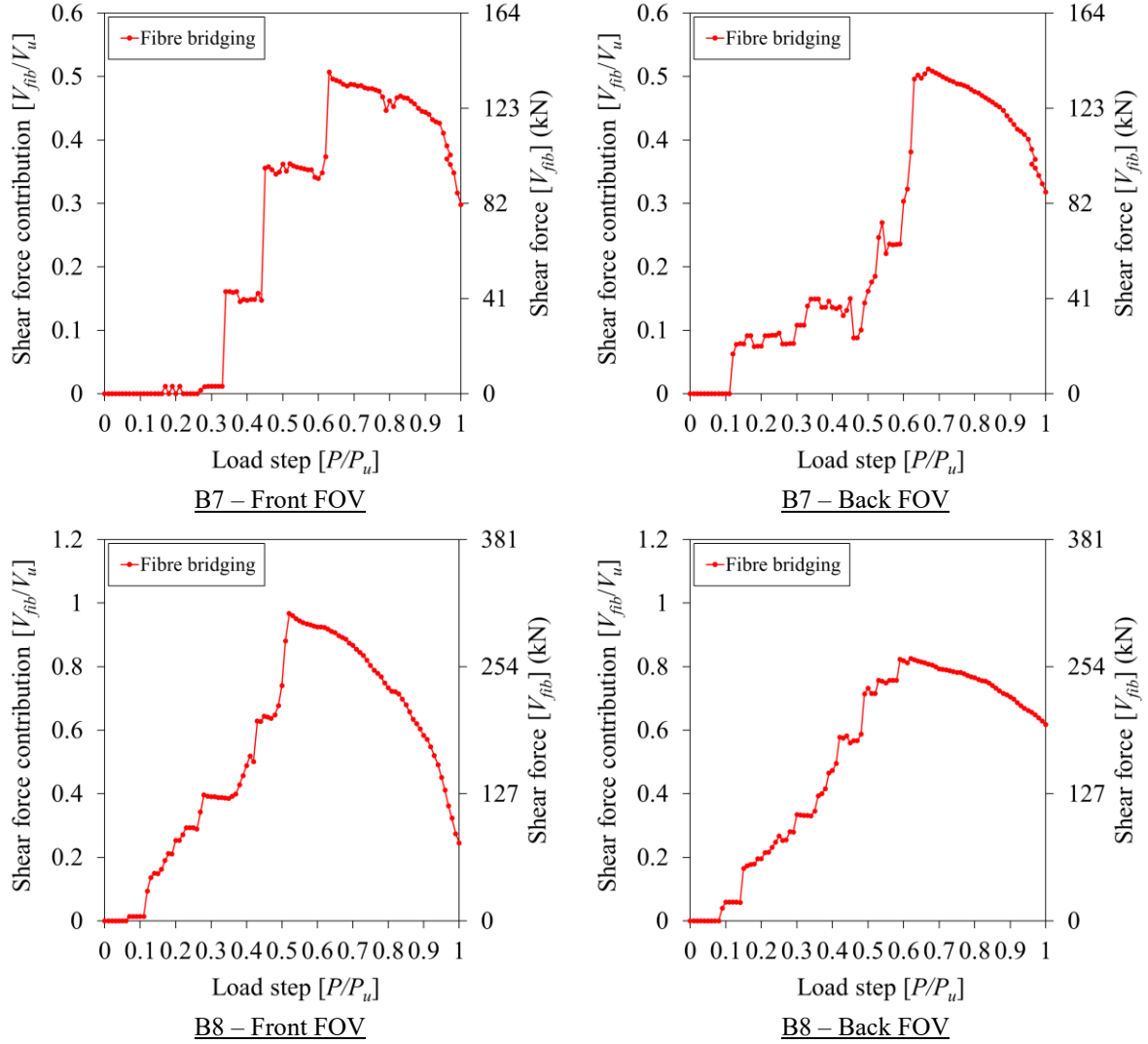


Figure 5.18 – Contribution of fibre bridging of group 2 specimens based on experimental post-cracking tensile stress of UHPC

5.3.4 DOWEL ACTION

Various theoretical models from the literature were used to estimate the shear force resisted by dowel action of the longitudinal reinforcement in tension (V_{dow}). The adopted theoretical models were those of Baumann and Rüsch (1970), Huber et al. (2020), Walraven and Reinhardt (1981), Millard and Johnson (1984) and He and Kwan (2001). Section 2.3.4 provides an overview of each model along with methods of calculation. Several input parameters that were recorded by the DIC system and data acquisition system (DAQ) were considered in the estimations of each specimen. The $\phi 20$ longitudinal reinforcement described in Section 4.2.2 was used in the beams and jackets of the specimens of groups 1 and 2.

Much like the approach for calculating the crack kinematics, virtual strain gauges were placed at several key locations using the DIC images to measure the required vertical and horizontal crack displacements throughout the experiment. For the models of Baumann and Rüschi (1970) and Huber et al. (2020), virtual gauges were placed at the intersections between the critical shear crack and uppermost layer of reinforcement to measure the shear displacement (Δ), or vertical crack displacement. These particular models also required virtual gauges to measure the vertical crack displacement (Δ_{stir}) associated with the point of intersection between the nearest stirrup that is a distance (f) away from the critical shear crack and the uppermost longitudinal reinforcement bar. Virtual gauges were also placed at the intersections between the critical shear crack and each layer of longitudinal reinforcement to measure the vertical crack displacements (Δ) required to calculate the shear resistance contribution of each individual layer according to the models of Walraven and Reinhardt (1981), Millard and Johnson (1984) and He and Kwan (2001). In addition, the model of Walraven and Reinhardt (1981) required the horizontal crack displacements (w) to be measured at these mentioned locations to determine the dowel action shear reduction factor (ξ), while the model of Millard and Johnson (1984) required the stress (σ_s) corresponding to the strain measurements of the longitudinal reinforcement recorded by the fixed strain gauges to determine the stress ratio (α) at each load step according to Eqn. 2.111.

The dowel action contribution was equal to the sum of the contributions of each individual layer of reinforcement in the RC beam and UHPC jacket if available for the models of Walraven and Reinhardt (1981), Millard and Johnson (1984) and He and Kwan (2001). These particular models do not consider the number of layers of reinforcement in their respective methods of calculation, therefore the contribution of each layer was treated separately. Of the aforementioned models, only the models of Millard and Johnson (1984) and He and Kwan (2001) limit the force due to dowel action of each layer of reinforcement by the plastic bending capacity of the reinforcement, with the former also considering the measured axial stress in the reinforcement. For the control RC beam specimens of groups 1 and 2, specimens B1 and B5, respectively, a single dowel action contribution was calculated for each beam using the methods of Baumann and Rüschi (1970) and Huber et al. (2020) described in Section 2.3.4 for RC beams with shear reinforcement. However, a set of assumptions were necessary to determine the dowel action contributions for the specimens with jackets of groups 1 and 2 according to the models of Baumann and Rüschi (1970) and Huber et al. (2020).

Seeing as the models of Baumann and Rüsch (1970) and Huber et al. (2020) consider the number of layers of reinforcement, the resulting contribution to shear resistance was equal to the sum of the contributions of the RC beam and UHPC jacket. To recapitulate, the dowel contribution of a RC beam according to both of these models (V_{da}) is a function of the dowel cracking load ($V_{da,cr}$). Additionally, the dowel contribution (V_{da}) is limited by the load carrying capacity of the stirrups ($V_{da,max}$). The dowel cracking load ($V_{da,cr}$) is the product of the concrete tensile strength (f_{ct}), effective width of the beam (b_n) and effective length of the concrete in tension below the reinforcement bars (l_{da}). The effective length (l_{da}) is a function of the moment of inertia (I_v) of the reinforcement bars and concrete area below the bars. Assuming a perfect bond between the beam and jacket, the contributions of the beam and jacket were calculated using the same moment of inertia (I_v) and resulting effective length (l_{da}). Seeing as the reinforcement bars in the beams and jackets of this research were not vertically aligned, the method of calculation for the moment of inertia was simplified as the beam and jacket were treated as unbonded and calculated separately. The moment of inertia (I_v), which considered the reinforcement bars and concrete covers in both the beam and jacket, was determined based on Figure 5.19 as well as the following equation:

$$I_v = (I_c + I_s)_{Beam} + (I_c + I_s)_{Jacket} \quad (5.11)$$

where I_c is the moment of inertia of the concrete calculated using Eqn. 2.94 and I_s is the moment of inertia of the longitudinal bars calculated using Eqn. 2.95. The resulting effective length (l_{da}) used to calculate the contributions of both the beam and jacket was determined using Eqn. 2.92. The effective width of the beam (b_n) was determined using Eqn. 2.91, and the effective width of the jacket (b_n) was calculated using the following equation:

$$b_n = 2 \cdot t_s \quad (5.12)$$

where t_s is the side thickness of the UHPC jacket. It was assumed that the dowel splitting crack, which initially forms just above the top layer of longitudinal reinforcement, passes through the RC beam and side layers of the UHPC jacket. Therefore, only the side layers of the UHPC jacket were considered to contribute to shear resistance due to dowel action. The sum of the dowel action contributions of the RC beam and UHPC jacket was assumed to be

limited by the load carrying capacity of the stirrups plus that of the side layers of the UHPC jacket. The limiting dowel action shear force ($V_{da,max}$) was calculated as follows:

$$V_{da,max} = (A_{sw} \cdot f_{u,sw}) + \left(\frac{f_{ct}}{b_n \cdot l_{da}} \right) \quad (5.13)$$

where f_{ct} and b_n refer to the tensile strength and effective width of the UHPC jacket, respectively. Figure 5.20 demonstrates the estimated dowel action contributions of the RC beam and UHPC jacket throughout the experiment for specimen B2 according to the model of Baumann and Rüschi (1970). For all specimens with jackets, the beam and jacket typically account for about 66% and 34% of the dowel action contribution to shear resistance, respectively.

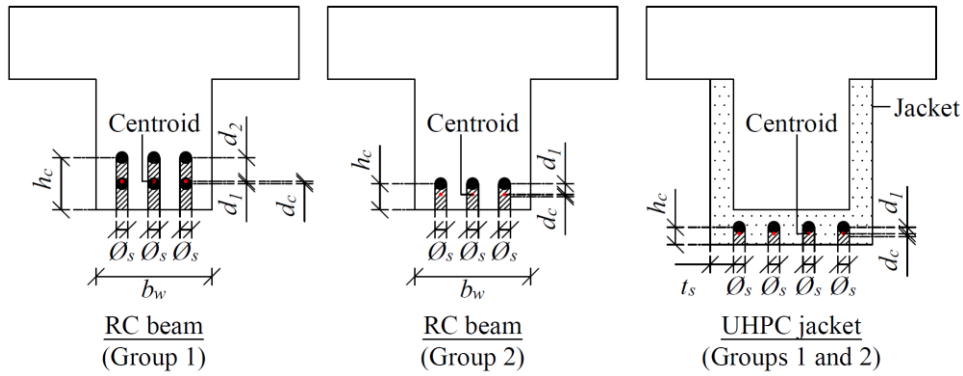


Figure 5.19 – Moment of inertia of longitudinal reinforcement bars and concrete area of RC beam and UHPC jacket. Adapted from Baumann and Rüschi (1970). Images reproduced with permission of the rights holder, German Committee for Structural Concrete (DAfStb).

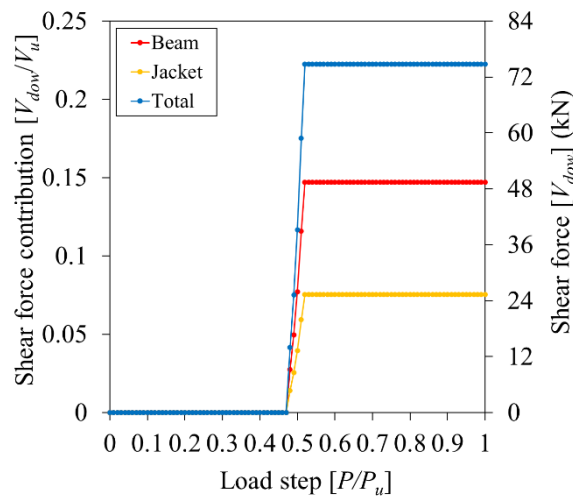


Figure 5.20 – Dowel action contributions of RC beam and UHPC jacket of specimen B2 based on model of Baumann and Rüschi (1970)

The estimated dowel action contributions to shear resistance for the specimens of groups 1 and 2 are presented in Figures 5.21 and 5.22, respectively. The results show the dowel action shear force (V_{dow}) and its contribution to the total shear resistance (V_{dow}/V_u) throughout each load step (P/P_u) until the ultimate load (P_u). The estimated results include both front and back FOVs of each specimen if available.

The dowel action contributions of the specimens of groups 1 and 2 in Figure 5.21 and 5.22, respectively, show a similar trend. The estimated dowel action contributions at the ultimate load ranged from 4.1% to 57.7% for the control specimens as well as the strengthened specimens. The model of Millard and Johnson (1984) gives the highest contributions for almost all specimens, with contributions appearing excessive and unrealistic when compared to the estimated dowel action contributions of similar RC beams from the literature (Campana et al., 2013; Huber et al., 2016; Pastore, 2020). The dowel action contributions given by the models of Walraven and Reinhardt (1981) and He and Kwan (2001) appear to be excessively high for some specimens like the front FOV of specimen B2, while they also appear quite low for others as in both FOVs of specimen B7. This trend was also observed in the dowel action contributions of Pastore (2020) for similar RC beams with shear reinforcement. The models of Baumann and Rüschi (1970) and Huber et al. (2020) give reasonable contributions for all specimens, with contributions at the ultimate load ranging around 20% to 40%. The model of Huber et al. (2020) gives comparatively higher contributions seeing as the dowel action shear force (V_{da}) for RC beams with transverse reinforcement is the sum of the dowel cracking load ($V_{da,cr}$) and stirrup contribution ($V_{da,s}$), which is calculated using Eqn. 2.100 when the vertical displacement (Δ) is greater than 0.08 mm. Unlike the approach of Huber et al. (2020), the Baumann and Rüschi (1970) model considers the dowel action shear force (V_{da}) to be equal to the maximum of the dowel cracking load ($V_{da,cr}$) and stirrup contribution ($V_{da,s}$) for RC beams with transverse reinforcement, which is calculated using Eqn. 2.96 when the vertical displacement (Δ) is greater than 0.08 mm. It can be seen in Figures 5.21 and 5.22, that both models give similar results up to a certain load step, which coincides with a vertical displacement (Δ) of 0.08 mm, where the contributions given by both models begin to then differ until the ultimate load is reached.

The dowel action contributions given by all of the considered models are dependent on the calculated vertical crack displacements (Δ) throughout loading. Therefore, reasonable differences in contributions between both FOVs of a single specimen are to be expected. The

largest percent differences between both FOVs at the ultimate load are seen in the contributions given by the model of Walraven and Reinhardt (1981). Percent differences at the ultimate load between both FOVs are excessively high as they range between 9.3% and 108.9%, which is equivalent to a difference of up to 120 kN. These unrealistic discrepancies are possibly linked to the dowel shear force reduction factor (ξ) that is considered by the model Walraven and Reinhardt (1981) that accounts for the axial tensile stress in the longitudinal reinforcement. The reduction factor (ξ) is a function of the calculated horizontal crack displacements (w), which differ for each FOV, hence likely causing relatively large discrepancies between the contributions of both FOVs. The percent differences of the dowel action contributions between both FOVs at the ultimate load are less for the models of Millard and Johnson (1984) and He and Kwan (2001), with differences ranging between 0.6% to 70.2%, which is equivalent to differences of 1 kN to 45 kN. There is no difference between the contributions at the ultimate load of both FOVS that are given by the model of Baumann and Rüschi (1970), and the percent differences are minimal as well overall between the contributions of both FOVs given by the model of Huber et al. (2020).

The dowel action contributions given by the model of Baumann and Rüschi (1970) were chosen to evaluate each specimen in Section 5.4. The model of Baumann and Rüschi (1970) is the most comprehensive model of those considered as it accounts for the diameter of the longitudinal reinforcement bars and the number of layers of reinforcement as well as the concrete cover. The model provides reasonable dowel action contributions that are consistent with similar beams from the literature, and the contributions at the ultimate load are also consistent between both FOVs for each specimen (Campana et al., 2013; Huber et al., 2016; Pastore, 2020). The contributions given by the model of Baumann and Rüschi (1970) were either the lower bound or fairly close to the lower bound of the considered dowel action models. The model was also recently adopted by Huber et al. (2016), Huber et al. (2020) and Pastore (2020) to estimate the dowel action contribution to shear resistance. The contributions of each FOV estimated using the model of Baumann and Rüschi (1970) were averaged in order to evaluate each specimen individually, and this is discussed later in Section 5.4.

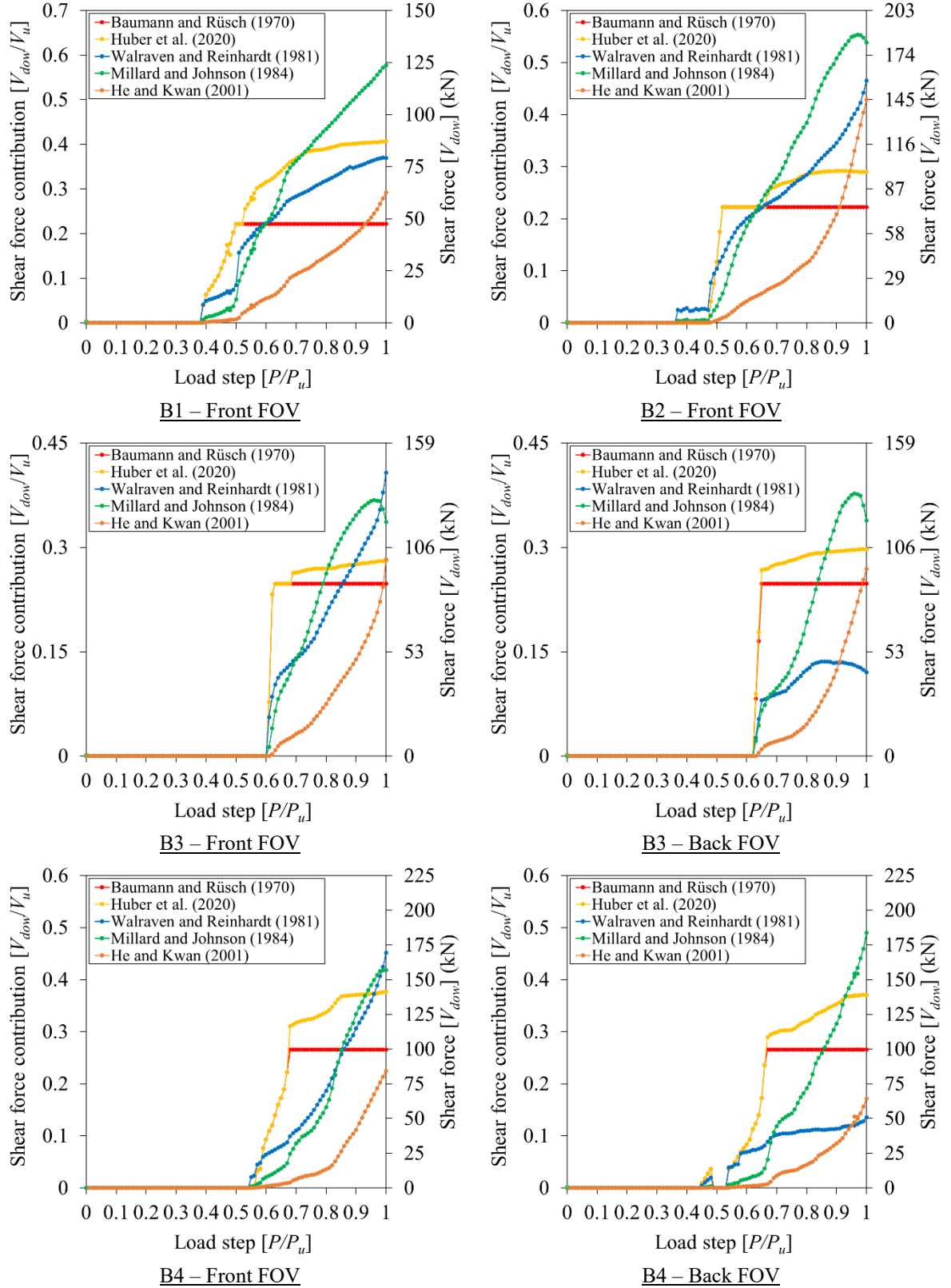
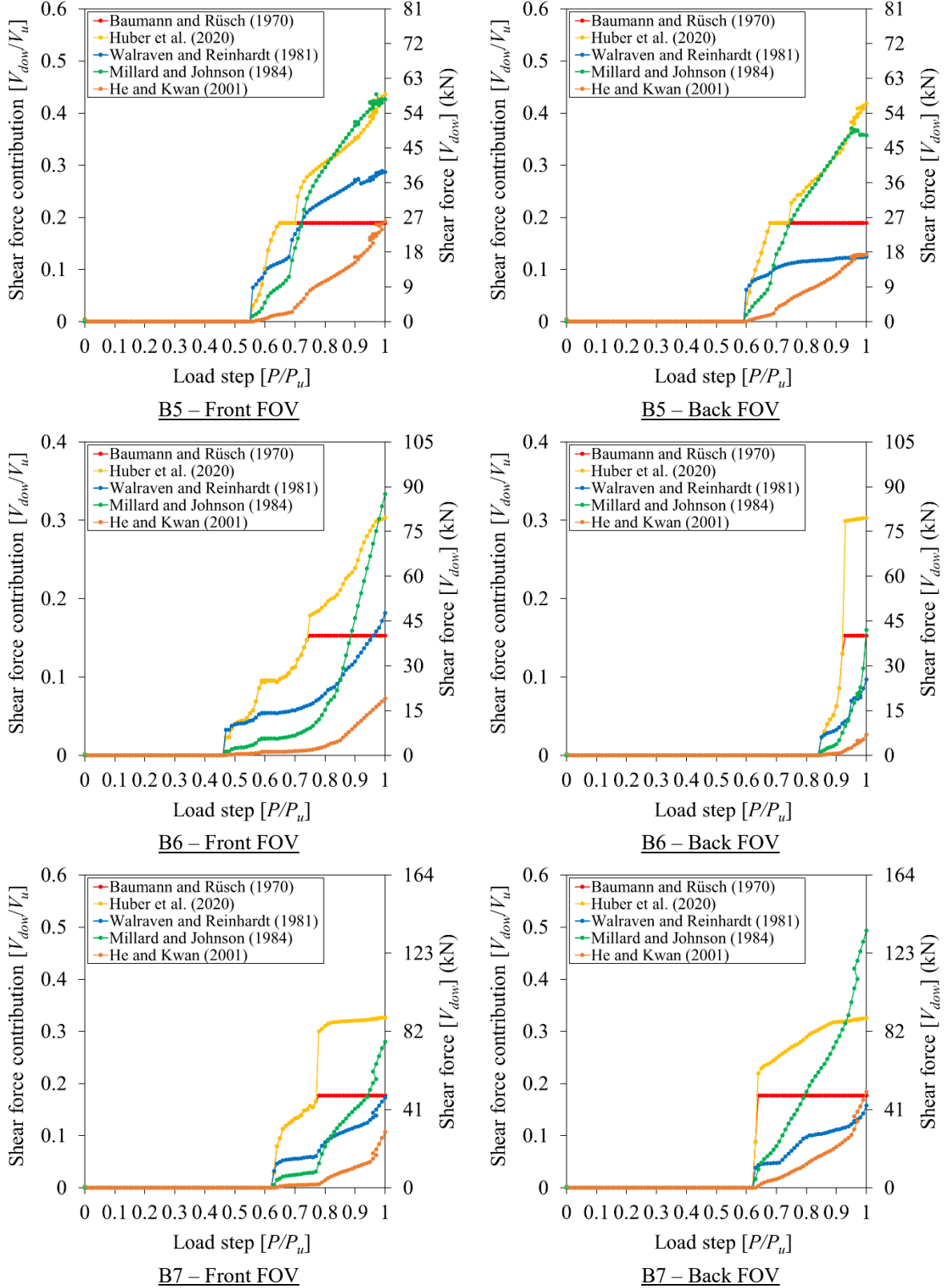


Figure 5.21 – Contribution of dowel action of group 1 specimens based on various theoretical models



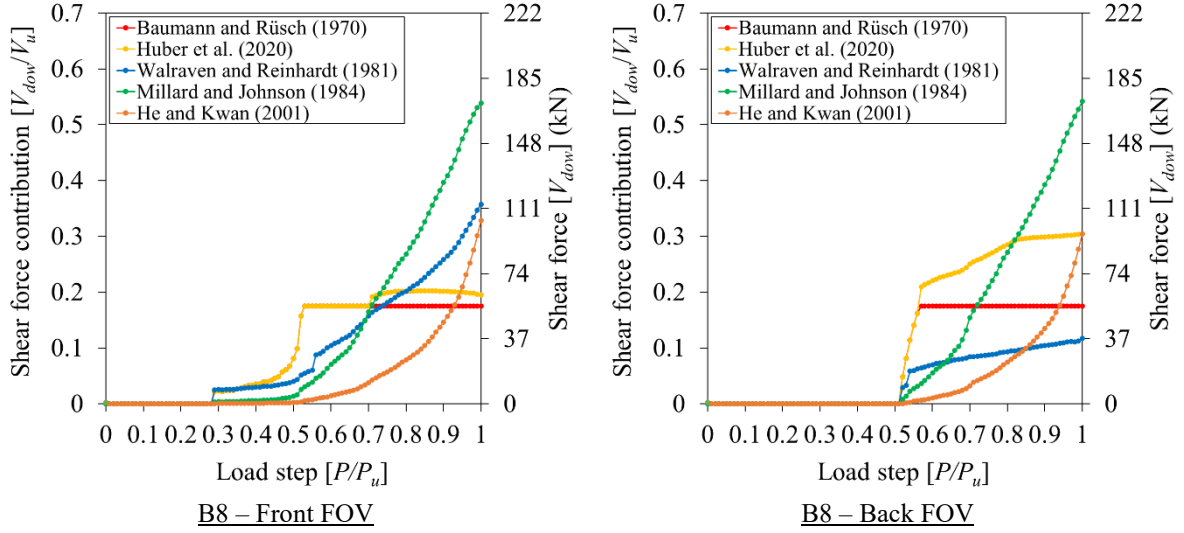


Figure 5.22 – Contribution of dowel action of group 2 specimens based on various theoretical models

5.3.5 STIRRUPS

The theoretical model of Sigrist (1995) was used to estimate the contribution to shear resistance of the stirrups crossing the critical shear crack (V_{stir}). The method of calculation for this constitutive model is explained in Section 2.3.5 along with the theoretical background. The stirrups contributing to shear resistance, or active stirrups, are numbered and highlighted in the free-body diagrams of each specimen shown in Figures 5.11 and 5.12. The contribution of the stirrups to the total shear resistance is equal to the sum of the contributions of each individual stirrup crossing the critical shear crack. Due to the difference in shape of the critical shear cracks featured in the front and back FOVs of each specimen, the active stirrups differ in each FOV of the beam for specimens B4 and B7 where two stirrups cross the critical crack in one FOV and a single stirrup crosses the critical crack in the other. Nonetheless, the contribution to shear resistance of the stirrups was estimated where possible for each available FOV of each specimen.

A couple of input parameters were required to determine the shear resistance of the stirrups according to Sigrist (1995). The vertical crack displacement (w_{sw}) located at the intersection between each contributing stirrup and the critical crack was required as an input to determine the stress resisted by each stirrup. The vertical crack displacements (w_{sw}) throughout the experiment were obtained by placing virtual gauges at each location of interest and using the DIC measurements. There were several cases where a virtual gauge could not be placed due to the location of interest being near the web/flange junction, which meant that the contribution

to shear resistance of certain stirrups could not be calculated. Cases where stirrup stresses could not be calculated were stirrup 1 in specimen B4 (Front FOV), stirrup 2 of specimen B6 (Back FOV) and stirrup 2 of specimen B8 (Back FOV) that are shown in Figures 5.11 and 5.12. The elastic strain hardening modulus (E_{sh}) of the transverse reinforcement was also required. It was determined using Eqn. 2.127 based on the experimental stress-strain response of the $\phi 6$ transverse reinforcement, and it was found to be equal to 820.41 MPa.

The shear resisted by the active stirrups was also determined using the strain measurements recorded by the fixed strain gauges to compare with the estimated value. The following equation was used to determine the shear resistance (V_{stir}):

$$V_{stir} = \sigma_{sw}(\varepsilon_{sw}) \cdot A_{sw} \quad (5.14)$$

where $\sigma_{sw}(\varepsilon_{sw})$ refers to the average experimental tensile stress-strain response of the $\phi 6$ transverse reinforcement presented in Figure 4.2, ε_{sw} is the strain measurement and A_{sw} is the area of the $\phi 6$ transverse reinforcement. Due to the difference in locations along the leg of each individual stirrup between where the stress was estimated and measured, the comparison between the estimated and measured results is only indicative. The locations of the strain gauges in relation to the points of intersection between each contributing stirrup and the critical shear crack are shown in the free-body diagrams of Figures 5.11 and 5.12 for each specimen.

The estimated contributions to shear resistance of the active stirrups for the specimens of groups 1 and 2 are presented in Figures 5.23 and 5.24, respectively. The results show the shear force of each stirrup (V_{stir}) and its contribution to the total shear resistance (V_{stir}/V_u) throughout each load step (P/P_u) until the ultimate load (P_u). The corresponding shear force of each stirrup determined using the fixed strain gauge measurements are also shown. The estimated results include both front and back FOVs of each specimen where available.

The estimated contributions of the stirrups in Figures 5.23 and 5.24 for the specimens of groups 1 and 2, respectively, show that all of the stirrups reached the yield point (38.08 kN). Pastore (2020) notes that yield can occur at crack widths of as little as 0.3 mm. Similar observations were made by Campana et al. (2013) and Huber et al. (2016). This was observed to be the case for most of the specimens of this research as well. Only a single stirrup was active for control

specimens B1 and B5, and their contributions to the total shear resistance at the ultimate load were equal to about 20% and 30%, respectively. This is reasonable when compared to similar RC beams from the literature (Campana et al., 2013; Huber et al., 2016; Pastore, 2020). The highest number of active stirrups was seen in specimen B2, and it included three active stirrups. Depending on the shapes of their critical shear cracks, either one or two stirrups were activated for all of the remaining specimens. The contributions of each stirrup accounted for about 12% and 15% of the total shear resistance for the strengthened specimens of groups 1 and 2, respectively. Figures 5.23 and 5.24 also show that the contributions given by the model of Sigrist (1995) match reasonably well overall with the shear force of each stirrup based on the fixed strain gauge measurements. The differences between the estimated and measured shear forces are likely due to their differences in location in relation to the critical shear crack opening. The contributions at the ultimate load given by the model Sigrist (1995) for both FOVs of a single specimen show minimal differences, although relatively larger differences can be observed throughout loading as in stirrup 1 shown in both FOVs of specimen B6. These discrepancies are dependent on the calculated vertical crack displacements (w_{sw}) throughout loading, which may differ for each FOV of a single specimen.

The contributions of the stirrups given by the model of Sigrist (1995) were chosen to evaluate each specimen in Section 5.4. The model provides reasonable contributions that are consistent with similar beams from the literature, and the contributions at the ultimate load are also consistent between both FOVs for each specimen (Campana et al., 2013; Huber et al., 2016; Pastore, 2020). The contributions of each FOV were averaged in order to evaluate each specimen individually, and this is discussed later in Section 5.4. For the particular case of specimen B4, the contribution of stirrup 1 was averaged using both FOVs and then doubled to account for the active stirrup appearing only in the front FOV (Stirrup 2) that could not be calculated as a virtual gauge could not be placed at the location of interest. The shear force based on the fixed strain gauge measurements indicates that stirrup 2 featured in the front FOV of specimen B4 reached its yield point, therefore increasing the contribution of stirrup 1 by two times was justified as stirrup 1 also yielded. The contributions of stirrup 2 of specimen B6 (FOV), stirrup 1 of specimen B7 (Back FOV) and stirrup 2 of specimen B8 (Front FOV) could only be calculated using a single FOV, therefore they were taken without averaging the contributions between both FOVs when evaluating each specimen in Section 5.4.

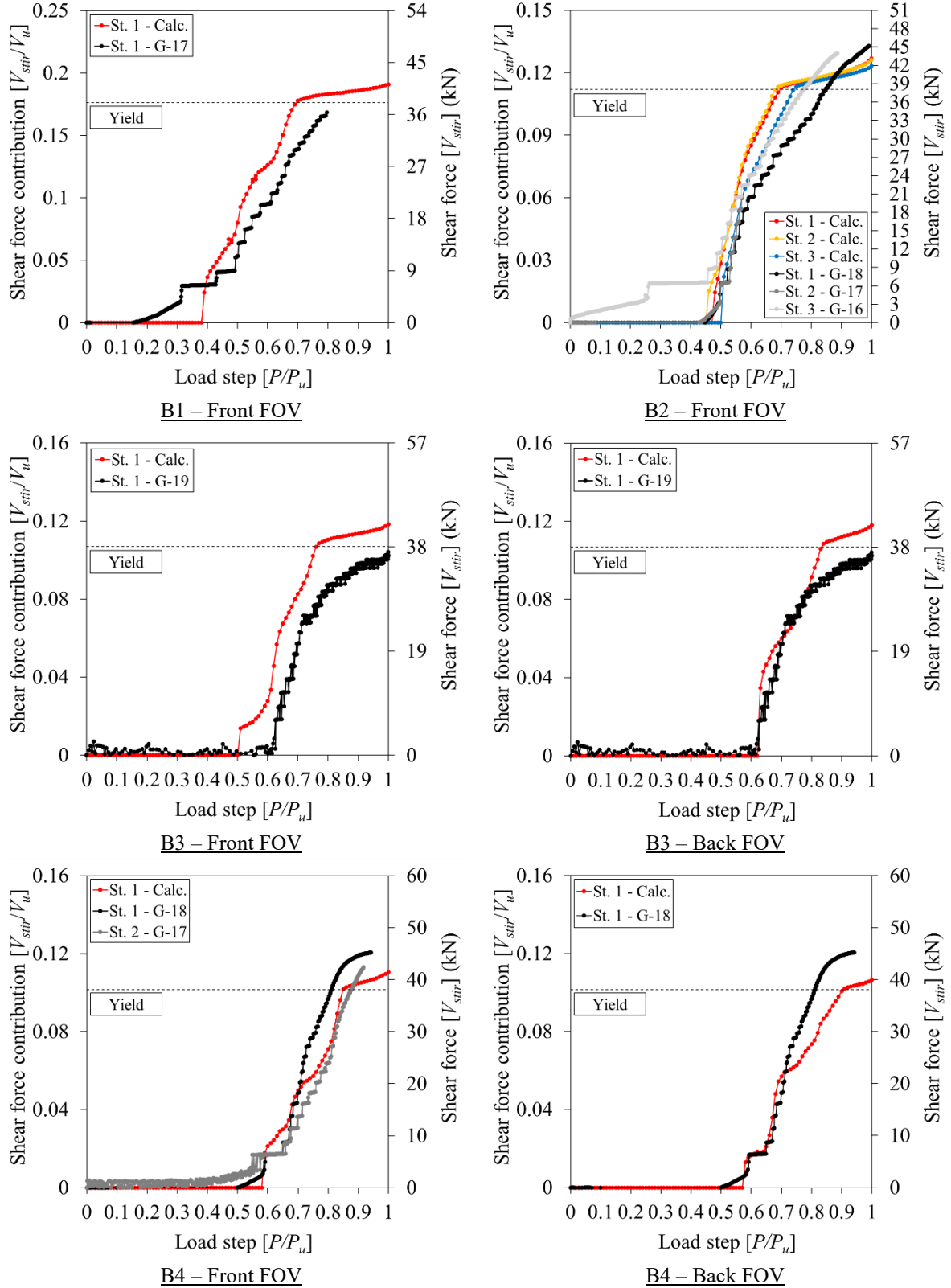
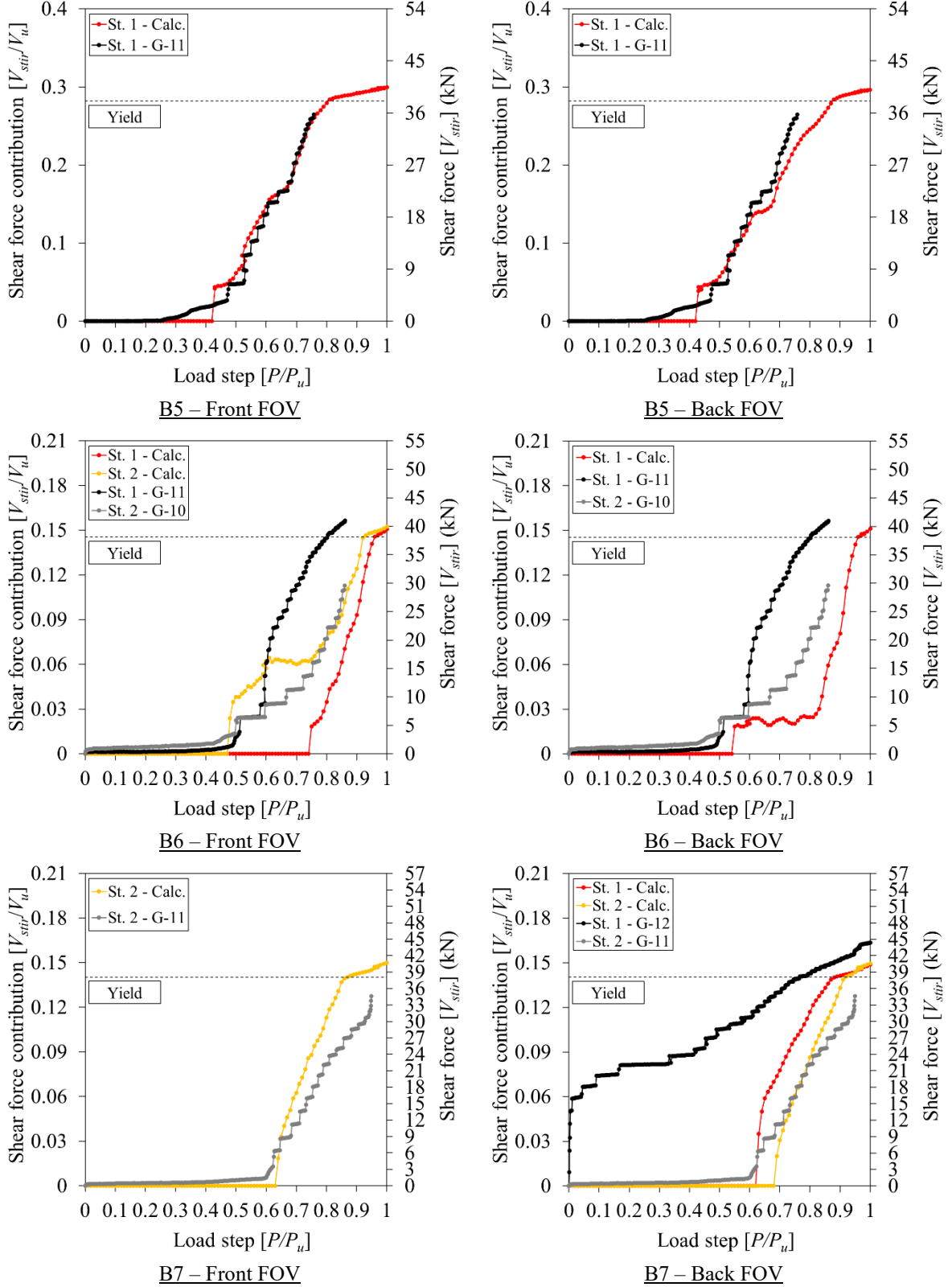


Figure 5.23 – Contribution of stirrups of group 1 specimens based on model of Sigrist (1995)



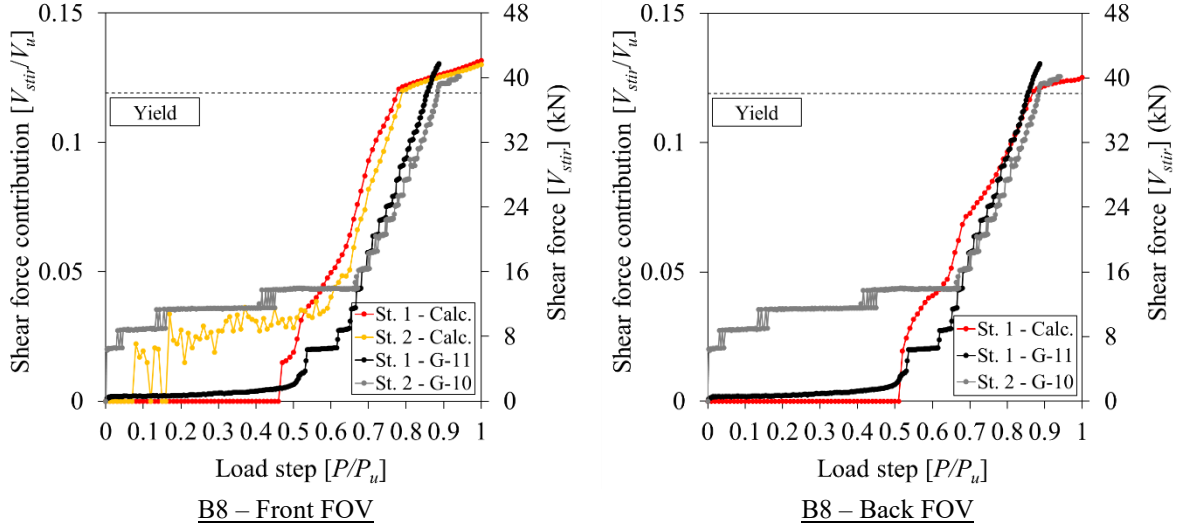


Figure 5.24 – Contribution of stirrups of group 2 specimens based on model of Sigrist (1995)

5.3.6 UNCRACKED COMPRESSION ZONE

The contribution to shear resistance of the uncracked compression zone (V_{cz}) was determined using the approach of Yang (2014) that is described in Section 2.3.6. It was also determined through vertical equilibrium of the internal shear forces shown in the free-body diagrams of Figures 5.11 and 5.12. The limitation of using vertical equilibrium to determine the contribution of the uncracked compression zone is that the result may be unreliable and questionable seeing as it is subject to errors in the estimations of the other shear resisting mechanisms. Therefore, the contribution given by vertical equilibrium represents the strength not explained by the other shear resisting mechanisms (Elwakeel et al., 2018). For comparison purposes, the ultimate shear strength of the uncracked compression zone was estimated for each specimen using the Multi-Action Shear Strength model (MASM) of Marí et al. (2015, 2016) and Cladera et al. (2016). The theoretical background of the MASM is described in Section 2.4.3 along with the method of calculation. The ultimate shear strength of the uncracked compression zone given by the MASM is an estimated upper bound for the contribution of the uncracked compression zone.

There are a couple of alternative approaches for determining the shear resisted by the uncracked compression zone as described in Section 2.3.6, however they were not applicable in this research. The method of calculation for the approach of Cavagnis et al. (2018a) requires the vertical and horizontal distances between the critical crack tip to the loading plate to be measured throughout the experiment using a DIC system in combination with virtual gauges.

This approach was not suitable given that virtual gauges could not be placed on the critical crack tips as they were located near the web/flange junction throughout the greater part of the experiment. The critical shear crack stemmed across the web area and reached the web/flange junction at relatively early loading stages for almost all specimens, and the critical crack only appeared to propagate into the flange after the ultimate load was reached and just before failure. The approach of Cavagnis et al. (2018b) was also not applicable due to an inadequate DIC image size that resulted in relatively inaccurate DIC measurements, with both Huber et al. (2016) and Pastore (2020) reporting a similar issue.

The shear resisted by the uncracked compression zone (V_{cz}) according to Yang (2014) was calculated using Eqn. 2.130. As described in Section 2.3.6, the model of Yang (2014) assumes that a crack spontaneously arises once the moment reaches the cracking moment where the vertical height of the crack (s_{cr}) remains constant until failure. The vertical height of the crack (s_{cr}) was determined using Eqn. 2.133. For the strengthened specimens, the reinforcement bars in the RC beam and UHPC jacket were included when determining the longitudinal reinforcement ratio (ρ_s), and the elastic modulus of NSC was considered when determining the modular ratio (n_e). It is important to note that the longitudinal reinforcement (ρ_l) and modular (α_e) ratios were calculated similarly when using the MASM to determine the ultimate shear strength of the uncracked compression zone of the strengthened specimens. The height of the compression zone (z_c) in the model of Yang (2014) was taken as the difference between the depth to the longitudinal reinforcement in tension and the vertical height of the crack (s_{cr}) as in Eqn. 2.131. Alternatively, the varying height of the compression zone (z_c) can be measured throughout the experiment using (DIC) as demonstrated by Pastore (2020). However, this approach was not suitable in this research given that virtual gauges could not be placed on the critical crack tips as they were located near the web/flange junction as previously mentioned.

The shear resisted by the uncracked compression zone (V_{cz}) according to vertical equilibrium was calculated using the following equation:

$$V_{cz} = V_{exp} - (V_{agg} + V_{fib} + V_{dow} + V_{stir}) \geq 0 \quad (5.15)$$

where V_{exp} is the experimental shear force, V_{agg} is the estimated shear force due to aggregate interlock, V_{fib} is the estimated shear force due to fibre bridging, V_{dow} is the estimated shear

force due to dowel action and V_{stir} is the shear force due to stirrups. The contribution to shear resistance of the uncracked compression zone was calculated for each individual specimen as the shear contributions of each available FOV were averaged throughout loading until the ultimate load was reached. The chosen contributions were given by the Rough Crack model of Gambarova and Karakoc (1983) for aggregate interlock, the experimental post-cracking stress response of UHPC for fibre bridging, the model of Baumann and Rüschi (1970) for dowel action, the model of Sigrist (1995) for stirrups. The contribution given by vertical equilibrium may result in a value of zero after the opening of the critical shear crack due to the sum of the other shear resisting mechanisms exceeding the experimental shear force.

The estimated contributions of the uncracked compression zone for the specimens of groups 1 and 2 are presented in Figures 5.25 and 5.26, respectively. The results show the shear force of the uncracked compression zone (V_{cz}) and its contribution to the total shear resistance (V_{cz}/V_u) throughout each load step (P/P_u) until the ultimate load (P_u). The contributions given by Yang (2014) and vertical equilibrium are quite different for most specimens. After the opening of the critical shear crack, the contributions given by the model of Yang (2014) are constant throughout loading until the ultimate load is reached. The contributions at the ultimate load given by the model of Yang (2014) range between 16.1% (B5) and 28.2% (B2, B3 and B4). The contributions of the strengthened specimens of group 1 are very close to one another as they each have the same amount of longitudinal reinforcement embedded in the beam and jacket, while those of group 2 were marginally different due to the different amount of longitudinal reinforcement provided in each jacket. The contributions given by the model of Yang (2014) are heavily dependent on the longitudinal reinforcement ratio seeing as Yang (2014) assumes that the flexural tension is resisted entirely by the longitudinal tensile reinforcement. The contributions at the ultimate load given by Yang (2014) appear to be reasonable overall when compared to similar beams from the literature (Campana et al., 2013; Huber et al., 2016; Pastore, 2020). Most of the contributions given by vertical equilibrium seem to be unrealistic throughout loading as there appears to be a sudden rise during the early stages of loading followed by a severe drop to zero percent for some specimens shortly after, followed by another sudden rise until the ultimate load is reached. This is due to the sum of the other shear resisting mechanisms exceeding the experimental shear force throughout certain load steps. The contributions at the ultimate load given by vertical equilibrium range between 11.2% and 51.7%, which correspond to specimens B8 and B1, respectively. Differences between the

contributions at the ultimate load given by the model of Yang (2014) and vertical equilibrium range between 19.2% and 74.0% for the specimens of group 1 and 28.2% and 71.8% for the specimens of group 2. In most specimens, the contribution at the ultimate load given by vertical equilibrium was greater than that given by the model of Yang (2014). The estimated contributions given by the model of Yang (2014) and vertical equilibrium do not exceed the ultimate shear strength of the uncracked compression zone given by the MASM. The contributions of the uncracked compression zone given by Yang (2014) were chosen to evaluate each specimen in Section 5.4.

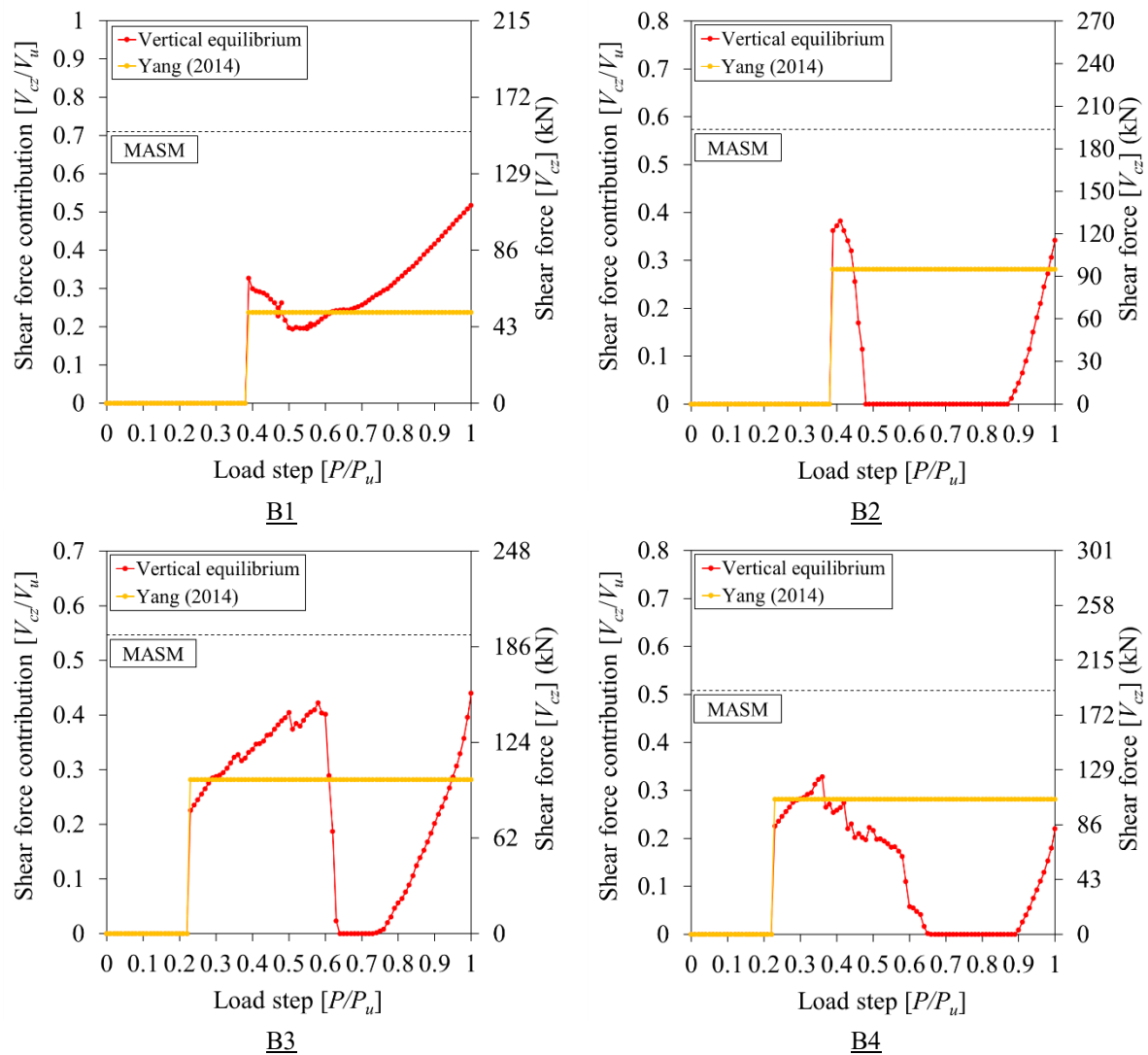


Figure 5.25 – Contribution of uncracked compression zone of group 1 specimens based on vertical equilibrium and Yang (2014)

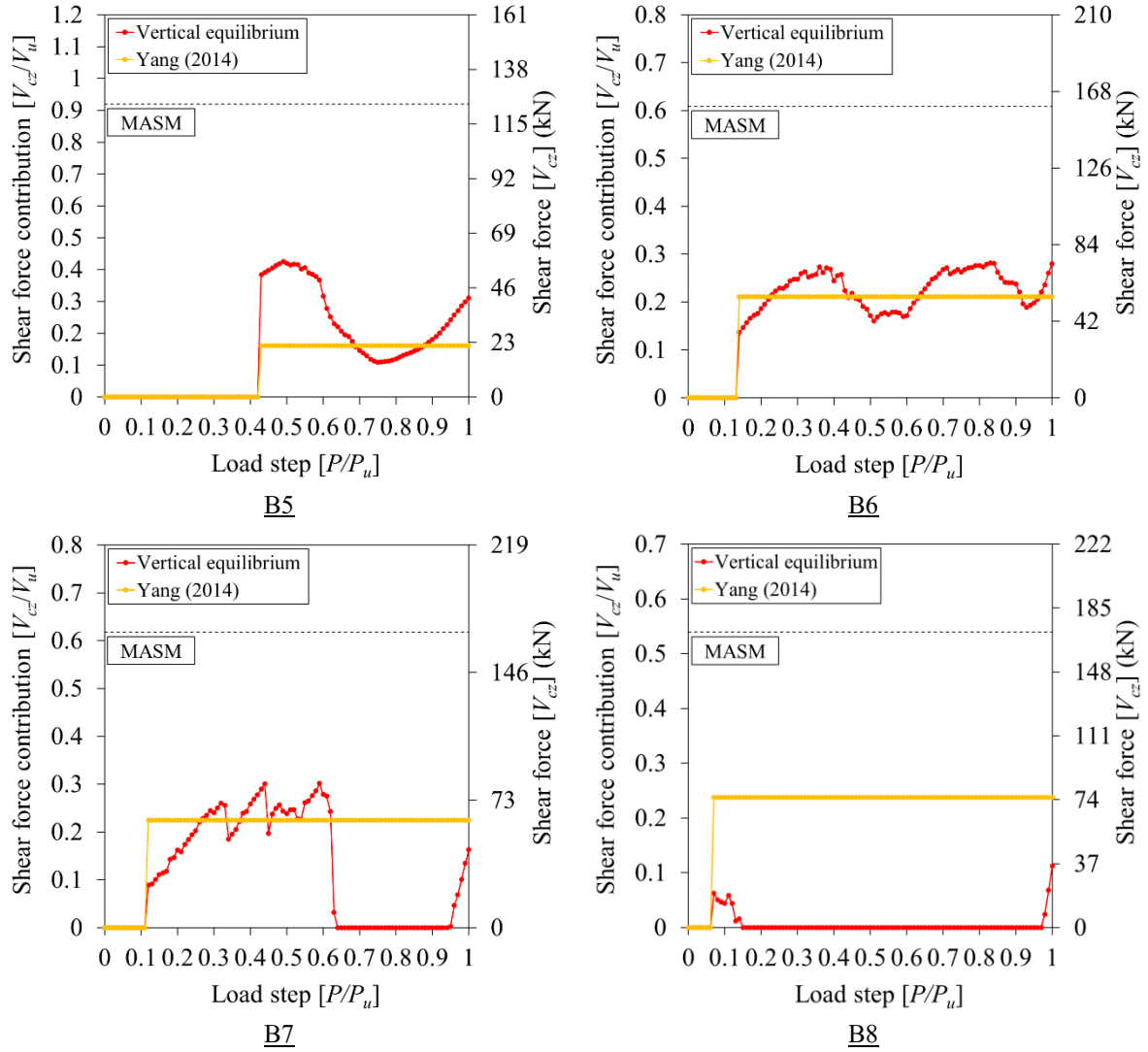


Figure 5.26 – Contribution of uncracked compression zone of group 2 specimens based on vertical equilibrium and Yang (2014)

5.4 EVALUATION OF SHEAR RESISTANCE MECHANISMS

5.4.1 INTRODUCTION

Each specimen was evaluated based on the estimated contributions of its shear resistance mechanisms to help identify the effect of the UHPC jackets. In order to evaluate each specimen accordingly, the results of a single model were selected to characterise each individual shear resistance mechanism. Based on the results of each individual shear resistance mechanism presented in Section 5.3, the models that were selected include the Rough Crack model of Gambarova and Karakoc (1983) for aggregate interlock, the experimental post-cracking stress response of UHPC for fibre bridging, the model of Baumann and Rüsç (1970) for dowel

action, the model of Sigrist (1995) for stirrups and the model of Yang (2014) for the uncracked compression zone. The shear resistance of each individual specimen was calculated by averaging the shear resistance contributions of each FOV where available.

5.4.2 GROUP 1 SPECIMENS

The sum of the contributions of the various shear resisting mechanisms are shown for each specimen of group 1 in Figure 5.27. The critical shear cracks begin to open around the load step $0.4P_u$ for specimens B1 and B2, and they begin to open around the load step $0.2P_u$ for specimens B3 and B4. For control specimen B1, the aggregate interlock contribution appears to be reasonably more dominant when compared to the specimens with jackets. It is evident that the aggregate interlock contribution is rather insignificant in the specimens with jackets. This is likely due to the crack openings that occurred in the specimens with jackets that were fairly large in relation to the coarse aggregate in the beams, which in turn reduces the contact between interlocked aggregate. The dowel action contribution seems to be reasonably significant for all specimens, with a similar level of contribution to the total shear resistance for all specimens as well. It appears that the uncracked compression zone contribution is significant throughout loading in control specimen B1, and it appears to be as equally significant for the specimens with jackets. The contributions of the stirrups vary between specimens due to the varying number of stirrups that are intercepted by the critical shear crack. The stirrup contribution of control specimen B1 is reasonable given that a single stirrup was active. The stirrup contribution of specimen B2 is exceptionally high due to the three active stirrups. Only a single stirrup was active for specimen B3, while two stirrups were active for specimen B4, which explains the comparatively higher stirrup contribution for specimen B4.

It is evident that the contribution of fibre bridging plays an important role throughout loading in the specimens with jackets. For specimen B2, the contribution of fibre bridging appears to be the most significant, comparatively, especially soon after the critical shear crack begins to open. The estimated fibre bridging contribution at its peak accounts for almost 100% of the experimental shear force at around the load step $0.5P_u$, which seems excessive given that there are other contributing shear resisting mechanisms as well. The fibre bridging contribution of specimen B2 reduces quickly after this load step until the ultimate load is reached due to the gradual widening of the critical shear crack. Nonetheless, the sum of the contributions of

specimen B2 are overestimated throughout loading as a result of the high fibre bridging contribution as well as the high stirrup contribution, which included three contributing stirrups. For specimens B3 and B4, the fibre bridging contributions at their peak both account for about 60% of their respective experimental shear forces at around the load steps $0.6P_u$ and $0.8P_u$, respectively. Much like specimen B2, the reduction in the fibre bridging contribution for specimens B3 and B4 is gradual as the ultimate load is reached and as the critical shear crack begins to widen. The sum of the contributions of specimens B3 and B4 become overestimated once the peak fibre bridging contribution occurs as in specimen B2. However, the sum of the contributions become much closer to the experimental shear force towards the latter stages of loading due to the reduction in the fibre bridging contribution caused by the widening of the critical shear crack.

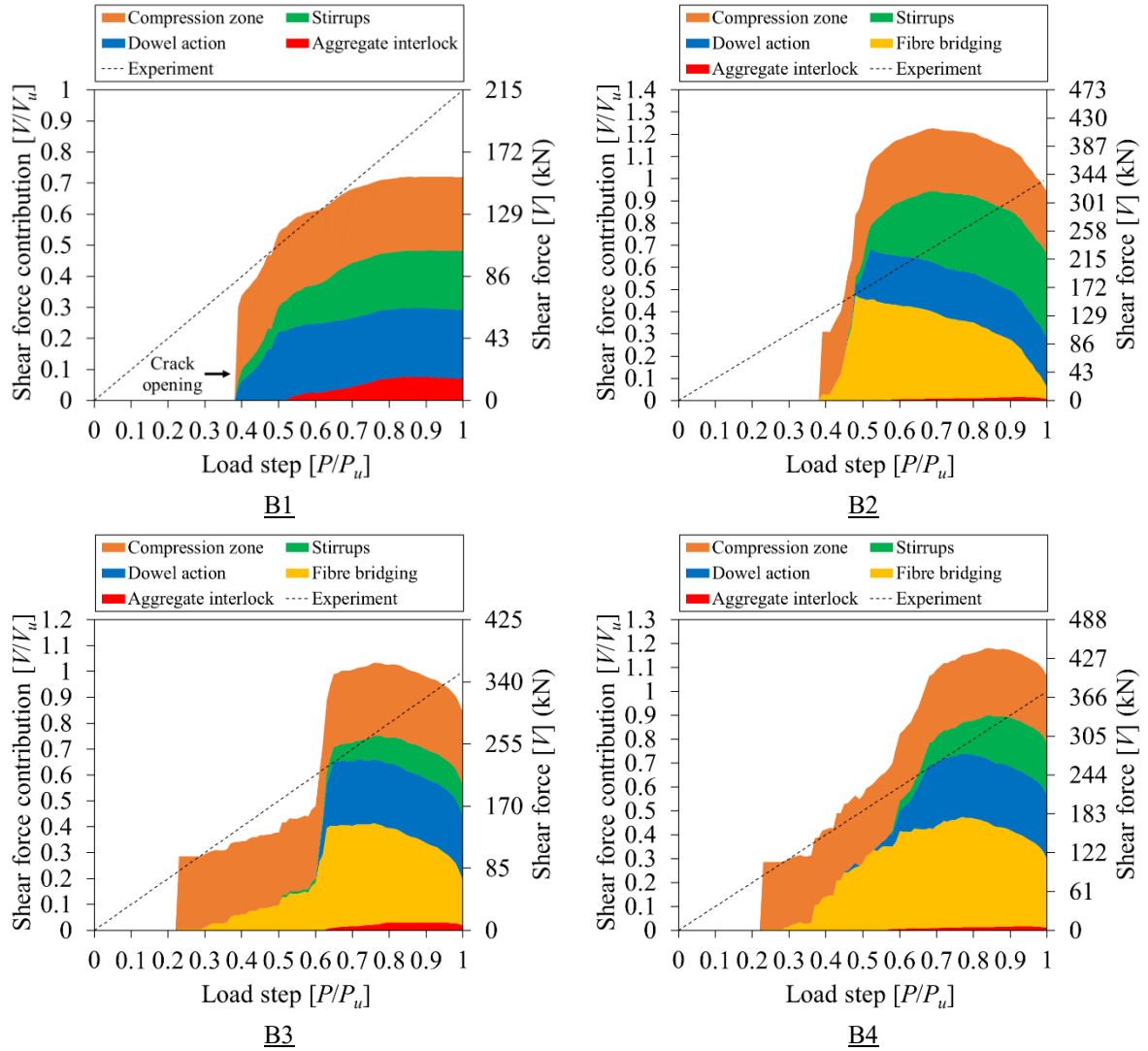


Figure 5.27 – Contributions of shear resistance mechanisms of group 1 specimens

The contributions of the various shear resisting mechanisms at the ultimate load are shown for each specimen of group 1 in Figure 5.28. The shear forces at the ultimate load of each shear resisting mechanism are presented in Table 5.1 along with the experimental shear forces (V_u). The differences between the shear forces of the strengthened specimens and control specimen B1 are also shown in Table 5.1. Table 5.1 includes the contribution of the uncracked compression zone (V_{cz}) given by vertical equilibrium for comparison purposes only as it represents the strength not explained by the other shear resisting mechanisms.

Figure 5.28 shows that there are inaccuracies in the sum of the contributions of each shear resisting mechanism at the ultimate load when compared to the experimental shear force. The sum of the contributions accounts for approximately 72.1%, 94.0%, 84.3% and 106.2% of the experimental shear force for specimens B1, B2, B3 and B4, respectively. This is equivalent to differences of 60.1 kN, 20.2 kN, 55.8 kN and 23.3 kN in specimens B1, B2, B3 and B4, respectively, that are not explained by the estimated contributions of each shear resisting mechanism. These inaccuracies are likely due to the estimated contributions of aggregate interlock and dowel action seeing as they are sensitive to the model choice as seen in Sections 5.3.2 and 5.3.4, respectively (Pastore, 2020). The inaccuracies may also be due to the estimated contribution of the uncracked compression zone as the results given by the chosen model of Yang (2014) and vertical equilibrium are quite different as shown in Table 5.1.

The aggregate interlock contribution of 7.0% seems reasonable for control specimen B1 given the amount of transverse reinforcement and the relatively large crack openings (Campana et al., 2013; Huber et al., 2016; Pastore, 2020). The aggregate interlock shear force (V_{agg}) was reduced up to 12.71 kN for the specimens with jackets, with contributions ranging between 0.7% and 2.0%. This is likely due to the fairly large crack openings that occurred, which helped reduce the contact between interlocked aggregate.

The dowel action contribution appears to be relatively similar for control specimen B1 and the jacketed specimens, with contributions ranging between 22.2% and 26.6%. The dowel action shear forces (V_{dow}) in Table 5.1 increase by 27.49 kN from control specimen B1 to specimen B2, which includes the 20 mm thick UHPC jacket with 4 ϕ 20 longitudinal reinforcement bars. To recapitulate, all of the jackets included a constant amount of longitudinal reinforcement bars. The dowel shear force (V_{dow}) increases by about 13 kN as the jacket side thickness

increases by 10 mm as in specimens B2 to B3 and in specimens B3 to B4. The increases in the dowel action shear forces between the control specimen and strengthened specimens are due to the additional longitudinal reinforcement bars embedded in the jacket as well as the side layers of the UHPC jacket.

The uncracked compression zone contribution for control specimen B1 is the lowest at 23.8%, and the contributions for the strengthened specimens are comparatively higher as they are all equal to 28.2%. The increase in the contribution of the uncracked compression zone from control specimen B1 to the strengthened specimens is equivalent to between 44.17 kN (B2) and 54.84 kN (B4). As discussed in Section 5.3.6, the contributions of the uncracked compression zone are heavily influenced by the longitudinal reinforcement ratio. Consequently, the uncracked compression zone contributions of the strengthened specimens do not significantly vary, and they are higher than that of the control specimen. On the other hand, the stirrup contributions vary due to the shapes of the critical shear cracks and the number of stirrups intercepted by the critical shear crack, with each active stirrup resisting about 40 kN.

Interestingly, the fibre bridging contribution is only 5.2% of the total shear resistance at the ultimate load for specimen B2, which is equivalent to 17.54 kN. However, the fibre bridging contributions significantly increase as the jacket side thickness increase. The fibre bridging contributions increase from 5.2% to 17.5% for the 20 mm to 30 mm jackets of specimens B2 and B3, respectively, which is equivalent to an increase of 44.32 kN. From the 30 mm to 40 mm thick jackets of specimens B3 and B4, respectively, the fibre contributions increase from 17.5% to 28.6%, which is equivalent to an increase of 45.63 kN. The increases in fibre contribution are reasonable given the increases in the jacket side thickness.

