

The effect of irregular seismic loading on the validity of the simplified liquefaction procedures

Efecto de cargas sísmicas irregulares en la validez de los procedimientos de licuefacción simplificados

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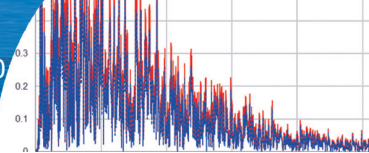
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Soil liquefaction has been one of the major hazards for civil engineering projects relating to earthquakes. The simplified liquefaction procedure which is used to assess liquefaction susceptibility in practice is still based on semi-empirical methods. These rely on the assumption that irregular seismic motions can be represented fully by an equivalent number of cycles of uniform stress amplitude, which is based on the peak acceleration measured at ground surface. Most methodologies used to calculate the equivalent number of cycles are based on Miner's damage concept developed for the fatigue analysis of metals. Several researchers have questioned the validity of this concept, as soils have a highly non-linear response. The present work investigates numerically the concept of the equivalent uniform amplitude cycles. Effective stress-based non-linear finite element analyses are performed with a modified bounding surface plasticity model that allows to realistically simulate liquefaction, reproducing the cyclic strength of sands accurately. The seismic response of a 15 m deep uniform level-ground sand deposit is simulated with full hydro-mechanical coupling to establish the benchmark extent of liquefaction zone. In parallel, the analyses are repeated assuming drained conditions to compute the irregular time-histories, which are then converted to an equivalent number of uniform amplitude cycles. The constant amplitude series are then applied in single element simple shear test simulations, with initial conditions those corresponding to the 7 m depth in the deposit. The results in terms of the predicted triggering of liquefaction are contrasted to the

Licuefacción sísmica ha sido uno de los mayores peligros para proyectos de ingeniería atribuidos a terremotos. Las metodologías comúnmente utilizadas para evaluar licuefacción siguen basándose en métodos semi-empíricos. El procedimiento simplificado para evaluar el potencial de licuefacción asume que los terremotos pueden ser representados por un número equivalente de ciclos uniformes, proporcionales a la aceleración superficial máxima de un registro. Las principales metodologías para calcular el número equivalente de ciclos están basadas en la regla de daño acumulado de Miner, desarrollada para el análisis de fatiga en metales. La aplicabilidad de este concepto en suelos ha sido ampliamente cuestionada, dado que los suelos tienen un comportamiento no lineal. El presente trabajo investiga numéricamente el concepto de ciclos de amplitud uniforme equivalentes. Se efectúan análisis de esfuerzos no lineales en elementos finitos utilizando un modelo constitutivo de plasticidad de superficie límite que permite simular realísticamente licuefacción, reproduciendo precisamente la resistencia cíclica de las arenas. Bajo un análisis hidro-mecánico acoplado, se estudia un depósito de arenas de 15 m de profundidad sujeto a solicitaciones sísmicas, a fin de establecer una referencia de los alcances de la licuefacción. En paralelo, se desarrollan análisis de respuesta drenados para obtener los registros irregulares de esfuerzo, los cuales son convertidos en registros cíclicos uniformes. Se efectúan simulaciones de ensayos de corte simple no drenado sometidos a los registros uniformes generados. Los resultados de los ensayos de corte simple se comparan



predictions of the fully coupled benchmark analyses at the corresponding depth to assess the validity of the Seed et al. (1975) methodology, based on Miner's cumulative damage concept.

Keywords: liquefaction, simplified procedure, finite element analyses, equivalent number of cycles, bounding surface plasticity

con los modelos en términos de licuefacción y se concluye sobre la validez de la metodología de Seed et al. (1975), basada en el concepto de daño acumulado de Miner.

Palabras clave: licuefacción, metodología simplificada, análisis de elementos finitos, número equivalente de ciclos, plasticidad de superficie límite

Introduction

Soil liquefaction constitutes one of the major hazards relating to earthquakes for civil engineering projects and since its acknowledgement at the beginning of the XX century (Hazen, 1920), it has been one of the main areas of focus in the field of geotechnical earthquake engineering.

