Original Research Article

International journal of spray and combustion dynamics

International Journal of Spray and Combustion Dynamics
0(0) 1–14
© The Author(s) 2016
Reprints and permissions:
sagepub.co.uk/journalsPermissions.nav
DOI: 10.1177/1756827716651791
scd.sagepub.com

SAGE

Entropy noise: A review of theory, progress and challenges

Aimee S Morgans and Ignacio Duran

Abstract

Combustion noise comprises two components: direct combustion noise and indirect combustion noise. The latter is the lesser studied, with entropy noise believed to be its main component. Entropy noise is generated via a sequence involving diverse flow physics. It has enjoyed a resurgence of interest over recent years, because of its increasing importance to aero-engine exhaust noise and a recognition that it can affect gas turbine combustion instabilities. Entropy noise occurs when unsteady heat release rate generates temperature fluctuations (entropy waves), and these subsequently undergo acceleration. Five stages of flow physics have been identified as being important, these being (a) generation of entropy waves by unsteady heat release rate; (b) advection of entropy waves through the combustor; (c) acceleration of entropy waves through either a nozzle or blade row, to generate entropy noise; (d) passage of entropy noise through a succession of turbine blade rows to appear at the turbine exit; and (e) reflection of entropy noise back into the combustor, where it may further perturb the flame, influencing the combustor thermoacoustics. This article reviews the underlying theory, recent progress and outstanding challenges pertaining to each of these stages.

Keywords

Entropy noise, combustion noise, aeroengine noise, thermoacoustic instability

Date received: 10 August 2015; accepted: 13 January 2016

Introduction

Combustion noise comprises two components: direct combustion noise and indirect combustion noise. Gas turbine geometries have a particular propensity to indirect combustion noise; it is becoming increasingly relevant to both aero-engine exhaust noise and gas turbine NO_x emissions. Indirect combustion noise comprises entropy noise and vorticity noise, with entropy noise believed to be the major component for combusting flows.

Unsteady heat release rate, such as arises from unsteady fuel combustion, leads to unsteady volume expansion. This acts as an acoustic "monopole" source,[1–6] generating acoustic waves known as "direct combustion noise." Unsteady heat release rate also generates temperature variations, known as "hot spots" or entropy waves, which may eventually go on to generate entropy noise. Entropy waves advect with the local flow velocity – they are swept downstream with the flow. In a non-accelerating flow, it is known that linear flow fluctuations can be decomposed into acoustic, entropy and vorticity waves, which do not interact.[7] This means that the entropy waves are silent. However, when entropy waves are accelerated, regions of fluid with different densities undergo a volume contraction, as shown diagrammatically in Figure 1. This invokes a fluctuating force, which acts as an acoustic "dipole" – a second source of noise.[1,8] It is this second noise source, resulting from entropy wave acceleration, that is known as "entropy noise" or "indirect combustion noise."[8]

Note that the term "indirect combustion noise" refers to the noise generated by the acceleration of both entropy and vorticity waves.[9] The acceleration of vorticity waves also generates sound, which may be relevant for azimuthal modes.[10] In combusting flows,

Department of Aeronautics, Imperial College, London, UK

Corresponding author:
Aimee S Morgans, Department of Aeronautics, Imperial College, London, UK.
Email: a.morgans@imperial.ac.uk

Creative Commons CC-BY-NC: This article is distributed under the terms of the Creative Commons Attribution-NonCommercial 3.0 License (http://www.creativecommons.org/licenses/by-nc/3.0/) which permits non-commercial use, reproduction and distribution of the work without further permission provided the original work is attributed as specified on the SAGE and Open Access pages (https://us.sagepub.com/en-us/nam/open-access-at-sage).

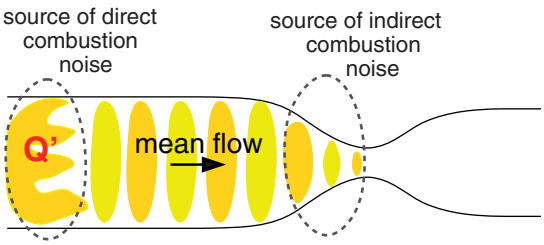

Figure 1. The two sources of combustion noise for a duct combustor with nozzle exit.

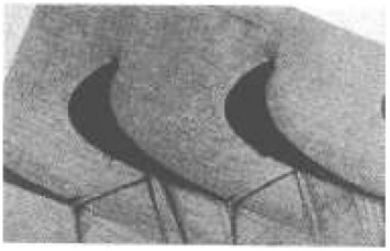

Figure 2. Stator exit shock waves.
Source: reproduced with permission from American Society of Mechanical Engineers.[11]

the former is likely to be more significant for two reasons: first, unsteady combustion generates large temperature fluctuations, and therefore entropy waves, and second, viscosity increases dramatically at large temperatures, dissipating large turbulent scales downstream from the flame. For this reason, the terms "indirect combustion noise" and "entropy noise" have sometimes been used interchangeably. Here, we use the term "entropy noise" for clarity, although the flow physics by which vorticity noise is generated is very similar.

A flow acceleration for converting entropy fluctuations into acoustic fluctuations may be provided by a nozzle (see Figure 1) or a turbine blade row (see Figure 2). Simple experiments on entropy noise tend to concentrate on the former, while gas turbine geometries exhibit the latter. The flow Mach number in a gas turbine combustor tends to be roughly 0.1–0.2 downstream of the flame; low enough to keep the flame attached to the burner. As the flow passes from the combustor through the first turbine stator row (the nozzle guide vane (NGV)), the flow Mach number increases rapidly, often to a condition very close to choked or Mach 1, as shown in Figure 2.[11] This rapid increase in Mach number, and hence strong acceleration, explains why gas turbine geometries have a strong propensity to entropy noise. Subsequent turbine blade rows also exhibit flow acceleration, with the potential for further generation of entropy noise.

When an entropy wave undergoes acceleration, one component of the generated entropy noise propagates back upstream toward the flame, while the other propagates downstream, away from the flame. The upstream propagating component will reach the flame, further contributing to the feedback loop that can lead to thermoacoustic instability. This will be further discussed in the section "Reflected component of entropy noise and its effect on combustion instability". For a gas turbine containing a series of turbine blade rows, both the downstream propagating acoustic wave and the entropy wave (which, for a one-dimensional flow, passes through the original blade row unaltered) go on to cause subsequent generation of acoustic waves as they are subject to further blade row accelerations.[12] Eventually, entropy noise will propagate from the turbine exit, contributing to exhaust noise in the case of aeroengines.

After much of the early work of the 1970's,[3,8,12–14] entropy noise has seen a resurgence in research interest over the last decade. This is for two primary reasons:

- Combustion noise is becoming an increasingly important component of aero-engine exhaust noise.[6,15] The downstream propagating component of combustion noise is emerging as an audible aero-engine noise source, particularly at take-off, largely due to reductions in jet noise.[16] The Advisory Council for Aeronautics Research in Europe (ACARE) is committed to reducing aircraft noise by 65% by 2050 compared with 2000 levels.[17] Such significant reductions require a range of noise sources to be reduced, including fan noise, airframe noise and combustion noise – they cannot be achieved by targeting jet noise alone.[16]
- Entropy noise affects the thermoacoustic stability of the combustor. This is true for both aero-engine and land-based power gas turbines. Achieving low NO_x combustion is important for air quality near power stations and airports – this is exemplified by the ACARE Flightpath 2050 target of reducing aviation NO_x emissions by 90% by 2050, compared with 2000 levels.[17] However, low NO_x requires lean premixed combustion, and this is highly susceptible to thermoacoustic instabilities, also known as combustion instabilities.[18–20] These arise due to a two-way coupling between the acoustic waves and unsteady heat release rate in the combustor, and are typically associated with destructive levels of self-excited oscillations. The presence of the reflected component of entropy noise affects this thermoacoustic behaviour.

The mechanism of entropy noise generation and the relevance of the downstream and upstream travelling components is summarised schematically in Figures 3 and 4.

