

Quantum discontinuity for massive spin 3/2 with a Λ term

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Abstract

We show that the recently demonstrated absence of the van Dam-Veltman-Zakharov discontinuity for massive spin 3/2 with a Λ term is an artifact of the tree approximation, and that the discontinuity reappears at one loop. As a numerical check on the calculation, we rederive the vanishing of the one-loop beta function for $D = 11$ supergravity on $AdS_4 \times S^7$ level-by-level in the Kaluza-Klein tower.

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I. INTRODUCTION

An old question is whether the graviton has exactly zero mass or perhaps a small but non-zero mass. This issue seemed to have been resolved by van Dam and Veltman [1] and, independently, Zakharov [2] when they noted that there is a discrete difference between the propagator of a strictly massless graviton and that of a graviton with mass M in the $M \rightarrow 0$ limit. The massless limit of a massive graviton then yields a bending of light by the sun which is only 3/4 of the Einstein value. A similar discontinuity appears in the massless limit of a massive spin 3/2 [3].

Subsequently, however, these discontinuities were called into question in [4,5] for the case of the graviton and in [6,7] for the case of the gravitino, by pointing out that they disappear if the background spacetime is anti-de Sitter (AdS) or more generally Einstein, satisfying

$$R_{\mu\nu} = \Lambda g_{\mu\nu}, \quad (1)$$

with a non-zero cosmological constant $\Lambda \neq 0$ provided $M^2/\Lambda \rightarrow 0$.

Yet in recent work [8] we have shown that the above disappearance of the discontinuity for the massive graviton is an artifact of the tree level approximation and that the discontinuity reappears at one loop as a result of going from five degrees of freedom to two¹. In this paper we exhibit a similar one-loop discontinuity for the massive gravitino, as a result of going from four degrees of freedom to two.

That a cosmological constant cannot cure the spin 3/2 discontinuity at the one-loop level is an observation that could have been made in 1978. Then it was shown that the gravitational axial anomaly for a Rarita-Schwinger particle is -21 times that of a Dirac particle in the massless case, but -20 times in the massless limit of the massive case [11,12]. Since the axial anomaly depends only on the Pontryagin density R^*R , moreover, it is completely unaffected by the presence of a cosmological constant.

The quantization of a single massive spin 3/2 field in the presence of a cosmological constant is treated in section II using the Stückelberg formalism [13,6] which introduces a massive spin 3/2 gauge invariance through the addition of an auxiliary spin 1/2 field. This approach allows us to carry over many of the same procedures used for a massless spin 3/2 field, and furthermore allows a simple comparison between massive and massless cases. In section III we compute the one-loop partition function and verify that a quantum spin 3/2 discontinuity arises.

In a previous paper on spin 2 [8], we made the important caveat that our results about quantum discontinuities apply to theories where the gauge symmetry is broken explicitly by the addition of a Pauli-Fierz mass term. We were agnostic about whether the problem is avoided if the graviton gets its mass through a dynamical mechanism of the kind appearing in the Karch-Randall braneworld [14,15], since this requires a separate treatment. A similar caveat applies to the spin 3/2 case. The supersymmetric Karch-Randall mechanism, which includes the case of spin 3/2, is discussed in [16] from the braneworld point of view and

¹Similar classical continuity but quantum discontinuity arises in the partially massless [9] limit as a result of going from five degrees of freedom to four [10].

we mention it again briefly in section V, postponing a more thorough analysis of the VVZ problem to a future publication.

In the spin 3/2 case there is a third possibility: the gravitino may acquire a mass through a conventional super-Higgs effect [17–20]. Here, as described in section III, the massive theory does limit smoothly to a massless theory with the same number degrees of freedom. In particular, the axial anomaly suffers no discontinuity. All this is true whether or not there is a cosmological constant. However, even for simple supergravity the massless theory is not the minimal one so the spin 3/2 analogue of the “wrong bending of light” feature continues to apply. Similarly, the dynamical mechanism for a massive spin 2 suggested in [21] has a smooth massless limit, but it limits to a tensor-scalar theory with the wrong bending of light. The Karch-Randall mechanism, on the other hand, is claimed to be both continuous and to limit to Einstein gravity [22].

Finally, in section IV, we collect the results for fields of spins ≤ 2 and examine the implication for massive supermultiplets. A check on the numerical calculations, which is of interest in its own right, is provided by the massive Kaluza-Klein tower arising from the $AdS_4 \times S^7$ compactification of $D = 11$ supergravity. We confirm that the one-loop beta function vanishes not only for the massless modes [23] but also for the massive Kaluza-Klein tower level by level [24,25].

II. STÜCKELBERG QUANTIZATION OF MASSIVE SPIN 3/2

We work in four dimensions with Euclidean signature $(+, +, +, +)$ and a cosmological constant Λ . As in [6,7], our starting point is the Rarita-Schwinger action for a massive spin 3/2 field coupled to a source

$$e^{-1}\mathcal{L}_{3/2} = -\frac{1}{2}\bar{\Psi}_\mu\gamma^{\mu\rho\nu}\nabla_\rho\Psi_\nu + \frac{M}{2}\bar{\Psi}_\mu\gamma^{\mu\nu}\Psi_\nu + \bar{\Psi}^\mu J_\mu. \quad (2)$$

In flat (or Ricci-flat) space, the massless equation (obtained by setting $M = 0$) is invariant under the spin 3/2 gauge transformation

$$\delta\Psi_\mu = \nabla_\mu\epsilon, \quad (3)$$

provided the source is conserved, $\nabla \cdot J = 0$. Gauge fixing and quantization of this system was previously carried out in [26].

