

Imperial College of Science, Technology and Medicine
Department of Mechanical Engineering

Material Development and Multi-Scale Analysis of Nonlinear Vibrations with Damping of Composite Blades

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A thesis submitted to Imperial College London for the
degree of Doctor of Philosophy

August 2022

Declaration of Originality

I hereby declare that the contents of this thesis are entirely my own work and are original. Any additional sources of information have been duly cited in the thesis. This PhD research project has been carried out in the Dynamics Group of the Department of Mechanical Engineering at Imperial College London and funded by the Scientific and Technological Research Council of Turkey (BIDEB 2213/A 2016/2 Scholarship for PhD Studies Abroad in the field of Material Technology). This thesis is original and has not been submitted for any other degree or qualification in this, or any other university and/or institution. Some parts of this thesis have been used in the following publications in which I am the first author.

- Tüfekci, M., Rendu, Q., Yuan, J., Dear, J. P., Salles, L., Cherednichenko, A. V. (2020). Stress and Modal Analysis of a Rotating Blade and the Effects of Nonlocality. Proceedings of the ASME Turbo Expo. 10B-2020, 1–12. <https://doi.org/10.1115/GT2020-14821>
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Dedication

I would like to dedicate this thesis to my family. None of my achievements would have been possible without them.

”Proportional damping is like suggesting that all men are linear combinations of Robert Redford and George Clooney.”

Sam Ibrahim

Abstract

This work focuses on manufacturing, characterisation, with a particular focus on the vibration damping capabilities, and modelling of nanocomposites that can be used as an improved matrix for designing composite aero-engine fan blades. For this purpose, epoxy is selected as the matrix material as it is one of the most commonly used matrix materials. The main improvement in the mechanical properties is achieved with the addition of various nanoreinforcements into the epoxy matrix which are namely fumed silica (FS), halloysite nanotube (HNT), silica and rubber nanoparticles. To characterise and model the effects of the nanoparticle reinforcements on the resultant composite structure, first, the effects of friction on the experimental test setups are investigated analytically. Then, the necessary samples are manufactured. The manufactured composite samples are put through experimental characterisation procedures, including scanning electron microscopy (SEM), Fourier transform infrared spectroscopy (FTIR), three-point bend tests, dynamic mechanical analysis (DMA) and Charpy impact tests. To understand the mechanisms behind the change in the mechanical behaviour of the composites with different reinforcing particles, numerical simulations are carried out. Initially, mean-field homogenisation (MFH) with Mori-Tanaka is performed to estimate the elastic properties of the nanoreinforcements, as their experimental characterisations are not possible in a straightforward and conventional way. Then finite element (FE) analyses are conducted on representative volume elements (RVE) for a more detailed, in-depth analysis of the mechanics of the composites. Considering these conventional methods do not take the size effects into account, the nonlocal continuum theory is employed to calculate the nonlocal stresses to investigate the size effects of the reinforcements. As another example, a fan blade made of material of nonlocal characteristics is presented. Finally, the characterised nonlinear dynamic properties are inserted into a numerical model. A cantilever beam made of these nonlinear composites is generated as an example, and its forced responses are plotted under the same levels of excitation in the frequency domain.

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List of Abbreviations

1F: First Flap Mode

1T: First Torsional Mode

2F: Second Flap Mode

3T: Third Flap Mode

AFTHB: Alternating Frequency-Time Harmonic Balance

CFRP: Carbon Fibre-Reinforced Polymers

CNT: Carbon Nanotube

CW: Chordwise Mode

DMA: Dynamic Mechanical Analysis

FE: Finite Element

FRF: Frequency Response Function

FS: Fumed Silica

FTIR: Fourier Transform Infrared Spectroscopy

HNT: Halloysite Nanotube

MD: Molecular Dynamics

MEMS: Microelectromechanical Systems

MFH: Mean Field Homogenisation

PBC: Periodic Boundary Conditions

PS: Pressure Side

RVE: Representative Volume Element

SEM: Scanning Electron Microscopy

SS: Suction Side

Chapter 1

Introduction

1.1 Background and Statement of the Problem

From the very early days of recorded history, people have sought the concept of producing energy without having to contribute first-hand to its generation. Aeolipile, which means "wind ball", of Heron, as shown in Figure 1.1, is one of the first examples/origins of steam-powered devices. Furthermore, windmills and watermills use the power of wind and water streams to mill the grains. Internal combustion engines entered the scene and took control of power generation later in history, almost two centuries ago. At about the same time, engineers and scientists began developing turbine engines, one of today's most used power production technologies.

As one of the most significant applications of turbine engines, the aero-engine is a rotating mechanical system designed to extract energy from a fluid flow. It converts the fluids' thermo-mechanical energy into rotary kinetic energy. The energy conversion occurs at first with the help of the blades. The fluid applies pressure on the frontal surface of the blades. There the fluid loses its thermal energy, pressure, and momentum.

Since the blades are arranged with certain forms and angles against the fluid flow, the sums of the pressure on the frontal surfaces, the pressure sides, of the blades create resultant forces and a moment that forces the component on that the blades are fixed, which is called the



Figure 1.1: A reconstruction of Aeolipile, one of the first known attempts of steam-powered machinery [1].

rotor, to rotate. This is quite a complicated system in terms of mechanics. The mechanical behaviour of the rotor is alone a potential subject for several research studies. Figure 1.2 shows an aero-engine structure schematically.

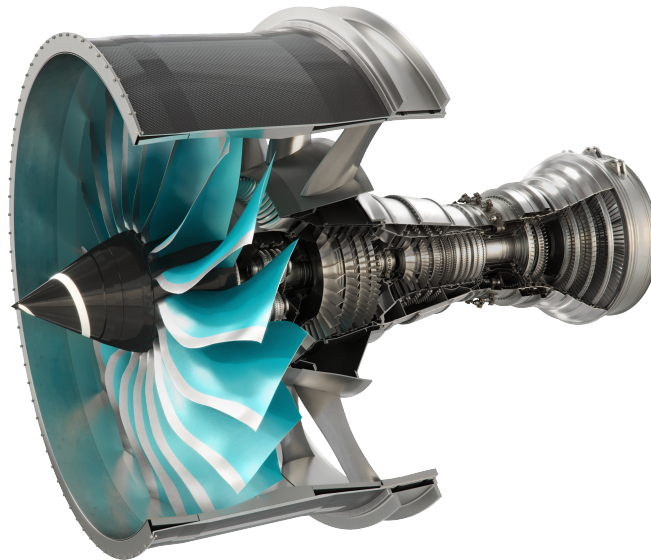


Figure 1.2: Schematic view of an aircraft engine (Rolls-Royce Ultrafan) [194].

The blades are usually the limiting factor when designing turbines and fans in terms of the thermomechanical properties of the materials from which the blades are built. Another reason for their low limits is, of course, their geometry. In order to get the most of the energy from the flow without causing significant turbulent losses, blades are designed as slim and delicate

as curved tapered thin-walled structures. To make them carry higher loads and operate at temperatures, which in most cases increase the efficiency of the turbine, their material has to be appropriately chosen.

Hence, the first thing that could be done to increase the strength/performance of the blades is to improve the material properties. Therefore, many different types of materials and material compositions are tested, such as super alloys (e.g. nickel-based superalloys), high technology coatings and nimonic.

Attempting to replace the traditional materials, modern fan blades are now being built more often from composite materials instead of traditional metallic materials and alloys, taking advantage of reduced weight and better dynamic properties, which is especially important for the aviation industry and applications. However, this introduces a series of challenges with regard to understanding their behaviour during operation depending on many parameters like manufacturing methods, macro-, micro- and even nanostructure. In composite materials, the reinforcing phases have a significant influence on the microstructure of the composite materials. Thus, it is essential to note that different reinforcements and nanostructures lead to different levels of changes in mechanical properties due to significant differences in their elastic properties and characteristic length scales. It is usually expected that some reinforcements are 'better' at adding damping than others. However, increased damping also affects the peak stress, as well as it can cause delamination. So, a damper microstructure should lead to different amounts of changes in individual mechanical properties, where some do not vary much. As a result, there would be a vast spectrum of scientific fields to work on developing optimal materials in terms of frequency response/damping vs strength. This would also be extremely interesting as it has not been investigated the relative importance of damping vs toughness and strength in a composite, at least for the range of parameters that are particularly important for turbomachinery applications. Figure 1.3 shows one of the composite fan blades that is being used in aircraft engines. This blade is designed and manufactured by Rolls-Royce for the engine series Ultrafan, mainly using carbon fibre-reinforced polymers (CFRP). Considering that the weakest component of CFRP composites is the polymeric matrix, improvements in the mechanical properties of the polymer-based matrix are essential. Therefore, this research

focuses on the problem of modifying the matrix to facilitate any further improvements on the fan blades of aero-engines.

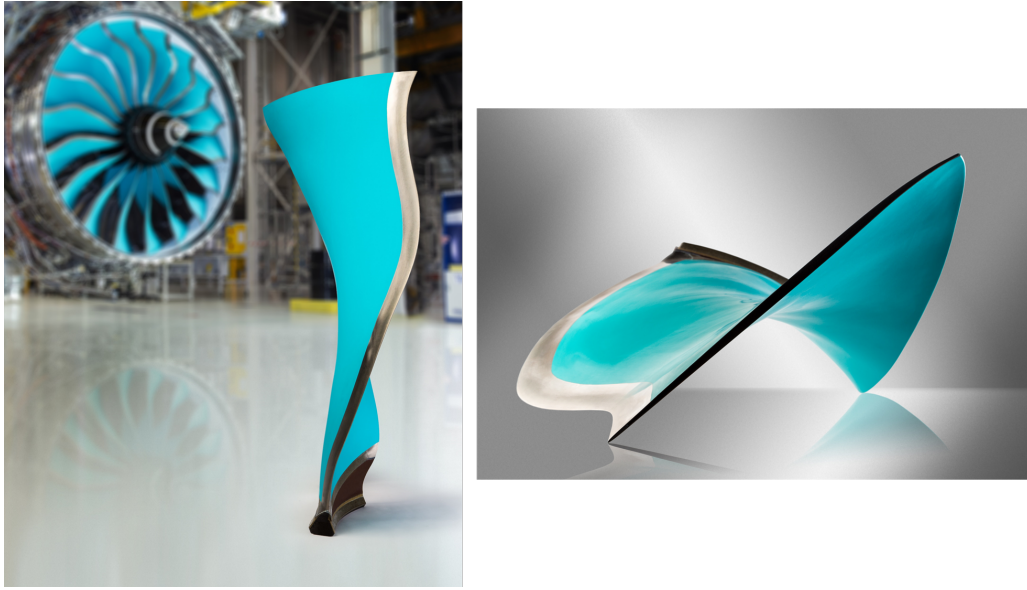


Figure 1.3: Composite fan blade [194].

1.2 Research Objectives

The in-depth analysis of the materials leads to the capability of defining/tailoring the best material types, components (matrix, reinforcement, reinforcement method and form etc.) and the best material model for the chosen materials that can be used in the desired application. The most reliable method to analyse the material's behaviour is performing experiments on the material samples. Based on the results acquired from the experiments, the material model is built, and the relevant parameters are tuned before moving on to modelling the behaviour of the applications' structure. The material modelling needs to determine the effective material behaviour using the effective properties. A point to focus on would be the nonlinear nature of stiffness and damping properties in the polymer-based composite structures, which significantly affect the overall behaviour of the structure made of the tailored composite material.

The main idea of the research is to design and manufacture the composite material to serve as the matrix resin for a possible carbon fibre-reinforced polymer (CFRP) composite performing a multi-scale analysis of the manufactured material as well as their experimental characterisation.

Thus, this research project consists of two main parts, experimental and numerical investigation of the mechanical properties of structures made of composite materials. Therefore, the goal of this study is to develop a suitable nanocomposite matrix for the rotor blade application and to develop an efficient numerical tool for multi-scale modelling of the nanocomposite material to provide the provisional data to model the nonlinear vibrations of the structures made of composite materials. This tool permits a better understanding of the dynamic response of composite structures, with a particular focus on the damping behaviour considering the composite material properties like microstructural parameters (volume fraction, reinforcement form and orientation etc.) with the help of experimental data, which is acquired from performing experiments on the manufactured composite materials.

It is aimed at the end of the research to have more understanding of the nonlinear nature of damping in composite materials, which helps the material and turbomachine communities in designing more efficient composite structures to be applied in turbomachinery.

1.3 Outline of the Thesis

To achieve the research goal in this study, first, a literature review of the state of art is presented. Using the knowledge in the literature review, the materials that are manufactured are selected using the rough range of parameters available and an approximation of the desired material properties is made on the nanoscale for the composite. To start the research process, epoxy resin reinforced with silica and rubber particles as well as halloysite nanotubes (HNT) are chosen. Following that, the selected materials are manufactured and put through experimental procedures. First, Fourier Transform Infrared Spectroscopy (FTIR) and Scanning Electron Microscopy (SEM) are performed. Then, the mechanical tests are performed on small beam samples to identify the nonlinear characteristics of the epoxy-based nanocomposites. These mechanical tests include Dynamic Mechanical Analysis (DMA), three-point bend and Charpy impact tests. Taking advantage of the data acquired from the experiments, micromechanical models of the composites are created to reproduce the experimental results and to understand

the stress state on the small scales using Mori-Tanaka Mean-Field Homogenisation (MFH) and Finite Element (FE) analysis using both classical and the nonlocal continuum models. Then, the last step of this research is the modelling of the structures made of these composite materials. The results of the micromechanical model are used on actual geometries (a beam and a fan blade). The geometries are created and used in the generation of numerical models, which use the characterised effective material properties.

A schematic organisation of the study is presented in Figure 1.4.

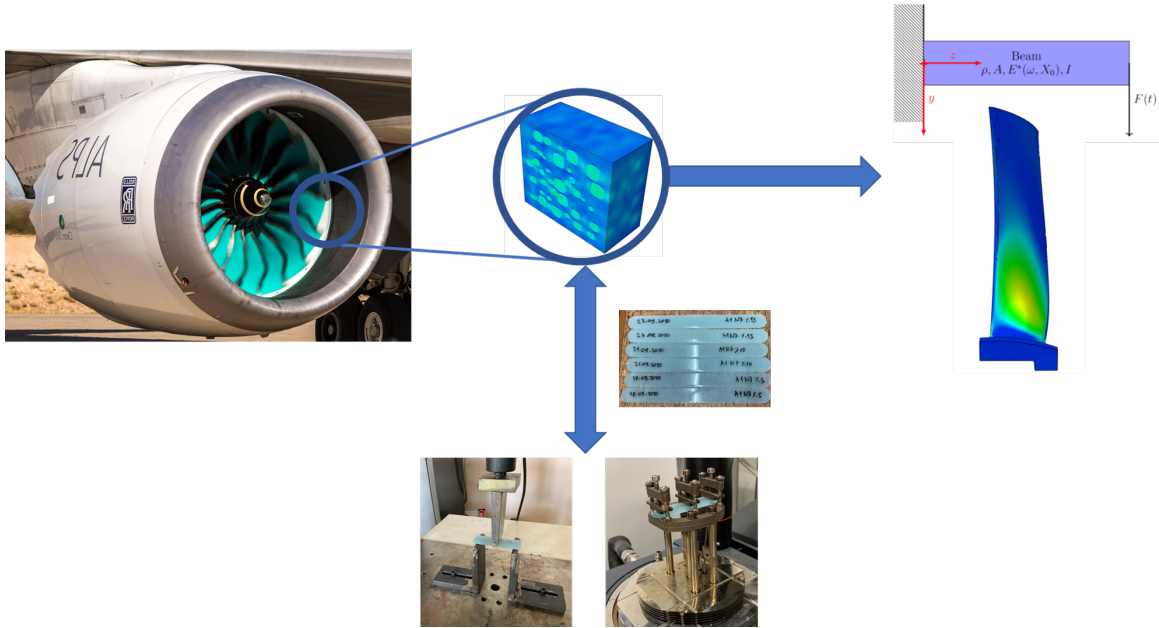


Figure 1.4: Schematic outline of the study [194].

In terms of research work, the study mainly includes the following sections:

- Literature review
- Analytical modelling of the effects of friction on the mechanical test setups with beams to predict the importance of frictional damping in the damping measurement data.
- Manufacturing the chosen composites and their experimental testing, gathering the material data for the necessary modelling of composite materials and rotor blades/beams to provide the basis for the numerical and theoretical models.
 - FTIR to inspect the chemical composition of the materials.

- SEM to examine the microstructure of the composites.
- Three-point bend tests to investigate the strain-rate dependence of the stiffness, strength and ductility.
- Charpy impact tests to explore the effects of the reinforcements on the impact behaviour.
- DMA to characterise the frequency and excitation amplitude dependence of the composites' stiffness and damping.
- Modelling of composites and composite structures
 - Multi-scale modelling of the manufactured composites that are put through the experimental procedures to reveal the mechanisms and the reasons behind their effective mechanical behaviours.
 - * MFH using the Mori-Tanaka approach to gain an understanding of the basic mechanics of the composites.
 - * FE modelling of the composites using Representative Volume Elements (RVE) to conduct a more detailed investigation to explain the effective mechanical properties with and without nonlocal effects.
 - Application to rotor blades/beams made of the chosen materials to explore the possible effects of the new nonlinear materials on the vibration behaviour.
- Conclusions and future remarks.

Chapter 2

Literature Review

2.1 Introduction

The main objective of this literature review is to understand the history and state of the art in various fields/topics that are relevant to this research. Considering the aim and the topics that contribute to the background of the idea of this research project, this literature survey is divided into specific sections. The literature review is done focusing on composite materials with a particular interest in the mechanical properties and modelling of composite materials that belong to the second and third sections of the project. This literature review mainly covers modelling, characterisation and design/applications of composite materials and structures.

2.2 Material Models in Composites

For the first section, some essential reviews on mathematical modelling techniques, including nonlinear elasticity, the theory of finite elements, nonlocal elasticity theory, and some appropriate numerical methods, are covered [70, 73].

In terms of the theory of elasticity, it is aimed to utilise the generalised theory of classical elasticity that makes the least assumptions and does not do any linearisation aiming to cover

as many cases in real-world problems, and materials [210]. Furthermore, it is a good preparation point to understand the theory of nonlocal continuum fields in depth [254, 253]. This field is recently of interest to many researchers in the scientific world since the technology is developing towards micro and nano-scales. The characteristics and the number/intensity of material flaws and irregularities change rapidly from macro-scale to nano-scale. This affects material properties dramatically. Hence it is explained why the modulus of elasticity of carbon nanotubes is around 1.2 TPa [151]. A more detailed review of this theory can be found in the following section and also in Chapter 5.

Since this study focuses on composite blades, nonlinearities may easily become significant. The sources of nonlinearities can be individual materials, discontinuous structures in the composites and large deformations. Materials in engineering are categorised based on how they are deformed when loaded. When deformation is expressed in terms of load, then the function directly makes the distinction of the materials being linear or nonlinear. It means if the material stiffness is constant, then the material characteristics are called linear, whereas if the stiffness depends on the deformation and the deformation rate etc., then it can be stated that the material has a nonlinear manner [176]. In many cases with polymers, it is not accurate enough to use a linear relation to represent the material behaviour, and therefore, the stiffness usually needs to be expressed as a function of strain and/or strain rate, as shown in the equation below.

$$\sigma = C\epsilon \quad (2.1a)$$

$$\sigma = C(\epsilon)\epsilon \quad (2.1b)$$

$$\sigma = C(\epsilon, \dot{\epsilon})\epsilon \quad (2.1c)$$

Here, σ stands for stress, ϵ , strain, $\dot{\epsilon}$, strain rate and C stiffness tensors.

The nonlinearities caused by the material behaviour can be expressed mathematically within materials' constitutional expressions [133, 210]. As a result, the function chosen to represent

the nonlinear behaviour becomes significant [174]. Thus, there are various ways to represent the material characteristics in the literature that scientists have developed over history. A loaded, perfectly elastic material is supposed to return to the same resting point when the load is removed, while plasticity causes the material to rest at a different position/form when the load is removed. Figure 2.1 visualises the behaviours of different material types when loading and unloading.

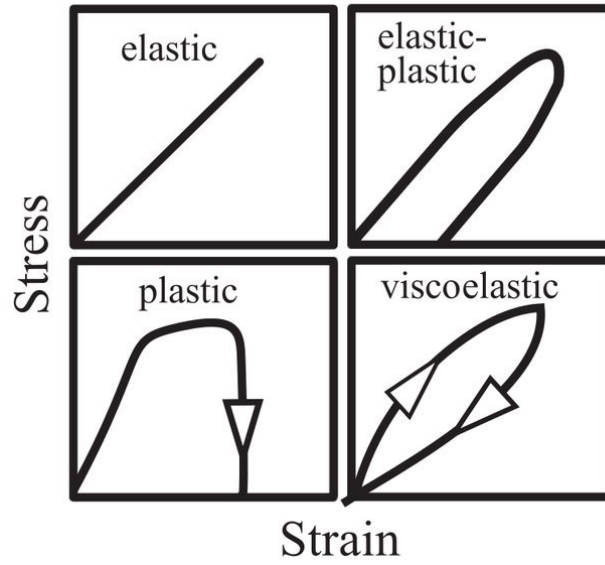


Figure 2.1: Types of material behaviour under loading and unloading [97].

In their constitutive equations, different material models employ different functions. These functions are selected based on the data acquired from material characterisation experiments that focus on mechanical behaviour under various loading scenarios [150]. Considering polymers are usually less-stiff materials compared to traditional materials, a different technique/model is necessary to predict their mechanics. Hyperelasticity is based on this less-stiffness assumption as well as incompressibility (Poisson's ratio $\nu = 0.5$), being independent of load history and pseudoelastic behaviour [206, 108]. One of the critical aspects of this model is that this material behaviour does not represent the damping behaviour of these materials. Another material modelling method is called viscoelasticity which describes the material behaviour as a combination of viscous and elastic behaviours [80, 179]. As it can be understood from the viscous behaviour, this modelling technique takes damping into account [217]. This damping can be represented with various forms/techniques such as linear viscoelasticity, nonlinear viscoelasticity, fractional

derivatives etc. [237, 33]. Damping as a material characteristic is discussed in a separate section within this chapter.

The discontinuities in composite materials are caused by the coexistence of the different phases / materials. Two aspects appear in the discontinuous behaviour, namely the different mechanical properties of the materials in the composite and the interface behaviour [227]. There are different ways to overcome the issues in modelling interface behaviour in composites. The first and most straightforward way can be that the materials in the composite are considered as rigidly bonded at the interface [46]. This way enables some analytical solutions to be done in some specific cases as it introduces good simplicity to the system, whereas its accuracy could be open to discussion. The second way to model the interface behaviour is to use the contact with friction approach. The forces at the interface are calculated, and friction behaviour is imposed accordingly [13]. As one of the most common ways is to insert a fictional third phase material with some equivalent damping and plasticity, if necessary, in order to represent the interface behaviour. Additionally, higher-order continuum theories are important since they are computationally less expensive, and they are relatively simple to formulate by adding some additional terms to the classical continuum theory [221, 52]. By those additional terms, nonlinearities and singularities such as crack tips and other sorts of discontinuities in a material structure, which are rather common in composites [188]. This type of approach can also be used in size dependencies in material properties and structures.

Further literature review on composite materials and their mechanics can be found among the following sections. Since this is a relatively new research field, there is still not a certain/unified and standardised roadmap to combine and use these theories for modelling the micro/nanostructure of composite materials [62, 168, 103]. One of the main aspects of modelling composites is the modelling of the interphase layer in micro-/nano-scale and its impact on the mechanical behaviour of the whole system [183, 184].

2.3 Mechanics in Small Scales

Based on the outline presented earlier, following the acquisition of the material data from the experimental section of the study, the modelling part of the study is the issue. Therefore, a literature review is also initialised for this field. The focus is kept on material modelling in micro and nano-scales.

Nano-scaled structures have great potential applications in many different fields such as nanorobots, micro gas turbines, all the structures that are related to microelectromechanical systems (MEMS) etc., as well as material modelling [123, 65]. At these scales, it is quite challenging to perform experiments on material specimens because of the extreme precision requirements. However, the available experimental results show that the mechanical properties, such as Young's modulus, are size dependent [213]. The predicted stiffness of a cantilever micro-beam is around a quarter of the measured stiffness values [161]. This size dependency leads to superior performance in comparison to classical bulk materials.

Considering the difficulties of experimental works in this field, better mathematical models are found to be much more helpful/feasible. These mathematical models must include small-scale effects caused by lattice spaces, dislocations, grain size etc. The micro/nanostructure of the materials are complex structures and behave in a complex manner, therefore, to be modelled using numerical tools [104, 46]. Based on the knowledge gathered from the reading on micro and nanostructure modellings, the materials are basically modelled using classical elastoplasticity theories in the literature [259, 162, 171, 251]. Also, the classical failure theories are used during the modelling of microstructure [84, 121, 178, 177]. Nevertheless, since the rubber reinforcement and epoxy matrix interaction has to be investigated at the micro- and nano-level, the classical continuum approach would not be perfectly accurate. Instead of classical continuum theory, a nonlocal theory of elasticity seems to be a better way of modelling the elastic behaviour of the composite material [41, 42, 43]. Nonlocal continuum theory is developed by Ahmed Cemal Eringen and is nowadays quite popular in the scientific environment.

The classical theory of elasticity relates the stress tensor of a point to the deformation and

strain tensors of the same point. It assumes the locality and claims that the stress state of a point is not affected by the deformations and strains of other points in the same continua [74, 69, 70, 67, 71, 68]. When travelling towards micro- or nano-scale, lattice spaces, atomic or intermolecular spaces become significant. A stress tensor of a point in the continuum is affected by the deformations and strains of all other points in the same continuum, as it is expressed in the following equation. The upper index "nl" stands for nonlocal.

$$\sigma_{ij}^{nl}(x) = \int_{\Omega} \alpha_{ijkl}(x, x') \varepsilon_{kl}(x') d\Omega \quad (2.2)$$

Here, ε denotes the strain tensor, σ is the stress tensor and α is the characteristic length.

It is possible to relate the characteristic lengths (the coefficients) with the stiffness and/or compliance matrix.

$$\alpha_{ijkl}(x, x') \rightarrow C \quad (2.3)$$

$$\sigma_{ij}^{nl}(x) = \int_{\Omega} \alpha(x, x') C_{ijkl} \varepsilon_{kl}(x') d\Omega \quad (2.4)$$

Following that, stress-strain relations are plugged into this nonlocal equation. So, the relation between the local and nonlocal stresses can be expressed as an integral equation. The upper index "l" stands for local.

$$\sigma_{ij} = C_{ijkl} \varepsilon_{kl} \quad (2.5)$$

$$\sigma_{ij}^{nl}(x) = \int_{\Omega} \alpha(x, x') \sigma_{ij}^l(x') d\Omega \quad (2.6)$$

This integral equation is analytically solvable in one and two dimensions [248, 27]. However, it is not easy to solve in a three-dimensional space.

Under some certain conditions, this integral equation can be transformed into a differential form which is much easier to solve [69].

$$(1 - \gamma^2 \nabla^2) \sigma^{nl} = \sigma^l \quad (2.7)$$

Here, γ is the size parameter which can also be expressed as ($\gamma = e_0 a$).

Classical continuum theory does not have an intrinsic length parameter which considers the effects of these explained size dependencies. This is the main reason why the classical elasticity theory fails to represent realistic results on such small scales. Thus, using the classical elasticity theory and for a reason related to the amount and characteristics and structure of material flaws, using the bulk material properties for modelling the microstructure should be discussed in terms of its accuracy and consistency. However, some studies use the classical elasticity theory and bulk material properties to model the microstructure, it is observed that they tend to stay close to the macro-scale [104, 46, 69]. The mathematics of nonlocal elasticity is based on integral equations, as it relates the deformations and strains of a field to the stress tensor of a point. Therefore, a mathematical summation is necessary. This integral equation is analytically solvable in only some certain cases [129]. This means, nanorods and nanobeams are analytically solved in many cases [20, 248]. Nevertheless, the equations have not been solved analytically in three dimensions so far. Only numerical approximations are possible with today's knowledge [209].

Also, in the literature, some studies formulate two-dimensional elements [193, 192]. However, to the best of the author's knowledge, there is no three-dimensional element formulated for nonlocal elasticity so far.

A plasticity theory is also applied using this nonlocal continuum theory [249, 250]. This theory is still being extended and developed so that it has built its own damping and failure theories [258]. The nonlocal theory is also being applied to nanocomposites giving out highly accurate results [66, 168]. As an alternative to failure for nanocomposites, peridynamics is a popular approach which could be applied in this study as well [236]. The molecular dynamics approach is also

one of the newest emerging approaches to nanocomposites [184, 236]. In order to understand the mechanics of materials in small-scale molecular dynamics (MD) is one of the most common ways in the modern scientific world [228]. MD simulations are performed numerically, and usually, they are computationally expensive. However, they provide results that are comparable with experiments. MD simulations provide excellent and consistent insight into interactions in atomic and molecular scales. MD theory is based on the interatomic potentials defined, aiming to simulate atomic and molecular interactions. The equations that are used to perform the MD simulations require multiple parameters which are coupled directly with the expression of the empirically defined bond energies. Equations determine the accuracy of the simulations. Figure 2.2 illustrates a schematic view of the molecular scale structure of epoxy nanocomposites.

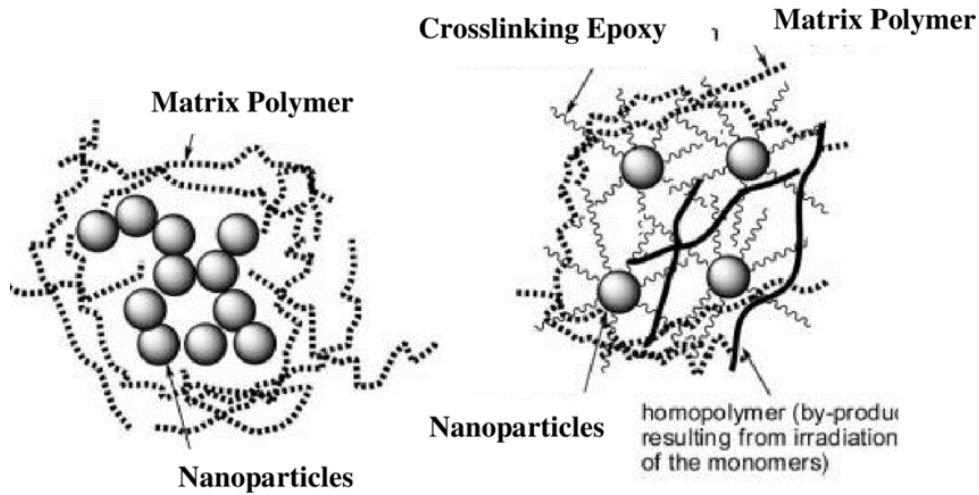


Figure 2.2: Rubber nanoparticles in the epoxy matrix and Their Reaction Schemes [12].

2.4 Vibration Damping

Damping is the energy dissipation from a vibrating structure. In damping, energy dissipation means that mechanical energy transforms into other forms of energy, such as heat; therefore, damping is a form of removing energy from the vibrating system. The type of energy transformed by damping depends on the system and the dissipation mechanism. The mechanical energy is converted into heat in most vibrating systems.

2.4.1 Material Damping Models

The specific ways of dissipation are dependent on the physical mechanisms in the structure. Since these physical mechanisms are very complicated physical processes, they are not totally understood yet. Damping in the structure depends on the mechanism in the given circumstances. In the nonlocal continuum theory, there are damping models integrated into these kinds of continua [136].

Aligned with the purpose of this study, the focus of this review is on the damping of the composite materials that can be used in manufacturing gas turbine blades of aero engines. In damping mechanisms of composites, there are two major aspects, individual material damping and damping of interfaces of phases, namely the reinforcements and the matrix [13, 238].

Damping affects the whole mechanical behaviour of structures in steady-state and transient situations [109]. Damping is one of the most crucial properties of a vibrational system, so it needs to be predicted accurately to avoid future problems and to get possible benefits during the design process [57]. Adhikari reviews and explains in detail some of the most commonly used and essential damping models [5]. The first class of damping in terms of simplicity would be linear damping [262]. Some most commonly used linear damping models are the linear viscous and linear viscoelastic damping models [144]. Viscous damping is usually better when modelling systems with a lesser degree of freedom, such as damping elements or external damping effects [233]. For material damping, the viscoelastic material model offers much more flexible and accurate modelling possibilities [23]. Linear viscoelastic damping is often expressed using the complex modulus approach [179]. This approach is based on adding a complex part to the Young and shear moduli of the material. Thus, damping is built directly using the stiffness.

$$E^* = E (1 + i\eta) \quad (2.8)$$

$$G^* = G (1 + i\eta) \quad (2.9)$$

Here, E and G are elasticity and shear moduli, respectively, while E^* and G^* are the complex modulus of elasticity and complex shear modulus, respectively. i is the complex number and η stands for the loss factor.

Building a direct relation between stiffness and damping is a common approach, such as the Rayleigh damping model, which follows a similar method. This model combines the mass and stiffness terms with their certain proportions α and β respectively [48]. α and β are assumed to be constants. The mass component of the damping matrix damps the low-frequency vibrations, while the stiffness component damps the high-frequency vibrations.

$$[D] = \alpha[M] + \beta[K] \quad (2.10)$$

Here, D is the damping matrix, K is the stiffness matrix and M is the mass matrix.

The mathematics must also be modified to increase the accuracy in modelling the inherent damping. The calculation process of the eigenvalues and eigenvectors of viscoelastic systems are also defined in detail in previous research that can be found in the literature [132, 7]. These two linear damping methods are often suitable for a specific range of frequency in which they are modulated. It is not guaranteed that these models would precisely represent the damping behaviour of the material or the structure. Although linear viscoelasticity is more accurate than viscous damping in the modelling of material damping, nonlinear viscoelasticity is even more accurate to this extent. However, it is computationally much more expensive compared to linear theories. For nonlinear damping, the challenge is to choose the suitable functions to represent the behaviour of real systems, just like the issue with the functions inserted into the constitutional equation of the materials [126]. Polynomials are the first type of function to represent nonlinearities. With sufficient terms, any type of function, even unknown functions, can be approximated [264]. But for some cases, there are better solutions that require less computational resources than polynomials [119]. The selected functions can be fitted to the experimental data to determine the necessary coefficients and to fully define the functions [205, 217]. Another type of modelling nonlinear behaviour of viscoelastic damping is to take

advantage of fractional derivatives of viscoelasticity [182, 89]. The investigation of the effect of the fractional derivatives on the vibrational behaviour of a mechanical system can be found in literature as well [208]. It is also possible to explain how fractional derivatives come into existence in the physical world of real materials [237].

As for the majority of nonlinear mechanics problems, an analytical solution is not always possible. Therefore, numerical approximations can be made [132, 219]. Excited with cyclic loads, steady-state responses of dissipative systems usually create closed curves. Therefore they are also called as hysteresis loops. The steady-state response of a viscoelastic system is shown in Figure 2.3 schematically. If the system is linear, the loop is fully represented with a perfect ellipse [113]. Any deviation from a perfect ellipse is usually caused by nonlinearities.

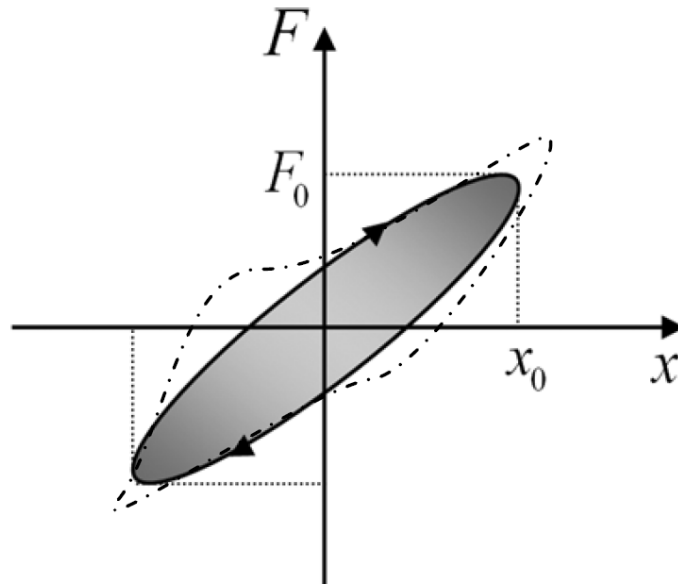


Figure 2.3: A typical viscoelastic hysteresis loop [113].

The primary function of damping in mechanical systems is to dissipate energy. Therefore, investigating the energy of a mechanical system is usually a common way for various types of mechanical analysis, and vibration analysis is not an exception. Since damping mechanisms basically subtract energy (usually converted to heat) from the system, the change in the total energy of the system could give a realistic idea for damping characteristics [187]. The dissipation of the total energy of the system is called an energy sink [186]. This approach is not only practical when characterising the damping itself, but it is also efficient when investigating the

effect of damping on the structures. Since discrete systems lead to sets of ordinary differential equations, they could be considered as simple models for theoretical modelling [22]. Thus, discrete systems are good starting points for an investigation of the effects of nonlinear damping on mechanical systems [229]. The next step for building more realistic and complicated systems would be one, and two-dimensional continuous systems, namely beams and plates/shells [137, 135, 24]. Beams are among the most common continuous systems on which the damping models/theories are tested and theoretically applied [3, 53]. There, partial differential equations are involved. The impact of nonlinear damping on continuous systems is more representative for real-life applications while they are more complicated to solve mathematically [9].

For the investigation of damping, there are different measures to interpret the effect of damping apart from the energy sink [5]. Free vibration behaviour of a mechanical system is one of the significant phenomena where the damping behaviour could be extracted by evaluating the ratios of successive peaks and the time for the vibration to fade out [208, 234]. Amplitude, period and phase lag between the force and displacement functions in a steady state forced vibration are also essential concepts on interpreting the damping characteristics of the systems [230, 125]. There is also some research that uses and evaluates the hysteresis curves of materials to obtain the damping characteristics [256, 13].

The damping characteristics of materials, especially polymers, depend on environmental hygrothermal conditions [36, 86, 35, 34]. The stiffness of polymeric materials is strongly dependent on temperature, so the damping properties [149]. It is a well-known fact that the stiffness of polymers decreases with increasing temperature while damping initially increases with increasing temperature until it reaches a peak and then converges to a lower bound value [148]. Additionally, previous research showed that stiffness and damping properties, even in the same ambient conditions, are not correlated with a direct linear dependency [255]. On the other hand, the damping properties of materials are related to the loading and boundary conditions, which also influence the stiffness of the system.

Amplitude and frequency of the exciting loads are considered the most dominant among many parameters [182, 57, 112, 232]. A nonlinear system's response differs from the excitations

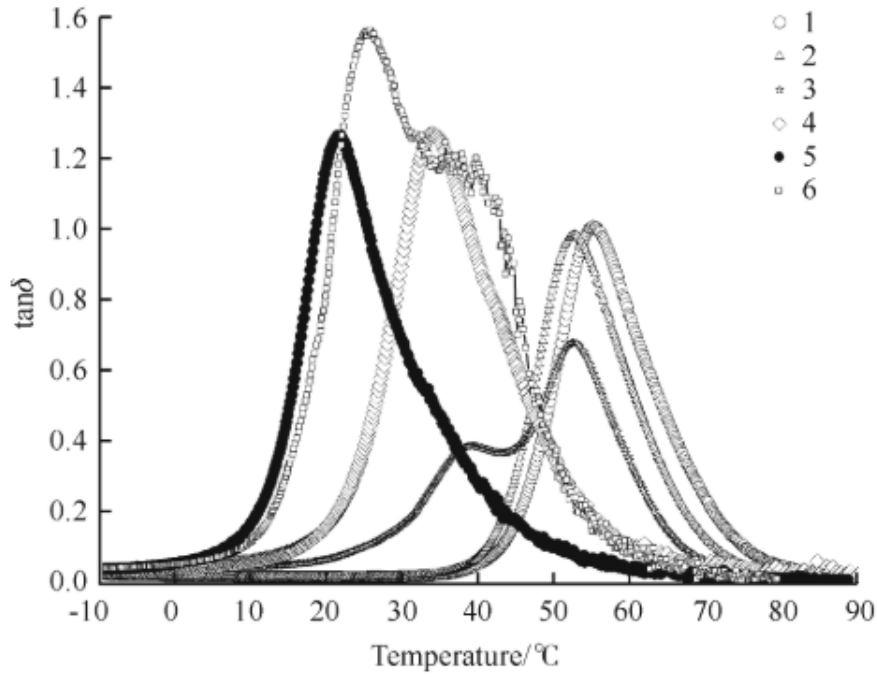


Figure 2.4: Temperature dependency of damping properties of epoxy [257].

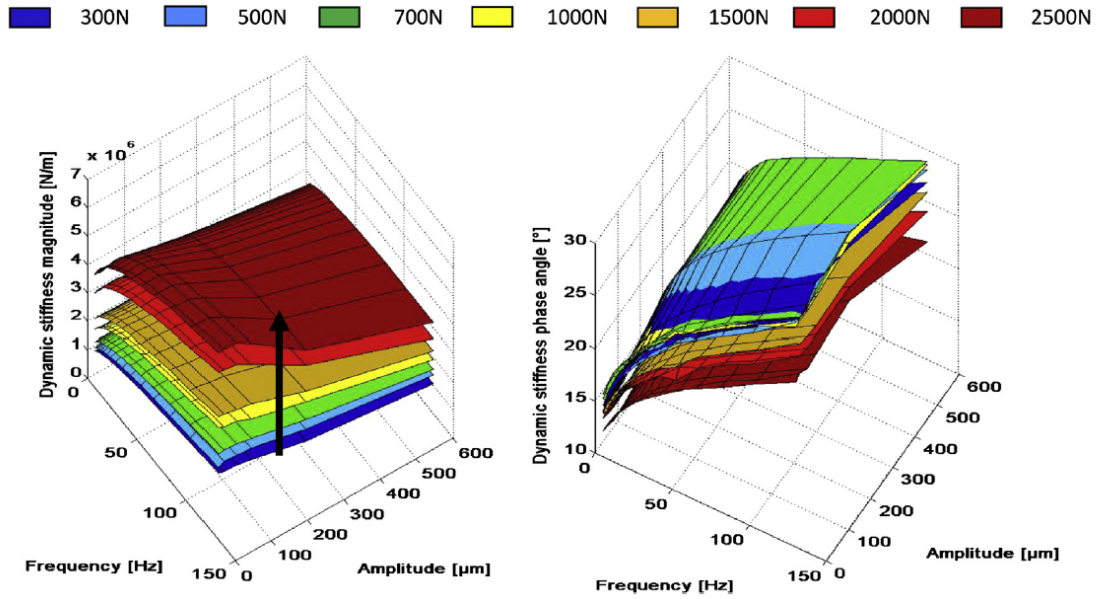


Figure 2.5: Frequency and amplitude dependency of damping properties [112].

of different frequencies and/or amplitudes [204]. Figure 2.4 represents a general overview of how damping properties of different types of epoxies change with temperature, and Figure 2.5 displays the dependency of damping properties on the frequency and amplitude of an elastomer. In Figure 2.4, $\tan(\delta)$ indicates the loss factor, which is earlier referred to as η .

2.4.2 Frictional Damping

The concept of damping could be observed in mechanical systems due to factors other than the material's own energy dissipation mechanism. Friction is one of the major factors in mechanical systems that dissipate energy [199, 226]. It is a common practice to add frictional dampers to the systems to reduce the amplitudes of displacements of vibrations [82]. However, not all types of frictional damping are desirable. Even in relatively simple cases like beams with imperfect fixtures, the effect of damping is so significant that the mechanical behaviour of beams changes due to the clamps [79]. Depending on the case, if the system is an assembly of multiple parts, such as bladed disks of aero-engines, with imperfect bonding at the interfaces, friction is expected to cause damping [127]. Figure 2.6 displays an example hysteresis loop caused by pure damping. On the particular example of aircraft engines, underplatform dampers are placed between disks and the blades [128]. Furthermore, this case includes the aerodynamic loads, which damp and/or excite vibrations [222, 64]. This particular example is important because there is a combination of inherent and external damping mechanisms.

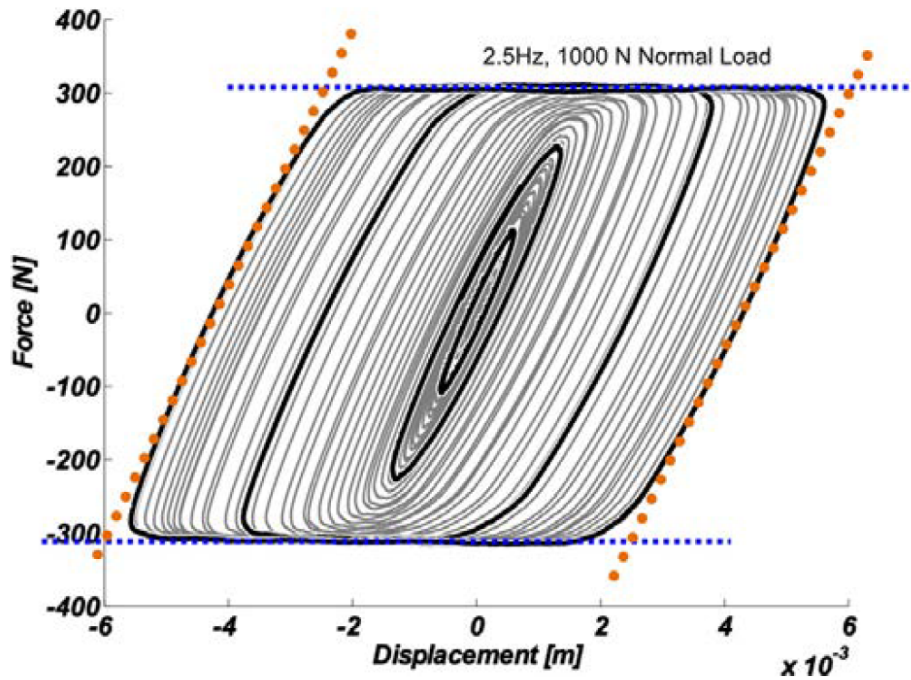


Figure 2.6: A typical frictional hysteresis loop [113].

2.4.3 Damping in Composites

Like all the traditional materials, damping in composites has been popular research over the recent decades. Although the mechanical behaviour of composites is more complex than the traditional materials, the same modelling techniques are used to predict and characterise the damping behaviour [106, 238, 153]. Particularly polymeric composites have superior damping properties. Therefore, they have been the focus of research in recent decades [83]. The majority of the previous research is based on experimental methods and applications attempting to have a deeper understanding of the damping characteristics of composites [255, 8, 154]. Experiments are designed and conducted considering the damping measures explained before. Some works focus on the free vibrations of composites and use the decay and fade-out times to evaluate damping as well as the free vibration frequency response functions [16, 220]. Forced vibration of composite structures/materials is also preferred by researchers to characterise the damping behaviour [269, 120]. To force a composite material specimen, dynamic mechanical analysers (DMA) offer a compact and fast experimental procedure enabling temperature, frequency and amplitude changes and their combinations to be done automatically. Additionally, the variety of fixtures allows researchers to create the stress and displacement states as they need. Among these fixtures, dual cantilever fixtures and three-point bend fixtures are commonly preferred because of their stiffening capability, which reduces the effect of friction introduced by the clamps [255]. On the contrary, some works published in the literature consider the dual cantilever fixture configuration to be somewhat disadvantageous because of clamping friction. Therefore, care has to be taken for the damping imposed by friction on the system. Except for experimental works, there are a few analytical formulations on how the damping works with composites [269, 238]. The majority of modelling work focusing on composite damping uses numerical techniques to simulate the dynamic behaviour [13, 53]. In the damping behaviour of composites, the interface is a major factor that affects the fundamental dynamical characteristics and introduces strong nonlinearity [30, 53, 59].

2.5 Composite Rotor Blades

Since this study is about the composite material development to fan blades, one of the critical things to do is to get a deeper understanding of the structure and what this kind of structure requires in terms of material properties and mechanical behaviour based on their operating conditions, and economic and engineering limitations. Therefore, the review papers are handled and are considered as the most valuable sources that are related to this study to any extent. Reviews on different types of composite materials are crucial to understanding the concepts and the historical development that is related to the study as well as they give hints where a research gap would be [151, 99].

A blade is a thin-walled structure, and studies have handled blades as beams [100]. Even though there have been both done, modelling the blades as shells and beams [138, 50, 244]. Both have advantages and disadvantages; like beams are simpler and computationally cheaper in many cases, they can be solved analytically, but shell models mostly represent the structure better than beam models [167].

Rotor blades have different sorts of loads simultaneously acting on them, and the blades also do not always operate under steady-state conditions [139]. Therefore, they are usually modelled using numerical methods under different case scenarios [100, 116, 115]. There are different modelling techniques developed for rotor blades that are made of composite materials taking into account all their advantages and weaknesses [166]. Shell models are the most common models, especially for composite laminated structures [138, 77]. Beam models, including thin-walled open cross-sectional models, are used, as well as different shell/plate models [50]. These theoretical models are based on many assumptions, and they require validation by experimental works. Thus, experimental works have been performed in the past to get a more reliable understanding of the rotor blades' mechanics as well as to develop better numerical tools [244]. After all, it is understood that the application of composites to rotor blades is more than the superposition of knowledge on composites and rotor blades. Considering this complex behaviour, it also needs to be stated that damping characteristics are also critical when modelling a composite material for rotor blade applications since excessive damping increases the stresses

significantly, whereas having less damping increases the vibration amplitudes.

While working on macro-scale, composites are relatively simple to handle. Nevertheless, when it comes to micromechanics or even nanomechanics of composites, the whole behaviour can be changed by some important new parameters that are introduced by the new scale [143]. Gaining more understanding of going towards nano-scale helps to grasp the mechanical behaviour of emerging new generation materials like nanocomposites in a more accurate way [150, 175]. Material modelling on small scales can be done in three ways: nonlocal continuum theory, molecular dynamics and multi-scale techniques, which are mixed methods of the two mentioned approaches [168, 175]. Modelling damping on small scales can be done using the same three main methods. Modelling of damping in the nonlocal continuum approach is based on typical nonlocal equations supported by the materials' constitutive equations as well as the appropriate damping functions. Starting from beams, Adhikari proposed two viscous damping types for nonlocal rods in axial vibrations, which are strain-rate-dependent and velocity-dependent damping functions [6]. As stated earlier, viscoelastic material modelling introduces a considerably accurate damping behaviour to a mechanical system [137, 135].

It is possible to approach composites as structures. After characterising the mechanical behaviour, specialised constitutive equations can be derived. Following the derivation of the constitutive equations, composites are evaluated as materials. This process is called homogenisation [185]. Homogenisation is based on calculating the mean of the field properties over a predefined domain [37, 39, 38]. Mori-Tanaka is a particularly effective method for homogenising particulate reinforced composites with nonlinear behaviour [175, 268, 61]. The mean-field estimations made using Mori-Tanaka homogenisation are also valid in small scales as well [150, 170, 189].

2.6 Strategy for the Research

This literature review has helped derivate the strategy followed for this research project by covering the relevant fields and revealing the research gaps in the literature. First, the mate-

rials are selected. Because of their variable/flexible properties and specific strength, polymeric composite materials are selected for the mentioned applications aviation industry [140]. Even though polymeric composites can have relatively high specific strength and stiffness, they are considered as soft materials [274]. It is also decided that the composite material is picked as epoxy-based reinforced with rubber, silica and HNT particle nanocomposite [181]. Rubber, HNT and silica reinforcements are selected to stiffen and toughen the polymer [40]. Apart from toughening, these types of reinforcements also affect the damping of the epoxy structure significantly [214]. Another reason for choosing this material combination is the low density and relatively considerable specific strength, as well as their lower risk of harming human health during the manufacturing process, unlike carbon nanotubes.

Following the selection of the materials, first, DMA test setups and their fixture configurations need to be analysed to understand if/how friction is involved in the results. Then, the nanocomposites are manufactured with consistent manufacturing procedures, which are followed by a detailed characterisation procedure to provide a basis for the validation of the numerical tools that are developed. As the next steps of the project, it is necessary to formulate a new novel three-dimensional solid nonlocal stress calculation with nonlocal material properties in order to model the representative volume of the composite material. As mentioned earlier, there is no exact solution to this three-dimensional nonlocal problem. Therefore, the Galerkin approach, approximate energy approach, or some similar polynomial-formed shape functions will be chosen to start the finite element formulation. Also, a new continuation tool to model the nonlinear vibration of the nonlinear material behaviour.

This developed strategy can be boiled down to the following four main ideas:

- Analyse the mechanics of damping measurement setups/fixture configurations of the DMA and ensure frictional effects are minimised.
- Manufacture the nanocomposites
- Conduct experimental characterisation on the manufactured composite samples.

- Perform the numerical analyses on the RVEs and on the composite structures, a cantilever beam made of nonlinear nanocomposites and a blade made of nonlocal material.

Chapter 3

Analytical Modelling of the Effects of Friction on Clamped Beams

3.1 Introduction

Beams/bars are the most straightforward continuous mechanical structures that are representative of real-world applications. This simplicity makes their mechanical behaviour relatively easier to predict. The beams/bars display predictable mechanical behaviour due to their simple mathematical formulations that make them easier to develop. The simplicity of their mathematical formulations makes it easier to develop analytical solutions [271]. This predictability makes these objects one of the most commonly used for experiments. Researchers use beams/bars in many experimental studies to test and compare hypotheses, and parameters like material properties [220]. In most experimental setups, the beams are subjected to contact either through clamping or jointed connections, which interact with the beam and change the stiffness and/or damping levels of the experimental system by introducing friction.

Damping can be considered a concept that is as old as vibration. Hence, it is one of the most important properties that is investigated in the literature [160]. Damping in a system does not always occur due to the material properties. It can be based on other structural reasons as well as the material itself [5]. Its importance lies in its effect on the dynamic behaviour of the

mechanical system [57].

As one of the most common external damping, friction must be considered [222]. Friction is one of the areas that has been actively researched for centuries [78]. A good example would be Parthenon. Between the marble drums of the columns of the Parthenon, there are wooden pinned joints instead of a grout or soil mixture. This reduces the stiffness and adds frictional damping to the structure. As a result, the stresses occurring during earthquakes are significantly reduced [152, 197].

The contribution of friction to damping can be investigated in many different ways, including analytical, numerical and experimental techniques. As a result, there are various modelling techniques for friction itself which are explained in relevant detail by Gagnon et al., and Pennestri et al. [88, 190]. Among these techniques, the expression of friction forces depending on slip velocity and/or displacement using functions such as Benson, Coulomb, and smooth Coulomb functions stand out in the literature [156, 105, 233].

It must be noted that friction leads to a nonlinear behaviour that introduces mathematical complexities, which makes the solution procedure more complicated [117]. Thus, nonlinear systems are commonly linearised to reduce the complexities. However, this hinders the accuracy of the solution to some extent [265, 233, 85]. Dealing with more complex models that are still analytically solvable increases the level of accuracy significantly [159, 158, 207]. As the industry develops, more complicated structures arise. More complex models become analytically unsolvable [230]. These different nonlinear models are solved by employing numerical methods such as harmonic balance, and finite element method [261, 240].

Even the most complicated nonlinear models use idealisations and simplification assumptions to make the governing equations solvable or at least computationally more reasonable. Therefore, imperfections like compliance of supports and friction at the contact interfaces of supports are usually neglected [79]. These imperfections may cause significantly different mechanical behaviour than the idealised model [200]. Some of these imperfections are hard to predict because of the lack of insight into their governing mechanisms. These imperfections can be handled as uncertainties [55].

Friction plays a crucial role as one of these imperfections since it may change the characteristics of a beam/bar-clamp-support system significantly [79, 261]. Thus, friction must be predicted and modelled accurately to be able to design systems that match the desired criteria. In most experimental test setups, friction can influence the results significantly. Prediction of the effects of friction is the primary concern/origin of this study. Contact is usually involved, especially in material damping measurement rigs where friction can contribute to the damping data. This phenomenon can be seen in dynamic mechanical analysis (DMA) test setups [111]. However, it must be avoided.

To avoid/minimise the effects of friction, one must predict it with reasonable accuracy. For this purpose, Shaw [222] explore the influence of dry friction on a dynamic system using a velocity-dependent, piece-wise function for friction force. Ionescu and Paumier [105] employ a slip displacement-dependent approach for elastic systems for static analyses. Ahn and Barber [10] handle the response contacts to cyclic loading. Riddoch et al. [207] deal with discrete systems that are subjected to frictional damping and base excitation. Marino et al. [159] handle the motion of a single degree of freedom system involving friction due to cyclic wall motion analytically. Marino and Cicirello [157] also investigate a similar system experimentally. They later develop an analytical model to determine the limits of the motion regimes of multiple degrees of freedom systems analytically [158]. Apart from discrete systems, there are studies that handle continuous systems like bars/beams subjected to friction. Asadi et al. [15] investigate a beam supported with a bolted structure where friction is observed, both analytically and experimentally. Ferri and Bindermann [79] solve multiple cases where there is friction introduced by different types of supports. Apart from the frictional supports, there are also studies that deal with the dynamics of the beams touching a wall that causes friction [261].

This study develops a new analytical solution to efficiently model the dynamical axial and torsional behaviour as well as the bending of bars/beams with frictional supports, which is essentially inspired by the DMA test setups [4, 243]. Since the formulation uses bars/beams with straight axes, it is assumed that axial and torsional as well as bending motions are decoupled. Therefore, these three motion types are handled in different subsections individually. This model is built taking into account frictional forces, which cause damping. It must be noted

that friction can also induce vibrations depending on the conditions that the system is in. In order to express the forces and moments at the supports, a Newtonian approach is used, and to model frictional contact, two separate approaches, namely, displacement and velocity-dependent formulations, are employed. These resultant contact reactions are reduced to concentrated forces/moments and are plugged into the governing bar/beam vibration equation as nonlinear external forces with relative slip and/or slip velocity dependence. The outcoming nonlinear partial differential equation is solved analytically using the Galerkin. In order to validate the model, a finite element model is built and solved with an alternating frequency-time scheme of harmonic balance (AFTHB) numerically [126]. The results of both analytical and numerical models are compared. Finally, the effects of friction on the dynamic behaviour of bars/beams are discussed. Based on the review of literature, there is no study published focusing on the analytical solutions for axial and torsional dynamics of bars and bending motion of frictionally clamped beams using these mentioned approaches.

