

ITERATIVE AND NON-LINEAR PROGRAMMING TECHNIQUES

APPLIED TO THE SOLUTION OF POWER SYSTEM PROBLEMS

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To my parents

To Fanny

To Sarita and Manuel

ABSTRACT

This thesis examines the steady state power system problem and introduces the author's investigations directed towards producing analysis methods which can be applied to a number of problems, such as the load flow, minimum loss and economic dispatching problems, for which no unified approach has been available in the past.

Load flow techniques are summarized and new iterative methods are introduced based on a mesh description of the system and on a hybrid admittance-impedance formulation. Criteria for the selection of reference and slack busses are presented.

Nonlinear programming techniques are discussed and are shown to provide a feasible mathematical context for the solution of not only the load flow problem but also the optimization problems involving minimum transmission losses and economic dispatching.

To extend the scope of nonlinear programming methods to large systems, decomposition techniques are developed and applied successfully to a number of systems.

Numerical results of actual systems are provided for all new methods introduced in this thesis.

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INTRODUCTION

A recent publication by A. Kusko¹ attempts to predict the pattern of development of present day power systems until the year 2030. Kusko states that power systems are today "operated with computer assistance, but with decision making by man ... computers are employed to regulate tie line flow and to obtain most economical system dispatching." For the 1970 - 1980 decade, Kusko predicts that computers will be used to "control and optimize the operation of all plants, interconnection flows and some loads, for least-cost operation. Computers will be programmed for decision making during emergency conditions, and to monitor incipient conditions that could lead to massive system failures." The period between 1980 and 2000 will see "computer-type equipment control all dispatching and protective system functions during normal and abnormal conditions, removing man from the decision-making loop." In his conclusions Kusko states that the limiting factor in the increasing role of computers to control power system operation is the slow development of sensors and techniques. He thus recognizes the need for more powerful techniques than the ones presently in use in power system design and operation.

It is clear to everyone that the rapid growth of power systems demands in itself the development of more powerful techniques to cope with the same problems that are being solved today. Kusko's prediction

goes much further as it forecasts that in 15 to 20 years time the complete operation of power systems will be automatized. Viewed from this perspective, the challenge in the years ahead is great.

The description of the complete operation of power systems involves thousands of nonlinear transient equations for which no practical methods of solution are available even on an off-line basis. To simplify matters, the transient and steady-state problems have been separated, and each, in turn, has been subdivided into different problems. Techniques have been developed to cope with the component problems but it is unfortunate that the solution of the individual problems do not constitute the solution of the original problem. We have seen that the transient problems of stability and the study of system variation that occur after some abnormality have been separated from the steady-state problems of load flows, short circuit calculations and economic dispatching. It is not only true that the solution of each individual problem does not take into consideration the conditions described in the others, but also, that many of the techniques employed today cannot optimize the solution of a single problem. For example, a load flow calculation may not yield adequate voltages and system changes are necessary to achieve them. Many load flow techniques cannot cope with the problem of producing the system which has adequate voltages in a direct way, in contrast with the practice of repeated calculations, as changes are

made, until an acceptable solution is found.

This thesis gives attention to the steady state power system problem and it is hoped that its contents will contribute towards making Kusko's predictions occur within the forecasted time.

Chapter 1 looks at the load flow techniques that are presently available. These techniques are all based on the use of the nodal description of systems. An investigation was carried out to study the feasibility of using a mesh description. This work is the subject of Chapter 2 in which it is shown that the mesh approach does possess some desirable characteristics. The hybrid combination of mesh and nodal, or more generally, of admittance and impedance methods is discussed in Chapter 3. The hybrid was found to be the best alternative in a number of cases. Next, in Chapter 4, the problem of the selection of the slack and reference busbars is investigated and practical criteria are given for their selection.

The first four chapters employ conventional mathematical methods for solution which have been used in power system work for the past 15 years. It was recognized that much of the work on load flows utilized the same mathematical methods, mainly the modified Gauss-Seidel iterative technique for the solution of a set of simultaneous nonlinear equations. The equations themselves changed from one method

to another. Recently, mathematical methods have been developed which convert the problem of a set of simultaneous nonlinear equations to one of finding the minimum of a function of these equations. Furthermore, these methods can be used to solve actual optimization problems. The use of these methods in power system problems seemed attractive as they contained the possibility of solving optimally a number of problems simultaneously. Chapter 5 describes these mathematical methods which are known as nonlinear programming methods. In Chapter 6, the load flow problem is solved by these methods, and in Chapter 7 minimum loss and economic dispatching operating conditions are determined subject to physical operating limitations which are automatically taken into account as the solution is produced, yielding a load flow solution at the optimum point as a by-product. The methods were used on a number of real systems ranging between 14 and 57 bus-bars. It was found that for bigger systems computation time increased to levels which made the approach unpractical.

To solve large systems, a search was made for decomposition techniques which would be appropriate to use in connection with the nonlinear programming methods. Chapter 8 presents the technique that was finally developed and adopted. Practical results shown indicate that these techniques reduce computation times by percentages that can be as high as 80 and 90, and that greater savings being obtained with larger systems.

Although this thesis closes with the present state of the use of optimization techniques in power systems, it is strongly felt that this field will develop considerably in the near future as more people become involved in these techniques.

CHAPTER 1
LOAD FLOW METHODS

1.1 General

In its most general form, the load flow problem involves the solution of a set of simultaneous nonlinear algebraic equations, which describe the steady state operation of an electric power system^{2,3}. Table 1.1 defines the load flow problem in terms of the physical components of a system, busses and transmission lines, and the electrical quantities which are known or unknown at the start of the problem, complex voltages and powers.

TABLE 1.1 Electrical Quantities in the Load Flow Problem

Point to be Investigated	Known Quantities	Unknown Quantities
generator bus	Magnitude of bus voltage; real power generated	Voltage angle; reactive power
load bus	Real and reactive power consumed	Magnitude and angle of bus voltage
reference-slack bus	Magnitude and angle of bus voltage	Real and reactive power
transmission system	Configuration and electrical characteristics	Real and reactive power flow in transmission lines

In general, the procedure to set up the equations of the load flow problem is to equate known quantities in terms of either the unknown quantities or other variables from which the unknown quantities can be calculated directly when a solution has been found. There is no one unique set of equations describing the system and indeed the difference between many methods lies in the particular set that it chooses to solve. Once a set of equations has been obtained, there are many possibilities as to the method of solution that can be adopted. Table 1.2 includes the principal methods available in the literature.

TABLE 1.2 Principal Load Flow Methods

A. Methods based on a Gauss-Seidel iterative solution

1. Glimm and Stagg
 2. Hale and Goodrich
 3. Carre
 4. Brameller and Denmead
 5. Gupta and Humphrey-Davies
 6. Brown, Carter, Happ and Person
 7. new methods presented in Chapters 2 and 3
-

B. Methods based on Newton's technique

1. Ward and Hale
2. Van Ness

B. continued

3. Van Ness and Griffin
4. Tinney and Hart
5. Feingold and Spohn

C. Method based on network flow techniques

1. Takahashi, Sekine and Umezu

D. Method based on geometric considerations

1. Long

E. Methods based on optimization techniques

1. new methods presented in Chapters 6 and 8

1.2 Methods based on a Gauss-Seidel iterative solution

The input current and power into a system node is given by the following set of equations,

$$I_m = \sum_{n=1}^N Y_{mn} V_n, \quad m = 1, 2, \dots, N \quad (1)$$

$$S_m = V_m I_m^*, \quad m = 1, 2, \dots, N \quad (2)$$

1.2.1 Method of Glimm and Stagg⁴

Glimm and Stagg substitute (1) into (2) and obtain equation (3),

$$V_m = \frac{\frac{P_m - jQ_m}{V_m^*} - \sum_{\substack{n=1 \\ n \neq m}}^N Y_{mn} V_n}{Y_{mm}} \quad (3)$$

If node m is a load bus, (3) is complete as it includes P_m and Q_m , which are the two conditions that must be satisfied. In the case of a generator bus, Q_m has to be calculated with the previous values of voltages and using the same equation (3). The resulting voltage V_m is multiplied by the ratio of the specified voltage magnitude to its actual magnitude. There is no equation for the reference-slack bus. Equation (3) is solved iteratively until the voltages V_n converge to a solution with a specified accuracy.

1.2.2 Method of Hale and Goodrich⁵

Equation (1) can be expanded and expressed in matrix form,

$$\begin{bmatrix} I_m \\ I_n \end{bmatrix} = \begin{bmatrix} Y_{mm} & Y_{mn} \\ Y_{nm} & Y_{nn} \end{bmatrix} \begin{bmatrix} V_m \\ V_n \end{bmatrix} \quad (4)$$

for a system of $m + n$ nodes. Equation (4) expresses input currents in terms of nodal voltages. It is possible to transform (4) so that a hybrid relation results in which currents and voltages are functions of both current and voltages. This is equation (5).

$$\begin{bmatrix} I_m \\ V_n \end{bmatrix} = \begin{bmatrix} Y_{mm} & D_{mn} \\ D_{nm} & Z_{nn} \end{bmatrix} \begin{bmatrix} V_m \\ I_n \end{bmatrix} \quad (5)$$

where

$$Y_{mm} = Y_{mm} - Y_{mn} Y_{nn}^{-1} Y_{nm}$$

$$D_{mn} = Y_{mn} Y_{nn}^{-1}$$

$$D_{nm} = -Y_{nn}^{-1} Y_{nm}$$

$$Z_{nn} = Y_{nn}^{-1}$$

Hale and Goodrich use equation (5) to solve the load flow problem. The first m equations are treated similarly to equation (3) while the last n equations are used in its present form.

1.2.3 Method of Carre⁶

Carre divides the system into several parts or blocks of tree form such that the interconnection branches are the branches of smallest admittances. It has been shown that under these conditions a successive block over-relaxation scheme has particularly good convergence. Each node of a block could be solved by equation (3) without updating a particular voltage until all nodes of a block have been covered. The voltages of a complete block are then updated and accelerated simultaneously. Instead of using (3) directly, Carre uses an optimally ordered Gaussian elimination scheme which requires a minimum of computation. Chapter 8 discusses Carre's work in greater detail.

1.2.4 Method of Brameller and Dennead⁷

Equation (5) is a partial inversion of equation (4). The complete inversion can be carried out. If the reference-slack equation is removed from (4), it is necessary to separate the admittance column due to the reference slack (subscript s) before inverting. The following matrix equation results,

$$V_n = Z_{nn} I_n - D_n V_s \quad (6)$$

where

$$D_n = Z_{nn} Y_s$$

Equation (6) is the basis for the method of Brameller and Dennead. It can be expanded and equation (2) substituted in the diagonal terms. The k^{th} equation will have the following form,

$$V_k = Z_{k1} I_1 + \dots + Z_{kk} (S_k^*/E_k^*) + \dots + Z_{kn} I_n - D_k V_s \quad (7)$$

which can be rewritten as,

$$|V_k|^2 - A_k V_k^* - B_k S_k^* = 0 \quad (8)$$

where

$$A_k = \sum_{n \neq k} (Z_{kn} I_n) - D_k V_s$$

$$B_k = Z_{kk}$$

Equation (8) is solved iteratively for the unknown voltages and these in turn are used to update the coefficients of (8).

1.2.5 Method of Gupta and Humphrey-Davies⁸

Generation and load requirements can be approximated by an equivalent admittance

$$Y = \frac{-S^*}{|V|^2_{\text{initial}}} \quad (9)$$

and included in the admittance matrix. S and V are not both completely known at the start of the problem. The current balance at each node is maintained by assuming that an injected current, I_{inj} , exists,

$$I_{\text{inj}} = \frac{S^*}{V^*} + YV \quad (10)$$

By inverting the admittance matrix an equation similar to (6) is obtained, where I_n is replaced by I_{inj} . The iterative process proceeds by alternating between equations (10) and (6). Generator reactive power is updated by multiplying it by the square of the ratio of desired to actual voltage magnitude.

1.2.6 Method of Brown, Carter, Happ and Person⁹

An alternative to the previous method consists of defining an

equivalent admittance only for load busses. Equation (10) will define an injected current for these busses. If the slack bus equation is not removed and the admittance matrix inverted, the result will be equation (11)

$$E_n = Z_{nn} I_N \quad (11)$$

where I_n includes the slack bus current, I_s . Before using (11), the slack bus current is calculated from the slack bus equation in (11) and is updated immediately after (11) has been used to obtain an improved value of a bus voltage.

1.3 Methods based on Newton's technique

1.3.1 Method of Ward and Hale¹⁰

Starting from equations (1) and (2), Ward and Hale arrive at (12) and (13) for generator busses and (12) and (14) for load busses.

$$P_m = R \left[(V_m + \Delta V_m) (I_m + Y_{mm} \Delta V_m)^* \right] \quad (12)$$

$$|V_m|^2 = |V_m + \Delta V_m|^2 \quad (13)$$

$$Q_m = \Im \left[(V_m + \Delta V_m) (I_m + Y_{mm} \Delta V_m)^* \right] \quad (14)$$

These equations are really expressing the change in the complex powers or voltage magnitude of a bus in terms of an increment

of its bus voltage alone and not as a function of the variation of all bus voltages. This amounts to neglecting the off-diagonal terms of the Jacobian of these equations. The iterative process consists of cyclically solving for ΔV_m for all busses until they become very small.

1.3.2 Method of Van Ness¹¹

All conditions required by the load flow problem can be met by solving the following set of equations formed by substituting (1) into (2) and forming the total differentials,

$$\begin{bmatrix} P_m \\ Q_m \end{bmatrix} = \begin{bmatrix} H_{mn} & N_{mn} \\ J_{mn} & L_{mn} \end{bmatrix} \begin{bmatrix} \Delta \delta_n \\ \Delta |V_n| \end{bmatrix} \quad (15)$$

where δ_n is the angle of bus voltage V_n . The coefficient matrix is not constant as its values depend on the voltages. For this reason, they must be calculated at every iteration. Calculation time would be extremely long if (15) is solved by inversion. To get around this Van Ness studied some simplifications of (15) by neglecting terms of the coefficient or Jacobian matrix.

1.3.3 Method of Van Ness and Griffin¹²

Equation (15) is given an adimensional right-hand vector by re-writing it as (16)

$$\begin{vmatrix} P_m \\ Q_m \end{vmatrix} = \begin{vmatrix} H_{mn} & N_{mn} \\ J_{mn} & L_{mn} \end{vmatrix} \begin{vmatrix} \Delta \delta_n \\ \Delta \frac{|V_n|}{|V_n|} \end{vmatrix} \quad (16)$$

By using an elimination method, (16) is solved without neglecting terms of the Jacobian.

1.3.4 Method of Tinney and Hart¹³

The direct implementation of an algorithm based on (16) is highly limited by computation time and storage. Tinney and Hart realized that the Jacobian matrix is a highly sparse matrix. In fact, off-diagonal terms of the H, N, J and L matrices are zero if the corresponding position of the admittance matrix is zero. To take advantage of sparsity two interdependent strategies are adopted:

1. Equations (16) are solved by triangularizing the Jacobian matrix and using a back-substitution process. The original Jacobian matrix is not calculated all at once but by one or two rows at a time and the triangularized matrix is stored as a column vector containing only non-zero elements.
2. An optional ordering of the equations is realized prior to the triangularization. This ordering aims at minimizing the non-zero terms after triangularization. This is accomplished by numbering the nodes so that at each step of the elimination the next node to be eliminated is the one having the fewest connected branches. It is necessary to simulate the

triangularization process to take into account the changes in the node-branch connections effected at each step.

1.3.5 Method of Feingold and Spohn¹⁴

Feingold and Spohn take equation (15) and place $J = N = 0$. Under these conditions they solve (15) iteratively.

1.4 Method based on network flow techniques

1.4.1 Method of Takahashi., Sekine and Umezu¹⁵

Network flow techniques are based on the proposition that for linear networks the flow is conservative and superposition can be used to satisfy the two Kirchoff-type conditions: 1) sum of the flows into a node must equal to zero; 2) a potential function exists between each two nodes such that the sum of these potentials around a closed loop is equal to zero. The procedure is to satisfy the first law arbitrarily and to superimpose the second law on the solution first obtained to arrive at the final solution. This is particularly useful if there are inputs and outputs specified at all nodes as these can be arbitrarily satisfied in the first stage and the superposition of the second law need only be done on the planar transportation network.

For load flow applications, the nonlinear problem is transformed

into a sequence of linear problems which are solved by network flow techniques.

Considering a Π -equivalent transmission line, the flows in the series portion are

$$P_{12} = |V_1| |V_2| \sin(\delta_1 - \delta_2)/Z^2/X + |V_1| (|V_1| - |V_2|)/Z^2/R \\ + 2 |V_1| |V_2| \sin^2((\delta_1 - \delta_2)/2)/Z^2/R \quad (17)$$

$$J_{12} = (|V_1| - |V_2|)/Z^2/X - |V_2| \sin(\delta_1 - \delta_2)/Z^2/R \\ + 2 |V_2| \sin^2((\delta_1 - \delta_2)/2)/Z^2/X \quad (18)$$

where

$$J_{12} = Q_{12}/|V_1|$$

$$Z^2 = R^2 + X^2$$

If the following approximation is assumed for the first term of (17),

$$\sin \Delta \delta \doteq \Delta \delta - 1/6 \Delta \delta^3$$

then (17) and (18) can be rewritten as,

$$P_{12} = f \delta_{12} + T \delta_{12} \quad (19)$$

$$J_{12} = f |V|_{12} + T |V|_{12} \quad (20)$$

where

$$f \delta_{12} = (\delta_1 - \delta_2)/Z \delta_{12} \quad (21)$$

$$f |V|_{12} = (|V_1| - |V_2|)/Z |V|_{12} \quad (22)$$

$$\begin{aligned} Z_{\phi_{12}} &= Z^2/X / |V_1| |V_2| \\ Z_{|V|_{12}} &= Z^2/X \end{aligned}$$

and

$$\begin{aligned} T_{\phi_{12}} &= |V_1| (|V_1| - |V_2|)/Z^2/R + |V_1| |V_2| (\phi_1 - \phi_2)^2 / 2Z^2/R \\ &\quad - V_1 V_2 (\phi_1 - \phi_2)^3 / 6Z^2/X \end{aligned}$$

$$\begin{aligned} T_{|V|_{12}} &= - |V_2| (\phi_1 - \phi_2)/Z^2/R + |V_2| (\phi_1 - \phi_2)^2 / 2Z^2/X \\ &\quad + |V_2| (\phi_1 - \phi_2)^3 / 6Z^2/R \end{aligned}$$

Equations (21) and (22) behave according to Ohm's law as they express the flow as a ratio of a potential difference to an impedance.

The procedure is as follows:

1. An initial complex voltage distribution is assumed.
2. As the flows f are conserved, at each node m

$$\begin{aligned} \Delta P_m &= P_m - \sum_n T_{mn} \\ \Delta Q_m &= Q_m - |V_m| \sum_n T_{|V|_{mn}} \end{aligned}$$

where ΔP_m and ΔQ_m will act as inputs or outputs of two different networks, the first defined by impedances $Z_{\phi_{12}}$ and the second by $Z_{|V|_{12}}$.

3. The flows are determined in two stages by superposition and equations (21) and (22) are used to calculate the voltage magnitude and angles.
4. Steps two and three are repeated until ΔP and ΔQ are arbitrarily small.

1.5 Method based on geometric considerations

1.5.1 Method of Long¹⁶

The load flow equations can be recognized to be the equations of circles when expressed in rectangular co-ordinates, $V_m = E_m + jF_m$

$$P_m = a_{11}(E_m^2 + F_m^2) + a_{12} E_m + a_{13} F_m \quad (23)$$

$$Q_m = a_{21}(E_m^2 + F_m^2) + a_{22} E_m + a_{23} F_m \quad (24)$$

$$|V|_m^2 = E_m^2 + F_m^2 \quad (25)$$

in addition to the above circles, the real and reactive losses are also circles of the form,

$$P \text{ losses} = a_{31}(E_m^2 + F_m^2) + a_{32} E_m + a_{33} F_m + a_{34} \quad (26)$$

$$Q \text{ losses} = a_{41}(E_m^2 + F_m^2) + a_{42} E_m + a_{43} F_m + a_{44} \quad (27)$$

Long considers four circles per bus, (23), (24), (26) and (27) for load busses and (23), (25), (26) and (27) for generator busses. If the square terms of two circles are eliminated, a straight line results. If the two circles intersect, the straight line is the common chord that passes through the intersection points. If the circles do not intersect, the resulting straight line is defined as the radical axis.

Three straight lines are obtained by taking the four circles of a bus and eliminating the square terms from three pairs of circles. Long recommends that straight lines be obtained from the P and Q circles, the P and P losses circles, and the Q and Q losses circles for a load bus. For generator busses the straight lines are formed from the P and $|V|^2$ circles, the P and P losses circles and the Q and Q losses circles. In both cases the result is:

$$A_{11} E_m + A_{12} F_m = C_1$$

$$A_{21} E_m + A_{22} F_m = C_2 \quad (28)$$

$$A_{31} E_m + A_{33} F_m = C_3$$

Equations (28) are solved for each node by a least squares process.

1.6 Assessment of load flow methods

Some time ago load flow methods were assessed in terms of storage and computing time. Today the importance of these two factors has been lessened by the use of large and fast computers although storage still causes some concern in relation to large interconnected systems. But there are other factors which seem to have been receiving much more attention lately: the stability of the method and the use of the method in other problems such as the minimum loss, optimum re-

active distribution, emergency conditions and economic dispatching problems, which are all optimal load flow problems. The stability of the method refers to its capability of arriving at a solution when applied to so-called difficult systems, if such a solution exists.

The method most widely used continues to be the Glimm and Stagg method for its ease in programming and its good storage characterisation. However, the trend is towards using other methods based on either an impedance matrix or Newton's technique due to its slow convergence and its inability to handle difficult systems.