In the presence of a cosmological constant, “masslessness” (in the sense of propagating reduced degrees of freedom) is no longer given by $M = 0$, but rather by $M^2 = -\Lambda/3$. Note that this is only possible in AdS; for future convenience we take $\Lambda = -3\bar{m}^2$ when specializing to the AdS case (so that masslessness corresponds to $M^2 = \bar{m}^2$). For the massless case, the combined action is invariant under the gauge transformation

$$\delta\Psi_\mu = \mathcal{D}_\mu\epsilon \equiv (\nabla_\mu + \tfrac{1}{2}M\gamma_\mu)\epsilon, \quad (4)$$

which, in the language of supergravity, is simply a supersymmetry transformation on the gravitino. However other values of mass, $M^2 \neq -\Lambda/3$, break this gauge invariance. To

see this, we note that the Lagrangian (2) may be written in terms of the “supercovariant derivative” $\mathcal{D}_\mu$ as simply

$$e^{-1}\mathcal{L}_{3/2} = -\frac{1}{2}\bar{\Psi}_\mu\gamma^{\mu\rho\nu}\mathcal{D}_\rho\Psi_\nu + \bar{\Psi}^\mu J_\mu. \quad (5)$$

The variation under the transformation (4) is

$$\begin{aligned} \delta(e^{-1}\mathcal{L}_{3/2}) &= -\frac{1}{4}\bar{\Psi}_\mu\gamma^{\mu\rho\nu}[\mathcal{D}_\rho, \mathcal{D}_\nu]\epsilon + \frac{1}{4}[\overline{\mathcal{D}_\rho}, \overline{\mathcal{D}_\mu}]\epsilon\gamma^{\mu\rho\nu}\Psi_\nu + \overline{\mathcal{D}^\mu}\epsilon J_\mu \\ &= \frac{1}{4}(\bar{\Psi}^\mu\gamma^\nu\epsilon - \bar{\epsilon}\gamma^\mu\Psi^\nu)(3M^2g_{\mu\nu} - R_{\mu\nu} + \frac{1}{2}g_{\mu\nu}R) - \bar{\epsilon}\mathcal{D}^\mu J_\mu. \end{aligned} \quad (6)$$

Substituting in the Einstein condition, $R_{\mu\nu} = \Lambda g_{\mu\nu}$, but keeping M and Λ independent, this turns out to be

$$\delta(e^{-1}\mathcal{L}_{3/2}) = \frac{\Lambda + 3M^2}{4}(\bar{\Psi} \cdot \gamma\epsilon - \bar{\epsilon}\gamma \cdot \Psi) - \bar{\epsilon}\mathcal{D}^\mu J_\mu. \quad (7)$$

This demonstrates that gauge invariance of the action demands both masslessness, $M^2 = -\Lambda/3$, and supercovariant conservation of the current, $\mathcal{D}^\mu J_\mu = 0$.

Although gauge invariance is lost for $M^2 \neq -\Lambda/3$, it may be restored using a Stückelberg approach [13]. In the present case, this amounts to the introduction of an auxiliary spin 1/2 field, χ , transforming as

$$\delta\chi = \epsilon, \quad (8)$$

so that the shifted quantity $\Psi'_\mu \equiv \Psi_\mu - \mathcal{D}_\mu\chi$ remains gauge invariant. Making the replacement $\Psi_\mu \rightarrow \Psi'_\mu$ in (2), we find

$$\begin{aligned} e^{-1}\mathcal{L}_{3/2} &= -\frac{1}{2}\bar{\Psi}_\mu\gamma^{\mu\rho\nu}\nabla_\rho\Psi_\nu + \frac{M}{2}\bar{\Psi}_\mu\gamma^{\mu\nu}\Psi_\nu + \bar{\Psi}^\mu J_\mu \\ &\quad - \frac{\Lambda + 3M^2}{4}[\bar{\chi}(\nabla + 2M)\chi + \bar{\Psi} \cdot \gamma\chi - \bar{\chi}\gamma \cdot \Psi] + \bar{\chi}\mathcal{D}_\mu J^\mu. \end{aligned} \quad (9)$$

While we have avoided explicit use of supergravity techniques, it is welcome to see that the Stückelberg spinor χ may be interpreted in a supergravity language as a Goldstino field associated with broken supersymmetry; it is the field eaten by the gravitino to provide its mass. Indeed, scaling $\chi \rightarrow \sqrt{2/(\Lambda + 3M^2)}\chi$ to give a canonical kinetic term, it is clear that χ couples to the gravitino via the Goldstino current, $\bar{\Psi}^\mu J_\mu^{(\chi)}$ where $J_\mu^{(\chi)} = \sqrt{(\Lambda + 3M^2)/2}\gamma^\mu\chi$. This current clearly vanishes in the limit $M^2 \rightarrow -\Lambda/3$.

For negative cosmological constant, this spin 3/2 Higgs mechanism may be interpreted in terms of a decomposition of AdS representations. Recall that for AdS_4 , with isometry group $SO(3, 2)$, representations are labeled by $D(E_0, s)$, where E_0 is the minimum energy and s is the spin. Generic unitary representations require $E_0 \geq s + 1$, with saturation corresponding to massless (shortened) representations. In particular, a massless graviton transforms as a $D(3, 2)$, while a massless gravitino transforms as a $D(5/2, 3/2)$. The Higgs mechanism in AdS is then expressed in terms of a massive representation becoming reducible in the zero-mass limit

$$D(s+1+\epsilon, s) \rightarrow D(s+1, s) \oplus D(s+2, s-1) \quad \text{as } \epsilon \rightarrow 0, \quad (10)$$

or $D(5/2+\epsilon, 3/2) \rightarrow D(5/2, 3/2) + D(7/2, 1/2)$ for the gravitino. To examine the connection between this decomposition and the Stückelberg Lagrangian (9), we make use of the relations between mass and E_0 for spin 1/2 and spin 3/2; these are generalizations of the Breitenlohner-Freedman bound [27] for higher spins and are given by $E_0 = \frac{3}{2} + |m_{1/2}/\overline{m}|$ for spin 1/2 and $E_0 = \frac{3}{2} + |m_{3/2}/\overline{m}|$ for spin 3/2 [28,29]. Reading off $m_{1/2} = 2M$ and $m_{3/2} = M$ from (9), and taking the massless limit $M \rightarrow \overline{m}$, we indeed find the values $E_0 = 7/2$ and $E_0 = 5/2$ for spins 1/2 and 3/2, respectively. This confirms that the mass terms in the Lagrangian are appropriate to an AdS Higgs mechanism.

