

ERRATA

- p. 8 Line 12 "vigorous" should read "rigorous".
- p. 11 Line 16 "CHARNAT" should read "CHARWAT".
- p. 22 Insert "Other typical conditions in the free stream are,
ionization fraction $\alpha = 0.2\%$, ion density $n_i = 5 \times 10^{18}/\text{m}^3$,
Debye length $\lambda_D = 10^{-3} \text{ m.m.}$ " after line 12.
- p. 34 Insert "pressure" after "pitot" in line 22.
- p. 42 Insert "value" after "stream" in line 9.
- p. 44 Insert "measured at different distances from the model. The shape
of these characteristics was" after line 26.
- p. 47 Insert "used" after "tubes" in line 18.
- p. 56 Line 9. "low" should read "bow".
- p. 57 Line 15. "new" should read "near".
- p. A2 Line 17. "Schmitt" should read "Schmidt".

A STUDY OF HIGH ENTHALPY RAREFIED HYPERSONIC FLOW

by

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ABSTRACT

The high speed flow of a monatomic gas about a cold flat plate has been examined experimentally in the intermediate regime between continuum and free molecular flows. Surface pressure and flow field density, obtained from an optical technique, are compared with direct simulation Monte-Carlo calculations of monatomic flows about flat plates at zero incidence. Measurements were also conducted with the plate at positive and negative incidence, for which there are at present no satisfactory theoretical predictions in the intermediate region.

The use of probes for examining the structure of the flow field is discussed, and they are shown to be unsatisfactory under the present conditions.

An analysis of the diffusion of species in a slip flow and a technique for measuring velocity in the free stream of an arc heated wind tunnel are described in appendices.

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NOTATION

A	U_e/U_∞
C	Chapman-Rubensin Constant, and Molecular Speed
D	Diameter, and Diffusion Coefficient
F(M)	Defined by equation (3.1)
H	Total Enthalpy
I	Current
Kn	Knudsen Number
L	Length
M	Mach Number
N	Number Flux
P	Pressure
R	Gas Constant
Re	Reynolds Number
S	Speed Ratio
Sc	Schmidt Number
T	Temperature
U	Velocity
$\bar{V}$	$M(C/Re_x)^{1/2}$
Z	α/α_e
e	Electron Charge
f(M)	Defined by equation (4.1)
f	Frequency
k	Catalytic Rate Constant
n	Number Density
s	Distance
u,v	Velocity Components
x	Streamwise Co-ordinate

y	Co-ordinate perpendicular to x.
α	Ionisation Fraction, and surface angle relative to x
γ	Ratio of Specific Heats
ζ	Defined by equation (A.3)
η	Co-ordinate Transform $\frac{\rho_e u_e}{(2\zeta)^{1/2}} \int \frac{\rho}{\rho_e} dy$
θ	Angle and $\zeta/3$
λ_∞	Free Stream Mean Free Path
λ_f	Distance Travelled by a Free Stream Molecule Before Striking a Body Molecule
λ_B	Distance Travelled by a Body Molecule Before Striking a Free Stream Molecule
μ	Viscosity
σ	Tangential Momentum Accommodation Coefficient
$\bar{\sigma}$	$(2-\sigma)/\sigma$
ρ	Density
ξ	$\int_0^x \rho_e \mu_e u_e dx$
$\bar{X}$	$u_\infty^3 (C/Re_x)^{1/2}$
Ω	Collision Cross Section

Subscripts

c	Cavity
e	Evaluated at edge of viscous layer
F.M.	Free Molecular
i	Ion
o	Isentropic Stagnation Value, and on page 31 only, Conditions at S.T.P.
s	Shock
w	Wall Value
∞	Free Stream

2 Conditions Behind a Rankine Hugoniot Shock

1. INTRODUCTION

The advent of space flight has generated extensive interest in the flow of rarefied gases about bodies at hypersonic speeds, particularly in the "transition regime" between free molecular flow and continuum flow. To date, the main practical requirement in space vehicle design has been for reliable drag and heat transfer data. However, if re-usable vehicles are to fly down to earth, as is proposed in the current "space shuttle", a much better understanding of the flow processes will be necessary in order to accurately predict the aerodynamic forces operating on the vehicle.

Part of the flight path of such a vehicle would be in the transition regime, and the problem is likely to be complicated by non-equilibrium real gas effects. It is most unlikely that an experimental simulation of all these factors will be achieved simultaneously and so it will be necessary to deduce conditions at the vehicle surface from calculations of the flow field about the vehicle.

The transition regime is also of academic interest, as a solution of Boltzman's transport equation is required for a complete description of the regime. This is a particularly challenging problem as neither small nor large Knudsen number assumptions may be employed, and the hypersonic nature of the flow precludes the use of linearisation techniques.

A detailed survey of current knowledge about transitional flows is presented in the next chapter, and the

need for an experimental study of the transitional flow of a monatomic gas about a body is indicated. In subsequent chapters, experiments in which a flat plate model was set at both positive and negative angles of incidence, whilst in an argon stream, are discussed. For the sake of clarity, much of the detailed description of apparatus and experimental technique is relegated to a series of appendices. Two related but separate studies are also presented as appendices.

2. SURVEY OF RAREFIED FLOW STUDIES

2.1. Early Developments.

The use of rockets for military and space research purposes has motivated much research in high speed rarefied flows during the last twenty years. Initially, interest was concentrated on obtaining information of direct aerodynamic utility, such as drag, rather than developing a detailed understanding of the flow field. This early work, and indeed most of that which has followed, may be classified as blunt body or slender body orientated, the relative emphasis on these two aspects being influenced by current fashions in rocket nose geometry. Potter⁵ conducted an extensive review of the work performed up to the date of his paper and included a discussion of experimental techniques as well as flow about bodies.

Within the slender body work, interest has been mainly concentrated on flow over a sharp flat plate parallel to the stream. With this geometry, the substantial flow disturbance caused by the viscous interaction of the plate with the flow may be studied separately from those effects arising directly from the displacement of the stream by the body surface. Ideally the work on flat plates should have been supplemented with studies of flows about very slender needles to assess the effects of three-dimensionality, but consideration of the experimental difficulties seems to have prevented this.

Studies of free molecular flow and inviscid and

viscous continuum flows were well advanced before serious thought was given to the transition regime in between. Indeed early efforts in this regime were mainly concerned with fitting smoothed curves to properties between the free molecular and inviscid limits.

Interest in rarefied gas flows was evident at least one hundred years ago, since in 1875 Kundt⁸ noted an anomaly in the rate of discharge from tubes at low densities, and this led Maxwell⁹ to consider conditions at the interface between a gas and a solid. Knudsen¹⁰ conducted extensive theoretical and experimental investigations of low speed free molecular flows which have been extended to high speed flows by several authors, including Tsien¹¹. There now exists a complete theory for free molecular flows and this is described in a number of standard texts, such as that by Paterson¹², who also provides an exhaustive list of references.

The essential assumptions of free molecular flow theory are that, i) the velocity distribution of the molecules is Maxwellian, ii) the incident and emergent molecular fluxes at surfaces do not interact and, iii) all physical dimensions are small compared to the mean free path. This last point is emphasized since it is often assumed that free molecular conditions must exist at the leading edge of a flat plate, although downstream influences may in fact preclude this.

With the assumptions listed above it is possible to calculate the flux of molecules to a surface. Hence with suitable assumptions about interaction processes occur-

ring at the surface, aerodynamic quantities such as pressure and heat transfer may be calculated. It is interesting to note that Newton also employed the last two assumptions listed, in the theory that bears his name. However the first assumption was replaced with that of zero temperature. This precludes lateral motion so the theory fails to predict the interaction of a surface with a parallel stream of molecules.

If hypersonic flow over a flat plate is considered far down stream, the flow model consists of a Mach line emanating from the leading edge and a very thin boundary layer at the wall, the intervening region being inviscid. Advancing towards the leading edge, the boundary layer thickness becomes significant compared with the distance from the plate surface to the Mach line, so the inviscid flow is deflected. This generates a thin shock wave of finite strength in lieu of the Mach line.

Lees¹³ distinguished two distinct regions, weak interaction and strong interaction, where the coupling between the shock wave and boundary layer are weak and strong, respectively. A simple consideration of average properties in the boundary layer, shows that in both regions, the appropriate scaling quantity is the interaction parameter $\bar{\chi}$, given by $\bar{\chi} = M_\infty^3 C_\infty^{1/2} / Re_{x_\infty}^{1/2}$.

Hayes and Probstein¹⁴ and Dorrance¹⁵ provide extensive discussions of weak and strong interaction in which the early theories are related and combined to give a comprehensive theory covering different values of wall temperature and specific heat ratio. These authors

note that in limited ranges the two interaction theories have been successfully compared with measured values of surface quantities, such as heat transfer and pressure, but as the leading edge is approached, the theories are in error since they predict infinite pressure and heat transfer.

Upstream of the strong interaction region there is a rarefied flow region in which the surface pressure rises to a plateau and then descends towards its free molecular value at the leading edge. Departure from strong interaction was observed experimentally by Schaaf¹⁶ and has been widely confirmed.

Talbot¹⁷ considered the Knudsen number behind the shock in the strong interaction region and noted that this was closely related to the rarefaction parameter, given by $\bar{V}_\infty = M_\infty C_\infty^{\frac{1}{2}} / Re_{x_\infty}^{\frac{1}{2}}$. This was used to correlate the departure of surface pressure data from that predicted by strong interaction.

It has been noted that this departure occurs at $\bar{V}_\infty \approx 0.15$. However, strong interaction theory does not apply unless $\bar{\kappa} \approx 10$, so it is inapplicable unless $M_\infty \approx 8$. Hence if $M \lesssim 8$, the flow changes directly from weak interaction to merging.

As the flow becomes more rarefied, slip occurs i.e. the velocity of the gas at the surface is no longer zero. Aroesty¹⁸, extended the strong interaction theory to include the effects of slip. Experiments by Wallace and Burke⁴ suggest that this extends the useful range of

the theory for the calculation of heat transfer and skin friction. There was little improvement in the case of surface pressure, probably because this is more critically dependent on flow structure.

The rarefied flow region upstream of the strong interaction region was subdivided into several regimes by Hayes and Probstein¹⁴, but as they were considering general body shapes, some of the regimes are indistinguishable or irrelevant to the flat plate case. However it is apparent that at least two major regimes exist, namely the merged layer and kinetic regimes.

2.2. The merged layer regime.

Initial experimental investigations of the merged layer regime consisted mainly of measurements of conditions at the surface. An attempt was made by Negamatsu and Sheer¹⁹ to determine shock shape from Schlieren photographs, although the results obtained have subsequently been shown to be incorrect.

Considerable disparity exists between the values of surface pressure obtained, even between experiments conducted under apparently similar conditions. This may now be explained in terms of the neglect, or incomplete application, of corrections for the static pressure tapping orifice effect, as will be discussed in section 2.4. However the early experiments provided qualitative information for comparison with theoretical models. Much of this work was reported at a series of symposia¹⁻⁷ up to that containing Potter's⁵ review paper which also references the major contributions appearing in other

publications.

Various approaches have been adopted in theoretical studies of the merged layer, mainly based on the Navier-Stokes equations. This may be criticised as the Navier-Stokes equations are only valid if the gas is in Maxwellian equilibrium, which may not be the case in the shock region. Non-equilibrium is likely to be dominant at the upstream end of the merged layer regime, as the flow consists mainly of particles from the free stream and those emitted by the body which have quite different properties. However the Navier-Stokes equations provide some understanding of the flow processes, and may be taken as a frame-work for a more vigorous kinetic theory as this becomes feasible.

One approach to the problem, contained in a series of separate papers by Oguchi^{2,3,20} and Probst^{21,22}, has been to separate the flow field into a boundary layer like region and shock region; conditions along the interface of these regions being matched.

In his most recent paper, Oguchi²⁰ employed shock layer equations derived from Navier-Stokes equations, in conjunction with the compressible boundary layer equations, to obtain results which are in qualitative but not quantitative agreement with experimental results. Probst²² extended this to include the effects of slip and shock curvature which produced much better agreement with experiment.

Chow²³ also obtained a solution by matching a boundary layer region to a shock region. He employed a

cubic-parabolic velocity profile in the boundary layer and so included slip directly in the analysis. Shock curvature was not considered but the results were again found to be in reasonable agreement with experiment.

As an alternative approach to the merged layer problem, various attempts have been made to solve the full Navier-Stokes equations for flow over a flat plate.

Butler²⁴ solved the time dependent Navier-Stokes equations numerically for a flat plate started impulsively from rest. The steady flow properties were obtained from the long time solution. Significant differences exist between these results and the available experimental results, which is probably due to the large mesh size employed. An improvement may be anticipated as computers having much greater capacity become available.

In separate studies Cheng et al⁶, and Rudman and Rubin²⁵, obtained numerical solutions to the steady state Navier-Stokes equations. This greatly reduces the amount of computation necessary. However, the equations are parabolic so the initial conditions have to be specified. It is also necessary to assume that the disturbed region is small, i.e. $\frac{y}{x} \ll 1$, but the shock angle,

$\Theta_s = 0(0.4)$, so the assumption is not well founded. However, the results obtained are in fair agreement with experiment; although, as in the case of the solutions obtained by matching, the comparison remains inconclusive.

The extension of the early theoretical models to

include a shock region of finite extent was suggested by several experimental studies in which the flow was examined with hot wire and pitot probes. Experiments reported by Becker et al⁵ and McCroskey et al²⁶, clearly demonstrated the existence of a finite shock region and showed that it merged with the boundary layer.

A technique proposed by Muntz²⁷, in which a beam of electrons locally excites fluorescence, has been widely used^{5,6,7,28,29} to measure density and temperature within the flow field. Most of these studies also include measurements in the kinetic regime.

Lillicrap²⁹ conducted a detailed comparison of the theories discussed above with measurements he made using the electron beam technique. He found that even the more complete theories given in references 22 and 25 gave an inaccurate prediction of flow properties in the shock region, although the trends are qualitatively correct to within $O(10\lambda_w)$ from the leading edge.

In view of this, the suitability of Navier-Stokes equations in the shock layer needs to be considered further. It is well known that Navier-Stokes equations do not provide a satisfactory description of the structure of normal shocks if $M \gg 1.5$, as discussed, for example, by Guiraud⁵. This is because at higher Mach numbers, the shock thickness ceases to be large compared with the mean free path and so non-equilibrium occurs. The limitation is not likely to be so drastic for oblique shocks, as the path length within the shock is greater, and so the number of molecular collisions is increased. Even

so, if it is assumed that the flow is approximately parallel to the surface, and that the maximum Mach number is proportional to path length in the shock, then Navier-Stokes equations are only likely to be satisfactory if $M \lesssim 4$, for a shock angle, $\theta_s = 0.4$. It is therefore unlikely that theories based on Navier-Stokes equations will accurately predict the shock region structure at high Mach numbers, and so a kinetic theory description is required.

2.3 Kinetic Regime

The region between the merged layer regime and the leading edge is generally known as the kinetic regime. The down stream limit is reached when the continuum equations used to describe the merged layer regime become applicable.

At the leading edge, free molecular flow is likely to occur and this led Charnat² to consider the effect of the collision of molecules emerging from the body, on the distribution of the incoming stream. Analytic results were obtained by restricting the discussion to the effect of the first collision. Clearly the method is not likely to be accurate, but it does predict the main trends of the transition region, such as a plateau in the surface pressure distribution.

In order to obtain an accurate description of the kinetic regime, it is necessary to obtain a solution to the Boltzman transport equation. Since this is an integro-differential equation, containing up to seven independent variables, with a complicated non linear

integral, the prospect of obtaining a complete solution is remote.

The classical Chapman-Enskog and moment methods have not proved useful in dealing with aerodynamically interesting problems as they do not yield solutions for high Mach number flow. Methods of obtaining solutions to Boltzmann's equation over a wide range of particular conditions are discussed in an extensive review by Guiraud⁵, who also discusses the evidence for the unique validity of the equation and the mathematical techniques employed in its solution.

A technique which has been successfully applied to a wide range of low speed problems consists of replacing the complicated collision integral with a more simple function. The most elementary of these model equations is the B.G.K. model, in which the collision integral is replaced by a term proportional to disturbance from Maxwellian equilibrium.

Huang and Hartley³⁰ employed the B.G.K. model to compute the flow over a flat plate at $M=1.5$, using finite difference methods. The results they obtained were in good agreement with experimental data, but when the calculations were repeated for $M=5$, the agreement with experiment was poor. This failure was thought to be due to the inability of the linear B.G.K. model to accommodate the non-linear situations arising in high speed flow. However Giddens et al.³¹ have since successfully used the method to predict the structure of a normal shock at $M=10$, so the reason for the failure of the flat

plate calculations remains uncertain.

An alternative method of obtaining information about the kinetic regime, by computational means, is to perform a numerical experiment in which the molecular interactions are prescribed. There are two distinct variations of this, so called, Monte-Carlo method. In the first, a test particle is followed through a cloud of particles whose distribution functions are presumed to be known from a previous iteration, the process being repeated many times until conditions become stable. The difficulty with the method is that the distribution functions for all the particles have to be stored, so the capacity of existing computers limits the application of the method to very simple problems.

The difficulty of storing distribution functions is avoided by following the motion of a large number of molecules, simultaneously, in the direct simulation method suggested by Bird³². This increases the complexity of the calculations, but as machine speed has increased much more rapidly than storage capacity, the direct simulation technique has proved to be much more successful. Bird³³ has also demonstrated that the direct simulation method is based on the same assumptions as Boltzman's equation, so the results obtained from the method constitute solutions to Boltzman's equation.

In a series of papers³⁴⁻³⁷, the results of direct simulation computations have been presented for simple shapes, flat plates, and cones. These results show good qualitative agreement with experimental data, including

that obtained from the electron beam surveys discussed at the end of the last section. Unfortunately, at present, it is only possible to conduct the calculations for monatomic gases. As the flow field surveys were made in a diatomic gas, accurate quantitative comparison is not meaningful. Calculations have been extended as far as $x/\lambda_\infty=90$, so that conditions in the kinetic regime and merged layer regime were calculated simultaneously.

Another development that has clarified understanding of the transition region was reported by Hamel and Cooper⁶. They demonstrated that the mean free paths relevant in the leading edge region are λ_f and λ_g , where the subscripts indicate the distance travelled by a free stream or body dominated molecule before collision with one of the other kind. It was noted that the plateaus of surface quantities occur at $x/\lambda_f=O(1)$, which indicates that λ_f is the appropriate streamwise scaling parameter. This also explains why large values of x/λ_∞ are reached before continuum behaviour is observed, since $\lambda_\infty \ll \lambda_f$. As might be expected, continuum behaviour starts at $x/\lambda_f=O(5)$, i.e. after a few relevant collisions have occurred.

In a recent paper El-Arsar and Chow³⁸ attempted to solve the Boltzman equation for the flat plate problem by specifying a bi-modal distribution function having components characteristic of the wall and free stream. The analysis necessarily contains many restrictive, and sometimes questionable assumptions, such as the use of

Maxwellian molecules, which suggests that it is at a preliminary stage. However, the results presented for some surface quantities are in reasonable qualitative agreement with existing data and it would be interesting to see the flow field structure arising from this analysis.

2.4. Surface pressure corrections.

In the preceding sections, comparisons of measured and calculated surface pressure have frequently been mentioned, and indeed many surface pressure measurements have been performed. Ideally surface pressure should be measured by a pressure transducer forming an integral part of the model surface, but as practical difficulties preclude this, pressure is usually measured by a transducer connected to the surface via a tube.

Clearly the hole in the model surface may interfere with the flow by changing the local accommodation coefficients. However, more serious errors are likely to arise because of thermal transpiration effects, as the model is not normally at the same temperature as the gas.

It is well known that if there is a temperature gradient along a tube containing gas at low density, then in the absence of a net mass flux, a pressure gradient also exists. A pressure difference will also occur at an orifice separating two regions of gas at different temperatures, as an orifice is essentially a short tube.

Two distinct semi-empiric methods have been developed to predict the corrections necessary to convert

measured pressure into surface pressure. Potter et al.⁴ proposed a semi-empiric correction scheme based on simultaneous measurements of surface and cavity pressures obtained in a static test rig. The results were presented in terms of local heat transfer and Knudsen number. Following a theoretical examination of the free molecular orifice, Kinslow and Potter⁷ presented a new correlation. This was based on a reference gas temperature at the wall and included the effects of shear stress.

Bartz and Vidal⁶ also measured surface pressure directly and compared it with cavity pressure, but in this case the experiments were conducted on a flat plate in a hypersonic flow. These results are also presented in terms of Knudsen number and gas temperature at the wall. Additional data, such as heat transfer rates, are required when applying these correction schemes.

Both the Potter and Bartz schemes predict large corrections at the leading edge so the corrected surface pressures are much greater than that predicted by free molecular theory.

Lillicrap³⁹ reconsidered both schemes and suggested that assumptions about the appropriate transfer rate and neglect of slip, respectively, might be incorrect. Surface pressure data was corrected under alternative assumptions and was found to behave more reasonably, although the two schemes yielded significantly different results. The results of a direct determination of surface pressure with an electron beam were also included, but as these measurements were conducted for values of

$\bar{V}_\infty \leq 0.2$ where corrections were small, this was inconclusive.

Hughes and de Leeuw⁴ developed a theory for a near free molecular pitot tube at various angles of incidence, including the case of a probe normal to the flow. Metcalf⁴⁰ has shown that this theory correlates flat plate data obtained from holes with a wide range of depths and diameters.

The basic source of difficulty in the various correction schemes is probably the fact that a complex flow structure occurs at the mouth of the measuring tube which is not amenable to analysis in transitional flow.

This problem may be avoided by adopting a more specific geometry. The equilibrium number flux density of particles in a cavity is given by $N_c = P_c / (2\pi RT_c)^{1/2}$. If such a cavity is connected to the model surface by a small sharp edged hole, then the number flux leaving the cavity will be the same as that incident on the model surface. Hence the number flux hitting the model surface may be obtained directly by measuring model temperature and cavity pressure. The hole must be small compared with the cavity, so that the number of incoming molecules is insufficient to disturb equilibrium. It must also have a diameter that is much less than the local mean free path, so that collisions between incoming and outgoing molecules are negligible.

Horstmann⁷ reported number flux measurements obtained in this way on a flat plate and wedge, in both monatomic and diatomic gases. He noted that in the case

of the flat plate, these fluxes were different and they were not in good agreement with the results of direct simulation Monte-Carlo calculations near the leading edge.

Lewis and Smith⁷ report similar number flux measurements in nitrogen and also predicted surface pressure from electron beam measurements of density at the cavity entrance. These surface pressure measurements were compared with tube measurements made under the same conditions and corrected with the techniques described above.

It was found that in the merged layer regime there was general agreement amongst the results. In the kinetic regime, tube data corrected using the Barty and Potter schemes indicated much higher pressures than the free molecular orifice measurements. However, tube data corrected by using the Hughes and de Leeuw theory was in agreement with the free molecular orifice measurements in both regimes, which suggests that the method may be widely applied.

2.5. Slender Bodies.

As stated earlier, much effort has been expended in attempts to understand rarefied flow over a flat plate and relatively little in understanding flow about slender bodies, although the latter are of more direct aerodynamic interest.

The case of strong viscous interaction on a wedge has been examined theoretically by several authors including Cheng⁴¹. Kemp⁴² showed that; provided a slight modification was added in Cheng's theory, much

experimental data obtained at high Mach number could be brought into agreement with the theory.

Several sets of measurements^{4,5,6}, of surface quantities showed the failure of the strong interaction theory for wedges near the leading edge and Vas et al.⁶ conducted flow field surveys to demonstrate that this failure is due to merging. They found that increase of incidence moved the point of merging forward, but were unable to correlate the departure from strong interaction with the normal rarefaction parameters. All of these measurements were conducted with $M > 9$, so it is not possible to tell if $M \geq 8$ is the necessary condition for strong interaction theory to apply, as it is for the flat plate.

At present there are no satisfactory theoretical predictions for the transition regime on a wedge. Rudman and Rubin²⁵ claim to have considered the effect of angle of incidence on a flat plate. Unfortunately they do not give any quantitative information, but state that their calculations show that the distance from the surface to the shock decreases with increasing incidence. Vogenitz³⁴ et al. used the Monte-Carlo method to compute the density field around a 40° wedge, but as this was only $3\lambda_\infty$ long, the information is of limited utility.

The Monte-Carlo method has been applied to long cones³⁷ and is substantially in agreement with the available experimental data. At a given distance from the leading edge, the flow field disturbance caused by a

slender cone is small compared with that on a flat plate because of the three-dimensionality of the flow. Zalskey and Vas⁷ conducted an experimental survey of flow about slender cones which confirmed that the disturbance is small, in agreement with the Monte-Carlo calculations.

2.6. Conclusions.

There have been many experimental investigations of the flow about a flat plate in a diatomic gas. These measurements suggest that the merged layer regime may be approximately represented by solutions to Navier-Stokes equations. It is not possible to distinguish whether the remaining disparities are due to i) inaccuracies in the, admittedly, difficult measurements ii) invalid assumptions used in obtaining solutions to the equations or iii) inadequacies of the Navier-Stokes equations when applied to rarefied gases.

