PRODUCTION OF STRANGE PARTICLES

by

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PREFACE

The work described in this thesis was carried out in the Department of Theoretical Physics, Imperial College, London between October 1960 and May 1964 under the supervision of Professor Abdus Salam.

Except where stated in the text, the work described is original and has not been submitted in this or any other University for any other degree. It consists of two related parts : (1) The Theory of Electroproduction of Strange Particles and (2) The Weak Production of Strange Particles.

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We study the production of strange particles and in particular the electroproduction and the weak production of K mesons. The electroproduction of strange particles is governed by the matrix element of the interaction $\gamma + N \rightarrow K + Y$, where γ is a virtual photon, and the main case of interest is in the energy region where a $P_{\frac{1}{2}}$ or an $F_{5/2}$ resonance is dominant. Using the method of Blankenbecler and Gartenhaus formulae are derived for solving the problem in this region. The weak production involves the matrix element of the interaction $W + N \rightarrow K + Y$, where W is a virtual intermediate boson for the weak interaction. Formulae of the electroproduction type are again derived for solving the problem in the resonance region.

INTRODUCTION

The discussion of the electroproduction and the weak production of strange particles follows naturally from the study of the photoproduction of strange particles. This extension has its analogue in the much discussed sequence of processes- the photoproduction, the electroproduction and the weak production of pions from nucleons.

The photoproduction of pions from nucleons played an essential role in determining the nature of the pion-nucleon interaction and in the assignment of the spin and parity of the higher pion-nucleon resonances[1,2]. The angular distribution data of the photoproduction of pions has also been exploited[3] to determine more accurately the value of the pion-nucleon coupling constant.

The electroproduction of pions extends the pion photoproduction process to that of production of pions by a virtual photon and thus opens up the analysis of the electromagnetic structure of nucleons and pions. In this case the theoretically interesting processes are confined to those experiments [4,5,6] in which the barycentric energies in the final pion-nucleon state are dominated by the first pion-nucleon resonance.

The theoretical approaches to the pion electroproduction problem have been many. Curtis[7] and Dalitz and Yennie[8] used an adapted version of the Weizsacker-Williams method and later Fubini, Nambu and Wataghin[9] applied dispersion relation techniques in the static limit

through an extension of the photoproduction theory of Chew, Goldberger, Low and Nambu[10] to obtain a first determination of the isovector form factor of the nucleon. This work was extended to the relativistic case by Blankenbecler, Gartenhaus, Huff and Nambu[11] using the method of Blankenbecler and Gartenhaus[12] to write down an approximate solution of the dispersion relations. In the framework of the Mandelstam representation Dennery[13] has used a completely covariant treatment making use of the Cini-Fubini approximation[14] to the Mandelstam representation and Barbour[15] used an extension of the N/D method to multichannel processes as proposed by Bjorken[16]. Salin[17] has further used a semi-phenomenological isobar model to obtain the neutron form factor in good agreement with the results of Stanford and Cornell[18].

Weak pion production - the production of a pion by high energy neutrino collisions with nucleons - introduces the axial vector current of weak interactions so that the axial vector form factors must be employed in its discussion. The conserved vector current hypothesis of Feynman and Gell-Mann[19] makes it possible to relate the vector part of weak pion production to the isovector part of pion electroproduction. Thus Dombey[20] has extended the electroproduction analysis of Blankenbecler, Gartenhaus, Huff and Nambu[11] and Dennery[21] has extended his earlier work on the electroproduction of pions.

Since the original investigations on the photoproduction of K mesons[22-24] the most extensively discussed process and the simplest experimentally is $\gamma + p \longrightarrow K^+ + \Lambda$. The recent work at Cornell [25,26] has yielded several angular distributions for this process in the energy range from 934 Mev. to 1160 Mev. Furthermore, the polarisation of the Λ at 90° in the barycentric frame for several energies up to 1160 Mev has been reported by the same group.[27] As was clear from early measurements, the production near threshold is consistent with the $K^+\Lambda$ system being in an s-state and at 1000 Mev, the appearance of polarization of the Λ and a gentle rise of the angular distribution in the K^+ forward direction indicate that higher partial waves are beginning to be excited. As the photon energy is increased the Λ polarization and the 90° cross section remain essentially constant[27] while the angular distribution becomes more peaked in the forward K^+ direction.[25,26] More recently[28] it has been shown that this behaviour is also obtained at a photon energy of 1200 Mev.

The K^+ photoproduction experiments are of theoretical interest as they provide, in principle at least, a model independent method of determining the KYN coupling constants by extrapolating the differential cross section to the one K-meson exchange pole in the forward direction. The hyperon exchange diagram gives rise to a pole in the backward direction and the residue of this pole is proportional to the hyperon magnetic moment. Hence information on the magnetic moment can be obtained from K meson photoproduction experiments.[29]

Several specific models[30-34] have been proposed recently for strange particle photoproduction. These models include contributions from the Born terms, the exchange of the K^* and various resonances. Fayyazuddin[30] considers low energy production with only s and p wave contribution through the Born terms, K^* meson and N^* - the first pion nucleon resonance. The model of Kuo[31] and Beauchamp and Holladay[32] incorporates a $P_{\frac{1}{2}}$ resonance that has been suggested by Kanazawa [35] to explain the bump in the total cross section for the associated production process $\pi^- + p \longrightarrow K^o + \Lambda$. The parameters in this model have been determined by fitting the angular distribution at low energies but it predicts the wrong sign for the Λ polarization and gives too flat an angular distribution at 1200 Mev.[27,36] The models of Hatsukade and Schnitzer[33] and Gourdin and Dufour[34] are similar to each other in that they take into account the second and third pion nucleon resonances, N^{**} and N^{***} respectively. The importance of the higher of these two states in associated production by pions has been discussed by Rimpault[37] who finds that a resonant $F_{5/2}$ state is consistent with a partial wave analysis of this reaction. Hoff[38] arrives at similar conclusions in his re-analysis of the same process.

The question as to whether, apart from the Born terms and the exchange of the K^*, the resonance in the $T = \frac{1}{2}$ channel is a $P_{\frac{1}{2}}$ or an $F_{5/2}$ is still an open one. This means that any discussion of the electroproduction and

weak production of strange particles must make full allowance for either possibility. Since the experimental situation would favour a Panofsky type of analysis, we employ the theoretical technique of Blankenbecler and Gartenhaus to derive the differential cross section for the region in which the resonance dominates the process- the two resonances being discussed simultaneously.

PART I

THE THEORY OF ELECTROPRODUCTION OF STRANGE PARTICLES.

i. NOTATION

We shall use the units $\hbar = c = 1$ and in these units, energy, momentum and mass have the units of a reciprocal length. The metric used will be

$$x_\mu = (x_o, x_1, x_2, x_3)$$

$$g_{oo} = 1 \quad g_{11} = g_{22} = g_{33} = -1$$

$$a_\mu b_\mu = a_o b_o - \underline{a} \cdot \underline{b}$$

$$\partial_\mu = \left(\frac{\partial}{\partial x_o} , - \frac{\partial}{\partial x_1} , - \frac{\partial}{\partial x_2} , - \frac{\partial}{\partial x_3} \right)$$

and the γ - matrices used have the following properties

$$\left\{ \gamma_\mu , \gamma_\nu \right\} = -2 g_{\mu\nu} \tag{1}$$

where γ_o is anti-hermitian and $\gamma_4 = i\gamma_o$, γ_1, γ_2 and γ_3 are hermitian. γ_5 is defined as

$$\gamma_5 = \gamma_1 \gamma_2 \gamma_3 \gamma_4 = -i \gamma_o \gamma_1 \gamma_2 \gamma_3 \tag{2}$$

and is also hermitian. The following representation will be used for the γ - matrices in the reduction to barycentric systems

$$\gamma_r = \begin{pmatrix} o & -i\sigma_r \\ i\sigma_r & o \end{pmatrix} \qquad r = 1, 2, 3,$$

$$i\gamma_o = \gamma_4 = \begin{pmatrix} 1 & o \\ o & -1 \end{pmatrix} \tag{3}$$

and

$$\gamma_5 = \begin{pmatrix} o & -1 \\ -1 & 0 \end{pmatrix}$$

We shall also put

$$\not{p} = i\gamma_\mu p_\mu \tag{4}$$

and the spinor u(p) satisfies the equation

$$(\not{p} - m)\, u(p) = 0 \tag{5}$$

with the normalization

$$\bar{u}(p)\, u(p) = 2m \tag{6}$$

For the reduction to the barycentric system we use the following explicit form for the Dirac spinor

$$u_r(p) = \frac{\not{p} + m}{(p_o + m)^{\frac{1}{2}}} \begin{pmatrix} \chi_r \\ 0 \end{pmatrix} \tag{7}$$

where χ_r is a two component spinor. Thus

$$u_r(p) = \frac{1}{(p_o + m)^{\frac{1}{2}}} \begin{pmatrix} p_o + m \\ \underline{\sigma} \cdot \underline{p} \end{pmatrix} \chi_r \tag{8}$$

Let the four-vectors r_1, r_2, p_1, p_2 and q correspond formally to the incoming particles (Figure Ia) and define as the three invariants

$$\begin{aligned} s &= (k + p_1)^2 = (q + p_2)^2 \\ u &= (q + p_1)^2 = (k + p_2)^2 \\ t &= (k + q)^2 = (p_1 + p_2)^2 \end{aligned} \tag{9}$$

where we have used $k = r_1 + r_2$. The four-vector k corresponds to the virtual photon.

Conservation of momentum gives

$$k + p_1 + q + p_2 = 0 \tag{10}$$

which implies that we can select only three independent four-vectors. These we take as k, p_1 and p_2 or k, q and $P = \frac{1}{2}(p_1 - p_2)$ whichever is the more convenient.

Now $k^2 = -\lambda^2$, $q^2 = m^2$, $p_1^2 = M_1^2$ and $p_2^2 = M_2^2$ where m, M_1, M_2 are the masses of the K-meson, nucleon and hyperon respectively. If the square of the virtual photon mass $(-\lambda^2)$ is taken as constant then there are only two independent scalars related by

$$s + u + t = m^2 + M_1^2 + M_2^2 - \lambda^2 \tag{11}$$

Each of the three invariants, s, u and t represent the square of the total energy in the barycentric system for the reactions

I $\quad k + p_1 \rightarrow -q - p_2$. $\quad$ ("γ" + N $\rightarrow$ K + Y)

$$\text{II}\quad k + p_2 \rightarrow -q - p_1 \qquad (\text{"}\gamma\text{"} + \bar{Y} \rightarrow K + \bar{N}) \tag{12}$$

$$\text{III}\quad k + q \rightarrow -p_1 - p_2 \qquad (\text{"}\gamma\text{"} + \bar{K} \rightarrow \bar{N} + Y)$$

and these three reactions must be considered together if one uses the Mandelstam representation.

In the barycentric system for reaction I we define

$$\begin{aligned} k &= (k_o\ ,\ \underline{k}) & p_1 &= (\varepsilon_1\ ,\ -\underline{k}) \\ -q &= (q_o\ ,\ \underline{q}) & -p_2 &= (\varepsilon_2\ ,\ -\underline{q}) \end{aligned} \tag{13}$$

giving the identities

$$\begin{aligned} 2Wk_o &= W^2 - M_1^2 - \lambda^2 & 2Wq_o &= W^2 + m^2 - M_2^2 \\ 2W\varepsilon_1 &= W^2 + \lambda^2 + M_1^2 & 2W_{\ 2} &= W^2 - m^2 + M_2^2 \end{aligned} \tag{14}$$

$$k^2 = k_o^2 + \lambda^2 \qquad\qquad q^2 = q_o^2 - m^2$$

and

$$s = W^2 = (\text{total barycentric energy for reaction I})^2$$

$$= (k_o + \varepsilon_1)^2 = (\varepsilon_2 + q_o)^2$$

$$\begin{aligned} u &= m^2 + M_1^2 - 2q_o\,\varepsilon_1 - 2qk\cos\vartheta \\ &= M_2^2 - \lambda^2 - 2k_o\,\varepsilon_2 - 2qk\cos\vartheta \end{aligned} \tag{15}$$

$$\begin{aligned} t &= M_1^2 + M_2^2 - 2\varepsilon_1\varepsilon_2 + 2qk\cos\vartheta \\ &= m^2 - \lambda^2 - 2q_ok_o + 2qk\cos\vartheta \end{aligned}$$

where $\cos\vartheta = \dfrac{\underline{q}\cdot\underline{k}}{q\ k}$ $\qquad q = |\underline{q}|$ $\qquad k = |\underline{k}|$

From (8) and (13) we have

$$u_N(p_1) = \frac{1}{(\varepsilon_1 + M_1)^{\frac{1}{2}}} \begin{pmatrix} \varepsilon_1 + M_1 \\ -\underline{\sigma}\cdot\underline{k} \end{pmatrix} \chi \tag{16}$$

and $$\bar{u}_Y(-p_2) = \chi^{\dagger} \frac{1}{(\varepsilon_2 + M_2)^{\frac{1}{2}}} (\varepsilon_2 + M_2 \ , \ \underline{\sigma}\cdot\underline{q})$$

In the barycentric system for reaction II we define

$$\begin{aligned} k &= (K_o\ ,\ \underline{K}) & -q &= (Q_o,\ \underline{Q}) \\ p_2 &= (E_2\ ,-\underline{K}) & -p_1 &= (E_1,-\underline{Q}) \end{aligned} \tag{17}$$

with the following identities

$$\begin{aligned} 2\omega K_o &= \omega^2 - M_2^2 - \lambda^2 & 2\omega Q_o &= \omega^2 - M_1^2 + m^2 \\ 2\omega E_1 &= \omega^2 + M_1^2 - m^2 & 2\omega E_2 &= \omega^2 + M_2^2 + \lambda^2 \\ K^2 &= K_o^2 + \lambda^2 & Q^2 &= Q_o^2 - m^2 \end{aligned} \tag{18}$$

and

$$u = \omega^2 = (\text{total barycentric energy for reaction II})^2$$
$$= (E_1 + Q_o)^2 = (E_2 + K_o)^2$$

$$s = M_1^2 - \lambda^2 - 2K_oE_1 - 2QK\cos\phi$$

$$= M_2^2 + m^2 - 2Q_oE_2 - 2QK\cos\phi \qquad (19)$$

$$t = m^2 - \lambda^2 - 2K_oQ_o + 2QK\cos\phi$$

$$= M_1^2 + M_2^2 - 2E_1E_2 + 2QK\cos\phi$$

where $\cos\phi = \dfrac{\underline{Q} \cdot \underline{K}}{QK}$ $\qquad Q = |\underline{Q}|$, $K = |\underline{K}|$

From (8) and (17) we have

$$v_N(-p_1) = -\frac{1}{(E_1 + M_1)^{\frac{1}{2}}} \begin{pmatrix} -\underline{\sigma} \cdot \underline{Q} \\ E_1 + M_1 \end{pmatrix} \chi \qquad (20)$$

and

$$\bar{v}_Y(p_2) = -\chi^{\dagger}\frac{1}{(E_2 + M_2)^{\frac{1}{2}}} \qquad (\underline{\sigma} \cdot \underline{K}, E_2 + M_2)$$

Finally, in the barycentric system for reaction III we define

$$k = (\kappa_o, \underline{\kappa}) \qquad -p_1 = (p_{10}, -\underline{p})$$

$$q = (\omega_o, -\underline{\kappa}) \qquad -p_2 = (p_{20}, \underline{p}) \qquad (21)$$

with the following identities

$$2\Omega\,\varkappa_o = \Omega^2 - \lambda^2 - m^2 \qquad 2\,\Omega p_{10} = \Omega^2 + M_1^2 - M_2^2$$

$$2\Omega\omega_o = \Omega^2 + \lambda^2 + m^2 \qquad 2\,\Omega p_{20} = \Omega^2 + M_2^2 - M_1^2 \tag{22}$$

$$\varkappa^2 = \varkappa_o^2 + \lambda^2 \qquad 4\,\Omega^2 p^2 = (\Omega^2 - \Delta^2)(\Omega^2 - 4\bar{M}^2)$$

where $2\bar{M} = M_1 + M_2$ $\qquad \Delta = M_2 - M_1$

and

$$t = \Omega^2 = (\text{total barycentric energy for reaction III})^2$$

$$= (\varkappa_o + \omega_o)^2 = (p_{10} + p_{20})^2$$

$$\begin{aligned} s &= M_1^2 - \lambda^2 - 2p_{10}\varkappa_o - 2p\varkappa\cos\psi \\ &= M_2^2 + m^2 - 2p_{20}\omega_o - 2p\varkappa\cos\psi \end{aligned} \tag{23}$$

$$\begin{aligned} u &= M_1^2 + m^2 - 2p_{10}\omega_o + 2p\varkappa\cos\psi \\ &= M_2^2 - \lambda^2 - 2p_{20}\varkappa_o + 2p\varkappa\cos\psi \end{aligned}$$

where $\cos\psi = \dfrac{\underline{p}\cdot\underline{\varkappa}}{p\varkappa}$ $\qquad p = |\underline{p}|$ $\qquad \varkappa = |\underline{\varkappa}|$

From (8) and (21) we have

$$v_{\overline{N}}(-p_1) = \frac{1}{(p_{10}+M_1)^{\frac{1}{2}}} \begin{pmatrix} -\underline{\sigma}\cdot\underline{p} \\ p_{10}+M_1 \end{pmatrix} \chi \tag{24}$$

and

$$\bar{u}_Y(-p_2) = \chi^\dagger \frac{1}{(p_{20}+M_2)^{\frac{1}{2}}} \quad (p_{20}+M_2\,,\ -\underline{\sigma}\cdot\underline{p})$$

iii CONSTRUCTION OF THE INVARIANT AMPLITUDES AND THE MANDELSTAM REPRESENTATION.

The S-matrix for electroproduction may be written as

$$S_{fi} = \delta_{fi} + i\,(2\pi)^4\,\delta^4\,(p_1+p_2+q+k)\,T_{fi} \qquad (25)$$

with $k = r_1+r_2$ and r_1, r_2 representing formally incoming electrons. To the first order in the electromagnetic coupling constant[8] we have

$$\langle -p_2, -q, -r_2 \,|T|\, p_1, r_1\rangle = \langle -p_2, -q \,|J_\mu|\, p_1\rangle\, e_\mu$$

with

$$e_\mu = e\,\frac{\bar{u}(-r_2)\gamma_\mu\, u(r_1)}{(r_1+r_2)^2} \qquad (26)$$

The matrix element $\langle -p_2, -q \,|J_\mu|\, p_1\rangle$ is the matrix element for the photoproduction of a K meson by a virtual photon of four momentum

$$k^2 = (r_1+r_2)^2 = -\lambda^2 \qquad (27)$$

It is this matrix element which has to be examined in detail and the separation (26) can be represented formally as in Figure Ib.

To construct the invariant amplitude we employ the method of Ball[39]. Since we are dealing with a process of first order in the electromagnetic coupling constant the T-matrix must be linear in e_μ. We can therefore write

$$T = M_\mu e_\mu \qquad (28)$$

and the most general M_μ that we can write is

$$M_\mu = k_\mu F_1 + q_\mu F_2 + P_\mu F_3 + i\gamma_\mu F_4$$

$$+ i\sigma_{\mu\nu}\, k_\nu\, F_5 \;+ i\sigma_{\mu\nu}\, q_\nu\, F_6 + i\sigma_{\mu\nu}\, P_\nu\, F_7 \qquad (29)$$

The F_i's are functions of the invariants which we can construct from the three independent four-vectors k, p_1, p_2 and γ_μ. Such scalars are s, u and t. Hence we can express each of the F's as

$$F = \sum_i f_i\,(s,u,t)\, O_i \qquad (30)$$

where

$$O_i = (1,\ \not{k},\ \not{p}_1,\ \not{p}_2,\ \not{k}\not{p}_1,\ \not{k}\not{p}_2,\ \not{p}_1\not{p}_2,\ \not{k}\not{p}_1\not{p}_2)$$

By the use of the commutation rules for the γ matrices and the Dirac equation

$$(\not{p}_1 - M_1)\, u(p_1) = 0$$

$$\bar{u}\,(-p_2)\,(-\not{p}_2 - M_2) = 0$$

we can reduce equation (30) to the form

$$F = a\,(s,u,t) + b(s,u,t)\,\not{k} \qquad (31)$$

A further application of the above techniques enables us to write

$$M_\mu = i\gamma_\mu\, B_1 + k_\mu B_2 + q_\mu B_3 + P_\mu B_4$$

$$+ \gamma_\mu\, \gamma.k\, B_5 + i\,\gamma.k\, k_\mu\, B_6 \qquad (32)$$

$$+ i\,\gamma.k\, q_\mu\, B_7 \;+\; i\,\gamma.k\, P_\mu B_8$$

Hence the T-matrix can now be written in terms of eight relativistic but not gauge invariant quantities N_i

$$T = \sum_{i=1}^{8} B_i\,(s,u,t)\,N_i \tag{33}$$

where

$$\begin{aligned} N_1 &= i\,\gamma.e & N_5 &= \gamma.e\ \gamma.k \\ N_2 &= k.e & N_6 &= i\,\gamma.k\ k.e \\ N_3 &= q.e & N_7 &= i\,\gamma.k\ q.e \\ N_4 &= P.e & N_8 &= i\,\gamma.k\ P.e \end{aligned} \tag{34}$$

Even though the vector e_μ (26) has both longitudinal and time-like components the Lorentz condition

$$k_\mu e_\mu = 0 \tag{35}$$

is still satisfied. Thus imposing gauge invariance on the above T-matrix i.e. replacing e_μ by k_μ and requiring the T-matrix to vanish, we have the two conditions

$$\begin{aligned} &(s - M_1^2)\,B_2 + (u-M_2^2)B_3 + (t-m^2)\,B_4 = o \\ &B_5 + (s - M_1^2)\,B_6 + (u-M_2^2)B_7 + (t-m^2)\,B_8 = o \end{aligned} \tag{36}$$

We are therefore left with six independent functions and the set we use is suggested by the Born terms derived in iv.

Thus for the KY parity even we write

$$T = \sum_{i=1}^{6} A_i^{+}\,M_i^{+} \tag{37}$$

with

$$M'_1 = \tfrac{1}{2}(\gamma, \gamma)$$

$$M'_2 = 2(P, q) + (P, k)$$

$$M'_3 = -i(\gamma, q)$$

$$M'_4 = -2i(\gamma, P) - M_1(\gamma, \gamma) \qquad (38)$$

$$M'_5 = (k, q)$$

$$M'_6 = -i(k, \gamma)$$

where we have used an abbreviation for the gauge-invariant combination

$$(a, b) = a.e\ b.k - a.k\ b.e$$

The A'_i's are related to the B_i's as follows

$$A'_1 = 2B_1 + 2M_1 (B_6 - B_7)$$

$$(t - m^2) A'_2 = 2(B_2 - B_3)$$

$$A'_3 = 2B_8 - B_6 - B_7$$

$$A'_4 = B_6 - B_7 \qquad (39)$$

$$(t - m^2) A'_5 = B_2 + B_3$$

$$A'_6 = -B_8$$

For the case of the KY parity odd the form of the T-matrix is the same as above with the inclusion of a factor $i\gamma_5$. Thus

$$T = \sum_{i=1}^{6} A_i \; i\gamma_5 \; M_i'$$

$$T = \sum_{i=1}^{6} A_i \; M_i \qquad (40)$$

where

$$M_i = i\gamma_5 \; M_i'$$

It is this form of the T-matrix that we use since there is now convincing evidence that the KY relative parity is odd[40].

The A_i's are functions of isotopic spin as well as of s, u and t and may be further decomposed into an isoscalar and into an isovector part, these designations referring to the character of the photon current. That is, the photon may be regarded as an isotopic scalar plus the third component of an isotopic vector. Denoting the isotopic index of the outgoing Σ hyperon by β, we have three isotopic spin invariants for the reaction $'\gamma' + N \rightarrow K + \Sigma$,

$$\begin{aligned} V_\beta^+ &= \tfrac{1}{2} \; (\tau_\beta \tau_3 + \tau_3 \tau_\beta) \; = \delta_{\beta 3} \\ V_\beta^- &= \tfrac{1}{2} \left[\tau_\beta \, , \tau_3 \right] \\ V_\beta^o &= \tau_\beta \end{aligned} \qquad (41)$$

Thus

$$A_i = A_i^{(+)} \; V_\beta^+ + A_i^{(-)} \; V_\beta^- + A_i^{(o)} \; V_\beta^o \qquad (42)$$

for Σ production.

The projection operators for isotopic spin 3/2 and $\frac{1}{2}$ are given by

$$\begin{aligned}(P_{3/2})_{\beta 3} &= \frac{1}{3}\ \left(2\delta_{\beta 3}\ - \tfrac{1}{2}\left[\tau_\beta\ ,\tau_3\ \right]\ \right) \\ &= \frac{1}{3}\ \left(2\ V_\beta^+\ - V_\beta^-\ \right)\end{aligned} \tag{43}$$

and

$$\begin{aligned}(P_{\frac{1}{2}})_{\beta 3}\ &=\ \frac{1}{3}\ \left(\delta_{\beta 3} + \tfrac{1}{2}\left[\tau_\beta\ ,\tau_3\ \right]\ \right) \\ &=\ \frac{1}{3}\ \left(V_\beta^+\ +\ V_\beta^-\right)\end{aligned} \tag{44}$$

We observe that $P3/_2$ and $P_{\frac{1}{2}}$ are linear combinations of the V_β^+ and V_β^- corresponding to the isovector part of the photon interaction. The scalar projection operator V_β^o contributes to the $I = \frac{1}{2}$ amplitude only.

As the Λ hyperon is an isoscalar we have the following isotopic spin invariants for the reaction "γ" + N $\rightarrow$ K + Λ

$$\begin{aligned}S^+\ &= \tau_3 \\ S^o\ &=\ 1\end{aligned} \tag{45}$$

and so

$$A_i = A_i^{(+)}\ S^+\ +\ A_i^{(o)}\ S^o \tag{46}$$

for Λ production.

The isotopic spin $3/_2$ is absent here and S^+ and S^o both contribute to the $I = \frac{1}{2}$ amplitude.

The valuos of the V's and S's for a particular reaction are given in Table I.

TABLE I.

Matrix elements of $V(\overset{+}{-}_{o})$ and $S(\overset{+}{o})$ for the possible charge configurations.

	V^+	V^-	V^o	S^+	S^o
"γ" + p → $K^+ + \Sigma^o$	1	0	1	-	-
"γ" + p → $K^o + \Sigma^+$	0	$\sqrt{2}$	$\sqrt{2}$	-	-
"γ" + p → $K^+ + \Lambda$	-	-	-	1	1
"γ" + n → $K^o + \Sigma^o$	1	0	-1	-	-
"γ" + n → $K^+ + \Sigma^-$	0	$-\sqrt{2}$	$\sqrt{2}$	-	-
"γ" + n → $K^o + \Lambda$	-	-	-	-1	1

The choice of the invariant spin matrices $N_i(34)$ does not introduce any kinematical singularities in the B_i, and so these amplitudes are taken to satisfy the Mandelstam representation(39).

Thus

$$B_i(s,u,t) = \frac{R_i(s)}{M_1^2 - s} + \frac{R_i(u)}{M_2^2 - u} + \frac{R_i(t)}{m^2 - t}$$

$$+ \frac{1}{\pi} \int_{(\mu+M_1)^2}^{\infty} \frac{\beta_i^{(1)}(s')\, ds'}{s' - s} + \frac{1}{\pi} \int_{(\mu+M_2)^2}^{\infty} \frac{\beta_i^{(2)}(u')du'}{u'-u}$$

$$+ \frac{1}{\pi}\int_{(\mu+m)^2}^{\infty} \frac{\beta_i^{(3)}(t')\,dt'}{t'-t} + \frac{1}{\pi^2}\int ds' \int du' \frac{b_i^{12}(s',u')}{(s'-s)(u'-u)}$$

$$+ \cdots\cdots\cdots + \cdots\cdots\cdots \quad . \qquad (47)$$

where the cuts are the same as in the usual Mandelstam representation due to the character of the virtual photon.

Since the amplitudes A_1, A_3 A_4 and A_6 contain no kinematical factors in their relation to the B_i's (39) they will have the same representation as the B_i's. The representations of the two remaining amplitudes, A_2 and A_5, can be constructed by combining the expressions for the amplitudes B_2 and B_3 according to combinations of (39).

In the framework of the Cini-Fubini (approximate) one-dimensional representation [14] we may write our dispersion integrals as

$$A_i(s,u,t) = B(t)\left[\frac{\Gamma_i(s)}{M_1^2 - s} + \frac{\Gamma_i(u)}{M_2^2 - u} + \Gamma_i(t)\right] + \frac{1}{\pi}\int \frac{a_i(s',t)\,ds'}{s'-s} + \frac{1}{\pi}\int \frac{b_i(u',t)\,du'}{u'-u} + \frac{1}{\pi}\int \frac{c_i(t',s)}{t'-t}\,dt' \qquad (48)$$

where

$$B(t) = 1 \quad \text{for } i = 1, 3, 4, 6.$$

$$B(t) = \frac{1}{t - m^2} \quad \text{for } i = 2, 5.$$

The spectral functions a_i , b_i and c_i can be obtained as the imaginary part of the amplitude for the reaction in channels I, II and III respectively. The poles arise from diagrams shown in Figure II and the residues of these poles, $\Gamma_i(s)$, $\Gamma_i(u)$ and $\Gamma_i(t)$ will be derived in the next section.

iv. CALCULATION OF RESIDUES OF POLES.