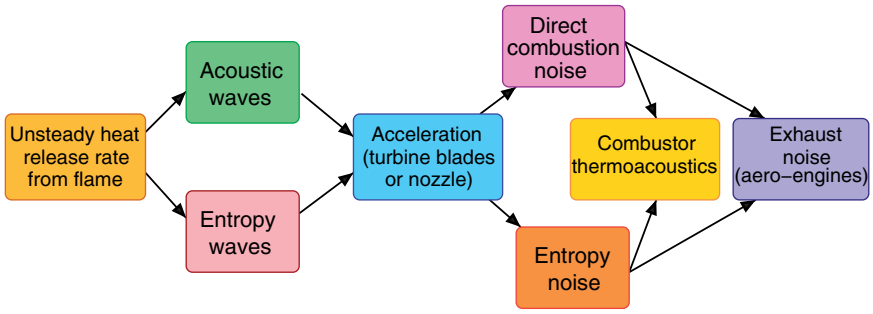

Figure 3. The two combustion noise components and the flow mechanisms by which they affect gas turbines.

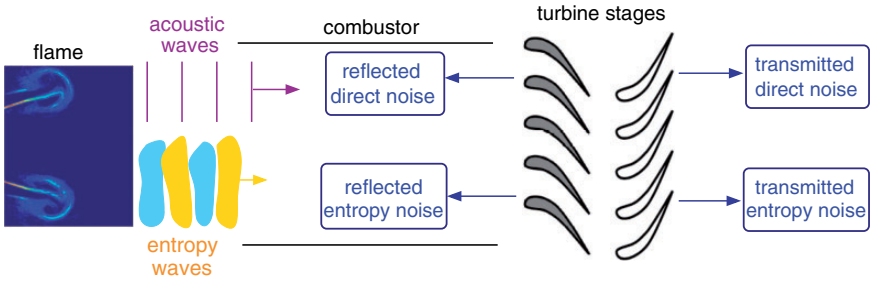

Figure 4. Schematic of flow perturbations in a gas turbine combustor.

Entropy waves are essentially temperature fluctuations, as will be shown in the Section "Generation of entropy waves at the flame". This means that attempts to experimentally characterise entropy noise via measurement of entropy waves are frustrated by the well-known problem that measuring temperature fluctuations at high frequencies is extremely challenging. Unsteady temperature measurements have been performed using either small fast-response thermocouples[21–24] or laser vibrometers.[23] The former technique is limited to frequencies below ~100 Hz (the cutoff frequency decreases with thermocouple wire thickness), and less harsh environments, while the latter tends to suffer contamination from rig vibration. Optical laser-based techniques are often the only viable option at moderate frequencies, severely hindering experimental progress. A further challenge is that of distinguishing between entropy noise and direct combustion noise, as unsteady heat release will always generate both.

In the context of thermoacoustic instabilities, an energy norm (the simplest being the well-known Rayleigh criterion[25]) is sometimes used as a marker for instability. The contribution of entropy waves to the energy norm has been considered by Cantrell and Hart,[26] Morfey[27] and more recently Nicoud and Poinsot.[28] However, the focus within this review is entropy noise; that is, the acoustic waves generated by downstream acceleration of entropy waves.

In this article, we review theory, recent contributions and outstanding challenges relating to entropy noise. The structure of the article will follow the chronological order of events implied in the schematic of Figure 4. That is, the first section will focus on the generation of entropy waves at the flame, the second on the advection of entropy waves through the combustor, the third on the acceleration of entropy waves to generate entropy noise, the fourth on the passage of waves through the turbine and the fifth on the reflection of entropy noise back into the combustor, where it can affect combustion instability.

Note that several recent reviews on combustion noise[6,15] span both: "direct combustion noise" and "entropy noise," generally with more detail on the former. As the phenomenon of entropy noise invokes a unique combination of flow physics, this review focusses specifically on elucidating this, bringing clarity to where progress has been made and where outstanding challenges remain.

Generation of entropy waves at the flame

The equation describing the conservation of energy in a compressible flow is;[29]

$$\rho\frac{D}{Dt}\left(e+\frac{1}{2}\mathbf{u}^2\right)=-\nabla\cdot(p\mathbf{u})+q+\nabla\cdot(k\nabla T)\cdots+\frac{\partial}{\partial x_j}\left(\tau_{ij}u_i\right) \quad (1)$$

where ρ, e, $\mathbf{u}$, p, q, k, T and τ_{ij} are the fluid density, internal energy per unit mass, velocity, pressure, heat release rate, thermal conductivity, temperature and viscous stress tensor, respectively.

Combining with the thermodynamic relation, $T\mathrm{d}s = \mathrm{d}h - \mathrm{d}p/\rho$, where s and h are entropy and enthalpy, respectively, the entropy in the flow is governed by the following equation:[29,30]

$$\rho T\frac{Ds}{Dt} = q + \nabla\cdot(k\nabla T) + \tau_{ij}\frac{\partial u_i}{\partial x_j} \tag{2}$$

Ds/Dt represents entropy advection, which will be discussed in the following section. The terms on the right-hand side act as entropy sources or sinks. They represent the effects of heat addition (such as that through chemical reactions, external heating, etc.), thermal gradients, and frictional heating. So, for example, heat addition constitutes an entropy source whilst heat removal reduces entropy.

From the preceding definition of entropy, it is possible to write $s = c_v\ln(p/\rho^\gamma)$. For small flow perturbations, fluid quantities can be decomposed into the sum of a mean $\overline{()}$ and fluctuating $()'$ component. Applying this to the definition of entropy, we find that $s' = c_v p'/\bar{p} - c_p\rho'/\bar{\rho}$, which when combined with the ideal gas relation gives

$$\frac{s'}{c_p} = \frac{T'}{\bar{T}} - \left(\frac{\gamma-1}{\gamma}\right)\frac{p'}{\bar{p}} \tag{3}$$

In or downstream of a combusting flow, the normalised temperature fluctuations will be much larger than the normalised pressure fluctuations, hence $s'/c_p \simeq T'/\bar{T}$, and entropy and temperature fluctuations are seen to be linearly related.

Returning to equation (2), which governs the generation and transportation of entropy, by linearising and assuming constant k we obtain,

$$\begin{aligned}\frac{\bar{D}s'}{Dt} = \frac{R}{\bar{p}}\Bigg[&q' + \nabla\cdot(k\nabla T') + \bar{\tau}_{ij}\frac{\partial u_i'}{\partial x_j} + \tau_{ij}'\frac{\partial \bar{u}_i}{\partial x_j}\cdots\\ &-\left(\frac{p'}{\bar{p}} + \frac{u_i'}{\bar{u}_i}\right)(\bar{q} + \nabla\cdot(k\nabla\bar{T}) + \bar{\tau}_{ij}\frac{\partial\bar{u}_i}{\partial x_j}\Bigg]\end{aligned} \tag{4}$$

The assumption of a non-diffusive flow is often made, in which viscous effects and temperature gradients are neglected. These are strong assumptions, whose validity in the context of combustion, with large temperature gradients and intensified viscosity, is uncertain. At low frequencies, the acoustic waves that propagate in a constant area duct will be plane and one-dimensional.[29,31] Assuming plane wave flow perturbations and non-diffusive flow throughout, equation (4) simplifies to the expression derived by Dowling et al.,[32]

$$\frac{\bar{D}s'}{Dt} = \frac{R\bar{q}}{\bar{p}}\left(\frac{q'}{\bar{q}} - \frac{p'}{\bar{p}} - \frac{u'}{\bar{u}}\right) \tag{5}$$

Note that generation of entropy waves arises from an unsteady heat release rate, irrespective of whether this is generated by premixed flame or non-premixed flame mechanisms. Karimi et al.[33] showed that the form of equation (5) acts as a low pass filter. An effective ''corner frequency'' exists, below which significant entropy generation occurs, and above which entropy generation diminishes and the main effect of unsteady heat release rate is to generate sound. The value of this corner frequency can vary widely.

