

SOME FIELD EXPERIMENTS ON
SEISMIC WAVE PROPAGATION.

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by

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ABSTRACT

Some small scale, near source seismic field experiments are described. A dropping weight source has been used for greater reproducibility, and particular attention was paid to the accuracy of recording the true ground motion. Geophones and amplifiers were designed and constructed so that the overall frequency response of the recording system, including the geophone-ground coupling, was substantially flat over the requisite frequency range (2.5 c/s to 150 c/s). An accurate versatile timing unit was constructed and adapted for use as a portable apparatus suitable for seismic refraction measurements.

Controlled experiments were performed to study a number of aspects of seismic wave propagation.

The results are particularly concerned with motion near the source and the frequency content and efficiency of different sources. Details are also given of dispersion and attenuation studies of the surface waves at a site situated on the London Clay. The phase velocity curve which was observed gave good agreement with the theoretical curve corresponding to the known conditions. The possibility of the use of surface waves for seismic prospecting is considered.

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Table I

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C H A P T E R 1
I N T R O D U C T I O N

1.1 Historical background

Most of the basic theory of seismic wave propagation was developed in the 19th and early 20th centuries, and was used to explain the main features of earthquake seismograms.

Poisson (1829) was the first to predict the existence of two types of plane waves in a homogeneous infinite elastic medium, and these waves were later shown by Stokes (1849) to be irrotational dilatation, or compressional, P waves, and equivoluminal distortion, or shear, S waves. Poisson's work was possible as a result of the development of the theory of elasticity from the initial enquiries of Galileo (1638) and Hooke (1678) in the 17th century to the situation in the 1820's. With the beginnings of definite methods of induction and hypothesis in the physical sciences, a fresh stimulus was provided by the introduction of Fresnel's (1821) theory of the transverse vibrations of light travelling through a medium. Navier (1827) gave the general differential equations of the equilibrium of an isotropic elastic solid, and Cauchy (1823,27) and later Poisson (1829) gave the equations of motion (see Love 1906).

Considering a semi-infinite elastic medium, Rayleigh (1885) showed that a plane wave may be propagated over the free surface with the amplitude of wave motion decreasing with depth. After the beginning of the routine recording of earthquakes in 1889, Oldham (1900) recognised on a seismogram the compressional, shear and Rayleigh waves which had been theoretically predicted. Lamb (1904) gave the solution for an impulsive disturbance applied along a line or at a point on the surface of a homogeneous semi-infinite elastic medium. He also gave solutions for internal sources as integrals which were later developed by Nakano (1925) and Lapwood (1949).

It was obvious that actual seismograms differed greatly from the simple P, S and R pulses following one another as given by Lamb's theory, in that each pulse was spread out into a train of waves, although there was usually a noticeable change in the character of the records roughly at the times expected (see Fig. 1). Knott (1908) and Zoppritz (1907) also noticed that there was considerable horizontal motion associated with the Rayleigh waves which had not been theoretically predicted. Love (1911) showed that some of these discrepancies could be quite well explained by allowing for the effect of a layer in the upper part of the medium. This model was

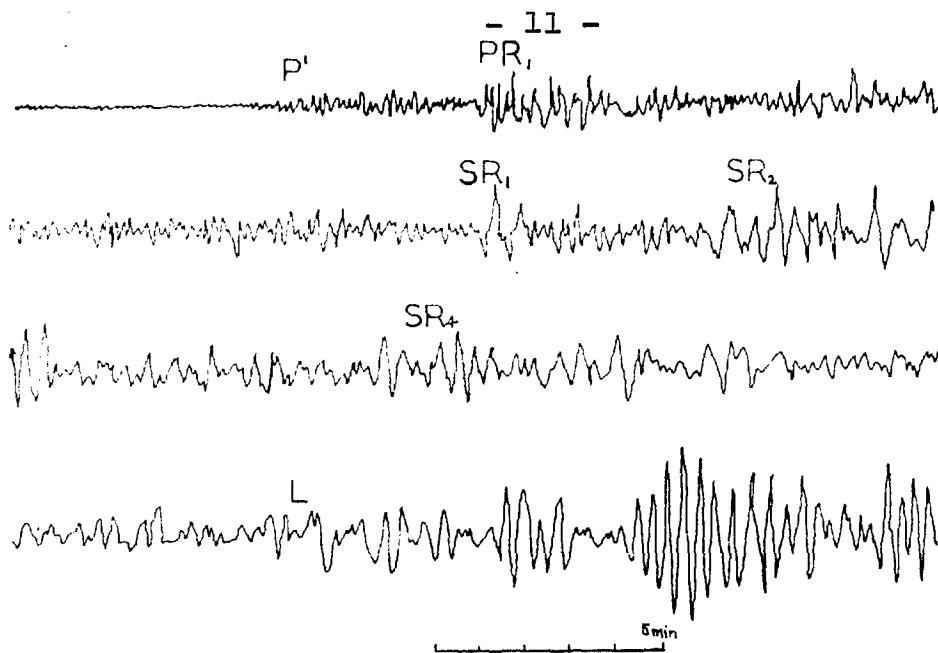


FIG. 1— A TYPICAL EARTHQUAKE SEISMOGRAM
SHOWING BODY (P & S) AND SURFACE (L) WAVES
(from B.S.S.A., vol. 26, p. 281, 1936)
R refers to reflections

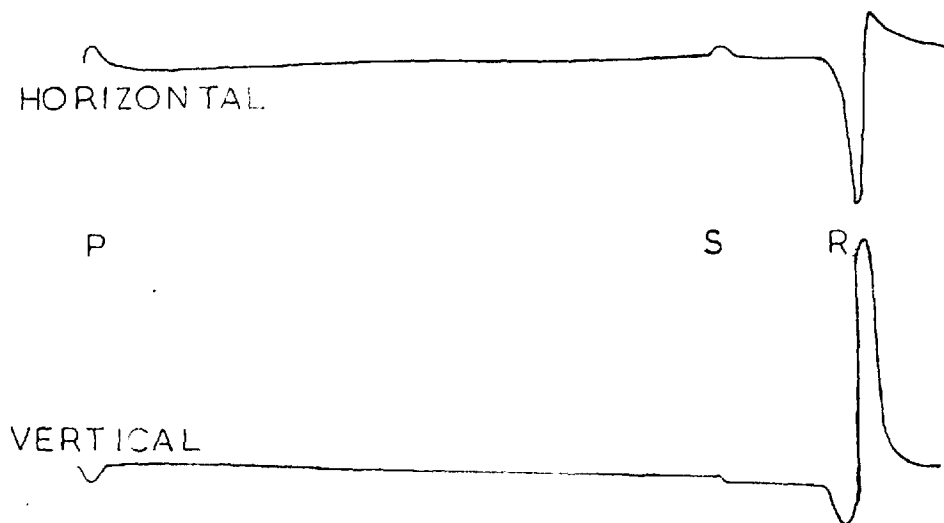


FIG 2— P, S & R PULSES, PREDICTED BY LAMB'S
THEORY DUE TO A DISTANT POINT IMPULSE
ON THE SURFACE.

(from Ewing, Jardetzky & Press, 1957 p. 64.
see also Sherwood, University of London, PhD Thesis, 1956, p 11
for details of scale.)

thought to be a better approximation to the conditions in the earth as a result of the observations of Mohorovicic (1909) indicating the existence of the crust. Love showed that surface waves, having a horizontal transverse wave motion, were possible provided that the shear wave velocity was greater in the lower medium. He further showed that the waves would be dispersive, and predicted that this was also true for Rayleigh waves in a one-layered semi-infinite medium: this difficult case was treated in greater detail by Jeffreys (1935). The transverse motion associated with the surface waves was thus explained, together with the broadening of the surface wave pulses due to dispersion, eventually giving the appearance of almost sinusoidal wave trains.

If allowance is further made for multiple shocks (Stoneley 1937), absorption of the higher frequencies (Jeffreys 1931a), curvature of the earth, the effects of more than one surface layer (Stoneley and Tillotson 1928, Jeffreys 1931b, Stoneley 1937, Newlands 1950 and Dorman 1959) and the presence of a low velocity stratum (Stoneley 1950), it is possible to give a very reasonable qualitative description of earthquake seismograms (see Jeffreys 1952 or Bullen 1959). One of the main features making a quantitative description difficult is the error introduced

by the recording technique, as most seismographs only respond to a narrow band of the spectrum of the actual ground motion. The two most important unexplained features on earthquake seismograms are the broadening of the body wave pulses to continuous wave trains, and the existence of the coda continuing some time after the surface waves (Jeffreys 1952, page 99). Jeffreys (1931c) considered a number of possible explanations of the body wave broadening and concluded that the effect of multiple reflections in the surface layers was a possible explanation, but later decided that, due to the local variation in record character, this was unlikely to be the complete explanation and that local scattering effects may be equally important. (see Jeffreys 1952, pages 99 and 109).

By 1914 a fairly comprehensive picture had been built up of the internal structure of the earth as a central core with an outer mantle of about the same radial thickness (about 3000 kms) covered by a crust varying in thickness from about 4 kms to about 60 kms, usually covered by sediments. Later, two, or possibly three, discontinuities in the crust were recognised; the core was shown to be liquid with a velocity discontinuity at about half its radial thickness and, with the determination of the

velocity distribution of P and S waves throughout the earth, it became possible to construct travel time curves for the various possible earthquake phases. Layers less than the thickness of the minimum wavelength on an earthquake record (about 1 km) are unlikely to affect the records, so that the sedimentary layers and local variations in crustal structure can usually be neglected and the standard time curves used throughout the world. Then the main problem in interpretation is not so much that of finding the subsurface structure but that of finding the time and position of the initial shock. ~~§~~ This is done by measuring the difference between the P and S arrival times at a number of stations and correlating the data from the different stations. Crustal structure is investigated by means of special studies on surface waves or near earthquakes.

1.2 Seismic surveying

The situation is very different for seismic surveying, a term which will be used to cover the seismic study of the sedimentary layers using artificial shocks. The position and time of the source are accurately known and the problem is to delineate the subsurface structure. Apart from the knowledge of the source, the problem is much

more complicated than that for earthquake seismology for a number of reasons. Firstly there are a large number of subsurface discontinuities of velocity and it is impossible to construct standard travel time curves, although an approximate approach to the problem may be made from well-log surveys. Secondly, the surface waves cover a large part of the record, necessitating filtering and consequent broadening of the body wave pulses. Thirdly, the discontinuities are so close that it is difficult to get a source with sufficient short wavelength energy to resolve them, due to the greater absorption of the higher frequencies by the earth.

As a result of the great interest, particularly of the oil companies, in the sedimentary layers, considerable effort has been expended in developing highly efficient recording techniques suitable for the higher frequency range, for recording the results of the seismic surveys. As surface waves and refraction methods only give information about two or three layers without excessive theoretical and experimental complication, reflection methods have mainly been used for examining many-layered structures. The seismic method has met with a reasonable degree of success, particularly in the early days when structures were simple enough to use refraction methods,

and has justified an increasing large expenditure. However, the situation has arisen when most of the large potentially oil-bearing structures have been found and it has become necessary to concentrate more on the smaller structures such as stratigraphic traps, etc. It has been suggested that the cost of seismic prospecting for oil has recently been approaching that of the amount saved by its use, which has led to a decline in the rate of expansion of the use of the seismic method by the oil industry (Clifford, 1958).

At the present time, apart from the use of deep boreholes which are expensive, the seismic method is still the most effective means of obtaining information of the deep structure of the sedimentary layers. However, since the introduction of the method in about 1925, the main advances have been instrumental rather than in the general principles of the method. Time and distance are still very largely the only variables which are used.

1.3 Summary of the present situation

It is obvious therefore that there is a need for research directed towards improving the general method of seismic prospecting for oil. A large amount of work is being done in America, where the main emphasis at the

moment seems to be on statistical correlation techniques and synthetic seismograms, and also in Russia and Europe where the emphasis is more on the use of record character as another useful parameter in addition to time and distance although there is also considerable interest in synthetic seismograms. There is also need for research in other seismological fields, for instance in determining the effects and detectability of atomic bombs and in the greater use of earthquake seismological records in general.

The basic theory of wave propagation needed for seismic surveying has of course been taken directly from the early work of earthquake seismologists, who used the classical theory of elasticity as a starting point, as already outlined. For the reasons stated above, there has been little emphasis on the theory or use of surface waves, and the majority of the theoretical and experimental advances of prospecting seismologists with regard to seismic wave propagation have been connected with body waves. In particular, conditions at the source, usually an explosive one, have been studied together with the effect of absorption which is important at the frequencies used in seismic prospecting, since it causes a broadening and change of shape of the pulses. (For general references see Anstey (1958), Soper (1957) and Ewing, Jardetzky and Press (1957).)

However, the theories have been developed well beyond their experimental verification, while at the same time there exists a number of features on seismograms which cannot be adequately explained. Probably the main reason for this situation is that the records are complicated because of the large number of variables in the experimental conditions and the recording technique, all of which may influence the record. A direct analysis of the records for any particular investigation is usually impossible, and the experimental conditions are usually not sufficiently well known to allow quantitative corrections to be applied for variables other than those being studied. The corrections, if they were possible, would be very tedious.

1.4 Aims of the present work

It is therefore obvious that an experimental technique is needed in which, firstly, variables may be controlled so that their effects may be studied, and secondly, the actual ground motion may be accurately measured, thereby removing the complication of instrumental distortion. The aim of the present work has been to develop and test apparatus for this purpose, and to carry out a number of controlled experiments designed to give a better understanding of seismic wave propagation, both as a matter of general

academic interest, and more particularly with a view to improving the seismic method of prospecting.

Model studies provide good control of experimental conditions, but involve the use of frequencies and materials not normally used in seismic surveys. Also, the source and receiver are not easily scaled down, and as a result it has been found in the past that model studies have been of limited usefulness. It was therefore decided to carry out a field study on a small scale, as the records closely resemble, in many aspects, those made at greater distances, but the operations involved are less expensive. A dropping weight source was chosen because it has been found to be reproducible, inexpensive and to give a pulse with a spectrum (2 to 250 c/s) roughly in the range found in seismic prospecting records using explosive sources at greater distances. With a view to simplicity and greater control, a uniform medium was sought and it was decided to use a 400' thick bed of London Clay near Ascot, Berkshire. The medium was thick enough and surprisingly uniform, with the water table almost at surface level. The first experiments were carried out on a two layered medium (about 25 feet of sand and 20 feet of saturated sand overlying the clay) as this had been used previously (Mason 1950) and was convenient for near source studies when the presence of water would have been a hindrance.

In the following chapters, details are given of the apparatus developed to record the ground motion, the testing of this apparatus, various field experimental techniques used and the results of a number of experiments using this apparatus.

The present work is intended as the beginning of a series of small scale field studies of seismic wave propagation, to be continued by D. Jenkinson in the Geophysics Department of Imperial College, using harder rocks.

The following is a list of some of the topics which may be studied, all of which may influence the character of a seismogram. They may be divided into those connected with the source, transmission and recording of the seismic waves.

Source

1. Generation of waves.
2. Percentage of wave types.
3. Pulse shapes for various sources.
4. Cavity effects and other shothole conditions; area of non-elastic behaviour.
5. Laws of attenuation in the immediate vicinity of the source.
6. Seismic efficiency of various sources.

Transmission

1. Change of pulse shape of the different wave types with distance.
2. Laws of reflection and refraction.
3. Dispersion of body and surface waves. More particularly, investigation of whether there is significant body wave dispersion. Also the usefulness of surface wave dispersion in studying subsurface structure.
4. Mechanism of attenuation and various models of earth friction.
5. Diffraction, scattering.

Recording (and interpretation)

1. Use of known ground pulses to test the overall response of the system.
2. Various methods of sorting out wave types and clarifying the record without distortion.
3. Design of low frequency, broad band, light weight borehole geophones.
4. Greater use of the information contained in the records.

These topics have been considered in varying degrees of detail as described in the following chapters.

CHAPTER 2

APPARATUS

2.1 Introduction

As stated in Chapter 1, the main requirement of the recording apparatus is that it should provide an undistorted record of the ground motion. The usual requirements that the record should be clear, with accurate time marks and sufficient signal to noise ratio, still hold.

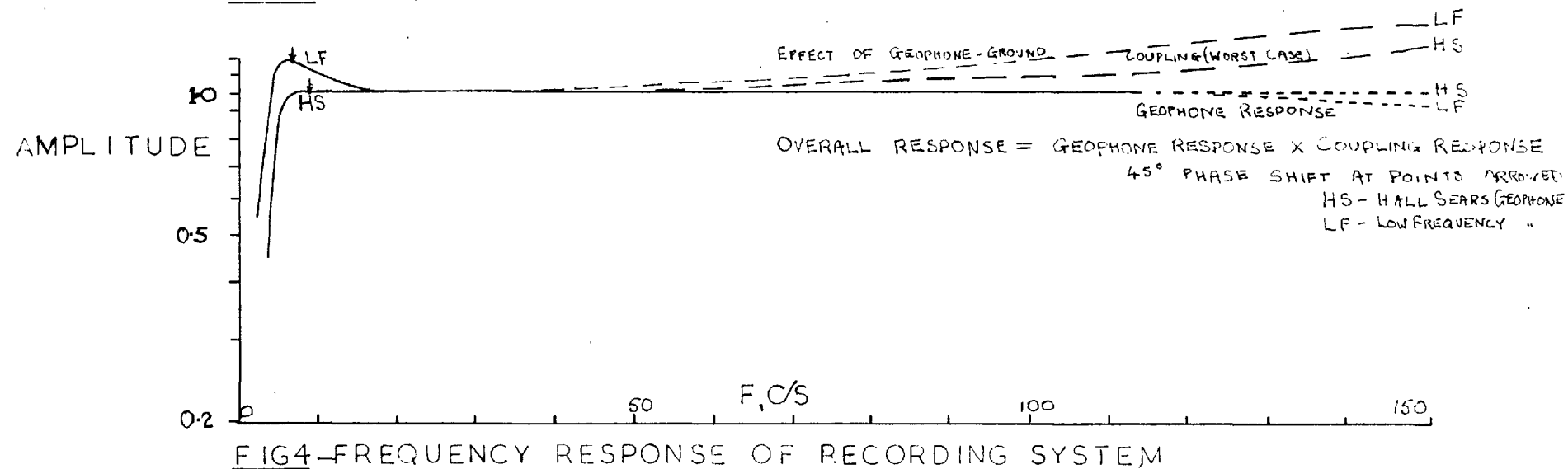
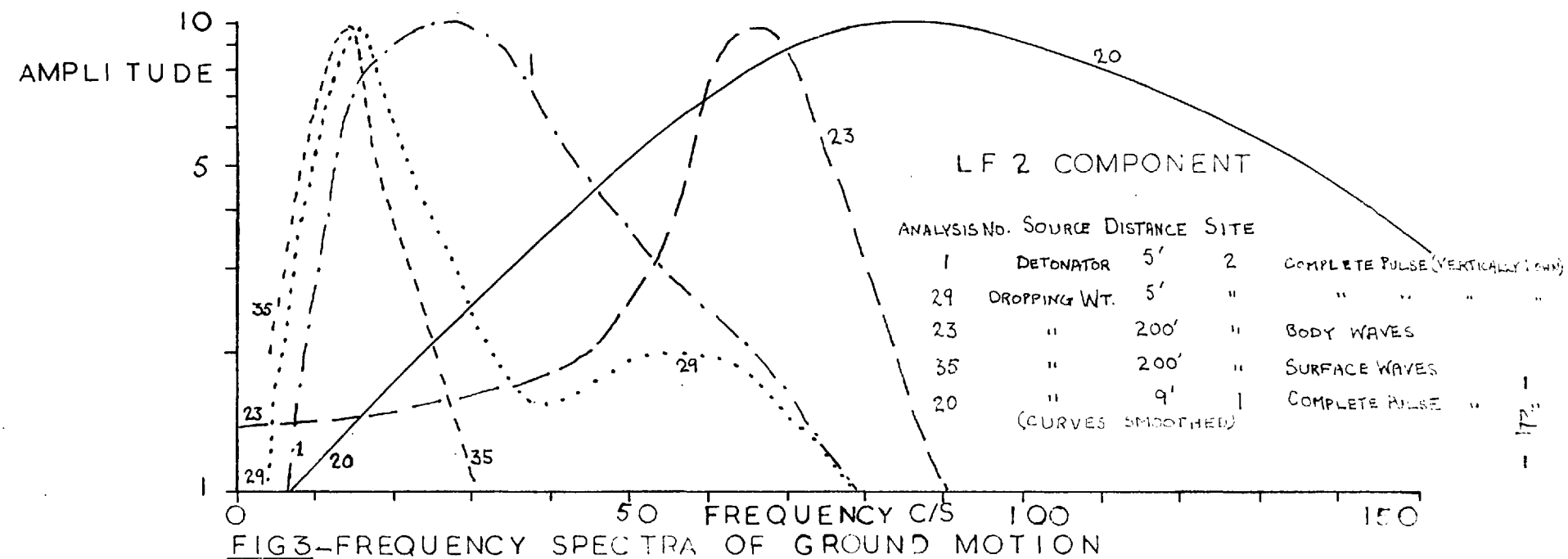
The requirement of accurate recording of the ground motion is equivalent to demanding that the amplitude frequency responses of each unit of the recording system, and the coupling between each unit, should be flat with zero phase shift over the range of frequencies covered by the spectra of the possible ground motions to be recorded. As the motions are transient, the spectra theoretically cover an infinite frequency band, but it was decided that a sufficiently accurate criterion would be as follows. The over-all frequency response should be within 3 d.b. (70 per cent) of the maximum at the upper and lower frequency limits, which would be defined as the frequencies at which the spectrum of the ground motion had dropped to 10 per cent of its maximum frequency component.

For dispersion and attenuation measurements, a flat

response is not essential, as ratios of amplitudes and differences of phases at various points are considered. It is important however that the response should not change at the different measuring points, and also that there is sufficient response at the frequencies of interest to give adequate accuracy of measurement. The best way of achieving this is to have a flat frequency response and to move the source rather than the geophone for consistency of response.

Figure 3 shows the spectrum of the ground motion which was expected, using the results of Fourier analyses of pulses very close to the source and at 200' from the source. These theoretically contain all the frequencies of interest, although the distribution of frequency components will move towards the lower frequency end with increasing distance from the source. Figure 4 shows the over-all frequency response of the recording system finally adopted, which is practically the geophone response limited at the upper frequency end by the geophone-ground coupling, and is seen to satisfy the above requirements. Figure 5 is a diagrammatic representation of the layout of the recording apparatus and Figures 6 and 7 show the equipment used for recording, and the weight dropping source.

A detailed discussion of the methods used to obtain a flat frequency response of each unit of the recording apparatus is given below. It will be seen that a satisfactory geophone response is the most difficult to achieve.



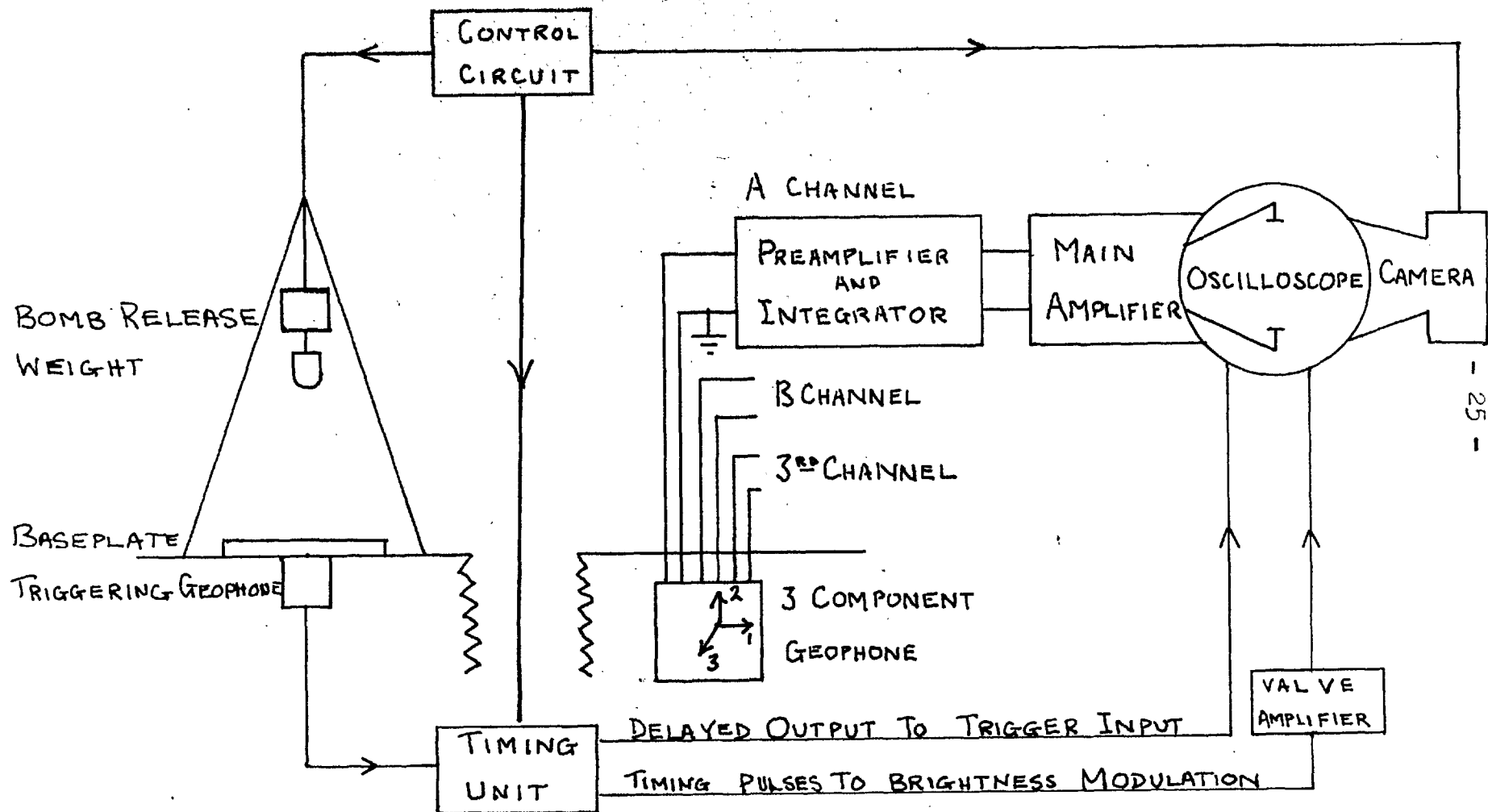


FIGURE 5 - LAY-OUT OF APPARATUS.

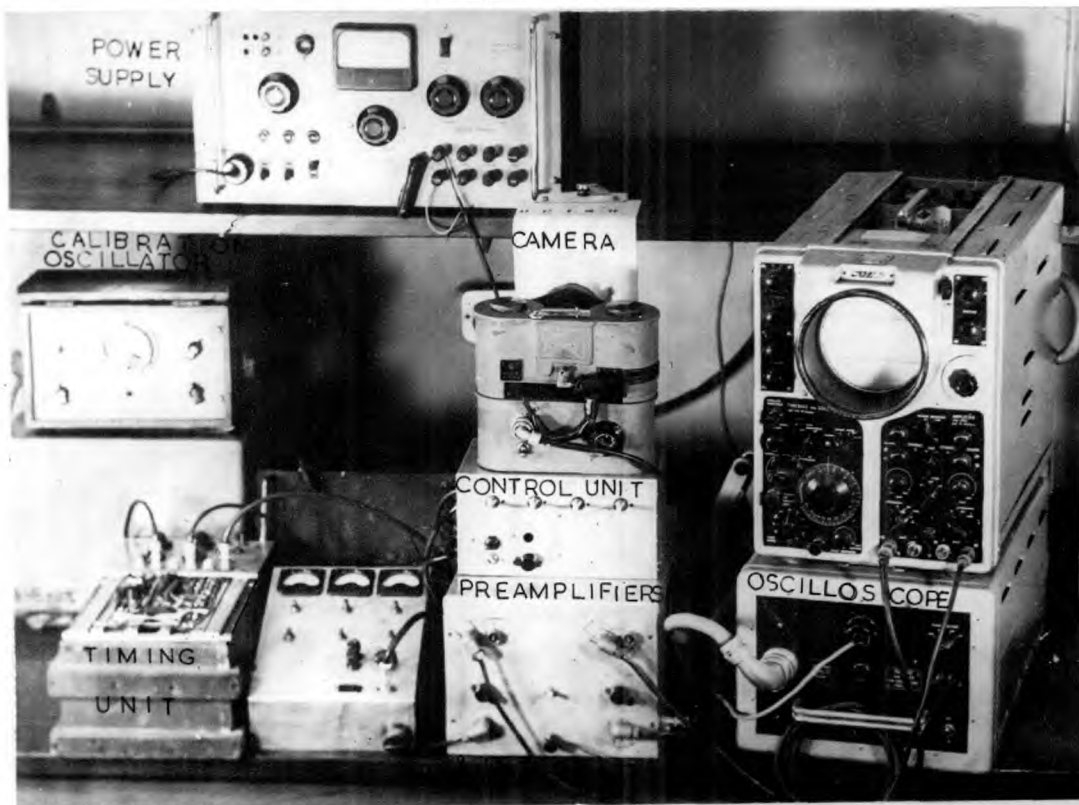


FIG. 6 — RECORDING APPARATUS



FIG 7 — WEIGHT DROPPING SOURCE

Of the couplings between the units, only that between the ground and geophone is likely to lead to error, as the remainder are electronic with an optical link between the oscilloscope spot and the recording paper. The question of geophone-ground coupling is dealt with in connection with geophone design.

Apparatus built by Mason (1951), for similar work was available and used for part of the preliminary experiments, but eventually was replaced due to inadequacy, poor state of preservation or general advances in instrumentation since its construction. The weight dropping equipment and calibration oscillator were retained for most of the experiments, but the weight release mechanism was eventually replaced due to wear.

2.2 Recording apparatus

The most important part of the recording apparatus is the instrument used to respond to the ground motion. This is variously called a pick-up, seismometer, geophone, seismograph, detector or various other names. Geophone is used for an electromagnetic instrument used for seismic surveying and seismograph for an earthquake seismological instrument. The means of magnifying and permanently recording the detector output is usually of less importance. It was decided to use geophones for the present work and as they respond to velocity, integrators were provided to give ground displacement if needed. It is

arguable that the integrators are unnecessary as displacement is no more diagnostic than ground velocity, but they were eventually retained in spite of the extra instrumental complication, as they help to simplify records. A single displacement pulse with one maximum has two velocity maxima (although a train of waves will not be simplified to the same extent). Also much of the theory is given in terms of displacement and the necessary differentiation is avoided.

Oscilloscopic display was chosen in preference to the use of a galvanometer because of its linear frequency response at high frequencies (in this case to 250 Kc/s.) A disadvantage is the limitation of the number of possible recording channels and extra bulk, but these are outweighed by the facts that only two or three geophones were available and that an oscilloscope was needed for other problems connected with the research. Expense precluded the use of the ideal system of a multichannel frequency-modulated magnetic tape recorder.

2.21 Geophones

Piezoelectric or electromagnetic detectors are usually used for seismic surveys as they are less bulky than seismographs and suited to the higher frequencies involved. Although it is perhaps easier to get good frequency response with piezoelectric crystals, none were known which gave sufficient output to make the pick-up sensitive enough in view of the low source energy and relatively low frequencies.

With future developments it may be well worth considering their use, but for the present work it was decided to use geophones.

2.211 Geophone requirements. There are a number of factors to be considered in geophone design, some of which give mutually contradictory requirements. The following are the most important:

- (a) Frequency response needs to be within 3 d.b. between about 2.5c/s and 150 c/s. These determine f_0 and f_1 , the geophone lower and upper natural frequencies.
- (b) The sensitivity K needs to be such that the electrical output $K \times \bar{u}_{\min}$ due to the minimum average ground noise level $\bar{u}_{\min}$ is of the order of the input noise level V_n of the pre-amplifier, (with geophone connected). i.e.
$$K \times \bar{u}_{\min} \sim V_n.$$

It is found that the electronic noise level is not less than 3 μ v peak to peak (p.p), and at the field sites at night time the minimum ground velocity is about 3×10^{-5} cm./sec. p.p. In some areas however it may be as low as 10^{-6} cm./sec. p.p. (Wilson, 1953, and Wolfe, 1942).

Hence it is necessary to have $K \sim 0.1$ V/cm./sec.

It was decided to make $K \sim 0.5$ v/cm./sec. if possible for use in quieter areas and also for greater definition

at night.

(c) Each geophone should be sufficiently small to enable it to go easily down a $4\frac{1}{2}$ " bore-hole, in case it is decided to make a bore-hole geophone in the future.

(d) The geophone should be watertight and fairly rugged, but need not be as rugged as commercial geophones, being intended solely for research purposes.

(e) The coil resistance, R_g , in parallel with any damping resistance, R_L , which may be used, should not be greater than 10 per cent of the input impedance of the preamplifier, which in this case means less than 2,000 ohms.

(f) For good geophone-ground coupling, the ratio of the weight to base area should satisfy the criteria given by Bycroft (1957). He has considered, 'the magnification caused by the partial resonance of the ground detector system'. Washburn and Wiley (1941) and Pasechnik (1952) have dealt with the problem in a more practical way and demonstrated its importance, but Bycroft's more thorough mathematical treatment was used when considering geophone design.

Bycroft gives the magnification M as

$$M = \frac{1}{[(1 + b a_0^2 f_1^2)^2 + (b a_0^2 f_2^2)^2]^{1/2}}$$

where f_1 and f_2 are the in phase and out of phase components of the ground displacement and are given in his

paper as functions of τ . The dimensionless parameters b, a_0 and τ are defined in terms of the shear and compressional wavelengths, λ_s and λ_p , as shown below.

$$b = \frac{m_0}{\rho r_0^3} \quad r_0 = \text{radius of the base plate}$$

$$a_0 = k r_0 = \frac{2\pi}{\lambda_s} \cdot r_0 \quad m_0 = \text{mass of geophone}$$

$$\text{and } \tau = \frac{\lambda_s}{\lambda_p} \text{ taken as } \frac{1}{\sqrt{3}}. \quad \rho = \text{ground density}$$

It is obvious that the minimum values of ground density and shear wavelength are the critical factors when deciding on geophone weight and overall physical dimensions for good geophone-ground coupling according to Bycroft. Also, from a common sense point of view, it would seem that, although a geophone can never measure what the motion would have been had the geophone not been at the measuring point, its motion will be a good approximation to this if its dimensions are small compared with the smallest wavelength to be measured, and its density is similar to that of the surrounding medium (so that it moves with the medium). These criteria are satisfied approximately (see table 1).

Fourier analyses of pulses recorded a few feet from sources at sites 1 and 2 indicate that the upper frequency limits of the pulses are about 200 c/s and 100 c/s respectively (see Fig. 3 and also Fig. 8 page 65, which shows velocity and displacement measurements of analysis 29, Fig. 3 in another context). It may be argued that the

recording system used introduced these limits rather than the characteristics of the pulse, but it seems that this is not likely to be so for the following reasons. Firstly there is a continuous fall from the peak near the low frequency end of the spectrum without any sharp cut off or resonance. In the latter case a resonance would appear in the spectra of all pulses. Secondly, different geophones give similar results, and thirdly, the spectra at site 2 are similar to those at site 1 but are centred on a lower frequency. This indicates that, as the system could record higher frequencies at site 1, ^(see p.93 for a description of sites) it should have been able to record them at site 2, with a suitably planted geophone, and their absence on the records indicates that they were not present in the true ground motion.

Table 1 gives the characteristics of the two three-component geophones used for most of the experiments and it may be seen that for the low frequency geophone (LF3C) $b = 5$ and for the Hall-Sears (HS3C) geophone $b = 30$. The HS geophone therefore may be expected to have a sharper resonance, but at a higher frequency. The values of minimum shear wavelength for 10 per cent amplitude magnification are 10' and 12' respectively, and their values for 45° phase shift are 3' and 2' respectively. Hence if the minimum shear wave velocity is taken as 500 ft./sec., there will be a 45° phase shift at about 200 c/s. As described later in section 3.32, when the geophones are placed

TABLE 1

Geophone	L.F.	H.S.
<u>Each component</u> (The range of variation between components is stated for L.F.)		
Coil resistance, R_g (ohms)	195 ± 4	215
Damping resistor used, R_L (ohms)	None (other than amplifier input)	512
Suspended mass, m (gms.)	182 ± 2	28.5
Lower natural frequency, f_0 (c/s) (as originally constructed)	$3 \pm .03$	4.5
Upper natural frequency, f_1 (c/s)	26 ± 1	110
Damping constant, h .	0.43 ± 0.15	0.60
Inductance, L	see Fig.28 (p.83)	2 μ H at 1 Kc/s (manufacturer's data)
Number of turns in coil, N	1670	1200
Wire used for coil	44 swg	no.38
Sensitivity, K (v./cm./sec.)	$0.50 \pm .01$	0.17 (with R_L)
<u>Three component unit</u>		
Mass, m_0 (lbs.)	35	6
Density (lbs./cu.ft. Multiply by 0.016 for gms./cc.)	155	145
Bycroft's non-dimensional factor b .	5	30
Base radius, r_0 (ins.)	5	1.5
Height (ins.)	5	12

on the ground, which corresponds to Bycroft's case of a rigid plate on a semi-infinite elastic medium, there is a resonance introduced at about this frequency which is removed when the geophones are buried with the replaced soil compacted. These calculations are very approximate and apply to the worst case of minimum velocity and density. From the considerations given above it seems safe to take the upper frequency limit due to geophone-ground coupling as about 150 c/s. which is seen to be adequate for good pulse reproduction. The upper frequency response limitations introduced by geophone-ground coupling shown in figure 4 are approximate and calculated, with the above considerations in mind, from the curves given in figure 2 of Bycroft's paper, p.929. The curves apply to the case of no packing of the geophone, but it would be possible to make a reasonable estimate of their shape when the geophone is packed by analogy with a damped resonant system (such as a galvanometer). The maxima would be roughly at the same values of a_0 , but the peak values of M would be reduced: however the exact shape would be difficult to estimate due to lack of knowledge of the damping constant.

2.212 Geophone theory. Above are stated the requirements of the geophone characteristics. Below the theory is given of the working of a geophone on which the design must be based. (see Dennison, 1953, Washburn, 1937).