3.2 Description of the Problem, Analysis and Formulation

This study focuses on straight bars/beams made of linear elastic isotropic and homogeneous materials with clamps introducing friction at the contact interfaces. The penalty contact algorithm is employed to model the contact. To investigate this problem, two different solutions are conducted, namely analytical and a method combining AFTHB and FE, for validation. In this section, the developed analytical formulations and methodologies are presented.

While developing the model, it is necessary to analyse the interaction between the clamps and the bar. It is assumed that the bar is made of linearly elastic, homogeneous and isotropic material without any inherent material damping, the vibration amplitude is small, the slip at the contact interfaces is small as well, and no contact separation occurs throughout the entire motion. The friction force is mainly defined by the dependency of relative slip and slip velocity. The coefficient of friction is assumed to be constant and independent of the slip distance and

velocity. Based on these assumptions, an analytical model is developed to analyse the effect of friction on the system behaviour. Furthermore, this model reduces the pressures and other types of distributed loads at the contact interfaces to concentrated point loads.

3.2.1 Model in Axial Motion

The first formulation concerns the axial vibrations of a bar as one of the most uncomplicated cases for bars. This system is representative of the tension-compression fixture configuration of a DMA experiment set up. The axial models consist of a clamped-clamped bar, including friction at the interfaces between the bar surfaces and the clamps on both ends of the bar. A free body diagram is displayed in Figure 3.1 to show the interaction forces, and some dimensions where μ is the coefficient of friction between the clamps and the bar, F_{NA} and F_{NB} are the clamp tightening forces, evaluated as normal forces for friction modelling. The subscripts A and B note the points where friction exists. The forces are defined as functions of the friction coefficient, slip and slip velocity and the normal force, which is the tightening force for this model. Based on this free-body diagram, an analytical model is formulated. Apart from the general assumptions enumerated earlier, this model is derived for distinct full-stick/full-slip (gross slip) motion regimes and assumes that the clamps are rigid and introduce tightening forces and friction as point loads.

The governing partial differential equation is the starting point for this analytical model. Relying on the assumptions described before, the wave equation is used given in Equation (3.1).

$$\rho A(z) \frac{\partial^2 w}{\partial t^2} = \frac{\partial}{\partial z} \left[AE(z) \frac{\partial w}{\partial z} \right] + f(z, t) \quad (3.1)$$

where L is the length of the bar which lies between the clamps, A is the cross-sectional area of the bar, E is the modulus of elasticity, ρ is the density of the bar material, $f(z, t)$ is the external distributed load and $w(z, t)$ is the axial displacement of the bar.

The nonlinear part of this equation is the force term. In Equation (3.2) the frictional forces are

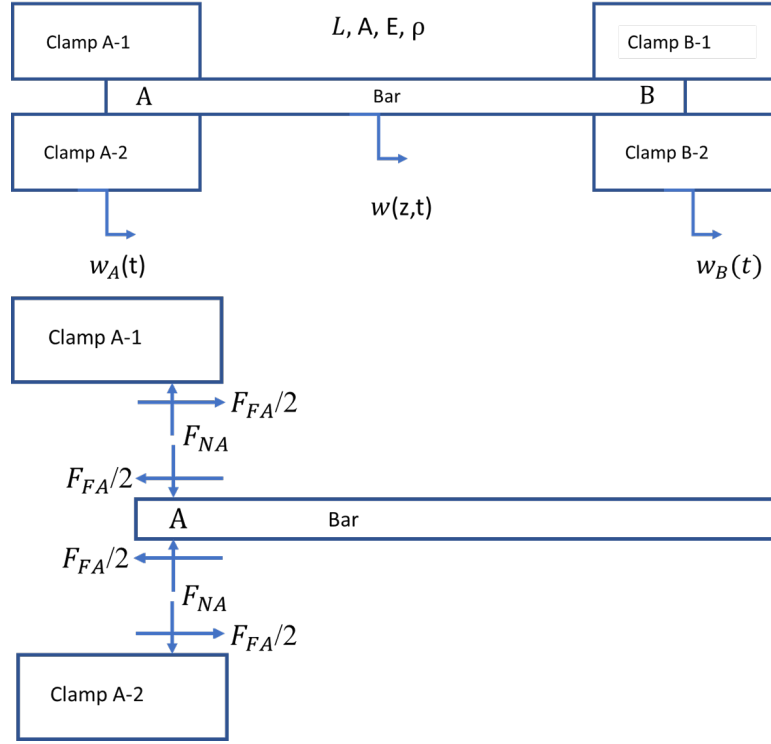


Figure 3.1: Axial forced vibration of a rod with frictional clamps.

expressed within the external load term.

$$f(z, t) = \delta(z) F_{FA} + \delta(z - L) F_{FB} \quad (3.2)$$

Here, $\delta(z)$ is the Dirac Delta function. Also, F_{FA} and F_{FB} denote the friction forces applied by the clamps A and B on the bar.

The friction depends on the relative slip or slip velocity dependency, which are denoted with w_{slip} and $\dot{w}_{slip}$ respectively. The relative slip and slip velocities are expressed with respect to their locations, namely, points A and B in Equations (3.3) and (3.4).

$$W_{A \text{ slip}} = w(0, t) - w_A(t), \quad W_{B \text{ slip}} = w(L, t) - w_B(t) \quad (3.3)$$

$$\dot{W}_{A \text{ slip}} = \dot{w}(0, t) - \dot{w}_A(t), \quad \dot{W}_{B \text{ slip}} = \dot{w}(L, t) - \dot{w}_B(t) \quad (3.4)$$

where $\dot{w}$ expresses the axial velocity of the bar, w_A , w_B , $\dot{w}_A$ and $\dot{w}_B$ express the displacement and velocity of the clamps respectively.

The interactions due to contacts appear in the form of distributed loads. Their resultants are the effective frictional loads and the effective frictional loads that are modelled as point loads/concentrated forces and/or moments. Acknowledging that this model aims to reduce the contact interactions into concentrated loads, the distributed loads need to be related to concentrated loads in terms of the contact parameters. The contact pressure p is the reason for the resulting normal force at the contact, and τ is the frictional shear stress which leads to the tangential frictional force when summed over the contact surface area A^* . The relations between the contact forces, pressure and shear are expressed in Equations (3.5) and (3.6).

$$F_{FA} = \iint_0^{l_A} \tau_A dA^*, \quad F_{FB} = \iint_0^{l_B} \tau_B dA^* \quad (3.5)$$

$$F_{NA} = \iint_0^{l_A} p_A dA^*, \quad F_{NB} = \iint_0^{l_B} p_B dA^* \quad (3.6)$$

The contact pressure and shear are related as a function of the friction coefficient, contact pressure and relative slip or slip velocity depending on the friction model used.

Finally, the governing equation of motion is transformed into Equation (3.7), which is a wave equation with a nonlinear force.

$$\rho A(z) \frac{\partial^2 w}{\partial t^2} = \frac{\partial}{\partial z} \left[AE(z) \frac{\partial w}{\partial z} \right] + \delta(z) F_{FA} + \delta(z - L) F_{FB} \quad (3.7)$$

3.2.2 Model in Torsional Motion

In this scenario, the friction involved in the torsional vibration of a bar is formulated. For the sake of simplicity, the model is formulated for a bar with a straight axis and a circular cross-

section. More specifically, this model is derived for cylindrical bars, not arbitrarily prismatic ones. The formulation of torsional vibrations of bars is based on the same assumptions and the same form of governing equation (Equation (3.8)) as the axial case. Figure 3.2 presents a simple cylindrical bar, fitted into pipes, which behave as clamps, with initial fitting/tightening pressures p_A and p_B along the lengths l_A and l_B , which are not considered in the free bar length L .

$$\rho I_p(z) \frac{\partial^2 \theta}{\partial t^2} = \frac{\partial}{\partial z} \left[G I_t(z) \frac{\partial \theta}{\partial z} \right] + m(z, t) \quad (3.8)$$

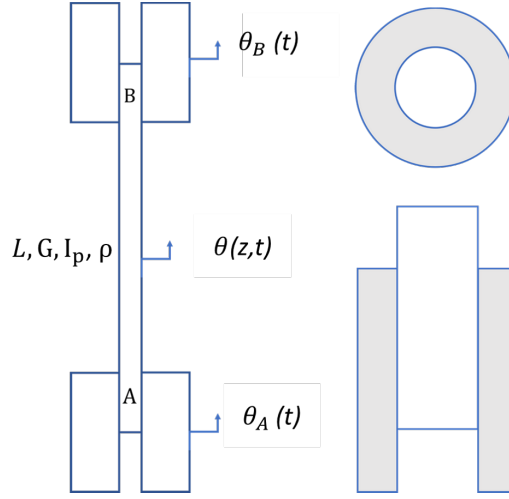


Figure 3.2: Circular bar fitted in pipes for clamping.

where I_t is the torsional moment of inertia, I_p is the polar area moment of inertia, G is the shear modulus of the material, $m(z, t)$ is the external distributed moment term, and θ is the twist angle per unit length of the bar. Considering that the bar has a circular cross-section, I_t becomes I_p .

Assuming the external loads as point loads, the load term becomes Equation (3.9).

$$m(z, t) = \delta(z) M_{FA} + \delta(z - L) M_{FB} \quad (3.9)$$

Here, M_{FA} and M_{FB} are the resultant moments applied on the bar by the fittings.

For further analysis, the loads acting on the bar are displayed in Figure 3.3. As mentioned earlier, p is the fitting pressure, while τ is the shear caused by the friction at the contact interface.

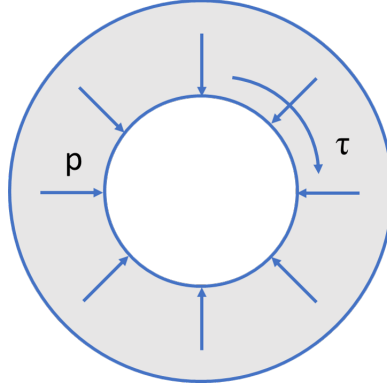


Figure 3.3: Contact pressure and tangential distributed load at the contact interface for fitted circular bar.

Using the relevant friction models, the relation between the shear and the contact pressure is the same as in the axial case given in Equations (3.5) and (3.6).

To calculate the resulting moment, the contact friction shear is to be integrated over the whole surface of contact. This integration is shown in Equation (3.10).

$$M_{FA} = \int_0^{l_A} \tau_A \pi D dz, \quad M_{FB} = \int_0^{l_B} \tau_B \pi D dz \quad (3.10)$$

Using the moment terms in the governing equation of motion, the expression is shown in Equation (3.11).

$$\begin{aligned} \rho I_p(z) \frac{\partial^2 \theta}{\partial t^2} = \frac{\partial}{\partial z} \left[G I_p(z) \frac{\partial \theta}{\partial z} \right] + \delta(z) \int_0^{l_A} \tau_A \pi D dz \\ + \delta(z - L) \int_0^{l_B} \tau_B \pi D dz \end{aligned} \quad (3.11)$$

3.2.3 Dual Cantilever Beam

The first case is inspired by the dual cantilever fixture configuration of a DMA experiment set up, and it is formulated considering the in-plane bending motion. Considering small displacements and the presence of simple bending motion, the typical equation of motion for bending of Euler-Bernoulli beams is used, and that is given in Equation (3.12). It is important to stress that no external load terms appear in this equation of motion since the friction is handled in the boundary conditions.

$$\rho A \frac{\partial^2 v}{\partial t^2} + EI(z) \frac{\partial^4 v}{\partial z^4} = 0 \quad (3.12)$$

A dual cantilever beam of the length $2L$, that is excited with an imposed displacement from the midpoint in the transverse direction is modelled. For simplicity, only half of the system is modelled. This means there is only one support at point A at $z = 0$ with two clamps, and the free end does not rotate but is subjected to a displacement. This can be done by taking advantage of the fact that the beam shows exact symmetry about an axis perpendicular to its tangent of the neutral axis at the mid-point located at $z = L$. So, modelling only half of the beam is sufficient to characterise its behaviour. A sketch for this beam-fixture assembly can be found in Figure 3.4.

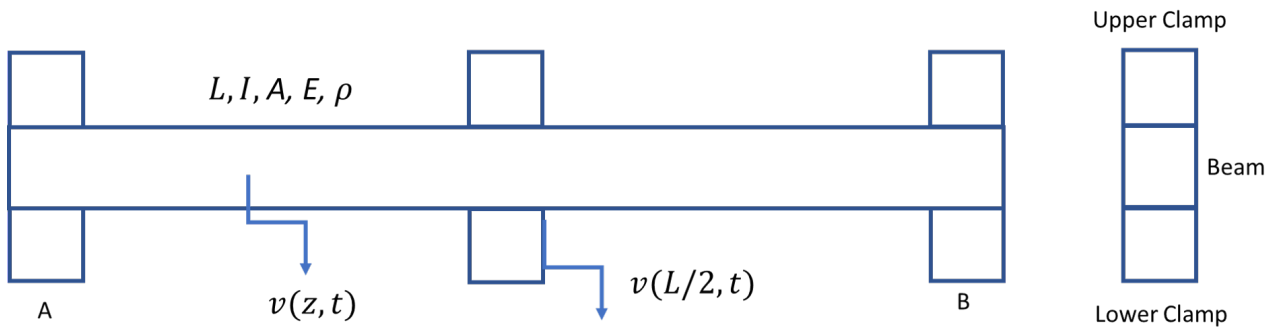


Figure 3.4: Schematic visualisation of DMA dual cantilever fixture configuration.

For this case, a free body diagram for a dual cantilever beam with frictional clamps is displayed in Figure 3.5 to show the interaction forces and some essential dimensions for the formulation.

Here, $F_{N \text{ top}}$ and $F_{N \text{ bottom}}$ are the clamp normal forces, including the tightening forces at the top and bottom clamps, respectively and $F_{F \text{ top}}$ and $F_{F \text{ bottom}}$ are the frictional shear forces at the contact interface between the clamps and the beam, h is the beam thickness, and e is the eccentricity between the resultant normal forces which roots from the non-uniform and different contact pressure distribution at the contact interfaces of the beam, upper and lower clamps. Like the axial and torsional models, this model also consists of a clamped beam system, including friction at the interfaces between the beam and the clamps.

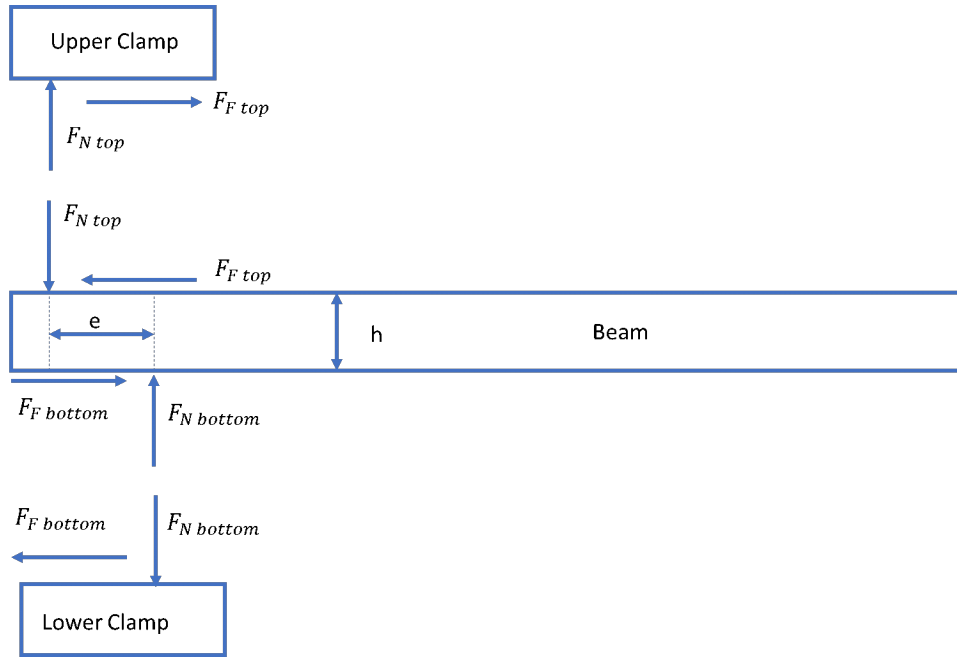


Figure 3.5: Free body diagram of a beam with frictional clamps.

Similar to the model in axial motion, the frictional forces can be defined as functions of the friction coefficient μ , the normal forces $F_{N \text{ top}}$, $F_{N \text{ bottom}}$ and the relative slip $V_{A \text{ top slip}}$, $V_{A \text{ bottom slip}}$ or slip velocities $\dot{V}_{A \text{ top slip}}$, $\dot{V}_{A \text{ bottom slip}}$.

Considering these clamps make an imperfect fixed (encastre) support, it is predicted that the reactions consist of moment, horizontal and vertical forces once the actual reactions shown in the free body diagram are simplified to point loads. The free-body diagram of the beam alone with the simplified resultant support reactions that root from the contact forces between the beam and the clamps is presented in Figure 3.6. Here, $M_{A \text{ bending}}$ denotes the resultant moment at point A.



Figure 3.6: Reaction forces and moments at the clamps.

The boundary conditions, where the clamps are, restrict only the translational degrees of freedom, namely longitudinal and transverse displacements. However, due to the imperfections caused by friction, the rotation is not entirely restricted. Instead, a solution-dependent reaction moment is imposed, and the midpoint of the beam has a constrained rotational degree of freedom. The reaction moment is a concentrated load representing the frictional interaction between the beam and the clamps. These boundary conditions can be expressed as in Equation (3.13).

$$v(0, t) = 0, \quad (3.13a)$$

$$M_{A \text{ bending}} = -EI(z) \left. \frac{\partial^2 v}{\partial z^2} \right|_{z=0}, \quad (3.13b)$$

$$v(L, t) = a \cos(\Omega t), \quad (3.13c)$$

$$\left. \frac{\partial v}{\partial z} \right|_{z=L} = 0 \quad (3.13d)$$

Here, a is the displacement amplitude, and Ω is the frequency of the clamp motion.

The reaction moment is the sum of two parts, as there are two different force couples in the axial and transverse directions. Let these parts of the reaction moment be referred to as M_N and M_F as given in Equation (3.14) where the subscripts N and F denote the directions of normal and friction forces respectively by which these moments are brought to existence.

$$M_{A \text{ bending}} = M_F + M_N \quad (3.14)$$

The first part of the reaction moment M_N is caused by the eccentric normal contact forces. The eccentricity is to be found between the top and bottom resultant normal contact forces due to different contact pressure distributions, which is a consequence of the bending deformation of the beam, as mentioned earlier. While the second part of the component of the reaction moment M_F roots from the existence of friction which depends on the normal strain or normal strain rate of the beam. This makes M_N conservative and M_F dissipative.

Taking a closer look at the normal contact forces and the mechanism that creates this moment M_N , one can notice that during the vibration of the beam, the normal forces at the top and the bottom beam-clamp interfaces are time-dependent and are not always equal to each other. The difference between these two normal contact forces balances the shear force that acts on the beam. Assuming the system is stable, summing the tightening forces with the shear force for each clamp, it becomes clear that, while the normal load increases on one clamp, the load on the other clamp relaxes. However, these individual normal forces are not directly the focus of interest for this formulation. Therefore, for the sake of simplicity, an average normal force, which can be approximated with the initial tightening force, is taken and denoted with F_N . Thus, the moment M_N can be essentially calculated by multiplying the eccentricity with the averaged normal force as given in Equation (3.15) where the eccentricity e is considered as the variable that controls the change in M_N since F_N is assumed as a constant.

$$M_N = F_N e(\phi, t) \quad (3.15)$$

An alternative form to express the non-dissipative first part of the reaction moment M_N would be to formulate a linear, rotational spring to calculate the value of the moment. There, the eccentricity is assumed to be proportional to the rotation of the cross-section ϕ and the rest of the parameters are included in the support stiffness. The spring formulation is expressed in Equation (3.16).

$$M_N = k_A \phi(0, t) = k_A \left. \frac{\partial v}{\partial z} \right|_{z=0} \quad (3.16)$$

As a result of changing normal contact forces $F_{N \text{ top}}$ and $F_{N \text{ bottom}}$, the frictional forces $F_{F \text{ top}}$ and $F_{F \text{ bottom}}$ are affected as well. The maximum frictional forces $\mu F_{N \text{ top}}$ and $\mu F_{N \text{ bottom}}$ have a similar trend as the normal forces. Note that μ represents the coefficient of friction for the contact and it is assumed to be constant throughout the whole process. Thus, the change of the normal forces directly affects the maximum frictional forces. Increasing the maximum frictional forces at one contact interface reduces the maximum frictional forces on the other interface for the same support depending on their respective normal forces. Hence, a similar averaging can also be done for the maximum frictional forces using the averaged coefficient of friction μ and the averaged normal forces F_N . The averaged frictional forces are indicated as F_F .

Using this assumption, the second and the last part of the support reaction moment, which roots from the frictional shear forces, can be formulated. This moment is simply calculated by multiplying the frictional shear force with the beam thickness as given in Equation (3.17). Since the friction forces can remove energy from the system, this part of the reaction moment is evaluated as dissipative.

$$M_F = F_F h \quad (3.17)$$

To clarify the definition of the frictional forces, slip in the system should be defined more precisely. As Euler-Bernoulli beam theory suggests, the cross-section of the beam is initially perpendicular to the axis and stays so after the deformation, which can be decomposed to translational and rotational motion. The translation of the cross-section is called the deflection, and the rotation of the cross-section is approximated with the first derivative of deflection with respect to the axis of the undeformed beam. The shear deformation of the beam is assumed to be small and neglected during the calculations. As a result, normal strain distribution due to bending deformation along the cross-section is linear, and the absolute value of the normal strain reaches its minimum at the neutral axis and its maxima at the boundaries, as displayed in Figure 3.7. Thus, the slip at the contact interfaces can be defined with the dependence on the normal strain at the boundaries of the beam and the support $\epsilon(0, t)$. Similarly, slip velocity can be correlated to the normal strain rate at the same location $\dot{\epsilon}(0, t)$. Since the clamps are

considered to be stationary, no complicated relative motion calculation is needed to express the slip. Considering the necessary concepts on which friction depends is the slip itself for Jenkins, but it is the slip velocity for the smooth Coulomb approach. The slip velocity is the first derivative of the slip with respect to time. Hence, using the relation between the slip and the maximum normal strain of the beam, the slip velocity is expressed in relation to the normal strain rate of the beam.

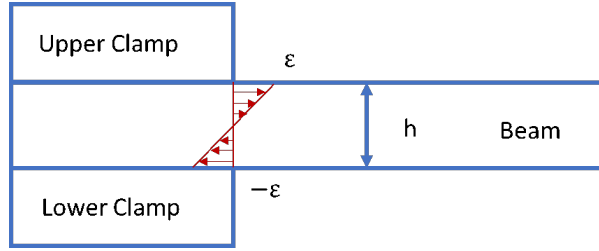


Figure 3.7: Axial strain and strain rate distribution along the cross-section at the clamps.

After all these simplifications, the boundary with clamps is visualised in Figure 3.8. To restrict the translation of the boundary, simple support is placed. To impose the solution-dependent moments, a rotational spring and a friction element are used.

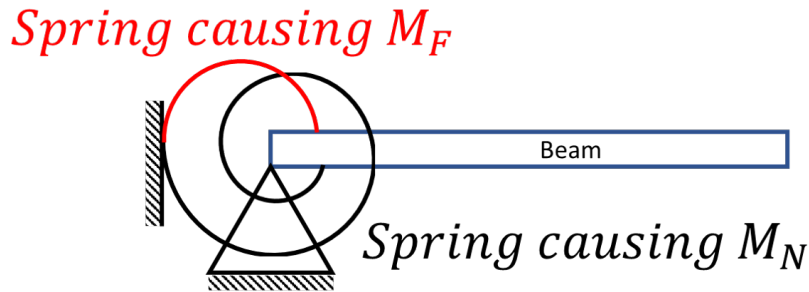


Figure 3.8: The boundary with the clamps.

Finally, the boundary conditions are rearranged. They read as in Equation (3.18).

$$v(0, t) = 0, \quad (3.18a)$$

$$\left. \frac{\partial^2 v}{\partial z^2} \right|_{z=0} = - \frac{F_F h + k_A \left. \frac{\partial v}{\partial z} \right|_{z=0}}{EI(z)}, \quad (3.18b)$$

$$v(L, t) = a \cos(\Omega t), \quad (3.18c)$$

$$\left. \frac{\partial v}{\partial z} \right|_{z=L} = 0 \quad (3.18d)$$

3.3 Analytical Solution

The axial and torsional formulations presented in Section 3.2 are solved using the Jenkins and the velocity-dependent friction models under some assumptions to simplify the system and make the solution procedure easier. It is assumed that the first vibration mode of the bar is the dominant to the response of the bar. Besides, it is considered that the clamps move with the same frequency and amplitude but in reverse directions, which causes the midpoint of the bar to be stationary despite the nonlinearity introduced by the frictional contact. This allows for the simplification of the model by evaluating only half of the bar. This simplified model is then nondimensionalised to generalise the solution procedure and the results.

3.3.1 Axial Model

The clamped bar model with moving clamps in the axial direction introduced in the previous section (Section 3.2.1) is studied in detail here. Referring to Figure 3.9a and Figure 3.9b, the frictional clamp is assumed to move with a harmonic displacement $a \cos(\Omega t)$ and to exert a contact force at the tip of the clamped free bar. This is half of the model described in the formulation section. Taking advantage of the symmetry assumption explained earlier, only half of the bar is representative of the actual system.

Nondimensionalisation

If one assumes that a constant cross-section and the imposed displacement at $z = 0$ to be equal to $a \cos(\Omega t)$, the equation of motion for the axial model, where $w(\bar{z}, t)$ is the axial displacement,

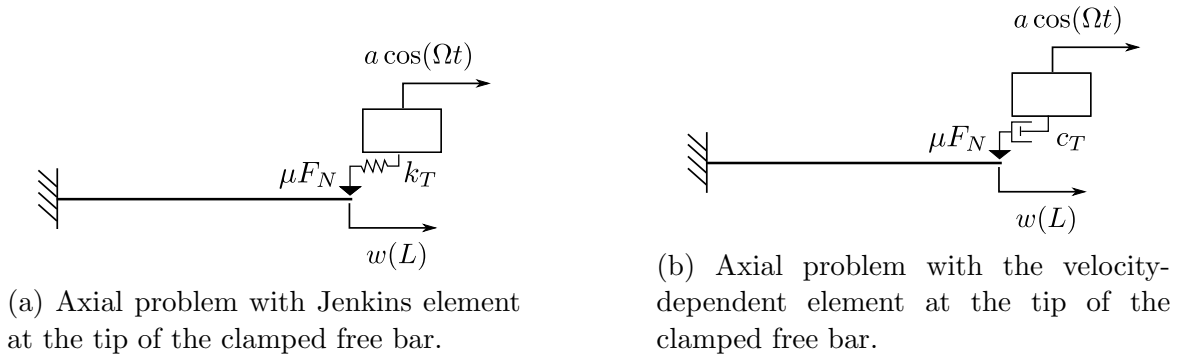


Figure 3.9: Axial problem with clamped and fixed and bar.

is transformed into Equation (3.19) which is a simplified version of Equation (3.7).

$$\rho A \frac{\partial^2 w}{\partial t^2} - EA \frac{\partial^2 w}{\partial z^2} = \delta(z - L) F_{NL}(w_{rel}, \frac{\partial w_{rel}}{\partial t}) \quad (3.19)$$

$$w(z, t)|_{z=0} = 0 \quad (3.20)$$

The previous equation is nondimensionalised using the following variables: $\tilde{w}$, $\tilde{t}$, $\tilde{\Omega}$, $\tilde{F}$, $\tilde{k}$, $\tilde{c}$, $\tilde{\kappa}$, $\tilde{\kappa}_0$, $\tilde{k}_T$, and $\tilde{c}_T$, obtained as:

$$\begin{aligned} m &= \rho AL ; \kappa = \omega \sqrt{\rho/E} ; \kappa_0 = \omega_0 \sqrt{\rho/E} ; F_0 = m \omega_0^2 a \\ k_0 &= m \omega_0^2 ; c_0 = m \omega_0 ; \tilde{w} = w/a ; \tilde{z} = z/L ; \tilde{t} = t \omega_0 \\ \tilde{\Omega} &= \Omega/\omega_0 ; \tilde{F} = F/F_0 ; \tilde{k} = k/k_0 ; \tilde{c} = c/c_0 ; \tilde{\kappa} = \kappa L \\ \tilde{\kappa}_0 &= \kappa_0 L ; \tilde{k}_T = k_T/k_0 ; \tilde{c}_T = c_T/c_0 \end{aligned} \quad (3.21)$$

where a is the amplitude of the clamp motion, m is the mass, A is the cross-sectional area and L is the length of the bar, ω_0 is the eigenvalue of the bar mode one wants to study, κ is the wavelength of the exciting frequency, κ_0 is the wavelength of the corresponding eigenfunction, F_0 is the characteristic force, k_0 is the characteristic stiffness, c_0 is the characteristic damping, k_T is the stiffness coefficient of the contact and c_T is the damping coefficient of the contact.

Finally, multiplying Equation (3.19) by the factor $1/(\rho A \omega_0^2 a)$ and making the scaled variables

appear reads as follows in Equation (3.22).

$$\ddot{\tilde{w}} - \frac{1}{\tilde{\kappa}_0^2} \tilde{w}'' = \delta(\tilde{z} - 1) \tilde{F}_{NL} \quad (3.22)$$

where $\ddot{\tilde{w}} = \frac{\partial^2 \tilde{w}}{\partial \tilde{t}^2}$, $\tilde{w}'' = \frac{\partial^2 \tilde{w}}{\partial \tilde{z}^2}$, and the transformation of the dirac delta from $\delta(z - L)$ to $\delta(\tilde{z} - 1)$ generated a factor L .

To solve this problem, it is assumed that the harmonic displacement of the clamp has a frequency around the first natural frequency of the bar. Hence, the ansatz is shown in Equation (3.23).

$$\tilde{w}(\tilde{z}, \tilde{t}) = \alpha \sin(\tilde{\kappa} \tilde{z}) \cos(\tilde{\Omega} \tilde{t} + \psi) \quad (3.23)$$

$\tilde{w}$ is the nondimensional displacement, α is the nondimensional amplitude of the bar motion over the nondimensional amplitude of the clamp motion and ψ is the phase.

The application Galerkin method i.e. the projection of Equation (3.22) onto the shape function $\sin(\tilde{\kappa}_0)$, leads to Equation (3.24)

$$\frac{1}{2} \alpha^2 (1 - \tilde{\Omega}^2) \cos(\tilde{\Omega} \tilde{t} + \psi) = \tilde{F}_{NL} \quad (3.24)$$

The nondimensional relative displacement of the bar by the clamp (at $\tilde{z} = 1$) is presented in Equation (3.25), and the amplitude of this relative displacement is expressed in Equation (3.26).

$$\tilde{w}_{rel}(\tilde{t}) = \cos(\tilde{\Omega} \tilde{t}) - \alpha \cos(\tilde{\Omega} \tilde{t} + \psi) \quad (3.25)$$

$$\alpha_{rel} = \sqrt{1 + \alpha^2 - 2 \alpha \cos(\psi)} \quad (3.26)$$

Jenkins Element

When using the Jenkins approach, there is a limit to the relative slip displacement up to which the frictional dissipation at the bar-clamp interface can be avoided. If the clamp is modelled with a Jenkins element, the stick limit condition for the amplitude yields Equation (3.27),

$$\tilde{k}_T \alpha_{rel}^* = \mu \tilde{F}_N \quad (3.27)$$

where α_{rel}^* is the relative amplitude at the stick limit, and if the actual amplitude of the bar does not exceed the limit, $\alpha_{rel} \leq \alpha_{rel}^*$ the contact force becomes linear because the clamp is in full-stick. Thus, F_{NL} becomes as presented in Equation (3.28).

$$\tilde{F}_{NL}(\tilde{t}) = \tilde{k}_T \tilde{w}_{rel}(\tilde{t}) \quad (3.28)$$

Under these conditions, one can solve Equation (3.24) in the unknown α and ψ by projection onto $\sin(\tilde{\Omega}\tilde{t} + \psi)$ and $\cos(\tilde{\Omega}\tilde{t} + \psi)$, that reads as shown in Equation (3.29).

$$\alpha = \frac{2\tilde{k}_T}{1 - \tilde{\Omega}^2 + 2\tilde{k}_T} \quad (3.29a)$$

$$\psi = 0 \quad (3.29b)$$

$$\alpha_{rel} = |1 - \alpha| = \left| \frac{1 - \tilde{\Omega}^2}{1 - \tilde{\Omega}^2 + 2\tilde{k}_T} \right| \quad (3.29c)$$

Imposing the condition $\alpha_{rel} \leq \alpha_{rel}^*$, which leads to the expression for α_{rel}^* in Equation (3.30), one can derive an expression for the range of $\tilde{\Omega} \in [\tilde{\Omega}^{*pre}, \tilde{\Omega}^{*post}]$ where the full-stick regime occurs and the values of the full-stick region limits. This is presented in Equation (3.31).

$$\alpha_{rel}^* = \frac{\mu \tilde{F}_N}{\tilde{k}_T} \quad (3.30)$$

$$\tilde{\Omega}^{*pre} = \sqrt{1 - 2 \tilde{k}_T \frac{\mu \tilde{F}_N}{\tilde{k}_T - \mu \tilde{F}_N}} \quad (3.31a)$$

$$\tilde{\Omega}^{*post} = \sqrt{1 + 2 \tilde{k}_T \frac{\mu \tilde{F}_N}{\tilde{k}_T + \mu \tilde{F}_N}}. \quad (3.31b)$$

Similarly, the corresponding vibration amplitudes are obtained as given in Equation (3.32).

$$\alpha^{*pre} = 1 - \frac{\mu \tilde{F}_N}{\tilde{k}_T} \quad (3.32a)$$

$$\alpha^{*post} = 1 + \frac{\mu \tilde{F}_N}{\tilde{k}_T} \quad (3.32b)$$

The slip condition is represented with a simplified approach. For the cases where there is slip, it is assumed that the system is excited with force, which has a maximum of $\mu \tilde{F}_N$. Considering the harmonic motion of the clamps, it can be stated that the excitation force is harmonic, and its amplitude is $\mu \tilde{F}_N$ as described in Section 3.4.1. Thus, the system becomes linear with a solution for this particular region.

Velocity-Dependent Element

Dissipation in the clamp is always present with the velocity-dependent model, but frictional dissipation only occurs if the relative slip velocity exceeds the stick limit. In this case, the stick limit condition reads as shown in Equation (3.33) where c_t is the damping coefficient of the frictional contact.

$$\tilde{c}_T \tilde{\Omega} \alpha_{rel}^* = \mu \tilde{F}_N \quad (3.33)$$

Because the clamp is in full-stick as Jenkins model if $\alpha_{rel} \leq \alpha_{rel}^*$, the contact force becomes linear because the clamp is in full-stick as Jenkins model. This is given in Equation (3.34).

$$\tilde{F}_{NL}(\tilde{t}) = \tilde{c}_T \dot{\tilde{w}}_{rel}(\tilde{t}) \quad (3.34)$$

Solving Equation (3.24) in the unknown α and ψ by projection onto $\sin(\tilde{\Omega}\tilde{t} + \psi)$ and $\cos(\tilde{\Omega}\tilde{t} + \psi)$, that yields the results which are shown in Equation (3.35).

$$\alpha = \frac{2 \tilde{c}_T \tilde{\Omega}}{\sqrt{(1 - \tilde{\Omega}^2)^2 + (2 \tilde{c}_T \tilde{\Omega})^2}}, \quad (3.35a)$$

$$\psi = \arctan \left(\frac{1 - \tilde{\Omega}^2}{2 \tilde{c}_T \tilde{\Omega}} \right), \quad (3.35b)$$

$$\alpha_{rel} = \frac{|1 - \tilde{\Omega}^2|}{\sqrt{(1 - \tilde{\Omega}^2)^2 + (2 \tilde{c}_T \tilde{\Omega})^2}} \quad (3.35c)$$

Imposing the condition $\alpha_{rel} \leq \alpha_{rel}^*$, one could derive an expression for the range of $\tilde{\Omega} \in [\tilde{\Omega}^{*pre}, \tilde{\Omega}^{*post}]$ where the full-stick regime occurs and the values of the full-stick region limits. To obtain a simplified expression, an approximation on the equation for α_{rel} can be done, assuming that the term $\tilde{c}_T \tilde{\Omega} \approx \tilde{c}_T \bar{\Omega}$ does not vary with $\tilde{\Omega}$. The expressions under this assumption are displayed in Equation (3.36).

$$\tilde{\Omega}^{*pre} = \sqrt{1 - \frac{2 \tilde{c}_T \bar{\Omega} \mu \tilde{F}_N}{\sqrt{(\tilde{c}_T \bar{\Omega})^2 - (\mu \tilde{F}_N)^2}}}. \quad (3.36a)$$

$$\tilde{\Omega}^{*post} = \sqrt{1 + \frac{2\tilde{c}_T \bar{\Omega} \mu \tilde{F}_N}{\sqrt{(\tilde{c}_T \bar{\Omega})^2 - (\mu \tilde{F}_N)^2}}}. \quad (3.36b)$$

Similarly, for the corresponding vibration amplitudes are presented in Equation (3.37).

$$\alpha^{*pre} = \alpha^{*post} = \sqrt{1 - \left(\frac{\mu \tilde{F}_N}{\tilde{c}_T \bar{\Omega}} \right)^2} \quad (3.37)$$

Similar to the case with the Jenkins model, for regimes other than full-stick, in other words, where slip occurs, the same assumption is made for harmonic force excitation.

3.3.2 Torsional Model

The bar model in torsional motion, described in the previous section (Section 3.2.2), is solved following the same steps for the axial case (Section 3.3.2). Considering that the form of the governing equation is the same as the axial model, the solution procedures are pretty similar. Besides, like the axial model, here, the clamps at both ends of the bar are assumed to move with the same frequency but in opposite directions obeying the harmonic displacement function $\theta_A \cos(\Omega t)$ and $\theta_B \cos(\Omega t - \pi)$. This enables one to model half of the bar to simplify the model by taking advantage of the stationary middle point of the bar. The motion of these clamps applies frictional forces on the bar that results in net moments, which force the bar into a torsion. As with the axial model, both Jenkins and velocity-dependent friction models are applied, and the analytical solutions are obtained.

Nondimensionalisation

Assuming a constant circular cross-section and the imposed displacement at $z = L$ to be equal to $\theta_A \cos(\Omega t)$, the equation of the torsional rotation $\theta(\bar{z}, t)$ is formed into Equation (3.38) using Equation (3.11). Here, θ_A is the rotation amplitude, and Ω is the frequency of the clamp

motion.

$$\rho I_p \frac{\partial^2 \theta}{\partial t^2} - G I_p \frac{\partial^2 \theta}{\partial z^2} = \delta(z - L) M_{NL} \quad (3.38)$$

For normalisation, the nondimensional variables $\tilde{r}$, $\tilde{\theta}$, $\tilde{t}$, $\tilde{\Omega}$, $\tilde{M}$, $\tilde{k}$, $\tilde{c}$, $\tilde{\kappa}$, $\tilde{\kappa}_0$, $\tilde{k}_T$ and $\tilde{c}_T$ are defined as documented in Equation (39) set:

$$\begin{aligned} r_0 &= r \theta_A, \quad J = \rho I_p; \quad \kappa = \omega \sqrt{\rho/G}; \quad \kappa_0 = \omega_0 \sqrt{\rho/G} \\ M_0 &= J \omega_0^2 L \theta_A; \quad k_0 = J \omega_0^2 L; \quad c_0 = J \omega_0 L; \quad \tilde{r} = r/r_0; \quad \tilde{\theta} = \theta/\theta_A \\ \tilde{z} &= z/L; \quad \tilde{t} = t \omega_0; \quad \tilde{\Omega} = \Omega/\omega_0; \quad \tilde{M} = M/M_0; \quad \tilde{k} = k/k_0 \\ \tilde{c} &= c/c_0; \quad \tilde{\kappa} = \kappa L; \quad \tilde{\kappa}_0 = \kappa_0 L; \quad \tilde{k}_T = k_T/k_0; \quad \tilde{c}_T = c_T/c_0 \end{aligned} \quad (3.39)$$

where the system parameters are the mass moment of inertia J , the cross-sectional polar area moment of inertia I_p and the rest of the parameters are defined as explained in the previous section where the axial case is solved.

Multiplying Equation (3.38) by the factor $1/(J \omega_0^2 \theta_A)$ and displaying the normalised variables, Equation (3.40) is obtained.

$$\ddot{\tilde{\theta}} - \frac{1}{\tilde{\kappa}_0^2} \tilde{\theta}'' = \delta(\tilde{z} - 1) \tilde{M}_{NL} \quad (3.40)$$

where $\ddot{\tilde{\theta}} = \frac{\partial^2 \tilde{\theta}}{\partial \tilde{t}^2}$, $\tilde{\theta}'' = \frac{\partial^2 \tilde{\theta}}{\partial \tilde{z}^2}$, and the Dirac Delta transforms from $\delta(z - L)$ to $\delta(\tilde{z} - 1)$ by generating a factor L .

Assuming that the clamps move following a harmonic rotation function with a range of frequency around the first torsional natural frequency of the bar. In the ansatz shown in Equation (3.41), the function $\tilde{\theta}$ stands for the nondimensional rotation and α for the nondimensional amplitude of the bar motion over the amplitude of the clamp motion.

$$\tilde{\theta}(\tilde{z}, \tilde{t}) = \alpha \sin(\kappa \tilde{z}) \cos(\tilde{\Omega} \tilde{t} + \psi) \quad (3.41)$$

Applying the Galerkin method i.e. the projection of Equation (3.40) onto the shape function $\sin(\frac{\pi \tilde{z}}{2})$, leads to Equation (3.42)

$$\frac{1}{2} \alpha^2 (1 - \tilde{\Omega}^2) \cos(\tilde{\Omega} \tilde{t} + \psi) = \tilde{M}_{NL} \quad (3.42)$$

The nondimensional relative rotation at $\tilde{z} = 1$ and its amplitude are given in Equation (3.43) and Equation (3.44), respectively.

$$\tilde{\theta}_{rel}(\tilde{t}) = \cos(\tilde{\Omega} \tilde{t}) - \alpha \cos(\tilde{\Omega} \tilde{t} + \psi) \quad (3.43)$$

$$\alpha_{rel} = \sqrt{1 + \alpha^2 - 2 \alpha \cos(\psi)} \quad (3.44)$$

Jenkins Element

Similar to the axial case, the stick limit condition is expressed in Equation (3.45), where α_{rel}^* is the nondimensional relative rotation amplitude at the stick limit. In the case where the amplitude remains below the limit, $\alpha_{rel} \leq \alpha_{rel}^*$ the contact force and consecutively, the moment becomes linear because the clamp stays in the full-stick regime.

$$\tilde{k}_T \alpha_{rel}^* = \mu \tilde{F}_N \tilde{r} \quad (3.45)$$

Hence, F_{NL} and M_{NL} become Equation (3.46).

$$\tilde{F}_{NL}(\tilde{t}) = \frac{\tilde{k}_T \tilde{\theta}_{rel}(\tilde{t})}{\tilde{r}} \quad (3.46a)$$

$$\tilde{M}_{NL}(\tilde{t}) = \tilde{k}_T \tilde{\theta}_{rel}(\tilde{t}) \quad (3.46b)$$

It then becomes possible to solve Equation (3.42) in the unknown α and ψ using the same philosophy as the axial case. The resulting expressions are shown in Equation (3.47).

$$\alpha = \frac{2\tilde{k}_T}{1 - \tilde{\Omega}^2 + 2\tilde{k}_T} \quad (3.47a)$$

$$\psi = 0 \quad (3.47b)$$

$$\alpha_{rel} = |1 - \alpha| = \left| \frac{1 - \tilde{\Omega}^2}{1 - \tilde{\Omega}^2 + 2\tilde{k}_T} \right| \quad (3.47c)$$

For the condition $\alpha_{rel} \leq \alpha_{rel}^*$, the expressions for the range of the full-stick regime and the limit values of the full-stick region $\tilde{\Omega} \in [\tilde{\Omega}^{*pre}, \tilde{\Omega}^{*post}]$ are found as well as the amplitudes. These are given in Equations (3.48) and (3.49).

$$\tilde{\Omega}^{*pre} = \sqrt{1 - 2\tilde{k}_T \frac{\mu\tilde{F}_N\tilde{r}}{\tilde{k}_T - \mu\tilde{F}_N\tilde{r}}} \quad (3.48a)$$

$$\tilde{\Omega}^{*post} = \sqrt{1 + 2\tilde{k}_T \frac{\mu\tilde{F}_N\tilde{r}}{\tilde{k}_T + \mu\tilde{F}_N\tilde{r}}}. \quad (3.48b)$$

$$\alpha^{*pre} = 1 - \frac{\mu\tilde{F}_N\tilde{r}}{\tilde{k}_T} \quad (3.49a)$$

$$\alpha^{*post} = 1 + \frac{\mu\tilde{F}_N\tilde{r}}{\tilde{k}_T} \quad (3.49b)$$

The final possible condition is when clamps fail to stick to the bar surface and slide. The same assumption is made in Section 3.3.1. This suggests that the bar is excited in the torsional direction with a harmonic moment and an amplitude of $\mu\tilde{F}_N\tilde{r}$ since the clamps rotate following a harmonic function.

Velocity-Dependent Element

Developing the model with the velocity-dependent formulation leads to the stick limit condition shown in Equation (3.50) where c_T is the damping coefficient of the frictional contact.

$$\tilde{c}_T \tilde{\Omega} \alpha_{rel}^* = \mu \tilde{F}_N \tilde{r} \quad (3.50)$$

$\alpha_{rel} \leq \alpha_{rel}^*$ makes the contact force linear because the clamp sticks fully similar to the Jenkins model. This is given in Equation (3.51).

$$\tilde{F}_{NL}(\tilde{t}) = \tilde{c}_T \dot{\tilde{\theta}}_{rel}(\tilde{t}) \tilde{r} \quad (3.51a)$$

$$\tilde{M}_{NL}(\tilde{t}) = \tilde{c}_T \dot{\tilde{\theta}}_{rel}(\tilde{t}) \quad (3.51b)$$

To solve Equation (3.42) for the unknown α and ψ by projection onto $\sin(\tilde{\Omega}\tilde{t}+\psi)$ and $\cos(\tilde{\Omega}\tilde{t}+\psi)$, that lead to the results presented in Equation (3.52).

$$\alpha = \frac{2 \tilde{c}_T \tilde{\Omega}}{\sqrt{(1 - \tilde{\Omega}^2)^2 + (2 \tilde{c}_T \tilde{\Omega})^2}}, \quad (3.52a)$$

$$\psi = \arctan \left(\frac{1 - \tilde{\Omega}^2}{2 \tilde{c}_T \tilde{\Omega}} \right), \quad (3.52b)$$

$$\alpha_{rel} = \frac{|1 - \tilde{\Omega}^2|}{\sqrt{(1 - \tilde{\Omega}^2)^2 + (2\tilde{c}_T\tilde{\Omega})^2}} \quad (3.52c)$$

Considering the condition $\alpha_{rel} \leq \alpha_{rel}^*$, the expressions for the range of the full-stick regime and the values of the full-stick region limits $\tilde{\Omega} \in [\tilde{\Omega}^{*pre}, \tilde{\Omega}^{*post}]$ can be derived. For a simplified expression, one can assume that the term $\tilde{c}_T\tilde{\Omega} \approx \tilde{c}_T\bar{\Omega}$ does not vary with $\tilde{\Omega}$. The expressions under these assumptions read as in Equation (3.53).

$$\tilde{\Omega}^{*pre} = \sqrt{1 - \frac{2\tilde{c}_T\bar{\Omega}\mu\tilde{F}_N\tilde{r}}{\sqrt{(\tilde{c}_T\bar{\Omega})^2 - (\mu\tilde{F}_N\tilde{r})^2}}}. \quad (3.53a)$$

$$\tilde{\Omega}^{*post} = \sqrt{1 + \frac{2\tilde{c}_T\bar{\Omega}\mu\tilde{F}_N\tilde{r}}{\sqrt{(\tilde{c}_T\bar{\Omega})^2 - (\mu\tilde{F}_N\tilde{r})^2}}}. \quad (3.53b)$$

The corresponding vibration amplitudes, shown in Equation (3.54), are derived similarly.

$$\alpha^{*pre} = \alpha^{*post} = \sqrt{1 - \left(\frac{\mu\tilde{F}_N\tilde{r}}{\tilde{c}_T\bar{\Omega}}\right)^2} \quad (3.54)$$

To solve the sliding condition, the same approach is used with the previous models. Taking advantage of the harmonic motion of the clamps, a harmonic moment with the amplitude of $\mu\tilde{F}_N$ is chosen to stand for the excitation moment.

3.3.3 Dual Cantilever Beam Model

The solution of the dual cantilever beam configuration with frictional clamps is conducted in this section. Here, the mid-point of the beam is forced to move with a harmonic displacement function $a \cos(\Omega t)$, and the identical clamps on both tips prevent the transverse deflection,

whereas the rotation is not fully constrained due to the possibility of slip at the contact interfaces.

One crucial thing to note is that the analytical solution is not straightforward and has not yet been possible with the velocity-dependent element. The expressions of the equation of motion with the necessary boundary conditions become too complicated to be analytically solvable. Therefore, using the physical analogy between the Jenkins elements and the velocity-dependent elements of the axial and torsional cases, the velocity-dependent element solution is not performed. It is expected to be similar to the Jenkins element as it was in the cases with the models in axial and torsional motions.

Nondimensionalisation

For the sake of simplicity and considering primarily the experiments are conducted this way, a beam with a constant rectangular cross-section and the imposed harmonic transverse displacement at $z = L$ $a \cos(\Omega t)$ are taken into consideration. As mentioned earlier, the system obeys the equation of motion given in Equation (3.12).

Normalisation is performed by defining the system parameters, m , the mass, A , the cross-sectional area and L , the length of the beam, ω_0 , the eigenvalue of the bending mode one wants to study, κ , the wavelength of the exciting frequency, κ_0 , the wavelength of the corresponding eigenfunction, F_0 , the characteristic force, k_0 , the characteristic stiffness, c_0 , the characteristic damping, k_T , the stiffness coefficient of the contact, c_T , the damping coefficient of the contact. Similarly, the reference parameters $\tilde{h}$, $\tilde{v}$, $\tilde{t}$, $\tilde{\Omega}$, $\tilde{F}$, $\tilde{k}$, $\tilde{c}$, $\tilde{\kappa}$, $\tilde{\kappa}_0$, $\tilde{k}_A$, $\tilde{k}_T$ and $\tilde{c}_T$ are defined taking advantage of the defined system parameters and their characteristic numbers as shown in Equation (3.55).

$$\begin{aligned}
m &= \rho AL ; \quad \kappa^2 = \omega \sqrt{\rho A / EI} ; \quad \kappa_0^2 = \omega_0 \sqrt{\rho A / EI} ; \quad F_0 = m \omega_0^2 a ; \\
k_{T0} &= m \omega_0^2 L^2 ; \quad k_{A0} = m \omega_0^2 L^3 ; \quad \tilde{h} = h/a ; \quad \tilde{v} = v/a ; \\
\tilde{z} &= z/L ; \quad \tilde{t} = t \omega_0 ; \quad \tilde{\Omega} = \Omega/\omega_0 ; \quad \tilde{F} = F/F_0 ; \quad \tilde{\kappa} = \kappa L^2 ; \\
\tilde{\kappa}_0 &= \kappa_0 L^2 ; \quad \tilde{k}_A = k_A/k_{A0} ; \quad \tilde{k}_T = k_T/k_{T0} ;
\end{aligned} \tag{3.55}$$

Finally, the nondimensional form of the equation of motion is obtained as in Equation (3.56).

$$\ddot{\tilde{v}} - \frac{1}{\tilde{\kappa}_0^4} \tilde{v}^{IV} = 0 \tag{3.56}$$

Like the previous cases, the Galerkin method is chosen to solve this problem, and the assumed solution is taken as the usual analytical solution for beams which is given in Equation (3.57). It is also assumed that the frequency range of the imposed harmonic displacement includes the first natural frequency of the beam. Here in the ansatz function, $\tilde{v}$ is the nondimensional displacement, α is the nondimensional amplitude of the imposed displacement at the mid-point of the beam and ψ denotes the phase.

$$\tilde{v}(\tilde{z}, \tilde{t}) = (C_1 \cos(\tilde{\kappa}\tilde{z}) + C_2 \sin(\tilde{\kappa}\tilde{z}) + C_3 \cosh(\tilde{\kappa}\tilde{z}) + C_4 \sinh(\tilde{\kappa}\tilde{z})) \cos(\tilde{\Omega}\tilde{t} + \psi) \tag{3.57}$$

The application of the Galerkin method requires plugging the ansatz function into Equation (3.56), which yields Equation (3.58).

$$\left(\frac{\tilde{\kappa}^4}{\tilde{\kappa}_0^4} - \tilde{\Omega}^2 \right) \tilde{v}(\tilde{z}, \tilde{t}) = 0 \tag{3.58}$$

Jenkins Element

The frictional dissipation only occurs with a Jenkins element if the rotation of the cross-section exceeds the stick limit, which is shown in Equation (3.59), where $\tilde{v}'(0, t)$, the dimensionless

rotation amplitude at the stick limit and $\tilde{v}'(0, t) \leq \tilde{V}'^*$ is the contact force at the interface between the beam and the clamps in full-stick.

$$\tilde{k}_T \tilde{V}'^* = \mu \tilde{F}_N \tilde{h} \quad (3.59)$$

It is worth noting that the slip here is related to the rotation of the beam cross-section at the clamp, and the frictional clamp reaction is actually a moment shown in Equation (3.17). Hence, plugging that relation into Equation (3.59) yields Equation (3.60) where $\tilde{v}' = \frac{\partial \tilde{v}}{\partial \tilde{z}}$ is the nondimensional rotation of the cross-section.

$$\tilde{F}_{NL}(\tilde{t}) = \tilde{k}_T \tilde{v}'^*, \quad (3.60a)$$

$$\tilde{M}_{NL}(\tilde{t}) = \tilde{F}_{NL}(\tilde{t}) \tilde{h} = \tilde{k}_T \tilde{h} \tilde{V}'^* \quad (3.60b)$$

Using these assumptions/conditions, the new boundary conditions can be written as in Equation (3.61).

$$\tilde{v}(0, \tilde{t}) = 0, \quad (3.61a)$$

$$\left. \frac{\partial^2 \tilde{v}}{\partial \tilde{z}^2} \right|_{\tilde{z}=0} = \tilde{\kappa}_0 \left(\tilde{F}_{NL} \tilde{h} + \tilde{k}_A \left. \frac{\partial \tilde{v}}{\partial \tilde{z}} \right|_{\tilde{z}=0} \right), \quad (3.61b)$$

$$\tilde{v}(\tilde{L}, \tilde{t}) = \alpha \cos(\tilde{\Omega} \tilde{t}), \quad (3.61c)$$

$$\left. \frac{\partial \tilde{v}}{\partial \tilde{z}} \right|_{\tilde{z}=\tilde{L}} = 0 \quad (3.61d)$$

Substituting the ansatz in Equation (3.57) into the boundary conditions Equation (3.61), the coefficients C_1 , C_2 , C_3 and C_4 are solved.

If the stick limit is reached, then, Equation (3.59) is used to replace F_{NL} with $\mu \tilde{F}_N$. Thus, $\tilde{V}'^*$ and $\mu \tilde{F}_N$ read as in Equation (3.62).

$$\tilde{V}'^* = \tilde{\kappa}(C_2 + C_4) \quad (3.62a)$$

$$\mu \tilde{F}_N = \tilde{k}_T \tilde{\kappa}(C_2 + C_4) \quad (3.62b)$$

Using these relations, the imposed displacement amplitude α^* and bending moment M_b^* , that correspond to the stick limit, are obtained as given in Equation (3.63).

$$\alpha^* = \mu \tilde{F}_N \frac{\tilde{\kappa}_0^4 (\tilde{k}_T \tilde{h} + \tilde{k}_A)(1 - \cos(\tilde{\kappa})\cosh(\tilde{\kappa})) + \tilde{\kappa}(\cos(\tilde{\kappa})\sinh(\tilde{\kappa}) - \sin(\tilde{\kappa})\cosh(\tilde{\kappa}))}{\tilde{k}_T \tilde{\kappa}^2 (\cos(\tilde{\kappa}) - \cosh(\tilde{\kappa}))} \quad (3.63a)$$

$$M_b^* = \mu \tilde{F}_N \frac{\tilde{k}_T \tilde{h} + \tilde{k}_A}{\tilde{k}_T} \quad (3.63b)$$

Finally, it is also important to note that the phase lag is found as zero in this case, as presented in Equation (3.64).

$$\psi = 0 \quad (3.64)$$

3.4 Numerical Modelling and Examples

3.4.1 Nonlinear Friction Force Definition for the Numerical Model

The friction models can be either defined in terms of the relative displacement between the two points in contact or the relative velocity. In both cases, the limit value that the nonlinear

force assumes for large relative motion coincides with the tangential friction force μF_N . At very small values of the relative motion, the contact is instead modelled as a force in linear relation with the chosen variable (either relative displacement or relative velocity).

