

Supplementary information

1. DCB- image tracking and error analysis

To evaluate the error performed by using the SEM images to measure the fracture energy via DCB tests, a Monte-Carlo analysis was performed. The analysis evaluates the cumulative effect of the errors on the different measurements obtained from the images: width of the beams, tracking of the beam edges, crack length and viewing angle due to the tilting of the stage (**Figures 1S and 2S**). It is important to note that tracking of the crack tip at the interphase was more challenging, probably due to defects present at the fibre-BN interface and surface step created due to the different milling rates between SiC and BN. This has been taken into account with the pixels of uncertainty used in the error analysis.

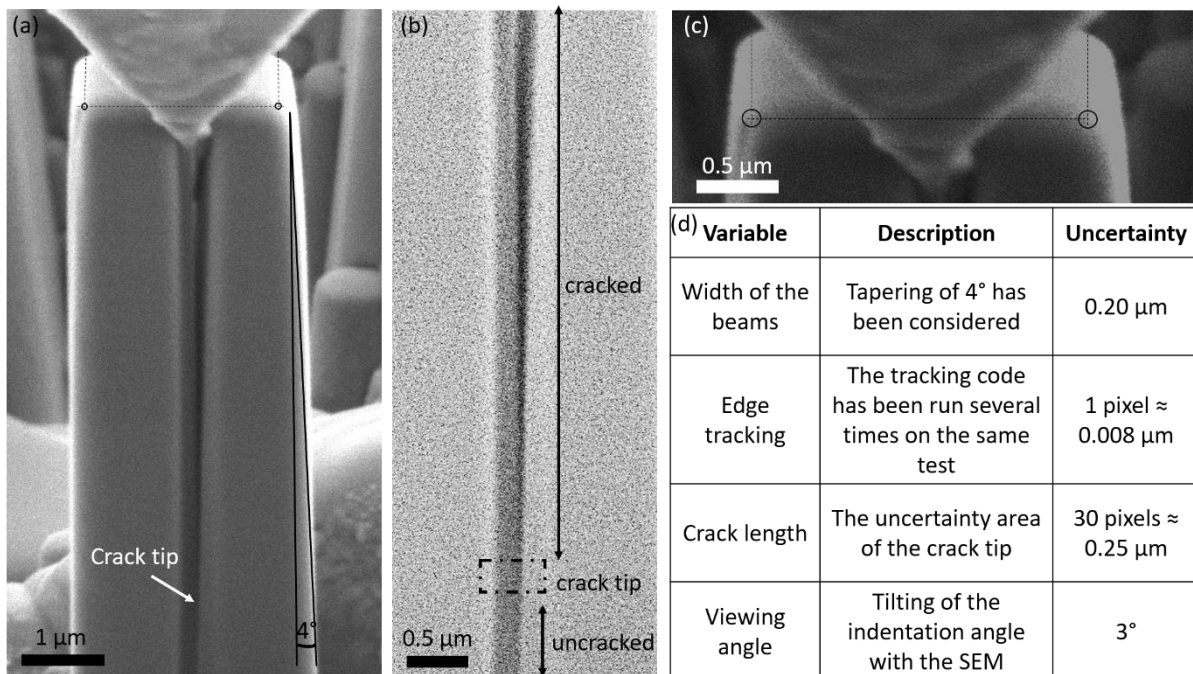


Figure 1S Error analysis on the DCB test performed at the interphase showing (a) an SEM frame of the DCB and its tapering, (b) tracking of the crack tip, (c) tracking point used for the displacement of the external edges and (d) the values of uncertainty used for the error propagation analysis.