In most practical applications, the flame can be assumed acoustically compact since the dimensionless frequency verifies $\Omega = fL_f/c \ll 1$, where L_f is the characteristic length of the flame, f the acoustic frequency and c the mean speed of sound. For linear, plane flow disturbances passing through an acoustically compact flame, as in Figure 4, Dowling[34] showed that the generated entropy wave is given by equation (6), where the subscripts c and h denote (cold) upstream and (hot) downstream of the flame, respectively, and $[]_c^h$ denotes the difference between the upstream and downstream variables:

$$\begin{aligned}s' = \frac{c_p(\gamma-1)}{\bar{\rho}_h\bar{u}_h\bar{c}_h^2}\Bigg(&Q' - \left[p'\bar{u} + \bar{\rho}\bar{u}^2u'\right]_c^h\cdots\\ &-\left[c_p\bar{T} + \frac{\bar{u}^2}{2}\right]_c^h\left(\bar{\rho}_c u_c' + \rho_c'\bar{u}_c\right)\Bigg)\end{aligned} \tag{6}$$

For low Mach number mean flows (and noting that the product $\bar{u}_h s'$ remains finite, even when $\bar{u}_h \to 0$), this simplifies to an expression depending only on perturbations of the unsteady heat release rate, Q' and the upstream velocity, u_c'.[34]

$$s' = \frac{c_p(\gamma-1)}{\bar{\rho}_h\bar{u}_h\bar{c}_h^2}\left(Q' - c_p\left(\bar{T}_h - \bar{T}_c\right)\bar{\rho}_c u_c'\right) \tag{7}$$

Equation (7) has been used to study the effect of the generated entropy on combustor thermoacoustics, both in analytical models[34,29] and in Helmholtz solvers.[35] Note that for Helmholtz solvers, which assume strictly zero mean flow, some interesting implications of the generated entropy wave being ''frozen'' in the flame region are discussed by Bauerheim et al.[36]

The expressions for the generated entropy wave have been written out explicitly above. It is also possible to deduce the entropy wave strength implicitly via a matrix method, as often occurs within low order

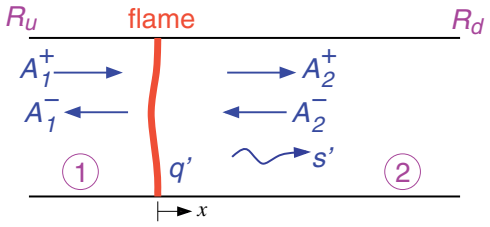

Figure 5. The 1-D wave strengths in a ducted flame combustor with mean flow from left to right. Flow parameters upstream and downstream of the flame are denoted by subscript c (cold) and h (hot), respectively. The upstream and downstream boundaries are characterised using pressure reflection coefficients R_u and R_d, respectively, as well as a downstream entropy reflection coefficient R_s.

network methods for combustor thermoacoustic stability analysis.[37,38] This can be illustrated through the example of the combustor in Figure 5. To derive these network models, the one-dimensional linearised Euler equations (LEE) of a homogeneous mean flow are recast as

$$\left(\frac{\partial}{\partial t}+\bar{u}\frac{\partial}{\partial x}\right)^2 p' - \bar{c}^2\frac{\partial^2 p'}{\partial x^2} = 0 \tag{8}$$

$$\left(\frac{\partial}{\partial t}+\bar{u}\frac{\partial}{\partial x}\right)u' = \frac{-1}{\bar{\rho}}\frac{\partial p'}{\partial x} \tag{9}$$

$$\left(\frac{\partial}{\partial t}+\bar{u}\frac{\partial}{\partial x}\right)s' = 0 \tag{10}$$

where x is the axial coordinate of the one-dimensional system. This decomposition gives two acoustic waves A^+ and A^-, propagating at speeds $u+c$ and $u-c$, respectively, and an entropy wave s' advecting at speed u, which is decoupled from the two acoustic waves.[29,37–39] Assuming there are no entropy waves upstream of the flame, there are then six unknowns – five wave strengths (the two acoustic waves either side of the flame, $A_c^+, A_c^-, A_h^+, A_h^-$, and the entropy wave downstream of the flame, s') and the unsteady heat release rate q'.

The pressure, density and velocity fluctuations either side of the flame can be written in terms of the wave strengths, with the entropy wave strength appearing only in the downstream density fluctuation:

$$\rho_h' = \frac{1}{\bar{c}_h^2}\left(A_h^+\left(t-\frac{x}{\bar{c}_h+\bar{u}_h}\right)+A_h^+\left(t+\frac{x}{\bar{c}_h-\bar{u}_h}\right)\right)\cdots - \frac{\bar{\rho}_h}{c_p}s'\left(t-\frac{x}{\bar{u}_h}\right) \tag{11}$$

This equation captures both the definition of entropy, and the fact that downstream of combustion entropy fluctuations advect with the flow.

Three equations are provided by the quasi-steady relations for the conservation of mass, momentum and energy across the flame,[40] two by the boundary conditions at either end of the combustor, and one by a "flame model" prescribing how q' depends on the upstream wave strengths.[41] There is then sufficient information to deduce all wave strengths, including the entropy wave strength, in response to input forcing of any other variable.[38]

Advection of entropy waves through the combustor

Between the flame, where they are generated, and the turbine inlet, where they are accelerated, entropy waves are subject to advection through the combustor flowfield. The transportation of entropy waves is governed by equation (2), where $q=0$ downstream of the flame. For acoustic waves, it is a good approximation that at low frequencies and in longitudinal combustors, only plane waves propagate (with plane and circumferential acoustic waves propagating in thin annular combustors).[29,31] However, the assumption of low-dimensionality for entropy waves is dubious; the mean velocity field varies spatially due to the viscous effects – boundary layers, wakes, etc. There is also the potential for large turbulent velocity fluctuations.

Despite this, the vast majority of entropy noise work has either assumed that (a) the advection process is 1-D, so that entropy waves reach the turbine inlet unaltered from the form in which they were generated at the flame, other than for an advective time delay or (b) that the turbulent flow field disperses or dissipates entropy waves sufficiently that their strength at combustor exit can be neglected. That is, models generally either assume that entropy waves are unaffected by the turbulent advection process, or that they fail to reach the combustor exit at all. This is despite the fact that simulations of full combustion chambers suggest that significant entropy waves may survive until combustor exit.[42]

Sattelmayer[43] was the first to propose an analytical model for the dispersion of scalar perturbations such as equivalence ratio and entropy fluctuations. The probability density function (PDF) of the variation in "residence time" or "delay time" between the flame and a cross-section downstream of the flame was shown to be the equivalent to the response at this location to an input impulse, $\delta(t)$, at the flame. For an experimental combustor, with the temperature fluctuations characterised using thermocouples and the velocity profile with Particle Image Velocimetry (PIV), this PDF was modelled as a rectangle with width $2\Delta\tau$ centred about the mean residence time τ_s, with height such that the integral remained unity. Both Sattelmayer[43] and Eckstein et al.[22] concluded that entropy waves were

likely to disperse before reaching the end of the combustor. Goh and Morgans[39] extended this modelling approach to include a dissipation factor, $k<1$, where k represents the integral of the impulse response.

To better understand the effect of advection within a turbulent flow on the entropy wave strength, Morgans et al.[30] studied the advection of a passive scalar in turbulent channel flow simulations using direct numerical simulation. Assuming transport in a constant area channel downstream of any heat sources, so that $q=0$, and for constant k and viscosity μ, equation (4) can be simplified, in non-dimensional form $\breve{(\,)}$, to

$$\breve{T}\left(\frac{\partial \breve{s}}{\partial \breve{t}}+\breve{u}_i\frac{\partial \breve{s}}{\partial \breve{x}_i}\right) = \frac{1}{\mathrm{Re}\,\mathrm{Pr}}\frac{\partial^2 \breve{T}}{\partial \breve{x}_i \partial \breve{x}_i}\cdots+\frac{(\gamma-1)M_{\mathrm{bulk}}^2}{\mathrm{Re}}\breve{\tilde{\tau}}_{ij}\frac{\partial \breve{u}_i}{\partial \breve{x}_j} \tag{12}$$

$\mathrm{Re}=\rho UL/\mu$ is the Reynolds number, $\mathrm{Pr}=\mathrm{c_p}\mu/k$ is the Prandtl number, $\tilde{\tau}_{ij}=\tau_{ij}/\mu$ and $\breve{\tilde{\tau}}_{ij}=\tilde{\tau}_{ij}/(U/L)$.