Many procedures have been developed over the years in order to assess the triggering of liquefaction in a sand deposit subjected to seismic loading. Different approaches exist: qualitative procedures, based on identifying ranges of particle size distribution (Tsuchida, 1970) or based on ageing and geomorphology (Youd and Perkins, 1978); semi-empirical procedures that rely on observations of ground surface manifestation of liquefaction and field testing (Seed and Idriss, 1971; Youd *et al.*, 2001; Idriss and Boulanger, 2008); numerical methods using advanced constitutive models which can realistically simulate pore water pressure build up and liquefaction (Papadimitiou and Bouckovalas, 2002; Taborda, 2011).

The semi-empirical procedures, which are based on information gathered from past earthquake events, prevail in practice. These procedures have evolved over the years incorporating different aspects of the dynamic performance of different types of soils and the nature of the seismic events, but all are in line with the original simplified procedure proposed by Seed and Idriss in the early 70's.

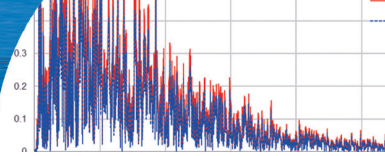
One of the key assumptions in the simplified procedure is the representation of the irregular loading patterns of the design earthquake by an equivalent number of uniform stress amplitude cycles (N_{eq}). This is captured through the magnitude scaling factors that are used to scale the seismic demand or the cyclic strength of soils for the calculation

of the factor of safety against liquefaction. Perhaps the most common procedure to count the equivalent number of cycles is that proposed by Seed and his colleagues (Seed *et al.*, 1975), consisting of applying weighting factors for stress levels of different percentage of the peak magnitude present in the irregular time-history. This method is based on the cumulative damage rule proposed by Miner (1945), originally developed to study low cycle fatigue conditions in metals.

Shen *et al.* (1978) carried out a series of cyclic undrained triaxial element tests on sand samples of different relative density (D_r), subjected to different random loading patterns. Through these they concluded that the resistance to liquefaction strongly depends on the loading path and sequence of loading cycles, particularly for the denser samples, suggesting that Miner's cumulative damage rule may not be applicable to soils undergoing large amplitude cycles during which the response is mainly non-linear elasto-plastic.

Recent laboratory studies on sand's liquefaction resistance (Azeitero *et al.*, 2012; Coehlo *et al.*, 2013) have shown that a similar dependency of the cyclic strength on the loading sequence exists even for loose samples. More recently, these laboratory findings were corroborated by finite element (FE) analyses, which further concluded that the equivalent number of cycles concept could lead to non-conservative results in terms of liquefaction resistance (Tsaparli, 2015).

The present work extends the study of Tsaparli *et al.* (2015) on the equivalent number of uniform amplitude cycles concept of the Seed *et al.* (1975) methodology, by investigating numerically, through use of the Imperial



College Finite Element Program ICFEP (Potts and Zdravkovic, 1999) the response of sand to liquefaction when subjected to three different types of ground motions of very different characteristics. An advanced constitutive model (Taborda *et al.*, 2014), which has been modified to accurately capture the variation of cyclic strength with the level of applied cyclic stresses (Tsaparli, 2017) further increases the robustness of this study in examining the applicability of the Miner's concept in soils.

Numerical procedure for the evaluation of the equivalent number of cycles concept

The general outline of the numerical method implemented to assess the simplified liquefaction procedure, focussing on the equivalent number of uniform amplitude cycles concept, consists of two types of FE site response analyses of a 15 m deep level ground fully saturated Nevada sand deposit, as well as single element simple shear test simulations with initial conditions those at 7 m depth in the deposit:

1. The first type of site response analysis uses full hydro-mechanical coupling to assess the depth of the liquefaction zone. This is used as benchmark in terms of triggering of liquefaction and is compared against the predictions of the empirical assessment (Idriss and Boulanger, 2008).
2. The second type of site response analysis assumes drained conditions to obtain the shear stress time history at mid-depth in the deposit. This excludes any effects of excess pore water pressure development and, in agreement with (Youd *et al.*, 2001), it is used for the conversion to an equivalent number of uniform amplitude cycles. The shear stress time-history at mid-depth is used rather than the acceleration time-history at ground surface, as is usually the case, for increased accuracy in the conversion to N_{eq} .
3. Single element FE simulations of undrained cyclic simple shear tests with initial conditions those at 7 m depth in the Nevada sand deposit are then carried out. These are subjected to the equivalent number of cycles as obtained from step 2. Based on Seed *et al.* (1975), the predictions of the single element tests in terms of triggering of liquefaction should agree with that

at mid-depth of the 15 m deep deposit, as predicted by the corresponding benchmark fully-coupled site response FE analysis (step 1 above).

