

IMPERIAL COLLEGE LONDON

Texture Characterisation in Metals using Diffuse Ultrasound

by

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Declaration of Originality

The content of this thesis is my own research work, which was completed under the supervision of Prof Michael J. S. Lowe and Dr Bo Lan. Appropriate references have been provided wherever the work of others has been used.

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02/10/25

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Abstract

In safety-critical engineering applications, such as aerospace components, the mechanical performance of polycrystalline metals is strongly influenced by crystallographic texture, which reflects the predominant orientation of grains. In Hexagonal Close-Packed (HCP) alloys for example, texture governs how properties such as strength vary with loading direction, giving rise to anisotropic behaviour, non-uniform stress distributions, and increased susceptibility to failure. Reliable texture measurement is therefore essential, particularly for components with complex geometries that cannot be readily characterised using conventional diffraction-based techniques limited to flat, well-prepared samples. Ultrasonic texture inversion offers bulk sensitivity by correlating directional wave velocities with texture, but its wider application is constrained by the difficulty of accurately measuring velocities in non-planar structures. This thesis develops a non-contact ultrasonic methodology based on diffuse field Green's Function (GF) reconstruction to determine travel times between two points, enabling bulk velocity measurement and crystallographic texture characterisation in complex engineering components.

The feasibility of using GF reconstruction for velocity measurement is demonstrated through experiments and simulations, providing insight into the principles governing velocity extraction from diffuse wavefields. Building on this, a robust measurement methodology is developed that incorporates signal processing to mitigate noise. The approach is validated using different receiver types and sample geometries, including curved surfaces, achieving a velocity error below 10 ms^{-1} , and demonstrating its potential for practical, non-destructive measurements in complex components.

Finally, the method is applied to a textured titanium specimen with curved geometry. Directional velocities reconstructed from diffuse fields are input into an established ultrasonic

inversion algorithm, and the resulting texture shows strong agreement with Electron Back Scatter Diffraction (EBSD) measurements. Quantitative comparison of the pole figures yields 92% similarity, demonstrating successful texture characterisation even in components with complex geometries. These findings highlight the potential of this approach for reliable in situ assessment, supporting manufacturing quality assurance of safety-critical HCP alloys and related materials.

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List of Acronyms

1B	One-Bit
2D	Two-Dimensional
3D	Three-Dimensional
CAD	Computer-Aided Design
CNC	Computer Numerical Control
CP Ti	Commercially Pure Titanium
DoF	Degrees of Freedom
EBS	Electron Back Scatter Diffraction
ED	Extrusion Direction
EMATs	Electro-Magnetic Acoustic Transducers
FE	Finite Element
FMC	Full Matrix Capture
FoM	Figure of Merit
GF	Green's Function
HCP	Hexagonal Close-Packed
MTRs	Microtextured Regions

NDE	Non-Destructive Evaluation
ND	Normal Direction
ODF	Orientation Distribution Function
PA	Phased Array
PCC	Pearson Correlation Coefficient
PDFs	Probability Density Functions
PLM	Polarised Light Microscopy
RMS	Root Mean Square
SAW	Surface Acoustic Wave
SCECs	Single-Crystal Elastic Constant
SDH	Side-Drilled Hole
SF	Spectral Flatness
SH	Spherical Harmonics
SNR	Signal-to-Noise Ratio
SRAS	Spatially Resolved Acoustic Spectroscopy
TD	Transverse Direction
TFM	Total Focusing Method
Ti-64	Titanium-6Al-4V
ToF	Time-of-Flight
TR	Time Reversal
XRD	X-Ray Diffraction
Zr	Zirconium

Contents

1	Introduction and Literature Review	29
1.1	Motivation	29
1.2	Literature review	31
1.2.1	Crystal anisotropy	32
1.2.2	How is texture presented?	37
1.2.3	Texture measurement methods	39
1.2.4	Diffuse field and Green's Function Reconstruction	44
1.3	Thesis Aim and Outline	46
1.4	Industrial Relevance	47
2	Exploration of Green's Function Reconstruction from Diffuse Fields for Velocity Measurement	49
2.1	Introduction	49
2.2	Green's Function Reconstruction Theories	50
2.3	Case Study I: Ultrasonic Imaging using Diffuse Field Full Matrix Capture .	52
2.3.1	Single array setup	53
2.3.2	Dual array setup	56
2.3.3	Effectiveness evaluation of noise-reduction reconstruction strategies	57
2.4	Case Study II: Velocity Measurement in a Rectangular Block: Role of Geometry in Wave Diffusion	62

2.5	Case Study III: Numerical-Based Analysis of Green's Function Reconstruction for Velocity Measurement	66
2.5.1	Defining the onset of the diffusive regime via simulation	67
2.5.2	Velocity measurement of different wave modes	69
2.6	Summary	72
3	Accurate Velocity Measurement from Diffuse Fields for Texture Characterisation	75
3.1	Introduction	75
3.2	Strategies for Accurate Green's Function Reconstruction	76
3.2.1	Optimisation of diffuse field parameters	77
3.2.2	Deconvolution of a source-dependent factor	82
3.2.3	One-bit normalisation	84
3.3	Phase or Group Velocity?	87
3.4	Experimental Implementation	89
3.4.1	Contact array receivers	90
3.4.2	Non-contact laser receivers	97
3.5	Evaluation of Velocity Measurement Method	102
3.6	Summary	103
4	Texture Characterisation via Velocity Inversion	105
4.1	Introduction	105
4.2	Texture Inversion Formula	106
4.3	Experimental Setup	108
4.4	Numerical Simulations	110
4.5	Experimental Results	118
4.6	Discussion	123
4.7	Summary	128

5 Conclusions 129

5.1 Thesis review 129

5.2 Future work 131

A Definition of Source-Dependent Term for Deconvolution 135

References 138

List of Publications 149

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List of Figures

- 1.1 Polycrystals with different textures, where arrows represent the orientation of the crystal stiffness tensor. **a** Random orientations result in no texture. **b** A strong preferred orientation to the right represents a strong texture. 30
- 1.2 Common crystallographic planes in a HCP single crystal. **a** Basal plane (0001) showing the ABAB stacking sequence and lattice constants a (in-plane) and c (axial). **b** Prismatic plane ($10\bar{1}0$). **c** Pyramidal plane ($10\bar{1}1$). 32
- 1.3 Comparison of phase and group velocity vectors in different media. **a** In an isotropic medium, phase velocity (V) and group velocity (V_g) are collinear, with wavefronts and energy flux propagating in the same direction. **b** In an anisotropic medium, phase and group velocity vectors diverge, with the group velocity indicating the direction of energy transport while the phase velocity remains normal to the wavefront (n). 37
- 1.4 Schematic of a pole figure generation. **a** Illustration of stereographic projection for the (0001) pole of a single HCP crystal. **b** The resulting pole figure, where the projected point (red dot) represents the orientation of the (0001) pole relative to the sample axes. 38
- 1.5 Illustration of Bragg diffraction for a monochromatic X-ray beam incident on parallel lattice planes with interplanar spacing d . When the Bragg condition $n\lambda = 2d \sin \theta$ is satisfied, constructive interference produces a diffracted beam emerging at angle θ relative to the lattice planes. 40

1.6	a Schematic of EBSD technique, which operates by detecting electrons backscattered from lattice planes in a sample tilted at approximately 70° . b A representative Kikuchi pattern recorded on the EBSD detector, where the intersecting bands arise from the diffraction of electrons by different crystallographic planes within the sample [1].	41
1.7	Schematic of SRAS technique which uses a laser-generated surface acoustic wave grating to map SAW velocities and infer crystallographic orientations across a sample surface [2].	42
1.8	Schematic of the ultrasonic texture measurement setup which relies on directional velocity measurements. a Immersion-based arrangement with velocities measured using a double through-transmission transducer configuration. b Illustration of how sample tilt and rotation are combined to achieve the required directional coverage. [3]	43
2.1	Schematic illustrating the setups used for coherent and diffuse field measurements. a illustrates the setup used for typical coherent field measurements, where the coherent response is captured at location j from a source excitation at location i . b illustrates the setup used for diffuse field measurements, where diffused fields at locations i and j are captured from a source excitation at location s	50
2.2	Schematic of experimental setup for ultrasonic imaging using diffuse FMC with one array transducer. The dashed outline denotes the region encompassed by the TFM image.	53
2.3	A typical time-domain diffuse signal, characterised by two parameters: transmission delay t_r and window length T , used to define the region of the diffuse field for GF reconstruction.	54

2.4	Comparison of TFM images obtained from both coherent and GF reconstruction method with one array transducer. a depicts the TFM image generated from conventional, coherent FMC with 64 elements. b displays TFM image generated from the reconstructed FMC dataset using diffuse fields from 64 source elements, with each signal averaged 16 times and using cross-correlation parameters $t_r = 50 \mu s$ and window length of $T = 60 \mu s$	54
2.5	Schematic of the experimental setup for ultrasonic imaging using diffuse FMC dataset with two array transducers. The dashed lines indicate the region encompassed by the TFM image.	56
2.6	Comparison of TFM images obtained from the GF reconstruction method using two array transducers, where one transducer operated as the source and the other as the receiver at a separate location. a depicts the TFM image generated with averaging over 32 sources. b displays the corresponding TFM image generated using a single source.	57
2.7	Schematic illustrating the three noise-reduction strategies. a Averaging over multiples sources. b Averaging over multiple non-overlapping time windows. c Increasing window length.	58
2.8	Plot of SNR against number of source averages. The red cross indicates the point of maximum SNR.	59
2.9	a Plot of SNR against number of time delays of window length $20 \mu s$. b Plot of SNR against window length. The red cross indicates the point of maximum SNR.	60

2.10	Schematic illustrating the effect of averaging across multiple sources on GF reconstruction compared with extending the window length for a single source. a With multiple source locations, a greater variety of propagation paths and scattering events contributes to a richer diffuse field. b When only the window length is increased for a single source, the same propagation path is reused, so path diversity remains limited. The thickness of the arrow line between the two receivers at location I and J represents the degree of convergence of the reconstructed GF.	61
2.11	Schematic of experimental setup for reconstructing velocities measured in different directions with array receivers.	63
2.12	Comparison of velocities measured from coherent signals in blue and reconstructed signals in orange across 32 measurement positions. a Normal incidence case. b Oblique incidence case.	64
2.13	Comparison of reconstructed first-arrival tonebursts at measurement angles of 0° in blue and 38° in orange.	65
2.14	Finite element model to identify the transition to diffuse fields and investigate the effect of receiver orientation on the type of velocity measured, with reference to a pitch-catch setup between locations I and J	67
2.15	Wavefield diffusion illustrated through angular histograms and statistical measures. a , b and c show polar probability density plots of propagation directions at $1 \mu s$, $10 \mu s$ and $60 \mu s$, respectively. d shows the temporal evolution of the normalised angular standard deviation of the Probability Density Functions (PDFs), with vertical red dashed lines indicating the times corresponding to the polar plots.	68
2.16	Comparison of coherent first-arrival toneburst and the respective reconstructed signal. a captures the longitudinal toneburst. b captures the shear toneburst.	70

3.1	Comparison of TFM images and time-domain signals obtained from both coherent and GF reconstruction methods. a depicts a TFM image generated from coherent signals, revealing a defect at a depth of 19 mm. b displays the corresponding TFM image generated using the GF reconstruction method. c compares the pulse-echo responses obtained by one element of the array from both methods, where the wave pulse at $6.5 \mu s$ represents the reflection from the defect. The red dashed lines indicate the x-coordinate where the pulse-echo response is obtained.	77
3.2	Comparison of the energy density curves of signals detected by a piezoelectric array element (in black) and a laser vibrometer (in red).	79
3.3	FE model to simulate the GF reconstruction process from diffuse fields. a shows the schematic of the sample modelled using FE. Diffuse fields were collected at location I and J to reconstruct the coherent response received at J from I . b compares the reconstructed signal as a result of only cross-correlation against the coherent response. c compares the reconstructed signal as a result of both cross-correlation and deconvolution against the coherent response.	83
3.4	Simulated diffuse signals, along with reconstructed GF, and their phase information with One-Bit (1B) normalisation. a overlays the signals before and after 1B normalisation. b displays two initially identical signals, with one experiencing attenuation. c compares the phase spectrum of the unattenuated signal and the binary form of the attenuated signal in b . d compares the reconstructed first-arrival from the unattenuated signal, attenuated signal and the binary form of the attenuated signal in b . e compares the phase spectrum of the reconstructed first-arrivals seen in d . .	85

3.5	Velocity measurement of longitudinal and shear vertical wave modes from GF. a shows an example of the time-domain GF after undergoing inverse Fourier Transform from the corresponding Three-Dimensional (3D) GF in the frequency domain. b compares the velocities of different wave modes calculated using arrival times extracted from time-domain signals, similar to the one in a , in different directions, to the respective group and phase velocity values from Christoffel's equation.	89
3.6	Schematic of experimental setup for reconstructing and validating velocities measured in different directions with array receivers	91
3.7	Coherent response of the 20th receiving array element and the respective reconstructed signal. a compares the reconstructed signal as a result of only cross-correlation against the coherent response. b compares the reconstructed signal as a result of both cross-correlation and deconvolution against the coherent response.	92
3.8	Experimental considerations for defining the source-dependent factor that requires deconvolution for accurate GF reconstruction. a provides a close-up view of the propagation path by the excitation waveform by an array element to define the source-dependent factor in the experimental setup. b shows the setup required to approximate the source-dependent factor. . .	93
3.9	Comparison of group velocity measured from coherent signals (in blue) and reconstructed signals (in orange) at 32 different measurement angles. The yellow data points (with axis scale on the right hand side) represent the percentage difference in mean velocity between the coherent and GF reconstruction method at the corresponding measurement angle.	96
3.10	Schematic of experimental setup for GF reconstruction with laser receivers. a illustrates the measurement setup by showing how measurements are conducted in five different directions using laser receivers. b depicts the synchronisation of the Phased Array (PA) controller with the OFV-5000 vibrometer controller to ensure simultaneous excitation from the array transducer and reception from the laser receivers.	98

3.11	Comparison of measured longitudinal group velocity from diffuse field reconstruction (orange data points) in a. five different normal incidence directions and b. one oblique direction with ground truth (solid grey line).	100
4.1	Step-by-step schematic provided to the manufacturer for producing the textured Commercially Pure Titanium (CP Ti) sample. Red dashed lines indicate the cuts made at different stages of the manufacturing process. The 14×7 mm surface exposed after the final cut was used for EBSD measurement. The sample system is defined by the Extrusion Direction (ED), TD and ND.	108
4.2	Schematic of the experimental setup used for GF reconstruction to measure velocities along 36 directions. Two laser receivers are positioned across the sample diameter to reconstruct the longitudinal velocity. Only the front-facing laser spot is shown due to the schematic orientation. The sample's z -axis is indicated, along with the polar angle (θ) and azimuthal angle (ϕ) used to define each measurement direction.	109
4.3	FE model to simulate the GF reconstruction process from diffuse fields and evaluate different excitation strategies considering the excitation path from the bottom flat surface of the wedge into the sphere. a shows the schematic of the sample modelled using FE. Diffuse fields were collected at location I and J to reconstruct the coherent response received at J from I . b compares the reconstructed signal against the coherent response, highlighting the longitudinal tone burst that is used to determine the wave velocity.	111
4.4	Snapshot of the wave field at $6.2 \mu\text{s}$ following excitation by a 3-element array source. At the curved wedge-sphere interface A, the incident longitudinal wave enters at an angle, resulting in mode conversion. This leads to the emergence of refracted longitudinal, shear, and surface wave modes within the CP Ti sphere.	112

4.5	Comparison of $FoM_{excitation}$ as a function of subaperture size for different excitation strategies: non-steered (blue), beam-steered (orange), and phase-shifted (yellow). Higher $FoM_{excitation}$ values indicate better reconstruction quality.	114
4.6	Close-up view of the wedge-sphere interface, illustrating how the incident angle of an array element excitation is defined. Snell's law is applied at this interface to determine the resulting wave modes generated in the titanium sphere, including refracted longitudinal, refracted shear, and the refracted surface wave generated from the longitudinal excitation in Rexolite.	115
4.7	Comparison of $FoM_{excitation}$ versus subaperture size, showing the effect of attenuation and noise on non-steered (green) and steered (pink) excitations, relative to the noise-free case. Higher values indicate better reconstruction quality.	117
4.8	Reconstructed signal between two laser receivers positioned at diametrically opposite ends of the CP Ti spherical sample along the measurement direction $\theta = 76.1949^\circ$, $\phi = 0^\circ$. The red box highlights the reconstructed longitudinal toneburst, which is used to determine the wave velocity.	119
4.9	Pole figures obtained from a GF reconstruction methodology and b EBSD, with EBSD results truncated to fourth SH order. The sample system is defined by the ED, TD, ND.	121
4.10	EBSD scan of a region from the 14×7 mm specimen, sectioned along the rod axis of the CP Ti rod from which the spherical sample was manufactured. The scan was acquired on the ED-TD plane, and the inverse pole figure colouring represents the crystallographic orientation of individual grains relative to the sample's TD.	122
5.1	Schematic overview of the research workflow, showing the progression from theoretical foundations and exploration work of Green's function reconstruction, through numerical and experimental validation of velocity extraction, to the final application in non-destructive texture characterisation.	130

5.2	Schematic illustrating how two single-axis lasers can be arranged to measure in-plane displacement at a point on the sample surface. The quantities shown are: d_{in} , the in-plane displacement; d_{out} , the out-of-plane displacement; and d_{45} , the displacement measured at a 45° angle, which contains contributions from both d_{in} and d_{out}	132
A.1	Schematic illustrating the setups used to describe the coherent response and the diffuse field correlation as a series of transfer functions. a illustrates the setup used to capture the diffuse fields at both location I and J from a source excitation at S to reconstruct the coherent response at J if there is a source at I . b illustrates the setup used to capture the coherent response at location J from a source excitation at location I	135

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List of Tables

4.1	Directional velocities of the CP Ti sample measured across 36 measurement directions, grouped by a 6×6 grid of polar (θ) and azimuthal (ϕ) angles. .	120
4.2	Single crystal elastic constants of CP Ti [4]. unit: GPa. $C_{66} = (C_{11}-C_{12})/2$	120
4.3	Sources of error and their potential impact of diffuse field GF velocity and texture measurements, along with experimental conditions under which they are assessed.	127

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Chapter 1

Introduction and Literature Review

1.1 Motivation

Metals are inherently polycrystalline, composed of many grains whose orientations collectively define the bulk behaviour of the material [5]. The statistical distribution of these orientations, known as crystallographic texture, is therefore a key factor governing anisotropic responses and has a dominant influence on the mechanical properties of polycrystals [6].

In recent decades, textures in HCP metals have attracted increasing attention, driven by the expanding use of this crystal structure in critical engineering applications. Titanium alloys are indispensable for aerospace structures [7–9], zirconium alloys (for example, Zircaloy) are widely used for nuclear fuel cladding [10], and magnesium alloys provide lightweight solutions for automotive and electronic components [11,12], where high specific strength, corrosion resistance, thermal stability, or low density are required. In such systems, macroscopic phenomena including directional variations in elastic modulus, yield strength, and fatigue resistance can often be traced directly to the underlying texture state of the HCP polycrystal [13–15]. Reliable texture measurement is therefore critical to predict the performance in these materials.

The strong anisotropy inherent to HCP crystal structures arises from their limited number of active slip systems, which encourages texture to develop during thermomechanical processing such as rolling, extrusion, forging, or plane-strain compression which impose

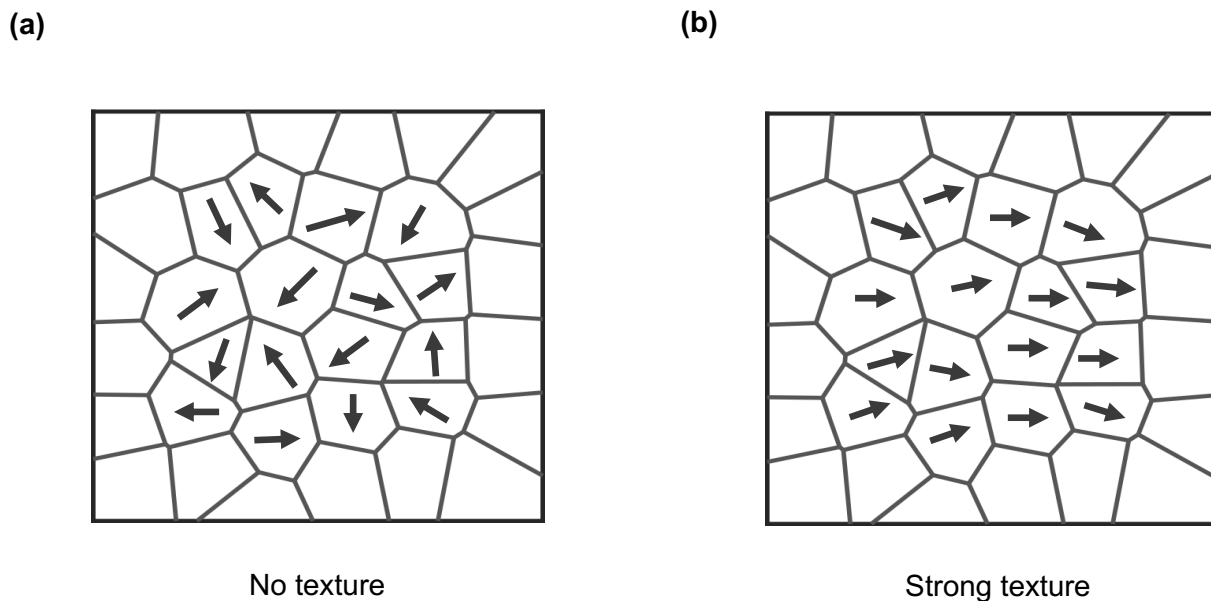


Figure 1.1: Polycrystals with different textures, where arrows represent the orientation of the crystal stiffness tensor. **a** Random orientations result in no texture. **b** A strong preferred orientation to the right represents a strong texture.

stresses that could align the crystal stiffness tensors of many grains along specific directions [5, 16, 17]. In titanium alloys, for instance, extrusion typically produces fibre textures, where many grains share a common orientation of their stiffness tensors relative to the processing direction [5], resulting in pronounced anisotropy in mechanical strength. Figure 1.1a illustrates a polycrystal with randomly oriented stiffness tensors and no texture, while Figure 1.1b shows a polycrystal with a strong preferred orientation, representative of a textured material. These processing routes are essential for producing safety-critical components with curved or irregular geometries, including turbine blades, pressure vessels, reactor tubing. In such final-form components, the resulting processing-induced textures are inherently difficult to assess because the complex, curved surfaces make many state-of-the-art texture measurement techniques impractical. In particular, these methods often require idealised sample geometries, flat surfaces, or physical sectioning to obtain meaningful measurements, which are incompatible with in-service, or high-value parts. Consequently, there is a critical need for techniques capable of measuring crystallographic texture directly on complex, curved surfaces and throughout the interior of bulk components.

To address this gap, ultrasonic waves provide a promising approach for texture character-

isation. They penetrate deeply into materials, giving information that is representative of the bulk, and their propagation is influenced by the underlying crystallographic structure, making them naturally sensitive to texture [18, 19]. Moreover, ultrasonic methods are non-destructive, require minimal surface preparation, and can be applied to components with complex geometries, as demonstrated by the use of laser-based ultrasound technique [20–23].

The motivation of this thesis is to bridge this gap by developing an ultrasonic-based methodology for bulk texture measurement that is non-contact, making it adaptable to complex geometries. Since texture plays a decisive role in the performance of safety-critical materials, establishing the capability to measure it reliably during or after manufacturing has the potential to greatly enhance quality assurance and inform material design for HCP alloys and related systems.

1.2 Literature review

With respect to the texture measurement problem, it is first necessary to establish the fundamental knowledge that underpins it. This starts with the fundamentals of HCP structures, their symmetry, and the resulting elastic and acoustic behaviour. Building on this foundation, the review then considers established texture measurement techniques such as X-ray diffraction and Electron Backscatter Diffraction, highlighting both their capabilities and their limitations when applied to bulk or irregularly shaped components. Alongside these, ultrasonic-based texture inversion is also an established method: it probes the bulk of the material and directly relates directional velocity measurements to crystallographic orientation. Although promising, its demonstrated use has so far been limited to immersion setups with parallel flat surfaces, even though the theoretical framework is inherently adaptable to arbitrary geometries. This limitation defines the challenge that the thesis seeks to address, while at the same time providing a solid foundation for further development. In this context, diffuse field approaches are introduced, with particular emphasis on GF reconstruction and its potential to evolve into a non-contact velocity measurement method for complex geometries, thereby enabling its extension to ultrasonic texture inversion.

1.2.1 Crystal anisotropy

This subsection provides a brief overview of HCP crystallography, with further details available in standard references such as [5].

HCP polycrystalline materials form the structural basis of many technologically important metals, including CP Ti, the α -phase of Titanium-6Al-4V (Ti-64), and Zirconium (Zr). The HCP structure is constructed from atomic layers arranged in a Two-Dimensional (2D) hexagonal network, stacked along the c -axis in an ABAB sequence such that every second basal plane is directly aligned, as illustrated in Figure 1.2a. The unit cell is characterised by two lattice parameters: the basal constant a , which sets the atomic spacing within the basal plane, and the axial constant c , which defines the interplanar separation between successive basal planes, also illustrated in Figure 1.2a.

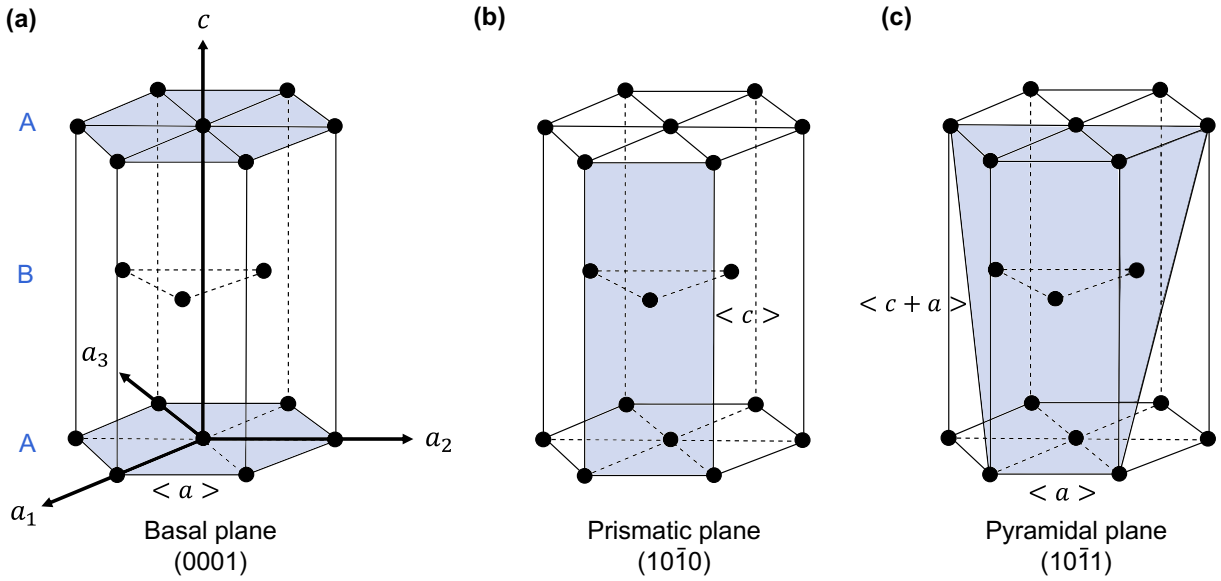


Figure 1.2: Common crystallographic planes in a HCP single crystal. **a** Basal plane (0001) showing the ABAB stacking sequence and lattice constants a (in-plane) and c (axial). **b** Prismatic plane (10 $\bar{1}$ 0). **c** Pyramidal plane (10 $\bar{1}$ 1).

In crystallography, atomic arrangements are described in terms of planes and their associated poles. A crystallographic plane is a flat, periodic arrangement of atoms, defined by its orientation relative to the crystal axes. In hexagonal systems such as HCP, this orientation is conventionally expressed using the four-index Miller-Bravais notation ($hkil$), which captures the three equivalent in-plane a -directions and the unique c -axis [24]. The corresponding crystallographic pole is the vector normal to the plane, representing its

orientation in space. In the HCP system, several families of planes are of primary importance: the basal planes $\{0001\}$, parallel to the hexagonal layers; the prismatic planes $\{10\bar{1}0\}$, oriented vertically and parallel to the c -axis; and the pyramidal planes $\{10\bar{1}1\}$, inclined to both the basal and prismatic planes. These primary planes are illustrated in Figure 1.2.

Plastic deformation in HCP crystals is primarily accommodated by slip on crystallographic planes and directions where atomic packing is relatively dense. Among these, the prismatic $\{10\bar{1}0\}$ planes usually provide the easiest slip mode and are considered the primary slip system. This preference arises because the c/a ratio in most HCP metals (≈ 1.58 for Ti and Zr, compared to the ideal 1.633) makes dislocation motion on prismatic planes energetically more favourable than on the basal $\{0001\}$ planes [25]. Basal slip may still occur, while pyramidal systems such as $\{10\bar{1}1\}$ are activated at higher stresses to accommodate additional deformation. At a fundamental level, this limited set of slip systems explains how preferred orientations develop during thermomechanical processing. The ability of a grain to deform is strongly controlled by the alignment of its slip planes and directions with the applied stress. Grains that are initially unfavourably oriented gradually rotate into orientations that permit easier dislocation glide, while those already well aligned deform more readily. Through this collective reorientation, crystallographic texture emerges, with specific poles, such as the c -axis in titanium alloys, becoming preferentially aligned according to the imposed processing stresses.

Among the planes in the HCP system, the (0001) basal plane and its associated c -axis pole are particularly significant, as they govern much of the mechanical and wave propagation behaviour. This arises from the underlying hexagonal symmetry, characterised by a six-fold rotational axis parallel to the c -axis, which makes all directions within the basal plane mechanically equivalent. At the same time, the elastic stiffnesses along the c -axis are significantly higher compared to those on the basal plane, preserving distinct mechanical and acoustic properties in that direction. Due to this symmetry, HCP crystals are classified as transversely isotropic. In general, the elastic behaviour of a crystal is described by the fourth-rank stiffness tensor C_{ijkl} , which has 81 components; symmetry considerations reduce this number to 21 independent constants for a fully anisotropic crystal. For HCP symmetry, the number of independent constants further reduces to only five, conveniently

expressed in Voigt notation as C_{11} , C_{12} , C_{13} , C_{33} , and C_{44} [26]. In this notation, the stiffness matrix of a hexagonal crystal takes the form:

$$\mathbf{C} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ C_{12} & C_{11} & C_{13} & 0 & 0 & 0 \\ C_{13} & C_{13} & C_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & C_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & C_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2}(C_{11} - C_{12}) \end{bmatrix}.$$

When placed in the broader context of crystal systems, HCP lies between the extremes of symmetry. On one side, cubic systems such as face-centred cubic and body-centred cubic are highly symmetric and therefore require only three independent elastic constants. On the other side, low-symmetry systems such as orthorhombic, monoclinic, and triclinic crystals demand many more constants to describe their elastic behaviour. HCP crystals fall between these cases; their symmetry is sufficient to reduce the elastic description, yet the unique c -axis still imparts a strong directional anisotropy.

These constants define how stresses and strains are coupled in different directions, offering a concise description of the elastic behaviour of the material. When introduced into the elastodynamic wave equation, they give rise to the Christoffel formulation, which governs the propagation of elastic waves in anisotropic media. Since ultrasound forms the basis for texture characterisation in this work, it is useful to briefly outline the derivation of this equation [27].

Wave propagation in elastic solids is governed by the elastodynamic wave equation

$$\rho \frac{\partial^2 u_i}{\partial t^2} = \frac{\partial \sigma_{ij}}{\partial x_j} \quad (1.1)$$

where i and j each take values from 1 to 3, x_j is the coordinate along the j -th axis, t is time, ρ is the mass density, u_i is the i -th component of the displacement vector, and σ_{ij}

denotes the stress tensor components. In anisotropic media, the stress is related to strain by the fourth-rank stiffness tensor C_{ijkl} through Hooke's law:

$$\sigma_{ij} = C_{ijkl} \epsilon_{kl}, \quad (1.2)$$

where the strain tensor components ϵ_{kl} are defined as

$$\epsilon_{kl} = \frac{1}{2} \left(\frac{\partial u_k}{\partial x_l} + \frac{\partial u_l}{\partial x_k} \right). \quad (1.3)$$

so that the governing equation becomes

$$\rho \frac{\partial^2 u_i}{\partial t^2} = \frac{1}{2} C_{ijkl} \left(\frac{\partial^2 u_k}{\partial x_j \partial x_l} + \frac{\partial^2 u_l}{\partial x_j \partial x_k} \right). \quad (1.4)$$

Since the stiffness is symmetric with respect to k and l , such that $C_{ijkl} = C_{ijlk}$, the equation simplifies to

$$\rho \frac{\partial^2 u_i}{\partial t^2} = C_{ijkl} \frac{\partial^2 u_l}{\partial x_j \partial x_k}. \quad (1.5)$$

Assuming a plane-wave solution of the form

$$u_i = A_i \exp \left(i \left(\xi n_j x_j - \omega t \right) \right), \quad (1.6)$$

where A_i is the polarisation vector, ξ is the wave number, ω the angular frequency, and n_j the j -th component of the unit vector n in the propagation direction, substitution into (1.5) yields

$$\rho\omega^2 A_i = C_{ijkl}\xi^2 n_j n_k A_l. \quad (1.7)$$

Introducing the phase velocity $v = \omega/\xi$ gives

$$\left(\Gamma_{il} - \rho v^2 \delta_{il}\right) A_l = 0, \quad (1.8)$$

where the Christoffel matrix is defined as

$$\Gamma_{il} = C_{ijkl} n_j n_k. \quad (1.9)$$

Equation 1.8 is commonly known as the Christoffel equation, an eigenvalue problem in which the stiffness tensor C_{ijkl} couples the propagation direction n with the phase velocity v and the corresponding particle polarisation A . Its three eigenvalues correspond to the squared phase velocities, while the eigenvectors define the associated polarisations. In anisotropic media these solutions appear as one quasi-longitudinal and two quasi-shear modes, with polarisations generally oblique to the propagation vector. For HCP crystals, the formulation further reveals that the longitudinal velocity is highest along the c -axis $[0001]$ and lower along directions in the basal plane $\langle 11\bar{2}0 \rangle$, reflecting the intrinsic elastic anisotropy between the c -axis and basal directions. In isotropic media, by contrast, the symmetry collapses the solutions into a single longitudinal mode and two indistinguishable shear modes, with polarisations strictly parallel or perpendicular to propagation. The Christoffel framework thus provides the essential basis for describing direction-dependent wave velocities and polarisations in anisotropic media.