We now turn to the issue of gauge fixing and quantization of the Stückelberg Lagrangian, (9). Before proceeding, we find it convenient to make the field redefinition [30]

$$\Psi_\mu = \phi_\mu - \frac{1}{2}\gamma_\mu \gamma \cdot \phi. \quad (11)$$

This is analogous to the decomposition of the linearized fluctuation $h_{\mu\nu}$ of the graviton into $\tilde{h}_{\mu\nu} = h_{\mu\nu} - \frac{1}{2}g_{\mu\nu}h$. Making this redefinition and performing the aforementioned rescaling of χ , we obtain

$$e^{-1}\mathcal{L}_{3/2} = -\frac{1}{2}\bar{\phi}^\mu(\nabla + M)\phi_\mu - \frac{1}{4}\bar{\phi} \cdot \gamma(\nabla - 2M)\gamma \cdot \phi - \frac{1}{2}\bar{\chi}(\nabla + 2M)\chi \\ + \frac{1}{2}\sqrt{\frac{\Lambda + 3M^2}{2}}(\bar{\phi} \cdot \gamma\chi - \bar{\chi}\gamma \cdot \phi) + \bar{\phi}^\mu J_\mu - \frac{1}{2}\bar{\phi} \cdot \gamma\gamma \cdot J + \sqrt{\frac{2}{\Lambda + 3M^2}}\bar{\chi}\mathcal{D}_\mu J^\mu. \quad (12)$$

Note that, because of the denominator in the last term, the supercurrent, J_μ , must be supercovariantly conserved in the massless limit.

It is instructive to examine the massless spin 3/2 field in flat space. In this case, (12) becomes simply

$$e^{-1}\mathcal{L}_{3/2} = -\frac{1}{2}\bar{\phi}^\mu\nabla \phi_\mu - \frac{1}{4}\bar{\phi} \cdot \gamma\nabla \gamma \cdot \phi + \bar{\phi}^\mu J_\mu - \frac{1}{2}\bar{\phi} \cdot \gamma\gamma \cdot J. \quad (13)$$

There has been a long history in quantizing the massless spin 3/2 field, especially as it pertains to supergravity theories. A convenient choice of gauge fixing would be to simply introduce the term

$$e^{-1}\mathcal{L}_{\text{gf}} = \frac{1}{4}\bar{\phi} \cdot \gamma\nabla \gamma \cdot \phi, \quad (14)$$

so that the resulting Feynman gauge Lagrangian has a Dirac-like form

$$e^{-1}\tilde{\mathcal{L}}_{3/2} = -\frac{1}{2}\bar{\phi}^\mu\nabla \phi_\mu + \bar{\phi}^\mu J_\mu - \frac{1}{2}\bar{\phi} \cdot \gamma\gamma \cdot J. \quad (15)$$

This gauge fixing of the spin 3/2 field is complicated by the fact that, in addition to the ordinary Faddeev-Popov ghosts, one must include a Nielsen-Kallosh ghost related to $\det \nabla$ which shows up in the Gaussian gauge fixing procedure [31,32]. This is avoided in the $\gamma \cdot \Psi = 0$ gauge [11,12].

Returning to the massive spin 3/2 field in a cosmological background, we find that the appropriate generalization of the gauge fixing term (14) is

$$e^{-1}\mathcal{L}_{\text{gf}} = -\frac{2M + \sqrt{M^2 - \Lambda}}{8\sqrt{M^2 - \Lambda}}\gamma \cdot \phi + \alpha\chi(\nabla - \sqrt{M^2 - \Lambda})(\gamma \cdot \phi + \alpha\chi), \quad (16)$$

where

$$\alpha = (2M - \sqrt{M^2 - \Lambda})\sqrt{\frac{2}{\Lambda + 3M^2}}. \quad (17)$$

The coefficients here are chosen so that the gauge fixed Lagrangian takes the simple quadratic form

$$\begin{aligned} e^{-1}\tilde{\mathcal{L}}_{3/2} = & -\frac{1}{2}\bar{\phi}_\mu(\nabla + M)\phi^\mu - \frac{1}{2}\bar{\xi}(\nabla + \sqrt{M^2 - \Lambda})\xi + \bar{\phi}^\mu J_\mu - \frac{1}{2}\bar{\phi} \cdot \gamma \gamma \cdot J \\ & + \frac{\alpha}{2}\sqrt{\frac{2}{\Lambda + 3M^2}}\bar{\phi} \cdot \gamma \mathcal{D}^\mu J_\mu + \sqrt{\frac{2 - \alpha^2}{\Lambda + 3M^2}}\bar{\xi} \mathcal{D}^\mu J_\mu, \end{aligned} \quad (18)$$

which may be compared to the massless flat space case of (15). Here ξ is a rescaled and shifted Stückelberg field

$$\xi = \sqrt{\frac{2}{2 - \alpha^2}}(\chi + \frac{1}{2}\alpha\gamma \cdot \phi). \quad (19)$$