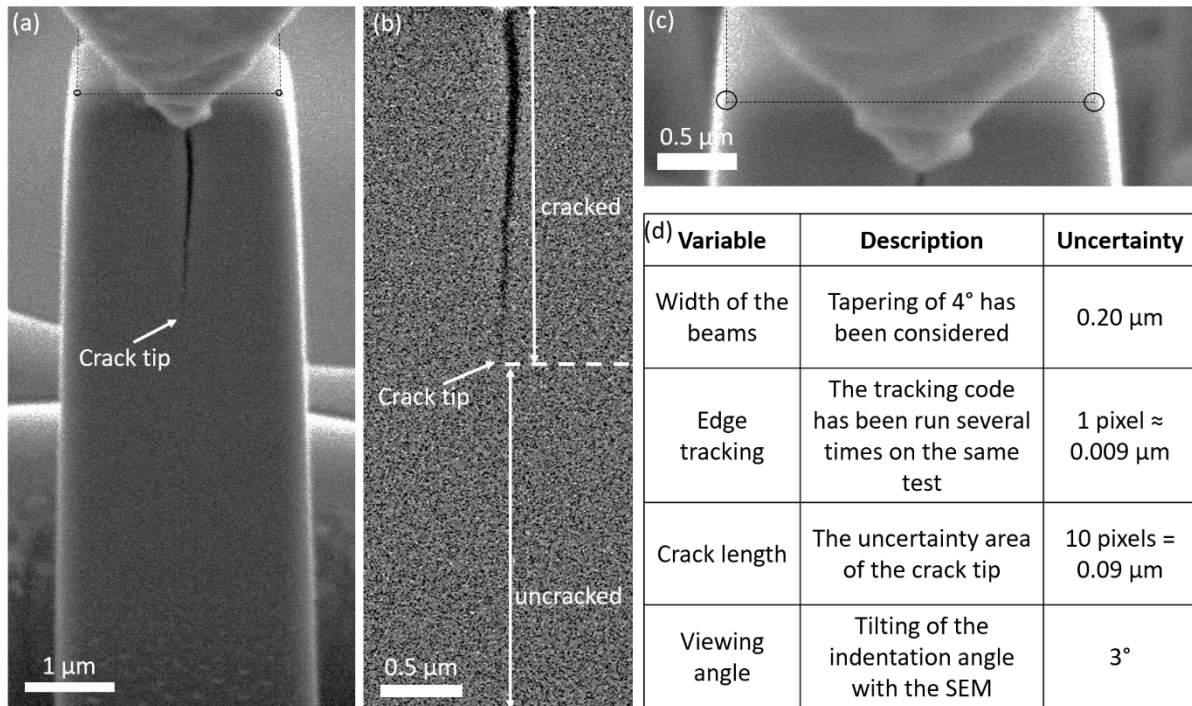


Figure 2S Error analysis on the DCB test performed in the fibre showing (a) an SEM frame of the DCB, (b) tracking of the crack tip area, (c) tracking point used for the displacement of the external edges and (d) the values of uncertainty used for the error propagation analysis.

The influence of the uncertainty on the calculation of the critical energy release rate has been shown in **Figure 3S**. For the interphase (**Figure 3Sa**) the error is reduced to less than 1 J/m² when working with larger cracks (3.5-5 µm as specified in the manuscript). The cracks used to measure the critical energy release rate of the fibres are shorter (**Figure 3Sb**) and the associated errors oscillate around 2 J/m².

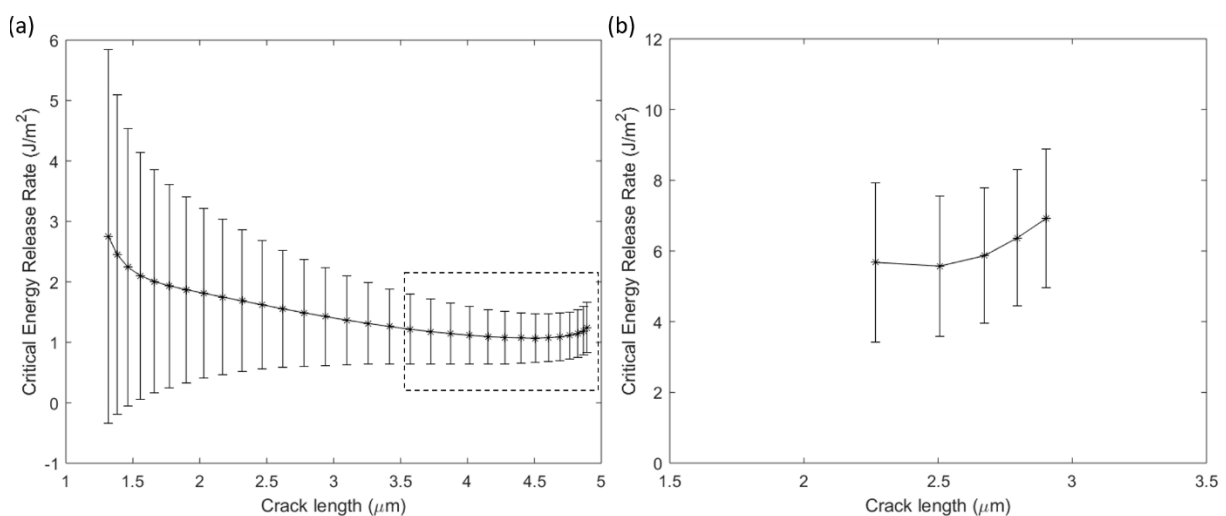


Figure 3S Values of fracture energy for (a) Interphase 2 and (b) Fibre 2 showing the overall effect of the uncertainty in the measurement of the width of the sample, tracking of the edges, crack length and interface tilting angle.