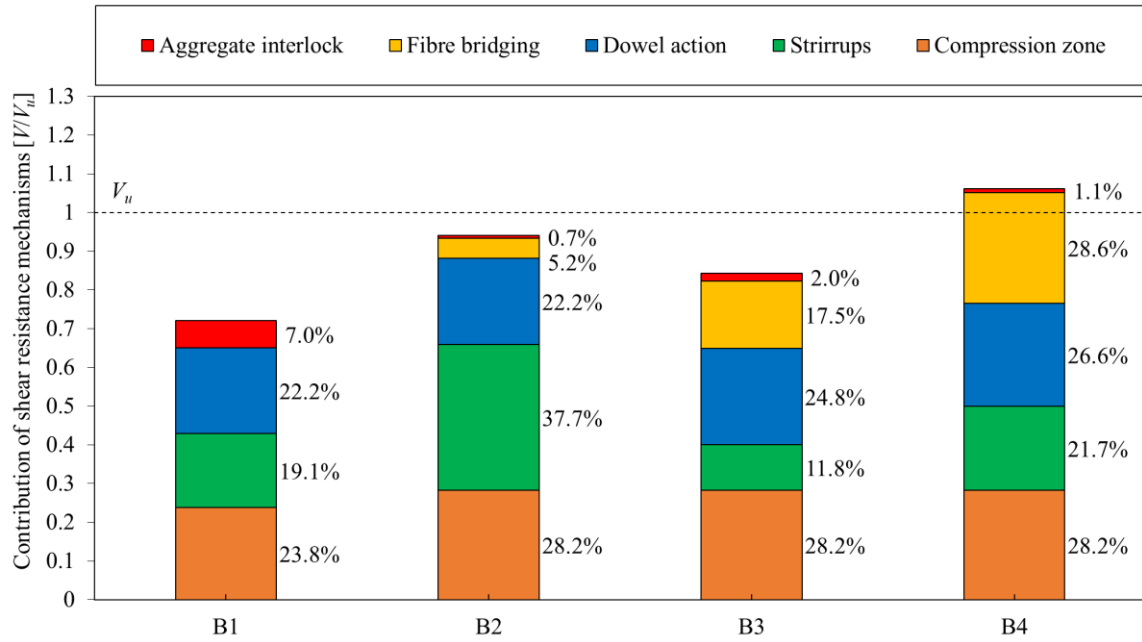


Figure 5.28 – Contributions of shear resistance mechanisms at the ultimate load of group 1 specimens

Table 5.1 – Contributions of shear resistance mechanisms at the ultimate load of group 1 specimens

Shear resistance mechanism		B1	B2	B3	B4
Aggregate interlock (V_{agg}) [kN]		15.12	2.4 (-12.71)	6.99 (-8.13)	4.06 (-11.06)
Fibre bridging (V_{fib}) [kN]		0	17.54 (+17.54)	61.86 (+61.86)	107.49 (+107.49)
Dowel action (V_{dow}) [kN]		47.65	75.14 (+27.49)	87.87 (+40.22)	99.90 (+52.25)
Strirrups (V_{stir}) [kN]		41.01	127.22 (+86.21)	41.89 (+0.88)	81.56 (+40.55)
Compression zone ^A (V_{cz}) [kN]		51.13	95.30 (+44.17)	99.97 (+48.84)	105.97 (54.84)
Compression zone ^B (V_{cz}) [kN]		111.18	115.53 (+4.35)	155.78 (+44.60)	82.66 (-28.52)
Experiment (V_u) [kN]		214.96	337.84 (+122.28)	354.39 (+139.43)	375.66 (+160.70)

[Note] – () Refers to the difference between the shear forces of the strengthened specimen and the control specimen;

^A Contribution of uncracked compression zone (V_{cz}) given by the model of Yang (2014); ^B Contribution of uncracked compression zone (V_{cz}) given by vertical equilibrium

5.4.3 GROUP 2 SPECIMENS

The sum of the contributions of the various shear resisting mechanisms are shown for each specimen of group 2 in Figure 5.29. The critical shear cracks begin to open at around the load step $0.4P_u$ for control specimen B5, and they begin to open much earlier for the specimens with jackets, which occurs around the load step $0.1P_u$. For control specimen B5, the aggregate

interlock contribution is quite significant compared to the companion specimens with jackets. As in the case of the strengthened specimens of group 1, this is likely due to the larger crack openings that occurred in the strengthened specimens of group 2, which reduced the contact between interlocked aggregate. The dowel action contribution seems reasonably significant for all specimens with the level of contribution to the total shear resistance similar for all specimens. The uncracked compression zone contribution is moderately dominant throughout loading for all specimens. The level of contributions of the stirrups to the total shear resistance are quite similar for all specimens. The stirrup contribution of control specimen B5 is reasonable given that only a single stirrup was active. The stirrup contributions of specimens B6, B7 and B8 are similar seeing as two stirrups were active for each specimen.

It appears that the contribution of fibre bridging is far more significant in the strengthened specimens of group 2 than those of group 1 that were discussed in Section 5.4.2. For specimen B6, the contribution of fibre bridging appears to be the least significant when compared to the other strengthened specimens of group 2. The peak fibre bridging contribution occurred at around the load step $0.5P_u$, and the fibre bridging contribution accounted for about 55% of the total shear resistance prior to gradually reducing as the ultimate load was reached. For specimens B7 and B8, the fibre bridging contributions seem excessively high at their peaks, which both occur around the load step $0.6P_u$. At their peaks, the fibre bridging contributions account for 80% and 148% of the experimental shear forces for specimens B7 and B8, respectively. These contributions seem overestimated, more so in the extreme case of specimen B8. The sum of the contributions of specimens B7 and B8 become overestimated once the peak fibre bridging contribution occurs. The fibre bridging contributions for all of the strengthened specimens gradually reduce after the load step corresponding to the peak fibre bridging contribution until the ultimate load is reached due to the gradual widening of the critical shear crack. Consequently, the sum of the contributions become much closer to the experimental shear force towards the latter stages of loading, specifically in specimens B7 and B8.

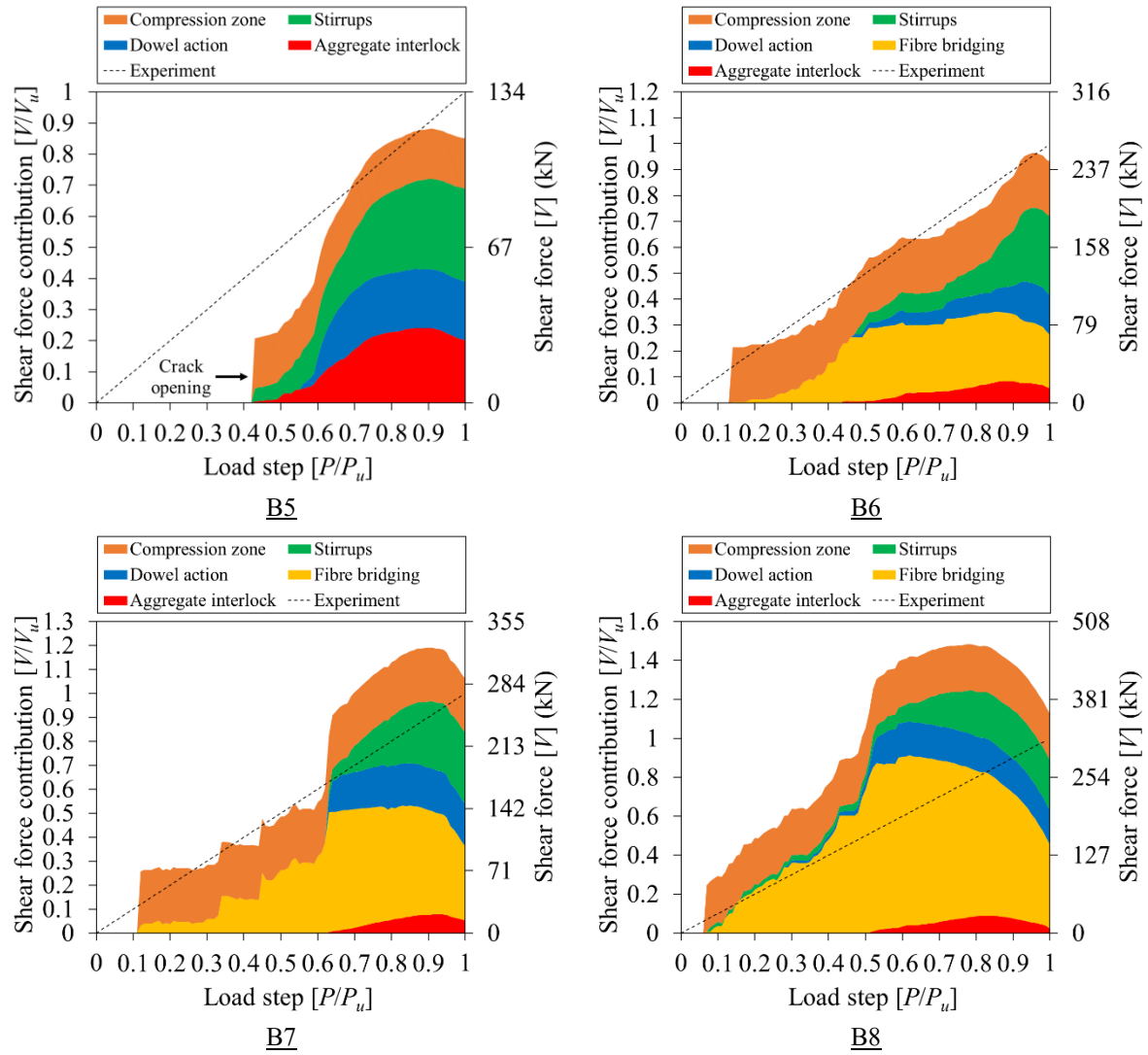


Figure 5.29 – Contributions of shear resistance mechanisms of group 2 specimens

The contributions of the various shear resisting mechanisms at the ultimate load are shown for each specimen of group 2 in Figure 5.30. The shear forces at the ultimate load of each shear resisting mechanism are presented in Table 5.2 along with the experimental shear forces (V_u). The differences between the shear forces of the strengthened specimens and control specimen B5 are also shown in Table 5.2. Table 5.2 includes the contribution of the uncracked compression zone (V_{cz}) given by vertical equilibrium for comparison purposes only as it represents the strength not explained by the other shear resisting mechanisms.

Figure 5.30 highlights the inaccuracies in the sum of the contributions of each shear resisting mechanism at the ultimate load when compared to the experimental shear force. The sum of the contributions accounts for approximately 85.0%, 93.1%, 106.2% and 112.6% of the experimental shear force for specimens B5, B6, B7 and B8, respectively. This is equivalent to

differences of 20.1 kN, 18.2 kN, 17.0 kN and 39.9 kN in specimens B5, B6, B7 and B8, respectively, that are not explained by the estimated contributions of each shear resisting mechanism. As discussed in Section 5.4.2, these inaccuracies are likely due to the estimated contributions of aggregate interlock and dowel action as they were found to be sensitive to the model choice. The inaccuracies may also be influenced by the estimated contribution of the uncracked compression zone seeing as the contributions given by the chosen model of Yang (2014) and vertical equilibrium are quite different as shown in Table 5.2.

The aggregate interlock contribution of control specimen B5 is 20.2%, which is significantly greater than the contribution of control specimen B1 of group 1 (7.0%). Both control specimens had similar fairly smooth crack profiles with regions of the critical shear cracks having steep inclination angles. Therefore, the differences in contributions may be due to differences in the amount of transverse reinforcement in the critical shear span as the RC beam specimens of group 2 have a lesser amount, comparatively. This may have caused the aggregate interlock action to be more dominant overall in the specimens of group 2 (Campana et al., 2013; Pastore, 2020). Much like the strengthened specimens of group 1, the aggregate interlock contributions of the strengthened specimens of group 2 were much less significant than that of the control specimen. The aggregate interlock shear force (V_{agg}) was reduced up to 19.81 kN for the specimens with jackets of group 2, with contributions ranging between 2.3% and 5.6%. This is likely due to the fairly large crack openings that occurred in the specimens with jackets that helped reduce the contact between interlocked aggregate.

The dowel action contribution of control specimen B5 is 18.9% (25.40 kN), which is less than that of control specimen B1 of group 1 that was equal to 22.2% (47.65 kN). This difference is caused by the RC beam specimens of group 1 having an additional layer of 3 ϕ 20 longitudinal reinforcement bars. Much like the specimens of group 1, the level of dowel action contributions of the specimens of group 2 appear to be relatively similar when comparing control specimen B5 to the specimens with jackets. The dowel action contributions range between 15.3% and 18.9%. The dowel action shear forces (V_{dow}) in Table 5.2 show increases of 14.86 kN, 23.00 kN and 30.31 kN when comparing specimens B6, B7 and B8, respectively, to control specimen B5. To recapitulate, the amount of longitudinal reinforcement embedded in the UHPC jackets varies as 3 ϕ 20, 4 ϕ 20 and 5 ϕ 20 longitudinal reinforcement bars were embedded in the jackets of specimens B6, B7 and B8, respectively. The increases in the dowel action shear forces

between the control specimen and strengthened specimens are likely due to the additional longitudinal reinforcement bars embedded in the jacket as well as the side layers of the UHPC jacket.

Much like the trend seen in the specimens of group 1, the uncracked compression zone contribution for the control specimen of group 2 is the lowest, and the contributions for the strengthened specimens are comparatively higher. The uncracked compression zone contribution of control specimen B5 (16.1%) is less than that of control specimen B1 of group 1 (23.8%). Figure 5.30 shows that the contributions of the uncracked compression zone are similar for the strengthened specimens as they are equal to 21.1% (55.2 kN), 22.5% (61.49 kN) and 23.8% (75.52 kN) for specimens B6, B7 and B8, respectively. The marginal increase in the uncracked compression zone contribution between the strengthened specimens is likely due to the increase in the amount of longitudinal reinforcement provided in the jacket.

Much like the dowel action contributions, the contributions of the stirrups are very similar when comparing control specimen B5 to the specimens with jackets. Although the shapes of the critical shear cracks vary, the same number of stirrups were active in the specimens with jackets. Only a single stirrup was intercepted by the critical shear crack for control specimen B5, while two stirrups were intercepted in each strengthened specimen.

The fibre bridging contribution is more dominant in the strengthened specimens of group 2 than group 1. The fibre bridging contribution of specimen B6, with the thinnest jacket, is 20.7% (54.46 kN). The fibre bridging contributions significantly increase as the jacket side thickness increase. The fibre bridging contributions increase from 20.7% to 30.8% for the 20 mm to 30 mm jackets of specimens B6 and B7, respectively, which is equivalent to an increase of 29.71 kN. From the 30 mm to 40 mm thick jackets of specimens B7 and B8, respectively, the fibre contributions increase from 30.8% to 43.1%, which is equivalent to an increase of 52.79 kN. Much like for the strengthened specimens of group 1, the increases in fibre contribution are reasonable given the increases in the jacket side thickness.

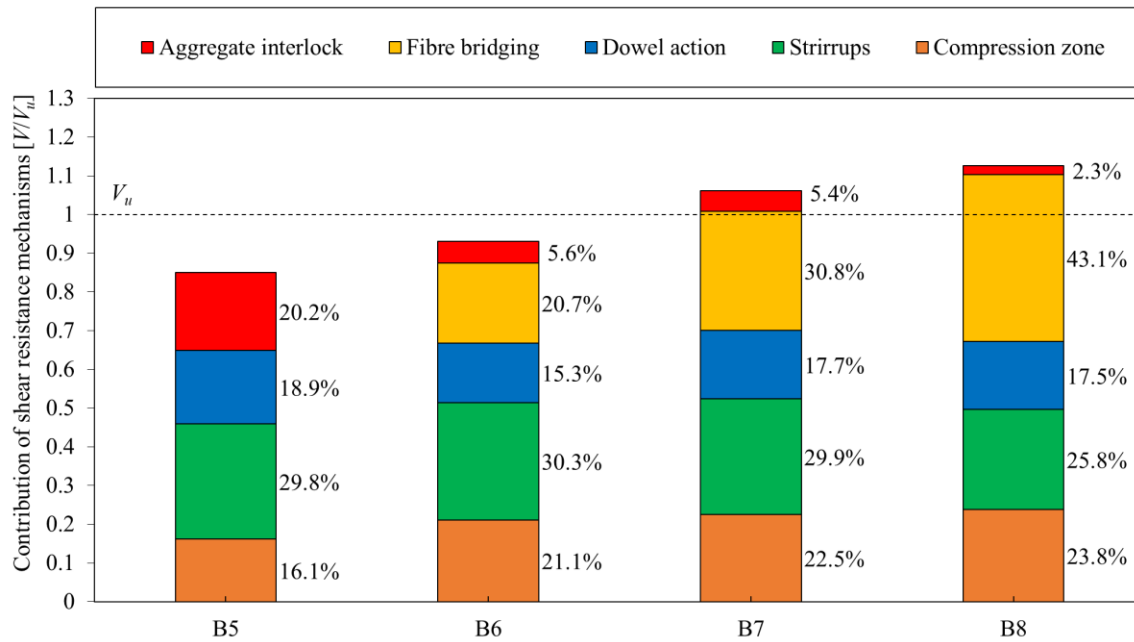


Figure 5.30 – Contributions of shear resistance mechanisms at the ultimate load of group 2 specimens

Table 5.2 – Contributions of shear resistance mechanisms at the ultimate load of group 2 specimens

Shear resistance mechanism		B5	B6	B7	B8
Aggregate interlock (V_{agg}) [kN]		27.12	14.82 (-12.30)	14.71 (-12.41)	7.31 (-19.81)
Fibre bridging (V_{fib}) [kN]		0	54.46 (+54.46)	84.17 (+84.17)	136.96 (+136.96)
Dowel action (V_{dow}) [kN]		25.40	40.26 (+14.86)	48.40 (+23.00)	55.71 (+30.31)
Strirrups (V_{stir}) [kN]		39.95	79.78 (+39.83)	81.64 (+41.69)	82.10 (+42.15)
Compression zone ^A (V_{cz}) [kN]		21.63	55.52 (+33.89)	61.49 (+39.86)	75.52 (53.90)
Compression zone ^B (V_{cz}) [kN]		41.71	73.74 (+32.03)	44.53 (+2.82)	35.61 (-6.10)
Experiment (V_u) [kN]		134.17	263.07 (+128.90)	273.45 (+139.28)	317.69 (+183.52)

[Note] – () Refers to the difference between the shear forces of the strengthened specimen and the control specimen;

^A Contribution of uncracked compression zone (V_{cz}) given by the model of Yang (2014); ^B Contribution of uncracked compression zone (V_{cz}) given by vertical equilibrium

5.5 CONCLUSIONS

This chapter covered the contributions of the shear resistance mechanisms of each beam specimen. The shear resistance mechanisms that were considered include aggregate interlock, fibre bridging, dowel action, stirrups and the uncracked compression zone. The aggregate interlock and stirrups resistance mechanisms were attributed to the RC beam, and the fibre

bridging resistance mechanism was attributed to the UHPC jacket. The dowel action shear resistance mechanism was attributed to both the RC beams and UHPC jackets as they were both embedded with longitudinal reinforcement in tension. The shear resistance mechanisms were estimated by substituting the calculated crack kinematics of the critical shear cracks into various models from the literature. The crack kinematics of each specimen's critical shear crack was calculated throughout the experiment using the DIC measurements of the cracked surfaces. The adopted method of calculation, which is described in Section 5.2.2, involved treating the investigated crack as two separate rigid bodies located on each side of crack with each rigid body behaving locally.

The main conclusions of this chapter are as follows:

- The crack opening and sliding displacement responses of the critical shear cracks featured in both FOVs of the specimens with jacket significantly vary for most cases.
- The Two-Phase models based on the crack kinematics of Walraven (1980), Guidotti (2010) and Ulaga (2003) significantly overestimate the aggregate interlock contributions in specimens with steep inclination angles where the crack sliding (s) is greater than or equal to the crack opening (w).
- The aggregate interlock contribution is linked to the amount of shear reinforcement. The aggregate interlock contribution at the ultimate load is significantly lower for control specimen B1 (7.0%) than control specimen B5 (20.2%), which includes a lesser amount of shear reinforcement.
- The aggregate interlock contribution is negligible throughout loading in the specimens with jackets. The aggregate interlock contribution at the ultimate load is less than 5.6% for all specimens with jackets.
- The dowel action shear force at the ultimate load increases due to the additional longitudinal reinforcement in the jacket and side thickness of the jacket.
- The uncracked compression zone contribution is more dominant in the specimens of group 1 than those of group 2.
- The stirrup contributions vary due to the shapes of the critical shear cracks and the number of stirrups intercepted by the critical shear crack.
- Fibre bridging is the dominant shear resisting mechanism in the jacketed specimens for most of the experiment. However, the fibre bridging contribution is less dominant near

the final load steps prior to the ultimate load as a result of the gradual widening of the critical shear crack. The peak fibre bridging contribution occurs before the ultimate load at around the load steps $0.5P_u$ to $0.8P_u$.

- The UHPC jacket does not only increase the shear resistance of the RC beam through fibre bridging as it also increases the contributions of other shear resistance mechanisms. This includes the dowel action, stirrups and the uncracked compression zone shear resistance mechanisms.

CHAPTER 6: NUMERICAL MODELLING

6.1 INTRODUCTION

This chapter covers the development and analysis of numerical models aimed at simulating the response of the beams tested in this research. The NLFEA software package ATENA was utilised in this research, and its main features are described in this chapter. Experimental results of RC, UHPC and strengthened beams from the literature were used to develop the adopted NLFEA approach before applying it to the beams tested in this campaign. The modelling approach was developed in an iterative trial and error process that involved calibrating key input parameters to achieve a reasonable fit between the numerically predicted and experimental results. The accuracy of the numerical predictions was assessed according to the load-displacement response, failure mode, reinforcement response and crack pattern. The results of the numerical analyses for the beams of this campaign were ultimately compared with the design code-based shear strength predictions discussed in Chapter 7.

6.2 NONLINEAR FINITE ELEMENT ANALYSIS WITH ATENA

6.2.1 INTRODUCTION

Only the main features of ATENA are described in this section, and more details can be found in Červenka et al. (2018). Similar material models were adopted for NSC and UHPC, and they both included a plasticity model in compression combined with a smeared crack model in tension. The material laws of UHPC were user-defined due to the complexity of modelling fibre reinforced concrete (FRC) in comparison to standard concrete. The steel reinforcement was modelled as discrete bars that were assumed to be in a uniaxial stress state and perfectly bonded with the surrounding concrete. The steel of the loading and support plates was modelled as elastic with a modulus of elasticity equal to 200 GPa. Isoparametric 3D solid finite elements were used to model the concrete and steel loading plates. Reinforcement bars were modelled with 1D truss elements. The solution method and convergence criteria that were adopted to iteratively solve the system of nonlinear equations are described in this section.

6.2.2 MATERIAL MODELLING IN ATENA

6.2.2.1 NORMAL STRENGTH CONCRETE

The fracture-plastic constitutive model depicted *CC3DNonLinCementitious2* was used to model the compressive and tensile behaviour of NSC. Elastic behaviour is modelled as isotropic with material properties remaining the same in all directions. The elastic behaviour is characterised in terms of the elastic modulus (E_c) and poisson's ratio (ν). The behaviour of cracked NSC is defined in terms of compressive crushing and tensile fracturing with the option of either a fixed or rotating smeared crack formulation. The combined fracture-plastic model allows compressive crushing and tensile fracturing to develop simultaneously.

A plastic constitutive model is used to simulate the compressive crushing behaviour of concrete. A new stress state is iteratively calculated at each load step. The plastic model uses the failure surface of Menétrey and Willam (1995) in combination with the compressive hardening/softening laws shown in Figure 6.1 (Červenka et al., 2018). The roundness of the failure surface can be modified by varying the eccentricity parameter (e) from 0.5 (sharp corners) to 1.0 (circular). The position of the failure surface moves along the hydrostatic axis depending on the strain hardening/softening parameter (Červenka et al., 2018). The hardening, or ascending, law is described in terms of strains while the softening, or descending, law is described in terms of displacements to decrease mesh dependency. The hardening law is initiated at the onset of concrete crushing (f'_{c0}), which is a user-defined parameter with a default value equal to twice the concrete tensile strength (f_t). The peak compressive stress (f'_c) arises at a plastic strain (ϵ_c^p) equal to the uniaxial compressive strength (f'_c) divided by the elastic modulus (E_c). For the softening law, equivalent plastic strains are converted into displacements by the characteristic length parameter (L_c), which is the projection of the element size in the direction of the minimum principal stress as shown in Figure 6.1 (Červenka et al., 2018). The displacement at which a complete loss of compressive strength occurs (w_d) is a user-defined parameter with a default value of 0.5 mm.

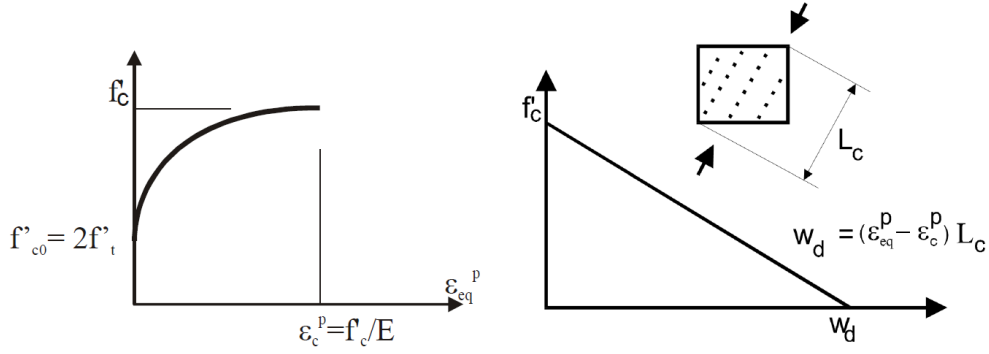


Figure 6.1 – Compressive hardening and softening laws of NSC in ATENA (Červenka et al., 2018). Image reproduced with permission of the rights holder, Červenka Consulting.

A fracture-energy based model is used to simulate the tensile cracking behaviour of concrete. The model uses the Rankine failure criterion in combination with the softening law shown in Figure 6.2 (Červenka et al., 2018). The stresses and strains are converted into the material directions, which correspond to the principal directions at the onset of cracking for the fixed crack formulation or principal directions for the rotated crack formulation. The adopted tensile softening law is an exponential function of the crack opening that is based on the formula of Hordijk (1991). The characteristic length (L_t) shown in Figure 6.2 converts the maximal fracturing strain in ATENA (ε_f) into crack opening. The characteristic length (L_t) is taken as the projected size of the element in the cracking direction.

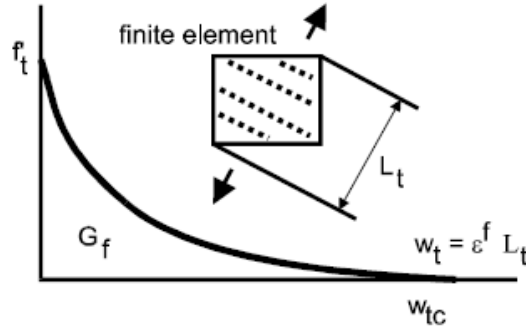


Figure 6.2 – Tensile softening law of NSC in ATENA (Červenka et al., 2018). Image reproduced with permission of the rights holder, Červenka Consulting.

CC3DNonLinCementitious2 allows either a fixed or rotating smeared crack formulation to be selected. Both models assume that cracks are uniformly distributed within the material volume, and that a crack forms when the principal stress exceeds the tensile strength of concrete (Červenka et al., 2018). In the fixed crack model, the crack direction is based on the principal stress direction at the onset of cracking. This direction is fixed and remains constant throughout the analysis. In the rotated crack model, the directions of the principal stress and strain are

parallel, and this direction and resulting cracks rotate throughout the analysis. The fixed crack parameter in ATENA, or locking level, should be set equal to 1.0 or zero to apply the fixed or rotated crack models. If an intermediary value is chosen, then the crack direction becomes fixed when the softening law reaches the product of the fixed crack parameter and the input tensile strength of concrete (f_t). The default fixed crack model was adopted in this research with the fixed crack parameter set equal to 1.0 based on the sensitivity studies discussed in Sections 6.3 and 6.4. The shear factor coefficient (s_F) is an influential input parameter in the fixed crack model. It relates the shear crack stiffness to the normal crack stiffness. A default value of 20 is given in ATENA, but it can be calibrated as discussed later in Sections 6.3 and 6.4. Prior studies suggest that it significantly influences the shear strength of beams, and its value is dependent on the amount of shear reinforcement (Pastore, 2020).

The fixed crack model assumes that the initial shear modulus of concrete (G_c) reduces due to a loss in shear stiffness as a crack opens. To account for this degradation, the initial shear crack modulus (G_c) is multiplied by a reduction factor, which is referred to as the shear retention factor (r_g). The shear retention factor (r_g) is a function of the user-defined shear factor coefficient (s_F), and it reduces as the maximal fracturing strain (ε_1) increases. The shear stress on the crack plane (τ) becomes equal to the reduced shear stress modulus (G) multiplied by the element orientation factor (γ), which considers the effects of skew meshes on the localisation of deformations in the failure state due to both compressive crushing and tensile cracking (Červenka et al., 2018).

The combined fracture-plastic constitutive model considers key interactions between the compressive and tensile behaviour models in order to accurately simulate concrete behaviour. The combined model assumes that the tensile strength degrades after concrete crushing occurs. The model also assumes that the compressive strength degrades after concrete cracking occurs in the perpendicular direction. The concrete compressive strength is reduced by a reduction factor (r_c) proposed by Bentz et al. (2006). The reduced concrete strength (σ_c) and reduction factor (r_c) are calculated as follows:

$$\sigma_c = r_c \cdot f_c \quad (6.1)$$

$$r_c = \frac{1}{0.8 + 170 \cdot \varepsilon_1} \quad \text{for } r_c^{lim} \leq r_c \leq 1.0 \quad (6.2)$$

where r_c^{lim} is the lower bound of the reduction factor (r_c), and it is an influential input parameter in ATENA. If the lower bound of the reduction factor (r_c^{lim}) is set equal to 1.0, then there will be no reduction in compressive strength after cracking. A default value of 0.8 is given in ATENA for the lower bound of the reduction factor (r_c^{lim}), but this value can be adjusted to achieve improved numerical predictions as discussed later in Section 6.3.

The main input parameters in ATENA that were initially used for NSC are summarized in Table 6.1. These input parameters include the mechanical properties of NSC such as compressive strength (f_c), tensile strength (f_t), elastic modulus (E_c) and fracture energy (G_f). These parameters can either be user-defined or automatically generated as a function of the input compressive strength or strength class according to either EC2 or fib MC 2010 (BSI, 2014; fib, 2013). In this research, the input compressive (f_c) and tensile (f_t) strength were taken as the measured mean compressive cylinder strength (f_{cm}) and mean splitting tensile strength ($f_{ctm,sp}$), respectively, while the fracture energy (G_f) was calculated using Eqn. 2.165 according to fib MC 2010 (fib, 2013). Other important parameters shown in Table 6.1 include the shear factor coefficient (s_F), aggregate size (d_g), plastic strain at compressive strength (ε_c^p), onset of concrete crushing (f_{c0}), critical compressive displacement (w_d), compressive strength reduction factor limit (r_c^{lim}) and eccentricity that defines the shape of the failure surface (e) amongst others. Of the input parameters shown in Table 6.1, the following were varied in the calibration of the adopted modelling procedure: elastic modulus (E_c), fracture energy (G_f), shear factor coefficient (s_F) and compressive strength reduction factor limit (r_c^{lim}). Table 6.1 also summarizes the final input parameters used for the NSC material model in the adopted modelling procedure of this research, which are based on the calibrations of the previously listed parameters discussed later in Sections 6.3 and 6.4.

Table 6.1 – Summary of initial and calibrated input parameters of NSC material model in ATENA

Parameter	Formula/Value	
	Initial	Calibrated
Compressive strength (f_c) [MPa]	$-f_{cm}^A$	$-f_{cm}$
Tensile strength (f_t) [MPa]	$f_{ctm,sp}^A$	$f_{ctm,sp}$
Elastic modulus (E_c) [MPa]	E_{cm}^A	$0.5 \cdot E_{cm}^C$
Fracture energy (G_f) [N/m]	$73 \cdot f_{cm}^{0.18A}$	$0.5 \cdot 73 \cdot f_{cm}^{0.18C}$
Poisson's ratio (ν)	0.2^B	0.2
Aggregate size (d_g) [mm]	d_g^A	10
Fixed crack	1.0^B	1.0
Shear factor coefficient (s_F)	20^B	$250 \text{ or } 50 \text{ (Group 1 or 2)}^C$
Plastic strain at compressive strength (ϵ_c^p)	-0.001^B	-0.001
Onset of concrete crushing (f_{c0})	$-2 \cdot f_t^B$	$-2 \cdot f_t$
Critical compressive displacement (w_d) [mm]	0.5^B	0.5
Compressive strength reduction factor limit (r_c^{lim})	0.8^B	0.5^C
Eccentricity (Shape of failure surface) (e)	0.52^B	0.52
Volume dilation plastic factor (β)	0^B	0
Density (ρ) [kg/m ³]	2300^B	2300
Coefficient of thermal expansion (α_r) [°C ⁻¹]	0.000012^B	0.000012

^A User-defined input value; ^B Default value; ^C Calibrated value

6.2.2.2 ULTRA HIGH PERFORMANCE CONCRETE

The fracture-plastic constitutive model depicted *CC3DNonLinCementitious2User* was used to define the compressive and tensile behaviour of UPHC. Unlike the model adopted for NSC, the model adopted for UHPC allows users to define the post-cracking compressive and tensile laws as well as the shear and concrete strength degradation laws. The adopted constitutive model for UHPC is based on a smeared crack formulation, where a fixed crack model was chosen (Sajdlová, 2016). The default input parameters in the *CC3DNonLinCementitious2User* model are intended for standard concrete and several modifications are required to account for the different mechanical behaviour of UHPC. The post-cracking compressive, tensile and degradation laws were defined in accordance with the recommendations of Sajdlová (2016) for modelling fibre reinforced concrete (FRC) in ATENA.

As described in Section 6.2.2.1 for NSC, the compressive crushing behaviour is simulated using a plastic constitutive model along with the failure surface of Menétrey and Willam (1995). The post-crushing compressive law for the *CC3DNonLinCementitious2User* model is shown in Figure 6.3. The compressive hardening and softening branches are defined in terms of the ratio of stress to compressive strength and the corresponding fracturing strain, with both branches depending on the number of user-defined points. The compressive softening behaviour initiates after exceeding the compression localisation onset (ε_{loc}^c). This user-defined input parameter equals the maximal fracturing strain at the start of the softening phase. When compression localisation occurs, the characteristic length (L_{ch}^c) used to determine the experimental strain is adjusted by the user-defined compressive crush band size (L_c). In this research, the compressive crush band size (L_c) was set equal to the default value of 100 mm (Sajdlová, 2016). The default compressive function of the *CC3DNonLinCementitious2User* model in ATENA was adopted where the hardening and softening branches are defined as bilinear and linear, respectively (Sajdlová, 2016). However, the strain corresponding to the end point of the softening branch, which is equal to -0.005841 by default, was multiplied by 100 to account for the increase in ductility due to the fibre reinforcement (Sajdlová, 2016). The input compressive function of UHPC used in this research is shown in Figure 6.4.

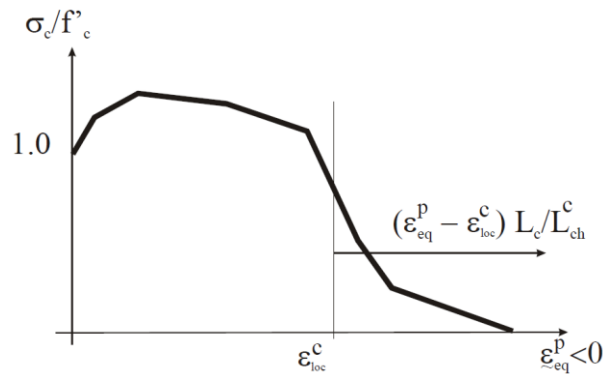


Figure 6.3 – Compressive function of *CC3DNonLinCementitious2User* in ATENA (Červenka et al., 2018).
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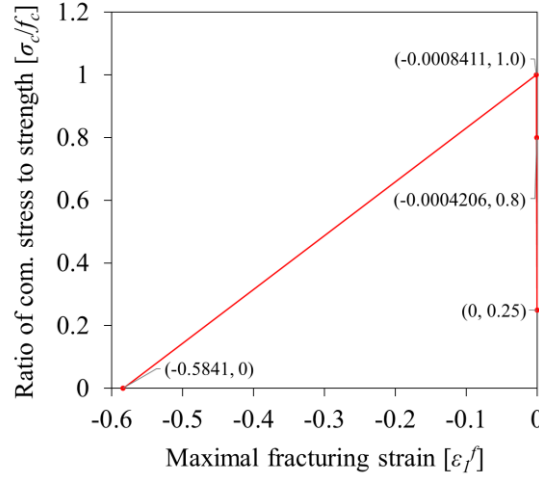


Figure 6.4 – Input compressive function of UHPC in ATENA (Sajdlová, 2016)

The tensile fracturing behaviour of UHPC was simulated based on the tensile stress-strain response given in NF P18-710 for thick UHPC members (AFNOR, 2016b). Where available, the given stress-strain response shown in Figure 6.5 was defined in terms of the post-cracking stress-crack opening response obtained from bending tests on notched prisms in accordance with NF P18-470 (AFNOR, 2016a, 2016b). As discussed in Section 2.2.5, the direct post-cracking stress-crack opening response of UHPC can be determined through the inverse analysis procedures given by NF P18-470 (AFNOR, 2016a). The following equations were used to derive the strains of the tensile stress-strain response of UHPC shown in Figure 6.5:

$$\varepsilon_{t,el} = \frac{f_{ctm}}{(\gamma_{cf} \cdot K) \cdot E_{cm}} \quad (6.3)$$

$$\varepsilon_{t,peak} = \varepsilon_{t,peak}^f + \varepsilon_{t,peak}^{el} = \frac{w_{peak}}{L_{ch}^t} + \frac{f_{ctm}}{KE_{cm}} \quad (6.4)$$

$$\varepsilon_{t,1\%} = \varepsilon_{t,1\%}^f + \varepsilon_{t,1\%}^{el} = \frac{w_{1\%}}{L_{ch}^t} + \frac{f_{ctm,1\%}}{KE_{cm}} \quad (6.5)$$

$$\varepsilon_{t,lim} = \frac{L_f}{4L_{ch}} \quad (6.6)$$

where $\varepsilon_{t,el}$ is the strain at the elastic limit, $\varepsilon_{t,peak}$ is the total strain at the peak stress, or post-cracking stress, $\varepsilon_{t,1\%}$ is the total strain corresponding to a crack width of $0.01H$ where H is the height of tested prism specimen after the deduction of the notch depth according to NF P18-470, $\varepsilon_{t,lim}$ is the ultimate strain, w_{peak} is the crack width, or opening, that corresponds to the

peak stress which was taken as 0.3 mm according to NF P18-710 if there was no post-cracking strain-hardening as in the UHPC of this research, $w_{1\%}$ is the crack width equal to $0.01H$, $f_{ctm,1\%}$ is the stress that corresponds to a crack width of $0.01H$, L_{ch}^t is the characteristic length which was taken as $2h/3$ according to NF P18-710 where h represents the height of the UHPC beam or jacket and L_f is the length of the longest fibre (AFNOR, 2016a, 2016b). The terms $\varepsilon_{t,peak}^f$ and $\varepsilon_{t,1\%}^f$ refer to the fracturing strains at peak stress and the stress corresponding to a crack width of $0.01H$, respectively, while the terms $\varepsilon_{t,peak}^{el}$ and $\varepsilon_{t,1\%}^{el}$ refer to the elastic strains, respectively. The reduction factor (K) was set equal to 1.0 when deriving the tensile stress-strain response of the modelled beams. Additional assumptions were necessary to derive the tensile stress-strain of the modelled beams from the literature when insufficient data were provided, and this is discussed in Section 6.3.3.

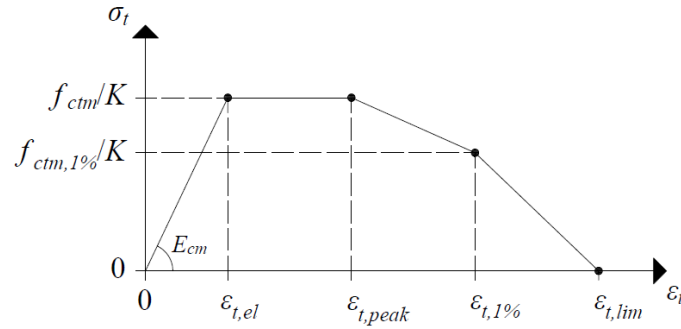


Figure 6.5 – Derived tensile stress-strain response of UHPC based on NF P18-710 (AFNOR, 2016b)

The tensile fracturing behaviour in ATENA was defined using the derived stress-strain response of Figure 6.5. Much like the input compressive function, the input tensile function in ATENA shown in Figure 6.6 depends on the number of user-defined points used to represent the post-cracking response. The post-cracking response was defined in ATENA in terms of the ratio of the tensile stress to the tensile strength and the corresponding fracturing strain (ε_1^f). Figure 6.7 shows how the input tensile function was defined in ATENA based on the derived tensile stress-strain response of Figure 6.5. The initial point of the input tensile function represents the end of the elastic phase as well as the beginning of the post-cracking phase. The end point of the tensile function was set equal to the ultimate strain ($\varepsilon_{t,lim}$) calculated using Eqn. 6.6. Tensile softening begins when the maximal fracturing strain (ε_1^f) is greater than or equal to the tensile localisation onset (ε_{loc}^t), which is the maximal fracturing strain at the initial point of the softening phase (see Figures 6.6 and 6.7). When tensile localisation occurs, the characteristic length (L_{ch}^t) that was used to derive the tensile stress-strain response of Figure

6.5 is adjusted by the user-defined tensile crack band size (L_t) according to the following equation:

$$\tilde{\varepsilon}_1^f = \begin{cases} \varepsilon_1^f & \text{for } \varepsilon_1^f < \varepsilon_{loc}^t \\ (\varepsilon_1^f - \varepsilon_{loc}^t) L_t / L_{ch}^t & \text{for } \varepsilon_1^f \geq \varepsilon_{loc}^t \end{cases} \quad (6.7)$$

where $\tilde{\varepsilon}_1^f$ is the fracturing strain that is used to determine the current tensile strength in ATENA from the input tensile function and ε_1^f is the strain calculated from the strain tensor at the finite element integration points (Červenka et al., 2018). This approach in ATENA accounts for the difference in experimental size and the size of the integration point, and it limits mesh dependency as it ensures the same fracture energy when large or small finite elements are used (Červenka et al., 2018). The tensile crack band size in ATENA (L_t) was set equal to the characteristic length (L_{ch}^t), which equals to $2h/3$ according to NF P18-710 where h represents the height of the UHPC beam or jacket (AFNOR, 2016b). Seeing as the tensile crack band size in ATENA (L_t) was set equal to the characteristic length (L_{ch}^t), the derived tensile stress-strain response was not scaled with respect to the mesh element size. This had the consequence that the solution was mesh dependent and did not converge with reducing mesh element size. Hence, the models were calibrated based on the adopted mesh element size that ultimately gives 10 elements per member thickness as suggested by Pryl and Červenka (2018) for quantitative results and regions subjected to bending. A similar issue of mesh dependency was encountered by Genikomsou and Polak (2015) when modelling punching shear failure of RC slabs in the NLFEA software package ABAQUS. Genikomsou and Polak (2015) justified not using very small elements by the observation that the mesh element size should not be too close to the aggregate size.

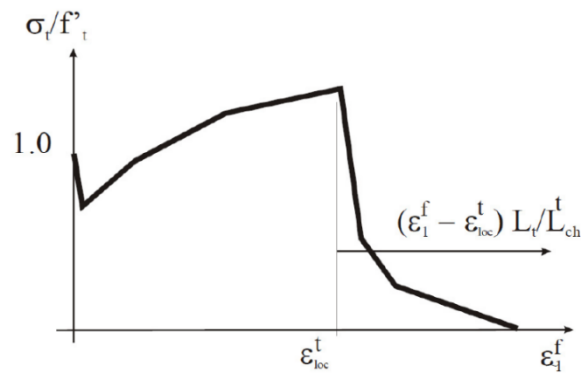


Figure 6.6 – Tensile function of *CC3DNonLinCementitious2User* in ATENA (Červenka et al., 2018). Image reproduced with permission of the rights holder, Červenka Consulting.

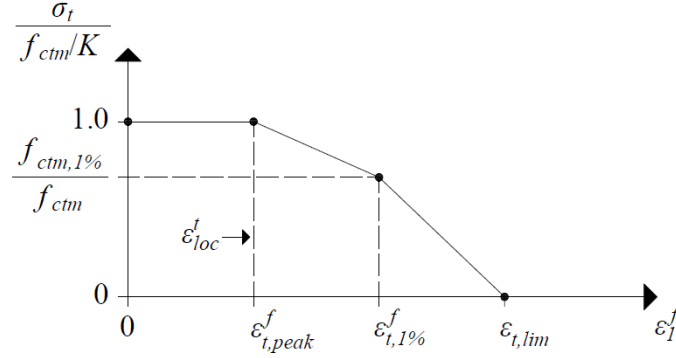


Figure 6.7 – Method of calculation for input tensile function of UHPC in ATENA based on NF P18-710 (AFNOR, 2016b)

The *CC3DNonLinCementitious2User* model allows the user to define the degradation laws due to cracking. The laws include the degradation of the shear retention factor and shear strength. The degradation of the shear retention factor (r_g) was defined in terms of the ratio of reduced to initial shear moduli and corresponding fracturing strain. The default input function in ATENA was used in this research (see Figure 6.8(a)) with the localisation onset parameter (ε_{loc}^{sh}) set equal to the default value of zero (Sajdlová, 2016). The degradation of the shear strength was defined in ATENA in terms of the ratio of shear stress to tensile strength and corresponding fracturing strain. The default input function shown in Figure 6.8(b) was used in this research (Sajdlová, 2016).

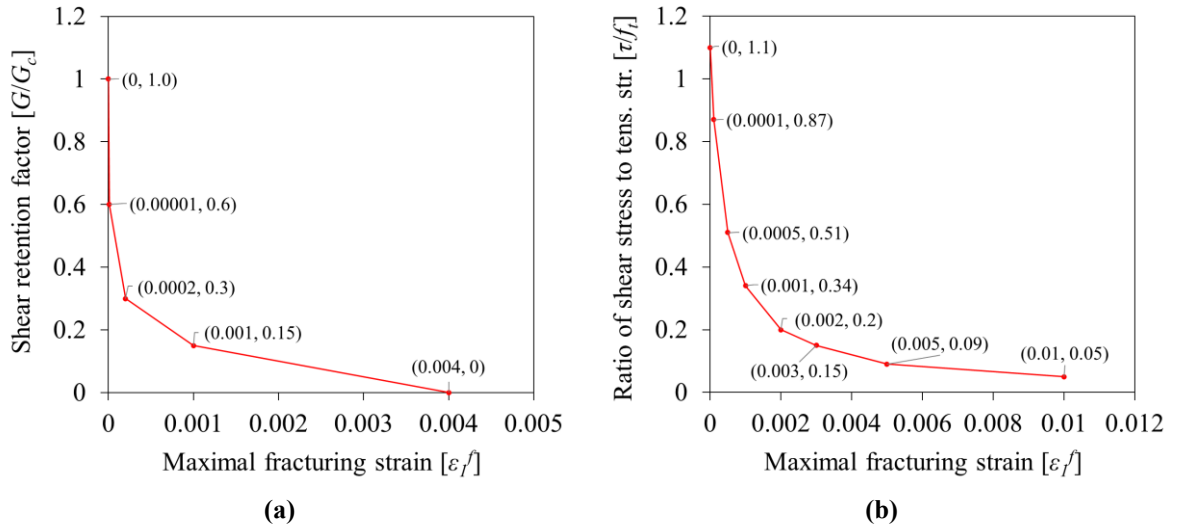


Figure 6.8 – Input degradation functions of shear retention and shear strength of UHPC in ATENA (Sajdlová, 2016): **(a)** Shear retention factor, **(b)** Shear strength

The degradation laws for the tensile and compressive strength reduction factors due to concrete crushing or fracturing were also defined in ATENA. The degradation of the tensile strength

was defined in terms of the ratios of tensile stress to tensile strength and compressive stress to compressive strength. The default input function shown in Figure 6.9(a) was used in this research (Sajdlová, 2016). The degradation of the compressive strength reduction factor (r_c) was defined in terms of the ratio of compressive stress to compressive strength and corresponding fracturing strain. In this research, the degradation of the compressive strength factor (r_c) was deactivated by setting the default function equal to a constant value of 1.0 as suggested by Sajdlová (2016) (see Figure 6.9(b)).

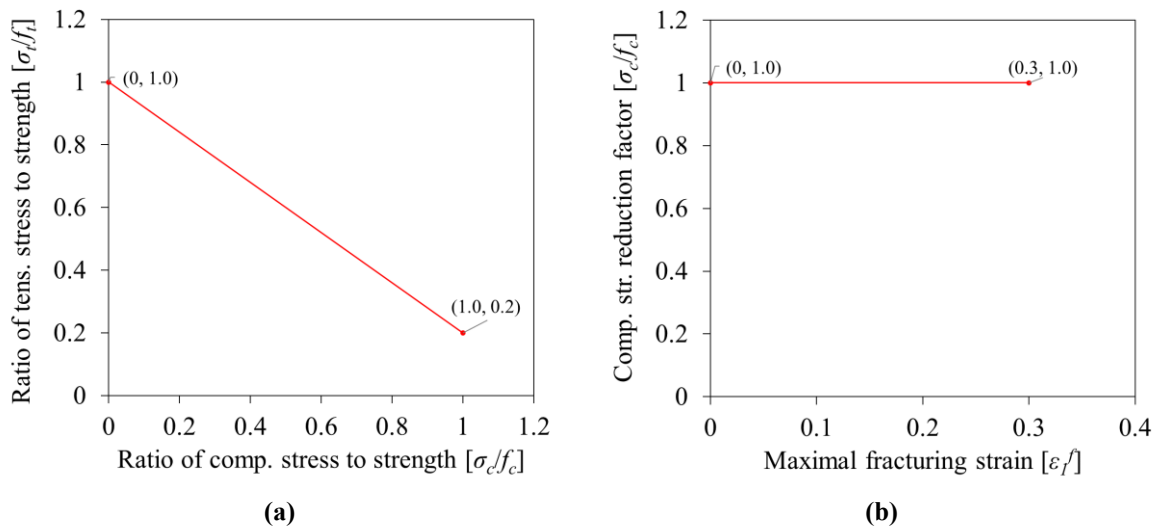


Figure 6.9 – Input degradation functions of tensile strength and compressive strength reduction factor of UHPC in ATENA (Sajdlová, 2016): **(a)** Tensile strength, **(b)** Compressive strength reduction factor

The input functions and parameters used to define UHPC in this research are presented in Table 6.2. These input parameters included the mechanical properties of UHPC such as compressive strength (f_c), tensile strength (f_t) and elastic modulus (E_c). The input compressive strength was taken as the mean compressive cylinder strength (f_{cm}), and the input tensile strength was taken as the mean direct tensile strength (f_{ctm}). A fixed crack model was adopted, and other important parameters shown in Table 6.2 include the aggregate size (d_g) and eccentricity that defines the shape of the failure surface (e) amongst others.

Table 6.2 – Summary of calibrated input parameters of UHPC material model in ATENA

Parameter			Formula/Reference/Value
Compressive strength	(f_c)	[MPa]	$-f_{cm}^A$
Tensile strength	(f_t)	[MPa]	f_{ctm}^A
Elastic modulus	(E_c)	[MPa]	E_{cm}^A
Poisson's ratio	(ν)		0.02 ^B
Aggregate size	(d_g)	[mm]	d_g^A
Fixed crack			1.0 ^B
Tension characteristic size	(L_t)	[mm]	$2h/3$ where h is height of UHPC beam or jacket ^A
Tension localisation onset	(ε_{loc}^t)		See Eqn. 6.4 for calculation of $\varepsilon_{t,peak}^f$ ^A
Tension function			See Figure 6.7 for method of calculation ^A
Compression characteristic size	(L_c)	[mm]	100 ^B
Compression localisation onset	(ε_{loc}^c)		-0.0008411 ^B
Compressive function			See Figure 6.4 ^A
Shear localisation onset	(ε_{loc}^{sh})		0 ^B
Shear retention factor degradation function			See Figure 6.8(a) ^B
Shear strength degradation function			See Figure 6.8(b) ^B
Tensile strength degradation function			See Figure 6.9(a) ^B
Comp. str. reduct. factor degradation function			See Figure 6.9(b) ^A
Eccentricity (Shape of failure surface)	(e)		0.52 ^B
Volume dilation plastic factor	(β)		0 ^B
Density	(ρ)	[kg/m ³]	2300 ^B
Coefficient of thermal expansion	(α_r)	[°C ⁻¹]	0.000012 ^B

^A User-defined input value; ^B Default value

6.2.2.3 STEEL REINFORCEMENT

The steel reinforcement was modelled as discrete reinforcement bars embedded in concrete. A perfect bond was assumed between the steel reinforcement and surrounding concrete. Due to this assumption, the strain in the steel reinforcement was equal to that in the surrounding concrete at the same level. The tensile behaviour of the steel reinforcement can be modelled in ATENA using either a bilinear or multi-linear stress-strain relationship as shown in Figure 6.10. A multi-linear relationship was adopted for all types of reinforcement bars used in this research to effectively capture the tensile response, which includes the linear elastic phase,

possible yielding plateau, strain-hardening phase and strain-softening phase prior to rupture. A bilinear relationship with strain-hardening was assumed for the modelled beams from the literature due to insufficient data as discussed in Section 6.3. The bilinear relationship was defined by the elastic modulus (E_s), yield stress (f_y) and yield strain (ε_y) along with the ultimate stress (f_u) and strain (ε_u) of the steel reinforcement. For the tested beams of this research, the multi-linear relationship was defined by the elastic modulus (E_s), yield stress (f_y) and yield strain (ε_y) along with up to six points following the yield point until the rupture of the reinforcement bar.

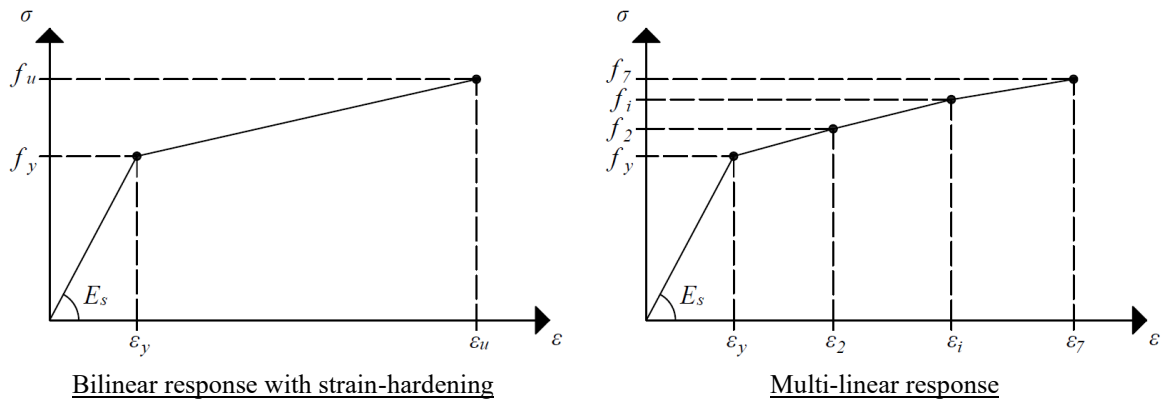


Figure 6.10 – Stress-strain response of steel reinforcement in ATENA. Adapted from Červenka et al. (2018).
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6.2.3 FINITE ELEMENT MESH

Solid isoparametric 3D elements were adopted for concrete due to their versatility for modelling materials with nonlinear behaviour (Červenka et al., 2018). The finite element mesh type and size of the loading apparatuses had no significant impact on the numerical results, so 4-noded tetrahedra elements (CCIsoTetra) were used as shown in Figure 6.11 with an assigned mesh size similar to that of the concrete finite elements for each respective model. All elements were integrated by Gauss integration at each of their respective integration points. NSC and UHPC were modelled using 8-noded cubic hexahedra elements as shown in Figure 6.11 (CCIsoBrick). To determine a suitable finite element mesh size for NSC and UHPC, sensitivity studies were conducted for the various developed numerical models as discussed in Section 6.3. The steel reinforcement bars were modelled using 2-noded truss elements (CCIsoTruss).

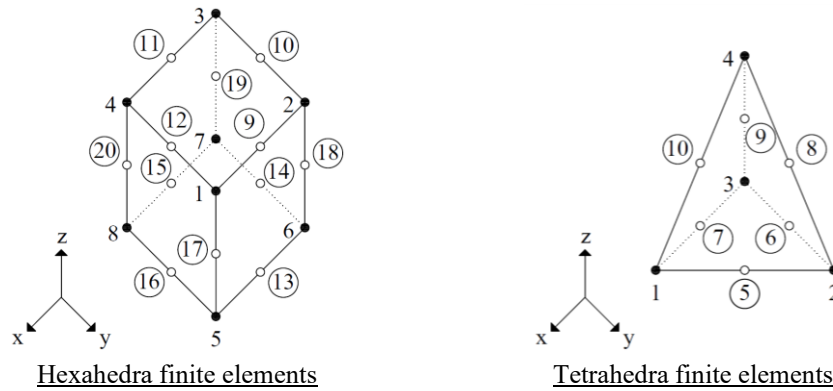


Figure 6.11 – Finite element types in ATENA. Adapted from Červenka et al. (2018). Images reproduced with permission of the rights holder, Červenka Consulting.

6.2.4 SOLUTION METHOD AND CONVERGENCE CRITERIA

To solve the systems of nonlinear equations for each analysis, force controlled loading was applied along with the arc-length solution method and default convergence criteria. The arc-length method offered in ATENA was selected as opposed to the alternative Newton-Raphson method seeing as it is more suitable for solving complex problems associated with material nonlinearity (Červenka et al., 2018). It is a robust solution method that avoids common instability issues such as snap back and snap through phenomena (Červenka et al., 2018). Unlike the Newton-Raphson method, the load conditions in the arc-length method vary during iterations within each load step, and it iteratively solves for both the load and displacement at each load step while the solution path remains constant (Červenka et al., 2018). This solution method is possible due to the mathematical introduction of an additional degree of freedom that is associated with the loading level. An iteration limit of 30 was adopted in this research to find a solution at each load step while using the default convergence criteria in ATENA. The default convergence criteria for the displacement, residual and absolute residual errors are equal to 1.0%, and the convergence criterion for the energy error equals 0.01%.

6.3 VALIDATING NLFEA APPROACH WITH BEAMS FROM THE LITERATURE

6.3.1 INTRODUCTION

The adopted NLFEA methodology was developed using the results of experiments from the literature. Several bending tests on beams from the literature with relatively similar geometry, loading arrangement and material mechanical properties to the beams of this research were

selected to validate the NLFEA approach. The experiments included RC, UHPC and strengthened beams that failed in both flexure and shear to ensure that the adopted NLFEA approach is capable of accurately simulating both flexural and shear behaviour. For simulating shear behaviour, slender beams failing in shear similar to the beams of this research were selected from the literature for validation. The UHPC beam specimens of Yang et al. (2019) that reportedly failed in flexure were considered to simulate the flexural behaviour of UHPC. The UHPC beams of Qi et al. (2016) that failed in shear were used to simulate the shear behaviour of UHPC. Al-Osta et al. (2017) and Alkhoury (2017) both tested a series of RC beams with UHPC jackets along with control RC beams. The beams of Al-Osta et al. (2017) that reportedly failed in flexure were considered to simulate the flexural behaviour of RC beams with and without jackets. The beams of Alkhoury (2017) that all reportedly failed in shear were considered to simulate the shear behaviour of RC beams with and without UHPC jackets.

The validation process was an iterative trial and error process aimed at establishing a set of consistent input parameters capable of achieving the best fit between the numerically predicted and experimental responses. This process began by calibrating, or changing, various input parameters for a single beam to achieve an acceptable numerical prediction. After having done so, the same set of input parameters were trialled on the following beams to see whether acceptable numerical predictions could be achieved as well. This process was repeated until a set of consistent input parameters were established that were capable of achieving acceptable numerical predictions for all of the considered beams. The accuracy of the numerical predictions was assessed according to the load-displacement response, failure mode, reinforcement response where available and crack pattern. Sensitivity studies were conducted to help assess the influence of specific parameters on the overall numerical response. The most influential input parameters that were calibrated namely included the elastic modulus (E_c), fracture energy (G_f), shear factor (s_F) and the compressive strength reduction factor limit (r_c^{lim}) of NSC. Sensitivity studies were also conducted on fine, medium and coarse mesh element sizes for the NSC beams and UHPC beams and jackets to determine suitable mesh sizes capable of achieving an acceptable numerical prediction while reducing computation time.

6.3.2 DEVELOPMENT OF NUMERICAL MODEL

The numerical models of the considered beams from the literature were developed based on their reported geometry and loading arrangements. The geometry of the loading apparatuses were assumed reasonable values if they were not reported based on the dimensions of each respective beam, drawings and photographs. Given that the design of each beam and loading arrangement were symmetrical, only half of each beam in the span direction was modelled to reduce computation time. The 3D model shown in Figure 6.12 of identical specimens RC-SB-3SJ and RC-EP-3SJ tested by Al-Osta et al. (2017) demonstrates how the numerical models were developed. A hexahedral grid was used to model the beam, jacket where available, embedded reinforcement bars and loading apparatuses as well as assigned boundary conditions and contact interactions.

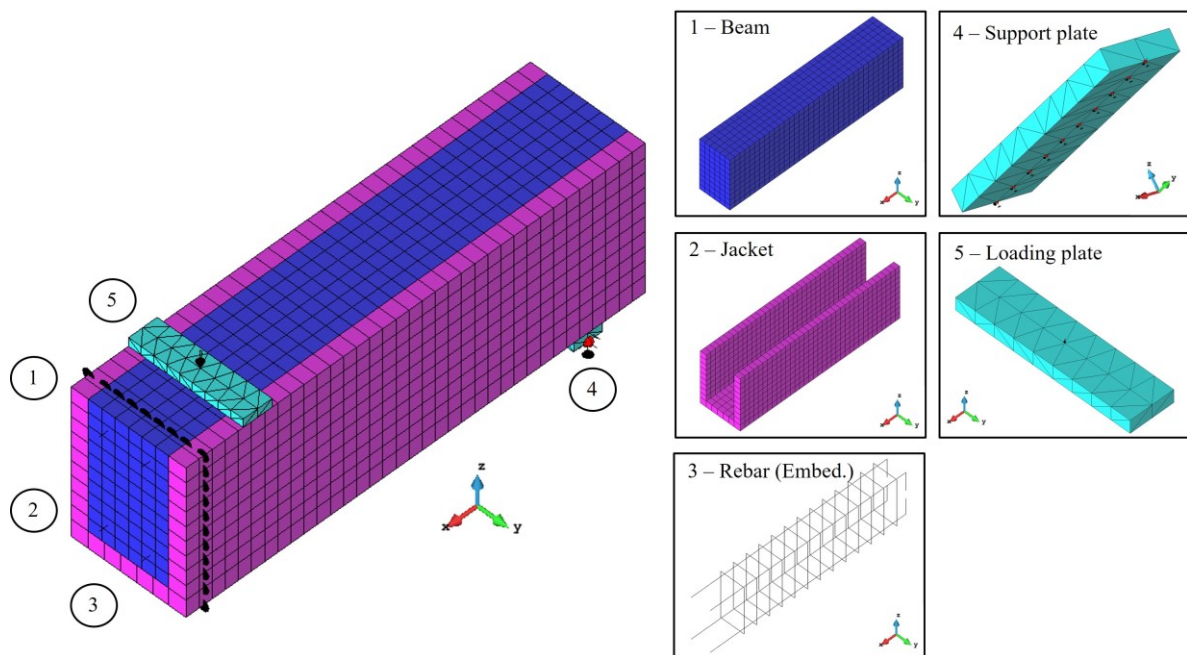


Figure 6.12 – 3D model of specimens RC-SB-3SJ and RC-EP-3SJ tested by Al-Osta et al. (2017) in ATENA

A set of boundary conditions and contact interactions were applied to numerically simulate the conditions of the physical experiments. Initially, a point load was applied on the top surface of the loading plate at the centre point. For the case where only half of the beam in the span direction is modelled, the support plate did not allow translation in the y- and z-directions while the entire plane of symmetry restrained translation in the x-direction. The conditions of the support plate were applied on its bottom surface at its centre line. For beams with jackets, the jacket was connected to the beam using a fixed contact surface based on the master and slave

technique. The loading apparatuses were also connected to the beams using the same technique as a perfect bond was assumed.

6.3.3 MATERIAL MODELLING AND ASSUMPTIONS

The validation process began with using the initial NSC material model summarized in Table 6.1 along with the calibrated UHPC material model summarized in Table 6.2. A bilinear relationship was adopted for the steel reinforcement as shown in Figure 6.10. The user-defined input parameters for the concrete and steel material models were based on the reported mechanical properties of each considered beam from the literature. The user-defined input parameters of NSC include the compressive (f_c) and tensile (f_t) strength, elastic modulus (E_c), fracture energy (G_f) and aggregate size (d_g). The user-defined input parameters of UHPC also include the compressive (f_c) and tensile (f_t) strength, elastic modulus (E_c) and aggregate size (d_g). The characteristic length in tension of UHPC (L_t), which is equivalent to the characteristic length in tension (L_{ch}^t) used to derive the tensile stress-strain response, is another user-defined input parameter along with the input tensile function and tension localisation onset (ε_{loc}^t). The steel reinforcement user-defined input parameters include the elastic modulus (E_s), yield stress (f_y) and yield strain (ε_y) along with the ultimate stress (f_u) and strain (ε_u).

Assumptions were necessary to establish the input compressive strength (f_c) and elastic modulus (E_c) of the NSC and UHPC material models when insufficient data were provided. It was necessary to convert the compressive cube strength of UHPC ($f_{cm,cu}$) reported by Qi et al. (2016) to cylinder strength (f_{cm}), and it was assumed that cylinder strength (f_{cm}) equals the cube strength ($f_{cm,cu}$) minus 15 MPa (AFNOR, 2016a). The elastic modulus of NSC (E_{cm}) was not reported by Al-Osta et al. (2017), therefore it was calculated using the following equation given by EC2 (BSI, 2014):

$$E_{cm} = 22000(f_{cm}/10)^{0.3} \quad (6.8)$$

The elastic modulus of UHPC (E_{cm}) was not reported by Alkhoury (2017), so it was assumed to be equal to a typical value 45 GPa (Fehling et al., 2014).

Assumptions were also necessary to determine the input tensile function of the UHPC material model for each considered beam or jacket. As described in Section 6.2.2.2, the input tensile function is based on the derived tensile stress-strain relationship given by NF P18-710 (AFNOR, 2016b). Ideally, the post-cracking stress-crack opening response obtained from notched prisms tested in accordance with NF P18-470 should be used to derive the tensile relationship (AFNOR, 2016a). Yang et al. (2019) performed such tests and provided the load-CMOD results, and they were used to derive the input tensile function of UHPC according to the procedure described in Section 6.2.2.2. Alkhoury (2017) and Qi et al. (2016) characterised the tensile strength of UHPC indirectly through bending tests, while Al-Osta et al. (2017) obtained the tensile strength directly using dumbbell-shaped specimens. Only the direct tensile strength of UHPC (f_{ctm}) was reported in these previously mentioned studies. Therefore, an alternative method to that described in Section 6.2.2.2 was required to ultimately determine the input tensile function of UHPC as the post-cracking stress-crack opening response was not given. The reported direct tensile strength (f_{ctm}) was assumed to correspond to a crack opening (w_{peak}) of 0.3 mm (AFNOR, 2016b). The tensile stress-strain response of UHPC shown in Figure 6.13(a) was then derived by determining the strain at the elastic limit ($\varepsilon_{t,el}$), total strain at peak stress ($\varepsilon_{t,peak}$) and ultimate strain ($\varepsilon_{t,lim}$) using Eqns. 6.3, 6.4 and 6.6, respectively. Figure 6.13(b) describes how the input tensile function was ultimately defined in ATENA. Although necessary, these assumptions may limit the accuracy of the numerical predictions as the shape of the load-displacement response is influenced by the input tensile function (Sajdlová, 2016).