Referring to figure 9 and neglecting eddy current damping in the former, the equations of motion of a geophone are:-

mechanical

$$m \frac{d^2}{dt^2} (x+u) = Ki - r\dot{x} - sx + F_{o\ell} - mg$$

The last two terms only occur for a vertical geophone, when it is arranged that

$$F_{o\ell} = mg$$

$$\text{giving } m\ddot{x} + r\dot{x} + sx - Ki = -m\ddot{u}, \quad (1)$$

and electrical

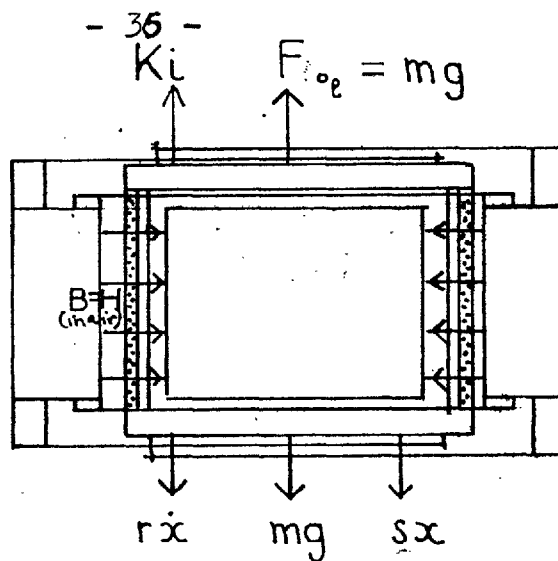
$$Ri = -Kx - L \frac{di}{dt} \quad (2)$$

where

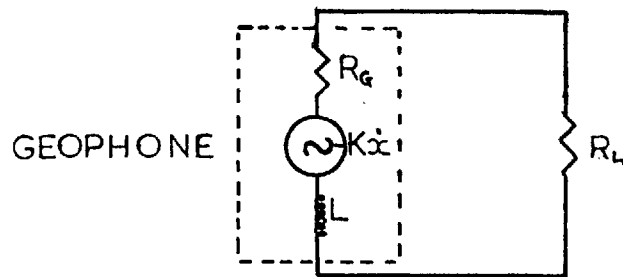
- | | |
|--|---|
| m = suspended mass, | x = displacement of mass w.r.t. case, |
| r = mechanical damping constant | u = displacement of case w.r.t. a point fixed in space, |
| s = spring constant, (or sum of spring constants if more than one spring acting) | K = sensitivity = lB |
| | l = length of wire = $2\pi aN$ |
| B = magnetic flux | a = radius of coil |
| N = number of turns | |
| R = total circuit resistance = $R_g + R_L$ | |
| R_g = geophone coil resistance | R_L = load resistance |
| L = coil inductance | i = current |
| $F_{o\ell}$ = the spring force introduced by prestressing (see 2.214(d)). | |

(1) and (2) give (neglecting r which is normally small)

$$\frac{d^3 i}{dt^3} + \frac{R}{L} \frac{d^2 i}{dt^2} + \left(\frac{s}{m} + \frac{K^2}{mL}\right) \frac{di}{dt} + \frac{sR}{mL} i = \frac{K}{L} \frac{d^3 u}{dt^3} \quad (3)$$

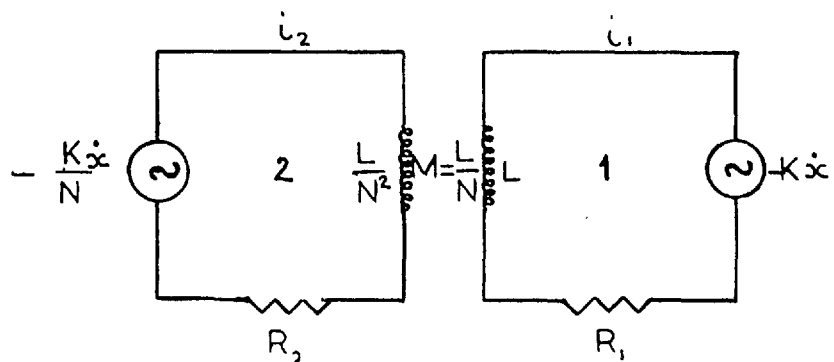


FORCES ACTING ON VERTICAL GEOPHONE



GEOPHONE CIRCUIT

FIGURE 9



CIRCUIT OF GEOPHONE WITH FORMER DAMPING

FIGURE 10

The voltage V across the geophone load $= R_L i$, hence

$$\frac{d^3 V}{dt^3} + \frac{R}{L} \frac{d^2 V}{dt^2} + \left(\frac{s}{m} + \frac{K^2}{mL} \right) \frac{dV}{dt} + \frac{sR}{mL} V = \frac{R_L K}{L} \frac{d^2 \dot{u}}{dt^2} \quad (4)$$

$$\text{It is usual to put } \omega_0^2 = \frac{s}{m}, \quad \omega_1^2 = \frac{s}{m} + \frac{K^2}{mL} \quad (5)$$

When L can be neglected, (4) gives

$$\frac{d^2 V}{dt^2} + \frac{K^2 dV}{mR dt} + \frac{s}{m} V = \frac{R_L}{R} K \frac{d^2 \dot{u}}{dt^2} \quad (6)$$

It is also usual to put $\frac{K^2}{mR} = 2h\omega_0$ where $h = 1$ for critical damping, and is known as the damping constant.

In the above theory, only the damping due to the load resistance was considered and the effects of the metallic former damping ignored. It was decided however to make resistive damping negligible in comparison to the former damping for loads above 20 Kohms, so that the damping should not depend on whether or not the preamplifiers were used. It is therefore necessary to consider the effects of former damping. Dennison (1953) gave a correction to allow for former damping, the theory of which is given below.

Figure 10 is a diagrammatic representation of a geophone with former damping. The former may be regarded as one turn of wire perfectly coupled to the coil and, as inductance is proportional to N^2 , its inductance $L_2 = \frac{L}{N^2}$ and $M = \sqrt{\frac{L \cdot L}{N^2}} = \frac{L}{N}$

The equations of motion become:-
mechanical

$$m\ddot{x} + sx - Ki_1 - \frac{K}{N} i_2 = m\ddot{u}, \quad (7)$$

electrical

$$- \frac{K\dot{x}}{N} = R_2 i_2 + \frac{L}{N^2} \frac{di_2}{dt} + \frac{L}{N} \frac{di_1}{dt} \quad (8)$$

$$\text{and } -K\dot{x} = R_1 i_1 + L \frac{di_1}{dt} + \frac{L}{N} \frac{di_2}{dt} \quad (9)$$

Eliminating i_2 and x , (7), (8) and (9) give

$$\frac{d^3 i_1}{dt^3} + \frac{R_1}{La} \frac{d^2 i_1}{dt^2} + \left(\frac{s}{m} + \frac{K^2}{mL} \right) \frac{di_1}{dt} + \frac{sR_1 i_1}{mLa} = \frac{K}{La} \frac{d^3 u}{dt^3} \quad (10)$$

which is the same as (3) if R is changed to $\frac{R_1}{a}$, where

$$a = 1 + \frac{R_1}{R_2 N^2}, \quad (11)$$

It may be seen by analogy with equation (6) that the sensitivity is unchanged and the $\ddot{u}$ term is divided by a .

A steady state solution for $\dot{u} = \dot{u}_0 e^{j\omega t}$ will be of the form $V = V_0 e^{j\omega t}$, where V_0 will in general be complex.

It may be shown that

$$V = - \frac{R_L}{R_1} K \dot{u}_0 \frac{\omega^2}{[(\omega_0^2 - \omega^2)^2 + [(\frac{\omega L}{R_1} + \frac{\omega L}{N^2 R_2})(\omega_1^2 - \omega^2)]^2}^{\frac{1}{2}} e^{j(\omega t - \phi)} \quad (12)$$

$$\text{where } \phi = \tan^{-1} \left[\frac{(\frac{\omega L}{R_1} + \frac{\omega L}{N^2 R_2})(\omega_1^2 - \omega^2)}{(\omega_0^2 - \omega^2)} \right]$$

$$= \tan^{-1} \left[\frac{\frac{K^2 \omega}{m} (\frac{1}{R_1} + \frac{1}{N^2 R_2})}{(\omega_0^2 - \omega^2)} + \omega L (\frac{1}{R_1} + \frac{1}{R_2 N^2}) \right] \quad (13)$$

= phase angle for negligible + inductive correction
inductance when $f > 20f_0$

The fraction of critical damping h is given from (10), when L is neglected by

$$2h\omega_0 = \frac{K^2}{mR} \left(1 + \frac{R}{N^2 R_2}\right) \quad (R_2 = \text{former resistance})$$

$$\text{hence } h = \frac{K^2}{2\omega_0 m} \left(\frac{1}{R} + \frac{1}{R_2 N^2}\right) \quad (14)$$

From (13), when $f > 20f_0$, and (14), it may be seen that

$$\frac{2h\omega\omega_0}{\tan\phi_f} = \frac{K^2}{mL_f} \omega_1^2 \frac{Lf_1}{L_f} \quad (15)$$

If f_u is the frequency ($>20f_0$) at which the response is 3 db down due to the inductive effect and $h \approx 0.5$, (15) gives

$$f_u \cdot f_0 \approx f_1^2 \frac{Lf_1}{Lf_u} \quad (16)$$

The upper natural frequency f_1 is therefore important and Terman (1943) gives

$L \approx 2\pi N^2 \frac{ad}{g}$ where d is the coil depth and g the width of the magnetic air gap.

$$\text{hence } f_1^2 = \frac{K^2}{4\pi^2 mL} \approx \frac{agB^2}{2\pi^2 md} \quad (17)$$

2.213 Geophone design

Equation (16) is the starting equation for designing a geophone with a given frequency response. It is complicated by the fact that L_{f_1}/L_{f_u} is not known, but its value is usually between 1 and 3 (2.5 in the present case, see figure 28, page 83). For the most difficult case when f_1 is greatest, it may be taken as 1 in calculating a value for f_1 . The geophone will then have a frequency response

at least as good as the specification. E.g. if $f_0 = 2.5$ c/s, then $f_1 = 25$ c/s., if $f_u = 250$ c/s.

Having calculated the value of f_1 needed, equation (17) gives the physical dimensions, but it is only approximate and does not allow for variation of L with f . B is made as large as possible, in the present case about 3,000 gauss, and m should be made as small as possible, but not less than that needed to make Brownian motion negligible (Wolf, 1942) which in this case is about 3.5 gms. The minimum value of m is also limited by the value of f_0 , since a light mass will need a small spring constant for a low value of f_0 , and this may lead to instability. Although it may appear that g should be large, this is not so, as this tends to reduce B , which must be large both because it is squared and because K should be as large as possible. This was confirmed by experiment. It should be noted that N does not enter the equation and therefore, to get a large K , N should be made as large as possible within the limits permitted by R_g and the size of the winding. The ratio a/d should be as large as possible within the limits of the size of the base and the fact that d/g should be at least 10 so that B does not vary substantially in the air gap.

It was not possible with the time and resources available to build a completely new geophone, and it was decided to modify an existing instrument. The Seismograph

Service Ltd., GAA geophone was chosen as it had approximately the required dimensions, was well made and available in reasonable quantities, due to the generosity of Messrs. Seismograph Service Ltd. For photographs of the geophone components see Mason (1951). The problems then were, firstly, to achieve a sufficiently low f_0 (a reduction from 30c/s to 3c/s was necessary) and secondly, to adjust the parameters to fit the requirements of 2.211. 2.214. Lower natural frequency f_0

The following is a list of the methods which were considered for reducing f_0 to the required value. (d), (e) and (f) are each described in greater detail later.

(a) Thinning and prestressing the manufacturer's spider springs. (Prestressing is the term normally used for forming a spring into any required initial shape, but preforming would be more descriptive). This method was used by Mason. The amount of prestressing needed to support the weight is given by $\delta = 25/f_0^2$ cms., which in this case is about 3cms. This was found to lead to instability and was not used. However, for the horizontal ^ogeophone, this method was possible as no prestressing was needed to compensate for the weight of the suspended mass.

(b) Use of the La Coste principle of suspension.

(c) Use of additional springs acting perpendicular to the direction of motion in compression against the mass, (see US Patent No.2, 788, 512). This would give a

negative spring constant, because for small displacements the spring force would have a component acting in the direction of motion away from the equilibrium position: i.e. proportional to $-x$. (b) and (c) were not tried sufficiently exhaustively to give conclusive results.

(d) Use of prestressed triangular springs.

(e) Use of integrated feedback.

(f) Use of a magnetic screw.

(d), (e) and (f) were found to work well and are considered below. The geophones which were finally employed in the experiments used (d) in combination with (f).

2.214(d). Prestressed triangular springs

A survey was made of the spring systems used in various geophones. The triangular springs used by Willmore (1950) seemed to provide a compact and stable means of obtaining a low value of f_0 , and, after prestressing into an arc of a circle, could be made plane by the application of a force at one end. The theory of the bending of a triangular spring is given below, (see figure 11).

If a beam is bent into an arc of a circle radius R , the bending moment at all points along the beam is given by $M = EI/R$ where E is Young's modulus and I is the geometrical moment of inertia of the cross section. If a beam is prestressed into an arc of a circle and then flattened, the reverse is true, and when the spring is

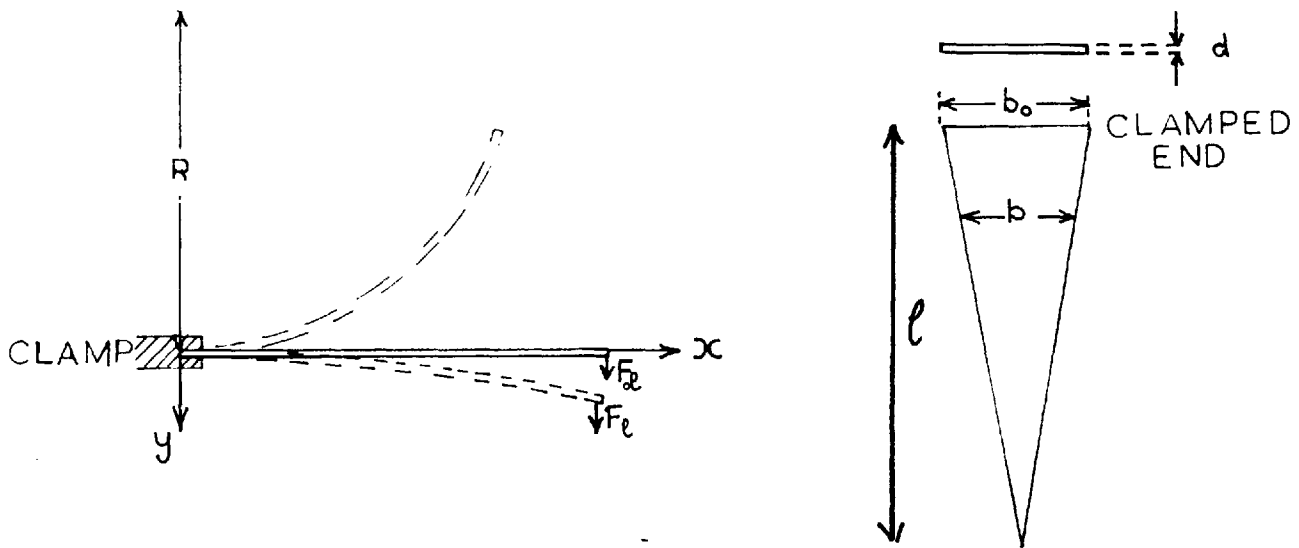


FIG. II — TRIANGULAR SPRINGS.

flat ($y=0$ for all x) the bending moment at a point distance x from the fixed end is given by

$$M_o = F_{o\ell} (\ell - x) = \frac{EI}{R} \quad (18)$$

where $F_{o\ell}$ is the force at the point x in the beam due to $F_{o\ell}$ acting at the end. This is the force needed to keep it flat and in this case is made equal to the weight of the geophone suspended mass. i.e. $F_{o\ell} = mg$

From (18) it may be seen that to keep R constant for all x , I must be proportional to $(\ell - x)$.

But $I = \frac{bd^3}{12}$ (see figure 11)

Therefore this may be done by making b proportional to $(\ell - x)$,

$$i.e. \quad b = b_o \frac{(\ell - x)}{\ell}$$

Hence a triangular spring prestressed into an arc of a circle may be made plane by the application of a force at the pointed end.

If the force acting on the end of the spring is increased to F_ℓ (when $y \neq 0$ for all x), and y is small, then to a first approximation

$$F_\ell (\ell - x) = \frac{EI}{R} + EI \frac{d^2 y}{dx^2} \quad (19)$$

(18) and (19) give

$$\left(\frac{F_\ell}{F_{o\ell}} - 1 \right) = R \frac{d^2 y}{dx^2}$$

Integrating and putting in the boundary conditions (i.e. the broad end is clamped) $y = 0$ and $\frac{dy}{dx} = 0$ at $x = 0$, it may be shown that

$$\left(\frac{F\ell}{F_0\ell} - 1\right) \frac{\ell^2}{2} = Ry$$

or

$$F\ell = \left(\frac{2Ry}{\ell^2} + 1\right) F_0\ell$$

The equation of motion for small vibrations of the beam about the equilibrium position is,

$$mg - \left(\frac{2Ry}{\ell^2} + 1\right) mg = m \frac{d^2y}{dt^2}$$

or

$$\frac{d^2y}{dt^2} + \frac{2Rgy}{\ell^2} = 0$$

hence

$$\omega_0^2 = \frac{2Rg}{\ell^2} = \frac{s}{m}, \quad f_0 = \frac{1}{2\pi\ell} \cdot \sqrt{2Rg} \approx \frac{5}{\ell} \cdot \sqrt{2R} \quad (20)$$

and

$$\frac{EI}{R} = mg(\ell - x) = \frac{E}{R} \frac{d^3}{12} \frac{(\ell - x)}{\ell} b_0$$

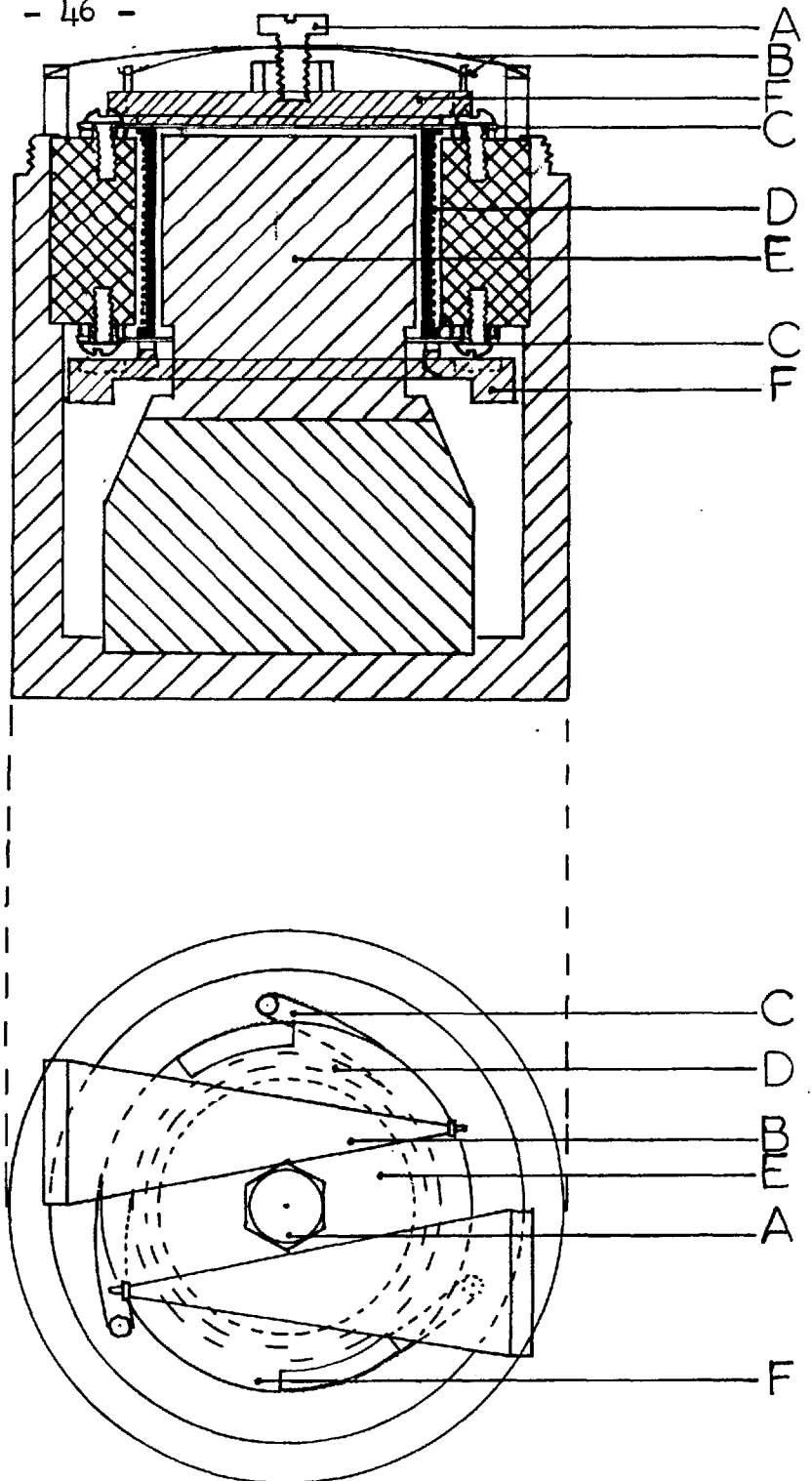
hence

$$R = \frac{Ed^3b_0}{12mg} \quad (21)$$

and

$$s = \frac{Ed^3b_0}{6\ell^3} \quad (22)$$

The triangular springs were designed using these formulae. After trying various designs, two springs were used as shown in figures 12 and 13. The dimensions of the springs eventually used were $b_0 = 2.0$ cms., $\ell = 5.3$ cms., $R = 1.0$ cms. and $d = 5.3 \times 2.54 \times 10^{-3}$ cms. (5.3 'thou') = 1.35×10^{-2} cms. Each of the two springs used was capable of supporting a mass of 91 gms., giving f_{0t} , the natural frequency with



- | | |
|-----------------------|-----------------------|
| A. MAGNETIC SCREW | D. COIL FORMER |
| B. TRIANGULAR SPRINGS | E. PERMANENT MAGNET |
| C. SPIDER SPRINGS | F. ADDITIONAL WEIGHTS |

FIG 12 — DIAGRAM OF ONE COMPONENT OF LF GEOPHON

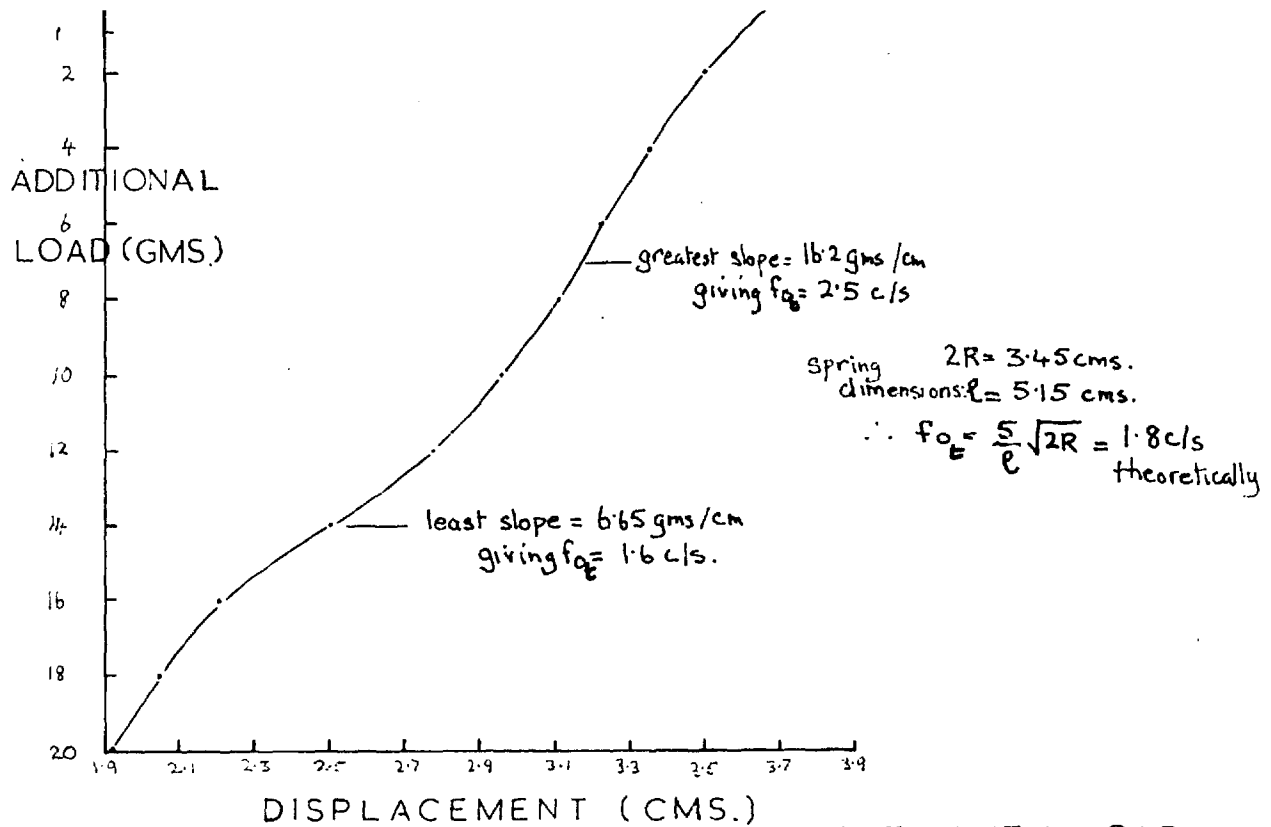


FIG 14—VARIATION OF DISPLACEMENT WITH LOAD
ON GEOPHONE TRIANGULAR SPRING
(not one of the springs actually used)

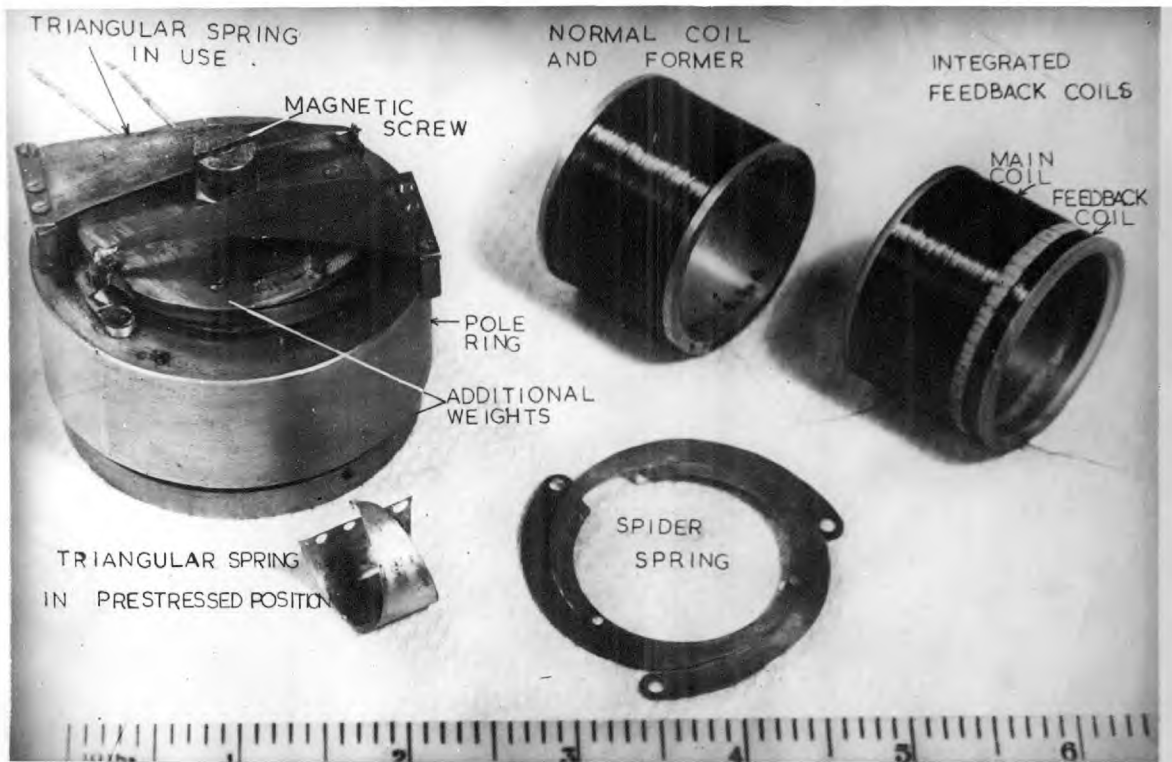


FIG 13— COMPONENTS OF LF GEOPHONE

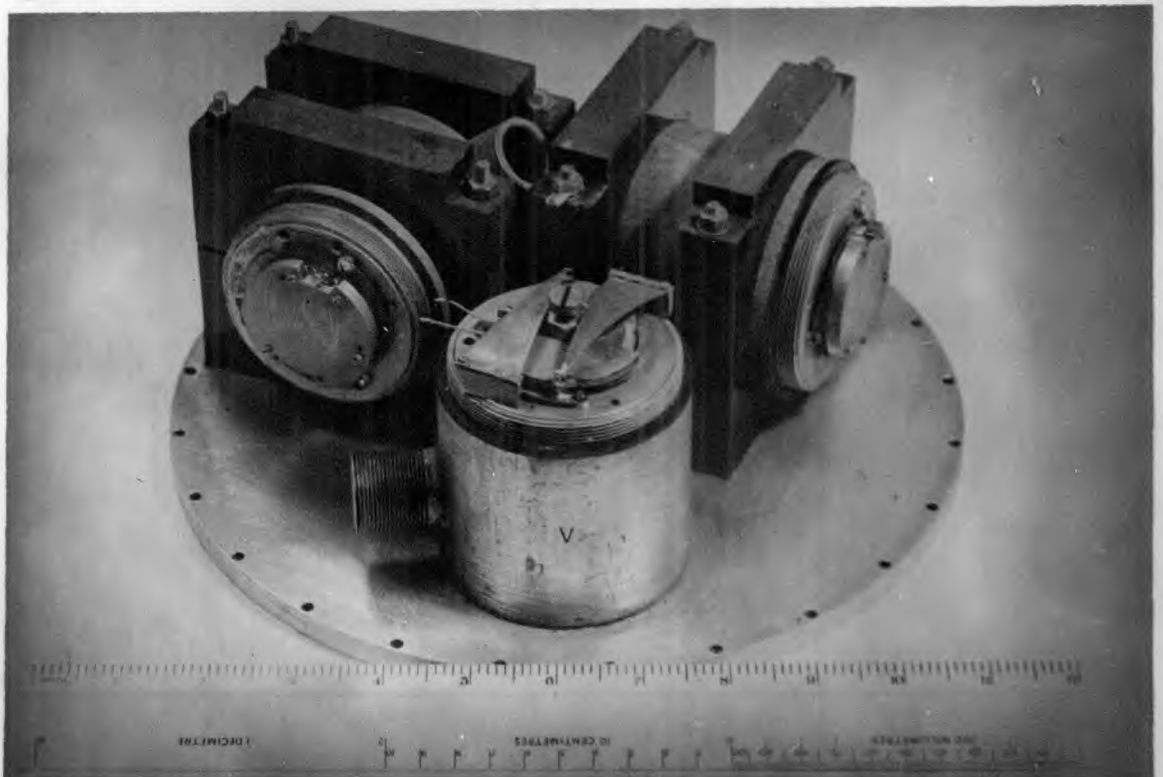


FIG.15— THE THREE COMPONENT LF GEOPHONE
(WITH TOP COVER REMOVED)

triangular springs alone, as about 2.5 c/s. The difference between this frequency and that given by equation (20) is due to variations in the spring constants. Figure 14 shows the variation of displacement with load for some of the triangular springs, indicating the large variations of s . This may be due to the method of cutting with a guillotine which produced slight ridges which were later filed down. It may be possible to use this particular property to produce a spring with a low spring constant, as the load it is capable of supporting is not affected and agrees well with the equation (21). When the triangular springs are more robust, such as those used by Willmore, the variations of s are not so great, and f_{ot} is given accurately by equation (20).

The spider springs were thinned to 2.4 thousandths of an inch giving a natural frequency with spider springs alone (f_{os}) of about 2.5c/s with $m = 182$ gms. Then

$$f_o = \sqrt{f_{os}^2 + f_{ot}^2} = 3.5 \text{ c/s.}$$

By use of the magnetic screw this was reduced to about 3c/s (see 2.217).

The increase in mass (from 35 gms. to 182 gms.) was effected by fixing brass discs to the ends of the former as shown in figures 12 and 13. When making the springs, triangles were cut to the required dimensions by a guillotine from sheets of beryllium-copper, then they were fixed around aluminium tubes machined to the correct diameter,

and heat treated by leaving them in a melt of equal proportions of potassium nitrate and sodium nitrite at 316°C ($\pm 4^{\circ}\text{C}$) for 3 hours ($\pm \frac{1}{2}$ hour).

The vertical component (LF2) of the low frequency geophone was constructed as described above.

The two horizontal geophones (LF1 or longitudinal and LF3 or transverse) presented the relatively simple problem of thinning the spider springs by dissolving in nitric acid, and increasing the mass to about 182 gms. to give a natural frequency of about 3c/s. The range of characteristics of each component of the LF geophone is shown in table 1.

The vertical and two horizontal geophones were mounted in a case (see figure 15). They were not mounted vertically permitting use in a borehole, as experiments showed that for accurate work the coupling between the walls and the geophone was not sufficiently good without the use of a special borehole geophone clamp. The horizontal mounting was more rigid for normal use. The geophones were covered with 'Araldite' or 'Preservite' to insulate them electrically from earth currents.

2.214(e). Integrated feedback geophone. Mason suggested (private communication) the possibility of using an integrated feedback device for reducing the natural frequency of a geophone. The spring restoring force is proportional to the displacement from the equilibrium position and the

geophone voltage output proportional to the velocity at frequencies well above the natural frequency f_0 . Hence a voltage proportional to the integral of the output appropriately feedback could lead to an effective negative spring constant. After building the geophone described in 2.215, this principle was tried and found to produce reasonable results. Figure 16 is a diagrammatic representation of a geophone with integrated feedback.

A fraction $a_1 V_0$, of the normal geophone output voltage is taken and integrated by means of a feedback integrating network, then fed back to a second, feedback, coil which is wound on the former used for the main coil. A feedback former with main and feedback coils is shown on the right of the photograph of geophone component parts in figure 13. The integrating network output (see fig. 16)

$$\begin{aligned} &= a_2 \int a_1 V_0 \cdot dt \\ &= a \int V_0 \cdot dt \quad \text{if } a = a_1 \cdot a_2. \end{aligned}$$

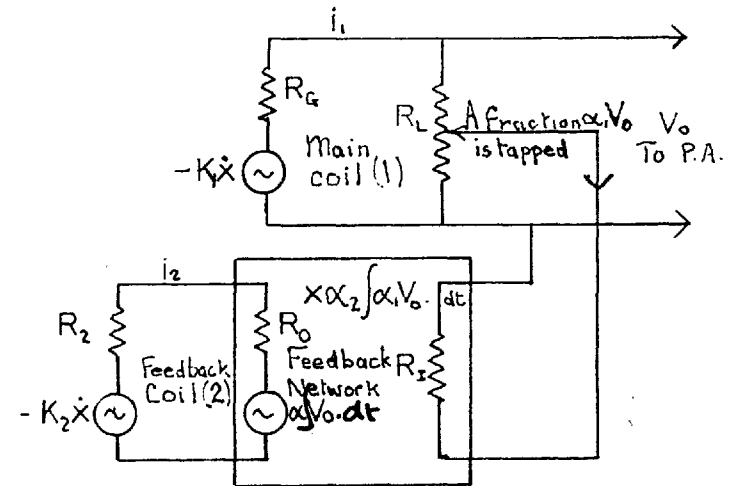
It is arranged that $R_I \gg a_1 R_L$.

Let $R_2 + R_0 = R_3$

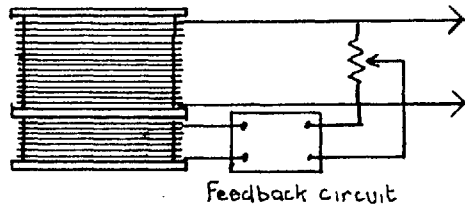
and $R_g + R_L = R$ as before.

Inductance will be neglected as the primary interest at the moment is with the low frequencies. Also former damping may be neglected as the coils were deliberately wound on a nylon former to make this possible.

FIGURE 16-DIAGRAM OF GEOPHONE WITH INTEGRATED FEEDBACK.



ARRANGEMENT OF FEEDBACK NETWORK.



17a. ARRANGEMENT OF COILS

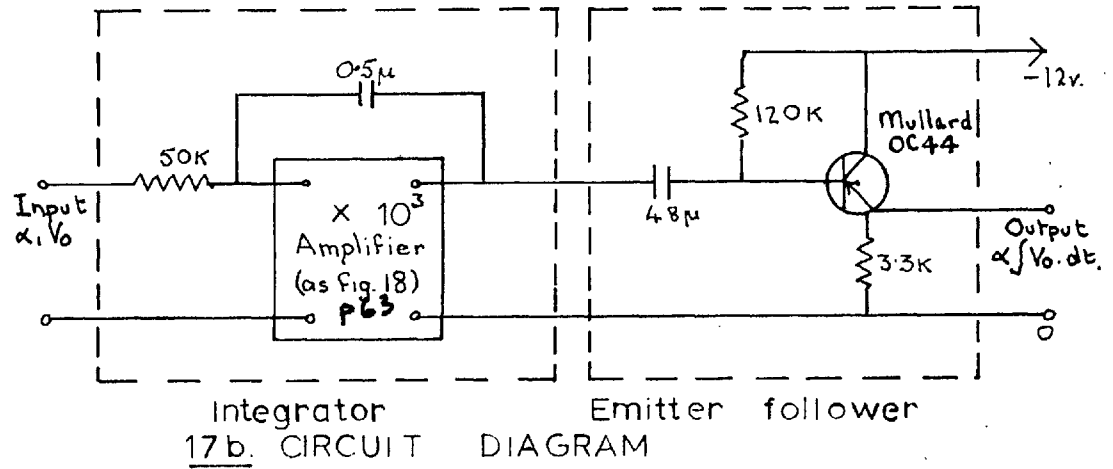


FIGURE 17 - FEEDBACK CIRCUIT.

Referring to figure 16, the equations of motion of the system are:-

$$\text{mechanical } m\ddot{x} + sx - K_1 i_1 + K_2 i_2 = - m\ddot{u}$$

$$\text{electrical, main coil } - K_1 \dot{x} = R i_1$$

$$\text{electrical, feedback coil } a \int V_o \cdot dt - K_2 \dot{x} = R_3 i_2$$

These give

$$\frac{d^2 V_o}{dt^2} + \left(\frac{K_1^2}{mR} - \frac{K_2^2}{mR_3} \right) \frac{dV_o}{dt} + \left(\frac{s}{m} - \frac{K_1 K_2 a R}{m R R_3} \right) V_o = \frac{R_3 K_1}{R} \frac{d^2 \dot{u}}{dt^2}, \quad (23)$$

indicating that the effect of feedback is to reduce the damping term by

$$\frac{K_2^2}{mR_3} \quad \text{and } \omega_o^2 \text{ by } \frac{K_1 K_2 a R_L}{mR R_3}$$

The geophone was designed making

$$\frac{K_1 K_2 a R_L}{mR R_3} \approx \frac{s}{m}$$

and $\frac{K_2^2}{mR_3} \leq \frac{1}{10} \frac{K_1^2}{mR}$ so that the effects of the change of damping could be neglected.

The feedback network which was constructed consisted of a transistor Miller integrator circuit (see 2.23), with a time constant $RC = \frac{1}{40}$, followed by an emitter follower to give a low impedance output R_o , (see figure 17).

The transfer function = $\frac{A}{1+ARCj\omega} \approx \frac{1}{j\omega RC} = \frac{a_2}{j\omega}$ when $ARC\omega \gg 1$.

Hence $a_2 = 40$ down to about 0.1 c/s. In this instance

$R_L = 2K$ ohms, $R = 3.45 K$ ohms and $R_3 = 330$ ohms. With a main coil of 3850 turns of 44s.w.g. enamelled copper wire, and the feedback coil of 455 turns of the same wire,

$$K_1 = 0.67 \text{ v/cm./sec. and } K_2 = 0.11 \text{ v/cm./sec.}$$

These give a feedback negative spring constant $\approx \frac{1}{2} \times 10^5$ dynes/cm., which may be increased by increasing a_2 .

It remains to be shown that the mutual coupling between the coils is negligible.

The induced e.m.f. in coil 1 due to the current in 2 is given by

$$\begin{aligned} V_c &= -M \frac{di_2}{dt} \\ &= -M \frac{d}{dt} \cdot \frac{s}{K_2} \int \dot{u} \cdot dt \\ &= -M \frac{s}{K_2} \cdot \dot{u} \text{ (constant term = 0)} \end{aligned}$$

and above f_0 ,

$$V_o = \frac{R_L}{R} K_1 u.$$

Therefore it is necessary to have $\frac{R_L K_1 K_2}{R m M} \gg \frac{M s \dot{u}}{K_2}$ for mutual coupling to be negligible.

$$\text{i.e. } \frac{R_L}{R} \frac{K_1 K_2}{M} \gg s$$

or the coupling natural frequency $\frac{\omega_c}{2\pi}$ should be such that

$$\omega_c^2 = \frac{R_L}{R} \frac{K_1 K_2}{m M} \gg \frac{s}{m} = \omega_o^2$$

$M = k \sqrt{L_1 L_2}$ where k is the coefficient of coupling, which in this case (see Langford-Smith, 'Radio Designers Handbook', p.447) is about 0.3.

By measurement of L_1 and L_2 it may be shown that $\omega_c^2 = 3 \times 10^4$ at 1 c/s, and if $f_o = 3.3$ c/s, the coupling is negligible.

In the trial geophone, the natural frequency without feedback of 4.9c/s was reduced to 3.3c/s with feedback.

One advantage of the use of this principle is that the required accuracy of levelling a horizontal geophone, which is proportional to f_0^2 (see 2.41), corresponds in this case to $\frac{a}{k}$ natural frequency without feedback and is not as critical as it would otherwise have been. This is because the feedback network does not respond to DC changes and only the geophone spring constant, tilt and weight of suspended mass determine the equilibrium position of the mass.

The limitations on the use of integrated feedback are, firstly, that the spring constant cannot be reduced below a value of the order of the original variations in its value which depend on the engineering and may be quite large, (figure 14). This tends to lead to instability. Secondly, it is necessary to have more complicated electronics, which may be built into the geophone or alternatively may be kept with the rest of the recording equipment thus necessitating an extra wire to the geophone. However this need not be used in the event of a fault when the system could be operated in the normal manner without integrated feedback. The operator could adjust the geophone natural frequency with a potentiometer - which would in effect act as a high pass filter.