A more detailed description of each stage of the adopted numerical procedure is given in the following subsections.

Strong motion data

The ground motions chosen as input at the base of the FE model are the east-west (EW) component of the 20th May 1986 Lotung motion, in Taiwan (Elgamal *et al.*, 1995), the EW component of the 20th September 1999 Chi-Chi motion, in Taiwan (Tsai and Huang, 2000) and the EW component of the 26th July 2003 Kawasaki motion, Japan (Okada *et al.*, 2003). Each ground motion has very different characteristics, representing a shock-type, a transitional type and a vibratory-type motion, respectively. The former exhibits a pronounced peak, while the latter is expected to more closely represent the uniform amplitudes cycles series. All three acceleration time-histories were recorded in downhole arrays and are representative of bedrock motions. Each ground motion was further scaled to two different levels: one strong enough to induce full liquefaction at a depth of 7 m depth, defined by an excess pore water pressure ratio, r_u , greater than 95%, where r_u is equal to the excess pore water pressures, Δu , over the initial mean effective stress, p'_{v0} , and one weaker to induce marginal liquefaction at a depth of 7 m, with a r_u less than 80%. Drained site response analyses undertaking full liquefaction and marginal liquefaction at 7 m should provide N_{eq} , and the corresponding single element analyses should reflect the same behaviour. Table 1 summarizes the main characteristics for each ground motion, while Figure 1 to Figure 3 show the acceleration time histories and corresponding Fourier spectra for each record.

Table 1: Characteristics of input ground motions

	Peak acceleration, m/s ²	Magnitude	Duration, s	Scaling factor	Type
Lotung, 1986	0.81	$M_L = 4.2$	36	x1.5 / x2.0	Shock
Chi-Chi, 2003	0.41	$M_w = 7.3$	110	x1.2 / x1.5	Transitional
Kawasaki, 1999	1.13	$M_w = 5.6$	106	x10 / x12	Vibratory

M_w : moment magnitude/ M_L : Richter local magnitude

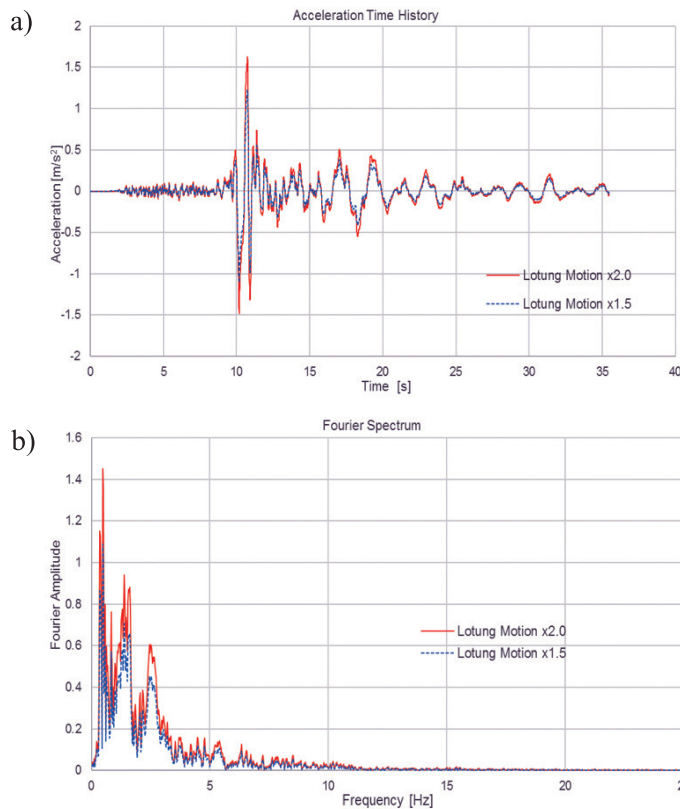


Figure 1: a) Acceleration record and b) Fourier spectrum for Lotung 1986 motion.