For fixed t the T-matrix is written as

$$T(s,u,t) = ie_\mu \int d^4x \; e^{-i(q-k)\frac{x}{2}} \left\langle -p_2 \left| \vartheta(x) \left[j(\tfrac{x}{2}) \, , \, j_\mu^\dagger(-\tfrac{x}{2}) \right] \right| p_1 \right\rangle \tag{49}$$

and the absorptive part is given by

$$\text{Im } T\,(s,u,t) = \tfrac{1}{2} e_\mu \int d^4x \; e^{-i(q-k)\frac{x}{2}} \left\langle -p_2 \left| \left[j\,(\tfrac{x}{2}), j_\mu^\dagger(-\tfrac{x}{2}) \right] \right| p_1 \right\rangle \tag{50}$$

so that

$$\text{Im } T_I = \tfrac{1}{2} e_\mu \int d^4x \; e^{-i(q-k)\frac{x}{2}} \left\langle -p_2 \left| j\,(\tfrac{x}{2})\, j_\mu^\dagger\, (-\tfrac{x}{2}) \right| p_1 \right\rangle \tag{51}$$

and

$$\text{Im } T_{II} = \tfrac{1}{2} e_\mu \int d^4x \; e^{-i(q-k)\frac{x}{2}} \left\langle -p_2 \left| j_\mu^\dagger\, (-\tfrac{x}{2}) j\,(\tfrac{x}{2}) \right| p_1 \right\rangle \tag{52}$$

These correspond to the two terms of the commutator and give the absorptive part for reaction I and II respectively. Introducing the complete set of states in (51) we have

$$\text{Im } T_I = \tfrac{1}{2} e_\mu \sum_{n,\gamma} \int d^4x \, e^{-i(q-k)\frac{x}{2}} \left\langle -p_2 \left| j\left(\tfrac{x}{2}\right) \right| n,\gamma \right\rangle \cdot \left\langle n,\gamma \left| j_\mu^\dagger\left(-\tfrac{x}{2}\right) \right| p_1 \right\rangle$$

$$= \tfrac{1}{2} e_\mu (2\pi)^4 \sum_{n,\gamma} \delta(p_n - k - p_1) \left\langle -p_2 \left| j(o) \right| n,\gamma \right\rangle \cdot \left\langle n,\gamma \left| j_\mu^\dagger(o) \right| p_1 \right\rangle \qquad (53)$$

where p_n is the four-momentum of the state $|n,\gamma\rangle$.

The lowest possible state with the same quantum numbers as $j_\mu^\dagger(o)\,|p_1\rangle$ is the single nucleon state which corresponds to Figure II(a). Inserting the single nucleon contribution gives

$$\text{Im } T_I = \pi\delta(s-M_1^2) \sum_\gamma e_\mu \left\langle -p_2 \left| j(o) \right| k+p_1\gamma \right\rangle \left\langle k+p_1\gamma \left| j_\mu^\dagger(o) \right| p_1 \right\rangle \qquad (54)$$

where $\sum_\gamma$ indicates the summation over the nucleon spin.

In the same way we take (52) and insert the complete set of states which results in the form

$$\text{Im } T_{II} = \tfrac{1}{2} e_\mu (2\pi)^4 \sum_{n,\gamma} \delta(p_n - q - p_1) \left\langle -p_2 \left| j_\mu^\dagger(o) \right| n\gamma \right\rangle \left\langle n\gamma \left| j(o) \right| p_1 \right\rangle \qquad (55)$$

The lowest possible state that has the same quantum numbers as $\left\langle -p_2 \right| j_\mu^\dagger(o)$ is the single hyperon state (Figure II(b)). Thus

$$\text{Im } T_{II} = \pi\delta(u - M_2^2) \sum_{\gamma} e_\mu \left\langle -p_2 \left| j_\mu^\dagger(o) \right| q + p_1 \gamma \right\rangle \cdot \left\langle q + p_1 \gamma \left| j(o) \right| p_1 \right\rangle \qquad (56)$$

where γ now denotes the summation over the hyperon spin.

Now

$\left\langle -p_2 \left| j(o) \right| k+p_1 \right\rangle$ is the KYN vertex,

$\left\langle k + p_1 \left| j_\mu^\dagger(o) \right| p_1 \right\rangle$ is the γNN vertex,

$\left\langle -p_2 \left| j_\mu^\dagger(o) \right| q + p_1 \right\rangle$ is the γYY vertex,

and $\left\langle q + p_1 \left| j(o) \right| p_1 \right\rangle$ is the KYN vertex.

Since the photon is the only virtual particle, we have, by general consideration of Lorentz and gauge invariance, the following forms for the vertices :

$$\left\langle -p_2 \left| j(o) \right| k+p_1 \right\rangle = \bar{u}(-p_2)\, i\gamma_5 g_Y\, u(k+p_1)$$

$$\left\langle q + p_1 \left| j(o) \right| p_1 \right\rangle = \bar{u}(q+p_1)\, i\gamma_5 g_Y\, u(p_1)$$

$$\left\langle k + p_1 \left| j_\mu^\dagger(o) \right| p_1 \right\rangle = \bar{u}(k+p_1)\, i \left[G_1 \gamma_\mu + G_2 \sigma_{\mu\nu} \left\{ (k+p_1) - (p_1) \right\}_\nu \right] u(p_1) \qquad (57)$$

and

$$\left\langle -p_2 \left| j_\mu^\dagger(o) \right| q+p_1 \right\rangle = \bar{u}(-p_2)\, i \left[G_1' \gamma_\mu + G_2' \sigma_{\mu\nu} \left\{ (-p_2) - (p_1+q) \right\}_\nu \right] u(p_1+q)$$

where g_Y is the renormalized KYN coupling constant and G_1, G_2 and G_1', G_2' are related to the renormalized electromagnetic form factors of the nucleon and hyperon respectively. The isotopic dependence of these form factors are as follows

NUCLEON FORM FACTORS.

$$
\begin{aligned}
G_1^{S_o} &= G_1^{V_o} = \tfrac{1}{2}e\left[F_{1p}(\lambda^2) + F_{1n}(\lambda^2)\right] \\
G_1^{S_+} &= G_1^{V_\pm} = \tfrac{1}{2}e\left[F_{1p}(\lambda^2) - F_{1n}(\lambda^2)\right] \\
G_2^{S_o} &= G_2^{V_o} = \tfrac{1}{2}\left[\mu_p F_{2p}(\lambda^2) + \mu_n F_{2n}(\lambda^2)\right] \\
G_2^{S_+} &= G_2^{V_\pm} = \tfrac{1}{2}\left[\mu_p F_{2p}(\lambda^2) - \mu_n F_{2n}(\lambda^2)\right]
\end{aligned}
\qquad (58)
$$

with the normalization

$$F_{1n}(o) = o$$

and

$$F_{2n}(o) = F_{1p}(o) = F_{2p}(o) = 1$$

HYPERON FORM FACTORS.

$$
\begin{aligned}
G_1'^{V_o} &= \tfrac{1}{2}e\left[G_1^{(+)}(\lambda^2) - G_1^{(-)}(\lambda^2)\right] \\
G_1'^{V_-} &= \tfrac{1}{2}e\left[G_1^{(+)}(\lambda^2) + G_1^{(-)}(\lambda^2)\right] \\
G_1'^{V_+} &= e\, G_1^{(T)}(\lambda^2) = G_1'^{S_+} \\
G_1'^{S_o} &= e\, G_1^{(\Lambda)}(\lambda^2)
\end{aligned}
\qquad (59)
$$

$$G_2^{'V}{}_o = \tfrac{1}{2}\left[\mu_+ G_2^{(+)}(\lambda^2) + \mu_- G_2^{(-)}(\lambda^2)\right]$$

$$G_2^{'V}{}_- = \tfrac{1}{2}\left[\mu_+ G_2^{(+)}(\lambda^2) - \mu_- G_2^{(-)}(\lambda^2)\right]$$

$$G_2^{'V}{}_+ = \frac{g_\Lambda}{g_\Sigma}\ \mu_T\ G_2^{(T)}(\lambda^2)$$

$$G_2^{'S}{}_o = \mu_\Lambda G_2^{(\Lambda)}(\lambda^2) \qquad G_2^{'S}{}_+ = \frac{g_\Sigma}{g_\Lambda}\ \mu_T\ G_2^{(T)}(\lambda^2)$$

where $\mu_{+,-,o}$ and μ_T are respectively the anomalous magnetic moments of the Σ^+, Σ^- and Σ^o and the transition moment associated with the $\gamma\Lambda\Sigma^o$ vertex. The form factors $G_{1,2}^{+,-,\Lambda,T}$ have their obvious meaning and are normalised as follows :

$G_1^{(\Lambda)}(o) = G_1^{(T)}(o) = 0$ and the rest are equal to unity at $\lambda^2 = 0$.

Hence from equations (54) and (56) using (57) we have

$$\text{Im } T_I = \pi\delta(s-M_1^2)\ \bar{u}(-p_2)\ i\gamma_5\ g_Y\ (\not{k} + \not{p}_1 + M_1)$$

$$ie_\mu\left[\ G_1\gamma_\mu + G_2\sigma_{\mu\nu}\ k_\nu\ \right]u(p_1) \qquad (60)$$

and

$$\text{Im } T_{II} = \pi\delta(u-M_2^2)\ \bar{u}(-p_2)\ ie_\mu\left[\ G_1'\gamma_\mu + G_2'\sigma_{\mu\nu}\ k_\nu\ \right]$$

$$(\not{p}_1 + \not{q} + M_2)\ i\gamma_5\ g_Y\ u\ (p_1) \qquad (61)$$

Using the standard techniques we can reduce these two expressions to forms involving only the invariant M_i. Thus

$$\text{Im } T_I = \pi\delta(s-M_1^2)\ \bar{u}(-p_2)\Big[G_1 M_1 + G_2\ (M_3 - M_4) - i\gamma_5\ G_1\ (\ q.\varepsilon - 2P.\varepsilon\ )\Big] g_Y\ u(p_1) \tag{62}$$

and

$$\text{Im } T_{II} = \pi\delta(u-M_2^2)\ \bar{u}(-p_2)\Big[(G_1' + 2\Delta G_2')\ M_1 - (M_3 + M_4)\ G_2' + i\gamma_5\ G_1'\ (2P.\varepsilon + q.\varepsilon)\Big] g_Y u(p_1) \tag{63}$$

For the reaction III (Figure II(c)) we have the following single K-meson pole contribution

$$\text{Im } T_{III} = \pi\delta(t-m^2)\ \bar{u}(-p_2)\Big[\ i\gamma_5\ (2q+k).\varepsilon \mathcal{J}_K\Big] u(p_1) \tag{64}$$

where $\mathcal{J}_K(\lambda^2)$ is the electromagnetic form factor of the K-meson. The isotopic dependence of $\mathcal{J}_K(\lambda^2)$ is as follows

$$\mathcal{J}_K^{V\ \pm}_{\ o} = \begin{pmatrix} \pm \\ + \end{pmatrix} \tfrac{1}{2} e \Big[\ F_K^{(+)}(\lambda^2) \begin{pmatrix} - \\ + \end{pmatrix} F_K^{(o)}(\lambda^2)\Big]$$

$$\mathcal{J}_K^{S\ (\pm)}_{\ \ (o)} = \tfrac{1}{2} e \Big[\ F_K^{(+)}(\lambda^2) \mp F_K^{(o)}(\lambda^2)\ \Big] \tag{65}$$

with $F_K^{V,S} = F_K^{(+)} \mp F_K^{(o)}$

We observe that in (62), (63) and (64) the electric charge terms, G_1, G_1' and $\mathcal{J}_K$ are not gauge invariant when taken individually whereas the magnetic terms are. The non gauge invariant terms can be made invariant by adding

the quantity

$$g_Y\, i\gamma_5 \left[\not{J}_K + G_1' - G_1 \right] \frac{k.e}{\lambda^2} \tag{66}$$

which is identically zero on account of the Lorentz condition (35).

Thus

$$\begin{aligned} \sum_{i=1}^{6} A_i(B.A)\, M_i \;=\; g_Y \Bigg[& \frac{G_1 M_1 + G_2 (M_3 - M_4)}{M_1^2 - s} \\ & + \frac{G_1 (M_5 + 2M_2)}{(M_1^2 - s)(t-m^2)} + \frac{(G_1' + 2\Delta G_2')\, M_1 - (M_3 + M_4)\, G_2'}{M_2^2 - u} \\ & + \frac{G_1' (2M_2 - M_5)}{(M_2^2 - u)(t-m^2)} + \frac{(G_1' - G_1 - \not{J}_K)\, 2M_5}{\lambda^2 (t-m^2)} \Bigg] \end{aligned} \tag{67}$$

and by comparing this expression with (48) we obtain the required residues of the poles, namely

$$\begin{aligned} \Gamma_1(s) &= g_Y G_1 & \Gamma_1(u) &= g_Y (G_1' + 2\Delta G_2') \\ \Gamma_2(s) &= 2g_Y G_1 & \Gamma_2(u) &= 2g_Y G_1' \\ \Gamma_3(s) &= g_Y G_2 & \Gamma_3(u) &= -g_Y G_2' \\ \Gamma_4(s) &= -g_Y G_2 & \Gamma_4(u) &= -g_Y G_2' \\ \Gamma_5(s) &= g_Y G_1 & \Gamma_5(u) &= -g_Y G_1' \\ \Gamma_6(s) &= 0 & \Gamma_6(u) &= 0 \\ \Gamma_i(t) &= 0 \quad i \neq 5 & \Gamma_5(t) &= \frac{2g_Y}{\lambda^2} (G_1' - G_1 - \not{J}_K) \,. \end{aligned} \tag{68}$$

v. ANGULAR MOMENTUM DECOMPOSITION.

In order to project out the angular momentum states of the Born terms we employ the following method. The T-matrix is first reduced to two-component Pauli spinors in the barycentric system of reaction I and the resulting Pauli amplitudes are then expanded in terms of electric, magnetic and longitudinal multipoles. These relations can be inverted and, with the aid of the inverse relations between the Dirac and Pauli amplitudes (A_i and $\mathcal{J}_i$ respectively), the required multipole projections of the Born terms can be performed. Finally, the isotopic projections of these $\mathcal{J}_{iR}(B.A)$ can be obtained by utilizing the isotopic dependence of the form factors given by equations (58), (59) and (65).

S-CHANNEL REDUCTION.

With the identification

$$\bar{u}_Y(-p_2)\left[\sum_{i=1}^{6} A_i M_i\right] u_N(p_1) = \chi_Y^{\dagger}\left[\sum_{i=1}^{6} \mathcal{J}_i f_i\right]\chi_N \qquad (69)$$

and the following forms of the f_i's

$$\begin{aligned} f_1 &= i\,\underline{\sigma}.\underline{a} & q^2 f_4 &= i\,\underline{\sigma}.\underline{q}\;\underline{q}.\underline{a} \\ qkf_2 &= \underline{\sigma}.\underline{q}\;\underline{\sigma}.(\underline{k}\wedge\underline{a}) & k^2 f_5 &= i\,\underline{\sigma}.\underline{k}\;\;\underline{k}.\underline{a} \\ qkf_3 &= i\,\underline{\sigma}.\underline{k}\;\;\underline{q}.a & qkf_6 &= i\,\underline{\sigma}.\underline{q}\;\;\underline{k}.\underline{a} \end{aligned} \qquad (70)$$

where $\underline{a}$ is the gauge invariant three vector

$$\underline{a} = \underline{e} - \frac{e_o}{k_o}\,\underline{k} \qquad (71)$$

we can relate the $\mathcal{J}_i$'s to the A_i's with the aid of the barycentric relations (13) and the explicit forms of the Dirac spinor given by (16). The resulting set of linear equations are as follows

$$\frac{\mathcal{J}_1}{O_1(W - M_1)} = F_1 = A_1 + \frac{q.k}{W - M_1}(A_3+A_4) + (W-M_1)A_4 + \frac{\lambda^2}{W - M_1} A_6$$

$$\frac{-O_1 \mathcal{J}_2}{qk(W +M_1)} = F_2 = A_1 - \frac{q.k}{W +M_1}(A_3+A_4) - (W-M_1)A_4 - \frac{\lambda^2}{W+M_1} A_6$$

$$\frac{- O_2 \mathcal{J}_3}{qk\ (W+M_1)} = F_3 = \frac{\beta}{W + M_1} A_2 + (A_3+A_4) - \frac{\lambda^2}{W + M_1} A_5$$

$$\frac{-\mathcal{J}_4}{q^2 O_2(W-M_1)} = F_4 = \frac{-\beta}{W - M_1} A_2 + (A_3+A_4) + \frac{\lambda^2}{W - M_1} A_5 \qquad (72)$$

$$\frac{O_2 \mathcal{J}_5}{k^2} = F_5 = - A_1 + \alpha A_2 - (W-M_1)A_4 - q.k\ A_5 - (W+M_1)\ A_6$$

$$\frac{\mathcal{J}_6}{qk\ O_2} = F_6 = - \frac{k_o}{\varepsilon_1+M_1} A_1 - \alpha A_2 + \frac{q.k}{\varepsilon_1+M_1}(A_3+ A_4) + q.k\ A_5 + \frac{k_o\ (W + M_1)}{\varepsilon_1 + M_1}(A_4- A_6)$$

where

$$\alpha = \frac{3}{2}\, q.k + Wk_o$$

$$\beta = 2Wk_o + \frac{3}{2}\lambda^2 \qquad (73)$$

$$o_1^2 = (\varepsilon_1+M_1)(\varepsilon_2+M_2) \qquad o_2^2 = \frac{(\varepsilon_1+M_1)}{(\varepsilon_2+M_2)}$$

To perform the multipole expension we proceed as follows. We expand into a complete set of states of orbital angular momenta l and among these states some will refer to states of total angular momentum $l+\frac{1}{2}$ and others $l-\frac{1}{2}$. These states are separated by the introduction of the projection operators

$$P_{l+} = \frac{l + 1 + \underline{\sigma} \cdot \underline{L}\, q}{2l + 1} \qquad P_{l-} = \frac{l - \underline{\sigma} \cdot \underline{L}\, q}{2l+1} \qquad (74)$$

where $\underline{Lq} = \frac{1}{i}\ \underline{q} \wedge \underline{\nabla}_{\underline{q}}$ is the angular momentum operator of the K-meson. A further classification pertaining to the photon state must also be included as some states transform as transverse pseudovectors (magnetic with parity $(-1)^{l+1}$), others as transverse vectors (electric with parity $(-1)^l$) and still others as longitudinal vectors (longitudinal with parity $(-1)^l$). There is no scalar radiation here because of the Lorentz condition (35). Thus we have

Magnetic	$\underline{\sigma}.\underline{q}\quad \underline{a}.\underline{L}_k$	(75)
Electric	$\underline{a}.\underline{k} \wedge \underline{L}_k$	
Longitudinal	$\underline{a}.\underline{k}$	
Scalar	a_o	

Finally, as the relative intrinsic parity of the initial to the final states is odd, we must include the factor $\underline{\sigma}_{-} \cdot \underline{q}$ on all terms.

Hence the required multipole expansion takes the form

$$\begin{aligned}\mathcal{J} = \sum_{l}\Big[& \frac{1}{i} P_{l+} \, \underline{a}\cdot\underline{L}_k \; M_l^+ + \frac{1}{i} P_{l-} \, \underline{a}\cdot\underline{L}_k \, M_l^- \\ & - \underline{\sigma}\cdot\underline{q} \, P_{l+} \; \underline{a}\cdot\underline{k}\wedge\underline{L}_k \; E_{l+1}^- \\ & + \underline{\sigma}\cdot\underline{q} \; P_{l-} \, \underline{a}\cdot\underline{k}\wedge\underline{L}_k \; E_{l-1}^+ \\ & - \frac{1}{i}(l+1) \, \underline{\sigma}\cdot\underline{q} \, P_{l+} \; \underline{a}\cdot\underline{k} \, L_{l+1}^- \\ & - \frac{1}{i} \, l \; \underline{\sigma}\cdot\underline{q} \, P_{l-} \, \underline{a}\cdot\underline{k} \, L_{l-1}^+ \Big] \, (2l+1) \, P_l(\cos\vartheta)\end{aligned} \tag{76}$$

where $P_l(\cos\vartheta)$ is the legendre polynomial of order l and the following notation is implied (Multipole)$^{\pm}_{l_K}$ absorbed giving a (KY) state of total angular momentum $j = l_K \pm \frac{1}{2}$ (for magnetic radiation $l_K = l$ and for electric and longitudinal radiations $l_K = l \pm 1$).

When the above operations are carried out and compared with

$$\mathcal{J} = \sum_{i=1}^{6} \mathcal{J}_i f_i \tag{77}$$

the required multipole expansion is obtained namely

$$\begin{aligned}\mathcal{J}_1 &= \sum \left[(l-1) M_{l-1}^+ + (l+2) M_{l+1}^- + E_{l+1}^- + E_{l-1}^+ \right] P_l'(x) \\ \mathcal{J}_2 &= \sum \left[(l+1) \, M_2^+ + l \, M_l^- \right] P_l'(x) \\ \mathcal{J}_3 &= \sum \left[-M_{l-1}^+ + M_{l+1}^- + E_{l+1}^- + E_{l-1}^+ \right] P_l''(x) \\ \mathcal{J}_4 &= \sum \left[M_l^+ - M_l^- - E_l^- - E_l^+ \right] P_l''(x)\end{aligned} \tag{78}$$

$$\mathcal{J}_5 = -\mathcal{J}_1 - x\mathcal{J}_3 + \Sigma\left[l\, L_{l-1}^{+} - (l+1)L_{l+1}^{-}\right] P_l'(x)$$

$$\mathcal{J}_6 = -x\mathcal{J}_4 + \Sigma\left[l\, L_l^{-} - (l+1)\, L_l^{+}\right] P_l'(x) \ .$$

The final stage in the s-channel reduction involves the inversion of the relations (72) and (78). The set of six linear relations (72) can be inverted in a straightforward manner to give

$$A_1 = \frac{W^2-M_1^2}{4W^2k_o}\left[(W+M_1 - \frac{\lambda^2}{W-M_1})\ F_1 + \frac{k_o}{\varepsilon_1+M_1}\ (W+M_1+\frac{\lambda^2}{W-M_1})\ F_2\right.$$

$$\left. - 2M_1 q.k\ \left(\frac{F_3}{W-M_1} + \frac{F_4}{W+M_1}\right) + \frac{2M_1\lambda^2}{W^2-M_1^2}\ (F_5+F_6)\right]$$

$$A_2 = \frac{W^2-M_1^2}{4W^2k_o}\ \frac{1}{q.k-\frac{1}{2}\lambda^2}\left[-\frac{\lambda^2}{W+M_1}(F_1+F_5) + \frac{k_o}{\varepsilon_1+M_1}\ \frac{\lambda^2}{W-M_1}\ F_2\right.$$

$$\left. + q.k\ (F_3 - F_4) + \frac{\lambda^2}{W-M_1}\ F_6\right] \qquad (79)$$

$$A_3 = \frac{W^2-M_1^2}{4W^2k_o}\left[-F_1 + \frac{k_o}{\varepsilon_1+M_1}\ \frac{W+M_1}{W-M_1}\ F_2 - \frac{\lambda^2}{W^2-M_1^2}\ (F_5+F_6)\right.$$

$$\left. + (2Wk_o+q.k)\ \left(\frac{F_3}{W-M_1} + \frac{F_4}{W+M_1}\right)\right]$$

$$A_4 = \frac{W^2-M_1^2}{4W^2k_o}\left[F_1 - \frac{k_o}{\varepsilon_1+M_1}\ \frac{W+M_1}{W-M_1}\ F_2 + \frac{\lambda^2}{W^2-M_1^2}\ (F_5+F_6)\right.$$

$$\left. - q.k\ \left(\frac{F_3}{W-M_1} + \frac{F_4}{W+M_1}\right)\right]$$

$$A_5 = \frac{\beta}{4W^2 k_o} \frac{W - M_1}{q.k - \frac{1}{2}^2} \left[-(F_1+F_5) + \frac{W + M_1}{W - M_1} F_6 + \frac{k_o}{\varepsilon_1+M_1} \frac{W+M_1}{W-M_1} F_2 + \frac{(W+M_1)\alpha}{\beta} (F_3 - F_4) \right]$$

$$A_6 = \frac{W^2 - M_1^2}{4W^2 k_o} \left[-(F_1+F_5+F_6) - \frac{k_o}{\varepsilon_1+M_1} F_2 + q.k \left(\frac{F_3}{W-M_1} + \frac{F_4}{W+M_1} \right) \right]$$

To invert the multipole expansion (78) we employ the orthogonality relations of the Legendre polynomials. The results are

$$M_l^+ = \frac{1}{2(l+1)} \int_{-1}^{+1} dx \left[\mathcal{J}_1 P_l(x) - \mathcal{J}_2 P_{l+1}(x) - \mathcal{J}_3 \frac{P_{l-1}(x) - P_{l+1}(x)}{2l+1} \right]$$

$$M_l^- = \frac{1}{2l} \int_{-1}^{+1} dx \left[-\mathcal{J}_1 P_l(x) + \mathcal{J}_2 P_{l-1}(x) + \mathcal{J}_3 \frac{P_{l-1}(x) - P_{l+1}(x)}{2l+1} \right]$$

$$E_l^+ = \frac{1}{2(l+1)} \int_{-1}^{+1} dx \left[\mathcal{J}_1 P_l(x) - \mathcal{J}_2 P_{l+1}(x) + \mathcal{J}_3 l \frac{P_{l-1}(x) - P_{l+1}(x)}{2l+1} + \mathcal{J}_4 (l+1) \frac{P_l(x) - P_{l+2}(x)}{2l+3} \right] \qquad (80)$$

$$E_l^- = \frac{1}{2l} \int_{-1}^{+1} dx \left[\mathcal{J}_1 P_l(x) - \mathcal{J}_2 P_{l+1}(x) - \mathcal{J}_3 (l+1) \frac{P_{l-1}(x) - P_{l+1}(x)}{2l+1} \right.$$

$$\left. - \mathcal{J}_4 \; l \; \frac{P_{l-2}(x) - P_l(x)}{2l-1} \right]$$

$$L_l^+ = \frac{1}{2(l+1)} \int_{-1}^{+1} dx \left[(\mathcal{J}_1 + x\mathcal{J}_3 + \mathcal{J}_5) P_l(x) + (x\mathcal{J}_4 + \mathcal{J}_6) P_{l+1}(x) \right]$$

$$L_l^- = \frac{1}{2l} \int_{-1}^{+1} dx \left[(\mathcal{J}_1 + x\mathcal{J}_3 + \mathcal{J}_5) P_l(x) + (x\mathcal{J}_4 + \mathcal{J}_6) P_{l-1}(x) \right]$$

This completes the s-channel reduction.

U-CHANNEL REDUCTION

In order to perform the reduction in the U-channel we introduce the S-matrix for the process $\gamma + \bar{Y} \rightarrow K + \bar{N}$. This process is equivalent to $\gamma + Y \rightarrow N + \bar{K}$ by invariance under charge conjugation and by the substitution rule we can obtain the S-matrix element for $\gamma + \bar{Y} \rightarrow K + \bar{N}$ from that of $\gamma + N \rightarrow K + Y$ by replacing $p_1, -p_2$, $u(p_1)$ and $\bar{u}(-p_2)$ by $-p_1$, p_2, $v(-p_1)$ and $\bar{v}(p_2)$ respectively. We then have the S-matrix in the form

$$S = i(2\pi)^4 \delta(k+p_1+q+p_2) \; \bar{v}(p_2) \; T \; v(-p_1)$$

with

$$T = \sum_{i=1}^{6} \tilde{A}_i \; (s,u,t) \; M_i \qquad (81)$$

Using the identification

$$\bar{v}(p_2)\left[\sum_{i=1}^{6} \tilde{A}_i M_i\right] v(-p_1) = \chi^{\dagger}_{\bar{Y}} \left[\sum_{i=1}^{6} \tilde{\mathcal{J}}_i \tilde{f}_i\right] \chi_{\bar{N}} \qquad (82)$$

with the $\tilde{f}_i$ defined by

$$\tilde{f}_1 = i\,\underline{\sigma}\cdot\underline{a} \qquad Q^2 \tilde{f}_4 = i\,\underline{\sigma}\cdot\underline{Q}\;\;\underline{Q}\cdot\underline{a}$$

$$QK\,\tilde{f}_2 = -\,\underline{\sigma}\cdot(\underline{K}\wedge\underline{a})\,\underline{\sigma}\cdot\underline{Q} \qquad K^2 \tilde{f}_5 = i\,\underline{\sigma}\cdot\underline{K}\;\underline{K}\cdot\underline{a} \qquad (83)$$

$$QK\,\tilde{f}_3 = -\,\underline{\sigma}\cdot\underline{K}\;\underline{Q}\cdot\underline{a} \qquad QK\,\tilde{f}_6 = i\,\underline{\sigma}\cdot\underline{Q}\;\underline{K}\cdot\underline{a}$$

We can relate the $\tilde{\mathcal{J}}_i$'s to the A_i's with the aid of (17) and (20). The required six linear relations are

$$\frac{\tilde{\mathcal{J}}_1}{\tilde{O}_1(\omega-M_2)} = \tilde{F}_1 = \tilde{A}_1 + \frac{q\cdot k}{\omega-M_2}(\tilde{A}_4-\tilde{A}_3) + (\omega+M_2-2M_1)\,\tilde{A}_4 - \frac{\lambda^2}{\omega-M_2}\,\tilde{A}_6$$

$$\frac{-\tilde{O}_1\,\tilde{\mathcal{J}}_2}{QK\,(\omega+M_2)} = \tilde{F}_2 = \tilde{A}_1 - \frac{q\cdot k}{\omega+M_2}(\tilde{A}_4-\tilde{A}_3) - (\omega+2M_1-M_2)\,\tilde{A}_4 + \frac{\lambda^2}{\omega+M_2}\,\tilde{A}_6 \qquad (84)$$

$$\frac{-\tilde{O}_2\ \tilde{J}_3}{QK\ (\omega + M_2)} = \tilde{F}_3 = \frac{\tilde{\beta}}{\omega + M_2}\tilde{A}_2 + (\tilde{A}_4 - \tilde{A}_3) + \frac{\lambda^2}{\omega + M_2}\tilde{A}_5$$

$$\frac{-\tilde{J}_4}{\tilde{O}_2\ Q^2(\omega - M_2)} = \tilde{F}_4 = \frac{-\tilde{\beta}}{\omega - M_2}\tilde{A}_2 + (\tilde{A}_4 - \tilde{A}_3) - \frac{\lambda^2}{\omega - M_2}\tilde{A}_5$$

$$\frac{\tilde{O}_2\ \tilde{J}_5}{K^2} = \tilde{F}_5 = -\tilde{A}_1 + \tilde{\alpha}\tilde{A}_2 - (\omega + M_2 - 2M_1)\tilde{A}_4 + q\cdot k\,\tilde{A}_5 + (\omega + M_2)\tilde{A}_6$$

$$\frac{\tilde{J}_6}{\tilde{O}_2\ QK} = \tilde{F}_6 = -\frac{K_o}{E_2 + M_2}\tilde{A}_1 - \tilde{\alpha}\tilde{A}_2 + \frac{q\cdot k}{E_2 + M_2}(\tilde{A}_4 - \tilde{A}_3) + \frac{K_o}{E_2 + M_2}(\omega - M_2 + 2M_1)\tilde{A}_4 - q\cdot k\,\tilde{A}_5 + \frac{K_o}{E_2 + M_2}(\omega + M_2)\tilde{A}_6$$

where

$$\tilde{\alpha} = \omega K_o + \tfrac{3}{2}\, q\cdot k$$

$$\tilde{\beta} = 2\omega K_o + \tfrac{3}{2}\lambda^2 \qquad (85)$$

$$\tilde{O}_1^2 = (E_1 + M_1)(E_2 + M_2) \qquad \tilde{O}_2^2 = \frac{E_2 + M_2}{E_1 + M_1}$$

We observe that the relations (84) could have been written down from the relations (74) by means of the substitutions

$$A_i \longrightarrow \pm \widetilde{A}_i \quad (+ \text{ for } i=1,2,4, \; - \text{ for } \; i = 3,5,6)$$

$$k_o \to K_o, \quad k \to K, \quad q \to Q, \quad W \to \omega, \quad \varepsilon_1 \to E_2 \; ,$$

$$\varepsilon_2 \to E_1, \quad \alpha \to \widetilde{\alpha}, \quad \beta \to \widetilde{\beta}, \quad O_1 \to \widetilde{O}_1 \; , \quad O_2 \to \widetilde{O}_2$$

and $M_1 \rightleftharpoons M_2$ except for the coefficient of $\widetilde{A}_4$ where the masses arise through the term M_4 of (38).