We are now in a position to compute the gauge fixed propagator evaluated between conserved sources. The tree-level amplitude takes the form

$$\begin{aligned} \mathcal{A} = & \bar{J}_\mu \left(g^{\mu\alpha} - \frac{1}{2}\gamma^\mu \gamma^\alpha - \frac{\alpha}{2}\sqrt{\frac{2}{\Lambda + 3M^2}}\bar{\mathcal{D}}^\mu \gamma^\alpha \right) [\nabla + M]_{\alpha\beta}^{-1} \\ & \times \left(g^{\beta\nu} - \frac{1}{2}\gamma^\beta \gamma^\nu + \frac{\alpha}{2}\sqrt{\frac{2}{\Lambda + 3M^2}}\gamma^\beta \mathcal{D}^\nu \right) J_\nu \\ & + \frac{2 - \alpha^2}{\Lambda + 3M^2}\bar{J}_\mu \bar{\mathcal{D}}^\mu [\nabla + \sqrt{M^2 - \Lambda}]^{-1} \mathcal{D}^\nu J_\nu. \end{aligned} \quad (20)$$

If we take current conservation to be $\mathcal{D}^\mu J_\mu = 0$ for arbitrary mass M , we find the simple result

$$\mathcal{A} = \bar{J}_\mu \frac{1}{\nabla + M} J^\mu + \frac{1}{2}\bar{J} \cdot \gamma \frac{\nabla + 2M}{\square - M^2} \gamma \cdot J, \quad (21)$$

which has a smooth limit, both for $M^2 \rightarrow 0$ and for $M^2 \rightarrow -\Lambda/3$ [7]. However, as explained in [7], we are certainly not allowed to modify the current conservation equation, provided we wish to make a proper comparison with the massless theory.

For comparison to the massless AdS limit, we note that proper current conservation takes the form $\bar{\mathcal{D}}^\mu J_\mu \equiv (\nabla^\mu + \frac{\bar{m}}{2}\gamma^\mu)J_\mu = 0$. In this case, we find $\mathcal{D}^\mu J_\mu = \frac{1}{2}(M - \bar{m})\gamma \cdot J$. Substituting this into (20), and noting that the squares of the Dirac operators are

$$\begin{aligned} -\Delta_{(1/2)} & \equiv [\nabla_{(1/2)}]^2 = \square - \frac{1}{4}R, \\ -\Delta_{(3/2)} & \equiv [\nabla_{(3/2)}]_{\mu\nu}^2 = (\square - \frac{1}{4}R)g_{\mu\nu} + \frac{1}{2}R_{\mu\nu\alpha\beta}\gamma^{\alpha\beta}, \end{aligned} \quad (22)$$

we now find

$$\mathcal{A} = \bar{J}_\mu \frac{1}{\not{\nabla} + M} J^\mu + \bar{J} \cdot \gamma \frac{(M + 2\bar{m})\not{\nabla} + (M^2 + 2M\bar{m} + 3\bar{m}^2)}{3(M + \bar{m})(\square - M^2)} \gamma \cdot J, \quad (23)$$

which agrees with [7], and contains the spin 3/2 version of the van Dam-Veltman-Zakharov discontinuity [6,7]. Namely, in the case where both M and $\bar{m}$ approach zero, we have

$$\mathcal{A} \sim \bar{J}_\mu \frac{1}{\not{\nabla}} J^\mu + \frac{M + 2\bar{m}}{3(M + \bar{m})} \bar{J} \cdot \gamma \frac{1}{\not{\nabla}} \gamma \cdot J, \quad (24)$$

which demonstrates the sensitivity to the order of limits. In particular, taking the massless AdS limit, $M \rightarrow \bar{m}$, we recover the expected factor of 1/2 in the second term relative to the first. But taking the flat space limit, $\bar{m} \rightarrow 0$, we find instead the discontinuous factor of 1/3.

III. THE ONE-LOOP PARTITION FUNCTION

The gauge fixing term (16), corresponding to the condition $\gamma \cdot \phi + \alpha\chi = b$ (with b a constant spinor), must be accompanied by a pair of Faddeev-Popov ghosts with action connected to the variation

$$\delta_\epsilon(\gamma \cdot \phi + \alpha\chi) = -\gamma^\mu \delta_\epsilon \Psi_\mu + \alpha \delta_\epsilon \chi = -(\not{\nabla} + \sqrt{M^2 - \Lambda})\epsilon. \quad (25)$$

In addition, there is a Nielsen-Kallosh ghost [31,32] yielding the determinant of $\not{\nabla} - \sqrt{M^2 - \Lambda}$, as can be read off directly from (16). Given these considerations in the ghost sector, it is now possible to compute the one-loop partition function for the massive spin 3/2 field. Collecting together the spin 3/2 field ϕ_μ and the Stückelberg spinor ξ with Lagrangian given by (18), as well as the Faddeev-Popov and Nielsen-Kallosh ghosts, the one-loop partition function becomes

$$Z[\Psi] \propto \frac{\text{Det}_{\text{red}}[\not{\nabla}_{(3/2)} + M] \text{Det}[\not{\nabla}_{(1/2)} + \sqrt{M^2 - \Lambda}]}{\text{Det}^2[\not{\nabla}_{(1/2)} + \sqrt{M^2 - \Lambda}] \text{Det}[\not{\nabla}_{(1/2)} - \sqrt{M^2 - \Lambda}]}, \quad (26)$$

where $\not{\nabla}_{(3/2)}$ acts on *reducible* states ϕ_μ .