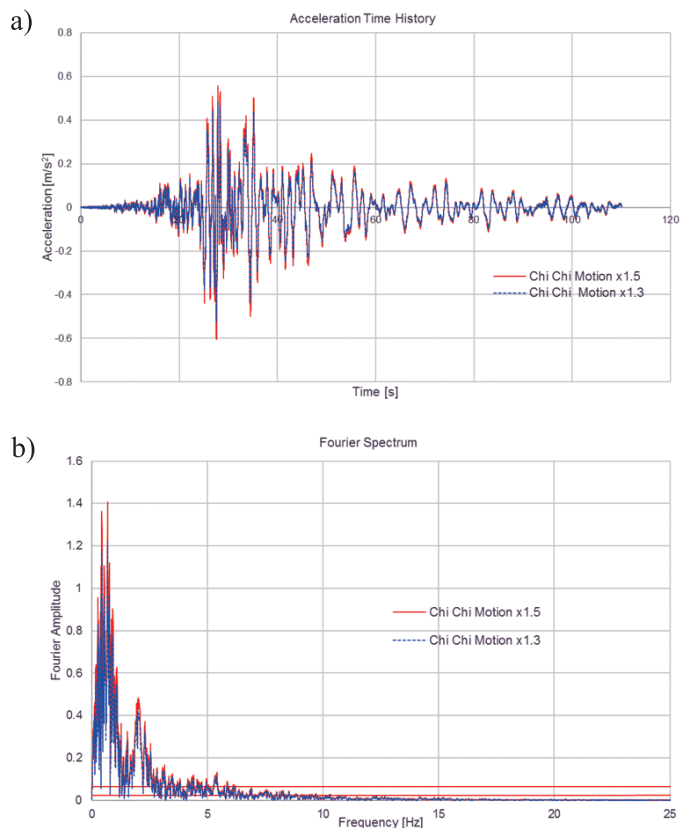


Figure 2: a) Acceleration records and b) Fourier spectrum for Chi-Chi 1999 motion.

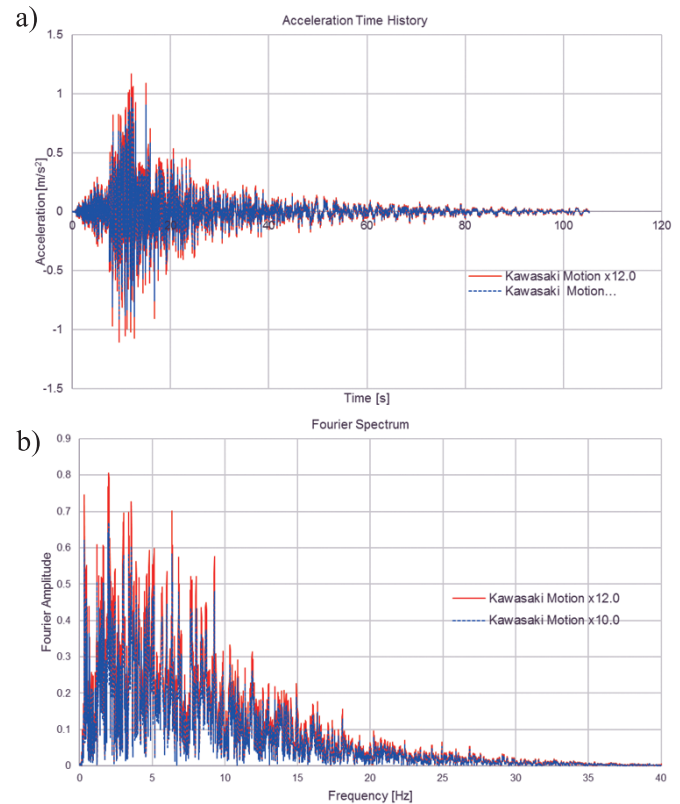


Figure 3: a) Acceleration records and b) Fourier spectrum for Kawasaki 2003 motion.

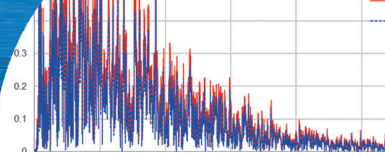
Methods for counting the number of cycles

Cycle counting methods are used to count the ranges or peaks of irregular time-histories. The rainflow range counting method (ASTM E1049, 2011) was utilised in this study to count the ranges of the shear stress time-histories, in accordance with the recommendations of Hancock and Bommer (2005). These ranges were then converted into peaks for the calculation of N_{eq} according to the Seed *et al.* (1975) procedure.