The multipole expansion for the u-channel may be obtained directly from (78) by invoking the substitutions

$$\mathcal{J}_i \to \widetilde{\mathcal{J}}_i$$

and

$$(\text{Multipole})_l^{\pm} \longrightarrow \widetilde{(\text{Multipole})}_l^{\pm}$$

We finally invert the relations (84) to complete the u-channel analysis.

$$\widetilde{A}_1 = \frac{\omega^2 - M_2^2}{4\,\omega^2 K_o} \Bigg[\left(\omega - M_2 + 2M_1 - \frac{\lambda^2}{\omega + M_2}\right) \widetilde{F}_1 + \frac{2M_1\,\lambda^2}{\omega^2 - M_2^2} \left(\widetilde{F}_5 + \widetilde{F}_6\right)$$

$$+ \frac{K_o}{E_2 + M_2} \; \frac{\omega + M_2}{\omega - M_2} \left(\omega + M_2 - 2M_1 + \frac{\lambda^2}{\omega + M_2}\right) \widetilde{F}_2$$

$$- 2M_1\, q.k \quad \left(\frac{\widetilde{F}_3}{\omega - M_2} + \frac{\widetilde{F}_4}{\omega + M_2}\right) \Bigg]$$

$$\tilde{A}_2 = \frac{\omega^2-M_2^2}{4\omega^2 K_o}\,\frac{1}{q\cdot k-\frac{1}{2}\lambda^2}\left[-\frac{\lambda^2}{\omega+M_2}(\tilde{F}_1+\tilde{F}_5)+\frac{K_o}{E_2+M_2}\,\frac{\lambda^2}{\omega-M_2}\,\tilde{F}_2 + q\cdot k\,(\tilde{F}_3-\tilde{F}_4)+\frac{\lambda^2}{\omega-M_2}\,\tilde{F}_6\right]$$

$$\tilde{A}_3 = \frac{\omega^2-M_2^2}{4\omega^2 K_o}\left[\tilde{F}_1-\frac{K_o}{E_2+M_2}\left(\frac{\omega+M_2}{\omega-M_2}\right)\tilde{F}_2+\frac{\lambda^2}{\omega^2-M_2^2}(\tilde{F}_5+\tilde{F}_6) -(q\cdot k+2\omega K_o)\left(\frac{\tilde{F}_3}{\omega-M_2}+\frac{\tilde{F}_4}{\omega+M_2}\right)\right]$$

$$\tilde{A}_4 = \frac{\omega^2-M_2^2}{4\omega^2 K_o}\left[\tilde{F}_1-\frac{K_o}{E_2+M_2}\left(\frac{\omega+M_2}{\omega-M_2}\right)\tilde{F}_2+\frac{2}{\omega^2-M_2^2}(\tilde{F}_5+\tilde{F}_6) - q\cdot k\left(\frac{\tilde{F}_3}{\omega-M_2}+\frac{\tilde{F}_4}{\omega+M_2}\right)\right] \qquad (86)$$

$$\tilde{A}_5 = \frac{\omega-M_2}{4\omega^2 K_o}\,\frac{\tilde{\beta}}{q\cdot k-\frac{1}{2}\lambda^2}\left[\tilde{F}_1+\tilde{F}_5-\frac{K_o}{E_2+M_2}\left(\frac{\omega+M_2}{\omega-M_2}\right)\tilde{F}_2 -\left(\frac{\omega+M_2}{\omega-M_2}\right)\tilde{F}_6-\frac{(\omega+M_2)\,\tilde{\alpha}}{\tilde{\beta}}(\tilde{F}_3-\tilde{F}_4)\right]$$

$$\tilde{A}_6 = \frac{\omega^2-M_2^2}{4\omega^2 K_o}\left[\tilde{F}_1+\tilde{F}_5+\tilde{F}_6+\frac{K_o}{E_2+M_2}\,\tilde{F}_2 - q\cdot k\left(\frac{\tilde{F}_3}{\omega-M_2}+\frac{\tilde{F}_4}{\omega+M_2}\right)\right]$$

T-CHANNEL REDUCTION

In order to perform the angular momentum decomposition of the t-channel we again use an identification

$$\bar{u}(-p_2)\left[\sum_{i=1}^{6} \bar{A}_i\, M_i\right] v(-p_1) = \chi^{\dagger}\left[\sum_{i=1}^{6} \bar{J}_i\, \bar{f}_i\right]\chi \qquad (87)$$

where the $\bar{f}_i$'s are defined by

$$\begin{aligned} p\,\bar{f}_1 &= i\,\underline{p}\cdot\underline{a} & \varkappa\bar{f}_4 &= \underline{\sigma}\cdot(\underline{\varkappa}\wedge\underline{a}) \\ p\,\bar{f}_2 &= \underline{\sigma}\cdot(\underline{p}\wedge\underline{a}) & \varkappa\bar{f}_5 &= i\,\underline{\varkappa}\cdot\underline{a} \\ p^2\varkappa\,\bar{f}_3 &= \underline{\sigma}\cdot\underline{p}\,\underline{p}\cdot(\underline{\varkappa}\wedge\underline{a}) & p\,\varkappa^2\bar{f}_6 &= \underline{\sigma}\cdot(\underline{p}\wedge\underline{\varkappa})\,\underline{\varkappa}\cdot\underline{a} \end{aligned} \qquad (88)$$

with

$$\underline{a} = \underline{e} - \frac{e_o}{\varkappa_o}\,\underline{\varkappa}$$

The use of (21) and (24) enables the above identification to be made and the relations between the $\bar{A}_i$'s and the $\bar{J}_i$'s are as follows

$$\begin{aligned} \frac{\bar{O}_1\,\bar{J}_1}{p(\Omega+2\bar{M})} = \bar{F}_1 &= \varkappa_o(\bar{A}_1+\Delta\,\bar{A}_4) - \frac{\Omega^2-\Delta^2}{2\Omega}\,(q.k+\tfrac{1}{2}k^2)\,\bar{A}_2 \\ &\quad + \frac{\Delta}{\Omega}\,(q.k\,\bar{A}_3 - k^2\bar{A}_6) \\ \frac{\bar{O}_1\,\Omega\bar{J}_2}{p\Delta(\Omega+2\bar{M})} = \bar{F}_2 &= \varkappa_o\,(\bar{A}_1+\Delta\bar{A}_4) + \frac{\Omega}{\Delta}\,(q.k\,\bar{A}_3 - k^2\bar{A}_6) \\ \frac{\bar{O}_1\,\bar{J}_3}{2p^2\varkappa} = \bar{F}_3 &= (\bar{A}_1+\Delta\,\bar{A}_4) + \Omega\bar{A}_4 \end{aligned} \qquad (89)$$

$$\frac{\bar{O}_1 \; \bar{\mathcal{F}}_4}{2\bar{M}\,\varkappa(p_{10}+M_1)(\Omega+\Delta)} = \bar{F}_4 = (\bar{A}_1 + \Delta\bar{A}_4) - \frac{\Omega^2}{2\bar{M}} \; \bar{A}_4$$

$$\frac{-\bar{O}_1 \; \bar{\mathcal{F}}_5}{(\Omega+2\bar{M})\varkappa} = \bar{F}_5 = [\, P.k\, \bar{A}_2 + (q.k + k^2)\, \bar{A}_5\,] \left(\frac{\Omega^2 - \Delta^2}{2\,\Omega}\right)$$

$$+ \frac{2\bar{M}\,\Omega\varkappa_o - 2\,\Delta P.k}{2\Omega} \; (\bar{A}_3 + \bar{A}_6)$$

$$\frac{-\bar{O}_1 \; \bar{\mathcal{F}}_6}{\varkappa^2 \; p} = \bar{F}_6 = \bar{A}_3 + \bar{A}_6$$

We now proceed with the angular momentum decomposition of the $\bar{\mathcal{F}}_i$ amplitudes. To facilitate the decomposition we introduce the helicity amplitudes as treated by Jacob and Wick[41]. The virtual photon has helicities $\lambda_\gamma = \pm 1, 0$ and the anti-nucleon and hyperon can take helicities $\lambda_{\bar{N}} = \pm\frac{1}{2}$. and $\lambda_Y = \pm\frac{1}{2}$ respectively. We select only the +1 and 0 helicities for the virtual photon since the case of $\lambda_\gamma = -1$ is completely equivalent to $\lambda_\gamma = +1$ by parity considerations of the resulting helicity amplitudes. The following coordinates for the three-vectors $\underline{p}$, $\underline{\varkappa}$ and $\underline{a}$ are also selected

$$\underline{p} = p\,\underline{e}_3 \qquad \underline{\varkappa} = \varkappa\cos\psi \; \underline{e}_3 - \varkappa\sin\psi \; \underline{e}_1$$

with

$$\underline{a} = \sqrt{\frac{i}{2}}\; (\cos\psi\,\underline{e}_1 + i\underline{e}_2 + \sin\psi\; \underline{e}_3) \text{ for } \lambda_\gamma = +1 \qquad (90)$$

and $\underline{a} = i\,\underline{e}_3$ for $\lambda_\gamma = 0$.

$\lambda_\gamma = +1$

Inserting (90) into (88) the complete amplitude $\bar{\mathcal{J}}^{\bar{\Pi}} = \sum_{i=1}^{6} \bar{F}_i \; \bar{\mathcal{J}}^{\bar{\Pi}}_i$ takes the form

$$\bar{\mathcal{J}}^{\bar{\Pi}}(\lambda_\gamma = +1) = -\frac{1}{\sqrt{2}} \begin{pmatrix} (\bar{\mathcal{J}}_1 - \bar{\mathcal{J}}_3 - \bar{\mathcal{J}}_4)\sin\psi & -(\bar{\mathcal{J}}_2 + \bar{\mathcal{J}}_4)(1+\cos\psi) \\ -(\bar{\mathcal{J}}_2 - \bar{\mathcal{J}}_4)(1-\cos\psi) & (\bar{\mathcal{J}}_1 + \bar{\mathcal{J}}_3 + \bar{\mathcal{J}}_4)\sin\psi \end{pmatrix} \tag{91}$$

which can be equated to the Pauli spinor form of the helicity amplitudes, namely

$$\bar{\mathcal{J}}^{\bar{\Pi}}(\lambda_\gamma) = \begin{pmatrix} -f^{\lambda_\gamma}_{\frac{1}{2}\frac{1}{2}} & f^{\lambda_\gamma}_{\frac{1}{2}-\frac{1}{2}} \\ -f^{\lambda_\gamma}_{-\frac{1}{2}\frac{1}{2}} & f^{\lambda_\gamma}_{-\frac{1}{2}-\frac{1}{2}} \end{pmatrix} \tag{92}$$

Thus

$$-\sqrt{2}\;\bar{\mathcal{J}}_1 = \frac{f^1_{-\frac{1}{2}-\frac{1}{2}} - f^1_{\frac{1}{2}\frac{1}{2}}}{\sin\psi}$$

$$\sqrt{2}\,\bar{\mathcal{J}}_2 = \frac{f^1_{\frac{1}{2}-\frac{1}{2}}}{1+\cos\psi} - \frac{f^1_{-\frac{1}{2}\frac{1}{2}}}{1-\cos\psi}$$

$$\sqrt{2}\,\bar{\mathcal{J}}_3 = -\sqrt{2}\,\bar{\mathcal{J}}_4 - \frac{f^1_{\frac{1}{2}\frac{1}{2}} + f^1_{-\frac{1}{2}-\frac{1}{2}}}{\sin\psi} \tag{93}$$

$$\sqrt{2}\,f_4 = \frac{f^1_{\frac{1}{2}\,-\frac{1}{2}}}{1+\cos\psi} + \frac{f^1_{-\frac{1}{2}\,\frac{1}{2}}}{1-\cos\psi}$$

The general expansion of the helicity amplitudes is given by

$$f_{\lambda_c\lambda_d,\,\lambda_a\lambda_b}(\vartheta,\phi) = \frac{1}{\sqrt{p\chi}} \sum_J (J+\tfrac{1}{2}) \left\langle \lambda_c\lambda_d \left| T^J(\Omega) \right| \lambda_a\lambda_b \right\rangle \cdot D^{J\,*}_{\lambda\mu}(\phi,\vartheta,-\phi)$$

for the process $a+b \to c+d$ with $\lambda = \lambda_a - \lambda_b$ and $\mu = \lambda_c - \lambda_d$. Hence in our case we have

$$\begin{aligned}
f^1_{\frac{1}{2}\frac{1}{2}} &= \frac{1}{\sqrt{p\chi}} \sum_J (J+\tfrac{1}{2})\; T^{++1}_J\; d^J_{10}(\psi) \\
f^1_{\frac{1}{2}-\frac{1}{2}} &= \frac{1}{\sqrt{p\chi}} \sum_J (J+\tfrac{1}{2})\; T^{+-1}_J\; d^J_{11}(\psi) \\
f^1_{-\frac{1}{2}\frac{1}{2}} &= \frac{1}{\sqrt{p\chi}} \sum_J (J+\tfrac{1}{2})\; T^{-+1}_J\; d^J_{1-1}(\psi) \\
f^1_{-\frac{1}{2}-\frac{1}{2}} &= \frac{1}{\sqrt{p\chi}} \sum_J (J+\tfrac{1}{2})\; T^{--1}_J\; d^J_{10}(\psi)
\end{aligned} \qquad (94)$$

where the energy dependent amplitudes $T^{\pm\,\pm 1}_J$ are the T-matrix elements for the transitions initiated by a photon of helicity +1 producing a hyperon of helicity $\pm\frac{1}{2}$ and an anti-nucleon of helicity $\pm\frac{1}{2}$ with total angular momentum J. The first superscript on T_J represents the hyperon helicity and the second that of the antinucleon. We can now use the explicit forms for the

$d^J_{\lambda\mu}(\psi)$ appearing above, namely

$$d^J_{10}(\psi) = \frac{1}{\sqrt{J\,(J+1)}} \sin\psi\; P'_J(\psi) \tag{95}$$

$$\frac{d^J_{11}(\psi)}{1+\cos\psi} = \frac{P_{J-1}}{2} - \frac{(1-\cos\psi)}{J} P'_{J-1} + \frac{(1-\cos\psi)^2}{2J\,(J+1)} P''_{J-1}$$

$$\frac{d^J_{1-1}(\psi)}{1-\cos\psi} = \frac{P_{J-1}}{2} + \frac{(1+\cos\psi)}{J} P'_{J-1} + \frac{(1+\cos\psi)^2}{2J(J+1)} P''_{J-1}$$

Inserting these oxpressions into (94) and using (93) we have

$$\begin{aligned}
\mathcal{J}_1 &= -\sum_J (J+\tfrac{1}{2})\,\beta^-_J\; P'_J \\
\mathcal{J}_2 &= \sum_J \left[\tfrac{1}{2}\,\alpha^-_J\,(JP''_{J+1}+(J+1)P''_{J-1}) - (J+\tfrac{1}{2})\,\alpha^+_J\,P''_J\right] \\
\mathcal{J}_3 &= -\sum_J \left[\tfrac{1}{2}\,\alpha^+_J\,(JP''_{J+1}+(J+1)P''_{J-1}) - (J+\tfrac{1}{2})(\alpha^-_J\,P''_J + \beta^+_J P'_J)\right] \\
\mathcal{J}_4 &= \sum_J \left[\tfrac{1}{2}\,\alpha^+_J\,(JP''_{J+1}+(J+1)P''_{J-1}) - (J+\tfrac{1}{2})\,\alpha^-_J\,P''_J\right]
\end{aligned} \tag{96}$$

where

$$\begin{aligned}
\alpha^{\pm}_J &= \frac{T^{+-1}_J \pm T^{-+1}_J}{J(J+1)\sqrt{2\,p\,\chi}} \\
\beta^{\pm}_J &= \frac{T^{++1}_J \pm T^{--1}_J}{\sqrt{J(J+1)}\sqrt{2p\chi}}
\end{aligned} \tag{97}$$

$\lambda_\gamma = 0$

We can immediately write down formulae analagous to (91), (93) and (94). They are

$$\bar{\mathcal{J}}(\lambda_\gamma = 0) = \begin{pmatrix} \bar{\mathcal{J}}_1 + \cos\psi\,\bar{\mathcal{J}}_5 & -\sin\psi\,\bar{\mathcal{J}}_4 + \cos\psi\sin\psi\,\bar{\mathcal{J}}_6 \\ \sin\psi\,\bar{\mathcal{J}}_4 - \cos\psi\sin\psi\,\bar{\mathcal{J}}_6 & \bar{\mathcal{J}}_1 + \cos\psi\,\bar{\mathcal{J}}_5 \end{pmatrix} \qquad (98)$$

$$\bar{\mathcal{J}}_1 + \cos\psi\,\bar{\mathcal{J}}_5 = f^{o}_{\frac{11}{22}}$$

$$\sin\psi\,(\bar{\mathcal{J}}_4 - \cos\psi\,\bar{\mathcal{J}}_6) = f^{o}_{-\frac{11}{22}} \qquad (99)$$

and

$$f^{o}_{\frac{11}{22}} = \frac{1}{\sqrt{p\chi}} \sum_J (J+\tfrac{1}{2})\, T^{++o}_J \; d^J_{oo}(\psi) \qquad (100)$$

$$f^{o}_{-\frac{11}{22}} = \frac{1}{\sqrt{p\chi}} \sum_J (J+\tfrac{1}{2})\, T^{-+o}_J \; d^J_{o-1}(\psi)$$

But we also have

$$d^J_{oo} = P_J(\psi)$$

and (101)

$$d^J_{,-1} = d^J_{1o}$$

so that we may write

$$\cos\psi\,\bar{\mathcal{J}}_5 = -\bar{\mathcal{J}}_1 + \Sigma\,(J+\tfrac{1}{2})\,\gamma^{(1)}_J\,P_J(\psi)$$

$$\cos\psi\,\bar{\mathcal{J}}_6 = \bar{\mathcal{J}}_4 + \Sigma\,(J+\tfrac{1}{2})\,\gamma^{(2)}_J\,P'_J(\psi) \qquad (102)$$

where

$$\gamma_J^{(1)} = \frac{T_J^{++o}}{\sqrt{p\chi}}$$

$$\gamma_J^{(2)} = \frac{T_J^{-+o}}{\sqrt{J(J+1)}\sqrt{p\chi}} \qquad (103)$$

We now analyse the amplitudes $\alpha^{\pm}$, $\beta^{\pm}$ and $\gamma^{(1,2)}$. The parity operator $\mathcal{P}$ on a state $|J\ M\ \lambda_a\ \lambda_b\rangle$ gives $\eta_a\,\eta_b(-1)^{J-s_a-s_b}\,|J\ M\ -\lambda_a\ -\lambda_b\rangle$ and so

$\frac{1}{\sqrt{2}}\left(|+\pm\rangle + |-\ \mp\rangle\right)$ have parity $(-1)^J$

$\frac{1}{\sqrt{2}}\left(|+\pm\rangle - |-\ \mp\rangle\right)$ have parity $(-1)^{J+1}$

and

$\frac{1}{\sqrt{2}}\left(|1o\rangle \pm |-1\ o\rangle\right)$ have parities $(-1)^J$ and $(-1)^{J+1}$ respectively.

This means we can form the following amplitudes under parity conservation

$$a_J^{\pm} = \frac{1}{\sqrt{2}}\left[\langle +-|\pm\langle -+|\right] T^J \frac{1}{\sqrt{2}}\left[|1\ o\rangle \pm |-1\ o\rangle\right] \qquad (104)$$

$$b_J^{\pm} = \frac{1}{\sqrt{2}}\left[\langle ++|\pm\langle --|\right] T^J \frac{1}{\sqrt{2}}\left[|1\ o\rangle \pm |-1\ \ o\rangle\right]$$

and also the equalities

$$\langle \pm\pm|T^J|1\ o\rangle = \langle \mp\mp|T^J|-1\ o\rangle$$

$$\langle \pm\mp|T^J|1\ o\rangle = \langle \mp\pm|T^J|-1\ o\rangle \qquad (105)$$

Hence on rearranging (104) we have

$$a_J^{\pm} = T_J^{+-1} \pm T_J^{-+1}$$
$$b_J^{\pm} = T_J^{++1} \pm T_J^{--1} \qquad (106)$$

where now the amplitudes a^+, b^+ represent transitions to final hyperon-antinucleon states of parity $(-1)^J$ and the amplitudes a^-, b^- represent transitions to final hyperon-antinucleon states of parity $(-1)^{J+1}$. The former are therefore magnetic while the latter are electric. The only further questions is that of determining the total spin of these four hyperon-antinucleon states.

It is a well known fact that for states labelled by the third component of spin $|++\rangle$, $\frac{1}{\sqrt{2}}\left[|+-\rangle + |-+\rangle\right]$ and $|--\rangle$ are triplet states (spin one) and $\frac{1}{\sqrt{2}}\left[|+-\rangle - |-+\rangle\right]$ is the singlet state (spin zero). With our labelling of the helicity states $|Y\bar{N}\rangle$ the $|+-\rangle$, $\frac{1}{\sqrt{2}}\left[|++\rangle + |--\rangle\right]$ and $|-+\rangle$ are triplet states and $\frac{1}{\sqrt{2}}\left[|++\rangle - |--\rangle\right]$ is singlet. Hence we may write

$$a_J^+ = M_{J1} \qquad b_J^+ = \mathcal{M}_{J1} \qquad (107)$$
$$a_J^- = E_{J1} \qquad b_J^- = \mathcal{E}_{J0}$$

where the first and second indices denote the total angular momentum and the total spin of the $Y\bar{N}$ pair in the final state.

For $\lambda_\gamma = 0$ (longitudinal transitions) we have

$$\mathcal{P}\,|00\rangle = (-1)^J\,|00\rangle$$

and hence

$$d_J^{(1,2)} = \sqrt{\tfrac{1}{2}}\left[\langle + \pm | + \langle - \mp |\right] T^J \quad | \, oo \rangle$$

with parity $(-1)^J$ in the final (triplet) state. Summarizing we have

$$\alpha_J^+ = \frac{T_J^{+-1} + T_J^{-+1}}{J(J+1)\sqrt{2p\varkappa}} = M_{J1} \quad \text{triplet} \quad (-1)^J$$

$$\beta_J^+ = \frac{T_J^{++1} + T_J^{--1}}{\sqrt{J(J+1)}\sqrt{2p\varkappa}} = \mathfrak{M}_{J1} \quad \text{triplet} \quad (-1)^J$$

$$\alpha_J^- = \frac{T_J^{+-1} - T_J^{-+1}}{J(J+1)\sqrt{2p\varkappa}} = E_{J1} \quad \text{triplet} \quad (-1)^{J+1} \qquad (108)$$

$$\beta_J^- = \frac{T_J^{++1} - T_J^{--1}}{\sqrt{J(J+1)}\sqrt{2p\varkappa}} = \mathfrak{E}_{Jo} \quad \text{singlet} \quad (-1)^{J+1}$$

$$\gamma_J^1 = \frac{T_J^{++o}}{\sqrt{p\varkappa}} = L_{J1} \quad \text{triplet} \quad (-1)^J$$

$$\gamma_J^2 = \frac{T_J^{-+o}}{\sqrt{J(J+1)}\sqrt{p\varkappa}} = \mathfrak{L}_{J1} \quad \text{triplet} \quad (-1)^J$$

vi THE K^* CONTRIBUTION.

The K^* resonance in the $K-\pi$ system at 880 Mev and $I=\frac{1}{2}$ [42] can contribute to the t-channel through Figure III. The spin of the K^* is now established to be unity [43] and its parity -1 [44]. If the spin of the K^* is zero, then it will not contribute to this reaction since zero spin (s-wave $K\pi$ resonance) would imply that the relative KK^* parity

is odd and by parity conservation the KK^* vertex could not exist.

We introduce the K^* by a pole at $t = M_*^2$ and since the K^* has isotopic spin $\frac{1}{2}$ it will contribute only to the $I=\frac{1}{2}$ component of the scattering amplitude. By Lorentz and gauge invariance the γKK^* vertex can be written as

$$\Lambda(\lambda^2)\, \varepsilon_{\mu\nu\pi\rho} \quad e_\nu \; k_\pi \; q_\rho \tag{109}$$

and the NYK^* vertex can be shown to be

$$\bar{u}(-p_2) i \left[g_{K^*} \gamma_\mu + G_{K^*} \sigma_{\mu\nu} (-p_1-p_2)_\nu \right] v(-p_1) \tag{110}$$

The vector boson propagator is given by

$$\frac{g_{\mu\nu} - \dfrac{Q_\mu Q_\nu}{M_*^2}}{M_*^2 - Q^2} \tag{111}$$

Thus the contribution T_{K^*} to the scattering matrix is given by

$$T_{K^*} = \delta(k+q-Q)\, \bar{u}(-p_2)\, \Lambda \varepsilon_{\mu\nu\pi\rho} \quad e_\nu k_\pi q_\rho$$

$$\frac{g_{\mu\sigma} - Q_\mu Q_\sigma / M_*^2}{M_*^2 - Q^2} \tag{112}$$

$$i \left[g_* \gamma_\sigma + G_* \sigma_{\sigma\varkappa} (-p_1-p_2)_\varkappa \right] v(-p_2)$$

Because of the complete antisymmetry of the $\varepsilon_{\mu\nu\pi\rho}$ and the fact that $Q = k+q$ the second term in the numerator of the propagator drops out. After a somewhat lengthy computation the result is

$$T_K^* = \frac{\Lambda}{M_*^2 - t} \bar{u}(-p_2) \Big[g_* (\Delta M_1 - M_4) + G \Big(tM_1 + \frac{(q.k + k^2)}{2\, q.k + k^2} M_2 - \Delta M_3 - \frac{2\, P.k}{2q.k + k^2} M_5 + \Delta M_6 \Big) \Big] v(-p_1) \qquad (113)$$

If we now introduce a factor

$$A_i^{K^*}(s,t) = \frac{\Gamma_i (K^*)}{M_*^2 - t} \qquad (114)$$

in the equation (48) then the residues of the K^* pole will be given by

$$\Gamma_1(K^*) = \Lambda \Big[g_* \Delta + G_* t \Big]$$

$$\Gamma_2(K^*) = \Lambda \Big[\frac{2\,(q.k + k^2)}{2\ q.k + k^2} G_* \Big]$$

$$\Gamma_3(K^*) = \Lambda \Big[-G_* \Delta \Big] \qquad \Gamma_4(K^*) = \Lambda \Big[-g_* \Big] \qquad (115)$$

$$\Gamma_5(K^*) = \Lambda \Big[\frac{-2\, P.k}{2q.k + k^2} G_* \Big]$$

$$\Gamma_6(K^*) = \Lambda \Big[\Delta\ G_* \Big]$$

The isotopic dependence of the form factor $\Lambda(\lambda^2)$ is given by

$$\left.\begin{aligned} \Lambda^{v+} &= \lambda_v \\ \Lambda^{v-} &= -\lambda_v \\ \Lambda^{v_o} &= \lambda_s \end{aligned}\right\} \quad \text{for K production} \tag{116}$$

and

$$\left.\begin{aligned} \Lambda^{s+} &= \lambda_v \\ \Lambda^{s_o} &= \lambda_s \end{aligned}\right\} \quad \text{for K production}$$

where we have used $\Lambda = \lambda_s + \tau_3 \lambda_v$ as the coupling constant for the γKK^* vertex.

vii EVALUATION OF $F_i(B.A)$ and $F_i(K^*)$

The task of evaluating the $F_i(B.A)$ and the $F_i(K^*)$ is quite simple. The $F_i(B.A)$ can be obtained by inserting the relations (68) into equation (72) with the aid of (48). The results are as follows (the subscript Y being omitted from g)

$$F_1(B.A.) = \frac{gG_1}{M_1^2 - s} + \frac{gG_1'}{M_2^2 - u} - \frac{(W-M_1)gG_2}{M_1^2 - s} + gG_2' \left(\frac{2M_2}{M_2^2 - u} - \frac{1}{W - M_1} \right)$$

$$F_2(B.A) = \frac{gG_1}{M_1^2 - s} + \frac{gG_1'}{M_2^2 - u} + \frac{(W+M_1)gG_2}{M_1^2 - s} + gG_2' \left(\frac{2M_2}{M_2^2 - u} - \frac{1}{W + M_1} \right)$$

$$F_3(B.A) = \frac{-2gG_1'}{(W+M_1)(M_2^2-u)} - \frac{2gG_2'}{M_2^2 - u} - \frac{2g\mathcal{J}_k}{(W+M_1)(m^2-t)}$$

$$F_4(B.A) = \frac{2gG_1'}{(W-M_1)(M_2^2-u)} - \frac{2gG_2'}{M_2^2 - u} + \frac{2g\mathcal{J}_k}{(W-M_1)(m^2-t)}$$

$$F_5(B.A) = \frac{gG_1}{\lambda^2} - \frac{gG_1'}{\lambda^2} + \frac{(W-M_1)\, gG_2}{M_1^2 - s} \qquad (117)$$

$$+ \frac{(W+M_1-2M_2)\, gG_2'}{M_2^2 - u} - \frac{2q.k\; g\mathcal{J}_k}{2\,(m^2 - t)}$$

$$F_6(B.A) = -gG_1\left(\frac{1}{\lambda^2} + \frac{W + M_1}{(\varepsilon_1+M_1)(M_1^2-s)}\right) + gG_1'\left(\frac{1}{\lambda^2} - \frac{W+M_1}{(\varepsilon_1+M_1)(M_2^2-u)}\right)$$

$$-gG_2\,\frac{k_o}{\varepsilon_1+M_1}\,\frac{W + M_1}{M_1^2 - s} - gG_2'\,\frac{(k_o(W+2M_2-M_1) + 2q.k)}{(\varepsilon_1+M_1)(M_2^2 - u)}$$

$$+ \frac{2q.k\; g\mathcal{J}_k}{\lambda^2\,(m^2 - t)}$$

In the same way the $F_i(K^*)$ can be evaluated by inserting the relations (115) and (72) with the aid of (114) and (48). The results are

$$F_1(K^*) = \frac{\Lambda}{M_*^2 - t}\left[-g\,(W-M_2 + \frac{q.k}{W-M_1}) + G\,(t - \frac{(q.k+k^2)\,\Delta}{W - M_1})\right]$$

$$F_2(K^*) = \frac{\Lambda}{M_*^2 - t}\left[g\ (W+M_2 + \frac{q.k}{W+M_1}) + G\ \left(t - \frac{(q.k + k^2)\,\Delta}{W + M_1}\right)\right]$$

$$F_3(K^*) = \frac{\Lambda}{M_*^2 - t}\left[-g + G\ (W - M_2 + \frac{\lambda^2}{W+M_1})\right]$$

$$F_4(K^*) = \frac{\Lambda}{M_*^2 - t}\left[-g - G\ (W+M_2 + \frac{\lambda^2}{W - M_1})\right] \qquad (118)$$

$$F_5(K^*) = \frac{\Lambda}{M_*^2 - t}\left[(W-M_2)g + G\ \left((W+M_1)(W+M_2) - m^2.\right)\right]$$

$$F_6(K^*) = \frac{\Lambda}{M_*^2 - t}\left[-g\ \frac{(W+M_2)k_o + q.k}{\varepsilon_1 + M_1} + G(u - M_2^2 + \lambda^2)\right.$$

$$\left. -G\ \frac{\left(k_o t + \Delta\,(q.k + k_o(W+M_1))\right)}{\varepsilon_1 + M_1}\right]$$

(45)

It has been shown that the ratio g/G is equal to the ratio $g_v/d(o)$ where g_v and $d(o)$ are the vector and induced vector coupling constants for the leptonic decay of the hyperon. $d(o)$ is usually neglected in comparison with g_v and so in what follows we set G equal to zero.

viii SOLUTION OF THE DISPERSION INTEGRALS.