2. DCB- Finite element modelling

The FE analysis performed for each DCB test of the interphase considered the three different phases of the CMC (fibre, BN and CVI SiC matrix), as shown in **Figure 4S**. Note that the crack propagates at the BN-fibre interface and that a tapering (4°) was considered.

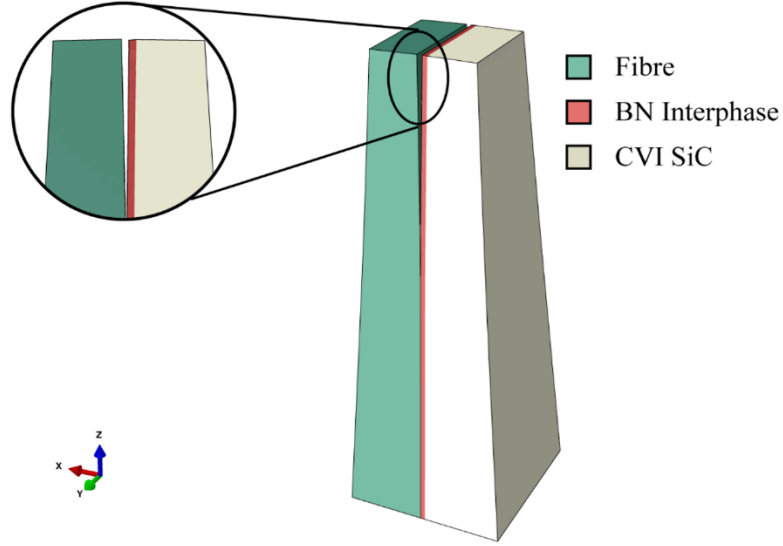


Figure 4S Schematic of the DCB used in the FE simulations showing the different phase: fibre, BN interphase and CVI SiC.

Mixed-mode Cohesive law:

The mixed-mode displacement is computed as:

$$\delta_m = \sqrt{\langle \delta_{11} \rangle^2 + \delta_{12}^2} \quad \text{Eq. A1}$$

$$\delta_{12} = \sqrt{\delta_{22}^2 + \delta_{33}^2}.$$

The Macaulay operator $\langle \cdot \rangle$ suppresses the damage initiation for compression acting on a plane normal to the contact surface. Damage initiation is then imposed using the following quadratic damage initiation criterion:

$$\sqrt{\left(\frac{\langle \sigma_{11} \rangle}{\sigma_I^{max}}\right)^2 + \left(\frac{\sqrt{\sigma_{21}^2 + \sigma_{31}^2}}{\sigma_{II}^{max}}\right)^2} = f. \quad \text{Eq. 1}$$

In this case σ_I^{max} and σ_{II}^{max} are the cohesive strengths in pure mode I and mode II respectively.

Finally, damage propagation is driven by the following power law criterion:

$$\left(\frac{G_I}{G_{IC}}\right)^\alpha + \left(\frac{G_{II}}{G_{IIC}}\right)^\alpha = 1 \quad \text{Eq. A3}$$

G_I and G_{II} are the energy release rate in mode I and mode II respectively and G_{IC} and G_{IIC} are the corresponding fracture toughness values. Traction components' degradation is controlled introducing an irreversible damage variable $D \in [0,1]$:

$$D = \max\left\{0, \min\left\{1, \frac{\delta_m - \delta_m^i}{\delta_m^f - \delta_m^i}\right\}\right\}. \quad \text{Eq. A4}$$

Superscripts 'i' and 'f' refer to the initiation and final failure of the mixed-mode displacement δ_m .

The implicit integration scheme requires the computation of a consistent Jacobian of the stiffness matrix:

$$d\sigma = \frac{\partial \sigma}{\partial \delta} \delta \delta = (1 - D)K d\delta - dDK \delta \Rightarrow \frac{\partial \sigma_i}{\partial \delta_j} = (1 - D)K_{ij} + \frac{\partial \sigma_i}{\partial D} \frac{\partial D}{\partial \delta_j}. \quad \text{Eq. A5}$$

- Comparison of the analytical solution and the FEM of the DCB test

Figure 5Sa compares the experimental final crack lengths with the ones calculated using FEM for each interphase tested. The fracture energies calculated using the analytical solution and FEM are compared **in Figure 5Sb**. The values obtained using both methods are very similar.