Such force is the contact force exerted by the clamp on the bar when the two components are considered to be in stuck conditions.

In both cases, $F_R(w_{rel}, \dot{w}_{rel})$ is defined as the frictional contact force, and the expression of the nonlinear friction force is written in Equation (3.65).

$$F_{NL}(w_{rel}, \dot{w}_{rel}) = \begin{cases} -\mu F_N & \text{if } F_R(w_{rel}, \dot{w}_{rel}) < -\mu F_N \\ +\mu F_N & \text{if } F_R(w_{rel}, \dot{w}_{rel}) > +\mu F_N \\ F_R(w_{rel}, \dot{w}_{rel}) & \text{else} \end{cases} \quad (3.65)$$

Jenkins Model

If the frictional contact force F_R is modelled with an elastic behaviour (displacement formulation, which is similar to a spring formulation), the resulting nonlinear element will coincide with a Jenkins element. Hence, the frictional contact force is given by Equation (3.66).

$$F_R = k_T(w_{rel} - \Delta w_{slip}) \quad (3.66)$$

where k_T is the tangential contact stiffness and Δw_{slip} is the cumulative relative displacement accumulated during the slip phase. Numerically the value of Δw_{slip} is obtained through a time discretisation of the period, solving for the variables and updating the value of Δw_{slip} at each time step of the slip phase. An analytical expression cannot be provided for the general case. However, it can be found if one assumes that the relative displacement is a purely harmonic function which is expressed in Equation (3.67).

$$w_{rel}(t) = a \cos(\Omega t). \quad (3.67)$$

If the value of $k_T a$ exceeds the friction limit μF_N a stick-slip cycle is observed. Else, the value of Δw_{slip} assumes the simple expression shown in Equation (3.68).

$$\Delta w_{slip} = -\text{sign}(\dot{w}_{rel})\left(a - \frac{\mu F_N}{k_T}\right) \quad (3.68)$$

Being $\dot{w}_{rel} = -a\Omega \sin(\Omega t)$, for the first half period, the sign of the velocity will be negative, and for the second half, positive. The frictional contact force is written in Equation (3.69).

$$F_R = \begin{cases} k_T a \cos(\Omega t) - (k_T a - \mu F_N) & \text{if } t \in [0, T/2] \\ k_T a \cos(\Omega t) + (k_T a - \mu F_N) & \text{if } t \in [T/2, T] \end{cases} . \quad (3.69)$$

Obviously, in the case of $k_T a < \mu F_N$, a full-stick cycle will occur. This means no dissipation is to be observed, and the frictional force is a purely elastic reaction.

The case of a generic stick-slip cycle is reported in Figure 3.10. One can notice that the four different conditions for the friction force are depicted with different colours, and the hysteresis loop generated is shown in the (b) plot. The area enclosed in the hysteresis loop, representing the dissipated energy, is equal to $2\mu F_N(a - \frac{\mu F_N}{k_T})$: for minimal values of the friction limit, the cycle will mostly consist of slip phases and the energy dissipated will be small, and for $a \leq \frac{\mu F_N}{k_T}$ the cycle will reduce to a pure stick cycle with no dissipation.

Another notable observation is that for the generic case reported in Figure 3.10 the equivalent stiffness introduced in the system by the friction element is lower than k_T due to the presence of the slip phases. In the extreme case of a pure stick cycle, this stiffness will equal k_T , whereas, in the extreme case of an almost full-slip cycle, it will tend to be zero.

Velocity-Dependent Model

If the frictional contact force is modelled with a viscous behaviour (velocity formulation, which is similar to a viscous damper formulation), the frictional force will simply be as follows in

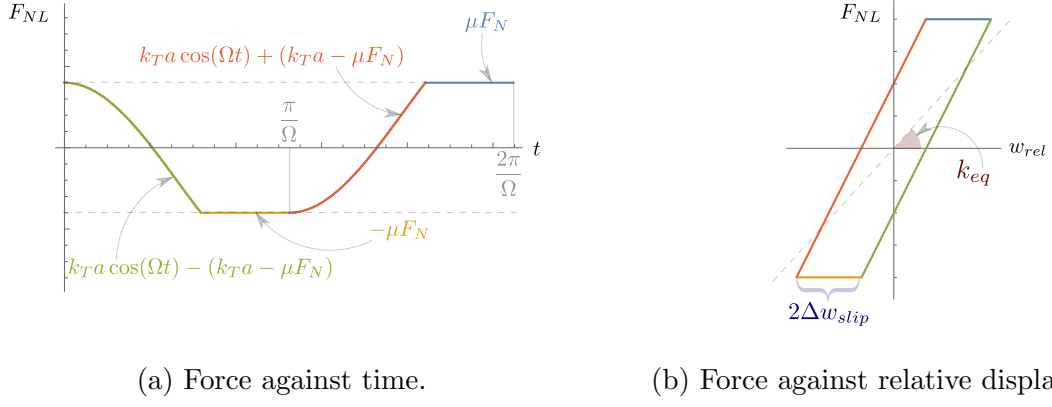


Figure 3.10: Simplified Jenkins element under the single harmonic assumption.

Equation (3.70).

$$F_R(\dot{w}_{rel}) = c_T \dot{w}_{rel} \quad (3.70)$$

where c_T is the viscous damping constant. Unlike the Jenkins model, this model is always dissipative, even in the case of a pure stick cycle. Under the single harmonic assumption, the generic stick-slip cycle assumes a simplified expression for the force. In Figure 3.11, the nonlinear friction force against time and the hysteresis loop are depicted.

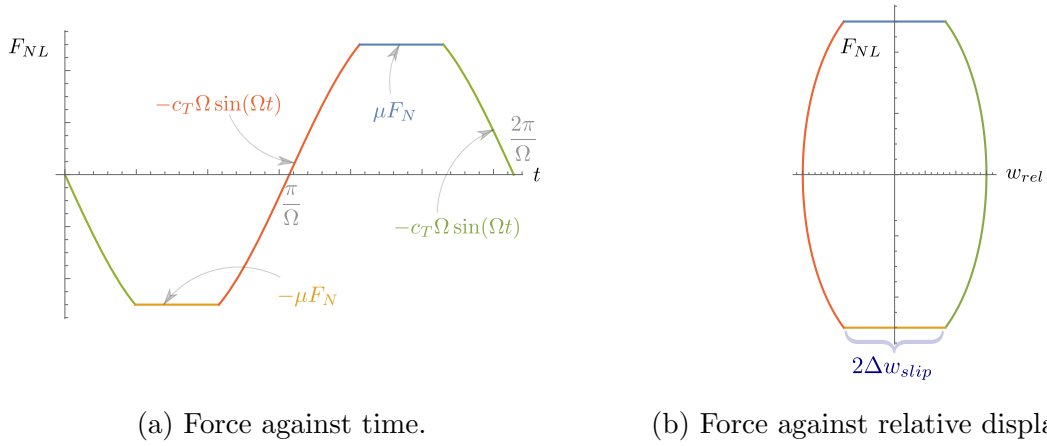


Figure 3.11: Simplified velocity-dependent element under the single harmonic assumption.

If the value of $c_T \Omega a$ is lower than μF_N , the element will reduce to a dashpot of value c_T . If $c_T \Omega a$ exceeds μF_N , a stick-slip cycle will occur. From the hysteresis loop, it must be noticed that this friction element does not introduce any stiffness into the system. Moreover, in the

generic case of a stick-slip cycle, the value of Δw_{slip} can also be derived as shown in Equation (3.71):

$$\Delta w_{slip} = \sqrt{a^2 - \left(\frac{\mu F_N}{c_T \Omega} \right)^2} \quad (3.71)$$

3.4.2 Numerical Examples

The developed analytical solutions are compared to the AFTHB numerical approach, which is a reliable methodology. Some specific cases for axial and torsional vibrations are picked and modelled using both Jenkins and velocity-dependent friction models. The numerical approach is briefly explained alongside the selected examples.

Axial Model: Description of the System

A one-dimensional finite element (FE) bar element is used to model the axial motion of the rod with frictional clamps. The dimension of the rod is given as a length of 70 mm, width of 14 mm and thickness of 4 mm. The material property of the rod is a density of 1400 kg/m^3 , Young's modulus of 3.5 GPa and Poisson's ratio of 0.38. In this FE model, half of the rod with symmetric boundary conditions and Clamp B is used to represent the whole model. There are 71 nodes in total, with only one degree of freedom for each node to model the axial motion of the rod. The clamp is assumed to be only applied on the last node. A base excitation is used to excite the clamp, and the displacement of the clamp, $u_B(t)$, is prescribed with a certain frequency Ω .

A one-dimensional Jenkins element with constant normal load is used to compute the friction force [231]. F_N is the normal load; k_T is the tangential contact stiffness; c_T is the viscous damping coefficient of the contact; μ is the friction coefficient. There are two contact statuses for this Jenkins element which are sticking and sliding. The equation used to calculate the friction force is given in Equation (3.72).

$$F_F(\Delta u(t)) = \begin{cases} k_T \Delta u(t), & \text{if } k_T \Delta u(t) \leq \mu F_N \\ \mu F_N, & \text{if } k_T \Delta u(t) > \mu F_N \end{cases} \quad (3.72)$$

Similarly, the friction forces depending on the contact conditions for the velocity-dependent element can be described in Equation (3.73).

$$F_F(\Delta \dot{u}(t)) = \begin{cases} c_T \Delta \dot{u}(t), & \text{if } c_T \Delta \dot{u}(t) \leq \mu F_N \\ \mu F_N, & \text{if } c_T \Delta \dot{u}(t) > \mu F_N \end{cases} \quad (3.73)$$

The equation of motion is given in Equation (3.74), where $\underline{\underline{\mathbf{M}}}$, $\underline{\underline{\mathbf{C}}}$ and $\underline{\underline{\mathbf{K}}}$ are the mass, viscous damping and stiffness matrices respectively. $\underline{U}(t)$ is the generalized displacement vector of the rod; $u_B(t)$ is the prescribed displacement of the left frictional clamp; F_F is the friction force calculated using Equations (3.72) and (3.73).

$$\underline{\underline{\mathbf{M}}} \ddot{\underline{U}}(t) + \underline{\underline{\mathbf{C}}} \dot{\underline{U}}(t) + \underline{\underline{\mathbf{K}}} \underline{U}(t) + F_F = 0 \quad (3.74)$$

The classic Harmonic Balance Method (HBM) with Alternating Frequency/Time (AFT) procedure is used to solve the Equation (3.74) in the frequency domain [44]. In HBM, all solutions in the time domain are discretised into Fourier series truncated to a certain order N_h , and shown in Equation (3.75) and Equation (3.76). Then, the equation of motion, which is originally formulated in the time domain (Equation (3.74)), can be rewritten in the frequency domain to yield Equation (3.77). The analytical derivation of friction force in the frequency domain is complicated and not efficient. Therefore, the AFT procedure is used to compute the necessary terms, including the contact friction force. In AFT, the solution expressed in the frequency domain $\tilde{\underline{U}}$ is transformed into the time domain using the inverse discrete Fourier transformation. Then, the friction force is calculated in the frequency domain. The forces based on displacement and velocity-dependent models are presented in Equations (3.72) and (3.73). Thus, the $\tilde{F}_F$ is obtained by applying inverse discrete Fourier transformation to F_F .

$$\underline{U}(t) = \sum_{h=0}^{Nh} \text{Real} \{ e^{i h \omega t} \bullet (\underline{U}^{h,c} - \underline{U}^{h,s_i}) \} \quad (3.75)$$

$$\tilde{\underline{U}} = \left[\underline{U}^{0,c}, \quad \underline{U}^{1,c} - \underline{U}^{1,s_i}, \quad \underline{U}^{h,c} - \underline{U}^{h,s_i} \right]^T \quad (3.76)$$

$$\underline{\underline{A}} \tilde{\underline{U}} + \tilde{F}_F (\tilde{u}_{71} - \tilde{u}_B) = 0 \quad (3.77)$$

To obtain the dynamic response of the bar-clamp system for different excitation frequencies Ω , a numerical continuation method with a secant predictor and arc-length corrector is used, which is known as an efficient tool to track the evolution of the results with respect to a particular chasing parameter. In this work, excitation frequency Ω is the chasing parameter. A secant predictor and pseudo-arc-length corrector is used [11].

Torsional Model: Description of the System

This case represents steel pipes with shrink fit. Therefore, the parameters are picked accordingly. The bar has a circular cross-section with a diameter of 7 mm, and the free length of the bar is 250 mm. The friction coefficient between the clamp and the bar is taken as 0.45. The material of the bar is taken as typical standard steel, which has the modulus of elasticity of 210 GPa, Poisson's ratio of 0.28 and density of 7800 kg/m³.

The same numerical method used in the axial motion example is used. The only difference is that the resultant loads of interest are the torsion moments, which also root from the contact friction as explained in Section 3.2.2.

Dual Cantilever Model: Description of the System

This example presents the DMA test setup with the dual cantilever fixture configuration. The fixtures of the DMA is known to be made of stainless steel and the beam samples are Based on

the results of Coulomb experiments carried out on the steel-epoxy material pairs, the coefficient of friction is assumed as 0.5. The steel properties are taken as presented in Section 3.4.2, and the epoxy beam sample sizes and the material properties are the same as in Section 3.4.2. Figure 3.12 displays the FE model that is used as the input for the AFTHB method.

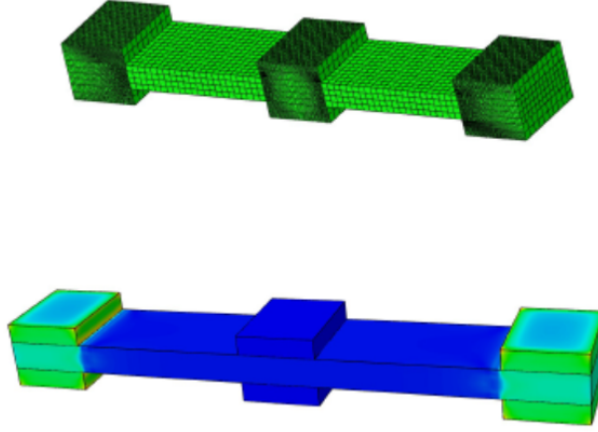


Figure 3.12: FE model of the dual cantilever fixture.

3.5 Results and Discussion

All the results presented within this study are nondimensionalised (frequency: $\tilde{\Omega}$, amplitude: α), including the results of the AFTHB. The described numerical cases are presented for an enhanced understanding of the effects of friction on the system behaviour.

As described previously, the analytical solutions are developed for the models considering full-stick and slip conditions, and therefore, there are two separate solutions. The combinations of these two represent the solution for the nonlinear bar-clamp system. The question is, which regime solution is valid at the particular point of the domain? The valid expression for a particular excitation frequency is the one that requires less frictional contact force, in other words, the one that yields the lower amplitude. Figure 3.13 shows some schematic curves for the required frictional force F_R , which reaches its minimum at the resonance frequency, and the maximum contact friction force F_{Fmax} against frequency assuming $F_{Fmax} = \mu F_N$.

In Figure 3.13a, the first case is displayed, where these curves do not intersect, which means the necessary frictional contact force is always greater than the maximum contact friction force. This will always result in slip and, consequently frictional dissipation along with the frequency interval of interest. The second case is visualised in Figure 3.13b where the required frictional contact forces are relatively lower so that there is only one intersection. In this case, the full-stick regime governs the system until the intersection frequency and slip regime takes over for the frequencies greater than that. Finally, as shown in Figure 3.13c, there can be two intersections of the presented curves. This leads to a domain divided into three regions, namely, a full-stick regime between the intersection points and slip before the first and beyond the second points of intersection.

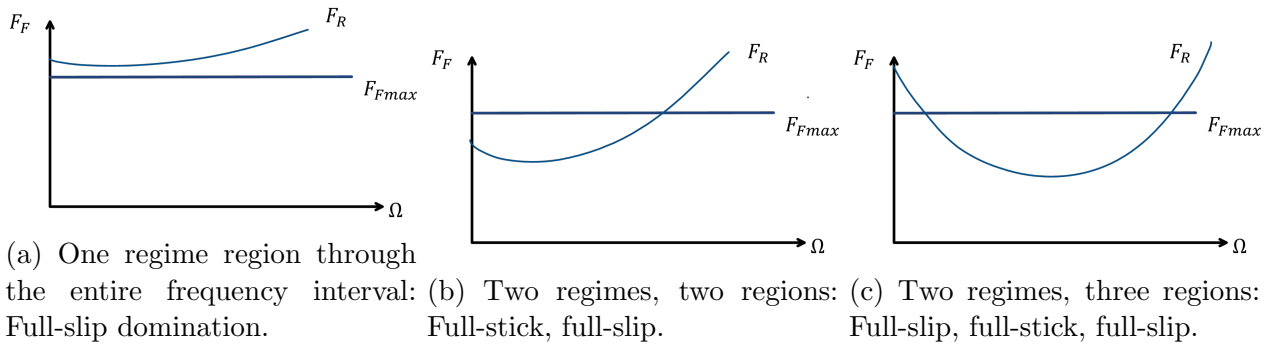


Figure 3.13: Existence of regimes and regions based on the contact forces.

3.5.1 Model in Axial Motion

Jenkins Element

The two separate solutions of the axially vibrating bar with Jenkins friction approach for full-stick and full-slip regimes are shown in a representative plot in Figure 3.14, which corresponds to the case displayed in Figure 3.13c. Re-emphasising that the valid solutions are ones that result in lower amplitudes for the same excitation frequency. The most significant feature of this representative plot in Figure 3.14 is that the full-stick regime occurs around the resonance where the minima of the required frictional contact forces are located. The frequency interval, where full-stick is prevalent, can be expanded or shrunk depending on the maximum friction force or the amplitude of the clamp motion.

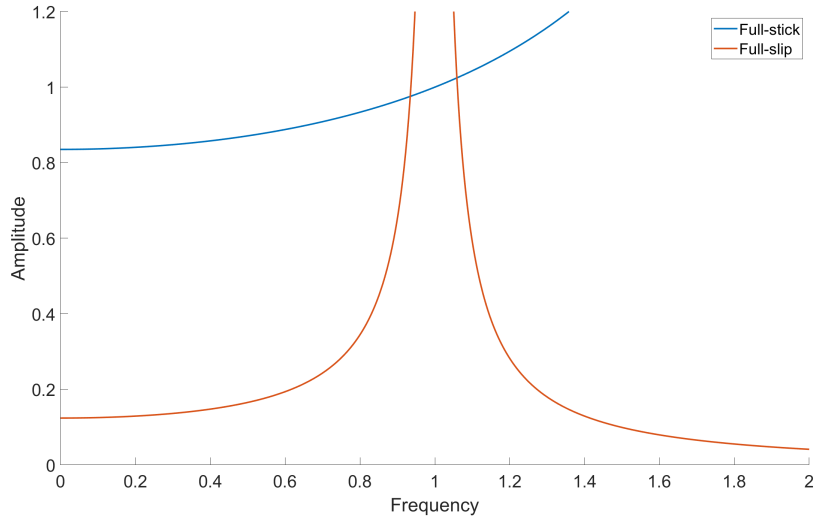


Figure 3.14: A sketch of the analytical solutions for the bar in axial motion with Jenkins model.

Figure 3.15 shows the analytical and numerical results together for the axially excited rod with the Jenkins friction approach, which assumes $\tilde{k}_T = 2.5$. Both analytical and numerical results display the same behaviour with a considerable level of accuracy. The analytical model predicts lower vibration amplitudes compared to the numerical model in the pre-resonance slip region. In the post-resonance slip region, however, analytical values are slightly greater than the results of the numerical analysis. This is judged to be caused by the representation of the friction forces by a simpler harmonic function which affects the dissipation characteristics leading to these differences in the predicted amplitudes. The gap between these two models is closed towards the resonance and becomes almost negligible in the stick regime. The only difference is that the numerical model predicts a small peak just before going back to sliding. The analytical model does not show that curvature before the peak, but both models predict the maximum amplitude with good precision. In the end, it can be stated that the results of both analytical and numerical models show good agreement.

Referring to the same figure (Figure 3.15), under constant normal load, increasing the clamp motion amplitude, which is expressed as W_A/L ratio, shrinks the full-stick regime dominance within the selected frequency interval; in other words, the slip region expands eventually causing more friction damping. The maximum amplitude is located towards the end of the post-resonance full-stick regime exceeding the clamp motion amplitude. This can be explained by

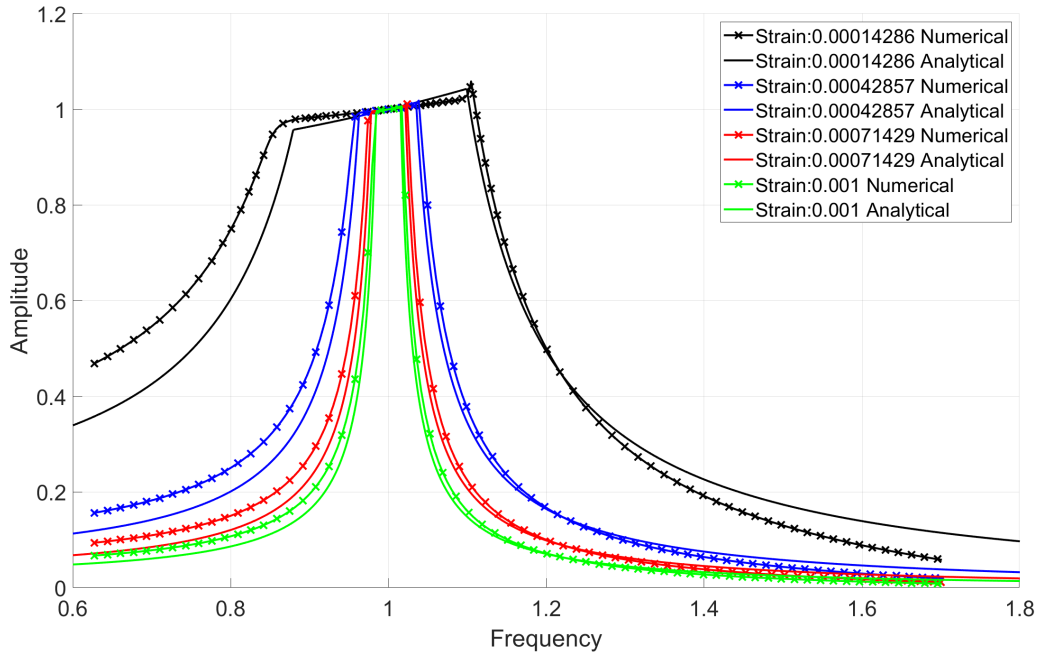


Figure 3.15: Comparison of analytical and numerical results for the axial case with Jenkins model from Section 3.4.2.

the presence of contact stiffness, which also controls the slope of the stick regime. The total stiffness of the system does not only consist of the stiffness of the bar, and the system is not directly excited with a simple harmonic force. The moving clamp interacts with the bar over the contact, which is described with a Jenkins element for this case. Hence, the amplitude of the bar may go slightly beyond the amplitude of the clamp motion. The small peak at the end of the stick region also occurs for the same reason.

Figure 3.16 displays the limit frequency and amplitudes values of the full-stick region depending on the maximum friction force, $\mu\tilde{F}_N$ with various contact stiffness coefficients $\tilde{k}_T$ under the same clamp motion amplitude. The first thing to note is that the stick region expands with increasing maximum friction force, $\mu\tilde{F}_N$. This expansion is greater in pre-resonance frequencies and less in the post-resonance region. For lower $\tilde{k}_T$ values, the curve has a greater slope and thus, the limit amplitudes pass beyond the value 1, which is the amplitude of the clamp motion. Meanwhile, increasing the tangential contact stiffness reduces the slope and leads to a flatter, full-stick region. This can be explained by the contact stiffness values considered greater than the bar's axial stiffness. Rising $\tilde{k}_T$ widens the gap between these stiffnesses and hinders the

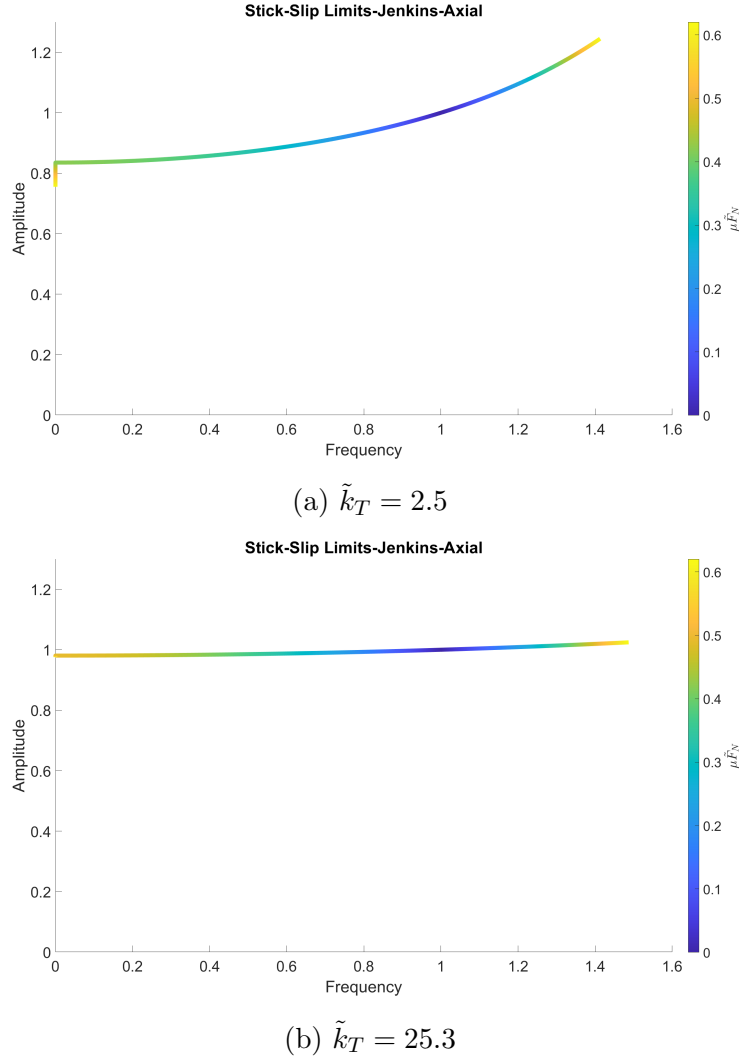


Figure 3.16: Expansion of the stick limit boundaries with $\mu\tilde{F}_N$. Limit amplitude against limit frequency for different values of $\tilde{k}_T$.

dynamic interaction of the contact. Thus, the overall behaviour of the system becomes more bar-dominated. The curves become flatter, and the interval of amplitude values, that approach closer to 1, is extended.

The stick limits shown in Figure 3.17 are plotted based on Equation (3.31), shows the minimum necessary limit frictional force, $\mu\tilde{F}_N$, under constant clamp motion amplitude. Based on this visualised data, the first thing to notice is that the minimum necessary limit friction force becomes zero at the resonance frequency. This is an expected result for a system with no damping source, which is always present, in other words, independent of stick and slip regimes of the contact friction. The only source of damping in this system is the frictional contact, which behaves elastically in a full-stick regime. Considering that this plot marks the border of

a damped (full-slip) and an undamped (full-stick) regime, a purely conservative elastic dynamic behaviour can be understood.

Moreover, in the static case where $\tilde{\Omega} = 0$, the minimum necessary limit frictional force has a finite value, and it decreases towards the resonance frequency, reaching zero at the resonance, as mentioned earlier. From the resonant frequency towards higher frequencies, the least necessary limit frictional force increases. Besides, in the pre-resonant region, the least necessary limit frictional force increases if the contact stiffness $\tilde{k}_t$ is increased. On the other hand, in the post-resonant frequencies, for the minimum necessary friction force, this trend is reversed.

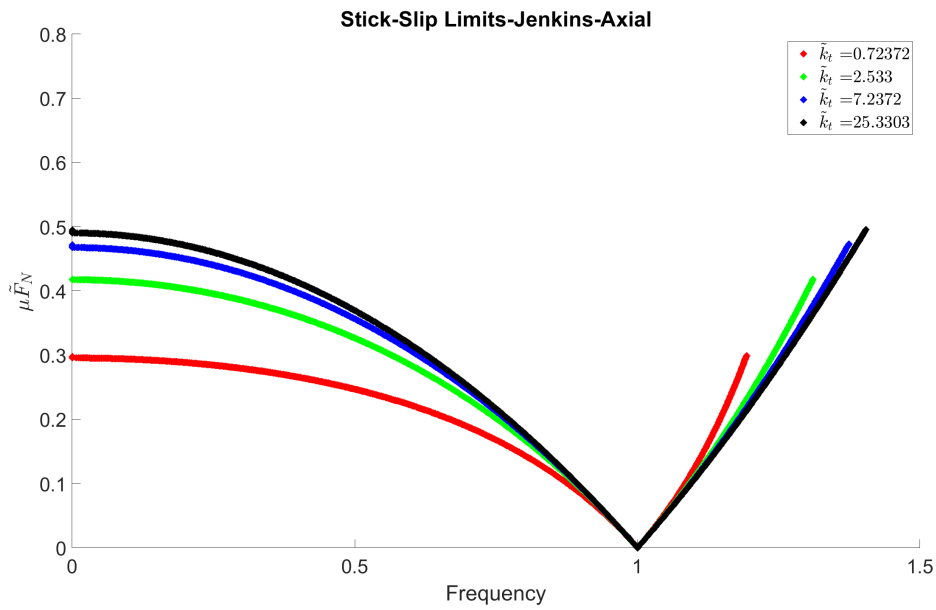


Figure 3.17: The least necessary $\mu\tilde{F}_N$ for the full-stick regime in axial Jenkins model.

Velocity-Dependent Element

The same case of the axially vibrating bar under the excitation of moving frictional clamps, described in Section 3.4.2, is also modelled with the velocity-dependent model.

Again, the solution procedure divides the motion into two regimes depending on the existence of slip at the contact interface between the bar and the clamp, like the model with Jenkins. The only difference is the calculation method of the friction force. A representative plot including the two contact conditions, stick and slip, is presented in Figure 3.18. A noticeable aspect in

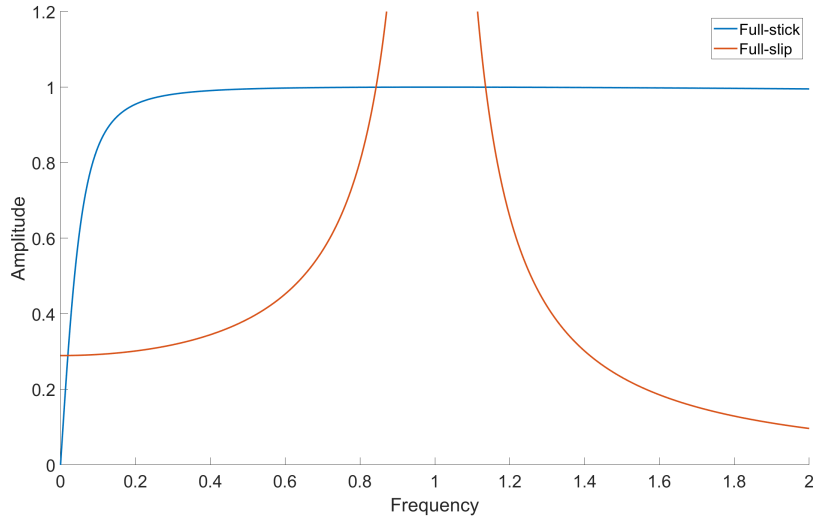


Figure 3.18: A sketch of the analytical solutions for bar in axial motion with velocity-dependent model.

this plot is that there is a tiny region for very low frequencies (around $\tilde{\Omega} = 0$) where clamps stick to the bar. Following this small stick region becomes a slip region with increasing frequency until the main stick region is located around the resonance. The amplitudes are almost fixed to the value of 1 in this second stick region. Therefore, it appears as a flat top in the plot. The second stick region is succeeded by another region dominated by the slip regime, which is similar to the case with the Jenkins model. Altogether, two regimes appear in two regions each. Thus, the solution consists of four regions.

To compare the results of the numerical and analytical approaches, Figure 3.19 is presented. During the calculations to generate this plot $\tilde{c}_T$ is taken as 7.7. Overall, the results of both methods are in good agreement. The amplitudes that the analytical model predicts are slightly smaller than the numerical predictions in the pre-resonance slip region. Later in the stick region around the resonance, the results agree almost perfectly. The amplitudes become 1 in the stick regime and remain constant throughout the region. In the post-resonance slip regime, the results start very similarly. However, with increasing excitation frequency, the results of the numerical model become smaller than their analytical correspondents. Since these trends are very similar to the Jenkins model, the reasons are expected to be the same, namely the error in damping caused by the representation of the contact forces in the slip regime.

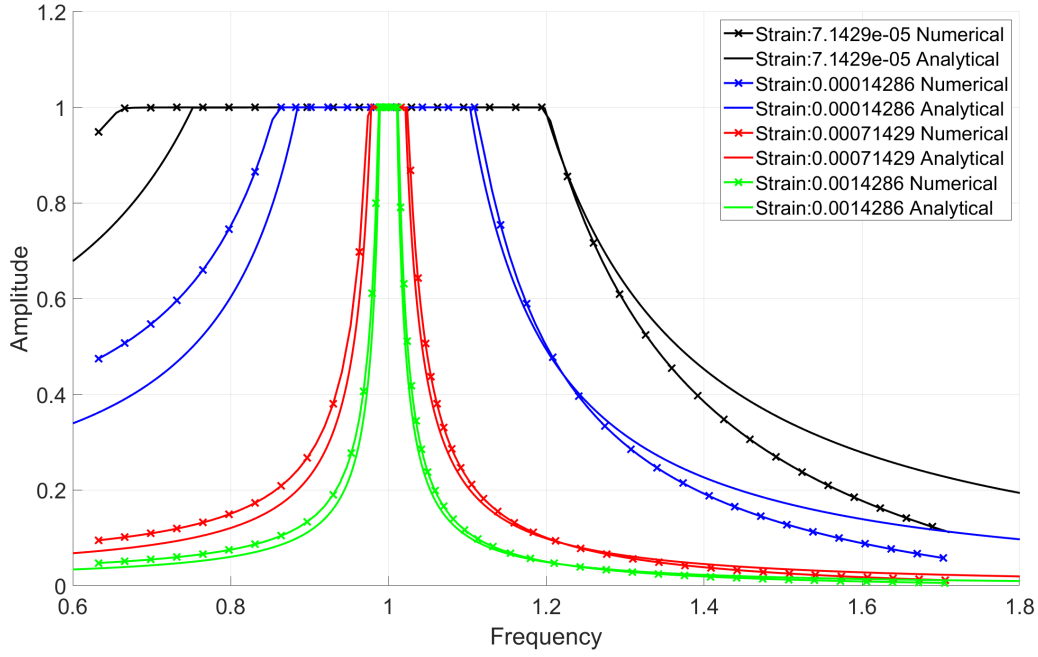


Figure 3.19: Comparison of analytical and numerical results for the axial case with the velocity-dependent model from Section 3.4.2.

Apart from these, Figure 3.19 points out that increasing clamp motion amplitude again shrinks the full-stick region. This is because increasing the clamp motion amplitude leads to greater required frictional contact forces. The friction forces attain the maximum value in larger intervals of frequency.

To understand the effect of the maximum frictional force on the limits of the stick regime, Figure 3.20 shows various $\tilde{c}_T$ values. For $\tilde{c}_T = 1.3$ (Figure 3.20a), an amplitude peak, which reaches its maximum value of $\alpha = 1$, is visible at the resonance frequency. The rest of the amplitude values are slightly below 1 for pre- and post-resonant frequencies. The curve in the pre-resonant region rises to 1 with a smaller gradient compared to the absolute value of the gradient of the falling amplitudes in the post-resonant region. Besides, the plots also show that greater $\tilde{c}_T$ values make the amplitude curve flatter equating all the values practically to 1.

In Figure 3.21 the least necessary frictional force is visualised using Equation (3.36) for this velocity-dependent model. The characteristics of the plots are pretty much the same, which could be expected. This figure again points out the limits of the conservative and dissipative regimes that are purely dependent on the status of the frictional contact. The condition that

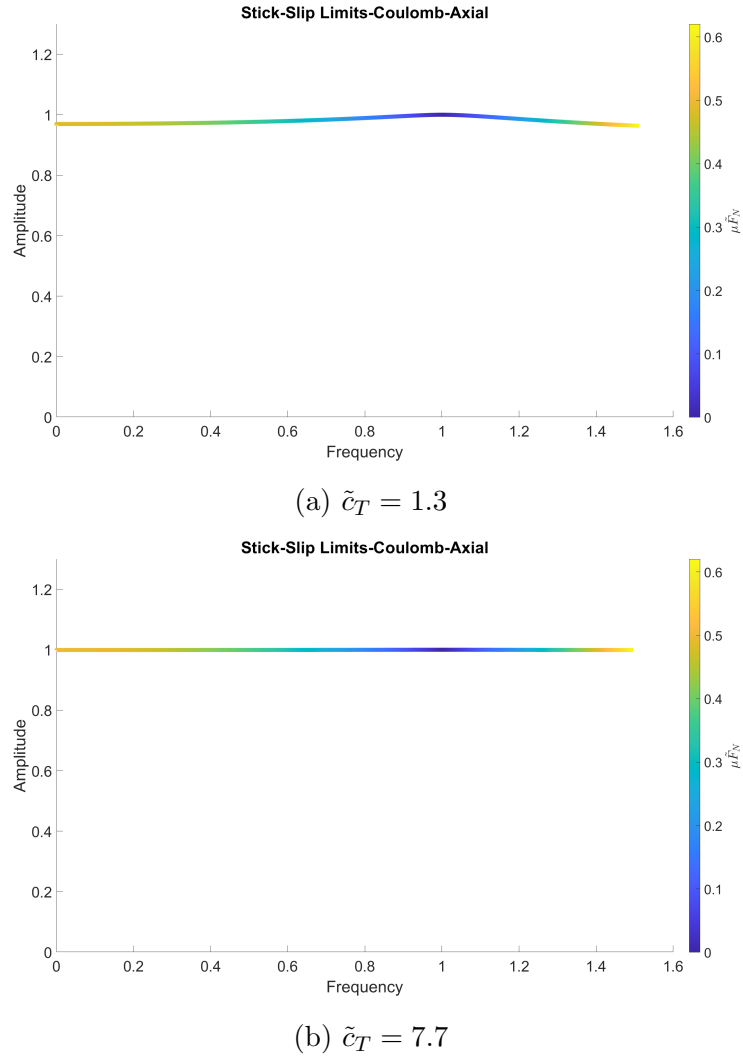


Figure 3.20: Enlargement of the stick limit boundaries with $\mu \tilde{F}_N$. Limit amplitude against limit frequency for different values of $\tilde{c}_T$.

ends one regime and starts the other is precisely the same as the Jenkins model. The difference in this model is the approach that the dissipative regime is formulated. The curves are shaped dominantly by the conservative elastic regime and the condition that separates them from the full-slip regime.

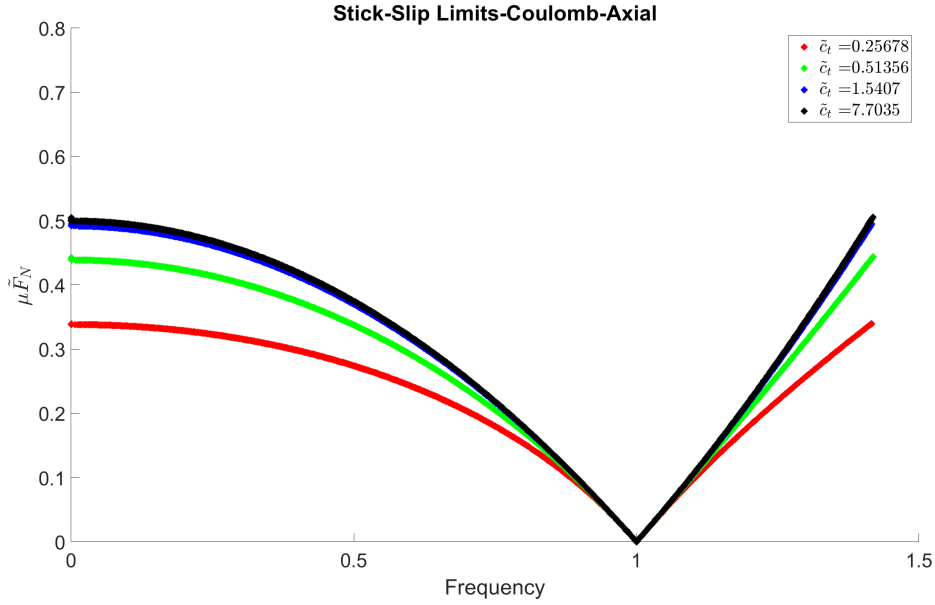


Figure 3.21: The least necessary $\mu\tilde{F}_N$ for the full-stick regime in the velocity-dependent model.

3.5.2 Model in Torsional Motion

Jenkins Element

Like the axial model, for the torsional bar model subjected to friction taken into account employing the Jenkins approach, the analytical solution is performed by decomposing the dynamics of the system in two regimes based on the contact conditions. The solutions of the stick and slip regimes are schematically shown in Figure 3.22. Following the same methodology, the valid solution is the one that provides the lower amplitude.

The results of the bar model with a circular cross-section fitted into a pipe, which is described in detail in Section 3.4.2, are visualised in Figure 3.23 while keeping $\tilde{k}_T = 0.9$ slightly softer than the axial case. The clamp motion amplitudes are defined over rotation per unit length, γ this time. All the comments made for the axial model are also valid for this torsional model, such as the expansion of the stick region with falling γ values and the differences between the analytical and numerical models. This is the consequence of the same characteristics of the governing equations. However, in the curve that is excited with the lowest γ , one can quickly notice that the stick region is dominant from the start of the frequency interval of interest ($\tilde{\Omega} = 0.6$) until the beginning of the post-resonance slip regime which is around $\tilde{\Omega} = 1.07$. This

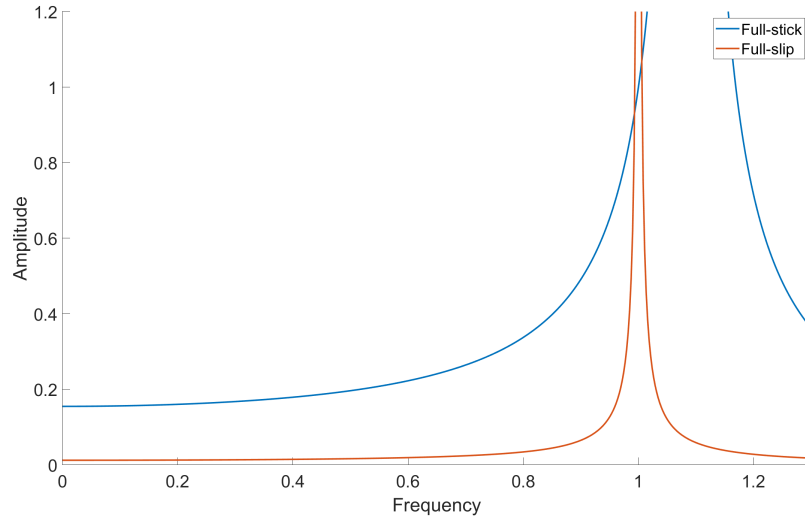


Figure 3.22: A sketch of the analytical solutions for the bar in torsional motion with Jenkins model.

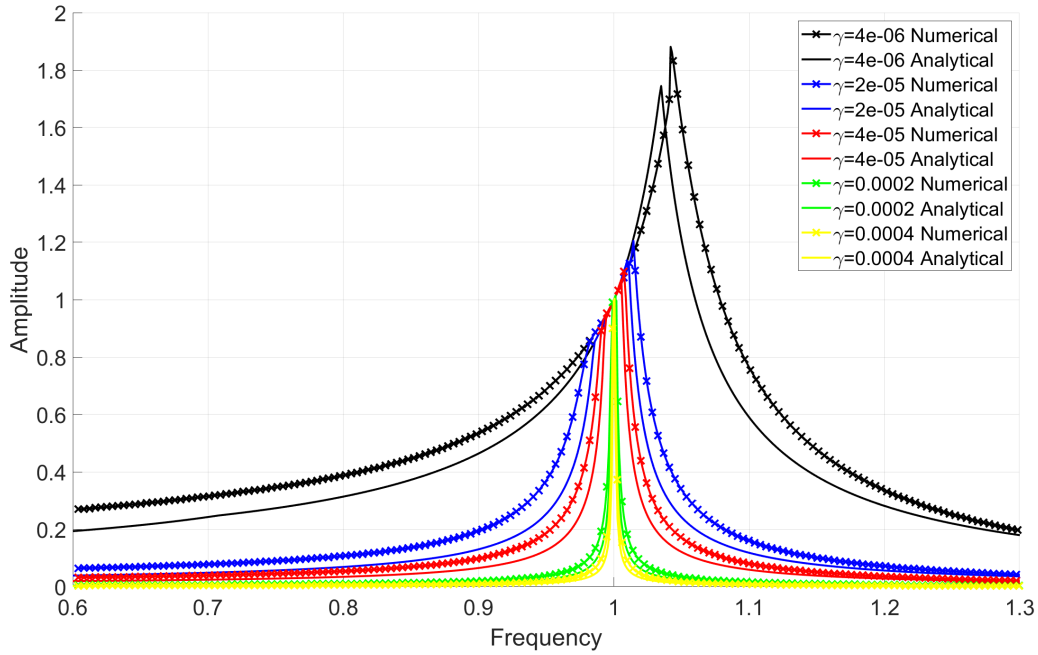


Figure 3.23: Comparison of analytical and numerical results for the torsional case with Jenkins model from Section 3.4.2.

is an example of the case described in Figure 3.13b. Here, the selected contact stiffness values are smaller than the ones in the axial model and closer to the bar stiffness. Therefore, the amplitude values tend to go beyond 1, which is caused by more interaction between the contact and the bar. The curve plotted for the smallest clamp motion amplitude almost reaches 2.

Figure 3.24 facilitates exploring the influence of the contact stiffness on the growth of the

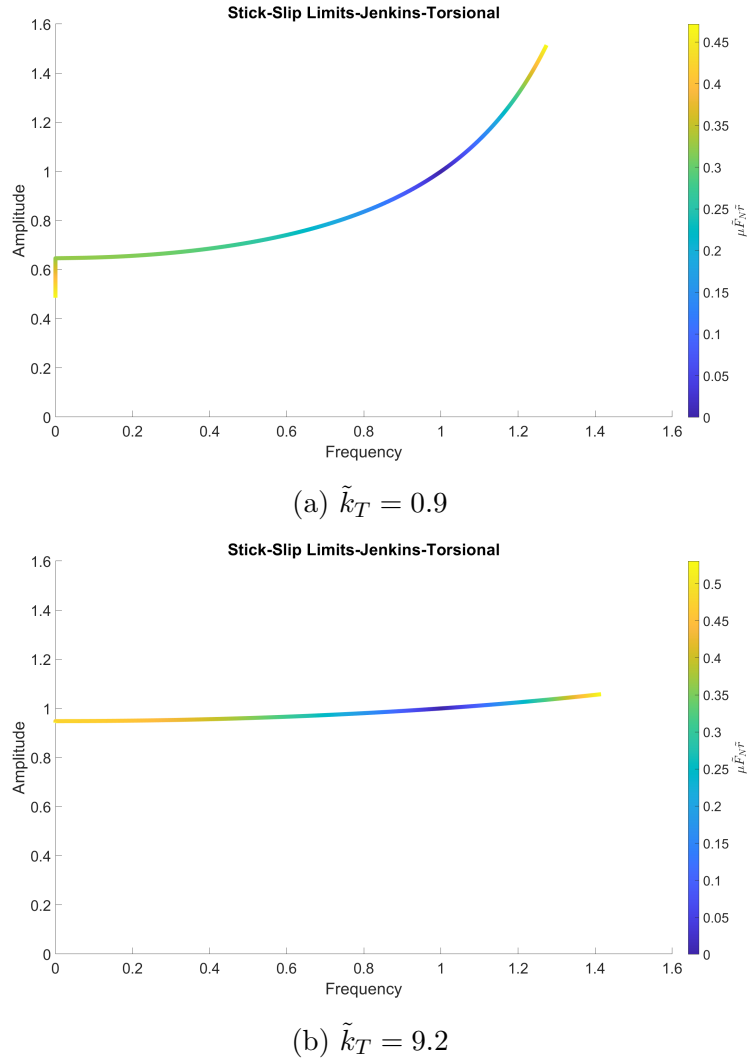


Figure 3.24: Growth of the stick limit boundaries with $\mu\tilde{F}_N$. Limit amplitude against limit frequency for different values of $\tilde{k}_T$.

stick region for the torsional bar with the Jenkins friction approach. Again, the stick region expands easier in the pre-resonant region towards smaller frequencies compared to the growth in the post-resonant frequencies. Considering that the contact is softer than the axial model, the curves have greater gradients and consequently, they rise to greater amplitude values. Stiffening the contact flattens the amplitude curve just as in the axial model.

Like the axial case with the Jenkins element, the minimum necessary frictional moment for a full-stick regime is displayed in Figure 3.25 using Equation (3.48). Knowing that the system characteristics of the torsional model with the Jenkins element are precisely the same as the axial model with the Jenkins element, one can explain this figure using the same perspective.

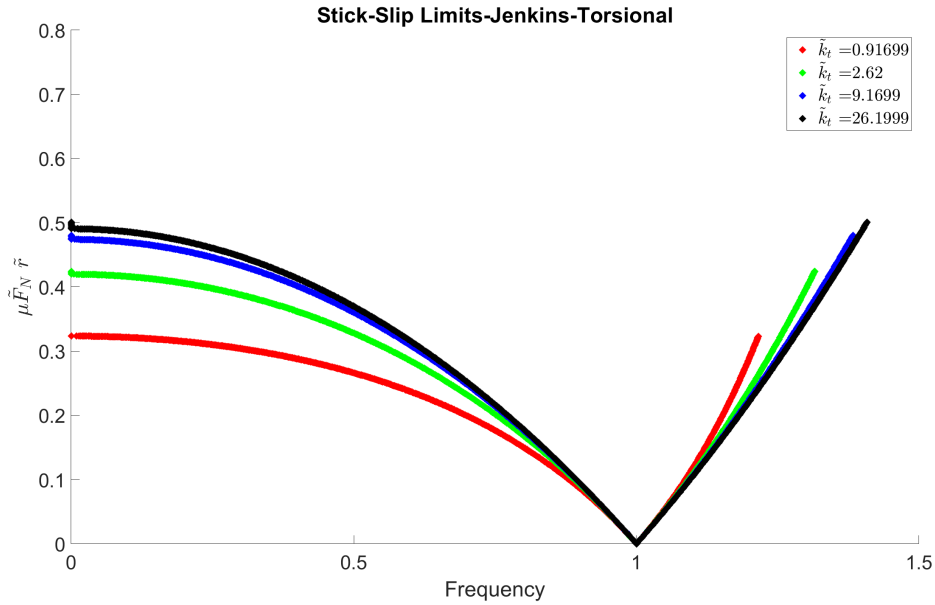


Figure 3.25: The least necessary $\mu \tilde{F}_N \tilde{r}$ for full-stick regime in torsional Jenkins model.

Velocity-Dependent Element

The results of the final analytical model built within the scope of this study are acquired by merging the two separate solutions as done in the previous models. These different solutions, which belong to the regimes of stick and slip contact conditions, are sketched in Figure 3.26. This sketch exhibits four regions of stick and slip identical to the axial model with a velocity-dependent friction approach. This is a natural consequence of closely related characteristics of the governing equations.

The results produced utilising the analytical and numerical models for a range of various clamp displacement amplitudes are presented in Figure 3.27. Here, $\tilde{c}_T$ is kept constant and selected as 0.1, which is very small compared to the examples of the axial case. Throughout the whole interval of frequencies of interest, both numerical and analytical solutions show very similar behaviours. However, the analytical solution predicts a motion with slightly less amplitude except for the resonance frequency, where both are equal to 1. The numerical approach suggests that the stick regime is almost a flat-top region around the resonance, whereas the analytical method yields a curve with a greater curvature which depends on $\tilde{c}_T$. The margin of error in both stick and slip regimes (in stick more dramatically) drops as the clamp motion amplitude

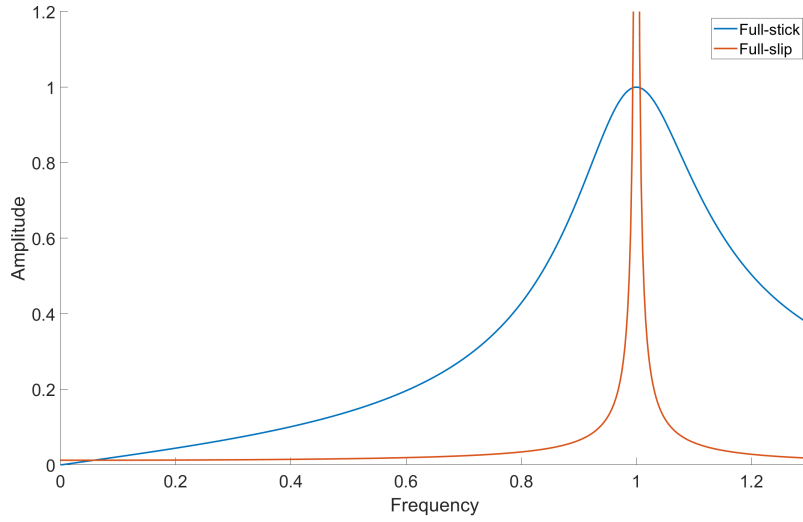


Figure 3.26: A sketch of the analytical solutions for the bar in torsional motion with the velocity-dependent model.

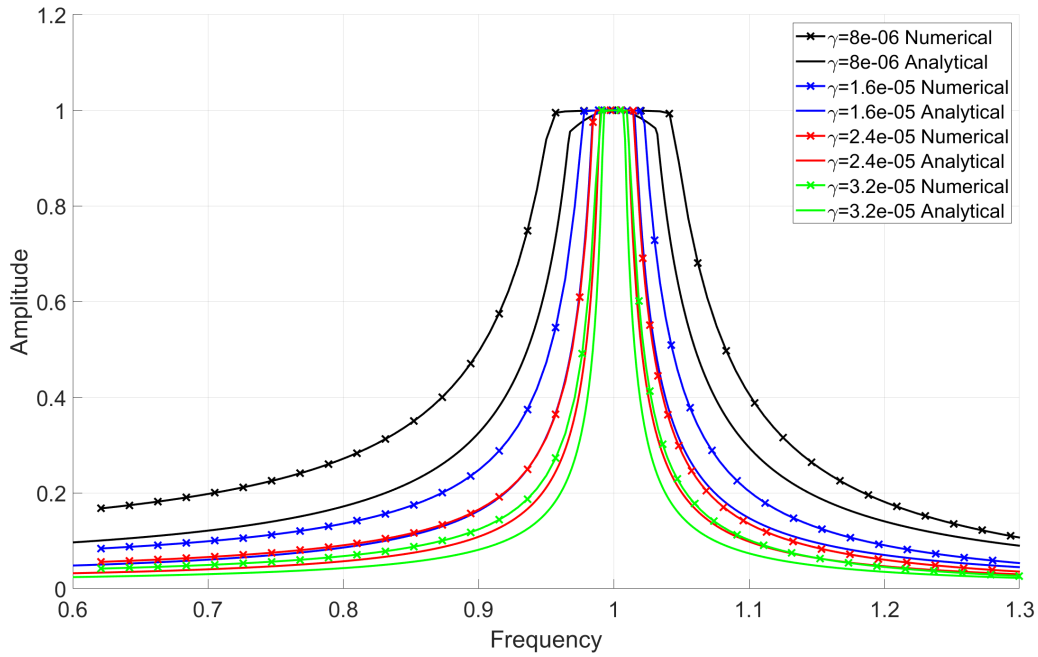


Figure 3.27: Comparison of analytical and numerical results for the torsional case with velocity-dependent model from Section 3.4.2.

increases. Comparing these results to the axial case, it is to be stated that another factor that affects the gap between the numerical and analytical approaches is the $\tilde{c}_T$, the damping coefficient of the contact, which controls the curvature of the plot in the stick regime. These models are in better agreement in the full-stick regimes for greater values of parameter $\tilde{c}_T$. As the $\tilde{c}_T$ rises, the amplitude response of the stick regime becomes flatter and the values closer

to the boundaries of the slip regime climb closer to 1. This behaviour can be seen in Figure 3.28 (3.28a and 3.28b), which is there to visualise the effects of $\mu\tilde{F}_N$ and $\tilde{c}_T$ on the boundaries of the stick and slip regions. Figure 3.28 also shows that the expansion of the stick region is easier towards smaller frequencies compared to growth in the positive direction along the axis of frequency.

