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# Relative amplitude preserved P–P and P–SV wave seismic sections

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**Abstract**

In seismic wave propagation, the amplitude decay along travel time comprises two components: the spherical divergence and the earth absorption effect in an inhomogeneous medium. In reservoir geophysics, both the P–P and P–SV wave seismic sections are required to be relative-amplitude preserved so that they can be used for comparative interpretation and analysis. Therefore, each of the following two processing steps, divergence correction and inverse $Q$ filtering, applied to both seismic sections should be relative-amplitude preserved. In this paper, the spherical divergence corrections on the P–P and P–SV wave sections are expressed in unified formulae in which the formula for the P–SV wave section differs from that for the P–P wave section by a factor $(1+\gamma)^{-1}$, where $\gamma$ is the ratio of P- to S-wave velocities, applicable to both the scale and power coefficients simultaneously. Divergence-corrected seismic sections are then used to estimate the earth $Q$ models, which in turn are used to design inverse $Q$ filters. Only when divergence correction and inverse $Q$ filtering, both of which are amplitude-related processing, applied to the P–P and P–SV wave sections are adequately accurate, the amplitude information contained in these seismic sections can be reliably used for lithological analysis in reservoir geophysics.

**Keywords:** spherical divergence, attenuation, amplitude preserving, divergence correction, earth $Q$ model, inverse $Q$ filter, converted wave seismic

## 1. Introduction

This paper attempts to address two issues in relation to amplitude-preserved data processing of converted wave seismics.

(1) When seismic waves propagate through an inhomogeneous medium, their amplitude decays along a ray path consisting of two components: the spherical divergence and the earth absorption effect. Thus, the first issue is that divergence correction alone in seismic processing is not sufficient to fully account for the amplitude decay with travel time, and inverse $Q$ filtering is also required to compensate for the amplitude attenuation due to the frequency-dependent energy absorption.
(2) The application of 'zero-offset' P–P and P–SV wave seismic sections to lithology interpretation is currently limited to the estimation of the S-wave to P-wave velocity ratio and is determined through the event correlation in the two seismic sections. To make further interpretative comparison quantitatively between the P–P and P–SV wave seismic sections, it is necessary that both sections are relative-amplitude preserved; this is the second issue.

To fulfil these basic requirements, one should make sure in the data processing that first the divergence correction to both

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sections is relative-amplitude preserved and then inverse $Q$ filtering is also relative-amplitude preserved.

The accuracy of divergence correction may be even more important, since the divergence-corrected seismic section is often used to estimate the earth $Q$ model (Wang 2004), which is needed for designing the inverse $Q$ filter (Wang 2002). The accuracy of the $Q$ model then directly influences the inverse $Q$ filtering result. Only when all of these amplitude-related processing steps applied on both the P–P and P–SV wave sections respectively are accurate enough can the amplitude information of these two types of seismic sections be used, in a comparative and joint manner, for lithological analysis and inversion in reservoir geophysics.

As regards a rigorous derivation of the spherical divergence, readers may refer to Červený and Ravindra (1971) and Červený (2001). In seismic data processing, however, Newman's (1973) 'tv-square' formula has been widely used for the divergence correction of a zero-offset, P–P wave seismic section. When a P–SV wave seismic section is used in comparison with the P–P wave section, a 'tv-square' equivalent formula is also required for the divergence correction of the zero-offset, P–SV wave seismic section.

We show in this paper that if the P–P wave divergence correction is represented as

$$A^{(\mathrm{pp})}(t) = \alpha t \exp[\beta t], \tag{1}$$

where $\alpha$ and $\beta$ are two generalized scaling coefficients and $t$ is the two-way travel time, the counterpart for the P–SV wave divergence correction may be expressed as

$$A^{(\mathrm{ps})}(t) = \alpha' t \exp[\beta' t], \tag{2}$$

where

$$\alpha' = \frac{\alpha}{1+\gamma}, \qquad \beta' = \frac{\beta}{1+\gamma}, \tag{3}$$

and

$$\gamma = \frac{v_\mathrm{p}}{v_\mathrm{s}} \tag{4}$$

is the ratio of P- to S-wave velocities.

After applying equations (1) and (2) to the P–P and P–SV wave sections respectively, we are able to estimate $Q$ values with respect to the two-way travel time directly from each of these sections. The $Q$ values are then used in designing inverse $Q$ filters, which are model based (Wang 2002). The results of inverse $Q$ filtering will show true amplitude variations of the P–P and P–SV waves, and provide reliable information for lithological interpretation and inversion.