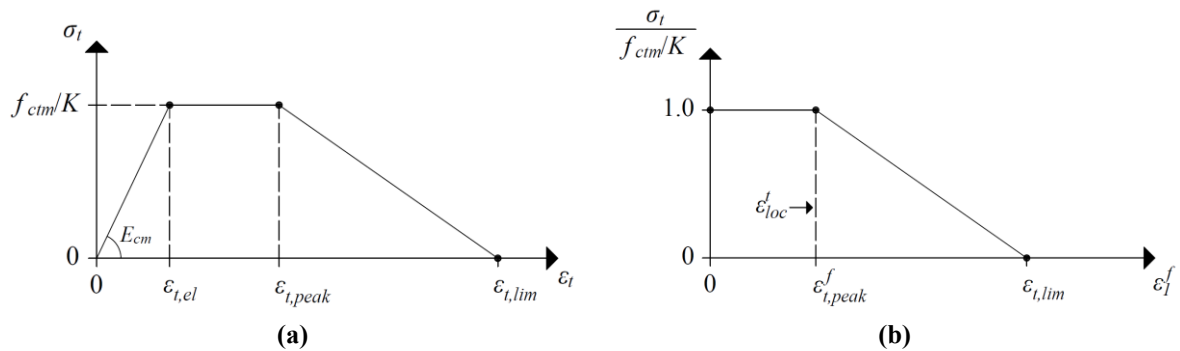


Figure 6.13 – Method of calculation for input tensile function of UHPC in ATENA for specimens tested by Alkhoury (2017), Al-Osta et al. (2017) and Qi et al. (2016): **(a)** Tensile stress-strain response of UHPC based on NF P18-710 (AFNOR, 2016b), **(b)** Input tensile function of UHPC in ATENA

A bilinear relationship with strain hardening was assumed for the steel reinforcement as shown in Figure 6.10 seeing as the tensile stress-strain responses were not reported by any of the

considered prior studies. The bilinear relationship was defined by a few user-defined input values, which include the elastic modulus (E_s), yield stress (f_y) and yield strain (ε_y) along with the ultimate stress (f_u) and strain (ε_u). When not reported, the ultimate strength (f_u) and strain (ε_u) were determined assuming grade B500C reinforcement (BSI, 2016). Therefore, the ultimate strength (f_u) and strain (ε_u) were set equal to $1.15f_y$ and 7.5%, respectively (BSI, 2016). Although grade B500B reinforcement is more common in practice, grade B500C is a more conservative assumption given that the ultimate strength (f_u) and strain (ε_u) are slightly higher. The elastic modulus (E_s) was set equal to a typical value of 200 GPa for all of the reinforcement bars in the considered beams as it was not reported. The assumptions related to the steel reinforcement may impact the numerical response, especially if the longitudinal or transverse reinforcement bars yield.

6.3.4 UHPC BEAMS FROM THE LITERATURE

6.3.4.1 FLEXURAL BEHAVIOUR

Nine UHPC beams tested by Yang et al. (2019) were used to replicate the flexural behaviour of UHPC. The geometry and reinforcement details of the considered beams are shown in Figure 6.14. The user-defined input parameters for the UHPC and steel reinforcement material models of each considered beam are presented in Tables 6.3 and 6.4, respectively. The beams included varying amounts of longitudinal reinforcement and a fixed amount of transverse reinforcement, and they all underwent symmetrical four-point bending tests. The longitudinal reinforcement ratios (ρ_l) were equal to 0.62% (R2), 0.93% (R3) and 1.24% (R4), and the shear reinforcement ratio (ρ_w) was fixed for all beams at 0.52%. Three different UHPC mixtures were used to manufacture the beams, which are referred to as FRC120, FRC150 and FRC180, and they differed in compressive and tensile strength as well as elastic modulus as shown in Table 6.3. The input tensile function used for each type of UHPC is presented in Figure 6.15. Although Yang et al. (2019) did not report on the response of the steel reinforcement, the reported crack patterns indicate flexural failure seeing as all of the beams featured a single vertical crack between the two loading plates at failure. Additionally, the load-displacement responses of the tested beams generally featured a displacement plateau.

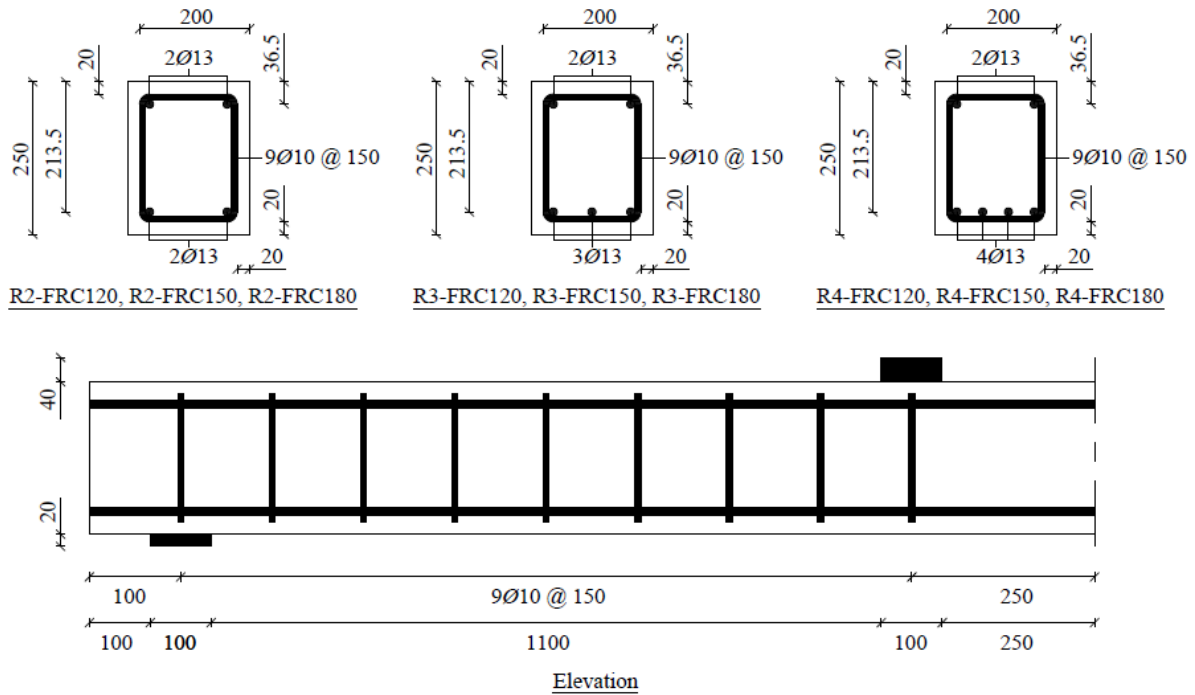


Figure 6.14 – Geometry and reinforcement details of beams tested by Yang et al. (2019) (Dimensions in mm)

Table 6.3 – Input parameters of UHPC in ATENA for beams tested by Yang et al. (2019)

Specimen	Mixture	Compressive strength (f_c) [MPa]	Tensile strength (f_t) [MPa]	Elastic modulus (E_c) [MPa]	Aggregate size (d_g) [mm]	Charact. length (L_t) [mm]	Localisation onset (ϵ_{loc}^t)
R2-FRC120	FRC120 ^A	-133.70	6.62	40150	0.5	166.67	0.00132
R2-FRC150	FRC150 ^A	-148.80	8.87	43220	0.5	166.67	0.00156
R2-FRC180	FRC180 ^A	-181.20	7.85	45140	0.5	166.67	0.00084
R3-FRC120	FRC120 ^A	-133.70	6.62	40150	0.5	166.67	0.00132
R3-FRC150	FRC150 ^A	-148.80	8.87	43220	0.5	166.67	0.00156
R3-FRC180	FRC180 ^A	-181.20	7.85	45140	0.5	166.67	0.00084
R4-FRC120	FRC120 ^A	-133.70	6.62	40150	0.5	166.67	0.00132
R4-FRC150	FRC150 ^A	-148.80	8.87	43220	0.5	166.67	0.00156
R4-FRC180	FRC180 ^A	-181.20	7.85	45140	0.5	166.67	0.00084

^A Fibre length (L_f) is 19.5 mm

Table 6.4 – Input parameters of steel reinforcement in ATENA for beams tested by Yang et al. (2019)

Diameter (ϕ) [mm]	Yield strength (f_y) [MPa]	Ultimate strength (f_u) [MPa]	Ultimate strain (ϵ_u) [%]	Elastic modulus (E_s) [MPa]
13	420	483 ^A	7.5 ^A	200000 ^B
10	500	575 ^A	7.5 ^A	200000 ^B

^A Assumed grade B500C steel (BSI, 2016); ^B Assumed value based on typical value of steel

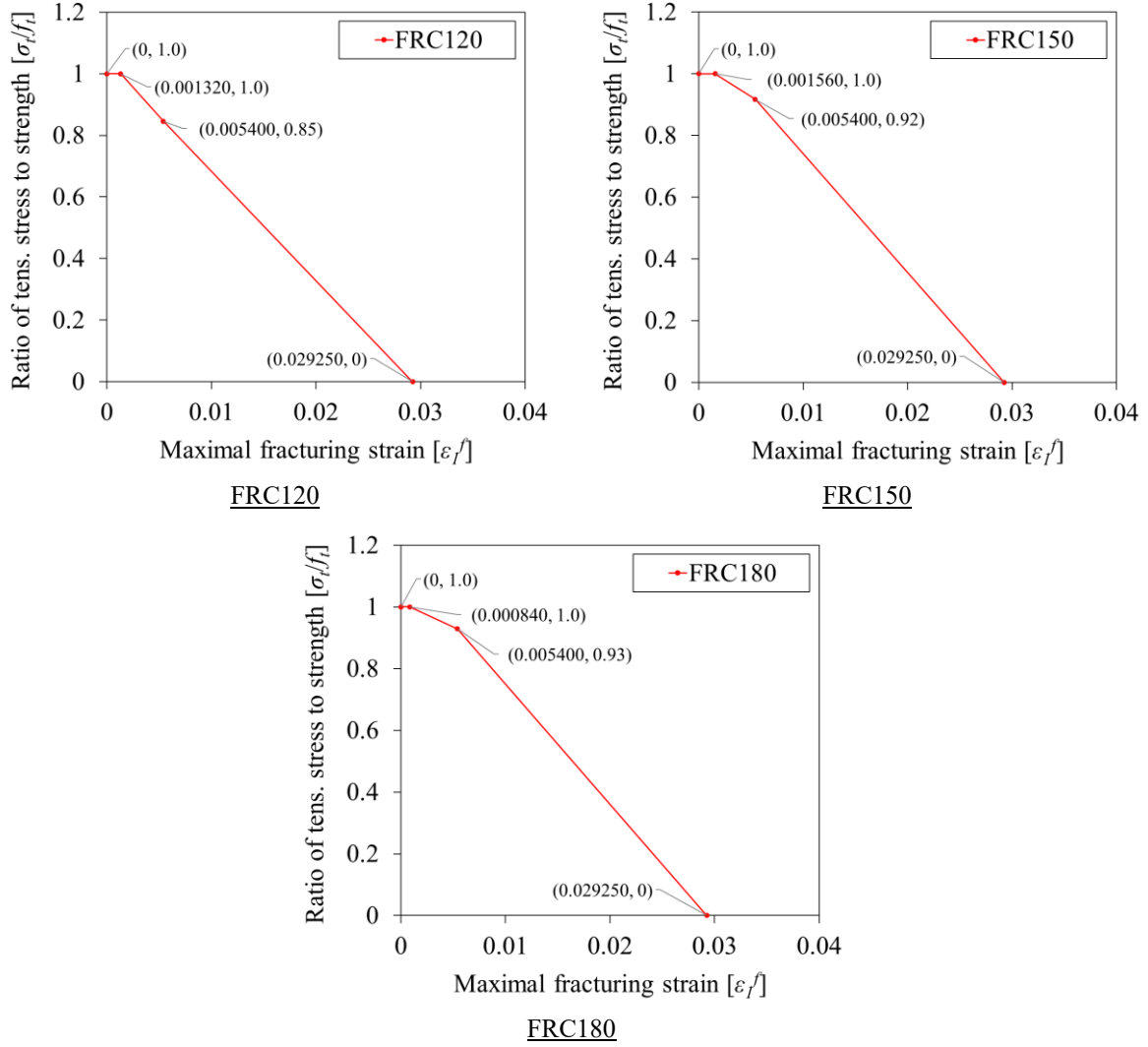


Figure 6.15 – Input tensile functions of UHPC in ATENA for beams tested by Yang et al. (2019)

A mesh sensitivity study was conducted on the numerical model of specimen R4-FRC120 to determine a suitable element size capable of achieving a computationally efficient yet accurate prediction. The mesh sizes shown in Figure 6.16 of the 8-noded hexahedra elements were equal to 16.67 mm, 25 mm and 50 mm, and along the height of the beam these produced 15, 10 and 5 elements, respectively. The influence of mesh size on the load-displacement response is shown in Figure 6.17 along with the experimental response. It is evident that the load-displacement response does not converge with a finer mesh size, which is likely due to the adopted fracturing behaviour of UHPC described in Section 6.2.2.2. However, the ultimate load is not significantly affected by mesh size. It also appears that the mesh sizes of 25 mm and 50 mm achieve the most accurate results when compared to the experimental results. Based on these findings, a medium mesh size of 25 mm was adopted for the beams of Yang et al. (2019).

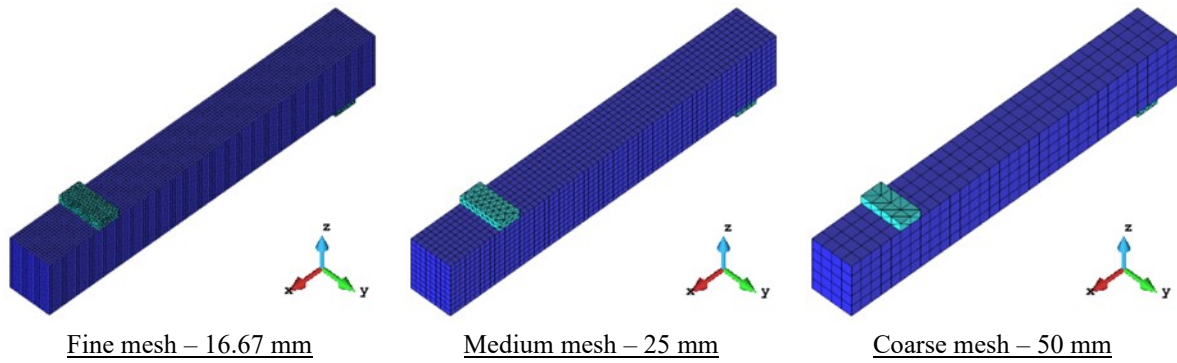


Figure 6.16 – Finite element mesh sizes for specimen R4-FRC120 tested by Yang et al. (2019)

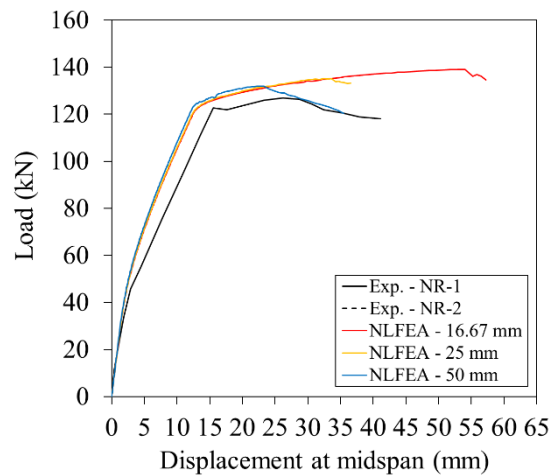
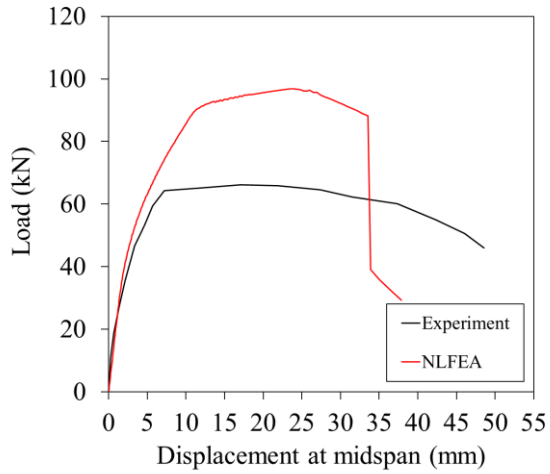
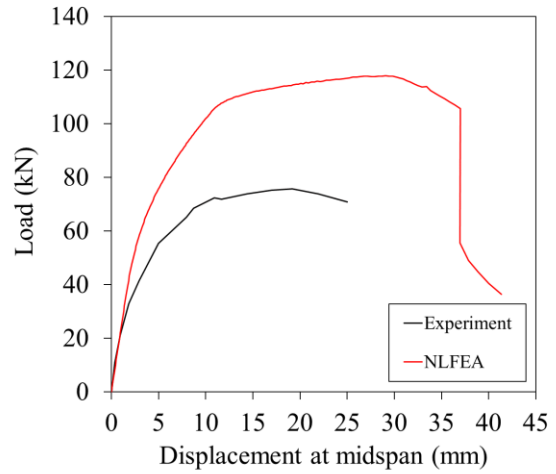


Figure 6.17 – Load-displacement response of specimen R4-FRC120 tested by Yang et al. (2019) using different mesh sizes

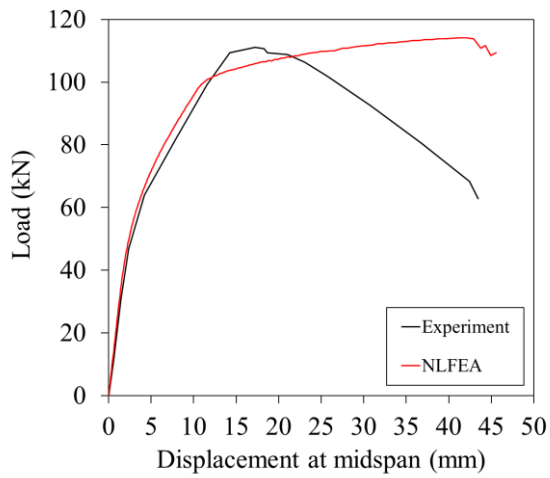
The numerical load-displacement responses of the beams tested by Yang et al. (2019) compare moderately well with the experimental results as shown in Figure 6.18. The numerical responses appear to accurately replicate both the initial cracking and ultimate loads for most of the beams as well as the relatively ductile flexural failure. Nonetheless, notable discrepancies between the numerical and experimental results can be seen for specimens R2-FRC120 and R2-FRC150. The shape of the numerical load-displacement responses is similar to that of the experimental responses, but the numerically predicted ultimate loads are much higher. Yang et al. (2019) reported that the tensile strength of UHPC based on the notched prisms does not appear to be representative of the tensile strength of UHPC used to manufacture these specific specimens. This is a rational assessment given that the numerically predicted ultimate load is influenced by the input tensile strength (f_t) while the shape of the load-displacement response is influenced by the input tensile function (Sajdlová, 2016).



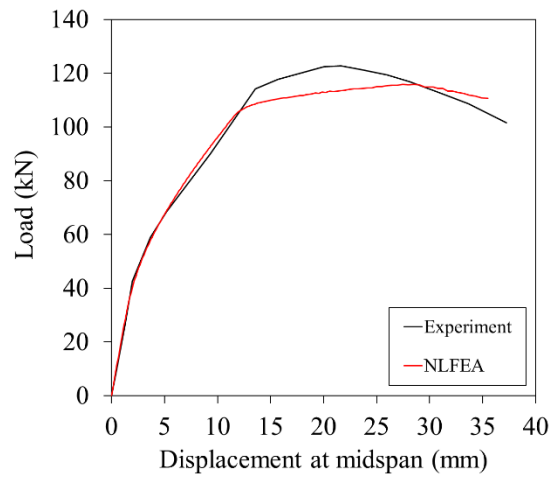
R2-FRC120



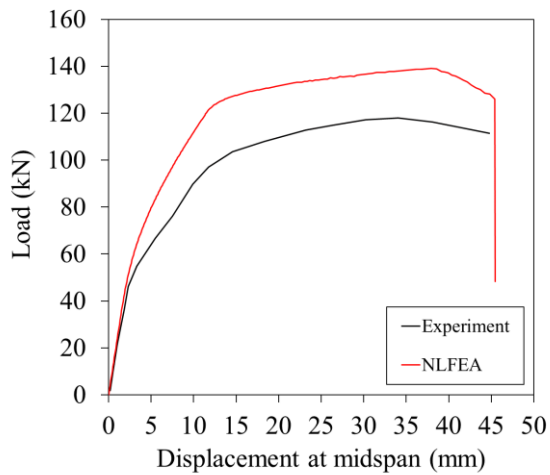
R2-FRC150



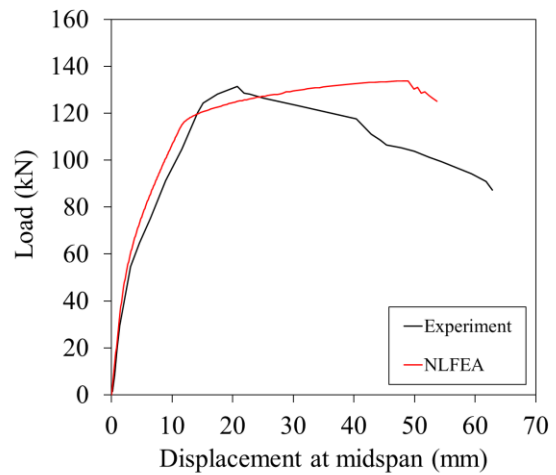
R2-FRC180



R3-FRC120



R3-FRC150



R3-FRC180

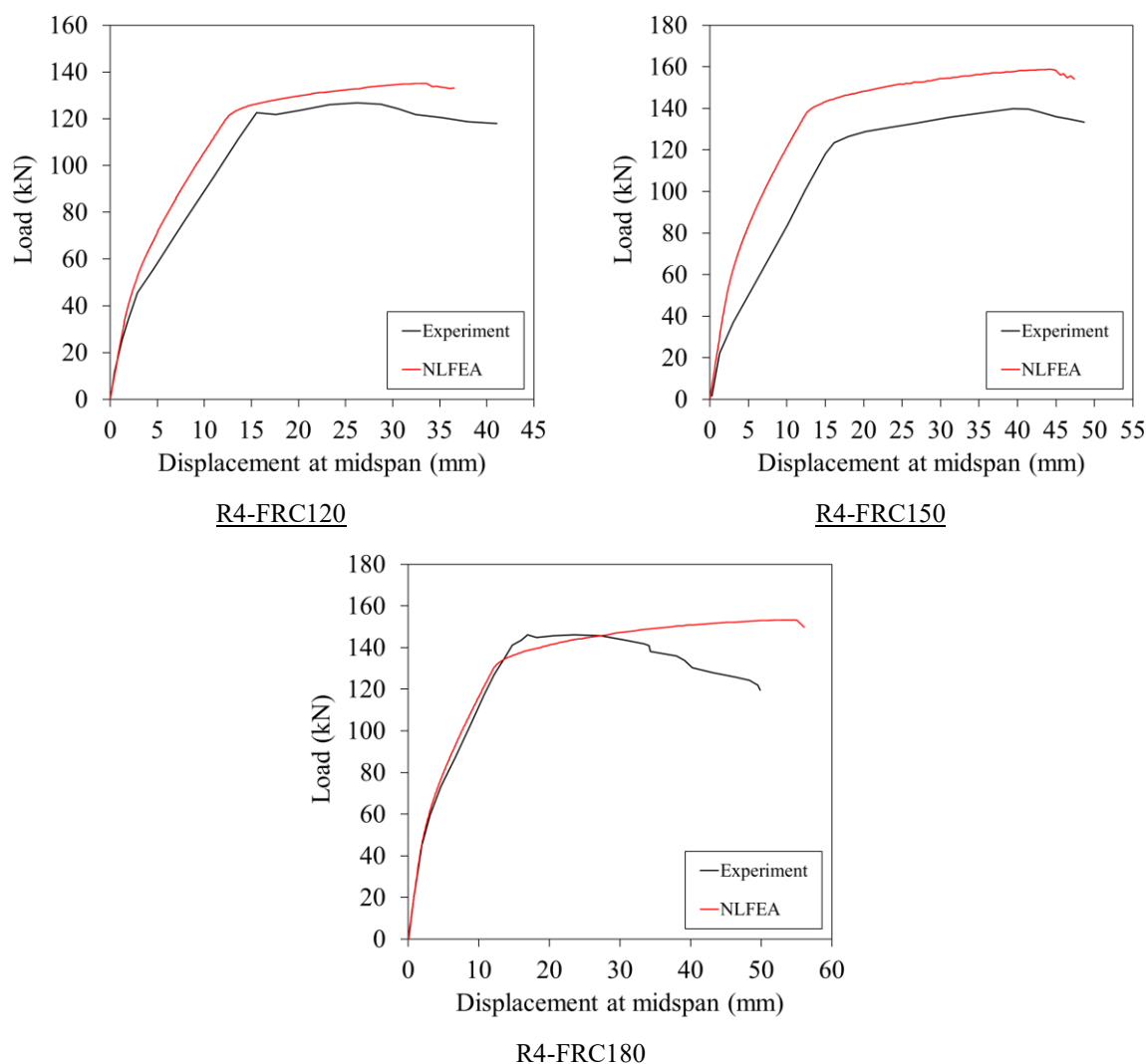


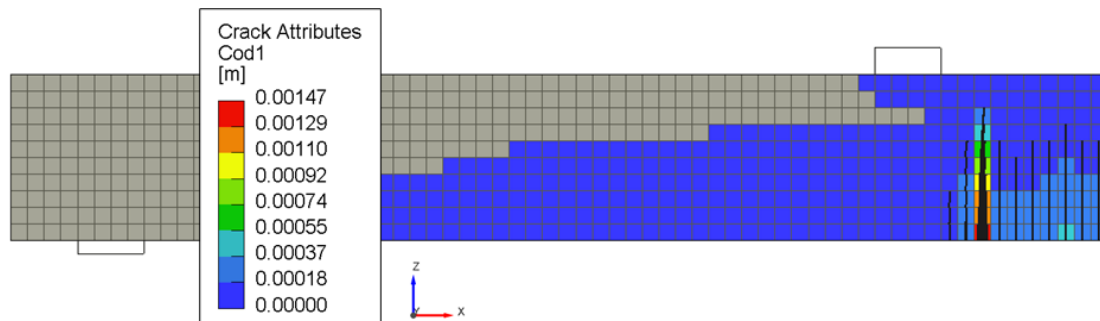
Figure 6.18 – Validated load-displacement responses of specimens tested by Yang et al. (2019)

A summary of the numerical and experimental ultimate loads and displacements is shown in Table 6.5 along with comparison ratios. It can be seen that the numerical models predict the ultimate loads moderately well given the average comparison ratio (P_{num}/P_{exp}) of 1.15. When excluding specimens R2-FRC120 and R2-FRC150, the average comparison ratio (P_{num}/P_{exp}) is equal to 1.06 with a low standard deviation of 0.08 which is fairly accurate. The relatively high average comparison ratio of the ultimate displacement ($\delta_{num}/\delta_{exp}$) of 1.58 suggests inaccuracies in the shape of the numerical load-displacement response for most beams. The minor discrepancies between the numerical and experimental load-displacement responses may be attributed to the assumptions related to the ultimate strength (f_u) and strain (ϵ_u) as well as the elastic modulus (E_s) of the longitudinal reinforcement seeing as all of the modelled beams failed in flexure. The input tensile stress-strain relationship of the longitudinal reinforcement heavily influences the flexural behaviour of beams.

Table 6.5 – Experimental and numerical ultimate loads and displacements for beams tested by Yang et al. (2019)

Specimen	Exp. load (P_{exp}) [kN]	Exp. disp. (δ_{exp}) [mm]	Num. load (P_{num}) [kN]	Num. disp. (δ_{num}) [mm]	Comparison ratio (P_{num}/P_{exp})	Comparison ratio ($\delta_{num}/\delta_{exp}$)
R2-FRC120	66.70	22.20	96.88	23.48	1.45	1.06
R2-FRC150	76.00	17.90	117.78	29.47	1.55	1.65
R2-FRC180	111.50	17.80	114.15	41.23	1.02	2.32
R3-FRC120	123.30	20.90	115.85	28.50	0.94	1.36
R3-FRC150	118.60	34.60	139.05	37.95	1.17	1.10
R3-FRC180	131.30	21.20	133.76	48.00	1.02	2.26
R4-FRC120	127.30	26.40	135.11	32.37	1.06	1.23
R4-FRC150	139.90	39.50	158.82	44.22	1.14	1.12
R4-FRC180	147.20	25.50	153.36	55.04	1.04	2.16
Average (Standard deviation)					1.15 (0.21)	1.58 (0.53)

The numerical results of the considered beams indicate flexural failure due to the crack pattern at failure and yielding of the longitudinal steel reinforcement. Although responses of the steel reinforcement were not reported, it is assumed to have yielded at midspan given that all of the specimens suffered flexural failure. The longitudinal reinforcement at midspan yielded shortly before failure for all modelled beam specimens, further highlighting the accuracy of the numerical models. The crack patterns at failure also matched the observations of Yang et al. (2019), where a single crack between the two loading points was reported to have developed prior to eventual failure. All the modelled beam specimens showed similar crack patterns, and an example of this is shown for specimen R4-FRC120 in Figure 6.19.

**Figure 6.19** – Crack pattern at failure of specimen R4-FRC120 tested by Yang et al. (2019)
(Minimum crack width of 0.1 mm is shown)

6.3.4.2 SHEAR BEHAVIOUR

Six beams tested by Qi et al. (2016) were used to replicate the shear behaviour of UHPC. The geometry and reinforcement details of the considered beams were identical, and they are shown in Figure 6.20. The user-defined input parameters for the UHPC and steel reinforcement material models of each considered beam are presented in Tables 6.6 and 6.7, respectively. The considered beams included 4.96% flexural reinforcement and no transverse reinforcement. Different UHPC mixtures (Mix A, B, D, E, F and G) were used to manufacture each of the considered beams, and they differed in compressive and tensile strength as well as elastic modulus as shown in Table 6.6. The input tensile function used for each type of UHPC is presented in Figure 6.21. Each beam underwent symmetrical four-point bending tests with a shear span to depth ratio (a_v/d) of 3.33, and they all reportedly failed in brittle shear. Qi et al. (2016) reported that the longitudinal reinforcement did not yield in any of the beams. The crack patterns at failure featured a critical inclined crack located between the loading and support plates.

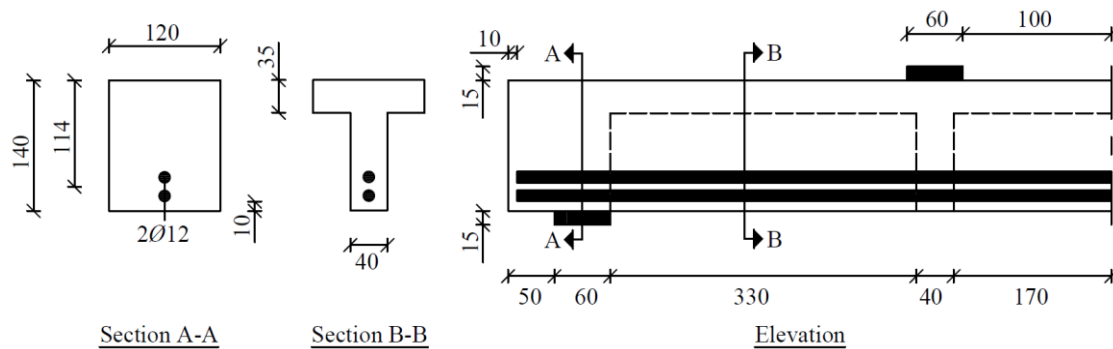


Figure 6.20 – Geometry and reinforcement details of beams tested by Qi et al. (2016) (Dimensions in mm)

Table 6.6 – Input parameters of UHPC in ATENA for beams tested by Qi et al. (2016)

Specimen	Mixture	Compressive strength (f_c) [MPa]	Tensile strength (f_t) [MPa]	Elastic modulus (E_c) [MPa]	Aggregate size (d_g) [mm]	Charact. length (L_t) [mm]	Localisation onset (ϵ_{loc}^t)
S1	Mix A ^A	-101.0 (-116.0)	7.1	42500	2.36	93.33	0.003214
S2	Mix D ^A	-81.3 (-96.3)	5.2	42400	2.36	93.33	0.003214
S3	Mix E ^A	-79.3 (-94.3)	6.1	47100	2.36	93.33	0.003214
S8	Mix F ^B	-105.5 (-120.5)	7.7	45600	2.36	93.33	0.003214
S9	Mix G ^C	-98.2 (-113.2)	6.5	47100	2.36	93.33	0.003214
S10	Mix B ^A	-102.1 (-117.1)	4.1	41800	2.36	93.33	0.003214

[Note] – () Refers to reported cube strength; ^A Fibre length (L_f) is 13 mm; ^B Fibre length (L_f) is 6 mm; ^C Fibre length (L_f) is 30 mm

Table 6.7 – Input parameters of steel reinforcement in ATENA for beams tested by Qi et al. (2016)

Diameter (ϕ) [mm]	Yield strength (f_y) [MPa]	Ultimate strength (f_u) [MPa]	Ultimate strain (ϵ_u) [%]	Elastic modulus (E_s) [MPa]
12	760.9	918.6	7.5 ^A	200000 ^B

^A Assumed grade B500C steel (BSI, 2016); ^B Assumed value based on typical value of steel

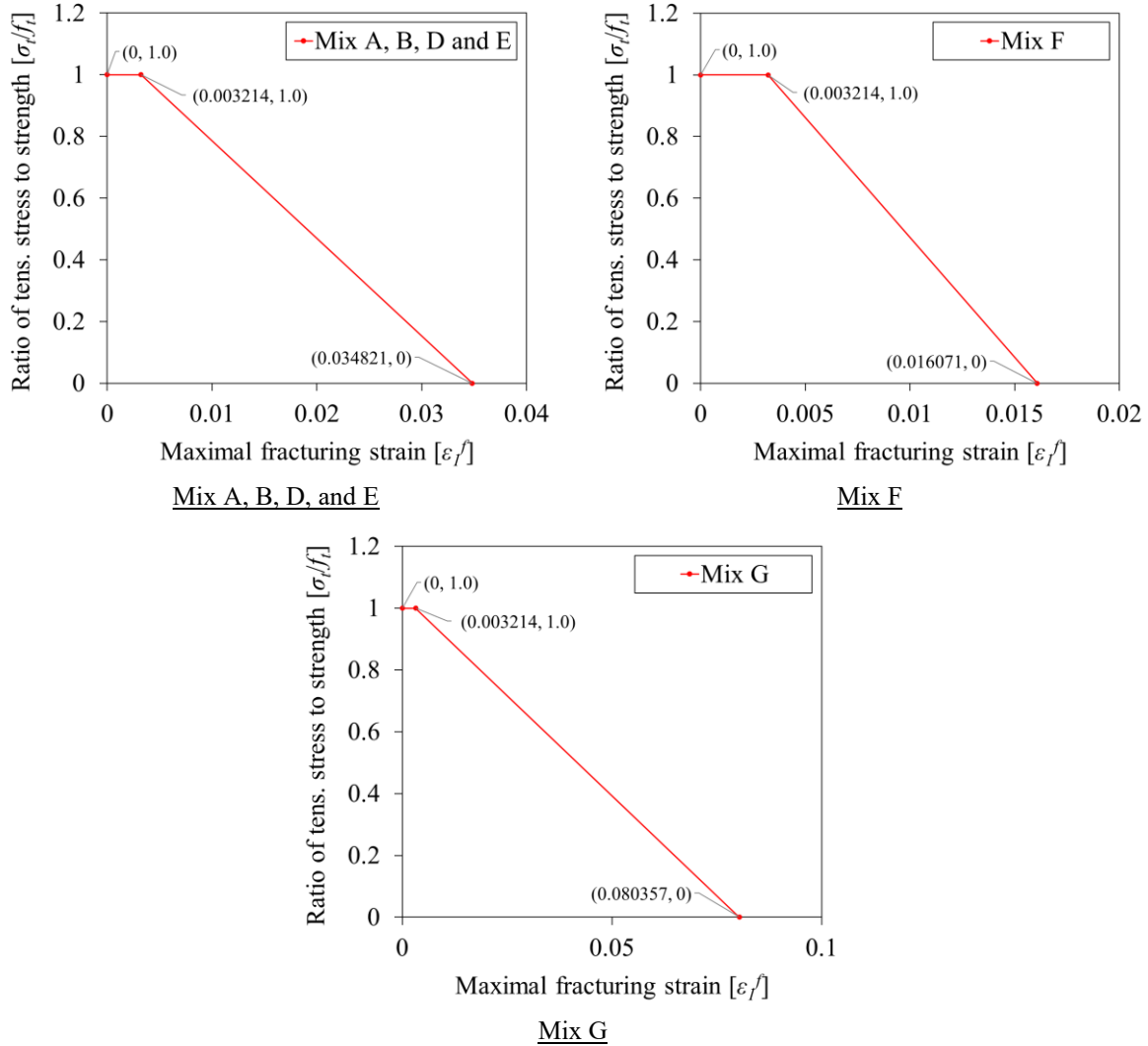


Figure 6.21 – Input tensile functions of UHPC in ATENA for beams tested by Qi et al. (2016)

A mesh sensitivity study was conducted on specimen S2. The mesh sizes of the 8-noded hexahedra elements were equal to 7 mm, 13 mm and 26 mm as shown in Figure 6.22, and along the height of the beam these produced 20, 10 and 5 elements, respectively. The influence of the mesh size on the numerical load-displacement response for specimen S2 is shown in Figure 6.23. It is evident that the difference is minimal between the numerical responses. The numerical responses do not appear to converge with a finer mesh size much like the modelled beams of Yang et al. (2019), which is possibly due to the adopted fracturing behaviour of

UHPC described in Section 6.2.2.2. The ultimate load is marginally affected by the varying mesh sizes with the finest mesh of 7 mm producing the highest ultimate load and the closest match to the experimental ultimate load. However, the difference between the ultimate loads given by the models with mesh sizes of 7 mm and 13 mm is marginal. Therefore, a medium mesh size of 13 mm was adopted for all beams tested by Qi et al. (2016) to reduce computation time, and this mesh size resulted in 10 elements along the height of the beam similar to the modelled beams of Yang et al. (2019).

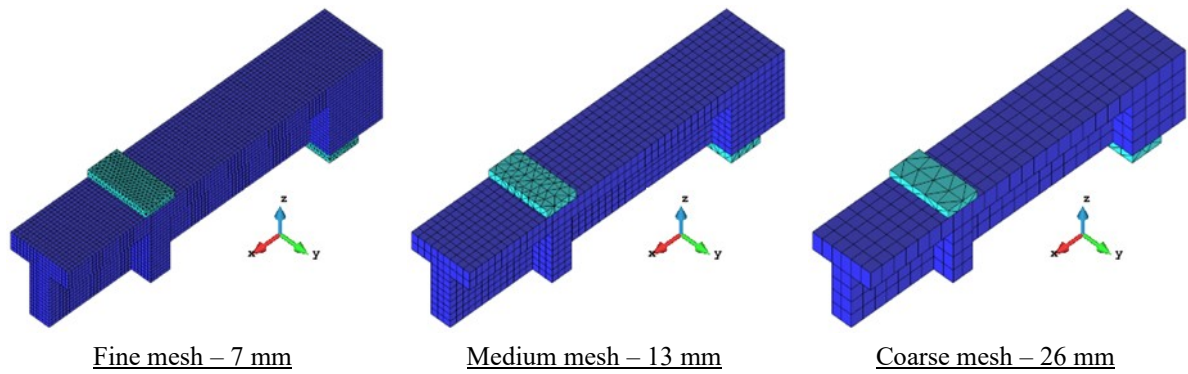


Figure 6.22 – Finite element mesh sizes for specimen S2 by Qi et al. (2016)

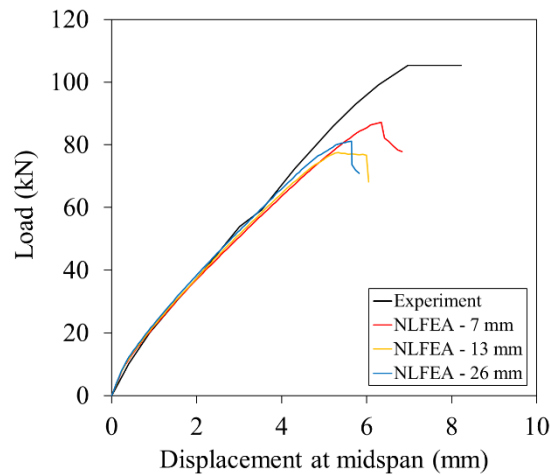
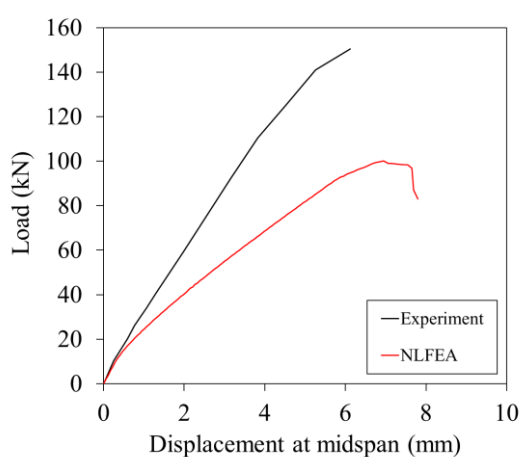
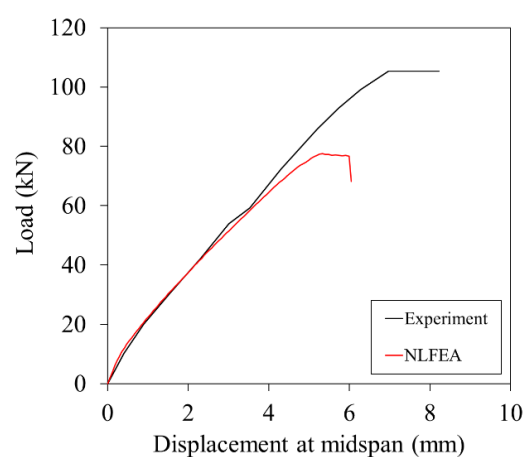
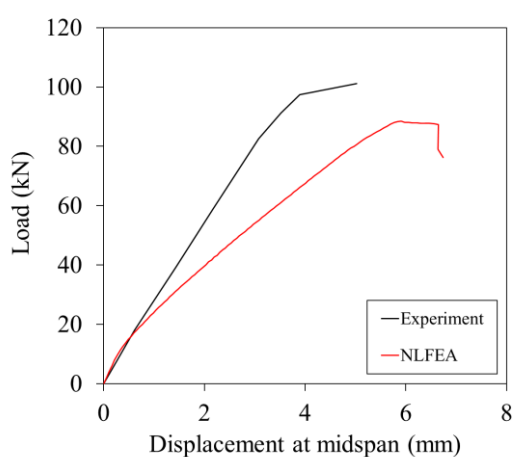
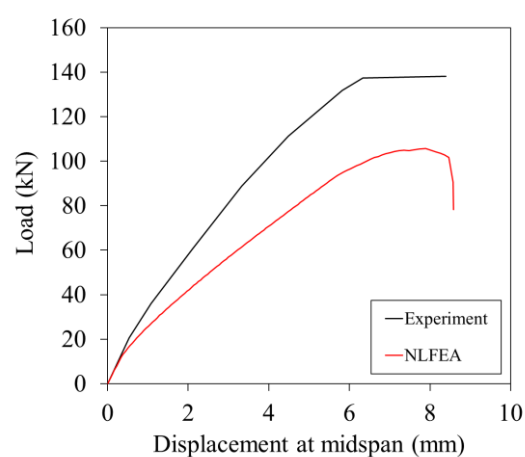


Figure 6.23 – Load-displacement response of specimen S2 tested by Qi et al. (2016) using different mesh sizes

Figure 6.24 compares the experimental and numerical load-displacement responses for the considered specimens of Qi et al. (2016). Much like the experimental responses, the numerical responses were brittle with failure occurring almost immediately after reaching the ultimate load, which suggests shear failure. The NLFEA captures the general shape of the experimental response, but for most beams the numerical response underestimates the experimentally observed strength and stiffness. Since the models consistently underestimate the measured

ultimate loads, these discrepancies could be due to the direct tensile strength of the UHPC being greater than reported by Qi et al. (2016). This would arise if the control specimens used to establish the UHPC properties were not representative of the tensile strength of UHPC used to manufacture the beams as was the case for specimens R2-FRC120 and R2-FRC150 of Yang et al. (2019). In addition, the derivation of the tensile stress-strain relationship of UHPC required several assumptions that are discussed in Section 6.3.3 such as taking the crack opening (w_{peak}) corresponding to the peak tensile stress (f_{ctm}) equal to 0.3 mm as it was not reported. Consequentially, this may have led to inaccuracies in the input tensile function in ATENA as well as discrepancies between the experimental and numerical responses.

S1S2S3S8

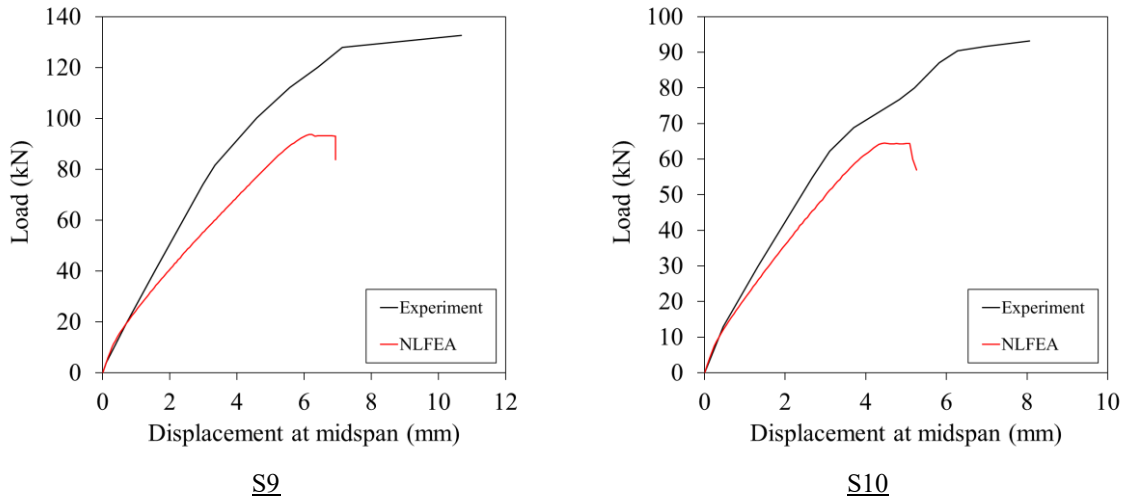


Figure 6.24 – Validated load-displacement responses of specimens tested by Qi et al. (2016)

To better understand the numerical predictions, the experimental (P_{exp}) and numerical (P_{num}) ultimate loads of each beam were compared with the ultimate loads (P_{calc}) given by SIA 2052 (EPFL, 2016). The latter were calculated as outlined in Section 2.4.4.2 seeing as the predicted shear strength given by SIA 2052 showed good agreement with test results of slender UHPC beams according to Yang et al. (2022). The comparisons are presented in Table 6.8. Both the numerical and calculated predictions consistently underestimate the measured ultimate load. The average and standard deviation are 0.71 and 0.07 for the comparison ratio (P_{num}/P_{exp}) and 0.72 and 0.08 for the comparison ratio (P_{calc}/P_{exp}). The similarity in statistics is due to the numerically predicted ultimate loads being almost identical to those given by SIA 2052 as highlighted by the average and standard deviation of the comparison ratio (P_{num}/P_{calc}) being equal to 1.02 and 0.05, respectively. The similarity of the NLFEA and SIA 2052 predictions suggests that the inaccuracies between the numerical and experimental ultimate loads may be linked to inaccuracies in the reported material strengths rather than the numerical models.

Table 6.8 – Experimental, numerical and calculated ultimate loads for beams tested by Qi et al. (2016)

Specimen	Experimental ultimate load (P_{exp}) [kN]	Numerical ultimate load (P_{num}) [kN]	Calculated ultimate load (P_{calc}) [kN]	Comparison ratio (P_{num}/P_{exp})	Comparison ratio (P_{num}/P_{calc})	Comparison ratio (P_{calc}/P_{exp})
S1	152.00	100.00	100.94	0.66	0.99	0.66
S2	104.20	76.41	73.93	0.74	1.05	0.71
S3	105.00	86.43	86.72	0.84	1.02	0.83
S8	137.80	107.25	109.47	0.77	0.97	0.79
S9	132.40	92.91	92.41	0.71	1.01	0.70
S10	94.20	65.98	58.29	0.68	1.11	0.62
Average (Standard deviation)				0.71 (0.07)	1.02 (0.05)	0.72 (0.08)

The numerical results reveal that the longitudinal reinforcement of specimens S1, S8 and S9 yielded at midspan shortly before failure. This was not the case in reality according to Qi et al. (2016) suggesting that the contribution to flexural resistance of the UHPC is underestimated by the NLFEA due to a mischaracterisation of the UHPC. The fact that these beams failed almost immediately after reaching the ultimate load in the numerical analysis (see Figure 6.24) is considered to indicate a combined flexural-shear mode of failure. The crack pattern at failure for all considered beams matched the observations of Qi et al. (2016), where an inclined crack between the loading and support plates was reported to have formed prior to eventual failure. All of the modelled beams had similar crack patterns as shown for specimen S2 in Figure 6.25. The experimental and numerical displacements are compared at the ultimate load in Table 6.9. The displacements should be assessed alongside Figure 6.24 which shows that the good comparison of ultimate displacements in Table 6.9 for beams S1, S3 and S8 results from the stiffness being underestimated in the NLFEA. Up to failure, the experimental and numerical load-displacement responses compare better for beams S2, S9 and S10, but the ultimate displacement is underestimated due to the ultimate load being underestimated. Overall, the correspondence between the experimental and numerical responses is considered reasonable given the uncertainty in the properties of the UHPC in the tested beams. The proposed modelling approach is further developed in subsequent sections.

Table 6.9 – Experimental and numerical ultimate displacements for beams tested by Qi et al. (2016)

Specimen	Experimental displacement (δ_{exp}) [mm]	Numerical displacement (δ_{num}) [mm]	Comparison ratio ($\delta_{num}/\delta_{exp}$)
S1	6.10	6.95	1.14
S2	8.20	5.34	0.65
S3	5.00	5.91	1.18
S8	8.40	7.89	0.94
S9	10.70	6.24	0.58
S10	8.00	4.47	0.56
Average (Standard deviation)			0.84 (0.28)

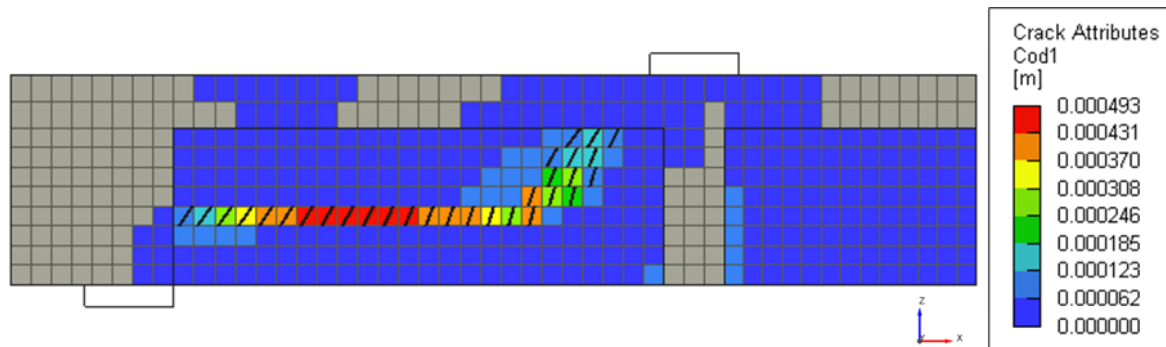


Figure 6.25 – Crack pattern at failure of specimen S2 tested by Qi et al. (2016)
(Minimum crack width of 0.1 mm is shown)

6.3.5 RC BEAMS WITH UHPC JACKETS FROM THE LITERATURE

6.3.5.1 FLEXURAL BEHAVIOUR

The beams of Al-Osta et al. (2017) were used to replicate the flexural behaviour of RC beams with and without UHPC jackets. Two control RC beams were considered along with two identical RC beams with UHPC U-shaped jackets. All beams were subjected to symmetrical four-point bending tests before failing in flexure. The geometry and reinforcement details of the control and strengthened beams are shown in Figure 6.26. The user-defined input parameters for the NSC, UHPC and steel reinforcement material models of each considered beam are presented in Tables 6.10, 6.11 and 6.12, respectively. The RC beams included 0.57% flexural reinforcement along with a shear reinforcement ratio (ρ_w) of 1.44%. The U-shaped jackets spanned across the entire length of the beams and were 30 mm in side and bottom thickness and contained no embedded reinforcement. The jackets were bonded to the beams using two different methods, namely casting against a rough sandblasted substrate and adhesively bonding against a smooth substrate for specimens RC-SB-3SJ and RC-EP-3SJ, respectively. The same type of UHPC was used to manufacture the jackets, and the input tensile function in ATENA is shown in Figure 6.27. Although the response of the steel reinforcement was not reported, the crack patterns indicate flexural failure as only vertical cracks between the two loading plates were observed for all beams. A displacement plateau was also observed in the load-displacement responses of all beams.

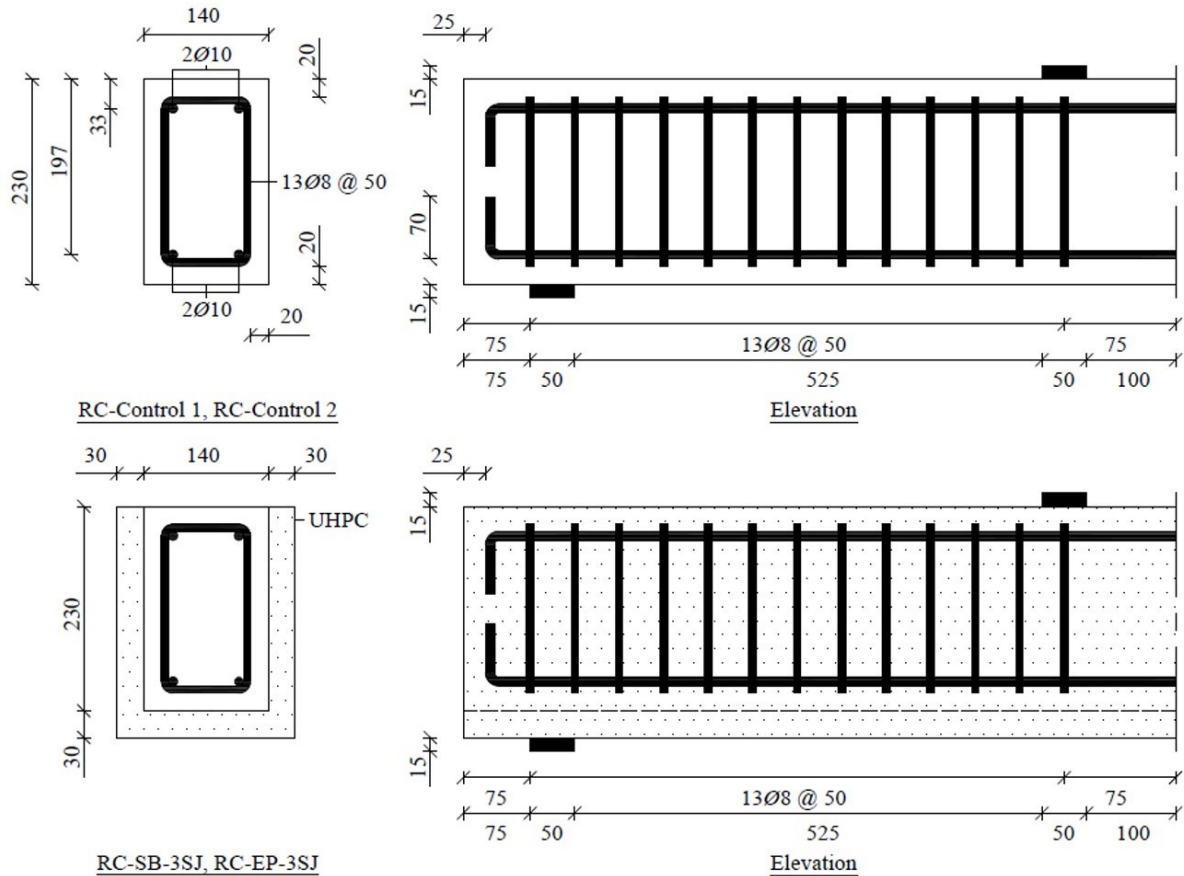


Figure 6.26 – Geometry and reinforcement details of beams tested by Al-Osta et al. (2017) (Dimensions in mm)

Table 6.10 – Input parameters of NSC in ATENA for beams tested by Al-Osta et al. (2017)

Specimen	Compressive strength (f_c) [MPa]	Tensile strength (f_t) [MPa]	Elastic modulus (E_c) [MPa]	Fracture energy (G_f) [N/m]	Aggregate size (d_g) [mm]	Onset of crushing (f_{c0})
RC-Control 1	-54.0	3.16	36487 (18243)	150 (75)	20.0	-6.32
RC-Control 2	-54.0	3.16	36487 (18243)	150 (75)	20.0	-6.32
RC-SB-3SJ	-54.0	3.16	36487 (18243)	150 (75)	20.0	-6.32
RC-EP-3SJ	-54.0	3.16	36487 (18243)	150 (75)	20.0	-6.32

[Note] – () Refers to reduced value based on calibration used in validated model

Table 6.11 – Input parameters of UHPC in ATENA for beams tested by Al-Osta et al. (2017)

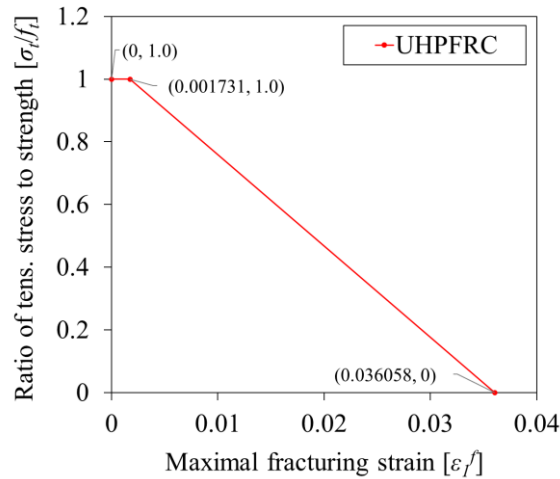
Specimen	Mixture	Compressive strength (f_c) [MPa]	Tensile strength (f_t) [MPa]	Elastic modulus (E_c) [MPa]	Aggregate size (d_g) [mm]	Charact. length (L_t) [mm]	Localisation onset (ϵ_{loc}^t)
RC-SB-3SJ	UHPRC ^A	-128.0	7.0	46000	2.5	173.33	0.001731
RC-EP-3SJ	UHPRC ^A	-128.0	7.0	46000	2.5	173.33	0.001731

^A Fibre length (L_f) is 25 mm

Table 6.12 – Input parameters of steel reinforcement in ATENA for beams tested by Al-Osta et al. (2017)

Diameter (ϕ) [mm]	Yield strength (f_y) [MPa]	Ultimate strength (f_u) [MPa]	Ultimate strain (ϵ_u) [%]	Elastic modulus (E_s) [MPa]
10	590	680	7.5 ^A	200000 ^B
8	590	660	7.5 ^A	200000 ^B

^A Assumed grade B500C steel (BSI, 2016); ^B Assumed value based on typical value of steel

**Figure 6.27** – Input tensile function of UHPC in ATENA for beams tested by Al-Osta et al. (2017)

Mesh sensitivity studies were conducted on the numerical models of the RC beams (RC-Control 1 and RC-Control 2) and strengthened beams (RC-SB-3SJ and RC-EP-3SJ). The numerical models of the RC and strengthened beams were used to determine suitable mesh sizes for the RC beam and UHPC jacket, respectively. The mesh sizes that were investigated for the RC beam were equal to 15 mm, 23 mm and 46 mm, which produced 15, 10 and 5 elements along the height of the beam, respectively. The mesh sizes that were investigated in the UHPC jacket were equal to 13 mm, 18 mm and 26 mm, which produced 20, 15 and 10 elements along the height of the jacket, respectively. The different mesh sizes used to investigate the UHPC jacket are shown in Figure 6.28, and the results of the sensitivity studies are shown in Figure 6.29. It is evident that the numerical response was not significantly affected by the mesh size of the RC beam, therefore a mesh size of 23 mm was adopted, which produced 10 elements along the height of the beam. The same observation can be made for the different mesh sizes of the UHPC jacket used in the modelled strengthened beam. It appears that the numerical responses do not converge with a finer mesh size much like the modelled UHPC beams in Section 6.3.4, which is likely due to the adopted fracturing behaviour of UHPC

described in Section 6.2.2.2. Therefore, a mesh size equal to 26 mm was adopted that produced 10 elements along the height of the jacket much like the RC beam.

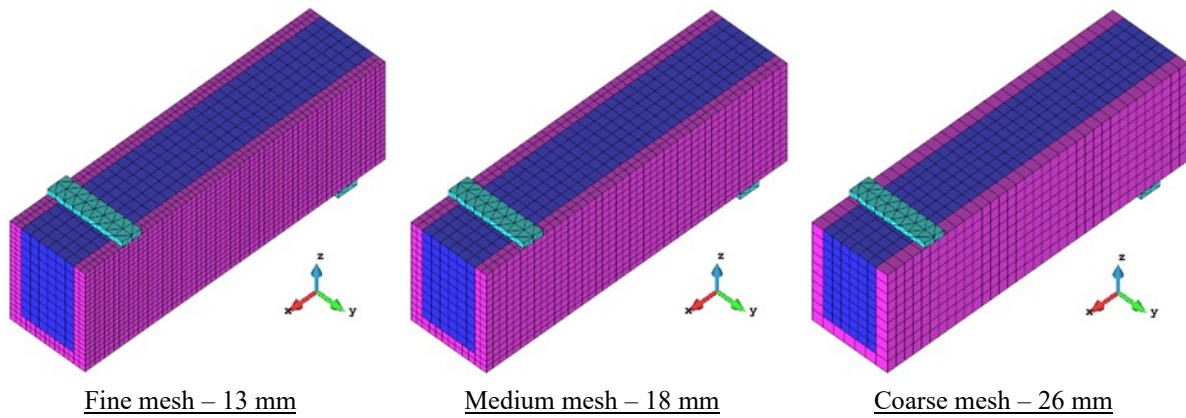
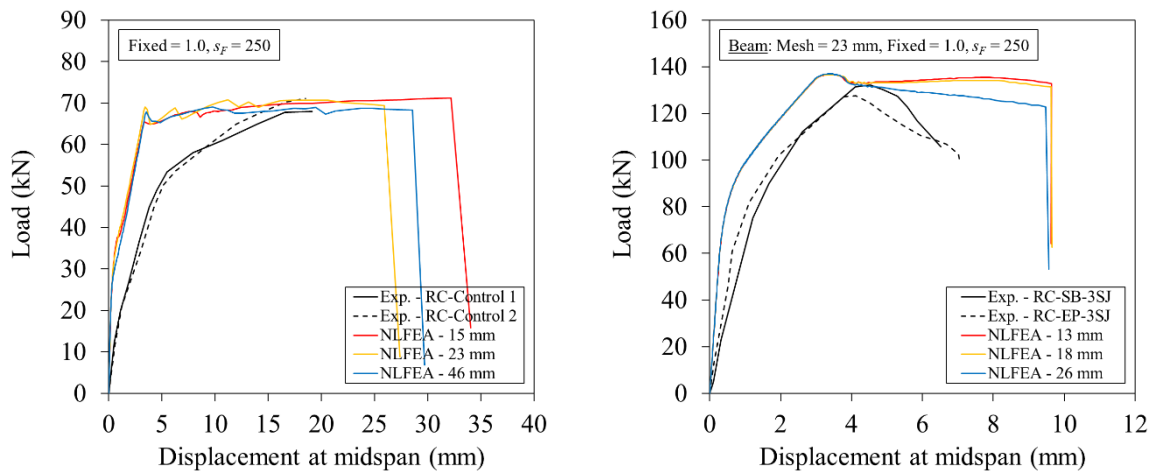


Figure 6.28 – Finite element mesh sizes for the UHPC jacket of specimens RC-SB-3SJ and RC-EP-3SJ tested by Al-Osta et al. (2017)



Mesh of RC beam – RC-Control 1 and RC-Control 2

Mesh of UHPC jacket – RC-SB-3SJ and RC-EP-3SJ

Figure 6.29 – Load-displacement response of beams tested by Al-Osta et al. (2017) using different mesh sizes for RC beam and UHPC jacket

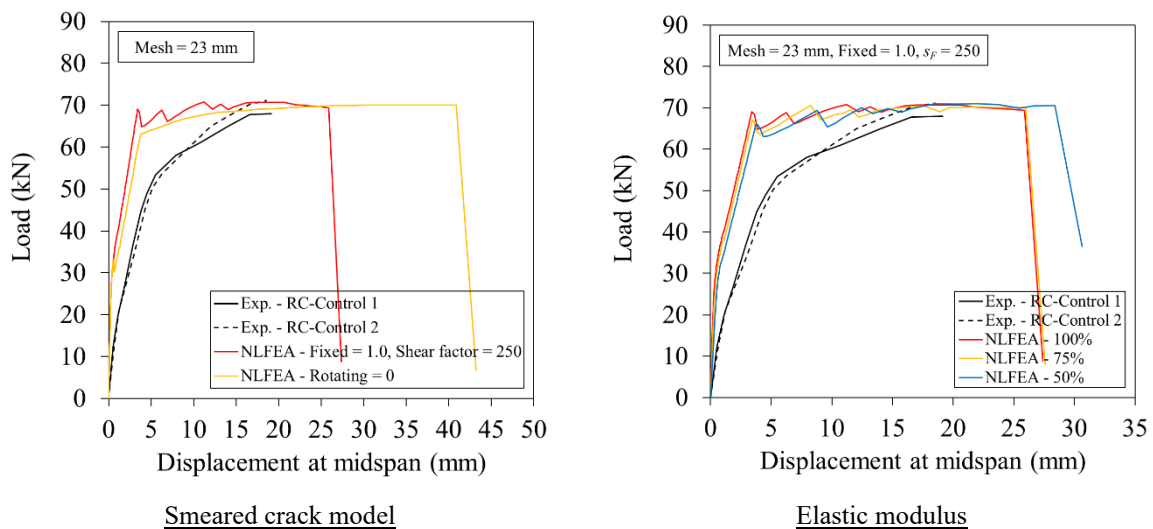
Sensitivity studies were also conducted on the numerical model of the RC beam (RC-Control 1 and RC-Control 2) to calibrate various input parameters of the NSC material model. The parameters that were investigated included the type of smeared crack model, initial shrinkage strain, elastic modulus (E_c), fracture energy (G_f) and compressive strength reduction factor limit (r_c^{lim}). Figure 6.30 shows the results of the sensitivity studies by comparing the numerical and experimental load-displacement responses for each investigated parameter.