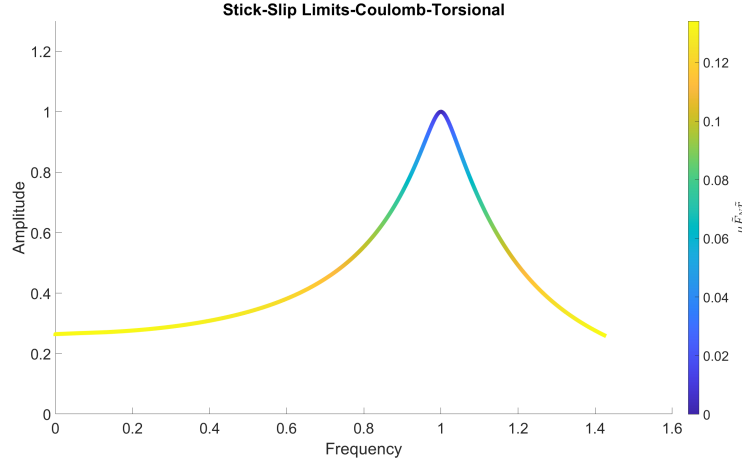
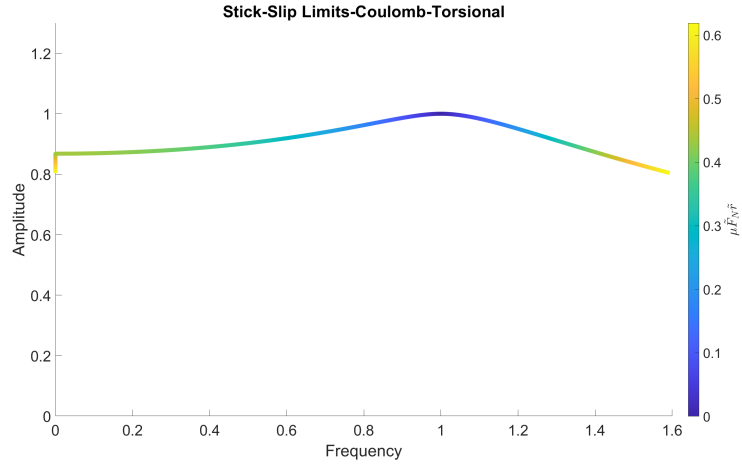
(a) $\tilde{c}_T = 0.1$ (b) $\tilde{c}_T = 0.8$

Figure 3.28: Enlargement of the stick limit boundaries with $\mu\tilde{F}_N$. Limit amplitude against limit frequency for different values of $\tilde{c}_T$.

Next, Figure 3.29 displays the relationship between the minimum required frictional moment for a full-stick regime based on Equation (3.48). Comparing this in every aspect, one can conclude that the plots are very similar to the ones presented in Section 3.5.1, and therefore, the interpretation needs to be made using the same approach.

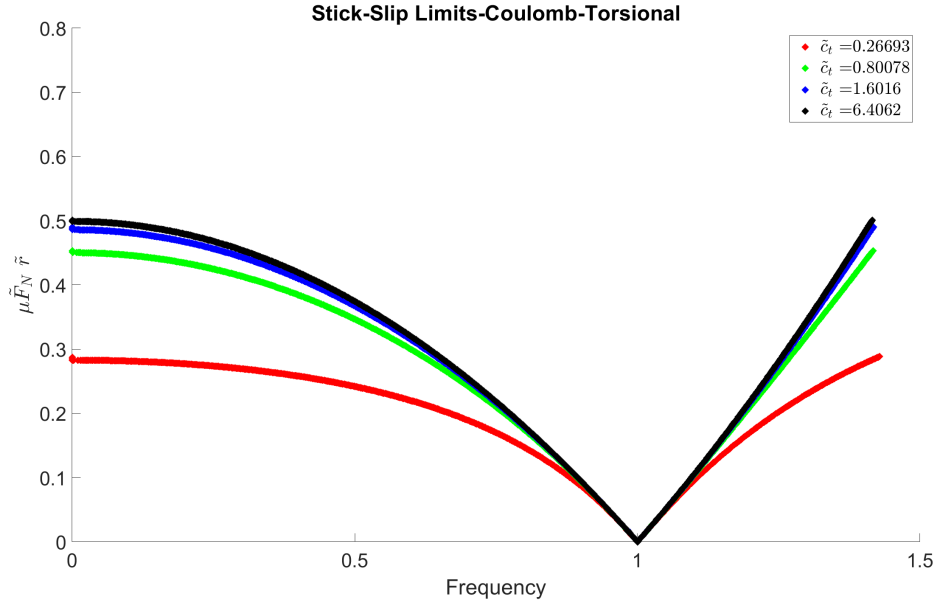


Figure 3.29: The least necessary $\mu \tilde{F}_N \tilde{r}$ for full-stick regime in torsional velocity-dependent model.

3.5.3 Dual Cantilever Beam Model

Jenkins Element

Following the same methodology as done in axial and torsional bar models, the dual cantilever beam in bending motion also has two separate solutions, namely, full-stick and full-slip. The one that yields the minimum amplitude is judged to be the dominant one for that frequency of excitation. The two separate solutions are represented in Figure 3.30 with the parameters of the actual DMA test setup at Imperial College London Department of Mechanical Engineering (TA DMA Q800) excited with the maximum displacement possible for the machine. This means the tangential contact stiffness is picked as $k_t = 10^6 \text{ N/m}$, the normal contact stiffness as $k_a = 10^8 \text{ N/m}$ and the coefficient of friction as 0.4.

The results of the model built to represent the dual cantilever fixture configuration of the DMA test setup are given in Figure 3.31. This figure is prepared with the parameters selected to fit the mentioned DMA rig. The data indicate that the physics of the system is quite similar to the cases of the bars with axial and torsional motions. However, the other models are subject to base excitation, whereas this dual cantilever beam is excited externally without the

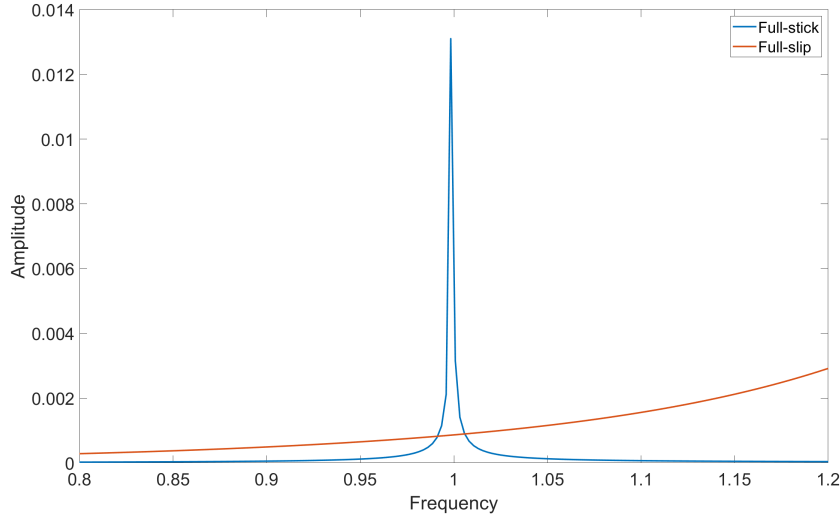


Figure 3.30: A sketch of the analytical solutions for the dual cantilever beam in bending motion with Jenkins model.

involvement of the frictional reactions, and the friction provides the support reactions only. Besides, here, the rotational degrees of freedom of the beam at the clamps are of interest which is already nondimensional. Thus, the y-axis (Amplitude (ϕ)) is not further processed. As seen from Figure 3.30, at lower and higher frequencies, the full-stick regime is dominant. Only around the resonance slip occurs. This is the exact opposite behaviour of what is happening with the axial and torsional cases. Nevertheless, the mechanics behind the phenomenon is the same. These two previous cases are excited with the base motion, and a system requires less force/moment around the resonance to move. Whereas in this case, friction is the restricting force, and the system is harder to restrict when excited around the resonance. Furthermore, the rotation of the beam's cross-section increases as the beam approaches its resonance which consequently increases the value of the term $\tilde{k}_T \tilde{h} \phi$ closer to the limit of the frictional slip. Once the slip takes over, energy dissipation starts, which reduces the amplitude of the rotational degree of freedom of the beam. Moreover, it is also observed that with increasing excitation levels, the rotation at the clamps also increases alongside the slope of the slip regime curves. Even though the slip regime also enlarges with increasing excitation levels, it is so small that it is hard to notice with the naked eye. Besides, the results of the analytical and numerical models are in good agreement. However, the analytical model slightly overshoots what the numerical model predicts with around 3 – 6% difference.

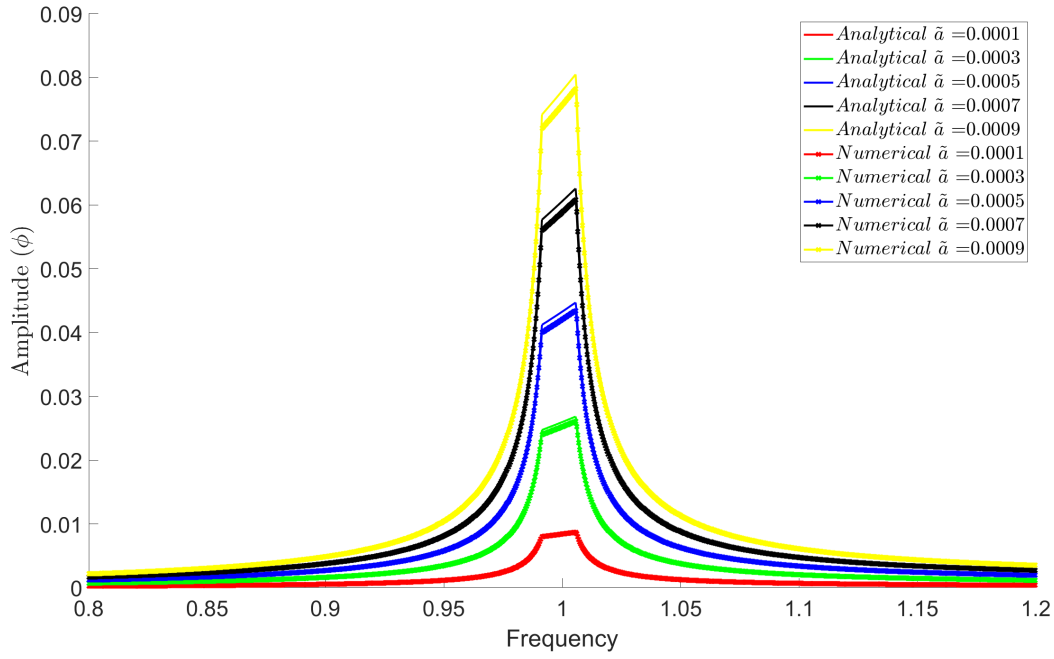


Figure 3.31: Comparison of analytical and numerical results for the dual cantilever beam case with Jenkins model from Section 3.4.2.

Figure 3.32 shows the effect of the tangential contact stiffness of the beam-clamp interface on the least necessary support moment caused by the friction. The data suggests a parallelism between the bending case and the other previous cases. This is an expected outcome, considering the same mechanism governs the system.

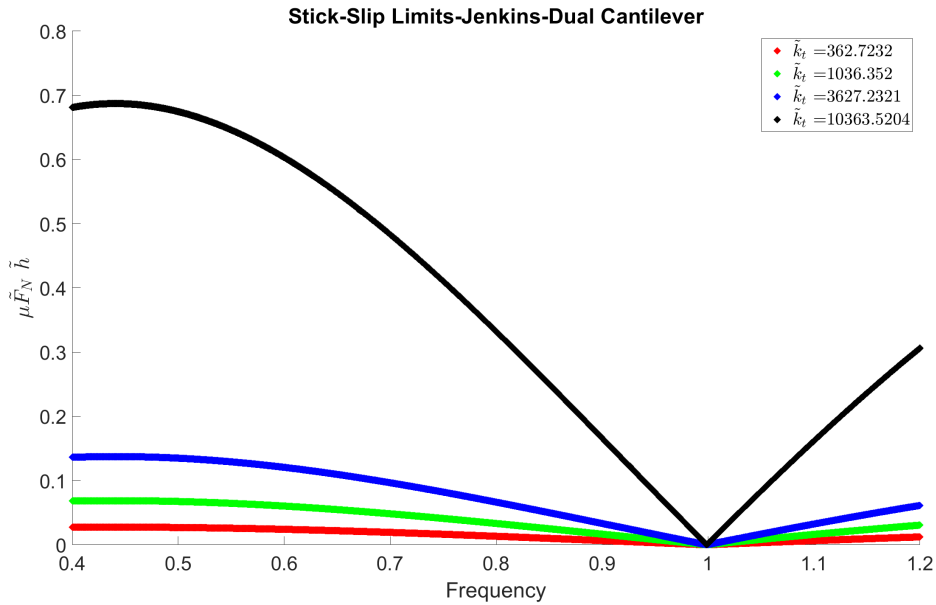


Figure 3.32: The least necessary $\mu \tilde{F}_N \tilde{h}$ for the full-stick regime in the dual cantilever beam with the Jenkins model.

3.6 Conclusions

This study presents analytical solutions to model the steady-state forced vibration behaviour of bars/beams with frictional clamps under axial and torsional motion. The reactions of the clamps are reduced into concentrated nonlinear loads and included in the equations of motion, taking advantage of the Dirac-delta function $\delta(z)$ for the axial and torsional cases while imposed as boundary conditions for the bending motion. The nonlinear loads caused by friction are modelled using two different models, namely, displacement-dependent (Jenkins) and velocity-dependent models. Using these models, the motion under the influence of friction is decomposed into two regimes of full-stick and full-slip. The equations of motion are then nondimensionalised and solved using the Galerkin method considering the two regimes as two separate solutions. These two separate solutions are combined to represent the mechanical behaviour of the bar/beam-clamp system. To understand how such systems are affected by friction, example cases are built, and the proposed analytical solution is conducted on those examples.

Alongside the analytical solution, a numerical method, AFTHB, which is flexible and requires fewer simplifications/assumptions, is performed on those examples. The results of the numerical and analytical models are compared to each other and are found to be in good agreement.

The results of both Jenkins and velocity-dependent models show that the stick regime governs the behaviour around the resonance since the necessary excitation forces reach their minima at the resonance. The interval where the stick regime is dominant is controlled by the maximum friction forces. The greater the maximum friction forces are, the larger the stick regime interval. Likewise, increasing the prescribed clamp motion amplitude shrinks the stick region. Here, the maximum friction forces are constant, but the required frictional forces climb.

The displacement amplitude of bars/beams with the Jenkins friction model exceeds the imposed clamp motion amplitude. The main reason is the interaction of the contact stiffness with the bar. The interaction becomes more significant if the contact stiffness is closer to the bar stiffness, and thus, the overshoot of the amplitude becomes greater. For stiffer contacts, the overshoot

becomes negligible, and the plot of the stick region becomes a flat line indicating there is no slip occurring.

The bar/beam-clamp system built using the velocity-dependent model generally shows similar behaviour. However, the peak amplitude value never exceeds 1 in this model. The stick regime may be found at two different locations, at very small frequencies, as well as around the resonance. The stick regime around the resonant frequency looks almost like a flat line for a larger contact damping coefficient and has a considerably significant curvature for the lower contact damping.

Finally, evaluating all three systems modelled (bars in axial and torsional motions and the dual cantilever beam in bending motion), the dual cantilever beam is proven to be a safer option regarding the involvement of friction in the motion. The stiffness of beams are much smaller compared to the axial and torsional stiffness of the bars. In bending, the gap between the stiffness of the clamps and the stiffness of the beams is much more significant, which reduces their interaction. Therefore, it is concluded that the dual cantilever fixture is more suitable for use in the DMA tests conducted for this study.

Chapter 4

Manufacturing and Characterisation of the Nanocomposites

4.1 Introduction

Nonlinearity can arise in homogeneous materials and nonhomogeneous materials like composites, particularly when subjected to large deformations [37]. One way of categorising the materials in engineering is based on how they are deformed when loaded. The deformation is expressed as a function of load, which directly makes the distinction of the materials being linear or nonlinear. Thus, if the stiffness of the material is constant, then the material characteristics are linear, whereas if the stiffness varies for the different stages of deformation and the deformation rates, then the material has a nonlinear manner [112]. In many cases with polymers, a linear relation does not accurately represent the behaviour of such material. For example, the nonlinear deformation of polymers can be affected by the presence of reinforcement [4].

The nonlinearities based on material characteristics can be expressed mathematically within the constitutive equation of the materials [168, 62]. Thus, there are different material models in the literature that have been used by researchers over the decades. Material models differ by functions used in the constitutive equations. These functions are generally based on experimental results of material characterisation focusing on mechanical behaviour under various loading

conditions [150]. Since polymers are usually more compliant materials with more damping capacity than other engineering materials, such as metals and ceramics, a different material model is required to represent their behaviour. The material modelling method, which includes the damping and/or strain-rate-dependent behaviour, is called viscoelasticity which describes the material behaviour as a combination of viscous and elastic. Due to the viscous behaviour, this modelling method adds damping and the strain-rate-dependency to the material characteristics [217, 218, 273]. This damping/strain-rate-dependence can be represented in various forms such as linear viscoelasticity, nonlinear viscoelasticity and fractional derivatives.

Polymer-based composites are known to have complicated characteristics, especially those with nanoreinforcements [235, 163, 202, 86, 153]. The major difference between conventional composites and nanocomposites is caused by the great surface area/volume ratio of the nanoreinforcements, which leads to larger interphase regions that tend to have a nonlinear characteristic [122, 96, 147]. The resultant behaviour can be even more complicated in the presence of multiple nanoreinforcements [180, 28, 107, 76]. Halloysite nanotubes (HNT), carbon nanotubes (CNT), fumed silica (FS), silica, and rubber nanoparticles are among the most common particles for the fabrication of nanocomposites [245, 4, 227, 181, 29, 272]. Experiments are preferred to be able to characterise the behaviour of composites, and when performed, are often found to be more reliable [110]. There are several methods to experimentally characterise the nonlinear dynamic behaviour. Kliem et al. use a vibration test setup to explore the dynamics of a cylindrical composite structure [124]. Xu and Gupta use dynamic mechanical analysis (DMA) and they develop a method to extract the storage modulus for various strain rates and temperatures [266]. Xu et al. again use DMA to characterise the elastic behaviour of nanocomposites at a range of strain rates and temperatures [267]. Esmaeeli et al. propose a new setup to measure the mechanical properties at high frequencies [75]. Pierro and Carbone also design a vibrational test setup using an impact hammer to extract the mechanical properties, which are then compared with the results obtained from DMA [191].

This chapter conducts detailed investigations of strain-rate-dependent mechanics and impact resistance of nanocomposites. Epoxy-based nanocomposites are manufactured with four different reinforcements, FS, HNT, silica (Nanopox F700) and rubber (Albipox 1000), as well as

the control samples made of unreinforced epoxy. To explore the synergistic behaviour of the reinforcements, FS and HNT are also combined together in equal masses. Silica and rubber are put together for the same purpose. To minimise the effects of manufacturing processes on the mechanics of the nanocomposites, a consistent manufacturing process is designed and employed for all sorts of materials. To visualise that there are no substantial irregularities in the material, scanning electron microscopy (SEM) and to ensure that the chemical composition is consistent, Fourier transform infrared spectroscopy (FTIR) are used on the manufactured materials. Afterwards, strain-rate-dependent mechanical properties are experimentally characterised with three-point bend tests at three different strain rates as well as with dynamic mechanical tests (DMA) in terms of frequency and strain amplitudes. Charpy impact tests are conducted to explore the impact resistance.

4.2 Material Preparation and Experimentation

To conduct this part of the study, epoxy and epoxy-based composite samples are manufactured. A scanning electron microscope (SEM) is used to examine the interior structures of these manufactured composite samples, which primarily determines the mechanical characteristics of those materials. The composites are also put through three-point bend tests at different deformation speeds to characterise the strain-rate-dependence of their mechanical properties. Finally, Charpy impact tests are performed to get an understanding of how ductility is affected by the reinforcements. In this section, manufacturing and testing procedures are described in detail.

4.2.1 Material Preparation and Manufacturing Process of the Samples

In terms of material groups, first epoxy samples are manufactured to be used as a benchmark to display the effects of the reinforcing materials on the mechanics of the nanocomposites. Epoxy resin MGS@ L 285 by Hexion is chosen along with the hardener MGS@ 285 by Hexion. Then,

FS Aerosil 200 by Evonik, HNT by Nanografi and their equal mass combinations are used to reinforce the epoxy matrix. For this material group, the reinforcements are added to the resin by 0.5%, 1%, 1.5% and 2% in terms of mass fraction (m_f) as previously researched in the literature [272, 31, 203]. In addition to those samples, Albipox 1000, Nanopox F700 by Evonik, which contain $m_f = 40\%$ rubber and silica respectively, and their equal mass mixtures are added to epoxy to manufacture these sets of composites. For this material group, the reinforcements are mixed in the epoxy resin by 5%, 10% and 15% (m_f). Studies with similar mass fractions can also be found in the literature [227, 181]. So, nine more sample sets are prepared. In total, 22 sample sets are manufactured that would include the control ($m_f = 0\%$) sample. Weighing is done using a Mettler-Toledo Standard Level Balances ME Precision Balances with a capacity of 5200g and precision between 0.001 – 0.01g. In each material manufacturing increment, 360g of material is prepared.

Preparation of Epoxy

The two components of the epoxy, the resin and the hardener, are brought together and mixed until the colour homogeneity is achieved. The mixing is done slowly to avoid any air bubbles coming into existence at room temperature in ambient conditions. The mixture is then moulded and put into the vacuum chamber for 2 hours. Afterwards, the mould is left in ambient conditions for 24 hours for curing. Finally, the samples are put into the incubator at 60°C for 15 hours for the post-curing process.

Preparation of FS and HNT-Reinforced Epoxy

Before starting the manufacturing process, the nanoparticles, namely FS and HNT, are put into an incubator and kept there for at least 8 hours to ensure the minimisation of harboured humidity between the particles. The FS particles have an average diameter of 12 nm, and the HNT particles have diameters between 30-70 nm and lengths between 1-3 μm . The particles are blended into the resin component of the epoxy by using an electromagnetic mixer at 50°C and 100 rpm for at least 2 hours. An ultrasonic homogeniser is used to ensure a better distribution

of particles and avoid agglomeration. During this process, the temperature of the mixture is controlled to prevent any chemical reactions that may be caused by excessive heating, which may strongly affect the mechanical properties of the epoxy resin. After, the mixture is degassed in the vacuum chamber for at least one hour. Following degassing, the hardener is mixed into the mixture slowly until colour homogenisation is achieved while trying to minimise the development of air bubbles. Then, the final moulded mixture is moulded and put into the vacuum chamber for two hours. Then, the same curing and post-curing processes are followed as done for the epoxy samples to achieve a consistent resin formation.

The mass composition of the composite described is presented in Table 4.1.

Table 4.1: Composition in mass of material depending on the nanoparticle reinforcements (mass is measured in g.).

Mass fractions (m_f)	Matrix mass [g]		Reinforcement mass [g]		
	Epoxy resin	Hardener	FS	HNT	FS and HNT
0.5%	255.86	102.34	1.80	1.80	1.80
1%	254.57	101.83	3.60	3.60	3.60
1.5%	253.29	101.31	5.40	5.40	5.40
2%	252.00	100.80	7.20	7.20	7.20

Preparation of Albipox 1000 and Nanopox F700 Mixed Epoxy

It is simpler to manufacture this group of nanocomposite materials compared to the previously explained nanocomposite materials since the ultrasound homogeniser is not needed in this process. In the final state of the composite, it is expected that the silica particles in Nanopox F700 would have an average diameter of 20 *nm* and a maximum diameter of 50 *nm*. The rubber particles (Albipox 1000) have equivalent diameters between 0.5 and 1 μm . Albipox 1000 and/or Nanopox F700 are directly mixed mechanically into the resin component of the epoxy in the desired amounts. The mixing procedure is performed at 100 rpm and 50°C for at least 4 hours. This is with the same degassing, mixing of hardener, moulding, curing, and post-curing processes followed to finalise the manufacturing of those samples.

The compositions of the manufactured composites are given in Table 4.2.

Table 4.2: Composition in mass of material depending on the reinforcements provided by Evonik (mass is measured in g.).

Mass fractions (m_f)	Matrix mass [g]		Reinforcement mass [g]		
	Epoxy resin	Hardener	Albipox 1000	Nanopox F700	Albipox 1000 and Nanopox 1000
5%	217.29	97.71	45.00	45.00	45.00
10%	177.43	92.57	90.00	90.00	90.00
15%	137.57	87.43	135.00	135.00	135.00

Some of the manufactured samples are shown in Figure 4.1.

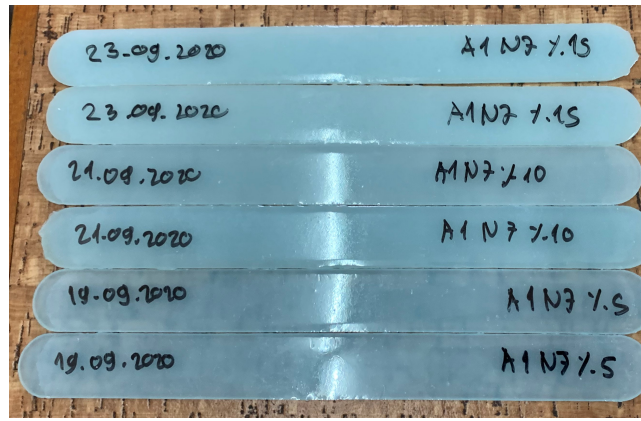


Figure 4.1: Some of the manufactured samples.

4.2.2 Scanning Electron Microscopy (SEM)

The internal structure of the composites strongly affects the mechanical properties of materials. Therefore, at least a basic understanding of the internal structure, which can be gained by SEM, is essential to be able to comment on the mechanical performance of materials. For this exact purpose, the internal structure of the manufactured composites is visualised using an SEM (FEI, Quanta Feg 250). In order to conduct the visualisation of material morphology using SEM, the material surfaces are first coated with gold/palladium (Au-Pd) to ensure conductivity. The images are acquired at 20 kV potential difference.

4.2.3 Fourier Transform Infrared Spectroscopy (FTIR)

Fourier transform infrared spectroscopy (FTIR) analyses in the mid-infrared spectrum (MIR-BETWEEN $4000\text{-}650\text{ cm}^{-1}$), where the most important peaks related to the polymeric matrix are expected, investigate the chemical groups that are on the surface of the nanocomposite samples. Thus, they are performed to examine the chemical composition of the manufactured materials. This aims to ensure all the changes in the mechanical properties are purely due to the presence of the reinforcing phases.

4.2.4 Three-Point Bend Tests

Considering polymers usually display complicated mechanical characteristics, which can be affected by numerous parameters, a simplistic yet accurate material model needs to be employed to predict their mechanical behaviour without creating computational difficulties. Strain rate is one of the dominant factors that form the models to be built on polymers. Therefore, in this study, the material behaviour is expected to be nonlinear. To understand the strain-rate-dependent mechanics of the manufactured composite materials, which are expected to have a nonlinear material behaviour, the material samples are put through three-point bend tests at three different strain rates, from a strain rate of 0.01 min^{-1} (sometimes referred to as quasi-static) up to 0.1 min^{-1} with equal increments based on the standard ASTM D7264 using the equipment Shimadzu AG-IS 50 kN which is displayed in Figure 4.2 [18]. The strain rate range chosen is based on equipment capability, as well as consideration of previous research at similar strain rates, so the experimental data are comparable [267, 101]. Thus, the effects of reinforcements on nonlinear material behaviour are explored.

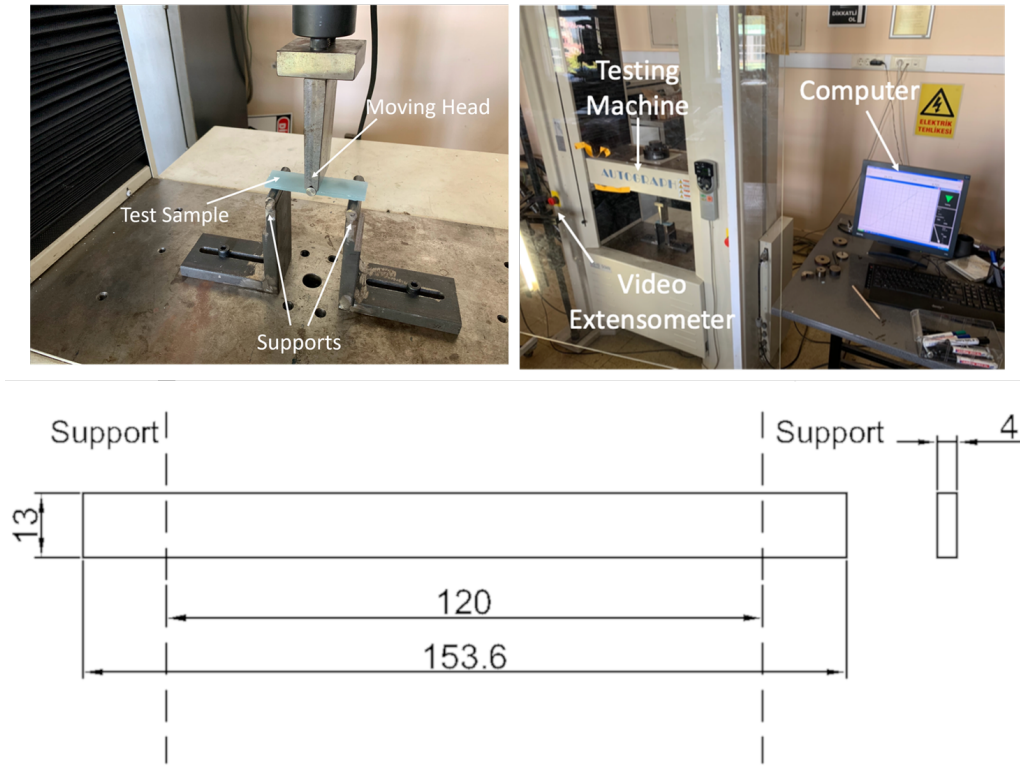


Figure 4.2: Three-point bend test setup and sample (Dimensions in mm).

4.2.5 Dynamic Mechanical Analysis (DMA)

As one of the most standardised methods, DMA is employed to characterise the dynamic properties of the manufactured epoxy-based nanocomposites. DMA is performed on a beam sample of the sizes 70 mm by 10 mm by 3 mm using the dual cantilever fixture on a TA DMA Q800. The dual cantilever beam configuration ensures that the required level of stiffness is achieved even with compliant materials such as these nanocomposites. The clamps at both ends of the beam sample are tightened, each with two screws with 8 in-lbs standardised torques. Hence, the friction at the beam-clamp interfaces does not contribute significantly to the measured damping data. That means, the beam and the clamps stay in the full-stick regime. The test is conducted by exciting the beam sample from the middle harmonically with a certain displacement amplitude and frequency. The excitation frequency goes up to 150 Hz while the strain amplitude for the beams goes up to 0.12%. During the test, the force response and the displacements are used to calculate the dynamic modulus of the material. As the measures of

stiffness and damping, storage modulus and loss factor are chosen. It is also important to note that all the tests are conducted multiple times for each set of samples to ensure repeatability and under constant room temperature considering these materials are temperature-sensitive. It is also important to note that the experiments are done multiple times (until at least three consistent data sets are acquired), so the repeatability of these experiments is also tested and ensured to be consistent.

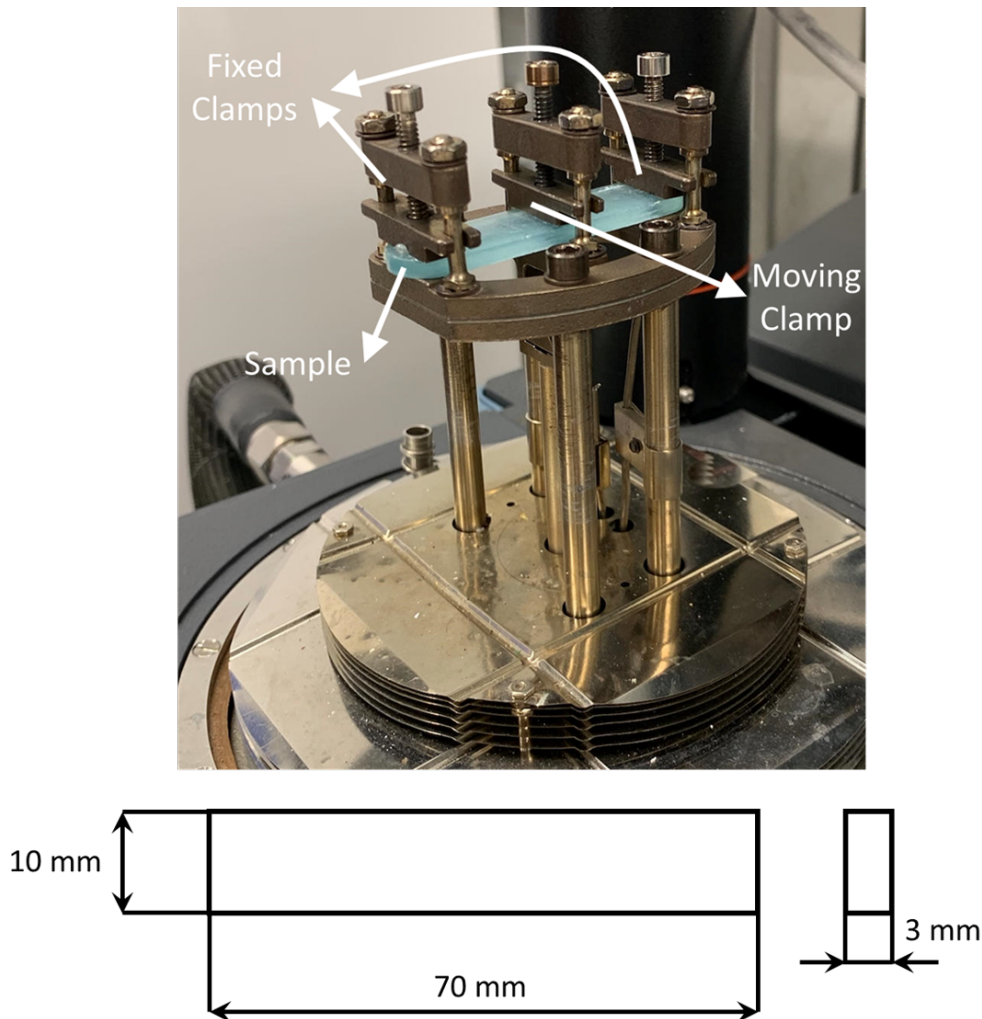


Figure 4.3: DMA test setup with a sample in the dual cantilever fixture and the dimensions of a standard sample.

4.2.6 Charpy Impact Tests

Following the characterisation of strain-rate-dependent mechanical properties of the nanocomposites, Charpy impact tests are conducted to compare the fracture energies of specimens with

different types and amounts of reinforcements. The effect of reinforcements on ductility is explored. Five samples from each of the 22 material groups are used to gather the necessary data from this test. The tests are performed with the setup shown in Figure 3 as described by the standard ASTM-D6110 [17].

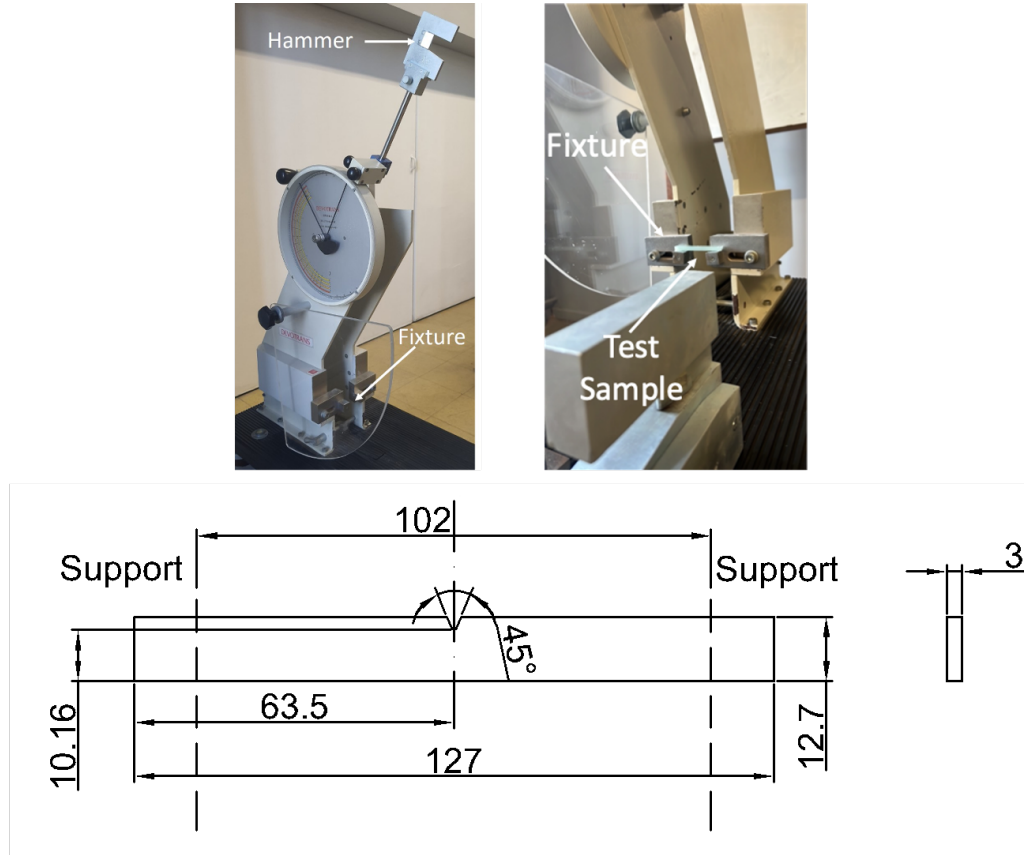


Figure 4.4: Charpy impact test setup and sample (Dimensions in mm).

4.3 Results and Discussions

4.3.1 Scanning Electron Microscopy (SEM)

The images acquired for each main reinforcement type are presented and interpreted starting from unreinforced epoxy, which is used as a reference for evaluating the mechanical properties of the nanocomposites. The SEM image of the surface of an unreinforced epoxy sample is given in Figure 4.5. The image obtained by magnifying the epoxy material by 10 000 times reveals that formations such as cracks and voids are not observed on the surface, and it displays a bulk/dense

structure. Thus, the hindering effects of cracks, voids or any similar sort of inhomogeneities are not expected in the results of the mechanical tests.

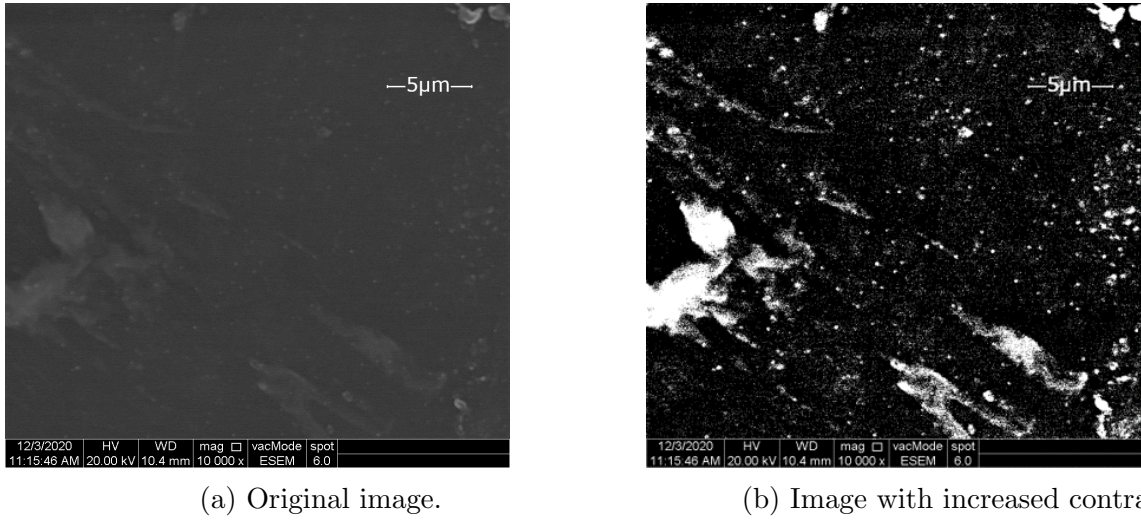


Figure 4.5: SEM image of an unreinforced epoxy sample showing homogeneous structure.

Based on the image shown in Figure 4.6 acquired from the sample reinforced with FS, it can be pointed out that the surface has a homogeneous structure and no undesirable material flaws, inhomogeneities, or cavities are visible. The FS particles are visible as white dots, which appear to have an average size of just above 10 nm . Clustering is also not observed, and the reinforcing particles display a relatively evenly spaced dispersion. These findings suggest that the variations in the mechanical performance of the composite will be strongly correlated to the presence of the reinforcing particles.

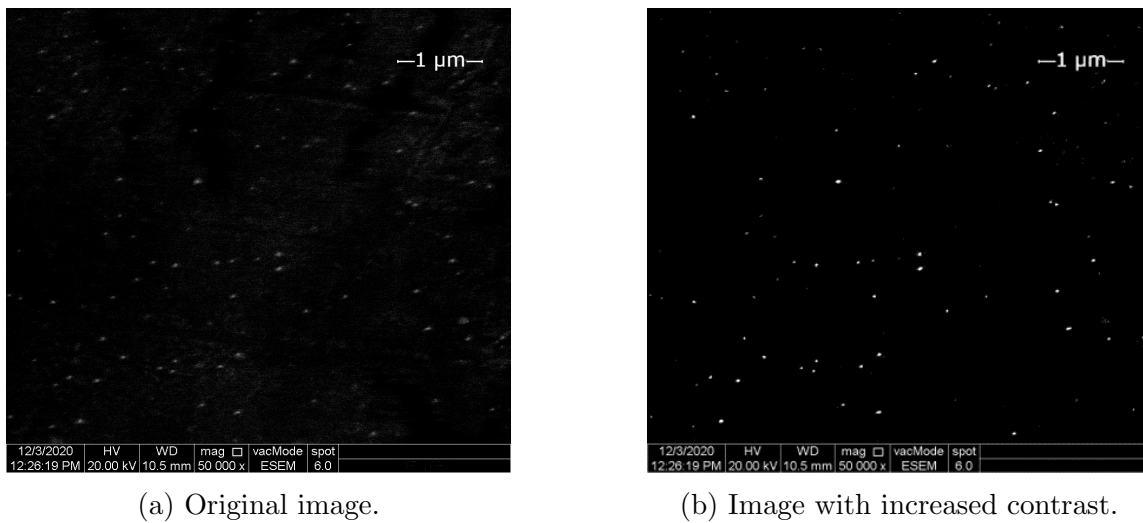
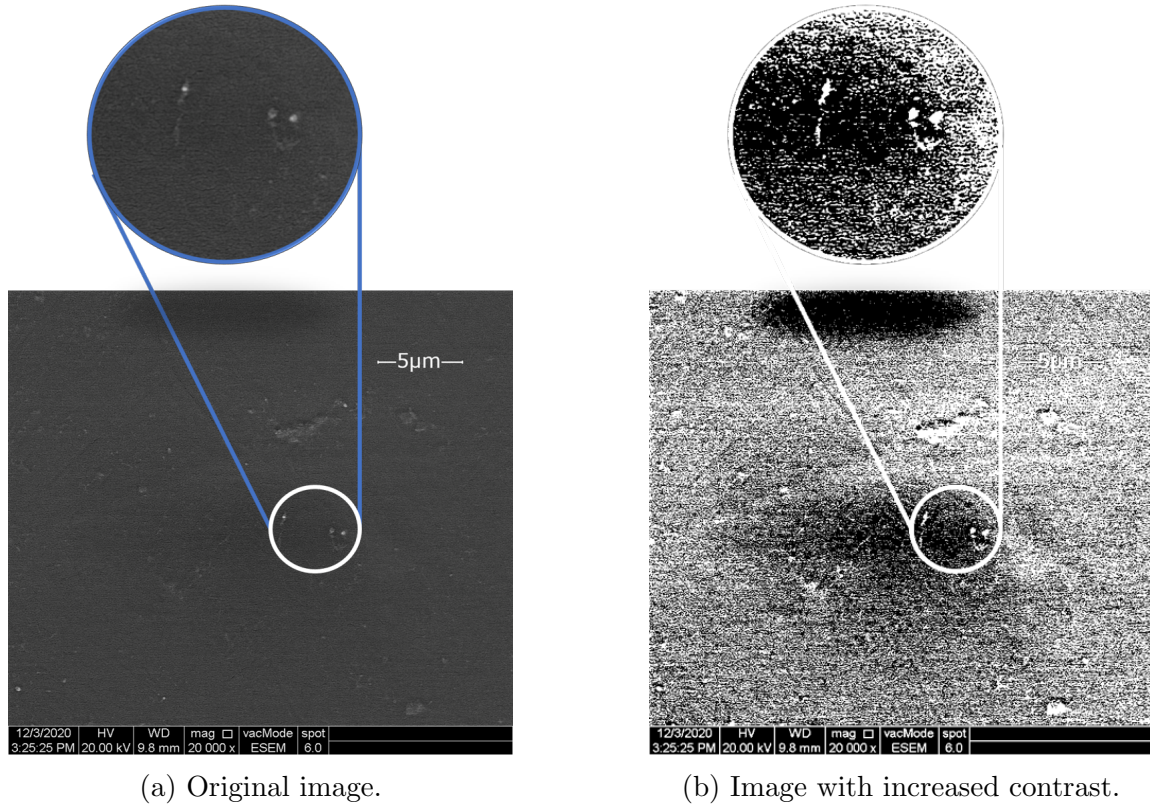


Figure 4.6: SEM image of an epoxy sample reinforced with FS (seen as dots with diameter of approximately $10 - 15\text{ nm}$).

The SEM image of an HNT-reinforced epoxy composite is presented in Figure 4.7. The HNT particles are visible as faint lines and dots. Compared to the sample with FS, the composite containing HNT displays a lesser homogenous structure. On the positive side, neither agglomeration nor major cavities or any sort of defects are visible, which strengthens the argument that the particles control the change of mechanics of the composite compared to unreinforced epoxy.



(a) Original image.

(b) Image with increased contrast.

Figure 4.7: SEM image of an epoxy sample reinforced with HNT (visible as dots and lines with diameter of approximately 30 – 70 nm and length of approximately 1 – 3 μm).

In Figure 4.8, the SEM image of epoxy with rubber addition is displayed. The formation of rubber is visible as grey sectors with a size below 1 μm, and the surface has superficial irregularities, whereas again, no major material defects such as clustering, cavities or cracks. Thus, one can expect no significant hindrance to mechanical properties due to material flaws caused by the manufacturing processes.

The last SEM image is gathered from the epoxy-based composite reinforced with silica and is given in Figure 4.9. The silica particles are seen as grey and white dots of the size 20 nm.

The surface appears to be relatively smooth and homogeneous without any notable material defects. Besides, it is clearly seen that the silica nanoparticle reinforcements have almost a uniform distribution. This substantiates the idea that the manufacturing of the composite is overall successful with respect to uniform mixing. The mechanical performance is predicted not to be impacted by non-uniform distributions of the reinforcement in the matrix.

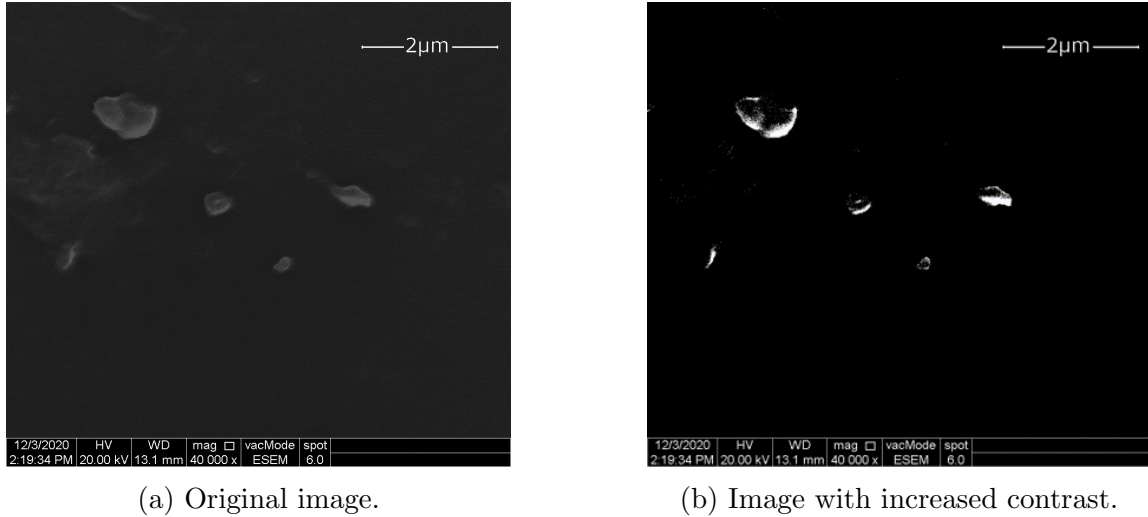


Figure 4.8: SEM image of an epoxy sample reinforced with rubber (seen as regions with lighter colour with diameter of approximately $0.5 - 1 \mu m$).

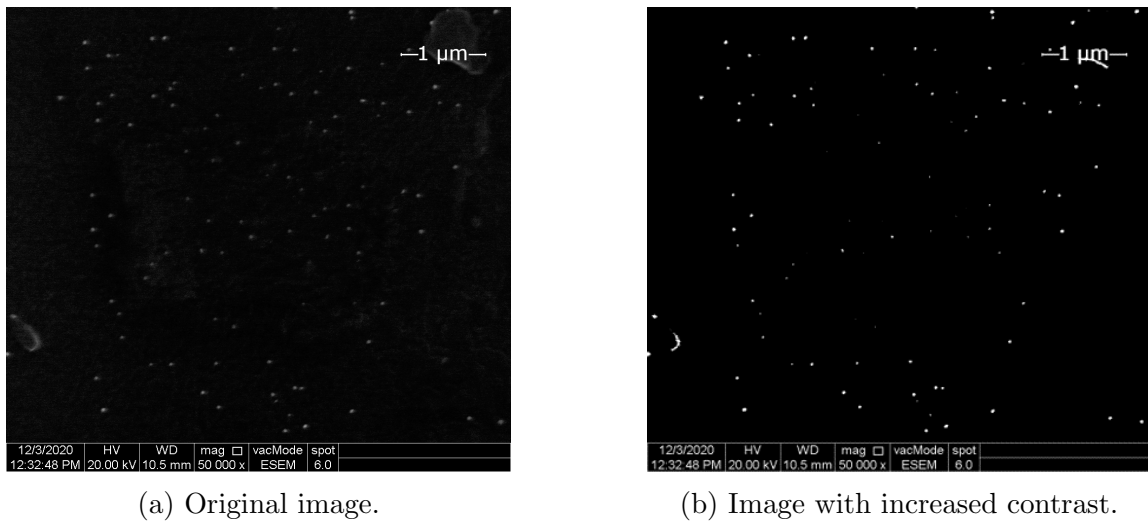


Figure 4.9: SEM image of an epoxy sample reinforced with silica (visible as dots diameter of approximately $20 - 30 nm$).

Finally, the SEM results suggest that the composites are manufactured overall without significant irregularities visible in their morphologies. Thus, comparing their mechanical behaviours based on the reinforcements becomes a valid approach.

4.3.2 Fourier Transform Infrared Spectroscopy

In the unreinforced epoxy, peaks occur at 2957, 2916, 2848, 1462, 1244, 826, 730, and 719 cm^{-1} as given in Table 4.3. The peak at 2957 cm^{-1} corresponds to the asymmetric vibration of CH_3 . While the peaks at 2916 and 2848 cm^{-1} correspond to $C - H$ stretching vibrations, the peak at 1462 cm^{-1} corresponds to the $C - H$ bending vibration of the methylene group. The peaks at 1244 and 826 cm^{-1} can be associated with the $C - O$ stretching vibration and the $C = C$ bending vibration of the aromatic structure, respectively. The peaks at 730 and 719 cm^{-1} occur due to the $C - H$ bending vibration [93, 155].

Table 4.3: Peaks appearing on FTIR spectrum for epoxy.

Wave number [cm^{-1}]	Vibration type
2957	CH_3 asymmetric
2916	$C - H$ stretching
2848	$C - H$ stretching
1462	$C - H$ bending
1244	$C - O$ stretching
826	$C = C$ bending
730	$C - H$ bending
719	$C - H$ bending

The plots obtained from the FTIR analyses performed on FS-reinforced epoxy samples are presented in Figure 4.10. With the addition of FS to the epoxy resin, the new peaks occurring at 1080, 1037 and 970 cm^{-1} prove the presence of FS in the epoxy. The peaks at 1080 and 1037 cm^{-1} can be explained with the asymmetric stretching or bending vibrations of $Si - O - Si$ and $Si - O$ bonds [131, 141], respectively, while the peaks at 970 cm^{-1} can be attributed to the $Si - OH$ vibration [141, 223, 225].

Figure 4.11 shows the FTIR results of HNT-reinforced epoxy composites. With HNT additive to epoxy, The water in the interlayer of HNT can be identified by the bending vibration peak seen at 1634 cm^{-1} [145, 211]. The peak corresponding to 1032 cm^{-1} can be attributed to the $Si - O$ stretching vibration [31, 26]. The large peak of $O - H$ stretching in the range of 3200-3600 cm^{-1} in the 0.5% HNT additive may be due to some impurities in the material. The Al bonds in the HNT's structure do not appear in the plots since it is expected to appear around 400 cm^{-1} , which falls out of the bounds of the investigated spectrum.

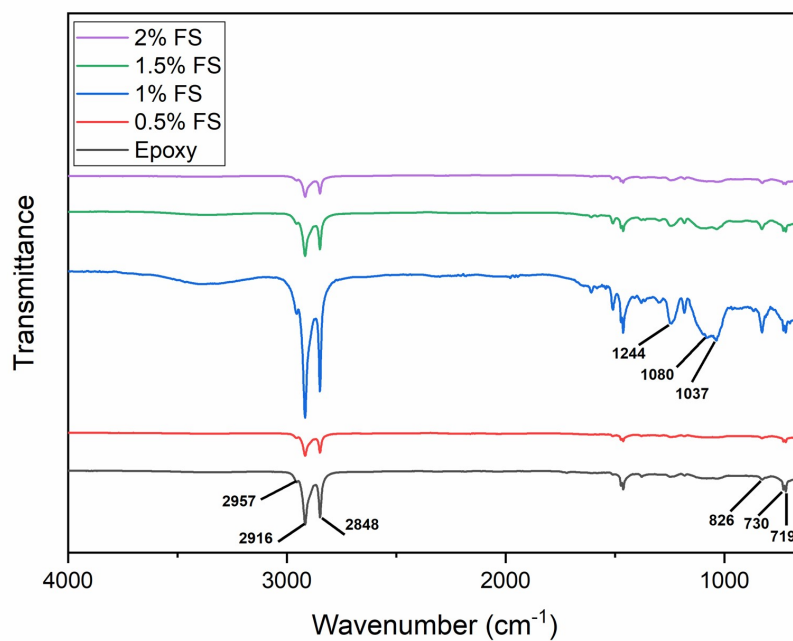


Figure 4.10: FTIR spectrum of FS-reinforced epoxy nanocomposites.

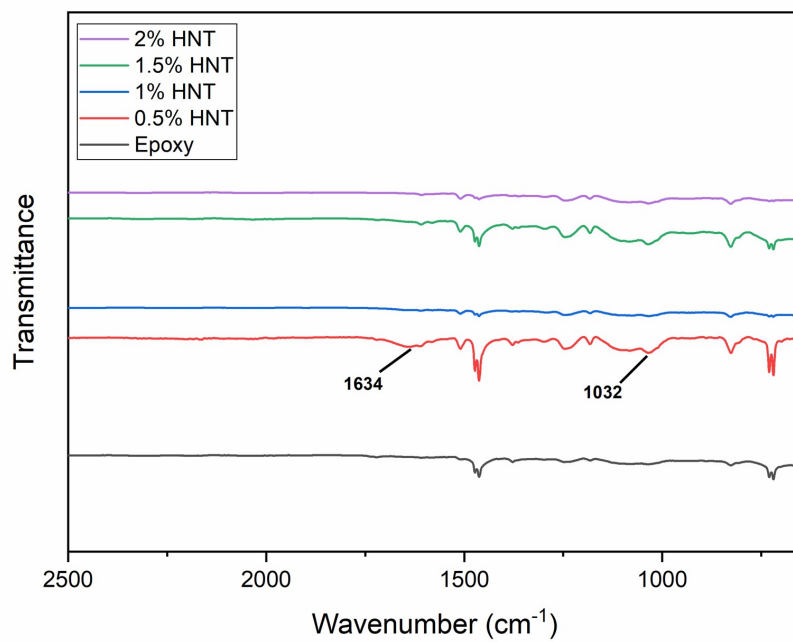


Figure 4.11: FTIR spectrum of HNT-reinforced epoxy nanocomposites.

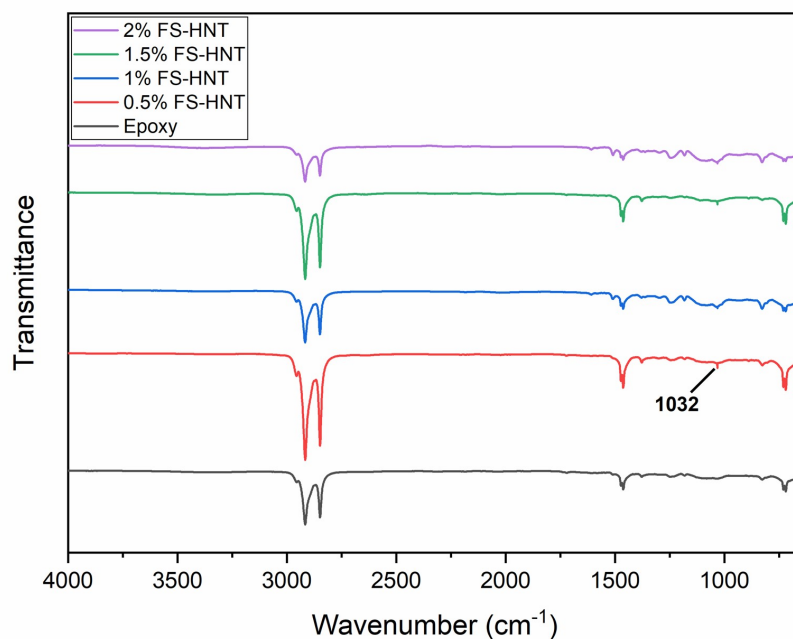


Figure 4.12: FTIR spectrum of FS and HNT-reinforced epoxy nanocomposites.

