

MAGNETIC EFFECTS ON DUST CHARGING AND
DYNAMICS IN PLASMAS

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ABSTRACT

This thesis documents the analytical and computational models developed for the purpose of understanding the influence of magnetic fields on dust behaviour in tokamak plasmas, vital for the safe and stable operation of future devices.

The Monte Carlo code Dust in Magnetised Plasmas (DiMPI) designed to investigate the influence of magnetic fields on dust rotation and charging is presented. Comparisons of DiMPI with other simulation results revealed a surprising insensitivity to the form of the potential and the collisionality of the plasma for the purposes of determining the equilibrium potential and net surface current for dust smaller than the Debye length. The dependence of the normalised potential on the temperature ratio, flow speed, dust size and magnetic field strength was explored and used to constrain a semi-empirical model for the plasma currents, suitable for implementation in dust tracking codes.

Simulations of the rotational dynamics of dust due to the collection of angular momentum revealed the importance of electrons at weak magnetic field strengths. These results and analytic estimates explain experimental observations in low temperature discharges of dust rotating anti-parallel to the magnetic field at $\sim 100\text{Hz}$. These improved models were incorporated into DTOKS-U and compared with fast-camera observations of carbon dust transport in DIII-D and breakup of molten tungsten observed over 103 discharges in JET. Both the importance of the strike point for limiting dust transport in DIII-D and the rotational breakup mechanism responsible for the observations in JET were identified. These results suggest that the higher magnetic fields, plasma temperatures and densities of ITER will alter charging but also enhance the breakup process, protecting the core plasma from acute impurity deposition, providing significant operational benefits.

DECLARATION

I declare that the contents of this thesis constitutes my own work unless otherwise referenced and appropriately indicated in the text.

London

SW7 2BZ, July 2020

Luke SIMONS

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PUBLICATIONS

As part of the work performed for completing my PhD, several documents both related and unrelated to the subject of this thesis have been published as journal articles and conference proceedings. A complete list of these works is given below:

1. J. E. Allen and L. Simons, *The Bennett pinch for non-relativistic electrons* J. Plasma Phys. 84, 1 (2018).
2. G. Abbas, J. E. Allen, M. Coppins, L. Simons and L. James, *A study of the propagation of a solitary wave along the magnetic field in a cold collision-free plasma* Phys. Plasmas 27, 042102 (2020).
3. L. Simons and M. Coppins, *Modeling of spherical dust charging in collisionless magnetized plasmas with DiMPL* Phys. Plasmas 27, 063702 (2020).
4. J. T. Holgate, L. Simons, Y. Andrew, C. K. Stavrou, and J. E. T. Contributors, *Spontaneous rapid rotation and breakup of metal droplets in tokamak edge plasmas* Eur. Phys. Lett. 127, 1 (2019).
5. C. Cowley, P. Fuller, Y. Andrew, L. James, L. Simons, M. Sertoli, S. Silburn, A. Widdowson, J.E.T. contributors, I. Bykov, D. Rudakov, T. Morgan, S. Brons, J. Scholten, J. Vernimmen, P. Bryant, and B. Harris, Phys. Rev. E 102, 43311 (2020).
6. L. Simons, J. Holgate, C.K. Stavrou, B. Dudson, M. Thomas, P. Bryant, B. Harris, S. Ratynskaia, T. Morgan, the Magnum-PSI Team and JET Contributors in 45th EPS Conf. Plasma Physics, EPS 2018 (Prague, 2018), P5.3007. URL: <http://ocs.ciemat.es/EPS2018PAP/pdf/P5.3007.pdf>
7. L. Simons, J. T. Holgate and M. Coppins, in 46th EPS Conf. Plasma Physics, EPS 2019 (Milan, 2019), P1.1062. URL: <http://ocs.ciemat.es/EPS2019ABS/pdf/P1.1062.pdf>

Item 3 and item 4 on the list are publications directly related to the subject of this thesis, corresponding to the work documented in section 4.2 and chapter 8 respectively. Each of those publications represent my own work and document a substantial portion of original material I have generated over the duration of completing my PhD. Portions of those works have been reprinted here with permission from the publishers and flourished with additional details. The 5th item is a recently accepted publication documenting the operation of the RIDAT code which was implemented in reconstructing trajectories identified in DIII-D as discussed in chapter 7. The remaining items provide details for two poster abstracts related to the presentation of preliminary work at the 45th and 46th EPS plasma physics conferences.

My own research and results gathered in support of this thesis are presented in sections 4.2, 5.1.2, 5.2, 5.3 and in chapters 6, 7 and 8.

*Things derive their being and nature by mutual dependence
and are nothing in themselves.*

— Widely attributed to Buddhist philosopher Nagarjuna c. 150 – c. 250 CE

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INTRODUCTION

*Not only is nothing good or ill but thinking makes it so,
but nothing is at all, except in so far as thinking has made it so.*

— Samuel Butler [1]

1.1 PLASMAS AND DUST

Though plasmas are ever-present in the universe, the use of the ancient Greek word, $\pi\lambda\alpha\sigma\mu\alpha$, in their designation is less than a century old, likely originating in a short conversation between Lewi Tonks and Irving Langmuir [2, 3]. At present, the name plasma is given to the ionised state of matter which exhibits long-range collective effects due to electromagnetic forces. Since its inception, the notion of a gas of charges has proven to have a wide range of useful theoretical applications, for example in envisaging electrons in metals through the free electron model [4] and quark-gluon plasmas in atomic nuclei [5]. The phase boundary between a plasma and the condensed matter state presents a particularly intriguing focal point for physical interactions that is surprisingly prevalent since dust particulates, in analogy with the everyday dust found in air, pervade plasmas. Dust in this context is broadly defined as solid or liquid matter existing at a length scale much smaller than the spatial scale of the plasma but much larger than the spatial scales of individual atoms and which is able to move freely within the plasma. The freedom of movement of dust is nearly all that discriminates the closely related topics of probe theory and dusty plasma physics.

Consequently, the interaction of plasmas with small particulate matter is an inevitable topic of scientific interest. For experimental plasmas, contact with the physical apparatus used for their creation or isolation invariably creates enduring microscopic solids which permeate the plasma. In the heliosphere, dust and ion mass densities are thought to be roughly equal [6] whilst small matter particulates exist within the solar system at many spatial scales [7]. At the largest possible scales, plasmas are known to be exceedingly common and form the dominant component of the universe [8] (excluding dark matter [9]). The cosmic abundance of dust, defined as the relative proportion of dust to other mass in the universe, is $(1 - 8) \times 10^{-6}$ at a red shift $\zeta \simeq 0 - 3$ [10], implying that the dust to stellar mass ratio is about 0.2% at $\zeta = 0$ [10]. Consequently, dust densities are considerable enough to have a major effect on depletion of refractory elements in stars, scattering of ultra-violet and visible light and influence directly star and galaxy formation in the intergalactic medium [11]. At a stretch, it may be claimed that this dusty plasma existing at the largest spatial scales constitutes around 87% of all baryonic matter [12].

Magnetic fields can have a substantial indirect impact on dust particle behaviour through altering the motion of charged particles in its vicinity, a feature which has

received little attention and remains poorly understood. Deviations in the floating potential and dynamical behaviour can affect the life-cycle of matter particulates and have ramifications for the macroscopic properties of the plasma. This thesis details advances made in understanding the effects of magnetic fields on isolated dust grains in collisionless plasmas through the development of novel simulation tools and models.

The context of this work is further established through exploring a number of relevant plasma environments. Of particular importance to this thesis is the historically important problem of dust particles in plasmas used for industrial manufacturing processes, the present day problem posed by interstellar dust as an anomalous microwave background to spectroscopic signals from astrophysical bodies and finally the small impurity particles in fusion plasmas which can present a serious threat to public safety.

1.2 SEMICONDUCTOR MANUFACTURING

In steady state, low temperature plasmas ($\sim 1\text{eV}$) there is insufficient energy to cause bond dissociation, meaning the solid or liquid matter of dust can endure indefinitely. Much of the initial interest in the field of dusty plasmas arose from this type of plasma with the discovery of the significant role contaminants had in degrading the industrial manufacture quality of semiconductor chips [13, 14]. Plasma processing techniques like etching and deposition are necessary to produce clean, sharp features on silicon wafers that are vital to, according to some estimates, two thirds of the technological infrastructure of fabrication plants [15] used by the billion dollar semi-conductor industry [16], upon which much of modern computing technology is based.

With the arrival of micro-electronics in the 1970's, following the unstoppable progression of Moore's law [17], the creation of reliable integrated circuits began to require immaculate precision in the creation of fine scale structures since the microscopic features were susceptible to short circuiting when contaminated with micrometre [18] and even nanometre scale solids [19]. The yield loss introduced during processing presented the greatest barrier to production with estimates made from models of manufacturing dynamic random access memory chips at the time suggesting that a reduction in production yield of 1%, potentially caused by contamination, would cause a 2% loss in profits [20]. This is remarkable considering that products would initially be introduced with production yields at the 10% to 30% level and only grow to 90% over a one to two year period [20].

As a result, the typical background density of particles (of sizes greater than $0.5\mu\text{m}$) in urban areas of around $35,000,000\text{m}^{-3}$ [21] was deemed no longer tolerable. For example, Intel states in their documentation "Transistors to Transformations", that: "Because particles of dust can ruin the complex circuitry on a chip," they permit only one such particle per cubic foot (0.03m^{-3}) [22]. The advent of clean rooms [23] eliminated this background problem whilst development of hygiene procedures for workers left a clean environment almost entirely dust free [24]. Even in these supposedly immaculate environments, many defects aris-

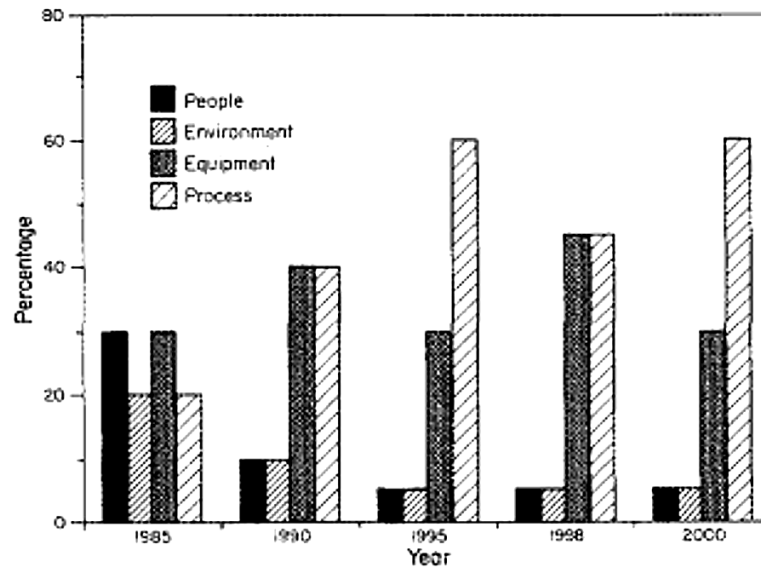


Figure 1: Defect allocation by source type showing the trend towards contamination arising from equipment and plasma processing [25]. Reprinted with permission from [20]. Copyright 1992, American Vacuum Society.

ing from an unknown source of micrometre particulates continued to occur, costing manufacturers millions in discarded electrical components.

Eventually, an intrinsic source of particulate contamination was uncovered accidentally, during routine measurements of the concentrations of reactive gases using laser induced fluorescence [26]. The planned measurements of fluorescence were overwhelmed by the incident light scattered from micron-sized dust particles electrostatically levitated above the wafers. Dust was found to be growing in-situ through nucleation around smaller defects in the plasma [27] and then aggregating further ions and molecules in a uniform crystalline growth process [26, 28]. The generation of this dust was identified as the dominant source of particulates, arising in a number of different plasma processing techniques [29], see figure 1. This dust rapidly accumulates a negative charge from collection of the highly mobile electrons and equilibrates above the sheath, balancing predominantly the time-averaged repulsive electrostatic field from the anode with gravity and the positive space charge of the sheath. Held in trapping fields generated by surface features of the silicon substrate, these particles accumulate in ordered levitating structures which fall onto the substrate once the plasma discharge has ceased and their charge dissipates. Ultimately, techniques were developed for manipulating the electrostatic fields of the substrate to avoid the contamination by guiding the dust spawned in the plasma safely away from the electronic components [30].

Since the 1980's, the use of low temperature partially ionised plasmas in industry for a variety of purposes has grown massively [31]. Though sources of impurities have been mostly removed from production through the use of sophisticated clean rooms and practices [32], contaminant particle control is an ongoing area of research [33], motivated by the huge costs to the \$468.8 billion dollar industry [16]. The notoriety earned by dusty plasmas here has been a spur for the field and led indirectly to progress in other areas.

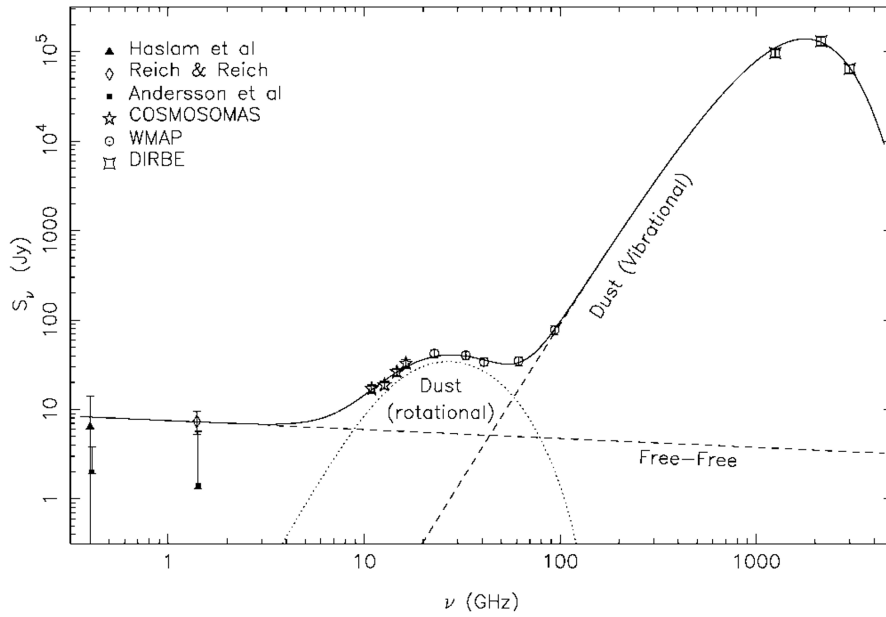


Figure 2: Spectral energy distribution of the Perseus Molecular Cloud [39] with data taken from early Planck results [41]. © AAS. Reproduced with permission from [39].

1.3 DUST IN SPACE

The presence of dust in the cosmos would have likely been noted by many ancient people who, in looking at the dark patches near Sagittarius at the centre of the milky way, noticed that the cosmos was unevenly illuminated. This was first recorded formally by Sir William Herschel, who believed these to be regions void of stars [34], with the true reason for occlusion not being associated with dust until the work of Agnes Clerke at the start of the 20th century [35]. The general existence of interstellar dust was conclusively established by Trumpler [36] in the 1930's, leading eventually to the derivation of a power-law for the dust size distributions from fitting interstellar extinction curves [37] and later modern models [38].

Subsequent iterations of these models for interstellar dust have been driven in part by the need to properly understand and model the anomalous microwave emission which forms a cosmological foreground in a number of galactic and extragalactic environments [39–41]. This astrophysical emission spectrum has been interpreted through adapting interstellar dust models to calculate the electric dipole radiation from spinning charged dust particles in magnetic fields [42, 43]. With inferred rotation frequencies of 10^9 Hz, these models are found to accurately replicate and explain the features of the emission spectrum, as shown by figure 2, though their refinement and direct measurement is a topic of ongoing research [44, 45].

Due to the robustness of their theoretical description across spacial scales, the physics used to describe dust in plasmas can also be applied in a similar manner to a range of larger astrophysical objects. The same physical considerations which have been used to characterise dust in the heliosphere [6] have been related to small debris which is common in space [7] and also for the charging of spacecraft in low earth orbit [46]. This is also exemplified by comets in the perihelion (less

than 1AU), where the abrasion of their surface and drag force felt are modelled in a similar way to those of a dust particle in a flowing plasma [47, 48]. These parallels also extend to connections in modelling important molecular reactions occurring on the surface of dust particles [49]. The propensity for these solid bodies to exhibit complex surface chemistry and harbour organic compounds necessary for life has even led to the gradual recognition and wider acceptance of the theory of panspermia, that the origins of life on earth may have extraterrestrial origins, as a credible philosophy [49, 50]. The broadly extensible simplifications of dusty plasma physics have resulted in the self-similar depiction of a variety of seemingly unrelated systems and applications in extraordinary environments.

1.4 FUSION PLASMAS

Perhaps most exceptional of these plasma environments is the plasma sustained by nuclear fusion reactions. Nuclear fusion is the process by which two light atomic nuclei react to form a heavier atom and is ultimately responsible for producing all elements heavier than hydrogen in the universe. During fusion reactions, a small difference in mass between the reactants and products resulting from the difference in binding energies of the elements produces energy. Harnessing a near limitless source of energy through this nuclear process has been a longstanding ambition of physicists following Eddington's hypothesis of nuclear fusion as the origin of stellar energy and prediction of its civic uses, nearly a century ago [51, 52]. The challenge of achieving ignition, a self-sustaining fusion reaction, through conventional means was summarised in the Lawson criterion as the requirement for maintaining a plasma at sufficiently high temperature and great enough density for fusion reactions to occur [53]. This objective remains and continues to motivate the design of modern devices.

1.4.1 Tokamak Design

The tokamak design principally comprises of a vacuum vessel with Plasma Facing Components (PFCs) which act to physically restrict plasma motion within a toroidal volume. The particles' motion is further inhibited by toroidal magnetic fields generated by solenoids which encircle the device, whilst a central solenoid is used to induce a current in the toroidal direction through the plasma. This plasma current generates its own poloidal magnetic field which produces a net helical magnetic field which is less vulnerable to some Magneto-HydroDynamic (MHD) instabilities [54] and limits the collisionless transport of particles. The efficient confinement of the core plasma enclosed by the Last Closed Flux Surface (LCFS) leads to steep gradients in temperatures and densities at the edge known as the pedestal. Inevitably however, plasma particles eventually escape the magnetic confinement through collisional processes to the region outside the LCFS known as the Scrape Off Layer (SOL).

The resulting exhaust of hot plasma is focused onto specially designed PFCs. The favoured material choice for use as PFCs across the inner wall are a combination of elements including tungsten, iron, molybdenum, graphite, beryllium and

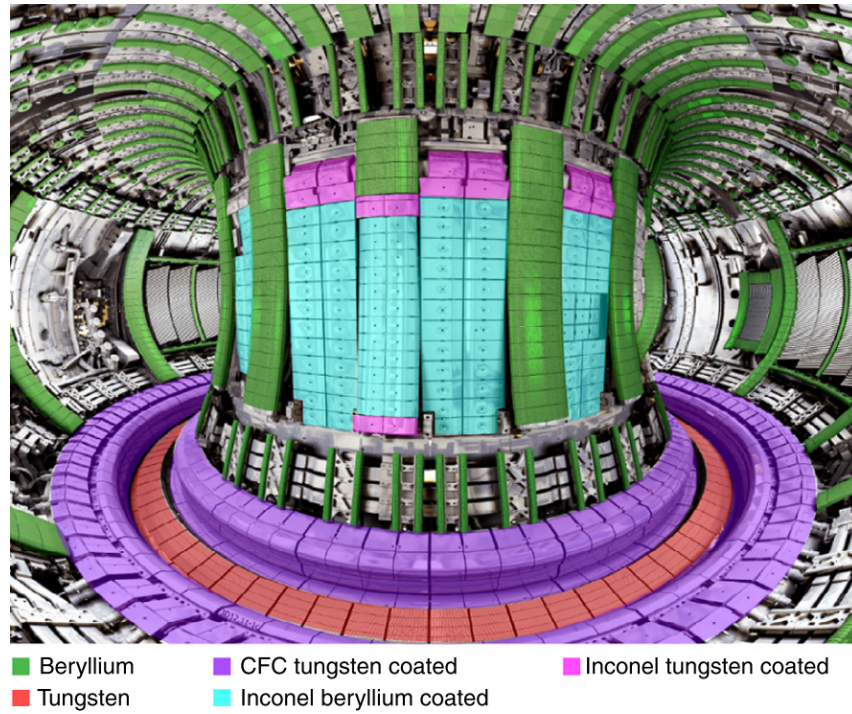


Figure 3: Image of JET inner walls colour coded by PFC material [55]. Reprinted from [55] with permission from Elsevier.

their alloys as is exemplified for the worlds largest tokamak the Joint European Torus (JET), see figure 3. These choices are made through a consolidated judgement of the material properties pertaining to radiation exposure, thermal fatigue and shock characteristics, phase boundaries, neutron irradiation resistance, chemical and physical erosion, cost and safety [56].

Prior to the advent of the divertor tokamak design and improved control of plasma instabilities, the major obstruction to fusion performance was the power losses caused by impurities from particulate contamination originating from the components physically constraining the plasma known as limiters [57]. It was quickly realised that "metallic limiters will not be possible in fusion devices and magnetic limiters (divertors) will have to be developed to reduce high-Z impurities" [58] as protruding physical limiters were far inferior for performance [59]. In general impurity contamination of the core plasma is a detrimental effect as it reduces the fuel density and consequently the probability of possible fusion collisions. This causes problems for long pulse operation where the effective confinement of particles in the core plasma causes the impurity content to steadily grow. These impurities must be expunged along with the helium ash by maintaining an active fuel cycle, typically achieved through neutral beam or pellet injection which accesses the particle density of the core plasma directly [60]. The central problem caused by impurities, sometimes expressed through the Murakami density limit [61], is that they radiate energy taken from the hot central plasma that is effectively removed from the system and absorbed by the inner wall. The extreme temperatures of the core plasma fully ionises most atoms in this region, which then emit energy through Brehmstrahlung radiation proportional to the square of the

particle charge, Z^2 , meaning heavier impurities radiate much greater amounts of energy. Particulate matter composed of the PFCs can be a large source of impurities and exacerbate the situation if they are able to penetrate into the core plasma [62].

This effect is particularly severe for tungsten [63] with $Z = 74$, such that a fractional proportion of 3×10^{-5} tungsten ions can increase the minimum Lawson criterion by around 20% [64] and is significant enough to cause plasma disruption [65]. For this reason, the first wall of tokamaks have historically been composed of lighter, heat resistant materials such as graphite. Modern tokamaks employ a magnetic field topology which allows the plasma wetted target area to be greatly enlarged over a region known as the divertor, see figure 4, mitigating the impurity transport to the core plasma and permitting high Z materials to be used for these surfaces. Due to the predilection of these materials to produce dust however, a host of safety concerns must be considered in designing modern tokamaks and in particular the International Thermonuclear Experimental Reactor (ITER) that have been summarised in a number of review articles [66–70].

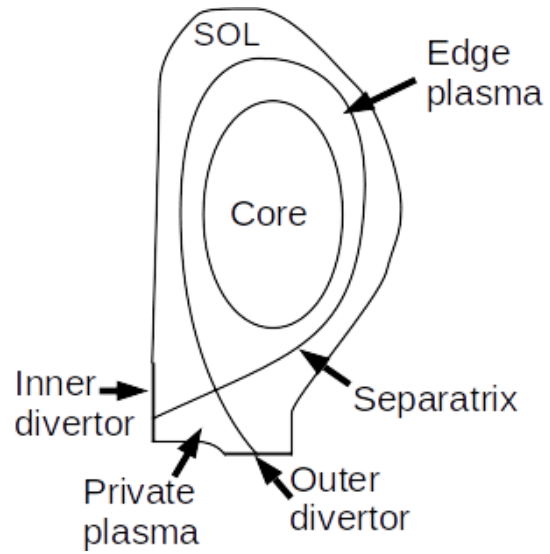


Figure 4: Schematic of a poloidal cross-section of a divertor tokamak indicating magnetic field geometry.

1.4.2 Operational and Safety Hazards of Dust

Disruptive events in tokamak plasmas are known to be correlated with, and occasionally caused by, the presence of dust. This has been directly observed by visual cameras in cases where the large forces acting on the vacuum vessel dislodge massive numbers of particulates. In JET, a comprehensive review of a decade of intentional and unintentional disruptions found the root cause of approximately 2% of these were dust, with the majority (18%) being due to impurity control problems [65]. Additionally, there is evidence to suggest that nano-particles can help facilitate multifaceted asymmetric radiation from the edge which can be involved in the chain of events leading to disruptions [71]. In experiments conducted with long pulse duration at the large helical device [72] and Tore Supra [73], it was found that sudden ejections of dust in the plasma impede stable operation. Finally, it has been noted that during the beginning of the discharge when the plasma is particularly vulnerable, the presence of dust can hinder start-up and lead to premature termination [62]. Dust is therefore an important contributor to the cause of some disruptions, particularly when allowed to alter the impurity concentration unchecked.

Additionally, the composition of dust can pose a more direct threat to worker and public safety. Beryllium has a long history in nuclear applications as a neutron reflector because of its low atomic mass and strong cross section for neutron scattering. The principal concern with using beryllium in a nuclear reactor however is beryllium ingestion leading to chronic beryllium disease as a result of its latent toxicity [74]. This risk is worsened by the threat of creating beryllium dust which can easily be inhaled during a vacuum failure event [70]. Graphite similarly has been a popular moderator material in nuclear fission power stations and is attractive for fusion applications because of its low mass and absence of a liquid phase. The propensity for chemical reactivity of carbon presents its own difficulties however, since it readily forms organic compounds through its reaction with hydrogen isotopes [75]. This is also the case for tritium, leading to a large portion of the fuel being retained by carbon dust particles which then pose a radiological hazard [76]. Both these issues are further complicated during a reactor leak as the region of contamination becomes extended and leaves the facility vulnerable to explosions resulting from fine hydrocarbon dust and hydrogen reacting with the oxygen in air [66]. For ITER and the nuclear regulators in France, the radioactivity of carbon dust was deemed to be prohibitively limiting, permitting fewer than 250 discharges were graphite to be used across every PFC, as summarised by figure 5 [67, 68]. Detailed considerations of the risks have led to the imposition of dust inventory limits for ITER to avoid the most dangerous consequences of dust [77]. Clearly, the production and survival of dust in Magnetically Confined Fusion (MCF) devices is a considerable concern which warrants a detailed understanding of their behaviour and origin.

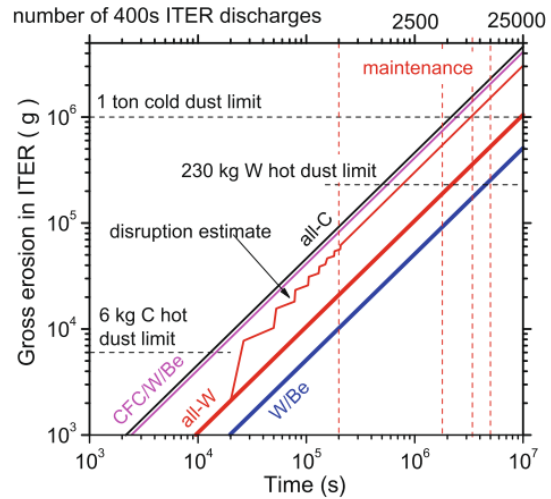


Figure 5: Times required to reach dust safety limits in ITER for the initial design with carbon fibre composite, all-C (black line), W and Be (magenta line), all-W (red line) and W divertor with Be first wall (blue line) PFC designs. Reprinted from [68] with permission from Elsevier.

1.4.3 Sources of Dust in Tokamaks

Dust production in tokamaks occurs as a result of the harsh heat loads which are placed on the exhaust region (divertor) and PFCs in combination with uncontrolled plasma events which redirect excessive stored plasma energy to vulnerable areas. The leading mechanisms for dust production are unipolar arcing, sputtering and melting of the surfaces [78]. An amount of the dust produced in this way is therefore anticipated to exist in the liquid state as molten metal droplets, in contrast to solid dust found in most other contexts.

Arcing refers to the short duration, high current electrical discharges that spark between the plasma and material surfaces predominantly during unstable events in the embedding discharge when initiating the plasma. Two types of arc tracks have been observed distinguished by their interaction with an externally applied magnetic field as unmagnetised and magnetised arc tracks [81]. Unmagnetised arc tracks are unlikely candidates for generating dust as they happen in glow discharges and follow random walks that cause only superficial damage. Eventually, as a result of the magnetic field generated by the arcs current, the arc contracts leaving a small cathode spot ($\sim 10\mu\text{m}$) which is cooled gradually by evaporation and melting [78]. Figure 6 shows an arc crater on W-coated graphite generated in a laboratory experiment demonstrative of this phenomena.

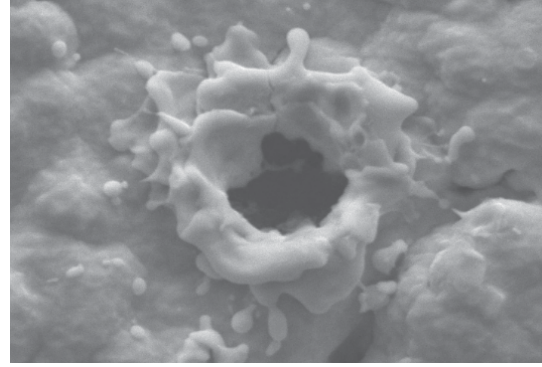


Figure 6: Arc crater on W-coated graphite cathode [79]. © IOP Publishing. Reproduced with permission. All rights reserved.

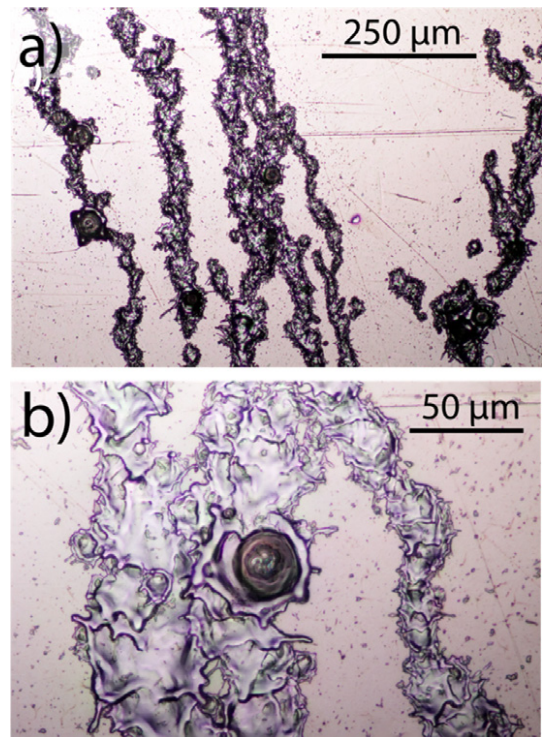


Figure 7: Optical microscopy of diagnostic mirror in JET with arc tracks (a) and evidence of melting due to arcing (b). Reproduced under creative commons act [80].

Magnetised arc tracks however cause substantial localised heating and erosion as the potential accelerates electrons from small surface protrusions into the plasma [81]. The deformities of the arc crater provide a protrusion for a subsequent arc to form around, producing tracks with dimensions in the range $10 - 200\mu\text{m}$ wide, $10\mu\text{m}$ deep and $\sim 10\text{mm}$ long [78, 81]. Examples of the damage to diagnostic mirrors caused by magnetised arc tracks in JET are shown in figure 7. The retrograde motion of arcs in the opposite sense to the Lorentz force is well documented and occurs due to the enhancement of the arc density and heating from the alignment of the two magnetic fields [78].

Though particle emission from arcs has not been observed directly in a tokamak plasma, when the process is replicated in laboratory experiments particle ejection is seen with droplets being up to a few micrometres in diameter [79], dependent on the surface composition. Tungsten coated graphite has been found to eject particles at modest velocities of $\sim 20\text{ms}^{-1}$ whilst pure graphite has a much higher ejection velocity $\sim 100\text{ms}^{-1}$, with no preferential ejection angle being observed [79]. Rudi-

mentary considerations of particle heating suggest that some massive remnants will survive long enough to deposit impurities into the core plasma [79].

The most dramatic dust production mechanism is the bulk melting of PFCs. In present day tokamaks, sophisticated power handling at the divertor means melting of tungsten (which at standard pressure takes place at 3422K) only occurs during off-normal transient events such as disruptions and Edge Localised Modes (ELMs) [78]. These intense plasma events melt and vaporise target material, generating a freely expanding neutral vapour which is ionised by the plasma. This vapour plasma reduces the energy flux somewhat and re-distributes the energy over the surrounding area whilst the melt layer underneath moves freely as soon as it is created due to the influence of various forces including gravity, surface tension, electromagnetism, plasma momentum and ablation recoil [83].

Thermal shocks of the intensity expected from mitigated ITER ELMs induce fatigue in tungsten and the developments of cracks that roughen surfaces and expose edges to further melting and droplet production [84]. Dedicated experiments on JET to study the effect of unmitigated ELMs in ITER demonstrated the potential for transient melting using specially designed divertor modules [55, 82, 85]. The special tungsten lamella, shown in figure 8, was intentionally raised at an angle to expose it to higher heat loads over several ELM discharges, leading to the formation of a melt-layer which moved under Lorentz forces [55]. Indirect evidence of droplets of diameter $\sim 80 - 100\mu\text{m}$ was found from the tungsten spectroscopy and vacuum-ultraviolet emission [55] that were also observed to move, coalesce and grow under successive plasma pulses [85]. Similar experiments of controlled melting of a tungsten rod on the ASDEX Upgrade tokamak found $\sim 100\mu\text{m}$ droplet production [86]. For ITER, the unavoidable occurrence of unmitigated disruptions are predicted to produce extreme heat fluxes capable of melting the tungsten divertor and inducing the chaotic release of tungsten droplets into the plasma.

The lower melting temperature of beryllium (1560K at standard pressure) makes it more vulnerable to such large, unanticipated heat fluxes. This has already led to the unintentional melting of the Be limiters in JET on a number of occasions [82]. Gradual melting following explorations of the power handling limits were found to produce melt layers similar to those of tungsten. These layers moved principally under Lorentz forces until reaching the material edge where a jet of streaming droplets was expelled into the plasma. In another unplanned experiment, disrup-

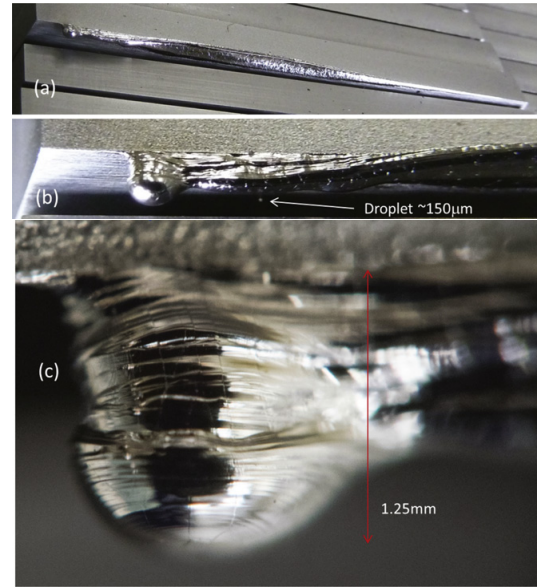


Figure 8: Melting of a tungsten divertor lamella (a) with a visible droplet (b) and evidence of material layering (c) [82]. © IOP Publishing. Reproduced with permission.

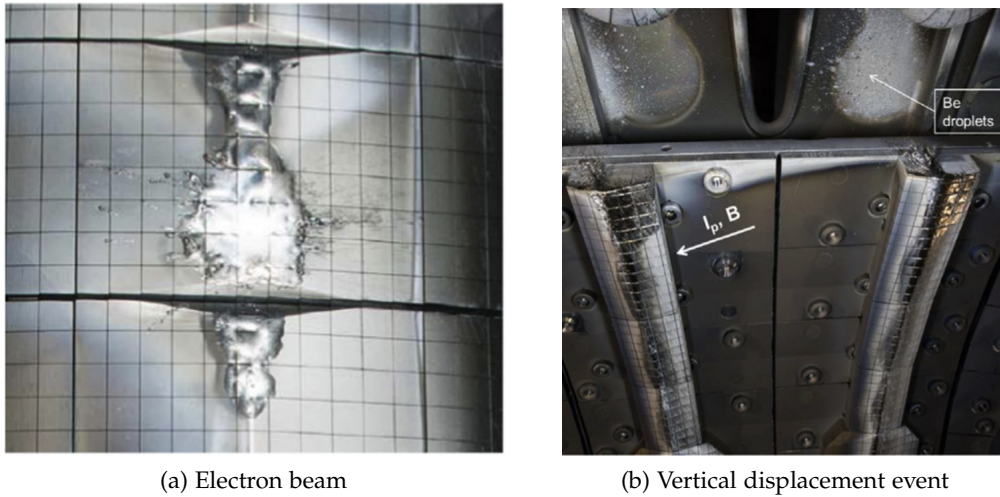


Figure 9: Images of unintentional melt damage of beryllium limiters by electron beams (9a) and during a vertical disruption (9b) creating beryllium droplets [82].
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tions leading to vertical displacement events were observed to quickly deposit 50 – 90% of the plasma thermal and magnetic energy on the upper dump plates [82], melting them and spraying molten beryllium on surrounding surfaces, see figure 9. This prompted a recent in depth predictive simulation of the implications for dust survival and redistribution following ITER disruptions [87]. A final serious and considerable source of dust is triggered by accidental intense local heating which can, for example, be caused by high energy electron beams. This releases huge amounts of energy in a relatively small area, creating melt patches (see figure 9) and ejected droplets with velocities of up to $\sim 10\text{ms}^{-1}$ [82] in this case. The uncontrolled exposure of PFCs to the plasma instabilities of future tokamaks has been predicted to not only alter the surface topology of the PFCs, affecting plasma behaviour of subsequent shots, but also to release metal droplets into the plasma that form a major contributor to in-vessel dust inventory.

A common source of solid dust is the flakes of embrittled material and co-deposited layers which develop over time and that become re-mobilised in subsequent plasma discharges. Energetic particles originating from the fusion plasma that are incident on the material PFCs erode material through elastic energy transfer to surface atoms in an atomic process known as sputtering [78]. The atoms released in this way rapidly ionise and are redeposited in other areas through a process called recycling, leading to the formation of layers tens of microns thick. For graphite especially, these layers can be weakly adhered and become detached during later discharges, allowing them to be transported further into the plasma, eventually being recovered as small flakes of material [88]. This is the dominant dust production mechanism for tokamaks with mostly graphite based PFCs, with average sizes in the range $(2.8 \pm 2.4)\mu\text{m}$ [66, 89]. Carbon flake detachment is further enhanced with the absorption of water molecules, which causes them to swell to thickness of over $100\mu\text{m}$ and later leave the surface [66]. An alternative source of flakes in tokamaks is the erosion of coatings such as tungsten on carbon fibre

composite tiles [90] or boron following wall conditioning [91]. These coatings can become brittle through regular plasma operation making detachment increasingly likely. The pieces resemble delaminated segments of the coating, with dimensions of up to $\sim 300\mu\text{m}$ in breadth and $\sim 30\mu\text{m}$ in depth [91]. The study of these creation mechanisms, combined with direct measurements of the creation of particulate matter, provide an integrated understanding of the dust life-cycle in a tokamak.

1.4.4 *Dust Diagnosis, Treatment and Modelling*

A variety of diagnostic methods have been employed for monitoring dust in tokamaks. Primarily, dust particle generation from PFC destruction is observed routinely by fast imaging visual cameras [92–94], accompanied in some instances by illumination via laser light scattering [94–96]. Estimates of velocities from dust tracking in two dimensions has revealed a log-normal velocity distribution with a mean velocity of $\sim 50\text{ms}^{-1}$ and maximum velocities of typically $\sim 200\text{ms}^{-1}$ [93, 97–100]. This is also in approximate agreement with velocity measurements from more sophisticated 3D reconstruction using stereoscopic camera views [97, 100]. Specialised modules for introducing extrinsic pre-characterised dust have provided evidence of re-mobilisation and correlated measurements with the effect on core impurity density [94]. However, measurements of dust in-situ remain challenging due to the high mobility of dust and the limited availability of dedicated diagnostics.

The majority of research has focused instead on the detailed analysis of the size distributions, morphology and composition of dust particles collected through vacuuming, filters or adhesive pads during shut down periods [80, 90, 91, 101–112]. Through the use of scanning electron microscopy and transmission electron microscopy it is known that dust sizes typically follow a log normal distribution with a mean diameter of between $\sim 1 - 10\mu\text{m}$ and a range between $10\text{nm} - 100\mu\text{m}$ [62, 106–108, 110]. Notably in Alcator C-Mod and in JET, the distribution of dust sizes was found to be far larger after melting experiments, with the most probable size being $\sim 50\mu\text{m}$ in ASDEX, the largest particles being almost 0.5mm [91] and dust sizes between $10\mu\text{m}$ and $50\mu\text{m}$ seen in JET [90, 111, 112]. Broadly speaking, dust is recovered either as re-solidified metal droplets which appear as splashes and spherical particles, see figure 10, figure 11 and figure 12, or as flakes and other irregularly shaped particles, shown in figure 13 [90, 91, 102, 103, 105, 107, 108, 110–112]. A massive diversity of dust compositions have been identified principally dependent upon the material design choice of the PFCs of a given tokamak. In ASDEX upgrade for example, spectroscopic techniques including energy dispersed x-ray, Raman and infrared absorption spectroscopy, identified homogeneous, tungsten dominated, porous spheroids, inhomogeneous, boron-dominated, tungsten conglomerate spheroids and carbon-dominated, inhomogeneous, irregularly shaped, porous flakes [107]. The rich variety of sizes and material properties of these particles incurs further difficulties when modelling them.

Though in present day MCF devices the quantities of dust are never prohibitively large, it is expected in ITER they will need to be actively removed to stay within the prescribed safety limits [68]. A number of mechanisms have been suggested

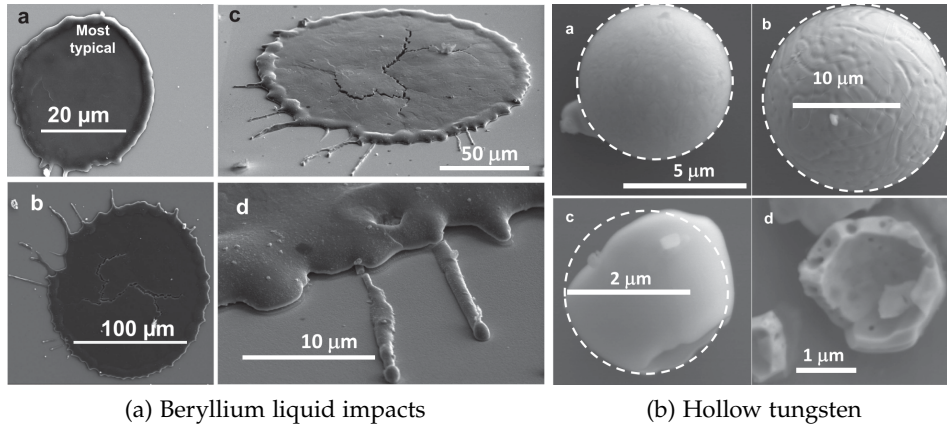


Figure 10: Micrographs of beryllium from inner wall (10a) and hollow tungsten spheres (10b) found in JET [111]. © IOP Publishing. Reproduced with permission.

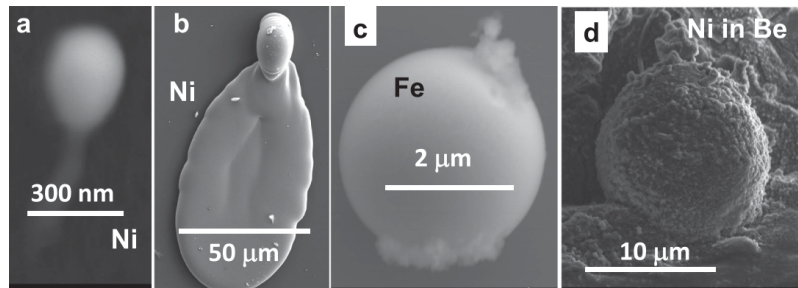


Figure 11: Nickel (inconel) (a,b), iron (c) and inhomogeneous nickel-beryllium (d) droplets recovered from JET [111]. © IOP Publishing. Reproduced with permission.

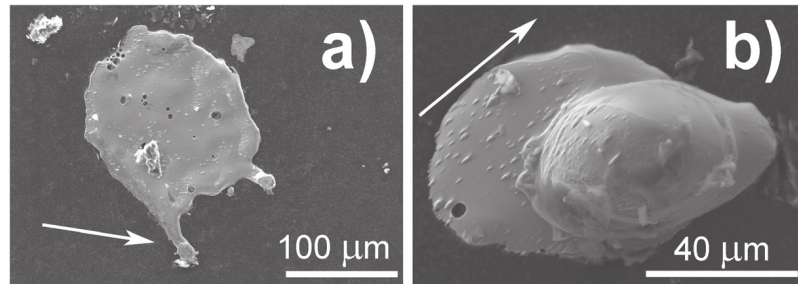


Figure 12: Molten steel droplet splashes (a,b) observed on limiter surface in TEXTOR, with arrows indicating direction of plasma flux [110]. © IOP Publishing. Reproduced with permission.

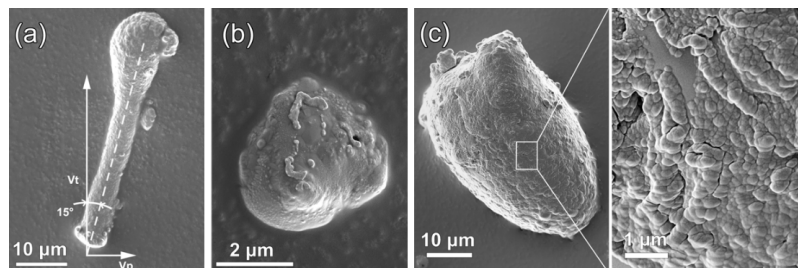


Figure 13: Irregularly shaped molybdenum and stainless-steel particles (a-c) recovered from EXTRAP T2R by adhesive tape [106]. © IOP Publishing. Reproduced with permission.

for this purpose which take advantage of the unique properties of dust particles in plasmas. Cyclone vacuum cleaning is a method used extensively on JET and other tokamaks during normal shutdown procedure for the purposes of cleaning and dust characterisation. However, the withdrawal of the divertor cassettes under atmospheric pressure through this approach will regrettably incur shutdown times of around 6 months [113]. An alternative method is to use fluids (liquids or gases) to agitate and flush the surfaces through the safety drain pipes in the divertor floor, though the loss of wall conditioning in this case will require at least a months recovery period [113]. The most promising technique suggests an electro-static conveyor, capable of performing whilst under vacuum and with magnetic fields present [77, 113]. The propensity for dust to accumulate a negative charge is then exploited by igniting a low density and temperature glow discharge in the device and introducing guiding electric fields to siphon them away [113]. In this manner, dust can be evacuated without the need for intervention inside the vacuum vessel.

To address the issues caused by dust, it is necessary to have a thorough understanding of dust behaviour, lifetimes and transport within a tokamak plasma. For this purpose, a range of simulation codes (DTOKS, DUMBO, DUCAD, DUSTT, DustTrack, MiGRAINe) for modelling dust in the SOL plasma of tokamaks have been used to interpret and predict experimental measurements [114–119]. Consequently, some progress has been made in explaining aspects of the experimental observations mentioned previously. Simulations performed with DUMBO were partially successful in reproducing visual camera observations of injected dust in the K-STAR tokamak, but with trajectories travelling toroidally having far shorter lifetimes in the simulations than observed experimentally and with only the free-fall behaviour of large dust accurately modelled [115]. Similarly, the reconstructed 3D trajectories of lithium dust in the NSTX tokamak were accurately replicated by DUSTT simulations when utilising an ad hoc shielding factor of ~ 50 to reduce the plasma heat fluxes and ignoring the small erratic motion of the dust [120]. Comparison of MiGRAINe simulations with observations of carbon and tungsten dust in TEXTOR concluded that trajectories were dictated primarily by their initial conditions when created, having inertia dominated motion [100]. Even with the dust temperature and size being free parameters, the experimental lifetimes here were found to be far longer than simulated lifetimes of even unrealistically large dust grains [100]. In the Mega-Ampere Spherical Tokamak, stereoscopic camera views allowed comparison of DTOKS simulations with 3D trajectories of both intrinsic and injected dust, identifying much greater acceleration of carbon relative to tungsten dust grains that was mirrored by simulations with the dust size again being a free parameter [121]. The limited success achieved so far by dust tracking codes highlights the ongoing need for well controlled experiments and more detailed modelling of dust charging and dynamics. In particular, dust in all of these simulations is assumed to be small enough $\sim 1\mu\text{m}$ to not influence the local plasma conditions, with physical properties similar to those of large samples of the material under standard conditions. Most importantly, the effects of external magnetic fields are entirely neglected in current models of dust charging, heating and drag forces. With the stronger magnetic fields of ITER and the recognition of the presence of large particles, these issues demand fresh attention.

THEORETICAL CONTEXT

*Ich habe das symbolische Rechnen
mit Stumpf und Stil verlernt.*

— Emmy Noether, Letter to Helmut Hasse (April 14, 1932) [122]

2.1 PLASMA SPECIES CHARGE AND INERTIA

By definition, plasmas consist of a number of freely moving negatively and positively charged particles [123, 124]. Though many exotic plasmas exist containing; negative ions, pair-ions and electron-positron pairs, “ordinary” plasmas such as MCF plasmas have only negatively charged electrons and positively charged ions as the principal plasma species and all plasmas under consideration in this thesis follow this standard. The difference in inertia of ions of mass m_i and electrons of mass m_e is a fundamental feature of “ordinary” plasmas and is the origin of many of their characteristics. The mobility of electrons and subsequent description of plasmas is built upon the explicit assumption that the ions mass $m_i \geq m_p = 1836.15m_e$, where m_p is the mass of a proton. This information is a pre-requisite for exploring the diverse range of phenomena exhibited even by these more common plasmas and any later theoretical deviation from this nominal case will be highlighted.

2.2 QUASI-NEUTRALITY

Perhaps the most fundamental concept to plasma physics is that of quasi-neutrality. The origins of the term plasma lies in the identification of a region with no-net electrostatic charge, yet through which electrical currents are conducted [124]. In analogy with a conducting metal, the abundant and unrestricted electrons permit currents to flow through a plasma when a potential difference is applied across it in spite of the overall charge being zero. This property of plasmas arises as a direct consequence of the ability of electrons to rapidly relocate to regions of excess electrostatic potential in an effort to negate it. Consequently, in any sufficiently large volume of plasma, approximate net charge neutrality is expected. The statement of quasi-neutrality is that the difference in the charge density of ions eZn_i of charge state Z and electrons of charge density en_e with number densities denoted as n_i and n_e respectively is negligible or equivalently $|Zn_i - n_e| \ll n_e$ [123, 125].

Extreme caution should be employed when speaking of quasi-neutrality since it is a term with an inherent scale of applicability. Clearly in the immediate neighbourhood of a charged particle instantaneously introduced into a plasma, quasi-neutrality will not hold since the occlusion of the electrostatic force of the charge will only occur over some distance and timescale determined by the mobility and density of electrons. Moreover, it is occasionally incorrectly concluded from quasi-

neutrality that in real physical plasmas $n_i = n_e$ identically. When comparing any reasonably large space containing an undisturbed plasma, $n_i \simeq n_e$ to a high degree of precision, but the typically huge order of magnitude of n_i means this is certainly not equivalent to the statement $n_i - n_e = 0$. Theoretically, this means the gradient of the electric field is undefined in the quasi-neutral region and the macroscopic displacement current is zero. Evidently electrical forces are central to the collective phenomena plasmas exhibit and they cannot be heedlessly thrown away with quasi-neutrality, even though the spatial and temporal average field of individual charges might be almost perfectly obscured by the plasma. The principal challenge is understanding the details of the electric and magnetic fields which arise from the motion of $N \gg 3$ charges in what is effectively an N-Body problem.

2.3 MAXWELL-BOLTZMANN STATISTICS

An approach to addressing the theoretical challenge of characterising a plasma, lies in a statistical treatment of the N body problem leading to a more manageable average description using macroscopic variables [126]. In Maxwell-Boltzmann statistics, classical particles in thermal equilibrium at temperature T have a probability density function, $f(E)$, which varies with the energy, E , of the particle

$$f(E) \propto e^{-\frac{E}{k_B T}}, \quad (1)$$

The energy of each particle of mass m is $E = \underline{p}^2/2m$ where $\underline{p} = (p_x, p_y, p_z)$ is the momentum vector for this particle. The constant of normalisation, C , is determined by integrating equation 1 over all energy states, or equivalently integrating over all particle momenta $\underline{p}$, and requiring that the probabilities sum to one [127],

$$\int f(E) dE = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} C e^{-\frac{p_x^2 + p_y^2 + p_z^2}{2mk_B T}} dp_x dp_y dp_z = C(\sqrt{2\pi mk_B T})^3 = 1, \quad (2)$$

yielding the normalised distribution function,

$$f(\underline{p}) = (2\pi mk_B T)^{-\frac{3}{2}} e^{-\frac{\underline{p}^2}{2mk_B T}}. \quad (3)$$

This can be re-expressed as a velocity probability density through the relationship

$$f(\underline{v}) d^3v = f(\underline{p}) \left(\frac{d\underline{p}}{d\underline{v}} \right)^3 d^3v, \quad (4)$$

where the velocity in Cartesian coordinates is $\underline{v} = (v_x, v_y, v_z)$ and the velocity space volume element is d^3v , meaning

$$f(v_x, v_y, v_z) = \left(\frac{m}{2\pi k_B T} \right)^{\frac{3}{2}} e^{-\frac{m(v_x^2 + v_y^2 + v_z^2)}{2k_B T}}. \quad (5)$$

This final well known equation will be used extensively in both the theoretical grounding and development to come. An important point to note here is that in order for the temperature to be well defined, our collection of freely moving non-interacting particles must be continually interacting in a confined region. In a kinetic treatment of plasmas, the influence of particle motion on other particles is generally modelled as discrete events, often by considering two particles in isolation in an event known as a binary collision. The collisions thermalise any distribution of particle velocities and cause it to tend towards a Maxwell Boltzmann distribution wherein all particles are in thermal equilibrium. Throughout this work, only weakly collisional plasmas are considered. This refers specifically to the regime in which densities are sufficiently low that the mean free path between collisions is larger than all other length scales but where collisions do still occur. This is necessary so that the particle distributions are thermalised ($T_i = T_e$) and can be considered to be well approximated by the Maxwell-Boltzmann velocity distribution function of equation 5 whilst permitting the neglect of collisions within the problem dimensions.

2.4 MOMENTS OF THE DISTRIBUTION FUNCTION

A crucial feature of the framework for describing plasmas through particle distribution functions is the possibility to calculate macroscopic plasma variables from their moments. These fluid variables give condensed information about the global character of the plasma encoded in the distribution function $f_s(\underline{r}, \underline{v})$ of species s , with subscript s used to distinguish quantities related to ions, $s = i$, and electrons, $s = e$ in this work. The k^{th} moment of the distribution function [123], $M_k(\underline{r})$, is defined as

$$M_k(\underline{r}) = \int \underline{v}^k f_s(\underline{r}, \underline{v}) d^3 \underline{v}, \quad (6)$$

with each integer value of k corresponding to a unique moment M_k providing a different perspective on an aspect of the distribution. The first few of these moments are important for subsequent calculations and are given here for a shifted Maxwell-Boltzmann velocity distribution with a constant mean flow velocity $\underline{U} = (U_x, U_y, U_z)$ as defined by,

$$f_s(\underline{v}, \underline{U}) = \left(\frac{1}{2\pi v_{T,s}^2} \right)^{\frac{3}{2}} e^{-\frac{(\underline{v} - \underline{U})^2}{2v_{T,s}^2}}, \quad (7)$$

where the root mean squared velocity $v_{T,s} = \sqrt{k_B T_s / m_s}$ has been introduced. The zeroth moment of the distribution function trivially returns unity, since it was normalised in equation 2 under this definition. The first moment of the distribution function relates to the particle flux $\underline{\Gamma}_s(\underline{r}, t) = n_s M_1(\underline{r})$, such that the one way flux of particles crossing the x-y plane in the positive z-direction is

$$\hat{\underline{z}} \cdot \underline{\Gamma}_s = n_s \left(\frac{1}{2\pi v_{T,s}^2} \right)^{\frac{3}{2}} \int_0^\infty \int_{-\infty}^\infty \int_{-\infty}^\infty \hat{\underline{z}} \cdot \underline{v} e^{-\frac{(\underline{v} - \underline{U})^2}{2v_{T,s}^2}} dv_x dv_y dv_z, \quad (8)$$

where the number density of plasma species s is represented by n_s . Completing this integral, the one way flux of particles is

$$\Gamma_{s,z}(U_z) = n_s v_{T,s} \left[\sqrt{\frac{1}{2\pi}} e^{-\frac{U_z^2}{2v_{T,s}^2}} + \frac{U_z}{2v_{T,s}} \left(1 + \operatorname{erf}\left(\sqrt{\frac{U_z^2}{2v_{T,s}^2}}\right) \right) \right], \quad (9)$$

which can be used to calculate the rate of transfer of particles through a plane with a mean flow velocity U_z . Setting $U_z = 0$ here gives the expected no flow result of $\Gamma_{s,z}(U_z = 0) = n_s v_{T,s} / \sqrt{2\pi}$. The second moment of the distribution relates to the rate of particle flow and can be used to describe the flow of momentum of the particles using the stress tensor [123]

$$\underline{T}_s = n_s \left(\frac{1}{2\pi v_{T,s}^2} \right)^{\frac{3}{2}} \int m_s \underline{v} \otimes \underline{v} e^{-\frac{(\underline{v} - \underline{U})^2}{2v_{T,s}^2}} d^3 \underline{v} \equiv \begin{pmatrix} T_{x,x} & T_{x,y} & T_{x,z} \\ T_{y,x} & T_{y,y} & T_{y,z} \\ T_{z,x} & T_{z,y} & T_{z,z} \end{pmatrix}. \quad (10)$$

The diagonal elements of the stress tensor provide the linear components of the rate of flow of momentum across a surface, so that in analogy with particle fluxes, the momentum flux parallel to the $\hat{z}$ direction can be written as

$$T_{z,z} = n_s \left(\frac{1}{2\pi v_{T,s}^2} \right)^{\frac{3}{2}} \int_0^\infty \int_{-\infty}^\infty \int_{-\infty}^\infty m_s v_z \cdot \underline{v} e^{-\frac{(\underline{v} - \underline{U})^2}{2v_{T,s}^2}} dv_x dv_y dv_z. \quad (11)$$

This rate of momentum transfer parallel to the direction of particle flow is identically the linear force exerted over the surface, in this case in the $\hat{z}$ direction. Similarly, the off-diagonal elements give shear stresses which relate to the viscosity of fluid mechanics and in the case of free-moving solids, the torques which lead to rotational motion.

2.5 BOLTZMANN RELATION

The electrostatic potential term present in the particle Hamiltonian has so far been ignored as inter particle interactions have been considered small in the case of $k_B T \gg e\phi$. Introducing instead a sufficiently large external charge at a fixed point into the gas of particles can produce a significant electric potential, $\phi(\underline{r})$, which will alter the macroscopic behaviour of the plasma [124]. For a set of particles that are in thermal equilibrium, a linear electrostatic potential term is added to adapt the Maxwell-Boltzmann Distribution of equation 5 to become

$$f(\underline{v}, \phi(\underline{r})) = \left(\frac{m}{2\pi k_B T} \right)^{\frac{3}{2}} e^{-\frac{m\underline{v}^2 + 2e\phi(\underline{r})}{2k_B T}}. \quad (12)$$

The zeroth moment of this distribution function unveils the boltzmann relation

$$n(\phi(\underline{r})) = n_0 e^{-\frac{e\phi(\underline{r})}{k_B T}} = n_0 e^{\chi(\underline{r})}, \quad (13)$$

describing the attenuation of the density of particles an infinite distance away $n_e(\underline{r} = \infty) = n_i(\underline{r} = \infty) \equiv n_0$ around the external point charge. An important quantity introduced here is the normalised potential,

$$\chi(\underline{r}) \equiv -\frac{e\phi(\underline{r})}{k_B T}, \quad (14)$$

defined as the ratio of the potential to the thermal energy. The minus sign included in the definition means positive normalised potentials are generated by negative charges, which is a more appropriate definition for dust in plasmas as shown later.

This derivation has an insightful physical interpretation in terms of the influence of a dust particle in an "ordinary" plasma. A variation in potential, such as might be caused by the negative equipotential surface of a solid boundary or dust particle, causes a reduction in the density of similarly charged particles in its vicinity in accordance with equation 13. For electrons surrounding negatively charged dust, this causes a reduction in their density relative to particles an infinite distance from the disturbing potential, as many have insufficient kinetic energy to remain close to the surface thus reducing their density.

2.6 DEBYE-HÜCKEL POTENTIAL

Maintaining a broad definition of plasmas, it is useful to acknowledge the origins of some of the foundational theory of plasma physics arising through studies of electrolytes [128]. Electrolytic solutions are close cousins of plasmas as they consist of free moving charged and neutral particles, albeit at the significantly higher densities of a fluid. When considering collections of charges in electrolytic solutions, it was recognised at an early stage that only for the lowest concentrations was it possible for the solution to be described ideally, ignoring interactions between ions [128] as was the approach of section 2.3. This breakdown of the approximation $k_B T \gg e\phi$ is clearly invalidated by the empirical observations of higher density electrolytic solutions where mutual interactions of ions become important. A theory to address this discrepancy was subsequently developed by Debye and Hückel who considered a single charged particle introduced in a collection of charged particles with spherical symmetry about the charges centre. Utilising the Boltzmann relation of equation 13 and Poissons equation [124], $\nabla^2 \phi(\underline{r}) = -\rho_c(\underline{r})/\epsilon_0 \epsilon_r$, with relative permittivity ϵ_r , the charge density is expressed as $\rho_c = en(\phi)$ to give the Poisson-Boltzmann equation

$$\nabla^2 \phi(\underline{r}) = -\frac{1}{\epsilon_0 \epsilon_r} \sum_s en_s e^{-\frac{eZ_s \phi_s}{k_B T}}, \quad (15)$$

where an arbitrary charge state Z_s of ions of species s has been permitted. Since the charges are weakly interacting, the limit $eZ_s \phi_s \ll k_B T$ is taken, allowing the Maclaurin series $\exp(x) \simeq 1 + x + \mathcal{O}(x^2)$ to be invoked, yielding

$$\nabla^2 \phi(\underline{r}) = -\frac{1}{\epsilon_0 \epsilon_r} \sum_s en_s \left(1 - \frac{eZ_s \phi_s}{k_B T}\right). \quad (16)$$

The first term is discarded by recognising that $\sum_s en_s = eZn_i - en_e \simeq 0$ through quasi-neutrality as described in section 2.2. At this point, what remains is a Helmholtz equation of the form

$$\nabla^2 \phi(r) = \frac{1}{\lambda_d^2} \phi(r), \quad \lambda_d^2 = \frac{\epsilon_0 \epsilon_r k_B T}{e^2 \sum_s n_s Z_s}, \quad (17)$$

in which λ_d is recognised as the Debye length. In an attempt to solve this equation analytically, a general solution using the characteristic-equation method of the form

$$\phi(r) = A_1 \frac{e^{-\frac{r}{\lambda_d}}}{r} + A_2 \frac{e^{\frac{r}{\lambda_d}}}{r}, \quad (18)$$

is introduced, with A_1 and A_2 being coefficients which are fixed by the boundary conditions. As $r \rightarrow \infty$, it must be that $\phi(r \rightarrow \infty)$ is finite, so it is required that $A_2 = 0$. In the opposite limit, of $r \rightarrow 0$ charges experience a potential almost purely due to the Coulomb potential of the charge, Q . The boundary conditions are thus summarised by the limits,

$$\lim_{r \rightarrow \infty} \phi(r) = 0, \quad \lim_{r \rightarrow 0} \phi(r) = \frac{Q}{4\pi\epsilon_0 r}. \quad (19)$$

Taking these two conditions together and applying them to 18 provides the solution

$$\phi(r) = \frac{Q}{4\pi\epsilon_0 r} e^{-\frac{r}{\lambda_d}}. \quad (20)$$

This expression suggests that point charges introduced into a quasi-neutral plasma will behave with a Coulomb like potential for $r \ll \lambda_d$ but at larger distance will be screened out over the characteristic Debye length scale λ_d in a region referred to as the sheath. There are several important caveats to this derivation which highlight the limitations of Debye-Hückel theory in its application to real world plasmas. Most prominently, in Taylor expanding the exponential term of equation 15 and taking only the first two terms, it has been implicitly assumed that $eZ_s \phi_s \ll k_B T$, limiting the applicability of this theory to scenarios where the plasma energy far exceeds the probe potential. This can be re-interpreted as meaning that if the plasma were cold and without thermal motion, the shielding would be perfect and no electric fields would permeate the plasma due to the initial disturbing charge. Additionally, a contradiction has been created in so far as our derivation of the Boltzmann relation assumed an insignificant potential with $k_B T \gg e\phi$ or equivalently particles kinetic energy is far greater than the potential. In reality, the entire sheath cannot be described in this simplistic manner.