It is evident in Figure 6.30 that the rotating smeared crack model for the RC control beam achieved a similar response to the default fixed smeared crack model. Both crack models predicted the ultimate load and ductile flexural failure relatively well, however the initial cracking load and overall shape of the numerical load-displacement response do not match well with the experimental results. The numerical response is also much stiffer initially than the experimental response. The inaccuracies in the initial stiffness of the numerical response may be attributed to the adopted boundary conditions or assumed elastic modulus of the longitudinal reinforcement (Pryl and Červenka, 2018). Based on this sensitivity study and the sensitivity study conducted on the shear critical RC beam of Alkhoury (2017) discussed later in Section 6.3.5.2, a fixed crack model with a shear factor coefficient equal to 250 was adopted for the beams of Al-Osta et al. (2017). The shear factor coefficient was observed to have no significant impact on the flexure critical RC beam of Al-Osta et al. (2017).

A few methods for improving the initial stiffness response in ATENA were investigated in this research. According to the Troubleshooting Manual of ATENA by Pryl and Červenka (2018), the initial stiffness is influenced by the boundary conditions of the model (i.e., loading apparatus and symmetry planes), elastic modulus of concrete and area and elastic modulus of the steel reinforcement bars. Additionally, the mesh size of the concrete member may impact the initial stiffness response as at least 6 to 10 elements per member thickness for quantitative results should be provided particularly in regions subjected to bending (Pryl and Červenka, 2018). Incorporating the effect of shrinkage is an alternative method to improve the initial stiffness as well as the overall accuracy of the numerically predicted load-displacement response in comparison to that of the test (Pryl and Červenka, 2018). Shrinkage can be considered by either reducing the tensile strength and/or fracture energy of concrete, with Pryl and Červenka (2018) suggesting a reduction between 10% to 50% of the initial tensile strength. Shrinkage can also be applied in ATENA as a constant initial volume change based on input strains in the x-, y- and z-directions (Pryl and Červenka, 2018). Based on the suggestions given by Pryl and Červenka (2018), reducing the elastic modulus of concrete as well as considering the effect of shrinkage were investigated in this research to improve the initial stiffness and overall accuracy of the numerically predicted load-displacement response. The effect of shrinkage was investigated by applying an initial shrinkage strain and reducing the fracture energy of concrete.

The results in Figure 6.30 show the effects of reducing the elastic modulus of concrete and incorporating shrinkage. A 50% reduction of the elastic modulus of concrete helped reduce the initial stiffness of the numerical response, although the effect is relatively minimal. The suggested shrinkage strains that were investigated ranged from 0.015% to 0.045% as suggested by Pryl and Červenka (2018), and the results shown in Figure 6.30 indicate that the initial stiffness in the numerical response is reduced by increasing the shrinkage strain, but the effect is relatively insignificant. The effect of reducing the fracture energy of concrete was also relatively insignificant as shown in Figure 6.30. Based on these observations as well as the observations of the modelled control RC beam (Nr. 5) of Alkhoury (2017) discussed later in Section 6.3.5.2, the best fitted parameters used in this research included a 50% reduction to both the elastic modulus and fracture energy.

The influence of the lower bound of the compressive strength reduction factor on the load-displacement was also investigated. There is no significant impact on the load-displacement response when the default value in ATENA of 0.8 is reduced to 0.65 and 0.5. However, a value of 0.5 was adopted in this research based on the results of the sensitivity study conducted on the shear critical RC beam of Alkhoury (2017), and this is discussed later in Section 6.3.5.2.



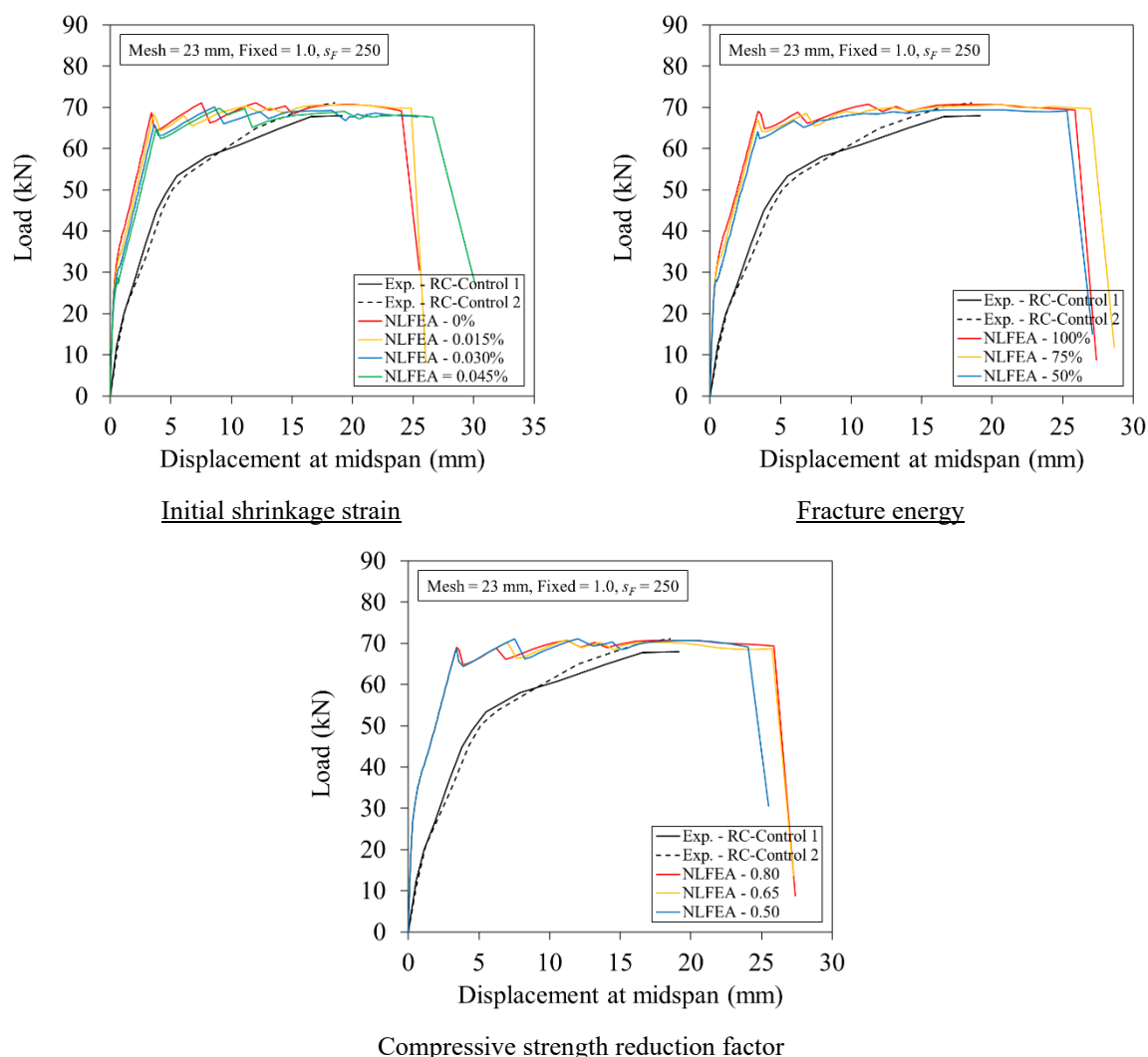


Figure 6.30 – Load-displacement response of specimens RC-Control 1 and RC-Control 2 tested by Al-Osta et al. (2017) using various input parameters for NSC material model

When using the calibrated parameters of the NSC and UHPC material models presented in Tables 6.1 and 6.2, the numerical and experimental results compare reasonably well overall as shown in Figure 6.31. For the control RC beams (RC-Control 1 and RC-Control 2), the numerical response accurately replicates both the initial cracking and peak load as well as the relatively ductile flexural failure. A notable discrepancy between the numerical and experimental results is in the numerical post-peak response which continues well past the point of experimental failure. The numerical response of the strengthened specimens (RC-SB-3SJ and RC-EP-3SJ) in Figure 6.31 also shows good agreement with the experimental responses. The initial cracking load appears to be replicated fairly well by the numerical model along with the peak load. However, the post-peak softening response of the numerical model is not as brittle as the strengthened beams, which may be due to the assumed grade of the reinforcement.

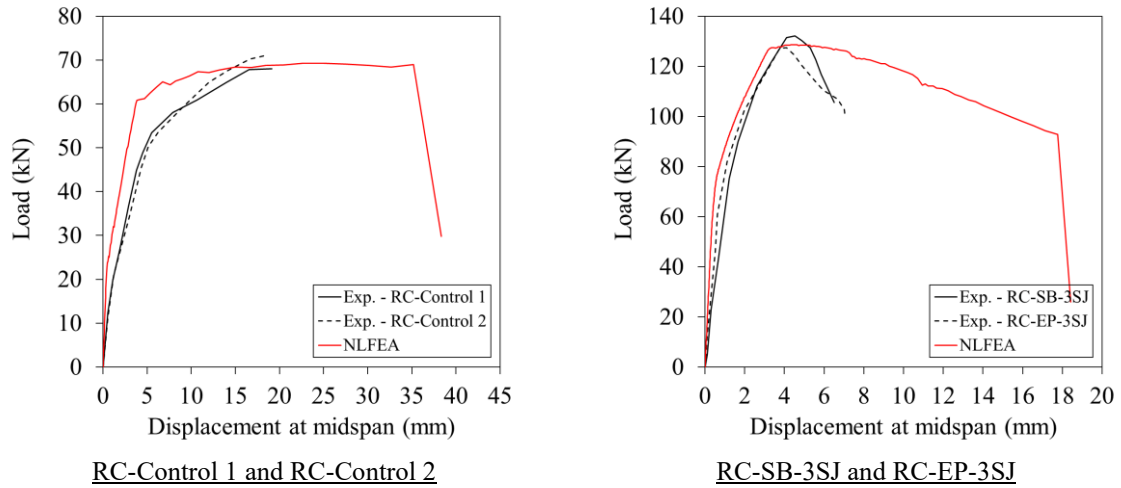


Figure 6.31 – Validated load-displacement responses for specimens tested by Al-Osta et al. (2017)

The results of the numerical models are acceptable overall for the beams of Al-Osta et al. (2017). A summary of the experimental and numerical ultimate loads and displacements are shown in Table 6.13. It can be seen that the numerically predicted ultimate loads and displacements for the RC beams and strengthened beams match reasonably well with the experimental results seeing as the average ultimate load (P_{num}/P_{exp}) and displacement ($\delta_{num}/\delta_{exp}$) comparison ratios are 1.01 and 0.90, respectively. Although the response of the longitudinal reinforcement was not reported, it is assumed that the reinforcement at midspan yielded given that all of the specimens suffered flexural failure. The reinforcement at midspan yielded shortly before failure in all of the modelled beams confirming that flexural failure occurred. The crack patterns at failure (see Figure 6.32) also matched the reported observations for the RC and strengthened beams. Several vertical cracks developed between the two loading plates prior to eventual failure for the RC beam. Only a single crack developed between the two loading points in the strengthened beam. The crack patterns at failure are almost identical in both the beam and jacket as a perfect bond was assumed between the two components.

Table 6.13 – Experimental and numerical ultimate loads and displacements for beams tested by Al-Osta et al. (2017)

Specimen	Exp. load (P_{exp}) [kN]	Exp. disp. (δ_{exp}) [mm]	Num. load (P_{num}) [kN]	Num. disp. (δ_{num}) [mm]	Comparison ratio (P_{num}/P_{exp})	Comparison ratio ($\delta_{num}/\delta_{exp}$)
RC-Control 1	68.00	19.10	69.29	22.62	0.98	0.84
RC-Control 2	71.14	19.10	69.29	22.62	1.03	0.84
RC-SB-3SJ	132.00	4.55	128.67	4.65	1.03	0.98
RC-EP-3SJ	129.00	4.35	128.67	4.65	1.00	0.94
Average (Standard deviation)					1.01 (0.02)	0.90 (0.07)

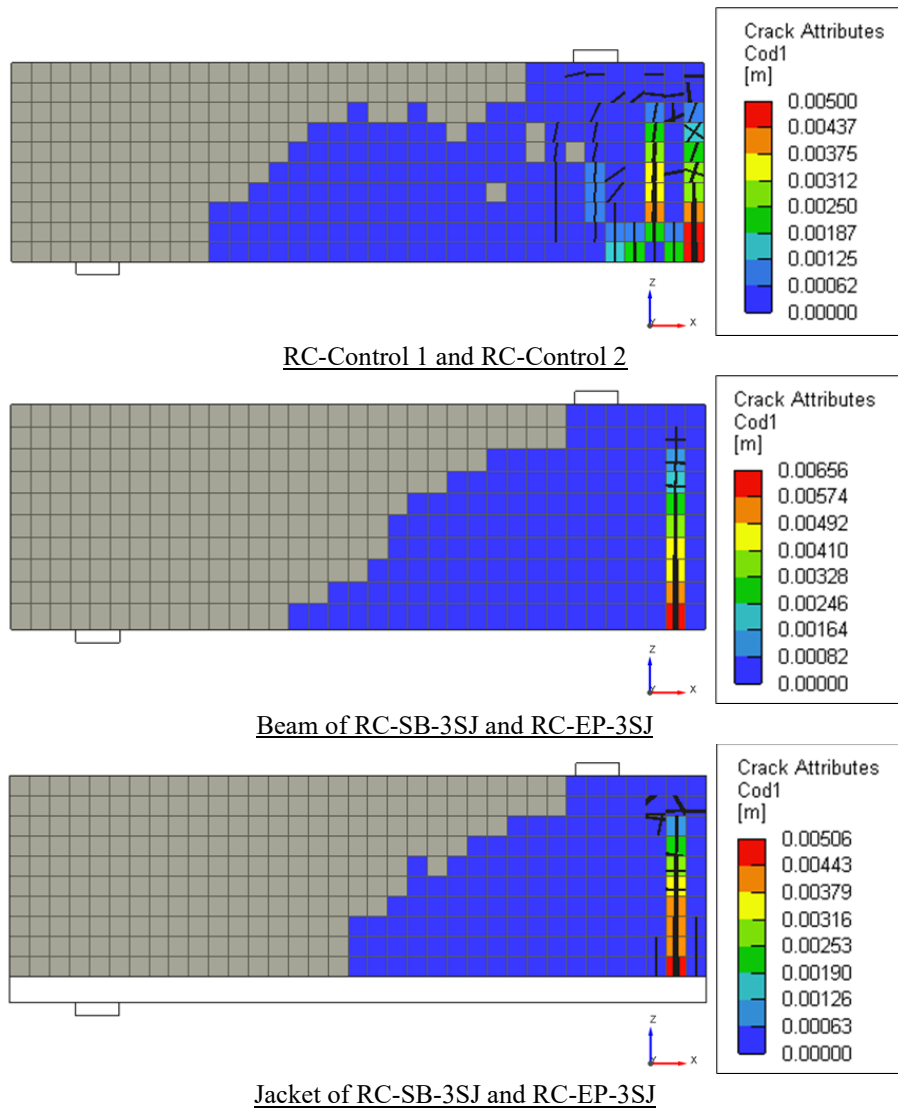


Figure 6.32 – Crack pattern at failure for beams tested by Al-Osta et al. (2017)
(Minimum crack width of 0.1 mm is shown)

6.3.5.2 SHEAR BEHAVIOUR

Beams tested by Alkhoury (2017) were used to replicate the shear failure of RC beams with and without UHPC jackets. A single control RC beam (Nr. 5) was considered along with two identical RC beams with UHPC U-shaped jackets (Nr. 6 and Nr. 7). The geometry and reinforcement details of the control and strengthened beams are shown in Figure 6.33. The user-defined input parameters for the NSC, UHPC and steel reinforcement material models of each considered beam are presented in Tables 6.14, 6.15 and 6.16, respectively. The beams were loaded symmetrically in four-point bending with a shear span to depth ratio (a_v/d) of 2.64. All of the tested beams failed in shear. The RC beams included 1.31% flexural reinforcement along with a shear reinforcement ratio (ρ_w) of 0.11%. The U-shaped jackets of strengthened specimens Nr. 6 and Nr. 7 spanned across the entire length of the beams, and they

were both 40 mm in side thickness and the bottom thickness was 40 mm and 60 mm for specimens Nr. 6 and Nr. 7, respectively. The strengthened beams were manufactured such that the U-shaped jacket reached the top surface of the beam at a spacing of 250 mm, and the jackets included embedded vertical steel dowels and transverse reinforcement in specimens Nr. 6 and Nr. 7, respectively. The jackets of specimens Nr. 6 and Nr. 7 also included varied amounts of embedded longitudinal reinforcement. Different batches of the same UHPC mixture were used to manufacture the jackets, and the input tensile function of UHPC in ATENA is shown in Figure 6.34 for each strengthened specimen. Inclined cracks were observed at failure between the two loading and support plates for all beams. Although the responses of the steel reinforcement were not reported, the observed crack pattern at failure along with the brittle post-peak load-displacement responses indicate shear failure occurred for all beams.

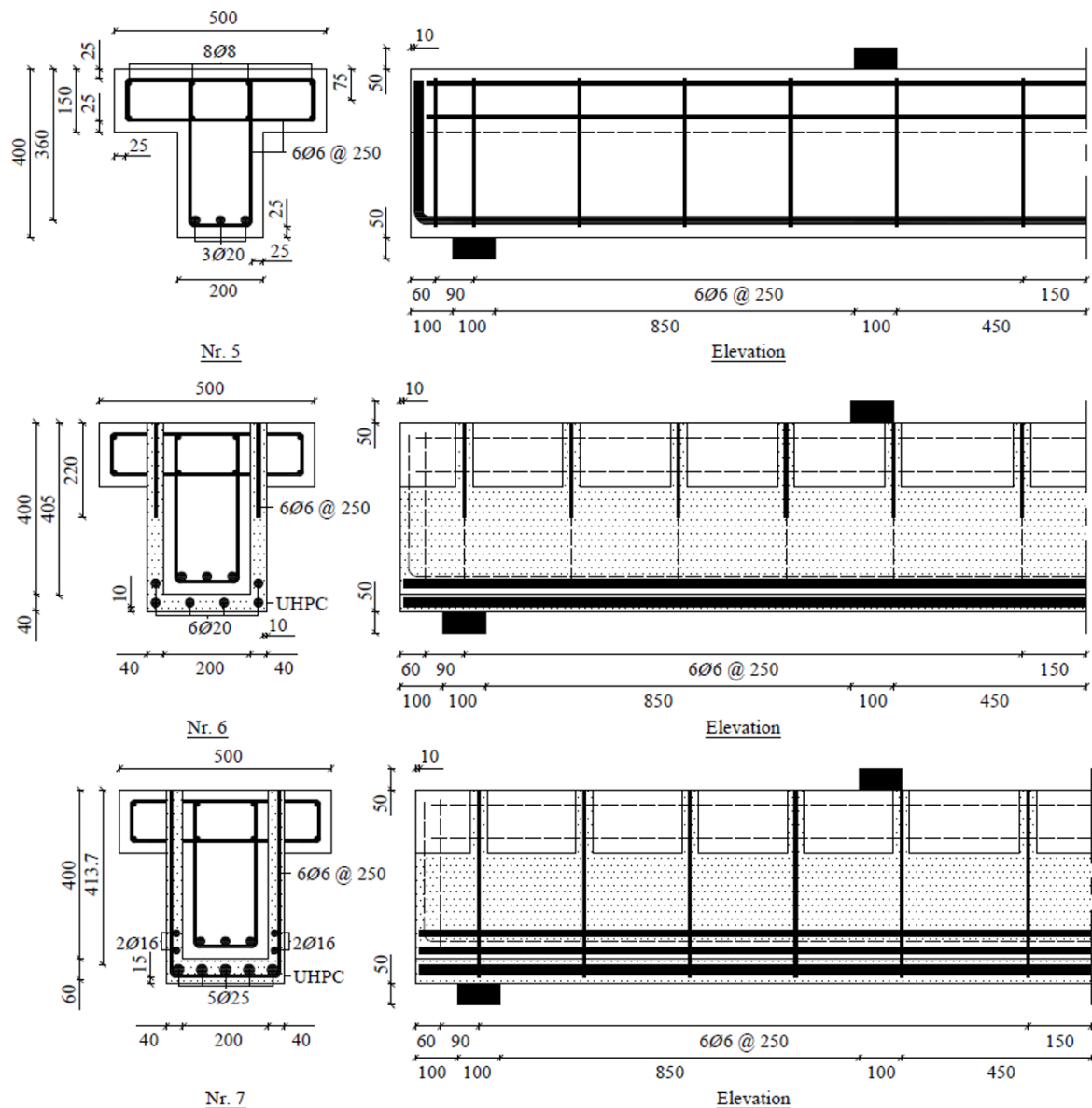


Figure 6.33 – Geometry and reinforcement details of beams tested by Alkhoury (2017) (Dimensions in mm)

Table 6.14 – Input parameters of NSC in ATENA for beams tested by Alkhoury (2017)

Specimen	Compressive strength (f_c) [MPa]	Tensile strength (f_t) [MPa]	Elastic modulus (E_c) [MPa]	Fracture energy (G_f) [N/m]	Aggregate size (d_g) [mm]	Onset of crushing (f_{c0})
Nr. 5	-17.1	1.61	13236 (6618)	122 (61)	8.0	-3.22
Nr. 6	-15.4	1.88	15658 (7829)	119 (60)	8.0	-3.76
Nr. 7	-17.1	1.61	13236 (6618)	122 (61)	8.0	-3.22

[Note] – () Refers to reduced value based on calibration used in validated model

Table 6.15 – Input parameters of UHPC in ATENA for beams tested by Alkhoury (2017)

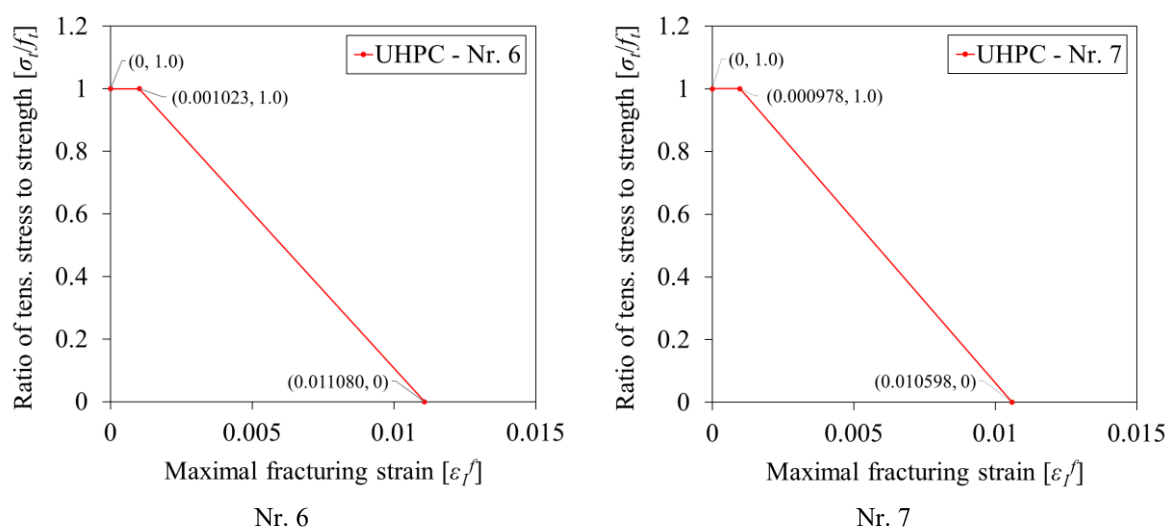
Specimen	Mixture	Compressive strength (f_c) [MPa]	Tensile strength (f_t) [MPa]	Elastic modulus (E_c) [MPa]	Aggregate size (d_g) [mm]	Charact. length (L_t) [mm]	Localisation onset (ϵ_{loc}^t)
Nr. 6	UHPC – Nr. 6 ^A	-167.4	7.0	45000 ^B	0.5	293.33	0.001023
Nr. 7	UHPC – Nr. 7 ^A	-167.4	7.0	45000 ^B	0.5	306.67	0.000978

^AFibre length (L_f) equals 13 mm; ^B Assumed value based on typical value of steel

Table 6.16 – Input parameters of steel reinforcement in ATENA for beams tested by Alkhoury (2017)

Diameter (ϕ) [mm]	Yield strength (f_y) [MPa]	Ultimate strength (f_u) [MPa]	Ultimate strain (ϵ_u) [%]	Elastic modulus (E_s) [MPa]
25	564.60	669.05	7.5 ^A	200000 ^B
20	518.10	640.30	7.5 ^A	200000 ^B
16 ^C	518.10	640.30	7.5 ^A	200000 ^B
10	529.70	609.30	7.5 ^A	200000 ^B
8	618.30	720.70	7.5 ^A	200000 ^B

^A Assumed grade B500C steel (BSI, 2016); ^B Assumed value based on typical value of steel; ^C Insufficient data, assumed same as $\phi 20$


Figure 6.34 – Input tensile functions of UHPC in ATENA for beams tested by Alkhoury (2017)

Mesh sensitivity studies were conducted on the models of the control RC beam (Nr. 5) and strengthened beam (Nr. 6) to determine suitable mesh sizes for the RC beam and UHPC jacket, respectively. The mesh sizes that were investigated for the control RC beam (Nr. 5) were equal to 20 mm, 40 mm and 80 mm, which produced 20, 10 and 5 elements along the height of the beam, respectively. The mesh sizes that were investigated for the UHPC jacket (Nr. 6) were equal to 22 mm, 30 mm and 40 mm, which produced 20, 15 and 10 elements along the height of the jacket, respectively. The different mesh sizes used to investigate the UHPC jacket are shown in Figure 6.35, and the results of the sensitivity studies are shown in Figure 6.36. It is evident that the load-displacement responses of the models with the finest (20 mm) and coarsest (80 mm) mesh sizes for specimen Nr. 5 are similar in shape, and they both overestimate the measured ultimate load. The model with the medium (40 mm) mesh size matches better with experimental ultimate load yet it fails prematurely due to nonconvergence. The mesh size for the model of specimen Nr. 5 does not appear to impact the overly stiff response throughout loading. When adopting a 40 mm mesh size for the RC beam, the numerical responses for specimen Nr. 6 match well with the experimental response, and the results are not significantly affected by the different mesh sizes for the UHPC jacket (see Figure 6.36). Therefore, a mesh size of 40 mm was adopted for both the RC beams and UHPC jackets for the specimens tested by Alkhoury (2017). It should be noted that the premature failure due to nonconvergence in specimen Nr. 5 is resolved when using the calibrated parameters for the NSC material model, and this is discussed later in this section.

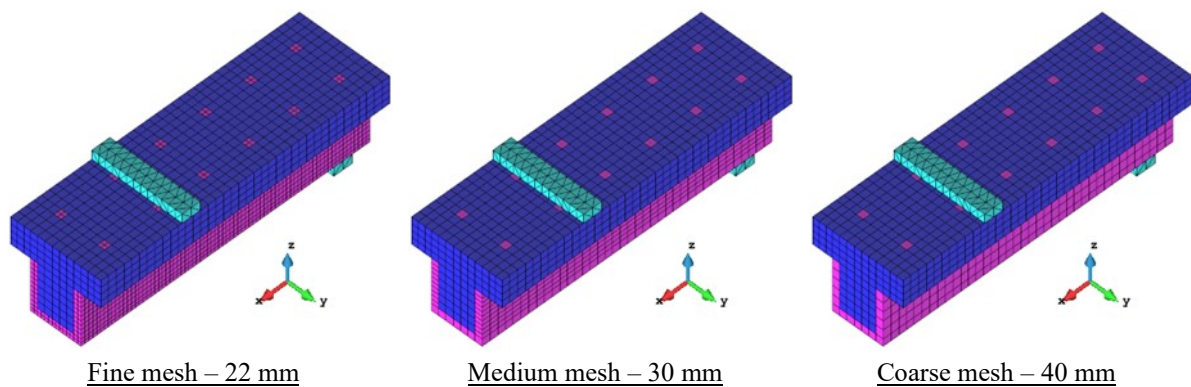


Figure 6.35 – Finite element mesh sizes for the UHPC jacket of specimen Nr. 6 tested by Alkhoury (2017)

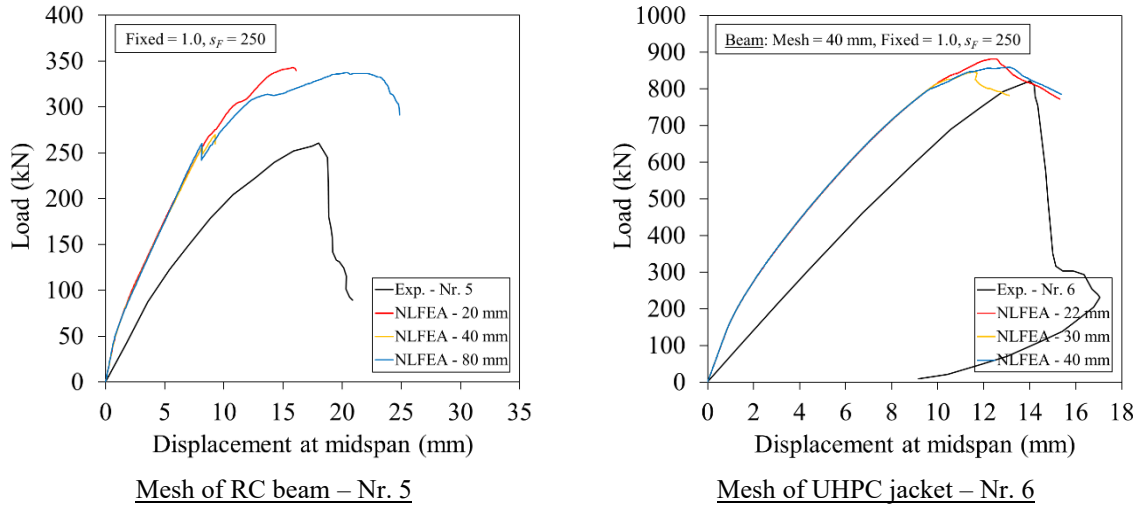


Figure 6.36 – Load-displacement response of beams tested by Al-Osta et al. (2017) using different mesh sizes for RC beam and UHPC jacket

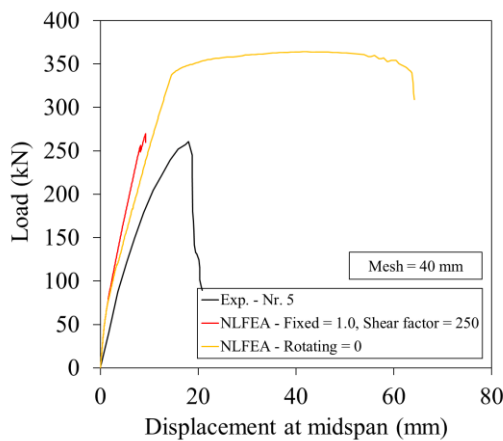
Sensitivity studies were conducted on the numerical model of the control RC beam (Nr. 5) to calibrate various input parameters of the NSC material model. The parameters that were investigated included the type of smeared crack model, shear factor coefficient (s_F), initial shrinkage strain, elastic modulus (E_c), fracture energy (G_f) and compressive strength reduction factor limit (r_c^{lim}). Figure 6.37 shows the results of the sensitivity studies by comparing the numerical and experimental load-displacement responses for each investigated parameter.

The rotating crack model gives a ductile response as shown in Figure 6.37. The predicted ultimate load given by the rotating crack model is also significantly overestimated, and it appears to incorrectly predict ductile flexural failure. For the fixed crack model, increasing the shear factor coefficient from the default value of 20 was observed to increase the ultimate load, however numerical models with a shear factor coefficient of 250 or less were found to abruptly fail prematurely due to nonconvergence. Premature failure due to nonconvergence does not occur when using the fixed crack model with shear factor coefficients equal to 500 and 1000 (see Figure 6.37). Additionally, the fixed crack model with shear factor coefficients equal to 500 and 1000 capture the shape and brittle shear failure of the experimental response relatively well despite the excessive stiffness, unlike the ductile response given with the rotating crack model.

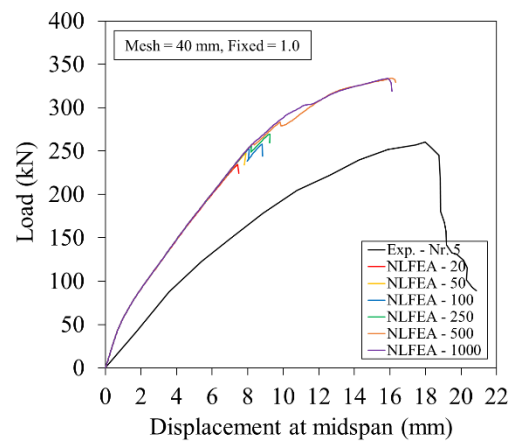
Reducing the elastic modulus of concrete as well as considering the effect of shrinkage were investigated to improve the initial stiffness and overall accuracy of the numerically predicted

load-displacement response for the control RC beam (Nr. 5). Figure 6.37 shows that a 25% and 50% reduction of the elastic modulus improves the initial stiffness of the numerical response while also resolving the premature failure due to nonconvergence that occurs when the non-reduced elastic modulus is adopted. The shrinkage strains that were investigated ranged from 0.015% to 0.045% as suggested by Pryl and Červenka (2018), and the results shown in Figure 6.37 indicate that the stiffness becomes reduced with increasing shrinkage strains, and premature failure due to nonconvergence no longer occurs. Reduction of the fracture energy was also investigated to consider the effects of shrinkage in addition to reducing the elastic modulus to help capture the initial stiffness of the beam more accurately (Pryl and Červenka, 2018). It appears that the stiffness in the numerical response is reduced as a result of reducing the fracture energy of concrete as shown in Figure 6.37. Based on these sensitivity studies, the best fitted parameters used in this research included a 50% reduction to both the elastic modulus and fracture energy in addition to a shear factor coefficient equal to 250.

The lower bound of the compressive strength reduction factor was also investigated. When the default value in ATENA is decreased from 0.8 to 0.5, premature failure due to nonconvergence no longer occurs, and the overall shape of the load-displacement response matches well with experimental response despite the difference in initial stiffness. A value of 0.5 was adopted in this research as it provided the best fit between the numerical and experimental results.



Smeared crack model



Shear factor

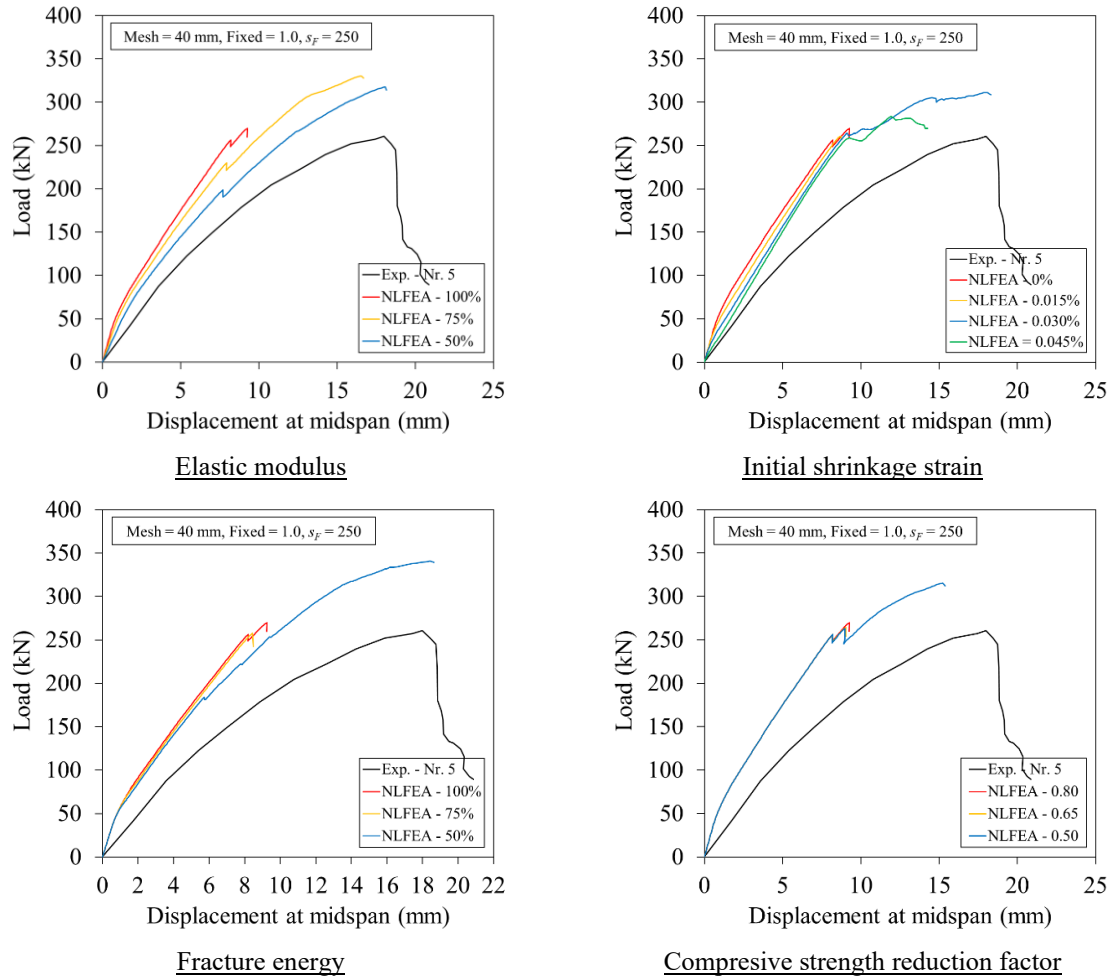


Figure 6.37 – Load-displacement response of specimen Nr. 5 tested by Alkhoury (2017) using different input parameters for NSC material model

When using the calibrated parameters of the NSC and UHPC material models presented in Tables 6.1 and 6.2, the numerical models achieved reasonably accurate results when compared to the experimental results as shown in Figure 6.38. The numerical responses appear to accurately capture the shape and ultimate load of the beams as well as the brittle shear failure. The shape of the numerical responses for all of the beams show that they are marginally stiffer than the experimental responses. However, this is considered a minor discrepancy given the material modelling assumptions related to the input tensile function and elastic modulus of UHPC as well as the grade and elastic modulus of the reinforcement described in Section 6.3.3.

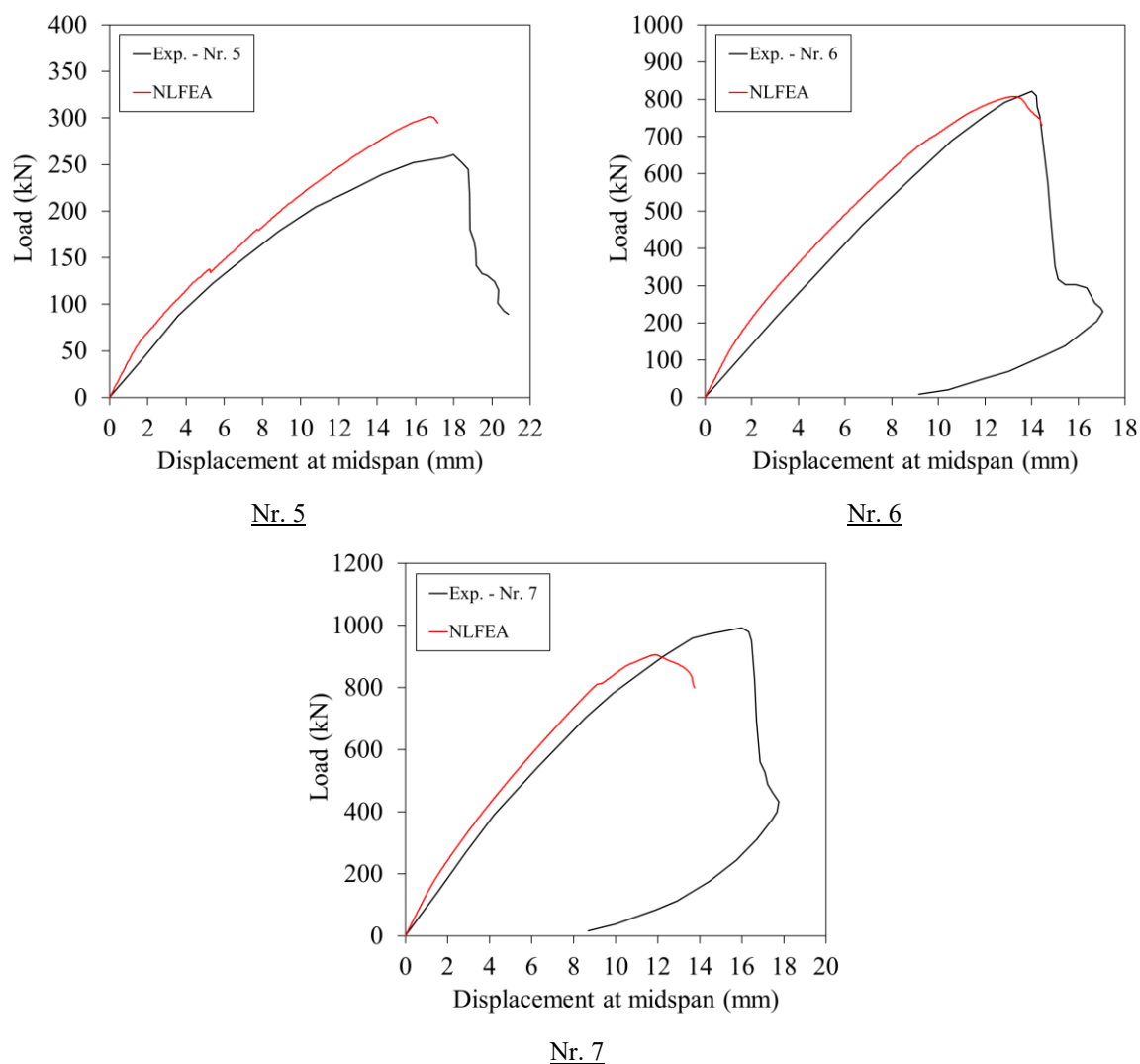


Figure 6.38 – Validated load-displacement responses for specimens tested by Alkhoury (2017)

The results of the numerical models are acceptable overall for the beams of Alkhoury (2017). A summary of the experimental and numerical ultimate loads and displacements are shown in Table 6.17. It can be seen that numerically predicted ultimate loads and displacements for the RC beams and strengthened beams match reasonably well with the experimental results seeing as the average ultimate load (P_{num}/P_{exp}) and displacement ($\delta_{num}/\delta_{exp}$) comparison ratios are 1.00 and 0.87, respectively. Although the response of the longitudinal reinforcement was not reported, it is assumed that the longitudinal reinforcement at midspan did not yield given that all of the beams failed in shear. The reinforcement remained elastic at midspan in all the modelled beams, which confirms shear failure occurred. The predicted crack patterns are shown in Figure 6.39 for the control RC beam (Nr. 5) and strengthened beam (Nr. 6), and they compared well with those reported. For all of the beams, an inclined crack developed between the loading and support plates prior to eventual failure. Much like the modelled beams of Al-

Osta et al. (2017) that are discussed in Section 6.3.5.1, the crack pattern of the jacket and beam are almost identical as a perfect bond was assumed.

Table 6.17 – Experimental and numerical ultimate loads and displacements for beams tested by Alkhoury (2017)

Specimen	Exp. load (P_{exp}) [kN]	Exp. disp. (δ_{exp}) [mm]	Num. load (P_{num}) [kN]	Num. disp. (δ_{num}) [mm]	Comparison ratio (P_{num}/P_{exp})	Comparison ratio ($\delta_{num}/\delta_{exp}$)
Nr. 5	270.00	18.00	301.41	16.71	1.12	0.93
Nr. 6	820.00	14.00	807.92	13.27	0.99	0.95
Nr. 7	1000.00	16.00	904.88	11.88	0.90	0.74
Average (Standard deviation)					1.00 (0.11)	0.87 (0.11)

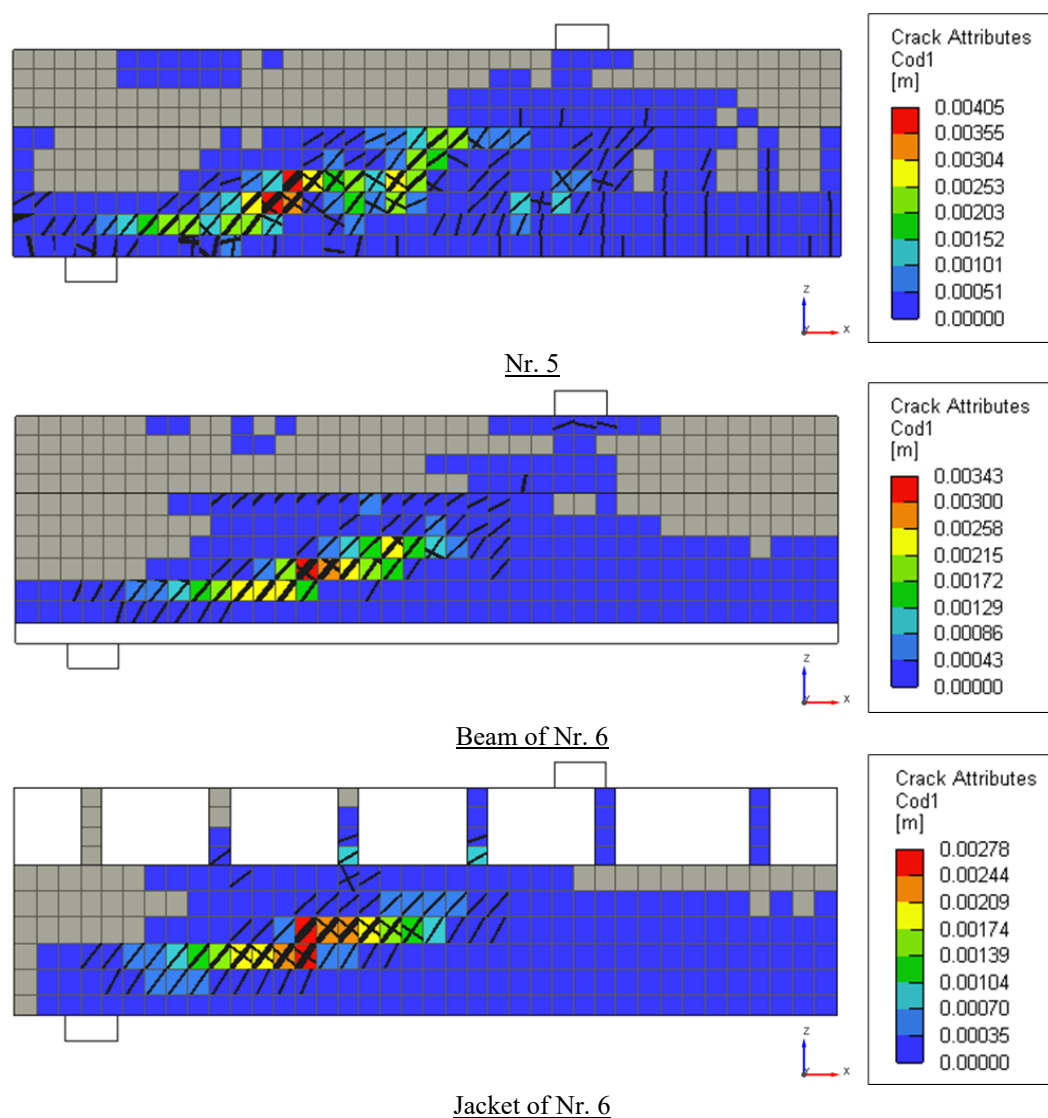


Figure 6.39 – Crack pattern at failure for specimens Nr. 5 and Nr. 6 tested by Alkhoury (2017)
(Minimum crack width of 0.1 mm is shown)

6.4 VALIDATING NLFEA APPROACH WITH BEAMS OF THIS RESEARCH

6.4.1 INTRODUCTION

Following the validation of the NLFEA approach with experiments from the literature, the beams tested in this research were modelled using the geometry and loading arrangement described in Chapter 3. Each 3D model considered the entire beam, as shown in Figure 6.40 for specimen B2. A point load was applied on the top surface of the loading plate at the centre point. The right support did not allow translation in the x-, y- and z-directions while the left support did not allow translation in the y- and z-directions, and these conditions were applied on the bottom surface of the support plates at their centre lines. For the beams with jackets, the jacket was connected to the beam using a fixed contact surface based on the master and slave technique, and the loading apparatuses were also connected to the beams using the same technique. The results of the mesh sensitivity studies described in Section 6.3.5 were used to determine the mesh sizes of the beams of this campaign seeing as the RC beams with UHPC jackets from the literature had fairly similar dimensions. Therefore, mesh sizes equal to 35 mm and 28 mm were used for the RC beams and UHPC jackets of this research, respectively, which produced 10 elements along each of their respective heights.

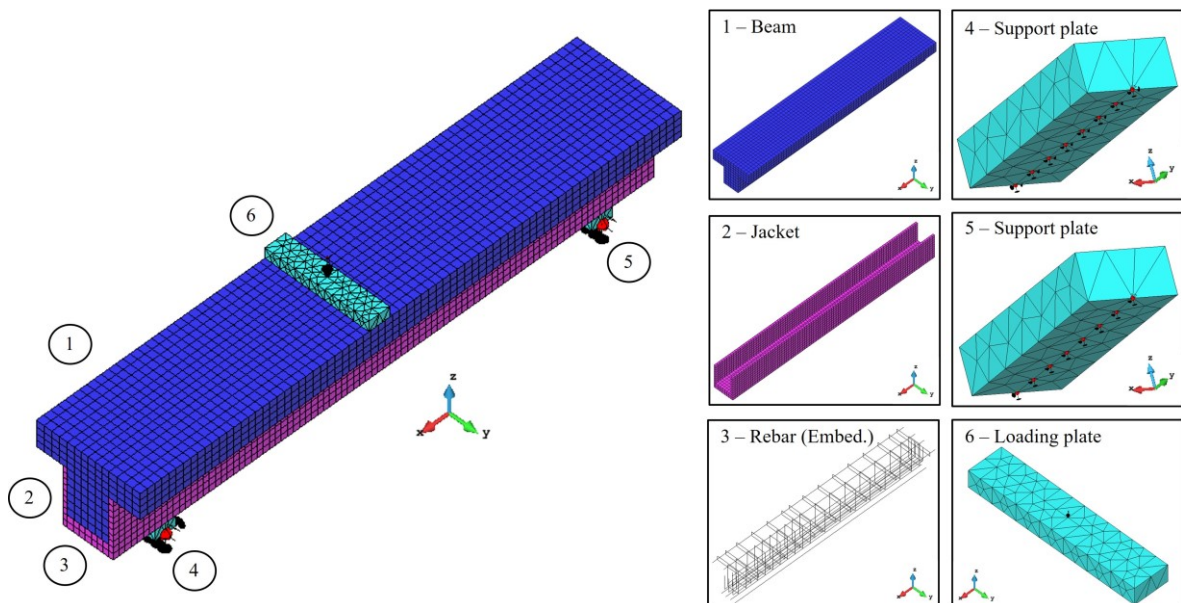


Figure 6.40 – Typical 3D model of beams of groups 1 and 2 in ATENA (Specimen B2)

The calibrated NSC and UHPC material models outlined in Tables 6.1 and 6.2, respectively, were adopted along with a multilinear relationship for the steel reinforcement. The user-defined

input values for the NSC, UHPC and steel reinforcement material models were based on the measured results presented in Chapter 4. The user-defined input values for the NSC, UHPC and steel reinforcement material models are summarised for each beam of this campaign in Tables 6.18, 6.19 and 6.20, respectively. To recapitulate, the calibrated parameters for the NSC material model included a fixed smeared crack model with a shear factor coefficient (s_F) of 250, 50% reduction of the elastic modulus (E_c), 50% reduction of the fracture energy (G_f) and a compressive strength reduction factor limit (r_c^{lim}) of 0.5. The maximum aggregate sizes (d_g) of the NSC and UHPC were 10 mm and 1.25 mm, respectively, and the length of the longest steel fibre (L_f) was 13 mm. The tensile stress-strain relationship of UHPC based on NF P18-710 along with the resulting input tensile function presented in Figure 6.41 were derived using the procedure described in Section 6.2.2.2 (AFNOR, 2016b). The input characteristic length (L_t) was equal to 190 mm and it is equivalent to the characteristic length (L_{ch}^t) that was taken as $2h/3$ according to NF P18-710 where h represents the height of the UHPC jacket. The input tension localisation offset of UHPC (ϵ_{loc}^t) was set equal to 0.001579 as it represents the strain at the initiation of the softening phase (see Figure 6.41).

Table 6.18 – Input parameters of NSC in ATENA for beam specimens of groups 1 and 2

Specimen	Compressive strength (f_c) [MPa]	Tensile strength (f_t) [MPa]	Elastic modulus (E_c) [MPa]	Fracture energy (G_f) [N/m]	Aggregate size (d_g) [mm]	Onset of crushing (f_{c0})
B1	-47.5	3.61	34998 (17499)	146 (73)	10.0	-7.22
B2	-47.0	3.56	34998 (17499)	146 (73)	10.0	-7.12
B3	-47.0	3.56	34998 (17499)	146 (73)	10.0	-7.12
B4	-46.4	3.52	34998 (17499)	146 (73)	10.0	-7.04
B5	-50.1	3.59	37497 (18749)	148 (74)	10.0	-7.18
B6	-50.5	3.45	37497 (18749)	148 (74)	10.0	-6.90
B7	-50.5	3.45	37497 (18749)	148 (74)	10.0	-6.90
B8	-50.9	3.30	37497 (18749)	148 (74)	10.0	-6.60

[Note] – () Refers to reduced value based on calibration used in validated model

Table 6.19 – Input parameters of UHPC in ATENA for beams specimens of groups 1 and 2

Specimen	Mixture	Compressive strength (f_c) [MPa]	Tensile strength (f_t) [MPa]	Elastic modulus (E_c) [MPa]	Aggregate size (d_g) [mm]	Charact. length (L_t) [mm]	Localisation onset (ϵ_{loc}^t)
B2	UHPC ^A	-136.0	6.39	39303	1.25	190	0.001579
B3	UHPC ^A	-132.6	6.39	39303	1.25	190	0.001579
B4	UHPC ^A	-134.3	6.39	39303	1.25	190	0.001579
B6	UHPC ^A	-124.4	6.39	39303	1.25	190	0.001579
B7	UHPC ^A	-127.9	6.39	39303	1.25	190	0.001579
B8	UHPC ^A	-132.0	6.39	39303	1.25	190	0.001579

^AFibre length (L_f) equals 13 mm

Table 6.20 – Input parameters of steel reinforcement in ATENA for beam specimens of groups 1 and 2

Dia. (ϕ) [mm]	Stress [MPa]							Strain [%]						Elastic mod. (E_s) [MPa]
	(f_y)	(f_2)	(f_3)	(f_4)	(f_5)	(f_6)	(f_7)	(ϵ_2)	(ϵ_3)	(ϵ_4)	(ϵ_5)	(ϵ_6)	(ϵ_7)	
20	548	549	590	626	648	622	560	0.015	0.03	0.05	0.097	0.14	0.163	195693
10	506	584	603	608	596	529	443	0.04	0.06	0.074	0.09	0.11	0.123	195485
6	673	788	832	854	834	696	-	0.015	0.12	0.226	0.24	0.255	-	159212

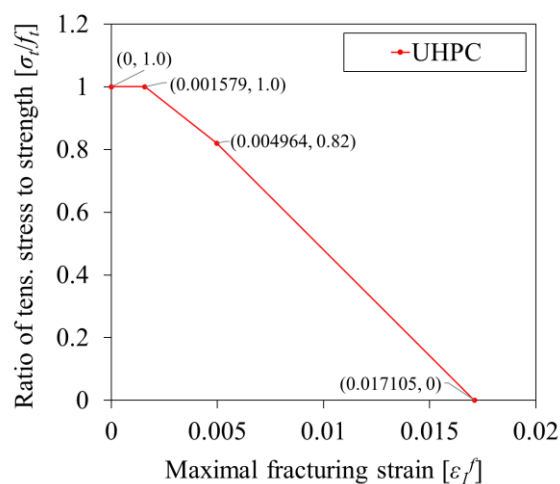
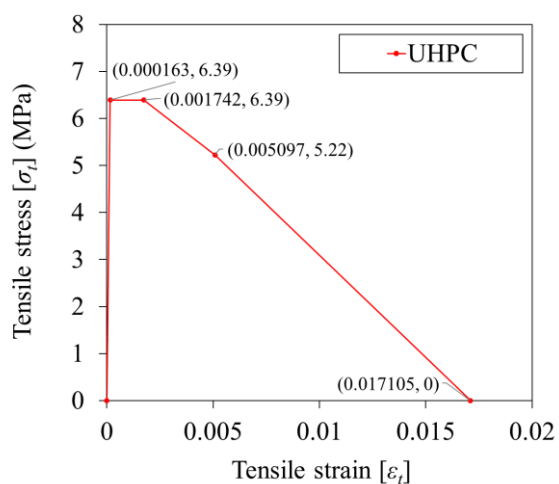
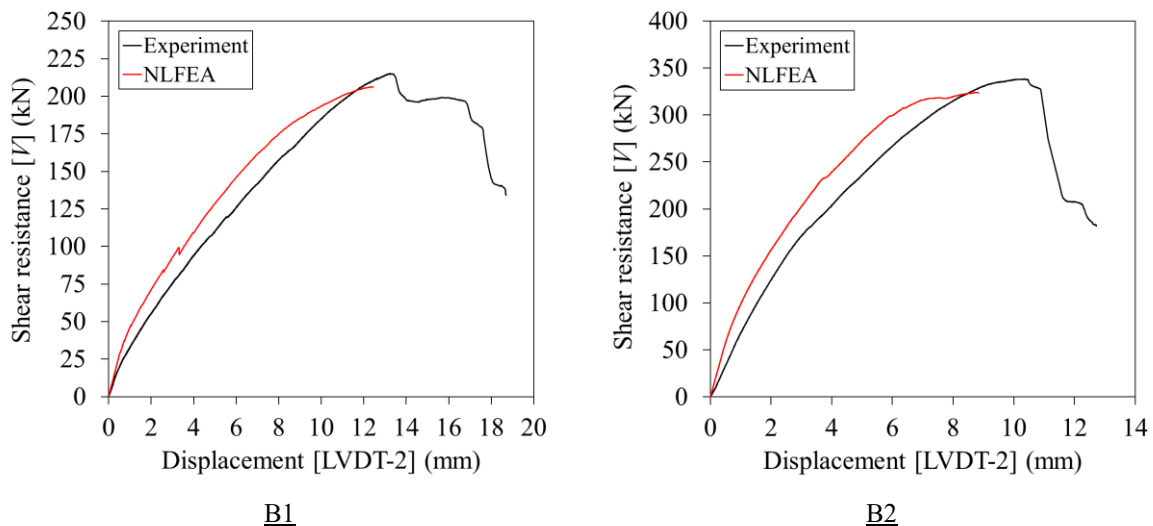


Figure 6.41 – Input tensile function of UHPC in ATENA for beam specimens of groups 1 and 2

6.4.2 NLFEA RESULTS OF BEAMS OF GROUPS 1 AND 2

6.4.2.1 GROUP 1 BEAMS

The numerical results compared reasonably well with the experimental results for the specimens of group 1 as shown in Figure 6.42. It is evident that the shape of the numerical load-displacement response and predicted ultimate load match reasonably well for control specimen B1 as well as specimen B2. The numerical responses of specimens B3 and B4 appear to be overly stiff in comparison to their respective experimental responses. Nonetheless, their numerically predicted ultimate loads remain quite close to their respective experimental ultimate loads. The numerical models replicate the brittle shear failure for all specimens. Much like the experiments, the longitudinal reinforcement did not yield (548 MPa) at the ultimate load as shown in the typical contour stress plots of Figure 6.43 for specimens B1 and B2. Much like the experiments, the typical crack pattern at the ultimate load consisted of a critical shear crack in the left span as shown in Figure 6.44 for specimens B1 and B2, with the critical shear crack overlapping between the jacket and beam. Despite the notable discrepancies associated with the numerical load-displacement responses, the results are reasonably acceptable overall as also shown by the results presented in Table 6.21 where the ultimate load (V_{num}/V_{exp}) and displacement ($\delta_{num}/\delta_{exp}$) average comparison ratios are equal to 0.98 and 0.79, respectively.



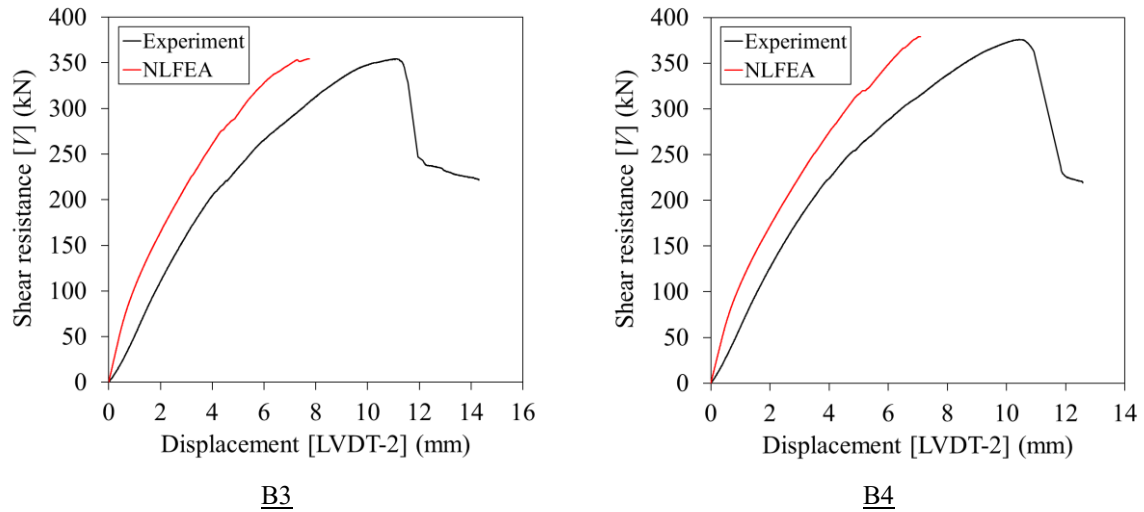


Figure 6.42 – Validated load-displacement responses of group 1 specimens

Table 6.21 – Experimental and numerical ultimate loads and displacements of group 1 specimens

Specimen	Exp. shear resistance (V_{exp}) [kN]	Experimental displacement (δ_{exp}) [mm]	Num. shear resistance (V_{num}) [kN]	Numerical displacement (δ_{num}) [mm]	Comparison ratio (V_{num}/V_{exp})	Comparison ratio ($\delta_{num}/\delta_{exp}$)
B1	214.96	13.34	206.13	12.40	0.96	0.93
B2	337.84	10.40	323.86	8.75	0.96	0.84
B3	354.39	11.11	354.37	7.72	1.00	0.70
B4	375.66	10.49	379.10	7.09	1.01	0.68
Average (Standard deviation)					0.98 (0.03)	0.79 (0.12)

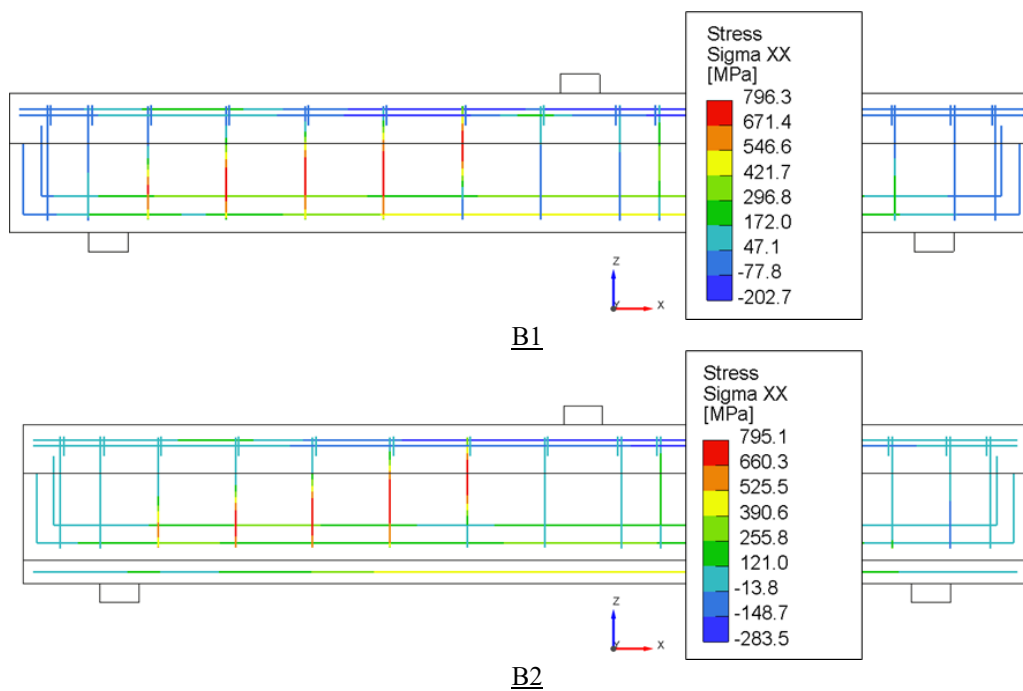


Figure 6.43 – Contour stress plot of the reinforcement at the ultimate load of the numerical models of specimens B1 and B2

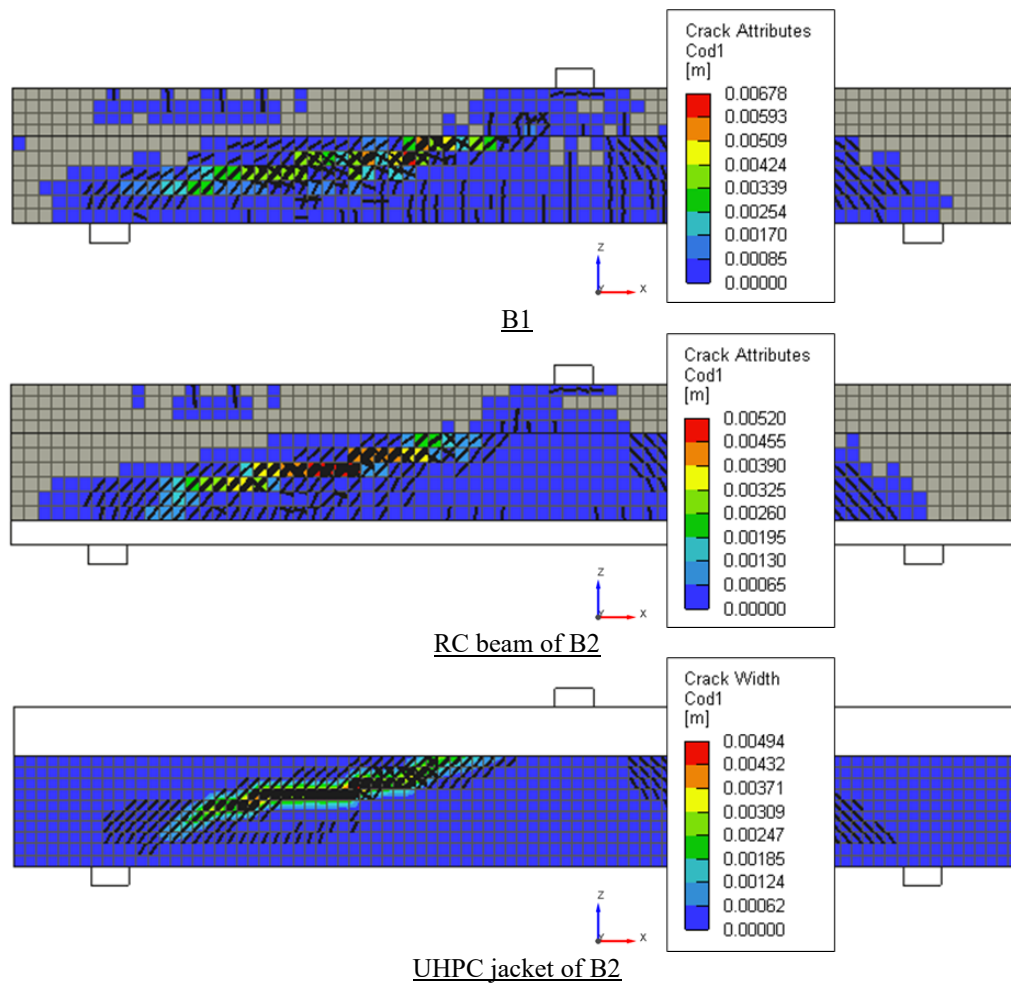


Figure 6.44 – Crack pattern at the ultimate load of the numerical models of specimens B1 and B2
(Minimum crack width of 0.1 mm is shown)

6.4.2.2 GROUP 2 BEAMS

The control RC beam of group 2 (B5) was initially modelled using the same NSC material model as used for the control RC beam of group 1 (B1). The numerical model of specimen B5 was surprisingly unable to replicate the combined flexural-shear mode of failure that occurred in the experiment seeing as flexural failure was predicted. It became evident that the applied NSC material model was overestimating the shear resistance of specimen B5, which caused it to be critical in flexure. Therefore, it was necessary to somehow reduce the shear resistance of the NSC material model for the control RC beam of group 2 only in order to accurately replicate the response of the control beam as well as the strengthened beams. A possible explanation for this reduction is that the spacing of the shear reinforcement in the critical span exceeds the maximum limit of $0.75d$ according to EC2, unlike the RC beams of group 1 and Alkhoury

(2017) (BSI, 2014). However, the NLFEA should ideally capture the effect of increased stirrup spacing without the modification of input parameters.