The results of the FTIR analysis conducted on epoxy samples reinforced with FS and HNT are given in Figure 4.12. The peaks occurring at 1097 and 1032 cm^{-1} can be attributed to the asymmetric stretching vibration of $\text{Si} - \text{O} - \text{Si}$ bonds originating from silica and HNT, respectively, to the $\text{Si} - \text{O}$ stretching vibration [141, 31].

In the FTIR spectra of rubber (Albipox 1000)-reinforced epoxy composites are shown in Figure 4.13. The results of 10% rubber (Albipox 1000) added epoxy, the peaks in the range of 3200-3600 cm^{-1} correspond to the $\text{O} - \text{H}$ stretching vibration caused by the wide peak water. No such impurities are found in the 5% and 15% Albipox 1000 additives. With the addition of Albipox 1000, the base resin of which is DGEBA, the peaks between 2957-2848 can be attributed to the $\text{C} - \text{H}$ stretching vibrations in CH_2 and the aromatic and aliphatic $\text{C} - \text{H}$ stretching vibrations. The peaks occurring at 1510, 1036 and 830 cm^{-1} show the aromatic $\text{C} - \text{C}$ stretch, the ether $\text{C} - \text{O} - \text{C}$ stretching and the $\text{C} - \text{O}$ stretching of the oxirane group, respectively. As a result of the addition of the rubber (Albipox 1000) to the epoxy, the presence of a new peak at 2121 cm^{-1} in the spectrum is clearly seen. The peak at 2121 cm^{-1} , which corresponds to the nitrile ($\text{C} \equiv \text{N}$) group of the rubber in Albipox 1000, confirms the presence of Albipox 1000 in the epoxy.

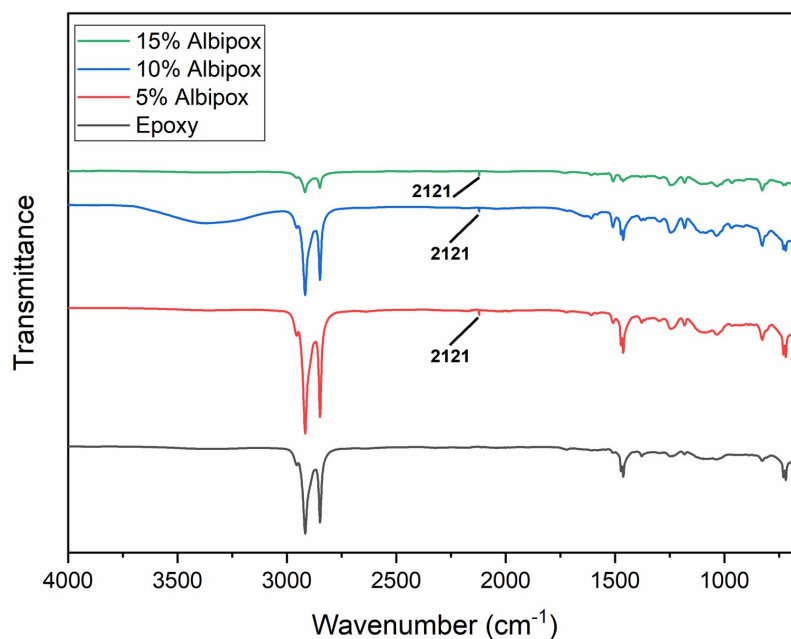


Figure 4.13: FTIR spectrum of rubber (Albipox 1000)-reinforced epoxy nanocomposites.

Figure 4.14 displays the FTIR results for silica (Nanopox F700)-reinforced epoxy nanocomposites. With the addition of Nanopox F700 to the epoxy, the new peak $Si-O$ stretching vibration occurred at 1036 cm^{-1} is successfully defined. The peak occurring at $3000\text{--}3600\text{ cm}^{-1}$ in the 5% silica (Nanopox F700) additive corresponds to the $O-H$ stretching vibration caused by the impurity in the material. No $O-H$ functional group of the material is found on the surface of the material with the contribution of 10% and 15% silica (Nanopox F700) to the epoxy.

As the last material group, the FTIR results of epoxy reinforced with both silica (Nanopox F700) and rubber (Albipox 1000) are presented in Figure 4.15. When the FTIR spectra of this material group are examined, the peak of the $Si-O$ stretching vibration, which is also seen only in the addition of Nanopox F700 to the epoxy, is observed at 1036 cm^{-1} . In addition, vibrations of the $C-O-C$ stretching of the ether occurring at 1036 cm^{-1} with the addition of Albipox 1000 to the epoxy are observed at 2121 cm^{-1} . The peaks became more evident as the Albipox 1000 & Nanopox F700 content increased (from 5% to 15%).

Finally, it needs to be pointed out that no chemical variations in the extent to affect the polymer's mechanical properties are found and therefore, it can be claimed that all the changes

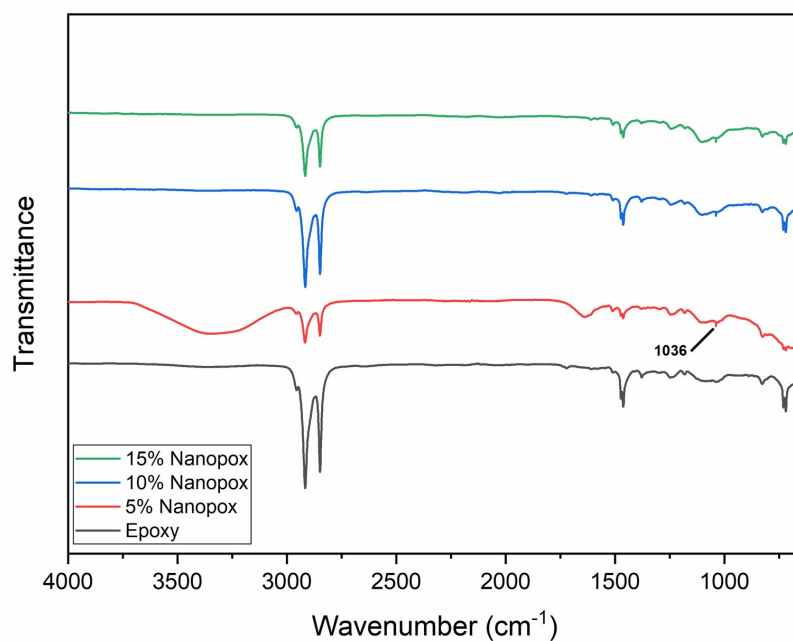


Figure 4.14: FTIR spectrum of silica (Nanopox F700)-reinforced epoxy nanocomposites.

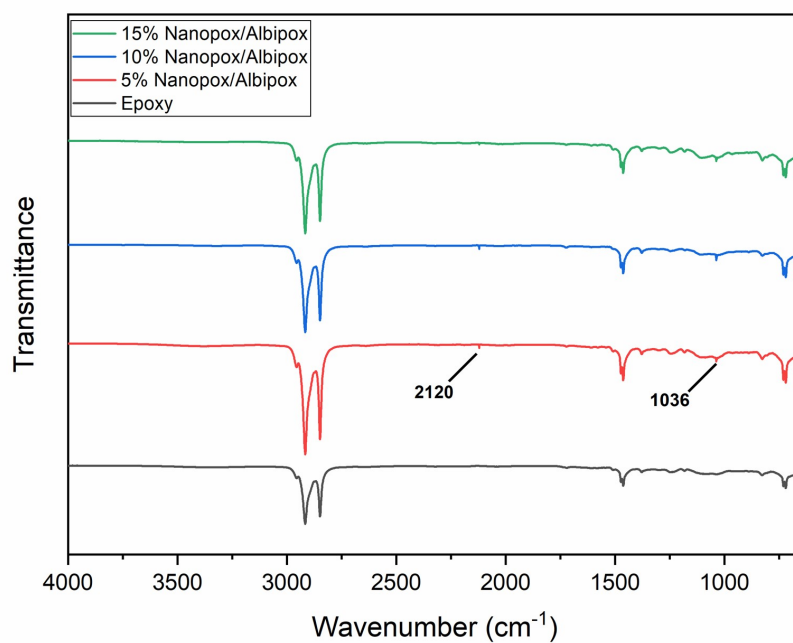


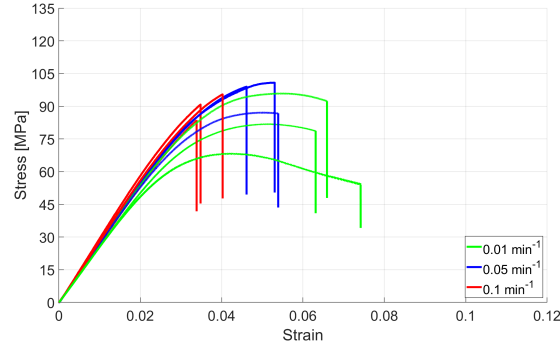
Figure 4.15: FTIR spectrum of rubber (Albipox 1000) and silica (Nanopox F700)-reinforced epoxy nanocomposites.

observed in the mechanics of the composites are mainly caused by the reinforcements. It is already unlikely since no chemical reactions/alterations are expected between the epoxy matrix and the reinforcing nanoparticles in the composites that are reinforced with FS and/or HNT. Nevertheless, also, the composites containing Nanopox F700 and Albipox 1000 do not display significant changes, which is initially judged as a plausible outcome considering they are provided in an epoxy-compatible polymeric dispersion.

4.3.3 Three-Point Bend Tests

The results of the three-point bend tests aim to characterise the rate-dependent mechanics of the manufactured epoxy-based composites and are presented in this section. Figures 4.16-4.21 exhibit the stress-strain plots of the unreinforced epoxy and the composites for the three strain rates, (0.01 min^{-1} , 0.05 min^{-1} and 0.1 min^{-1}). The curves of unreinforced epoxy given in Figure 4.16a suggest that the unreinforced epoxy acts stiffer and becomes more brittle, which leads to failure at a lower deformation, particularly at higher strain rates. Furthermore, the strength is also found to be greater at higher deformation rates.

The data in Figure 4.16 and Table 4.4 suggests that both tensile strength and Young's modulus rise with increasing mass fraction of FS reinforcement, as well as with increasing strain rate, particularly at the highest mass fraction. It is also important to note that the elongation at break also increases with more FS particles present. The material is observed to behave in a more brittle way with an increasing strain rate.



(a) Unreinforced epoxy.

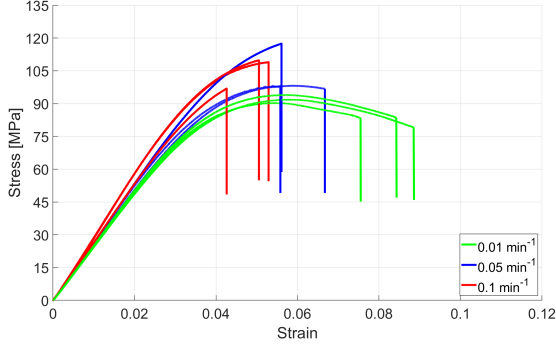
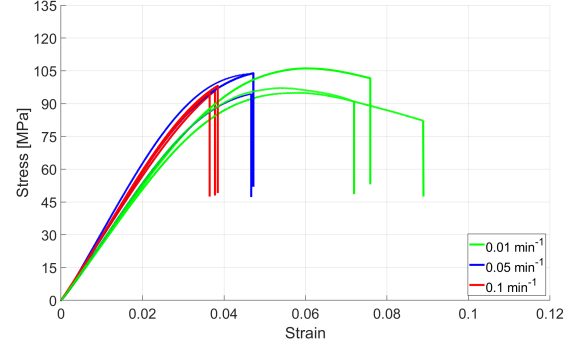
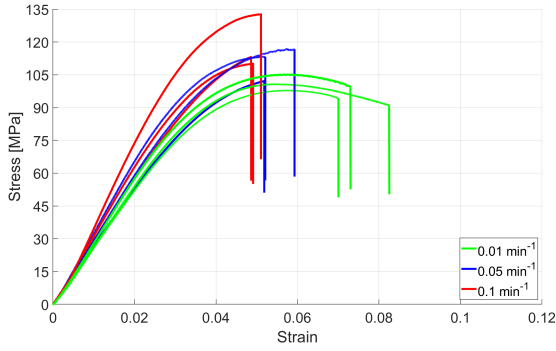
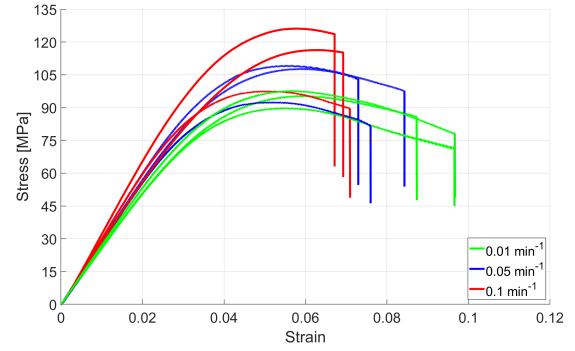
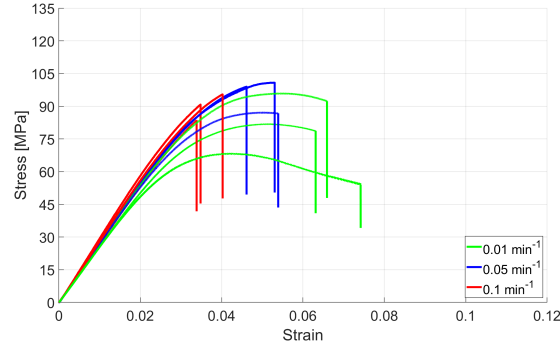
(b) $m_f = 0.5\%$ FS-reinforced epoxy.(c) $m_f = 1\%$ FS-reinforced epoxy.(d) $m_f = 1.5\%$ FS-reinforced epoxy.(e) $m_f = 2\%$ FS-reinforced epoxy.

Figure 4.16: Stress-strain curves of FS-reinforced epoxy.

Table 4.4: Averaged results of the three-point bend tests of epoxy reinforced with FS.

Modulus of Elasticity [MPa]	$m_f = 0$	$m_f = 0.5\%$	$m_f = 1\%$	$m_f = 1.5\%$	$m_f = 2\%$
0.01 min^{-1}	2447.3	2461.2	2544.8	2454.2	2511.0
0.05 min^{-1}	2491.6	2499.1	2496.6	2732.0	2660.7
0.1 min^{-1}	2561.2	2577.4	2737.4	2796.5	2856.5
Flexural Strength [MPa]	$m_f = 0$	$m_f = 0.5\%$	$m_f = 1\%$	$m_f = 1.5\%$	$m_f = 2\%$
0.01 min^{-1}	77.8	88.6	92.4	92.3	92.6
0.05 min^{-1}	94.8	93.9	96.2	102.3	99.3
0.1 min^{-1}	92.1	96.3	98.8	106.6	109.7
Elongation at Break	$m_f = 0$	$m_f = 0.5\%$	$m_f = 1\%$	$m_f = 1.5\%$	$m_f = 2\%$
0.01 min^{-1}	0.0823	0.0844	0.0878	0.0781	0.0927
0.05 min^{-1}	0.0635	0.0612	0.0515	0.0604	0.0821
0.1 min^{-1}	0.0396	0.0511	0.0406	0.0492	0.0697

The stress-strain curves of epoxy nanocomposites reinforced with HNT are presented in Figure 4.17. The extracted data from these curves are given in Table 4.5. The tensile strength and rigidity increase with the mass fraction of HNT. A similar trend is also noticed at higher rate deformations. The change in these parameters is more marked than for nanocomposites reinforced with FS. On the other hand, the elongation at break values drop with increasing strain rates. Meanwhile, the addition of HNT influences the elongation at break in a more complicated way. The elongation at break peaks when the composite includes $m_f = 0.5\%$ HNT and then drops slightly when more HNT is added. The elongation at break hits the minimum under the greatest strain rate and in the absence of HNT. Therefore, it appears as if the behaviour is controlled by HNT at lower mass fractions whereas with increasing amount of reinforcements, FS takes over dominance of the behaviour regarding elongation at break.



(a) Unreinforced epoxy.

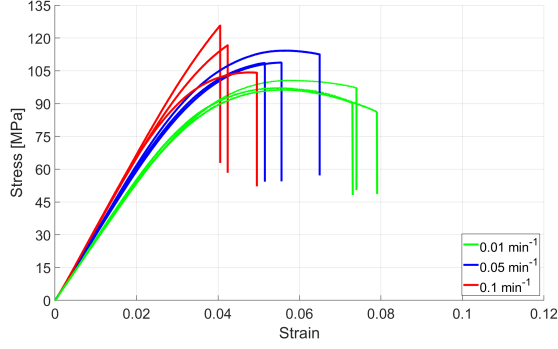
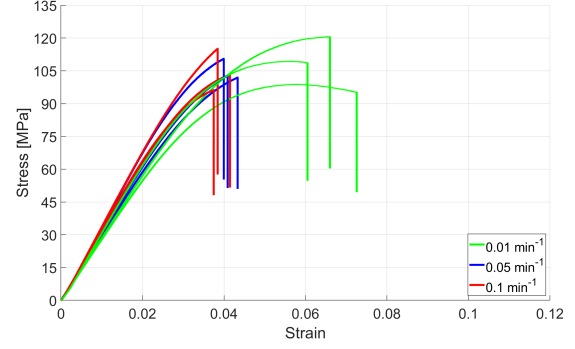
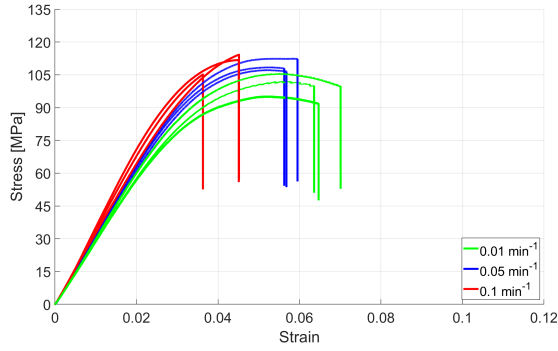
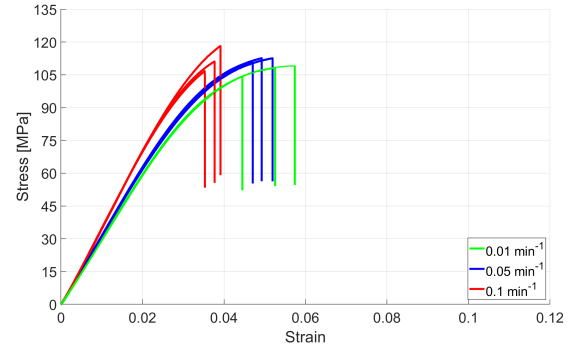
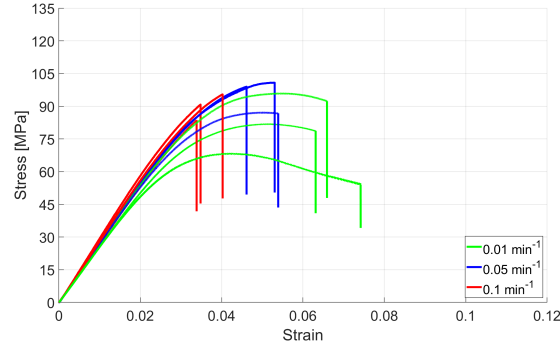
(b) $m_f = 0.5\%$ HNT-reinforced epoxy.(c) $m_f = 1\%$ HNT-reinforced epoxy.(d) $m_f = 1.5\%$ HNT-reinforced epoxy.(e) $m_f = 2\%$ HNT-reinforced epoxy.

Figure 4.17: Selected stress-strain curves of HNT-reinforced epoxy.

Table 4.5: Averaged results of the three-point bend tests of epoxy reinforced with HNT.

Modulus of Elasticity [MPa]	$m_f = 0$	$m_f = 0.5\%$	$m_f = 1\%$	$m_f = 1.5\%$	$m_f = 2\%$
0.01 min^{-1}	2447.3	2738.0	2760.5	2816.5	2826.5
0.05 min^{-1}	2491.6	2931.4	2944.4	2949.9	3068.4
0.1 min^{-1}	2561.2	3064.0	3006.3	3148.0	3211.2
Flexural Strength [MPa]	$m_f = 0$	$m_f = 0.5\%$	$m_f = 1\%$	$m_f = 1.5\%$	$m_f = 2\%$
0.01 min^{-1}	77.8	98.0	104.8	103.3	105.7
0.05 min^{-1}	94.8	106.6	102.8	107.1	109.2
0.1 min^{-1}	92.1	118.0	104.1	108.5	112.9
Elongation at Break	$m_f = 0$	$m_f = 0.5\%$	$m_f = 1\%$	$m_f = 1.5\%$	$m_f = 2\%$
0.01 min^{-1}	0.0823	0.0752	0.0666	0.0703	0.0529
0.05 min^{-1}	0.0563	0.0666	0.0412	0.0594	0.0504
0.1 min^{-1}	0.0396	0.0481	0.0384	0.0419	0.0387

Aiming to further investigate the synergistic effects of FS and HNT inclusions together, Figure 4.18 and Table 4.6 are presented, which show the stress-strain graphs of the epoxy-based nanocomposites that are reinforced with FS and HNT inclusions in equal masses and put through the experimental procedure with different strain rates. Based on the data, the composite is found to be stronger and stiffer with the growing amount of reinforcement along with rising strain rates. The findings reveal that the elongation at break falls with the pace of strain speeding up. These magnitudes of growth in both strength and stiffness are found mostly in between the results of the composite reinforced with FS and the one with HNT. While all these describe monotonic trends, the mass fraction of reinforcement-elongation at break has a more complex relation. Elongation at break reaches a local maxima at $m_f = 0.5\%$ mass fraction of the reinforcements, and then it drops at $m_f = 1.5\%$ reinforcement constant (RC), where it rises again. This behaviour can be understood by referring to the previously explained composites, which are reinforced with either FS or HNT. The peak at $m_f = 0.5\%$ is a characteristic of the composite reinforced with HNT, while the monotonically increasing trend is the composite type reinforced with FS. Therefore, one can claim that the behaviour is controlled by HNT at lower mass fractions whereas with increasing amount of reinforcements, FS takes over the domination of the behaviour regarding elongation at break.



(a) Unreinforced epoxy.

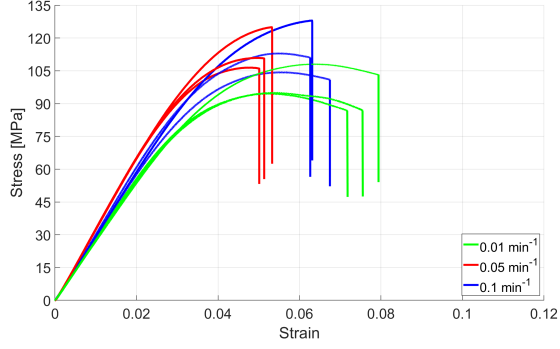
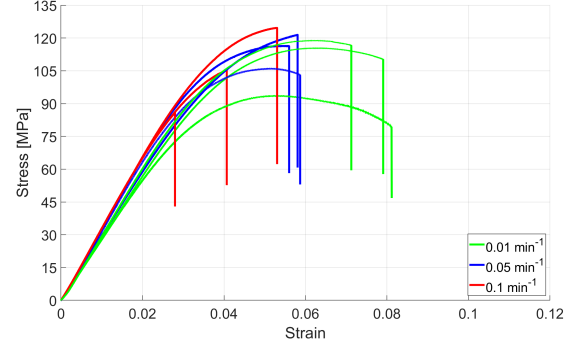
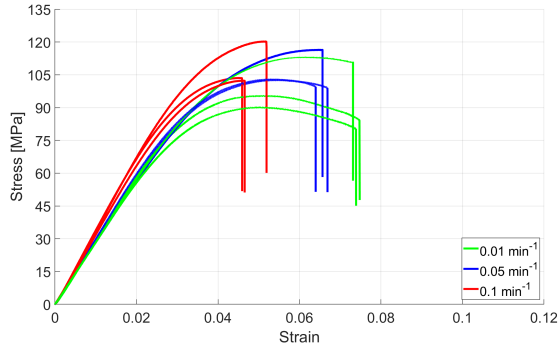
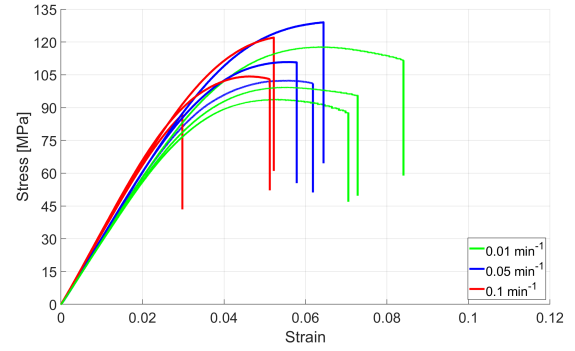
(b) $m_f = 0.5\%$ FS and HNT-reinforced epoxy.(c) $m_f = 1\%$ FS and HNT-reinforced epoxy.(d) $m_f = 1.5\%$ FS and HNT-reinforced epoxy.(e) $m_f = 2\%$ FS and HNT-reinforced epoxy.

Figure 4.18: Selected stress-strain curves of FS and HNT-reinforced epoxy.

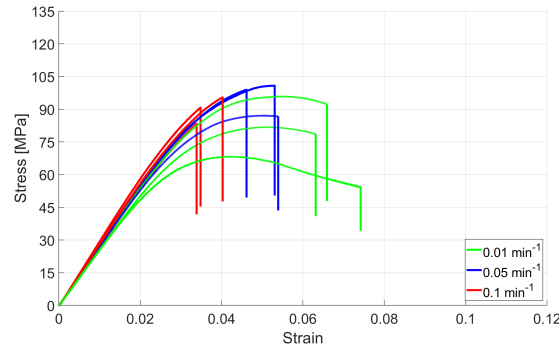
Table 4.6: Averaged results of the three-point bend tests of epoxy reinforced with FS and HNT.

Modulus of Elasticity [MPa]	$m_f = 0$	$m_f = 0.5\%$	$m_f = 1\%$	$m_f = 1.5\%$	$m_f = 2\%$
0.01 min^{-1}	2447.3	2694.4	2737.1	2806.6	2853.4
0.05 min^{-1}	2491.6	2839.2	2914.5	2857.7	2893.3
0.1 min^{-1}	2561.2	3036.5	3073.8	3128.9	3217.8
Flexural Strength [MPa]	$m_f = 0$	$m_f = 0.5\%$	$m_f = 1\%$	$m_f = 1.5\%$	$m_f = 2\%$
0.01 min^{-1}	77.8	97.1	107.4	94.9	106.7
0.05 min^{-1}	94.8	107.8	110.6	108.5	113.8
0.1 min^{-1}	92.1	111.4	108.9	110.9	103.2
Elongation at Break	$m_f = 0$	$m_f = 0.5\%$	$m_f = 1\%$	$m_f = 1.5\%$	$m_f = 2\%$
0.01 min^{-1}	0.0823	0.0745	0.0778	0.0748	0.0776
0.05 min^{-1}	0.0563	0.0632	0.0567	0.0682	0.0633
0.1 min^{-1}	0.0396	0.0531	0.0436	0.0509	0.0481

The stress-strain results of three-point bend tests conducted on samples made of epoxy reinforced with rubber (Albipox 1000) particles are presented in Figure 4.19. This material group differs significantly from others due to the relative stiffness of the reinforcing phase. In all other material groups, the reinforcing material is stiffer and stronger than the matrix material. The first major difference is that the strength alongside stiffness drop with increasing rubber particle content in the composite. Secondly, both strength and stiffness build up with increasing strain rates. Consequently, they reach the maximum values with unreinforced epoxy under 0.05 min^{-1} . Although the material becomes weaker and more compliant with more rubber particles, the ductility is significantly increased. As a result, elongation at break peaks at the most rubber content under 0.01 min^{-1} loading. The elongation at break values are above 0.10, which is greater compared to the composites reinforced with FS, which also display increasing ductility. It must be noted that the amounts of reinforcements used in this group of composites are significantly more than the mass of reinforcements used in composites with FS.

The stress-strain data of the next group of epoxy-based composites with a different type of nano-silica (Nanopox F700) is given in Figure 4.20. The compiled data from the stress-strain curves are presented in Table 4.8. The general tendency is that the material stiffens, and the ultimate flexural stress increases with rising deformation speeds and reinforcement presence which is similar to the behaviour observed in material groups reinforced with FS. The only difference to be noted is a slight drop in stiffness at $m_f = 5\%$ silica reinforcement. Also, the fall

of the elongation at break values with increasing strain rates exhibit similar behaviour to the composites with FS. However, moving on to the elongation at break, it becomes clear that a similar trend to the FS-reinforced epoxy is no longer evident. The values of elongation at break hit their minimum when the reinforcement is at $m_f = 5\%$ and then climbs up at $m_f = 10\%$ RC and then falls again at $m_f = 15\%$ silica content.



(a) Unreinforced epoxy.

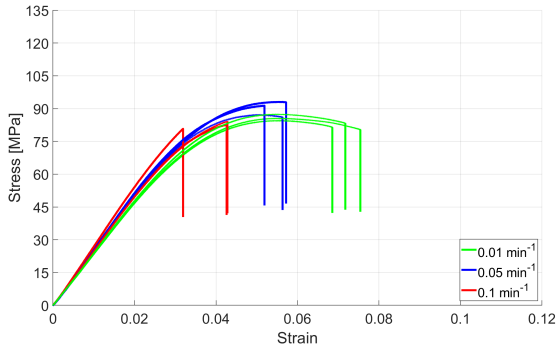
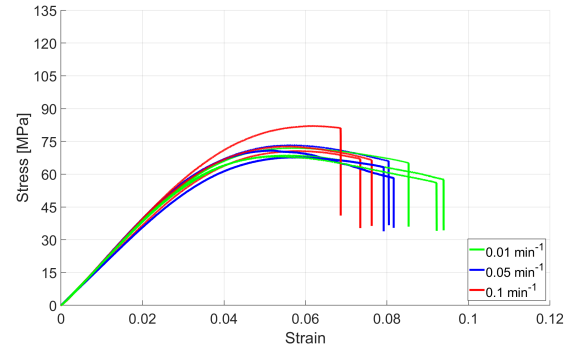
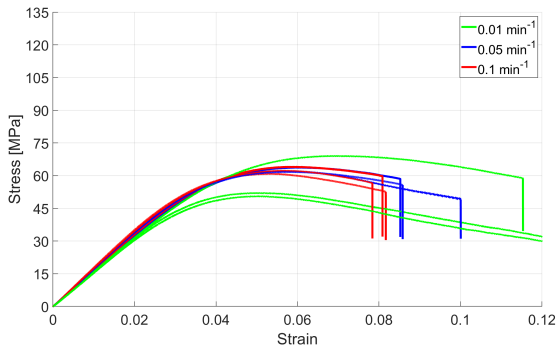
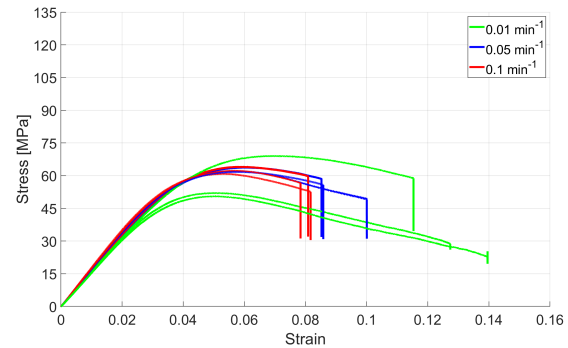
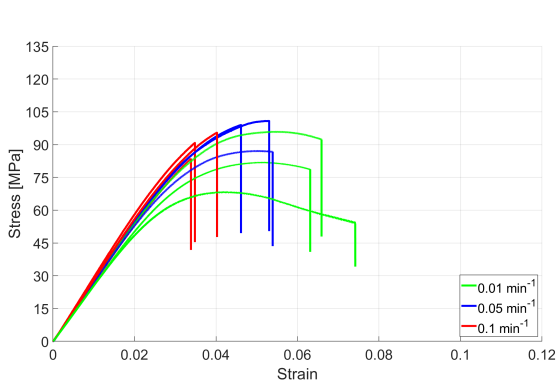
(b) $m_f = 5\%$ rubber (Albipox 1000)-reinforced epoxy.(c) $m_f = 10\%$ rubber (Albipox 1000)-reinforced epoxy.(d) $m_f = 15\%$ rubber (Albipox 1000)-reinforced epoxy (with limited axis).(e) $m_f = 15\%$ rubber (Albipox 1000)-reinforced epoxy.

Figure 4.19: Selected stress-strain curves of rubber (Albipox 1000)-reinforced epoxy.

Table 4.7: Averaged results of the three-point bend tests of epoxy reinforced with rubber (Albipox 1000).

Modulus of Elasticity [MPa]	$m_f = 0$	$m_f = 5\%$	$m_f = 10\%$	$m_f = 15\%$
0.01 min^{-1}	2447.3	2305.0	1823.9	1532.9
0.05 min^{-1}	2491.6	2349.0	1856.0	1657.2
0.1 min^{-1}	2561.2	2411.5	1871.6	1728.0
Flexural Strength [MPa]	$m_f = 0$	$m_f = 5\%$	$m_f = 10\%$	$m_f = 15\%$
0.01 min^{-1}	77.8	85.3	71.4	58.2
0.05 min^{-1}	94.8	87.1	73.1	61.1
0.1 min^{-1}	92.1	85.4	77.3	62.3
Elongation at Break	$m_f = 0$	$m_f = 5\%$	$m_f = 10\%$	$m_f = 15\%$
0.01 min^{-1}	0.0823	0.0721	0.0917	0.1271
0.05 min^{-1}	0.0563	0.0532	0.0812	0.0905
0.1 min^{-1}	0.0396	0.0388	0.0733	0.0814



(a) Unreinforced epoxy.

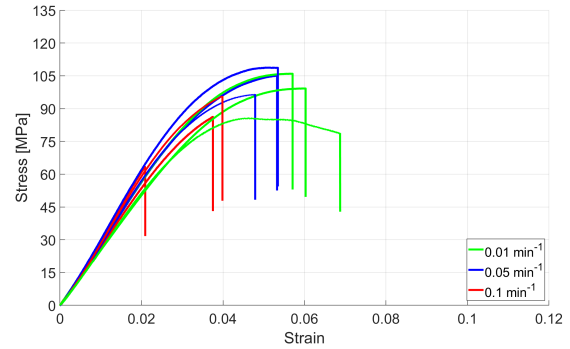
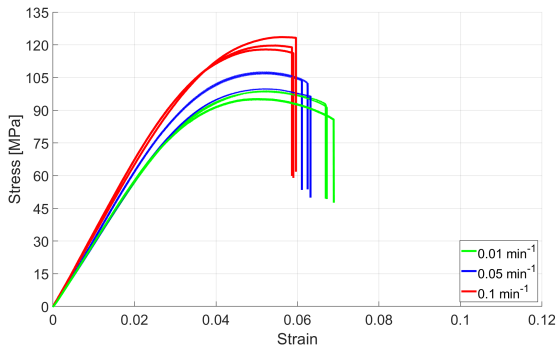
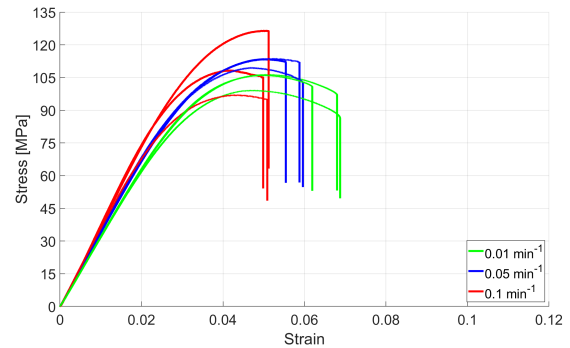
(b) $m_f = 5\%$ silica (Nanopox F700)-reinforced epoxy.(c) $m_f = 10\%$ silica (Nanopox F700)-reinforced epoxy.(d) $m_f = 15\%$ silica (Nanopox F700)-reinforced epoxy.

Figure 4.20: Selected stress-strain curves of silica (Nanopox F700)-reinforced epoxy.

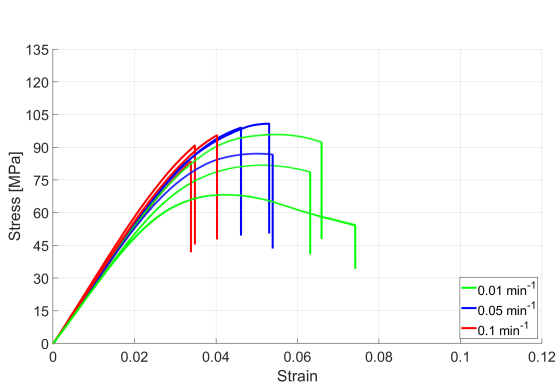
Table 4.8: Averaged results of the three-point bend tests of epoxy reinforced with silica (Nanopox F700).

Modulus of Elasticity [MPa]	$m_f = 0$	$m_f = 5\%$	$m_f = 10\%$	$m_f = 15\%$
0.01 min^{-1}	2447.3	2399.2	2761.0	3122.1
0.05 min^{-1}	2491.6	2470.3	2838.5	3357.1
0.1 min^{-1}	2561.2	2788.6	3286.6	3428.0
Flexural Strength [MPa]	$m_f = 0$	$m_f = 5\%$	$m_f = 10\%$	$m_f = 15\%$
0.01 min^{-1}	77.8	96.4	94.2	100.7
0.05 min^{-1}	94.8	100.6	103.2	109.8
0.1 min^{-1}	92.1	84.2	121.1	110.6
Elongation at Break	$m_f = 0$	$m_f = 5\%$	$m_f = 10\%$	$m_f = 15\%$
0.01 min^{-1}	0.0823	0.0543	0.0688	0.0671
0.05 min^{-1}	0.0563	0.047637	0.0634	0.0578
0.1 min^{-1}	0.0396	0.041137	0.0588	0.0527

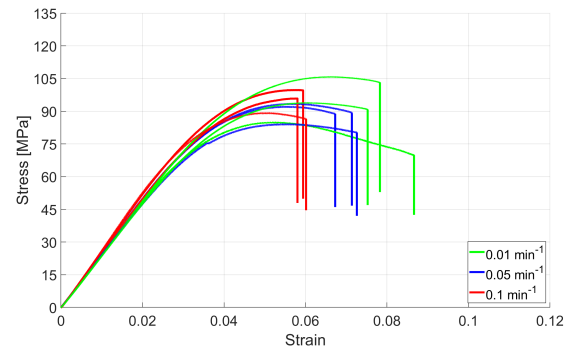
The results of the final set of materials, epoxy-reinforced with rubber and silica (Albipox 1000 and Nanopox F700) are shown in Figure 4.21 and Table 4.9. These are the stress-strain curves obtained from the three-point bend experiments. As the rest of the material groups, this one also displays the same behaviour with increasing strain rate. Specifically, the material's strength and stiffness values rise, whereas the elongation at break falls with faster deformations. However, the addition of reinforcements influences material behaviour in a more complicated manner. The stiffness declines and hits its minimum at $m_f = 5\%$ reinforcement and under 0.01 min^{-1} loading. Adding more reinforcements to the composite, the stiffness starts to ascend slightly, though, the flexural strength shows an entirely different characteristic. It peaks under 0.1 min^{-1} and decays with lowering strain rates. However, the RC does not significantly affect the materials' strength. Meanwhile, elongation at break has its peak where the loading is 0.01 min^{-1} , and the reinforcement mass fraction is $m_f = 10\%$. This complex behaviour of the three mechanical properties can be explained by the synergistic effect of the two reinforcing particles having opposite characteristics and effects. For instance, the stiffness drops at lower reinforcement masses with increasing particle content. This is parallel to what is observed in composites with Albipox 1000. With more particles in the material, Young's modulus recovers after reaching the trough, suggesting that the silica influence overcomes rubber dominance. A similar explanation can be given for the trends in elongation at break. The elongation at break values of composite mixed with Nanopox F700 reach a maximum at $m_f = 10\%$ RC in

terms of mass which is followed by a fall with increasing reinforcements. Whilst the composite that consists of Albipox 1000 exhibits an increasing trend against the rubber content. Hence, coming together, these two particles lead to the characteristic described reaching its maximum at the RC of $m_f = 10\%$.

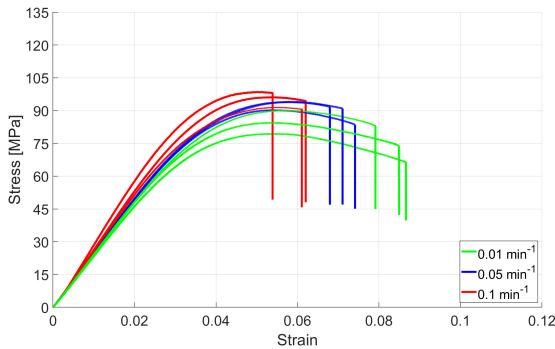
The results presented in this section so far are found to be in good agreement to the previous research. The results in the literature also suggest that the material behaves stiffer and the elongation at break values drop with increasing strain rates [267, 101]. Also, the changes of the stiffnesses and the ductilities of the composites with the addition of silica and HNT can be found [163, 181, 31].



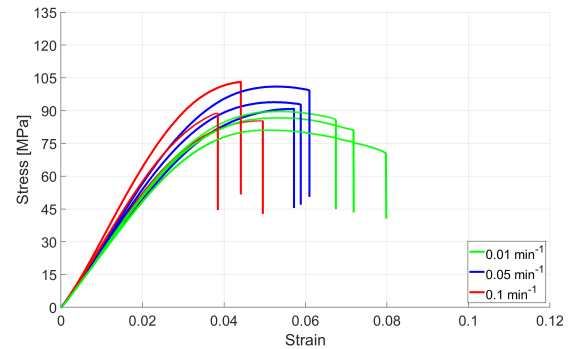
(a) Unreinforced epoxy.



(b) $m_f = 5\%$ rubber and silica (Albipox 1000 and Nanopox F700)-reinforced epoxy.



(c) $m_f = 10\%$ rubber and silica (Albipox 1000 and Nanopox F700)-reinforced epoxy.



(d) $m_f = 15\%$ rubber and silica (Albipox 1000 and Nanopox F700)-reinforced epoxy.

Figure 4.21: Stress-strain curves of rubber and silica (Albipox 1000 and Nanopox F700) reinforced epoxy.

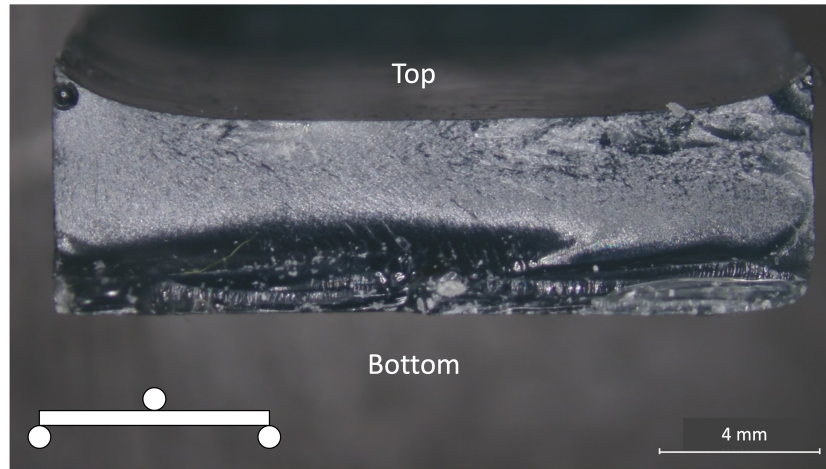
Table 4.9: Averaged results of the three-point bend tests of epoxy reinforced with rubber (Albipox 1000) and silica (Nanopox F700).

Modulus of Elasticity [MPa]	$m_f = 0$	$m_f = 5\%$	$m_f = 10\%$	$m_f = 15\%$
0.01 min^{-1}	2447.3	2344.0	2354.8	2362.0
0.05 min^{-1}	2491.6	2379.0	2439.5	2417.5
0.1 min^{-1}	2561.2	2442.0	2502.0	2447.5
Flexural Strength [MPa]	$m_f = 0$	$m_f = 5\%$	$m_f = 10\%$	$m_f = 15\%$
0.01 min^{-1}	77.8	93.3	80.2	80.9
0.05 min^{-1}	94.8	89.2	88.9	92.1
0.1 min^{-1}	92.1	92.1	92.6	90.7
Elongation at Break	$m_f = 0$	$m_f = 5\%$	$m_f = 10\%$	$m_f = 15\%$
0.01 min^{-1}	0.0823	0.0811	0.0858	0.0747
0.05 min^{-1}	0.0563	0.0723	0.0727	0.0579
0.1 min^{-1}	0.0396	0.0598	0.0594	0.0435

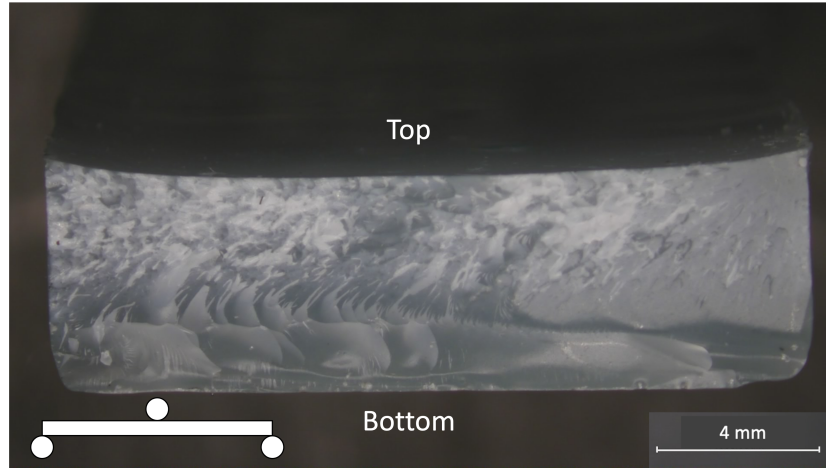
Overall, the findings of the three-point bend tests with different strain rates are found to be compatible with the literature. With more rigid particles like silica and HNT, it is expected that the shear band yielding and plastic void growth due to debonding of the particles are the primary toughening mechanisms [40, 47, 102]. In addition, stiffer and stronger particles contribute to the effective stiffness of the material, but due to the debonding, the contribution of the particles to the strength of the material remains limited. Whereas the presence of the rubber particles lead to the activation of a different toughening mechanism due to better interface bonding properties with the epoxy matrix and lower stiffness. Cavitation initiates within the rubber particle, and the interface debonding is usually not observed [104, 181]. Thus, plastic void growth initiating in the particles is expected to be the dominant toughening mechanism alongside the localised plastic shear bands. Apart from these, the effects of the strain rate also show good agreement with the literature. A decline of ductility with rising strain rates is observed, similar to the research available in the literature [2].

Finally, the fracture surfaces of one sample from each material group with different reinforcement types displaying opposite characteristics in terms of ductility and stiffness are shown in Figure 4.22. $m_f = 1\%$ HNT-reinforced (Figure) 4.22a and $m_f = 10\%$ rubber-added (Albipox 1000) 4.22b epoxy samples are selected since HNT increases the stiffness and reduces the ductility and rubber has the opposite effect. The images show that the fracture surface of the brittle material (HNT-reinforced epoxy) is brighter and more reflective and the cross-section is

less deformed/warped than the ductile material which is a consequence of the high stiffness of the HNT particles that contribute to the effective stiffness of the composite.



(a) Fracture surface of the three-point bend sample reinforced with HNT.



(b) Fracture surface of the three-point bend sample reinforced with rubber (Albipox 1000).

Figure 4.22: Fracture surfaces of the selected three-point bend samples.

4.3.4 Dynamic Mechanical Analysis

The results of the DMA tests are presented in this section. First of all, to perform an initial examination on the gathered data, a stress-strain plot, which shows the repetitive cycles of loading and unloading, is discussed. The stress-strain plot, which reveals the hysteresis curves acquired during the experimentation of $m_f = 5\%$ silica-reinforced (Nanopox F700) sample under 1 Hz, is presented in Figure 4.23. The data shows that the stress-strain plot of the

nanocomposite material displays a very slight curvature, which forms a cycle and points out a noticeable hysteresis at this excitation frequency and in this range of strain amplitude. It is important to emphasise that the hysteresis is mainly caused by the material's inherent damping and not the frictional effects introduced by the clamps as discussed in Chapter 3. In frictional cases, it is expected that the curves have lower slopes or even flat tops and bottoms towards the both ends of the motion. In the curves that are plotted using the data obtained from the experiments, such a change in slope is not observed. This is a sign showing, that the frictional damping either does not exist or is negligible. This leads to the conclusion of the energy dissipation by friction can be ignored. Besides, it is to be stated that the curves obtained from the experiment have very sharp forms of a typical viscoelastic solid, which means the damping properties are not as high as the materials that are used to illustrate viscoelastic damping such as neoprene rubber and polymethylmethacrylate [54]. The main reason for this can be explained as the temperature effect on the polymer. The temperature that these experiments are conducted at is in the glassy region of the epoxy polymer, where the viscoelastic properties are not as dominant as they are in the rubbery region.

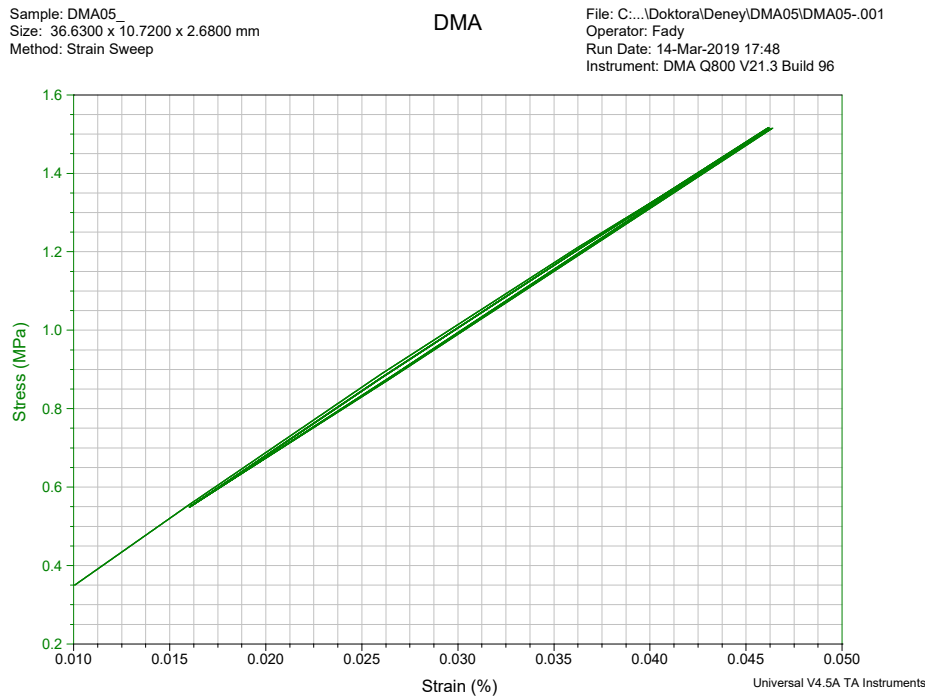
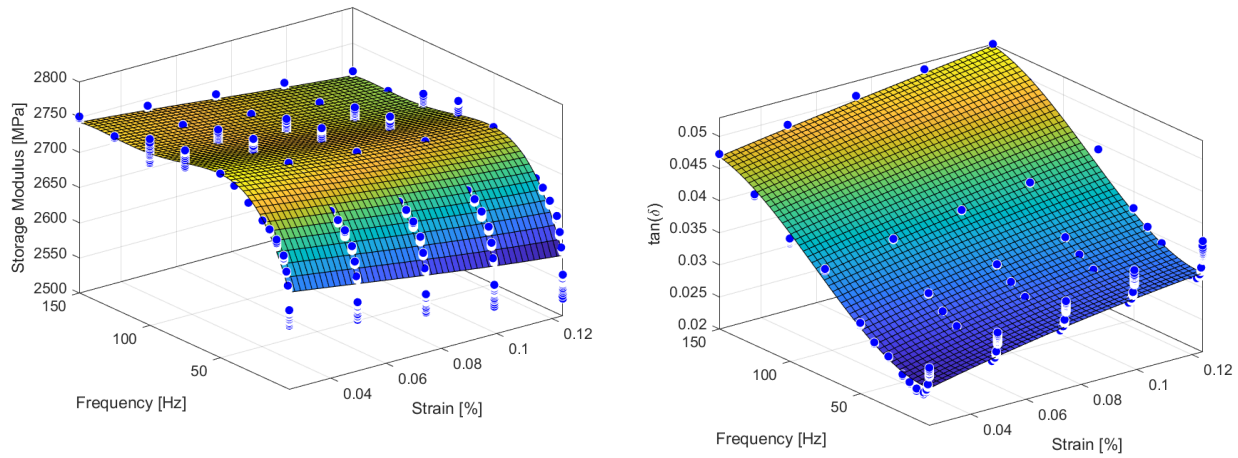


Figure 4.23: Hysteresis curves for $m_f = 5\%$ silica-reinforced (Nanopox F700) at 1 Hz.

The results of DMA tests are plotted in Figures 4.24-4.30 which display considerable nonlinear

behaviours of all the material groups.

Figure 4.24 shows the data acquired from the tests performed on the unreinforced epoxy samples. The variation of storage modulus of epoxy is found to be around 10% within the investigated range of amplitude and frequency. The storage modulus, however, is not as sensitive to the strain amplitude of the vibration as it is to the excitation frequency. Increasing amplitudes lead to a slight drop in the storage modulus values, while with increasing frequency, the storage modulus rises, showing a saturation characteristic. On the other hand, the damping is affected more dramatically. The loss factor almost doubles as the strain amplitude and frequency increase. The loss factor of epoxy changes similarly with both of the parameters, which is a result of the viscoelasticity of the material. The variation of both of the material properties clearly reveal the nonlinear nature of the polymer. Increasing the excitation frequency and/or amplitude linearly increases the loss factor. Also, the excitation frequency has a slightly more significant effect on the loss factor compared to the strain amplitude.



(a) Variation of storage modulus of unreinforced epoxy.

(b) Variation of loss factor of unreinforced epoxy.

Figure 4.24: Variation of dynamic mechanical properties of unreinforced epoxy with strain amplitude and frequency.

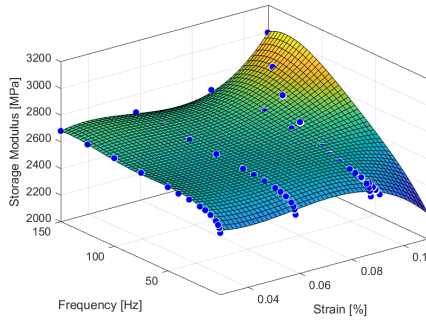
The results of the DMA tests conducted on FS-reinforced epoxy nanocomposites are presented in Figure 4.25. The results suggest that the addition of FS up to $m_f = 1\%$ increases the storage modulus and damping capabilities noticeably. The addition of 0.5% mass of silica stiffened the material approximately by 5%, whereas it increased the damping capabilities by up to 20% compared to unreinforced epoxy. Both properties reach their peaks with the presence

of $m_f = 1\%$ FS. This increase in the storage modulus can be explained by the stiffening effect of the particles, while the damping is mainly related to the effect of the interfaces created between the matrix and the nanoparticles as well as the local plastic deformations in the matrix around the reinforcing particles which are caused by the triaxial stress concentrations. Even though there is a slight drop in the storage modulus and the loss factor with an increasing amount of FS in the epoxy resin, they still have enhanced storage moduli and loss factors compared to the unreinforced epoxy. The drop in the storage modulus can be explained by the increased number of stress concentration locations that display plastic deformation to hinder the capacity of storing elastic potential energy. It is also possible to state that the viscoelastic properties are influenced by the same region, which leads to a drop in the loss modulus. Overall, for the FS-reinforced epoxy nanocomposites, the storage modulus shows a similar trend by changing approximately 10 – 15% with respect to amplitude and frequency of excitation, whereas the damping is affected by around 45%.

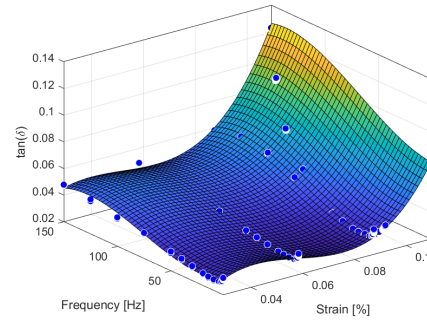
Figure 4.26 shows the results of the DMA tests performed on HNT-reinforced epoxy samples. Based on the results, it can be claimed that the presence of HNT increases the storage modulus and the damping capability of the epoxy matrix. These two properties are best improved when the mass fraction of HNT is at $m_f = 1\%$. Knowing that HNT particles are stiffer than FS, it is an expected outcome that the storage modulus rises by larger margins. Similar to FS-reinforced epoxy samples, the addition of particles in larger amounts, both mechanical properties display relatively minor drops. Considering the sharp tips of the HNT particles, it is possible to predict that these sharp tips lead to greater stress concentrations. Thus, the material's capability of storing the elastic potential energy is hindered by the localised plastic deformations. In the presence of more particles, these plastic deformations overcome the stiffening effect of the particles and reduce the storage modulus. Similar to the FS-reinforced samples, the plastic deformations influence the viscoelastic properties to the fall in the loss modulus values is observed. Moreover, the strain amplitude of vibration has a limited effect on the storage modulus. In all HNT-reinforced samples, a slight drop of the storage modulus is observed against increasing strain amplitudes. This fall is more significant compared to the FS-reinforced epoxy samples due to the increased magnitude of localised plastic deformations

in the matrix. Similar to the composites reinforced with FS, frequency is more influential on the storage modulus with quite parallel trends. Whereas, this is not the case for the loss modulus. Strain amplitude leads to a rapid increase in the damping capabilities of the material after approximately 0.1% strain amplitude. However, in the measured frequency range, no such rapid change is observed.