Measurements also show that the kinetic region is qualitatively predicted by the direct simulation Monte-Carlo method. Unfortunately this method is at present only capable of dealing with monatomic gases, so that direct comparison is not possible. The limited experimental results at present available for monatomic gases consist only of measurements of conditions at the surface and these show slight differences from the Monte-Carlo calculations.

There has been little theoretical consideration of wedge flows in the rarefied flow regime although the results of several experimental investigations are available.

The Monte-Carlo method appears to offer the prospect of predicting flow about slender bodies from the leading edge, back to the classical viscous interaction regime, although only for monatomic gases. Conversely, the vast majority of experimental information has been obtained from diatomic gas flows.

Clearly there is a need for further experimental investigation of monatomic flows about slender bodies, of which flat plates and wedges form representative examples. It is for this reason that the measurements described in the next two chapters were undertaken.

3. APPARATUS

3.1. The wind tunnel

The experiments described in the next chapter were conducted in an arc heated blow down wind tunnel which is shown schematically in figure 1.

Argon gas is heated by being passed through an electric arc in a "Plasmadyne" AM-4 plasma jet head assembly and this raises the temperature of the gas to $O(7000^{\circ}K)$, at a typical mass flow rate of 1 gr/sec. The gas comes to equilibrium in a settling chamber where it has a pressure of $O(2 \times 10^4 N/m^2)$. A contoured nozzle having an exit diameter of 75m.m. is then used to produce a Mach 7 flow, at a pressure of $O(10 N/m^2)$, with a corresponding mean free path of $O(1m.m.)$.

After traversing the model, the gas is collected by a diffuser and exhausted to a dump tank at a pressure $O(5 \times 10^2 N/m^2)$. Much of the pressure recovery occurs in the nozzle shock and model flow field as the ratio of the pressure at the diffuser exit to that in the working section is $O(6)$. The dump tank is exhausted to the atmosphere with a "Roots" pump backed by a "vane" pump.

All components in contact with the flow were water cooled, as was the working section, since convective and radiative heating were severe.

When originally constructed, the wind tunnel had a nozzle with an exit diameter of 20m.m. The performance of the facility in this configuration is described in references 43-47. The development of the facility for use in the present low density studies is described in

Appendix 3.

3.2. Recalibration of the plasma jet

Following the increase in Mach number, it was necessary to recalibrate the plasma jet. Conditions in the free stream were extensively investigated at Mach 2.7. as discussed in Appendix 4 and these measurements suggest that the free stream conditions may be adequately calculated from the reservoir conditions. In order to do this, the free stream Mach number must be measured. It is also necessary to verify that the flow is frozen and has an inviscid core. Pitot and Langmuir probe measurements were conducted to establish the conditions in the free stream.

3.2.1. Pitot pressure measurements

A 20m.m. outside diameter, water cooled probe was used to scan the flow at the nozzle exit. It was verified that both the Reynold's number correction, discussed by Potter⁵ and the thermal transpiration correction measured by Bartz⁶, were negligible for the probe used. The flow contained an inviscid core of about 40m.m. diameter at the nozzle exit, and was bounded by a nozzle lip shock which reached the axis about 50m.m. down stream of the nozzle exit. Within the inviscid core, the Mach number was constant to 1% both radially and axially. This was calculated from the ratio of pitot pressure to reservoir pressure using a suitably modified value of γ . The nozzle exit centre line Mach number was also measured over a range of power levels and was found to vary between 6.1. and 6.7.

3.2.2. Langmuir Probe measurements

The positive ion flux in the free stream, was measured with a negatively biased cylindrical probe, placed transverse to the stream at the nozzle exit. The probe consisted of 0.1m.m. diameter tungsten wire, with sintered aluminium oxide tubes to confine the current collection region to the inviscid core. Whilst in use, the probe was red hot which kept the tungsten surface clean, so avoiding the surface contamination problems that commonly arise with Langmuir probe measurements. A transverse probe was used, in preference to the conventional spherical or axial cylindrical probe as it is more robust, this being necessary in fast flowing, relatively dense plasma.

The ion current collected by the probe is,

$$I_1 = eDLU_n F(M)/4 \quad (3.1)$$

where $F(M)$ is a Mach number dependent term determined by Clayden⁴⁸.

For this equation to be applicable, it is necessary that the probe be free molecular, and also that the probe collects only those ions which would have entered the volume occupied by the probe. Under normal operating conditions the Knudsen number of the cylinder, $Kn_D = 0(5)$, so that a free molecular flow assumption is valid.

When a body is placed in a plasma it becomes surrounded by a region of charge non-neutrality, known as a plasma sheath. This results from the difference in the mobility of ions and electrons. It was verified

that the Debye length was negligible compared with the probe diameter. As the Debye length is approximately the same as the sheath thickness, the probe area was not increased by the sheath.

Another possible reason for collecting more ions than those entering the probe volume, is that they may be attracted from some distance away from the probe by the high negative potential necessary to repel electrons. This effect was eliminated by extending the high voltage slope into the zero voltage region.

The results of several measurements are shown in figure 20 and are compared with calculations assuming frozen isentropic flow. It will be seen that agreement is good, providing considerable support for this method of predicting free stream conditions. At low power levels significant ionization persists, indicating that the gas does not come fully into equilibrium in the settling chamber. The effect of this non-equilibrium on free stream properties should be slight, as it represents a very small ionization fraction.

At the upper power levels, which were limited by melting of the ceramic tubes, the flow appears to have a higher Mach number than that indicated by the pitot measurements. This may well have been the case, as the large pitot tube used may have caused some tunnel blockage.

3.3. The model

3.3.1. Description

A water cooled copper model was constructed as

shown by the continuous line in figure 2. Copper was used as it has a relatively high melting point and large thermal conductivity. Cooling water was pumped through the model at 0.4 MW/m^2 . It was necessary to provide cooling close to the leading edge to prevent melting.

A set of surface pressure taps were provided along the model centre line. These taps were connected to a bank of pressure transducers outside the working section. Small thermo-couples were soldered into the model surface, along side the first three pressure taps from the leading edge.

The leading edge wedge angle was 10° . This was a compromise between the need to avoid under surface interference and the practical difficulties of very slender models, as discussed in section 3.3.2.

The upper surface of the model was ground flat and this produced a leading edge thickness of, $0(0.01 \text{ m.m.})$. Tarnish was removed with very fine emery cloth before making measurements.

Side plates were used to support the model in the working section. These also prevented communication, via the sides, of the upper and lower surfaces. The upper surface of the model was 6 m.m. below the nozzle centre line to minimise the region of shock impingement, whilst retaining the model in the nozzle inviscid core flow. This arrangement also helps to reduce the effect of any flow irregularities produced by the nozzle, as these are focussed on the centre line.

Before proceeding to use the model for detailed

flow studies, preliminary measurements and calculations were conducted. These establish that, in a limited region, the flow about the model corresponds to that about a semi-infinite flat plate in a uniform flow.

3.3.2. Leading edge effects

Leading edge bluntness is a possible source of disturbance to the flow. However, Joss⁷ has shown that provided the leading edge thickness is less than $\lambda_\infty/4$, little disturbance occurs. Since for the present model the leading edge thickness was of order $0.02 \lambda_\infty$, no interference was anticipated.

Another possible source of difficulty is the necessarily finite under surface wedge angle. Surface pressure is generally sensitive to flow conditions and so a preliminary model was used to examine the effect of wedge angle on surface pressure.

The surface pressure was measured at a station 12m.m. from the leading edge, with a McLeod gauge which was reached via a liquid nitrogen vapour trap. Triangular section shims of different angles were attached to the bottom surface to vary the wedge angle between 10° and 50° . The variation of surface pressure with wedge angle is shown, circled, in figure 3. Free stream pressure was calculated assuming a Mach number of 6.5.

A small pitot tube was attached to the underside of the model, 10m.m. below the leading edge. The pitot pressure was then used in conjunction with plenum chamber pressure, to obtain local free stream pressure. This was to eliminate any effect of the wedges on the

free stream conditions, or any differences between runs. Measurements were repeated, using the reference pitot tube, with wedge angles of 10° and 20° . These are shown as crosses in figure 3. It will be seen that the wall pressure decreases with smaller angle. Even at 10° there is no indication that this trend has ceased. However, the minimum realistic wedge angle appears to be 10° , since it is necessary to provide water cooling and pressure taps near the leading edge. So this angle was necessarily adopted for the final model described above.

3.3.3. Trailing edge effects

Initially the model had the dotted outline shown in figure . Measurements of surface pressure indicated that the nozzle lip shock was hitting the back of the model as the pressure rose rapidly at the last two stations. The back of the model was relieved so that it had the continuous outline shown in figure 2. This reduced the pressure at the last pressure tap. However, as the operating conditions were varied, shock impingement sometimes recurred. This behaviour appeared to be completely erratic.

Clearly there is some risk that the high pressure generated by shock impingement might be propagated upstream in a similar manner. So a model length of 65m.m. was adopted as a compromise.

At 40m.m. from the leading edge, cutting back the model produced no measurable effect. The intermittent shock impingement was also undetectable at this station. Since the furthest from the leading edge at which there

is a distinguishable region of inviscid flow, external to the model flow field, is about 25m.m., the limitation is quite acceptable.

3.3.4. Flow two-dimensionality

In order to verify that the model flow was two-dimensional, pitot pressure profiles of the flow field were determined, using the apparatus described in the next section. These profiles were measured at the centre line and 6m.m. from the centre line. The measurements were conducted at 20m.m. from the leading edge. The profiles were indistinguishable, indicating that the flow was two-dimensional.

3.4. Probe dipping equipment

A simple device was built to enable probes to be briefly dipped in the model flow field. At higher power levels this is necessary as blunt bodies melt unless they are water cooled. Provision of water cooling up to the tip is not possible, as it would make the probes too large for use in the model flow field.

The device was constructed about a polished steel rod which passed through the base of the working section via a double "O-ring" seal. Probes were mounted on top of the rod. Guides within the working section constrained the rod to move vertically and prevented rotation. The rod was hand driven by a lever external to the working section.

An adjustable stop controlled the depth to which the probe entered the flow. The position of the rod was monitored by a linear potentiometer. It was verified

that this potentiometer was linear to 1% over a distance of 20m.m.

This simple dipping system proved to be entirely satisfactory, provided the guides were well lubricated. It was capable of injecting a probe into the flow and removing it again in about one second.

A water cooled pitot probe was constructed from thin walled stainless steel tubing, having a cross sectional area of 12m.m.² facing into the flow. This was tipped with a 2m.m. outside diameter thin walled alumina tube, which was 40m.m. long.

Pressure was measured with a Statham PM731TC transducer. This was contained in a water cooled jacket and mounted within the working section. The pitot tube was connected to the transducer by a flexible stainless steel tube.

3.5. Range of operating conditions

The two independently variable operating parameters of the plasma jet are mass flow and power level, which in turn control free stream conditions such as mean free path. Mass flow and power level were varied with the model in the working section to find the limits of operation.

At low mass flow rate the arc became unstable, and at high flow rates the pumping system was inadequate so the nozzle flow stalled. As the normal scaling parameters varied little with mass flow, it was decided to use only one flow rate of 1.1 gr/s.

At very low power the nozzle again stalls as the

nozzle exit pressure is too low for the diffuser. At high power, the inviscid core in the nozzle flow becomes small. The tunnel was operated within these limits to give non-dimensional enthalpies (H/RT_w) of 40, 60 and 90, the corresponding mean free paths being about 0.4m.m., 0.7m.m. and 1.0m.m., respectively.

The thermocouples mentioned in section 3.3.1. were used to determine model wall temperature. The variation of the ratio of wall temperature to free stream stagnation temperature, along the model, is shown in figure 4. For the operating conditions established above, it will be seen that the model is "cold walled."

4. MEASUREMENTS

4.1. Surface Pressure

4.1.1. Experimental Procedure

The model described in the last chapter was used to measure surface pressure on a flat plate and on wedges.

The surface pressure taps were connected to a set of "Pace" type P1 variable reluctance pressure transducers, having a range of 0-lp.s.i.d. These transducers were buried in polystyrene beads to minimise thermal drift. They were provided with by-pass valves, capable of connecting the measuring and reference sides of the transducer whilst isolating the transducer from the model, so that the transducer output could be set to zero when the plasma jet was operating. The transducers were driven by "Pace" carrier-demodulator units, type CD12, the outputs of these units being connected to a digital voltmeter, via a selector switch.

Calibration was performed with the transducers connected to the pressure taps on the model. Various pressures were obtained in the working section by bleeding gas into the dump tank at different rates, with the Kinney pump running. Working section pressure was measured with a McLeod gauge so the transducers were calibrated directly against this gauge, whilst in situ. This eliminates errors arising from any small leaks on the relatively long pipes connecting the transducers to the pressure tapings.

The transducers were calibrated before and after

each set of pressure measurements. These calibration points are linear and repeatable to about $\pm 2\%$ at the operating pressure of $0(60\text{N/m}^2)$. As a straight line was drawn through about five calibration points, obtained both before and after making measurements, the calibration error is likely to be less than 1% .

Conditions in the free stream were established using the dippable pitot probe described in section 3.4. The tip of the probe was aligned with the leading edge of the model and was dipped to a distance of 12m.m. above the model surface. As the model surface is 6m.m. below the nozzle exit centre line, any non-uniformity in the core flow would be compensated for, provided the flow is symmetric. Hence all measurements are effectively based on conditions at the model leading edge. The output of the pitot probe transducer was displayed on an oscilloscope to verify that the rise time of the system was much less than the dwell time of the probe in the flow. Normally the transducer output was displayed on a digital volt meter.

The correction for thermal transpiration was estimated and found to be negligible. However the Reynold's number correction discussed by Potter⁵ proved to be substantial. Potter showed that the ratio of real to measured pitot pressure was correlated over a wide range by the quantity $\text{Re}_{2D}(\rho_2/\rho_\infty)^{\frac{1}{2}}$.

The correction data compiled by Pollard⁶ was, in fact, used as this has a greater range than that previously available. The maximum pitot pressure corrections

were 0(15%), which produces corrections to the calculated free stream pressure of 0(20%). However the data used for the corrections was obtained under similar conditions to the present measurements and was well correlated. It is therefore estimated that the likely error in free stream pressure will be less than 0(2%).

Surface pressure was measured only at the first three holes as there were only three reliable pressure transducers available, and the data from the last hole was of doubtful value because of trailing edge effects.

A consistent measuring procedure was adopted. The transducers were initially calibrated and the plasma jet was then run at one of the standard operating conditions described in section 3.5. When the plasma jet was running stably, the pitot pressure was measured and the surface pressure transducers were zeroed. After the probe had been removed, surface pressure was noted when the transducer readings had become constant. Surface pressure and pitot pressure measurements were repeated twice more. Conditions in the plasma jet were maintained as near constant as possible, throughout, and the resulting variations in pitot were generally less than 2%.

Variation in the observed surface pressure was normally less than $\frac{1}{2}\%$, probably because the length of pipe between the model and the transducer served to integrate short term variations in running conditions. As an average of three, almost repeatable measurements was taken for both the surface pressures and the pitot

pressures, the error likely in these measurements is negligible, apart from the possible calibration errors discussed above.

The process was repeated for the other two standard operating conditions and the transducers were then recalibrated. A complete run took about two hours, excluding the time taken to calibrate the transducers. This rather laborious approach is felt to be justified as much of the published data is badly scattered and not sufficiently accurate to provide a critical test of the various theories.

The possibility of operating the model at incidence was investigated. It was found that at angles of $\pm 2^\circ$, the point of impingement of the nozzle shock was moved forward. At 10° incidence the nozzle shock reached the leading edge of the model. In the case of positive incidence, this failure is thought to be due to the generation of high pressures on the upper surface that are not contained by side plates. At negative incidence the diffuser is incapable of providing adequate recovery from the low pressures occurring on the upper surface of the model.

Uncorrected surface pressure is shown for incidence of $+5^\circ$ and -5° in figure 5. Clearly the third pressure tap in each case is strongly affected by shock impingement. In order to estimate the likely effect of this on data from the second hole, a comparable shock was induced on to the model when at zero incidence by bleeding gas into the dump tank. It will be seen that the

pressure rise at the third hole is similar to that at incidence. Measurements were also made at zero incidence without shock impingement and the results are shown as solid symbols in figure 5. These are displaced slightly to the right in order to avoid confusion, but were in fact obtained at the same distance from the leading edge.

It will be seen that the data obtained from the first two holes is little affected by the shock on the flat plate and the same will be assumed to be true for the model at incidence, within the limits, $-5^\circ \leq \alpha \leq +5^\circ$.

4.1.2. Results

The flat plate surface pressure data is replotted in figure 6. The thermo-molecular pressure produced by the temperature difference between the model and transducers was estimated, using the data of Kinslow and Arney⁶, and was found to be negligible. No corrections for orifice effects were applied to the data in figure 6, as the available methods are not felt to be sufficiently reliable. It was estimated that if the Bartz and Vidal scheme had been employed, then the correction factor would have varied between 20% and 50%.

For the purposes of comparison, the uncorrected data obtained by Metcalf et al.⁶ in a nitrogen flow, under otherwise similar conditions, is included. The surface pressure observed in the present work is greater than that observed by Metcalf, continuing the trend suggested by viscous interaction theory. Other than this, the main features of the pressure distribution are

similar to those obtained before.

Strong interaction, calculated from the theory given by Dorrance¹⁵, does not appear to have any significant region of relevance. First order weak interaction, calculated from the theory given Hayes and Probst¹⁴, fails at $\bar{V}_\infty > (0.2)$, as before.

In figure 7, the departure from weak interaction is shown for wedges, as well as the flat plate. The pressure on the wedges was calculated by linearly adding the pressure produced on a flat plate by weak interaction, to that generated by a wedge in the absence of viscous interaction. Cheng⁴¹ has shown that this is a valid procedure for weak interaction. Inviscid wedge pressure was calculated from the tangent wedge approximation expressed as follows,

$$\frac{P_{\text{wedge}}}{P_\infty} = \frac{1+M^2}{4} \alpha^2 \left\{ \gamma + 1 + \left[\frac{(\gamma + 1)}{4} \right]^2 + \frac{1}{M^2 \alpha^2} \right\}^{1/2}$$

The alternate positive or negative sign depends on whether α is positive or negative, respectively.

Stollery⁴⁹ has shown that the tangent wedge approximation is applicable to viscous interaction on a variety of body shapes.

Since the surface pressure is independently related to wedge angle and viscous interaction, it is unlikely that any simple correlating parameter exists. However Raat and Pasuik⁵⁰ showed that departure from strong interaction on a wedge and cone are approximately correlated by $\bar{V}_\infty$ and so this parameter is again used to obtain qualitative information.

It will be seen that the data is rather scattered by this incomplete correlation and possibly also by the neglect of orifice corrections. The main feature which emerges is that, as the wedge angle increases, the effect of rarefaction is diminished and confined to a region closer to the leading edge. Similarly for negative angles of incidence, the point of departure from viscous interaction is moved aft.

At positive incidence the surface pressure appears to rise slightly above that predicted by weak interaction. This could occur because of inaccuracies in the method of predicting weak interaction. Alternatively it might be due to a small region of strong interaction occurring because of the increased shock strength on a wedge at positive incidence.

In order to avoid the use of unsatisfactory orifice corrections, the measured pressures were reduced to number fluxes, as discussed in section 2.4. Initially the pressures were converted to those in an equivalent cavity using the theory of Hughes and Leeuw. This assumes near free molecular conditions, which do not occur in the present case as the Knudsen number for the holes, $Kn_D = 0(0.3)$. However Metcalf et al.⁴⁰ found that even for small Knudsen numbers the theory was quite satisfactory and so it should be applicable in the present case.

Speed ratio at the model surface must be known to apply the Hughes and de Leeuw theory. Vidal and Bartz⁶ showed that to first order,

$$\frac{U_G}{U_\infty} = \frac{D_F}{4} \left(\frac{\pi(\gamma-1)}{\gamma^{\frac{1}{2}}} \right)^{\frac{1}{2}} \left(\frac{h_G}{H_\infty} \right)^{\frac{1}{2}} \quad (4.1)$$

where,

$$D_F = C_F \left(\frac{\rho_\infty U_\infty^2}{P_w} \right),$$

and the subscript, G, indicates conditions in the gas at the model wall. If the usual hypersonic approximation, $H_\infty \approx \frac{U_\infty^2}{2}$, is employed then equation 4.1 reduces to,

$$S_G = D_F \left(\frac{5\pi}{32} \left(\frac{\gamma-1}{\gamma} \right) \right)^{\frac{1}{2}} \quad (4.2)$$

This equation was used in conjunction with the skin friction data quoted by Wallace and Burke⁴ to estimate the speed ratio at the wall. It is not suggested that this method would give an accurate prediction of speed ratio. However the relationship between tube and orifice pressure is not strongly dependent on speed ratio. Hence inaccuracies in the speed ratio should not lead to significant errors in the predicted orifice pressure.

The orifice pressures (P_c) were used to calculate number flux from,

$$\dot{N} = P_c / (2\pi RT_c)^{\frac{1}{2}},$$

where T_c is the temperature in the orifice, as determined from the thermo-couples embedded in the model.

The number flux to a flat plate is shown in figure 8 and compared with other measurements and calculations. λ_f and $\dot{N}_{PM}$ were used as scaling parameters as these are thought to be the most relevant quantities in the kinetic and merged layer regimes as discussed in section 2.3.

Hamel and Cooper⁶ showed that,

$$\lambda_{fB} = \left(\Omega_{fB} n_{\infty} \frac{C_{\infty}}{C_B} \right)^{-1},$$

where Ω_{fB} is the collision cross section. If the molecules behave as hard spheres, then the collision cross section is not velocity dependent. From simple kinetic

theory, $\lambda_{\infty} = \frac{1}{\sqrt{2}} \cdot \frac{1}{n_{\infty} \Omega}$, hence for hard sphere molecules,

$\lambda_f = \sqrt{2} \cdot \sqrt{\frac{T_{\infty}}{T_w}} \cdot \lambda_{\infty}$. The free molecular number flux to the model surface was calculated from,

$$\dot{N}_{FM} = n_{\infty} \left(\frac{RT_{\infty}}{2\pi} \right)^{\frac{1}{2}} \left[e^{-(S \sin \alpha)^2} + \sqrt{\pi} (S \sin \alpha) (1 + \text{erf}(S \sin \alpha)) \right].$$

This equation is demonstrated in the discussion of free molecular flows presented by Hayes and Probstein¹⁴. In the case of a flat plate $\alpha=0$ and the equation reduces to,

$$\dot{N}_{FM} = n_{\infty} \left(\frac{RT_{\infty}}{2\pi} \right)^{\frac{1}{2}}.$$

The results of the Monte Carlo calculations quoted in figure 8 appear in references 36 and 37, and also as a private communication in the paper by Horstman⁷.

This paper also reports experimental results obtained using helium.

Unfortunately the scaling in figure 8 is such that it is difficult to relate the present results to those obtained previously. The present data was in the merged layer regime, rather than the kinetic regime, as $\lambda_f \gg 5$. As discussed previously, λ_f should be appropriate in both regimes, but this does not appear to be the case. The data was therefore replotted against $\frac{x}{\lambda_{\infty}}$, as shown in figure 9. It will be seen that the previous results are qualitatively very similar to the current

data. Quantitative comparison is less satisfactory as the number flux measured at $M=7$ is similar to that calculated at $M=13$. Normally one would anticipate a smaller number flux at the lower Mach number. However the trend is the same as that exhibited by Horstman's data which shows that at a given Mach number, the experimentally determined number fluxes are greater than the calculated values.

The surface pressure data on wedges has also been reduced to number fluxes as shown in figure 10. Again, there is no simple correlating quantity available, so the number flux data is scattered, as the pressure data was. The solid lines are rather optimistic, particularly at negative incidence, but they do reveal the main features of the data.

At negative incidence, the number flux is substantially greater than that predicted by free molecular theory, presumably because the shock deflects molecules down on to the model surface. When positive incidence is applied, there is little effect initially. However as the incidence is increased, the relative number flux drops. This may be because the proportion of free stream molecules which reach the surface without collision is increased and hence conditions become similar to those in a free molecular flow.

4.2. Flow field study, using probes

Langmuir probes and pitot probes were used in conjunction with wall pressure measurements, in an attempt to determine density and velocity profiles within the

model flow field.

4.2.1. Theory

Pitot pressure may be related to dynamic pressure by, $\rho u^2 = P_0' f(M)$. (4.1)

The equation for positive ion current to a Langmuir probe, i.e. equation (3.1), may also be re-arranged to

$$\text{yield } \rho u = \frac{4I_1}{eDL\alpha F(M)} \quad (4.2)$$

Taken together, equations (4.1) and (4.2) may be used to yield ρ and u separately. The ionisation fraction, α , is frozen at its free stream except very close to the wall, as discussed in appendix 5.