At present, the method receiving most attention for its speed, its ability to handle large systems, its flexibility in extending its scope to deal with optimal conditions and its inherent stability, is Newton's technique as developed by Tinney and Hart. A recent paper¹⁷ has shown that not all matters concerning its extension to optimal solutions have been resolved and work is being done currently to overcome these difficulties.

In the following chapters, several new load flow methods will be introduced and compared to some of the ones described in this chapter. An assessment of the new methods will be made after they are presented.

CHAPTER 2

THE MESH APPROACH

2.1 General

A set of network equations describing a power system can be more readily used for the efficient solution of the load flow problem by computer if they satisfy the following conditions:-

- (a) the equations follow a definite pattern
- (b) the voltage and current variables in the equations are readily interpretable in terms of the boundary conditions usually imposed on power system nodes.
- (c) the number of equations are as few as possible.

The exclusive use of node equations in the past has been due to the fact that they satisfy all conditions but mesh equations can also be formulated in such a way that they satisfy the three conditions equally well, as will now be shown.^{18,19,20}

In a power system, some meshes can be formed exclusively by the transmission branches. The remaining meshes are formed by shunt branches from nodes to neutral. To reduce the number of meshes, all passive shunt elements due to transmission lines, off-nominal tap transformers and static capacitors are initially disconnected from the system, their effect being considered later.

To satisfy the three above mentioned conditions, the following procedure of writing the mesh equation is adopted: (i), all generators and loads are replaced by ideal voltage sources. The system is thus reduced to a passive transmission system with only ideal voltage sources connected from the neutral node to all system nodes. (ii), a tree is formed by removing branches from the system in such a way that finally no closed mesh exists and no branch or node is left completely disconnected from the tree. A flow diagram for this tree forming program is shown in fig. 2.1. The number of branches or "links" removed in arriving at the tree defines the number of mesh equations. The addition of a link to the system forms a mesh through the tree.

In this procedure, all voltage sources are considered as links, with the exception of the one source connected to the voltage reference node. Some branches of the transmission system will also be links. For every link, a mesh equation is now formed starting from the neutral node, advancing through the reference voltage source, possibly through several branches of the tree, through the link and returning to neutral. There is then one and only one mesh current per link. In the case of transmission links, these will be a zero voltage term in the mesh equation. In the case of a voltage source link, the voltage term in the mesh equation will consist of the difference between the reference and source voltages.

If there are k transmission links and n voltage source links, where n is also the number of nodes minus one, and if E_s is the voltage of the reference source, the mesh equations will take the following form:

$$\begin{vmatrix} 0 \\ E_s - E_n \end{vmatrix} = \begin{vmatrix} Z_{kk} & Z_{kn} \\ Z_{nk} & Z_{nn} \end{vmatrix} \begin{vmatrix} I_k \\ I_n \end{vmatrix} \quad (1)$$

For reasons to be discussed later, I_k is eliminated from equation (1).

$$E_s - E_n = Z'_{nn} I_n \quad (2)$$

where

$$Z'_{nn} = Z_{nn} - Z_{nk} Z_{kk}^{-1} Z_{kn} \quad (3)$$

Equation (2) is a set of mesh equations having a definite pattern and is equal in number to the set of node equations describing the same system. The voltages of equation (2) are node voltages. The mesh currents of equation (2) differ from the nodal currents because the passive shunt elements have been removed before the formation of the equations. However, this presents no problem as the usual nodal restrictions can be easily interpreted in terms of those mesh currents, as shown in the following section. The shunt elements will not add equations to the formulation of the problem, therefore, the mesh formulation satisfies all these conditions

stated at the beginning of this section.

2.2. The Iterative Process

The load flow requires the solution of equation (2) subject to certain restrictions depending on whether a node is a P, Q, or a P, V busbar. To relate the node complex voltamperes to the mesh currents, the effect of the shunt impedances has to be taken into account. If Z_j is the shunt impedance of node j , the relation between the mesh current and the complex voltamperes for the j^{th} bus is

$$I_j = (E_j/Z_j) - (S_j/E_j)^* \quad (4)$$

Several methods of solution have been tried for this problem and are explained below.

1. A solution was attempted by the simultaneous use of equations (1) and (4). As the I_k currents are transmission link currents, they are determined as a function of the difference of the voltages at the terminals of the link. This led to considerable error during the early stages of the iterative process and resulted in divergence. The method was therefore unsuccessful.

2. Convergence for some systems was obtained by using equation (2) to determine the voltage variables, followed by the use of equation (4) for the current variables. Accelerated block substitution and

successive over-relaxation were tried but with some systems neither method produced satisfactory results. In these unsuccessful cases, the solution started to converge but at some point became oscillatory with no further signs of convergence or divergence.

3. Satisfactory convergence was obtained by solving equation (2) for current variables. For a typical node j

$$I_j = \frac{(E_s - E_j) - \sum_{i \neq j} Z_{ji} I_i}{Z_{jj}} \quad (5)$$

Equation (4) can be rewritten in the following form:

$$|E_j|^2 - A_j E_j^* - B_j S_j^* = 0 \quad (6)$$

where A_j and B_j are defined as,

$$A_j = Z_j I_j$$

and

$$B_j = Z_j$$

As shown in reference 7, the solution of equation (6) takes two different forms depending on whether the node is a generator or load busbar. The outstanding characteristic of equation (6) is that it incorporates the P , V and P , Q restrictions in its solution of the

voltage variables.

The calculation of reactive generation for a P, V bus requires a division by the real part of B_j , which may have a value of zero. In that case, equation (4) can be solved for the imaginary part of S_j .

4. An approach similar to the "zero residual current" method of reference⁷ was also tried with success. Equation (4) is substituted into the diagonal term of equation (2).

$$E_j = E_s - Z_{ji} I_i - \dots - Z_{jj} \left(\frac{E_j}{Z_j} - \frac{S_j^*}{E_j^*} \right) - \dots - Z_{jn} I_n \quad (7)$$

The above equation can be put into the form of equation (6), where the coefficients A_j and B_j are now given by:

$$A_j = \frac{E_s - \sum_{i \neq j} Z_{ji} I_i}{1 + Z_{jj}/Z_j}$$

$$B_j = \frac{Z_{jj}}{1 + Z_{jj}/Z_j}$$

2.3 Numerical Examples

In this section, numerical results of the solution of several systems using method 4 above are presented. Method 3, although successful, converged very slowly and was abandoned in favour of method 4. Therefore, throughout the rest of this paper, the term "mesh approach" will imply the use of method 4.

Table 2.1 gives the results of four systems solved by three different approaches; the mesh approach of this chapter, the nodal admittance⁴ and nodal impedance⁷ methods. Listings of computer programs used can be found in Power System Report No. 79, Imperial College.

For a meaningful comparison, the voltage precision index was set at 0.0001 and the VAR precision index at 0.001 for all methods of solution. Furthermore, acceleration studies were performed on all three methods. An acceleration factor of 1.6 was used to obtain the results of Table 2.1 for the nodal admittance method; complex acceleration factors and different real acceleration factors for in phase and quadrature components of voltages produced only slight improvements.

Neither the mesh impedance nor the nodal impedance methods respond to acceleration, their optimum factor being very close to unity.

This characteristic of the two impedance methods is very useful, as the power system engineer is relieved of the burden of choosing an optimum acceleration factor.

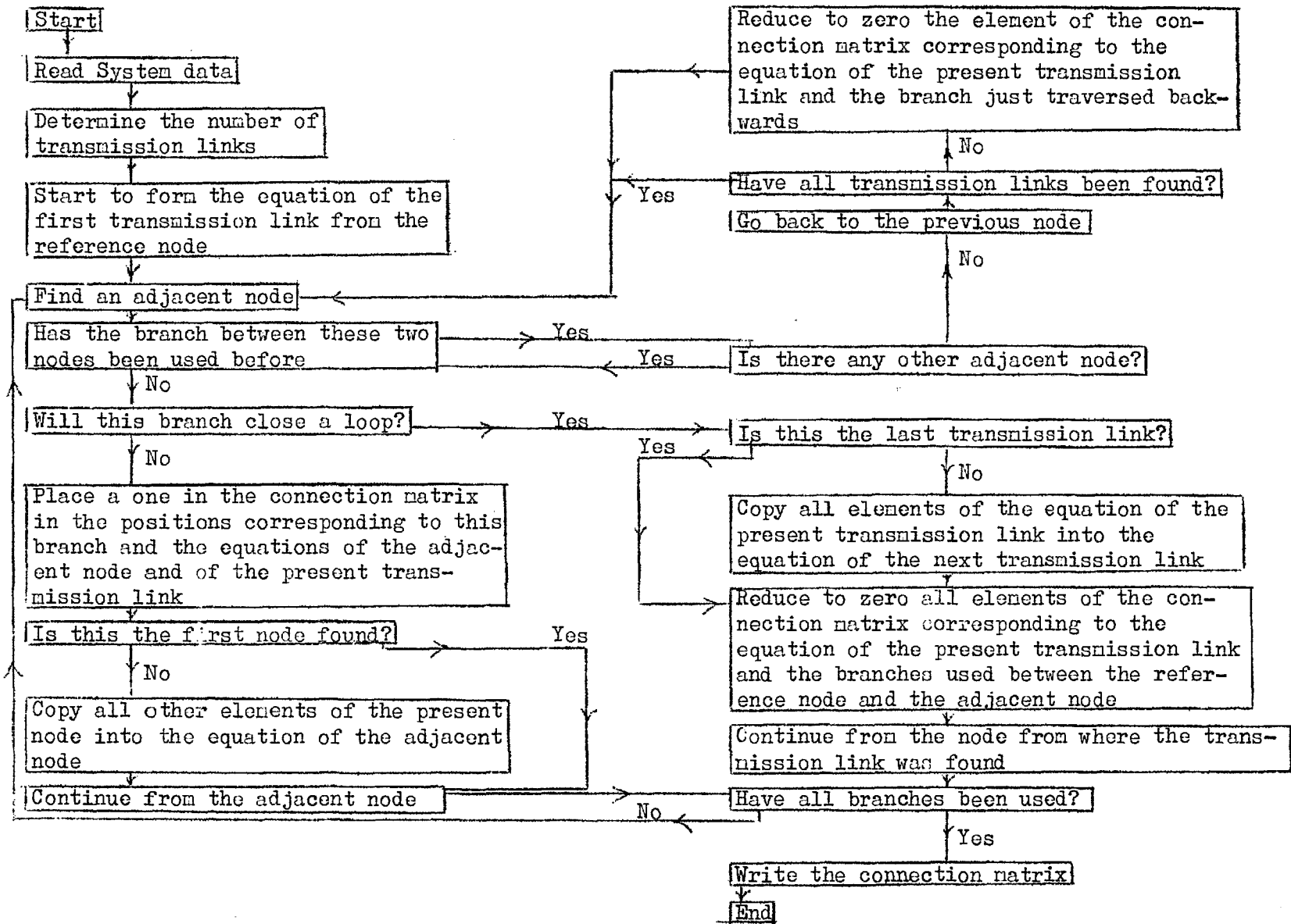
Systems A, C and D are the IEEE standard test systems. System B is a highly interconnected high voltage system. Table 2.1 shows the similarity of response of the two impedance methods, these being faster for systems A, C and D, but slower for system B, compared to the nodal admittance method. In all cases, the same bus was chosen as reference and slack when applying different methods. For a variety of cases solved by the two impedance approaches, on average the mesh impedance method involved 10 to 25 per cent fewer iterations than the nodal impedance method, although there were cases where the nodal impedance method was slightly faster.

2.4 The Mesh Impedance Matrix

The mesh impedance matrix can be assembled by the computer. Initially, the system tree is formed and all the links in the system determined. As a result, the paths of all the mesh currents in equation (1) are found. A computer flow diagram that will accomplish this task is shown in Fig. 2.1. The resulting information is stored as a matrix C^t whose elements have values of 0 and 1.

System	Number of nodes	Number of generators	Number of Iterations			
			Mesh Impedance Method	Nodal Impedance Method	Nodal Admittance Method	Hybrid Method
A	14	5	7	7	20	9
B	26	24	87	87	33	8
C	30	6	10	9	49	9
D	57	7	16	16	65	12

TABLE 2.1 Comparison of convergence of mesh, nodal impedance
and admittance and hybrid methods.



Each row of C^t represents a mesh equation and each column a branch of the transmission system. The following equation relates the impedance of individual branches to the mesh impedance matrix of equation (1)

$$Z_{\text{mesh}} = C^t Z_{\text{branch}} C \quad (8)$$

This triple multiplication need not be carried out, as Z_{mesh} can be determined by inspection of C^t . The diagonal elements of Z_{mesh} are formed by summing the impedance of branches that have an entry of "one" in the corresponding row of C^t . An off-diagonal term of Z_{mesh} in position (j,k) is formed by summing the impedance of branches whose columns have an entry of "one" in both rows j and k . As Z_{mesh} is symmetrical, only a triangular portion of it has to be determined. Furthermore, if the columns of C^t are ordered in such a way that all columns due to link branches are grouped together, no calculations need be done with the sub-matrix thus formed, as, by definition, a link will only affect the diagonal element of the equation it forms.

Having obtained equation (1), Z_{kk} can be inverted taking into account its symmetry and sparsity. Storage can be reduced by the use of a general term to calculate Z'_{nn} of equation (2). Rewriting equation (3),

$$Z'_{nn} = Z_{nn} - Z_{kn}^t Y_{kk} Z_{kn} \quad (9)$$

the general term (j,k) can be written as

$$Z_{jk} = Z_{jk} - \sum_{i=1}^n \sum_{n=1}^n Z_{ij} Y_{in} Z_{nk} \quad (10)$$

A complete program was written in Fortran IV to perform all the above mentioned operations. On the average, three times as much time was required in the derivation of Z'_{nn} directly from equation (3) compared with the method outlined above.

2.5 System Outages and System Losses

Studies on actual systems require the solution of a number of variations on a base case. The mesh impedance matrix must be changed whenever an alteration is introduced in the system configuration. The most efficient method to perform these changes is to treat Z'_{nn} as the inverse of some matrix whose elements are directly recognizable in terms of line impedances. Such a matrix is the nodal admittance matrix with some terms removed.

The following equation is the admittance equation for a system of $n + 1$ nodes, where the particular equation for the reference bus has been removed.

$$I'_n = Y_{n,n+1} E_{n+1} = Y_s E_s + (Y_a U) E_n + Y'_{nn} E_n \quad (11)$$

In equation (11), the effect of the reference node and the shunt

admittances have been separated from the other terms of the admittance equation which are all contained in $Y'_{nn} E_n$. The column matrix Y_s is related to Y'_{nn} ,

$$Y_s = - Y'_{nn} [1] \quad (12)$$

Replacing the Y_s term of equation (11) by (12) and re-arranging,

$$I_n = (Y_a U) E_n - I'_n = Y'_{nn} (E_s - E_n) \quad (13)$$

Inserting Y'_{nn} to get Z_{nn} equation (13) is reduced to (14),

$$E_s - E_n = Z_{nn} I_n \quad (14)$$

which is the same as equation (2). To alter the mesh impedance matrix when system changes are introduced, it is only necessary to use the procedures that reflect in the inverse matrix the changes introduced to the original matrix. As only the knowledge of the change is required, the original admittance matrix need not be determined.

To determine the system losses at the end of a load flow calculation, the following equation can be used,

$$\text{LOSSES} = \text{REAL} \left(E_n^H I_n + E_s^* I_n [-1] \right) \quad (15)$$

2.6 The terms "Mesh" and "Nodal"

The previous section showed the close relationship existing between the mesh impedance and the nodal admittance matrix. This may cause some concern as to the appropriateness of calling the present method a mesh approach. The argument could be advanced that the method described is really a nodal method in disguise. Section 2.4 described the method by which mesh equations are written and how some of the mesh currents are eliminated. If, furthermore, the mesh impedance matrix is manipulated, the nodal admittance matrix would result. The conclusion could then be reached that nodal methods are really mesh methods in disguise. In fact, for any network the mesh matrix can be obtained from the nodal and vice-versa. The doubt arises here only because the matrices are of the same order. This, however, was artificially imposed through the method adopted (i.e. suppressing the shunt impedances and considering their effect without adding to the number of equations) so that condition 3 of section 2.1 was satisfied.

Another point of view, is that node equations are usually solved for voltage variables whilst mesh equations are solved for current variables. However, as was shown in section 2.2, starting from a set of mesh equations, a solution can be obtained by solving for either voltage or current variables. Strictly speaking, the terms "mesh approach" and "node approach" are valid only for linear

systems. In the non linear load flow problem, neither the mesh nor the node equations are solved by themselves. Equations bringing in terminal voltage and load conditions are solved simultaneously with the network equations. The resulting composite set of equations are no longer mesh or nodal equations. Nevertheless, as the present method determines a set of mesh equations, and as nodal approaches use a set of nodal equations, the terms "mesh" and "nodal", do appear to carry some significance.

2.7 Conclusion

This chapter has demonstrated the feasibility of using the mesh approach in the formulation of the load flow problem. This method, based on the mesh impedance matrix, exhibits characteristics very similar to the nodal impedance matrix methods. The mesh impedance method for the systems studied, reaches convergence on average in 15% less iterations than the nodal impedance method.

CHAPTER 3

THE HYBRID APPROACH

3.1 The hybrid method

A powerful hybrid approach exhibiting very rapid convergence has been developed. The hybrid is formed from a combination of impedance and admittance approaches. Throughout this section, the term "impedance" approach will signify either the mesh impedance or the nodal impedance method, as when using the hybrid approach, no significant variations were found to occur when substituting one impedance method for the other.

A load flow solution was commenced with an impedance method, and at an early stage in the iterative process, the impedance method was replaced by the nodal admittance method. Table 2.1, Chapter 2, shows a comparison of results obtained with such a hybrid with the precision index for voltage and VARS set at the same level as for the other methods. This approach exhibits not only a fast convergence but also an overall number of iterations largely independent of system size and of the response of the impedance and admittance methods when used separately.

An examination of the rate of convergence of the variables of the load flow problem, using an impedance approach, shows that the volt-

age variables converge to about two decimal places in very few iterations. On the other hand, the reactive generation, or Q variables, converge at a slower rate. The nodal admittance approach exhibits a completely different convergence characteristic. Convergence is slow for both voltages and Q's. However, if the nodal approach is provided with voltages that are already near to their correct value, Q is found to converge very rapidly. This last observation provides the basis for the powerful hybrid impedance/admittance combination. The hybrid possesses the salient properties of the two approaches; the rapid initial voltage convergence of impedance methods and the fast Q convergence of the admittance method. These properties are seen to be characteristic of the methods and not of the systems to which they are applied.

The explanation of the fast convergence of the hybrid approach, lies in the calculation of Q. Nodal admittance methods use the following equation to determine Q,

$$Q_j = \text{IMAGINARY PART} \left(V_j^* \sum_i (V_j - V_i) Y_{ji} \right) \quad (1)$$

The summation term involves the difference between pairs of terminal voltages. This difference is very sensitive to the precision of the individual voltages. For this reason, the admittance method has in general a slow Q convergence characteristic. One of the reasons for

slow voltage convergence is the use that is made in the calculation of Q values from equation (1). To obtain further evidence of this adverse effect on voltage convergence, a procedure was developed which does not use equation (1) to calculate Q . The nodal equation for bus j ,

$$I_m = \frac{S_j^*}{E_j^*} = Y_{j1}E_1 + \dots + Y_{jj}E_j + \dots + Y_{jn}E_n \quad (2)$$

can be put into the form of equation (6), Chapter 2, where the A_j and B_j coefficients now are,

$$A_j = \frac{-\sum_{i \neq j} Y_{ji} E_i}{Y_{jj}}$$

and

$$B_j = 1/Y_{jj}$$

The solution of equation (6), Chapter 2, gives simultaneously values for voltages and for Q . Convergence was about 20 per cent faster with this approach than with the conventional nodal admittance solution for a number of cases tested, using the same acceleration factor and precision. The value of Q calculated at each iteration is not used in any other calculation.

The hybrid method possesses such a fast natural convergence that very little benefit will result if an optimum acceleration

factor is used in the nodal admittance part of the method. This relieves the user of the burden, and sometimes the difficulty, associated with the admittance methods, of choosing an optimum acceleration factor.

To confirm the properties which give rise to the outstanding behaviour of the impedance/admittance hybrid, the following hybrid methods were tried, all of which proved less successful than the one already described.

(i) A hybrid in the reverse order. The solution was commenced with an admittance approach and continued with an impedance approach. In all cases, the number of iterations in that part of the solution which used the impedance approach was about the same as the number of iterations that the impedance approach took when applied by itself. In other words, the use of the admittance method to start the process was totally immaterial to the impedance approach. It is apparent that the slow convergence of the admittance approach provided the impedance method with a starting point which was no better than the assigned initial starting values.

(ii) A hybrid in which some nodes were solved with only admittance coefficients, and the rest of the nodes with impedance coefficients. Having no objective criteria for the segregation of

busses into the two groups, several combinations were tried. Results were always intermediate between those obtained with either method applied individually.

3.2 Conclusion

The hybrid approach developed in this chapter, exploits the salient characteristics of the impedance and admittance methods. The impedance methods are characterized by fast convergence of the voltage variables, but slow convergence of Q . The nodal admittance method is characterized by equally slow convergence of both the voltage and Q variables. However, if the nodal admittance approach is provided with voltages near the correct value, both the voltage and Q variables converge rapidly. The impedance/admittance hybrid exhibits very favourable characteristics. Due to its inherent rapid convergence, acceleration factors are not so sensitive and this relieves the power system engineer of the burden of making an optimum choice for such a factor. Finally, it is a combination of two popular, well-known and widely programmed approaches.