Phase velocity refers to the speed at which a constant phase point on the wavefront propagates through the medium. Beyond this, group velocity is used to describe the transport of elastic wave energy, and it can be expressed in terms of the phase velocity, as derived in [28]:

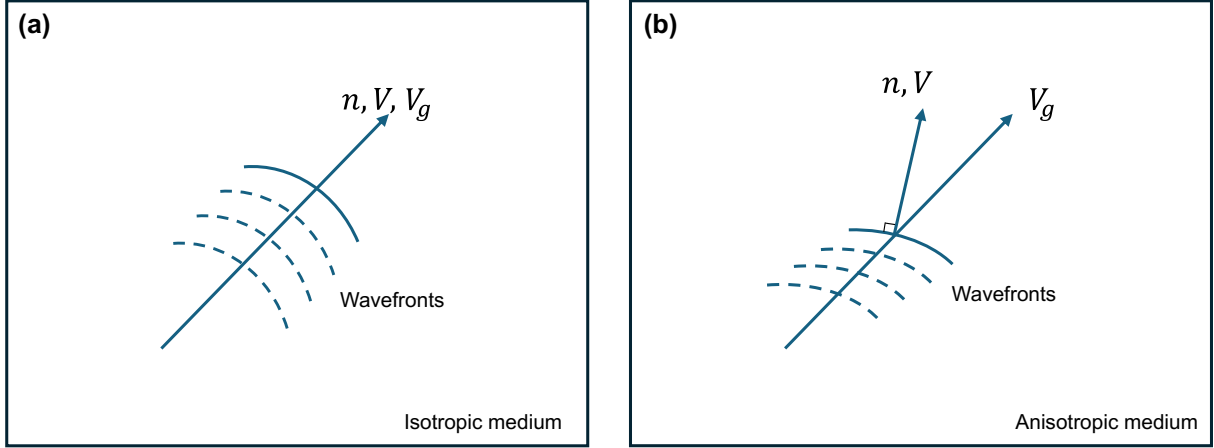


Figure 1.3: Comparison of phase and group velocity vectors in different media. **a** In an isotropic medium, phase velocity (V) and group velocity (V_g) are collinear, with wavefronts and energy flux propagating in the same direction. **b** In an anisotropic medium, phase and group velocity vectors diverge, with the group velocity indicating the direction of energy transport while the phase velocity remains normal to the wavefront (n).

$$V_{gi} = \frac{C_{ijkl} n_k p_l p_j}{\rho v} \quad (1.10)$$

where p is the polarisation vector and V_g is the group velocity. In isotropic media, phase and group velocities are collinear, so that wavefronts and energy flux propagate in the same direction as shown in Figure 1.3a. In anisotropic media, however, the two vectors diverge: the phase velocity remains normal to the wavefront, while the group velocity defines the actual direction of energy transport as shown in Figure 1.3b. This divergence between phase and group velocity is a key concept in ultrasonic studies of anisotropic medium, and the concept will be revisited in the §3.3.

1.2.2 How is texture presented?

The Orientation Distribution Function (ODF) and pole figures are among the most widely used methods for representing crystallographic texture. A clear understanding of these concepts is essential for the later chapter of this thesis, which will focus on texture characterisation. Therefore, the key background information on both representations is summarised here.

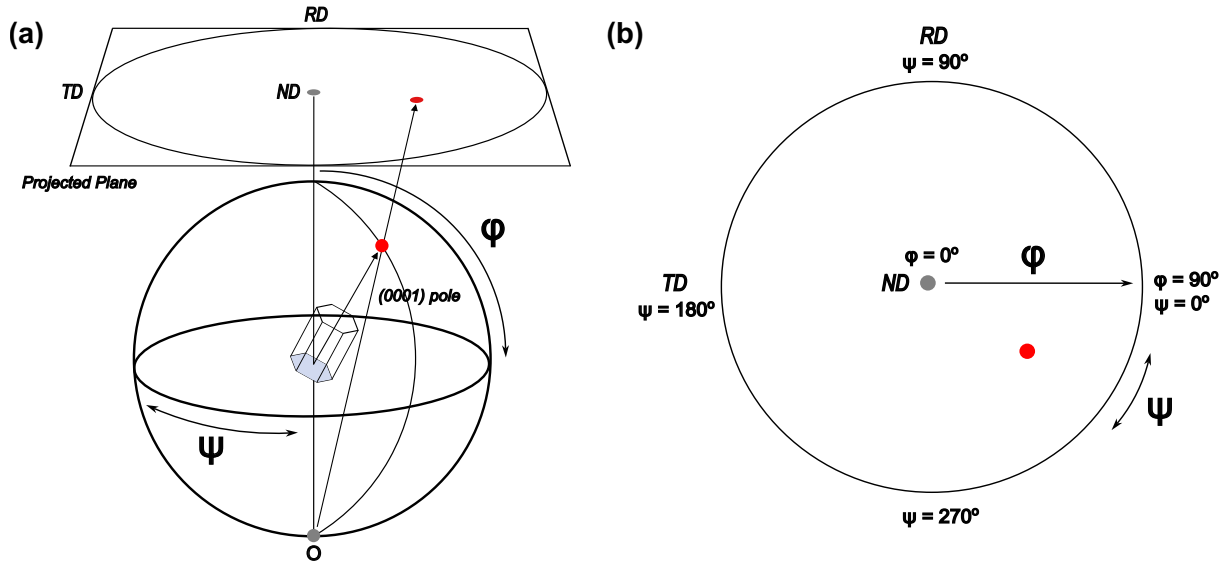


Figure 1.4: Schematic of a pole figure generation. **a** Illustration of stereographic projection for the (0001) pole of a single HCP crystal. **b** The resulting pole figure, where the projected point (red dot) represents the orientation of the (0001) pole relative to the sample axes.

A pole figure illustrates the distribution of a crystallographic pole relative to the sample coordinate system. Figure 1.4 demonstrates this concept and the stereographic projection principle used in its construction. The sample is typically described by a right-hand coordinate system, such as the Rolling Direction, Transverse Direction (TD), and Normal Direction (ND) in rolled sheets or extruded rods. Consider a single HCP crystal within this framework where the orientation of a lattice vector, such as the (0001) pole, can be described by the angular coordinates ϕ and ψ . As shown in Figure 1.4a, the vector is first mapped onto a unit sphere in the sample frame and then stereographically projected onto a horizontal plane. This projection produces a single point on the pole figure, as illustrated as a red dot in Figure 1.4b. Repeating this process for the same pole across all grains in the aggregate yields a distribution of points, the pole figure, which reflects the overall crystallographic texture of the sample [17]. In practice, pole figures are displayed either as scattered point maps or as contour plots showing pole density. Since a single pole figure provides only partial information, multiple pole figures are required to fully characterise the crystallographic orientation distribution.

The Orientation Distribution Function (ODF) provides a complete mathematical description of crystallographic texture by quantifying the statistical distribution of crystal orientations in a polycrystalline material. Each crystallographic orientation is represented

as a point in 3D space and characterised by three Euler angles (α , β , and γ), where the sequence of rotations is defined according to established conventions [16, 29]. Formally, the ODF is expressed as a probability density function $f(\alpha, \beta, \gamma)$, which gives the likelihood of finding a crystal with a specific orientation. It can be visualised as an ODF diagram and is often expanded mathematically using harmonic functions [16, 29]. From this representation, secondary quantities, including pole figure contour plots, can also be derived.

1.2.3 Texture measurement methods

Texture is a key factor in determining material behaviour, and a variety of techniques have been developed to characterise it. This subsection provides a brief review of several commonly used methods for texture measurement. These can be categorised into two distinct approaches to measuring orientations. The first approach measures the statistical distribution of crystallographic orientations by averaging over a large volume of the polycrystalline aggregate, using techniques such as X-Ray Diffraction (XRD) and neutron diffraction; and requires sampling a sufficiently large number of grains to obtain meaningful texture information. The second approach determines the orientations of individual crystals, as in EBSD, and can also be used for texture measurement provided that a significant number of grains across the area of interest are analysed, allowing the overall orientation distribution to be inferred while retaining spatial resolution of individual grains.

XRD texture measurements rely on the interactions of X-rays with electrons in a crystal lattice. As illustrated in Figure 1.5, a monochromatic X-ray beam of wavelength λ is incident on parallel lattice planes separated by an interplanar spacing d . When the Bragg condition $n\lambda = 2d \sin \theta$ (where n is an integer) is satisfied, constructive interference occurs, resulting in a diffracted beam that emerges at an angle θ relative to the lattice planes. In a single crystal, the orientation of a specific lattice plane is determined by rotating the sample in a goniometer until that plane is in reflection condition, i.e. aligned so the Bragg condition is met [17]. In a polycrystalline sample, only those grains correctly oriented contribute to the diffracted intensity at a given sample orientation, making the

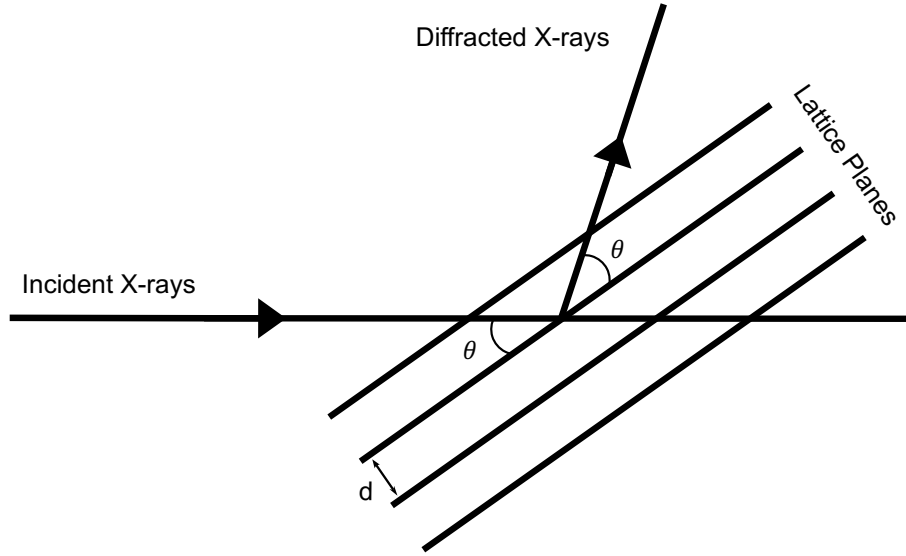


Figure 1.5: Illustration of Bragg diffraction for a monochromatic X-ray beam incident on parallel lattice planes with interplanar spacing d . When the Bragg condition $n\lambda = 2d\sin\theta$ is satisfied, constructive interference produces a diffracted beam emerging at angle θ relative to the lattice planes.

recorded intensity proportional to the volume fraction of crystallites in reflection geometry. Systematic variations in the diffracted intensity as the sample is rotated allow the ODF to be derived, which quantifies the crystallographic texture, and the generation of pole figures. As the penetration depth of X-rays is limited, conventional XRD primarily probes the near-surface region, making it highly effective for thin films, coatings, or surface treatments. With the use of synchrotron X-ray sources and the development of techniques such as 3D XRD [30] and Diffraction Contrast Tomography (DCT) [31], crystallographic measurements can be extended into the bulk. However, the accessible sample volume remains limited to the millimetre scale.

Neutron diffraction is also governed by Bragg's law, but in this case, it relies on the interaction of neutrons with atomic nuclei in a crystal lattice. This requires either nuclear reactors to produce thermal neutrons or accelerators to drive spallation sources [32]. As neutrons are electrically neutral and interact with matter through the nuclear force rather than the electromagnetic force, their interaction with atomic nuclei is relatively weak. This necessitates longer counting times but also provides the advantages of relatively high penetration and low absorption, enabling bulk measurements on samples with dimensions of several centimetres [17]. This makes neutron diffraction particularly suited for bulk

texture measurements of modest-sized engineering components that are inaccessible to X-rays. However, its inability to penetrate very large parts with thickness of several tens of centimetres remains a limitation for many engineering applications, and its use is further constrained by the high cost and the scarcity of suitable facilities.

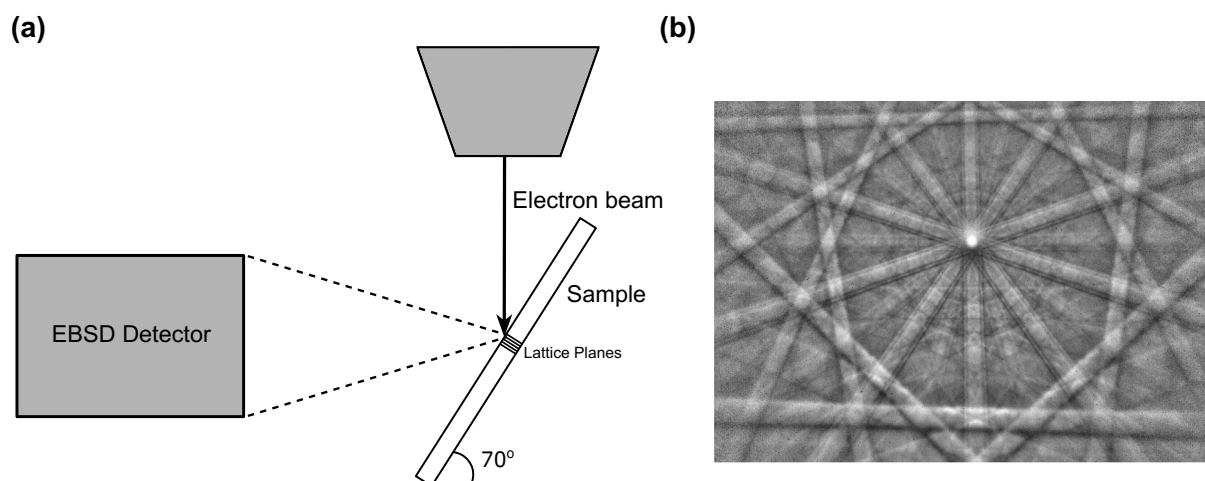


Figure 1.6: **a** Schematic of EBSD technique, which operates by detecting electrons backscattered from lattice planes in a sample tilted at approximately 70° . **b** A representative Kikuchi pattern recorded on the EBSD detector, where the intersecting bands arise from the diffraction of electrons by different crystallographic planes within the sample [1].

The two diffraction-based techniques discussed so far, XRD and neutron diffraction, provide statistically averaged information on crystallographic orientations over relatively large sampling volumes, but they do not resolve the spatial distribution of orientations at the grain scale. By contrast, EBSD, which is based on electron diffraction within a scanning electron microscope, enables spatially resolved mapping of individual grains, albeit in a more localised region [17]. Figure 1.6a illustrates the working principle of EBSD. Like XRD and neutron diffraction, it is fundamentally based on Bragg's law. When a focused electron beam strikes a tilted crystalline sample, electrons penetrate only a shallow depth of a few tens of nanometres before being backscattered [33]. As these electrons exit the crystal, they are diffracted by multiple families of lattice planes that satisfy the Bragg condition at that point. The constructive interference of these diffracted beams produces Kikuchi bands, pairs of bright and dark lines that correspond to different crystallographic planes, as shown in Figure 1.6b. Each EBSD pattern therefore encodes contributions from many lattice planes, and the angular relationships of these bands uniquely define the local crystal orientation. The patterns are projected onto a

phosphor screen, captured by a Complementary Metal-Oxide Semiconductor (CMOS) or Charge-Coupled Device (CCD) detector (represented by the detector in Figure 1.6a), and indexed against reference crystallographic data to extract orientation information. By raster-scanning the electron beam, a pattern is acquired and analysed at every step, generating orientation maps that resolve the crystallographic texture and microstructure at the grain scale [34].

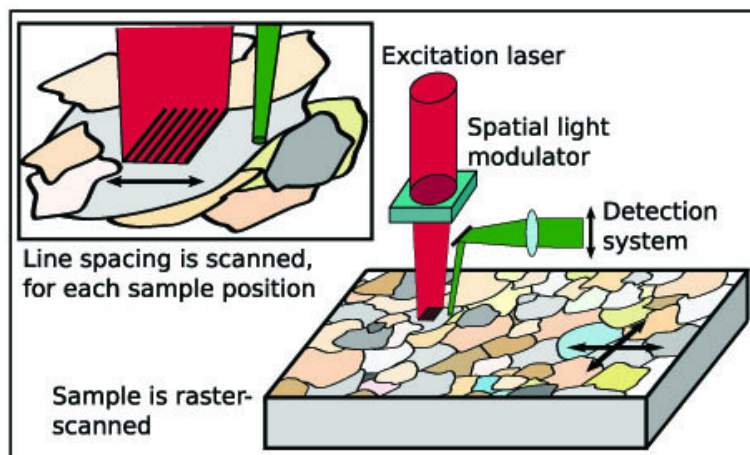


Figure 1.7: Schematic of SRAS technique which uses a laser-generated surface acoustic wave grating to map SAW velocities and infer crystallographic orientations across a sample surface [2].

The key limitations of EBSD are its relatively slow scanning speed, which restricts measurements to small, highly localised regions, and the need for meticulous surface preparation before measurement [32]. Spatially Resolved Acoustic Spectroscopy (SRAS) provides an effective alternative, overcoming these limitations. The working principle of SRAS is illustrated in Figure 1.7: an excitation laser generates a Surface Acoustic Wave (SAW) on the sample using a grating of regularly spaced lines. Due to crystal anisotropy, SAW velocities vary with propagation direction. By mapping these velocities across multiple directions, the crystallographic orientation of the sample surface can be inferred. This enables SRAS to characterise orientations over larger areas and much more rapidly than EBSD [2, 18]. However, unlike EBSD, SRAS infers crystal orientation indirectly through acoustic wave velocities, which are governed by the elastic stiffness tensor of the material. Because of crystallographic symmetry, multiple distinct orientations can exhibit identical elastic responses, rendering them acoustically indistinguishable. Consequently, SRAS cannot always uniquely resolve the full crystal orientation, but instead identifies it

up to the symmetry class of the material.

Polarised Light Microscopy (PLM) is also a cheaper and quicker alternative to EBSD that is worth highlighting [35]. PLM exploits the optical anisotropy of crystals, whereby the refractive index varies with crystallographic direction. When polarised light passes through or reflects from these materials, it undergoes birefringence, causing changes in polarisation state that depend on the local crystal orientation. By analysing these changes using crossed polarisers, orientation-dependent contrast can be generated, allowing individual grains and texture variations to be visualised. This makes PLM well suited to materials with optically anisotropic crystal structures, such as titanium alloys with a HCP structure. In contrast, cubic materials are more optically isotropic due to their high crystallographic symmetry, making PLM less effective for texture characterisation in cubic systems.

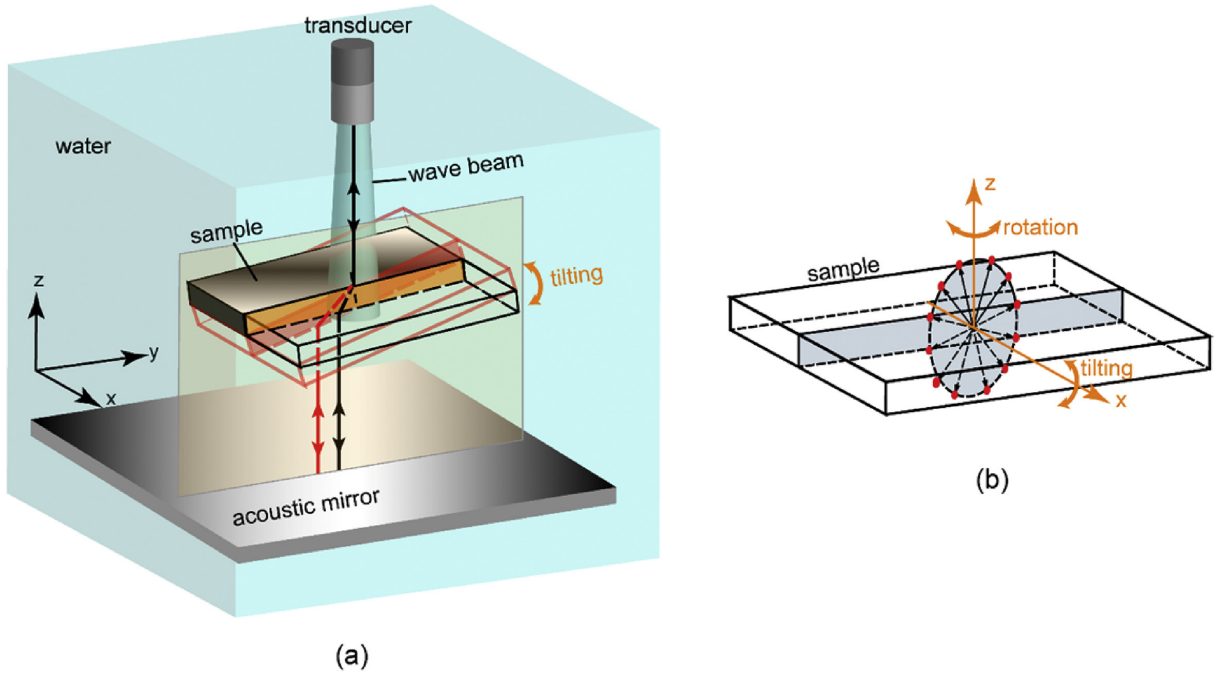


Figure 1.8: Schematic of the ultrasonic texture measurement setup which relies on directional velocity measurements. **a** Immersion-based arrangement with velocities measured using a double through-transmission transducer configuration. **b** Illustration of how sample tilt and rotation are combined to achieve the required directional coverage. [3]

Considering that XRD, EBSD, SRAS and PLM measurements are typically limited to near-surface regions, the textures they measure may not be representative of the bulk material. Neutron diffraction, on the other hand, provides volumetric information but is generally expensive and restricted to sample sizes of a few centimetres. To address

these limitations, a cost effective, acoustic based, volumetric and non-destructive method for crystallographic texture characterisation has emerged [3]. This technique relies on inversely retrieving the bulk texture from ultrasonic wave speed measurements along different directions through the material volume, based on the well-established correlation between experimentally observed anisotropy of wave speeds and crystallographic texture [36].

Experimentally, ultrasonic texture inversion has been demonstrated using a double through-transmission configuration within an immersion setup, as illustrated in Figure 1.8a. In this arrangement, the sample is tilted and rotated to measure longitudinal wave velocities along a finite set of propagation directions, as shown in Figure 1.8b, and the resulting velocity data are then used to inversely reconstruct the crystallographic texture. Since ultrasonic waves penetrate the bulk, this approach inherently probes a representative sample volume, making it an accessible alternative to neutron diffraction for bulk texture characterisation. However, the double through-transmission method is restricted to samples with flat, parallel surfaces [37,38], and its sensitivity to surface curvature and deviations from parallelism has been highlighted in subsequent studies [39]. By contrast, the broader ultrasonic texture inversion framework can, in principle, be applied to samples of arbitrary geometry, provided that directional wave velocities are determined. The velocity accuracy targeted in this work is approximately 10 ms^{-1} , informed by the immersion-based method reported in [3]. This flexibility highlights its potential for texture monitoring in engineering components with complex shapes. To fully realise this potential, however, a robust methodology must be established for acquiring directional velocities in complex geometries at the required level of accuracy, thereby enabling the inversion formulas to be effectively applied for crystallographic texture characterisation.

1.2.4 Diffuse field and Green's Function Reconstruction

Components with complex geometries present significant challenges. Curved surfaces, for example, complicate source-receiver alignment and can give rise to scattering, reflections, and mode conversions at such boundaries, which obscure the direct propagation path between source and receiver. These effects make measurements particularly susceptible

to distortions, potentially compromising the reliable extraction of directional velocities.

This motivated the use of diffuse field Green’s Function (GF) reconstruction as a fundamentally different measurement paradigm. In a diffuse field, the amplitudes and directional distributions of waves are effectively random, and the phases of individual waves are uncorrelated [40]. Despite this apparent randomness, diffuse fields retain statistical properties that encode information about the medium. Seminal studies have shown that cross-correlating diffuse responses at two receiver locations reconstructs the Green’s Function between them [41, 42]. Crucially, this approach does not rely on a single, well-defined propagation path or precise source-receiver alignment. Instead, the Green’s Function emerges from the ensemble of multiple scattered wave paths, provided that sufficient scattering occurs for the field to become diffuse. As a result, geometric complexity, which severely hinders coherent measurements, naturally promotes multiple reflections and mode conversions that accelerate the diffusion process and enhance GF reconstruction. Complex and curved geometries are therefore not a limitation but an intrinsic advantage of this approach. Its application has been highly successful in seismology for studying the structure of the Earth’s crust [43–45], largely because it does not require a coherent wave source, overcoming the challenges of relying on explosions or natural earthquakes for imaging [46, 47]. The method has also been applied in Non-Destructive Evaluation (NDE), for example in defect imaging [48, 49] and in mitigating early-time saturation effects that obscure near-surface defects [50–53]. In these NDE-related work, GF reconstruction was primarily used for imaging purposes. However, since an image is formed through the coherent summation of many individual signals, this implies that the method can, in principle, be extended beyond imaging to reconstruct the individual waveforms themselves. The aim of reconstructing these individual signals is to extract directional velocities by measuring the Time-of-Flight (ToF) of the first-arrival between two points. In imaging, the superposition of multiple scattered arrivals naturally provides an averaging mechanism that enhances robustness. Transitioning to velocity measurement, however, which does not benefit from such averaging, requires far more precise determination of arrival times and remains unexplored.

Importantly, GF reconstruction imposes no restrictions on the type of receiver, making it compatible with non-contact sensors such as laser-based detectors, well suited for complex

geometries. Combined with the fact that diffuse fields can be generated not only from deterministic excitations but also from random fluctuations such as thermal noise [42], the method offers significant versatility. These features provide flexibility in source-receiver configurations and open up the possibility of a new wave velocity measurement technique in complex geometries. Motivated by these findings, this thesis pursues the development and application of GF reconstruction for accurate velocity measurement.

1.3 Thesis Aim and Outline

This thesis aims to gain a deeper understanding of diffuse fields, and harness their potential advantages to develop an accurate wave velocity measurement technique that enables reliable texture inversion of components with complex geometry in a non-contact manner. Acknowledging that complex geometry can be defined in many manners, for the scope of this project, it has been defined as *curved geometry*. To progressively identify the challenges and demonstrate how they are addressed, the thesis is organised as follows:

Chapter 2 presents the fundamental investigations into diffuse field GF reconstruction. It explores early case studies involving experimental imaging, initial experimental attempts at extracting wave velocity from diffuse fields and supporting simulations designed to improve understanding of the reconstruction process. Together, these studies form the basis for developing a robust velocity measurement method.

Chapter 3 builds on the insights from Chapter 2 to establish a methodology for accurate longitudinal wave velocity measurement using diffuse fields in a quasi-2D setup, employing several strategies to ensure reliable reconstruction. This methodology is validated experimentally in two scenarios; one using array receivers and another, more challenging setup using laser receivers, to demonstrate the feasibility of a non-contact approach.

Chapter 4 extends the validated method to full 3D velocity acquisition. Using a custom experimental setup, wave speeds are measured along multiple propagation directions and subsequently used for texture inversion. The reconstructed textures are benchmarked against EBSD data to evaluate the accuracy and reliability of the approach for characterising crystallographic texture in samples with complex geometries.

Chapter 5 summarises the key findings and contributions of the thesis and outlines potential directions for future research in the diffuse field-based texture characterisation technique.

1.4 Industrial Relevance

The industrial relevance of this research is well illustrated by the aerospace industry, where the crystallographic texture of fracture-critical titanium components such as fan blades, compressor discs, and hubs plays a central role in determining structural integrity and service life. These components are manufactured through thermomechanical processing routes including billet conversion, forging, and heat treatment, which are designed to achieve demanding mechanical properties and complex geometries. However, these same processes can give rise to Microtextured Regions (MTRs), or macrozones, in which clusters of α -phase grains of HCP structure in titanium alloys share similar crystallographic orientations. These MTRs can dominate the local mechanical response and significantly increase susceptibility to cold dwell fatigue, a failure mechanism in which sustained loading at maximum stress promotes early subsurface crack initiation and accelerated faceted crack growth. The presence of MTRs has been shown to increase crack growth rates by up to 100 times [35]. In the 1970s-1990s, Ti-64 was considered insensitive to cold dwell fatigue at the time of engine certification, including for the GP7270 engine. This assumption was challenged by the 2017 Airbus A380 engine failure over Greenland [54], in which unfavourable MTRs within the titanium fan hub were identified as a key contributing factor. Given that Ti-64 remains the alloy of choice for “cold” engine components operating below approximately 200 °C, this event highlighted a critical gap in the industry’s ability to reliably detect and manage texture-related microstructural risk in large components with complex geometries.

Current approaches to texture characterisation rely predominantly on EBSD, which offers high spatial resolution and accurate local orientation measurements. However, EBSD is inherently destructive, limited to small, flat specimens, and unsuitable for volumetric inspection of full-scale components. In parallel, ultrasonic inspection is already embedded within manufacturing pipelines but is primarily focused on defect detection, such as

inclusions or cracks, using pulse-echo methods. These inspections are typically confined to early manufacturing stages, such as raw billets, or require intermediate machining to sonic-friendly geometries to enable testing [35]. As a result, conventional ultrasonic methods provide limited insight into texture evolution and are impractical for inspecting fully machined components, where MTRs may still form during the final stages of manufacturing.

The research presented in this thesis addresses this industrial challenge by developing a diffuse field Green's function-based ultrasonic methodology for bulk velocity measurement and texture characterisation in components with complex geometries. By reconstructing directional wave velocities from diffuse ultrasonic fields, the method enables the inference of dominant crystallographic orientations within the material volume without requiring destructive sectioning or geometry simplification. This capability makes the approach compatible with curved surfaces and realistic component geometries encountered in both manufacturing and service.

Importantly, the proposed methodology offers a dual industrial benefit. When applied during manufacturing, it enables monitoring of texture evolution at different stages of thermomechanical processing, providing feedback that can be used to optimise forging and heat-treatment parameters to reduce the formation of unfavourable MTRs while performing routine quality control. When applied to finished or in-service components, it enables non-destructive texture assessment in complex geometries, supporting more accurate predictions of remaining service life. Together, these capabilities provide a comprehensive, non-destructive insight into the development and state of microstructural texture to support both process optimisation and life-cycle assessment of critical components.

Chapter 2

Exploration of Green's Function Reconstruction from Diffuse Fields for Velocity Measurement

2.1 Introduction

This chapter presents a series of exploratory studies aimed at building a robust understanding of GF reconstruction from diffuse fields and assessing its feasibility for ultrasonic velocity measurement. It begins with a technical overview of GF reconstruction theory, followed by a series of case studies that progressively build insight into the practical considerations for implementing this approach. The first case study experimentally examines ultrasonic imaging using full matrix capture from diffuse fields, using it to evaluate the effectiveness of established reconstruction strategies such as source averaging and time averaging. The second experimental case study investigates velocity measurement on a rectangular block, demonstrating the critical role of sample geometry in promoting effective wave diffusion. The final simulation-based case study explores how numerical modelling can be used to identify the onset of a diffusive regime and to understand how receiver orientation influences the type of wave velocity that can be reconstructed from diffuse fields in a pitch-catch setup. Collectively, these studies establish guiding experimental principles for velocity measurement from diffuse fields, providing a foundation

for the subsequent experimental developments presented in later chapters. A practical checklist is also provided to guide readers who wish to implement the method for the first time. A part of this chapter has been adapted from publication [P1], which is an open access article under the CC BY license.

2.2 Green's Function Reconstruction Theories

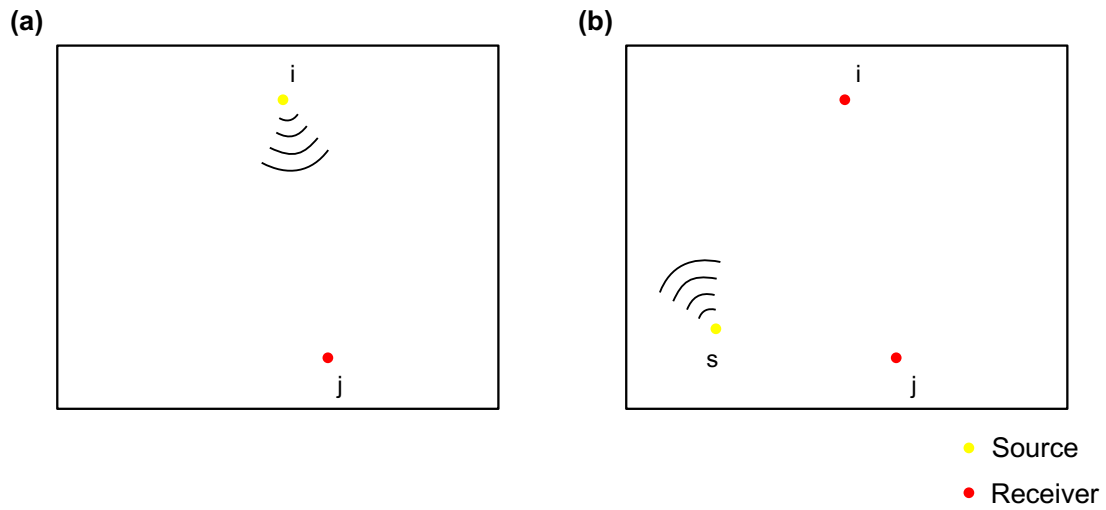


Figure 2.1: Schematic illustrating the setups used for coherent and diffuse field measurements. **a** illustrates the setup used for typical coherent field measurements, where the coherent response is captured at location j from a source excitation at location i . **b** illustrates the setup used for diffuse field measurements, where diffused fields at locations i and j are captured from a source excitation at location s .

For conventional coherent field measurements, the response between two points is obtained directly by transmitting a transient pulse from one location and receiving it at another. Referring to Figure 2.1a, this corresponds to sending a pulse at location i and recording the signal at location j , giving the direct ballistic arrival along that path. This is the familiar basis of ultrasonic NDE, where coherent signals are generated and detected using, for example, piezoelectric transducers or laser interferometers.

In contrast, diffuse field measurements do not rely on a single direct path but instead exploit multiply scattered waves that eventually sample all possible propagation directions. As illustrated in Figure 2.1b, a third location s is introduced to act as a source, exciting the medium such that diffuse fields are recorded at both i and j . While the individual

signals at each receiver may appear random, their cross-correlation retains the coherent information and can, in principle, reconstruct the same Green's Function between i and j that would have been obtained from a direct coherent measurement.