In supersonic flows the Mach number dependent quantities $F(M)$ and $f(M)$ change slowly with Mach number and are very weakly dependent on Mach number if $M > 2$.

Within the viscous layer, $\frac{P_w}{P_\infty} = O(1)$, so that the local Mach number may be obtained approximately from pitot pressure and wall pressure. Mach number in the shock layer may then be approximately obtained by extrapolating the viscous layer data through to the free stream value.

Alternatively the Mach number at some distance from the wall may be obtained by assuming that the free stream total enthalpy is conserved. The flow equations,

$$\frac{u^2}{2} + C_p T = C_p T_0 \quad \text{and} \quad \frac{T_0}{T} = \frac{1 + \gamma - 1 M^2}{2},$$

may be manipulated to yield,

$$M = u((\gamma - 1)(C_p T_0 - \frac{u^2}{2}))^{-\frac{1}{2}} \quad (4.3)$$

If total enthalpy is conserved then $C_p T_0 = H_\infty$, and

equation (4.3) may be used to calculate M .

Having estimated local Mach number, $F(M)$ and $f(M)$ may be obtained and substituted in equations (4.1) and (4.2). The resulting dynamic pressure (ρu^2) and number flux (ρu) are used to find ρ and u . Local temperature ($\pm \frac{P_w}{R\rho}$), viscosity and Reynolds number may now be obtained successively and used to calculate the pitot pressure rarefaction correction. The complete process may be iterated to obtain more accurate values of ρ and u .

Rapid and accurate closure of these iterations requires that the quantities $F(M)$ and $f(M)$ vary slowly with Mach number, as is the case for $M > 1$. The speed ratio at the wall was calculated, in section 4.1.2., to determine number flux. Since $S = (\gamma/2)^{1/2} M$, the Mach number was also obtained. It was found that $M = O(1)$ at the model wall, so that $M > 1$ for most of the model flow field.

4.2.2. Experiments

Pitot probe and Langmuir probe measurements were made using the dipping apparatus described in section 3.5. The cylindrical pitot probe tip was replaced by a rectangular section ceramic tip having a cross section of 3m.m. x 1m.m. Hence the probe still had an acceptable rise time, as before, but the spacial resolution was improved. The Langmuir probe again consisted of a fine tungsten wire with a collection region defined by ceramic tubes. This collection region was 7m.m. long. The probe was mounted transversely to the flow and parallel to the model surface.

Smooth motion of the probes through the flow was achieved by driving them with a screw thread, via the potentiometer. The output of the potentiometer was applied to the X plates of a storage oscilloscope and the probe signal was applied to the Y plates, so that profiles could be obtained directly. Photographs of the oscilloscope display were used to obtain a permanent record of the results.

It was noted when making pitot probe measurements, that there was a slight difference in the traces obtained when the pitot probe travelled from the free stream down on to the model surface, rather than from the model surface out to the free stream. This hysteresis was due to lag in the measuring system and the two traces bracket the correct value.

A further investigation was performed in which the probe was allowed to remain at a specific distance above the model for several seconds, so that the correct value of pitot pressure was obtained. It was found that these values were close to those obtained when moving the probe away from the model. This is reasonable as vacuum systems generally fill more rapidly than they empty.

It was possible that the response of the Langmuir probe might be affected by the model causing a distortion of the electric field associated with the probe. To investigate this, the voltage-current characteristic was identical except when the probe was less than 1.0m.m. above the model surface and the bias was also less than two volts. Hence the probe is virtually unaffected by

the model.

As the slope of the characteristics is constant, the ratio of ion current collected at a given voltage to that collected at zero voltage, is the same in the model flow field as in the free stream i.e.,

$$\frac{I_i}{I_{i\infty}}(V=0) = \frac{I_i}{I_{i\infty}}(V=-7).$$

It is therefore not necessary to plot the characteristics at each point in the flow. A bias of -7 volts was used as this ensures that the probe is operating on the slope rather than the "knee" of the characteristic.

In order to avoid any effects arising from variations in free stream conditions occurring whilst a traverse of the flow field was made, a fixed Langmuir probe was installed 5m.m. below the leading edge of the model. The output of the fixed probe was used with a voltage multiplier, operating in the "divide" mode, to normalise the output from the movable probe. Unfortunately at the higher powers, the fixed probe melted, so this system could not be employed. However at low power there was no detectable difference between the normalised and non-normalised profiles, so failure to normalise the probe output does not appear to be important.

The result of a Langmuir probe and pitot probe traverse is shown reduced to velocity and density profiles in figure 11. One iteration was performed to correct for Reynolds number effects on the pitot pressure and the correction was found to be very small. Monte-Carlo calculation results from reference 52 are included for the purposes of comparison.

It will be seen that profiles obtained experimentally are not satisfactory. The increase in the velocity above that in the free stream, followed by a decrease when moving further away from the model are highly improbable.

These measurements were conducted at 12m.m. from the leading edge. The Mach number obtained from the wall pressure and pitot pressure is shown as a series of circles in figure 13. As the shock region is approached, the indicated value of Mach number drops, as the procedure becomes invalid. Hence the measured Mach number at the wall was phased to conditions in the free stream using the dotted line shown. It will be seen that $M > 2$ for most of the flow field and hence $F(M)$ and $f(M)$ were virtually constant. The Mach number was also calculated using equation 4.3 and the results are shown as crosses in figure 13. The erratic behaviour of the velocity caused the calculated Mach number to vary wildly and hence the method is not useful.

Further probe measurements were conducted at different positions and under different operating conditions. The density and velocity profiles obtained again exhibited improbable behaviour including, in some cases, an overshoot on the velocity profiles. There was no apparent pattern in this behaviour although the results obtained at a given condition were reproducible.

In view of the careful consideration of the applicability of the probes these gross distortions of the profiles were surprising. However they could not be

accounted for by experimental error as considerable care was exercised in making the measurements.

It was not apparent whether the combined technique failed because of the pitot probe measurements, Langmuir probe measurements or both. Further tests were conducted to try to establish the source of the errors.

Pitot pressure profiles were obtained using a tip that was 60m.m. long, so that the probe support was beyond the end of the model. As no difference was observed, the probability of interference between the probe support and the flow field is eliminated. The pitot probe was normally pitched 5° downward, with respect to the model surface and free stream flow. Measurements were made with the probe parallel to the flow, but again there was no discernable difference in the profile obtained.

In the case of the Langmuir probe, an error might have been introduced by the ceramic tubes to locate the collection region. These tubes presumably generate local shocks which strike the wire. They might also become electrically conducting so extending the collection region. The probe was used in the free stream with collection lengths varying between 2m.m. and 12m.m., the output being normalised against the fixed probe. It was found that the current collected was directly proportional to collection length, even at 2m.m. Hence the presence of end effects induced by the ceramic tubes may be eliminated.

A further possible source of difficulty in making

the Langmuir probe measurements was considered. It is well known in classical electrostatic theory, that a charged body induces a corresponding charge of opposite sign on an earthed conducting body.

As the Langmuir probe is negative, it could induce a positive charge on the model which would tend to deflect ions away from the model. To eliminate this, the model was electrically isolated from the working section and different voltages were applied. It was found that the profiles were unaffected by model voltage even when this exceeded that of the probe, so it is unlikely that the induced charge would have any effect.

As no other reason could be found, it is suggested that the erratic behaviour of the velocity and density profiles may be due to the disturbance caused to the flow by the probes themselves. This seems unlikely as the probes are small compared with the extent of the flow field, and also, most of the flow field is supersonic, so that disturbances should not be propagated upstream.

Considered separately, both the Langmuir probe and pitot probe profiles appear quite reasonable. They are smooth and comparable in shape to the previously reported flow field surveys discussed in section 2.3.

However it has subsequently been established, by performing additional independent measurements, that the probes were interfering with the flow, and this conclusion will be discussed later in section 4.3.4.

4.3. Optical determination of density

As the probe measurements proved to be unsatisfactory, an alternative method of determining conditions in the flow field without introducing probes was sought. The application of the electron beam technique was considered, but this would have been difficult because of the considerable self luminosity of the flow in the plasma jet. It was therefore decided to use the self luminosity of the flow to determine density.

4.3.1. Meta-stable atoms

The cause of the high luminosity of the plasma jet is the decay of atoms from meta-stable states. These meta-stable atoms are generated in the arc region by the highly energetic particles present in that region. Brewer⁵¹ has shown that under conditions similar to those in the plasma jet, the presence of meta-stable atoms increases the luminosity by some five orders of magnitude.

Adcock⁴⁶ verified that almost all the luminosity in the plasma jet is due to the presence of meta-stable atoms, by adding a trace of hydrogen to the flow. This greatly reduced the intensity of light emitted, but did not affect the ionization fraction, so luminosity is not due to ionic recombination. The reduction in luminosity must therefore be due to quenching of meta-stable atoms.

The reason for the decay of the meta-stable atoms is not certain. Spectroscopic measurements of temperature using the relative line intensity method appear to yield the electron temperature rather than the neutral

particle temperature, as discussed by Adcock. This suggests that the decay of the meta-stable atoms is associated with electron impact. However the half-life of the meta-stable states may be comparable to the transit time through the nozzle, so decay may be spontaneous.

The amount of light emitted will depend on the density of meta-stable atoms present and if the ratio of these to all atoms is constant, it will also be proportional to gas density. Hence a measurement of light intensity may be used to determine the density of the gas.

When the plasma jet is operating it may be observed that the luminosity in the inviscid core at the nozzle exit is constant and it has been confirmed by the photographic measurements described below that this is so to 0(1%). The gas density in the core is also constant to 0(1%) as has been verified by the probe measurements. It is therefore apparent that the ratio of meta-stable atoms to all atoms is constant. Since the extent of the model flow field is less than that of the core, the fraction of atoms in meta-stable states should be essentially constant in the model flow field. This will not be necessarily the case if, either, the decay processes are different, or excitation occurs.

The temperature in the flow field, even behind the shock, is less than 1000°K so that excitation will not occur. As discussed in appendix 6, the electron density is likely to be the same in the shock region as in the free stream. Conditions generally are similar in the

two regions so the decay processes are likely to be the same.

The light intensity in the shock region should therefore be proportional to local gas density.

Close to the model, the light intensity may cease to be proportional to gas density, as many of the atoms in this region will have undergone a collision with the model wall. Such a collision may cause non-radiative decay of a meta-stable atom, so the fraction of atoms in meta-stable states is likely to be reduced.

4.3.2. Photographic technique

The intensity of the light emitted from the flow field was measured photographically. This method enables the entire flow field to be viewed at one instant, which is useful as it is difficult to reproduce and maintain conditions exactly in the plasma jet. It also enjoys the advantage of simplicity.

A plate camera with a small depth of focus was used to localise the observation to the model centre line. The camera was set up to give a magnification of about one, and it had an aperture of $f/1.1$. Under these conditions, the depth of focus was 0(lm.m.).

Whilst in the camera, part of the film was masked. Subsequently in the dark room, a set of different exposures were applied to the masked area to provide a direct calibration of the film. In order to avoid stray light effects, observation was restricted to the blue region of the spectrum, where much of the argon emission occurs. Blue sensitive film was used and ultra-violet

light was excluded by placing a glass sheet between the camera and the working section window. The calibration strip was produced with a cold cathode light source as this emits strongly in the blue region.

The pictures were recorded on Ilford N5-31 sheet film and this developed in Microphen. This combination produced reasonably fine grain pictures. Whilst being developed, the film was continuously agitated to minimise grain size and ensure a uniform development across the film.

Since the piece of film containing the flow field picture is calibrated directly, the calibration should be accurate.

The film was scanned with a photo-densimeter, the output being plotted on a chart. As the densimeter had a slit of 0.2m.m. x 0.05m.m. and the slit was aligned in the flow direction in which the flow properties vary slowly, local definition was very sharp. A calibration curve relating film transmittance to length of time exposure (and hence amount of light received), was constructed for each film. Eight calibration times were used and there was little scatter in the results. Flow field scans were converted into density profiles using the calibration curves.

As the local density was comparable to that in the free stream, it was possible to operate in a reasonably linear region of the response of the film. These measurements should therefore be accurate to a few per cent.

Spacial accuracy was also ensured by including

markers in the photographs.

4.3.3. Density Profiles

The result of a set of scans made on one piece of film is shown in figure 14. Monte-Carlo calculations from reference 52 are included for comparison.

It will be seen that the calculated and measured profiles are similar in shape except close to the wall. The reason for the apparently low value of measured density at the wall may be that the meta-stable atoms are quenched by collision with the wall. Hamel and Cooper⁶ suggested that the extent of the region occupied primarily by body dominated atoms is $O(\lambda_g)$. However, if quenching by collision with the model wall is the reason for the apparently low density at the wall, the extent of the body dominated atoms is greater than $O(\lambda_g)$. Away from the wall the agreement between the profiles is good.

It may also be noted that the shock starts slightly ahead and below the model leading edge, presumably because of the presence of the under surface wedge. The density is also slightly greater at this position than it is until well down stream. As stated earlier, the leading edge of the model should be sufficiently sharp to preclude leading edge bluntness effects. The high density ahead of the model must, therefore, arise from upstream propagation resulting from diffuse reflection at the model.

The peak densities obtained before are plotted with data obtained under different operating conditions in figure 15. The results of measurements conducted in

nitrogen using an electron beam under otherwise similar conditions are also included. These results were taken from reference 53.

It will be seen that the shock in the present measurements using argon is slightly steeper and stronger than that measured by Lillicrap⁵³ in nitrogen. This continues the trend predicted by viscous interaction theory in which shock angle and strength increase with γ . For example, in the case of weak interaction on a cold wall, the slope of the boundary layer edge is proportional to $(\gamma-1)$. One would anticipate that shock angle will be related to γ in a similar fashion.

The present results are in reasonable agreement with the Monte-Carlo calculations, although the values of density maxima are a little higher. It is felt that the close agreement previously obtained between Lillicrap's measurements and the Monte-Carlo calculations may have been fortuitous in view of the fact that they were conducted at different values of γ .

Flow field density maps derived from results obtained at different conditions are shown in figure 16. It appears that at a given Knudsen number the shock is slightly more rarefied when the mean free path is greater, although there is no obvious explanation for this. The shock constitutes an increasing proportion of the flow field although the shock thickness decreases as the leading edge is approached.

Density ratio across the shock did not reach that for a Rankin-Hugoniot shock within the range of the

present measurements. For the observed shock angle and Mach number, $\rho_2/\rho_0 = 2.7$ for a Rankine-Hugoniot shock.

Density maps were constructed for the flow about the model when it was set at incidence to the flow, as shown in figure 17. The crosses show the position of the observed density peak and are included to indicate the amount of scatter obtained, which in fact was slight. It will be seen that with the limited range of incidence applied, there was little difference in the observed structure. The maximum density of the shock was slightly higher when at positive incidence but its position was virtually unchanged. At negative incidence the shock strength is again very similar, but the shock angle with respect to the model is increased significantly. The shock angle with respect to the flow is, however, similar to that at zero incidence.

4.3.3. The effect of probes on the flow

In view of the difficulties encountered in section 4.2., photographs were taken whilst probes were present in the flow. This was to determine whether the probes were affecting the flow. These photographs were reduced to density profiles as before. The measurements were conducted at the lower power level, to prevent the probes from melting while being photographed.

Two probe positions (A and B), as shown in figure 18, were employed, corresponding approximately with the centres of the shock and viscous layers. The probes were those described in section 4.2.2. Only one probe was introduced into the flow at a time.

It was found that due to the presence of either probe in the flow, the density profiles were slightly distorted but the values of density maxima were changed substantially. This effect extended far upstream of the probes e.g. when the pitot tube was at A, the density at 2m.m. from the leading was 15% greater than when the probe was absent. The flow disturbance could not be attributed simply to the flow fields of the probes since these would be confined to the low shock stand-off distance. For a wide range of physical conditions this is $O(d/2)$, which is $O(0.5\text{m.m.})$ in the case of the largest of the probes used.

It was difficult to draw any conclusions from the miscellaneous disturbed profiles and so the position of the density maxima were plotted as in figure 18. Clearly the pitot probe has a drastic effect on the flow which depends on the position of the probe within the flow. The Langmuir probe apparently has less effect on the shock position, although, as stated before, it does disturb the density profiles a long way up stream.

As a final check, density profiles obtained from photographs were considered separately with pitot probe and Langmuir probe profiles to obtain velocity profiles. This necessitated the assumptions that, $\rho u^2 \propto P_0$ for a pitot tube, and $\rho u \propto I$ for the Langmuir probe, which should be satisfactory away from the wall. In both cases the resultant velocity profiles were distorted and this again indicates that both probes interfere substantially with the flow.

5. Discussion

5.1. Flow about a flat plate

The present measurements were confined essentially to the merged layer regime. The measured pressure downstream approaches that predicted by weak interaction theory, but does not quite reach this limit. The density jump across the shock is also less than that predicted by the Rankin-Hugoniot shock relations, at the observed shock angle. Further, the density profiles do not contain a region of constant density between the shock and the boundary layer. This again indicates that the boundary layer and shock were merged.

At the upstream end, the number flux data reach a plateau, but do not actually drop towards the leading edge as would be expected if free molecular flow occurs at the leading edge. Measurements were therefore not conducted in the new free molecular region which occurs very close to the leading edge. However the peak of the plateau does appear to occur at $\lambda_f = 0(1)$, as suggested by the analysis of Hamel and Cooper⁶.

The trends in the surface number flux data were better correlated with those of the Monte-Carlo calculations and the experiments performed by Horstman⁷, when X/λ_∞ replaced X/λ_f as the length scale. It will be seen from figure 9 that the shapes of the number flux distributions are very similar when plotted against X/λ_∞ , but as the experiments and calculations were performed under different conditions, some differences in the absolute values of number flux were to be

anticipated. The successful use of X/λ_∞ as the length scale for the number flux distribution is in contrast with the results reported by Lewis and Smith⁷, who found that experimental data obtained in both the near free molecular region and the merged layer region were correlated by X/λ_f . However Lillicrap⁵³ successfully used the parameter, MC/Re_{x_∞} , to correlate shock strength data. Since $MC/Re_{x_\infty} \approx 0.8C/\gamma^{1/2} \cdot \lambda_\infty/X$ and both C and γ are $O(1)$, the parameter used by Lillicrap is approximately the inverse of X/λ_p .

As the present comparison is between data covering a wider range of wall temperature and Mach number, it imposes a greater task on the correlating parameter, which is probably why X/λ_f proved to be less satisfactory. It therefore appears that X/λ_∞ is generally more suitable than X/λ_∞ , except in the near free molecular region. However, as Vogenitz³⁷ discussed, the various mean free paths may be approximately related by Mach number, which does not drastically affect the flow structure in the merged layer regime. It is therefore unlikely that there will be a startling difference between the two.

The structure of the flow field was similar at different operating conditions. This is not surprising since the wall is very cold, and also both the relevant temperature ratios (T_w/T_0 and T_w/T_∞) are virtually the same in all the measurements. Further, the Mach number range was moderate.

The general features of the flow are similar to

those previously observed in nitrogen. It consists of a relatively thick shock in contact with a viscous layer that extends to the wall. The shock is nearly straight, but decreases slowly in thickness and strength as the leading edge is approached. However, it constitutes an increasing proportion of the flow and is dominant at the leading edge.

The density maximum emanates from a position a little below and ahead of the leading edge, because the lower surface of the model has a finite angle relative to the free stream. Unfortunately the scale of the measurements was such as to preclude accurate measurements, at the leading edge. However, no significant disturbance was apparent at distances greater than $2\lambda_\infty$ up stream of the leading edge, i.e. the disturbance was less than 0(2%).

The shock was steeper and stronger than that observed in nitrogen flows under similar conditions. It produced larger surface pressures and number fluxes.

As discussed in section 4.3.3., the theory for weak interaction on a cold flat plate suggests that the slope of the boundary layer edge is proportional to $(\gamma-1)$. At high Mach number, the oblique shock relations indicate that the shock angle is related to the wedge angle by a term proportional to $(\gamma+1)$. Hence, if viscous interaction produces a shock supported by the boundary layer, the shock angle, θ_s , is proportional to $(\gamma+1)(\gamma-1)$. The shock angle in argon can therefore be nearly twice that in nitrogen.

Similarly, the pressure ratio across the shock can be related to specific heat as follows, $\frac{P_1 - P_2}{P_1} \sim 8(\gamma - 1)(\gamma^2 - 1)$. This suggests that the pressure jump in argon will be nearly four times higher than that in nitrogen.

These calculations are based on the extreme assumption that the wall is completely cold. Under the present conditions, the shock is also substantially weakened by merging. Even so, some increase in shock angle and strength might be anticipated, and this appears to be the case.

When comparing the present results with those obtained from Monte-Carlo calculations performed under similar conditions, it is found that the shock positions are very similar, but the density rise is slightly higher in the present case. The number flux to the wall is also slightly higher, in agreement with the results of the measurements made by Horstman⁷ in a monatomic gas.

When the Monte-Carlo calculations were performed, some rather uncertain assumptions about physical processes, (such as surface reflection and molecular interaction), were unavoidable. Small discrepancies are therefore likely to occur until sufficient experience has been gained to specify these quantities precisely. However, the agreement between the calculations and the measurements is certainly sufficiently close to indicate that the Monte-Carlo technique is sound.

It will be noted that both the calculations and

experiments were conducted with monatomic gases. The comparison is therefore more meaningful than those previously conducted between the calculations and experiments performed in diatomic nitrogen.

The present measurements have been discussed and compared to the results of Monte-Carlo calculations with the aid of density profiles, flow field density maps, and graphs showing the position and value of the maximum density. These latter two quantities are particularly useful as they may be measured accurately and unambiguously and dependent on the complete flow structure. Taken together, they give the best simple representation of the flow field and for this reason many of the results of the Monte-Carlo calculations are presented in this form.

Since the Monte-Carlo calculations were performed for the complete flow field, simultaneously, there is no obvious advantage in considering the interrelated shock layer and viscous layer regions separately. However, in earlier studies the regions have been considered separately and this will be repeated for the sake of completeness. Close to the wall, the present density measurements may be unreliable because of the quenching of meta stable atoms by collision with the wall, and hence only the shock layer will be considered.

The shock position can be defined as the position of maximum slope, i.e. point of inflection, in the density profiles as shown in figure 19. If a tangent to the profile is drawn at this point, the intersection of the

tangent with the values of free stream and peak density may be used to define a shock thickness (Δs) also shown in figure 19.

It will be seen that the shock position of the present data is reasonably correlated by the parameter $M^2 C_\infty (T_w/T_\infty)^{1/2} / Re_\infty$ suggested by Shorenstein²². A best fit line drawn through data obtained by Lillicrap⁵³ in nitrogen, under otherwise similar conditions is included. The results of Monte-Carlo calculations also performed under similar conditions and obtained from reference⁵² are shown. Unfortunately this information had to be obtained from small reproductions of complete density profiles and so are not likely to be accurate.

As before, it will be seen that the shock in argon is rather steeper than that observed in nitrogen. The argon shock also appears to be slightly steeper than that indicated by the Monte-Carlo calculations, although the likely errors in extracting this information from the referenced paper precludes any definite conclusions.

The shock layer thickness is also shown in figure 19. It will be seen that the shock region in argon constitutes an increasing proportion of the flow field as the leading edge is approached, but not to quite the same extent as in nitrogen. The same trend occurs when the present measurements are compared with the Monte-Carlo calculations. This slight disparity is not thought to be significant as none of the measurements are likely to be accurate very close to the leading edge, further, $\bar{V}_\infty$ may not be a satisfactory scaling parameter at a dis-

tance of a few mean free paths from the leading edge.

5.2. Flow about a wedge

The structure of the flow field is not greatly changed when the model is set at small angles of incidence to the flow. At positive incidence the shock is slightly curved but the position of the shock relative to the model is little affected. The shock strength is also very similar to that which occurs at zero incidence, although the density increased more rapidly near the leading edge. This is consistent with the surface pressure measurements which show that the departure from weak viscous interaction occurs nearer to the leading edge when the model is at positive incidence.

When the model is at negative incidence, the shock strength is again similar to that on a flat plate, but the departure from weak interaction is moved back from the leading edge. There was, however, a region of increased density close to the leading edge as at positive incidence. In the case of negative incidence, this was probably due to the enhanced under surface wedge angle. At positive incidence, under-surface effects become less significant and vanish when the incidence is half the leading edge wedge angle.

At negative incidence the shock is straight, but the shock angle relative to the model is greater than at zero incidence. However, the angle of the shock relative to the free stream is virtually the same as at zero incidence, so the shock appears to emanate from the leading edge, and the flow field expands to support

it.

It was found that as incidence was increased in the positive sense, the number flux at the surface decreased towards that predicted by free molecular theory. This appears inconsistent, as it might be anticipated that the flow would become less rarefied with increased incidence, so the number flux would become less like that predicted by free molecular theory.

However, at positive incidence, the fraction of atoms reaching the surface directly from the free stream, without suffering collisions increases with incidence, so the surface number flux becomes more like that in free molecular flow.