The shear factor coefficient (s_F) of the NSC material model was progressively reduced in order to find the best fit to the experimental response of specimen B5. Figure 6.45 shows that the shear factor coefficient (s_F) reduces the shear resistance of the beam causing a comparatively less ductile failure. Pastore (2020) also found that the shear strength of beams was significantly affected by the value of the shear factor coefficient (s_F), and that its value may not be applicable for all cases such as beams with and without shear reinforcement. Figure 6.45 shows that a shear factor coefficient (s_F) of 50 achieves the most accurate numerical response. The combined flexural-shear mode of failure is also replicated as the reinforcement response and crack pattern described later in the section indicate as well. Based on these results, a shear factor coefficient (s_F) of 50 was adopted for the NSC material model of group 2 only, rather than the value of 250 that is adopted for group 1.

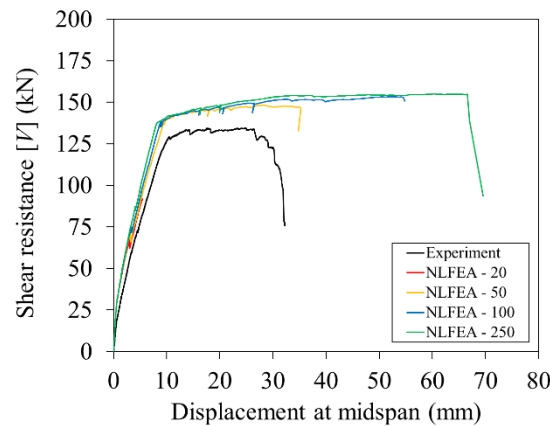


Figure 6.45 – Load-displacement response of specimen B5 with different shear factor coefficients

When using the calibrated NSC material model of group 2, the numerical results were reasonably accurate overall when compared to the experimental results. The comparisons between the numerical and experimental load-displacement responses are shown in Figure 6.46. It is evident that the shape of the numerical response matches reasonably well for control specimen B5 although the initial cracking and ultimate loads are marginally overestimated. The numerical responses of strengthened specimens B6, B7 and B8 appear to be overly stiff in comparison to their respective experimental responses. Nonetheless, their numerically predicted ultimate loads remain quite close to their respective experimental ultimate loads. The numerical model of control specimen B5 replicated the combined flexural-shear failure mode,

and the brittle shear modes of failure of the strengthened specimens were also replicated. The longitudinal reinforcement reached its yielding point (548 MPa) at the ultimate load in control specimen B5 as shown in the reinforcement contour stress plot in Figure 6.47. The typical reinforcement response of the strengthened specimens is shown in Figure 6.47 for specimen B6 where it is evident that the longitudinal reinforcement did not quite reach the yielding point (548 MPa). The crack pattern of control specimen B5 featured significant flexural and shear cracks similar to the experiment (see Figure 6.48). The typical crack pattern at the ultimate load for the strengthened specimens featured a critical shear crack in the left span as shown in the RC beam and UHPC jacket of specimen B6 in Figure 6.48. Despite some discrepancies with the experimental load-displacement responses, the numerical results are reasonably acceptable overall as also shown in Table 6.22 where the ultimate load (V_{num}/V_{exp}) and displacement ($\delta_{num}/\delta_{exp}$) average comparison ratios are equal to 1.07 and 1.06, respectively.

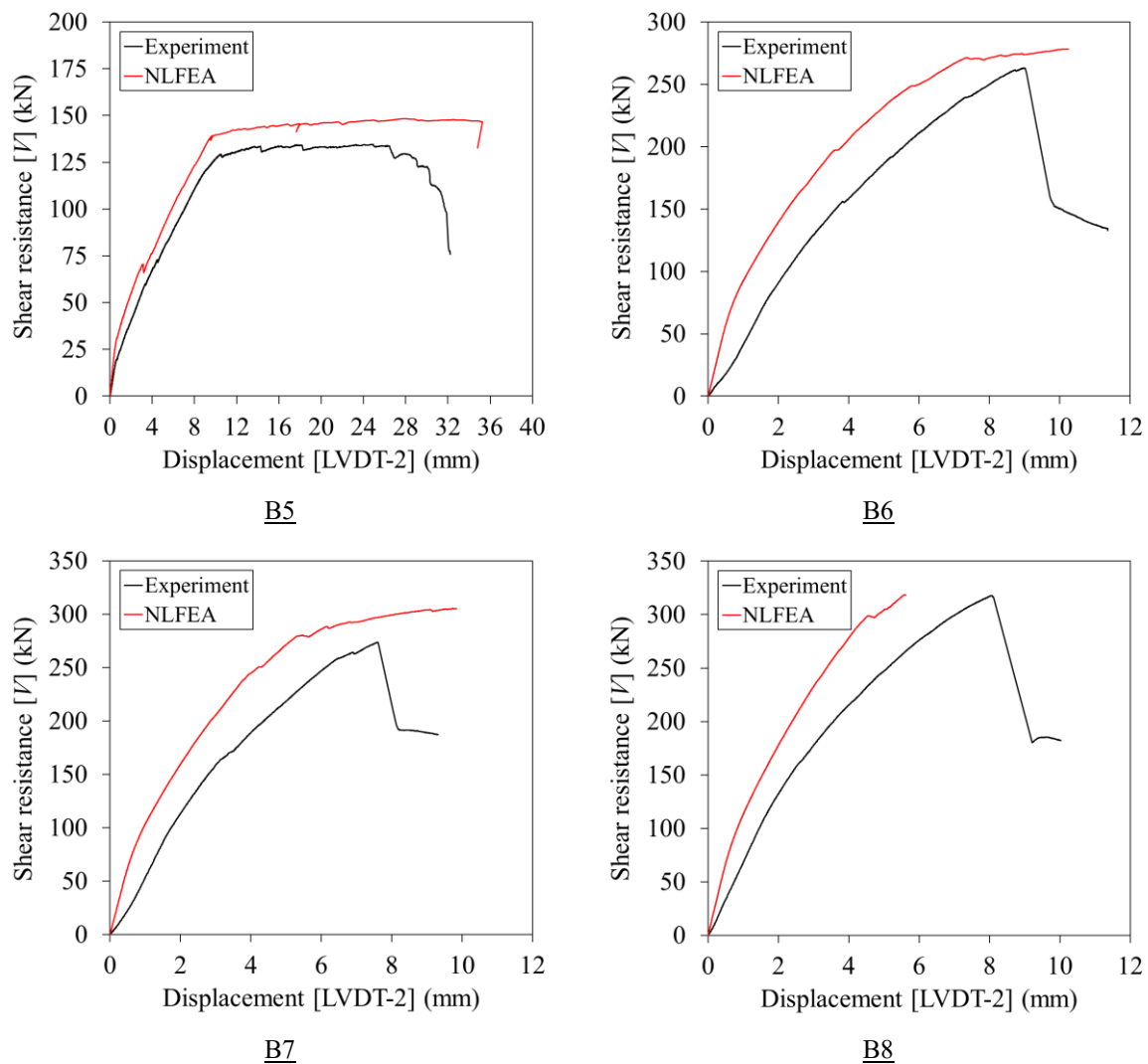
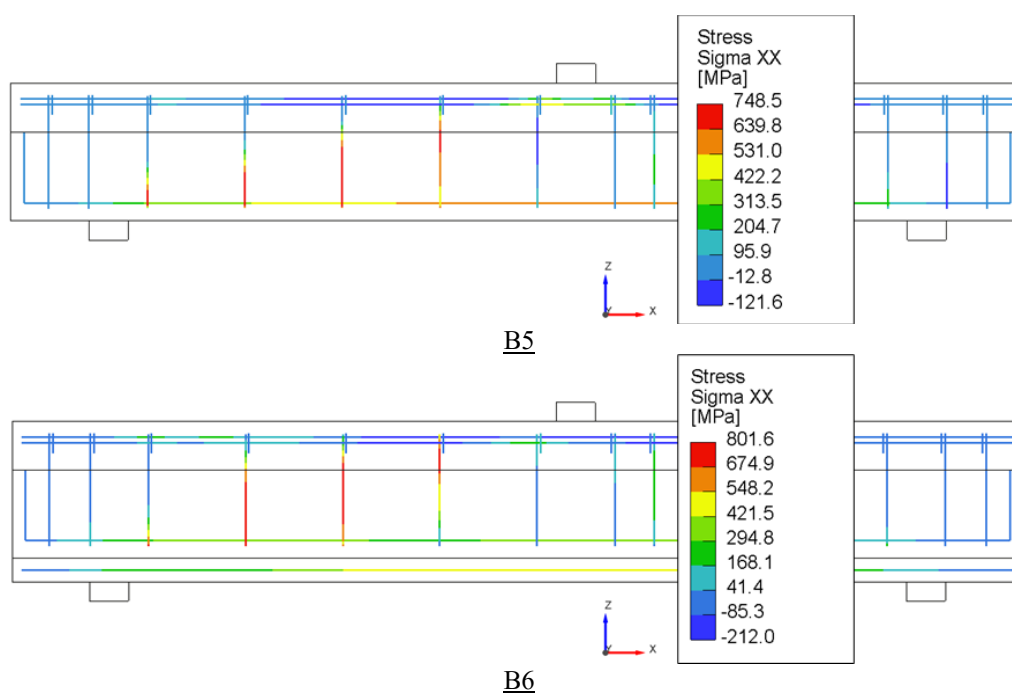
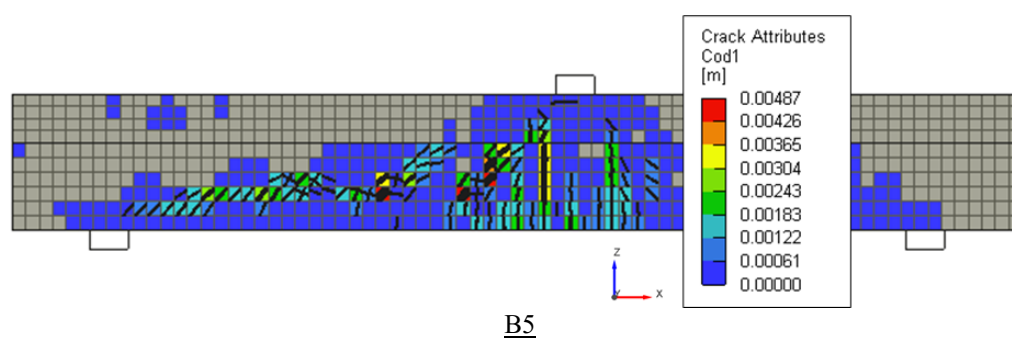


Figure 6.46 – Validated load-displacement responses of group 2 specimens

Table 6.22 – Experimental and numerical ultimate loads and displacements of group 2 specimens

Specimen	Exp. shear resistance (V_{exp}) [kN]	Experimental displacement (δ_{exp}) [mm]	Num. shear resistance (V_{num}) [kN]	Numerical displacement (δ_{num}) [mm]	Comparison ratio (V_{num}/V_{exp})	Comparison ratio ($\delta_{num}/\delta_{exp}$)
B5	134.17	24.92	148.32	28.02	1.11	1.12
B6	263.07	8.98	278.44	10.17	1.06	1.13
B7	273.45	7.60	305.21	9.80	1.12	1.29
B8	317.69	8.06	318.63	5.60	1.00	0.70
Average (Standard deviation)					1.07 (0.05)	1.06 (0.26)

**Figure 6.47** – Contour stress plot of the reinforcement at the ultimate load of the numerical models of specimens B5 and B6

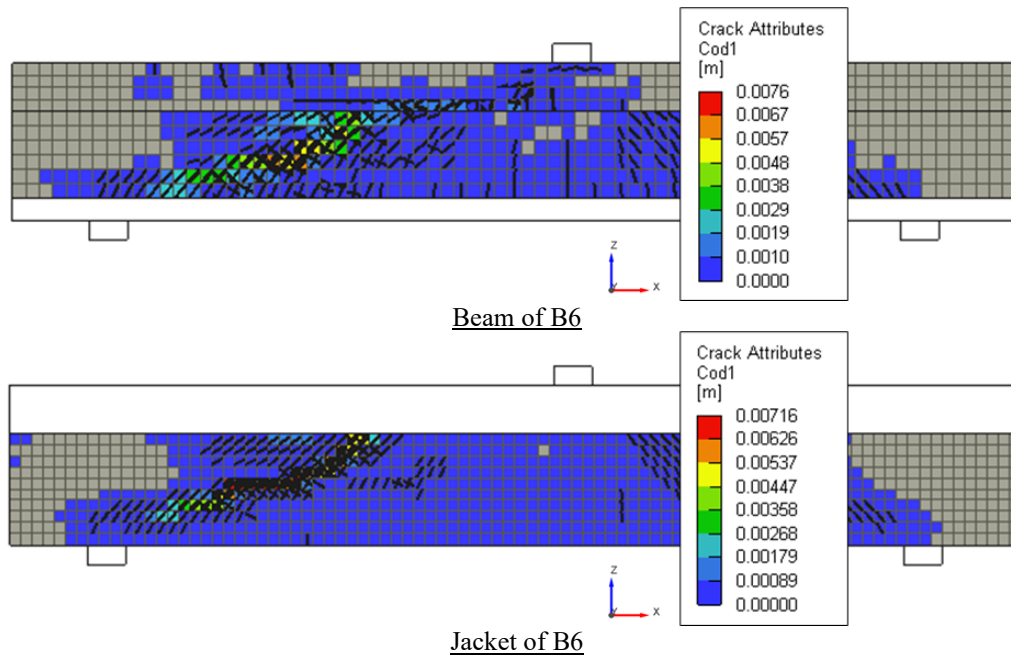


Figure 6.48 – Crack pattern at the ultimate load of the numerical models of specimens B5 and B6 (Minimum crack width of 0.1 mm is shown)

6.5 CONCLUSIONS

This chapter describes the development and application of the NLFEA approach used to model the tested beams of this research. Relevant features of the adopted NLFEA software package ATENA are described along with the adopted input parameters. The combined fracture-plastic model offered in ATENA was adopted for both NSC and UHPC. Some adjustments from the default input parameters were required to realistically simulate the responses of the considered beams with reasonable accuracy. The *CC3DNonLinCementitious2User* material model in ATENA used for UHPC required minimal calibration as it allowed for the post-cracking compressive, tensile and degradation laws to be user-defined. The tensile law of UHPC was derived using the tensile stress-strain response given by NF P18-710 (AFNOR, 2016b). Table 6.2 summarizes the adopted parameters and material laws for UHPC. The NLFEA approach developed and applied in this research was validated using the experimental results of this research along with results from the literature of tested RC, UHPC and strengthened beams. The validation process involved calibrating, or changing, various input parameters in the material models, mainly that of NSC, to achieve acceptable numerical predictions. The developed NLFEA approach can be used in future investigations to study other important test parameters that were not considered in the experimental campaign of this research, which only included the side thickness of the jacket and amount of longitudinal reinforcement in the jacket.

The effect of having different types of UHPC (i.e., limited or significant post-cracking strain-hardening behaviour) on the shear behaviour of UHPC strengthened beams can be investigated numerically along with the effect of having transverse reinforcement embedded in the jacket. It is important to note that parametric studies were not carried out in this research due to time constraints as discussed later in Section 8.3.

The default *CC3DNonLinCementitious2* material model in ATENA used for NSC required the calibration of a few key parameters. The influence of the shear factor coefficient (s_F) in the fixed smeared crack model was investigated, and it was observed that it significantly impacts the shear strength of beams. Increasing the shear factor coefficient (s_F) increases the shear strength of beams, and its value was found to be case dependent based on the amount of shear reinforcement. A shear factor coefficient (s_F) of 250 was adopted for the beams of group 1, and a value of 50 was adopted for the beams of group 2. To reduce the initial stiffness of the numerical response and incorporate the effect of shrinkage, 50% reductions of the elastic modulus (E_c) and fracture energy (G_f) were found to be necessary to help replicate the responses of the beams more accurately. The compressive strength reduction factor limit (r_c^{lim}) was also investigated, and a value of 0.5 was adopted in this research as it provided the best fit between the numerical and experimental results. A summary of the best fitted parameters of NSC material model used for the beams of this campaign is shown in Table 6.1.

CHAPTER 7: CODE PREDICTIONS AND DESIGN RECOMMENDATIONS

7.1 INTRODUCTION

This chapter investigates the application of design methods for RC and UHPC beams to the assessment of RC beams with UHPC jackets. This is not straightforward since as shown in Chapter 5 the peak shear resistances of the RC beam and UHPC jacket are not developed simultaneously. Currently, there is no agreed method for calculating the shear strength of RC beams with UHPC jackets (Liu and Charron, 2023). Initially, prediction equations from RC and UHPC design codes and the literature are used to calculate the ultimate shear strength of RC beams with UHPC jackets by straightforward superposition. The ultimate flexural strength is also calculated to enable determination of the predicted mode of failure. It is shown that this leads to an overestimate of shear resistance. Next, the experimentally determined contributions of each shear resisting mechanism from Chapter 5 are used to study the interaction between the shear resistances of the RC beams and UHPC jackets. Based on the comparisons between the design code predictions with the experimental results and assessment of the contributions of each shear resisting mechanism determined from crack kinematics in Chapter 5, recommendations are suggested for the design of RC beams with UHPC jackets. These recommendations are assessed for the beams tested in this campaign as well as a database of RC beams with UHPC jackets from the literature.

7.2 METHODS OF CALCULATION FOR SHEAR AND FLEXURAL STRENGTH

7.2.1 INTRODUCTION

Methods of calculation for the ultimate shear and flexural strength of RC beams with UHPC jackets are presented in this section. The methods of calculation are applied to the beams tested in this campaign as well as those in the developed database from the literature. The ultimate shear strength of a strengthened RC beam is commonly based on the principles of superposition where the contributions of the RC beam and strengthening material are added together as in recognized retrofitting design guides for fibre composite materials (ACI, 2017; The Concrete Society, 2012). A similar approach is initially adopted in this chapter, where the ultimate shear strength of the RC beam and UHPC jacket are calculated individually using design methods for each material described in Section 2.4. The ultimate flexural strength is also calculated through cross-sectional analysis based on simplified material models given by RC and UHPC

design recommendations. Shear failure is predicted when the shear strength is less than the flexural strength and vice versa. This approach was found to overestimate the shear resistance of the beams tested in this campaign. Subsequently, the principal of adding peak shear resistances of the RC beam and UHPC jacket is modified based on the assessment of the shear resisting mechanisms in Chapter 5. The amended assessment method is shown to give safe shear strengths for the beams tested in this campaign as well as beams in a database assembled by the author.

7.2.2 ULTIMATE SHEAR STRENGTH

Superposition is adopted in this research to determine the ultimate shear strength of RC beams with UHPC jackets. A similar approach is adopted by national and international retrofitting design guides for fibre composite materials such as The Concrete Society Technical Report No. 55 and ACI 440.2R-17 (ACI, 2017; The Concrete Society, 2012). Applying the superposition approach to determine the ultimate shear resistance of a jacketed beam involves adding the contributions of the unstrengthened RC beam and UHPC jacket together, while limiting each respective contribution to the shear strength of the concrete web. This is not straightforward since the shear resistance provided by the jacket depends on the width of the diagonal shear cracks. In the case of a softening material, like the UHPC developed in this campaign, the shear resistance of the jacket reduces once the crack width exceeds a limiting value. Consequently, the shear resistance provided by the jacket can peak prior to shear failure of the strengthened beam as shown in Chapter 5. In this case, the peak contributions of the RC beam and UHPC jacket to shear resistance do not occur simultaneously as assumed in The Concrete Society Technical Report No. 55 for RC beams strengthened with fibre composite materials (The Concrete Society, 2012).

Initially, to determine an upper bound to the shear strength of the jacketed beams, the design ultimate shear strengths of the RC beam and UHPC jacket were calculated individually and added. The shear resistance of the RC beam was calculated using the equations given by EC2 (Eqn. 2.137), fib MC 2010 LoAIII (Eqn. 2.140) and ACI 318-19 (Eqn. 2.151) that are presented in Sections 2.4.2.1, 2.4.2.2 and 2.4.2.3, respectively (ACI, 2019; BSI, 2014; fib, 2013). The Multi-Action Shear Strength model (MASM) proposed by Marí et al. (2015, 2016) and Cladera et al. (2016) was also used to determine the shear resistance of the RC beams (Eqn. 2.159), and the equations of this model are presented in Section 2.4.3.1. The shear resistance of the RC beams was also calculated using the equations of EC2 plus the contribution of the flange

outstands calculated using the MASM. The purpose of this combined method was to account for the shear strength contribution of the flange outstands in the T-beams seeing as it is not directly accounted for by reinforced concrete design codes such as EC2 (BSI, 2014). The contribution of the flange was equal to difference in shear strength between the T-shaped and rectangular cross-sections that were both calculated using Eqn. 2.159 according to the MASM. The shear resistance of the UHPC jacket was calculated using the equations given by NF P18-710 (Eqn. 2.172), SIA 2052 (Eqn. 2.183) and JSCE 2006 (Eqn. 2.187) that are presented in Sections 2.4.4.1, 2.4.4.2 and 2.4.4.3, respectively (AFNOR, 2016; EPFL, 2016; JSCE, 2006).

The truss angles of the transverse and fibre reinforcement truss models initially varied as each angle was selected according to each of the RC and UHPC design codes, respectively. The truss angle differs according to each considered design method as described in Section 2.4. The adopted angles used to calculate the ultimate shear resistance are presented in Table 7.1. The adopted truss angles of EC2, fib MC 2010 LoAIII and the MASM represent the minimum truss angle. The truss angle for the fibre reinforcement was set at 30° as it is the minimum recommended angle according to NF P18-710, SIA 2052 and JSCE 2006. The choice of 30° is also based on the observed behaviour of the beams tested in this campaign. Alternatively, this angle could be taken as 45° based on the recommendation of NF P18-710 that suggests the angle between the principal compression stress and the longitudinal axis should be calculated at the neutral axis using elastic analysis (AFNOR, 2016b). The maximum angle for the transverse and fibre reinforcement truss models is equal to 45°, therefore the estimated ultimate shear resistance using the minimum angles can be viewed as the upper bound, with that using 45° as the lower bound. The principle of adopting different angles for the truss models in the RC beam and UHPC jacket is consistent with the approach adopted in The Concrete Society Technical Report No. 55 (The Concrete Society, 2012). However, as discussed in Section 7.3.5 this assumption is inconsistent with the observed failure cracks in the beams of this campaign. Consequently, the adoption of the same angle for both truss models is also evaluated.

Table 7.1 – Transverse and fibre reinforcement truss angles according to various design codes and models that were used when calculating the ultimate shear resistance of RC beams with UHPC jackets

Design variable	Transverse reinforcement				Fibre reinf.
	EC2 ^A	fib MC 2010 ^B	ACI 318-19 ^C	MASM ^D	NF/SIA/JSCE ^E
Truss angle (θ) [Degrees]	21.8	Eqn. 2.144	45.0	Eqn. 2.166	30.0

^A EC2 (BSI, 2014); ^B fib MC 2010 LoAIII (fib, 2013); ^C ACI 318-19 (ACI, 2019); ^D MASM (Cladera et al., 2016; Mari et al., 2015, 2016); ^E NF P18-710, SIA 2052, JSCE 2006 (AFNOR, 2016b; EPFL, 2016; JSCE 2006)

The shear resistance contribution of the fibre reinforcement is a function of the residual tensile strength of UHPC (σ_{res}) according to the considered UHPC design recommendations. It represents the average stress along the critical shear crack at the ultimate load. It was determined using the method of calculation suggested by SIA 2052 in Eqn. 2.184 due to its relative simplicity and ease of use, where it is simply taken as the average of the post-cracking tensile strength at the elastic limit and the post-cracking tensile strength. The following equation was used in this research to calculate the residual tensile strength (σ_{res}):

$$\sigma_{res} = \frac{f_{ctm} + f_{ctm,el}}{2} \quad (7.1)$$

where f_{ctm} is the post-cracking tensile strength and $f_{ctm,el}$ is the post-cracking tensile strength at the elastic limit. An alternative method given by NF P18-710 discussed in Section 2.4.4 suggests that the residual tensile stress ($\sigma_{Rd,f}$) can be calculated according to Eqn. 2.175 that is based on the tensile stress-crack opening response obtained indirectly using the procedures of NF P18-470 (AFNOR, 2016a, 2016b). The residual tensile stress ($\sigma_{Rd,f}$) according to Eqn. 2.175, is effectively the average stress between the stresses corresponding to a crack opening of zero (i.e., elastic limit) and the crack opening limit (w^*), which is equal to the crack opening obtained in bending at the ultimate limit state with axial force on the end fibre (w_u) if it exceeds a crack opening of 0.3 mm (AFNOR, 2016b). The Japanese recommendations suggests a similar approach to NF P18-710 for determining the residual tensile strength of UHPC (f_{vd}), which can be calculated using Eqn. 2.193. A crack opening limit (w_{lim}) of 0.3 mm is suggested by JSCE 2006 (JSCE, 2006).

Several considerations were made when applying the shear design equations presented in Section 2.4 to calculate the ultimate shear strength of RC beams with UHPC jackets. The width of the UHPC jacket was accounted for by setting the width of the beam (b_w) in the equations presented in Section 2.4.4 equal to twice the width of the side layer of the jacket ($2t_s$). The height of the jacket was taken as the height between the top and bottom surfaces of the jacket, which may differ from the overall height of the beam as in the T-beams tested in this campaign. The effective depth (d) of the jacket was considered as the distance from the top surface of the jacket to the centroid of the longitudinal reinforcement embedded in the jacket, with the shear lever arm equal to $0.9d$ according to all considered design codes (AFNOR, 2016b; EPFL, 2016; JSCE, 2006). For the jackets without longitudinal reinforcement as in most featured in the test

database, the effective depth (d) was set equal to $7h/8$ as suggested by NF P18-710 (AFNOR, 2016b). All material reduction factors were set equal to 1.0, and this includes the reduction factors of NF P18-710 ($K = 1.25$) and JSCE 2006 ($\gamma_c = 1.3$) that consider the global mechanical effects of fibre orientation on the tensile strength of UHPC (AFNOR, 2016b; JSCE, 2006).

7.2.3 ULTIMATE FLEXURAL STRENGTH

7.2.3.1 INTRODUCTION

The procedure for estimating the ultimate flexural strength of RC beams strengthened with UHPC jackets was based on cross-section analysis along with the following assumptions:

- Plane cross-sections remain plane while bending;
- There is no slip between the NSC beam and UHPC jacket;
- There is no slip between the longitudinal reinforcement and surrounding concrete;
- The strain in the longitudinal reinforcement and concrete are equal at the same level;
- The strain in the extreme compression fibre (ϵ_{cu}) is equal to 0.0035;
- The tensile contribution of the NSC is neglected.

In addition to the previously listed assumptions, the parabolic-rectangular stress block given by fib Bulletin No. 90 was adopted for both NSC and UHPC in compression (fib, 2019). The rectangular stress block of SIA 2052 was adopted for UHPC in tension (EPFL, 2016). Alternative and less simplified compressive and tensile stress distributions for concrete are given by EC2, fib MC 2010, ACI 318-19, NF P18-710, SIA 2052 and JSCE 2006. Rather than adopting simplified material models to characterise the compressive and tensile behaviour of concrete, a more exact approach would involve the use of the measured compressive and tensile responses. The concrete stress distributions adopted in this research were chosen due to their relative simplicity and ease of use. Material reduction factors were set equal to 1.0, and this includes the reduction factors of NF P18-710 ($K = 1.25$) and JSCE 2006 ($\gamma_c = 1.3$) that consider the global mechanical effects of fibre orientation on the tensile strength of UHPC (AFNOR, 2016b; JSCE, 2006).

The method of calculation for the flexural strength is dependent on the cross-section of the RC beam, which was either rectangular or T-shaped for the beams considered in this chapter. The estimations began by first iteratively determining the depth to the neutral axis (x) using strain compatibility and equilibrium of internal compressive and tensile forces. The following general equation represents the equilibrium of the internal compressive and tensile forces for a RC beam strengthened with a UHPC jacket that includes embedded longitudinal reinforcement:

$$F_{sc} + F_c + F_{c,u} = F_s + F_{s,u} + F_{t,u} \quad (7.2)$$

where F_s is the internal force of the longitudinal tensile reinforcement in the beam, F_{sc} is the internal force of the longitudinal compressive reinforcement, $F_{s,u}$ is the internal force of the longitudinal tensile reinforcement in the jacket, F_c is the internal compressive force of the NSC, $F_{c,u}$ is the internal compressive force of the UHPC and $F_{t,u}$ represents the internal tensile force of the UHPC. The predicted flexural strength was then estimated by multiplying the internal forces by their respective distances to the neutral axis. The following general equation was used to estimate the flexural strength (M_{calc}) of a RC beam strengthened with a UHPC jacket:

$$M_{calc} = F_{sc}d_{sc} + F_cd_c + F_{c,u}d_{c,u} + F_sd_s + F_{s,u}d_{s,u} + F_{t,u}d_{t,u} \quad (7.3)$$

where d_s is the distance to the neutral axis from the longitudinal tensile reinforcement in the beam, d_{sc} is the distance to the neutral axis from the longitudinal compressive reinforcement, $d_{s,u}$ is the distance to the neutral axis from the longitudinal tensile reinforcement in the jacket, d_c is the distance to the neutral axis from the internal compressive force of NSC, $d_{c,u}$ is the distance to the neutral axis from the internal compressive force of UHPC and $d_{t,u}$ is the distance to the neutral axis from the internal tensile force of UHPC.

7.2.3.2 RECTANGULAR RC BEAMS WITH UHPC JACKETS

The cross-section analysis for rectangular RC beams strengthened with a UHPC U-shaped jacket is illustrated in Figure 7.1 which shows the cross-section, material strain and stress distributions as well as internal forces and their respective distances to the neutral axis. The same procedure was used for rectangular RC beams strengthened with UHPC side jackets in test database, but the contribution of the bottom layer of the UHPC jacket was neglected.

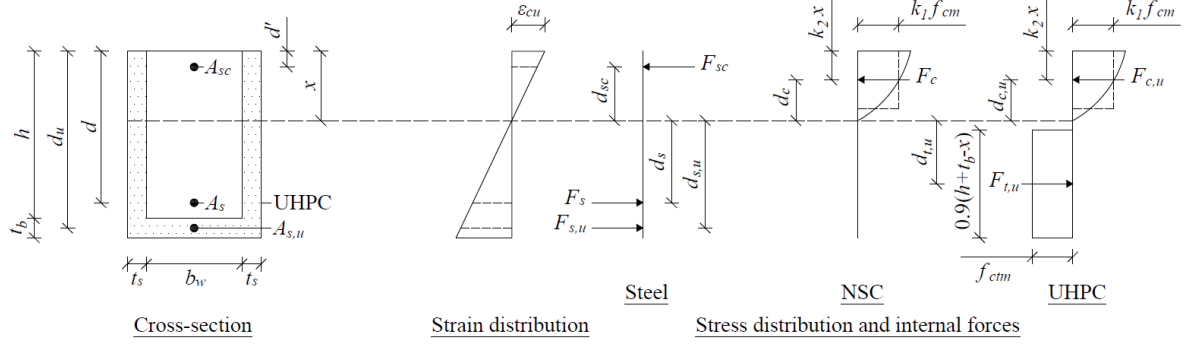


Figure 7.1 – Cross-section analysis of rectangular RC beam strengthened with UHPC U-shaped jacket

Initially, the depth to the neutral axis (x) was iteratively determined using Eqn. 7.2. The internal forces of the steel reinforcement, NSC and UHPC were then calculated using the following set of equations:

$$F_s = A_s f_s \quad (7.4)$$

$$F_{sc} = A_{sc} f_{sc} \quad (7.5)$$

$$F_{s,u} = A_{s,u} f_{s,u} \quad (7.6)$$

$$F_c = k_1 f_{cm} b_w x \quad (7.7)$$

$$F_{c,u} = 2k_1 f_{cm} t_s x \quad (7.8)$$

$$F_{t,u} = f_{ctm} (1.8(h + t_b - x)t_s + b_w t_b) \quad (7.9)$$

where A_s is the area of longitudinal tensile reinforcement in the beam, A_{sc} is the area of longitudinal compressive reinforcement, $A_{s,u}$ is the area of longitudinal reinforcement in the jacket, b_w is the width of the beam, h is the height of the beam, t_s is the thickness of the side layer of the UHPC jacket, t_b is the thickness of the bottom layer of the UHPC jacket, f_{cm} is the mean concrete compressive cylinder strength, and f_{ctm} is the mean direct post-cracking tensile strength of UHPC. The factor related to the average stress of the parabolic-rectangular stress block in compression of concrete (k_1) was determined using the following equation given by fib Bulletin No. 90:

$$k_1 = \begin{cases} 1000\varepsilon_c(0.5 - \frac{1000\varepsilon_c}{12}) & \text{for } \varepsilon_c \leq 0.002 \\ 1 - \frac{2}{3000\varepsilon_c} & \text{for } 0.002 \leq \varepsilon_c \leq 0.0035 \end{cases} \quad (7.10)$$

where ε_c was equal to the strain in the extreme compression fibre (ε_{cu}) (fib, 2019). The stresses in the longitudinal tensile reinforcement (f_s) and compressive reinforcement (f_{sc}) in the RC beam as well as the stress in the longitudinal tensile reinforcement in the UHPC jacket ($f_{s,u}$) were determined using the following set of equations:

$$f_s = \min\left\{\frac{\varepsilon_{cu}E_{sm}(d-x)}{x}, f_{ym}\right\} \quad (7.11)$$

$$f_{sc} = \min\left\{\frac{\varepsilon_{cu}E_{sm}(x-d')}{x}, f_{ym}\right\} \quad (7.12)$$

$$f_{s,u} = \min\left\{\frac{\varepsilon_{cu}E_{sm}(d_u-x)}{x}, f_{ym}\right\} \quad (7.13)$$

where d is the depth to longitudinal tensile reinforcement in the RC beam, d' is the depth to longitudinal compressive reinforcement, d_u is the depth to the longitudinal tensile reinforcement in the UHPC jacket, E_{sm} is the mean elastic modulus of the respective reinforcement bar and f_{ym} is the mean yield strength of the respective reinforcement bar. The distances from the internal forces to the neutral axis were then calculated after the depth to the neutral axis (x) was iteratively determined. The distances were calculated using the following set of equations:

$$d_s = d - x \quad (7.14)$$

$$d_{sc} = x - d' \quad (7.15)$$

$$d_{s,u} = d_u - x \quad (7.16)$$

$$d_c = x - k_2x \quad (7.17)$$

$$d_{cu} = d_c = x - k_2 x \quad (7.18)$$

$$d_{t,u} = \begin{cases} h + t_b - x - \left[\frac{0.81 t_s (h + t_b - x)^2 + 0.5 b_w t_b^2}{1.8 t_s (h + t_b - x) + b_w t_b} \right] & \text{for U-shaped jacket} \\ 0.55(h - x) & \text{for side jacket} \end{cases} \quad (7.19)$$

where k_2 is a factor related to the centroidal position of the parabolic-rectangular stress block in compression for concrete according to fib Bulletin No. 90, and it was calculated using the following equation:

$$k_2 = \begin{cases} \frac{8 - 1000 \varepsilon_c}{4(6 - 1000 \varepsilon_c)} & \text{for } \varepsilon_c \leq 0.002 \\ \frac{1000 \varepsilon_c (3000 \varepsilon_c - 4) + 2}{2000 \varepsilon_c (3000 \varepsilon_c - 2)} & \text{for } 0.002 \leq \varepsilon_c \leq 0.0035 \end{cases} \quad (7.20)$$

where ε_c was equal to the strain in the extreme compression fibre (ε_{cu}) (fib, 2019). The flexural strength (M_{calc}) was finally estimated using Eqn. 7.3, where it equals the summation of the internal forces multiplied by their respective distances to the neutral axis.

7.2.3.3 RC T-BEAMS WITH UHPC JACKETS

The cross-section analysis for RC T-beams strengthened with UHPC U-shaped jackets is shown in Figure 7.2. The cross-section, material strain and stress distributions as well as internal forces and their respective distances to the neutral axis are shown in Figure 7.2. Estimating the flexural strength of the T-beams depends on the position of the neutral axis in relation to the flange and web. The neutral axis may lie either in the flange or web as shown in Figures 7.2(a) and 7.2(b), respectively. When the neutral axis lies in the flange, the UHPC only contributes to tensile resistance at the ULS. On the other hand, the UHPC contributes to both compressive and tensile resistance when the neutral axis lies in the web.

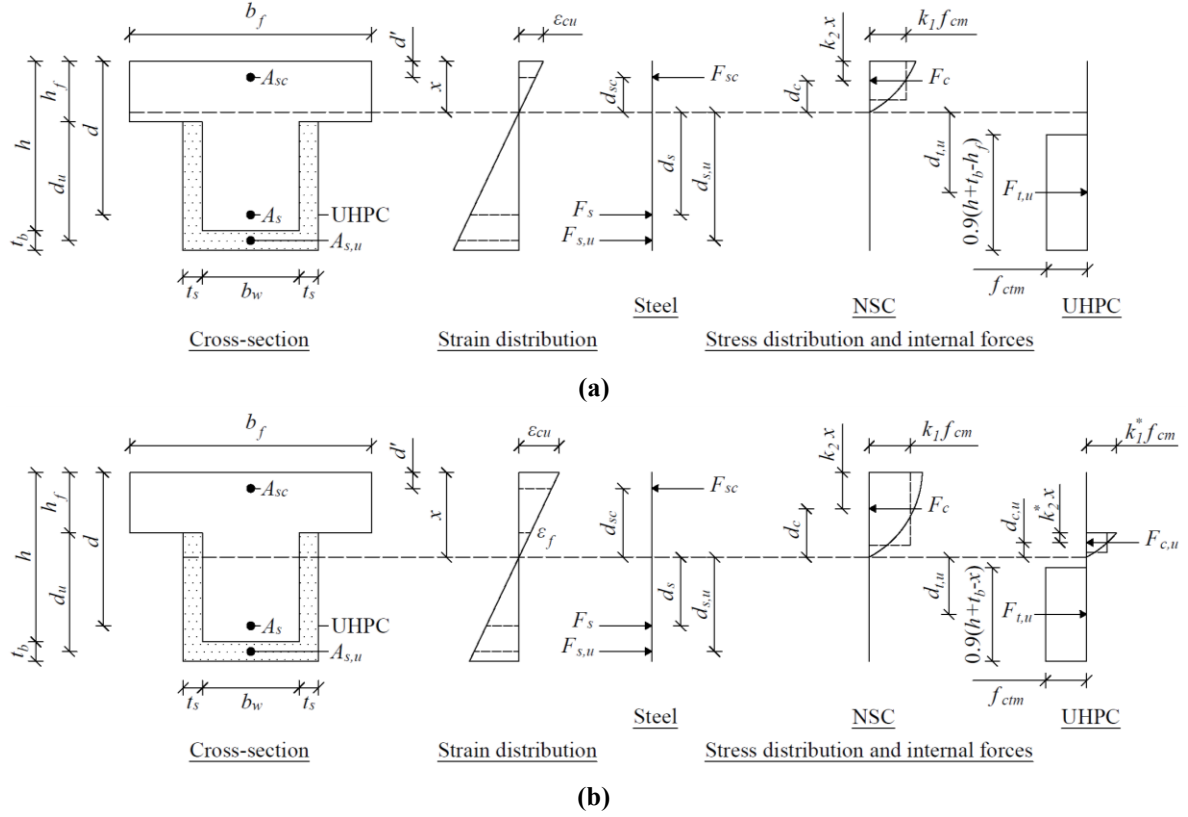


Figure 7.2 – Cross-section analysis of RC T-beam strengthened with UHPC U-shaped jacket: **(a)** Neutral axis lies in flange, **(b)** Neutral axis lies in web

Initially, it was assumed that the neutral axis lies in the flange, where Figure 7.2(a) applies. The internal forces were calculated, and Eqn. 7.2 was used to iteratively determine the depth to the neutral axis (x). If the determined depth to the neutral axis (x) was less than or equal to the height of the flange (h_f), then the initial assumption was correct along with the determined depth to neutral axis (x). If the determined depth to the neutral axis (x) was found to be greater than the height of the flange (h_f), then the neutral axis lies in the web, which means that the initial assumption was incorrect. Therefore, the cross-section analysis of Figure 7.2(b) applies, and the internal forces were recalculated to iteratively determine the correct depth to the neutral axis (x).

The internal forces of the longitudinal tensile reinforcement in the beam (F_s), longitudinal compressive reinforcement (F_{sc}) and longitudinal tensile reinforcement in the jacket ($F_{s,u}$) were calculated using Eqns. 7.4, 7.5 and 7.6, respectively. The remaining internal forces were calculated using the following set of equations:

$$F_c = \begin{cases} k_1 f_{cm} b_f x & \text{for } x \leq h_f \\ \sum F_{c,i}; F_{c,1} = k_1 f_{cm} b_f x; F_{c,2} = -k_1^* f_{cm} (b_f - b_w)(x - h_f) & \text{for } x > h_f \end{cases} \quad (7.21)$$

$$F_{c,u} = \begin{cases} 0 & \text{for } x \leq h_f \\ 2k_1^* f_{cm} t_s (x - h_f) & \text{for } x > h_f \end{cases} \quad (7.22)$$

$$F_{t,u} = \begin{cases} f_{ctm} (1.8(h + t_b - h_f)t_s + b_w t_b) & \text{for } x \leq h_f \\ f_{ctm} (1.8(h + t_b - x)t_s + b_w t_b) & \text{for } x > h_f \end{cases} \quad (7.23)$$

where b_f is the width of the flange. When the neutral lies in the web ($x > h_f$), the internal compressive force of the NSC (F_c) was calculated by splitting the force into two components ($F_{c,1}$ and $F_{c,2}$). One component ($F_{c,1}$) represents the entire rectangular area of concrete under compression including the areas beneath the flange outstands where no concrete is present, and the other ($F_{c,2}$) represents the rectangular areas beneath the flange outstands where no concrete is present. The factor related to the average stress of the parabolic-rectangular stress block in compression of concrete (k_1^*) was calculated using Eqn. 7.10 with the strain (ε_c) equal to the strain at the web/flange junction (ε_f). The strain at web/flange junction (ε_f) was calculated using the following equation:

$$\varepsilon_f = \frac{\varepsilon_{cu}(x - h_f)}{x} \quad (7.24)$$

The stresses in the longitudinal tensile reinforcement in the RC beam (f_s) and longitudinal compressive reinforcement (f_{sc}) are calculated according to Eqns. 7.11 and 7.12, respectively. The stress in the longitudinal tensile reinforcement in the UHPC jacket ($f_{s,u}$) was determined using the following equation:

$$f_{s,u} = \min \left\{ \frac{\varepsilon_{cu} E_{sm} (d_u + h_f - x)}{x}, f_{ym} \right\} \quad (7.25)$$

where d_u is the depth to longitudinal tensile reinforcement in the UHPC jacket from the web/flange junction of the T-beams as shown in Figure 7.2. After determining the depth to the

neutral axis (x), the distances from the internal forces to the neutral axis were then calculated. The distances from the internal forces to the neutral axis of the longitudinal tensile reinforcement in the RC beam (d_s) and compressive reinforcement (d_{sc}) were calculated using Eqns. 7.14 and 7.15, respectively. The following set of equations were used to calculate the remaining distances from the internal forces to the neutral axis:

$$d_c = \begin{cases} x - k_2 x & \text{for } x \leq h_f \\ d_{c,1} = x - k_2 x; d_{c,2} = (x - h_f)(1 - k_2^*) & \text{for } x > h_f \end{cases} \quad (7.26)$$

$$d_{s,u} = d_u + h_f - x \quad (7.27)$$

$$d_{c,u} = (x - h_f)(1 - k_2^*) \quad (7.28)$$

$$d_{t,u} = \begin{cases} h + t_b - x - \left[\frac{0.81t_s(h + t_b - h_f)^2 + 0.5b_w t_b^2}{1.8t_s(h + t_b - h_f) + b_w t_b} \right] & \text{for UJ and } x \leq h_f \\ h + t_b - x - \left[\frac{0.81t_s(h + t_b - x)^2 + 0.5b_w t_b^2}{1.8t_s(h + t_b - x) + b_w t_b} \right] & \text{for UJ and } x > h_f \\ 0.55h + 0.45h_f - x & \text{for SJ and } x \leq h_f \\ 0.55(h - x) & \text{for SJ and } x > h_f \end{cases} \quad (7.29)$$

where the abbreviations UJ and SJ refer to RC beams strengthened with U-shaped and side jackets, respectively. When the neutral lies in the web ($x > h_f$), the flexural resistance of the NSC was calculated by multiplying each split force component ($F_{c,1}$ and $F_{c,2}$) by its respective distance to the neutral axis ($d_{c,1}$ and $d_{c,2}$). The factor that is related to the centroidal position of the parabolic-rectangular stress block in compression for concrete (k_2^*) was calculated using Eqn. 7.20 with the strain (ε_c) equal to the strain at the web/flange junction (ε_f). The flexural strength (M_{calc}) was finally estimated using Eqn. 7.3 by adding the internal forces multiplied by their respective distances to the neutral axis.

7.3 PREDICTIONS FOR BEAM SPECIMENS OF THIS RESEARCH

7.3.1 INTRODUCTION

The methods of calculation of the ultimate shear and flexural strength described in Section 7.2 were applied to the beams tested in this campaign in order to evaluate the methods and suggest

improvements. It should be noted that the adopted design recommendations for RC and UHPC were developed for beams of a single material rather than strengthened beams. Consequently, the assumption of superposition requires investigation. The calculated ultimate shear and flexural strength and predicted failure modes of the specimens of groups 1 and 2 are compared with the experimental results. Comparisons are made between the components of shear resistance calculated with the design codes and estimated from the calculated crack kinematics in Chapter 5. The effects of the jacket side thickness, UHPC tensile strength and transverse and fibre reinforcement truss angles on the ultimate shear strength of the jacketed specimens is also assessed by comparing the shear strengths given by the design codes with the test results.

The ultimate shear strength of the control beams of groups 1 and 2 was calculated using the equations given by EC2, fib MC 2010 LoAIII and ACI 318-19 as well as the MASM (ACI, 2019; BSI, 2014; Cladera et al., 2016; fib, 2013; Marí et al., 2015, 2016). An alternative method was also applied to account for the flange contribution to shear strength. In this case, the total shear strength was taken as the shear strength given by EC2 plus the flange outstands contribution from the MASM. The ultimate shear strength of the jacketed beams of groups 1 and 2 was calculated using the superposition approach, where the contribution of the control beam was added to the contribution of the jacket given by NF P18-710, SIA 2052 and JSCE 2006 (AFNOR, 2016b; EPFL, 2016; JSCE, 2006). The flexural strength of the control and strengthened specimens of groups 1 and 2 was calculated using the cross-section analysis procedures for T-beams described in Section 7.2.3.3. The obtained material properties of each beam specimen presented in Chapter 4 were considered when calculating the ultimate shear and flexural strength. The geometry, reinforcement details and loading arrangement of the control RC beams and strengthened beams are presented in Tables 7.2 and 7.3, respectively.

Table 7.2 – Geometry, reinforcement details, mechanical properties and loading arrangement of control RC beams of groups 1 and 2

Specimen	Width (b_w) [mm]	Depth (d) [mm]	Height (h) [mm]	Flange width (b_f) [mm]	Flange height (h_f) [mm]	Compr. strength (f_{cm}) [MPa]	Long. reinf. ^A (ρ_l) [%]	Shear reinf. ^B (ρ_w) [%]	Load ratio (a_v/d)
B1	200	282.5	350	500	125	47.49	3.34	0.14	3.89
B5	200	305	350	500	125	50.14	1.55	0.11	3.61

^A Longitudinal reinforcement ratio (ρ_l) equals to $A_s/b_w d$; ^B Shear reinforcement ratio (ρ_w) equals to $A_{sw}/b_w s$

Table 7.3 – Geometry, reinforcement details and mechanical properties of UHPC jackets of groups 1 and 2

Specimen	Side thickness (t_s) [mm]	Bottom thickness (t_b) [mm]	Depth (d) [mm]	Compr. str. beam (f_{cm}) [MPa]	Compr. str. jack. (f_{cm}) [MPa]	Tensile strength (f_{ctm}) [MPa]	Tensile str. el. ($f_{ctm,el}$) [MPa]	Residual strength (σ_{res}) [MPa]	Long. reinf. (A_s)
B2	20	60	255	46.96	135.97	6.39	8.58	7.49	4 ϕ 20
B3	30	60	255	46.96	132.55	6.39	8.58	7.49	4 ϕ 20
B4	40	60	255	46.44	134.26	6.39	8.58	7.49	4 ϕ 20
B6	20	60	255	50.53	124.44	6.39	8.58	7.49	3 ϕ 20
B7	30	60	255	50.53	127.92	6.39	8.58	7.49	4 ϕ 20
B8	40	60	255	50.91	132.04	6.39	8.58	7.49	5 ϕ 20

7.3.2 EVALUATION OF ULTIMATE SHEAR STRENGTH

7.3.2.1 CONTROL RC BEAMS

Table 7.4 presents the ratio of calculated to experimental ultimate shear and flexural strength (V_{calc}/V_{exp}) for the control RC beams of groups 1 (B1) and 2 (B5), which failed in shear and combined flexure-shear, respectively. There is considerable variation in the shear strengths given by the considered design models. For specimen B1, the shear strengths given by EC2, fib MC 2010 LoAIII and ACI 318-19 are similar with the ratios (V_{calc}/V_{exp}) ranging between 0.56 and 0.61. The significant underestimate in shear strength is thought to be due to neglect of the flange contribution, which appears greatest for specimen B1. The shear strength given by the MASM, which considers the flange contribution to shear strength, matched the experimental shear strength of specimen B1 well with the ratio (V_{calc}/V_{exp}) equal to 1.05. The EC2 shear strength prediction improved significantly when increased by the flange contribution from the MASM giving a ratio (V_{calc}/V_{exp}) equal to 0.83. Specimen B1 was predicted as shear critical according to the design models.

Much like specimen B1, the shear strengths given by EC2, fib MC 2010 LoAIII and ACI 318-19 for specimen B5 are similar. In this case, the calculated shear strengths given by the design codes were all relatively close to the experimental shear strength. The ratio (V_{calc}/V_{exp}) given by the design codes ranged from 0.78 to 0.90 with ACI 318-19 being most accurate followed by EC2 and fib MC 2010 LoAIII. The shear strength was significantly overestimated by the MASM with the ratio (V_{calc}/V_{exp}) equal to 1.36. The combined shear strength given by EC2

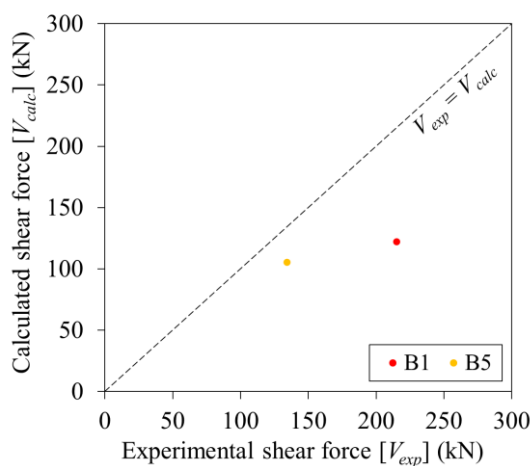
and the flange contribution of the MASM matched reasonably well with experimental shear strength, with the ratio (V_{cac}/V_{exp}) equaling to 1.15. Specimen B5 was predicted as shear critical according to the design codes, but critical in flexure according to the MASM and the combined EC2 and MASM method. This is feasible given that the specimen failed in combined flexure-shear.

Figure 7.3 compares the calculated shear strengths given by the various design models with the experimental shear strengths. The open markers in Figure 7.3 indicate that the plotted shear strength is less than the flexural strength implying a mismatched failure mode. This occurs for specimen B5 where the calculated shear strengths of the MASM and combined EC2 and MASM methods in Figure 7.3 were less than the calculated flexural strengths, and the predicted flexural failure mode did not match the experimental failure mode of combined flexure-shear. Overall, the best predictions are given by the combined EC2 and MASM method as shown in Table 7.4. and Figure 7.3.

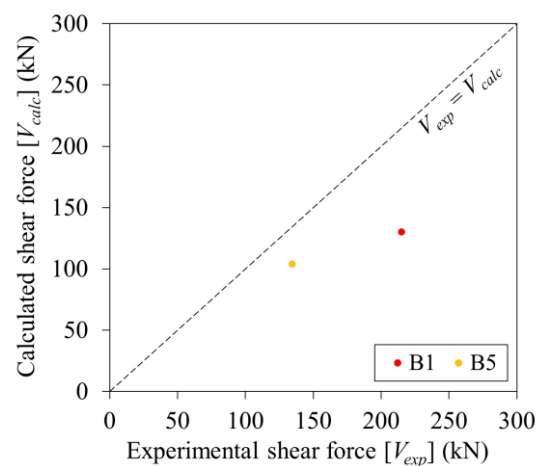
Table 7.4 – Experimental and calculated ultimate shear strength of control RC beams of groups 1 and 2

Specimen	Experimental ult. shear force (V_{exp}) [kN]	Calculated ultimate shear and flexural strength (V_{calc}/V_{exp})					
		EC2 ^A	fib MC 2010 ^B	ACI 318-19 ^C	MASM ^D	EC2 & MASM ^E	Flexure ^F
B1	214.96	0.57	0.61	0.56	1.05	0.83	1.04
B5	134.17	0.79	0.78	0.90	1.36	1.15	0.96
Average (Standard dev.)		0.68 (0.15)	0.69 (0.12)	0.73 (0.24)	1.20 (0.22)	0.99 (0.22)	1.00 (0.06)

^A EC2 (BSI, 2014); ^B fib Model Code 2010 LoAIII (fib, 2013); ^C ACI 318-19 (ACI, 2019); ^D MASM (Cladera et al., 2016; Mari et al., 2015, 2016); ^E EC2 with MASM contribution of flange; ^F Shear force (V_{calc}) equals to the calculated ultimate flexural strength (M_{calc}) divided by the shear span (a_v)



Eurocode 2



fib Model Code 2010 LoAIII

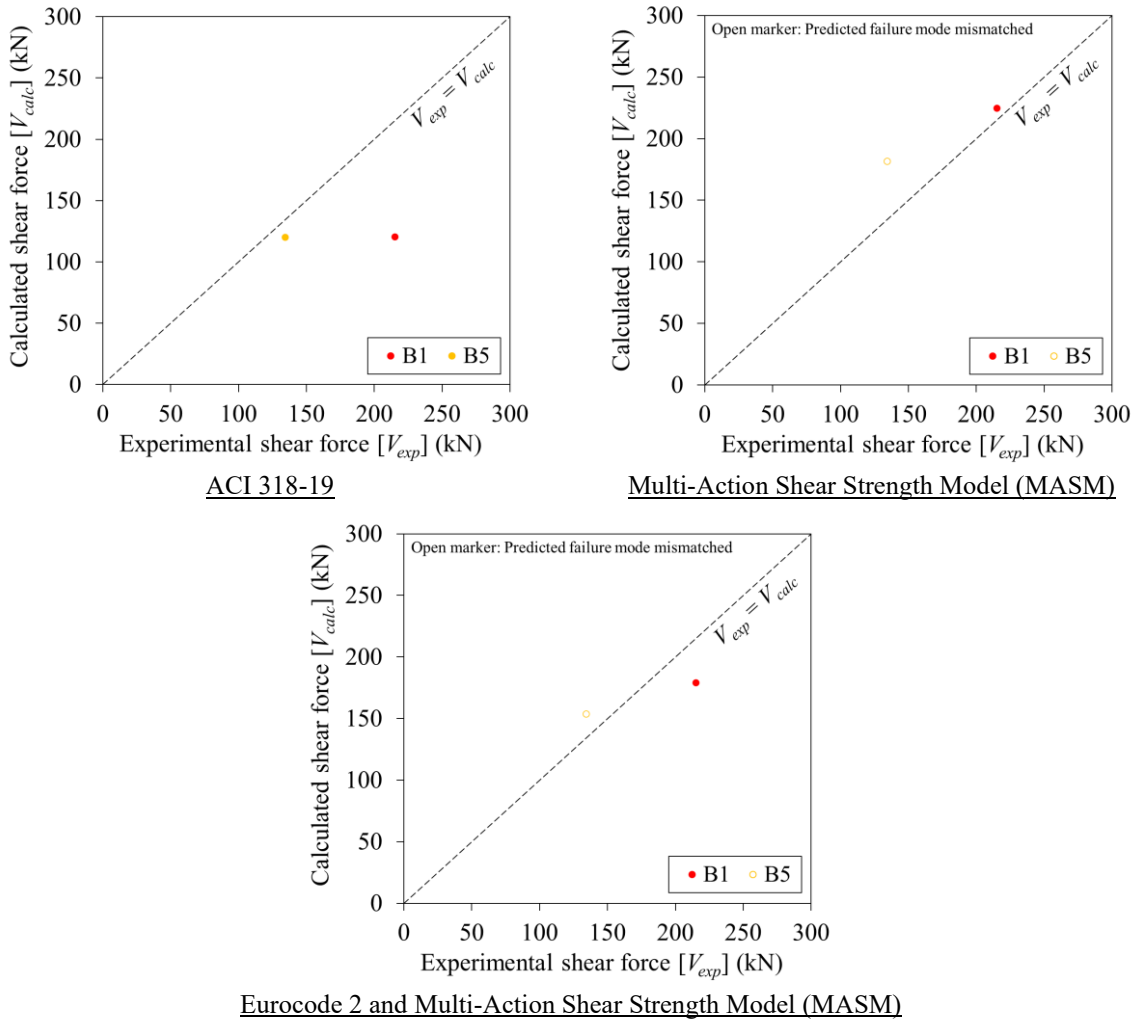


Figure 7.3 – Experimental and calculated shear strength of control RC beams of groups 1 and 2

7.3.2.2 RC BEAMS WITH UHPC JACKETS

Table 7.5 presents the ratios of calculated to experimental ultimate shear and flexural strength (V_{calc}/V_{exp}) for the strengthened specimens of groups 1 (B2, B3 and B4) and 2 (B6, B7 and B8), which all failed in shear. The contribution to shear strength of the UHPC jacket given by NF P18-710, SIA 2052 and JSCE 2006 was compared with the contribution of the jacket in the tests. The latter was calculated as the difference in experimentally determined shear strength of the jacketed and control specimens. The calculated ultimate shear strengths of the strengthened specimens were also compared with the test results.

The predicted jacket contribution is overestimated on average when compared to the test results, with the degree of overestimation depending on the side thickness of the jacket. The calculated contributions of the 20 mm thick jackets of groups 1 (B2) and 2 (B6) matched well

with the test results with the ratios (V_{calc}/V_{exp}) for the jackets ranging across the design codes from 0.97 to 1.16 for specimen B2 and 0.92 to 1.10 for specimen B6. The jacket contribution is overestimated for the 30 mm thick jackets of groups 1 (B3) and 2 (B7) with the ratios (V_{calc}/V_{exp}) ranging across the codes between 1.28 and 1.53. The jacket contribution is overestimated even more for the 40 mm jackets with the ratios (V_{calc}/V_{exp}) ranging from 1.48 to 1.78 and 1.30 to 1.55 for specimens B4 and B8, respectively. The shear strength predictions of SIA 2052 were the lowest since SIA 2052 does not include a contribution to shear strength from the UHPC unlike NF P18-710 and JSCE 2006, which both gave similar and higher predictions. Table 7.5 indicates that the predictions of SIA 2052 are the most accurate with the average comparison ratio (V_{calc}/V_{exp}) for all beams equal to 1.21.

As discussed, the degree of overestimation of the jacket contribution at the ultimate load increases with jacket side thickness. This is highlighted for groups 1 and 2 in Figure 7.4. The shear strength contribution of the jacket given by SIA 2052 was predicted to increase by 50% and 100% when increasing the side thickness from 20 mm to 30 mm and 40 mm, respectively, as illustrated in Figure 7.4. This was not the case at the ultimate load according to the test results of both groups 1 and 2. For the strengthened specimens of group 1, increasing the side thickness from 20 mm (B2) to 30 mm (B3) and 40 mm (B4) increased the shear strength contribution of the jacket by 13% and 31%, respectively. For the strengthened specimens of group 2, increasing the side thickness from 20 mm (B6) to 30 mm (B7) and 40 mm (B8) increased the shear strength contribution of the jacket by 8% and 42%, respectively. The trendlines in Figure 7.4 highlight the significant difference between the predicted and experimental shear strength enhancement provided by the varying side thickness of the UHPC jackets of both groups 1 and 2.

The shear strength of the jacketed specimens was calculated by adding the contribution of the RC beam to that of the UHPC jacket. The contribution of the RC beam was taken as the shear strength given by EC2 along with the contribution of the flange outstands given by the MASM as it gave the best fit on average for the control beams of groups 1 (B1) and 2 (B5). The calculated shear strengths of the jacketed specimens of groups 1 and 2 were compared with the experimental shear strengths, with the results presented in Table 7.5. The calculated shear strength was generally overestimated, which is largely due to the UHPC design codes overpredicting the shear strength contribution of the jackets at the ultimate load. Figure 7.5

compares the calculated shear strength given by the various design models with the experimental shear strength. The overestimation of the shear strength contribution of the jackets is evident for all of the considered UHPC design codes. Comparison with the predicted flexural strengths shows that shear failure is predicted for all the strengthened specimens according to SIA 2052. Flexural failure was predicted for a few strengthened specimens when using the jacket contributions of NF P18-710 and JSCE 2006 due to their significant overestimations. These specimens are highlighted with open markers in Figure 7.5, which indicate that the plotted shear strength is less than the flexural strength.

Overall, the best predictions for the jacketed specimens are given when the RC beam contribution was calculated using the combined EC2 and MASM method, and the jacket contribution was calculated using SIA 2052. The average ratio (V_{calc}/V_{exp}) for this method was equal to 1.08 with a relatively low standard deviation of 0.13 as shown in Table 7.5.