The development of load flow programs by users has involved considerable effort and money. Some users have developed nodal impedance methods, others nodal admittance methods. Some possess programs with both approaches. The majority of these users will not be prepared to abandon well tried programs for new methods claiming better results.

For such users, the hybrid may be considered an improvement with the minimum outlay of capital. Furthermore, if the computer facilities have provisions for dividing programs into several parts, one of which is only present in the computer core at any one time, the programming of the hybrid method is considerably simplified involving a minimum of alterations to existing programs.

CHAPTER 4

THE REFERENCE AND SLACK BUSES

4.1 The Choice of a Reference-Slack Bus

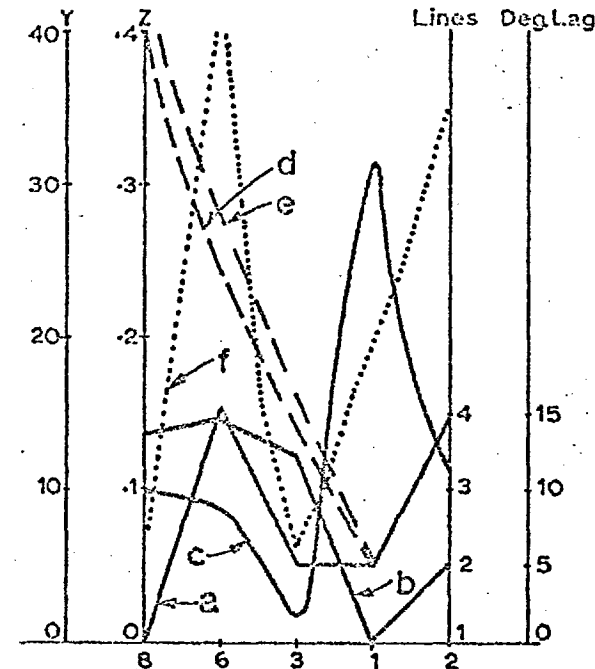
The problem of reference-slack (r-s) bus choice is a very important one. For some systems, an unfortunate selection may lead to a non-convergent solution, a fortunate one to a solution within few iterations.

A dependable criterion for the choice of r-s bus location has been found. The present section describes this criterion and compares it with several rules of thumb that have been suggested in the literature as guidance towards an optimum choice. The most commonly used rules for r-s selection recommend that the chosen bus should

1. have a large number of lines connected to it
2. have a voltage which leads all other voltages of the system
3. be a generator with a relatively large infeed to the system

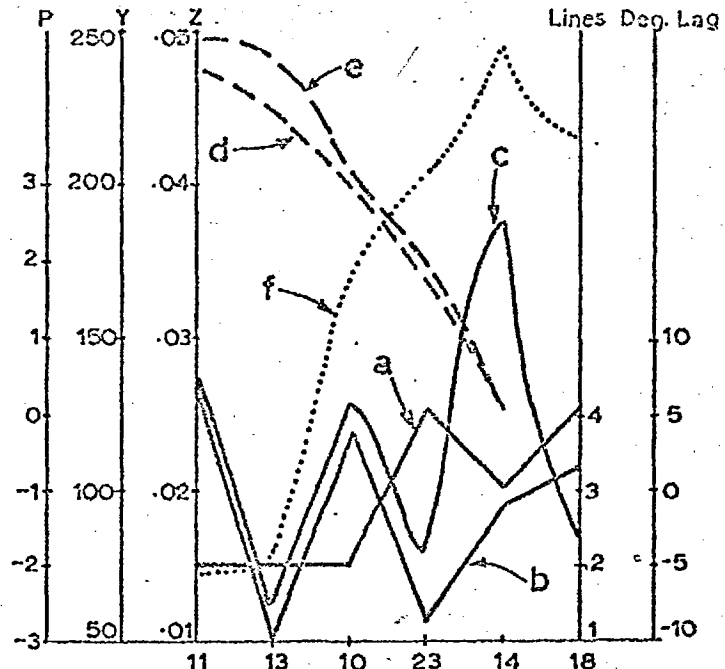
The results of an extensive investigation that was carried out to study the problem of the choice of r-s bus are summarized in Figures 4.1 and 4.2. Figures 4.1.1 to 4.1.4 reveal that the variation of the number of iterations plotted against the chosen r-s bus follows a similar pattern for all three methods studied, i.e. the nodal admittance, nodal impedance and mesh impedance methods. In other words, Figs. 4.1.1 to 4.1.4 show that the effect of the r-s bus is independent of the method of solution and only depends on the system characteristics. Therefore, a criterion for the selection of the r-s bus is equally valid for any method of solution.

Figures 4.1.1 to 4.1.4 are extended in the y-axis up to the maximum number of iterations that was allowed for each system. In some cases, this maximum number was reached without convergence and the plot is therefore incomplete. These figures also illustrate that the convergence characteristics of the mesh impedance and nodal impedance methods are different. For system B, in figure 4.1.2, the mesh impedance method is considerably faster than the nodal impedance method, for a wide range of r-s bus location. Figure 4.1.4 shows a marginal improvement of the mesh impedance over the nodal impedance approach. However, Figs. 4.1.1 and 4.1.3 are examples of systems in which the nodal impedance is the fastest of the two impedance methods.



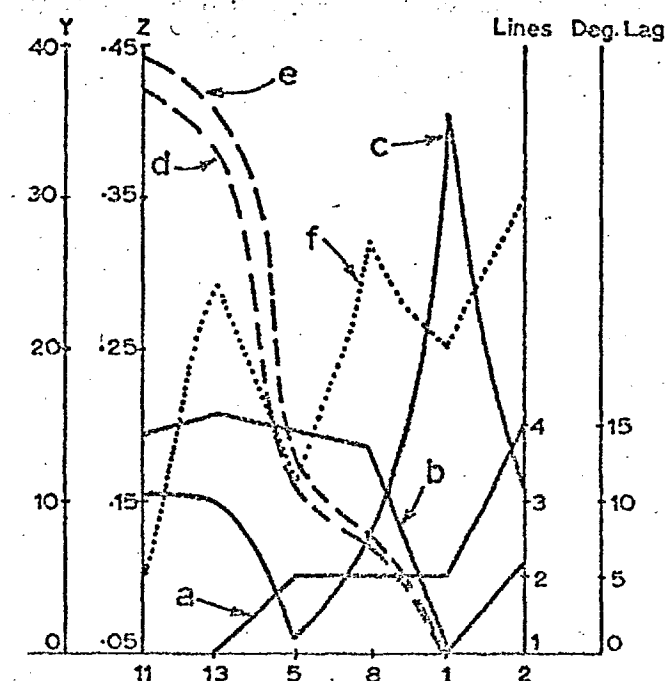
Busses in ascending order of merit

Fig 4.2.1 System A



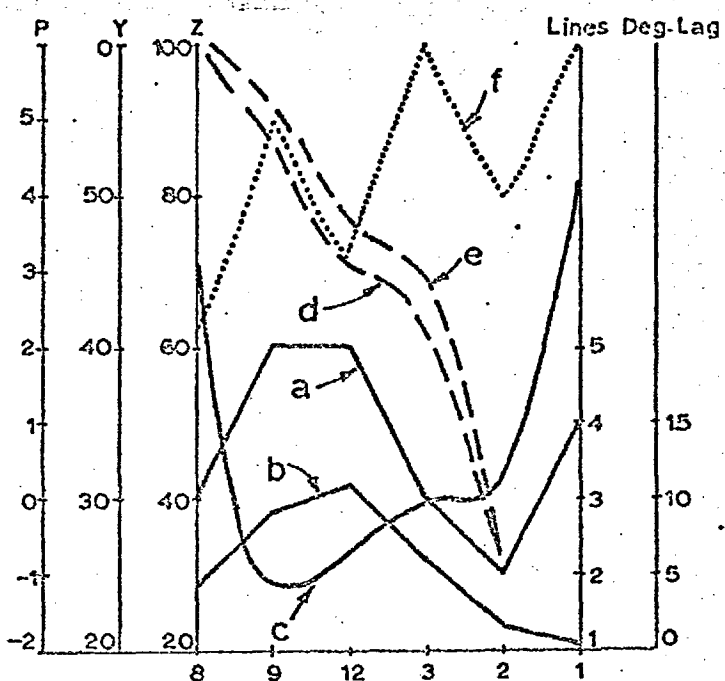
Busses in ascending order of merit

Fig 4.2.2 System B



Busses in ascending order of merit

Fig 4.2.3 System C



Busses in ascending order of merit

Fig 4.2.3 System D

a = LINES CONNECTED TO BUS c = POWER INFEEED TO BUS e = DIAGONAL NODAL IMPEDANCE MATRIX VALUE
b = ANGLE OF BUS VOLTAGE d = DIAGONAL MESH MATRIX f = DIAGONAL NODAL ADMITTANCE MATRIX VALUE

Figure 4.2 Comparison of performance of optimum r-s bus choice criteria

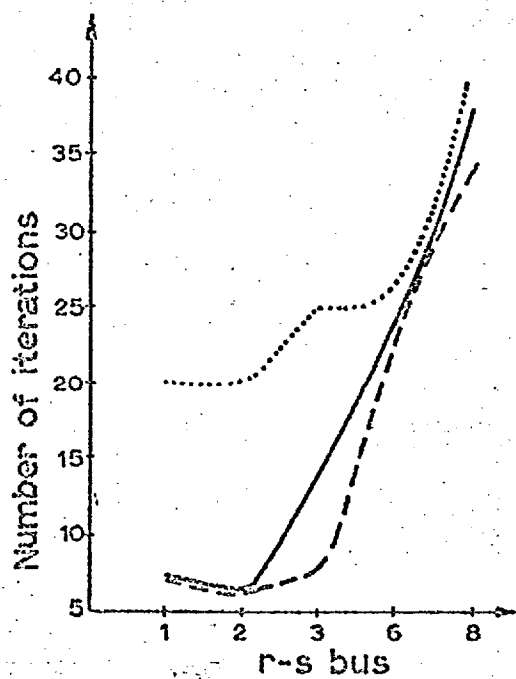


Fig. 4.1.1 System A

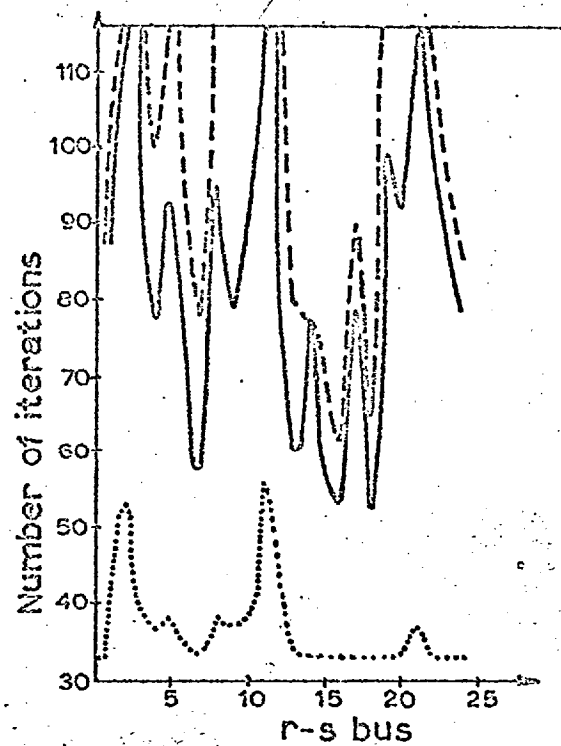


Fig. 4.1.2 System B

— MESH IMPEDANCE METHOD
 - - - NODAL IMPEDANCE METHOD
 NODAL ADMITTANCE METHOD

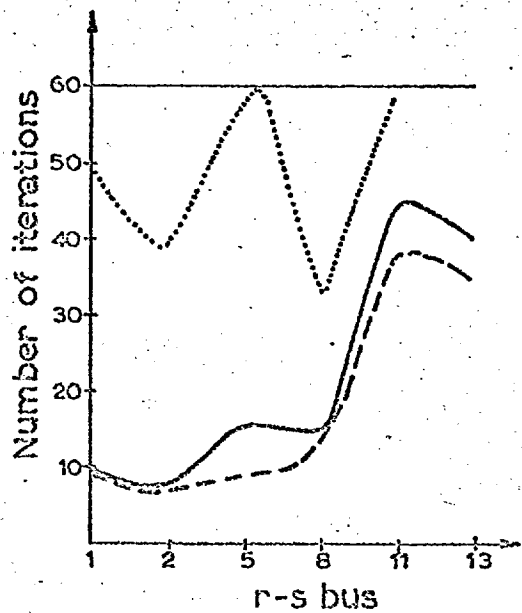


Fig. 4.1.3 System C

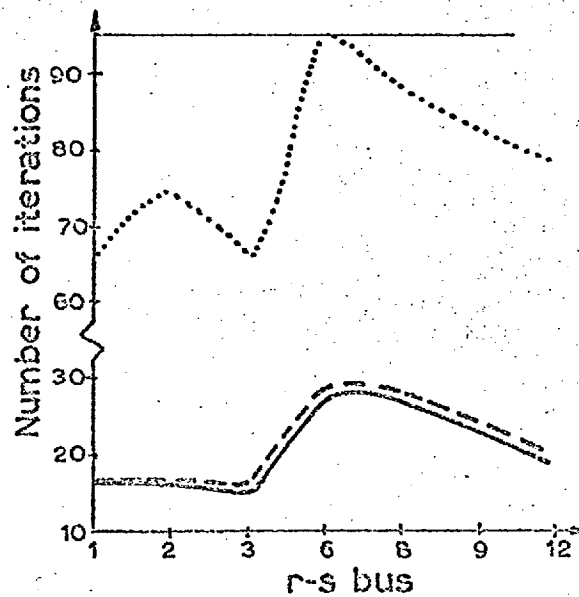


Fig. 4.1.4 System D

Figure 4.1 - Comparison of system response to various methods of solution with change in r-s bus location

Figs. 4.2.1 to 4.2.4 shows plots of (a) number of lines connected to the bus, (b) angle of bus voltage, (c) power in-feed to bus. The magnitude of the diagonal elements of (d) the mesh impedance, (e) the nodal impedance and (f) the nodal admittance matrices are also shown. The above quantities are plotted against nodes in order of merit, i.e. the number of the node that requires the maximum number of iterations is located on the first division of the x-axis, followed by the node of the next order of merit, etc. With this plot, any of the quantities (a) to (f) that exhibits a continuously diminishing or increasing rate for all systems examined, could form a criterion of optimum r-s bus selection.

From the graphs, it is clear that no evidence whatsoever exists that the rules of thumb, listed at the beginning of this section, are dependable criteria. The diagonal element of the admittance matrix exhibits an unreliable trend, better choices of r-s bus corresponding to higher values of admittance. However, it is clear from the graphs, that the most consistent and dependable criterion is the value of the diagonal element of both the mesh and nodal impedance matrices. The smallest value of impedance corresponds to the optimum position of the r-s bus.

The systems of Fig. 4.2 possess a wide range of diagonal impedance values ranging approximately from a minimum of 0.02 for system B

to a maximum of 100 for system D. In all cases the criterion proved reliable and easy to implement.

The values of impedance elements given in figures 4.2.1 to 4.2.4 are those obtained with node 1 chosen as the r-s bus. The diagonal elements will then show where the optimum position lies. The impedance matrix can then be transformed, following the methods described in Appendices 4.4.1 and 4.4.2. The transformation is quite fast, involving less steps in the case of the mesh matrix than in the case of the nodal impedance matrix. It must be stressed here that the order of merit of nodes accorded by the values of the diagonal impedance coefficients does not alter when different nodes are chosen as reference in the calculation of the initial matrix.

4.2 The Use of Different Reference and Slack Busses

The reference and slack bus functions do not necessarily have to be allotted to the same bus. The criterion for r-s bus selection may indicate a P,Q bus as optimum choice. Clearly, such a bus cannot be used for slack purposes as it contains no generation. It has been found that an acceptable procedure for such a case is to allot the reference bus function to the P,Q bus selected by the above criterion and then to select the slack bus by using the same criterion among P,V busses only. An arbitrary selection of slack bus location can lead to very poor convergence, even if an optimum position for the reference bus is used.

4.3 Conclusion

A reliable criterion has been found for guidance in the choice of the highest merit swing bus. This criterion is based on selecting a swing/reference bus, the node with the lowest self-impedance in the impedance matrix methods. The nodal admittance matrix does not yield a completely reliable criterion and other suggested criteria in the literature have been shown to have no validity.

4.4 Appendix to Chapter 4

4.4.1 "Change of reference" transformation for the mesh method

Let j be the old reference bus and k the new reference. The k^{th} equation is,

$$E_j - E_k = \sum_{\substack{m=1 \\ m \neq j}}^n Z_{km} I_m \quad (1)$$

The current at bus j is related to the other bus currents by the relation,

$$I_j = - \sum_{\substack{m=1 \\ m \neq j}}^n I_m \quad (2)$$

Substituting (2) in (1) to eliminate I_k ,

$$E_k - E_j = \sum_{\substack{m=1 \\ m \neq k, j}}^n (Z_{kk} - Z_{km}) I_m + Z_{kj} I_j \quad (3)$$

This is the equation for the previous reference bus, referred now to the new bus k. I_k is then eliminated from the rest of the mesh equations in the same manner. The E_j term of these equations is eliminated by substituting the relation for E_j contained in equation (3). The 1th equation takes the following form, referred to the new bus k,

$$E_k - E_1 = \sum_{\substack{m=1 \\ m \neq j, k}}^n (Z_{kk} + Z_{lm} - Z_{lk} - Z_{km}) I_m + (Z_{kk} - Z_{lk}) I_j \quad (4)$$

4.4.2 "Change of reference" transformation for the nodal impedance method

Let j be the old reference bus and k the new reference. The kth equation is,

$$E_k = \sum_{\substack{m=1 \\ m \neq j}}^n (Z_{km} I_m) - D_k E_j \quad (5)$$

The current at bus k is related to the other bus current and voltages by the relation,

$$I_k = - \sum_{\substack{m=1 \\ m \neq k}}^n I_m + \sum_{m=1}^n Y_{mk} E_m \quad (6)$$

where Y_m is the total shunt admittance at node m .

Substituting for E_m in equation (6) and in the normal nodal impedance equations (Equation 6 of Chapter 1) and re-arranging the resulting relation gives

$$I_k = \sum_{\substack{m=1 \\ m \neq k, j}}^n (A_m - 1) I_m + A_k I_k - A_j E_j + Y_j E_j - I_j \quad (7)$$

$$\text{where, } A_m = \sum_{\substack{l=1 \\ l \neq j}}^n Y_l Z_{lm}$$

$$A_k = \sum_{\substack{l=1 \\ l \neq j}}^n Y_l Z_{lk}$$

$$A_j = \sum_{\substack{l=1 \\ l \neq j}}^n Y_l D_l$$

Equation (7) can be reduced to the following form,

$$I_k = \sum_{\substack{m=1 \\ m \neq j, k}}^n C_m I_m + C_j I_j + C_k E_j \quad (8)$$

$$\text{where, } C_m = \frac{A_m - 1}{1 - A_k}$$

$$C_j = \frac{-1}{1 - A_k}$$

$$C_k = \frac{Y_j - A_j}{1 - A_k}$$

Substituting (8) into (5), I_k can be eliminated,

$$E_j = \sum_{\substack{m=1 \\ m \neq j,k}}^n \left(\frac{Z_{km} + Z_{kk} C_m}{D_k - Z_{kk} C_k} \right) I_m + \left(\frac{Z_{kj} C_j}{D_k - Z_{kk} C_k} \right) I_j - \left(\frac{1}{D_k - Z_{kk} C_k} \right) E_k \quad (9)$$

This is the equation for the old reference bus. It will now appear as,

$$E_j = \sum_{\substack{m=1 \\ m \neq j,k}}^n (Z_{jm} I_m) + Z_{jj} I_j - D_j E_k \quad (10)$$

Substituting I_k in all the other nodal impedance equations using (5) and eliminating E_j by use of (10) the transformation is completed. The 1th equation takes the following form referred to the new bus k,

$$\begin{aligned}
E_1 = & \sum_{\substack{m=1 \\ m \neq j,k}}^n (Z_{1m} - Z_{1k}C_m + C_{jm}(Z_{1k}C_k - D_1))I_m + (Z_{1k}C_j + Z_{jj} \\
& (Z_{kk}C_k - D_1))I_j - D_j(Z_{1k}C_k - D_1)E_k
\end{aligned} \tag{11}$$

CHAPTER 5

NONLINEAR PROGRAMMING

5.1 Statement of problem

The term nonlinear programming is used to classify those methods in mathematical programming which optimize nonlinear problems. There are many types of nonlinear problems and special methods deal with particular cases. The general nonlinear problem can be stated as follows:

minimize $f(x)$ subject to the constraints

$$\left. \begin{array}{l} g_i(x) \geq 0; \quad i = 1, \dots, m \\ h_j(x) = 0; \quad j = 1, \dots, p \end{array} \right\} \quad (1)$$

where x is an n -dimensional vector.

5.2 Unconstrained minimization

Particular types of nonlinear problems can be obtained from (1). If $m = p = 0$, the problem is reduced to the unconstrained minimization of $f(x)$. The classical approach to obtain the minimum is to solve the simultaneous set of equations defined by

$$\frac{\partial f(x)}{\partial x} = 0 \quad (2)$$

The use of equation (2) can be very impracticable. The resulting set of equations could themselves be nonlinear and numerical procedures would have to be used to solve them. The conditions expressed in (2) are necessary but not sufficient.

5.2.1 Method of steepest descents

The gradient of $f(x)$, evaluated at some initial point x_0 , points in the direction of maximum change of $f(x)$ around x_0 . Let the gradient be $g(x)$, then the direction $-g(x)$ is the best direction to take to minimize $f(x)$, around an infinitesimally small region about x_0 . The step from x_0 to x_1 is determined from the relation

$$x_1 = x_0 - \alpha g(x_0) \quad (3)$$

for some scalar parameter α . This parameter is sometimes chosen arbitrarily, but it can be chosen as α_n , the step that will minimize $f(x)$ along the given direction. Methods for finding α_n will be discussed later.