Evaluation of the fermion determinants is most conveniently performed by first squaring the Dirac operators corresponding to spin 1/2 and spin-3/2 fields. For Dirac fermions, we have simply

$$\begin{aligned} \text{Det}[\not{\nabla}_{(1/2)} + \sqrt{M^2 - \Lambda}] &= \text{Det}^{\frac{1}{2}}[\not{\nabla}_{(1/2)} + \sqrt{M^2 - \Lambda}] \text{Det}^{\frac{1}{2}}[\not{\nabla}_{(1/2)} - \sqrt{M^2 - \Lambda}] \\ &= \text{Det}^{\frac{1}{2}}[\Delta(\tfrac{1}{2}, 0) + \Delta(0, \tfrac{1}{2}) + M^2 - \Lambda], \end{aligned} \quad (27)$$

and

$$\begin{aligned} \text{Det}_{\text{red}}[\not{\nabla}_{(3/2)} + M] &= \text{Det}_{\text{red}}^{\frac{1}{2}}[\not{\nabla}_{(3/2)} + M] \text{Det}_{\text{red}}^{\frac{1}{2}}[\not{\nabla}_{(3/2)} - M] \\ &= \text{Det}_{\text{red}}^{\frac{1}{2}}[\Delta(\tfrac{1}{2}, 1) + \Delta(1, \tfrac{1}{2}) + M^2], \end{aligned} \quad (28)$$

where $\Delta(A, B)$ are the Laplacians acting on irreducible (A, B) representations of the Lorentz group [11,12]. In particular, $\Delta(\frac{1}{2}, 0)$ and $\Delta(\frac{1}{2}, 1)$ are the chiral components of the operators given in (22). In terms of these determinants, the partition function, (26), becomes

$$Z[\Psi] \propto \text{Det}_{\text{red}}^{\frac{1}{2}}[\Delta_{(3/2)} + M^2] \text{Det}^{\frac{1}{2}}[\Delta_{(1/2)} + M^2 - \Lambda] \\ \times \text{Det}^{-1}[\Delta_{(1/2)} + M^2 - \Lambda] \text{Det}^{-\frac{1}{2}}[\Delta_{(1/2)} + M^2 - \Lambda]. \quad (29)$$

Note that the appropriate operators are given by the non-chiral combinations $\Delta_{(1/2)} = \Delta(\frac{1}{2}, 0) + \Delta(0, \frac{1}{2})$ and $\Delta_{(3/2)} = \Delta(\frac{1}{2}, 1) + \Delta(1, \frac{1}{2})$. Because the ghost and Stückelberg determinants have the same form, there is a partial cancellation in the partition function. In addition, we note from (22) that $\Delta_{(3/2)} = \Delta_{(1/2)} g_{\mu\nu} - \frac{1}{2} R_{\mu\nu\alpha\beta} \gamma^{\alpha\beta}$. Thus when acting on a pure gamma-trace field $\phi_\mu = \gamma_\mu \chi$, we find (in an Einstein background)

$$\bar{\chi} \gamma_\mu [\Delta_{(3/2)}^{\mu\nu} + M^2 g^{\mu\nu}] \gamma_\nu \chi = 4 \bar{\chi} [\Delta_{(1/2)} + M^2 - \Lambda] \chi, \quad (30)$$

so that, up to a constant

$$\text{Det}_{\text{red}}[\Delta_{(3/2)} + M^2] = \text{Det}[\Delta_{(3/2)} + M^2] \text{Det}[\Delta_{(1/2)} + M^2 - \Lambda]. \quad (31)$$

where $\Delta_{(3/2)}$ on the right hand side acts on gamma-traceless spin-3/2 fields, $\gamma \cdot \phi = 0$. As a result, the one-loop effective action for the spin-3/2 field in an Einstein background takes on the simple form

$$\Gamma^{(1)}[\Psi] = -\ln Z[\Psi] = -\frac{1}{2} \ln \text{Det}[\Delta_{(3/2)} + M^2] + \frac{1}{2} \ln \text{Det}[\Delta_{(1/2)} + M^2 - \Lambda]. \quad (32)$$

For comparison, in the strictly massless case, $M^2 = \bar{m}^2$, there is no Stückelberg field, and the partition function has the form

$$Z[\Psi]_{\text{massless}} \propto \frac{\text{Det}_{\text{red}}[\nabla_{(3/2)} + \bar{m}]}{\text{Det}^2[\nabla_{(1/2)} + 2\bar{m}] \text{Det}[\nabla_{(1/2)} - 2\bar{m}]}. \quad (33)$$

This leads instead to an effective action

$$\Gamma^{(1)}[\Psi]_{\text{massless}} = -\frac{1}{2} \ln \text{Det}[\Delta_{(3/2)} + \bar{m}^2] + \ln \text{Det}[\Delta_{(1/2)} + 4\bar{m}^2]. \quad (34)$$

The difference in these two expressions reflects the fact that in the massive case the Stückelberg (or Goldstino) spinor provides an additional two degrees of freedom to the gravitino. This provides the correct counting of degrees of freedom, namely two for the massless case and four for the massive one.

The actual forms for the determinants of the above operators may be computed in a heat-kernel expansion. We focus on the coefficient functions $b_k^{(\Delta)}$ in the expansion

$$\text{Tr} e^{-\Delta t} = \sum_{k=0}^{\infty} t^{(k-4)/2} \int d^4 x \sqrt{g} b_k^{(\Delta)}, \quad (35)$$

which were calculated in [11,12] for the general operators $\Delta(A, B)$ and generalized in [33,23] to allow for a cosmological constant. In particular,

$$\begin{aligned}
180(4\pi)^2 b_4(\tfrac{1}{2}, 0) &= -\frac{7}{4} R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} + 12\Lambda^2 - \frac{15}{4} R_{\mu\nu\rho\sigma}^* R^{\mu\nu\rho\sigma}, \\
180(4\pi)^2 b_4(1, \tfrac{1}{2}) &= \frac{219}{4} R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} - 24\Lambda^2 + \frac{285}{4} R_{\mu\nu\rho\sigma}^* R^{\mu\nu\rho\sigma},
\end{aligned} \tag{36}$$