The method of integrated feedback was not used eventually, mainly because the springs were not sufficiently well made to have constant values of s . A 70 per cent

reduction in f_0 entails a 90 per cent reduction in s , so that s must not vary by more than about 5 per cent in this case over the greatest displacement to be recorded. However the principle could be very useful when used with good springs.

2.214(f) Magnetic screw

The introduction of a steel screw on the moving coil system above the fixed magnet was used to provide a non-linear negative spring constant (see Figs. 12 and 13). A similar idea was used by Willmore (1950) as a zeroing device. The effect of the non-linearity is shown below to be negligible.

If d is the separation of the screw and the permanent magnet, d_0 the equilibrium position and x the displacement from the equilibrium position,

$$d = d_0 + x.$$

Let the force on the screw due to the induced magnetism be

$$F = f(d) = f(d_0 + x) \text{ then,}$$

$$\delta f = f'(d) \cdot \delta d = f'(d) \cdot x.$$

If x is small this gives the value of the negative spring constant. For non-linearity effects to be negligible, it is sufficient to require the variation in $f'(d)$ to be less than 5 per cent at the maximum displacement.

$$\delta(f') = f'' \cdot \delta d$$

so that it is necessary to have

$$\frac{\delta(f')}{f'} = \frac{f''}{f'} , \quad .\delta d \ll 0.05 \text{ for } \delta d \leq 10^{-2} \text{ cms.}$$

If l is the length of the screw and $+m$ and $-m$ the induced pole strengths, F may be written in the form

$$F \approx m^2 \frac{(2d + l)l}{d^2 (d+l)^2}$$

Although this is an approximation, it is reasonable to assume an approximate form of F as

$$F \propto \frac{1}{d^n} \quad \text{where } n \approx 3$$

then $\frac{\delta(f')}{f'} \approx \frac{n+1}{d} \cdot \delta d \approx 4 \text{ per cent when } d \approx 1 \text{ cm.}$

After a number of trials, a $\frac{1}{2}$ " long 2 B.A. steel screw was used. The natural frequency could be reduced from 3.5c/s to 2.2c/s, but was kept at about 3c/s for greater stability. However after some months field work it became necessary to make a slight reduction in m (of a few gms.) to allow for a change in the zero position of the mass, possibly due to fatigue of the springs. It was decided to over-compensate for this effect to allow for further changes, as the geophone was to be buried making it inaccessible. This decision was unfortunate as the consequent change in the zero position changed the value of f_0 to about 4 c/s (see Fig. 14), and the increased distance of the magnetic screw from the geophone permanent magnet was such as to make the reduction in frequency which it effected negligible. As a result of this effect, the geophone was used for some of the experiments with f_0 at 4 c/s and not 3 c/s. The zero positions of the horizontal geophones were arranged so that they had similar values of f_0 (see Fig. 27). The

existence of this undesirable effect is another argument in favour of carefully manufactured good quality springs.

2.215. Upper natural frequency f_1 . Coils were wound with various numbers of turns varying from a few hundred to about 20,000 and the variations of inductance with frequency measured for the various coils clamped in position (see 2.41).

Equation (14) gives

$$h = \frac{K^2}{2\omega_0 m} \left(\frac{1}{R} + \frac{1}{R_2 N^2} \right).$$

Applying the criterion that resistive damping should be negligible for load resistances greater than 20 K ohms, and using 44 s.w.g. copper wire which is about the thinnest which is easy to wind, it is necessary to have

$$\frac{1}{R} \leq \frac{1}{10 R_2 N^2}.$$

For 44 s.w.g. copper wire, $R_g \approx 0.4$ N ohms, hence the condition is

$$\frac{1}{(R_L + 0.4N)} \leq \frac{1}{10 R_2 N^2}$$

giving

$$N \leq 2,000 \text{ (as } R_2 \approx 5.5 \times 10^{-4} \text{ ohms, see 2.41)}$$

as the criterion for the number of turns needed. The coils used by the manufacturers had 1,670 turns and were therefore used.

These gave $K = 0.5$ volts/cm./sec., $L = 0.53$ Henries at 25c/s.

Now

$$f_1^2 = \frac{K^2}{4\pi^2 mL} \approx 25\text{c/s}$$

and $f_u \approx \frac{1}{2h} \frac{f_1^2}{f_0} \cdot \frac{Lf_1}{Lf_u} \approx 450 \text{ c/s}$,
 considering the worst case when $f_0 = 4.1 \text{ c/s}$.

Thus the low frequency geophone frequency response should be within 3 d.b. from about 2.6 c/s to about 450 c/s.

(When $f_0 = 4.1 \text{ c/s}$, $h = 0.43$). The upper frequency limit may be in error by as much as about 50 per cent, but this is not important as the geophone-ground coupling probably determines the upper frequency limit of the recording system.

2.216 Geophones used.

Two different types of three-component geophones were used in the majority of the experiments: both seemed from practical and theoretical considerations to have adequate frequency responses and to give adequate coupling with the ground. The low frequency geophone was constructed according to the principles described above and is illustrated in Figs. 12, 13 and 15. As described in 2.214(f), its natural frequency varied from 3 c/s to 4 c/s during different experiments, with the upper frequency response theoretically 3 d.b. down at about 450 c/s. Unfortunately there was not sufficient time to make a borehole clamp for this geophone and its three mutually perpendicular geophones were therefore mounted in a horizontal plane and not in a vertical line, as was originally intended for the borehole geophone. The former method was found to be less susceptible to resonances caused by the lack of

rigidity of the clamping in the case.

For measurements in a borehole, a commercial Hall-Sears three-component geophone was used and packed with a probe to give better coupling with the walls. Each component had a lower natural frequency of 4.5 c/s and a theoretical inductive 3 d.b. upper frequency limit in the kilocycle range. Although the simple theoretical approximations used in this calculation probably do not hold at these higher frequencies, because other effects become more important, it is safe to assume that the upper frequency limit is certainly greater than 450 c/s.

By reference to table 1 it may be seen that the lower frequency limit is determined by f_0 and is about 2.6 c/s (3 d.b.), while the upper frequency limit is determined by the geophone-ground coupling and, although this is not accurately known, it is seen by section 2.211 to be about 150 c/s.

The Hall-Sears geophone was used in boreholes and at all other times the LF geophone was used in preference because of its lower natural frequency, greater sensitivity (0.5 v/cm/sec as opposed to 0.17 v/cm/sec) and better shielding from 50 c/s pick-up because of the thicker metallic case.

Single standard GAA geophones ($f_0 = 30$ c/s) were used as triggering geophones because of their ruggedness and also to investigate the effects of a limited frequency

response.

Mason's two-component geophone was used in the preliminary experiments ($f_0 \approx 10$ c/s, $f_1 = 57$ c/s, $K = 1.5$ v./cm./sec.) and also 3 Willmore geophones, but these were found by experiment to give poor coupling with the ground and also to have insufficient high frequency response ($f_0 = 1$ c/s $f_1 \approx 10$ c/s $f_u \approx 70$ c/s $K = 1$ v./cm./sec.)

2.22 Preamplifiers

The most obvious requirements of the preamplifiers are:-

- (1) Flat frequency response with zero phase shift from at least 1c/s to 1000c/s.
- (2) Input noise level as low as possible (of the order of a few microvolts if possible).
- (3) Sufficient gain to make the lowest signal levels at least ten times greater than the noise level of the main amplifier or integrator; in this case greater than 100.

They should be transistorised for possible incorporation into the case of the geophone, thus reducing the relative effect of cable noise. They should also be simple to use without complicated power supplies or setting up procedures.

Among the various amplifiers considered were those used by Mason (1951) and the Solartron A.A.621 D.C. amplifiers. None were found which satisfied the requirements sufficiently well.

The special transistor amplifiers finally used were developed in conjunction with Mr. J. Dempsey of Elliot Brothers (London) Ltd., Boreham Wood. The circuit diagram of one amplifier followed by a second amplifier used as an integrator is shown in figure 18. The approximate characteristics of the amplifier are given below:-

Frequency response:-

0.5 c/s to 20 K c/s (3 d.b.). This is the unfiltered frequency response. High and low pass filters are set as shown in figures 18 and 19 to be 3 d.b. down at 0.16 c/s and 1000 c/s respectively.)

Gain : 1000 approximately

Noise level referred to input:

4 μ .v.p.p.(but varying with each amplifier and increasing with time to about 7 μ .v.p.p.)

Input impedance : 20 K ohms

Output impedance: 30 K ohms

Output swing : $\pm$ 23v.p.p.

Power supplies : + 100v, (3 m.a.), very smooth,
and -100v. (10 m.a.) These may

be reduced to 12v with some increase in noise and reduction of gain and output swing.

The frequency response of the amplifiers is shown in figure 19 with and without the filters which are set at 0.16 c/s and 1000 c/s. 3 d.b. down.

PREAMPLIFIER

INTEGRATOR

TRANSISTORS - OC 44

RESISTORS - 1%, 1/2w.

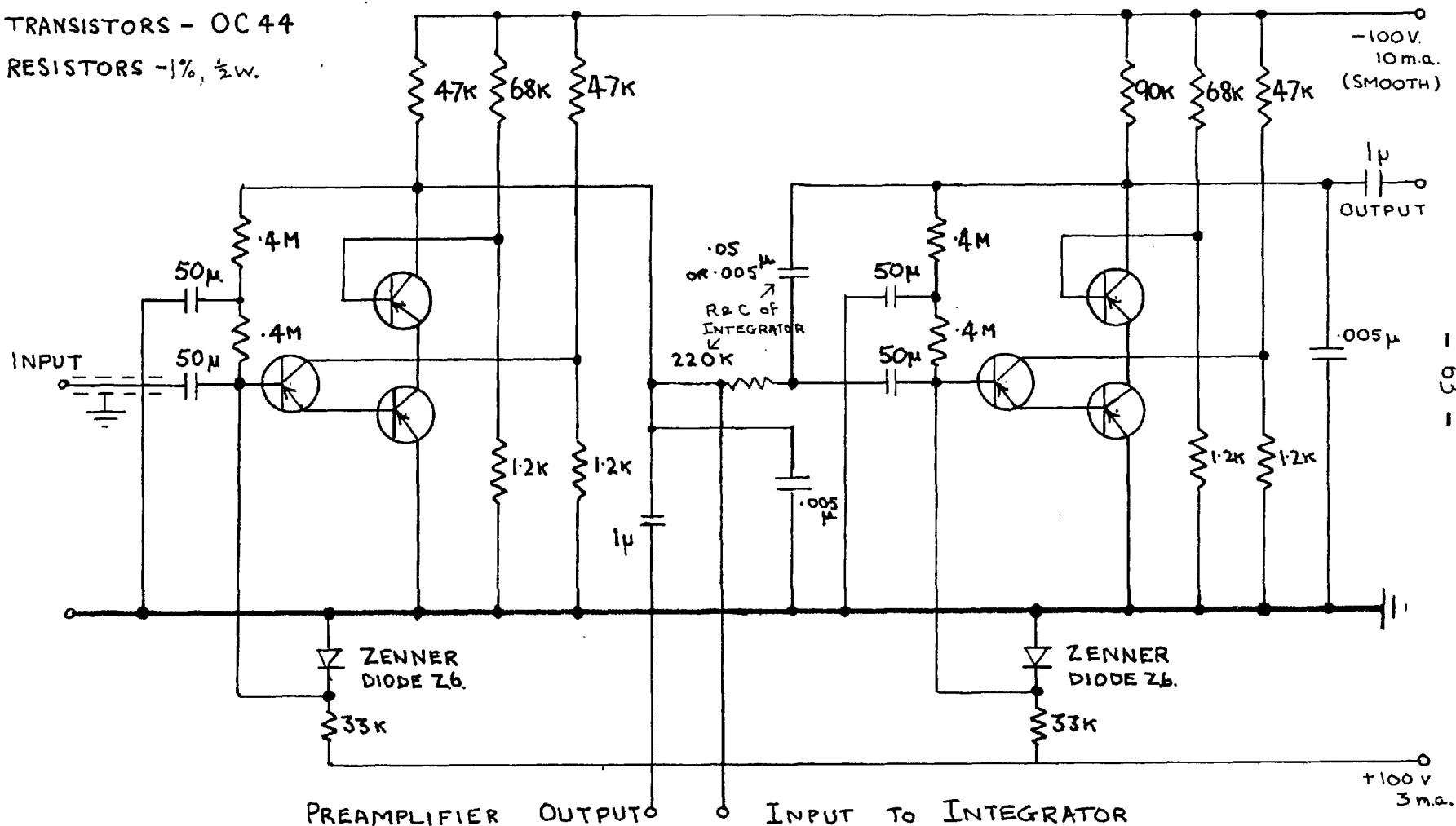


FIGURE 18 - PREAMPLIFIER AND INTEGRATOR

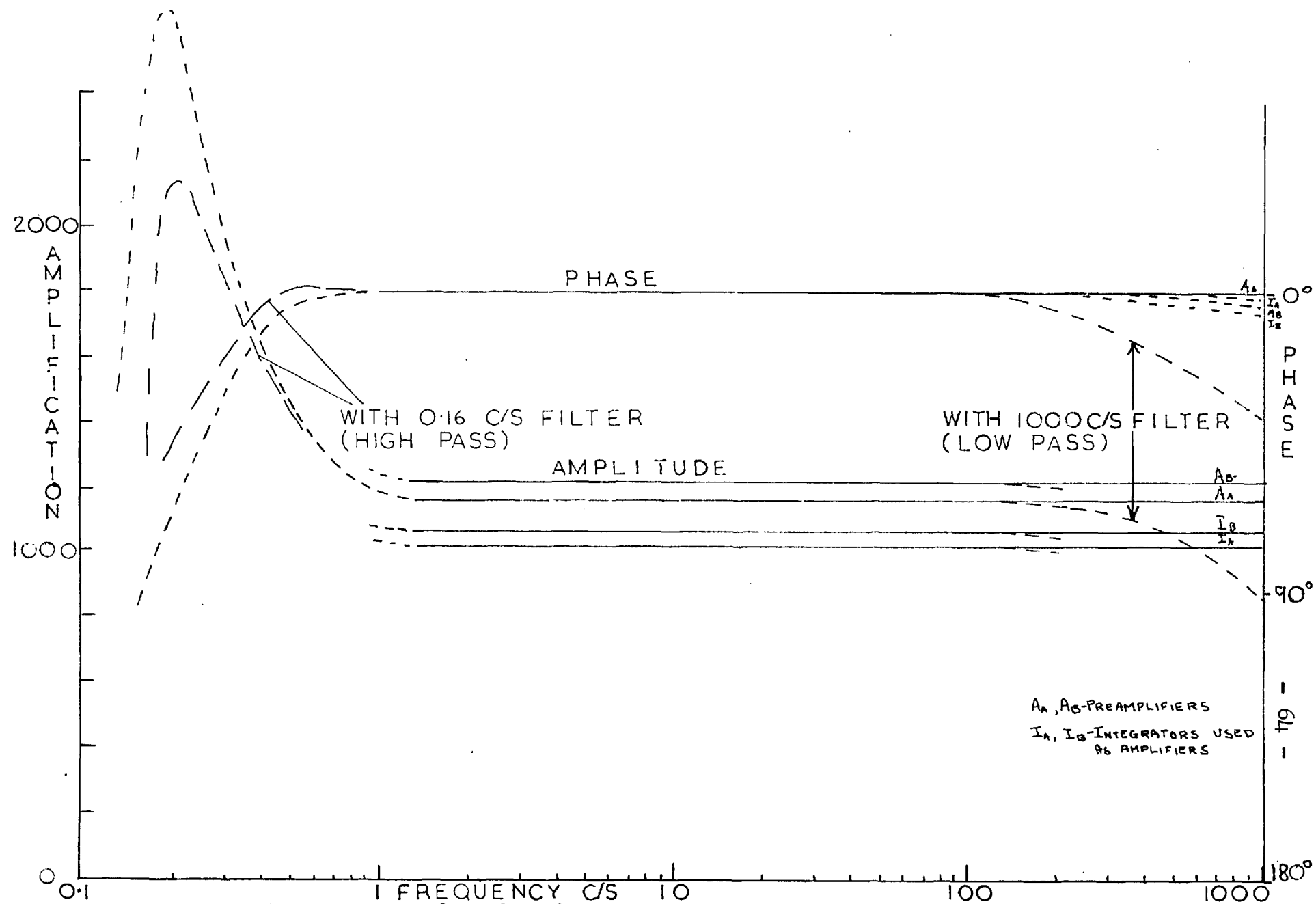


FIG 19-FREQUENCY RESPONSE OF PREAMPLIFIERS

- 65 -

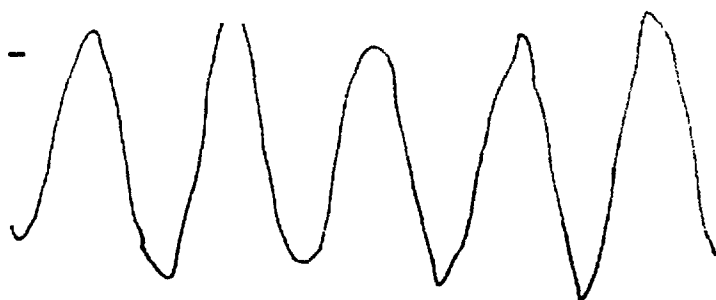


FIG. 20-TRACING OF PHOTOGRAPH OF 10 μ v. 40 C/S SINE WAVE AFTER AMPLIFICATION BY PREAMPLIFIER

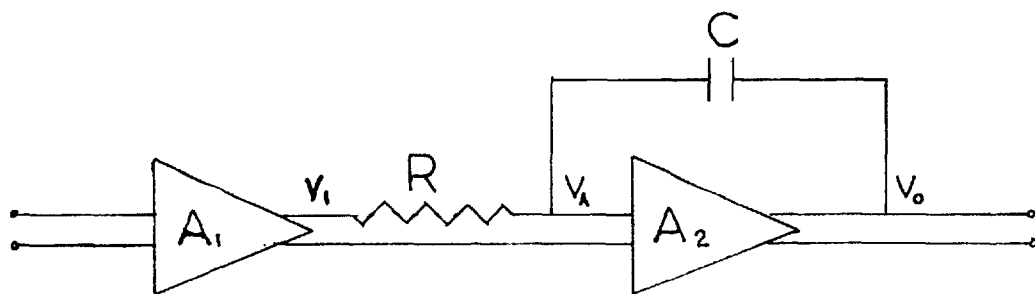


FIG 21 - MILLER INTEGRATOR WITH PREAMPLIFIER

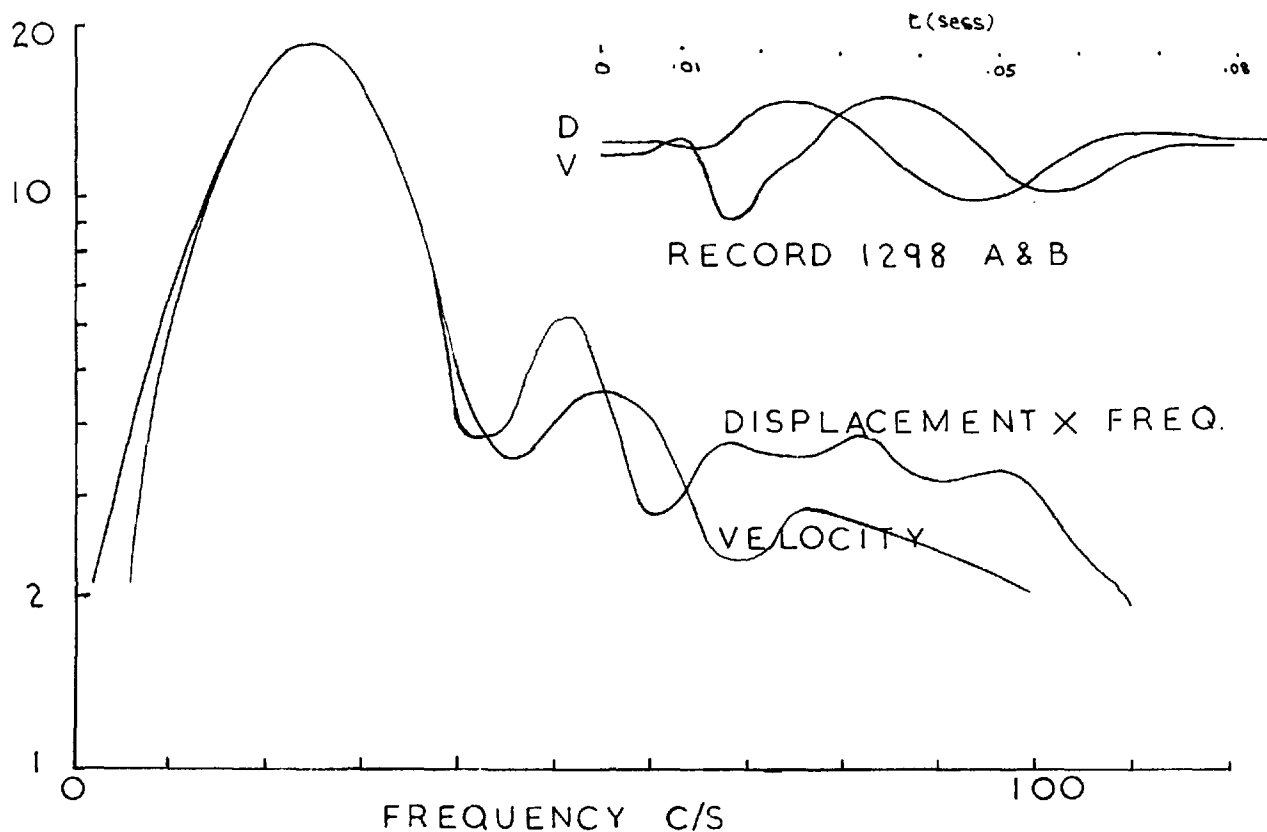


FIG 8-FOURIER ANALYSES OF VELOCITY AND DISPLACEMENT MEASUREMENTS OF THE SAME PULSE (SHOWN)

The noise level is as low as any reported in the literature for this frequency range (see Nightingale, 1958, who gives $4\mu\text{v.p.p.}$ from 10 to 2.300 c/s as the lowest value consistently attained with matching of Mullard EF86 valves). It was achieved by good shielding and earthing, and by selecting and matching the transistors, using twelve selected from a batch of 13. It is believed that at least one commercial firm has since developed a transistor amplifier which will have a lower noise level when finally produced. In time the transistors became noisier, rising to $7\mu\text{v.p.p.}$ average noise level after two years' use. Figure 20 shows a $10\mu\text{v.p.p.}$ 40 c/s signal photographed when the amplifiers were first constructed.

With the relatively short cables used in the present work, it was not found necessary to build the preamplifiers into the geophone case.

Four amplifiers were used, two with the integrators and two as preamplifiers.

2.23. Integrators

A simple R.C. integrator followed by an amplifier, and a Miller integrator (Miller U.S.A. Bureau of Standards paper 351 and Blumlein, 1942) were both tried. The latter was used and is shown with the preamplifier schematically in figure 21, and the circuit diagram is shown in figure 18.

For an infinite input impedance to the amplifier, so that all the current goes through the feedback condenser, it may be shown that

$$\frac{1}{R} (V_A - V_1) = C \frac{d}{dt} (V_O - V_A)$$

and

$$V_O = -A_2 V_A$$

These give

$$-\frac{V_1}{R} = C \frac{d}{dt} (V_O + \frac{V_O}{A_2}) + \frac{V_O}{RA_2} \quad (24)$$

For sinusoidal voltages

$$V_O = \frac{A_2}{[1+j\omega RC(A_2+1)]} V_1 \quad (25)$$

and if $j\omega RCA_2 \gg 1$

$$\begin{aligned} V_O &\approx -\frac{1}{j\omega RC} V_1 \\ &\approx -\frac{1}{RC} \int V_1 dt \end{aligned}$$

The effects of finite input impedance may be shown to be negligible in this case (see Wass, 1955).

Using the preamplifier described in 2.22 it is necessary to have $\omega RC \gg 0.01$.

One of the difficulties with the use of integrators is that low frequency components of the input noise level are magnified in comparison to the higher frequencies and appear at the output as undesirable low frequency variations of the output level. It is therefore best to apply as much low frequency filtering as possible when working with low signal levels, as long as the low frequency

response of the system is not affected. 1.6 c/s (3 d.b. down) was the greatest low frequency cut off used (having a 0.1 μ .f. blocking capacitor on the output to the oscilloscope) and 0.16 c/s was used for greater signal levels when noise was less noticeable (1 μ .f. blocking capacitor).

At 1.6 c/s therefore, it is necessary to have

$$RC \geq 0.001$$

and at 0.16 c/s

$$RC \geq 0.01$$

R was kept at 0.2 meg ohms and C allowed the value 0.005 μ .f. or 0.05 μ .f. with transfer functions of $10^3 \int V_1 dt$ above 1.6 c/s and $10^2 \int V_1 dt$ above 0.16 c/s.

The combination of A_1 and A_2 as shown in figure 21 was found to be better than having A_1 following the integrator or combined with A_2 inside the integrator.

In figure 8, page 65 Fourier analyses are shown of two records of the same pulse. One was recorded straight (giving velocity) while the other is the analysis of the output of the same geophone recorded on the second channel but integrated this time (giving displacement). The components of the displacement pulse are multiplied by frequency and a correction made for the different gain settings. The agreement is a measure of the accuracy of the integration (and also of the Fourier analyses).

The two preamplifiers and two integrators used for the A and B beams of the oscilloscope were all housed in

one box between the geophones and the oscilloscope. (see figure 5).

2.24 Oscilloscope

As no three-beam oscilloscopes were available, it was decided to use the Nagard DT 103 double-gun model which has D.C. amplifiers with a gain of 10^5 , a noise level of about 50 μ .v.p.p. and bandwidth up to 250Kc/s, giving a sensitivity of 0.25 m.v./cm. It has the convenient features that the YB amplifiers may be switched to the XA plates for Lissajous figure type of presentations, while at the same time the time-base may be used to control the length of duration of the brightening of the spot after it has been triggered on by a pulse from the timing unit. This facility was used to display two-component pictures of the ground motion. A G.E.C. tube was later fitted to give a smaller spot size.

Timing marks were put onto the record as small breaks in the trace of the oscilloscope spot by means of the timing pulses which were passed from the timing unit via a valve amplifier to the brightness modulation input. It is necessary to have a 60v. amplitude to completely darken the spot, which was the reason for the use of a valve and not a transistor amplifier. A 0.01 μ f and 1 meg ohm RC circuit across the output of the amplifier was used to vary the time break width.

2.25 Camera

A Shackman AC 2/25 Oscilloscope recording camera was chosen mainly because of the reasonable cost and continuous range of speeds available up to 34 inches per second, found to be constant and reproducible to within 1 per cent at a given speed. A possible future improvement to the apparatus may be the use of a better quality camera with facilities for using wider film and the Polaroid-Land fast processing technique for single shots.

2.26 Timing unit

Mason (1951) used an electromagnetically maintained vibrating reed working from the mains 50 c/s supply. This is guaranteed by the Electricity Authority to be accurate to within 1 per cent and was in fact found to be accurate to within about 0.3 per cent at most times. However, for the timing pulses to be accurate throughout one second to an accuracy of one millisecond, it is necessary to have a timing standard accurate to 1 in 10^3 . This accuracy may be achieved with some difficulty using L - C oscillator circuits but for greater accuracy it is necessary to go to crystal oscillators. In view of the complication of the timing circuit which was envisaged, it was decided to use a crystal standard providing a very versatile accurate timing unit suitable for a variety of other uses in addition to those connected with the present work.

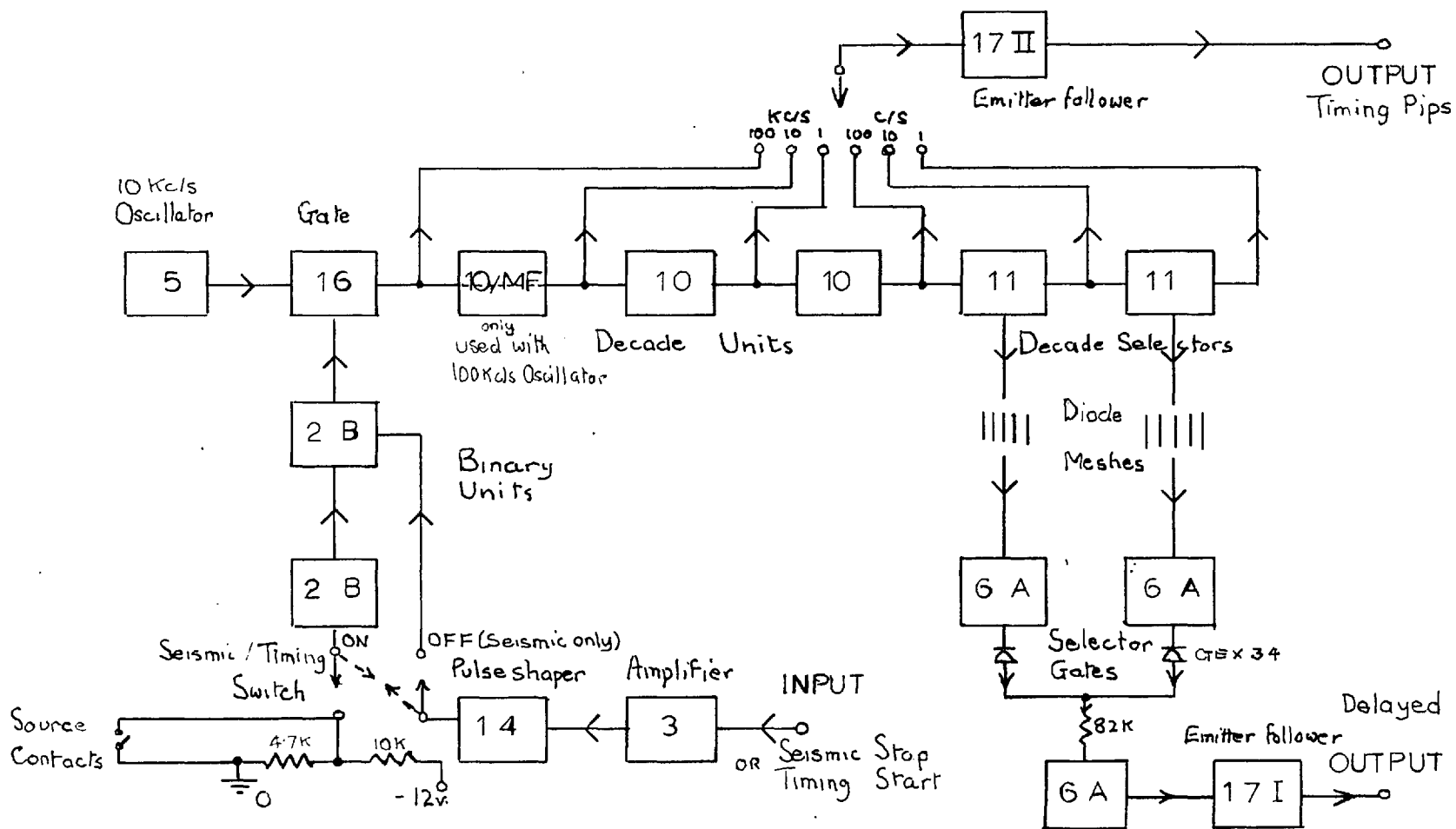


FIGURE 2 2 - BLOCK DIAGRAM OF TIMING UNIT.

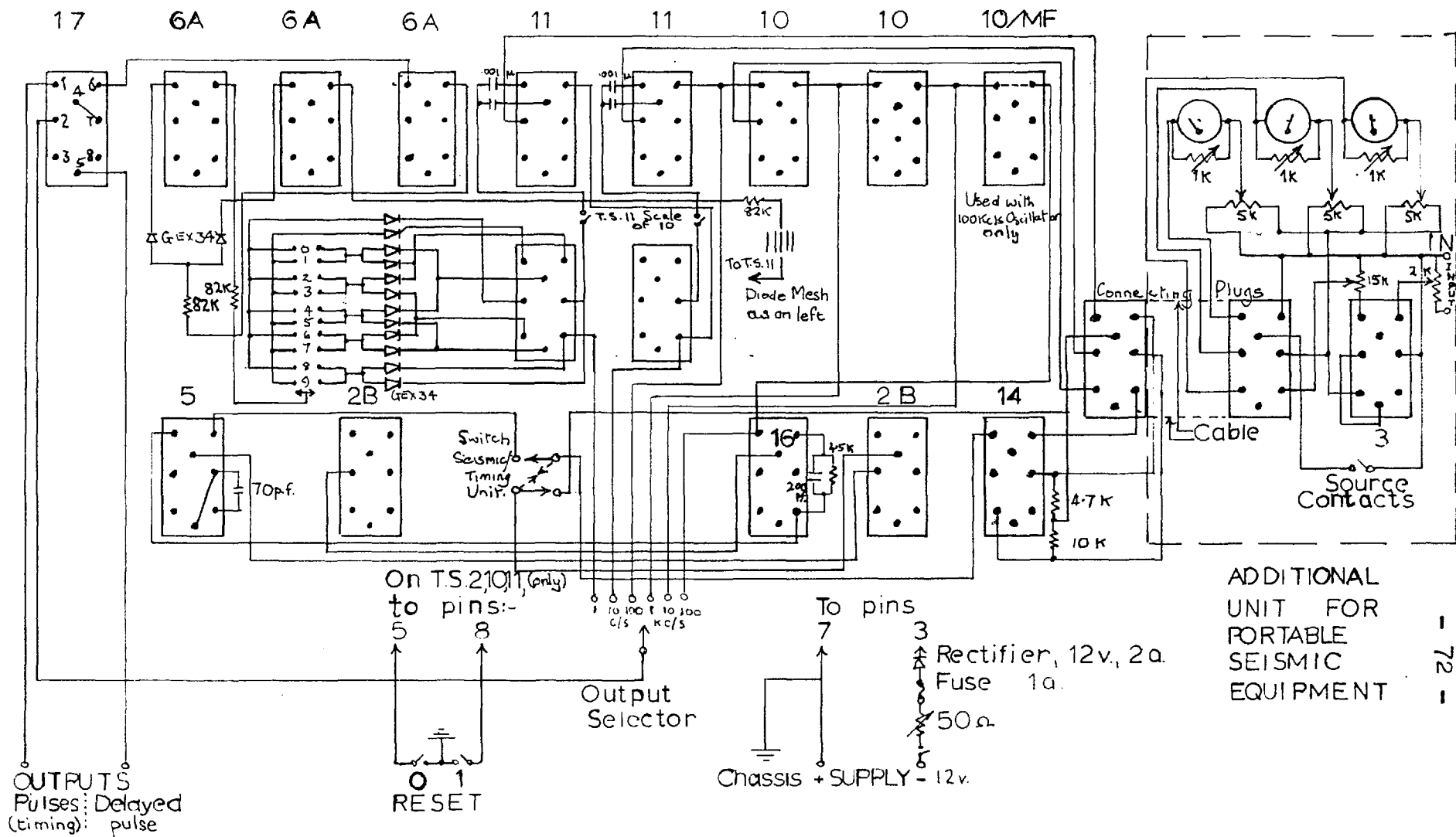


FIGURE 23 - WIRING DIAGRAM OF TIMING UNIT.

The requirements of the timing unit were that it should provide timing pulses every 0.1, 0.01 and 0.001 sec. accurate to at least 1 in 10^3 and, for use in the experimental method described in 3.1, also provide an output delayed with respect to the input for a time up to about 1 second, which may be set, with the above accuracy, to the nearest 0.01 seconds.

A unit was built to do this using the Venner Electronics Ltd., plug-in units, and is shown in figures 22 and 23 and also as part of photographs 6 and 25. ^(p. 28) The T.S.5 crystal oscillator provides rectangular timing pulses, accurate to 3 in 10^6 , which are gated by the T.S.16 electronic gate and passed into the T.S.10 and T.S.11 decade dividers and selectors. (The prefix 'T.S.' has been omitted in figures 22 and 23). After successive divisions by ten, outputs are available giving pulses every 1, 0.1, 0.01, 0.001 and 0.0001 sec., and at the same time, the T.S.11's are used in conjunction with T.S.6 selector gates and diode meshes to give the required delayed output, the delay time being set to the nearest 0.01 sec. with the crystal accuracy by two selector knobs. The working of the gate is controlled by a pulse from the triggering geophone which is first amplified by a T.S.3 wide band amplifier and then passed through a T.S.14 pulse shaper and T.S.2B binary unit before being passed to the T.S.16. Records nos. 452-62 (not illustrated) show that

this can be done with an accuracy such that the timing pulses start less than 1 millisecond after the weight hits the plate. The second T.S.2B binary unit shown in figures 22 and 23 was added later for use with the first in connection with the portable seismic application of the equipment described in 2.31. The use of two binary units in this manner, known as 'interlock action', ensures that only the first and not subsequent pulses open and close the gate when applied to the appropriate on and off points. The timing pulses were passed to the brightness modulation of the oscilloscope via a valve amplifier as described in 2.24. The amplifier used was made by adapting the amplifiers used by Mason in his triggering and timing circuit (see Mason 1951, p.27).

The timing unit is transistorised, portable and uses 0.2 amps from a small 12v Ni - Fe battery.

2.27 Control circuit

This is shown in figure 24 and is used to control automatically the sequence of operation of the experiments.

Switch A is the master switch which, (a) resets the timing circuit for future use (so that the beam may not be triggered off before the experiment by random seismic noise), (b) switches on the shutter or motor circuit (depending on the position of switch C) if switch B is up, otherwise does not affect this circuit, and (c) operates slug relay (1) after a time up to about 1 second, depending

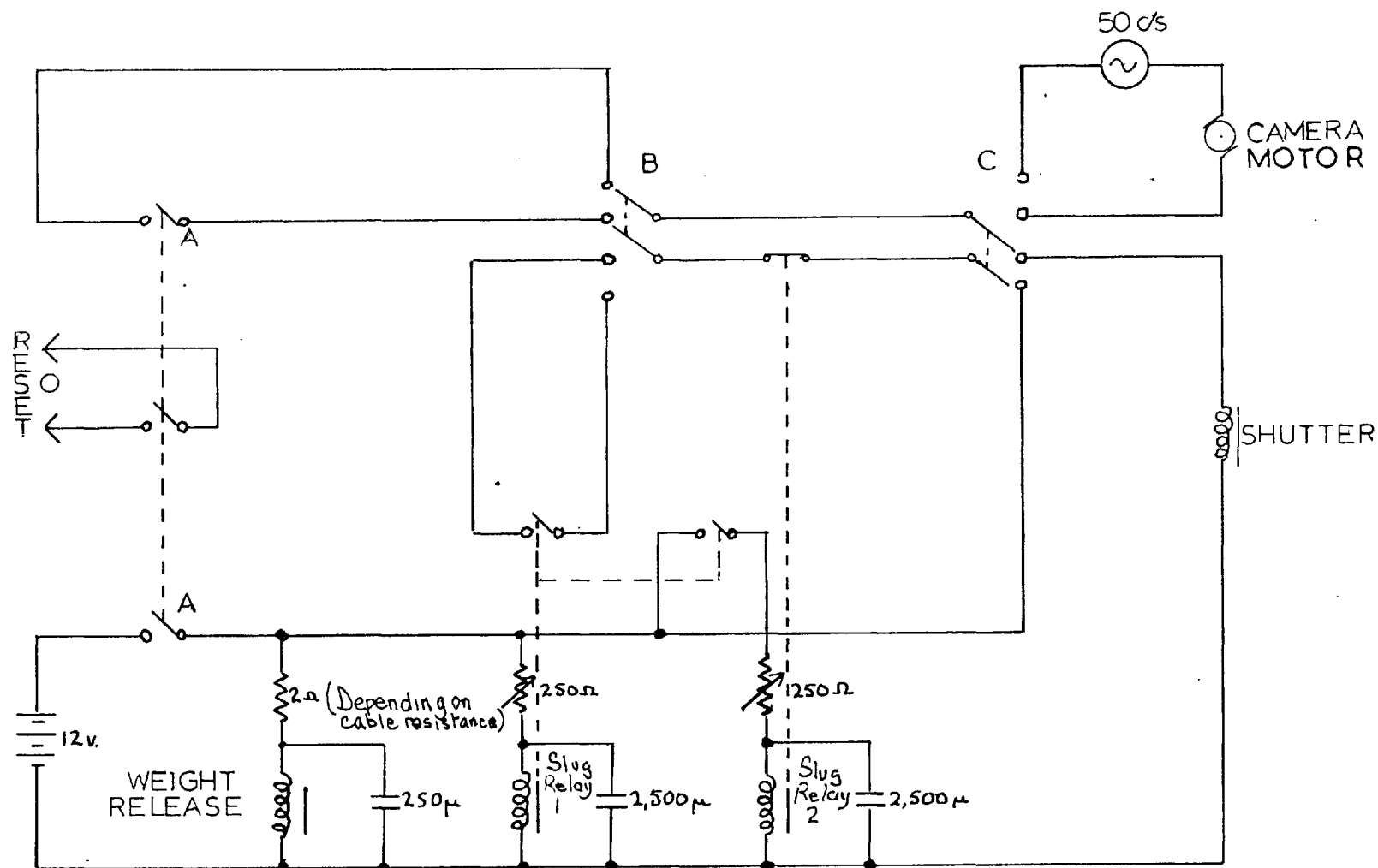


FIGURE 24 - CONTROL CIRCUIT.

on the value of the variable resistance in its circuit. When this relay operates, it (a) works the shutter or motor according to the positions of switches B and C as before, and (b) operates slug relay (2) after a time up to about 1 second depending on the variable resistance in its circuit. This second relay then turns off the shutter or motor.