At point x_1 , the gradient $g(x_1)$ is evaluated and a new point x_2 is found in the same manner as above. This process is continued until the minimum is found.

For a general nonlinear function $f(x)$, the gradient may not

point in the general direction of the minimum and the method may have a strong oscillatory convergence behaviour.

5.2.2 Methods of conjugate directions

A desirable feature of minimization methods is that they should be quadratically convergent. This means that for quadratic functions it is guaranteed, apart from round-off errors, that the minimum will be located in a finite number of iterations, usually equal to the number of variables, n . The important decision is the determination of directions of minimization, p_i , such that quadratic convergence is assured. A set of conjugate directions, that define vectors p_i that are conjugate to the matrix of second derivatives, has this property. The step from a point x_i to a point x_{i+1} along the direction p_i is determined by the relation,

$$g_{i+1}^T p_i = 0 \quad (4)$$

$$x_{i+1} = x_i + \alpha_i p_i \quad (5)$$

Proof of quadratic convergence

If a quadratic function $f(x)$ is expanded around the minimum h ,

$$f(x) = f(h) + \frac{1}{2}(x - h)^T G(x - h) \quad (6)$$

The gradient is

$$g(x) = G(x - h) \quad (7)$$

By equation (5),

$$x_n = x_{j+1} + \sum_{i=j+1}^{n-1} \alpha_i p_i \quad (8)$$

for any j in $0 \leq j \leq n-1$. It then follows

$$g_n = g_{j+1} + \sum_{i=j+1}^{n-1} \alpha_i G p_i \quad (9)$$

If both sides of (9) are multiplied by p_j , and equation (4) is used,

$$g_n^T p_j = \sum_{i=j+1}^{n-1} \alpha_i p_i^T G p_j \quad (10)$$

Now, if the directions p are conjugate to G , then

$$p_i^T G p_j = 0 \text{ for } i \neq j \quad (11)$$

It then follows from (10) that

$$g_n^T p_j = 0 \quad (12)$$

and as the p_j are independent,

$$g_n = 0 \quad (13)$$

Equation (7) then shows that

$$x_n = h \quad (14)$$

This completes the proof as it shows that the minimum is found at the n th step, starting from any initial point x_0 .

5.2.3 Method of conjugate gradients (Fletcher and Reeves)²¹

The method of conjugate gradients chooses directions p satisfying the following relation,

$$p_{i+1} = -g_{i+1} + \beta_i p_i \quad (15)$$

where

$$\beta_i = \frac{|g_{i+1}|^2}{|g_i|^2}$$

It can be shown that this direction is conjugate to G .

The algorithm is as follows:

1. x_0 is arbitrary; $g_0 = g(x_0)$; $p_0 = -g_0$; $i = 0$
2. x_{i+1} = position of minimum of $f(x)$ on the line through x_i in the direction p_i .
3. $g_{i+1} = g(x_{i+1})$; $\beta_i = g_{i+1}^2 / g_i^2$; $p_{i+1} = -g_{i+1} + \beta_i p_i$
4. steps 2 and 3 are repeated until convergence is detected,
 $i = i + 1$.

Methods of minimizing $f(x)$ along a linear direction (step 2) will be discussed later.

5.2.4 Variable metric method (Fletcher-Powell)^{22,23}

Equation (7) shows the vector difference between the minimum h and a point x is

$$h - x = -G^{-1}g(x) \quad (16)$$

which says that for a quadratic function f , with gradient g and Hessian G^{-1} evaluated at some point x , h can be found in one step. For non-quadratic functions, h would not be the true minimum but the starting point for another evaluation of (16) which would eventually converge to the minimum.

In the Fletcher-Powell method, the G^{-1} matrix is not evaluated directly. Instead, it is built up iteratively by an algorithm that

ensures that once the Hessian is ready, the minimum has also been found. For clarity, the matrix in which the Hessian is being formed will be called H .

The algorithm is as follows:

1. x_0 is arbitrary; $g_0 = g(x_0)$; $H_0 =$ unit matrix; $i = 0$
2. $p_i = -H_i g_i$
3. Obtain α_i such that $f(x_i + \alpha_i p_i)$ is a minimum with respect to α . Call $x_{i+1} = x_i + \sigma_i$, where $\sigma_i = \alpha_i p_i$
4. update H by $H_{i+1} = H_i + A_i + B_i$, where the A_i and B_i are matrices whose coefficients are,

$$A_{jk} = \frac{\sigma_j \sigma_k}{\sigma_i^T y_i}$$

$$B_{jk} = \frac{(H_i y_i)_j (y_i^T H_i)_k}{y_i^T H_i y_i}$$

and

$$y_i = g_{i+1} - g_i$$

5. Repeat steps 2 to 4 with $i = i + 1$ until convergence is detected.

It can be shown that the chosen directions are conjugate to G for quadratic functions and so the method has quadratic convergence.

5.2.5 Variance method (Davidon)²⁴

The variance method is a variation of the variable metric method designed to require less function and gradient evaluations. The algorithm is as follows:

1. x_0 is arbitrary; $g_0 = g(x)$; $f_0 = f(x_0)$; $H_0 =$ unit matrix;
 $i = 0$. α and β are real numbers satisfying $0 < \alpha < 1 < \beta$
2. $x_{i+1} = x_i - H_i g_i$; f_{i+1} and g_{i+1} computed at x_{i+1}
3. Define $r = H_i g_{i+1}$ and $\rho = g_{i+1}^T r$; if ρ is sufficiently small a solution has been found at x_{i+1} ; if not continue with 4.
4. Define $\gamma = -g_i^T r / \rho$
 if $-1 \leq \gamma \leq \frac{\alpha}{1-\alpha}$, define $\lambda = \alpha$
 if $-\frac{\beta}{\beta-1} \leq \gamma < -1$, define $\lambda = \beta$
 if neither are true, $\lambda = -\frac{1}{\gamma+1} + 1$
5. $H_{i+1} = H_i + (\lambda - 1) r r / \rho$
6. If $f_i \leq f_{i+1}$, define $x_{i+1} = x_i$; $f_{i+1} = f_i$; $g_{i+1} = g_i$
 Set $i = i + 1$ and go back to step 2

As can be noticed from steps 2 and 6, no linear search for the minimum along a line is made and a fixed unit step is taken.

The H matrix is updated and the new point is used only if the function there is smaller than at the starting point of the iteration. If the function is not smaller, the same starting point is used together with the improved H matrix.

5.2.6 Methods that do not require derivatives

Powell²⁵ and Zangwill²⁶ have developed algorithms which are gradient type methods but which do not require the explicit evaluation of derivatives. In general, these methods should be used only when derivatives cannot be directly evaluated as they are weaker than true gradient methods. No more will be said here about these methods as all applications in later chapters used gradient methods.

5.2.7 Linear minimizations²³

Most methods that have been mentioned previously require that the minimum of a function f in the direction p be found. The problem can be stated as that of finding a value of α that will minimize

$$z(\alpha) = f(x + \alpha p) \quad (17)$$

and therefore the derivative of z with respect to α is set to zero,

$$z'(\alpha) = p^T g(x + \alpha p) = 0 \quad (18)$$

Equation (18) is normally not available explicitly. Instead, given a point x and an α , (18) evaluates the derivative of $f(x + \alpha p)$ in the direction p .

First, a step a is chosen as

$$a = \text{minimum of } (1, -2(f'_x - f'_0)/z'_x) \quad (19)$$

where f'_0 is an estimate of the minimum. A step of size a is taken as $y = x + ap$ and z'_y evaluated to find if there has been a change of sign with respect to z'_x . A change of sign, from negative z'_x to positive z'_y , indicates that the minimum point is contained within these two points. If there is no change of sign, a new step a is taken and again its derivative computed. This process is continued until the minimum is enclosed. The last two points, which must fall on either side of the minimum, will be renamed w and y , and a cubic interpolation is used to locate α . The distance from x to w and from x to y will be called α_w and α_y . Then

$$\alpha = \alpha_y - \frac{(\alpha_y - \alpha_w)(z'_y + r - s)}{z'_y - z'_w + 2r} \quad (20)$$

where

$$r = (s^2 - z'_w z'_y)^{\frac{1}{2}}$$

and

$$s = 3 \frac{f_w - f_y}{(\alpha_y - \alpha_w)} + z'_w + z'_y$$

The point $x + \alpha p$ is accepted as the minimum point if the function there, f_α , is smaller than f_w and f_y . If this is not the case, the interpolation is repeated using x and one of the two previous points, chosen such that the minimum is enclosed. This is easily decided by computing z'_α , as its sign will define on which side of the minimum point it falls.

5.2.8 Solution of systems of nonlinear equations^{27,28}

The solution of a system of equations $f_i = 0$, $i = 1, \dots, m$, where f is a function of n variables x is a frequent and important problem. In this section a review is made of several methods available for solutions of this type of problem. In all cases, the problem will be transformed into one in which the least squares norm will be minimized,

$$F(x) = f^T f = (f_1^2 + f_2^2 + \dots + f_n^2) \quad (21)$$

The gradient of F with respect to x is

$$g(x) = 2 J^T f \quad (22)$$

where J = matrix of first partial derivatives (Jacobian).

5.2.8.1 Direct minimization

The unconstrained minimization methods described above can be used to minimize $F(x)$. Equation (22) can be used to evaluate the gradient.

5.2.8.2 Least Squares solution

If the number of equations is larger or smaller than the number of variables, the solution may not always be unique, but nevertheless, if $F(x)$ is minimized to a value of zero, a solution has been found. In general, for any m and n the following method could be employed (J_i^+ is the generalized inverse of J).

1. x_0 is arbitrary; $i = 1$
2. compute f_i , J_i and $p_i = -J_i^+ f_i$
3. set $x_{i+1} = x_i + \alpha_i p_i$ choosing α_i so that $\alpha_i > 0$ and $F(x_{i+1})$ is the minimum of $F(x)$ in the direction p_i through x_i .
4. set $i = i + 1$ and repeat from 2 until convergence is obtained.

Step 2 is the important step in the algorithm. If the original equations were linear, then

$$f(x) = Jx - b = 0 \quad (23)$$

and it can be shown that

$$x + \Delta x = x - J^+ f(x) \quad (24)$$

is a solution of (23). In the general nonlinear case, (24) is an approximation to the minimum and the new point obtained should be used as the starting point of a further evaluation of (24). This is step 2 in the algorithm.

The generalized inverse is an extension of the concept of an inverse for matrices which are singular or rectangular. If J is an $m \times n$ matrix, then the generalized inverse of J is an $n \times m$ matrix, denoted by J^+ which satisfies the equations:

$$JJ^+J = J; \quad J^+JJ^+ = J^+; \quad (J^+J)^T = J^+J; \quad (JJ^+)^T = JJ^+ \quad (25)$$

Methods are available²⁸ for computing J^+ given J .

Returning to the algorithm, if $m \geq n$ and $\text{rank}(J) = n$, then J^+ becomes $(J^+J)^{-1}J^T$ and the method reduces to the generalized least squares method. If the case where $m = n$ is considered, J^+ becomes J^{-1} and the method becomes the well known Newton's method.

5.2.9 Assessment of unconstrained minimization methods

The method of steepest descent to find a minimum is by far the slowest of the methods described in this chapter. Computationally, the conjugate gradient method requires about the same amount of work for iteration and is much more powerful. The best method available is the Fletcher-Powell method whose main disadvantage is its need to store the Hessian matrix it builds up. Several comparisons have recently been made⁵¹ to determine the performance of these methods with several difficult functions and they point to the Fletcher-Powell method as the one which performed most consistently. These methods were also applied for comparison purposes to the power system load flow problem and are described in the following chapter. Results showed that the conjugate gradient was about 10 times faster than steepest descents and that Fletcher-Powell was 10 times faster than the conjugate gradient.

Davidon's variance method²⁴ has just been recently published. Convergence has been theoretically proved and shown to be nearly quadratic, requiring about half the function and gradient evaluations of the Fletcher-Powell method. In practice, the author of this thesis has found that the method runs into convergence problems when the algorithm was programmed for a standard test function given by

$$f(x_1, x_2) = 100 (x_2 - x_1^2)^2 + (1 - x_1)^2$$

Results given in Table 5.1 show that the method does converge, but with difficulty. The process several times reaches a point where no further improvements are obtained. To jump this point, the Hessian matrix being built up was discarded and returned to a unit matrix. This amounts to restarting the process several times. The conclusion reached is that more work is needed on the variance algorithm before it can compete with the Fletcher-Powell method.

TABLE 5.1 Convergence characteristic of the Variance Method*

Iteration	Function	x_1	x_2
0	24.200	-1.200	1.000
5	4.485	-1.117	1.250
19	3.227	-0.759	0.539
4	3.059	-0.749	0.564
9	2.449	-0.532	0.251
4	2.316	-0.522	0.276
20	0.697	0.198	0.016
30	0.248	0.513	0.274
4	0.229	0.521	0.271
20	0.045	0.916	0.819
30	10^{-3}	0.987	0.973
35	10^{-6}	1.000	1.000
37	10^{-14}	1.000	1.000

* This characteristic should be compared with the 19 steps of the Fletcher-Powell method which converges to the same values.

5.3 Constrained minimizations

5.3.1 Method of Lagrange Multipliers

The classical approach to the constrained minimization problem is the method of the Lagrange Multipliers. The problem is (1) with $m = 0$, $p > 0$. It requires the solution of the constraints $h_j(x) = 0$ together with

$$\frac{\partial f}{\partial x} + \sum_{j=1}^p \lambda_j \frac{\partial h_j(x)}{\partial x} = 0 \quad (26)$$

The solution of this set of equations can be quite difficult.

5.3.2 Method of Kuhn and Tucker

An extension of the method of Lagrange multipliers, introduced by Kuhn and Tucker, deals with problem (1) with $m > 0$ and $p > 0$. The method requires the solution of the constraints $h_j(x) = 0$ together with

$$\left. \begin{aligned} \frac{\partial f}{\partial x} + \sum_{j=1}^p \lambda_j \frac{\partial h_j(x)}{\partial x} - \sum_{i=1}^m \mu_i \frac{\partial g_i(x)}{\partial x} &= 0 \\ \mu_i g_i(x) &= 0, \quad i = 1, \dots, m \\ \mu_i &\geq 0, \quad i = 1, \dots, m \end{aligned} \right\} \quad (27)$$

and the exclusion equations,

As before, it may be very difficult to solve these equations.

5.3.3 Method of Fiacco-McCormick^{30, 31, 32}

The basic concept behind the method of Fiacco-McCormick, and other methods described later in this chapter, is to transform the original constrained optimization problem into a sequence of unconstrained optimization problems in order to make use of the minimization methods discussed in section 5.2. No further reference will be made in this section to the unconstrained method used to perform the actual minimizations.

Carroll²⁹ first proposed a method which Fiacco-McCormick later developed and gave rigorous proof. It deals with problem (1) with $m > 0$ and $p = 0$. The method is to minimize the unconstrained function

$$P(x, r_k) = f(x) + r_k \sum_{i=1}^n 1/g_i(x) \quad (28)$$

with a particular value of r_k , with $k = 1$, starting from an initial x that satisfies all the inequality constraints. When the minimum of $P(x, r_1)$ has been found, $r_2 < r_1$, is chosen, and $P(x, r_2)$ is minimized. In the limit, as k increases, the minimum of $P(x, r_k)$ and $f(x)$ will be the same. The method is called the Sequential Unconstrained Minimization Technique (SUMT).

The concept behind (28) is that if an initial point is chosen such that all $g_i(x) > 0$, then $1/g_i(x)$ will act as a boundary repulsion term as its value greatly increases as $g_i(x)$ approaches zero. In practice, it is necessary to ensure that a finite step taken in the minimization process does not cross the boundary thus missing the repulsion altogether. This is detected by a $g_i(x)$ term becoming negative.

The obtaining of an initial feasible point such that all $g_i(x) > 0$ can be quite difficult. Fiacco-McCormick use SUMT for this purpose. If the chosen initial point violates some constraints, it means that some g_i have negative values. One by one, these violated constraints can be maximized (by minimizing the negative of the constrained function) subject to the non-violation of the constraints that are currently satisfied. The violated constraint is placed in the expression for $f(x)$ in (28) and the satisfied constraints are placed in $1/g_i$. This process is continued until all constraints are satisfied.

Fiacco-McCormick proposed that the initial value of r_k be chosen from the relationship

$$r_1 = \frac{-\sum_j \frac{\partial}{\partial x_j} \cdot \frac{\partial e}{\partial x_j}}{\left| \frac{\partial e}{\partial x} \right|^2} \quad (29)$$

where

$$e = \sum_{i=1}^m 1/g_i(x)$$

provided r_1 is greater than zero. If a negative result is obtained, f is reduced by taking steps of given lengths in the direction of the negative gradient of f and evaluating r_1 at each step until a positive value is found.

Predictions of the final minimum point can be made on the basis of the two preceeding minimizations. If the prediction does not satisfy the convergence conditions imposed on the minimum point, a prediction of the minimum of the following process can be made and used as the starting point for that minimization.

By solving the corresponding dual problem, Fiacco-McCormick found that at the end of a minimization process the upper bound of the distance to the absolute minimum is

$$E(x, r_k) = r_k \sum_{i=1}^m 1/g_i(x) \quad (30)$$

As the values of $g_i(x)$ are not known at the start of iteration k , (30) can only be used as a means for detecting convergence of SUMT.

5.3.4 Method of Lootsma^{33, 34}

Lootsma searched for a transformation that would provide an upper bound of the distance to the absolute minimum at the end of each minimization process independent of x . Such a relation is

$$E(x, r_k) = d \quad (31)$$

A function corresponding to (31) is

$$L(x, r_k) = f(x) - r_k \sum_{i=1}^m \ln g_i(x) \quad (32)$$

provided that d is chosen as

$$d = m \cdot r_k \quad (33)$$

The value of d can be pre-specified and thus the optimal value of $f(x)$ can be approximated with a prescribed accuracy in one minimization process.

Transformation (32) is also of a boundary repulsion type and this requires an initial point which is feasible. The method can be accelerated by using extrapolation techniques.

5.3.5. Discussion on the Fiacco-McCormick and Lootsma methods

The author has found that the following points cause difficulties and concern when applying one or both of these methods:

1. The method for obtaining the initial value of r_k can break down or produce quite unsatisfactory values. If (29) produces a negative value and the recommended steps are taken it is quite possible that x might drift to a point outside the feasible regions, or produce values that are either too small or too big. If r_k is too big the constraints are weighed very heavily and the minimization will not work on reducing $f(x)$. The result is a convergence to a wrong solution. Too small an r_k causes the constraints to drift towards the boundaries specially in the case where $f(x)$ is smaller outside the feasible region. A large number of minimization steps are then necessary to locate the solution.
2. The acceleration and extrapolation techniques tend to fail, specially in large problems where the constraints are critical. This happens because these techniques often determine a point which is outside the feasible region.
3. The method for locating an initial feasible point is often time consuming as it can take more time than the actual solution of the problem.

The following comments deal with suggestions on the three points just raised:

1. An adequate choice of r_k is one that initially balances the two terms of (28) so that they have comparative weights during the minimization process. Kraynak³⁵ has proposed that after a number of minimization steps slightly greater than the number of variables, r_1 can be recalculated according to

$$r_1 = E / \max_i \left| \frac{\partial (1/g_i)}{\partial x} \right| \quad (34)$$

where E is the acceptable convergence tolerance for the gradient of the objective function.

2. If extrapolation techniques are found for a particular problem to lead to unfeasible points, they can be abandoned.
3. To obtain an initial feasible point for a particular problem use should be made of any special knowledge about the problem that is available. In many cases, an initial feasible point can be directly chosen. In others, it can be directly found by specifying a set of values b_i for the inequalities that lie within the constraint region. An initial point satisfying these values can then be obtained by minimizing.

$$F(x) = \sum_{i=1}^m (g_i(x) - b_i)^2, \quad b_i > 0 \quad (35)$$

This amounts to solving a set of simultaneous nonlinear equations, $g_i(x) = b_i$. It is not necessary to solve (35) completely as all that is required is that all $g_i(x) > 0$ and the process can be stopped as soon as this is fulfilled. Some precautions are necessary when applying (35) as there may not be any values of x that satisfy (35). If there are more constraints than variable, then an overdetermined situation exists and a least squares solution results. If there are less constraints than variables, then an underdetermined situation exists and some variables can be arbitrarily fixed. In all cases, a sensible choice of b_i will make the application of (35) worthwhile.

5.3.6 Inclusion of equality constraints

The use of (35) to obtain a starting point paves the way to other possibilities and interpretations. The inequality constraints, $g_i(x) \geq 0$ are treated in (35) as equality constraints, $h_i(x) = b_i$. Problem (1) with $m = 0$, $p > 0$ could be transformed into the minimization of

$$P(x) = f(x) + \sum_{i=1}^p (h_i(x) - b_i)^2 \quad (36)$$

The unconstrained minimization of (36) is sufficient to obtain the unconstrained minimum of $f(x)$, provided a state is not reached

in the minimization process where a variation in x to reduce the summation term is balanced by an equal increase in $f(x)$. Under those circumstances, the minimization process would find that $P(x)$ had been minimized at a point where the equality constraints are not satisfied. To go beyond this point, the balance has to be upset. This can be achieved by amplifying the summation term to:-

$$P(x, r_k) = f(x) + 1/r_k \sum_{i=1}^p (h_i(x) - b_i)^2 \quad (37)$$

It will be noticed that (36) and (37) are equal when $r_k = 1$. After minimizing $P(x, r_1)$, r can be changed to $r_2 < r_1$, and the minimization process repeated until the summation term is reduced to an acceptable level.