They defined any loss in entropy wave strength caused by spatial variations in the advective operator on the left-hand side as shear-dispersion, and any loss in strength due to (negative) contributions from the right-hand side as dissipation. They found that entropy waves are not dissipated by the turbulent flow, only undergoing shear-dispersion due to spatial variations in the mean velocity. Turbulent fluctuations have negligible effect. The impulse response characterising the entropy wave strength at combustor exit in response to an impulse, $\delta(t)$, at the flame, was found to be better approximated by a Gaussian profile than a rectangle, as shown in Figure 6 and equation (13), where A_δ is the height of the Gaussian and $\Delta\tau_s=1/(\sqrt{\pi}A_\delta)$.

$$\frac{s'_{\mathrm{exit}}(t)}{c_p}=\sigma_{\mathrm{exit}}(t)=A_\delta e^{-\pi A_\delta^2(t-\tau_s)^2}=\frac{1}{\sqrt{\pi}\Delta\tau_s}e^{-\left(\frac{t-\tau_s}{\Delta\tau_s}\right)^2} \tag{13}$$

This is convenient as it also yields a Gaussian form for the flame-to-turbine frequency response. The features of this Gaussian can be deduced directly from the mean turbulent velocity profile. Morgans et al.[30] concluded that, for typical gas turbine geometries, entropy wave dispersion would be weak enough for significant entropy wave strength to remain at combustor exit. This Gaussian model has been embedded in at least one low-order network tool for thermoacoustic instability modeling.[38] As yet, the effect of more complex swirl-stabilised combustor flow features, such as vortex cores and recirculation zones, on the advection process, has not been studied. Because of the importance of the advection process on the eventual noise generated, this is an area likely to see further work in the future.

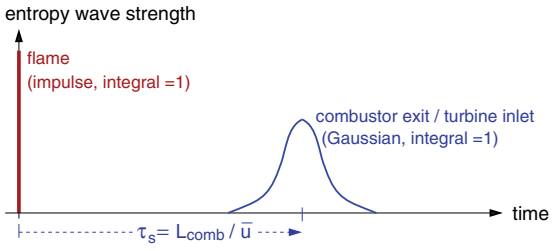

Figure 6. The effect of advection within a turbulent flow on the entropy wave strength.

Acceleration of entropy waves to generate entropy noise

The phenomenon of sound generation due to flow inhomogeneities was qualitatively described by Lord Rayleigh in chapter XV of his 1896 book. It was almost 80 years later that Morfey[44] established a mathematical framework for this. The concept of "excess density" introduced by Lighthill[45] – the density fluctuation component not associated with acoustic waves was used as a basis for explaining how the acceleration of non-uniform density perturbations generated sound.

$$\rho_e=\rho'-\frac{p'}{\bar{c}^2} \tag{14}$$

These entropy inhomogeneities were incorporated into Lighthill's acoustic analogy,[46] and the entropy noise generated in low Mach number flows predicted using Green's function approaches.[47,48] More recent work[49] has combined the Green's function approach with rapid distortion theory and numerical simulations under low Mach number conditions.

To address entropy wave acceleration to higher Mach numbers, the complications of the Green's function technique were avoided by performing one-dimensional analyses of the LEE for "compact nozzles." The now seminal work of Marble and Candel[8] assumed that the perturbation wavelengths were much longer than the spatial extent of the nozzle area change (i.e. the nozzle length). Linear, low frequency perturbations are then effectively accelerated through an abrupt area change in a quasi-steady manner, as shown in Figure 7.

The quasi-steady form of the LEE allows the normalised mass flow rate fluctuation, $\dot{m}'/\bar{\dot{m}}$, stagnation temperature fluctuation, $T_t'/\overline{T_t}$, and entropy fluctuation, s'/c_p, to be matched across the area change, i.e. between the nozzle inlet (subscript 1) and outlet (subscript 2):

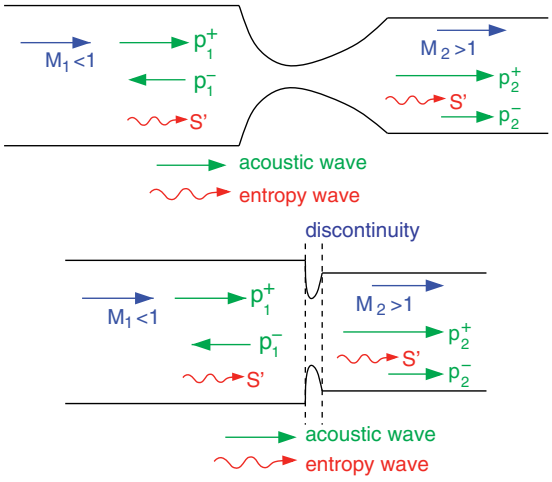

Figure 7. The waves associated with flow acceleration through a supercritical nozzle: true geometry (left) and compact assumption (right).

$$\left(\frac{\dot{m}'}{\bar{\dot{m}}}\right)_1 = \left(\frac{\dot{m}'}{\bar{\dot{m}}}\right)_2$$
$$\left(\frac{T_t'}{\overline{T_t}}\right)_1 = \left(\frac{T_t'}{\overline{T_t}}\right)_2 \qquad (15)$$
$$\left(\frac{s'}{c_p}\right)_1 = \left(\frac{s'}{c_p}\right)_2$$

For a choked nozzle, these matching conditions collapse to a boundary condition that can be applied separately to the upstream and downstream flows. For both choked ("supercritical")[8] and subsonic ("subcritical")[8,13] nozzles, reflection and transmission coefficients for the (plane) acoustic waves generated when either plane acoustic or entropy waves are incident on the nozzle are provided, in terms of the mean flow Mach numbers either side of the nozzle. It was this work that first introduced the concepts of "direct" and "indirect" combustion noise.

The Marble and Candel[8] work mainly applied to linear perturbations in the zero-frequency limit. Leyko et al.[50] compared direct noise and entropy noise for a simple nozzle acceleration, analytically and using Euler simulations. They found that the compact assumption holds up to acoustically non-dimensional frequencies of approximately 0.2, and that for typical large nozzle accelerations (high subsonic, $M > 0.7$, or supersonic Mach numbers), entropy noise will substantially exceed the direct noise contribution.

Much recent work has focussed on relaxing the assumptions made by Marble and Candel. For example, Huet and Giauque[51] extended the analysis to non-linear perturbations in the compact limit, while several studies have provided motivation for relaxing the low-frequency, compact assumption.[50,52] Marble and Candel[8] themselves considered a linear mean velocity profile in a one-dimensional nozzle (requiring a specific nozzle geometry) and solved this using a hypergeometric solution. This approach was later extended by Moase et al.[53] and Giauque et al.[54] to piecewise-linear mean velocity profiles, with which any nozzle geometry could be discretised using linear mean velocity segments.

An alternative approach by Stow et al.[55] extended the Marble and Candel method to higher frequencies for general nozzle geometries, subject to the nozzle being choked. An asymptotic expansion in non-dimensional frequency was applied to the LEE for a choked nozzle. This yielded an equivalent nozzle length, allowing a first-order correction to the phase for the reflection coefficient of choked nozzles. The method was implemented in a thin annular configuration, allowing for the prediction of reflected entropy noise in both plane and circumferential modes. The "equivalent nozzle length" method was further developed by Goh and Morgans,[56] who obtained the phase correction of the transmission coefficient of a choked nozzle (with and without a shock wave) for both acoustic and entropy waves in the case of 1-D plane waves.