We employ the method of Blankenbecler and Gartenhaus[12] to solve the one-dimensional dispersion relations in the energy region in which the $P_{\frac{1}{2}}$ resonance or the $F_{5/2}$ resonance is dominant. This is the method that has been applied to the electroproduction of pions by Blankenbecler, Garthenhaus, Huff and Nambu[11] and to the weak production of pions by Dombey[20] in the energy region where the first pion-nucleon resonance ($P_{3/2}$ resonance) is dominant. This approach is quite general and can be applied here with only minor changes.

It is assumed that the resonance- the $P_{\frac{1}{2}}$ or the $F_{5/2}$ - in the s-channel dominates the one dimensional dispersion integrals so that the results are confined to those experiments in which the barycentric energies in the s-channel (the barycentric energies of the final KY state) are near the resonance energy. No assumptions concerning the effects of recoil are made nor does the method employ a partial wave expansion. In the case of photoproduction at low energies an expansion of the amplitudes in inverse powers of the nucleon mass may be presumed to converge rapidly since the only terms that occur of order M^{-1} are M^{-1} and $(W-M)\,M^{-1}$, which are small. In the case of electroproduction there is an additional quantity available, namely the square of the invariant momentum transfer given up by the electron, λ^2, and terms like $\lambda^2(W-M)^{-1}\,M^{-1}$ also appear. Such terms are not small since wide angle scattering of the electron can produce the resonance energy in the s-channel even with $\lambda^2 \simeq M^2$. Thus

the recoil expansion is inconvenient and the method assumes no such expansion.

We assume that the amplitudes in the dispersion relations have a certain known phase and that the contributions from the u-channel can be neglected in the spirit of the K-meson photoproduction analysis. Thus from (48) we may write

$$A_i(s,t) = A_i^{B.A}(s,t) + A_i^{K^*}(s,t) + \frac{1}{\pi}\int_{s_o}^{\infty} \frac{\mathrm{Im}\, A_i(s',t)\, ds'}{s' - s} \qquad (119)$$

where $s_o = (M_1 + \mu)^2$.

By a slight extension of the method of Omnes[46] the following formal solution of (119) may be written

$$A_i(s,t) = B_i(s,t) + \frac{\exp(\rho + i\delta)}{\pi}\int_{s_o}^{\infty} \frac{e^{-\rho(s')} \sin\delta(s')\, B_i(s',t)}{s' - s}\, ds'$$

$$\text{where} \quad \rho(s) = \frac{1}{\pi} P \int_{s_o}^{\infty} \frac{\delta(s')\, ds'}{s' - s} \qquad (120)$$

and

$$B_i(s,t) = A_i(B.A) + A_i(K^*)$$

A first approximation of (120) can be written as

$$A_i(s,t) = B_i(s,t) + \frac{\exp(\rho + i\delta)}{\pi}\int_{s_o}^{\infty} \frac{e^{-\rho(s')} \sin\delta(s')\, B_{iR}(s',t)\, ds'}{s' - s} \qquad (121)$$

where $B_{iR}(s,t)$ are the resonance projections of the Born approximation $B_i(s,t)$. Further, for s in the resonance region we may expand the exponential terms as

$$e^{\rho(s) - \rho(s')} = 1 + a\,(s' - s) + \cdots\cdots$$

Under the integral the second term will be small compared to the unity so that near the resonance we have

$$A_i(s,t) = B_i(s,t) + \frac{\exp i\delta(s)}{\pi}\, P \int_{s_o}^{\infty} \frac{\sin\delta(s')\, B_{iR}(s',t)\, ds'}{s' - s} \tag{122}$$

The principal value integral gives the enhancement to the resonant state and, in general, will give the enhancement for a resonance of width Γ. But in the resonance region we know that a Breit-Wigner formula approximates the dispersion integral hence we expect the form

$$A_i(s,t) = B_i(s,t) + \frac{B_{iR}(s,t)}{1 - s/s_R - i\Gamma} \tag{123}$$

or in the resonance region

$$A_i(s,t) = \frac{B_{iR}(s,t)}{1 - s/s_R - i\Gamma} \tag{124}$$

It is this form that we are interested in and our next task is the determination of the resonance projections of the Born approximation.

We write

$$B_{iR}(s,t) = A_{iR}^{B.A}(s,t) + A_{R}^{K^*}(s,t) \qquad (125)$$

Now from (72) we see that the amplitude $\mathcal{J}_i$ are linearly related to the amplitudes A_i and as it is more convenient for the differential cross section evaluation to work in the barycentric system of channel I the expression of interest will be

$$\mathcal{J}_i = \frac{\mathcal{J}_{iR}}{1 - s/s_R - i\Gamma} \qquad (126)$$

where again $\mathcal{J}_{iR}$ represents the sum of the resonance projections of the amplitudes $\mathcal{J}_i(B.A)$ and $\mathcal{J}_i(K^*)$ given by (117) and (118) respectively.

ix. THE DIFFERENTIAL CROSS SECTION

Recalling the definition of the S- and T-matrices, (25) and (26), and the normalization of the spinors (6) the cross section can be written in invariant form as follows

$$d\sigma = \frac{1}{(2\pi)^5} \frac{1}{32\,F} \delta^{4}(P_i - P_f) \frac{d^3p_2}{p_{20}} \frac{d^3q}{q_o} \frac{d^3r_2}{r_{20}} |m|^2 \qquad (127)$$

where F is the flux given by

$$F^2 = (p_1 \cdot r_1)^2 - m_e^2 M_1^2 \qquad (128)$$

and $|m|^2 = \sum_{\text{spins}} |T|^2$

Since we require the cross section for inelastic electron scattering, that is, only the final electron being observed, we must integrate out the final hyperon and meson momenta. This is easily performed by noting that the quantity

$$|\mathcal{M}|^2 \frac{d^3p_2}{p_{20}} \frac{d^3q}{q_o} \delta^4(p_2+q+k+p_1) \qquad (129)$$

is a Lorentz scalar and may be evaluated in any frame, and in particular in the barycentric frame of the final hyperon and meson. It is therefore identical to

$$|\mathcal{M}|^2 \frac{q^2\, dq}{\varepsilon_2\, q_o} \delta(\varepsilon_2 + q_o - W)\, d\Omega_{\hat{q}}$$

which can be integrated to the form

$$\frac{q}{W}|\mathcal{M}|^2 \, d\Omega \, \hat{q}$$

If we now express the final electron integration in the overall laboratory frame and introduce capital letters for laboratory quantities then the differential cross section for inelastic electron scattering becomes

$$\frac{d^2\sigma}{dR_2\, d\Omega_{\hat{R}_2}} = \frac{1}{(2\pi)^5} \frac{R_2}{32\, M_1 R_1} \langle \mathcal{M}^2 \rangle \qquad (130)$$

where the electron mass has been neglected where convenient and the quantity $\langle \mathcal{M}^2 \rangle$ is defined by

$$\langle \mathcal{M}^2 \rangle = \int \frac{q}{W} |\mathcal{M}|^2 \, d\Omega \quad \hat{q} \tag{131}$$

It is this quantity that has to be determined and for the first step in its evaluation we recall equations (69), (70) and (71). If we write

$$\mathcal{J} = \mathcal{J}_\alpha \epsilon_\alpha \qquad (\alpha = 1,2,3) \tag{132}$$

with

$$\begin{aligned} \mathcal{J}_\alpha = {} & i\sigma_\alpha \mathcal{J}_1 + i \, \underline{\sigma}\cdot\hat{\underline{q}} \, \hat{k}_\alpha \left(\mathcal{J}_2 + \mathcal{J}_6 - \frac{k^2}{k_o^2}\left(x\mathcal{J}_4 + \mathcal{J}_6\right)\right) \\ & + i \, \underline{\sigma}\cdot\hat{\underline{k}} \, \hat{q}_\alpha \mathcal{J}_3 + i \, \underline{\sigma}\cdot\hat{\underline{q}} \, \hat{q}_\alpha \mathcal{J}_4 \\ & - i \, \underline{\sigma}\cdot\hat{\underline{q}} \, \underline{\sigma}\cdot\hat{\underline{k}} \sigma_\alpha \mathcal{J}_2 + i \, \underline{\sigma}\cdot\hat{\underline{k}} \, \hat{k}_\alpha \left(\mathcal{J}_5 - \frac{k^2}{k_o^2}\left(\mathcal{J}_1 + x\mathcal{J}_3 + \mathcal{J}_5\right)\right) \end{aligned} \tag{133}$$

and

$$\epsilon_\mu = \frac{e \, \bar{u}(-r_2) \, \gamma_\mu \, u(r_1)}{k^2}$$

then

$$|\mathcal{M}|^2 = T_{\alpha\beta} \epsilon_{\alpha\beta} \tag{134}$$

where

$$T_{\alpha\beta} = \tfrac{1}{2} \text{ Trace } (\mathcal{J}_\alpha \mathcal{J}_\beta^\dagger)$$

and

$$\epsilon_{\alpha\beta} = \tfrac{1}{2} \text{ Trace } (\epsilon_\alpha \epsilon_\beta^\dagger) \tag{135}$$

The electron contribution $\epsilon_{\alpha\beta}$ may be easily computed to give

$$\epsilon_{\alpha\beta} = \frac{2e^2}{\lambda^4}\left[2r_{1\alpha}\,r_{1\beta} - k_\alpha\,r_{1\beta} - r_{1\alpha}k_\beta + \tfrac{1}{2}\lambda^2\delta_{\alpha\beta}\right] \tag{136}$$

and the tensor $T_{\alpha\beta}$ can be written in a simple form as

$$T_{\alpha\beta} = \sum_{i=1}^{4} T_i \; L^i_{\alpha\beta} \tag{137}$$

with

$$L^1_{\alpha\beta} = \delta_{\alpha\beta} \qquad L^2_{\alpha\beta} = \hat{k}_\alpha\hat{k}_\beta$$

$$L^3_{\alpha\beta} = \hat{q}_\alpha\hat{q}_\beta \qquad L^4_{\alpha\beta} = \hat{q}_\alpha\hat{k}_\beta$$

It should be noted that since $\epsilon_{\alpha\beta}$ is symmetric the tensor $T_{\alpha\beta}$ should also be symmetric. This means that $L^4_{\alpha\beta}$ should really be symmetrized. However, when the contraction (134) is performed the resulting expression will be correct. The saturation of the tensor $T_{\alpha\beta}$ with the tensor $\epsilon_{\alpha\beta}$ gives

$$\begin{aligned}|\mathcal{M}|^2 = \frac{2e^2}{\lambda^4}\Bigg[& \left(2\,\underline{r}_1^2 - 2\underline{k}\cdot\underline{r}_1 + \tfrac{3}{2}\lambda^2\right) T_1 \\ & + \frac{1}{k^2}\left(2\,\underline{r}_1\cdot\underline{k}\;\underline{r}_1\cdot\underline{k} - 2\underline{k}^2\,\underline{r}_1\cdot\underline{k} + \tfrac{1}{2}\lambda^2\underline{k}^2\right) T_2 \\ & + \frac{1}{q^2}\left(2\,\underline{r}_1\cdot\underline{q} - 2\underline{q}\cdot\underline{k}\;\underline{r}_1\cdot\underline{q} + \tfrac{1}{2}\lambda^2\,\underline{q}^2\right) T_3 \\ & + \frac{1}{qk}\left(2\underline{r}_1\cdot\underline{k}\;\underline{r}_1\cdot\underline{q} - \underline{q}\cdot\underline{k}\;\underline{r}_1\cdot\underline{k} - \underline{k}^2\,\underline{r}_1\cdot\underline{q} + \tfrac{1}{2}\lambda^2\,\underline{q}\cdot\underline{k}\right) T_4\Bigg]\end{aligned} \tag{138}$$

This means that the differential cross section takes the form

$$\frac{d^2\sigma}{dR_2\, d\Omega} = \frac{e^2}{(2\pi)^5}\,\frac{R_2}{16M_1R_1}\,\frac{q}{W\lambda^4}\int\left[\begin{array}{l}\text{Quantity in square}\\ \text{brackets in (138)}\end{array}\right] d\Omega_{\hat{q}} \qquad (139)$$

and the integration implies that the unit vector $\hat{\underline{q}}$ must be taken over the whole of its three dimensional space. In order to perform this integration we take $\hat{\underline{k}}$ fixed and the angles α, ϑ, Ψ and Φ are defined by the configuration of Figure IV. The relevant formulae for the integration implied in (139) are

$$d\Omega_{\hat{q}} = \sin\vartheta\, d\vartheta\, d\Phi$$

and

$$\cos\alpha = \cos\vartheta\cos\Psi + \sin\vartheta\,\sin\psi\cos\Phi$$

The $d\Phi$ integration can now be performed to give

$$\int\left[(138)\right] d(\cos\vartheta)\, d\Phi = 4\pi\int_{-1}^{+1} F(\cos\vartheta)\, d(\cos\vartheta) \qquad (140)$$

where

$$F(\cos\vartheta) = T_1(\rho_1+\rho_2) + T_2\,\rho_1 + T_3(\cos^2\vartheta\rho_1 + \tfrac{1}{2}\sin^2\vartheta\rho_2) + T_4\,\rho_1\cos\vartheta$$

and

$$\rho_1 = r_1^2\cos^2\psi - r_1 k\cos\psi + \tfrac{1}{4}\lambda^2 \qquad (141)$$
$$\rho_2 = r_1^2\sin^2\psi + \tfrac{1}{2}\lambda^2$$

We have yet to define the four quantities T_i (i=1,2,3,4) and as they do not occur explicitly in the final form of $F(\cos\vartheta)$ we omit to give them. Instead we write

$$F(x) = A(\xi\rho_1+\rho_2) + B(\xi\rho_1) + C(\xi\rho_1 x^2 + \tfrac{1}{2}(1-x^2)\rho_2) + D(x\xi\rho_1) \tag{142}$$

with

$$A = |\mathcal{F}_1|^2 + |\mathcal{F}_2|^2 - 2x\,\mathrm{Re}\,(\mathcal{F}_1\mathcal{F}_2^*)$$

$$B = -|\mathcal{F}_2|^2 + |\mathcal{F}_5|^2 + |\mathcal{F}_6|^2 + 2\mathrm{Re}\,(\mathcal{F}_1\mathcal{F}_5^* - x\mathcal{F}_2\mathcal{F}_5^* + x\mathcal{F}_5\mathcal{F}_6^*) \tag{143}$$

$$C = |\mathcal{F}_3|^2 + |\mathcal{F}_4|^2 + 2\mathrm{Re}\,(\mathcal{F}_1\mathcal{F}_4^* + \mathcal{F}_2\mathcal{F}_3^* + x\mathcal{F}_3\mathcal{F}_4^*)$$

$$C = 2\mathrm{Re}\big[\mathcal{F}_1\mathcal{F}_2^* + \mathcal{F}_1\mathcal{F}_3^* + \mathcal{F}_1\mathcal{F}_6^* + \mathcal{F}_2\mathcal{F}_5^* + \mathcal{F}_3\mathcal{F}_5^* + \mathcal{F}_4\mathcal{F}_6^* - (\mathcal{F}_2\mathcal{F}_3^* - \mathcal{F}_3\mathcal{F}_6^* - \mathcal{F}_4\mathcal{F}_5^*)\big]$$

and

$$\xi = \frac{\lambda^4}{k_o^4}$$

Since we are only interested in a $P_{\frac{1}{2}}$ resonance and an $F_{5/2}$ resonance in the s-channel we keep only the relevant multipoles as given in (78) and insert them into the functions A,B,C and D of (143). The final

integration over cosϑ can then be performed. Thus for the two cases we have

$P_{\frac{1}{2}}$ RESONANCE.

$$\mathcal{J}_1 = \mathcal{J}_3 = \mathcal{J}_4 = \mathcal{J}_5 = 0$$

and $\quad \mathcal{J}_2 = M_{1-} \qquad \mathcal{J}_6 = L_{1-}$

Thus $\quad F(x) = \rho_2 M_{1-}^2 + \xi\rho_1 L_{1-}^2$

and
$$\int_{-1}^{+1} F(x)\,dx = 2\left[\rho_2 (M_{1-})^2 + \xi\,\rho_1 (L_{1-})^2\right] \qquad (144)$$

$F_{5/2}$ RESONANCE.

$$\begin{pmatrix} \mathcal{J}_1 \\ \mathcal{J}_2 \\ \mathcal{J}_3 \\ \mathcal{J}_4 \\ \mathcal{J}_5 \\ \mathcal{J}_6 \end{pmatrix} = \begin{pmatrix} 12x & 3x & 0 \\ \frac{9}{2}(5x^2-1) & 0 & 0 \\ 3 & 3 & 0 \\ -15x & -15x & 0 \\ -15_x & -6x & -9x \\ 15x^2 & 15x^2 & \frac{9}{2}(5x^2-1) \end{pmatrix} \begin{pmatrix} M_{3-} \\ E_{3-} \\ L_{3-} \end{pmatrix} \qquad (145)$$

Inserting these valuos in (142) and (143) we have

$$F(x) = \rho_2 \left(\frac{45}{4} + \frac{54}{4}\, x^2 + \frac{45}{4}\, x^4\right) (M_{3-})^2$$

$$+ \rho_2 \left(\frac{9}{2} + \frac{-54}{2}\, x^2 - \frac{45}{2}\, x^4\right) (E_{3-})^2$$

$$+ \xi\rho_1 \left(\frac{9}{2}\right)^2 (1 - 2x^2 + 5x^4)\ (L_{3-})^2$$

$$+ \rho_2 \left(-\frac{9}{2} + 81x^2 - \frac{15^2}{2}\, x^4\right) (M_{3-}E_{3-})$$

and so

$$\int F(x)\, dx = 18\left[\rho_2 \left(2M_{3-}^2 + E_{3-}^2\right) + 3\xi\rho_1\, L_{3-}^2\right] \qquad (146)$$

x. NUMERICAL ANALYSIS

In order to use the quantities given by equations (144) and (146) to compute the differential cross section we must project out of the Born terms and K^* terms, given by equations (117) and (118) respectively, the relevant multipoles. These are M_{1-} and L_{1-} for the $P_{\frac{1}{2}}$ resonance and M_{3-}, E_{3-} and L_{3-} for the $F_{5/2}$ resonance. We treat the two cases simultaneously and from the relations (80) we have

$$M_{1-} = \tfrac{1}{2} \int_{-1}^{+1} dx \left[-\mathcal{J}_1 P_1(x) + \mathcal{J}_2 P_0(x) + \tfrac{1}{3}\mathcal{J}_3 (P_0(x) - P_2(x))\right] \qquad (147)$$

$$L_{1-} = \tfrac{1}{2} \int_{-1}^{+1} dx \left[(\mathcal{J}_1 + x\mathcal{J}_3 + \mathcal{J}_5)\, P_1(x) + (x\mathcal{J}_4 + \mathcal{J}_6)\, P_0(x)\right]$$

and

$$M_{3-} = \frac{1}{6}\int_{-1}^{+1} dx\left[-\mathcal{J}_1 P_3(x) + \mathcal{J}_2 P_2(x) + \frac{1}{7}\mathcal{J}_3(P_2(x) - P_4(x))\right]$$

$$E_{3-} = \frac{1}{6}\int_{-1}^{+1} dx\left[\mathcal{J}_1 P_3(x) - \mathcal{J}_2 P_2(x) - \frac{4}{7}\mathcal{J}_3(P_2-P_4) - \frac{3}{5}\mathcal{J}_4(P_1-P_3)\right] \qquad (148)$$

$$L_{3-} = \frac{1}{6}\int_{-1}^{+1} dx\left[(\mathcal{J}_1+x\mathcal{J}_3+\mathcal{J}_5)\,P_3(x) + (x\mathcal{J}_4+\mathcal{J}_6)\,P_2(x)\right]$$

It can be seen from equations (117) and (118) that, with the aid of (15), the denominators in the expression for $F_i(B.A)$ and $F_i(K^*)$ which are $\cos\vartheta$ dependent can be written as

$$\frac{1}{M_2^2 - u} = \frac{1}{2qk}\,\frac{1}{a+x} \qquad a = \frac{\lambda^2 + 2k_o\,\varepsilon_2}{2qk}$$

$$\frac{1}{m^2 - t} = \frac{1}{2qk}\,\frac{1}{b-x} \qquad b = \frac{\lambda^2 + 2q_o k_o}{2qk} \qquad (149)$$

$$\frac{1}{M_*^2 - t} = \frac{1}{2qk}\,\frac{1}{c-x} \qquad c = b + \frac{M_*^2 - m^2}{2qk}$$

On performing the integrations in (147) and (148) the following functions occur

$$\Psi_n(\alpha) = \int_{-1}^{+1}\frac{P_n(x)dx}{\alpha+x} \qquad \Phi_n(\alpha) = \int_{-1}^{+1}\frac{P_n(x)\,dx}{\alpha-x} \qquad (150)$$

$$\psi_n(\alpha) = \int_{-1}^{+1}\frac{xP_n(x)dx}{\alpha+x} \qquad \phi_n(\alpha) = \int_{-1}^{+1}\frac{xP_n(x)\,dx}{\alpha-x}$$

which may be expressed in terms of the Legendre functions of the second kind, namely

$$Q_n(\alpha) = \tfrac{1}{2} \int_{-1}^{+1} \frac{P_n(x)\, dx}{\alpha - x} \tag{151}$$

Thus

$$\begin{aligned} \Psi_n(\alpha) &= 2(-1)^n Q_n(\alpha) & \Phi_n(\alpha) &= 2Q_n(\alpha) \\ \psi_n(\alpha) &= 2(-1)^{n-1} \alpha Q_n(\alpha) & (n \geq 1) & \\ \phi_n(\alpha) &= 2\alpha Q_n(\alpha) & (n \geq 1) & \\ \psi_o(\alpha) &= 2Q_1(\alpha) & \phi_o(\alpha) &= 2Q_1(\alpha) \end{aligned} \tag{152}$$

The expressions for the relevant multipoles as given by (147) and (148) can now be evaluated in terms of the Q_n's and each multipole will, in general, consist of a sum of the six form factors $G_1, G_1', G_2, G_2', \mathcal{J}_K$ and Λ with certain coefficients which are W and λ^2 dependent. It is convenient to rearrange these linear sums and write each multipole as

$$\text{Multipole} = \sum_{i=1}^{6} \rho_i \, M_i(W, \lambda^2) \tag{153}$$

with

$$\rho_i = \left[g_Y G_1,\ g_y \mu,\ g_y G_1',\ g_y \mu_2,\ g_y \mathcal{J}_K,\ g\Lambda \right] \tag{154}$$

where

$$\mu_1 = \frac{G_1}{2M_1} + G_2 \qquad \mu_2 = \frac{G_1'}{2M_2} + G_2'$$

We now list the expressions $M_i(W,\lambda^2)$ of (153). They are

$$M_{1-}(\rho_1) = -\tfrac{1}{2}\ \alpha_6$$

$$M_{1-}(\rho_2) = \tfrac{1}{2}\ \alpha_7$$

$$M_{1-}(\rho_3) = \tfrac{1}{2}\ \alpha_8 - \alpha_3 \cdot \tfrac{1}{3}(Q_o - Q_2) \qquad (155)$$

$$M_{1-}(\rho_4) = -\tfrac{1}{2}\ \alpha_9 + \alpha_2 Q_1 - \alpha_3 Q_o + \alpha_4 \cdot \tfrac{1}{3}(Q_o - Q_2)$$

$$M_{1-}(\rho_5) = \alpha_5 \tfrac{1}{3}(Q_o - Q_2)$$

$$M_{1-}(\rho_6) = -(2qk)^{-1}\left[(\gamma_1 + c\gamma_2 + \gamma_4)\, Q_1 + \gamma_3 Q_o + \gamma_5 \tfrac{1}{3}(Q_o - Q_2)\right]$$

$$L_{1-}(\rho_1) = -\sigma_2 - (2M_1)^{-1}\sigma_1$$

$$L_{1-}(\rho_2) = \sigma_1$$

$$L_{1-}(\rho_3) = \tfrac{1}{2}\sigma_4 + \sigma_{14} Q_o + \sigma_{15} Q_1 \qquad (156)$$

$$L_{1-}(\rho_4) = -\sigma_5 Q_o + \sigma_{16} Q_1$$

$$L_{1-}(\rho_5) = -\sigma_9 Q_o - \sigma_{17} Q_1$$

$$L_{1-}(\rho_6) = (2qk)^{-1}\left[(\gamma_{13}+c\gamma_{14}+\gamma_{16})\,Q_1+\gamma_{15}Q_0\right]$$

and

$$\begin{aligned}
M_{3-}(\rho_1) &= M_{3-}(\rho_2) = 0\\
M_{3-}(\rho_3) &= -\tfrac{1}{3}\,\alpha_1\cdot\tfrac{1}{7}\,(Q_2-Q_4)\\
M_{3-}(\rho_4) &= \tfrac{1}{3}\left[\alpha_2Q_3-\alpha_3Q_2+\alpha_4\tfrac{1}{7}\,(Q_2-Q_4)\right] \qquad (157)\\
M_{3-}(\rho_5) &= \tfrac{1}{3}\,\alpha_5\cdot\tfrac{1}{7}\,(Q_2-Q_4)\\
M_{3-}(\rho_6) &= (5qk)^{-1}\left[(\gamma_1+c\gamma_2)\,Q_3+(\gamma_3+c\gamma_4)\,Q_2\right.\\
&\qquad\left.+\gamma_5\cdot\tfrac{1}{7}\,(Q_2-Q_4)\right]
\end{aligned}$$

$$\begin{aligned}
E_{3-}(\rho_1) &= E_{3-}(\rho_2) = 0\\
E_{3-}(\rho_3) &= \tfrac{1}{3}\left[4\alpha_1\cdot\tfrac{1}{7}\,(Q_2-Q_4)-3\sigma_6\cdot\tfrac{1}{5}\,(Q_1-Q_3)\right]\\
E_{3-}(\rho_4) &= \tfrac{1}{3}\left[-\alpha_2Q_3+\alpha_4Q_2-4\alpha_4\cdot\tfrac{1}{7}\,(Q_2-Q_4)\right. \qquad (158)\\
&\qquad\left.+3\sigma_{13}\cdot\tfrac{1}{5}\,(Q_1-Q_3)\right]\\
E_{3-}(\rho_5) &= -\tfrac{1}{3}\left[4\,\alpha_5\cdot\tfrac{1}{7}\,(Q_2-Q_4)-3\alpha_{16}\cdot\tfrac{1}{5}\,(Q_1-Q_3)\right]\\
E_{3-}(\rho_6) &= (6qk)^{-1}\left[(\gamma_1+c\gamma_2)\,Q_3+(\gamma_3+c\gamma_4)\,Q_2\right.\\
&\qquad\left.-3\gamma_{11}\tfrac{1}{5}\,(Q_1-Q_3)+4\gamma_5\cdot\tfrac{1}{7}\,(Q_2-Q_4)\right]
\end{aligned}$$

$$L_{3-}(\rho_1) = L_{3-}(\rho_2) = 0$$

$$L_{3-}(\rho_3) = \frac{1}{3}\left[\sigma_3 Q_3 + (\sigma_6 - \tfrac{1}{2}\alpha_8)\, aQ_2 + \sigma_{14} Q_2\right]$$

$$L_{3-}(\rho_4) = \frac{1}{3}\left[(a\alpha_4 - \alpha_2 - \sigma_{12})\, Q_3 + (\tfrac{1}{2}\alpha_9 - \sigma_{13})\, aQ_2 - \sigma_5 Q_2\right] \qquad (159)$$

$$L_{3-}(\rho_5) = -\frac{1}{3}\left[(b\sigma_7 - \sigma_{10})\, Q_3 + (\sigma_9 - b\sigma_8)\, Q_2\right]$$

$$L_{3-}(\rho_6) = (6qk)^{-1}\left[(\gamma_{13} + c\gamma_{14})\, Q_3 + (\gamma_{15} + c\gamma_{16})\, Q_2\right]$$

where the arguments of the $Q_n(\alpha)$ are given as $\alpha = a$ for ρ_1, ρ_2, ρ_3 and ρ_4, $\alpha = b$ for ρ_5, and $\alpha = c$ for ρ_6.