Hence the weight is dropped and the camera or shutter not operated until the weight has nearly reached the ground (about 0.15 sec. beforehand) thus saving film or paper. The camera may however be started directly the weight is dropped and is automatically turned off when the experiment has finished.

A 12v battery is used, and although the timings are not very accurate (10 per cent) the circuit has proved to be very useful.

2.28 Weight dropping equipment

Mason's tripod and bomb release were used, the latter being later replaced due to wear. The weights used were $1\frac{1}{2}$, 50 and 300 pounds. The weight falls onto the ground or more usually onto a baseplate weighing 75 or 800 pounds. A triggering geophone is buried directly below the plate or point of impact with the ground. Towards the end of the series of experiments a winch was fitted onto the front of a Land Rover and used for weight lifting.

2.3 Additional apparatus

In addition to the apparatus described above which is essentially for recording, the additional equipment described below was used.

2.31 Portable seismic equipment

Mooney and Kaasa (1958) describe a transistorised portable seismic equipment which was demonstrated at Silwood Park on March 11th, 1959, at the time when the timing unit (2.26) was being constructed. By the addition of a box containing three meters to indicate the time, and with a little re-wiring, the timing unit was adapted for use as a portable seismic equipment very similar to Mooney's but with the advantages that the reading is directly on the decimal system, not on the binary system, to a greater accuracy. The three figure reading may be taken to the nearest millisecond or, more usually, to the nearest tenth of a millisecond. The circuit is shown in figures 22 and 23, and a photograph of the complete equipment, including hammer and base-plate, ready for use is shown in figure 25. It would be better if the additional plug-in unit for adaption to the portable seismic application of the apparatus were incorporated in the main timing unit box as this would be more convenient and not much more bulky. (That shown on the right of the timing unit in Fig. 25 is larger than it need be).

The working is essentially that the gate is opened by

FIG. 25

PORTABLE
SEISMIC EQUIPMENT

the initial shock caused by an explosion or hammering a plate and is closed by the first arrival at the geophone which is amplified and shaped as described in 2.26. The number of 10 K c/s timing pulses passed through the gate between the opening and closing times is counted and indicated on the dials. The amplifier gain is adjusted by means of input and output potentiometers to be just below the value needed for the ground noise level to close the gate. Two T.S.3 amplifiers are usually used beyond about 150 feet, and it is best for them to have their own power supply. With increasing distance from the source (depending on the signal to noise ratio), the triggering point on the first arrival pulse becomes later with respect to the beginning of the pulse because of the slower rise time, and eventually is only reached on a later pulse of greater amplitude, from the surface waves for example. The magnitudes of these errors, which increase with the distance from the source, are difficult to estimate. Nevertheless, the equipment was useful for a number of refraction surveys and velocity logs when it was possible to obtain an independent check of the first break times from the normal records and fill in the details with the portable seismic equipment (see 3.51). The agreement was found to be good (see for example the time distance graph of site 2)

2.32 Curve follower

A double beam curve follower developed by Dash (1959) was employed to change straight records into two-component records. Two straight records of two mutually perpendicular components of the amplitude of ground motion recorded against time in the usual manner were traced onto separate strips of paper and passed into the curve follower which produced push-pull electrical outputs of a few volts on either side of a 200 volt D.C. level which were passed to the XA and YA plates of the oscilloscope. The resulting picture of a section of the ground motion was photographed.

One advantage of the system was that the curves were followed slowly, thus permitting a particular section of the record to be studied closely. The main difficulty was the inaccuracy due to timing, tracing and following the curves. A similar system without the need of the curve follower could be used if the records were on magnetic tape.

Usually the two-component pictures were obtained by direct recording at the time of the experiment as described in 3.1.

2.33 Photographic equipment

The recording hut was used also as a dark room so that records could be developed immediately for quick analysis. Kodak DX80R replenisher used as an ultra rapid developer, AM33 rapid liquid fixer and Kodak hypo-cleaning

agent were used for processing, and the records could be examined in the light after about one minute. Kodak RP30 recording paper was used, though RP35 would be a little better but it was necessary, at that time, to order in fairly large quantities. A number of different films were tried (Kodak R60, R55, R20 and Ilford 5B52) and eventually R60 was used as this permitted a duller, and therefore smaller spot. This fact outweighed the disadvantage of having to work in almost complete darkness as the important factor for good definition is the ratio of spot to screen size. For enlargements Kodak D19b developer and WSG4S extra hard Bromide printing paper were used.

2.34 Cables

Four-core coaxial cable (B.I.C.C. '4D') was used for the geophone leads and found to have a sufficiently low capacity (giving 10^0 phase shift at 800 c/s under working conditions) and to provide good 50 c/s shielding.

2.35 Hardware

In addition to the usual spades etc. used for work of this nature, 1" , 4" and 6" augers were used for making boreholes for geophone burying, velocity logs and geological interpretation. The greatest depth augered was 18.5 feet with the 4" auger (in sand, at the bottom of a 10' hole to find the water table at site 1).

2.4 Equipment Testing

Various means were used for testing the apparatus

described above, both when under construction and in the field.

2.41 Pulse testing

A simple circuit was made for measuring the geophone response to a step function. The response curve can theoretically, by Fourier analysis and measurement of voltage levels, be used to give the complete frequency response and sensitivity, but in fact was only used to give f_0 and h which can be obtained simply from the shape of the curve. This was because Fourier analysis would not give accurate information about the high frequencies, and voltage measurements on an oscilloscope are only accurate to about 3 per cent.

By solving the differential equation (6) with the particular initial conditions of a step function, it may be shown that the ratio of amplitudes of successive peaks of the damped vibration resulting from a step function when $h < 1$ is given by

$$\frac{V_1}{V_2} = e^{\frac{h\pi}{\sqrt{1-h^2}}}$$

and if
$$L_1 = \log_e \frac{V_2}{V_1}$$

then
$$h = \frac{L_1}{(\pi^2 + L_1^2)^{\frac{1}{2}}}$$

and the voltage is zero at times given by

$$t_n = \frac{n\pi}{\omega_0(1-h^2)^{\frac{1}{2}}}$$

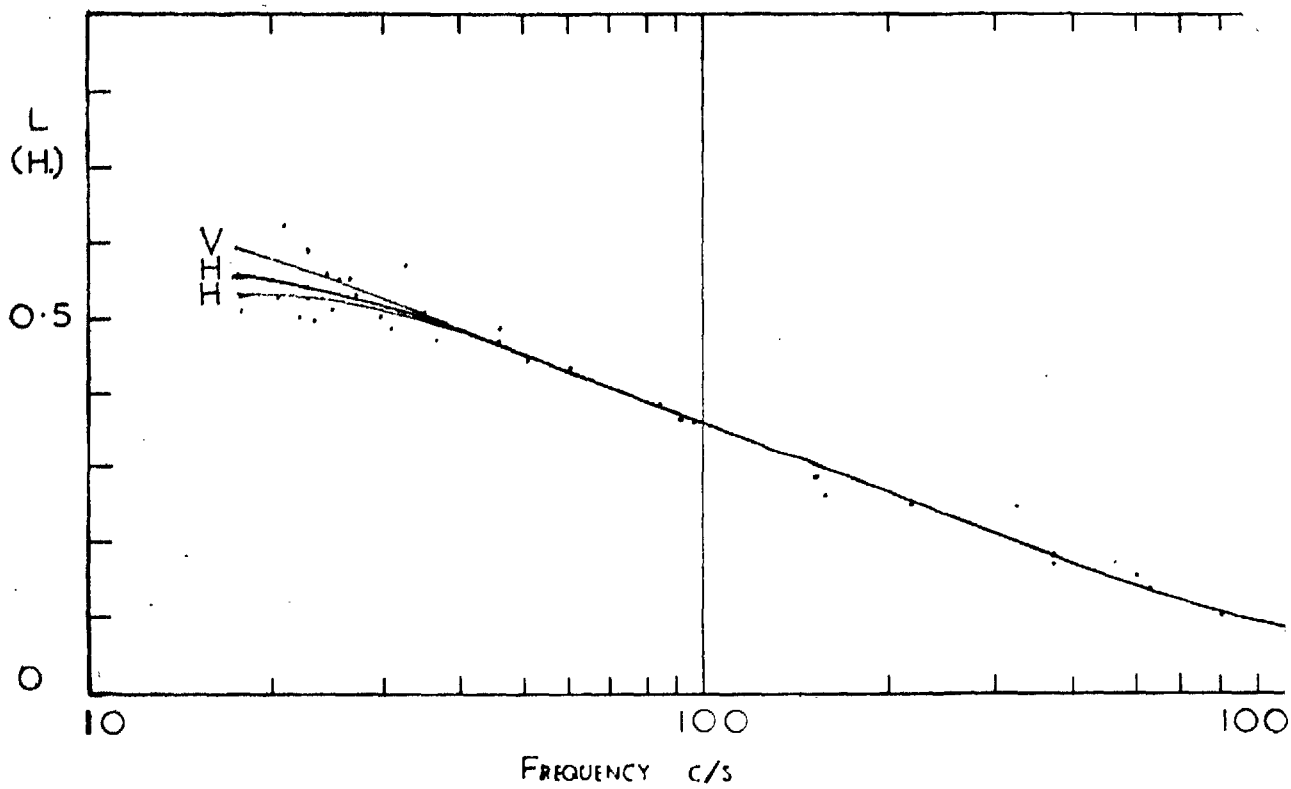


FIG 28—VARIATION OF INDUCTANCE WITH FREQUENCY FOR LF VERTICAL AND 2 HORIZONTAL GEOPHONE

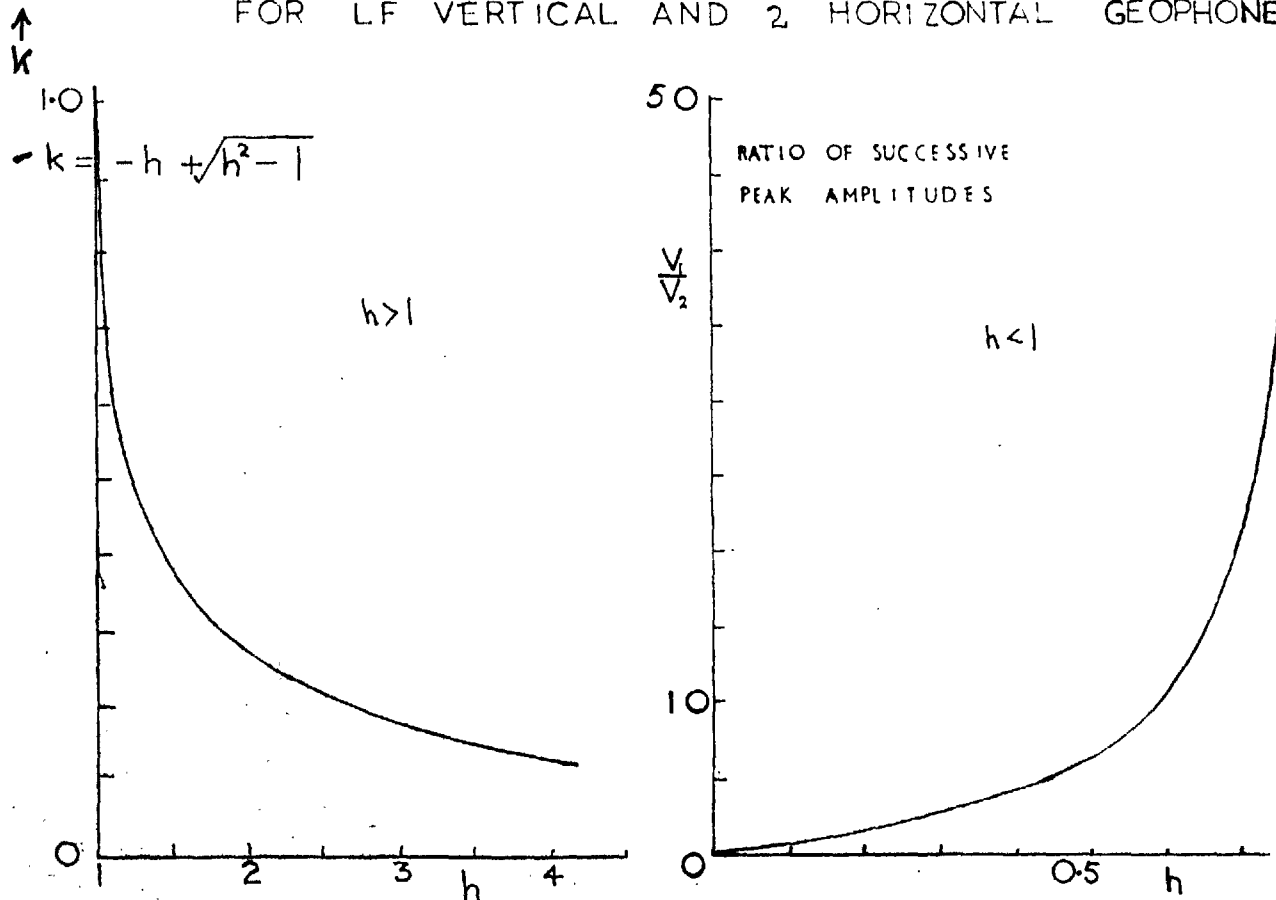


FIG 26—CURVES FOR DETERMINATION OF h FROM GEOPHONE RESPONSE TO A STEP FUNCTION

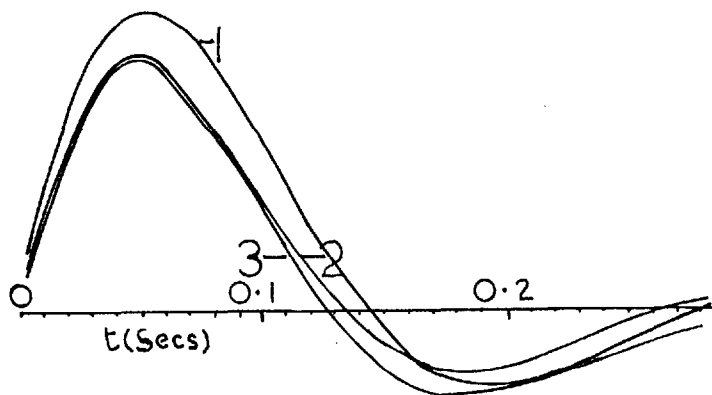


FIG 27 - PULSE TEST OF LF GEOPHONE
IN POSITION

(TRACING OF RECORD 1073)

$$\frac{V_1}{V_2} = 4.75 \pm 0.2$$

$$\therefore h = 0.43 \pm 0.015 \text{ (fig. 2b)}$$

$$f_0 = \frac{1}{2t_0(1-h^2)^{\frac{1}{2}}} \\ = 4.15 \pm 0.12 \text{ c/s.}$$

(AVERAGE VALUES FOR THIS AND
OTHER RECORDS FOR DIFFERENT INPUTS)

When $h > 1$, the decay is ~~nonoscillatory~~ of the form

$$V = V_0 e^{-h\omega_0 t} (e^{+\sqrt{h^2-1}\omega_0 t} - e^{-\sqrt{h^2-1}\omega_0 t})$$

and beyond the time when the second term is negligible, which is roughly at the maximum output voltage, it may be seen that

$$\begin{aligned} \frac{V_t}{V_{t+\Delta t}} &= \frac{V_0 e^{-k\omega_0 t}}{V_0 e^{-k\omega_0(t+\Delta t)}} \quad \text{where } k = -h + \sqrt{h^2-1} \\ &= e^{k\omega_0 \Delta t} \end{aligned}$$

$$\text{If } L_2 = \log_e \left(\frac{V_t}{V_{t+\Delta t}} \right) = k\omega_0 \Delta t$$

$$\text{then } k = \frac{L_2}{\omega_0 \Delta t}$$

and hence h may be calculated from the value of k which is found by measuring L_2 for a particular Δt , knowing ω_0 . For simplicity, a curve of k against h was prepared and used to give h for a given k . Curves of V_1/V_2 against h were also drawn and used to find h by measuring V_1/V_2 when $h < 1$. Figure 26 gives these curves.

Pulse tests were applied during field experiments before using the low frequency geophone to ensure that the coils were free, as it may be shown that a horizontal geophone must be levelled to within an accuracy of ϕ° , where ϕ is given by $\phi \approx \frac{f\omega^2 d}{0.4}$ and d is the maximum tolerable displacement of the coil from the zero position permitted by the stop. ϕ in this case is about 2° , which is difficult to achieve under field conditions. The

possibility of applying a DC correcting current to the coil was considered, but eventually found to be unnecessary. It should be noted that the small displacements from the true equilibrium position of the suspended mass of a horizontal geophone due to inaccurate levelling may affect the natural frequency, f_0 . This is because the springs are working at a different point on the load/displacement graph (see figure 14). This fact was used to adjust the responses of the three components of the LF3 C geophone to be similar (see 2.214(f)).

Figure 27 shows photographs of pulse tests of the three components of the low frequency geophone in position for measurement showing the degree of similarity of the responses and the calculation of f_0 and h . Tests were usually carried out at a displacement of about 0.01 cms., which was about the greatest likely to be measured during the experiments, and also with a displacement such that the output was only a little more than the ground noise level. Results similar to those shown in figure 27 were obtained for both positive and negative pulses of various amplitudes.

In the laboratory, R_2 was measured by plotting R_L against h for values of h greater and less than 1.

Using the formula

$$h = \frac{K^2}{2\omega_0^2 m} \left(\frac{1}{(R_L + R_g)} + \frac{1}{R_2 N^2} \right),$$

R_2 could be found from the intercept, and was found to be 5.5×10^{-4} ohms. This value of R_2 gave values of h which agreed well with those found experimentally for geophones with windings similar to the one on the geophone used for finding R_2 . However for different windings the theoretical and experimental values of h differed by as much as 20 per cent. For the LF geophone $h = 0.43$ experimentally, which agrees fairly well with the theoretical values of 0.35 and 0.44 which are found by taking f_0 as 4.15 c/s and 3.3 c/s respectively.

2.42 Other geophone parameters

The frequencies at which the geophone voltage and current were in phase when driven by a sinusoidal voltage input gave f_0 and f_1 . By measuring the DC current needed to keep the coil in the zero position for various values of an added mass it is possible to find a value for K . The application of this method is difficult and less accurate for the horizontal ^ogeophones.

As stated above, the variation of L with f was measured by a resonance method using various known series condensers (see figure 28). Knowing L , f_1 , f_0 and m , a value can then be calculated for K . This was found to be the most accurate way of measuring K , giving a standard error of about 2 per cent, and had the advantage that it could be used equally well for the horizontal geophone.

2.43 Shake-table measurements

Two shake-tables were built, one very rigid and one very elastic (see Washburn, 1942). The rigid shake-table was found to be unsatisfactory because of inherent resonances, but the elastic shake-table worked well and is shown in figure 29 for a horizontal geophone.

A small Goodmans vibrator capable of producing a 21b. thrust was bolted rigidly to the geophone under test and the whole suspended horizontally or vertically, as required, from the frame. The geophone was also bolted rigidly to another Goodmans vibrator which was fixed rigidly to the frame, and this was used as the detector. The output of this vibrator used in reverse gave the motion of the geophone case with respect to a point fixed in space, as long as the frequencies used were well above the natural frequency of the vibrator when loaded with the geophone which was true in this case. The vibrator produced a motion of the geophone (and itself) which was measured by the detector. The geophone output was compared on an oscilloscope with the measured geophone motion by means of a Lissajous figure presentation to give the amplitude and phase characteristics of the geophone. The results of the shake-table measurements were found to agree quite well with the calculated frequency response for Mason's geophone (see figure 30), and the frequency responses of the other geophones were therefore assumed

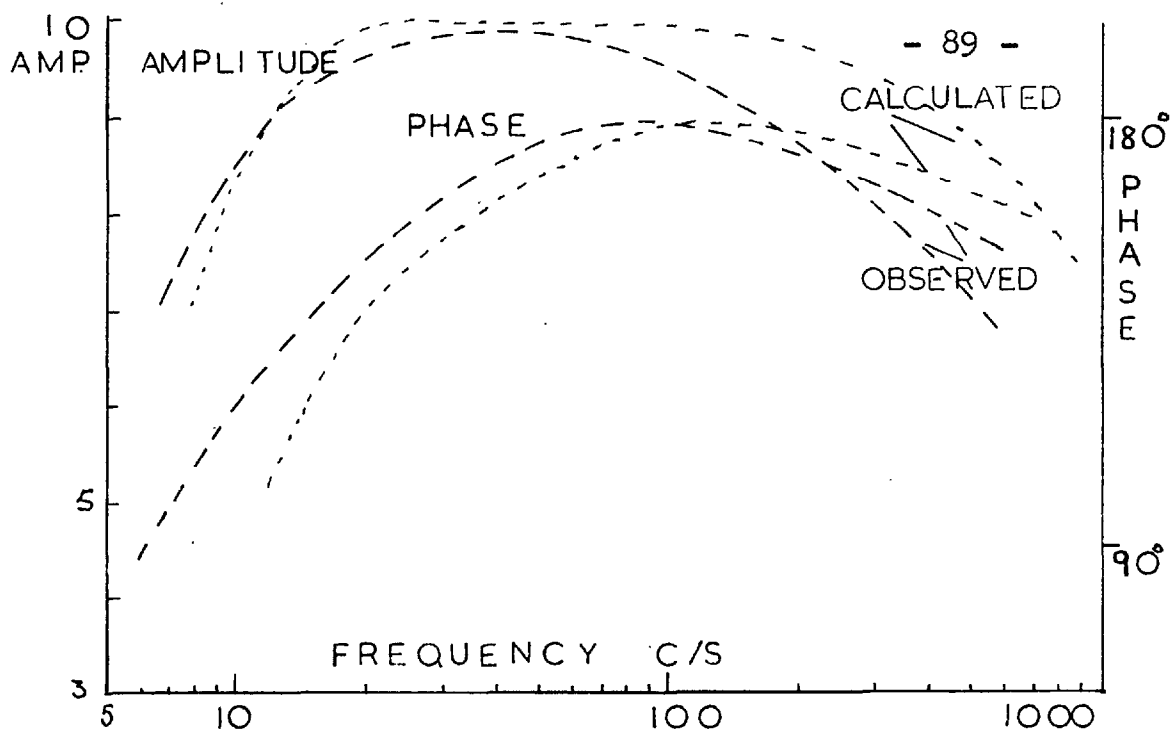


FIG30 - SHAKETABLE MEASUREMENT OF FREQUENCY RESPONSE OF MASON'S GEOPHONES

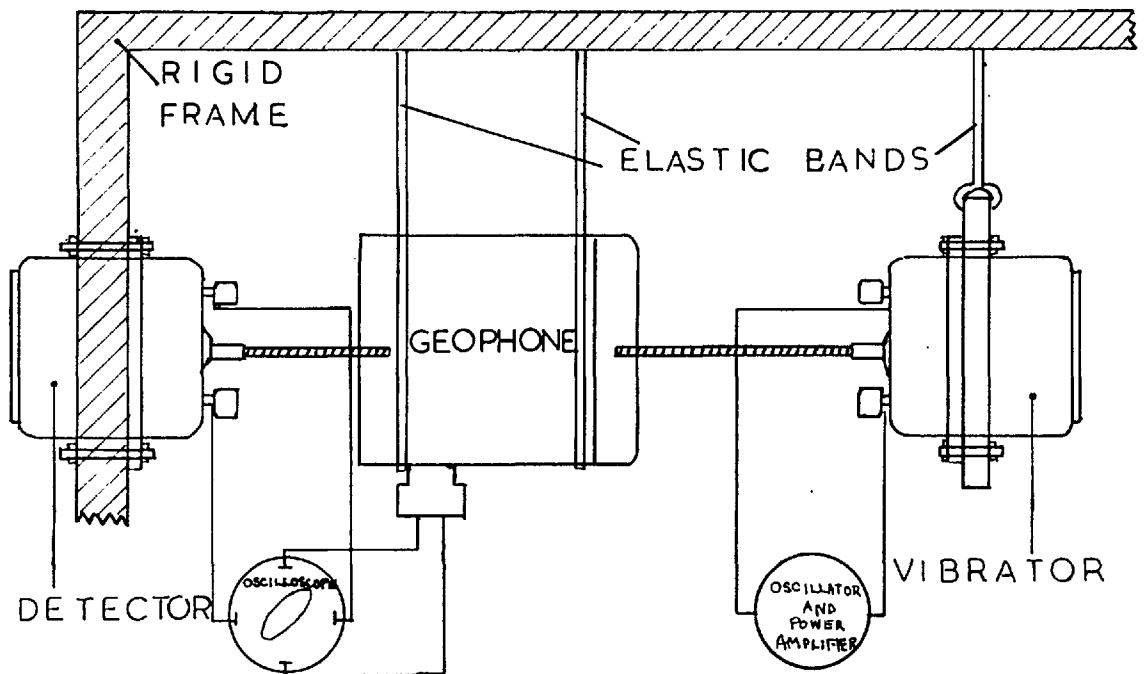


FIG29 - ELASTIC SHAKE - TABLE USED FOR TESTING MASON'S GEOPHONES

from the calculated values.

An advantage of the shake-table is that it can be used to show up isolated resonances due to the construction, both electrically or when examined with a stroboscopic light source driven by a supply working at the same frequency as a current used to drive the geophone.

If a larger Goodmans vibrator with the necessary power amplifier were available, a shake table similar to that described above could be built to test the complete response of the recording system, including the geophone-ground coupling, by placing the geophone in a box of earth which is shaken.

2.44 Calibration oscillator

This useful piece of apparatus was built by Mason (1951) and used to measure noise levels and gains. It provides voltage outputs between 10 μ .v. and 10 m.v. at 40.2 c/s. The outputs were compared with a standard cell and found to be accurate to within 3 per cent, although this measurement did not allow for possible changes in the oscilloscope amplifier gain between 40 c/s and D.C.

The outputs however were only at the correct levels when the battery was not run down, and, during analysis of results, some inconsistencies in sensitivities were found. These were probably due to variations of the calibration oscillator output when used in the field, and a mains operated voltage standard would be far more satisfactory.

CHAPTER 3

EXPERIMENTS

This chapter deals with the experimental techniques which were used, and describes the main experiments at the two sites. A number of additional experiments were carried out to supplement the main results, and are also described here.

3.1 Experimental Method

For the main experiments various weights were dropped according to the particular study and energy required, usually using a base-plate for better reproducibility of results. When the base-plate had bedded down after a few drops, the results of repeated experiments were so similar that differences were almost indistinguishable. The 50 lb. weight was used whenever possible as it gave a good pulse when dropped from about 4 to 7 feet onto either base-plate.

The geophones were buried by hand at the depth required, and either the source or the geophones moved along the traverse. It was more usual to move the source due to the difficulty of levelling the low frequency geophone under field conditions (see 2.41), and the fact that the geophones had to be packed by hand as the coupling was not good at the bottom of an auger hole. It was also possible by this means to obtain a constant frequency response unaffected by changes in coupling.

The sequence of operations of an experiment was controlled by the control circuit. After each experiment the film or paper was cut off and processed. The experiments were usually repeated three times and recorded on photographic paper in the normal manner as a variation of amplitude of two of the components with time. In addition, about fifteen two-component records were taken on film, the three components again being taken in three pairs of two. Sometimes only one component was recorded on each of the three normal records on paper because this facilitated the technique of compounding two-component pictures using the curve follower as described in 2.32. Both velocity and displacement were usually recorded in the normal manner on paper, and only velocity for the two-component records on film.

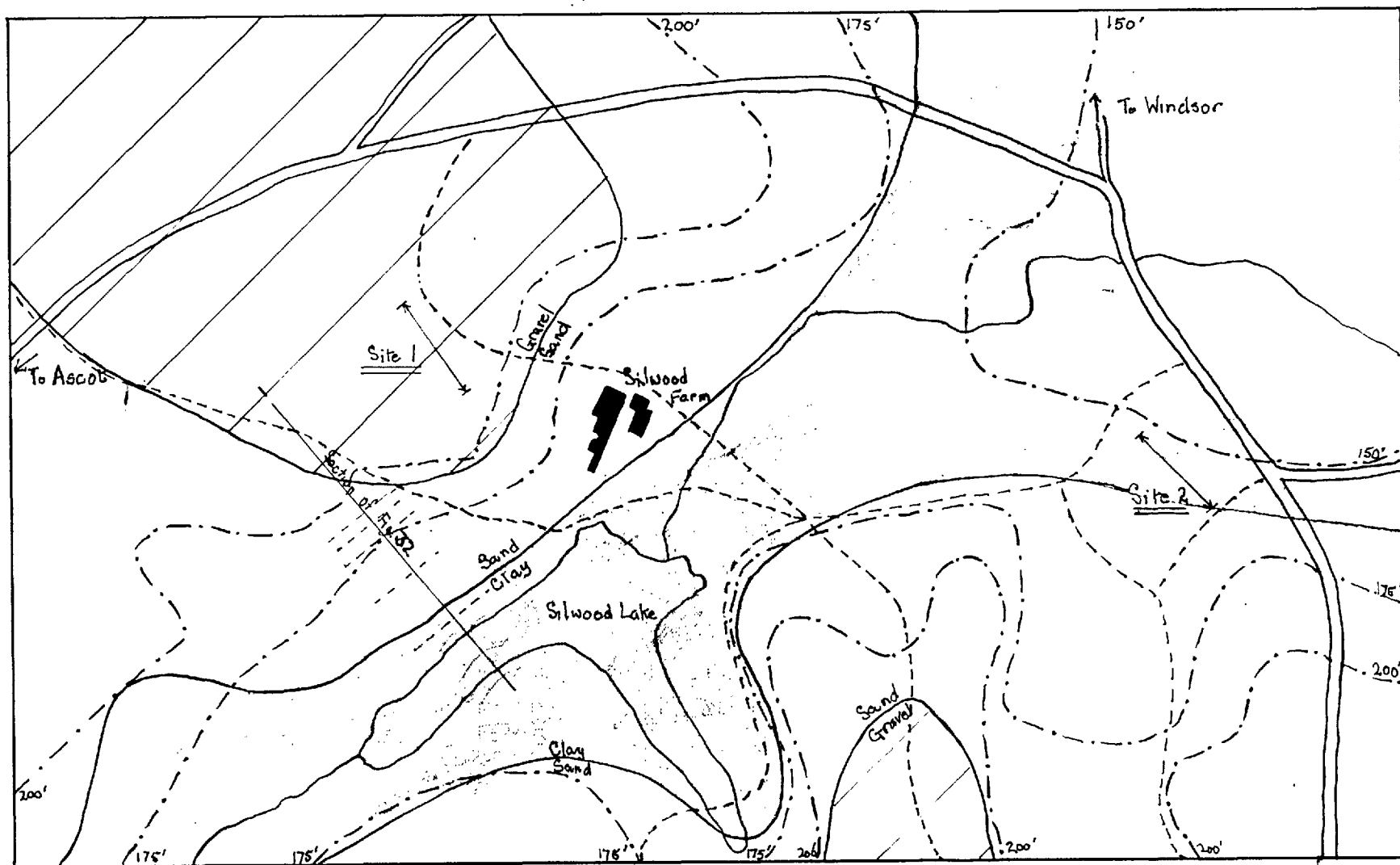
By setting the delay of the timing unit to any particular value picked from the straight records, it was possible to delay the brightening of the oscilloscope spot with respect to the time of impact ($t=0$) by any time up to 0.99 sec. It was also possible to set the time for which the spot stayed bright by means of the oscilloscope time base, not with great accuracy (5 per cent), but the exact duration could be read from the timing marks on the record. Hence it was possible to select any particular section of

the record desired, and this technique was used extensively to simplify the otherwise complicated two-component pictures. Each drop therefore had to be repeated about 20 times to get a complete picture of the motion, and hence the need for a base-plate to give good reproducibility. The gain or source energy could be increased to study a particular section of the record more closely.

When working beyond about 200 feet, the experiments were usually done at night because of the lower ground noise level at that time. The amplitude of ground motion produced is roughly proportional to the square root of the energy, and a reduction by a factor m of the amplitude of ground noise permits a reduction by m^2 of the energy. This is a strong argument for choosing experimental sites in quiet places.

3.2 Experimental Sites

The experiments were carried out in the ground of the Imperial College Field Station, Silwood Park, Ascot, Berkshire. On the higher ground over much of the area the geology consists of 45 feet of Bagshot Sands overlying about 400 feet of London clay, both with negligible dip. At site 1 approximately 5' of gravel and 2' of top soil overlie the pure sand at the surface and the water table



Geology taken from Geological Survey sheet
269 (Drift) with minor corrections near sites 1 & 2.

FIG. 31.— MAP SHOWING GEOLOGY AND EXPERIMENTAL SITES AT SILWOOD PARK.

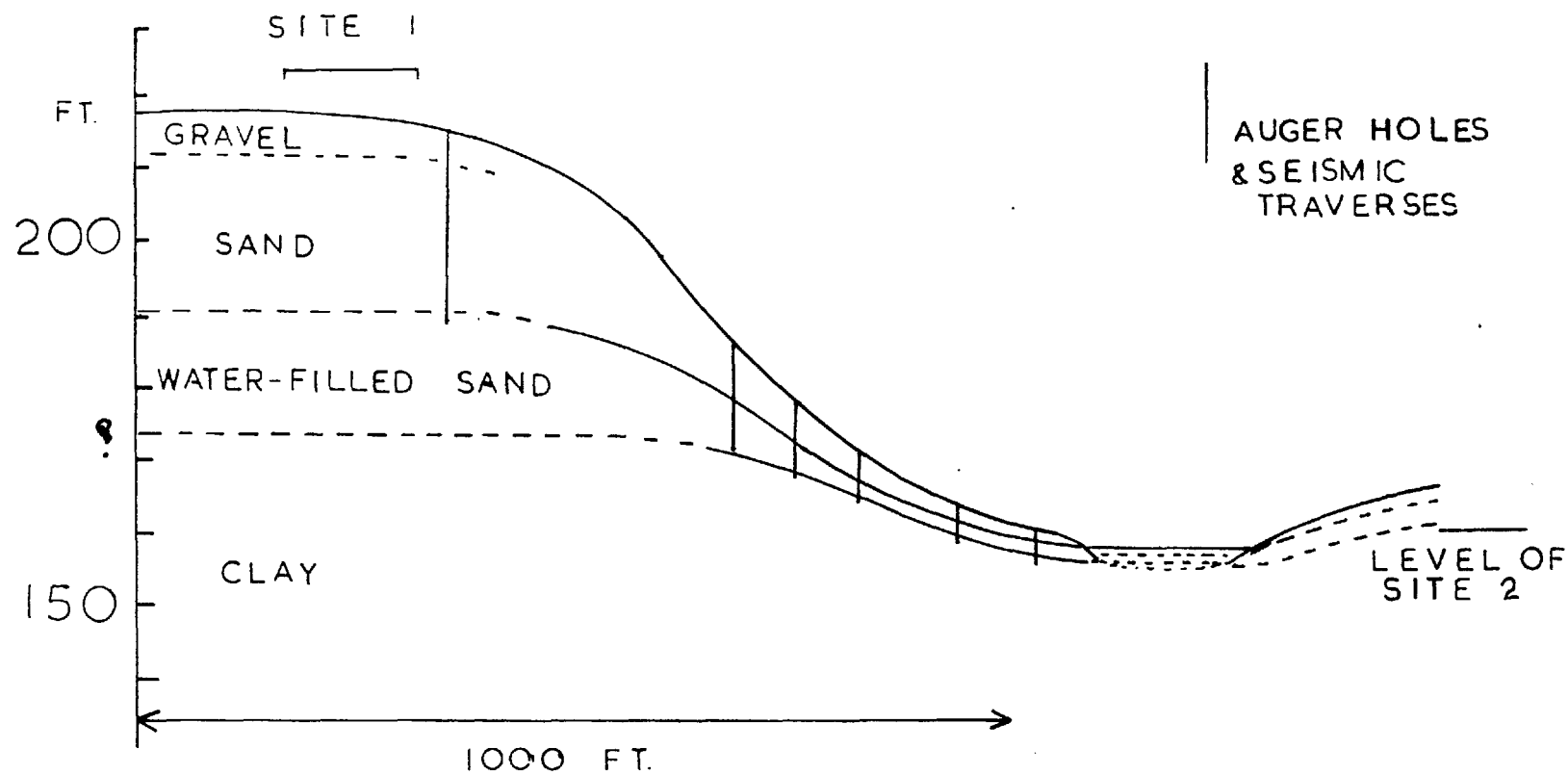


FIG 32 - GEOLOGICAL SECTION OF HILLSIDE AT SILWOOD PARK.

was at 26 feet at the time of the experiments. River erosion has cut into the sand and in the lowest parts of Silwood Park the clay is exposed below the top soil with the water table at a depth of about 2' during the experiments on this site.

The two experimental sites chosen were on the sand (Site 1) and on the clay (Site 2). Figure 31 shows the positions of the traverses and the geology of the area. Site 1 was chosen as it had been used by Mason (1951) and was suitable for near source investigations, and Site 2 because it was thought to be ^{an} unusually uniform medium.

The transition from sand to clay was traced along one of the hillsides with the portable seismic equipment, confirmed where possible by auger holes, thus giving a geological cross-section of the area (see Fig. 32).

3.3 Site 1 Experiments

The preliminary experiments at this site were mainly to test equipment. It was possible to decide on requirements and test the suitability of each piece of apparatus as it was built.

3.31 Noise Levels

Overhead 50 c/s power cables induced currents into the ground which flow ed through the geophone and gave rise to a large 50 c/s noise level particularly when the soil was damp. This could be almost completely eliminated by insulating the geophone from the ground and, together with good earthing and shielding, the 50 c/s noise level was kept below the level of the amplifier noise. At one time, a balancing circuit with double-sided geophones and amplifiers was tested, but the capacitors needed for phase balancing introduced significant phase shifts and the method was abandoned. A hum-bucking coil was also tried, but was not used as the electromagnetic field in the geophone seemed to vary too much to allow sufficiently good cancellation to be achieved.

The variation of noise level with time of day and depth of geophone was investigated.

3.32 True Ground Motion

The first experiments were concerned with the accuracy of recording ground motion. Various types of geophones were buried next to one another and their records compared. Those used were the Willmore, GAA, Hall-Sears, Mason and low frequency geophones. The effects of loose and tight

packing, different frequency responses and filtering were investigated. By this means it was possible to obtain similar records from two geophones with sufficiently wide frequency responses, well packed near one another. The effects of a tunnel or hole made for the purpose of burying the geophones were investigated to determine whether they significantly affected the records and needed to be filled in before the experiments. The conditions needed to obtain what seemed to be a reproducible true record of the ground motion were thus found.

3.33 Source Pulses

The pulses originating from various types of plate and weight were investigated and a picture of the pulse arising from each type of source obtained. The bounce times of the weight were measured for various drop heights, weights and base-plates so that the source could be chosen to have a bounce time after any motion of interest on the record. The 800 lbs. base-plate was usually used for producing a large bounce time.

The variation of pulse shape and amplitude with depth and source energy was studied. This was used to obtain the variation of amplitude of ground motion with energy at a given depth, and with depth for a given energy, and also

to indicate where the amplitudes were great enough to introduce non-linearities. The graph of ground velocity versus source energy gave a straight line at low source energies, but velocities were found to fall below the line for energies greater than a critical value at which it was assumed that the behaviour of the ground was non-elastic.

Fourier analysis was used to give the change in frequency components of the pulse.

3.34 Motion near the Source

The dry sand above the water table at Site 1 was convenient for studies of motion near the source. By use of a tunnel which was dug 10' in from the bottom of a 10' hole, it was possible to measure the passage of the pulse travelling vertically through undisturbed ground. To do this, the geophone was buried at the end of the tunnel and well packed with sand. The weight was then dropped onto a plate approximately 10' vertically above.