A powerful approach which can be used to predict the entropy noise generated at any frequency, and for both choked and subcritical nozzles, was recently developed by Duran and Moreau.[57] The starting point is the full set of LEE, in one spatial dimension.

$$\left[\frac{\partial}{\partial t} + \bar{u}\frac{\partial}{\partial x}\right]\left(\frac{p'}{\gamma\bar{p}}\right) + \bar{u}\frac{\partial}{\partial x}\left(\frac{u'}{\bar{u}}\right) = 0 \qquad (16)$$

$$\left[\frac{\partial}{\partial t} + \bar{u}\frac{\partial}{\partial x}\right]\left(\frac{u'}{\bar{u}}\right) + \frac{\bar{c}^2}{\bar{u}}\frac{\partial}{\partial x}\left(\frac{p'}{\gamma\bar{p}}\right)\cdots + \left[2\frac{u'}{\bar{u}} - (\gamma-1)\frac{p'}{\gamma\bar{p}} + \frac{s'}{c_p}\right]\frac{d\bar{u}}{dx} = 0 \qquad (17)$$

$$\left[\frac{\partial}{\partial t} + \bar{u}\frac{\partial}{\partial x}\right]\left(\frac{s'}{c_p}\right) = 0 \qquad (18)$$

Note that the entropy term appears as a "source" in the momentum equation (equation (17)) when there is a velocity gradient. A change of variables is applied, such that the variables which are conserved in the compact, zero-frequency limit, $\dot{m}'/\bar{\dot{m}}$, $T_t'/\overline{T_t}$ and s'/c_p (see equations (15)), replace pressure, $p'/(\gamma\bar{p})$, velocity, $u'/\bar{u}$ and entropy, s'/c_p, in equations (16) to (18). The LEE can then be expressed in matrix form as,

$$\frac{d[I]}{d\left(\frac{x}{L}\right)} = \Omega\,\hat{A}\left(\frac{x}{L}\right)[I]$$

$$[I] = \left[\frac{\dot{m}'}{\bar{\dot{m}}}, \frac{T_t'}{\bar{T}_t}, \frac{s'}{c_p}\right]^T \tag{19}$$

where Ω is non-dimensional frequency, x is distance along the nozzle, L is the nozzle length and $\hat{A}$ is a 3×3 matrix containing mean flow variables, which depend on the mean axial coordinate only. Note that in the compact limit ($\Omega = 0$) equation (19) is equivalent to the jump conditions of equation (15).

Duran and Moreau showed that equation (19) can be solved using Magnus expansions.[57] Thus, the transfer functions, including the entropy noise transfer functions, of any quasi one-dimensional nozzle geometry up to any frequency, for both supercritical and subcritical nozzles, can be predicted. The effectiveness of this approach for predicting the real and imaginary parts of the nozzle admittance (i.e. the inverse of the impedance, $Y = 1/Z = \bar{\rho}\bar{c}(u'/p')_0$), for three different nozzle geometries is shown in Figure 8. The nozzle geometries are described by Duran and Moreau[57] and Zinn et al.;[52] they have the same inlet Mach number and a choked outlet, but with different convergent lengths. Although this comparison shows the effect of acoustic waves rather than entropy waves incident on the nozzle, the correct frequency dependence is clearly captured. Note that the compact assumption would predict only the zero frequency point. This approach has recently been extended to circumferential modes in annular combustors, where it correctly predicted that accelerating a circumferential entropy wave generates a vorticity wave, as well as acoustic waves.[58]

Experimental measurements of entropy noise and direct combustion noise for a nozzle acceleration are important to validate the models and prediction tools. However, there have been very few experimental studies, due to the challenges posed by fast temperature measurements. The Entropy Wave Generator (EWG) rig, developed by the German Aerospace Centre (DLR) and shown in Figure 9,[23] focussed specifically on the generation of noise by the acceleration of entropy waves, while avoiding the harsh environments created by combustion. It used an electrical heating device to generate pulsed heat release rate fluctuations, with the resulting temperature pulse measured using a laser vibrometer and "fast" thermocouple. The heated flow then passed through a convergent-divergent nozzle, providing flow acceleration. Both subsonic and supersonic flows with a shock wave could be achieved. The difference in time between the acoustic wave propagation and the entropy wave advection could be used to provide evidence of entropy noise.

When operating the EWG under supersonic nozzle conditions, measurements agreed with analytical[8] and numerical[59] predictions, confirming that entropy noise dominates over direct combustion noise. Under subsonic conditions, analytical studies by Howe[60] and numerical studies[54,61] suggest that the direct noise mechanism, combined with imperfect downstream reflections, is the main source of the measured noise, and that entropy noise is less significant.

As yet, models for comparing direct combustion noise and entropy noise have not fully combined state-of-the-art models for entropy wave generation and advection with those for entropy wave acceleration. It would be interesting to see how the combustion noise sources compare when these models are combined.

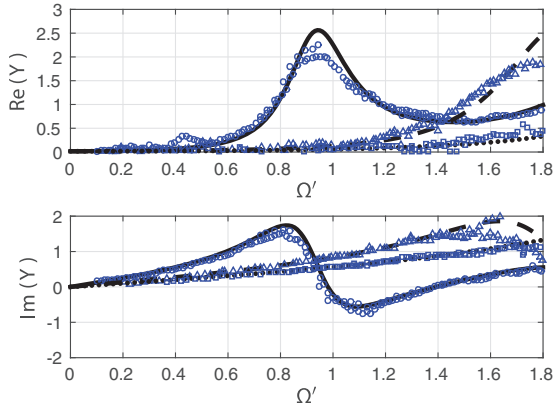

Figure 8. The acoustic admittance, Y of three different nozzle geometries, as measured experimentally[52] (○, Δ and □) and predicted[57] (_____, – – – and · · ·). Ω' is a non-dimensional frequency.
Source: reproduced with permission.[57]

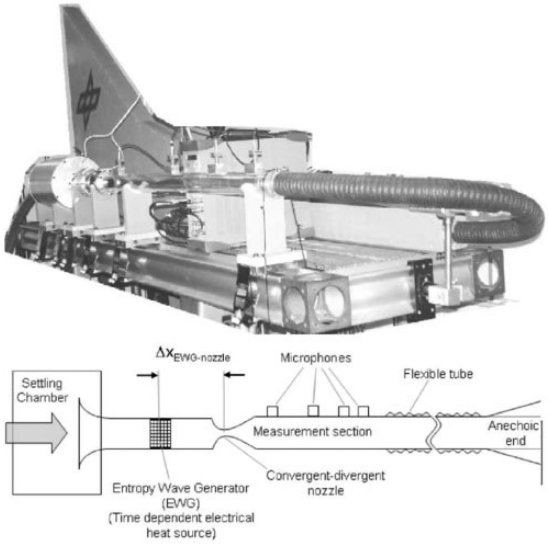

Figure 9. The Entropy Wave Generator experimental facility.
Source: reproduced with permission from Elsevier.[23]

Passage of entropy noise through turbine

In a real gas turbine geometry, entropy waves are accelerated not through a nozzle, but through a series of blade rows. The mean flow turns as it passes through each blade row, meaning that there are at least two velocity and momentum components as well as a fluctuating blade row force.

Cumpsty and Marble[12] proposed a method for solving the blade row problem using an actuator disk method, which built on the compact approach of Marble and Candel.[8] This considered flow fluctuations to be plane both upstream and downstream of the row (although with an angle change across the row), undergoing a discontinuous jump in strength across the row, as shown in Figure 10.

For a stator row, the mass flow rate, stagnation enthalpy (temperature) and entropy are matched between the blade row inlet and outlet. The fourth condition comes from either applying the Kutta condition at the blade row exit for subsonic flow or applying choked condition for supersonic flow. The Kutta condition fixes the flow deflection angle at the blade row exit to match that of the blade trailing edges, and is usually relaxed using a small parameter β. The system of equations then reads,

$$\begin{aligned}
\left(\frac{\dot{m}'}{\dot{m}}\right)_1 &= \left(\frac{\dot{m}'}{\dot{m}}\right)_2 \\
\left(\frac{T_t'}{T_t}\right)_1 &= \left(\frac{T_t'}{T_t}\right)_2 \\
\left(\frac{s'}{c_p}\right)_1 &= \left(\frac{s'}{c_p}\right)_2 \\
\theta_2' &= \beta\theta_1'
\end{aligned} \tag{20}$$

The procedure is identical for a rotor row, except that rotating frame quantities such as rothalpy are used instead. A vector of wave strengths can be defined as

$$\mathbf{v} = \begin{bmatrix} \text{upstream-propagating acoustic wave strength} \\ \text{downstream-propagating acoustic wave strength} \\ \text{entropy wave strength} \\ z\text{-vorticity wave strength} \end{bmatrix}, \tag{21}$$

where the z-direction is out of the page. For subsonic flow, the equations relating the wave strengths either side of the first stage in Figure 11, between stations (1) and (2), can be deduced from equation (20) and written in matrix form as $M_1\mathbf{v}_1 = M_2\mathbf{v}_2$, where M_1 and M_2 are 4×4 invertible matrices, given in Cumpsty and Marble.[12,14]

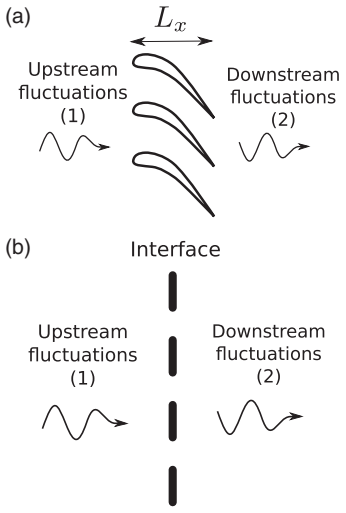

Figure 10. The waves associated with flow acceleration through a blade row: true geometry (left) and compact assumption (right).