At negative incidence, the presence of the shock serves to deflect many more atoms down on to the surface than would reach it by the normal free molecular processes. The number flux is therefore much greater than that predicted by free molecular theory. This effect is enhanced by the fact that the shock position relative to the free stream is constant at negative incidence. If the shock were to follow the body, with increasing negative incidence the shock would become more masked from the free stream by the body, and hence less able to deflect free stream atoms down on to the body.

In brief, the flow field structure was not greatly affected by the small range of angles of incidence available. This is not surprising since these angles are small compared with relevant angles in the flow, such as shock angle which are $O(30^\circ)$.

As discussed previously, there are not, at present, any adequate theoretical predictions of conditions in merged layer wedge flows and so it was not possible to make a comparison. The present results would be useful for such a comparison if the calculations were performed.

5.3. Use of probes in rarefied flows

Previously, probes have been used to obtain qualitative information in low density flows. Pitot probes were successfully used to demonstrate the onset of merging and also to approximately establish the position of the shock. The profiles obtained are comparable with those calculated, although it is only the early calculations which explicitly quote results for pitot pressure profiles. Unfortunately these calculations are found to be unsatisfactory in other respects, as they are based on crude physical models of the flow field and neglect important phenomena such as slip. They cannot be used to provide a quantitative check on the measurements.

In the present case, it is only when the mass flux and momentum flux are combined to yield velocity and density, that anomalies appear.

It is perhaps, profitable to consider the fate of observations which were reduced with a similar scheme proposed by McCroskey⁵⁴. In that case, momentum flux was combined with energy flux obtained from a heat transfer probe, to obtain density and velocity profiles. Only one density profile was reported. This was found to be in moderately good agreement with a density pro-

file obtained using an electron beam. The measurements were conducted at $\bar{V}_\infty = 0.2$, so they were only just in the merged layer regime.

Unfortunately, the corresponding velocity profile was not presented, presumably because of irregularities in the profile, so it is not possible to verify that the velocity profile was satisfactory. In the present case, some of the density profiles were reasonable, but they were always accompanied by irrational velocity profiles.

There have been no further reports of the application of the apparently promising techniques proposed by McCroskey. It is therefore concluded that difficulties similar to those at present, were encountered.

The photographs, taken with the probes in the flow, showed that both of the present probes cause significant disturbance to the flow and this is thought to be the reason for the distorted velocity and density profiles obtained from probe measurements. The extent of flow disturbance is rather surprising in the case of the Langmuir probe, as this was near free molecular. However, the flow field associated with the probe support was unavoidably more extensive and this presumably caused the interference observed from the glow intensity measurements.

It therefore appears that probes are an unsatisfactory method of obtaining quantitative information in the merged layer regime, even when very small. They can, however, be used to obtain a general qualitative understanding of the flow structure.

6. CONCLUSIONS

- a) Probes may not be used reliably for flow field studies in the merged layer regime even when the probes are near free molecular, because of the inevitable interference of the probe support.
- b) Under the conditions of the present experiments, the luminosity in the flow field is proportional to local gas density, except close to the model wall where the meta-stable atoms responsible for the flow luminosity are quenched by collision with the wall.
- c) The present measurements are essentially in agreement with the results of Monte-Carlo calculations conducted for flow about a flat plate, under similar conditions. It would be of interest to see the calculations extended to flow about a wedge since the measurements provide comparative data for this case.
- d) In the merged layer regime, the flow fields in monatomic and diatomic gases are qualitatively very similar. However, quantitative comparison reveals that the shock layer is slightly steeper and stronger in a monatomic gas, as suggested by weak interaction theory.
- e) When the angle of incidence is varied over a small range about zero, the flow field structure is little changed, but surface quantities, such as number flux and pressure, may be substantially modified.

APPENDIX 1Diffusion of species in an ionized slip flowA.1.1. Introduction

At one time, the possibility of making heat transfer measurements in a rarefied flow was considered. As the flow in the plasma jet is partially ionized, significant heat transfer can arise from recombination of the ions and electrons. Attempts to estimate this heat transfer revealed that solutions of the diffusion equations are available only for boundary layers without slip. It was therefore decided to perform an analysis which includes the effect of slip.

In the event, the projected measurements were not undertaken. Heat transfer arising from both fluid dynamic heating and recombination is little affected by low density effects unless $\bar{V} \gg 0(1)$, as shown below. For conditions in the plasma jet this corresponds to a distance from the leading edge of $O(1\text{m.m.})$. However, as the preliminary measurements reported in section 3.3. showed, a very slender model is required. Therefore it was not possible to use calorimetric, or other bulky heat transfer measuring systems, sufficiently close to the leading edge to be of interest. Thin film heat transfer gauges are also unsuitable, as they depend on change of temperature with time, whereas the plasma jet is continuous running.

The analysis was conducted for an ionized flow but is equally applicable to a dissociated flow. The

latter is of interest as high speed, high altitude, slender vehicles are likely to have a partially dissociated flow field. Wall recombination heat transfer in a dissociated slip flow about a flat plate was discussed by the present author in reference 55.

As discussed in appendix 5, the flow field is frozen except close to the model. Hence the recombination heat transfer depends on the diffusion controlled concentration of ions and electrons at the model surface. Analytic solutions of the diffusion equations have been obtained by several authors including Chambré and Acrivos⁵⁶, who considered the case of a flat plate, and Freeman and Simpkins⁵⁷ who extended this to flow over a wedge. In obtaining these solutions it was assumed that the layer in which diffusion occurs is thin compared with the boundary layer and thus the analysis is strictly valid in the limit of Schmitt number (Sc) tending to infinity. Consistent with this assumption a Page and Falkner velocity distribution without slip was used to determine the flow within the diffusion layer.

In rarefied flows considerable velocity slip can occur in the vicinity of a leading edge and under these circumstances an alternative to the linear Page and Falkner variation of velocity can be used. Provided the Schmitt number and the slip velocity at the wall are both large, the velocity within the diffusion region will be approximately constant.

Using this assumption, the wall recombination heat transfer may be calculated.

A.1.2. Analysis

The equation for the conservation of species in a two-dimensional flow without gas phase reactions is given by Dorrance¹⁵ as,

$$\rho u \frac{\partial \alpha}{\partial x} + \rho v \frac{\partial \alpha}{\partial y} = \frac{\partial}{\partial y} \left(\rho D_{12} \frac{\partial \alpha}{\partial y} \right) \quad (\text{A.1})$$

The recombination rate at the wall will be assumed to be proportional to the concentration and the concentration gradient at the wall will then be given by Fick's law as

$$D_w \frac{\partial \alpha}{\partial y} \Big|_w = k_w \alpha_w \quad (\text{A.2})$$

k_w being the wall catalytic rate constant.

Freeman⁵⁷ solved equation (A.1) together with those for momentum and energy by using the Illingworth transformations and the streamwise co-ordinate transformation,

$$\zeta = \frac{k_w Sc (2\xi)^{1/2}}{\mu_w u_e} \quad (\text{A.3})$$

for the case of a 'cold wall' when $\frac{\rho \mu}{\rho_e \mu_e}$ and Sc can reasonably be taken to be constant and $\frac{\rho_e}{\rho_w} \ll 1$. Under these conditions the momentum and energy equations are those of a chemically inert gas and are decoupled from the concentration equation (A1) which can be re-written as,

$$\frac{\partial^2 Z}{\partial \eta^2} + Sc \cdot f \cdot \frac{\partial Z}{\partial \eta} - Sc \zeta \frac{\partial f}{\partial \eta} \cdot \frac{\partial Z}{\partial \zeta} = 0, \quad (\text{A.4})$$

with the boundary conditions

$$\begin{aligned} \frac{\partial Z}{\partial \eta} (\zeta, 0) &= \zeta Z(\zeta, 0) \\ Z(\zeta, \infty) &= 1 \end{aligned}$$

Z is reduced mass fraction α/α_e which for the frozen flow is α/α_∞ .

Equation (A.4) can be solved if we specify the velocity distribution within the diffusion layer i.e. we

feed in a solution to the decoupled momentum and energy equations. Adopting the Page and Falkner approximation and including slip

$$f(\eta) = (u_w/u_e)\eta + \frac{1}{2}\eta^2 f''(0) \quad (A.5)$$

This assumption implies that the diffusion layer is thin compared with the viscous layer i.e. $Sc \gg 1$. For the case of no slip, Freeman⁵⁸ has compared this result with a more detailed analysis for $Sc = O(1)$ and shown that no serious error is introduced by using the Page and Falkner approximation even when $Sc \leq 0.5$. It would appear that the method is insensitive to the requirement of $Sc \gg 1$.

For the present analysis we will consider a boundary layer in which significant slip occurs i.e. $\frac{u_w}{u_e} = O(1)$ and as $f''(0)$ is also $O(1)$ the first term on the right hand side of (A.5) dominates as η is small in the diffusion layer.

In physical terms the present assumption implies that within the thin diffusion layer adjacent to the wall, the velocity is essentially constant and equal to the slip velocity. This requires the slip velocity to be large and thus the solution is asymptotically valid as the leading edge is approached.

The value of u_w is needed for the calculation and is obtained from the boundary layer equation assuming Maxwellian slip, i.e.

$$u_w = \bar{\sigma}(\pi/2)^{\frac{1}{2}} \frac{\mu}{(\rho\mu)^{\frac{1}{2}}} \frac{\partial u}{\partial y} \Big|_w \quad (A.6)$$

where $\bar{\sigma} = \frac{2-\sigma}{\sigma}$; σ being the tangential momentum accommodation coefficient.

Substitution of $\frac{u_w}{u_e} \eta$ for $f(\eta)$ in equation (A.3) using equation (A.6) to eliminate $\frac{u_w}{u_e}$ yields,

$$Z\eta\eta + \frac{1}{\bar{\sigma}} \cdot (Z\eta\eta/\zeta - Z\zeta) = 0 \quad (\text{A.7})$$

where

$$\frac{1}{\bar{\sigma}} = \frac{\left(\frac{\pi}{2}\right)^{\frac{1}{2}} \sigma \text{Sc}^2 k_w f''(0)}{(2RT_w)^{\frac{1}{2}}}$$

and is assumed to be constant.

The problem may be simplified by a further change of variables. If $\xi = \eta\zeta$, equation (A.7) may be reduced to,

$$\zeta \frac{\partial^2 Z}{\partial \xi^2} = \frac{1}{a} \frac{\partial Z}{\partial \xi} \quad (\text{A.8})$$

Finally, the streamwise co-ordinate may be rescaled such that $\Theta = \frac{\xi^3}{3}$, and equation (A.8) becomes,

$$\frac{\partial^2 Z}{\partial \Theta^2} = \frac{1}{a} \frac{\partial Z}{\partial \Theta} \quad (\text{A.9})$$

This is subject to the boundary conditions,

$$\partial Z / \partial \xi = Z_w \text{ when } \xi = 0, \text{ from equation (A.4)} \quad (\text{A.10})$$

$$\text{and } Z \rightarrow 1 \text{ at the leading edge.} \quad (\text{A.11})$$

The problem is now formally identical to that of time dependent diffusion in a half space and hence may be solved by the application of Laplace transforms.

If $\tilde{Z}(P, \xi)$ is the Laplace transform of $Z(\Theta, \xi)$, then the transform of equation (A.9), with equation (A.11) substituted is,

$$a \frac{\partial^2 \tilde{Z}}{\partial \xi^2} = P \tilde{Z} - 1,$$

the solution of which is

$$\tilde{Z} = \frac{1}{P} + C \exp\left(-\left(\frac{P}{a}\right)^{\frac{1}{2}} \xi\right). \quad (\text{A.12})$$

The constant C is eliminated by differentiating equation (A.12) and substituting the result into the transform of equation (A.10) and this yields,

$$Z = \frac{1}{P} - \frac{Z_w(\frac{a}{P})^{\frac{1}{2}}}{P} \exp\left(-(\frac{p}{a})^{\frac{1}{2}} \xi\right).$$

Inverting this, with the aid of the table of transforms edited by Erdelyi⁵⁹, gives,

$$Z = 1 - Z_w \left(2 \left(\frac{a\theta}{\pi} \right)^{\frac{1}{2}} \exp\left(\frac{-\xi^2}{4a\theta}\right) - \xi \operatorname{erfc}\left(\frac{\xi}{2(a\theta)^{\frac{1}{2}}}\right) \right). \quad (\text{A.13})$$

It will be seen that at large values of ξ , with θ finite, corresponding to large distances from the wall, the solution is invalid, since equation (A.13) reduces to $Z \approx 1 + \xi Z_w$, which is impossible as $Z \leq 1$. This failure is not surprising, since the assumption that the velocity is constant, at its wall value, is obviously violated far from the wall.

No such difficulty exists at the wall as $\xi = 0$; then equation (A.13) reduces to,

$$Z_w = \frac{1}{1 + \left(\frac{4a\theta}{\pi}\right)^{\frac{1}{2}}}. \quad (\text{A.14})$$

A.1.3. Discussion

As the formulation of the present work is different from that preceding it, a direct comparison in terms of ζ or θ is not possible. However as the present analysis is concerned with wall concentration in a rarefied flow, it is appropriate to compare the solutions in terms of the viscous interaction parameter, $\bar{V}_\infty$.

Equation (A.14) may be expressed as,

$$Z_w = \left[1 + \left[\frac{16}{3} \frac{Sc}{f''(0)} \frac{k_w^2}{RT_w} \right]^{\frac{1}{2}} \left[\frac{1}{\pi \gamma} \frac{C_\infty \mu_e}{A^2 \mu_w} \frac{\rho_e u_e}{\rho_\infty u_\infty} \right]^{\frac{3}{4}} \frac{1}{\bar{V}_\infty^{\frac{3}{2}}} \right]^{-1}, \quad (A.15)$$

where

$$\bar{V}_\infty = \frac{C_\infty M_\infty}{(R_{ex\infty})^{\frac{1}{2}}};$$

$$C_\infty = \frac{T_\infty \mu_w}{T_w \mu_\infty},$$

and

$$\Lambda = \frac{u_e}{u_\infty}.$$

Freeman's results for a flat plate are presented as series solutions in terms of $\frac{J}{K}$ and $\frac{K}{S}$ where,

$$K = \frac{2}{3} \left(\frac{1}{6} Sc f''(0) \right)^{\frac{1}{3}} \frac{\Gamma(\frac{2}{3})}{\Gamma(\frac{4}{3})}.$$

The streamwise co-ordinate may be expressed in terms of the viscous interaction parameter as,

$$\frac{J}{K} = \frac{3}{2} \frac{\Gamma(\frac{4}{3})}{\Gamma(\frac{2}{3})} \left(\frac{6 Sc^2}{f''(0)} \right)^{\frac{1}{3}} \left(\frac{2 C_\infty \mu_e}{A^2 \gamma \mu_w} \frac{\rho_e u_e}{\rho_\infty u_\infty} \frac{k_w^2}{RT_w} \right)^{\frac{1}{2}} \frac{1}{\bar{V}_\infty} \quad (A.16)$$

In order to compare the various solutions, it is necessary to assign suitable values to the quantities in equations (A.15) and (A.16). In the case of a cold, highly catalytic wall, these might have the values,

$$\frac{C_\infty \mu_e \rho_e u_e}{A^2 \gamma \mu_w \rho_\infty u_\infty} = 1 \text{ and } \frac{k_w}{(RT_w)^{\frac{1}{2}}} = 1.$$

An adiabatic, weakly catalytic wall may be represented by,

$$\frac{C_\infty \mu_e \rho_e u_e}{A^2 \gamma \mu_w \rho_\infty u_\infty} = \frac{1}{4} \quad \text{and} \quad \frac{k_w}{(RT_w)^{\frac{1}{2}}} = \frac{1}{4}.$$

The results for these two representative cases are

compared in figures A1 and A2 where the effect of different accommodation coefficients is also shown. If the reflection of incident molecules at the wall is completely diffuse $\bar{\sigma} \triangleq 1$ and if 30% of the molecules undergo specular reflection $\bar{\sigma} \triangleq 2$.

It will be seen that close to the leading edge, the wall concentration predicted by the present theory is higher than that indicated in the earlier work, which is reasonable since one would expect the increased bulk motion produced by wall slip to enhance the concentration. Down stream the wall velocity becomes small and ceases to be representative of the velocity in the diffusion region. Thus the neglect of those ionized species transported at the higher velocities which occur away from the wall, causes the prediction of wall concentration to be too low. The present theory, and that of Freeman are likely to be correct in the upstream and downstream limits, respectively, but both underestimate the wall concentration in the intermediate region.

The effect of slip appears to be significant mainly in the rarefied regime corresponding to fairly large values of $\bar{V}_\infty$. In this regime the present theory should be valid, since equation (A.1) is a fundamental gas dynamic conservation equation, and although boundary layer type transformations are employed to solve it, the boundary layer assumptions are not made.

In the weakly catalytic wall case, neglect of slip is not serious as the wall concentration does not fall much below its free stream value until $\bar{V}_\infty < o(1)$ which

is the lower limit of applicability of the present theory. However, in the highly catalytic wall case, the difference is significant as the present theory provides estimates of wall concentration which are 20% greater than those obtained from continuum theory. The heat transfer arising from the recombination of the ionized species at the wall in rarefied slip flow will be increased correspondingly and may be calculated from

$$\dot{q} = \rho_w k_w h_R \propto Z_w, \quad (\text{A.17})$$

where h_R is the energy of recombination, and Z_w is evaluated from equation (A.15).

APPENDIX 2Velocity MeasurementsA.2.1. Previous studies

Conditions in the plasma jet were previously studied, as discussed in appendix 4. The reservoir conditions were well established, but conditions in the free stream were less certain. It was therefore decided to attempt a more accurate measurement of velocity, this being a directly measurable, fundamental free stream property.

Bohn⁶⁰ measured velocity in a low power plasma jet by observing the Doppler shift of spectral lines emitted by excited atoms moving with the stream. These measurements had to be conducted at low electron densities, so that Stark effects were not significant. Velocity has also been determined by measuring the time of flight of random disturbances produced by the arc. Authors following this approach include Simpkins⁴⁵, Park⁴⁴ and Winkler⁶¹ who used photo-multipliers, and Fuhs⁶², who used magnetic search coils. Winkler also tried Langmuir probes. In order to produce a more easily observed disturbance, Simpkins repeated his measurements using a capacitor discharge between electrodes in the stream to obtain a luminosity pulse. All of these techniques exhibit scattered data, probably because the disturbances do not travel at stream velocity.

Buckingham⁶³ produced a sinusoidal variation in stream luminosity by modulating the power to the arc heater. Observing the variation via a rotating mirror,

produced an apparent change in frequency directly dependant on stream velocity. The method minimises doubts about whether the induced disturbances are travelling at stream velocity as any components failing to do so should be self cancelling, owing to the asymmetry of the wave.

Unfortunately the method is of limited accuracy, and may only be applied to a stable arc, which usually necessitates operating at a power level below that of interest for aerodynamic facilities.

A.2.2. Experimental procedure

In view of the difficulties discussed above, the time of flight method was modified. As in Buckingham's experiment, a sinusoidal variation in intensity was obtained by modulating the power supplied to the arc, but the velocity was determined by measuring the difference in phase of the light detected by photo-multipliers at two points along the stream. The velocity (U) is then simply given by:

$$U = 2\pi \frac{fs}{\theta}$$

where θ = phase difference (in radians) between stations.

f = modulation frequency.

s = separation of the stations (50cm.).

As the measurement is quasi-continuous, extensive filtering may be employed to eliminate random disturbance effects. This was in fact necessary as a signal to noise ratio of about 0.3 was obtained.

The stream intensity modulation was produced by

by-passing some of the current carried by the arc with the shunt shown in figure A.3. The shunt dissipated 2Kw (about 5% of the arc power) and was operated at about 3KHz. Operation was restricted to bursts of a few seconds, as the components were considerably over run. Backed P.K.P., N.P.N. transistors were used in the current gain stages, so that a full 30v peak to peak signal was available to the ten power by-pass transistors with a 30v line.

The layout of the detection system is also shown in figure A.3. Light from two stations in the stream was detected with photo-multipliers. The resultant signals were passed through high selectivity filters to remove unwanted noise and then through logic elements to give an output dependent on the difference in phase of the light at the two stations. Detailed discussion of the filters and subsequent detection system is deferred to section A.2.3. below. It was, unfortunately, not possible to lock the filters to the shunt oscillator, as the low velocity of the gas through the plenum chamber eliminates any phase correlation between the arc and the free stream modulation. Initially the filters were adjusted to give maximum gain and the same phase shift. Two signals of known phase difference, derived from a simple R-C phase shifter, were then used to calibrate the complete detection system. Calibration was performed before and after each set of measurements and an average value used. The change in calibration was at most 5%. Any differential drift of the phase change

produced by the filter circuits occurring between calibrations, was minimised by taking the mean of two readings obtained at a given condition, using a reversing switch.

It was also necessary to determine the enthalpy of the stream, since velocity depends primarily upon enthalpy and Mach number. The enthalpy (H) was calculated from,

$$H = \frac{\eta(E-W)}{\dot{m}}$$

where E = output of the power supply

W = power by-passed by the shunt

$\dot{m}$ = argon mass flow rate = 1.1g/s

η = head efficiency = 0.35, the mean of measured data given by Adcock⁴⁶

A.2.3. Filters and phase shift measurement

In order to separate the 3kHz modulation signal from the noise, and a 300Hz ripple originating in the arc power supplies, two stages of filtering were used. A parallel-T notch filter, having a transmission of 1% at 300Hz and 95% at 3kHz, suppressed much of the 300Hz signal and was followed by a highly selective active filter incorporating "Computing Techniques" operational amplifiers, as in figure A.4. These were used to doubly integrate an initial signal (say θ) which was then added to the input signal, with some signal derived from an intermediate stage, and passed round a feed-back loop. Equating signals at either end of the loop gives rise to the expression, $\ddot{\theta} + A_3(A_1A_2k_1\dot{\theta} + A_1^2k_2\theta + f(t)) = 0$, where A_1 , A_2 and A_3 are the transfer functions of the operational elements, k_1 and k_2 are the ratios of the

resistive dividers and $f(t)$ is the input signal. Clearly, damped, tuned or oscillatory operation may occur, depending on the values of transfer functions and resistance ratios. With the components shown, the filter remained stable with values of Q and of gain equal to 200, and had a tunable range of 2.5 to 3.2 kHz. The Q was predominantly controlled by k_1 , and the tuned frequency by k_2 .

The sinusoidal outputs from the filters were converted to square waves, using S.G.S. - Fairchild $\mu A710$ comparators as very high gain amplifiers. These have a rise time of 40ns and give peak output from 3mv, so that the 10v, 3kHz input was faithfully converted to a square wave. Following the squaring, a simple cross-over network constructed from Motorola mcB46p "Nand" logic elements was used to generate rectangular pulses of a width corresponding to the phase difference of the initial signals. These rectangular pulses were then fed into a quasi-integrator of 100ms time constant, the mean output of this being proportional to the phase difference of the initial signals. The integrator output was measured on a digital voltmeter.

A.2.4. Discussion

Velocity has been measured over a range of enthalpies with the method described above. The results are shown in figure A5 and compared with earlier values, Park (A) and Simpkins (B) calculated the free stream conditions from those in the reservoir. Park⁴⁴ numerically solved the full flow equations, including a rate

equation for ionic relaxation and Simpkins⁴⁵ used a Mollier chart, assuming isentropic, frozen flow and an exit Mach number of 2.8, as derived from pitot measurements. Both the rate constant used by Park and the simplifying assumptions about real gas effects used by Simpkins are rather uncertain. However, it will be seen that the present measurements lie between these theoretical predictions. Also they improve upon Simpkins' badly scattered data obtained by the "random disturbance" method, but preclude the results he obtained with the "induced disturbance" method. This suggests that any method of measuring velocity which depends on observing non-periodic disturbances is suspect, since these may propagate at a velocity other than that of the stream. One would expect rearward travelling disturbances to be generated at the same time as forward travelling disturbances, giving a spread of velocity measurements about the true value. It will in fact be seen that the velocities measured by both the random disturbance and induced disturbance methods are consistently high.

In the case of random disturbances, the rearward travelling components would not be observed as they are generated in the slow, subsonic, flow of the arc region, and, being at least sonic, they would travel upstream and be lost. The large scatter occurring in the random disturbance method may be due to the wide range of disturbance strength arising from random arc fluctuations. However, the method is intrinsically inaccurate, as it

is necessary to measure time differences that are much smaller than the rise time of the disturbances.

In order to generate a more sharply defined disturbance, Simpkins discharged a capacitor through electrodes placed normal to the free stream. This produced intense cylindrical disturbances, which appear to have been propagated down the stream at a Mach number comparable to that of the stream itself. Again the very low values of velocity corresponding to rearward travelling components were not observed. This is probably because by the time the slow moving rearward components reach the observer, they lie on a much larger diameter cylinder than the fast components, and hence are heavily attenuated. The slow components are also more liable to attenuation by diffusive and other dissipative mechanisms. It is thus concluded that observation of non-periodic disturbances tends always to measure speeds higher than the true stream velocity.