## 2. Spherical divergence corrections

Spherical divergence is usually one of the major factors causing travel-time-dependent amplitude changes. Seismic energy spreads out so that the wave decreases in strength, although the total energy in the wavefield does not change. If the earth medium were homogeneous, the amplitude weakening would be inversely proportional to the travel distance. However, because velocity generally increases with depth, ray-path curvature makes the wave spread out more and thus makes the decrease in amplitude greater. For a stack (i.e. zero-offset) seismic trace, the effect of spherical divergence may be expressed as (Newman 1973)

$$D = \frac{v_0}{t v_\mathrm{rms}^2}, \tag{5}$$

where $v_0$ is the velocity at the source position and $v_\mathrm{rms}$ is the rms velocity varying with respect to the two-way time.

For a P–P wave zero-offset trace, the correction for spherical divergence is then given by

$$A^{(\mathrm{pp})}(t) = \frac{t v_{\mathrm{p,rms}}^2(t)}{v_0}. \tag{6}$$

This is the so-called 'tv-square' formula, which shows that for a normal incidence ray path through a horizontally layered model, the ray-amplitude compensation is proportional to $t v_\mathrm{rms}^2$.

If we assume the P-wave rms velocity $v_\mathrm{p,rms}$ to be a linear function

$$v_{\mathrm{p,rms}}(t) = v_0\big(1 + \tfrac{1}{2}\beta t\big), \tag{7}$$

where $\beta$ is the velocity gradient with respect to the one-way time (equivalently, the depth), equation (6) may be approximated as

$$A^{(\mathrm{pp})}(t) = v_0 t\big(1 + \tfrac{1}{2}\beta t\big)^2 \tag{8}$$

$$\approx v_0 t \exp[\beta t]. \tag{9}$$

For a zero-offset P–SV wave, we assume that the travel path of the downgoing P wave is equal to the travel path of the upgoing SV wave reflected from an interface. Denoting the travel time of the downgoing P-wave as $t_\mathrm{p}$ and the travel time of the upgoing SV-wave as $t_\mathrm{s}$, the total travel time is expressed as $t = t_\mathrm{p} + t_\mathrm{s}$. The correction for spherical divergence is then given by

$$A^{(\mathrm{ps})}(t = t_\mathrm{p} + t_\mathrm{s}) = 2A_{\mathrm{down}}^{(\mathrm{p})}(t_\mathrm{p}), \tag{10}$$

where $A_{\mathrm{down}}^{(\mathrm{p})}(t_\mathrm{p})$ is the divergence correction for the downward travel path, and the factor 2 is given because of the return path equal to the downgoing travel path. This factor of 2 also means that the divergence correction for the return path can compensate only for a fixed 6 dB loss. Following O'Brien and Lucas (1971), the divergence correction of the downward P-wave travel path is given by

$$A_{\mathrm{down}}^{(\mathrm{p})}(t_\mathrm{p}) = \frac{t_\mathrm{p} v_{\mathrm{p,rms}}^2(t_\mathrm{p})}{2v_0}. \tag{11}$$

For a zero-offset P–SV wave trace, the downward travel time is

$$t_\mathrm{p} = \frac{1}{1+\gamma} t. \tag{12}$$

Finally, we obtain the formula for the P–SV wave divergence correction as

$$A^{(\mathrm{ps})}(t) = \frac{t_\mathrm{p} v_{\mathrm{p,rms}}^2(t_\mathrm{p})}{v_0} \tag{13}$$

$$= \frac{v_0 t}{1+\gamma}\left(1 + \frac{\beta}{2(1+\gamma)} t\right)^2 \tag{14}$$

$$\approx \frac{v_0 t}{1+\gamma} \exp\left[\frac{\beta}{1+\gamma} t\right]. \tag{15}$$

Equations (13), (14) and (15) are the counterparts of equations (6), (8) and (9), respectively.