It is also easy to generalize the results (by summing over chiralities) to encompass the reducible Dirac combinations $\Delta_{(1/2)}$ and $\Delta_{(3/2)}$ that are of present interest. For the b_4 coefficients, we find

$$\begin{aligned}
180(4\pi)^2 b_4^{(1/2)} &= -\frac{7}{2} R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} + 24\Lambda^2, \\
180(4\pi)^2 b_4^{(3/2)} &= \frac{219}{2} R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} - 48\Lambda^2.
\end{aligned} \tag{37}$$

Given the above b_4 coefficients, it is simple to extend these results to cover the relevant massive operators, $\Delta_{(3/2)} + M^2$ and $\Delta_{(1/2)} + M^2 - \Lambda$. As was done in [12,10], we note that for constant X , the b_4 coefficient for the operator $\Delta - X$ is given by the sum $b_4^{(\Delta-X)} = b_4 + Xb_2 + \frac{1}{2}X^2b_0$. We obtain the resulting coefficients

$$\begin{aligned}
180(4\pi)^2 b_4^{(\Delta_{(1/2)}+M^2-\Lambda)} &= -\frac{7}{2} R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} + 144\Lambda^2 - 480\Lambda M^2 + 360M^4, \\
180(4\pi)^2 b_4^{(\Delta_{(3/2)}+M^2-\Lambda)} &= \frac{219}{2} R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} - 48\Lambda^2 + 1440\Lambda M^2 + 1080M^4.
\end{aligned} \tag{38}$$

These b_4 coefficients are perfectly smooth functions of M^2 . Thus we may compare the results for the massive gravitino in the limit $M^2 \rightarrow -\Lambda/3$

$$\begin{aligned}
180(4\pi)^2 b_4^{(\text{massive})} &= 180(4\pi)^2 \left[b_4^{(\Delta_{(3/2)}+M^2)} - b_4^{(\Delta_{(1/2)}+M^2-\Lambda)} \right] \\
&\rightarrow 113 R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} - 752\Lambda^2,
\end{aligned} \tag{39}$$

which clearly differs from the pure massless ($M^2 = -\Lambda/3$) result

$$\begin{aligned}
180(4\pi)^2 b_4^{(\text{massless})} &= 180(4\pi)^2 \left[b_4^{(\Delta_{(3/2)}+\overline{m}^2)} - 2b_4^{(\Delta_{(1/2)}+4\overline{m}^2)} \right] \\
&= \frac{233}{2} R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} - 1096\Lambda^2.
\end{aligned} \tag{40}$$

Even for a constant curvature background,

$$R_{\mu\nu\rho\sigma} = (\Lambda/3)(g_{\mu\nu}g_{\rho\sigma} - g_{\mu\rho}g_{\nu\sigma}), \quad R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} = \frac{8}{3}\Lambda^2, \tag{41}$$

there is no cancellation. This demonstrates that, at the quantum level, there remains a distinction between a pure massless spin-3/2 field and the massless limit of a massive one. The difference in the b_4 coefficients may be completely attributed to the presence of a single additional Stückelberg spinor generating the additional degrees of freedom for a massive field. Note also that by subtracting the b_4 for opposite chiralities we recover the result that the gravitational axial anomaly for a spin 3/2 particle is -21 times that of a spin 1/2 particle in the massless case, but -20 times in the massless limit of the massive case [11,12], a result that is insensitive to the presence of the cosmological constant.

Note, however, that these discontinuities are essentially only one between theories with different numbers of degrees of freedom. In a spontaneously broken supergravity theory, where the gravitino picks up a mass, it does so by eating a Goldstino field that would be recovered in the unbroken limit. In this case no such discontinuity arises, since the original theory necessarily contains spin-1/2 degrees of freedom in addition to the gravitino, one combination of which will eventually provide the appropriate Goldstino combination when supersymmetry is broken.

IV. MASSIVE SUPERMULTIPLETS

The above computation of the b_4 coefficient for massive spin-3/2, along with the spin-2 results of [8] now allow us to complete the picture for all spins ≤ 2 . For AdS representations $D(E_0, s)$ with $s \leq 2$, the formal expressions for the one-loop partition functions are given in Table I for the massive case and Table II for the massless case. We note that, for spins $s = 1, 3/2$ and 2 , these expressions are compatible with the AdS Higgs mechanism, (10), in the sense that no discontinuity arises in the massless limit, provided the proper spin $s - 1$ Goldstone field is included.

Following the discussion of the previous section, we may determine the appropriate b_4 coefficients for both massive and massless fields in the Einstein background. The resulting coefficients are given in Table III. This now allows us to examine some implications for massive supermultiplets. We begin with $N = 1$ AdS supergravity, with supergroup $OSp(1|4)$. Excluding the supersingleton, we may evaluate the contribution to the b_4 coefficients from the various supermultiplets. For the Wess-Zumino multiplet

$$\mathcal{D}(E_0, 0) = D(E_0, 0) + D(E_0 + \tfrac{1}{2}, \tfrac{1}{2}) + D(E_0 + 1, 0), \quad (42)$$

we find

$$180(4\pi)^2 b_4(E_0, 0) = \frac{15}{4} R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} + \Lambda^2 [-20 + 60(E_0 - 1)^2], \quad (43)$$

while for massless multiplets

$$\mathcal{D}(s + 1, s) = D(s + 1, s) + D(s + \tfrac{3}{2}, s + \tfrac{1}{2}) \quad (s > 0), \quad (44)$$

we have

$$180(4\pi)^2 b_4(s + 1, s) = \begin{cases} -\frac{45}{4} R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} - 60\Lambda^2 & s = \tfrac{1}{2} \text{ (Maxwell)}, \\ -\frac{285}{4} R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} + 500\Lambda^2 & s = 1 \text{ (gravitino)}, \\ \frac{615}{4} R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} - 1540\Lambda^2 & s = \tfrac{3}{2} \text{ (graviton)}. \end{cases} \quad (45)$$