2.7 SHEATH STRUCTURE

Though it is apparent from the preceding sections that electrons will respond to local disturbances in potential to obscure their effect over a length scale λ_d , it is still unclear what effect the sheath has on ions. The influence of a surface, which

is charged negatively under "ordinary" plasma conditions, might be expected to accelerate ions provided the screening by electrons does not entirely obscure the surface potential.

The motion of ions is investigated by treating the simpler one dimensional planar case of an equipotential surface in Cartesian coordinates given by the y - z plane at $x = 0$ [129]. Of particular interest is the behaviour of ions during steady state, meaning when the flux of ions is at all points equal and continuous. In this case, the density, $n_i(x)$, and velocity, $U_i(x)$, of ions at a position x can be related to their respective values an infinite distance away from the equipotential surface n_0 , U_0 such that

$$n_0 U_0 = n_i(x) U_i(x). \quad (21)$$

The ions are assumed to be cold and so have a fixed initial energy, meaning the change in energy of ions travelling from infinity is

$$U_i(x)^2 = U_0^2 - \frac{2e\phi(x)}{m_i}. \quad (22)$$

Next, the Boltzmann relation given by equation 13 and Poisson's equation are combined with equation 21 to give

$$\frac{d^2\phi(x)}{dx^2} = -\frac{e(n_e(x) - n_i(x))}{\epsilon_0} = -\frac{en_0}{\epsilon_0} \left(e^{\frac{e\phi(x)}{k_B T_e}} - \frac{U_0}{U_i(x)} \right). \quad (23)$$

This equation can be re-written in a much more elegant form by normalising distances to the Debye length, $\xi = x/\lambda_d$, normalising speeds to the thermal speed $u_0 = U_0/v_{T,i}$ and potentials using $\chi(\xi)$ defined in equation 14 so that

$$\frac{d^2\chi(\xi)}{d\xi^2} = \left(1 + \frac{2\chi}{u_0} \right)^{-\frac{1}{2}} - e^{-\chi}. \quad (24)$$

Multiplying both sides of this equation by $d\chi(\xi)/d\xi$, this equation can be expressed as an integral of the form

$$\int_0^{\xi_0} \frac{d\chi(\xi)}{d\xi} \frac{d^2\chi(\xi)}{d\xi^2} d\xi = \int_0^{\xi_0} \frac{d\chi(\xi)}{d\xi} \left(1 + \frac{2\chi}{u_0} \right)^{-\frac{1}{2}} - \frac{d\chi(\xi)}{d\xi} e^{-\chi} d\xi. \quad (25)$$

The boundary conditions are fixed at the sheath edge ($\xi = 0$) where the quasi-neutral plasma exists such that $\chi = 0$ and where the electric field can be assumed to be zero such that $d\chi/d\xi = 0$. Evaluating the integral under these conditions yields

$$\frac{1}{2} \left(\frac{d\chi(\xi)}{d\xi} \right)^2 = u_0^2 \left[\sqrt{1 + \frac{2\chi}{u_0}} - 1 \right] + e^{-\chi} - 1. \quad (26)$$

Since the left hand side of this equation is a squared quantity, the right side of the equation must be positive. Taylor expanding the right hand side of equation 26, the first non-zero term is the quadratic term

$$\frac{1}{2} \chi^2 \left(1 - \frac{1}{u_0^2} \right) \geq 0. \quad (27)$$

This leaves the final condition [129], that

$$u_0^2 \geq 1. \quad (28)$$

With the normalisation scheme used, this states that the velocity of particles at the point at which the potential is zero, is given by $U_0 = \sqrt{k_B T_e / m_i}$. This derivation highlights the need for a pre-sheath structure around an equipotential surface in a plasma since there must be some mechanism for the ions to accelerate up to a speed U_0 before reaching the sheath edge. Ions are seen to be accelerated to at least the sound speed before entering the sheath, after which they continue to hurtle towards the material surface. Were the ions to have not accelerated to this speed, the sheath edge by definition would be closer to the surface since this defines the region of positive space charge.

EXISTING THEORIES DESCRIBING DUST PLASMA INTERACTIONS

*As far as the laws of mathematics refer to reality,
they are not certain; and as far as they are certain,
they do not refer to reality.*

— Albert Einstein, Prussian Academy of Sciences, Berlin, 27 Jan. 1921 [130]

3.1 SIMPLIFYING ASSUMPTIONS

Dust grains are a focal point for a range of subtle and diverse elementary plasma physics concepts. Matter at solid or liquid density presents a discontinuity in electrical properties which the plasma must accommodate. Charged particles reaching a solid density boundary will either be absorbed or reflected whilst the boundary itself will emit charged and neutral particles into the plasma with fluxes dependent on its material characteristics and the incident primary particles. Invariably, the surface will retain some of the incident charge, extending its influence further into the plasma. Collectively, these effects are theoretically consolidated into macroscopic currents, forces and flows of energy through the boundary that in turn alter material properties in a dynamic cycle. In this manner, the conceptualisation of material surfaces as sources and sinks of particles is a pragmatic way to categorise the intricate codependency of these particle interactions.

While detailed calculations are necessary to correctly tally all the particle fluxes in a microscopic sense, there are some useful and accurate approximations that simplify the problem. For a conducting metal surface in a relatively high temperature plasma, almost all ions and electrons which come into contact with the surface will contribute their charge completely. This is because most sputtered ions have insufficient energy to escape the surface completely and only a small fraction of ions are backscattered [131]. This permits the simplifying assumption that all charged particles that reach the surface transfer their full charge to the material. The mass contribution is more ambiguous as a significant number of neutralised particles may re-enter the plasma through a process known as surface recombination, an obvious example for this being the neutralisation of atomic hydrogen on plasma facing components in a tokamak [132]. Alternatively, the produced neutral may also be trapped by the material [131], leading to the formation of co-deposited layers as mentioned in Section 1.4.3.

Another approximation used for conducting dust grains is the absence of internal currents or temperature gradients. The validity of approximating dust grains as perfect thermal and electrical conductors can be demonstrated for tungsten from its electrical and thermal resistivity. The timescale for charge relaxation, τ_c , is calculated for the potential difference between two faces of an ideal parallel plate capacitor, $\tau_c = \epsilon_0 \eta$, with resistivity η . At room temperature (293K), the resistivity

of tungsten is $\eta = 56n\Omega m$ [133], meaning that the timescale is extremely rapid $\tau_c \sim 4 \times 10^{-18} \text{ s}$. Even at higher temperatures and for less conductive materials like graphite, the charge relaxation time is much faster than any other timescale. The thermal conductivity of tungsten, $115 \text{ W m}^{-1} \text{ K}^{-1}$ [134], allows a temperature asymmetry of $\Delta T = 2\Xi a_d / 115 \text{ K}$ to be maintained for heat flux Ξ and a dust grain of radius a_d . For typical heat fluxes in the SOL of a tokamak being $\Xi = 10 \text{ MW m}^{-2}$, this yields a temperature difference of $\Delta T = 1.7 \times 10^4 (2a_d) \text{ K}$ over a dust grain of width $2a_d$. Even at higher temperatures when thermal conductivity can decrease by an order of magnitude, a large particle of width $2a_d \simeq 1 \text{ mm}$ would sustain only small temperature gradients of $\sim 10 \text{ K}$. From these estimations, it is appropriate to approximate metal dust grains as being isothermal and forming equipotential surfaces as expected of highly conductive bodies at equilibrium.

When modelling dust particulates in plasmas, it is important to distinguish between the circumstance in which they are mutually interacting and when they exist effectively in isolation within the plasma. This can be quantified through assessing the number of dust particles capable of interacting with one another within a given volume. In analogy with the plasma parameter, a dust parameter $\Lambda_d = 4\pi\lambda_d^3 n_d / 3$ is defined, with the dust number density n_d . A dusty plasma is then associated with $\Lambda_d \gg 1$, whilst $\Lambda_d \ll 1$ refers to isolated, non-interacting dust grains. In the first case, referred to as a dusty or complex plasma, their presence adds a layer of detail to plasma behaviour. Dust contributes another charged species to the plasma and alters its elementary characteristics with the dust in some cases forming a regular structure or dusty plasma crystal [135]. In this way, early research of dusty plasma physics has uncovered an array of dusty plasma phenomena with parallels in classical plasmas such as dust acoustic waves [136]. The enhanced behaviour of dusty plasmas itself constitutes a field of study with many peculiar observations [137–140]. The focus here however will be purely on the intricacies of plasma interaction with individual dust particles, where $\Lambda_d \ll 1$.

To constrain the problem further, approximations about externally applied magnetic fields are required. The influence of magnetic fields on dust is quantified through the magnetisation parameter $\beta_i \equiv a_d / \rho_{T,i}$, which is the ratio of the dust radius to the thermal gyro-radius of ions, $\rho_{T,i} = mv_{T,i} / eB$, travelling at the thermal velocity, $v_{T,i}$, perpendicular to a magnetic field of magnitude B . In the SOL of a tokamak where $B = 2 \text{ T}$ and $T_i = 10 \text{ eV}$, hydrogen ions have $\rho_{T,i} = 161 \mu\text{m}$ such that for small dust grains $a_d = 1 \mu\text{m}$, the ion magnetisation parameter is $\beta_i = 0.006$. Unless otherwise stated, magnetic fields are ignored by only considering small dust in weak magnetic fields with $\beta_i \ll 1$. Extending existing theories to incorporate the dependence of plasma particle fluxes on magnetic fields through the parameter β_i will be the central topic of this thesis.

3.2 DUST CHARGING BEHAVIOUR

A long-standing theoretical problem fundamental to understanding dust motion in plasmas is the determination of the floating potential of a conducting object immersed in a plasma [141]. Knowledge of the total surface charge of a dust grain is necessary to understand the most important effects on its motion, for example

the magnitude of the Lorentz force. The time dependence of this surface potential relative to timescales affecting the dust is negligible, since the charging timescale is determined by the mobility of electrons which is approximated by the plasma frequency $\sim 5 \times 10^{10} \text{ Hz}$ in a tokamak plasma density of $n_e = 10^{18} \text{ m}^{-3}$. For this reason, focus is given to determining only the normalised, equilibrium surface potential, defined as

$$\chi_a \equiv -\frac{e\phi_a}{k_B T_e}. \quad (29)$$

This equation is identical to equation 14 with the electrostatic potential evaluated at the surface of the dust grain, $\phi_a = \phi(r = a_d)$, for the electrons.

The universal method for establishing closed form expressions for this kind of problem relies upon an implementation of Kirchoff's current law. This prescribes that the sum of the currents to the dust grain are zero, such that

$$0 = \sum_k I_k(\chi_a), \quad (30)$$

where $I_k(\chi_a)$ are the current sink and source terms to the surface due to phenomena k . The current terms arising from the different plasma species are formulated through the first moment of the distribution function, by integrating the distribution function for a particular species, $f_s(\underline{r}, \underline{v})$, over the spherical surface of the dust grain and taking the product with the charge, Ze , and density n_s of the corresponding charged particle source,

$$I_s = Zen_s \int \int \underline{v} f_s(\underline{r}, \underline{v}) d^3v dS. \quad (31)$$

The normalised surface potential, χ_a , is determined from equation 30 numerically via an appropriate root finding algorithm and in special cases re-expressing the equation in terms of the Lambert W function. The complex dependency of the currents on the disturbed plasma close to the equipotential dust surface make solving equation 31 difficult. To attain tractable analytic expressions, the regime of interest must be isolated by limiting dependencies including the plasma drift velocity U , the Debye length relative to the dust size, magnetic field and dust geometry. Most simple expressions for the equilibrium potential necessitate approximations of all but one of these parameters. The solutions to these equations and the limiting approximations necessary to retain a symbolic expression for different combinations of source and sink terms are explored in the following sections.

3.2.1 Small Stationary Dust, Orbital Motion Limited

The Orbital Motion Limited (OML) theory is the primitive dust charging model upon which many other variants are adapted from. Thematically for dusty plasmas, it was derived independently first for probes in laboratory plasmas [141] and later dust in space plasmas [142]. Critically, this model is valid only for small dust grains $a_d \ll \lambda_d$ in the absence of externally imposed electromagnetic fields and which are stationary with respect to the plasma.

The model is constructed by considering the limiting impact parameter, b_c , of attracted charged particle orbits approaching from infinity such that a particle is absorbed by the dust grain with grazing incidence, as shown in figure 14. Conservation of angular momentum, L , is used to express this critical impact parameter, b_c , in terms of the ratio of its initial velocity an infinite distance away, $v(r = \infty) \equiv v_\infty$, to the velocity when it grazes the surface $v(r = a_d) \equiv v_a$,

$$L = m_i b_c v_\infty = m_i a_d v_a. \quad (32)$$

The conservation of total energy, E , of the particle relates the exchange of initial kinetic energy with the final electrostatic potential energy at the surface ϕ_a ,

$$E = \frac{1}{2} m_i v_\infty^2 = \frac{1}{2} m_i v_a^2 + Ze\phi_a. \quad (33)$$

By combining equation 32 and equation 33 the dependence of the impact parameter, b_c , on the velocity at the surface v_a is eliminated. The effective cross-sectional area for absorption of particles approaching the dust grain from infinity,

$$\sigma_{OML} = \pi b_c^2 = \pi a_d^2 \left(1 - \frac{2Ze\phi_a}{m_i v_\infty^2} \right), \quad (34)$$

is then used to circumvent the complexity in evaluating the integral of equation 31 at the dust surface where the plasma is disturbed. Taking advantage of the spherical symmetry of the problem, the velocity, $\underline{v}$, is transformed to a spherical coordinate system using the differential velocity element $d^3v = \sin(\theta) |\underline{v}|^2 dv d\theta d\psi$ so that the ion current to a negatively charged dust grain is written as

$$I_i = \pi Z e n_i \int_0^{2\pi} \int_0^\pi \int_0^\infty f_i(\underline{r}, \underline{v}) b_c^2 |\underline{v}|^3 \sin(\theta) dv d\theta d\psi, \quad (35)$$

recognising that $|\underline{v}| \equiv v = v_\infty$ as this integral is evaluated over a surface far from the dust grain. At the circular surface an infinite distance from the dust grain, it is appropriate to use the Maxwell-Boltzmann speed distribution,

$$f(v) = \left(\frac{m}{2\pi k_B T} \right)^{\frac{3}{2}} e^{-\frac{mv^2}{2k_B T}}, \quad (36)$$

similar to equation 5. Evaluating the integral of equation 35 using equation 36, the ion current is

$$I_i = 4\pi a_d^2 Z e n_i \sqrt{\frac{k_B T_i}{2\pi m_i}} \left(1 - \frac{Ze\phi_a}{k_B T_i} \right). \quad (37)$$

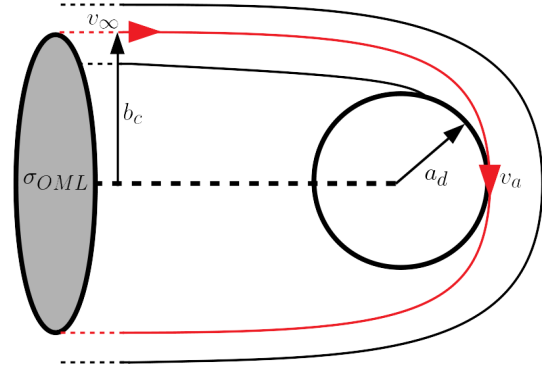


Figure 14: Schematic of ion trajectories that are collected over a circular area, σ_{OML} , for all impact parameters smaller than, b_c , with initial velocity, v_∞ , and final velocity, v_a .

Introducing the temperature ratio $\tau = T_i/T_e$, this is written in normalised units as

$$I_i = I_{0,i} \left(1 + \frac{Z\chi_a}{\tau} \right), \quad \chi_a \geq 0. \quad (38)$$

Here, $I_{0,i} \equiv \pi a_d^2 Z e n_i \langle v_{T,i} \rangle$ is the ion thermal current which is the current passing through a spherical surface of area $4\pi a_d^2$ due to a one way flux $\Gamma_i = n_i \langle v_{T,i} \rangle / 4$ of particles each with a charge Ze and where $\langle v_{T,i} \rangle = \sqrt{8k_B T_i / \pi m_i}$ is the mean velocity magnitude. This formulation most clearly demonstrates that the expected thermal current, $I_{0,i}$, through a spherical surface at the plasma potential is enhanced by a factor $(1 + Z\chi_a/\tau)$ due to the dust grain potential $\chi_a > 0$. For $T_i > T_e$, this effect is less severe as the ions have a greater kinetic energy relative to the potential, reducing the cross section of collection of equation 34.

For electrons approaching a repulsive potential, the critical impact parameter equals the dust radius $b_c = a_d$ since even particles with smaller impact parameters may be reflected if they don't have sufficiently large kinetic energy to overcome the potential barrier, $v_\infty \geq \sqrt{2e\phi_a/m_e}$. Instead, consider the result of completing the integral of equation 31 with an electron density determined by the Boltzmann relation given by equation 13. In this case, the electron current is

$$I_e = -I_{0,e} e^{-\chi_a}, \quad \chi_a \geq 0, \quad (39)$$

where $I_{0,e} = \pi a_d^2 e n_e \langle v_{T,e} \rangle$ with $\langle v_{T,e} \rangle = \sqrt{8k_B T_e / \pi m_e}$ defines the electron thermal current.

For a dust grain which attains a positive charge, the arguments used to derive the electron and ion current reverse [140]. Though this is not expected in an "ordinary" plasma as described in section 2.1, electron emission processes discussed later are prevalent for high temperature dust in fusion plasmas and can lead to positively charged dust. In this case, following similar arguments gives

$$\begin{aligned} I_e &= -I_{0,e} (1 - \chi_a), & \chi_a \leq 0, \\ I_i &= I_{0,i} e^{\frac{Z\chi_a}{\tau}}, & \chi_a \leq 0, \end{aligned} \quad (40)$$

for the ion and electron currents to a positively charged dust grain. The OML current is seen to be dependent on whether particles experience an attractive or repulsive potential because of the effect this has on the distribution function. Here, for the sake of brevity, only the solutions for negatively charged dust grains are discussed further.

To determine the surface potential, the final step must be completed and Kirchhoff's law, equation 30, used to equate the currents of equations 38 and 39

$$\frac{Z n_i v_{T,i}}{n_e v_{T,e}} \left(1 + \frac{Z\chi_a}{\tau} \right) = e^{-\chi_a}. \quad (41)$$

Given the quasi-neutrality of the plasma at infinity, $Z n_i \simeq n_e$, and defining the mass ratio as $\mu \equiv m_i/m_e$ this equation is simplified further to become

$$\sqrt{\frac{\tau}{\mu}} \left(1 + \frac{Z\chi_a}{\tau} \right) = e^{-\chi_a}. \quad (42)$$

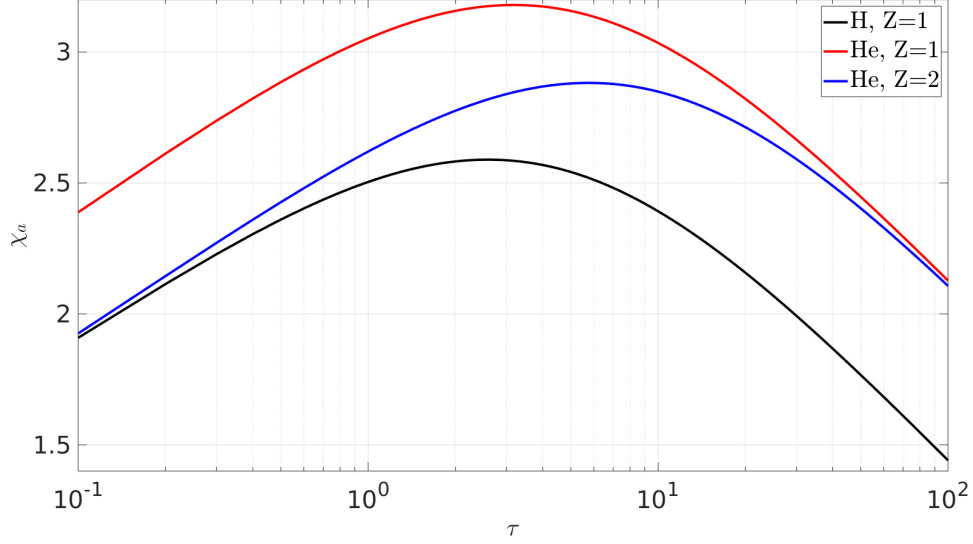


Figure 15: Normalised surface potential, χ_a , as a function of temperature ratio, τ , for dust in hydrogen and helium plasmas with different ionisation Z .

The solution of equation 42 can be determined numerically through root-finding methods or re-expressed as a closed-form solution of the Lambert W function [143]

$$\chi_a = W\left(\frac{\sqrt{\mu\tau}}{Z}e^{\frac{\tau}{Z}}\right) - \frac{\tau}{Z}. \quad (43)$$

Importantly, the OML model predicts that the potential is independent of the size of the dust grain and so supports the extension of dusty plasma physics concepts to similar systems of different length scales. Figure 15 shows the variation in $\chi_a(\tau, Z, \mu)$ for a hydrogen plasma and two helium plasmas with $Z = 1$ and $Z = 2$. A useful reference value to consider is the result for an isothermal, fully ionised hydrogen plasma with $\mu = m_p/m_e \simeq 1836.15$, $Z = 1$ and $\tau = 1$ in which case $\chi_a = -2.504$.

Reviewing the derivation of this model following [144], it hinges on the initial assumption that there exists a critical impact parameter for which a particle will be incident at the spherical surface with grazing incidence. Though this may superficially appear to be a sensible assumption, this can be demonstrated to not be universally applicable through careful examination of the effective potential. Combining equations 32 and 33, the radial velocity v_r is written as

$$v_r(r) = \sqrt{\frac{2}{m_i}(E - Ze\phi(r)) - \frac{L^2}{m_i^2 r^2}}. \quad (44)$$

If a collected ion has $v_r = 0$ at some radius greater than the dust radius $r_a \geq a_d$, then the initial assumption that the limiting orbit is collected with grazing incidence at the surface is contradicted. The range of applicability of OML is therefore defined by the requirement that $v_r(r_a) > 0$ for all relevant particle energies E and

distances r_a . Setting $v_r(r_a) = 0$ and evaluating equation 44 for some radius larger than the dust radius,

$$\frac{2}{m_i}(E - Ze\phi(r_a)) = \frac{L^2}{m_i^2 r_a^2}. \quad (45)$$

The substitution $L = 2m_i a_d^2 (E - Ze\phi_a)$ is made based on conservation of angular momentum,

$$E - Ze\phi(r_a) > (E - Ze\phi_a) \frac{a_d^2}{r_a^2}. \quad (46)$$

For a Maxwellian velocity distribution, this must be the case for all E so that a sufficient condition for $E = 0$ is

$$\frac{\phi(r_a)}{\phi_a} > \frac{a_d^2}{r_a^2}, \quad (47)$$

meaning that the validity of OML relies upon the radial dependence of the potential. For an unshielded electric charge, a Coulomb potential is expected meaning that $\phi(r) \propto \phi_a/r$ and $\forall r \in r \geq a_d$, equation 47 is satisfied. However, from the discussion of section 2.5 and 2.6 it is clear that the screening effect in a plasma will invariably alter this potential in some manner. If instead the potential is approximated by the Debye-Hückel potential of equation 20, for $r \gg a_d$ the comparison becomes

$$\frac{a_d - r}{a_d e^{-\frac{r}{\lambda_d}}} > \frac{a_d^2}{r^2}. \quad (48)$$

This will contradict the condition on the potential as the exponential term will decrease faster than r^{-2} at larger r as first highlighted by Allen et al. [145]. This is best visualised by plotting orbits of the attracted species at a fixed energy with varying impact parameter in the vicinity of a charged sphere for different forms of the electrostatic potential. This is shown in figure 16 with trajectories closest to the critical impact parameter marked in red. It is clear that the grazing incidence approximation is valid for a Coulomb potential but not for a Debye-Hückel potential where some absorption radius exists at $a_d < r$. The consequences for OML of denying the existence of absorption radii is not limited to making it incompatible with Debye-Hückel potentials as this has been shown more comprehensively to prohibit alternative realistic forms of $\phi(r)$ [144].

Another limitation of OML results from electron depletion close to the dust grain surface, which is not accounted for correctly by the Boltzmann relation in equation 39. This has been shown to cause violation of the OML condition of equation 47 for $\tau \leq 1$ [146]. This is true for any ratio of a_d/λ_d so that the potential around even a small body will decrease more rapidly than $1/r^2$ at some large distance.

Despite the self-contradictory nature of the assumptions, OML is a remarkably robust model which provides accurate estimates of experimental measurements of the floating potential of small spherical conductors in collisionless plasmas. Investigations of the currents and floating potentials of Langmuir probes, closely related

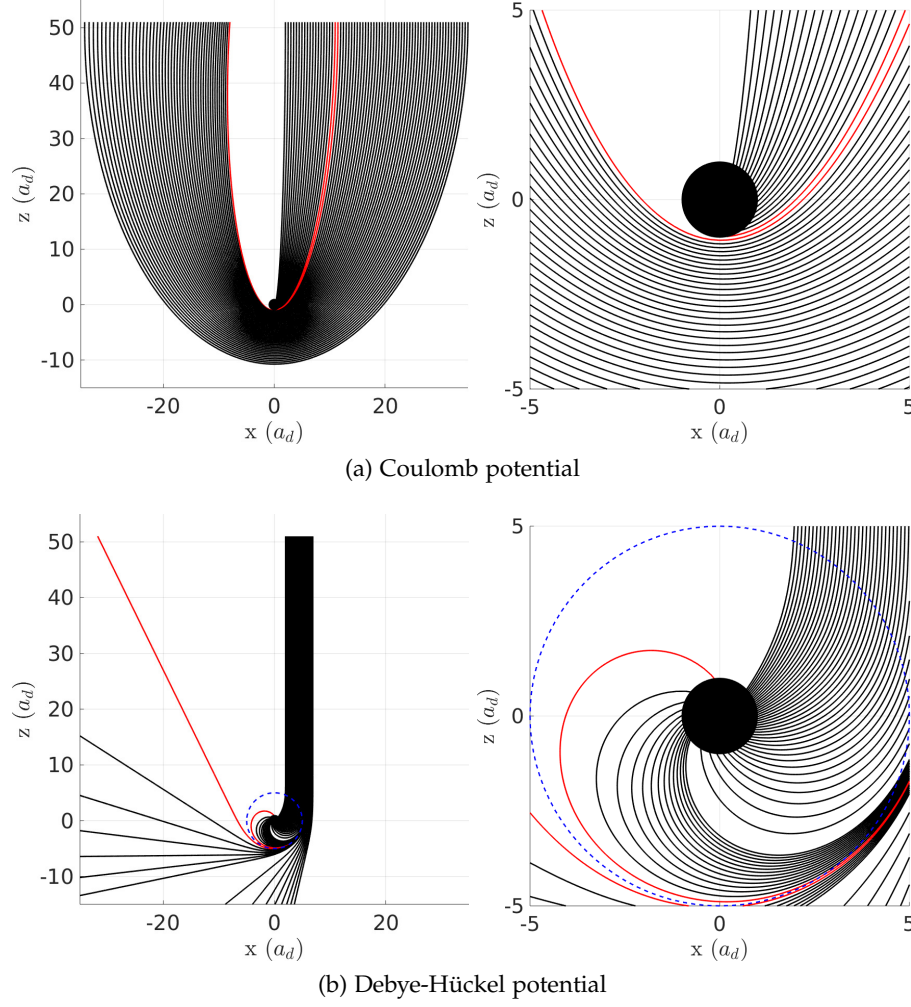


Figure 16: Representative orbits of particles in Coulomb (16a) and Debye-Hückel (16b) potentials showing the validity of the assumption of grazing incidence for potentials that vary more weakly than r^{-2} .

analogues to dust grains, agree well with OML theory [147, 148]. More challenging measurements of the charge of isolated dust grains in similar environments have also observed good agreement with OML theory [149, 150], though even low neutral pressures can influence charging due to ion-neutral collisions, causing departure from OML predictions [151].

The OML theory is perhaps used most profitably as a tool for validating simulation codes [152–155]. When compared with the results of fluid codes with low flow speeds ($U \leq v_{T,i}/2$) [156], magnetic field strengths [152, 155–157] and reasonably small dust size relative to Debye length $a_d \leq \lambda_d$ [153, 154, 158], OML predicts the floating potential to a reasonable approximation. For these reasons, OML is used as the de facto standard for bench-marking simulation results and comparing more involved theoretical treatments.

3.2.2 Large Stationary Dust, Modified OML

To address the flaws of OML, the declaration of the existence of a critical impact parameter and interrelated grazing incidence condition must be re-examined with the awareness of absorption radii. The Modified OML theory [158–160] (MOML) attempts to take such steps to predict the floating potentials for the regime of dust grains much larger than the plasma screening length. The assumptions of MOML about absorption radii is the antithesis of OML as it acknowledges the presence of an effective absorption radius within the sheath. This is formally stated through the assumption that in the limit $a_d/\lambda_d \rightarrow \infty$, known as the thin sheath limit, every ion that passes the sheath edge at a radius r_s will be collected by the dust grain. The mechanics of the OML theory are then used to provide a solution in terms of the ion current at the surface defined by the sheath-edge,

$$\sqrt{\frac{\tau}{\mu}} \left(1 + \frac{Z\chi_s}{\tau} \right) = e^{-\chi_a}, \quad (49)$$

where the potential at the sheath edge is $\chi_s \equiv -e\phi(r = r_s)/k_B T_e$ or equivalently equation 14 evaluated at r_s . The same formulation for the electron current due to a normalised surface potential, χ_a , is maintained, as electrons behave in a similar manner to the OML case as they still experience a repulsive potential. The task remaining is to determine the potential at the sheath edge, χ_s , relative to the dust surface potential, χ_a . The ion current density, J_i , through the sheath is

$$J_i = eZn_i(r = a_d)v_a = eZn_i(r = r_s)v_{se}, \quad (50)$$

where $v_{se} = v(r = r_s)$ is the velocity of ions at the sheath edge. This is true since the explicit assumption of MOML is that the current at the sheath edge is equivalent to the current to the dust surface. Since this is applied in the thin sheath limit, the spherical dust surface and sheath are essentially planar and the ion velocity at the sheath edge will be determined by some Bohm criterion similar to that of section 2.7. However because OML assumes warm ions, it is inappropriate to substitute the cold ion result of equation 28. There is no generalised Bohm condition for $0 < T_i$, so in order to estimate v_{se} particles that enter the sheath are assumed to travel at the sound speed as was shown to be appropriate by the work of Reimann [129]

$$v_{se} = \sqrt{\frac{k_B T_e + \gamma_c k_B T_i}{m_i}}, \quad (51)$$

with γ_c being the ratio of specific heat capacities with a nominal value of $\gamma_c = 5/3$. At the sheath edge, quasi-neutrality holds so that the ion and electron densities are roughly equivalent,

$$n_i(r = r_s) = n_0 e^{-\chi_s}. \quad (52)$$

Combining equations 50, 51 and 52, the ion current density is

$$J_i = eZn_0 e^{-\chi_s} \sqrt{\frac{k_B T_e + \gamma_c k_B T_i}{m_i}}, \quad (53)$$

while the electron current density is simply given by the normal OML expression

$$J_e = en_0 e^{-\chi_a} \sqrt{\frac{k_B T_e}{2\pi m_e}}. \quad (54)$$

Equating the ion and electron current densities of equations 53 and 54,

$$Zen_0 e^{-\chi_s} \sqrt{\frac{k_B T_e + \gamma_c k_B T_i}{m_i}} = en_0 e^{-\chi_a} \sqrt{\frac{k_B T_e}{2\pi m_e}}, \quad (55)$$

which can be rearranged to give

$$\chi_s = \chi_a + \frac{1}{2} \ln \left[\frac{2\pi}{\mu} (1 + \gamma_c \tau) \right]. \quad (56)$$

Replacing the sheath potential, χ_s , in equation 49 with the expression for the sheath potential given by equation 56, yields the final expression

$$\sqrt{\frac{\tau}{\mu}} \left(1 + \frac{Z}{\tau} \left(\chi_a + \frac{1}{2} \ln \left[\frac{2\pi Z^2}{\mu} (1 + \gamma_c \tau) \right] \right) \right) = e^{-\chi_a}. \quad (57)$$

Which in affinity with the OML theory be expressed analytically as

$$\chi_a = W \left(\frac{\sqrt{2\pi\tau(1 + \gamma_c \tau)}}{Z} e^{\frac{\tau}{Z}} \right) - \frac{\tau}{Z} - \frac{1}{2} \ln \left(\frac{2\pi}{\mu} (1 + \gamma_c \tau) \right). \quad (58)$$

The MOML theory applies only in the regime of a thin sheath where the modified Bohm criterion is valid. Consequently, regardless of the size of the dust grain relative to the Debye length, the potential predicted by MOML will always differ to that of OML. Additionally, there is a drawback in having to introduce the factor γ_c which lacks a concrete definition or means of determination and acts as a free parameter. The issues encountered in these theories demonstrate the subtleties in accommodating for intermediate length sheath structures surrounding dust grains when estimating dust surface potential.

3.2.3 Small Moving Dust, Shifted OML

A simple extension to the OML theory known as the shifted OML or SOML theory considers the charging of small dust grains with small relative velocities with respect to the background plasma [161, 162]. The motion of dust within an "ordinary" plasma primarily affects the ion current rather than electron current since small relative velocities will be more significant as compared with the mean thermal ion speed. Additionally, it is more often possible in equilibrium plasmas for the ions to move with an average velocity whilst the electrons remain Maxwellian. Therefore, to accommodate a plasma flowing at a relative velocity, U , the Maxwell-Boltzmann velocity distribution for ions is replaced with a flow shifted Maxwell-Boltzmann distribution of equation 7 in the OML derivation of section 3.2.1. The equations

dictating conservation of energy and angular momentum are left unchanged so that continuing from equation 35,

$$I_i = Z n_i \pi \left(\frac{m_i}{2\pi k_B T_i} \right)^{\frac{3}{2}} \int_0^{2\pi} \int_0^\pi \int_0^\infty b_c^2 v_r^3 e^{-\frac{m_i(v_r^2 + U^2 - 2v_r U \cos(\theta))}{2k_B T_i}} \sin(\theta) dv_r d\theta d\phi. \quad (59)$$

This is simplified by making the substitution $u = \cos(\theta)$ so that the solution to this in terms of the normalised plasma flow velocity $\tilde{u} \equiv U/\sqrt{2v_{T,i}}$ is

$$I_i = I_{0,i} \left(F_1(\tilde{u}) + F_2(\tilde{u}) \frac{Z\chi_a}{\tau} \right), \quad (60)$$

where

$$F_1(\tilde{u}) = \frac{1}{2} e^{-\tilde{u}^2} + \frac{\sqrt{\pi}(1 + 2\tilde{u}^2)\text{erf}(\tilde{u})}{4\tilde{u}} \quad (61)$$

and

$$F_2(\tilde{u}) = \frac{\sqrt{\pi}}{2\tilde{u}} \text{erf}(\tilde{u}). \quad (62)$$

Greater insight into the abstract function $F_1(\tilde{u})$ of equation 61 is gained by comparing it with the one-way flux of particles of equation 9. By making this comparison, $F_1(\tilde{u})$ is then seen to be the flow dependent portion of the charged flux to the dust which is independent of the surface potential. Examining the limit of $\tilde{u} \rightarrow 0$ and noting that

$$\lim_{\tilde{u} \rightarrow 0} \frac{\text{erf}(\tilde{u})}{\tilde{u}} = \frac{2}{\sqrt{\pi}}, \quad (63)$$

equation 60 is seen to reproduce the OML result for the ion current, given by equation 38. Taking the previous expression for the OML electron current, equation 39, the transcendental equation for the potential is written

$$\sqrt{\frac{\tau}{\mu}} \left(F_1(\tilde{u}) + F_2(\tilde{u}) \frac{Z\chi_a}{\tau} \right) = e^{-\chi_a}. \quad (64)$$

The analytic solution to which may once more be written in terms of the Lambert W function

$$\chi_a = W \left(\frac{\sqrt{\mu\tau}}{ZF_2(\tilde{u})} e^{\frac{\tau F_1(\tilde{u})}{ZF_2(\tilde{u})}} \right) - \frac{\tau F_1(\tilde{u})}{ZF_2(\tilde{u})}. \quad (65)$$

The most important caveat to this extension of the OML theory is the assumption of a spherically symmetric potential. The results of various Particle In Cell (PIC) code simulations have demonstrated that the flow of ions past a sphere disturbs the spherical symmetry of the potential and therefore the fundamental assumptions of OML [156]. It is surprising that in spite of this, the predictions of SOML accurately match the results of PIC codes for $\tau \geq 1$ and $\lambda_d \geq a_d$ when the potential distribution is already not spherically symmetric [156].

3.2.4 Large Moving Dust, Shifted MOML

The same procedure for extending OML theory to encompass flow, can be completed for the modified OML theory [159] of section 3.2.2. Taking the SOML current balance of equation 64, and replacing the surface potential, χ_a , on the left hand side of the equation (which represents the potential dependence of the ion current) with the sheath potential, χ_s , of equation 56,

$$\sqrt{\frac{\tau}{\mu}} \left(F_1(\tilde{u}) + \frac{ZF_2(\tilde{u})}{\tau} \left(\chi_a + \frac{1}{2} \ln \left[\frac{2\pi}{\mu} (1 + \gamma_c \tau) \right] \right) \right) = e^{-\chi_a}. \quad (66)$$

This has an analytical solution of the form

$$\chi_a = W \left(\frac{\sqrt{2\pi\tau(1 + \gamma_c\tau)}}{ZF_2(\tilde{u})} e^{\frac{\tau F_1(\tilde{u})}{ZF_2(\tilde{u})}} \right) - \frac{\tau F_1(\tilde{u})}{ZF_2(\tilde{u})} - \frac{1}{2} \ln \left[\frac{2\pi}{\mu} (1 + \gamma_c\tau) \right]. \quad (67)$$

This equation can be applied in the thin sheath limit to estimate the normalised surface potential as a function of normalised flow velocity $\tilde{u}$, temperature ratio, τ and mass ratio, μ , for a given value of γ_c . This is plotted for hydrogen and singly ionised helium as a function of temperature ratio τ and normalised flow velocity $\tilde{u}$ in figures 17 and 18 by solving OML, SOML and SMOML given by equation 42, equation 64 and equation 66 respectively. It is worthwhile noting that in the limit of $\tilde{u} \rightarrow \infty$ the SMOML and SOML solutions converge. This occurs because terms containing $F_2(\tilde{u})$ tend to zero in both these limits, which results in the charged flux to the dust becoming purely geometrical and independent of χ_a . However, the same vulnerability to non-spherically symmetric potentials of SOML also affects SMOML similarly, so only small relative velocities are theoretically permissible, $\tilde{u} \leq 1$.

3.2.5 Orbital Motion Theory

To solve the potential for arbitrary dust sizes, it is necessary to employ Orbital Motion theory (OM) [163], wherein the potential as a function of distance from the surface is determined. Solving OM involves calculating the potential and ion distributions with Vlasov and Poissons' equations, using the Boltzmann relation of equation 13 to describe electron densities, as well as conservation of ion energy and angular momentum. This theory has been used to attain solutions for the potential distribution surrounding perfectly spherical dust grains in collisionless plasmas with a stationary Maxwellian distribution for ions [164].

This theory yields an expression that cannot be expressed in closed form and requires substantial computational expenditure to solve. It provides little additional explanation for the dependence of the equilibrium surface potential of dust on the plasma conditions and dust size and is hence utilised principally for the precise determinations of the full potential distribution. For most purposes, the implementation of OM theory is considered to be exorbitant and for dust tracking codes in particular, the added detail would invoke an unjustifiable computational cost.

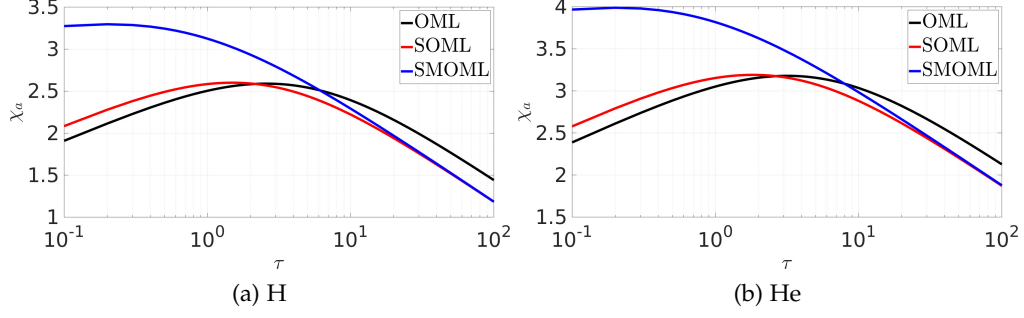


Figure 17: Normalised potential, χ_a , as a function of temperature ratio, τ , for dust in hydrogen (17a) and helium (17b) plasmas with $Z = 1$ and $\tilde{u} = 1$.

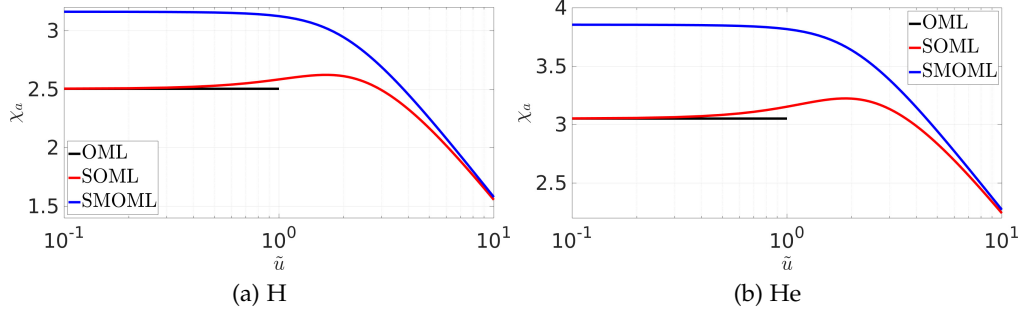


Figure 18: Normalised potential, χ_a , as a function of normalised flow speed, $\tilde{u}$, for dust in hydrogen (18a) and helium (18b) plasmas with $Z = 1$ and $\tau = 1$.

3.2.6 Electron Emission Processes

The leading order effect on the surface potential of dust grains after direct collection of charged fluxes are the thermally driven emission of electrons, the photoelectric emission of electrons and the secondary electrons yielded by electron impact. For dust grains in tokamak plasmas, the current due to electrons liberated through the photoelectric effect is negligible as compared with the plasma currents and is not discussed further for this reason.

3.2.6.1 Thermionic Electron Emission, TEE

Heated conductors emit electrons [165] from the high energy tail of the population that are able to escape the confining potential. This current term is of particular interest for modelling dust in tokamaks with dust tracking codes because of the significant current emitted by metal dust at high temperatures. The emitted current, I_{tee} , from a conducting dust grain at uniform temperature, T_d , due to this process is described by the Richardson-Dushman formula [166–168],

$$I_{tee} = 4\pi a_d^2 A_R \lambda^T T_d^2 e^{-\frac{eW_f}{k_B T_d}}. \quad (68)$$

Here, A_R is the Richardson constant, λ^T is a material specific factor accounting for electron reflection and scattering whilst W_f is the work function of the material.

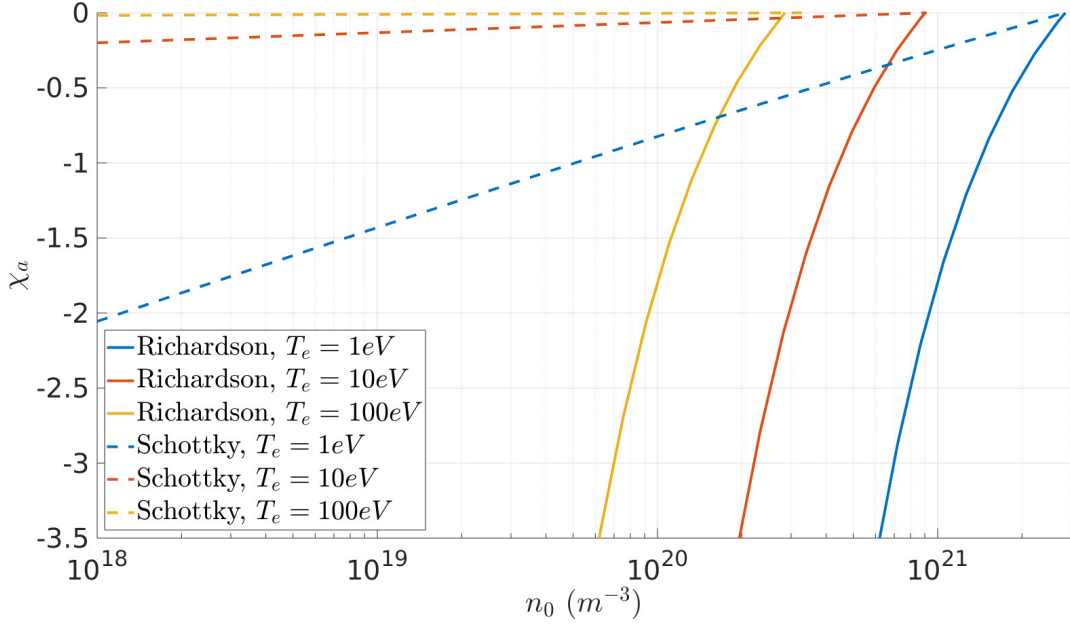


Figure 19: Normalised surface potential, χ_a , of tungsten at $T_d = 3500\text{K}$ as a function of plasma density, n_0 , for different temperatures, $T_e = T_i$, with electron emission. The surface potential is calculated by solving equation 30 with for an OML electron and ion current from equation 40 with the Richardson-Dushman formula of equation 68 for thermionically emitted electrons (solid) as compared with using the Schottky corrected formula of equation 69 for the emitted electrons (dashed).

However, equation 68 makes no account for the influence of the surface potential on the emitted current which can lower or raise the potential barrier for electrons. A correction introduced by Schottky [169] accounting for the electric field at the emitting surface with surface potential ϕ_a ,

$$I_{tee} = 4\pi a_d^2 A_R \lambda^T T_d^2 e^{-\left(\frac{eW_f + e\phi_a}{k_B T_d}\right)}, \quad (69)$$

provides a more accurate model for the electron emission from a charged dust particle. This theoretical description breaks down for nanoscale size objects, as the curvature of the surface becomes important and can cause deviations from the planar theory [170, 171]. Importantly, this current can become larger than the collected electron current for dust composed of tokamak relevant materials at temperatures $T_d \geq 1500\text{K}$.

The comparative effect of including the Schottky correction factor is demonstrated in figure 19 for tungsten at its melting temperature $T_d = 3500\text{K}$ in a $T_i = T_e$ hydrogen plasma over a range of densities and temperatures relevant for the SOL of a tokamak. The normalised surface potential, χ_a , is calculated from the solution to equation 30 with current terms calculated from OML theory for ions and electrons, equation 40, using additionally either equation 68 or equation 69 to model the current of thermionically emitted electrons. Since the plasma current scales with plasma density, increasing the plasma density decreases the magnitude of the thermionic current relative to the plasma currents and leads to less negative values of χ_a . When using equation 69, larger electron temperatures lead to greater

fluxes of plasma particles since $T_i = T_e$ and therefore more negatively charged dust. This demonstrates the expectation for high temperature dust to maintain a positive surface charge.

3.2.6.2 Secondary Electron Emission, SEE

In contrast to thermionic electron emission which occurs passively, independent of the presence of a plasma, secondary electrons require a high energy source of primary particles to escape from the material. Following the derivation by Dekker, the Sternglass semi-empirical formula [172, 173] for the average secondary electron yield is

$$\delta_{see} = \int Y_{see}(x, E_0) P_{sec}(x) dx, \quad (70)$$

where $P_{sec}(x)$ is the probability for a secondary particle to escape from the surface of the material and $Y_{see}(x, E_0) dx$ is the average number of secondary particles produced per incident particle with energy E_0 at a depth x into the surface in a layer of thickness dx . The sensitivity of $Y_{see}(x, E_0)$ on the material specific properties means it is difficult to determine an analytical formula for δ_{see} and is for this reason modelled by an empirical formula in DTOKS [114], see appendices 10.1.

3.2.7 Particle Backscattering

There is a possibility that primary particles incident on the dust surface collide directly with a nucleus and rebound elastically without transferring their charge. Theoretically, this problem is approached by integrating the Rutherford backscattering differential cross section over backscattered angles and energies of the particle distribution. In practice, the fraction of back-scattered particles is often calculated from fits to empirical results as they offer a faster and sufficient description of the average behaviour [114]. The analogous phenomena for electrons as primary particles is relevant for electron temperatures greater than 30eV and for positively charged dust. In fusion plasmas with high temperature positively charged dust due to electron emission, electron backscatter coefficients of up to 0.32 can be reached [174]. Accurately accounting for these backscattering effects is therefore important for determining not only the surface potential but also the plasma heat fluxes.

3.3 DUST POWER BALANCE

The charged fluxes onto the dust grain also deliver heat fluxes which are in dynamic competition with other mechanisms that dissipate heat energy from the surface. The specific heat capacity, c_v , of a material defines the amount of heat energy, ΔH , required to change the temperature of a mass, M_d , by an amount ΔT_d through $M_d c_v = \Delta H / \Delta T_d$ for small ΔH . If this heat energy is supplied over a time period Δt , then taking the limit of Δt being infinitely small the total power, P_t , is

$$\lim_{\Delta t \rightarrow 0} \frac{\Delta H}{\Delta t} = \frac{dH}{dt} \equiv P_t = \sum_k P_k = c_v M_d \frac{dT_d}{dt}, \quad (71)$$

where P_k are the various terms of the power flux to the dust grain due to process k . In this section, the most relevant theories contributing to the dust power balance in the SOL of a tokamak are introduced.

The thermalisation of incident ions with the dust grain is typically the source of most thermal energy [62]. Ions contribute not just the thermal energy from being in thermal equilibrium with the distant plasma, but importantly also the potential energy of their electrostatic attraction and bonding. The power contributed from the kinetic energy of plasma particles in thermal equilibrium with the undisturbed plasma is expressed using the second moment of the distribution function, from equation 6. Implementing the theoretical framework of the SOML model for $\lambda_d \gg a_d$ of section 3.2.3, this is integrated over the collection area which is given by the OML cross section σ_{OML} from equation 34,

$$P_s^{KE} = \int \frac{m_s v^2}{2} n_s v f_s(\mathbf{r}, \mathbf{v}, t) \sigma_{OML} d^3 v, \quad (72)$$

giving a heating power, P_s^{KE} [62]. For SOML ions, this integral is performed for a shifted Maxwellian velocity distribution, equation 7, giving

$$P_i^{KE} = \pi a_d^2 n_i v_{T,i} k_B T_i \frac{\sqrt{2}}{4} \left[\frac{2}{\sqrt{\pi}} \left[5 + 2 \left(\tilde{u}^2 + \frac{Z\chi_a}{\tau} \right) \right] e^{-\tilde{u}^2} + \frac{1}{\tilde{u}} \left[3 + 12\tilde{u}^2 + 4\tilde{u}^4 + 2 \frac{Z\chi_a}{\tau} (1 + 2\tilde{u}^2) \right] \text{erf}(\tilde{u}) \right] \chi_a \geq 0. \quad (73)$$

However, this does not include the additional energy contributed from ion acceleration in the sheath. This is given simply by $\phi_a I_i$, with I_i taken from the ion currents previously derived in section 3.2. Following on this example in the SOML limit, $\lambda_d \gg a_d$, for a flowing plasma, this is taken from equation 60 as

$$\phi_a I_i = \left(\frac{\chi_a k_B T_i}{e\tau} \right) \frac{I_{0,i}}{2} \left[e^{-\tilde{u}^2} + \frac{\sqrt{\pi} \text{erf}(\tilde{u})}{\tilde{u}} \left(\frac{1 + 2\tilde{u}^2}{2} + \frac{Z\chi_a}{\tau} \right) \right]. \quad (74)$$

Adding equation 73 and equation 74 together, gives the total power of incident ions. For simplicity, this is given here in the limit of no relative velocity, $\tilde{u} \rightarrow 0$, using equation 63 such that

$$P_i = 4\pi a_d^2 n_i v_{T,i} k_B T_i \sqrt{\frac{2}{\pi}} \left[1 + \frac{Z\chi_a}{\tau} + \frac{Z^2 \chi_a^2}{2\tau^2} \right], \quad \chi_a \geq 0. \quad (75)$$

In addition to this power, the recombination of plasma ions on the dust grain surface also transfers a significant amount of energy [125]. For each captured ion, the bond formation energy plus the energy taken to capture an electron by the dust is released. For the recombination of two hydrogen ions to form a hydrogen molecule for example, the recombination energy imparted is $E_{rec} = (13.6 + 2.2/2)\text{eV} = 14.7\text{eV}$ arising from the ionisation energy, 13.6eV, and half the bond energy of molecular hydrogen, 2.2eV. This produces an extra source of thermal energy,

$$P_{rec} = \frac{E_{rec} I_i}{e}, \quad (76)$$

from recombination which is proportional to the ion current.

Meanwhile for electrons, the small relative flow speed can almost always be neglected [62] as mentioned earlier, allowing equation 5 to be used with 72 instead

$$P_e = 2\sqrt{2}\pi a_d^2 n_e v_{T,e} k_B T_e (2 + \chi_a) e^{-\chi_a} \quad \chi_a \geq 0. \quad (77)$$

Heat energy gained from electron collection can be large for positively charged dust grains when the fluxes are much higher and is also enhanced in strong magnetic fields [87].

The power delivered by neutral particles [62], though often negligible in tokamak plasmas with low neutral densities, can be calculated from equation 72 as

$$P_n^{KE} = \pi a_d^2 n_n v_{T,n} k_B T_n \frac{\sqrt{2}}{4} \left[\frac{2}{\sqrt{\pi}} (5 + 2\tilde{u}^2) e^{-\tilde{u}^2} + \frac{1}{\tilde{u}} (3 + 12\tilde{u}^2 + 4\tilde{u}^4) \text{erf}(\tilde{u}) \right], \quad (78)$$

by replacing the OML cross-section with the cross section of an uncharged spherical dust grain $\sigma_{OML}(Z = 0) = 4\pi a_d^2$. This is equivalent to setting $\chi_a = 0$ in equation 73, which removes all potential dependant terms.

In balance with the heat fluxes from primary plasma particles, there exist several mechanisms which generally cause heat loss from the dust grain. The power emitted as electromagnetic radiation can be modelled using the Stefan-Boltzmann law [175]

$$P_{rad} = 4\pi a_d^2 \epsilon(a_d, T_d) \sigma_B (T_d^4 - T_a^4), \quad (79)$$

where σ_B is the Stefan-Boltzmann constant, $\epsilon(a_d, T_d)$ is the emissivity and T_a is the ambient temperature.

Finally, the electron emission processes mentioned previously in relation to charging also export heat energy. For thermionic electron emission, this is calculated from equation 69, with each electron stripping the dust of energy $H_d + W_f$ [114] where $H_d = 2k_B T_d$ and W_f is the work function of the dust such that

$$P_{tee} = -(H_d + eW_f) I_{tee} / e. \quad (80)$$

For secondary electrons, the spectrum of emission energies has a sensitive dependence on the energy of the primary particle, but for an average energy $E_{see} \simeq 3\text{eV}$ [114], the power flux is estimated as

$$P_{see} = -\delta_{see} I_e E_{see} / e. \quad (81)$$

Figure 20 shows the dependence of the different power flux terms given by these equations for a $1\mu\text{m}$ tungsten dust grain in a stationary, isothermal, hydrogen plasma with $T_a = 300\text{K}$, $n_i = n_e = 10^{18}\text{m}^{-3}$ and $n_n = 10^{16}\text{m}^{-3}$ with a fixed normalised surface potential, $\chi_a = 2.50$, calculated from the OML solution as explained in section 3.2.1. The heating due to neutrals is always small due to their relatively low density. At low plasma temperatures, neutral recombination competes with both the thermal radiation and thermionic electron emission. As the plasma temperature increases, the dust temperature increases such that the power loss from thermionic electron emission balances the heating from the flux of ions and electrons. This demonstrates the relative magnitudes of the different power fluxes typically expected.

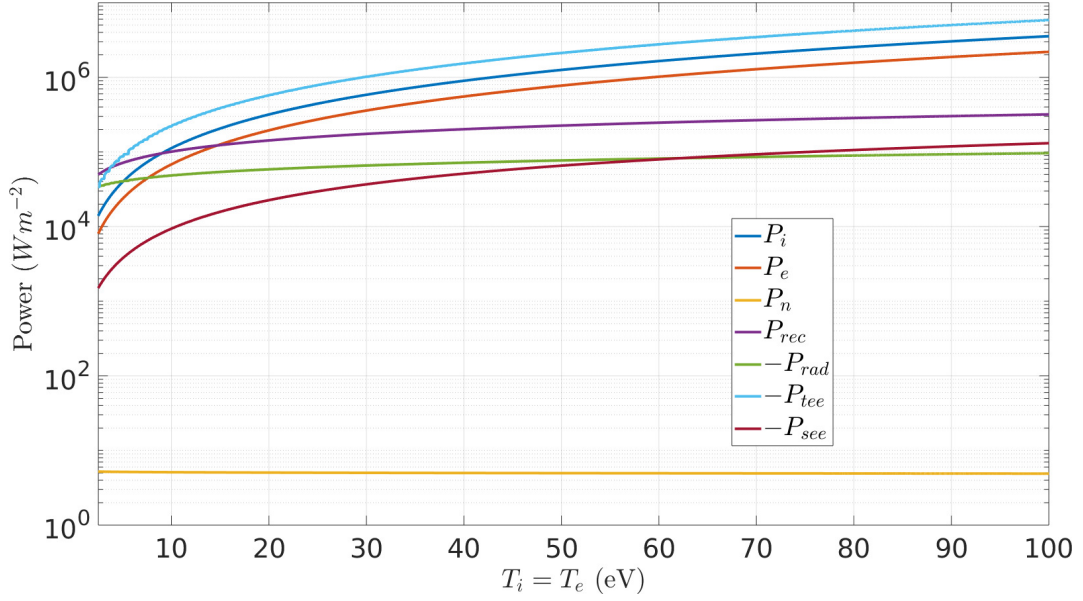


Figure 20: Magnitude of power flux terms at surface of tungsten dust from equations 75, 76, 77, 78, 79, 80 and 81 as a function of plasma temperature, $T_i = T_e$.

3.4 DUST FORCES

A number of forces, $\underline{F}_k$, compete to control dust dynamics in a plasma, which is described by the system of dynamical equations

$$\begin{aligned} \frac{d^2 \underline{r}(t)}{dt^2} &= \frac{1}{M_d} \sum_k \underline{F}_k, \\ \frac{d \underline{r}(t)}{dt} &= \underline{v}_d(t). \end{aligned} \quad (82)$$

Here, $\underline{r}(t)$ is the position in an arbitrary coordinate system for dust with velocity $\underline{v}_d(t)$ being acted upon by various forces $\underline{F}_k$ at time t . The body force due to gravity is taken simply as $\underline{F}_g = M_d \underline{g}$ where $\underline{g}$ is the constant acceleration of the gravitational field. Similarly for a spherical, conducting dust grain that is small compared with the length scale of variation in the electromagnetic field, the Lorentz force, $\underline{F}_L = Q_d (\underline{E} + \underline{v} \times \underline{B})$, gives the force due to the interaction of the electric field, $\underline{E}$, and magnetic field, $\underline{B}$, with the dust charge, Q_d . The remaining forces relevant to dust motion in tokamak plasmas arise principally from the exchange of particle momenta.

3.4.1 Hybrid Ion and Neutral Drag

The most important contribution to surface forces for negatively charged dust grains arises from interactions with ions and neutral atoms. Dust grains travelling with a velocity relative to the plasma background experience a net drag force due to collection, $\underline{F}_{i,c}$, and deflection, $\underline{F}_{i,d}$ of ions [138]. The expression for the force is adapted from the second moment of the distribution function, equation 6, with

the effective area of impact taken from the OML cross section σ_{OML} , equation 34, plus a component due to the deflected momentum cross section $\sigma_s(v)$, such that

$$\underline{F}_i = \underline{F}_{i,c} + \underline{F}_{i,d} = n_i m_i \left[\int \underline{v} \cdot \underline{v} \sigma_{\text{OML}} f_s(\underline{r}, \underline{v}) d\underline{v} + \int \underline{v} \cdot \underline{v} \sigma_s(v) f_s(\underline{r}, \underline{v}) d\underline{v} \right]. \quad (83)$$

The force component due to deflection of particles can be described through the binary collision approach [176], which calculates the sum of the momentum transfer of ballistic ion trajectories in the electrostatic field of the dust grain. For an isotropic potential distribution the momentum transfer cross section is found from integrating the deflection angle across impact parameters. Issues arise when considering the upper limit of this integral for a Coulomb potential as it is necessary to truncate the integration at some impact parameter larger than the plasma screening length, λ . This gives rise to the effective Coulomb logarithm, $\ln(\Lambda_{\text{BC}}) = \ln(1 + mv^2\lambda/|Q_d|e)$, which quantifies the degree of influence of deflected particles through the ratio of the Coulomb radius, $|Q_d|e/mv^2$, to the plasma screening length, λ , sometimes referred to as the scattering parameter

$$S_c(v) = \frac{|Q_d|e}{mv^2\lambda}. \quad (84)$$

The scattering parameter provides an estimate for the strength of the coupling of the ions to the dust grain. In the Hybrid model [176], this is used to express the momentum transfer cross section as

$$\sigma_s(v) = 4\pi\lambda^2 S_c(v)^2 \ln(\Lambda_{\text{BC}}). \quad (85)$$

Completing the integral of equation 83 using a shifted Maxwellian velocity distribution, equation 7, and the cross-sections of equation 85 and equation 34 gives the total ion drag force as

$$\begin{aligned} \underline{F}_i = & \sqrt{2\pi}a_d^2 n_i m_i v_{T,i}^2 \\ & \left[\sqrt{\frac{\pi}{2}} \text{erf}\left(\frac{\underline{u}_i}{\sqrt{2}}\right) \left[1 + u_i^2 + \left(1 - \frac{1}{u_i^2}\right) (1 + 2\chi_a \tau) + \frac{4\chi_a^2 \tau^2}{u_i^2} \ln(\Lambda_{\text{BC}}) \right] \right. \\ & \left. + \frac{1}{u_i} [1 + 2\chi_a \tau + u_i^2 - 4\chi_a^2 \tau^2 \ln(\Lambda_{\text{BC}})] e^{-\frac{u_i^2}{2}} \right] \hat{\underline{u}}_i, \end{aligned} \quad (86)$$

expressed as a function of the normalised ion flow velocity $\underline{u}_i = (\underline{U}_i - \underline{v}_d)/v_{T,i}$.

For neutral particles, no deflection is possible and only a force due purely to scattering of collected particles remains, which is calculated in the same way as the first term of equation 83 with $\sigma_{\text{OML}} = 4\pi a_d^2$

$$\begin{aligned} \underline{F}_n = & \sqrt{2\pi}a_d^2 n_n m_n v_{T,n}^2 \\ & \left[\sqrt{\frac{\pi}{2}} \text{erf}\left(\frac{\underline{u}_n}{\sqrt{2}}\right) \left[2 + u_n^2 - \frac{1}{u_n^2} \right] + \frac{1}{u_n} e^{-\frac{u_n^2}{2}} [1 + u_n^2] \right] \hat{\underline{u}}_n. \end{aligned} \quad (87)$$

As expected, this equation has an identical form to equation 86 with $\chi_a = 0$ and is equivalent to Epstein drag [177].