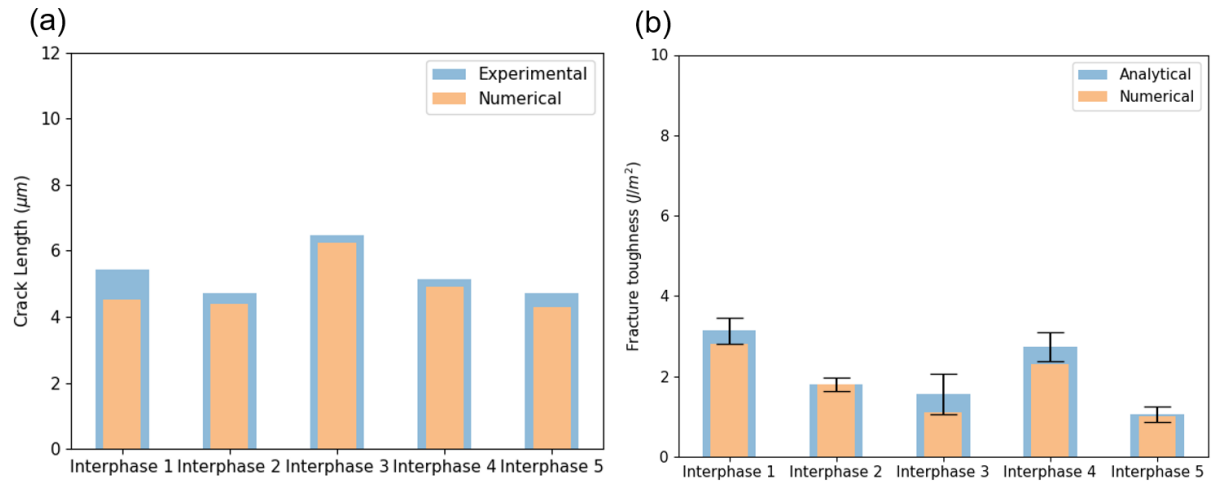


Figure 5S Results obtained with the Finite Element analysis. (a) Comparison between the experimental and calculated final crack lengths. (b) Critical energy release rates calculated using the analytical solution and FEM.

- Effect of compression in the mode mixity

The mode mixity due to compression of the beams was studied with the FE model concluding that the measured values of vertical load during crack propagation are very small (≈ 0.12 mN, see **Figure 6a**) and do not influence on the mode mixity or stress field at the crack tip, as show in **Figure 6b**.

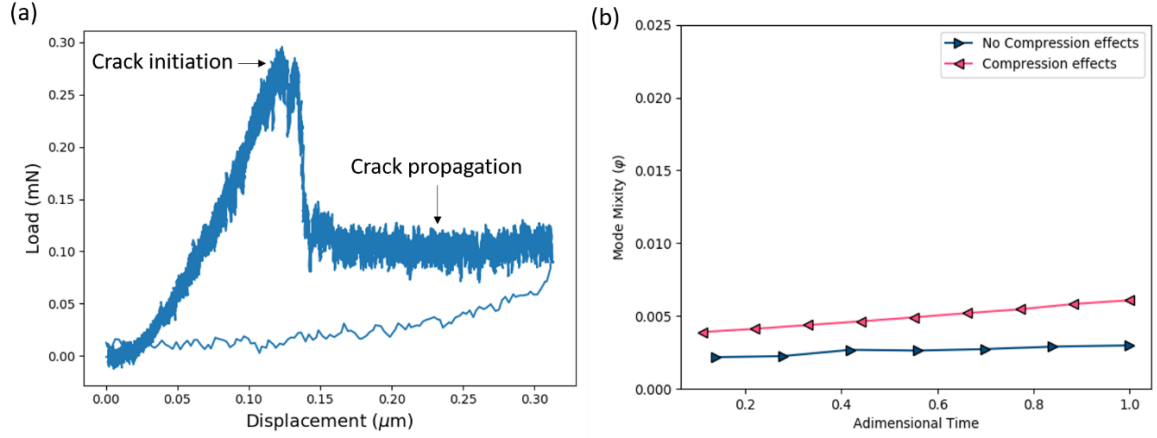


Figure 6S (a) Vertical load measured by the load cell during a DCB test showing the crack initiation and crack propagation events. (b) Comparison of the mode mixity when having a compression effect in the DCB (obtained with the FE simulation).