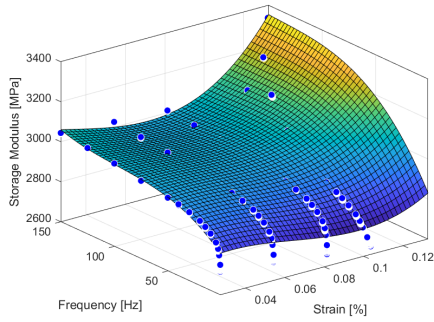
The measured dynamic mechanical properties of nanocomposites reinforced with FS and HNT in equal masses are shown in Figure 4.27. The data reveals the synergistic effects of the co-existence of these two types of particles in the epoxy matrix. Adding only $m_f = 0.5\%$ of particles into the matrix increases the storage modulus radically. However, further additions do not bring significant marginal stiffening benefits to the composite. The almost-linear drop of the storage modulus with increasing strain amplitudes and the saturation characteristics of the storage modulus with rising frequencies are also observed in these samples, as seen in FS-reinforced and HNT-reinforced composites. When it comes to the damping characteristics, a more moderate behaviour is seen. Both frequency and amplitude have almost a monotonically increasing linear correlation to the loss modulus. Also, it needs to be pointed out that the sudden increase in the loss modulus after 0.1% strain is not noticeable.



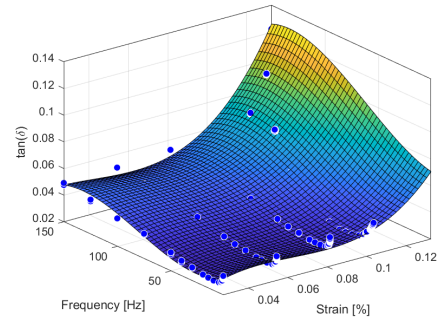
(a) Variation of storage modulus of $m_f = 0.5\%$ FS-reinforced epoxy.



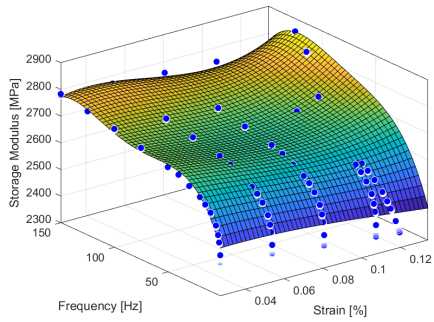
(b) Variation of loss factor of $m_f = 0.5\%$ FS-reinforced epoxy.



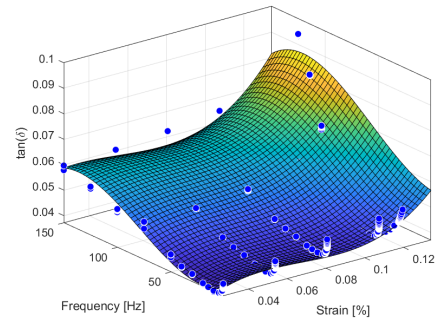
(c) Variation of storage modulus of $m_f = 1\%$ FS-reinforced epoxy.



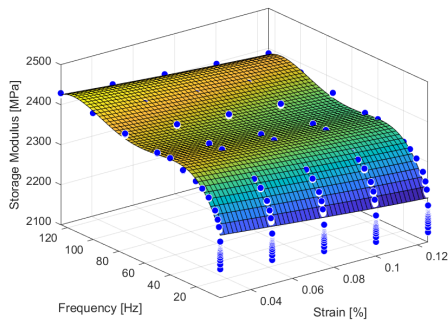
(d) Variation of loss factor of $m_f = 1\%$ FS-reinforced epoxy.



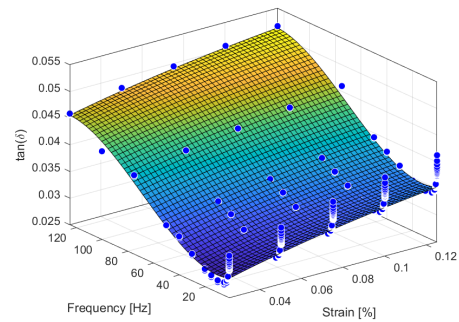
(e) Variation of storage modulus of $m_f = 1.5\%$ FS-reinforced epoxy.



(f) Variation of loss factor of $m_f = 1.5\%$ FS-reinforced epoxy.

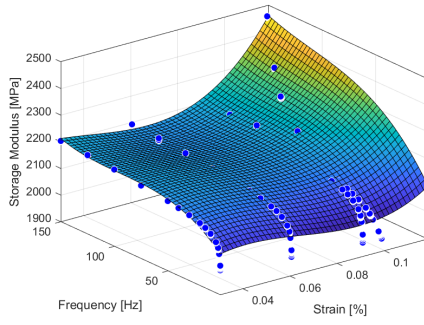


(g) Variation of storage modulus of $m_f = 2\%$ FS-reinforced epoxy.

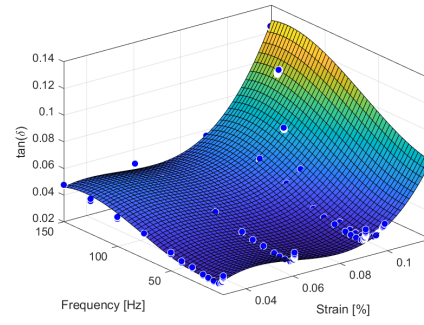


(h) Variation of loss factor of $m_f = 2\%$ FS-reinforced epoxy.

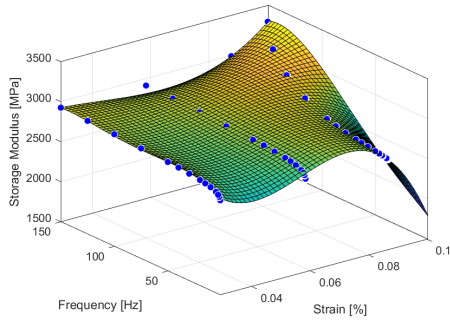
Figure 4.25: Variation of dynamic mechanical properties of FS-reinforced epoxy with strain amplitude and frequency.



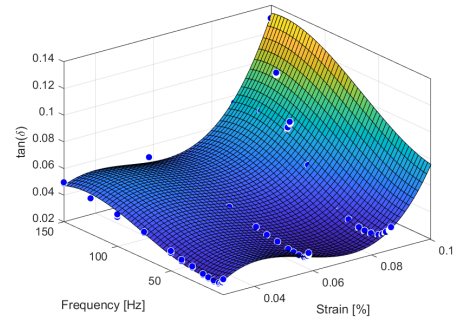
(a) Variation of storage modulus of $m_f = 0.5\%$ HNT-reinforced epoxy.



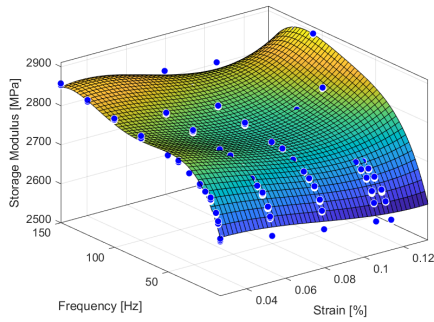
(b) Variation of loss factor of $m_f = 0.5\%$ HNT-reinforced epoxy.



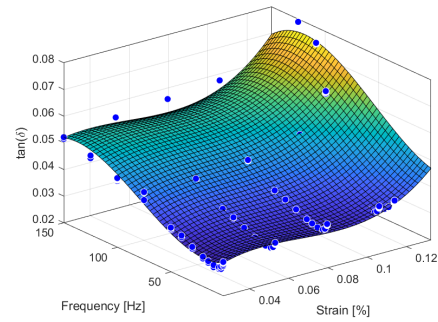
(c) Variation of storage modulus of $m_f = 1\%$ HNT-reinforced epoxy.



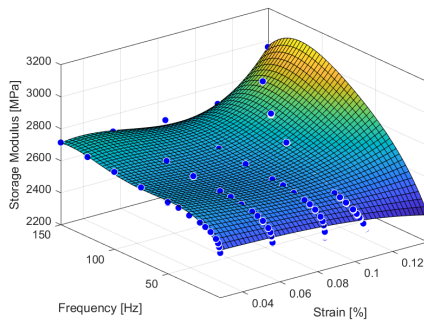
(d) Variation of loss factor of $m_f = 1\%$ HNT-reinforced epoxy.



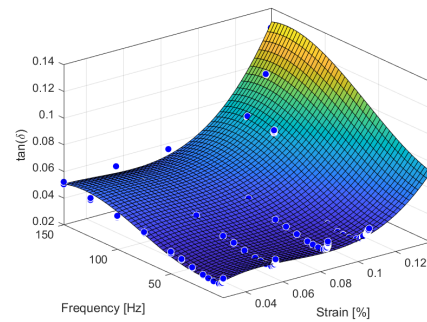
(e) Variation of storage modulus of $m_f = 1.5\%$ HNT-reinforced epoxy.



(f) Variation of loss factor of $m_f = 1.5\%$ HNT-reinforced epoxy.

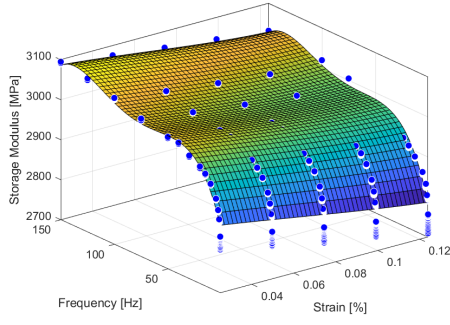


(g) Variation of storage modulus of $m_f = 2\%$ HNT-reinforced epoxy.

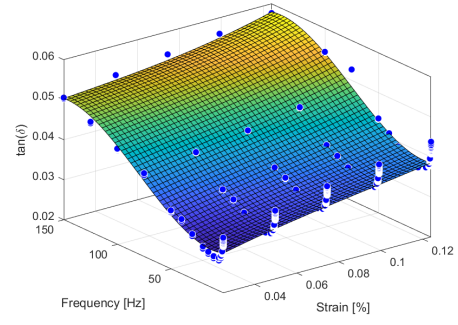


(h) Variation of loss factor of $m_f = 2\%$ HNT-reinforced epoxy.

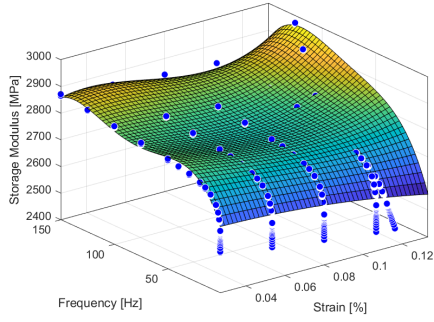
Figure 4.26: Variation of dynamic mechanical properties of HNT-reinforced epoxy with strain amplitude and frequency.



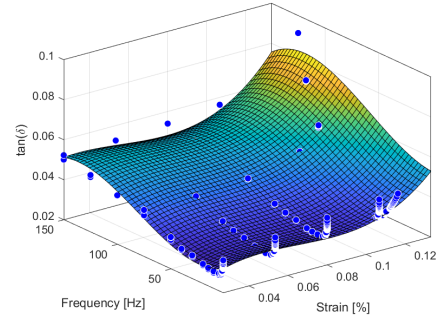
(a) Variation of storage modulus of $m_f = 0.5\%$ FS and HNT-reinforced epoxy.



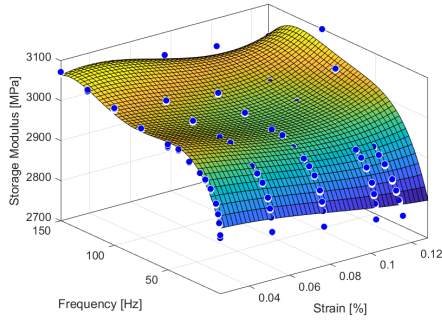
(b) Variation of loss factor of $m_f = 0.5\%$ FS and HNT-reinforced epoxy.



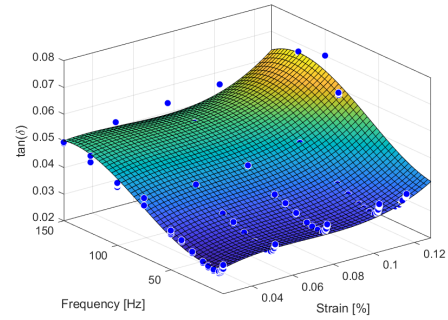
(c) Variation of storage modulus of $m_f = 1\%$ FS and HNT-reinforced epoxy.



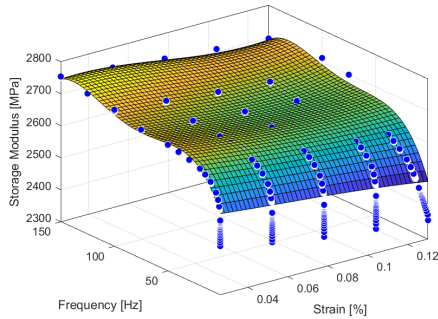
(d) Variation of loss factor of $m_f = 1\%$ FS and HNT-reinforced epoxy.



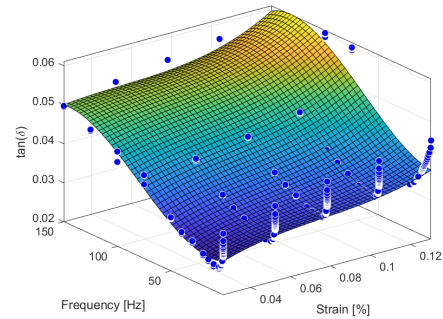
(e) Variation of storage modulus of $m_f = 1.5\%$ FS and HNT-reinforced epoxy.



(f) Variation of loss factor of $m_f = 1.5\%$ FS and HNT-reinforced epoxy.



(g) Variation of storage modulus of $m_f = 2\%$ FS and HNT-reinforced epoxy.



(h) Variation of loss factor of $m_f = 2\%$ FS and HNT-reinforced epoxy.

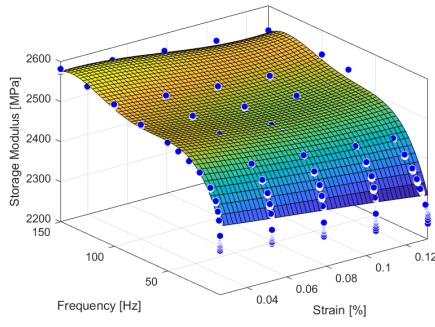
Figure 4.27: Variation of dynamic mechanical properties of FS and HNT-reinforced epoxy with strain amplitude and frequency.

The data acquired from the DMA tests of epoxy-based composites containing rubber particles is visualised in Figure 4.28. Unlike any other composites presented within the scope of this study, this composite group displays a significant drop in storage modulus, which is related to the lower stiffness of added rubber particles compared to the epoxy matrix. On the other hand, the general trend of the storage modulus of each sample exhibits the general characteristics that other material groups also display. The storage modulus falls slightly with increasing strain amplitudes, and it increases in a saturating manner with increasing frequency, but the change is more significant compared to the strain amplitudes. Meanwhile, the case with the damping of these material groups is slightly more complicated. While the samples with $m_f = 5\%$ and $m_f = 15\%$ rubber particle content have almost a linearly increasing correlation to increasing strain amplitudes of vibration excitation as well as frequency, the sample with $m_f = 10\%$ RC shows the jump to reach the peak damping capacity, as seen in the previous materials, of loss modulus around 0.1% strain amplitude. Also, it is fair to say that the damping capacity is lost significantly with the addition of $m_f = 15\%$ rubber particles. Thus, adding 5% more rubber to $m_f = 10\%$ does not help improve the damping capabilities of the composites and also reduces the storage modulus even more.

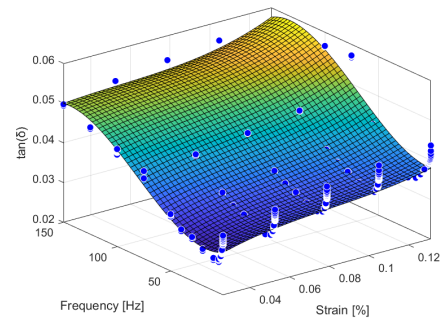
The results of the DMA tests of silica (Nanopox F7000)-reinforced are given in Figure 4.29. The results show that the storage moduli of each sample possess the same trends and characteristics seen in the previous material groups with increasing strain amplitude and frequency. The storage modulus increases and converges to certain values when the vibration frequency builds up, and climbing strain amplitudes leads to some small reductions of the storage moduli of the composites investigated. Overall, the storage modulus values of these silica-reinforced composites are greater compared to the rest of the groups of composites. Considering the amount of silica added to the epoxy matrix and the stiffness of these silica particles, this is a naturally predictable outcome. Besides, a more noticeable rise of the storage modulus is visible for vibrational motions with larger frequencies and strain amplitudes. Likewise, the loss factors for the tested samples reach their peaks under excitations with larger amplitudes and higher frequencies. The loss factors of these samples remain relatively small unless both of the parameters of amplitude and frequency are not simultaneously closer to their upper

boundaries of their ranges of interest. The reason for this behaviour can be explained by the nonlinear viscoelastic (strain-rate-dependent) nature of the nanocomposite material which is mainly caused by the interphase between the particles and the matrix. Nevertheless, the loss factor generally climbs mildly with rising strain amplitude and frequency. The storage modulus as well as the loss factor, reach their maximum in the presence of $m_f = 5\%$ silica. This peak is also the maximum among the entire material samples investigated within the scope of this study.

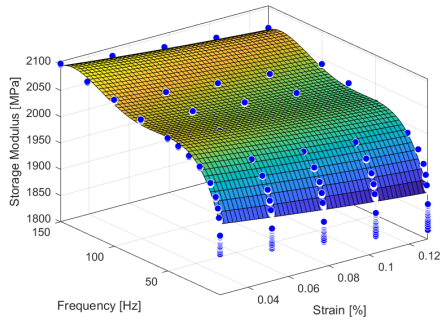
The final set of DMA results that belong to epoxy reinforced with rubber (Albipox 1000) and silica (Nanopox F700) in equal masses are presented in Figure 4.30. The material group shows moderate behaviour, which lies between only silica (Nanopox F700) and only rubber (Albipox 1000)-reinforced composites. The storage modulus declines slightly with increasing vibration amplitude and rises and converges with rising frequencies. Whilst the loss factor is influenced by frequency and strain amplitude in a very similar manner as silica-reinforced epoxies are affected in terms of both the level and trends. The storage modulus almost remains the same up until the RC exceeds $m_f = 10\%$, and then a small fall is observed when the silica content is $m_f = 15\%$. Unlike the storage modulus, the loss factor reaches its maximum when the RC is only $m_f = 5\%$, and as the RC is increased, the loss modulus gradually declines by a small margin while preserving the general trends with respect to the changes in frequency and amplitude.



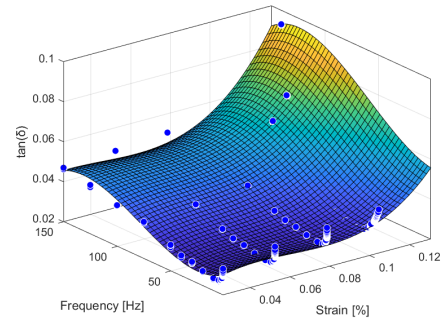
(a) Variation of storage modulus of $m_f = 5\%$ rubber (Albipox 1000)-reinforced epoxy.



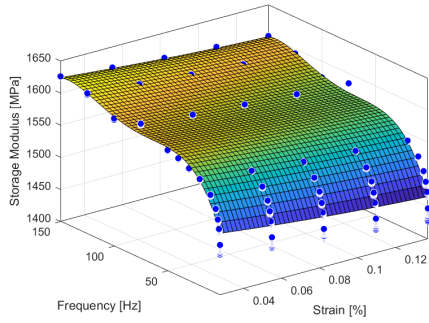
(b) Variation of loss factor of $m_f = 5\%$ rubber (Albipox 1000)-reinforced epoxy.



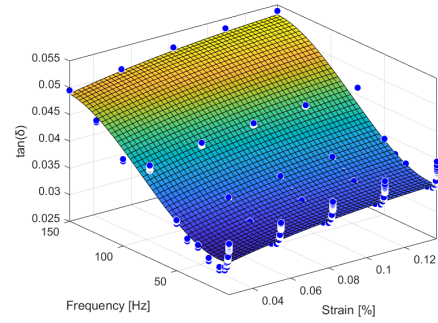
(c) Variation of storage modulus of $m_f = 10\%$ rubber (Albipox 1000)-reinforced epoxy.



(d) Variation of loss factor of $m_f = 10\%$ rubber (Albipox 1000)-reinforced epoxy.

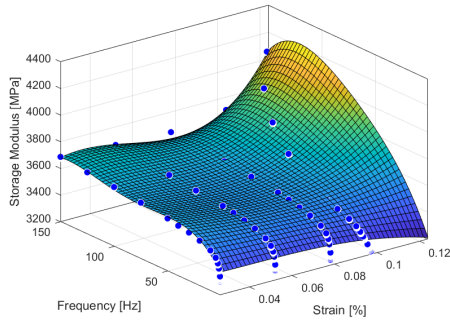


(e) Variation of storage modulus of $m_f = 15\%$ rubber (Albipox 1000)-reinforced epoxy.

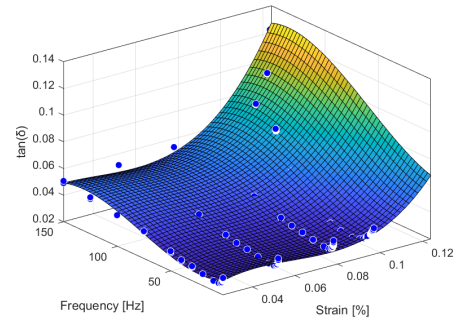


(f) Variation of loss factor of $m_f = 15\%$ rubber (Albipox 1000)-reinforced epoxy.

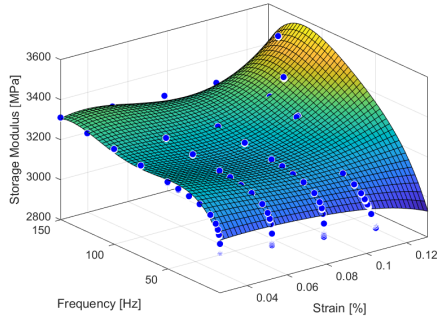
Figure 4.28: Variation of dynamic mechanical properties of rubber (Albipox 1000)-reinforced epoxy with strain amplitude and frequency.



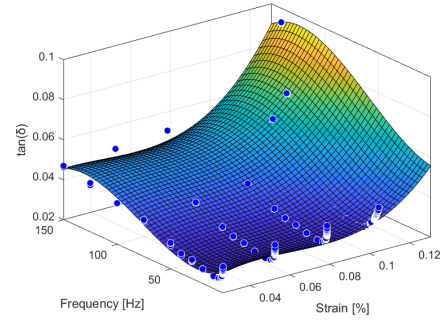
(a) Variation of storage modulus of $m_f = 5\%$ silica (Nanopox F700)-reinforced epoxy.



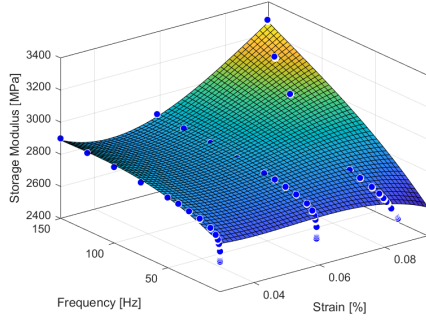
(b) Variation of loss factor of $m_f = 5\%$ silica (Nanopox F700)-reinforced epoxy.



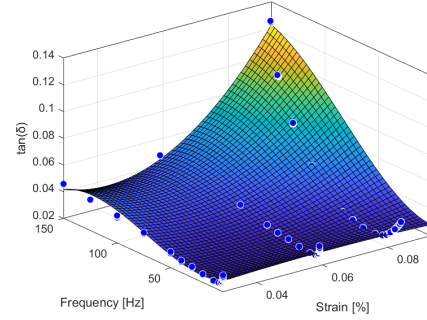
(c) Variation of storage modulus of $m_f = 10\%$ silica (Nanopox F700)-reinforced epoxy.



(d) Variation of loss factor of $m_f = 10\%$ silica (Nanopox F700)-reinforced epoxy.

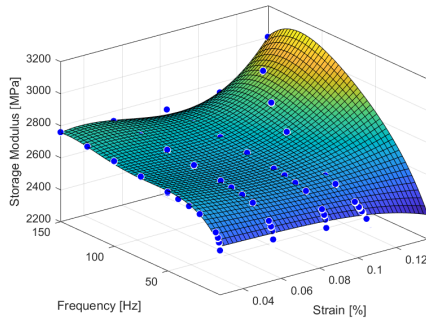


(e) Variation of storage modulus of $m_f = 15\%$ silica (Nanopox F700)-reinforced epoxy.

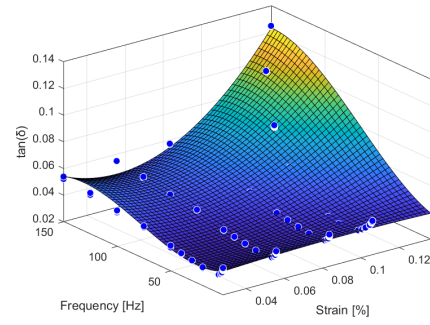


(f) Variation of loss factor of $m_f = 15\%$ silica (Nanopox F700)-reinforced epoxy.

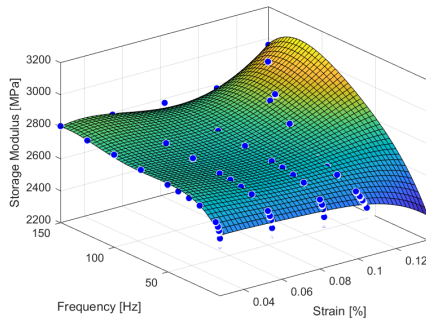
Figure 4.29: Variation of dynamic mechanical properties of silica (Nanopox F700)-reinforced epoxy with strain amplitude and frequency.



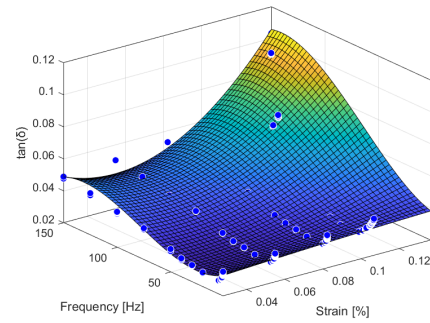
(a) Variation of storage modulus of $m_f = 5\%$ rubber (Albipox 1000) and silica (Nanopox F700)-reinforced epoxy.



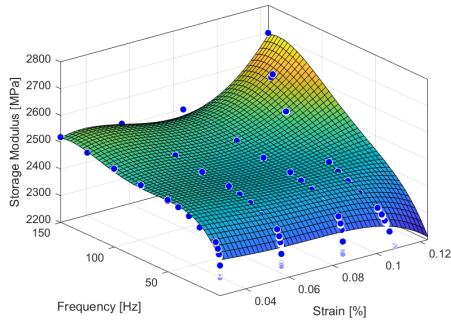
(b) Variation of loss factor of $m_f = 5\%$ rubber (Albipox 1000) and silica (Nanopox F700)-reinforced epoxy.



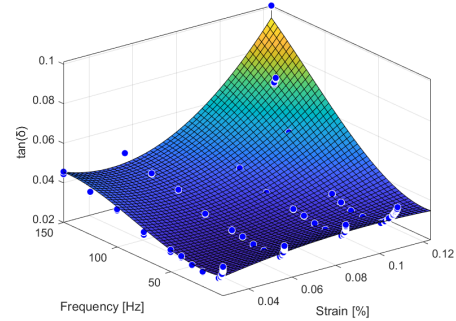
(c) Variation of storage modulus of $m_f = 10\%$ rubber (Albipox 1000) and silica (Nanopox F700)-reinforced epoxy.



(d) Variation of loss factor of $m_f = 10\%$ rubber (Albipox 1000) and silica (Nanopox F700)-reinforced epoxy.



(e) Variation of storage modulus of $m_f = 15\%$ rubber (Albipox 1000) and silica (Nanopox F700)-reinforced epoxy.



(f) Variation of loss factor of $m_f = 15\%$ rubber (Albipox 1000) and silica (Nanopox F700)-reinforced epoxy.

Figure 4.30: Variation of dynamic mechanical properties of rubber (Albipox 1000) and silica (Nanopox F700)-reinforced epoxy with strain amplitude and frequency.

4.3.5 Charpy Impact Tests

After investigating the strain rate-dependent mechanics of the epoxy and epoxy-based composites, the impact behaviour of those materials is also investigated, and the results are presented

in this section. The measured fracture energy for FS and HNT-reinforced epoxy is shown in Figure 4.31. The data in Figure 4.31 suggests that the addition of FS, even in small amounts, increases the fracture energy dramatically. It is observed that adding more FS into epoxy resin makes the fracture energy rise almost in a linear manner. On the other hand, HNT-doped epoxy shows a distinctively different characteristic. It can be claimed that the presence of HNT slightly increases the fracture energy, but the trend is not linear between the mass of HNT reinforcement and the resulting fracture energy. The relation looks rather more like saturation behaviour. The presence of $m_f = 0.5\%$ HNT appears to be enough to bring the fracture energy to its upper boundary. The final curve in Figure 4.31 displays the fracture energy measured from the epoxy samples reinforced with FS and HNT, which demonstrates the synergistic effect caused by the presence of two different particles in the resin. The results show that adding these two particles into epoxy has a more significant synergistic influence on the materials' fracture energy. The fracture energy rises by almost 50% with the presence of a $m_f = 0.5\%$ mass fraction of those particles. The fracture energy keeps increasing if more particles are involved in the composite. However, the growth rate of the fracture energy drops with increasing mass fractions of particles. Thus, the effect of both particles becomes visible, the almost-linear increasing trend of FS and the saturation behaviour in the presence of HNT.

Similarly, Figure 4.32 shows the impact test results of the epoxy reinforced with silica and rubber. In these material groups, the change of fracture energy depending on the inclusion mass fraction appears similar when compared to the previous composites. The fracture energy increases with increasing inclusion content. The rise, however, occurs at a declining rate. The results suggest silica reinforcement is the most preferable reinforcement type for improving energy absorption under impact. Rubber also provides some improvement in terms of fracture energy which is slightly less than silica. Using both rubber and silica particles in the composite leads to an intermediate behaviour between only silica-reinforced and only rubber-added composites with respect to fracture energies.

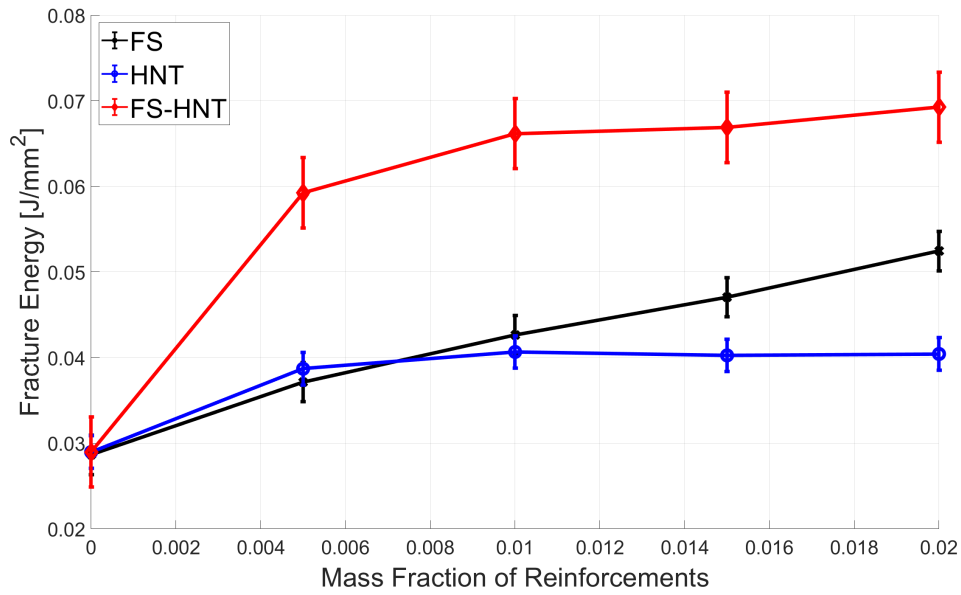


Figure 4.31: Fracture energy from Charpy testing of composites reinforced with FS and/or HNT for increasing mass fraction.

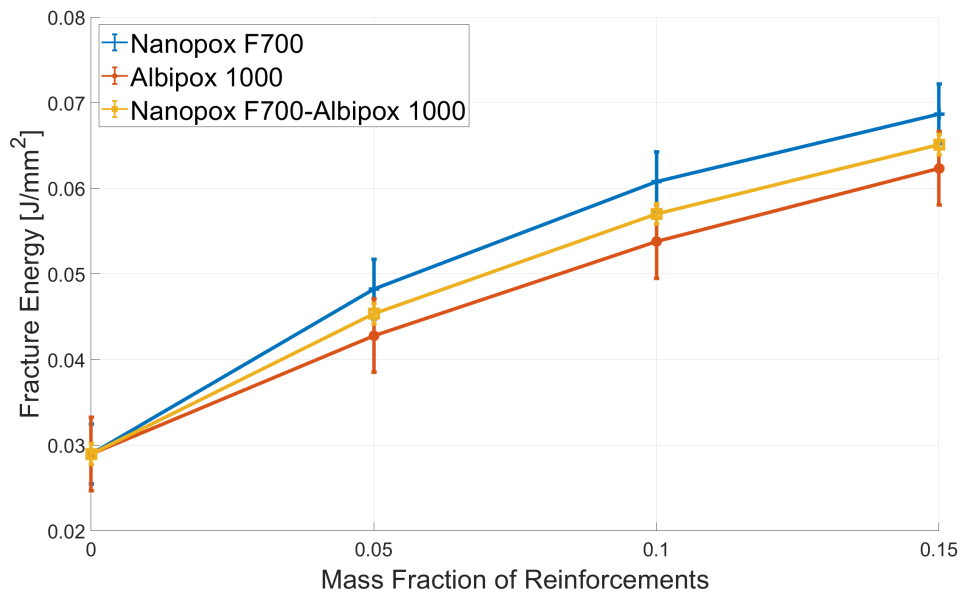
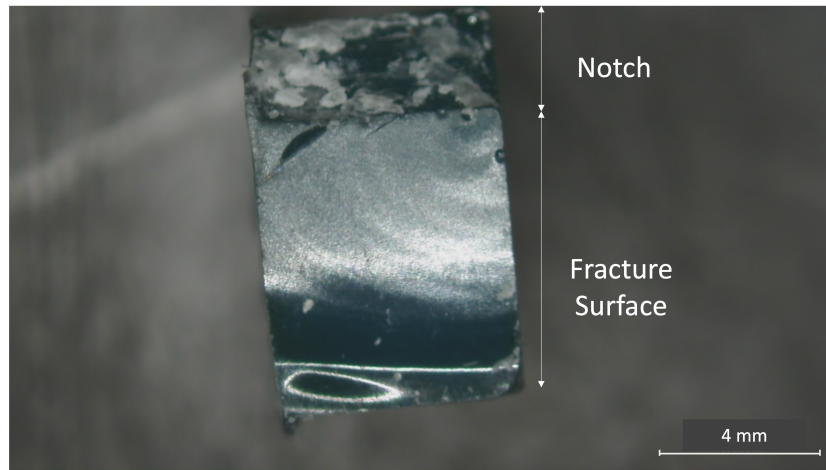


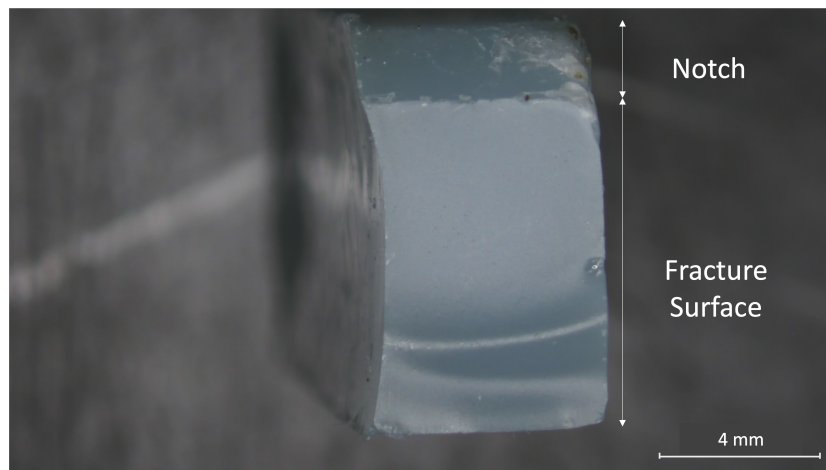
Figure 4.32: Fracture energy from Charpy tests of composites reinforced with silica and/or rubber (Nanopox F700 and Albipox 1000) for increasing mass fraction.

Generally, the findings of the Charpy impact tests are in good agreement with the results presented in the literature. Similar comments as in Section 4.3.3 can be made regarding the toughening mechanisms of the composite materials depending on the type of particles added as reinforcements.

Figure 4.33 exhibits the cross-sections of the fracture surfaces of the samples reinforced with $m_f = 1\%$ HNT and $m_f = 10\%$ rubber (Albipox 1000). These material groups are selected for the same reasons as explained in Section 4.3.3. As observed for the three-point bend cases, the fracture surface of the more ductile material, which is epoxy with rubber (Albipox 1000) particles, is deformed more and is duller compared to the fracture surface of the HNT-reinforced epoxy composite sample.



(a) Cross-section and the fracture surface of the Charpy sample reinforced with HNT.



(b) Cross-section and the fracture surface of the Charpy sample reinforced with rubber (Albipox 1000).

Figure 4.33: Cross-sections and the fracture surfaces of the selected Charpy samples.

4.4 Conclusions

This chapter of this study focuses on the manufacturing and strain-rate-dependency, dynamic mechanical property characterisations of various nanocomposites as well as their impact performance.

First, a consistent manufacturing procedure is developed to manufacture all the material groups, namely FS, HNT, FS & HNT, Albipox 1000, Nanopox F700 and Albipox 1000 & Nanopox F700. Reinforced epoxy control samples with unreinforced epoxy are used for comparison to show the effects of the inclusions. The manufactured samples are put through experimental procedures, including SEM, FTIR, three-point bend tests under three different strain rates, DMA and Charpy impact tests. SEM images suggest that no significant material flaws exist in the manufactured samples. The results of the FTIR reveal that there is no significant variation in chemical composition among the materials. The DMA tests point out that the presence of various nanoreinforcements can significantly affect the storage modulus and the damping capabilities of the material. In terms of improving the stiffness damping simultaneously, silica (Nanopox F700) reinforcement is found as the most successful. However, HNT-reinforced epoxies also yielded well-improved results in stiffness and damping compared to the unreinforced epoxy and other material groups. Also, the results of the three-point bend show that the materials tend to be stiffer and more brittle under greater strain rates. Except for the rubber inclusions, which weaken and soften the material, all reinforcements increase the strength and stiffness of the composite materials. Other than HNT, all the inclusions lead to more ductile behaviour. In terms of impact performance, the material performances are evaluated based on two main groups: the first group consists of FS and/or HNT-reinforced epoxy, and the second group is formed by silica and/or rubber-reinforced epoxy. For the first group, the results show that FS is the most successful reinforcement when it comes to toughening. The addition of more FS, contributes more to the impact performance of the composite material. Whereas HNT has a completely different behaviour. The presence of HNT in the smallest amount increases the toughness, but more addition of HNT may not always help. The combination of FS and HNT in equal masses exhibits a different behaviour than the previous two groups. The presence of

HNT and FS in small amounts contributes significantly to the toughness, which is explained by the synergistic effects. However, adding more HNT and FS contributes little. In the second main group, where Evonik composites are, the materials show a more linear regarding the RC content and impact performance. Silica reinforcement toughens the composite material more than the rubber, and their equal mass mixture lies in between.

Finally, it is seen that using multiple types of inclusions, with different mechanical characteristics, can be helpful in designing with filled epoxy composite materials due to synergistic effects. Each matrix-particle combination has their unique mechanics, and each particle influences the mechanics of the matrix/composite they are added into in a unique way. Thus, the contribution of the particles on different mechanical properties (stiffness, damping, toughness, ductility etc.) is different. Therefore, it is concluded that, based on the requirements of the specific application for which the materials are tailored, the synergistic effects of the presence of multiple particles can be useful.

Chapter 5

Modelling the Mechanics of the Composites and Composite Structures

5.1 Introduction

The idea of using power without humans being involved in its generation is an idea which has been chased after starting from ancient times. Aeolipile of Heron is one of the ancestors of steam-powered mechanisms. Later, almost two centuries back from today, internal combustion engines stepped onto the stage and dominated power generation. Nearly in the same period of time, engineers and scientists started work on one of the most common power generation methods nowadays, turbine engines. While the future of internal combustion engines is in the discussion, with a promising future, gas turbine engines still attract attention so that even nanoturbines are being researched [90].

As the first component of an aero-engine, the fan is crucial. The fan is responsible for the air intake, which is directly related to the efficiency of the engine. Furthermore, the fan contributes to 80% of the thrust generated by the engine [14]. Increasing the radius of the fan also increases the thrust [58]. Seeking more thrust brings problems with it. The geometry of the fan blade must satisfy conditions in two different fields, aerodynamics and solid mechanics. The blade must be long and slender to provide more thrust and better efficiency while being able to

withstand the loads acting on it, which are aerodynamic loads, centrifugal loads and contact at the tip and the root in steady-state conditions [252, 212, 146]. Therefore, the design of the geometry of the rotor blades is a multidisciplinary iterative process [114, 51]. The blades are primarily designed and formed based on aerodynamic or similar fluid dynamics calculations. Then the designs are checked with structural analysis. There are different structural modelling techniques for the blades. Based on the geometry, the first technique that comes to mind is to model the blade as a shell [138].

Nevertheless, in some cases, shells introduce relatively complicated expressions to be solved and are usually computationally expensive. Hence, a simpler structure is used as an alternative modelling method, namely beams [271, 242]. However, simple beam formulations may not be accurate enough for detailed fan blade modelling. Thus, beam formulations are modified to increase the accuracy of the representation of blades by taking the thin-walled structure of the blades into account [201]. Furthermore, reduced-order modelling is another modelling technique that is applied to the fan blades as well [270]. Even though there is research to make improvements on the fan blade geometry, it is a fact that the shape of the blade is restricted [45]. The reason for the restriction of the geometry, the design of the blade is mainly controlled by the fluid flow. So, the design should not disturb the flow, which would lead to a reduction in thrust and efficiency. Hence, the design does not offer much flexibility for the solid mechanics part of the system design.

With restricted geometry, engineers tend to do research on improvement on the materials for the rotor blades [275]. Traditional metallic materials are limited for further improvements in weight reduction and aerodynamic performance on rotor blades with significant centrifugal loads. Larger densities of materials cause large stresses in rotating parts. Therefore, higher specific strengths are required from the material. Whereas modern engineering materials such as foams and composites have much more flexible and tailorable mechanical properties, relatively better specific strength, and are lightweight compared to traditional materials [263, 260]. For instance, polymer-based composites, due to their lightweight, significant vibration damping capabilities and tailorable mechanical properties, are becoming more widely used and are replacing the traditional materials in many engineering systems like automotive, aircraft struc-

tures, and renewable energy applications. However, introducing composite materials into these new fields of applications leads to new challenges related to variability and sensitivity of mechanical properties. Therefore, the reasons behind the sensitivity to the parameters of interest and the variability of the mechanics need to be understood and characterised. Nevertheless, mechanical modelling of those modern materials might require more advanced techniques than classical continuum theory due to their nonhomogeneous internal structures [19, 195]. The internal structure of such modern materials is usually inhomogeneous. They are either composed of multiple materials/phases and/or include cavities. Using different materials in the same structure helps reduce weight without losing much stiffness and strength. Whereas cavity inclusion might increase the stiffness and strength of the material under certain conditions with strain hardening mechanism [94].

Those inhomogeneities limit the capabilities of the classical continuum theory to handle the media as bulk and homogeneous materials. Classical continuum theory essentially uses a volume element to derive the equilibrium equations involving stresses, body forces and inertial loads where this volume element is infinitesimal. Whereas in composites or foams, the volume element has a minimum size to be representative [169]. However, it is still possible under certain assumptions.

Some studies use the data acquired from the virtual experiments performed using their material models [276]. However, the mechanisms that control and shape that complicated mechanical behaviour are often difficult to interpret. It is thought that the internal structure of the material is among the most important parameters that control the mechanics of such composites. Several theories, such as molecular dynamics and nonlocal continuum theory, explore the mechanics on small scales [147, 247, 32, 168, 63]. Therefore, multi-scale modelling approaches are developed to investigate the small-scale structures' effective mechanics, such as nanocomposites [98, 276]. Mori-Tanaka mean-field homogenisation (MFH) is among the most common and convenient methods of multi-scale modelling [170]. It is proven to work well with particulate reinforced nanocomposites [150, 175]. The FE method (FEM) is another numerical modelling approach that can be used to perform multi-scale analyses on nanocomposites. The stress analysis is conducted by generating representative volumetric elements (RVE) of the nanocomposites and

applying periodic boundary conditions (PBC) to the generated elements, and then the effective mechanical properties of the materials can be computed [243, 246, 165, 130, 95]. The FE model usually produces more accurate results than Mori-Tanaka or any other MFH methods. However, its computational cost is also greater [25, 173].

The classical continuum theory's lack of intrinsic scale and limitations brings out the need to have a scale dependency for a continuum theory since the classical theory has no consideration of the scale [142]. Mindlin is one of the first researchers who claimed the scale effect influences material behaviour [164]. This is followed by formulating another advanced continuum theory evaluating the scale effect in the atomic/molecular scale. This is called the nonlocal continuum theory developed by A. Cemal Eringen [72]. Nonlocal continuum theory is one of the most advanced continuum theories in the literature that are successful in modelling such inhomogeneous media with good accuracy [67]. One important thing to note about the nonlocal continuum theory applied to micro/nano continua, it is possible to validate the results of nonlocal continuum theory with other methods such as molecular dynamics [69, 21].

To formulate accurately representative structures using nonlocal continuum theory is more challenging in comparison to the classical continuum theory. This is mainly because of the introduction of complexities by the fundamental nonlocal integral equation. Therefore, it is more common to work with simpler formulations such as beams, plates, and shells [240, 239, 20, 215]. To solve those formulations, usually, numerical methods are employed, such as the FE method [241]. It is also possible to derive specific element formulations for relatively more complicated cases the same way it is used for classical applications with the nonlocal terms added [195, 193, 192, 87].

This chapter looks into the problem of modelling the mechanics of nanocomposite materials and structures. For this purpose, three numerical tools are developed. The first one is capable of performing Mori-Tanaka MFH as well as generating the RVEs reinforced with elliptical and spherical inclusions and then applies the PBC on the sides of the RVEs before finally calculating the effective material properties of the composite using stress-averaging techniques. The second one calculates the nonlocal mechanical behaviour of classical FE models post-processing the

discrete stress/strain fields calculated using the integral form of the nonlocal continuum theory. The third tool is a continuation tool that calculates the frequency response functions of systems that have material nonlinearity.

First, the classical continuum theory is employed, and Mori-Tanaka MFH and FE analyses are performed on the generated RVEs of the composites manufactured. This also helps further interpret the results of the experimental procedures presented in the previous chapter (Chapter 4). The numerical simulations conduct a more detailed stress analysis and characterise the mechanics of the composites prepared. These numerical simulations both analyse RVEs with appropriately shaped inclusions. Thus, the main contribution of this study is to observe the effects of the mentioned nanoreinforcements on the epoxy-based composites experimentally.

Furthermore, the mechanisms that change the mechanical properties of the composites are revealed through numerical analysis. This is followed by the post-processing procedure developed to calculate the nonlocal stresses using the integral form of the nonlocal continuum constitution equation after being discretised. It is important to emphasise that the materials are considered weakly nonlocal.

After these static calculations, a fan blade test case is generated, and the modal analysis of a 3D prestressed fan blade structure using the FE method is presented. A full-scale fan blade model is chosen as the test case to represent the rotor blade for a modern high bypass ratio turbofan engine. The blade is considered to be made of an isotropic, linear elastic material of weak nonlocal characteristics. The loads and the boundary conditions are arranged so that they are representative of steady-state operating conditions. The stress analysis of the nonlocal blade is done in two main steps. The first step is the calculation of local stresses in the blade with the classical FE procedure. In the second step, the local stresses are post-processed, and the nonlocal stress components are calculated by numerical evaluation of the integral equation of nonlocal continuum theory. The local stress components of each point in the blade are multiplied with the nonlocal kernel function and integrated over the domain. The results of local and nonlocal stresses are compared as well as the natural frequencies. Even though the industrial applications of such materials do not currently exist, nanocomposites, nanophase

alloys or any other material which is likely to have nonlocal behaviour might become usable in the future [134, 60].

Finally, for the third tool, a cantilever beam test case is built with effective nonlinear material properties that are characterised using the DMA tests. Surface functions are fitted to the acquired forced vibration data with amplitude and frequency dependence for each material. Then, these functions are inserted into the stiffness term of the equation of motion (EOM), and the nonlinear frequency responses are plotted using the appropriate continuation method. Thus, the effects of each particle on the frequency response and the material nonlinearity are discussed from another perspective.

5.2 Modelling with the Classical (Local) Continuum Theory

The modelling works employing the classical continuum theory are explained in detail in this section. First, the material modelling is presented, including the MFH approach and the FE analysis. Then, the FE model of the aero-engine fan blade is described.

5.2.1 Material Modelling

Theoretical modelling is also conducted using MFH and FE methods to investigate the mechanics of the prepared composites. The MFH is employed to predict the effective mechanical properties of the composites and their individual components, whereas the FE method is used to predict the micromechanical behaviour as well as mechanical properties at the macro-level.

Mean-Field Homogenisation with Mori-Tanaka Method

As one of the most common, computationally convenient and accurate methods to predict the mechanical behaviour of the composites, the Mori-Tanaka MFH method is employed within the

scope of this study. To predict the effective mechanical properties of the reinforcing particles and, as a result, the manufactured composites, Mori-Tanaka homogenisation is performed [170]. The elastic moduli of the particles are iterated until the error between the experimental and numerical approaches is minimised. The epoxy matrix and all the inclusions are taken as linear elastic. The interfaces between the matrix and the particles are modelled using the rigid bonding approach for the sake of simplicity.

Furthermore, except for HNT, which is taken into the calculations as ellipsoids, the rest of the inclusions are presumed as spheres. The dimensions for the specific inclusions can also be found in the corresponding technical data-sheets as well as the literature [4, 181]. Thus, a basis is built for a more detailed numerical investigation of the mechanics of the nanocomposites with the FE method.

Finite Element Analysis with Representative Volume Elements

Inheriting the assumptions, as well as the knowledge from the computationally faster method, Mori-Tanaka MFH, the FE models of the nanocomposites, are built using RVEs with PBC under the uniaxial tensile strain of 0.01 imposed on one surface. These RVEs are subjected to uniaxial tension so that the stresses in small-scale as well as the effective mechanical properties could be calculated. As the first step, a FE model of an RVE with randomly sized and positioned reinforcement particles is prepared. The random positions and orientations of the nanoparticles are arranged so that none of the particles intersects with each other or the boundaries of the RVE [196, 224, 198, 81]. This approach can be valid as long as the RVE parameter is chosen appropriately and the reinforcement volume fraction must not be too high, so tight packing is not necessary and does not occur. Therefore, the RVE is constructed as a cube with a side length at least five times the largest particle dimension so that the RVE converges to the actual material behaviour [92, 49]. Alongside the careful selection of safe RVE parameters, the analyses are conducted for each material set five times to ensure reliable and repeatable results. Figure 5.1 shows a section view sample of an RVE generated for this study.

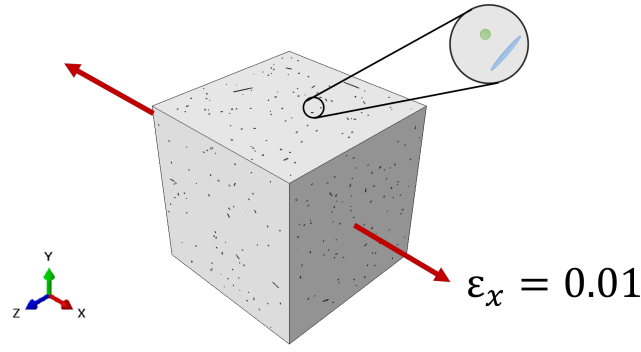


Figure 5.1: A section view of a generated RVE with spherical and ellipsoidal inclusions.

5.2.2 Finite Element Modelling of the Fan Blade

A FE study is conducted to model the behaviour of the representative blade structure. The modelling process starts with the generation of the CAD, which is followed by meshing and preprocessing. Finally, the analysis is run, and the stresses are displayed and also exported for further post-processing. This section describes this part (until post-processing) of the study in some detail.

Modelling of the blade starts with preparing the CAD. The overall geometry and the main dimensions are kept similar to a real fan blade, as shown in Figure 5.2. Hence, it is expected to be a representative solid model. The CAD is drawn in Solidworks and exported in universal CAD formats for further processing in the FE software. The prepared solid model is shown in Figure 5.3. In Figure 5.3, SS stands for the suction side and PS for the pressure side. The blade and the disc are considered a single-body system for simplicity to remove contact nonlinearities from the system. The contact introduces friction to the system. This changes the static stress and displacements significantly. Hence it is also to be stated that dynamic behaviour is also affected dramatically [212].

Following the preparation of the solid model, the CAD data is migrated to the software Hypermesh for mesh generation, where cyclic symmetry is applied at the boundaries of the disc sector. Since the geometry is relatively complicated, quadratic tetrahedral elements are used to mesh the structure. 47 419 elements and 78 266 nodes are generated as the result of the meshing process. Doubling the node numbers increased the stress levels by up to 3%. There-



Figure 5.2: A fan assembly of Trent 500 displayed in the Department of Mechanical Engineering at Imperial College London.

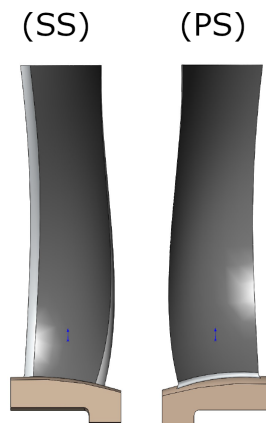


Figure 5.3: CAD model of the blade.

fore, it is judged that convergence is achieved. Taking note that the shear behaviour is to be captured by inserting two rows of quadratic elements throughout the blade thickness. Figure 5.4 displays the final mesh. Pre-processing is done in the same software. The material is taken as a titanium alloy Ti-6Al-4V, which has an elastic modulus of 110 GPa , Poisson's ratio of 0.31 and a density of 4600 kg/m^3 . The boundary conditions and the loads are selected to represent the steady-state operating conditions with caution. Hence, the rotational speed is taken as 2000 rpm . The steady-state aerodynamic loads consist of the pressure due to the airflow. The

aerodynamic pressure distribution is calculated by using CFD computations.



Figure 5.4: Blade model after mesh processing.

Static pressure on the blade surface is obtained through steady Reynolds-Averaged Navier-Stokes (RANS) computations. The software relies on the edge-based finite-volume method to solve 3D compressible RANS equations on unstructured grids with Spalart-Allmaras turbulence model [216]. The chosen operating point corresponds to the peak efficiency of the low-speed fan at the nominal rotational speed. The pressure distribution on the pressure side is given in Figure 5.5.

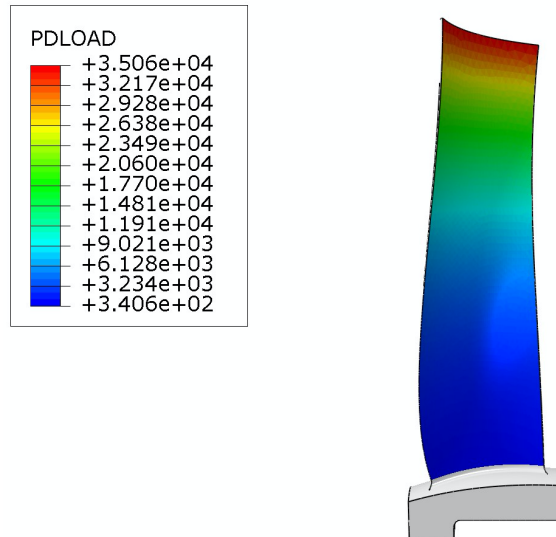


Figure 5.5: Pressure distribution on the pressure side (Units in $[Pa]$)

After preprocessing, the model is exported from Hypermesh and imported to Abaqus. Using Abaqus, a static analysis is run to obtain the stresses in the blade. The obtained results are

visualised and exported for further processing. The exported stress data is used to calculate the nonlocal stress components.

The prestressed model is put through the procedure for the modal analysis, and natural frequencies and mode shapes are obtained. The effect of stress on natural frequencies is investigated.

5.3 Modelling with Nonlocal Continuum Theory

The nonlocal continuum theory assumes that the stresses of the individual points depend on the strains of all the points that lie within the whole domain. This dependency is defined over a kernel/attenuation function that peaks when the distance between the points is zero and approaches infinity when the distance between the points increases. There are three essential constants to define the attenuation function: material constant and intrinsic and extrinsic lengths. This section covers basic details of the nonlocal continuum theory applied within the scope of this study, some details of the attenuation function and how it is to be chosen as well as the calculation of inertial properties. The flowchart shown in Figure 5.6 summarises the strategy.