In general for sub-sonic drift velocities, the contribution is less significant for electrons because at equilibrium, as discussed previously, the flux of each charged species is equal but with ions having a far larger mass and so much greater momentum flux. For point like dust, this can be expressed algebraically as the ratio $F_i/F_e \sim \sqrt{m_i/m_e} (T_e/T_i)^{\frac{3}{2}} (u_i/u_e)$ where u_e is the normalised electron drift velocity. Only weakly ionised plasma environments with constant, externally imposed electric fields such as low temperature discharge plasmas have the conditions necessary for $u_e \gg u_i$ and for electron drag to become significant [178]. This is never the case in the plasma environments under consideration here.

3.4.2 Other Forces

Other forces that can be important in tokamaks are harder to characterise precisely and are extremely sensitive to local plasma conditions and dust surface properties. A dust grain placed in a plasma with strong thermal gradients will collect faster moving particles from the hotter side asymmetrically, leading to a net force [62]. This thermal force can be significant in tokamaks for ions moving along magnetic field lines and is proportional to the temperature gradient parallel to the magnetic field. However, this force requires a background plasma environment which is not isotropic over the dust length scale which violates the core assumptions inherent in the models discussed so far for small dust.

Another force dependant on temperature gradients is the rocket force, which occurs when a portion of a dust grain surface is of a different temperature to other regions of the surface, causing asymmetric ablation [179]. The expulsion of these particles causes a large acceleration in the opposite direction in analogy with the force produced by rocket engines. This force is approximated from the asymmetric flux of evaporated particles, that are assumed to leave with a velocity equal to the thermal velocity of dust grain atoms. Though this phenomena can be important for dust composed of an agglomeration of impurities, it is dubious whether it is possible for conducting dust which has such high thermal conductivity to maintain the thermal gradients, as was described in section 3.1.

Finally, for magnetically susceptible ferromagnetic dust, there exists the possibility for forces arising from the interaction of external magnetic fields with the induced magnetic field of the dust grain. In a tokamak, this force has been found to be proportional to both the induced magnetic field and the toroidal magnetic field [180]. Though modern tokamak walls have far fewer PFCs constructed from materials containing ferrous metals which are vulnerable to induced magnetic fields, some components use inconel alloys, see figure 3, meaning iron is still an element found in dust inventories. For iron, this force can become more significant than gravity for gradients in the magnetic field strength greater than 0.03Tm^{-1} [180]. However, this force will also be neglected since the dust considered and modelled here is always comprised of non-ferrous metals.

DUST CHARGING IN MAGNETIC FIELDS

*We have seen that computer programming is an art,
because it applies accumulated knowledge to the world,
because it requires skill and ingenuity, and especially
because it produces objects of beauty.*

— Donald E. Knuth, Computer Programming as an Art [181]

4.1 THEORY OF CHARGING IN A MAGNETIC FIELD

The presence of an external magnetic field, $\mathbf{B}(\mathbf{r})$, alters the behaviour of a dust grain both through influencing the surrounding plasma and directly interacting with the dust grains charge. In addition to the Lorentz forces mentioned already in section 3.4, the motion of a conducting dust grain in a magnetic field can generate an electric dipole moment in the $\mathbf{v} \times \mathbf{B}$ direction due to the hall effect [182]. More significant effects occur indirectly through alteration of the trajectories of ions and electrons, consequently affecting the sheath structure, plasma fluxes to the surface and ultimately the surface potential. Even in the limit $a_d \ll \lambda_d$, the problem is extremely complex and it is not possible to express a closed form solution for the surface potential. In this section (section 4.1) the existing techniques and approximations used for modelling the equilibrium surface potential of a dust grain in a hydrogen plasma with a constant uniform magnetic field when $a_d < \lambda_d$ are reviewed. Subsequently in section 4.2, the development of the Dust in Magnetised Plasmas (DiMPI) code and solutions for these challenging problems describe new progress in this research area.

4.1.1 Geometrical Considerations

To approach this problem, the simplest case of an uncharged spherical surface maintained at the plasma potential is first considered. Following the derivation of Whipple [182], the geometry of the problem is established as shown in figure 21 with a constant and spatially uniform magnetic field parallel to the z-axis, $\mathbf{B} = B\hat{z}$. The plasma particle velocity in Cartesian coordinates is decomposed into components parallel, $v_{\parallel} = v_z$, and perpendicular, $v_{\perp}^2 = v_x^2 + v_y^2$, to the magnetic field so that the equations describing the position of a particle are given by

$$x = x_1 + \rho_{\perp} \cos\left(\frac{\omega_s z}{v_{\parallel}} + \psi_0\right), \quad (88)$$

and

$$y = y_1 + \rho_{\perp} \sin\left(\frac{\omega_s z}{v_{\parallel}} + \psi_0\right). \quad (89)$$

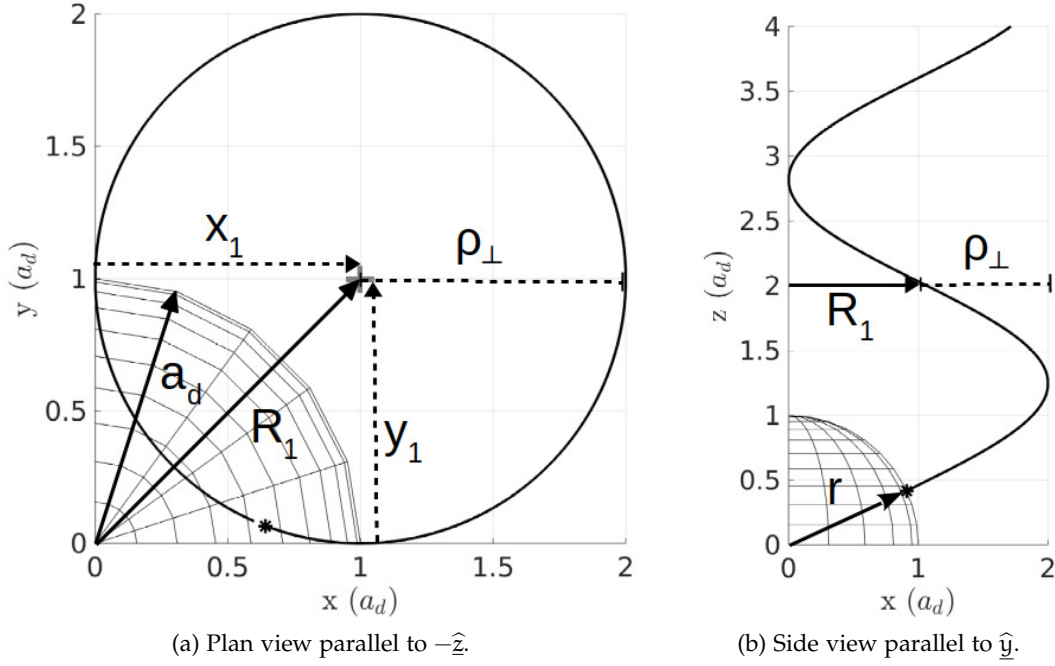


Figure 21: Ion gyro-orbits (solid black line) for $\rho_\perp = a_d$, $v_\parallel = \omega_s a_d$ and $x_1 = y_1 = a_d$ calculated by solving equations 88 and 89 for $\psi_0 = 3.8$ from plan (21a) and side (21b) views. The asterisk indicates the intersection point of the trajectory with the spherical surface calculated using equation 90 at a spherical radial coordinate $r^2 = a_d^2 = 1 = R^2 + z^2$.

The geometry of helical particle orbits and the spherical dust grain surface is most clearly described by defining both a cylindrical radial distance from the z axis, $R^2 = x^2 + y^2$, in two dimensions in addition to a three dimensional, spherical radial distance $r^2 = x^2 + y^2 + z^2$ from the origin. Here, $R_1^2 = x_1^2 + y_1^2$, is the cylindrical radial co-ordinate of the guiding centre of a gyro-orbit of radius $\rho_\perp = m_s v_\perp / eB$ with initial phase angle ψ_0 and gyro-frequency $\omega_s = eB/m_s$. The solutions to equations 88 and 89 produce helical trajectories corresponding to gyro-motion of plasma particles as shown in figure 21b. A spherical dust particle of radius a_d centered on the origin will collect a plasma particle if at any point the displacement from the origin, r , of that particle is less than or equal to the dust radius $r \leq a_d$. Substituting equations 88 and 89 into the equation for a sphere of radius $a_d = |r| = \sqrt{x^2 + y^2 + z^2}$, an equation for the point of intersection is derived

$$a_d^2 - z^2 = R_1^2 + \rho_\perp^2 + 2R_1 \rho_\perp \cos\left(\frac{\omega_s z}{v_\parallel} + \psi_0\right). \quad (90)$$

Setting $\omega_s z / v_\parallel = 2\pi$ in equation 91 defines a minimum wavelength, $\lambda_m = 2\pi v_\parallel / \omega_s$, such that particles having wavelengths smaller than the sphere have helical trajectories that intersect the sphere multiple times. To better understand this interaction, it is instructive to first consider the case when $2a_d \leq \lambda_m$ so that it is equivalently described by the intersection between a cylindrical and spherical surface (which is given by equation 90 when $\omega_s z / v_\parallel + \psi_0 \rightarrow \psi_0$). This line forms either one or two closed surfaces on the sphere as demonstrated in figure 22, with the critical case marking the transition between the two. Particles with

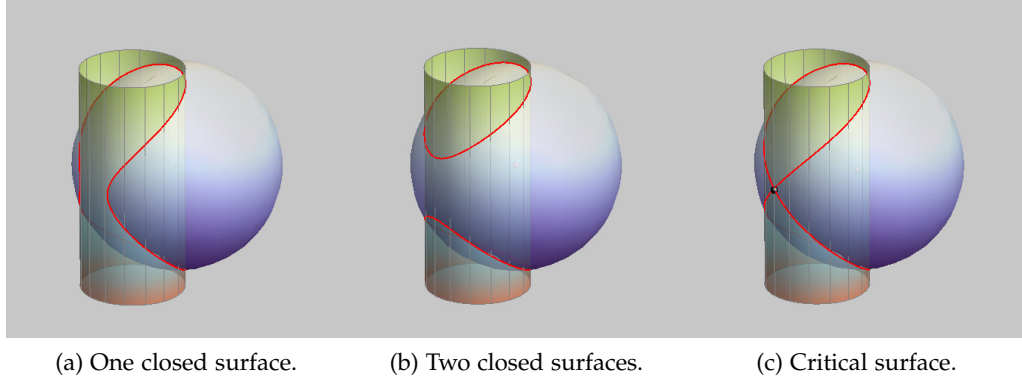


Figure 22: Possible types of interaction boundaries between a sphere and a cylinder with one (22a) or two (22b) closed surfaces and the critical case (22c). Reproduced from [183].

$(R_1 - \rho_\perp)^2 > a_d^2$ cannot be collected because no point of the cylindrical surface intersects the sphere, as is evident from figure 21a. Orbits initially directed towards the dust with $|R_1| + |\rho_\perp| < |a_d|$ form two closed surfaces on the spherical surface and must be collected irrespective of the initial phase as is shown in figure 22b. For particles which exist in the annulus of allowed radii between these two conditions defined by the condition $|\rho_\perp - a_d| < R_1 < |\rho_\perp + a_d|$, a single closed surface is formed as shown in figure 22a, wherein particles may or may not be collected depending upon the particles initial conditions.

The term $\omega_s z / v_\parallel$ inside the cosine function acts to limit the range of values of z and ψ_0 for which intersection of a helical trajectory with the spherical surface occurs. In this case, it is possible for the line of intersection to no longer form a closed surface as is shown in figure 23b. The critical phase angle, ψ_c , defines the limiting phase angle at which a collected particle grazes the spherical surface tangentially. Rearranging equation 90 for the critical phase angle at the corresponding critical height, z_c , the critical phase is defined as

$$\psi_c(z_c) = \cos^{-1} \left(\frac{a_d^2 - z_c^2 - R_1^2 - \rho_\perp^2}{2R_1 \rho_\perp} \right) - \frac{\omega_s z_c}{v_\parallel}. \quad (91)$$

The critical height, z_c , at which a particle is collected is determined from differentiating equation 90 with respect to z ,

$$z_c = \frac{R_1 \rho_\perp \omega_s}{v_\parallel} \sin \left(\frac{\omega_s z_c}{v_\parallel} + \psi_c \right), \quad (92)$$

under the condition that particles reaching the surface do so with grazing incidence. Substituting the expression for ψ_c from equation 91 into equation 92, the following equation is obtained after some algebraic manipulation

$$z_c^2 = a_d^2 - 2 \left(\frac{v_\parallel}{\omega_s} \right)^2 - R_1^2 - \rho_\perp^2 + 2 \sqrt{\left(\frac{v_\parallel}{\omega_s} \right)^2 \left(\left(\frac{v_\parallel}{\omega_s} \right)^2 + R_1^2 + \rho_\perp^2 - a_d^2 \right) + R_1^2 \rho_\perp^2}. \quad (93)$$

Using equation 91, equation 93 and the minimum wavelength, λ_m , it is possible to determine the range of z and corresponding ψ_0 for which a plasma particle with

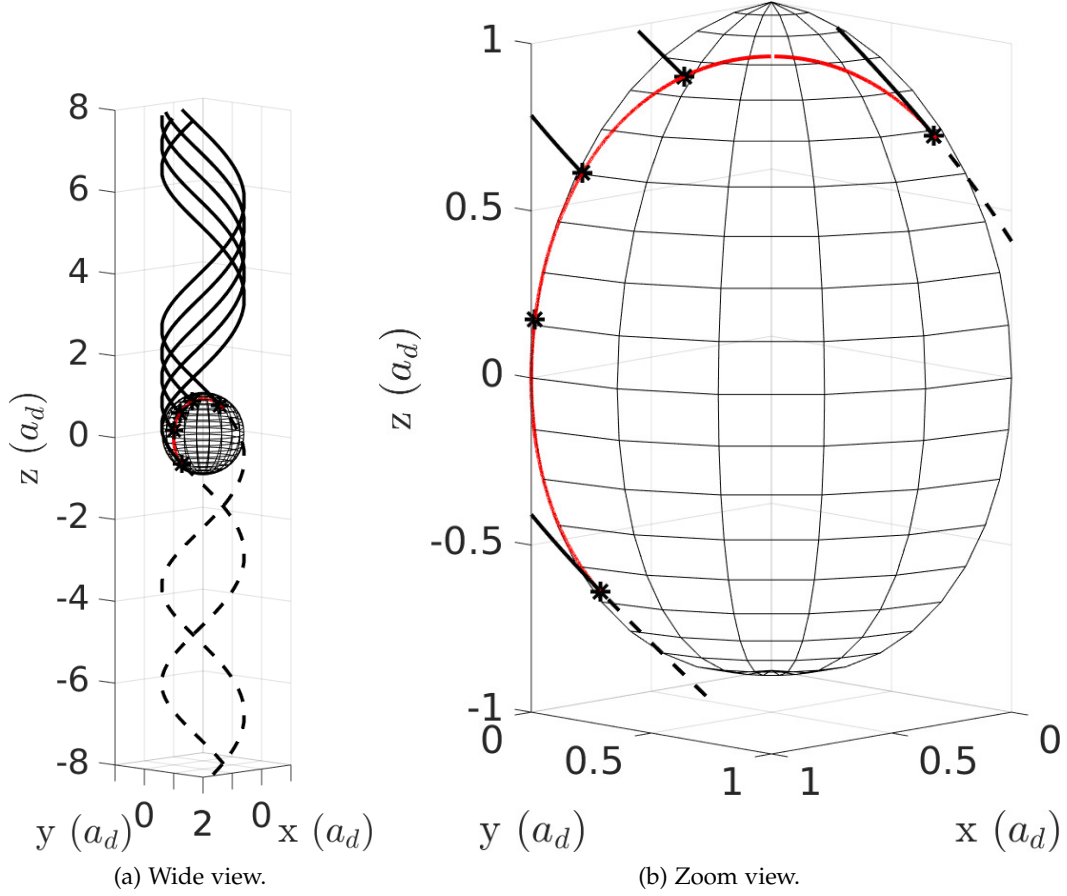


Figure 23: Gyro-orbits of ions (solid black lines) for $\rho_{\perp} = a_d$, $v_{\parallel} = \omega_s a_d$ and $x_1 = y_1 = a_d$ from solving equation 88 and equation 89 for five equally spaced values of phase in the range $|\psi_c| \leq |\psi_0| \leq \pi$. Asterisks indicate the interaction points for each value of ψ_0 calculated using equation 90. The red line indicates all solutions of equation 90 within the the limiting trajectories (black dashed line) having ψ_c and z_c calculated from equation 91 and equation 93 respectively.

given initial conditions collides with the dust grain. Figure 23 shows the trajectories of five ions with $\rho_{\perp} = a_d$, $v_{\parallel} = \omega_s a_d$ and $x_1 = y_1 = a_d$ over a range of values of ψ_0 between the two limiting trajectories defined by ψ_c . For a given set of initial conditions; R_1 , ψ_0 , $v_{\perp}$ and $v_{\parallel}$, the fate of a particle can therefore be determined a priori from the solution of equation 91 using equation 93 and the condition that particles with $-\psi_c < \psi_0 < \psi_c$ cannot be collected.

4.1.2 Current Through a Spherical Surface

Employing the geometrical effects discussed in the previous section, a partial analytic solution for the current to an uncharged sphere can be obtained [182]. The expression for the current is related to equation 6 by the velocity space volume element $d^3v = 2\pi v_{\perp} dv_{\perp} dv_{\parallel}$ and is calculated from the total current passing through two distant surfaces oriented perpendicular to the magnetic field. At these surfaces,

the particles can be assumed to be described by a Maxwell-Boltzmann velocity distribution equivalently related to equation 5. The current is then expressed as,

$$I_s = 2\pi Z e n_s \left(\frac{1}{2\pi v_{T,s}^2} \right)^{\frac{3}{2}} \iint v_{\parallel} v_{\perp} e^{-\frac{(v_{\parallel}^2 + v_{\perp}^2)}{2v_{T,s}^2}} dv_{\perp} dv_{\parallel} dS, \quad (94)$$

where $v_{T,s} = \sqrt{k_B T_s / m_s}$ is the thermal velocity of a given species as before. This expression is further simplified in a number of ways. Firstly, the vertical symmetry about the plane $z = 0$, means that the integral over the lower surface is equivalent to the integral over the upper surface. Next, the geometrical considerations previously described are applied to split the integral into different parts corresponding to different regions of the distant surface with surface element $d\mathbf{S} = dR_1 (R_1 d\psi_0) \hat{\mathbf{z}}$ such that

$$I_s = \frac{4\sqrt{2} Z e n_s}{\sqrt{\pi} v_{T,s}^3} \left[\int_0^{a_d - \rho_{\perp}} \int_0^{\omega_s a_d} \int_0^{\infty} \int_0^{2\pi} G d\psi_0 dv_{\parallel} dv_{\perp} dR_1 + \int_{|\rho_{\perp} - a_d|}^{\rho_{\perp} + a_d} \int_0^{\infty} \left[\int_0^{v_m} \int_0^{2\pi} G d\psi_0 dv_{\parallel} + 2 \int_{v_m}^{\infty} \int_{\psi_c}^{\pi} G d\psi_0 dv_{\parallel} \right] dv_{\perp} dR_1 \right], \quad (95)$$

where

$$G(R_1, v_{\perp}, v_{\parallel}) = R_1 v_{\perp}^2 v_{\parallel} e^{-\frac{(v_{\parallel}^2 + v_{\perp}^2)}{2v_{T,s}^2}}. \quad (96)$$

The first term of equation 95 contains trajectories of the type shown in figure 22b with $0 \leq \rho_{\perp} \leq a_d - R_1$, which are collected by the dust for all $0 \leq \psi_0 \leq 2\pi$ and $0 \leq v_{\parallel} \leq \infty$. The second term calculates the current due to collection of particles with $v \leq v_m \equiv \lambda_m \omega_s / 2\pi$, since all orbits of this kind are collected as the helices defining their orbits intersect the sphere multiple times. The third term accounts for the remaining current for particles with $v_m \leq v_{\parallel}$ for which only orbits with $\psi_c \leq \psi_0 \leq \pi$ are collected. Solving equation 95 with equation 96 provides an equation for the normalised current [182]

$$\frac{I_s}{I_{0,s}} = \frac{1}{2} \left[1 + \frac{\sqrt{2\pi}}{\beta_s} \left(2 - \operatorname{erf} \left(\frac{\beta_s}{\sqrt{2}} \right) \right) + \frac{2}{\beta_s^2} \left(1 - e^{-\frac{\beta_s^2}{2}} + \frac{8Y(\beta_s)}{\pi} \right) \right]. \quad (97)$$

The magnetisation parameter, $\beta_s = a_d / \rho_{T,s}$, has been introduced here for a thermal gyro-radius $\rho_{T,s} = v_{T,s} / \omega_s$, along with $I_{0,s} = \pi a_d^2 Z e n_s \langle v_{T,s} \rangle$, the thermal current for species $s = i, e$ previously introduced in section 3.2.1. The last term in this equation contains an undetermined function $Y(\beta_s)$ which is a numerical result from the contribution of the final term of equation 95 containing ψ_c , as this integral has no analytical solution. Evidently, even for an uncharged surface fixed at the plasma potential $\chi_a = 0$, it is not possible to extract an analytical solution without further approximations.

4.1.3 Exact Current to a Charged Sphere

Accounting for the potential distribution surrounding a charged dust grain complicates the matter further, making calculations extremely computationally expensive. As a result, rapid and reliable determinations of the charged particle fluxes, though desirable, have so far been limited in scope.

The exact current to a small ($a_d \ll \lambda_d$), stationary, spherical probe with a spherically symmetric potential distribution was first calculated by Sonmor and Laframboise (SL) [184] in a comprehensive paper on the topic. The current was formulated in a similar manner to Whipple, equation 94, from twice the charged flux crossing a distance reference plane $z = z_0 < 0$ that is later collected by the dust

$$I_s = 2q \int \int f(\mathbf{r}, \mathbf{v})(\mathbf{v} \cdot \hat{\mathbf{z}}) \Theta(a_d - |\mathbf{r}|_{\min}) d^3\mathbf{v} dS. \quad (98)$$

Here, $f(\mathbf{r}, \mathbf{v})$ is the distribution function at the distant surface and Θ is the Heaviside step function, which hides the complexity of the problem by taking the value 1 for orbits that intersect the sphere and zero otherwise. The evaluation of $\Theta(a_d - |\mathbf{r}|_{\min})$ is determined by calculating $|\mathbf{r}|_{\min}$, the minimum value of $|\mathbf{r}(t)|$ for all t as the solution of the initial value problem defined by the equations of motion for the charged particles. This statement of the problem can be further abstracted through employing symmetry to reduce the dimensionality however it cannot in general be solved analytically. Instead, further limiting conditions are necessary to derive analytic estimates for the upper and lower bound of the current [184, 185].

4.1.3.1 Helical Bound of Current

The problem is greatly simplified by assuming that particle trajectories are unaffected by the dust potential when moving close to the surface [184]. To arrive at this estimate, the Heaviside step function $\Theta(a_d - |\mathbf{r}|_{\min})$ must be re-expressed into a form that can be readily evaluated. In this case, particle orbits are assumed to be helical and remain unaffected by the surface potential in a region defined by the reference plane $z = z_0 \leq 0$. Before reaching the plane $z = z_0$, attracted particles gain an amount of energy $e\phi_a$ due to acceleration through the dust potential. A Maxwell-Boltzmann velocity distribution of the form of equation 5 is assumed for attracted particles with a positive total energy $E > mv^2/2 + e\phi_a$. This assumption excludes absorption radii as discussed in section 3.2.1, limiting this treatment to the case $a_d \ll \lambda_d$.

In this circumstance, referred to as the helical limit, the condition that the particle has sufficient energy to be collected, $0 \leq E(|\mathbf{r}| = a_d)$ and the geometric information of section 4.1.1 are employed to alter the integral expression for the current of equation 98. The condition that orbits have sufficient energy to overcome the potential of the sphere is formulated in the Heaviside function $\Theta(mv_{\perp}^2 + mv_{\parallel}^2 - 2e\phi_a)$. As was introduced in section 4.1.1, the geometrical requirement for an orbit to be collected is found by averaging across all phases $0 \leq \psi_0 \leq (\psi_c, \pi)$ so that

$$\int_0^{\psi_c(z_c)} \frac{d\psi_0}{\pi} = \min\left(1, \frac{\psi_c(z_c)}{\pi}\right). \quad (99)$$

The product of these two terms provides an alternative expression for the collection condition in the helical limit,

$$\Theta(a_d - |r|_{\min}) \equiv \Theta(mv_{\perp}^2 + mv_{\parallel}^2 - 2e\phi_a) \int_0^{\psi_c(z_c)} \frac{d\psi_0}{\pi}, \quad (100)$$

summarising the requirements for particles following helical trajectories to be collected by the spherical dust surface with positive total energy. Re-expressing $\Theta(a_d - |r|_{\min})$ in equation 98 using equation 100 and incorporating the potential $\chi_a = e\phi_a/k_B T_s$ in the Maxwell-Boltzmann velocity distribution for particles, the attracted species current is

$$I_s = \frac{nq}{v_{T,s}^3 \sqrt{2\pi}} e^{\chi_a} \int_0^{2\pi} \int_0^{\infty} \int_0^{\infty} \int_0^{\infty} v_{\parallel} v_{\perp} \Theta(mv_{\perp}^2 + mv_{\parallel}^2 - 2e\phi) \\ e^{-\frac{m(v_{\perp}^2 + v_{\parallel}^2)}{2k_B T}} \int_0^{\psi_c(z_c)} R_1 d\psi_0 dR_1 dv_{\perp} dv_{\parallel} d\theta. \quad (101)$$

At this point, it is useful to introduce a normalisation scheme to clarify the dependency of the current on the variables of particle motion. Normalising distances to the dust radius, a_d , and time to the inverse of the gyro-frequency, $1/\omega_s$, the dimensionless quantities b , s , h and D are defined by

$$R_1 = a_d b, \quad \rho_{\perp} = a_d s, \quad v_{\parallel} = \frac{a_d h}{\omega_s}, \quad D = \sqrt{\frac{2\chi_a}{\beta_s^2}}, \quad \beta_s = \frac{a_d}{\rho_{T,s}}. \quad (102)$$

The rescaled potential D was arrived at from re-arranging the equation for total energy, $m\omega_s^2 a_d^2 (s^2 + h^2) + 2e\phi_a = 0$ under the new normalisation scheme. Resurrecting the previous arguments of section 4.1.1, it is once again possible to split the integral into parts representative of orbits that are collected irrespective of ψ_0 with $|\rho_{\perp}| + |R_1| \leq a_d$, see figure 22b, and those in the annulus $|\rho_{\perp} - a_d| \leq R_1 \leq \rho_{\perp} + a_d$ which retain a dependence on ψ_0 , see figure 23,

$$\frac{I_s}{I_{0,s}} = \frac{\beta_s^4}{\sqrt{2\pi}} e^{\chi_a} \int_0^{\infty} \int_0^{\infty} sh \Theta(s^2 + h^2 - D^2) e^{-\frac{\beta_s^2 (s^2 + h^2)}{2}} \\ \left[\Theta(1-s) \int_0^{1-s} b db + \int_{|s-1|}^{s+1} b \int_0^{\psi_c(z_c)} d\psi_0 db \right] ds dh. \quad (103)$$

For repelling potentials, D in equation 103 is replaced by 0, which recovers the normal exponential dependence on potential of the current. Taking the limit of zero magnetic field strength $\underline{B} \rightarrow 0$ and evaluating equation 103 for ions,

$$\frac{I_i}{I_{0,i}} = 1 + \chi_a / \tau, \quad (104)$$

the OML ion current of equation 38 is recovered for $Z = 1$. This analytical limit of the current incorporating the influence of the magnetic field on the geometrical interaction of particle orbits with the dust surface [184] is therefore seen to be an extension of the OML theory of section 3.2.1.

4.1.3.2 Adiabatic Lower Bound on Current

To arrive at an analytical solution for the current as a function of magnetic field strength, further approximations about particle motion are required. In the adiabatic limit [184], this is achieved by assuming that particle acceleration in the dust potential occurs solely in the direction parallel to $\underline{z}$, leading to an estimate for the lower bound of the current. This is referred to as the adiabatic limit since it is assumed that the sum of the potential and vertical kinetic energy $mv_{\parallel}^2/2 + e\phi$ of a plasma particle becomes an adiabatic invariant when the gyro-radius is much smaller than the scale length of the potential. This alters the constraint requiring positive total particle energy of equation 103 from $mv_{\parallel}^2 + mv_{\perp}^2 + e\phi_a$ to $mv_{\parallel}^2 + e\phi_a$ with the transformation $\Theta(s^2 + h^2 - D^2) \rightarrow \Theta(h - D)$. The integral equation can then be evaluated in various magnetic field limits to attain an analytical solution.

In the high magnetic field limit, $\beta \rightarrow \infty$, where $\rho_{\perp} \ll a_d$, all orbits intersect the sphere multiple times meaning that the result of equation 99 is 1. In this limit, equation 101 becomes

$$\frac{I_s}{I_{0,s}} = \frac{\beta_s^4}{\sqrt{2\pi}} e^{\chi_a} \int_0^{\infty} \int_0^{\infty} s h e^{-\frac{\beta_s^2(s^2 + h^2)}{2}} \Theta(h - D) \left(\frac{(1-s)^2}{2} \Theta(1-s) + 2s \right) ds dh. \quad (105)$$

Solving this integral equation provides the following analytic approximation for the current,

$$\frac{I_s}{I_{0,s}} = \frac{1}{2} \left[1 + \frac{2}{\beta_s^2} \left(1 - e^{-\frac{\beta_s^2}{2}} \right) + \frac{\sqrt{2\pi}}{\beta_s} \left(1 + \operatorname{erfc} \left(\frac{\beta_s}{\sqrt{2}} \right) \right) \right]. \quad (106)$$

This equation is identical to equation 97 without the undetermined term $Y(\beta_s)$ which has been removed through the assumptions of the adiabatic lower bound. For $\beta_s \rightarrow 0$, the current becomes $I_s/I_{s,0} = 1$ whilst for $\beta_s \rightarrow \infty$, the current tends to $I_s/I_{s,0} = 1/2$. Evaluating equation 103 under the same conditions but in the opposite magnetic field limit of $\underline{B} \rightarrow 0$, the current is

$$\frac{I_s}{I_{0,s}} = 1 - \frac{\sqrt{\pi\chi_a}}{2} \operatorname{erfc}(\sqrt{\chi_a}) e^{\chi_a}. \quad (107)$$

This gives $I_s/I_{s,0} = 1$ for $\chi_a \rightarrow 0$ and $I_s/I_{s,0} = 1/2$ for $\chi_a \rightarrow \infty$. This is a result of the fact that in the adiabatic limit, particles remain tied to field lines such that they are only collected over two circular areas totaling $2(\pi a_d^2)$. The theoretical limits discussed provide a useful background and comparison with simulation results.

4.1.4 Magnetic Envelope

A useful and simplifying conceptualisation of the motion of particles is achieved through visualisation of the boundary of possible paths in the cylindrical $R - z$ plane referred to here as the magnetic envelope. This defines a boundary containing the possible trajectories accessible to a particle for a given set of initial

conditions. This boundary is built upon the conservation of energy and particle canonical momentum, L , parallel to the cylindrical axis. Following the description formulated by Rubinstein and Laframboise [185], the Hamiltonian from equation ?? is adapted for the motion of a particle of charge Ze in cylindrical coordinates $\underline{r} = (R, \theta, z)$, with velocity $\underline{v} = (v_R, v_\theta, v_\parallel)$, total energy

$$E = \frac{m}{2}(v_R^2 + v_\theta^2 + v_\parallel^2) + e\phi(R, z), \quad (108)$$

and canonical angular momentum

$$L = mRv_\theta + \frac{eBR^2}{2}. \quad (109)$$

Combining equations 108 and 109, the total energy can be re-written as

$$E = \frac{m}{2}(v_R^2 + v_z^2) + e\phi(R, z) + \frac{mR^2}{2} \left(\frac{L}{mR^2} - \frac{\omega}{2} \right)^2. \quad (110)$$

The canonical angular momentum of particles whose gyro-orbits encircle the z -axis is $L = -m\omega R_1^2/2$ whilst those that are non-encircling have $L = +m\omega R_1^2/2$, as is shown in [185]. Substituting $L = \pm m\omega R_1^2/2$ into equations 110,

$$E = \frac{m}{2}(v_R^2 + v_\parallel^2) + e\phi(R, z) + \frac{m\omega R^2}{8} \left(\pm \frac{R_0^2}{a_d^2} - 1 \right)^2, \quad (111)$$

where $R_0 = \sqrt{|R_1^2 - \rho_\perp^2|}$ is the radial distance from the central axis to the position of the particle. At the boundary of the magnetic envelope, $v_R = v_\parallel = 0$ which defines the quantity $\delta(R, z)$ that describes the shape of the magnetic envelope

$$\delta^2(R, z) = \frac{8(E - e\phi(R, z))}{m\omega R^2} = \frac{R_0^2}{a_d^2} \left(\frac{R_0}{R} \mp \frac{R}{R_0} \right)^2. \quad (112)$$

The two positive solutions for R from equation 112 are

$$\begin{aligned} \frac{R_m}{a_d} &= \frac{1}{2} \left[\mp \delta \pm \sqrt{\delta^2 \pm \frac{4R_0^2}{a_d^2}} \right], \\ \frac{R_M}{a_d} &= \frac{1}{2} \left[\delta + \sqrt{\delta^2 \pm \frac{4R_0^2}{a_d^2}} \right], \end{aligned} \quad (113)$$

with R_m being the inner boundary closest to the central axis and R_M the outer boundary such that $R_m \leq R_M$. Trajectories of attracted particle species within a central Coulomb electric field and a constant uniform magnetic field were plotted by SL alongside their magnetic envelopes [184, Figure 2]. Trajectories terminate either at infinity or on the spheres surface, experiencing any number of reflections and exchanging momentum in the process. Pinches in the magnetic envelope lead to a strong sensitivity in initial conditions and non-linearity in particle orbits.

4.2 DUST IN MAGNETISED PLASMAS (DIMPL)

As has been shown in the previous section, no general analytic solution exists for the flux of particles to spherical conducting surfaces in the presence of a magnetic field, even when the surface has no net charge. The potential as a function of magnetic field strength must either be calculated through numerical integration of equation 98 or estimated based on results of particle-in-cell simulations [155, 186]. Due to the challenges of numerical modelling, only a small number of studies have explored the dependence of forces [156], torques [117] and heat fluxes [187] on magnetic field strength. The Dust in Magnetised Plasma (DiMPI) Code was created to rapidly attain solutions for these currents and forces for isolated conducting dust grains in plasmas with external magnetic fields [188].

DiMPI is a Monte-Carlo code that tracks particles in a Cartesian coordinate system through a cylindrical geometry with a spherically symmetric electric and constant, uniform magnetic field. The background plasma flows at an arbitrary velocity parallel to the magnetic field in the $\hat{z}$ direction. Simulations assume a singly charged ion and electron plasma, having a collisional length scale much larger than all other length scales.

4.2.1 Boundary Conditions

A cylindrical simulation domain is used as the spherical symmetry of the problem is broken by the linear magnetic field and flow. A schematic of the simulation domain is shown in figure 24. Unlike the theoretical description presented in section 4.1.2, DiMPI accounts for particles arriving from both $z = \infty$ and $z = -\infty$ to allow for cases with a flowing plasma. Particles are injected uniformly across two circular planes oriented perpendicular to $\hat{z}$, at a height $\pm z_0$ from the origin. The infinite planes defined by $z = \pm z_0$ and the spheroidal surface of the dust grain centred around the origin define the simulation boundaries. The dust grain is assumed to be a perfectly conducting spheroid of charge Q_d .

For large z_0 , the electrostatic potential at the injection surface, $\chi_a a_d / z_0$, is assumed to be equal to the plasma potential and the distribution function is unaffected by the dust electric field. In this case, the distribution of velocities parallel to the circular surface $f(v_x, v_y)$ can be approximated by a Maxwell-Boltzmann distribution as given by equation 5. Perpendicular to the plane however, the distribution

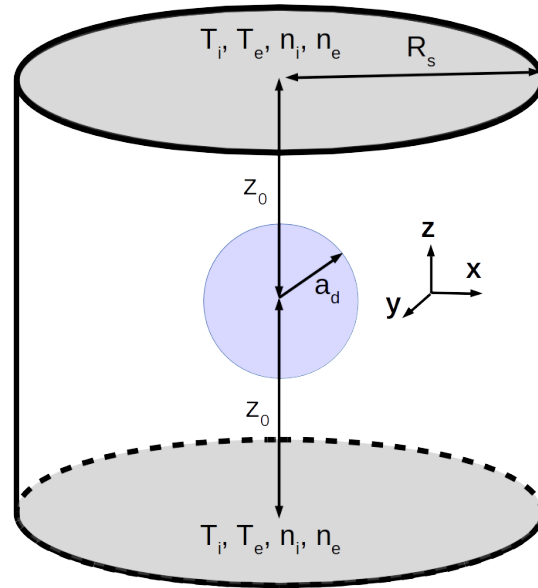


Figure 24: Schematic of simulation domain in Cartesian coordinates. Reprinted from [188], with the permission of AIP Publishing.

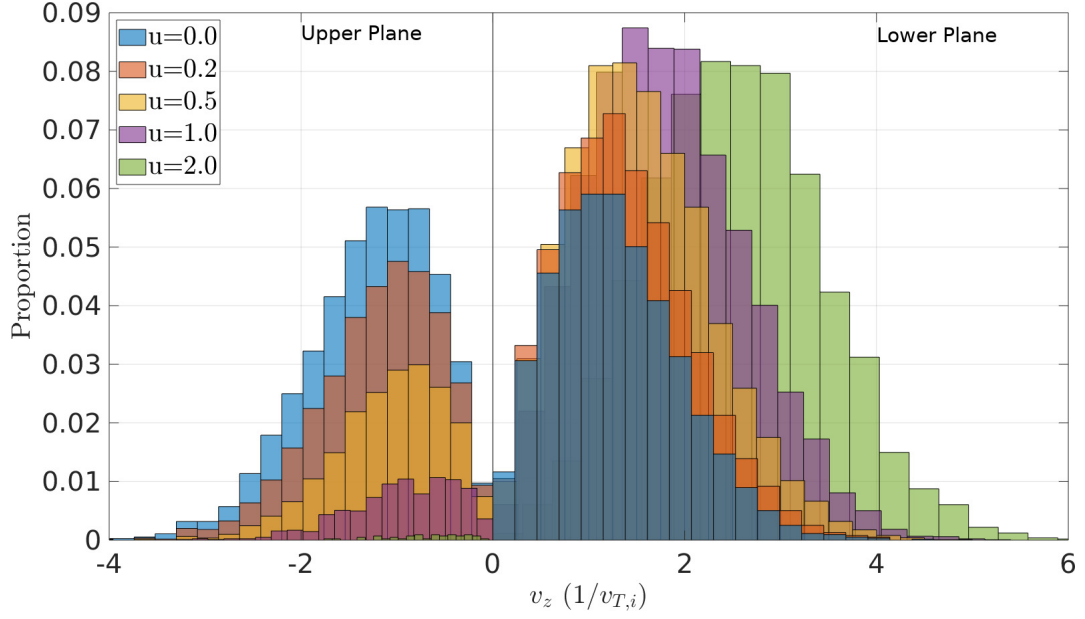


Figure 25: Normalised velocity distribution of ions injected over the circular boundaries at the upper, $z = z_0$, and lower, $z = -z_0$, planes for different normalised drift velocities $u = U/v_{T,i}$. Reprinted from [188], with the permission of AIP Publishing.

differs due to the preferential passage of particles with higher velocities. For this reason, the flux of particles is calculated using equation 9 with the particle velocities sampled from a shifted Maxwellian distribution following the method of Makkonen [189],

$$f(v_z) = \frac{n_0 v_z}{\Gamma_s(U)} \sqrt{\frac{m_s}{2\pi k_B T_s}} e^{-\frac{m(v_z - U)^2}{2k_B T_s}}. \quad (114)$$

Figure 25 shows the v_z velocity distribution for particles for different flow velocities $u = U/v_{T,i}$ originating from both the upper and lower plane.

Only particles with an initial radial position on the order of the gyro-radius or smaller are considered. This is justified since at large initial impact parameters, only a small proportion of particles have a perpendicular velocity component large enough to be collected. For this reason, the circular injection planes for ions and electrons are defined with areas $A_s = \pi R_s^2$ and radii $R_s = a_d + \kappa$ where κ is a simulation parameter fixed at runtime. Typically, the value of κ is calculated as a multiple of the species gyro-radius and Debye length, $\kappa = b(\rho_{T,s} + \lambda_d)$, with impact factor b . The probability of injecting an ion, $P_i^\pm$, or electron, $P_e^\pm$, at the upper (+) or lower (-) surfaces of areas A_i and A_e with an initial vertical position $\pm z_0$ is calculated from the ratio of the flux of particles of that species through that surface to the total flux,

$$\begin{aligned} P_i^\pm &= \frac{\Gamma_i(\mp U) A_i}{A_i(\Gamma_i(U) + \Gamma_i(-U)) + A_e(\Gamma_e(U) + \Gamma_e(-U))}, \\ P_e^\pm &= \frac{\Gamma_e(\mp U) A_e}{A_i(\Gamma_i(U) + \Gamma_i(-U)) + A_e(\Gamma_e(U) + \Gamma_e(-U))}. \end{aligned} \quad (115)$$

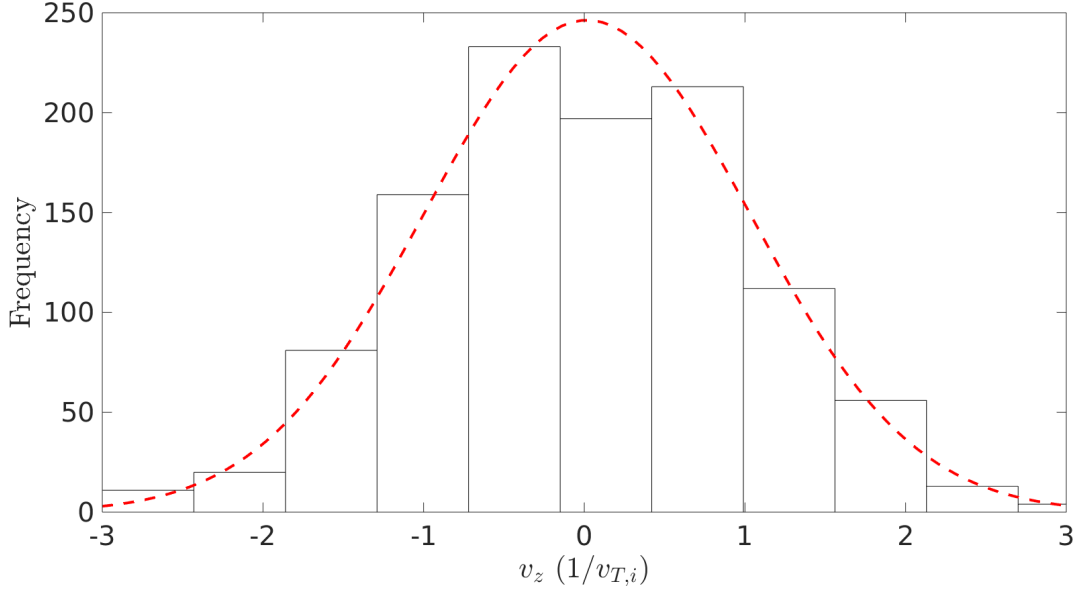


Figure 26: Velocity distribution of ions at the $z = 0$ plane fit to a Maxwellian distribution.

where $\Gamma_s(\mp U)$ is given by equation 9. The particle velocity is then sampled from the velocity distribution given by equation 5 and equation 114, with background flow velocity U . After specifying the initial conditions, ions and electrons are injected at random with probabilities given by equation 115 with a random initial position uniformly distributed over the circular areas A_s .

Without an electrostatic potential to alter the motion of particles, the particle velocity distribution at every point inside the simulation domain is Maxwellian. Figure 26 shows the vertical velocity distribution of particles at the $z = 0$ plane, demonstrating the validity of the injection method.

4.2.2 Solving the Equations of Motion

The motion of non-relativistic charged particles traversing a region with a central electric field $\underline{E}(\underline{r}(t)) = E(\underline{r}(t))\hat{r}$ due to the charged spherical dust grain and a constant uniform magnetic field $\underline{B} = B\hat{z}$ is given by the Lorentz force law. Using the initial conditions $\underline{r}(t_0)$ and $\underline{v}(t_0)$ at $t = t_0$ provided by the boundary conditions, the position after a time step Δt is calculated via numerical integration using the Boris algorithm [190]. The limiting time scale of the simulation is determined from the smallest of the inverse of the electron gyro frequency m_e/eB , the thermal transit time of the dust grain $a_d/v_{T,i}$ and the sheath transit time $\lambda_d/v_{T,i}$. The time step, Δt , is set as a fraction $\gamma_0 < 1$ of this time scale, where $\gamma_0 = 0.01$ is the default value for this simulation parameter.

As described in section 2.6, in the limit where the Debye length is much greater than the size of the sphere $1 \ll \lambda_d/a_d \equiv \tilde{\lambda}_d$, the potential of the sphere is unshielded close to the surface and the ions and electrons experience a bare Coulomb electric field,

$$\underline{E}(\underline{r}(t), t) = \frac{Q_d}{4\pi\epsilon_0|\underline{r}(t)|^2}\hat{r}. \quad (116)$$

For large dust sizes, the dust charge is screened by the plasma over a length scale comparable to the Debye length and the electric field is approximated instead by

$$\underline{E}(\underline{r}(t), t) = \frac{Q_d}{4\pi\epsilon_0|\underline{r}(t)|} e^{-\frac{|\underline{r}(t)| - a_d}{\lambda_d}} \left(\frac{1}{|\underline{r}(t)|} + \frac{1}{\lambda_d} \right) \hat{\underline{r}}, \quad (117)$$

arising from a Debye shielded potential following equation 20.

The system of equations is solved explicitly until one of three termination conditions are satisfied. If at any point, $|\underline{r}(t)| \leq a_d$, then the particle is considered to be collected and contribute fully its charge and momentum to the dust grain. If a particle reaches a point where $|z| > z_0$, the particle leaves the simulation domain and the lost momentum and charge are recorded. This information is stored for use in verification of charge conservation and for calculating momentum transfer to the dust.

As was discussed previously in section 4.1.4, attracted particle species can become transiently trapped in the magnetic envelope, incurring excessive computational cost. When determining the charging behaviour, a maximum number of reflections parallel to the magnetic field is defined. Particles of the attracted species which exceed this number of reflections are arbitrarily assumed to return to infinity, leading to an underestimate of their current. This estimate provides an increasingly accurate but intensive calculation for larger values of the maximum number of reflections. The default value for this parameter for the attracted species is set to 15 as this has been shown in previ-

ous work to provide an accurate estimate within $< 2\%$ of the ion current to a sphere [184, Figure 6], though this parameter can be altered freely. For the repelled species, one reflection predetermines a trajectory to return to infinity and is a sufficient condition to determine their current and improve simulation performance.

Figure 27 shows examples of plasma particle trajectories produced by DiMPL that are close to a negatively charged dust grain in an isothermal plasma. The smaller inertia of electrons (blue) allows them to be significantly accelerated by the dust potential and in many cases reflected. For the same reason, electrons also have smaller gyro-radii than ions, with $\rho_{T,e}\sqrt{\tau\mu} = \rho_{T,i}$.

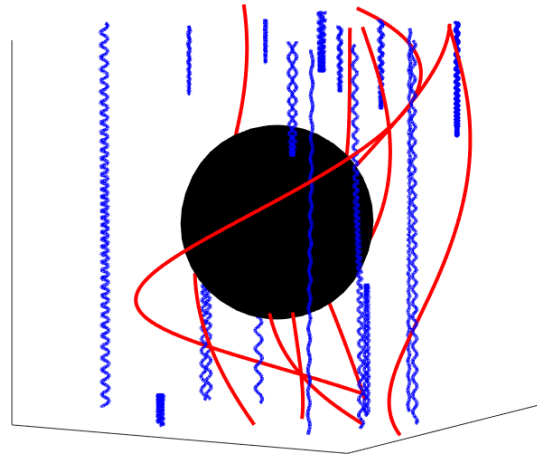


Figure 27: Example ion (red) and electron (blue) orbits close to the charged sphere. Reprinted from [188], with the permission of AIP Publishing.

4.2.3 Method

Each ion or electron trajectory is solved independently across a number of processors in parallel and simultaneously, without a global record of time. Calculations are performed for an isothermal hydrogen plasma with mass ratio $\mu = m_i/m_e$

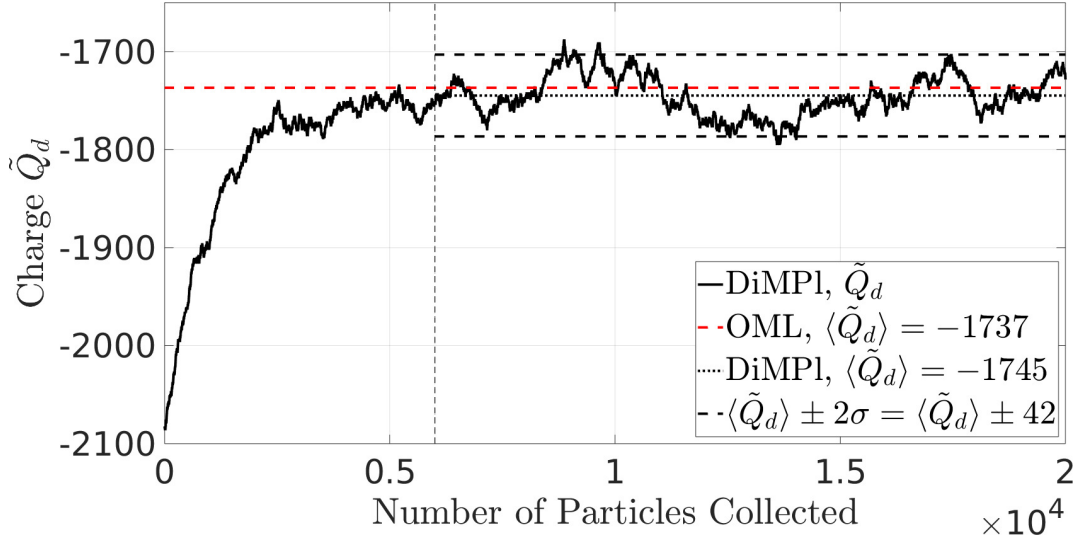


Figure 28: Normalised dust charge, $\tilde{Q}_d$, as a function of number of particles collected, with a self-consistent surface charge (black solid) as compared with the theoretical OML result (red dashed) from equation 42. The vertical line at 6000 collected particles indicates the start of the equilibrium measurement with mean (black dotted) and two standard deviations (black dashed). Reprinted from [188], with the permission of AIP Publishing.

where m_i and m_e are the hydrogen ion and electron masses respectively. The default simulation parameters used were $b = 5$, $z_0 = 50a_d$ and $\gamma_0 = 0.01$ with default plasma parameters $T_i = T_e = 1\text{eV}$ and $n_0 = 10^{18}\text{m}^{-3}$ unless otherwise stated. When calculating the floating potential of the dust grain, the potential varies dynamically as charges are collected. After a transitory equilibration phase, a time independent measurement of the equilibrium charge can be made, as shown in figure 28 for $\beta_i = 0.01$. The mean potential is calculated after reaching equilibrium, indicated by the dashed vertical black line, from the average normalised charge $\langle \tilde{Q}_d \rangle$ where $Q = e\tilde{Q}_d$. The uncertainty on the mean decreases with increasing number of particles collected. The results form a non-Markovian time series since a given measurement of the equilibrium charge, $\tilde{Q}_d$, is correlated with subsequent and previous measurements of charge, affecting the interpretation of the standard errors, as described in detail in the appendices 10.2. For this reason, the standard deviation can be taken as an over-estimate of the standard error on the mean, simultaneously representing the expected natural charge fluctuations.

The speed of convergence to equilibrium can be enhanced by weighting plasma particles with a fictitious charge, SZe , and having this weighting, S , vary throughout the simulation. This is useful when simulating large dust particles, which must collect many more charged particles to attain the same surface potential as small dust particles. Weighting the charge of ions and electrons by the same amount increases the amplitude of both currents and consequently the rate of approach to equilibrium without altering the equilibrium value. By default, the scale of the charge is set based on an expected equilibrium surface potential, χ_a , of order $\mathcal{O}(1)$, so that the initial weighting is set by $1 = e\phi_a/k_B T_e = eQ_d/(4\pi\epsilon_0 a_d k_B T_e)$ with $Q_d = eS$ so that $S = 2\pi\epsilon_0 a_d k_B T_e / e^2$. However, this additional weighting increases the charge fluctuation so that a measurement of the equilibrium solution becomes

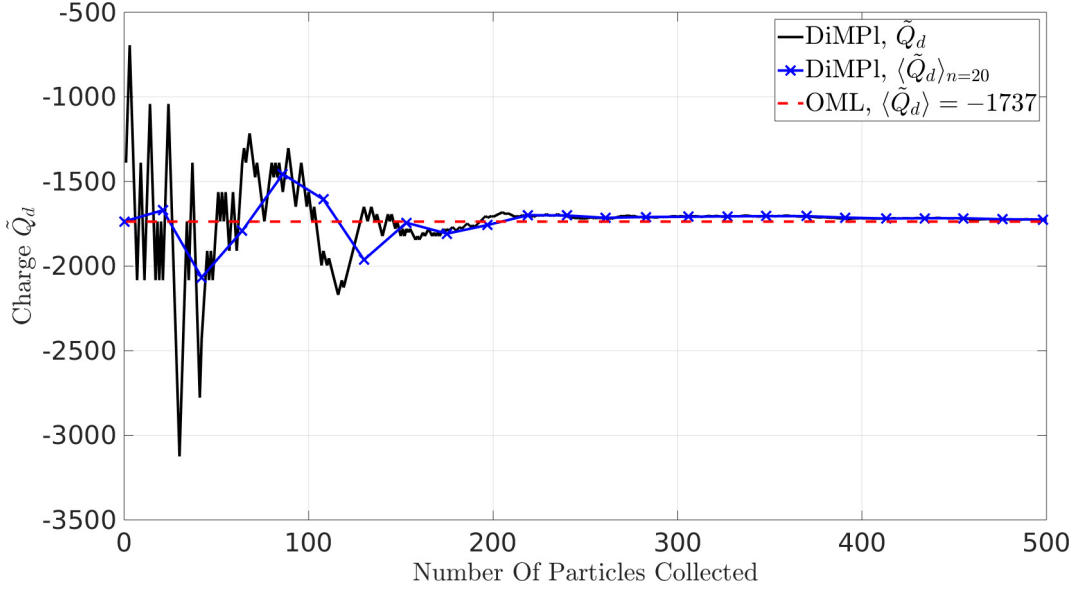


Figure 29: Normalised dust charge, $\tilde{Q}_d$, as a function of number of particles collected, with a self-consistent surface potential (black) and variable charge scale as compared with the theoretical OML result (red dashed) from equation 42. Convergence is achieved after collecting a total of 400 particles with S halving after collecting $N = 20$ particles if the sign of the difference in average charge, $\langle \tilde{Q}_d \rangle_{n=20}$ (blue) changed.

increasingly imprecise. For this reason, the mean charge and change in charge for a group of particles is calculated. If the sign of the difference in charge changes between groups, then for a sufficiently large group this is indicative of an approach to an equilibrium solution and the charge weighting is halved, $S \rightarrow S/2$. This process can be tailored by altering the number of charges in a group and the number of particles collected in the final group that are used to calculate the equilibrium solution as described previously. An example of this technique is shown in figure 29, with the same equilibrium measurement as in figure 28 performed 5 times faster. Through this technique, the efficiency and precision of the measurement is improved and more challenging regimes are made accessible.

It may appear from figure 28 that a numerical solution of the time dependent equation $I(t) = dQ(t)/dt$ has been performed. Upon collection of an ion or electron, the dust surface potential is updated instantaneously and all other calculations being performed in parallel are sensitive to this change in potential. Because the rate of collection of ions and electrons is not related to a physical quantity but depends upon the computational time for a trajectory, estimates of the time period over which charge is accumulated by the dust grain cannot be determined directly. However, for a sufficiently large number of injected particles and a fixed surface potential, the time averaged flux at the dust surface can be assumed to be constant and related to the flux of the number of particles at the outer surface to calculate the time averaged behaviour. This was exploited to measure the time averaged evolution of quantities by fixing the dust surface potential in analogy with a biased probe.

From the particle flux at the outer boundary, an artificial simulated time, t_G , can be defined from the time taken for G_s particles of a particular species to cross the outer boundary, $\Gamma_s A_s = G_s/t_G$. In the case where $u = 0$,

$$t_G = \frac{G_s}{\sqrt{2\pi} R_s^2 n_0 v_{T,s}}, \quad (118)$$

can be calculated for a given G_s in the simulation, all other parameters being specified as initial conditions. The currents, I_s , are calculated by counting the amount of charge collected by the sphere, eC_s , and the time to collect that many particles t_G using equation 118,

$$\tilde{I}_s = \frac{I_s}{I_{s,0}} = \frac{eC_s}{t_G} \frac{1}{I_{s,0}} = \frac{C_s R_s^2}{2G_s a_d^2}, \quad (119)$$

where as before $I_{s,0} = 2\sqrt{2\pi} a_d^2 e n_0 v_{T,s}$ is the thermal current through a spherical surface. The measurement error is dependent on the number of particles recorded. At a fixed surface potential the dust grain acts like a biased probe and can be benchmarked against previous numerical results from probe theory as discussed in the following section.

4.2.4 Verification & Validation

To ratify the implementation of the DiMPL algorithm, tests are performed to check that expected physical criteria are met and results are benchmarked against predicted theoretical and previous numerical results.

First, the degree of conservation of energy of particles was investigated. In the absence of any electric fields, the percentage deviation of the energy of ions is found to be of the order of machine precision $\sim 10^{-15}$. The change in energy when using the Boris algorithm is bounded, so local truncation errors dominate in this case meaning smaller time steps produce a greater deviation from energy conservation. When an electric field is present, the variance in energy of an ion orbit is sensitively dependent on the forces over a particle trajectory however the greatest errors are found to be smaller than $\sim 1\%$ for $\gamma_0 = 0.01$. Another metric used to characterise the precision of calculated trajectories is the displacement from the origin of collected particles which is expected to equal the radius of the sphere, $|\vec{r}| = 1$. Irrespective of the dust charge, the mean absolute error of the final position of particles was found to be proportional to time-step size, with $\gamma_0 = 0.01$ corresponding to absolute error of less than 1% in final position.

Figures 30a and 30b show examples of ion orbits calculated by DiMPL in the x-z and x-y plane with different impact factors $b = 2.0$ & 3.0 and a constant injection velocity $v_{\parallel} = 0.2v_{T,i}$ directed in the $-\hat{z}$ direction close to a negatively charged sphere with a Coulomb electric field using equation 116. For particles with a large impact parameter, the electric field displaces the gyro-centre and redistributes an amount of the particles momentum perpendicular to $\hat{z}$. At some intermediate impact factor, the magnetic envelope calculated from equation 113 and previously described in section 4.1.4 grows, leading to trapped orbits. Particles exhibit guiding centre motion with a clear $\underline{E} \times \underline{B}$ drift and reflect multiple times along the

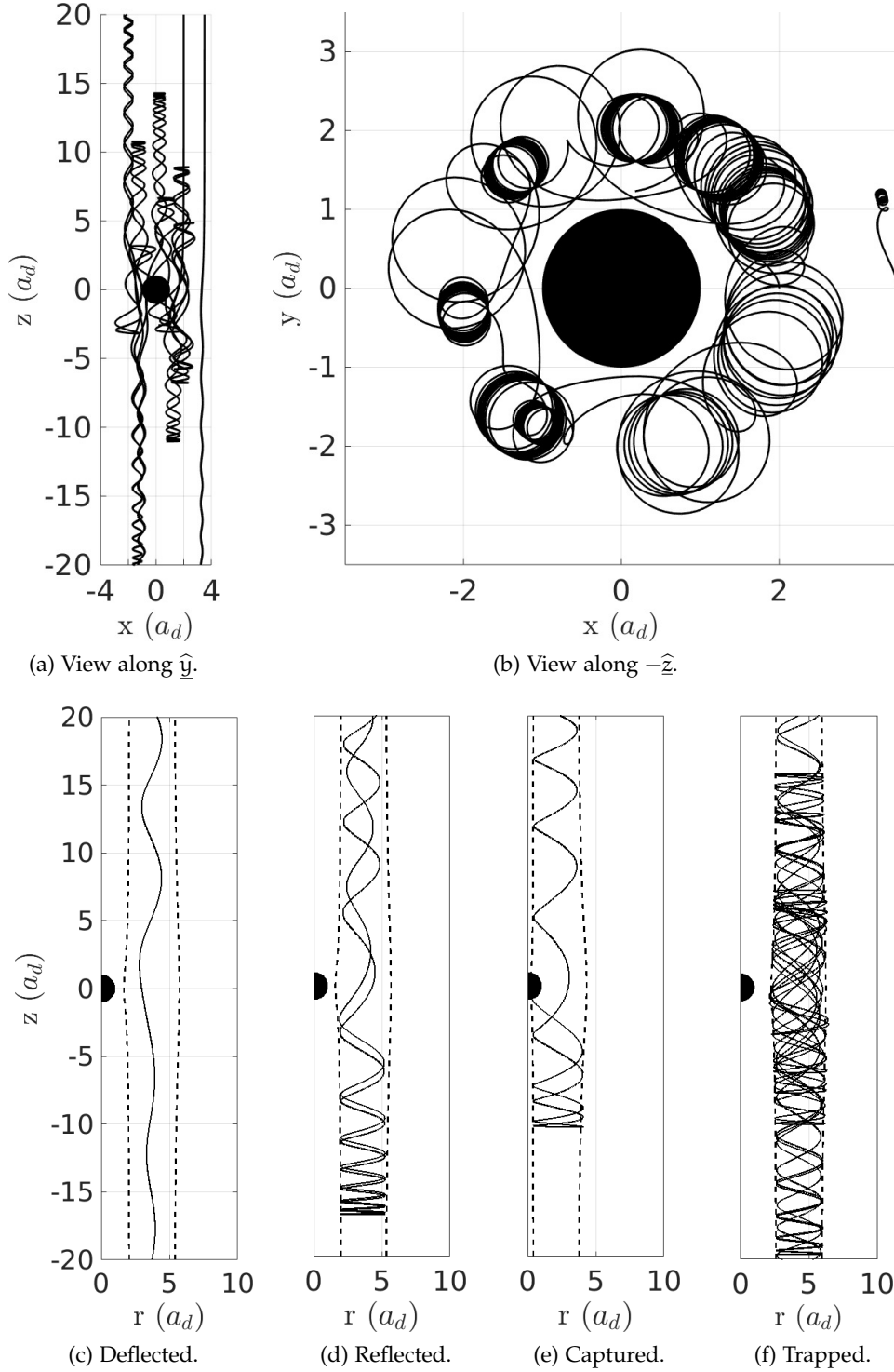


Figure 30: Example ion orbits (solid lines) in the vicinity of a negatively charged sphere (black circle) with fixed initial velocities $v_{\parallel} = -0.2v_{T,i} \hat{z}$ (30a and 30b) and randomly directed initial velocities demonstrating the confining magnetic envelopes calculated from equation 113 (black dashed lines) of deflected (30a), reflected (30d), captured (30e) and transiently trapped (30f) ions for $\beta_i = 0.5$. The trajectories here can be compared directly with previous work [184, Figure 2]. Reprinted from [188], with the permission of AIP Publishing.