This equivalence forms the foundation of the diffuse field approach. To achieve convergence toward the true coherent response, averaging should be performed over many independent source positions s , where positions separated by at least half a wavelength can be considered independent [41]. In the frequency domain, the expression takes a form similar to that presented in [41]:

$$C_{j,i}(\omega) \approx \frac{R_{j,i}(\omega)}{F^*(\omega)} = \frac{1}{N} \sum_{s=1}^N \frac{D_{i,s}(\omega) D_{j,s}^*(\omega)}{F^*(\omega)} \quad (2.1)$$

The early time coherent signal is $C_{j,i}(\omega)$ which is received at location j and from an excitation at i , and ω represents the angular frequency. $R_{j,i}(\omega)$ is the diffuse field correlation, which can be calculated by multiplying a diffuse field term $D_{i,s}(\omega)$, with the conjugate of another $D_{j,s}^*(\omega)$. Here, $D_{i,s}(\omega)$ and $D_{j,s}(\omega)$ are the diffuse field data collected at i and j , respectively. The approximation of $C_{j,i}(\omega)$ is obtained through the division of $R_{j,i}(\omega)$ by a source-dependent factor, $F^*(\omega)$. s denotes the indices of the source elements used to generate diffuse fields and N is the total number of spatially distinct source elements used. The diffuse fields data $D_{i,s}(\omega)$ and $D_{j,s}(\omega)$ are collected after some transmission delay t_r and within a window length T .

As demonstrated in [41] and evident in Equation 2.1, the diffuse field correlation contains $F^*(\omega)$, a source-dependent factor that needs to be deconvolved to achieve an accurate reconstruction. When a deterministic transient pulse is used as an excitation source, the frequency spectrum of the resultant diffuse field will be limited by the frequency characteristic of the pulse, instead of being a flat spectral density over a wide range of frequencies. Consequently, the diffuse field correlation will inherently contain a transmission component that is related to the excitation source. Depending on the reconstruction aim, which in this work is the Green's Function convolved with the excitation pulse, the relevant source-dependent factor needs to be deconvolved to ensure high-accuracy reconstruction. Details about the source-dependent factor and the deconvolution process will be discussed

further in §3.2.2 and §3.4.1.

2.3 Case Study I: Ultrasonic Imaging using Diffuse Field Full Matrix Capture

As outlined in §1.2.4, previous studies have applied diffuse field reconstruction in NDE for defect imaging. The imaging approach of interest combines Full Matrix Capture (FMC) with Total Focusing Method (TFM). FMC acquires the complete set of transmitter–receiver signals for an array of elements, recording the response of each receiver to each excitation. This dataset is then processed using TFM, which synthetically focuses at every point in the sample to produce high-resolution images that can reveal internal features or defects. In this case study, the objective was to first build a fundamental understanding of how this can be achieved by drawing on the experimental framework presented by Potter *et al.* [50]. From their work, the time-domain formulation was adopted as the basis for reconstruction, and, more importantly, their reported parameter values were used as guidance. This was particularly valuable because diffuse field reconstruction is not straightforward; its success depends on careful selection of parameters such as the transmission delay t_r and window length T , which must be optimised to balance noise and diffusion effects. The corresponding reconstruction equation used in [50] to obtain the coherent full matrix, essentially Equation 2.1 expressed in the time domain, is:

$$h_{i,j}(t) \approx g_{i,j}(\tau) = \frac{1}{N} \sum_{s=1}^N \int_{t_r}^{t_r+T} d_{s,i}(t) d_{s,j}(t + \tau) dt \quad (2.2)$$

Here, $h_{i,j}(t)$ is the conventional coherent full matrix, $g_{i,j}(\tau)$ is the reconstructed full matrix, i and j are the transmitting and receiving elements respectively, N is the total number of transmitting elements (with each element treated as an independent source because the element pitch meets the requirement of half a wavelength), $d_{s,i}$ is the diffuse field received by element i from source s , $d_{s,j}$ is the corresponding signal for element j , t_r denotes the delay after transmission before commencing diffuse field acquisition, while T

represents the duration of the time window used to capture the diffuse signal.

2.3.1 Single array setup

The objective here is to follow the single array configuration used in [50] to compare imaging results obtained under coherent excitation with those reconstructed from diffuse fields, to demonstrate the feasibility of the latter. Experiments were conducted on an ISO 7963 MAB (M) 1018 steel Olympus TB1065-1 calibration block measuring $75 \times 43 \times 25$ mm and containing a 5 mm diameter Side-Drilled Hole (SDH), as shown in Figure 2.2. This specimen was selected due to its wide availability and well-characterised, elastically isotropic behaviour, as such calibration blocks are routinely used as reference standards in ultrasonic measurements. A 64-element Sonaxis array (10 MHz nominal centre frequency; 0.3 mm pitch) was employed for both coherent and diffuse field measurements, driven by a Peak NDT Micropulse FMC controller. The array was gel-coupled to the surface nearest to the SDH to capture it as part of the generated TFM image. Measurements were performed using an excitation voltage of 100 V and the maximum gain setting of 70 dB. Data points were recorded at 50 MHz sampling frequency and each dataset was averaged over 16 repeats to improve Signal-to-Noise Ratio (SNR).

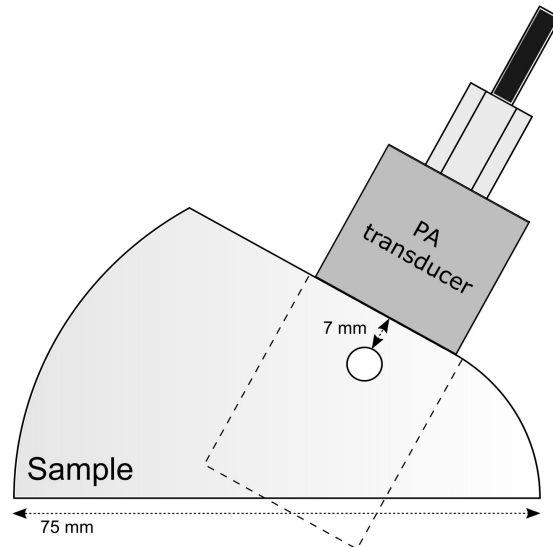


Figure 2.2: Schematic of experimental setup for ultrasonic imaging using diffuse FMC with one array transducer. The dashed outline denotes the region encompassed by the TFM image.

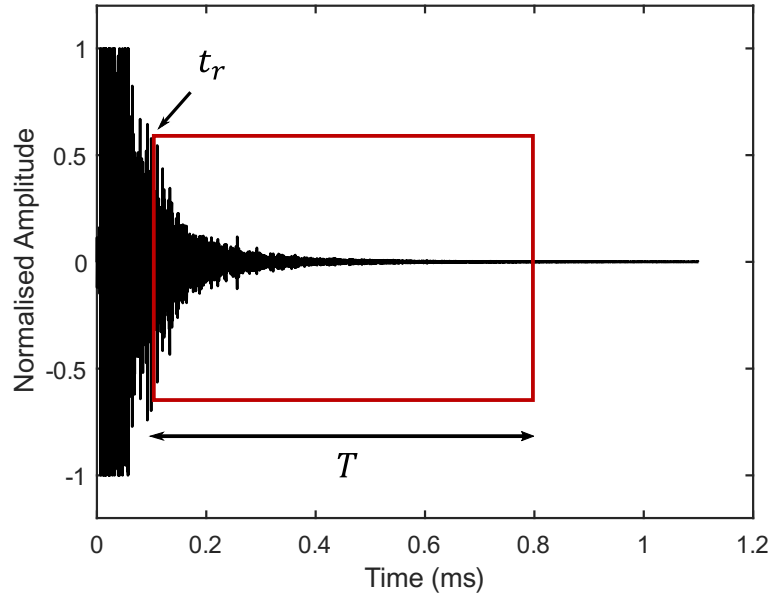


Figure 2.3: A typical time-domain diffuse signal, characterised by two parameters: transmission delay t_r and window length T , used to define the region of the diffuse field for GF reconstruction.

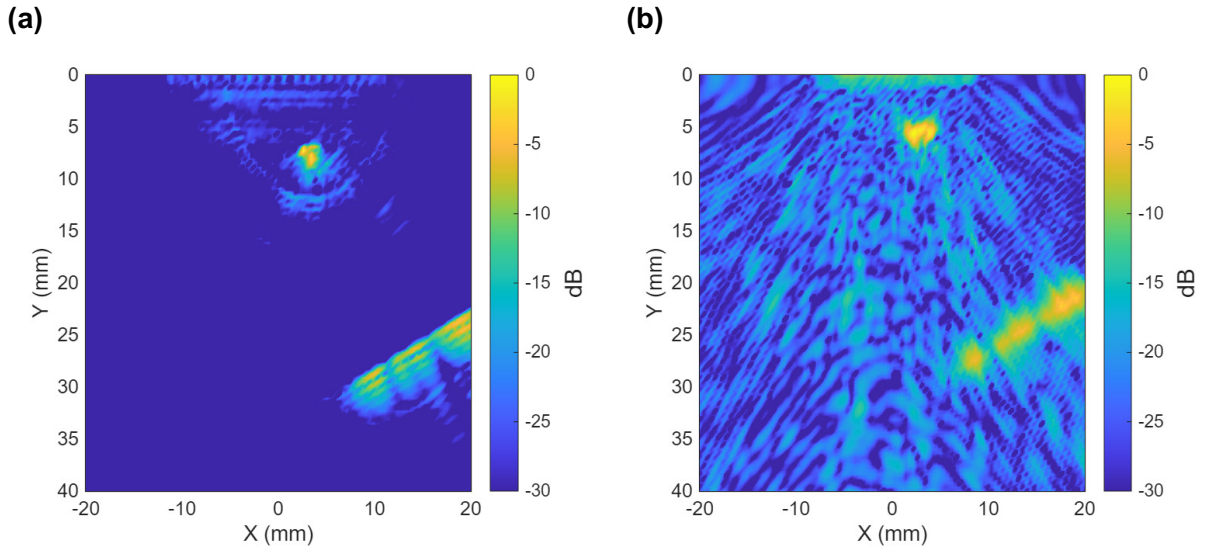


Figure 2.4: Comparison of TFM images obtained from both coherent and GF reconstruction method with one array transducer. **a** depicts the TFM image generated from conventional, coherent FMC with 64 elements. **b** displays TFM image generated from the reconstructed FMC dataset using diffuse fields from 64 source elements, with each signal averaged 16 times and using cross-correlation parameters $t_r = 50 \mu\text{s}$ and window length of $T = 60 \mu\text{s}$.

For coherent measurements, a conventional FMC dataset was acquired to validate the reconstruction results. In this configuration, each array element was sequentially excited while all other elements recorded the received signals, yielding the complete set of

transmitter-receiver combinations for the coherent FMC dataset. For diffuse field measurements, each of the 64 array elements ($N = 64$) was sequentially excited while all elements recorded the resulting signals over a longer time duration to capture the diffuse wavefields. Rather than being directly assembled, the corresponding FMC dataset was reconstructed by cross-correlating the diffuse field signals between all element pairs, following Equation 2.2, thereby generating the full set of transmitter–receiver combinations. A representative time-domain diffuse signal is shown in Figure 2.3, highlighting the much longer time scales (milliseconds) characteristic of diffuse fields compared to the microsecond ranges typical of conventional NDE signals. The figure also illustrates how t_r and T are defined to delimit the portion of the signal used for GF reconstruction. Defining appropriate diffuse field parameters for GF reconstruction is not a trivial task. For the present study, the reconstructed FMC dataset was obtained using $t_r = 50 \mu\text{s}$ and $T = 60 \mu\text{s}$. In the subsequent section and chapter, a more detailed discussion will be presented on tools and strategies that can assist in determining suitable parameter choices for different experimental conditions.

The delay parameter τ was converted to seconds by dividing the sampling frequency, yielding $h_{i,j}(t)$ to get the matrix. Subsequently, the TFM algorithm was applied to both coherent and reconstructed FMC dataset to generate the TFM images shown in Figure 2.4. The TFM image obtained from diffuse field reconstruction, Figure 2.4b, captures both the SDH and the backwall (at depths of approximately 7 mm and 27 mm, respectively), consistent with the coherent TFM image shown in Figure 2.4a. This agreement confirms the feasibility of diffuse field-based imaging. The diffuse field image, however, exhibits a higher noise level, reflecting the inherent SNR issue of the method. Since only a finite number of averages can be performed in practice, the reconstruction, which relies on ensemble averaging, inevitably retains a level of reconstruction noise and therefore remains noisier than direct coherent measurements. A slight spatial offset of SDH and backwall between the two images was also observed, which is expected, as Equation 2.2 used in this study does not account for the deconvolution of the source-dependent factor described in §2.2.

2.3.2 Dual array setup

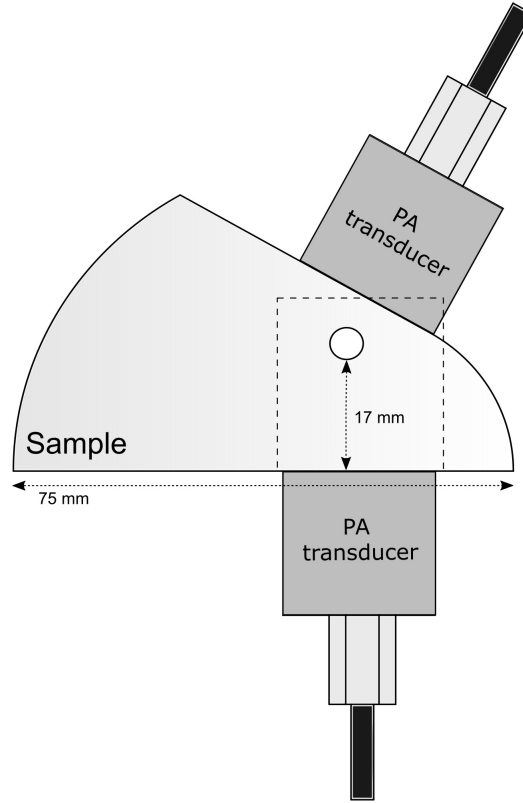


Figure 2.5: Schematic of the experimental setup for ultrasonic imaging using diffuse FMC dataset with two array transducers. The dashed lines indicate the region encompassed by the TFM image.

Experiments were subsequently extended to investigate ultrasonic imaging with the sources and receivers positioned at different locations. An Olympus 32-element array transducer, with a nominal centre frequency of 10 MHz and 0.3 mm pitch, was positioned at the top of the sample, with its individual elements acting as sources. The receivers were the first 32 individual elements of the Sonaxis transducer described in §2.3.1. Both transducers were interfaced with the Micropulse FMC array controller. The Olympus array was gel-coupled to the surface nearest the SDH, and the Sonaxis array was gel-coupled to the adjacent surface, as illustrated in Figure 2.5. Measurements were conducted using a 100 V excitation voltage, maximum gain of 70 dB, and 32000 data points sampled at 50 MHz, with each dataset averaged over 16 acquisitions to enhance the SNR.

Using the procedure described in §2.3.1 with $N = 32$, $t_r = 50 \mu\text{s}$ and a window length of T

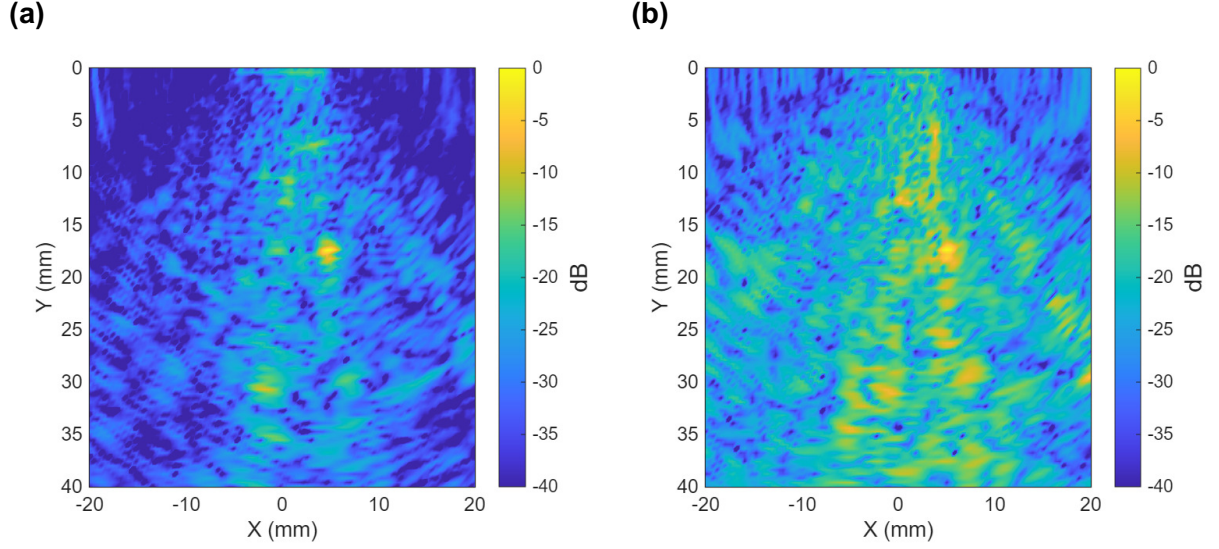


Figure 2.6: Comparison of TFM images obtained from the GF reconstruction method using two array transducers, where one transducer operated as the source and the other as the receiver at a separate location. **a** depicts the TFM image generated with averaging over 32 sources. **b** displays the corresponding TFM image generated using a single source.

$= 60 \mu s$ in Equation 2.2, the resulting TFM image is shown in Figure 2.6a. Although the backwall is absent, the SDH is visible as a yellow hotspot at a depth of approximately 17 mm, similar to the distance between the SDH and the sample surface where the receiver array was positioned. This demonstrates that diffuse field imaging remains effective even when the source and receiver are positioned at different locations on the sample. Figure 2.6b shows the TFM image obtained using only a single source instead of all 32. Without averaging over multiple source locations, the image exhibits significantly higher reconstruction noise, highlighting the benefit of multi-source averaging as a noise-reduction method for GF reconstruction.

2.3.3 Effectiveness evaluation of noise-reduction reconstruction strategies

The comparison presented in §2.3.2 raises the question of what strategies can be employed for noise suppression during the reconstruction process to enhance the reconstruction quality, and more importantly how effective these strategies are. As outlined in [41], accurate GF reconstruction depends on increased averaging, which can be achieved in

2. Exploration of Green's Function Reconstruction from Diffuse Fields for Velocity Measurement

different ways. An averaging strategy can be defined as any procedure that increases the effective amount of diffuse field information contributing to the reconstruction, thereby reducing uncorrelated noise. To explore and better understand the relative effectiveness of such approaches, three strategies that were suggested by [41] were examined in this study: averaging over multiple source positions (hereafter termed *source averaging*), averaging over multiple non-overlapping time windows of a fixed length, and extending the overall window length.

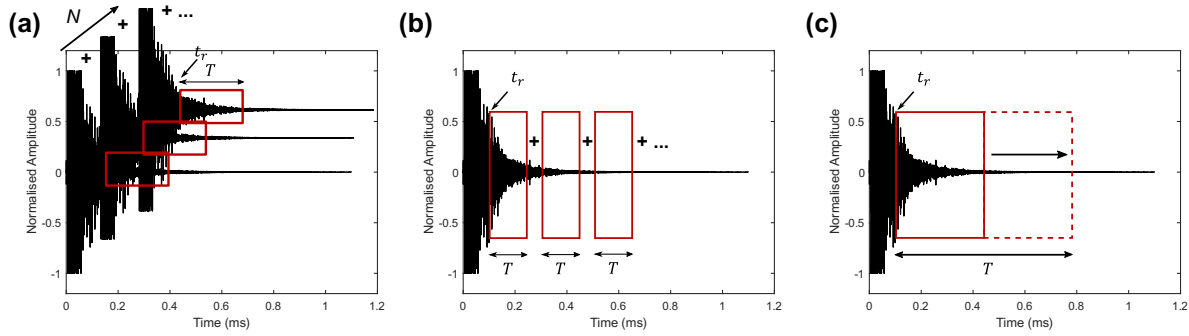


Figure 2.7: Schematic illustrating the three noise-reduction strategies. **a** Averaging over multiples sources. **b** Averaging over multiple non-overlapping time windows. **c** Increasing window length.

The same setup shown in Figure 2.5 was used, and strategies were evaluated using the reconstructed TFM image such as the one seen in Figure 2.6a. The evaluation metric was the SNR of the SDH, hereafter referred to as the defect, calculated from TFM images generated with different sets of diffuse field parameters (N , t_r , T), as dictated by the strategy under investigation. The corresponding effects of each strategy, illustrated schematically in Figure 2.7, are:

1. Averaging over multiple source positions: N was varied, with t_r and T fixed.
2. Averaging over multiple non-overlapping time windows: t_r was incremented in discrete steps, with N fixed and T constant per window.
3. Increasing window length: T was varied, with N and t_r fixed.

Similar to Figure 2.6, TFM images were displayed on a decibel scale, with the defect normalised to 0 dB and the background appearing as negative values as a result of this normalisation. The SNR was then calculated as follows:

1. Identify the location of the defect.
2. Define a region surrounding the defect.
3. Calculate the Root Mean Square (RMS) of all dB values outside the defect region to obtain the average background level.
4. Compute the SNR (in dB) as the difference between the defect amplitude (0 dB by normalisation) and the RMS background level.

For fair comparison, the reference parameters were $N = 1$, $t_r = 50 \mu\text{s}$, and the total window length of $T = 60 \mu\text{s}$. Thereafter, the effectiveness of each noise-reduction strategy was quantified by expressing the improvement in SNR, relative to the reference SNR obtained with these parameters, as a percentage increase.

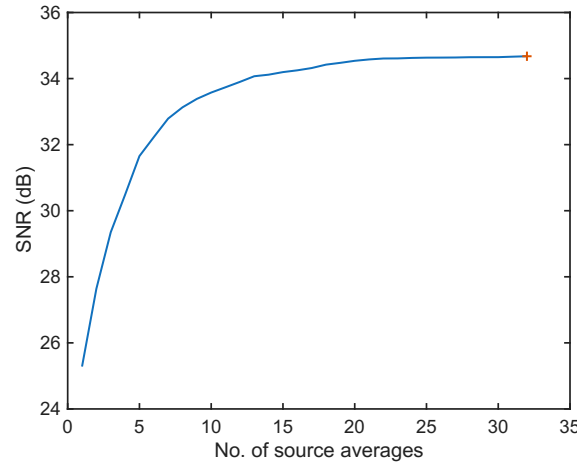


Figure 2.8: Plot of SNR against number of source averages. The red cross indicates the point of maximum SNR.

The first strategy examined was averaging over multiple source positions. As shown in Figure 2.8, SNR increases with N but plateaus beyond a certain point, indicating diminishing returns. The maximum SNR achieved was 34.68 dB, corresponding to a 37.19% increase.

The second and third strategies both involve time averaging using a source, but differ in how the recorded signals are processed. This study therefore also serves as means to conduct a direct comparison of these approaches to assess whether one processing

2. Exploration of Green's Function Reconstruction from Diffuse Fields for Velocity Measurement

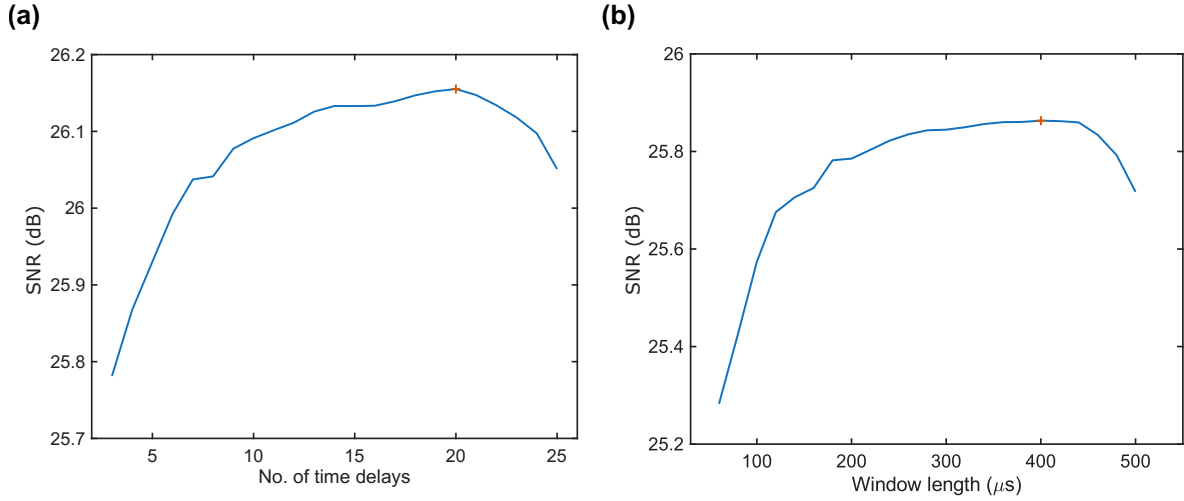


Figure 2.9: **a** Plot of SNR against number of time delays of window length 20 μs . **b** Plot of SNR against window length. The red cross indicates the point of maximum SNR.

method offers superior reconstruction performance. The second strategy investigated was averaging over multiple non-overlapping time windows, which can be seen as an indirect way of controlling the total window length T . Each window had a fixed length of $T = 20 \mu s$, with the smallest number analysed being three (total $T = 60 \mu s$). As shown in Figure 2.9a, the SNR exhibits a concave trend, increasing to a maximum at 20 time delays (corresponding to a total duration of $T = 400 \mu s$) before declining. The decline arises because later delays contain only weak diffuse signals, which contribute less to the cross-correlation, thereby reducing the reconstructed defect amplitude. The third strategy, increasing the window length (Figure 2.9b) exhibited a trend similar to that of the second strategy, as shown in Figure 2.9a. The optimal window length was found to be $T = 400 \mu s$, consistent with the result from the second strategy. With a starting delay of $t_r = 50 \mu s$, this corresponds to an end time of $450 \mu s$ for diffuse field capture. This is notably shorter than in [50], where capture began at 0.1 ms. The discrepancy is attributed to material and frequency differences because their study used aluminium at 5 MHz, whereas this study used stainless steel at 10 MHz, in which diffuse fields attenuate more rapidly to the noise floor.

In terms of maximum SNR improvement, averaging over time windows yielded a 1.47 % increase, while increasing window length achieved a 2.25 % increase. Averaging over multiple source positions remained the most effective strategy, outperforming both by

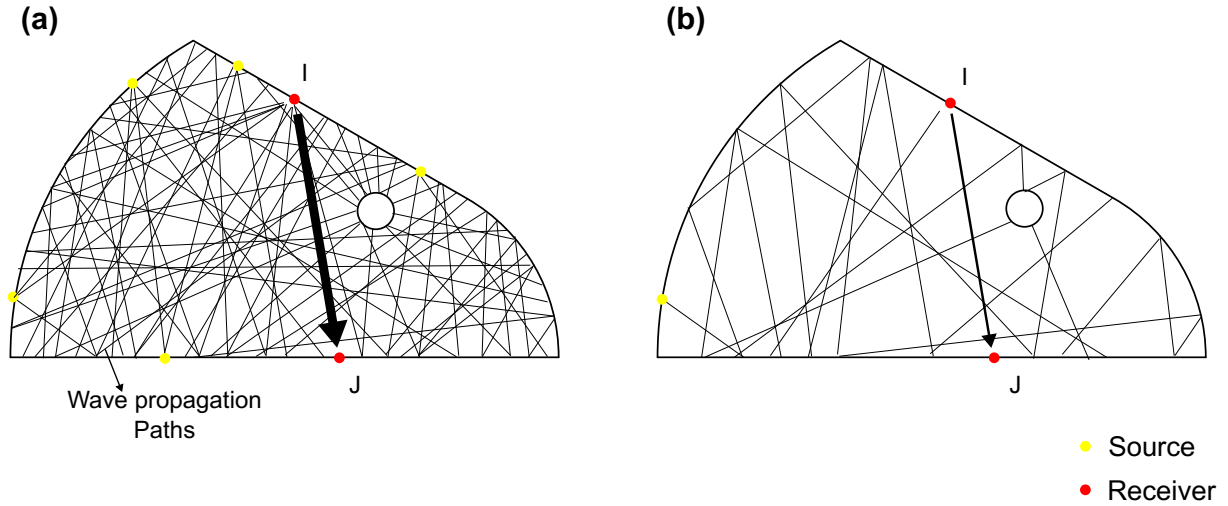


Figure 2.10: Schematic illustrating the effect of averaging across multiple sources on GF reconstruction compared with extending the window length for a single source. **a** With multiple source locations, a greater variety of propagation paths and scattering events contributes to a richer diffuse field. **b** When only the window length is increased for a single source, the same propagation path is reused, so path diversity remains limited. The thickness of the arrow line between the two receivers at location *I* and *J* represents the degree of convergence of the reconstructed GF.

a significant margin. This is because each source location generates a distinct set of propagation paths and scattering events within the medium, so multiple sources enrich the ensemble of wave interactions in the diffuse field, as illustrated in Figure 2.10a. The added path diversity increases the likelihood that waves travel between the two receivers along independent routes, which accelerates the convergence of the GF estimate during cross-correlation, yielding to lower noise and more accurate reconstruction. Under ideal conditions, SNR increases proportionally to $\sqrt{N}$, assuming fully independent sources, a perfectly diffuse wavefield, and only uncorrelated noise [55]. In practice, the SNR observed in the present study may plateau due to several factors. One potential cause is that the sources are restricted to a relatively small area of the sample because of the array configuration, which could lead to partially correlated wavefields rather than fully independent realisations. Another possible factor is instrumental noise, such as electronic interference, which cannot be reduced by averaging and may therefore impose a practical ceiling on the maximum SNR that can be achieved. As a result, while the initial increase roughly follows $\sqrt{N}$, adding more sources beyond a certain point yields negligible improvement. In contrast, Figure 2.10b shows that when only a single source is used, extending

the observation window or averaging over time windows does not introduce new wave propagation paths. Instead, it merely reinforces the same existing path, providing little additional independent statistical information and therefore yielding a smaller improvement in SNR. Of the two approaches for regulating the total time window, all subsequent implementations adopted direct increases in T , which proved slightly more effective and simpler to implement.

Overall, when examining the SNR plateau observed with increasing source averages or the peak SNR obtained through extending the window length, a natural question is whether these points can be predicted in advance from known factors such as wave frequency and sample geometry. In practice, however, the location of these optima, or the corresponding optimised parameter values, depends on multiple interacting factors such as sample size, attenuation, and measurement noise, making them difficult to determine a priori. Instead, they are more reliably identified through practical evaluation of diffuse signals under the specific conditions of the study. Further discussion on strategies for defining an appropriate T , corresponding to the optimised window length, is given in §3.2.1.

2.4 Case Study II: Velocity Measurement in a Rectangular Block: Role of Geometry in Wave Diffusion

Following the work on ultrasonic imaging using diffuse fields and with an improved understanding of GF reconstruction, the next step was to extend the methodology to velocity measurements. The aim of this work was to demonstrate accurate velocity measurement on a simple, rectangular block, a geometry commonly encountered in NDE applications. The experimental setup is illustrated in Figure 2.11. The sample was an isotropic 1018 steel block of thickness 20 mm. A quasi-2D configuration was employed using two Imasonic 64-element array transducers, each with a nominal centre frequency of 10 MHz, a pitch of 0.25 mm, and an element length of 10 mm. Near their midpoints, each element could be approximated as a line source. The transducers were driven by a Verasonics Vantage 64 array controller, which provided a maximum excitation voltage of 96 V; this

was fully utilised to maximise the SNR of the diffuse signals. Data was recorded at a 50 MHz sampling frequency and subsequently upsampled to 500 MHz to improve time-domain resolution for velocity estimation. Gel couplant was applied between each array transducer and the sample surface.

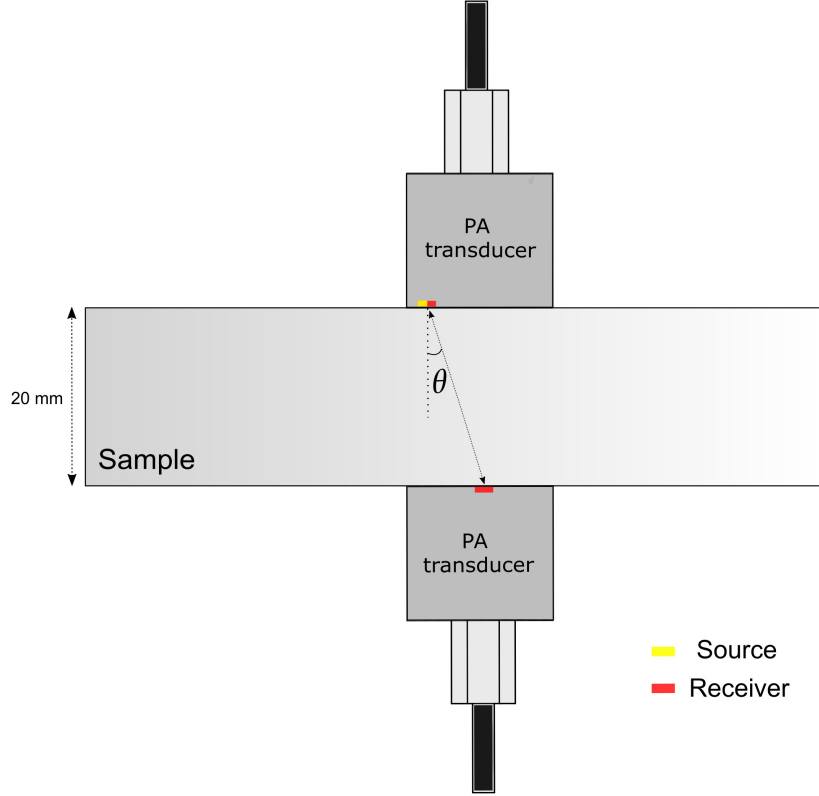


Figure 2.11: Schematic of experimental setup for reconstructing velocities measured in different directions with array receivers.

In this experiment, 32 unique transducer pair positions were used. The same set of 32 pairs was employed as source-receiver pairs for coherent measurements and as receiver-receiver pairs for diffuse field measurements. As in §2.3.1, coherent measurements were used to validate the GF reconstruction method, both for normal and oblique incidence velocity measurements.