Table 7.5 – Experimental and calculated ultimate shear strength of strengthened beams of groups 1 and 2

Specimen	Experimental		Calculated ultimate shear and flexural strength (V_{calc}/V_{exp})						
	Total (V_{exp})	Jacket ^A (V_{exp})	NF P18-710 ^B		SIA 2052 ^C		JSCE 2006 ^D		Flexure ^E
	[kN]	[kN]	Total ^E	Jacket	Total ^E	Jacket	Total ^E	Jacket	Total
B2	337.84	122.87	0.95	1.16	0.88	0.97	0.94	1.14	1.30
B3	354.39	139.43	1.11	1.53	1.01	1.28	1.10	1.51	1.26
B4	375.66	160.70	1.23	1.78	1.11	1.48	1.22	1.75	1.20
B6	263.07	128.89	1.12	1.10	1.03	0.92	1.11	1.08	1.18
B7	273.45	139.27	1.34	1.53	1.21	1.28	1.32	1.51	1.33
B8	317.69	183.52	1.37	1.55	1.22	1.30	1.35	1.53	1.31
Average (Standard deviation)			1.19 (0.16)	1.44 (0.26)	1.08 (0.13)	1.21 (0.22)	1.17 (0.15)	1.42 (0.25)	1.27 (0.06)

^A Ultimate shear strength of the jacket equals to the difference in shear strength between the jacketed beam and control beam; ^B NF P18-710 (AFNOR, 2016b); ^C SIA 2052 (EPFL, 2016); ^D JSCE 2006 (JSCE, 2006); ^E Calculated ultimate shear strength of jacketed specimens includes contribution of RC beam given by EC2 with contribution of flange given by MASM; ^F Shear force (V_{calc}) equals to the calculated ultimate flexural strength (M_{calc}) divided by the shear span (a_v)

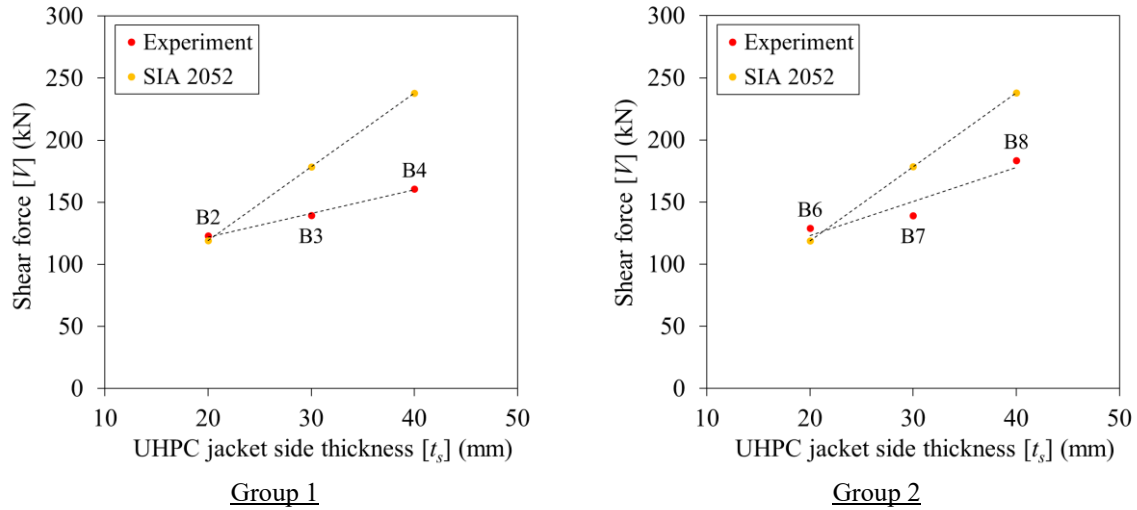
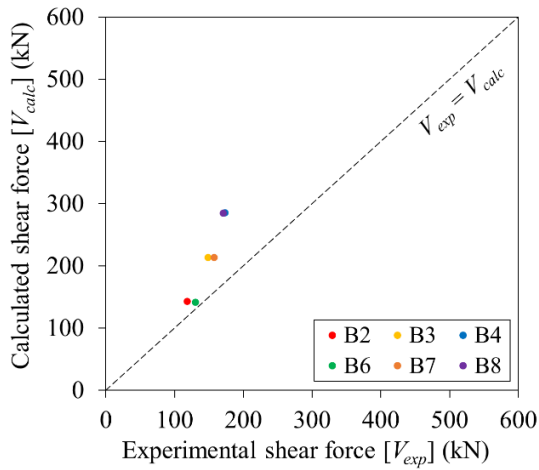
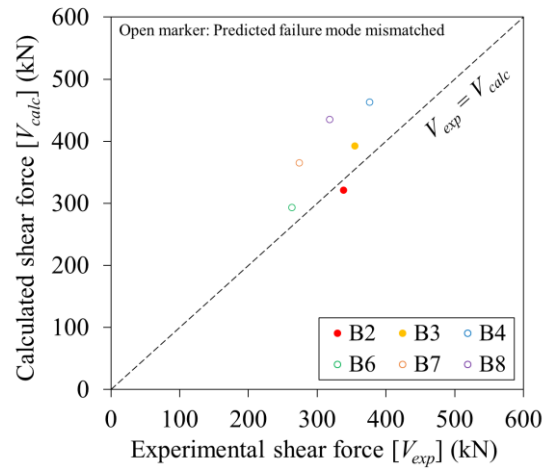


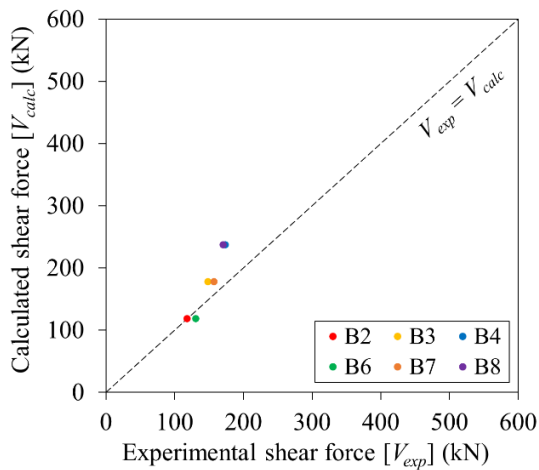
Figure 7.4 – Effect of UHPC jacket side thickness on the experimental and calculated ultimate shear strength of the strengthened beams of groups 1 and 2



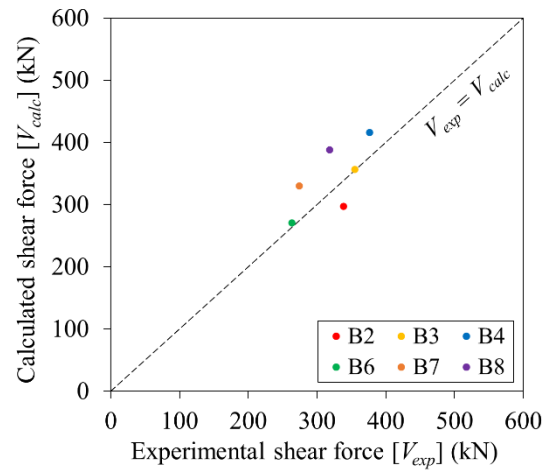
NF P18-710 – Jacket



NF P18-710 with EC2 and MASM – Beam with jacket



SIA 2052 – Jacket



SIA 2052 with EC2 and MASM – Beam with jacket

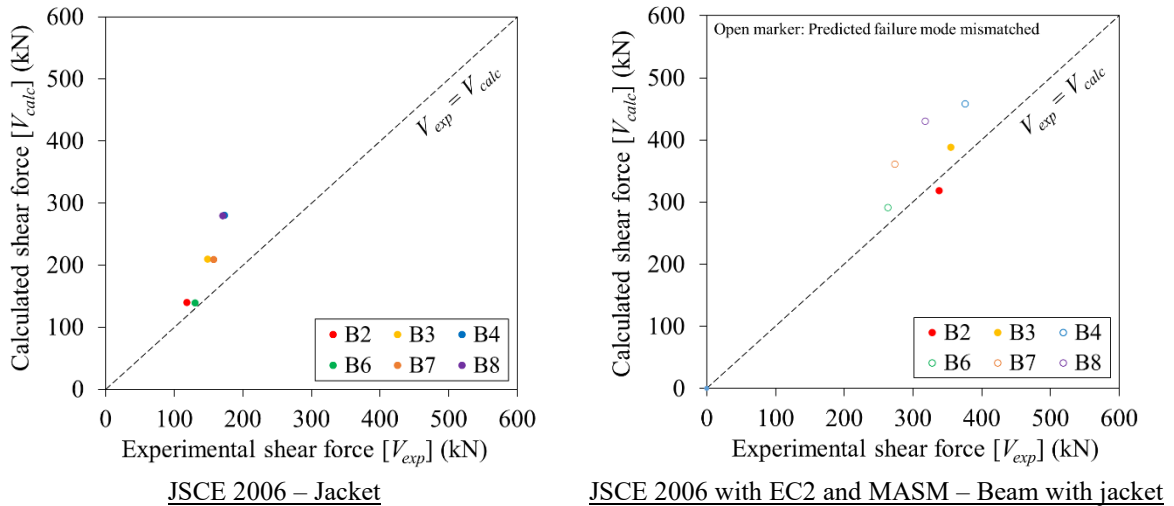


Figure 7.5 – Experimental and calculated ultimate shear strength of strengthened beams of groups 1 and 2

7.3.2.3 COMPARISON WITH NLFEA RESULTS

Table 7.6 presents the NLFEA results from Chapter 6 compared with the calculated shear strengths given by the combined EC2, MASM and SIA 2052 method and the calculated flexural strengths. The NLFEA results compare well with the test results, especially for control specimens B1 and B5 as well as specimens B7 and B8. The combined EC2, MASM and SIA 2052 method is in reasonable agreement with the test results for the specimens with thinner jackets (B2, B3 and B6), but overestimates the strengths of specimens B4, B7 and B8 with thicker jackets. All of the specimens, apart from specimen B5, which failed in combined flexure-shear, were predicted to be shear critical as observed. This failure mode was replicated by the numerical model of specimen B5.

Table 7.7 shows the relative contributions to flexural strength of the longitudinal reinforcement in tension and compression, NSC in compression and UHPC in tension. The contribution of UHPC in compression was equal to zero seeing as the depth to the neutral axis was found to lie in the flange for all specimens. It is evident that the majority of the flexural strength is provided by the reinforcement in tension as it accounts for approximately 70% to 90% of the total flexural strength.

Table 7.6 – Experimental, numerical and calculated ultimate shear and flexural strength of specimens of groups 1 and 2

Specimen	Experimental		Numerical		Calculated		
	Ult. shear force (V_{exp}) [kN]	Failure mode	Ult. force (V_{num}/V_{exp})	Failure mode	Ult. shear strength ^A (V_{calc}/V_{exp})	Ult. flexural strength ^B (V_{calc}/V_{exp})	Failure mode
B1	214.96	Shear	0.96	Shear	0.83	1.04	Shear
B2	337.84	Shear	0.96	Shear	0.88	1.30	Shear
B3	354.39	Shear	1.06	Shear	1.01	1.26	Shear
B4	375.66	Shear	1.08	Shear	1.11	1.20	Shear
B5	134.17	Flexure-Shear	1.11	Flexure-shear	1.15	0.96	Flexure
B6	263.07	Shear	1.01	Shear	1.03	1.18	Shear
B7	273.45	Shear	1.13	Shear	1.21	1.33	Shear
B8	317.69	Shear	0.93	Shear	1.22	1.31	Shear

^A Contributions of RC beam, compression zone of flange and UHPC jacket given by EC2, MASM and SIA 2052, respectively (BSI, 2014; Cladera et al., 2016; EPFL, 2016; Mari et al., 2015, 2016); ^B Shear force (V_{calc}) equals to the calculated ultimate flexural strength (M_{calc}) divided by the shear span (a_v)

Table 7.7 – Contributions to calculated ultimate flexural strength of specimens of groups 1 and 2

Specimen	Calculated ult. flexural strength (M_{calc}) [kN-m]	Contribution to calculated ultimate flexural strength				
		Beam rebar in compression ($F_{sc}d_{sc}/M_{calc}$) [%]	NSC in compression ($F_c d_c/M_{calc}$) [%]	Beam rebar in tension ($F_s d_s/M_{calc}$) [%]	Jacket rebar in tension ($F_{s,u} d_{s,u}/M_{calc}$) [%]	UHPC in tension ($F_{t,u} d_{t,u}/M_{calc}$) [%]
B1	268.63	0.04	11.58	88.38	0	0
B2	528.36	0.89	17.99	37.13	37.46	6.52
B3	535.11	0.93	18.40	36.35	36.78	7.54
B4	541.03	1.00	19.02	35.45	36.04	8.50
B5	154.77	1.21	7.39	91.41	0	0
B6	373.26	0.07	9.99	34.47	44.84	10.63
B7	437.70	0.22	11.42	28.35	49.60	10.40
B8	501.08	0.39	12.89	23.88	52.69	10.15

7.3.3 EVALUATION OF RESISTING MECHANISMS IN STRENGTHENED BEAMS

The experimentally derived components of shear resistance at peak load from Chapter 5 were compared with those calculated for each considered design method. The results of the comparisons are shown for the control RC beams of groups 1 (B1) and 2 (B5) in Figures 7.6

and 7.7, respectively. The individual contributions to shear resistance differ for EC2, fib MC 2010 LoAIII and ACI 318-19, but the total shear resistances are similar. The total shear resistances predicted by these codes range from 56% to 61% of the measured resistance for specimen B1 and 78% to 90% for specimen B5. The codes overestimate the stirrup contributions for specimens B1 and B5, relative to the experimentally derived contributions, especially for EC2 where shear is assumed to be entirely resisted by stirrups. In fib MC 2010 LoAIII and ACI 318-19 shear is assumed to be resisted partly by concrete and partly by shear reinforcement. The concrete term in these codes includes contributions from all the shear resisting mechanisms apart from stirrups. Concrete makes a significant contribution to shear resistance according to fib MC 2010 LoAIII and ACI 318-19, but significantly less than derived experimentally from consideration of aggregate interlock, dowel action and the flexural compression zone. The MASM predicts the measured shear resistance of specimen B1 well but the strength of specimen B5 is overestimated by around 36%. The MASM stirrup contributions match the experimentally derived results very well for both specimens B1 and B5 but the dowel action and cracked concrete web contributions are less than those experimentally derived. The uncracked compression zone contributions given by the MASM are significantly greater than those experimentally derived. The MASM compression zone contribution appears to be an overestimate particularly for specimen B5 though it should be noted that this beam is predicted to fail in flexure. The total shear resistance given by the combined EC2 and MASM method matches the test results for specimens B1 and B5 best overall, but the individual components of shear resistance do not compare well with the experimentally determined ones. It should be noted that the flange contribution in the combined EC2 and MASM method just includes the outstands of the flange.

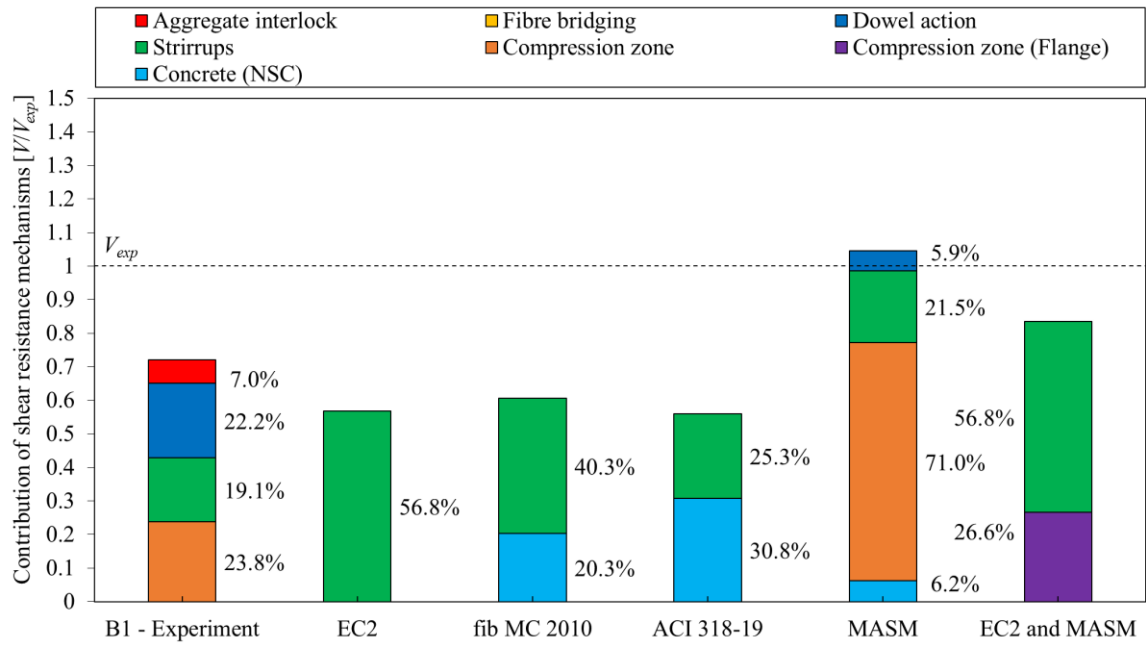


Figure 7.6 – Experimental and calculated shear resistance mechanisms of specimen B1

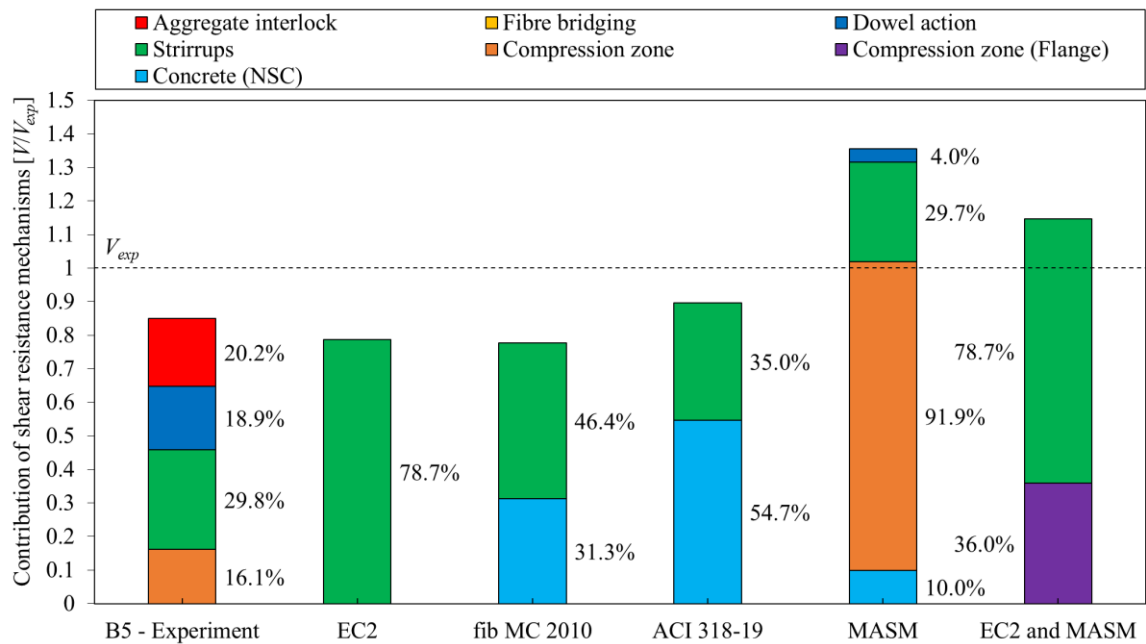


Figure 7.7 – Experimental and calculated shear resistance mechanisms of specimen B5

The comparisons of the contributions of the experimentally derived and calculated shear resistance mechanisms for strengthened specimens B2, B3, B4, B6, B7 and B8 are presented in Figures 7.8 to 7.13. The shear resistance contributions of the RC beam determined using the combined EC2 and MASM method are shown in Figures 7.8 to 7.13 along with the contributions of the UHPC jacket given by NF P18-710, SIA 2052 and JSCE 2006 (AFNOR, 2016b; EPFL, 2016; JSCE, 2006). The calculated fibre contribution is the same for all the

design codes since the same truss angle and residual tensile strength were adopted in each method. There is a small UHPC matrix contribution in NF P18-710 and JSCE 2006 but not SIA 2052. The calculated fibre contribution is significantly greater than experimentally determined at the ultimate load. According to the design codes, the fibre bridging contribution accounts for a minimum of 37% (B2) to a maximum of 60% (B8) of the total calculated shear resistance, with the contribution of the UHPC matrix making up no more than 10% and the contributions of the RC beam making up the remaining amount.

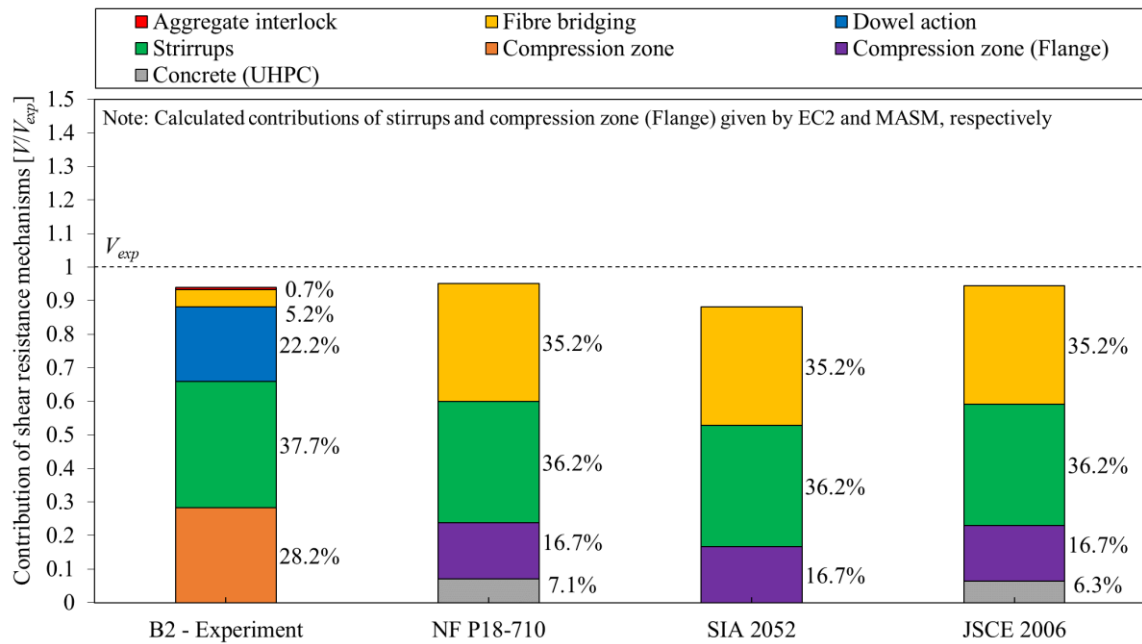


Figure 7.8 – Experimental and calculated shear resistance mechanisms of specimen B2

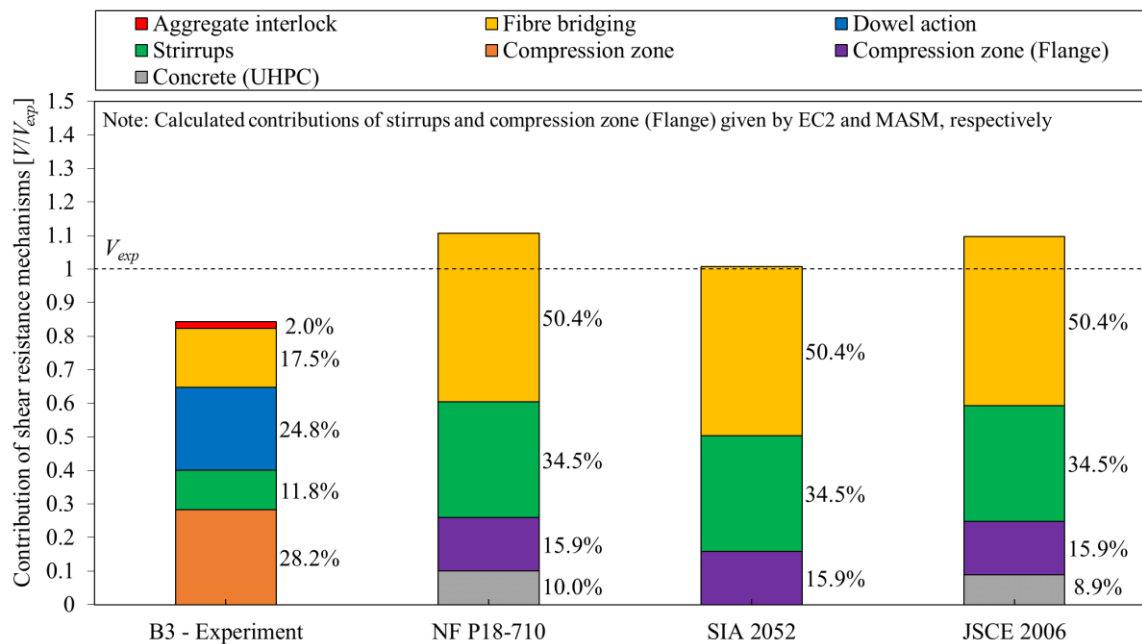


Figure 7.9 – Experimental and calculated shear resistance mechanisms of specimen B3

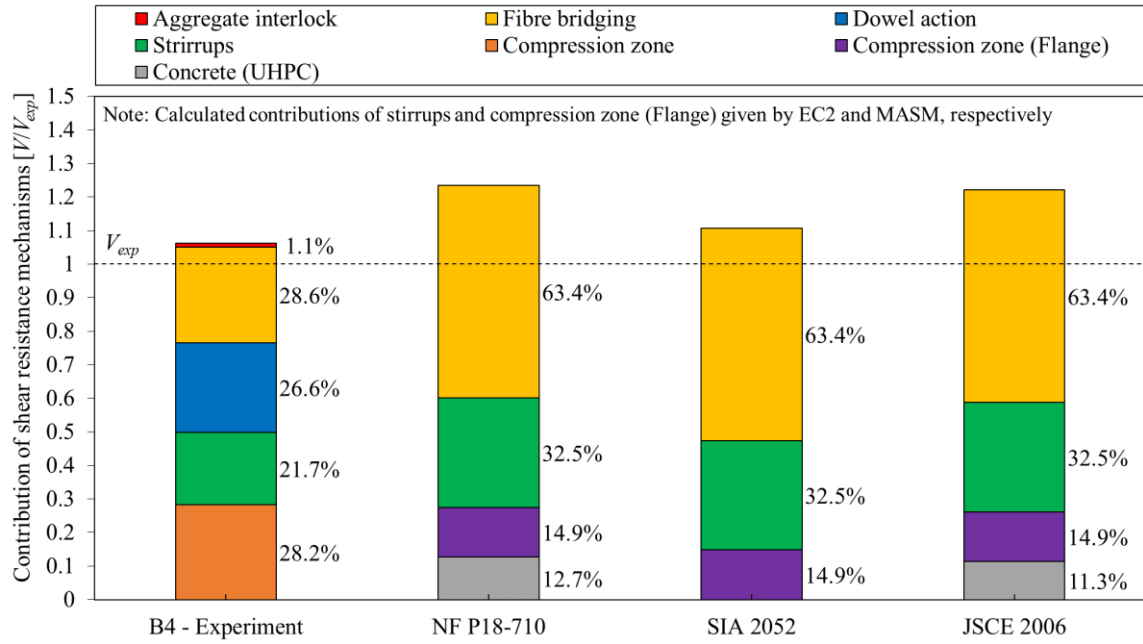


Figure 7.10 – Experimental and calculated shear resistance mechanisms of specimen B4

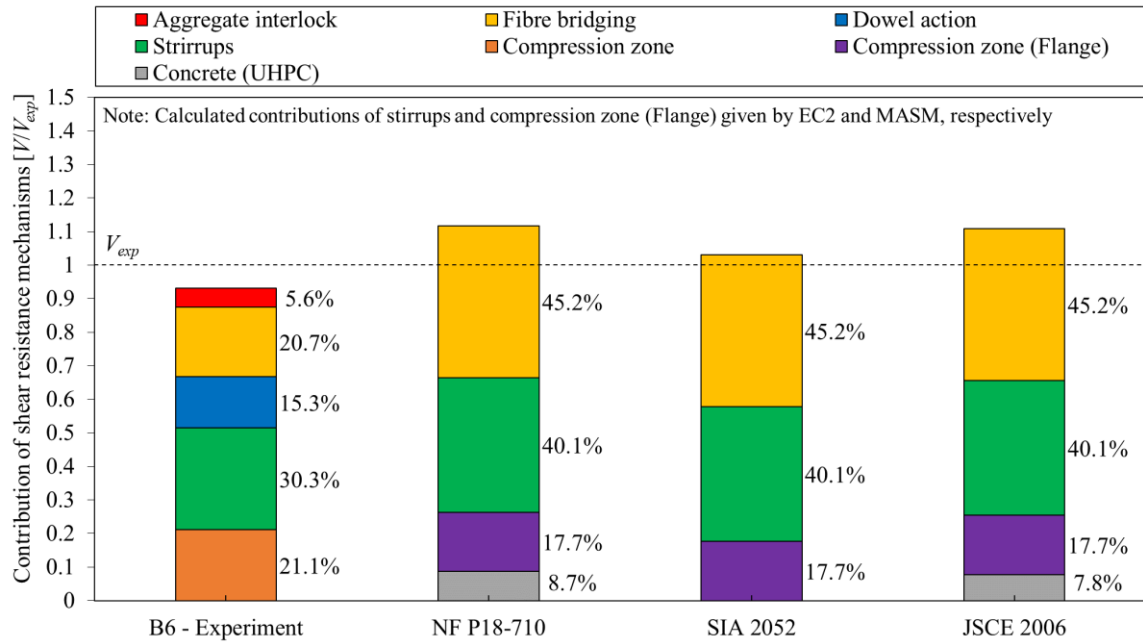


Figure 7.11 – Experimental and calculated shear resistance mechanisms of specimen B6

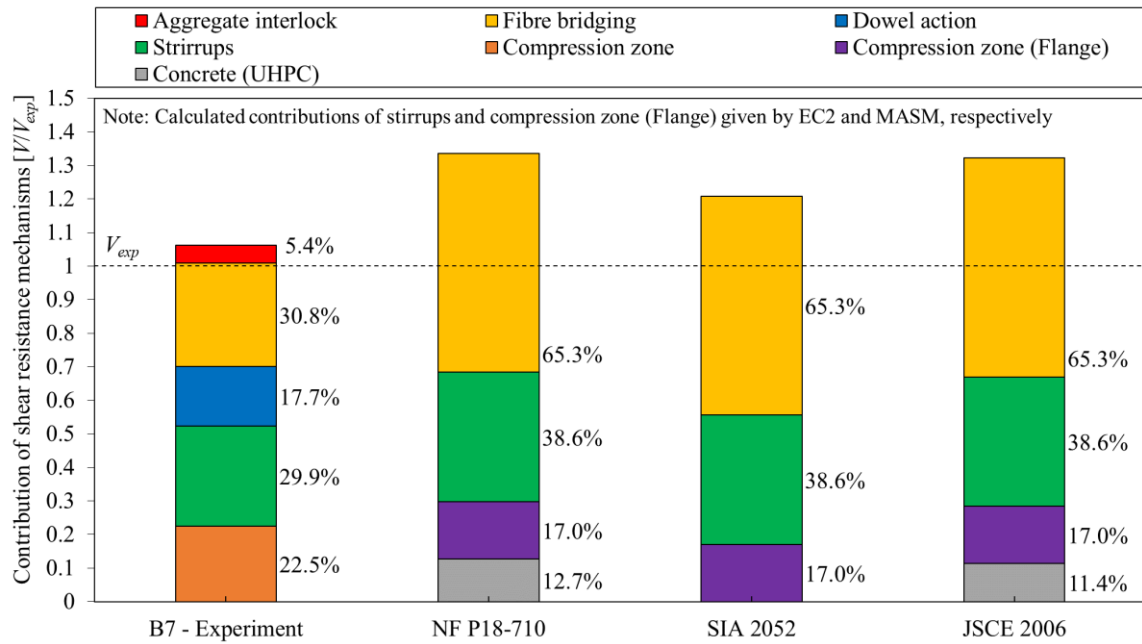


Figure 7.12 – Experimental and calculated shear resistance mechanisms of specimen B7

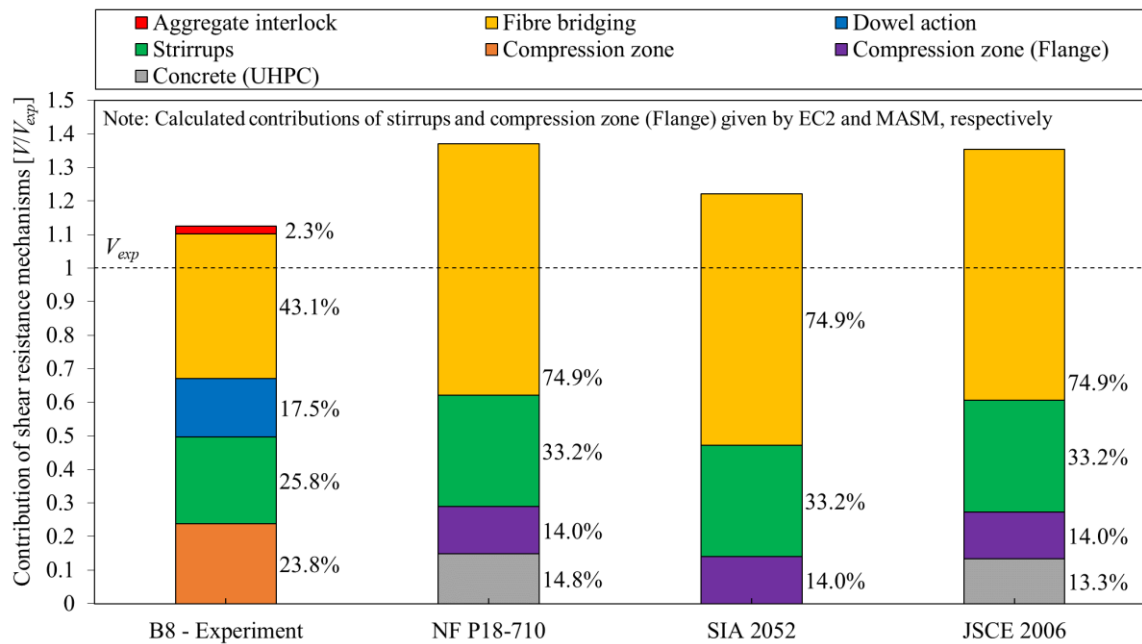


Figure 7.13 – Experimental and calculated shear resistance mechanisms of specimen B8

7.3.4 EVALUATION OF RESISTANCE MECHANISMS IN UHPC JACKETS

Shear resistance was calculated in Section 7.3.2.2 assuming that the peak shear resistance contributions of the transverse reinforcement in the RC beam and fibre reinforcement in the UHPC jacket occur simultaneously. This contradicts the test observation that indicate the peak jacket contribution, or fibre bridging contribution, occurs well before the ultimate load is

reached for all of the jacketed specimens tested in this campaign. The peak fibre bridging contribution was observed to occur before the ultimate load at around the load steps of $0.5P_u$ to $0.8P_u$ where the critical shear crack opening displacements based on the calculated crack kinematics in Chapter 5 are less than 0.5 mm. The crack opening displacements at the ultimate load are between a minimum of 1.25 mm and maximum of 4.18 mm, which correspond to specimens B6 and B2, respectively. The adopted residual strength (σ_{res}) was the average of the post-cracking tensile strength at the elastic limit and the post-cracking tensile strength, which corresponds to a crack opening displacement of 0.3 mm according to the test results. It should be noted that for strain-softening UHPC, the highest tensile resistance occurs at low crack opening displacements and vice versa. Therefore, the residual strength (σ_{res}) used to estimate the shear resistance contribution of the fibre reinforcement at the ultimate load is significantly overestimated as the crack opening displacements at the ultimate load are far greater than that assumed in the calculations.

Figure 7.14 shows the average tensile stress and crack opening responses along the length of the critical shear cracks of the jacketed specimens throughout each load step (P/P_u) until the ultimate load (P_u) that were derived based on the calculated crack kinematics. The average tensile stress at the ultimate load along the critical shear crack ranges between 0.88 MPa and 4.60 MPa, and the adopted residual tensile strength (σ_{res}) of 7.49 MPa was found to be about four times greater on average when considering all specimens. The tensile stress at peak jacket contribution ranges between 6.41 MPa to 7.00 MPa apart from specimen B7 where it was 5.86 MPa. While the average tensile stresses at peak jacket contribution are relatively close to the adopted residual tensile strength (σ_{res}), they are still less than the adopted design value. This is likely due to slight non-uniformity in the crack opening displacements along the length of the critical shear cracks as the crack opening displacements at the crack ends were generally narrower as shown in the crack kinematics results presented in Section 5.2.3.

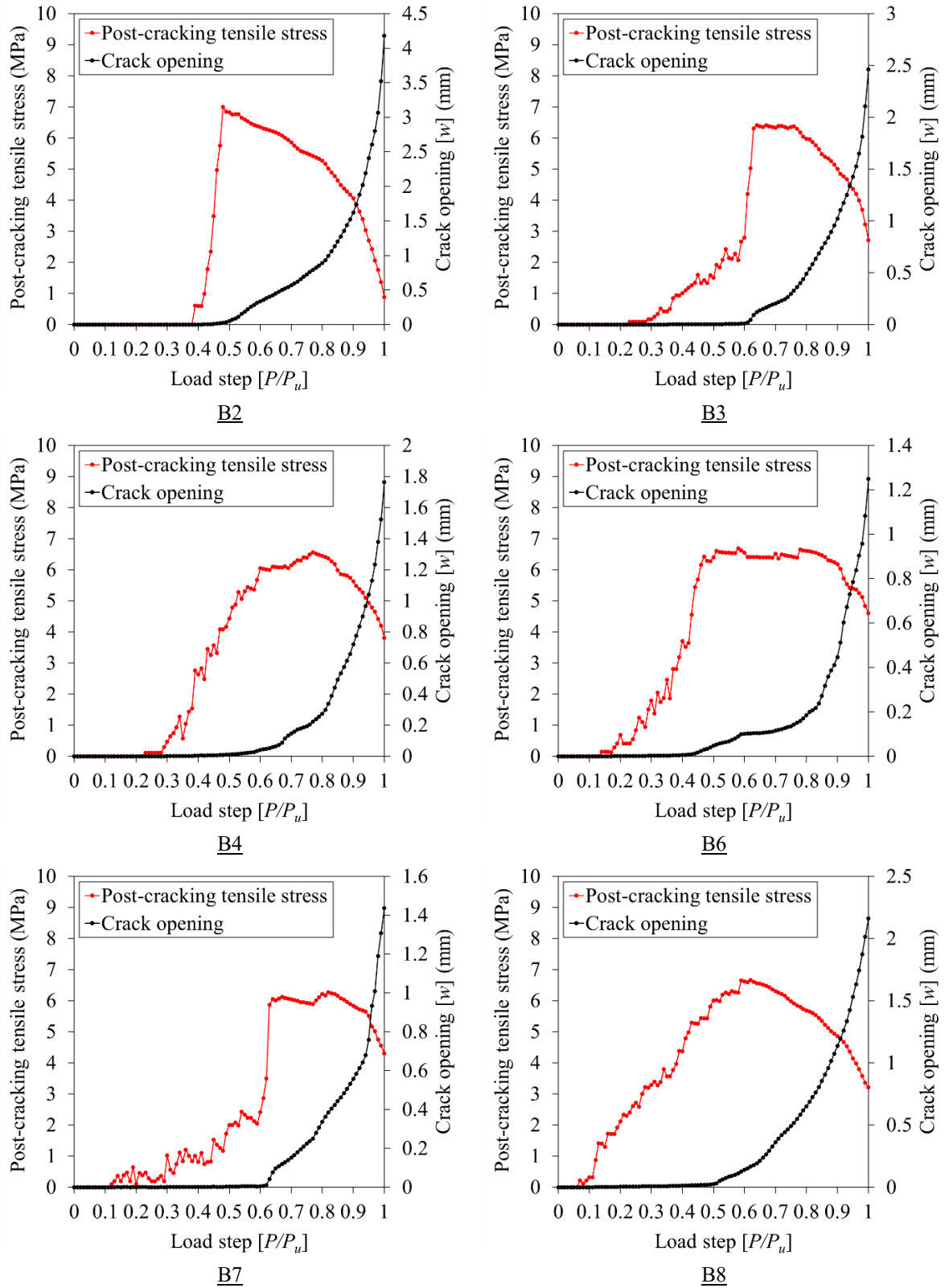


Figure 7.14 – Tensile stress and crack opening response along the critical shear crack throughout loading based on the calculated crack kinematics for the strengthened specimens of groups 1 and 2

A breakdown of the experimentally derived contributions to shear resistance for the UHPC jackets at the load step corresponding to peak jacket contribution and at the ultimate load are

presented in Table 7.8. The contributing shear resistance mechanisms in the UHPC jackets included fibre bridging in addition to dowel action of the longitudinal reinforcement embedded in the jacket. At the load step corresponding to peak jacket contribution, fibre bridging accounts for up to 86% of the total shear resistance as in specimen B2 and as little as 53% as in specimen B3. Fibre bridging is the most dominant mechanism overall at the load step corresponding to peak jacket contribution, while the dowel contribution of the longitudinal reinforcement is generally less than 10%. Interestingly, as the load is increased following the peak fibre bridging contribution, the fibre bridging contribution decreases while the remaining resistance mechanisms in both the beam and jacket remain fairly constant apart from the uncracked compression zone. The fibre bridging contribution becomes much less dominant at the ultimate load. A reduction of up to 75% occurred in specimen B2, which was caused by the exceptional widening of the critical shear crack that led to fibre pull-out in addition to the yielding of all three active stirrups. The fibre bridging contribution reduced between approximately 11% and 46% for the remaining strengthened specimens due to the widening of the critical shear crack, while the dowel action contribution did not significantly change between load steps corresponding to peak jacket contribution and the ultimate load.

Table 7.8 – Experimentally derived contributions of UHPC jackets of strengthened specimens of groups 1 and 2 at the load step corresponding to peak jacket contribution and at the ultimate load

Specimen	Load step corresponding to peak contribution of jacket				Ultimate load			
	Load step (P/P_u)	Fibre bridging (V_{fib}) [kN]	Dowel action (V_{dow}) [kN]	Shear force (V_{exp})	Load step (P/P_u)	Fibre bridging (V_{fib}) [kN]	Dowel action (V_{dow}) [kN]	Shear force (V_{exp})
B2	0.48	160.49 (0.86)	14.09 (0.08)	187.28	1.00	17.54 (0.05)	25.45 (0.08)	337.84
B3	0.64	140.10 (0.60)	31.81 (0.14)	235.44	1.00	61.86 (0.17)	38.17 (0.11)	354.39
B4	0.77	173.39 (0.53)	50.90 (0.16)	326.67	1.00	107.49 (0.29)	50.90 (0.14)	375.66
B6	0.51	74.56 (0.56)	1.92 (0.01)	134.16	1.00	54.46 (0.21)	13.94 (0.05)	263.07
B7	0.63	137.10 (0.80)	7.10 (0.04)	172.27	1.00	84.17 (0.31)	21.42 (0.08)	273.45
B8	0.59	278.03 (0.76)	29.26 (0.08)	367.67	1.00	136.96 (0.43)	29.26 (0.09)	317.69

[Note] – () refers to comparison ratio between experimentally derived contribution and experimental shear resistance (V_{fib}/V_{exp} or V_{dow}/V_{exp})

The UHPC jacket shear resistance that was experimentally derived from the calculated crack kinematics ($V_{fib+dow}$) at the load step corresponding to peak jacket contribution and the ultimate load was compared to the calculated ultimate shear resistance (V_{calc}), and the results are presented in Table 7.9. The experimentally derived shear resistance of the UHPC jacket

($V_{fib+dow}$) in Table 7.9 considers the contributions of fibre bridging (V_{fib}) and dowel action (V_{dow}), while the calculated shear resistance (V_{calc}) only includes the contribution of fibre reinforcement. It is evident that the calculated shear resistance matches reasonably well overall with the experimentally derived shear resistance at the load step corresponding to peak jacket contribution for all strengthened specimens. The average comparison ratio ($V_{calc}/V_{fib+dow}$) is equal to 1.06 yet the standard deviation is relatively high at 0.32. Notably, the calculated shear resistance is underestimated when compared to the experimentally derived shear resistance at the peak jacket contribution for specimens B2 and B8 as their respective ratios ($V_{calc}/V_{fib+dow}$) were equal to 0.68 and 0.77. On the other hand, the calculated shear resistance is overestimated for specimens B6 and B7 with their ratios ($V_{calc}/V_{fib+dow}$) equal to 1.56 and 1.24, respectively. The comparisons between the experimentally derived and calculated UHPC jacket shear resistances at the ultimate load did not match well overall as seen in Table 7.9. The calculated shear resistance of the UHPC jacket was significantly overestimated for all specimens as the average ratio ($V_{calc}/V_{fib+dow}$) is equal to 1.82. The significant overestimation is related to the decline in the fibre bridging contribution caused by the widening of the cracks and subsequent fibre pull-out as the ultimate load is reached.

Table 7.9 – Experimentally derived and calculated contributions of UHPC jackets of strengthened specimens of groups 1 and 2 at the load step corresponding to peak jacket contribution and at the ultimate load

Specimen	Jacket side thickness (t_s) [mm]	Derived force at peak contrib. ^A ($V_{fib+dow}$) [kN]	Derived force at ultimate load ^A ($V_{fib+dow}$) [kN]	Calculated force (V_{calc}) [kN]	Comparison at peak contribution ($V_{calc}/V_{fib+dow}$)	Comparison at ultimate load ($V_{calc}/V_{fib+dow}$)
B2	20	174.58	42.99	119.01	0.68	2.77
B3	30	171.91	100.03	178.52	1.04	1.78
B4	40	224.29	158.39	238.03	1.06	1.50
B6	20	76.48	68.40	119.01	1.56	1.74
B7	30	144.19	105.59	178.52	1.24	1.69
B8	40	307.29	166.22	238.03	0.77	1.43
Average (Standard deviation)					1.06 (0.32)	1.82 (0.48)

^A Shear resistance of jacket ($V_{fib+dow}$) was taken as sum of the fibre bridging (V_{fib}) and dowel action (V_{dow}) contributions to shear resistance that were experimentally derived

7.3.5 KEY FINDINGS AND SHEAR DESIGN RECOMMENDATIONS

The best shear strength predictions obtained by superimposing the ultimate shear strengths of the RC beam and UHPC jacket are given by the combined EC2, MASM and SIA 2052 method for the beams of this research. However, predictions are unsafe particularly for jacket side thicknesses of 30 mm and 40 mm as in specimens B3, B4, B7 and B8. The adopted design method considered the minimum truss angle of 21.8° for the transverse reinforcement given by EC2 and the minimum truss angle of 30° for the fibre reinforcement suggested by the UHPC design codes (AFNOR, 2016b; EPFL, 2016; JSCE, 2006). The use of the minimum truss angles in the adopted method represents the upper bound for the calculated shear resistances of the RC beam and UHPC jacket, and it implies that the peak contributions of the RC beam and UHPC jacket occur simultaneously. This is inconsistent with the experimentally derived fibre reinforcement contributions that indicate the peak jacket contribution occurs well before the ultimate load as highlighted in Section 7.3.4. Therefore, superposing the upper bound and peak shear resistances of the RC beam and UHPC jacket as in the adopted calculation method is not justified. Although adopting different angles for the truss models of the transverse and fibre reinforcement is consistent with the approach adopted in The Concrete Society Technical Report No. 55, this assumption is inconsistent with the observed failure cracks in the beams of this campaign (The Concrete Society, 2012). The crack depth measurements and observations following the partial jacket removal after failure discussed in Section 4.6 suggest a shared truss angle for the transverse and fibre reinforcement truss models may be more appropriate.

An alternative method for predicting the shear strength of the jacketed beams of this research was evaluated to address the previously mentioned inconsistencies between the combined EC2, MASM and SIA 2052 method and the experimental observations. The aim of this evaluation is to recommend the use of a safe design method for predicting the ultimate shear strength for RC beams with UHPC jackets. The proposed method involves adding the shear strength of the transverse reinforcement given by EC2 to that of the fibre reinforcement given by SIA 2052, with both truss models having a shared angle equal to 30° . By increasing the truss angle of the transverse reinforcement from the minimum prescribed angle given by EC2 of 21.8° to the same angle as the UHPC jacket of 30° , the ultimate shear strength of the RC beam is reduced. Therefore, the reduced contribution of the RC beam is added to the upper bound contribution of the UHPC jacket. Another approach would be to add a reduced jacket contribution to the

strength of the RC control beam, however this is more complex since it requires an estimate of crack width in the UHPC jacket at failure to define the residual tensile strength. The contribution of the flange outstands given by the MASM was not considered in the proposed method when determining the shear resistance of the RC beam. The contribution of the flange outstands was originally considered in combination with EC2 to account for the shear resistance provided by the flange outstands that the design codes neglect. However, it is unconventional in design codes to combine design methods with different basis together such as EC2 and the MASM. The proposed method considered the following equation to predict the ultimate shear strength of a RC beam with a UHPC jacket:

$$V_{calc} = V_{R,s} + V_{R,f} = A_{sw} f_{ywm} z \cot \theta / s + A_{fv} \sigma_{res} \cot \theta \quad (7.30)$$

where $V_{R,s}$ is the shear resistance contribution of the transverse reinforcement in the beam and jacket where available, $V_{R,f}$ is the shear resistance contribution of the fibre reinforcement in the jacket, A_{sw} is the area of transverse reinforcement, f_{ywm} is the mean yield strength of the transverse reinforcement, z is the shear lever arm taken as $0.9d$, s is the spacing of the transverse reinforcement, σ_{res} is the residual tensile strength of UHPC calculated using Eqn. 7.1 and θ is the truss angle of the transverse and fibre reinforcement taken as 30° . The area of the jacket cross-section over which fibre reinforcement contributes to shear resistance (A_{fv}) is equal to $2t_s z$, where t_s refers to the side thickness of the jacket and z is the shear lever arm that is equal $0.9d$ for jackets with longitudinal reinforcement or $7h/8$ for jackets without longitudinal reinforcement (AFNOR, 2016b). The effective depth (d) and height (h) of the jacket are considered when determining the shear lever arm (z) of the jacket. The shear resistance contribution of the transverse reinforcement ($V_{R,s}$) is limited by the shear strength of the concrete web ($V_{Rd,max}$) calculated using Eqn. 2.138 according to EC2 (BSI, 2014). The shear resistance contribution of fibre reinforcement ($V_{R,f}$), which is referred to as $V_{Rd,U}$ in SIA 2052, is also limited by the shear strength of the UHPC web as it must satisfy the condition given in Eqn. 2.186 according to SIA 2052 (EPFL, 2016).

The calculated shear strength of each strengthened specimen based on the proposed and original combined EC2, MASM and SIA 2052 methods were compared with the experimental shear strength, and the results are presented in Table 7.10. Control specimens B1 and B5 were reduced from ratios (V_{calc}/V_{exp}) of 0.83 and 1.15 to 0.57 and 0.79, respectively, when comparing the calculated shear strengths given by the originally adopted and proposed design

methods due to neglecting the contribution of the flange outstands. More importantly, the predicted shear strengths given by the proposed method are safe for all of the jacketed beams as they are less than the experimental shear strengths. The predicted shear strengths given by the proposed method are more conservative for the jacketed beams of group 1 than those of group 2. Additionally, the predicted shear strengths given by the proposed method are more conservative for the specimens with thinner jackets as in specimens B2 and B6 than those with thicker jackets as in specimens B3, B4, B7 and B8. Figure 7.15 graphically compares the calculated shear strengths given by the proposed method with experimental shear strengths for the control and strengthened specimens of groups 1 and 2. Overall, the recommended design method which uses the equations of EC2 and SIA 2052 for the transverse and fibre reinforcement contributions, respectively, along with a shared truss angle of 30° gives safe ultimate shear strength predictions for the beams of this research.

Table 7.10 – Experimental and calculated ultimate shear and flexural strength of beam specimens of groups 1 and 2 according to original and proposed shear design methods

Specimen	Experimental		Calculated (Original) ^A			Calculated (Proposed) ^B			Calculated
	Ult. shear force (V_{exp}) [kN]	Fail. mode ^C	Ultimate shear strength		Fail. mode ^C	Ultimate shear strength		Fail. mode ^C	Ult. flexural strength ^D (V_{calc}/V_{exp})
			(V_{calc}) [kN]	(V_{calc}/V_{exp})		(V_{calc}) [kN]	(V_{calc}/V_{exp})		
B1	214.96	S	179.31	0.83	S	122.19	0.57	S	1.04
B2	337.84	S	297.61	0.88	S	203.66	0.60	S	1.30
B3	354.39	S	357.11	1.01	S	263.17	0.74	S	1.26
B4	375.66	S	416.04	1.11	S	322.67	0.86	S	1.20
B5	134.17	F-S	153.79	1.15	F	105.53	0.79	S	0.96
B6	263.07	S	271.05	1.03	S	192.12	0.73	S	1.18
B7	273.45	S	330.56	1.21	S	251.63	0.92	S	1.33
B8	317.69	S	388.19	1.22	S	311.14	0.98	S	1.31
Average (Standard deviation)			-	1.06 (0.15)	-	-	0.77 (0.14)	-	1.20 (0.13)

^A Original combined EC2, MASM and SIA 2052 method with different truss angles for the transverse and fibre reinforcement truss models (BSI, 2014; Cladera et al., 2016; EPFL, 2016; Mari et al., 2015, 2016); ^B Proposed combined EC2 and SIA 2052 method with the same truss angle for the transverse and fibre reinforcement truss models, and control RC beams calculated using EC2 with minimum truss angle (BSI, 2014; EPFL, 2016); ^C F and S refer to flexural and shear failure modes, respectively; ^D Shear force (V_{calc}) equals to the calculated ultimate flexural strength (M_{calc}) divided by the shear span (a_v)

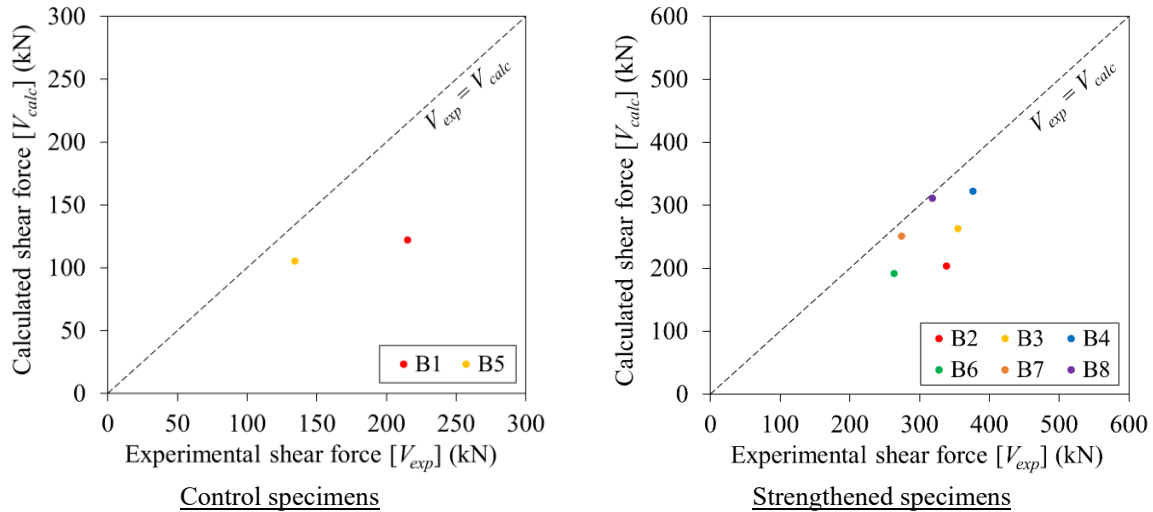


Figure 7.15 – Experimental and calculated ultimate shear strength of the specimens of groups 1 and 2 according to proposed shear design method

7.4 PREDICTIONS FOR BEAM SPECIMENS FROM THE LITERATURE

7.4.1 INTRODUCTION

A database was developed for bending test results from the literature related to RC beams with UHPC jackets (Alkhoury, 2017; Al-Osta et al., 2017; Bahraq et al., 2019; Choi, 2016; Daugevičius et al., 2015; Hong and Lim, 2023; Liu and Charron, 2023; Martinola et al. 2010; Paschalis, 2017; Wang et al., 2013). The purpose of the database was to further assess the RC and UHPC design code equations by comparing the measured and predicted shear and flexural strengths. Key data such as specimen geometry, reinforcement details, mechanical properties and loading arrangement were collected in order to calculate the ultimate shear and flexural strength for each specimen in the database using the methods described in Section 7.2. A general description of each specimen in the test database is given in Table B.1, while Table B.2 compares the measured and predicted shear and flexural strengths. The test database contains a total of 14 unstrengthened rectangular or T-shaped RC beams with transverse reinforcement and 36 RC beams strengthened with either a side or U-shaped UHPC jacket as shown in Figure 2.27. The specimens were loaded in either three- or four-point bending until failure. Beam specimens with a shear span to effective depth ratio (a_v/d) of less than 2.0 were not considered as the strength of these beams is increased by arching action. Some test results from the literature were omitted from the test database due to insufficient data. However, in some cases

beams were included if sufficient information was provided to allow their strength to be calculated with a few consistent and rational assumptions.

7.4.2 TEST DATA AND ASSUMPTIONS

The data collected for each specimen in the database includes the geometry, reinforcement details and material mechanical properties as presented in Table B.1. The rectangular RC beams ranged from 1120 mm to 4550 mm in length, 100 mm to 300 mm in width and 200 mm to 500 mm in height. The T-beams ranged from 3000 mm to 3300 mm in length, 200 mm to 250 mm in web width, 450 mm to 500 mm in flange width, 100 mm to 150 mm in flange height and 400 mm in total height. The UHPC jackets ranged from 7 mm to 60 mm in side thickness and 20 mm to 60 mm in bottom thickness, and normally extended over the complete length of the beam. For the strengthened specimens of Alkhoury (2017) and Wang et al. (2013), the jacket configurations were not uniform over the entire length of the beam as highlighted in Figure 2.28. Therefore, cross-sections A-A and B-B shown in Figure 2.28 were considered when calculating the ultimate shear and flexural strength, respectively. The compressive cylinder strength of NSC and UHPC ranged between 17.1 MPa to 65.0 MPa and 112.9 MPa to 210.0 MPa, respectively, and the direct post-cracking tensile strength of UHPC ranged between 5.0 MPa to 12.7 MPa. The ratios of the longitudinal and shear reinforcement in the beams ranged between 0.29% to 5.52% and 0.11% to 1.44%, respectively. The jacketed specimens of Alkhoury (2017) were the only specimens that featured embedded longitudinal and transverse reinforcement in the jackets, and their details are provided in Table B.1. The shear span to depth ratios (a_v/d) of the bending tests ranged from a minimum of 2.0 to a maximum of 5.2.

When insufficient data were provided, assumptions were necessary to calculate the ultimate shear and flexural strength of the specimens in the database. These assumptions mainly related to the geometry of the specimen and mechanical properties of the steel reinforcement and concrete as described below. The assumptions were as follows:

- The concrete cover of the longitudinal reinforcement in tension and compression was equal to 25 mm in order to determine the depth to the longitudinal reinforcement if not given;

- The vertical spacing between longitudinal reinforcement bars was equal to 25 mm in order to determine the depth to the longitudinal reinforcement if not given;
- The yield strength of the longitudinal and transverse reinforcement (f_{ym}) was equal to a typical value of 500 MPa if not given;
- The elastic modulus of the longitudinal and transverse reinforcement (E_{sm}) was equal to a typical value of 200 GPa if not given;
- The compressive strength of NSC and UHPC was equal to the compressive cylinder strength (f_{cm}) if not specified;
- The compressive cylinder strength of NSC (f_{cm}) was equal to 0.8 times the reported compressive cube strength ($f_{cm,cu}$) if not given (BSI, 2014);
- The compressive cylinder strength of UHPC (f_{cm}) was equal to the reported compressive cube strength ($f_{cm,cu}$) minus 15 MPa if not given (AFNOR, 2016a);
- The direct tensile strength of NSC (f_{ctm}) was equal to the splitting tensile strength ($f_{ctm,sp}$) if given, otherwise it was calculated as follows according to EC2 (BSI, 2014):

$$f_{ctm} = \begin{cases} 0.30 \cdot f_{cm}^{2/3} & \text{for } f_{cm} \leq \text{C50/60} \\ 2.12 \cdot \ln(1 + (f_{cm}/10)) & \text{for } f_{cm} > \text{C50/60} \end{cases} \quad (7.31)$$

- The direct tensile strength at the elastic limit of UHPC ($f_{ctm,el}$) was equal to the reported direct tensile strength (f_{ctm}) if not given;
- The elastic modulus of NSC (E_{cm}) was calculated using Eqn. 6.8 as suggested by EC2 if not given (BSI, 2014);
- The elastic modulus of UHPC (E_{cm}) was equal to a typical value of 45 GPa if not given (Fehling et al., 2014);
- The fracture energy of NSC (G_f) was calculated using Eqn. 2.165 as suggested by fib MC 2010 if not given (fib, 2013).

7.4.3 EVALUATION OF ULTIMATE SHEAR STRENGTH FOR DATABASE

7.4.3.1 CONTROL RC BEAMS IN DATABASE

The ultimate shear strength of the control RC beams in the database that failed in shear was calculated using the equations of the various design codes and models described in Section 7.2.2. Only six of the RC control beams failed in shear. These comprised three rectangular beams and three T-beams (Alkhoury, 2017; Bahraq et al., 2019; Choi, 2016; Hong and Lim, 2023; Liu and Charron, 2023; Wang et al., 2013).

Table 7.11 presents the ratio of calculated to experimental shear strength (P_{calc}/P_{exp}) for the control RC beams in the database that reportedly failed in shear. EC2 slightly overestimates the shear strength of the rectangular beams with an average ratio (P_{calc}/P_{exp}) for all beams equal to 1.17. The standard deviation is high at 0.64. The strength predictions given by fib MC 2010 LoAIII and ACI 318-19 are more conservative with average ratios (P_{calc}/P_{exp}) equal to 0.85 and 0.82, respectively, both with relatively low standard deviations. The design code predictions are reasonable for the T-beams, with EC2 most accurate and ACI 318-19 most conservative as shown in Table 7.11. The shear strength predictions given by the MASM match well with the test results of the rectangular beams, but significantly overpredict the shear strength of the T-beams for which the ratio (P_{calc}/P_{exp}) equals 1.52 with a relatively high standard deviation of 0.31. The overestimation of the MASM appears to be due to the overestimation of the flange contribution. When the contribution flange outstands calculated with the MASM is included in the EC2 shear strength predictions for T-beams, the ratio (P_{calc}/P_{exp}) equals 1.31 with a high standard deviation of 0.61.

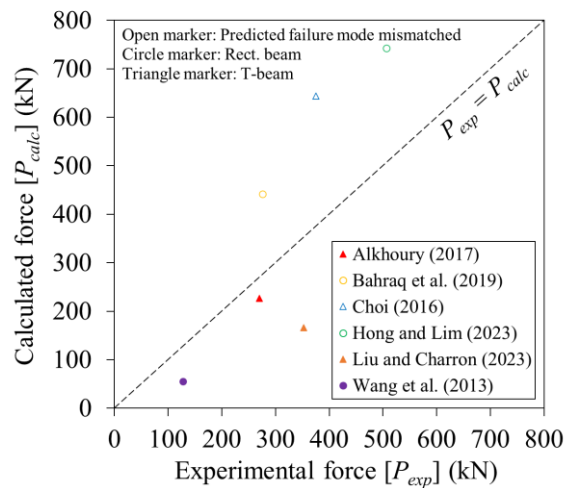
Figure 7.16 compares the experimental and calculated shear strengths for the control beams that failed in shear. Shear failure was not predicted for the beams depicted with open markers in Figure 7.16 as the plotted shear strength is greater than the flexural strength. Overall, the calculated ultimate shear strengths agreed fairly well with the measured strengths although a number of beams had mismatched failure modes. Due to the small number of RC beams in the database it is not possible to draw any general conclusions about the relative accuracy of EC2, fib MC 2010 LoAIII, ACI 318-19 and the MASM. Previously, Sagaseta and Vollum (2011) showed that the accuracy of the EC2 design method depends on the shear reinforcement ratio.

The method tends to be overly conservative for high flexural reinforcement ratios combined with low shear reinforcement ratios but can be on the unsafe side for high shear reinforcement ratios. This is consistent with the significant underestimate of strength for specimen B1 of this campaign where the flange appears to have significantly contributed to shear resistance. In general, the procedure of adding the MASM flange contribution to the EC2 shear resistance is not justified.

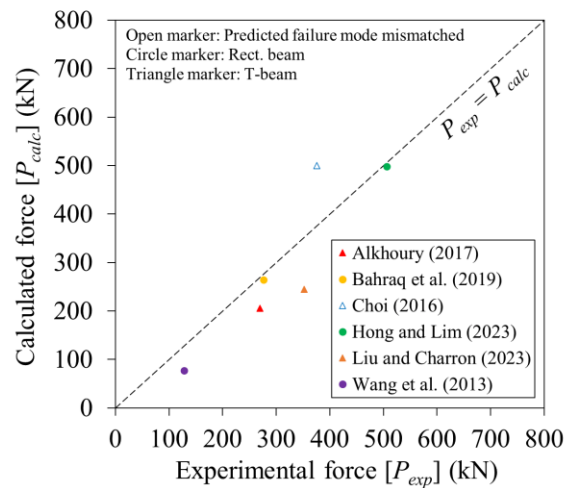
Table 7.11 – Experimental and calculated ultimate shear strength of control specimens in database that reportedly failed in shear

Beam cross-section	Calculated ultimate shear strength (P_{calc}/P_{exp})					Number of specimens
	EC2 ^A	fib MC 2010 ^B	ACI 318-19 ^C	MASM ^D	EC2 and MASME ^E	
Rectangular	1.17 (0.64)	0.85 (0.21)	0.82 (0.24)	1.07 (0.25)	1.17 (0.64)	3
T-shaped	1.01 (0.64)	0.93 (0.35)	0.82 (0.32)	1.52 (0.31)	1.31 (0.61)	3

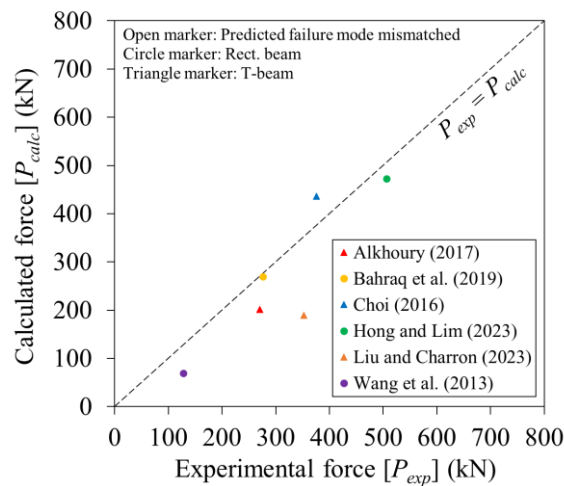
[Note] – () refers to standard deviation; ^A EC2 (BSI, 2014); ^B fib Model Code 2010 LoAIII (fib, 2013); ^C ACI 318-19 (ACI, 2019); ^D MASM (Cladera et al., 2016; Marí et al., 2015, 2016); ^E EC2 and MASM contribution of flange



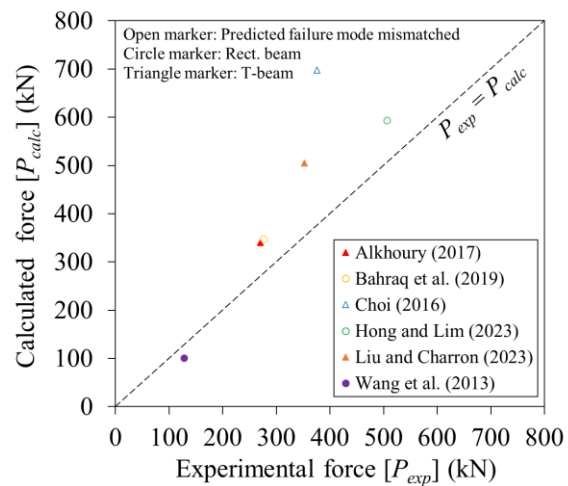
Eurocode 2



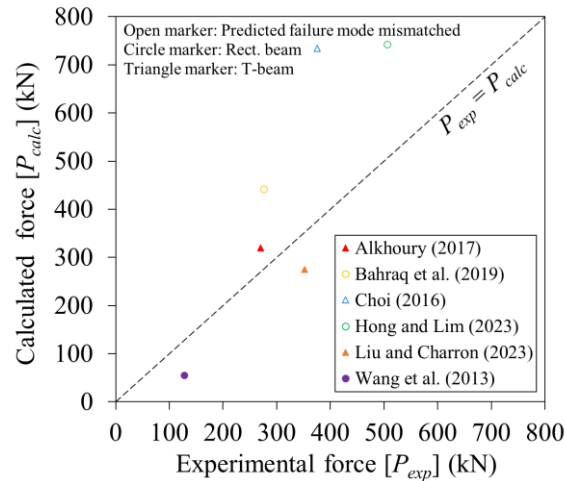
fib Model Code 2010 LoAIII



ACI 318-19



Multi-Action Shear Strength Model (MASM)



Eurocode 2 and Multi-Action Shear Strength Model (MASM)

Figure 7.16 – Experimental and calculated shear strength of control specimens in database that reportedly failed in shear

7.4.3.2 RC BEAMS WITH UHPC JACKETS IN DATABASE

Only eight of the strengthened specimens in the database failed in shear. These comprised three T-beams with side jackets and five T-beams with U-shaped jackets (Alkhoury, 2017; Liu and Charron, 2023). The measured and calculated UHPC jacket contributions to shear strength were compared. As previously, the experimental jacket contributions were assumed to equal the difference in shear strength between the jacketed and control specimens. The average comparison ratios (P_{calc}/P_{exp}) for the jacket contribution to shear strength are presented in Table 7.12. It is evident that the design codes significantly overestimate the jacket contributions at the ultimate load. The average comparison ratios (P_{calc}/P_{exp}) ranged between 2.22 and 2.70 along with excessively high standard deviations. The jacket contributions given by SIA 2052 are the least followed by the contributions given by JSCE 2006 and NF P18-710. This is due to SIA 2052 only considering the fibre reinforcement contribution to shear strength, while NF P18-710 and JSCE 2006 also consider the contribution of UHPC as explained in Section 7.3. Figure 7.17 graphically shows the large discrepancies between the experimental and calculated jacket contributions. It is notable that the jacket contribution is most severely overestimated for the jackets with the thickest side and bottom layers of Liu and Charron (2023). Additionally, Liu and Charron (2023) reported debonding in all of the T-beams with U-shaped jackets (TS-U25, TS-U50 and TS-U50A), which likely impacted their ultimate shear strengths with a single specimen (TS-U50) reportedly failing prematurely and this is discussed later in greater detail in Section 7.5.