It will be seen that the present results exhibit greater scatter than would be expected from the measuring procedure used. This is because the filters, although highly selective, were unable to completely suppress the effects of noise. This was verified by adding high noise levels to the calibration signal. Comparable scatter was produced, but by taking the mean of several readings the correct results were obtained. Further increase in the selectivity of filters would make stabilisation of phase shift extremely difficult. Alternatively the modulation of the power supplied to

the arc could be substantially increased, but this would, in effect, mean measuring the velocity in an unsteady jet.

APPENDIX 3Increase in the operating Mach number of the plasma jet

Following the velocity measurements described in appendix 2 and reference 64, it was decided to increase the operating Mach number from 2.7 to 7, the corresponding Reynolds number being reduced from 30/m.m. to 10/m.m. This was to enable the facility to be used for studies in the rarefied flow regime, and also to increase the working area of the jet.

A larger nozzle was made, having an exit diameter of 110m.m. and a length of 240m.m. The necessary contour was determined by adding the displacement thickness, calculated from an empiric correlation devised by Burke⁶⁵, to the inviscid contour. This consisted of a cubic profile at the throat, matching into a conical expansion section of 10° half angle, and a turning section, computed from a program devised by D. Petty⁶⁶, which was intended to bring the flow parallel to the axis of the nozzle. An over all length of 240m.m. was adopted as this corresponded to the position at which the core cross section was greatest.

A diffuser was installed to assist pressure recovery, as the Kinney vacuum pump would have been unable to accommodate the mass flow rate at the low pressures corresponding to the higher Mach number flow. The minimum value of mass flow was determined by the onset of arc instability. Working at a Reynolds number of 4/m.m., Boylan⁶⁷ obtained an efficiency of 20% in an open jet diffuser, efficiency being defined as the ratio of

pressure recovered by a diffuser to that recovered by a normal shock under the same conditions. In order to obtain the necessary pressure recovery in the plasma jet, an efficiency of 40% would be required, and hence a variable geometry centre body diffuser was used. This should have been much more efficient than an open diffuser.

The combination of nozzle and diffuser was not satisfactory. At first the flow in the nozzle was established, as was evidenced by the uniform luminosity across the flow, but after a few seconds the flow collapsed to an intense core. Fine strands of asbestos attached to the nozzle wall revealed that back flow was occurring. The diffuser was operated with the centre body and throat in a range of positions and also without centre body and throat, but was unable to maintain the flow. A series of cylindrical inserts were placed in the nozzle to determine the maximum diameter of nozzle exit at which distributed flow could be maintained. This was found to be 80m.m. but the flow contained a shock generated by the intersection of the insert with the nozzle, and so another contoured nozzle of 75m.m. exit diameter was made.

The contour for the new nozzle was derived by adding half the predicted displacement thickness to the original inviscid profile, and then scaling so that the exit diameter was 75m.m. with a length of 120m.m. This rather crude procedure was adopted, in the absence of more reliable information, to provide a smooth contour.

The massive displacement thickness correction predicted by the Burke correlation was almost certainly excessive. It probably arose in the attempt to extrapolate a correlation based on information gained in high Mach number, high Reynolds number facilities, to conditions in the plasma jet. More recently Potter⁶⁸ has calculated boundary layer development from the boundary layer equations and successfully compared the results with experiments, even for thick boundary layers. This would provide a more acceptable method for any future nozzle design, although it would require considerable effort to develop a similar program.

The new nozzle again failed to maintain the flow and several diffuser geometries were tried without success. Probably the reason for the satisfactory operation of the 80m.m. diameter insert was that the strong shock, generated by its intersection with the nozzle wall, provided adequate pressure recovery. This of course, could not happen in a shock free, contoured nozzle. However the new nozzle did maintain the flow for about two minutes, which enabled the additional pumping capacity required, to be determined. The rate of rise of pressure in the dump tank in between the diffuser outlet and the pump, was measured with a transducer, while the flow was being maintained. Since the volume (V) of the dump tank is known, the effective pumping speed ($\dot{V}$) of the dump tank can be calculated from,

$$\dot{V} = \frac{V}{P} \frac{dP}{dt}$$

From this expression it was calculated that an additional

pumping capacity of 2000 m³/hr was necessary to maintain the flow. In order to achieve this, a "Ruvac 46" Rootes pump, having a nominal speed of about 3500 m³/hr, was installed. This was backed by the existing Kinney vane pump, and the combination proved to be capable of indefinitely maintaining the flow from the 75m.m. nozzle.

The diffuser system was optimised with a model in the working section. A series of static pressure taps were installed along the diffuser to establish the position of maximum pressure recovery and the diffuser was then terminated at this point. Various centre body geometries were tried but these proved to be either ineffective or deleterious. Diffuser performance was relatively insensitive to position and length.

The final geometry is indicated in figure 1. The flow emerging from the 75m.m. exit diameter nozzle has a free jet length of 130m.m. in which the model is placed. The diffuser has a bell intake, with a 30° semi-vertex angle, to catch gas displaced by the model. It is 1m. long and has an internal diameter of 90m.m.

APPENDIX 4Flow properties at Mach 2.7

When the plasma jet was operating at Mach 2.7, extensive efforts were made to determine the free stream conditions, as discussed in references 43-47. Also, the specific case of velocity measurements are described in appendix 2 of this thesis and reference 64.

Initially, conditions in the settling chamber were established. Gas mass flow rate and settling chamber pressure were noted. Total enthalpy was then calculated either from the throat area or the energy supplied to the gas. The latter is the difference between the electrical energy supplied to the arc and the heat removed by the cooling water. Other gas properties were then determined from a Mollier chart.

Adcock and Plumtree, working with argon and nitrogen, respectively, measured temperature in the settling chamber by comparing the intensity of appropriate spectral lines. Adcock also used the width of these lines to determine the ionization fraction. These measurements, and also those of settling chamber pressure, were found to be consistent with the energy balance determination of reservoir conditions.

Once the Mach number is known, it should be possible to predict the free stream conditions from those in the settling chamber. However, at the power levels used, the gas is significantly ionized, so the expansion of the gas through the Laval nozzle gives rise to

non-equilibrium effects. Park attempted a complete solution of the nozzle flow equations. The extensive assumptions necessary to make the problem tractable, coupled with uncertainties about the rate constants for recombination, makes assessment of the results difficult. It was therefore felt to be necessary to measure free stream properties directly.

Simpkins conducted a pitot survey at the nozzle exit, which showed that the flow had an inviscid core and a frozen Mach number of 2.7. Repetition of the spectroscopic measurements suggested that the temperature in the free stream was rather high. This was attributed to the effect of electrons which retain a temperature comparable to that in the settling chamber. The ionization fraction appears to have been similar to that in the settling chamber, but these measurements were subject to particularly bad experimental scatter, as the electron density in the free stream is rather low for spectroscopic measurement. The results of the measurements of free stream velocity, described in appendix 2, also suggest that the flow is frozen.

Fope⁶⁹ and Witte⁷⁰, have used total enthalpy probes in comparable facilities and found that the enthalpy was conserved. This would also be expected to apply in the present facility, as the calculations by Park show that the heat transfer to the nozzle wall is small, and radiative losses are negligible.

In addition to direct measurements of free stream properties, the interaction of the stream with various

bodies has been observed. Simpkins measured the shock stand off distance on a cylinder, and the surface pressure on a flat plate. Park measured the heat transfer rate to a cone and hemisphere, as well as stand off distance. Again the results were badly scattered, because of the erratic operation of the plasma jet, but were in general agreement with calculations based on the assumption that the flow was frozen and the enthalpy preserved.

Taken together, the results of all these measurements suggest that, after being heated in the arc, the gas is well mixed and in equilibrium in the settling chamber. The ionization or dissociation fraction then rapidly freezes, i.e. becomes constant, and remains at virtually the settling chamber value, right through to the free stream. Since the core flow is also inviscid and isenthalpic, it is reasonable to use the usual isentropic flow equations to determine free stream conditions from those in the settling chamber.

It is however, necessary to modify the ratio of specific heat, γ . Ionization or dissociation generate more particles, which, in the classic sense, increase the number of degrees of freedom. This effect may be included by using an effective ratio of specific heat given by,

$$\gamma = \frac{n_1(1+\alpha) + n_2(1-\alpha) + 2}{n_1(1+\alpha) + n_2(1-\alpha)}$$

where n_1 and n_2 are the number of degrees of freedom available in translational and non-translational modes, respectively. α is the ionization or dissociation fraction.

For a monatomic gas, $\Gamma = \frac{5+3\alpha}{3(1+\alpha)}$.

APPENDIX 5Ionisation fraction in the model flow field

In order to interpret the results of the Langmuir probe surveys, it is necessary to make simplifying assumptions about the ionization fraction. As it is frozen in the free stream, it could also remain frozen in the model flow field. There are three likely ways in which the ionization fraction may be affected:-

- (i) Generation of ions in the shock region
- (ii) Gas phase recombination between shock and body
- (iii) Catalytic recombination at the body

Case i is unlikely as the temperature recovery across an oblique, merged region shock would not be more than a few hundred degrees, whereas a temperature of several thousand degrees is necessary to produce significant ionization of argon. Similarly case (ii) may be neglected as the recombination rate depends primarily on density and within the flow field the density is comparable to that in the free stream where the ionization fraction is completely frozen. In fact freezing appears to occur soon after the gas leaves the plenum chamber, where the density is much higher than in the free stream or model flow field. Case (iii), however, requires more careful consideration.

In the analysis of wall concentration of ionized species given in appendix 1, the following expression (equation A13) was obtained for concentration close to the wall,

$$Z = 1 - Z_w \left(2 \left(\frac{\alpha \theta}{\pi} \right)^{\frac{1}{2}} \exp \left(\frac{-\xi^2}{4 \alpha \theta} \right) - \xi \operatorname{erfc} \left(\frac{\xi}{2 (\alpha \theta)^{\frac{1}{2}}} \right) \right)$$

At the diffusion layer thickness, δ , defined as that distance from the wall at which $Z = 1 + 2w/2$, this equation reduces to

$$2 \exp \left(-\frac{Y}{4} \right) - \pi^{\frac{1}{2}} Y^{\frac{1}{2}} \operatorname{erfc} \left(\frac{Y^{\frac{1}{2}}}{2} \right) = 1, \text{ where } Y = \frac{\xi^2}{4 \theta}$$

This may be solved graphically to yield a single value solution of $Y = 0.715$. Replacing ξ and θ with physical space co-ordinates in the solution for Y yields,

$$\delta^2 = \frac{0.54}{\bar{S} \bar{c} \gamma^{\frac{1}{2}}} \left(\frac{T_e}{T_w} \right)^{\frac{3}{2}} \frac{\mu_w^2}{P_w \rho_w} \frac{1}{V_{\infty}^3}$$

For the conditions occurring at $H/RT_0 = 60$, when $x = 20\text{m.m.}$, $\delta = 0.6\text{m.m.}$

However under these conditions, the neglect of the increased bulk motion occurring away from the wall, causes the prediction of wall concentration to be half its true value and would be expected to overestimate diffusion layer thickness by a comparable amount. Hence, $\delta \approx 0.3\text{m.m.}$, and the effects of recombination at the wall will only be significant for $y < 0(1\text{m.m.})$. This is a similar limitation to that imposed by finite probe size and the requirement that $M > 0(2)$ in order to use the data reduction scheme proposed in section 4.2.

APPENDIX 6The mean free path of electrons

The mean free path travelled by an electron before striking an atom is, $\lambda_{ea} = (n_a \Omega_{ea})^{-1}$, as given in many standard texts, such as that by Jeans⁷¹.

Jaffrin⁷² showed that for $T_e < 10^4$ °K, the collision cross section for electrons with argon atoms may be represented by, $\Omega_{ea} = (0.4 - 0.55 \times 10^{-4} T_e + 0.6 \times 10^{-8} T_e^2) \times 10^{-20} \text{ m}^2$.

The electron temperature in the free stream of the plasma jet was found by Adcock to be approximately 4000°K. Since the electron temperature is frozen, this value will apply in the present case. Hence $\Omega_{ea} = O(3 \times 10^{-21} \text{ m}^2)$. Combining this with the free stream density yields,

$$\lambda_{ea} = O(10^{-1} \text{ m}).$$

A typical length in the flow field, such as distance from the leading edge is $O(10^{-2} \text{ m})$, hence

$\lambda_{ea}/X = O(10)$. The electrons will therefore be effectively "free molecular" with respect to the flow field, so the electron number density in this region should be similar to that in the free stream.

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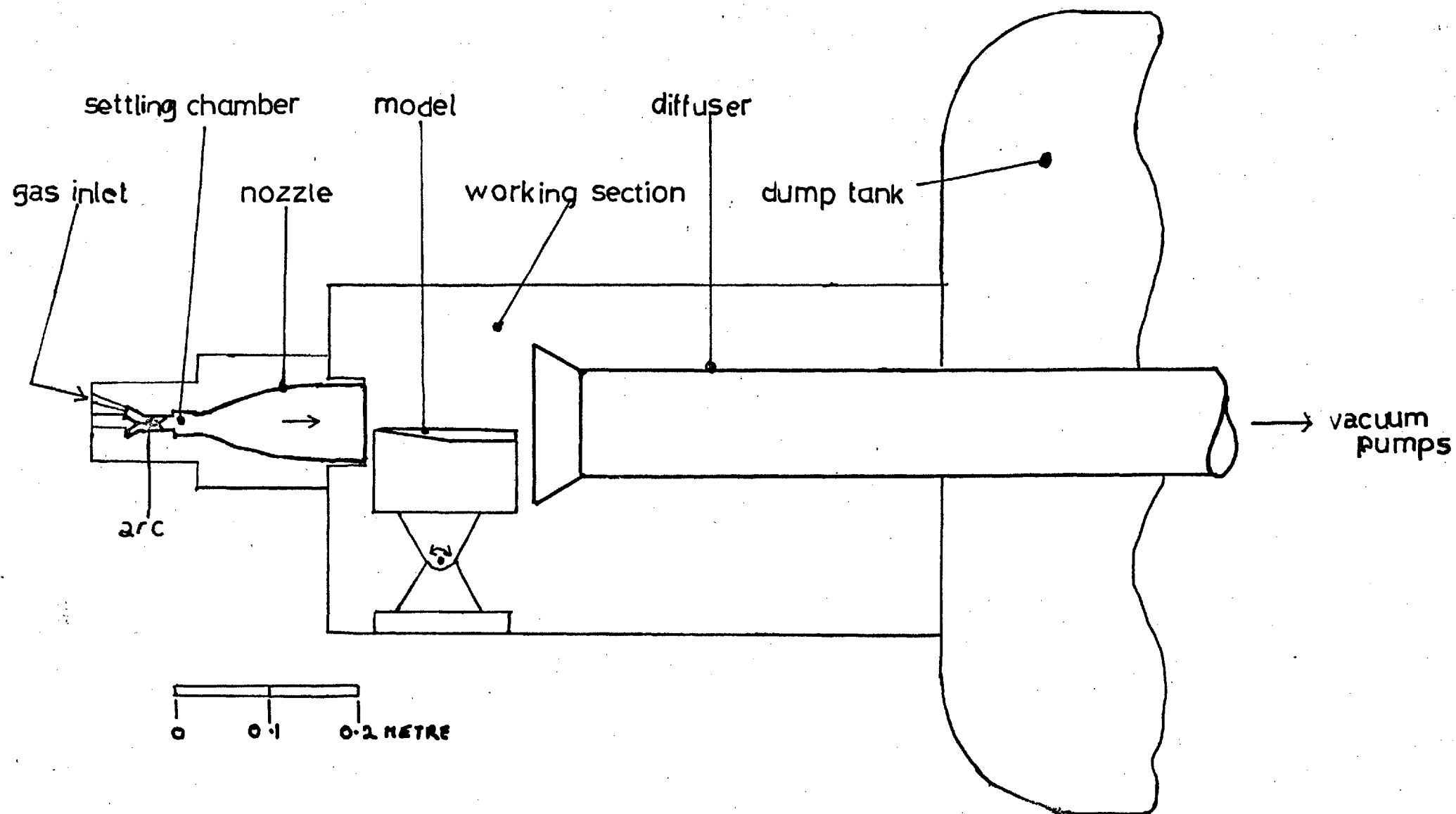