For the normal incidence case, each of the 32 measurements corresponded to a pulse-echo configuration in which a single array element acted as both source and receiver. The coherent measurements employed the first 32 elements of the transducer located at the top of the sample. In the diffuse field measurements, the same elements were used solely as receivers, while all 64 elements of the top transducer acted as sources to generate the

2. Exploration of Green's Function Reconstruction from Diffuse Fields for Velocity Measurement

diffuse field ($N = 64$).

For the oblique incidence case, each of the 32 measurements corresponded to a specific angle θ , defined as the angle between the normal to the emitting element on the top transducer and the direct line connecting it to the receiving element on the bottom transducer, as illustrated in Figure 2.11. Coherent measurements used a single top transducer element as the source and the first 32 elements on the bottom transducer as receivers. In the diffuse field measurements, the top transducer element was configured solely as a receiver, while the first 32 elements of the bottom transducer remained as receivers. All 64 elements of the same bottom transducer were used as sources ($N = 64$). For all cross-correlation calculations, the diffuse fields were defined with a transmission delay $t_r = 0.1$ ms and a window length $T = 300$ μ s, as defined in Equation 2.2.

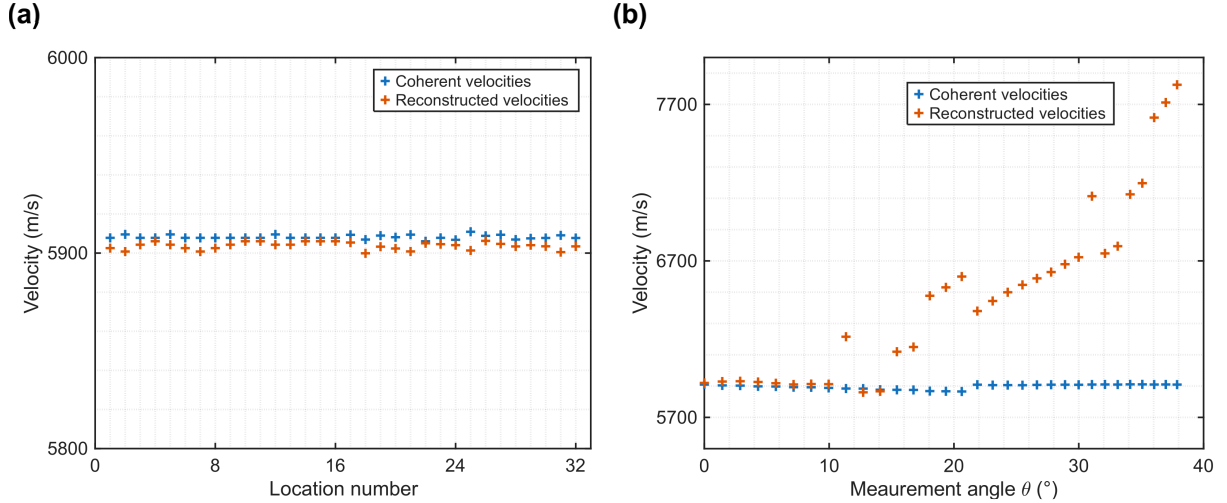


Figure 2.12: Comparison of velocities measured from coherent signals in blue and reconstructed signals in orange across 32 measurement positions. **a** Normal incidence case. **b** Oblique incidence case.

Velocity at normal incidence was computed for all 32 measurement locations using both coherent and reconstructed backwall echoes. Following standard practice, the velocity was obtained by dividing the travel distance (i.e. twice the block thickness) by the ToF between two successive echoes. The ToF was determined from the time lag corresponding to the maximum of the cross-correlation between the two echoes, after upsampling to an effective sampling rate of 1 GHz to improve temporal resolution. As shown in Figure 2.12a, the velocities derived from GF reconstruction closely match those from coherent measurements. Quantitatively, the coherent dataset exhibits a mean velocity of 5908.2

ms^{-1} with a standard deviation of 1.01 ms^{-1} , while the reconstructed dataset yields a mean velocity of 5903.7 ms^{-1} with a standard deviation of 1.88 ms^{-1} . The difference between the mean values is 4.5 ms^{-1} (or 0.075%), indicating good agreement between the two measurements that is below the required accuracy of 10 ms^{-1} for texture inversion as outlined in §1.2.3.

The oblique incidence configuration used a pitch-catch arrangement, with travel time determined from the ToF between the transmitted and received pulses. Source-receiver distances were calculated from the sample thickness and the known transducer pitch. As shown in Figure 2.12b, agreement on the order of tens of ms^{-1} is observed for measurement angles below 10° . At larger angles, the reconstructed velocities increase sharply, with discrepancies reaching up to 2000 ms^{-1} , rendering these measurements unreliable.

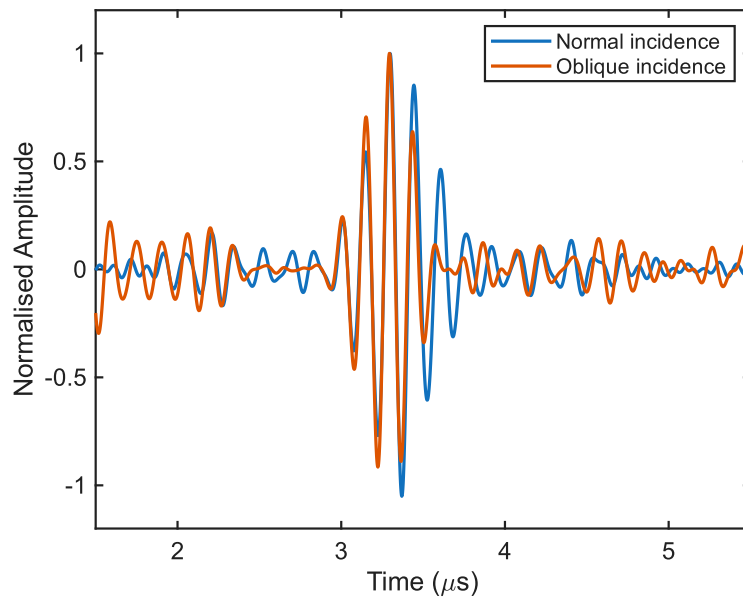


Figure 2.13: Comparison of reconstructed first-arrival tonebursts at measurement angles of 0° in blue and 38° in orange.

The source of this inaccuracy was traced to persistent reverberations within the parallel-faced steel block. Excitation at the top surface generated long lasting back-and-forth reflections between the top and bottom surfaces, creating a highly coherent wavefield dominated by repeated echoes. This limited the diversity of propagation paths and reduced scattering, preventing the formation of a truly diffuse field. For oblique incidence, accurate GF reconstruction requires multiple distinct propagation paths. However, in

practice, the fields employed for cross-correlation did not attain a truly diffuse character, being dominated by reverberations rather than the intended oblique propagation paths, as evident in Figure 2.13. The reconstructed first-arrival toneburst at a measurement angle of 38° arrived at the same time as that at 0° (normal incidence), rather than later as expected from the longer propagation path. Conversely, these reverberations were advantageous for the normal incidence case, where repeated backwall echoes were reinforced during the cross-correlation process, accelerating the emergence of the GF.

It is important to emphasise that the poor performance in oblique directions is not a limitation of the GF reconstruction method per se, but rather a consequence of the specific measurement conditions, particularly the parallel surface geometry of the sample, which hinders rapid diffuse field formation. As demonstrated in the following chapter, introducing non-parallel geometries promotes enhanced scattering and mode conversion, leading to more diffuse wavefields and enabling successful GF reconstruction even for oblique propagation directions. This highlights the critical role of sample geometry in the generation of diffuse fields, an important consideration when implementing GF reconstruction. In particular, nonparallel surfaces are advantageous because they enhance scattering and path diversity, thereby strengthening diffuse field reconstruction. This makes the method especially well suited for texture characterisation in complex geometries, where parallel faces are uncommon and coherent measurements become more challenging.

2.5 Case Study III: Numerical-Based Analysis of Green's Function Reconstruction for Velocity Measurement

In this case study, numerical simulations were utilised both to identify the transition to diffuse fields and to examine how receiver orientation influences the type of wave velocity that can be reconstructed through diffuse field GF reconstruction in a pitch-catch configuration. Simulations are particularly valuable here because they provide controlled access to parameters and wavefield information that are difficult or impossible to isolate

experimentally, thereby offering deeper insight into the underlying mechanisms of the method.

2.5.1 Defining the onset of the diffusive regime via simulation

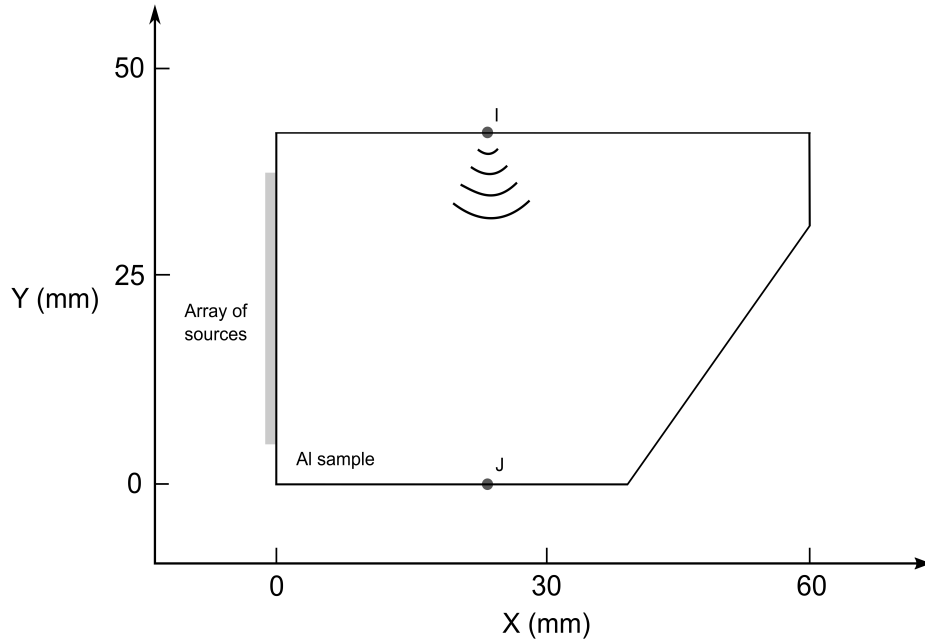


Figure 2.14: Finite element model to identify the transition to diffuse fields and investigate the effect of receiver orientation on the type of velocity measured, with reference to a pitch-catch setup between locations I and J .

As discussed in §2.3.3, care must be taken in selecting the transmission delay t_r . If t_r is too large, only weak diffuse signals remain, which contribute little to GF reconstruction. Conversely, if t_r is chosen too small, the wavefields may not yet have reached a diffuse state, and the correlation process will be adversely affected as well. Numerical simulation can be used to estimate the onset of the diffusive regime, providing a basis for optimising the choice of t_r . A truly diffuse wavefield exhibits equipartition, with energy equally distributed among all directions. This can be evaluated using the angular distribution of nodal displacements obtained from a Finite Element (FE) model.

To demonstrate this, a 2D FE model was developed in Pogo [56], consisting of an isotropic aluminium block of 45 mm height (along the y-axis) with a wedge cut at the side

2. Exploration of Green's Function Reconstruction from Diffuse Fields for Velocity Measurement

to promote scattering, as discussed previously in §2.4. The dimensions of the block is comparable to the experimental samples tested in earlier sections and provides a rough estimate of the time delay t_r required for the diffuse field to develop over such dimensions, serving as a reference for selecting GF reconstruction parameters. A schematic of the model is shown in Figure 2.14. A 64-element, 5 MHz linear array with 0.6 mm pitch was modelled and positioned on the left surface. In this demonstration, the 32nd element was used as the excitation source, modelled as a 5-cycle Hann-weighted sine wave applied as a perpendicular force over a 0.5 mm aperture. While a single element suffices for the purpose of this illustration, a robust and statistically reliable estimate of the onset of diffusive regime should, in practice, require repeating the procedure across all elements and averaging the results.

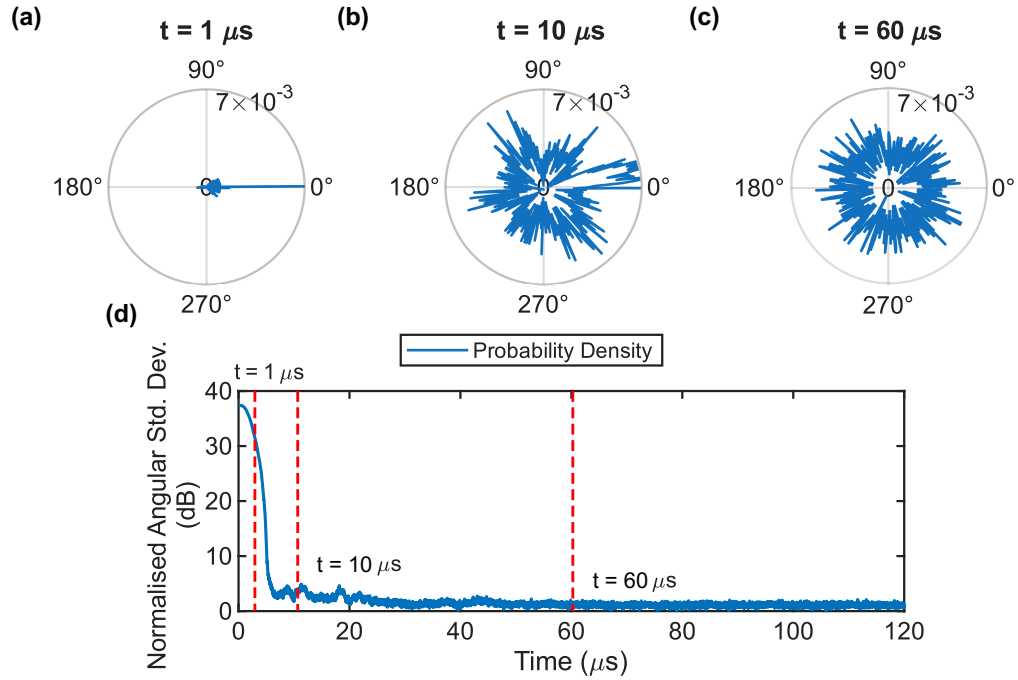


Figure 2.15: Wavefield diffusion illustrated through angular histograms and statistical measures. **a**, **b** and **c** show polar probability density plots of propagation directions at $1 \mu s$, $10 \mu s$ and $60 \mu s$, respectively. **d** shows the temporal evolution of the normalised angular standard deviation of the PDFs, with vertical red dashed lines indicating the times corresponding to the polar plots.

To monitor the transition towards equidistribution of propagation directions, the x - and y -displacements of the nodes were extracted from the model and used to calculate instantaneous propagation angles at each node and time step. These angles were grouped into 500 bins to form angular PDFs of propagation direction, which were then quantified

using the normalised standard deviation as a measure of directional bias where large values indicate strong anisotropy, while low, stable values reflect a diffuse field. Figure 2.15 illustrates this evolution. At the early stage of $1\ \mu\text{s}$, the wavefield exhibits strong directionality, with probability density concentrated around 0° along the x -axis, consistent with excitation from the left side of the model as shown in Figure 2.14. The corresponding normalised standard deviation is high, approximately 38 dB, as seen in Figure 2.15d. With increasing time, this value decays, indicating a progressively more random distribution of propagation directions. At $10\ \mu\text{s}$, the normalised standard deviation decreases markedly to approximately 4 dB. The corresponding polar plot in Figure 2.15b shows that the propagation directions are more randomised than in Figure 2.15a, although some wave energy remains preferentially concentrated near 0° along the x -axis. At later times, from around $60\ \mu\text{s}$ onwards, the probability density is broadly distributed across all angles with minimal fluctuations. This yields a low normalised standard deviation of approximately 0.1 dB and a polar plot characteristic of energy equipartition, as shown in Figure 2.15c, marking the onset of a well-diffused wavefield. This evolution of angular distribution provides a practical means of identifying a suitable t_r , as monitoring when the angular distribution stabilises ensures that the cross-correlation process involves wavefields in the diffusive regime. In the present case, this threshold occurs beyond about $60\ \mu\text{s}$, though its precise value will depend on factors such as sample dimensions, geometry, and material properties.

2.5.2 Velocity measurement of different wave modes

As noted in §1.2.3, the texture inversion framework employed in this work requires the measurement of wave velocities along different directions within a sample. These directional velocities can be typically obtained from pitch-catch setups, where the measured value is calculated using the ToF of the first-arrival toneburst. Accordingly, this configuration is adopted in the velocity measurement methodology with diffuse fields, and the present study is framed within the context of pitch-catch setups, focusing specifically on the first-arrival toneburst. In the conventional coherent case, the measured wave mode and velocity is dictated by the polarisation direction of the source (i.e. the direction

2. Exploration of Green's Function Reconstruction from Diffuse Fields for Velocity Measurement

of force applied by the piezoelectric element). In the corresponding diffuse field setup, one would naturally assume that the receiver orientation would now become the dictating factor. To illustrate this distinction and explore its implications for texture measurements, numerical simulations were performed to compare diffuse field reconstructions obtained with different receiver orientations.

For this demonstration, the same model described in §2.14 was employed. The entire 64-element array was employed to generate the diffuse fields ($N = 64$), which were then recorded at two receiver locations, denoted I and J . The recordings were averaged and cross-correlated in the time domain using a window starting at $60 \mu\text{s}$ with a duration of 1.9 ms , yielding a reconstructed response between the two points. Results were then compared directly with the coherent pitch-catch response simulated between the same receiver pair, as seen in Figure 2.16.

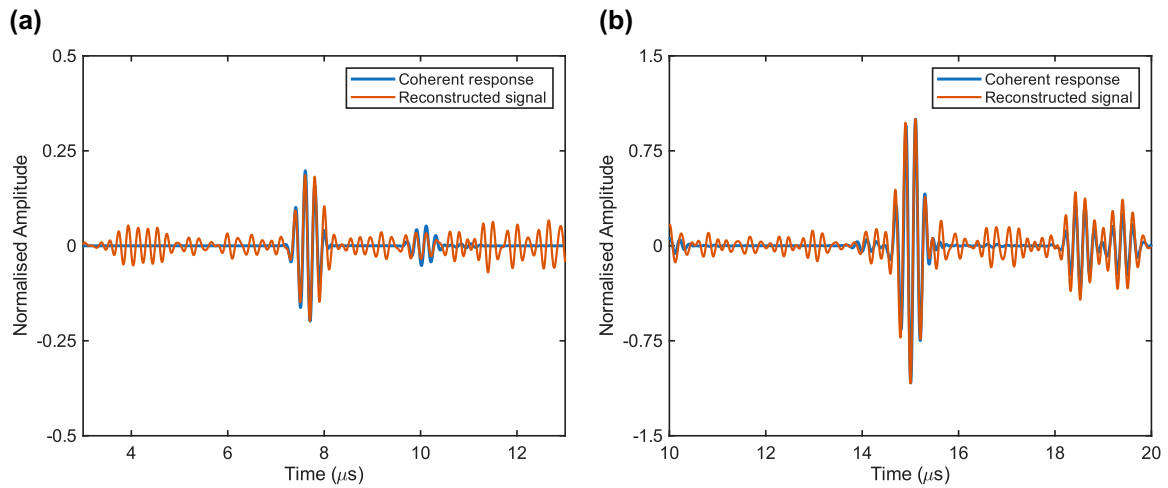


Figure 2.16: Comparison of coherent first-arrival toneburst and the respective reconstructed signal. **a** captures the longitudinal toneburst. **b** captures the shear toneburst.

In Figure 2.16a, a longitudinal source at I implemented as a force normal to the surface in the y -direction and a receiver at J measuring the out-of-plane or y -displacements results in a coherent first-arrival toneburst near $8 \mu\text{s}$. When the diffuse field reconstruction is conducted with the receiver at I oriented normal to the surface, retrieving the y -component, the longitudinal first-arrival is well recovered and closely matches the coherent response. This configuration mirrors the experiments conducted in §2.3 and §2.4, where only longitudinal transducers with maximum excitation and reception sensitivity along the normal directions were employed. In Figure 2.16b, setting the reception direction parallel to the

surface and retrieving the in-plane or x -displacements yields a reconstruction that matches the coherent shear response obtained with an in-plane force applied along x , recovering the shear first-arrival toneburst at around $15\ \mu s$ due to the lower shear velocity, even though the diffuse fields were generated by longitudinal transducers. This shows that, in diffuse field reconstructions, the orientation of one receiver, which is effectively reconstructed as the polarisation direction, relative to the line connecting the two receivers (the direction along which the GF is reconstructed) determines the wave mode of the first-arrival, and therefore the type of velocity measured in a pitch-catch configuration. In the present simulation, the geometry of the model is such that out-of-plane motion corresponds to longitudinal waves, while in-plane motion corresponds to shear waves. For other orientations, such as a 45° tilt of receiver at I relative to the flat surface, both longitudinal and shear waves would be generated, propagating along orthogonal directions, and either mode could be detected depending on the location of the second receiver. Importantly, this does not mean that a given orientation isolates a particular wave mode of the reconstructed GF. Diffuse fields contain multiple modes due to scattering-induced conversions from both macroscopic geometry and microscopic grain structures; the cross-correlation simply extracts the displacement component aligned with the receiver sensitivity. Therefore, while an out-of-plane measurement primarily yields longitudinal velocity by isolating the first-arrival, the later part of the reconstructed signal can still contain other modes.

Overall, in the diffuse field reconstructions, the first-arrival wave mode, and therefore the velocity type (longitudinal or shear), is determined by the orientation of one receiver relative to the line connecting both receivers rather than the wave mode of the source used to generate diffuse fields. Consequently, any source can in principle be used, provided it excites the frequency band of interest. This flexibility has been demonstrated in previous work where compressed air was successfully employed to generate diffuse fields for GF reconstruction [57, 58], representing a key advantage over coherent measurements. Another advantage is the ability to measure different velocity types simply by adjusting the displacement measurement. In practice, this can be achieved using laser receivers: conventional out-of-plane detectors, or more advanced 3D detectors capable of capturing in-plane displacements, enable the measurement of both longitudinal and shear velocities for texture characterisation. To date, directional longitudinal velocities for texture inver-

sion have been measured using the double through-transmission setup, but shear velocity measurements at certain angles have been restricted by critical angle effects. Overcoming this limitation would enable directional shear velocity measurements and extend texture characterisation to less-symmetrical crystal systems, such as tetragonal structures [3].

2.6 Summary

After a technical overview of GF reconstruction theory, the chapter presents three case studies which provides practical guidance to help establish the feasibility of GF reconstruction from diffuse fields for ultrasonic velocity measurement. The takeaways from each case studies are:

Case Study I: Maximising the number of distinct source positions should be prioritised, as it is the most effective noise-reduction strategy. For setting the total window length, the observation window was extended directly rather than averaging across multiple time delays, as this approach proved both more effective and simpler to implement. In implementing GF reconstruction, material and excitation effects should also be considered where using specimens with lower attenuation and lower excitation frequency slows signal decay and extends the duration of usable diffuse signals.

Case Study II: Geometries with nonparallel boundaries accelerate the development of a diffuse wavefield by avoiding normal incidence reverberations, and enhancing mode conversion and path diversity. Consequently, Green’s Function reconstruction is most robust on complex components with curved, tapered, or irregular surfaces, since these geometries promote scattering and path diversity that accelerate diffusion. This is advantageous for the goal of reconstructing velocities along multiple directions in such geometries.

Case Study III: The use of a numerical model has been demonstrated as a tool to define the onset of a diffusive regime to optimise the choice of t_r . In addition, in the context of implementing diffuse field reconstructions for velocity measurements with a pitch-catch setup, the wave mode of the first-arrival is governed primarily by the receiver’s orientation rather than the source that generates the diffuse field. This decoupling not only provides flexibility in excitation but it also enables potential implementation of a non-contact

method using laser receivers to measure both longitudinal and shear velocities by using out-of-plane displacement for the former and in-plane displacement for the latter.

Together, these takeaways establish guidelines for accurate reconstruction of diffuse fields for potentially velocity extraction and lay the groundwork for the subsequent experimental chapters. Importantly, the GF reconstruction method shows strong promise for ultrasonic velocity measurement, as evidenced by the close agreement between reconstructed and coherent results across the three case studies.

As a supplementary aid, a checklist is included below for readers who may wish to apply the method. The points are presented in order of importance, reflecting a logical sequence that should be followed when implementing diffuse field GF reconstruction.

1. Begin with geometry

- Assess the sample shape. Geometries with nonparallel boundaries, curvature, or tapers tend to accelerate diffusion, whereas flat slabs with parallel faces can delay it.
- Where possible, choose or design a geometry that promotes scattering and path diversity.

2. Check material properties

- Evaluate the attenuation of the material. Low-damping materials, such as aluminium, preserve longer diffuse fields.
- For materials with high attenuation, a lower operating frequency should be selected to lengthen the usable diffuse field.

3. Plan source positions

- Use as many distinct spatial source positions as possible, since this provides the most effective means of reducing noise.
- Aim for at least several dozen sources, with 50 to 100+ being desirable if feasible.

- Phased arrays provide a practical starting point, as they inherently offer many controlled source locations within a single device.

4. Select t_r and T

- Select t_r to coincide with the onset of the diffuse regime, which varies with sample size, frequency, and scattering characteristics. As a rough guide, values on the order of $t_r \approx 0.1$ ms have been used in similar studies, but this must be refined for the specific case under investigation.
- (If possible) Adopt numerical modelling, as demonstrated in Case Study III, to better estimate an appropriate choice of t_r by monitoring the angular distribution of propagation directions.
- Increase T , which typically improves reconstruction as long as the diffused signals remain above the noise floor. See §3.2.1 for further discussion on the optimisation of T .

Chapter 3

Accurate Velocity Measurement from Diffuse Fields for Texture Characterisation

3.1 Introduction

This chapter presents the development and validation of a methodology for accurate longitudinal velocity measurement using GF reconstruction from diffuse ultrasonic fields. Building on prior case studies, the focus is to achieve the precision necessary for texture inversion, which requires more stringent reconstruction accuracy. While imaging applications has more tolerance towards unoptimised parameters, velocity extraction demands careful control of experimental and processing parameters. To achieve this, the chapter outlines several key strategies implemented to maximise reconstruction accuracy, including optimisation of diffuse field parameters, a deconvolution of source-dependent factor, and one-bit normalisation. A dedicated section is added to evaluate whether the reconstructed velocity corresponds more closely to group or phase velocity. The methodology is then validated through two quasi-2D experimental setups. The first uses array receivers to establish the experimental framework and signal processing pipeline. The second, more challenging approach employs laser-based ultrasonic detection on a curved sample, introducing additional complexities such as reduced signal amplitude and variable surface

roughness. Despite these challenges, the aim is to investigate whether accurate measurements of 10 ms^{-1} , to achieve successful texture inversion, can still be obtained under such conditions. Demonstrating feasibility in this scenario would provide evidence of the method's robustness and extend its potential application to the non-contact inspection of components with complex or curved geometries, which are common in practical engineering contexts. A section will also be included to discuss the extension of the GF reconstruction implementation to texture measurements and other potential applications. This chapter is adapted from publication [P1].

3.2 Strategies for Accurate Green's Function Reconstruction

Achieving high reconstruction accuracy is crucial for accurate velocity measurements, as deviations in waveform or a poor SNR can impede the extraction of accurate arrival times essential for velocity calculations. This challenge is particularly pronounced when compared to the reconstruction of TFM images. For TFM, each point on the image is generated by summing all the data from each transmitter-receiver combination, acting as an averaging method to enhance the SNR of the image. This property allows the GF reconstruction method to still be effective for defect detection using TFM images, even with a set of unoptimised parameters, as depicted in Figure 3.1a and b. The former illustrates a TFM image generated using coherent signals, revealing the presence of a defect at a depth of 19 mm. The latter shows the corresponding image generated using the GF reconstruction method. Despite the poorer SNR, the defect in the reconstructed image can still be identified due to this averaging principle.

However, this principle cannot be leveraged when reconstructing individual signals. Upon examining the pulse-echo response at the location where the reflection from the defect is expected, as seen in Figure 3.1c, and comparing the two signals, it becomes apparent that the reflection signal at $6.5 \text{ }\mu\text{s}$ from the defect is much more challenging to distinguish from the reconstruction noise. Essentially, this indicates that in the context of velocity measurement, accurate measurements using the ToF of transient pulses would not be

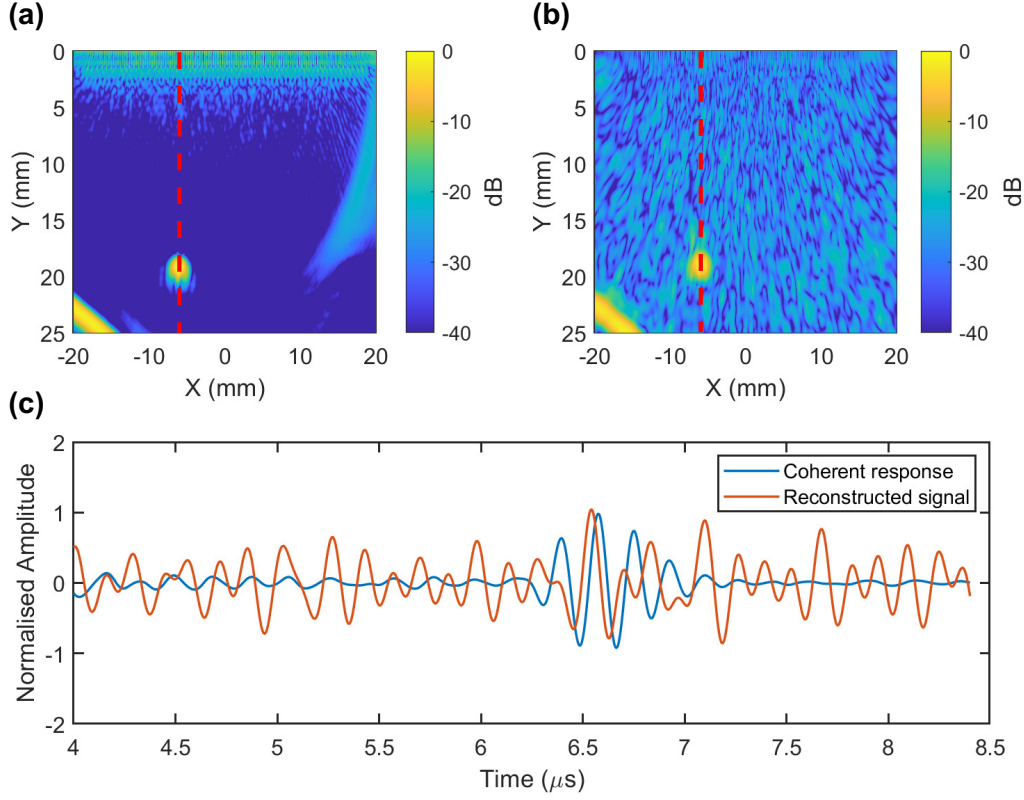


Figure 3.1: Comparison of TFM images and time-domain signals obtained from both coherent and GF reconstruction methods. **a** depicts a TFM image generated from coherent signals, revealing a defect at a depth of 19 mm. **b** displays the corresponding TFM image generated using the GF reconstruction method. **c** compares the pulse-echo responses obtained by one element of the array from both methods, where the wave pulse at $6.5 \mu s$ represents the reflection from the defect. The red dashed lines indicate the x-coordinate where the pulse-echo response is obtained.

achievable using the current approach. Therefore, it becomes crucial to understand how to optimise the parameters or incorporate effective techniques when necessary to maximise reconstruction accuracy. The subsequent subsections will delve into the different methods to achieve this.

3.2.1 Optimisation of diffuse field parameters

Signals recorded at the two locations inevitably contain unwanted noise, such as electronic interference stemming from equipment and laser speckle noise caused by surface roughness, which may obscure the diffuse fields. Generally, it is desirable for the diffuse fields to exhibit larger amplitudes than the noise to achieve better reconstruction quality. This

noise contributes to the generation of reconstruction noise during the cross-correlation process, and a higher level of reconstruction noise ultimately leads to a decline in the overall quality of the reconstruction.

With reference to §2.3, by optimising t_r , T and source averages N , reconstruction noise can be reduced to achieve better reconstruction quality. For the choice of t_r , it should correspond to the time at which the field becomes diffuse. Numerical modelling, as demonstrated in §2.5, provides a practical tool to identify this onset of the diffusive regime.

For the selection of N and T , their qualitative relationship with reconstruction quality, indicated by the SNR of the reconstructed Green's Function, has been well-established and widely accepted in various studies [59–61], and also confirmed in §2.3. It is generally recommended that increasing the number of source locations N and extending the window length T enhances reconstruction quality. In practice, N is typically determined by utilising all the individual elements of an array transducer as independent sources, while T is usually defined by the longest duration before the energy dissipates below the noise floor, usually on the order of milliseconds. A more quantitative tool for assessing whether the first-arrival will emerge within the practical limits of N and T can be derived from the Figure of Merit (FoM) defined in [62], which is denoted here as $FoM_{arrival}$:

$$FoM_{arrival} = \frac{1}{2\sqrt{\pi}} \frac{NT}{\Delta} \frac{c^2}{r^2\bar{\omega}^2} \quad (3.1)$$

where N represents the number of independent source locations, T denotes the window length, Δ is given by $2.2/(\text{half-power bandwidth})$, c is the wave velocity, r is the distance between the passive receivers and $\bar{\omega}$ is the nominal wave frequency.

If the calculated $FoM_{arrival}$ based on the defined parameters exceeds unity, the arrival is expected to be detectable. While this derivation is based on a 3D open system, it is still applicable to this study, which considers a closed system. In fact, this FoM serves as a conservative estimate, as boundary reflections aid in the diffusion of wavefields, potentially enhancing reconstruction accuracy and making it easier to satisfy the criterion.

When aiming to maximise T for reconstruction accuracy, a longer T results in longer com-

putation times and, due to energy dissipation, the amplitude of diffuse fields at later times might become too small, making them indistinguishable from unwanted noise. Utilising these signals would not contribute to the reconstruction of the GF. Therefore, there is a time threshold beyond which these signals should not be used. It is recommended to use a spectral energy density curve derived from a diffuse signal to determine this threshold for the optimisation of T . This curve illustrates the change in ultrasonic spectral energy over time and provides a more reliable indication of ultrasonic diffuse fields compared to the time-domain signal.