The overestimation in the jacket contribution is a function of the adopted residual tensile stress at shear failure and the choice of truss angle. The latter was taken as 30° throughout as specified in SIA 2052 but could alternatively be taken as 45° for NF P18-710 as discussed in Section 7.2. The overestimate may have been due to taking the residual tensile strength of UHPC as that at the elastic limit, which was assumed to be equal to the peak tensile strength for the jackets of Alkhoury (2017) and Liu and Charron (2023). Additionally, the contribution of the jacket is likely to have peaked prior to the strengthened beams reaching their peak strength as found for the beams tested in this campaign.

The calculated ultimate shear strength of the strengthened specimens, rather than just jacket, were compared with the test results. The RC beam contribution to shear strength was calculated using the combined EC2 and MASM method for the beams of Alkhoury (2017) and Liu and Charron (2023) as this method provided the best fit to their respective control beams. The jacket contribution to shear strength was calculated using each considered UHPC design code. The average ratios (P_{calc}/P_{exp}) for the strengthened specimens are presented in Table 7.12. The calculated strengths are significantly overestimated with average ratios (P_{calc}/P_{exp}) ranging between 1.60 and 1.78 along with standard deviations between 0.44 and 0.49. The overestimations are mostly due to overestimate of the jacket contributions as previously discussed. The calculated shear strength given by EC2, MASM and SIA 2052 are the closest to the test results, followed by the shear strength given EC2, MASM and JSCE 2006 along with EC2, MASM and NF P18-710. Figure 7.17 shows the large discrepancies between the experimental and calculated shear strength given by the various UHPC design codes, and the predicted failure modes that did not match that of the tests are indicated with open markers.

Table 7.12 – Experimental and calculated ultimate shear strength of strengthened specimens in database that reportedly failed in shear

Beam cross-section	Jacket configuration	Calculated ultimate shear strength (P_{calc}/P_{exp})						Number of specimens
		NF P18-710 ^A		SIA 2052 ^B		JSCE 2006 ^C		
		Total ^D	Jacket ^E	Total ^D	Jacket ^E	Total ^D	Jacket ^E	
T-shaped	Side	1.79 (0.44)	2.70 (0.80)	1.63 (0.39)	2.39 (0.71)	1.68 (0.41)	2.49 (0.74)	3
T-shaped	U-shaped	1.78 (0.49)	2.55 (1.20)	1.60 (0.46)	2.22 (1.11)	1.69 (0.44)	2.38 (1.08)	5

[Note] – () refers to standard deviation; ^A NF P18-710 (AFNOR, 2016b); ^B SIA 2052 (EPFL, 2016); ^C JSCE 2006 (JSCE, 2006); ^D Calculated ultimate shear strength of jacketed specimens includes contribution of RC beam given by EC2 with contribution of flange given by MASM; ^E Calculated ultimate shear strength of jacket compared with experimental shear strength of jacket, which equals to difference in strength between the jacketed beam and control beam

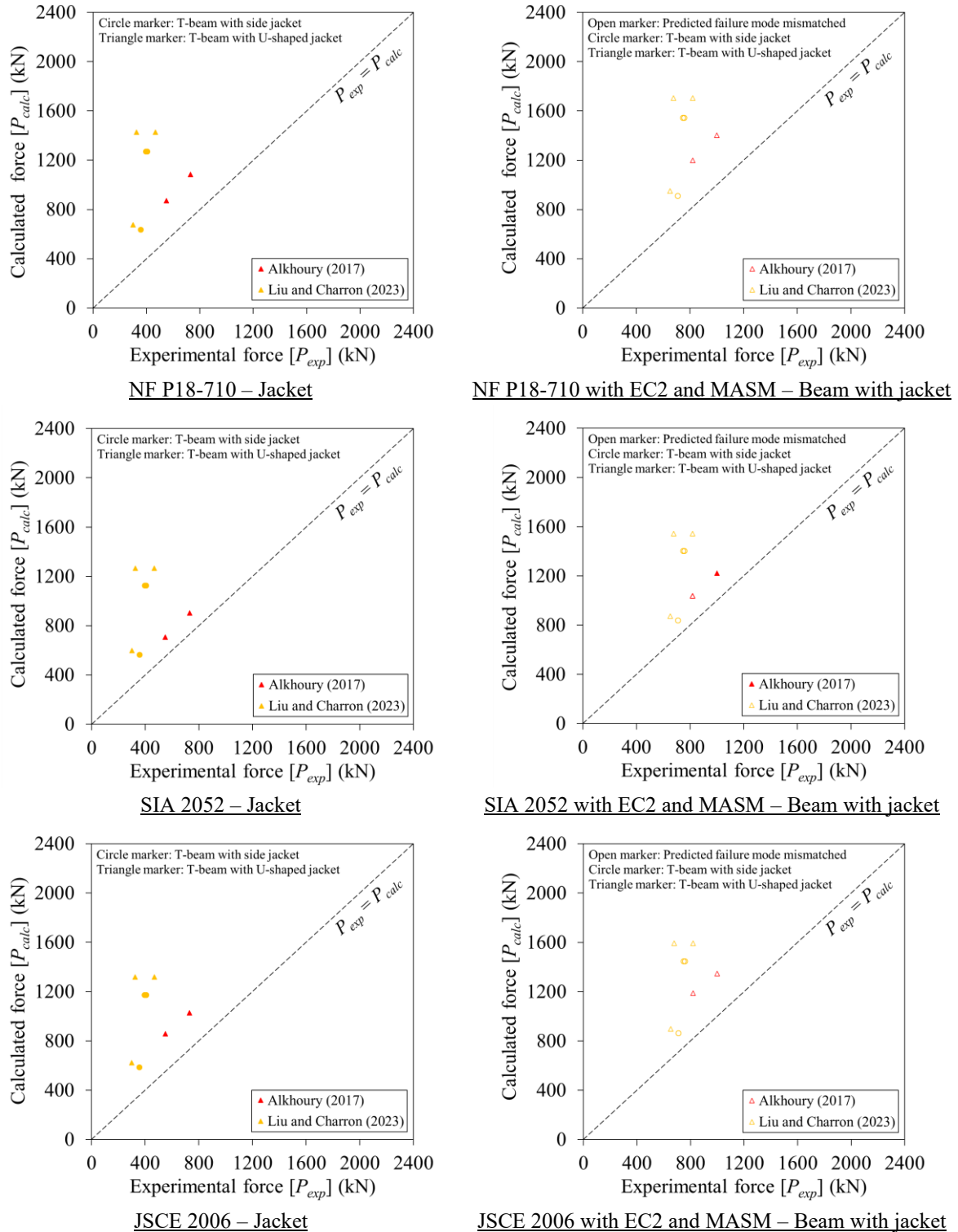


Figure 7.17 – Comparison between experimental and calculated ultimate shear strength of strengthened specimens in database that reportedly failed in shear

7.4.4 EVALUATION OF ULTIMATE FLEXURAL STRENGTH FOR DATABASE

The ultimate flexural strengths of the control and strengthened specimens in the test database were calculated using the cross-section analysis procedures described in Section 7.2.3. The calculated flexural strengths matched reasonably well with the test results of the control and

strengthened specimens in the database that failed in flexure. There were a total of 8 control RC beams with rectangular cross-sections and 28 RC beams with either side or U-shaped UHPC jackets that failed in flexure (Al-Osta et al., 2017; Bahraq et al., 2019; Choi, 2016; Daugevičius et al., 2015; Hong and Lim, 2023; Martinola et al. 2010; Paschalis, 2017; Wang et al., 2013).

Table 7.13 presents the ratios of calculated to experimental ultimate flexural strength (P_{calc}/P_{exp}) for the control and strengthened specimens in the test database that reportedly failed in flexure. It is evident that the calculations match relatively well with the ultimate flexural strength on average for the control RC beams and rectangular RC beams with U-shaped jackets as the comparison ratios (P_{calc}/P_{exp}) are equal to 0.90 and 1.09, respectively, with relatively low standard deviations. The rectangular RC beams with side jackets and T-beams with U-shaped jackets are marginally overpredicted as their comparison ratios (P_{calc}/P_{exp}) are equal to 1.17 and 1.28, respectively. Flexural failure is predicted accurately for all beam specimens that failed in flexure in the test database according to the calculations. Figure 7.18 compares the experimental and calculated flexural strength for all control and strengthened beam specimens in the test database that failed in flexure. It is evident that the ultimate flexural strengths calculated using the cross-section analysis procedures described in Section 7.2.3 match the test results relatively well overall despite a few strengthened specimens that were overpredicted, primarily those of Choi (2016) and Hong and Lim (2023). Relatively accurate predictions were achieved as the flexural strength is generally dictated by the yield strength of the longitudinal reinforcement. The minor discrepancies between the calculated and test results may be related to assuming a yield strength of 500 MPa for the longitudinal reinforcement when it was not reported, or the adoption of oversimplified material stress-strain relationships for NSC and UHPC in the cross-section analysis.

Table 7.13 – Experimental and calculated ultimate flexural strength of specimens in database that reportedly failed in flexure

Specimen type	Beam cross-section	Jacket configuration	Calc. flexural strength (P_{calc}/P_{exp})	Number of specimens
Control	Rectangular	-	0.90 (0.13)	8
Strengthened	Rectangular	Side	1.17 (0.12)	10
	Rectangular	U-shaped	1.09 (0.18)	16
	T-shaped	U-shaped	1.28 (0.12)	2

[Note] – () refers to standard deviation

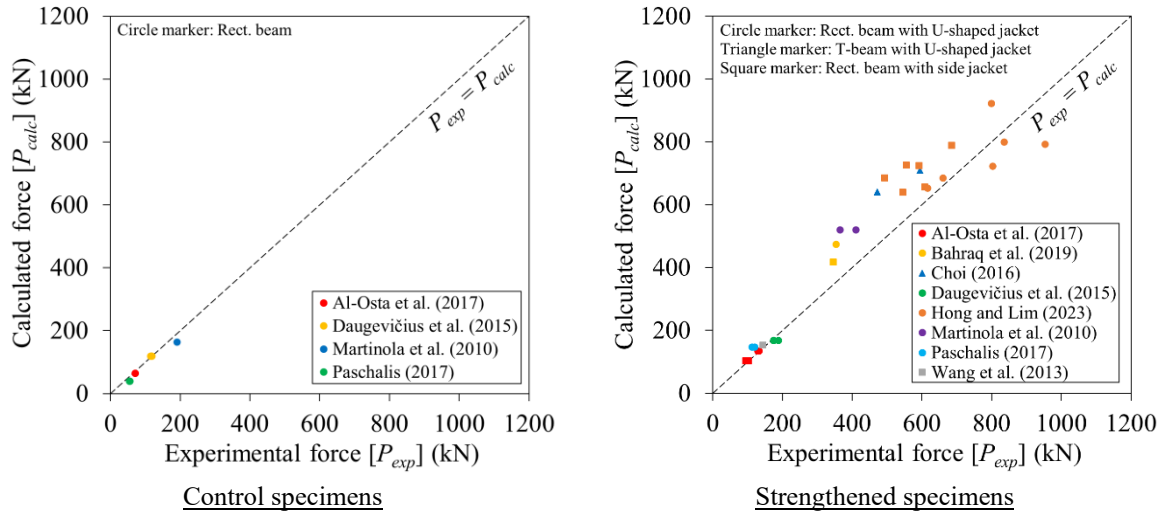


Figure 7.18 – Experimental and calculated ultimate flexural strength of specimens in database that reportedly failed in flexure

7.5 KEY FINDINGS AND SHEAR DESIGN RECOMMENDATIONS

The best estimates of the shear strength of the RC control beams in the database where the jacketed beams failed in shear are given by the combined EC2 and MASM method. The predicted shear strength of the jacketed beams in these series are overestimated when the full calculated jacket strength is added to that of the RC beam (Alkhoury, 2017 and Liu and Charron, 2023). The overestimate, which also applies to the beams of this campaign, arises since straight superposition of the RC beam and jacket strengths assumes that the full resistance of each is reached simultaneously. Analysis of the shear resisting mechanisms in Chapter 5 shows that this is not the case. The overestimate also arises from the adoption of the minimum allowable truss angle of 21.8° for the RC beam simultaneously with the minimum truss angle of 30° suggested by the UHPC design codes (AFNOR, 2016b; EPFL, 2016; JSCE, 2006).

To address this, Section 7.3.5 suggests adopting a truss angle of 30° for both the RC beam and UHPC jacket. This approach was also applied to the jacketed beams in the test database in order to investigate its effectiveness. In this approach, the contribution of the jacket is calculated using SIA 2052. The MASM contribution of the flange outstands is not considered in the proposed method since it is not generally applicable to RC beams where the accuracy of the EC2 design method for shear varies with the flexural reinforcement ratio (Sagaseta and Vollum, 2011).

The calculated shear strengths based on the proposed and original combined EC2, MASM and SIA 2052 method were compared with the experimental shear strengths for the specimens of Alkhoury (2017) and Liu and Charron (2023). The results are presented in Table 7.14. The comparison ratios (P_{calc}/P_{exp}) of the control specimens of Alkhoury (2017) (Nr. 5) and Liu and Charron (2023) (TS-R) were reduced from 1.19 and 0.78 to 0.84 and 0.47, respectively, when using the EC2 alone due to the exclusion of the flange outstand contribution. The predicted shear strengths given by the proposed method for the jacketed beams of Alkhoury (2017) are much closer to the test results with ratios (P_{calc}/P_{exp}) of 1.05 and 1.06 for specimens Nr. 6 and Nr. 7. The minor overestimate may result from the assumed direct tensile strength of UHPC. The predicted shear strengths given by the proposed method for the jacketed beams of Liu and Charron (2023) remain significantly overestimated especially for the specimens with the thickest jackets as in TS-W50, TS-W50A, TS-U50 and TS-U50A. No longitudinal reinforcement bars were placed in the jackets of these beams, which are predicted to fail in flexure rather than shear as reported, although anchors were fixed onto the sides of the jacket that reportedly increased the ductility of the failure mode. It should be noted that the bottom layer of the specimens with U-shaped jackets was stopped 200 mm short of the supports since the aim was to examine realistic strengthening configurations involving the possibility of jacket delamination at the extremities (Liu and Charron, 2023). This may have adversely affected the bond between the beams and the jackets. Beam TS-U50, in which the stirrups did not yield, is reported to have failed prematurely with a combination of separation and shear cracks. The stirrups yielded in TS-U25 and TS-U50A which is reported as indicating shear failure. However, it should be noted that the shear resistance of RC beams can increase significantly after first yield of the stirrups due to rotation of the shear struts at failure as observed in Chapter 5 for the beams of this campaign. Some debonding of the jacket appears to have occurred in specimens TS-U25 and TS-U50A. The use of the direct tensile strength of UHPC in the calculation of the jacket contribution is likely to have also contributed to the overestimations in shear strength. Liu and Charron (2023) shows that the post-cracking tensile response of UHPC softens immediately after cracking, which suggests the tensile strength at the elastic limit and at peak are not equal as assumed in the calculations.

Figure 7.19 graphically compares the calculated shear strengths given by the proposed method with experimental shear strengths for the control and strengthened beams of Alkhoury (2017) and Liu and Charron (2023). Table B.3 presents the comparisons between the shear strength

predictions based on the proposed and original shear design methods with test results for all of the specimens in test database. Table 7.15 compares the predicted ultimate strength based on the proposed and original methods with the experimental strengths for the specimens of Alkhoury (2017) and Liu and Charron (2023), where the predicted ultimate strength was taken as the least of the calculated shear and flexural strengths. The average comparison ratio (P_{calc}/P_{exp}) of 1.02 given by the proposed method is less than that given by the original method which is equal to 1.16 although the standard deviation is slightly higher. Overall, the proposed method appears to give reasonable estimates of the ultimate load of the jacketed beams in Table 7.15, but the correct failure mode is not always correctly predicted. There is clearly a need for more experimental testing to develop a definitive design method for UHPC strengthened beams.

Table 7.14 – Experimental and calculated ultimate shear and flexural strength of specimens of Alkhoury (2017) and Liu and Charron (2023) according to original and proposed shear design methods

Author	Specimen		Experimental		Calculated (Original) ^A		Calculated (Proposed) ^B		Calculated
	ID	Type ^C	Ult. force (P_{exp}) [kN]	Fail. mode ^D	Ult. shear strength (P_{calc}/P_{exp})	Fail. mode ^D	Ult. shear strength (P_{calc}/P_{exp})	Fail. mode ^D	Ult. flexural strength ^E (P_{calc}/P_{exp})
Alkhoury (2017)	Nr. 5	CON	270.00	S	1.19	S	0.84	S	1.26
	Nr. 6	UJ	820.00	S	1.26	F	1.05	S	1.24
	Nr. 7	UJ	1000.00	S	1.22	S	1.06	S	1.36
Liu and Charron (2023)	TS-R	CON	352.10	S	0.78	S	0.47	S	2.14
	TS-W25	SJ	710.00	S	1.18	F	0.95	S	1.12
	TS-W50	SJ	748.50	S	1.87	F	1.66	F	1.11
	TS-W50A	SJ	759.10	F-S	1.85	F	1.63	F	1.10
	TS-U25	UJ	652.20	S	1.34	F	1.09	S	1.29
	TS-U50	UJ	677.40	S	2.28	F	2.04	F	1.38
	TS-U50A	UJ	819.60	S	1.88	F	1.69	F	1.14

^A Original combined EC2, MASM and SIA 2052 method with different truss angles for the transverse and fibre reinforcement truss models (BSI, 2014; Cladera et al., 2016; EPFL, 2016; Mari et al., 2015, 2016); ^B Proposed combined EC2 and SIA 2052 method with the same truss angle for the transverse and fibre reinforcement truss models, and control RC beams calculated using EC2 with minimum truss angle (BSI, 2014; EPFL, 2016); ^C CON, SJ and UJ refer to control RC beams and RC beams with side and U-shaped jackets, respectively; ^D F and S refer to flexural and shear failure modes, respectively; ^E Shear force (V_{calc}) equals to the calculated ultimate flexural strength (M_{calc}) divided by the shear span (a_v)

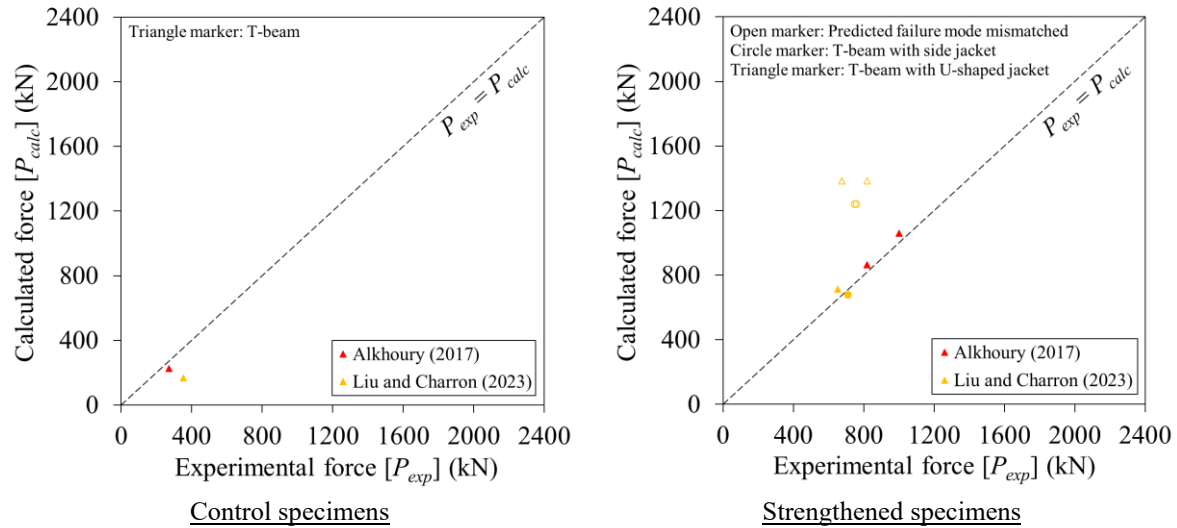


Figure 7.19 – Experimental and calculated ultimate shear strength of specimens of Alkhoury (2017) and Liu and Charron (2023) according to proposed shear design method

Table 7.15 – Experimental and calculated ultimate strength of specimens of Alkhoury (2017) and Liu and Charron (2023) according to original and proposed shear design methods

Author	Specimen		Experimental		Calculated (Original) ^A		Calculated (Proposed) ^B	
	ID	Type ^C	Ultimate force (P_{exp}) [kN]	Failure mode	Ultimate strength (P_{calc}/P_{exp})	Failure mode	Ultimate strength (P_{calc}/P_{exp})	Failure mode
Alkhoury (2017)	Nr. 5	CON	270.00	Shear	1.19	Shear	0.84	Shear
	Nr. 6	UJ	820.00	Shear	1.24	Flexure	1.05	Shear
	Nr. 7	UJ	1000.00	Shear	1.22	Shear	1.06	Shear
Liu and Charron (2023)	TS-R	CON	352.10	Shear	0.78	Shear	0.47	Shear
	TS-W25	SJ	710.00	Shear	1.12	Flexure	0.95	Shear
	TS-W50	SJ	748.50	Shear	1.11	Flexure	1.11	Flexure
	TS-W50A	SJ	759.10	Flexure-Shear	1.10	Flexure	1.10	Flexure
	TS-U25	UJ	652.20	Shear	1.29	Flexure	1.09	Shear
	TS-U50	UJ	677.40	Shear	1.38	Flexure	1.38	Flexure
	TS-U50A	UJ	819.60	Shear	1.14	Flexure	1.14	Flexure
Average (Standard deviation)					1.16 (0.16)	-	1.02 (0.24)	-

^A Original combined EC2, MASM and SIA 2052 method with different truss angles for the transverse and fibre reinforcement truss models (BSI, 2014; Cladera et al., 2016; EPFL, 2016; Mari et al., 2015, 2016); ^B Proposed combined EC2 and SIA 2052 method with the same truss angle for the transverse and fibre reinforcement truss models, and control RC beams calculated using EC2 with minimum truss angle (BSI, 2014; EPFL, 2016); ^C CON, SJ and UJ refer to control RC beams and RC beams with side and U-shaped jackets, respectively

7.6 CONCLUSIONS

This chapter compares the measured strengths of the beams tested in this research with the predictions of design codes and numerical analysis. There is no agreed method for calculating

the shear strength of RC beams with UHPC jackets (Liu and Charron, 2023). Superposition was used to calculate the shear strength of the jacketed beams. The complication here is that the peak resistances of the RC beam and jacket are not reached simultaneously as shown in Chapter 5. The strength of the RC beam was calculated with EC2, fib MC 2010 LoAIII, ACI 318-19 and the MASM (ACI, 2019; BSI, 2014; Cladera et al., 2016; fib, 2013; Marí et al., 2015, 2016). Unlike the other considered methods the MASM explicitly includes the contribution of the flange outstands to shear strength. The shear resistance of the UHPC jacket was estimated using the equations given by NF P18-710, SIA 2052 and JSCE 2006 (AFNOR, 2016b; EPFL, 2016; JSCE, 2006). The minimum truss angles prescribed in the considered RC and UHPC design codes were initially adopted in this analysis. Direct superposition of the individually calculated design strengths of the RC beam and UHPC jacket was found to overestimate shear strength particularly for thicker jackets. However, reasonable estimates of the peak jacket contribution, which develops prior to the ultimate load of the jacketed beam, were obtained with SIA 2052. Reasonably good estimates of flexural strength are obtained with cross-sectional analysis by combining the contributions of the jacket and longitudinal reinforcement. This is unsurprising since apart from making the reinforcement bars in the jacket act compositely with the beam, the jacket contributed relatively little to flexural strength of the beams tested in this campaign. Based on the assessment of the shear resisting mechanisms in Chapter 5, a design method was suggested for predicting the shear strength of RC beams with UHPC jackets. The method involves adopting a shared truss angle, not less than 30° , for both the RC beam and UHPC jacket. The shear resisting contributions of the RC beam and UHPC jacket are then calculated with EC2 and SIA 2052 (see Eqn. 7.30), respectively, neglecting the MASM flange contribution. A developed database of 36 RC beams with UHPC jackets from the literature was used to further validate the proposed methodology. Overall, the calculated capacities compare reasonably with the measured strengths though flexural failure is predicted to occur in a few cases where shear failure was reported.

Detailed conclusions of this chapter are as follows:

- The most accurate strength predictions of the beams tested in this campaign were obtained with NLFEA.
- The ultimate flexural strength predicted using the cross-section analysis procedures described in Section 7.2.3 show reasonably good agreement with the test results for the 28 strengthened beams in the test database.

- The considered design code equations tend to underestimate the shear strength of the RC control beams from this campaign and the test database. This appears to be due to the contribution of the flange not being considered explicitly. In test series where the jacketed beams failed in shear, the best estimate of the shear strength of the control RC beam is given by EC2 when enhanced by a contribution for the flange outstands calculated with the MASM.
- The increase in shear strength of the RC beam due to the UHPC jacket is overestimated by NF P18-710, SIA 2052 and JSCE 2006 for both the beams of this research and the database in Appendix B. The predictions of SIA 2052 are lowest since it does not include a contribution to shear strength from the UHPC unlike NF P18-710 and JSCE 2006.
- The degree of overestimation in the estimated UHPC jacket contribution to shear strength is dependent on its side thickness. For the jackets of this campaign, the jacket contribution was predicted to increase by 50% and 100% when increasing the side thickness from 20 mm to 30 mm and 40 mm, respectively. However, test results of the strengthened specimens of group 1 showed that increasing the side thickness from 20 mm (B2) to 30 mm (B3) and 40 mm (B4) increased the ultimate shear strength contribution of the jacket by only 13% and 31%, respectively. For the strengthened specimens of group 2, increasing the side thickness from 20 mm (B6) to 30 mm (B7) and 40 mm (B8) increased the shear strength contribution of the jacket by only 8% and 42%, respectively.
- The residual tensile strength of UHPC adopted in the UHPC design codes is about four times that experimentally derived along the critical shear crack at the ultimate load for the beams of this campaign. However, the peak jacket contributions determined in Chapter 5 are similar to those calculated with the code methods.
- The use of different truss angles for the transverse (21.8°) and fibre (30°) reinforcement is inconsistent with the observed failure cracks in this campaign. Observations following the partial jacket removal discussed in Section 4.6 suggest that a shared truss angle for the transverse and fibre reinforcement truss models is more appropriate.
- Based on the analysis of shear resisting mechanisms in Chapter 5, a more conservative method for predicting the ultimate shear strength of RC beams with UHPC jackets was proposed. The method involves adding the shear contributions of the transverse reinforcement given by EC2 to that of the fibre reinforcement given by SIA 2052 as in

Eqn. 7.30, with both truss models having a shared minimum truss angle of 30° . Theoretically, this approach is broadly equivalent to adding the RC beam contribution at peak jacket contribution to the peak jacket contribution. An alternative approach would be to add a reduced jacket contribution to the strength of the RC control beam. This approach is more complex since it requires an estimate of crack width in the UHPC jacket at failure to define the residual tensile strength.

- The proposed shear strength prediction method gives safe predictions for the beams of this research. The strengths for the jacketed beams in the test database are reasonably well estimated if the ultimate load is taken as the least of the estimated shear and flexure strengths. In the case of Liu and Charron (2023), the failure mode of some beams is predicted to be flexural rather than shear as reported. In these cases, the reported failure mode is based on yielding of the stirrups in the RC beam. This seems inconclusive since stirrups typically yield in RC beams well before failure with resistance increasing subsequently due to strut rotation. Hence, where the measured ultimate load is close to the estimated flexural capacity, yet shear failure is reported, the actual failure mode seems debatable. Further laboratory testing is required to refine the proposed design method but based on the available data it seems reasonable.

CHAPTER 8: CONCLUSIONS

8.1 OVERVIEW

This thesis investigates the strengthening of reinforced concrete (RC) beams in shear with ultra-high performance concrete (UHPC) jackets. UHPC is a cementitious material made up of fine granular constituent materials, high binder content and low water to cement ratio that normally includes high strength steel fibres that combine to give compressive and tensile strengths in excess of 150 MPa and 6 MPa, respectively. Several studies have investigated the behaviour of RC beams with UHPC jackets, but very little experimental work has focused on jacketed RC beams failing in shear as emphasis was more on flexural failure (Alkhoury, 2017; Al-Osta et al., 2017; Bahraq et al., 2019; Choi, 2016; Daugevičius et al., 2016; Hong and Lim, 2023; Liu and Charron, 2023; Martinola et al., 2010; Paschalis, 2017; Meda et al., 2012; Martins et al., 2021; Said et al., 2022; Sakr et al., 2019; Wang et al., 2013, 2015a, 2015b). At present, there is no agreed method for calculating the shear strength of RC beams with UHPC jackets (Liu and Charron, 2023). This research investigates the shear behaviour of RC T-beams with UHPC U-shaped jackets through laboratory testing and nonlinear finite element analysis (NLFEA). The contribution of each shear resisting mechanism is assessed in the tested beams from the kinematics of the critical shear cracks. The applicability of prediction equations from national and international RC and UHPC design codes and recommendations is evaluated using the results of the experimental program of this research along with those from the literature. Improvements to the available ultimate shear strength prediction equations for RC beams strengthened with UHPC jackets are assessed and suggested. Detailed conclusions relating to each chapter of the thesis are presented in this chapter along with suggestions for future research.

8.2 DETAILED CONCLUSIONS

8.2.1 LITERATURE REVIEW

The literature review describes the historical development of UHPC along with its general composition, production and characterisation of its material properties. The shear resistance mechanisms of reinforced concrete beams as well as UHPC beams are described, and various theoretical models from the literature are presented that can be used to estimate the contribution

of each shear resisting mechanism to the total shear resistance of a given beam. The shear design methods of national and international RC and UHPC design codes and recommendations are presented, with focus on Eurocode 2, fib Model Code 2010, ACI 318-19 and NF P18-710 amongst others (ACI, 2019; AFGC, 2013; AFNOR, 2016b; BSI, 2014; EPFL, 2016; fib, 2013; JSCE, 2006). Observations and findings from tested RC beams with UHPC jackets from the literature are discussed, with the gaps in knowledge highlighted.

The main conclusions from the literature review are as follows:

- Properly characterising the direct tensile strength of UHPC, which is a key design parameter, is essential. Direct tensile test methods can be used, but are sensitive to specimen shape and loading arrangement, which often result in premature failure near the constrained ends due to unwanted stress concentration (Benson and Karihaloo, 2005; Denarié et al., 2017; EPFL, 2016; Graybeal and Baby, 2013; Maltais et al., 2018; Shen and Brühwiler, 2018; Wille et al., 2014; Zhou and Qiao, 2018). Measuring the direct tensile strength of UHPC indirectly is often preferred. This involves performing bending tests on small-scale prism specimens and applying inverse analysis to determine the equivalent direct tensile strength (Baby et al., 2012; Denarié et al., 2017; Martínez, 2017). Annex D of NF P18-470 provides comprehensive procedures for determining the direct tensile strength of UHPC indirectly, including specimen production, loading test arrangements, monitoring techniques and an inverse analysis method (AFNOR, 2016a).
- Despite the lack of consensus on a theory describing shear behaviour, the research community agrees that there are up to six mechanisms that contribute to shear resistance in reinforced concrete beams (ASCE-ACI Committee 445, 1998). For RC beams with transverse reinforcement, the shear resistance mechanisms mainly include aggregate interlock, dowel action, shear reinforcement and the uncracked compression zone. Fibre bridging is the main shear resisting mechanism in diagonally cracked UHPC, where aggregate interlock does not significantly contribute to shear resistance due to the absence of coarse aggregate. Several theoretical models for each shear resistance mechanism are available in the literature, which can be used to estimate the contribution of each action to the total shear resistance. Researchers have successfully measured the contribution of each shear resistance mechanism in RC beams, but not

UHPC strengthened RC beams, by using the latest developments in non-contact optical monitoring techniques such as digital image correlation (DIC) (Campana et al., 2013; Huber et al., 2016; Pastore, 2020). The contributions of each shear resistance mechanism are based on the crack kinematics and shape of the critical shear crack as well as its location. Recently developed mechanical based models such as the Multi-Action Shear Strength Model (MASM) can also be used to assess shear strength according to all of its resisting mechanisms unlike design codes (Cladera et al., 2016; Mari et al., 2015, 2016). In addition to mechanical models, alternative methods can be used to assess the ultimate shear strength of RC beams such as lower-bound plasticity based stress field models and the strut-and-tie method (STM) as well as compatibility based methods such as the Modified Compression-Field Theory (MCFT) model.

- Reinforced concrete design codes typically define shear strength in terms of one or two resistance mechanisms rather than directly accounting for all shear resistance mechanisms (ASCE-ACI Committee 445, 1998). Design codes for RC beams typically use idealised truss models to represent the contribution of shear reinforcement to shear resistance. The truss model relies heavily on the adopted truss angle, which is a variable for EC2 and fib MC 2010 that ranges between a minimum value and 45° and it is fixed at 45° for ACI 318-19 (ACI, 2019; BSI, 2014; fib, 2013). Design codes such as fib MC 2010 and ACI 318-19 also consider the contribution of concrete to shear resistance (ACI, 2019; fib, 2013).
- UHPC design codes and recommendations assume that the shear strength is mainly resisted by the fibre reinforcement, which is calculated based on a truss model with a suggested minimum truss angle of 30° (AFGC, 2013; AFNOR, 2016b; EPFL, 2016; JSCE, 2006). According to AFGC 2013 and NF P18-710, the truss angle should be determined through an elastic calculation of the uncracked section (AFGC, 2013; AFNOR, 2016b). Therefore, the inclination angle is equal to 45° for UHPC beams without axial load or prestressed reinforcement along with UHPC beams with transverse reinforcement (AFGC, 2013; AFNOR, 2016b). UHPC design codes consider the contribution of the transverse reinforcement using the same truss model, although the equations given by JSCE 2006 do not consider the contribution of the transverse reinforcement (AFGC, 2013; AFNOR, 2016b; EPFL, 2016; JSCE, 2006). Design codes and recommendations such as AFGC 2013, NF P18-710 and JSCE 2006 also consider

the contribution of the UHPC matrix, although SIA 2052 does not (AFGC, 2013; AFNOR, 2016b; EPFL, 2016; JSCE, 2006).

- Several studies investigated the behaviour of RC beams with UHPC jackets. Test results showed increases in strength of up to 120% and 270% due to the addition of UHPC jackets without and with embedded reinforcement, respectively (Alkhoury, 2017; Paschalis, 2017). Very little experimental work focused on jacketed RC beams failing in shear as emphasis was more on flexural failure. Researchers have yet to develop a mechanical model for the shear design or assessment of a RC beam with UHPC jacket (Liu and Charron, 2023).

8.2.2 EXPERIMENTAL CAMPAIGN

The experimental program of this research focused primarily on the testing of two groups of four medium-scale RC T-beams strengthened with UHPC U-shaped jackets. Each group of beams consisted of one control RC beam and three identical RC beams with UHPC U-shaped jackets that varied in geometry and amount of embedded longitudinal reinforcement. The beams were designed to fail in shear to investigate the impact of the UHPC jackets on shear behaviour, with focus primarily on the effect of the jacket on each shear resistance mechanism, angle of the critical shear crack and shear strength enhancement through the side layers. The specimens were subjected to three-point bending tests with asymmetrical type monotonic loading until failure with a critical shear span to depth ratio (a_v/d) greater than 3.5. Each beam was fitted with linear variable displacement transducers, inclinometers, load cells and strain gauges. A digital image correlation (DIC) system by LaVision was also used to monitor the cracked surfaces of each specimen throughout the experiment (LaVision, 2019). The DIC results were used to calculate the kinematics of the critical shear cracks and hence the contributions of each shear resistance mechanism. The non-proprietary UHPC used to manufacture the jackets was developed by the author through trial mixing and testing. Composite cylinder specimens were also manufactured to characterise the interfacial bond between the normal strength concrete (NSC) and UHPC through splitting tensile tests in order to ensure that the bond capacity of the jacketed specimens was adequate.

The control RC beam of group 1 (B1) was reinforced with 3.34% flexural reinforcement. The critical shear span was reinforced with 6 mm diameter stainless steel stirrups at 250 mm

spacing. Group 2 consisted of a control RC beam (B5) with 1.55% flexural reinforcement, and the critical shear span was reinforced with 6 mm diameter stainless steel stirrups at 200 mm spacing. The three identical RC beams of groups 1 (B2, B3 and B4) and 2 (B6, B7 and B8) were strengthened with UHPC jackets with side thicknesses of 20 mm, 30 mm and 40 mm, respectively, and a bottom thickness of 60 mm. All of the jackets of group 1 were reinforced with $4\phi 20$ longitudinal reinforcement bars, and the reinforcement in the jackets of group 2 was $3\phi 20$ in specimen B6, $4\phi 20$ in specimen B7 and $5\phi 20$ in specimen B8.

The main conclusions from the experimental campaign of this research are as follows:

- The relatively high composite split cylinder tensile strength results and failure modes, which occurred along the interfacial bond and in the NSC substrate, indicate that the interfacial bond between the NSC and UHPC was at least as strong as the NSC.
- In group 1, the control RC beam (B1) and the strengthened beams (B2, B3 and B4) failed in shear.
- In group 2, the control RC beam (B5) failed in combined flexure-shear, while the jacketed beams (B6, B7 and B8) failed in shear.
- In group 1, the increase in the ultimate load due to jacketing was 57.16% (296.83 kN) for specimen B2 with a 20 mm thick jacket and 74.76% (379.86 kN) for specimen B4 with a 40 mm thick jacket.
- In group 2, the increase in the ultimate load due to jacketing was 96.06% (308.32 kN) for specimen B6 with a 20 mm jacket and 136.78% (448.66 kN) for specimen B8 with a 40 mm jacket.
- The development of the critical and secondary shear cracks in the strengthened specimens of groups 1 and 2 varied in the front and back face of the UHPC jacket throughout the test. However, the critical and secondary shear cracks mostly overlapped at the ultimate load.
- The critical shear cracks formed near the centre of the critical span in the specimens of group 1 apart from specimen B3 where the crack formed much closer to the support. The critical shear cracks of all the specimens of group 2 formed around the middle of the critical span.
- The critical shear crack was steepest at 34° in the control specimen of group 1 (B1). The crack inclination varied in the strengthened beams of group 1. The flattest crack

inclination angle of 19° was in specimen B2. The critical shear crack inclination angles were equal to 32° and 33° for the front and back FOVs of specimen B3, respectively. Due to the differences in crack development in both faces of the jacket of specimen B4, the critical shear crack inclination angles were equal to 31° and 21° for the front and back FOVs, respectively.

- The steepest critical shear crack inclination angle was equal to 38° for the control specimen of group 2 (B5). The crack inclination angles varied for the strengthened specimens of group 2 as in group 1. The flattest inclination angles were equal to 21° and 20° for the critical shear cracks in the front and back FOVs of specimen B8, respectively. The crack inclination angles were equal to 29° and 26° for the critical shear cracks in the front and back FOVs of specimen B6, respectively. The inclination angles were equal to 30° and 29° for the critical shear cracks in the front and back FOVs of specimen B7, respectively.
- After the tests, the depths of the critical shear cracks were measured, and some jackets were partially removed. It was concluded that the critical shear cracks on the jackets passed through to the beams, which indicates monolithic performance.

8.2.3 SHEAR RESISTANCE MECHANISMS

The contribution of each shear resisting mechanism to the overall shear resistance of the tested beams of this campaign was determined from crack kinematics obtained with DIC. The adopted method of calculation involved treating the investigated crack as two separate rigid bodies located on each side of crack with each rigid body behaving locally. The contributions of each shear resistance mechanism were estimated by substituting the calculated crack kinematics into various models from the literature. The shear resistance mechanisms that were considered include aggregate interlock, fibre bridging, dowel action, stirrups and the uncracked compression zone. The aggregate interlock and stirrup resistance mechanisms were attributed to the RC beam, and fibre bridging resistance to the UHPC jacket. Dowel action was assessed for both the RC beams and UHPC jackets as both included embedded longitudinal reinforcement in tension.

The main conclusions from the calculated crack kinematics and contributions of each shear resistance mechanism for the beams of this research are as follows:

- Significant differences were observed in the crack opening and sliding displacements measured in each face of the jackets of the strengthened specimens.
- Based on evaluations of the individual shear resistance mechanisms, the models that were finally adopted for assessment were the Rough Crack model of Gambarova and Karakoc (1983) for aggregate interlock, the experimental post-cracking stress response of UHPC for fibre bridging, the model of Baumann and Rüschi (1970) for dowel action, the model of Sigrist (1995) for stirrups and vertical equilibrium of the internal shear forces for the uncracked compression zone.
- The aggregate interlock contribution is linked to the transverse reinforcement ratio. The aggregate interlock contribution at the ultimate load is significantly lower for control specimen B1 (7.0%) with 6 mm diameter stirrups at 200 mm spacing than control specimen B5 (20.2%), with 6 mm diameter stirrups at 250 mm spacing.
- The aggregate interlock contribution is negligible throughout loading in the specimens with jackets. The aggregate interlock contribution at the ultimate load is less than 5.6% for all specimens with jackets.
- The dowel action shear force at the ultimate load increases due to the additional longitudinal reinforcement in the jacket and side thickness of the jacket.
- The uncracked compression zone contribution is more dominant in the specimens of group 1 than those of group 2, which may be related to the higher longitudinal reinforcement ratio in the RC beam of group 1.
- The stirrup contributions vary due to the shapes of the critical shear cracks and the number of stirrups intercepted by the critical shear crack.
- Fibre bridging was initially the dominant shear resisting mechanism in the jacketed specimens. However, it became less dominant towards the ultimate load as a result of the gradual widening of the critical shear crack. The peak fibre bridging contribution occurs before the ultimate load at around the load steps $0.5P_u$ to $0.8P_u$.
- The UHPC jacket increased shear resistance of the RC beam not only through fibre bridging but also through enhancing the contributions of dowel action, stirrups and the uncracked compression zone.

8.2.4 NUMERICAL MODELLING

Nonlinear finite element analysis (NLFEA) was carried out with the aim of developing a modelling procedure capable of simulating the response of the beams tested in this campaign. The NLFEA was carried out in the software package ATENA. The modelling approach was developed in an iterative trial and error process that involved calibrating key input parameters mainly related to element mesh size and concrete to predict the ultimate load and failure mode with reasonable accuracy in comparison to the experimental results. The NLFEA approach was validated using the experimental results of this research along with results from the literature of tested RC, UHPC and strengthened beams.

The combined fracture-plastic model offered in ATENA was adopted for both NSC and UHPC. The default *CC3DNonLinCementitious2* material model in ATENA used for NSC required the calibration of a few key parameters to accurately simulate the response of the considered beams. Shear strengths and failure modes obtained using the fixed smeared crack model were found to depend significantly on the adopted shear factor. Increasing the shear factor increases the shear strength of beams, with the optimum value being case dependent as it is based on the amount of shear reinforcement as found by Pastore (2020). A shear factor coefficient of 250 was adopted for the beams of group 1, and 50 for the beams of group 2. The compressive strength reduction factor limit was also investigated, and a value of 0.5 was adopted in this research as it provided the best fit between the numerical and experimental results.

The adopted material model for UHPC was *CC3DNonLinCementitious2User*. It required little calibration of key parameters as it allowed for the post-cracking compressive, tensile and degradation laws to be user-defined. The default post-cracking compressive and degradation laws required minimal calibration as they were mainly based on default values as recommended by Sajdlová (2016). The post-cracking tensile law adopted in ATENA was derived using the tensile stress-strain response given by NF P18-710 for thick UHPC members at the ultimate limit state, which is based on the experimentally obtained load-CMOD response (AFNOR, 2016b).

The numerical results of the tested beams of this research compared reasonably well overall with the experimental results. The ultimate shear strength predictions for the specimens of

group 1 were quite close to the experimental results as the comparison ratios (V_{num}/V_{exp}) were equal to 0.98 on average. For the specimens of group 2, the comparison ratios (V_{num}/V_{exp}) were equal to 1.07 on average. The numerical models replicated the brittle shear failure mode for all of the specimens of group 1 as well as the strengthened specimens of group 2. The combined flexure-shear failure mode of the control specimen of group 2 (B5) was also simulated with reasonable accuracy. The reinforcement responses and crack patterns at the ultimate load generally compared well with the test results.

8.2.5 CODE PREDICTIONS AND DESIGN RECOMMENDATIONS

The measured strengths of the beams tested in this research were compared with design code-based predictions and numerical analysis. In this study, the strength of the RC beam was calculated with EC2, fib MC 2010 LoAIII, ACI 318-19 and the MASM (ACI, 2019; BSI, 2014; Cladera et al., 2016; fib, 2013; Marí et al., 2015, 2016). Of these methods, only the MASM explicitly includes the contribution of the flange outstands to shear strength. The best estimate of the shear strength of the RC control beams of this study was obtained by combining the shear resistance from EC2 with the flange outstand contribution from the MASM. This approach was adopted for evaluating the test results but is not recommended for design as its accuracy is dependent on the provided shear and flexural reinforcement. The shear resistance of the UHPC jacket was estimated using the equations given by NF P18-710, SIA 2052 and JSCE 2006 (AFNOR, 2016b; EPFL, 2016; JSCE, 2006). The minimum truss angles prescribed in the considered UHPC design codes were initially adopted in this analysis. Reasonable estimates of the peak jacket contribution, which develops prior to the ultimate load of the jacketed beam, were obtained with SIA 2052. However, direct superposition of the individually calculated design strengths of the RC beam and UHPC jacket was found to overestimate shear strength, particularly for thicker jackets, since the peak resistances of the RC beam and UHPC jacket do not occur simultaneously. Reasonably good estimates of flexural strength are obtained with cross-sectional analysis by combining the contributions of the jacket and longitudinal reinforcement. This is unsurprising since apart from making the reinforcement bars in the jacket act compositely with the beam, the jacket contributed relatively little to the flexural strength of the beams tested in this campaign. Based on the assessment of the shear resisting mechanisms, a design method was suggested for predicting the shear strength of RC beams with UHPC jackets. The method involves adopting a shared truss angle, not less than

30°, for both the RC beam and UHPC jacket. The shear resisting contributions of the RC beam and jacket are then calculated with EC2 and SIA 2052, respectively, neglecting the MASM flange contribution. A developed database of 36 RC beams with UHPC jackets from the literature was used to further validate the proposed methodology. Overall, the calculated capacities compare reasonably with the measured strengths though flexural failure is predicted to occur in a few cases where shear failure was reported.

The conclusions from this study are as follows:

- The most accurate strength predictions of the beams tested in this campaign were obtained with NLFEA.
- The ultimate flexural strength predicted through cross-section analysis show reasonably good agreement with the test results for the 28 strengthened beams in the test database.
- The considered design code equations tend to underestimate the shear strength of the RC control beams from this campaign and the test database. This appears to be due to the contribution of the flange not being considered explicitly. In test series where the jacketed beams failed in shear, the best estimate of the shear strength of the control RC beam is given by EC2 when enhanced by a contribution for the flange outstands calculated with the MASM.
- The increase in shear strength of the RC beam due to the UHPC jacket is overestimated by NF P18-710, SIA 2052 and JSCE 2006 for both the beams of this research and the database. The predictions of SIA 2052 are lowest since it does not include a contribution to shear strength from the UHPC unlike NF P18-710 and JSCE 2006.
- The degree of overestimation in the estimated UHPC jacket contribution to shear strength is dependent on its side thickness. For the jackets of this campaign, the jacket contribution was predicted to increase by 50% and 100% when increasing the side thickness from 20 mm to 30 mm and 40 mm, respectively. However, test results of the strengthened specimens of group 1 showed that increasing the side thickness from 20 mm (B2) to 30 mm (B3) and 40 mm (B4) increased the ultimate shear strength contribution of the jacket by only 13% and 31%, respectively. For the strengthened specimens of group 2, increasing the side thickness from 20 mm (B6) to 30 mm (B7) and 40 mm (B8) increased the shear strength contribution of the jacket by only 8% and 42%, respectively.

- The residual tensile strength of UHPC adopted in the UHPC design codes is about four times that experimentally derived along the critical shear crack at the ultimate load for the beams of this campaign. However, the peak jacket contributions are similar to those calculated with the code methods.
- The use of different truss angles for the transverse (21.8°) and fibre (30°) reinforcement is inconsistent with the observed failure cracks in this campaign. Observations following the partial jacket removal suggest that a shared truss angle for the transverse and fibre reinforcement truss models is more appropriate.
- Based on the analysis of shear resisting mechanisms, a more conservative method for predicting the ultimate shear strength of RC beams with UHPC jackets was proposed. The method involves adding the shear contributions of the transverse reinforcement given by EC2 to that of the fibre reinforcement given by SIA 2052, with both truss models having a shared minimum truss angle of 30° . Theoretically, this approach is broadly equivalent to adding the RC beam contribution at peak jacket contribution to the peak jacket contribution.
- The proposed shear strength prediction method gives safe predictions for the beams of this research. The strengths for the jacketed beams in the test database are reasonably well estimated if the ultimate load is taken as the least of the estimated shear and flexure strengths. The failure mode of some beams is predicted to be flexural rather than shear as reported as in the beams of Liu and Charron (2023). In these cases, the reported failure mode is based on yielding of the stirrups in the RC beam. This seems inconclusive since stirrups typically yield in RC beams well before failure with resistance increasing subsequently due to strut rotation. Hence, where the measured ultimate load is close to the estimated flexural capacity, yet shear failure is reported, the actual failure mode seems debatable. Further laboratory testing is required to refine the proposed design method but based on the available data it seems reasonable.

8.3 SUGGESTIONS FOR FUTURE RESEARCH

This research investigated the shear behaviour of two groups of RC beams with UHPC U-shaped jackets that varied in geometry and embedded longitudinal reinforcement. The number of tested jacketed beams was limited to six to ensure time- and cost-effectiveness, which in turn limited the number of test parameters that could be varied and studied. In general, further

laboratory testing is required to ultimately refine the proposed design method, particularly for jacketed beams that fail in shear. In this regard, the following tests could be performed:

- Testing shear critical RC beams with UHPC U-shaped jackets that vary only in the tensile class of UHPC would be interesting. The influence of the post-cracking strain-softening, limited strain-hardening and significant strain-hardening behaviour on shear strength could be examined. The non-proprietary UHPC used to manufacture all of the jackets of this research had post-cracking strain-softening behaviour.
- Testing shear critical RC beams with UHPC U-shaped jackets that include embedded transverse reinforcement would also be of interest. The shear resisting mechanisms can be assessed to investigate the impact of the transverse reinforcement.
- Further research could include the testing of non-slender RC beams with UHPC jackets that are shear critical where the shear span to depth ratio (a_v/d) is less than 2.5. This research only focussed on studying the shear behaviour of slender beams. It would be interesting to assess the behaviour of such beams as their crack patterns may differ. It would also be of interest to study the validity of the shear enhancement factors given by RC and UHPC design codes such as EC2 and NF P18-710 against the test results as the strength of non-slender beams differs from slender beams due to arching action (AFNOR, 2016b; BSI, 2014). From the literature, only two non-slender RC beams with UHPC side jackets failed in shear (Wang et al., 2013, 2015b).

The proposed design method for predicting the ultimate shear strength of RC beams with UHPC jackets that involved the use of a shared truss angle is broadly equivalent to adding a reduced RC beam contribution to the peak jacket contribution. Based on the assessment of the shear resistance mechanisms, the contribution of the UHPC jacket was reduced at the ultimate load rather than the contribution of the RC beam as the proposed method suggests. Despite the estimates of the proposed method showing good agreement overall with the limited amount of available test data, it would be interesting to investigate an alternative approach that is equivalent to adding the reduced jacket contribution to the peak contribution of the RC beam. This approach is more complex than that proposed in this research as it requires an estimate of crack width in the UHPC jacket at the ultimate load. The development of a design method for RC beams with UHPC jackets through strut-and-tie modelling could also be investigated.

Nonlinear finite element analysis (NLFEA) was carried out in this research to ultimately simulate the response of the beams tested in this campaign, with the developed modelling approach showing good agreement with test results. Due to time constraints, the developed modelling approach was not used to examine other test parameters that were not considered in the experimental program of this research. It would be interesting to numerically model the suggested experiments listed above to investigate how the shear behaviour of jacketed beams is influenced by the post-cracking tensile behaviour of UHPC, transverse reinforcement in the UHPC jacket and different types of RC beams.

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APPENDIX A: DIC RESULTS OF BEAMS OF GROUPS 1 AND 2

A.1 GROUP 1 SPECIMENS

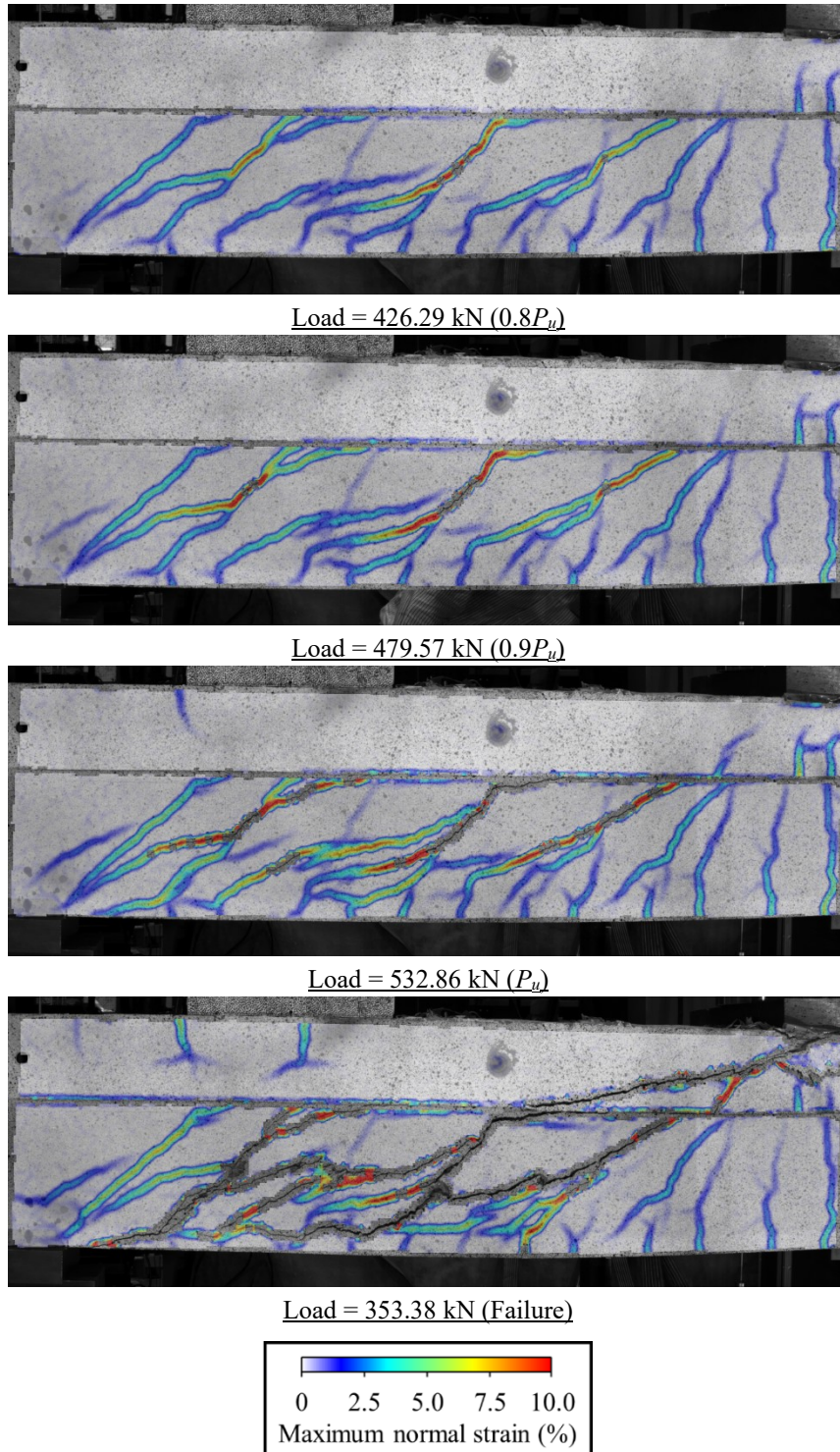


Figure A.1 – Crack pattern and strain state of specimen B1 (Front FOV) throughout the latter stages of loading until failure

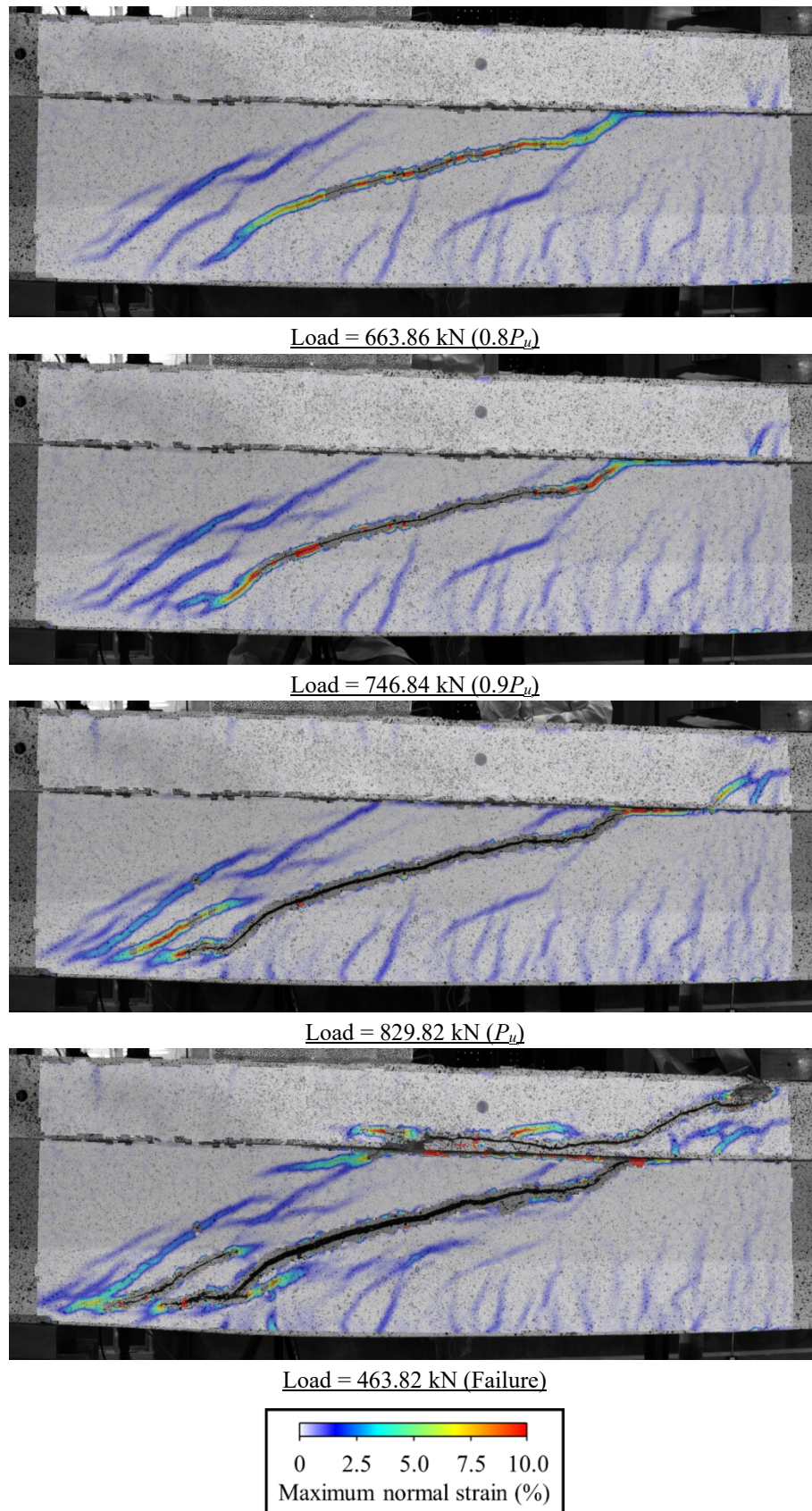


Figure A.2 – Crack pattern and strain state of specimen B2 (Front FOV) throughout the latter stages of loading until failure

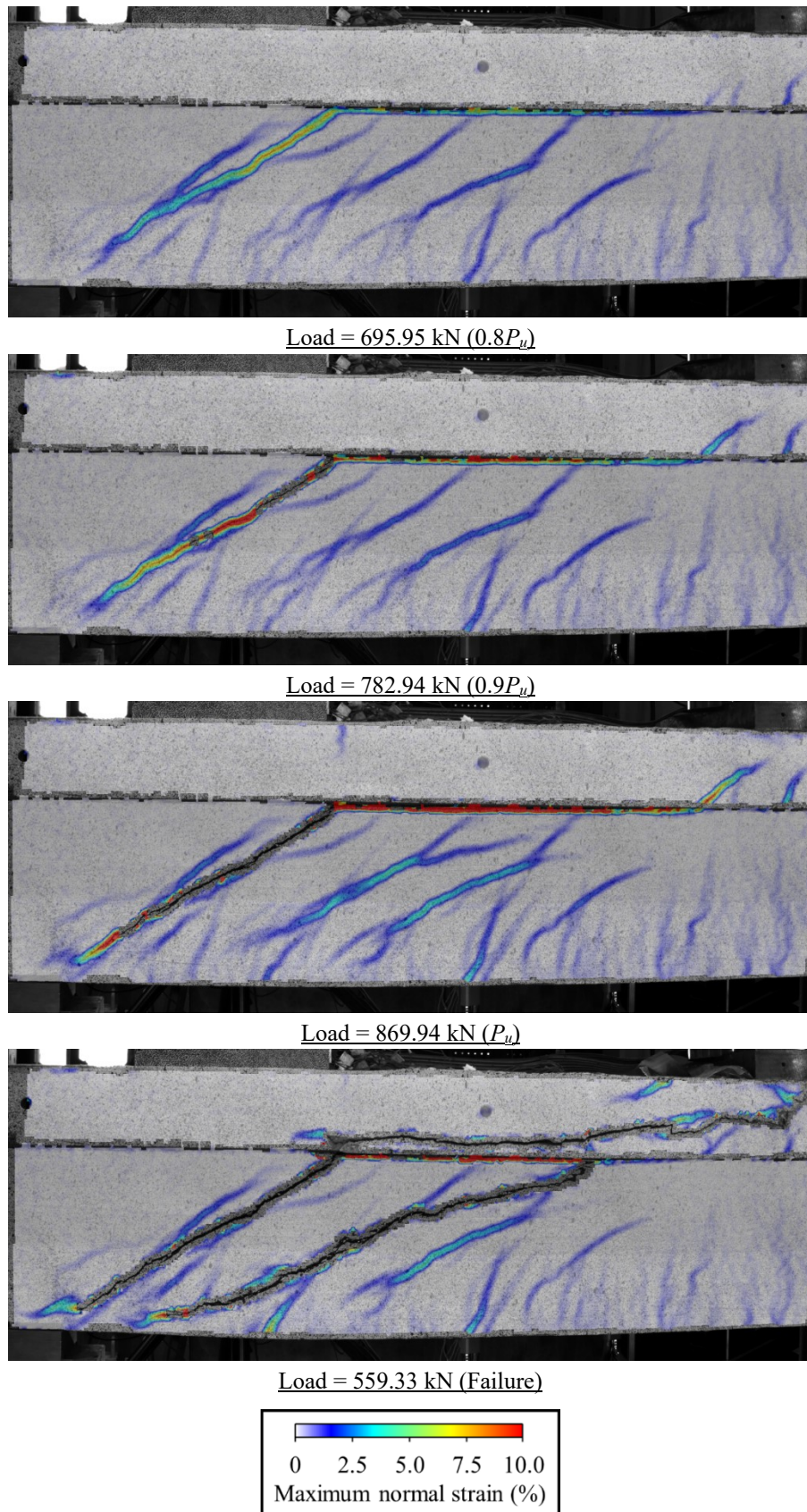


Figure A.3 – Crack pattern and strain state of specimen B3 (Front FOV) throughout the latter stages of loading until failure

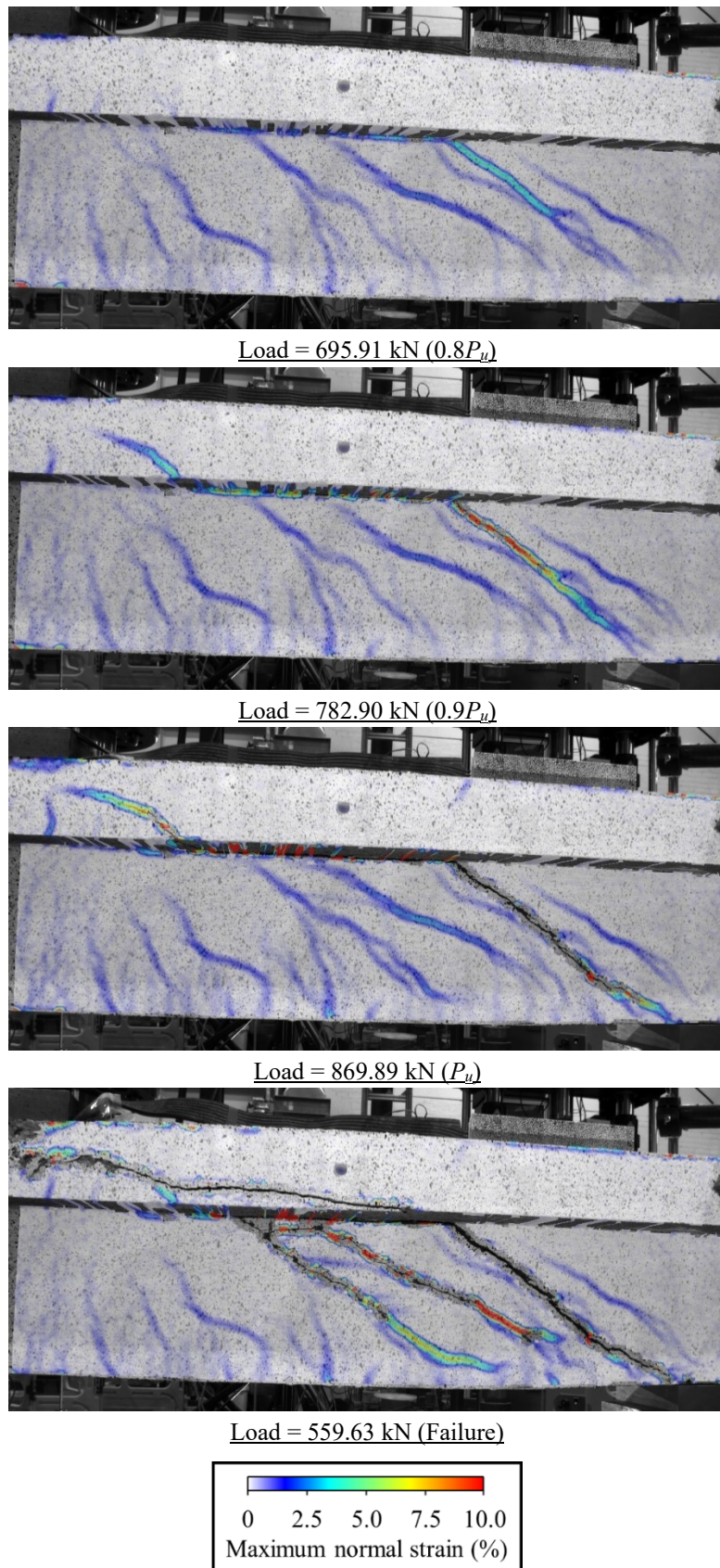


Figure A.4 – Crack pattern and strain state of specimen B3 (Back FOV) throughout the latter stages of loading until failure