FIG.1 The plasma jet

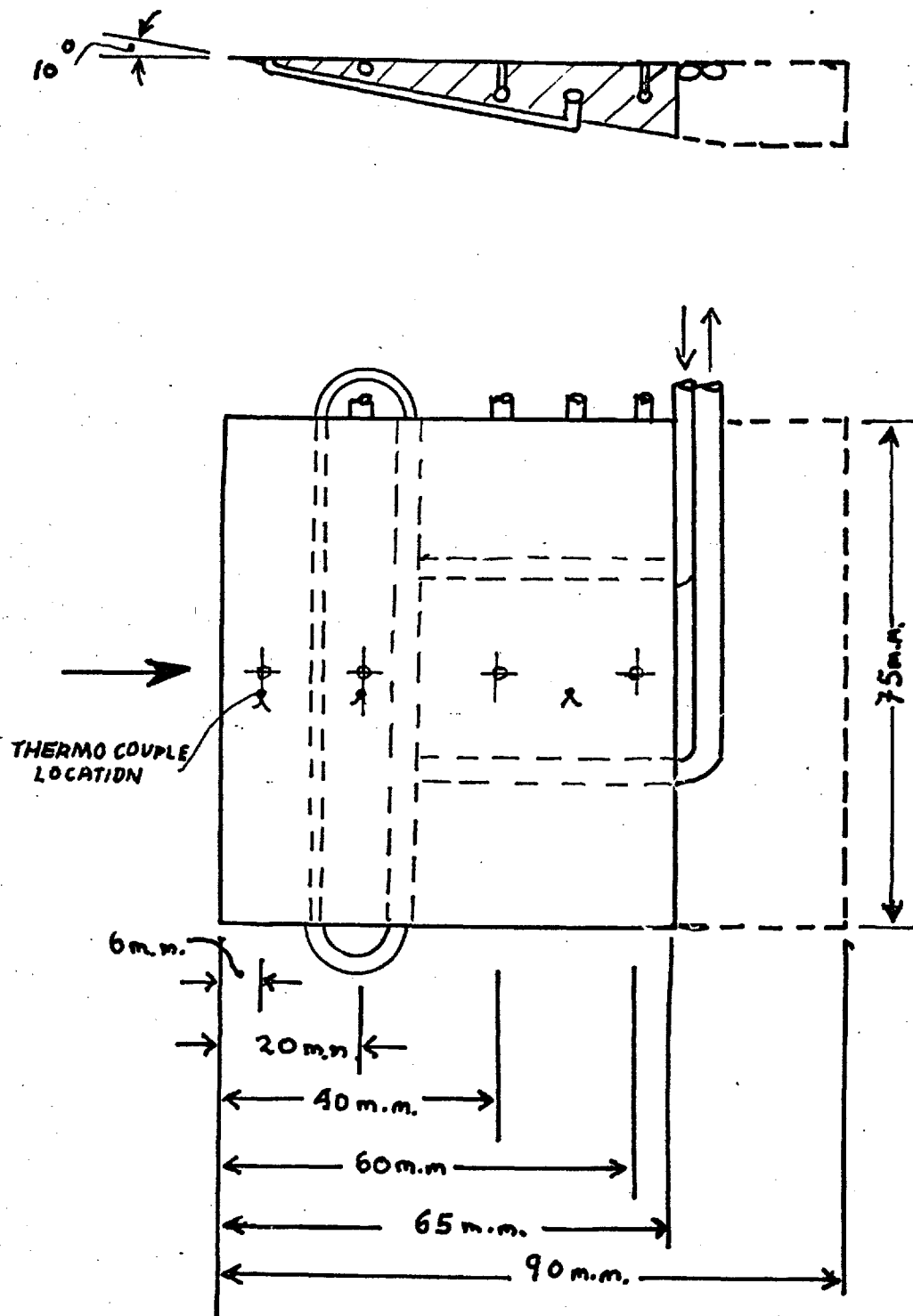


FIG. 2. Model

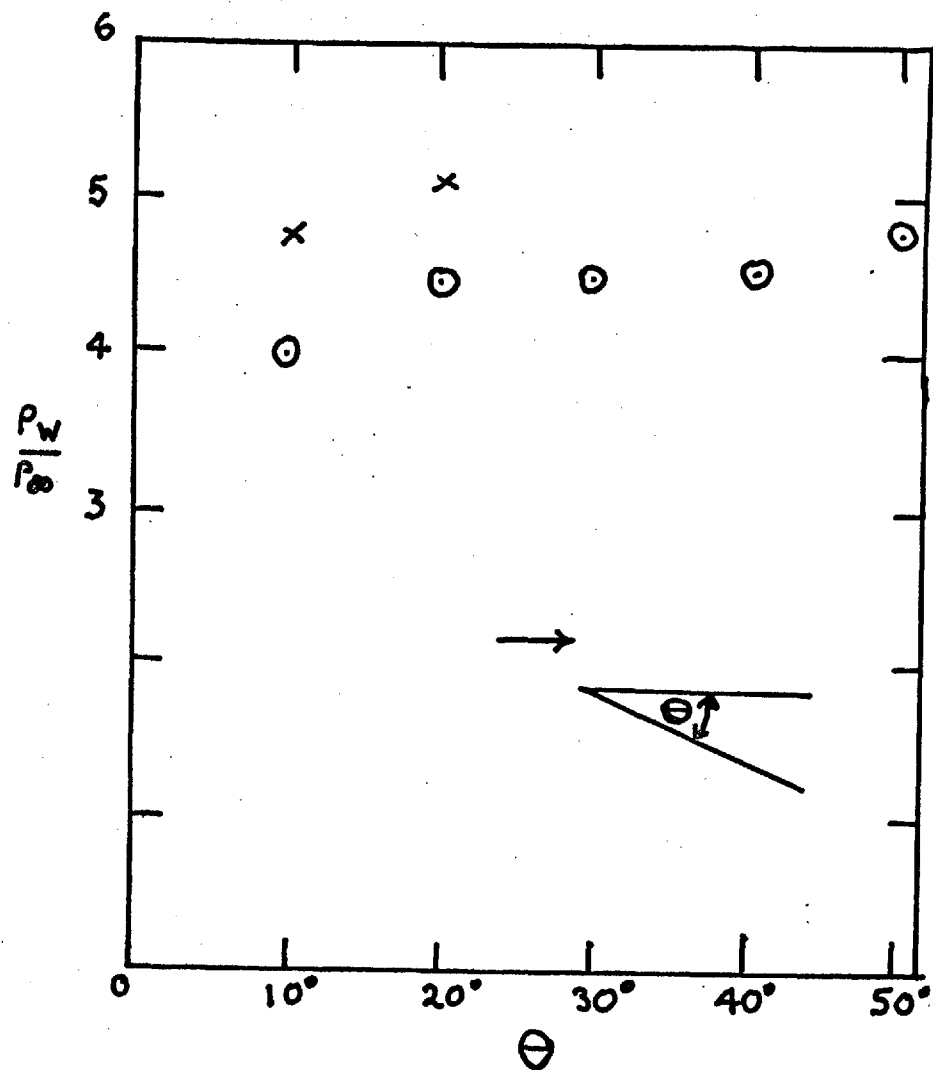


FIG. 3 Under surface wedge effect

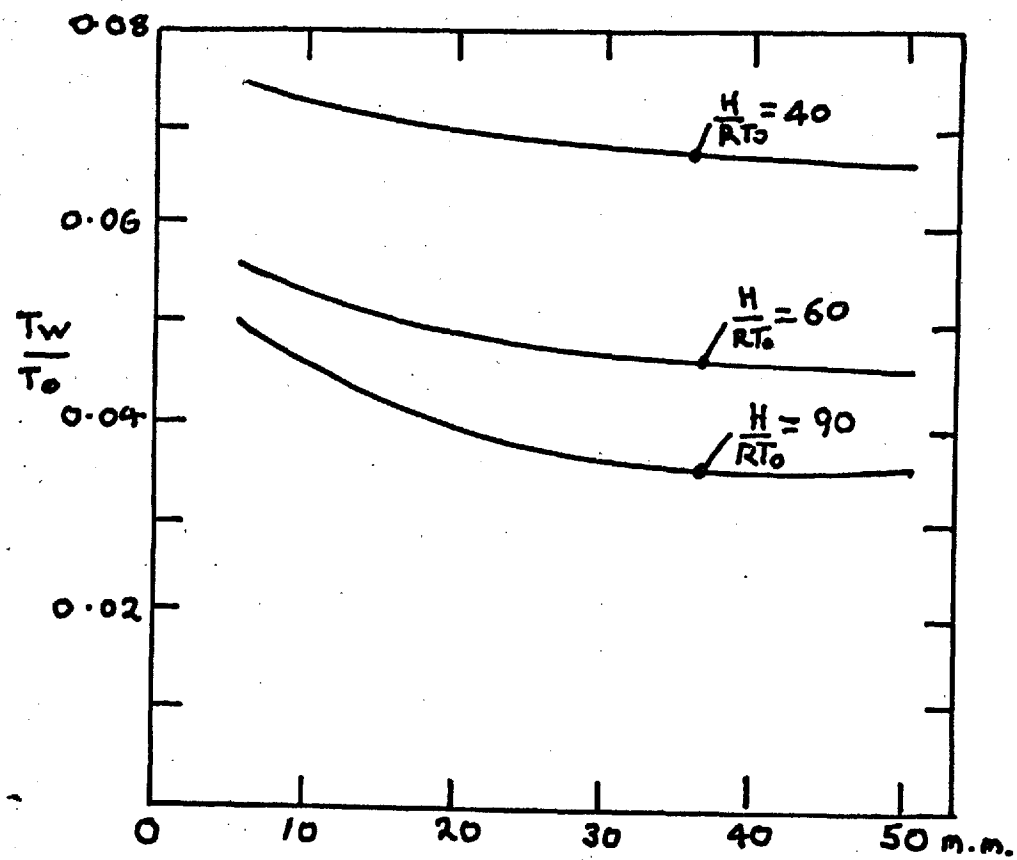


FIG. 4 Wall temperature

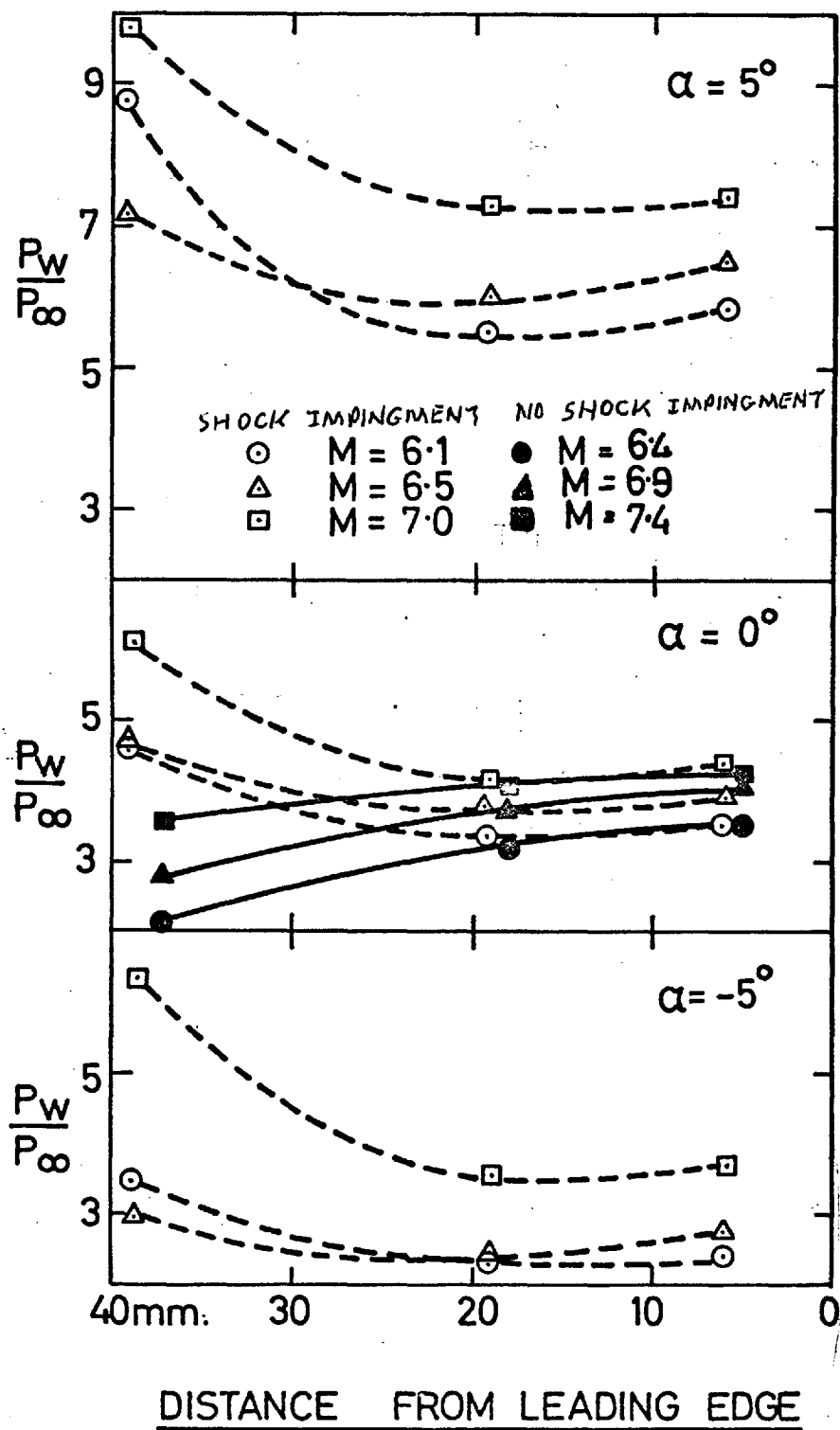


FIG. 5 Shock impingement

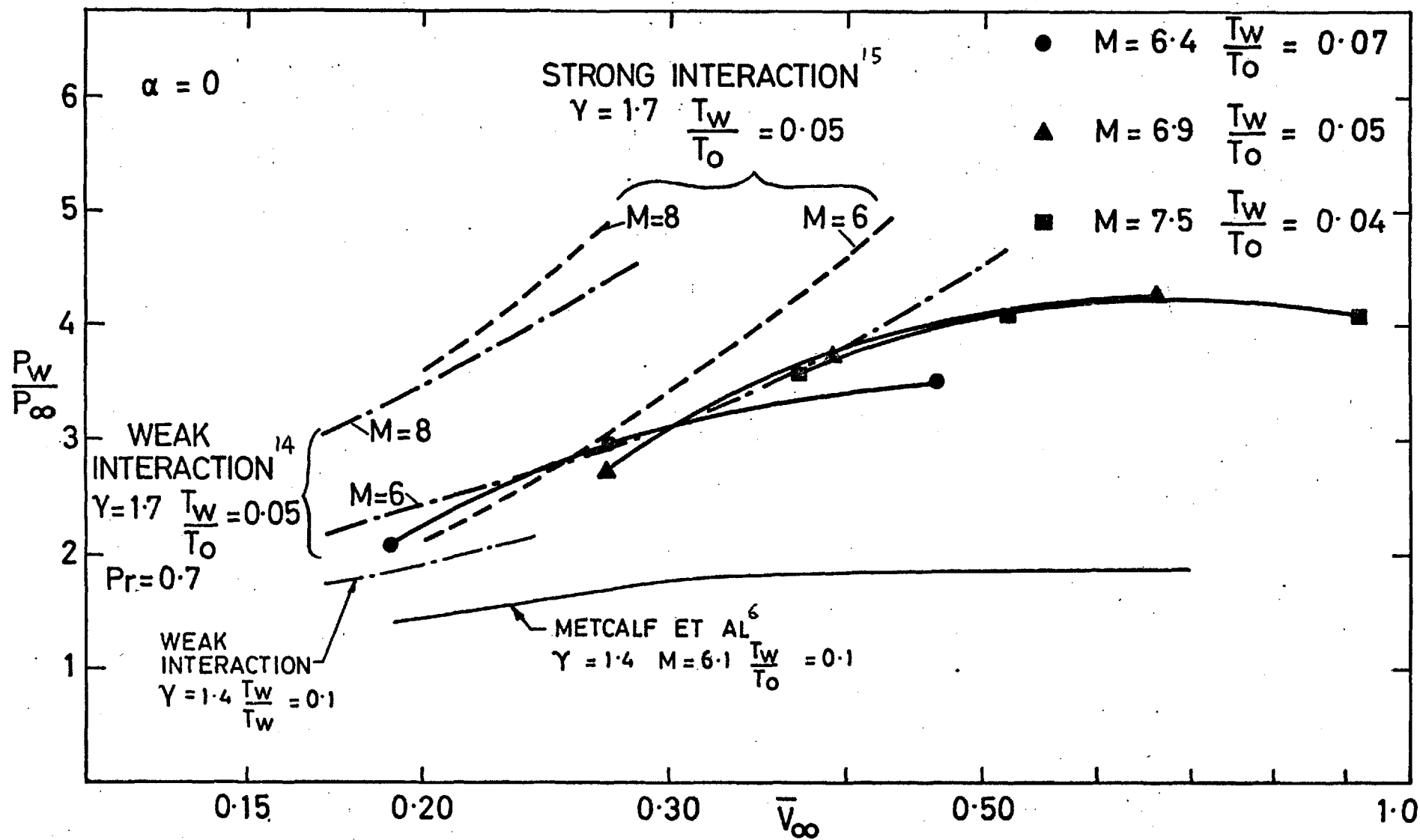


FIG. 6 Pressure on a flat plate

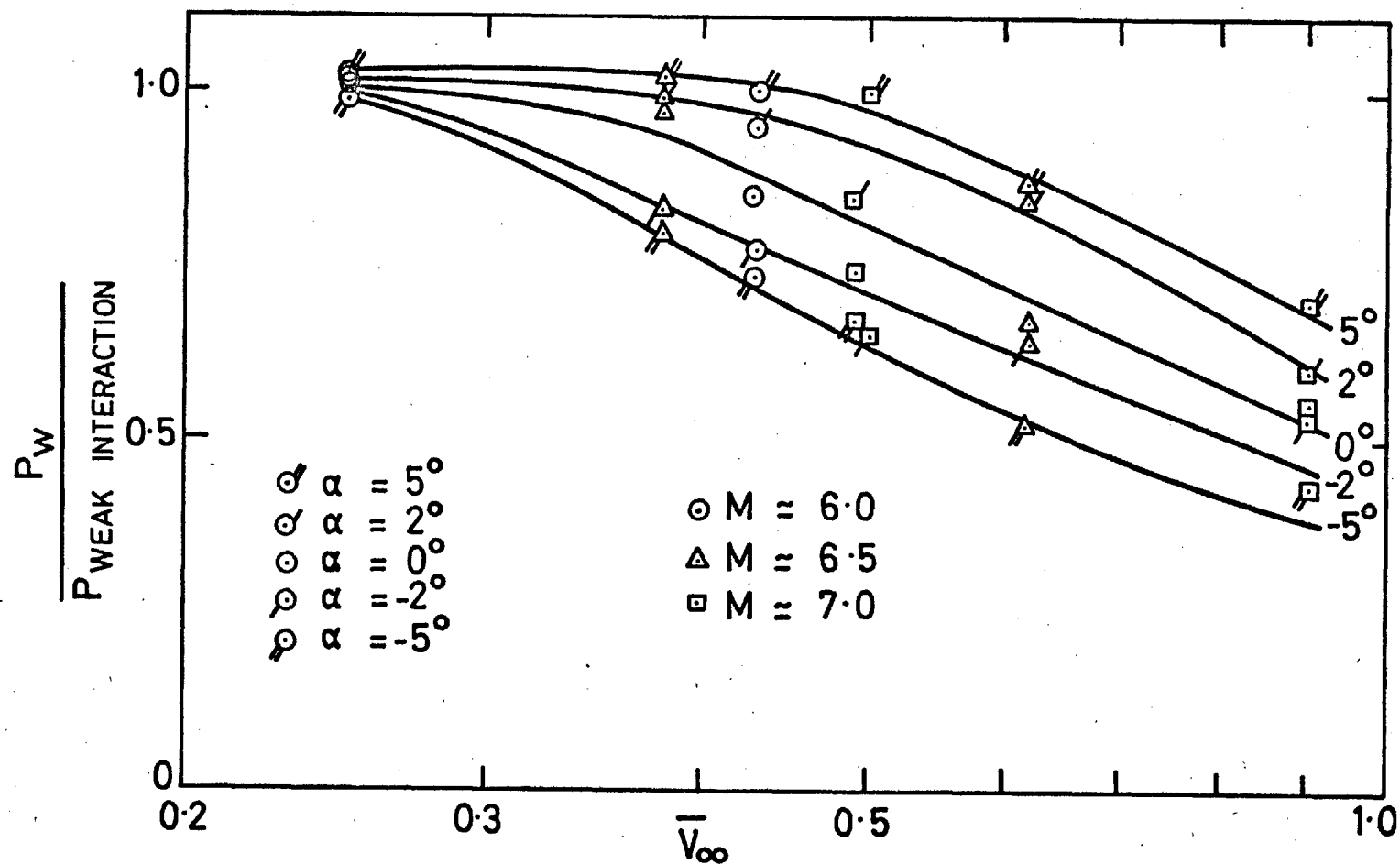


FIG. 7 Pressure on wedges

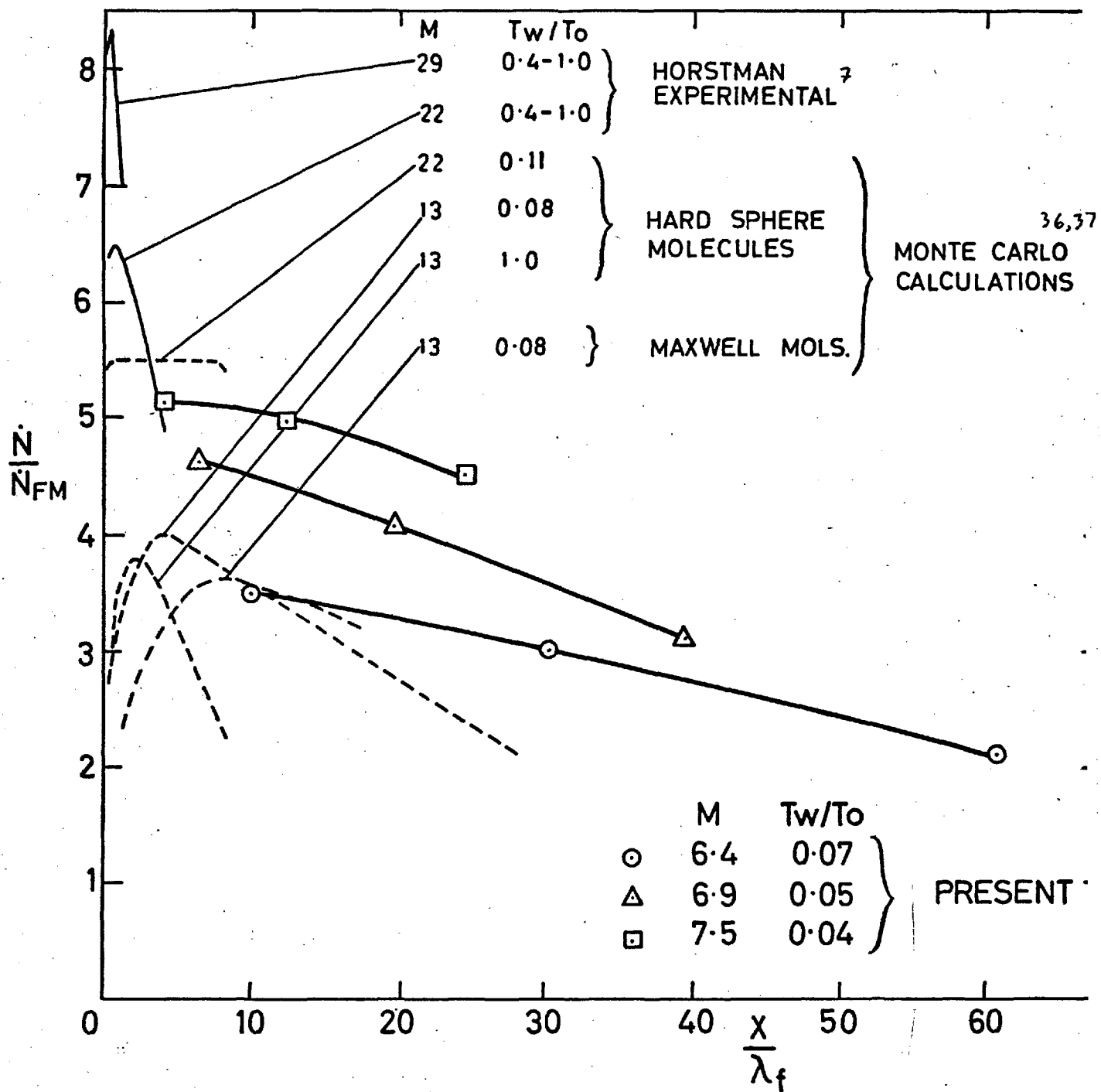


FIG 8 Number flux on a flat plate

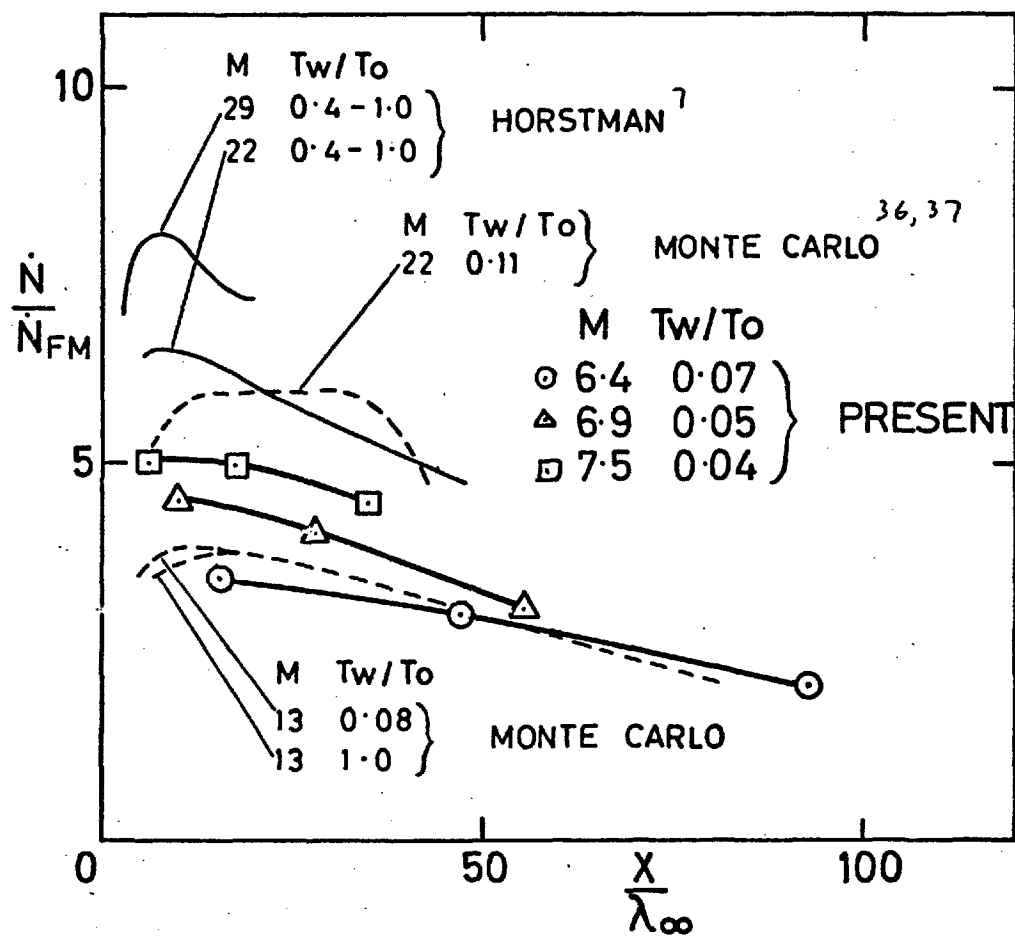


FIG.9 Number flux on a flat plate

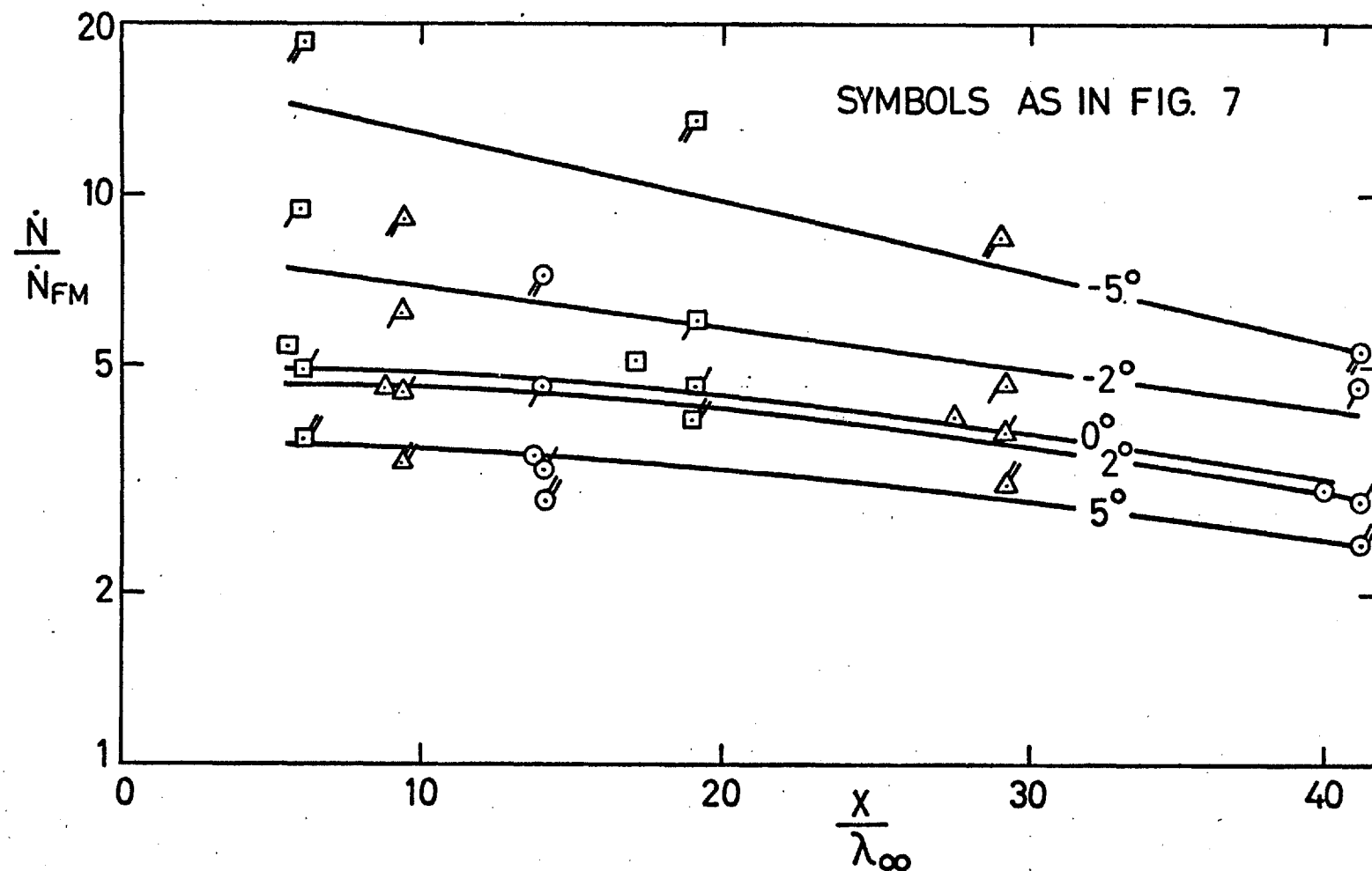


FIG. 10 Number flux on wedges

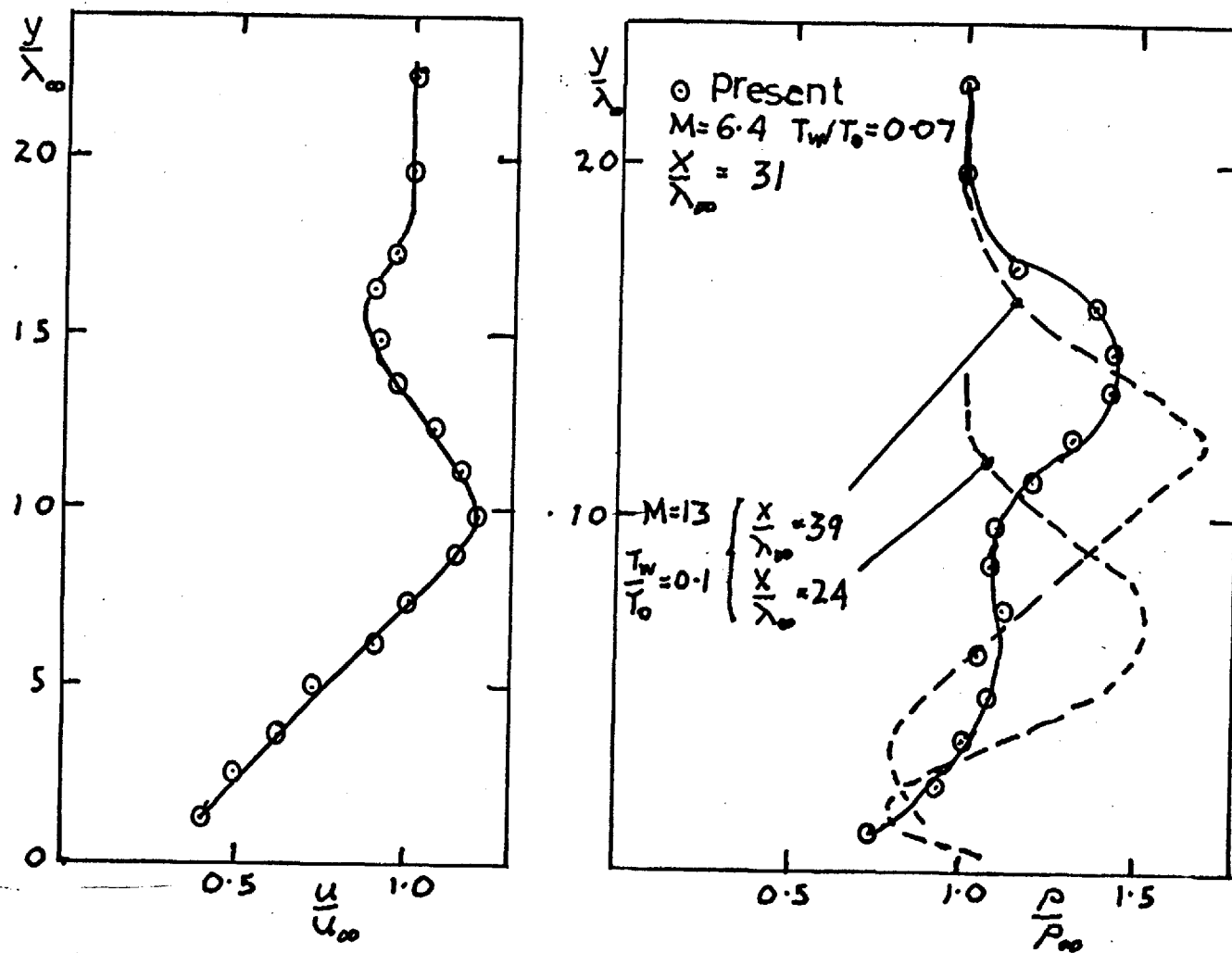


FIG. 11. Profiles from probes

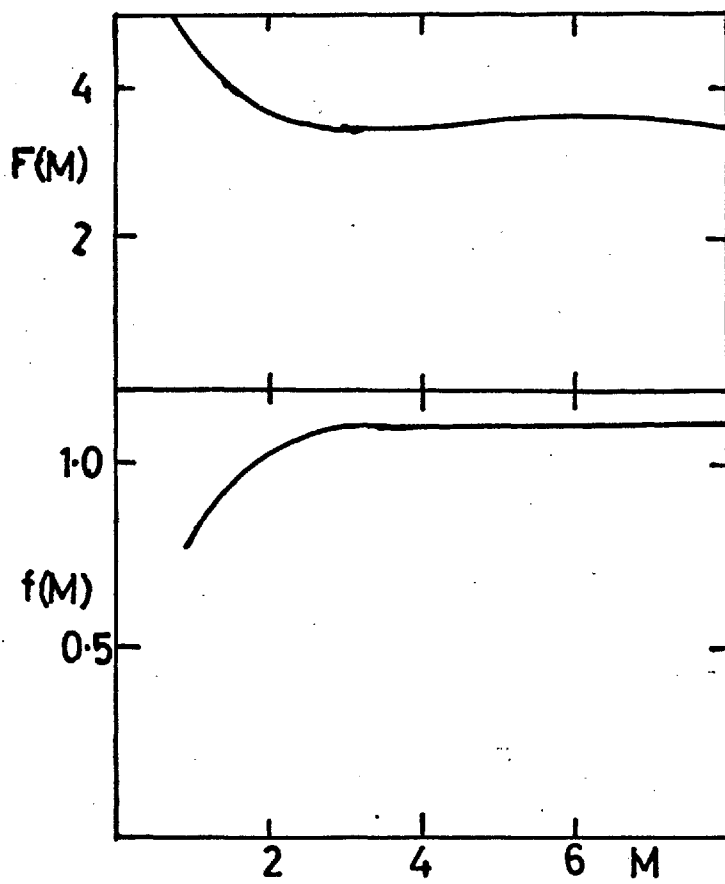


FIG.12.

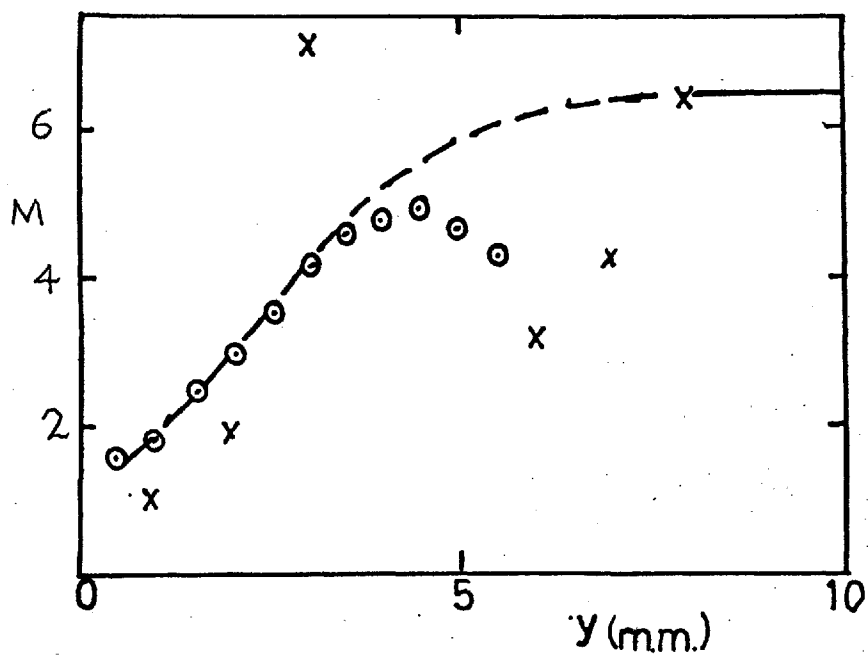