5.3.1 Integral Form of Nonlocal Elasticity

To explain the nonlocal elasticity theory, it is a good start to give the relation between the classical and nonlocal elasticity in cartesian coordinates [69]. In Equation (5.1), the stress-strain relation of a linear elastic material within the boundaries of the classical theory of elasticity is given. The relation between the local and the nonlocal elasticity theories is given in Equation (5.2).

$$\sigma_{ij}^l = C_{ijkl} \epsilon_{kl}^l \quad (5.1)$$

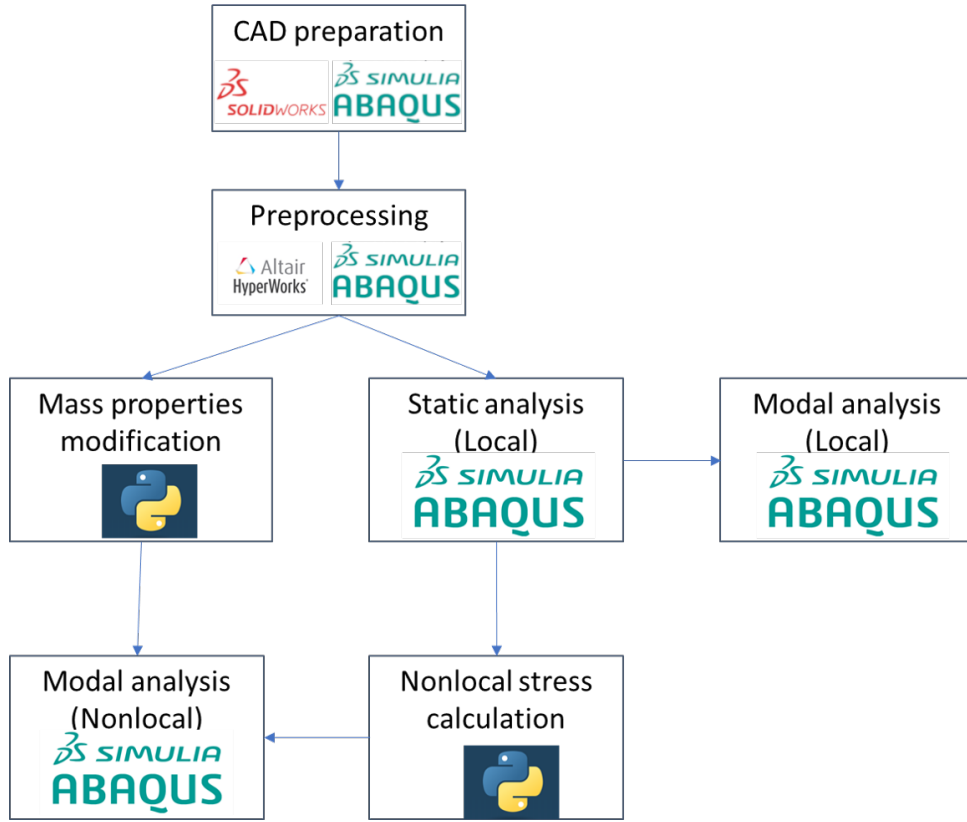


Figure 5.6: A schematic outline of the study.

$$\sigma_{ij}^{nl}(\mathbf{x}) = \int_{\Omega} \alpha_{ijkl}(\mathbf{x}, \mathbf{x}', \tau) \epsilon_{kl}^l(\mathbf{x}) d\Omega \quad (5.2)$$

In this equation, C_{ijkl} represents the stiffness tensor of the fourth-order, σ_{ij}^l , the local stress tensor, σ_{ij}^{nl} , the nonlocal stress tensor, ϵ_{ijkl} represents the classical/local strain tensor and α_{ijkl} a kernel tensor of the fourth order from which the kernel function is to be built. Also, $\mathbf{x}'$, $\mathbf{x}$ are the field points and the points of interest, respectively. These concepts are used in the definition of the kernel function, which is handled in the following section. The superscripts nl and l refer to nonlocal and local, respectively. In isotropic continua, it is possible to distribute the kernel tensor uniformly to the components of the stiffness tensor, and a scalar function remains as a factor within the integral. This is called the kernel function. Executing the distribution, the equation becomes as shown in Equation (5.3).

$$\sigma_{ij}^{nl}(\mathbf{x}) = \int_{\Omega} \alpha(\mathbf{x}, \mathbf{x}', \tau) C_{ijkl} \epsilon_{kl}^l(\mathbf{x}') d\Omega \quad (5.3)$$

Here C_{ijkl} represents the stiffness tensor of an isotropic and linear elastic material whereas α becomes the scalar kernel function. Combining the Equations (5.1) and (5.2) it is possible to get the relation between the local and nonlocal stresses for a linear elastic material. The form in the Equation (5.4) is obtained. This is called the integral form of nonlocal elasticity [69].

$$\sigma_{ij}^{nl}(\mathbf{x}) = \int_{\Omega} \alpha(\mathbf{x}, \mathbf{x}', \tau) \sigma_{ij}^l(\mathbf{x}') d\Omega \quad (5.4)$$

This integral is evaluated for each point of interest by scanning the entire domain. The point of interest ($\mathbf{x}$) is fixed while the points of the field ($\mathbf{x}'$) are floating, and so the field is scanned. During this scanning process, for each field point, the distance between the point of interest and the field point is calculated and used as input for the evaluation of the integral. The numerical evaluation of the integral is shown in Figure 5.7.

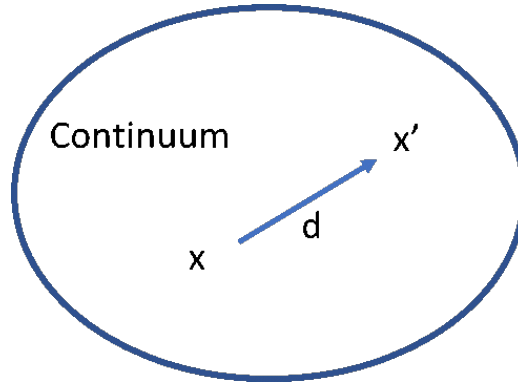


Figure 5.7: Schematic evaluation of the integral

5.3.2 The Kernel Function and its Constants

The kernel function, also called the attenuation function, is the key to nonlocal continuum theory. It is defined as a function of the distance between field points, which are noted as $\mathbf{x}'$, and the point of interest, which are noted as $\mathbf{x}$, as well as the media itself. So the value of α changes while scanning the domain for integration, as it depends on the distance between points. Moreover, structural characteristics also have a dominant effect on the function. Those structural factors can be summarised as follows:

- γ : Size parameter
- e_0 : Material constant
- a : Characteristic intrinsic length
- l : Extrinsic length
- τ : Nonlocal constant

Some of those concepts have relations which are shown in Equations (5.5) and (5.6).

$$\gamma = e_0 a \quad (5.5)$$

$$\tau = \gamma/l = e_0(a/l) \quad (5.6)$$

The nonlocal function α must satisfy the conditions below:

- When $\tau \longrightarrow 0$, α must converge to the form of a Dirac delta function, which means the local and nonlocal elasticity theories converge to each other and calculate the same results for the same cases.
- Nonlocal tensor shows symmetry with respect to the field $\mathbf{x}$.
- The nonlocal modulus reaches its peak where $\mathbf{x} = \mathbf{x}'$ and approaches towards 0 as the distance between the point and the field points increases.
- In an infinite space domain Ω , the integral of α in the domain is equal to 1. This is called the normalisation condition and is shown in Equation (5.7).

$$\int_{\Omega} \alpha(\mathbf{x}, \mathbf{x}', \tau) d\Omega = 1 \quad (5.7)$$

- If calculations are to be done in a finite domain V which is embedded in Ω , the normalisation condition turns out to be the way shown in Equation (5.8).

$$\int_{\Omega} \alpha(\mathbf{x}, \mathbf{x}', \tau) dV' = 1 \quad (5.8)$$

As mentioned earlier, some of the kernel function constants are material dependent and known, such as e_0 . However, a and l are chosen based on the structure. The nonlocal constants in this equation are chosen, taking care of the normalisation constant.

This normalisation condition, alongside the limit condition of the kernel function, when the distance goes to infinity, $\tau \rightarrow 0$ also leads to a practical concept. A radius can be defined where the nonlocal function has a significant effect on the actual stresses.

Alternatively, the nonlocal size parameter γ can also be used in the calculations, which is the multiplication of the intrinsic length a and the material constant e_0 . The chosen attenuation function, which is a Gaussian function, is given in Equation (5.9) [248]. It is also to be noted that the attenuation function has the dimensions of $force * length^{-5}$.

$$\alpha(\mathbf{x}, \mathbf{x}', \tau) = \frac{1}{2\gamma} e^{-\frac{(|\mathbf{x}-\mathbf{x}'|)^2}{l^2}} \quad (5.9)$$

5.3.3 Calculation of Inertial Properties and Nonlocal Stresses

Proceeding to the mass/inertia properties, inertia terms are modified by generating the relevant nonlocal component and summing it with the classical local inertial terms [172, 164]. Kinetic energy per unit volume T is expressed in Equation (5.10).

$$T = \frac{1}{2} \rho \dot{u}_j^l \dot{u}_j^l + \frac{1}{2V'} \int_{V'} \rho^{nl} \dot{u}_j^n l \dot{u}_j^n l dV' \quad (5.10)$$

Here ρ is the density of the material, and $\dot{u}$ is the velocity in local and nonlocal theories to be

distinguished based on the superscripts. The relation between local and nonlocal components of the density is given in Equation (5.11).

$$\rho^{nl} = \rho^l \gamma^2 \quad (5.11)$$

Plugging this relation into the Equation (5.10).

$$T = \frac{1}{2} \rho \dot{u}_j^l \dot{u}_j^l + \frac{1}{2V'} \gamma^2 \int_{V'} \rho^l \dot{u}_j^n l \dot{u}_j^n l dV' \quad (5.12)$$

As the next step, the total energy is calculated by integrating through the finite domain V . That integration is formulated in Equation (5.13).

$$T = \int_V \left[\frac{1}{2} \rho \dot{u}_j^l \dot{u}_j^l + \frac{1}{2V'} \gamma^2 \int_{V'} \rho^l \dot{u}_j^n l \dot{u}_j^n l dV' \right] dV \quad (5.13)$$

The local stresses are calculated with the classical FE model used. The local stress data is exported. This discrete data is numerically integrated to obtain the nonlocal stresses. Here, it is essential to note that the local strain field is assumed to be the same as the nonlocal strain field, and the system is assumed to be weakly nonlocal.

5.3.4 Numerical Examples

In this section, the nonlocal parameters for each example handled are explained.

Representative Volume Element Examples

It is crucial to emphasise that all the cases created with the RVEs are all individual since the reinforcement particle sizing and positioning are done randomly. Therefore, the nonlocal parameters that are imposed on each RVE have to be different in terms of numerical values. However, the nonlocal size parameter γ is chosen for each case based on the RVEs and inclusions'

geometric properties. The extrinsic length is selected as the RVE parameter, and the intrinsic length is chosen as the average reinforcement diameter (for HNT, the mid-diameter of the ellipsoid in the axial direction). The material constant e_0 is determined based on the literature [91, 118, 56]. Overall the nonlocal size parameters γ for the epoxy and rubber come down to a range of approximately 0.08-0.13, while for FS, silica and HNT, the size parameters lie in the range of 0.15-0.21, with HNT having the strongest nonlocality.

Aero-Engine Fan Blade Example

In this example of the nonlocal fan blade, the extrinsic length is chosen as the blade's average thickness, around 5 mm. Therefore l is assumed to be 5 mm. Thus, γ becomes roughly 0.15.

5.4 Forced Vibration of Nanocomposite Elastic Beams with Material Nonlinearity

To serve as an example and a model to compare the variety of nonlinearities caused by the reinforcements, a cantilever Euler-Bernoulli beam case is picked, which is excited from its free end (displayed in Figure 5.8) around its first bending mode. The beam has a rectangular cross-section (width $b = 30 \text{ mm}$, height $h = 5 \text{ mm}$ and length $L = 208 \text{ mm}$) and is assumed to be made of the nonlinear nanocomposite materials manufactured and characterised within the scope of this study. Considering the excitation is mono-harmonic, and only the first bending mode is of interest, the beam can be reduced to a single degree of freedom (SDOF) system as shown in Figure 5.9 by calculating and using the equivalent mass, stiffness and consequently damping properties. These equivalent stiffness/damping properties are then expressed as $k_{eq} = 3EI/L^3$ while the equivalent mass can be written as $m_{eq} = (33 \text{ } m_{beam})/140$. The dynamic properties extracted as a result of the DMA tests are included within the stiffness and damping expressions as polynomial functions with the frequency, (ω) , and amplitude, (X_0) , dependence. To integrate this into the typical EOM of an SDOF system with a viscous damper, the following

Table 5.1: Selection of the most distinctive material samples from each material group for the nonlinear vibration calculations.

Material groups	RC
FS	1 %
HNT	1 %
FS and HNT	0.5 %
Rubber (Albipox 1000)	15%
Silica (Nanopox F700)	5 %
Rubber(Albipox 1000) and silica (Nanopox F700)	5 %

steps can be taken. From each material group (reinforcement type and/or combination) the most distinctive ones compared to epoxy are selected regarding both stiffness and damping properties to make the effects of the different particles more obvious in the results. The selected materials are shown in Table 5.1. First, the EOM of the system can be written as in Equations (5.14) and (5.15) considering the viscous damper and dynamic modulus approaches separately.

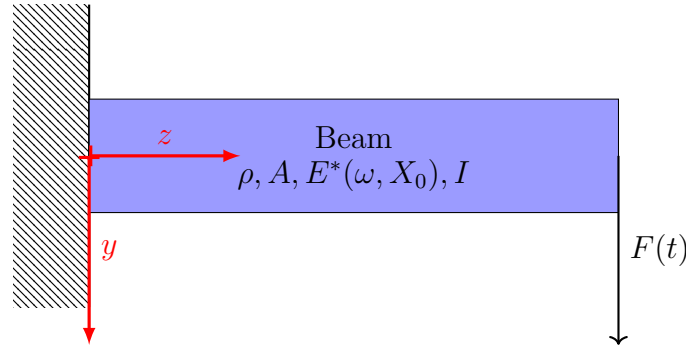


Figure 5.8: Cantilever Euler-Bernoulli beam system example.

$$k(X_0, \omega)y + c(X_0, \omega)\dot{y} + m\ddot{y} = F(t) \quad (5.14)$$

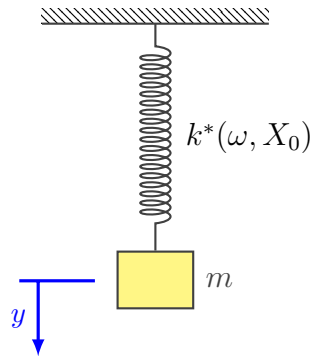


Figure 5.9: Reduced mechanical system.

$$k^*(X_0, \omega)y + m\ddot{y} = F(t) \quad (5.15)$$

Here, t indicates time, and y is the coordinate of the point of interest. Furthermore, k denotes the stiffness, c damping, m the mass of the SDOF system and $F(t)$ the excitation force, while k^* expresses the dynamic stiffness. Dynamic stiffness possesses two parts, as shown in Equation (5.16), the real and complex parts; thus, it is also called complex stiffness.

$$k^*(X_0, \omega) = k'(X_0, \omega)(1 + i\eta(X_0, \omega)) \quad (5.16)$$

i is the complex number, k' , the coefficient of the real part, is responsible for the elastic potential energy storage, whereas η , the loss factor, is the coefficient of the complex part, and it is the concept that leads to damping. This also explains why η stands for the loss factor.

Next, assuming a typical solution, as shown in Equation (5.17), for the EOM, the response of the SDOF system is modelled.

$$x(t) = X_0 e^{i\omega t} \quad (5.17)$$

Knowing these two equations in Equations (5.14) and (5.15) represent the same system, it is possible to state that they are supposed to be identical. Taking advantage of this equality, one can express the viscous damping in terms of the loss factor. This expression is presented in Equation (5.18).

$$c(X_0, \omega) = \frac{k'(X_0, \omega)\eta(X_0, \omega)}{\omega} \quad (5.18)$$

As mentioned earlier, the stiffness of the SDOF system is obtained by calculating the equivalent stiffness of the cantilever beam excited at its tip with a concentrated force. Thus, the relation of the dynamic stiffness, which, again, depends on the amplitude and the frequency of the

vibration motion, to the dynamic modulus of elasticity (E^*) is given below, in Equation (5.19).

$$k^* = k^*(X_0, \omega) = \frac{3E^*(X_0, \omega)I_x}{L^3} \quad (5.19)$$

Here, L is the beam's length, and I_x is the area moment of inertia of the cross-section of the beam. Similarly, the equivalent mass, which is a constant unlike the stiffness term, should also be presented as in Equation 5.20, where m_{eq} denotes the equivalent mass and m_{beam} is the actual mass of the beam. The mass of a constant cross-sectioned beam can be expressed in terms of the density, ρ , cross-sectional area, A and the length of the beam, L .

$$m_{eq} = m_{beam} \frac{33}{140} = \rho AL \frac{33}{140} \quad (5.20)$$

Similar to the dynamic stiffness, the dynamic modulus of elasticity should also be formulated, as in Equation (5.21) with the complex part to preserve the nature of the dynamic stiffness, considering that the equivalent stiffness only depends on one material property which is the modulus of elasticity. The equivalent stiffness depends on all the other parameters, which are geometric properties.

$$E^*(X_0, \omega) = E'(X_0, \omega)(1 + i\eta(X_0, \omega)) \quad (5.21)$$

Plugging in the relation between the dynamic stiffness and the beam properties (Equations (5.19) and (5.21)) into the EOM with the dynamic stiffness approach in Equation (5.15), the final form of the EOM is derived which is presented in Equation (5.22).

$$\frac{3E'(X_0, \omega)(1 + i\eta(X_0, \omega))I_x}{L^3}y + m\ddot{y} = F(t) \quad (5.22)$$

Transforming the assumed solution in Equation (5.17) using sine and cosine functions, the solution assumed becomes as in Equation (5.23).

$$x(t) = C_1 \cos(\omega t) + C_2 \sin(\omega t) \quad (5.23)$$

C_1 and C_2 here denote the unknowns of the assumed solution function.

Then, the differential EOM is transformed into a set of two algebraic equations as shown in Equation (5.24).

$$kC_1 - \omega^2 m C_1 = F_0 \quad (5.24a)$$

$$kC_2 - \omega^2 m C_2 = 0 \quad (5.24b)$$

The vibration amplitude, X_0 can be expressed in terms of the unknown coefficients of the assumed function, which leads to Equation (5.25).

$$X_0 = \sqrt{C_1^2 + C_2^2} \quad (5.25)$$

Finally, remembering that the DMA tests characterise the material properties based on the strain amplitudes, the strain-load relation, given in Equation (5.26), for a cantilever Euler-Bernoulli beam is used to convert the DMA data into a usable form in the model formulated in this section.

$$\epsilon = \frac{FLh}{2I_x E} \quad (5.26)$$

Here, h denotes the beam thickness, and E is the modulus of elasticity of the beam material.

The calculated results are presented in a nondimensionalised form. The frequencies are normalised with the first bending mode (approximately 19,9588 Hz) of the beam, assuming the beam is made of the unreinforced epoxy ($E = 2.5 \text{ GPa}$, $\nu = 0.38$ and $\rho = 1400 \text{ kg/m}^3$) having

the static modulus of elasticity (which makes the structure linear) while the vibration amplitudes are nondimensionalised compared to the static displacement of the unreinforced epoxy beams.

5.5 Results and Discussion

5.5.1 Representative Volume Element Examples

Calculation of Local Stresses

As discussed in Section 3, MFH (Mori-Tanaka) studies are first conducted on each composite group to determine the elastic properties of the reinforcements. Finally, the approximated values of the elastic modulus for the particles are determined. The Young's moduli for FS, HNT, rubber (Albipox 1000) and silica (Nanopox F700) are found to be 70 *GPa*, 1000 *GPa*, 200 *MPa* and 10 *GPa* respectively. These values are employed for all the numerical analyses conducted for this study.

Determining the elastic properties of the particles, the homogenised mechanical properties of the composites are calculated for each material group under each strain rate using both Mori-Tanaka and FE methods. Figures 5.11-5.19 present the theoretically calculated effective moduli of elasticity and compare them to the experimentally determined elastic moduli of the composites. A view of the stress analysis results of an RVE reinforced with FS is presented in Figure 5.10. Moreover, the data in Figure 5.11 suggests that for FS-reinforced epoxy nanocomposites, both numerical methods yield results that are in good agreement with the experimental results showing a deviation of approximately 7%. However, FE delivers a more accurate representation compared to MFH. FE models have less simplifying assumptions compared to the MFH, and FE conducts a more detailed stress analysis [25, 173]. Therefore, this would be an expected outcome of the analyses conducted. It is noteworthy that FE is computationally significantly more expensive than MFH, and the error increases with increasing strain rates. This is explained by the nonlinear behaviour of the composite interphase regions, which affect the behaviour of the

composite more with increasing strain rates, whereas the numerical models do not consider the nonlinear interphase regions [122]. Based on the FE analyses, the stress reaches its maximum in the inclusions, which means the inclusions play a role in bearing the load. More importantly, significant triaxial stress concentrations, which lead to local plastic deformations that initiate in the polar region of the spherical inclusions, occur in the matrix. It is also observed that the magnitude of stress concentrations mainly depends on the size of the inclusion as well as the sizes of the nearest inclusions and the distances to the nearest inclusions. The stress concentrations around the inclusions in the matrix interact with each other depending on the distances between the particles of interest. The closer the particles, the stronger the interaction. This emphasises the importance of a uniform distribution of the positions of particles.

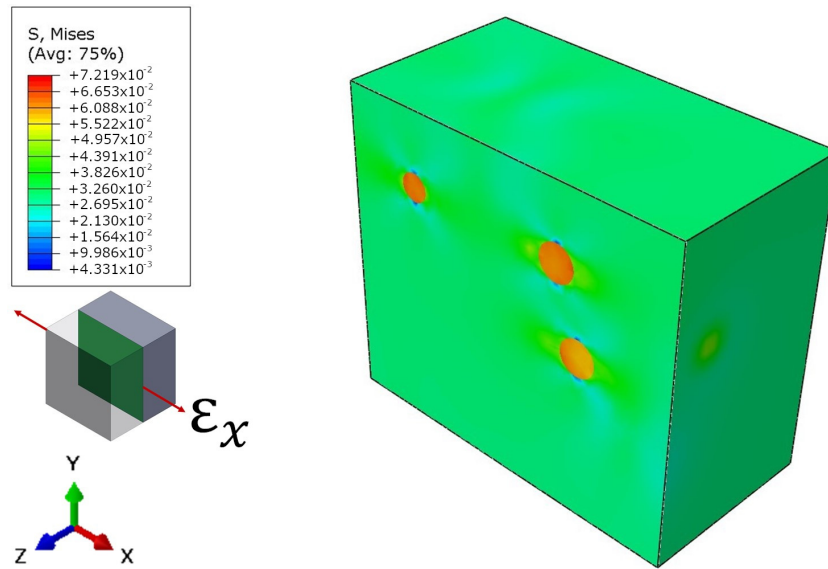
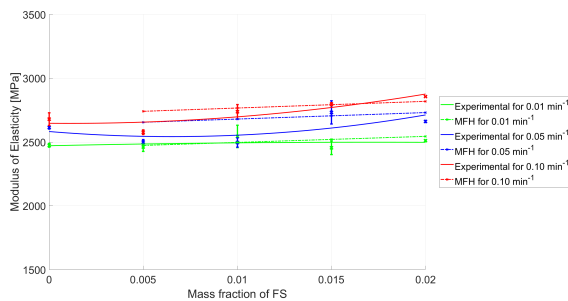
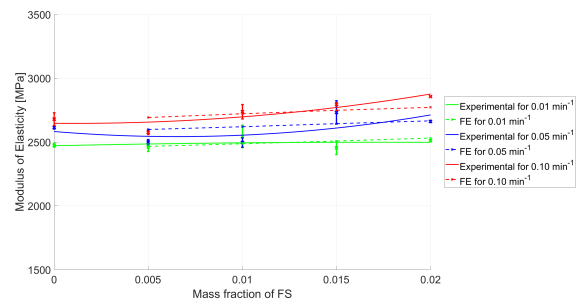


Figure 5.10: A section view of an RVE with spherical FS inclusions with an average diameter of 12 nm taken between 10 – 15 nm (Stress unit: GPa).



(a) Experimental versus MFH.



(b) Experimental versus FE

Figure 5.11: Comparison of experimental results for modulus of elasticity with the numerical simulations of FS-reinforced epoxy.

Figure 5.12 presents a section plot of the stress analysis performed with FE methodology. Here, it is necessary to note that the scales are significantly different between the resolution of the image and the size of the HNT particles. Figure 5.13 presents the results of HNT-reinforced epoxy composites and compares them to the experimental results. The maximum error between the numerical and the experimental results is around 9%. Here, the results are in good agreement. Especially the results of MFH and FE match well. However, the magnitude of error is more significant compared to the composites with FS, and the deviation of error between the numerical and experimental results does not follow a perfect trend. The reason for this is the deviations in the experimental results. The results of both numerical methods follow a similar trend, as mentioned earlier. However, the experimental results display less of this trend, and therefore, the error grows. The FE results show that the maximum stresses are in the inclusions and these stresses are greater when compared to the ones observed in FS inclusions. This is due to the greater elastic modulus of HNT over FS.

Moreover, the stress concentrations in the matrix occur around the tips of the ellipsoidal inclusions. Due to the geometry and greater difference in stiffness, the concentrations display larger stress values. Also, due to the sharper geometry of the tip of the HNT inclusions, the stress triaxiality is stronger, which explains the more brittle behaviour. Unlike the case with FS, there are some additional parameters that are decisively influential on stress concentration. The size of the HNT inclusions is one of those parameters. Since these are considered ellipsoidal particles, the size affects the radius of curvature at the tip. The radius of curvature for the aspect ratios that HNTs have is already relatively small and therefore has a limited effect. The orientation of the HNT particles relative to the uniaxial loading direction is also one of those parameters. If the HNT inclusion lies perpendicular to the loading direction, the stress concentration tends to be smaller, and if the HNT inclusion lies parallel to the loading direction, the stress concentration gets its peak value possible for that size of HNT inclusion. It is also worth mentioning that if the HNT particle orientation is parallel to the uniaxial loading direction, its contribution to the load-bearing capacity of the material in that direction is maximum. Thus, the maximum stress possible is observed. However, if the orientation is perpendicular to the loading direction, the load that the HNT particle carries will be less and therefore, the stress

in the HNT particle is less than what it would be if the orientation were parallel to the load.

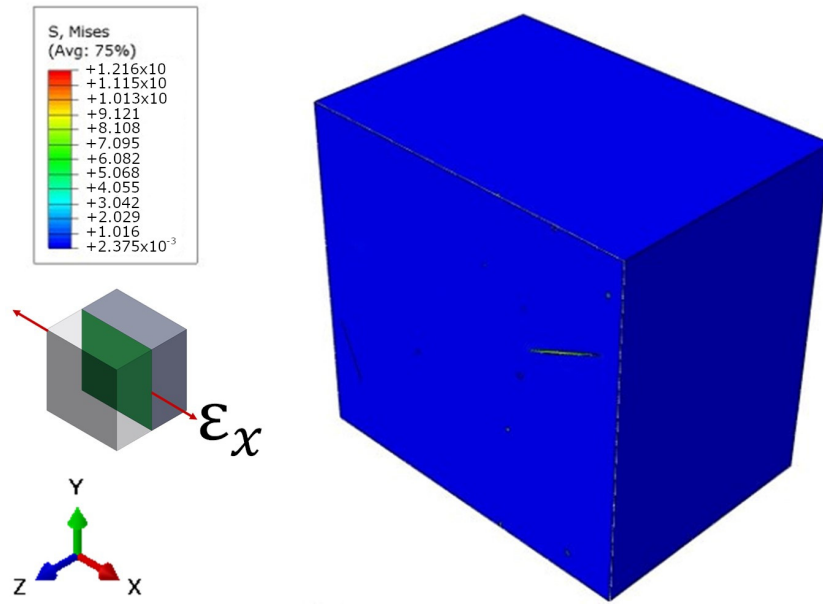


Figure 5.12: A section view of an RVE with ellipsoidal HNT inclusions with diameters from 30 nm to 70 nm and lengths from 1 μm to 3 μm (Stress unit: GPa).

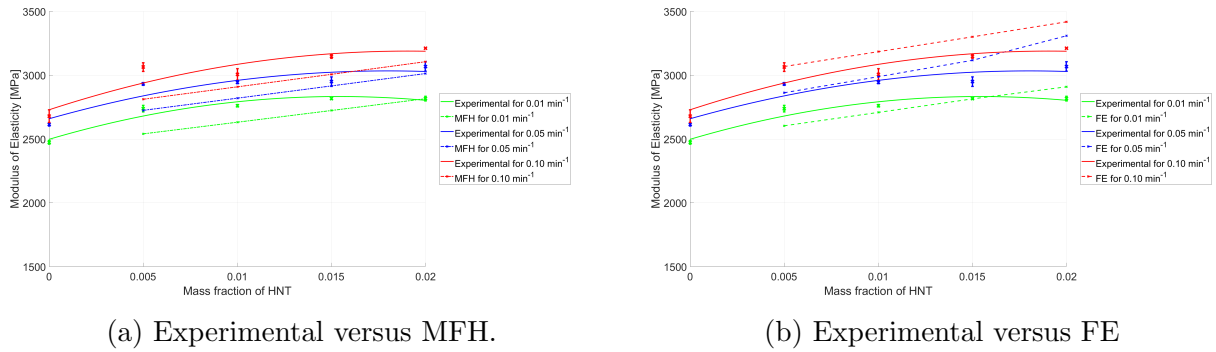
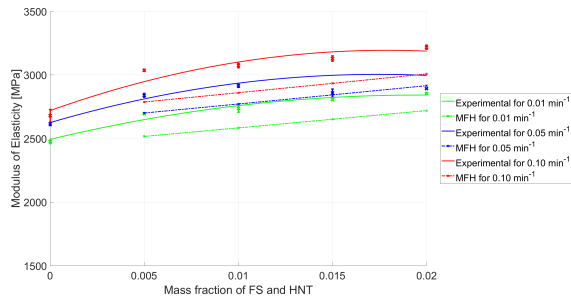


Figure 5.13: Comparison of experimental results for modulus of elasticity with the numerical simulations of HNT-reinforced epoxy.

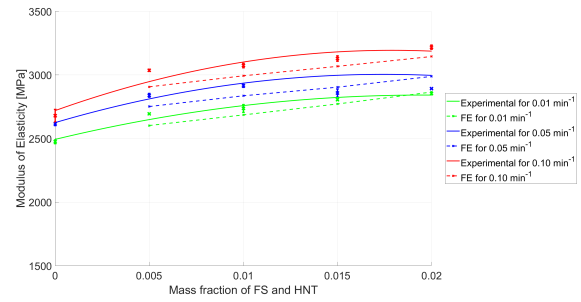
Furthermore, Figure 5.14 displays experimentally determined and numerically calculated Young's moduli results for the FS and HNT reinforcement combination. The results are in good agreement with the maximum error of slightly less than 8%. Similar to the previous results, FE yields less error compared to MFH, which means it is capable of capturing the synergistic effect of multiple reinforcing phases better than MFH. The magnitude of error rises with growing strain rates for the same reason as with FS-reinforced epoxy, the nonlinear interphase region. The nonlinearity of the interphase region becomes more significant with increasing strain rates, and

therefore it influences the overall behaviour of the composite. The FE stress analyses indicate that the presence of both particles in the resin shows an intermediate behaviour compared to the two other individual cases. The presence of FS particles mostly reduces the interaction of stress concentrations at the tips of HNT. Thus, the stress concentrations interact in a more mild manner which reduces the brittle behaviour. The presence of HNT increases the effective stiffness of the composite.

Looking at another material group, Figure 5.16 contains the results of experimental and numerical procedures. The maximum error between the numerical and experimental results is just below 19%. Unlike others, this material group displays a different behaviour in terms of error variation between the numerical and experimental. Overall, the error increases with the rising rubber inclusion content. The main reason is that the rubber reinforcement is more compliant than the matrix. Therefore, the stress peaks in the regions adjacent to the inclusions around the equatorial region of the inclusions in the matrix cause local plastic deformations [104]. Thus, the inclusions do not really contribute to the load-carrying capacity of the composite and weaken it instead. The rubber particles act like voids to this extent. Thus, the local plastic deformations make ground and visibly affect the effective mechanics of the composite. These findings are supported by the FE stress analyses. It must be emphasised that Albipox 1000 contains a resin which is compatible with most varieties of epoxies. Although it is compatible with the epoxy resin used in this study, it does not have to possess the exact same mechanical characteristics. This resin can be one of the reasons for the increasing error with increasing Albipox 1000 content. Figure 5.15 shows a representative view of stress analysis of an RVE containing rubber particles.



(a) Experimental versus MFH.



(b) Experimental versus FE

Figure 5.14: Comparison of experimental results for modulus of elasticity with the numerical simulations of FS and HNT-reinforced epoxy.

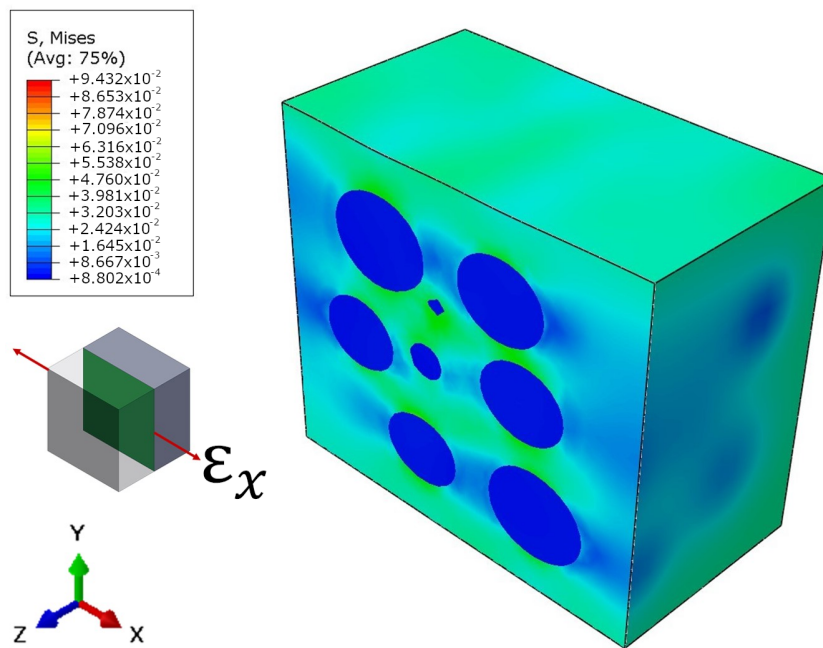
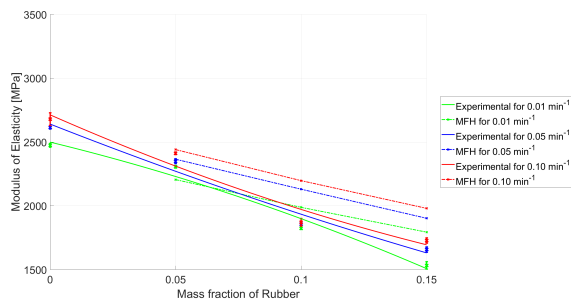
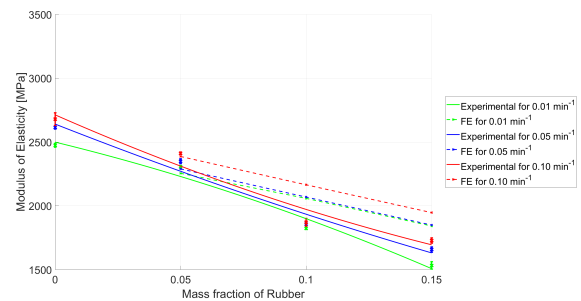


Figure 5.15: A section view of an RVE with spherical rubber inclusions with diameters between 500 nm and 1 μm (Stress unit: GPa).



(a) Experimental versus MFH.



(b) Experimental versus FE

Figure 5.16: Comparison of experimental results for modulus of elasticity with the numerical simulations of rubber-added epoxy.

The effective moduli of elasticity of the composites reinforced with silica (Nanopox F700) are given in Figure 5.18. Overall the results agree and show similar trends with a maximum error of approximately 13%. Similar to the other cases MFH and FE show a better match against the experimental results. The relatively high error can be explained by the resin that Nanopox F700 has. This can also be the reason for the lower modulus of elasticity predicted for silica particles. Compared to FS, silica particles of Nanopox F700 are more compliant since the numerical models only include the stiffness of inclusions as a variable and do not consider any other resin that could be mixed into the composite. Other than these variances, the same comments made on FS-reinforced epoxy can be made for this material group as well. An RVE is given in Figure 5.17 to show a representative stress distribution for silica-reinforced epoxy nanocomposites.

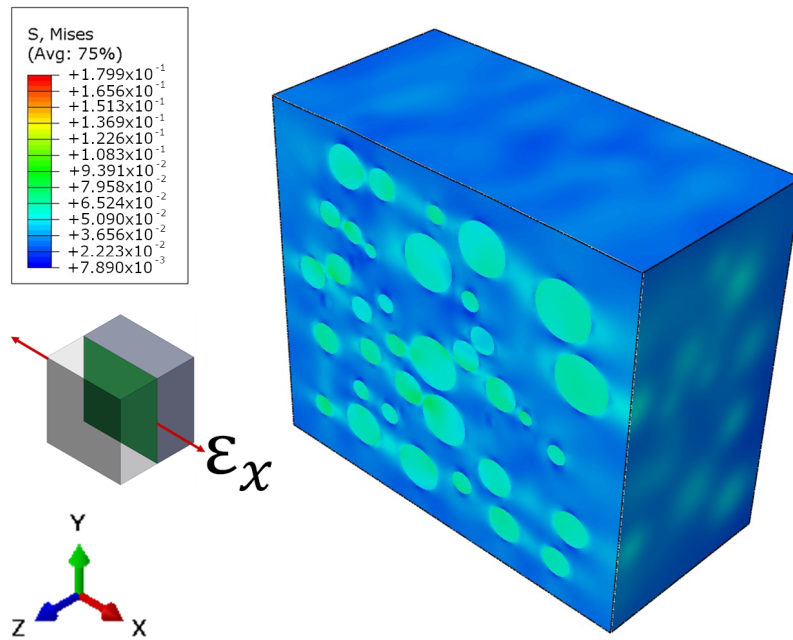
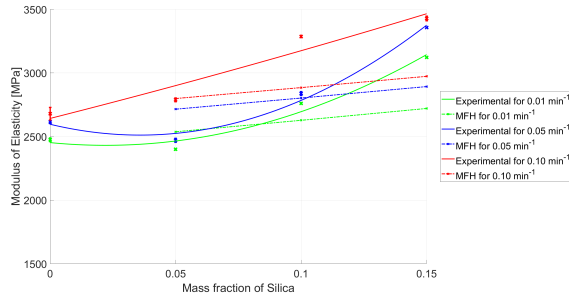
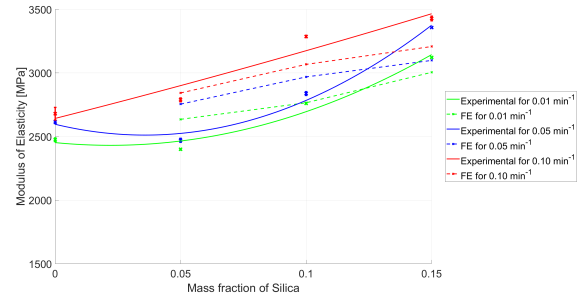


Figure 5.17: A section view of an RVE with spherical silica inclusions with an average diameter of 20 nm taken between 5 – 50 nm (Stress unit: GPa).

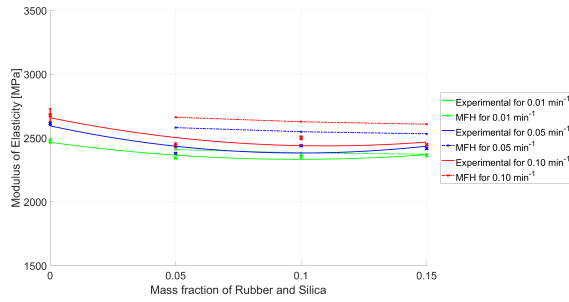


(a) Experimental versus MFH.

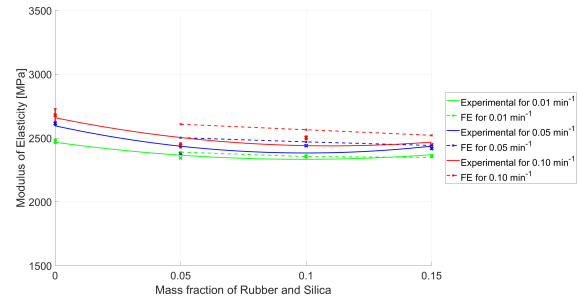


(b) Experimental versus FE

Figure 5.18: Comparison of experimental results for modulus of elasticity with the numerical simulations of silica-reinforced epoxy.



(a) Experimental versus MFH.



(b) Experimental versus FE

Figure 5.19: Comparison of experimental results for modulus of elasticity with the numerical simulations of silica and rubber-reinforced epoxy.

Finally, to comment on the synergistic effect of rubber and silica particles, Figure 5.19 is presented. The maximum error between the numerical and experimental results is approximately 9%. The error increases with increasing strain rates and decreases with increasing RC. The increasing error with increasing strain rate can be explained by the interphase nonlinearity, which is not included in the numerical models. Furthermore, the decreasing error with increasing RC is expected to be caused by the synergistic effect of the existence of two different particles with opposite mechanical characteristics. The stress analyses show that the silica particles have higher stresses and the rubber particles have lower stresses. Similar to the other cases, the stress concentrations occur in the equatorial regions of the rubber inclusions and interact based on the size and distance between the particles. For the silica inclusions, the stress concentrations appear in the polar regions, and the stress distribution is affected the same way by the interactions of stress concentrations.

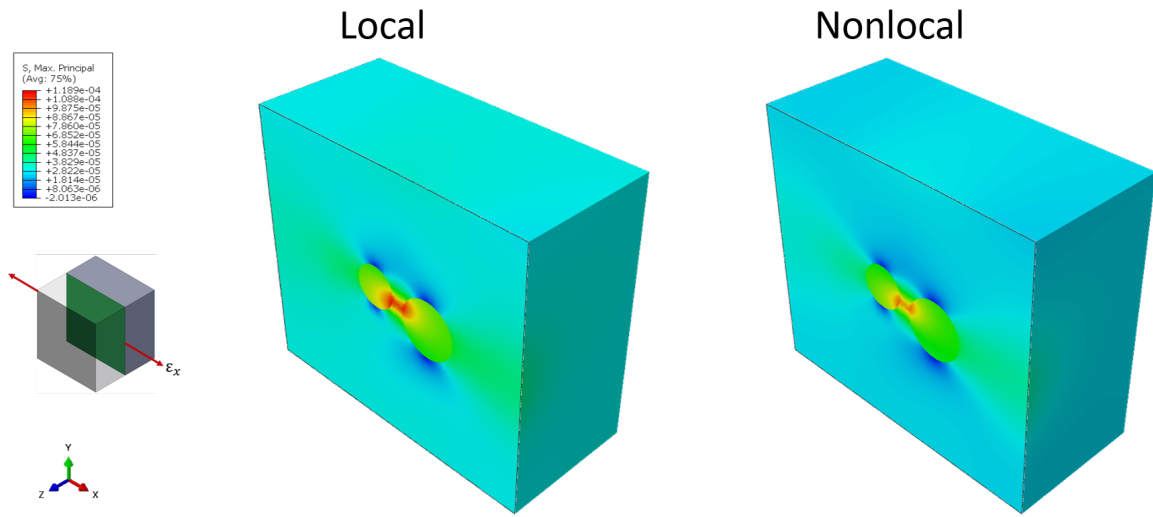
Calculation of Nonlocal Stresses

The maximum principal stress distribution of the composite reinforced with FS is shown in Figure 5.20a. It can be seen from the figure that the stress concentrations occur around the reinforcing FS particles due to caused discontinuity. Relatively uniform stress distributions are observed in the matrix away from the particles. The maximum stress in the matrix appears on the boundaries of the FS particles. The compression stresses are observed around the boundaries of the particle since the moduli of elasticity and Poisson's ratios of the matrix and particle materials are quite different. But the magnitudes of the compressive stresses are much smaller compared to the tension seen around the particles and in the particles. The maximum stress values in the matrix are read near the particles, and for the nonlocal model, the stresses are lower by approximately 5 – 10% overall than those for the local model. Thus, in the model employing the nonlocal theory, the softening effect is to be seen for both the particles and the matrix material. The stress distributions of both models are almost the same, but for the local model, the stress concentrations occur on the boundaries of the particles, and the matrix is greater than in the nonlocal model.

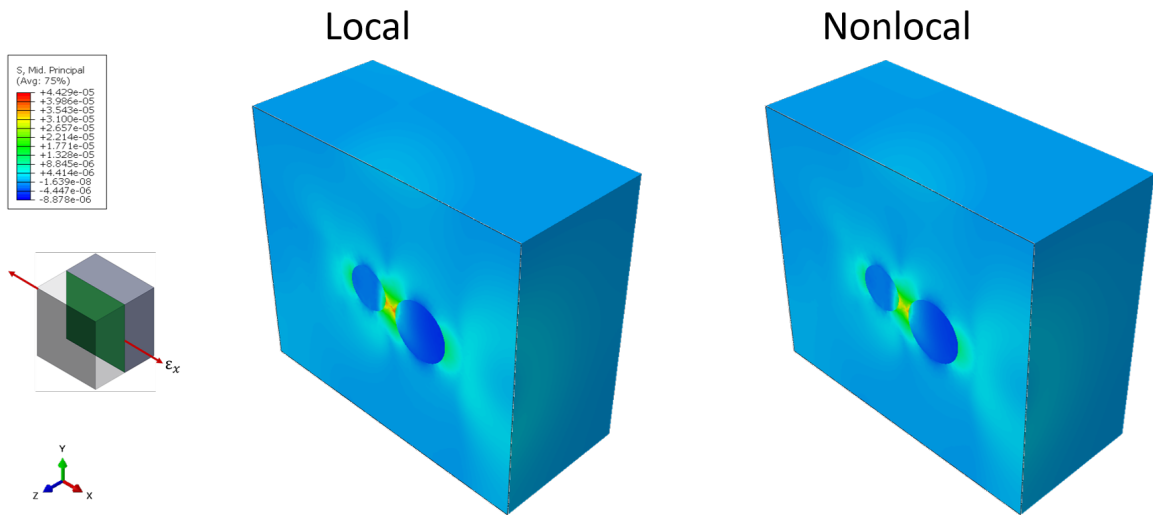
Figure 5.20b shows the middle primary stress distribution of the composite reinforced with FS. The stress concentrations, which originate due to discontinuity in the material, are visible as tension in the surroundings and particularly between the closest FS particles. The stress distributions of both models change to compression and are largely uniform in the matrix far from the particles. The maximum stress in the epoxy matrix is observed between the neighbouring particles. The stress values both in the matrix and the rubber particles are higher in the local model by roughly 5 – 10%, compared to the nonlocal model. Thus, in the results calculated with the nonlocal theory, a softening is observed for both the inclusions and the epoxy. The trends of stress distributions of both models are almost identical with the mentioned shift and the stress concentrations are found to be less sharp.

Figure 5.20c displays the minimum principal stress distribution of the FS-reinforced composite. Tension-form stress concentrations are found in the area nearby the FS particles, similar to the maximum and middle principal stress distributions. Further away from the particles, the

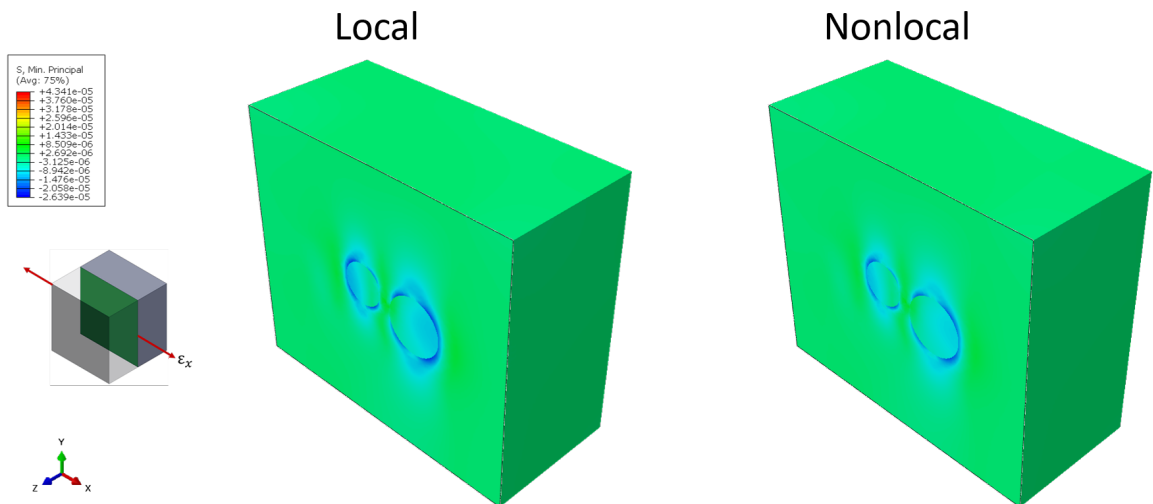
stress turns into compression and becomes relatively uniform. The maximum stresses emerge in form of tension in the epoxy matrix around particles that are close enough where their stress concentrations can interact. The stresses in the particles are compression. Similar to the other principal stress results, the differences between the local and the nonlocal stresses are around 5 – 10%. The softening of the nonlocal continua is also observed here, so the greater stresses belong to the local model. Again, the stress distribution trends are quite close to each other. Only the variations in the nonlocal model are more moderate compared to the local model.



(a) Maximum principal stresses.



(b) Middle principal stresses.



(c) Minimum principal stresses.

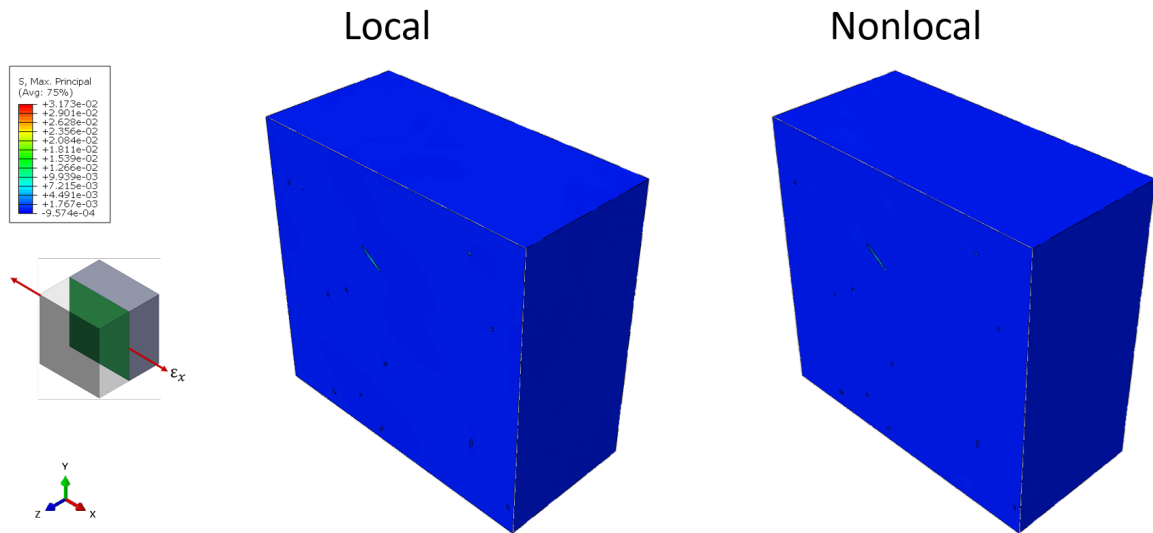
Figure 5.20: Comparison of local and nonlocal continuum theories over principal stresses with the numerical simulations of FS-reinforced epoxy (Stress unit: *GPa*).

Figure 5.21a reveals the maximum principal stress distribution of the composite reinforced with HNT. It is observed that the stress concentrations arise around the reinforcing particles, especially around the tips of the HNT inclusions. The stress distribution is relatively uniform in the matrix far enough from any particle remaining incomparably smaller than the stresses around the inclusions and in the inclusions, while the stress concentration is observed in the matrix around the particles for both RVEs, as expected. The maximum stress can be read in the HNT particles. The maximum principal stress on the boundaries of the particles and the matrix as well as the for the local model, is greater by approximately 10% than that for the nonlocal model. So, in nonlocal theory, the stiffening effect is observed for the particles, while the softening effect is seen for the matrix material. This can be explained by different nonlocal constants of the particles and the matrix. The characteristics of the stress distributions of both models are very similar. However, some significant stress concentrations are observed on the boundaries of the particles and the matrix for the local model, while for the nonlocal model, the stress distribution around the boundaries is more uniform. The maximum principal stresses in the particles are parallel to their axis. The stresses in the particles also increase as the axis is oriented towards the loading direction.

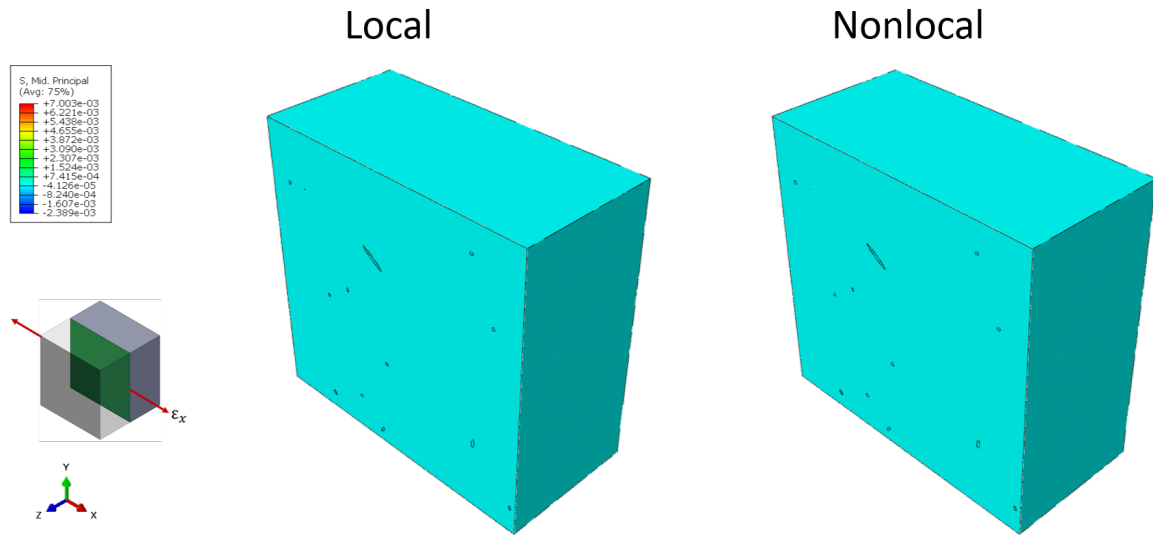
The middle principal stress distribution of the composite with HNT reinforcement is given in Figure 5.21b. While the stress distribution is somewhat homogeneous in the matrix far from any reinforcing particles, as would be expected, stress concentrations are seen around the reinforcing particles, HNT. The HNT particles exhibit the highest values of the middle principal stress. The middle principal stresses for the local model are greater, by about 10%, than those for the nonlocal model, similar to the maximum principal stress on the boundaries of the particles and the matrix for the local model. When compared to the nonlocal model, the stresses in the particles for the local model are around 10% lower. In the nonlocal theory, the softening effect is seen for the matrix material and the stiffening effect is seen for the particles, like the maximum principal stress case. In contrast to the nonlocal model, which has a more uniform stress distribution across the boundaries, the local model exhibits stress concentrations on the interfaces of the particles and the matrix. The middle principal stress is seen to be 5–10 times less than the maximum principal stress.

Figure 5.21c shows the minimum principal stress distribution of the HNT-reinforced epoxy nanocomposite. Similar to the stress distributions of other principal stresses, the stress concentrations are observed around the reinforcing particles, HNT. In the matrix far from the particles, the stress distribution is rather uniform. Nevertheless, the matrix surrounding the particles exhibits stress concentrations around the particle-matrix interfaces. The maximum stress value appears in the HNT particles, similar to the other principal stresses. Overall the stresses in the matrix are greater by about 10 % in the local model than the nonlocal one. Whereas, the HNT particles display greater stresses in the nonlocal model by a similar margin compared to the particles in the local model. The nature of the stress distribution remains similar in both models. On the other hand, the nonlocal model yields stress concentrations of lesser magnitude.

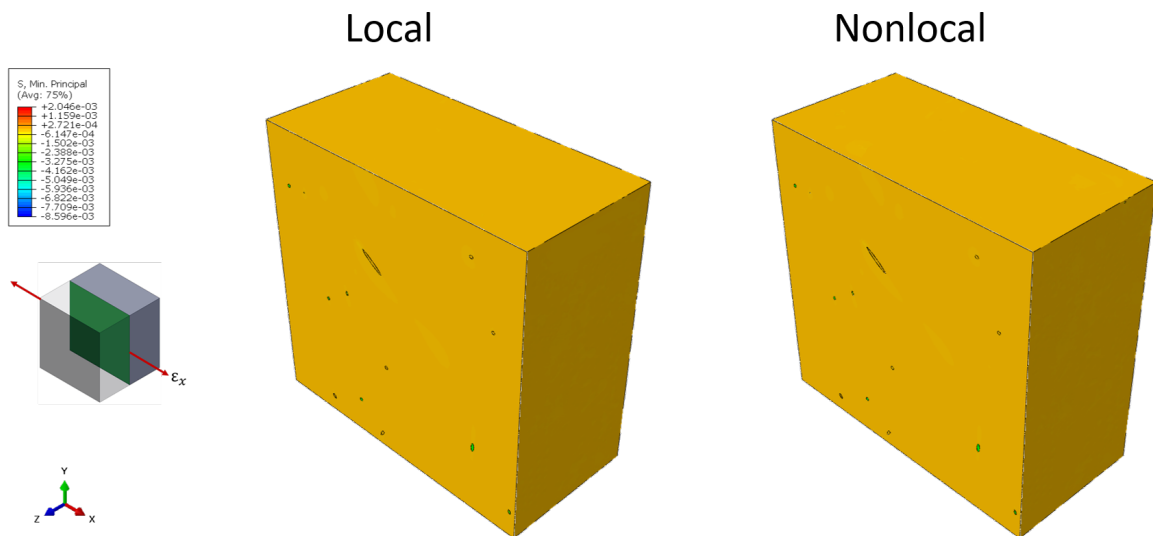
In these images, the black dots and the lines indicate the boundaries of the particle reinforcements. The HNT particles cannot appear like other reinforcing particles used within the scope of this study due to their very high aspect ratio. This hinders their visibility in Figure 5.21 since the resolution of the figure cannot precisely capture the particles.