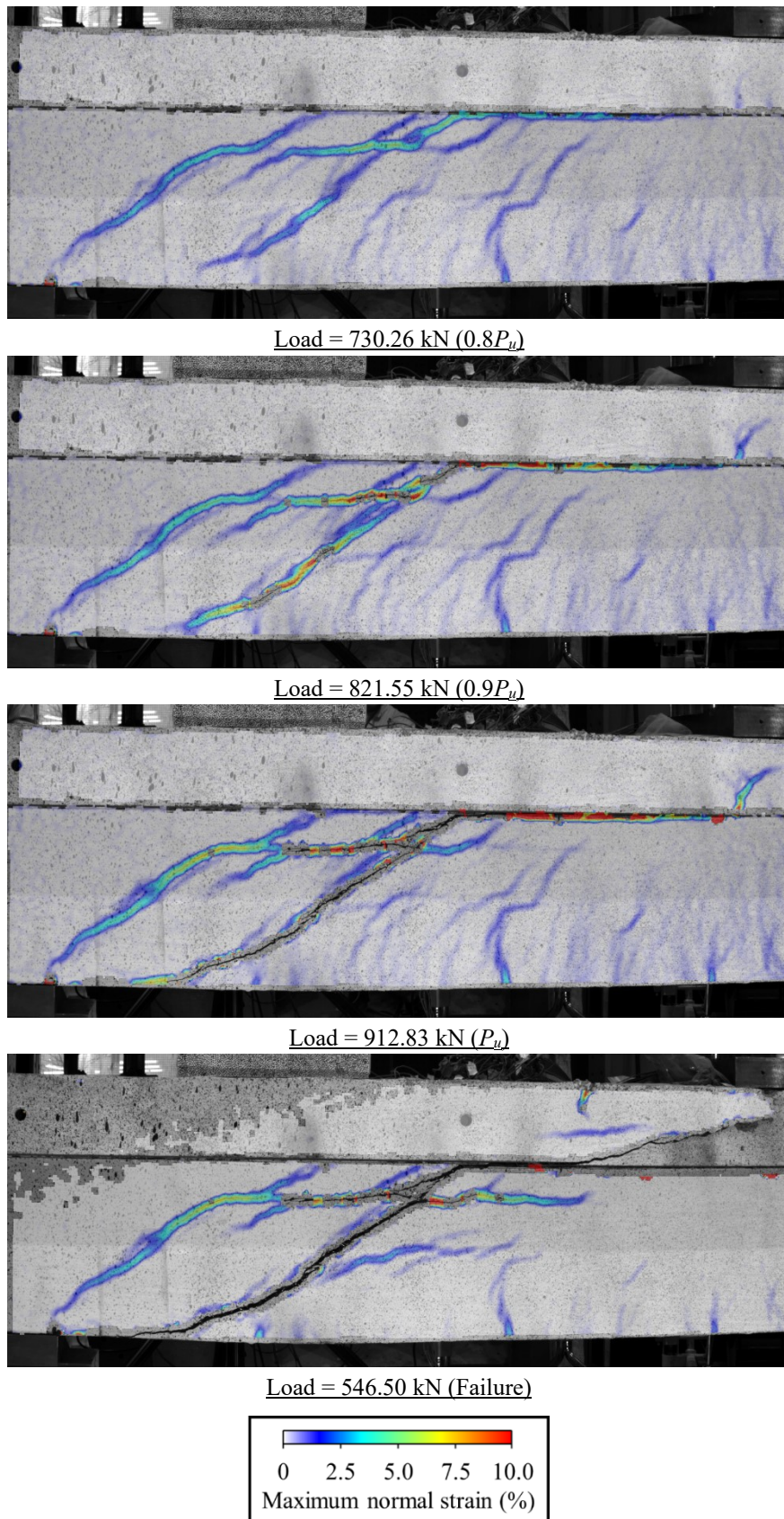


Figure A.5 – Crack pattern and strain state of specimen B4 (Front FOV) throughout the latter stages of loading until failure

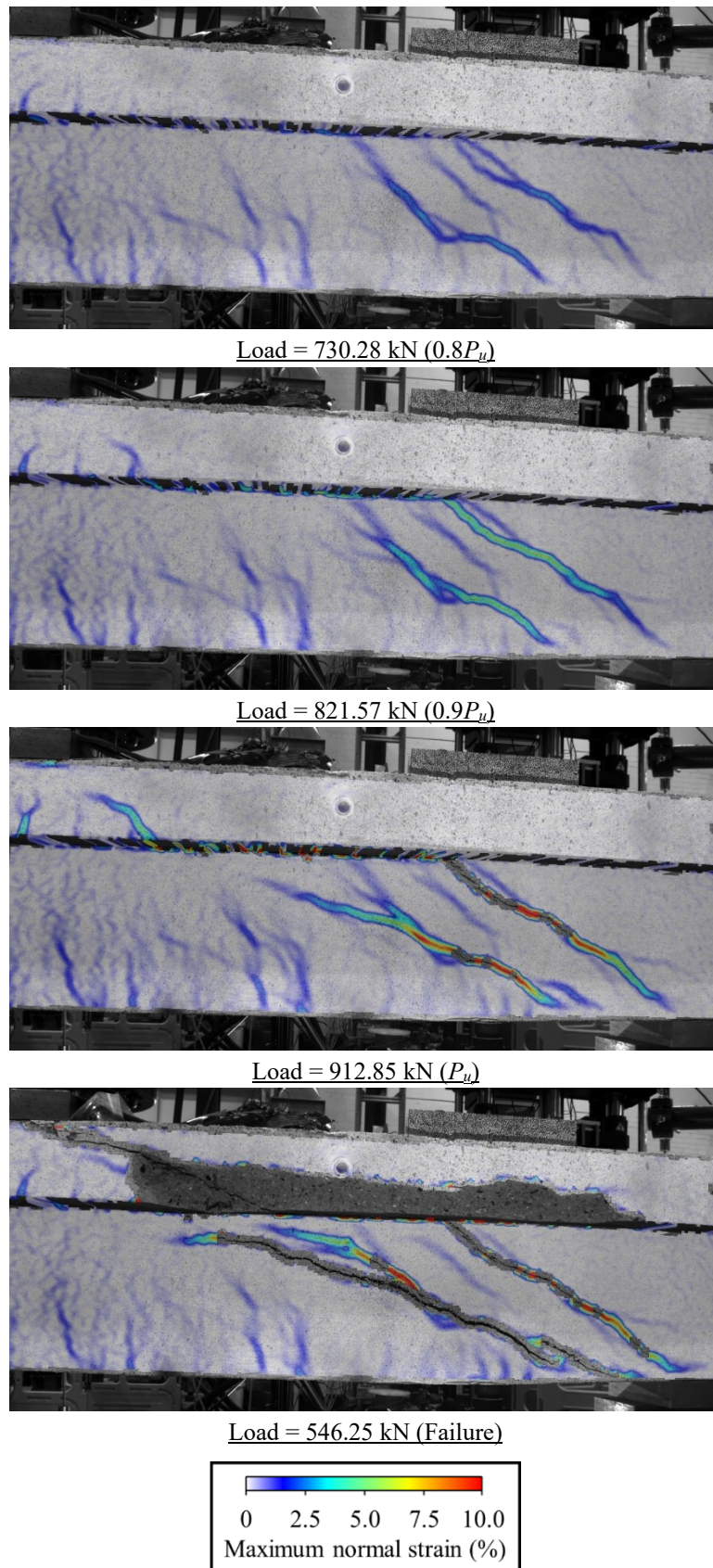


Figure A.6 – Crack pattern and strain state of specimen B4 (Back FOV) throughout the latter stages of loading until failure

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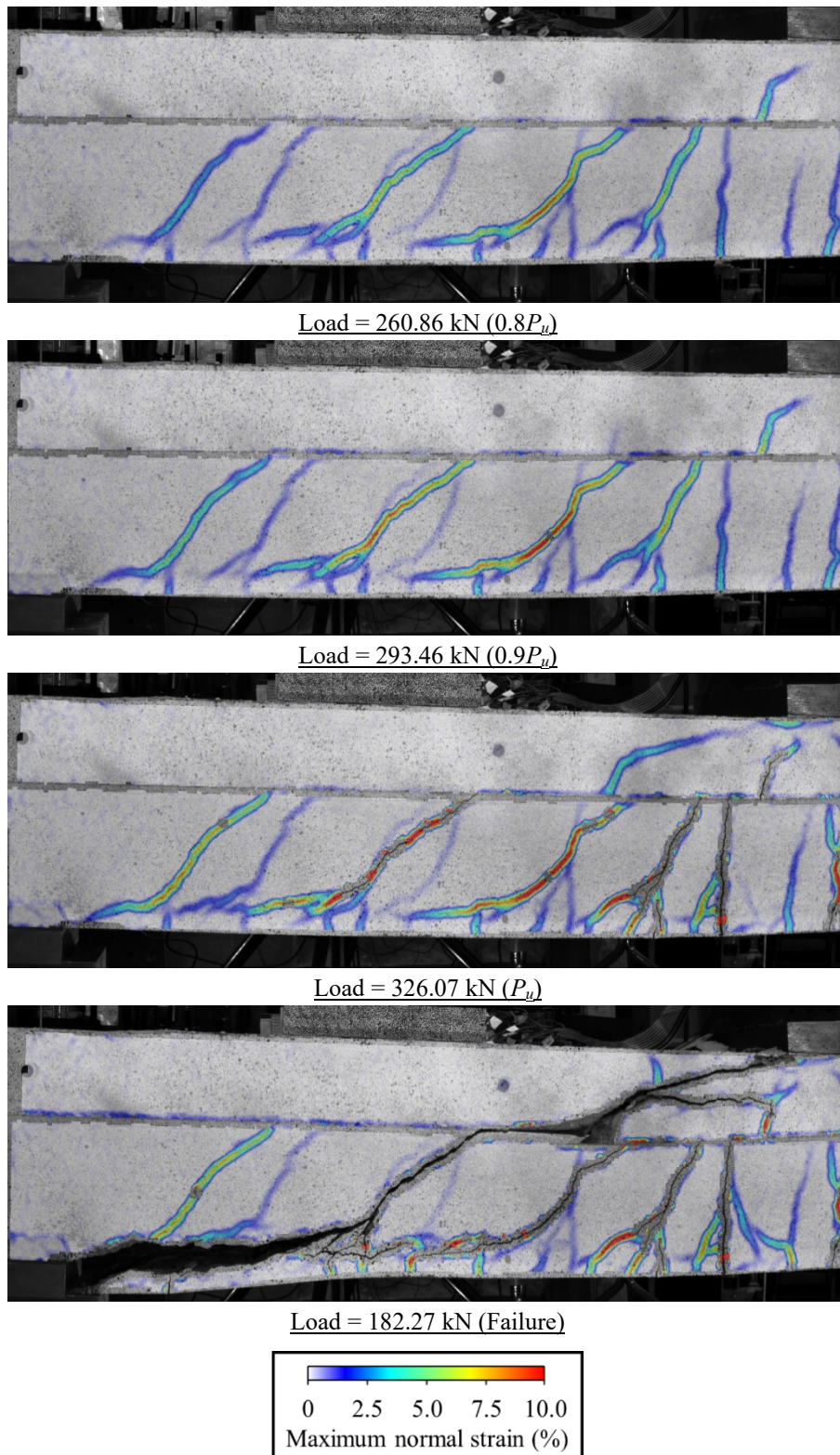


Figure A.7 – Crack pattern and strain state of specimen B5 (Front FOV) throughout the latter stages of loading until failure

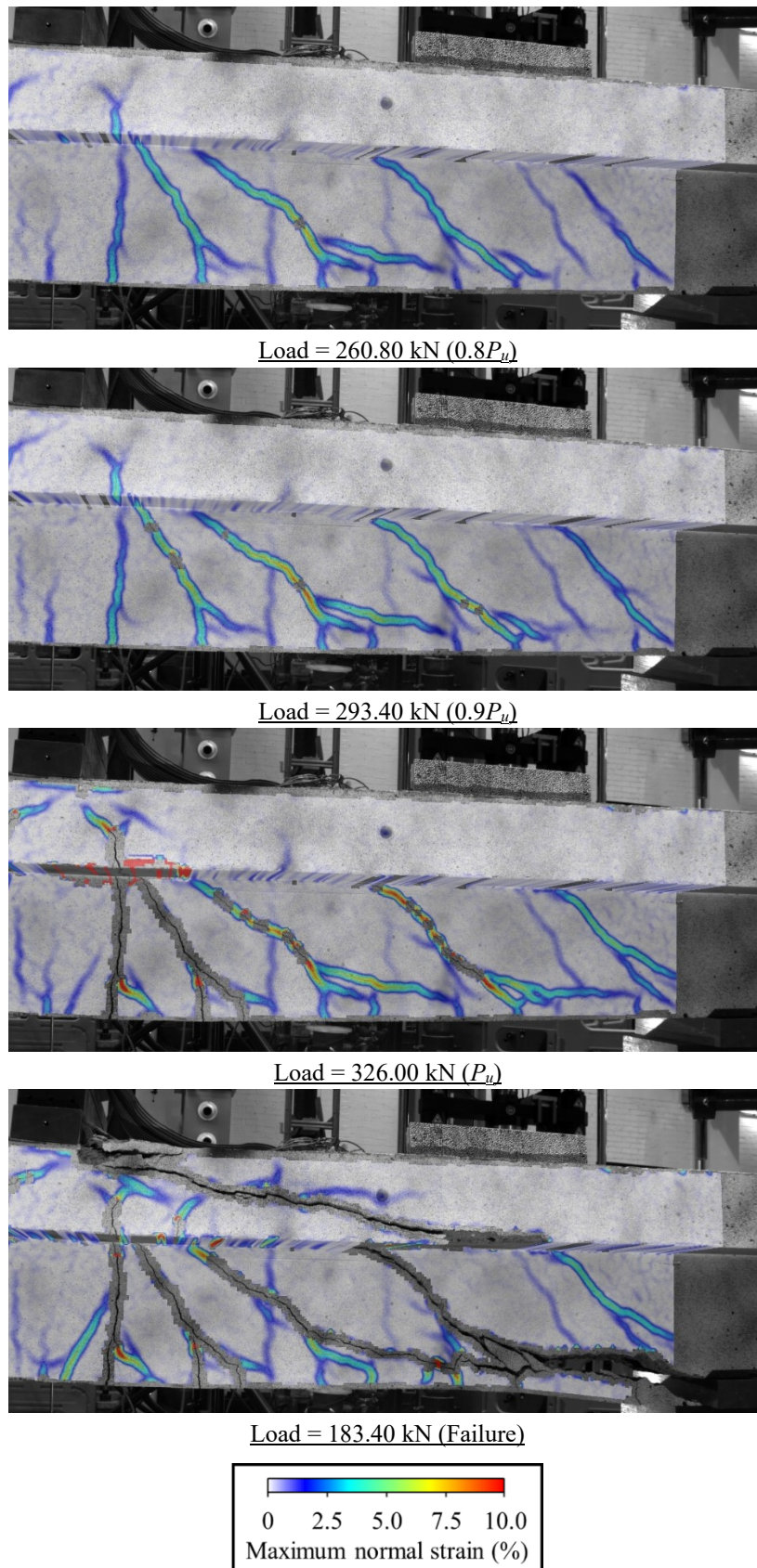


Figure A.8 – Crack pattern and strain state of specimen B5 (Back FOV) throughout the latter stages of loading until failure

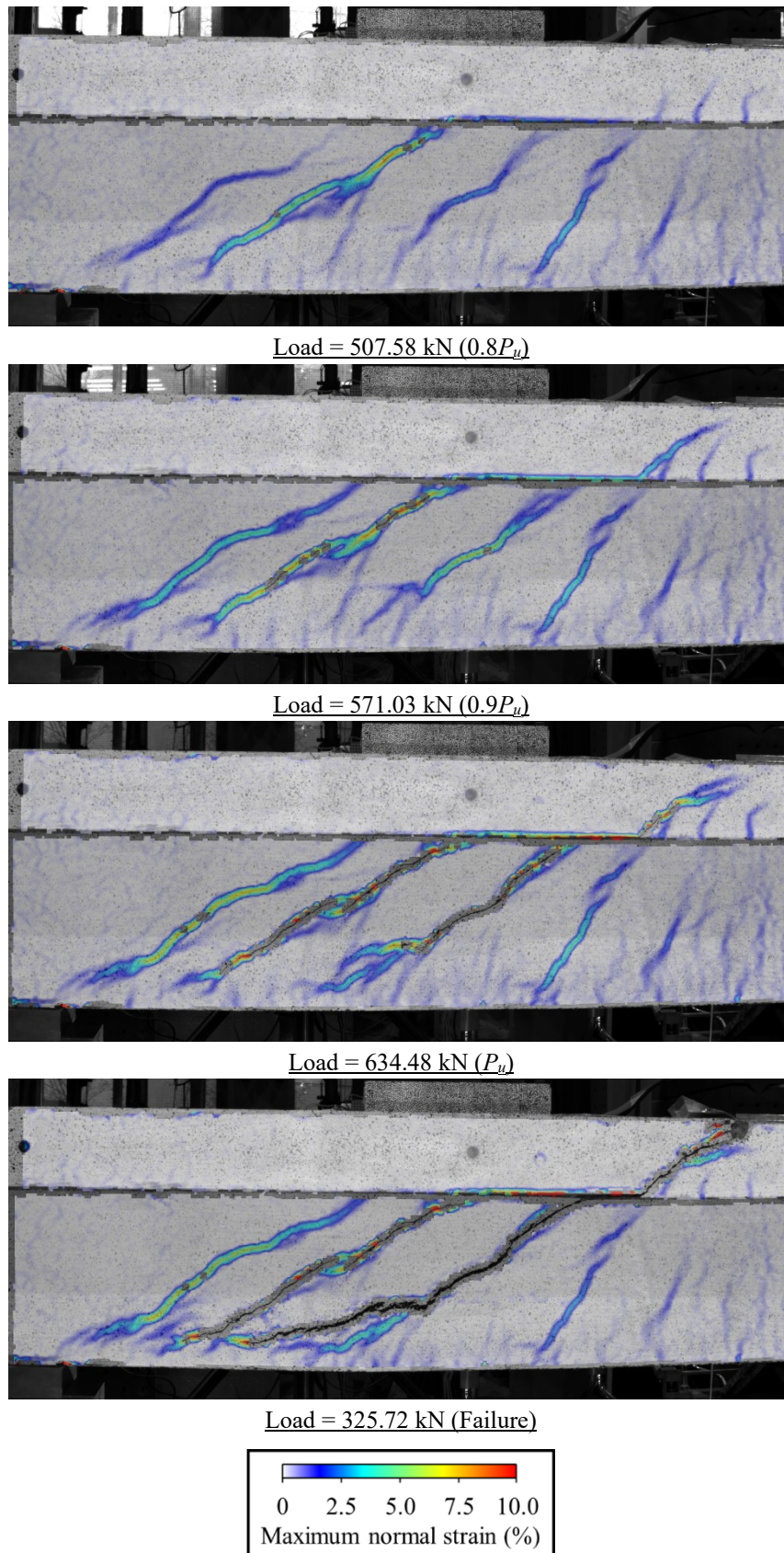


Figure A.9 – Crack pattern and strain state of specimen B6 (Front FOV) throughout the latter stages of loading until failure

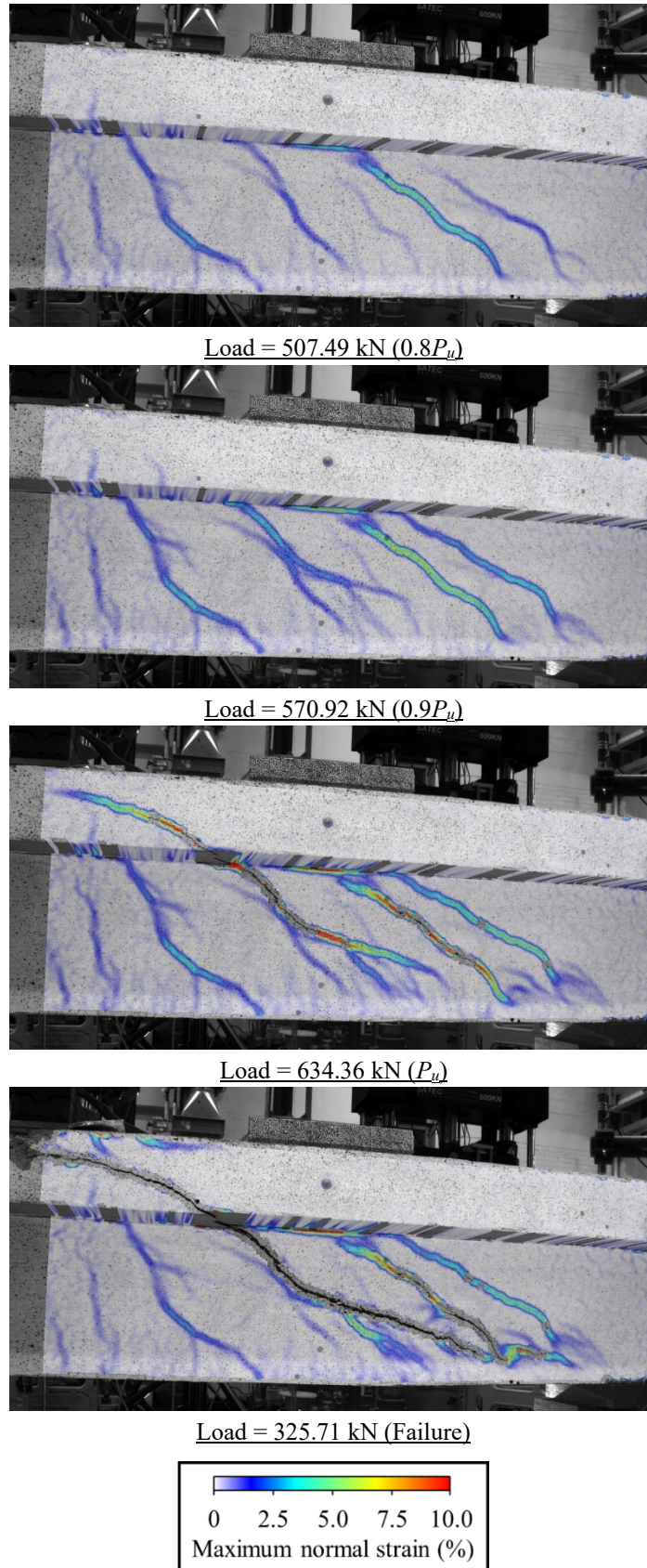


Figure A.10 – Crack pattern and strain state of specimen B6 (Back FOV) throughout the latter stages of loading until failure

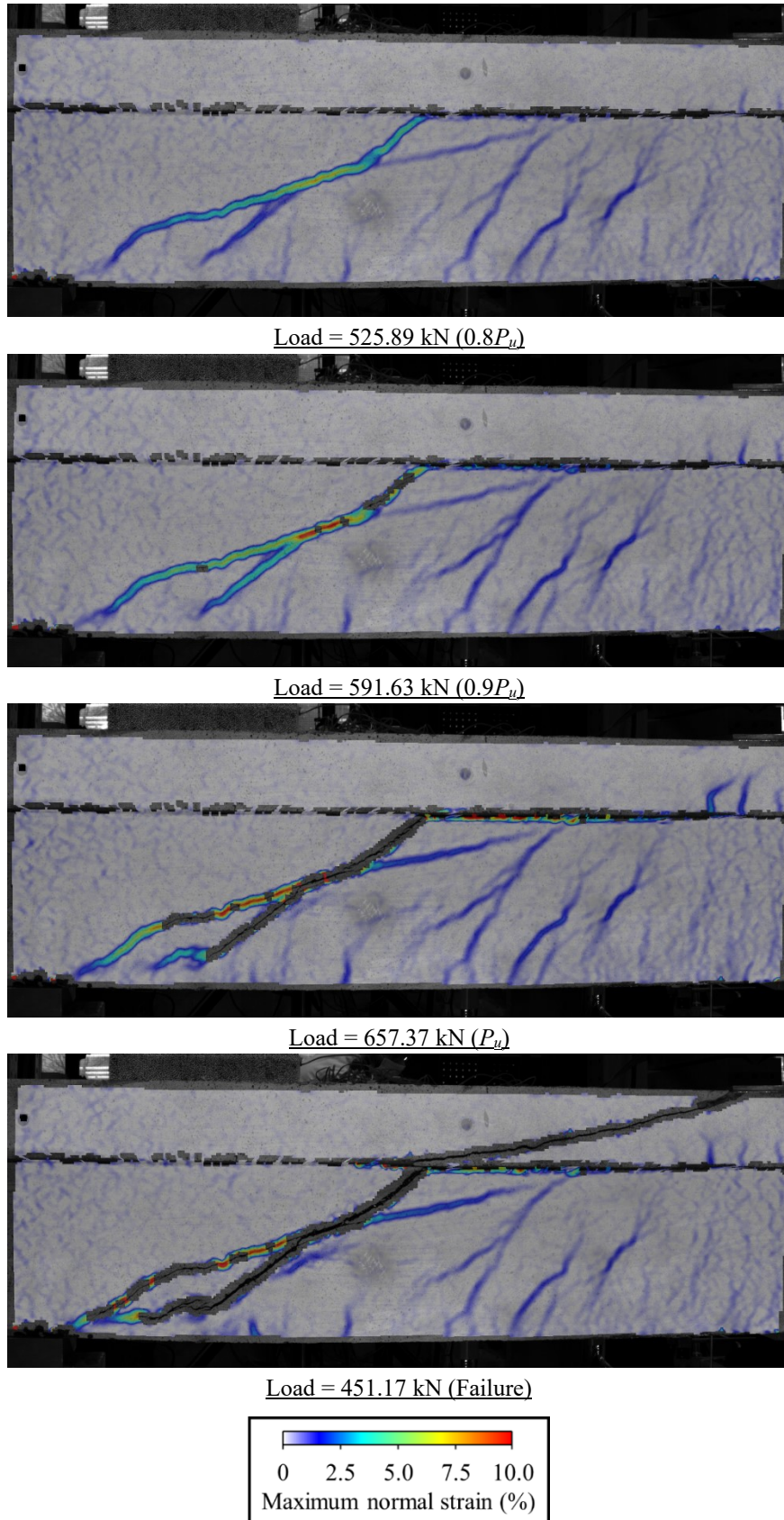


Figure A.11 – Crack pattern and strain state of specimen B7 (Front FOV) throughout the latter stages of loading until failure

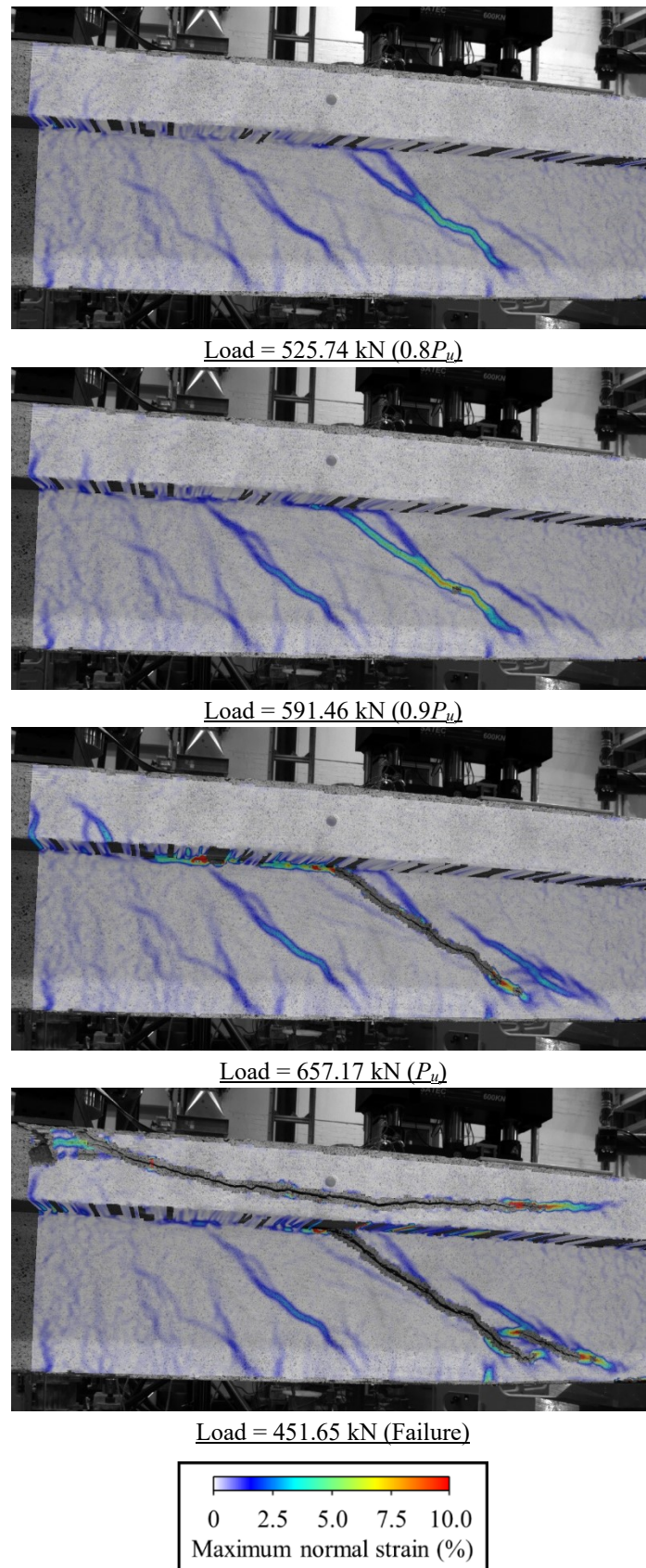


Figure A.12 – Crack pattern and strain state of specimen B7 (Back FOV) throughout the latter stages of loading until failure

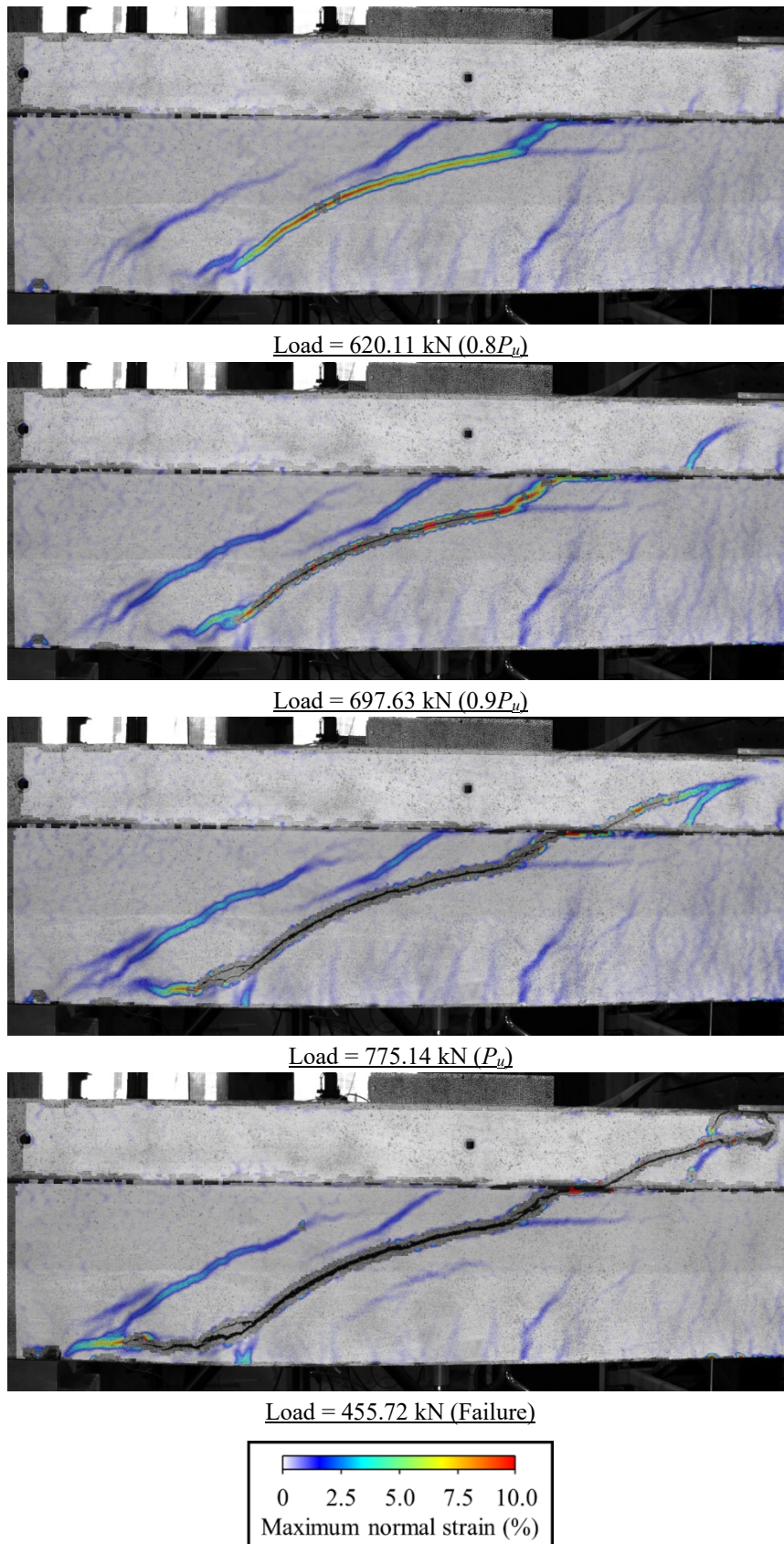


Figure A.13 – Crack pattern and strain state of specimen B8 (Front FOV) throughout the latter stages of loading until failure

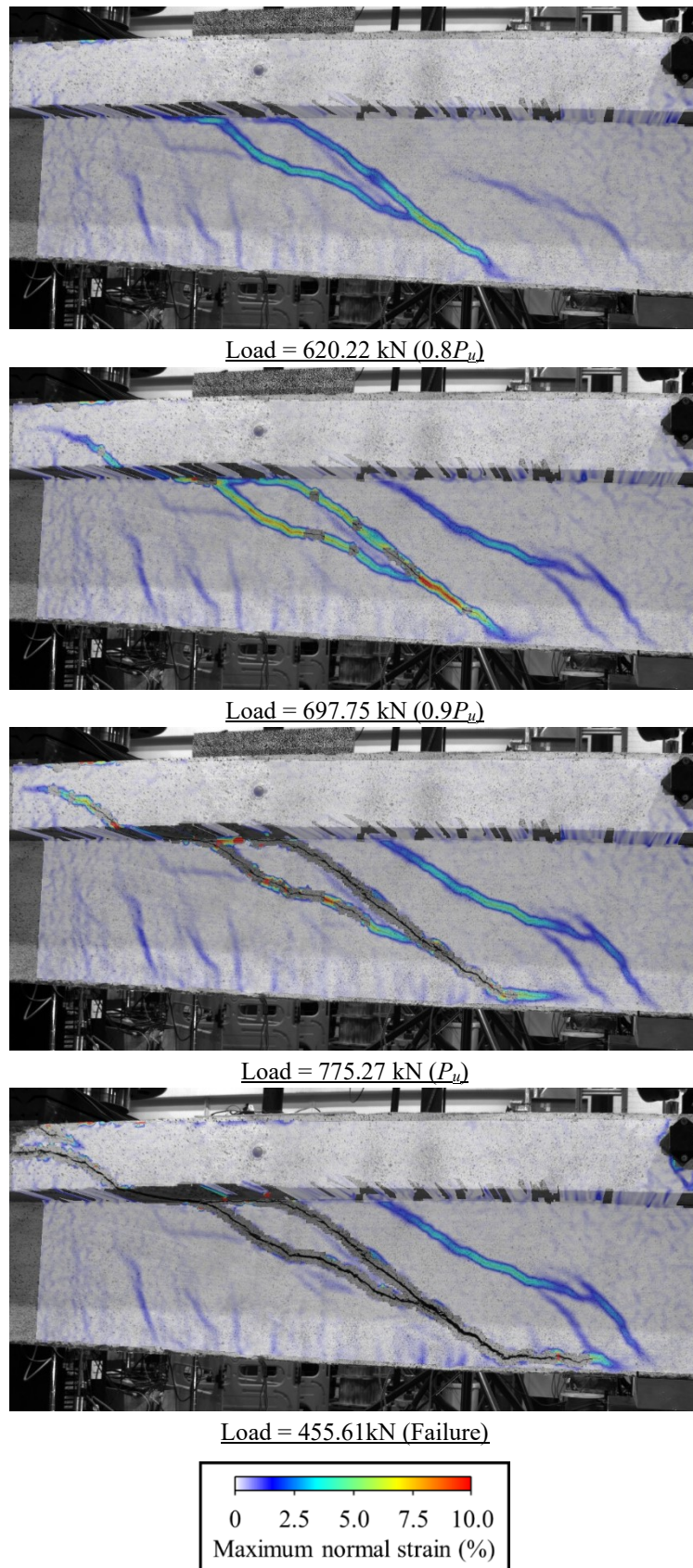


Figure A.14 – Crack pattern and strain state of specimen B8 (Back FOV) throughout the latter stages of loading until failure

APPENDIX B: TEST DATABASE OF BEAMS FROM THE LITERATURE

Tables B.1, B.2 and B.3 are presented in the following pages.

APPENDIX B: TEST DATABASE OF BEAMS FROM THE LITERATURE

Table B.1 – Geometry, reinforcement details, mechanical properties and loading arrangement of beams in database

Author	Specimen		RC beam								UHPC jacket						Load
	ID	Type ^A	Width	Height	Flange width	Flange height	Length	Comp. str.	Long. reinf. ^B	Shear reinf. ^C	Side thick.	Bot. thick.	Comp. str.	Tens. str.	Tens. str. el.	Resid. str.	Ratio ^D
			(b_w) [mm]	(h) [mm]	(b_f) [mm]	(h_f) [mm]	(l) [mm]	(f_{cm}) [MPa]	(ρ_l) [%]	(ρ_w) [%]	(t_s) [mm]	(t_b) [mm]	(f_{cm}) [MPa]	(f_{ctm}) [MPa]	($f_{ctm,el}$) [MPa]	(σ_{res}) [MPa]	
Alkhoury (2017)	Nr. 5	CON	200	400	500	150	2900	17.10	1.31	0.11	-	-	-	-	-	-	2.64
	Nr. 6 ^E	UJ	200	400	500	150	2900	15.40	1.31	0.11	40	40	167.40	7.00	7.00	7.00	2.64
	Nr. 7 ^F	UJ	200	400	500	150	2900	17.10	1.31	0.11	40	60	167.40	7.00	7.00	7.00	2.64
Al-Osta et al. (2017)	RC-Control 1	CON	140	230	-	-	1400	54.00	0.57	1.44	-	-	-	-	-	-	2.92
	RC-Control 2	CON	140	230	-	-	1400	54.00	0.57	1.44	-	-	-	-	-	-	2.92
	RC-SB-2SJ	SJ	140	230	-	-	1400	54.00	0.57	1.44	30	0	128.00	7.00	7.00	7.00	2.92
	RC-EP-2SJ	SJ	140	230	-	-	1400	54.00	0.57	1.44	30	0	128.00	7.00	7.00	7.00	2.92
	RC-SB-3SJ	UJ	140	230	-	-	1400	54.00	0.57	1.44	30	30	128.00	7.00	7.00	7.00	2.92
	RC-EP-3SJ	UJ	140	230	-	-	1400	54.00	0.57	1.44	30	30	128.00	7.00	7.00	7.00	2.92
Bahraq et al. (2019)	CT-2.0	CON	140	230	-	-	960	65.00	2.34	0.60	-	-	-	-	-	-	2.00
	SB-2SJ-2.0	SJ	140	230	-	-	960	65.00	2.34	0.60	30	0	136.40	8.70	8.70	8.70	2.00
	SB-3SJ-2.0	UJ	140	230	-	-	960	65.00	2.34	0.60	30	30	136.40	8.70	8.70	8.70	2.00
Daugevičius et al. (2015)	S4U	UJ	100	200	-	-	1800	32.40	1.76	1.01	20	20	112.90	5.73	5.73	5.73	3.43
	S5U	UJ	100	200	-	-	1800	32.40	1.76	1.01	20	20	112.90	5.73	5.73	5.73	3.43
	S6U	UJ	100	200	-	-	1800	32.40	1.76	1.01	20	20	112.90	5.73	5.73	5.73	3.43
	S7	CON	100	200	-	-	1800	32.40	1.76	1.01	-	-	-	-	-	-	3.43
	S8	CON	100	200	-	-	1800	32.40	1.76	1.01	-	-	-	-	-	-	3.43
	S9	CON	100	200	-	-	1800	32.40	1.76	1.01	-	-	-	-	-	-	3.43

APPENDIX B: TEST DATABASE OF BEAMS FROM THE LITERATURE

Author	Specimen		RC beam								UHPC jacket						Load
	ID	Type ^A	Width	Height	Flange width	Flange height	Length	Comp. str.	Long. reinf. ^B	Shear reinf. ^C	Side thick.	Bot. thick.	Comp. str.	Tens. str.	Tens. str. el.	Resid. str.	Ratio ^D
			(b_w) [mm]	(h) [mm]	(b_f) [mm]	(h_f) [mm]	(l) [mm]	(f_{cm}) [MPa]	(ρ_l) [%]	(ρ_w) [%]	(t_s) [mm]	(t_b) [mm]	(f_{cm}) [MPa]	(f_{ctm}) [MPa]	($f_{ctm,el}$) [MPa]	(σ_{res}) [MPa]	(a_v/d)
Choi (2016)	CON	CON	200	400	450	100	2494	24.70	2.86	0.52	-	-	-	-	-	-	3.15
	U2	UJ	200	400	450	100	2494	24.70	2.86	0.52	20	20	164.00	9.50	9.50	9.50	3.15
	U4	UJ	200	400	450	100	2494	24.70	2.86	0.52	40	40	164.00	9.50	9.50	9.50	3.15
Liu and Charron (2023)	TS-R	CON	250	400	500	120	2600	29.80	5.52	0.13	-	-	-	-	-	-	3.25
	TS-W25	SJ	250	400	500	120	2600	29.80	5.52	0.13	25	0	150.80	12.70	12.70	12.70	3.25
	TS-W50	SJ	250	400	500	120	2600	29.80	5.52	0.13	50	0	150.80	12.70	12.70	12.70	3.25
	TS-W50A	SJ	250	400	500	120	2600	29.80	5.52	0.13	50	0	150.80	12.70	12.70	12.70	3.25
	TS-U25	UJ	250	400	500	120	2600	29.80	5.52	0.13	25	25	150.80	12.70	12.70	12.70	3.25
	TS-U50	UJ	250	400	500	120	2600	29.80	5.52	0.13	50	50	150.80	12.70	12.70	12.70	3.25
	TS-U50A	UJ	250	400	500	120	2600	29.80	5.52	0.13	50	50	150.80	12.70	12.70	12.70	3.25
Martinola et al. (2010)	R/C	CON	300	500	-	-	4350	22.00	0.29	0.22	-	-	-	-	-	-	2.37
	R/C str.	UJ	300	500	-	-	4350	22.00	0.29	0.22	40	40	162.00	11.80	11.80	11.80	2.37
	R/C rep.	UJ	300	500	-	-	4350	22.00	0.29	0.22	40	40	162.00	11.80	11.80	11.80	2.37
Paschalis (2017)	P1	CON	150	200	-	-	2000	24.72	0.98	0.70	-	-	-	-	-	-	5.19
	P2	CON	150	200	-	-	2000	24.72	0.98	0.70	-	-	-	-	-	-	5.19
	3SJ1	UJ	150	200	-	-	2000	24.72	0.98	0.70	40	50	121.50	11.50	11.50	11.50	5.19
	3SJ2	UJ	150	200	-	-	2000	24.72	0.98	0.70	40	50	121.50	11.50	11.50	11.50	5.19
Wang et al. (2013)	N2.5	CON	150	240	-	-	1200	19.00	2.01	0.13	-	-	-	-	-	-	2.50
	U2.5	SJ	150	240	-	-	1200	19.00	2.01	0.13	7	0	210.00	10.80	10.80	10.80	2.50

APPENDIX B: TEST DATABASE OF BEAMS FROM THE LITERATURE

Author	Specimen		RC beam								UHPC jacket						Load
	ID	Type ^A	Width (b_w) [mm]	Height (h) [mm]	Flange width (b_f) [mm]	Flange height (h_f) [mm]	Length (l) [mm]	Comp. str. (f_{cm}) [MPa]	Long. reinf. ^B (ρ_l) [%]	Shear reinf. ^C (ρ_w) [%]	Side thick. (t_s) [mm]	Bot. thick. (t_b) [mm]	Comp. str. (f_{cm}) [MPa]	Tens. str. (f_{ctm}) [MPa]	Tens. str. el. ($f_{ctm,el}$) [MPa]	Resid. str. (σ_{res}) [MPa]	Ratio ^D (a_v/d)
Hong and Lim (2023)	CS	CON	200	400	-	-	2494	36.30	2.72	0.48	-	-	-	-	-	-	3.00
	S1.5T2	SJ	200	400	-	-	2494	36.30	2.72	0.48	20	0	151.10	9.90	9.90	9.90	3.00
	S1.5T4	SJ	200	400	-	-	2494	36.30	2.72	0.48	40	0	151.10	9.90	9.90	9.90	3.00
	S1.5T6	SJ	200	400	-	-	2494	36.30	2.72	0.48	60	0	151.10	9.90	9.90	9.90	3.00
	U1.5T2	UJ	200	400	-	-	2494	36.30	2.72	0.48	20	20	151.10	9.90	9.90	9.90	3.00
	U1.5T4	UJ	200	400	-	-	2494	36.30	2.72	0.48	40	40	151.10	9.90	9.90	9.90	3.00
	U1.5T6	UJ	200	400	-	-	2494	36.30	2.72	0.48	60	60	151.10	9.90	9.90	9.90	3.00
	S0.5T2	SJ	200	400	-	-	2494	36.30	2.72	0.48	20	0	158.40	5.00	5.00	5.00	3.00
	S0.5T4	SJ	200	400	-	-	2494	36.30	2.72	0.48	40	0	158.40	5.00	5.00	5.00	3.00
	S0.5T6	SJ	200	400	-	-	2494	36.30	2.72	0.48	60	0	158.40	5.00	5.00	5.00	3.00
	U0.5T2	UJ	200	400	-	-	2494	36.30	2.72	0.48	20	20	158.40	5.00	5.00	5.00	3.00
	U0.5T4	UJ	200	400	-	-	2494	36.30	2.72	0.48	40	40	158.40	5.00	5.00	5.00	3.00
	U0.5T6	UJ	200	400	-	-	2494	36.30	2.72	0.48	60	60	158.40	5.00	5.00	5.00	3.00

^A CON, SJ and UJ refer to control beams and beams with side and U-shaped jackets, respectively; ^B Longitudinal reinforcement ratio (ρ_l) equals to $A_s/b_w d$; ^C Shear reinforcement ratio (ρ_w) equals to $A_{sw}/b_w s$;

^D Considers the depth to the longitudinal reinforcement in the beam; ^E Jacket of specimen Nr. 6 includes embedded longitudinal reinforcement ($A_s = 6\phi 20$); ^F Jacket of specimen Nr. 7 includes embedded longitudinal ($A_s = 5\phi 25$ and $4\phi 16$) and transverse ($A_{sw} = \phi 6$ at 250 mm) reinforcement

APPENDIX B: TEST DATABASE OF BEAMS FROM THE LITERATURE

Table B.2 – Experimental and calculated ultimate shear and flexural strength of beams in database

Author	Specimen		Experimental		Calculated									
	ID	Type ^A	Ultimate force (P_{exp}) [kN]	Failure mode ^B	Ultimate shear and flexural strength (P_{calc}/P_{exp})									Failure mode ^{B, L}
					EC2 ^C	fib MC 2010 ^D	ACI 318-19 ^E	MASMF	EC2 and MASMG	NF P18-710 ^{H, I}	SIA 2052 ^{J, I}	JSCE 2006 ^{K, I}	Flexure	
Alkhoury (2017)	Nr. 5	CON	270.00	S	0.84	0.76	0.75	1.26	1.19	-	-	-	1.26	S
	Nr. 6	UJ	820.00	S	0.28	0.25	0.24	0.44	0.40	1.46	1.26	1.45	1.24	F
	Nr. 7	UJ	1000.00	S	0.23	0.21	0.20	0.34	0.32	1.40	1.22	1.35	1.36	S
Al-Osta et al. (2017)	RC-Control 1	CON	70.00	F	8.49	6.16	3.82	7.57	8.49	-	-	-	0.92	F
	RC-Control 2	CON	70.00	F	8.49	6.16	3.82	7.57	8.49	-	-	-	0.92	F
	RC-SB-2SJ	SJ	102.00	F	5.83	4.23	2.62	5.19	5.83	8.96	8.41	8.89	1.00	F
	RC-EP-2SJ	SJ	95.00	F	6.26	4.54	2.82	5.58	6.26	9.62	9.03	9.55	1.08	F
	RC-SB-3SJ	UJ	132.00	F	4.50	3.27	2.03	4.01	4.50	7.24	6.76	7.18	1.02	F
	RC-EP-3SJ	UJ	129.00	F	4.61	3.34	2.07	4.11	4.61	7.41	6.92	7.35	1.04	F
Bahraq et al. (2019)	CT-2.0	CON	276.00	S	1.60	0.96	0.98	1.26	1.60	-	-	-	1.24	F
	SB-2SJ-2.0	SJ	346.00	F-S	1.28	0.76	0.78	1.01	1.28	2.39	2.22	2.37	1.20	F
	SB-3SJ-2.0	UJ	353.00	F	1.25	0.75	0.76	0.99	1.25	2.49	2.30	2.46	1.34	F
Daugevičius et al. (2015)	S4U	UJ	174.53	F-S	1.47	1.13	0.75	1.47	1.47	2.45	2.26	2.42	0.97	F
	S5U	UJ	187.63	F-S	1.36	1.05	0.70	1.37	1.36	2.28	2.10	2.26	0.90	F
	S6U	UJ	173.53	F-S	1.48	1.14	0.76	1.48	1.48	2.46	2.27	2.44	0.97	F
	S7	CON	114.50	F	2.24	1.73	1.15	2.24	2.24	-	-	-	1.04	F
	S8	CON	117.13	F	2.19	1.69	1.12	2.19	2.19	-	-	-	1.02	F
	S9	CON	117.73	F	2.18	1.68	1.12	2.18	2.18	-	-	-	1.01	F

APPENDIX B: TEST DATABASE OF BEAMS FROM THE LITERATURE

Author	Specimen		Experimental		Calculated									
	ID	Type ^A	Ultimate force (P_{exp}) [kN]	Failure mode ^B	Ultimate shear and flexural strength (P_{calc}/P_{exp})									Failure mode ^{B, L}
					EC2 ^C	fib MC 2010 ^D	ACI 318-19 ^E	MASMF	EC2 and MASMG	NF P18-710 ^{H, I}	SIA 2052 ^{J, I}	JSCE 2006 ^{K, I}	Flexure	
Choi (2016)	CON	CON	375.40	S	1.72	1.33	1.16	1.86	1.96	-	-	-	1.54	F
	U2	UJ	471.80	F-S	1.36	1.06	0.92	1.48	1.56	2.38	2.26	2.37	1.36	F
	U4	UJ	594.20	F	1.08	0.84	0.73	1.17	1.24	2.63	2.42	2.61	1.19	F
Liu and Charron (2023)	TS-R	CON	352.10	S	0.47	0.70	0.54	1.43	0.78	-	-	-	2.14	S
	TS-W25	SJ	710.00	S	0.23	0.35	0.27	0.71	0.39	1.28	1.18	1.21	1.12	F
	TS-W50	SJ	748.50	S	0.22	0.33	0.25	0.67	0.37	2.06	1.87	1.93	1.11	F
	TS-W50A	SJ	759.10	F-S	0.22	0.32	0.25	0.67	0.36	2.04	1.85	1.91	1.10	F
	TS-U25	UJ	652.20	S	0.25	0.38	0.29	0.77	0.42	1.46	1.34	1.38	1.29	F
	TS-U50	UJ	677.40	S	0.25	0.36	0.28	0.75	0.41	2.52	2.28	2.35	1.38	F
	TS-U50A	UJ	819.60	S	0.20	0.30	0.23	0.62	0.34	2.08	1.88	1.94	1.14	F
Martinola et al. (2010)	R/C	CON	190.00	F	3.64	1.82	2.77	2.45	3.64	-	-	-	0.86	F
	R/C str.	UJ	410.00	F	1.69	0.84	1.29	1.14	1.69	5.56	5.08	5.50	1.27	F
	R/C rep.	UJ	365.00	F	1.90	0.95	1.44	1.28	1.90	6.25	5.71	6.18	1.43	F
Paschalis (2017)	P1	CON	55.18	F	4.42	2.78	2.75	3.99	4.42	-	-	-	0.72	F
	P2	CON	53.98	F	4.52	2.84	2.81	4.08	4.52	-	-	-	0.73	F
	3SJ1	UJ	119.20	F	2.05	1.29	1.27	1.85	2.05	7.98	7.31	7.29	1.24	F
	3SJ2	UJ	112.00	F	2.18	1.37	1.35	1.97	2.18	8.49	7.78	7.76	1.32	F
Wang et al. (2013)	N2.5	CON	128.00	S	0.43	0.60	0.54	0.79	0.43	-	-	-	1.05	S
	U2.5	SJ	144.00	F	0.38	0.54	0.48	0.70	0.38	1.19	1.07	1.18	1.06	F

APPENDIX B: TEST DATABASE OF BEAMS FROM THE LITERATURE

Author	Specimen		Experimental		Calculated									
	ID	Type ^A	Ultimate force (P_{exp}) [kN]	Failure mode ^B	Ultimate shear and flexural strength (P_{calc}/P_{exp})									Failure mode ^{B, L}
					EC2 ^C	fib MC 2010 ^D	ACI 318-19 ^E	MASM ^F	EC2 and MASM ^G	NF P18-710 ^{H, I}	SIA 2052 ^{J, I}	JSCE 2006 ^{K, I}	Flexure	
Hong and Lim (2023)	CS	CON	506.10	S	1.47	0.98	0.93	1.17	1.47	-	-	-	1.11	F
	S1.5T2	SJ	608.90	F	1.22	0.82	0.78	0.97	1.22	2.05	1.93	2.03	1.08	F
	S1.5T4	SJ	555.50	F	1.34	0.90	0.85	1.07	1.34	3.15	2.89	3.12	1.31	F
	S1.5T6	SJ	685.30	F	1.08	0.73	0.69	0.87	1.08	3.28	2.97	3.25	1.15	F
	U1.5T2	UJ	659.50	F	1.13	0.75	0.72	0.90	1.13	1.93	1.81	1.91	1.04	F
	U1.5T4	UJ	835.60	F-S	0.89	0.60	0.57	0.71	0.89	2.21	2.03	2.19	0.96	F
	U1.5T6	UJ	798.60	F	0.93	0.62	0.59	0.74	0.93	3.10	2.80	3.06	1.16	F
	S0.5T2	SJ	545.60	F	1.36	0.91	0.87	1.09	1.36	1.89	1.76	1.88	1.17	F
	S0.5T4	SJ	493.90	F	1.50	1.01	0.96	1.20	1.50	2.68	2.39	2.64	1.39	F
	S0.5T6	SJ	592.70	F	1.25	0.84	0.80	1.00	1.25	2.72	2.36	2.68	1.22	F
	U0.5T2	UJ	616.00	F-S	1.21	0.81	0.77	0.96	1.21	1.70	1.58	1.69	1.06	F
	U0.5T4	UJ	803.20	F-S	0.92	0.62	0.59	0.74	0.92	1.72	1.52	1.70	0.90	F
	U0.5T6	UJ	952.70	F-S	0.78	0.52	0.50	0.62	0.78	1.83	1.57	1.80	0.83	F

^A CON, SJ and UJ refer to control RC beams and RC beams with side and U-shaped jackets, respectively; ^B F and S refer to flexural and shear failure modes, respectively; ^C EC2 (BSI, 2014); ^D fib MC 2010 LoAIII (fib, 2013); ^E ACI 318-19 (ACI, 2019); ^F MASM (Cladera et al., 2016; Mari et al., 2015, 2016); ^G EC2 and MASM contribution of flange; ^H NF P18-710 (AFNOR, 2016b); ^I Calculated ultimate shear strength of strengthened specimens includes contribution of RC beam given by EC2 with contribution of flange given by MASM; ^J SIA 2052 (EPFL, 2016); ^K JSCE 2006 (JSCE, 2006); ^L Predicted failure mode based on lesser value between calculated shear (EC2 or EC2 with MASM and SIA 2052) and flexural strength; ^M Shear strength includes contribution of transverse reinforcement given by NF P18-710 as JSCE 2006 does not provide guidance for calculating the shear strength of UHPC members with transverse reinforcement

APPENDIX B: TEST DATABASE OF BEAMS FROM THE LITERATURE

Table B.3 – Experimental and calculated ultimate shear and flexural strength of beams in database according to original and proposed shear design methods

Author	Specimen		Experimental		Calculated (Original) ^A			Calculated (Proposed) ^B			Calculated
	ID	Type ^C	Ultimate force (P_{exp}) [kN]	Failure mode ^D	Ultimate shear strength (P_{calc}/P_{exp})		Failure mode ^D	Ultimate shear strength (P_{calc}/P_{exp})		Failure mode ^D	Ult. flexural strength (P_{calc}/P_{exp})
					EC2 and MASM	EC2, MASM and SIA 2052		EC2	EC2 and SIA 2052		
Alkhoury (2017)	Nr. 5	CON	270.00	S	1.19	-	S	0.84	-	S	1.26
	Nr. 6	UJ	820.00	S	0.40	1.26	F	0.19	1.05	S	1.24
	Nr. 7	UJ	1000.00	S	0.32	1.22	S	0.16	1.06	S	1.36
Al-Osta et al. (2017)	RC-Control 1	CON	70.00	F	8.49	-	F	8.49	-	F	0.92
	RC-Control 2	CON	70.00	F	8.49	-	F	8.49	-	F	0.92
	RC-SB-2SJ	SJ	102.00	F	5.83	8.41	F	5.83	8.41	F	1.00
	RC-EP-2SJ	SJ	95.00	F	6.26	9.03	F	6.26	9.03	F	1.08
	RC-SB-3SJ	UJ	132.00	F	4.50	6.76	F	4.50	6.76	F	1.02
	RC-EP-3SJ	UJ	129.00	F	4.61	6.92	F	4.61	6.92	F	1.04
Bahraq et al. (2019)	CT-2.0	CON	276.00	S	1.60	-	F	1.60	-	S	1.24
	SB-2SJ-2.0	SJ	346.00	F-S	1.28	2.22	F	0.88	1.83	F	1.20
	SB-3SJ-2.0	UJ	353.00	F	1.25	2.30	F	0.87	1.92	F	1.34
Daugevičius et al. (2015)	S4U	UJ	174.53	F-S	1.47	2.26	F	1.47	2.26	F	0.97
	S5U	UJ	187.63	F-S	1.36	2.10	F	1.37	2.10	F	0.90
	S6U	UJ	173.53	F-S	1.48	2.27	F	1.48	2.27	F	0.97
	S7	CON	114.50	F	2.24	-	F	2.24	-	F	1.04
	S8	CON	117.13	F	2.19	-	F	2.19	-	F	1.02
	S9	CON	117.73	F	2.18	-	F	2.18	-	F	1.01

APPENDIX B: TEST DATABASE OF BEAMS FROM THE LITERATURE

Author	Specimen		Experimental		Calculated (Original) ^A			Calculated (Proposed) ^B			Calculated
	ID	Type ^C	Ultimate force (P_{exp}) [kN]	Failure mode ^D	Ultimate shear strength (P_{calc}/P_{exp})		Failure mode ^D	Ultimate shear strength (P_{calc}/P_{exp})		Failure mode ^D	Ult. flexural strength (P_{calc}/P_{exp})
					EC2 and MASM	EC2, MASM and SIA 2052		EC2	EC2 and SIA 2052		
Choi (2016)	CON	CON	375.40	S	1.96	-	F	1.72	-	F	1.54
	U2	UJ	471.80	F-S	1.56	2.26	F	1.20	1.90	F	1.36
	U4	UJ	594.20	F	1.24	2.42	F	0.95	2.14	F	1.19
Liu and Charron (2023)	TS-R	CON	352.10	S	0.78	-	S	0.47	-	S	2.14
	TS-W25	SJ	710.00	S	0.39	1.18	F	0.16	0.95	S	1.12
	TS-W50	SJ	748.50	S	0.37	1.87	F	0.15	1.66	F	1.11
	TS-W50A	SJ	759.10	F-S	0.36	1.85	F	0.15	1.63	F	1.10
	TS-U25	UJ	652.20	S	0.42	1.34	F	0.18	1.09	S	1.29
	TS-U50	UJ	677.40	S	0.41	2.28	F	0.17	2.04	F	1.38
	TS-U50A	UJ	819.60	S	0.34	1.88	F	0.14	1.69	F	1.14
Martinola et al. (2010)	R/C	CON	190.00	F	3.64	-	F	3.64	-	F	0.86
	R/C str.	UJ	410.00	F	1.69	5.08	F	1.17	4.56	F	1.27
	R/C rep.	UJ	365.00	F	1.90	5.71	F	1.31	5.12	F	1.43
Paschalis (2017)	P1	CON	55.18	F	4.42	-	F	4.42	-	F	0.72
	P2	CON	53.98	F	4.52	-	F	4.52	-	F	0.73
	3SJ1	UJ	119.20	F	2.05	7.31	F	2.05	7.31	F	1.24
	3SJ2	UJ	112.00	F	2.18	7.78	F	2.18	7.78	F	1.32
Wang et al. (2013)	N2.5	CON	128.00	S	0.43	-	S	0.43	-	S	1.05
	U2.5	SJ	144.00	F	0.38	1.07	F	0.26	0.98	S	1.06

APPENDIX B: TEST DATABASE OF BEAMS FROM THE LITERATURE

Author	Specimen		Experimental		Calculated (Original) ^A			Calculated (Proposed) ^B			Calculated
	ID	Type ^C	Ultimate force (P_{exp}) [kN]	Failure mode ^D	Ultimate shear strength (P_{calc}/P_{exp})		Failure mode ^D	Ultimate shear strength (P_{calc}/P_{exp})		Failure mode ^D	Ult. flexural strength (P_{calc}/P_{exp})
					EC2 and MASM	EC2, MASM and SIA 2052		EC2	EC2 and SIA 2052		
Hong and Lim (2023)	CS	CON	506.10	S	1.47	-	F	1.47	-	F	1.11
	S1.5T2	SJ	608.90	F	1.22	1.93	F	0.84	1.55	F	1.08
	S1.5T4	SJ	555.50	F	1.34	2.89	F	0.93	2.48	F	1.31
	S1.5T6	SJ	685.30	F	1.08	2.97	F	0.75	2.64	F	1.15
	U1.5T2	UJ	659.50	F	1.13	1.81	F	0.78	1.47	F	1.04
	U1.5T4	UJ	835.60	F-S	0.89	2.03	F	0.62	1.75	F	0.96
	U1.5T6	UJ	798.60	F	0.93	2.80	F	0.64	2.51	F	1.16
	S0.5T2	SJ	545.60	F	1.36	1.76	F	0.94	1.34	F	1.17
	S0.5T4	SJ	493.90	F	1.50	2.39	F	1.04	1.93	F	1.39
	S0.5T6	SJ	592.70	F	1.25	2.36	F	0.87	1.97	F	1.22
	U0.5T2	UJ	616.00	F-S	1.21	1.58	F	0.83	1.21	F	1.06
	U0.5T4	UJ	803.20	F-S	0.92	1.52	F	0.64	1.24	F	0.90
	U0.5T6	UJ	952.70	F-S	0.78	1.57	F	0.54	1.33	F	0.83

^A Original combined EC2, MASM and SIA 2052 method with different truss angles for the transverse and fibre reinforcement truss models (BSI, 2014; Cladera et al., 2016; EPFL, 2016; Mari et al., 2015, 2016);

^B Proposed combined EC2 and SIA 2052 method with the same truss angle for the transverse and fibre reinforcement truss models, and control RC beams calculated using EC2 with minimum truss angle (BSI, 2014; EPFL, 2016); ^C CON, SJ and UJ refer to control RC beams and RC beams with side and U-shaped jackets, respectively; ^D F and S refer to flexural and shear failure modes, respectively

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2.18	82	Baumann, T. and Rüsch, H. (1970) <i>Tests for the analysis of dowel action of flexural reinforcement of reinforced concrete beams (in German: Versuche zum Studium der Verdübelungswirkung der Biegezugbewehrung eines Stahlbetonbalkens)</i> . Berlin, Germany, Ernst & Sohn.	German Committee for Structural Concrete (DAfStb)	25/10/2023	Yes	Written permission by Prof. Udo Wiens on 25/10/2023
2.19	83	Huber, P., Huber, T. and Kollegger, J. (2016) Investigation of the shear behavior of RC beams on the basis of measured crack kinematics. <i>Engineering Structures</i> . 113, pp. 41–58. doi:10.1016/j.engstruct.2016.01.025.	Elsevier	25/10/2023	Yes	25/10/2023 License number 5655950541561

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2.21	87	He, X.G. and Kwan, A.K.H. (2001) Modeling dowel action of reinforcement bars for finite element analysis of concrete structures. <i>Computers & Structures</i> . 79(6), pp. 595–604. doi:10.1016/S0045-7949(00)00158-9.	Elsevier	25/10/2023	Yes	25/10/2023 License number 5655970328789
2.22	88	Campana, S., Ruiz, M.F., Anastasi, A. and Muttoni, A. (2013) Analysis of shear-transfer actions on one-way RC members based on measured cracking pattern and failure kinematics. <i>Magazine of Concrete Research</i> . 65(6), pp. 386–404. doi:10.1680/mac.12.00142.	Emerald Publishing Limited	26/10/2023	Yes	26/10/2023 License number 1410729-1
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2.24	92	Cavagnis, F., Fernández Ruiz, M. and Muttoni, A. (2018a) A mechanical model for failures in shear of members without transverse reinforcement based on development of a critical shear crack. <i>Engineering Structures</i> . 157, pp. 300–315. doi:10.1016/j.engstruct.2017.12.004.	Elsevier	25/10/2023	Yes	25/10/2023 License number 5655970701767
2.26	100	Cladera, A., Marí, A., Bairán, J.M., Ribas, C., Oller, E. and Duarte, N. (2016) The compression chord capacity model for the shear design and assessment of reinforced and prestressed concrete beams. <i>Structural Concrete</i> . 17(6), pp. 1017–1032. doi:10.1002/suco.201500214.	John Wiley and Sons	25/10/2023	Yes	25/10/2023 License number 5655970909108
3.28	152	Espeche, A.D. and León, J. (2011) Estimation of bond strength envelopes for old-to-new concrete interfaces based on a cylinder splitting test. <i>Construction and Building Materials</i> . 25(3), pp. 1222–1235. doi:10.1016/j.conbuildmat.2010.09.032.	Elsevier	25/10/2023	Yes	25/10/2023 License number 5655971065747

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