In a similar manner, by assuming compact relations across blade rows and non-compact transport between rows, the interactions across and between all of the blade rows in Figure 11 can be written as

$$M_1\mathbf{v}_1 = M_2\mathbf{v}_2, \quad M_3\mathbf{v}_3 = M_4\mathbf{v}_4, \quad \mathbf{v}_2 = T\mathbf{v}_3 \tag{22}$$

where the wave strengths between blade rows are related by a 4×4 matrix T, which follows from wave propagation/advection. The combined effect of several blade rows is captured via manipulation of the matrices. For example, equations (22) can be combined to give a relation between the wave strengths entering and leaving the system: $\mathbf{v}_4 = M_4^{-1}M_3(M_2T)^{-1}M_1\mathbf{v}_1$. Typically, the inlet entropy, z-vorticity and downstream-travelling acoustic waves along with the exit upstream-travelling acoustic wave are known. The upstream-propagating acoustic wave at the inlet, together with the exit entropy, vorticity and downstream-travelling acoustic waves must be deduced. The procedure used to solve the system of equations involves arranging the matrix equations to obtain a 4×4 system that can be solved.[12,14] The method can be extended to many blade rows; the matrix equation to be solved will always collapse to a 4×4 system.

Cumpsty and Marble[12,14] used this method to predict low-frequency engine core noise, finding that their predictions agreed well with acoustic data collated from the Rolls-Royce Spey 512, Olympus 593 and Pratt and Whitney JT8D-9 gas turbines. This provided circumstantial evidence that entropy noise affects real gas turbines. The analytical method was also validated using numerical simulations by Leyko et al.[62] and Duran and Moreau,[63] showing that the method is able to predict

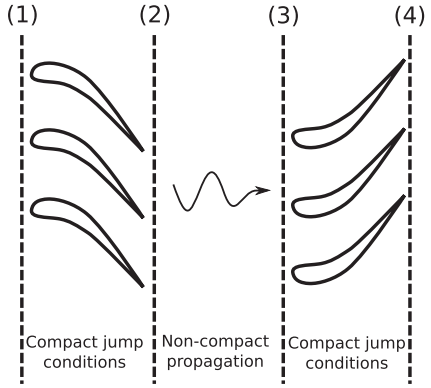

Figure 11. Schematic showing the variables stations used when tracking wave strengths across and between blade rows.

the generation of entropy noise correctly up to a reduced frequency of $\Omega = fL/c \sim 0.1$, where here L is the characteristic axial length of the stator blade.

More recent work on turbine stages performed by Miles[64] used a coherence/time delay analysis to show that entropy noise is likely to be dominant at low frequencies. Tam et al.[65] suggested that direct combustion noise dominates for auxiliary power units, although these are significantly different from turbofan engines. Simple single stator-rotor simulations also suggest that entropy noise is important, with predictions from compact models being accurate at low frequencies.[63] Shear-dispersion attenuation mechanisms, as described in the context of the combustor in the section "Advection of entropy waves through the combustor", continue to provide attenuation within the turbine.[62] The TEENI project[15] underway currently in Europe will look in detail at the noise components present in turboshaft engine full-scale tests.

One challenge that has been addressed for nozzle flows, but not yet for blade row flows, is the development of frequency-dependent predictive tools. Mishra and Bodony[66] performed a numerical blade row study, which showed that compact actuator disk methods are only accurate at low frequencies. However, the current state-of-the-art for predicting the entropy noise generated by blade row acceleration continue to assume that the blade row is compact, even though this assumption has now been fully relaxed for nozzle flows.

Reflected component of entropy noise and its effect on combustion instability

Unsteady heat release rate at the flame generates acoustic waves and entropy waves. Both may result in acoustic waves returning to further perturb the flame, giving the potential for thermoacoustic instability.[21,29,67,68] Many of the early tools for predicting thermoacoustic combustor modes accounted only for the direct acoustic waves, neglecting acoustic waves generated by entropy wave acceleration. This is justifiable for many of the laboratory-scale combustors, such as Rijke tubes, which have open ends and so do not exhibit entropy wave acceleration.[31] However, for real gas turbine geometries or more realistic combustor rigs with constricted downstream boundaries, both sources of upstream travelling acoustic wave should be accounted for.[21,24,69,70]

The mechanisms by which the directly generated acoustic waves may return to the flame (e.g. acoustic wave propagation downstream → acoustic wave reflection → acoustic wave propagation upstream) generally exhibit much faster timescales than mechanisms involving entropy noise, due to the "slow" nature of the advection between the flame and combustor exit. It is therefore unsurprising that several analytical, experimental and numerical studies have identified that entropy noise introduces extra low frequency thermoacoustic combustor modes.[21,29,67,71–74] For example, aero-engine combustors with fuel-spray atomisers are particularly susceptible to a low-frequency oscillations in the range 50–120 Hz at idle and sub-idle conditions, commonly called "rumble." In many cases, these are likely to be associated with entropy noise modes.[75,76] This occurs for two main reasons: first, fuel-spray atomisers are likely to generate larger mixture ratio fluctuations, and therefore larger entropy waves. Second, the combustion process in a fuel spray is much slower, since it involves atomisation, evaporation and mixing of the fuel prior to combustion. This means that the time-delay of the flame is likely to be larger, and therefore more likely to couple with a low-frequency mode such as a mixed entropy-acoustic mode, rather than a purely acoustic one (which presents a larger frequency, due to the low Mach number in the combustion chamber).

However, for many combustors, the growth rates associated with the low frequency entropy noise modes are so low that they are highly unlikely affect the combustor thermoacoustics in any meaningful way. Dowling and Stow[29] showed that when including the entropy wave in the analysis, the number of modes increases, but most of them remain stable, as seen in Figure 12. Goh and Morgans[39] performed an analytical study, which investigated the effect of entropy noise on the "most unstable" modes of several model combustors – these were modes present even in the absence of entropy noise. They found that entropy noise could affect these most unstable modes in a variety of possible ways. They could:

- Destabilise otherwise stable modes.
- Stabilise otherwise unstable modes.

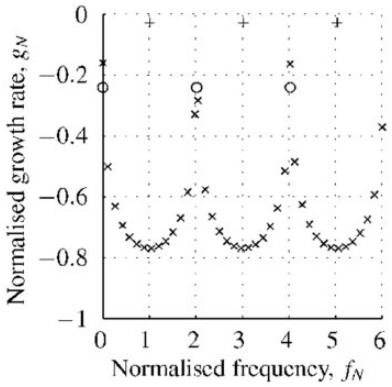

Figure 12. Frequencies and growth rates of resonate modes of a duct with choked outlet: °, choked inlet with entropy and vorticity waves included; x, choked inlet with entropy and vorticity waves assumed to dissipate before combustor exit.
Source: reproduced with permission from the authors.[29]

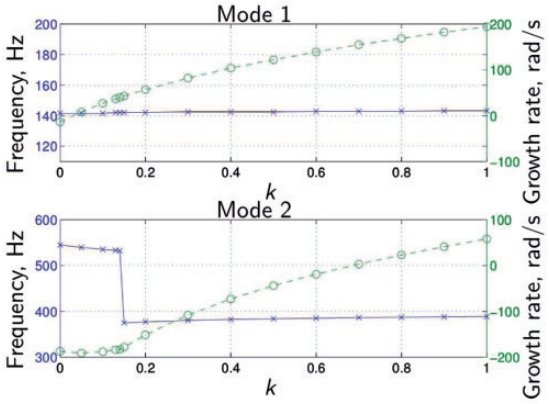

Figure 13. The frequency and growth rate of the two most unstable modes of a model combustor as a function of k, the proportion of generated entropy wave accounted for. x= frequency, °= growth rate.
This is a modified version of a figure in ref.[30]

- Cause mode switching – a change in the dominant unstable mode.
- Cause an acoustic-entropic instability in which heat release rate amplitudes stay small, even when the acoustic and entropy wave amplitudes successively increase.