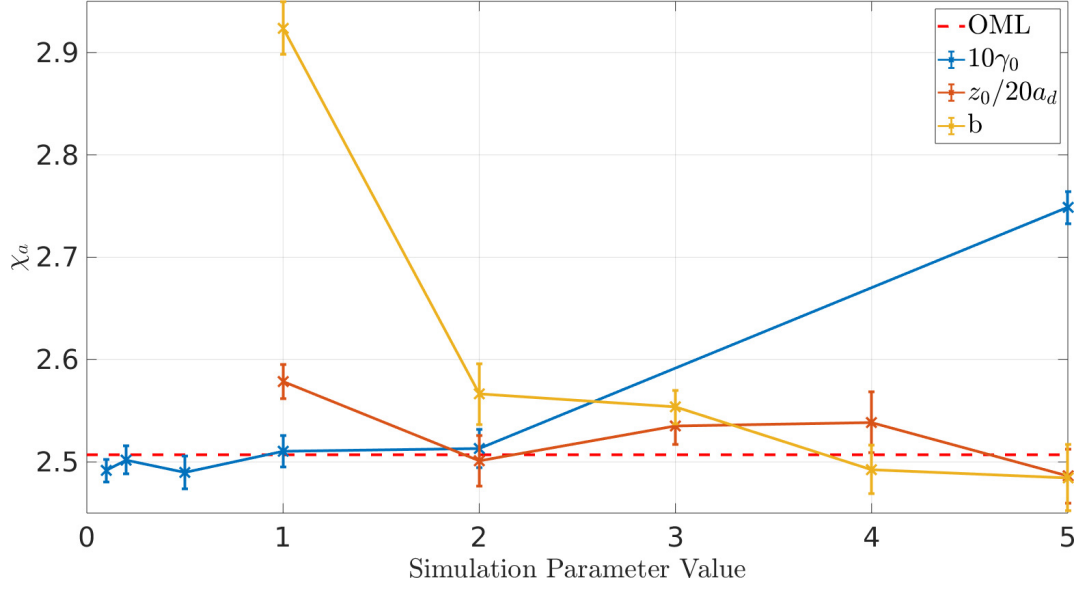


Figure 31: Normalised potential as a function of simulation parameters γ_0 , b , z_0 as compared with the theoretical prediction of OML (red dashed) given by equation 42 with default values $\gamma_0 = 0.01$, $b = 5$ and $z_0 = 50.0a_d$.

$\hat{z}$ axis. For smaller impact factors, the guiding centre motion becomes disrupted by the strong electric potential gradients and the magnetic envelope intersects the sphere making collection of particles possible. These figures demonstrate the types of collisionless orbital motion produced in DiMPL, in agreement with previous observations [152, 184] and the adherence to the theoretical boundaries defined by conservation of energy and canonical angular momentum. The tests described so far provide evidence for the correct implementation of the simulation method.

The results of DiMPL were benchmarked against existing theoretical and numerical calculations in a series of test cases. The equilibrium normalised potential for small dust, $\tilde{\lambda}_d \rightarrow \infty$, without flow or a magnetic field is expected to be well approximated by OML theory as calculated from equation 42. Results for a hydrogen plasma with $T_i = T_e$ have already been shown to reproduce equilibrium dust grain charges consistent with OML predictions in figure 28. The dependency of the accuracy of this result as a function of time step factor γ_0 , impact factor b and injection plane height z_0 is shown in figure 31. The associated simulation times needed to acquire these results shown in figure 32 have a strong dependence on time step factor γ_0 and linear dependence on b and z_0 . These tests have been used to determine optimal generic values for the simulation parameters, $\gamma_0 = 0.01$, $b = 5$ and $z_0 \geq 50a_d$ for $\chi_a = -2.5$. Note that the optimal choice of b and z_0 will depend upon χ_a and the magnitude of χ_a relative to β_i .

The case of flowing plasmas in the low magnetic field limit were also investigated and compared with the predictions of SOML theory previously described in section 3.2.3. Figure 33 shows agreement between the predicted normalised potential of DiMPL and SOML as a function of normalised flow speed using simulation parameters $b = 5$ and $z_0 = 101a_d$.

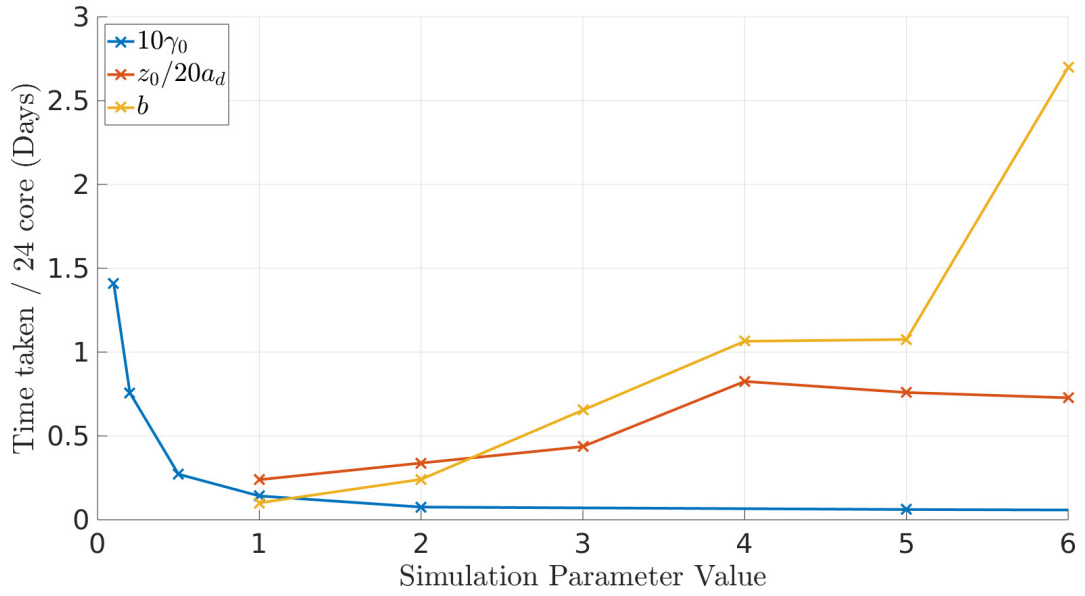


Figure 32: Time taken in days to produce results of figure 31 using 24 core nodes as a function of simulation parameters γ_0 , b and z_0 with default values $\gamma_0 = 0.01$, $b = 5$ and $z_0 = 50.0a_d$. Decreasing γ_0 increases simulation time exponentially whilst increasing z_0 or b increases simulation time linearly.

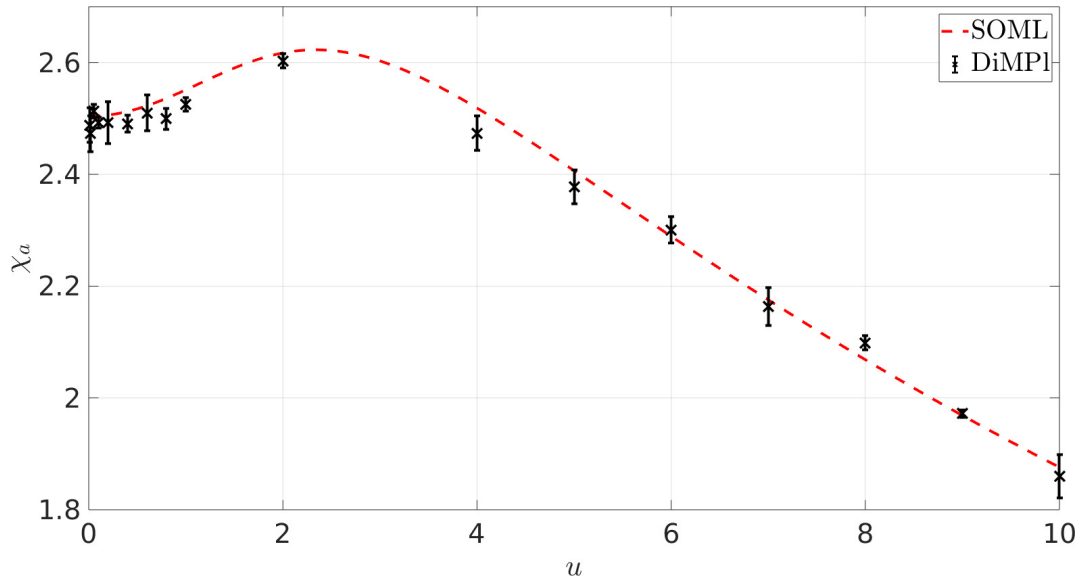


Figure 33: Normalised potential, χ_a , as a function of flow velocity (markers) for a Coulomb potential, $\bar{\lambda}_d = \infty$, using equation 116 with $\beta_i = 0.01$ as compared with the results of SOML theory (dashed line) from equation 64. Adapted from [188], with the permission of AIP Publishing.

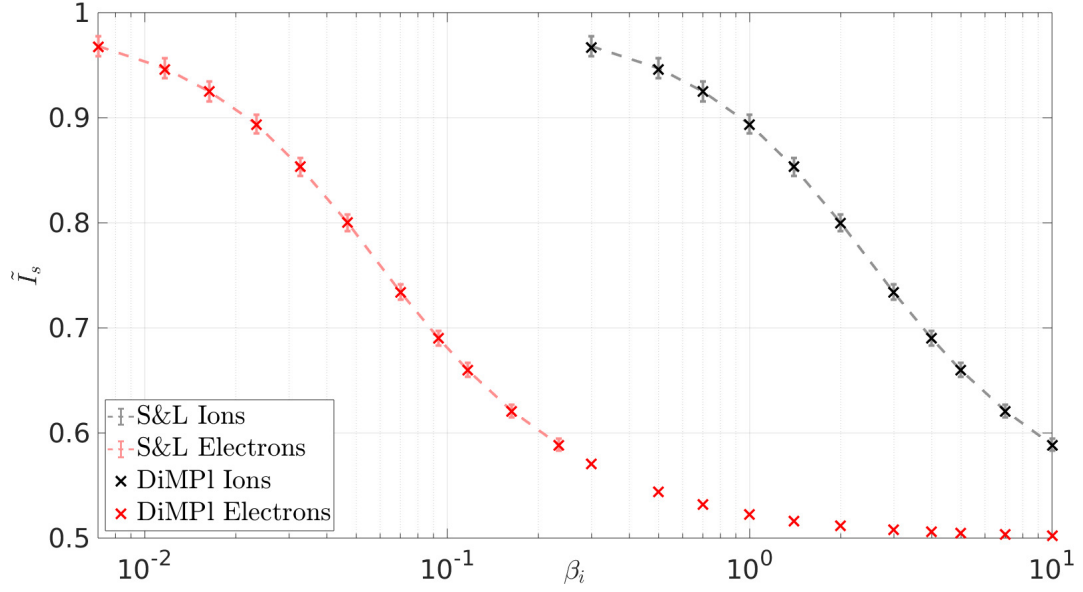


Figure 34: Ion and electron currents normalised to their respective thermal currents calculated using equation 119 as a function of the ion magnetisation parameter, β_i , for electrons (red markers) and ions (black markers) and compared with previous numerical results [182, Figure 17][184, Table 1] (faded dashed) for an uncharged sphere. Adapted from [188], with the permission of AIP Publishing.

Using equation 119, the currents of the attracted and repelled species as a function of fixed dust potential and magnetic field strength were measured and compared to previous numerical results of SL [184, Table 1]. Without an electrostatic potential due to the dust grain, simulation performance is greatly improved by placing the injection planes very close to the sphere at $z_0 = 1.01a_d$ allowing a larger impact factor $b = 10.0$ to be used. The results for uncharged dust grains shown in figure 34, show excellent agreement with these previous numerical results [182, 184] for both electrons and ions over the full range of ion magnetisation parameter. Additionally, the expected theoretical high field, $I = I_{s,0}/2$, and low field, $I_s = I_{s,0}$, limits are reproduced, see section 4.1.3.2. The method of solution is symmetric for both ions and electrons with the normalised currents, $\tilde{I}_s$, calculated in DiMPL, validating the implementation of each species. The largest values of ion magnetisation parameter in a tokamak are expected to be around $\beta_i = 2$ for large dust $a_d = 100\mu\text{m}$ in the SOL of a hydrogen plasma with $B = 2\text{T}$ and $T_i = 1\text{eV}$. For dust in a tokamak, figure 34 demonstrates that the electron current is expected to be significantly affected for even small dust while only large dust with $a_d \geq 50\mu\text{m}$ has an ion current that is appreciably affected by the magnetic field.

Figure 35 shows the variation of the attracted species current for different fixed floating potentials and magnetic field strengths as compared with the results of SL [184, Table 1]. Good agreement within measurement error is observed for $\beta_i = 1.0, 1.4$ & 3.0 with table 3 giving the full range of results in the appendices 10.3. The replication of the previous results of SL and others [182, 184] grants confidence in the use of DiMPL for calculating the equilibrium floating potential and time averaged behaviour using the methods described in section 4.2.3.

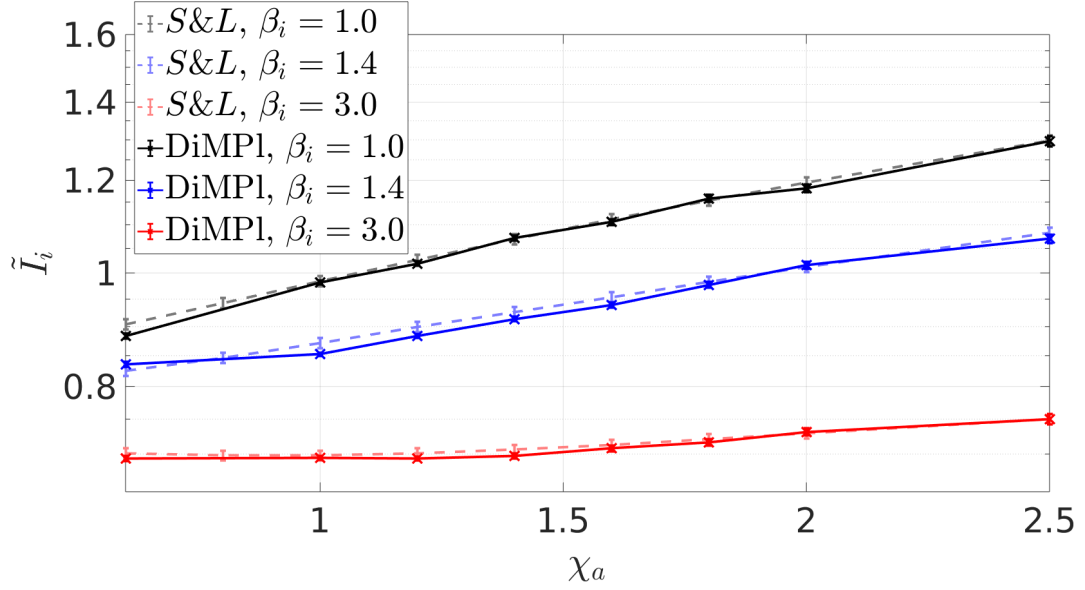


Figure 35: Ion currents, $\tilde{I}_i$, as a function of the normalised floating potential, χ_a , for different values of ion magnetisation parameter, β_i , displayed in different colours for DiMPL (solid crosses) and compared with previous numerical results (faded dashed) [184, Table 1]. Reprinted from [188], with the permission of AIP Publishing.

4.2.5 Limitations of the Debye-Hückel Potential

The imposition of a Debye-Hückel potential in DiMPL provides only an approximate description to the potential distribution surrounding a large dust grain. Consequently, this does not model the self-consistent potential distribution formed around a dust grain when it is large compared with the plasma shielding length scale. Due to the internal assumptions, the Debye-Hückel model is only strictly valid when $e\phi \ll k_B T_e$, $1 \ll \tilde{\lambda}_d$, $u = 0$ and $\beta_i = 0$. The model makes no account of a pre-sheath structure and is relevant for only stationary or equilibrium ions. In the absence of magnetic fields, the breakdown of the model has been observed by PIC codes in flowing plasmas leading to non-spherically symmetric potentials [156, Figure 3] and radial potential profiles that differ from Debye-Hückel potentials [191, Figure 7] for $\tilde{\lambda}_d \geq 1$, $\tau = 1$ and $u = 1$. However it has also been noted that excellent agreement with OML predictions of the surface potential is maintained even with the asymmetric, non Coulombic potentials [156, Figure 2].

The accuracy of imposing a Debye-Hückel potential for different dust sizes with normalised Debye lengths $\tilde{\lambda}_d$ in the low magnetic field limit, $\beta_i = 0.01$, was tested by comparison to previous numerical solutions [144] of orbital motion theory [163, 164, 192, 193], as shown in figure 36 for $\tau = 1.0$ and $\tau = 0.1$. The parameter $\tilde{\lambda}_d$ was varied in DiMPL by altering the dust grain size whilst keeping all other parameters fixed. The floating potential calculated using DiMPL for different values of $\tilde{\lambda}_d$ shows excellent agreement for $\tilde{\lambda}_d \geq 0.5$ with $\tau = 1.0$ and for $\tilde{\lambda}_d \geq 5.0$ with $\tau = 0.1$. At larger dust sizes, the potential calculated by DiMPL is over estimated, with the over estimation being worse for small values of τ . The large dust limit

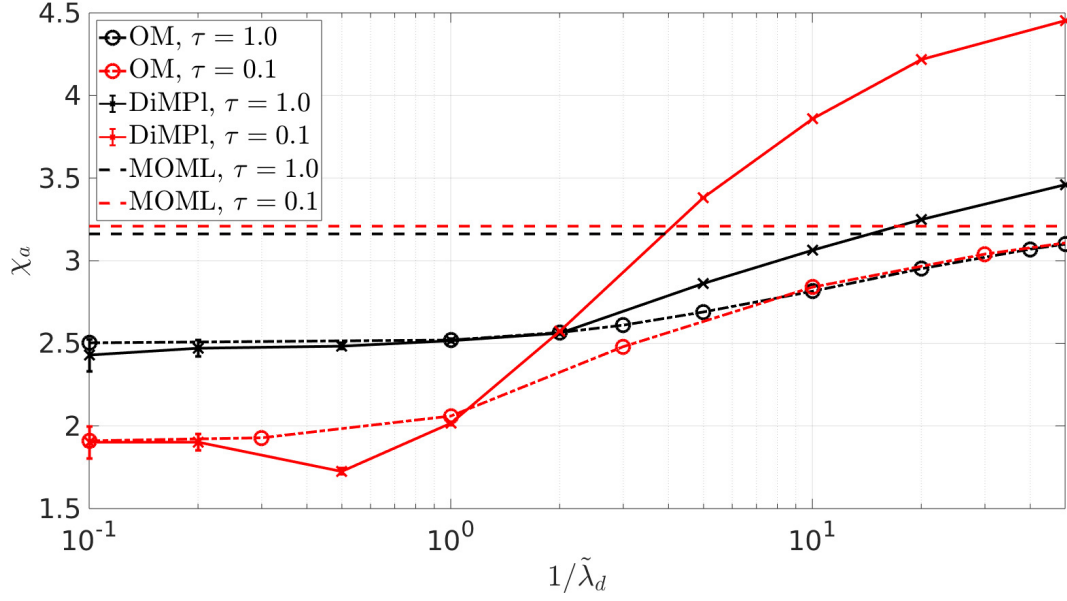


Figure 36: Normalised potential, χ_a , as a function of the inverse normalised Debye length, $1/\tilde{\lambda}_d$, for ion magnetisation $\beta_i = 0.01$ with $\tau = 1.0$ (black solid) and $\tau = 0.1$ (red solid) as compared with results of Kennedy (dash dotted) [144] and the MOML result (dashed) from equation 57.

predicted by MOML theory, section 3.2.2 equation 58, is approached by the orbital motion results whilst the Debye-Hückel potential in DiMPl exceeds this prediction.

The agreement between these results for the dust surface potential for small dust demonstrates the validity of approximating the potential using a Debye-Hückel model in a $T_i = T_e$ plasma when $\tilde{\lambda}_d \geq 0.5$ and no magnetic fields are present, $\beta_i \ll 1$. The results generated depart from the expected values at approximately the same point at which the OML approximation of section 3.2.1 becomes invalid. The agreement up to this point however demonstrates the insensitivity of the surface potential to the exact potential distribution. Furthermore, the implementation of a Debye-Hückel potential permits much faster computational timescales since the condition that the electrostatic potential at the boundary, $\chi(R, z = z_0)$, be small is satisfied with smaller values of z_0 . The Debye-Hückel potential distribution is therefore useful to expedite calculations for large dust and to demonstrate the insensitivity of the surface potential to the radial form of the potential.

4.2.6 Stationary Dust

In this section, results acquired with DiMPl for the charging behaviour of spherical dust in magnetic fields are presented, beginning with the case of small stationary dust. The presence of a magnetic field is known to alter the angular distribution of the particle flux across the spherical surface, causing particles to become collected preferentially at the poles. The distribution of the normalised final vertical position, $\tilde{z} = z/a_d$, of ions and electrons when collected was calculated and plotted in figure 37 for different magnetic field strengths. For larger magnetic field strengths, the collection position becomes asymmetric, indicating that particles are being col-

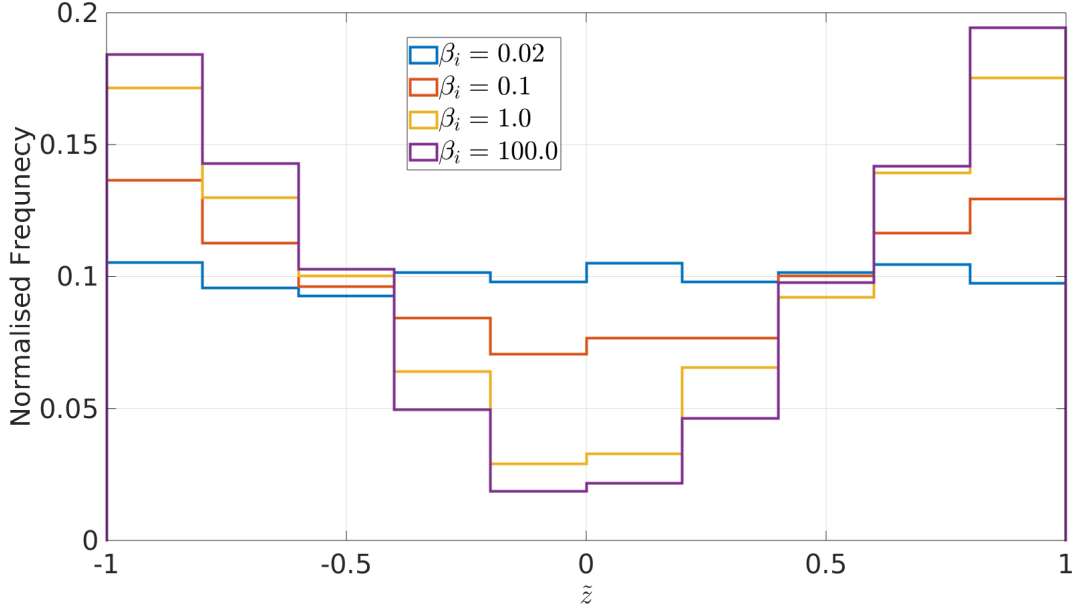


Figure 37: Distribution of the $\tilde{z}$ coordinate of particles at collection for different ion magnetisations, β_i .

lected preferentially at the poles of the sphere in good agreement with previous observations [186, Figure 7]. This geometric phenomena leads to net surface currents as charge relocates to form an equipotential surface within the conductor in addition to causing asymmetric collection of momentum as is discussed in more detail in chapter 5.

The potential as a function of temperature ratio, τ , of small dust was calculated with DiMPL using a Coulomb electric field in the low magnetic field regime, $\beta_i \leq 0.2$ and compared to the theoretical predictions of OML, given by equation 43. The temperature ratio was varied by changing T_i whilst keeping all other parameters fixed with $z_0 = 100.0a_d$ and $b = 3.0$. This value of z_0 should be proportionally larger for small T_i since the ratio of the electrostatic potential energy, $e\phi(R, z)$, to kinetic energy, $\sim k_B T_i$ at the boundary, $e\phi_a a_d / k_B T_i z_0 = \chi_a a_d / \tau z_0$, is no longer small. Consequently, a systematic error in the results occurs which was corrected for by systematically increasing the potential by a factor of $\chi_a a_d / z_0 \tau$ in plotting the results of figure 38. After this transformation, the dependence of surface potential on temperature ratio of DiMPL shows good agreement with OML theory predictions for $\beta_i = 0.01$. The effect of increasing magnetic field is to increase the potential at small τ and decrease it at large τ . At high values of τ , the reduction in electron current relative to ion current for $\beta_i \leq 0.2$ causes a decrease in the potential. At low values of τ however, the reduced electron current due to the magnetisation of electrons is opposed by their relatively large thermal energy as compared with ions, causing an increase in potential. For small ion magnetisation parameters $\beta_i \leq 0.2$, the effect of the magnetic field is simply to shift the dependence on τ of the potential to smaller values of τ and slightly decrease the potential peak.

Focusing on the case of an isothermal plasma, the results of DiMPL are compared with the PIC code results of Lange [155] and novel measurements made with the

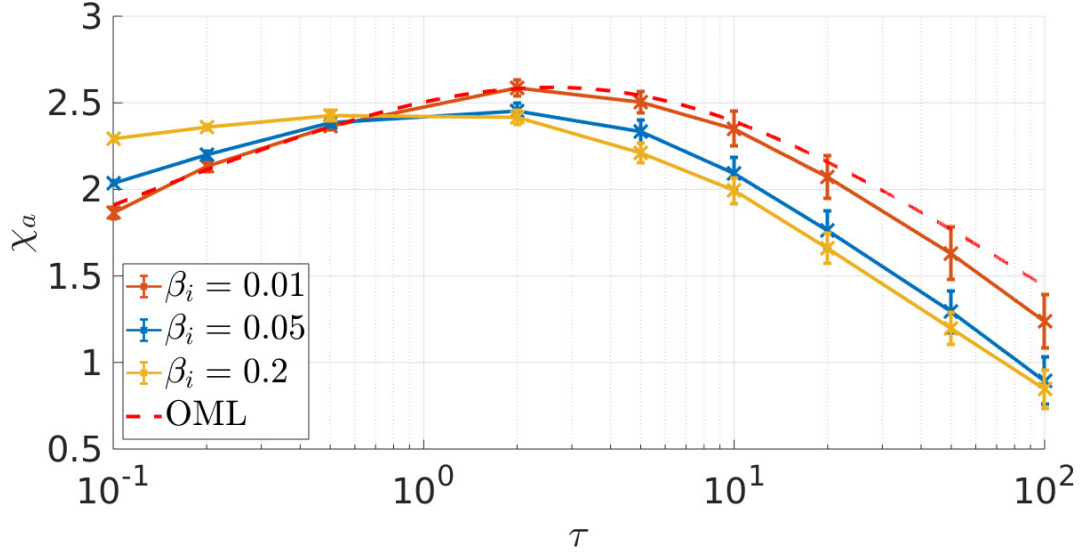


Figure 38: Normalised potential, χ_a , as a function of temperature ratio, τ , for different values of ion magnetisation β_i . Agreement with OML predictions given by equations 43 is seen for $\beta_i = 0.01$.

plasma octree code, *pot* [152] for different dust sizes. Similar to DiMPL, *pot* is a N-body Monte Carlo code developed at Imperial that simulates electrons and ions in proximity to a charged, conducting spheroid [143]. The benefit of *pot* is that it solves the entire N-body problem, making the result inherently self-consistent in its implementation of collisional and sheath effects. The principle assumption is that electrons and ions arrive from the distant plasma uniformly distributed over a spherical surface with a Maxwellian velocity distribution. Each individual particle is tracked in a constant uniform magnetic field and an electric field sensitive to the influence of all other particles in the simulation domain as well as the central charged spherical dust grain. The recent PIC code results of Lange presented have been decreased by 5% and plotted with error bars reflective of this reported systematic error [155].

The floating potential as a function of ion magnetisation parameter was investigated over a range $0.01 \leq \beta_i \leq 10.0$ using a Coulomb electric field and a Debye-Hückel electric field given by equation 117 with $\tilde{\lambda}_d = 15$. In figure 39, these results are compared against the results of *pot* for $\tilde{\lambda}_d = 15$ as well as the PIC code results of Lange [155] for $\tilde{\lambda}_d = 9.598$. In *pot*, this value of $\tilde{\lambda}_d$ was attained by changing the plasma density whilst maintaining $n_i = n_e$ and $T_i = T_e$.

All three codes predict the same qualitative trend for $1 \ll \tilde{\lambda}_d$, with a slight reduction in potential around $\beta_i \simeq 0.15$ followed by a gradual increase to much higher potentials for $\beta_i > 1$. The simulations therefore also reproduce the trend expected from theoretical predictions of the variation in ion and electron currents with increasing magnetic field, as is exemplified by figure 34. All results replicate the theoretical OML result at low ion magnetisation $\beta_i \leq 0.05$ of $\chi_a = 2.504$. The data of *pot* for $\tilde{\lambda}_d = 15$ is in agreement with both implementations of the potential field in DiMPL over the examined range of β_i . When using a Coulomb potential in DiMPL, $\tilde{\lambda}_d = \infty$, the results produced are very similar to the case of $\tilde{\lambda}_d = 15$,

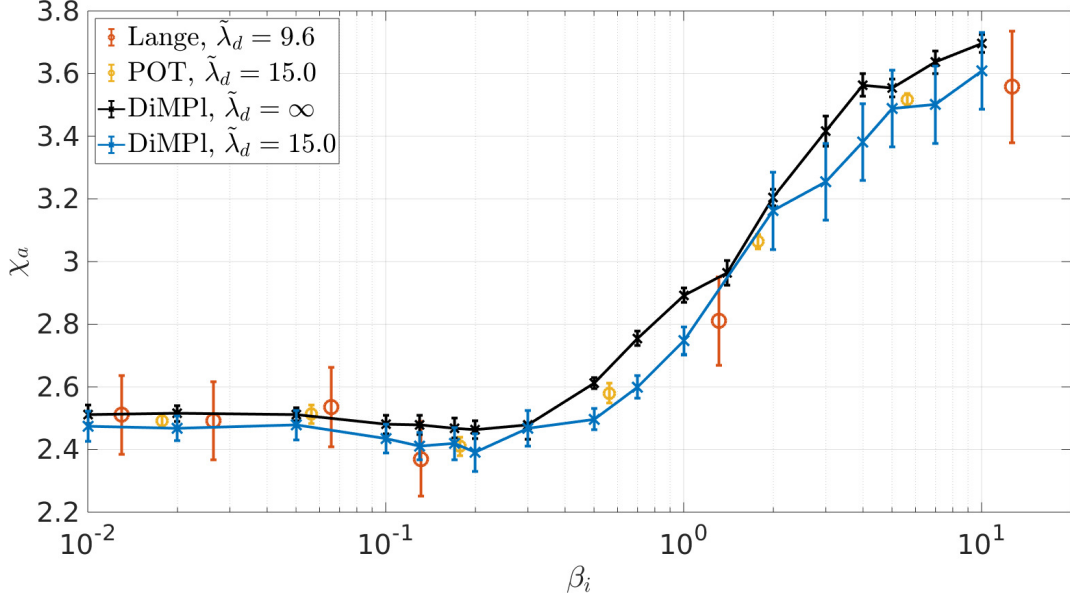


Figure 39: Normalised potential, χ_a , as a function of the ion magnetisation parameter, β_i , as measured by DiMPl for a Debye-Hückel potential using equation 117 (blue) and DiMPl using equation 116 (black), pot [152] for a normalised Debye length of $\tilde{\lambda}_d = 15$ (yellow), alongside the results of Lange [155] for $\tilde{\lambda}_d = 9.6$. Reprinted from [188], with the permission of AIP Publishing.

though the potential is slightly elevated, particularly at higher β_i . This is most likely caused by the deviation from the plasma potential at the injection plane in the Coulomb case, $\propto \chi_a a_d / z_0$, with height $z_0 = 101 a_d$ which is less impactful with a rapidly decaying potential. Additionally, the Debye-Hückel potential reduces the attractive force affecting ions at large distances meaning fewer ions are deflected towards and potentially captured by the dust grain.

The floating potential of larger dust was calculated by DiMPl for normalised Debye lengths of $\tilde{\lambda}_d = 3$ and $\tilde{\lambda}_d = 1$ over the same range of ion magnetisation parameters, $0.01 \leq \beta_i \leq 10.0$. These results are plotted in figure 40 and are compared against the results of pot for fixed dust grain radii $a_d = 10 \mu\text{m}$ with $\tilde{\lambda}_d = 3$ and $\tilde{\lambda}_d = 1$ in addition to the PIC code results of Lange [155] for $\tilde{\lambda}_d = 1.9$ and $\tilde{\lambda}_d = 1.0$ and SCEPTIC [186] for $\tilde{\lambda}_d = 2.6$. For medium sized dust, $\tilde{\lambda}_d \simeq 1.9 - 3$, agreement is seen between the four simulations for $\beta_i \leq 0.1$, whilst for larger β_i , the results of pot start to deviate to lower values of normalised potential χ_a . For large dust, $\tilde{\lambda}_d \simeq 1$, this feature is further enhanced, with the results of DiMPl and Lange continuing to agree whilst pot deviates to increasingly smaller potentials in the range $0.1 \leq \beta_i \leq 10$.

Overall, the dependence of χ_a on β_i was found in general to be relatively insensitive to dust size for isothermal plasmas when $1 \leq \tilde{\lambda}_d$ in agreement with previous observations [156]. This is also in good agreement with the results of Lange who measured minimal variation in surface potentials for dust sizes $\tilde{\lambda}_d \leq 0.5$ [155]. Table 1 shows the results for various dust sizes with simulation parameters, $b = 5.0$ and $z_0 = 5\lambda_d$. A small difference occurs at greater magnetic field strengths, where the potential decreases for larger dust. The observed similarity between results can

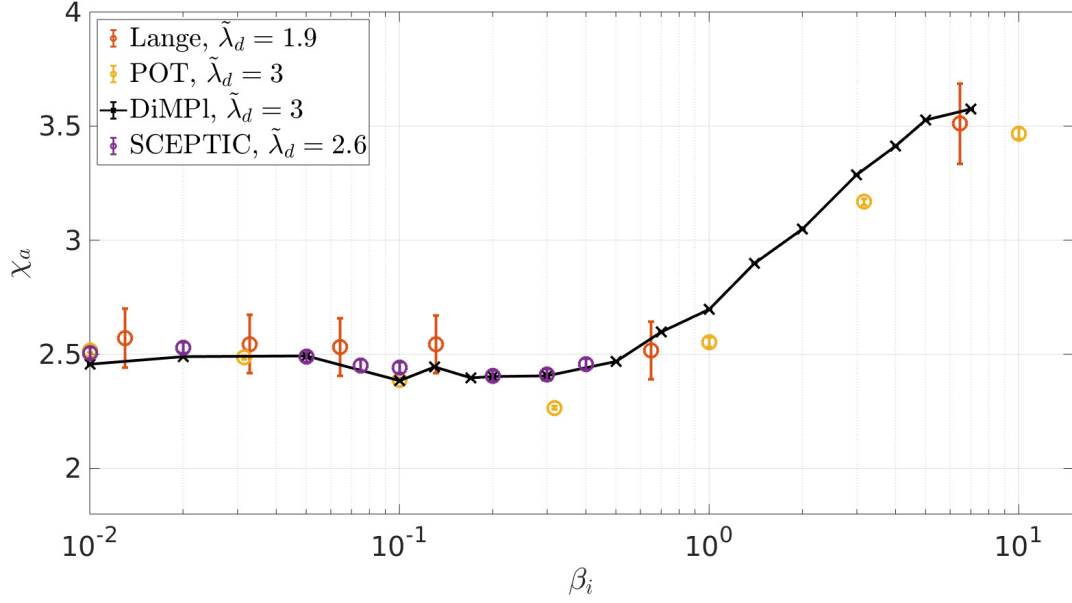
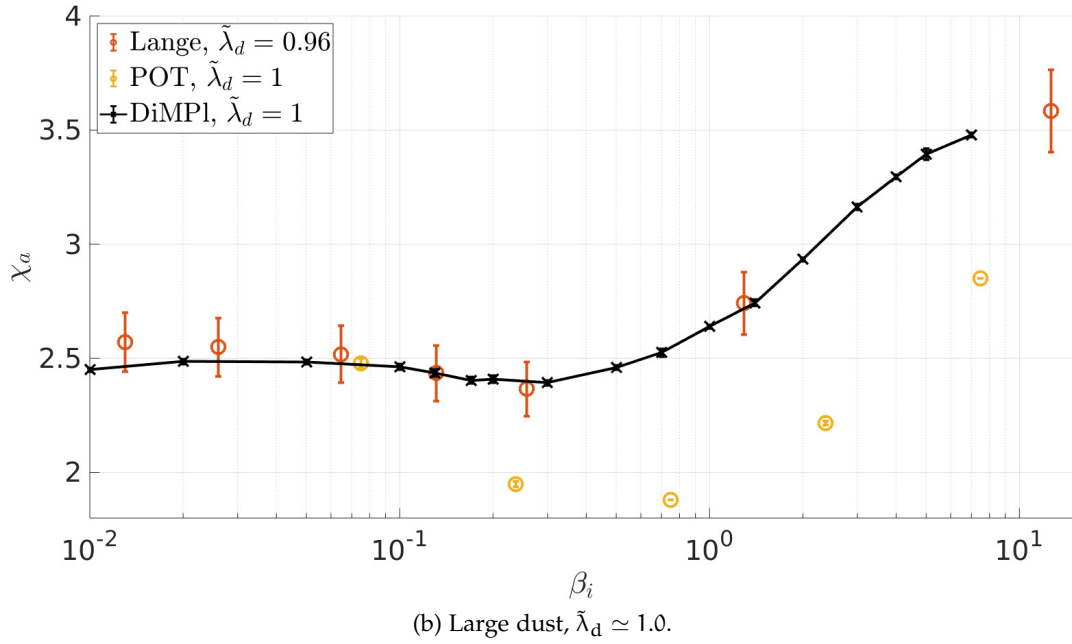


Figure 40: Normalised surface potential, χ_a , as a function of ion magnetisation parameter, β_i , as measured by SCEPTIC [157] (purple) for $\tilde{\lambda}_d = 2.6$, Lange [155] (orange) for $\tilde{\lambda}_d = 1.9$, (40a) pot [152] (yellow) and DiMPL (black) both with $\tilde{\lambda}_d = 3$, for medium sized dust (40a). For large dust, (40b), the results of pot and DiMPL for $\tilde{\lambda}_d = 1$ are plotted alongside the results of Lange [155] with $\tilde{\lambda}_d = 0.96$, (40b).

		$\tilde{\lambda}_d$			
		1.0	2.0	5.0	10
β_i	0.05	2.471 ± 0.002	2.439 ± 0.006	2.47 ± 0.01	2.46 ± 0.01
	0.5	2.454 ± 0.002	2.454 ± 0.003	2.48 ± 0.01	2.45 ± 0.04
	5.0	3.407 ± 0.003	3.487 ± 0.007	3.5 ± 0.5	-

Table 1: Normalised surface potential, χ_a , with standard deviations for different values of ion magnetisation parameter, β_i and normalised Debye lengths $\tilde{\lambda}_d$.

be understood to be a continuation of the insensitivity of the radial form of the potential distribution for $0.5 \leq \tilde{\lambda}_d$ in the case with no external magnetic field as was shown previously in figure 36. The presence of a magnetic field therefore does not appear to substantially affect the floating potential when $\tilde{\lambda}_d$ is of the order of the dust size or larger.

With a magnetic field being present, some previous PIC code results have shown that the potential distribution ceases to be spherically symmetric, particularly for cold plasmas with small electron Debye lengths [157]. A possible source of the discrepancy of the pot results may arise from the self-consistent determination of the potential distribution which is approximated by a Debye-Hückel potential distribution in DiMPL. The potential distribution as measured in pot in the vicinity of a dust grain for different values of $\beta_i = 0.316, 1.0, 3.16$ and 10 is displayed in figure 41. A rapidly falling radial potential is observed which is approximately spherically symmetric, regardless of magnetic field strength. This result supports the implementation of a spherically symmetric potential distribution in DiMPL in this regime and suggests an alternative explanation for the discrepancy in results is necessary.

A small degree of asymmetry was observed in the potential distribution parallel and anti-parallel to the magnetic field axis as measured by pot. The source of this asymmetry is not well understood, though it was observed similarly when flipping the direction of $\hat{\mathbf{B}}$. Additionally, two discontinuities in the potential distribution occur at around $4a_d$ and $7a_d$. These peculiarities and the disagreement of pot measurements of equilibrium surface potential with the results of other PIC code results suggest differences exist in the computational method.

The radial potential averaged across all polar angles as measured by pot for $\beta_i = 0.316, 1.0, 3.16$ is compared to a Coulomb potential distribution in figure 42. The self-consistent potential distribution is well approximated by a Coulomb potential with a maximal deviation of 10%. The greatest deviation occurs over moderate distances $1 \leq \tilde{R} \leq 6$ where there is a significant dip in potential. This shows that the dust potential is not substantially screened by the plasma for $\lambda_d \geq 3.0$ in pot. The self-consistent electrostatic potential distribution established in pot therefore appears similar to the approximation of a spherically symmetric Coulomb potential in DiMPL and unlikely to be the source of large differences in results between the two codes.

Given the similarities in the simulation methodology of pot and DiMPL, it is possible to isolate the most likely causes of the discrepancy in the results for the

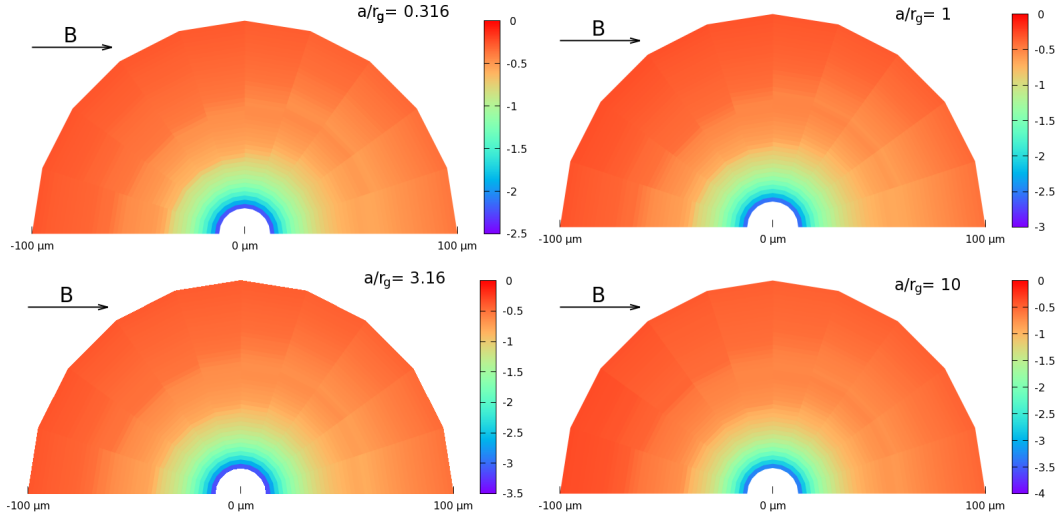


Figure 41: Polar plots of electrostatic potential measured by pot in the vicinity of dust grains with radii $a_d = 10\mu\text{m}$ using $\tilde{\lambda}_d = 3$ and ion magnetisation parameter values of $\beta_i = 0.316, 1, 3.16$ and 10 (credit Joshua Holgate).

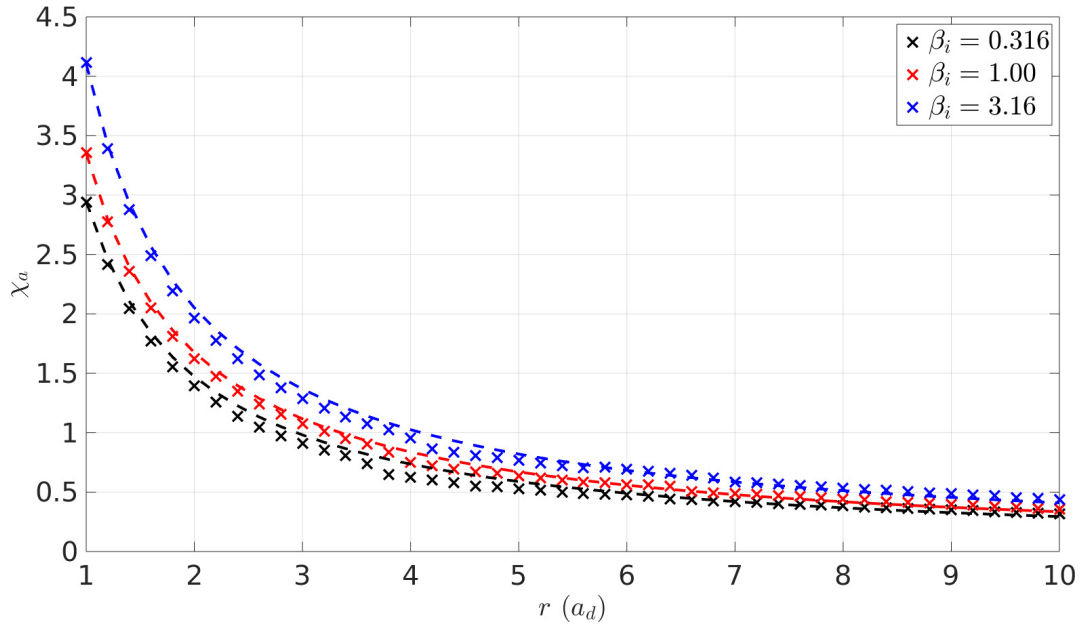


Figure 42: Radial electrostatic potential distribution measured by pot averaged across all angles and compared with a Coulomb potential with the same surface potential for $\beta_i = 0.316, 1.0$ and 3.16 .

surface potential for $\tilde{\lambda}_d = 3$ and $\tilde{\lambda}_d = 1$ presented in figure 40. The principal differences in implementation are the self-consistent determination of the potential distribution and presence of collisions in pot which are absent in DiMPL. Considering the approximate spherical symmetry of the potential observed by pot, shown in figure 41 this is unlikely to be the dominant cause of the discrepancy. Since pot predicts substantially different surface potentials for a dust grain with $\tilde{\lambda}_d = 3.0$ compared with $\tilde{\lambda}_d = 15.0$ in the range $0.1 \leq \beta_i \leq 1.0$ whilst maintaining an approximately Coulombic potential distribution suggests the radial dependence of the potential distribution cannot fully explain the discrepancy either. Additionally, the agreement between the results of Lange, SCEPTIC and DiMPL for $\tilde{\lambda}_d \simeq 1.9 - 3$ and $\tilde{\lambda}_d \simeq 1$ imply that the different determinations of surface potential should arise from differences between these three codes and pot. For these reasons, it is postulated that either collisional effects or unknown errors in the computation or analysis of the results of pot are the most probable cause for the observed differences in results.

4.2.7 Flowing Plasma

The floating potential of small dust grains in flowing plasmas with magnetic fields is an active and valuable area of research with many important applications including the charging of dust grains in the SOL of tokamaks. Few results for this regime exist, with the most recent results of the PIC code SCEPTIC presenting the ion currents as a function of probe potential for $\beta_i \leq 1$ and a range of dust sizes $10^{-3} \leq \tilde{\lambda}_d \leq 10^2$ [157, Figure 7]. The authors of that work highlighted that the "last step, left for future publication, consists of coupling the ion and electron collection theories to obtain the dust floating potential" [157], an objective which is achieved by DiMPL.

Figure 43 shows the variation of the floating potential of small dust, $\tilde{\lambda}_d = \infty$ for a range of ion magnetisation parameters $0.01 \leq \beta_i \leq 3.0$ and normalised flow speeds $u = 0.01, 0.2, 0.5$ & 1.0 as measured by DiMPL. The dependence on β_i is similar to the no flow case, with the ion current decreasing at large β_i as it becomes inhibited by the magnetic field, generating larger surface potentials. The dependence on flow velocity for all β_i is relatively weak, though at higher values of magnetisation parameter $0.2 \leq \beta_i$, the potential does decrease slightly with increasing flow speed. This can be explained through the additional kinetic energy of ions at larger flow speeds which enables them to reach the dust in spite of the strong confining magnetic field. The additional momentum of ions with $0 \leq u$ initially directed parallel to $\hat{\mathbf{B}}$ is quickly redistributed in other directions through reflections in the magnetic envelope, described in section 4.1.4, similar to the example orbit shown in figure 30d. With a greater amount of kinetic energy, the dimensions of the magnetic envelope increase, allowing more ion orbits to reach the sphere, thus making the surface potential more positive. For all flow velocities, the same dip in potential is observed around $\beta_i = 0.15$ as was identified previously for dust in a stationary plasma. However, for larger flow velocities, $u = 0.5$ and $u = 1.0$, the feature in figure 43 spans a greater range $0.1 \leq \beta_i \leq 0.5$. This can also be best explained from the greater kinetic energy of ions which remain relatively

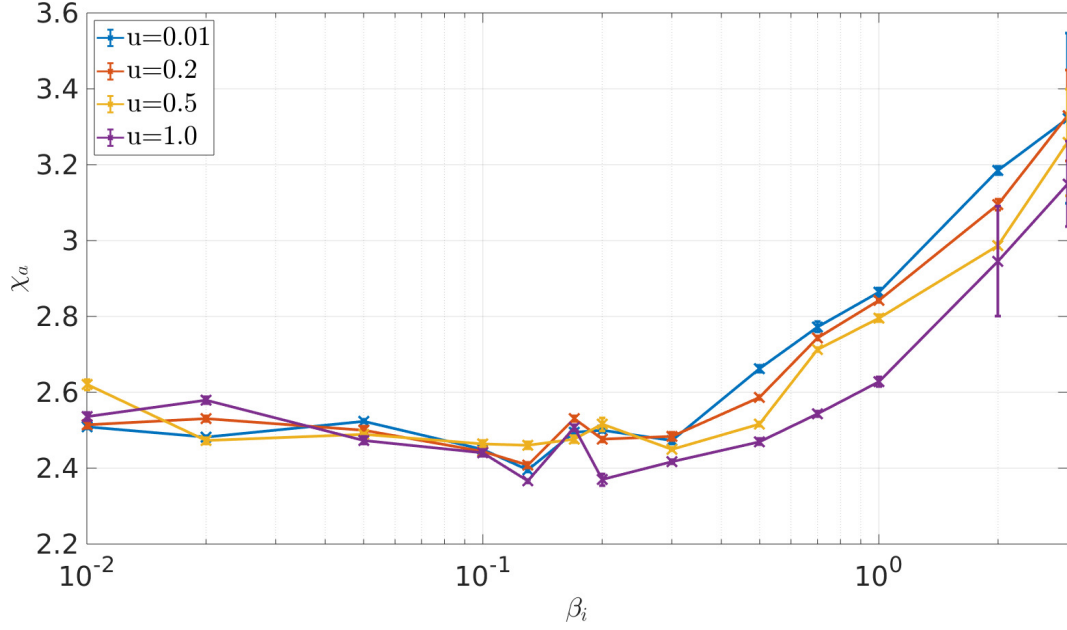


Figure 43: Normalised potential, χ_a , as a function of ion magnetisation, β_i , for different values of flow velocity $u = 0.01, 0.2, 0.5, 1.0$ as measured by DiMPL for a Coulomb potential $\tilde{\lambda}_d = \infty$.

unaffected until stronger magnetic fields are applied. A jump in potential observed at $\beta_i = 0.17$ for $u = 1.0, 0.2$, & 0.01 might indicate more sensitive dependence on magnetic field strength at this crucial point where electrons begin to become significantly affected by the magnetic field.

The results of DiMPL for $\tilde{\lambda}_d = 1$, $\tilde{\lambda}_d = 2$ and $\tilde{\lambda}_d = 5$ shown in figure 44 suggest that the Debye-Hückel potential does not alter the surface potential dramatically for different values of $\tilde{\lambda}_d$ when $1 \leq \tilde{\lambda}_d$, particularly at low flow speeds, $u \leq 0.2$. At large magnetic field strengths, the potential decreases with increasing flow velocity as was observed previously for the Coulomb potential, $\tilde{\lambda}_d = \infty$, case and the overall trend of the dependence of χ_a on β_i and u is similar to that observed in the small dust limit, see figure 43.

There are however a few noticeable features arising from the variation in $\tilde{\lambda}_d$ which become more severe for large dust. The potential appears to approach a smaller high field limit and rounds off more gradually between $1 \leq \beta_i \leq 10$ for $\tilde{\lambda}_d = 1$ as compared with $\tilde{\lambda}_d = 5$. This is most likely a result of the reduced effective range of the Debye-Hückel potential for small $\tilde{\lambda}_d$, meaning ions that are already bound to magnetic field lines cannot be easily displaced. This effect is less severe for ions initially with some large flow speed as even a weak interaction with the dust potential can lead to a re-distribution of their kinetic energy and escaping their previous magnetically dominated trajectory.

The normalised floating potential varies very weakly as a function of u for weak magnetic field strengths, $\beta_i \leq 0.2$, with a similar variation to the SOML model of section 3.2.3 for $\tilde{\lambda}_d = 2$ and $\tilde{\lambda}_d = 5$. In general for $\beta_i \leq 0.2$, the effect of increasing size appears to further inhibit the effect of flow to the point of being nearly independent of flow speed for $\tilde{\lambda}_d = 1.0$ and $\beta_i \leq 0.1$. Electrons that are

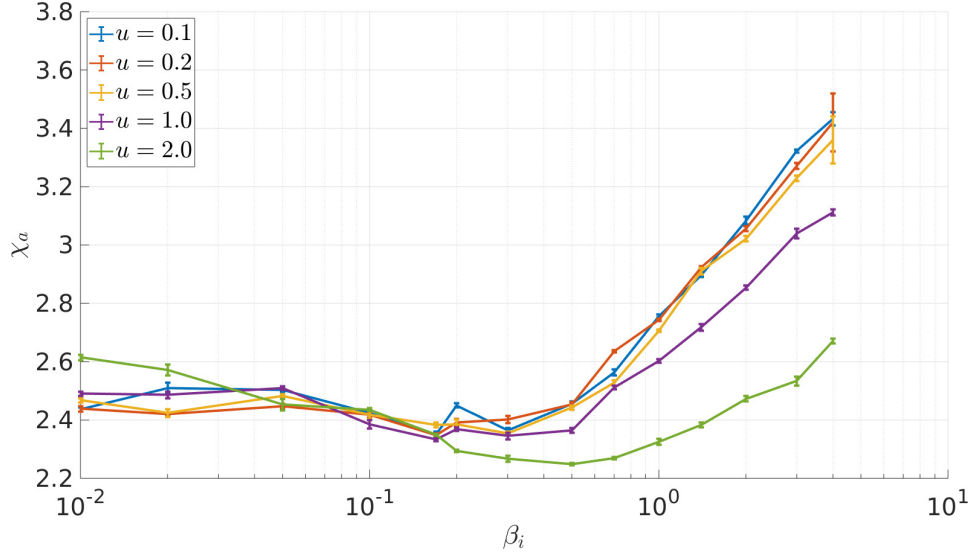
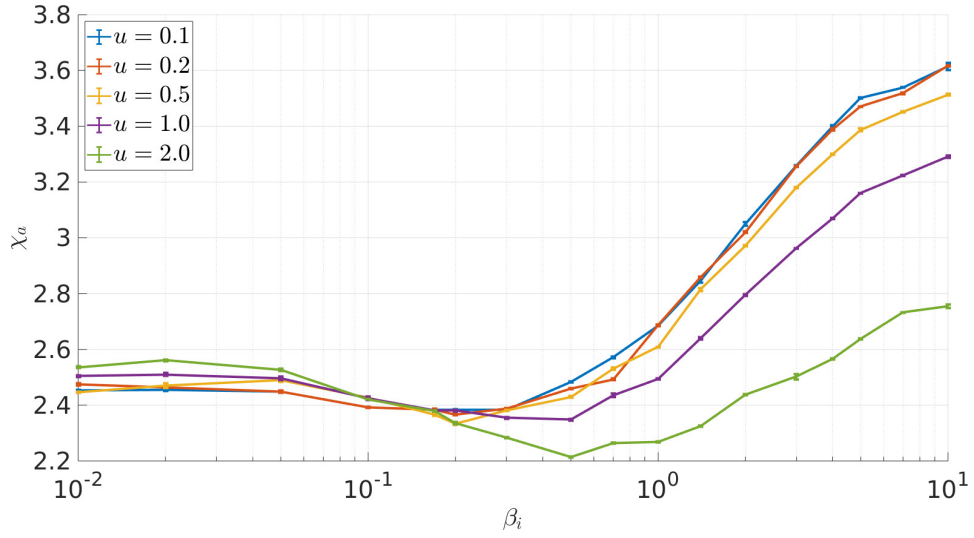
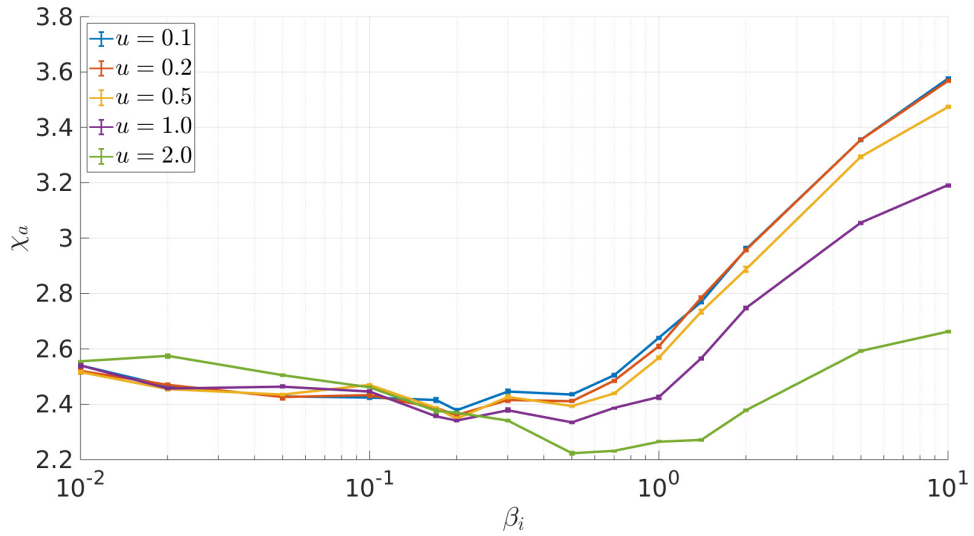
(a) $\tilde{\lambda}_d = 5$.(b) $\tilde{\lambda}_d = 2$.(c) $\tilde{\lambda}_d = 1$.

Figure 44: Normalised potential, χ_a , as a function of ion magnetisation parameter, β_i , for a range of values of flow velocity, u , for $\tilde{\lambda}_d = 5$ (44a), $\tilde{\lambda}_d = 2$ (44a) reprinted from [188], with the permission of AIP Publishing, and $\tilde{\lambda}_d = 1$ (44a) with $z_0 = 3.0\lambda_d$ and $b = 5.0$.

strongly magnetised at $\beta_i = 0.1$ are less sensitive to the form of the potential as only the population with enough energy to reach the dust are collected. They are therefore expected to behave similarly irrespective of $\tilde{\lambda}_d$ as they remain Boltzmann distributed with motion principally parallel to $\underline{B}$. Furthermore, it is likely that the kinetic energy of most ions without flow is already sufficient for their magnetic envelopes, as described in section 4.1.4, to encompass the dust grain at these low magnetic field strengths. So with an impact factor limited by their gyro-radius, the greater flow speed parallel to $\underline{B}$ does not advantage ions to be collected.

4.2.8 Semi-empirical Formulation of the Floating Potential

Though the equilibrium surface potential of dust grains can be determined using DiMPL over a day long timescale, there is a need for more immediate estimates for use in dust tracking codes that must perform such calculations repeatedly. For this reason, a semi-empirical expression for the potential as a function of ion magnetisation is formulated for small dust, $1 \ll \tilde{\lambda}_d$.

Previous theoretical work and the results presented here indicate that the magnetic field first affects the electron current when $\beta_e \simeq 1$ and at higher field strengths the ion current when $\beta_i \simeq 1$, reducing both to a half of their low field limit values [152, 155, 157], see figure 34. In the limit $a_d \ll \rho_{T,e}, \rho_{T,i}$, the OML result for the ion and electron current to a negatively charged sphere given by equations 38 and 39 is recovered. In the opposite magnetic field limit, $\rho_{T,e}, \rho_{T,i} \ll a_d$, all plasma species follow trajectories along magnetic field lines with only the projected area of the sphere along the z axis accessible. Reintroducing the thermal current $I_{0,s} = \pi a_d^2 n_s \langle v_{T,s} \rangle$ of species $s = i, e$, the high field limit ion and electron currents are

$$\begin{aligned} I_i(\beta \rightarrow \infty) &= \frac{I_{0,i}}{2}, \\ I_e(\beta \rightarrow \infty) &= \frac{I_{0,e}}{2} e^{-\chi_a}, \end{aligned} \quad (120)$$

as mentioned in section 4.1.3.2. The current of each species is assumed to vary smoothly from the low to high field limit such that

$$\begin{aligned} I_i &= I_e(\beta \rightarrow \infty) + \frac{g(\beta_e)}{2} I_e(\beta \rightarrow 0), \\ I_e &= I_i(\beta \rightarrow \infty) + \frac{g(\beta_i)}{2} I_i(\beta \rightarrow 0), \end{aligned} \quad (121)$$

where $g(\beta_s)$ is a fitting function. The form $g(\beta_s) = e^{-a_s \beta_s^{b_s}}$ is assumed since this sigmoidal function has the required properties of $g(\beta_s \rightarrow 0) = 1$ and $g(\beta_s \rightarrow \infty) = 0$. The free parameters a_s and b_s take different values for ions a_i, b_i and electrons a_e, b_e . Introducing this fitting function into equation 121, the equations for the current become.

$$\begin{aligned} I_i &= \frac{I_{0,i}}{2} \left[\left(1 + \frac{2Z\chi_a}{\tau} \right) e^{-a_i \beta_i^{b_i}} + 1 \right], \\ I_e &= \frac{I_{0,e}}{2} e^{-\chi_a} \left[e^{-a_e \beta_e^{b_e}} + 1 \right]. \end{aligned} \quad (122)$$

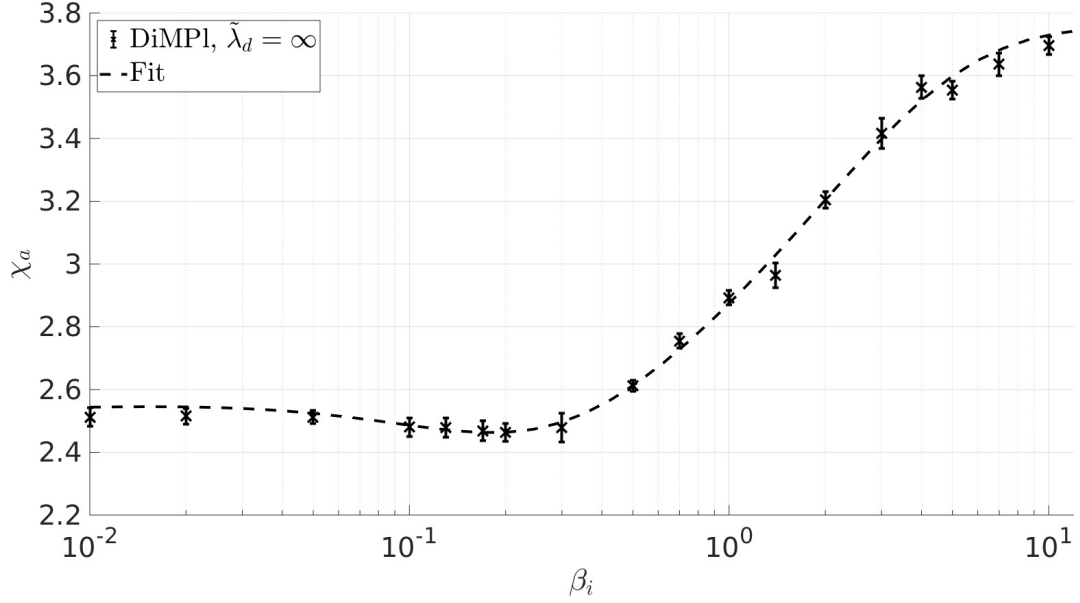


Figure 45: Fit from solving equation 30 for χ_a with currents given by equation 122 to the normalised surface potential calculated by DiMPL for a Coulomb potential, $\tilde{\lambda}_d = \infty$, as a function of the ion magnetisation parameter, β_i . Reprinted from [188], with the permission of AIP Publishing.

These currents then have the desired property of approaching the correct theoretical dependence in the limit $\beta_i \rightarrow 0$ and $\beta_i \rightarrow \infty$. Additionally, the electron and ion currents of equation 122 can be fit well to the previous numerical results of SL [184] over the range $0.3 \leq \beta_i \leq 10.0$ and $0 \leq \chi_a \leq 5.0$.