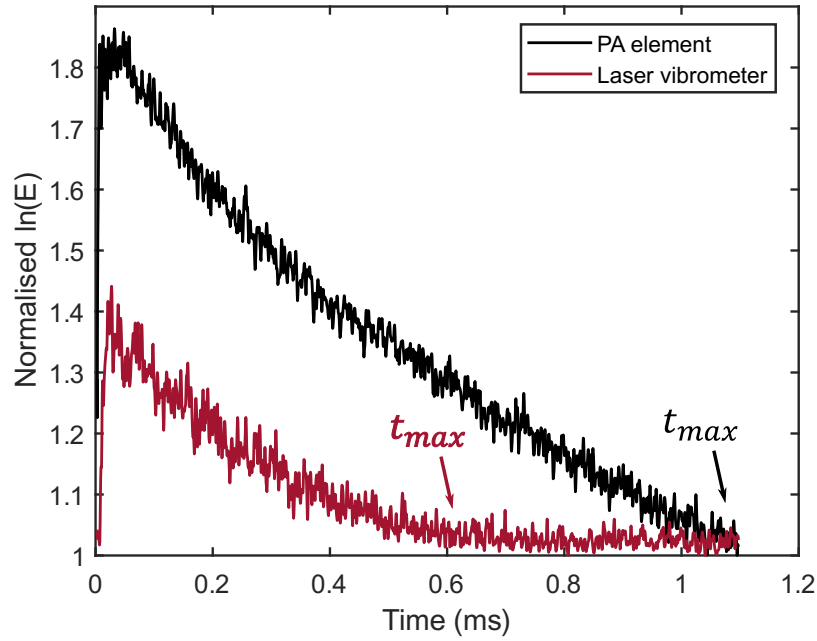


Figure 3.2: Comparison of the energy density curves of signals detected by a piezoelectric array element (in black) and a laser vibrometer (in red).

Conducting standard time-frequency analysis on a time-domain diffuse signal, as depicted in Figure 2.3, allows for the generation of the corresponding spectral energy density curve shown in Figure 3.2. The process involves the following steps as seen in [63–65]:

1. Divide time-domain diffuse signal into overlapping time windows of length Δt and apply a Hanning window on the signals within each window. In this study, Δt is defined as 3 μs , with a 90% overlap between consecutive windows.

The selection of Δt is based on Equation 4 in [63], which describes the inverse relationship between fluctuations of the measured energy density and the square

root of the product of the overlapping time window length and frequency bandwidth. Specifically, $\Delta t = 3 \mu\text{s}$ corresponds to defining the frequency bandwidth as $\Delta f = 6 \text{ MHz}$, and allowing for energy density fluctuations to be relatively low at 20%. To enhance temporal resolution, a larger window overlap of 90% is used, ensuring greater continuity between successive computations. However, this overlap does not influence the energy fluctuations.

2. Calculate the discrete-time Fourier transform (DTFT) for each time window and square it.
3. Determine the spectral density of each time window in a certain frequency band by summing the power spectral density in that bandwidth with Δf centred around the central frequency f_c . In this study, Δf is defined as 6 MHz and f_c as 5 MHz, as dictated by the absolute bandwidth of the transducer and excitation waveform.
4. Calculate the logarithm of the spectral density of each time window, normalised by the minimum, and plot it against the central time of each respective time window. The normalisation standardises the baseline across receivers, allowing their relative sensitivities to be compared, and provides a consistent and quantifiable reference for identifying the end of the time window over which the diffuse field can contribute meaningfully to GF reconstruction.

Figure 3.2 shows the respective energy density curves for different receivers. The black curve corresponds to an averaged signal received from a piezoelectric array element, while the red curve is from a laser vibrometer. Both signals originate from a 5 MHz nominal frequency tone burst generated by a single array element. The curves exhibit a similar shape, reflecting the ultrasonic energy density that experiences a sharp and rapid ascent to a peak before descending due to energy dissipation.

Smaller amplitudes indicate weaker contributions from the corresponding diffuse fields to the reconstruction of the GF. The point at which the normalised amplitude reaches 1 indicates the time t_{max} , beyond which unwanted noise begins to dominate the diffuse fields. It should be clarified that the normalised amplitude does not represent the absolute noise level; rather, it provides a practical indicator for when the diffuse fields become

difficult to detect. Consequently, any signal used beyond t_{max} will not contribute to the GF reconstruction. An important note is that the signals used to plot the energy density curves are only averaged over repetitions and not averaged over N . While this approach is sufficient for the qualitative analysis presented here, aimed at demonstrating how parameters such as T can be optimised, the curves should ideally be generated from diffuse field signals that are averaged over both repetitions and N in GF reconstruction implementations.

Overall, there is no universally established standard for defining these time points, as they depend on factors such as material microstructure and sample geometry. The proposed method, based on energy density curves, has proven to be effective in guiding parameter choices in this study. When selecting T , a larger value is recommended to enhance reconstruction quality, but the sum of t_r and T should not exceed t_{max} . It is important to emphasise that this approach serves as a practical guideline rather than a definitive standard.

These curves also offer valuable insights into the difference in sensitivity of ultrasonic vibrations between a piezoelectric array element and a laser vibrometer. A higher peak on the graph reflects better sensitivity to ultrasound signals, underscoring the comparatively limited sensitivity of the lasers, which would result in lower SNR of signals and consequently, poorer reconstruction accuracy when lasers are utilised. However, the decision regarding which type of receiver to use is not solely based on SNR. Despite the higher SNR associated with the contact transduction method, it requires the use of a couplant to excite ultrasound in the material. This presents a significant obstacle to accurate velocity measurements, as the thickness of the couplant is uncontrollable. Even small uncertainties on the micron scale can result in velocity measurement errors exceeding 150 ms^{-1} , which is unacceptable, especially in contexts such as crystallographic texture analysis, as demonstrated by Lan *et al.* [3]. By contrast, laser interferometry provides a pathway towards a fully non-contact implementation, enabling remote measurements in environments that are difficult for conventional contact-based sensors, such as high-temperature manufacturing conditions. This opens up significant possibilities for measurements across various manufacturing steps. Overcoming the inherent challenge of lower sensitivity is crucial for achieving this, and techniques to address this issue will be discussed in §3.4.2.

3.2.2 Deconvolution of a source-dependent factor

As mentioned in §2.2, the diffuse field correlation contains a source-dependent factor that requires deconvolution to achieve accurate GF reconstruction. In this study, the Time Reversal (TR) symmetry interpretation of GF reconstruction, as used in [66, 67], was employed to determine this factor $F^*(\omega)$:

$$F^*(\omega) = W^*(\omega) \cdot H^*(\omega) \cdot H(\omega) \cdot G_{S,S}^*(\omega) = \hat{T}F(\omega) \quad (3.2)$$

where ω represents the angular frequency, $W(\omega)$ represents the excitation waveform, $H(\omega)$ and $H^*(\omega)$ are, respectively, the transfer function of the source used to generate the diffuse field and its time reversed form, both of which are only determined by the intrinsic properties of the source transducer, $G_{S,S}(\omega)$ denotes the GF at source location S , $\hat{T}$ is the TR operator, and $\hat{T}F(\omega)$ is an alternative representation of the source-dependent factor.

Derivation for $F^*(\omega)$ can be found in Appendix A and is based on the aim of reconstructing the coherent response, which is the convolution of the GF with the excitation pulse. However, if the objective is to reconstruct the GF, the factor to be removed would be $F^*(\omega) \cdot W(\omega)$ as mentioned by Derode *et al.* [66]. The derivation assumes a single source, so if multiple sources are utilised for averaging, $F^*(\omega)$ will represent the averaged value. Assuming $H^*(\omega)$ approximates the receiver transfer function, $F(\omega)$ which is represented by $W(\omega) \cdot H(\omega) \cdot H^*(\omega) \cdot G_{S,S}(\omega)$, describes the early-time pulse-echo response of the source. Therefore, the source-dependent factor $\hat{T}F(\omega)$ represents the early-time time reversed pulse echo response of the source.

In order to gain more understanding about the reconstruction process from diffuse fields, especially the deconvolution step, a FE model was generated using Pogo [56]. The model replicated a ISO 7963 miniature calibration block [68], and its schematic is illustrated in Figure 3.3a. Diffuse fields that were generated by an array of 64 sources, were collected at locations I and J and averaged to reconstruct the coherent response between the two locations. Each source was modelled through a force applied perpendicular to the surface, to nodes within a 0.5 mm aperture. The nodes were forced using a 5-cycle Hann-weighted

sine wave, with a central frequency of 5 MHz.

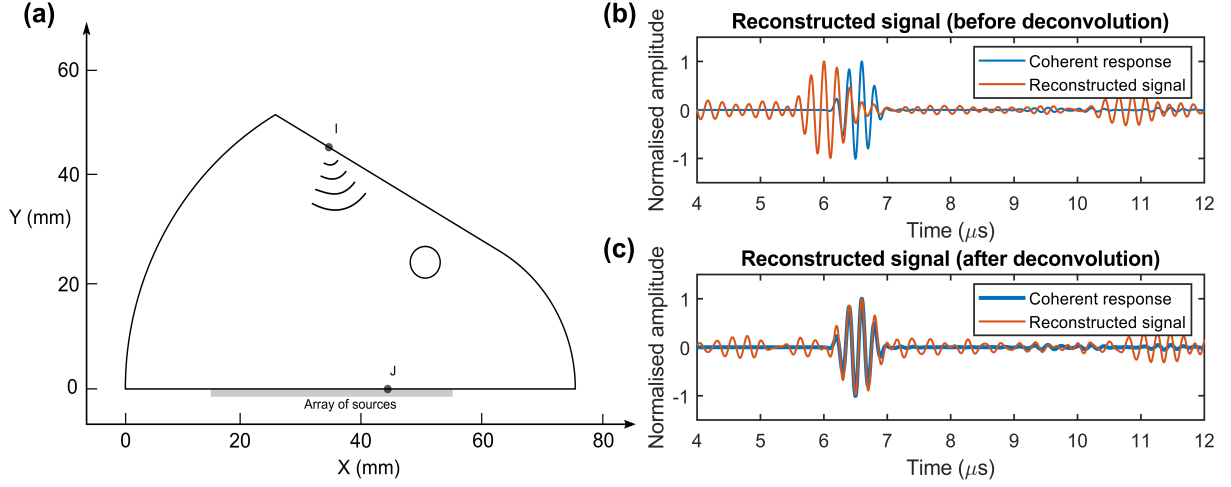


Figure 3.3: FE model to simulate the GF reconstruction process from diffuse fields. **a** shows the schematic of the sample modelled using FE. Diffuse fields were collected at location *I* and *J* to reconstruct the coherent response received at *J* from *I*. **b** compares the reconstructed signal as a result of only cross-correlation against the coherent response. **c** compares the reconstructed signal as a result of both cross-correlation and deconvolution against the coherent response.

The cross-correlation of diffuse fields received at locations *I* and *J* was calculated in the time domain and compared with the corresponding coherent response between the two locations. The comparison between both signals is illustrated in Figure 3.3b. The main feature of the direct pitch-catch waveform which is the first-arrival is clearly present in the correlation as seen from the orange signal. However, it is also apparent that the reconstructed first-arrival occurs at a different time compared to the coherent response depicted in blue and appears to have a different waveform shape. Notably, the first-arrival pulse occurs roughly 1 μs earlier in time in this case. This equates to a velocity difference of about 700 ms⁻¹ which will render velocity measurement based on arrival time to be extremely inaccurate. This discrepancy arises not from a delay in the coherent response but due to the presence of the source-dependent factor $\hat{T}F(\omega)$.

Therefore, the deconvolution of $\hat{T}F(\omega)$ is necessary for accurately reconstructing the coherent response. Referring to Equation 3.2, $F(\omega)$ can be simplified to $W(\omega) \cdot G_{ss}(\omega)$ in this case, given the absence of transducer characteristics, as excitation was defined by applying forces on the source nodes and responses were retrieved by recording the displacements of the receiver nodes. Since $W(\omega) \cdot G_{ss}(\omega)$ describes the pulse-echo response, it was obtained as the signal retrieved after designating the nodes for excitation as re-

ceivers. Following this, a TR operation was applied before converting the signal to the frequency domain to obtain $\hat{T}F(\omega)$. Deconvolution was then performed by transforming the diffuse field correlation to the frequency domain to obtain $R_{j,i}(\omega)$ and dividing it by $\hat{T}F(\omega)$, as seen in Equation 2.1. The outcome, represented in orange, is juxtaposed with the coherent response in Figure 3.3c. The significantly improved agreement with the coherent response clearly highlights the importance of the deconvolution step in attaining an accurate estimation of the coherent response.

It is crucial to emphasise that obtaining $F(\omega)$ in experimental GF reconstruction is more challenging, as it depends on the properties of the source transducer and the experimental setup. Consequently, establishing a reliable method to account for and accurately determine $F(\omega)$ is imperative. This will be addressed in §3.4, which covers the experimental implementation of this method.

3.2.3 One-bit normalisation

As discussed in §3.2.1, the amplitude of diffuse fields decreases over time due to energy decay, resulting in lower-amplitude signals that contribute less to the GF reconstruction process [69]. This phenomenon reduces the effective window length for reconstruction, consequently impacting the accuracy of the reconstruction process. To address this issue, temporal normalisation techniques can be employed to mitigate the impact of diminishing signal amplitudes over time.

Temporal normalisation is a commonly applied procedure in the preprocessing of seismic noise data before conducting cross-correlation in seismology [46,55,70]. Simply performing a basic cross-correlation between noise records from two stations can disproportionately emphasise the most energetic segments of the noise, such as signals from unanticipated earthquakes, or down-weight the less energetic ones. The use of temporal normalisation aims to reduce the influence from such non-uniform noise sources on seismic diffuse fields (e.g. ambient noise, scattered coda waves) utilised for reconstructing GF. In this context, the method is applicable for mitigating the energy decay issue, since its fundamental mechanism is to give more weights to the smaller-amplitude signals.

The study by Bensen *et al.* explores five main temporal normalisation methods, including

1B normalisation and running-absolute-mean normalisation [71]. The latter’s implementation is detailed in the paper, with 1B actually being a simplified form of this method that retains only the signs of the signals, as seen in Figure 3.4a. Despite its simplicity, several studies have demonstrated that 1B effectively eliminates spikes in seismic noise and enhances the SNR of cross-correlation functions [46,47,57]. Therefore, 1B was chosen for its straightforward implementation and documented efficacy in enhancing SNR, as also demonstrated in acoustic experiments conducted by Larose *et al.* [67].

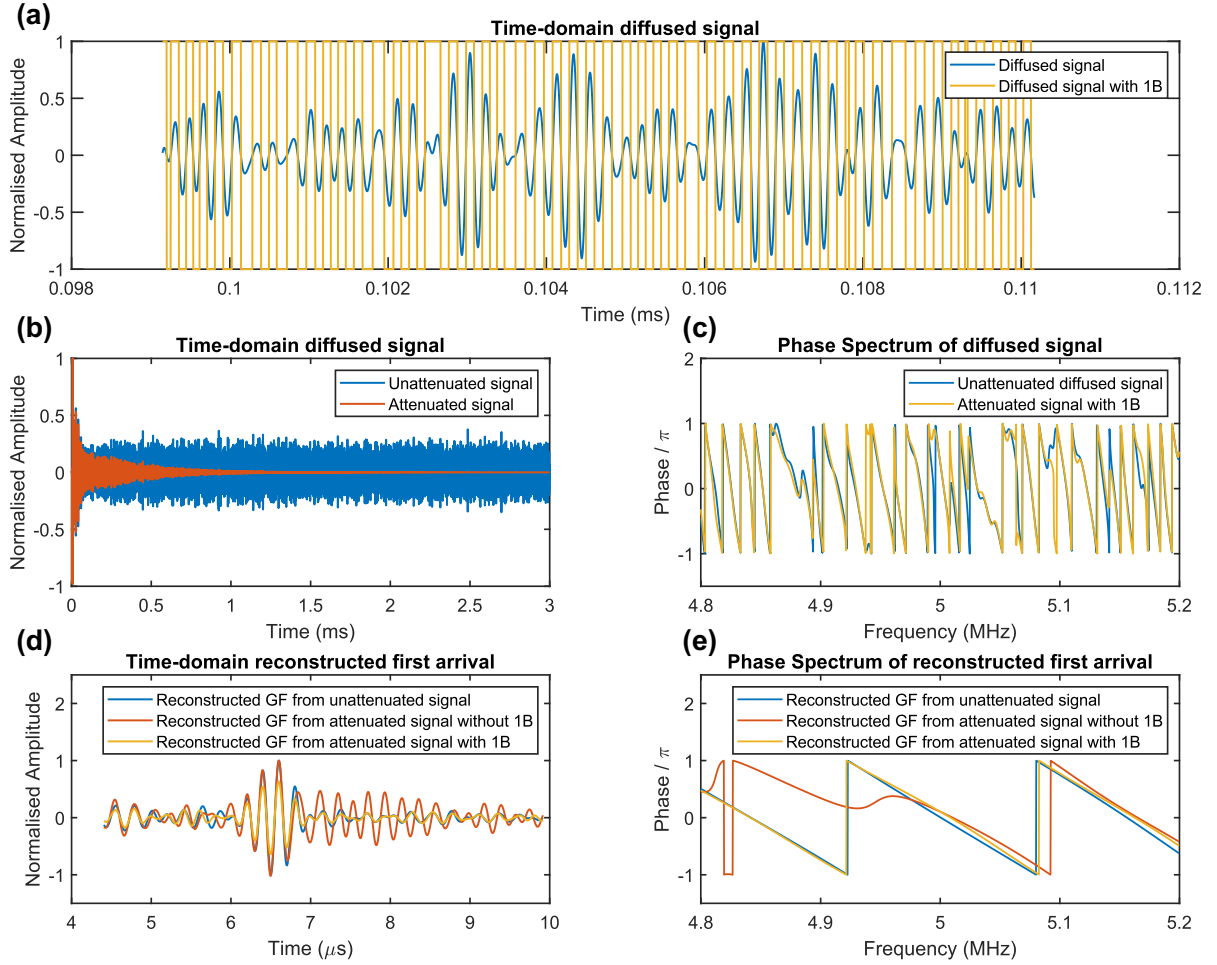


Figure 3.4: Simulated diffuse signals, along with reconstructed GF, and their phase information with 1B normalisation. **a** overlays the signals before and after 1B normalisation. **b** displays two initially identical signals, with one experiencing attenuation. **c** compares the phase spectrum of the unattenuated signal and the binary form of the attenuated signal in **b**. **d** compares the reconstructed first-arrival from the unattenuated signal, attenuated signal and the binary form of the attenuated signal in **b**. **e** compares the phase spectrum of the reconstructed first-arrivals seen in **d**.

To fundamentally grasp the workings of 1B normalisation and its impact on GF recon-

struction, a simulation-based demonstration is provided using the signals shown in Figure 3.4. All signals within Figure 3.4 were generated using the same FE model that was introduced in §3.2.2. In Figure 3.4b, two signals are presented: the blue signal represents an unattenuated diffuse signal, whereas the orange signal undergoes dissipation. This dissipation is introduced by incorporating a mass proportional damping coefficient of 6000 s^{-1} when specifying the material properties of the model. This step is undertaken to emulate the energy decay encountered in real-life experiments. Moving to Figure 3.4c, the phase spectrum of the unattenuated diffuse signal and that of the attenuated signal, post-conversion, are displayed. The phase spectrum of the binary signals is shown to closely resemble that of the unattenuated signal, demonstrating that the phase information is effectively preserved through the conversion process. This preservation of phase information extends to the resultant GF emerging from the correlation, as evident in both Figure 3.4d and Figure 3.4e where the time-domain first-arrivals and their phase spectrum respectively exhibit good agreement with each other. This observation aligns with the understanding that one-bit noise correlation has the same phase as the raw noise correlation [72]. Both figures also serve to highlight the poorer reconstruction quality associated with shorter effective window length when 1B is not applied. As shown in Figure 3.4d, the first-arrival waveform remains identifiable but the SNR is lower due to the presence of larger amplitude reconstruction noise (in the region of 7 to 9 μs) after the first-arrival. Referring to its phase spectrum in Figure 3.4e, depicted in orange, it is evident that the phase spectrum does not agree well with the case when 1B is applied.

The findings illustrate that 1B normalisation enables decayed signals with lower amplitudes to make a more substantial contribution to GF reconstruction, thereby accelerating the convergence toward GF reconstruction within the confines of the same parameter T . This highlights its significance in the experimental implementation of GF reconstruction especially for low-SNR scenarios such as when laser receivers are used.

It is important to note that 1B normalisation is a nonlinear operation that preserves only the sign of the signal, introducing sharp temporal transitions that generate additional high-frequency spectral components beyond the original bandwidth. If these components exceed the Nyquist frequency of the sampling system, aliasing may occur, introducing distortions in the measured spectrum. In the simulations, these distortions were present

and noticeable from frequencies as low as around 10 MHz up to 100 MHz while the bandwidth of interest is 3.25 to 7.25 MHz, centred around the nominal excitation frequency of 5 MHz. However, it is worth noting that these amplitude variations are relatively small and generally do not affect the accuracy of the reconstructed GF or velocity measurement results. To further ensure that the reconstructed GF remains focused on the desired frequency range relevant to the velocity measurements, low- or band-pass filters can be incorporated based on the specific bandwidth of interest.

3.3 Phase or Group Velocity?

To conduct velocity measurement from the reconstructed GF, it is essential to clarify the type of velocity being measured, i.e. group or phase. These two are the same for isotropic non-dissipative materials but can be drastically different for anisotropic or strongly absorbing materials. The interest here is in anisotropic materials, normally metals with low absorption. Therefore, the GF is considered in the context of an anisotropic medium, where group and phase velocities differ depending on the propagation direction, as introduced in §1.2.1.

This section compares a theoretical model of 3D GF in anisotropic material with theoretical calculations of both the group and phase velocity from Christoffel's equation under the plane wave assumption. This comparison aims to demonstrate whether the velocity measured from the GF matches the phase velocity or the group velocity. The theoretical model considered here is the 3D GF for anisotropic solids presented by Wang and Achenbach [73]. In the paper, the 3D GF corresponds to the displacement field in the x_p direction produced by an impulsive point load applied at the origin in the x_k direction. The expression for the 3D frequency domain GF shown in [73] is:

$$g_{pk}(\mathbf{x}, \omega) = \frac{i}{8\pi^2} \int_{|\mathbf{n}|=1} \sum_{m=1}^3 \frac{k_m E_{pm} E_{km}}{2\rho c_m^2} e^{ik_m |\mathbf{n} \cdot \mathbf{x}|} dS(\mathbf{n}) + \frac{1}{2\pi^2} Re \int_0^\pi \sum_{m=1}^3 \left[\frac{A_{pk}(\mathbf{k})}{\partial_{k_3} D(\mathbf{k})} \frac{sgn(x_3)}{\mathbf{k} \cdot \mathbf{x}} \right]_{k_3=k_3^m} d\theta \quad (3.3)$$

3. Accurate Velocity Measurement from Diffuse Fields for Texture Characterisation

where $\mathbf{x}$ is the location of the receiver, ρ is the density of the material, $m = 1, 2, 3$ represents the three different wave modes. For each wave mode, c_m is the phase velocity and k_m is the wave number, E_{pm} and E_{km} corresponds the polarisation amplitude in the x_p direction and x_k direction respectively. Other terms in the equation, such as $A_{pk}(\mathbf{k})$ and $D(\mathbf{k})$, requires the use of Christoffel tensor. Further details on solving for these terms can be found in the paper.

The medium selected was a single crystal cubic steel with density of 7850 kgm^{-3} . The elastic constants used to define the medium are given in units of GPa, by [74]:

$$C_{11} = 21.83, C_{12} = 21.75, C_{44} = 11$$

In this medium, notable variations exist in the group and phase velocities of longitudinal and shear wave modes across different directions, with the differences between them spanning the range from zero to hundreds of ms^{-1} . This variability makes it possible to identify the type of velocity being measured. For the theoretical calculation of Green's Functions using Equation 3.3, a circular array of receivers was defined in the XZ plane, positioned 10 mm away from the origin source, with each receiver separated by 5° . To optimise computation time, only the first quadrant (i.e. 0 to 90°) was considered, leveraging the four-fold symmetry axes inherent in a cubic crystal structure. Equation 3.3 was used to calculate the GF in the frequency domain along 19 directions. Subsequently, the results were transformed into time-domain signals using inverse Fourier Transform. An illustrative result is shown in Figure 3.5a. The signal represents the time-domain Green's function between the source and a receiver of the array, from which the different wave modes can be identified. The ToF of each mode is determined from the corresponding peak in the signal, and these times are then used to calculate the associated velocities. It is noteworthy from Figure 3.5a that the shear horizontal mode was present; however, as the group and phase velocities of this wave mode remain constant across all directions in this plane, it was not considered in the analysis.

In Figure 3.5b, the velocity values are superimposed with the longitudinal and shear vertical group and phase velocities obtained through Christoffel's equation. It is important

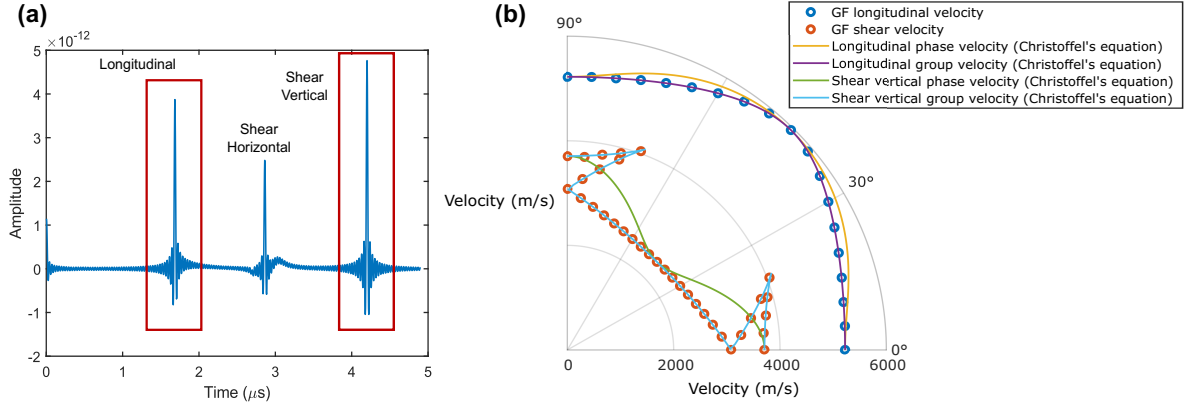


Figure 3.5: Velocity measurement of longitudinal and shear vertical wave modes from GF. **a** shows an example of the time-domain GF after undergoing inverse Fourier Transform from the corresponding 3D GF in the frequency domain. **b** compares the velocities of different wave modes calculated using arrival times extracted from time-domain signals, similar to the one in **a**, in different directions, to the respective group and phase velocity values from Christoffel's equation.

to highlight that all phase velocity values are plotted according to their respective phase velocity directions, while group velocity values are plotted in accordance with their group velocity directions. The close correspondence between the values computed using the 3D GF equation and the group velocity for both longitudinal and shear modes establishes that the velocity measured from GF corresponds to the group velocity. This observation highlights that when utilising a point-to-point measurement method where one point serves as a source and another as the reception to obtain the GF, the measured velocity is the group velocity.

While the samples employed in this study are lossless and isotropic, exhibiting identical group and phase velocities in all directions, the measured velocity is specifically designated as the group velocity for clarity.

3.4 Experimental Implementation

This section describes the implementation of GF reconstruction for velocity measurements using different types of receivers, namely array and laser receivers. The experiment that uses array transducers as both a source and receiver is prioritised as the initial step due to its ability to generate coherent signals for validation and its higher receiver sensitivity

as compared to lasers. This approach establishes the experimental configuration, data acquisition, and signal processing needed before applying the methodology to the more challenging implementation of using laser receivers for velocity measurement on a sample with curved surfaces.

3.4.1 Contact array receivers

To assess the feasibility of accurately reconstructing coherent responses at two arbitrary locations from diffuse fields and subsequently determining corresponding velocity values, a laboratory experiment was conducted. It utilised a calibration block that was intentionally heat-treated to be homogeneous and isotropic. Given the critical role of geometry in promoting wave diffusion, it was also selected for its nonparallel surfaces. Array transducers were employed for their dual capability as both a source and a receiver. This allowed velocity measurements in various directions using both coherent signals and reconstructed GF where the coherent velocities were used for validation. Although the use of couplant introduced measurement errors, the same transducer placement was maintained for both coherent and diffuse field measurements. As a result, couplant-related errors affected both cases equally, ensuring that any observed differences in velocity arose from the methods themselves, thereby allowing accurate validation of the reconstructed results. The use of array transducers also enabled averaging over multiple source locations through their individual elements, which was crucial for achieving higher reconstruction accuracy, as discussed in §2.3 and §3.2.1.

The schematic depicting the experimental setup can be seen in Figure 3.6. Although the broader focus of this work is on HCP materials, an isotropic sample was required to validate the fundamental aspects of the GF reconstruction and velocity extraction methodology. This is most easily ensured using a standard calibration block, and steel is typically the most commonly available material for such blocks. Accordingly, a TB1065-1 ISO 7963 miniature calibration block from Olympus, made of 1018 steel with a thickness of 25 mm and a 5 mm diameter hole, was used for these experiments. A quasi-2D configuration was employed using two Imasonic 64-element array transducers with a nominal frequency of 5 MHz, a 0.6 mm pitch, and a 15 mm element length. Near its middle, each

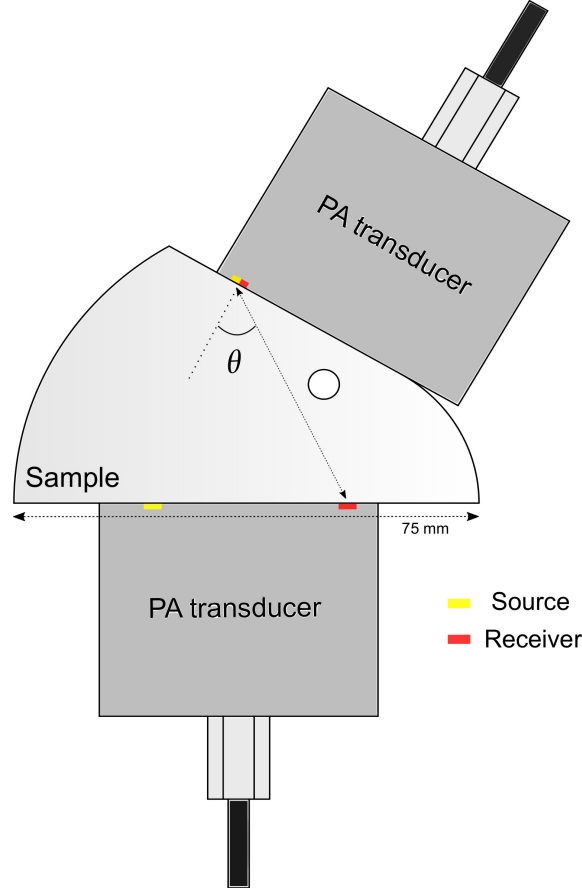


Figure 3.6: Schematic of experimental setup for reconstructing and validating velocities measured in different directions with array receivers

element can be approximated as a line source. The transducers were interfaced with a Verasonics Vantage 128 PA PA controller, which provided a maximum excitation voltage of 96 V. This maximum voltage was fully utilised to enhance the SNR of the diffuse signals. Data points were initially recorded at a 25 MHz sampling frequency and were later upsampled to 1 GHz by zero-padding the signal's spectrum in the frequency domain to enhance time-domain resolution. Gel couplant was applied between the array transducer and the sample surface. The experiment was conducted in a stable air-conditioned laboratory environment where temperature variations were below 2 °C to minimise the impact of temperature drift on GF reconstruction.

In this experiment, 32 unique transducer pair positions were used. The same set of 32 pairs was employed as source-receiver pairs for coherent measurements and as receiver-receiver pairs for dsubdiffuse field measurements. Each of these 32 measurements corresponds to a specific measurement angle denoted as θ . This angle was defined as the angle between the

3. Accurate Velocity Measurement from Diffuse Fields for Texture Characterisation

normal line extending from the array element at the top and the direct line connecting the two array elements, as illustrated in Figure 3.6. Using the same transducer placements, different array configurations were used for coherent and diffuse field measurements. For coherent measurements, the configuration consisted of a source element at the top transducer along with 32 alternating receiver elements at the bottom transducer, indexed as 1st, 3rd, 5th, and so on, up to the 63rd element. For diffuse field measurements, the element at the top transducer was reconfigured to function as a receiver, while all 64 elements of the bottom transducer were employed as sources to generate diffuse fields ($N = 64$). For the cross-correlation calculation, the diffuse fields were defined with a value of t_r set at 0.1 ms and a T of 300 μ s, following the procedures outlined in §3.2.1, where $FoM_{arrival}$ was validated to exceed unity.

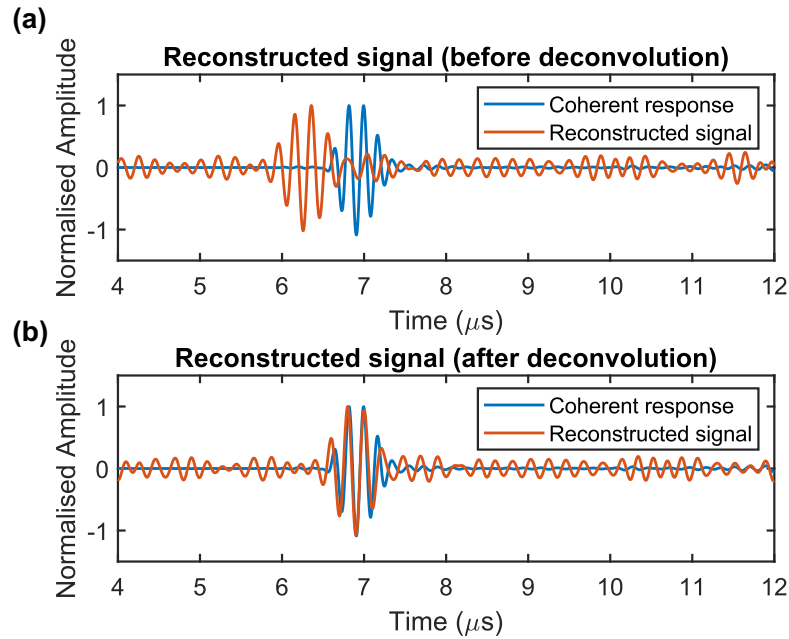


Figure 3.7: Coherent response of the 20th receiving array element and the respective reconstructed signal. **a** compares the reconstructed signal as a result of only cross-correlation against the coherent response. **b** compares the reconstructed signal as a result of both cross-correlation and deconvolution against the coherent response.