More illuminating, perhaps, is the E_0 dependence of the b_4 coefficient for a generic massive $N = 1$ multiplet

$$\mathcal{D}(E_0, s) = D(E_0, s) + D(E_0 + \tfrac{1}{2}, s + \tfrac{1}{2}) + D(E_0 + \tfrac{1}{2}, s - \tfrac{1}{2}) + D(E_0 + 1, s) \\ (E_0 > s + 1; s > 0). \quad (46)$$

For spins less than two, we find

$$180(4\pi)^2 b_4(E_0, s) = \begin{cases} -\frac{15}{2} R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} + \Lambda^2[-20 - 120(E_0 - \frac{1}{2})(E_0 - \frac{3}{2})] & s = \frac{1}{2} \text{ (massive vector),} \\ -\frac{315}{4} R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} + \Lambda^2[240 + 180E_0(E_0 - 2)] & s = 1 \text{ (massive spin-}\frac{3}{2}\text{),} \\ 75 R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} + \Lambda^2[-760 - 240(E_0 + \frac{1}{2})(E_0 - \frac{5}{2})] & s = \frac{3}{2} \text{ (massive spin-2).} \end{cases} \quad (47)$$

In particular, potential terms proportional to E_0^3 and E_0^4 are absent in the above. This indicates that $N = 1$ supersymmetry provides a partial cancellation in the one-loop partition function, and furthermore suggests that complete cancellation may be obtained in N -extended supersymmetry.

This possibility for a cancellation among b_4 coefficients is most readily seen in the case of $N = 8$ AdS supergravity. To see this, we first recall that the one-loop divergences related to the b_4 coefficient may be canceled by the introduction of counterterms [34]

$$\gamma^{(s)} = \int d^4x \sqrt{g} b_4^{(s)} = A\chi + B\delta, \quad (48)$$

where χ is the Euler character and $\delta = \frac{1}{12\pi^2} \int d^4x \sqrt{g}$. The same combinations determine the trace anomaly. It was shown in [23] that, for the massless $SO(8)$ gauged $N = 8$ supergravity multiplet, the total B coefficient vanishes, *i.e.* the cosmological constant Λ is not renormalized. Since Λ is related to the $SO(8)$ coupling constant e by $3e^2 = -8\pi G\Lambda$, where G is Newton's constant, this implies a vanishing one loop beta function $\beta(e)$.

It was further shown in [24,25], using zeta function methods, that the vanishing of $\beta(e)$ continues to hold even for the massive Kaluza-Klein tower arising from the round seven-sphere compactification of eleven dimensional supergravity. Having obtained in this paper the massive spin $3/2$ b_4 coefficient in the presence of a cosmological constant, we now have the complete set for spins $0 \leq s \leq 2$, as given in Table III. So we can perform a novel calculation of the beta function directly from the b_4 coefficients. Using the well known spectrum shown in Table IV [28,29], and the b_4 coefficients of Table III, we obtain (at level n)

$$180(4\pi)^2 b_4^{(N=8)} = -450 d(n, 0, 0, 0) R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} \quad (49)$$

(valid for $n \geq 0$) where

$$\begin{aligned} d(a_1, a_2, a_3, a_4) = & (1 + a_1)(1 + a_2)(1 + a_3)(1 + a_4) \left(1 + \frac{a_1 + a_2}{2}\right) \left(1 + \frac{a_3 + a_2}{2}\right) \left(1 + \frac{a_4 + a_2}{2}\right) \\ & \left(1 + \frac{a_1 + a_3 + a_2}{3}\right) \left(1 + \frac{a_1 + a_4 + a_2}{3}\right) \left(1 + \frac{a_3 + a_4 + a_2}{3}\right) \\ & \left(1 + \frac{a_1 + a_3 + a_4 + a_2}{4}\right) \left(1 + \frac{a_1 + a_3 + a_4 + 2a_2}{5}\right) \end{aligned} \quad (50)$$

is the Weyl formula for the dimension of the $SO(8)$ representation given by Dynken label (a_1, a_2, a_3, a_4) .

This result confirms that the total beta function vanishes, level by level, for the entire massive Kaluza-Klein tower. In fact, recalling that the massive tower is obtained by tensoring the $(n, 0, 0, 0)$ representation (related to appropriate spherical harmonics on S^7) with the massless supergraviton multiplet and Higgsing the result, (49) is naturally interpreted as a contribution from $d(n, 0, 0, 0)$ copies of the massless multiplet, each one of which carries $180(4\pi)^2 b_4 = -450 R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma}$.

V. CONCLUSION

We have seen that a cosmological constant cannot cure the VVZ problem for spin 3/2 at the quantum level since loop diagrams care that a massive gravitino has four degrees of freedom and not two. This is in keeping with a similar result for spin 2 [8].

An interesting question is whether these spin 2 and spin 3/2 discontinuities appear in the Karch-Randall braneworld [14]. According to [22], the spin 2 (and, by implication, the spin 3/2) discontinuity is absent. If we try to analyze this from a brane, as opposed to bulk, perspective we see that the graviton acquires a mass by eating a massive spin one bound state of the CFT living on the brane [15], a phenomenon peculiar to AdS. In the supersymmetric case, the whole graviton supermultiplet acquires a mass by eating a supermultiplet of bound states of the SCFT ($N = 4$ Yang-Mills in the maximally supersymmetric case) [16]. However, the massless limit is much more subtle here because this is also the limit in which the AdS brane becomes Minkowski and for which, therefore, the bound states no longer appear. Moreover, the propagators are no longer simply given by the inverses of the Δ operators given in [8] and in section II of this paper. We intend to return to this issue elsewhere.