3. Push Out- image tracking and error analysis

The error in the measurement of fibre displacement using image tracking on the push out tests has been analysed by comparing the displacement of the indenter tip measured with the displacement sensor and the image tracking algorithm during a monotonic push out test (**Figure 7S**). The result is used in a Monte-Carlo based error analysis of the Liang and Hutchinson equation (**Eq. 8** in the manuscript). The uncertainty on the displacement of the fibre was considered to be $0.07 \mu\text{m}$ which corresponds to twice the error measured in **Figure 7Sb** as the fibre displacement is obtained by subtracting the displacements of the tip and the sample. The calculated fracture energy is $3.0 \pm 0.5 \text{ J/m}^2$.

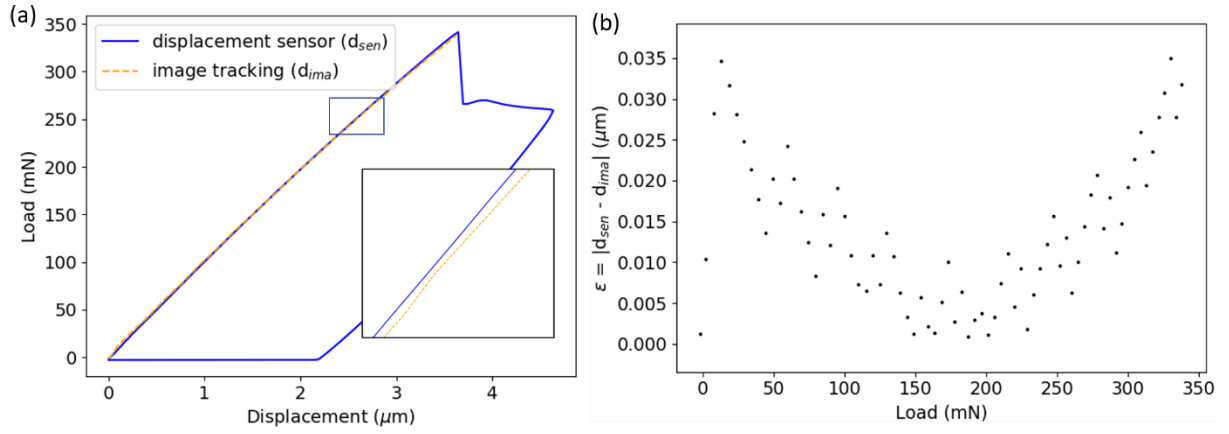


Figure 7S (a) Comparison of the load-displacement curve of the tip obtained using the displacement data acquired by the piezoelectric/displacement sensor (d_{sen} , continuous blue) and using image tracking (d_{ima} , discontinuous orange). The inset shows a higher magnification view of the two curves (b) Error on the measurement of the displacement.

4. Push out- Calibration factor:

The calibration factor, C , was calculated using the load-displacement curve of a monotonic push out test in the same sample. As shown in **Figure 8S** a correction factor of $C = 0.275 \mu\text{m}^{-2}$ gives a good agreement with the data obtained with image tracking.

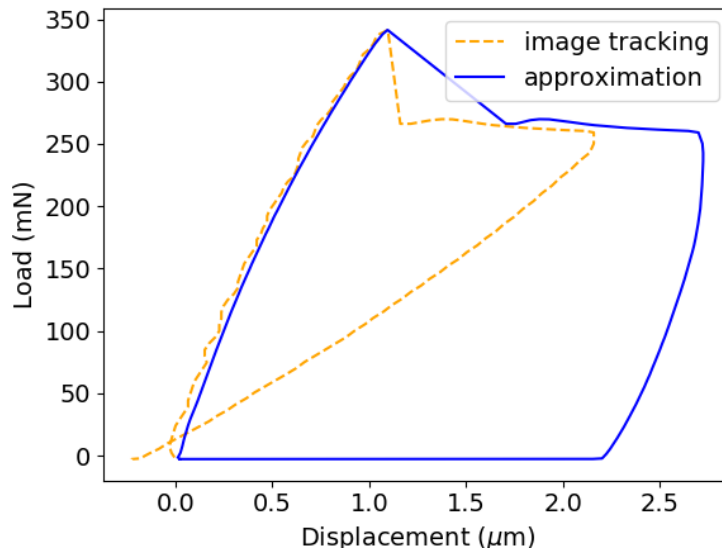


Figure 8S Load vs fibre displacement plots obtained with image tracking (discontinuous orange) and based on the approximation of Liang and Hutchinson with a $C = 0.275 \mu\text{m}^{-2}$ (continuous blue) for a monotonic push out test.