3.35 Traverse Measurements

A traverse was completed up to 300 feet. Measurements were made with the geophone at a depth of 12 feet with the source moved to distances of 50, 100, 200 and 300 feet.

Mason completed a traverse on a similar site over the same

distance with the geophone on the surface taking readings at 20 foot intervals. Recordings were taken in the present experiments on the surface at distances of 20 and 300 feet to check the agreement with Mason's results.

3.4 Site 2 Experiments

The experiments at site 2 were similar to those at Site 1; noise levels, geophone-ground coupling and motion near the source for pulse shapes and amplitude studies were investigated, but the main emphasis was on the traverse records. The reason for this was that the geology was thought to be simpler so that the results from this traverse would be simpler to interpret and provide a starting point for the more difficult interpretation of the results of the traverse records at Site 1. In fact, this did not prove to be entirely the case, as the records from this site took longer to interpret than had been expected, and a detailed interpretation of the traverse results at Site 1 will not be given, but may be attempted at a later date.

The 50 lbs. weight dropped from 4 and 7 feet onto the 800 lbs. base-plate was used for all the traverse experiments at Site 2. The 300 lbs. weight and 75 lbs. base-plate were occasionally used to get more energy at greater

distances or for comparison of sources. Measurements were made at 0, 15, 50, 100, 200 and 300 feet with the LF3C geophone buried by hand at 6' and 5' and with the HS3C geophone in a borehole at 15', packed as tightly as possible with a probe. For comparison between explosive and dropping weight sources detonators were exploded about 6' below the ground surface and recorded with the 5' and 15' geophones at 0', 15', 50', 100', 150', 200', 250' and 300'.

Detonators were also exploded in a borehole and in a 3' cavity just below the surface to investigate the effects of source conditions. At each geophone-source combination, records were taken as described in 3.1 (except that two-component pictures were only taken for some of the experiments using detonators).

A velocity log to a depth of 14' and a portable seismic traverse to 300' were completed using the portable seismic equipment to give the velocity conditions (see Figs. 34 and 36 pp 105 & 112 ~~42~~).

3.5 Supplementary Experiments

3.51 Portable seismic equipment surveys

The portable seismic equipment was used for refraction surveys and velocity logs. The positions of the refraction surveys are marked on the geological map of Silwood Park, Fig. 31. They are two long surveys at Sites 1 and 2 up to

220 feet and 300 feet respectively, and a number of shorter traverses up to about 100 feet to find the geological structure of the hillside shown in Fig. 32 and when "reverse shooting" at site 2 to determine dip. These surveys were used to give velocities and depths by the refraction method. Each geophone location had to be surveyed to the nearest inch as the recording points were closely spaced and times were measured to the nearest tenth of a millisecond for the purpose of showing up small variations in velocity.

The variations of velocity down auger holes at sites 1 and 2 were also measured. These were confirmed by normal records at a number of depths.

3.52 Density Logs

Using the Soil Mechanics Ltd. gamma-ray density logger, the density variation with depth was measured at sites 1 and 2 down to 18 feet, to an accuracy which is claimed to be one per cent.

3.53 Soil Analysis

With the help of Dr. Ambrascoys of the Soil Mechanics department of Imperial College, water content, particle size distribution and elastic constant measurements, were made at both sites using the holes which were dug for burying the geophones.

C H A P T E R 4

RESULTS AND INTERPRETATION

4.1 Introduction

More than a thousand normal paper records and a larger number of two-component particle motion records were taken at the two experimental sites, using the methods described in Chapter 3 and the apparatus described in Chapter 2.

The experiments were not concerned with any particular line of research, but were designed to study as many as possible of the main topics of interest outlined at the end of Chapter 1. By this means it was hoped to be able to decide which topics were most suitable for study, and to make suggestions for future investigations which might seem promising.

Although it was expected that a large proportion of the results would not be studied thoroughly, it was decided to take a comprehensive set of records at both sites while the equipment was in use in the field. It is hoped that some of the aspects which have been dealt with only briefly may be studied in detail at a later date, using the records which are available.

One topic of particular interest to seismologists at the present time is the dispersion and attenuation of body and surface waves. For this reason, and also because a computer was available to perform the Fourier analyses,

this subject received the greatest attention in the analysis of the traverse records at Site 2 (see Sections 4.9 and 4.10). At Site 1, the traverse records were mainly considered in a qualitative manner (Section 4.8) and greater emphasis was placed upon the more limited topics of noise levels (Section 4.4), the true ground motion (Section 4.5), source pulses (Section 4.6) and motion near the source (Section 4.7).

First, however, the results are given of a number of supplementary experiments to determine the velocity conditions and obtain quantitative measurements of the properties of the earth materials (Section 4.2). In Section 4.3, some details are given of the method of analysis, particularly in connection with the Fourier analyses.

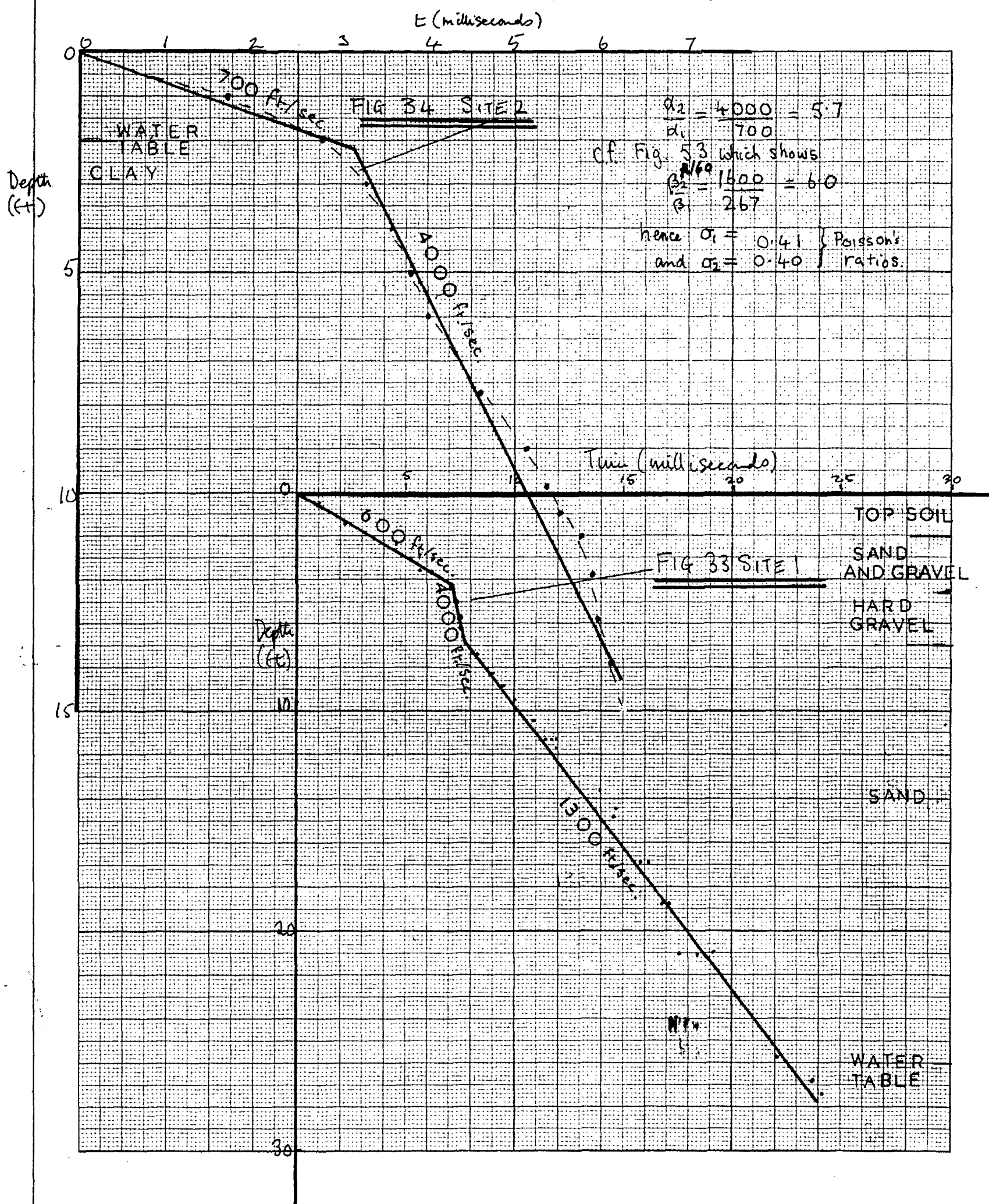
4.2 Supplementary Results

Figures 33 and 34 show the variations of vertical travel time with depth as measured in bore-holes using the portable seismic equipment. The results of the density logs are given in Figures 35 and 36.

4.21 Velocity and Density Logs

The vertical travel times are accurate and reproducible to about the nearest 0.0002 sec., and as

105
 FIGS. 33 & 34 - TIME DEPTH CURVES, SITES 1 & 2.
 (for vertical travel time)



the total travel time at 14' at Site 2 is only about 0.006 secs., the accuracy of the Site 2 measurements may be taken as about 4 per cent. The velocities found give reasonable agreement with the refraction results (Figures 37 and 38) both with regard to velocities and depths. The bore-hole velocities tend to be lower than those found by refraction, but if these values are used in the calculation of the depth by refraction, the depths give even better agreement with the values which have been determined more accurately by other means.

4.22 Soil Analysis

Figure 39 gives the particle size distribution graphs for Site 1. Those for Site 2 were not recorded. The positions from which the samples were taken are shown in Figure 40. It may be seen that the sand is particularly uniform.

A number of experiments were carried out to determine Young's Modulus E by loading a small plate which was carefully placed on the surface of the sand and measuring the resulting deflection. The method, however, did not prove sufficiently accurate. The range of values of E determined at Site 1 was 250 to 400 kgm/cm².

The water content, w , at Site 1 was also measured by weighing a carefully taken sample before and after drying

FIGS. 35 & 36

DENSITY DETERMINATIONS

FIG. 36 - CLAY SITE(2) FIG. 35-SAND SITE(1)

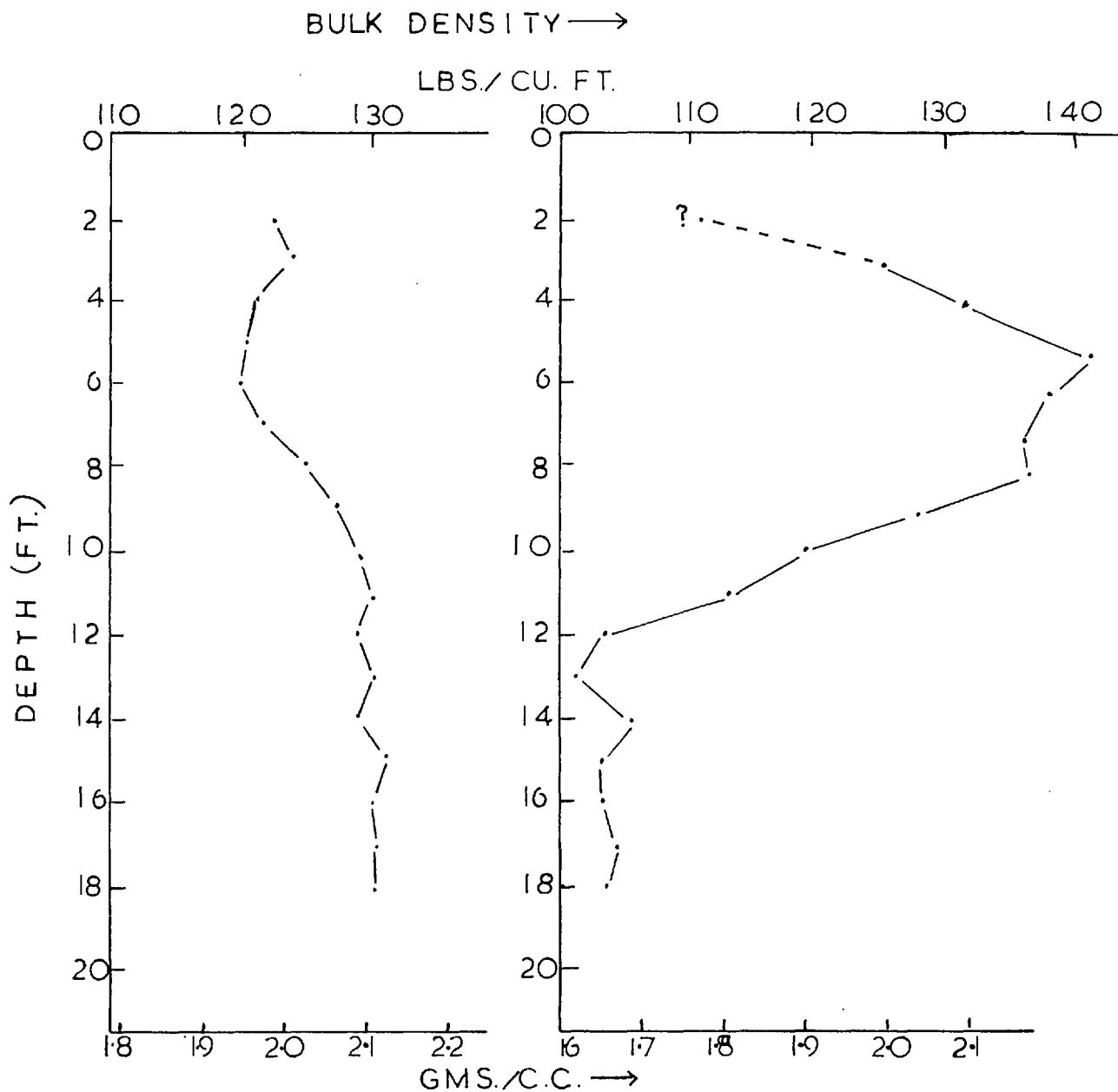


FIGURE 39

Particle Size Distribution

Location SITE 1

Sample Nos. 123 & 4

Date of Test FEB. 60

Description SAMPLES FROM SAND AND GRAVEL IN 11' HOLE AT SITE 1

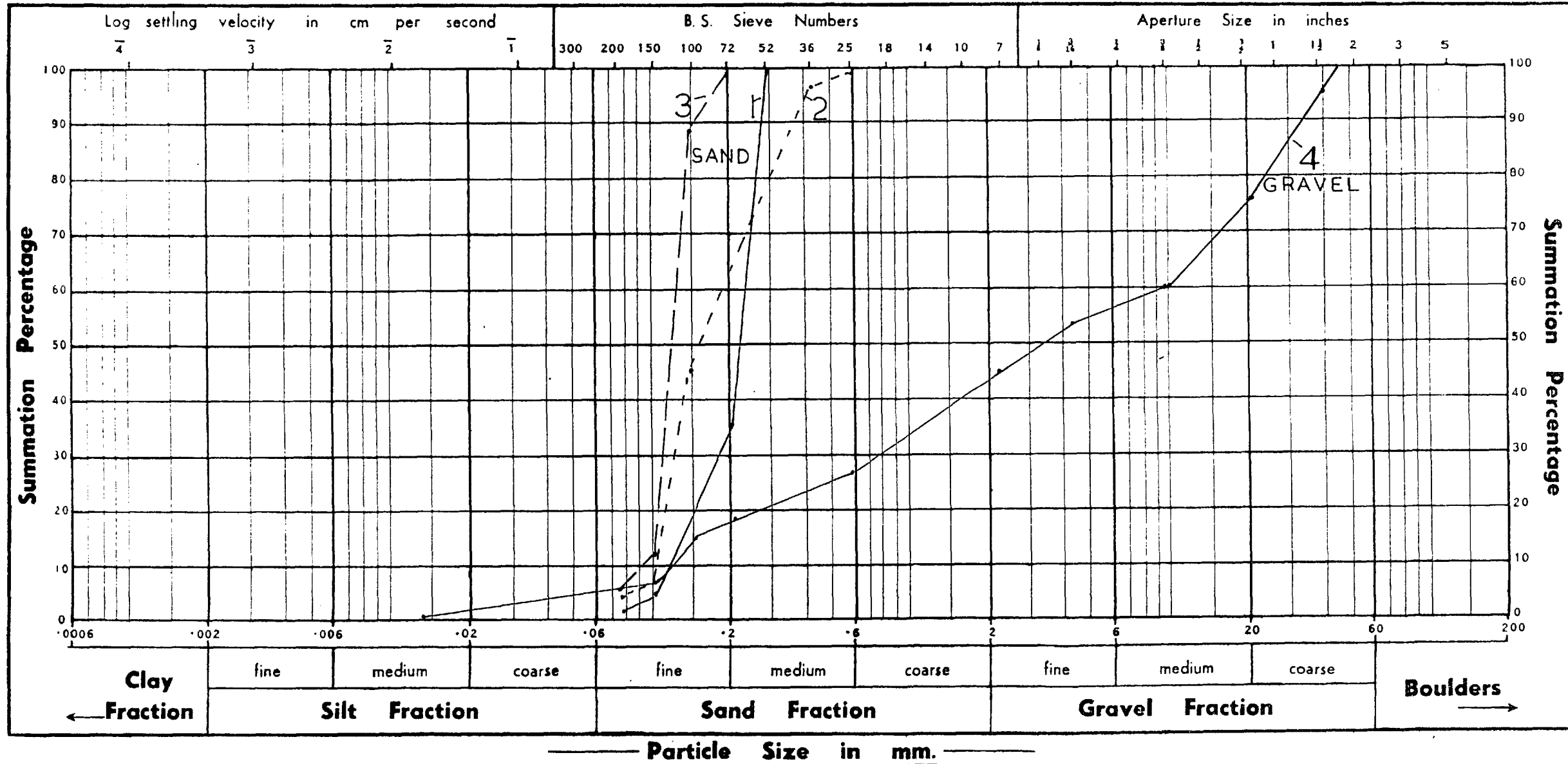
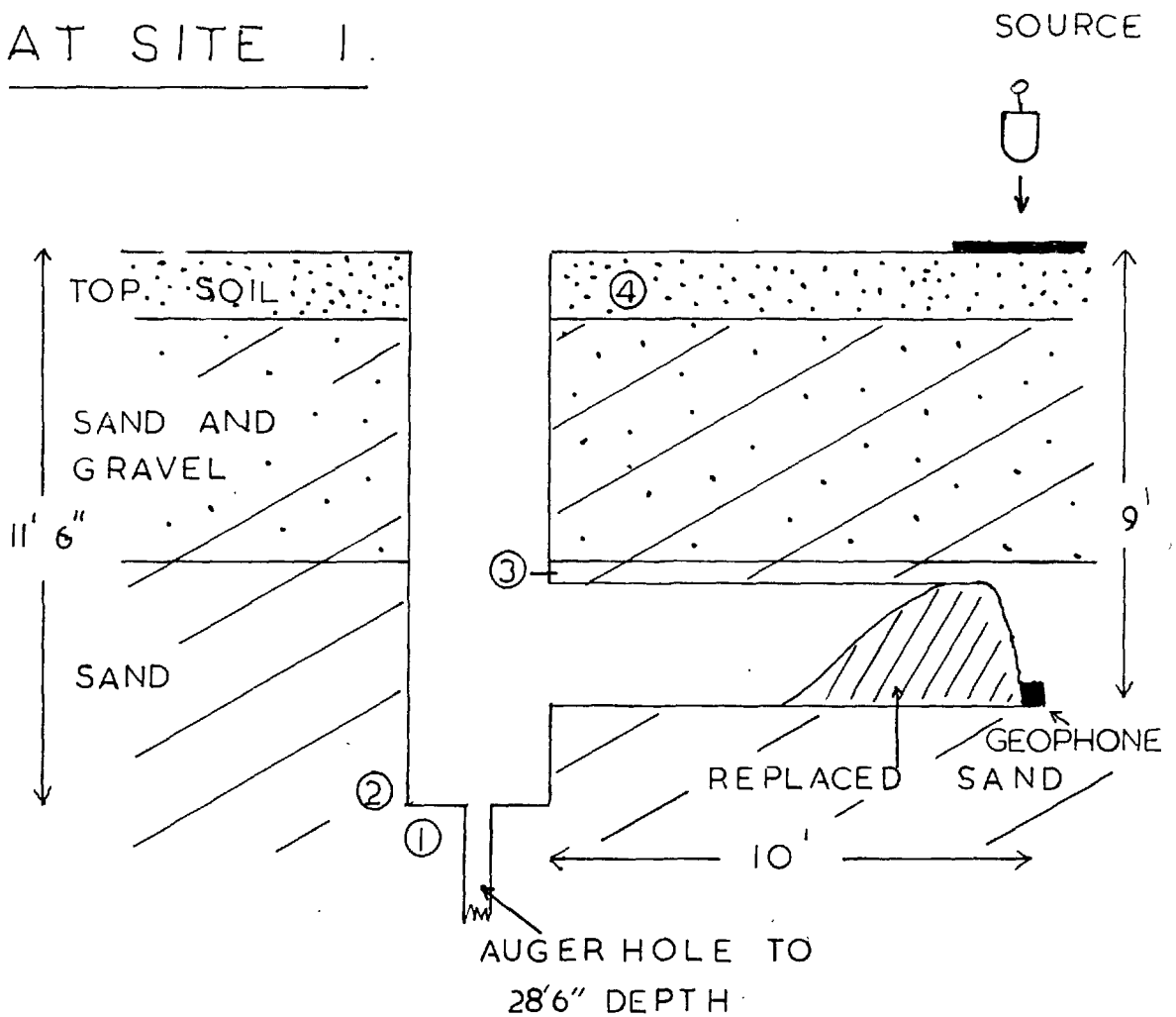


FIG. 40

EXPERIMENTAL LAYOUT OF HOLE AND
TUNNEL USED FOR MEASUREMENTS
AT SITE 1.



SOIL ANALYSIS PARTICLE SIZE
DISTRIBUTION SAMPLES MARKED ① ② ③ & ④

in an oven. The water content was found to be 6 per cent in the body of the sand at a depth of approximately 13'6" below the surface, although the value increased to 28 per cent immediately below the gravel bed at an approximate depth of 7 feet. This is because the finer sands which were found at this depth (see Figures 39 and 40) were less permeable to water and gave rise to a thin layer of saturated sands. Calculations of the degree of saturation, S , may be made from these figures in conjunction with other data regarding bulk density, void ratio, etc. It is found that when $w = 6$ percent $S = 24$ per cent, and when $w = 28$ per cent $S = 99$ per cent.

4.23 Time-distance Graphs

These were determined using the portable seismic equipment and are shown in Figures 37 and 38. In Figure 38 a number of relevant times picked from the normal records at Site 2 are shown. It may be seen that there is good agreement between the P wave velocities found by the two methods. Besides the air wave and P waves shown in Figure 38, a possible S wave velocity in the lower medium is also shown. This was determined both from the transverse motion measurements which were indefinite and open to misinterpretation, and also from the initial motion of the long period Love and Rayleigh waves. Another

FIG. 37 TIME DISTANCE GRAPH AT SITE 1

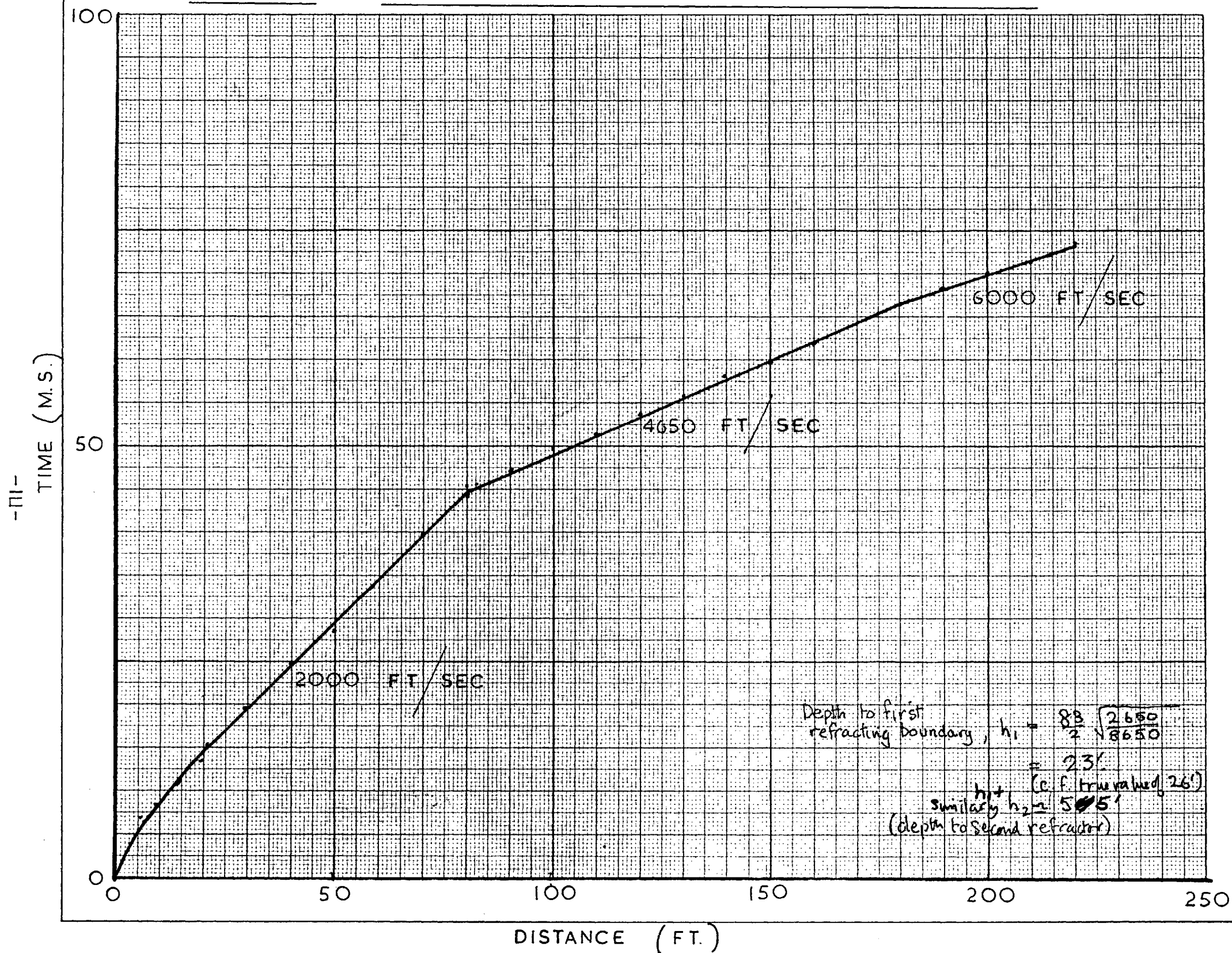
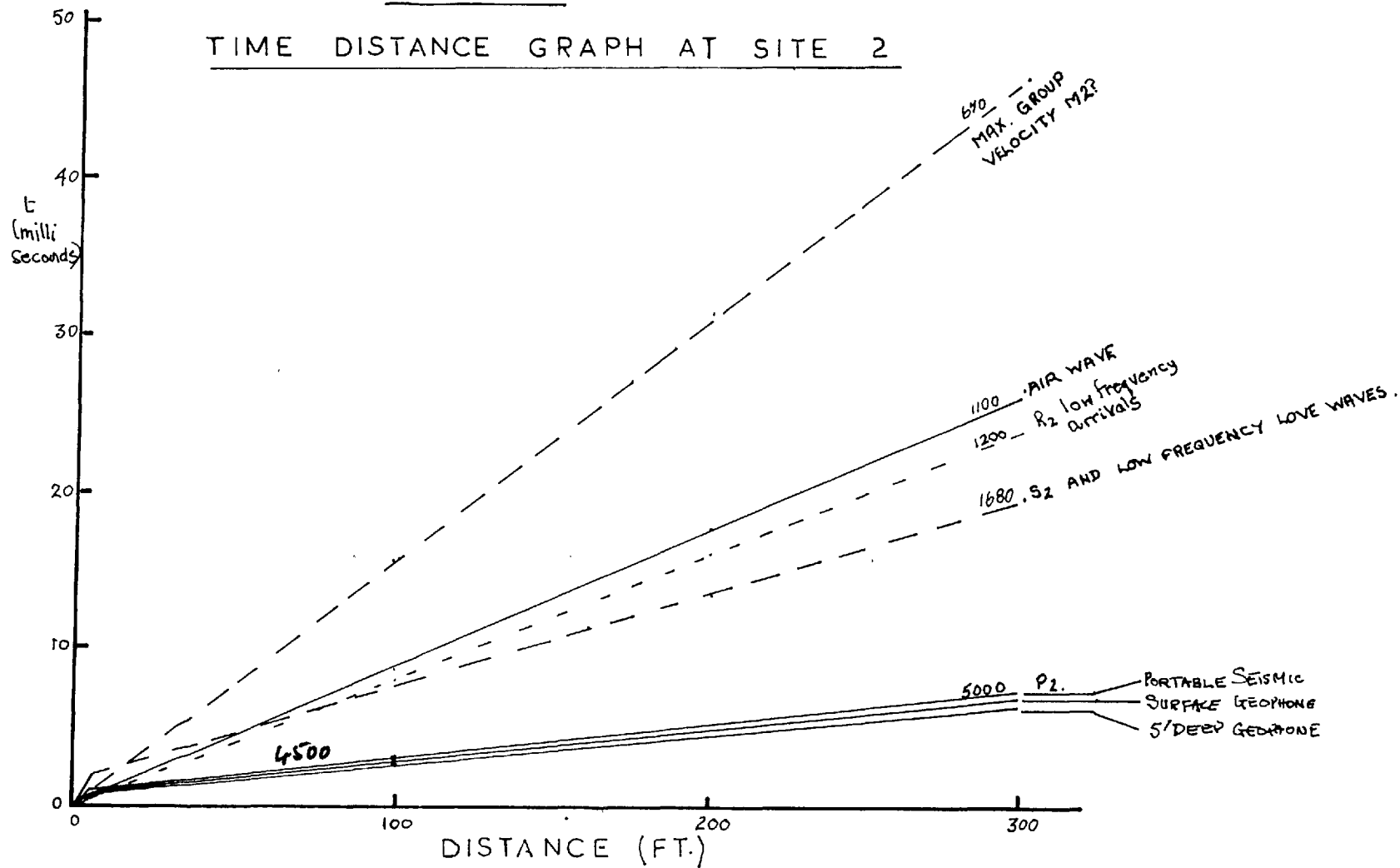


FIG. 38

TIME DISTANCE GRAPH AT SITE 2



particularly marked feature on the records was the initial arrival times of the higher modes of the Love and Rayleigh waves. These may be seen clearly in Figure 44, e.g. on record 134C at 0.197 sec. These initial arrivals probably correspond to maximum values of the group velocities of the higher modes and, for example, the value for the Rayleigh waves is in reasonable agreement for the maximum group velocity of the M_2 wave, given by Kanai (1951, p.52, Fig. 29) as $2.4 \times \beta_1$ when $\beta_2/\beta_1 = \sqrt{20}$.

The depths calculated from the refraction results for the P waves are 23' and approximately 55' at Site 1 and 3.8' and approximately 15' at Site 2. These agree with interpretations of the geology arrived at by other means apart from the 15' boundary at Site 2 between media of velocities of 4,500 ft/sec. and 5,000 ft/sec. whose existence was not confirmed.

The times of the peaks and troughs of the surface waves which were used for computing the group velocity data graphically, were determined from the normal records but are not shown here. They may be easily seen from the photographs of some of the records, Figures 48 and 49.

4.3 Methods of Analysis

4.31 General

Amplitude measurements were made on the photographic

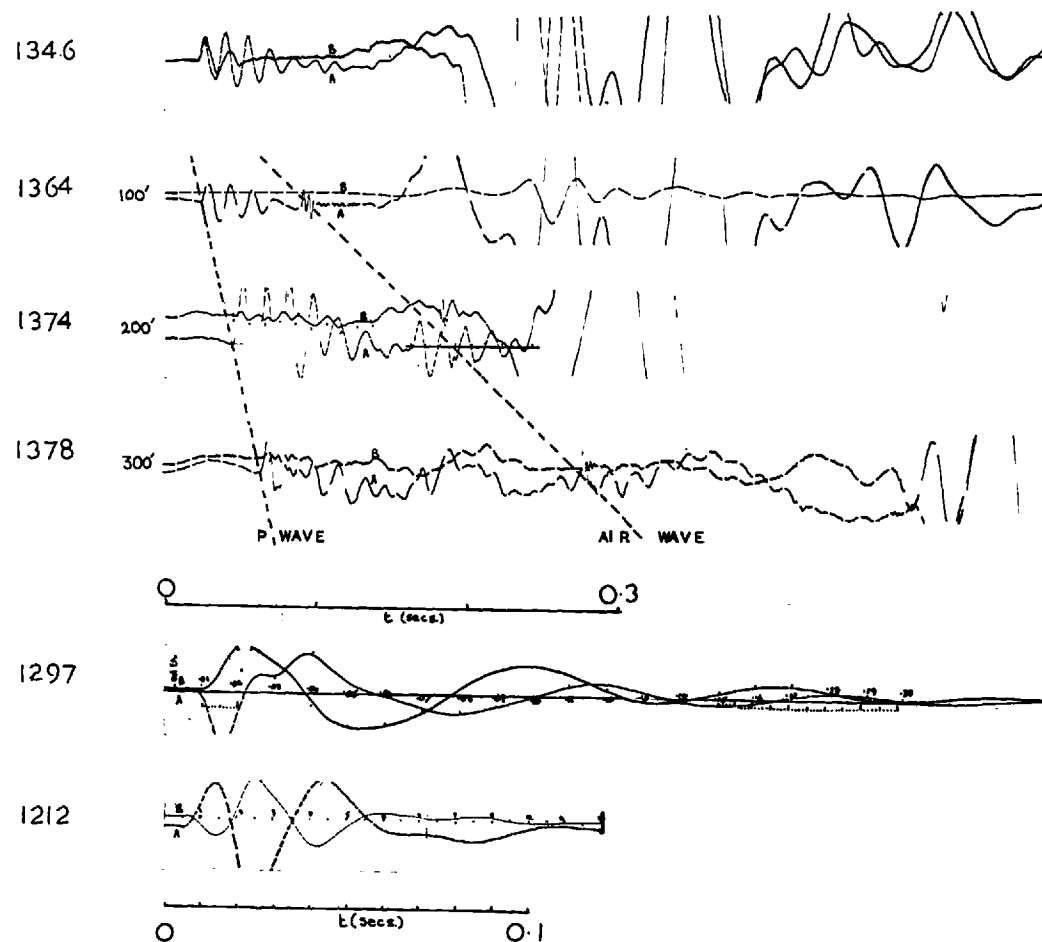


FIG. 44. RECORDS OF BODY WAVES SITE 2.

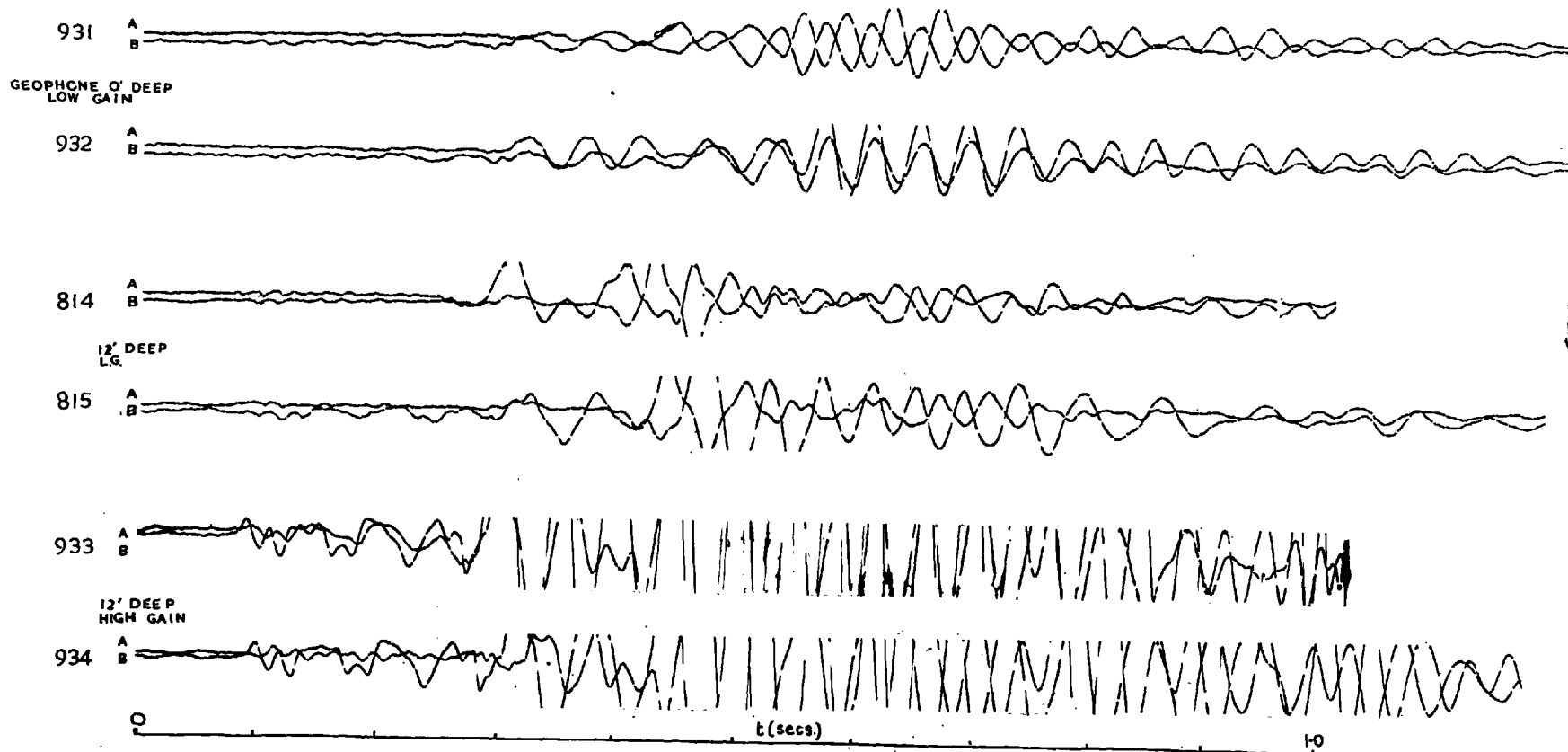


FIG. 48. EXAMPLES OF 300' RECORDS AT SITE I.

RECORDS 931,2; 814,5; 933,4.