Equation (37) is very nearly in the form later proposed by Fiacco-McCormick³⁶ in a method that extends their original formulation to include equality constraints. They propose adding a term to (37) to take into account the inequality constraints.

$$P(x, r_k) = f(x) + r_k \sum_{i=1}^m 1/g_i(x) + 1/r_k \sum_{i=1}^p (h_i(x) - b_i)^2 \quad (38)$$

The great disadvantage of (38) is the requirement of an interior point for the inequality constraints.

5.3.7 Method of Zangwill³⁷

It would be desirable to modify (37) in a different way than (38) to remove the interior point requirement. This was actually done by Zangwill and later by Fiacco-McCormick. Zangwill proposes the following transformation

$$P(x, r_k) = f(x) + 1/r_k \sum_{i=1}^p (h_i(x) - b_i)^2 + 1/r_k \sum_{j=1}^m (g_j(x) - b_j)^2 \quad (39)$$

where

$$b_i = 0$$

$$b_j = g_j(x) \text{ if } g_j(x) \geq 0 \text{ (this term can be removed)}$$

$$b_j = 0 \text{ if } g_j(x) < 0$$

Equation (39) does not require an initial interior point. It does not work on the basis of a boundary repulsion but on an "outside" - "in" concept. The inequality constraints are not continuously present as those that are satisfied are immediately removed. The b_j definition is a step function. This can lead in some cases to convergence problems if some inequality constraints keep jumping back and forth across the boundary. In practice this seldom happens and the method behaves quite satisfactorily.

5.3.8 Method of Powell³⁸

The main disadvantage of all methods mentioned in this section so far is that they depend on successive values of the positive

parameter r_k being chosen such that

$$\lim_{\substack{k \rightarrow \infty \\ k}} r_k = 0 \quad (40)$$

or in the boundary repulsion concept that $g_i(x)$ tends to 0 as r_k is decreased if the constraints are critical. In both cases, there is a tendency to reach large numbers for $1/g_i(x)$ or for $1/r_k$ which makes the minimization process quite sensitive to changes in the variables that can cause many difficulties and a resulting large number of steps.

The above methods depend on the cancellation of the constraint terms from the various transformations so that in the end not only do the minimum of the original and transformed functions occur at the same point but their values are equal. Powell recognized that what is necessary is for the minimums to occur at the same point. If a transformation could be found in which relation (40) was not necessary, as long as the minimum occurs at the same point, it does not really matter if the functions are different. Powell found such a function as:-

$$F(x, r, s) = f(x) + \sum_{i=1}^p r_i (h_i(x) + s_i)^2 + \sum_{j=1}^n r_j (g_j(x) + s_j)^2 \quad (41)$$

where the constraints are defined as

$$h_i(x) = 0$$

$g_j(x) \geq 0$ (this term is present as $g_j(x) = 0$ only if the constraint is violated)

The procedure is to choose some initial values for the r and s variables and to vary s after the first minimization process in such a way that the next minimization process will tend to satisfy better the constraints. Such a relation is found to be

$$s_{k+1} = s_k + \text{constraint value} \quad (42)$$

In (42) the intention is that to the previous value of s is added the corresponding h_i or g_j constraint. This process is continued until its effect becomes small. For those h_i or g_j that are not converging fast enough their corresponding r term is increased and again attention is concentrated on the s term for further improvements. Powell recommends that whenever an r term is increased by a **certain factor**, its corresponding s factor is decreased by the same factor so that the product of s and r remains constant. This minimizes the disturbance caused by the increased r term. An interesting feature of the method is that an adequate control of the rate of convergence is obtained.

The method provides a greater liberty in the selection of the initial r factors as their effect is not as sensitive compared with the methods previously discussed. This is not to say that any value

can be selected. If values that are too large are chosen, convergence to a wrong result occurs; if too small, too many iterations are necessary. The difference is that the gap between small and big values is larger. The larger the initial value can be selected for r , the faster is the overall convergence.

Figure 5.1 is a flow diagram for the method. Only equality constraints are assumed for simplicity, although inequalities are treated in the same way, as explained above.

CHAPTER 6

NONLINEAR PROGRAMMING LOAD FLOW METHODS

6.1 General³⁹

The load flow problem involves the solution of a system of non-linear equations. For the solution to be unique, one equation is specified for each variable. In general, if the solution of the set

$$g_i(x) = b_i, \quad i = 1, n, \quad x = n\text{-dimensional vector} \quad (1)$$

is desired, the minimum of the following function coincides with the solution,

$$f(x) = \sum_{i=1}^n (g_i(x) - b_i)^2 = \sum_{i=1}^n G_i^2 \quad (2)$$

where

$$G_i = g_i(x) - b_i$$

As unconstrained minimization process can be used to reduce $f(x)$ to zero. In the investigation of this ~~thesis~~, gradient methods were used for the minimization. These methods require the partial derivatives of (2),

$$\frac{\partial f}{\partial x} = \sum_{i=1}^n 2 G_i \frac{\partial G_i}{\partial x} \quad (3)$$

6.2 Nodal Admittance Solution

The following equations (4), (5) and (6) present the G terms of (2) for a typical node j. These are the load flow equations expressed as voltage magnitude, real and reactive voltampere mismatches respectively.

$$G_{j1} = (E_j^2 + E_{n+j}^2)^{\frac{1}{2}} - V_j \quad (4)$$

$$G_{j2} = \left(\sum_{k=1}^n (D_{jk}(E_j E_k + E_{n+j} E_{n+k}) - B_{jk}(E_j E_{n+k} - E_{n+j} E_k)) \right) - P_j \quad (5)$$

$$G_{j3} = \left(\sum_{k=1}^n (-B_{jk}(E_j E_k + E_{n+j} E_{n+k}) - D_{jk}(E_j E_{n+k} - E_{n+j} E_k)) \right) - Q_j \quad (6)$$

where

n = number of nodes

D_{jk}, B_{jk} = nodal admittance matrix coefficients

E_j, E_{n+j} = complex voltage of node j

V_j, P_j, Q_j = desired voltage magnitude, real and reactive
voltamperes for node j

Two of the equations (4), (5) and (6) are selected for each node depending on whether the node is a load bus (equations (5) and (6)) or generator bus (equations (4) and (5)) and equation (2) is used.

The reference-slack node does not require an equation. This nodal admittance load flow function is then ~~minimized~~ using the complex voltages as variables, until it approaches zero with some specified degree of accuracy. Alternatively, each individual mismatch G can be checked until all are smaller than a prescribed amount.

The partial derivatives of each G term with respect to each voltage variable are needed in equation (3). These are given in the following equations (7) to (18), where $GG_{ji,k}$ stands for the partial derivative of G_{ji} with respect to voltage E_k ,

$$GG_{j1,k} = E_k / (E_k^2 + E_{n+k}^2)^{\frac{1}{2}}, j = k \quad (7)$$

$$GG_{j1,k} = 0, j \neq k \quad (8)$$

$$GG_{j1, n+k} = E_{n+k} / (E_k^2 + E_{n+k}^2)^{\frac{1}{2}}, j = k \quad (9)$$

$$GG_{j1, n+k} = 0, j \neq k \quad (10)$$

$$GG_{j2, k} = 2D_{jk}E_k + \sum_{\substack{i=1 \\ i \neq k}}^n (D_{ki}E_i - B_{ki}E_{n+i}), j = k \quad (11)$$

$$GG_{j2, k} = D_{jk}E_j + B_{jk}E_{n+j}, j \neq k \quad (12)$$

$$GG_{2, n+k} = 2D_{jk}E_{n+k} + \sum_{\substack{i=1 \\ i \neq k}}^n (D_{ki}E_{n+i} + B_{ki}E_i), j = k \quad (13)$$

$$GG_{2, n+k} = D_{jk}E_{n+j} - B_{jk}E_j, j \neq k \quad (14)$$

$$GG_{j\bar{j},k} = -2B_{jk}E_k - \sum_{\substack{i=1 \\ i \neq k}}^n (D_{ki}E_{n+i} + B_{ki}E_i) \quad , j = k \quad (15)$$

$$GG_{j\bar{j},k} = D_{jk}E_{n+j} - B_{jk}E_j \quad , j \neq k \quad (16)$$

$$GG_{j\bar{j},n+k} = -2B_{jk}E_{n+k} + \sum_{\substack{i=1 \\ i \neq k}}^n (D_{ki}E_i - B_{ki}E_{n+i}) \quad , j = k \quad (17)$$

$$GG_{j\bar{j},n+k} = -B_{jk}E_{n+j} - D_{jk}E_j \quad , j \neq k \quad (18)$$

6.3 Nodal Impedance Solution

The load flow equation (2) can also be constructed from the nodal impedance equations. The problem is in many respects the dual of the previous one if nodal currents are used as variables instead of nodal voltages. As the boundary conditions do not change, the situation does not become the exact dual.

Taking the (subscript r) terms corresponding to the reference voltage in the admittance equations to the opposite side of the equations, then

$$I'_j = I_j - Y_{jr}V_r \quad (19)$$

The voltage and voltampere equations become,

$$V_j = \sum_i Z_{ji}I'_i \quad (20)$$

$$Z_j = I_j' * V_j + (Y_{jr} V_r) * V_j \quad (21)$$

Equations (20) and (21) are the required equations to calculate the voltage magnitude and complex voltampere mismatches. The actual expressions for the mismatches and their derivatives are given below. The notation will be defined by the following relations, where n does not include the reference node:

$$I_j' = E_j + jE_{n+j} \quad (\text{these are the variables}) \quad (22)$$

$$V_j = V3_j + jV4_j \quad (\text{same as equation (20)}) \quad (23)$$

$$Z_{jk} = D_{jk} + jB_{jk} \quad (\text{impedance coefficients}) \quad (24)$$

$$(Y_{jr} V_r) * = A1_j + jA2_j \quad (25)$$

The mismatches are:

$$G_{ji} = (V3_j^2 + V4_j^2)^{\frac{1}{2}} - V_j \quad (26)$$

$$G_{j2} = (E_j + A1_j) V3_j + (E_{n+j} - A2_j) V4_j - P_j \quad (27)$$

$$G_{j3} = (E_j + A1_j) V4_j - (A2_j - E_{n+j}) V3_j + Q_j \quad (28)$$

where V_j , P_j and Q_j are the desired voltage magnitude, real and reactive voltamperes for node j .

By use of equations (26), (27) and (28), the load flow function

(2) is constructed by taking the appropriate two equations per node. The corresponding partial derivatives needed for equation (3) are the following equations (29) to (38) where $GG_{ji,k}$ stands for the partial derivative of G_{ji} with respect to current E_k ,

$$GG_{j1,k} = (D_{jk} V_{3j} + B_{jk} V_{4j}) / (V_{3j}^2 + V_{4j}^2)^{\frac{1}{2}} \quad (29)$$

$$GG_{j1, n+k} = (D_{jk} V_{4j} - B_{jk} V_{3j}) / (V_{3j}^2 + V_{4j}^2)^{\frac{1}{2}} \quad (30)$$

$$GG_{j2,k} = 2D_{jk} E_j + A1_j D_{jk} - A2_j B_{jk} + \sum_{i=1}^n (D_{ki} E_i - B_{ki} E_{n+i}) \quad , j = k \quad (31)$$

$$GG_{j2,k} = D_{jk} E_j + B_{jk} E_{n+j} + A1_j D_{jk} - A2_j B_{jk} \quad , j \neq k \quad (32)$$

$$GG_{j2, n+k} = 2D_{jk} E_{n+j} - A1_j B_{jk} - A2_j D_{jk} + \sum_{i=1}^n (D_{ki} E_{n+i} + B_{ki} E_i) \quad , j = k \quad (33)$$

$$GG_{j2, n+k} = D_{jk} E_{n+j} - B_{jk} E_j - A1_j B_{jk} - A2_j D_{jk} \quad , j \neq k \quad (34)$$

$$GG_{j3,k} = -2 B_{jk} E_j - A1_j B_{jk} - A2_j D_{jk} - \sum_{i=1}^n (D_{ki} E_{n+i} + B_{ki} E_i) \quad , j = k \quad (35)$$

$$GG_{j3,k} = - B_{jk} E_j + D_{jk} E_{n+j} - A1_j B_{jk} - A2_j D_{jk} \quad , j \neq k \quad (36)$$

$$GG_{j3, n+k} = - 2B_{jk} E_{n+j} - A1_j D_{jk} + A2_j B_{jk} + \sum_{i=1}^n (D_{ki} E_i - B_{ki} E_{n+i}) \quad , j = k \quad (37)$$

$$GG_{j3,n+k} = -B_{jk}E_{n+j} - D_{jk}E_j - A1_j D_{jk} - A2_j B_{jk}, j \neq k \quad (38)$$

6.4 Performance of Methods

Computer programs were written by the author for both the nodal admittance and impedance formulations of the load flow problem (for listings of the programs refer to Power System Report No. 82, Imperial College). Systems solved include the IEEE test systems and other systems whose responses with some of the standard methods were known. Figure 6.1 presents solution characteristics on the IEEE systems. All cases were started from a flat voltage profile of $1 + j0$ and taken to a high solution precision to study the complete response of the present approaches. The Fletcher-Powell method was used in all cases discussed because it was found to be much more powerful than other methods. Preliminary tests showed that Fletcher-Powell was about 10 times faster than the conjugate gradient (Fletcher-Reeves) method and about 100 times faster than steepest descents in minimizing the 14 bus IEEE system load flow function. A recent publication by Wallach⁴⁰ favours the conjugate gradient method over others, but it seems that he did not experiment with Fletcher-Powell.

The following observations were made:

- i. Convergence is fast for the first few Fletcher-Powell steps, but it has a slower middle section and a very steep final section.

ii. The impedance method exhibits a much faster convergence characteristic than the admittance method. This is accounted for by the greater coupling between the variables in the impedance method, due to the fullness of the impedance matrix, contrasted with the loose coupling in the admittance method, caused by the sparsity of the admittance matrix.

iii. The load flow function is the sum of the squares of the mismatches. The individual mismatches do not all decrease at the same rate.

iv. The nonlinear programming convergence characteristic is totally different from normal iterative approaches, where a high voltage precision is necessary to obtain acceptable mismatches. It was observed that mismatches decreased at about the same rate as the precision of individual voltages. The present method varies voltages in order to reduce mismatches, whereas the normal iterative methods correct each individual voltage assuming that all others are already correct and without any control on the mismatches.

Further tests on the IEEE systems and other systems have revealed other characteristics of the nonlinear programming approach.

- i. The starting value for voltages can have an important effect on the rate of convergence.
- ii. The position of the reference bus has a slight effect on convergence for the admittance approach. The same is not true for the impedance method. A real system, which had diverged with normal nodal admittance iterative methods, was selected to study the response to different reference-slack bus positions, using both nonlinear programming methods. The admittance approach produced results ranging from 32 steps to 43 steps for different reference bus positions while the impedance approach produced results that ranged from 25 steps to more than 150 steps.
- iii. Convergence tests were also made by running many cases on a single system, varying its parameters and conditions from one case to the next. Loadings were increased, negative line reactances were introduced, the ratio of long to short line impedances for lines terminating on the same bus was increased and generators were connected through very low impedance lines. Convergence was obtained in many cases without difficulties. In other cases, the minimization proceeded until a minimum value, different from zero, was obtained for the load flow function. This means that there was no solution to the set of nonlinear equations that described the load flow problem. This in itself

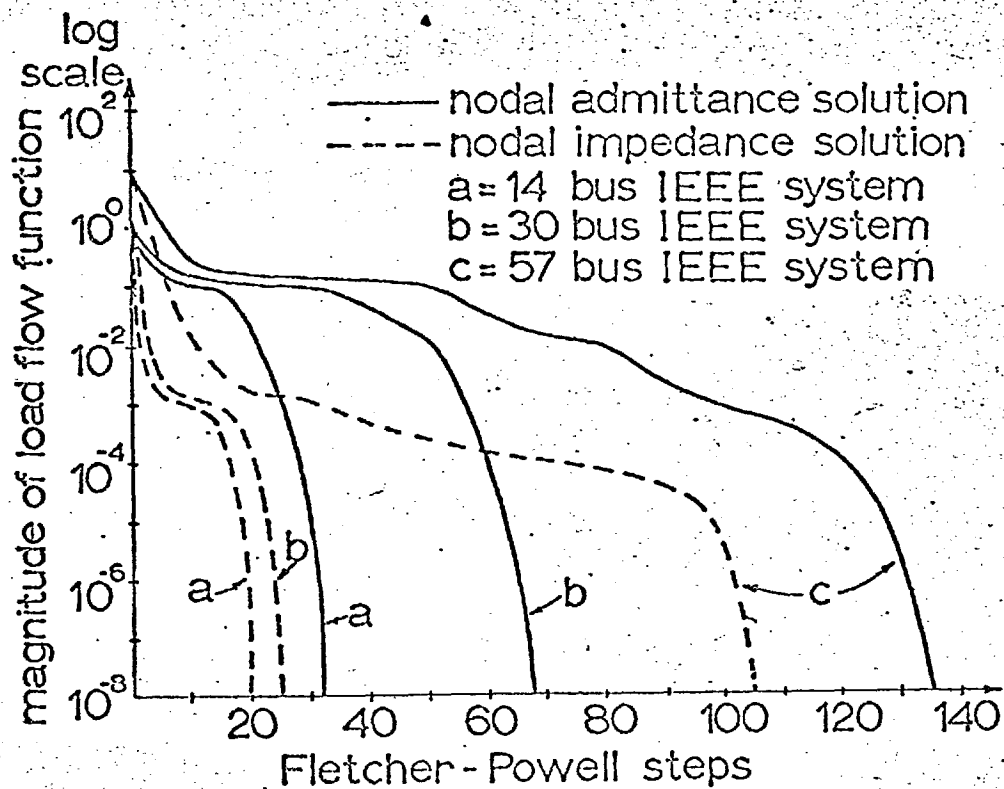


Fig. 6.1 Convergence comparison of admittance and impedance solutions of the load flow problem

is a very useful result as it indicates a situation with no solution. When conventional techniques fail to converge, there is no indication as to whether divergence has been due to the method or to the nonexistence of a solution. Conditions were imposed on the system which placed it very near to the limit of existence of a solution, the minimization process deteriorated significantly but managed to obtain a solution.

iv. Another property of the nonlinear programming method is that it does not require any acceleration factor. It is known that nodal admittance iterative methods are extremely sensitive to the choice of such a factor, poor choices giving divergent results. The necessity of the use of such factors are to compensate for inherent errors in the calculation of corrections. The present method provides corrections which always produce better values for the variables. This is evidenced by the negative slopes of the curves in fig. 6.1.

6.5 Assessment of the nonlinear programming approach

From the results and observations already made, it appears that the nonlinear programming approach to the load flow problem has some desirable characteristics. Initial convergence is fast. This is shown by the steep first part of all curves of fig. 6.1 and is specially true for the impedance approach. Very high degrees of accuracy do not demand an inordinate number of iterations, as shown

by the very steep final part of the curves of fig. 6.1. In spite of these favourable characteristics, fig. 6.1 shows that the number of steps required increases at a rate similar to the increase in system size. The time per iteration depends highly on system size and is greater for the impedance approach than in the admittance formulation. This difference is caused by the sparseness of the admittance Jacobian, while the impedance Jacobian is a full matrix. System size also becomes a limiting factor for storage since a full Hessian has to be calculated and stored at each Fletcher-Powell step.

The general conclusion that can be drawn is that the nonlinear programming approach is seriously limited by size. Tearing techniques can be employed to extend the application of the approach for larger systems. These are presented in Chapter 8.

CHAPTER 7

OPTIMAL LOAD FLOWS

7.1 General

The load flow problem, as defined in Chapter 6, consists of a set of simultaneous nonlinear equations with as many variables as there are equations. Within the region of interest, only one solution can be found. This formulation demands that certain conditions be imposed on the solution, actually one condition per equation, such as the values of the voltage magnitude or real or reactive voltamperes at different nodes. These conditions act as constraints on the possible values of the variables of the problem, usually the complex voltages at nodes. In fact, in the load flow problem, normal conditions constrain the voltages to only one combination. The knowledge of the uniqueness of a solution simplifies the problem and has made possible the development of a great number of methods which are only capable of finding that solution.

In the practical case, such stringent conditions are not desired. For example, the voltage magnitude of a certain generator node may have a range of acceptable values. Mathematically, conditions such as this one can be expressed as an inequality constraint, as opposed to equality constraints used generally. A problem may contain both equality and inequality constraints. Formulated in this way, the load flow problem has a range of possible solutions any one of

which may be acceptable. This is the basis for an optimal load flow solution. Now, a choice can be made from the set of possible solutions to locate the one solution which optimizes one aspect of the operation or design of the system, such as, the most economic solution or the one that produces minimum transmission losses. Here, the methods presented in Chapter 5 can be used to locate the optimal solution. These methods have already been used for other problems such as the optimization of a system under emergency conditions⁴¹.

7.2 The minimum loss problem

The minimum loss problem has been solved in various ways. The first approach⁴² was to solve the economic dispatching problem minimizing transmission losses at the same time. This ignored the possibility of minimizing losses by an optimal use of the reactive capabilities of the system as a whole. It is in this second point of view that the minimum loss problem will be considered. One of the first attempts⁴³ was to minimize losses by varying one voltage variable at a time in an iterative cycle and performing several load flow computations. Bauman⁴⁴ derived an approximate minimum loss equation which replaces one of the equations of the load flow problem. The load flow thus computed will approximate to the minimum loss situation. Zollenkopf⁴⁵ makes use of variable transformer taps to minimize losses.