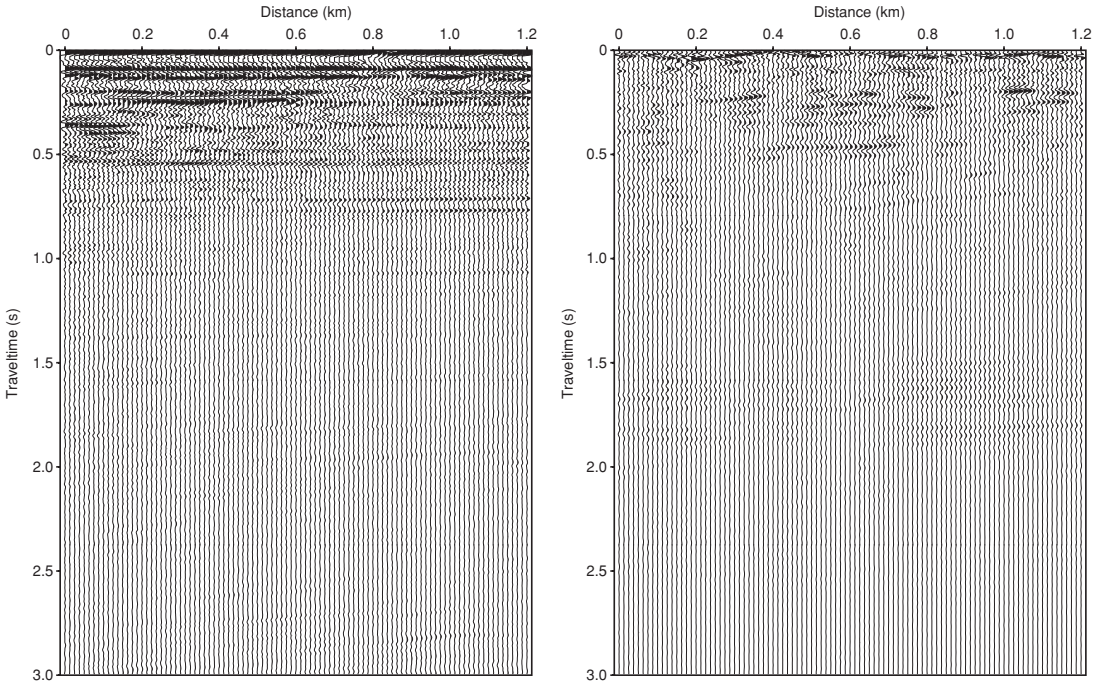



**Figure 1.** The P–P wave seismic section (left) and the P–SV wave seismic section (right) with spherical divergence correction, which show that the spherical divergence correction alone is not sufficient to fully account for the amplitude decay with travel time.

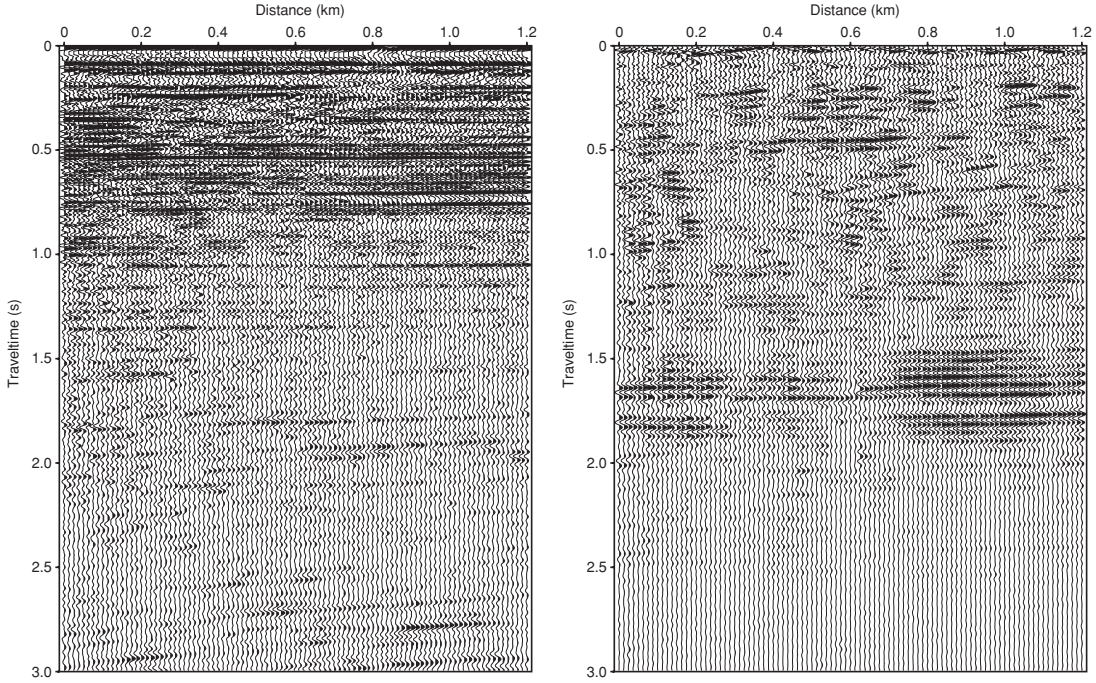



**Figure 2.** The P–P wave section (left) and the P–SV wave section (right) after inverse $Q$ filtering, which show true amplitude variations of the P–P and P–SV waves and thus provide reliable information for further geological and lithological interpretation.