See p 151 for details

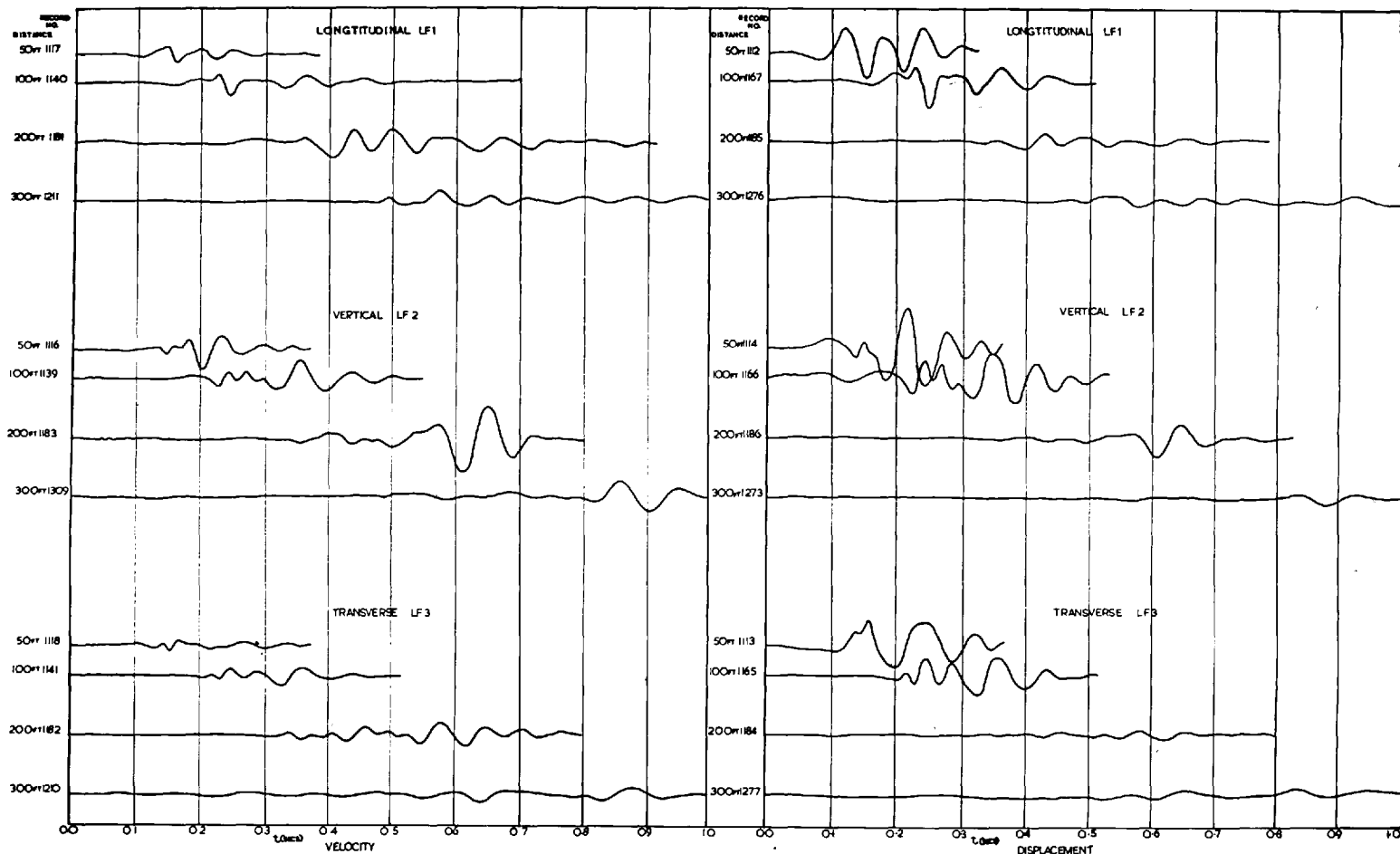


Fig 48 RECORDS OBTAINED FROM LF3C GEOPHONE (SITE 2, 5m DEEP, LOW GAIN, 50m FROM 4m OR 7m)

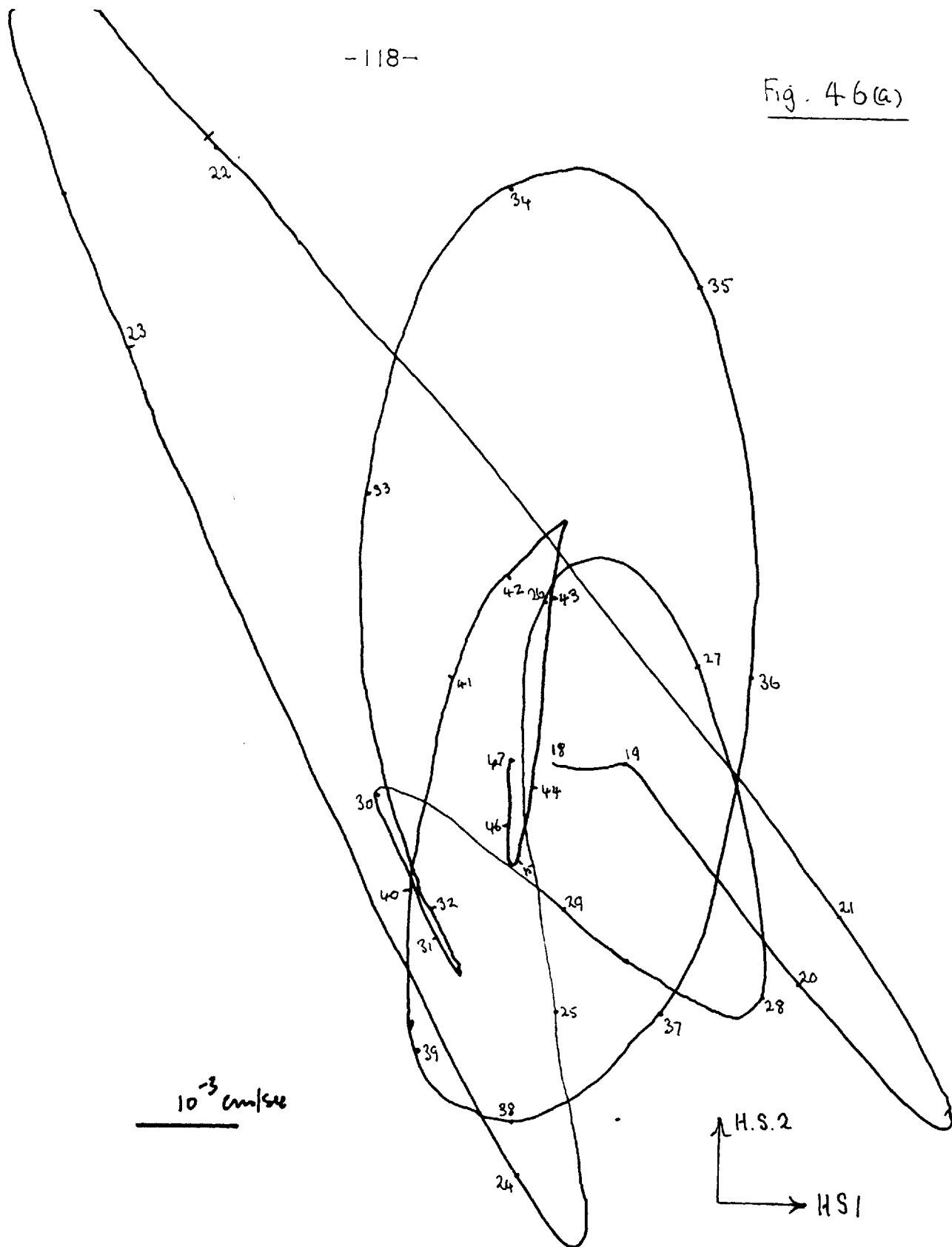
recording paper, or on enlargements of the films, to the nearest 0.01 cm. Using enlargements of a number of two component pictures of a small portion of the record it was possible to unravel details of a record of the complete motion. Figures 46 and 47 show some examples of two-component particle motion pictures obtained in this manner.

4.32 Fourier Analyses

Mr. Micholson of the Mathematics Department of Imperial College very kindly wrote a programme for performing Fourier analyses, using the London University Ferranti "Mercury" electronic digital computer.

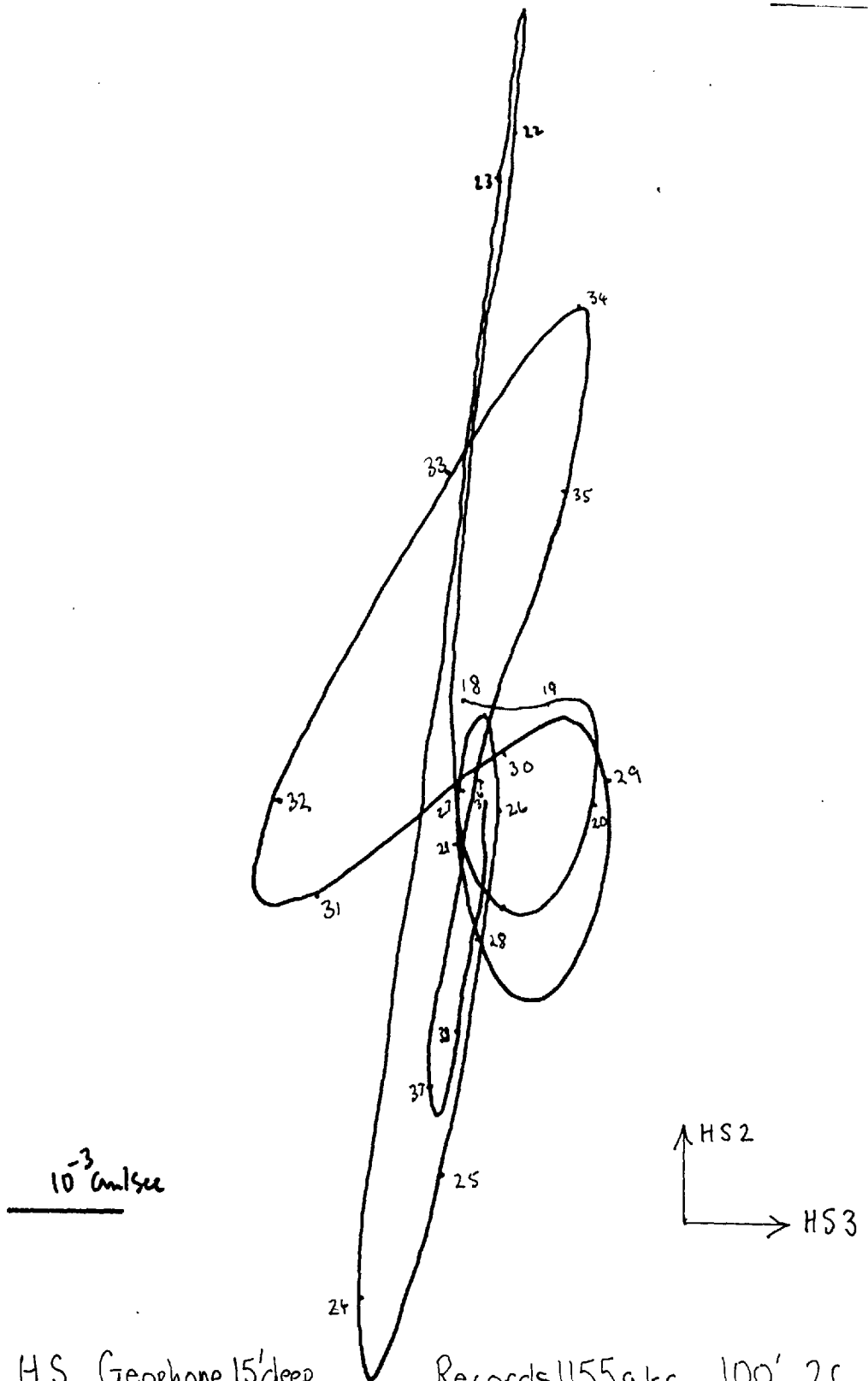
The amplitudes were read at the appropriate time intervals from the records (a process known as "digitising" the data) and then punched on to paper tape using a Creed tape punch which converts numbers into a five-hole code in a form suitable for presenting to the computer. The data tapes were taken to the London University Computer Unit, 40 Gordon Square, N.W.1., where the computer is situated, and the actual analysis performed there. A total of about four hours of computing time was used. It would be possible to reduce the computer time in future by about 30 per cent by re-writing the programme. The computer could also be programmed to do more of the analysis of the results, now

Fig. 46(a)



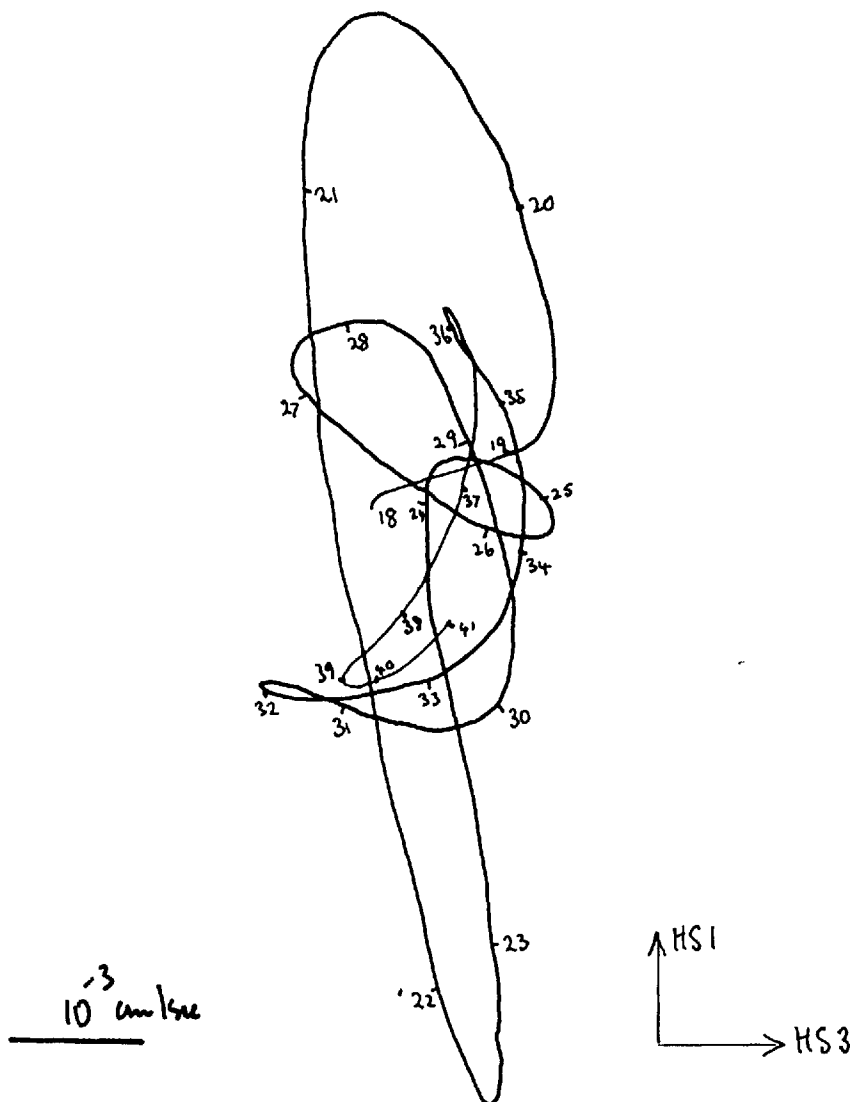
For geophone directions see p. 153
 HS Geophone 15' deep. Records 1153a, b, c. 100' 2c
 time unit - 0.01 sec.

Fig. 46(b)

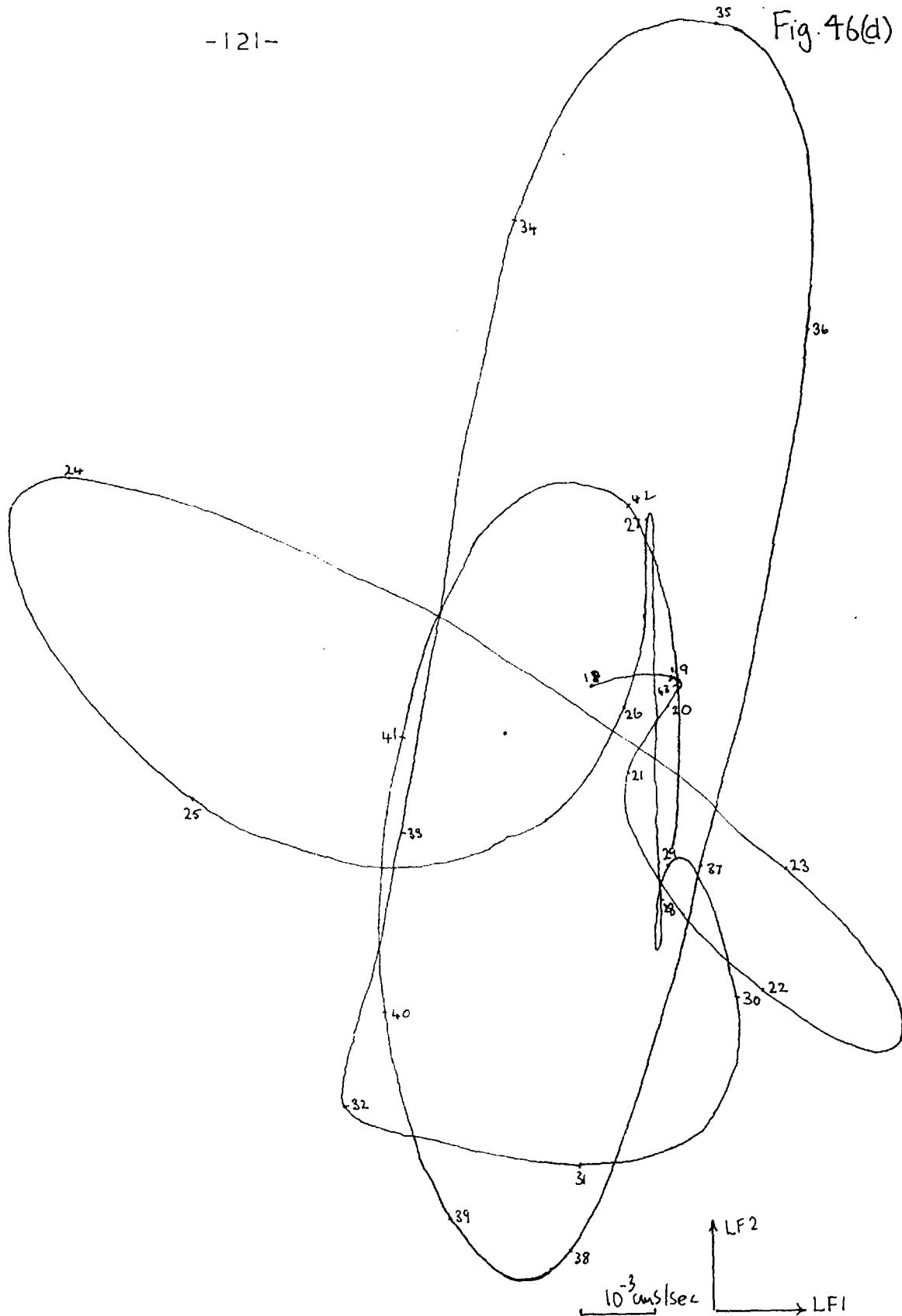


HS Geophone 15' deep.

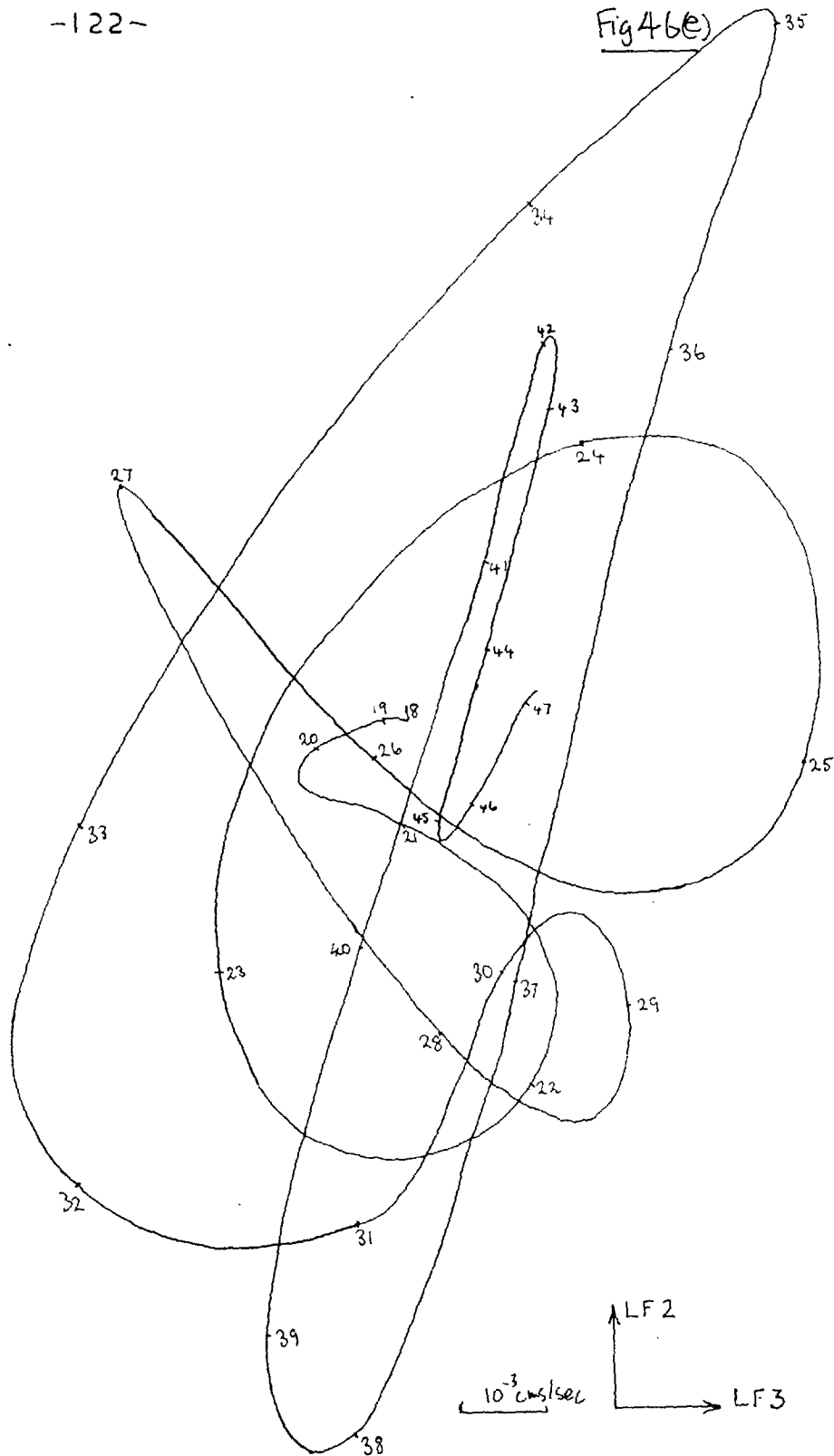
Records 1155 a, b, c. 100' 2c
time unit - 0.01 sec.



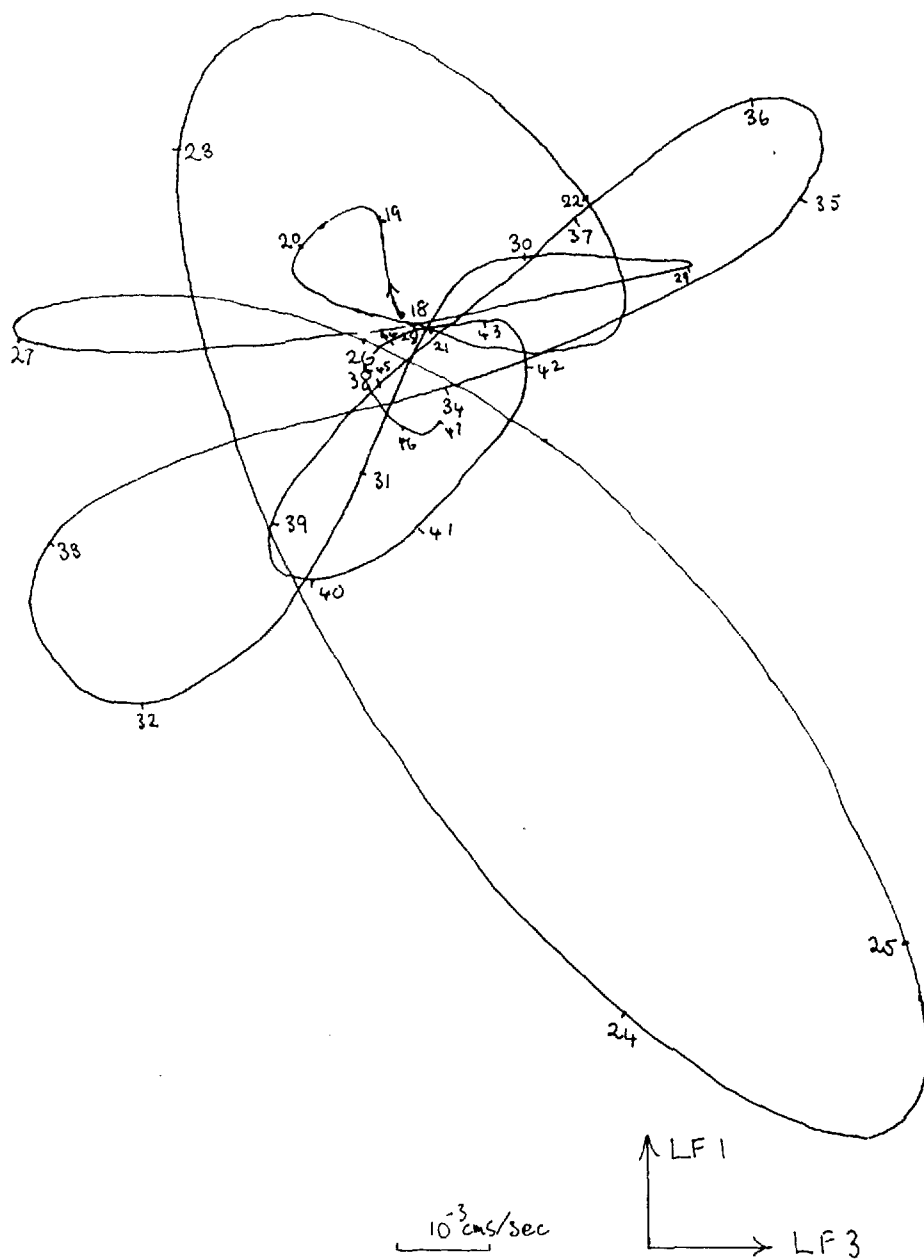
HS Geophone 15' deep. Records 1157e, f. 100' 20
time unit - 0.01 sec.



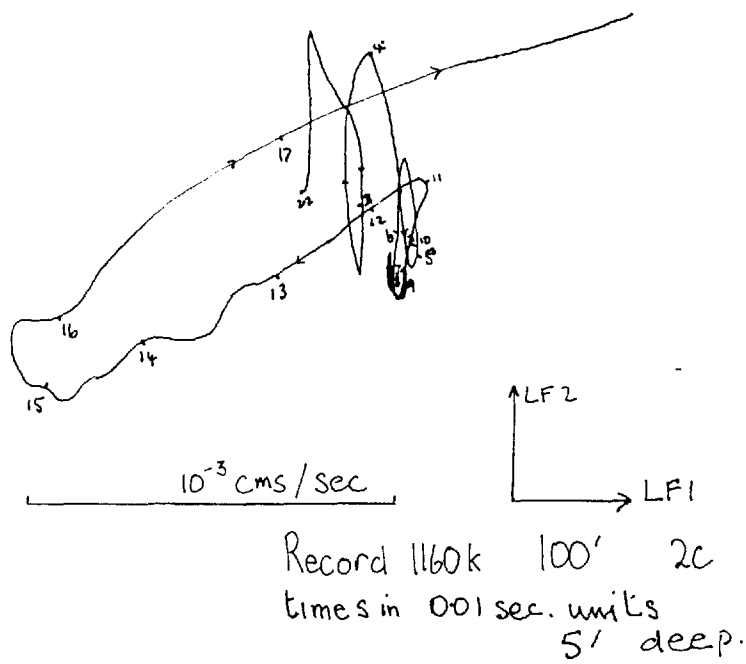
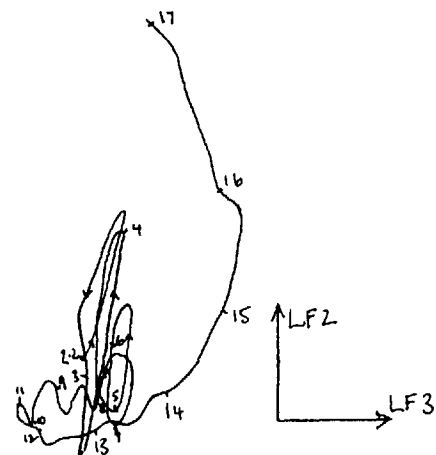
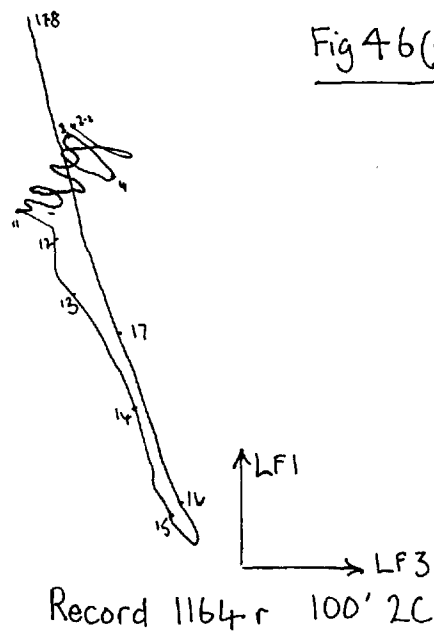
Record nos. 1159 h&j , 100', 2C.
5' Deep

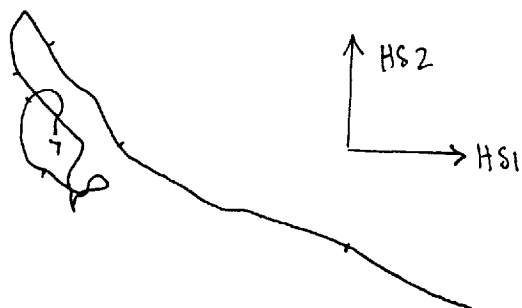


Record nos. 1162 m & n 100', 2c.
5' deep

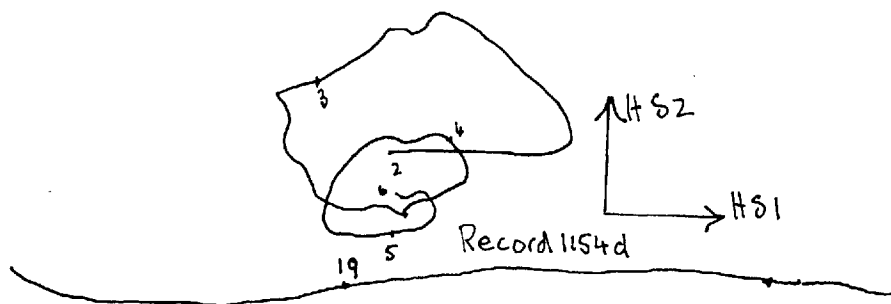


Record nos. 116302q 100' 2c
5' deep.

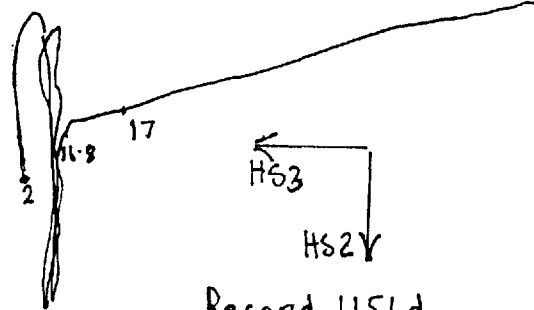




Record 1154e



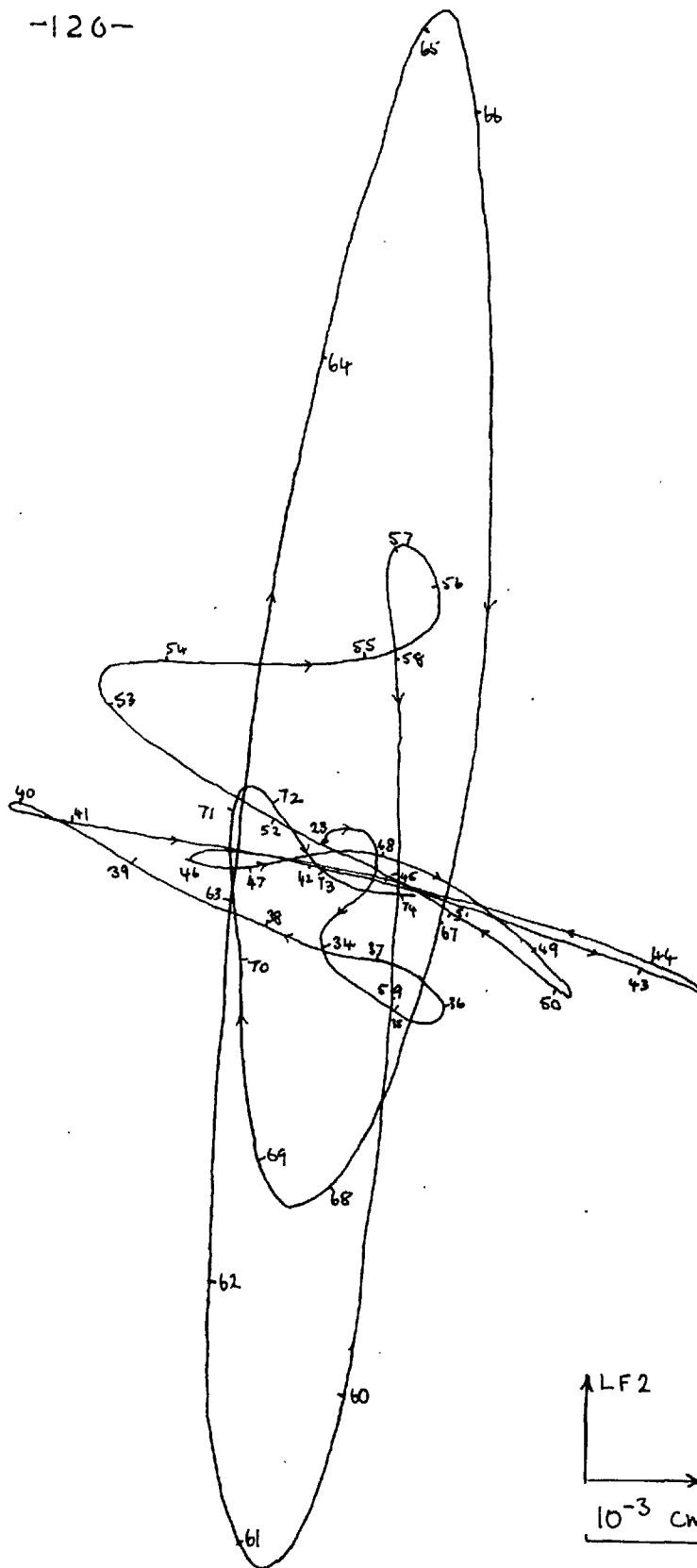
Record 1154d



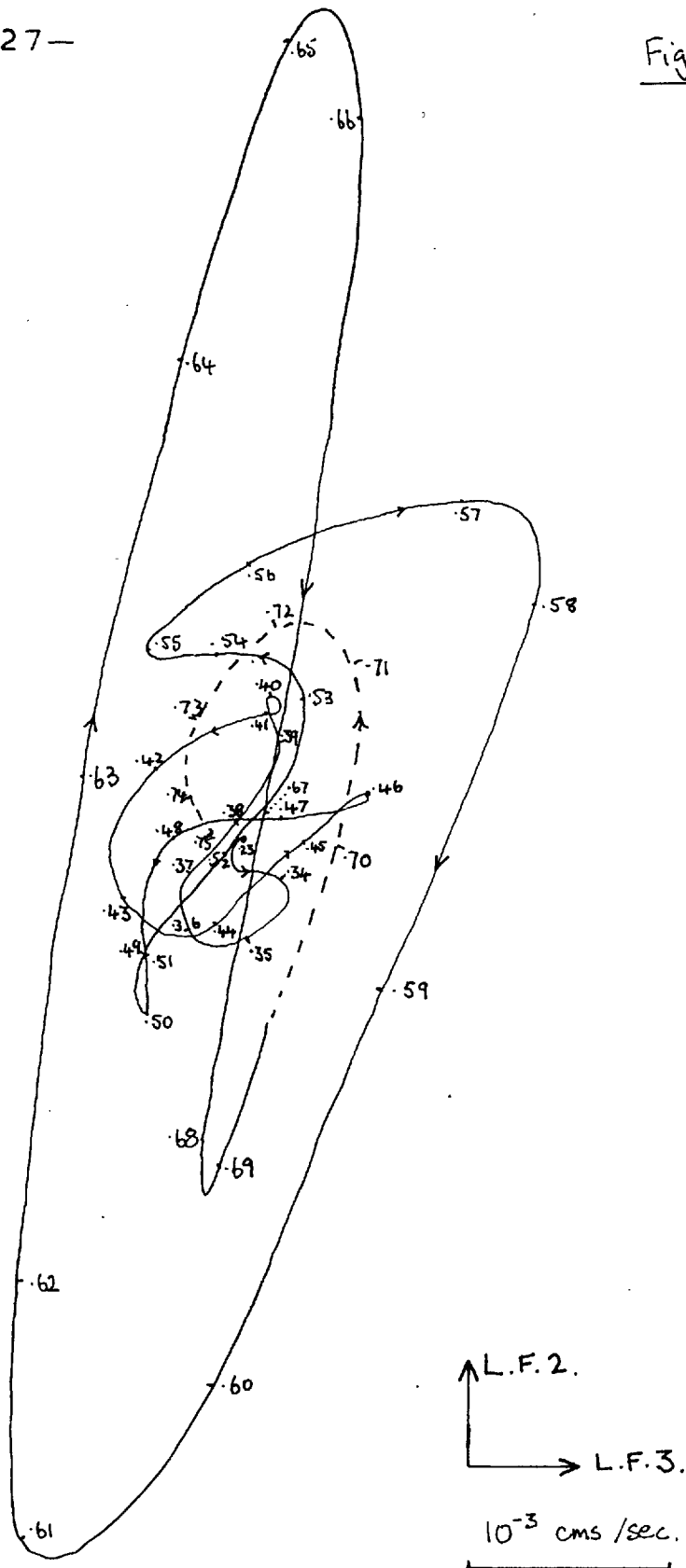
Record 1156d

100', 2-Component
15' Deep.

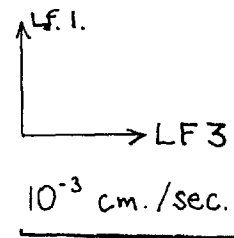
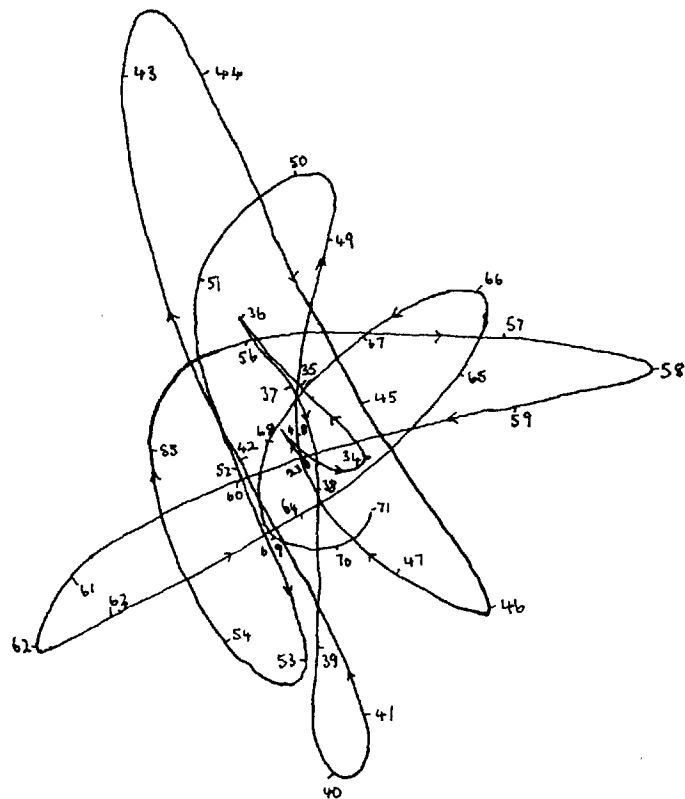
10^{-4} cm/sec



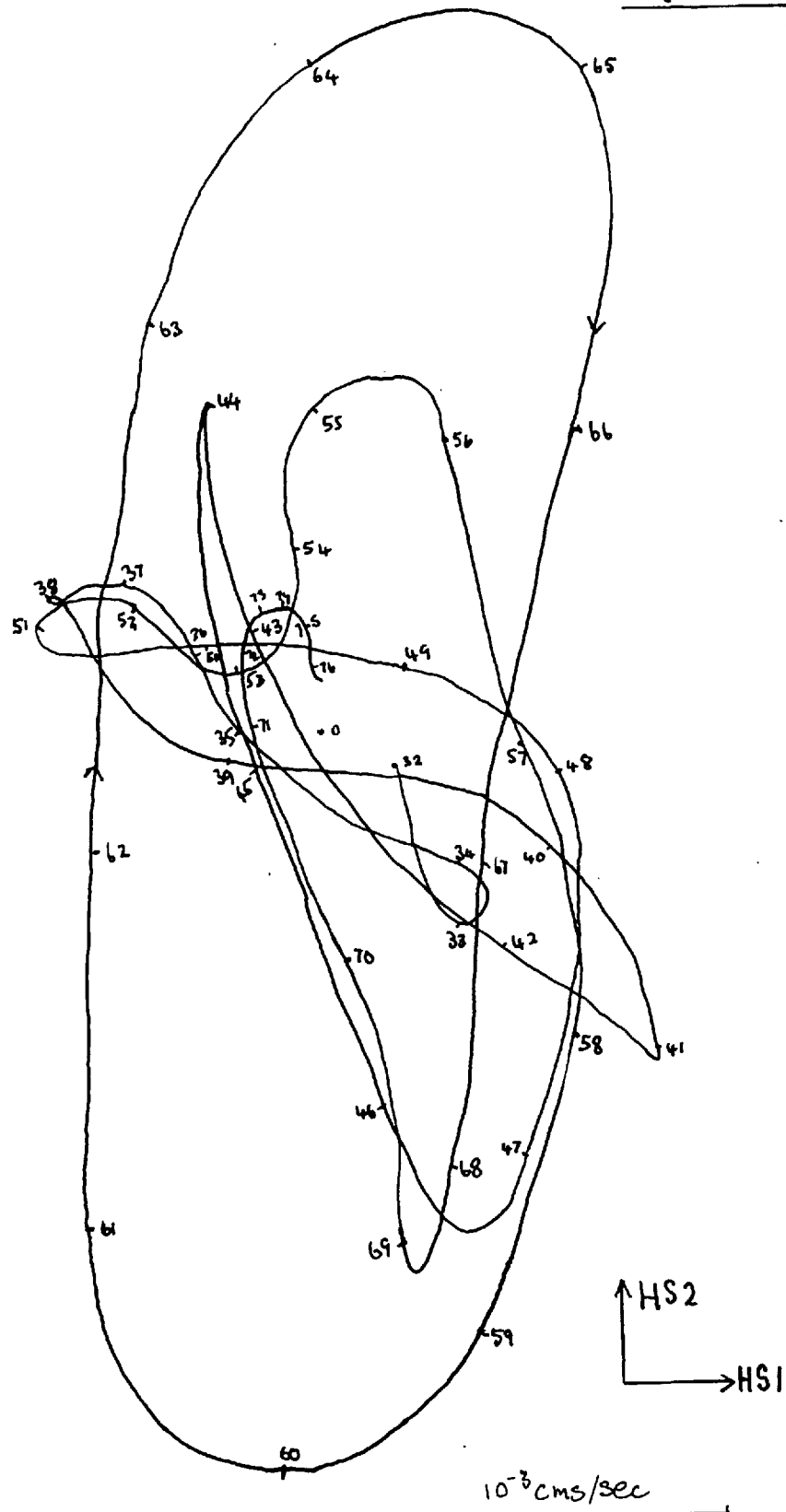
Record 1188a 200', 2-component.
from side, geophone ←
5' Deep.