The coefficients of the $Q_n(\alpha)$ are as follows

$$\alpha_1 = \frac{W + M_1 - 2M_2}{2M_2 O_2} \qquad \alpha_2 = \frac{2M_2 O_1 (W - M_1)}{2qk}$$

$$\alpha_3 = \frac{2M_2 (W + M_1)}{2O_1} \qquad \alpha_4 = \frac{W + M_1}{O_2}$$

$$\alpha_5 = \frac{1}{O_2} \qquad \alpha_6 = \frac{2qk}{2M_1 O_1}$$

$$\alpha_7 = \frac{2qk(W + M_1)}{O_1 (W - M_1)} \qquad \alpha_8 = \frac{2qk}{2M_2 O_1}$$

$$\alpha_9 = \frac{2qk}{O_1}$$

$$\sigma_1 = \frac{M_2 k_o}{\alpha_2} \qquad \sigma_2 = \frac{k_o \alpha_7}{2\lambda^2}$$

$$\sigma_3 = \left(\frac{k^2}{2qk} - a \right) \alpha_1 \qquad \sigma_4 = \frac{2qk\, O_2}{\lambda^2}$$

$$\sigma_5 = \frac{k_o (W+2M_2-M_1) - 2q_o k_o}{2O_1}$$

$$\sigma_6 = \frac{2q^2 O_2 (W+2M_2 - M_1)}{2qk \quad 2M_2} \qquad \sigma_7 = \frac{k_o^2}{O_2 \lambda^2}$$

$$\sigma_8 = \frac{q^2 k_o^2 O_2}{qk\, \lambda^2} \qquad \sigma_9 = \frac{q_o k_o O_2}{\lambda^2}$$

$$\sigma_{10} = \frac{q_o k_o k^2}{qk\, \lambda^2 O_2} \qquad q_{11} = \frac{W+M_1}{2 O_1}$$

$$\sigma_{12} = \frac{k^2 (W-2M_2+M_1)}{2qk\, O_2} \qquad \sigma_{13} = \frac{2q^2 (W-M_1) O_2}{2qk}$$

$$\sigma_{14} = \frac{\sigma_5}{2M_2} - \sigma_{11} \qquad \sigma_{15} = \sigma_3 + \sigma_6 - \tfrac{1}{2} \alpha_8$$

$$\sigma_{16} = \tfrac{1}{2} \alpha_9 + \alpha_4 a - \alpha_2 - \sigma_{12} - \sigma_{13}$$

$$\sigma_{17} = b \sigma_7 - \sigma_{10} - \sigma_8$$

$$\gamma_1 = -O_1\left[(W-M_1)(W-M_2) - q_o k_o\right]$$

$$\gamma_2 = -qkO_1 \qquad \gamma_3 = \frac{qk}{O_1}\left[(W+M_1)(W+M_2) - q_o k_o\right]$$

$$\gamma_4 = \frac{q^2k^2}{O_1} \qquad \gamma_5 = -\frac{qk}{O_2}(W+M)$$

$$\gamma_{13} = \frac{k^2(W-M_2)}{O_2} + \gamma_1 \qquad \gamma_{14} = qk\left(\frac{W+M_1}{O_2} - O_2\right)$$

$$\gamma_{15} = \frac{qk\,k_o\,O_2}{\varepsilon_1 + M_1}\left[q_o - (W-M_2)\right]$$

$$\gamma_{16} = q^2 O_2\left[(W-M_1) - \frac{k^2}{\varepsilon_1+M_1}\right] \qquad (160)$$

$$\gamma_{11} = O_2(W-M_1)q^2 \qquad \alpha_{16} = \frac{q}{k}O_2$$

It is apparent that the above expressions are far from easy to manipulate in any differential cross section formula and in order to proceed further we resort to a numerical analysis. We calculate the $M(\rho_i)$ for a fixed value of W and a range of λ^2. With $m_\pi = 1$ the particle masses are $M_1 = 6.80$, $M_2 = 8.10$ (for Λ) $M_* = 6.55$ and $m = 3.58$. The $Q_n(\alpha)$ are determined from their expressions in terms of the hypergeometric function - the general formula being

$$Q_n(\alpha) = \frac{\Gamma(\frac{1}{2})\,\Gamma(n+1)}{2^{n+1}\,\Gamma(n+\frac{3}{2})}\;\alpha^{-n-1}\;F(\tfrac{1}{2}n+\tfrac{1}{2},\tfrac{1}{2}n+1;n+\tfrac{3}{2};\;\frac{1}{\alpha^2}) \qquad (161)$$

so that

$$\alpha Q_0(\alpha) = 1 + \frac{1}{3\alpha^2} + \frac{1}{5\alpha^4} + \frac{1}{7\alpha^6} + \frac{1}{9\alpha^8} + \cdots\cdots\cdots$$

$$3\alpha^2 Q_1(\alpha) = 1 + 3\left[\frac{1}{5\alpha^2} + \frac{1}{7\alpha^4} + \frac{1}{9\alpha^6} + \frac{1}{11\alpha^8} + \cdots\cdots\right]$$

$$15\alpha^3 Q_2(\alpha) = 2\left[1 + \frac{6}{7\alpha^2} + \frac{5}{7\alpha^4} + \frac{20}{33\alpha^6} + \frac{75}{143\alpha^8} + \cdots\cdots\right]$$

$$35\alpha^4 Q_3(\alpha) = 2\left[1 + \frac{10}{9\alpha^2} + \frac{35}{33\alpha^4} + \frac{140}{143\alpha^6} + \frac{35}{39\alpha^8} + \cdots\cdots\right]$$

$$315\alpha^5 Q_4(\alpha) = 8\left[1 + \frac{15}{11\alpha^2} + \frac{210}{143\alpha^4} + \frac{210}{143\alpha^6} + \frac{315}{221\alpha^8} + \cdots\cdots\right]$$

The fixed value of W is taken as 12 and $\lambda^2 = 5, 10\ (10)\ 50$. The results for Λ production are presented in Tables 2 and 3 for the $P_{\frac{1}{2}}$ and $F_{5/2}$ multipoles respectively.

Now the multipoles occuring in equation (144) and (146) are determined by the sum (153) so that the form of the vector ρ_i given by (154) is still required. In the framework of the method described in (viii) the values already obtained are exact and as the determination of ρ_i involves a complet

TABLE 2

m \ χ^2	5	10	20	30	40	50
$M(p_1)$	-0.0268	-0.0288	-0.0325	-0.0358	-0.0388	-0.0416
$M(p_2)$	1.319	1.418	1.598	1.760	1.904	2.046
$M(p_3)$	0.0132	0.0144	0.0167	0.0188	0.0209	0.0230
$M(p_4)$	0.1759	0.1666	0.1419	0.1114	0.0718	0.0197
$M(p_5)$	0.1180	0.1153	0.1099	0.1048	0.0997	0.0946
$M(p_6)$	-0.9474	-0.9578	-0.9622	-0.9535	-0.0387	-0.9263
$L(p_1)$	-1.0397	-0.5390	-0.2792	-0.1851	-0.1400	-0.0973
$L(p_2)$	0.2712	0.2759	0.2755	0.2643	0.2438	0.2165
$L(p_3)$	1.0030	0.5015	0.2426	0.1505	0.1108	0.0697
$L(p_4)$	-0.3182	-0.3144	-0.2969	-0.2700	-0.2486	-0.2222
$L(p_5)$	-0.8840	-0.3947	-0.1561	-0.0813	-0.0440	-0.0280
$L(p_6)$	0.5577	0.5698	0.5838	0.5890	0.5840	0.5762

TABLE 3

λ^2 / ($\times 10^{-3}$)	5	10	20	30	40	50
$M_{3-}(\rho_3)$	-0.0050	-0.0059	-0.0076	-0.0093	-0.0108	-0.0119
$M_{3-}(\rho_4)$	0.278	0.311	0.362	0.394	0.390	0.383
$M_{3-}(\rho_5)$	0.260	0.246	0.219	0.195	0.173	0.155
$M_{3-}(\rho_6)$	-1.84	-1.98	-2.18	-2.29	-2.34	-2.37
$E_{3-}(\rho_3)$	-0.655	-0.681	-0.718	-0.742	-0.754	-0.760
$E_{3-}(\rho_4)$	0.621	0.386	-0.098	-0.577	-0.814	-1.36
$E_{3-}(\rho_5)$	1.06	0.906	0.690	0.548	0.502	0.388
$E_{3-}(\rho_6)$	-0.707	-0.626	-0.485	-0.373	-0.264	-0.183
$L_{3-}(\rho_3)$	0.312	0.305	0.282	0.252	0.220	0.180
$L_{3-}(\rho_4)$	-1.73	-1.75	-1.73	-1.64	-1.48	-1.31
$L_{3-}(\rho_5)$	-0.273	-0.209	-0.128	-0.082	-0.054	-0.035
$L_{3-}(\rho_6)$	1.75	1.78	1.79	1.76	1.71	1.61

knowledge of the nucleon, hyperon and K meson form factors we employ an approximation. We set all form factors equal to their values at $\lambda^2 = o$ times the factor

$$F(\lambda^2) = \left[1 + \frac{a^2 \lambda^2}{12}\right]^{-2} \qquad (162)$$

with $a = 0.8$ fermi.

In this way we do not take into account the splitting between $F_{1p}(\lambda^2)$ and $F_{2p}(\lambda^2)$ and the non-zero value of $F_{1n}(\lambda^2)$ [47]. We are still left with the problem of the hyperon magnetic moments. As they are unknown the only guides for fixing their values are the symmetry theories. We shall use the values predicted by the octet model of unitary symmetry of Gell-Mann and Ne'eman [48]. Even here, there are three possible couplings of pseudoscalar meson to baryon, that is, pure F-type, pure D-type, and a mixture of D and F. The most favoured coupling is a mixture of D and F, and we select the couplings in such a way that $g_\Lambda / g_\Sigma = -\sqrt{3}$ with g_Λ negative and $g_\Sigma^2 / 4\pi \sim \left(\frac{\mu}{m}\right)^2 \frac{g_{\pi NN}^2}{4\pi} \sim 1.2$. The same type of coupling as that for the pseudoscalar meson with the baryon is used for K^* so that we write $\lambda_{v,s}$ of (116) as

$$\frac{\lambda^2_{v,s}}{4\pi} = \frac{e^2}{M_*^2} \frac{F^2_{v,s}}{4\pi} \qquad (163)$$

where $F_{v,s}$ is now dimensionless. We set $F_s = 0$ and select F_v in such a

$$F_v^2 \frac{g^2_{K^*N\Sigma}}{4\pi} = 4.3 \qquad (164)$$

The values of the hyperon magnetic moments are given by

$$\mu_\Lambda = \tfrac{1}{2}\mu_n \qquad \mu_T = \frac{\sqrt{3}}{2}\mu_n$$

with

$$\mu_p = 1.79 \quad \text{and} \quad \mu_n = -1.91.$$

Thus the isotopic parts of the vector ρ_i may be written as

$$\rho^{s_+}_i = \frac{e\, g_\Lambda}{2} F(\lambda^2) \left[1,\ 0.346,\ 0,\ -0.141,\ 1,\ 0.578 \right] \qquad (165)$$

$$\rho^{s_o}_i = \frac{e g_\Lambda}{2} F(\lambda^2) \left[1,\ 0.0647,\ 0,\ -0.141,\ 1,\ 0 \right]$$

with the aid of (58), (59), (65) and (116).

The only further parameters to be fixed are the positions and widths of the two resonances in the expression (126). Taking the $P_{\frac{1}{2}}$ resonance at 1700 Mev with full width 60 Mev[31] and the $F_{5/2}$ resonance at 1680 Mev with full width 100 Mev we have the values 12.36 and 12.23 for W_R and, 0.0362 and 0.0604 for Γ, respectively.

The differential cross section may now be written (from (139)) as

$$\frac{d^2\sigma}{dR_2 d\Omega} = \frac{e^2}{4\pi} \quad \frac{g_\Lambda^2}{4\pi} \quad \frac{1}{\Gamma^2} \quad \frac{qR_2 \left[F(\lambda^2)\right]^2}{8\pi M_1 R_1 W \lambda^4} \cdot L(\lambda^2) \qquad (166)$$

where $L(\lambda^2)$ is given by the functions on the right hand side of (144) and (146) and ρ_i in (153) is replaced by the expression in square brackets in (165).

Until now we have only considered the virtual process $\gamma N \rightarrow K\Lambda$ and in order to compute the kinematical factors of (166) and the functions ρ_1 and ρ_2 of $L(\lambda^2)$ we have to relate the laboratory quantities of the $e\gamma e$ vertex to the barycentric quantities.

With a Panofsky type of experiment the quantities R_1, R_2 and α_L are programmed together in such a way that W remains fixed (at 12μ in our case) and λ^2 varies. By considering the invariants $(p_1+r_1+r_2)^2$ and $(r_1+r_2)^2$ in the laboratory frame we can relate R_1, R_2 and α_L to W and λ^2. The relations are

$$W^2 = M_1^2 - 2R_1R_2 (1-\cos\alpha_L) + 2M_1 (R_1-R_2) \qquad (167)$$

and

$$\lambda^2 = 2R_1R_2 (1-\cos\alpha_L)$$

where α_L is the laboratory scattering angle.

In the quantities of (141) only ψ and r_1 are unknown in terms of

W and λ^2 since all others are given by the relations (14). The required expressions are

$$r_1 = \frac{2M_1R_1 - \lambda^2}{2W} \tag{168}$$

and

$$r_1 k \cos\psi = k_o r_1 + \tfrac{1}{2}\lambda^2$$

In what follows we fix a_L at 60^o. Thus the relations (167) will give R_1 and R_2 in terms of W and λ^2 for the required ranges. Typical curves are presented in Figure V. The quantities required in (144), (146) and (166) are presented in Table 4 and the relevant multipoles for the production of $K^o\Lambda$ and $K^+\Lambda$ via the $P_{\frac{1}{2}}$ or $F_{5/2}$ resonances are presented in Table 5. Finally the differential cross sections are prosented in Table 6 and in Figures VI and VII.

Thus with the aid of the Blankenbecler and Gartenhaus method of solving the dispersion integrals in the energy region of the dominant resonance and the form factor approximation employed in obtaining a numerical estimate of the differential cross sections we arrive at the results presented in Table 6.

It is evident that the first result of this analysis lies in the large difference (about 10^3 cm^2/Mev) between the $P_{\frac{1}{2}}$ and the $F_{5/2}$ differential cross sections - a result that could have been anticipated on general

TABLE 4

λ^2	5	10	20	30	40	50
ρ_1	0.910	1.39	1.68	1.52	1.25	0.880
$\xi\rho_1$	0.102	0.777	6.09	21.5	59.6	140
ρ_2	2.80	6.02	13.2	20.7	28.4	36.2
ρ_3	1.39	2.57	6.39	9.95	13.6	17.2
R_1	8.17	9.03	10.6	11.9	13.2	14.4
R_2	0.612	1.11	1.89	2.52	3.04	3.48
$C(\lambda^2)$	1146	298.8	50.47	13.92	4.917	2.033

where $C(\lambda^2) = \dfrac{q\, R_2 \left[F(\lambda^2) \right]^2}{8\pi M_1 V R_1 \lambda^4}$ in 10^{-9}

TABLE 5

$P_{\frac{1}{2}}$		5	10	20	30	40	50
$K^+\Lambda$	M^2	0.0160	0.0238	0.0459	0.0770	0.1157	0.1636
	L^2	11.05	1.785	0.1131	0.00006	0.0195	0.0548
	$L(\lambda^2)$	2.341	3.062	2.586	3.190	8.909	27.22
$K^o\Lambda$	M^2	0.0314	0.0241	0.0115	0.0032	0.00006	0.0015
	L^2	0.1588	0.1654	0.1721	0.1719	0.1648	0.1550
	$L(\lambda^2)$	0.2080	0.5476	2.400	7.530	19.64	43.60
10^{-6}	$F5/_2$	5	10	20	30	40	50
$K^+\Lambda$	M^2	0.3969	0.5476	0.8538	1.092	1.246	1.364
	E^2	2.372	1.796	1.277	1.082	1.166	1.124
	L^2	0.9025	1.254	1.613	1.742	1.690	1.513
	$L(\lambda^2)$	164.5	330.8	1238	3243	7312	13980
$K^o\Lambda$	M^2	1.132	1.309	1.588	1.753	1.831	1.877
	E^2	0.1665	0.1310	0.0784	0.0467	0.0234	0.0112
	L^2	1.024	1.059	1.071	1.035	0.9761	0.8668
	$L(\lambda^2)$	123.1	342.4	1123	2528	5027	9022

TABLE 6

$\frac{d^2\sigma}{dR_2 d\Omega}$	$P_{\frac{1}{2}}(10^{-37})$		$F_{5/2}$ (10^{-40})	
λ^2	$K^+\Lambda$	$K^0\Lambda$	$K^+\Lambda$	$K^0\Lambda$
5	599	53.1	15.0	11.3
10	204	36.4	7.88	8.15
20	29.1	29.0	4.98	4.52
30	9.89	23.3	3.60	2.81
40	9.76	21.5	2.87	1.97
50	12.3	19.7	2.26	1.46

considerations of the angular momentum barriers involved. Once the photoproduction analysis settles the question as to whether the dominant resonance is a $P_{\frac{1}{2}}$ or an $F_{5/2}$ the relevant coupling constants and magnetic moments will be available.

The second result, and the more important, can be obtained by noting that the expressions (153) and (154) for the multipoles will give a general form of the differential cross section which is a homogeneous quadratic in the form factors with known (constant) coefficients. Thus, when the electro-kaon production data from a Panofsky type of experiment and a firm knowledge of the hyperon magnetic moments become available this form of the differential cross section will enable a determination of the unknown form factors to be performed, in exactly the same manner as has been carried out by Salin[17] in his determination of the neutron form factor from the electro-pion production data.

PART II

THE WEAK PRODUCTION OF STRANGE PARTICLES

i INTRODUCTION.

To extend the analysis of Part I to the weak production of strange particles, that is, the production of a K meson by high energy neutrino collisions with nucleons, we require three working hypotheses. They are the Universal Fermi Interaction (U.F.I), the intermediate boson hypothesis and the Conserved Vector Current hypothesis (C.V.C.) *.

In order to explain the remarkable universality of the vector coupling constants in μ-decay and in β-decay (remarkable, since strong interactions clearly play an important role in the latter but not in the former) Feynman and Gell-Mann[19] proposed for the weak interaction Lagrangian the following expression

$$\mathcal{L} = \frac{G}{\sqrt{2}} \; J_\mu J_\mu^\dagger \qquad (169)$$

where J_μ stands for the sum of the leptonic current

$$\bar{\nu}\,\gamma_\mu\,(1+\gamma_5)e \;+\; \bar{\nu}'\gamma_\mu\,(1+\gamma_5)\mu$$

and the strangeness conserving $j_\mu = j_\mu^V + j_\mu^A$ and the strangeness violating $s_\mu = s_\mu^V + s_\mu^A$ currents of the strongly interacting particles. The C.V.C. hypothesis makes it possible to relate the vector current j_μ^V to the isovector part of the electromagnetic current by a rotation in isotopic spin space.

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* For an elucidative account of Weak Interactions we refer the reader to the review article of J.C. Taylor[49].

Thus the current j_μ^V is proportional to the + component of the total isotopic current $\underline{J}_\mu$ the isovector electromagnetic current being proportional to the z component of $\underline{J}_\mu$.

Similar assumptions cannot be made for the axial vector current j_μ^A for if it were conserved the decay rate of the charged pion would vanish. The treatment of j_μ^A by Goldberger and Treiman[50] and others [51] assumes a partially conserved current and the essence of their treatment lies in the assumption that the one pion pole dominates the unsubtracted dispersion relation satisfied by the quantity $\langle | \partial_\mu j_\mu^A | \rangle$. Although this could be employed here in our analysis it is rendered unnecessary as we write down the vertices associated with the various poles on general invariance grounds. The resulting form factors are treated as in Part I.

Since the leptons interact weakly in a vector (or axial vector) combination with the strongly interacting particles it is possible to introduce non-locality in the form of an intermediary charged vector boson $W^\pm$ in analogy with mesons and photons in strong and electromagnetic interactions [52]. In such a case we write

$$\mathcal{L} = g\sqrt{2}\ J_\mu \phi_\mu^\dagger \qquad (170)$$

where J_μ is the total weak current, ϕ_μ is the field operator corresponding to W and g is a coupling constant. If the expressions for the amplitude for neutron β-decay obtained from (169) and (170) are compared then

$$g^2 = \frac{G M^2}{2\sqrt{2}} \qquad (171)$$

where M is the mass of the intermediate vector boson and it turns out that

$$G = \frac{10^{-5}}{\sqrt{2}\, M_1^2} \qquad (172)$$

We assume that W exists, though by (171) the results of the current-current theory are reproduced by letting $M \to \infty$, that is, we have the limit

$$\frac{g^4}{(\lambda^2 + M^2)^2} \to \frac{G^2}{8} \qquad (173)$$

as $M \to \infty$.

Thus the results of Part I can be extended to weak production; the virtual photon being replaced by the virtual vector boson and the current J_μ (26) having an axial vector part as well as a vector part - the vector part given by the C.V.C. hypothesis.

ii. THE SCATTERING AMPLITUDE.

We follow the procedure of I iii and relate the T-matrix to the S-matrix by

$$S_{fi} = \delta_{fi} + i(2\pi)^4 \delta^4 (p_1 + p_2 + q + k)\, T_{fi}$$

To first order in the weak interaction we assume that the leptons interact

with the strongly interacting particles via an intermediate boson of mass M, illustrated formally in Figure VIII. The strong current consists of a vector and an axial vector part and we write

$$\langle -p_2, -q, -r_2 \left| T \right| p_1\ r_1 \rangle = \langle -p_2\ -q \left| J_\mu \right| p_1 \rangle\ \varepsilon_\mu \qquad (174)$$

where

$$\varepsilon_\mu = g^2\sqrt{2}\ \frac{\bar{u}(-r_2)\ \gamma_\mu\ (1+\gamma_5)\ u(r_1)}{(r_1+r_2)^2-M^2} \qquad (175)$$

and

$$J_\mu = V_\mu + A_\mu \qquad (176)$$

Now $\langle -p_2\ -q \left| J_\mu \right| p_1 \rangle$ is the matrix element for the weak production of a K-meson by the virtual weak vector boson of four-momentum

$$k^2 = (r_1+r_2)^2 = -\lambda^2$$

and it is this matrix element that we have to analyse in detail. If we employ the decomposition (176) and define the quantities $\mathcal{M}_V$ and $\mathcal{M}_A$ by

$$\mathcal{M}_V = \langle -p_2 - q \left| V_\mu \right| p_1 \rangle\ \varepsilon_\mu \qquad (177)$$

and

$$\mathcal{M}_A = \langle -p_2 - q \left| A_\mu \right| p_1 \rangle\ \varepsilon_\mu \qquad (178)$$

then

$$\mathcal{M} = \mathcal{M}_V + \mathcal{M}_A$$

will be the matrix element in question.

In the case of the final lepton being an electron we can neglect the mass of the electron and it is easily seen that the vector ε_μ will obey the supplementary condition

$$k_\mu \varepsilon_\mu = 0 \tag{179}$$

Throughout the subsequent analysis we treat the final lepton quite generally and only set $m_e = 0$ for the differential cross section computation.

By the C.V.C. hypothesis we can relate the matrix element $\mathfrak{M}_V$ to that obtained in the electroproduction analysis and the essence of this part lies in the decomposition of $\mathfrak{M}_A$ in the spirit that $\mathfrak{M}_V$ was decomposed in PART I.

We write for the axial vector part the following relativistic invariant decomposition

$$-i\mathfrak{M}_A = \sum_{i=1}^{8} A_i m_i \tag{180}$$

with the eight invariant forms given by

$$m_1 = \tfrac{1}{2}(\gamma.q\,\gamma.\varepsilon - \gamma.\varepsilon\,\gamma.q) \qquad m_5 = i\,\gamma.k\;2P.\varepsilon$$

$$m_2 = 2P.\varepsilon \qquad m_6 = i\gamma.k\;\; q.\varepsilon$$

$$m_3 = q.\varepsilon \qquad m_7 = k.\varepsilon$$

$$m_4 = i\gamma.\varepsilon \qquad m_8 = i\gamma.k\;k.\varepsilon \tag{181}$$

iii CALCULATION OF RESIDUES OF POLES.

We adhere to the notation of I iv and discuss the various graphs of Figure IX separately.

Single Nucleon Term.

By general considerations of invariance the WNN vertex : $\left\langle k+p_1 \left| j_\mu^+(o) \right| p_1 \right\rangle$ may be written as

$$\bar{u}(k+p_1)i\left[G_1\gamma_\mu + G_2\sigma_{\mu\nu}k_\nu + G_3\gamma_\mu\gamma_5 + iG_4k_\mu\gamma_5\right]u(p_1) \qquad (182)$$

so that the contribution of Figure IX(a) is given by

$$\text{Im } T_I = \pi\delta(s-M_1^2)\bar{u}(-p_2)i\gamma_5 g_Y \ (\not{k}+\not{p}_1+M_1)$$
$$i\varepsilon_\mu\left[G_1\gamma_\mu+G_2\sigma_{\mu\nu}k_\nu+ G_3\gamma_\mu\gamma_5 +iG_4\gamma_5k_\mu\right]u(p_1) \qquad (183)$$

Now the first two terms in the square bracket of (183) have been dealt with in I iv and contribute to $\mathcal{M}_\nu$ so by (62) we write

$$\text{Im } T_I^\nu = \pi\delta(s-M_1^2)\bar{u}(-p_2)g_Y\left[G_1M_1 + G_2\ (M_3-M_4)\right.$$
$$\left. - i\gamma_5\ G_1\ (q.\varepsilon - 2P.\varepsilon)\right]u(p_1) \qquad (184)$$

We are left with the evaluation of the last two terms in (183). Using the m_i's we can write

$$\text{Im } T_I^A = i\pi\delta(s-M_1^2)\ \bar{u}(-p_2)\ g_y\left[G_3(m_1-m_3) + \Delta G_3 m_4\right.$$
$$\left. + G_4m_8\right]u(p_1) \qquad (185)$$

Hence the residues of the single nucleon pole term are given as follows

$$\Gamma_s(A_1) = g_y G_1 \qquad \Gamma_s(A_4) = -g_y G_2$$
$$\Gamma_s(A_2) = 2g_y G_1 \qquad \Gamma_s(A_5) = g_y G_1 \qquad (186)$$
$$\Gamma_s(A_3) = g_y G_2 \qquad \Gamma_s(A_6) = 0$$

for the vector amplitudes, which are identical with the expressions (68), and

$$\Gamma_s(A_1) = g_y G_3 \qquad \Gamma_s(A_5) = 0$$
$$\Gamma_s(A_2) = 0 \qquad \Gamma_s(A_6) = 0 \qquad (187)$$
$$\Gamma_s(A_3) = -g_y G_3 \qquad \Gamma_s(A_7) = 0$$
$$\Gamma_s(A_4) = g_y \Delta G_3 \qquad \Gamma_s(A_8) = g_y G_4$$

for the axial vector amplitudes.

Single Hyperon Term.

The contribution of the single hyperon graph, Figure IX(b) can be written down immediately as

$$\text{Im } T_{II} = \pi\delta(u-M_2^2)\bar{u}(-p_2)i\varepsilon_\mu \Big[G_1' \gamma_\mu + G_2' \sigma_{\mu\nu} k_\nu + G_3' \gamma_\mu \gamma_5 + i G_4' k_\mu \gamma_5 \Big] (\not{p}_1 + \not{q} + M_2)\, i\gamma_5 \varepsilon_y\, u(p_1) \qquad (188)$$

The first two terms in square brackets above contribute to Im T_{II}^V and have been evaluated in I iv giving

$$\text{Im } T^{V}_{II} = \pi\delta\,(u-M_2^2)\bar{u}(-p_2)g_y\Big[\;(G_1' + 2\Delta G_2')\;M_1$$

$$- (M_3-M_4)\;G_2' + i\gamma_5\;G_1'\;(2P.\varepsilon + q.\varepsilon)\Big]\,u(p_1) \qquad (189)$$

The contribution of the remaining terms in (188) to the axial vector part is as follows

$$\text{Im } T^{A}_{II} = \pi\delta\,(u-M_2^2)\;\bar{u}(-p_2)\;g_y(-i)\Big[(m_1+m_3)\;G_3'$$

$$- \Delta m_4\;G_3' + m_8\;G_4'\Big]\,u(p_1) \qquad (190)$$

and identifying this expression with $\mathcal{M}_A$ (180) we have the following residues from the single hyperon pole

$$\Gamma_u(A_1) = g_y\;\;(G_1' + 2\,\Delta G_2')$$

$$\Gamma_u(A_2) = 2g_y\;G_1' \qquad (191)$$

$$\Gamma_u(A_3) = \Gamma_u\,(A_4) = \Gamma_u(A_5) = -g_y G_2'$$

$$\Gamma_u(A_6) = 0$$

for the vector amplitudes, and

$$\Gamma_u(A_1) = -g_y G_3' \qquad \Gamma_u(A_5) = 0$$

$$\Gamma_u(A_2) = 0 \qquad \Gamma_u(A_6) = 0 \qquad (192)$$

$$\Gamma_u(A_3) = g_y G_3' \qquad \Gamma_u(A_7) = 0$$

$$\Gamma_u(A_4) = g_y\,\Delta G_3' \qquad \Gamma_u(A_8) = g_y G_4'$$

for the axial vector amplitudes.