To determine the optimal values of the free parameters, the floating potential calculated from equating the currents of equation 122 are fitted to the results of DiMPL for a Coulomb potential, $\tilde{\lambda}_d \rightarrow \infty$ previously shown in figure 39. For the values $a_i = 0.23$, $b_i = 1.56$, $a_e = 0.56$ and $b_e = 1.0$ this fits the data with an $R^2 = 0.996$ and root mean squared error of 0.006 as shown in figure 45. This produces an accurate description of the surface potential dependence on magnetic field strength parameterised through β_i for $\tilde{\lambda}_d = \infty$.

The semi-empirical model was been applied in predicting the results of DiMPL previously presented in figure 38 for the surface potential as a function of τ . Figure 46 demonstrates that the model accurately recovers the trend in the dependence across τ and for all β_i investigated and provides an accurate estimate for the value of the normalised surface potential for most values of τ and β_i . Additionally, the model has been further found to accurately predict the results of Lange for the mass ratio $\mu = 100$ with $\tau = 1$ and $\tau = 0.1$.

This simple model offers a quick method for determining the equilibrium surface potential of small dust grains in magnetic fields which is remarkably accurate over a large region of the parameter space of β_i , τ and μ . Additionally, it recovers the expected theoretical result in the limit $\beta_i \rightarrow \infty$ and $\beta_i \rightarrow 0$, providing a powerful predictive tool useful for example in dust tracking codes.

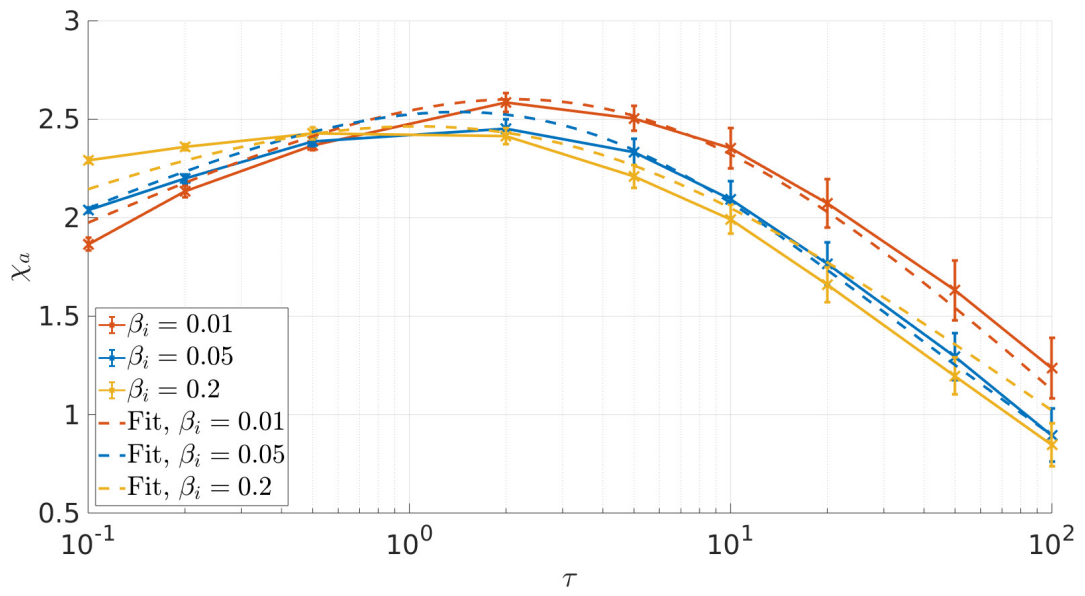


Figure 46: Semi-empirical model (dashed) from solving equation 30 using the currents of equation 122 alongside the normalised potential calculated by DiMPL (solid markers) as a function of temperature ratio τ for $\beta_i = 0.01, 0.05$, & 0.2 from figure 38.

*Only words and conventions can isolate us
from the entirely undefinable something
which is everything.*

— Alan Wilson Watts, *The Wisdom of Insecurity* [194]

5.1 THEORETICAL DESCRIPTION

Magnetic fields not only alter the currents of charged particles collected by dust grains, they also affect the torque at the dust surface. Understanding the balance of these torques for droplets in tokamak plasmas is extremely important, as the rotational instability offers an explanation for observations of particle breakup in the divertor of JET [195] as described later in chapter 8. Though generally receiving far less attention than its linear counterpart, multiple theories have been proposed that describe processes contributing to the rotational dynamics of dust particles in plasmas. These theories account for torque due to shear plasma flow [196], asymmetrical radiation to the grain [197], rocket forces from asymmetric ablation [198] and the interaction between external electric fields and an insulating grain with a dipole potential distribution [199, 200]. This final process occurs in the plasma sheath, and has been theorised to cause dust particles to spin at up to $6 \times 10^5 \text{ s}^{-1}$ in low-temperature argon plasmas [199]. These models are not considered further as they rely upon an asymmetry in either the dust grain surface topology, the plasma flow or the electrostatic potential distribution for rotational motion to occur. Instead, focus is given to perfectly spherical, stationary dust grains immersed in plasmas permeated by constant and uniform magnetic fields, beginning with the previous elementary treatment of Krasheninnikov [201]. Note that the momentum transfer due to the deflection of particles in the dust grain potential is not considered as the dust potential is considered to be purely central. Section 5.1.1 summarises the relevant existing research whilst section 5.2 and 5.3 describes the authors own work in this area.

5.1.1 Existing Theoretical Description

Krasheninnikov [201] identified two particularly important mechanisms that affect the rotational dynamics of dust grains in this regime which are described here in more detail. Both of these theories depend upon a phenomena referred to as geometric shadowing, wherein the fluxes of particles over the dust surface become asymmetrically distributed as a result of the geometry of helical trajectories that are collected by a spherical surface as discussed briefly in section 4.1.1.

5.1.1.1 Asymmetric Ion Momentum Collection

The first mechanism depends upon the direct collection of angular momentum from impacting particles with geometrical shadowing of orbits leading to a net transfer of angular momentum, L , parallel or anti-parallel to $\underline{B}$. Gyro-orbits of particles that enclose the axis passing through the dust grain centre parallel to $\underline{B}$ always contribute a net angular momentum in the same direction. In the coordinate system used here, this axis is equivalent to the z -axis, since $\underline{B}$ is defined parallel to the z -axis and the dust grain is centered on the origin, as in section 4.1.1. For example, ion gyro-orbits that enclose the z -axis always contribute angular momentum in the direction anti-parallel to $\underline{B}$ as shown in figure 47. Orbits that do not enclose the axis may contribute angular momentum in either sense, but are preferentially collected with angular momentum in the opposite sense to those that encircle the z -axis. This is demonstrated in figure 48 where the collection of ions in this example always contributes angular momentum in the $\hat{\underline{B}}$ direction. From figures 47 and 48, it is also clear that the problem is vertically symmetric when reflected across the x - y plane at $z = 0$, meaning ion orbits originating from $z_0 = \pm\infty$ with $v_z = \mp eBa_d/m$ are preferentially collected with angular momentum in the same sense.

Krasheninnikov considered the net angular momentum transfer due to the collection of ions arising from the geometrical shadowing of helical orbits having only the type of interaction shown in figure 48 [201]. In particular, for the infinitely strong magnetic field limit, $\beta_i \rightarrow \infty$, the gyro-radii of ions tend to zero $\rho_\perp/a_d \rightarrow 0$. Since only particles with an impact parameter $b \leq \rho_\perp$ encircle the z -axis, this limit removes the contribution of orbits of the type shown in figure 47. Additionally, in the limit of $v_\parallel \omega_s/a_d \rightarrow 0$ (which is implied by an isotropic velocity distribution with $\rho_\perp/a_d \rightarrow 0$), ion trajectories intersect the spherical surface multiple times and initially with grazing incidence in the x - y plane, as is shown in figure 49. This means ions will be collected irrespective of their initial phase, ψ_0 , with an angular momentum $\underline{L} = m_i \underline{r} \times \underline{v} = m_i |\underline{R}_0| v_\perp \hat{\underline{z}}$, where the cylindrical radial distance $\underline{R}$ is again distinguished from the spherical distance $\underline{r}$ and R_0 is the cylindrical radial coordinate of impact.

Using these approximations, the torque in the high field limit, $\beta_i \rightarrow \infty$, is estimated. The gyro-orbits of ions are infinitely small in this limit such that they are effectively tied to field lines. The flux of ions $\Gamma = n_i \langle v_{T,i} \rangle / 4$ to the dust surface is therefore the same as through the projected circular cross section, $A = 2\pi a_d^2$,

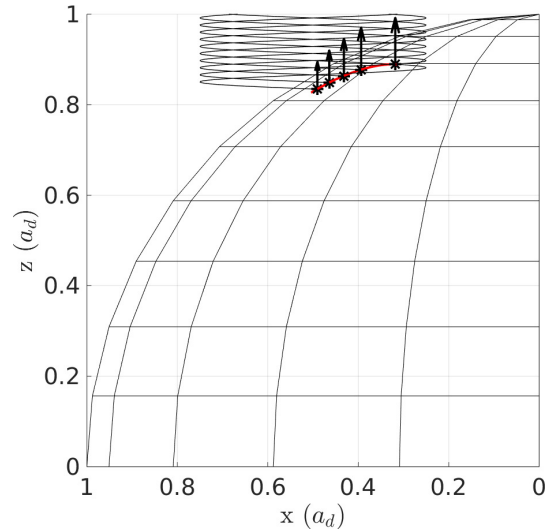


Figure 49: Example ion orbits for $\rho_\parallel = 2\pi a_d/100$ demonstrating the enhanced geometric shadowing effect for $\rho_\perp = a_d/4$, $x_1 = y_1 = a_d/2$ with orbits incident over a small range of z .

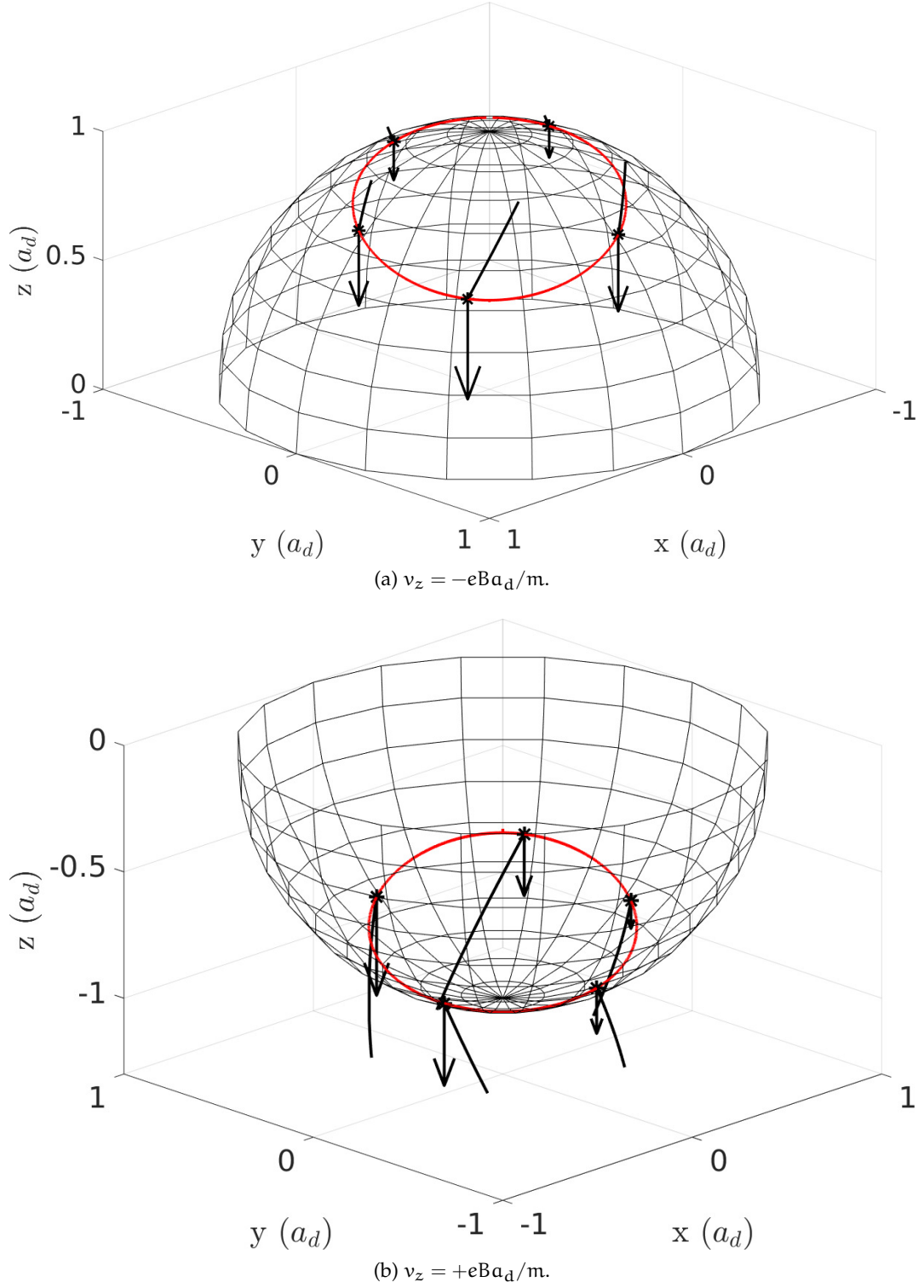


Figure 47: Gyro-orbits of ions (black line) for $\rho_{\perp} = a_d/2$ and $x_1 = y_1 = a_d/5$ following equations 88 and 89 for $\psi_0 = 0, \pi/2, \pi, 3\pi/2, 2\pi$ for $v_z = -eBa_d/m$ (47a) and $v_z = +eBa_d/m$ (47b). Asterisks indicate the interaction points calculated using equation 90, with arrows indicating the relative magnitude of the angular momentum calculated from $\mathbf{R} \times \mathbf{v}_{\perp}$. The intersection boundary (red line) for $0 \leq \psi_0 \leq 2\pi$ was calculated from equation 90, with the angular momentum transfer observed to be symmetric across the $z = 0$ plane.

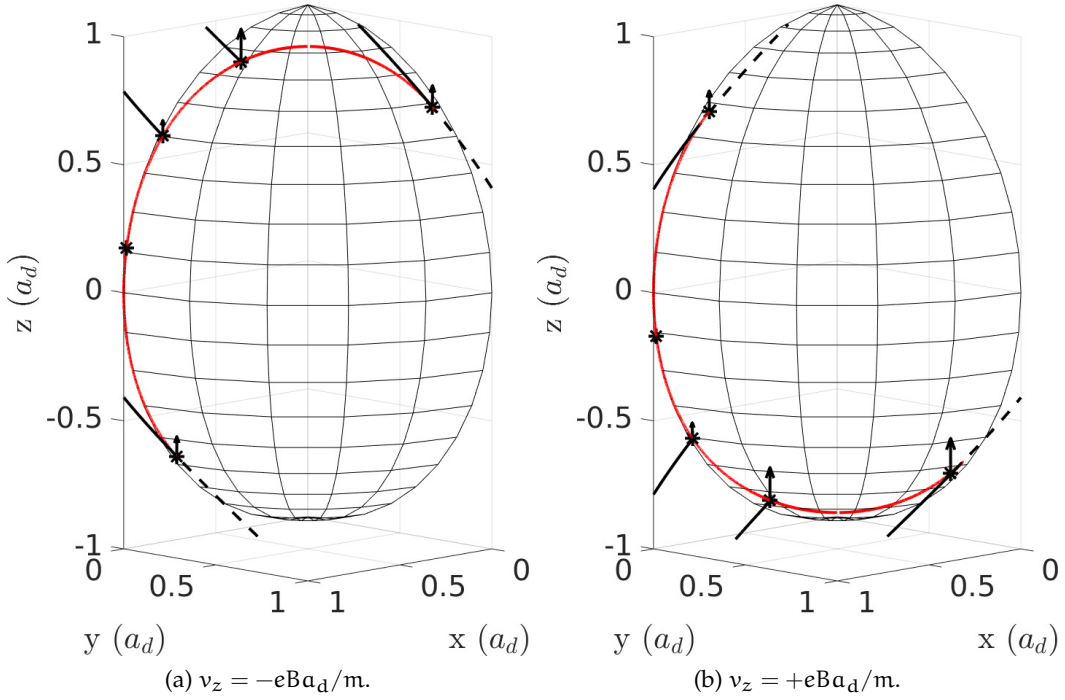


Figure 48: Figure 23b with $v_z = -eBa_d/m$ reproduced with arrows indicating the direction and relative magnitude of angular momentum transfer (48a) and the same case with reversed velocity direction $v_z = +eBa_d/m$ (48b) demonstrating the symmetry of angular momentum transfer for particles arriving from $z = \pm\infty$.

in the distant plasma parallel to $\hat{B}$. Ions are collected across the entire surface but with an average radial distance from the z -axis equal to half the dust grain radius $\langle R_0 \rangle = a_d/2$. The average velocity of incident ions is estimated by the thermal velocity, meaning that they are collected with an average angular momentum of approximately $\langle L \rangle \simeq m_i a_d v_{T,i}/2$. This was used to estimate the torque, Υ , as $\Upsilon(1 \ll \beta_i) \simeq \pi a_d^2 \Gamma \langle L \rangle$ such that

$$\Upsilon(1 \ll \beta_i) = \Upsilon_{0,i} \equiv n_0 k_B T_i a_d^3. \quad (123)$$

This suggests that the torque in the high field limit $1 \ll \beta_i$ is a finite value, $\Upsilon_{0,i}$, independent of β_i , which is referred to as the high field limit torque.

For the opposite limit, $\beta_i \ll 1$, the problem is far more complicated since collection of particles is dependent upon the initial phase, ψ_0 , and their motion is further influenced by the charge of the dust. To simplify this, particles are assumed to be unaffected by the dust potential so that they continue to follow helical trajectories until they impact the dust grain. Revisiting the geometrical arguments of section 4.1.1, it is clear from figure 21a and figure 22b that for $\beta_i \ll 1$ where generally $a_d \ll \rho_\perp$, there are few orbits of the type that form two closed surfaces on the sphere as the average gyro-radius greatly exceeds the radius of the sphere. This leaves only ions with impact parameters in the annulus between $|a_d - \rho_\perp| \leq R_1 \leq a_d + \rho_\perp$ mentioned previously able to be collected. Interestingly, orbits with $|a_d - \rho_\perp| \leq R_1$ contribute angular momentum in the opposite sense to those with $R_1 \leq a_d + \rho_\perp$ for $a_d \leq \rho_\perp$ if undisturbed by the dust grain poten-

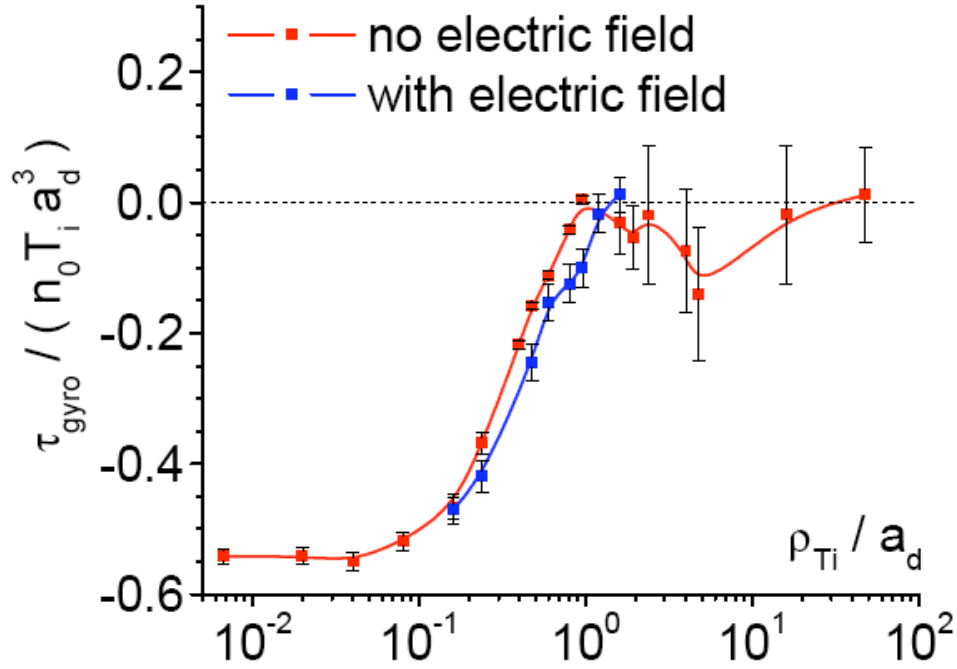


Figure 50: DiMag PIC code results of torque normalised to the high field limit torque $\Upsilon_{0,i}$ of equation 123 as a function of the inverse of the ion magnetisation parameter $\rho_{Ti}/a_d = 1/\beta_i$. A constant torque of $\Upsilon \simeq -0.55\Upsilon_{0,i}$ is observed at small ρ_{Ti}/a_d (large β_i) in disagreement with the high field limit torque of equation 123 whilst zero torque is measured at large ρ_{Ti}/a_d (small β_i). Reproduced under creative commons act [202].

tial. This effect is enhanced for $v_{\parallel}\omega_s \ll a_d$ as was shown in figure 49 and in this case ions are once again collected with grazing incidence in the x-y plane with $\langle L \rangle \simeq m_i v_{\perp} a_d$. Under these considerations, the torque in the regime $\beta_i \ll 1$ was estimated by the following integral equation,

$$\Upsilon(\beta_i \leq 1) \sim \int_{v_{\parallel}\omega_s \leq a_d} m_i |v_{\parallel}| v_{\perp} a_d^3 f(\underline{v}) d\underline{v} \simeq \Upsilon_{0,i} \beta_i, \quad (124)$$

with the integral being performed over $v_{\parallel}\omega_s \leq a_d$. Since equation 124 is only valid in the regime $\beta_i \ll 1$, this implies that $\Upsilon(\beta_i \leq 1) < \Upsilon(1 \ll \beta_i)$ such that the torque increases with increasing field strength.

This theory was evaluated through comparison with PIC code simulations investigating the angular momentum accumulated by a dust grain through collection of ion momentum. Figure 50 shows Dust in Magnetic Field (DiMag) simulations of the torque experienced by a dust grain at the floating potential and an uncharged dust grain for $\tau = 10.0$ [202]. The constant torque at high β_i of approximately $\Upsilon \simeq 0.55\Upsilon_{0,i}$ is in agreement with the approximate theory, $\Upsilon(\beta_i \rightarrow \infty) \simeq \Upsilon_{0,i}$ used in attaining equation 123 and does decrease as expected for increasingly weak magnetic field strengths as suggested by equation 124. Additionally, the insubstantial effect the dust grain potential had on the example investigated suggests that the theory could be extended to incorporate charged dust grains as well.

5.1.1.2 Asymmetric Current Distribution

The second theory postulated that the asymmetric flux of charged particles to the dust surface generates a torque through the $\mathbf{J} \times \mathbf{B}$ force [201]. For intermediate magnetic field strengths when the electron gyro-radius is comparable or smaller than the dust grain size, $1 \leq \beta_e$, but the ion gyro radius is still far larger, $\beta_i \ll 1$,

the surface current density loses its spherical symmetry. For $1 \ll \tau$, this effect is enhanced and electron trajectories can be approximated as being in the limit $1 \ll \beta_e$ so that they are tied to field lines. In this regime, the ion surface current density continues to be approximately spherically symmetric whilst the electron current is preferentially collected at the poles, as is illustrated in figure 51. This asymmetry in the distribution of plasma particles collected across the surface was observed in DiMPL simulations previously in figure 37. The internal currents that act to maintain the equipotential surface as a result of this asymmetry interact with the external magnetic field through the Lorentz force and generate a torque in the $\hat{\mathbf{B}}$ direction. The torque imparted through this mechanism is therefore seen to be in the same sense as caused by ion collection as described in the previous section [201].

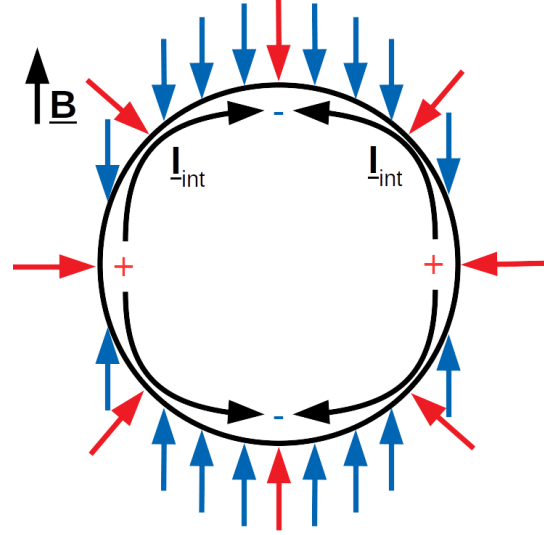


Figure 51: Cross-section demonstrating the geometry of internal currents, I_{int} , generated by the asymmetric flux of ions (red) and electrons (blue) over the spherical surface.

An estimate for the torque comes from the Lorentz force acting on the net internal current, $I_{\text{int}} \sim en_0 v_{T,i} a_d^2$, from the edges of the sphere to the axis,

$$\Upsilon_{\mathbf{j} \times \mathbf{B}} \sim \Upsilon_{0,i} \beta_i. \quad (125)$$

These models for the net angular momentum transfer to a dust grain resulting from the geometrical shadowing of particle trajectories have received little attention since this work and have not been substantiated further than the estimates of equations 123, 124 and 125.

5.1.2 Theoretical Model

Based on the analytical treatment of currents in section 4.1.3 and the estimates presented in section 5.1.1.1, a more detailed theoretical description for the rotational dynamics of a dust grain as a function of magnetic field strength is developed. Focus is given here to torques due to direct collection of angular momentum from plasma particles as was described in section 5.1.1.1 since this mechanism generates torques for all values of β_i and is applicable to experiments involving insulating dust grains. The torque due to current asymmetries discussed in section 5.1.1.2 can

be measured with DiMPI as was demonstrated in figure 37 and is the subject of future work. The model developed here is used to provide insight into experimental measurements of dust rotation in low temperature discharge plasmas and the numerical results of DiMPI.

5.1.2.1 *Simplifying Approximations*

Several of the core assumptions made in calculating the linear forces on a dust grain are also necessary when considering rotational dynamics. In particular, it is assumed that an ion or electron which impinges at a particular point on the dust grain surface contributes the full angular momentum with which it was incident. The mass contribution is neglected as it is small and expected to be balanced by recycling which is spherically symmetric and therefore will not affect the rotational dynamics greatly. The plasma is considered to be collisionless without background plasma flows as they extend the parameter space unnecessarily. The problem is solved for the steady state behaviour of a dust grain fixed at the floating potential without considering alterations to the local magnetic field due to rotation of the charged sphere.

A summary of the geometrical limits that predetermine the direction in which torque is transferred to the dust grain for helical trajectories is given in figure 52. The geometry of the collection of particles is identical to that of figure 21 but with the extra consideration for the sign of $\underline{R} \times \underline{v}_\perp$ at the surface being necessary. Figure 52a demonstrates that ion orbits that enclose the axis must satisfy $R_1 \leq \rho_\perp$ and always contribute angular momentum in the opposite direction to $\underline{B}$ when collected. For ions that do not enclose the axis with $\rho_\perp \geq a_d$, shown in figure 52b, the direction is dependent on the initial phase. For $v_\parallel \omega_s \ll a_d$ however, ion orbits of the type shown in figure 52b will be preferentially collected with angular momentum parallel to $\underline{B}$ due to the geometrical shadowing effect as was seen in figure 49.

An important distinction of the present theoretical treatment is the inclusion of the angular momentum imparted due to the collection of electrons. The negative charge of electrons and the reversed sense of their gyro-motion means their behaviour mirrors that of ions such that they contribute angular momentum in the opposite sense to ions for each case identified in figure 52. Electron orbits that enclose the z axis will therefore contribute angular momentum parallel to $\underline{B}$, in the opposite sense of ions as is seen by comparing figure 47a and figure 53. Similarly for orbits that do not enclose the z -axis as shown in figure 54, though the angular momentum contributed depends on the initial phase in this case, the direction is always inverted for electrons as compared with ions. From this symmetry it is expected that the torque due to collection of electrons will have a similar dependence on the electron magnetisation parameter, β_e , but will be of opposite sign to the torque due to ions which scales with the ion magnetisation parameter, β_i .

5.1.2.2 *Outlining the Problem*

Using these assumptions and the geometrical considerations employed in attaining the currents in section 4.1, the magnitude of the torque is calculated from the

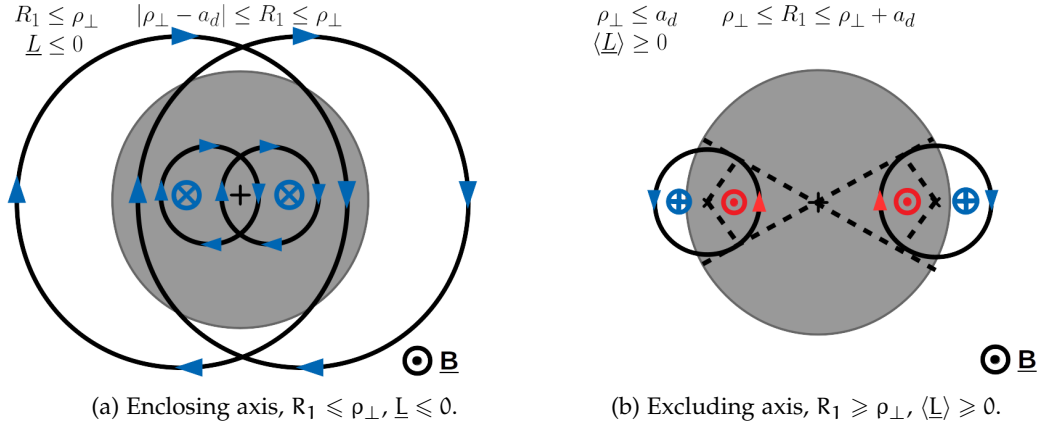


Figure 52: Plan view of ion gyro-orbits with arrows indicating the direction of motion and dashed lines indicating points where $\underline{\mathbf{R}} \times \underline{\mathbf{v}} = 0$. Gyro-orbits of ions that enclose the z axis (52a) are always incident with $L \leq 0$, whilst for those that exclude the z axis (52b) the sign of L is dependent on ψ_0 and $v_\parallel$ with the geometrical shadowing effect causing $\langle L \rangle \geq 0$ for $v_\parallel \omega_s \leq a_d$.

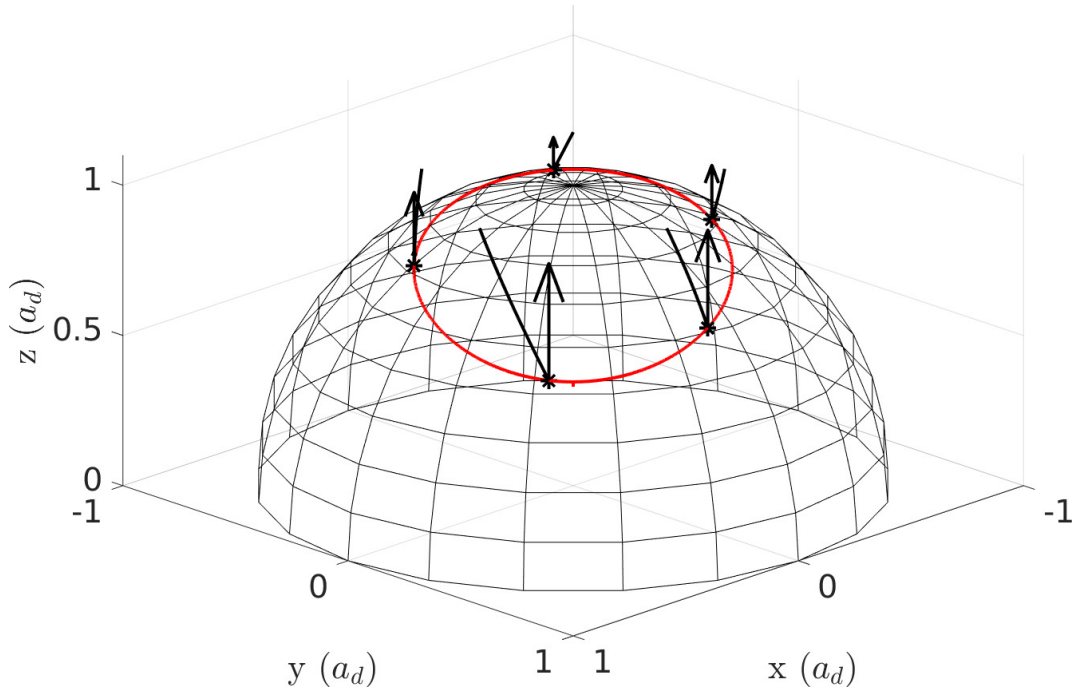


Figure 53: Gyro-orbits of electrons for $\rho_\perp = a_d/2$ and $x_1 = y_1 = a_d/5$ following equations 88 and 89 for $\psi_0 = 0, \pi/2, \pi, 3\pi/2, 2\pi$ for $v_z = -eBa_d/m$. Asterisks indicate the interaction points calculated using equation 90, with arrows indicating the relative magnitude of the angular momentum calculated from $\underline{\mathbf{R}} \times \underline{\mathbf{v}}_\perp$. The intersection boundary for $0 \leq \psi_0 \leq 2\pi$ calculated from equation 90 is indicated in red. Electrons with $R_1 \leq \rho_\perp$ contribute L always in the positive sense.

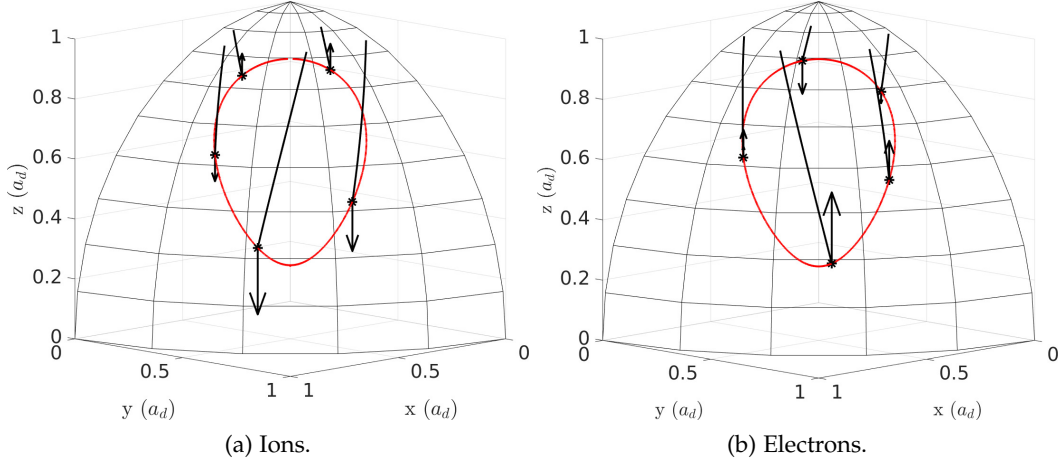


Figure 54: Gyro-orbits of ions (54a) and electrons (54b) where $\rho_{\perp} = a_d/4$, $v_{\parallel}\omega_s = 2\pi a_d$ and $x_1 = y_1 = a_d/2$ for both species, with five values of ψ_0 . The sign of L changes dependent on ψ_0 . Electrons contribute L in the opposite sense to ions for corresponding values of ψ_0 .

second moment of the distribution function, equation 6, integrated over the dust grain surface. Here, it is assumed that the only non-zero component of the stress tensor corresponds to the transfer of angular momentum, L , parallel to $\underline{B}$ since the spherical symmetry is broken by the magnetic field in this direction. This is once again challenging to evaluate at the dust surface where the plasma is disturbed and is instead calculated by considering the flux of collected particles over a distant surface in analogy with the charging theories described in section 4.1.3. Assuming a Maxwell-Boltzmann velocity distribution from equation 12, the torque is then expressed as the flux of angular momentum collected over the surface element dS ,

$$\Upsilon = 2n_0 \left(\frac{1}{2\pi v_T^2} \right)^{\frac{3}{2}} \int_S \int_0^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} L v_z e^{-\frac{mv^2 + 2e\phi}{2k_B T}} dv_x dv_y dv_z dS. \quad (126)$$

As was shown before, in figures 47 and 48 the problem is vertically symmetric along the $x - y$ plane at $z = 0$ so that the torque imparted by all particles incident approaching from $z_0 = +\infty$ will equal the torque of particles approaching from $z_0 = -\infty$. Particles incident on the surface of a dust grain rotating with an angular velocity $\Omega_d(t)$ at a point R_0 contribute an angular momentum

$$L = mR_0[v_{\theta} - R_0\Omega_d(t)]. \quad (127)$$

The velocity $v_{\theta} - R_0\Omega_d(t)$ is the velocity of particles in the $\hat{\theta}$ direction, v_{θ} , relative to the velocity of the dust grain surface at this point, $R_0\Omega_d(t)$, in this direction. Since the angular momentum of incident particles given by equation 127 has a term proportional to $\Omega_d(t)$ and a term independent of $\Omega_d(t)$, it is anticipated that the torque of equation 126 will be of the form

$$\Upsilon \equiv \frac{dL}{dt} = K \frac{d\Omega_d(t)}{dt} = \alpha - \gamma\Omega_d(t), \quad (128)$$

where K is the moment of inertia of the dust and the constants α and γ have useful physical interpretations. The term α is referred to as the initial torque since it is the torque that acts on a dust grain with zero angular velocity, $\Omega_d(t) = 0$. The second term, $\gamma\Omega_d(t)$ is called the resistive torque as it acts in opposition to the initial torque with a magnitude that increases with $\Omega_d(t)$. Finally, the coefficient γ/K is referred to as the equilibration rate since it determines the time scale required to reach the equilibrium condition, $\alpha = \gamma\Omega_d(t)$. The time dependence of this equation arises solely due to the term containing the dust angular velocity, $\Omega_d(t)$, with the solution to this first order differential equation being of the form

$$\Omega_d(t) = \frac{\alpha}{\gamma} \left(1 - e^{-\gamma t/K} \right). \quad (129)$$

The ratio, α/γ , is equal to the equilibrium rotation rate, $\Omega_d(t \rightarrow \infty) = \Omega_{eq} = \alpha/\gamma$. The task remaining is to calculate α and γ using equation 126 to characterise the rotational dynamics of the sphere.

5.1.2.3 High Field Limit

In the limit of extremely high magnetic field strengths where $\rho_\perp \ll a_d$, the previous result of equation 123 [201] can be re-derived more thoroughly. As stated earlier, in this limit the motion of particles is inhibited perpendicular to the magnetic field as their Larmor radius is exceedingly small. Particles with $a_d + \rho_\perp \simeq a_d \leq R_1$ therefore miss the sphere as the magnetic field prevents particles from being displaced from their magnetic field lines. For this reason, the surface integration of equation 126 is given by $dS = R_1 dR_1 d\theta$, with radial limits from 0 to a_d . As a result of the geometry of the sphere and the helical orbits, particles are always collected with grazing incidence in the x-y plane such that $v_\theta = v_\perp$ at the dust grain surface as was shown in figure 49. A collected ion will transfer an amount of angular momentum given by equation 127, where $R_0 = R_1 - \rho_\perp$ is the radial coordinate of intersection on the surface relative to the impact parameter of the guiding centre R_1 . Re-expressing the velocity as components parallel, $v_\parallel$, and perpendicular, $v_\perp$, to $\underline{B}$, equation 126 takes the form

$$\gamma_i(\beta_i \rightarrow \infty) = 2n_0 \left(\frac{1}{2\pi v_{T,i}^2} \right)^{\frac{3}{2}} \int_0^{2\pi} \int_0^{a_d} \int_0^\infty \int_0^\infty v_\parallel m_i R_0 (v_\perp - R_0 \Omega_d) e^{-\frac{(v_\parallel^2 + v_\perp^2)}{2v_{T,s}^2}} (2\pi v_\perp dv_\perp dv_\parallel) (R_1 dR_1 d\theta). \quad (130)$$

In the high field limit, $R_0 \simeq R_1$ so that simplifying this integral and evaluating the integral over θ and R_1 the result is

$$\gamma_i(\beta_i \rightarrow \infty) = \frac{2\sqrt{2\pi} n m_i a_d^3}{v_{T,i}^3} \int_0^\infty \int_0^\infty v_\perp v_\parallel \left(\frac{v_\perp}{3} - \frac{a_d \Omega_d}{4} \right) e^{-\frac{(v_\parallel^2 + v_\perp^2)}{2v_{T,s}^2}} dv_\perp dv_\parallel. \quad (131)$$

This integral is evaluated and expressed in terms of the normalised ion torque, $\gamma_{0,i} = n_0 k_B T_i a_d^3$, from equation 123 as

$$\gamma_i(\beta_i \rightarrow \infty) = \gamma_{0,i} \left[\frac{2\pi}{3} - \frac{\sqrt{2\pi} a_d \Omega_d}{2v_{T,i}} \right]. \quad (132)$$

From this result, the theoretical values $\alpha = 2\pi\Upsilon_{0,i}/3$ and $\gamma = \sqrt{2\pi}a_d\Upsilon_{0,i}/2v_{T,i}$ are identified, giving an equilibrium rotation rate, torque and spin time in approximate agreement with the previous estimates [201] of equation 123 discussed in section 5.1.1. Equation 132 is of the form of equation 128 and can be used to express the time dependence of the dust angular velocity as in equation 129 due to ion impact as

$$\Omega_d^{(i)}(t) = \frac{2\sqrt{2\pi}v_{T,i}}{3a_d} \left[1 - e^{-\frac{15\sqrt{2}n_0v_{T,i}m_i}{16\sqrt{\pi}a_d\rho_d}t} \right]. \quad (133)$$

Here the moment of inertia of a sphere of uniform density $K = 2M_d a_d^2/5$ and a dust mass density, ρ_d , has been introduced. The predicted initial torque is $\alpha \sim \Upsilon_{0,i}$, with an equilibrium rotational velocity, $\alpha/\gamma \sim v_{T,i}/a_d$, which is dictated by the thermal motion of ions relative to the dust size. The parameter $\gamma/K \sim n_0 v_{T,i} m_i / a_d \rho_d$ is expressed in terms of the dust mass as in equation 133. The torque in this high field limit is independent of the magnetic field strength, suggesting a tendency to a torque, equilibration rate and equilibrium rotation rate independent of β_i as $\beta_i \rightarrow \infty$.

Electrons also contribute, though they are repelled by the negative potential of the dust grain. As mentioned earlier and demonstrated in figure 54, electrons will be preferentially collected with angular momentum directed in the opposite sense to ions due to the geometry of the intersection of their helical orbits with the spherical surface, with $v_\theta = -v_\perp$. Since they are also tied to field lines for $\beta_e \rightarrow \infty$, the arguments affecting their torque are identical but with a density modified by a Boltzmann factor

$$\Upsilon_e(\beta_e \rightarrow \infty) = -\Upsilon_{0,e} e^{-\chi_a} \left[\frac{2\pi}{3} - \frac{\sqrt{2\pi}a_d\Omega_d}{2v_{T,e}} \right], \quad (134)$$

where $\Upsilon_{0,e} = n_0 k_B T_e a_d^3$ is the electron high field limit torque. For an isothermal plasma $T_i = T_e = T_0$, the high field limit torques $\Upsilon_{0,i} = \Upsilon_{0,e} = \Upsilon_0$ are equal so that equation 132 and equation 134 can be combined to attain the total torque as

$$\Upsilon(B \rightarrow \infty) = \Upsilon_0 \left(\frac{2\pi}{3} - \frac{\sqrt{2\pi}a_d\Omega_d}{2v_{T,i}} - e^{-\chi_a} \left[\frac{2\pi}{3} - \frac{\sqrt{2\pi}a_d\Omega_d}{2v_{T,e}} \right] \right). \quad (135)$$

This demonstrates the dominance of ions in determining the rotational dynamics in the high magnetic field limit for $\Upsilon_{0,e} e^{-\chi_a} \leq \Upsilon_{0,i}$. It is worthwhile noting however that for a dust grain with $\chi_a \rightarrow 0$, the contribution to torque by ions and electrons cancels exactly in the $B \rightarrow \infty$ high field limit.

5.1.2.4 Intermediate Fields, Adiabatic Limit

Outside the high magnetic field limit, the problem is much more difficult to solve as the potential of the dust is now able to influence particle trajectories significantly and cannot be sidestepped as easily. Additionally, the grazing incidence condition is not automatically valid since the gyro-radius of particles will be much larger than the sphere. Consequently, many approximations are required in order to attain an analytical solution. Following the approach in deriving the helical limit of

the current in section 4.1.3.1, the second moment of the distribution function is evaluated to provide an integral equation for the torque similar to equation 101. Re-introducing the normalisation scheme used previously, see section 4.1.3.1, the torque due to species s is

$$\begin{aligned} \gamma_s = & 2\sqrt{2\pi}n_a^2v_{T,s}\beta_s^4 \\ & \int_0^\infty \int_0^\infty \int_0^\infty \text{sh}\Theta(s^2 + h^2 - D^2)e^{\chi_a - \frac{\beta_s^2(s^2 + h^2)}{2}} \\ & \int_0^{\psi_c(z_c)} bL(h, s, b)d\psi_0 ds dh db. \end{aligned} \quad (136)$$

The integral over the initial phase ψ_0 between 0 and $\psi_c(z_c)$ is evaluated by substituting equation 99 as before.

The adiabatic approximation used in deriving the lower bound for current in section 4.1.3.2 can be applied to arrive at an estimate for the torque. Again, these approximations apply in the regime where $\rho_\perp$ and a_d are much smaller than the potential scale length and lead to considering particle trajectories that are accelerated by the surface potential only parallel to $\underline{B}$ up to some reference plane $z = -z_0$ such that it is possible to make the substitution $\Theta(s^2 + h^2 - D^2) \rightarrow \Theta(h - D)$. Under these conditions, equation 136 becomes,

$$\begin{aligned} \gamma_s = & 2\sqrt{2\pi}n_a^2v_{T,s}\beta_{T,s}^4 \\ & \int_0^\infty \int_0^\infty \int_0^\infty \text{sh}\Theta(h - D)e^{\chi_a - \frac{\beta_{T,s}^2(s^2 + h^2)}{2}} \int_0^{\psi_c(z_c)} bL(h, s, b)d\psi_0 ds dh db. \end{aligned} \quad (137)$$

To evaluate the innermost part of the integral, it is necessary to make further approximations about the geometry of the angular momentum imparted by helical orbits incident over the spherical surface.

When the magnetic field is finite, the geometric shadowing of orbits may be less pronounced, leading to cancellation as was shown in figure 52b and a large proportion of particles missing the sphere, reducing the overall torque. As was discussed previously and summarised in figure 52, it can be seen that for $\rho_\perp \leq a_d$ the torque due to ion collection is negative for $1 - s \leq b \leq s$ and positive for $s \leq b \leq s + 1$ whilst for $\rho_\perp \geq a_d$ it is negative for $s - 1 \leq b \leq s$ but still positive for $s \leq b \leq s + 1$. Ion orbits that don't enclose the z -axis in the impact parameter range $1 - s < b$ will only experience significant preferential collection if $v_z < a_d\omega_i$. This introduces an additional constraint on the vertical velocity expressed as $\Theta(1 - h)$. Note that this implicitly assumes that $\sqrt{\chi_a/\beta_i^2} \leq 1$ or equivalently $\chi_a \leq \beta_i$, which agrees with the initial ordering of length scales. Making an upper estimate of the torque, it is assumed that all particles are collected irrespective of their initial phase and are collected with grazing incidence in the x - y plane $v_\theta = v_\perp$. The angular momentum of ions incident on a stationary sphere, $\Omega_d = 0$, that is parallel to $\underline{B}$

is $L(s, b) = m_i a_d^2 \omega_i (b - s)s$, meaning the inner part of the integral equation 137 pertaining to the geometry of collected orbits is

$$\begin{aligned} sm_i a_d^2 \omega_i \int_0^\infty \int_0^{\Psi_c(z_c)} b(b-s) d\psi_0 db &= sm_i a_d^2 \omega_i \Theta(1-h) \\ &\left(\Theta(1-s) \left[-\int_{1-s}^s b(b-s) db + \int_s^{s+1} b(b-s) db \right] + \right. \\ &\left. \Theta(s-1) \left[\int_s^{s+1} b(b-s) db - \int_{s-1}^s b(b-s) db \right] \right). \end{aligned} \quad (138)$$

Evaluating this part of the integral,

$$\begin{aligned} sma_d^2 \omega \int_0^\infty \int_0^{\Psi_c(z_c)} b(b-s) d\psi_0 db &= sma_d^2 \omega \\ &\left(\Theta(1-s) \left[\frac{2}{3} - s + 2s^2 - \frac{2s^3}{3} \right] + \Theta(s-1)s \right), \end{aligned} \quad (139)$$

and introducing equation 139 into equation 137, the remaining integral is

$$\begin{aligned} \gamma_i(\Omega_d = 0) &= 2\sqrt{2\pi} m_i a_d^3 v_{T,i}^2 \beta_i^5 e^{\chi_a} \\ &\int_0^\infty \int_0^\infty s^2 h e^{\chi_a - \frac{\beta_i^2(s^2 + h^2)}{2}} \Theta(h-d) \Theta(1-h) \\ &\left(\Theta(1-s) \left[\frac{2}{3} - s + 2s^2 - \frac{2s^3}{3} \right] + \Theta(s-1)s \right) ds dt. \end{aligned} \quad (140)$$

Evaluating this integral gives the following result for the torque

$$\begin{aligned} \gamma_i(\Omega_d = 0) &= 2\sqrt{2\pi} \gamma_{0,i} (1 - e^{\chi_a - \frac{\beta_i^2}{2}}) \left[\frac{\sqrt{2\pi}}{3} \operatorname{erf}\left(\frac{\beta_i}{\sqrt{2}}\right) + \frac{2\left(e^{-\frac{\beta_i^2}{2}} - 3\right)}{3\beta_i} + \right. \\ &\left. \frac{3\sqrt{2\pi} \operatorname{erf}\left(\frac{\beta_i}{2}\right)}{\beta_i^2} - \frac{16\left(1 - e^{-\frac{\beta_i^2}{2}}\right)}{3\beta_i^3} \right]. \end{aligned} \quad (141)$$

In the low field limit, $\beta_i \rightarrow 0$, for $\chi_a = 0$ this suggests that the initial torque, $\alpha = 2\sqrt{2\pi} \gamma_{0,i} \beta_i$, is linearly dependent on β_i . In the high field limit, $\beta_i \rightarrow \infty$, for $\chi_a = 0$, the initial torque, $\alpha = 4\pi/3$, is independent of β_i . The other term that must be calculated due to the relative velocity of the dust surface when $0 \leq |\Omega_d|$ contributes an angular momentum $L = -ma_d^2(b-s)^2\Omega_d$. Here, the integral is performed across all values of v_z since orbits that are not preferentially collected will still contribute to the resistive torque. In this case the inner part of the integral related to the geometry is

$$\begin{aligned} ma_d^2 \Omega_d \int_0^\infty \int_0^{\Psi_c(z_c)} b(b-s)^2 d\psi_0 db &= ma_d^2 \Omega_d \\ &\left(\Theta(1-s) \left[-\int_{1-s}^s b(b-s)^2 db + \int_s^{s+1} b(b-s)^2 db \right] + \right. \\ &\left. \Theta(s-1) \left[\int_s^{s+1} b(b-s)^2 db - \int_{s-1}^s b(b-s)^2 db \right] \right). \end{aligned} \quad (142)$$

Repeating the same process used in attaining equation 141, equation 142 is substituted into equation 137 to obtain

$$\begin{aligned} \gamma_i(\Omega_d > 0) - \gamma_i(\Omega_d = 0) = & -\sqrt{2\pi}\gamma_{0,i} \frac{a_d \Omega_d}{v_{T,i}} \left[1 - \frac{4\sqrt{2\pi}\text{erf}\left(\frac{\beta_i}{\sqrt{2}}\right)}{3\beta_i} \right. \\ & \left. + \frac{8(6 - e^{-\frac{\beta_i^2}{2}})}{3\beta_i^2} - \frac{12\sqrt{2\pi}\text{erf}\left(\frac{\beta_i}{\sqrt{2}}\right)}{\beta_i^3} + \frac{64(1 - e^{-\frac{\beta_i^2}{2}})}{3\beta_i^4} \right]. \end{aligned} \quad (143)$$

When $\chi_a = 0$ and in the low field limit, $\beta_i \rightarrow 0$, this yields $\gamma = \sqrt{\pi}\gamma_{0,i}\beta_i^2 a_d/v_{T,i}\sqrt{2}$ whilst for $\beta_i \rightarrow \infty$, $\gamma = \sqrt{2\pi}\gamma_{0,i}a_d/v_{T,i}$. Taking together equations 141 and 143 provides an upper bound for the torque acting on dust due to the impact of charged particles.

For electrons, identical arguments apply but with the sense of angular momentum contribution reversed, as was shown in figure 53 and figure 54, and with $\Theta(t - d) \rightarrow 1$ as was the case for the currents. With these changes, similar expressions to equation 141 and equation 143 are produced such that

$$\begin{aligned} \gamma_e(\Omega_d = 0) = & 2\sqrt{2\pi}\gamma_{0,e}e^{-\chi_a} \left[\frac{\sqrt{2\pi}}{3}\text{erf}\left(\frac{\beta_e}{\sqrt{2}}\right) + \frac{2(e^{-\frac{\beta_e^2}{2}} - 3)}{3\beta_e} \right. \\ & \left. + \frac{3\sqrt{2\pi}\text{erf}\left(\frac{\beta_e}{\sqrt{2}}\right)}{\beta_e^2} - \frac{16(1 - e^{-\frac{\beta_e^2}{2}})}{3\beta_e^3} \right], \end{aligned} \quad (144)$$

and

$$\begin{aligned} \gamma_e(\Omega_d > 0) - \gamma_e(\Omega_d = 0) = & -\sqrt{2\pi}\gamma_{0,e} \frac{a_d \Omega_d}{v_{T,e}} e^{-\chi_a} \left[1 - \frac{4\sqrt{2\pi}\text{erf}\left(\frac{\beta_e}{\sqrt{2}}\right)}{3\beta_e} \right. \\ & \left. + \frac{8(6 - e^{-\frac{\beta_e^2}{2}})}{3\beta_e^2} - \frac{12\sqrt{2\pi}\text{erf}\left(\frac{\beta_e}{\sqrt{2}}\right)}{\beta_e^3} + \frac{64(1 - e^{-\frac{\beta_e^2}{2}})}{3\beta_e^4} \right]. \end{aligned} \quad (145)$$

Taking the sum of equations 141, 143, 144 and 145 provides an upper bound estimate for the total torque experienced by a dust grain due to collection of both ions and electrons as a function of the plasma conditions and magnetisation parameter.

To model experiments performed in weakly ionised plasmas with a significant neutral background discussed later in section 5.3, further resistance to spinning motion due to neutral drag must be accounted for. For neutrals, the torque will be independent of magnetic field strength and is estimated by,

$$\gamma_n = -\gamma_{0,n} \sqrt{\frac{\pi}{2}} \frac{\Omega_d a_d}{v_{T,n}}, \quad (146)$$

which is equivalent to equation 143 when $\beta_i \rightarrow 0$ with $T_i \rightarrow T_n$ and $n_0 \rightarrow n_n$. However, the neutral torque $\gamma_{0,n} = n_n k_B T_n a_d^3$ and neutral thermal velocity $v_{T,n}$ can differ greatly from the corresponding parameters for ions and electrons.

5.2 DUST IN MAGNETISED PLASMAS (DIMPL) SIMULATIONS

The rotational dynamics of spherical conductors were modelled with DiMPI for dust with a fixed surface potential in a fully ionised hydrogen plasma. The dependence of α and γ on the ion magnetisation parameter, β_i , was examined and interpreted using the theoretical model developed in section 5.1.2.

5.2.1 Simulation Methodology

Extensions to the methodology of DiMPI described in section 4.2.3 are required to explain the calculations performed to ascertain the simulated rotational motion of dust. Once the termination condition for collected particles $|\mathbf{r}(t)| \leq a_d$ is satisfied, plasma particles are removed from the simulation and their relative angular momentum, $\mathbf{L} = m_s \mathbf{R} \times (\mathbf{v} - \mathbf{R} \times \mathbf{\Omega}_d)$, is added to the dust grain, updating the dust rotational velocity, $\mathbf{\Omega}_d$, which is initially set to zero.

A convenient normalisation scheme for measurement of torques, α , and equilibration time scale, K/γ , is established based on the ion plasma parameters

$$\tilde{\alpha} \equiv \frac{\alpha}{\gamma_{0,i}} = \frac{\alpha}{a_d^3 n_i k_B T_i}, \quad \frac{1}{\tilde{\gamma}} \equiv \frac{M_i n_i v_{T,i}}{a_d \rho_d} \frac{K}{\gamma}, \quad \tilde{t} \equiv \frac{\gamma_{i,0}}{K} t = \frac{M_i n_i v_{T,i}}{\rho_d a_d} t. \quad (147)$$

The torque from collection of plasma particles of equation 128 is independent of the dust grain mass. As a result, the initial torque, α , and the parameter, γ , are independent of the dust mass allowing the dust density to be chosen as a free parameter. This was exploited to optimise simulation performance by altering the moment of inertia, $K = 2M_d a_d^2/5$, of the solid, spherical dust grain. Decreasing the density of dust grains decreased the time taken for simulations to reach equilibrium whilst simultaneously increasing the magnitude of random fluctuations from the stochastic particle collection process, reducing the overall precision of measurements. Preliminary tests found an unphysical dust grain mass density of $\rho_d = 5 \times 10^{-5} \text{ kg m}^{-3}$ gave a suitable trade-off between speed and precision. To improve the precision of measurements further, multiple runs were performed with different initial seeds for each value of β_i and χ_a and averaged, as shown in figure 55. The rotation rate is seen to approach an equilibrium rotation rate, Ω_{eq} , as expected, with an initial slope that is approximately linear, $\Omega_d(t \rightarrow 0) \simeq \alpha/K$.

The mean angular velocity, $\langle \Omega_d \rangle$, parallel to each orthogonal direction as a function of normalised time was calculated using equation 118. The z component of angular velocity, $\langle \Omega_{d,z} \rangle$, was fit to a function of the form of equation 129, with $\tilde{\alpha}$ and $\tilde{\gamma}$ being free parameters as is demonstrated for $\beta_i = 1$ and $\chi_a = 0$ in figure 56. This fit converged for $\tilde{\alpha} = -0.2917 \pm 0.0001$ and $\tilde{\gamma} = 4.404 \pm 0.001$ with $R^2 = 0.9999$. Using this method, the values of $\tilde{\alpha}$ and $\tilde{\gamma}$ were extracted from the average of a number of independent simulations performed with unique seeds, with the uncertainties in these values based on the quality of the fit. The results for the other two orthogonal directions $\Omega_{d,x}$ and $\Omega_{d,y}$ found zero time averaged rotational motion. The rotational motion in the directions perpendicular to $\mathbf{B}$ appears to be a form of Brownian motion with fluctuations around zero that grow larger for greater values of γ/K . Having demonstrated in figure 56 that significant rota-

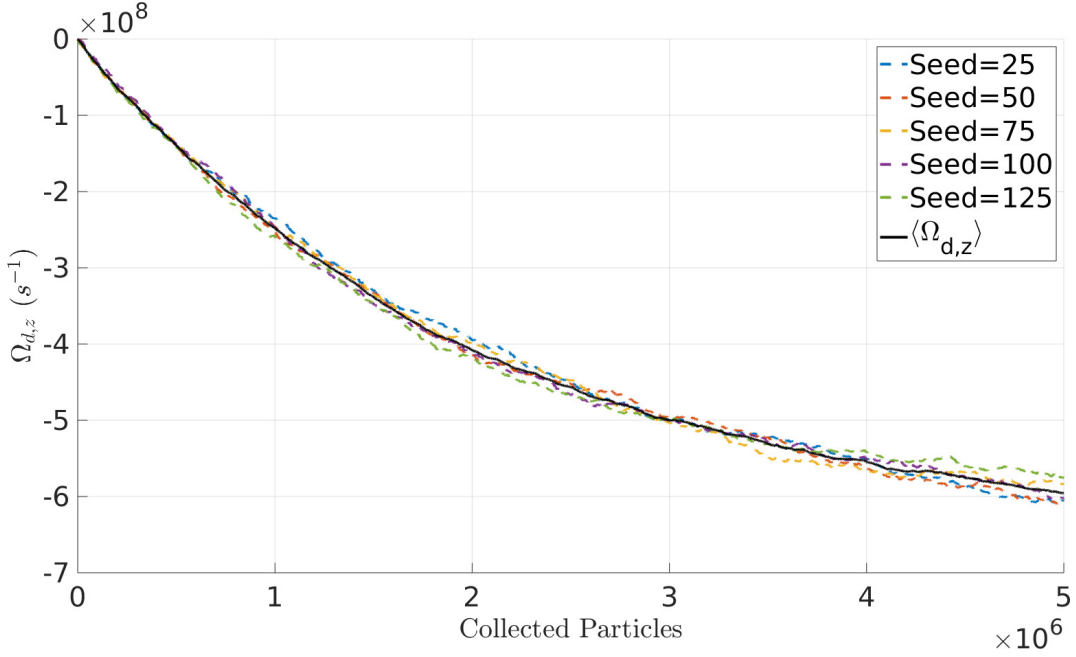


Figure 55: Dust angular velocity parallel to $\underline{B}$, $\Omega_{d,z}$, as a function of number of particles collected for five different random seeds and their average, $\langle\Omega_{d,z}\rangle$ for $\beta_i = 1.0$ and $\chi_a = 0$.

tional motion occurs exclusively in the direction parallel to z , subsequently only the component parallel to $\underline{B}$ will be of interest so that $\Omega_d \equiv \Omega_{d,z}$ is assumed.

An alternative approach to measuring α and γ involved calculating the equilibrium rotation rate and initial torque independently. To obtain the equilibrium rotational velocity, $\Omega_{eq} = \alpha/\gamma$, the dust mass density is dynamically reduced throughout the simulation as equilibrium is approached in analogy to the way in which the equilibrium charge was measured in figure 29. The initial dust density is determined from the condition $K/\Upsilon_{0,i} = 1000$, so that, $\rho_d \simeq M_d n_i v_{T,i}/(1000 a_d)$. As with charging, the mean angular velocity of the dust after collecting groups of N particles is then calculated. If the sign of the mean angular velocity of the dust after collecting N particles changes, the value of dust density was doubled $\rho_d \rightarrow 2\rho_d$ effectively halving the angular momentum imparted by collected particles. This method is demonstrated in figure 57 for $\beta_i = 1.0$ and $\chi_a = 1.0$ using averages over $N = 500$ particles. This method provides a much faster approach to equilibrium than possible in the method demonstrated in figure 55 and obtains the same result to a desired accuracy. To measure the initial torque, α , separate simulations were performed with the dust mass density fixed at a large value that meant $\gamma\Omega_d \ll \alpha$ for the duration of the simulation. Equation 128 then reduces to $\Upsilon = \alpha$ and the value α can be related directly to the rate of change of angular velocity and extracted from a linear fit. This technique is demonstrated in figure 58 with an accurate measurement of α being made with dust having the density of solid tungsten $\rho_d = 19600 \text{ kg m}^{-3}$. This fitting method was found to converge well for all values of β_i and χ_a investigated, verifying the compatibility of the time dependence of the theoretical model used in deriving equation 129 with the steady state assumptions of DiMPL simulations.