As an example, the frequencies and growth rates of the two most unstable modes for a model combustor are shown in Figure 13, as a function of a parameter, k, representing the amount of entropy wave accounted for. $k=0$ represents no entropy waves reaching the end of the combustor, and hence no entropy noise, while $k=1$ represents entropy waves reaching the combustor in the form that they were generated at the flame, other than for an advective time delay.[39] It can be seen that as the proportion of entropy waves accounted for is increased, the growth rates of both modes transition from being negative to positive, representing a change in combustor stability from stable to unstable. The frequencies are not strongly affected, other than for a mode switch in the second most unstable mode.

In typical gas-turbine geometries, dimensionless entropy waves are usually larger than dimensionless acoustic waves.[50] This means that, as the thermoacoustic instability grows in amplitude, the entropy wave is likely to experience non-linear effects sooner, governing (in combination with the non-linearity of the flame) the evolution of the thermoacoustic mode and the limit cycle amplitude. The non-linear effect of indirect combustion noise has been studied only recently by Huet and Giauque,[51] and its effect on the feedback loop and on the non-linear instability remains an open question. The effect of entropy noise on thermoacoustic instability when accounting for realistic entropy wave advection within the combustor and non-linear effects is an area likely to see further work in the near future.

Conclusions

Entropy noise is believed to be the major component of indirect combustion noise for combusting flows. It involves a diverse range of flow physics, and has enjoyed a recent resurgence of interest, because of its increasing importance to aero-engine exhaust noise and a recognition that it can affect gas turbine combustion instabilities. Entropy noise occurs when unsteady heat release rate generates temperature fluctuations (entropy waves), and these are subsequently accelerated to generate noise which propagates both upstream and downstream of the acceleration zone. Because gas turbines consist of a low Mach number combustor flow, in which unsteady heat release (and hence entropy waves) are generated, followed by a strong flow acceleration through the turbine rows, they have a particular propensity to entropy noise.

This article has described the flow physics associated with the five stages, which are key to the entropy noise phenomenon:

- Generation of entropy waves by unsteady heat release rate (typically at the flame).
- Advection of these entropy waves through the combustor (typically with negligible acceleration).
- Acceleration of entropy waves through either a nozzle or a blade row to generate entropy noise.
- Passage of the entropy noise through a succession of turbine blade rows (including subsequent acceleration of the entropy wave though downstream blade rows to generate more entropy noise) to appear at the turbine exit.

- Reflection of entropy noise back into the combustor, where the relevant acoustic waves may further perturb the heat release rate at the flame, influencing the combustor thermoacoustic modes.

It has reviewed underlying theory, recent progress and outstanding challenges pertaining to each of these five stages.

A long-standing challenge, which spans all of the stages and has yet to be overcome, is associated with measuring temperature variations at moderate frequencies. The lack of simple methods available for unsteady temperature measurements has hindered both fundamental experimental studies, and entropy noise measurement at larger scales. This includes distinguishing between direct and entropy noise, as direct noise and entropy waves are always generated in tandem. For entropy wave generation, a fundamental open question concerns the validity of the widespread assumption of non-diffusive flow. For entropy wave advection, the main outstanding challenges pertain to the effect of real combustor hydrodynamic flow features on the advection process: these have not yet been considered in any detail. For both entropy noise generation and passage of entropy noise through blade row stages, powerful predictive tools for the noise generated by a nozzle acceleration have been developed, but analytical methods for noise generation by a blade row acceleration remain restricted to those employing a zero frequency assumption. Thus, entropy noise generation by blade row acceleration is an area ripe for future research. It is also worth noting that there are unexplored opportunities in combining predictive advances across the different flow physics. For example, the frequency dependencies of the separate entropy wave generation, advection and acceleration processes have been estimated, but not as yet combined to assess the overall frequency dependence of entropy noise generation. This is especially important in assessing the effect of entropy noise on the thermoacoustic stability of combustors. Finally, the relative importance of the two indirect noise mechanisms, entropy noise and vorticity noise, in real three-dimensional geometries sustaining higher order modes, is only just beginning to be quantified.

Declaration of conflicting interests

The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

Funding

The author(s) disclosed receipt of the following financial support for the research, authorship, and/or publication of this article: The authors are grateful for financial support from the European Research Council Starting Grant ACOULOMODE (2013-18).

References

1. Crighton DG, Dowling AP, Williams JEF, et al. Thermoacoustic sources and instabilities, chap. 13. In: *Modern methods in analytical acoustics*. Berlin: Springer-Verlag, 1992.
2. Strahle WC. On combustion generated noise. *J Fluid Mech* 1971; 49: 399–414.
3. Strahle WC. Combustion noise. *Prog Energy Comb Sci* 1978; 4: 157–176.
4. Chiu HH and Summerfield M. Theory of combustion noise. *Acta Astro* 1974; 1: 967–984.
5. Hassan HA. Scaling of combustion generated noise. *J Fluid Mech* 1974; 66: 445–453.
6. Dowling AP and Mahmoudi Y. Combustion noise. *Proc Comb Inst* 2014; 35: 65–100.
7. Chu BT and Kovasznay LSG. Non-linear interactions in a viscous heat-conducting compressible gas. *J Fluid Mech* 1958; 3: 494–514.
8. Marble FE and Candel SM. Acoustic disturbance from gas non-uniformities convected through a nozzle. *J Sound Vib* 1977; 55: 225–243.
9. Kings N and Bake F. Indirect combustion noise: noise generation by accelerated vorticity in a nozzle flow. *Int J Spray Combust Dyn* 2010; 2: 253–266.
10. Li L and Sun X. Effect of vorticity waves on azimuthal instabilities in annular chambers. *Combust Flame* 2015; 162: 628–641.
11. Mee DJ, Baines NC, Oldfield MLG, et al. An examination of the contributions to loss on a transonic turbine blade in a cascade. *ASME J Turbomach* 1992; 114: 155–162.
12. Cumpsty NA and Marble FE. The interaction of entropy fluctuations with turbine blade rows; a mechanism of turbojet engine noise. *Proc R Soc Lond A* 1977; 357: 323–344.
13. Bohn MS. Response of a subsonic nozzle to acoustic and entropy disturbances. *J Sound Vib* 1977; 52: 283–297.
14. Cumpsty NA and Marble FE. Core noise from gas turbine exhausts. *J Sound Vib* 1977; 54: 297–309.
15. Duran I, Moreau S, Nicoud F, et al. Combustion noise in modern aero-engines. *Aerosp Lab J* 2014; 7: 1–11.
16. NASA Glenn Research Center. NASA facts: Making future commercial aircraft quieter. Technical Report FS-1999-07-003-GRC, NASA, 1999.
17. ACARE. Flightpath 2050: Europe's vision for aviation. Technical Report, European Commission, 2011.
18. Poinsot T and Veynante D. *Theoretical and numerical combustion*. Toulouse: CERFACS, 2012.
19. Swaminathan N and Bray KNC. *Turbulent premixed flames*. Cambridge University Press, 2011.
20. Candel S. Combustion dynamics and control: Progress and challenges. *Proc Combust Inst* 2002; 29: 1–28.
21. Keller JJ, Egli W and Hellat J. Thermally induced low-frequency oscillations. *Zeitschrift für angewandte Mathematik und Physik ZAMP* 1985; 36: 250–274.
22. Eckstein J, Freitag E, Hirsch C, et al. Experimental study on the role of entropy waves in low-frequency oscillations for a diffusion burner. In: *Proceedings of ASME Turbo Expo*, GT2004-54163, 2004.