The time-domain diffuse field correlation was first calculated at all 32 measurement angles. Figure 3.7a shows the comparison of the reconstruction signals against the corresponding coherent response at one measurement angle. The result is similar to that in Figure 3.3b because $\hat{T}F(\omega)$ has yet to be deconvolved. Expanding on the deconvolution step introduced in §3.2.2, it was essential to recognise the distinctions in its implementation

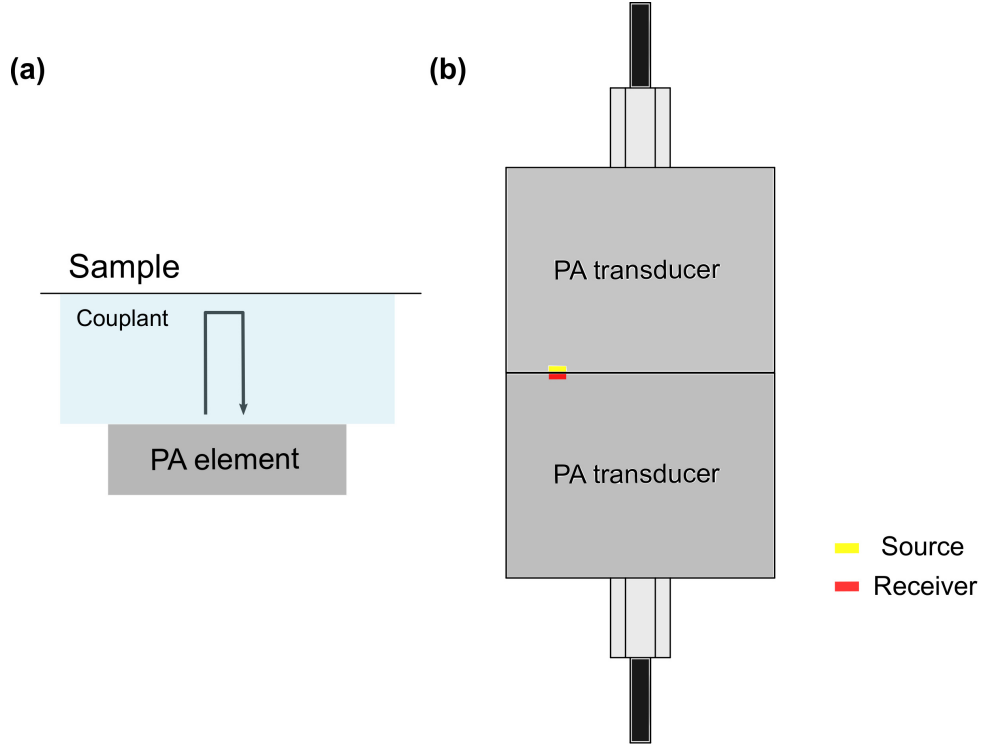


Figure 3.8: Experimental considerations for defining the source-dependent factor that requires deconvolution for accurate GF reconstruction. **a** provides a close-up view of the propagation path by the excitation waveform by an array element to define the source-dependent factor in the experimental setup. **b** shows the setup required to approximate the source-dependent factor.

between simulations and experiments, specifically in acquiring $F(\omega)$. Unlike the FE model where transducer characteristics were not present, experiments required the consideration of the transducer functions $H(\omega)$ and $H^*(\omega)$. Ideally, all terms in $F(\omega)$ can be easily obtained from the early-time pulse echo response, comprising of the reflected pulse from the sample boundary as shown in Figure 3.8a. However, this reflected pulse is often obscured by the element ring down. To address this, the pulse-echo response was approximated as a phase-reversed excitation signal, with the phase reversal resulting from reflection at the flat sample boundary. The excitation signal was obtained using the configuration shown in Figure 3.8b, which incorporates the transducer characteristics $H(\omega)$ and $H^*(\omega)$. The setup involved positioning two identical transducers, which were used as sources in the diffuse field measurement, facing each other, with one functioning as a transmitter and the other as a receiver. The excitation pulse used had to be identical to the one employed during the measurement. The resultant measured signal was the excitation signal and can be described by $W(\omega) \cdot H(\omega) \cdot H^*(\omega)$, by assuming that $H^*(\omega)$ represented the receiver

function. Therefore, the only adjustment required to obtain $W(\omega) \cdot H(\omega) \cdot H^*(\omega) \cdot G_{S,S}(\omega)$ is the application of a phase reversal on the signal. Subsequently, a time reversal operation was applied to obtain $\hat{T}F(\omega)$. This procedure ensured an accurate representation of the experimental conditions, which is important since $\hat{T}F(\omega)$ depended on these conditions.

It is worth noting that $G_{S,S}(\omega)$ is spatially dependent as it varies with each source location. In this study, the uniform surface finish and flat geometry across all source positions allow the assumption of minimal variation, thereby simplifying the acquisition of $F(\omega)$. Importantly, $F(\omega)$ essentially represents the reflected excitation pulse from the sample boundary and does not depend on the material properties themselves. As a result, even in anisotropic materials, which are the main focus of this work, the procedure remains applicable, provided that any spatial variation introduced by non-uniform geometries is properly accounted for.

However, the inevitable presence of reconstruction noise posed a challenge in applying deconvolution, given its high sensitivity to noise. To tackle this, the Wiener filter approach for deconvolution [75,76] was implemented. This filter was designed to minimise the mean squared error between the estimated signal and the true signal, ensuring the best balance between bandwidth and SNR. In the frequency domain, the Wiener deconvolution can be expressed as [76]:

$$\tilde{X}(\omega) = \frac{Z^*(\omega)Y(\omega)}{|Z(\omega)|^2 + \lambda} \quad (3.4)$$

where $\tilde{X}(\omega)$ is the reconstructed coherent response, $Z(\omega)$ is the source-dependent factor or $\hat{T}F(\omega)$, $Y(\omega)$ is the diffuse field correlation and λ is a regularisation factor used to avoid ill condition division in the determination of the filter and to mitigate the presence of high-frequency noise artefacts that might arise after the deconvolution.

To optimise the regularisation parameter λ in the Wiener filter for deconvolution, a parametric study approach was employed. This method enabled the identification of an optimal λ that balances noise suppression and signal preservation. A composite FoM was formulated by incorporating SNR and Spectral Flatness (SF) of the reconstructed signal.

SNR, defined by the ratio of peak amplitude of the first-arrival to the reconstruction noise level, was used to assess the overall fidelity of the signal, while SF helped detect high-frequency oscillations that could indicate instability or artefacts in the deconvolution result. This composite FoM was defined as:

$$FoM_{deconvolution} = \frac{SNR}{1 + 20 * SF} \quad (3.5)$$

which was constructed to penalise low SNR and excessive oscillations, thereby ensuring a stable and reliable deconvolution outcome. The parametric study involved varying λ within a pre-defined range, using a commonly adopted reference value of $10^{-2}|Z(\omega)|_{max}^2$ [77] as a midpoint. For each λ , the corresponding $FoM_{deconvolution}$ was calculated and the optimal λ was determined by maximising the FoM.

The result after deconvolution is shown in Figure 3.7b. It was evident that the reconstructed first-arrival now agreed more closely with the coherent one. There were still slight discrepancies in the waveform but these could be due to insufficient averages of different source locations or inaccurate estimation of transducer functions in the definition of $\hat{T}F(\omega)$ or both.

Following this, the velocity was computed at all 32 measurement angles using both coherent and reconstructed first-arrivals, and a comparison was conducted. Velocity was determined from the phase difference between the transmitted and received pulses, extracted via Fourier transforms, by dividing the product of the source-receiver distance and angular frequency by this phase difference. The distances between each source-receiver pair were readily known because the array transducers were aligned with reference marks on the sample, each of which had a predetermined position [68]. This alignment allowed for the accurate determination of the distances used in the velocity calculations.

Twenty repetitions of each set of 32 velocity measurements were conducted, and the mean values, along with standard deviations represented by error bars, were calculated for every measurement angle. The outcomes are visually depicted in Figure 3.9. Additionally, the figure illustrates the calculated difference in mean velocity between both methods at the

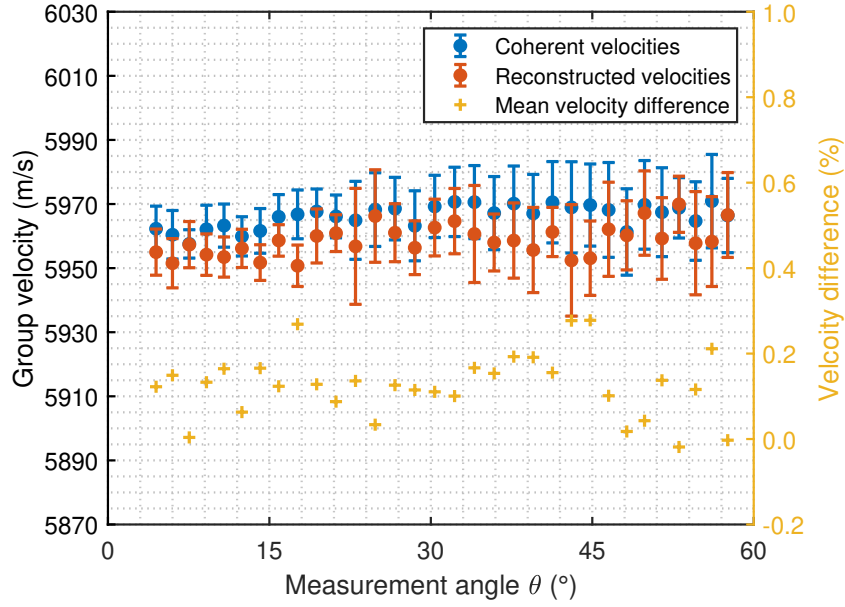


Figure 3.9: Comparison of group velocity measured from coherent signals (in blue) and reconstructed signals (in orange) at 32 different measurement angles. The yellow data points (with axis scale on the right hand side) represent the percentage difference in mean velocity between the coherent and GF reconstruction method at the corresponding measurement angle.

corresponding angles. The mean velocity difference in percentage fluctuates within the range of -0.02 to 0.3%, with a mean value of 0.12%. This corresponds to a velocity difference of approximately 7 ms^{-1} , which satisfies the accuracy requirement of 10 ms^{-1} for successful texture inversion. The standard deviations of individual velocities from both methods hover around 0.17%, possibly due to fluctuations in experimental implementations. The deviations appear to increase with measurement angle θ , a trend that can be attributed to a directivity pattern of the phased array element. As shown in Figure 3.9, larger θ values correspond to measurement angles deviating further from the surface normal, where the element's sensitivity decreases. This reduction in directional sensitivity can lead to a lower SNR, resulting in higher measurement uncertainty at larger angles. Note that this increased uncertainty at larger angles is not a consequence of the GF reconstruction methodology as the equipartition characteristic of diffuse field ensures uniform energy distribution in all directions. Instead, this phenomenon arises from the characteristic of the receiver used. This limitation in directional sensitivity is less relevant to laser receivers, as they could behave more like point detectors with a relatively omnidirectional directivity pattern.

Overall, these findings illustrate a substantial resemblance in measured velocities when employing the GF reconstruction method, as opposed to relying on the coherent response. Notably, this similarity spans a wide angular range of normal incidence to 53° , suggesting that the GF reconstruction method utilising diffuse fields provides an accurate approximation of coherent measurements at arbitrary locations. Furthermore, this applicability is not limited to phased array transducers but should extend to various types of receivers. The next subsection demonstrates this with a laser interferometer-based implementation.

3.4.2 Non-contact laser receivers

In this aspect of the study, the methodology is applied to laser receivers, considering the unique advantages of lasers. The use of lasers in this methodology enables localised point-to-point wave speed measurements, a feat not achievable with conventional methods employing contact or immersion transducers. This capability is particularly advantageous for applications requiring fine spatial resolution and precision. Furthermore, the non-contact nature of lasers enhances the methodology's applicability to components with various complex geometries, such as additive manufactured parts, and allows for potential measurements in challenging environments like manufacturing processes.

The experiment was designed to measure wave velocity along different directions and validate the results against the known velocity of an isotropic Aluminium 6082 block. This material was chosen for its low acoustic noise, making it ideal for a proof of concept study. The block had a diameter of 70 mm and a thickness of 25 mm, featuring curved surfaces for directional velocity measurements and flat surfaces to accommodate array transducers as sources. This allowed for the generation of a diffuse field and averaging over multiple source locations to enhance reconstruction accuracy. The experimental setup, which is quasi-2D, is shown in Figure 3.10. In this setup, the same 5 MHz array transducers were used for excitation, while two Polytec OFV-505 laser vibrometer sensor heads were employed for reception. As depicted in Figure 3.10a, the sensors were placed on opposite ends of the sample, with their inspection points positioned across the sample diameter. Each sensor interfaced with an OFV-5000 vibrometer controller equipped with a DD-300 displacement decoder to output displacement signals. These signals were transmitted to a

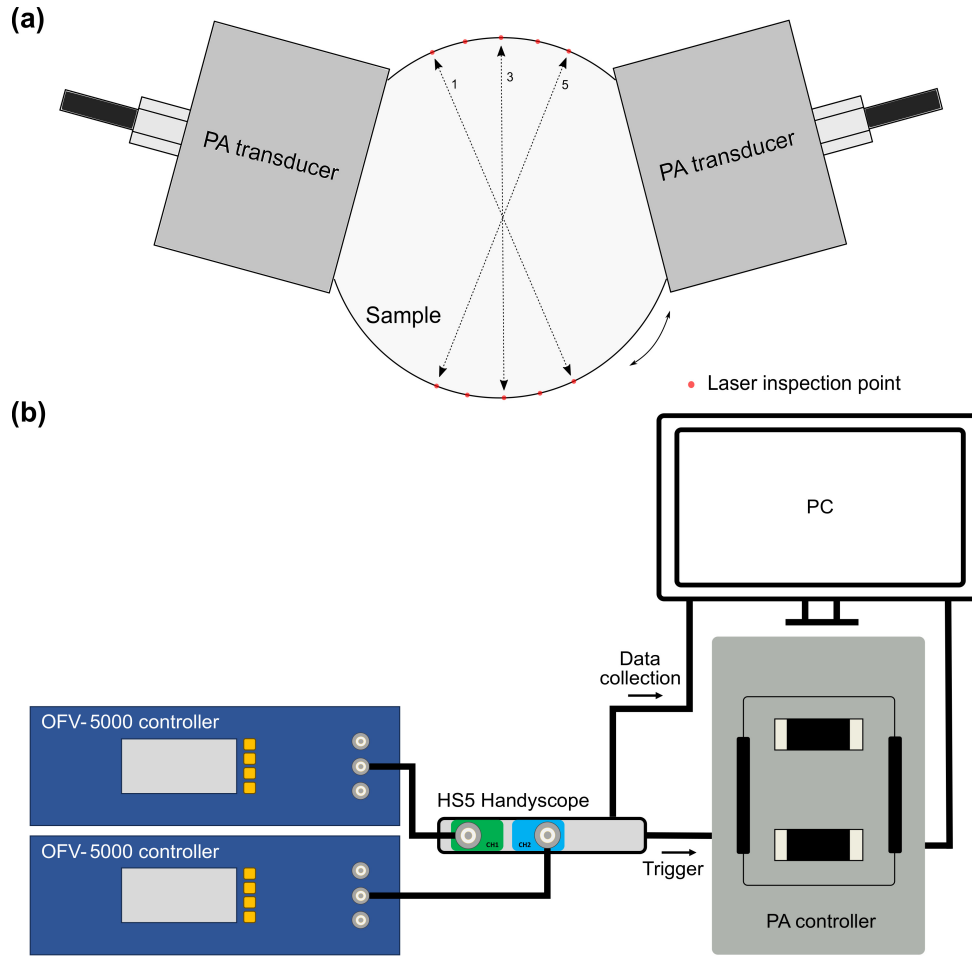


Figure 3.10: Schematic of experimental setup for GF reconstruction with laser receivers. **a** illustrates the measurement setup by showing how measurements are conducted in five different directions using laser receivers. **b** depicts the synchronisation of the PA controller with the OFV-5000 vibrometer controller to ensure simultaneous excitation from the array transducer and reception from the laser receivers.

computer via an HS5 Handyscope USB oscilloscope, which also generated a trigger signal for the PA controller. This ensured synchronisation between transducer excitation and laser reception, as illustrated in Figure 3.10b. Data points were captured at a sampling frequency of 25 MHz and were also upsampled to 1 GHz to enhance time-domain resolution. Gel couplant was applied between the array transducer and the sample surface. The experiment was conducted in a stable laboratory environment where temperature variations were below 2 °C to minimise the impact of temperature drift on GF reconstruction.

As seen in Figure 3.2, the sensitivity of lasers to ultrasound is much lower compared to piezoelectric array elements. This significantly impacts the reconstruction quality of the GF. Therefore, several modifications were implemented in the experiment to address this

issue. Firstly, each set of diffuse signals underwent 2000 repetitions and averaging before storage. This extensive averaging aims to effectively reduce the unwanted random noise present in the signal. Secondly, each source consisted of four array elements that were simultaneously excited to introduce more energy into the sample, thereby improving the SNR of the diffuse signals. Specifically, elements 1, 2, 3, and 4 were excited for the initial measurement, followed by elements 2, 3, 4, and 5 for the subsequent measurement, and so forth. Given the presence of 128 elements from 2 array transducers, this slightly reduced the number of source averages from 128 to 122 ($N = 122$). However, the simultaneous excitation of array elements resulted in a more focussed beam of higher directivity, necessitating a longer duration before the wave energy reaches a diffused state. As a result of this phenomenon and the need for longer window length to improve the reconstruction accuracy as mentioned in §3.2.1, the reconstruction process required the use of diffuse signals at later times. Therefore, the diffuse fields were defined with a value of t_r set at 0.2 ms and a much longer T of 1800 μ s. In this case, there is no need to define a value larger than 1800 μ s because, the energy density curves showed that signals beyond this point in time are dominated by noise. Lastly, to account for the substantial amount of energy dissipation present within the 1800 μ s window length, 1B normalisation was applied to the diffuse signals before correlation to accelerate convergence towards GF reconstruction. Subsequently, a bandpass filter with cut-off frequencies of 2 and 8 MHz was applied to minimise higher-frequency quantisation errors, as discussed in §3.2.3.

Both laser beams were aligned perpendicular to the sample surface to capture out-of-plane displacement and the laser inspection points were adjusted by rotating the aluminium sample with a rotation stage. This setup facilitated the reconstruction of the coherent response between the two points, enabling the determination of the arrival time of the longitudinal wave and, consequently, the measurement of the longitudinal velocity in five different directions as depicted in Figure 3.10a.

Each set of diffuse signals underwent the same procedure outlined in §3.4.1 to calculate the velocity in each measurement direction and compared against the ground truth. Due to the curvature of the measurement surface and the localised nature of each laser measurement, achieving high-accuracy contact measurements at precisely the same spot for ground truth validation is challenging. Consequently, velocity values from pulse-echo

3. Accurate Velocity Measurement from Diffuse Fields for Texture Characterisation

measurements and the material's isotropic characteristic were used to calibrate the laser results. The ground truth velocity was determined using the same 5 MHz array transducer to capture pulse-echo signals in the out-of-plane direction of Figure 3.10a. To account for velocity variations, the array transducer was rotated and each element was used to obtain measurements at 128 different locations on the top flat surface of the sample. At each location, the velocity was calculated by measuring the ToF between two consecutive back-wall echoes, a well-established and highly accurate method, while considering the sample's thickness. The ground truth velocity was then defined as the average of these calculated values, resulting in 6427.6 ms^{-1} . Given the relatively small standard deviation of 7.4 ms^{-1} , this can be regarded as a reliable estimate of the ground truth velocity.

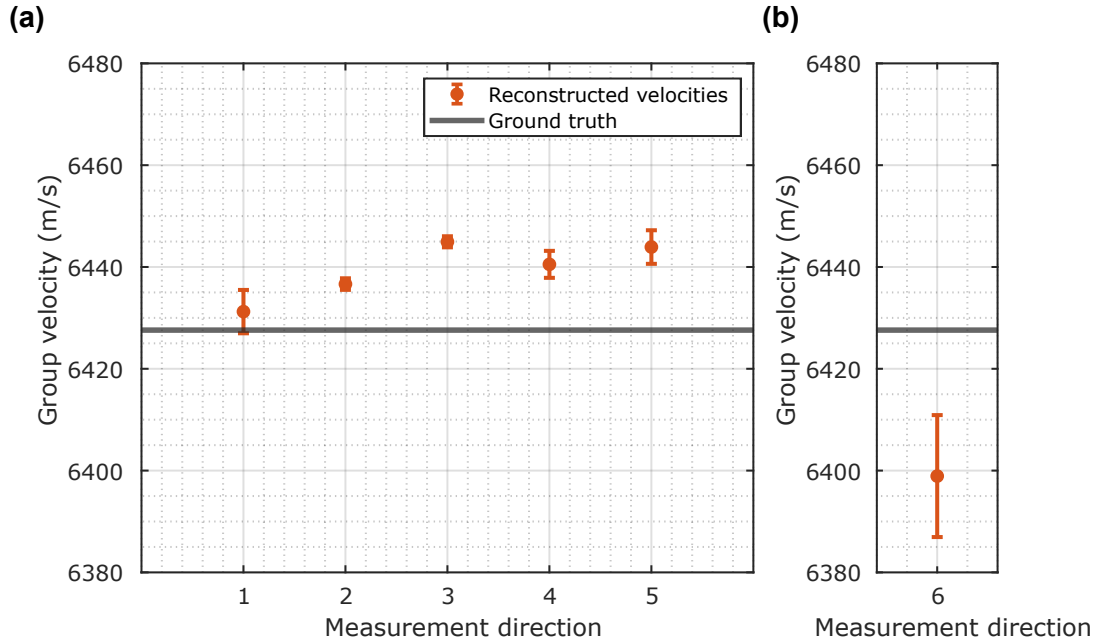


Figure 3.11: Comparison of measured longitudinal group velocity from diffuse field reconstruction (orange data points) in **a.** five different normal incidence directions and **b.** one oblique direction with ground truth (solid grey line).

Three repetitions were conducted in each of the measurement directions from one to five. The mean values, along with standard deviations represented by error bars, were calculated and the outcomes are visually depicted in Figure 3.11a. The average measured velocity using the GF reconstruction method in each direction deviates by approximately 10 ms^{-1} (0.18%) from the ground truth value of 6427.6 ms^{-1} , thereby satisfying the accuracy requirement for texture inversion. The standard deviations hover around 4 ms^{-1} or 0.06%,

likely due to fluctuations in experimental implementations. Additionally, all reconstructed velocities are higher than the ground truth, for which there are possible explanations. Although the sample block is isotropic, slight differences in velocity measured in different directions could exist. Therefore, the velocity measured in the plane of the sample surface, corresponding to the view in Figure 3.10a, using the lasers, could differ from that measured out-of-plane, which was used to determine the ground truth. Alternatively, it could result from inaccurate positioning of laser inspection points such that the actual distance between the points are less than the diameter of the block. Lastly, the discrepancy could also stem from an unoptimised regularisation factor used during the deconvolution step, leading to errors in determining the ToF for velocity measurements. Nonetheless, this discrepancy remains below the required error threshold, indicating that the velocities measured using the GF reconstruction method achieve the desired accuracy.

A point to note is that all five measurement directions were normal incidence, which passes through the centre of the sample. This standardisation was done to define the distance between the two inspection points as a value that can be determined more accurately, specifically the diameter of the sample, for velocity calculations. To validate the flexibility of lasers for wave speed measurements on complex geometries and in oblique directions, the coherent response between the top inspection point of direction one and the bottom inspection point of direction three, as seen in Figure 3.10a, was also reconstructed. Subsequently, the averaged velocity (based on three repetitions) was calculated in that direction, which is defined as direction six. The result for the oblique direction can be seen in Figure 3.11b, which shows that the averaged measured velocity deviates from the ground truth by around 30 ms^{-1} . This deviation is acknowledged to be larger compared to the normal incidence measurement. However, this inaccuracy is likely to stem from the challenge of accurately measuring the distance between the inspection points, which could result in a corresponding measurement error of about 0.3 mm. Overall, these results conclude that accurate velocity measurements in arbitrary directions of a material with curved geometry can be achieved using non-contact laser receivers.

3.5 Evaluation of Velocity Measurement Method

The results presented in the previous section demonstrate that GF reconstruction from diffuse fields enables travel-time estimation between two points with the accuracy required for texture inversion, as validated using different receiver types. Building on these findings, the discussion now turns to the extension of this method to texture characterisation, while considering its strengths and current limitations. Further potential applications and extensions of the velocity measurement methodology are also outlined.

Building on its versatility with different types of receivers, the diffuse field approach has demonstrated accurate velocity measurements on curved samples using laser receivers, which is particularly significant given the prevalence of curved geometries in industrial components. Using laser receivers for reconstruction also eliminates the need for high-power lasers for wave generation, thereby lowering equipment costs and avoiding associated safety requirements. Moreover, because laser receivers share a directivity pattern with piezoelectric elements [78], they overcome the unfavourable directivity linked to thermo-optical laser generation, thereby improving the practicality of implementation. When combined with the flexibility in source excitation inherent in GF reconstruction, these advantages underscore the method's potential and adaptability for in-situ texture characterisation of manufactured components.

A number of practical considerations also arise when extending the method to texture measurement. Scaling from the relatively small samples studied here to larger geometries poses challenges, as diffuse fields develop more slowly, requiring careful optimisation of parameters such as N and T to ensure reliable convergence of the reconstructed Green's Function. Environmental stability is another important factor. While temperature drift was tightly controlled in the laboratory, field applications are more prone to fluctuations that can cause phase drifts, signal decorrelation, and loss of accuracy. Implementations with laser receivers, while proven effective, require extensive averaging due to their lower sensitivity, which in turn increases measurement times to achieve sufficient SNR. This issue points towards possible avenues for improvement, including the use of more sensitive detectors, enhanced surface reflectivity, or adjustments to excitation frequency and amplitude to increase signal strength.

Beyond texture applications, the methodology can be extended in several directions. One promising avenue is the reconstruction of shear waves through measurement of in-plane rather than out-of-plane displacements at laser inspection points in a pitch-catch configuration, as illustrated in the numerical case study in §2.5. This adaptation could provide deeper insights into material characterisation and enhance applicability to components with complex geometries, owing to the higher sensitivity of shear waves to microstructural features. Since some non-destructive methods rely on phase velocity to determine elastic constants [28], extending the methodology from group to phase velocity also represents a promising direction for future application.

3.6 Summary

This chapter explores the potential of employing GF reconstruction from ultrasonic diffuse fields for accurate velocity measurements on components with curved geometries. This application significantly increases the complexity compared to imaging applications, due to the sensitivity of arrival time to velocity measurements. Therefore, highly accurate reconstruction in terms of good waveform accuracy and minimal reconstruction noise must be achieved. This involved establishing a set of necessary steps and techniques, which included the utilisation of spectral energy density curves to guide the optimisation of diffuse field parameters such as window length and time delay, the deconvolution of an unwanted source-dependent factor to achieve the correct waveform at the accurate arrival time and the implementation of 1B signal normalisation to expedite convergence to GF reconstruction within the same window length to account for energy decay in experiments. Subsequently, the capability for velocity measurements at the level of accuracy required for texture inversion (i.e. 10 ms^{-1}) was demonstrated using phased array receivers on a flat sample and, in a more challenging implementation, using laser receivers on a curved surface. These results establish the practical feasibility of the approach and motivate its extension to full texture characterisation, which is developed in the following chapter.

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Chapter 4

Texture Characterisation via Velocity Inversion

4.1 Introduction

This chapter implements the previously validated velocity measurement methodology, based on GF reconstruction from ultrasonic diffuse fields, to a full 3D acquisition for texture characterisation. A custom experimental setup is developed to measure directional longitudinal wave velocities across a spherical, textured titanium sample. The spherical geometry is deliberately chosen to represent complex, curved surfaces that are frequently encountered in industrial applications and that pose challenges for conventional ultrasonic velocity measurement approaches. To complement and refine the GF reconstruction process, finite element simulations are employed to systematically assess different excitation strategies using phased array elements, with a particular focus on improving the reconstruction accuracy of the longitudinal wave component. Strategies considered include conventional non-steered excitation, beam steering, and phase-shifted excitation. The experimentally acquired directional velocities are subsequently processed through a texture inversion framework to estimate the sample's crystallographic texture, which is then benchmarked against EBSD measurements performed on an off-cut specimen from the same sample. The chapter concludes with a discussion on the agreement between ultrasonic and EBSD-derived textures and explores the implications of this methodology

for practical texture characterisation of polycrystalline materials with complex geometry. This study is being prepared for publication [P2].

4.2 Texture Inversion Formula

In this section, the texture inversion formula, developed by Lan *et al.* [3, 79], used in this study is briefly outlined, with emphasis placed on the underlying theoretical basis and, more importantly the specific inputs required for texture characterisation. The formula is developed based on a grain-average model, in which the polycrystal velocity arises from waves traversing multiple grains, each contributing differently depending on its orientation relative to the propagation direction. This leads to an expression of the polycrystal velocity (v) as a spherical convolution between the ODF (ω) and the single-crystal velocity variation function (k), which serves as the kernel of the convolution and can be calculated directly from the Single-Crystal Elastic Constant (SCECs). In the Spherical Harmonics (SH) domain, this convolution simplifies to a pointwise multiplication of the corresponding SH coefficients. In the defined context of hexagonal crystals due to the use of titanium, whose single crystal is transversely isotropic about the c -axis, the expression further reduces to a convolution between the c -axis orientation distribution (i.e. the (0001) pole distribution) and the single-crystal velocity variation function [79]:

$$V_{lm} = W_{lm0} K_{l0} \quad (4.1)$$

where V_{lm} are the SH coefficients of the polycrystal velocity variation, K_{l0} the coefficients derived from the SCECs, and W_{lm0} the coefficients of the ODF. The V_{lm} and K_{l0} are expanded in standard SH bases over the polar angle θ and azimuthal angle ϕ , while the ODF is expanded in Wigner-D functions of two Euler angles (α, β) , reduced from three by hexagonal symmetry [79].

Equation 4.1 allows both forward prediction of polycrystal velocities from a known texture and inverse reconstruction of texture from measured velocities. In this study, the focus

is on the latter, specifically on retrieving the ODF, which characterises the complete texture of a polycrystal. In [79], although only the texture inversion was conducted by de-convolving the single crystal phase velocity from the polycrystal phase velocity function, it was highlighted that group velocity could also be used. Consequently, since the GF reconstruction method provides polycrystal group velocity as demonstrated in §3.3, the inversion remains valid as long as the kernel is likewise defined in terms of group velocity.

To obtain the ODF coefficients W_{lm0} , the polycrystal velocity coefficients V_{lm} must first be computed from velocity measurements in discrete directions. This is carried out using a Gaussian-Legendre integration scheme [79, 80]:

$$V_{lm} = \frac{\pi}{N} \sum_{i=0}^{2N-1} \sum_{j=0}^{N-1} v(\theta_i, \phi_j) Y_{lm}^*(\theta_i, \phi_j) \omega_N \quad (4.2)$$

where N is the order of the integration scheme, θ_i are the Gaussian-Legendre polar nodes, ϕ_j are $2N$ equiangular azimuthal angles, Y_{lm}^* are the conjugates of the SH basis functions, and ω_N are the Gaussian weights.

In the same work [79], it was shown that higher-order ODF coefficients (W_{lm0} with $l > 4$) have a negligible influence on polycrystal velocities. Truncation at the fourth order therefore provides sufficient accuracy for texture inversion in HCP crystals, yielding very good quantitative agreement. Accordingly, this study limits the determination of ODF coefficients to the fourth expansion order. Implementing this truncation requires setting $N = 6$ in the Gaussian-Legendre scheme, which corresponds to velocity measurements in 6×12 discrete directions. By exploiting antipodal symmetry, the measurement set is reduced to six polar angles (21.1769° , 48.6078° , 76.1949° , 103.805° , 131.392° , and 158.823°) combined with six azimuthal angles (0° , 30° , 60° , 90° , 120° , and 150°).

The objective of this study is to employ the GF reconstruction methodology to obtain velocity values $v(\theta_i, \phi_j)$ in these 36 discrete directions. These measured velocities are then used within the inversion framework to reconstruct the texture (i.e. the ODF). Specifically, the discrete data are first processed through the numerical integration in

Equation 4.2 to compute the coefficients V_{lm} , which are then substituted into Equation 4.1 to derive the ODF coefficients W_{lm0} .

4.3 Experimental Setup

This section provides a step-by-step account of the experimental setup, outlining the material and engineering design choices informed by GF reconstruction principles, and shows how the system components were integrated to obtain diffuse field reconstructions of wave velocity along the 36 directions defined in §4.2.

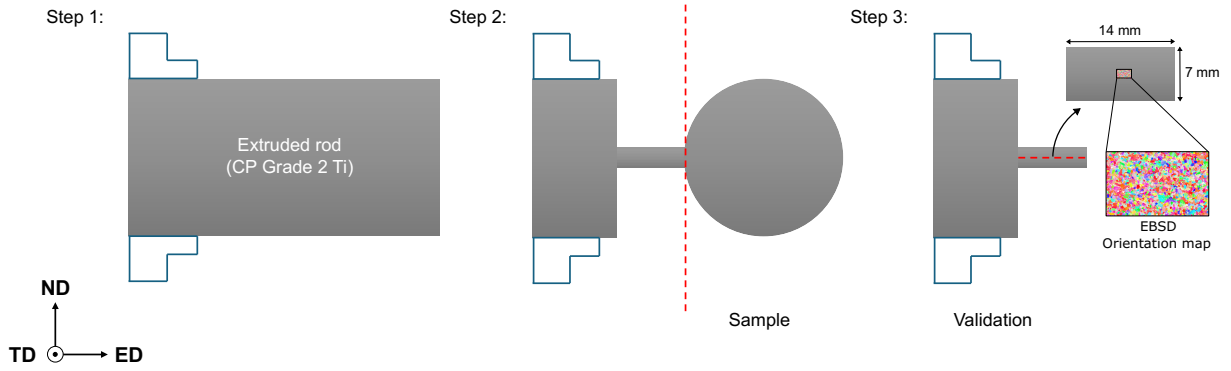


Figure 4.1: Step-by-step schematic provided to the manufacturer for producing the textured CP Ti sample. Red dashed lines indicate the cuts made at different stages of the manufacturing process. The 14×7 mm surface exposed after the final cut was used for EBSD measurement. The sample system is defined by the ED, TD and ND.