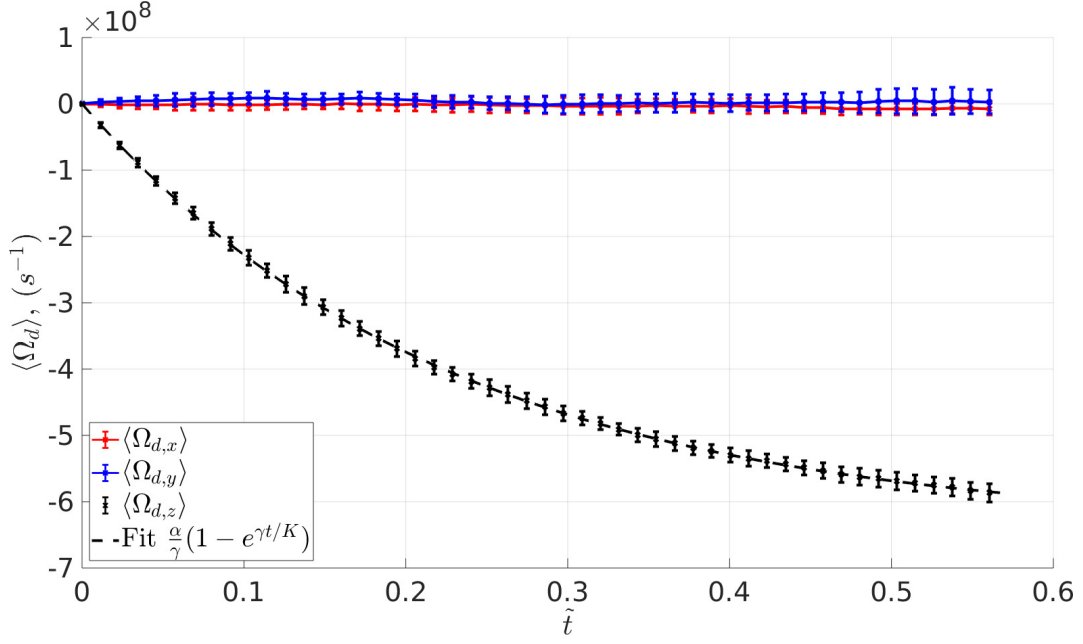


Figure 56: Dust angular velocity, $\underline{\Omega}_d = (\Omega_{d,x}, \Omega_{d,y}, \Omega_{d,z})$, in each orthogonal direction as a function of normalised time, $\tilde{t}$, for $\beta_i = 1$ and $\chi_a = 0$ plotted with standard deviations. Equation 129 is fit to the results for $\langle \Omega_{d,z} \rangle$ with $\tilde{\alpha} = -0.2917 \pm 0.0001$ and $\tilde{\gamma} = 4.404 \pm 0.001$ with $R^2 = 0.9999$.

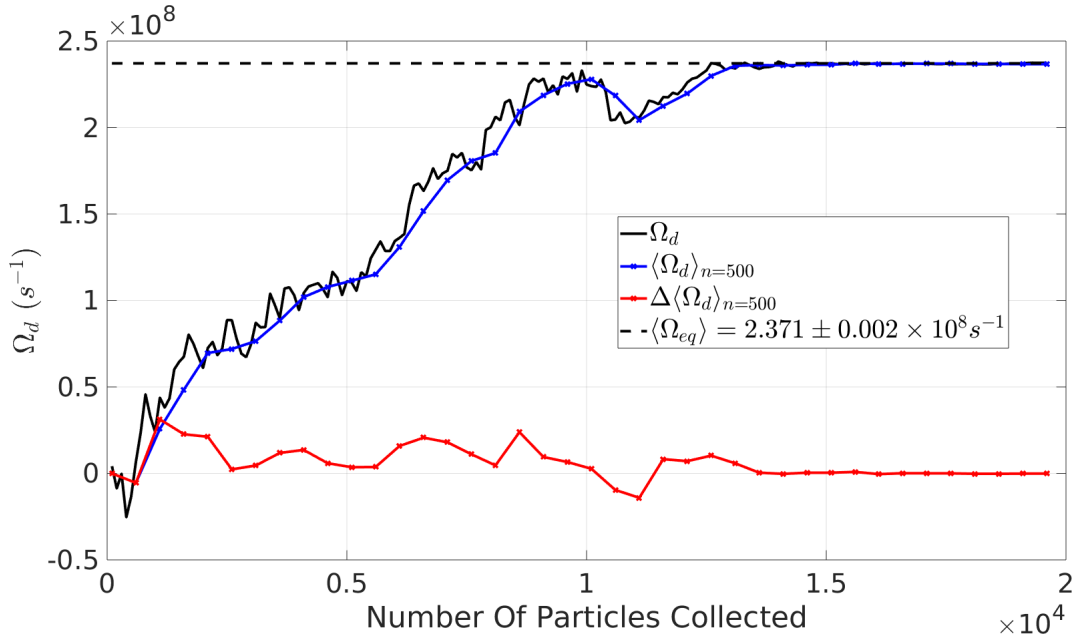


Figure 57: Dust angular velocity parallel to magnetic field, Ω_d , (black solid), average of angular velocity for 500 particles (blue), difference in angular velocity between averages (red) and measured equilibrium rotation speed (black dashed) as a function of number of particles collected for $\beta_i = 1.0$ and $\chi_a = 1.0$.

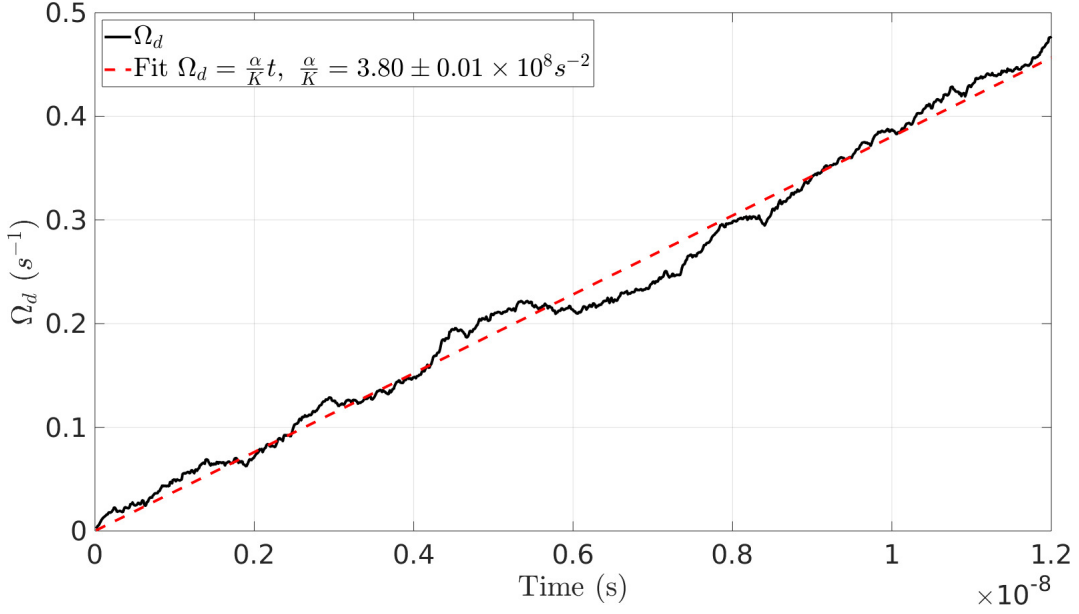


Figure 58: Linear fit $\Omega_d = \alpha t/K$ to the dust angular velocity, Ω_d , as a function of time for $\beta_i = 1$ and $\chi_a = 1.0$ with $\alpha/K = 3.80 \pm 0.01 \times 10^8 s^{-2}$ having $R^2 = 0.9903$.

5.2.2 Dust at the Plasma Potential

To provide comparison with theory and previous numerical results, the case of spheres with surface potentials fixed at the plasma potential, $\chi_a = 0$, were first investigated. Simulations were performed for a spherical dust grain of radius $a_d = 1 \mu m$ and density $\rho_d = 10^{-5} kg m^{-3}$ in a $T_i = T_e$ hydrogen plasma without background flow. Due to the absence of the electric field, the injection plane could be positioned at a height $z_0 = 1.0001 a_d$ with impact factor $b = 10$.

Figure 59 shows the normalised initial torque, $\tilde{\alpha}$, due to impact of ions and electrons in isolation and in combination calculated in separate simulations for a range of values of ion magnetisation parameter, β_i . The initial torque due to ions is directed parallel to $\underline{B}$ and anti-parallel to $\underline{B}$ for electrons. The results indicate that the dependence of the torque due to ions on β_i is identical in magnitude but of opposite sign to the dependence of the torque due to electrons on β_e . In the limit $\beta_i \rightarrow 0$, no torque is experienced by the dust due to collection of either species as expected from equation 141 and the symmetry of the problem. For low ion magnetisation $\beta_i \leq 1$, the torque due to electrons gradually increases until reaching a constant value of $\alpha \simeq -0.55\Upsilon_0$ for $\beta_i \simeq 0.5$. At this point, the torque due to ion collection increases up to $\alpha \simeq 0.55\Upsilon_0$ at $\beta_i \simeq 10.0$ having the opposite sign. The net effect with both species is for the torque to be directed in the sense caused by electron collection anti-parallel to $\underline{B}$, reaching a peak at a value equivalent to the electron high field limit torque at $\beta_i \simeq 0.5$. As the ions begin to counteract this, the torque gradually decreases until the initial net torque of both species cancel in the limit $\beta_i \rightarrow \infty$. This feature matches the predictions of section 5.1.2.3 and specifically equation 135 for the relative effect of electrons and ions for a sphere fixed at the plasma potential. However, the magnitude of the high field limit initial torque $\alpha = 0.55\Upsilon_0$ is substantially smaller than the theoretical infinite field value

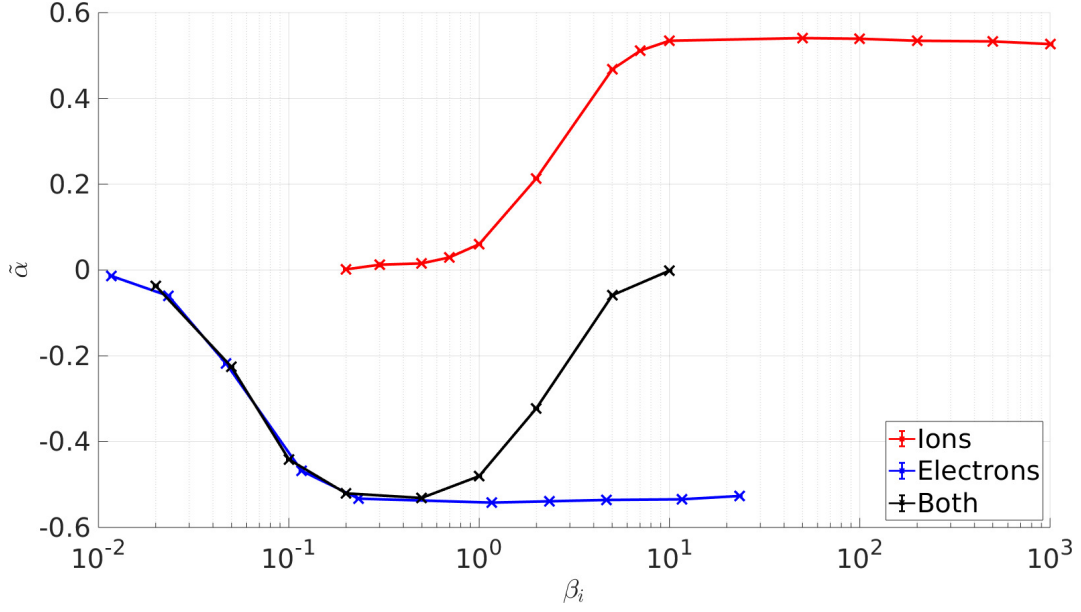


Figure 59: Normalised initial torque, $\tilde{\alpha}$, due to collection of electrons, ions and both in combination as a function of ion magnetisation parameter β_i .

$\alpha \simeq 2\pi\gamma_0/3$ or even the high field limit of equation 141. However, the high field limit initial torque for ions does agree with the previous PIC code results shown in figure 50. In general, the theoretical description recovers the trend and order of magnitude of the torque but provides an overestimate of the magnitude of α . This is not surprising since the approximations of grazing incidence and total particle collection made were expected to overestimate the torque generated.

The corresponding equilibrium rotation rates, α/γ , are plotted in figure 60 as a function of ion magnetisation parameter, β_i . The equilibrium rotation rate, $\Omega_{eq} = \alpha/\gamma$, when considering torque due only to collection of electrons is approximately $\sqrt{\mu}$ larger than that for ions, being of the order of $v_{T,e}/a_d$ in the high field limit (as opposed to $\sim v_{T,i}/a_d$ for ions). This is because the resistance to the rotation of the dust is similarly $\sqrt{\mu}$ smaller than the ion resistance as is seen in equation 135. When both ions and electrons are accounted for, the equilibrium rotation rate is dictated at weak magnetic fields, $\beta_i \leq 1$, by the initial torque of electrons as hampered by the ions. At magnetic field strengths of $\beta_i \simeq 0.5$ when the electron torque is greatest but ions remain relatively unaffected, a peak is reached that is slightly smaller than the ion high field equilibrium rotation rate. This slight decrease occurs because of the additional resistance to spinning motion provided by electrons, which further decreases the maximum equilibrium rotation rate below that measured when only accounting for ions. At larger magnetic field strengths, $\beta_i = 10.0$, the ion and electron initial torques begin to cancel as they become equal and opposite in magnitude as was shown in figure 59, further decreasing the equilibrium rotation rate. These results support theoretical descriptions for which γ is governed by the heaviest plasma species, in this case ions, and α driven by electrons for $\beta_i \leq 1$ and in competition between ions and electrons for $1 \leq \beta_i$.

From the normalised equilibration timescale, $1/\tilde{\gamma}$, shown in figure 61, it is clear that ions control the behaviour of this term. For electrons in isolation, the spin up

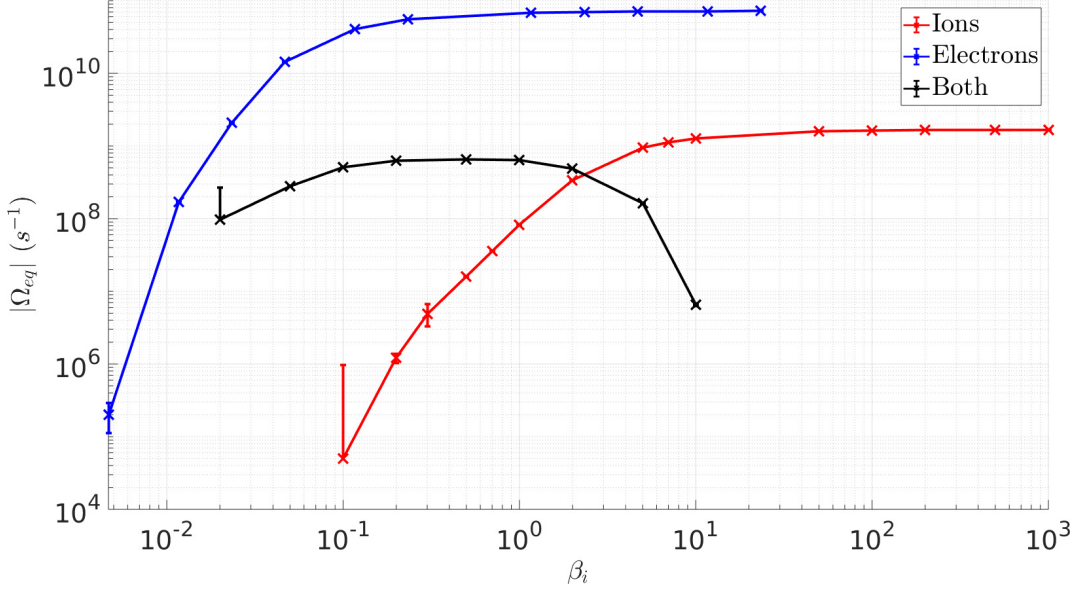


Figure 60: Magnitude of equilibrium rotation rate, $|\Omega_{eq}|$, due to collection of electrons, ions and both in combination as a function of ion magnetisation parameter β_i .

time is slower by a factor of $\sqrt{\mu}$ than ions. In the high field limit, the equilibration rate for ions appears to be $1/\tilde{\gamma}(\beta_i \rightarrow \infty) \simeq 0.5$ suggesting approximate agreement with the high field limit $1/\tilde{\gamma}(\beta_i \rightarrow \infty) = 15\sqrt{2}/16\sqrt{\mu} \simeq 0.75$ from equation 133 but substantially larger than that calculated from equation 143. For high magnetic field strengths, $50 \leq \beta_i$, the equilibration timescale decreases. Accounting for both ions and electrons, the timescale is seen to be very similar to the case of ions in isolation and both ions and electrons, suggesting that ions play a dominant role in controlling the equilibration rate, γ/K . For both species together with $\beta_i \leq 0.5$, the spin up time plateaus in conjunction with the equilibrium rotation speed as the torque resisting the motion is still dominated by weakly magnetised ions whilst electrons provide the same magnitude of torque. The uncertainty in measurements for ions for $\beta_i \leq 0.5$ and electrons for $\beta_i \leq 0.01$ are large as the equilibrium rotation rate is small and of comparable size to the stochastic fluctuations for the density chosen.

To summarise these results, simulations in DiMPl show significant rotational motion along the z-axis parallel to $\underline{B}$ as a result of collisions of plasma particles with the dust surface. Through examining the influence of ions and electrons separately in isothermal plasmas, the net torque was shown to be anti-parallel to $\underline{B}$ as a result of impact of electrons for $\beta_i \leq 1$ when $\chi_a = 0$, with the time scale and equilibrium rotation rate controlled by the resistive ion drag. In the high field limit, the net initial torque is zero as the initial torque of ions and electrons cancel exactly. These observations and the dependence of α and γ on β_i agree with the predictions of the theoretical model developed in section 5.1.2 in the various magnetic field limits, though the theoretical values overestimate the torque magnitudes substantially.

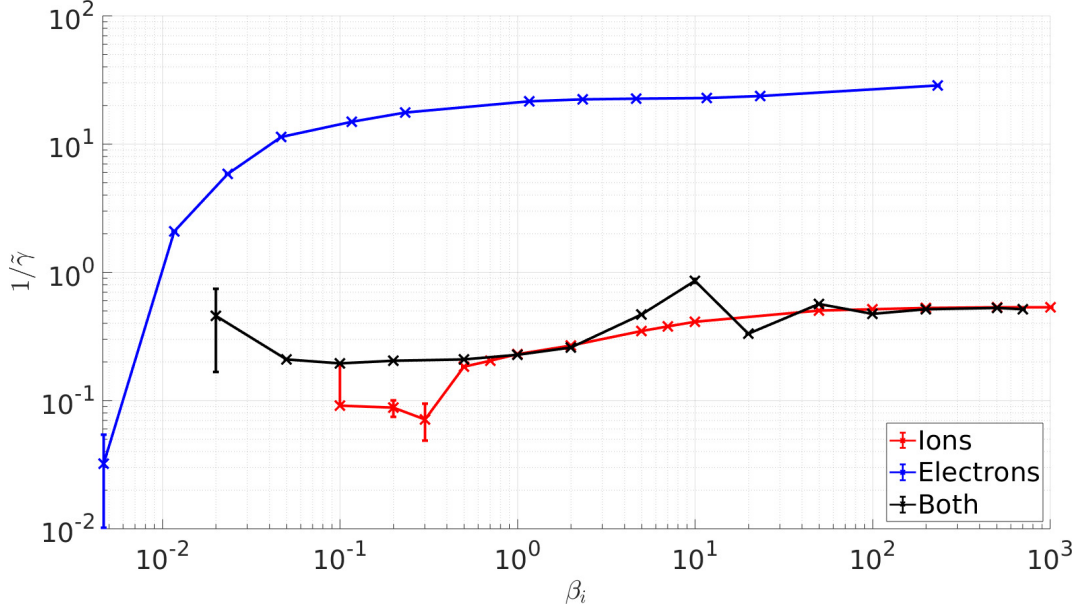


Figure 61: Normalised equilibration timescale, $1/\tilde{\gamma}$, due to collection of electrons, ions and both in combination as a function of ion magnetisation parameter β_i .

5.2.3 Dust at Fixed Potential

The more realistic case of a charged dust grain was also investigated using DiMPL. This was performed firstly with a Debye-Hückel potential with $\tilde{\lambda}_d = 1$ and a fixed surface potential from $\chi_a = -5$ and $\chi_a = 5$ over a range of ion magnetisation parameters from $\beta_i = 0.01$ to $\beta_i = 10.0$. This parameter scan was performed to understand the relative influence of ions and electrons as well as the influence of these parameters on the rotational motion for a range of physically realistic normalised potentials and values of ion magnetisation parameter. The range of values of ion magnetisation parameter were chosen based on the limitations of computational times for simulations with $\beta_i \leq 0.01$ when the injection surface is large and for $\beta_i \geq 100.0$ when the electron gyro frequency becomes limiting.

The equilibrium rotation rate, $\Omega_{eq} = \alpha/\gamma$, as a function of ion magnetisation parameter β_i is shown in figure 62 for negatively charged dust with five different values of normalised surface potential. Comparing these results with the uncharged case of figure 60, several important features are identified. The dust grain rotation rate increases with increasing ion magnetisation parameter from $0.1 \leq \beta_i \leq 2$ and is directed in the sense caused by collection of angular momentum from ions. Ions dominate the rotational dynamics in this case because the presence of negative charge on the dust grain greatly increases their flux to the surface relative to electrons. The magnitude of the peak in rotation speed increases approximately linearly with increasing χ_a but still occurs at $\beta_i \simeq 2$. At large ion magnetisation parameters, $2 \leq \beta_i$, the equilibrium rotation rate tends to zero as the flux of ions to the surface is also reduced with increasing ion magnetisation parameter. The onset of rotational dynamics at comparatively weak magnetic field strengths $\beta_i \simeq 0.1$ and the broadening of the peak with increasing χ_a suggest that the potential enhances the torque asymmetry which is still present even for very low β_i .

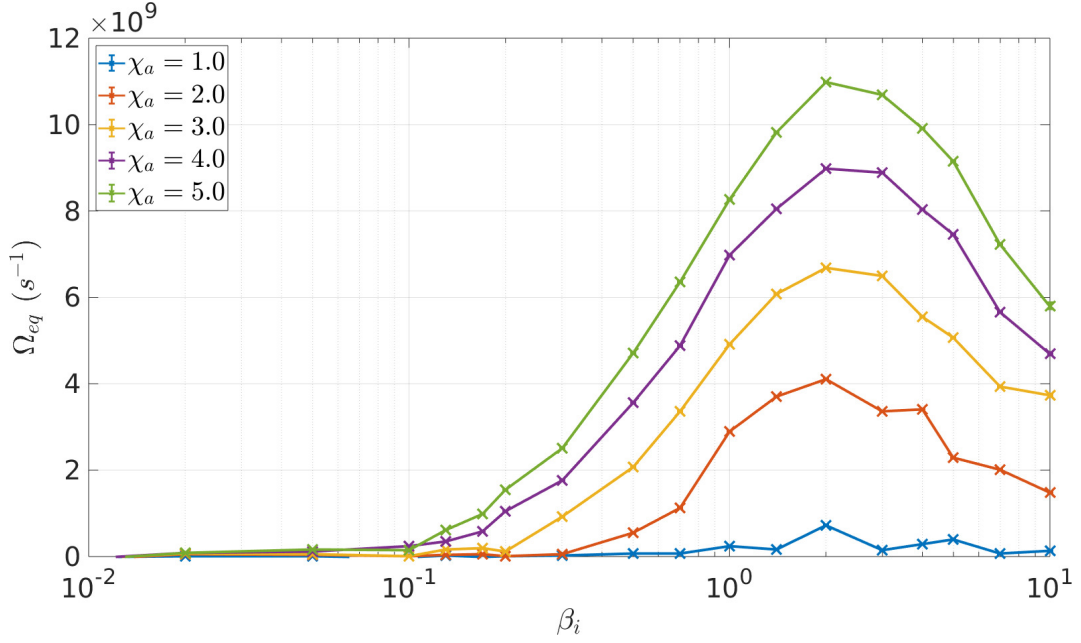


Figure 62: Equilibrium rotation rate, Ω_{eq} , of negatively charged dust as a function of ion magnetisation parameter, β_i , for five different normalised potentials, $0 < \chi_a$, with $\tilde{\lambda}_d = 1$

From the normalised initial torques, $\tilde{\alpha}$, shown figure 63, it is clear that the surface potential has a significant effect. A smooth peak is observed at $\beta_i \simeq 1$ which also increases in magnitude for greater surface potentials. Interestingly, the peak in initial torque occurs at smaller values of ion magnetisation parameter, β_i , than the peak in equilibrium rotational speed, unlike the uncharged case. This is best explained as a consequence of the total flux of ions decreasing for $1 \leq \beta_i$, simultaneously decreasing γ and influencing Ω_{eq} at this point. The initial torque with a self-consistent surface potential, $\chi_a(\beta_i)$, taken from the results of DiMPL from section 4.2.6 is similar to the results with a fixed normalised surface potential of $\chi_a = 3$ but with a more gradual increase towards the peak at $\beta_i = 1$. Comparing these results to figure 59, the peak torque magnitude is seen to be greater than the uncharged case for all χ_a . The negative potential increases the kinetic energy of impacting ions, increasing both α and γ by similar amounts, which explains the similarity in peak equilibrium rotation speeds observed between the charged and uncharged case.

This feature is highlighted by the variation in equilibration timescale, $1/\tilde{\gamma}$, with normalised ion magnetisation parameter, β_i , shown in figure 64. The time scale grows longer for greater χ_a but is actually smaller than the uncharged case, figure 61, for $\chi_a \leq 2$. The peak in the equilibration time shifts to larger β_i for increasing χ_a , with a steep cut off at $\beta_i \simeq 1$ for $\chi_a = 5$. This is indicative of a transition away from a regime where $1/\tilde{\gamma}$ is influenced by electrons at low χ_a as was seen in the uncharged case to one where it is dependent purely on ion momentum. The sharp drop at $\beta_i = 1$ suggests that at this point most ion orbits experience significant geometric shadowing and hence are preferentially collected. Fluctuations in the equilibration timescale at $\beta_i = 0.13, 0.17$ observed for all χ_a suggest a sensitive dependence of the rotational dynamics on these values of β_i . This corresponds

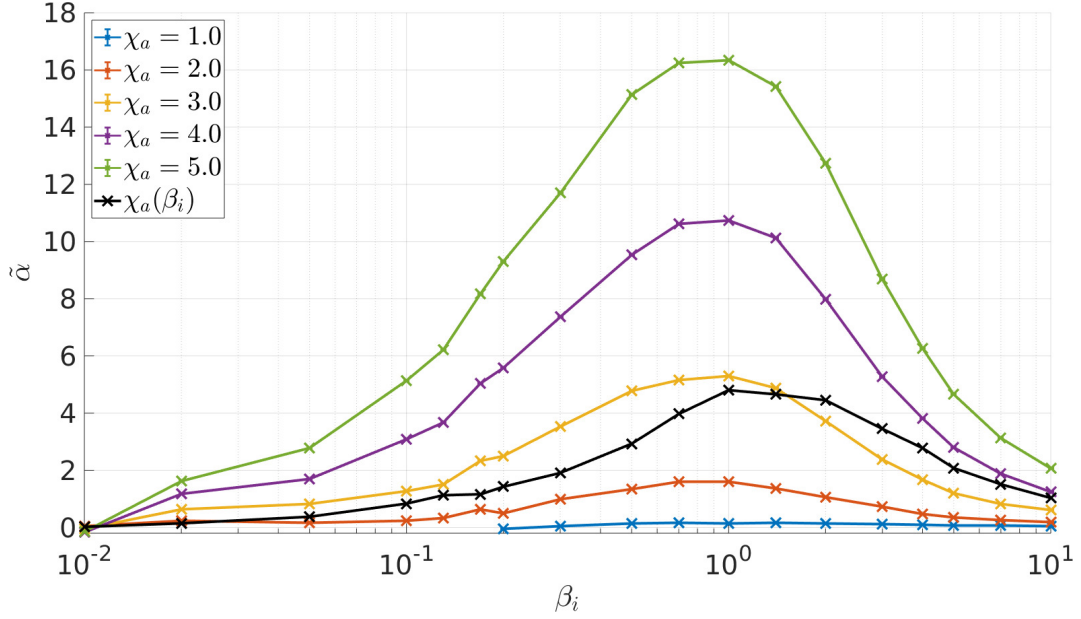


Figure 63: Normalised initial torque, $\tilde{\alpha}$, acting on negatively charged dust as a function of ion magnetisation parameter, β_i for five different normalised potentials, $0 < \chi_a$ and for a self-consistent surface potential $\chi_a(\beta_i)$ with $\tilde{\lambda}_d = 1$.

with the characteristic ion magnetisation parameter value at which the electron and ion currents begin to compete as was seen with charging in section 4.2.6, figure 39.

Though in ordinary plasmas, dust is expected to acquire a negative charge, the high temperatures of dust in tokamak environments is anticipated to lead to significant thermionic emission of electrons which can produce dust with a small positive potential, as was shown in section 3.2.6.1. Consequently, the rotational dynamics of dust with a range of positive surface potentials was also investigated.

The equilibrium rotation frequency, Ω_{eq} , of positively charged dust shown in figure 65 is directed in the sense due to collection of electron angular momentum. This result is expected since the positive surface potential greatly increases the flux of electrons to the surface whilst reducing the ion flux. Comparing the results with the uncharged case of figure 60 and the negatively charged dust in figure 62, the most significant rotational motion occurs at comparatively small values of ion magnetisation parameter $0.01 \leq \beta_i \leq 1$. This range of β_i suggests rotational motion can be induced at much weaker magnetic field strengths than previously anticipated, having implications for a much wider range of plasma environments.

Figure 66 shows the normalised initial torque, $\tilde{\alpha}$, experienced by positively charged dust also peaks at small values of ion magnetisation parameter $\beta_i \simeq 0.02$. The magnitude of the peak torque is approximately equivalent to the peak torque of negatively charged dust for all χ_a and also increases linearly with increasing potential. Interestingly, the uncharged case indicates a smooth transition as the potential changes sign from a peak in torque at low $\beta_i \sim 0.02$ due to electron collection to a peak at large $\beta_i \sim 1$. The torque at larger magnetic field strengths decreases as a result of the reduction in electron flux for $0.1 \leq \beta_i$ and the counteracting torque of ions.

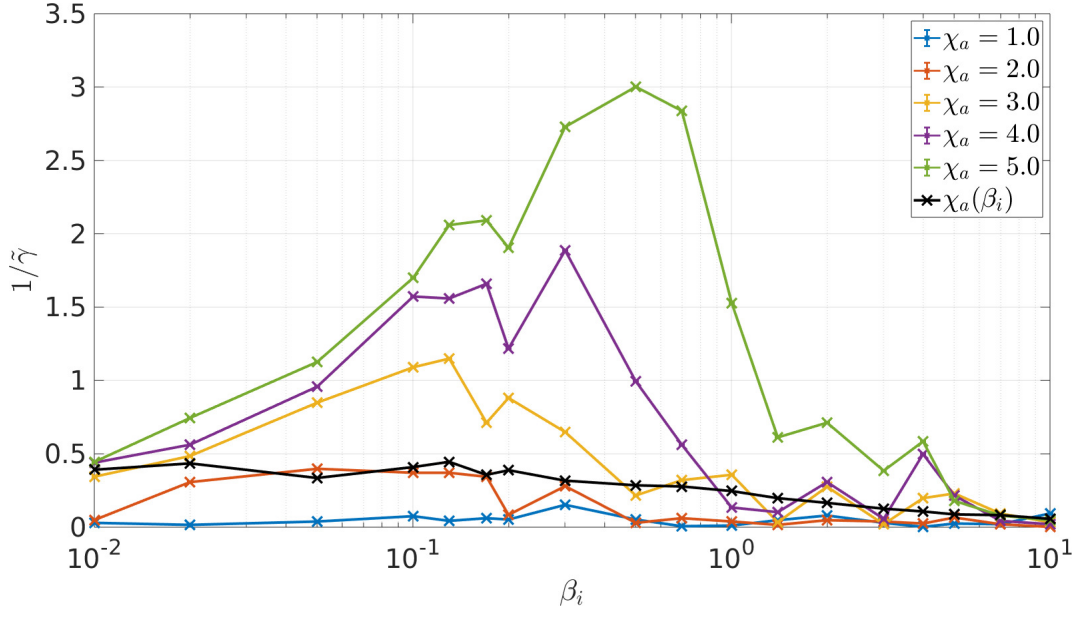


Figure 64: Spin equilibration timescale, $1/\tilde{\gamma}$, of negatively charged dust as a function of β_i for five different normalised potentials, $0 < \chi_a$, and for a self-consistent surface potential $\chi_a(\beta_i)$ as measured by DiMPL with $\tilde{\lambda}_d = 1$.

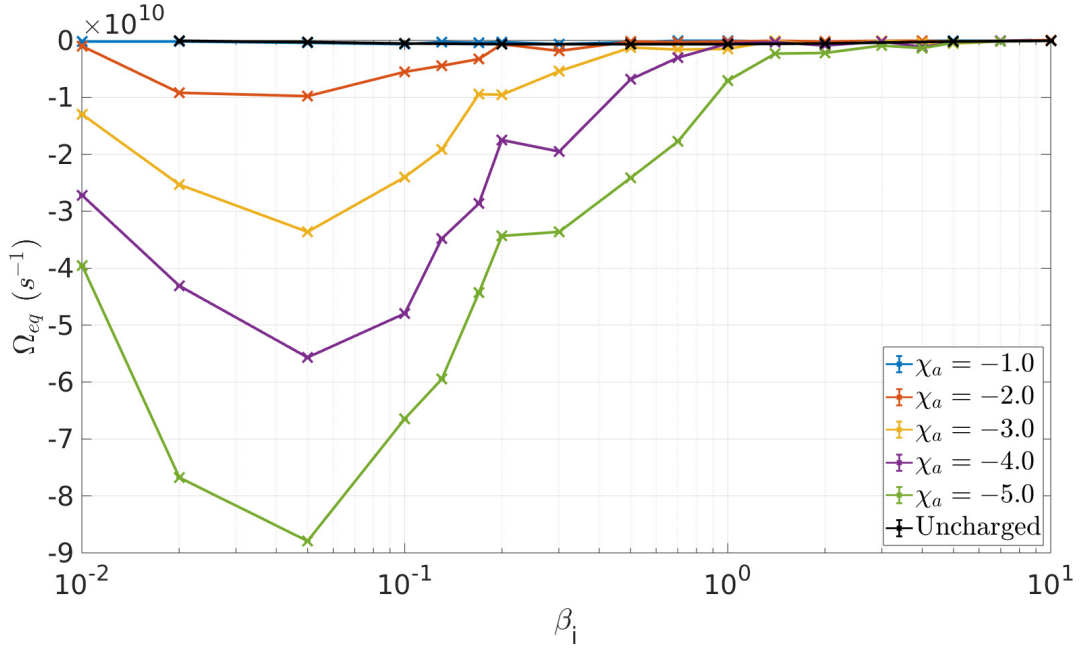


Figure 65: Equilibrium rotation rate, Ω_{eq} , of positively charged dust as a function of ion magnetisation parameter, β_i for five different normalised potentials, $\chi_a < 0$, with $\tilde{\lambda}_d = 1$ as compared with the uncharged case of figure 60.

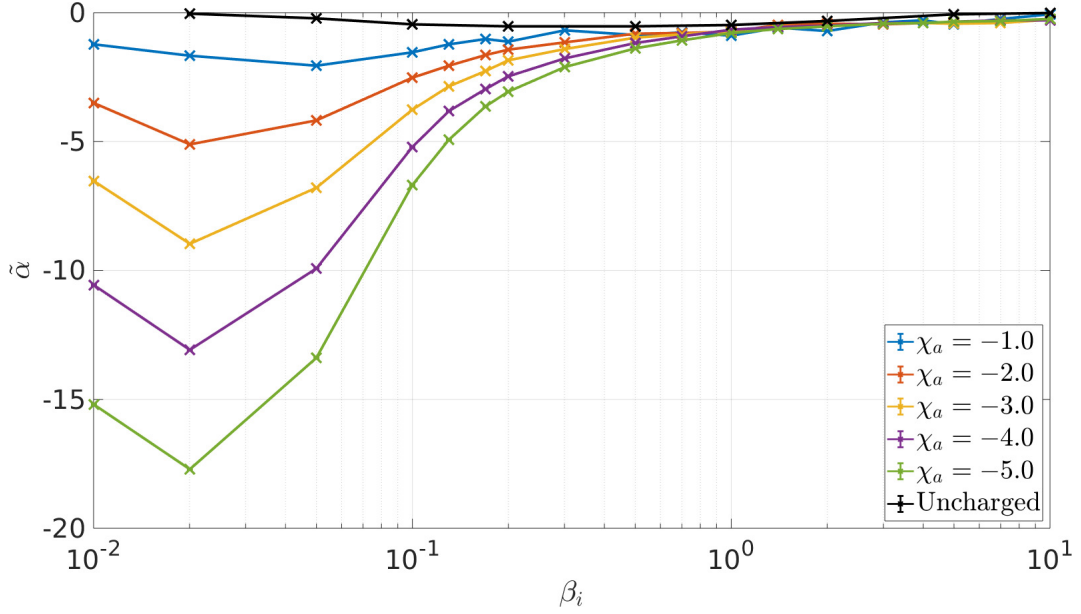


Figure 66: Normalised initial torque, $\tilde{\alpha}$, acting on positively charged dust as a function of ion magnetisation parameter, β_i for five different normalised potentials, $\chi_a < 0$, with $\tilde{\lambda}_d = 1$ as compared with the uncharged case of figure 59.

Finally, the normalised equilibration time scale, $1/\tilde{\gamma}$, for rotational motion as a function of ion magnetisation for positively charged dust is shown in figure 67. The results for $\chi_a = -1.0$ have been omitted due to the large uncertainties in these measurements. This timescale is faster than for negatively charged dust for all χ_a for $\beta_i \leq 1$ with a gradual decrease for $1 < \beta_i$. This increase most likely arises from the growing influence of ions in dictating the equilibration rate, γ/K , at larger magnetisations. Overall, the initial torques and equilibrium rotation rates for positively charged dust mirror those of negatively charged dust, causing rotational motion in the sense due to electron collection which is most significant for $\beta_i \simeq 0.02$.

The results of DiMPl simulations of uncharged dust are in agreement with the previous numerical estimates presented in section 5.1.1 and remain smaller than the theoretical upper bound for torque established in section 5.1.2. The crucial feature in determining the direction and threshold magnetic field strength for rotational motion of a dust grain is the surface potential. Negatively charged dust experiences the greatest net torques for $\beta_i \simeq 1.0$ due to collection of angular momentum from ions. This causes the dust to reach fast equilibrium rotation velocities of the order of $a_d/v_{T,i}$. However, for positive potentials, dust is found to rotate in the sense due to electron impact for very small values of ion magnetisation parameter of $\beta_i \simeq 0.02$ or equivalently $\beta_e \simeq 1$, with comparable rotation frequencies.

These results indicate that asymmetric collection of plasma particles by charged dust grains can lead to substantial rotational motion. In particular, dust grains acquire significant angular momentum at much lower ion magnetisation parameters than previously predicted. Detailed investigation of the equilibration timescale suggests a dynamic competition between the torque imparted by ions and electrons which becomes more significant for weaker negative surface potentials that

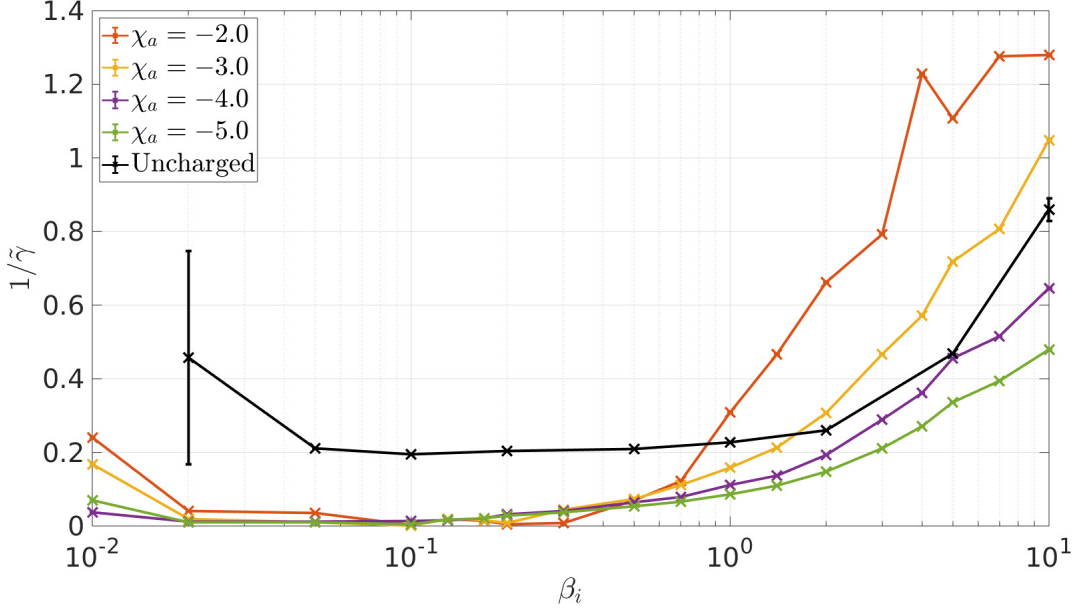


Figure 67: Spin equilibration timescale, $1/\tilde{\gamma}$, of positively charged dust as a function of β_i for four different normalised potentials, $\chi_a < 0$, with $\tilde{\lambda}_d = 1$ as compared with the uncharged case of figure 61.

has been ignored previously. These simulations indicate dust rotational frequencies measured in GHz are achieved over an equilibration time scale of $K/\gamma \simeq a_d \rho_d / m_i n_i v_{T,i}$ which for tungsten dust in a tokamak SOL plasma environment ($a_d = 10^{-6} \text{m}$, $\rho_d = 19600 \text{kgm}^{-3}$, $m_i = m_h$, $n_i = 10^{19} \text{m}^{-3}$, $T_i \simeq 10 \text{eV}$) would correspond to $K/\gamma \simeq 40 \text{s}$. Though this equilibrium rotation frequency may not be achieved in the lifetime of a dust grain ($\sim 10 \text{ms}$) in a tokamak, the large initial torques can produce significant rotational motion even over this shorter timescale. This model has substantial consequences for the dynamics of dust grains in tokamaks that have so far been ignored in previous modelling efforts.

5.3 COMPARISON WITH EXPERIMENTAL OBSERVATIONS

Simulations of rotational motion in DiMPI and the model developed in section 5.1.2 have been applied to describe experiments conducted in low temperature discharges investigating the rotational dynamics of isolated dust grains in stationary plasmas with constant uniform magnetic fields. In a low temperature argon discharge experiment [203], glass dust grains of radii $10 \mu\text{m} - 15 \mu\text{m}$ in a $B = 0.096 \text{T}$ field were observed to rotate predominantly in the direction anti-parallel to $\underline{B}$ with rotation frequencies of $\sim 50 - 100 \text{s}^{-1}$ [203, Figure 6]. For this experiment, no torque due to internal currents as described in section 5.1.1.2 is expected as the dust is composed of an insulating material. For the experimental plasma conditions reported ($n_e = 10^{14} \text{m}^{-3}$, $T_e = 3 \text{eV}$ and pressure 0.2Torr) and assuming $T_i = T_n \simeq 0.03 \text{eV}$ then the ion gyro-radius is expected to be $\rho_{T,i} = 1.1 \text{mm}$ such that $\beta_i \simeq 0.009$. This low value of ion magnetisation parameter suggests the mag-

netic field was too weak for ions to induce a torque but sufficient for electrons with $\beta_e \simeq 2.4$.

Simulations performed by DiMPl show that the torque due to collection of electrons exceeds the torque due to ions under these plasma conditions, causing the dust grain to spin in the paramagnetic direction, anti-parallel to the magnetic field $\underline{B}$. The torque due to electrons estimated from equation 144 and equation 145 in the limit $\beta_e \rightarrow 0$ is

$$\gamma_e = \sqrt{2\pi}\gamma_{0,e}\beta_e e^{-\chi_a} \left(2 - \frac{\beta_e}{2} \frac{a_d \Omega_d}{v_{T,e}}\right). \quad (148)$$

Using the OML equation 43 for the normalised surface potential in an argon plasma gives $\chi_a \simeq 2.4$. Note that even for this repulsive potential, $\gamma_{0,i}\beta_i \leq \gamma_{0,e}\beta_e e^{-\chi_a}$, meaning the magnitude of ion torque is relatively small. Due to the substantial neutral background, it is necessary to account for the drag force due to collisions of the dust grain with neutral particles. Adding together equations 148 and 146, the total torque is

$$\gamma = \gamma_{0,e} \sqrt{\frac{\pi}{2}} \left(4\beta_e e^{-\chi_a} - \frac{a_d \Omega_d}{v_{T,e}} \left[\beta_e^2 e^{-\chi_a} + \frac{n_n}{n_e} \sqrt{\frac{T_n m_n}{T_e m_e}}\right]\right). \quad (149)$$

Since $n_e \ll n_n$, the term proportional to β_e^2 can be neglected. For the same reason, it is not necessary to include the contribution to resistive torque by ions as the neutral particle density far exceeds the ion density. From the ratio of the two remaining terms, α/γ , the equilibrium rotation rate is found to be

$$\Omega_{eq} = \frac{4\omega_e e^{-\chi_a}}{\frac{n_n}{n_e} \sqrt{\frac{T_n m_n}{T_e m_e}}}. \quad (150)$$

The neutral density $n_n \simeq 6.6 \times 10^{21} \text{m}^{-3}$ is estimated approximately from the pressure with the mass of argon being $m_n = 39.948 m_p$, giving a value $\Omega_{eq} \simeq 41.4 \text{s}^{-1}$ with $\gamma/K \simeq 10 \text{s}$. This is in excellent agreement with experimental observations which find rotational rates of the same order of magnitude over similar equilibration time scales. Crucially, the angular momentum imparted by electrons causes rotation in the direction anti-parallel to $\underline{B}$ which is also in agreement with the experimental observations.

In a similar experiment, Dzlieva [204] observed the spinning motion of dust grains in a neon discharge. Spherical glass dust grains of radii $10 \mu\text{m}$ would rotate at $\sim 90 \text{Hz}$ in the paramagnetic direction but importantly only above a threshold magnetic field strength of $B = 0.01 \text{T}$. A measure of the collisionality of the plasma is given by the ratio of the electron gyro-frequency, ω_e , and the electron collision frequency, ν_e , which is estimated by

$$\nu_e \simeq \frac{5.8 \times 10^{-6} n_e \ln(\Lambda_{ee})}{T_e^{3/2}}, \quad (151)$$

with $\ln(\Lambda_{ee}) \simeq 8.75$ [205]. Calculating the equilibrium rotation rate using equation 150, with the plasma parameters stated [204], $n_e = 10^{14} - 10^{15} \text{m}^{-3}$, $T_e = 3 - 4 \text{eV}$,

pressure of $p = 0.4\text{Torr}$ and $T_i = T_n = 0.1\text{eV}$, the predicted equilibrium rotation rate is, $\Omega_{eq} \simeq 56\text{s}^{-1}$, which is comparable to the experimentally observed rotation rate. Furthermore, for the plasma parameters stated, the magnetic field strength required for $\omega_e/\nu_e \simeq 1$ is $B \simeq 0.006\text{T}$ which may explain the threshold behaviour observed as collisions would frequently disturb gyro-orbits below this magnetic field strength. This model and the results of DiMPI therefore offer the most complete and simple description of these experimental observations.

SIMULATIONS OF DUST IN TOKAMAKS

*Cautious Dædalus will apply his theories where he feels most confident ...
Icarus will strain his theories to the breaking-point until the weak joints gape.*

— Arthur S. Eddington, *The Internal Constitution of the Stars* [51]

6.1 INTRODUCTION

Dust poses a direct threat to the safe operation of tokamaks, as was discussed in section 1.4.2, yet dust tracking codes vital to understanding and preventing the impact of dust on stable operation, mostly ignore the consequences of magnetic fields on their dynamics. From the previous two chapters however (chapter 4 and chapter 5), it is clear that magnetic fields have a significant impact on the dust equilibrium surface potential and rotational dynamics. In this chapter, the research and implementation of physical models addressing these issues in the Dust in Tokamaks (DTOKS) code is discussed and new aspects of the problem are investigated.

The original Dust in TOKamaks (DTOKS) code developed at Imperial College London [206], simulates the forces, currents, heating and mass exchange experienced by an isolated conducting sphere in a collisionless plasma. The code has previously been implemented in modelling dust in the Mega Amp Spherical Tokamak [121], JET [109] and ITER [207, 208]. The upgraded Dust in TOKamakS (DTOKS-U) code has overhauled the original, simplifying the operation and implementation of different physics best suited to coping with a broader range of tokamak plasma conditions. A complete list of the major changes made to DTOKS is given in the appendices 10.4. In this chapter, the computational method is described and the impact of the most relevant improvements for modeling dust in magnetic fields is explored in more detail. The addition of rotational motion to DTOKS-U and related models for fluid instabilities, guided by the analysis of chapter 5, are discussed later in section 8.3. The description of the behaviour of molten metals has been further enhanced by incorporating the effects of evaporation as described in section 6.2.2. Finally, the effect of more accurate models accounting for the variability of standard values with temperature and size is described in section 6.2.4.

6.2 DUST IN TOKAMAKS UPGRADE (DTOKS-U)

DTOKS-U simulates dust traversing a cylindrical coordinate system, (R, θ, z) , with the central axis aligned with the toroidal axis of the tokamak as shown in figure 68. The description of the plasma environment is provided as an input, operating either in a *ContinuousPlasma* mode or a *PlasmaGrid* mode. In the *ContinuousPlasma* mode the plasma is considered to be uniform and infinite in extent so that the dust grain experiences only a fixed plasma environment without spatial variation. For a tokamak plasma, the *PlasmaGrid* mode is used with the plasma parameters

provided as an input over a two dimensional, regular, rectangular grid in the R-z plane. The plasma data in both cases includes information about the electrostatic potential and magnetic field strength in addition to the temperatures, densities and flow velocities of each plasma species across this space. This plasma data is most often provided by plasma fluid codes such as HERMES [209] or EDGE2D [210] which are in turn informed by measurements made by plasma diagnostics.

A few simplifying approximations are made about the plasma conditions which are generic to all dust transport codes. The plasma is assumed to be toroidally symmetric, with the flow velocity of ions and electrons purely parallel to the magnetic field. The plasma conditions are constant, since simulations are performed for steady state plasma operation where the plasma conditions remain approximately constant over the duration of a dust grain

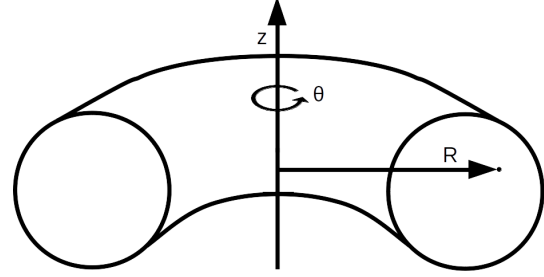


Figure 68: Schematic of coordinate system used by DTOKS-U for tracking dust grains.

lifetime ($\sim 10\text{ms}$). Further simplifying assumptions about the properties of the dust grain are also employed. The dust is taken to be a homogeneous, spherical sample of a material with a single elemental or chemical constituent, having physical properties equal to those of the same material at room temperature and pressure, unless otherwise stated. Finally, most models assume that the dust radius is much smaller than the plasma Debye length. Together, these simplifications of the problem permit the dust to be treated as a point particle, described by the system of equations introduced in chapter 3.

6.2.1 Governing Equations

To determine the behaviour of dust grains in a tokamak plasma, DTOKS-U solves three non-linear, partial differential equations explicitly in time. These equations for the particle motion, heating and mass loss are solved along with a fourth time independent transcendental equation for the potential. The disparity in timescales between charging and other physical processes justifies the assumption of calculating the surface potential time independently as was justified in section 3.1. When solving the equations simultaneously, it was previously assumed in DTOKS that the heating and dynamical timescales were roughly equivalent, meaning they can be solved in an interchangeable order. This can lead to unnecessary computational expense in solving the system of equations. In DTOKS-U, a characteristic timescale for both the heating and dynamical behaviour is calculated. If these two timescales differ by less than a factor of two, they are both solved in a single time step over the shorter of the two timescales. Otherwise, the faster of the two processes is solved in a series of smaller steps before completing a single larger step for the slower process. In either case, the charging equation is solved at the start of every time step.

The dust surface potential, ϕ_a , is determined by solving Kirchoff's law, equation 30, using the false position method, using combinations of the currents described in section 3.2. To correctly configure a simulation, a single ion and electron current must be specified. By default, simulations assume small dust in a tokamak SOL environment with electron and ion currents given by the magnetised ion and electron currents specified in section 4.2.8 and accounting for thermionic and secondary electron emission with terms given by equations 69, 70 and 122. If the dust grain acquires a positive charge due to electron emission processes, the semi-empirical formulation for the current dependence on magnetic field strength is abandoned and the OML electron and ion currents, equation 40, are used. Equation 70 for the current due to secondary electron emission is solved through the yield δ_{sec} as a fraction of the total electron current $I_{see} = \delta_{sec} I_e$ in the same manner as performed previously in DTOKS [114] and as is described in appendices 10.1.

For a dust grain experiencing a total force F_d , the equation of motion, equation 82 as described in section 3.4, is solved explicitly using a 4th order Runge-Kutta method. By default, the terms accounted for in this equation are gravitational and Lorentz forces as well as ion drag, given by equation 86. Optionally, the neutral drag force given by equation 87 may also be included. The timestep used in solving the equation of motion, Δt_m , is calculated from the minimum of three timescales of the motion, $\Delta t_{m,1}$, $\Delta t_{m,2}$ & $\Delta t_{m,3}$. The first of these timescales, $\Delta t_{m,1}$, defines a time-step that is small enough to ensure that the particle does not cross a grid cell of width Δl in a single step. Secondly, to avoid rapid acceleration causing large discontinuous changes in velocity, a limit on the acceleration magnitude of 0.01 ms^{-2} is used to define a timescale for resolving accelerations, $\Delta t_{m,2}$. Finally, for a dust grain with a large charge to mass ratio, the dust gyro-motion when travelling perpendicular to $\hat{B}$ must be resolved. These limiting timescales are expressed through the equations

$$\Delta t_{m,1} = \frac{0.01 \text{ ms}^{-1}}{\left| \frac{d\mathbf{v}_d}{dt} \right|}, \quad \Delta t_{m,2} = \frac{\Delta l}{2|\mathbf{v}_d|}, \quad \Delta t_{m,3} = \frac{M_d}{e|\mathbf{B}|}. \quad (152)$$

The timestep for the equation of motion, Δt_m , is determined from the minimum of these three different timescales up to a motional accuracy $0 < A_m \leq 1$ such that

$$\Delta t_m = A_m \min(\Delta t_{m,1}, \Delta t_{m,2}, \Delta t_{m,3}). \quad (153)$$

The equation of motion is then solved using this explicit method to provide the position and velocity of the particle after a time step Δt_m . If the particle enters a new grid cell in this time, the timescales of motion and heating are re-evaluated and the surface potential is updated before proceeding with computing the trajectory.

As explained in section 3.3, the evolution of the dust temperature, T_d , given by equation 71, is determined from the net power flux at the surface, P_t , which is calculated from a combination of the various heat flux terms. By default, heat transfer due to ion and electron bombardment, thermal radiation, recombination and both thermionic and secondary electron emission will be accounted for in simulations of dust transport in tokamaks as formulated by equations 73, 74, 76,

77, 79, 80 and 81. This equation is solved explicitly using a 4th order Runge-Kutta method. The time step, Δt_h , over which the heating model is solved is

$$\Delta t_h = \frac{c_v M_d A_h}{P_t}, \quad (154)$$

which is determined from the requirement that the temperature change in one step is smaller than a specified temperature accuracy A_h .

6.2.2 Evaporation

In addition to the models discussed, a model for evaporation has been introduced which provides an alternative description for the mechanism of mass loss and an additional heat loss term for dust grains in the liquid phase. Mass loss is modelled through two different mechanisms, with heat energy directly breaking bonds at the boiling temperature, T_b , and evaporation through the liquid phase $T_m < T_d < T_b$ where T_m is the melting temperature of the dust material. The mass loss due to boiling and evaporation respectively are described by

$$\begin{aligned} \frac{dM_d}{dt} \Big|_{T_d=T_b} &= \frac{P_t}{L_v}, \\ \frac{dM_d}{dt} \Big|_{T_d \geq T_m} &= m_a \Gamma_v, \end{aligned} \quad (155)$$

where L_v is the latent heat of vapourisation and m_a is the atomic mass of the element. The high energy atoms at the surface which are able to escape the confining potential through evaporation can be modelled with the Hertz-Knudsen formula [211] for the evaporated particle flux

$$\Gamma_v = \frac{1}{4\pi a_d^2} \frac{dN_d}{dt} = \frac{P_v(T_d)}{\sqrt{2\pi M_d k_B T_d}}. \quad (156)$$

Here, N_d is the number of atoms which are evaporated from the dust grain surface. The vapour pressure P_v is typically given by an empirical relation known as the Antoine equation [212] having the form

$$P_v(T_d) = 10^{B_1 - \frac{B_2}{T_d}}, \quad (157)$$

with constants B_1 and B_2 [213]. Figure 69 shows the vapour pressure of tungsten, iron and beryllium over their liquid temperature ranges. This provides the dominant variation with temperature as compared with the weak dependance of $\sqrt{T_d}$ in the denominator of equation 156. Atoms evaporated from the surface are assumed to carry away an amount of thermal energy from the dust, $H_d \simeq 2k_B T_d$, plus the atomic bond energy E_b of the material so that the power flux is $(H_d + E_b)\Gamma_v$. Both equation 155 and equation 71 are evaluated over the same timescale given by equation 154 to determine the evolution of the mass and temperature of the dust grain.

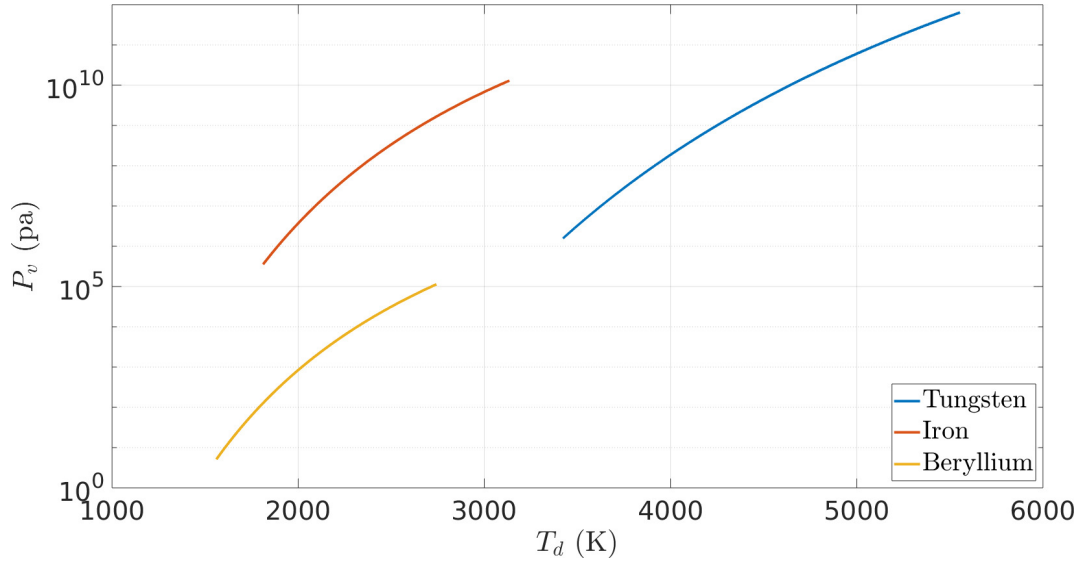


Figure 69: Vapour pressure, P_v , as a function of dust temperature, T_d , from solving equation 157 for tungsten, iron and beryllium [213].

6.2.3 Plasma Background

For the *PlasmaGrid* mode, the plasma parameters for each grid cell are read from a file and stored. The nearest integer cell coordinates (i, k) to the dust are tracked and used to determine if the dust particle has exited the simulation domain. With plasma parameters stored in grid cells, the dust grain experiences discontinuities in local plasma conditions when transitioning across grid cell boundaries. Though using a smaller grid cell spacing, Δl , mitigates these discontinuities, it also increases the memory and computational time required to solve the equation of motion, see equation 152. For this reason, the plasma parameters at the dust location are instead interpolated from the four closest grid cell centres to provide a weighted mean value at the dust location.

The value of the electric field $\underline{E}(i, k) = \underline{E}_r(i, k) + \underline{E}_z(i, k)$ is calculated from the difference in potential $\phi(i, k)$ between neighbouring grid cells. In this way, the radial, $\underline{E}_r(i, k)$, and vertical, $\underline{E}_z(i, k)$, electric field components are defined as,

$$\begin{aligned}\underline{E}_r(i, k) &= \frac{\phi(i+1, k) - \phi(i-1, k)}{2\Delta l_r} \hat{r} \\ \underline{E}_z(i, k) &= \frac{\phi(i, k+1) - \phi(i, k-1)}{2\Delta l_z} \hat{z}.\end{aligned}\tag{158}$$

In addition to this information about the plasma, a series of points defining a polygon in the R-z plane may be used to specify separately the wall boundary of the device and the core plasma boundary. For particles incident on a wall boundary representative of a tokamak PFC, there are many possible realistic outcomes including adhesion, rolling or reflection, each of which depend sensitively on the material properties, angle of incidence, speed and size of the dust at impact [119]. In DTOKS-U, the simplifying assumption is made that all particles incident on a wall boundary are specularly reflected from the surface, altering only the direction

of the dust motion. Meanwhile, particles which enter inside of the region defined by the core boundary are assumed to be instantaneously vapourised. Often this is a necessary consideration when utilising plasma parameters generated by SOL fluid plasma codes which provide no solution for the core plasma.

The termination conditions imposed on simulations differ for the *ContinuousPlasma* mode and the *PlasmaGrid* mode. Simulations are halted if:

- In the *ContinuousPlasma* mode, either the net power, P_t , to the surface is identically zero or the temperature gradient with time is small whilst no mass loss processes are occurring. In this case, the dust grain is assumed to have reached a stable thermal equilibrium.
- In the *PlasmaGrid* mode, the nearest integer cell coordinates of the dust grain are outside the simulation domain.
- In the *PlasmaGrid* mode, the dust enters inside of the region defined by the core boundary data file.
- In either the *ContinuousPlasma* or *PlasmaGrid* mode, the mass falls below 10^{-17} kg. In this case the dust grain is considered to have completely disintegrated and to be of insignificant size for consideration.
- In either the *ContinuousPlasma* or *PlasmaGrid* mode, an electrostatic or rotational instability occurs whilst the dust grain is in the liquid phase.

This last termination condition assumes that the fluid instability of molten metal dust leads to a fluid breakup process as a result of the disruption to the liquid surface, described in more detail later in section 8.3.

6.2.4 Variation of material properties

The material properties used by dust tracking codes are normally assumed to remain fixed at the values relevant for bulk samples of the material, whilst the realistic value can in fact vary significantly over the relevant temperature and size scales. Variations in the emissivity, thermal expansivity and heat capacity are crucial for determining heat fluxes and equilibrium temperatures which in turn determine the ablation rate and lifetime of the dust grain. The dependency of the heat capacity and thermal expansion on temperature, and of the emissivity on the size and temperature of the material have been considered for four fusion relevant elements; beryllium, carbon, iron and tungsten.