In this chapter, two different approaches are discussed in greater detail; the nonlinear programming approach³⁹ and the hybrid classical and nonlinear programming approach given by Donnell and Tinney⁴⁶. The problem is as follows:

minimize transmission losses,

$$f = \sum_{j=1}^n (\text{Re} (V_j * (Y_{j1} V_1 + \dots + Y_{jn} V_n))) \quad (1)$$

subject to constraints on the voltage magnitude, real and reactive voltamperes of:-

$$|V|_{\text{lower}} \leq |V| \leq |V|_{\text{upper}} \quad (2)$$

$$P_{\text{lower}} \leq P \leq P_{\text{upper}} \quad (3)$$

$$Q_{\text{lower}} \leq Q \leq Q_{\text{upper}} \quad (4)$$

7.3 The economic dispatching problem

There are several approaches to the economic dispatching problem that take into consideration transmission losses. The B-constants method⁴² was the first widely used approach. With this method, several assumptions and approximations are made to obtain the B-constants equation. The B constants thus computed are assumed to be valid for a range of system loading conditions. Brownlee⁴² introduced a method which assumed constant X/R ratio for system impedances. Squires⁴⁷ produced a method based on the nonvariation of the inphase component of generator voltages which involved the

solution of the set of non-linear equations of the Lagrangian minimum point conditions. Carpentier⁴⁸ was the first to make a general and exact formulation of the dispatching problem. His solution involved the application of the Kuhn and Tucker theorem. Recently, Dopazo, Klitin, Stagg and Watson⁴⁹ have presented a method based on an iterative process which includes the use of the Lagrangian method, a gradient method and a load flow method.

The nonlinear programming method³⁹ and the method of Dorniel and Tinney⁴⁶ mentioned in the previous section in relation to the minimum loss problem are both applicable to the economic dispatching problem when defined as follows:

minimize production cost

$$f(P(V)) = \sum_{j=1}^m C_j(P_j(V)) \quad (5)$$

subject to constraints (2), (3) and (4)

when

C_j = generation costs of each of m generating stations.

7.4 Application of nonlinear programming methods

7.4.1 Numerical results

The methods of Fiacco-McCormick, Lootsma, Zangwill and Powell

were used by the author to solve the minimum loss and economic dispatching problems. In all cases the actual minimization process was performed by the Fletcher-Powell method. The Fiacco-McCormick and Lootsma methods require an initial feasible point. Although the Zangwill and Powell methods do not require a feasible starting point, it has been found by the author that for the power system applications considered it is advantageous to start from a nearly feasible point as this accelerates convergence. The simplest way to locate a feasible point is to use a load flow method. In the examples that will be presented, the Fletcher-Powell method was used as described in Chapter 2. This corresponds with the recommendation made in Chapter 1 in connection with equation (35).

The use of the Fletcher-Powell method for an initial load flow does not complicate programming. Firstly, the Fletcher-Powell method can be written as a general subroutine (see Power System Report No. 81, Imperial College) that can be used equally well by the load flow and the optimization sections of the program. Secondly, the squared terms which make up the load flow function come in during the optimization process. Thus, all that is required of the optimization program, to act as a load flow program, is some trivial organizational statements.

The results in Tables, 7.1, 7.2 and 7.3 were obtained with computations using all four methods mentioned. These three cases were considered without equality constraints for the benefit of the Fiacco-McCormick and Lootsma methods and correspond to the IEEE 14 bus

system. At the initial load flow solution point transmission losses were 0.1329 p.u. Table 7.1 shows that a reduction of 14.1% was achieved by optimizing losses within the limits imposed. It should be noted that this reduction is partly accomplished by the fact that the system moved to the upper load limits corresponding to a smaller total load. In Table 7.2, generator 2 was given a wider range, while all other limits remained intact. The results produced a reduction of 30.1% from the initial point losses. Compared with Table 7.1, a 16% reduction in losses was achieved by allowing for a different distribution of generation.

For the economic dispatching case in Table 7.3, the following generation cost straight line equations were assumed:

$$C_1 = 1.5 + 1.1P_1$$

$$C_2 = 2.1 + 0.6P_2$$

These tables also show a marked tendency towards a flat voltage profile which is to be expected as this minimizes losses.

Tables 7.4 and 7.5 were obtained with Powell's method, using equality constraints on the same 14 bus IEEE system as the previous tables.

Table 7.1 Minimum Loss Solution for IEEE 14-bus System with Close Limits on Power Generation of bus 2

bus	voltages	V lower	V	V upper	P lower	P	P upper	Q lower	Q	Q upper
1	1.0621+j0.0	1.058	1.0621	1.062	1.6	2.1301	3.0	-0.5	-0.3034	0.5
2	1.0501-j0.0847	1.0	1.0535	1.1	0.17	0.1898	0.19	0.0	0.2833	0.8
3	1.0006-j0.2112	1.0	1.0227	1.1	-0.95	-0.9300	-0.93	-0.4	0.0418	0.4
4	1.0198-j0.1702	1.0	1.0339	1.1	-0.49	-0.4599	-0.46	0.037	0.0372	0.04
5	1.0250-j0.1450	1.0	1.0352	1.1	-0.078	-0.0740	-0.074	-0.02	-0.0200	-0.012
6	1.0684-j0.2504	1.0	1.0973	1.1	-0.12	-0.1007	-0.1	-0.3	0.1166	0.4
7	1.0562-j0.2251	1.0	1.0800	1.1	-0.02	0.0195	0.02	-0.01	0.0062	0.01
8	1.0746-j0.2257	1.0	1.0980	1.1	-0.02	0.0199	0.02	-0.2	0.1124	0.4
9	1.0506-j0.2523	1.0	1.0804	1.1	-0.31	-0.2804	-0.28	-0.18	-0.1416	-0.14
10	1.0475-j0.2545	1.0	1.0779	1.1	-0.11	-0.0701	-0.07	-0.07	-0.0417	-0.04
11	1.0547-j0.2532	1.0	1.0847	1.1	-0.045	-0.0255	-0.025	-0.02	-0.0172	-0.016
12	1.0517-j0.2597	1.0	1.0833	1.1	-0.07	-0.0544	-0.05	-0.018	-0.0173	-0.014
13	1.0472-j0.1593	1.0	1.0788	1.1	-0.14	-0.1200	-0.12	-0.06	-0.0597	-0.056
14	1.0299-j0.2661	1.0	1.0637	1.1	-0.16	-0.1301	-0.13	-0.06	-0.0491	-0.04

transmission losses = 0.1143 p.u.

System for Wide Limits* on Power Generation of
bus 2.

bus	voltages	V	P	Q
1	1.0621+j0.0	1.0621	1.6000	-0.4155
2	1.0630-j0.0619	1.0648	0.6919	0.3493
3	1.0154-j0.1905	1.0331	-0.9300	0.0474
4	1.0309-j0.1508	1.0418	-0.4600	0.0370
5	1.0348-j0.1271	1.0425	-0.0740	-0.0128
6	1.0758-j0.2284	1.0998	-0.1000	0.0877
7	1.0655-j0.2042	1.0848	0.0200	0.0082
8	1.0810-j0.2039	1.1000	0.0200	0.0949
9	1.0603-j0.2314	1.0853	-0.2800	-0.1421
10	1.0568-j0.2335	1.0823	-0.0700	-0.0425
11	1.0630-j0.2317	1.0879	-0.0250	-0.0191
12	1.0600-j0.2376	1.0863	-0.0500	-0.0177
13	1.0553-j0.2377	1.0818	-0.1200	-0.0598
14	1.0395-j0.2455	1.0681	-0.1300	-0.0467

transmission losses = 0.0929 p.u.

* |V|, P and Q limits are the same as for Table 7.1, with the only exception of bus 2, whose P limits have been changed to 0.1 and 1.5 respectively.

14-bus System*.

bus	voltages	V	P	Q
1	1.0618+j0.0	1.0618	0.7735	-0.1608
2	1.0623-j0.0192	1.0625	1.5000	0.0000
3	1.0227-j0.1554	1.0344	-0.9300	0.0799
4	1.0340-j0.1183	1.0408	-0.4500	0.0370
5	1.0368-j0.0974	1.0414	-0.0740	-0.0194
6	1.0822-j0.1970	1.1000	-0.1000	0.1038
7	1.0705-j0.1711	1.0841	0.0200	0.0085
8	1.0866-j0.1704	1.0998	0.0200	0.0985
9	1.0659-j0.1987	1.0842	-0.2800	-0.1400
10	1.0625-j0.2011	1.0814	-0.0700	-0.0442
11	1.0690-j0.1998	1.0875	-0.0250	-0.0198
12	1.0665-j0.2065	1.0863	-0.0500	-0.0167
13	1.0616-j0.2064	1.0814	-0.1200	-0.0596
14	1.0441-j0.2128	1.0655	-0.1300	-0.0597

cost of production = 5.3509

* |V|, P, Q limits are the same as for Table 7.2 with the exceptions of busses 1 and 2, whose lower P limits were changed to 0.1 and 1.2 respectively.

Table 7.4 Minimum Loss Solution for IEEE 14-bus system including Equality Constraints

bus	voltages	$ V $	M*		M*		M*
1	1.0580+j0.0	1.0581	0	2.4731	1	-0.2264	1
2	1.040-j0.097	1.0442	1	0.1900	1	0.2994	1
3	0.986-j0.235	1.0135	1	-0.9500	0	0.0877	1
4	1.000-j0.196	1.0189	1	-0.4900	0	0.0370	0
5	1.007-j0.168	1.0209	1	-0.0780	0	-0.0200	0
6	1.041-j0.302	1.0838	1	-0.1200	0	0.1641	1
7	1.030-j0.275	1.0663	1	-0.0200	0	-0.0100	0
8	1.062-j0.287	1.1000	1	-0.0200	0	0.2099	1
9	1.016-j0.504	1.0603	1	-0.3100	0	-0.1800	0
10	1.010-j0.307	1.0556	1	-0.1100	0	-0.0700	0
11	1.020-j0.306	1.0654	1	-0.0450	0	-0.0200	0
12	1.020-j0.313	1.0669	1	-0.0700	0	-0.0180	0
13	1.015-j0.312	1.0618	1	-0.1400	0	-0.0600	0
14	0.991-j0.320	1.0108	1	-0.1600	0	-0.0600	0

* Limits as on Table 7.1

M = 1 inequality constraints

M = 0 equality constraint corresponding to lower limits on Table 7.1

Transmission losses = 0.1507

Table 7.5 Economic Dispatching of IEEE 14-bus System Including Equality Constraints

bus	voltages	$ V $	M*	P	M*	Q	M*
1	1.058+j0.0	1.0580	0	2.4738	1	-0.1889	1
2	1.038-j0.096	1.0423	1	0.1899	1	0.2489	1
3	0.986-j0.235	1.0137	1	-0.9500	0	0.1079	1
4	0.999-j0.196	1.0176	1	-0.4900	0	0.0370	0
5	1.006-j0.301	1.0196	1	-0.0780	0	-0.0200	0
6	1.037-j0.301	1.0808	1	-0.1199	0	0.1535	1
7	1.029-j0.276	1.0652	1	-0.0200	0	-0.0100	0
8	1.061-j0.287	1.0998	1	-0.0199	0	0.2162	1
9	1.014-j0.304	1.0587	1	-0.3100	0	-0.1800	0
10	1.008-j0.307	1.0537	1	-0.1100	0	-0.0700	0
11	1.018-j0.306	1.0630	1	-0.0450	0	-0.0200	0
12	1.017-j0.313	1.0640	1	-0.0700	0	-0.0180	0
13	1.012-j0.312	1.0590	1	-0.1400	0	-0.0600	0
14	0.988-j0.320	1.0386	1	-0.1600	0	-0.0600	0

* Limits as on Table 7.1

M = 1 inequality constraints

M = 0 equality constraints corresponding to lower limits on Table 7.1

Cost of production = 6.4351 p.u.

Table 7.6 presents a minimum loss calculation on the 30-bus IEEE system. Results show a reduction of losses by 27% with respect to the initial point.

Table 7.7 indicates the final values of the r_i and s_i terms of equation (41) Chapter 5. An initial value of 20 was given to all r_i and of 0.0 to all s terms. These results confirm Powell's statement as to the capability of the method of achieving convergence by varying s_i and not increasing r_i greatly.

Table 7.8 summarizes the performance of the method in the same case already described in Tables 7.6 and 7.7. Iteration 0 corresponds to the ordinary load flow initial point calculation. A maximum number of 80 Fletcher-Powell steps was allowed during each separate minimization process. Results show that after the first iteration a point was produced which was already very close to the correct solution. In fact, Table 7.8 shows that a practical solution has been obtained by the third iteration.

At the start of each iteration the Hessian matrix has always been returned to unity in all the above examples. It was recently detected that a much better convergence characteristic is obtained if, at the start of each iteration, the Hessian left over from the previous iteration is used. For the first iteration a unit approximation to the Hessian is still used. Table 7.9 presents the type

of convergence obtained for a dispatching problem on the IEEE 30-bus system. There were both equality and inequality constraints present. A reduction in the number of iterations is obtained compared to the case of Table 7.8. More important, however, is the great reduction in Fletcher-Powell steps per iteration. This implies a considerable reduction in overall computation time. The case of Table 7.9 took less than 2 minutes in an IBM 7094 computer.

Table 7.6 Minimum Loss Solution for IEEE 30-bus System

voltages		V	V	V	P	P	P	Q	Q	Q	bus
real	reactive	lower		upper	lower		upper	lower		upper	
1.0625	-0.	1.058	1.0620	1.062	0.10	2.1985	3.50	-0.300	-0.2999	0.100	1
1.04858	-0.08604	0.9	1.0521	1.1	0.17	0.1900	0.19	0.100	0.2872	0.500	2
1.03136	-0.12074	0.9	1.0384	1.1	-0.03	-0.0100	-0.01	-0.015	-0.0100	-0.010	3
1.02210	-0.14480	0.9	1.0323	1.1	-0.09	-0.0600	-0.06	-0.018	-0.0149	-0.014	4
0.99764	-0.22611	0.9	1.0229	1.1	-0.95	-0.9300	-0.93	0.100	0.1389	0.500	5
1.01748	-0.17158	0.9	1.0318	1.1	-0.01	0.0100	0.01	-0.01	-0.0047	0.100	6
1.00132	-0.20107	0.9	1.0213	1.1	-0.24	-0.2100	-0.21	-0.12	-0.1012	-0.100	7
1.01479	-0.18312	0.9	1.0312	1.1	-0.31	-0.2899	-0.29	-0.100	0.0402	0.300	8
1.05468	-0.22164	0.9	1.0777	1.1	-0.01	0.0101	0.01	-0.010	0.0099	0.010	9
1.04896	-0.24437	0.9	1.0771	1.1	-0.07	-0.0400	-0.04	-0.030	-0.0100	-0.010	10
1.07690	-0.22435	0.9	1.1000	1.1	-0.01	0.0099	0.01	0.000	0.1180	0.400	11
1.06036	-0.23945	0.9	1.0871	1.1	-0.12	-0.1000	-0.10	-0.090	-0.0600	-0.060	12
1.07327	-0.24105	0.9	1.1000	1.1	-0.01	0.0100	0.01	-0.100	0.0108	0.400	13
1.04653	-0.24859	0.9	1.0757	1.1	-0.07	-0.0500	-0.05	-0.018	-0.0140	-0.014	14
1.04348	-0.24873	0.9	1.0727	1.1	-0.09	-0.0700	-0.07	-0.040	-0.0101	-0.010	15
1.04942	-0.24338	0.9	1.0773	1.1	-0.05	-0.0199	-0.02	-0.020	-0.0161	-0.016	16
1.04398	-0.24625	0.9	1.0726	1.1	-0.10	-0.0801	-0.08	-0.070	-0.0500	-0.050	17
1.03427	-0.25332	0.9	1.0648	1.1	-0.04	-0.0199	-0.02	-0.011	-0.0073	-0.007	18
1.03129	-0.25459	0.9	1.0623	1.1	-0.11	-0.0800	-0.08	-0.036	-0.0328	-0.032	19
1.03522	-0.25242	0.9	1.0656	1.1	-0.03	-0.0100	-0.01	-0.009	-0.0051	-0.005	20
1.03769	-0.24807	0.9	1.0669	1.1	-0.19	-0.1600	-0.16	-0.115	-0.1102	-0.110	21
1.03869	-0.24787	0.9	1.0679	1.1	-0.01	0.0100	0.01	-0.010	0.0097	0.010	22
1.03507	-0.25004	0.9	1.0648	1.1	-0.04	-0.0200	-0.02	-0.020	-0.0140	-0.014	23
1.03051	-0.25001	0.9	1.0604	1.1	-0.10	-0.0700	-0.07	-0.070	-0.0650	-0.065	24
1.03195	-0.24009	0.9	1.0595	1.1	-0.01	-0.0100	0.01	-0.010	0.0101	0.010	25
1.02024	-0.24031	0.9	1.0482	1.1	-0.05	-0.0200	-0.02	-0.030	-0.0179	-0.018	26
1.03543	-0.23336	0.9	1.0614	1.1	-0.01	0.0100	0.01	-0.010	0.0100	0.010	27
1.01566	-0.17957	0.9	1.0314	1.1	-0.01	0.0100	0.01	-0.010	0.0096	0.010	28
1.01846	-0.24489	0.9	1.0475	1.1	-0.04	-0.0100	-0.01	-0.011	-0.0059	-0.006	29
1.00533	-0.25572	0.9	1.0373	1.1	-0.12	-0.0900	-0.09	-0.022	-0.0160	-0.016	30

Minimum Loss = 0.12884 p.u.

Table 7.7 Final r_i and s_i values⁺ in Powell's Method for example of Table 7.6

bus	V upper		P upper		Q upper	
	r_i^*	$s_i \times 10^{-4}$	r_i^*	$s_i \times 10^{-4}$	r_i^*	$s_i \times 10^{-4}$
1	2	-2.5	0	0	0	0
2	0	0	2	-0.53	0	0
3	0	0	2	-0.91	1	-0.346
4	0	0	2	-0.57	0	0
5	0	0	2	-0.91	0	0
6	0	0	2	-1.00	0	0
7	0	0	2	-0.93	0	0
8	0	0	2	-1.30	0	0
9	0	0	2	-0.83	2	-4.720
10	0	0	3	-0.15	2	-0.796
11	1	-3.0	3	-1.20	0	0
12	0	0	2	-1.02	1	-3.530
13	1	-2.7	2	-0.76	0	0
14	0	0	2	-0.88	1	-0.209
15	0	0	2	-1.37	2	-1.900
16	0	0	2	-0.91	2	-0.006
17	0	0	2	-1.34	1	-1.470
18	0	0	2	-1.40	1	0
19	0	0	3	-0.07	1	0
20	0	0	2	-1.85	2	0
21	0	0	2	-2.06	2	-0.847
22	0	0	2	-2.25	2	-0.162
23	0	0	3	-0.20	2	-0.295
24	0	0	2	-1.65	2	-0.706
25	0	0	2	-1.35	2	-0.683
26	0	0	2	-1.32	2	-1.370
27	0	0	2	-1.30	2	-0.840
28	0	0	2	-1.19	0	0
29	0	0	2	-1.12	1	-2.870
30	0	0	2	-1.25	1	-3.360

+ Lower |V|, P and Q constraints were never violated and so their corresponding r_i and s_i remained at their initial values of 20 and 0.0 respectively

* The numbers in this column indicate the number of times that each r_i was multiplied by 10. Initial $r_i = 20$.

Table 7.8 Convergence characteristic of Powell's Method

Iteration	Maximum Mismatch	Losses p.u.	Fletcher- Powell steps
0		0.176453	69
1	0.00591	0.119630	80
2	0.00596	0.119803	23
3	0.00142	0.127880	80
4	0.00147	0.127893	4
5	0.00083	0.128770	49
6	0.00050	0.128812	55
7	0.00047	0.128810	3
8	0.00111	0.128848	25
9	0.00046	0.128845	8
10	0.00030	0.128845	24
11	0.00030	0.128846	2
12	0.00013	0.128839	26
13	0.00011	0.128839	16
14	0.00009	0.128841	11

Table 7.9 Convergence Characteristic for IEEE 30-bus Example
Using Powell's Method Without Returning Hessian to Unity

Iteration	Maximum Mismatch	Cost of production	Fletcher- Powell steps
0		7.72773	69
1	0.01601	5.93475	72
2	0.00907	6.06781	15
3	0.00141	6.24745	39
4	0.00062	6.24787	7
5	0.00036	6.24787	4
6	0.00116	6.24742	9
7	0.00012	6.24772	7
8	0.00012	6.24772	1
9	0.00005	6.24788	6
10	0.00005	6.24788	1

7.4.2 Evaluation of minimization methods

The Fiacco-McCormick, Lootsma and Zangwill methods have been used on the constrained load flow, minimum loss and economic dispatching problems described in previous sections. The Fletcher-Powell method was used in all cases to perform the actual minimization.

The Fiacco-McCormick original method produced satisfactory results in many cases. One of the problems encountered was in applying their recommended method for evaluating the initial value of the parameter r . Negative values were obtained for several unconstrained steps to minimize $f(x)$ and a constraint was then violated before a positive r value had been found. Under these conditions the method breaks down. As already mentioned, their acceleration scheme invariably violated constraints and could not be implemented. It was finally decided to choose r such that the product of r and the summation of the inverse of the constraints was initially about 10% of the value of $f(x)$. No acceleration was included in the process.

The Lootsma method in general converged faster than the Fiacco-McCormick method. With this method r was chosen so that a certain upper bound in the error of the minimization process is guaranteed.

However, the greatest limitation of both the Fiacco-McCormick and Lootsma methods proved to be the requirement to work always from within the constraint space. Many functional constraints have very sharp peaks for $1/g_i(x)$. These peaks can be completely missed by the minimization process which works on the basis of finite steps. They are only detected by the change of sign as the constraint $g_i(x) \geq 0$ is violated. When this occurs, a smaller step in the minimization process must be taken. For problems with hundreds of constraints, many of which are very critical, several trials are necessary at each point to find a step size that does not violate constraints. Computation wise, this can be a very lengthy process. The author is not aware of any way of avoiding this situation.