Single K-meson Term.

The K-meson contributes only to the vector amplitudes and so by (68) we have

$$\Gamma_t(A_5) = \frac{2g_v}{\lambda^2} \quad (G_1' - G_1 - \mathcal{J}_K) \tag{193}$$

$$\Gamma_t(A_i) = 0 \quad i \neq 5$$

and

$$\Gamma_t(A_i) = 0 \quad i = 1,\ldots,8 \text{ for the axial vector amplitudes.}$$

The isotopic spin dependence of the amplitudes for the process "W" + N $\rightarrow \Sigma +$ K will be similar to that of the associated production process $\pi +$ N $\rightarrow \Sigma +$ K. If β is the isotopic state of the W and α that of the produced Σ then the isotopic invariants are

$$A^+_{\alpha\beta} = \delta_{\alpha\beta}$$

$$A^-_{\alpha\beta} = \tfrac{1}{2}\left[\tau_\alpha, \tau_\beta\right]$$

so that

$$A_i = A^+_i A^+_{\alpha\beta} + A^-_i A^-_{\alpha\beta} \tag{195}$$

for Σ production.

The projection operators for total isotopic spin $\frac{1}{2}$ and 3/2 are given by

$$(P_{3/2})_{\alpha\beta} = A^+_{\alpha\beta} - A^-_{\alpha\beta}$$

$$(P_{\frac{1}{2}})_{\alpha\beta} = A^+_{\alpha\beta} + 2A^-_{\alpha\beta}$$

For the process "W" + N → Λ+ K there is only one isotopic invariant τ_β so that

$$A_i = A_i^\beta \tau_\beta \qquad (196)$$

for Λ production.

iv. Angular Momentum Decomposition.

The methods of I v are adapted to decompose the amplitudes into multipoles. In the electroproduction case the multipoles that occured were magnetic, electric and longitudinal - the scalar multipoles being eliminated by the Lorentz condition $k.\varepsilon = 0$ (35). In weak production the Lorentz condition only holds for an electron being produced and in order to decompose the amplitudes quite generally we include the scalar multipoles. The Lorentz condition will only be imposed in the differential cross section computation.

With the identification

$$\bar{u}(-p_2)\mathcal{H}_A u(p_1) = i\chi^\dagger \mathcal{J}_A \chi \qquad (197)$$

where

$$\mathcal{J}_A = \sum_{i=1}^{8} \mathcal{J}_i \Sigma_i$$

and

$$q\Sigma_1 = \underline{\sigma}.\underline{q}\,\underline{\sigma}.\underline{\varepsilon} \qquad k\Sigma_5 = \underline{\sigma}.\underline{\varepsilon}\,\underline{\sigma}.k$$

$$q k^2\Sigma_2 = \underline{\sigma}.\underline{q}\,\underline{\sigma}.\underline{k}\,\underline{k}.\underline{\varepsilon} \qquad q\Sigma_6 = \underline{q}.\underline{\varepsilon} \qquad (198)$$

$$q^2 k\Sigma_3 = \underline{\sigma}\cdot\underline{q}\ \underline{\sigma}\cdot k\ \underline{q}\cdot\underline{\varepsilon} \qquad k\,\Sigma_7 = \underline{k}\cdot\underline{\varepsilon}$$

$$q\,k\Sigma_4 = \underline{\sigma}\cdot\underline{q}\cdot\ \underline{\sigma}\cdot\underline{k}\ \ \varepsilon_o \qquad \Sigma_8 = \varepsilon_o$$

we can express the Dirac amplitudes A_i in terms of the Pauli amplitudes $\mathcal{F}_i$ with the help of the relations (16). The resulting expressions are

$$F = \frac{\mathcal{F}_1}{q\,0_2} = (W+M_2)\,A_1 + A_4$$

$$F_2 = \frac{0_1\mathcal{F}_2}{q\,k^2} = -A_2 + A_7 + (W+M_1)\,(A_5 - A_8)$$

$$F_3 = \frac{0_1\mathcal{F}_3}{q^2\,k} = -A_1 - A_2 - A_3 + (W+M_1)\,(A_5+A_6)$$

$$F_4 = \frac{0_1\mathcal{F}_4}{qk} = -(\varepsilon_2+M_2)\,A_1 - (\varepsilon_1+\varepsilon_2)\,A_2 + q_o A_3$$

$$+ A_4 - k_o\,A_7 + (W+M_1)\Big[(\varepsilon_1+\varepsilon_2)\,A_5$$

$$- q_o\,A_6 + k_o\,A_8\Big]$$

$$F_5 = \frac{0_2\mathcal{F}_5}{k} = (W - M_2)\,A_1 + A_4 \qquad (199)$$

$$F_6 = \frac{\mathcal{F}_6}{q\,0_1} = A_1 + A_2 + A_3 + (W-M_1)\,(A_5+A_6)$$

$$\mathbb{F}_7 = \frac{\mathcal{F}_7}{k\,O_1} = A_2 - A_7 + (W-M)\,(A_5-A_8)$$

$$\mathbb{F}_8 = \frac{\mathcal{F}_8}{O_1} = (\varepsilon_2 - M_2)\,A_1 + (\varepsilon_1+\varepsilon_2)\,A_2 - q_o\,A_3$$

$$+ A_4 + k_o A_7 + (W-M_1)\left[(\varepsilon_1+\varepsilon_2)\,A_5 - q_o\,A_6 + k_o\,A_8\right]$$

These expressions can be inverted, giving

$$2WA_1 = \mathbb{F}_5 - \mathbb{F}_1$$

$$(2W)^2\ A_2 = (W+M_1)\left[\mathbb{F}_8 + q_o\mathbb{F}_6 + k_o\,\mathbb{F}_7 - \mathbb{F}_5\right] - (W-M_1)\left[\mathbb{F}_4 + q_o\,\mathbb{F}_3 + k_o\,\mathbb{F}_2 - \mathbb{F}_1\right]$$

$$(2W)^2\ A_3 = (W+M_1)\left[\mathbb{F}_1 + (W+\varepsilon_2)\,\mathbb{F}_6 - \mathbb{F}_8 - k_o\,\mathbb{F}_7\right] + (W-M_1)\left[\mathbb{F}_4 - \mathbb{F}_5 + k_o\,\mathbb{F}_2 - (W+\varepsilon_2)\,\mathbb{F}_3\right]$$

$$2W\ A_4 = (W-M_2)\,\mathbb{F}_1 + (W+M_2)\,\mathbb{F}_5 \qquad (200)$$

$$(2W)^2\ A_5 = -\mathbb{F}_1 + \mathbb{F}_4 - \mathbb{F}_5 + \mathbb{F}_8 + q_o\,(\mathbb{F}_3 + \mathbb{F}_6) + k_o\,(\mathbb{F}_2 + \mathbb{F}_7)$$

$$(2W)^2 A_6 = F_1 - F_4 + F_5 - F_8 + (W+\varepsilon_2)(F_3+F_6) - k_o (F_2+F_7)$$

$$(2W)^2 A_7 = (W+M_1)\left[F_8 - (W+\varepsilon_1) F_7 + q_o F_6 - F_5\right] - (W - M_1)\left[F_4 + q_o F_3 - (W+\varepsilon_1) F_2 - F_1\right]$$

$$(2W)^2 A_8 = - F_1 + F_4 - F_5 + F_8 + q_o(F_3+ F_6) - (W+ \quad_1) (F_2+ F_7)$$

The multipole analysis of I.v follows through with but slight modification. The vector part is identical with that for the photon and the treatment of the axial vector part is carried out as follows.

We note that for a vector boson inducing a magnetic transition of order l, the parity of the initial state is $(-1)^{l+1}$, whereas for an axial vector boson it is $(-1)^l$. Now an initial state of total angular momentum $l+\frac{1}{2}$ will lead to a final state of orbital angular momentum $l_f=l$ in the vector case. As the relative intrinsic parity of the initial state to the final state is even for the axial vector boson (which will require that an additional factor of $\underline{\sigma}.\underline{q}$ be included on the left of the summation in (76))the final state will have orbital angular momentum $l_f = l+1$ and therefore total angular momentum $l_f - \frac{1}{2}$. The electric, longitudinal and

scalar transitions can be treated along similar lines. The overall result is that the following replacements must be effected in (76):

$$(\text{Multipole})^{\pm}_{l_K} \longrightarrow (\text{Multipole})^{\mp}_{l_K \pm 1} \tag{201}$$

so that the decomposition of $\vec{J}_A$ into multipoles is given by

$$\begin{aligned}\vec{J}_A = \Sigma_l \Big[& \frac{1}{i}\, \underline{\sigma}\cdot\underline{q}\; P_l^+ \;\; \underline{e}\cdot\underline{L}_k \, \mathcal{M}^-_{l+1} \\ & + \frac{1}{i}\;\; \underline{\sigma}\cdot\underline{q} \;\; P_l^- \;\; \underline{e}\cdot\underline{L}_k \mathcal{M}^+_{l-1} \\ & - P_l^+ \;\; \underline{e}\cdot\underline{k} \wedge \underline{L}_k \, \mathcal{E}^+_l + P_l^- \;\; \underline{e}\cdot\underline{k} \wedge \underline{L}_k \mathcal{E}^-_l \\ & - \frac{1}{i} P_l^+ \; \underline{e}\cdot\underline{k} \; (l+1) \mathcal{L}^+_l - \frac{1}{i} P_l^- \;\; \underline{e}\cdot\underline{k} \; l \mathcal{L}^-_l \\ & - \frac{1}{i} \;\; P_l^+ \;\; e_o \; (l+1) \mathcal{S}^+_l - \frac{1}{i} P_l^- \;\; e_o \; l \mathcal{S}^-_l \Big] \\ & \cdot (2l+1)\, P_l(\cos\vartheta)\end{aligned} \tag{202}$$

On performing the operations indicated in the summation above and identifying the final expression with (197) and (198) the required multipole decomposition of the $\vec{J}_i$ is obtained:

$$\vec{J}_1 = \sum_l \Big[(l-1)\mathcal{M}^-_l + \mathcal{E}^-_l + (l+2)\mathcal{M}^+_l + \mathcal{E}^+_l \Big] P^1_l(x)$$

$$\mathcal{F}_2 = -\mathcal{F}_1 - x\mathcal{F}_3 + \Sigma_l \left[l\mathcal{J}_l^- - (l+1)\mathcal{J}_l^+ \right] P_l'(x)$$

$$\mathcal{F}_3 = \Sigma_l \left[-\mathcal{E}_l^- - \mathcal{M}_l^- + \mathcal{E}_l^+ + \mathcal{M}_l^+ \right] P_l''(x)$$

$$\mathcal{F}_4 = \Sigma_l \left[l\mathcal{S}_l^- - (l+1)\mathcal{S}_l^+ \right] P_l'(x) \qquad (203)$$

$$\mathcal{F}_5 = -\Sigma_l \left[l\mathcal{M}_l^- P_{l-1}' + (l+1)\mathcal{M}_l^+ P_{l+1}' \right]$$

$$\mathcal{F}_6 = \Sigma_l \left[(\mathcal{M}_l^- - \mathcal{E}_l^-) P_{l-1}'' - (\mathcal{E}_l^+ + \mathcal{M}_l^+) P_{l+1}'' \right]$$

$$\mathcal{F}_7 = -\mathcal{F}_5 - x\mathcal{F}_6 + \Sigma_l \left[(l+1)\mathcal{J}_l^+ P_{l+1}' - l\mathcal{J}_l^- P_{l-1}' \right]$$

$$\mathcal{F}_8 = \Sigma_l \left[(l+1)\mathcal{S}_l^+ P_{l+1}' - l\mathcal{S}_l^- P_{l-1}' \right]$$

These expressions can be inverted to give

$$\mathcal{M}_l^- = \frac{1}{2l} \int_{-1}^{+1} dx \left[\mathcal{F}_1 P_{l-1} + \mathcal{F}_5 P_l - \mathcal{F}_3 \frac{P_{l-2} - P_l}{2l-1} \right]$$

$$\mathcal{M}_l^+ = \frac{1}{2(l+1)} \int_{-1}^{+1} dx \left[-\mathcal{F}_1 P_{l+1} - \mathcal{F}_5 P_l + \mathcal{F}_3 \frac{P_l - P_{l+2}}{2l+3} \right]$$

$$\mathcal{E}_l^- = \frac{1}{2l} \int_{-1}^{+1} dx \left[\mathcal{F}_1 P_{l-1} + \mathcal{F}_5 P_l + \mathcal{F}_3 (l-1) \frac{P_{l-2} - P_l}{2l-1} + \mathcal{F}_6 \, l \, \frac{P_{l-1} - P_{l+1}}{2l+1} \right]$$

$$\mathcal{E}_l^+ = \frac{1}{2(l+1)} \int_{-1}^{+1} dx \Big[\mathcal{F}_1 P_{l+1} + \mathcal{F}_5 P_l - \mathcal{F}_3 (l+2) \frac{P_l - P_{l+2}}{2l+3} - \mathcal{F}_6 (l+1) \frac{P_{l-1} - P_{l+1}}{2l+1} \Big]$$

$$\mathcal{L}_l^- = \frac{1}{2l} \int_{-1}^{+1} dx \Big[(\mathcal{F}_1 + x\mathcal{F}_3 + \mathcal{F}_2) P_{l-1} + (\mathcal{F}_5 + x\mathcal{F}_6 + \mathcal{F}_7) P_l \Big]$$

$$\mathcal{L}_l^+ = \frac{1}{2(l+1)} \int_{-1}^{+1} dx \Big[(\mathcal{F}_1 + x\mathcal{F}_3 + \mathcal{F}_2) P_{l+1} + (\mathcal{F}_5 + x\mathcal{F}_6 + \mathcal{F}_7) P_l \Big]$$

$$S_l^- = \frac{1}{2l} \int_{-1}^{+1} dx \Big[\mathcal{F}_4 P_{l-1} + \mathcal{F}_8 P_l \Big]$$

$$S_l^+ = \frac{1}{2(l+1)} \int_{-1}^{+1} dx \Big[\mathcal{F}_4 P_{l+1} + \mathcal{F}_8 P_l \Big] \qquad (204)$$

Finally, using equation (187), (192), (194) and (199) we can write down expressions for $\mathbb{F}_i(B.A)$. They are

$$\Gamma_s (\mathbb{F}_1) = -(W+M_1) g_Y G_3$$

$$\Gamma_s (\mathbb{F}_2) = -(W+M_1) g_Y G_4$$

$$\Gamma_s (\mathbb{F}_3) = 0 = \Gamma_s (\mathbb{F}_6)$$

$$\Gamma_s (\mathbb{F}_4) = -(W+M_1) g_Y G_3 + (W+M_1) k_o g_Y G_4 \qquad (205)$$

$$\Gamma_s (\mathbb{F}_5) = (W-M_1) g_Y G_3$$

$$\Gamma_s\ (\mathbb{F}_7) = -(W-M_1)\ g_Y\ G_4$$

$$\Gamma_s\ (\mathbb{F}_8) = (W-M_1)\ g_Y\ G_3 + (W-M_1)\ k_o\ g_Y\ G_4$$

$$\Gamma_u\ (\mathbb{F}_1) = (W+2M_2-M_1)\ g_Y\ G_3'$$

$$\Gamma_u\ (\mathbb{F}_2) = (W+M_1)\ g_Y\ G_4'$$

$$\Gamma_u\ (\mathbb{F}_3) = 2g_Y G_3' = -\Gamma_u(\mathbb{F}_6) \qquad (206)$$

$$\Gamma_u\ (\mathbb{F}_4) = \left[(W+2M_2-M_1)-2q_o\right] g_Y G_3' - k_o(W+M_1)\ g_Y G_4'$$

$$\Gamma_u\ (\mathbb{F}_5) = -(W-2M_2+M_1)\ g_Y\ G_3'$$

$$\Gamma_u\ (\mathbb{F}_7) = (W-M_1)\ g_Y\ G_4'$$

$$\Gamma_u\ (\mathbb{F}_8) = -\left[(W-2M_2+M_1)-2q_o\right] g_Y G_3' - k_o(W-M_1) g_Y G_4'$$

and

$$\Gamma_t\ (\mathbb{F}_i) = 0 \qquad i = 1, \ldots..8. \qquad (207)$$

This completes the angular momentum decomposition for the s-channel. As the analysis of the u and t channels does not enter into the subsequent work we omit to give it.

v. THE DIFFERENTIAL CROSS SECTION

We treat the case of the final lepton being an electron and so the supplementary condition will reduce the number of effective amplitudes from eight to six in the axial vector part. The results of I. ix equations (127) to (135) can be taken over for the present analysis if we employ the following substitutions:

$$\text{equation (132)} \rightarrow \mathcal{J} = (\mathcal{J}_V + i\,\mathcal{J}_A)_\alpha \; e_\alpha \tag{208}$$

$$\text{equation (133)} \rightarrow \mathcal{J}^{\,V}_{\alpha} = \text{expression (133)}$$

$$\mathcal{J}^{\,A}_{\alpha} = \underline{\sigma}\cdot\underline{q}\;\sigma_\alpha \mathcal{J}_1 \cdot$$

$$+ \underline{\sigma}\cdot\underline{q}\;\underline{\sigma}\cdot\underline{k}\;k_\alpha\;(\mathcal{J}_2 + \frac{k}{k_o}\mathcal{J}_4) + \underline{\sigma}\cdot\underline{q}\;\underline{\sigma}\cdot\underline{k}\;q_\alpha \mathcal{J}_3$$

$$+ \underline{\sigma}\cdot\underline{k}\;\sigma_\alpha \mathcal{J}_5 + q_\alpha \mathcal{J}_6 + k_\alpha\;(\mathcal{J}_7 + \frac{k}{k_o}\mathcal{J}_8) \tag{209}$$

where we have used the supplementary condition $k.e = 0$ to eliminate the terms proportional to e_o.

Hence

$$|\mathcal{M}|^2 = T_{\alpha\beta}\; e_{\alpha\beta} \tag{210}$$

where now

$$T_{\alpha\beta} = \tfrac{1}{2}\,\text{Trace}\Big[\mathcal{J}^{V}_{\alpha}\,\mathcal{J}^{V\,*}_{\beta} + \mathcal{J}^{A}_{\alpha}\,\mathcal{J}^{A*}_{\beta} \tag{211}$$

$$+ i\,\mathcal{J}^{A}_{\alpha}\mathcal{J}^{V*}_{\beta} - i\,\mathcal{J}^{V}_{\alpha}\mathcal{J}^{A*}_{\beta}\Big]$$

and

$$e_{\alpha\beta} = \frac{2\,g^4}{(\lambda^2 + M^2)^2}\ \text{Trace}\left[-\not{r}_1 \gamma_\beta (1+\gamma_5)\not{r}_2 \gamma_\alpha (1+\gamma_5) \right] \qquad (212)$$

This trace can easily be performed and gives

$$e_{\alpha\beta} = \frac{16 g^4}{(\lambda^2+M^2)^2}\left[2\, r_{1\alpha} r_{1\beta} - k_\alpha r_{1\beta} - k_\beta r_{1\alpha} + \tfrac{1}{2}\lambda^2 \delta_{\alpha\beta} + i\varepsilon_{\alpha p\beta q} k_p r_{1q} \right] \qquad (213)$$

where the Greek indices run over 1, 2 and 3 and the Roman indices over o,1,2, and 3.

When the tensor contraction (210) is finally executed the resulting expression arises from symmetric and antisymmetric products. In the tensor $e_{\alpha\beta}$ the first three terms are symmetric with respect to the interchange $\alpha \longleftrightarrow \beta$ whereas the last term is antisymmetric. Denoting these by $e^S_{\alpha\beta}$ and $e^A_{\alpha\beta}$ and the corresponding symmetric and antisymmetric parts of $T_{\alpha\beta}$ by $T^S_{\alpha\beta}$ and $T^A_{\alpha\beta}$ the contraction (210) becomes

$$|\mathcal{M}|^2 = T^S_{\alpha\beta}\ e^S_{\alpha\beta} + T^A_{\alpha\beta}\ e^A_{\alpha\beta} \qquad (214)$$

After some lengthy calculation of the traces the following expressions are obtained

$$T^S_{\alpha\beta} = (\alpha_o + \alpha_1)\,\delta_{\alpha\beta} + (\beta_o + \beta_1)\,\hat{k}_\alpha \hat{k}_\beta$$

$$+ \quad (\gamma_o+\gamma_1)\,\hat{q}_\alpha\hat{q}_\beta \quad + \quad (\delta_o+\delta_1)\,\tfrac{1}{2}\,(\hat{q}_\alpha\hat{k}_\beta + \hat{q}_\beta\hat{k}_\alpha)$$

$$- \hat{k}\,\hat{q}_\mu\Big[\tfrac{1}{2}\,(\varepsilon_{\alpha\lambda\mu}\,\hat{k}_\beta + \varepsilon_{\beta\lambda\mu}\,\hat{k}_\alpha)\;2\,\mathrm{Im}\,(m_o)$$

$$+\tfrac{1}{2}\,(\varepsilon_{\alpha\lambda\mu}\hat{q}_\beta + \varepsilon_{\beta\lambda\mu}\,\hat{q}_\alpha)\;2\mathrm{Im}\,(m_1)\Big] \qquad (215)$$

and

$$T^A_{\alpha\beta} = i\,(e_o+e_1)\,\tfrac{1}{2}\,(\hat{q}_\alpha\hat{k}_\beta - \hat{q}_\beta\hat{k}_\alpha)$$

$$+ i\,\varepsilon_{\alpha\beta\gamma}\,(l_o\,\hat{k}_\gamma + l_1\hat{q}_\gamma)$$

$$+ i\hat{k}_\lambda\hat{q}_\mu\Big[\tfrac{1}{2}\,(\varepsilon_{\alpha\lambda\mu}\hat{k}_\beta - \varepsilon_{\beta\lambda\mu}\,\hat{k}_\alpha)\;2\mathrm{Re}\,(m_o)$$

$$+ \tfrac{1}{2}\,(\varepsilon_{\alpha\lambda\mu}\,\hat{q}_\beta - \varepsilon_{\beta\lambda\mu}\hat{q}_\alpha)\;2\mathrm{Re}\,(m_1)\Big] \qquad (216)$$

where

$$\alpha_o = |\mathcal{J}_1|^2 + |\mathcal{J}_2|^2 - 2x\;\mathrm{Re}\,(\mathcal{J}_1\mathcal{J}_2^*)$$

$$\beta_o = \left[\left(\frac{k}{k_o}\right)^4 - 2\left(\frac{k}{k_o}\right)^2\right]\alpha_o + \left[1 - \left(\frac{k}{k_o}\right)^2\right]^2 B$$

$$+ x^2\left(\frac{k}{k_o}\right)^4\gamma_o + \left(\frac{k}{k_o}\right)^2\left[1 - \left(\frac{k}{k_o}\right)^2\right]2\,x\;\mathrm{Re}\,(\tau_1)$$

$$\gamma_o = |J_3|^2 + |J_4|^2 + 2\mathrm{Re}\left[J_1 J_4^* + J_2 J_3^* + x J_3 J_4^*\right]$$

$$\delta_o = -2\left(\frac{k}{k_o}\right)^2 x\gamma_o + \left[1 - \left(\frac{k}{k_o}\right)^2\right] 2\mathrm{Re}\ (\tau_1)$$

(217)

$$e_o = \left[1 - \left(\frac{k}{k_o}\right)^2\right] 2\mathrm{Im}\ (\tau_1)$$

$$\tau_1 = J_1 J_6^* + J_2 J_1^* + J_2 J_5^* + J_3 J_1^* + J_3 J_5^*$$

$$+ J_4 J_6^* + x\ (J_3 J_6^* - J_3 J_2^* + J_4 J_5^*)$$

$$B = -|J_2|^2 + |J_5|^2 + |J_6|^2 + 2\mathrm{Re}\ (J_1 J_5^* - x J_2 J_5^*$$

$$+ x J_5 J_6^*)$$

for the VV part

$$\alpha_1 = |J_1|^2 + |J_5|^2 + |J_8|^2 + 2x\ \mathrm{Re}\ (J_1 J_5^*)$$

$$\beta_1 = \left|J_2 + \frac{k}{k_o} J_4\right|^2 + \left|J_7 + \frac{k}{k_o} J_8\right|^2$$

$$+ 2\mathrm{Re}\left[\left\{J_1 + 2x J_5 + x\left(J_7 + \frac{k}{k_o} J_8\right)\right\}\left(J_2 + \frac{k}{k_o} J_4\right)^*\right.$$

$$\left. + J_5\left(J_7 + \frac{k}{k_o} J_8\right)^*\right]$$

$$\gamma_1 = |\mathcal{F}_3|^2 + |\mathcal{F}_6|^2 + 2\mathrm{Re}\,(\mathcal{F}_1 \mathcal{F}_6^* - \mathcal{F}_3 \mathcal{F}_5^* + x\mathcal{F}_3 \mathcal{F}_6^*)$$

$$\delta_1 = 2\mathrm{Re}\,(\tau_2) \qquad (218)$$

$$\theta_1 = 2\mathrm{Im}\,(\tau_2)$$

$$\tau_2 = \mathcal{F}_3 \mathcal{F}_1^* + \mathcal{F}_1 \mathcal{F}_5^* - \mathcal{F}_5 \mathcal{F}_1^* + (2x\mathcal{F}_3 + \mathcal{F}_6)\mathcal{F}_5^*$$

$$+ (\mathcal{F}_1 + x\mathcal{F}_3 + \mathcal{F}_6)\left(\mathcal{F}_7 + \frac{k}{k_o}\mathcal{F}_8\right)^*$$

$$+ (\mathcal{F}_3 - \mathcal{F}_5 + x\mathcal{F}_6)\left(\mathcal{F}_2 + \frac{k}{k_o}\mathcal{F}_4\right)^*$$

for the AA part, and

$$l_o = 2\mathrm{Re}\left[\mathcal{F}_1 \mathcal{F}_5^* - \mathcal{F}_2 \mathcal{F}_1^* - 2x\mathcal{F}_2 \mathcal{F}_5^*\right]$$

$$l_1 = 2\mathrm{Re}\left[\mathcal{F}_1 \mathcal{F}_1^* + \mathcal{F}_2 \mathcal{F}_5^*\right]$$

$$m_o = \mathcal{F}_1\left(\mathcal{F}_2 + \frac{k}{k_o}\mathcal{F}_4\right)^* + \mathcal{F}_2\left(\mathcal{F}_7 + \frac{k}{k_o}\mathcal{F}_8\right)^* \qquad (219)$$

$$+ \mathcal{F}_1\left[\mathcal{F}_5 - \left(\frac{k}{k_o}\right)^2 (\mathcal{F}_1 + x\mathcal{F}_3 + \mathcal{F}_5)\right]^*$$

$$- \mathcal{F}_5\left[\mathcal{F}_2 + \mathcal{F}_6 - \left(\frac{k}{k_o}\right)^2 (x\mathcal{F}_4 + \mathcal{F}_6)\right]^*$$

$$m_1 = \mathcal{F}_1 \mathcal{F}_3^* + \mathcal{F}_2 \mathcal{F}_6^* + \mathcal{F}_1 \mathcal{F}_3^* - \mathcal{F}_5 \mathcal{F}_4^*$$

for the VA and AV parts.