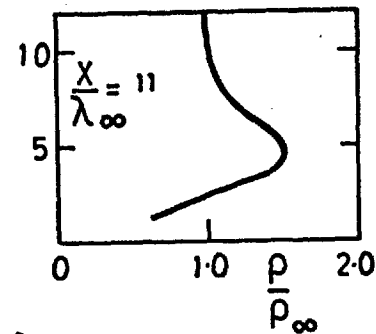
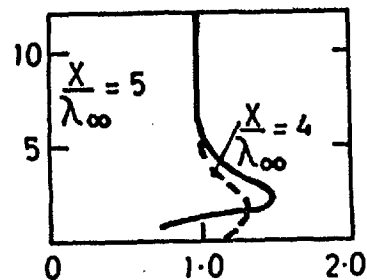
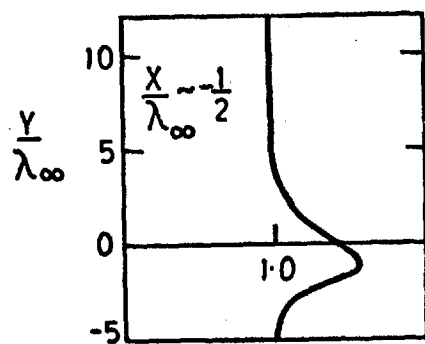


FIG.13. Flow field mach number



	T_w/T_0	M	λ_∞
PRESENT	0.05	6.4	0.61 mm.
MONTE CARLO ⁵²	0.15	5.5	

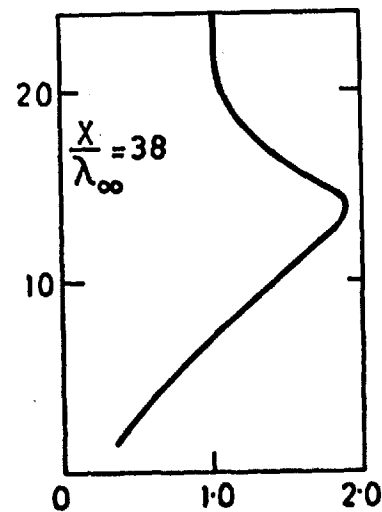
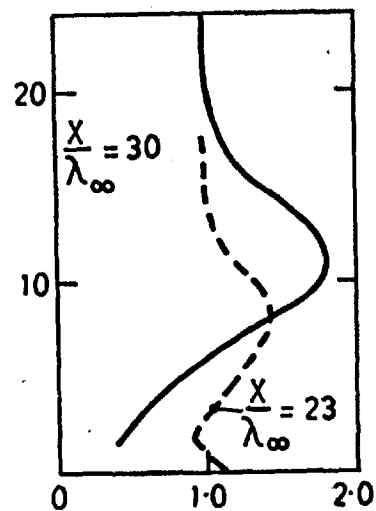
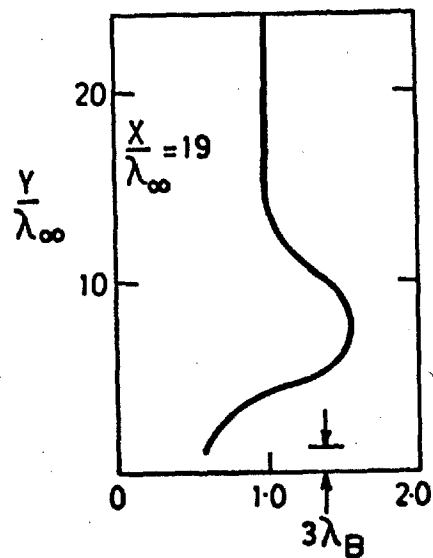


FIG. 14 Density profiles

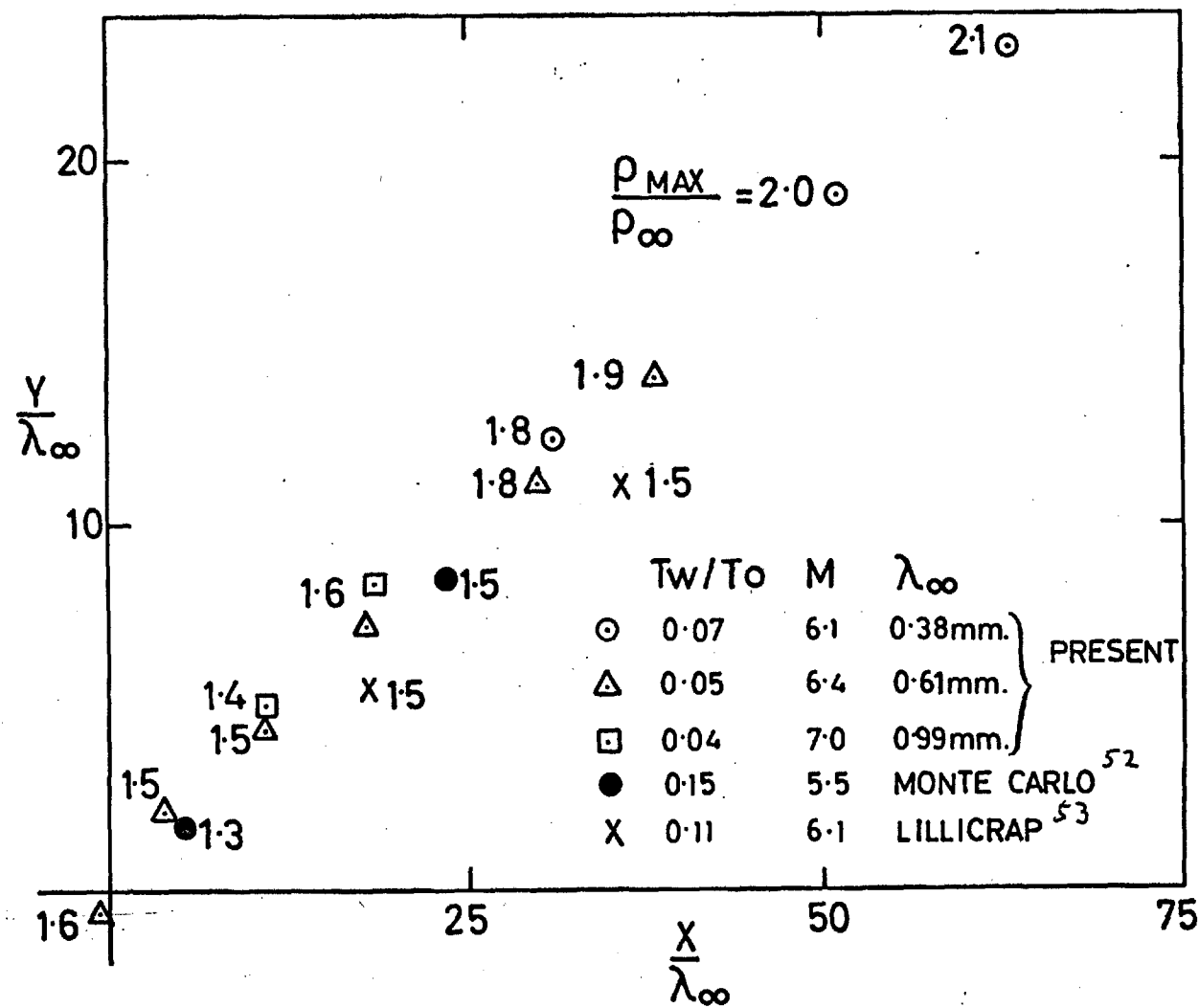


FIG.15 Density maxima

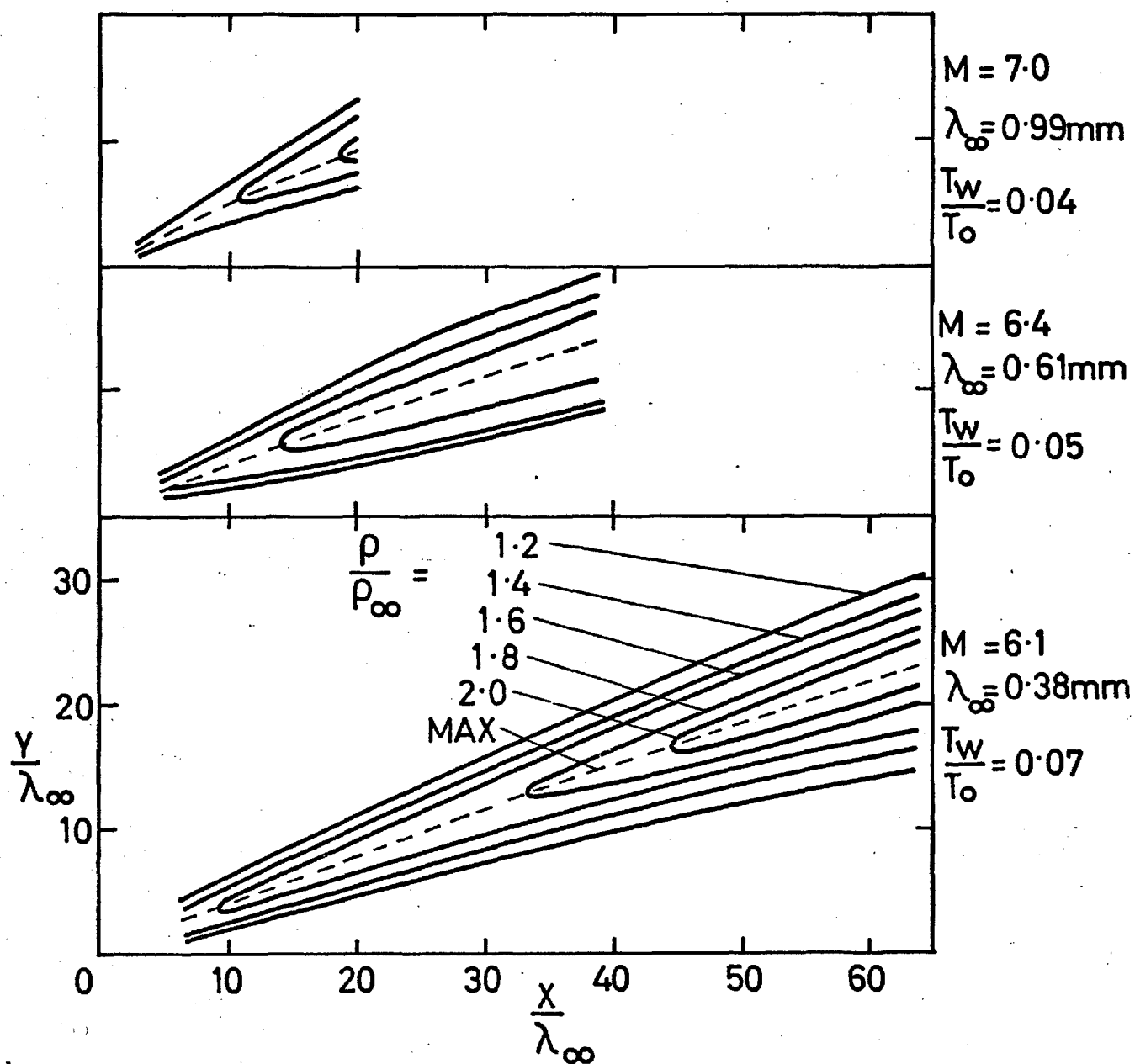


FIG. 16 Density about a flat plate

$M \approx 6.5$ $\lambda_\infty \approx 0.65\text{mm}$ $\frac{T_w}{T_o} \approx 0.05$

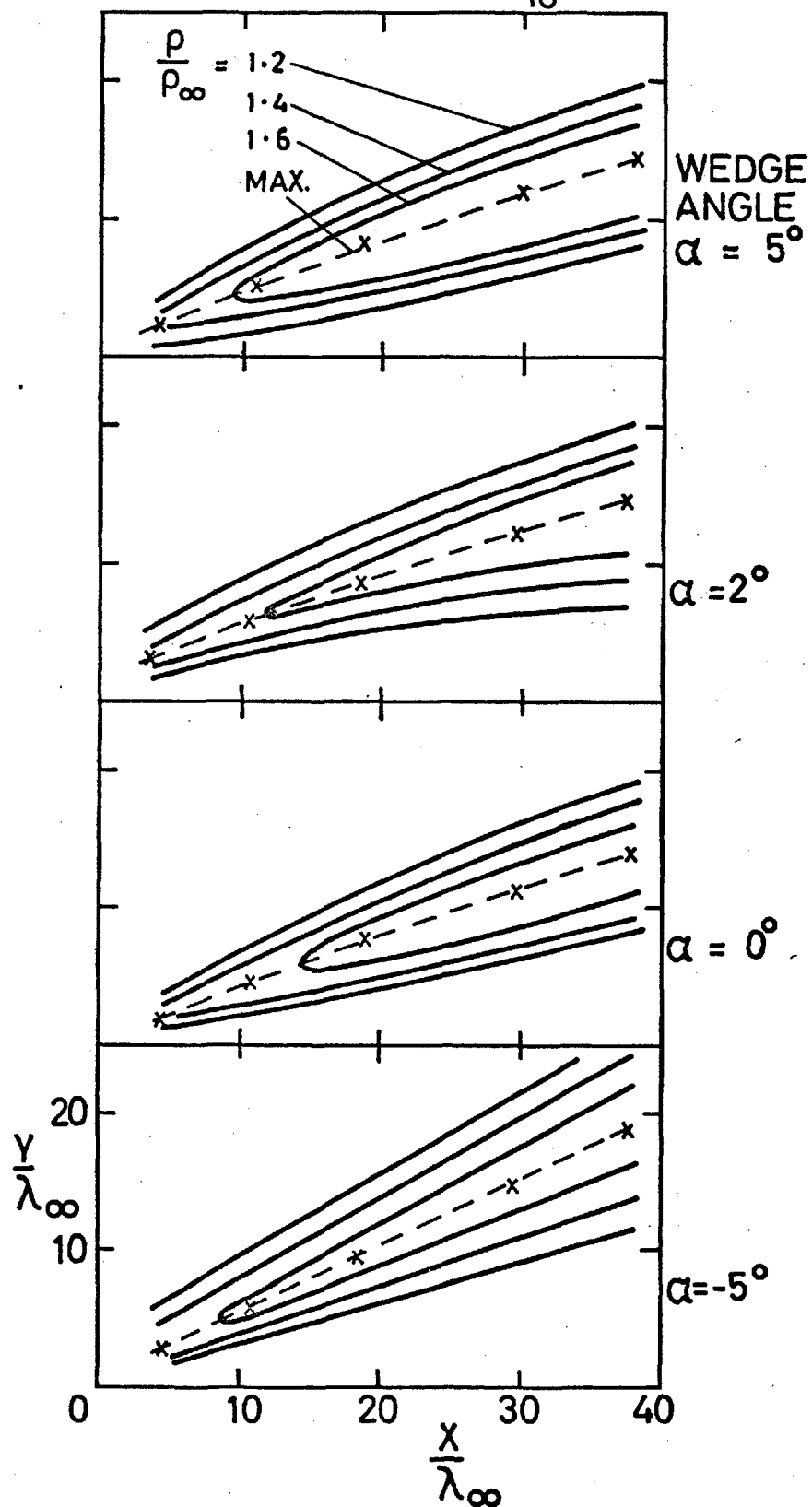


FIG. 17 Density about wedges

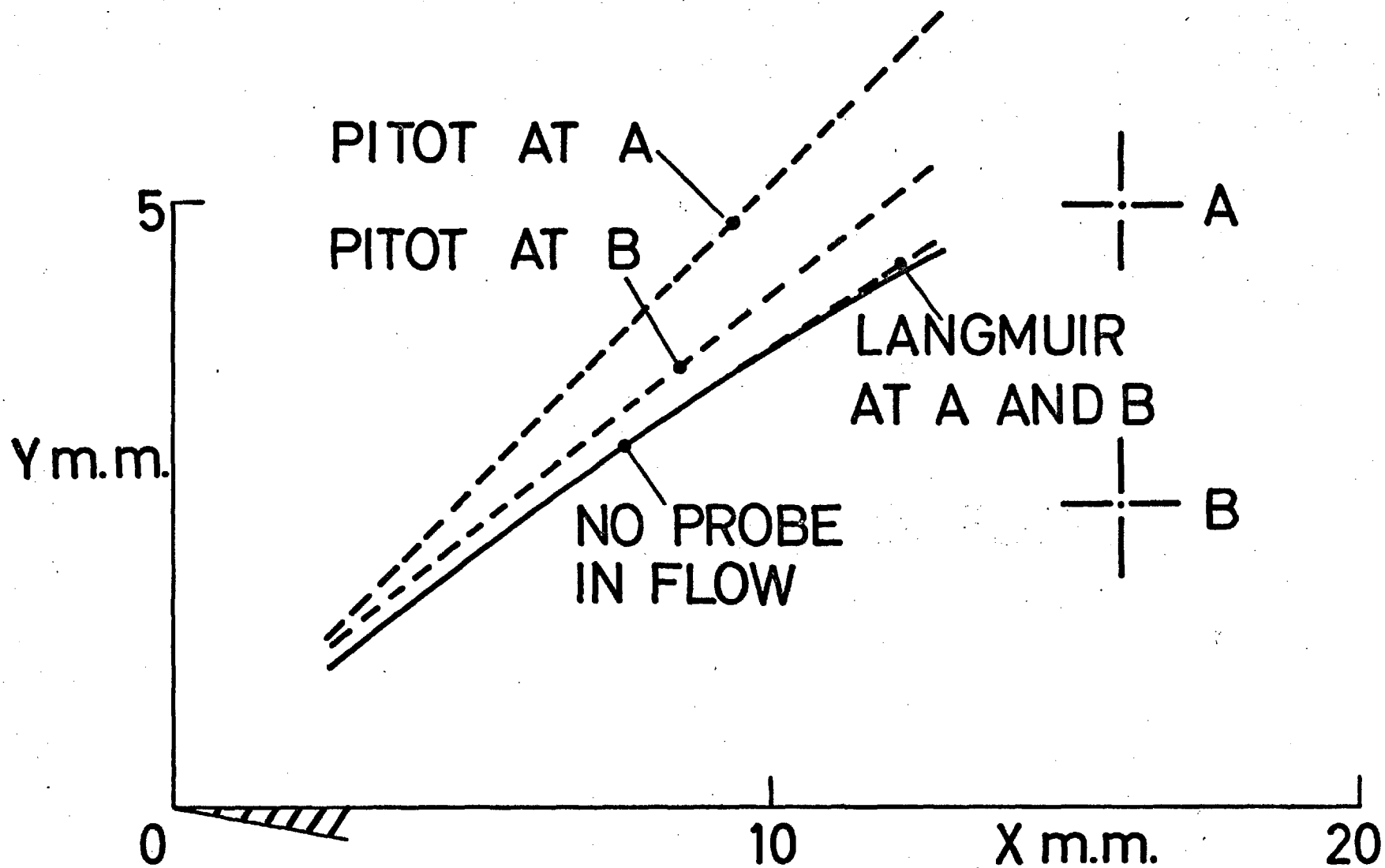


FIG. 18 THE EFFECT OF PROBES ON THE POSITION OF $\frac{\rho_{\text{MAX.}}}{\rho_{\infty}}$