With Zangwill's method, the difficulties encountered with the Fiacco-McCormick methods disappear. As the method works from both the interior and the exterior of the constraint space, there is no concern for violated constraints during the minimization process. Computation time is reduced considerably by the fact that all constraints are satisfied at any one time are removed from the calculations and are not considered again until they are violated. The most sensitive parameter in Zangwill's method is the selection of the r factor. If a value that is too small is selected to start the process, the method may converge prematurely without completely terminating the minimization of the desired function. Values

that are too small cause slow convergence. The Fiacco-McCormick and Lootsma methods suffer from this fault too.

The essence of Powell's method is to eliminate the sensitiveness of the other methods with regard to the r factor, while retaining all the advantages of Zangwill's method. The results of Section 7.4.1 show evidence that complete control is retained in approaching the minimum, even when many constraints become effective, without increasing the r factor to very high values. In fact, in Powell's method, as evidenced from Table 7.7, the majority of r values lie between 20 and 2000 which compares with values of between 10^6 and 10^{10} in some cases with the other three methods. The conclusion is drawn that Powell's method together with the Fletcher-Powell minimization constitute the most powerful and reliable solution to the optimal load flow problem found by the author to date.

7.5 Method of Dommel and Tinney⁴⁶

This method combines the classical Lagrangian approach with a gradient minimization scheme and has been found suitable for application to large systems. The problem will be reformulated for greater clarity:-

minimize $f(x, u)$, with u as independent variables
 subject to equality constraints $g(x, u) = 0$ and
 inequality constraints $h(x, u) \leq 0$, there being an
 equal number of x variables and equality constraints.

7.5.1 Solution without inequality constraints

The solution is based on the Lagrangian conditions:

$$g(x, u) = 0 \quad (6)$$

$$\frac{\partial f}{\partial x} + \left(\frac{\partial g}{\partial x} \right)^T (\lambda) = 0 \quad (7)$$

$$\frac{\partial f}{\partial u} + \left(\frac{\partial g}{\partial u} \right)^T (\lambda) = 0 \quad (8)$$

The method solves (6) by starting with an assumed set of u variables and then evaluating the partial derivatives of g with respect to x at the solution point. Eq. (7) is then used to evaluate λ . Equations (6) and (7) are now satisfied but not (8). Equation (8) is really the gradient vector to f , ∇f . At the optimal solution, the ∇f must be reduced to zero, which is what equation (8) implies. This is accomplished by choosing a new set of u variables, such that

$$u_{\text{new}} = u_{\text{old}} - C \cdot \nabla f \quad (9)$$

where C is a real factor.

With this new vector u , the whole process is repeated until (8) is satisfied with some degree of precision.

Newton's method with optimal ordering taking sparsity into account is used to solve (6). The solution by this method yields, in factorial form, the Jacobian matrix which is computationally equivalent to the transposed inverse of itself. The transposed inverse of the Jacobian is needed for the direct solution of (7) for the vector λ .

7.5.2 Solution including inequality constraints

To include the effect of inequality constraints, two types of constraints are considered, parameter and functional constraints.

Parameter constraints are constraints in the values of the variable u ,

$$u_{\min} \leq u \leq u_{\max} \quad (10)$$

These are considered in relation to equation (9). If by adding $(-C \cdot \nabla f)$ to u_{old} , one of the limits of (10) is violated, the limit value is taken as u_{new} . It can be easily shown that this approach to parameter inequality constraints corresponds to the exclusion equations of the Kuhn and Tucker method, as described in equation (27), Chapter 5.

Functional constraints, $h(x, u) \leq 0$ are more difficult to handle. The recommended approach is to penalize the f function,

$$f \text{ with penalties} = f(x, u) + \sum_j r_j (h_j(x, u))^2 \quad (11)$$

where a particular penalty term is added only if the constraint is violated.

The critical part of the algorithm is the step described by equation (9). Dommel and Tinney make a tentative recommendation which has worked successfully on a number of problems.

- 1) use (9) with the factor C chosen so that the minimum along the ∇f line is located.
- 2) whenever a gradient component changes sign from iteration $(n-1)$ to (n) , its parameter is assumed to be close to the solution and C is chosen as

$$C_i = \left(\frac{\partial^2 f}{\partial u_i^2} \right)^{-1} \quad (12)$$

where,

$$\frac{\partial^2 f}{\partial u_i^2} = \frac{\left(\frac{\partial f}{\partial u_i} \right)^{(n-1)} - \left(\frac{\partial f}{\partial u_i} \right)^{(n)}}{u_i^{(n-1)} - u_i^{(n)}} \quad (13)$$

El Abiad and Jaines⁵⁰ have proposed a different method of obtaining the u variables than equation (9), but their method has not yet been tried on large systems.

7.5.3 Power system applications

Donnell and Tinney consider the load flow equations are expressed in polar form:

$$|V_k| e^{-j\theta_k} \sum_{n=1}^n (D_{kn} + jB_{kn}) |V_n| e^{j\theta_n} = P_k - jQ_k \quad (14)$$

for each node k. The simultaneous mismatch equations to be solved are the real and reactive equalities of (14) for each load bus and the real equality of (14) for each generator bus. This set of equations are the equality constraints $g(x, u) = 0$, i.e. equation (6). The x variables are the load flow variables, that is, $|V|$ and θ for load nodes and θ for generator nodes. The u variables are then specified conditions on (14), P and Q for load busses, P and $|V|$ for generator busses and $|V|$ and θ for the slack bus. For a particular application, some of the u variables may remain unvaried throughout the optimization process.

The load flow mismatch equations are then solved by Newton's method, as explained in Section 5.2.8.1.

7.6 Comparison between the nonlinear programming and the Dommel and Tinney approaches

The nonlinear programming approach takes the whole problem in its entirety while Dommel and Tinney split the problem into parts. With nonlinear programming the bus voltages are always the problem variables while all other conditions are considered as equality and inequality constraints. With Dommel and Tinney, the equality constraints are satisfied by means of Newton's method while the actual minimization is performed on the u variables that are allowed to vary. The number of varying u variables is usually much smaller than the number of bus voltage variables, thus giving Dommel and Tinney an inherent possibility of having the problem size dependent on the number of varying u variables instead of dependent on system size as in the nonlinear programming approach. The method of Dommel and Tinney has been found quite critical and sensitive in the application of equation (9), as mentioned in Section 3.5.2. The nonlinear programming method is very stable and converges rapidly.

The nonlinear programming approach is a more general method and can be used to solve a wide variety of problems. The method of Dommel and Tinney is applicable only to those problems in which the number of equality constraints are equal to the number of dependent variables x .

CHAPTER 8

DECOMPOSITION TECHNIQUES

8.1 General

The ever increasing size of present day power systems impose great burdens on analytical methods now in use due to storage capacity of computers and large computation times. These circumstances do not apply only to the load flow problem but, in general, to all power system problems. Methods that will decompose large systems into smaller sections, and only work on the sections to arrive at a solution, seem to be the only way of coping with this situation.

The work of Kron,⁵² through his theory of "diakoptics", is based on the above statements. Kron considers a system actually torn into parts (thus the term "tearing" used frequently), each part being solved separately and a transformation on the torn solutions applied to obtain the overall solution. For nonlinear problems, the process becomes iterative. Happ⁵³ in the United States, and Brameller⁵⁴ in Great Britain have made use of the theory of diakoptics on a number of problems.

In control system applications⁵⁵ ideas similar to diakoptics have appeared under the general term "decomposition techniques".

The controversy exists as to whether diakoptics is only a particular

technique for partitioning equations together with an algebraic manipulation of certain terms or if it is a physical approach. Recently Laughton⁵⁶ has shown that tearing is a form of partitioning.

The author of this thesis thinks that diakoptics involves a partitioning of the equations of a physical system together with the creation of a set of new variables and a set of equations on these new variables which express the relation between the torn parts. Decomposition is based on these same ideas. Thus, as far as this author is concerned, decomposition techniques and diakoptics are one and the same thing. Certain forms of algebraic partitioning can be considered as decomposition techniques. When algebraic partitioning becomes just a short hand notation it is evidently not a decomposition technique. It could be that the controversy has arisen because partitioning does not always imply decomposition.

As far as the load flow problem is concerned, a recent paper⁵⁷ by Happ et al. has applied a decomposition technique specially designed for use with their z-matrix load flow method. Dy-Liacco⁵⁸ has developed a different decomposition algorithm which is independent of the load flow method used to obtain the partial solutions of the torn parts. Carre⁶ also has applied decomposition to the load flow problem in order to reduce computation time.

Carre's work⁵⁹ shows that decomposition can be considered as a technique specially suited for successive block over-relaxation where each torn section is considered as a block. Carre's main conclusions are that:

1. the actual physical places where the system is torn apart have an important effect on overall convergence to the solution.
2. The partial solutions can be subject to over relaxation.
3. The partial solutions need not be accurate solutions at all times.

The work presented in this thesis follows closely the work of Carre and uses it in connection with the nonlinear programming load flow method³⁹. While Carre has chosen trees as his torn networks, the present method does admit loops.

8.2 Method of Andretich, Brown, Happ and Persen⁵⁷

This method is the first method to appear in the literature in which the load flow problem is solved by decomposition techniques. It proposes that the system be decomposed by cutting branches to isolate different sections from one another. The impedance matrix of each section is formed, together with matrices which contain information on how the sections are interconnected and on the relationship between the various sections and the one that contains

the swing bus. The reader is referred to the original publication for computational details. It is wished to stress here that the method cuts branches and that it requires additional information in the form of interconnection matrices apart from the impedance matrix of each section. No acceleration was tried by the authors but it can be expected from the work of Carre⁵⁹ that block acceleration would be successful.

8.3 Method of Dy-Liacco⁵⁸

Dy-Liacco proposes that the system be decomposed by cutting through nodes instead of branches. The advantage of cutting nodes is that the interconnection requirements become more simple. All this is required is for the voltages of each torn node to be the same. An electrical decomposition is accomplished by connecting, to each node that will be torn, a voltage source. When the currents through these voltage sources are zero, and all other conditions are met, the solution has been found. The solution algorithm is as follows:

1. voltage sources are assumed at all nodes that will be torn and values are given to them.
2. Each isolated electrical section is solved by a convenient load flow method. The torn nodes come in as fixed voltage nodes in the same way as a reference node.
3. The currents through the voltage sources are computed. If

they are small a solution has been found. If not, the voltage of the sources is varied to force its currents to be zero and the process returns to step 2.

Again the reader is referred to the original publication for computational details. It is stressed here that the method cuts nodes and that it requires additional computation to adjust the introduced voltage sources. No acceleration was tried but again it can be expected, from the work of Carre⁵⁹ that it would be successful.

8.4 The proposed method

While the two previous methods tear either branches or nodes, a third possibility exists in which essentially branches are cut by a double cut of the two terminal nodes of the branch and not the branch itself. The idea of the second method, of isolating through voltage sources, is introduced by considering that only the cut node on the far side of the interconnection branch will act as a source; while the near node will be a normal node of the section which contains it. Fig. 8.1 shows the decomposition technique which results in an overlapping of the subsystems. A decision has been made

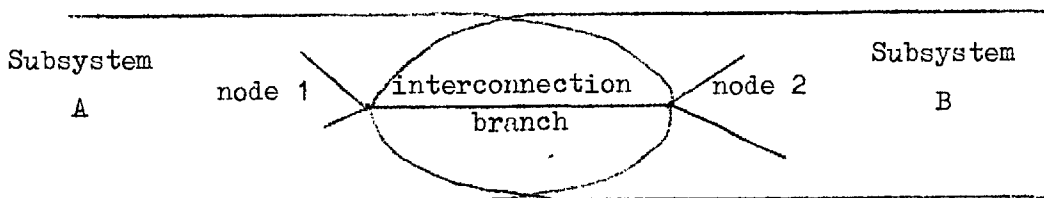


Fig. 8.1 Representation of decomposition technique

to cut through the branch between nodes 1 and 2. If this could be done as described in the method Section 8.2, subsystem A would contain node 1 and subsystem B would contain node 2. Under the present scheme, subsystem A contains node 1 and 2, node 1 appearing as a normal node while node 2 appears as a constant voltage node. The reverse is true for subsystem B. When subsystem A is solved initially node 2 remains constant at its initial value. When subsystem B is solved, node 1 remains at the value that it took when subsystem A was solved and node 2 takes on a new value. This value of node 2 is used as a constant when A is solved again. It is noted that the interconnection branch is part of both subsystems. There is no need for any additional computation apart from the load flow solutions of each subsystem. The method thus proceeds iteratively from subsystem to subsystem until all conditions are simultaneously met.

8.4.1 The partitioning of the system

Carre⁵⁹ has shown that the actual places which are chosen for cuts are of considerable importance in relation to the speed of convergence. The author of this thesis has found that convergence improvements by a factor of 4 or 5 are possible by choosing different subsystem sets of a particular system. It is thus recognized that the partitioning problem is one that demands as much attention as the study of different decomposition techniques. Carre and Varga have shown that the overriding criterion for partitioning is that it should be made at places

where the weakest couplings exist. In electrical systems, the branches with highest reactances or smaller susceptances can be chosen for interconnection. Carre⁶ has devised an algorithm which partitions a system into tree sections by cutting at the weakest branches. The reasons for using tree sections is his use of optimal ordering of the nodes of each tree, which results in a minimum of computation in his solution of each section. By using the nonlinear programming method for the load flow solution of each section, no particular advantage is obtained by using tree sections. The partitioning will thus follow Carre in that it will choose the weakest interconnections, although loops will be allowed in each section.

Carre's algorithm is as follows: the trees will be built up by considering branches one by one. First the line with highest susceptance, $B_{\max}$, is taken. Following this branch, the susceptance values are scanned and the branches whose susceptances exceed a value of $f \cdot B_{\max}$, with $f = 2/3$, are chosen. Next, the ones that exceed $f^2 \cdot B_{\max}$ are taken, and so on, until all branches have been used. For each successive branch (i, j) considered, the branch and its terminal nodes are assigned to the same tree if and only if there are no other branches in the system connecting any node of the partially formed tree containing node i to the partially formed tree containing node j . Neutral connections and branches terminating at the slack bus are not considered. This algorithm leads to the possibility of finding trees made up of only one node.

The algorithm used with the nonlinear programming differs from the above one in two ways: (1) during the first scanning, where branches whose susceptance is higher than $f.B \max$ are considered, the rules given above are by-passed and all of these branches form parts of trees disregarding the fact that some loops may be formed; (2) no tree made up of a single node is allowed. If the algorithm forms one, the node is incorporated to a tree through the branch of highest susceptance. If there are other branches between the same node and the same tree, these too are assigned to the tree, disregarding the fact that they produce loops within the tree.

Both Carre's algorithm and the one just described will usually lead to some partitions containing large number of nodes. As will be shown through the examples of the next sections, with the use of non-linear programming to solve the partitioned load flows it is particularly advantageous to have the number of nodes in each tree limited to a certain maximum number. To accomplish this large trees are examined, in order of increasing branch susceptances, to search for possible places to subdivide them into smaller trees within the acceptable range.

8.4.2 The nonlinear programming solution

Once the system has been partitioned as described in Section 8.4.1, the nodal admittance nonlinear programming load flow program³⁹ is used to solve the partitioned sections, treating the interconnections with

the overlapping decomposition technique described at the beginning of this section 8.4. All nodes on the far side of interconnection branches appear as slack busses. The original system slack bus is kept, but there is no need to define extra slack busses in each subsystem. It is pointed out that there is no need for subsystems to contain both generation and load nodes. As already explained, the partitions are chosen on the grounds of having weak interconnections and not on the type of nodes.

It has been found greatly advantageous to store the Hessian matrix of each subsystem to be used as the initial approximation to the Hessian during the following iteration. Normally, the Hessian matrix is always given an initial value of a unit matrix. The result of using the Hessian of the previous iteration for a particular subsystem, is an acceleration in the convergence to a solution during the current iteration. As a solution would have been found anyway had a unit Hessian been used, the total number of iterations for the whole system is not reduced. The savings occur in the time per iteration, which is reduced by 50 to 70 per cent in some instances.

In Section 8.1 it was mentioned that decomposition by weak interconnections is a technique specially suited for successive block over relaxation. This has also been confirmed in the present case. As soon as a subsystem has been solved, the voltage variables are accelerated

with respect to their values at the start of the iteration. Reductions by as much as 50% in the number of iterations are possible by using over relaxation.

It is not necessary to solve completely each subsystem in every iteration. The process will converge if only a fixed maximum number of Fletcher-Powell steps, say 4, are allowed for each subsystem. In any case, the final iterations usually require 2 or 1 steps and sometimes even $\frac{1}{2}$ of a step. A complete step requires a minimum of two function evaluations. Sometimes, after the function has been evaluated at a point along the direction of minimization, it is found that the function value is already within the tolerance demanded and there is no need to perform an interpolation to locate the minimum. As only one function evaluation has been made when this occurs, it seems appropriate to call this $\frac{1}{2}$ a step. Experience with different systems has shown that there is no advantage gained by completely solving each subsystem during the first few iterations. This is to be expected, as the overall error due to the errors in the interconnection voltages has a far greater effect than what can be obtained by a very precise subsystem solution based on those voltages. As the process continues on the overall solution is approached, the reverse becomes true. There thus appears to be an optimum moment where a precise subsystem solution should be demanded. In practice, good results have been obtained by allowing only 4 Fletcher-Powell steps for each

subsystem during the first 3 iterations, and from then on demanding the load flow function to be less than the specified mismatch tolerance. After 6 iterations, the tolerance on the load flow function is reduced by a factor of 100.

A solution has been found when the load flow function meets a tolerance for all subsystems simultaneously.

Fig. 8.2 presents a flow diagram of the nonlinear programming decomposition load flow method.

8.4.3 Numerical solution of an actual system

The IEEE standard 30-bus system was chosen to illustrate the application of the decomposition method. Figs. 8.3 and 8.4 present the structure of the subsystems. In Fig. 8.3, the system has been decomposed into 4 sections of 8, 6, 10 and 5 nodes. The 10 bus subsystem was further subdivided into 2 sections of 7 and 3 nodes in Fig. 8.4, to study the effect of having a greater number of sections with less nodes per section.

Table 8.1 presents the characteristics of the method on both systems. The effect of fewer subsystems is a reduction in the number of iterations, with an increase in overall solution time, due to an increase in the time per Fletcher-Powell step with subsystem size.

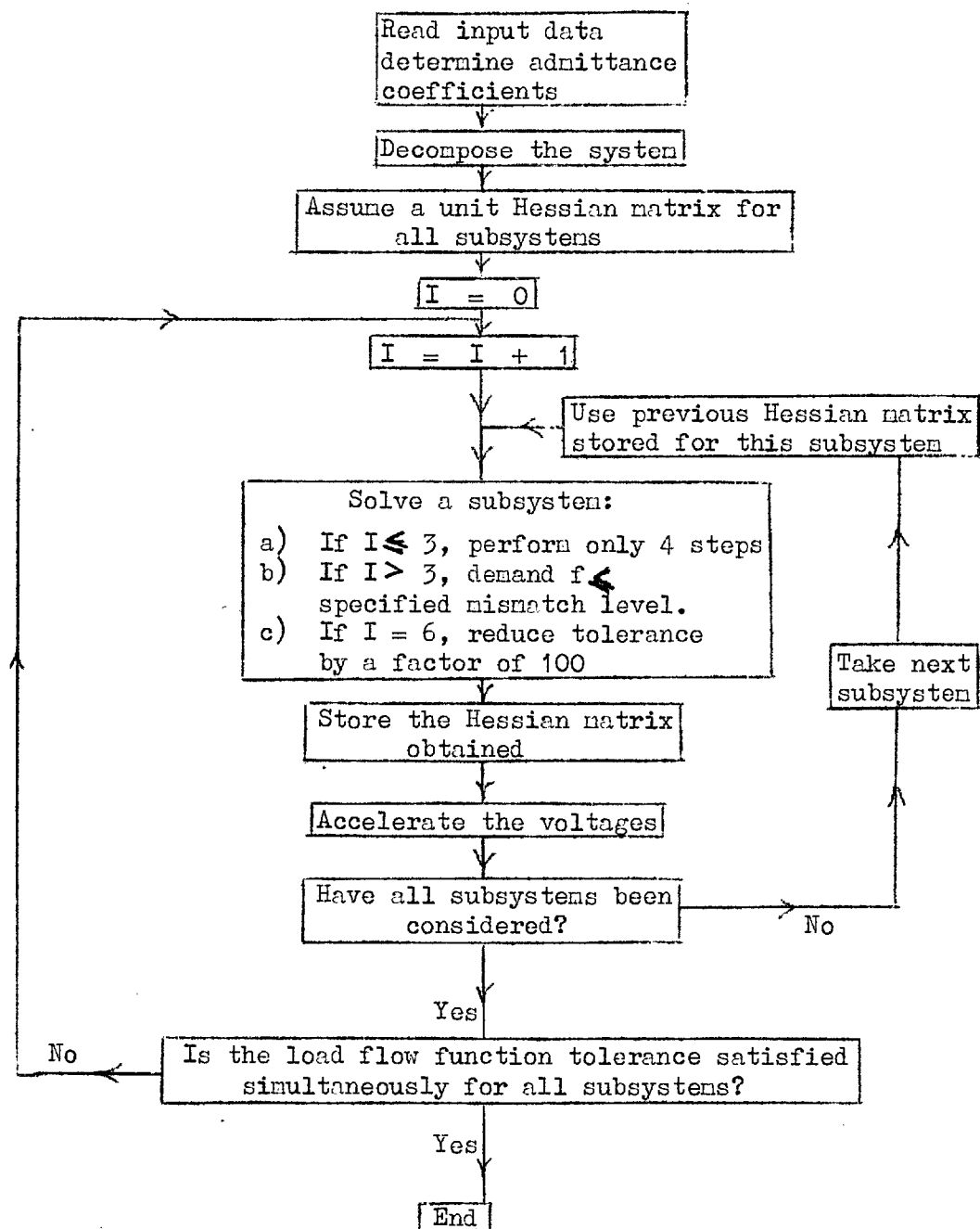


Fig 8.2 Flow diagram of nonlinear programming decomposition load flow method.