Equivalent number of cycles

Earthquake loading patterns are extremely irregular. The representation of earthquake seismic motions as a series of uniform amplitude cycles allows to simplify the conduction of element tests in the laboratory, which are carried out with the aim to investigate the response of soil under seismic conditions.

To convert an irregular time-history to an equivalent number of uniform amplitude cycles, the resulting peaks, as obtained in accordance to the previous section, are used following the process of application of weighting factors of Seed *et al.* (1975), described in detail in Tsaparli *et al.* (2015).



Finite element analyses

As aforementioned, site response FE analyses simulating a 15 m deep level ground fully saturated Nevada sand deposit, with a relative density of 40% are carried out. Two types of site response analyses are performed: the first one is the benchmark case that uses full hydro-mechanical coupling to assess the depth of the liquefaction zone. The second one assumes drained conditions in order to obtain the shear stress time history at mid-depth in the deposit, which is then used for the conversion to an equivalent number of uniform amplitude cycles.

The 3 different input motions, each scaled to two different levels, along with the two types of site response analyses resulted in a total of 12 site response FE analyses. Their characteristics are summarized in Table 2, along with the predictions of the peak ground acceleration (PGA) and the maximum shear stress, τ_{xy} , registered at 7 m depth in the drained analyses, as well as the maximum excess pore water pressure ratio, r_u , predicted at 7 m depth in the fully coupled analysis.

Subsequently, single element FE simulations of undrained cyclic simple shear tests with initial conditions those at 7 m depth in the Nevada sand deposit are carried out, under different confining pressures, representative of the different depths of the soil deposit. These are subjected to the equivalent number of cycles as obtained from the drained site response analyses, under uniform amplitudes at different percentages of the peak shear stress in the time-history. Based on Seed *et al.* (1975), the predictions of the single element tests in terms of triggering of liquefaction

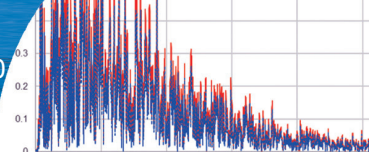
should agree with those at 7 m depth in the Nevada sand deposit, as predicted by the corresponding benchmark fully-coupled site response FE analysis. An excess pore water pressure ratio over 95% was once again used as the criterion to define the occurrence of liquefaction.

The mechanical behaviour of sand is modelled using a bounding surface plasticity model, originally implemented in ICFEP in three-dimensional stress space by Taborda (2011) and Taborda *et al.* (2014). The modifications to the flow rule and the tensor representing the evolution of soil fabric for accurate prediction of cyclic strength at varying CSRs and overburden effects, as well as better compliance following a dilative response and subsequent unloading, as described in Tsaparli (2017), have also been included. This consists of a significant improvement to Tsaparli *et al.* (2015), as accurate modelling of cyclic strength is of outmost importance when it comes to the type of study presenting herein. The model requires as input 32-parameters and it was calibrated based on extensive element testing on Nevada sand. More details can be found in Tsaparli (2017).

The mesh generated for the site response analyses, of the 15 m depth sand deposit, consists on a column of 60 elements stacked up vertically. The type of element to be used corresponds to 8-noded quadrilateral elements with dimensions of 25 cm per side. The selection of the size of the elements was based on the recommendations of Bathe (1996) for 8 noded elements, in order to ensure that the mesh is fine enough, to avoid the filtering of short wave-length waves.

Table 2: Summary of the site response FE analyses

Motion	Scale factor	Motion type	Analysis	D_r , %	Duration, s	PGA, m/s ² (drained)	Max r_u , % at 7 m depth	Max τ_{xy} , kPa at 7 m depth
Lotung	1.5	Shock	Coupled	40	35.46		79.64	-
Lotung	1.5	Shock	Drained	40	35.46	2.540	-	29.20
Lotung	2.0	Shock	Coupled	40	35.46		100.00	-
Lotung	2.0	Shock	Drained	40	35.46	3.350	-	30.20
Chi-Chi	1.3	Transitional	Coupled	40	110.07		75.98	-
Chi-Chi	1.3	Transitional	Drained	40	110.07	1.106	-	11.60
Chi-Chi	1.5	Transitional	Coupled	40	110.07		100.00	-
Chi-Chi	1.5	Transitional	Drained	40	110.07	1.244	-	13.20
Kawasaki	10.0	Vibratory	Coupled	40	105.30		74.15	-
Kawasaki	10.0	Vibratory	Drained	40	105.30	1.194	-	10.00
Kawasaki	12.0	Vibratory	Coupled	40	105.30		100.00	-
Kawasaki	12.0	Vibratory	Drained	40	105.30	1.274	-	11.30