A spherical sample, 40 mm in diameter, was made from CP Ti Grade 2, which has a pure hexagonal crystal structure, and has low acoustic noise, making it an ideal material for this proof of concept study. To intentionally introduce texture, the spherical sample was machined from an extruded CP Ti rod. The extrusion process aligns the crystal stiffness tensors of grains perpendicular to the extrusion direction, thereby inducing a preferred crystallographic orientation. The manufacturing procedure, illustrated in Figure 4.1, was provided to the manufacturer as step-by-step instructions. The spherical curvature was achieved via Computer Numerical Control (CNC) machining (Step 2), leaving a stem that connected the sphere to the remaining rod. A subsequent cut separated the sample, producing a flat surface on the sphere by design that defined the z -axis in alignment with the original rod axis, as shown in Figure 4.2. Finally, the remaining stem was cut along

the rod axis (Step 3) to produce a 14×7 mm specimen for EBSD analysis to validate the induced texture.

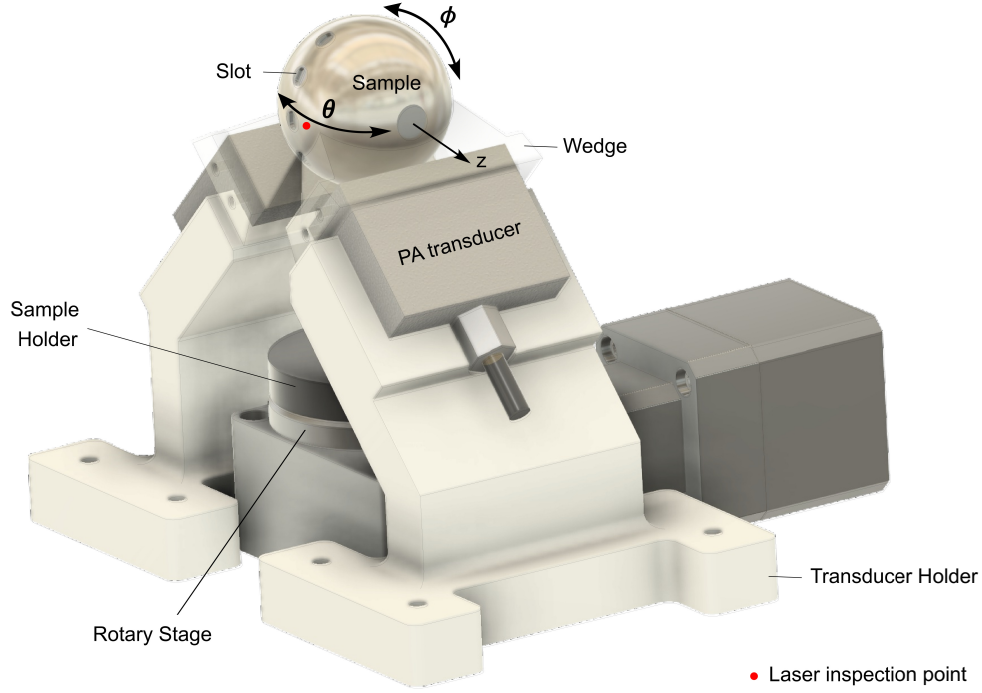


Figure 4.2: Schematic of the experimental setup used for GF reconstruction to measure velocities along 36 directions. Two laser receivers are positioned across the sample diameter to reconstruct the longitudinal velocity. Only the front-facing laser spot is shown due to the schematic orientation. The sample's z -axis is indicated, along with the polar angle (θ) and azimuthal angle (ϕ) used to define each measurement direction.

A schematic of the full setup is shown in Figure 4.2. The spherical geometry of the sample ensures standardised propagation distances across all directions, with laser receivers positioned at diametrically opposite points on the surface to reconstruct the coherent response and determine wave velocity from the first-arrival. This configuration also replicates the complex curved surfaces commonly encountered in industrial components.

Velocity measurements along multiple directions were achieved by machining six equally spaced slots at 30° intervals around the sphere's equator, enabling manual rotation in the azimuthal direction. Polar angle adjustments, accurate to four decimal places, were finely controlled using a Newmark RT-2-10 motorised rotary stage. These experimental components are depicted in Figure 4.2.

Wave excitation was achieved using the same array transducers mentioned in §3.4.2. However, because the spherical sample lacks flat surfaces for direct coupling, custom

Rexolite wedges were designed to interface the flat transducer faces to the curved surface. Rexolite 1422 was selected for its low acoustic attenuation compared to other polymers such as acrylic or fused silica. Gel couplant was used at both the transducer-wedge and wedge-sample interfaces to ensure effective acoustic transmission.

Signal reception was performed using two Polytec laser vibrometer sensor heads (specifications provided in §3.4.2), aligned perpendicular to the sphere surface to detect out-of-plane displacement. To maintain consistent excitation and reception conditions, several additional components were designed and 3D-printed. These included a custom sample holder mounted to the motorised rotary stage, which featured a protruding tip to engage the machined slots and enable stable azimuthal rotation, and two transducer holders designed to maintain a fixed height and alignment of the wedges against the sphere surface without interfering with the laser reception, as seen in Figure 4.2.

The experiment was conducted in a temperature-controlled environment with fluctuations kept below 2°C to minimise thermal drift during GF reconstruction. Synchronisation of excitation, laser detection, and data acquisition followed the same protocol outlined in §3.4.2 and is illustrated in Figure 3.10b.

4.4 Numerical Simulations

Numerical simulations were carried out in this study with two main objectives. First, they provided a reference pitch-catch signal for comparison with the experimentally reconstructed waveforms. Second, and more critically, they enabled investigation of wave propagation at the wedge-sphere interface, where the wave emitted from the flat transducer surface enters the curved geometry of the spherical sample. This interaction is a key factor, as it directly influences the characteristics of the diffuse field and, in turn, the quality of the GF reconstruction. To explore this, a 2D FE model was developed using Pogo [56]. A 2D model was chosen over of a full 3D implementation due to the prohibitive computational cost of the latter, which would require solving over 0.5 trillion Degrees of Freedom (DoF), compared with 8.7 million DoF in 2D. While the 2D simplification sacrifices out-of-plane scattering, full mode density, and some path diversity inherent to

3D geometries, the restriction to a single plane enhances scattering, making GF reconstruction more robust for generating the reference signal. Crucially, it still preserves the essential wave interactions at the wedge-sphere interface and the qualitative development of diffuse fields, making it sufficient to meet the two objectives of this study: providing a reference signal and enabling investigation of wave propagation into curved geometries.

The model includes an isotropic CP Ti sphere with a 40 mm diameter and a Rexolite wedge used in the experimental setup. A schematic of the model is shown in Figure 4.3. The material was modelled as isotropic rather than anisotropic, since the focus of the simulation was on wave transmission across the wedge-sample interface, rather than capturing the effects of crystallographic anisotropy.

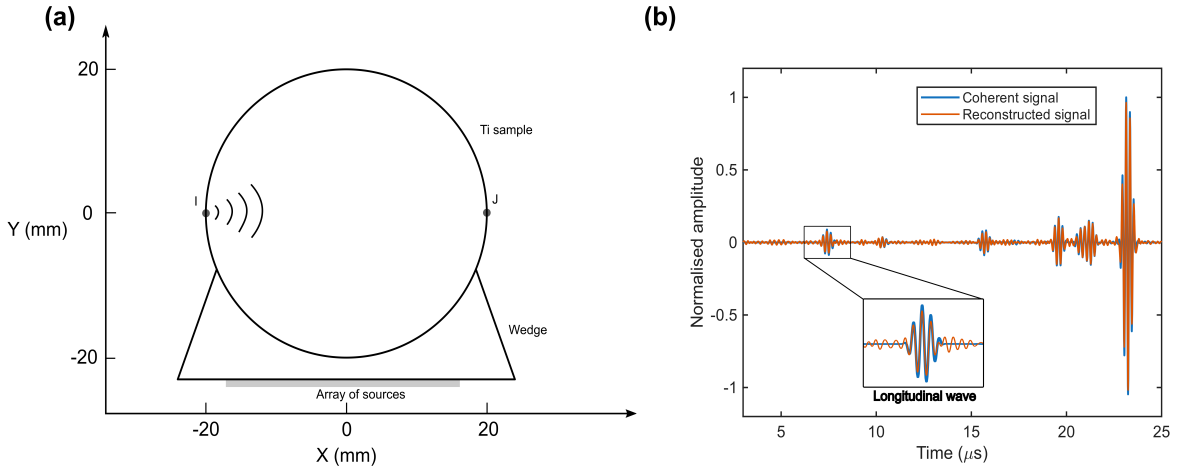


Figure 4.3: FE model to simulate the GF reconstruction process from diffuse fields and evaluate different excitation strategies considering the excitation path from the bottom flat surface of the wedge into the sphere. **a** shows the schematic of the sample modelled using FE. Diffuse fields were collected at location *I* and *J* to reconstruct the coherent response received at *J* from *I*. **b** compares the reconstructed signal against the coherent response, highlighting the longitudinal tone burst that is used to determine the wave velocity.

Since only a 2D model was used, the simulation was limited to a single 5 MHz linear array consisting of 64 sources with a pitch of 0.6 mm to generate the diffuse fields. These fields were recorded at two receiver locations, denoted as *I* and *J*, and subsequently averaged over all sources to reconstruct the coherent response between them. Each source was modelled through a force applied perpendicular to the surface, to nodes within a 0.5 mm aperture, using a 5-cycle Hann-weighted sine wave with a central frequency of 5 MHz. The cross-correlation of the diffuse fields received at locations *I* and *J* was computed in

4. Texture Characterisation via Velocity Inversion

the time domain, using a time window starting at 0.1 ms with a duration of 1.9 ms. The resulting reconstructed signal was then compared against the directly simulated coherent response between the same two points. This comparison is shown in Figure 4.3b.

Two key observations can be made from Figure 4.3b. First, with this setup, the direct pitch-catch arrival, corresponding to the longitudinal wave at around $7\text{ }\mu\text{s}$, can be successfully reconstructed. The reconstructed tone burst, shown in orange, aligns closely with the coherent response in both arrival time and waveform shape, as shown in blue. Second, the amplitude of the longitudinal arrival is relatively low compared to other signal components, such as the SAW observed at $23\text{ }\mu\text{s}$. This makes it more susceptible to reconstruction noise and underscores the importance of carefully tuning the GF reconstruction parameters to ensure the longitudinal arrival can be reliably extracted for velocity measurement.

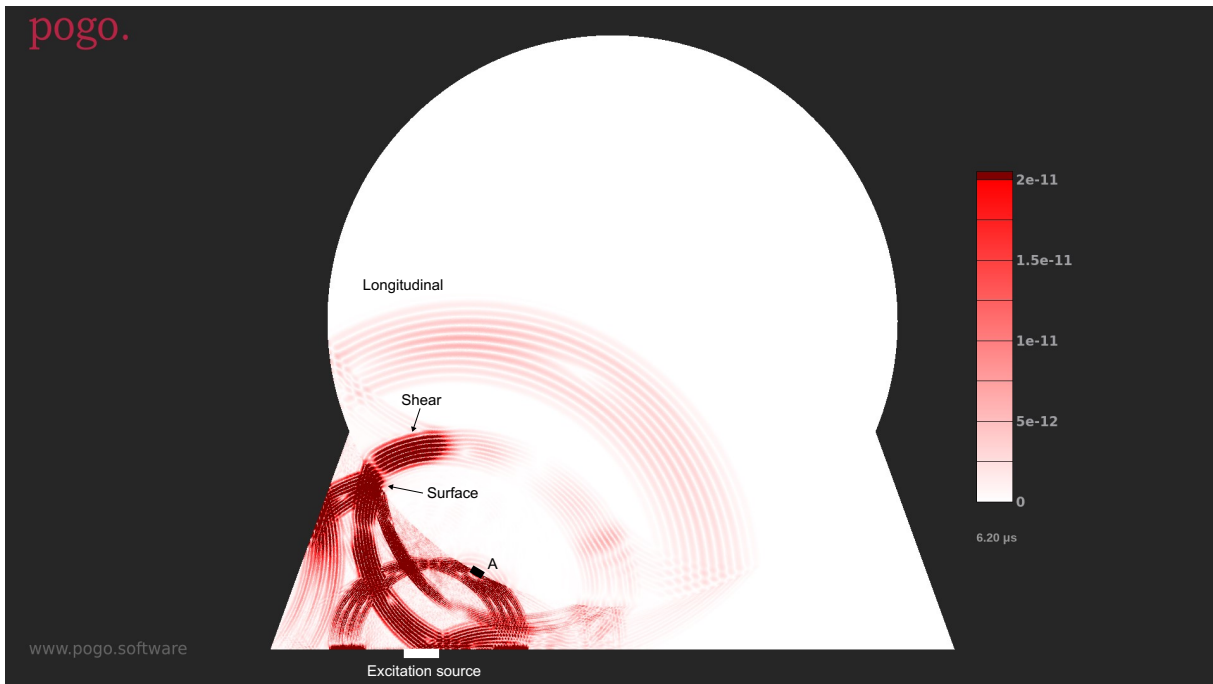


Figure 4.4: Snapshot of the wave field at $6.2\text{ }\mu\text{s}$ following excitation by a 3-element array source. At the curved wedge-sphere interface A, the incident longitudinal wave enters at an angle, resulting in mode conversion. This leads to the emergence of refracted longitudinal, shear, and surface wave modes within the CP Ti sphere.

Noise was not included in the FE model; however, experimental observations in §3.4.2 indicate that laser sensitivity to ultrasound is relatively low, resulting in received signals with high noise levels. To resolve this problem, each source was modified to become a

multi-element subaperture that was excited to introduce more energy into the sample. In preparation for a similar experimental implementation in this study, this excitation strategy was modelled, and the effect of the curved surface on wave propagation into the sample is shown in Figure 4.4. The simulation results show that, due to the curvature of the spherical surface, the excited longitudinal wave arrives at the wedge-sphere interface at an angle. This oblique incidence leads to refraction and, more importantly, mode conversion, where part of the wave energy is converted into shear and surface waves in addition to the refracted longitudinal wave. This observation suggests that such mode conversion could degrade the reconstruction quality of the longitudinal component of the Green's Function, highlighting the need to investigate whether modifications to the excitation method, leveraging the control available over individual array elements, can improve reconstruction accuracy.

In this numerical study, two excitation strategies were investigated on their effectiveness to enhance the reconstruction quality of the longitudinal wave. The first strategy involved beam steering, where time delays were applied across the subaperture elements to reduce the angle of incidence at the curved wedge-sphere interface. Specifically, the delays were calculated such that the centre of the resultant longitudinal wavefront arrives tangent to the interface, ensuring perpendicular propagation at the point of incidence and thereby reducing mode conversion. The second strategy employed a phase-shifted subaperture excitation, in which relative phase delays were applied across the active elements to suppress surface wave generation. Owing to the flat-to-curved excitation geometry, the effective spacing between neighbouring excitation points projected onto the spherical titanium surface does not correspond to the SAW wavelength at 5 MHz in CP Ti. The relative phase delays were therefore calculated to compensate for this geometric mismatch. This phase modification aims to drive adjacent surface wave contributions into antiphase, resulting in destructive interference that suppresses the formation of unwanted surface waves on the spherical sample.

Subaperture sizes ranging from one to ten elements were systematically investigated for the beam-steered configuration, while sizes from three to six elements were chosen for the phase-shifted excitation to provide a representative subset. To ensure a fair comparison across different excitation configurations, all subapertures were centred at the same spa-

4. Texture Characterisation via Velocity Inversion

tial location, and the total number of subapertures used for source averaging was fixed at 55 for each case, corresponding to the maximum achievable with the ten-element subaperture. For each subaperture size, the performance of the beam-steered and phase-shifted configurations was evaluated both relative to one another and against their corresponding non-steered counterparts, which served as reference cases.

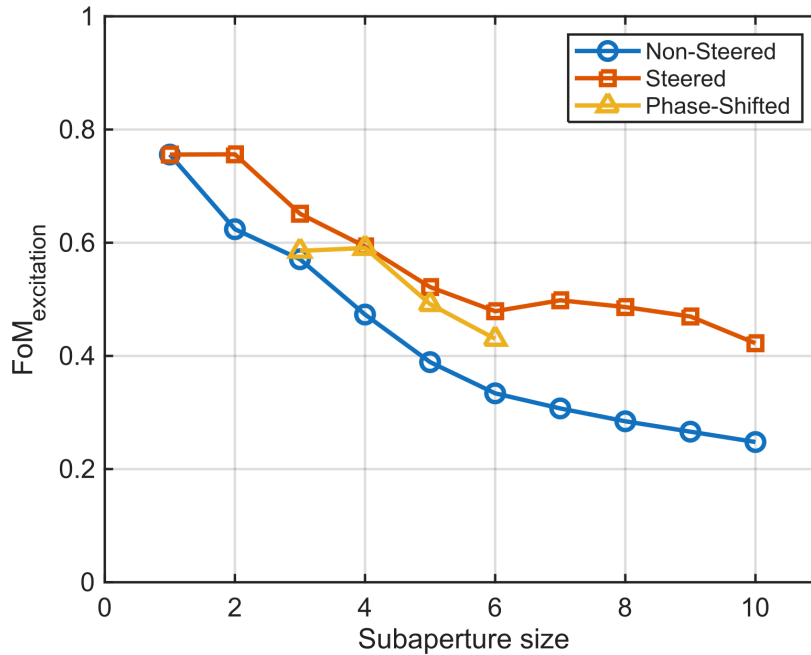


Figure 4.5: Comparison of $FoM_{excitation}$ as a function of subaperture size for different excitation strategies: non-steered (blue), beam-steered (orange), and phase-shifted (yellow). Higher $FoM_{excitation}$ values indicate better reconstruction quality.

To quantitatively evaluate the effectiveness of each excitation strategy, a composite FoM, $FoM_{excitation}$, was formulated by incorporating two key metrics: SNR and correlation value (corrVal) of the reconstructed tone burst. SNR is defined as the ratio of the peak amplitude of the first-arrival to the background noise level in the reconstruction, reflecting signal fidelity. The corrVal, computed as the Pearson Correlation Coefficient (PCC) [81] between the reconstructed and reference coherent tone burst, quantifies waveform similarity and time-shift errors. The composite FoM is expressed as:

$$FoM_{excitation} = SNR * corrVal \quad (4.3)$$

This formulation penalises reconstructions with low SNR, significant time shifts, and shape distortions. The FoM was normalised such that the ideal value of 1 corresponds to the case using all 64 individual elements as sources.

Figure 4.5a shows the variation of $FoM_{excitation}$ across different subaperture sizes for non-steered, steered, and phase-shifted excitations, represented in blue, orange, and yellow, respectively. Higher values of $FoM_{excitation}$ correspond to better reconstruction quality. All three curves exhibit a decreasing trend with increasing subaperture size, consistent with the established understanding that larger subapertures generate more directional wave fields, which reduce the rate of wave diffusion within the sample. A less diffused field diminishes the effectiveness of GF reconstruction, thereby lowering $FoM_{excitation}$.

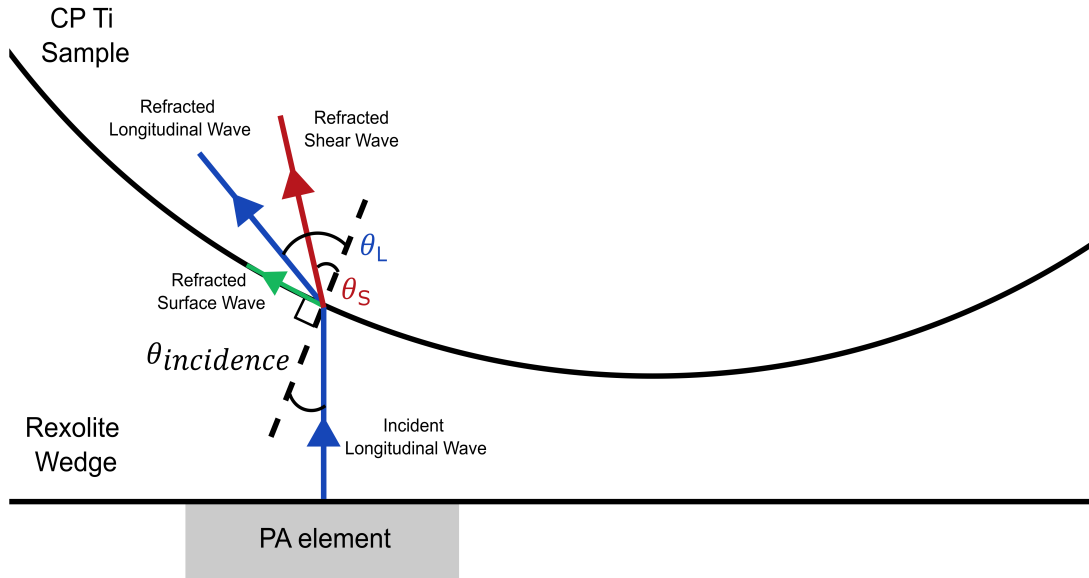


Figure 4.6: Close-up view of the wedge-sphere interface, illustrating how the incident angle of an array element excitation is defined. Snell’s law is applied at this interface to determine the resulting wave modes generated in the titanium sphere, including refracted longitudinal, refracted shear, and the refracted surface wave generated from the longitudinal excitation in Rexolite.

Comparing the blue and orange curves, it is evident that beam steering improves the reconstruction quality across all subaperture sizes and confirms the hypothesis that mode conversion at the curved wedge-sphere interface degrades reconstruction performance. Beam steering aligns the wavefront to enter the sphere closer to normal incidence, reducing the angle of incidence and thus minimising mode conversion. From a wave mode perspective,

4. Texture Characterisation via Velocity Inversion

the mode-converted surface wave does not contribute to reconstruction, as the GF reconstruction method relies on scattering within the volume of the sample, rather than along its surface. Mode-converted shear waves, although bulk modes, contribute only indirectly through subsequent scattering that partially redistributes energy into longitudinal modes, because GF reconstruction is inherently mode-specific. In addition, considering that shear waves propagate more slowly, scattering is less efficient than for longitudinal waves, further weakening their role in reconstructing longitudinal components and resulting in a lower $FoM_{excitation}$.

Further insight can be obtained by examining the excitation generated by an individual array element and using Snell's law to determine the resultant wave modes generated at the wedge-sphere interface. With reference to Figure 4.6, a longitudinal wave incident at angle $\theta_{incidence}$ in Rexolite gives rise to transmitted longitudinal and shear waves in titanium, as well as the possibility of a SAW along the interface. The condition is expressed by:

$$\frac{\sin \theta_{incidence}}{v_{Rex}} = \frac{\sin \theta_L}{v_{Ti}^L} = \frac{\sin \theta_S}{v_{Ti}^S} = \frac{\sin 90^\circ}{v_{Ti}^{SAW}}, \quad (4.4)$$

where $v_{Rex} \approx 2330 \text{ ms}^{-1}$ is the longitudinal velocity in Rexolite, and $v_{Ti}^L \approx 6100 \text{ ms}^{-1}$ and $v_{Ti}^S \approx 3200 \text{ ms}^{-1}$ are the longitudinal and shear velocities in CP Ti. The surface wave velocity in titanium, $v_{Ti}^{SAW} \approx 2940 \text{ ms}^{-1}$, corresponds to the limiting case of $\theta = 90^\circ$. From these values, three critical angles can be identified: the longitudinal critical angle at approximately 22° , the shear critical angle at approximately 47° , and the surface-wave critical angle at approximately 52° . By calculating the incidence angle $\theta_{incidence}$ for each array element from its position and wedge geometry, the resultant wave modes in the titanium sphere can be predicted. This analysis shows that, of the 64 array elements, around ten generate only reflections, six excite surface waves, 22 generate primarily shear waves, and only 36 contribute to longitudinal propagation (alongside some shear waves). In other words, approximately half of the array elements excite longitudinal waves into the Ti sample effectively. Beam steering reduces $\theta_{incidence}$ toward normal incidence, ensuring that more elements excite longitudinal modes directly and thereby enhancing the quality

of Green's Function reconstruction.

Another important observation is that the difference in performance between the non-steered and beam-steered cases widens as the subaperture size increases. This may be attributed to improved beam formation and steering control with more elements, leading to better alignment of the wavefront with the curved interface and hence better preservation of longitudinal energy. The yellow curve representing phase-shifted excitation shows some improvements over the non-steered case but remains slightly less effective than full beam steering. This strategy successfully suppresses surface waves via destructive interference but does not address the issue of shear wave generation, which continues to limit reconstruction quality compared to beam-steered excitation.

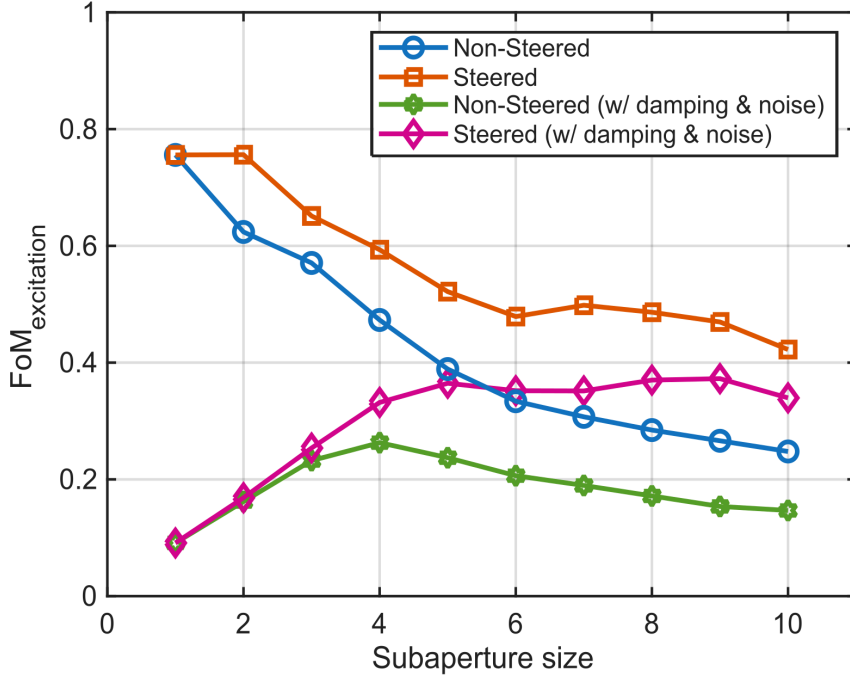


Figure 4.7: Comparison of $FoM_{excitation}$ versus subaperture size, showing the effect of attenuation and noise on non-steered (green) and steered (pink) excitations, relative to the noise-free case. Higher values indicate better reconstruction quality.

Attenuation and noise were not included in the FE model; however, in realistic scenarios, both effects are unavoidable and significantly impact the diffuse signals by reducing their amplitude and limiting the effective time window before the essential diffuse information is obscured by the noise floor. As a result, the overall contribution to GF reconstruction will be weakened. Here, the generated $FoM_{excitation}$ curves are further employed as a tool to illustrate the need for larger subapertures to overcome the effective noise level,

while also highlighting the qualitative trend in reconstruction quality. To demonstrate this, a mass-proportional damping coefficient of 1500 s^{-1} was applied to simulate energy decay observed in experiments, and 20 dB bandlimited white Gaussian noise in the 2 to 8 MHz range, set below the maximum signal amplitude from a one-element subaperture, was added to represent the noise levels typically encountered with laser vibrometers. The results, shown in Figure 4.7, indicate that attenuation and noise reduce $FoM_{excitation}$, with the greatest impact observed for the smallest subaperture sizes. In particular, for phase-shifted excitation (pink curve), the trend reverses; instead of decreasing, the FoM rises with increasing subaperture size and eventually plateaus. This highlights the importance of using larger subapertures to increase the effective signal level for better reconstruction quality. Importantly, this analysis serves as a qualitative illustration of the effects of attenuation and noise, which cannot be easily reproduced experimentally, rather than serving as a prescription of exact experimental parameters.

4.5 Experimental Results

Drawing on the numerical study in the previous section, which provided a qualitative indication that experimental conditions would require subapertures larger than approximately five elements, a parametric study was conducted experimentally using subapertures of five to twelve elements to identify the optimal configuration. The experiments confirmed the need for larger subapertures, with ten elements found to yield the best performance. Consequently, each source in the experimental implementation was defined as a ten-element subaperture with beam steering applied. Given the presence of 128 elements from 2 array transducers, this reduced the number of source averages from 128 to 110 ($N = 110$). The diffuse fields were defined with a value of t_r set at 0.2 ms and a much longer T of 2800 μs . In this case, there is no need to define a value larger than 2800 μs because the energy density curves showed that signals beyond this point in time are dominated by noise. Each set of diffuse signals underwent 2000 repetitions and averaging before storage. This extensive averaging aims to effectively reduce the unwanted random noise present in the signal. Lastly, to account for the substantial amount of energy dissipation present within the 2800 μs window length, 1B normalisation was applied to the

diffuse signals before correlation to accelerate convergence towards GF reconstruction. Subsequently, a bandpass filter with cut-off frequencies of 2 and 8 MHz was applied to minimise higher-frequency quantisation errors, as discussed in §3.2.3.

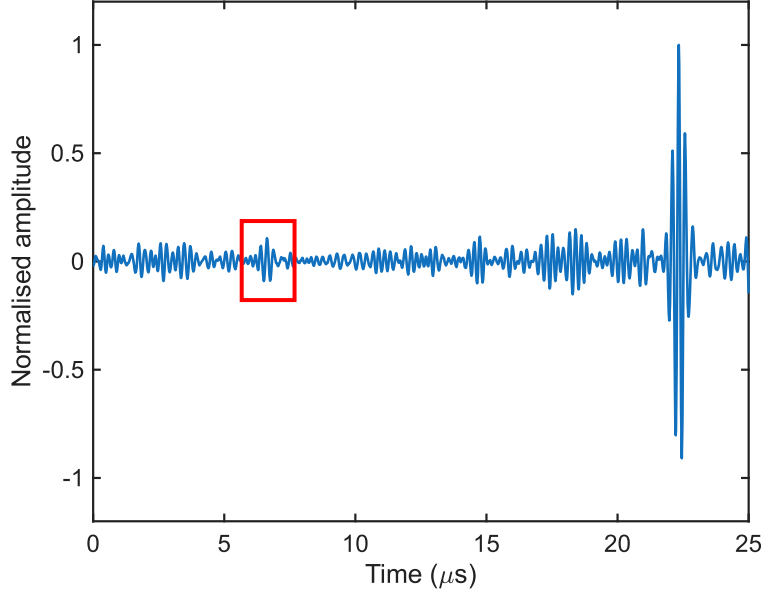


Figure 4.8: Reconstructed signal between two laser receivers positioned at diametrically opposite ends of the CP Ti spherical sample along the measurement direction $\theta = 76.1949^\circ$, $\phi = 0^\circ$. The red box highlights the reconstructed longitudinal toneburst, which is used to determine the wave velocity.

The result after GF reconstruction is illustrated by Figure 4.8, which presents the reconstructed signal in the measurement direction of $\theta = 76.1949^\circ$ and $\phi = 0^\circ$. A surface wave arrival is observed at approximately $23 \mu\text{s}$, consistent with the simulated result in Figure 4.3b. The key feature is the reconstructed longitudinal toneburst at around $7 \mu\text{s}$, which lies within the expected $\pm 0.5 \mu\text{s}$ temporal window centred at $6.6 \mu\text{s}$ when accounting for texture-induced velocity variations. A possible concern with steered or time-delayed excitations is that the different delays applied to each source might interfere with the phase of the reconstructed GF. In practice, this effect is negligible because, for a given source, the same delay is imposed on the signals recorded at both receivers. This results only in a common timing shift, which cancels out during cross-correlation. As a consequence, the reconstruction depends on the relative arrival times of the diffuse field at the receivers and is unaffected by the absolute timing of the excitation source [82].

With this validation established, reconstructed signals across all required measurement

4. Texture Characterisation via Velocity Inversion

directions were subsequently processed using the procedure outlined in §3.4 to determine the directional velocities. This yielded a complete grid of 36 data points, rounded to two decimal places, as presented in Table 4.1. Analysis of these results reveals trends consistent with the expected influence of extrusion-induced texture. During extrusion of titanium, the material undergoes substantial plastic deformation. At the microscopic level, this deformation takes place mainly by dislocation slip along the prismatic planes, which have the lowest critical resolved shear stress and therefore represent the easiest path for dislocations to move under the applied extrusion stress [83]. This deformation elongates grains along the extrusion axis and promotes alignment of crystallographic c -axes predominantly perpendicular to it [84]. Because the c -axis represents the stiffest direction in HCP titanium, ultrasonic wave velocities are maximised along directions normal to the extrusion axis, while propagation parallel to the extrusion axis exhibits comparatively lower velocities. When a spherical sample is subsequently machined from the rod, the extrusion axis defines the z -axis of the sphere, and the volumetric texture at the centre of the sphere is expected to be preserved. Consistent with this, Table 4.1 shows lower average velocities of 6039.30 ms^{-1} at $\theta = 21.1769^\circ$ and 158.823° , directions aligned closer to the z -axis, and higher velocities of 6159.54 ms^{-1} at $\theta = 76.1949^\circ$ and 103.805° , directions where the c -axes of grains preferentially orient.

Table 4.1: Directional velocities of the CP Ti sample measured across 36 measurement directions, grouped by a 6×6 grid of polar (θ) and azimuthal (ϕ) angles.

θ	21.1769°	48.6078°	76.1949°	103.805°	131.392°	158.823°
$\phi = 0^\circ$	6040.21	6153.61	6174.02	6176.69	6108.70	6067.42
$\phi = 30^\circ$	6106.98	6041.91	6180.16	6139.91	6118.22	6009.85
$\phi = 60^\circ$	5985.51	6096.18	6159.36	6163.01	6101.11	6023.99
$\phi = 90^\circ$	6051.04	6136.47	6115.08	6138.49	6135.36	6074.16
$\phi = 120^\circ$	6040.96	6045.89	6183.50	6180.77	6096.68	6044.46
$\phi = 150^\circ$	6014.70	6138.08	6167.41	6163.38	6148.80	6015.89

Table 4.2: Single crystal elastic constants of CP Ti [4]. unit: GPa. $C_{66} = (C_{11} - C_{12})/2$

C_{11}	C_{12}	C_{13}	C_{33}	C_{44}
143.4	92.0	69.0	180.7	46.7

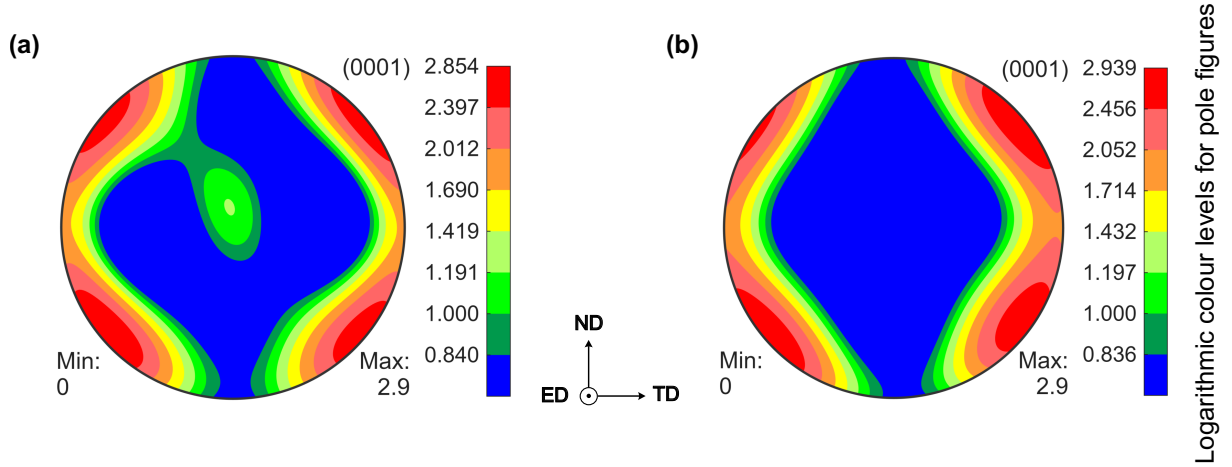


Figure 4.9: Pole figures obtained from **a** GF reconstruction methodology and **b** EBSD, with EBSD results truncated to fourth SH order. The sample system is defined by the ED, TD, ND.