Record 1189 fzd. 200' 2-component
looking towards source
5' Deep



Record 1191 h. 200', 2-component
from below, geophone $\downarrow$
5' Deep



H.S. geophone 15' deep. Record 1192 d 200' 2C
 Times in 0.01 sec. units. 15' Deep

that a procedure for interpretation has been developed, and may then be made to plot the results.

Computer output tapes, "played back" at the Mathematics department research laboratory, were converted into results' data sheets. Figure 50 is an example of the results of the Fourier analyses of the vertical component of the Rayleigh Mode at Site 2 as recorded on the LF 2 geophone at a depth of 5'. The harmonics in this example are of a fundamental of 1 c/s so that the numerical value of the harmonic is equal to the frequency. The harmonic labelled "0" refers to the value of the constant term.

A repeating function $f(t)$ may, subject to certain conditions which are fulfilled in this case, be represented by the sum of a series of sine and cosine terms of varying amplitudes, and of frequencies which are multiples of the fundamental frequency $1/T$, where T is the time of repetition of the function. Hence $f(t)$ may be written as

$$f(t) = \frac{1}{2} a_0 + \sum_{r=1}^{\infty} a_r \cos \frac{2\pi r t}{T} + \sum_{r=1}^{\infty} b_r \sin \frac{2\pi r t}{T}$$

= $F(t)$, the Fourier series of $f(t)$.

The values of the amplitude coefficients a_r and b_r may be found by multiplying $f(t)$ by $\cos \frac{2\pi r}{T} \cdot t$ or $\sin \frac{2\pi r}{T} \cdot t$, for any particular harmonic r , and integrating from $-T/2$ to $+T/2$.

FIG 50 - HARMONIC ANALYSES

OF VERTICAL COMPONENT OF RAYLEIGH WAVE, RECORDED BY LF2 (5' DEEP.)

		50'		100'		200'		300'	
FREQUENCY		AMPLITUDE	PHASE	AMPLITUDE	PHASE	AMPLITUDE	PHASE	AMPLITUDE	PHASE
0		0.50	180	0.83	180	2.13	0	1.17	0
1		1.04	-113	1.42	-55	3.48	-144	2.22	-29
2		1.31	-46	0.81	60	1.59	54	1.82	-64
3		1.56	36	0.45	107	1.56	166	1.10	-123
4		1.96	103	1.09	-178	3.60	-6	1.37	122
5		2.26	-178	1.97	-69	5.80	-168	3.37	60
6		2.32	-102	3.09	46	9.19	35	5.96	17
7		2.28	-35	4.45	166	14.08	-114	8.66	-24
8		2.58	24	5.96	-72	19.59	105	11.06	-63
9		3.46	88	7.60	52	24.47	-33	12.79	-102
10		4.63	161	9.41	178	27.74	-168	13.56	-141
11		5.71	-123	11.27	-53	28.99	59	13.28	180
12		6.57	-47	12.77	79	28.32	-74	12.00	140
13		7.37	28	13.37	-147	26.18	155	9.96	100
14		8.36	102	12.70	-11	23.10	25	7.49	59
15		9.54	179	10.72	125	19.55	-103	4.99	16
16		10.57	-101	7.85	-101	15.83	129	2.83	-34
17		10.98	-19	4.89	24	12.10	2	1.40	-101
18		10.49	65	3.13	127	8.49	-125	1.07	169
19		9.11	146	3.29	-132	5.13	106	1.19	104
20		7.20	-136	3.75	-9	2.27	-25	1.06	59
21		5.49	-67	3.58	126	0.18	170	0.71	21
22		4.73	-6	2.87	-97	1.12	-107	0.28	-3
23		4.84	59	1.96	40	1.64	110	0.17	78
24		4.96	132	1.18	174	1.71	-35	0.39	70
25		4.60	-150	0.73	-59	1.62	171	0.48	41
26		3.77	-74	0.60	67	1.48	21	0.46	10
27		2.81	-2	0.63	-156	1.19	-125	0.35	-24
28		2.12	59	0.68	-16	0.77	96	0.23	-69
29		1.90	116	0.68	119	0.32	-24	0.17	-145
30		1.89	179	0.61	-114	0.32	-75	0.26	146
31		1.83	-113	0.54	1	0.64	162	0.38	102
32		1.69	-44	0.55	108	0.85	22	0.45	65
33		1.56	23	0.65	-139	0.85	-121	0.43	31
34		1.49	91	0.73	-18	0.63	92	0.32	-0
35		1.40	161	0.73	105	0.31	-85	0.16	-18
36		1.29	-128	0.65	-133	0.45	57	0.11	43
37		1.17	-58	0.54	-19	0.78	-100	0.26	40
38		1.09	13	0.50	88	0.92	112	0.35	10
39		1.02	86	0.52	-160	0.86	-38	0.36	-24
40		0.92	162	0.51	-40	0.73	162	0.27	-59
41		0.78	-122	0.43	80	0.67	-2	0.14	-92
42		0.62	-47	0.33	-167	0.64	-160	0.02	-15
43		0.50	23	0.27	-62	0.54	44	0.13	-3
44		0.44	94	0.25	44	0.41	-114	0.19	-39
45		0.40	173	0.22	150	0.33	86	0.19	-78
46		0.33	-98	0.22	-110	0.25	-56	0.15	-120
47		0.24	2	0.27	-3	0.19	-160	0.09	-168
48		0.16	118	0.32	121	0.28	108	0.05	136
49		0.14	-115	0.32	-107	0.42	-18	0.04	94

It may be shown that

$$a_r = \frac{2}{T} \int_0^T f(t) \cdot \cos \frac{2\pi r}{T} t \cdot dt.$$

and

$$b_r = \frac{2}{T} \int_0^T f(t) \cdot \sin \frac{2\pi r}{T} t \cdot dt.$$

Alternatively, the amplitude and phase of each component may be evaluated using

$$A_r = \sqrt{a_r^2 + b_r^2}$$

$$\psi_r = \tan^{-1} \left(\frac{b_r}{a_r} \right)$$

In the programme which was used, the sums of the products of the sine and cosine terms with the values of $f(t)$ at a number of discrete values of t were evaluated. This strictly speaking is a harmonic analysis rather than a Fourier analysis, but the first approximates to the second when a sufficiently large number of points on the curve is taken. The extra time to ~~actually~~ evaluate the integrals would not be justified.

A number of pulses whose spectra were known were analysed to measure the accuracy of the analysis. The pulses chosen were (a) a square wave using 100 points, (b) two of the pulses given by McDonal et. al. (1958), and (c) a pulse which had previously been analysed using a desk calculating machine. Case (b) was chosen because it was of interest to find details of the amount of

dispersion of the body wave pulses given by McDonal. Good agreement was found in cases (b) and (c). More quantitatively, for case (a), it was found for example that the 31st harmonic of the fundamental frequency of the square wave differed from its theoretical value by an amount equal to 0.57 per cent of the amplitude of the maximum harmonic component (the fundamental in this case). The phase differed from the theoretical value by an amount which agreed exactly with the value which was worked out for this particular degree of approximation.

There are a number of factors to be considered when deciding upon the optimum number of points n and the time interval, t . These are firstly that the time taken by the computer is roughly proportional to n^2 when $n > 50$, and secondly that the construction of the computer limits the possible value of n so that it is necessary to have $n < 478$. Thirdly, since at least 4 points are needed to define one cycle sufficiently accurately, the highest frequency which may be investigated is approximately $\frac{1}{4 \cdot \Delta t}$. Finally, the lowest harmonic $\frac{1}{n \cdot \Delta t}$ is determined by both the requirements of the amplitude and phase spectra.

The amplitude of the fundamental frequency must be below that of the lowest frequency of interest in the amplitude spectrum of the pulse. For the phase spectrum, the harmonics should be chosen sufficiently close together

to define the phase of each harmonic uniquely. If the phase between harmonics differs by more than about 180 degrees, it is easily possible to make an error in the integral number of cycles for that particular phase angle, as the computer results can only give the fraction of a cycle.

It was necessary to analyse again some of the spectra used for the surface wave dispersion studies because of the possibility of misinterpretation of the phase. The closely spaced points shown in Figure 51 are the results of the second analysis, and the more widely spaced points were retained once the direction of the phase/frequency line had been uniquely determined, because the first analyses were more accurate for the higher frequencies.

A total of 87 Fourier (or harmonic) analyses were performed, 31 of body wave pulses, 55 of surface waves, both of complete records and parts of records for particular phases, and 1 of a square wave.

4.4 Noise Levels

Measurements were made from time to time of the seismic noise of the ground, and some approximate figures obtained for its variation with time of day and with depth of geophone.

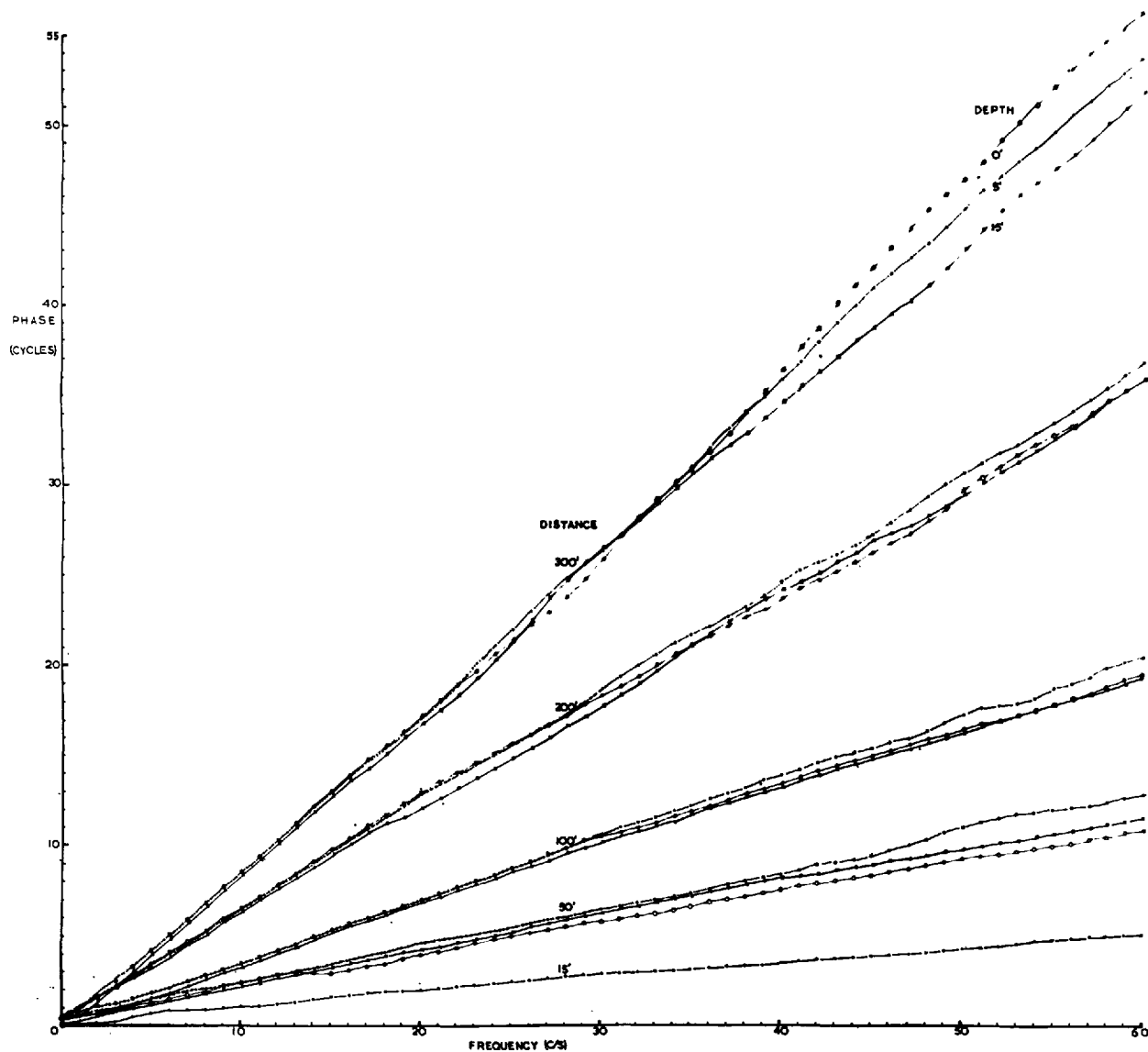


FIG.51 *see p.156*

PHASE SPECTRA

FREQUENCY / PHASE

ANALYSIS NOS 05-0104 & 6972

DEPTH
 HSE17 —
 LF2(5) —
 LF2(5) —

The approximate R.M.S.noise levels at the two sites were

maximum	=	3×10^{-5}	cm/sec.
average	=	3×10^{-4}	cm/sec.
minimum	=	3×10^{-5}	cm/sec.

The average noise was lower by a factor of approximately 2 at a depth of 15'. The minimum noise levels were found at night between 1 a.m. and 6 a.m. and appeared to be mainly in the frequency range of 5 c/s to 10 c/s. The maximum noise levels were caused by aeroplanes which gave approximately 3 times the maximum noise consistently found from other sources, such as cars on nearby roads. Aeroplane noise was approximately 3 times greater on the vertical component than on the horizontal components, and had a dominant frequency of about 100 c/s with beats, probably caused by the aeroplanes having more than one engine.

The question of noise is also important for questions of determining the detectibility of nuclear explosions, and its variation with position and depth would be worth studying closely by prospecting seismologists to allow a reduction in charge size.

4.5 True Ground Motion

As described in Section 3.32, the unsaturated sand, overlain by a 5' bed of sand and gravel under the topsoil at Site 1, was utilised for a series of experiments to study the conditions needed to record the true ground motion. A number of geophones were employed and the effects of instrumental filtering, geophone packing and geophone frequency response were investigated. Figure 40 illustrates the experimental layout.

Weights were dropped on to the surface and the ground motion recorded by geophones placed at the end of a tunnel. By this means the geophones and their packing could be changed after each experiment without disturbing the ground through which the waves had to pass.

The results of a number of experiments recording the motion produced by a 50 lbs. weight dropped from about 2' on to the ground are shown in Figure 41.

4.51 Instrumental Filtering Effects

Records (a) to (d) were taken with two similar standard G.A.A. vertical geophones (frequency response 30-500 c/s.) They were placed near one another and well compacted with sand. The output from one of them was passed through various filters while the other output was

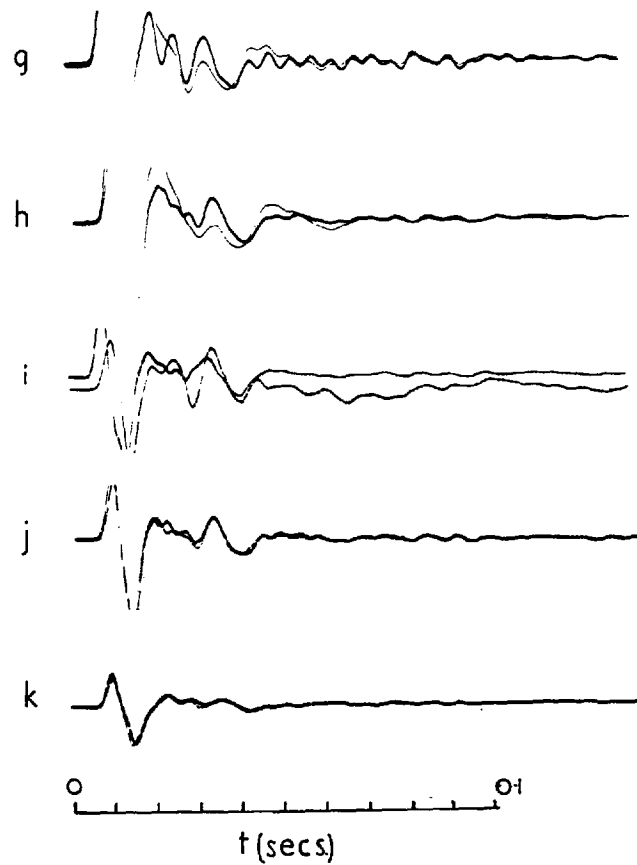
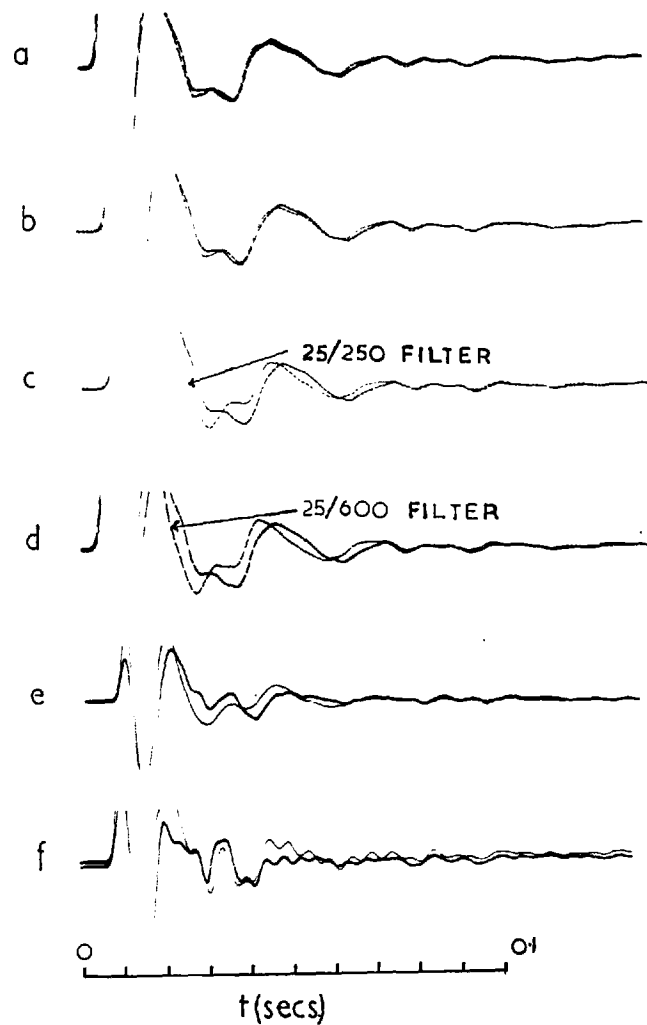


FIG. 41. SITE 1 RECORDS TO DETERMINE
THE TRUE GROUND MOTION.

RECORD NOS. BETWEEN 179 & 241

recorded on a wide filter setting (0 c/s to 250 Kc/s).

The filter settings chosen for the first geophone were as follows:-

Record (a) 0-250 Kc/s. This shows the similarity of pulses.

" (b) 0.16-600 c/s. Showing that the normal preamplifier setting does not affect the pulse shape.

" (c) 25-250 c/s. This shows the marked change in pulse shape.

" (d) 25-600 c/s. The similarity to (c) suggests that the 250 c/s high frequency filter used in (c) had little effect on the pulse shape.

Successive high and low frequency filtering at 10 c/s intervals was applied and the effects noted. It was found that 170 c/s was the lowest high frequency filter to affect the pulse noticeably. Figure 41 (e) shows the record using a 50 c/s to 170 c/s filter (3 d.b. limits given unless otherwise stated) on both channels. The darker line in the figure in this case is the record of a vertical Hall-Sears geophone, while the lighter line of greater amplitude corresponds to one of the original GAA geophones. The difference between the HS record and that shown in (j), which is ^a similar record unfiltered, should be noted.

4.52 The Effect of Packing

Records (f) to (h) illustrate the effect of packing. The HS and GAA geophones were placed, uncompacted, on the sand at the end of the tunnel and records taken with a 0.16 c/s to 600 c/s broad filter. (f) is the record with the geophones uncompacted, (g) with slight packing, and (h) with tighter packing. It may be seen that the light packing produces more resonance than no packing.

4.53 Geophone Frequency Response

Records (a) to (h) show that two geophones with different frequency responses and the same filter setting, or similar geophones with different filter settings, give very different records of the ground motion.

Figure 41 (i) - (k) shows records taken using the HS vertical and LF2 geophones with wide instrumental filters. These two geophones have frequency responses which, although different, are theoretically flat over the necessary range (see Figures 3 and 4). Record (i) was taken with the geophones not packed and (j) when they were both well packed. The resulting similarity is a good indication that this is the true ground motion. (k) is a record of a pulse of greater energy taken on a lower gain setting.

During the experiment, Mason's two-component geophone and a number of Willmore geophones were used when investigating the effects of packing. It was not found possible to obtain similar results from any pair of them.

The effects of filling in the tunnel were studied. It was found that the record did not change noticeably in character after the tunnel had been filled in up to about 3 feet from the end at which the geophone was placed (see shading on Figure 40).

The geophone was also put at the bottom of the hole shown in Figure 40 at a depth of 12' and the hole was filled in 2' at a time. Records were taken with the source on the surface at a distance of 200', at each different hole depth. Theoretically wavelengths less than about 3 times the hole diameter should be affected and this was found to be approximately true.

Records 818-839 (not illustrated) indicated that the hole had some slight effect on the records, which was most noticeable with the higher frequency surface waves.

4.6 Source Pulses

The different pulses produced by a number of different sources were studied. In Figure 44, the records nos. 1297A and 1212A are respectively of a 50 lbs. weight

falling from 4 feet on to an 800 lbs. base plate, and of a detonator exploded 6 inches below the surface of the ground, both being recorded using the LF2 geophone 5' below the surface vertically underneath the source.

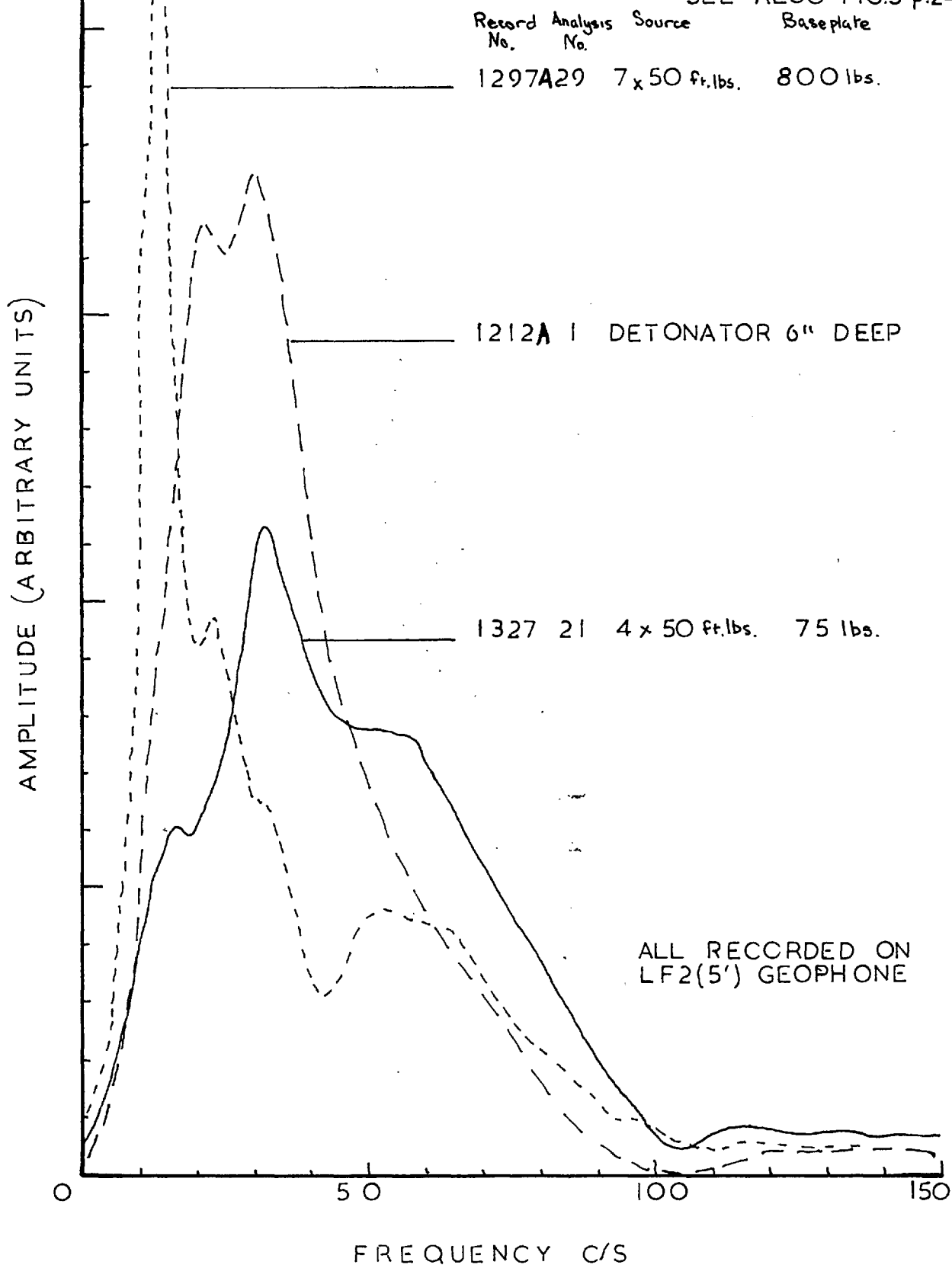
The spectra of these pulses are shown in Figure 42 together with the spectra of a pulse recorded similarly using a 75 lbs. base plate (record 1327).

The difference between the spectra of the pulses using two different base plates is very pronounced and shows a marked difference in pulse shapes. The greater high frequency components of the pulse produced with the 75 lbs. base plate results in its greater efficiency in producing body waves which were recorded at the traverse points. This is because the transmission properties of the ground acted as a band pass filter for frequencies around 55 c/s.

The explosive pulse had an unexpectedly low dominant frequency (30 c/s.), with a large transverse motion (record 1212B). Record 1297B is a displacement measurement of the pulse of which record 1297A gives the velocity. By careful examination of records 1212 and 1297, it may be seen that the first 4 milliseconds of the motion in both cases is of small amplitude (1.5 and 5 per cent of the maximum amplitudes respectively), but in a direction

FIG 42 AMPLITUDE SPECTRA OF PULSES RECORDED
5' VERTICALLY BELOW THE SOURCE. SITE 2

SEE ALSO FIG.3 p.24



towards the source. The reason for this phenomenon, which was noticed on a number of records at both sites, is not clear.

4.7 Motion near the Source

4.71 Amplitude Studies

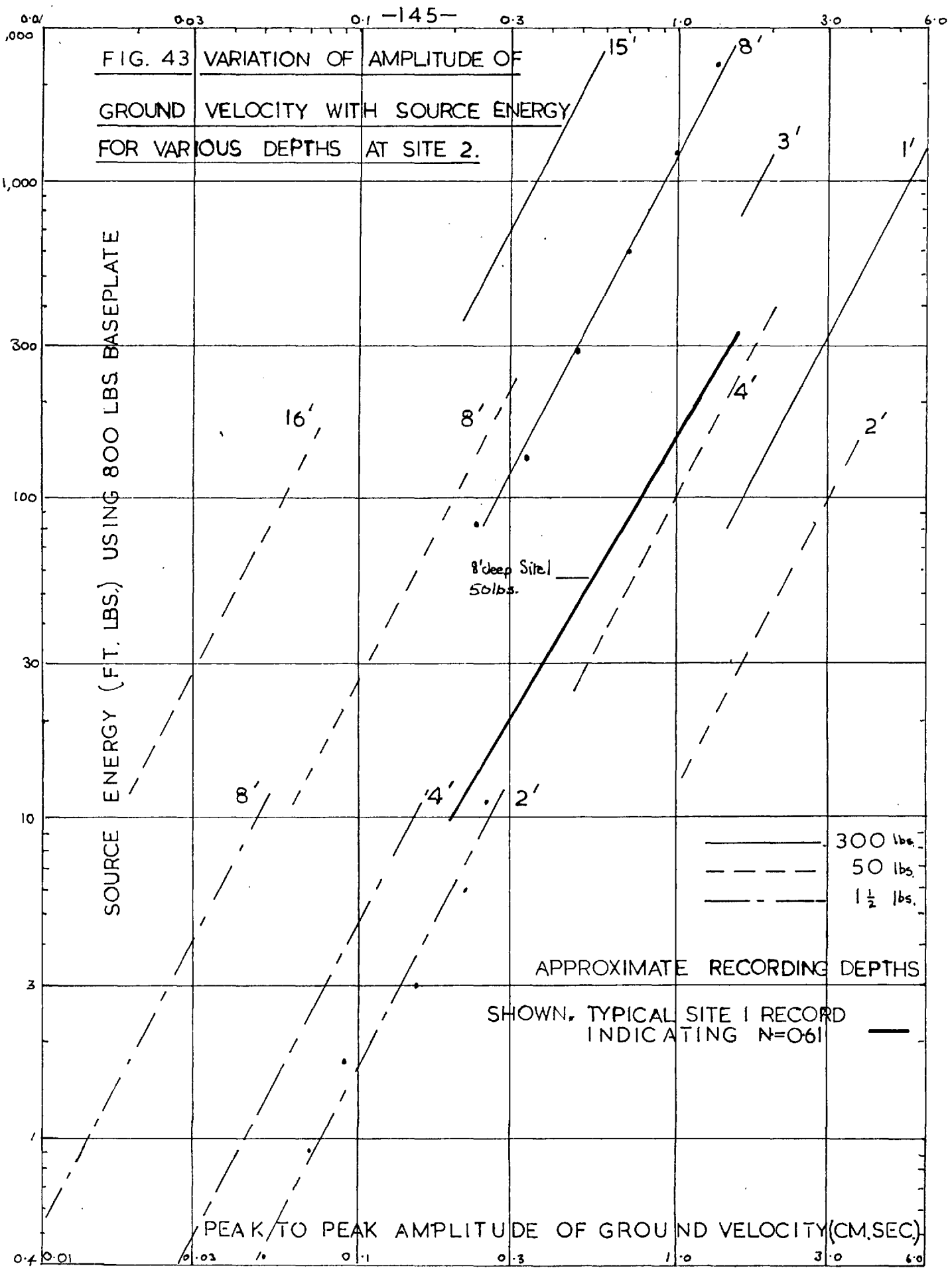
Amplitude studies of the motion near the source were made at both sites by recording pulses due to different sources at the surface, using geophones at various depths in a bore-hole down to a maximum depth of 16 feet.

It may be assumed that the amplitude of particle velocity, A , is connected with the energy of the source E by a law of the form

$$A = K E^N$$

The theoretical value of N at large distances from the source is 0.5 and it was of interest to find how close to the source this applied. The law of variation of amplitude with depth was also considered.

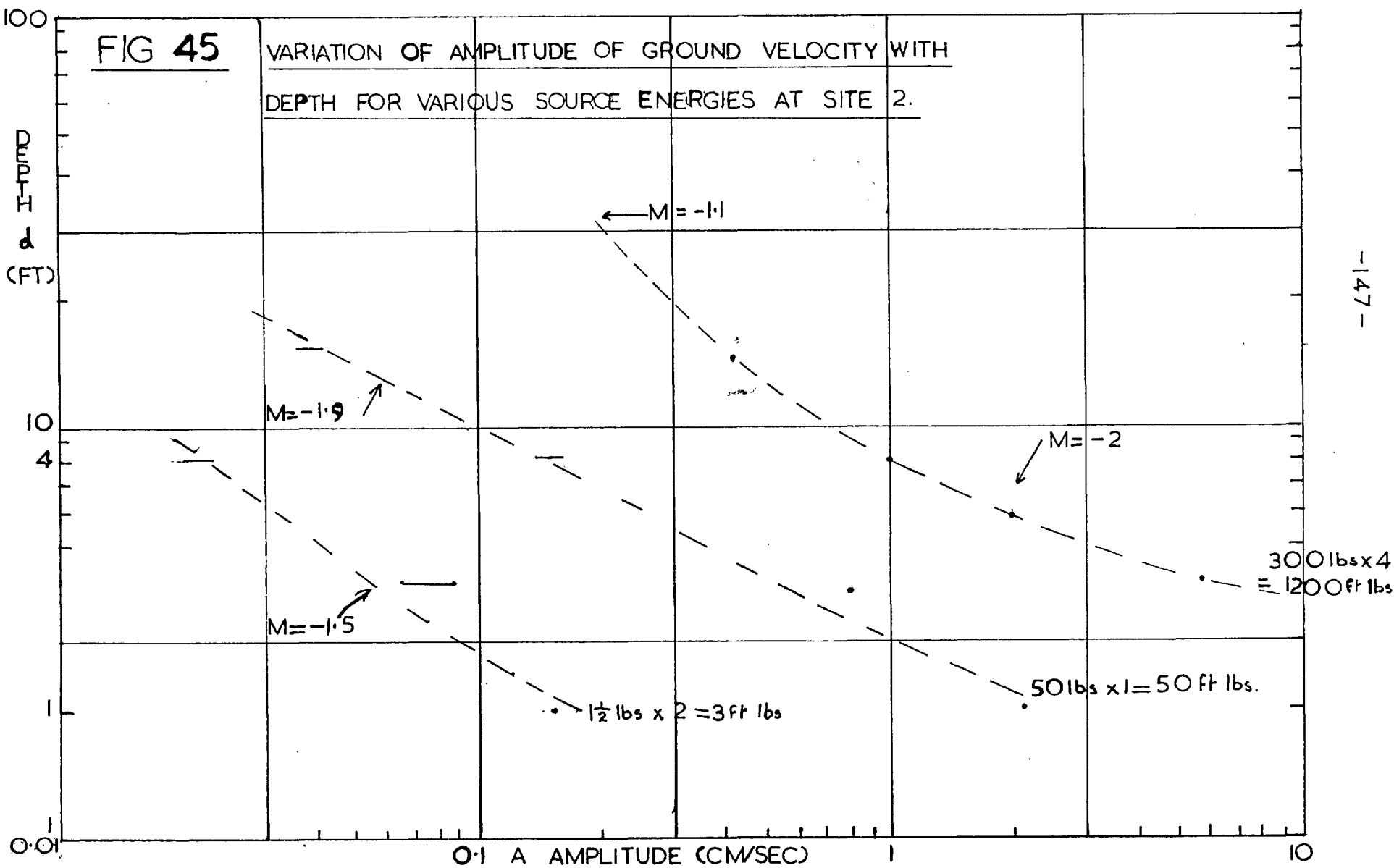
The curves of variation of amplitude with energy for various geophone depths at Site 2 are given in Figure 43, a similar series of curves was obtained at Site 1 but the values of the constants N and K were different. A typical curve from Site 1 for the 50 lbs. weight is shown in Figure 43.



The Site 2 curves were obtained using the 800 lbs. plate and the Site 1 curves using the 75 lbs. plate. An experiment was carried out to obtain a conversion factor for changing from one plate to another to be used in energy calculations. The factor was found to be 2 ± 0.5 ; This was the ratio of energies using the 800 lbs. and 75 lbs. baseplates to give the same ground amplitude.

The values of N determined from Figure 43 are all approximately equal to 0.5 ($\pm .03$), whereas at Site 1 the values for N for the ~~three~~ different weights were $0.6 \pm .01$ for the 50 lbs. weight and $0.25 \pm .03$ for the 300 lbs. weight. There is little flattening of the curves at higher ground amplitudes due to non-linearities at the range of working of the present experiments.

It was also found that the larger weights were more effective because of their greater weight to baseplate weight ratio. The relative effectiveness of the 300, 50 and $1\frac{1}{2}$ lbs. weights were in the ratio 1.6:1.3:1. The calculations needed to give this ratio are theoretically complex, but for the 300 lbs. to 50 lbs. weights ratio, the simple factor of $\frac{800 + 300}{800 + 50} = 1.3$ is in fairly good agreement with that found. Figure 45 shows three graphs of the variation of ~~the~~ ^{of ground velocity} ~~ground~~ amplitude with depth for source energies of 3, 50 and 1,200 ft. lbs.



The amplitude variation with depth, d , is assumed to obey a law of the form

$$A = Cd^M$$

For the 1,200 ft. lbs. energy, the curves are seen to tend towards the theoretical value of $M = -1$ as d increases, but for small d and for the other curves the amplitude is seen to die away more rapidly with depth.

4.72 Energy Relationships

The numerical values obtained from the amplitude studies described above are only very approximate, because of the variations in the materials of the ground near the surface, bad geophone coupling with the bore-hole walls and numerous other variables. However, this seemed the best means of obtaining a knowledge of the energy relationships between the various theoretical source energies and the energy which is actually converted into seismic energy in the ground, a problem which is always difficult.

Considering the case of 50 lbs. weight dropped from 1' on to the 800 lbs. base plate, the peak to peak amplitude of ground velocity at a depth of 2' is found to be 2 cm/sec., and at 15' to be 0.05 cm/sec.

The energy of the source, 50 ft. lbs. using the 800 lbs. baseplate is equivalent to 25 ft. lbs. when a 75 lbs. baseplate is used. As it has been found that the use of the 75 lbs. baseplate produces about the same amplitude as when a weight is dropped directly on to the ground, this is taken as the source energy.

Source energy $\approx$ 25 ft. lbs. $\approx$ 34 joules.

The computation of the total seismic energy from records of the ground velocity variation with depth is difficult. The approximate approach used was to consider the source to radiate uniformly in all directions and to consider the energy to be spread over a hemisphere of radius equal to the pulse length (30' at this depth). The value of velocity was assumed to be constant over this volume and equal to the RMS value at 15'. This assumption no doubt leads to considerable error. A more elegant method would involve integration over a number of hemispherical shells. ^{sec left} Using a ground density of 2 gms/cc the approximate seismic energy is found to be 50 joules. In spite of the approximations made, this indicates that a large part of the source energy is converted into seismic energy from a dropping weight source.

A comparison was made of the efficiencies of dropping weights and explosive sources.

Considering measurements made at a depth of 5' below the surface with the sources vertically above, from record 1212, Figure 44, the amplitude of vertical ground velocity is found to be 1.2 cm/sec. This corresponds to a 50 lbs. weight dropped from about 2'6" on to the large plate.

The relative effectiveness were also compared by measurements from surface wave records. The maximum amplitudes of the P and R waves are given below for records taken at 300' and 50' with the LF2 Geophone 5' deep, both for detonators exploded 6" below the surface and also for dropping weights giving roughly the same amplitudes.

Distance	Source	Record Number	Max. Amp. of P wave, cm/sec. (p.p.)	Max. Amp. of R wave, cm/sec. (p.p.)	$\frac{R \text{ amp.}}{P \text{ amp.}}$
300'	Detonator	1231	2.0×10^{-4}	8.0×10^{-4}	4
300'	7x50 ft.lbs.	1206 1209	4.7×10^{-4}	32×10^{-4}	7
50'	Detonator	1240	6×10^{-3}	42×10^{-3}	7
50'	4x50 ft.lbs.	1121	1.3×10^{-3}	77×10^{-3}	60

These results indicate that a detonator gives about the same seismic energy as a 50 lbs. weight dropped from

a height of 2' but the figures above give a measure of the variations from this figure with distance.