The boundary conditions in the site response analysis consisted in tied degrees of freedom for all node along the vertical boundaries of the mesh to ensure 1D conditions (Zienkiewicz *et al.*, 1988). The vertical displacement was fixed by restricting vertical movements from the bottom boundary of the mesh. For each motion, the acceleration was transformed into incremental, and it was applied that way at the base of the mesh.

A modified Newton-Raphson integration scheme was used, with a sub-stepping stress point algorithm based on the non-linear solver (Potts and Zdravkovic, 1999). The integration scheme used was the generalized alpha method of Chung and Hulbert (1993) based on a spectral radius of $\rho_{\infty} = 0.818$. A time step of $\Delta t = 0.01$ s was found to be small enough to achieve an accurate solution for all the motions. This combination would allow to minimize the numerical damping and the period elongation.

Results

Based on the fully coupled analysis Lotung x1.5 motion

was selected to marginally liquefy at a depth of 7 m with a pore water pressure ratio r_u of 79.64%. Nevertheless, the associated single element (SE) analyses with lower CSR evidenced liquefaction ($r_u = 95\%$), while higher CSR did not, as the equivalent number of cycles were not enough to build up pore pressure to trigger liquefaction.

The coupled site response analysis using the Lotung x2.0 motion presented full liquefaction ($r_u = 100\%$) up to a depth of 9 m approximately. Surprisingly, the SE analyses yield the same results in terms of liquefaction of Lotung x1.5 motion, only lower CSR reached liquefaction. The results for both motions are presented in Table 3.

The Chi-Chi x1.3 motion was selected to marginally liquefy ($r_u = 75.98\%$), the SE analyses presented liquefaction on the lower CSR (higher number of cycles) only, similar to Lotung motions. On Chi-Chi x1.5, all SE analyses had enough cycles to reach the r_u criterion, in conformity to the associated site response analysis that showed full liquefaction ($r_u = 100\%$) at the analysed depth. The results for both motions are presented in Table 4.

Table 3: Lotung motion single element DSS test results for equivalent number of cycles.

Lotung x1.5 motion – Marginal liquefaction					Lotung x2.0 motion – Full liquefaction				
% of max. τ_{xy}	Max τ_{xy} , kPa	CSR	Equivalent N° of cycles	Cycles to $r_u = 95\%$	% of max. τ_{xy}	Max τ_{xy} , kPa	CSR	Equivalent N° of cycles	Cycles to $r_u = 95\%$
0.65	19.0	0.43	1.57	-	0.65	19.6	0.44	0.64	-
0.40	11.7	0.26	41.73	2.50	0.4	12.1	0.27	16.99	2.50
0.50	14.6	0.33	7.30	2.00	0.5	15.1	0.34	2.97	2.00
0.70	20.4	0.46	1.25	-	0.7	21.1	0.48	0.51	-
0.80	23.4	0.53	0.91	-	0.8	24.2	0.55	0.37	-
0.90	26.3	0.59	0.65	-	0.9	27.2	0.61	0.27	-
1.00	29.2	0.66	0.52	-	1	30.2	0.68	0.21	-

Table 4: Chi-Chi motion single element DSS test results for equivalent number of cycles.

Chi Chi x1.3 motion – Marginal liquefaction					Chi Chi x1.5 motion – Full liquefaction				
% of max. τ_{xy}	Max τ_{xy} , kPa	CSR	Equivalent N° of cycles	Cycles to $r_u = 95\%$	% of max. τ_{xy}	Max τ_{xy} , kPa	CSR	Equivalent N° of cycles	Cycles to $r_u = 95\%$
0.65	7.5	0.17	5.70	5.50	0.65	8.6	0.19	6.00	4.00
0.40	4.6	0.10	151.97	14.50	0.4	5.3	0.12	159.95	10.55
0.50	5.8	0.13	26.60	9.00	0.5	6.6	0.15	27.99	7.00
0.70	8.1	0.18	4.56	4.50	0.7	9.2	0.21	4.80	3.50
0.80	9.3	0.21	3.32	-	0.8	10.6	0.24	3.50	3.00
0.90	10.4	0.24	2.37	-	0.9	11.9	0.27	2.50	2.50
1.00	11.6	0.26	1.90	-	1	13.2	0.30	2.00	2.00

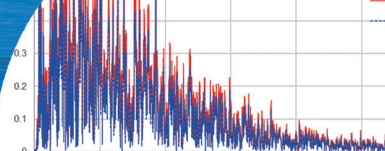


Table 5: Kawasaki motion single element DSS test results for equivalent number of cycles.