23. Bake F, Richter C, Mühlbauer B, et al. The entropy wave generator (EWG): A reference case on entropy noise. *J Sound Vib* 2009; 326: 574–598.
24. Hield PA and Brear MJ. Comparison of open and choked premixed combustor exits during thermoacoustic limit cycle. *AIAA J* 2008; 46: 517–526.
25. Rayleigh JWS. *The theory of sound*. vol. 2. Dover Publications, 1945.
26. Cantrell RH and Hart RW. Interaction between sound and flow in acoustic cavities: mass, momentum, and energy considerations. *J Acoust Soc Am* 1964; 36: 697–706.
27. Morfey CL. Acoustic energy in non-uniform flows. *J Sound Vib* 1971; 14: 159–170.
28. Nicoud F and Poinsot T. Thermoacoustic instabilities: should the Rayleigh criterion be extended to include entropy changes? *Combust Flame* 2005; 142: 153–159.
29. Dowling AP and Stow SR. Acoustic analysis of gas turbine combustors. *J Prop Power* 2003; 19: 751–764.
30. Morgans AS, Goh CS and Dahan JA. The dissipation and shear dispersion of entropy waves in combustor thermoacoustics. *J Fluid Mech* 2013.
31. Dowling A and Ffowcs Williams JE. *Sound and sources of sound*. Ellis Horwood, 1983.
32. Dowling AP, Hooper N, Langhorne PJ, et al. Active control of reheat buzz. *AIAA J* 1988; 26: 783–790.
33. Karimi N, Brear MJ and Moase WH. Acoustic and disturbance energy analysis of a flow with heat communication. *J Fluid Mech* 2008; 597: 67–89.
34. Dowling AP. The calculation of thermoacoustic oscillations. *J Sound Vib* 1995; 180: 557–581.
35. Motheau E, Selle L and Nicoud F. Accounting for convective effects in zero mach number thermoacoustic models. *J Sound Vib* 2014; 333: 246–262.
36. Bauerheim M, Nicoud F and Poinsot T. Theoretical analysis of the mass balance equation through a flame at zero and non-zero mach numbers. *Combust Flame* 2015; 162: 60–67.
37. Stow SR and Dowling AP. A time-domain network model for nonlinear thermoacoustic oscillations. *ASME J Eng Gas Turb Power* 2009.
38. Li J and Morgans AS. Time domain simulations of nonlinear thermoacoustic behaviour in a simple combustor using a wave-based approach. *J Sound Vib* 2015; 346: 345–360.
39. Goh CS and Morgans AS. The influence of entropy waves on the thermoacoustic stability of a model combustor. *Comb Sci Tech* 2013; 185: 249–268.
40. Dowling AP. Nonlinear self-excited oscillations of a ducted flame. *J Fluid Mech* 1997; 346: 271–290.
41. Lieuwen T. Modeling premixed combustion-acoustic wave interactions: A review. *J Prop Power* 2003; 19: 765–781.
42. Franzelli B, Riber E, Gicquel LYM, et al. Large eddy simulation of combustion instabilities in a lean partially premixed swirled flame. *Comb Flame* 2012; 159: 621–637.
43. Sattelmayer T. Influence of the combustor aerodynamics on combustion instabilities from equivalence ratio fluctuations. *ASME J Eng Gas Turb Power* 2003; 125: 11–19.
44. Morfey C. Amplification of aerodynamic noise by convected flow inhomogeneities. *J Sound Vib* 1973; 31: 391–397.
45. Lighthill MJ. On sound generated aerodynamically, I: General theory. *Proc Royal Soc A* 1952; 211: 564–587.
46. Howe MS. Contributions to the theory of aerodynamic sound, with application to excess jet noise and the theory of the flute. *J Fluid Mech* 1975; 71: 625–673.
47. Howe MS. The generation of sound by aerodynamic sources in an inhomogeneous steady flow. *J Fluid Mech* 1975; 67: 597–610.
48. Williams JEF and Howe MS. The generation of sound by density inhomogeneities in low mach number nozzle flows. *J Fluid Mech* 1975; 70: 605–622.
49. Bodony D. Scattering of an entropy disturbance into sound by a symmetric thin body. *Phys Fluids* 2009; 21.
50. Leyko M, Nicoud F and Poinsot T. Comparison of direct and indirect combustion noise mechanisms in a model combustor. *AIAA J* 2009; 47: 2709–2716.
51. Huet M and Giauque A. A nonlinear model for indirect combustion noise through a compact nozzle. *J Fluid Mech* 2013; 733: 268–301.
52. Zinn BT, Bell WA, Daniel BR, et al. Experimental determination of three-dimensional liquid rocket nozzle admittances. *AIAA J* 1973; 11: 267–272.
53. Moase WH, Brear M and Manzie C. The forced response of choked nozzles and supersonic diffusers. *J Fluid Mech* 2007; 585: 281–304.
54. Giauque A, Huet M and Clero F. Analytical analysis of indirect combustion noise in subcritical nozzles. *ASME J Eng Gas Turb Power* 2012; 134.
55. Stow SR, Dowling AP and Hynes TP. Reflection of circumferential modes in a choked nozzle. *J Fluid Mech* 2002; 467: 215–239.
56. Goh CS and Morgans AS. Phase prediction of the response of choked nozzles to entropy and acoustic disturbances. *J Sound Vib* 2011; 330: 5184–5198.
57. Duran I and Moreau S. Solution of the quasi-one-dimensional linearized euler equations using flow invariants and the magnus expansion. *J Fluid Mech* 2013; 723: 190–231.
58. Duran I and Morgans AS. On the reflection and transmission of circumferential waves through nozzles. *J Fluid Mech* 2015; 773: 137–153.
59. Leyko M, Moreau S, Nicoud F, et al. Numerical and analytical modelling of entropy noise in a supersonic nozzle with a shock. *J Sound Vib* 2011; 330: 3944–3958.
60. Howe MS. Indirect combustion noise. *J Fluid Mech* 2010; 659: 267–288.
61. Duran I, Moreau S and Poinsot T. Analytical and numerical study of combustion noise through a subsonic nozzle. *AIAA J* 2013; 51: 42–52.
62. Leyko M, Duran I, Moreau S, et al. Simulation and modelling of the waves transmission and generation in a stator blade row in a combustion-noise framework. *J Sound Vib* 2014; 333: 6090–6106.
63. Duran I and Moreau S. Numerical simulation of acoustic and entropy waves propagating through turbine blades. In: *AIAA/CEAS aeroacoustics conference*, AIAA2013-2102, 2013.

64. Miles JH. Time delay analysis of turbofan engine direct and indirect combustion noise sources. *J Prop Power* 2009; 25: 218–227.
65. Tam CKW, Pastouchenko NN, Mendoza J, et al. Combustion noise of auxiliary power units. In: *AIAA/CEAS aeroacoustics conference,* AIAA-2005-2829, 2005.
66. Mishra A and Bodony D. Evaluation of actuator disk theory for predicting indirect combustion noise. *J Sound Vib* 2013; 332: 821–838.
67. Polifke W, Paschereit CO and Döbbeling K. Constructive and destructive interference of acoustic and entropy waves in a premixed combustor with a choked exit. *J Acoust Vib* 2001; 6: 135–146.
68. Brear MJ, Nicoud F, Talei M, et al. Disturbance energy transport and sound production in gaseous combustion. *J Fluid Mech* 2012; 707: 53–73.
69. Muthukrishnan M, Strahle WC and Neale DH. Separation of hydrodynamic, entropy, and combustion noise in a gas turbine combustor. *AIAA J* 1978; 16: 320–327.
70. Morgans AS and Annaswamy AM. Adaptive control of combustion instabilities for combustion systems with right-half plane zeros. *Comb Sci Tech* 2008; 180: 1549–1571.
71. Zhu M, Dowling AP and Bray KNC. Self-excited oscillations in combustors with spray atomizers. *ASME J Eng Gas Turbines Power* 2001; 123: 779–786.
72. Giuliani F, Gajan P, Diers O, et al. Influence of pulsed entries on a spray generated by an airblast injection device: An experimental analysis on combustion instability processes in aeroengines. *Proc Combust Inst* 2003; 29: 91–98.
73. Motheau E, Nicoud F and Poinsot T. Mixed acoustic entropy combustion instabilities in gas turbines. *J Fluid Mech* 2014; 749: 542–576.
74. Hochgreb S, Dennis D, Ayranci I, et al. Forced and self-excited instabilities from lean premixed, liquid-fuelled aeroengine injectors at high pressures and temperatures. In: *Proceedings of ASME Turbo Expo*, GT2013-95311, 2013.
75. Hubbard S and Dowling AP. Acoustic instabilities in premix burners. *AIAA paper* 2272, 1998.
76. Umurhan OM. Exploration of fundamental matters of acoustic instabilities in combustion chambers. Center for Turbulent Research, Annual Briefs, 1999, pp.85–98.