The chemical energy in each detonator was 1,220 calories or 5,100 joules. This must be multiplied by a factor of between 3 and 10 to allow for the energy of the exploding aluminium case. Calculations using these figures indicate that a dropping weight is about 1,000 times as efficient as an explosion in producing seismic energy. It must be stressed that these figures are only accurate within an order of magnitude.

4.8 Traverse Measurements at Site 1

Figure 48 shows examples of records taken at 300' distance at Site 1. The details of the records are given below.

Record No.	Geophone depth	Source energy (75 lb. baseplate)	Components	
			A	B
931	0'	50 x $2\frac{1}{2}$ ft.lbs.	HS3	HS2
932	0'	50 x $2\frac{1}{2}$ ft.lbs.	HS1	HS2
814	12'	300 x $2\frac{1}{2}$ ft.lbs.	HS1	HS3
815	12'	300 x $2\frac{1}{2}$ ft.lbs.	HS3	HS2
933	12'	300 x 7 ft.lbs.	HS1	HS2
934	12'	300 x 7 ft.lbs.	HS3	HS2

One cm. on the original records is equivalent to 10^{-3} cm./sec. amplitude of ground motion. Because of the photo reduction used for producing Figure 48, 1 cm. of the original has been reduced to approximately 0.43 cms.

The Rayleigh waves which dominate the records of the motion at the surface are shown by two-component pictures to have true elliptical particle motion, although the major axes of the ellipses are inclined in most cases at angles up to 45 degrees to the vertical. It is obvious that they are much more dispersed than at 300' at Site 2 (Figure 49), which indicates that the largest frequency components of the amplitude spectrum have wavelengths which fall on the steep part of the group velocity curve. This is to be expected as the dispersion is caused by a layer which is considerably thicker than that at Site 2.

The records of motion at a depth of 12' show a more marked change in the record character, as compared with the surface motion, than at Site 2. The Rayleigh waves are much reduced in amplitude, while the arrivals between 0.25 sec. and 0.50 sec. are only changed in shape. The first arrival of the P wave at 0.090 sec. agrees well with the time distance graph of Figure 37 and lends support to the 6,000 ft./sec. refraction velocity tentatively shown there beyond 180'.

No further details of the results of the traverse experiments at Site 1 will be given here.

4.9 Dispersion and Attenuation of Surface Waves at Site 2

4.91 Introduction

For reasons which were stated in the introduction to this chapter it was decided to concentrate on the study of the dispersion and attenuation of surface waves at Site 2. Even in this relatively narrow field it was necessary to confine most attention to one mode of one wave type. That chosen was the primary or Rayleigh mode of the Rayleigh wave. This is the best developed feature of the records at Site 2 which are shown in Figure 49.

Figures 46(a) to (f) and 47 (a) to (d) show two-component pictures of the total surface wave motion at depths of 5' and 15'. Records 46 (g) and (h) give examples of body wave motions at 100', and also show the direction of first motion of the surface wave low frequency components.

The geophone component directions are:-

1. Longitudinally away from the source.
2. Vertically upwards.
3. Transversely to the left as seen by an observer with his back to the source.

It may be seen that the particle motion has changed from the retrograde elliptical motion found at the surface to prograde elliptical motion at depths as shallow as 5 ft. as may be expected for a layer about 4' thick (Sezawa and Kanai, 1935).

The greater the distance from the source, the more are the different modes separated from one another. There will never be complete separation except at very low frequencies below the higher mode cut off frequencies, when the lowest frequency components of the first mode will arrive first, but may be affected by body waves. The first four records of Figure 44 show the initial arrivals of the lowest frequency components of the first mode of the Rayleigh wave. Study of these arrival times may be used to give the Rayleigh wave velocity in the lowest medium. The amplitude spectra of the Rayleigh waves are good indication of the amount of overlapping of higher modes. When this occurs interference between different modes shows up as maxima and minima in the amplitude spectra. Figure 54 shows that there is some interference of the higher modes at high frequencies and this increases at closer distances to the source. (p.163)

At distances less than 50' it was not possible to separate the modes by inspection of the records.

4.98 Methods of Study of Surface Wave
Dispersion and Attenuation

Most work in the study of dispersion of surface waves has been done by earthquake seismologists. They are more concerned with dispersion than attenuation which is in any case very low at the frequencies which they consider (Bullen, 1959, p.98).

There are a number of methods used to study the dispersion of surface waves. These are listed below.

For phase velocities:

1. By Fourier analysis (Sato, 1955)
2. By other forms of spectral analysis (Ewing et. al. 1959)
3. The methods used by Press (1956) in which a wave is followed over the area studied by recording at points separated by distances roughly equal to the wavelength of the wave. It is necessary to have a well-dispersed wave train, and for the period of the wave not to change significantly.
4. From observed group velocity curves when the shear wave velocities are known.

For group velocities:

5. The method usually used is the graphical peak and trough method. The peaks and troughs are numbered

serially, and their arrival times plotted against serial number. The slope of the resulting curve at a particular arrival time gives the period for that arrival time.

6. Indirectly from an observed phase velocity curve.

Because of the undispersed nature of the surface waves for this particular combination of layer thickness, source pulse and layer parameters, Method 1 was most suitable and also more accurate than Method 5 which was also used. The Fourier analysis phase spectra were used for dispersion and the amplitude spectra for attenuation computations.

4.93 Dispersion

Curves of the phase ϕ were plotted against frequency f . Figure 51/^{p.135} shows the curves obtained for the vortical component of the Rayleigh mode recorded at distances of 50', 100', 200' and 300' by geophones at depths of 0', 5' and 15'. For records at greater distances, the Fourier analysis harmonics would have to be taken at closer intervals for unambiguous determination of the integral number of cycles in the phase values.

The determination of phase velocities was at first carried out using the phase differences at each frequency

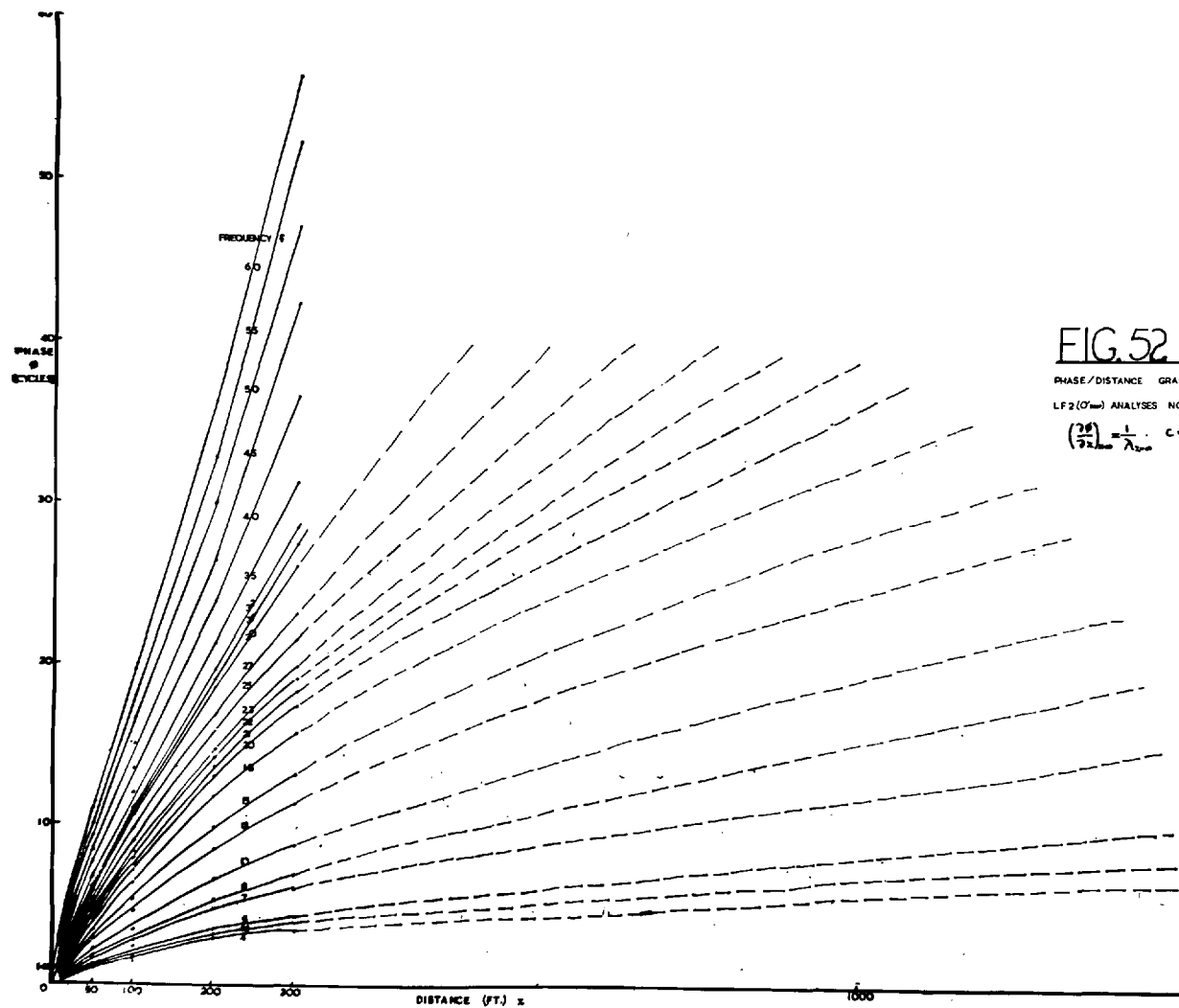
between the 200' and 300' phase/frequency curves and using the formula

$$\lambda = 100 \times \frac{2\pi}{\Delta\psi} = \frac{100}{\Delta\phi}$$

Where ψ = the phase angle in radians, and ϕ in cycles.

Then $c = f \cdot \lambda$

The phase velocity curves thus determined did not agree even approximately with the known conditions. It was realised that the reason for this was that the theoretical curves are worked out for plane waves, and this approximation does not apply in this case. The phase angles were then plotted against distance for a number of selected frequencies. The phase/distance curves for the records taken by the geophones at the surface are shown in Figure 52. Smooth curves were drawn for each frequency and the curves were produced until it was estimated that they had flattened and the slope of the flat part of the curves was taken. This was assumed to be the slope at infinity. The method was again inaccurate, but the only one that appeared to be suitable. However, the curves for frequencies beyond 30 c/s had become flat within 300' and also an approximate value was known by other methods of the shear wave velocity in the lower medium giving the slope of the lowest frequency curve.



The wavelength is given by the relationship

$$\frac{\partial \phi}{\partial x} = \frac{1}{\lambda}$$

Values of c plotted against $1/\lambda$ are shown in Figure 53. The total variation of values of c determined for the three depths is shown in Figure 53. It varies from 35% at the lowest frequencies to about 5% at the highest frequencies. It should be noted that the values determined for the highest frequencies showed significant differences in the curves for the three depths of the recording point which were not within the experimental error, but this result might have been expected (see Jeffreys 1935).

The theoretical dispersion curves given by Kanai (1951) for a one-layered medium, when Poisson's relationship holds true, were drawn on log-log graph paper. Values were ^{extrapolated} ~~interpolated~~ from Kanai's curves for values of β^2/β_1 of 5, 6 and 7 which are greater than the greatest value ($\sqrt{20}$) given by Kanai. The best fit was obtained with the curve for $\beta^2/\beta_1 = 6$ by sliding a tracing of the observed curve on similar graph paper over the master curves. This then gave $\beta_1 = 270$ ft/sec., $\beta_2 = 1,600$ ft/sec. and $H = 3.6'$. These are in agreement with the value of H found by refraction of $3.8'$, or a similar thickness of

FIG. 53—RAYLEIGH WAVE PHASE VELOCITY DISPERSION CURVES (RAYLEIGH MODE M_{11})

FROM RECORDS:- LF2(0'DEEP) •

LF2(5'DEEP) ○

HS 2(15'DEEP) ×

○ — AVERAGE OF 0' & 5'

OBSERVED CURVE COMPARED WITH THEORETICAL CURVES FOR CASES:-
 a) $\frac{\beta_2}{\beta_1} = 6$, $\beta_1 = 270$ ft/sec, $H = 3.6$ ft. & b) $\frac{\beta_2}{\beta_1} = 4.47$, $\beta_1 = 300$ ft/sec, $H = 4.4$ ft.

1385, 50'
 1360, 100'
 1366, 200'
 1383, 300'

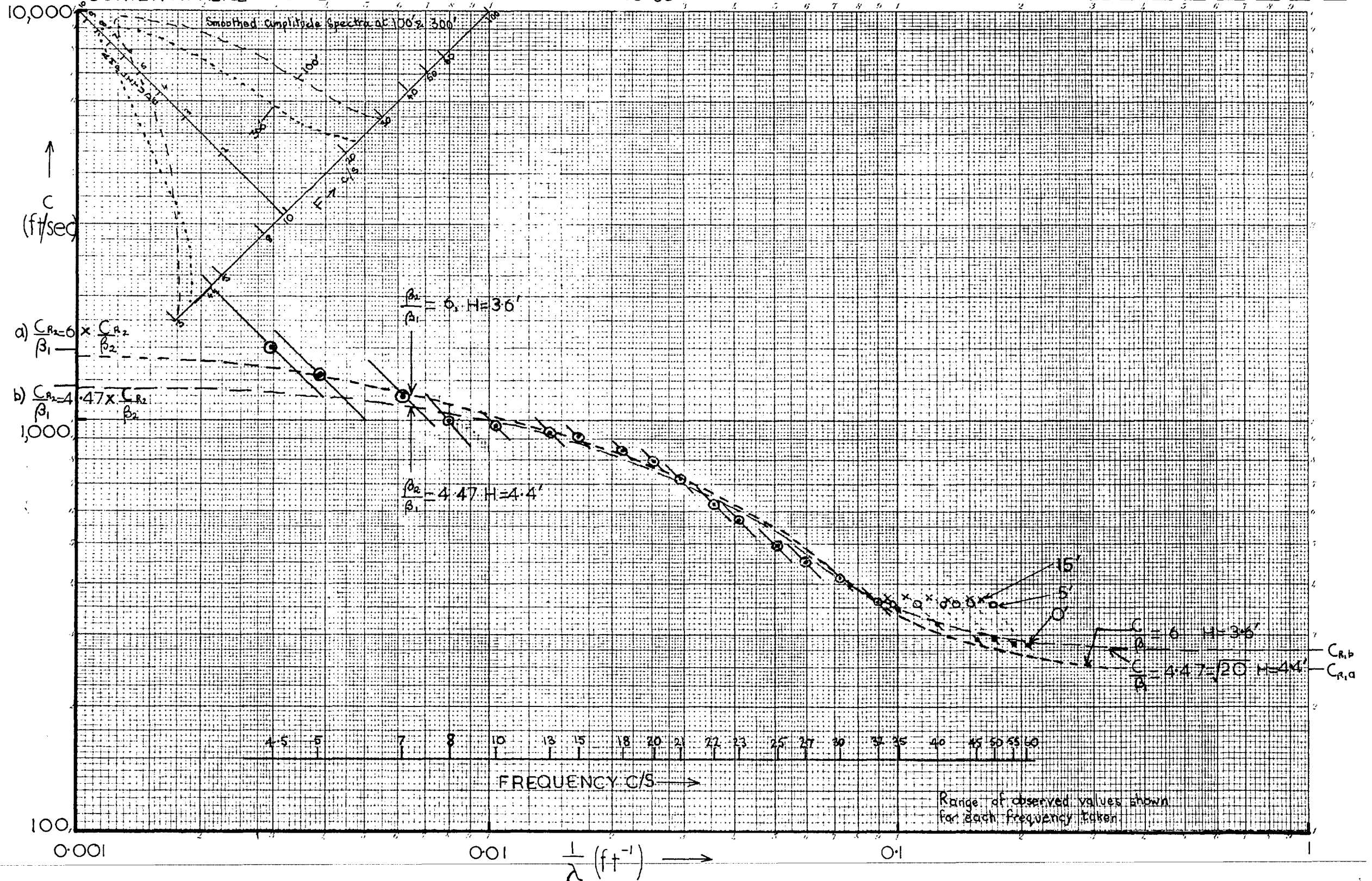
1116, 50'
 1139, 100'
 1183, 200'
 1209, 300'

1122, 50'
 1045, 100'
 1176, 200'
 1267, 300'

FOURIER ANALYSES:- 69-72

61-64

65-68



the low velocity layer given by Figure 34. The value of β_2 found from Figure 38 using the first arrival times of the low frequency components of the Rayleigh wave is 1,200 ft/sec. In Figure 53, the curve corresponding to Kanai's case of $\beta_2/\beta_1 = 4.47$ is also shown. This gives a value of H of 4.4'. Deviations from the theoretical curve may be explained by the fact that α/β is not equal to $\sqrt{3}$, the value taken for the calculation of the theoretical curves. β_2 directly determined (Figure 38) is 1600 ft/sec. or 1200 ft/sec

The analysis of higher modes of the Rayleigh wave and of the Love wave modes was also attempted but not completed. The phase velocity curve determined for the primary mode of the Love wave agreed well at the lower frequencies with the theoretical curve which was calculated from the nomogram given by Sato (1953). The phase velocity at the higher frequencies was greater than the theoretical value. From the records of Figure 49 it may be seen that a large proportion of the higher mode energy of both Rayleigh and Love waves must be contained in frequencies which lie on the part of their group velocity curves corresponding to inverse dispersion, because the higher frequencies arrive first. The maximum group velocities thus obtained agree approximately with the theoretical values.

Attenuation of Surface Waves

The vertical component records of the Rayleigh mode used for the dispersion studies described above were also used to study the attenuation of surface waves. Reliable measurements were only obtained at frequencies at which there were amplitude components not less than about 5 per cent of the maximum.

Three factors which made interpretation difficult were, firstly the small variations in the gain setting at different distances as mentioned in Section 2.44. Secondly, the higher modes interfered at the higher frequencies. This interference was least for the surface geophone, as may be expected from theoretical considerations, and hence the LF2 (0') geophone gave the most reliable results. Thirdly, the usual correction for the geometrical divergence appeared to be inadequate at distances less than about 100'. Figure 54 shows the amplitude spectra of the Rayleigh mode recorded at a depth of 5'. The corrections for geometrical spreading and differing gains is not shown. The interference of the higher modes shows up clearly in this case.

The attenuation which was computed from these curves for the narrow frequency band which does not show marked interference is shown in Figure 55 curve 1. The

FIG 54 ATTENUATION OF RAYLEIGH MODE

CURVES SHOW AMPLITUDE OF EACH HARMONIC, NOT CORRECTED FOR GEOMETRICAL DIVERGENCE, SENSITIVITY OR DROP HEIGHT (LF 2.5')

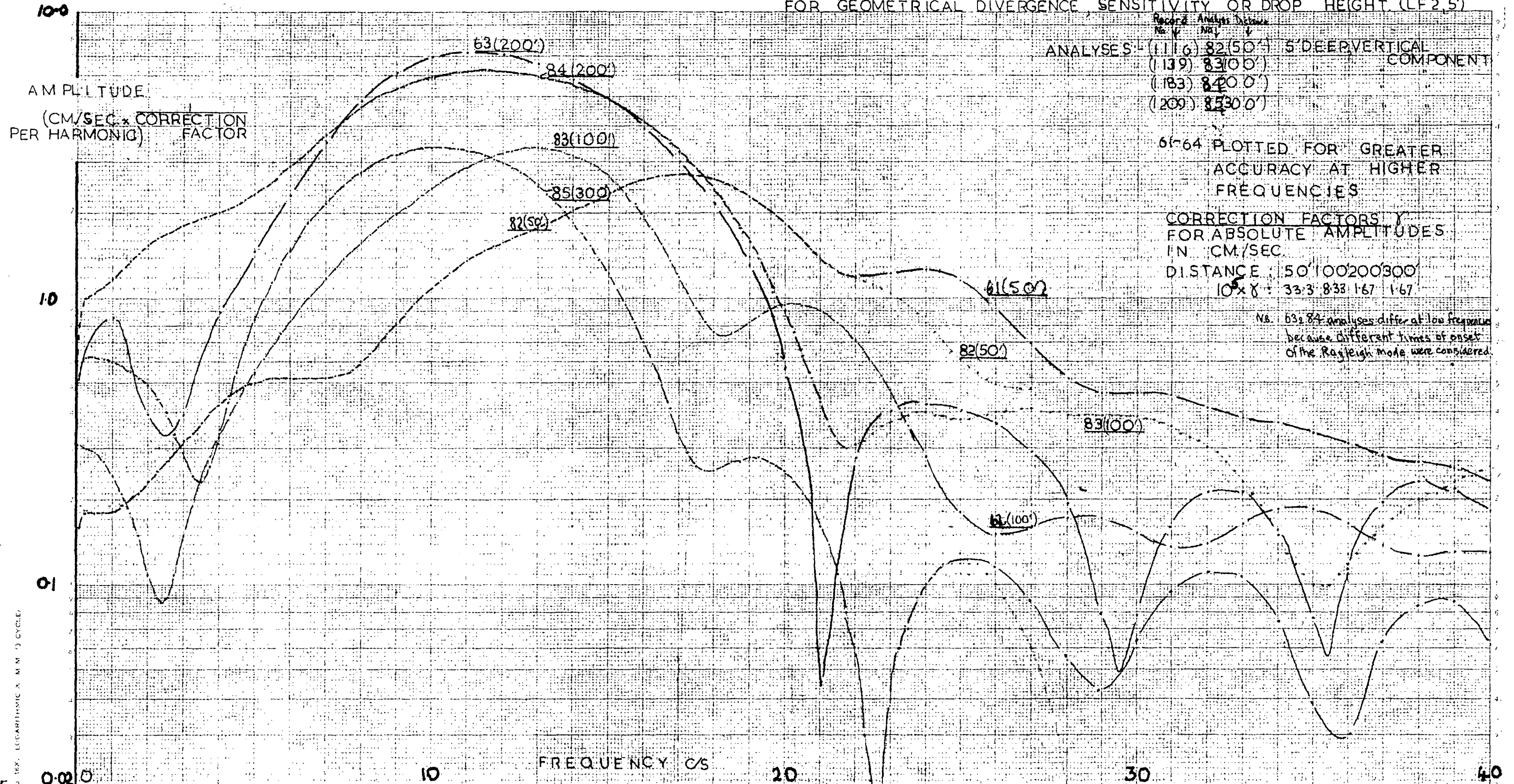
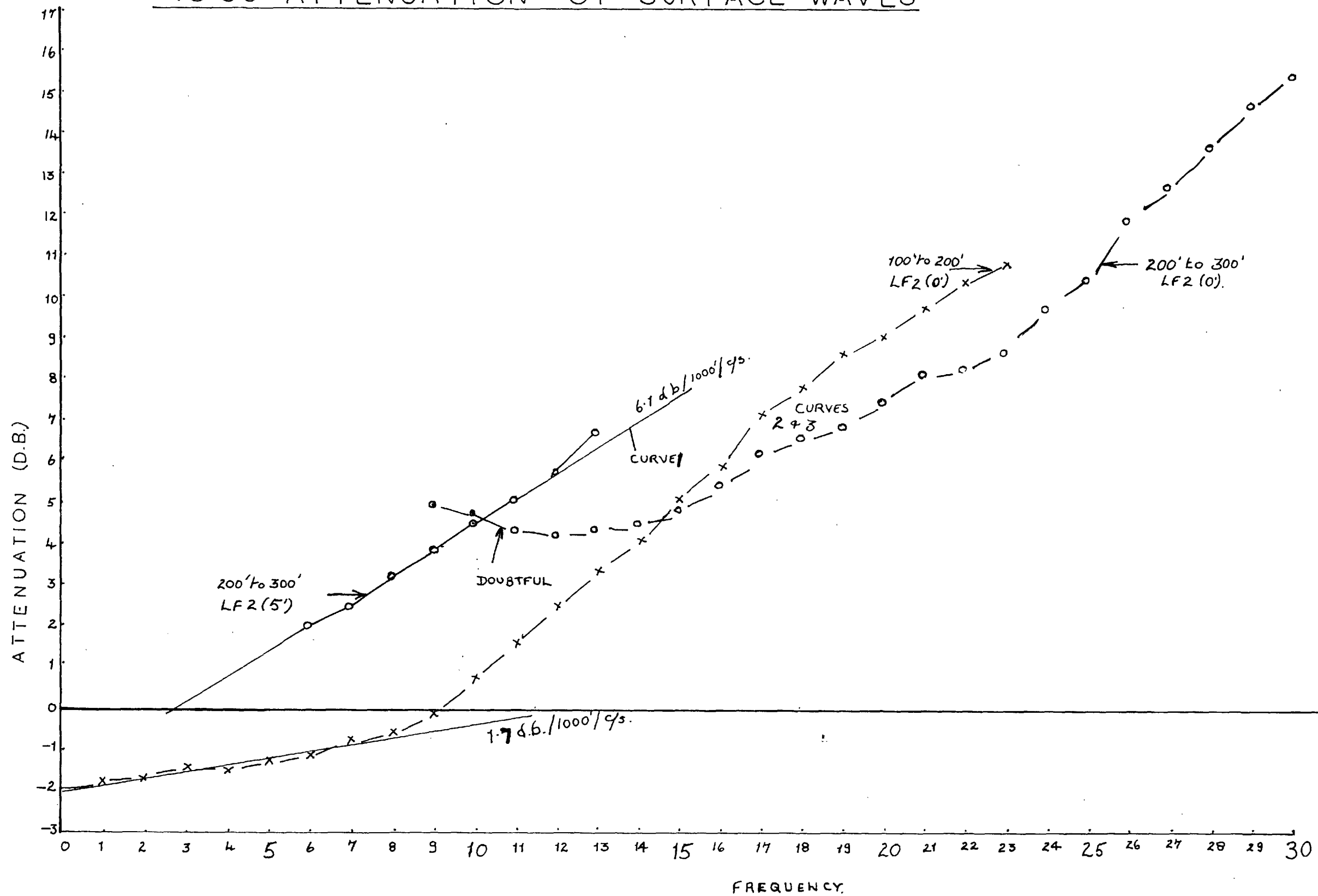


FIG. 55-ATTENUATION OF SURFACE WAVES



same figure also shows the calculations made using the records of the geophone at the surface (curves 2 and 3). With a certain amount of smoothing out of the spectra to eliminate the effects of higher mode interference, the measurements were extended to higher frequencies as shown.

The values found for attenuation are in some cases less than 0. This is the result of the differing gain settings. The attenuation is shown to exhibit a linear variation with frequency having an average value of about 6 d.b. per 1,000' per c/s. beyond about 10 c/s, and a less reliable figure of 1.7 d.b. per 1,000' per c/s. is found below about 10 c/s. McDonal et. al. (1958, p.424) give a value of attenuation of shear waves at 1.0 d.b. per 1,000' per c/s. Their figures apply to the much harder Pierre shale and the measurements were made at a depth of several hundred feet. In view of these facts, the agreement is within reason.

The cause of the reduction of attenuation at lower frequencies may be due to the fact that the longer wavelengths penetrate deeper into the medium and give a measure of the attenuation of the deeper materials which might be expected to be less than the value for those near the surface. The wavelength at 10 c/s is 100'.

4.10 Attenuation of Body Waves

The filtering effects of multiple reflections within the surface layers was calculated using a theory based on a method given by Officer (1958). For a layer 3.8' thick, and with the average velocity conditions which have been determined above and below the layer lower boundary, the filter was shown to have a pass band from approximately 50 c/s to 100 c/s (and also lower and higher pass bands). The exact calculations were very sensitive to variations of α_1/H and it was eventually decided that it was not possible to calculate an accurate correction for the filtering effects due to multiple reflection. No allowance had been made for other layers, and the attenuation of the low frequency components of the pulses shown in Figure 42 was not explained.

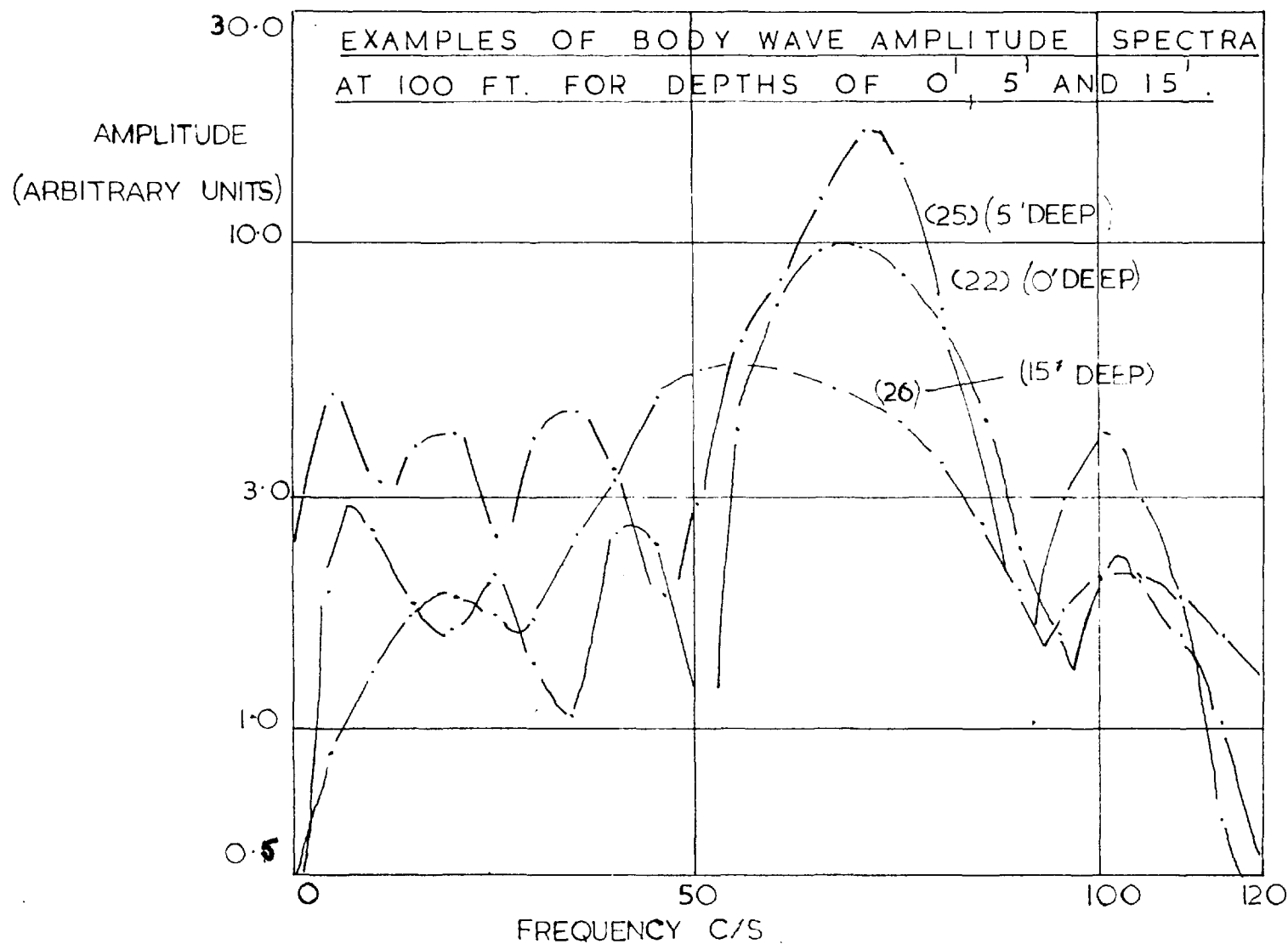
Examples of the body wave pulses are given in Figure 44. They show the "ringing" effect which indicates a narrow frequency spectrum. In particular, record 1346 is of two records at a distance of 100' from the source. Curve A was produced by the geophone at 5' while curve B was a record of the output of a geophone at 15' deep. The spectra of these pulses is shown in Figure 56 together with that of a similar pulse recorded at the surface (record 1364 A). The selective absorption of

frequencies on either side of the pass band central frequency of 70 c/s even in passing from 15' to 5' is well shown. The pass band in this instance is approximately 50 c/s to 80 c/s.

Records 1364, 1374, 1378 are of surface records of body waves at distances of 100', 200' and 300' using the 75 lbs. baseplate at the source. The airwave is well developed on the surface records.

With greater refinement of the methods of applying the necessary corrections for determination of body wave attenuation from these experiments, it may later be possible to obtain the order of magnitude of its value for comparison with the surface wave measurements.

FIG. 56



C H A P T E R 5

SUMMARY AND CONCLUSIONS

5.1 Summary

In Chapter 1 the historical background of the present work has been reviewed and it has been pointed out that it is desirable that more of the information which is contained in a seismic record be used.

Controlled experiments have therefore been carried out designed to obtain a better understanding of the processes involved.

In Chapter 2 the design of recording apparatus is described. The aim has been to develop a high fidelity system which will give a faithful record of the motion of the ground so that corrections do not have to be applied for instrumental distortion. Special broadband geophones and amplifiers were designed and constructed and a method is also described of applying integrated feedback to reduce the effective natural frequency of a geophone. This could possibly be applied in the future for getting wider geophone frequency response.

An accurate timing unit was also constructed which had the particular feature of providing an equally accurate delayed output for use when recording two-component pictures of particle motion. By applying

this technique it was possible to study in detail any particular section of the record in the form of particle motion diagrams without the complication of the overlapping pattern caused by the remainder of the record. A number of other pieces of equipment were constructed and used to control experiments and help with the recording process in general.

The field procedure is described in Chapter 3 together with the various experiments which were carried out. The first site was used to develop this experimental technique and test apparatus as it was constructed. Most of the experiments were very near the source and were concerned with measurements of true ground motion, etc. Although a traverse was completed to 300', the results were not interpreted in detail because it was decided that the experimental site conditions were too complicated. The main traverse results were computed from Site 2 experiments on the clay. This represented a much simpler medium which consisted of about 4' of low velocity material ~~overlapping~~ about 400' of clay. Refraction measurements indicated that the velocity of the clay increased from about 4,500 ft/sec. to about 5,000 ft/sec. at a depth of about 15'.

In working out the results of these experiments a digital computer was used to perform Fourier analyses. This was found to be of considerable help for attenuation and dispersion measurements and also for studying the frequency content of a number of pulses from dropping weights and explosions.

A large number of experiments were performed to cover as broad as possible a range of the research topics listed in Chapter 1. This was done because it would have been impracticable to return to the field site to make further measurements, and also so that it could be decided which were likely to justify detailed study by small scale field work of this nature.

In Chapter 4 the results are presented of a small proportion of the experiments described in Chapter 3. These fall into two main groups. Firstly, general results of measurements of noise levels, source types and the conditions affecting true ground motion are described, together with the results of the near surface amplitude variations, with depth and source energy. These experiments were mostly carried out at Site 1 and are not directly applicable to seismic prospecting.

The second group of results from Site 2 are concerned with the study of the attenuation and dispersion of body

and surface waves. It is concluded that reliable results cannot be obtained regarding the body waves without accurate knowledge of the near surface conditions and detailed calculations to allow for their effects. These could probably be done at a later date using the computer but the effort involved may not be justified unless the near surface attenuation is of interest. Otherwise it would be possible to design experiments which would make the corrections simpler as long as measurements could be made in boreholes with good borehole geophones in a manner similar to that used by McDonal et. al. (1958).

The experiments were particularly suited to the study of surface waves and the analysis of the primary mode of the Rayleigh wave is given in detail, both with respect to attenuation and dispersion. Once the method of dealing with the dispersion of surface waves had been decided upon, the results were found to be in good agreement with the theory. With more time, analysis of the higher modes of the Rayleigh waves and also of the Love wave modes should prove to be instructive.

Group velocity curves obtained in the usual manner from the phase velocity curves determined at a particular distance by taking the slope of the phase/distance curves at that distance gave results in reasonable agreement with

those found graphically by the peak and trough method.

5.2 Conclusion

A method has been developed for carrying out small scale field studies on seismic wave propagation with particular emphasis on the control of the experimental conditions and the accuracy of recording the ground motion. Although from the point of view of interpretation the results are hampered by the near source and near surface nature of the problem and consequent complication which they introduce, it has been possible to gain a good knowledge of a number of aspects of the seismic process and more particularly to work out in detail dispersion and attenuation figures for the primary mode of the Rayleigh wave.

The most significant feature emerging from the results is the possibility of the use of surface wave dispersion as a means of studying subsurface structure.

A number of authors (Sato (1959): Press and Takeuchi (1960): Press (1960): Dorman, Ewing and Oliver (1960) and Bruno, Nafe and Oliver (1960)) have recently enthused upon the possibility of applying this method to the study of crustal structures, and from the results of these experiments it seems that use could be made in seismic

prospecting to delineate smaller structures, particularly as it is possible to obtain accurate dispersion curves over a wider frequency range than that used by earthquake seismologists.

Some of the advantages and disadvantages of the use of this method are given below.

Advantages:

1. It permits the use of a much greater portion of the seismic record than that used at the present time. For this reason, more information can be obtained from any one record. For phase velocity measurements it is only necessary to record at two points which should be sufficiently close together to enable phase velocities to be determined uniquely at all frequencies used. For group velocities only one recording point is needed. Because of these facts, the method may be suitable for use in areas of difficult topography.

2. Because of the slower reduction in amplitude of surface waves relative to body waves due to geometrical divergence, and their consequent greater relative amplitude with increasing distance, surface waves studies do not require such large source energies as those needed for body waves. If records were taken at two gain settings, then the refraction first breaks could also be recorded,

and, with filtering, reflections could also be identified. To obtain sufficient signal to noise ratio in the latter case it would be necessary to increase the charge size or work in particularly quiet areas.

3. The possibility of broadband recording and the use of computers for analysis of results should enable very accurate dispersion curves to be obtained when the two recording distances are far enough from the source. Allowance for curvature of wave fronts could be made as described above, as the time of origin of the shock is exactly known in seismic prospecting. Alternatively, the time of origin need only be known roughly (within a minute) as long as the time scales at the two nearby stations are accurately correlated.

4. The Rayleigh wave dispersion curves depend on ρ , α , β and H of each layer, so that when a curve is found that agrees with the theoretical curves, a knowledge of Poisson's ratio may also be obtained, thus giving an idea of the type of materials.

In view of the fact that amplitude spectra would also be obtained from the analysis, attenuation measurements might be used to give information regarding the nature of the rocks and possibly the variation of attenuation with wavelength and hence with depth. Interpretation of the

Love wave data depends only on β , ρ and H . The possibility exists of finding as much about the nature of the subsurface rock as the use of the seismic method permits. In this connection the applicability to soil mechanics might be considered.

5. Average values of the velocity variations are given by dispersion methods. It has been found that Rayleigh waves sound deeper structures than Love waves of the same period (Dorman, Ewing and Oliver (1960) p.105). The group velocity data gives the large scale average between the shot and recording points, whereas the phase velocity data gives smaller scale average values between the two recording points.

6. A velocity decrease with depth may be detected by this method.

7. Small scale as well as large scale measurements could be made. The scale factor between the present experiments and the earthquake case is about 1 to 3×10^4 .

Disadvantages:

1. The main disadvantage is the difficulty of interpretation. This has hitherto prohibited the use of surface waves, but with the greater use of computers it is now possible to calculate dispersion curves for the most complicated cases and also to compute the observed curves very accurately.

The problem of interpretation is still essentially indirect and non-unique. The method would be to prepare a large number of theoretical curves of increasing degrees of accuracy so that one could step by step eliminate those which do not apply. Other methods of eliminating inapplicable solutions could be used, such as the knowledge of the P and S wave velocities possibly measured by other methods, or in other areas.

Press (1960) talks of the problems of quantifying the uniqueness. For Love waves, less variables are involved and it may be possible eventually to develop a more direct interpretational method, which could be used in conjunction with the more difficult problem of interpretation of the Rayleigh wave data (possibly for more than one mode).

2. The results will always be less accurate and less detailed than the refraction or reflection results. There are no means at the moment of determining the angle of dip of the strata, other than by using the variations of depth found between a number of pairs of recording points.

3. In some cases it would be necessary to record over greater distances than for refraction work because of the need to use plane waves for simpler interpretational

procedures. Greater distances are also needed for separation of the higher modes.

4. It would be necessary to have good low frequency geophones. For a regional survey, for example, the frequencies needed to define the steep part of the phase velocity curve might be roughly between 1 and 1/10 cycles per sec. (which is also the region of maximum micro-seismic noise).

It is suggested therefore that the possibility of using surface waves for seismic prospecting should be further examined.

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- Note: J.G.R. - Journal of Geophysical Research.
- B.S.S.A. - Bulletin of the Seismological Society of America.
- B.E.R.I.T. - Bulletin of the Earthquake Research Institute (Tokyo).
- M.N.R.A.S., G.S. - Monthly Notices of the Royal Astronomical Society, Geophysical Supplement.