From the experimentally measured velocities, texture inversion was carried out through the data processing workflow described in §4.2. The discrete velocity values were first used in the numerical integration of Equation 4.2 to compute the spherical harmonic expansion coefficients V_{lm} . These were then combined with known single-crystal K_{l0} values (calculated from the SCECs listed in Table 4.2) in the spherical convolution Equation 4.1 to obtain the ODF coefficients W_{lm0} . The resulting coefficients were subsequently imported into the MTEX toolbox [85] to reconstruct the truncated fourth order ODF and generate the (0001) pole figure, shown in Figure 4.9a.

To validate the texture results obtained from ultrasound, EBSD measurement was performed on the 14×7 mm specimen, sectioned along the rod axis as illustrated in Figure 4.1. This sectioning results in an EBSD scan plane corresponding to the ED–TD plane. EBSD analysis began with careful sample preparation to obtain a smooth, deformation-free surface. This was achieved by first grinding the specimen using SiC paper up to 4000 grit, followed by electro-polishing at -40°C in a solution of 90% methanol and 10% perchloric acid for one minute. The prepared sample was then mounted in a Tescan CLARA Scanning Electron Microscope equipped with an EBSD detector and tilted to approximately 70° to maximise backscattered electron yield. Kikuchi diffraction patterns generated by the electron beam were captured by the detector and indexed using AZtec-Crystal software to determine the crystallographic orientations across a scanned region of 2×1 mm with a step size of $1 \mu\text{m}$. EBSD mapping on the ED–TD plane provides

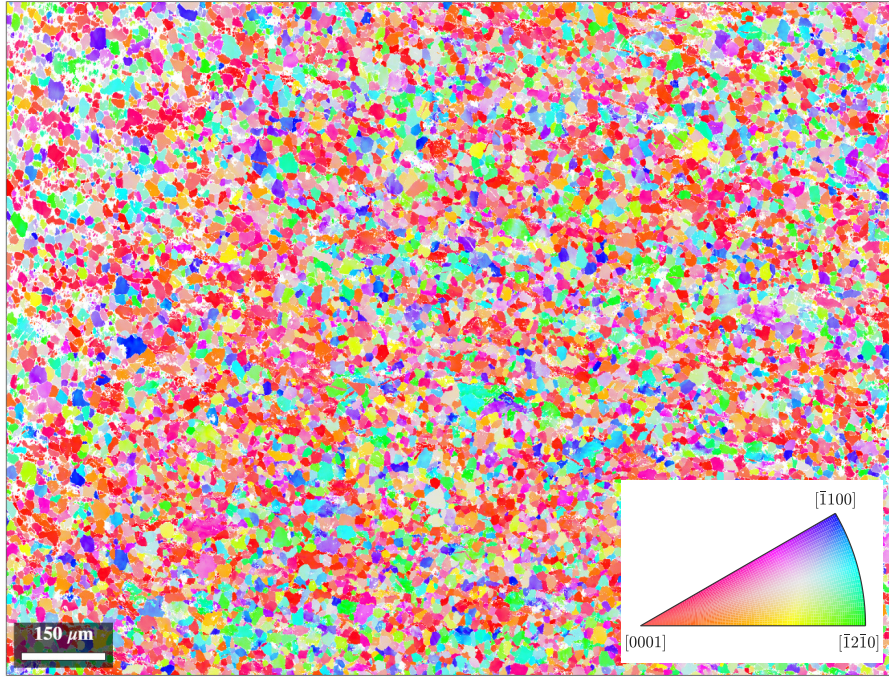


Figure 4.10: EBSD scan of a region from the 14×7 mm specimen, sectioned along the rod axis of the CP Ti rod from which the spherical sample was manufactured. The scan was acquired on the ED-TD plane, and the inverse pole figure colouring represents the crystallographic orientation of individual grains relative to the sample's TD.

the full crystallographic orientation of each intersecting grain, enabling grain alignment relative to the TD to be evaluated independently of the scan plane. The resulting 2D EBSD orientation map, coloured with respect to the TD, is presented in Figure 4.10. The crystallographic orientation dataset obtained from EBSD was truncated to the fourth order before reconstructing the corresponding (0001) pole figure using MTEX, shown in Figure 4.9b.

Comparing the two pole figure plots in Figure 4.9, the ultrasonic GF-based pole figure reproduces the dominant texture features observed in the EBSD reference truncated to fourth order. The intensities, shown on a logarithmic scale, indicate the degree of orientation preference relative to a random texture, which corresponds to a log-scaled intensity of zero. The values demonstrate good agreement between the two methods, with the similarity between pole figures to be quantified as 92% similarity using PCC. A strong concentration of (0001) poles is observed along the ND-TD plane, together with a depletion along ED, consistent with the extruded titanium texture where the c -axes lie predominantly perpendicular to ED [84]. For the GF-based measurement, the pre-

cise orientations of ND and TD on the spherical sample were not physically marked; consequently, the reconstructed pole figure in Figure 4.9a was rotated to align the pole concentration with the EBSD-derived reference in Figure 4.9b. This rotation should not affect the overall interpretation of the GF reconstruction methodology. Minor variations in the ND-TD plane orientation may occur, but the pole figures consistently show the main texture pattern, with strong alignment in the ND-TD plane and reduced intensity along ED. These dominant features are preserved, highlighting the robustness of the GF-based velocity measurement and texture inversion methodology. A minor discrepancy remains in the GF-based result as a residual hotspot near the GF-axis, visible as a green lobe at the centre of Figure 4.9a. However, this region still corresponds to relatively low texture intensity, so the deviation has little overall influence. This localised difference may also reflect the fundamental difference between the techniques: EBSD captures orientations at the surface, whereas GF reconstruction reflects a volumetric response. Additional sources of discrepancy may arise from slight mismatches between the single crystal elastic constants used in the inversion and those of the actual material, as well as experimental uncertainties in the measured velocities such as small positioning errors in the laser measurements, where a misalignment of only 0.5 mm can introduce differences of approximately 70 ms^{-1} . Despite localised deviations, the overall texture characteristics, including the dominant alignment in the ND-TD plane, the relative randomness along ED, and the general intensity distribution, are effectively captured. Overall, these results demonstrate that the GF reconstruction-based velocity measurement methodology delivers accurate velocities to enable reliable texture inference in textured samples with complex geometries.

4.6 Discussion

The results presented in the previous section highlight the effectiveness of the GF reconstruction method from diffuse fields for accurately reconstructing directional longitudinal velocities for texture inversion. Building on this core capability, several key considerations and opportunities arise for extending the methodology to more general and practical applications.

A limitation of the present implementation is the time required for data acquisition, primarily due to the limited sensitivity of the laser receiver, which necessitated extensive signal averaging to obtain accurate reconstructions. In addition, laser vibrometers are inherently sensitive to environmental vibrations, which can further degrade the SNR, particularly at low amplitudes. This sensitivity makes measurements susceptible to external disturbances and increases the need for repeated averaging. For each measurement direction, approximately 2000 signal averages were required per source, corresponding to an acquisition time of about one hour per source. With over 110 different source positions used, this resulted in substantial total acquisition times when extended across all measurement directions. To reduce this burden, several strategies can be considered. As discussed in §3.5, one option is to employ laser systems with higher sensitivity, which would directly lower the required number of averages. Another approach is to maximise the Pulse Repetition Frequency (PRF), ensuring that the time interval between excitations is close to the diffuse field duration, thereby accelerating acquisition without signal overlap. Furthermore, convergence studies can be performed to identify the minimum number of signal averages beyond which reconstruction quality shows negligible improvement; in this study, such an analysis enabled a reduction from 3000 to 2000 averages. A similar optimisation could be applied to the number of source positions, which was not explored here but offers further potential to reduce acquisition time. Due to the time-intensive nature of the experiment, the present study is based on a single demonstration performed on one sample, and the current validation therefore remains limited in scope. Future work should focus on repeating the measurements multiple times on the same specimen to assess repeatability and robustness, and subsequently extending the investigation to a wider range of samples to evaluate reproducibility and general applicability.

Another consideration concerns the spatial accuracy of laser receiver placement, which becomes especially critical when extending the method to arbitrary geometries. Accurate knowledge of the spatial positions of the laser receivers is critical for reliable velocity extraction from GF reconstruction, as the measured travel times must be divided by the separation distance between detection points to calculate velocity. Any misalignment in receiver positioning introduces a direct error in this distance and therefore produces an inaccurate velocity estimate. For example, in the present 40 mm spherical sample, a 1 mm

positional error corresponds to a velocity offset of approximately 150 ms^{-1} , underscoring the sensitivity of the method to geometric accuracy. In this study, such errors were minimised by the use of a spherical geometry with well-defined symmetry, CNC machining tolerances of $\pm 0.05 \text{ mm}$, and stable mounting on a rotation stage. Receiver locations were further verified through physical marking of laser inspection spots and caliper measurements. Extending the approach to arbitrary, irregular geometries presents greater challenges, as symmetry cannot be relied upon to define receiver locations. In such cases, one possible solution is to capture the surface geometry and place it in a global coordinate system, for example using a Computer-Aided Design (CAD) model or a 3D scan of the part, and then reference the laser detection points against this model, so their precise positions on the part are known. For practical application, automated laser positioning systems, such as galvanometer mirrors or robotic arms, could be used to ensure accurate and consistent placement of detection points on curved surfaces. Without such measures, errors in spatial positioning may become as limiting to inversion accuracy as uncertainties in the arrival time measurements themselves, highlighting a key consideration for future application of the method to engineering-scale components.

In addition, the flexibility of the method in terms of excitation strategies offers opportunities to adapt it for different practical scenarios, including fully non-contact configurations that are more suitable for in-process applications. In this experiment setup, however, the excitation remains a contact mode with a flat surface, which introduces additional complexity in generating diffuse fields, as discussed in §4.4. One possible option for excitation is the use of Electro-Magnetic Acoustic Transducers (EMATs), which can directly generate longitudinal waves in a non-contact configuration. When paired with laser vibrometry for detection, EMATs would enable a fully non-contact experimental setup suitable for in-process monitoring. A drawback, however, is that EMATs typically produce weaker signals than piezoelectric transducers [86], leading to lower SNR that may limit the accuracy of GF reconstruction. Other options include high-energy pulsed lasers or arrays of pin transducers in contact mode. Overall, the methodology is versatile: the excitation source can be tailored to balance practicality, signal strength, and measurement needs, while laser detection retains the advantages of non-contact operation on complex geometries, such as continuous monitoring during manufacturing.

4. Texture Characterisation via Velocity Inversion

Another aspect is the inevitable presence of surface waves in the reconstructed signals, as shown in Figure 4.3b and 4.8. Although they were not required for texture characterisation in this study and were deliberately suppressed through the excitation strategy described in §4.4, their reconstruction remains unavoidable in practice. Crucially, the same GF reconstruction framework can retrieve both bulk and surface wave components simultaneously, enabling crystallographic texture characterisation to be complemented by the possible detection of surface-breaking defects within a single measurement framework. Finally, the capability of this method can be extended by reconstructing shear velocities, which broaden its applicability to lower-symmetry crystal systems for texture inversion. As discussed in the numerical case study in §2.5, measuring pure in-plane particle motion at the laser inspection points, rather than only the out-of-plane component used here, provides a route to reconstruct the single shear velocity present in isotropic materials. In anisotropic materials, each propagation direction supports two distinct quasi-shear modes (quasi-shear vertical and quasi-shear horizontal). While their polarisations are orthogonal to each other, they are generally oblique to the propagation direction and therefore contain both in-plane and out-of-plane components. Consequently, accurate separation of the two requires access to at least two orthogonal displacement components at each point, which in practice calls for detectors with multi-axis sensitivity. Experimentally, this requirement can be satisfied by using a 3D scanning laser vibrometer to capture orthogonal displacement components at each receiver position. If such a commercial multi-axis system is unavailable, two single-axis vibrometers mounted at known angles can be combined to recover the necessary vector components.

Even with appropriate equipment, several technical challenges remain and using diffuse field parameters that worked for longitudinal reconstruction may not be optimal for shear recovery. Shear waves propagate more slowly than longitudinal waves, which reduces their scattering efficiency and delays the onset of diffusion. This implies that a larger t_r is required before the wave field becomes sufficiently diffuse, and a longer time window T must be captured to ensure contributions from multiple propagation paths for reliable reconstruction. Consequently, longer diffuse fields must be recorded before sufficient energy redistribution occurs, which in turn makes the reconstruction more sensitive to noise. This reinforces one of the central challenges identified throughout this work: con-

trolling the noise floor. Among the strategies investigated, increasing spatial averaging over source positions proved the most effective, though enlarging subapertures or raising the excitation voltage can also help by boosting the effective signal level. At a deeper level, the choice of excitation mode is also important. While scattering eventually redistributes energy among wave modes in a diffuse field, starting with purely longitudinal excitation means shear energy only arises through mode conversion, which is a gradual process. Direct shear excitation, by contrast, introduces shear energy immediately, potentially yielding stronger shear contributions during the cross-correlation process and improving shear reconstruction. A more promising route is to excite shear waves directly, which can be achieved using EMATs or high-power lasers, though each approach carries its own limitations.

Table 4.3 summarises the key sources of error and the experimental conditions under which accurate velocity and texture measurements were achieved using the diffuse field GF methodology. It serves as a practical specification of the limits within which the technique has been demonstrated to operate reliably.

Table 4.3: Sources of error and their potential impact of diffuse field GF velocity and texture measurements, along with experimental conditions under which they are assessed.

Source of Error	Effect on Measurement	Operational Limit
Temperature variation	Causes decorrelation of diffuse signals and changes source transfer function	$\leq 2^\circ$
Distance between receivers	Directly affects velocity calculation	± 0.050 mm tolerance
Surface roughness	Increases laser scattering and measurement noise	$R_a = 0.8 \mu m$
Movements and vibrations	Adds measurement noise	Mounted on 3D scanning frame

In summary, these considerations lay the groundwork for future implementations aimed at continuous, in-process texture monitoring and broaden the potential of diffuse field methods in both research and industrial applications.

4.7 Summary

This chapter investigates the application of the new validated velocity measurement methodology, based on GF reconstruction from diffuse fields, to ultrasonic texture characterisation of a sample with curved geometry. The work builds on insights gained from the foundational case studies and initial experimental trials described earlier, encompassing the full workflow from the design of a textured sample to an experimental setup that enables diffuse field excitation for GF reconstruction and 3D velocity measurements along 36 defined directions. As wave excitation from flat phased array surfaces into a curved sample introduces additional complexity, numerical simulations were first performed to optimise the excitation strategy for generating diffuse fields, and, crucially, for ensuring successful reconstruction of the desired longitudinal component. The experimentally measured directional velocities were then supplied to an established texture inversion algorithm to estimate the crystallographic texture of the titanium sample. Comparison of the resulting pole figures with those obtained from EBSD on an off-cut of the same sample shows very good agreement, demonstrating the capability of diffuse field GF reconstruction to achieve accurate texture characterisation in samples with complex geometry.

Chapter 5

Conclusions

5.1 Thesis review

Accurate velocity measurement and subsequent texture characterisation on curved geometries have been investigated and experimentally demonstrated using ultrasonic diffuse fields, advancing the development of a non-destructive bulk texture measurement technique for metals with complex geometries. This progression builds on the systematic studies presented from Chapter 2 to Chapter 4, in which the theoretical foundations and initial feasibility of using GF reconstruction for velocity measurement are progressively developed, experimentally validated, and ultimately translated into a practical framework for texture characterisation, as summarised in Figure 5.1.

In Chapter 2, following the technical overview of diffuse field GF reconstruction, three case studies were presented that demonstrated its feasibility for ultrasonic velocity measurement. These studies provided essential guidelines: maximising the number of distinct source positions to achieve effective noise reduction; recognising the role of geometry, where nonparallel boundaries accelerate diffusion and promote robust GF convergence; and demonstrating the value of numerical modelling for identifying the onset of diffusion and optimising reconstruction parameters. Additionally, the case studies showed that, in pitch-catch configurations, the reconstructed wave-mode velocity is governed primarily by the relative orientation of the receivers with respect to one another. Collectively, these findings laid the theoretical and practical groundwork for the subsequent experimental

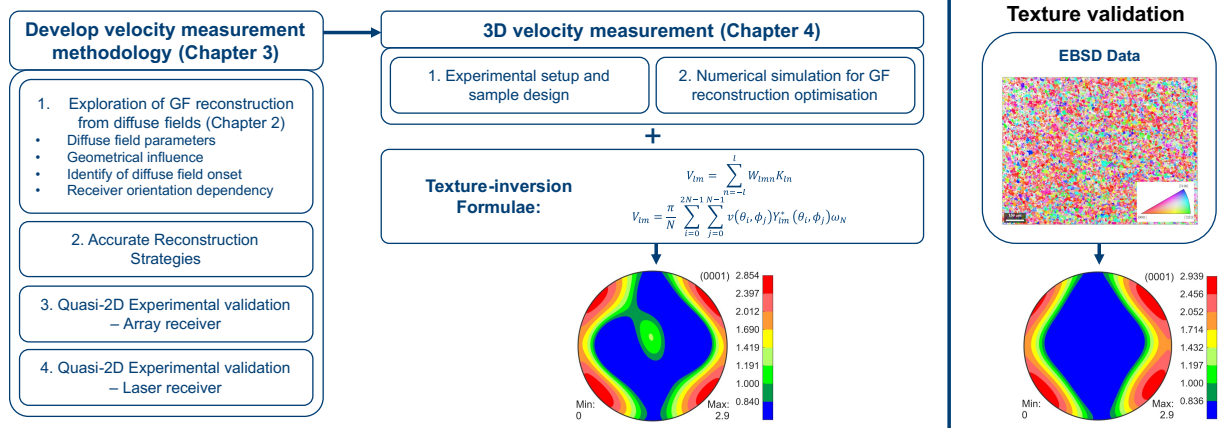


Figure 5.1: Schematic overview of the research workflow, showing the progression from theoretical foundations and exploration work of Green’s function reconstruction, through numerical and experimental validation of velocity extraction, to the final application in non-destructive texture characterisation.

investigations.

Chapter 3 extended this foundation to the challenge of accurate velocity measurement in samples with arbitrary geometries, where high-fidelity waveform reconstruction and low noise are critical. Several practical adaptations were introduced, including the use of spectral energy density curves for parameter optimisation, deconvolution to remove source-dependent effects, and 1B signal normalisation to account for energy decay and accelerate convergence. These strategies enabled robust reconstruction, which was experimentally validated using phased array receivers on flat samples and laser receivers on curved geometries, demonstrating the adaptability of the methodology across diverse experimental configurations.

The methodology was then applied in Chapter 4 to ultrasonic texture characterisation of a curved titanium specimen. The complete workflow encompassed textured sample design, diffuse field excitation supported by numerical simulations to refine excitation strategies and ensure recovery of the longitudinal mode, 3D velocity measurements along 36 orientations, and texture inversion using the experimentally obtained velocities. The retrieved pole figures showed excellent agreement with those obtained by EBSD from the same material, confirming that diffuse field GF reconstruction can achieve accurate texture characterisation in curved, non-planar components.

The contributions of this thesis can be summarised in two key advances. First, it estab-

lishes diffuse field GF reconstruction as a reliable method for accurate ultrasonic velocity measurement, expanding its role beyond imaging in NDE. More importantly, it demonstrates the feasibility of implementing the method with laser receivers, by overcoming their sensitivity limitations to enable non-contact measurement on complex geometries. Across both phased array and laser-based configurations, the velocity measurements achieved an inaccuracy of less than 10 ms^{-1} . Crucially, this accuracy was required to enable texture inversion in an HCP titanium sample, with the reconstructed texture in excellent agreement with the EBSD result. Together, these advances provide a robust foundation for integrating diffuse field GF reconstruction into manufacturing pipelines, enabling non-destructive, volumetric, and geometry-independent texture assessment. Such capabilities directly address the industrial need to monitor microstructural evolution in critical titanium components, allowing unfavourable orientations to be identified and mitigated during thermomechanical processing, improving structural reliability while reducing reliance on conservative service life assumptions and costly inspection regimes [35].

5.2 Future work

Following this thesis, three natural extensions may be pursued to further advance diffuse field GF reconstruction for ultrasonic texture characterisation.

The first extension concerns the measurement of shear velocities. Numerical modelling in §2.5 demonstrated that, in diffuse field GF reconstruction, the orientation of the receiver, which effectively acts as the source, determines the wave mode of the first-arrival and thus the type of velocity measured in a pitch-catch configuration. Out-of-plane displacement measurements predominantly reconstruct longitudinal velocities, while in-plane displacements enable the recovery of shear velocities. Although the present study has focused on longitudinal waves, extending the method to shear velocity measurement through in-plane displacement detection will be essential for applications in lower-symmetry crystal systems.

This requires access to orthogonal displacement components at each receiver position. One option is to use a 3D laser vibrometer such as the Polytec PSV-500-3D HV. Alternatively,

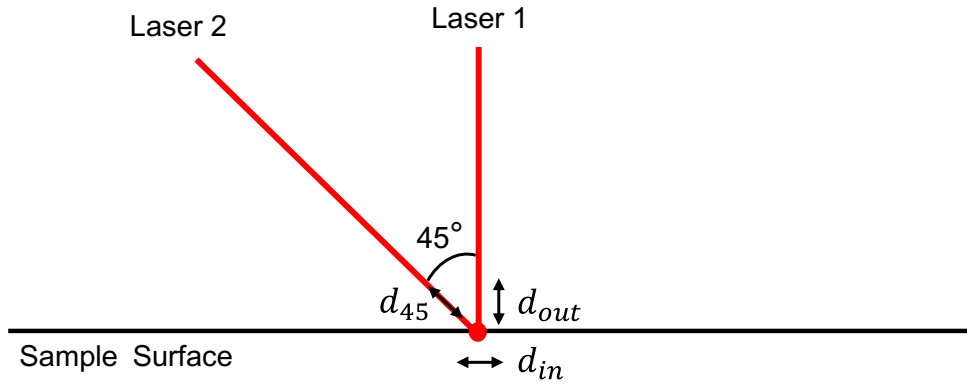


Figure 5.2: Schematic illustrating how two single-axis lasers can be arranged to measure in-plane displacement at a point on the sample surface. The quantities shown are: d_{in} , the in-plane displacement; d_{out} , the out-of-plane displacement; and d_{45} , the displacement measured at a 45° angle, which contains contributions from both d_{in} and d_{out} .

two single-axis vibrometers can be arranged at known angles so that their measurements, when combined, can be decomposed into orthogonal components, as illustrated in Figure 5.2. This allows the in-plane displacement to be isolated using the mathematical basis:

$$d_{45} = d_{out} \cos(45^\circ) + d_{in} \sin(45^\circ) \quad (5.1)$$

$$d_{in} = \frac{d_{45} - d_{out} \cos(45^\circ)}{\sin(45^\circ)} \quad (5.2)$$

Equation 5.1 shows that the 45° -oriented laser measures a displacement d_{45} , which is a projection of both the out-of-plane (d_{out}) and in-plane (d_{in}) components. With d_{out} obtained from a normal-incidence single-axis laser, Equation 5.2 then allows d_{in} to be isolated from d_{45} .

In practice, however, significant challenges remain. At oblique incidence of 45° , only a small fraction of light is reflected back into the detector, resulting in poor SNR and often unusable signals. To address this, 3M Scotchlite 7610 retro-reflective tape has been tested to enhance light return. While it does improve reflectivity, the tape also introduces

additional attenuation and damping, particularly at frequencies above 2 MHz, so its overall effect on GF reconstruction at the tested 5 MHz and practical usability requires further investigation. Moreover, the added thickness of the tape complicates calibration, as it must be accurately accounted for in velocity measurements.

Additionally, Ardrox 9D1B reflective spray was tested. As the solvent evaporates, it leaves behind a dried powdery layer of fine particles that produces diffuse (non-specular) scattering. This allows some reflected light to return to the vibrometer even at oblique incidence, yielding a modest improvement. However, the SNR of the diffuse field signals remained weak overall, suggesting that further investigation is required. This also points to the possibility that introducing controlled surface roughness could be beneficial, as scattered light increases the fraction detected by the vibrometer. In this regard, sandblasting the surface may be a potential next step, although it would be considered a destructive method since it permanently alters the surface.

Alternatively, a more practical path forward may be to reduce excitation frequency to 1-2 MHz, where the noise level is lower. This approach would necessitate the use of larger phased array transducers with wider pitch, which in turn places practical considerations on sample design and geometry to ensure sufficient source averaging and maintain reconstruction robustness.

The second extension is to broaden the methodology beyond HCP systems to other crystal classes, particularly cubic materials, which underpin many technologically important alloys such as steels, aluminium, and nickel. In this thesis, the inversion framework (Equation 4.1) was formulated by simplifying the ODF to c-axis pole distributions, an assumption appropriate for HCP metals. In contrast, cubic systems require full treatment of all three Euler angles. Lan *et al.* subsequently generalised the spherical convolution method to cubic crystals [80], and this approach has since been validated experimentally in stainless steel [3], while still employing the Gaussian-Legendre integration scheme (Equation 4.2) with 36 directional velocity measurements. Building on the velocity measurement framework developed in this thesis, adapting diffuse field GF reconstruction to cubic systems would be a critical next step towards enabling texture characterisation across a wider spectrum of engineering alloys.

5. Conclusions

The third extension is to implement GF reconstruction beyond simplified spherical samples, where receiver distances are fixed across the diameter, to larger and more complex geometries that lack such symmetry. A key challenge in this context is the spatial accuracy of laser receiver placement, since misalignment directly affects the distances used for velocity calculation. This could be mitigated by capturing the surface geometry with CAD models or 3D scans and registering measurement points within a global coordinate system, potentially supported by automated laser steering systems such as galvanometer mirrors or robotic arms for accurate positioning. Beyond receiver placement, scaling to large components also leads to slower diffusion, which may require adjustments such as source design or excitation frequency. Overcoming these challenges will be critical to realising the practical application of GF reconstruction to engineering-scale components.

Appendix A

Definition of Source-Dependent Term for Deconvolution

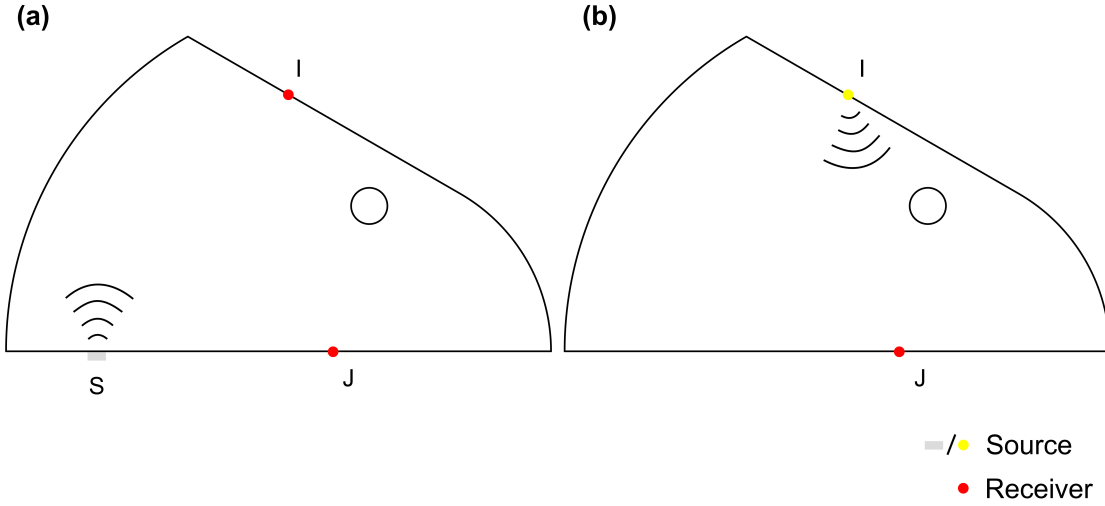


Figure A.1: Schematic illustrating the setups used to describe the coherent response and the diffuse field correlation as a series of transfer functions. **a** illustrates the setup used to capture the diffuse fields at both location I and J from a source excitation at S to reconstruct the coherent response at J if there is a source at I . **b** illustrates the setup used to capture the coherent response at location J from a source excitation at location I .

Referring to Figure A.1a, the cross-correlation of diffuse fields received at points I and J

A. Definition of Source-Dependent Term for Deconvolution

from a source at S , denoted as $R_{J,I}(\omega)$, can be represented as a series of transfer functions. To do so, the diffuse fields received at I and J are first defined as $D_{I,S}(\omega)$ and $D_{J,S}(\omega)$, respectively:

$$\begin{aligned} D_{I,S}(\omega) &= W(\omega) \cdot H(\omega) \cdot G_{I,S}(\omega) \cdot T_1(\omega) \\ D_{J,S}(\omega) &= W(\omega) \cdot H(\omega) \cdot G_{J,S}(\omega) \cdot T_2(\omega) \end{aligned}$$

Thereafter, the diffuse field correlation $R_{J,I}(\omega)$ can be defined as:

$$\begin{aligned} R_{J,I}(\omega) &= D_{I,S}^*(\omega) \cdot D_{J,S}(\omega) \\ &= W^*(\omega) \cdot H^*(\omega) \cdot G_{I,S}^*(\omega) \cdot T_1^*(\omega) \cdot W(\omega) \cdot H(\omega) \cdot G_{J,S}(\omega) \cdot T_2(\omega) \end{aligned} \tag{A.1}$$

where $W(\omega)$ represents the excitation waveform, $T_1(\omega)$ is the transfer function of the receiver at location I , $T_2(\omega)$ is the transfer function of the receiver at location J , and $H(\omega)$ is the transfer function of the source at location S . $G_{I,S}(\omega)$ and $G_{J,S}(\omega)$ denote the Green's Function between locations I and S , and between locations J and S , respectively. The asterisk superscript (*) denotes the complex conjugate operation for time-reversed representations.

Figure A.1b illustrates the configuration used to capture the coherent response at point J from a source at point I , denoted as $C_{J,I}(\omega)$, which is the signal to be reconstructed from the setup depicted in Figure A.1a. The coherent response can be expressed as:

$$C_{J,I}(\omega) = W(\omega) \cdot T_1(\omega) \cdot G_{J,I}(\omega) \cdot T_2(\omega) \tag{A.2}$$

where $W(\omega)$ is the excitation waveform used by the source to generate the diffuse field in Equation A.1, $T_1(\omega)$ is the transfer function of the source at location I , $T_2(\omega)$ is the transfer function of the receiver at J , and $G_{J,I}(\omega)$ is the GF between locations I and J .

With reference to Derode's paper [66]:

$$G_{I,S}^*(\omega) \cdot G_{J,S}(\omega) = G_{J,I}(\omega) \cdot G_{S,S}^*(\omega) \quad (\text{A.3})$$

where $G_{S,S}^*(\omega)$ represents the early-time, time-reversed GF at location S , which in the frequency domain is equivalent to its complex conjugate.

Substituting Equation A.3 into the Equation A.1, the equation that describes the cross-correlation of diffuse fields can be re-written as:

$$R_{J,I}(\omega) = W^*(\omega) \cdot H^*(\omega) \cdot T_1^*(\omega) \cdot W(\omega) \cdot H(\omega) \cdot T_2(\omega) \cdot G_{J,I}(\omega) \cdot G_{S,S}^*(\omega) \quad (\text{A.4})$$

By comparing Equation A.4 to Equation A.2, the terms that must be removed for the diffuse field correlation to yield the coherent response can be identified, which can be defined by $F^*(\omega)$:

$$F^*(\omega) = \frac{R_{J,I}(\omega)}{C_{J,I}(\omega)} = W^*(\omega) \cdot H^*(\omega) \cdot H(\omega) \cdot G_{S,S}^*(\omega) \quad (\text{A.5})$$

Here, the time-reversed receiver function $T_1^*(\omega)$ is assumed to be equivalent to the source function $T_1(\omega)$.

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List of Publications

Journal publications

- [P1] M. Png, M. Huang, M. Bahreman, C. M. Kube, M. J. S. Lowe, and B. Lan, “Accurate wave velocity measurement from diffuse wave fields,” *NDT & E Int*, vol. 156, 2025.
- [P2] M. Png, M. Huang, M. Bahreman, C. M. Kube, M. J. S. Lowe, and B. Lan, “Volumetric texture measurement of HCP metals from diffuse wave fields,” In preparation for submission to *IJMS*.

Conference presentations

- [C1] M. Png, M. J. S. Lowe, and B. Lan, “Material texture characterisation in metallic curved components using diffused ultrasound”, at *59th Annual British Conference on Non-Destructive Testing*, 6-8 Sep 2022, Telford, UK.
- [C2] M. Png, M. Huang, M. Bahreman, C. M. Kube, M. J. S. Lowe, and B. Lan, “Material texture characterisation in metallic curved components using diffused ultrasound”, at *31st ASNT Research Symposium*, 26-30 Jun 2023, Columbus, USA.
- [C3] M. Png, M. Huang, M. Bahreman, C. M. Kube, M. J. S. Lowe, and B. Lan, “Accurate velocity measurement through Green's Function Reconstruction from diffused fields”, at *32nd ASNT Research Symposium*, 24-28 Jun 2024, Pennsylvania, USA.