Kawasaki x10.0 motion – Maginal liquefaction					Kawasaki x12.0 motion – Full liquefaction				
% of max. τ_{xy}	Max τ_{xy} , kPa	CSR	Equiva- lent N° of cycles	Cycles to $r_u = 95\%$	% of max. τ_{xy}	Max τ_{xy} , kPa	CSR	Equivalent N° of cycles	Cycles to $r_u = 95\%$
0.65	6.5	0.15	18.08	7.00	0.65	7.5	0.17	16.77	5.50
0.40	4.0	0.09	482.13	23.00	0.4	4.6	0.10	447.07	14.50
0.50	5.0	0.11	84.37	12.00	0.5	5.8	0.13	78.24	9.00
0.70	7.0	0.16	14.46	6.00	0.7	8.1	0.18	13.41	4.50
0.80	8.0	0.18	10.55	5.00	0.8	9.3	0.21	9.78	3.50
0.90	9.0	0.20	7.53	4.00	0.9	10.4	0.24	6.99	3.00
1.00	10.0	0.22	6.03	3.00	1	11.6	0.26	5.59	2.50

Finally, the Kawasaki motion, the most vibratory of the three studied records, both x10.0 and x12.0 scaling factors, exhibited liquefaction in all SE analyses. In general, results are very conservative, but not accurate, particularly for Kawasaki x10.0 motion. The results of Kawasaki motions that marginally and fully liquefy are presented in Table 5.

In parallel, the results of the fully coupled benchmark analyses at 7 m depth were checked against the predictions of the empirical simplified procedure (Idriss and Boulanger, 2008). The results obtained in the simplified procedure are similar to the SE analyses. Similarly, non-conservative results have been found for shock type motions.

Conclusions

The Seed *et al.* methodology and Miner's concept of cumulative damage for soils has been studied numerically through a benchmark study. The first impression of the single element DSS simulations results was that for the same motion, the equivalent number of cycles at different stress levels by Seed *et al.* (1975) methodology, the response was quite different. This observation was particularly accentuated for the Lotung shock type motions, associated with a reduced number of cycles.

Previous research (Biondi *et al.*, 2004), observed the equivalent cycles methodologies do not comply good enough for very low CSR, while for high CSR, has been proved how conservative is showing that is not critical. This observation does not agree with what was observed particularly on the shock type motion.

In general, the simplified procedure presents very conservative results to the benchmark site response analyses, but in agreement with the SE analyses, for the analysed shock type motion safety factors over 1.0, were

obtained on motions that exhibited full liquefaction on the coupled site response analyses.

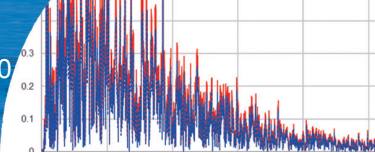
In a liquefaction point of view, the Seed *et al.* (1975) methodology has evidenced to be motion dependant. Shock type motions have evidenced non-conservative results while for vibratory type motions the findings are the opposite, very conservative.

Future research should include a detailed study on the performance of the bounding surface plasticity model on boundary value problems using high density on material properties. This would require an extensive advanced laboratory testing using high relative densities.

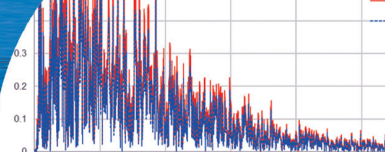
Further work should be carried out, considering new energy-based approaches instead of equivalent number of cycles, to perform liquefaction assessments. A recent investigation has shown better and more accurate methods to capture the irregular effects of seismic loading (Pan and Yang, 2019), instead of the early presented equivalent methodology (Seed *et al.*, 1975).

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