The quantity $|\mathcal{M}|^2$ can now be formed by contracting the relevant parts of (213) with (215) and (216). Thus

$$\frac{(\lambda^2 + M^2)^2}{16g^4} |\mathcal{M}|^2 = (\alpha_o+\alpha_1)\left[2\underline{r}_1^2 - 2\underline{k}\cdot\underline{r}_1 + \tfrac{3}{2}\lambda^2 \right]$$

$$+ (\beta_o+\beta_1)\frac{1}{\underline{k}^2}\left[2(\underline{k}\cdot\underline{r}_1)^2 - 2\underline{k}^2\,\underline{k}\cdot\underline{r}_1 + \tfrac{1}{2}\lambda^2\,\underline{k}^2 \right]$$

$$+ (\gamma_o+\gamma_1)\frac{1}{\underline{q}^2}\left[2(\underline{q}\cdot\underline{r}_1)^2 - 2\underline{q}\cdot\underline{k}\;\underline{q}\cdot\underline{r}_1 + \tfrac{1}{2}\lambda^2\,\underline{q}^2 \right]$$

$$+ (\delta_o+\delta_1)\frac{1}{qk}\left[2\underline{q}\cdot\underline{r}_1\;\underline{k}\cdot\underline{r}_1 - \underline{q}\cdot\underline{k}\;\underline{k}\cdot\underline{r}_1 - \underline{k}^2\;\underline{q}\cdot\underline{r}_1 + \tfrac{1}{2}\lambda^2\,\underline{q}\cdot\underline{k} \right]$$

$$- 2\,\mathrm{Im}\,(m_o)\,\frac{1}{qk^2}\left[\underline{r}_1\cdot\underline{k}_\wedge\underline{q}\;(2\underline{k}\cdot\underline{r}_1 - \underline{k}^2) \right]$$

$$-2\,\mathrm{Im}\,(m_1)\,\frac{1}{q^2k}\left[\underline{r}_1\cdot\underline{k}_\wedge\underline{q}\;(2\underline{q}\cdot\underline{r}_1 - \underline{q}^2) \right]$$

$$-(e_o + e_1)\frac{1}{qk}\left[k_o\;(\underline{r}_1\cdot\underline{k}_\wedge\underline{q}) \right]$$

$$-2l_o\,\frac{1}{k}\left[\underline{k}^2\;r_{10} - \underline{r}_1\cdot\underline{k}\;k_o \right]$$

$$-2l_1\,\frac{1}{q}\left[\underline{q}\cdot\underline{k}\;r_{10} - \underline{q}\cdot\underline{r}_1\;k_o \right]$$

$$-2\,\mathrm{Re}\,(m_o)\,\frac{k_o}{qk^2}\left[\underline{q}\cdot\underline{k}\;\underline{k}\cdot\underline{r}_1 - \underline{k}^2\;\underline{q}\cdot\underline{r}_1 \right]$$

$$- 2 \text{ Re } (m_1) \frac{1}{q^2 k} \left[r_{10} \ (\underline{q}\cdot\underline{k}\ \underline{q}\cdot\underline{k} - \underline{q}^2 k^2) \right.$$

$$\left. -k_o \ (\underline{q}\cdot\underline{k}\ \underline{q}\cdot\underline{r}_1 - \underline{k}\cdot\underline{r}_1\ \underline{q}^2) \right] \qquad (220)$$

Hence the differential cross section takes the form (c.f. (139))

$$\frac{d^2\sigma}{dR_2 d\Omega} = \frac{g^4}{(2\pi)^5} \frac{R_2 q}{2M_1 R_1 W} \frac{1}{(\lambda^2+M^2)^2} \int \left[\text{R.H.S. of (220)} \right] d\Omega_q \qquad (221)$$

To perform the integration indicated above we first of all note that the scalar and triple scalar products occuring in (220) can be expressed tricnometrically with the aid of the angles defined in Figure IV. Thus

$$d\Omega = d(\cos\vartheta)\, d\Phi$$

$$\cos\alpha = \cos\vartheta \cos\psi + \sin\vartheta \sin\psi \cos\Phi$$

$$\sin\alpha \sin\omega = \sin\vartheta \sin\Phi$$

and

$$\underline{q}\cdot\underline{r}_1 = q\, r_1 \cos\alpha \qquad \underline{k}\cdot\underline{r}_1 = k r_1 \cos\psi$$

$$\underline{q}\cdot\underline{k} = qk\cos\vartheta$$

$$\underline{r}_1 \underline{k} \underline{q} = r_1 k q \sin\psi \sin\alpha \sin\omega = r_1 k q \sin\psi \sin\vartheta \sin\Phi$$

so that all quantities under tho integral in (221) can be expressed in terms of the angles ψ, ϑ, and Φ. The integration over Φ can be performed

immediately since the functions of (217), (218) and (219) are independent of Φ. Integrating over Φ and redefining the integral of (221) as

$$\int_{-1}^{+1} 4\pi F(\cos\vartheta)\ d(\cos\vartheta)$$

we have

$$\begin{aligned} F(\cos\vartheta) = {} & (\alpha_o + \alpha_1)(\rho_1+\rho_2) + (\beta_o+\beta_1)\rho_1 \\ & + (\gamma_o+\gamma_1)(\cos^2\vartheta\rho_1 + \tfrac{1}{2}\sin^2\vartheta\,\rho_2) \\ & + (\delta_o+\delta_1)\cos\vartheta\,\rho_1 - 2l_o\,\rho_3 - 2l_1\cos\vartheta\,\rho_3 \\ & + 2\,\mathrm{Re}\,(m_1)\sin^2\vartheta\,\rho_3 \end{aligned} \tag{222}$$

where ρ_1 and ρ_2 are given by (141) and

$$\rho_3 = \tfrac{1}{2} r_1 (k - k_o\cos\psi) \tag{223}$$

For convenience we decompose the function $F(\cos\vartheta)$ into parts arising from pure vector, pure axial vector and vector-axial vector interference terms. Thus

$$F(\vartheta) = F_{VV}(\vartheta) + F_{AA}(\vartheta) + F_{VA}(\vartheta) \tag{224}$$

and the three functions are defined in Tables 7, 8 and 9 respectively.

TABLE 7

	$\mathcal{H}_1^*$	$\mathcal{H}_2^*$	$\mathcal{H}_3^*$	$\mathcal{H}_4^*$	$\mathcal{H}_5^*$	$\mathcal{H}_6^*$
$\mathcal{H}_1$	$\mathcal{M}\rho_1 + \rho_2$	$-x\rho_2$	$\mathcal{M}x\rho_1$	$\mathcal{M}x^2\rho_1$ $+\frac{1}{2}(1-x^2)\rho_2$	$\mathcal{M}\rho_1$	$\mathcal{M}x\rho_1$
$\mathcal{H}_2$	$-x\rho_2$	ρ_2	$\frac{1}{2}(1-x^2)\rho_2$	0	0	0
$\mathcal{H}_3$	$\mathcal{M}x\rho_1$	$\frac{1}{2}(1-x^2)\rho_2$	$\mathcal{M}x^2\rho_1$ $+\frac{1}{2}(1-x^2)\rho_2$	$\mathcal{M}x^3\rho_1$ $+\frac{1}{2}x(1-x^2)\rho_2$	$\mathcal{M}x\rho_1$	$\mathcal{M}x^2\rho_1$
$\mathcal{H}_4$	$\mathcal{M}x^2\rho_1$ $+\frac{1}{2}(1-x^2)\rho_2$	0	$\mathcal{M}x^3\rho_1$ $+\frac{1}{2}x(1-x^2)\rho_2$	$\mathcal{M}x^2\rho_1$ $+\frac{1}{2}(1-x^2)\rho_2$	$\mathcal{M}x^2\rho_1$	$\mathcal{M}x\rho_1$
$\mathcal{H}_5$	$\mathcal{M}\rho_1$	0	$\mathcal{M}x\rho_1$	$\mathcal{M}x^2\rho_1$	$\mathcal{M}\rho_1$	$\mathcal{M}x\rho_1$
$\mathcal{H}_6$	$\mathcal{M}x\rho_1$	0	$\mathcal{M}x^2\rho_1$	$\mathcal{M}x\rho_1$	$\mathcal{M}x\rho_1$	$\mathcal{M}\rho_1$

TABLE 8

	$\mathcal{F}_1^*$	$\mathcal{F}_2^* + \frac{k}{k_0}\mathcal{F}_4^*$	$\mathcal{F}_3^*$	$\mathcal{F}_5^*$	$\mathcal{F}_6^*$	$\mathcal{F}_7^* + \frac{k}{k_0}\mathcal{F}_8^*$
$\mathcal{F}_1$	$\rho_1 + \rho_2$	ρ_1	$x\rho_1$	$x(\rho_1 + \rho_2)$	$x^2\rho_1 + \frac{1}{2}(1-x^2)\rho_2$	$x\rho_1$
$\mathcal{F}_2 + \frac{k}{k_0}\mathcal{F}_4$	ρ_1	ρ_1	$x\rho_1$	$x\rho_1$	$x^2\rho_1$	$x\rho_1$
$\mathcal{F}_3$	$x\rho_1$	$x\rho_1$	$x^2\rho_1 + \frac{1}{2}(1-x^2)\rho_2$	$x^2\rho_1 - \frac{1}{2}(1-x^2)\rho_2$	$x^3\rho_1 + \frac{1}{2}x(1-x^2)\rho_2$	$x^2\rho_1$
$\mathcal{F}_5$	$x(\rho_1 + \rho_2)$	$x\rho_1$	$x^2\rho_1 - \frac{1}{2}(1-x^2)\rho_2$	$\rho_1 + \rho_2$	$x\rho_1$	ρ_1
$\mathcal{F}_6$	$x^2\rho_1 + \frac{1}{2}(1-x^2)\rho_2$	$x^2\rho_1$	$x^3\rho_1 + \frac{1}{2}x(1-x^2)\rho_2$	$x\rho_1$	$x^2\rho_1 + \frac{1}{2}(1-x^2)\rho_2$	$x\rho_1$
$\mathcal{F}_7 + \frac{k}{k_0}\mathcal{F}_8$	$x\rho_1$	$x\rho_1$	$x^2\rho_1$	ρ_1	$x\rho_1$	ρ_1

TABLE 9

	$\mathcal{F}_1^*$	$\mathcal{F}_2^* + \frac{k}{k_0}\mathcal{F}_4^*$	$\mathcal{F}_3^*$	$\mathcal{F}_5^*$	$\mathcal{F}_6^*$	$\mathcal{F}_7^* + \frac{k}{k_0}\mathcal{F}_8^*$
$\mathcal{F}_1$	$-2x\rho_3$	0	$(1-x^2)\rho_3$	$-2\rho_3$	0	0
$\mathcal{F}_2$	$2\rho_3$	0	0	$2x\rho_3$	$(1-x^2)\rho_3$	0
$\mathcal{F}_3$	$(1-x^2)\rho_3$	0	0	0	0	0
$\mathcal{F}_4$	0	0	0	$-(1-x^2)\rho_3$	0	0
$\mathcal{F}_5$	0	0	0	0	0	0
$\mathcal{F}_6$	0	0	0	0	0	0

As we are interested only in the $P_{\frac{1}{2}}$ and $F_{5/2}$ resonances we proceed with the cosθ integration by retaining only the relevant multipoles in (203). For the $P_{\frac{1}{2}}$ resonance they are M_{1-}, L_{1-} and $\mathcal{E}_{1-}$, $\mathcal{S}_{1-}$ for the vector and axial vector amplitudes respectively. For the $F_{5/2}$ resonance they are M_{3-}, E_{3-}, L_{3-} and $\mathcal{M}_{3-}$, $\mathcal{E}_{3-}$, $\mathcal{S}_{3-}$ for the vector and axial vector amplitudes respectively. It should be noted that the supplementary condition allows the elimination of either the scalar of the longitudinal multipoles. This can be seen by noting that the combinations $\mathcal{F}_2 + \frac{k}{k_o}\mathcal{F}_4$ and $\mathcal{F}_7 + \frac{k}{k_o}\mathcal{F}_8$ modify the multipole expansion (203) and thereby introduce the quantity $\mathcal{L} + \frac{k}{k_o}\mathcal{S}$ which we denote by $\mathcal{S}$.

$P_{\frac{1}{2}}$ resonance.

For the axial vector amplitudes we have

$$\mathcal{F}_1 = \mathcal{E}_{1-} \qquad \mathcal{F}_2 + \frac{k}{k_o}\mathcal{F}_4 = -\mathcal{E}_{1-} + \frac{k}{k_o}\mathcal{S}_{1-}$$

$$\mathcal{F}_3 = \mathcal{F}_5 = \mathcal{F}_6 = \mathcal{F}_7 + \frac{k}{k_o}\mathcal{F}_8 = 0$$

and

$$\mathcal{F}_2 = M_{1-} \qquad \mathcal{F}_6 = L_{1-}$$

$$\mathcal{F}_1 = \mathcal{F}_3 = \mathcal{F}_4 = \mathcal{F}_5 = 0$$

for the vector amplitudes.

We therefore have that

$$F_{vv}(x) = \rho_2 (M_{1-})^2 + \xi\rho_1 (L_{1-})^2$$

$$F_{AA}(x) = \rho_2 (\xi_{1-})^2 + \rho_1 \left(\frac{k}{k_o} \zeta_{1-}\right)^2 \qquad (225)$$

$$F_{VA}(x) = 4\rho_3 (M_{1-})(\xi_{1-})$$

Thus

$$\int_{-1}^{+1} F(x)\,dx = 2\Big[\Big\{ \rho_2 (M_{1-})^2 + \xi\rho_1 (L_{1-})^2 \Big\} + 4\rho_3 (M_{1-})(\xi_{1-}) + \Big\{ \rho_2 (\xi_{1-})^2 + \rho_1 \left(\frac{k}{k_o}\zeta_{1-}\right)^2 \Big\} \Big] \qquad (226)$$

$F_{5/2}$ resonance.

For the axial vector amplitudes we have

$$\begin{pmatrix} \mathcal{F}_1 \\ \mathcal{F}_3 \\ \mathcal{F}_5 \\ \mathcal{F}_6 \\ \mathcal{F}_2 + \frac{k}{k_o}\mathcal{F}_4 \\ \mathcal{F}_7 + \frac{k}{k_o}\mathcal{F}_8 \end{pmatrix} = \begin{pmatrix} 3(5x^2-1) & \frac{3}{2}(5x^2-1) & 0 \\ -15x & 15x & 0 \\ -9x & 0 & 0 \\ 3 & -3 & 0 \\ 3 & -\frac{3}{2}(15x^2-1) & \frac{9}{2}(5x^2-1) \\ 6x & 3x & -9x \end{pmatrix} \begin{pmatrix} M_{3-} \\ \xi_{3-} \\ \frac{k}{k_o}\zeta_{3-} \end{pmatrix} \qquad (227)$$

and for the vector amplitudes the required expression is given by (145).

From (146) we already have that

$$\int_{-1}^{+1} F_{VV}(x)\,dx = 18\left[\rho_2\left\{2(M_{3-})^2+(E_{3-})^2\right\} + 3\rho_1\,(L_{3-})^2\right]$$

For the pure axial vector function $F_{AA}(x)$ we have from table.

$$F_{AA}(x) = \rho_2\left[\frac{9}{2} + 27x^2 - \frac{45}{2}\,x^4\right](\mathcal{M}_{3-})^2$$
$$+ \rho_2\left[\frac{9}{2}\,(1 - 18x^2 + 25x^4)\right](\mathcal{M}_{3-})(\mathcal{E}_{3-})$$
$$+ \rho_2\left[\frac{45}{4} + \frac{54}{4}\,x^2 + \frac{45}{4}\,x^4\right](\mathcal{E}_{3-})^2$$
$$+ \rho_1\left[\frac{81}{4}\,(1 - 2x^2 + 5x^4)\right]\left(\frac{k}{k_o}\,\mathcal{S}_{3-}\right)^2$$

Thus

$$\int_{-1}^{+1} F_{AA}(x)dx = 18\left[\rho_2\left\{(\mathcal{M}_{3-})^2 + 2\,(\mathcal{E}_{3-})^2\right\} + 3\rho_1\left(\frac{k}{k_o}\,\mathcal{S}_{3-}\right)^2\right] \quad (228)$$

For the interference function $F_{VA}(x)$ we have

$$F_{VA}(x) = 9\rho_3\left[(25x^4 - 18x^2+1)\left\{(M_{3-})(\mathcal{M}_{3-})-(\mathcal{E}_{3-})(E_{3-})\right\}\right.$$
$$+ 2\,(5x^4 - 6x^2 + 1)\,(E_{3-})\,(\mathcal{M}_{3-})$$
$$\left. + (5\,x^4 + 6x^2 + 5)\,(M_{3-})\,(\mathcal{E}_{3-})\right]$$

so that

$$\int_{-1}^{+1} F_{VA}(x)\,dx = 72\,\rho_3\left[2\,(M_{3-})\,(\mathcal{E}_{3-}) - (E_{3-})\,(\mathcal{M}_{3-})\right] \qquad (229)$$

Collecting terms together we have for the $F_{5/2}$ resonance

$$\begin{aligned}\int_{-1}^{+1} F(x)\,dx = 18\Big[& \rho_2\left\{2(M_{3-})^2 + (E_{3-})^2\right\} + 3\xi\rho_1\,(L_{3-})^2 \\ & + \rho_2\left\{2\,(\mathcal{E}_{3-})^2 + (\mathcal{M}_{3-})^2\right\} + 3\rho_1\left(\frac{k}{k_o}\mathcal{S}_{3-}\right)^2 \\ & + 4\rho_3\left\{2\,(M_{3-})(\mathcal{E}_{3-}) - (E_{3-})\,(\mathcal{M}_{3-})\right\}\Big] \end{aligned} \qquad (230)$$

vi. NUMERICAL ANALYSIS.

In the framework of our analysis the numerical analysis requires the form of the multipole projections of the Born terms $\mathbb{F}_i(B.A)$ given by (205) and (206). The relevant multipoles are $\mathcal{E}_{1-}$ and $\mathcal{S}_{1-}$ for the $P_{\frac{1}{2}}$ resonance and $\mathcal{M}_{3-}, \mathcal{E}_{3-}$ and $\mathcal{S}_{3-}$ for the $F_{5/2}$ resonance. By equations (204) we have

$$\mathcal{E}_{1-} = \tfrac{1}{2}\int_{-1}^{+1} dx\left[\mathcal{F}_1 P_o + \mathcal{F}_5 P_1 + \mathcal{F}_6\,\frac{1}{3}\,(P_o - P_2)\right]$$

$$\begin{aligned}\frac{k}{k_o}\mathcal{S}_{1-} = \tfrac{1}{2}\int_{-1}^{+1} dx\Big[& \left\{\mathcal{F}_1 + x\mathcal{F}_3 + \left(\mathcal{F}_2 + \frac{k}{k_o}\mathcal{F}_4\right)\right\} P_o \\ & + \left\{\mathcal{F}_5 + x\mathcal{F}_6 + \left(\mathcal{F}_7 + \frac{k}{k_o}\mathcal{F}_8\right)\right\} P_1\Big]\end{aligned}$$

$$\mathcal{M}_3 = \frac{1}{6} \int_{-1}^{+1} dx \left[\mathcal{J}_1 P_2 + \mathcal{J}_5 P_3 - \mathcal{J}_3 \frac{1}{5} (P_1 - P_2) \right]$$

$$\mathcal{E}_{3-} = \frac{1}{6} \int_{-1}^{+1} dx \left[\mathcal{J}_1 P_2 + \mathcal{J}_5 P_3 + \mathcal{J}_3 \frac{2}{5} (P_1 - P_3) + \mathcal{J}_6 \frac{3}{7} (P_2 - P_4) \right]$$

$$\frac{k}{k_o} \mathcal{S}_{3-} = \frac{1}{6} \int_{-1}^{+1} dx \left[\left\{ \mathcal{J}_1 + x \mathcal{J}_3 + \left(\mathcal{J}_2 + \frac{k}{k_o} \mathcal{J}_4 \right) \right\} P_2 + \left\{ \mathcal{J}_5 + x \mathcal{J}_6 + \left(\mathcal{J}_7 + \frac{k}{k_o} \mathcal{J}_8 \right) \right\} P_3 \right] \quad (231)$$

With the aid of the relations in I.x. and the following linear decomposition of the multipoles

$$\text{Multipole} = \sum_{i=1}^{3} \sigma_i \mathcal{M}_i \quad (232)$$

with

$$\sigma_i = (g_Y G_3 , g_Y G_3') \quad (233)$$

the integrations implied in (231) can be performed. The $\mathcal{M}_i$ are given by

$$\mathcal{E}_{1-}(\sigma_1) = \delta_1$$

$$\mathcal{E}_{1-}(\sigma_2) = \delta_2 Q_o + \delta_3 Q_1 - \delta_4 \frac{1}{3} (Q_o - Q_2)$$

$$\frac{k}{k_o}\mathcal{S}_{1-}(\sigma_1) = \delta_1 + \delta_2$$

$$\frac{k}{k_o}\mathcal{S}_{1-}(\sigma_2) = (\delta_6 - \delta_7)\, Q_o + (\delta_8 - \delta_9 - \delta_{10} - a\delta_4)\, Q_1 \qquad (234)$$

and

$$\mathcal{M}_{3-}(\sigma_1) = 0$$

$$\mathcal{M}_{3-}(\sigma_2) = \frac{1}{3}\left[\, \delta_2 Q_2 + \delta_3 Q_3 + \delta_{10}\, \frac{1}{5}\, (Q_1 - Q_3) \,\right]$$

$$\mathcal{E}_{3-}(\sigma_1) = 0$$

$$\mathcal{E}_{3-}(\sigma_2) = \frac{1}{3}\left[\delta_2 Q_2 + \delta_3 Q_3 - 2\delta_{10}\, \frac{1}{5}\, (Q_1 - Q_3) - 3\,\delta_4\, \frac{1}{7}\, (Q_2 - Q_4)\,\right]$$

$$\frac{k}{k_o}\mathcal{S}_{3-}(\sigma_1) = 0$$

$$\frac{k}{k_o}\mathcal{S}_{3-}(\sigma_2) = \frac{1}{3}\left[\, (\delta_6 - \delta_7 - a\delta_{10})\, Q_2 + (\delta_8 - \delta_9 - a\,\delta_4)\, Q_3 \,\right] \qquad (235)$$

where

$$\delta_1 = \frac{q O_2}{W - M_1} \qquad \delta_2 = \frac{q O_2\, (W + 2M_2 - M_1)}{2qk}$$

$$\delta_3 = \frac{k\,(W - 2M_2 + M_1)}{O_2\, 2qk} \qquad \delta_4 = \frac{2q\, O_1}{2qk}$$

$$\delta_3 = \frac{k\,(W - 2M_2 + M_1)}{O_2\ 2qk} \qquad \delta_4 = \frac{2q\,O_1}{2qk}$$

$$\delta_5 = \frac{qk^2}{k_o\ O_1(W-M_1)} \qquad \delta_6 = \frac{W+2M_2-M_1}{2qk}\left(qO_2 + \frac{qk^2}{k_o O_1}\right)$$

$$\delta_7 = \frac{2q_o\ qk^2}{2qk\ k_o\ O_1} \qquad \delta_8 = \frac{W-2M_2+M_1}{2qk}\left(\frac{k}{O_2} + \frac{kO_1}{k_o}\right)$$

$$\delta_9 = \frac{2q_o k\ O_1}{2qk\ k_o} \qquad \delta_{10} = \frac{2\,q^2\,k}{2qk\ O_1} \qquad (236)$$

Again taking μ=1 the functions above can be tabulated for the specific values W= 12 and λ^2 = 5, 10(10)50. The results are presented in Tables 10(a) and 10(b) for the $P_{\frac{1}{2}}$ and $F_{5/2}$ resonance respectively.

We have now to decide what the form factors in (233) should be. The vector part is fully determined by the conserved vector current theory so that the quantities of I.x. can be taken over with but slight modification. For the axial vector part considered here the form factors are again set as

$$a(\lambda^2) = a(o)\ F(\lambda^2)$$

with $F(\lambda^2)$ given by I.x.

For the form factor $G_3(\lambda^2)$ we know that $G_3(o) = -\,{}^{G_A}/_{G} = 1.2$ and for $G_3'(\lambda^2)$ we set it equal to unity at $\lambda^2 = 0$, although ideally this

TABLE 10(a)

$P_{\frac{1}{2}}$	5	10	20	30	40	50
$\mathcal{E}_{1-}(\sigma_1)$	0.2347	0.2363	0.2396	0.2428	0.2464	0.2490
$\mathcal{E}_{1-}(\sigma_2)$	0.2005	0.1989	0.1958	0.1928	0.1898	0.1869
$\frac{k}{k_o}\mathcal{S}_{1-}(\sigma_1)$	0.3158	0.3461	0.3845	0.4471	0.5332	0.6506
$\frac{k}{k_o}\mathcal{S}_{1-}(\sigma_2)$	0.2090	0.2102	0.2141	0.2208	0.2450	0.2662

TABLE 10(b)

$10^{-3}F_{5/2}$	5	10	20	30	40	50
$\mathcal{M}_3(\sigma_1)$	0	0	0	0	0	0
$\mathcal{M}_{3-}(\sigma_2)$	0.5492	0.6097	0.6828	0.8054	0.8927	0.9448
$\mathcal{E}_{3-}(\sigma_1)$	0	0	0	0	0	0
$\mathcal{E}_{3-}(\sigma_2)$	0.0832	0.0937	0.1128	0.1294	0.1444	0.1572
$\frac{k}{k_o}\mathcal{S}_{3-}(\sigma_1)$	0	0	0	0	0	0
$\frac{k}{k_o}\mathcal{S}_{3-}(\sigma_2)$	0.0660	0.0722	0.0825	0.0905	0.1126	0.1570

coupling should be considered as a parameter of the analysis. It turns out that the qualitative features are reproduced for a wide range of values of $G_3^1(o)$. The results for the process $W^+ + n \rightarrow K^+ + \Lambda$ for the two cases are presented in Table 11 and graphically in Figure X .

TABLE 11.

$\left(\frac{d^2\sigma}{dR_2 d\Omega} \right)_{\text{Resonance}}$ [Numbers in brackets denote power of 10^{-1}]

λ^2	5	10	20	30	40	50
$P_{\frac{1}{2}}$	2.87(42)	5.13(42)	6.14(42)	5.88(42)	6.51(42)	6.85(42)
$F_{5/2}$	.289(46)	1.00(46)	1.96(46)	3.11(46)	3.71(46)	3.98(46)

FIGURES

FIGURE I(a) — The four-vector diagram for the electroproduction of K mesons : $e(r_1)+N(p_1) \rightarrow e(-r_2)+Y(-p_2)+K(-q)$.

FIGURE I(b) — The electro-kaon production diagram to first order in the electromagnetic interaction.

FIGURE II. — The diagrams which give rise to poles.

FIGURE III. — The Feynman diagram for K^* exchange.

FIGURE IV. — The polar diagram for the definition of the angles occuring in the differential cross section formula.

FIGURE V. — Curves of constant barycentric energy W and constant invariant momentum transfer λ^2 as a function of the initial and final electron energies R_1 and R_2.

FIGURE VI. — The $P_{\frac{1}{2}}$ differential cross sections for $K^+\Lambda$ and $K^o\Lambda$ electroproduction in 10^{-35} cm^2/Mev.

FIGURE VII. — The $F_{5/2}$ differential cross sections for $K^+\Lambda$ and $K^o\Lambda$ electroproduction in 10^{-39} cm^2/Mev.

FIGURE VIII. — The Weak production diagram to first order in the weak interaction.

FIGURE IX. — The diagram which give rise to poles in the weak production.

FIGURE X. — The $P_{\frac{1}{2}}$ and $F_{5/2}$ differential cross sections for the process $\nu + n \rightarrow e^- + K^+ + \Lambda$ with units of 10^{-42} and 10^{-46} cm^2/Mev respectively.