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TABLES

spin	massive partition function
0	$\det^{-\frac{1}{2}}[\Delta(0,0) + E_0(E_0 - 3)]$
1/2	$\det^{\frac{1}{2}}[\Delta_{(1/2)} + (E_0 - \frac{3}{2})^2]$
1	$\det^{\frac{1}{2}}[\Delta(0,0) + (E_0 - 1)(E_0 - 2)] \det^{-\frac{1}{2}}[\Delta(\frac{1}{2}, \frac{1}{2}) + (E_0 - 1)(E_0 - 2)]$
3/2	$\det^{\frac{1}{2}}[\Delta_{(3/2)} + (E_0 - \frac{3}{2})^2] \det^{-\frac{1}{2}}[\Delta_{(1/2)} + (E_0 - \frac{3}{2})^2 + 3]$
2	$\det^{\frac{1}{2}}[\Delta(\frac{1}{2}, \frac{1}{2}) + E_0(E_0 - 3) + 6] \det^{-\frac{1}{2}}[\Delta(1,1) + E_0(E_0 - 3) + 6]$

TABLE I. One loop partition functions for massive fields of spins ≤ 2 . The E_0 values are given in units of $\overline{m}$.

spin	E_0	massless partition function
0	1 or 2	$\det^{-\frac{1}{2}}[\Delta(0,0) - 2\overline{m}^2]$
1/2	3/2	$\det^{\frac{1}{2}}[\Delta_{(1/2)}]$
1	2	$\det[\Delta(0,0)] \det^{-\frac{1}{2}}[\Delta(\frac{1}{2}, \frac{1}{2})]$
3/2	5/2	$\det^{\frac{1}{2}}[\Delta_{(3/2)} + \overline{m}^2] \det^{-1}[\Delta_{(1/2)} + 4\overline{m}^2]$
2	3	$\det[\Delta(\frac{1}{2}, \frac{1}{2}) + 6\overline{m}^2] \det^{-\frac{1}{2}}[\Delta(1,1) + 6\overline{m}^2] \det^{-\frac{1}{2}}[\Delta(0,0) + 6\overline{m}^2]$

TABLE II. One loop partition functions for massless fields. Gauge invariance leads to a modified ghost structure for spins 1, 3/2 and 2 compared to Table I. The fermions are taken to be Dirac.

spin		$180(4\pi)^2 b_4$
0		$R_{\mu\nu\rho\sigma}^2 + \Lambda^2[36 + 40E_0(E_0 - 3) + 10E_0^2(E_0 - 3)^2]$
1/2		$-\frac{7}{4}R_{\mu\nu\rho\sigma}^2 + \Lambda^2[12 - 40(E_0 - \frac{3}{2})^2 + 20(E_0 - \frac{3}{2})^4]$
1	$E_0 = 2$	$-13R_{\mu\nu\rho\sigma}^2 - 48\Lambda^2$
	$E_0 > 2$	$-12R_{\mu\nu\rho\sigma}^2 + \Lambda^2[-132 + 30E_0^2(E_0 - 3)^2]$
3/2	$E_0 = 5/2$	$\frac{233}{4}R_{\mu\nu\rho\sigma}^2 - 548\Lambda^2$
	$E_0 > 5/2$	$\frac{113}{2}R_{\mu\nu\rho\sigma}^2 + \Lambda^2[-96 - 320(E_0 - \frac{3}{2})^2 + 40(E_0 - \frac{3}{2})^4]$
2	$E_0 = 3$	$212R_{\mu\nu\rho\sigma}^2 - 2088\Lambda^2$
	$E_0 > 3$	$200R_{\mu\nu\rho\sigma}^2 + \Lambda^2[2460 - 1000(E_0^2 - 3E_0 + 6) + 50(E_0^2 - 3E_0 + 6)^2]$

TABLE III. b_4 coefficients for fields of spins ≤ 2 . Note that here the fermions are taken to be Majorana.

spin	$SO(8)$ irrep	E_0
2^+	$(n, 0, 0, 0)$	$3 + n/2$
$\frac{3}{2}^{(1)}$	$(n, 0, 0, 1)$	$\frac{5}{2} + n/2$
$\frac{3}{2}^{(2)}$	$(n - 1, 0, 1, 0)$	$\frac{7}{2} + n/2$
$1^{-(1)}$	$(n, 1, 0, 0)$	$2 + n/2$
1^+	$(n - 1, 0, 1, 1)$	$3 + n/2$
$1^{-(2)}$	$(n - 2, 1, 0, 0)$	$4 + n/2$
$\frac{1}{2}^{(1)}$	$(n + 1, 0, 1, 0)$	$\frac{3}{2} + n/2$
$\frac{1}{2}^{(2)}$	$(n - 1, 1, 1, 0)$	$\frac{5}{2} + n/2$
$\frac{1}{2}^{(3)}$	$(n - 2, 1, 0, 1)$	$\frac{7}{2} + n/2$
$\frac{1}{2}^{(4)}$	$(n - 2, 0, 0, 1)$	$\frac{9}{2} + n/2$
$0^{+(1)}$	$(n + 2, 0, 0, 0)$	$1 + n/2$
$0^{-(1)}$	$(n, 0, 2, 0)$	$2 + n/2$
$0^{+(2)}$	$(n - 2, 2, 0, 0)$	$3 + n/2$
$0^{-(2)}$	$(n - 2, 0, 0, 2)$	$4 + n/2$
$0^{+(3)}$	$(n - 2, 0, 0, 0)$	$5 + n/2$

TABLE IV. The spectrum of supergravity on the round seven-sphere. For $n = 0$ or 1 , states with negative labels are not present.

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