(a) Maximum principal stresses.



(b) Middle principal stresses.



(c) Minimum principal stresses.

Figure 5.21: Comparison of local and nonlocal continuum theories over principal stresses with the numerical simulations of HNT-reinforced epoxy (Stress unit: GPa).

The maximum principal stress distribution of the epoxy-based composite reinforced with rubber is shown in Figure 5.22a. It can be seen from the figure that the stress concentrations emerge around the reinforcing rubber particles. However, the rubber particles are more compliant than the epoxy matrix. Therefore, here, the maximum stress values appear in the matrix instead of the particles, unlike in other materials. The maximum stress values in the particles' neighbourhood and the overall stress distribution for the nonlocal model are lower, by approximately 10%, than the stresses calculated with the local theory. Thus, it should be pointed out that the softening effect is observed for both the particles and the matrix material in the model using the nonlocal theory. Another thing to emphasise is that with the nonlocal theory, the stress concentrations are reduced compared to the local theory. Apart from these, the stress distribution natures of both models are pretty similar.

The middle principal stress distribution of the rubber-added epoxy is presented in Figure 5.22b. Because of the discontinuities brought on by the inclusions, stress concentrations can be identified in the form of tension close to the rubber particles. Away from a particle, the stress transforms into compression and is relatively consistent throughout the matrix. The epoxy matrix bears the most stress. The maximum stress values for the local model are greater, by around 5–10%, in both the matrix and the rubber particles than in the nonlocal calculations. The softening effect is seen in both the particles and the matrix materials in nonlocal theory. The stress distributions of the two models are just about equivalent, but for the local model, the stress concentrations happen in tension on the boundaries of the particles and the matrix, whereas for the nonlocal model, the stress distribution occurs in compression and is more continuous in the matrix away from the boundaries.

Figure 5.22c displays the minimum principal stress distribution of the rubber-added epoxy. Like the middle principal stress distribution, stress concentrations are observed as tension near the rubber particles. The stress distribution becomes relatively uniform in the matrix far away from the particles. The maximum stress occurs in the epoxy matrix, which is identified as tension. The stresses in the particles are also tension, but they are much less than the stresses observed in the matrix as the particles are softer. The maximum stress values both in the matrix and the rubber particles for the local model are higher, by approximately 5–10%, in comparison to

the nonlocal model. Again, the softening effect of the nonlocal theory is to be pointed out for both the particles and the matrix phases, which leads to the natural conclusion of the nonlocal model showing more consistent stress distribution and weaker stress concentrations.

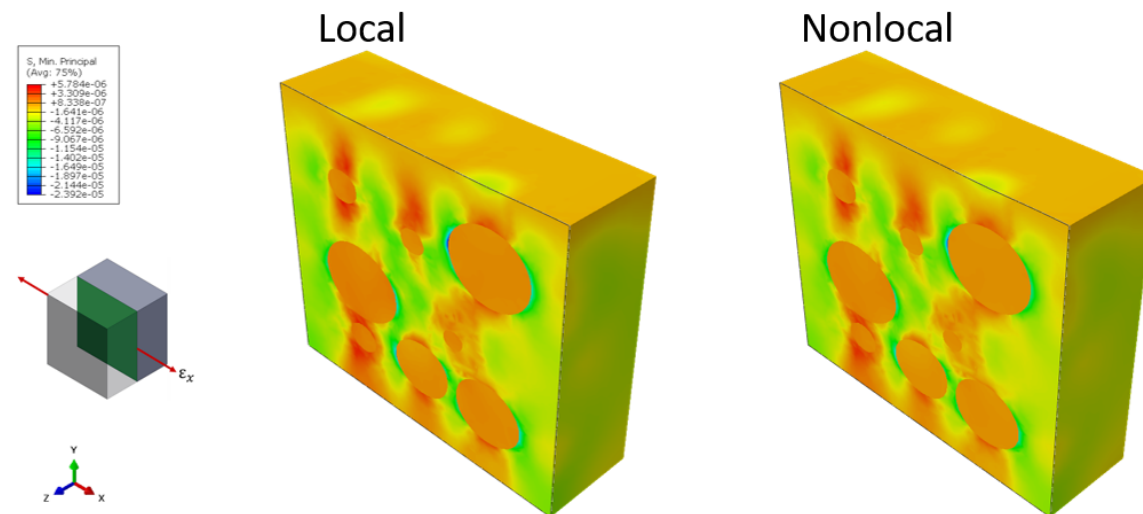
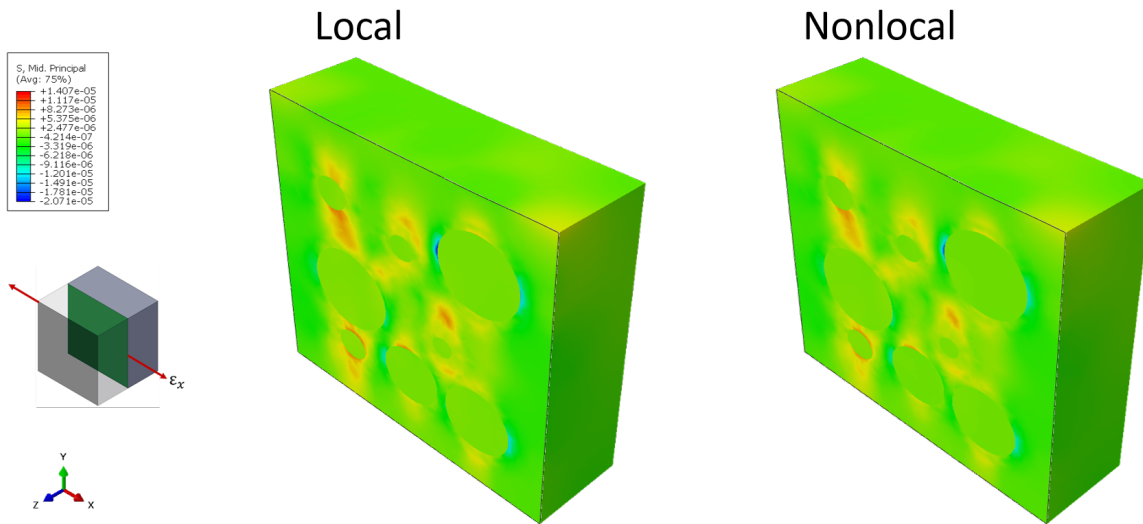
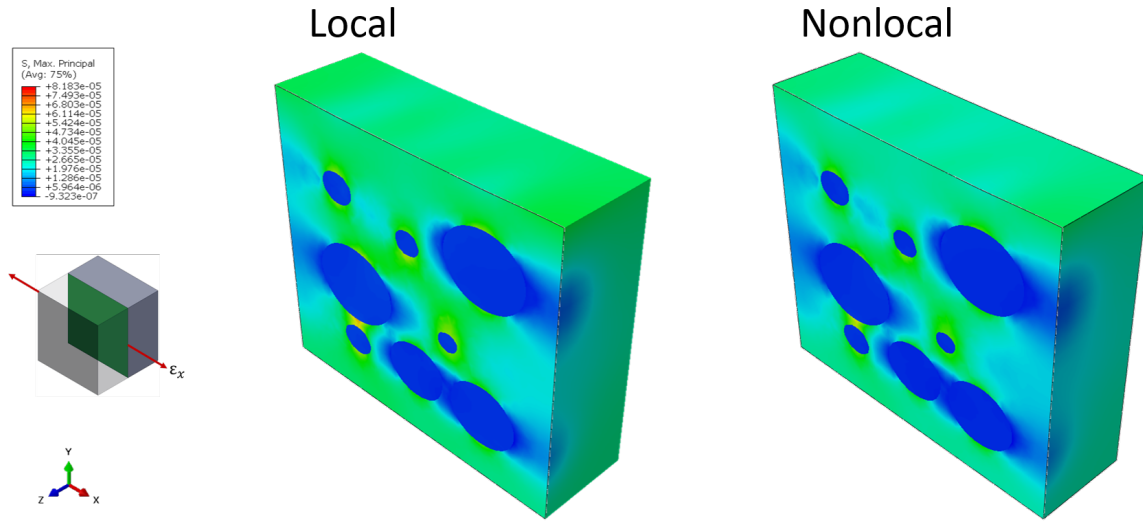


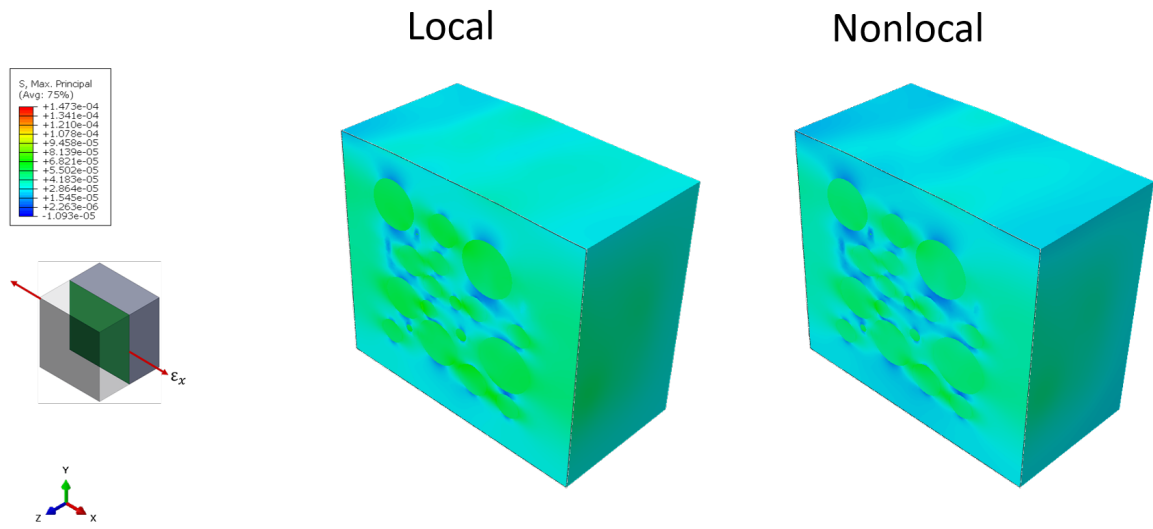
Figure 5.22: Comparison of local and nonlocal continuum theories over principal stresses with the numerical simulations of rubber (Albipox 1000)-reinforced epoxy (Stress unit: GPa).

Figure 5.23a presents the maximum principal stress distribution of the composite reinforced with silica (Nanopox F700). The stress concentrations are observed around the reinforcing silica particles. The matrix material displays a relatively uniform stress distribution if the points of interest are far enough from the particles. Whilst the stress concentrations appear in the matrix around the particles, as is expected and seen in the cases with FS and HNT-reinforced composites. The maximum stresses are observed in the nano-silica particles and around them as tension. Both models display similar characters in terms of the trends in that the stress is distributed. But the maximum principal stresses at the interfaces of the particles and the matrix for the local model are roughly 10% higher than the nonlocal model, while the stress in the particles for the local model is approximately 10% higher than that calculated for the nonlocal model.

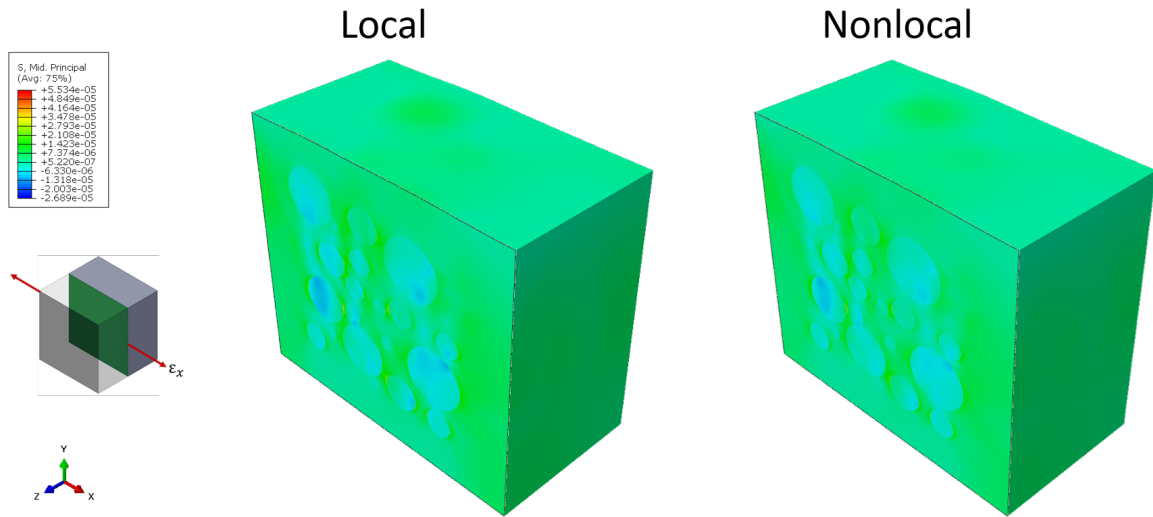
The middle principal stress distribution of the silica (Nanopox F700)-reinforced epoxy is shown in Figure 5.23b. This time, the stress concentrations are observed as compression in the regions near the silica particles. The stress is in tension form and relatively uniform in the matrix away from any particle. The maximum stress occurs in tension in the epoxy matrix. The maximum stress values both in the matrix and the rubber particles for the local model are higher, roughly 5 – 10%, than those for the nonlocal model. Here, another example of the softening of the material under nonlocality is observed, which is valid for both phases, namely the matrix and the reinforcements. Other than the softening, both local and nonlocal models exhibit similar characters in terms of the distribution of stress.

The minimum principal stress distribution of the epoxy-based nanocomposite reinforced with silica (Nanopox F700) is given in Figure 5.23c. The stress concentrations are observed as tension in the neighbourhood of the rubber particles, which occur due to discontinuities caused by the reinforcements, like in other cases with FS and silica reinforcements. The stress distribution changes from tension to compression and becomes more uniform in the matrix further away from the particles. The maximum stress occurs in tension in the epoxy matrix. In the particles, there are compressive stresses. The maximum stress values for the local model are roughly 5–10% higher than those computed for the nonlocal model in both the matrix and the rubber particles. The softening effect of the nonlocal theory is seen in both the particles and the matrix

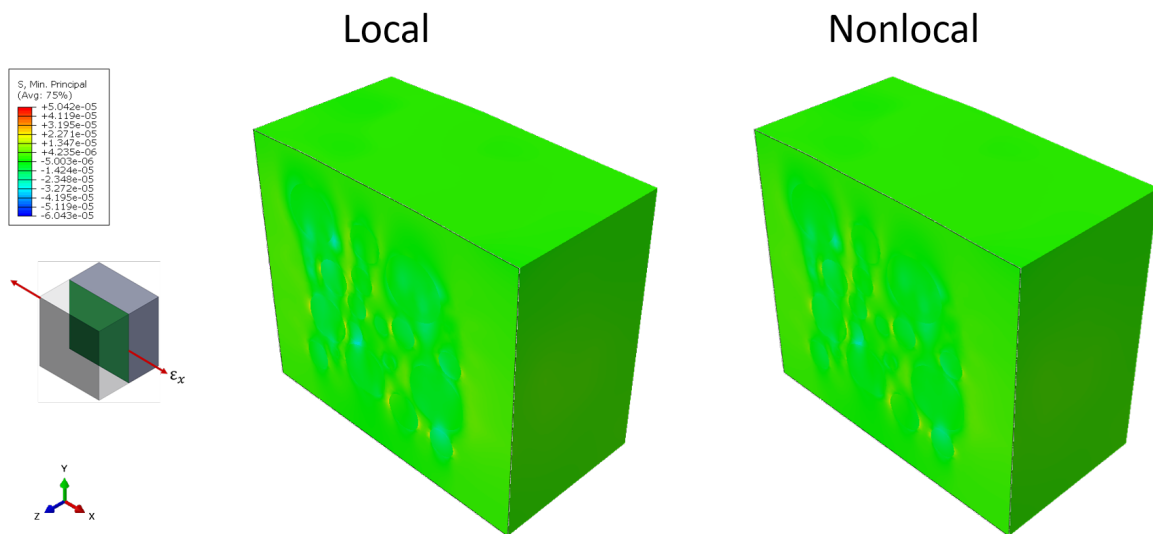
components of the composite. The stress distributions of the two models are nearly matching. However, for the local model, the stress concentrations are in tension at the interfaces of the particles and the matrix. Whereas for the nonlocal model, the stress distribution in the matrix outside of the boundaries of the particles is in compression and is relatively uniform.



(a) Maximum principal stresses.



(b) Middle principal stresses.



(c) Minimum principal stresses.

Figure 5.23: Comparison of local and nonlocal continuum theories over principal stresses with the numerical simulations of silica (Nanopox F700)-reinforced epoxy (Stress unit: GPa).

5.5.2 Aero-Engine Fan Blade Example

Calculation of Local Stresses

The FE model is used to calculate the stress and strain distribution of the blade under quasi-static loading. The blade is found to be safe under the prescribed loads and boundary conditions due to the quasi-static analysis. In other words, the maximum principal stress is below the standard range of yield stress values of such materials. However, to evaluate long-term safety, extensive research on the fatigue characteristics of the blade structure is required.

Figure 5.24 shows the logarithmic maximum principal strain distribution. The maximum logarithmic principal strain is approximately 0.5 %. The strain values increase significantly towards the root around the trailing edge of the blade, especially on the pressure side. For the rest of the blade, the strain field is relatively smooth.

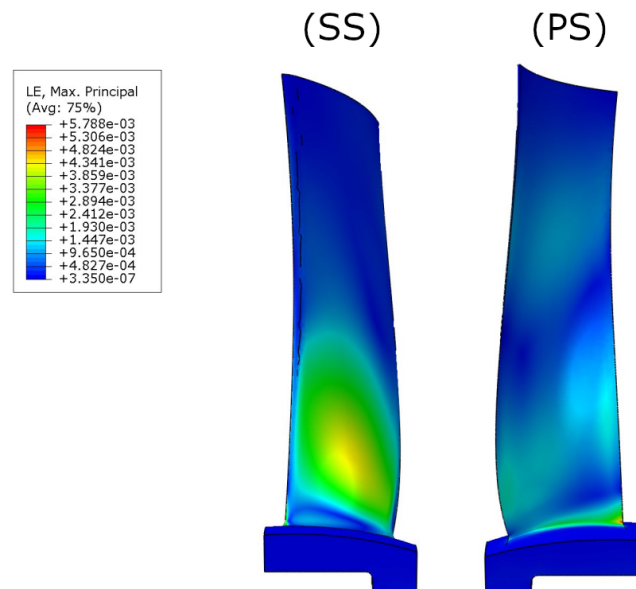


Figure 5.24: Logarithmic maximum principal strain distribution.

Similar comments can be made on the maximum principal stress distribution. The stress distribution is presented in Figure 5.25. The stress concentrates around the same region, on the pressure side, around the root, and at the trailing edge. The maximum value lies around 638 MPa, which is safely below the yield strength of the material Ti-6Al-4V titanium, which is

supposed to be around 1 GPa [45]. The existence of that stress concentration can be explained by loads and geometry. The centrifugal load causes tension as well as pressure causing bending, which leads to more tension on the pressure side. Due to the non-uniform pressure distribution and geometry, there is a resultant twisting moment. So there are shear stress components resulting from torsional deformation. In terms of geometry, the trailing edge is sharper than any other edge on this solid model. Higher curvature, of course, creates higher stresses [45]. This combination generates a greater value of principal stress. In the design process of a real fan blade, this stress concentration must have been observed. Therefore, the thickness around the root, especially at the edges, increases significantly, as shown in Figure 5.26.

Furthermore, there is a region on the suction side where relatively higher stresses are observed. This is interpreted to occur due to the higher curvature of the blade, which causes stiffer bending behaviour. Additionally, the twist introduces shear stress of significant magnitude as well. Therefore, the principal stresses are higher, whereas lower stress values are observed around the root at the suction side. This is caused by the introduction of stresses of opposite characters by the external loads. The centrifugal loads introduce tension around that region, whereas the pressure leads to compression due to bending. These two opposite characteristics of stresses reduce the resultant stress value.

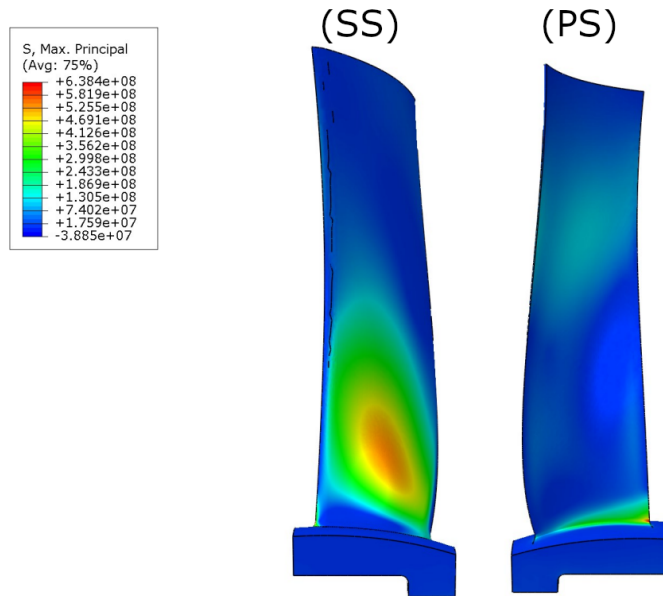


Figure 5.25: Local maximum principal stress distribution (Units in $[Pa]$).



Figure 5.26: Thickness change around the root.

Calculation of Nonlocal Stresses

As explained in some detail in the previous section, the chosen attenuation function is plugged into the normalisation equation, and the necessary constants are calculated. Figure 5.27 is the plotted normalised attenuation function used to calculate the nonlocal stresses. This function fulfils each and every condition, which is described by the nonlocal continuum theory.

To introduce a practical concept, the expression "influence radius" is used in this study. This concept describes the maximum distance between the field points and the points of interest, at which the nonlocal attenuation function has practically nonzero values to make significant changes in the resultant nonlocal stress when not included in the integral. It is to be observed that there is a practical influence radius, which is around two times the extrinsic length l . To note that l is chosen as 5 mm based on the average thickness value of the blade. So the radius where this integration makes a significant difference lies at about 12.5 mm.

The stress data is exported from the FE simulation of the blade and imported into Python for further processing. The integral given in Equation (5.4), is discretised, and the local stresses are put through that triple integration process being multiplied with the attenuation function. The results of the nonlocal stress components are taken, and the maximum principal stresses

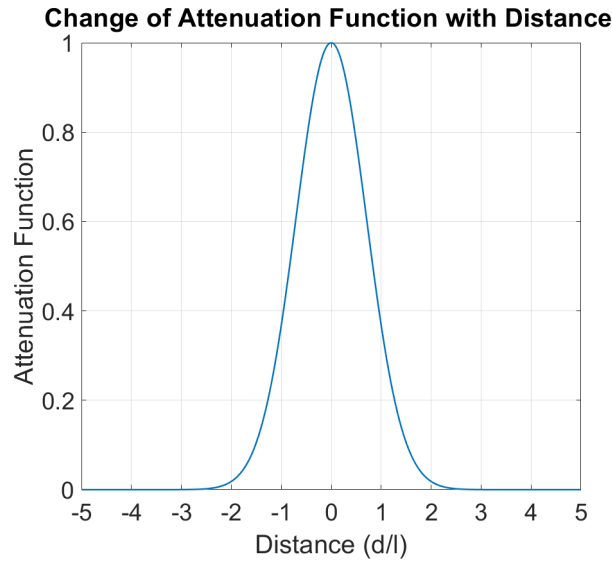


Figure 5.27: Nonlocal attenuation function in the near neighbourhood.

are calculated. Figure 5.28 displays the nonlocal principal stresses.

The distribution is generally similar, but the values of stress generally increase. This is visible in Figure 5.28. The maximum stress increases by more than 20%. The maximum principal nonlocal stress peak is calculated as 789 MPa , whereas the respective local value is 638 MPa .

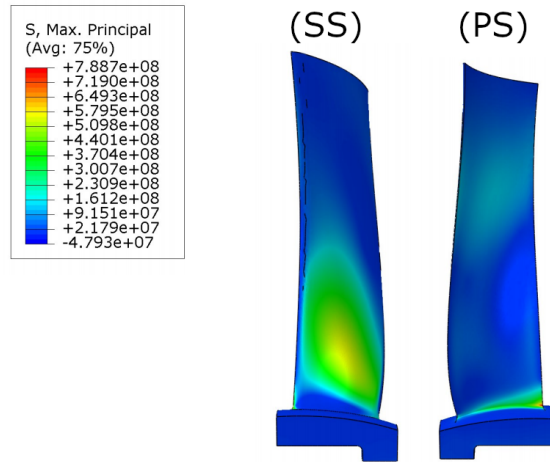


Figure 5.28: Nonlocal maximum principal stress distribution (Units in $[Pa]$).

It is also to be noted that there is no constant ratio between local and nonlocal stresses, as this is a relatively complicated integral operation. The ratio of nonlocal and local stresses grows with the magnitude of the local stresses. This increases the difference between nonlocal and

local stresses around stress concentrations. In other words, the stress concentration factors increase. Figure 5.29 displays the maximum principal stress distributions for both local and nonlocal theories around the stress peak near the root on the pressure side. Figure 5.30 and 5.31 are plotted for better visualisation of this fact of this phenomenon. The local and nonlocal stress values that are taken from the local FE points are compared to the nonlocal stresses taken from the same locations. Figure 5.30 is taken from the stress concentration region, whereas Figure 5.31 is taken from where the stress levels are significantly lower. It is to be seen that the increase is more significant around the stress concentration than at a location where stresses are less. This also affects the design of the blade, as the safety inspections are usually done with the maximum values.

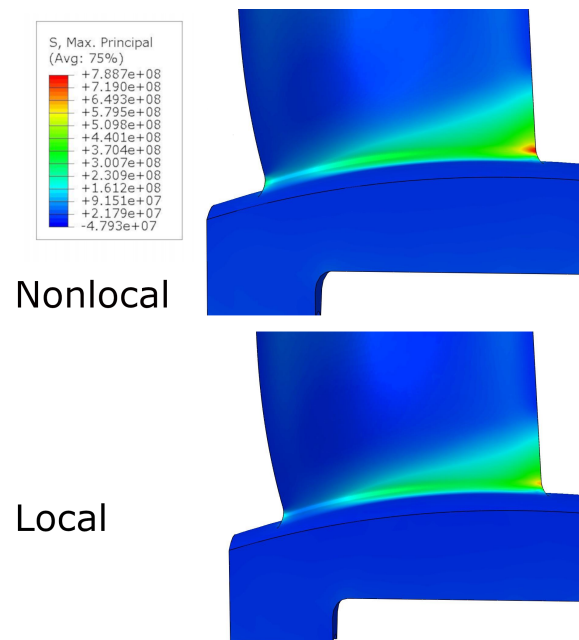


Figure 5.29: Comparison of local and nonlocal maximum principal stress distributions near root stress concentration (Units in [Pa]).

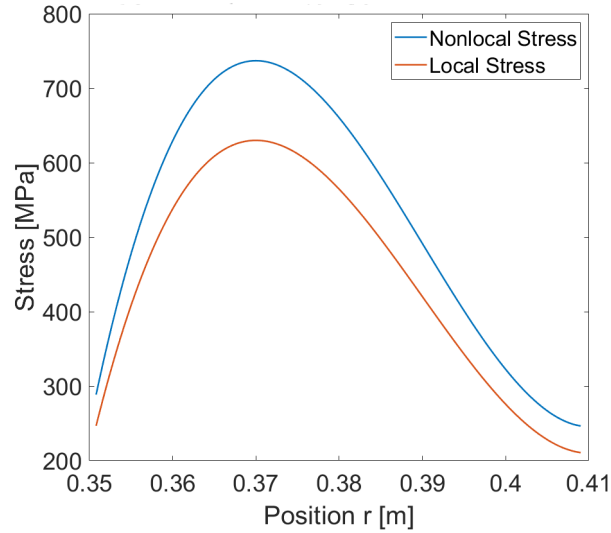


Figure 5.30: Comparison of local and nonlocal maximum principal stress distributions near root stress concentration.

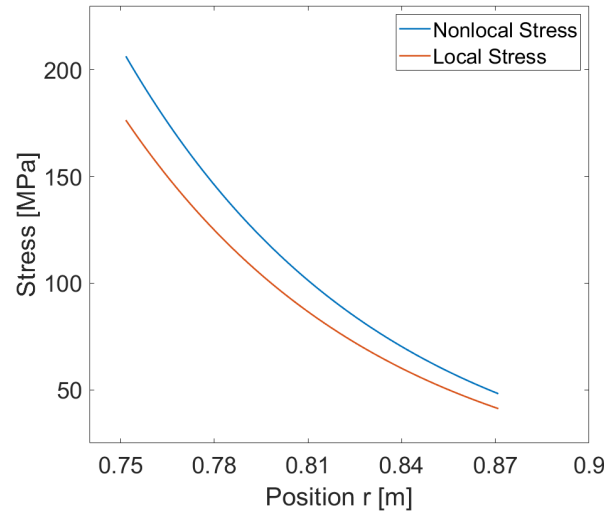


Figure 5.31: Comparison of local and nonlocal maximum principal stress distributions near the tip.

Unlike the results of the stress analyses of the RVEs, which exhibit a softening behaviour with reduced stress after nonlocal calculations are performed, this blade displays greater nonlocal stress compared to its local stresses. The primary reason for that is the different strain fields and geometries of the structure, as well as the considerable difference in their loading conditions. The blade has asymmetric static pressure fields and centrifugal body forces acting on it, whereas

the RVEs are subjected to uniaxial strain. The resultant stress/strain fields of the RVEs are much simpler compared to the one that the fan blade has, even though the PBC is applied and there are inhomogeneities like inclusions in the matrix of the RVEs. Thus, the nonlocal integral yields different effective behaviours for these two distinct cases.

Modal Analysis of the Blade

After recalculating the mass properties to improve the accuracy of the nonlocal dynamic calculations and with the obtained nonlocal stress field, it is possible to run some modal analyses on the blade with nonlocal properties. In both local and nonlocal cases, the stress fields are kept as well as the boundary conditions. With varying rotational speeds and pressure distributions, as the pressure distribution mainly depends on the rotation speed of the fan, modal analyses are run. The first five natural frequencies and mode shapes are included in this manuscript. The first five mode shapes of the blade are presented in Figure 5.32.

The comparison of natural frequencies depending on the rotation speed is presented in Figure 5.33. It is to be seen that the frequencies decrease as the nonlocality takes effect. Therefore, the lower natural frequencies belong to the results of calculations with nonlocal theory fields. The nonlocal stiffness and mass matrices both are greater than the local properties. Even though a frequency shift occurs. This is because of the components of the mass matrix grow more significantly in comparison to the components of the stiffness. However, the difference is approximately between 2 and 5%. This can be explained by the weak nonlocal characteristics of the material considered in this study.

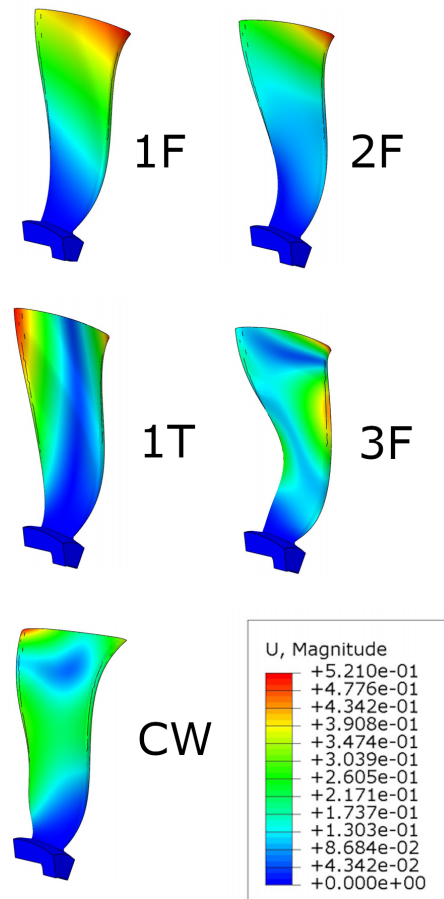


Figure 5.32: Mode shapes of the blade

Here 1T, 2T, and 3T stand for the first, second and third flap modes, respectively. Meanwhile, 1T denotes the first torsional mode and CW is for the chordwise vibration mode.

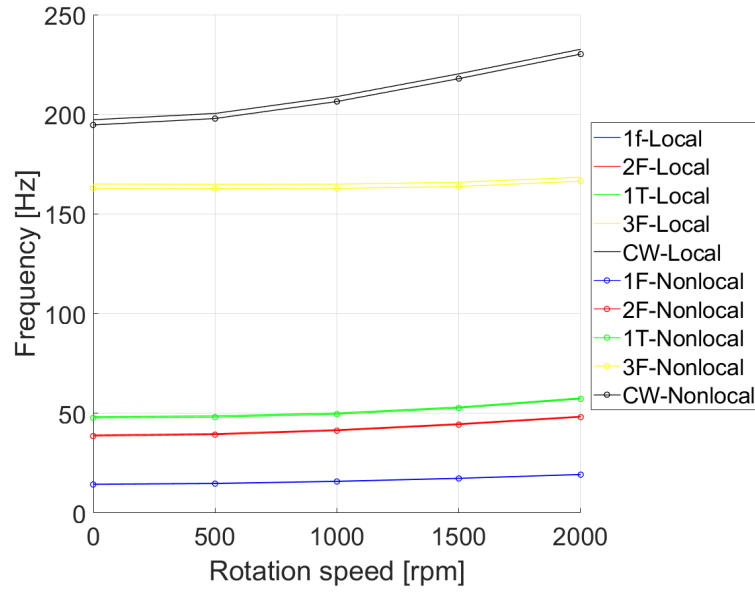


Figure 5.33: Comparison of the natural frequencies calculated with local and nonlocal theories.

5.5.3 Forced Vibration of Nanocomposite Elastic Beams with Material Nonlinearity

As described earlier, the results are plotted after being nondimensionalised to facilitate easy comparison between the materials reducing the number of parameters involved. The vertical axis is normalised to the natural frequency of a typical pure epoxy, and the horizontal axis is normalised to the static deflection due to the force applied.

Figure 5.34 presents the frequency response functions (FRF) for the pure epoxy. The acting forces change from 2 N to 10 N . It can be seen from Figure 5.34, that for smaller forces, it seems linear. Nevertheless, if the force increases, it shows slight softening with increasing amplitude, then hardens slightly with decreasing amplitude which can be explained by the rise of damping for these amplitudes and frequencies.

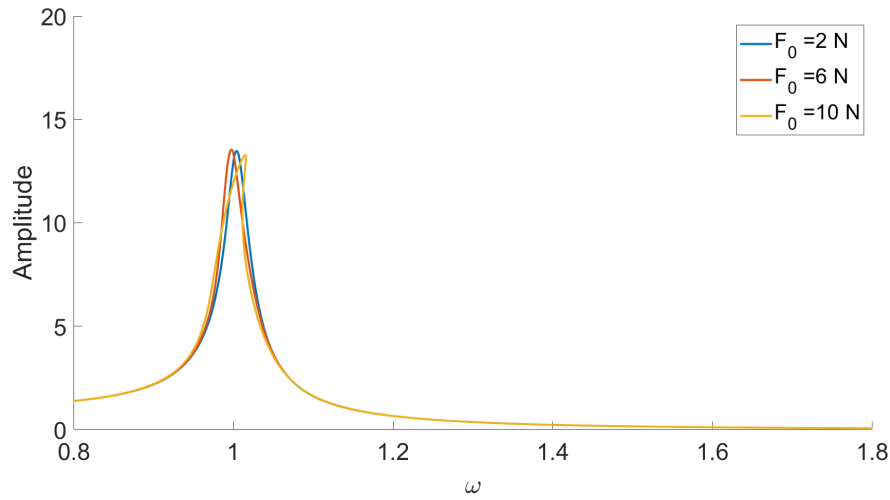


Figure 5.34: Frequency responses of the cantilever beam made of unreinforced epoxy.

The FRF for a composite material which is epoxy reinforced with FS particles, is shown in Figure 5.35. The results suggest that adding FS particles into epoxy increases the natural frequency by about 10%. If the acting force on the composite materials changes from 2 N to 10 N , the vibration amplitude slightly increases, and the composite material shows a softening behaviour for these amplitude and frequency intervals.

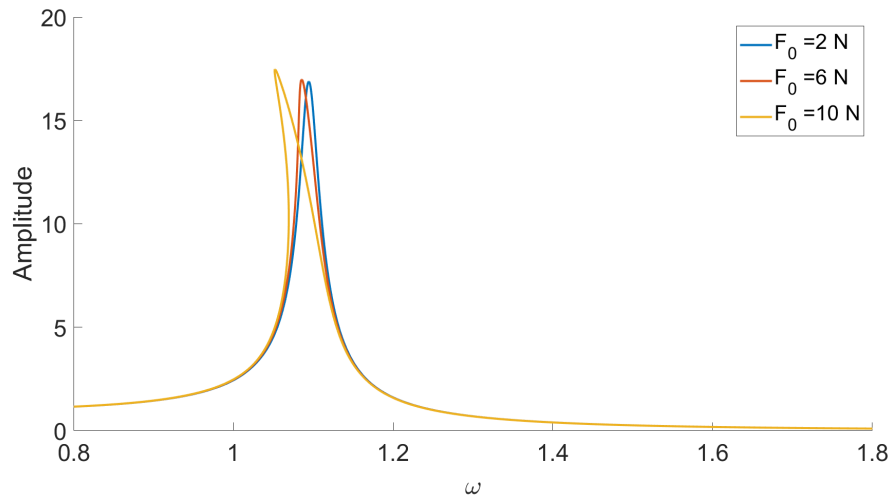


Figure 5.35: Frequency responses of the cantilever beam made of epoxy reinforced with FS.

The FRF for HNT-reinforced composite material is given in Figure 5.36. The data indicate that adding HNT particles into the epoxy considerably increases the natural frequency. Even

for a smaller excitation force, a significant nonlinearity (hardening effect) is observed, and the amplitudes are still as high as those of the pure epoxy, which can be explained by the changes in stiffness and a slight increase in damping with the addition of HNT. This hardening effect becomes much more significant when the excitation force is increased. Then, with the increasing damping coefficient for the higher frequencies, the vibration amplitude also falls considerably.

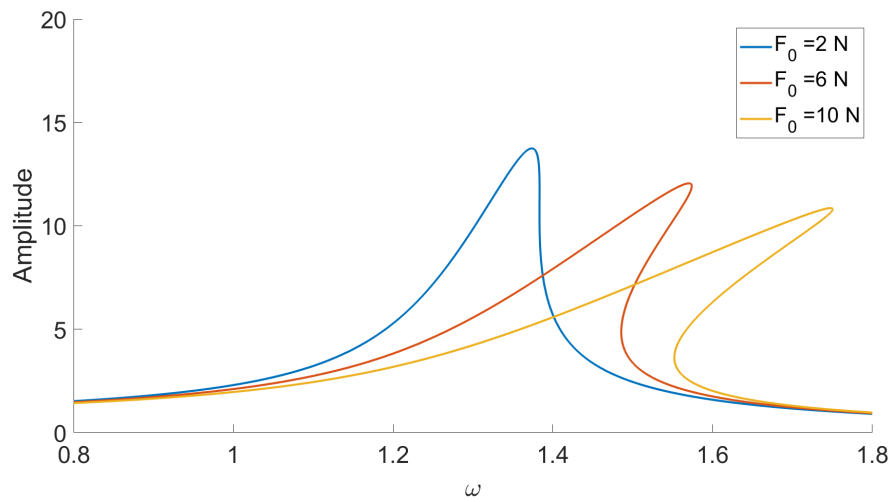


Figure 5.36: Frequency responses of the cantilever beam made of epoxy reinforced with HNT.

Figure 5.37 presents the FRF for the epoxy composite reinforced with both FS and HNT particles. The natural frequency slightly rises in the presence of HNT and FS particles. By increasing the acting force on the composite beam from 2 N to 10 N , the vibration amplitude moderately decreases, and the composite beam shows a significant hardening effect. This can be seen as the synergetic effects of the reinforcement materials.

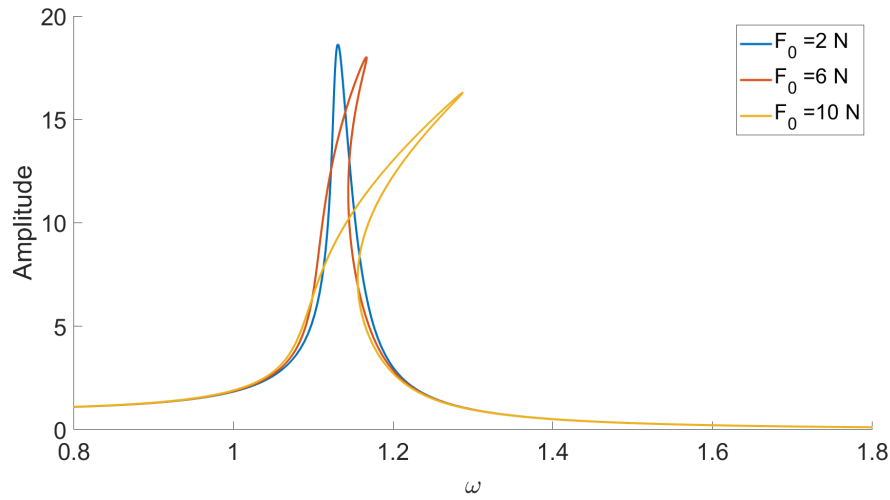


Figure 5.37: Frequency responses of the cantilever beam made of epoxy reinforced with FS and HNT.

Figure 5.38 shows the FRF for a composite material which is epoxy reinforced with rubber (Albipox 1000) particles. As shown in Figure 5.38, adding rubber particles into epoxy decreases the natural frequency, as is expected. The acting forces change from 2 N to 10 N. If the acting force on the composite material increases, the amplitude of the vibration also builds up, and the softening effects become more noticeable.

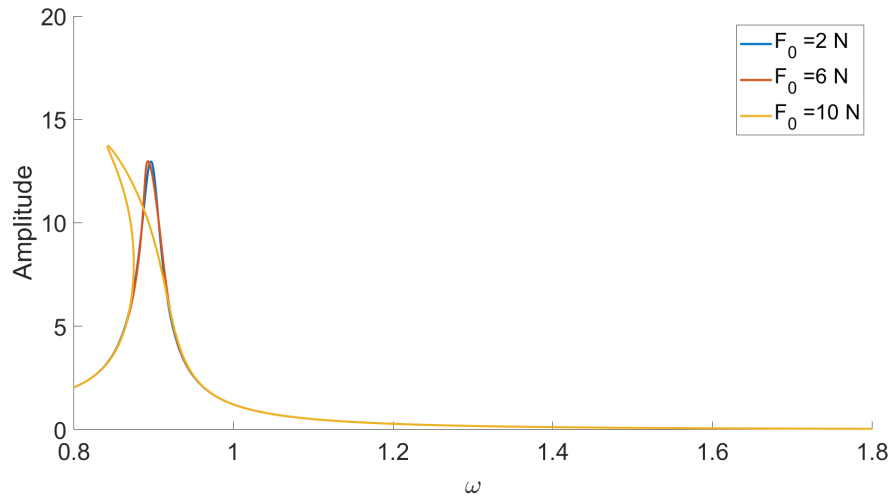


Figure 5.38: Frequency responses of the cantilever beam made of epoxy reinforced with rubber (Albipox 1000).

In Figure 5.39, the FRF for epoxy reinforced with silica (Nanopox F700) particles is given. The

results point out that by adding silica particles into the epoxy, the natural frequency increases significantly (more than 15% compared to unreinforced epoxy), and the vibration amplitude substantially increases. The vibrational behaviour is almost linear for a smaller excitation force. If the force increases from 2 N to 10 N , the amplitude of the vibration slightly decreases, and the composite material shows a significant hardening effect.

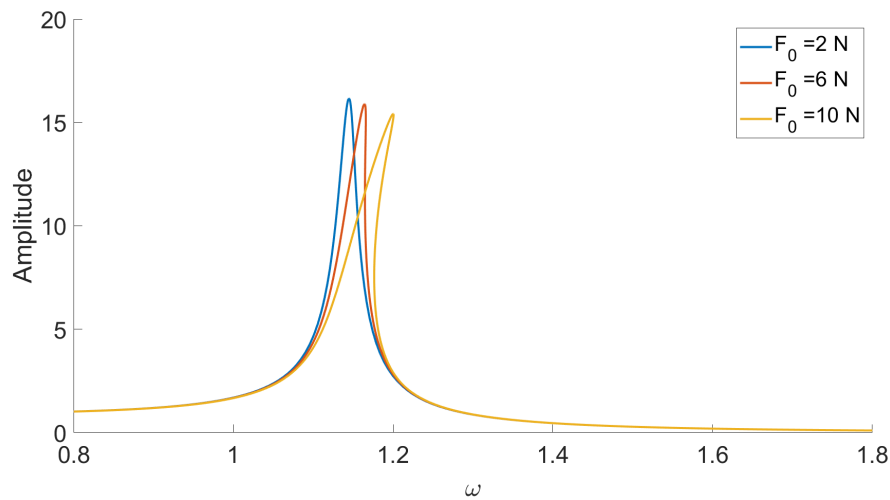


Figure 5.39: Frequency responses of the cantilever beam made of epoxy reinforced with silica (Nanopox F700).

The FRF for the epoxy reinforced with both nano rubber and nano-silica particles is given in Figure 5.40. It can be seen from the figure that adding nano silica particles into the epoxy reinforced with rubber particles increases the natural frequency slightly. The acting forces change from 2 N to 10 N . If the acting force on the composite material increases, the vibration amplitude slightly decreases, and the composite material shows a significant hardening effect. This can be considered as the synergetic effects of the reinforcement materials.

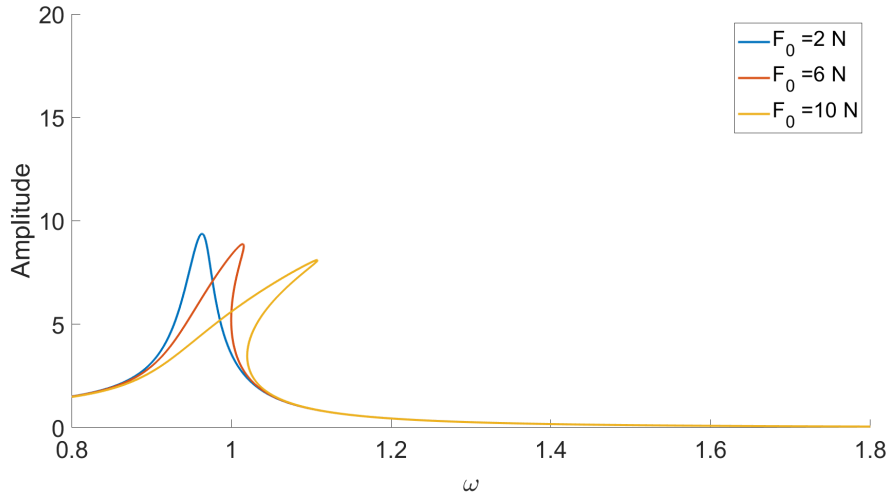


Figure 5.40: Frequency responses of the cantilever beam made of epoxy reinforced with rubber (Albipox 1000) and silica (Nanopox F700).

5.6 Conclusions

In this chapter, material modelling with MFH and FE methods is performed as well as the stress and modal analysis of a representative structure of an aero-engine's fan blade utilising the FE method. To do the FE analyses, local and nonlocal theories are employed, and their results are compared. Furthermore, based on the data gathered from the DMA experiments, nonlinear material models are built and inserted into the EOM of a cantilever beam to investigate the effect of the reinforcing particles on the material nonlinearity and the system's nonlinearity.

Following the experimental studies, numerical simulations are performed employing MFH and FE methods using the classical continuum theory. First, initial MFH simulations are run to determine the effective mechanical properties of the inclusions based on the experimental results. Then, using those determined mechanical properties, the effective mechanical properties of the composites are calculated and compared to the experimental results. However, the FE results tend to deliver a more accurate representation compared to the MFH results. Overall, the addition of FS offers optimal mechanical behaviour with respect to maximum stresses that could be withstood. Also, the addition of both FS and HNT has a synergistic effect in not only optimising the maximum stress before breakage but also the impact energies. Overall, the

results are in good agreement, and the stress analyses can reasonably explain the mechanical behaviour of these nanocomposite materials.

Then the FE model of the fan blade structure is constructed with again the classical continuum theory. The pressure and the centrifugal loads are applied to calculate the stress and the first five natural frequencies under these prestressed conditions.

The nonlocal stresses are obtained for these two cases by post-processing the local stresses using the discretised version of the integral equation of the nonlocal continuum theory. At the same time, the mass properties are recalculated by generating the nonlocal terms and adding those to the classical mass properties.

The RVEs display a slight softening under the same strain field once the nonlocal stress calculations are conducted. The discontinuous behaviour becomes milder, and the magnitudes of the stress concentrations are reduced.

The first five natural frequencies are found to be slightly lower in nonlocal theory in comparison to the local theory. This can also be called a softening. However, the nonlocal theory calculated the stresses to be relatively higher than the local theory. But it is crucial to note that the strongly nonlocal a system gets, those changes get more dramatic. Therefore the new generation materials that obey the constitutive formulations of nonlocal theory are to be handled in engineering designs with more care and caution.

Finally, the cantilever beam case shows the influence of the reinforcing particles on the dynamics of the beam by changing the material characteristics. The samples made of epoxy with silica and rubber reinforcements show a considerable increase in damping without having a shift in the first peak frequency. Whereas HNT-reinforced epoxy displays significant stiffening nonlinearity as well as some rise damping. This case study shows that depending on the system requirements, modifying the material can have a great impact on the overall system behaviour. This can be an important parameter to take care of during the design process of the engineering systems.

Chapter 6

Conclusions

6.1 Summary of Thesis Achievements

The primary aim of this study is to present a detailed theoretical and experimental investigation of the mechanics of the nanocomposite materials that can be used as a matrix for composite fan blades of aero-engines to reveal the effects of different types and amounts of reinforcements. Considering that each chapter has its own standalone conclusions section, instead of going into the details, in this chapter, an overview of the achievements and contributions of each chapter is summarised.

First, in Chapter 3, a new one-dimensional continuous analytical formulation is developed to model the effects of friction on the beam-clamp systems, which also represent the DMA test setup used for damping and stiffness measurements of the materials. The formulated model is analytically solved using the Galerkin approach by separating the motion into two distinct regimes, namely full-stick and full-slip. The more dominant regime is assumed as the *reigning* solution, and in the case of the dominance of the slip regime, one can expect that the friction also contributes to the energy dissipation alongside the intrinsic damping of the materials of which the system consists. Thus, the involvement of friction in the energy dissipation is predicted, which helps design and arrange the parameters for the experimental setups. Taking advantage of the analytical approach, the computational cost is much lower compared to any

other numerical method like harmonic balance combined with finite element analysis. Though, the assumptions like solving the motion using two discrete regimes that have no *grey* zone like the ones where stick-slip motion occurs, the model is shown to capture the frequency response quite well compared to the results of the numerical analyses with less simplifying assumptions.

Next, in Chapter 4 the selected materials are manufactured with an *optimised* and consistent process. The manufacturing is followed by a detailed and structured experimental characterisation. First, SEM and FTIR are conducted to visualise the microstructure of the composite and to ensure that the chemical composition remains consistent so that any changes that are observed as results of the mechanical tests purely depend on the reinforcements themselves rather than any other sorts of chemical alterations. Then, the mechanical tests, consisting of three-point bend with different strain rates, DMA and Charpy impact tests, are carried out with the dynamic properties of the composites in the spotlight. FS, HNT, FS & HNT, silica (Nanopox F700)-reinforced composites stiffen and toughen with the presence of the reinforcements while rubber (Albipox 1000) added epoxy matrix becomes more compliant, but due to increased ductility, toughening is observed. Unlike others, rubber (Albipox 1000) & silica (Nanopox F700)-reinforced epoxy becomes slightly compliant when the reinforcements are added, but the change is not significant. This material group also showed toughening behaviour, which lies in between the composites reinforced with one of these reinforcements individually. All the reinforcement types and combinations considered within the scope of this study contributed to the damping capabilities of the materials. Silica (Nanopox F700)-reinforced epoxy, as well as HNT-reinforced epoxy, stand out by a small margin in terms of their efficiency in increasing the damping capacity of the material.

Finally, in Chapter 5 the numerical modelling work on the mechanics of the nanocomposites conducted for this study is presented, including the multi-scale modelling with local and multi-phase nonlocal continuum approaches as well as the nonlinear vibration analysis of a cantilever beam structure based on the data acquired during the DMA testing of the nanocomposite samples. Initially, to predict the mechanical properties of the reinforcing nanoparticles, Mori-Tanaka MFH procedures are run considering experimental characterisation of the nanoparticle mechanics is not a straightforward procedure and requires very specific pieces of equipment.

The determined mechanical properties are used as input for the FE analyses performed on the RVEs. For the FE analyses, a Python script is prepared to generate the RVEs with the spherical and/or ellipsoidal particulate reinforcements. The script applies the PBC and the uniaxial normal strain of 0.01 on one surface of the RVE. Using the stress averaging approach, the effective elastic properties of the composite materials are calculated. To be able to include the size effects of the nanoreinforcements, the nonlocal continuum theory is employed for a more *accurate* stress analysis. Therefore, another script is written that reads the stresses/strains from the FE models and calculates the nonlocal integral with the predefined attenuation functions. It is important to note that the systems that can be represented with this approach must be *weakly nonlocal*. Apart from the RVEs, a fan blade made of nonlocal characteristic material is also modelled with the same approach and computer programs. These are the first examples of three-dimensional nonlocal stress analyses in the literature. It also needs to be stressed that this numerical method can also handle multiphase materials/composites without a problem. As the last step of the modelling works, a cantilever beam test case is generated and reduced to an SDOF system since only the first bending mode is of interest. The beam material is assumed to have a nonlinear viscoelastic constitution equation, and the nonlinear behaviour is defined with respect to vibration frequency and strain amplitude using the DMA results. This model is used to explain the different effects of different reinforcements on the mechanics of the composite materials/structures.

6.2 Perspective on Possible Future Works

This study mainly handles the mechanics of the nanocomposites that can be used as a matrix material for (possibly) fibre-reinforced hybrid composites and points out the effects of various reinforcements on the characteristics of the materials while exploring the multi-scale physics behind the phenomena. Even though this work contributes to the field of multi-scale modelling and characterisation of nanocomposites, there are still infinite ways to expand the knowledge in the field and gain a better understanding of the mechanics of nanocomposite materials and structures.

For the investigation of friction on beam-clamp systems, a unified solution can be derived instead of separating the solution into two distinct regimes. This unified solution should be able to capture the stick-slip, unlike the presented model. Moreover, one can include multiple modes in the assumed solution, which should increase the accuracy of such a nonlinear system.

In terms of materials, different particle reinforcements and combinations can be employed as well as different matrix materials which can be influenced by the particles significantly divergent compared to epoxy. Another aspect that is important for most of the applications is fatigue behaviour. The fatigue behaviour can be investigated in a future study. It is also important to study fibre-reinforced (chopped or continuous) hybrid composites. The synergistic effects of the fibres and the nanoreinforcements can be explored with a similar multi-scale approach to understand the interaction between the scales.

For experimentation, an approach like [154, 153] presented can be utilised. Thus, it would be possible to replace the DMA test setup, which is not a piece of common equipment, since this method would require more common pieces of equipment like shakers, laser vibrometers etc. Also, by using this method, larger-scale structures can be tested instead of a small beam sample which handles the damping as a material property and cannot take the structural dependence of damping into account. This way, the structural dependence of damping can be eliminated.

For the sake of modelling, a new roadmap can be drawn for a hybrid MFH involving the differential and/or integral constitutive equation of the nonlocal continuum theory. The nonlocal formulation can be integrated into the MFH approach, which should close the gap between the mechanics on small scales and the effective mechanical properties on the macro-scale, taking advantage of being computationally very efficient. Besides, a nonlocal FE formulation can be made instead of post-processing the local results, which limits the scope to the weakly nonlocal materials. Using the nonlocal background, the fatigue behaviour as well as the vibration of such composites can be investigated in more detail, which would yield more accurate models for future applications. Also, in the future, it could be possible to investigate the effect of the variation of the nonlocal modulus on the behaviour of the system. It is also aimed to include the nonlocal damping properties for the mechanical systems, as the damping also needs to be

reviewed as the mass and stiffness properties. Including damping in engineering calculations is crucial for accurate predictions of mechanical characteristics and behaviour of the systems. Furthermore, frequency-domain modelling also needs to be done to be able to analyse the dynamics of nonlocal systems.

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