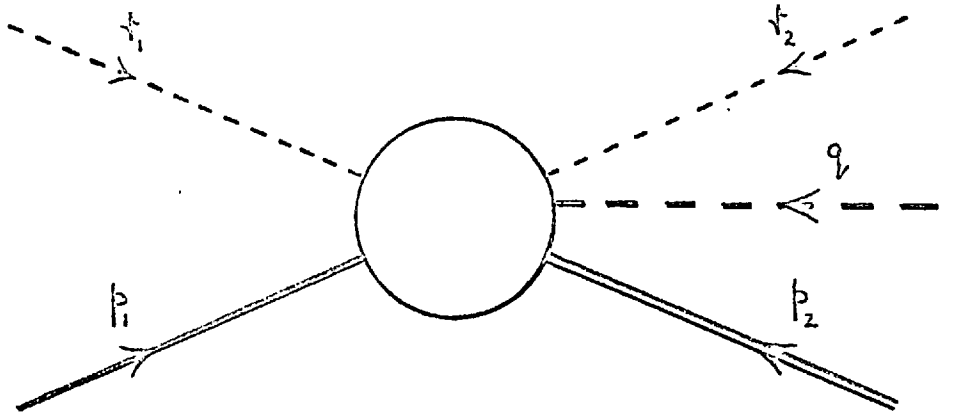


FIGURE 1(a)

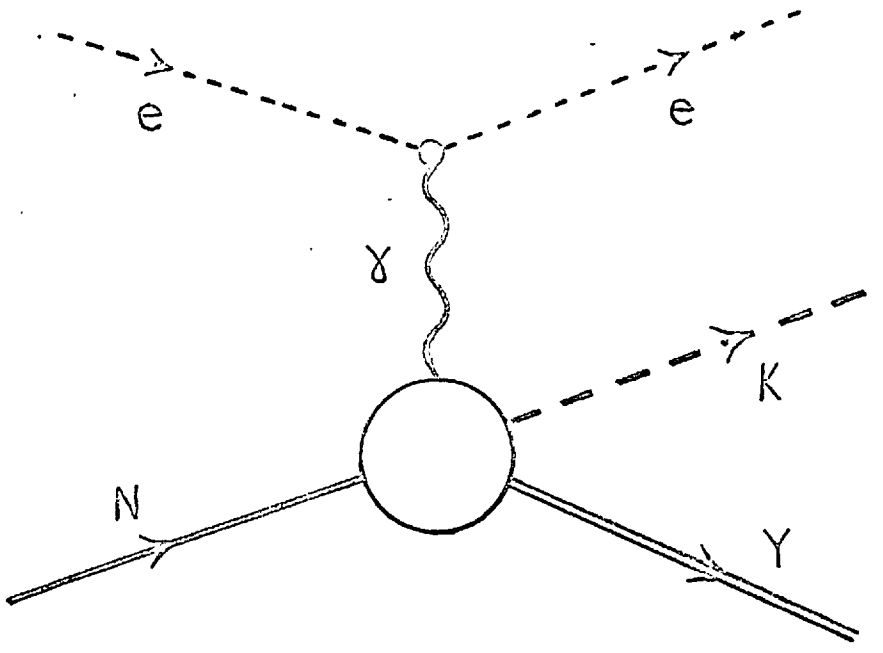


FIGURE 1(b)

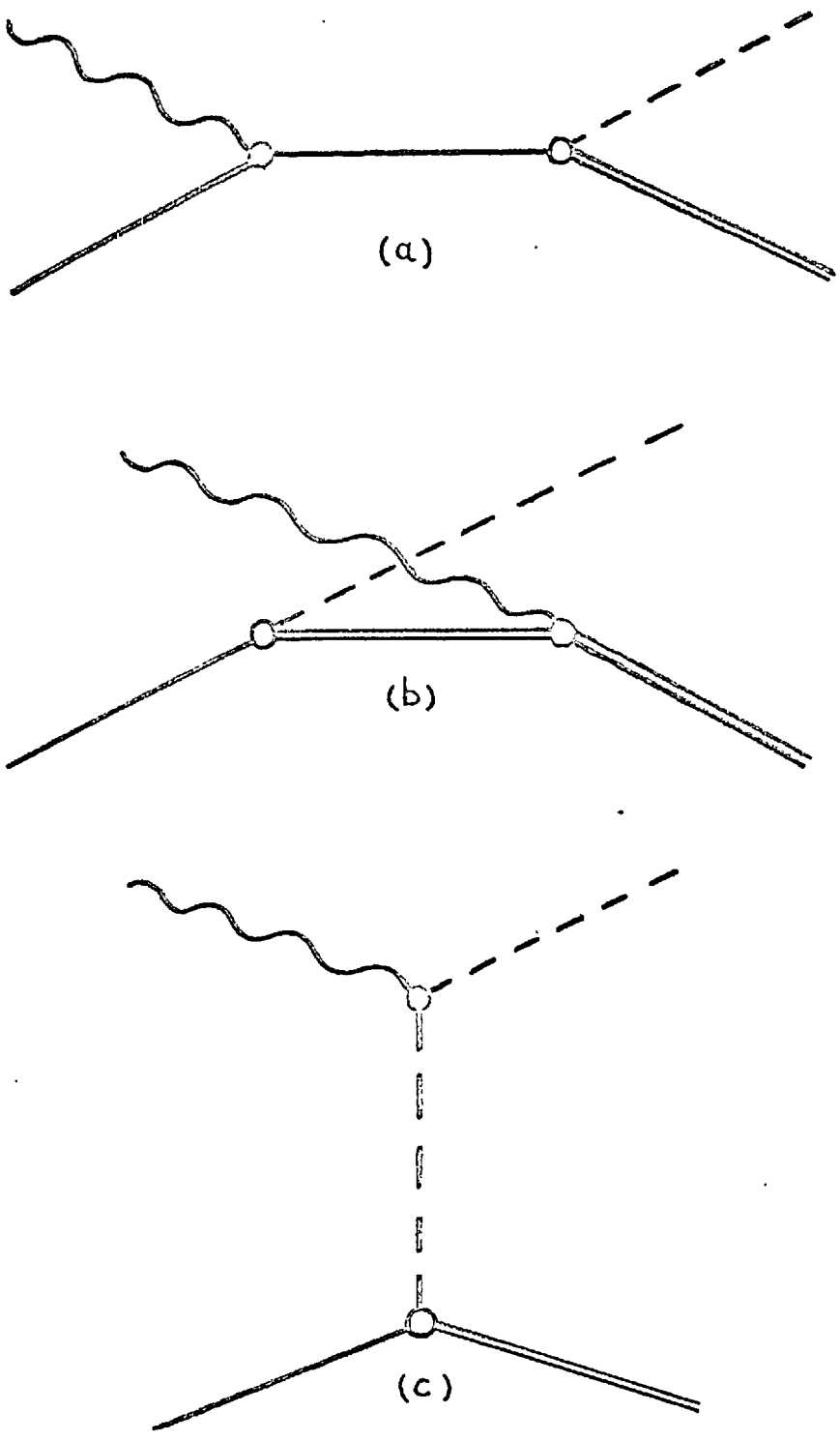


FIGURE II

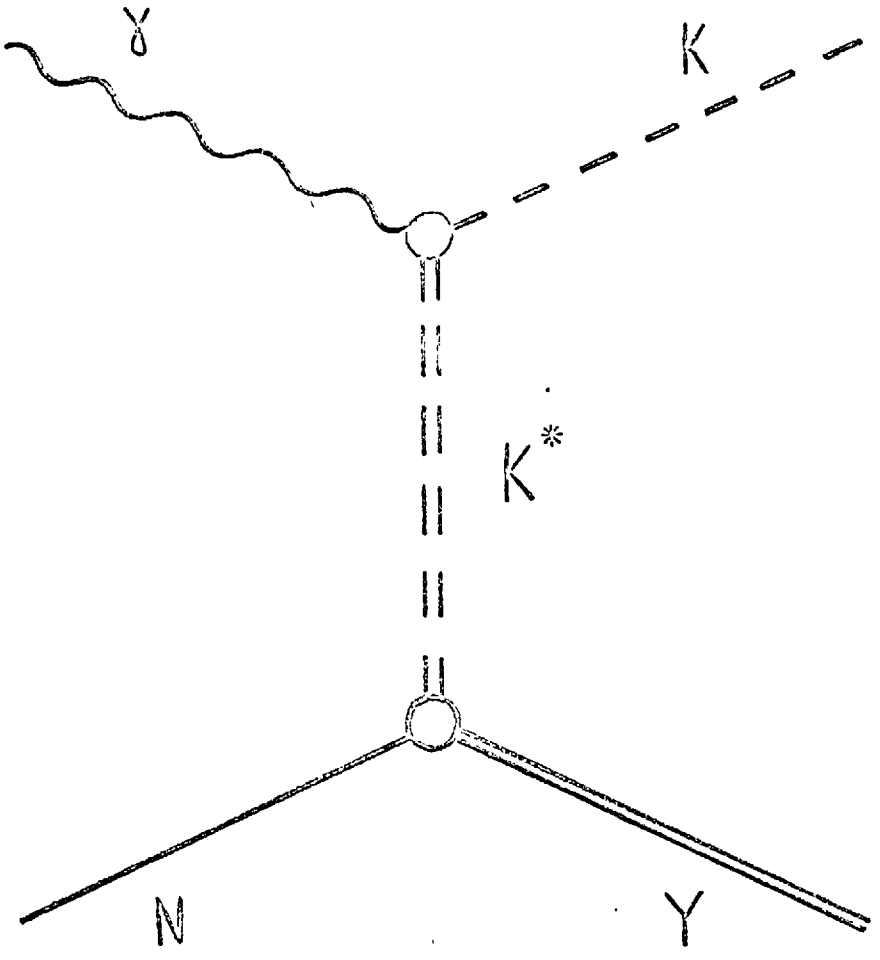


FIGURE III

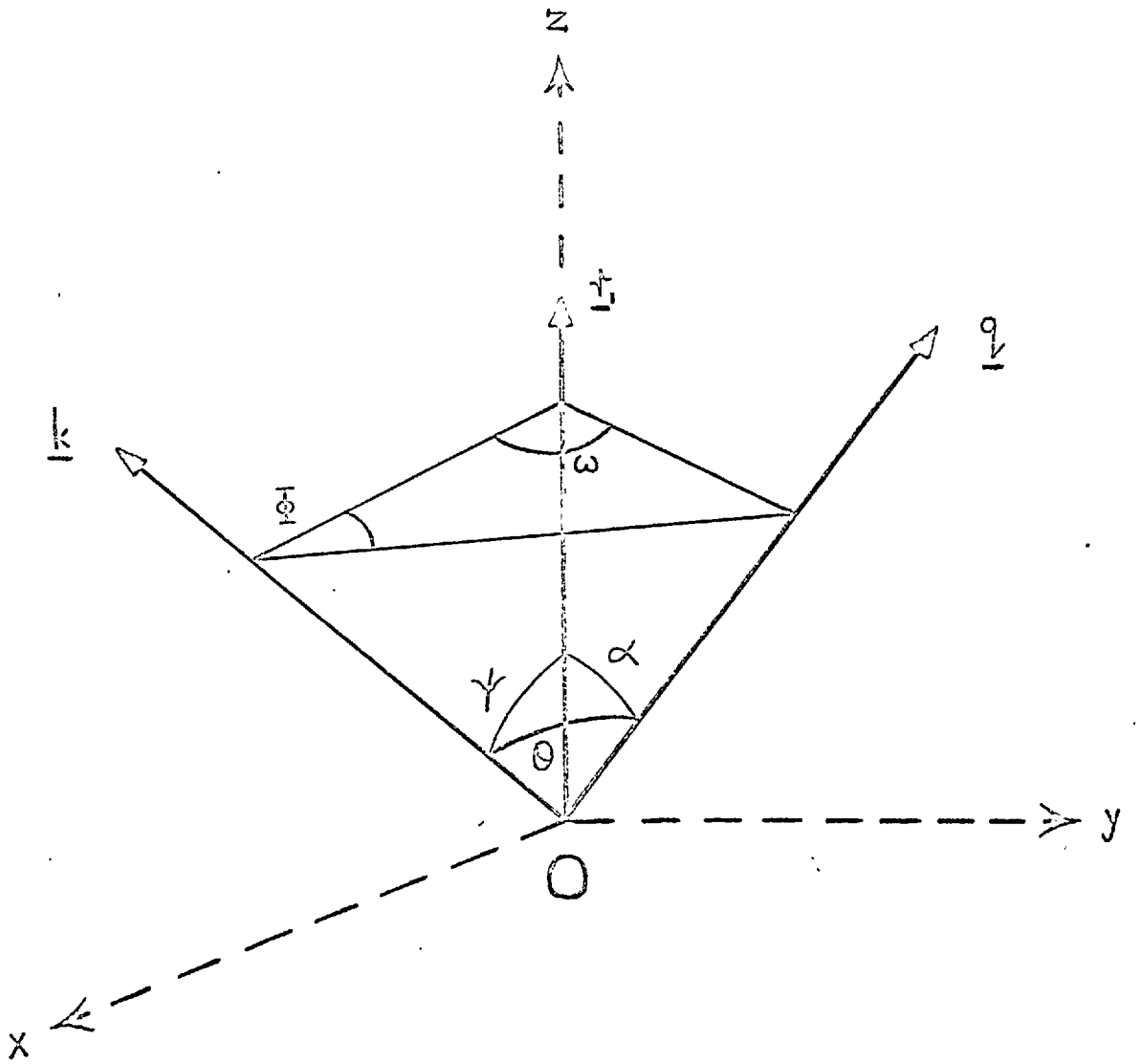


FIGURE IV

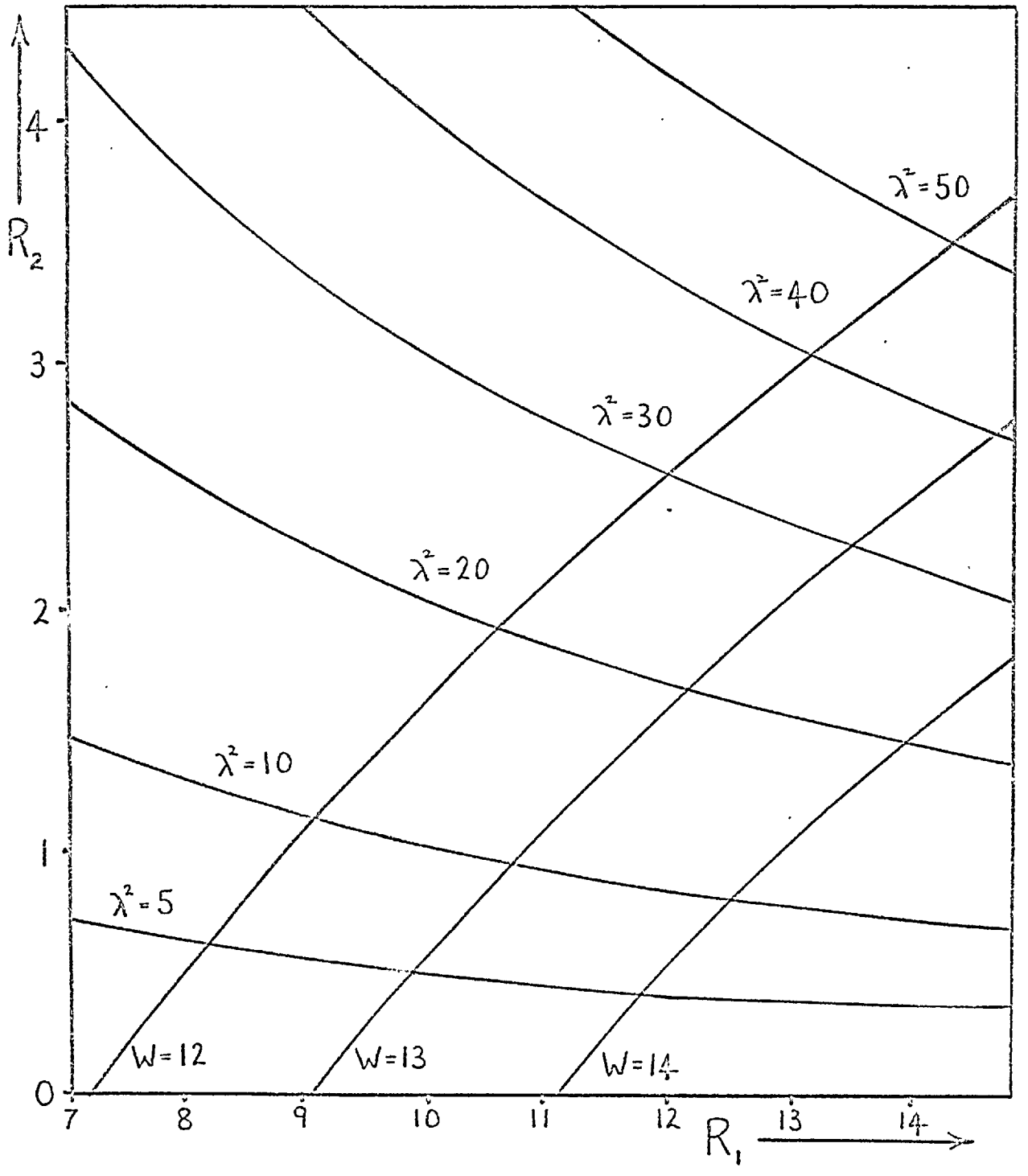

R_2
4
3
2
1
0
7
8
9
10
11
12
13
14
R_1
$\lambda^2 = 50$
$\lambda^2 = 40$
$\lambda^2 = 30$
$\lambda^2 = 20$
$\lambda^2 = 10$
$\lambda^2 = 5$
W = 12
W = 13
W = 14

FIGURE V

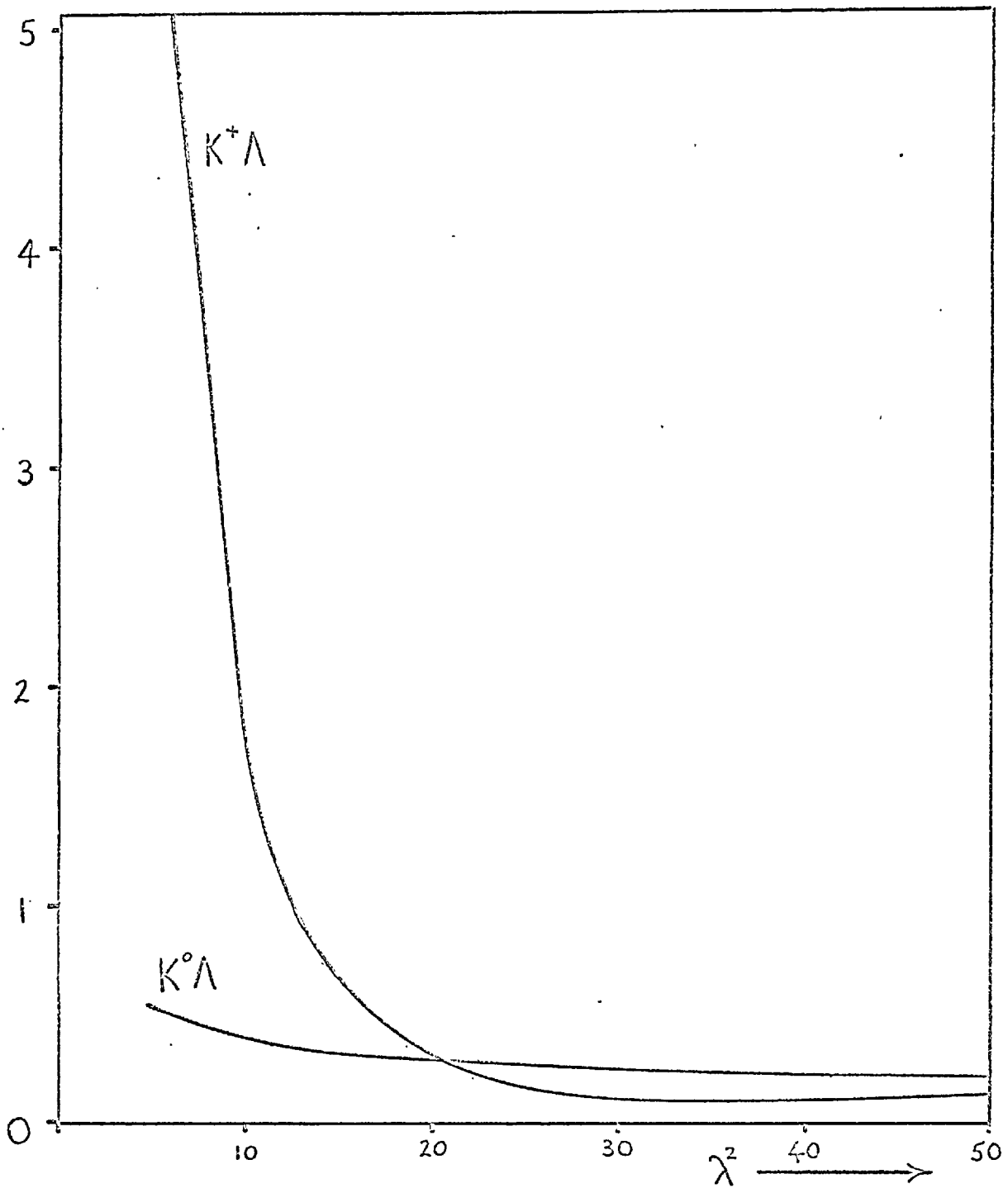


FIGURE VI

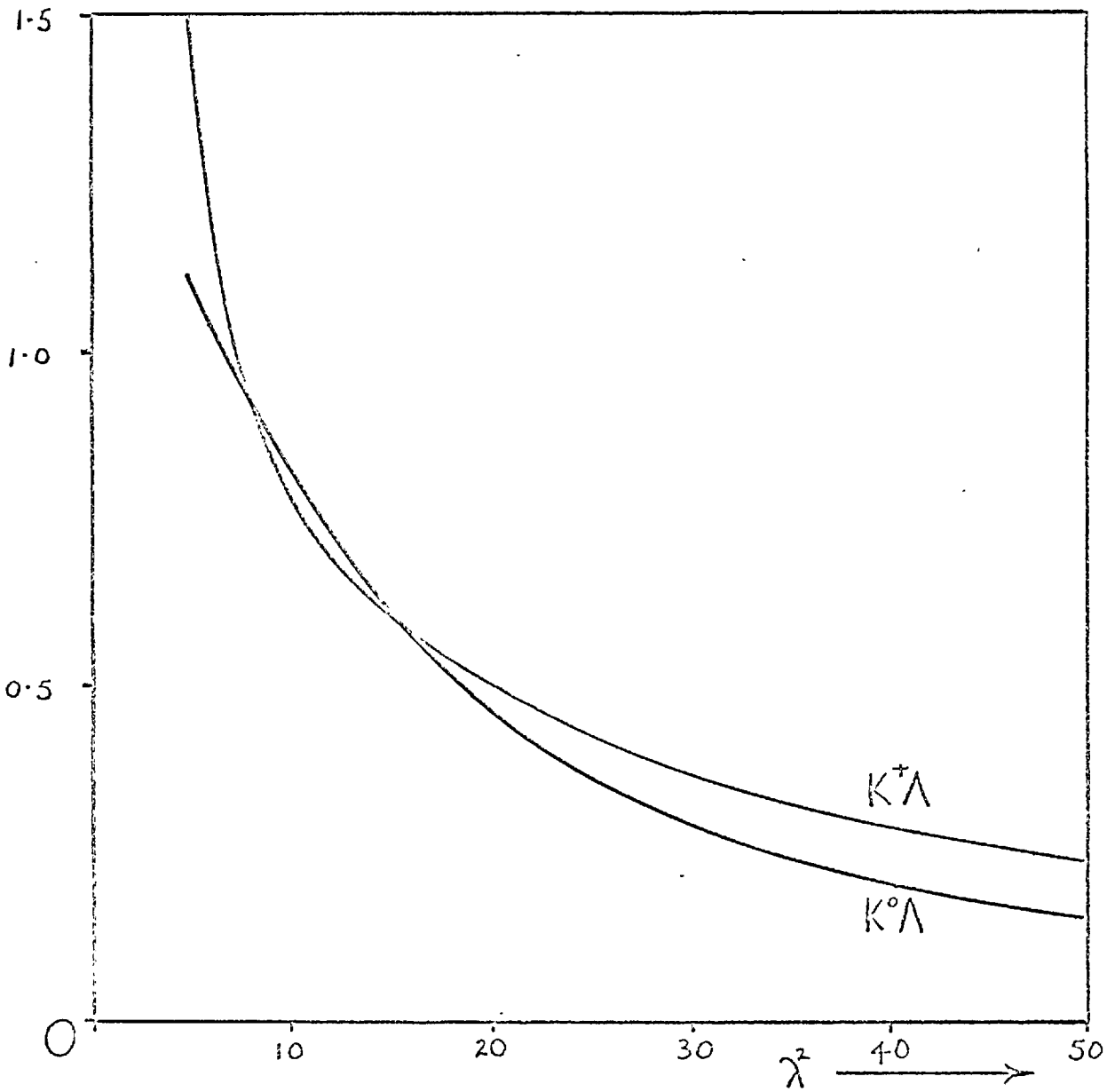


FIGURE VII

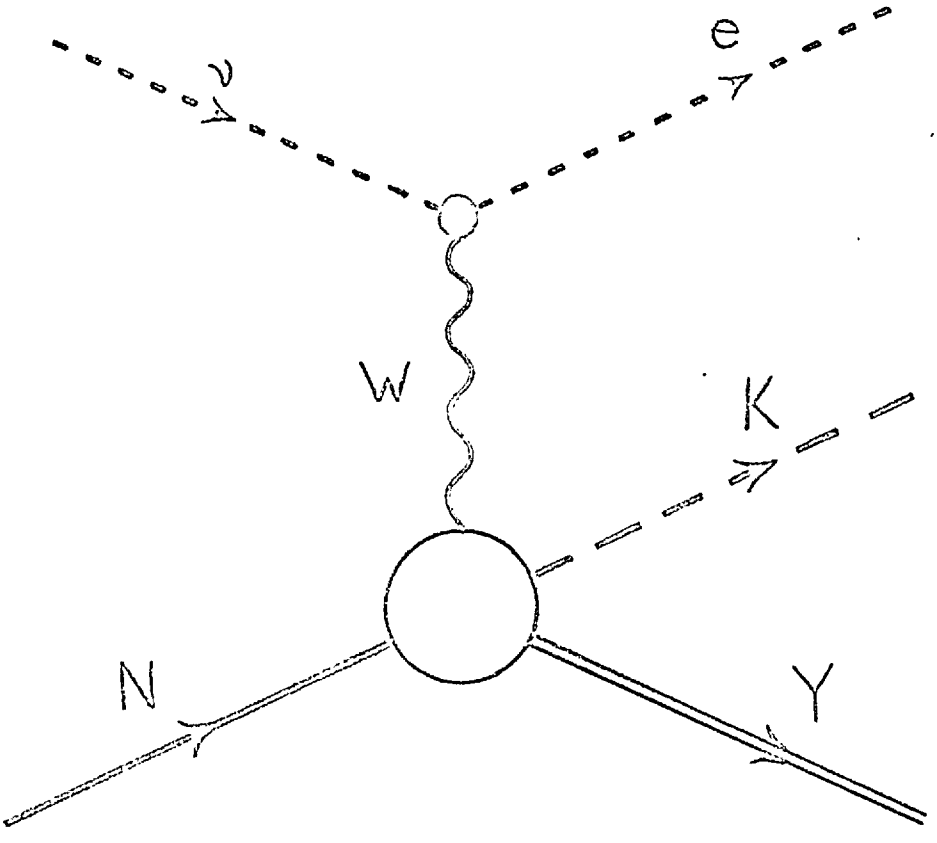
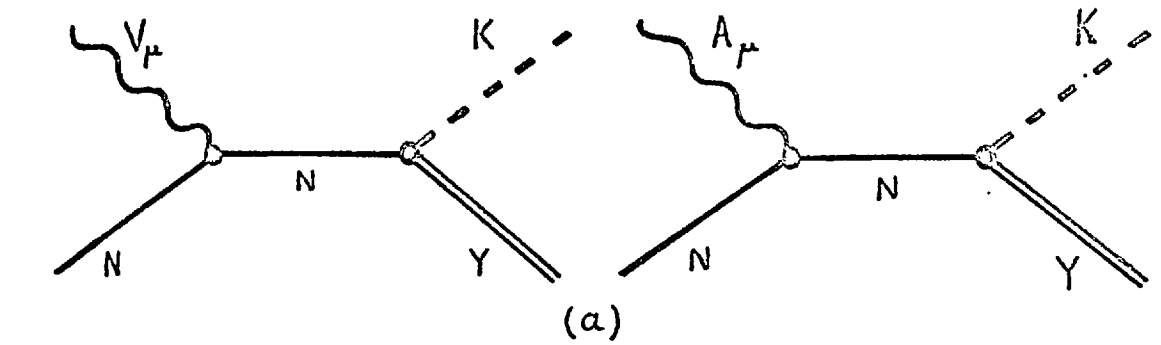
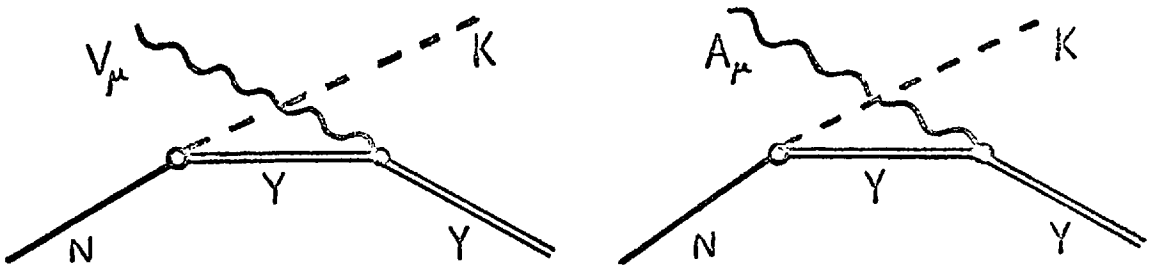


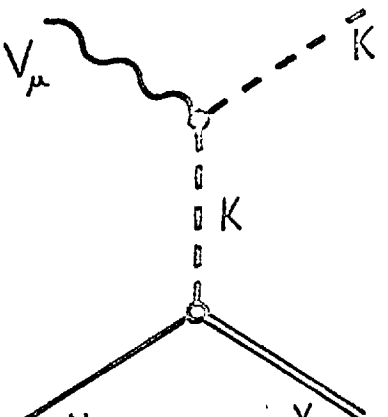
FIGURE VIII



(a)



(b)



(c)

FIGURE IX

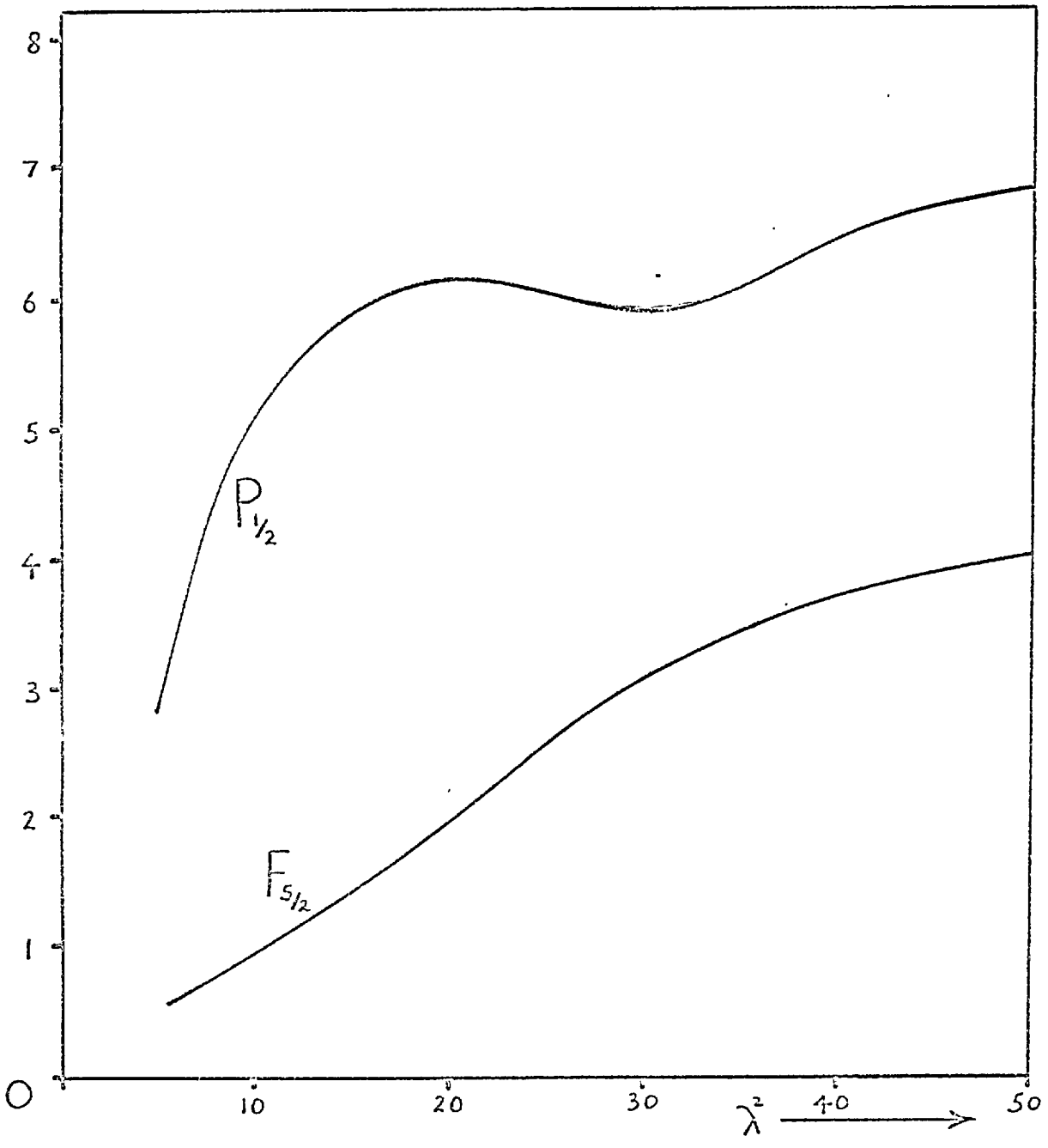


FIGURE X

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