At high temperatures, one of the most dominant heat loss terms is blackbody radiation from the surface, given by equation 79. A constant emissivity is a poor approximation when considering objects of varying temperatures and sizes comparable to the peak wavelength of emitted radiation [214, 215]. To calculate the emissivity, a tool developed by Wiscombe [216] was employed to generate a look-up table over the phase space of temperature and dust grain radii, as described in appendices 10.5. The emissivity was calculated in steps of 1K for temperatures from $280\text{K} \leq T_d \leq T_B$ for each material, with a pseudo-logarithmic step in radius $0.01\mu\text{m} \leq a_d \leq 100\mu\text{m}$. Figure 94 shows $\epsilon(a_d, T_d)$ for carbon which varies

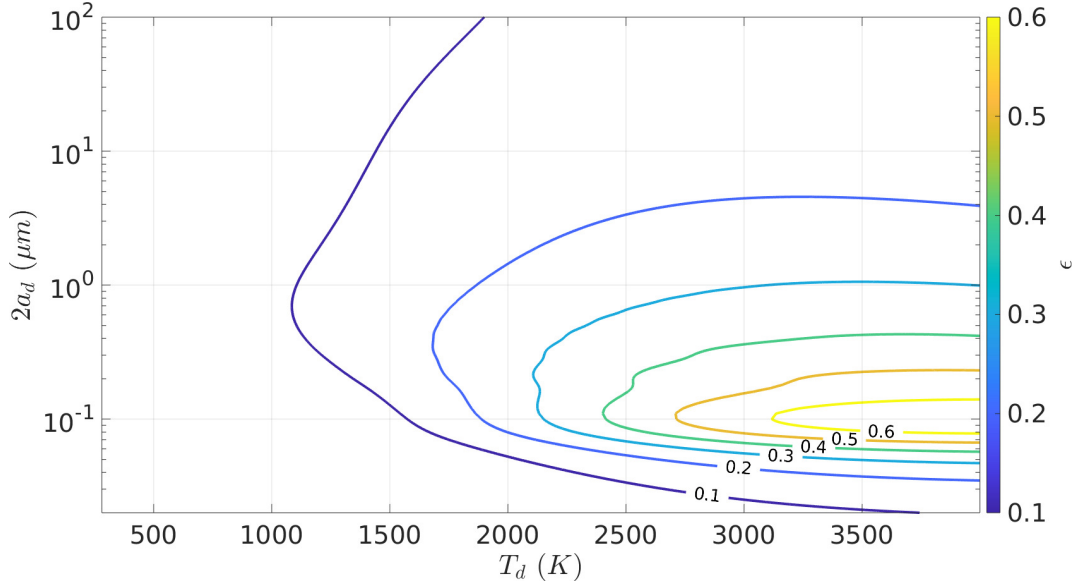


Figure 70: Emissivity, ϵ , of carbon as a function of dust diameter, $2a_d$, and temperature, T_d .

by several orders of magnitude over the parameter space with the other relevant elements given in the appendices 10.5. In general, the emissivity of small metal spheres ($0.01\mu\text{m} \leq a_d \leq 1\mu\text{m}$) is overestimated at low and modest temperatures ($300\text{K} \leq T_d \leq 2000\text{K}$) compared with using the values relevant for bulk solid samples. This more accurate estimation of the emissivity therefore leads to a slightly higher prediction of the equilibrium temperature as compared with a constant value.

The thermal expansion of materials can also have a significant effect on the dynamics of dust, as an increased surface area proportionally increases the magnitude of thermal and momentum fluxes. The expansivity of materials in DTOKS-U is modeled through the semi-empirical formula,

$$\frac{a_d(T_d)}{a_d(T_d = 0)} = 1 + C_1 T_d + C_2 T_d^2 + C_3 T_d^3 + C_4 T_d^4, \quad (159)$$

where C_1 , C_2 , C_3 and C_4 are material specific constants [217–219] and $a_d(T_d = 0)$ is the unexpanded radius. Figure 71 shows the variation of the linear expansion coefficient for different dust temperatures with the maximum change in size typically being around $\sim 30\%$. For carbon however, the thermal expansion is severe causing size changes of up to $\sim 400\%$. Discontinuities in the expansivity occur as a result of phase changes, with different phases modelled using equation 159 with different sets of coefficients. The larger radii of thermally expanded dust enhances the influence of collection forces such as ion and neutral drag since it increases the area for collection for the same mass. This also has an impact on heating timescales by increasing the magnitude of the heat flux and causing a faster approach to equilibrium. Finally, though the size of the dust grain is not anticipated to affect the charging behaviour provided $a_d \ll \lambda_d$ is maintained, it may indirectly through changes in temperature and consequently altering the thermionic electron current and overall current balance.

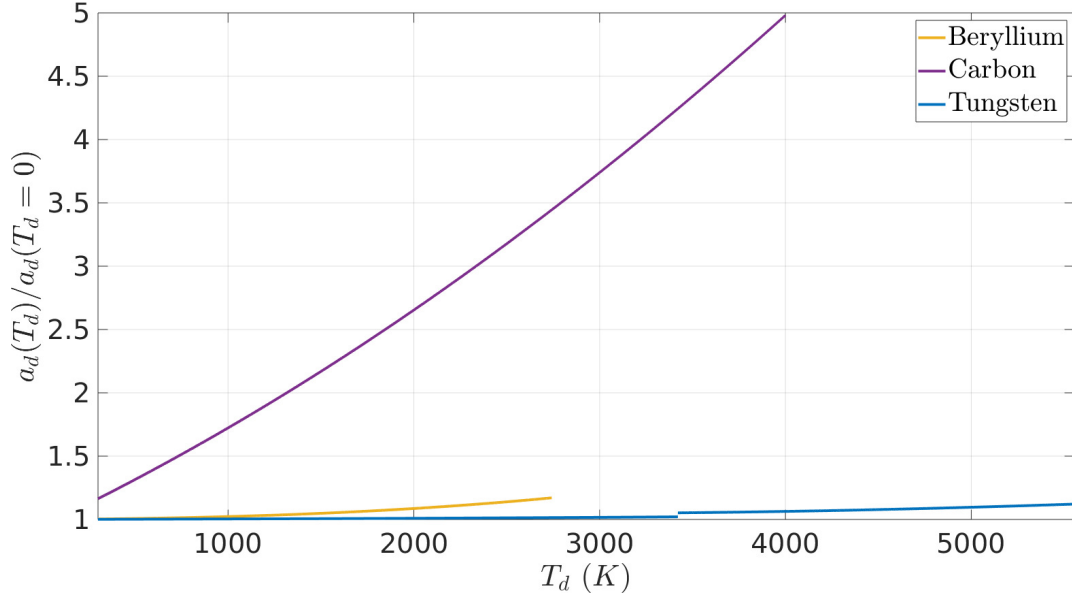


Figure 71: Dependency of the linear expansion coefficient on dust temperature, T_d , for beryllium [217], carbon [219] and tungsten [218] spheres.

Similarly the value for heat capacity used to model a dust grain not only dictates the conversion of thermal energy into a bulk temperature change but can also indirectly affect charging. As with the thermal expansion, the heat capacity is approximated by a semi-empirical function of the dust temperature,

$$C_v(T_d) = D_1 + D_2 T_d + D_3 T_d^{-1} + D_4 T_d^{-2} + D_5 T_d^{-3} + D_6 T_d^{-4}. \quad (160)$$

Using empirical constants D_1, D_2, D_3, D_4, D_5 and D_6 , relevant for tungsten [218], iron [220], graphite [221] and beryllium [222], figure 72 shows the temperature dependant heat capacity given by equation 160 for each of these materials. Discontinuities occur for all the materials when they undergo a phase transition, indicating a step change in the thermal properties of the material.

The variation in the material properties with temperature introduce feedback effects when conflated with the dependency of the other properties of the dust on size and temperature. Figure 73 shows an example of the temperature evolution of a carbon dust particle when modelling the temperature and size dependence of the material properties as compared with a reference simulation in which all three values are fixed ($\epsilon = 0.7$, $C_v = 0.846 \text{ kJ kg}^{-1} \text{ K}^{-1}$ and $a_d(T_d) = a_d(T_d = 0 \text{ K})$). For the purposes of testing, simulations were performed in the *ContinuousPlasma* mode for a carbon dust grain with a radius of $1 \mu\text{m}$ at an initial temperature $T_d = 280 \text{ K}$ in a SOL hydrogen plasma with $n_i = n_e = 10^{18} \text{ m}^{-3}$, $T_i = T_e = 50 \text{ eV}$ and $\underline{B} = (0, 0, 1) \text{ T}$. The smaller value of emissivity of carbon dust over this temperature range and at this size as shown in figure 70 causes a slight increase in the equilibrium temperature seen in figure 73. The heat capacity of carbon is greater at temperatures larger than room temperature, causing the approach to equilibrium to be more gradual however the large thermal expansivity of graphite dust overcompensates, leading to a more rapid approach to equilibrium. The combined effect of modelling the variation of all three values increases the equilibrium temperature by 200 K and

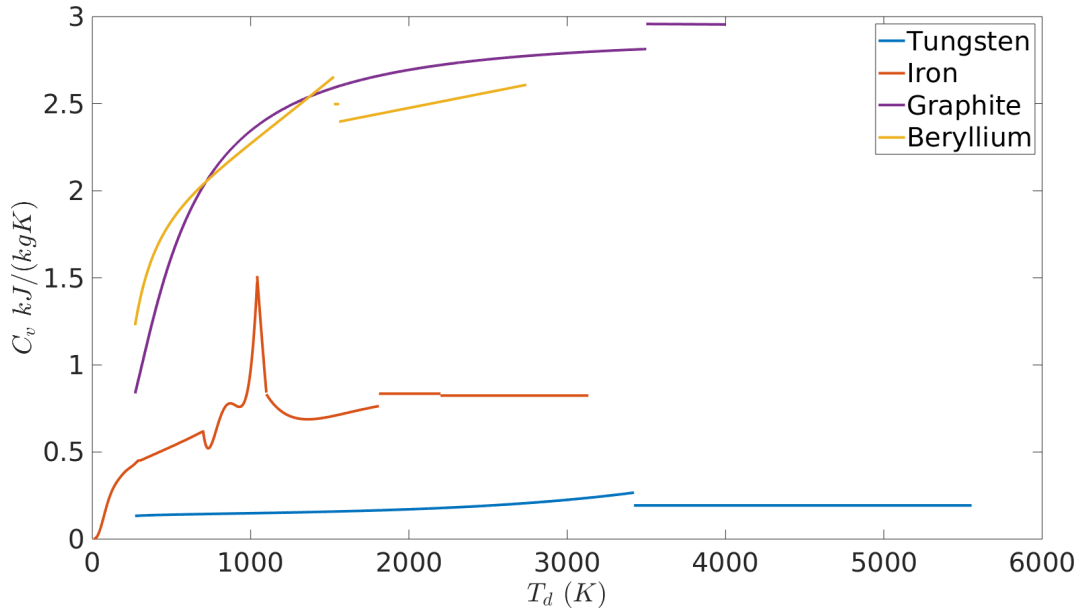


Figure 72: Dependency of heat capacity, C_v , on temperature, T_d , for tungsten [218], iron [220], graphite [221] and beryllium [222]. Discontinuities arise during phase changes.

halves the time taken to reach thermal equilibrium. This demonstrates the importance of accurately modelling material properties which significantly alter dust ablation rates and lifetimes.

6.3 EXAMPLE CASE STUDY

Later in chapter 7, carbon dust trajectories observed by fast imaging cameras inside of the DIII-D tokamak are analysed and compared with DTOKS-U simulations. Here, a simplified example of a carbon dust grain of radius $a_d = 1\mu\text{m}$ at an initial temperature $T_d = 300\text{K}$ released from the lower divertor of the DIII-D tokamak is explored to demonstrate the behaviour of DTOKS-U. The behaviour simulated by DTOKS-U is compared to a reference case employing constant values for material properties as in figure 73 and with the potential calculated with the OML electron and ion current terms of equation 39 and equation 38 respectively instead of the magnetised currents of equation 122. The plasma parameters used have been generated by OEDGE simulations [223] based on experimental observations of plasma shot S167353. The simulated plasma density and electron temperature used for DIII-D are shown in figure 74 as well as the wall boundary defining the inner wall of the device. This shows the characteristic peaked temperatures and densities in the core of a tokamak with a lower single null, H-mode plasma. The plasma parameters in the undefined region close to the outer wall were extrapolated from the edge plasma conditions. For this example, the graphite dust is injected at the lower divertor plate at $\underline{r} = (1.6, 0, -1.25)$ where dust is commonly observed in DIII-D with a velocity, $\underline{v}_d = (0, 0, 50.0)$, directed vertically upwards. This velocity was chosen as the plasma temperature and density steadily increase towards the core,

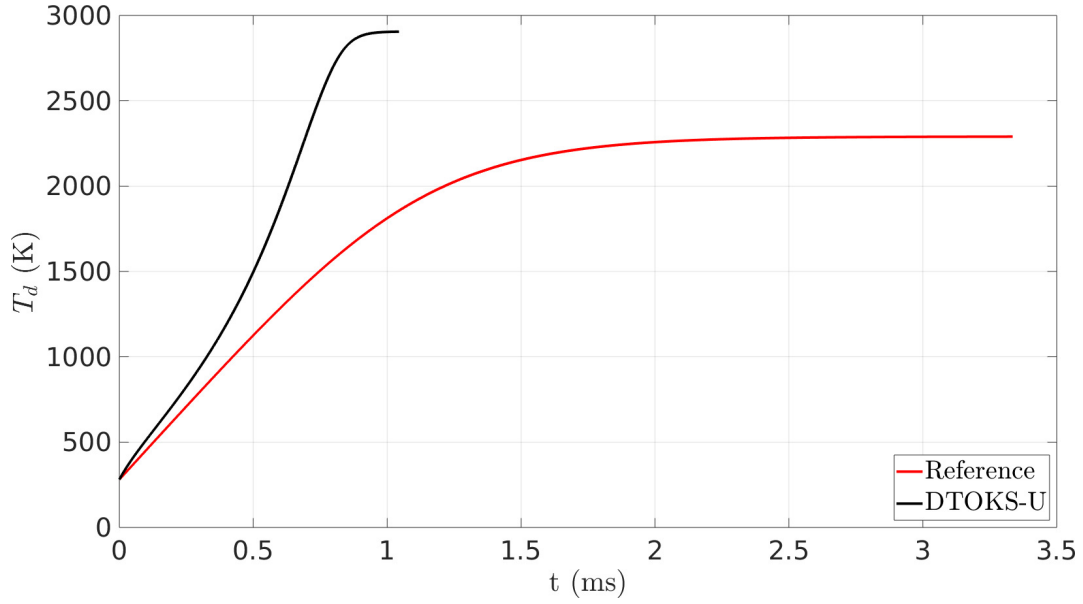


Figure 73: Temperature evolution of a carbon dust grain of radius $a_d = 1\mu\text{m}$ in a *ContinuousPlasma* background when using fixed reference values (red) as compared with empirical models for estimating the material properties (black). The reference case uses fixed values of $C_v = 0.846\text{kJkg}^{-1}\text{K}^{-1}$ and $\epsilon = 0.7$ without accounting for thermal expansion.

causing the dust grain to experience progressively more intense fluxes of charge and heating, offering a simplified test case under a range of plasma conditions.

Figure 75 shows the simulated trajectories superimposed on a contour plot of the electron temperature for a poloidal and toroidal cross section of DIII-D. In the DTOKS-U simulation, the dust grain terminated before the reference case when the dust mass fell below the lower limit of 10^{-17}kg , in the edge plasma where the electron temperature is approximately $\simeq 15\text{eV}$ and the plasma density is $5 \times 10^{18}\text{m}^{-3}$. The trajectories in each case are similar with motion principally in the vertical direction and a small amount of toroidal motion, though the reference case is slightly shorter. From the temporal evolution of the dust velocity in cylindrical coordinates shown in figure 76a, the particle is observed to experience substantial acceleration in the toroidal direction parallel to $\underline{B}$ as well as a slight deceleration of its initial vertical velocity, whilst the radial velocity component remains relatively unaffected. The various forces that contribute to the acceleration experienced by the dust grain are distinguished in figure 76b. The ion drag force is seen to cause acceleration that is orders of magnitude larger than gravity or the Lorentz force once the particle reaches the edge of the SOL for $t > 1.5\text{ms}$. The ion drag force steadily increases with time as the dust grain enters the edge plasma where the plasma density, temperature and flow velocity are greater. The magnitude of the Lorentz force varies discontinuously as the dust charge varies with the local plasma conditions.

The temperature and normalised potential of the dust grains as a function of time for the two sets of models are also shown in figure 76. In either case, the temperature shown in figure 76c gradually increases from 300K to its boiling temperature at 4000K with a steep gradient $t \simeq 7\text{ms}$ corresponding to where the plasma heating increases rapidly. The equilibrium temperature calculated by DTOKS-U

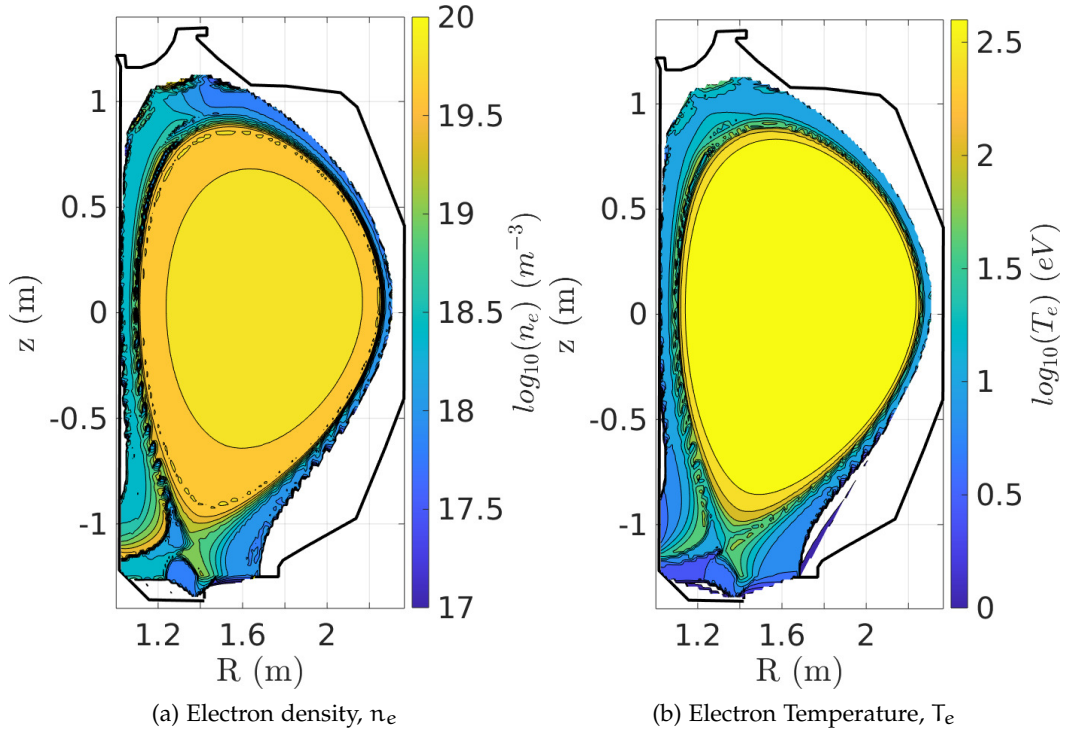
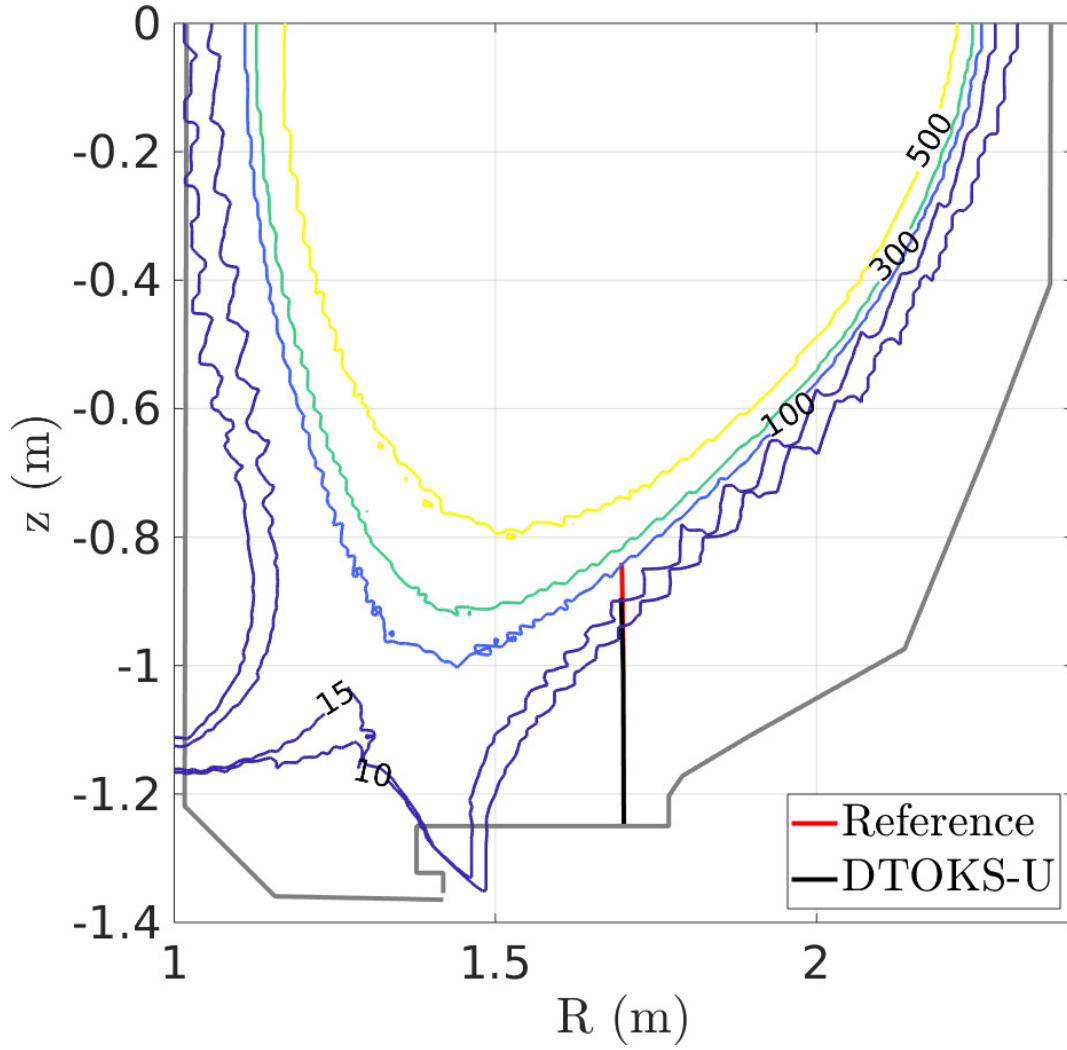


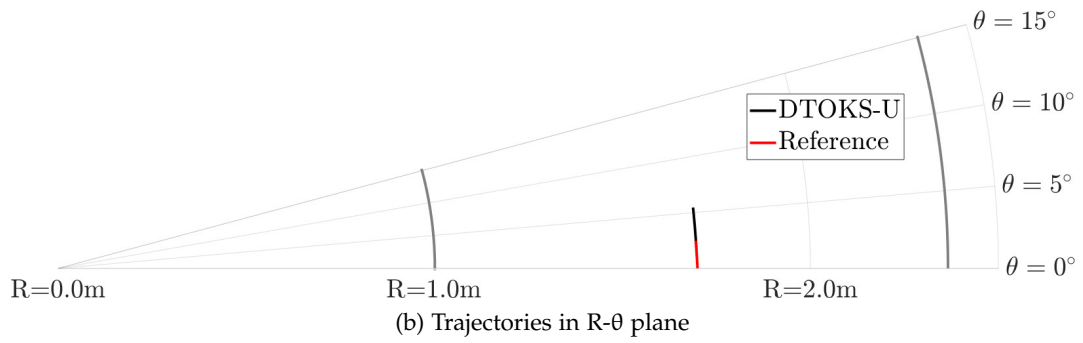
Figure 74: Poloidal cross section of DIII-D, showing the electron density, 74a and temperature, 74b within the wall boundary (black) produced by OEDGE, credit G. Xu [223].

is consistently higher and lifetimes markedly shorter than the reference case as a result of more accurately modelling the material properties. Once the temperature increases above around 1500K, the current of thermionic electrons becomes significant and the potential begins to decrease. When the plasma heat fluxes increase again at $t \simeq 7\text{ms}$, the electron emission becomes significant enough to cause the dust grain to charge positively. At this point, the ion heating becomes extremely strong and causes rapid heating and boiling of the carbon dust over a fraction of a millisecond.

The details of this are demonstrated more clearly in figures 77 and 78 which show the contributions of the various current and heating terms respectively as a function of time for the DTOKS-U simulation. Initially for $t < 7\text{ms}$ when the dust grain is at low temperatures, the current balance is dominated by the ion and electron current as shown in figure 77, with a small contribution from secondary electron emission. When the dust reaches the hotter plasma region after 7ms however, the temperature begins increasing rapidly causing the thermionic electron emission to overtake the ion current. Once the potential becomes positive, the ion current is reduced and the charging behaviour is controlled almost entirely by incident electrons and electron emission processes. The positive charge of the dust impedes the thermionic electron emission process by presenting a potential barrier that prevents the dust grain from acquiring larger positive potentials, leading to a new equilibrium solution at weak positive potentials, as was demonstrated in figure 19 of section 3.2.6.1.



(a) Trajectories in R-z plane with contours of electron temperature in eV



(b) Trajectories in R-θ plane

Figure 75: Trajectories of dust for a reference case using fixed values for material properties and OML charging model (red) as compared with empirical models for estimating the material properties and magnetised charging model (black). Figure 75a shows the motion in the R-z plane with the wall boundary (grey) and contours of the electron temperature whilst figure 75b shows the trajectory in the R-θ plane bounded by the maximum and minimum radial coordinate of the wall boundary (grey).

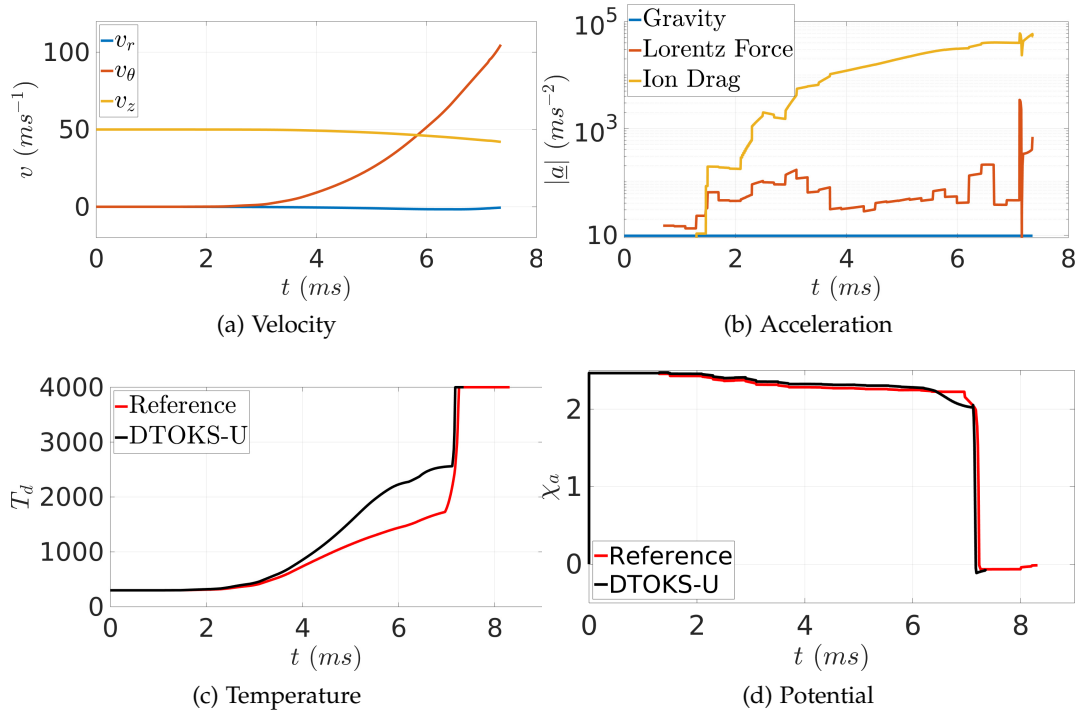


Figure 76: Evolution of velocity components (76a) and contributions to acceleration (76b) for example carbon dust grain traversing DIII-D plasma simulated by DTOKS-U shown in figure 75 alongside the dust temperature (76c) and equilibrium floating potential (76d) evolution simulated by DTOKS-U (black) as compared to a reference case (red).

Inspecting the relative contributions of the heating terms in figure 78, initially at the lowest temperatures the surface recombination drives heating along with the plasma heat fluxes as impaired by secondary electron emission. At approximately $t \simeq 4.5$ ms, thermal radiation becomes significant and causes the temperature to plateau at around $t \simeq 6$ ms as seen in figure 76c. Once the dust grain charges positively, the electron heating begins to play a dominant role that is only partially cancelled by cooling due to the thermionic electron emission and thermal radiation terms, leading to the sublimation and eventual destruction of the dust grain.

This example demonstrates the type of behaviour characteristic of DTOKS-U simulations of carbon dust grains in the SOL of a tokamak plasma. In particular, it is observed that at low temperatures, the dust charges negatively with the current and thermal balance dominated by the plasma fluxes and secondary electron emission. At much higher temperatures, the dust charges positively as a result of the strong thermionic electron emission, leading to elevated electron heat fluxes. The dust motion on the other hand is independent of the dust temperature and is dictated principally by the ion drag force, causing acceleration parallel to $\underline{B}$.

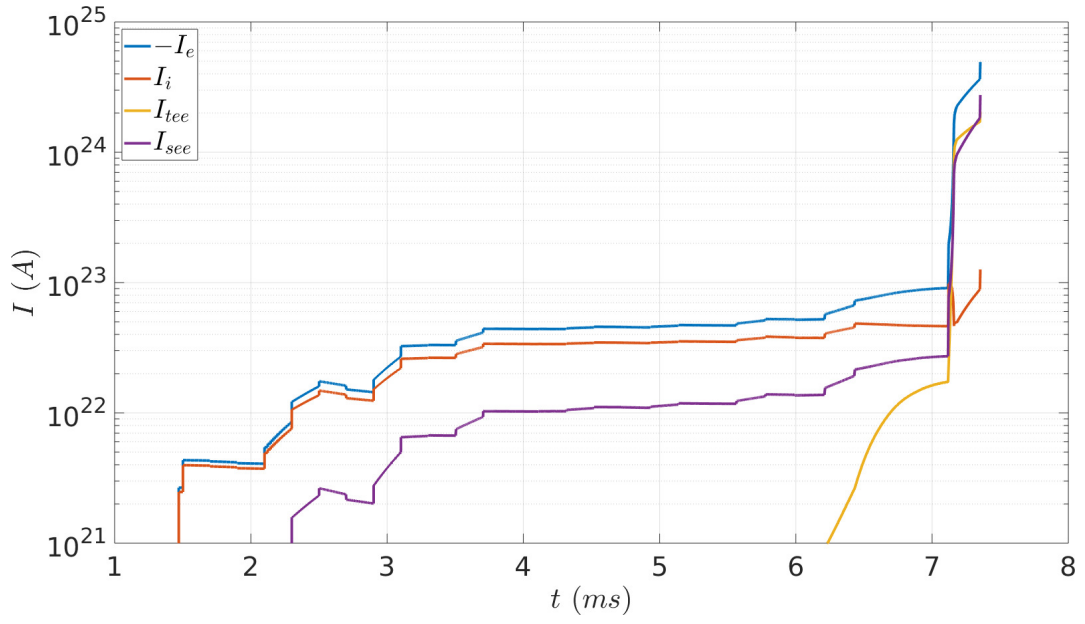


Figure 77: Contributions of different current terms calculated using equations 38, 39, 40, 69, 70 and 122 to the current balance of the carbon dust grain simulated by DTOKS-U inside DIII-D plasma.

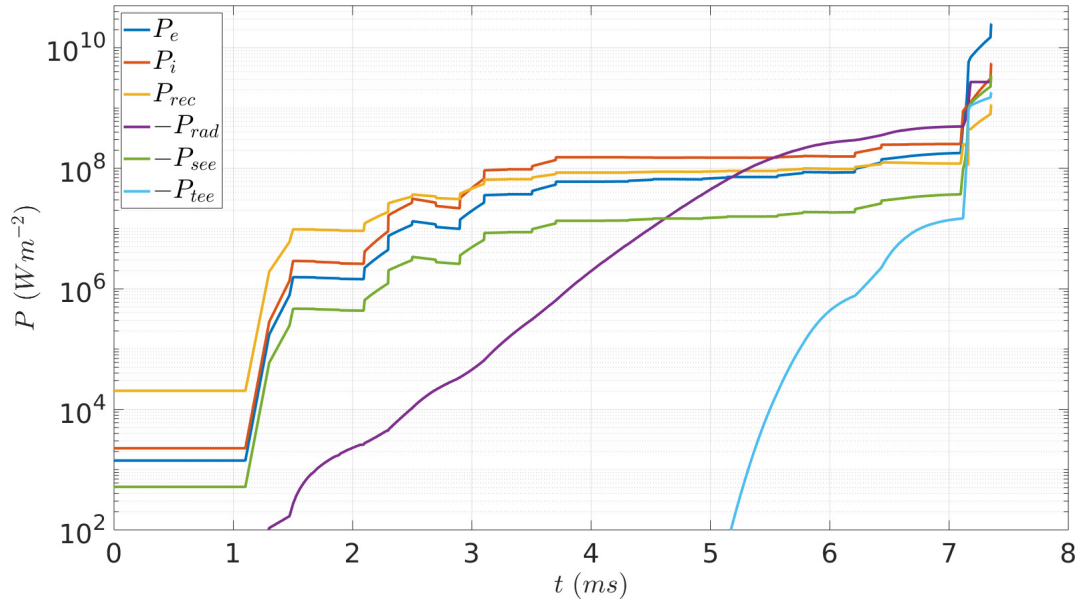


Figure 78: Contributions of different heat terms calculated using equations 73, 74, 76, 77, 79, 81 and 80 to the heating of the carbon dust grain simulated by DTOKS-U inside DIII-D plasma.

RECONSTRUCTING DUST TRAJECTORIES IN DIII-D

*there is only a mixing and then separating of what was mixed,
but by mortal men these processes are named "beginnings."*

— Empedocles, Empedocles: The Extant Fragments [224]

7.1 OVERVIEW

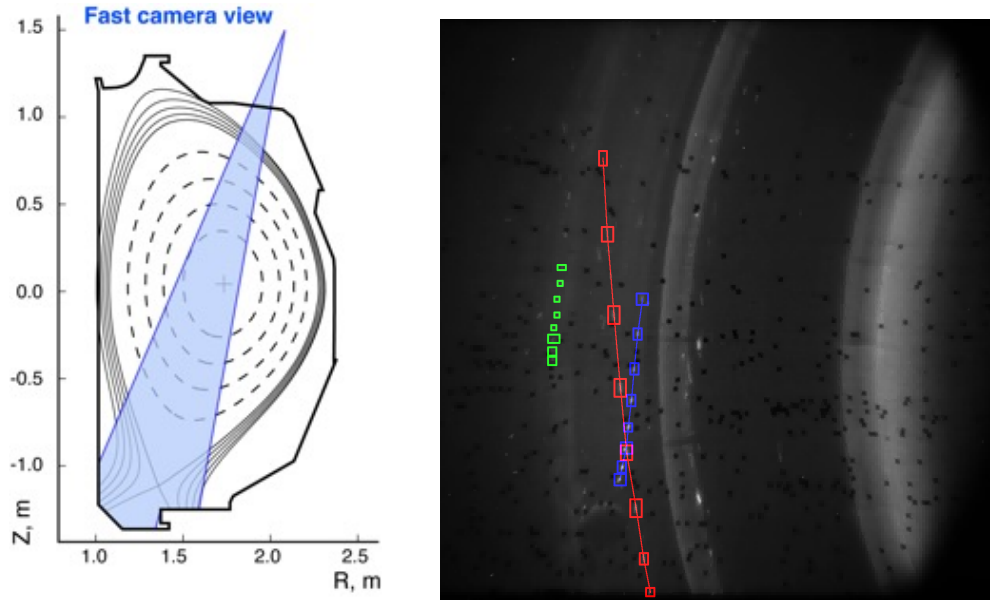
The DTOKS-U code has been applied in modelling novel observations of dust particle motion in the divertor of the DIII-D tokamak. Characterising particles velocity distributions and lifetimes is vital for understanding their capability for distributing impurities and affecting safe and stable operation of the machine. Simulations of the experiment offer insight into the physical models necessary to explain observations and calculations of the impurities deposited that can in turn inform plasma simulations.

Dust particle trajectories were observed in two plasma discharges labelled S167342 and S167345 by a single visual imaging diagnostic camera with a pixel resolution of 368×512 and frame rate of 5kHz. The sight-lines of the imaging system shown in figure 79a illustrate the camera field of view directed towards the divertor floor. Without additional information from an orthogonal view, only the motion of particles perpendicular to the camera line of sight is measured, with particles assumed to be moving along the divertor floor. Figure 79b shows seven superimposed frames from the same shot with the trajectory of three example particles highlighted. The white band visible in the centre corresponds to the location of the outer divertor leg strike point, with the other white band on the right hand side being the inner strike point. The black spots shown in figure 79b are dead channels which were ignored for the purpose of reconstruction. The dust particles emit thermal radiation which is observed as bright spots or streaks depending on the frame rate of the camera and the speed of the dust.

7.2 ANALYSIS OF VISUAL CAMERA DATA

The Robust Impurity Detector And Tracker, RIDAT¹, code was developed to automatically measure dust particle positions as recorded by visual cameras using a machine learning algorithm [225]. The algorithm operates by first performing background subtraction using the mean intensity from images prior to the appearance of dust. A cut is then performed across all frames to identify pixels that exceed a specified threshold intensity corresponding to hot spots and dust. A subset of this data was then used to train a machine learning algorithm employing Gaussian naive Bayes classifiers by identifying individual dust particles across sets of three

¹ <https://github.com/cydcowley/RIDAT>



(a) Poloidal cross-section of DIII-D demonstrating approximate magnetic field topology and field of view of fast imaging visual cameras.

(b) Seven superimposed raw images from S167342 with three trajectories tracked over 49 concurrent frames highlighted in red, green and blue every 7 frames.

Figure 79: Schematic of the field of view (79a) and example data (79b) from the fast imaging visual camera.

consecutive frames in the shot. The machine learning algorithm was re-trained on over 200 dust particles for each shot to overcome variations in noise, brightness and dust track characteristics.

Typically, dust particles were observed by the visual camera as streaks as a result of the frame rate as compared with the velocity of particles across the frame. For this reason, both the beginning and end points of particle tracks were identified from the threshold intensity and used to define variables used in training. The brightness of tracks was in many cases comparable to the level of noise making identification of dust challenging. It is also noted that the tracks varied in length due to variations in particle velocity, brightness and changes in proximity to the camera. The mean position of the start and end points in a single frame were used as an estimate for the particle position in that frame. This was then used to make measurements of velocities from the known frame rate of the camera.

A raw total of 932 tracks were identified; 66 in S167342 and 866 from S167345. In S167345, 829 of the tracks recorded were visible following a plasma termination event, so that only 37 of the particle tracks in this discharge travelled under nominal plasma conditions before the disruption. After also omitting tracks with observation times shorter than 1ms, equivalent to five frames, figure 80 shows a polar plot of the positions of the remaining 53 dust particles, 33 from S167342 and 20 from S167345. The position in world coordinates was calibrated based on the known width of the probe port, 0.073m, and the measured pixel width, 120px, visible in figure 79b. In general, particles are observed to follow trajectories with a

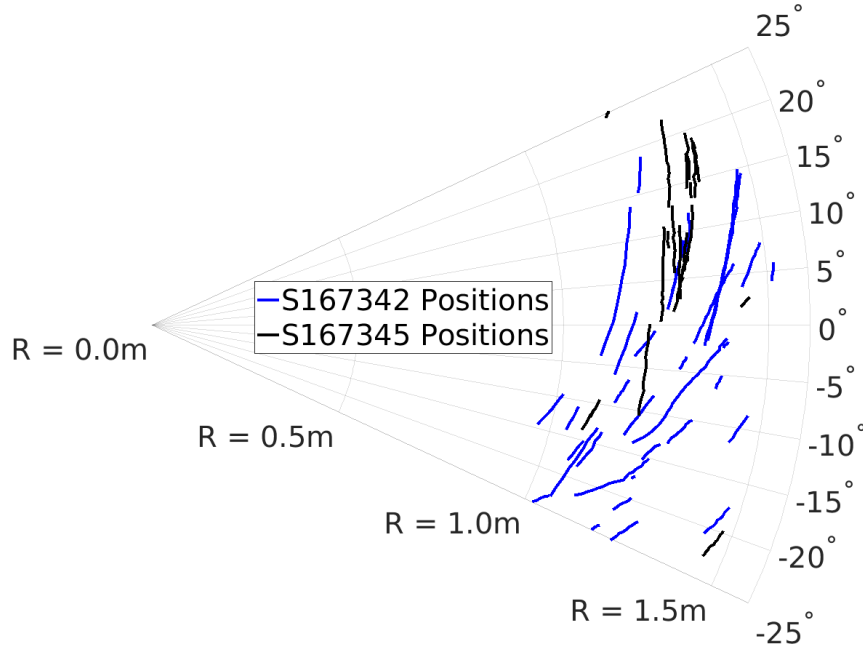


Figure 80: Dust grain trajectories in S167342 (blue) and S167345 (black) in the $R - \theta$ plane with angular position measured with respect to camera, reconstructed using RIDAT.

significant speed in the $-\theta$, toroidal direction in the same sense as the toroidal ion flow. The small erratic movements of the observed dust occurs as a result of the limited pixel resolution of the visual imaging camera. The majority of dust trajectories are within the private plasma region, $1.0\text{m} \leq R \leq 1.4\text{m}$, and travel in a straight line before disappearing at the edge of the camera view or when approaching the outer strike point at $R \simeq 1.45$.

The speeds of the particles displayed in figure 81 are calculated from the change in position of particles between frames and the frame rate of the camera. The radial speed was fit with a normal distribution whilst the toroidal speeds are fit with a log-normal distribution as shown in figure 81. These speeds are in good agreement with previous measurements of dust grain speeds in DIII-D [98] and as described previously in section 1.4.3. The mean observation time measured for these particles was $2.1\text{ms} \pm 0.4\text{ms}$ which provides a lower bound for the particle lifetime, since this timescale is also affected by particle brightness and occlusion from the camera view. Few particles experienced significant acceleration, with the majority traversing across the field of view with a constant speed.

7.3 DTOKS-U SIMULATIONS

Predictive modelling of the lifetimes and dynamics of micrometre sized carbon dust grains was performed, implementing the improved models for potential and non-constant standard values as described previously in section 6.2.1. The speed distributions shown in figure 81 were used to randomly generate the initial radial and angular velocities of 200 graphite particles in DTOKS-U. The initial temperature of 300K was chosen, with initial positions at a fixed height just above the

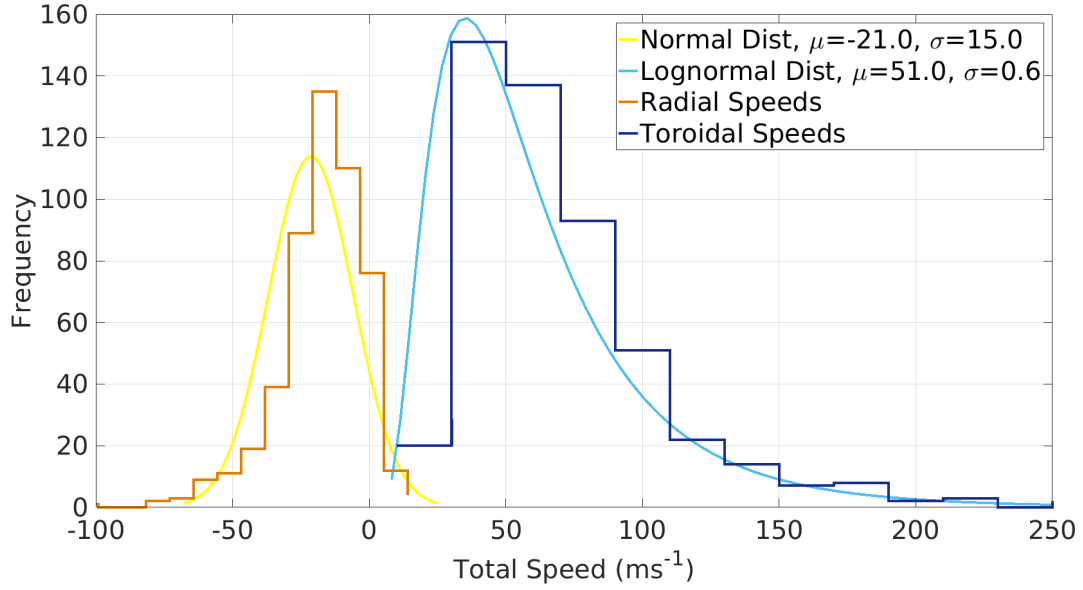


Figure 81: Toroidal and radial speed distributions of dust observed in DIII-D for shots S167342 and S167345.

divertor floor, $z = -1.24\text{m}$, and a randomly generated radial coordinate uniformly distributed between $1.1\text{m} \leq R \leq 1.4\text{m}$ taken from the observed positions of particles seen in figure 80. The initial radii of particles were selected randomly through inverse transform sampling of the size distribution previously recorded [98], where the fraction of particles of a particular size were found to be proportional to $\propto a_d^{-1.4}$. An upper limit on dust size of $100\mu\text{m}$ was set to ensure the dust grain sizes were never significantly greater than the plasma Debye length. The plasma background parameters used were taken from OEDGE simulations [223] of a similar representative shot, S167353, as mentioned earlier and are shown in figure 74.

The trajectories of the simulated particles in the poloidal plane are shown in figure 82, overlaid on the profile of the electron temperature at $z = -1.25\text{m}$. Particles with an initial radial position located within the strike point, $R \simeq 1.4\text{m}$, have much shorter tracks relative to other dust particles. Almost all dust grains traverse the plasma in straight lines having inertia dominated trajectories and terminate in the plasma close to the strike point where the plasma densities and temperatures are peaked. Larger particles with greater velocities are able to cross the separatrix without being disintegrated. The dashed grey lines at the radial positions $R = 1.03\text{m}$ and $R = 1.77\text{m}$ indicate the PFC boundary at the divertor, with particles reflected following collisions with this boundary. Those which have a shallow angle of incidence on the outer divertor wall are able to reflect multiple times surviving for extended periods in the edge plasma.

Figure 83 shows the the average temperature evolution experienced by the simulated dust grains in the first 10ms. The heating from the initial temperature of 300K to $\sim 2200\text{K}$ is extremely similar for all simulated dust grains since all particles experience intense plasma heating regardless of initial location which is only balanced by the cooling due to thermionic electron emission and thermal radiation at higher temperatures, as was the case for the example explored in section 6.3. Sub-

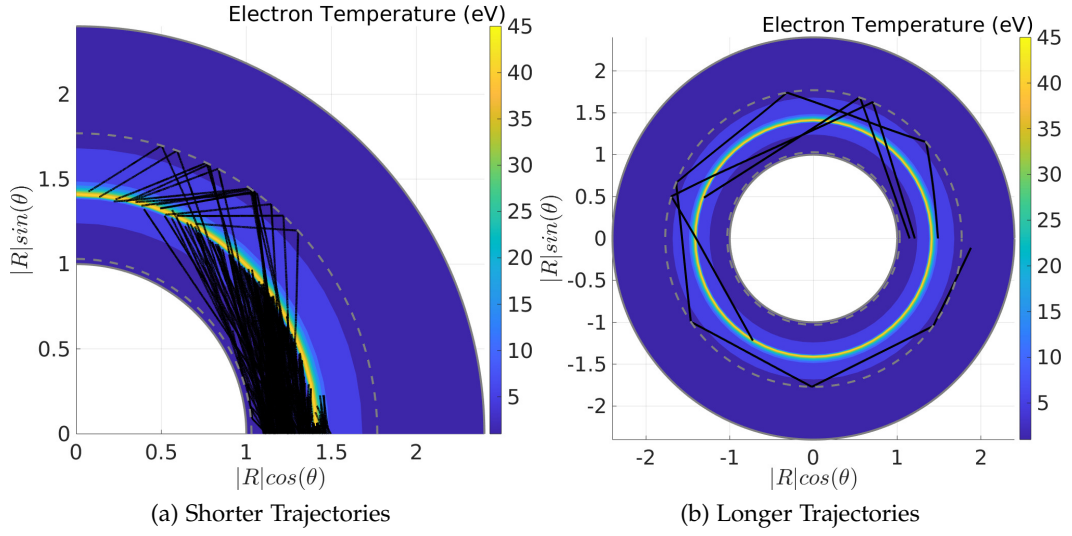


Figure 82: Simulated dust grain trajectories in the poloidal plane overlaid on the electron temperature profile at $z = -1.25\text{m}$ with the divertor PFC boundary marked (grey dashed) at the radial positions $R = 1.03\text{m}$ and $R = 1.77\text{m}$. Short tracks (82a) terminate at the outer divertor leg where the electron temperature peaks whilst longer tracks (82b) reflect multiple times from the outer PFC divertor wall in the edge plasma.

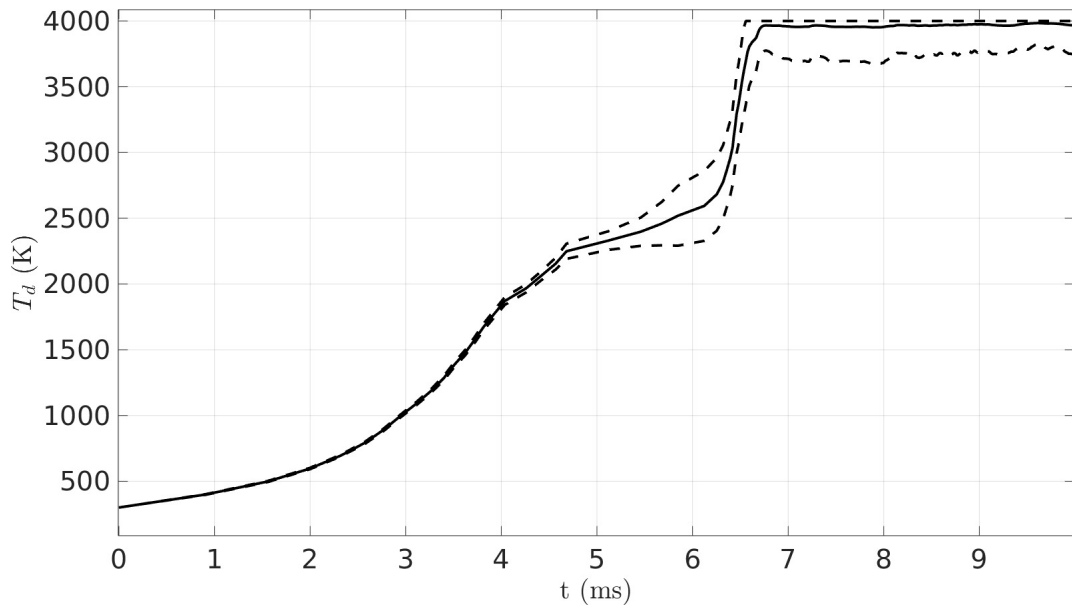


Figure 83: Average temperature evolution of simulated dust (solid) with standard deviations (dashed) over the first 10ms.

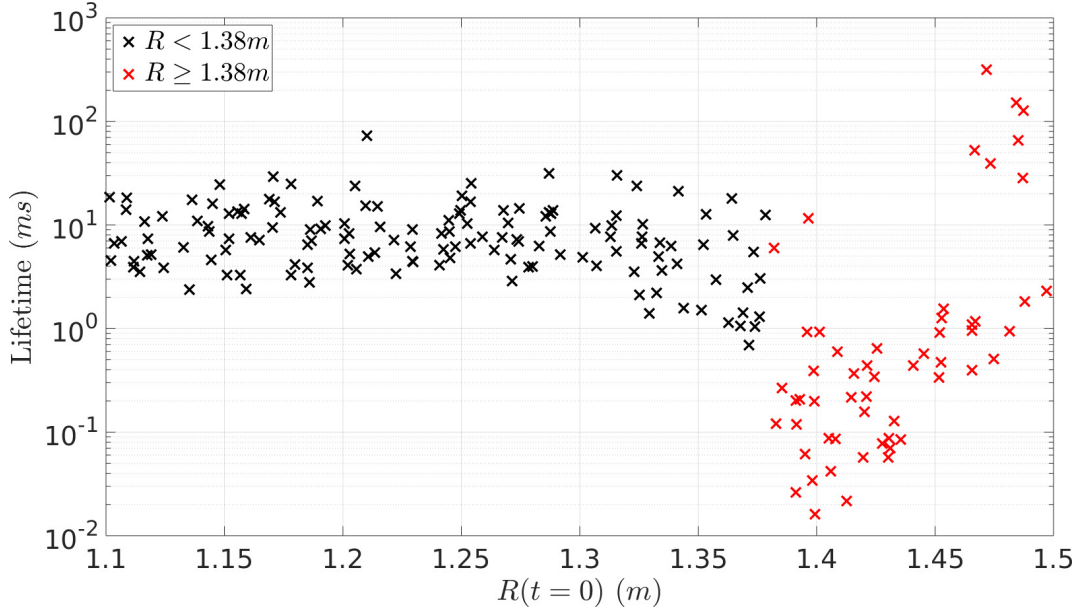


Figure 84: Dependence of the simulated lifetimes of particles on the initial radial position, $R(t=0)$. Particles with initial radial positions close to $R = 1.4\text{m}$ have substantially shorter lifetimes due to the intense plasma heating at this location.

sequently at disparate times when they reach regions of plasma with greater heat fluxes, the temperature rises until it reaches the boiling temperature, $T_d = 4000\text{K}$, causing the dust grain to ablate. Two distinct equilibria in temperature are identified, corresponding to regions in the edge plasma far from the strike point with $T_d \simeq 2500\text{K}$ and within the strike point where the dust temperature is fixed at the boiling temperature, $T_d \simeq 4000\text{K}$.

A large range of lifetimes between $16.2\mu\text{s}$ and 316ms were observed as a result of the variety in plasma conditions experienced and dust sizes. The mean simulated lifetime recorded was 10.8ms , with a standard deviation of 27ms . Figure 84 shows the correlation between the simulated lifetime of particles and their initial radial position in the simulation. Particles with an initial radial position closer to $R = 1.4\text{m}$ have shorter lifetimes in comparison to trajectories with $R(t=0) < 1.38$ as they are more likely to have trajectories that remain within the intense plasma region close to the strike point. Several dust grains with $R(t=0) > 1.45\text{m}$ survive for an extremely long time and correspond to trajectories as were shown in figure 82b that rebound from the outer divertor wall and remain in the relatively cool edge plasma.

The dependence of the dust lifetime on the initial conditions was further investigated for the two sets of particles distinguished in figure 84. The lifetimes of particles with $R(t=0) < 1.38\text{m}$ were found to be correlated with the initial toroidal velocity, as shown in figure 85b with a correlation coefficient of -0.4718 which is attributed to the fact that this speed dictates the time required to reach the separatrix. Conversely, particles with $R(t=0) \geq 1.38\text{m}$ have lifetimes which are independent of initial toroidal velocity, with a correlation coefficient 0.0389 , but are primarily dependent on the initial size of the dust grain, see figure 85b, as this affects the total mass that must be ablated. This highlights the importance of the

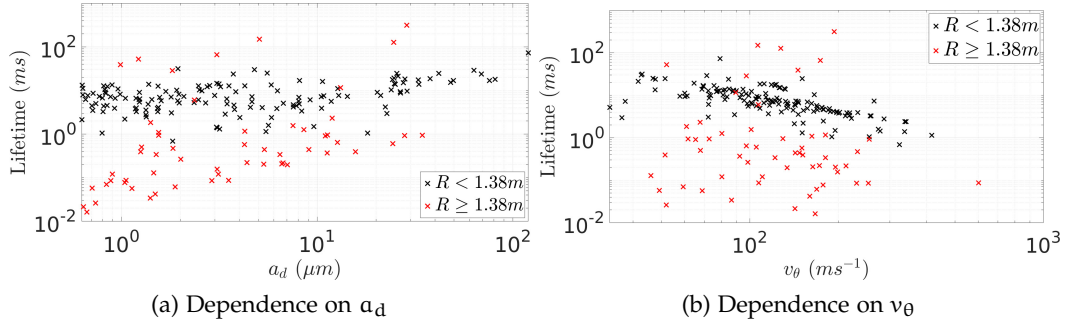


Figure 85: Dependence of the simulated lifetimes of particles on the initial radius, a_d , (85b) and initial toroidal velocity, v_θ , (85b) with particles distinguished by their initial radial position.

separatrix for the survivability of dust grains in the divertor, with dust outside the separatrix that is not visible to the camera surviving for extended periods of time.

Figure 86 shows the average mass of carbon atoms deposited across the poloidal cross section of DIII-D for the 200 carbon dust grains simulated in DTOKS-U. The majority of the mass was deposited near to the divertor floor as expected, with a large amount of the dust being destroyed close to the separatrix. A small number of the dust grains outside the separatrix were able to reach larger vertical positions closer to the core after reflecting from the outer PFCs. These deposit a small amount of mass much closer to the core plasma and could have more significant implications for the radiation energy balance in the edge plasma. Overall however, they represent an insignificant fraction of the total mass of carbon impurities deposited and their motion is sensitively dependent on the initial conditions and the type of collision the dust experiences with the local geometry of the PFCs.

Interestingly, the experimental observations of figure 79b, similarly observed many particle tracks that terminate in the annulus corresponding to the outer strike point location. The heating models and description of the plasma therefore appear to provide a sufficiently accurate description to replicate this key observation. The discrepancy between the mean simulated lifetime of 10.8ms and the observed lifetime of $2.1\text{ms} \pm 0.4\text{ms}$ is most likely a result of the fact that particles disappear outside of the camera view before being destroyed by the plasma. In general, the ion drag force experienced by simulated dust particles is too weak to maintain a velocity directed in the toroidal direction, causing particles to drift to larger radial positions. This is also the case for many particles observed by the camera as exemplified by figure 79a, suggesting a drift of dust towards larger radial positions after being created. However, without exact knowledge of the dust size or measuring the dust motion in 3D, it is not possible to make quantitative conclusions about the accuracy of the ion drag or other force models.

Through combining the output of DTOKS with the visualisation software Calcam [226], synthetic diagnostic data for the fast imaging visual cameras was generated. Comparing the tracks in figure 87, qualitative agreement is observed with the synthetic data. In particular, the simulated paths show similar radial and toroidal acceleration to experimental trajectories as well as similar track lengths. Both experimental and simulated particle tracks appear to often terminate at the divertor lip

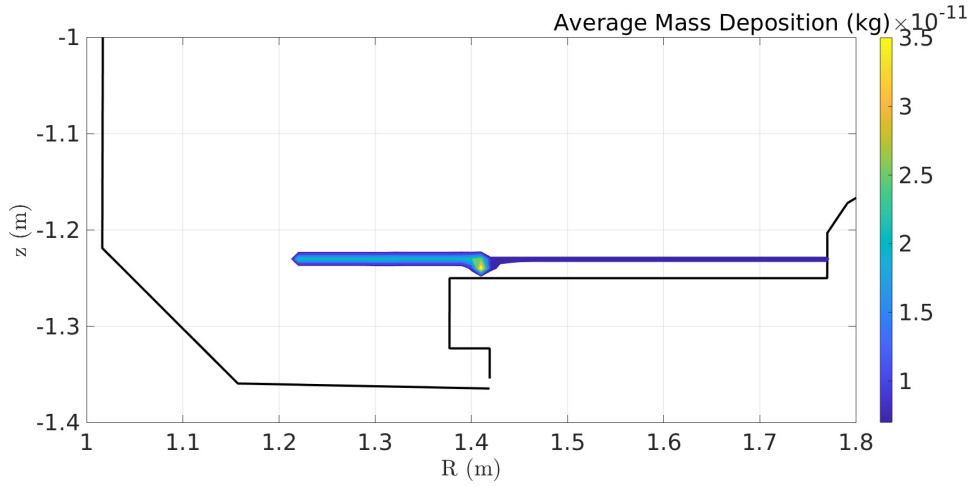


Figure 86: Average impurity deposition profile for randomly generated carbon dust in DIII-D divertor.

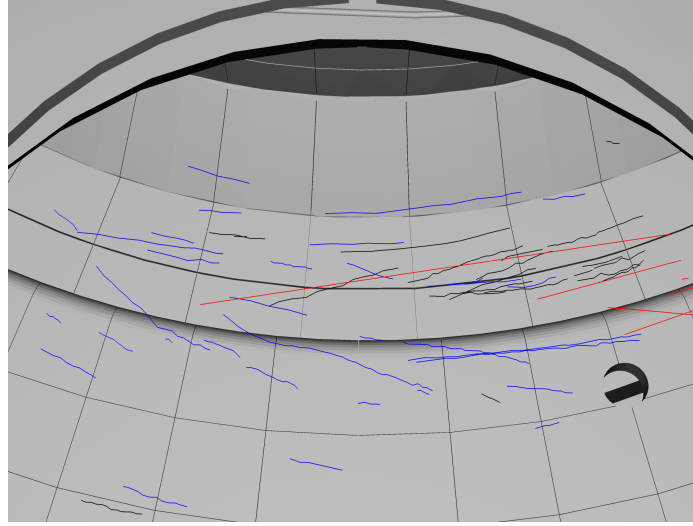


Figure 87: Synthetic visual camera data rendered using Calcam [226] showing dust grain tracks as simulated with DTOKS-U (red) as compared with reconstructed trajectories from S167342 (blue) and S167345 (black) identified using RIDAT.

close to the strike point. However, many of the simulated trajectories are substantially longer than any observed dust and tend to travel in a fixed direction meaning particle motion does not remain in the toroidal direction. The ion drag force model for carbon dust of this size and at this speed therefore appears to be substantially smaller than the magnitude experienced. This analysis demonstrates the utility of DTOKS-U for explaining dust behaviour, predicting impurity deposition profiles and its capability for predictive modelling and reconstruction of visual diagnostic data.

*If we eliminate the particles,
we also eliminate the need to count them.
That is our goal.*

— Gary S. Selwyn [30]

8.1 OVERVIEW

To explain observations of dust breakup in DIII-D [94, Figure 10] reported in a major review of dust in tokamaks, it was suggested that “the spinning solid grains and liquid droplets also can be subject to disintegration by centrifugal forces” [62, p.31]. Having also suggested several other unsubstantiated hypotheses to explain these observations, it was concluded that “further studies are required to clarify this issue” [62, p.31]. The potential for liquid metal dust grains to become rotationally unstable has significant implications for dust transport in tokamaks and the subsequent deposition of impurities which has so far remained largely unexplored. In this chapter, the recent observations of breakup of dust particles in JET are reported and the results are analysed using simulations performed by DTOKS-U [195].

8.2 EXPERIMENTAL OBSERVATIONS

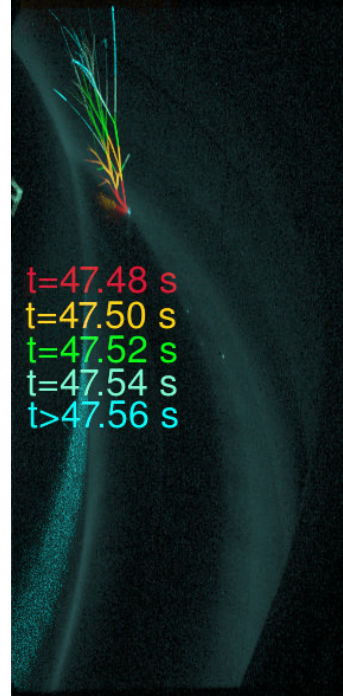
Superimposed images of concurrent frames in figure 88 show three examples of dust grain breakup in JET observed by the IR divertor-view camera. In each of these examples, a single dust particle released from the divertor floor was found to break up into two sub-particles, both of which then split again, in a repeated bifurcation process. The 20ms exposure time of the camera and the speed of dust particles means they were recorded as streaks, as was the case for dust observations in chapter 7.