$\circ$ 6.1
 $\triangle$ 6.4
 $\square$ 7.0
 $\bullet$ 5.5
 --- 6.0

} PRESENT
 } MONTECARLO⁵²
 } LILLICRAP⁵³

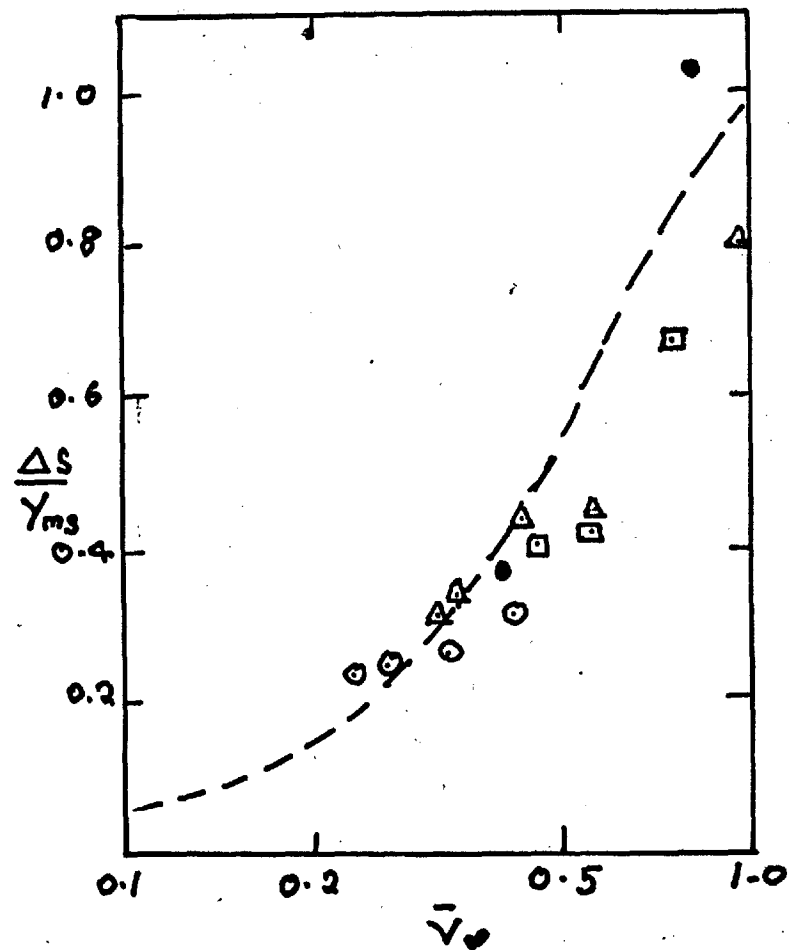
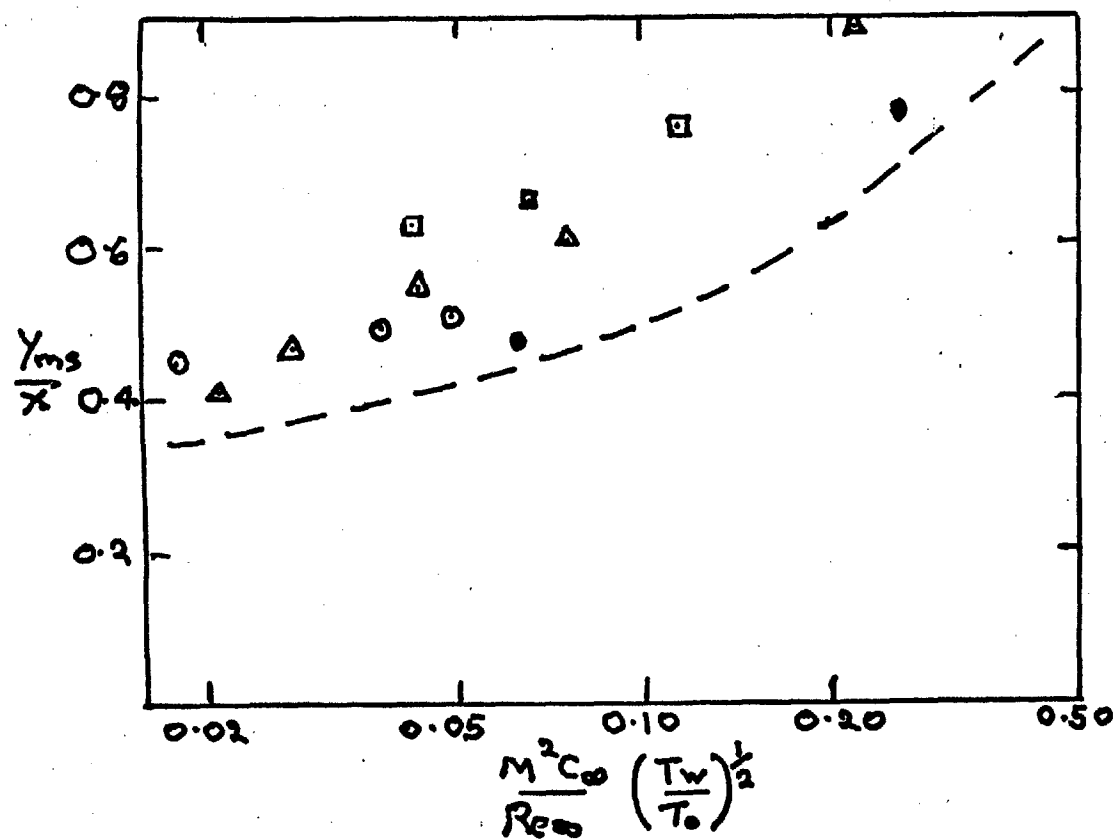
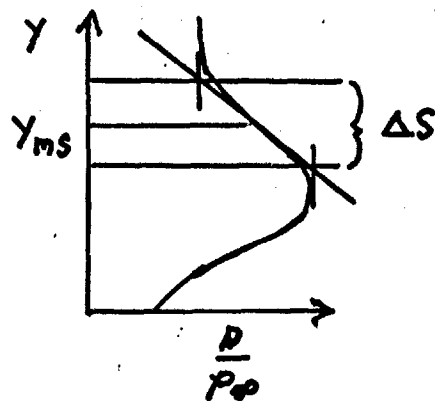


FIG. 19 Shock position and thickness

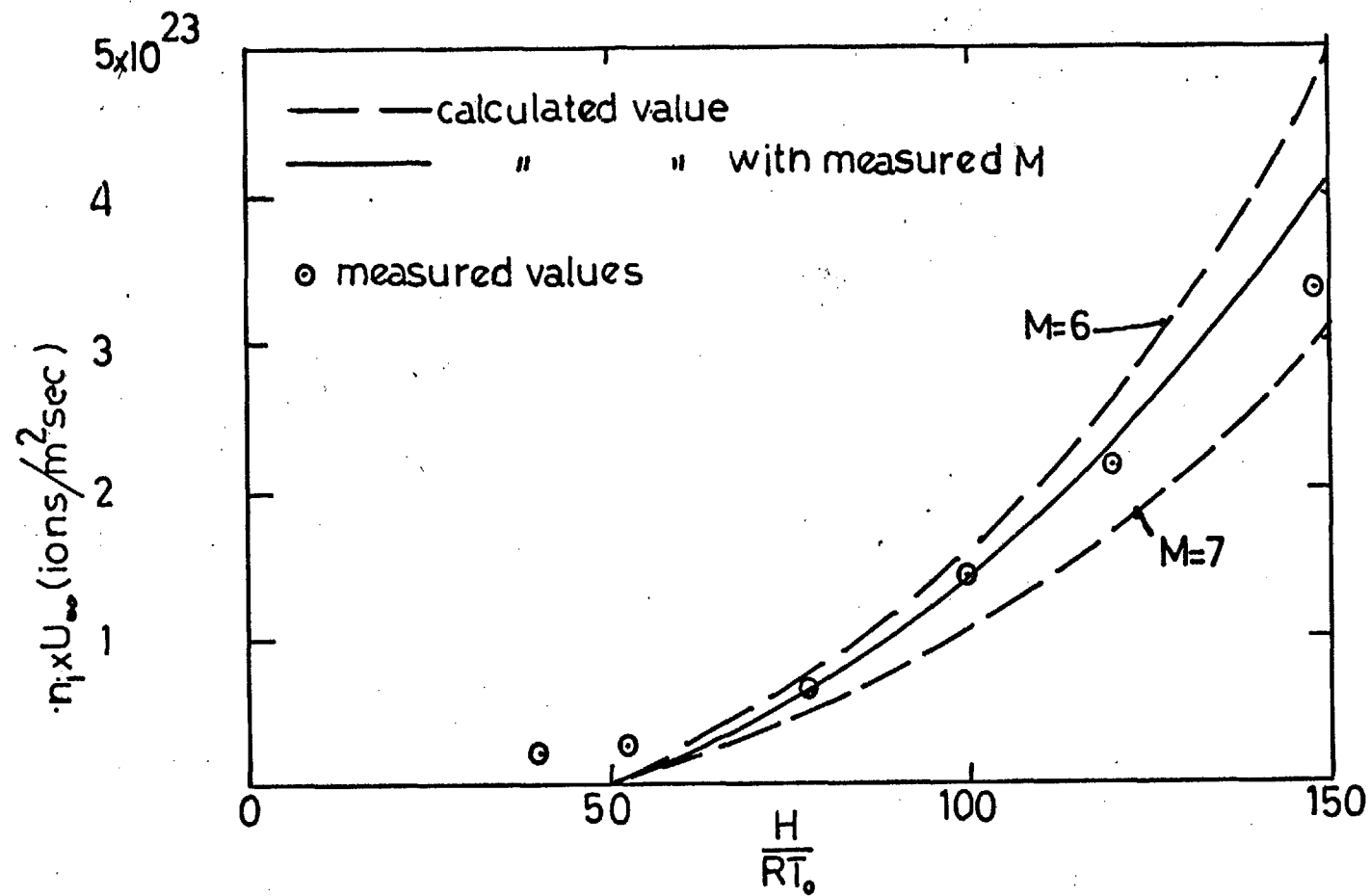
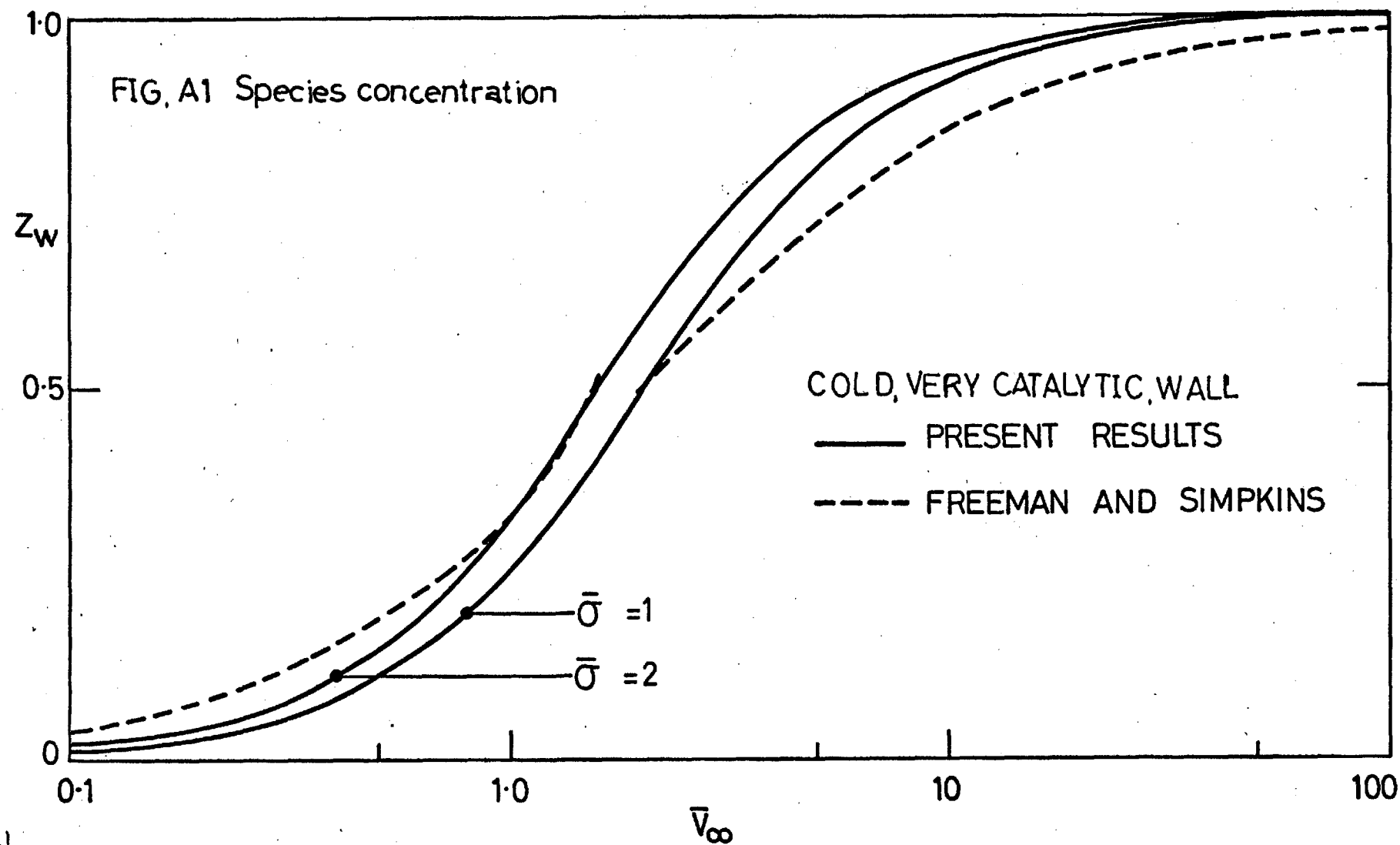


FIG.20

Ion flux in the free stream.



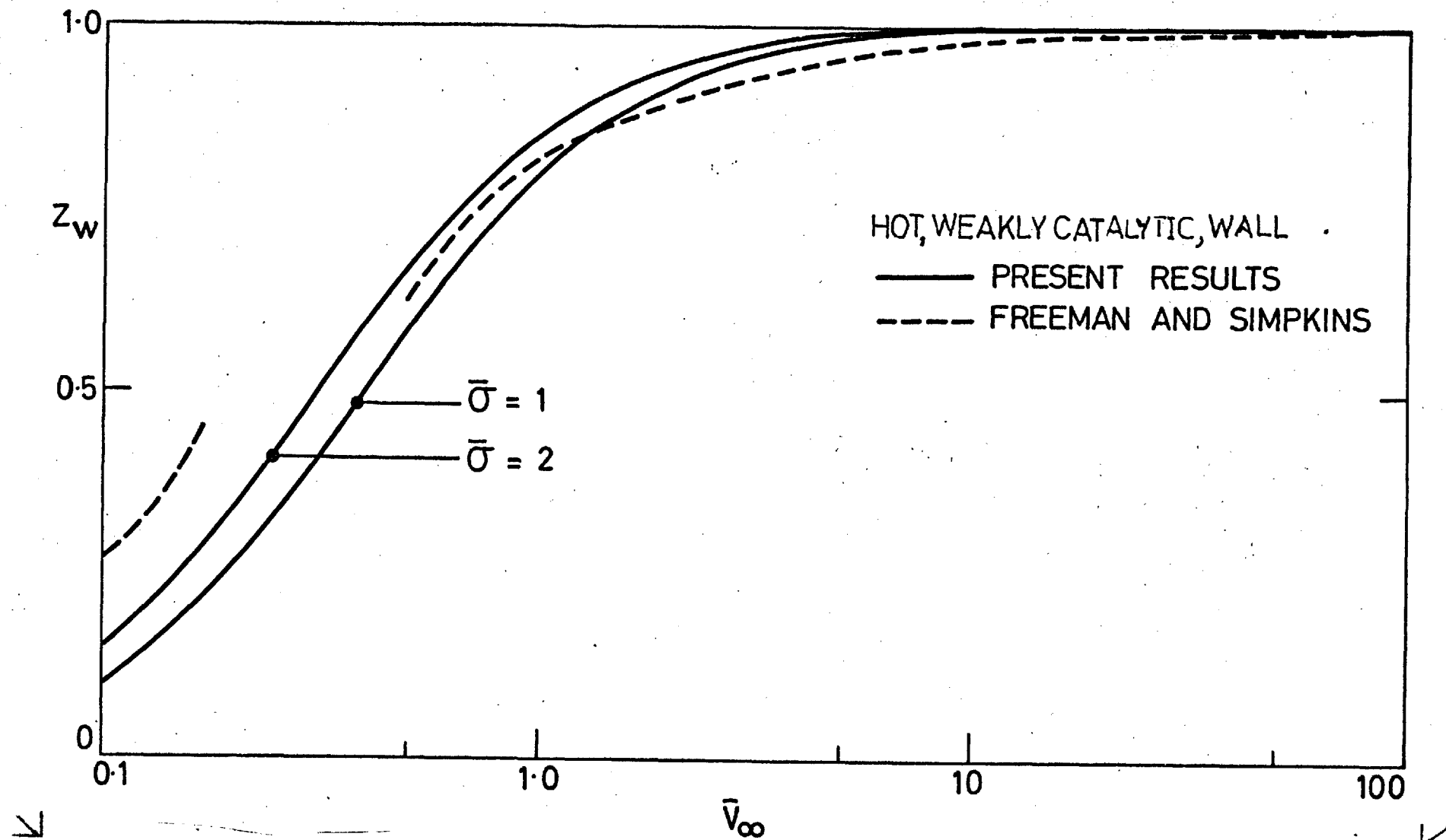


FIG A2 Species concentration

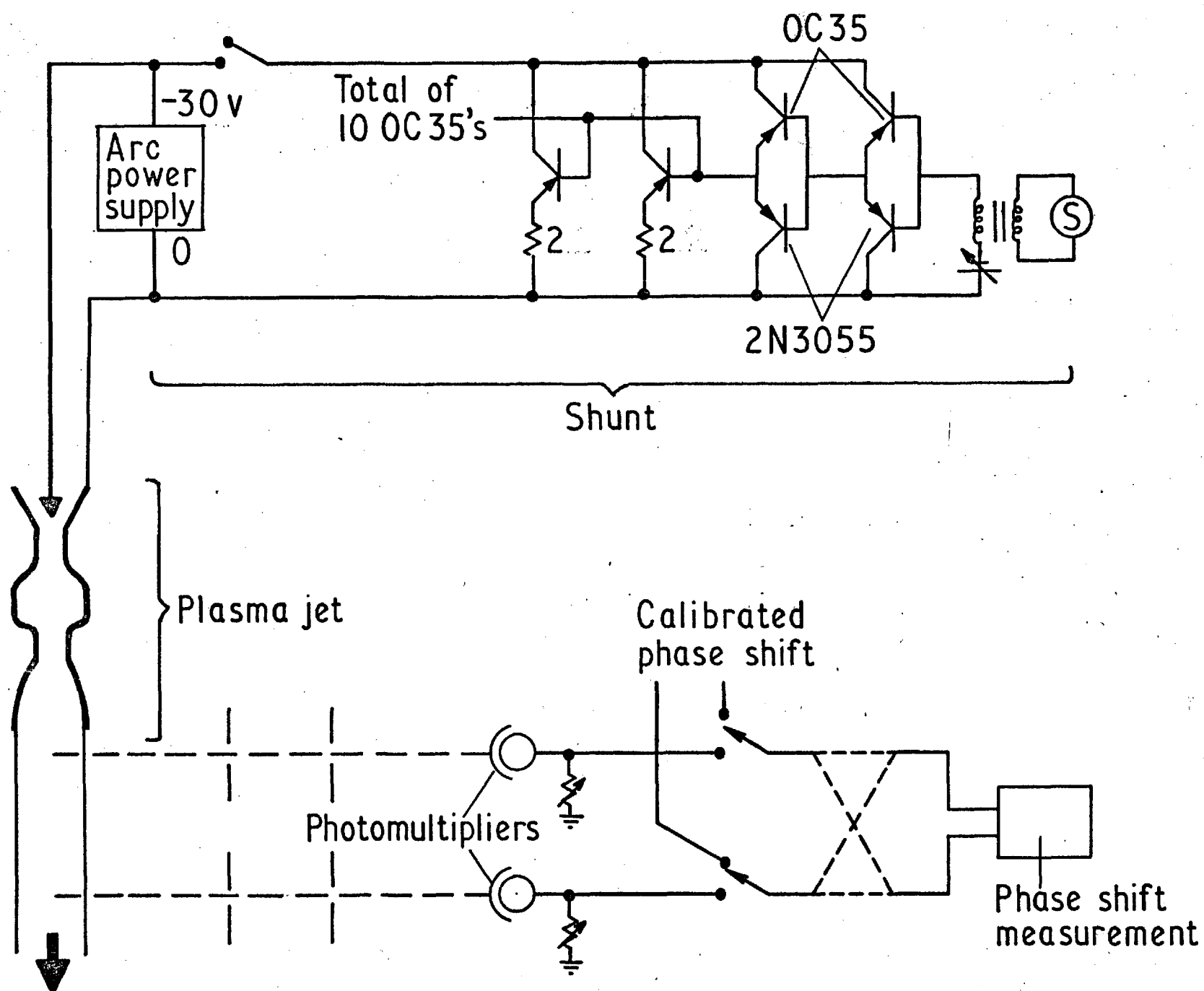


FIG A3 Velocity measuring system

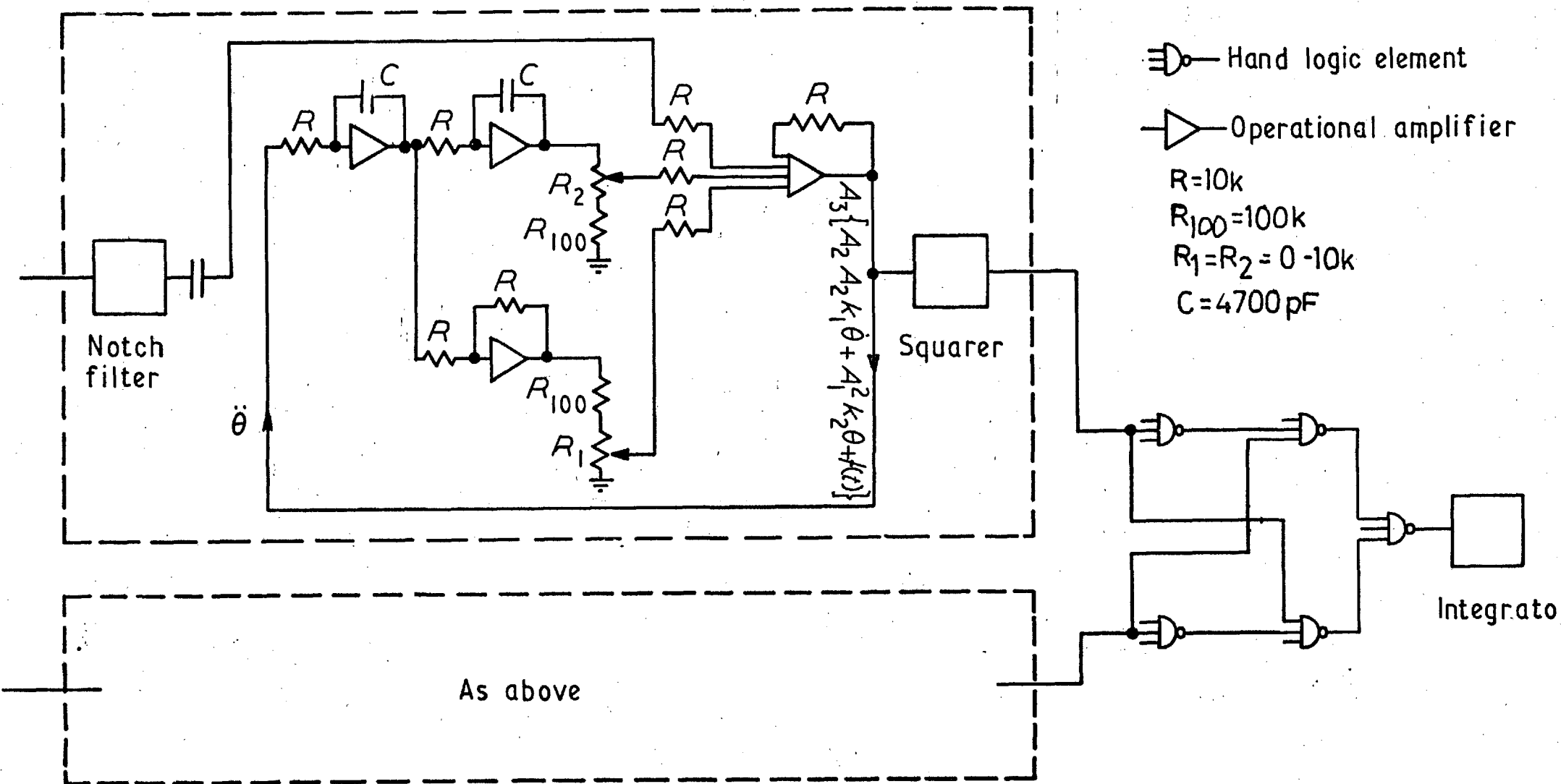


FIG. A4 Phase difference detection