Further cases confirmed this result. The number of subsystems is limited by the number of weak interconnections. This implies that systems cannot be subdivided arbitrarily so that the statement of the advantage in a greater number of subsystems is really bounded by the possible interconnection places. If a strong link is chosen as an interconnection to reduce subsystem size, a reduction in overall solution time will not occur.

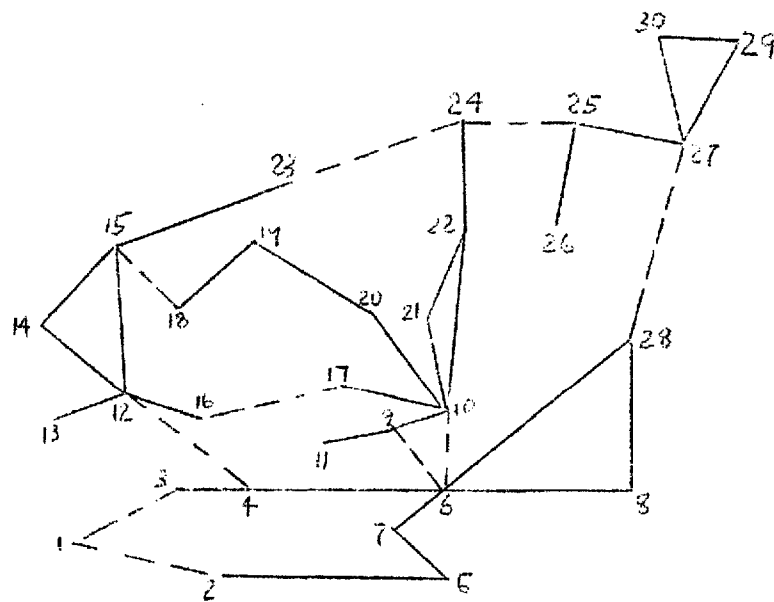


Fig. 8.3 Decomposition of IEEE 30-bus system into four subsystems

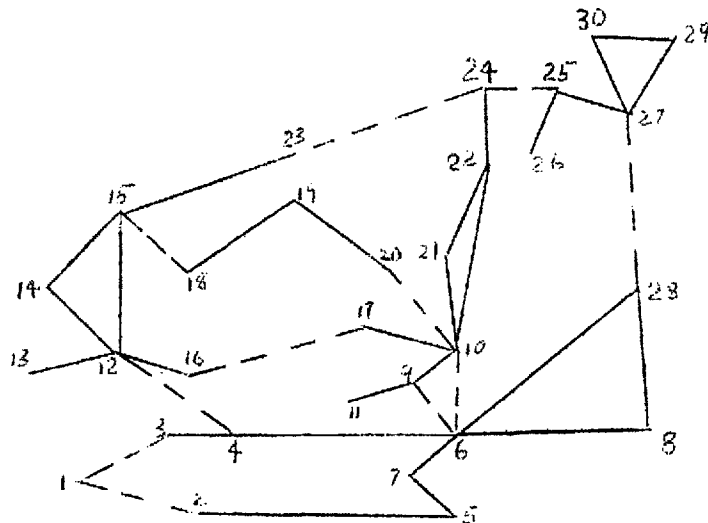


Fig. 8.4 Decomposition of IEEE 30-bus system into five subsystems

Table 8.1 Solution characteristics⁺ of decomposition method on the
systems of Figs. 8.3 and 8.4. Acceleration factor = 1.3

Iteration	System of fig. 8.3			System of fig. 8.4		
	Subsystem*	Initial load flow function	Number of steps	Subsystem*	Initial load flow function	Number of steps
1	1(8)	1.4776	4	1(8)	1.4776	4
	2(6)	0.0546	4	2(6)	0.0546	4
	3(10)	0.1541	4	3(7)	0.1396	4
	4(5)	0.0199	4	4(3)	0.0152	4
3				5(5)	0.0199	4
	1	0.4702	4	1	0.5398	4
	2	0.0331	4	2	0.0593	4
	3	0.2893	4	3	0.2365	4
4	4	0.0417	4	4	0.0644	2
				5	0.0543	4
	1	0.2255	16	1	0.3753	16
	2	0.0493	4	2	0.1875	4
6	3	0.3079	15	3	0.4987	8
	4	0.0455	4	4	0.1315	2
				5	0.0333	3
	1	0.1615	4	1	0.0687	4
8	2	0.1481	4	2	0.0788	3
	3	0.1173	3	3	0.0902	3
	4	0.0105	2	4	0.0218	2
				5	0.0057	2
10	1	0.0147	4	1	0.0096	4
	2	0.0172	2	2	0.0107	2
	3	0.0156	2	3	0.0123	2
	4	0.0012	2	4	0.0032	2
10				5	0.0008	2
	1	0.0019	4	1	0.0016	3
	2	0.0021	3	2	0.0018	2
	3	0.0021	1	3	0.0021	1
	4	0.0002	1	4	0.0006	1
10				5	0.0002	1

Table 8.1 (continued)

Iteration	System of fig. 8.3			System of fig. 8.4		
	Subsystem*	Initial load flow function	Number of steps	Subsystem*	Initial load flow function	Number of steps
12	1	0.0002	3	1	0.0003	2
	2	0.0003	1	2	0.0003	1
	3	0.0002	1	3	0.0004	1
	4	0.0000	1	4	0.0001	1
				5	0.0000	$\frac{1}{2}$
13	1	0.0000	1	1	0.0001	1
	2	0.0000	1	2	0.0001	1
	3	0.0000	1	3	0.0001	1
	4	0.0000	1	4	0.0000	$\frac{1}{2}$
				5	0.0000	$\frac{1}{2}$
14				1	0.0000	$\frac{1}{2}$
				2	0.0000	$\frac{1}{2}$
				3	0.0000	$\frac{1}{2}$
				4	0.0000	$\frac{1}{2}$
				5	0.0000	$\frac{1}{2}$

* The number of nodes in each subsystem appears in parenthesis

+ System of fig. 8.3 took about 15% more time than the one of
fig. 8.4 to reach a solution.

Finally, the effect of acceleration was studied on the systems of figs. 8.3 and 8.4 and results are presented in fig. 8.5. A unit of time was chosen as the time that the nonlinear programming approach took to solve the 30-bus system without decomposition. Fig. 8.5 shows the great reduction in computer time that is obtained with decomposition. The system of fig. 8.4 still converges faster than the one of fig. 8.3 for different block acceleration factors.

In conclusion, it has been shown that the decomposition technique has the following characteristics:

- i - A considerable reduction in computer time is obtained which makes possible the extension of the nonlinear programming method to larger systems.
- ii - Through decomposition, a big reduction in storage requirements is obtained. The nonlinear programming method requires the formation and storage of the Hessian matrix which is a full matrix of size equal to twice the number of nodes. Obviously, this matrix limits system size due to storage demands. Decomposition makes possible for the method to handle larger systems than would otherwise be able.
- iii - Block successive over relaxation factors can be successfully applied and have an important effect on convergence.

- iv - The actual decomposition of the system can be done automatically with the algorithm provided. It is advantageous to keep subsystem size as small as possible using only weak interconnections.
- v - The decomposition technique used requires no work between iterations. The effect of other subsystems on the one being solved is transnitted automatically as the solution of each system takes place.

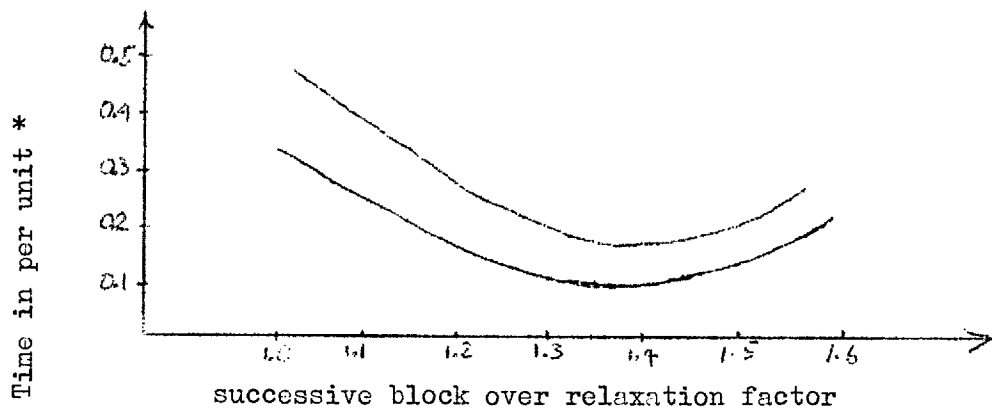


Fig. 8.5 Effect of acceleration on convergence time

* unit of time = solution time of 30-bus system
without decomposition using the nonlinear
programming method.

CHAPTER 9

CONCLUSIONS

The original contributions of this thesis can be summarized as follows:

1. development of a mesh impedance iterative load flow method which has been shown to converge well.
2. development of a hybrid impedance-admittance load flow method. This method produced very fast convergence with some systems, compared with other methods tried.
3. development of criteria for selection of reference and slack busses specially suitable for Gauss-Seidel type load flow methods.
4. use of nonlinear programming methods to solve the load flow problem. The Fletcher-Powell²³ method was found to be the most powerful method to minimize a load flow function consisting of the sum of the squares of the mismatches in voltage and power at each node. The actual mismatch terms were formulated using both the admittance and impedance matrix coefficients.
5. use of nonlinear programming methods to solve optimal load flow problems, such as the minimum loss and economic dispatching. By these methods, the optimal problem is converted into a sequence of unconstrained minimization problems which can be solved by the Fletcher-Powell method. The method

of Powell³⁸ was found to be the best method.

6. development of decomposition techniques to extend the non-linear programming solution of the load flow problem to large systems. By these techniques, not only are the computer storage requirements reduced substantially but also reductions in computation time by factors of up to 80% are obtained.

For each of the above six topics described, digital computer programs were written and methods were applied to actual power systems. Listings of these programs can be found in Power System Reports 79, 81 and 82 of the Department of Electrical Engineering, Imperial College of Science and Technology.

The first five subjects of the above list have been already presented in the form of 3 papers,^{20, 39, 60} and a fourth is being planned on decomposition techniques. The material of Chapters 5, 6 and 7 has been put together as a review paper.⁶¹ A correspondence paper¹⁹ announcing the investigation summarized in Chapters 2, 3 and 4 has also appeared.

The work on nonlinear programming has shown the feasibility of the approach to the solution of power system problems. With the large size of present day systems, these methods are still not practical. The requirement that the Hessian matrix of the Fletcher-Powell

method must be of size equal to twice the number of nodes is the most important barrier to be removed before the method can be of general use for large systems. This requirement demands a computer storage which increases rapidly with system size and an equally increasing computation time. If a method is developed which would be at least as powerful as the Fletcher-Powell method but without the Hessian matrix requirement, a great improvement would result.

In the absence of a better method than the Fletcher-Powell one, the only possibility of making the nonlinear programming approach practical for large systems lies in the development of decomposition techniques. Chapter 8 is the start of this work. The 30-bus and 57-bus systems which had been solved without decomposition were solved in a small fraction of the time by making use of a decomposition technique. Further work is being done to write a computer program with this approach that would permit the solution of systems in the 300 to 500 node range.

The next step would be to extend the decomposition techniques to the optimal load flow problem. The encouraging results with the load flow problem indicate that this would be of great importance.

Other possibilities in the future may lie in the use of optimization methods outside of the scope of nonlinear programming, such as network

flow techniques.

As mentioned in the Introduction of this thesis, the work has dealt with steady state power system problems. There is a need for the extension of mathematical programming methods to transient problems.

The future will almost certainly bring the overall automation and optimization of the operation of power systems. It is hoped that this thesis will make its small contribution towards this goal.

REFERENCES

1. A. Kusko, "A prediction of power system development, 1968 to 2030," IEEE SPECTRUM, vol. 5, April, 1968, pp. 75 - 80.
2. A. M. Sasson and F. J. Jaimes, "Digital methods applied to power flow studies," IEEE TRANS. PAS-86, 1967, pp. 860 - 867.
3. M. A. Laughton and M. W. Humphrey Davies, "Numerical techniques in solution of power system load flow problems," PROC. IEE, vol. 111, 1964, pp. 1575 - 1588.
4. A.F.Glimm and G.W.Stagg, "Automatic calculations of load flows," TRANS. AIEE, vol. 76, pt. III, 1957, pp. 817-825.
5. H.W.Hale and R.W.Goodrich, "Digital computation of power flow: some new aspects," ibid, vol. 78, 1959, pp. 919-924.
6. B.A.Carre, "Solution of load flow problems by partitioning systems into trees," IEEE Winter Power Meeting, paper number 68 TP 112, 1968.
7. A.Braneller and J.K.Denmead, "Some improved methods for digital network analysis," PROC. IEE, vol. 109A, 1962, pp.109-116.
8. P.P.Gupta and M.W.Humphrey Davies, "Digital computers in power system analysis," ibid, vol. 108A, 1961, pp.383-404.
9. H.E.Brown, G.K.Carter, H.H.Happ and C.E.Person, "Power flow solution by impedance matrix iterative method," IEEE TRANS. PAS-82, 1963, pp. 1 - 9.

10. J.B.Ward and H.W.Hale, "Digital computer solution of power flow problems," TRANS. AIEE, vol. 75, pt. III, 1956, pp. 398-404.
11. J.E.Van Ness, "Iteration methods for digital load flow studies," ibid, vol. 78, Pt. III, 1959, pp.583-588.
12. J.E.Van Ness and J.H.Griffin, "Elimination methods for load flow studies," ibid, vol. 80, pt. III, 1961, pp. 99-303.
13. W.F.Tinney and C.E.Hart, "Power flow solution by Newton's method," IEEE TRANS. PAS-86, 1967, pp.1449-1461.
14. D.Feingold and D. Spohn, "Bounded variables load flow problems," PROC. 2nd PSCC, Stockholm, 1966, Part 2, Paper 4.3.
15. K.Takahashi, Y. Sekine, T. Unezu, "Network-flow method applied to load flow calculation," IEEE Winter Power Meeting, paper number 68 TP 95, 1968.
16. R.W.Long, "Inherent constraints control convergence - a new method in the long look at load flow," IEEE Summer Power Meeting, paper number 68 TP 662, 1968.
17. L.P.Hajdu, J. Peschon, W.F.Tinney, D.S. Piercy, "Optimum load-shedding Policy for Power Systems," IEEE PAS-87, 1968, pp. 784-795.
18. A.M.Sasson, "Mesh impedance matrix solution of load flow problems," IEEE Winter Power Meeting, 1967, paper number 31 PP 67-2.

19. L.L.Freris and A.M.Sasson, "Investigation on the load flow problem," PROC. IEE (correspondence) vol. 114, p. 1960, 1967.
20. L.L.Freris and A.M.Sasson, "An investigation of the load flow problem," (to be published in the Proc. IEE 1968).
21. R.Fletcher and C.M.Reeves, "Function minimization by conjugate gradients," THE COMPUTER JOURNAL, Vol. 7, pp. 149-154, 1964.
22. W.C.Davidon, "Variable metric method for minimization," A.E.C. RESEARCH AND DEVELOPMENT REPORT, ANL-5990, 1959.
23. R.Fletcher and M.J.D.Powell, "A rapidly convergent descent method for minimization," THE COMPUTER JOURNAL, vol. 6, pp. 163-168.
24. W.C.Davidon, "Variance algorithm for minimization," *ibid*, vol. 10, pp. 406-410.
25. M.J.D.Powell, "An efficient method for finding the minimum of a function of several variables without calculating derivatives," *ibid*, vol. 7, pp. 155-162, 1964.
26. W.I.Zangwill, "Minimizing a function without calculating derivatives," *ibid*, vol. 10, 293-296.
27. M.J.D.Powell, "A method for minimizing a sum of squares of non-linear functions without calculating derivatives," *ibid*, vol. 7, pp. 303-307.

28. R.Fletcher, "Generalized inverse methods for the best least squares solution of systems of non-linear equations,"
ibid, vol. 10, pp. 392-399.
29. C.W.Carroll, "The created response surface technique for
optimizing nonlinear restrained systems," OPERATIONS
RESEARCH, vol. 9, pp. 169-184, 1961.
30. A.V.Fiacco, G.P.McCormick, "The sequential unconstrained
minimization technique for nonlinear programming, a
primal-dual method," MANAGEMENT SCIENCE, vol. 10,
pp.360-366, 1964.
31. A.V.Fiacco, G.P.McCormick, "Computational algorithm for the
sequential unconstrained minimization technique for
nonlinear programming," ibid, pp. 601-617, 1964.
32. A.V.Fiacco, G.P.McCormick, "Programming under nonlinear con-
straints by unconstrained minimization: a primal-
dual method," RESEARCH ANALYSIS CORPORATION, TP-96, 1963.
33. F.A.Lootsma, "Logarithmic programming: a method of solving
nonlinear programming problems," PHILLIPS RES. REPTS.
vol. 22, pp. 329-344.
34. F.A.Lootsma, "Extrapolation in logarithmic programming,"
ibid, vol. 23. pp. 108-116.
35. T.E.Dy-Liacco, Discussion to Reference 19

36. A.V.Fiacco, G.P.McCormick, "The sequential unconstrained minimization technique for convex programming with equality constraints," RESEARCH ANALYSIS CORPORATION, TP-155, 1965.
37. W.I.Zangwill, "Non-linear programming via penalty functions," MANAGEMENT SCIENCE, vol. 13, pp. 344-358, 1967.
38. M.J.D.Powell, "A method for non-linear constraints in minimization problems," CONFERENCE ON OPTIMIZATION, University of Keele, March, 1968.
39. A.M.Sasson, "Nonlinear programming for the load flow, minimum loss and economic dispatching problem, IEEE, Paper No. 69 TP 673 given at the Chicago Summer Power Meeting, 1968.
40. Y.Wallach, "Gradient methods for load flow problems," IEEE PAS, vol. 87, May, 1968, pp. 1314-1318.
41. T.E.Dy-Liacco, S.K.Mitter, R.K.Babickas and T.J.Kraynak, "Multi-level approach to power system decision problems - the optimizing level," POWER INDUSTRY COMPUTER APPLICATIONS CONFERENCE, Pittsburg, May, 1967.
42. L.K.Kirchmayer, Economic Operation of Power Systems.
New York: Wiley, 1958.
43. H.M.Smith, Tong Shih-Yung, "Minimizing power transmission losses by reactive-volt-ampere control," IEEE PAS, vol. 82, pp. 542-544, 1963.

44. R.Bauman, "Power flow solution with optimal reactive flow,"
ARCHIV FUER ELEKTROTECHNIK, vol. 48; pp. 213-224, 1963.
45. K.Zollenkopf, "Load flow calculation using less-minimization
techniques," PROC. IEE, vol. 115, pp. 121-127, 1968.
46. H.W.Dommel, W.F.Tinney, "Optimal power flow solutions,"
Paper given at IEEE Winter Power meeting, New York, 1968.
47. R.B.Squires, "Economic dispatch of generation directly from
power system voltages and admittances," AIEE Trans.,
Pt.III, vol. 59, pp. 1235, Feb. 1961.
48. J.Carpentier, "Contribution to the economic dispatch problem,"
BULLETIN DE LA SOCIETE FRANCAISE DES ELECTRICIENS,
vol. 8, pp. 431-447, August, 1962.
49. J.F.Dopazo, O.A.Klitin, Stagg, Watson, "An optimization
technique for real and reactive power allocation,"
Proc. IEEE, vol. 55, Nov. 1967.
50. A.H.El-Abiad, F.J.Jaines, "A method for optimum scheduling
of power and voltage magnitude," IEEE, Paper No.
68 TP 674 given at the Chicago Summer Power Meeting, 1968.
51. M.J.Box, "A comparison of several current optimization methods
and the use of transformations in constrained problems,"
COMPUTER JOURNAL, vol. 8, pp. 67-75, 1966.
52. G.Kron, "Diakoptics", Macdonald, London, 1963.
53. H.H.Happ, "Z diakoptics", Proceedings of the 2nd power systems
computations conference, RIT, Stockholm, 1966, Part 2,
paper 4, 16.

54. A.Brameller, "The application of diakoptics to network analysis," *ibid*, paper 4.6.
55. C.B.Brosilow, L.S.Lasdon, J.O.Pearson, "Feasible optimization methods for interconnected systems," American Institute of Chemical Engineers.
56. M.A.Laughton, "Decomposition techniques in power-system network load-flow analysis using the nodal-impedance matrix," *Proc. IEE*, Vol. 115, No. 4, April, 1968.
57. R.G.Andretich, H.E.Brown, H.H.Happ, C.E.Person, "The piece-wise solution of the impedance matrix load flow," *IEEE 68 TP 93*, 1968, Winter Power Meeting, 1968.
58. T.E.Dy-Liacco, "Control of power systems via the multi-level concept," Ph.D. Thesis, Case Western Reserve University, June, 1968.
59. B.A.Carre, "The partitioning of network equations for block iteration," *Computer Journal*, vol. 8, 1966, pp.84-97.
60. A.M.Sasson, "Combined use of the Powell and Fletcher-Powell nonlinear programming methods for optimal load flows," (submitted for *IEEE Winter Power Meeting*, 1969).
61. A.M.Sasson, "Nonlinear programming applications to power systems," *IFORS-HELORS CONFERENCE on Operational Research in Electrical Power Systems*, Athens, 1968.