A survey of dust events spanning campaigns *c35_restart* to *c38* (shots #88687 to #92774) was performed using the JET Users Video Imaging Library (JUVIL) software, focusing on 169 experiments where dust was observed as recorded by the visual systems operator¹. These 169 shots are investigated further here, with 103 experiments found to contain at least one example of dust exhibiting breakup as shown in figure 88. A detailed analysis of 7 of these shots found 91 individual breakup events and revealed a pattern in their behaviour. Table 2 shows the number of tracks and cumulative number of branches observed in shot #86124 for each frame of the breakup event corresponding to figure 88c. From analysis of these shots, splitting was found to exclusively produce pairs of particles of

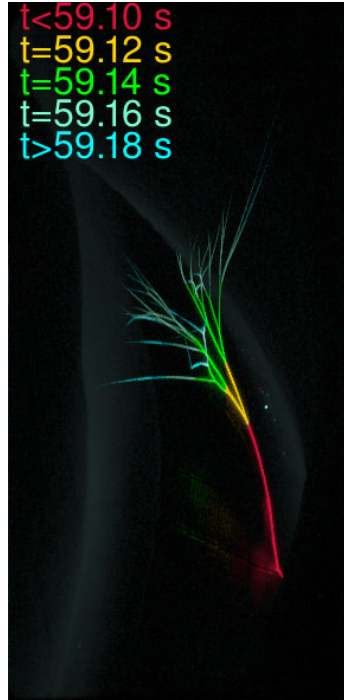
¹ JET Contributors, See the author list of “X. Litaudon et al. 2017 Nucl. Fusion 57 102001”



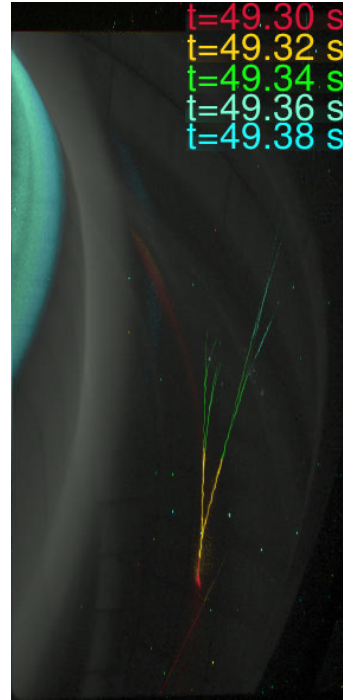
(a) Divertor view



(b) Shot #85974, Frames 525-531



(c) Shot #86124, Frames 1104-1111



(d) Shot #89472, Frames 616-620

Figure 88: Example observations of breakup of particles in JET by KL-11 divertor view camera system [227] with a 20ms exposure time, over wavelengths 390-2500nm. Figure 88a shows the field of view of divertor camera with vessel lights turned on. The branched tracks of shot #85974 (88b), #86124 (88c) and #89472 (88d) are colour coded by successive frames, with the plasma pulse beginning at 40s. Reproduced from [195].

Time (s)	Frame	Number of Tracks	Cumulative Number of Branches
59.04	1103	0	0
59.06	1104	1	0
59.08	1105	1	0
59.10	1106	1	0
59.12	1107	2	1
59.14	1108	21	11
59.16	1109	26	21
59.18	1110	15	23
59.20	1111	9	23
59.22	1112	4	23

Table 2: Number of tracks visible and the cumulative number of branches in each frame at corresponding times for shot #86124 as shown in figure 88c.

approximately equivalent brightness and with an average breakup time scale of $12(\pm 1)$ ms.

Investigating the breakup event in shot #85974 in more detail, information about the composition and position of the dust is gained from the bolometry measurements presented in figure 89. A peak in the tungsten intensity at ~ 7.5 s at the radial coordinate 2.7m corresponds exactly to the time and location of the event observed by the divertor view camera shown in figure 88b (note that the plasma pulse begins at 40s). This signal is most likely a result of the strong radiative emission from tungsten atoms after the dust particle matter has dispersed and ablated. The absence of a peak in the beryllium spectroscopic signal at a similar time and location suggests that it is composed mostly of tungsten. For shot #85974, this influx of tungsten impurities eventually lead to loss of plasma stability and termination of the discharge through disruption.

In JET, only hollow tungsten spheres, shown previously in figure 10b, have been recovered as opposed to solid spheres, which is most likely evidence of tungsten coatings that experience partial melting [111]. At the same time, there is also an absence of collected dust from the lower divertor tile 5, despite the fact that the material identified in the camera observations originates from this location. For this reason, it is likely that an unknown mechanism acts to destroy or transport dust in this region before it is deposited.

The fracture mechanisms of solid materials is expected to produce any number of fragments with a random orientation that is dependent on the internal crystal or aggregate structure of the solid. The predictable nature of the breakup events is therefore more intuitively explained through the hydrodynamic instability of a liquid metal surface. This is consistent with the evidence of molten metal dust collected from inside the vessel as described in section 1.4.3 and shown in figure 8.

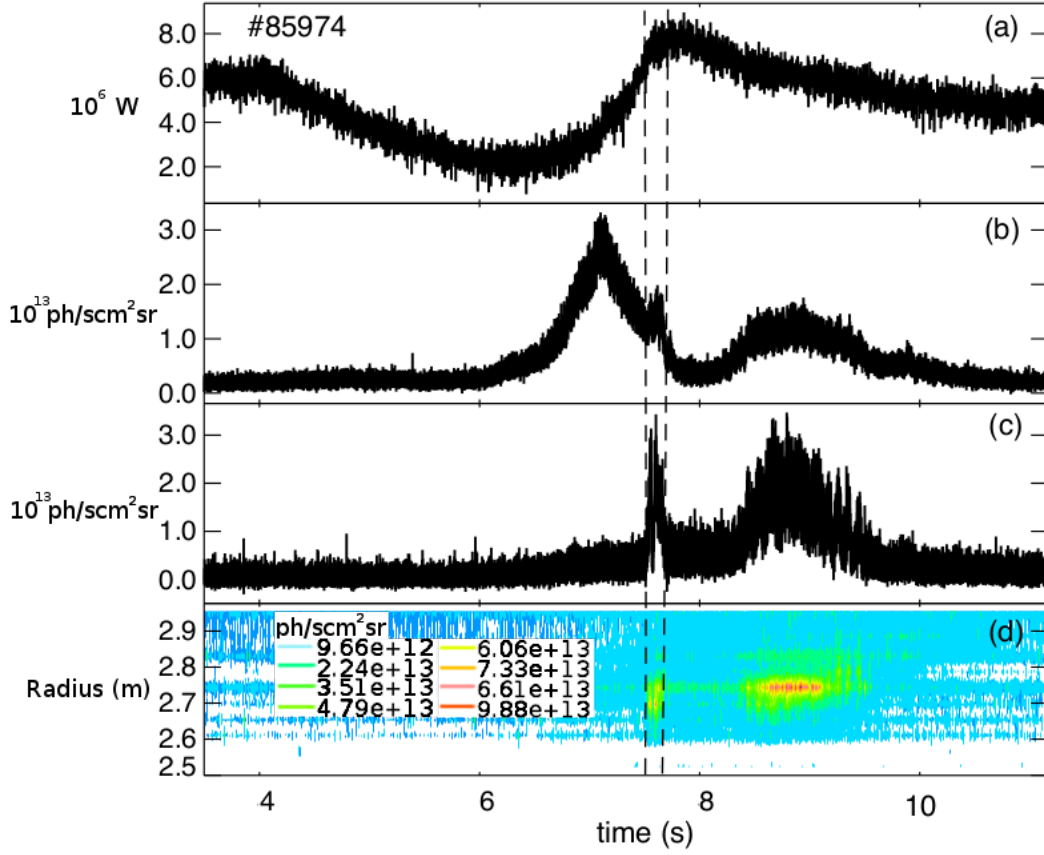


Figure 89: Total radiated power as measured by vertical viewing bolometer (a), spectroscopy of beryllium 527 nm line (b) and tungsten 401 nm line (c) and 2d tungsten intensity for shot #85974 (d). A sharp peak in (c) is observed around $t=7.5$ s at a major radius $r=2.7$ m (d) corresponding to the tracks of figure 88b. Reproduced from [195].

8.3 FLUID INSTABILITIES

In previous chapters, it was shown that dust grains in tokamaks are charged objects which experience rapid rotational motion whilst in the liquid phase. Experimental observations in DIII-D and other tokamaks, have shown evidence of molten metal dust grains undergoing breakup into smaller fragments [94]. This raises the question as to whether the liquid metal droplets are able to remain stable against both electrostatic and rotational instabilities. This is a particularly important phenomena as it can severely limit the lifetime of dust in tokamaks and alter the subsequent distribution of impurities.

Figure 90 shows the simulated shape of liquid droplets preceding an electrostatic and rotational instability. With excess charge, the simulated droplet elongates into two conical tips, from which jets of sub-droplets are ejected in agreement with electrostatically unstable droplets observed in experiments [228]. For rapidly rotating droplets on the other hand, a two-lobed fissioning process occurs, resulting in two equally sized droplets leaving with oppositely directed velocities in the centre of mass frame of the parent droplet from which they were produced. Theoretical models for both of these fluid instabilities are implemented in the DTOKS-U code.

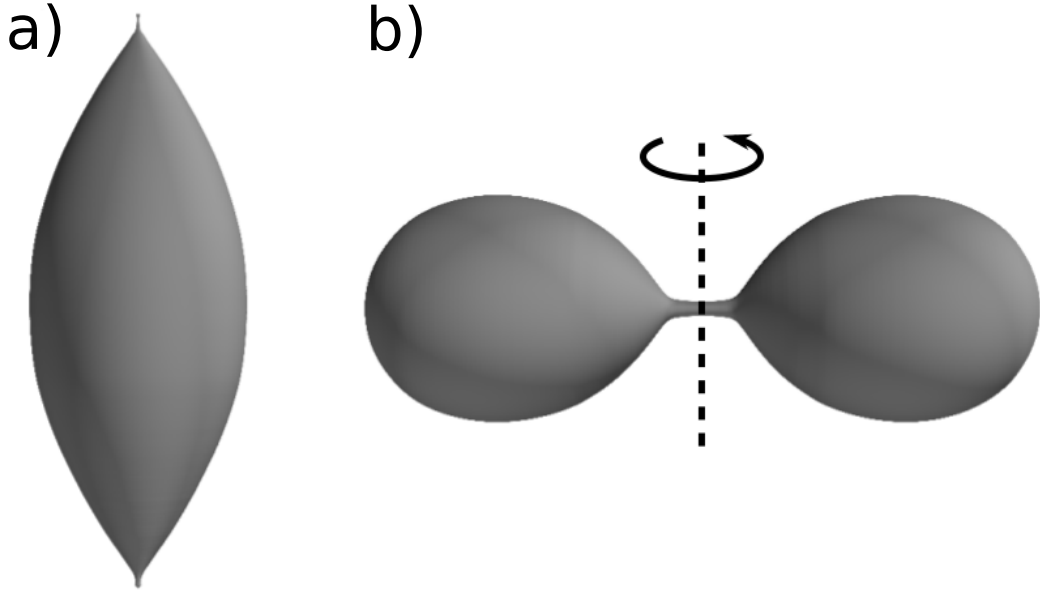


Figure 90: The shapes of unstable droplets immediately before disruption due to (a) excess charge or (b) rotational motion. Charged droplets break up by forming two conical tips from which fine jets of sub-droplets are ejected. Rotation causes two-lobed breakup into a pair of equally sized droplets, adapted with permission from Ref. [229].

8.3.1 Electrostatic Breakup

In the same way liquid droplets in vacuum with excess charge become hydrodynamically unstable, molten metal dust in tokamak plasma environments are vulnerable to electrostatic disruption [230]. If a spherical liquid droplets charge becomes great enough for electrostatic repulsive forces to overcome surface tension, Rayleigh instabilities cause it to eject charge and a small amount of matter. In the case of dust in plasmas this is a completely destructive process as it continues to recharge more rapidly than the mass loss rate [230]. This feature holds true for any size independent charging model and is dependant only on the fluid surface tension, ι , and the surface potential ϕ_a . This places a lower limit, on the minimum size of liquid metal droplets which is given by

$$\frac{\epsilon_0 \phi_a^2}{4\iota} \leq a_d. \quad (161)$$

In DTOKS-U, this is modelled by providing a dynamic lower limit on molten dust grain radii dependant on ι and ϕ_a given by equation 161. If the dust size falls below this lower bound, it is assumed to form a Rayleigh instability and be instantly destroyed.

8.3.2 Rotational Breakup

The large angular velocities, $\sim 10^9 \text{s}^{-1}$, acquired by dust grains under tokamak conditions postulated in chapter 5 can lead to fluid instabilities caused by centrifugal

forces. The maximum value of the dimensionless angular velocity, Ω_c , for which a fluid sphere in vacuum begins to go unstable in vacuum as verified by theory [231], simulation [232] and experiment [233] is given by

$$\Omega_c = 0.56 \sqrt{\frac{8\iota}{\rho_d a_d^3}}, \quad (162)$$

where ρ_d is once again the dust mass density. For a droplet of molten tungsten of radius $a_d = 10\mu\text{m}$, having surface tension $\iota = 2.33\text{Nm}^{-1}$ [234] and mass density $\rho_d = 19600\text{kgm}^{-3}$, the critical angular velocity is $\Omega_c = 1.73 \times 10^7\text{s}^{-1}$. This value is smaller than the predicted equilibrium rotation rates, Ω_{eq} , shown in figures 62 and 66, providing the possibility for these liquid droplets to become unstable. Given that $\Omega_c \ll \Omega_{eq}$ in most cases, it is possible to take a linear approximation, $\gamma t/K \ll 1$, of equation 129, so that

$$\frac{d\Omega_d}{dt} \simeq \frac{\alpha}{K}. \quad (163)$$

In the SOL of a tokamak, it is anticipated that the ion magnetisation parameter will be between $0.001 \leq \beta_i \leq 2.0$ depending on dust size, plasma temperature and magnetic field strength. It is also likely that the high temperatures of molten metals that are vulnerable to this type of rotational instability will be positively charged as a result of the substantial thermionic electron emission, as was mentioned in section 3.2.6.1 and shown in figure 19. For this reason, it is assumed that the dominant torque arises from collection of electrons and is given by the low field limit of equation 141 so that $\alpha \simeq \Upsilon_{0,e}\beta_e = n_0 k_B T_e a_d^3 \beta_e$. For this torque and the moment of inertia of a uniform sphere, $K = 8\pi a_d^5 \rho_d / 15$, equation 163 becomes

$$\frac{d\Omega_d}{dt} \simeq \frac{e|\underline{B}|n_0}{\rho_d a_d} \sqrt{\frac{k_B T_e}{m_e}}. \quad (164)$$

In DTOKS-U, this equation is solved using an explicit Euler method over the same timescale as the force model described in section 6.2.1. When the critical rotational velocity of equation 162 is exceeded, $\Omega_c \leq \Omega_d$, for dust in the liquid phase, rotational breakup of the dust grain occurs. After rotational disruption occurs, it is assumed that two new particles form each with half the mass of the parent particle and $\Omega_d = 0$. The rotational energy of the parent particle is therefore assumed to be fully converted into kinetic energy of translational motion, giving the two new droplets oppositely directed velocities of magnitude $\sim \Omega_d a_d$, randomly orientated in the plane defined by $\hat{\underline{v}}_d \times \hat{\underline{B}}$. This stochastic description of the consequences of a rotational instability on dust dynamics provides a useful qualitative comparison with experimental observations and statistical estimates of the distribution of impurities.

The theoretical models for rotational and electrostatic breakup of liquid metal droplets used in DTOKS-U lead to very different physical outcomes for the molten metal dust. A droplet which experiences a rotational instability whilst in motion will produce two sub-droplets with oppositely directed velocities in the centre of mass frame, which will appear as a forked track in the lab frame. The electrostatic instability in contrast occurs for extremely small (10–100 nm) droplets and

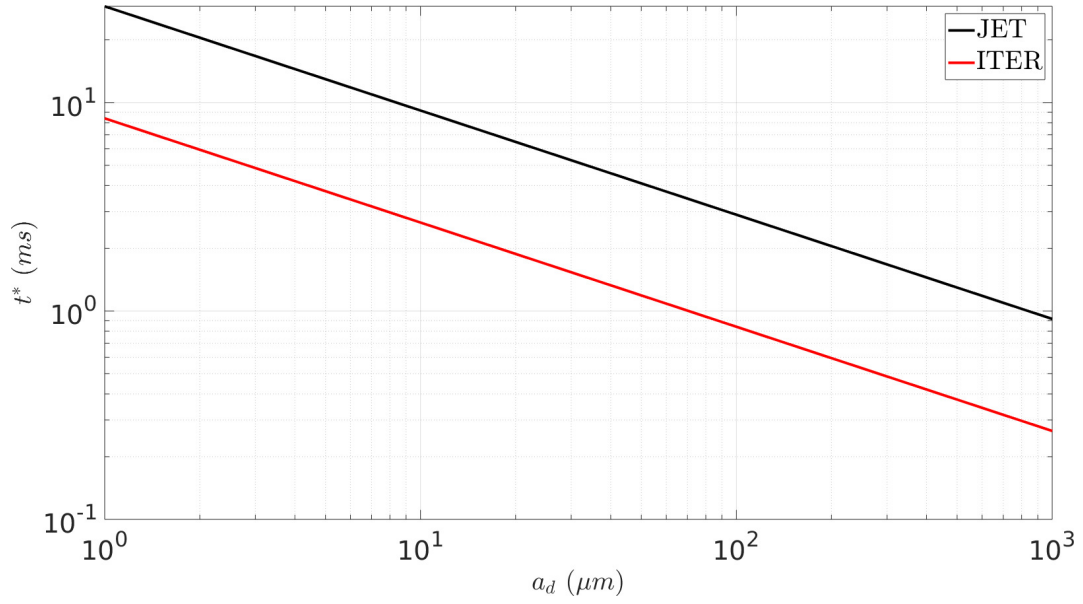


Figure 91: Estimates of the bifurcation timescale of tungsten droplets calculated using equation 165 in constant JET (black) and ITER (red) like tokamak edge plasmas conditions [235, Table 1 & 3] for different droplet radii.

causes complete annihilation on a timescale of ~ 1 ns once the process has begun. Considering the geometry of droplets ejected after a rotational instability occurs as shown in figure 90, it is clear that the forked trajectories in figure 88 are most simply explained through the rotational instability of the molten metal surface. The rotational instability is therefore expected to dominate for larger dust grains ($0.5\mu m < a_d$) that are visible to the IR camera.

Combining the rotational stability limit given by equation 162 with the linear approximation for the dust angular velocity of equation 164 provides an estimate of the bifurcation timescale,

$$t^* \sim \frac{0.56}{eBn_0v_{T,e}} \sqrt{\frac{8\epsilon\rho_d}{a}}. \quad (165)$$

The rotational breakup timescale of equation 165 is plotted in figure 91 for a range of sizes of tungsten droplets in the edge plasmas of JET and ITER, with magnetic fields of 2.5T and 5.3T respectively, using typical SOL plasma parameters [235, Table 1 & 3]. This shows how tungsten droplets with sizes $a_d \sim 1$ – $100\mu m$ survive for corresponding times of $t \sim 25$ – 1 ms in JET before they split into a pair of sub-droplets due to rotational disruption, in good agreement with the measured average breakup time scale and the times of figure 88. Rotational breakup is the only phenomena for which these accurate predictions of the qualitative breakup dynamics and the quantitative timescales of experimental observations can be made.

8.4 SIMULATED RECONSTRUCTION

DTOKS-U has been employed for the purpose of modelling the observed breakup phenomena, focusing on the rotational breakup event observed in shot #85974

examined in detail in figure 88b and 89. The location of the hot spot from which the dust originates is measured at a tokamak coordinate $(R,z)=(2.7,-1.6)$ m. The dust particle is assumed to be a $50\mu\text{m}$ tungsten droplet initially at the melting temperature of tungsten, 3500K, positioned at the same location at the corner of tile 5 and with an initial velocity of $\mathbf{v}_d = (v_r, v_\theta, v_z) = (-20, 10, 50)$ m s⁻¹. The background plasma generated with the EDGE2D/EIRENE code [210] is based on the magnetic field topology of a similar shot (#90414) with plasma parameters based on plasma diagnostic measurements for shot #85974.

The resulting simulated trajectory shown in figure 92 is rendered in the computer model of JET from the perspective of the KLBD-11 camera using CAL-CAM [226] and coloured by the stage of the breakup, allowing direct comparison with figure 88b. Good qualitative agreement with the fast cameras in JET is seen, with comparable track lengths and breadth of the feature seen. Note that from equation 165, the smaller radius of droplets produced by the repeated breakup process might be expected to lead to an increase in the breakup timescale. However, as the droplet travels towards the last closed flux surface from the edge, the product $n_0 v_{t,e}$ increases and over-compensates for the reduction in radius. In particular, the density increases greatly across the pedestal causing the flux of electrons to the surface to increase, generating larger torques. The lifetime of the original droplet and each sub droplet created in the chain was measured with the longest lifetime being 8.2ms and having mean lifetime of 4.8ms. The survival times were found to drop as the droplets approached the core plasma.

The confirmation of the hypothesis of dust breakup in tokamaks as a result of the rotational instability of liquid droplets, which is in part supported by the agreement with DTOKS-U simulations, has several important implications. Firstly, the rotation rate necessary for a tungsten droplet to experience a rotational instability is approximately two orders of magnitude greater than the rotation rates of the order of KHz observed in previous laboratory plasma experiments [203, 204, 236, 237]. In fact, laboratory observations of objects spinning at over 10^9s^{-1} in any context have only recently been attained through optical trapping of nanoparticles under vacuum [238, 239]. The proof of the rotational breakup hypothesis would therefore entail the measurement of dust rotation in plasmas at unprecedented speeds. Additionally, observation of spontaneous spinning at over 10^5s^{-1} provides significant support for the related theories and observations of anomalous microwave radiation from spinning dust in clouds as described in section 1.3, currently based on phenomena that have not yet been observed directly. Finally, the rotational breakup of droplets in tokamak plasmas is expected to lead to the rapid destruction of metal dust particles in the SOL of future devices such as ITER, since they operate at higher densities and temperatures with far stronger magnetic fields. This will most likely result in the impurities released from the ablation of dust being dispersed over a wider area closer to the wall than would otherwise be anticipated, a beneficial outcome for mitigating their impact on the core plasma.

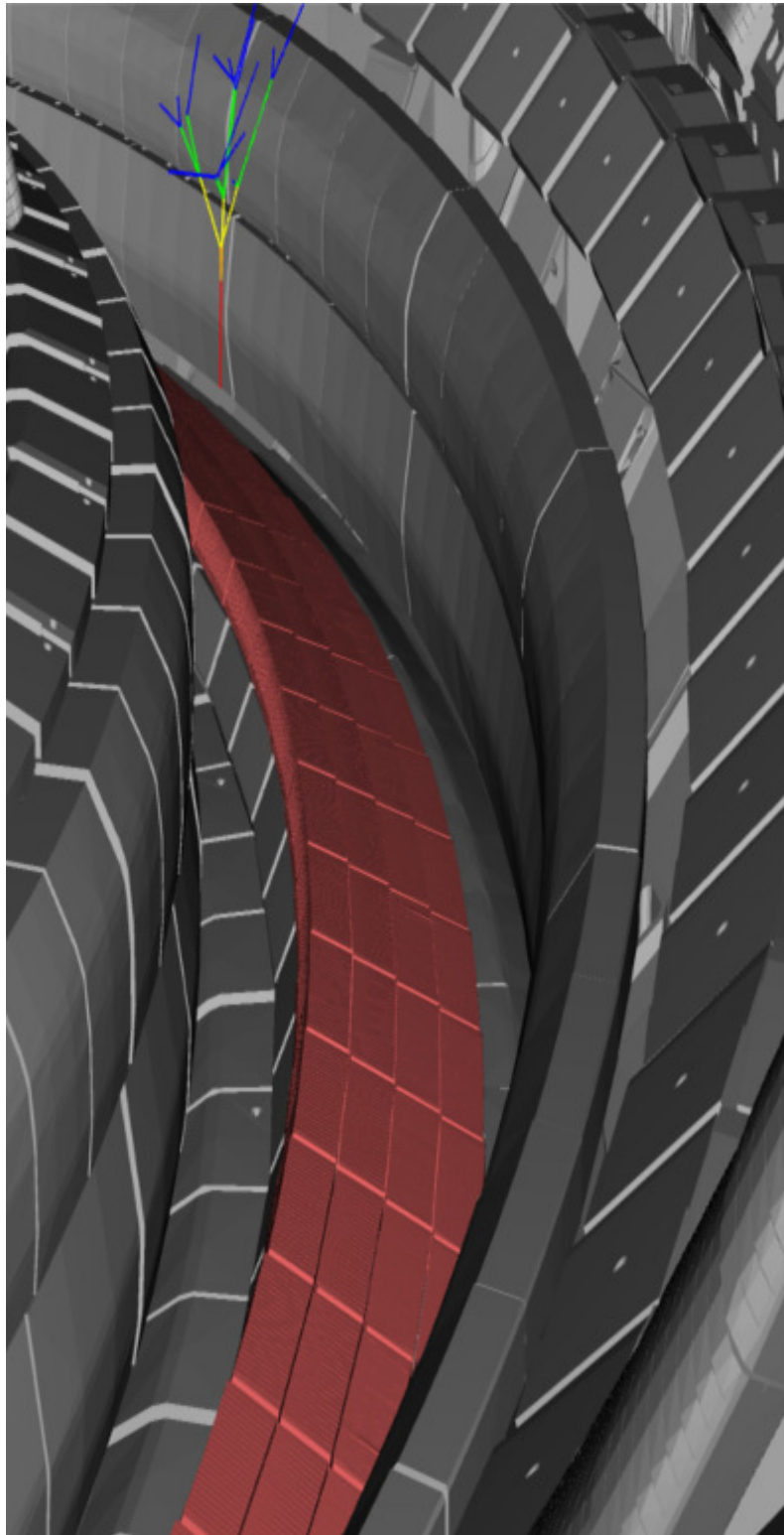


Figure 92: Simulated trajectory with initial radius of $a_d = 50\mu\text{m}$ and temperature $T_d = 3500\text{K}$ generated using DTOKS-U reconstructing breakup of tungsten droplet as observed by JET KL11-P1DB divertor view camera, see figure 88b. Reproduced from [195].

CONCLUSION

This thesis documents the development of computational and theoretical models for calculating the effect of magnetic fields on dust behaviour in tokamak plasmas. Here, the findings are summarised and suggestions are made for directions of future work in the many areas where unanswered questions remain.

9.1 SUMMARY

In chapter 1, the importance of dust was established in the context of its impact on semiconductor manufacturing, astronomical observations and fusion plasmas. For ITER, dust was shown to be a serious and limiting operational hazard arising inevitably in tokamaks through damage to the inner walls following unmitigated ELMs and plasma instabilities.

Chapter 2 and chapter 3 summarised the contemporary theoretical understanding, foundational to describing the interaction between dust and plasmas.

The methodology of the DiMPI code was presented in chapter 4 and validated against theoretical estimates and PIC code results for the charging behaviour of spherical dust in collisionless plasmas. The remarkable insensitivity of the surface potential to the form of the potential distribution favours the method employed by DiMPI and reveals the weak dependence of the net particle flux on the form of the potential. For dust in flowing plasmas with a magnetic field, these new results revealed a strong dependence at high magnetic field strengths, due to the additional kinetic energy of particles that are only weakly inhibited by the magnetic field. A semi-empirical model for the surface potential allows the dust potential to be accurately estimated as a function of ion magnetisation parameter β_i for the first time in dust transport codes.

Chapter 5 provided a comprehensive description of the rotational dynamics of dust smaller than the Debye length due to the impact of plasma particles in a magnetic field. The results of DiMPI identified rotational motion in the direction along the axis defined by the magnetic field with the contribution from electrons found to be more significant than previously documented. Modelling of positively charged dust, as is anticipated to be the case for high temperature dust in tokamaks, found unprecedented rotational motion at low values of ion magnetisation parameter as a result of electron impact, leading to large equilibrium rotation velocities. Using this novel mechanism and the results of DiMPI simulations, it was possible to explain experimental observations of rotating glass spheres in low temperature discharges.

The implementation of these new models into the dust transport code DTOKS-U, along with improvements to the physical models, were demonstrated in chapter 6 with a case study of a simulated dust grain in the DIII-D tokamak. The careful modelling of the temperature and size dependence of the material properties were

found to significantly alter the dust grain lifetime and equilibrium temperature in this case.

Finally, in chapter 7 and chapter 8, the simulation and modelling tools developed were applied to experimental observations of dust in tokamak environments. Chapter 7 describes analysis of experimental observations of dust in DIII-D from shot S167342 and S167345 made with visual cameras; providing information about the ranges of observed positions, the speed distributions and the mean observation time 2.1 ± 0.4 ms. This was used to parameterise simulations performed by DTOKS-U of dust in DIII-D using a plasma profile generated with OEDGE, providing a predicted impurity density profile, a mean simulated lifetime of 10.8 ms and demonstrating the novel method for reconstructing synthetic visual camera diagnostic data using CALCAM. The results of the experiment and simulations revealed the importance of the strike point of the outer divertor leg in preventing dust transport across the divertor floor. The study of dust events in JET spanning campaigns c35_restart to c38 (shots #88687 to #92774) described in chapter 8 found of 169 shots with visible dust, 103 shots contained at least one example of dust breakup. Detailed analysis of 7 of these shots found 91 individual breakup events and found splitting exclusively produced pairs of particles. The average breakup time scale of 12 ± 1 ms and qualitative features of the breakup were related to the hydrodynamic instability of molten tungsten droplets and the experimental observation were replicated by DTOKS-U simulations that incorporated models for rotational motion and charging influenced by the magnetic field.

9.2 EVALUATION AND IMPACT

A number of fundamental insights into the behaviour of dust in plasmas with magnetic fields have been made through this work. The reproduction of the results of Orbital Motion theory for dust as large as the Debye length in isothermal plasmas by DiMPI verifies the assumptions of the computational method in spite of the known inaccuracies of the assumed form of the potential [156]. This is further backed by the robust spherical symmetry of the potential distribution for different magnetic field strengths and dust sizes as measured by pot and is ultimately cemented through the comparison of the simulation results of pot [152], Lange [155] and SCEPTIC [157] with DiMPI. This confirms the appropriateness of assuming the radial form of the potential in DiMPI, which permits faster calculations of surface fluxes than provided by PIC codes, whilst maintaining a similar precision $< \sim 1\%$ in solutions. Together these observations support extending the application of these simplifying assumptions further in theoretical and computational calculations than previously anticipated. In particular, they imply that for dust grains smaller than the Debye length, the net particle flux is predominantly determined by particle motion close to the surface, with the details of the pre-sheath structure influencing only the energy of particles before they approach close to the dust surface.

The modelling and simulation of dust rotational motion has uncovered the importance of the angular momentum transferred through the collection of electrons. Consequently, positively charged dust is expected to experience significant rota-

tional motion at low, previously unprecedented, values of β_i . The confirmation of this mechanism for rotational motion in low temperature discharge experiments and the indirect observations of this rotational motion as the explanation for the rotational breakup of dust observed frequently in JET therefore supports theories of the anomalous microwave background which depend upon hypothesis of dust spinning at 10^9 Hz. In addition, the rotational breakup observed in JET entails rotation rates two orders of magnitude greater than previous observations in plasmas. Collectively, the results of the analysis of simulation, theory and experiment underline the historically overlooked importance of rotational motion for the stability and behaviour of dust grains. This also hints at an exploitable physical mechanism that could be used to destroy molten dust in tokamaks before it becomes a hazard.

Novel reconstruction techniques developed for the analysis of experimental data taken in JET and DIII-D offer a new purpose for dust transport codes which have struggled with the limitations of validation against an incomplete experimental understanding. Simulations of carbon dust in DIII-D has revealed the importance of the strike point location for limiting graphite transport across the divertor floor. The demonstration of the effectiveness of RIDAT as an automated tool for identifying dust trajectories in DIII-D, marks the start of long anticipated broader statistical analysis of dust in tokamaks, necessary for accurate predictions of operational timescales for ITER. Meanwhile the pairing of simulated trajectories of DTOKS-U with the visualisation software CALCAM exhibits the potential for dust tracking codes to generate predictive simulations that are representative of visual diagnostic data, allowing for a direct comparison with experiment. This method was instrumental in conclusively illustrating the rotational breakup mechanism in JET and provides an invaluable research tool for scrutinising simulations of dust transport.

9.3 ONGOING AND FUTURE WORK

The progress made by this work is accompanied by many more open questions which are highlighted here, along with plans for ongoing research.

As ever, there is a need for the physical models to incorporate a wider range of phenomena to address the complexity of physical reality. Section 1.4.4 of the introduction showcases the need to model non spherical dust such as flakes composed of a conglomerate of substances, which will also entail re-examining the material and electrical properties of dust more carefully. To extend simulations to incorporate pellets and larger dust grains, the influence on local plasma conditions of the fluxes of emitted electrons and neutral particles from the dust surface must be accounted for, along with relaxing the collisionless approximation. There is also a demand for simulated plasma conditions that are time dependent in order to replicate the conditions of H-mode plasmas with ELMs that are known to produce dust. In many areas, the framework for investigating these features has now been established through the development of DTOKS-U and DiMPI.

Even at present, there remain several uncharted avenues immediately available for future work with DiMPI. The variation in the ion drag force with magnetic field strength remains a largely unexplored but accessible area that is of great importance given the dominance of this force in determining dust trajectories in tokamak

environments. Additionally, the torque due to internal $\mathbf{J} \times \mathbf{B}$ forces as described in section 5.1.1.2 is readily assessable by DiMPI given that the dependence of the current density at the surface with magnetic field strength has been presented already. The transiently trapped particles though not initially or uniquely observed in DiMPI have yet to be investigated in detail and present interesting questions about the effect they have on the potential distribution. Similarly trapping of particles as a result of collisions and the effect of magnetic fields on electron emission are interesting emerging areas of study that could be addressed with minor enhancements to DiMPI.

Conversely, the accurate determination of the plasma surface fluxes as a function of β_i for intermediate and large dust sizes, $\tilde{\lambda}_d \geq 1$ remains an intractable problem for DiMPI because the form of the potential distribution around a spherical conductor of arbitrary size relative to the sheath length scale is not known. Therefore, to extend the applicability of DiMPI to larger dust sizes and flowing plasmas, the potential distribution surrounding that dust grain must be appropriately characterised. This is also the case for investigating non-spherically symmetric dust which is anticipated to have a correspondingly asymmetric potential distribution.

Experimental observations of dust continue to be the most essential and challenging forefront of research. As mentioned briefly in section 1.4.4, experiments of dust motion in tokamaks have struggled to control enough extraneous variables to make meaningful interpretations, having incomplete information about dust size, composition and local plasma conditions. As a result, the ion drag force has not been conclusively validated against experimental observation for the tokamak edge plasma regime in spite of the dominant role it plays in dust dynamics as was highlighted in section 6.3, figure 76b. Additionally, though it is clear that the forces and floating potential of dust grains are affected by external magnetic fields and will be substantially altered by the strong magnetic fields of ITER, direct experimental measurements of this effect in tokamak conditions have yet to be performed.

To address this concern, a dedicated experiment has been performed in the linear plasma device, Magnum-PSI, using pre-characterised dust released into plasma conditions replicating those of the SOL of a tokamak. The experimental objective was to measure the ion drag force under different plasma conditions through observing dust illuminated by scattered laser light with both a visual and IR camera positioned with perpendicular viewing angles. Due to the limited range of magnetic field strengths explored however, no measurements with significant values of ion magnetisation parameter were made and for this reason the results have not been presented here. Nevertheless, the analysis of the experimental plasma conditions and simulations of the dust motion is the subject of immediate ongoing work with preliminary results yielding measurements of the ion drag force and providing insight into the accuracy of ion drag force models. This effort emphasises the continued need for experiments to characterise the effect of magnetic fields on dust behaviour in collisionless plasmas.

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APPENDICES

10.1 CALCULATING SECONDARY ELECTRON YIELD

The average yield from secondary electron emission, δ_{see} , is calculated in DTOKS and DTOKS-U from a semi-empirical formula that is unique for each element [114]. The equations for carbon, tungsten, iron, beryllium and lithium are

$$\delta_{see}^C = -0.849\log(T_e)^3 + 0.1149\log(T_e)^2 + 0.7428\log(T_e) - 1.341, \quad (166)$$

$$\delta_{see}^W = -0.0765\log(T_e)^3 + 0.1521\log(T_e)^2 + 0.724\log(T_e) - 1.4755, \quad (167)$$

$$\delta_{see}^{Fe} = -0.0903\log(T_e)^3 + 0.1813\log(T_e)^2 + 0.6368\log(T_e) - 1.2668, \quad (168)$$

$$\delta_{see}^{Be} = -0.0906\log(T_e)^3 + 0.1018\log(T_e)^2 + 0.717\log(T_e) - 1.4729, \quad (169)$$

$$\delta_{see}^{Li} = -0.0648\log(T_e)^3 - 0.1358\log(T_e)^2 + 0.9887\log(T_e) - 1.3253, \quad (170)$$

respectively. The dependence of δ_{see} on the electron temperature, T_e , in eV for these elements is shown in figure 93.

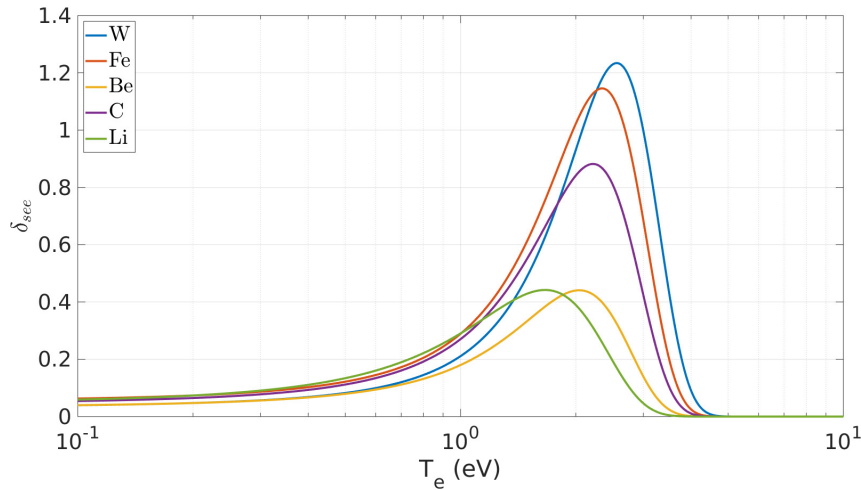


Figure 93: Semi-empirical formulation of the secondary electron emission yield, δ_{see} as a function of electron temperature, T_e , for carbon, tungsten, iron, beryllium and lithium from the equations in section 10.1.

10.2 STANDARD ERROR OF SEQUENTIAL MEASUREMENTS

To correctly calculate the uncertainty for non-Markovian stochastic processes, the correlation between concurrent measurements must be accounted for. The covariance, Cov , of a measurement of charge, Q_n , with subsequent measurements Q_{n+i} is

$$\text{Cov}(Q_n, Q_{n+i}) = \sum_{n=0}^{n=N^{\text{coll}}-i} (Q_n - \langle Q_d \rangle)(Q_{n+i} - \langle Q_d \rangle), \quad (171)$$

where $\langle Q_d \rangle$ is the mean charge. The auto-correlation coefficient, ρ_Q is calculated as the average of the covariance of Q_n with Q_{n+i} for all i divided by the variance

$$\rho_Q = \frac{1}{\sigma^2} \sum_{i=0}^{i=N^{\text{coll}}} \frac{\text{Cov}(Q_n, Q_{n+i})}{n} \quad (172)$$

Which provides the standard error, σ_{se} , from the uncertainty σ by

$$\sigma_{se} = \sqrt{\frac{1 + \rho_Q}{1 - \rho_Q}} \sigma \quad (173)$$

Typically, the correction factor in DiMPI measurements was found to be approximately $\sigma_{se} \simeq 1.1$ which does not increase the magnitude of the standard error significantly enough to change the order of magnitude of the estimated uncertainty.

10.3 ION CURRENT RESULTS

		β_i									
		0.3	0.5	0.7	1.0	1.4	2.0	3.0	4.0	5.0	7.0
χ_a	0.6	1.227	1.113	0.993	0.894	0.845	0.772	0.702	0.697	0.633	0.620
	1.0	1.469	1.249	1.141	1.001	0.869	0.810	0.709	0.678	0.646	0.604
	1.2	1.588	1.341	1.210	1.043	0.904	0.827	0.711	0.682	0.641	0.603
	1.4	1.629	1.372	1.200	1.072	0.913	0.814	0.698	0.664	0.630	0.587
	1.6	1.759	1.472	1.274	1.106	0.939	0.828	0.708	0.653	0.638	0.589
	1.8	1.851	1.523	1.336	1.159	0.977	0.842	0.716	0.659	0.626	0.584
	2.0	1.971	1.655	1.436	1.182	1.015	0.877	0.731	0.666	0.626	0.585
	2.5	2.169	1.778	1.537	1.297	1.070	0.921	0.750	0.681	0.628	0.589

Table 3: Normalised ion current, $\tilde{I}_i$, for different values of normalised surface potential, χ_a , and ion magnetisation parameter, β_i , as measured by DiMPI with maximal uncertainties of ± 0.01 .

10.4 LIST OF CHANGES TO DTOKS

The changes made to DTOKS include the addition of

- A *ContinuousPlasma* mode for testing purposes under spatially homogeneous plasma conditions as described in section 6.2.3
- A fourth order interpolation procedure for calculating the plasma parameters when performing simulations with a plasma grid as input
- Capability for parallelisation of computation for dust grains with different initial conditions
- Run time configuration capability for simulations through config4cpp package and command line input
- Optional input of wall and core boundary with specular reflection and termination occurring at each respectively as described in section 6.2.3
- Timescales for the force and heating model which are used to solve each equation to different user specified accuracies, discussed in section 6.2.1
- Models accounting for the variability of standard values with temperature and size, discussed in section 6.2.4
- The false position method in place of the Newton Rhapsod method for calculating the equilibrium potential
- Current terms due to OML electrons of equation 39, OML ions of equation 38, SOML ions of equation 60, MOML ions of equation 53, Schottky corrected thermionic electron emission of equation 69, magnetised electron and ion currents equation 122 as described in section 6.2.1
- Force terms due to neutral drag of equation 87 and the hybrid model of equation 86
- Heating terms due to neutral particles of equation 78, evaporative cooling as described in section 6.2.2 and Schottky corrected thermionic electron emission of equation 80
- A model for rotational breakup of liquid metals following the theoretical model discussed in chapter 5, implemented as described in section 8.3.2
- A model for electrostatic breakup of liquid metals, described in section 8.3.1
- Physical properties for simulating dust composed of beryllium, deuterium, lithium and molybdenum.

10.5 EMISSIVITY

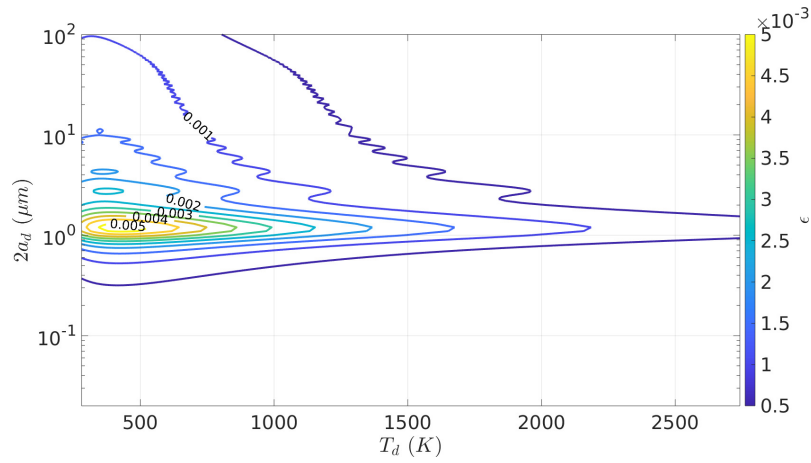
Here the equations used to determine the emissivity of a conducting sphere as a function of temperature and size using the Wiscombe code [216] are derived following the method of Rosenberg et al. [215, 240]. The emissivity of a material, $\epsilon(a_d, T_d)$, as a function of the size, a_d , and temperature, T_d , is defined by the total radiated power flux from the surface, P_{rad} , as

$$\epsilon(a_d, T_d) = \frac{P_{rad}(a_d, T_d)}{\sigma_B T_d^4}. \quad (174)$$

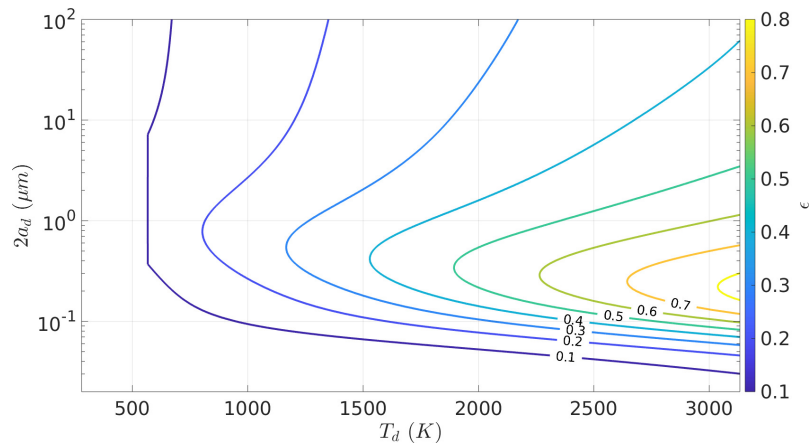
Here, σ_B is a constant whilst the dependence of the total radiated power flux $P_{rad}(a_d, T_d)$ on dust size and temperature must be determined. The thermal radiation emitted from a spherical body is defined as

$$P_{rad}(a_d, T_d) = \pi \int_{\lambda_c}^{\infty} Q_{abs}(X, n_1, n_2) S_p(\lambda, T_d) d\lambda, \quad (175)$$

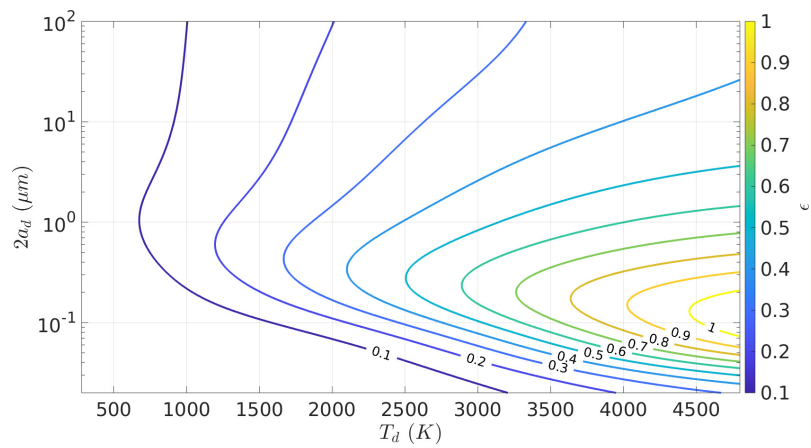
where Q_{abs} is the absorption efficiency of the sphere as a function of the size parameter $X = 2\pi a_d/\lambda$ and complex refractive index $n = n_1 + in_2$. The Planck function, $S_p(\lambda, T_d)$, gives the intensity of emitted radiation as a function of both temperature and wavelength of radiation $\lambda_c \leq \lambda \leq \infty$ above some critical wavelength λ_c . For $Q_{abs} = 1$, this provides the result for the emissivity of a perfect black body. The absorption efficiency, $Q_{abs}(X, n_1, n_2)$ determines the susceptibility of a dust grain to absorb radiation of a particular wavelength and is calculated by the Wiscombe code [216]. Values of $Q_{abs}(X, T_d)$ obtained for spherical dust of different sizes and temperatures were used to evaluate equation 175 by numerical integration using a Simpson method, from which values of $\epsilon(a_d, T_d)$ were calculated using equation 174. The emissivities calculated in this way as a function of the dust diameter, $2a_d$ and temperature, T_d are provided in figure 94 for beryllium, iron and tungsten.



(a) Beryllium.



(b) Iron.



(c) Tungsten.

Figure 94: Dependency of the emissivity of beryllium 94a, iron 94b and tungsten 94c on dust diameter, $2a_d$, and temperature, T_d .

ACRONYMS

DiMag	Dust in Magnetic Field
DiMPI	Dust in Magnetised Plasmas
DTOKS	Dust in TOKamakS
DTOKS-U	Dust in TOKamakS - Upgrade
DUCAD	DUst Characterization And Dynamics
DUMBO	DUst Migration in a plasma BOundary
DUSTT	DUST Transport
DUSTTrack	DUST Track
EDGE2D	EDGE plasma modeling code in 2 Dimensions
EIRENE	Neutral Monte Carlo plasma physics code
ELM	Edge Localised Mode
ITER	International Thermonuclear Experimental Reactor - "The Way"
JET	Joint European Torus
LCFS	Last Closed Flux Surface
MCF	Magnetically Confined Fusion
MHD	Magneto-Hydrodynamics
MiGRAINE	Migration of GRAINs in fusion devices
MOML	Modified Orbital Motion Limited
OEDGE	Onion-skin modelling EIRENE DIVIMP for EDGE analysis
OM	Orbital Motion
OML	Orbital Motion Limited
PIC	Particle In Cell
PFC	Plasma Facing Component
pot	Plasma Oct-Tree
RIDAT	Robust Impurity Detection And Tracking
SCEPTIC	Specialized-Coordinate Electrostatic Particle and Thermals In Cell

SEE	Secondary Electron Emission
SL	Sonmor & Laframboise
SMOML	Shifted Modified Orbital Motion Limited
SOL	Scrape Off Layer
SOML	Shifted Orbital Motion Limited
TEE	Thermionic Electron Emission
Tokamak	Toroidal chamber with Magnetic coils

SYMBOLS

In approximate alphabetical order:

a_d	Radius of spherical dust
a_s	First fitting coefficient, for species s
A_e	Area of injection surface for electrons, $A_e = \pi R_e^2$
A_i	Area of injection surface for ions, $A_i = \pi R_i^2$
A_s	Area of injection surface for species s , $A_s = \pi R_s^2$
A_h	Heating accuracy
A_m	Motional accuracy
A_R	Richardson constant
A_1	First coefficient of characteristic equation
A_2	Second coefficient of characteristic equation
b	Normalised impact parameter of gyro-centre, $b = R_1/a_d$
b_c	Critical impact parameter
b_s	Second fitting coefficient, for species s
B	Magnetic field
B_{1-2}	Constants of Antoine equation for vapour pressure
c_v	Specific heat capacity at constant volume
C	Constant of normalisation for Boltzmann distribution, $C = (2\pi m k_B T)^{-\frac{3}{2}}$
C_{1-4}	Constants of semi-empirical formula for expansivity
Cov	Covariance
D	Normalised potential, $D = \sqrt{2\chi_a/\beta_s^2}$
D_{1-5}	Constants of semi-empirical formula for heat capacity
E	Energy of particles, $E = \underline{p}^2/2m$
E_0	Energy of incident particle
E_{rec}	Recombination energy
E_{see}	Heating energy carried by secondary electrons
E_b	Energy of dust atomic bond
$\underline{E}$	Electric field
f	Particle distribution function
f_s	Particle distribution function of species s
$\underline{F}_g$	Force due to gravity
$\underline{F}_i$	Force due to ion drag

$\underline{E}_{i,c}$	Force due to collection of ions
$\underline{E}_{i,d}$	Force due to deflection of ions
$\underline{E}_k$	Force due to source k
$\underline{E}_L$	Lorentz Force, $\underline{E}_L = Q_d(\underline{E} + \underline{v} \times \underline{B})$
$\underline{E}_n$	Force due to neutral drag
F_1	Functional form of flow dependent part of SOML flux
F_2	Functional form of flow independent part of SOML flux
g	A fitting function, $g(\beta_s) = e^{-a_s \beta_i^{b_s}}$
$\underline{g}$	Acceleration due to gravity
G	Functional integrand, $G = R_1 v_\perp^2 v_\parallel e^{-(v_\parallel^2 + v_\perp^2)/2v_{T,s}^2}$
G_s	Number of particles generated of species s
h	Normalised distance travelled parallel to $\underline{B}$ in one gyro-period, $h = v_\parallel \omega_s / a_d$
H	Heat energy
H_d	Thermal energy of dust atom, $H_d = 2k_B T_d$
i	Index of plasma grid cell coordinate in R direction
I_e	Electron current
I_i	Ion current
I_s	Current due to collection of species $s = i, e$
$\tilde{I}_s$	Normalised current, $\tilde{I}_s = I_s / I_{s,0}$ of species s
I_{int}	Net internal current, $I_{int} \sim en_0 v_{T,i} a_d^2$
I_k	Current with physical source k
I_{tee}	Thermionically emitted electron current
$I_{0,e}$	Electron thermal current, $I_{0,e} \equiv \pi a_d^2 Z n_e \langle v_{T,e} \rangle$
$I_{0,i}$	Ion thermal current, $I_{0,i} \equiv \pi a_d^2 Z n_i \langle v_{T,i} \rangle$
$I_{0,s}$	Thermal current, $I_{0,s} \equiv \pi a_d^2 Z n_s \langle v_{T,s} \rangle$, of species s
J_i	Ion current density
J_e	Electron current density
k	Index of plasma grid cell coordinate in z direction
K	Moment of inertia
Δl	Plasma grid cell width
Δl_r	Plasma grid cell width in R direction
Δl_z	Plasma grid cell width in z direction
L	Angular momentum
L_v	Latent heat of vapourisation
m	Mass of particle
m_i	Mass of ion

m_e	Mass of electron
m_p	Mass of hydrogen
m_s	Mass of species s
M_k	Moment k of the distribution function for integer k
M_d	Dust mass
M_a	Atomic mass
n	Number density of particles
n_d	Number density of dust
n_e	Number density of electrons
n_i	Number density of ions
n_s	Number density of species s
n_0	Plasma density
N	Number of particles
N_d	Number of atoms evaporated from dust grain surface
$\underline{p}$	Momentum, $\underline{p} = (p_x, p_y, p_z)$, of particle
$p_e^\pm$	Probability of injecting an electron at the upper (+) or lower (-) surface
$p_i^\pm$	Probability of injecting an ion at the upper (+) or lower (-) surface
P_e	Heating power due to collection of electrons
P_i	Heating power due to collection of ions
P_k	Power due to source k
P_{rad}	Heating power due to thermal radiation
P_{rec}	Recombination power, $P_{rec} \equiv E_{rec} I_i / e$
p_s^{KE}	Heating power due to thermalisation of species s
P_{see}	Heating power due to secondary electron emission
P_{sec}	Probability for a secondary particle to escape the surface of a material
P_t	Total power
P_{tee}	Heating power due to thermionic electron emission
P_V	Vapour pressure
Q	Charge of a body
$\tilde{Q}$	Normalised charge, $\tilde{Q}_d = Q_d / e$
$\langle \tilde{Q} \rangle$	Mean normalised charge
$\langle \tilde{Q} \rangle_n$	Mean normalised charge for N particles
Q_d	Dust charge
Q_{abs}	Absorption efficiency
$\underline{r}$	Position vector, $\underline{r} = (x, y, z) = (r, \theta, \phi) = (R, \theta, z)$, of a point in space
r_a	A radial distance greater than dust radius $r_a > a_d$

$ \underline{r} _{\min}$	Minimum value of $ \underline{r}(t) $ for all t
r_s	Position of the sheath edge
R	Cylindrical radial co-ordinate of guiding centre, $R^2 = x^2 + y^2$
R_m	Solution for inner boundary of magnetic envelope
R_M	Solution for outer boundary of magnetic envelope
R_s	Radius of circular injection plane $R_s = a_d + \kappa$
R_0	Final cylindrical radial co-ordinate of guiding centre of gyro orbit
R_1	Initial cylindrical radial co-ordinate of guiding centre of gyro orbit, $R_1^2 = x_1^2 + y_1^2$
s	Normalised gyro-radius of particle, $s = \rho_{\perp}/a_d$
S	Charge weighting
S_c	Scattering Parameter, $S_c(v) = Z e^2/mv^2\lambda$
S_p	Plank function
t	Time
$\tilde{t}$	Normalised equilibration time, $\tilde{t} \equiv \Upsilon_{i,0}t/K$
t_G	Artificial simulated time, $t_G = G_s/\sqrt{2\pi}R_s^2n_0v_{T,s}$, to generate G_s particles
t_0	Initial time
t^*	Bifurcation timescale, $t^* = 0.56\sqrt{8\iota\rho_d/a_d e^2 B^2 n_0^2 v_{T,e}^2}$
$\Delta t_{m,1}$	Timescale for large acceleration, $\Delta t_{m,1} \equiv 0.01\text{ms}^{-1}/ d\underline{v}_d/dt $
$\Delta t_{m,2}$	Timescale for crossing grid cell, $\Delta t_{m,2} \equiv \Delta l/2 \underline{v}_d $
$\Delta t_{m,3}$	Timescale for gyro-motion, $\Delta t_{m,3} \equiv M_d/e \underline{B} $
Δt_m	Timescale for equation of motion, $\Delta t_m = A_m \min(\Delta t_{m,1}, \Delta t_{m,2}, \Delta t_{m,3})$
Δt_h	Timescale for heating equation, $\Delta t_h = c_v M_d A_h/P_t$
T	Temperature of particles
T_a	Temperature of ambient environment
T_b	Boiling temperature
T_d	Dust temperature
T_e	Temperature of electrons
T_i	Temperature of ions
T_m	Melting temperature
T_s	Temperature of species s
$\underline{T}_s$	Stress tensor of species s
$T_{i,j}$	Component of the stress tensor, $\underline{T}$
$\underline{u}$	Normalised flow velocity, $\underline{u} = \underline{U}/v_{T,i}$ with respect to RMS velocity, $v_{T,i}$
$\underline{u}_e$	Normalised ion flow velocity, $\underline{u}_e = (\underline{U}_e - \underline{v}_d)/v_{T,e}$
$\underline{u}_i$	Normalised ion flow velocity, $\underline{u}_i = (\underline{U}_i - \underline{v}_d)/v_{T,i}$

u_0	Normalised speed, $u_0 = U_0/v_{T,i}$, at equipotential surface
$\underline{U}$	Flow velocity, $\underline{U} = (U_x, U_y, U_z)$, of plasma
$\underline{U}_i$	Ion flow velocity relative to dust
U_0	Speed at equipotential surface
$\underline{v}$	Velocity, $\underline{v} = (v_x, v_y, v_z) = (v_r, v_\theta, v_\psi) = (v_R, v_\theta, v_z)$, of particle
v_a	Velocity at dust surface at radial distance a_d
v_d	Velocity of dust
v_m	Minimum velocity, $v_m = \lambda_m \omega_s / 2\pi$
$v_{T,s}$	Root mean squared velocity, $v_{T,s} = \sqrt{k_B T_s / m_s}$, of species s
$\langle v_{T,e} \rangle$	Mean electron thermal velocity magnitude, $\langle v_{T,e} \rangle = \sqrt{8k_B T_e / \pi m_e}$
$\langle v_{T,i} \rangle$	Mean ion thermal velocity magnitude, $\langle v_{T,i} \rangle = \sqrt{8k_B T_i / \pi m_i}$
$\langle v_{T,s} \rangle$	Mean electron thermal velocity magnitude, $\langle v_{T,s} \rangle = \sqrt{8k_B T_s / \pi m_s}$
v_{se}	Velocity at sheath edge
$\tilde{u}$	Normalised flow velocity, $\tilde{u} = U / \sqrt{2k_B T_i / m_i}$ with respect to, $\sqrt{2}v_{T,i}$
$v_{\parallel}$	Velocity parallel to $\underline{B}$, $v_{\parallel} = v_z$
$v_{\perp}$	Velocity perpendicular to $\underline{B}$, $v_{\perp}^2 = v_x^2 + v_y^2$
v_{∞}	Spherical radial component of velocity at infinity
W_f	Work function
X	Size parameter, $X = 2\pi a_d / \lambda$
Y	Undetermined function of β_s , $Y(\beta_s)$
Y_{see}	Average number of secondary particles produced per incident particle
z_0	Fixed $x - y$ plane at height $z = z_0$
z_c	Critical height corresponding to critical phase angle
Z	Ion charge state
Z_s	Ion charge state of species s
α	Initial torque
$\tilde{\alpha}$	Normalised initial torque, $\tilde{\alpha} \equiv \alpha / \Upsilon_{0,i}$
β_e	Electron magnetisation parameter, $\beta_e \equiv a_d / \rho_{T,e}$
β_i	Ion magnetisation parameter, $\beta_i \equiv a_d / \rho_{T,i}$
β_s	Magnetisation parameter, $\beta_s \equiv a_d / \rho_{T,s}$, of species s
δ	Shape function for magnetic envelope
δ_{see}	Average secondary electron emission yield
ϵ	Material specific emissivity
ϵ_r	Relative permittivity
γ	Resistive torque rate
$\tilde{\gamma}$	Normalised equilibration parameter, $\tilde{\gamma} \equiv M_i n_i v_{T,i} K / a_d \rho_d \Upsilon$

γ_0	Simulation parameter determining the size of the time step
γ_c	Ratio of specific heats
$\underline{\Gamma}$	Flux of particles, $\underline{\Gamma} = (\Gamma_x, \Gamma_y, \Gamma_z)$
$\underline{\Gamma}_s$	Flux of particles, $\underline{\Gamma}_s = (\Gamma_{s,x}, \Gamma_{s,y}, \Gamma_{s,z})$, of species s
Γ_v	Evaporated particle flux
ι	Surface tension
κ	Simulation parameter determining the impact parameter $\kappa = b(\rho_{t,s} + \lambda_d)$
λ	Plasma screening length
λ^t	Material specific factor accounting for electron reflection and scattering
λ_d	Debye length, $\lambda_d^2 = \epsilon_0 \epsilon_r k_B T / e^2 n_0 Z_0$
$\tilde{\lambda}_d$	Normalised debye length, $\tilde{\lambda}_d = \lambda_d / a_d$
λ_c	Critical wavelength
λ_m	Minimum wavelength for multiple intersection, $\lambda_m = 2\pi v_{ } / \omega_s$
Λ_d	Dust parameter, $\Lambda_d \equiv 4\pi\lambda_d^3 n_d / 3$
Λ_{BC}	Coulomb logarithm, $\ln(\Lambda_{BC}) = \ln(1 + mv^2\lambda / Z e^2)$
$\ln(\Lambda_{ee})$	Electron-electron coulomb logarithm, $\ln(\Lambda_{ee}) = 8.75$
μ	Ion to electron mass ratio, $\mu \equiv m_i / m_e$
ν_e	Electron collision frequency, $\nu_e \simeq 5.8 \times 10^{-6} n_e \ln(\Lambda_{ee}) / T_e^{3/2}$
η	Electrical resistivity
ϕ	Electrostatic potential
ϕ_a	Potential of spherical surface at radial distance a_d
ϕ_s	Electrostatic potential, ϕ , of particle of species s
ψ_0	Initial phase angle
ψ_c	Critical phase angle for grazing incidence
ρ_d	Dust mass density
ρ_Q	Auto-correlation coefficient
$\rho_{T,e}$	Electron thermal gyro-radius, $\rho_{T,e} = mv_{T,e} / eB$
$\rho_{T,i}$	Ion thermal gyro-radius, $\rho_{T,i} = mv_{T,i} / eB$
$\rho_{T,s}$	Thermal gyro-radius, $\rho_{T,s} = mv_{T,s} / eB$, of species s
$\rho_{\perp}$	Gyro-radius of particle, $\rho_{\perp} = m_s v_{\perp} / eB$
σ_B	Stefan Boltzmann constant
σ_{OML}	OML cross section, $\sigma_{OML} = \pi b_c^2$
σ_s	Scattering cross section, $\sigma_s = 4\pi\lambda^2 S_c(v)^2 \ln(\Lambda_{BC})$
σ_{se}	Standard error
τ	Ion to electron temperature ratio, $\tau \equiv T_i / T_e$
τ_c	Charge relaxation timescale, $\tau_c = \epsilon_0 \eta$

Θ	Heaviside step function
Υ	Torque
Υ_s	Torque due to species s
$\Upsilon_{0,s}$	Thermal torque, $\Upsilon_{0,s} = n_s k_B T_s a_d^3$, due to species s
$\Upsilon_{\underline{j} \times \underline{B}}$	Torque, $\Upsilon_{\underline{j} \times \underline{B}} \sim \Upsilon_{0,i} \beta_i$, due to $\underline{j} \times \underline{B}$ force
ω_s	Gyro-frequency, $\omega_s = eB/m_s$ of species s
Ω_c	Critical angular velocity, $\Omega_c = 0.56 \sqrt{8\epsilon/\rho_d a_d^3}$
$\underline{\Omega}_d$	Dust angular velocity
Ω_{eq}	Equilibrium rotation rate, $\Omega_{eq} = \alpha/\gamma$
χ	Normalised potential, $\chi = -e\phi(\underline{r})/k_B T$
χ_a	Normalised potential of spherical dust surface, $\chi_a \equiv -e\phi_a/k_B T_e$
χ_s	Normalised potential at the sheath edge, $\chi_s = \chi(r = r_s)$
ξ	Normalised distance, $\xi \equiv x/\lambda_d$, with respect to debye length, λ_d
Ξ	Power flux
ζ	Red shift

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Author/Editor	Kungl. Svenska vetenskapsakademien.	Start Page	014070
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Thu 23/07/2020 12:36

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