The velocity $v_0$ at the recording surface is assumed to be a constant and can be dropped from the divergence-correction formulae (8)–(9) and (14)–(15). It is equivalent to scaling down the $\alpha$ coefficient in equations (1) and (2) simultaneously.

## 3. Divergence correction followed by inverse $Q$ filtering

To demonstrate the use of divergence correction formulae, we managed to obtain a pair of P–P and P–SV wave stack sections which had no divergence correction applied. In the example

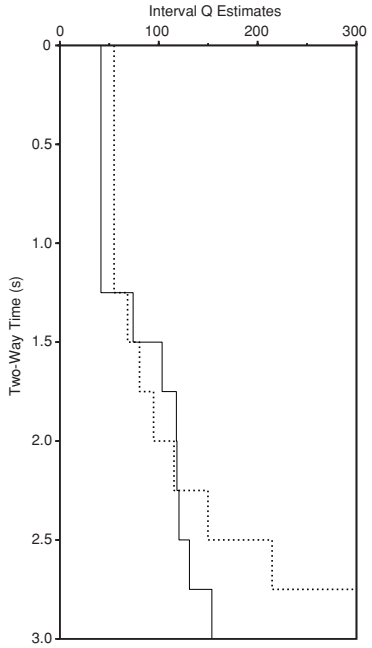



**Figure 3.** The time-variant earth $Q$ models estimated from the P–P wave (dashed line) and from the P–SV wave (solid line) seismic sections shown in figure 1. These $Q$ functions are used to design inverse $Q$ filters which are applied to figure 1, producing the final seismic sections in figure 2.

below, we first apply the divergence correction to each of them, then estimate $Q$ models from them, and finally perform inverse $Q$ filtering to them.

Figure 1 shows the P–P wave and P–SV wave seismic sections, after spherical divergence correction. In the P–P wave divergence correction formula (1), we use $\alpha = 1$ and $\beta = 0.3$. In the P–SV wave divergence correction formula (2), we use $\alpha' = 0.366$ and $\beta' = 0.11$. The latter is equivalent to using $\gamma = \sqrt{3}$, the ratio of P- to S-wave velocities. The corrections of spherical divergence are not sufficient to fully account for the amplitude decay with travel time.

Figure 2 shows the corresponding seismic sections after inverse $Q$ filtering. The inverse $Q$ filtered results show true amplitude variations of the P–P and P–SV waves, and thus provide reliable information for further geological and lithological interpretation. The inverse $Q$ filter technique used here is the so-called stabilized inverse $Q$ filtering algorithm, proposed by Wang (2002). It can perform phase-correction (correcting for velocity dispersion) and amplitude-compensation (compensating for energy absorption) simultaneously, without boosting the ambient noise. Readers may refer to Wang (2003) for a quantitative analysis of the effectiveness of the stabilized inverse $Q$ filtering, and to Guo and Wang (2004) for more applications.

In this paper, two inverse $Q$ filters, applied to the P–P and P–SV wave sections, respectively, are designed on the basis of two different earth $Q$ models, as depicted in figure 3, where the dashed line is the $Q$ model for the P–P wave section and the solid line is the $Q$ model for the P–SV wave section. These two time-variant $Q$ models are estimated directly from the P–P and P–SV wave sections, respectively, shown in figure 1, in which the spherical divergence has been corrected. As expected, the P–SV wave interval-$Q$ values in general are smaller than the P–P wave interval-$Q$ values. For $Q$ analysis on reflection seismic sections, readers may refer to Wang (2004).

## 4. Conclusions

Investigation into the amplitude-preserved data processing of both the P–P and P–SV wave seismic sections shown in this paper suggests that

(1) formulae for the spherical divergence corrections to the P–P and P–SV wave seismic sections may be unified. The formula for the P–SV wave section differs from that for the P–P wave section by a factor $(1 + \gamma)^{-1}$, applicable to both the scale and power coefficients simultaneously.
(2) the correction of spherical divergence alone is not sufficient to fully account for the amplitude decay with travel time, and inverse $Q$ filtering is also required to compensate for the energy absorption in the inhomogeneous media.

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